A Hardy Cross Approach for Hydraulic Modelling of Water Pipe Networks.

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ABSTRACT

Whenever there are substantial variations in the quantity of demands within a metropolitan water network, it is necessary to assess the pipe network to aid the water utilities in decision making. Variability in demand exists every time new industries or residences are connected to the network. In cases where no analyses are done prior to making new connections, unnecessarily huge funds are incurred and use of unreasonably bigger pipes is inevitable, some of which may stay redundant. The present study aims at developing a user-friendly numerical hydraulics model for analysing compound pipe networks. The model was developed using the V-Model approach, written in visual basic language to resolve the elementary pipe system equations using the improved Hardy Cross method. This program examines steady-state flows, head losses, flow velocities, and pressures for single, two, three, and four loop water distribution networks. The four-loop example represents the entire network of the case study area in consideration. The comparative study conducted on results from the program and EPANET indicated consistency in the results as coefficient of determinant, R², for all the computed variables was approximately unity (1). The Root Mean Square Error (RMSE) and Mean Bias Error (MBE) were found to be reasonably so small. Therefore, it can be concluded from the statistical analysis that the model is reliable for the analysis of a water network consisting of 1, 2, 3, and 4 closed loops.

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INTRODUCTION

Water is an essential resource for the existence of all life forms on earth (Sonaje & Joshi, 2015). It plays voluminous central roles such as navigation, irrigation, power production, recreation, machine cooling and raw material cleaning in factories and receiving wastewater (Ahmed, 1997). Today, this resource is delivered to communities either through looped, branched or combined pipe networks which are one of the principal infrastructure assets of the general public (Poulakis, Valougeorgis, & Papadimitriou, 2003). These networks are interconnections of various components such as transmission pipes, distribution pipes, service connection pipes, pumps, joints, valves, and fire hydrants supplying water to consumers in recommended quantities with adequate pressure (Esiefarierhnre & Effiong, 2014). A study conducted by WHO/UNICEF (joint Water Supply and Sanitation Monitoring Programme) in 2015 exposed that the percentage of the world’s population with access to developed drinking water sources propagated from 76% to 91% from 1990 to 2015, and the population share with access to piped water on their buildings grew from 44% to 58%.

Despite the fact that piped water supply is regularly regarded as the criterion of improved water supply (Erickson, 2016). Its reliability is at the mercy of the pressure needed to provide and supply a sufficient quantity of water to the end-users. The ever-increasing population has led to perpetual growth in water demand and low pressures in pipe networks. As more people get connected to the network, the water authorities are tasked to transit from branching arrangement with dead ends to grid configuration with loops to increase the pressure heads in quotas of a municipality (including institutional, industrial, business and commercial areas). This invokes rigorous and precise analysis of the required flow rates and sustainable pressures to deliver an adequate quantity of water at a lower cost. Simple branched network problems can be resolved by hand calculation. Conversely, compound networks with complex loops need additional effort even for steady-state flow situations (Lansey & Mays, 1999). Today, reliable commercial hydraulic network software suites available are unaffordable, particularly in underdeveloped countries. Besides, their usage has been a real test as it calls for cutting-edge computer knowledge and skills or acquaintance with a particular software package (Tigkas, Vangelis, & Tsakiris, 2015). For the past few years, Engineers engrossed in software development have focused more on the numerical code (the computation engine), ignoring the ease of use of the final product (Khezzar, Harous, & Benayoune, 2000). Consequently, most firms involved in the design, construction and operation of water distribution networks (WDN) resort to manual calculation. The manual calculation is susceptible to mistakes and is time-wasting. Using a computer model to calculate and analyse hydraulic networks will help to save much time. In the calculation process, computers are less vulnerable to errors (Kurniawan, 2009). Therefore, this study aims to develop a user-friendly numerical hydraulics model to efficiently analyse and evaluate the cost of pipes in a complex network.
COMPONENTS OF HYDRAULIC NETWORKS

A pipe network in Figure 1 is perceived as a network consisting of many elements. Table 1 summarises the various components that make up a water network system.

Table 1: Hydraulic Network Components

| Components | Description |
|------------|-------------|
| Pipe       | A pipe is the principal network element, viewed as a circular closed conduit for supplying water under pressure to the end-users. |
| Pump       | A hydraulic device used to increase water pressure within the system. |
| Storage tank | Used to maintain continuous water supply by storing water during low demand periods and releasing it at peak demands. |
| Node       | A connection point where pipes join together within the network. |
| Valve      | Pressure and flow regulator within the network. |

Figure 1: Water Network Components

Source: (Rossman, 2000)

Existing Hydraulic Models for Water Supply Networks

There are some existing computer models that can be applied to solve a set of equations defining flow in pipe networks and to mimic a number of flow control devices. Some useful educational packages also do exist for analysing smaller network problems. Table 2 presents some existing hydraulic models used for solving flow network problems.
Table 2: Existing Hydraulic Models Applied to Solve Flow Network Problems

