Universality of the Unruh effect

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ABSTRACT

In this paper we prove the universal nature of the Unruh effect in a general class of weakly non-local field theories. At the same time we solve the tension between two conflicting claims published in literature. Our universality statement is based on two independent computations based on the canonical formulation as well as path integral formulation of the quantum theory.

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1 Introduction

As mentioned in the literature [1], the non-locality has its importance at various levels. On the one hand, the non-locality may originate from a fundamental theory like string theory [3] or from the quantum spacetime [2]. On the other hand, the non-locality may come out as the low-energy phenomena after integrating out the higher momentum degrees of freedom, just as realized in the quantum effective action. Finally, the nature could be intrinsically non-local and the non-local field theory will turn out to be the most appropriate framework to describe it [4, 5, 6, 7]. In all these cases, non-local theories urge us to revisit the well-established results found in local field theory. In fact, many papers have followed this route in non-commutative field theories (see [8] for a review), which could be regarded as a kind of non-local theories.

One of the recent renewed interests in non-local theories is due to a class of super-renormalizable or finite quantum gravity models (see for examples, [7, 9]). These non-local theories have improved behaviour in the ultraviolet regime and it can be proved on the base of rigorous power counting arguments that there is class of non-local gravitational theories super-renormalizable at quantum level. The good behaviour at quantum level and the perturbative unitarity provide us a good motivation to investigate non-local theories. Contrary to the non-commutative field theory, these theories preserve the Lorentz symmetry and do not violate the macroscopic causality. Here, the non-locality may be represented by a length scale $\ell$ and one may ask what is the effect of the non-local scale $\ell$ in various settings.

To see whether there are non-local effects in the quantum field theory setup, we here focus on a toy model presenting non-local modifications only on the field theory propagator, while the interactions are the usual local ones. Therefore, one can ask whether the non-locality and the related scale $\ell$ do affect the results of the free field theory quantization. One of those effects is the Unruh one [10] seen by a uniformly accelerating observer. Recently, in the non-local theory, whose non-locality is given by a particular entire function, the Unruh effect appeared with opposite outcomes: a significant modification in [11] and no modification [12]. In order to resolve the tension between the two results, we here revisit the Unruh effect in a non-local theory of “Gaussian exponential type”. However, the result applies to any weakly non-local field theory.

This paper is organized as follows. In section two, we briefly review the Unruh effect by employing the canonical quantization for a specific non-local model. In section three, we revisit the Unruh-DeWitt detector method for the Unruh effect and we show the irrelevance of the non-locality scale $\ell$ to the Unruh effect. In section four, we confirm our results by implementing a field redefinition in the Lagrangian of the theory. We summarize our results in the section five.
2 Review: The canonical method

The Unruh effect is well-established as the quantum phenomenon seen by a uniformly accelerating (Rindler) observer. The Rindler observer confined in the Rindler wedge of the whole Minkowski spacetime cannot access the other parts of Minkowski spacetime and so, he/she has to consider the Minkowski vacuum as an entangled mixed state. After computation, it turns out that the Minkowski vacuum could be regarded as a thermal state with its temperature $T_{DU}$ known as the Davies-Unruh (DU) temperature. The DU temperature is given by the proper acceleration $a$ of the observer or the detector as

$$T_{DU} = \frac{a\hbar}{2\pi k_B}.$$  

Here, we will set $c = \hbar = k_B = 1$ as usual and take the signature as $\eta_{\mu\nu} = (-, +, +, +)$. One way to derive $T_{DU}$ is to use the canonical quantization and adopt the Bogoliubov transformation between the Minkowski Fock space and the Rindler Fock space [13]. In the following, we adopt the argument given in [12] in order to fix our conventions and viewpoints. We would like to mention some potential loopholes appeared in this approach.

