Golubev and Zaikin reply: In a Comment\cite{Aleiner0} Aleiner, Altshuler and Gershenson (AAG) stated that we “found that “zero-point fluctuations of electrons” contribute to the dephasing rate $1/\tau_z$”. We made no such claims. Obviously, the eigenmodes of any quantum system preserve their coherence. On the other hand, single-particle properties of an interacting system – which are not directly expressed in terms of eigenmodes – can display a finite decoherence length $L_\varphi$ in the low-temperature limit. An example is provided by a particle coupled to a harmonic oscillator bath, which we discuss explicitly below. This is also the case for the weak localization correction to the conductivity in the presence of interaction, where we found $\mathcal{L} = L_\varphi$ to remain finite at $T \to 0$. AAG furthermore concluded that our procedure is “simply wrong” because “some contributions were lost” during “uncontrollable” semiclassical averaging. This is not the case, as demonstrated by the fact that our approach fully reproduces the results of AAG if analyzed on a perturbative level. The advantage of our formulation is that we can proceed beyond.

The conductivity and the electron density matrix $\rho$ can be expressed in terms of a path integral

$$ \mathcal{D}\mathbf{r} \int \mathcal{D}\mathbf{p} \exp(iS_0 - iS_0' - iS_R - iS_I), $$

where $S_0$ and $S_0'$ represent the electron action on the two parts of the Keldysh contour, while $iS_R + S_I$ accounts for the interaction. We reproduce AAG’s results on a perturbative level if we expand the path integral (1) up to first order in $iS_R + S_I$. To show the equivalence we perform (see Appendix A for details) some exact transformations starting from the formula for the conductivity of AAG, making use of the expression for the Green-Keldysh function

$$ G^{A \to A}_{R \to R'}(t) = \int dt' [G^{A \to A}_{R \to R'}(t') - G^{A \to A}_{R \to R'}(t)][1 - 2\rho_{R',R}]. $$

The result (of either approach) consists of the two groups of terms:

- $G^A G^A G^A$ and $G^A G^A G^A(1 - 2\rho) G^A$. Terms of the type $G^R G^A G^R G^A$ (claimed to be lost\cite{Aleiner0} in our calculation) vanish due to the causality principle. Thus no diagrams or paths are missing in our analysis.

The crucial difference between AAG’s and our procedures is that we do not expand in $iS_R + S_I$, but rather evaluate the full path integral (1). The expressions $S_R$ and $S_I$ are non-local in time and are not small at low $T$ even if the interaction is weak. Hence an expansion of the path integral (1), which is equivalent to a Golden-rule-type perturbation theory in lowest order in the scattering processes, becomes insufficient at low $T$. The only approximation we employ is an RPA expansion of the effective action. This yields tractable expressions for $S_R$ and $S_I$, which include (within RPA) processes of all orders. Averaging over disorder plays no important role at this point: our results and those of AAG are different already before averaging.

The difference between AAG’s and our approaches can be illustrated by the example of a quantum particle (with mass $m$ and coordinate $q$) interacting with a Caldeira-Leggett (CL) bath of oscillators (with coupling strength $\gamma$ and high-frequency cutoff $\omega_c$). Its reduced density matrix $\rho(q_1, q_2)$ is also determined by a path integral of the type (1). An expansion in the interaction $iS_R + S_I$ in the long-time limit yields Fermi’s Golden rule with conservation of the bare energy. Within this approximation, a particle with initial energy $E \to 0$ at $T = 0$ cannot exchange energy and, hence, phase coherence is preserved. This is the qualitative argument of Ref.\cite{Aleiner0}, which refers to a situation where the total energy of the system is fixed to be the sum of energies of the noninteracting particles. On the other hand, if one does not expand but evaluates the path integral (1) exactly, one obtains\cite{Aleiner0} in the long-time limit, independent of the initial conditions, $\rho(q_1, q_2) \propto \exp(-m\langle E\rangle q_1 q_2^2)$. Here, the expectation value of the kinetic energy of the interacting particle at $T = 0$ is $\langle E\rangle = \gamma \ln(\omega_c/\gamma) > 0$. These results can also be derived by an exact diagonalization of the initial model; however, they cannot be obtained from the Golden rule approach. The off-diagonal elements of the density matrix decay on a length scale $\sim 1/\sqrt{m\langle E\rangle}$. Due to interaction this length scale is finite even at $T = 0$: it would diverge if one would assume $\langle E\rangle \sim T$. The latter applies to the (obviously coherent) eigenmodes, but physical quantities which are expressed in the basis $q$ will be sensitive to the decay of the density matrix $\rho(q_1, q_2)$ as a function of $q_1 - q_2$. We encountered a similar situation while describing the interacting electron system. A more detailed discussion is given in Appendix B.

Concerning experiments we can say that our results are in a quantitative agreement with various experiments, especially in the quasi-1-dimensional case. The disagreement reported by AAG is observed in strongly disordered systems with short mean free paths down to several or even one (!) Ångström. Such systems are well beyond the applicability of our quasiclassical theory. An extended discussion of the experiments is given in Appendix C.

We are grateful to our numerous colleagues and friends for support.

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APPENDIX A:

Here we will compare and analyze the expressions for the weak localization correction to the conductivity in the presence of interaction obtained in our paper and in Ref. 5. The authors raised an extensive critique of our results which includes qualitative arguments, "highlighting errors" in our calculation and comparison with experiment. The existence of an explicit error in our calculation would be the most serious argument of AAG against our theory. Therefore our first task will be to demonstrate that the AAG statement about the mistakes in our calculation is not correct. Below we will demonstrate that on a perturbative level our result is equivalent to that of Ref. 5.

Before coming to a more technical part of our analysis it would be useful to briefly remind the reader about the main steps of our calculation. An attempt to examine our derivation “step by step” has been already made in Sec. 6 of Ref. 5. Unfortunately the information contained there is incomplete. In fact from the reader might get an impression that what we do essentially boils down to two trivial technical steps (performing the Hubbard-Stratonovich transformation and representing the Green functions in terms of the path integral) and one (“incorrect”) disorder average. The actual procedure is richer both formally and physically.

Our analysis consists of the two main steps.

1. The first step is to reformulate the initial many-body problem with interaction in terms of a single quantum particle interacting with an effective quantum environment. Here we indeed use the Hubbard-Stratonovich transformation, but it is only a simple technical tool. A somewhat less trivial task is to derive a formally exact equation of motion for the single electron density matrix $\rho$ in the presence of interaction (eqs. (24-25) of Ref. 5):

$$i\frac{\partial \rho_V}{\partial t} = [H_0 - eV_+, \rho_V] - (1 - \rho_V) \frac{eV_+}{2} \rho_V - \rho_V \frac{eV_-}{2} (1 - \rho_V),$$

(A1)

The matrix $\rho$ is obtained after averaging of $\rho_V$ over the fluctuating fields $V_\pm$ carried out with the (again formally exact) effective action $S[V_+, V_-]$ derived by integrating out electronic degrees of freedom. No approximations have been made so far.

Although the expression for $S[V_+, V_-]$ is too complicated to deal with, some important observations can be made already at this stage. Namely, the fluctuating field $V_+$ enters the equations just like an external field whereas the field $V_-$ enters in a qualitatively different manner. Fluctuations of the field $V_-$ are essentially responsible for dephasing.

In order to proceed further we make the first approximation: we evaluate the effective action $S[V_+, V_-]$ within RPA. The action $S$ is now quadratic in $V_\pm$ and contains the dielectric susceptibility $\epsilon(\omega, k)$ of the effective environment. After that we easily integrate out the fields $V_\pm$ and arrive at the influence functional $F$ for interacting electrons in a disordered metal. This completes the first part of our analysis. We can only add that this approach can be also used in the situations when approximations other than RPA are more appropriate. In those cases the action $S[V_+, V_-]$ and the influence functional should be modified accordingly.

2. As a result of our derivation we arrived at the problem of a quantum particle in a random potential in the presence of the effective environment described by the influence functional $F$. The Fermi statistics and the Pauli principle are explicitly accounted for in the expression for $F$. The kinetic energy of a particle $E$ is counted from the Fermi energy $\mu$ and the states with $E < 0$ are forbidden. The second step of our analysis is to investigate the quantum dynamics of such a particle. According to the general principles this stage the actual physical nature of the environment already plays no important role, any environment yields the same effect as long as it is described by the same influence functional. One can also develop better qualitative understanding of the phenomenon with the aid of simpler models for the environment. In this sense the experience gained in the Caldeira-Leggett-type of models (see e.g.) is of particular interest and will be used in the Appendix B. Now we only mention one important feature of all these problems: the results for physically measurable equilibrium quantities do depend on the high frequency cutoff $\omega_c$ of the effective environment. This dependence has nothing to do with the excitation of the environment oscillators, it exists even if the expectation values are calculated in the true ground state of the whole system. We will return to this point further below.

Let us now come to a more technical part of this Appendix. We are going to test our expression for the influence functional at the level of the Golden-rule-type perturbation theory employed by the authors. Since at the first stage of our calculation we only used RPA (the same approximation was used in) we have to recover all the same diagrams as in Ref. 5. However, AAG state that it is not the case.
We proceed in two steps. We first transform the AAG result for the conductance and demonstrate that by virtue of the causality principle one can completely remove the terms of the type $G^R G^A G^R G^A$. We will arrive at the Eqs. (A14, A15) which are exactly equivalent to the result 1. Our second step is to expand our expression for the conductance correction in the interaction terms. This will lead us to the Eq. (A24) which is identical to (A13). The reader not interested in technical details can skip the technical part and continue reading after the Eq. (A24).

