Kepley, Shane; Mireles James, J. D.
Chaotic motions in the restricted four body problem via Devaney’s saddle-focus homoclinic tangle theorem. (English) Zbl 1422.37062 J. Differ. Equations 266, No. 4, 1709-1755 (2019).

The authors introduce the mathematical formulation of the planar Circular Restricted Four Body Problem (CRFBP), consisting of three masses $0 < m_3 \leq m_2 \leq m_1 < 1$, such that $m_1 + m_2 + m_3 = 1$, and a forth massless particle (like an asteroid or space craft in co-rotating coordinates). They focus on the dynamics of the massless particle, when it moves in the gravitational field of the first three particles.

The paper contains several results. They review the parameterization method for invariant manifolds, and use the power series methods for the solution of the resulting partial differential equations.

The authors propose a change of variables which transforms the CRFBP to a polynomial system of seven differential equations. They “prove the existence of chaotic motions in an equilateral planar CRFBP, establishing that the system is not integrable. The proof works by verifying the hypotheses of a topological forcing theorem for Hamiltonian vector fields on $\mathbb{R}^4$ which hypothesizes the existence of a transverse homoclinic orbit in the energy manifold of a saddle focus equilibrium”. “Additionally, the method is constructive and yields additional byproducts such as the locations of transverse connecting orbits, quantitative information about the invariant manifolds, and bounds on transport times”. “Finally, the paper concludes with several appendices containing additional technical details”.

The paper combines theoretical and numerical results for the CRFBP. It is well written and contains many references.

Reviewer: Maria Gousidou-Koutita (Thessaloniki)

MSC:

37N05 Dynamical systems in classical and celestial mechanics
37J45 Periodic, homoclinic and heteroclinic orbits; variational methods, degree-theoretic methods (MSC2010)
37J15 Symmetries, invariants, invariant manifolds, momentum maps, reduction (MSC2010)
37J25 Stability problems for finite-dimensional Hamiltonian and Lagrangian systems
70F07 Three-body problems
70F10 n-body problems
70H14 Stability problems for problems in Hamiltonian and Lagrangian mechanics

Keywords:
circular restricted four body problem; parameterization method; invariant manifolds

Full Text: DOI arXiv

References:
[1] Darwin, G. H., Periodic orbits, Acta Math., 21, 1, 99-242, (1897) · Zbl 28.0851.03
[2] Moulton, F. R.; Buchanan, D.; Buck, T.; Griffin, F. L.; Longley, W. R.; MacMillan, W. D., Periodic Orbits, Number Publication, vol. 161, (1920), Carnegie Institution of Washington
[3] Strömgren, E., Connaissance actuelle des orbites dans le probleme des trois corps, Bull. Astron., 9, 87-130, (1934) · Zbl 0012.12801
[4] Castelli, Roberto; Lessard, Jean-Philippe, Rigorous numerics in Floquet theory: computing stable and unstable bundles of periodic orbits, SIAM J. Appl. Dyn. Syst., 12, 1, 204-245, (2013) · Zbl 1293.37033
[5] G. Shearing, PhD thesis, University of Manchester, 1960.
Szebehely, V.; Flandern, T. V., A family of retrograde orbits around the triangular equilibrium points, Astron. J., 72, 2, 198-201, (1967)

Arioli, Gianni; Barutello, Vivina; Terracini, Susanna, A new branch of Mountain Pass solutions for the choreographical 3-body problem, Nonlinearity, 11, 2, 37-75, (1998) · Zbl 0924.70015

Zbrowski, A.; Ziogas, P. D., The restricted problem of three bodies, I, Mat.-Fys. Skr. Danse Vid. Selsk., 3, 1, (1965), 53 pp · Zbl 0131.21305

Henrard, Jacques, Proof of a conjecture of E. Strömgren, Celestial Mech., 7, 449-457, (1973) · Zbl 0258.70005

