To the theory of non-local non-isothermal filtration in porous medium

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Abstract. A new approach to the theory of non-local and non-isothermal filtration based on the mathematical apparatus of fractional order derivatives is developing. A solution of the Cauchy problem for the system of equations of non-local non-isothermal filtration in fractional calculus is obtained. Some applications of the solutions obtained to the problems of underground hydrodynamics (fracturing and explosion) are considered. A computational experiment was carried out to analyze the solutions obtained. Graphs of pressure and temperature dependences are plotted against time.

1. Introduction
Despite the historical prescription of increased interest in the study of fluid motion in porous media [1] the filtering theory is far from being completed. Unsolved fundamental problems are still present. This is due to the multiphase porous structures. Non-local memory effects, strong spatial correlation and self-organization are typical for such structures that realize to a complex nature of heat and mass transfer. In addition to traditional approaches in the filtration theory development the mathematical methods of quantum field theory are applied [2,3]. Attempts are being made to use the self-organization theory with application of the methods of deterministic chaos [4].

The research of heat and mass transfer processes in complex media demands of the efficient media method. There are two approaches to insert the efficient media. The first one is to proceed from the microscopic equations with subsequent averaging [5]. The averaging procedure there plays a key role and determines the final shape of the macroscopic equations. The second approach is based on the applications of the macroscopic equations with the different relations between generalized forces and corresponding streams. One of such relations is the Fourier’s law for heat transfer processes and the Darcy’s law for fluid transfer process. These two laws determine the resulting view of heat and mass transfer equations.

The newest stage of development of the theory of filtration is associated with the account of effects of memory and spatial correlations in complex systems, including porous media. The particular interest in this regard is the development of the theory of non-local non-isothermal filtration on the basis of fractional calculus methods [6–8]. The fundamental difference between the non-local non-isothermal filtration and earlier known filtration theories is the appearance...
of new parameters. These new parameters are the rates of the derivatives of fractional order in coordinate and time. They result to appearance of a new range of solutions that allow developing more adequate mathematical models of filtration process. In this paper, we evolve a non-isothermal filtration theory based on fractional calculus proposed in [9–14]. The particular solutions for the problems of hydraulic fracturing and explosion are considered. So the obtaining solutions could be interesting for a wide spectrum of applications.

2. Classical non-isothermal filtration

The study of non-isothermal liquid filtration processes in the porous media includes the Navier–Stokes equation of motion of a viscous fluid, general heat equation and the equation of continuity. The necessity of using the Navier–Stokes equation for the flow of fluid in fractured porous media is associated with the fact that the viscosity forces play a very important role. Fluid flow in porous media is a flow around a lot of obstacles with a complex surface structure (e.g., a fractal structure). Therefore practical use of the Navier–Stokes equation becomes impossible. So one usually the hydrodynamic equations in the form of the Euler equations are taking as a starting point, but fictitious mass forces are added to actually existing mass forces [15]. The presence of the porous medium is taken into account by the Darcy law:

\[
\mathbf{v} = -\frac{k}{\mu} \nabla P,
\]

where \( v \) is a speed of filtration, \( \tilde{k} \) is the penetrability of a porous medium, \( \mu \) is the dynamic viscosity of the fluid, \( P \) is the pressure. Using the Darcy law 1 in the Euler equation we obtain expression for the velocity of the fluid

\[
\mathbf{v} = \frac{k}{\mu} (\mathbf{X} - \frac{1}{\rho} \nabla P),
\]

where \( \rho \) is a density of the fluid, \( \mathbf{X} \) is the actually existing mass force.

With the account of (2) in the continuity equation we obtain the equation for liquid filtration

\[
\frac{\partial}{\partial x} \left( \frac{k}{\mu} \frac{\partial P}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{k}{\mu} \frac{\partial P}{\partial y} \right) + \frac{\partial}{\partial z} \left( \frac{k}{\mu} \frac{\partial P}{\partial z} \right) = \frac{m}{\partial t} - g \frac{\partial}{\partial z} \left( \frac{k}{\mu}^2 \right).
\]

Here

\[
\frac{\partial}{\partial t} = \frac{d}{dt} \frac{\partial P}{\partial t} + \frac{d}{\partial T} \frac{\partial T}{\partial t},
\]