Let us consider the simplest non-local field theory given by the following kinetic Lagrangian operator with gaussian form factor,

$$\mathcal{L} = -\frac{1}{2} \phi(x) e^{-\frac{\ell^2}{2} (\square_x)} \phi(x).$$  \hspace{1cm} (1)

Then, the field equation could be written as

$$e^{-\frac{\ell^2}{2} (\square_x)} \phi(x) = 0.$$  \hspace{1cm} (2)

Note that the conventional local case can be recovered by taking the non-local scale $\ell$ to be zero. Hereafter, we denote the local scalar theory by introducing the super/sub-script (0) as

$$\mathcal{L}_{(0)} = -\frac{1}{2} \phi(x) (\square_x) \phi(x).$$  \hspace{1cm} (3)

In order to investigate the Unruh effect by the Bogoliubov transformation, it is necessary to adopt the canonical quantization in the Minkowski and the Rindler spaces, respectively. To proceed in this way, we quantize the the Lagrangian (1) canonically. As was emphasized in [12], the non-local entire function does not change the pole structure of the propagator at perturbative level [14, 15] and thus, the homogenous solution $\phi(x)$ to the equation of motion (2) is unique and given by

$$\phi(x) = \int d^3p \left[ a_p u_p(x) + a_p^\dagger u_p^*(x) \right].$$  \hspace{1cm} (4)

At the classical level, the coefficients of $a_p$ and $a_p^\dagger$ correspond to the initial data of $\phi$ to determine the evolution of the system.
Now, one may adopt the canonical quantization approach taken in [14, 16, 17]. This approach provides the simple quantization rule

\[ [a_p, a_p^\dagger] = \delta^{(3)}(p - p'), \quad [a_p, a_{p'}] = [a_p^\dagger, a_{p'}^\dagger] = 0. \] (5)

We admit that this quantization might not have the clear meaning as in the local theory, since the equal-time commutator of the field and its momentum, which was used to derive Eq. (5), is not well-defined in the non-local theory. Moreover, the non-locality does not allow us to convert easily the relation between momentum and velocity, which gives rise to difficulties when the Lagrangian is Legendre-transformed to the canonical Hamiltonian. However, as was done in [16], one may take the canonical quantization without introducing the equal-time commutators and the canonical Hamiltonian\(^1\). Moreover, one may consider (5) as the fundamental quantization rules by analog with those in the local theory [18, 19]. These quantization rules are also acceptable from the viewpoint that the position-space non-locality in (1) is “local” in the momentum space. This means that the non-locality in the position space is just the UV modification of the propagator in the momentum space.

As explained in [20, 21], the Unruh effect may be understood through the Bogoliubov transformation of annihilation/creation operators in between Minkowski and Rindler spaces. Considering arguments stated in [12], the unchange of the Unruh effect or the DU temperature originates from the commutation relations in (5). For instance, the Klein-Gordon inner product for \(\phi(x)\)'s can be defined as usual without any modifications and so all steps for the Bogoliubov transformation are identical with those in the local theory.

3 The Unruh-DeWitt detector method

In this section, we revisit the Unruh-DeWitt detector approach for the model of (1). The Unruh-DeWitt detector [22] is a uniformly accelerating hypothetical detector with two energy levels and its interaction with the surrounding field is given by

\[ \mathcal{L}_{\text{int}} = g M(x(\tau)) \phi(x(\tau)), \] (6)

where \(\tau\) denotes the proper time of the uniformly accelerating detector.

In the detector approach to the Unruh effect, a physical quantity to compute is the response function \(F(\omega)\) defined by

\[ F(\omega) = \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\tau' e^{-i(\tau-\tau')} \langle 0_M | \phi(x(\tau)) \phi(x(\tau')) | 0_M \rangle, \] (7)

\(^1\)In Ref. [16], the Hamiltonian was read off from the time component of energy-momentum tensor, not through the Legendre transformation.
where $|0_M\rangle$ denotes the Minkowski vacuum. Here, we assumed that the Minkowski vacuum can be defined through the canonical quantization.