Abraham, Ralph H., Chaostrophes, intermittency, and noise, (Chaos, Fractals, and Dynamics. Chaos, Fractals, and Dynamics, Guelph, Ont., 1981/1983. Chaos, Fractals, and Dynamics. Chaos, Fractals, and Dynamics, Guelph, Ont., 1981/1983, Lecture Notes in Pure and Appl. Math., vol. 98, (1985), Dekker: Dekker New York), 3-22 · Zbl 0577.58003

Deveney, Robert L., Blue sky catastrophes in reversible and Hamiltonian systems, Indiana Univ. Math. J., 26, 2, 247-263, (1977) · Zbl 0362.58006

Lerman, L. M., Behavior of a Hamiltonian system in a neighborhood of a transversal homoclinic saddle-focus trajectory, Uspekhi Mat. Nauk, 44, 2(266), 233-234, (1989)

Shilnikov, Leonid Pavlovich; Shilnikov, Andrey L.; Turaev, Dmitry V., Showcase of blue sky catastrophes, Internat. J. Bifur. Chaos Appl. Sci. Engrg., 24, 8, Article 1440003 pp., (2014) · Zbl 1300.34100

Piotosik, Antonis D., Infinite Feigenbaum sequences and spirals in the vicinity of the Lagrangian periodic solutions, Celestial Mech. Dynam. Astronom., 108, 2, 187-202, (2010), with supplementary material available online · Zbl 1223.70049

Sicardy, B., Stability of the triangular Lagrange points beyond Gascheau’s value, Celestial Mech. Dynam. Astronom., 107, 1-2, 145-155, (2010) · Zbl 1218.70074

Deveney, Robert L., Homoclinic orbits in Hamiltonian systems, J. Differential Equations, 21, 2, 431-438, (1976) · Zbl 0343.58005

Mireles James, J. D., Validated numerics for equilibria of analytic vector fields: invariant manifolds and connecting orbits, (van den Berg, Jan Bouwe; Lessard, Jean-Philippe, Rigorous Numerics in Dynamics. Rigorous Numerics in Dynamics, Proceedings of Symposia in Applied Mathematics, vol. 74, (2018)), 1-55 · Zbl 1409.65110

Kalies, William D.; Kepley, Shane; Mireles James, J. D., Analytic continuation of local (un)stable manifolds with rigorous computer assisted error bounds, SIAM J. Appl. Dyn. Syst., 17, 1, 157-202, (2018) · Zbl 1409.65110

Lessard, Jean-Philippe; Mireles James, Jason D.; Reinhardt, Christian, Computer assisted proof of transverse saddle-to-saddle connecting orbits for first order vector field equations, J. Dynam. Differential Equations, 26, 2, 267-313, (2014) · Zbl 1351.37107

Mireles James, J. D.; Mischauik, Konstantin, Rigorous a-posteriori computation of (un)stable manifolds and connecting orbits for analytic maps, SIAM J. Appl. Dyn. Syst., 12, 2, 957-1006, (2013) · Zbl 1330.37029

Leandro, Eduardo S. G., On the central configurations of the planar restricted four-body problem, J. Differential Equations, 226, 1, 323-351, (2006) · Zbl 1097.70011

Barros, Jean F.; Leandro, Eduardo S. G., The set of degenerate central configurations in the planar restricted four-body problem, SIAM J. Math. Anal., 43, 2, 634-661, (2011) · Zbl 1315.70005

Barros, Jean F.; Leandro, Eduardo S. G., Bifurcations and enumeration of classes of relative equilibria in the planar restricted four-body problem, SIAM J. Math. Anal., 46, 2, 1185-1203, (2014) · Zbl 1391.70025

Arioli, Gianni, Periodic orbits, symbolic dynamics and topological entropy for the restricted 3-body problem, Comm. Math. Phys., 231, 1, 1-24, (2002) · Zbl 1016.70007