We start from reproducing the AAG expression for the correction to the conductivity due to electron-electron interaction which they split into two terms $\delta \sigma_{\alpha \beta} = \delta \sigma_{\text{dep}}^{\alpha \beta} + \delta \sigma_{\text{int}}^{\alpha \beta}$, where

$$
\delta \sigma_{\text{dep}}^{\alpha \beta} = -\frac{i}{16} \int \frac{dr_1 dr_2 dr_3 dr_4}{V} \int \frac{d \omega}{2 \pi} \left( \frac{d}{d \epsilon} \tanh \frac{\epsilon}{2T} \right) \left( \coth \frac{\omega}{2T} + \tanh \frac{\epsilon - \omega}{2T} \right) \left[ L_{12}^R(\omega) - L_{34}^A(\omega) \right] \times \left\{ 2 \hat{J}_{ij} \left[ G_{12}^R(\epsilon) - G_{12}^A(\epsilon) \right] \hat{J}_{ij} \left[ G_{23}^A(\epsilon - \omega) G_{41}^A(\epsilon) - G_{23}^R(\epsilon) G_{41}^R(\epsilon) \right] + \ldots + \alpha \leftrightarrow \beta \right\},
$$

(A2)

$$
\delta \sigma_{\text{int}}^{\alpha \beta} = -\frac{i}{8} \int \frac{dr_1 dr_2 dr_3 dr_4}{V} \int \frac{d \omega}{2 \pi} \left( \frac{d}{d \epsilon} \tanh \frac{\epsilon}{2T} \right) \tanh \frac{\epsilon - \omega}{2T} \times \left\{ \hat{J}_{ij} \left[ G_{12}^R(\epsilon) - G_{12}^A(\epsilon) \right] \hat{J}_{ij} \left[ G_{23}^A(\epsilon - \omega) G_{41}^A(\epsilon) - G_{23}^R(\epsilon) G_{41}^R(\epsilon) \right] \left[ G_{34}^R(\epsilon - \omega) L_{34}^A(\omega) - G_{34}^A(\epsilon - \omega) L_{34}^R(\omega) \right] + \ldots + \alpha \leftrightarrow \beta \right\}.
$$

(A3)

For simplicity we keep the same notations as in Ref. G^{R(A)} are the retarded (advanced) Green functions for noninteracting electrons and $L^{R(A)}$ are photon propagators. Here $\ldots$ stands for the terms containing two $\omega$-dependent Green functions. Such terms do not contribute to $1/\tau_\phi$ neither in Ref. 1 nor in our analysis II and therefore these terms will be ignored further below. For our purposes it will be convenient to rewrite (A2, A3) in the form

$$
\delta \sigma_{\alpha \beta} = -\frac{i}{16} \int \frac{dr_1 dr_2 dr_3 dr_4}{V} \int \frac{d \omega}{2 \pi} \left( \frac{d}{d \epsilon} \tanh \frac{\epsilon}{2T} \right) \coth \frac{\omega}{2T} \left[ L_{12}^R(\omega) - L_{34}^A(\omega) \right] \times \left\{ 2 \hat{J}_{ij} \left[ G_{12}^R(\epsilon) - G_{12}^A(\epsilon) \right] \hat{J}_{ij} \left[ G_{23}^A(\epsilon - \omega) G_{41}^A(\epsilon) - G_{23}^R(\epsilon) G_{41}^R(\epsilon) \right] + \ldots + \alpha \leftrightarrow \beta \right\} - \frac{i}{16} \int \frac{dr_1 dr_2 dr_3 dr_4}{V} \int \frac{d \omega}{2 \pi} \left( \frac{d}{d \epsilon} \tanh \frac{\epsilon}{2T} \right) \tanh \frac{\epsilon - \omega}{2T} \times \left\{ 2 \hat{J}_{ij} \left[ G_{12}^R(\epsilon) - G_{12}^A(\epsilon) \right] \hat{J}_{ij} \left[ G_{23}^A(\epsilon - \omega) L_{34}^A(\omega) - G_{23}^R(\epsilon) G_{41}^R(\epsilon) L_{34}^R(\omega) \right] \left[ G_{34}^R(\epsilon - \omega) - G_{34}^A(\epsilon - \omega) \right] + \ldots + \alpha \leftrightarrow \beta \right\}.
$$

(A4)

We observe that the factor $\tanh \frac{\omega}{2T}$ enters in this expression together with the difference $[G_{34}^R(\epsilon - \omega) - G_{34}^A(\epsilon - \omega)]$. This combination is just the Keldysh function

$$
G^K(\epsilon, r_1, r_2) = \tanh \frac{\epsilon}{2T} \left[ G^R(\epsilon, r_1, r_2) - G^A(\epsilon, r_1, r_2) \right] = \tanh \frac{\epsilon}{2T} \left[ \frac{1}{\epsilon + \mu - H + i0} - \frac{1}{\epsilon + \mu - H - i0} \right].
$$

(A5)

This function can also be rewritten as follows

$$
G^K(\epsilon, r_1, r_2) = \tanh \frac{\epsilon}{2T} \sum_\lambda \left[ \frac{1}{\epsilon - \xi_\lambda + i0} - \frac{1}{\epsilon - \xi_\lambda - i0} \right] \Psi_\lambda(r_1) \Psi_\lambda^*(r_2)
$$

$$
= \tanh \frac{\epsilon}{2T} \sum_\lambda (-2\pi i) \delta(\epsilon - \xi_\lambda) \Psi_\lambda(r_1) \Psi_\lambda^*(r_2)
$$

$$
= \sum_\lambda (-2\pi i) \left( \tanh \frac{\xi_\lambda}{2T} \right) \delta(\epsilon - \xi_\lambda) \Psi_\lambda(r_1) \Psi_\lambda^*(r_2)
$$

$$
= \sum_\lambda \left( \tanh \frac{\xi_\lambda}{2T} \right) \left[ \frac{1}{\epsilon - \xi_\lambda + i0} - \frac{1}{\epsilon - \xi_\lambda - i0} \right] \Psi_\lambda(r_1) \Psi_\lambda^*(r_2)
$$

$$
= \int dr' \left[ G^R(\epsilon, r_1, r') - G^A(\epsilon, r_1, r') \right] \left( \delta(r' - r_2) - 2\rho(r', r_2) \right),
$$

(A6)

where $\xi_\lambda, \Psi_\lambda$ are respectively the eigenvalues and the eigenfunctions of the Hamiltonian $\hat{H} - \mu$; $\rho(r', r_2)$ is the equilibrium single electron density matrix, $\tilde{\rho} = 1/(\exp((\hat{H} - \mu)/T) + 1)$. In a similar manner one obtains
\[
\left( \frac{d}{d\epsilon} \tanh \frac{\epsilon}{2T} \right) [G^R(\epsilon, \mathbf{r}_1, \mathbf{r}_2) - G^A(\epsilon, \mathbf{r}_1, \mathbf{r}_2)] = 2 \int d\mathbf{r} \frac{\partial \rho(\mathbf{r}, \mathbf{r}')}{\partial \mu} [G^R(\epsilon, \mathbf{r}', \mathbf{r}_2) - G^A(\epsilon, \mathbf{r}', \mathbf{r}_2)]. \tag{A7}
\]

We also introduce the evolution operator \( \bar{U}(t) = \exp(-i(\bar{H} - \mu)t) \) which is defined both for positive and negative times. The functions \( G^R \) and \( G^A \) are related to this operator by means of the following equations:

\[
G^R(t, \mathbf{r}_1, \mathbf{r}_2) = -i\theta(t)U(t, \mathbf{r}_1, \mathbf{r}_2); \quad G^A(t, \mathbf{r}_1, \mathbf{r}_2) = i\theta(-t)U(t, \mathbf{r}_1, \mathbf{r}_2). \tag{A8}
\]

Now let us write down the two equivalent forms of the Keldysh Green function in the real time representation. We find from (A5):

\[
G^K(t, \mathbf{r}_1, \mathbf{r}_2) = \int_{-\infty}^{+\infty} dt' \frac{-iT}{\sinh(\pi T(t - t'))} [G^R(t', \mathbf{r}_1, \mathbf{r}_2) - G^A(t', \mathbf{r}_1, \mathbf{r}_2)]
\]

\[
= - \int_{-\infty}^{+\infty} dt' \frac{T}{\sinh(\pi T(t - t'))} U(t', \mathbf{r}_1, \mathbf{r}_2), \tag{A9}
\]

and from (A4) we get

\[
G^K(t, \mathbf{r}_1, \mathbf{r}_2) = \int d\mathbf{r} [G^R(t, \mathbf{r}, \mathbf{r}') - G^A(t, \mathbf{r}, \mathbf{r}')] (\delta(\mathbf{r} - \mathbf{r}_2) - 2\rho(\mathbf{r}', \mathbf{r}_2))
\]

\[
= -i \int d\mathbf{r}' U(t, \mathbf{r}, \mathbf{r}') (\delta(\mathbf{r}' - \mathbf{r}_2) - 2\rho(\mathbf{r}', \mathbf{r}_2)). \tag{A10}
\]