At this stage, we emphasize that the two-point function in (7) is not the Feynman propagator of $\Delta_F(x,x')$, but the positive-frequency Wightman function $G^+(x,x')$. Note that these two functions satisfy the inhomogeneous and homogeneous equations, respectively, as

$$e^{-\frac{\ell^2}{2}\Delta x}(-\square_x)\Delta_F(x,x') = \delta^4(x-x'), \quad e^{-\frac{\ell^2}{2}\Delta x}(-\square_x)G^+(x,x') = 0,$$

while both preserve translational symmetry as

$$\Delta_F(x,x') = \Delta_F(x-x') \quad \text{and} \quad G^+(x,x') = G^+(x-x').$$

A big difference between two equations in (8) is the presence or not of the source $\delta^4(x-x')$. Another distinguishing property between $\Delta_F$ and $G^+$ is

$$\Delta_F(-x) = \Delta_F(x), \quad G^+(-x) = G^-(x) \neq G^+(x),$$

where $G^-(x)$ is the negative-frequency Wightman function. It is worth to note that a relevant quantity is not the response function (7) but the rate of response function given by

$$\dot{F}(\omega) = \int_{-\infty}^{\infty} d\Delta \tau \ e^{-i\Delta \tau} G^+(x(\tau) - x(\tau')).$$

Using the Unruh-DeWitt detector method, it was shown that the Unruh effect is significantly changed by the non-locality [11]. This change is interpreted as a kind of the UV-IR mixing phenomena and the entangling aspect is destroyed by the non-locality. Here, we point out that this result is inconsistent with the section two in [12]. The authors in [12] argued that the point-interacting Unruh-DeWitt detector is not adequate for describing the non-local theory and thus, the results obtained from the detector are not physically meaningful in non-local theories. However, it is worth to remember that the Unruh-DeWitt detector method is taken in the inertial frame only, without resorting to the Rindler-Fulling or non-inertial frame quantization scheme. Therefore, the claims for the inadequacy of the Unruh-DeWitt detector seem misleading because it was based on the standard inertial frame quantization. Moreover, all the information about non-locality or equivalently non point-interacting Unruh-DeWitt detector is already defined in the theory that consists of the Lagrangians $\mathcal{L}$ and $\mathcal{L}_{\text{int}}$.

In the following, we revisit the Unruh-DeWitt detector method and we prove that the results are completely consistent with those given in section two of [12]. Now, let us first follow the steps to obtain the position-space propagator given in [11]. It is straightforward to obtain the Feynman propagator in the momentum space directly from the non-local action

$$\tilde{\Delta}_F(p) = e^{-\frac{\ell^2}{2}p^2}{p^2 - i\epsilon}.$$ 

(11)
Since there are some convergence issues in the direct computation in the Lorentzian signature, we take the Euclidean signature version to compute the propagator. We may avoid this signature issue using the form factor \( e^{-\ell^2 \Box^2} \), but we focus on our example for simplicity in the presentation, since we can eventually make an analytic continuation of the amplitudes. Notice that we rotated \( p_0 = ip_E \) in Eq. (11), while below in Eqs. (14) and (15), we rotate \( x_0 = -it_E \) only.

Introducing the Schwinger parametrization of the propagator

\[
e^{-\ell^2 \Box^2} = \int_0^\infty ds \, e^{-sp^2},
\]

one obtains the non-local propagator in the position-space [11]

\[
\Delta_F(x) = \frac{1}{4\pi^2 x^2} \left(1 - e^{-\frac{x^2}{2\ell^2}}\right). \tag{12}
\]