\( m \)—is the porosity of the medium. The heat conduction equation has the form

\[
\rho T \left( \frac{\partial s}{\partial t} + \mathbf{v} \nabla s \right) = \nabla \cdot (\lambda \nabla T) + \frac{\eta}{2} \left( \frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i} - \frac{2}{3} \delta_{ik} \frac{\partial v_i}{\partial x_k} \right)^2 + \frac{\zeta}{2} (\nabla \cdot \mathbf{v})^2.
\]

Here \( s \) is the entropy, \( \rho \) is the density, \( T \) is the temperature, \( \lambda \) is the heat conductivity, \( \eta, \zeta \) are the shear and volume viscosities. Calculating the entropy derivative, one should consider pressure to be constant (while temperature and density change). In result we have

\[
\frac{\partial s}{\partial t} = \left( \frac{\partial s}{\partial T} \right)_P\frac{\partial T}{\partial t} = \frac{1}{T} \frac{\partial T}{\partial t},
\]

\[
\nabla s = \left( \frac{\partial s}{\partial T} \right)_P \nabla T.
\]

Here \( c_p \)—specific heat at constant pressure. In result heat conduction equation has the following form

\[
\frac{\partial T}{\partial t} + \mathbf{v} \nabla T = \nabla \cdot (\lambda \nabla T) + \frac{\eta}{2 \rho c_p} \left( \frac{\partial v_i}{\partial x_k} + \frac{\partial v_k}{\partial x_i} - \frac{2}{3} \delta_{ik} \frac{\partial v_i}{\partial x_k} \right)^2 + \frac{\zeta}{2 \rho c_p} (\nabla \cdot \mathbf{v})^2.
\]
where $\chi$—is the thermal diffusivity. Similarly to the case of Navier–Stokes equation, fluid viscosity account we perform by the transition to effective medium. In result heat conduction equation takes the form
\[
\frac{\partial T}{\partial t} - \frac{\tilde{k}}{\mu} \nabla P \nabla T = \nabla \cdot (\chi \nabla T),
\]
Thus, the system of equations for describing non-isothermal filtration in porous media takes the form
\[
\frac{\partial}{\partial x} \left( \frac{\tilde{k}_p \partial P}{\mu \partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\tilde{k}_p \partial P}{\mu \partial y} \right) + \frac{\partial}{\partial z} \left( \frac{\tilde{k}_p \partial P}{\mu \partial z} \right) = m \left( \frac{d \rho}{dP} \frac{\partial P}{\partial t} + \frac{d \rho}{dT} \frac{\partial T}{\partial t} \right) - g \frac{\partial}{\partial z} \left( \frac{\tilde{k}_p^2}{\mu} \right),
\]
(3)
\[
\frac{\partial T}{\partial t} - \frac{\tilde{k}}{\mu} \nabla P \nabla T = \nabla \cdot (\chi \nabla T).
\]
(4)
Assuming $\tilde{k}$ and $\mu$ to be constant values, we obtain the following system of correlated equations
\[
\frac{\partial P}{\partial t} - \frac{\tilde{k}}{m \mu} \frac{dP}{d \rho} \left[ \frac{\partial}{\partial x} \left( \rho \frac{\partial P}{\partial x} \right) + \frac{\partial}{\partial y} \left( \rho \frac{\partial P}{\partial y} \right) + \frac{\partial}{\partial z} \left( \rho \frac{\partial P}{\partial z} \right) \right] - 2g \rho \frac{\tilde{k}}{m \mu} \frac{\partial P}{\partial z} = \tilde{a} \frac{\partial T}{\partial t},
\]
(5)
\[
\frac{\partial T}{\partial t} - \nabla \cdot (\chi \nabla T) = \frac{\tilde{k}}{\mu} \nabla T \nabla P.
\]
(6)
Next we will investigate the special case.
Using the decomposition for the density and the entropy we obtain
\[
\rho(x, t) = \rho_0 (1 + \tilde{b} (T(x, t) - T_0) + \tilde{a} (P(x, t) - P_0)),
\]
\[
s(x, t) = s_0 + \frac{c_P}{T} (T(x, t) - T_0) + \frac{\tilde{b}}{\rho_0} (P(x, t) - P_0).
\]
Here $\tilde{a}$ is the isothermal compressibility factor, $\tilde{b}$ is the thermal expansion coefficient. As a result, we obtain the following system of equations
\[
\frac{\partial P(x, t)}{\partial t} - d_P(x, t) \frac{\partial^2 P(x, t)}{\partial x^2} + V_P(x, t) \frac{\partial P(x, t)}{\partial x} = \frac{\tilde{b}}{\tilde{a}} \frac{\partial T(x, t)}{\partial t},
\]
(7)
\[
\frac{\partial T(x, t)}{\partial t} - d_T(x, t) \frac{\partial^2 T(x, t)}{\partial x^2} + V_T(x, t) \frac{\partial T(x, t)}{\partial x} = \frac{\tilde{b} T_0}{c_P \rho_0} \frac{\partial P(x, t)}{\partial t}.
\]
(8)
Here
\[
d_P(x, t) = \frac{\tilde{k}}{m \mu \tilde{a}}, \quad d_T(x, t) = \frac{\lambda}{c_P \rho},
\]
\[
V_P(x, t) = \frac{\tilde{k}}{m \mu} \left( \frac{\partial P(x, t)}{\partial x} - \frac{\tilde{b}}{\tilde{a}} \frac{\partial T(x, t)}{\partial x} \right), \quad V_T(x, t) = -\frac{\tilde{k}}{\mu} \frac{\partial P(x, t)}{\partial x}.
\]
The system of equations (7, 8) together with the state equation $\rho = \rho(P, T)$ are the basis of the classical theory of non-isothermal filtration.
3. Non-local non-isothermal filtration

