Theoretical and observational constraints on regularized $4D$ Einstein-Gauss-Bonnet gravity

Jia-Xi Feng,1,2, * Bao-Min Gu,1,2, † and Fu-Wen Shu1,2,3, ‡

1Department of Physics, Nanchang University, Nanchang, 330031, China
2Center for Relativistic Astrophysics and High Energy Physics, Nanchang University, Nanchang, 330031, China
3Center for Gravitation and Cosmology, Yangzhou University, Yangzhou, China

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Regularized Einstein-Gauss-Bonnet (EGB) theory of gravity in four dimensions is a new attempt to include nontrivial contributions of Gauss-Bonnet term. In this paper, we make a detailed analysis on possible constraints of the model parameters of the theory from recent cosmological observations, and some theoretical constraints as well. Our results show that the theory with vanishing bare cosmological constant, $\Lambda_0$, is ruled out by the current observational value of $w_{de}$, and the observations of GW170817 and GRB 170817A as well. For nonvanishing bare cosmological constant, instead, our results show that the current observation of the speed of GWs measured by GW170817 and GRB 170817A would place a constraint on $\alpha$, a dimensionless parameter of the theory, as $-7.78 \times 10^{-16} \leq \alpha \leq 3.33 \times 10^{-15}$.

I. INTRODUCTION

The first detection of gravitational waves (GWs) by LIGO/Virgo [1] begins to have a profound impact on our understanding of the nature. They provide new powerful ways to explore physics of the Universe. GW170817 [2], the first detected GW event with electromagnetic counterparts, extensively enriched the ways. From then on, a new era of multimeessenger GW astronomy has begun. Fermi Gamma-Ray Burst Monitor [3] and the International Gamma-Ray Astrophysics Laboratory [4] observed a gamma ray burst GRB 170817A after 1.74 $\pm$ 0.05s, on which a range of constraint on the speed of GWs can be obtained. Particularly, with the assumption that the GW signal was emitted at most 10s before the GRB signal, one can obtain a bound on the velocity of the GWs, namely, $-3 \times 10^{-15} \leq \frac{v_{gw}}{c} - 1 \leq 7 \times 10^{-16}$ [5].

The observations of GW events [1, 2, 6–9] in recent years, of course, support the validity of Einstein’s theory enough. However, whether alternative theories of gravity which can do equally well as Einstein’s theory can be constructed or not? This point has attracted a large number of researchers to study, such as scalar-tensor theories [10–20], vector-tensor theories [21], and so on. With more and more GW events to be detected in the future, it is expected that constraints on the speed of GWs will be more and more stringent. This makes it an effective tool to test the alternative theories of gravity [18–28].

Hence, we are paying attention to modified theories of gravity. One of the most elegant modifications is the Einstein-Gauss-Bonnet gravity. It is generally discussed that the extension of higher derivatives by adding the polynomial invariants of the Riemann tensor to the Einstein-Hilbert action is admitted in Einstein’s gravity. The field equations which involve four derivatives may lead to renormalizability, while the theory contains an inevitable ghostlike massive graviton [29]. It was found that the Gauss-Bonnet (GB) term, which is a quadartic combination of the Riemann curvature tensor, keeps the equations at second-order in the metric and hence is free of the ghost [30, 31]. In four or lower dimensions, however, these specific combinations of tensor polynomials either vanish or become total derivative. The trivialness in four dimensions excludes it as a more realistic model.

In [32], a new theory called 4 dimensional Einstein-Gauss-Bonnet (4D EGB) gravity was proposed. It considers a $D \rightarrow 4$ limit of the D-dimensional Gauss-Bonnet gravity by rescaling the GB dimensional coupling constant $\alpha \rightarrow \hat{\alpha}/(D - 4)$. The idea is to introduce the divergent coefficient to cancel the vanishing contribution of $\mathcal{G}$ in four dimensions, in a manner that is conceptually similar to the dimensional regularization procedure used in quantum field theories. The goal of this is to produce a nontrivial gravity theory in four dimensions that includes a non-vanishing contribution from the Gauss-Bonnet term. A large number of relevant works has been done in the past few months [33–63]. However, the resulting theory has been questioned a lot. The theory is found to be not well defined in the limit $D \rightarrow 4$ [64–69]. Moreover, the vacua of the model are unstable or ill-defined too [70]. To overcome this, several regularization schemes have been proposed [66–69, 71]. This generally leads to a scalar-tensor gravity, being a subclass of Hordenski theories [72].