Arioli, Gianni, Branches of periodic orbits for the planar restricted 3-body problem, Discrete Contin. Dyn. Syst., 11, 4, 745-755, (2004) · Zbl 1090.37070

Wilczak, Daniel; Zgliczynski, Piotr, Heteroclinic connections between periodic orbits in planar restricted circular three-body problem—a computer assisted proof, Comm. Math. Phys., 234, 1, 37-75, (2003) · Zbl 1055.70005

Capinski, Maciej J.; Zgliczynski, Piotr, Transition tori in the planar restricted elliptic three-body problem, Nonlinearity, 24, 5, 1395-1432, (2011) · Zbl 1218.37074

Arioli, Gianni; Barutello, Vivina; Terracini, Susanna, A new branch of Mountain Pass solutions for the choreographical 3-body problem, Comm. Math. Phys., 268, 2, 439-463, (2006) · Zbl 1118.70009

Celletti, Alessandra; Chierchia, Luigi, A computer-assisted approach to small-divisors problems arising in Hamiltonian mechanics, (Computer Aided Proofs in Analysis. Computer Aided Proofs in Analysis, Cincinnati, OH, 1989. Computer Aided Proofs in Analysis, Cincinnati, OH, 1989, Dekker: Dekker New York), 1-55 · Zbl 0577.58003

Edited by FIZ Karlsruhe, the European Mathematical Society and the Heidelberg Academy of Sciences and Humanities
© 2022 FIZ Karlsruhe GmbH
Burgos-García, Jaime; Delgado, Joaquín, On the “blue sky catastrophe” termination in the restricted three-body problem, SIAM J. Appl. Dyn. Syst., 11, 4, 1723-1753, (2012) · Zbl 1264.37008

Burgos-García, Jaime; Roldán, Pablo, Existence of a center manifold in a practical domain around \$L_1\$ in the restricted three-body problem, SIAM J. Appl. Dyn. Syst., 11, 1, 285-318, (2011) · Zbl 1235.37028

Burgos-García, Jaime J.; Wasieczko-Zajac, Anna, Geometric proof of strong stable/unstable manifolds with application to the restricted three-body problem, Topol. Methods Nonlinear Anal., 46, 1, 363-399, (2015) · Zbl 1365.34079

Burgos-García, Jaime J.; Gidea, Marian; de la Llave, Rafael, Arnold diffusion in the planar elliptic restricted three-body problem: mechanism and numerical verification, Nonlinearity, 30, 1, 329-360, (2017) · Zbl 1359.37121

Lessard, Jean-Philippe; Mireles James, J. D.; Ransford, Julian, Automatic differentiation for Fourier series and the radii polynomial approach, Phys. D, 334, 174-186, (2016)

Kapela, Tomasz; Simó, Carles, Computer assisted proofs for nonsymmetric planar choreographies and for stability of the Eight, Nonlinearity, 20, 5, 1241-1255, (2007) · Zbl 1115.70008

Kapela, Tomasz, \(\text{textit{(N)}}\)-body choreographies with a reflectional symmetry—computer assisted existence proofs, (EQUADIFF 2003, (2005), World Sci. Publ.; World Sci. Publ. Hackensack, NJ), 999-1004 · Zbl 1116.70019

Kapela, Tomasz; Simó, Carles, Rigorous KAM results around arbitrary periodic orbits for Hamiltonian systems, Nonlinearity, 30, 3, 965-986, (2017) · Zbl 1412.70024

Kapela, Tomasz; Zgliczyński, Piotr, The existence of simple choreographies for the \(\text{textit{(N)}}\)-body problem—a computer-assisted proof, Nonlinearity, 16, 6, 1899-1918, (2003) · Zbl 1060.37023

Burgos-García, Jaime; Delgado, Joaquín, On the “blue sky catastrophe” termination in the restricted four-body problem, Celestial Mech. Dynam. Astronom., 117, 2, 115-136, (2013) · Zbl 1293.70050