Analogously we obtain

\[
\left( \frac{d}{d\epsilon} \tanh \frac{\epsilon}{2T} \right) [G^R(\epsilon, \mathbf{r}_1, \mathbf{r}_2) - G^A(\epsilon, \mathbf{r}_1, \mathbf{r}_2)] \Rightarrow \int_{-\infty}^{+\infty} dt' \frac{T(t - t')}{\sinh(\pi T(t - t'))} [G^R(t', \mathbf{r}_1, \mathbf{r}_2) - G^A(t', \mathbf{r}_1, \mathbf{r}_2)]
\]

\[
= \int_{-\infty}^{+\infty} dt' \frac{-iT(t - t')}{\sinh(\pi T(t - t'))} U(t', \mathbf{r}_1, \mathbf{r}_2); \tag{A11}
\]

and

\[
2 \int d\mathbf{r} \frac{\partial \rho(\mathbf{r}, \mathbf{r}')}{\partial \mu} [G^R(\epsilon, \mathbf{r}', \mathbf{r}_2) - G^A(\epsilon, \mathbf{r}', \mathbf{r}_2)] \Rightarrow 2 \int d\mathbf{r}' \frac{\partial \rho(\mathbf{r}, \mathbf{r}')}{\partial \mu} [G^R(t, \mathbf{r}', \mathbf{r}_2) - G^A(t, \mathbf{r}', \mathbf{r}_2)]
\]

\[
= -2i \int d\mathbf{r}' \frac{\partial \rho(\mathbf{r}, \mathbf{r}')}{\partial \mu} U(t, \mathbf{r}', \mathbf{r}_2). \tag{A12}
\]

It is easy to observe that the eqs. (A9A11) contain the integral over time which does not enter the eqs. (A10A12). It is this additional time integration that leads to violation of the normal time ordering at the level of the perturbation theory and is responsible for the appearance of the diagrams \( G^R G^A G^R G^A \). The interpretation of such diagrams in terms of the path integral is not possible. However, if one uses the other form of the same functions (A10A12) the normal time ordering is automatically restored, the combinations \( G^R G^A G^R G^A \) dissappear due to the causality principle and the path integral interpretation of the remaining terms of the perturbation theory can be made.

We emphasize that all the above transformations are exact and have the advantage that in the final expressions only the propagators depend on the frequencies \( \epsilon \) and \( \omega \) (except for the factor \( \frac{1}{2T} \) in \( \delta \rho_{\alpha\beta} \)). This allows one to use the analytical properties of the propagators related to the causality principle. Namely, \( G^R(\epsilon) \) and \( L^R(\omega) \) have no singularities in the upper half-plane, while \( G^A(\epsilon) \) and \( L^A(\omega) \) are analytic functions in the lower half-plane. Making use of these properties one can easily prove the identities

\[
\int d\omega L^R(\omega) G^A(\epsilon - \omega) \equiv 0, \quad \int d\epsilon G^A_{12}(\epsilon) G^A_{23}(\epsilon) G^A_{34}(\epsilon - \omega) G^A_{41}(\epsilon) \equiv 0,
\]

\[
\int d\omega L^A(\omega) G^R(\epsilon - \omega) \equiv 0, \quad \int d\epsilon G^R_{12}(\epsilon) G^R_{23}(\epsilon) G^R_{34}(\epsilon - \omega) G^R_{41}(\epsilon) \equiv 0. \tag{A13}
\]
Consider e.g.

\[ \int d\omega \mathcal{L}^R(\omega)G^A(\epsilon - \omega). \]

Since both functions \( \mathcal{L}^R(\omega) \) and \( G^A(\epsilon - \omega) \) are regular in the upper half-plane, the integral vanishes. Alternatively, we can write \( \int d\omega \mathcal{L}^R(\omega)G^A(\epsilon - \omega) = 2\pi \int dt \exp(i\epsilon t)\mathcal{L}^R(t)G^A(t) \) and note that \( \mathcal{L}^R(t) \equiv 0 \) for \( t < 0 \) due to the causality principle, while \( G^A(t) \equiv 0 \) for \( t > 0 \) and the integral is identically equal to zero. Analogously one can prove all the other identities (A13).

The corrections to the conductivity can now be considerably simplified:

\[
\delta\sigma_{\alpha\beta} = -\frac{e^2}{2} \int \frac{dr_1dr_2dr_3dr_4dr_5}{V} \int \frac{d\omega d\epsilon}{2\pi dT} \frac{e^{-i\omega t+ikr}}{L_4^A(\omega) - L_4^A(\omega) + 1} \times 
\left\{ \begin{array}{l}
\tilde{\gamma}_a \left[ \frac{R_{15}^A(\epsilon)}{2\pi} \frac{\partial \rho_{15}^A}{\partial \mu} \right] \tilde{j}_b \bar{G}^{A_2}_{23}(\epsilon)G^A_{41}(\epsilon)G^A_{41}(\epsilon) + \tilde{\gamma}_a \left[ \frac{R_{15}^A(\epsilon)}{2\pi} \frac{\partial \rho_{15}^A}{\partial \mu} \right] \tilde{j}_b \bar{G}^{A_2}_{23}(\epsilon)G^A_{41}(\epsilon)G^A_{41}(\epsilon) + \cdots + \alpha \leftrightarrow \beta \\
-\frac{i}{4} \int \frac{dr_1dr_2dr_3dr_4dr_5dr_6}{V} \int \frac{d\omega d\epsilon}{2\pi dT} \left\{ -\tilde{\gamma}_a \frac{R_{15}^A(\epsilon)}{2\pi} \frac{\partial \rho_{15}^A}{\partial \mu} \tilde{j}_b \bar{G}^{A_2}_{23}(\epsilon) \left[ G^A_{36}(\epsilon - \omega) \right] (1 - 2\rho)_{64} \right\} G^A_{41}(\epsilon)L_{44}^A(\omega) + \cdots + \alpha \leftrightarrow \beta \}
\right\}.
\]

We observe that the terms of the type \( \tilde{\gamma}_a \frac{R_{15}^A(\epsilon)}{2\pi} \frac{\partial \rho_{15}^A}{\partial \mu} \tilde{j}_b \bar{G}^{A_2}_{23}(\epsilon) \left[ G^A_{36}(\epsilon - \omega) \right] (1 - 2\rho)_{64} \) do not enter the expression (A14) at all. For later purposes it will be useful to rewrite the above expression in the form of the time integral:

\[
\delta\sigma_{\alpha\beta} = -\frac{e^2}{2} \int \frac{dr_1dr_2dr_3dr_4dr_5}{V} \int dt_1 \int dt_2 \int dt_3 \times 
\left\{ \left[ j_a U_{15}(t_1) \frac{\partial \rho_{15}^A}{\partial \mu} j_b U_{23}^+(t_3) \right] \left[ j_3 (t_2 - t_3) U_{34}^+(t_2 - t_3) \right] U_{41}^+(t_1 - t_2) \\
+ \tilde{\gamma}_a \left[ j_{15}^A(t_1) \frac{\partial \rho_{15}^A}{\partial \mu} \right] \tilde{j}_b \bar{G}^{A_2}_{23}(\epsilon) \left[ j_3 (t_2 - t_3) U_{34}^+(t_2 - t_3) \right] U_{41}^+(t_1 - t_2) + \cdots + \alpha \leftrightarrow \beta \}
\right\}.
\]

where

\[
R(t, r) = \int \frac{d\omega d\epsilon}{(2\pi)^4} \frac{4\pi}{k^2 \epsilon(\omega, k)} e^{-i\omega t + ikr} = -\frac{1}{e^2} \mathcal{L}^R(t, r) = -\frac{1}{e^2} \mathcal{L}^A(-t, r),
\]

\[
I(t, r) = \int \frac{d\omega d\epsilon}{(2\pi)^4} \frac{4\pi}{k^2 \epsilon(\omega, k)} \coth \left( \frac{\omega}{2T} \right) e^{-i\omega t + ikr} = \frac{1}{2e^2} \int \frac{d\omega d\epsilon}{(2\pi)^4} \coth \left( \frac{\omega}{2T} \right) \left[ L^R(\omega, k) - L^A(\omega, k) \right] e^{-i\omega t + ikr}.
\]

Now we will demonstrate that the equation (A13) can be obtained within the path integral formalism. The formal expression for the conductivity has the form

\[
\sigma = \frac{e^2}{3m} \int dt \int dr_1dr_2 \left( \nabla r_1f \left( \nabla r_2f \right) \right) \left( J(t; r_1, r_2) \right) \rho_0(r_1, r_2).
\]

The kernel \( J \) is given by the path integral over electron coordinates and momentums \( r_1(t), p_1(t) \) and \( r_2(t), p_2(t) \) corresponding respectively to the forward and backward parts of the Keldysh contour. The explicit expression for this kernel reads:

\[
J(t; t', r_1f, r_2f; r_1i, r_2i) = \int \frac{Dp_1}{2\pi} \int \frac{Dp_2}{2\pi} \int \frac{Dp_1}{2\pi} \int \frac{Dp_2}{2\pi} \times \exp \left\{ iS_0[r_1, p_1] - iS_0[r_2, p_2] - iS_R[r_1, p_1, r_2, p_2] - S_I[r_1, r_2] \right\}
\]

\[
\text{A17}
\]
where

\[
S_0[r, p] = \int_{t'}^t dt'' \left( p r' - \frac{p^2}{2m} - U(r) \right); \tag{A18}
\]

\[
S_R[r_1, p_1, r_2, p_2] = \frac{e^2}{2} \int_{t'}^t dt_1 \int_{t'}^t dt_2 \left\{ R(t_1 - t_2, r_1(t_1) - r_1(t_2)) \right\} \left[ 1 - 2n(p_1(t_2), r_1(t_2)) \right] \]

\[
- R(t_1 - t_2, r_2(t_1) - r_2(t_2)) \left[ 1 - 2n(p_2(t_2), r_2(t_2)) \right]
\]

\[
+ R(t_1 - t_2, r_1(t_1) - r_2(t_2)) \left[ 1 - 2n(p_2(t_2), r_2(t_2)) \right]
\]

\[
- R(t_1 - t_2, r_2(t_1) - r_1(t_2)) \left[ 1 - 2n(p_1(t_2), r_1(t_2)) \right]; \tag{A19}
\]

and

\[
S_I[r_1, r_2] = \frac{e^2}{2} \int_{t'}^t dt_1 \int_{t'}^t dt_2 \left\{ I(t_1 - t_2, r_1(t_1) - r_1(t_2)) + I(t_1 - t_2, r_2(t_1) - r_2(t_2)) \right\}
\]

\[
- I(t_1 - t_2, r_1(t_1) - r_2(t_2)) - I(t_1 - t_2, r_2(t_1) - r_1(t_2)) \right\}. \tag{A20}
\]

The functions \( R(t, r) \) and \( I(t, r) \) were already defined above.