In this work, we will perform a detailed analysis of cosmological perturbations of the regularized model around the FRW universe. The speed of tensor modes can be read off from these perturbative equations. Then we look for possible constraints on the coupling constant $\alpha$ of the regularized model through latest observational constraints on the speed of GWs from GW170817 [2] and GRB 170817A [5]. Our results show that, for the theory with $\Lambda_0 = 0$, where $\Lambda_0$ is the bare cosmological constant, there are two contradictions: one is the theoretical requirements of the model are contradicted with the current observational value of $w_{de}$. The other is the constraints imposed from GW170817 and GRB 170817A disagree with the current cosmological constraint on the ratio of energy densities between dark energy and matter, $\frac{\Omega_{de}}{\Omega_m}$. Therefore, the theory $\Lambda_0 = 0$ is ruled out. The case with $\Lambda_0 \neq 0$, however, receives a constraint from the speed of GWs measured by GW170817 and GRB 170817A, explicitly, $-7.78 \times 10^{-16} \leq \hat{\alpha} \leq 3.33 \times 10^{-15}$. To see whether the scalar perturbations will give more stringent constrains or not,
we discuss scalar perturbations as well.

The rest of the paper is organized as follows. In section II, we briefly review the regularized Einstein-Gauss-Bonnet theory in $D$ dimensions with cosmological constant. After applying it to the FRW universe, a set of dynamical equations are obtained, followed by a set of cosmological solutions. In section III, we perform linear perturbation analysis around FRW background. The quadratic action and the velocity of gravitational waves are obtained. In section IV, we apply the observational constraints from GW170817 and GRB 170817A to restrict the coupling constant $\hat{\alpha}$ of the model. In section V, the constrains from the scalar perturbations which may be more stringent are discussed as well. A brief concluding remark is drawn in the last section.

II. REGULARIZED EINSTEIN-GAUSS-BONNET THEORY IN FOUR DIMENSIONS

The action of Einstein-Gauss-Bonnet theory in $D$ dimensions with cosmological constant is

$$S = \int_{\mathcal{M}} d^D x \sqrt{-g} \left( R - 2\Lambda_0 + \alpha G \right) + S_m, \quad (1)$$

where $\alpha$ is a coupling constant, $S_m$ is the action associated with matter field, and the Gauss-Bonnet term is

$$G = R^2 - 4R_{\mu\nu} R^{\mu\nu} + R_{\mu\nu\alpha\beta} R^{\mu\nu\alpha\beta}. \quad (2)$$

The idea of $[32]$ is to construct a nontrivial theory by considering a replacement $\alpha \rightarrow \frac{\hat{\alpha}}{(D-4)}$. This, however, turns out to be questionable in many aspects. In particular, it was found the theory defined in this way has no well-defined limit $[64–70]$. The way to fix this pathology is to perform a regularization. There are several regularization schemes in the literatures, such as the Kaluza–Klein-reduction procedure $[66, 67]$, the conformal subtraction procedure $[68, 69]$, and ADM decomposition analysis $[71]$. The first two approaches give rise to the same regularized action, which is of the following form

$$S = \int_{\mathcal{M}} d^4 x \sqrt{-g} \left[ R - 2\Lambda_0 + \hat{\alpha} \left( 4G^{\mu\nu} \nabla_\mu \phi \nabla_\nu \phi - \phi G \right) 
+ 4 \Box \phi (\Box \phi)^2 + 2(\Box \phi)^4 \right] + S_m, \quad (3)$$

where $\phi$ is a scalar field inherent from $D$ dimensions. It is introduced by Kaluza–Klein reduction of the metric $[66, 67]
$$ds_D^2 = ds_4^2 + e^{2\phi} d\Omega_{D-4}^2.$$ or by conformal subtraction $[68, 69]$ where the subtraction background is defined under a conformal transformation $g_{ab} \rightarrow e^{2\phi} g_{ab}.$

Varying with respect to the metric, we can get the field equations

$$G_{\mu\nu} + \Lambda_0 g_{\mu\nu} = \hat{\alpha} \hat{\mathcal{H}}_{\mu\nu} + T_{(m)}^{\mu\nu}, \quad (4)$$

where $T_{(m)}^{\mu\nu}$ is the energy-momentum tensor of the matter field, considering the matter context of the universe is a perfect fluid, so that the energy-momentum tensor take the form $[73]$

$$T_{(m)}^{\mu\nu} = (\rho_m + p_m) U_\mu U_\nu + p_m g_{\mu\nu}, \quad (5)$$

where $\rho_m$, $p_m$ and $U_\mu$ are respectively energy density, pressure and four-velocity of the fluid. And