| Software      | Capability                                                                                                                                  | Developer          | Reference                                                                                     |
|---------------|----------------------------------------------------------------------------------------------------------------------------------------------|--------------------|-----------------------------------------------------------------------------------------------|
| KYPIPE2       | This mimics pressure flow in pipe networks including water distribution, irrigation. It as well handles a tree-like pipe network with dead-ends. Output from the program includes; flow rates, velocities, head losses, junction pressures, energy grade line elevations and water surface elevations in the storage tanks. Additionally, the pump heads and the valve losses can be presented. It uses the LT Algorithm. | D. J. Wood         | (Brater, King, Lindell, & Wei, 1996)                                                         |
| EPANET        | A computer model written in C programming language for performing an extended-period simulation of hydraulic and water quality behaviour within pressurised pipe networks. EPANET uses the gradient algorithm proposed by Todini and Pilati (1988) for hydraulic analysis. A network in EPANET is represented by; links (pipes), junction nodes, pumps, valves, and storage tanks or reservoirs (figure 2-1). Indeed, EPANET is able to perform water quality analysis as well as water age tracing. | L. A. Rossman,     | (Brater, King, Lindell, & Wei, 1996; Rossman, 2000; Sonaje & Joshi, 2015)                    |
| Water CAD V8i | A hydraulic model which implements Gradient Algorithm with an array of functionalities and advancements in GUI. It is capable of hydraulic and water quality analysis, steady-state and extended period simulations, strong data management along with AutoCAD and GIS integrations. | Bentley’s Haestad Methods | (Sonaje & Joshi, 2015)                                                                         |
| FLOWMASTER    | A general-purpose application for simulating fluid flow in complex pipe networks. The model mimics the real situation by offering a mathematical representation of individual network components and joining them at nodes according to user need. The model can also analyse heat transfer within the pipeline. | Amtech (UK) Limited. | (Brater, King, Lindell, & Wei, 1996)                                                         |

In most existing hydraulic network simulation models such as EPANET2, after manually estimating the base demand for a given the population at every network node, the demand can then be fed into the computer model to compute actual discharges in every pipe in the network. There is time wastage in the course of manual evaluation of base demand before the data could be keyed into the computer program. However, in this research thesis, the currently developed user-friendly numerical hydraulic model has the capability to compute for the user the base demand at every network node given the available projected population figures at each junction node. This will...
save much time the designer would waste trying to
determine the base demand with a hand calculator.

**Pipe Network Analysis**

A hydraulic network analysis model is a very
important decision-making tool for assessing the
sufficiency of a pipe network. The solution to the
steady-state flow network problem is directed by
two basic hydraulic principles: (1) the conservation
of mass at nodes; and (2) the conservation of energy
around the loops (Lee, 1983). The conservation of
mass at nodes uses linear algebraic equations, while
the energy conservation around the closed loops is
based on non-linear equations written in terms of
flow rate. The non-linear equations require special
solution techniques with rigorous iterative steps.
Studying complex pipe networks involves
undertaking the method of approach stated below.

**Step 1: Defining pipe properties (length, diameter,
roughness coefficient) and node elevation.**

\[ \sum_{n=1}^{p_{ij}} Q_{nj} = q_{nj} \quad (j = 1,2, \ldots NN) \]  

Where; \( Q_{nj} \) = Flow rate into node \( j \) from pipe \( n \); \( q_{nj} \) = Base demand at node \( j \);

**H-Equations:** The H-Method involves resolving for
Heads at junctions as unknowns \( (H_{j}) \). If we initially
consider the elevation of the energy line or
hydraulic grade line all through a network as the
fundamental set of unknown variables, then a set of
\( H \)-equations can be derived and resolved

\[ Q_{ij} = \left( \frac{h_{ij}}{K_{ij}} \right)^{1/n_{ij}} = \left[ \frac{(H_{i} - H_{j})/K_{ij}}{n_{ij}} \right] \]  

Where; \( Q_{ij} \) and \( K_{ij} \), Connote flow rate and loss coefficient for the pipe from node \( i \) to node, \( j \).

**\( \Delta Q \)-Equations:** These equations regard the loop
corrective discharges or \( \Delta Q \)'s as the prime
unknowns. These equations can be expressed in the
following form for each loop and path (Khamkham, 2000; Larock, Jeppson, & Watters, 2000).

\[ \sum K_{i}[Q_{oi} \pm \sum \Delta Q_{ki}]^{n_{i}} = 0 \]  

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Identifying Pipe Network Solution Approaches

There are different analysis techniques offered to compute discharges and pressures or head losses all through the pipe network. The three widely applied solution techniques in water networks include; Newton-Raphson, Linear Theory and Hardy Cross.

Newton-Raphson (NR) Method: NR scheme is a legendary method found in most of the mathematics textbooks of numerical analysis. The practical application of the NR technique is seen in solving simple and intricate water supply systems. It is said to have a “quadratic convergence” compared to other iterative schemes, which exhibit a linear convergence (Gerald & Wheatley, 2004; Lee, 1983). Just like cross’s (1936) method, the NR method requires an initial assumption of unknown variables or a reference point. The choice of an initial guess is so relevant in determining the speed of convergence of the NR scheme. The NR expression can be formulated either from the graph or the Tailor series expansion theorem. For simplicity, we shall adopt the graphical method in Figure 2 for its derivation. If $x_i$ is the initial estimate that is near to the root of the function, $f(x) = 0$, drawing a tangent to the curve at $f(x_i)$, then point $x_{i+1}$ where the tangent intersects with the x-axis would become the next approximation. The gradient of the curve will be given by the gradient of line tangent to the curve.

$$\text{Gradient} = \tan \emptyset_i = f'(x_i) = \left[ \frac{f(x_i) - 0}{x_i - x_{i+1}} \right]$$

(4)

Resolving eq. (4) generates the Newton-Raphson formula

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)}$$

(5)

Figure 2: Geometric Interpolation of Newton’s Method
\[
\frac{f(x_i)}{f'(x_i)} = \Delta x
\]  

(6)

\(x_i\) is the known initial \(x\)-value; \(f(x_i)\) denotes the value of the function at \(x_i\); \(f'(x_i)\) is the slope or gradient of the graph above at \(x_i\) also written as \(\frac{f(x)}{dx}\) and \(x_{i+1}\) signifies the next \(x\)-value.