In order to obtain the perturbative result \( \text{(A13)} \) from the formally exact expression \( \text{(A16)} \) one needs to expand the kernel \( J \) \( \text{(A17)} \) in \( isR + S_I \). In the first order one obtains eight different terms. Again we will consider only the terms contributing to \( 1/\tau_\omega \), i.e. the terms containing \( R(t_1 - t_2, r_1(t_1) - r_1(t_2)) \), \( R(t_1 - t_2, r_2(t_1) - r_2(t_2)) \), \( I(t_1 - t_2, r_1(t_1) - r_1(t_2)) \) and \( I(t_1 - t_2, r_2(t_1) - r_2(t_2)) \). Diagrammatically, these contributions are described by the first two diagrams shown in Fig.4 of Ref. \( \text{[3]} \). Four other terms which relate two different branches of the Keldysh contour and contain both \( r_1 \) and \( r_2 \), e.g. of the type \( I(t_1 - t_2, r_1(t_1) - r_2(t_2)) \), are described by the last diagram in Fig. 4 of Ref. \( \text{[3]} \). These terms give the contributions containing two Green functions depending on the frequency \( \omega \), and were denoted as ... in \( \text{(A15)} \). Such terms are fully reproduced within our method as well, however we will not consider them here for the sake of simplicity.

The correction to the kernel \( J \) due to the term \( I(t_1 - t_2, r_1(t_1) - r_1(t_2)) \) has the form

\[
\delta J^{(1)}(t, t'; r_{1f}, r_{2f}; r_{1i}, r_{2i}) = -e^2 \int dt_3 \int dt_2 \int_{t'}^t d\tau_1 \int_{t'}^t d\tau_2 \int \mathcal{D}r_1 \int \mathcal{D}r_2 \int \mathcal{D}p_1 \int \mathcal{D}p_2 \times
\]

\[
\times U_{r_{2f}, r_{2i}}^+(t - t')U_{r_{1f}, r_{1i}}(t - t_3)I_{34}(t_3 - t_2)U_{34}(t_3 - t_2)U_{4r_{1i}}(t_2 - t'). \tag{A21}
\]

Here we made use of a simple property of a path integral:

\[
\int \mathcal{D}r \int \mathcal{D}p f(t', r, p) \exp \{ iS_0[r, p] \} = \int \mathcal{D}r' U(t - t''; r_f, r_{r'}) f(t'', r_{r''}) U(t'' - t; r_{r''}, r_f), \tag{A22}
\]

which holds for an arbitrary function \( f(t'', r(t'')) \). Actually in deriving \( \text{(A21)} \) the property \( \text{(A22)} \) was used twice because the function of two arguments \( I(t_2 - t_3, r_1(t_2) - r_1(t_3)) \) enters under the integral \( \text{(A21)} \). Already at this stage one can observe the similarity between the expression \( \text{(A21)} \) and the second term in the expression \( \text{(A13)} \). To establish the equivalence between these two expressions the following steps are in order: i) after substituting the result \( \text{(A21)} \) into the expression for the conductivity \( \text{(A16)} \) and applying the current operator \( j = (ie/m)(\nabla r_{1f} - \nabla r_{2f}) \) one puts \( r_{1f} = r_{2f} = r_2, r_{1i} = r_1, r_{2i} = r_5 \); ii) one denotes \( t - t' \) \( t_1, t - t_2 \) \( t_2, t - t_3 \) \( t_3 \); iii) one introduces an...
additional integration \( \int dr_4/V \) which is just averaging of the expression (A16) over the sample volume and iv) one transforms the effective initial density matrix as follows

\[
(r_{1i} - r_{2i}) \rho_0(r_{1i}, r_{2i}) = i \sum_{\lambda_1, \lambda_2} \frac{\langle \Psi_{\lambda_1} | p | \Psi_{\lambda_2} \rangle}{m} \frac{\delta(\xi_{\lambda_1}) - \delta(\xi_{\lambda_2})}{\xi_{\lambda_1} - \xi_{\lambda_2}} \Psi_{\lambda_1}(r_{1i}) \Psi_{\lambda_2}^*(r_{2i}) \simeq -i \frac{\partial}{\partial \mu} \frac{\partial \rho(r_{1i}, r_{2i})}{\partial \mu}.
\]

(A23)

After these transformations one can immediately observe the equivalence of the results obtained by means of two methods and on the level of the perturbation theory. The terms arising from the real part of the action \( S_R \) can be transformed analogously, the only difference in this case is the presence of an additional factor \((1 - 2\rho)_{64} \) related to the term \( 1 - 2n(p, r) \) in the expression (A13). Finally we get

\[
\delta \sigma = -\frac{e^3}{3} \int \frac{dr_1 dr_2 dr_3 dr_4 dr_5}{V} \int_0^{+\infty} \int_0^{t_1} \int_0^{t_2} \int_0^{t_2} \delta \rho \frac{\partial \rho}{\partial \mu} U_{52}(t_1) \hat{J}_{23}(t_3) \left[ I_{34}(t_2 - t_3) U_{34}^+(t_2 - t_3) \right] U_{41}^+(t_1 - t_2)
\]

\[
+ \frac{\partial \rho}{\partial \mu} U_{52}(t_1) \hat{J}_{23}(t_3) \left[ I_{34}(t_2 - t_3) U_{34}^+(t_2 - t_3) \right] U_{41}(t_1 - t_2) + \ldots \}
\]

\[
- \frac{i e^3}{6} \int \frac{dr_1 dr_2 dr_3 dr_4 dr_5 dr_6}{V} \int_0^{+\infty} \int_0^{t_1} \int_0^{t_2} \int_0^{t_2} \delta \rho \frac{\partial \rho}{\partial \mu} U_{52}(t_1) \hat{J}_{23}(t_3) \left[ R_{34}(t_2 - t_3) U_{36}^+(t_2 - t_3) (1 - 2\rho)_{64} \right] U_{41}^+(t_1 - t_2)
\]

\[
+ \frac{\partial \rho}{\partial \mu} U_{52}(t_1) \hat{J}_{23}(t_3) \left[ R_{34}(t_2 - t_3) U_{36}^+(t_2 - t_3) (1 - 2\rho)_{64} \right] U_{41}(t_1 - t_2) + \ldots \}
\]

(A24)

In order to verify complete equivalence of (A13) and (A24) one should a) replace the operator \( e\hat{p}/m \) by \( \hat{J} \); b) adjust the factor 3 by observing that (A24) and (A13) are the corrections respectively to to the scalar conductivity and the conductivity tensor (in the isotropic case one has \( \delta \sigma = (\delta \sigma_{xx} + \delta \sigma_{yy} + \delta \sigma_{zz})/3 \)) and c) adjust another factor 2 having in mind symmetrization of (A13) with respect to indices \( \alpha \) and \( \beta \). Also, one should keep in mind that the operator \( \partial \rho_{15}/\partial \mu \) commutes with the evolution operator \( U_{52} \). This completes the proof of equivalence of the results (A13) and (A24).

Thus the AAG statement that within our analysis “only some paths were selected by hand” is false. As it was explained above the paths presented e.g. in Fig. 10c of Ref. cannot appear in the path integral, they are forbidden by the causality principle. Unfortunately AAG did not indicate the direction of the electron motion (e.g. by arrows) in their Fig. 10c. Otherwise it would be completely clear that at some parts of this path the electron moves backward in time. Such diagrams can appear from direct multiplication of the Keldysh matrices, but not in the path integral.

In any case the technical issue with “missing diagrams” is settled and now we can come to the central question: if there are no calculational errors what causes the difference between the results of Refs. and ? As it was already discussed AAG proceed perturbatively in the interaction. They state that we do the same: according to AAG our procedure “is nothing but a perturbative expansion” and our results are “purely perturbative”. If this were true, our final results could be compared directly indeed. However, this is not true. Of course, in some sense we also proceed perturbatively when we expand the exact effective action \( S[V_+ \hat{p}] \) in powers of \( V_+ \) in the exponent. But this is just RPA, and it has nothing to do with the Golden-rule-type perturbation theory developed in . In this sense our procedure is essentially nonperturbative and (within RPA) includes processes in all orders.

On a slightly more formal level we can reformulate the difference as follows. The conductance (A14) is determined by the path integral (A17). As it was demonstrated above the AAG perturbative procedure is equivalent to expanding \( J \) up to the first order in \( iS_R + S_I \), integrating over time in the infinite limits (this yields energy conservation) and averaging the result over disorder within the quasiclassical approximation. Our procedure does not involve the expansion in \( iS_R + S_I \). We just evaluate the complete path integral within the quasiclassical approximation taking into account all saddle point paths and average over disorder. The latter average is quite trivial because the part of \( S_0 \) which could be important for dephasing disappears already before averaging.