$$\hat{\mathcal{H}}_{\mu\nu} = 2R(\nabla_\mu \nabla_\nu \phi - \nabla_\mu \phi \nabla_\nu \phi) + 2G_{\mu\nu} \left( (\nabla \phi)^2 - 2\Box \phi \right) + 4G_{\nu\alpha} (\nabla^\alpha \nabla_\mu \phi - \nabla^\alpha \phi \nabla_\mu \phi)$$
$$+ 4G_{\mu\nu} \phi \nabla_\mu \nabla_\nu \phi - \nabla_\mu \phi \nabla_\nu \phi + 4G_{\mu\nu\alpha\beta} (\nabla^\alpha \nabla_\beta \phi - \nabla^\alpha \phi \nabla_\beta \phi)$$
$$+ 4\Box \phi \nabla_\mu \nabla_\nu \phi - g_{\mu\nu} \left[ 2R(\Box \phi - (\nabla \phi)^2) + 4G_{\nu\alpha} (\nabla_\alpha \nabla_\beta \phi - \nabla_\alpha \phi \nabla_\beta \phi) + 2(\Box \phi)^2 \right.$$
$$\left. - (\nabla \phi)^4 + 2 \nabla_\beta \nabla_\alpha \phi (2 \nabla^\alpha \phi \nabla^\beta \phi - \nabla^\beta \nabla^\alpha \phi) \right] \quad (6)$$

By varying with respect to the scalar field, we get

$$\frac{1}{8} \hat{G} = R^\mu\nu \nabla_\mu \phi \nabla_\nu \phi - G^\mu\nu \nabla_\mu \phi \nabla_\nu \phi - \Box \phi (\Box \phi)^2$$
$$+ (\nabla_\mu \nabla_\nu \phi)^2 - (\Box \phi)^2 - 2 \nabla_\mu \phi \nabla_\nu \phi \nabla^\mu \nabla^\nu \phi \quad (7)$$

The trace of the field equations (4) is found to satisfy

$$R + \frac{\hat{\alpha}}{2} \hat{G} - 4\Lambda_0 = -T, \quad (8)$$

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1 This action belongs to a subclass of the Horndeski gravity $[10, 74]$ with $G_2 = 8 \delta X^2 - 2\Lambda_0, G_3 = 8 \delta X, G_4 = 1 + 4 \delta X$ and $G_5 = 4 \delta \ln X$ (where $X = -\frac{1}{2} \nabla_\mu \phi \nabla^\mu \phi$).
where \( T = -\rho_m + 3p_m \).

Assuming that the line-element describing by spatially-flat Friedmann-Robertson-Walker (FRW) metric is
\[
ds^2 = -dt^2 + a^2(t)(dx_1^2 + dx_2^2 + dx_3^2),
\]
then taking a direct calculation, we show that the equations of motion become
\[
3H^2 = \rho_{GB} + \rho_m + \rho_\Lambda, \\
2\dot{H} + 3H^2 = -\rho_{GB} - \rho_m - \rho_\Lambda,
\]
where the energy density, pressure of the GB term and the cosmological constant term are defined as
\[
\rho_{GB} \equiv \left(3\dot{\phi}^4 + 12\dot{\phi}^2 H + 18\dot{\phi}^2 H^2 + 12\dot{\phi}^3 H^3\right)\dot{\alpha}, \\
p_{GB} \equiv \left[\dot{\phi}^4 - 4\dot{\phi}^2 \ddot{\phi} - 4\dot{\phi}^2 (H + H^2) - 2\dot{\phi}^2 H^2 \\
-8\dot{\phi}^2 H - 8\dot{\phi}(H + H^2)H - 4\ddot{\phi} H^2\right]\dot{\alpha}, \\
\rho_\Lambda \equiv \Lambda_0, \\
p_\Lambda = -\Lambda_0.
\]
And the scalar field equation, which is equivalent to \( \nabla^\mu H_{\mu\nu} = 0 \), reduces to,
\[
\partial_i[(a\ddot{\phi} + \dot{\phi})^2] = 0, \\
\]
which can be solved simply by
\[
\dot{\phi} = -H, \\
\]
or
\[
\dot{\phi} = -H + \frac{A}{a},
\]
where \( H \) is the Hubble parameter, dot denotes differentiation with respect to \( t \), \( A \) is the integration constant. The latter solution with \( A = 0 \) corresponds to the former which is the isotropically expanding solution, and these solutions are similar with [66, 67].