**Linear Theory Method (LTM):** The Linear Theory Method (LTM) (Wood & Charles, 1972) solves a set of \(Q\)-equations at once after linearising the system of non-linear equations. To avoid manual initialisation, an initial estimate of 1.0 \(cfs\) (0.0283 \(m^3/s\) or 28.31 \(l/s\)) for each pipe was an assumption suggested by Wood and Charles (1972) when applying LTM. Furthermore, for the first iteration, a constant velocity value of 1 \(m/s\) may perhaps be assumed for all network pipes. Linear theory (LT) converts the non-linear energy equations into linear by estimating the pressure drop in each pipe as:

\[
h_{Lj} = [K_j Q_{jo}^{m-1}] Q_j = K'_j Q_j
\]

(7)

Where; \(Q_{jo} = \) Estimated discharge in line \(j\).

**Hardy Cross (1936) Method:** One of the primary and oldest renowned, widely used analysis approaches is the Hardy Cross (1936) method. This method of course, was initially proposed for manual calculation especially for networks with few loops before the birth of digital computers. The distinct advantage this method has over the rest is the ability to perform simple arithmetic while self-adjusting the initially guessed flow values in each duct. Cross’ (1936) work has been the most cited pieces of work but with slight understanding by the writers. Hardy Cross (1936) invented two methods—that is the “method of balancing heads” and the “method of balancing flows”. The “method of balancing heads” gained popularity and was accepted by industries until the late 1960s.

**Method of Balancing Heads:** This technique balances the initially guessed discharges in each network pipe founded on the loop-continuity equations \([\Sigma Q_{in} = \Sigma Q_{out}]\). The point is that an initial guess of flows in the network which satisfy continuity must be provided. Then, proceed to calculate the counterbalancing flow \([\Delta Q = \frac{-\Sigma K Q_o^{m-1}}{\Sigma m K |Q_o^{m-1}|}]\) and use it to make corrections to the initial flows \([Q_{New1} = Q_o + \Delta Q_{loop1}]\). The iterative process is carried on until the counterbalancing flow decreases to within an acceptable range. Usually, continuity must be maintained at the nodes and the discharges are successively modified to satisfy the zero-sum of head loss around the loops.

**Convergence Criteria**

In pipe network analysis using numerical algorithms, iterations are continued until the defined convergence criterion is achieved. Generally, there are four criteria for convergence that can be useful to determine the acceptability of a solution.
1. Based on $\Delta Q$ for each loop

This criterion is commonly implemented in HC and NR algorithms. The solution convergences once the absolute value of all corrective flows $\Delta Q$ is less than defined tolerance, i.e., $\Delta Q_i < TOL$.

2. Based on $\Delta H_i$

This is commonly used in HC and NR algorithms. The solution convergences once the absolute value of all corrective heads $\Delta H_i$ is less than the defined tolerance, i.e., $\Delta H_i < TOL$.

$$\left(\frac{Q_{\text{new}} - Q_{\text{old}}}{Q_{\text{new}}}\right) \times 100\% < 0.5\%$$ (8)

Network Cost

Network cost is found by totalling the cost of each conduit. The total cost can be mathematically stated as (Sadafule, Hiremath, & Tuljapure, 2013):

$$C = \sum_j C_j \times [L_jD_j]$$ (9)

Where; $C_j$: Cost per unit length of pipe $j$ with diameter $D_j$ and $L_j$ is the Length of pipe $j$.

Program Validation Methods

Verifying the performance of the model requires conducting statistical analysis. The analysis schemes include; the coefficient of determination ($R^2$), the root mean square error (RMSE), and the mean bias error (MBE). RMSE measures the variation of predicted figures around the observations. The smaller the RMSE, the more precise is the approximation. MBE is a representation of the mean deviation of the predicted values from the respective observations. The smaller the MBE, the more superior is the model performance (Maitha, Assi, & Hassan, 2011).

The expressions for the aforementioned statistical parameters are:

$$R^2 = 1 - \frac{\sum (q_{\text{obs}} - q_{\text{cal}})^2}{\sum (q_{\text{obs}} - q_{\text{cal}})^2}$$ (10)

$$RMSE = \sqrt{\frac{1}{N} \sum_{k=1}^{N}(q_{\text{cal}} - q_{\text{obs}})^2}$$ (11)

$$MBE = \frac{1}{N} \sum_{k=1}^{N}(q_{\text{cal}} - q_{\text{obs}})$$ (12)
METHODS AND MATERIALS

The overall methodological approach used was according to V-model\(^1\) in Figure 3, the structure of program design that encompasses; the project definition (Requirement’s specification, Architecture design, Module design), implementation (coding) and the project test and integration (Integration, test, and verification, System verification and validation, operation and maintenance). The material used was Visual Basic Language.

**Figure 3: V-Model Methodology of Application Design**

![V-Model Methodology of Application Design](image)

**System Requirements**

During this first phase, the system requirements were established to determine the feature set. Both functional and non-functional requirements were identified. The functional requirements describe the task of a system or its modules. This considers the system’s ability to; compute the flow rates, head losses, flow velocities within the pipe network, pressure heads at the network nodes and cost network pipelines. Non-functional requirements include; reliability (is the system able to give only correct output?), usability (is the system user friendly?), efficiency (is the system able to run without any intervention, thus maintaining the correct outputs ever?), performance requirements (is the system fast in executing its algorithm to provide results in case of any looping?)