Papadakis, K. E., Families of three-dimensional periodic solutions in the circular restricted four-body problem, Astrophys. Space Sci., 361, 4, Article 120 pp., (2016)

Burgos-García, Jaime, Families of periodic orbits in the planar Hill’s four-body problem, Astrophys. Space Sci., 361, 11, Article 353 pp., (2016)

Papadakis, K. E., Families of asymmetric periodic solutions in the restricted four-body problem, Astrophys. Space Sci., 361, 12, Article 377 pp., (2016)

Blazevski, D.; Ocampo, C., Periodic orbits in the conic circular restricted four-body problem and their invariant manifolds, Phys. D, 241, 13, 1158-1167, (2012) · Zbl 1293.70006

Mireles James, J. D.; Murray, Maxime, Chebyshev-Taylor parameterization of stable/unstable manifolds for periodic orbits: implementation and applications, Internat. J. Bifur. Chaos Appl. Sci. Engrg., 27, 14, Article 1730050 pp., (2017), 32 pp · Zbl 1382.34053

Álvarez-Ramírez, Martha; Barrabés, E., Transport orbits in an equilateral restricted four-body problem, Celestial Mech. Dynam. Astronom., 121, 2, 191-210, (2015)

Alvarez Ramírez, Martha; Vidal, Claudio, Dynamical aspects of an equilateral restricted four-body problem, Math. Probl. Eng., Article 181360 pp., (2009) · Zbl 1194.70008

Gidea, Marian; Burgos, Melissa, Chaotic transfers in three- and four-body systems, Phys. A, 328, 3-4, 360-366, (2003) · Zbl 1098.70014

She, Zhiukun; Cheng, Xuhua; Li, Cuiping, The existence of transversal homoclinic orbits in a planar circular restricted four-body problem, Celestial Mech. Dynam. Astronom., 115, 3, 299-309, (2013) · Zbl 1342.70032

She, Zhiukun; Cheng, Xuhua, The existence of a Smale horseshoe in a planar circular restricted four-body problem, Celestial Mech. Dynam. Astronom., 118, 2, 115-127, (2014) · Zbl 1300.70009

Cheng, Xuhua; She, Zhiukun, Study on chaotic behavior of the restricted four-body problem with an equilateral configuration, Internat. J. Bifur. Chaos Appl. Sci. Engrg., 27, 2, Article 1750026 pp., (2017) · Zbl 1362.70015

Álvarez Ramírez, Martha; Delgado, Joaquín; Vidal, Claudio, Global regularization of a restricted four-body problem, Internat. J. Bifur. Chaos Appl. Sci. Engrg., 24, 7, Article 1450092 pp., (2014) · Zbl 1300.70009

Burgos-García, Jaime; Gidea, Marian, Hill’s approximation in a restricted four-body problem, SIAM J. Appl. Dyn. Syst., 11, 1, 285-318, (2012) · Zbl 1362.70015

Lee, John M., Introduction to Smooth Manifolds, Graduate Texts in Mathematics, vol. 218, (2013), Springer: Springer New York, 43-51 · Zbl 0683.58021
[66] Lanford, Oscar E., Computer-assisted proofs in analysis, Mathematical Physics, VII. Mathematical Physics, VII, Boulder, Colo., 1983. Mathematical Physics, VII. Mathematical Physics, VII, Boulder, Colo., 1983, Phys. A, 124, 1-3, 465-470, (1984) · Zbl 0599.58036

[67] van den Berg, J. B.; Lessard, J. P., Rigorous numerics in dynamics, Notices Amer. Math. Soc., 62, 9, 1057-1061, (2015) · Zbl 1338.68301

[68] Rump, Siegfried M., Verification methods: rigorous results using floating-point arithmetic, Acta Numer., 19, 287-449, (2010) · Zbl 1323.65046

[69] Calèrè, X.; Fontich, E.; de la Llave, R., The parameterization method for invariant manifolds. I. Manifolds associated to non-resonant subspaces, Indiana Univ. Math. J., 52, 2, 283-328, (2003) · Zbl 1034.37016