### III. THE SPEED OF GRAVITATIONAL WAVES

To study the gravitational waves, let us consider the linear tensor perturbations of the FRW metric,
\[
ds^2 = -dt^2 + a^2(t)\left(\delta_{ij} + h_{ij}\right)dx^i dx^j,
\]
where the tensor \( h_{ij} \) satisfies the transverse-traceless condition, \( \partial_i h_{ij} = 0 = \delta^{ij} h_{ij} \). Then the linear order field equation of \( h_{ij} \) can be expressed as
\[
\beta_1 \ddot{h}_{ij} + \left(\beta_1 + 3H\beta_1\right)\dot{h}_{ij} - \beta_2 \frac{\nabla^2}{a^2} h_{ij} = 0.
\]
The coefficients \( \beta_1 \) and \( \beta_2 \) are defined as
\[
\beta_1 = 1 - 2\dot{\phi}^2 - 4\dot{\phi}\ddot{\phi}H, \\
\beta_2 = 1 + 2\dot{\phi}\ddot{\phi} - 4\dot{\phi}^2.
\]
The corresponding quadratic action is
\[
S_h = \int dt d^3x a^3 \left[\beta_1 \dot{h}_{ij}^2 - \frac{\beta_2}{a^2} \left(\nabla h_{ij}\right)^2\right].
\]
To avoid ghost and gradient instability [74], the two coefficients \( \beta_1 \) and \( \beta_2 \) should be positive, namely, \( \beta_1 > 0 \) and \( \beta_2 > 0 \). This imposes constraints on the coupling constant \( \dot{\alpha} \), and we will recall these constraints in next section.

It is more convenient to make the Fourier transformation and write the tensor perturbation \( h_{ij}(\vec{x}, t) \) as
\[
h_{ij}(\vec{x}, t) = \int \frac{d^3k}{(2\pi)^3} \left[h_{ij}^+ (t) A_{ij}^+ + h_{ij}^- (t) A_{ij}^-\right] \exp \left(ik \cdot \vec{x}\right),
\]
where \( k^i A^+_{ij} = A^-_{ii} = 0, A^-_{ij} A^+_{ij} = 2\delta_{ij} \), and the superscript \( \text{"}+\text{"} \) stands for the \( \text{"}+\text{"} \) or \( \text{"}\times\text{"} \) polarizations. In terms of the Fourier modes, we have
\[
h_k^+ + (3 + \alpha_M) H h_k^+ + \frac{c_{gw}^2k^2}{a^2} h_k^- = 0,
\]
where
\[
\alpha_M = \frac{\beta_1}{H\beta_1}, \\
c_{gw}^2 = \frac{\beta_2}{\beta_1}.
\]
\( c_{gw} \) is actually the propagation speed of the gravitational waves, \( \alpha_M \) is a dimensionless parameter which describes the the running of the effective Planck mass. We see that there are modifications to the Hubble friction and the gravitational wave speed. Using the expressions of \( \beta_1 \) and \( \beta_2 \), we get
\[
\alpha_M = \frac{-4\dot{\phi}\ddot{\phi} - 4\dot{\phi}\ddot{\phi}H - 4\dot{\phi}\ddot{\phi}}{H(1 - 2\dot{\phi}^2 - 4\dot{\phi}\ddot{\phi})}, \\
c_{gw}^2 = \frac{1 + 2\dot{\phi}^2 - 4\dot{\phi}\ddot{\phi}}{1 - 2\dot{\phi}^2 - 4\dot{\phi}\ddot{\phi}}.
\]
Recalling the solution (13), we find that both \( \alpha_M \) and \( c_{gw} \) are functions of \( H \) and its derivative, whose form depends on cosmological models as we will see in the next section.

### IV. THE EFFECT OF THE SPEED OF GRAVITATIONAL WAVES IN THE GAUSS-BONNET THEORY

In this section we would like to consider possible constraints on the Gauss-Bonnet coefficient \( \dot{\alpha} \) from the current cosmological and gravitational waves’ observations.

Substituting the solution (13) into \( \beta_1, \beta_2 \) and equations (23), we have
\[
\beta_1 = 1 + 2\dot{\alpha} H^2, \\
\beta_2 = 1 + 2\dot{\alpha} H^2 + 4\dot{\alpha} \ddot{H},
\]
and
\[
\alpha_M = \frac{4\dot{\alpha} \ddot{H}}{1 + 2\dot{\alpha} H^2}, \\
c_{gw}^2 = \frac{4\dot{\alpha} \ddot{H}}{1 + 2\dot{\alpha} H^2}.
\]
This expression of the propagation speed of the tensor modes is the same as the form in [71]. And the energy density and pressure of the GB term are given by
\[
\rho_{GB} = -3\dot{\alpha}H^4, \\
p_{GB} = \dot{\alpha}H^2(3H^2 + 4\dot{H}),
\] (28)
then the equations of motion (10) become
\[
3H^2 = \rho_m - 3\dot{\alpha}H^4 + \Lambda_0, \\
2\dot{H} + 3H^2 = -p_m - \alpha H^2(3H^2 + 4\dot{H}) + \Lambda_0. 
\] (29)

In the rest part, two cases will be discussed: one is \(p_m = 0\) while \(\Lambda_0 \neq 0\), the other is \(p_m = 0\) and \(\Lambda_0 = 0\). However, the latter case is found to have two contradictory points which will be discussed latter.