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\(^1\) https://www.tutorialspoint.com/sdlc/sdlc_V_model.htm
Program Architecture

The program consisting of welcome form, user login form, main menu form and computation forms illustrated in Figure 4 was written in a simple programming language called VB6.0. The first form that loads after the program is started is called the welcome form. The next form that opens after clicking the start button on the welcome form is called the main menu form. This is where the user performs loops and head loss formula selection. After the user has selected a specific head loss formula and the loop to be analysed, the model entreats computation form for a particular loop and head loss formula. The computation form allows the user to enter pipe properties.

Implementation/Coding

At this phase, just halfway through the stages along with the V-Model program development methodology, definite coding and implementation started. During this period, ample time was allotted to transform all the preceding steps into a coded working model. In this study, an improved Hardy Cross Algorithm (Epp & Fowler, 1970) was modelled and implemented in VB6.0.

Modelling Improved Hardy Cross Algorithm

Generally, Hardy Cross (HC) method adheres to two (2) important laws: [1] the net flow at each junction must be equal to zero (0). [2] The head loss around a closed loop must be equal to zero (0). The algorithm operates on an initial guess of discharges in each conduit that must fulfil the first law of Kirchhoff. There is a number of iterative steps executed in the algorithm until both conservation principles are satisfied. One important feature to take note of is that, in the original HC method, the corrective flows \(\Delta Q_{Cor}\) are then determined separately and applied to obtain the next flow rates. However, the rationale of separately obtaining corrective flows somehow renders the algorithm slow.

Figure 4: System Architecture Hierarchy

![System Architecture Hierarchy Diagram]

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The new HC method increases the convergence speed and simultaneously resolves the entire closed-loop pipe network (Brkic, 2011). The new HC (Epp & Fowler, 1970) adopts a matrix technique to simultaneously solve for unknown corrective flow rates. In the original HC, the unknown flow modification factor is determined as:

\[ \Delta Q_k = \frac{\sum_{j=1}^{n} k_j Q_{oj}^n}{\sum_{j=1}^{n} n [k_j Q_{oj}^{n-1}]} \]  
\[ \text{(13)} \]

From Darcy-Weisbach formula;

\[ k_j = \frac{f_j L_j}{2gD_j A_j^2} \]  
\[ \text{(14)} \]

Where; \( L_j \): Length of pipe j; \( D_j \): Diameter of pipe j, and \( f_j \): Darcy-Weisbach friction factor of pipe j.

After obtaining the flow adjustment factor (\( \Delta Q \)), it should be added to the initial flows to determine the new flows.

\[ Q_{\text{New1}} = Q_o + \Delta Q_{\text{loop1}} \]  
\[ \text{(15)} \]

In the case of pipes common in both loops, the correction formula in Equation (16) will be applied.

\[ Q_{\text{New1}} = Q_o + \Delta Q_{\text{loop1}} - \Delta Q_{\text{loop2}} \]  
\[ \text{(16)} \]

When implementing the HC algorithm, for the starting successive iterations, conservation laws are likely not to be satisfied as the computed \( \Delta Q \) will not make pressure drop around the closed-loop zero albeit, it will reduce it closer to zero than it was in the previous iteration. Expressing the original HC method (1936) in a matrix arrangement (Brkic, 2011) would produce:

\[
\begin{bmatrix}
\sum n \frac{h_{11}}{Q_{11}} & 0 & 0 & 0 & \ldots & 0 \\
0 & \sum n \frac{h_{22}}{Q_{22}} & 0 & 0 & \ldots & 0 \\
0 & 0 & 0 & 0 & \ldots & 0 \\
\vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\
0 & 0 & 0 & 0 & \ldots & \sum n \frac{h_{mk}}{Q_{mk}}
\end{bmatrix}
\begin{bmatrix}
\Delta Q_1 \\
\Delta Q_2 \\
\Delta Q_k \\
\Delta Q_{\text{loop1}} \\
\Delta Q_{\text{loop2}} \\
\Delta Q_k
\end{bmatrix}
= 
\begin{bmatrix}
\sum h_1 \\
\sum h_2 \\
\ldots \\
\sum h_k
\end{bmatrix}
\]
\[ \text{(17)} \]

Epp & Fowler (1970) revised the original HC method (Cross, 1936) by substituting the zeroes (0) in the off-diagonal of Equation 17 with the first derivative of the head loss for pipes common in two loops Equation (18). The work of (Epp & Fowler, 1970) led to what is today commonly known as the “modified HC Algorithm”, capable of simultaneously solving for the flow rectification factor.
Developing a VB6.0 Program

There are a number of steps that were undertaken to create a VB6.0 project:

**Step 1:** The researcher used the forms onto which the object controls like buttons, picture boxes, among others, were drawn.

**Step 2:** After which the form module was used to write the program or source code.

**Step 3:** The program was then compiled using a VB compiler (a special program that transforms programs written in a high-level language such as VB into machine code (low-level language) that a computer comprehends). The compiler examines a program written in a language such as VB and transforms it into a form that is readable by a computer system (Khamkham, 2000).