[70] Calèrè, X.; Fontich, E.; de la Llave, R., The parameterization method for invariant manifolds. II. Regularity with respect to parameters, Indiana Univ. Math. J., 52, 2, 329-360, (2003) · Zbl 1034.37017

[71] Calèrè, X.; Fontich, E.; de la Llave, R., The parameterization method for invariant manifolds. III. Overview and applications, J. Differential Equations, 218, 2, 444-515, (2005) · Zbl 1101.37019

[72] Haro, À.; de la Llave, R., A parameterization method for the computation of invariant tori and their whiskers in quasi-periodic maps: numerical algorithms, Discrete Contin. Dyn. Syst. Ser. B, 6, 6, 1261-1300, (2006), (electronic) · Zbl 1296.37048

[73] Haro, A.; de la Llave, R., A parameterization method for the computation of invariant tori and their whiskers in quasi-periodic maps: rigorous results, J. Differential Equations, 228, 2, 530-579, (2006) · Zbl 1102.37017

[74] Haro, À.; de la Llave, R., A parameterization method for the computation of invariant tori and their whiskers in quasi-periodic maps: explorations and mechanisms for the breakdown of hyperbolicity, SIAM J. Appl. Dyn. Syst., 6, 1, 142-207, (2007), (electronic) · Zbl 1210.37062

[75] Haro, Àlex; Canadell, Marta; Figueras, Jordi-Lluís; Luque, Alejandro; Mondelo, Josep-Maria, The Parameterization Method for Invariant Manifolds: From Rigorous Results to Effective Computations, Applied Mathematical Sciences, vol. 195, (2016), Springer: Springer Cham · Zbl 1372.37002

[76] Van den Berg, J. B.; Mireles James, J. D.; Reinhardt, Christian, Computing (un)stable manifolds with validated error bounds: non-resonant and resonant spectra, J. Nonlinear Sci., 26, 1055-1095, (2016) · Zbl 1360.37176

[77] de la Llave, R.; González, A.; Jorba, À.; Villanneva, J., KAM theory without action-angle variables, Nonlinearity, 18, 2, 855-895, (2005) · Zbl 1067.37081

[78] Steffensen, J. F., On the differential equations of Hill in the theory of the motion of the moon, Acta Math., 93, 169-177, (1955) · Zbl 0065.07504

[79] Rabe, Eugene, Determination and survey of periodic Trojan orbits in the restricted problem of three bodies, Astron. J., 66, 500-513, (1961)

[80] Drprit, André; Price, J. F., The Computation of Characteristic Exponents in the Planar Restricted Problem of Three Bodies, 1-84, (1965), Boeing Scientific Research Laboratories, Mathematics Research Laboratory, Mathematical Note (Number 415)

[81] Knuth, Donald E., The Art of Computer Programming, vol. 2: Seminumerical Algorithms, Addison-Wesley Series in Computer Science and Information Processing, (1981), Addison-Wesley Publishing Co.: Addison-Wesley Publishing Co. Reading, Mass. · Zbl 0477.65002

[82] Murray, Maxime; Mireles James, J. D., Chebyshev-Taylor parameterization of stable/unstable manifolds for periodic orbits: implementation and applications, Internat. J. Bifur. Chaos Appl., 27, 14, (2017) · Zbl 1382.34053

[83] van den Berg, Jan Bouwe; Breden, Maxime; Lessard, Jean-Philippe; Murray, Maxime, Continuation of homoclinic orbits in the suspension bridge equation: a computer-assisted proof, J. Differential Equations, 264, 5, 3086-3130, (2018) · Zbl 1405.34037

This reference list is based on information provided by the publisher or from digital mathematics libraries. Its items are heuristically matched to zbMATH identifiers and may contain data conversion errors. It attempts to reflect the references listed in the original paper as accurately as possible without claiming the completeness or perfect precision of the matching.