**A. \(p_m = 0, \Lambda_0 \neq 0\)**

Now let us focus on the case where \(p_m = 0\) and \(\Lambda_0 \neq 0\), describing current acceleration of the universe. Constraints on \(\dot{\alpha}\) will be performed at great length in what follows.

First, the energy density of matter can be obtained by the equation of motion (29)
\[
\rho_m = -2\dot{H}(t) - 4\dot{\alpha}H^3\dot{H}(t). 
\] (30)
Defining a dimensionless parameter \(\tilde{\alpha}\)
\[
\tilde{\alpha} = \dot{\alpha}H^2, 
\] (31)
then the ratio of the current value of the energy densities between dark energy and matter is given by
\[
\frac{\Omega_{de}}{\Omega_m} = \frac{\rho_{GB} + \rho_\Lambda}{\rho_m} = \frac{3H_0^2 - \rho_m}{\rho_m} = \frac{3H_0^2 + 2\dot{H}(t_0) + 4\dot{\alpha}H(t_0)}{-2H(t_0) - 4\dot{\alpha}H(t_0)},
\] (32)
where \(\rho_{de} = \rho_{GB} + \rho_\Lambda\), and \(H_0\) is the Hubble constant in present universe. Meanwhile, the current equation of state parameter for dark energy is of the form
\[
w_{de} = \frac{\rho_{GB} + \rho_\Lambda}{\rho_m} = -2\dot{H}(t_0) - 3H_0^2 \frac{3H_0^2 + 2\dot{H}(t_0) + 4\dot{\alpha}H(t_0)}{3H_0^2 + 2\dot{H}(t_0) + 4\dot{\alpha}H(t_0)},
\] (33)
where \(w_{de} = \frac{\rho_{GB} + \rho_\Lambda}{\rho_m}\). Current cosmological observation suggests that the ratio is approximately \(\Omega_{de}/\Omega_m = 7/3\) [75], we hence get
\[
\dot{H}(t_0) = -\frac{9H_0^2}{20(1 + 2\tilde{\alpha})}. 
\] (34)

In what follows we would like to show that four possible constraints on the model parameters \(\tilde{\alpha}\) can be imposed, theoretically and observationally.

- **Constraints from \(\beta_1\) and \(\beta_2\)**

In section III we have shown that \(\beta_1 > 0\) and \(\beta_2 > 0\) should be satisfied so that the theory is free of ghost and gradient instability. From eqs. (24), (25) and (34) we have
\[
\beta_1 = 1 + 2\tilde{\alpha}, \\
\beta_2 = 1 + 2\tilde{\alpha} - \frac{9\tilde{\alpha}}{5(1 + 2\tilde{\alpha})^2}. 
\] (35) (36)
The positivity of \(\beta_i\) forces \(\tilde{\alpha}\) to be
\[
\tilde{\alpha} > -0.5. 
\] (37)

- **Constraints from \(w_{de}\)**

Substituting (34) into (33), we then get
\[
w_{de} = -\frac{7 + 20\tilde{\alpha}}{7 + 14\tilde{\alpha}}. 
\] (38)
This indicates that the observational value of \(w_{de}\) will place new constraint on the parameter \(\tilde{\alpha}\). Current cosmological observation shows that \(w_{de}\) is bounded by \(-1.1 < w_{de} < -0.9\) [75]. Inserting this into (38) one has
\[
-0.0945946 < \tilde{\alpha} < 0.152174. 
\] (39)

- **Constraints from \(\alpha_M\)**

The current cosmological constraints on \(\alpha_M\) is \(-0.434 < \alpha_M < 0.945\) (the parametrization I) at 95% confidence level [76]. Eqs. (26) and (34) lead to
\[
\alpha_M = -\frac{9\tilde{\alpha}}{5(1 + 2\tilde{\alpha})^2}, 
\] (40)
which implies
\[
\tilde{\alpha} < -1.28104, 
\] (41)
or
\[
\tilde{\alpha} > -0.195155. 
\] (42)
However, (37) shows that the constraint (41) should be abandoned.