**Step 4:** Eventually, the program was executed, tested on a case study pipe network and validated.

Algorithm Flow Chart

As aforementioned, the algorithm implemented in this study is an improved HC algorithm to solve single (1), two (2), three (3) and four (4) loops. Unlike in the previous studies such as (Yengale, Wadhai, & Khode, 2012; Demir, Yetilmezsoy, & Manav, 2008) where the user is required to first approximate the initial flow rates, in this developed model, the user just needs to enter the population or base demand data at the network nodes and per capita water demand. The program then automatically guesses the initial discharges for each pipe in the network while maintaining continuity. The model also requires the user to key in the nodes elevation data, pipe length, diameter, and loss coefficients.

Following that, the program proceeds to compute the actual flow rates in pipe \( k \), pressure head at node \( I \), velocity in pipe \( k \), head losses in pipe \( k \) and total head losses around loop \( j \). The model then checks whether or not the number of iterations is equal to the maximum iteration as in Figure 5.
Figure 5: Flow Chart of HC Algorithm for Multiple Loops

Testing, Verification & Validation

All the modules developed in the implementation phase were integrated into a single system. Several testing techniques were employed and these include testing of each unit, integration testing of combined units and post-integration of the entire system where errors and failures were noticed and corrected.

Unit Testing: This method of separately testing individual modules helped in correcting all errors (bugs) in the code to make certain each module fulfils its functional requirements.

System Integration Testing: This process was used to ensure that the individual modules were properly integrated and working in harmony.

One of the central requirements for any software acceptability is reliability and accuracy. To ensure this, the model was tested on a case study water network. The output from the model was compared...
with the EPANET software solution to determine any discrepancies.

In order to assess the reliability of the built numerical hydraulic model and prove whether there is any fundamental inclination in its performance, a statistical study including the coefficient of determination ($R^2$), RMSE and MBE were adopted. The higher the $R^2$, the more accurate is the estimation.

**COMPUTER PROGRAM DESCRIPTION AND TESTING**

**Model Validation**

The numerical model was validated using hydraulic parameters for the Water Distribution Network (WDN) of Mbale town (Uganda) as indicated in Figure 6. The study area is about 2$km^2$ traced at about 10°4′49.4″ “N and 34°10′2″ “E with a pipe network of about 3 km within Mbale town water service area. The overall land area coverage of Mbale district is 519 $km^2$ with a pipe network of around 305 km extending over a radius of 25 km within the water service area and a consumer base of 11,824 connections. Presently, 4,200 $m^3$ of water per day is the average demand from the served places. Piped water is supplied to the clients through a delivery network consisting of 300 mm GMS trunk mains which is reduced into 9 inch and 6-inch GI pipes.

**Computer Program**

The computer program herein developed has been written in VB6.0 language to solve for flow rates, piezometric heads, head losses, and velocities using the improved HC algorithm. Essentially the computer model reads input data defining the network links (pipes) and junction nodes. A number of key things need to be noted about this hydraulic model: (1) Depending on the data at the user’s disposal, the program can read in either node population data and compute the base demand plus assumed initial discharge in each pipe or it can read in base demand data at the nodes and then compute the initial discharge for each pipe in the closed-loop, (2) One type of liquid, essentially water is applied to this model, (3) The head loss equations for simulation are: Darcy-Weisbach and Hazen-Williams formulae and (4) Computation continue until a tolerance of 0.00001 is achieved.

**Model Input Variables**

The data requirements for the program are as follows:

- **Pipe data:** For each pipe in the closed-loop network, diameter, length, and pipe roughness are required. If the pipe has any minor loss device, specifically a valve, the number of valves and the value of minor loss coefficient needs to be fed in.

- **Junction data:** First, the population or base demand ($m^3$/s) data is keyed in by the user. If for example, the user enters population data at the nodes, the software will go ahead to compute the base demand plus initially assumed flow rates. The unit for base demand is in cubic meters per second ($m^3$/s). In cases where you are faced with the external flow (Base Demand) into the junction node, then a negative (-) sign should be assigned to it by the user. Second, the elevation data at each node is fed in. This data is required to proceed with pressure head calculation. However, classroom examples without elevation data can still be solved by the model, but the user eventually receives “WARNING messages”, which can be ignored. The elevation is in meters and the calculated pressure heads are also in meters of water.
Figure 6: Case Study for model Verification and testing.

Starting/Running the Program

- Go to windows search and type “MHC-NET” (Modified Hardy Cross-Network).

- Click on its icon to execute the program. Figure 7 will appear on the screen. It is called the “welcome” window.

- When you click on the “Info” button, the form/window in Figure 8 below is displayed. This window gives the user an introduction to the program, that is, what the program is all about, and the developer.

- Prior to performing analysis, click on the “Start” button and the “Main Menu” form (Figure 9) below will appear. Selection of data type, head loss formula and loop to work with is done from this window.

- Click on the “Back” button on the “Main Menu” window to go back to the “Welcome” window (Figure 7).
Click “Next” after selecting a particular loop, data type and head loss formula to execute network analysis. The computation form will then appear with a message dialog box informing the user that the current inputs are just default values. Therefore, the user needs to input new data to perform analysis (Figures 10a-d).