One might think that both procedures should give the same result as long as the interaction is weak and the terms \( iS_R + S_I \) are small. Here, however, one should be cautious: these terms are defined by nonlocal in time expressions (A13, A20) and involve integrals over the two times. These terms are never small at low \( T \) as long as time is not
small. Therefore in general one cannot expand and should evaluate the whole path integral no matter how weak the interaction is.

Thus not only the equivalence of the results of refs. 3 and 9 on the perturbative level but also an important difference between these results is established. In both papers the conductance of a disordered metal (A10) in the presence of interaction was calculated. In our paper the full path integral (A17) was evaluated whereas the analysis is equivalent to the first order expansion of this path integral in $iS_R + S_I$. It is exactly due to this reason the results and differ. The average over disorder emphasized in 3 is not important at this stage: the results are different already before averaging.

We can also add that for simplicity in Ref. 3 we did not consider the effect of magnetic field $H$. This effect can be trivially incorporated into our analysis by adding the term $ciA(r)$ to the electron Lagrangian, $A$ is the corresponding vector potential. The whole procedure remains the same except the action $S_0$ will now depend on $A$. Averaging over disorder at the last stage of the calculation will yield the equations (2.42) from 3 which define the dependence of the magnetoconductance curves on $H$. This part of the calculation is standard and was discussed in details e.g. in 5.

We would only like to emphasize that even in the presence of the magnetic field $H$ one is not allowed to expand the path integral $J(A17)$ in powers of $iS_R + S_I$ for all relevant fields $\tau_H \gg \tau_e$, where $\tau_H$ is the dephasing time due to the magnetic field, see eqs. (2.9) of Ref. 3.

Having established the important formal difference between the procedures 3 and 9 we could, in principle, already conclude our consideration. At this stage one does not even need to introduce and discuss such concepts as “decoherence”, “dephasing time” etc. Just viewing $\tau_e$ as a formal parameter extracted in a standard way from the magnetoconductance measurements one immediately observes that the perturbative calculation 3 yields $\tau_e \rightarrow \infty$ at $T \rightarrow 0$, while if one evaluates the whole path integral (A17) (which contains all the perturbative terms) plus infinitely many other terms which contribution is not smaller than that kept in 3 one obtains a finite result at low $T$ in agreement with many experiments. We believe, however, that it is important not only to establish the formal difference between the two approaches 3 and 9 but also to understand the physical reasons for this difference to occur. The corresponding discussion is presented below.

### APPENDIX B:

Let us first recall the qualitative arguments of AAG concerning the effect of quantum decoherence 3. “Consider a quantum particle which moves in the environment of harmonic oscillators: each oscillator is characterized by its frequency $\omega$. The result of the collision of the particle with an oscillator depends on the relation between $\hbar \omega$ and the temperature.” AAG first consider the case $\hbar \omega \ll T$ and find that in this case “the probability for the inelastic collision is substantial”. Then they write: “The situation for $\hbar \omega \gg T$ is quite different. Indeed, up to the exponentially small terms the oscillator is in the ground state and the particle has the energy smaller than $\hbar \omega$”. Here we already have two questions: (i) it remains unclear whether the interacting or noninteracting ground state of the system is meant and (ii) it is also not clear whether the whole system is in equilibrium at all. But let us first finish the citation from 3: “Therefore, the energy transfer is forbidden by the energy conservation and the collision is elastic. Therefore, there is no difference whatsoever with the collision with such an oscillator and with the quenched disorder, which definitely does not dephase”.

Here we definitely agree with the last part of this sentence, namely quenched disorder indeed cannot dephase. The remaining part emphasizes the main physical difference between 3 and our work. The AAG arguments apply if one considers a scattering problem in which the total energy of the system is a sum of those for the noninteracting particle and the oscillators and it (the energy) is conserved during the whole process. Then, if initially all the oscillators were in their noninteracting ground states and the particle energy was small, in the end this particle will have the same energy because none of the oscillators can be excited. Hence, the collision is elastic and no dephasing takes place.

We, however, are interested in another physical situation. Namely, we would like to describe the properties of a quantum particle which is in equilibrium with an (infinite) bath of oscillators or close to this equilibrium. In this case the energy exchange between the particle and the bath is possible and the total energy of this interacting system is different (in our case larger) as compared to a sum of energies of noninteracting particles. Then for various physical situations one can demonstrate (see e.g. 5) that interaction of a quantum particle with other quantum degrees of freedom is qualitatively different from the effect of a static potential. Since the system with harmonic oscillators is simple enough one can illustrate the difference between the two physical situations by means of an exact calculation. This will be done below.

The density matrix of a quantum particle $q$ interacting with other quantum degrees of freedom can be represented in the form
\[ \rho(q_f, q'_f) = \int dq dq' J(q_f, q'_f; q, q', 0) \rho_i(q, q'), \]  

(81)

where \( \rho_i(q, q') \) is the initial density matrix and

\[ J = \int_{q_n}^{q_f} dq_1 \int_{q'_n}^{q'_f} dq_2 \exp(iS_0[q_1(t)] - iS_0[q_2(t)] - iS_R[q_1, q_2] - S_I[q_1, q_2]). \]  

(B2)

Here \( S_0[q] \) is the action of a noninteracting particle, while the terms \( S_{R,I} \) account for interaction and define the influence functional for an effective quantum environment. It is easy to observe that the Eq. (B2) has essentially the same form as the Eq. (A17). If the environment consists of harmonic oscillators with frequencies \( \omega_n \) and unity masses, the influence functional can be easily found provided one assumes bilinear in coordinates interaction between the particle and the oscillators. Defining \( q_+ = (q_1 + q_2)/2 \) and \( q_- = q_1 - q_2 \) one readily obtains

\[ S_R = \sum_n C_n^2 \int_0^t dt_1 dt_2 q_-(t_1) \sin(\omega_n(t_1 - t_2))q_+(t_2), \]  

(83)

\[ S_I = \sum_n C_n^2 \coth(\frac{\omega_n}{2T}) \int_0^t dt_1 dt_2 q_-(t_1) \cos(\omega_n(t_1 - t_2))q_-(t_2), \]  

(84)

where \( C \) is a constant which governs the strength of interaction. At this stage the problem (B3,B4) is qualitatively analogous to one we arrive in our analysis of interacting electrons in a disordered metal after we employ an RPA expansion of the exact effective action (cf. eqs. (A17,A20)). In a model with oscillators we deal with the quadratic effective action from the very beginning.

Let us assume \( C \) to be very small and try to find the transition probability

\[ W_{ij} = \int dq dq' dq dq' \psi_i(q) \psi_i^*(q) J(q_i, q'_i; t; q, q', 0) \psi_j(q) \psi_j^*(q) \]  

(85)

from some initial state with the wave function \( \psi_i(q) \) and the energy \( E_i \) to some other orthogonal state \( \psi_j(q) \), \( E_j \) just by expanding the kernel \( J \) in powers of \( C \) and we only reproduce the result:

\[ W_{ij} = \text{Re} \sum_n \frac{\pi C_n^2 |\langle i | q_j \rangle|^2}{2\omega_n} \int dt \int dt' e^{-i(E_j - E_i)(t-t')} \int \frac{d\omega}{2\pi} e^{-i\omega(t-t')} \left( \coth \frac{\omega}{2T} + 1 \right) \delta(\omega_n - \omega) \delta(\omega_n + \omega) \]  

(86)

Integration over \( t-t' \) in the infinite limits yields the delta-function \( \delta(E_j + \omega - E_i) \) which ensures the energy conservation. At \( T \to 0 \) only positive \( \omega \) contribute and the second delta-function in the square brackets fails. Thus in this case the transitions are only possible if \( E_j + \omega_n - E_i = 0 \), i.e. for \( E_i \leq E_j \) the transition never happens because the system has no energy to excite the oscillator \( \omega_n \). Thus it always stays in its initial state, i.e. \( W_{ij} = 0 \) and the quantum coherence is never lost. At nonzero \( T \) and \( E_i \leq E_j \) we have \( W_{ij} \neq 0 \), but only oscillators with small frequencies \( \omega_n \lesssim T \) can take part in the transitions, so that \( W_{ij} \) is small as long as \( T \) is sufficiently low.

The above simple example just illustrates the qualitative arguments of the authors presented above. Exactly the same physical reasons are behind the cancellation of diagrams demonstrated in [13]. This cancellation will always take place at \( T \to 0 \) within the Golden-rule-type perturbation theory just because of energy conservation. The only difference is that instead of the combination “coth +1” for Bose particles there appears the combination “coth - tanh” in the case of electrons. This is well known and was also rederived from our formalism (see Sec. 5 of Ref. [9]).

Thus the AAG calculation [13] is fully consistent with their qualitative arguments. Both describe the same physical situation: the total energy of the system “particle+oscillators” is equal to its value without interaction and fixed during the whole process.