- **Constraints from GW170817 and GRB 170817A**

Thanks to the first detection of an electromagnetic counterpart (GRB 170817A) to the gravitational wave signal (GW170817), we have a new powerful way in testing theories of gravity. It is well known this event gave rise to a new stringent bound on the speed of GWs has been suggested by using the GW170817 and the GRB 170817A [5]
\[
-3 \times 10^{-15} \leq c_{gw} - 1 \leq 7 \times 10^{-16}. 
\] (43)
On the other hand, from Eqs. (27) and (34) we have
\[
c_{gw}^2 = 1 - \frac{9\tilde{\alpha}}{5(1 + 2\tilde{\alpha})^2}. 
\] (44)
This places more constraints on the parameter $\tilde{\alpha}$

$$-7.78 \times 10^{-16} \leq \tilde{\alpha} \leq 3.33 \times 10^{-15}, \quad (45)$$
on

or

$$\tilde{\alpha} \leq -3.21 \times 10^{14}, \quad (46)$$

or

$$\tilde{\alpha} \geq 7.50 \times 10^{14}. \quad (47)$$

However, the bound (46) should be abandoned due to $\beta_1, \beta_2 > 0$ as shown in (37), and the bound (47) is also invalid due to (39).

In summary, combining all these constraints, the latest observations of the speed of GWs from GW170817 and GRB 170817A impose the most stringent one, which is

$$-7.78 \times 10^{-16} \leq \tilde{\alpha} \leq 3.33 \times 10^{-15}. \quad (48)$$

Note that the expression of $\Lambda_0$ can be obtained from Eqs. (29) and (34) as follow

$$\Lambda_0 = \left(3H_0^2 + 4H(t_0)\right)\tilde{\alpha} + 2H(t_0) - 3H_0^2,
= \frac{3}{10} \left(10\tilde{\alpha} - 13\right)H_0^2, \quad (49)$$

which shows that these two parameters are not independent.

**B. $p_m = 0, \Lambda_0 = 0$**

In this subsection, let us turn to consider the case where the theory has vanishing bare cosmological constant, namely, $p_m = 0$ and $\Lambda_0 = 0$.

From (29), it is straightforward to show that $H(t)$ and $\rho_m$ are, respectively, given by

$$H(t) = -\frac{3(1 + \tilde{\alpha})H^2}{2(1 + 2\tilde{\alpha})}, \quad (50)$$

$$\rho_m = 3H^2(1 + \tilde{\alpha}), \quad (51)$$

where, again, we have introduced a dimensionless parameter $\tilde{\alpha} = \alpha H^2$.

Just like what we did in $\Lambda_0 \neq 0$ case, four possible bounds on the model parameter $\tilde{\alpha}$ can be obtained.

- **Constraints from $\beta_1$ and $\beta_2$**

  In the present case, $\beta_1$ and $\beta_2$ become

  $$\beta_1 = 1 + 2\tilde{\alpha}, \quad (52)$$

  $$\beta_2 = 1 + 2\tilde{\alpha} - 6\tilde{\alpha} \cdot \frac{1 + \tilde{\alpha}}{1 + 2\tilde{\alpha}}. \quad (53)$$

  The requirement that $\beta_1 > 0$ thus place a constraint on $\tilde{\alpha}$ as

  $$-0.5 < \tilde{\alpha} < 0.366025. \quad (54)$$

  

- **Constraints from $w_{de}$**

  The expression for $w_{de}$ now becomes

  $$w_{de} = \frac{1}{\alpha} \left(1 - \frac{1 + \tilde{\alpha}}{1 + 2\tilde{\alpha}}\right), \quad (55)$$

  

  The current bound on $w_{de}$ is $-1.1 < w_{de} < -0.9$ [75] implies that

  $$-1.0556 < \tilde{\alpha} < -0.954545. \quad (56)$$

  

  Clearly, this result contradicts with (54), a theoretical requirement to guarantee the theory is free from ghost and instabilities. This strongly suggests that the model with vanishing bare cosmological constant is ruled out from current cosmological observations. In what follows, we will show another evidence to support this statement.

- **Constraints from $\alpha_M$**

  From (27) and (50) one gets

  $$\alpha_M = \frac{6\tilde{\alpha}(1 + \tilde{\alpha})}{(1 + 2\tilde{\alpha})^2}. \quad (57)$$

  

  Again we use the current cosmological constraints of $\alpha_M$, $-0.434 < \alpha_M < 0.945$ [76], then we get

  $$-1.09311 < \tilde{\alpha} < -0.89163, \quad (58)$$

  or

  $$-0.10837 < \tilde{\alpha} < 0.0931124. \quad (59)$$

  Clearly, the bound (58) should be abandoned due to $\beta_1, \beta_2 > 0$.