One may be tempting to read off the Wightman function \( G^+(x) \) from the Feynman propagator \( \Delta_F(x) \) because these are related to each other\(^2\). For example, in the local theory (3), the relation is given by

\[
\Delta_{F}^{(0)}(x) = \Theta(t_E)G_{(0)}^+(x) + \Theta(-t_E)G_{(0)}^-(x), \tag{13}
\]

where \( \Delta_{F}^{(0)}(x) \) and \( G_{(0)}^+(x) \) denote the Feynman propagator and the positive (negative)-frequency Wightman function in the local case, respectively. Explicitly, the (Euclidean) Wightman functions can be found through the Wick rotation to be

\[
G_{(0)}^+(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2\omega_p} e^{-\omega_p t_E + ip \cdot x}, \tag{14}
\]

\[
G_{(0)}^-(x) = \int \frac{d^3p}{(2\pi)^3} \frac{1}{2\omega_p} e^{\omega_p t_E - ip \cdot x} \tag{15}
\]

with \( \omega_p \equiv \sqrt{p^2} \). Adopting the same derivation of the Wightman function from the propagator in the non-local case, one arrives at the misleading conclusion that there are huge changes in the Unruh effect [11]. Technically, this huge difference came from the disappearance of the pole in the position-space Feynman propagator (12) by sending \( x \to 0 \). It is meaningful to note that the disappearance of the pole in the position space is different from the one in the local case (\( \ell = 0 \)), but it induces a wrong derivation of the Wightman function.

Now, we are in a position to present the robustness of the Unruh-DeWitt detector method. Contrary to the local theory, the non-local Feynman propagator could not be related to the time

\(^2\)See for example [23]. Our convention for the propagator: \(-i\Delta_{F}^{(0)}(x - x') = \langle 0|T(\phi(x)\phi(x'))|0 \rangle\).
ordering of two fields. Indeed, the direct computation leads to

\[ \Delta_F(x) = \int \frac{d^4p}{(2\pi)^4} \Delta_F(p)e^{ip\cdot x} = \int \frac{d^4p}{(2\pi)^4} e^{-\frac{p^2}{2\pi}}e^{ip\cdot x} \]

\[ = e^{\frac{\ell^2}{2}\nabla^2} \left[ \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2}e^{ip\cdot x} \right] = e^{\frac{\ell^2}{2}\nabla^2} \Delta_F^{(0)}(x), \tag{16} \]

where we drop the \( e \)-prescription because we work in the Euclidean space temporarily. The non-local operator \( e^{\frac{\ell^2}{2}\nabla^2} \) could be represented in terms of the kernel \( K \) as

\[ e^{\frac{\ell^2}{2}\nabla^2} \Delta_F^{(0)}(x) = \int d^4y K(x-y)\Delta_F^{(0)}(y), \quad K(x-y) = \frac{1}{(2\pi\ell^2)^2} e^{-\frac{(x-y)^2}{2\ell^2}}. \tag{17} \]

Using the (Euclidean) D’Alambertian \( \Box = \partial_{t_E}^2 + \nabla^2 \) together with (13)-(15), one finds that (16) takes the form

\[ \Delta_F(x) = \int \frac{d^3p}{(2\pi)^3} \left[ \frac{1}{\sqrt{2\pi}t_E} \text{Erfc} \left( \frac{t_E}{\sqrt{2\ell}} \right) G_p^+(x) + \frac{1}{\sqrt{2\pi}t_E} \text{Erfc} \left( \frac{t_E}{\sqrt{2\ell}} \right) G_p^-(x) \right]. \tag{18} \]

Here the complementary error function \( \text{Erfc}(z) \) and \( G_p^\pm(x) \)'s are defined by

\[ \text{Erfc}(z) \equiv 1 - \text{Erf}(z) = 1 - \frac{2}{\sqrt{\pi}} \int_0^z dt e^{-t^2}, \quad G_p^\pm(x) \equiv \frac{1}{2\omega_p} e^{\mp\omega_p t_E + p\cdot x}. \tag{19} \]