For the transport processes in porous media the account of non-local effects in time and coordinate becomes important. Darcy’s law generalization, where non-local effects are accounted in the form of integral relations, leads to integral-differential filtration equations. The solving of such equations is associated with some fundamental difficulties. Memory effects account (time non-locality) and spatial correlations (space non-locality) using traditional approaches bring to the appearance of the integral operator in differential equations, where the operator kernel has the information about the nature of non-locality. To solve such equations integral operators are represented as a series of differential operators with an increasing rate of differentiation, and with the small parameter are limited to a few terms of series. In the absence of a small parameter, this approach is counterproductive. Finally the obtained equations are not always able to solve.

A new stage in the development of non-isothermal filtration theory is associated with the mathematical apparatus of integral-differentiation of fractional order–fractional calculus. Equations of non-local non-isothermal filtration in fractional calculus have the form [16]

\[
\frac{\partial^\alpha P(\xi, \tau)}{\partial \tau^\alpha} - D_P(\xi, \tau) \frac{\partial^\beta P(\xi, \tau)}{\partial \xi^\beta} + V_P(\xi, \tau) \frac{\partial^\gamma P(\xi, \tau)}{\partial \xi^\gamma} = \frac{P(\xi, 0)}{\Gamma(1-\alpha)\tau^\alpha} + \frac{\tilde{b}}{a} \frac{\partial^\alpha T(\xi, \tau)}{\partial \tau^\alpha},
\]

\[
\frac{\partial^\alpha T(\xi, \tau)}{\partial \tau^\alpha} - D_T(\xi, \tau) \frac{\partial^\beta T(\xi, \tau)}{\partial \xi^\beta} + V_T(\xi, \tau) \frac{\partial^\gamma T(\xi, \tau)}{\partial \xi^\gamma} = \frac{T(\xi, 0)}{\Gamma(1-\alpha)\tau^\alpha} + \frac{\tilde{b}}{a} \frac{\partial^\alpha P(\xi, \tau)}{\partial \tau^\alpha}.
\]

Here

\[
D_P(\xi, \tau) = \frac{l_0^2 k_0 \rho(\xi, \tau)}{t_0 m \mu a}, \quad D_T(\xi, \tau) = \frac{\lambda}{c_P \rho},
\]

\[
V_P(\xi, \tau) = \frac{k_0}{m \mu} \left( \frac{\partial^\beta P(\xi, \tau)}{\partial \xi^\beta} - \frac{\tilde{b}}{a} \frac{\partial^\alpha T(\xi, \tau)}{\partial \tau^\alpha} \right), \quad V_T(\xi, \tau) = -\frac{l_0 \tilde{k}}{\mu} \frac{\partial^\alpha P(\xi, \tau)}{\partial \tau^\alpha}.
\]

