Analysis of Possible Quantum Metastable States in Ballistic Graphene-based Josephson Junctions

Joseph G. Lambert, Steve Carabello, and Roberto C. Ramos
Department of Physics, Drexel University, Philadelphia, PA 19104 USA

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Graphene is a relatively new material (2004) made of atomic layers of carbon arranged in a honeycomb lattice. Josephson junction devices are made from graphene by depositing two parallel superconducting leads on a graphene flake. These devices have hysteretic current-voltage characteristics with a supercurrent branch and Shapiro steps appear when irradiated with microwaves. These properties motivate us to investigate the presence of quantum metastable states similar to those found in conventional current-biased Josephson junctions. We present work investigating the nature of these metastable states for ballistic graphene Josephson junctions. We model the effective Washboard potential for these devices and estimate parameters, such as energy level spacing and critical currents, to deduce the design needed to observe metastable states. We propose devices consisting of a parallel on-chip capacitor and suspended graphene. The capacitor is needed to lower the energy level spacing down to the experimentally accessible range of 1-20 GHz. The suspended graphene helps reduce the noise that may otherwise come from two-level states in the insulating oxide layer. Moreover, back-gate voltage control of its critical current introduces another knob for quantum control. We will also report on current experimental progress in the area of fabrication of this proposed device.

I. INTRODUCTION

Since its isolation in 2004, the electronic properties of graphene have inspired a flourishing field of research. Graphene is a two-dimensional, hexagonal lattice of carbon atoms that effectively contains relativistic Dirac charge carriers. Its characteristic linear dispersion relation where the upper and lower energy bands meet has led to novel and interesting physics and applications.

Recent studies have demonstrated the robust and reproducible phenomenon of the Josephson effect in devices consisting of two superconducting leads contacted by flakes of graphene that are 1-4 layers thick. Clear evidence of multiple Andreev reflections illustrates that such graphene junctions behave as SNS junctions. The critical current can be tuned by modulating a back-gate voltage. A dc-SQUID has been constructed using graphene, demonstrating several advantages. These include how graphene is straightforward to fabricate, and electrically contact with high-transparency electrodes. The current-phase relation has been predicted and measured.

The appearance of the Josephson effect in graphene junctions motivates us to investigate potential metastable quantum states within the washboard potential wells. Such states have been extensively studied in conventional junctions for over 25 years and have been used as basis states for superconducting quantum computing.

II. DEVICE FABRICATION AND CHARACTERIZATION

To fabricate devices, we prepare graphene using the mechanical exfoliation technique onto Si/SiO₂ (300nm) substrates. We first locate and identify single-layer and multi-layer graphene specimens using optical microscopy. We confirm and characterize the number of layers using Raman spectroscopy. An example of a typical Raman spectrum that we collect for single-layer graphene is shown in Fig. 1.

![Example Raman spectrum of our single-layer graphene specimens using 514.5 nm laser excitation. The G peak at ~ 1580 cm⁻¹ is the E₂g optical mode due to in-plane lattice vibrations and the 2D peak at ~ 2700 cm⁻¹ is the overtone of the D peak due to second-order double resonance. The ratio of the peak heights, and the positions of the peaks are signatures of the thickness of the graphene flake.](image-url)
ing proximity effect\textsuperscript{2}, the areas of graphene underneath
the leads become superconducting. We have fabricated
devices, an example of which is depicted in Fig. 2(a).
This device has a room temperature, 2-probe resistance
of 5.8 kΩ, which is evidence of good electrical contact.

![Image of graphene device](image)

**FIG. 2:** (a) Optical image of one of our fabricated graphene
devices that is not suspended. The lead separation is
$L = 200$ nm and the length of the leads in contact with graphene is
$W = 700$ nm. (b) Schematic of our future graphene device
with suspended graphene. The graphene and leads are ele-
vated from the substrate only for visual clarity.

Charge carrier mobility in graphene is reduced by sub-
strate induced scatterers, such as charges trapped under
the graphene and rippling of the graphene due to the
roughness of the substrate\textsuperscript{14}. Removal of the SiO$_2$ sub-
strate should also reduce the effects of anomalous two-
level systems that plague superconducting qubits\textsuperscript{15–17}. In
order to achieve ballistic transport in graphene, fu-
ture devices will consist of suspended graphene, as in Fig.
2(b)\textsuperscript{18–20}. Suspension of the graphene will be achieved
by etching the SiO$_2$ with hydrofluoric acid (HF). HF so-
lution is targeted underneath the graphene by capillary
action\textsuperscript{21}.

The back-gate is provided by the doped Si substrate.
For the case of non-suspended graphene, the back-gate
and graphene are separated by the 300 nm of SiO$_2$ as an
insulating dielectric, and for suspended graphene, they
are separated by vacuum. The on-chip shunting capaci-
tor will have an interdigitated design, which will be fab-
ricated with the device leads during a single deposition.