We would like to mention that the final expression (18) was derived in Euclidean signature and there are other contributions in the first and fourth quarter of the complex energy plane when rotating back to Minkowski space. However, the naive Wick rotation may be incorrect in this case. In deriving the propagator \( \Delta_F(x) \) (18), we have used the two relations:

\[ e^{\frac{\ell^2}{2}\nabla^2} (\theta(t_E)e^{\mp\omega_p t_E}) = \int_{-\infty}^{\infty} dt_E' \frac{1}{\sqrt{2\pi}t_E} e^{\frac{(t_E-t_E')^2}{2\ell^2}} \theta(t_E'), \]

\[ = e^{\frac{\ell^2}{2}\nabla^2} e^{\mp\omega_p t_E} \frac{1}{2} \text{Erfc} \left( \frac{t_E}{\sqrt{2\ell}} \right), \]

\[ e^{\frac{\ell^2}{2}\nabla^2} (e^{ip\cdot x}) = e^{-\frac{\ell^2}{2}\nabla^2} e^{ip\cdot x}. \tag{20} \]

It is important to note that the propagator (18) could also be derived by adopting the Schwinger parametrization.

As a check, we remind that \( \text{Erfc}(z) \) reduces to the \( \theta \)-function in the limit \( \ell \to 0 \) as

\[ \lim_{\ell \to 0} \frac{1}{2} \text{Erfc} \left( \frac{t_E}{\sqrt{2\ell}} \right) = \theta(\pm t_E), \]

and the positive-frequency Wightman function takes a simple form

\[ G^+(x) = \int \frac{d^3p}{(2\pi)^3} G_p^+(x), \tag{21} \]
which is the same form of the standard Wightman function (14) for the local theory. Note that our Wightman function (21) satisfies the required properties for the Wightman function given by (8) and (9). Moreover, it is consistent with an expression obtained through the canonical quantization in the section two. Therefore, we insist that the Wightman function of the non-local theory (1) should be read as

\[ G^\pm(x) = G^\pm_{(0)}(x). \]  

(22)

Furthermore, we argue that the Unruh effect remains unchanged because the non-local Wightman function takes the same form as in the \( \ell = 0 \) local case.

Finally, we stress that the consistency of the Feynman propagator (18) can be checked again from the micro-causality violation of the propagator [24]. The micro-causality violation states that the relation between the Feynman propagator and the Wightman function for the non-local case \( \ell \neq 0 \) cannot be the same as the one for the local case \( \ell = 0 \). Although a relation (13) for the local theory is required by the micro-causality, it does not constrain the case of the non-local theory. For this purpose, employing the \( \theta \)-function representation

\[ \theta(\pm t_E) = i \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} e^{\pm i\omega t_E}, \]

one finds

\[ e^{\ell t^2 E^2/2} \theta(t_E)e^{\mp i\omega t_E} = e^{\ell^2 t^2 \omega^2/2} e^{\mp i\omega t_E} \left[ i \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{1}{\omega + i\epsilon} e^{\pm i\omega t_E} e^{-\ell^2 t^2 \omega^2 + i\ell^2 \omega \omega_2} \right]. \]

This gives us another representation of the complementary error function as

\[ \text{Erfc} \left( \frac{\ell \omega_2}{\sqrt{2}} \right) = 2i \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} \frac{1}{\omega + i\epsilon} e^{\pm i\omega t_E} e^{-\ell^2 t^2 \omega^2 + i\ell^2 \omega_2 \omega}. \]  

(23)

Making use of this representation together with the Taylor expansion

\[ F(\omega, \omega_2) \equiv e^{-\ell^2 \omega^2 + i\ell^2 \omega_2 \omega} = 1 - \ell^2 \omega^2 + i\ell^2 \omega_2 \omega + \cdots, \]

one can rewrite the non-local Feynman propagator (18) as

\[ \Delta_F(x) = \theta(t_E)G^\pm_{(0)}(x) + \theta(-t_E)G^\pm_{(0)}(-x) + \Delta_{\text{nc}}(x). \]

Here \( G^\pm_{(0)}(x) \)'s are the Wightman functions (14) and (15) and \( \Delta_{\text{nc}}(x) \) denotes a micro-causality violating term. Actually, \( \Delta_{\text{nc}}(x) \) is composed of an infinite number of contact terms derived in [24]

\[ \Delta_{\text{nc}}(x) = - \sum_{m \geq 1} \frac{i^{m-1}}{m!} \frac{\partial^m \delta(t_E)}{\partial t_E^m} \left[ D^+(x) - D^-(x) \right], \]  

