Research on Global Stability Design of Lower Flange of Steel-concrete Composite Beams under Negative Bending Moment

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Abstract. The instability problem of steel-concrete composite beams will occur in the negative bending area, and different codes get different results for beams bearing negative bending moment on stability. In this paper, the MATLAB is used to analyse the parameter sensitivity of span, height-thickness ratio of web, concrete slab reinforcement ratio based on Eurocode 4 and China Code for Design of Steel and Concrete Composite Bridges. Through a large number of data analysis, the influence degree of each parameter on the stability is determined, and the results of the two codes are compared, which provides a basis for the design of the stability in the negative moment zone of steel-concrete composite beams.

1. Introduction
Steel-concrete composite beams are widely used in industrial buildings, civil buildings and bridge structures because of their reasonable stress, convenience in construction and strong spanning ability. The upper flange of the composite steel beam is constrained by the concrete slab, and the slab can be regarded as reliable support, so there is no stability problem in the upper flange. In the vicinity of the support, the composite beam is subjected to a negative bending moment, and instability may occur because of the compression of lower flange. When the load distribution makes a span subject to more negative bending moments, the instability will be more dangerous than the strength and local stability in some long-span structures.

At present, the results calculated by various theoretical methods are still quite different. How to choose a calculation method which not only satisfies certain precision, but also is relatively simple to calculate is an urgent problem to be solved. In this paper, the European code EC4 and the Code for Design of Steel and Concrete Composite Bridges based on two classical theoretical methods are compared and analyzed, which provides a basis for engineers to choose the section and the calculation method of global stability in negative moment zone.

2. Two classical theories
The stability of the lower flange in the negative moment zone of steel-concrete composite beams without lateral braces is regulated by national codes mainly based on two theories. One is the theory of beams on elastic foundation theory, the other is the inverted U type frame theory.

2.1 Beams on elastic foundation theory
In the method of beams on elastic foundation, the concrete slab and the upper flange are equivalent to the elastic foundation, the restraint stiffness of the upper part of web is equivalent to the spring stiffness, and the compression zone of the lower flange and the web is equivalent to the compression beam on the elastic foundation. The solution of classical beam on elastic foundation method is based

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¹ It is an error in the manuscript to mix superscript numbers with regular text. It should be corrected to Shan Wang, Litao Cheng* and Yuanqing Yu.
on the assumption that the axial force is invariant along the beam and the deformation of concrete slab is not considered\[3\]. The schematic diagram is shown in Figure 1:

![Figure 1. Calculation diagram of beams on elastic foundation theory](image1)

Typical code for calculating the stability of lower flange using beams on elastic foundation method is GB 50917-2013, Code for Design of Steel and Concrete Composite Bridges. The calculation method of elastic critical moment in the code is:

\[ M_{cr} = \frac{N_c W_{os}}{A_{sk} + A_{cr}} / 2 \]  

(1)

\[ N_{cr} = \frac{\pi^2 E_s I_{shy}}{l_0^2} \]  

(2)

2.2 Inverted U type frame theory

In this method, considering the contribution of cracked concrete slabs to lateral stiffness, the restrained stiffness is divided into two parts: web lateral restraint and cracked concrete slab lateral stiffness. The restraints on the lower flange and web are equivalent to torsional restraints,\[4\] and the Eurocode 4 first put the method into code.\[5\],[6] This method is also adopted in the Specifications for Design and Construction of Highway Steel-concrete Composite Bridges (JTG/T D64-01-2015)\[7\]. The schematic diagram is shown in Figure 2:

![Figure 2. Calculation diagram of inverted U type frame method](image2)

The calculation method of elastic critical moment is:

\[ M_{cr} = \frac{k_s C_s}{L} \sqrt{(G I_{as} + k_s \frac{L^2}{\pi^2}) E I_{efc}} \]  

(3)

After calculating the elastic critical moment, both methods check the stability by calculating the lateral torsional buckling reduction coefficient of composite beams. Therefore, this paper evaluates the calculation results of the stability bearing capacity of composite beams in the code of EC4 and the Code for Design of Steel-concrete Composite Bridges by means of the lateral torsional buckling reduction coefficient $\chi_{LT}$.

3. Parameter sensitivity analysis

For the steel-concrete composite I-beam with high and narrow cross-section, there are many factors that affect its mechanical performance under negative bending moment. In the formula, the influence of factors on the results should also be reflected, including the ratio of reinforcement, height to thickness ratio, span, residual stress and so on. This chapter compares and analyses the results of the two codes in various factors. The section representation is as follows: lower flange width (mm) - lower flange thickness (mm) - web height (mm) - web thickness (mm) - upper flange width (mm) - upper flange thickness (mm). The calculation model is a continuous girder mid span model with uniformly distributed load. The bending moment diagram is shown in figure 3:
3.1 Span
For the free compression rod, the span has a great influence on the stability bearing capacity. The larger the span is, the easier it is to lose its stability. The steel-concrete composite beam is a structure with continuously constrain, and its instability half-wavelength is smaller than that of the free compression bar, so the impact of span changes on stability requires further analysis. When the length of negative moment zone is too small, the instability of negative moment zone of composite beams is mainly local instability, which is not the content of this section. Therefore, this section takes the span of 6000mm~40000mm to compare the Code for Design of Steel-concrete Composite Bridges with the European standard EC4.