It is obvious, however, that the formalism allows to go beyond the simple Golden-rule-type perturbation theory and to provide a full description of an interacting system. In order to demonstrate that let us perform a simple Gaussian integral in (B2,B4) exactly for the case of a free quantum particle with a mass \( m \). Just for the sake of convenience we will assume a continuous spectrum of the oscillators

\[ \sum_n \frac{\pi C_n^2}{2\omega_n} \delta(\omega_n - \omega) \delta(\omega_n + \omega) = \eta \omega^\beta, \quad |\omega| < \omega_c, \]  

(87)
where \( \omega_c \) defines the high frequency cutoff and \( \beta \) is a positive number. The exact results for the kernel \( J \) are well known and are presented elsewhere. Here we only focus on the most important terms. One finds

\[
J \propto \exp \left[ -m f_1(t) q_i^2 - m f_2(t) (q_f - q_i)^2 + \ldots \right],
\]

where \( \gamma = \eta/m \) and \( q_{\pm i/f} \) are the initial/final values of \( q_{\pm} \). For the Ohmic bath \( (\beta = 1) \) one obtains

\[
f_1(t) = \gamma \int_0^t ds \int_0^{\omega_c} \frac{d\omega}{2\pi} \coth \left( \frac{\omega}{2T} \right) e^{-i\omega(x-s')} = \gamma T t + \gamma \ln \frac{1 - e^{-2\pi T t}}{2\pi(T/\omega_c)}. \tag{B9}
\]

In the long time limit the function \( f_2 \) coincides with the equilibrium value of the average kinetic energy of the particle \( \langle E \rangle = \langle m q_i^2/2 \rangle ::

\[
f_2 = \langle E \rangle = \gamma \int_0^{\omega_c} \frac{d\omega}{2\pi} \coth \left( \frac{\omega}{2T} \right) \omega^2 \approx \frac{\gamma}{2\pi} \ln \frac{\omega_c}{\gamma} + \frac{T}{\pi} \arctan \frac{T}{\omega_c}. \tag{B10}
\]

We observe that the particle \( q \) loses its coherence due to interaction with the bath of oscillators. Indeed in the long time limit we have \( f_1(t) \gg f_2 \) and the kernel \( \langle E \rangle \) effectively reduces to

\[
J \rightarrow \frac{1}{L} e^{-m f_2 q_i^2} \delta(q_{-i}), \tag{B11}
\]

where \( L \) is the system size. Any perturbation of the density matrix will relax to the same equilibrium form

\[
\rho(q_1, q_2) = (1/L) e^{-\langle q_1 q_2 \rangle^2 / L_d^2}, \quad L_d^2 = 1/m f_2 \tag{B12}
\]

which is not sensitive to the initial phase. As a result of interaction with the bath even at \( T \to 0 \) the off-diagonal elements of the equilibrium density matrix \( \langle E \rangle \) decay on a typical length scale \( L_d \sim 1/\sqrt{\eta \ln(\omega_c/\gamma)} \) set by interaction. The average value of the kinetic energy of the particle \( \langle E \rangle \) explicitly enters the expressions \( \langle B8 \rangle \) which in the limit \( T \to 0 \) describe the true ground state properties of the system. The same results can be obtained within the imaginary time technique, or just by an exact diagonalization of the initial Hamiltonian of the system “particle+oscillators”. From the latter work it is particularly transparent that all the results depend on \( \omega_c \) just because all the oscillators (including the high frequency ones) “take part” in the diagonalization. By no means this implies excitation of such oscillators. Rather one can say that in the presence of interaction the noninteracting energy levels of the oscillators acquire a finite width and they can exchange energy with a particle in arbitrarily small portions. As a result of this exchange the particle energy is distributed as

\[
w(E) = \int \frac{dp}{2\pi} \delta \left( E - \frac{p^2}{2m} \right) f(p) \propto \exp(-E/2\langle E \rangle),
\]

with the average value \( \langle E \rangle \) given by eq. \( \langle B10 \rangle \). Here we defined

\[
f(p) = L \int dq_- \rho(q_-) e^{-ipq_-},
\]

where \( \rho(q_-) \) is the equilibrium density matrix \( \langle B12 \rangle \) and \( q_- = q_1 - q_2 \).

In order to avoid misunderstandings we would like to emphasize that we (on purpose) work in basis of “noninteracting” eigenstates of the system. It is obvious that the full wave function of the total system as well as each of the eigenmodes obtained by an exact diagonalization always stay coherent. However, since the behavior of the particle \( q \) (and not that of the eigenmodes) is of interest for us, the reduced density matrix \( \rho(q_1, q_2) \) should be studied. The decay of the off-diagonal elements of \( \rho \) on the length scale \( \sim L_d \) just implies that the bath in some conventional sense “measures” the particle position. In principle the off-diagonal elements of \( \rho(q_1, q_2) \) (and thus the coherence of the particle \( q \)) can be suppressed completely \( (L_d \text{ tends to zero if one e.g. chooses } \omega_c \to \infty) \), while the eigenmodes of the total system obviously remain fully coherent. Thus it is quite useless to discuss the presence or absence of
quantum coherence in the interacting many-body system without discussing which quantity is actually calculated and/or measured in experiments.

Let us also note that the particular dependence of the results on the high frequency cutoff \( \omega_c \) is fully determined by the spectrum of the bath \( B_{\omega,k} \). E.g. for \( \beta < 1 \) for most of the physical quantities of interest the cutoff does not enter at all, for \( \beta = 1 \) this dependence is logarithmic, while for \( \beta > 1 \) one gets a power-law dependence on \( \omega_c \). In the latter case also the decay of perturbations of the density matrix to its equilibrium value at \( T = 0 \) is faster than logarithmic. All these features have been extensively studied and we will not go into more details here.

We conclude that the presented above Golden rule approach fails to reproduce all the features obtained from the exact solution. E.g. at \( T = 0 \) the Golden rule approach yields \( \langle E \rangle = \eta 0 = 0 \), while the correct result is given by eq. (31), according to this approach the initial state does not decay as a result of interaction, while in reality it does (cf. eq. (36)), according to eq. (30) no coherence can be lost at \( T = 0 \), while actually it is lost and even in equilibrium the off-diagonal elements of the density matrix decay on a typical length scale \( \sim L_d \) set by interaction. The Golden rule approach cannot correctly describe both the ground state properties of an interacting system as well as their low-lying excitations (cf. e.g. Ref. 6 where such properties were derived by means of an exact diagonalization).

The example with a free damped quantum particle is not unique, of course. One can also consider e.g. a degenerate two-level system and observe that interaction with the CL bath leads to the effective renormalization of the transition amplitude \( \Delta \) between the two levels.

\[
\Delta \to \Delta = \Delta (\omega_c / \omega) \propto \alpha \eta
\]

where \( \alpha \) and \( \eta \) depend on the high frequency cutoff \( \omega_c \). Again this is the property of the true ground state of an interacting system which explicitly depends on the high frequency cutoff \( \omega_c \) even for very small \( \alpha \). Again it is impossible to obtain the above result for \( \Delta \) within the Golden rule approach which at \( T = 0 \) gives \( \Delta_0 - \Delta = \alpha 0 = 0 \).

Coming back to the problem of interacting electrons in a disordered metal we observe exactly the same situation. The Golden-rule-type perturbation theory is not sufficient at low \( T \) just because it cannot correctly account for the ground state properties of an interacting system. Proceeding perturbatively one starts from a noninteracting system and imposes energy conservation. Obviously no dephasing can occur at \( T = 0 \) because the electron energy \( \langle E \rangle \) is zero in this case. But this simply means that the energy of an interacting system lower than it is in its true ground state.

The presence of interaction this energy is not zero \( \langle E \rangle > 0 \) even at \( T = 0 \). Although we do not calculate this energy explicitly within our analysis it is obvious that the scale for \( \langle E \rangle \) set by interaction and the high frequency cutoff of the effective environment will enter. As in the above example of the exactly solvable CL model this dependence has nothing to do with excitation of high frequency oscillators. This is just the property of the true ground state of an interacting system.

Thus we see that our problem has essentially the same qualitative features as the CL model. Of course, there exist also physical differences between these problems. For instance, there seems to be no quantity in the CL model which would be completely analogous to the decoherence time \( \tau_\phi \) measured in the weak localization experiments. One can, however, qualitatively compare the length \( L_d \) in the CL model with the decoherence length \( L_\phi \sim \sqrt{D \tau_\phi} \). Actually both quantities agree qualitatively if one establishes the correspondence between the interaction parameters and the bath spectra of these problems. More importantly, in both problems quantum decoherence appears as a result of energy exchange between the particle and the effective environment. [We emphasize again that we are working in the basis of noninteracting electrons which is obviously not the basis of the eigenstates in the presence of interaction.] In both problems the effect cannot be captured within the Golden rule approach where this energy exchange is forbidden at \( T = 0 \).

In order to illustrate this point again let us “forbid” this exchange in our results by hand. Formally it implies that one should take the limit \( \omega \to 0 \) in the expression for the dielectric susceptibility of the effective environment in which case \( \text{Im} \ 1 / \epsilon (\omega, k) \) vanishes. Obviously this procedure is not justified both physically and mathematically, but we just use it for illustration. For the CL model one can e.g. integrate over \( s - s' \) in \( B_{\omega,k} \) in the infinite limits, get \( \delta (\omega) \) and as a result recover only the first term \( \gamma T t \) while the second will be missing. Then in the limit \( T \to 0 \) one would observe no decay of the initial density matrix which would be incorrect. Analogously at \( T = 0 \) one would get \( \langle E \rangle = 0 \) which would be incorrect too. The correct procedure is – as it was done above – to integrate over frequencies keeping the time finite and only then to send it to infinity.

Analogously if one (just by hand) substitutes the time integral in eq. (71) of Ref. 3 by the delta function \( \delta (\omega) \) in order to provide the limit \( \text{Im} \ 1 / \epsilon (\omega, k) \to 0 \) (no energy exchange) one obtains

\[
\frac{1}{\tau_\phi} \sim T \sigma_d^2 \int \frac{d^3 k}{(2\pi)^3} \frac{\omega}{k^2} \delta (\omega)
\]

The result is again zero at \( T = 0 \), but again the \( T \)-independent term will be missing as in the above case of the CL model. [Note, however, that for large \( T \) one can recover the correct result proceeding in such a way. This is achieved if one cuts the integral at the lower limit at \( k \sim 1 / L_\phi \sim 1 / \sqrt{D \tau_\phi} \). Although the above procedure is not rigorous it clearly illustrates the physical difference between the results and]
APPENDIX C:

In order to complete the critique of our theory AAG presented a collection of various experimental results and reported a strong disagreement with our theory. In some 3d systems the disagreement was found to be 4 to 5 orders of magnitude. This must look as a strong argument against our theory.