### III. THEORY

The quantum metastable states we are investigating
are analogous to those existing within the wells of the
tilted washboard potential of a current-biased Josephson
junction\textsuperscript{22,23}. This potential has the form

$$U(\gamma) = -\frac{\Phi_0}{2\pi} (I_0 \cos(\gamma) + I_b \gamma)$$

where $\gamma$ is the gauge-invariant phase difference, $\Phi_0 = h/2\pi$ is the flux quantum, $I_0$ is the critical current, and $I_b$ is the bias current. For a junction with capacitance $C$, this system is analogous to a phase particle of mass $m = C(\Phi_0/2\pi)^2$ oscillating in a local well with plasma frequency\textsuperscript{23}

$$\omega_p = \omega_0 \left[1 - \left(\frac{I_b}{I_0}\right)^2\right]^{1/4}$$

where $\omega_0 = \sqrt{2\pi I_0/\Phi_0 C}$. At bias current $I_b \sim I_0$, the
energy spacing between the ground and the first excited
states is $\hbar\omega_{01} \approx \hbar\omega_p$\textsuperscript{23}.

The current-phase relationship, the critical current, and $I_0 R_N$ product for ballistic graphene junctions were calculated by Titov and Beenakker\textsuperscript{24}. Specifically, at the Dirac point, these equations are analytic:

$$I(\gamma) = \frac{e\Delta_0}{\hbar} \frac{2W}{\pi L} \cos(\gamma/2) \text{arctanh}[\sin(\gamma/2)]$$

$$I_0 = 1.33 \frac{e\Delta_0}{h} \frac{W}{\pi L}$$

$$I_0 R_N = 2.08 \frac{\Delta_0}{e}$$

Here, $\Delta_0$ is the superconducting energy gap of the leads, $W$ is the length of the leads in contact with the graphene flake, and $L$ is the lead separation. These equations were derived assuming $L \ll W, \xi$ where $\xi = \hbar v/\Delta_0$ is the superconducting coherence length, and $v$ is the velocity of charge carriers. According to the RCSJ model, the equation of motion of the “phase particle” is\textsuperscript{23}

$$I_b = I(\gamma) + \frac{e\Phi_0}{w\pi} \gamma + \frac{\Phi_0}{2\pi R} \dot{\gamma}.$$
The SGS potential \( U(\gamma) \) is plotted alongside the conventional washboard potential \( U \) in Fig. 3(a). For the same critical current and capacitance, the wells of the SGS potential are shallower. This would result in more closely spaced energy levels.

From (7) we calculate the plasma frequency about the local minimum of the well, \( \gamma_{\text{min}} \), for ballistic graphene junctions. This is plotted and compared to (2) in Fig. 3(b).

\[
\omega_p = \left( \frac{2\pi I_0}{1.33\Phi_0 C} \right)^{\frac{1}{2}} \times \left[ 1 - \sin\left( \frac{\gamma_{\text{min}}}{2} \right) \tanh^{-1} \left( \sin\left( \frac{\gamma_{\text{min}}}{2} \right) \right) \right]^\frac{1}{2}. \tag{8}
\]

The superconducting gap for bulk Al is \( \Delta_0 = 180 \mu \text{eV} \); however, experimentally determined values\(^\text{7}\) in typical devices ranged from 90 to 120 \( \mu \text{eV} \). Using the approximation \( \Delta_0 \approx 100 \mu \text{eV} \), we estimate a critical current of \( I_0 \approx 100 \) nA for the device shown in Fig. 2(a) \( (L = 200 \text{ nm}, W = 700 \text{ nm}) \). The intrinsic capacitance of the device is small, \( \sim 10^{-16} \) F. Using the estimated critical current and (2) gives \( f_p = \omega_p / 2\pi \approx 300 \) GHz, in qualitative agreement with experimental results\(^\text{6,8}\). We can lower this frequency to \( f_p \approx 5 \) GHz by adding a 0.1 pF shunting capacitor.

**IV. CONCLUSION**

We have calculated the washboard potential and plasma frequency of SGS junctions. Our calculations show that ballistic graphene junctions are essentially similar to conventional junctions, with the added benefit of a back-gate voltage control of the critical current. We are testing this conclusion with a series of experiments such as current-voltage measurements, microwave resonant activation, and quantum tunneling\(^\text{25}\) experiments. We will also measure the graphene junctions in both the classical and quantum regimes. The crossover temperature, \( T_c = \hbar \omega / 2\pi k_B \), distinguishes these two regimes\(^\text{26}\).

At current biasing near the critical current, the estimated plasma frequency is 100 GHz, and with a shunting capacitor of 0.1 pF, it is 5 GHz. These plasma frequencies have crossover temperatures of approximately 800 mK and 40 mK, respectively. For the higher plasma frequencies, multi-photon processes will be used to enhance the escape of the phase particle\(^\text{27}\). We will tune these experiments with the back-gate voltage to study this unique graphene/superconductor system.