(24)

where \( D^\pm(x) \) are defined by

\[ D^\pm(x) = \int \frac{d^3 p}{(2\pi)^3} F^{(m)}(\omega_2, \omega_2) G^\pm_{p}(x), \quad F^{(m)}(\omega_2, \omega_2) \equiv \frac{\partial^m}{\partial \omega_2^m} F(\omega, \omega_2) \bigg|_{\omega = \omega_2}. \]  

(25)

The presence of \( \Delta_{\text{nc}}(x) \) implies the micro-causality violation in the non-local theory.
4 Field redefinition method and Universality

In this section, we would like to confirm our claims that the Unruh effect remains unchanged in the model (1) by using field redefinition method. For this purpose, we consider a massive scalar field coupled to a detector

\[ S[\phi] = \int d^4x \left[ -\frac{1}{2} \frac{\partial^2}{\partial x^2}(-\Box + m^2)\phi(-\Box + m^2)\phi \right] + g \int d\tau M(x(\tau))\phi(x(\tau)). \]  

(26)

Considering weak non-locality only, we can move the form factor in the interaction term by introducing a field redefinition of

\[ \tilde{\phi} = e^{\frac{\partial^2}{\partial x^2}(-\Box + m^2)}\phi. \]

At quantum level, the Jacobian of the transformation is just a constant. The action takes the form

\[ S[\tilde{\phi}] = \int d^4x \left[ -\frac{1}{2} \frac{\partial^2}{\partial x^2}(-\Box + m^2)\tilde{\phi}\right] + g \int d\tau M(x(\tau))e^{\frac{\partial^2}{\partial x^2}(-\Box + m^2)}\tilde{\phi}(x(\tau)). \]  

(27)

Noting that the propagator is the conventional one of a local theory and it satisfies the usual Källen-Lehman representation, its form is given by

\[ G^+(x - x') = \int \frac{d^4p}{(2\pi)^4} e^{ip(x-x')}\theta(p_0)\delta(p^2 + m^2), \]  

(28)

or making the dependence of \( \tau \) explicitly as

\[ G^+(x(\tau_1) - x(\tau_2)) = \int \frac{d^4p}{(2\pi)^4} e^{ip(x(\tau_1) - x(\tau_2))}\theta(p_0)\delta^4(p^2 + m^2). \]  

(29)

The detector response function reads as

\[ \hat{F}(\omega) = \int_{-\infty}^{\infty} d\tau \int_{-\infty}^{\infty} d\tau' e^{-i\omega(\tau - \tau')} e^{-\frac{\partial^2}{\partial x^2}(-\Box_x(\tau) + m^2)} \frac{e^{\frac{\partial^2}{\partial x^2}(-\Box_x(\tau') + m^2)}}{e^{\frac{\partial^2}{\partial x^2}(-\Box_x(\tau) + m^2)}} G^+(x(\tau) - x(\tau')). \]  

(30)

The response rate function is also given by the general expression

\[
\dot{\hat{F}}(\omega) = \int_{-\infty}^{\infty} d\Delta \tau' e^{-i\omega \Delta \tau} \frac{e^{\frac{\partial^2}{\partial x^2}(-\Box_x(\tau) + m^2)}}{e^{\frac{\partial^2}{\partial x^2}(-\Box_x(\tau) + m^2)}} \frac{e^{\frac{\partial^2}{\partial x^2}(-\Box_x(\tau') + m^2)}}{e^{\frac{\partial^2}{\partial x^2}(-\Box_x(\tau') + m^2)}} G^+(x(\tau) - x(\tau'))
\]

\[ = \int_{-\infty}^{\infty} d\Delta \tau' e^{-i\omega \Delta \tau} \int \frac{d^4p}{(2\pi)^4} e^{-\frac{\partial^2}{\partial x^2}(p^2 + m^2)} e^{-\frac{\partial^2}{\partial x^2}(p^2 + m^2)} e^{ip(x(\tau) - x(\tau'))}\theta(p_0)\delta^4(p^2 + m^2)\]

\[ = \int_{-\infty}^{\infty} d\Delta \tau' e^{-i\omega \Delta \tau} \int \frac{d^4p}{(2\pi)^4} e^{ip(x(\tau) - x(\tau'))}\theta(p_0)\delta^4(p^2 + m^2). \]  