However, looking closer at the experimental data one observes that in many systems chosen by AAG for comparison the mean free path is several or even one (!) Angstrom. Such strongly disordered systems are well beyond the applicability of our quasiclassical theory. Hence, for such systems the comparison cannot be carried out at all. On top of that, many systems have the granular structure. Our analysis cannot be directly applied to such systems as well, for instance because the simple formula for the noise spectral density

\[
\frac{\langle |V_{\omega,k}|^2 \rangle}{a^{3-d}} = \frac{\omega \coth \frac{\omega}{\tau}}{\sigma \epsilon d^2},
\]

used in our theory works only for relatively small frequencies. [Here \(d\) is the effective dimension and \(a\) is a film thickness in 2d or a square root of a wire cross section area in 1d.] In such systems the high frequency cutoff \(\omega_c\) in (\(C1\)) is determined not by the elastic scattering rate but rather by the effective capacitance of metallic grains. Examples will be analyzed in the first part of this Appendix.

In the second part of this Appendix we will discuss several experiments in which the saturation of \(\tau_{\varphi}\) was observed. All these experiments are in a quantitative agreement with our theory. It appears that no alternative explanation of these experiments exists at the moment.

It was always implied that our analysis is applicable for good metals with high conductivity and relatively low elastic scattering rate. Then and only then one can use a simple formula (\(C1\)) up to frequencies \(\sim 1/\tau_e\). In such complicated and strongly disordered systems as granular percolating systems and metallic glasses one typically has \(1/\tau_e \sim 10^{15} \div 10^{16}\) Hz. This corresponds to energies \(\sim 10^9 K\) where the simple approximation (\(C1\)) does not work due to various reasons. In a general case the noise power spectrum is expressed by the formula

\[
\frac{\langle |V_{\omega,k}|^2 \rangle}{a^{3-d}} = \text{Im} \left( \frac{-4\pi}{k^2 \epsilon(\omega,k)} \right) \coth \left( \frac{\omega}{2T} \right)
\]

which should be used for the comparison with experiments. Below we will demonstrate how much the noise can be reduced (and hence \(\tau_{\varphi}\) increased) in many systems discussed in \(\ref{sec:4c}\) if one uses a realistic model for \(\epsilon(\omega,k)\). For 1d and 2d systems some possible sources of the noise reduction have been already analyzed in Sec. 4c of Ref. \(\ref{sec:3}\).

Let us e.g. consider an experimental work \(\ref{sec:3}\) where the decoherence time \(\tau_{\varphi}\) was measured in 3d Cu granular percolating systems. According to the inverse inelastic time in this system is \(1/\tau_e \sim 2 \times 10^{15}\) Hz and the elastic mean free path (extracted from the diffusion coefficient) is \(l \sim 4 \times 10^{-8}\) cm. It is clear that our analysis cannot be applied to systems with such small values of \(l\) because the quasiclassical approximation should not really work there. But for a moment we ignore this fact and concentrate on the expression for \(\epsilon(\omega,k)\). The authors\(\ref{sec:3}\) use our formula (81) from Ref. \(\ref{sec:3}\) obtained from the Eq. (\(C1\)) and reported 5 orders of magnitude difference with the measured value for \(\tau_{\varphi}\). But the formula (\(C1\)) definitely cannot be applied for granular materials already at frequencies much lower than \(10^{15}\) Hz! The effect of grain capacitances at not very low \(\omega\) is crucially important and drastically changes the frequency dependence of \(\epsilon\). Using the standard model for a granular metal one easily arrives at the conclusion that the high frequency cutoff \(\omega_c\) in (\(C1\)) never exceeds \(\omega_c \sim 1/\tau_e\) and can be even smaller if the grain self-capacitance dominates over the intergrain capacitance (here \(C\) is the typical grain capacitance and \(R\) is the typical intergrain resistance). In order to get a rough estimate for \(\omega_c\) in this case we make use of the typical grain size \(\sim (1 \div 3) \times 10^{-6}\) cm and the resistivity \(\rho \approx 7 \times 10^{-5}\) Ω cm reported by the authors\(\ref{sec:3}\). A reasonable estimate for \(R\) would be \(R \sim 100\) Ω. Although the grain capacitance is difficult to estimate from the grain size we can guess that the corresponding (renormalized) charging energy \(E^*_C \lesssim E_C = e^2/2C\) should not exceed \(E^*_C \sim (1\) K (most probably \(E^*_C\) is even smaller because the effect of capacitance renormalization should be quite strong in \(\ref{sec:3}\)). This yields an upper limit estimate for the high frequency cutoff \(\omega_c \sim 10^{11} \div 10^{12}\) Hz (most probably even lower). As a result the estimate for the decoherence rate \(1/\tau_{\varphi}\) extracted by AAG from our simple formula should be reduced by a factor \(\sim (\omega_c/\tau_e)^{3/2} \sim 10^{-5} \div 10^{-6}\). We see that already such a simple estimate allows to completely remove 5 orders of magnitude disagreement reported by AAG for the experiment\(\ref{sec:3}\). An analogous conclusion can be reached in the case of other granular systems, e.g.\(\ref{sec:3}\). By saying that we do not want to claim a good agreement of our theory with these experiments. Rather we want to emphasize that a simple approximation (\(C1\)) (and hence our simple formulas derived on its basis) cannot be applied e.g. for granular systems at frequencies exceeding \(\omega_c \ll 1/\tau_e\). The whole theory should be substantially modified in this case.

We believe that the same conclusion can be made for metallic glasses. There the problem to construct a reasonable model for the dielectric susceptibility appears to be even more complicated. Such systems are usually strongly...
is not quite clear to which extent the glasses can be considered as homogeneous materials. In some cases granular structure was reported, so that some properties of metallic glasses can be similar to those of granular materials. These systems are clearly different from simple metals in many respects and therefore the simple approximation (C1) can hardly be applied already at moderately high frequencies, certainly well below 10^{16} Hz. As in the case of granular metals the effective cutoff \( \omega_c \sim 10^{12} \div 10^{13} \) Hz would allow to completely remove the disagreement in all cases. But – we repeat – the systems are very complicated and it is not clear whether our formulas can be applied even at \( \omega \lesssim \omega_c \).

Another very important condition used in our theory is \( p_F l \gg 1 \) or, equivalently, \( \epsilon_F \tau_c \gg 1 \), where \( \epsilon_F \) is the Fermi energy. Only provided this condition is satisfied the quasiclassical diffusion picture can be applied. Again in the experiments with 3d and 2d metallic glasses this condition is not satisfied. In all the experiments which cannot be described by our formulas the parameter \( \epsilon_F \tau_c \) is smaller than 5. Thus the simple quasiclassical approximation used in our paper does not work in these cases. An interesting illustration of this point can be extracted from the experimental data. In this work the two 2DEG samples with \( \epsilon_F \tau_c \sim 3 \) and one sample with \( \epsilon_F \tau_c \sim 45 \) were studied. For the strongly disordered samples the measured \( \tau_c \) at low temperatures exceeds the value predicted by our theory by the factor \( \sim 200 \div 300 \), while for a weakly disordered sample the agreement between the theory and the experiment is very good (\( \tau_c^{exp}(T_{min} \sim 300\text{mK}) = 1.5 \times 10^{-10} \) sec, \( \tau_c^{theor}(T = 0) = 2 \times 10^{-10} \) sec).

Yet another problem is the accuracy of independent measurements of the parameters of such complicated systems as strongly disordered granular materials and metallic glasses. Many authors estimate the elastic scattering time \( \tau_e \) using the value for the density of states obtained within the free electron model. Komori et al. extracted the value \( \tau_e \) for disordered Cu films directly from the measured magnetoresistance curves and arrived at much higher values for \( \tau_c \) than obtained by another method. If we use the estimate we will immediately conclude that our theory works well in the case of disordered Cu films. Here we do not want to discuss which way of evaluation of \( \tau_c \) is better. Rather we would like to emphasize that in many cases the accuracy of determination of the system parameters is clearly insufficient for quantitative comparison with theoretical predictions.

It is also interesting to note that in the experiments with 3d metallic glasses the saturation of \( \tau_c(T) \) was observed at relatively high temperatures, \( T \sim 1 \div 4 \) K depending on the experiment. According to the authors, this saturation can be explained neither by heating nor by the effect of magnetic impurities. If so, what could be an alternative explanation for this effect? If, following the authors, one adopts a free electron model of metallic glasses and estimates the values of \( \sigma, \tau_c \) and other parameters, one immediately arrives at the conclusion that the simple Drude formula (C1) is not applicable in this case and the noise should be greatly reduced. If, furthermore, one assumes (again following the authors) that in the metallic glasses the effect of electron-phonon interaction is more important than that of electron-electron interaction, then the experimental data should be compared with the eq.(82) of Ref. 3. The results are summarised in the Table I:

| Sample   | 1/\(\tau_c\), Hz | \(p_F l\) | \(E_F \tau_c\) | \(\sigma\), Hz | \(c\), km/sec | \(1/\tau_c^{exp}(T = 0)\), Hz | \(1/\tau_c^{\text{el-ph}}(T = 0)\), Hz |
|----------|------------------|----------|---------------|---------------|--------------|----------------|-----------------|
| Cu_{70}Al_{30} | 3 \times 10^{15}  | 4        | 2             | 2.9 \times 10^{15} | 6            | 1 \times 10^{10} | 6 \times 10^{10} |
| Cu_{50}Y_{50} | 3.2 \times 10^{15} | 6        | 3             | 5.7 \times 10^{15} | 2            | 4 \times 10^{9} | 1 \times 10^{9}  |
| Y_{40}Si_{60} | 1.1 \times 10^{16} | 2        | 1             | 1.8 \times 10^{15} | 2            | 1 \times 10^{9} | 3 \times 10^{11} |

TABLE I.