The lateral torsional buckling reduction coefficient of composite beams with different cross-sections is calculated and analysed when the span changes. The results are as figure 4~figure 9:

![Figure 4. Section of 650-56-1605-16-200-20](image1)
![Figure 5. Section of 700-56-1760-20-600-34](image2)
![Figure 6. Section of 750-56-2360-24-600-34](image3)
![Figure 7. Section of 800-58-2358-24-650-34](image4)

Figure 3. Moment diagram of composite beams
According to the above calculation, the difference between the two design methods is relatively large when the span is small. With the increase of the span, the trend of the reduction coefficient $\chi_{LT}$ gradually slows down, which shows that when the span increases to a certain extent, the stability of steel-concrete composite beams will not be significantly affected by the increase or decrease of the span. When the span is large, the difference between the beams on elastic foundation method and the inverted U-shaped frame method is small.

3.2 Height to thickness ratio of web

The web height-thickness ratio has an important influence on the stability of steel beams. For composite beams, the web provides out-of-plane constraints. When the web can provide sufficient lateral stiffness at a relatively small height-thickness ratio, the composite beams will not destabilize until they reach full-section plasticity. In this section, the upper and lower flange sections are selected, and the ratio of height to thickness is changed by changing the web thickness. The upper and lower flange are selected as Table 1:

| lower flange width | lower flange thickness | upper flange width | upper flange thickness |
|--------------------|------------------------|--------------------|------------------------|
| 650                | 56                     | 600                | 20                     |
| 850                | 56                     | 650                | 34                     |

The results are shown in Figure 10–Figure 11:

(a) Code for Design of Steel-concrete Composite Bridges
(b) EC4

Figure 10. Section of 650-56-1605-X-600-20
It can be seen from the above calculation that the instability reduction coefficient has almost a linear relationship with the high-thickness ratio. When the span is larger, the slope of the reduction coefficient and the high-thickness ratio is larger too, that is, the greater the span is, the greater the stability is affected by the high-thickness ratio. When the height to thickness ratio is reduced by about 40%, the reduction factor is increased by about 12%. This indicates that it is very effective to improve the stability of the lower flange of composite beams by reducing the height to thickness ratio. When the ratio of height to thickness is small enough, the lateral instability of steel-concrete composite beams occur after yielding, and there is no need for lateral restraint member to ensure the stability of the lower flange in the negative moment zone.

3.3 Reinforcement ratio of concrete slab

In the section of composite beams subjected to negative bending moment, the concrete cracks before the section reach the ultimate bearing capacity, and its contribution to the stability bearing capacity of the lower flange can be neglected, only the reinforcement bars in the plate are considered. The EC4 code considers the effect of longitudinal reinforcement on the flexural rigidity and neutral axis position of the section and the effect of transverse reinforcement on the out-of-plane bending stiffness. But the Code for Design of Steel and Concrete Composite Bridges regards the concrete slab as complete rigidity without considering the influence of reinforcement ratio on stability.

3.3.1 Longitudinal reinforcement. The method provided by EC4 is used to calculate the variation of the reduction coefficient of the stability bearing capacity of the four sections under different longitudinal reinforcement ratios. As shown in figure 12.
On the one hand, the increase of longitudinal reinforcement ratio will increase the stiffness of steel beam, on the other hand, it will lead to the neutral axis shift and increase the compression zone. From the above calculation and analysis, it can be seen that the adverse effect of the neutral axis shift is greater than the beneficial effect of the section stiffness increase. The section reduction coefficient will be slightly reduced with the increase of reinforcement ratio, but the reduction range is very small. The reduction coefficient will be reduced about 1% when the reinforcement ratio is doubled.

3.3.2 Transverse reinforcement. In the EC4 calculation method, the out-of-plane bending stiffness of cracked concrete slabs is provided by transverse bars. The effect of transverse reinforcement ratio on stability is shown in Figure 13:

![Figure 13. Influence of transverse reinforcement ratio on stability](image)

From the calculation, it can be seen that the bending stiffness provided by reinforcing bars is more than $10^4$ times larger than that provided by web in the range of 0.4% ~ 0.69% reinforcement ratio, and the lateral stiffness of composite beams is mainly controlled by web stiffness, which has little relationship with transverse reinforcement. Therefore, it is reasonable for beams on elastic foundation method to make the concrete slab equivalent to rigid.

4. Conclusion
In this paper, the factors affecting the stability of the lower flange of the steel-concrete composite beam in the negative moment zone are analysed by a large number of examples. It is found that with the increase of the span, the change of the stability reduction coefficient is gradually gentle. The results calculated by the two codes are quite different when the span is small, and the difference is small when the span is large. The ratio of height to thickness is the main factor affecting the stability of composite beams in negative bending moment zone. The buckling reduction coefficient is almost linear with the ratio of height to thickness. Reducing the ratio of height to thickness can effectively improve the stability of composite beams. The reinforcement ratio of concrete slab is the secondary factor affecting the stability of composite beams, and it cannot significantly affect the stability of composite beams by increasing it.

References
[1] Zhu PR. (1989) Design Principle of Steel-concrete Composite Beams. China Architecture & Building Press, Beijing.
[2] Ministry of Housing and Urban-Rural Construction of the People's Republic of China. (2013) Code for Design of Steel and Concrete Composite Bridges GB50917-2013, China Planning Press, Beijing.
[3] Chen SM. (1997) Stable solution of elastic foundation method for lateral buckling of continuous composite beam. Industrial Construction, 27(2):29-32.
[4] Johnson R P, Fan C. (2015) Distortional lateral buckling of continuous composite beams. International Journal of Solids & Structures, 39(11):2939-2963
[5] Anderson D. (2014) Eurocode 4 - Design of composite steel and concrete structures. Springer Berlin Heidelberg, heidelberg.

[6] Hendy C R, Johnson R, Gulvanessian H. (2006) Eurocode 4: Design of Composite Steel and Concrete Structures. Part 2 General Rules for Bridges

[7] China highway planning and Design Institute Co., Ltd. (2015) Specifications for Design and Construction of Highway Steel-concrete Composite Bridge JTG/T D64-01—2015, China Communications Press, Beijing.