(31)

Importantly, we observe that \( \dot{\hat{F}}(\omega) \) is independent of the form factor \( e^{\frac{\partial^2}{\partial x^2}(-\Box + m^2)} \) and gives us the same result of the local theory. Therefore, there are no modifications due to the non-locality. It indicates that the Unruh effect is not modified when implementing a field redefinition in (26).
We note that the analysis in this section is independent of the specific form factor as long as it is a weakly non-local entire function. Indeed, in all the formulas in this section we can replace the Gaussian form factor with the exponential of a general entire function, namely
\[ e^{-\frac{\ell^2}{2\Box}} \rightarrow e^{-\frac{1}{2}H(\Box\ell^2)}, \]
and nothing change if \( H(0) = 0 \). \( H(\Box) \) could be a polynomial of any entire function, while we fixed \( m^2 = 0 \) for the sake of simplicity. Therefore, in this section we proved the universality of the Unruh effect in weakly non-local field theories.

It deserve to be notice that the Jacobian of the field redefinition (4) is just a constant and, therefore, does not affect the scattering amplitudes at any order in the loop perturbative expansion. Moreover, (4) does not change the spectrum of the theory introducing or including extra poles in the propagator.

The non-local field theory is well defined in the path integral formalism and all the interactions are analytically well-defined operators obtained when expanding the action in perturbations around the selected vacuum. Weakly non-local or quasi-polynomial theories share most the properties of local theories with two or higher derivative. It seems that everything is very standard. Eventually, it is the canonical formulation that should be further investigated, but actually we do not need it.

5 Conclusion

In this work, we have revisited the Unruh effect in the simplest non-local theory (1) that captures all the features of a large class of weakly non-local field theories. By considering the non-local model specified by the exponential of the d’Alembertian operator, we have confirmed that various methods adopted in exploring the Unruh effect give us the same results of the local scalar theory, unlike to the drastic change found in [11]. It turns out that the Unruh effect is not modified in the non-local model (1) respect to the local field theory (3). Consequently, the Davies-Unruh temperature is unchanged.

More concretely, we have revisited the Unruh-DeWitt detector method and we have found that it leads to the same results as those from the canonical approach. Contrary to the previous claims [12], we have shown that the detector model is robust and can be used to describe the Unruh effect correctly. Even though we have focused on a non-local scalar theory of Gaussian exponential type, our main conclusion would hold for any weakly non-local or quasi polynomial theory [7].

Furthermore, we have checked our claim by using the field redefinition approach. Indeed, all the scattering amplitudes are invariant under weakly non-local field redefinition that turns the propagator into the local one, but leads to changing the interaction vertexes. We mention
that the theories before and after field redefinition are identical at perturbative level and at any order in the loop expansion.

Finally, we would like to point out the reason why there is a significant modification in the Newtonian potential \( V(r) = -GM\text{Erf}(r/2l)/r \) [25, 26], whereas there is no modifications in the Unruh effect. This can be simply understood by looking at the Feynman propagator and the Wightman function that involve the potential and the Unruh effect respectively. Indeed, the Feynman propagator \( \Delta_F(x, x') \) determining the potential satisfies a Poisson-like equation with a Delta source, which becomes a Gaussian-like source in the non-local case. On the other hand, the Wightman function determining the Unruh effect satisfies the Laplace equation without source. The last statement supports our claim strongly that the Unruh effect is not modified in the non-local model.

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