Here \(c\) is the sound velocity. Taking into account the uncertainty of the experimental parameters as well as the accuracy of the theory, we can conclude that the agreement between theory and experiment is good.

The electron-phonon formulas also well describe the experiments in 2d Mg films. The results are given in the Table II:

| Sample | 1/\(\tau_c\), Hz | \(p_F l\) | \(E_F \tau_c\) | \(\sigma\), Hz | \(c\), km/sec | \(1/\tau_c^{exp}(T_{max})\), Hz | \(1/\tau_c^{\text{el-ph}}(T = 0)\), Hz |
|--------|------------------|----------|---------------|---------------|--------------|----------------|-----------------|
| Mg3    | 1.5 \times 10^{10} | 1.63    | 0.8           | 1.6 \times 10^{15} | 6            | 9 \times 10^{9} | 6 \times 10^{10} |
| Mg4    | 6.1 \times 10^{15} | 3.55    | 1.8           | 3.6 \times 10^{15} | 6            | 7 \times 10^{9} | 7 \times 10^{9}  |
| Mg5    | 2.16 \times 10^{15} | 10      | 5             | 1 \times 10^{16} | 6            | 1.5 \times 10^{8} | 5.8 \times 10^{8} |

TABLE II.

Again the agreement between the theory and the experiment is reasonable. Thus in the above cases our theory provides a natural explanation for the saturation of \(\tau_c\) at low temperatures. No alternative explanation is known to us at the moment.

The saturation of the decoherence time extracted from the magnetoconductance measurements was by now observed
in very many experiments. Typically it occurs already at relatively high temperatures of order 1 K. Although in some cases it can be due to extrinsic factors and/or magnetic impurities these reasons definitely cannot explain the saturation effect in all cases. Just for an illustration let us consider three experiments from those quoted in Ref. 5.

The decoherence time saturation was observed in Ref. 2. In Ref. 2 the authors argued that this saturation can be due to the effect of external noise. Let us not discuss this explanation but just accept it for a moment. AAG concluded that the external noise may effectively destroy the phase coherence without heating the sample if the resistance of the latter is much smaller than the quantum resistance 24 KΩ. In the experiment the resistance was smaller, so in that case the necessary condition was satisfied.

The decoherence time saturation was also observed in the experiments. In this case the resistance was at the MΩ level, so that according to the authors external noise cannot be the reason for this saturation. At the same time the authors observed a rapid increase of the resistance which they interpreted as a crossover to the strongly localized regime. As this crossover occurs close to the temperature where \( \tau_\phi \) starts saturating the authors concluded that this saturation is not meaningful because \( \tau_\phi \) makes little sense in the strongly localized regime. Although this argument is essentially linked to the interpretation of the resistance increase as a Thouless crossover, and although even in this case we do not quite understand why the dephasing length \( L_\phi \) extracted from the magnetoresistance measurements should stay constant in the strongly localized regime being 3 times smaller than the localization length, for a moment let us accept this argument of the authors too.

The third experiment we are going to discuss is one by Pooke et al. 23. These authors also observed the decoherence time saturation in the temperature range \( T \lesssim 1 \) K in three samples with resistances 120, 240 and 360 KΩ. According to AAG external noise cannot cause this saturation (unless it produces overheating) since the resistance of each of these 3 samples was much bigger than 24 KΩ. Although the authors did not present the resistance curves one can hardly expect the Thouless crossover to take place in their measurements in the temperature range 0.1 \( \div \) 1 K: e.g. for the 120 KΩ sample the dephasing length \( L_\phi \) saturates at the level nearly 10 times smaller than the localization length. We believe that under these conditions it is already quite difficult to argue that in the experiments the “saturated” part of the curve \( \tau_\phi(T) \) is meaningless. Since the effect of magnetic impurities and heating are excluded in this case one should think about yet another explanation of the effect specifically for the experiments. We are not aware of any proposal in this direction at the moment.

Now let us recall that the results of all 3 experiments are in a quantitative agreement with our theory (see Fig. 5). The parameters in these experiments differ by several orders of magnitude, but for practically all samples the agreement is within a numerical factor of order one. We believe this agreement can hardly be interpreted as a simple coincidence. Rather we can assume that this agreement strongly supports the validity of our calculation which offers the same explanation for these three and many other experiments and makes it unnecessary to search for a special explanation for each of them separately. In some cases the agreement is very good, for other experiments it is only qualitative, but this is rather a question of finding a proper model for \( \epsilon(\omega, k) \) and/or the high frequency cutoff \( \omega_c \) for the effective environment for this or that particular system. Different experiments are carried out on physically very different structures and it would be naive to expect that there can exist a unique formula which alone could quantitatively describe all existing weak localization experiments. But the saturation effect at low temperatures appears to be a unique intrinsic property of very many systems, and this effect is naturally explained by our theory.

Finally, let us briefly discuss the experiments where the crossover to the insulating behavior was observed. We believe that the experimental results in 2d structures quoted by the authors by no means can be used as an argument against our theory. In all these experiments the crossover to the insulating state was observed for strongly disordered systems (\( \epsilon_F \tau_c \) was always of order one or only slightly larger). This is well beyond the applicability range of our theory.

Furthermore, at least in some cases (e.g. Ref. 25) the films could have the granular structure on the nanometer scale. If this is the case the insulating crossover can be also explained in terms of the Coulomb blockade of electrons on grains which does not require any phase coherence of these electrons at all. It is well known and was demonstrated experimentally (see e.g. Ref. 5) that this effect occurs in granular metals and arrays at temperatures \( T \lesssim E_C^* \), where \( E_C^* \) is the effective (renormalized) charging energy of the grain. If the resistance of a granular array or film (or, equivalently, the intergrain resistance) is (roughly) above the quantum resistance unit \( R_q \approx 24 \) KΩ, the capacitance renormalization is not important and the bare charging energy \( E_C \) sets the temperature scale for the metal-to-insulator crossover \( T_c \sim E_C \). The estimate \( E_C \approx 10 \) K is reasonable for the grain size in the nanometer range and is consistent with the experimental data. If the intergrain resistance is lower than \( R_q \) the effective charging energy gets strongly renormalized due to charge fluctuations (see e.g. Ref. 5). As a result one gets \( E_C^* \approx E_C \exp(-A G R_q) \), where \( G \) is the film resistance and \( A \) is constant. For ordered square 2d granular arrays one has \( A = 2 \), and \( A \) is definitely bigger in a disordered case. This might also explain the exponential dependence of \( T_c \) on \( G R_q \) reported e.g. in Ref. 27. We can also quote recent theoretical results by Nazarov 26 who demonstrated that charging effects may persist in disordered conductors even without grains and tunnel barriers. All these facts as well as some other experimental details (like e.g. an obvious inconsistency of the experimental data with the standard scenario of the orthogonal-to-unitary transition
in the magnetic field) suggest that interactions can play quite an important role in the above experiments. It is not our purpose to discuss this issue in more details here. But in any case it is quite clear that the above experimental data can hardly be interpreted as contradicting to our theory.

The same conclusion can be made concerning the experiments where the metal-to-insulator crossover in quasi-1d structures was reported and interpreted as the evidence for the Thouless crossover to the strongly localized regime at low $T$. It is clear, however, that the “noninteracting” scenario of this crossover cannot be applied in this case. For instance, at the crossover temperature $T \sim 1 \text{ K}$ the measured value of $T \tau_\varphi$ was found to be $T \tau_\varphi \sim 0.3 \div 0.5$ depending on the sample and remained of order one even deep in the weak localization regime (e.g. at $T \sim 10 \text{ K}$). As it was already discussed above the dephasing length $L_\varphi$ was found to saturate at the value 3 times smaller than the “noninteracting” localization length. All that implies that interaction effects play a very important role in the experiments. This was also acknowledged by the authors of the experiments. If so, the physical origin of the crossover can be debated. It is not clear how the Thouless scenario should be modified in the presence of strong interaction. There exist other mechanisms of localization which are solely due to interaction. E.g. at $T = 0$ a quantum particle can get localized in a periodic potential if it interacts with an Ohmic bath of oscillators. In the subohmic case it is localized even in the absence of any potential. Also the actual role of Coulomb-blockade-type of effects in disordered conductors should be understood better. All these effects do not involve the phase coherence of electrons and therefore would be compatible with the low temperature saturation of the decoherence length $L_\varphi$ predicted by our theory and found in experiments.

Summarizing our discussion of the experiments we can conclude that in most cases discussed the comparison with our theory cannot be carried out at all because the corresponding systems are very strongly disordered (the typical value of the mean free path is in the range $l \sim 10^{-8} \text{ cm}$) and therefore cannot be described by our quasiclassical theory. For other experiments the agreement is reasonable and can be improved further if a realistic model for $\epsilon(\omega, k)$ is chosen. In many cases, especially for quasi-1d systems, our theory is in a quantitative agreement with experiments. Further experiments are needed to study the effect of low temperature saturation of $\tau_\varphi$ in more details.

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