Figure 7: Welcome Window Interface

![Welcome Window Interface](image)

Figure 8: Introduction Form

![Introduction Form](image)

This program was designed to compute the Actual Flow Rates (Qactual) in each pipe in a closed loop pipe network using THE MODIFIED HARDY CROSS METHOD (MHC). The program is limited to ONLY four types of network which ARE: single (1) loop, two (2) loops, three (3) loops and (4) four loops. The MHC (Epp and Fowler, 1970) is an iterative technique to determine flow in a closed loop pipe network by making corrections to the Assumed Flow (Qassumed) Rates until the corrective flows (QCorrection) are negligible. The software uses Darcy-Weiszbach (HL=K’Q^2) and Hazen Williams (HL=K’Q^1.852) formulas to compute Major Head Losses in pipes and Modified Barr (1981) Equation to compute the friction factor. Minor Losses are also considered. The data required for the Model to run ARE: Node Inputs (i.e. Elevation (m), Population and Base Demand (m3/s)) and Link Inputs (i.e. Pipe Diameter (m), Length (m), Pipe Roughness, and Minor Loss Coefficients). The Program has been built to compute the Base Demand (BD) and Assumed Flow Rates (Qassumed) automatically given the node population thus saving time the user would have wasted in manually calculating the Base Demand and guessing flow in each pipe. The manual evaluation of the cost of the network pipes has been solved as this model is able to estimate the Total Pipe Costs.
Figure 9: Main menu Interface
Figure 10: Computation Interface for Loops

(a) Computation Interface for Loops

(b) Computation Interface for Loops

(c) Computation Interface for Loops

(d) Computation Interface for Loops
• Enter both the node and the link data into the computation interface, then click the “Run” button to perform the loop analysis.

**NB:** In cases where you are faced with the external flow (Base Demand) into the junction node, then a negative (-) sign should be assigned to it by the user when keying in the data.

**Figure 11: Results table**

| Pipe (m³/s) | HL (m) | V (m/s) | D (m) | f | BD (m³/s) |
|-------------|--------|---------|-------|---|-----------|
| 1           | 0.0616 | 1.791   | 1.25  | 0.250 | 0.019 | 0.0000 |
| 2           | 0.0376 | 2.070   | 1.20  | 0.200 | 0.020 | 0.0445 |
| 3           | 0.0132 | 5.530   | 1.39  | 0.110 | 0.017 | 0.0303 |
| 4           | 0.0171 | 5.810   | 1.80  | 0.110 | 0.016 | 0.0244 |

Q = Actual Flow Rates  
HL = Head Losses  
V = Velocity  
D = Diameter  
f = Friction Factor  
BD = Base Demand

**Figure 11:** Results table

- After running the analysis, you may want to know how much your network pipes will cost. By clicking the “Pipe Cost” button, the “Pipe cost” interface will pop up. Enter the cost per unit length, then click “cost” to perform costing in Figure 12.

- The analysis results are displayed in the table of results interface. After running the analysis, the “Results Table” interface will turn yellow in colour in Figure 11. The negative (-) flow output is okay. It describes a flow that is in the anti-clockwise direction. The flow in the clockwise direction maintains a positive (+) sign convention.
Figure 12: Pipe Cost Interface

Model Testing

The testing of the built program was conducted on a single (1), two (2), three (3) and Four (4) loop case study area water network. The four-loop network represents the case study area. The different networks for each loop with the collected input data are presented in Appendix 1, 2, and 3. The figure of per capita demand used in the analysis was an average of 100 l/ca/d according to the Ministry of Water and Environment (2013) design manual.

RESULTS AND DISCUSSION

This section aims at discussing and comparing the results of the developed user-friendly hydraulic model with EPANET hydraulic tool. The first section presents and compares the results in a tabular format for one (1), two (2), three (3) and four (4) network problems.

Results Comparison

The comparison of results produced from the program and EPANET software is done through Tables 3-6 as follows:

Table 3: Results of a single [1] Loop Network from the Program and EPANET

| Pipe | Program Output | EPANET Output |
|------|----------------|---------------|
|      | QNew (m³/s)   | HL (m) | V (m/s) | QNew (m³/s) | HL (m) | V (m/s) |
| 1    | 0.0616         | 1.791  | 1.25    | 0.0616       | 1.791  | 1.25    |
| 2    | 0.0376         | 2.07   | 1.20    | 0.0376       | 1.907  | 1.20    |
| 3    | 0.0132         | 5.53   | 1.39    | 0.0132       | 5.491  | 1.39    |
| 4    | 0.0171         | 5.81   | 1.80    | 0.0171       | 5.775  | 1.80    |

**Dynamic Pressure**

| Program Output | EPANET Output |
|----------------|---------------|
| Node           | Pressure Head [m] | Pressure Head [m] |
| 1              | 6.60          | 6.60            |
From Table 3, it can be clearly seen that the discharge results from both the program and EPANET software are consistent. The velocities are within the recommended range of (0.6 - 3.0) m/s. The pressures at the nodes are within the allowable range (2 – 60 bars or 2 m – 60 m) (Ministry of Water and Environment, 2013). The head losses calculated in pipes 1 & 2 are 0.006 m/m and 0.008 m/m respectively. These values are less than the maximum allowable (0.01 m/m). Pipes 3 & 4 have 0.015 and 0.024 head loss per unit length values which are slightly higher than the recommended maximum allowable value.