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1 K. S. Novoselov, A. K. Geim, S. V. Morozov, D. Jiang, Y. Zhang, S. V. Dubonos, I. V. Grigorieva, and A. A. Firsov, Science 306, 666 (2004).
2 K. S. Novoselov, A. K. Geim, S. V. Morozov, D. Jiang, M. I. Katsnelson, I. V. Grigorieva, S. V. Dubonos, and A. A. Firsov, Nature 438, 197 (2005).
3 H. B. Heersche, P. Jarillo-Herrero, J. B. Oostinga, L. M. K. Vandersypen, and A. F. Morpurgo, Nature 446, 56 (2007).
4 H. B. Heersche, P. Jarillo-Herrero, J. B. Oostinga, L. M. K. Vandersypen, and A. F. Morpurgo, Solid State Communications 143, 72 (2007).
5 H. B. Heersche, P. Jarillo-Herrero, J. B. Oostinga, L. M. K. Vandersypen, and A. F. Morpurgo, Eur. Phys. J. Special Topics 148, 27 (2007).
6 X. Du, I. Skachko, and E. Y. Andrei, Phys. Rev. B 77, 184507 (2008).
7 F. Miao, W. Bao, H. Zhang, and C. N. Lau, Solid State Communications 149, 1046 (2009).
8 C. Girit, V. Bouchiat, O. Naaman, Y. Zhang, M. F. Crommie, A. Zettl, and I. Siddiqi, Nano Letters 9, 198 (2009).
9 C. Girit, V. Bouchiat, O. Naaman, Y. Zhang, M. F. Crommie, A. Zettl, and I. Siddiqi, physica status solidi (b) 246, 2568 (2009).
10 C. Chialvo, I. C. Moraru, D. J. V. Harlingen, and N. Mason (2010), arXiv/1005.2630.
11 J. M. Martinis, M. H. Devoret, and J. Clarke, Phys. Rev. Lett. 55, 1543 (1985).
12 P. Blake, E. W. Hill, A. H. C. Neto, K. S. Novoselov, D. Jiang, R. Yang, T. J. Booth, and A. K. Geim, Applied Physics Letters 91, 063124 (2007).
13 A. C. Ferrari, J. C. Meyer, V. Scardaci, C. Casiraghi, M. Lazzeri, F. Mauri, S. Piscanec, D. Jiang, K. S. Novoselov, S. Roth, et al., Phys. Rev. Lett. 97, 187401 (2006).
14 J.-H. Chen, C. Jang, M. Ishigami, S. Xiao, W. Cullen, E. Williams, and M. Fuhrer, Solid State Communications 149, 1080 (2009).
15 K. B. Cooper, M. Steffen, R. McDermott, R. W. Simmonds, S. Oh, D. A. Hite, D. P. Pappas, and J. M. Martinis, Phys. Rev. Lett. 93, 180401 (2004).
16 J. M. Martinis, K. B. Cooper, R. McDermott, M. Steffen, M. Ansmann, K. D. Osborn, K. Cicak, S. Oh, D. P. Pappas, R. W. Simmonds, et al., Phys. Rev. Lett. 95, 210503 (2005).
17 T. A. Palomaki, S. K. Dutta, R. M. Lewis, A. J. Przybysz, H. Paik, B. K. Cooper, H. Kwon, J. R. Anderson, C. J. Lobb, F. C. Wellstood, et al., Phys. Rev. B 81, 144503 (2010).
18 K. Bolotin, K. Sikes, Z. Jiang, M. Klima, G. Fudenberg, J. Hone, P. Kim, and H. Stormer, Solid State Communications 146, 351 (2008).
19 X. Du, I. Skachko, F. Duerr, A. Luican, and E. Y. Andrei, Nature 462, 192 (2009).
20 K. I. Bolotin, F. Ghahari, M. D. Shulman, H. L. Stormer, and P. Kim, Nature 462, 196 (2009).
21 X. Du, I. Skachko, A. Barker, and E. Y. Andrei, Nat Nano 3, 491 (2008).
22 R. C. Ramos, M. A. Gabrud, A. J. Berkley, J. R. Anderson, C. J. Lobb, and F. C. Wellstood, IEEE Trans. Appl. Supercond. 11, 998 (2001).
23 H. Xu, Ph.D. thesis, University of Maryland College Park (2004).
24 M. Titov and C. W. J. Beenakker, Phys. Rev. B 74, 041401 (2006).
25 S. Washburn, R. A. Webb, R. F. Voss, and S. M. Faris, Phys. Rev. Lett. 54, 2712 (1985).
26 M. Tinkham, Introduction to Superconductivity (McGraw-Hill, New York, 1975).
27 A. Wallrath, T. Duty, A. Lukasenko, and A. V. Ustinov, Phys. Rev. Lett. 90, 037003 (2003).