- **Constraints from the speed of GWs**

  Using the bound on the speed of GWs [5],

  $$-3 \times 10^{-15} \leq c_{gw} - 1 \leq 7 \times 10^{-16}, \quad (60)$$

  and the expression of $c_{gw}^2$ of this case

  $$c_{gw}^2 = 1 - \frac{6\tilde{\alpha}(1 + \tilde{\alpha})}{(1 + 2\tilde{\alpha})^2}, \quad (61)$$

  we find the following bounds

  $$-1 \times 10^{-15} \leq \tilde{\alpha} + 1 \leq 2 \times 10^{-16}, \quad (62)$$

  or

  $$-2.33 \times 10^{-16} \leq \tilde{\alpha} \leq 1.0 \times 10^{-15}. \quad (63)$$

  It is obvious that the bound (62) is not allowed because of (54).
In summary, if we put the inconsistency obtained from the constraint of \( w_{de} \) aside, we naively have a stringent bound (63). However, we should be very careful here. If we take the current cosmological observations into consideration, we find there is an inconsistency in this case. Particularly, the current cosmological observations put a severe constraint on the ratio of energy densities between dark energy and matter (i.e. \( \frac{\Omega_{de}}{\Omega_m} \sim \frac{7}{3} \)). Direct computation shows that the present case leads to the following ratio

\[
\frac{\Omega_{de}}{\Omega_m} = \frac{\rho_{GB} + 0}{\rho_m} = \frac{3H_0^2 - \rho_m}{\rho_m} = \frac{-\tilde{\alpha}}{1 + \tilde{\alpha}}, \tag{64}
\]

which is much much less than \( 7/3 \) after combining the result (63). This provides another evidence, in addition to the one given in (54) and (56), for the inconsistency of the model with vanishing bare cosmological constant. Hence, we conclude that the model in question does not admit a cosmological solution with vanishing bare cosmological constant, \( \Lambda_0 = 0 \).

V. THE SCALAR PERTURBATIONS

Now let us consider the scalar perturbations. We choose the unitary gauge, in which the fluctuation of the scalar field vanishes and all of the fluctuations are described by that of the spacetime metric. The line element is assumed as

\[
ds^2 = -(1 + 2\Phi)dt^2 + 2\beta \xi dt dx^i + a^2(1 + 2\Psi)\delta_{ij} dx^i dx^j. \tag{65}
\]

Varying the action with respect to \( \Phi \) and \( \xi \) leads to two constraints, corresponding to the energy and momentum constraints,

\[
\beta_1 \dot{\Phi} + \gamma_1 \Phi = 0, \tag{66}
\]
\[
\gamma_1 \frac{\nabla^2 \xi}{a^2} - \beta_1 \frac{\nabla^2 \Psi}{a^2} - 3\gamma_1 \ddot{\Phi} + \gamma_2 \Phi = 0. \tag{67}
\]

Here we introduce two coefficients,

\[
\gamma_1 = 6\alpha H^2 \dot{\phi} + 6\alpha H \dot{\phi}^2 + 2\alpha \dot{\phi}^3 - H, \tag{68}
\]
\[
\gamma_2 = 3H^2 - 2\Lambda_0. \tag{69}
\]

The variation of \( \Psi \) then gives a nontrivial field equation

\[
-H\beta_2 \nabla^2 \xi - \beta_1 \nabla^2 \xi + 3a^2 \beta_1 \dot{\Phi} - \beta_2 \nabla^2 \Psi + \gamma_3 \Psi = 0 \tag{70}
\]

where \( \gamma_3 \) and \( \gamma_4 \) are defined by

\[
\gamma_3 = 3a\dot{\phi}(2\beta_1 + \beta_2), \tag{72}
\]
\[
\gamma_4 = 3a^2(3H\gamma_1 + \gamma_1). \tag{73}
\]

Using the two constraints we can eliminate \( \Phi \) and \( \xi \), and get the equation for \( \Psi \),

\[
\sigma_1 \dot{\Psi} + (3H\sigma_1 + \sigma_4)\Psi - \frac{\sigma_2}{a^2} \nabla^2 \Psi = 0. \tag{74}
\]

The corresponding quadratic action is

\[
S_{\Psi} = \int dt d^3 x a^3 \left( \sigma_1 \dot{\Psi}^2 - \frac{\sigma_2}{a^2} \nabla^2 \Psi^2 \right). \tag{75}
\]

The coefficients \( \sigma_1 \) and \( \sigma_2 \) are defined as

\[
\sigma_1 = 3\beta_1 + \frac{\beta_2^2}{\gamma_1}, \tag{76}
\]
\[
\sigma_2 = -\beta_2 \left( 1 + \frac{H \beta_1}{\gamma_1} \right) - \beta_1 \frac{d}{dt} \left( \frac{\beta_1}{\gamma_1} \right). \tag{77}
\]