Table 4: Results of a 2 Loop Network from the Program and EPANET

| Pipe | Program Output | EPANET Output |
|------|----------------|---------------|
|      | QNew (m³/s)   | HL (m) | V (m/s) | QNew (m³/s) | HL (m) | V (m/s) |
| 1    | 0.0362        | 1.918  | 1.15    | 0.070        | 2.305  | 1.43    |
| 2    | 0.0118        | 4.485  | 1.24    | 0.0362       | 1.769  | 1.15    |
| 3    | 0.0141        | 4.105  | 1.48    | 0.0118       | 4.466  | 1.24    |
| 4    | 0.0359        | 3.720  | 2.03    | 0.0141       | 4.087  | 1.48    |
| 5    | 0.0059        | 0.726  | 0.62    | 0.0359       | 3.724  | 2.03    |
| 6    | 0.0362        | 1.918  | 1.15    | 0.0059       | 0.729  | 0.62    |
| 7    | 0.0068        | 1.110  | 0.72    | 0.0068       | 1.091  | 0.72    |

Dynamic Pressure

| Node | Program Output | EPANET Output |
|------|----------------|---------------|
|      | Pressure Head [m] | Pressure Head [m] |
| 1    | 6.60            | 6.60          |
| 2    | 5.30            | 5.30          |
| 3    | 4.58            | 4.58          |
| 4    | 17.47           | 17.48         |
| 5    | 18.20           | 18.21         |
| 6    | 32.68           | 32.68         |

From Table 4, the discharge results from both the program and EPANET software are the same, which shows the reliability of the model. The velocities in all pipes are within the recommended range (0.6-3.0) m/s. The pressure at all nodes for a two-loop network are within the acceptable range (2 bars – 60 bars) according to the Ministry of Water and Environment (2013) design guidelines for a water supply network.
From Table 5, the discharge from the program and EPANET are the same, and the velocities except in pipe 10 are within the recommended range (0.6-3.0) m/s. The pressure at all nodes is acceptable according to (Ministry of Water and Environment, 2013) design guidelines manual.
From Table 6, the discharge from the program and EPANET are similar, and the velocities (except in pipe 6) are within the recommended range (0.6-3.0) m/s. The pressure at nodes 2 and 3 are below the minimum recommended value that is 2.0 m or 2 bars (Ministry of Water and Environment, 2013). The head loss per unit length of pipe is higher in pipes 3, 4, 5, 8 and 12.

**Statistical Analysis of Results**

From the statistical analysis Tables 7, the Coefficient of Determinant ($R^2$), Root Mean Square Error (RMSE), and Mean Bias Error (MBE) for both discharge and Velocity results from program and EPANET has been found to be: 1.000, 0.000, and 0.000 respectively for single (1), two (2), three (3) and four (4) loop networks. It should be noted that velocity is calculated from ($Q_{Discharge}$/Area). Therefore, if the discharge results from the program and EPANET agree, then velocities will automatically be the same for the same pipe diameter. This shows the reliability of the program in computing flows and velocities.

Likewise, from the statistical analysis (Table 7), it can be seen that the Coefficient of Determinant ($R^2$) is practically higher. Rounding off ($R^2$) to 2 decimal places makes $R^2$ approximately 1.000. The higher the ($R^2$), the better is the performance of the model. The Root Mean Square Error (RMSE) and Mean Bias Error (MBE) table 5-6 above can be clearly observed to be reasonably small values. It is stated that the lower the RMSE and MBE, the better is the performance of the model. The reasonably minimal variability in head loss results resulted in the above statistical analysis output. However, this does not affect the final actual flow results. There are about three (3) reasons that could be attributed to the very small variability in head losses which are: (1) the convergence speed (dependent on initial assumed flows), (2)- the numerical algorithm implemented by both models to solve the system of non-linear equations, (3)- the implemented friction factor ($f$) formula. EPANET implements the “Gradient Algorithm” and (Swamee & Jain, 1976) friction factor equation (Rossman, 2000) while the developed model implements improved hardy cross (Epp & Fowler, 1970) and modified Barr (1981) friction factor equation. Nonetheless, the statistical analysis outputs show that the model is reliable.

In general, the Coefficient of Determinant ($R^2$) is 1.000 for the considered loops. The values of the Root Mean Square Error (RMSE), Mean Bias Error (MBE) are rationally minimal. Small values of RMSE and MBE indicate good performance of the model. Therefore, it can be concluded from the statistical study conducted between EPANET and the program that the developed model is reliable and can be trusted in analysing flows in a closed-loop water network.
Table 7: Statistical Analysis of Results from the Program and EPANET

| Statistical Method                  | Discharge Loops | Velocity Loops |
|------------------------------------|-----------------|----------------|
|                                    | I    | II   | I    | II   | I    | II   | I    | II   |
| Coefficient of Determination (R²)  | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 | 1.000 |
| Root Mean Square Error (RMSE)      | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |
| Mean Bias Error (MBE)              | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Statistical Method                  | Head Loss Loops | Pressure Loops |
|------------------------------------|-----------------|----------------|
|                                    | I    | II   | I    | II   | I    | II   | I    | II   |
| Coefficient of Determination (R²), | 0.998 | 0.998 | 0.999 | 0.999 | 1.000 | 1.000 | 1.000 | 1.000 |
| Root Mean Square Error (RMSE)      | 0.086 | 0.058 | 0.07  | 0.048 | 0.016 | 0.006 | 0.042 | 0.017 |
| Mean Bias Error (MBE)              | 0.059 | 0.031 | 0.041 | 0.025 | 0.01  | 0.003 | 0.034 | 0.012 |

CONCLUSION

In the face of the challenges encountered during the hydraulic model development, the User-Friendly Numerical Hydraulic model for analysing complex pipe networks using modified hardy cross algorithm has undergone complete and successful design, implementation and testing to meet the earlier stated objectives. The model has been limited to analysing one (1), two (2), three (3), and four (4) closed loops. For the purposes of model validation, a comparative study was conducted on the outputs from the program with EPANET. The statistical analysis revealed the model validity since the discharge results from both EPANET software and the developed model showed no variations.

It should be noted that the developed hydraulic model may have a limited application for a compound pipe network. For the future advancement of the model, the following recommendations are presented: Develop a model that implements modified HCM for both available population and the base demand data. That is to say, for the base demand entering and leaving the junction node. Adopt other existing algorithms like; Newton-Raphson and Linear theory methods to conduct a comparative study for the further advancements of this study. Extending the present model to solve hydraulic grid of any number of loops greater than four. Develop the graphical interface for the current model. Extend the present model to handle water quality modelling plus extended period hydraulic analysis. Extend the present program to model a network with a pump and pseudo-loops.

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**APPENDIX**

**Appendix 1: Pipe Network**

**A single loop, 4 Pipe Network Problem**

**2 Loop, 6 Pipe Network Problem**
A 3 Loop, 10 Pipe Network Problem

A 4 Loop, 12 Pipe Network Problem
### Appendix 2: Pipe Input Data

| Pipe | Pipe Type | Pipe Length [m] | Pipe Diameter [mm] | No. of Valves | Minor Loss Coefficient | Pipe Roughness [mm] | Pipe Unit Cost [$] |
|------|-----------|-----------------|--------------------|---------------|------------------------|---------------------|-------------------|
| **Pipe Input Data for a Single [1] Loop Network Problem** | | | | | | | |
| 1 | GI | 297 | 200 | - | - | 0.15 | 61 |
| 2 | UPVC | 262 | 90 | 1 | 2.3 | 0.0015 | 25 |
| 3 | UPVC | 370 | 90 | - | - | 0.0015 | 25 |
| 4 | UPVC | 245 | 90 | - | - | 0.0015 | 25 |
| **Pipe Input Data for a 2 Loop Network Problem** | | | | | | | |
| 1 | GI | 297 | 250 | - | - | 0.15 | 77 |
| 2 | GI | 262 | 200 | 1 | 2.3 | 0.15 | 61 |
| 3 | UPVC | 370 | 110 | - | - | 0.0015 | 38 |
| 4 | UPVC | 245 | 110 | - | - | 0.0015 | 38 |
| 5 | GI | 128 | 150 | - | - | 0.15 | 46 |
| 6 | UPVC | 209 | 110 | - | - | 0.0015 | 38 |
| 7 | UPVC | 243 | 110 | - | - | 0.0015 | 38 |
| **Pipe Input Data for a 3 Loop Network Problem** | | | | | | | |
| 1 | GI | 262 | 200 | 1 | 2.3 | 0.15 | 61 |
| 2 | GI | 297 | 250 | - | - | 0.15 | 77 |
| 3 | UPVC | 245 | 110 | - | - | 0.0015 | 38 |
| 4 | UPVC | 370 | 110 | - | - | 0.0015 | 38 |
| 5 | UPVC | 149 | 110 | - | - | 0.0015 | 38 |
| 6 | UPVC | 100 | 110 | - | - | 0.0015 | 38 |
| 7 | UPVC | 416 | 110 | - | - | 0.0015 | 38 |
| 8 | UPVC | 180 | 110 | - | - | 0.0015 | 38 |
| 9 | UPVC | 200 | 90 | - | - | 0.0015 | 25 |
| 10 | UPVC | 400 | 90 | - | - | 0.0015 | 25 |
| **Pipe Input Data for a 4 Loop Network Problem** | | | | | | | |
| 1 | GI | 297 | 250 | - | - | 0.15 | 77 |
| 2 | GI | 262 | 200 | 1 | 2.3 | 0.15 | 61 |
| 3 | UPVC | 370 | 110 | - | - | 0.0015 | 38 |
| 4 | UPVC | 245 | 110 | - | - | 0.015 | 38 |
| 5 | GI | 128 | 150 | - | - | 0.15 | 46 |
| 6 | UPVC | 209 | 110 | - | - | 0.0015 | 38 |
| 7 | UPVC | 243 | 110 | - | - | 0.0015 | 38 |
| 8 | UPVC | 100 | 90 | - | - | 0.0015 | 25 |
| 9 | UPVC | 234 | 90 | - | - | 0.0015 | 25 |
| 10 | UPVC | 100 | 110 | - | - | 0.0015 | 38 |
| 11 | UPVC | 416 | 110 | - | - | 0.0015 | 38 |
| 12 | UPVC | 149 | 110 | - | - | 0.0015 | 38 |

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Appendix 3: Node Input Data

| Node | Population | Elevation [m] |
|------|------------|---------------|
| **Node Input Data for a Single [1] Loop Network Problem** | | |
| 1 | 0 | 48 |
| 2 | 10000 | 47 |
| 3 | 6400 | 30 |
| 4 | 5000 | 20 |
| **Node Input Data for a 2 Loop Network Problem** | | |
| 1 | 0 | 48 |
| 2 | 4000 | 47 |
| 3 | 6110 | 44 |
| 4 | 2400 | 30 |
| 5 | 4000 | 30 |
| 6 | 5000 | 20 |
| **Node Input Data for a 3 Loop Network Problem** | | |
| 1 | 0 | 48 |
| 2 | 1000 | 20 |
| 3 | 0 | 17 |
| 4 | 2100 | 14 |
| 5 | 2100 | 15 |
| 6 | 2400 | 9 |
| 7 | 4000 | 30 |
| 8 | 10000 | 47 |
| **Node Input Data for a 4 Loop Network Problem** | | |
| 1 | 0 | 48 |
| 2 | 4000 | 47 |
| 3 | 6110 | 44 |
| 4 | 0 | 30 |
| 5 | 2400 | 22 |
| 6 | 2100 | 9 |
| 7 | 2100 | 17 |
| 8 | 1000 | 20 |
| 9 | 4000 | 30 |