Similar to the tensor modes, the coefficients \( \sigma_1 \) and \( \sigma_2 \) should be positive to avoid ghost and gradient instability. let us follow the method in section IV. By defining a dimensionless variable \( \tilde{\Lambda} = \Lambda / H_0 \), \( \tilde{H}(t_0) \) can be obtained from the current value of \( \Omega_{de} / \Omega_m \)

\[
\tilde{H}(t_0) = \frac{9 + 4\alpha \tilde{A}^4}{20(1 + 2\tilde{\alpha})} H_0^2, \tag{78}
\]

which recovers the results in case A of section IV as \( \Lambda = 0 \). In addition, now we have

\[
\beta_1 = 1 - 2\alpha(1 - \tilde{A})^2 + 4\alpha(1 - \tilde{A}),
\]
\[
\beta_2 = \frac{5 + 6\alpha(11 + 10\Lambda^2) + 20\alpha^2(1 + \tilde{A}^2 - 2\tilde{A}^4)}{5 + 10\alpha},
\]
\[
\gamma_1 = \left( -6\alpha(1 - \tilde{A}) + 6\alpha(1 - \tilde{A})^2 - 2\alpha(1 - \tilde{A})^3 - 1 \right) H_0,
\]
\[
\gamma_2 = 6 \left( \frac{9}{5} - \tilde{\alpha} \right) H_0^2. \tag{79}
\]

Using \( \sigma_1 > 0 \) and \( \sigma_2 > 0 \), we get the constraints on \( \tilde{A} \) as shown in Fig 1.

![Fig. 1. Constraints on \( \tilde{\alpha} \) and \( \tilde{A} \) from theoretical bounds.](image)

VI. CONCLUSIONS AND DISCUSSIONS

In this paper, we give detailed analysis about the theoretical and observational constraints on the regularized 4D EGB theory. Our analysis is based on linear perturbation around the
FRW universe and is limited to the tensor modes such that we can deal with the gravitational waves. For these modes, the fluctuations of the scalar field $\phi$ are decoupled, and a set of linear perturbation equations are obtained, through which the speed of GWs can be read off.

Our results can be divided into two classes according to whether the bare cosmological constant $\Lambda_0$ is vanishing or not. For $\Lambda_0 = 0$, we find that theoretical requirements of the model are contradicted with the current observational results, indicating that the theory of this case should be ruled out. We make the conclusion from two strong evidences: one is from the theoretical contradiction with the current observations of $w_{de}$, the other comes from the huge (about 15 orders of magnitude) deviations between constraints from GW170817 and GRB 170817A and constraints from the current cosmological model are contradicted with the current observational results, and matter, $\Omega_{\text{de}}/\Omega_{\text{m}}$.

For $\Lambda_0 \neq 0$, however, one can place a stringent constraint on the dimensionless model parameter $\tilde{\alpha}$ (and $\Lambda_0$, since $\Lambda_0$ and $\tilde{\alpha}$ are not independent in this case as shown in (49)). Compared to theoretical and cosmological constraints (values of $w_{de}$ and $\alpha_M$), the constraint from the speed of GWs measured by GW170817 and GRB 170817A is much more stringent. Specifically, it is given by $-7.78 \times 10^{-16} \leq \tilde{\alpha} \leq 3.33 \times 10^{-15}$. One may expect that including the integration constant $A$ in (14) may give more stringent constraint. Indeed, for some values of $A$, $\tilde{\alpha}$ is forced to zero. To see this explicitly, let us follow what we did in section IV. From (78), we obtain that

$$w_{de} = -\frac{21 + 60\tilde{\alpha} - 40\tilde{\alpha}A^4}{21(1 + 2\tilde{\alpha})},$$
$$\alpha_M = \frac{4\tilde{\alpha}A^2}{1 - 2\tilde{\alpha}(A^2 - 1)},$$
$$c_{gw}^2 = 1 - \frac{9\tilde{\alpha} + 40\tilde{\alpha}^2A^4 - 20\tilde{\alpha}(1 + 2\tilde{\alpha})A^2}{5(1 + 2\tilde{\alpha})(1 + 2\tilde{\alpha} - 2\tilde{\alpha}A^2)}. \quad (80)$$

Using the constraints obtained in section IV and section V, we can make a plot (Fig. 2). From this plot we find that although for $|A| \lesssim 1.35$ we have less stringent constraints $|\tilde{\alpha}| < 3 \times 10^{-14}$, for $|A| \gtrsim 1.35$, however, $\tilde{\alpha}$ is restricted to zero.

**Note added:** After this work was completed, we learned a similar work [77], which appeared in arXiv a few days before. Their work focused on $\Lambda_0 = 0$ case, which should be ruled out due to inconsistency between theoretical requirements and current observations as suggested in our present work.

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