Derivatives of fractional order, included in equations (9, 10) are defined as follows

\[
\frac{\partial^\alpha T(\xi, \tau)}{\partial \tau^\alpha} = \frac{1}{\Gamma(1-\alpha)} \frac{\partial}{\partial \tau} \int_0^\tau \frac{T(\xi, z)}{(\tau-z)^{\alpha}} \, dz - \frac{T(\xi, 0)}{\Gamma(1-\alpha)\tau^\alpha},
\]

\[
\frac{\partial^\beta T(\xi, \tau)}{\partial \xi^\beta} = \frac{1}{2\Gamma(2-\beta) \cos(\frac{\beta}{2}(2-\beta))} \frac{\partial^2}{\partial \xi^2} \int_{-\infty}^{\infty} \frac{T(\xi', \tau)}{|\xi' - \xi|^{\beta-1}} \, d\xi'.
\]

Here \(|\xi| < \infty, \tau > 0, 0 < \alpha \leq 1, 1 < \beta \leq 2, \tau = t/t_0, \xi = x/x_0\) are the dimensionless time and coordinate, \(t_0\) and \(x_0\) are the characteristic time and the scale, \(\Gamma(x)\) is the Euler gamma function. It is important to note that \(\alpha = 1\) and \(\beta = 2\) case corresponds to classical derivatives, which can be considered as a one limit case of our generalization. Caputo derivative (11) accounts memory (non-locality in time), Riesz derivative (12) accounts spatial correlation (spatial non-locality).

System of equations (9, 10) together with the state equation \(\rho = \rho(P, T)\) describes the processes of non-local non-isothermal filtration on the basis of fractal calculus. In the general case this system of equations is not possible to solve due to its non-linearity. For the qualitative analysis of the impact of non-locality in time and space accounting we have analyzed one-dimensional case of the Cauchy problem for an unbounded domain. Considering the linear approximation in the perturbation of temperature and pressure we obtain the following system
of equations:

\[
\frac{\partial^n P(\xi, \tau)}{\partial \tau^n} - D_P(\xi, \tau) \frac{\partial^2 P(\xi, \tau)}{\partial \xi^2} = \frac{P(\xi, 0)}{\Gamma(1 - \alpha) \tau^\alpha} + \frac{\tilde{b}}{\tilde{a}} \frac{\partial^n T(\xi, \tau)}{\partial \tau^n},
\]

(13)

\[
\frac{\partial^n T(\xi, \tau)}{\partial \tau^n} - D_T(\xi, \tau) \frac{\partial^2 P(\xi, \tau)}{\partial \xi^2} = \frac{T(\xi, 0)}{\Gamma(1 - \alpha) \tau^\alpha} + \frac{\tilde{b}}{\tilde{c}_P \rho_0} \frac{\partial^n T(\xi, \tau)}{\partial \tau^n}.
\]

(14)

To solve the system of equations (13, 14) we have applied the Laplace transform on time and Fourier transform on coordinate:

\[
P_LF(k, s) = \int_0^\infty \text{d}t e^{-st} \int_{-\infty}^{\infty} \text{d}ξ e^{-\xi k} P(\xi, t).
\]

The result in an algebraic system of equations whose solution has the form:

\[
P_LF(k, s) = \frac{P_F(k, 0)}{s^{1-\alpha}(s^\alpha + D_P(k))} + \frac{\tilde{b}}{\tilde{a}} \frac{s^\alpha}{s^\alpha + D_P(k)} \frac{T_F(k, 0)}{s^{1-\alpha}(s^\alpha + D_T(k))},
\]

(15)

\[
T_LF(k, s) = \frac{T_F(k, 0)}{s^{1-\alpha}(s^\alpha + D_T(k))} + \frac{\tilde{b}}{\tilde{c}_P \rho_0} \frac{\int_{-\infty}^{\infty} \text{d}k T_F(k - k', 0)}{s^\alpha (s^\alpha + D_P(k'))},
\]

(16)

Performing inverse Fourier and Laplace transforms, we finally obtain

\[
P(\xi, \tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \text{d}k \int_{-\infty}^{\infty} \text{d}\xi' e^{ik(\xi' - \xi)} P(\xi', 0) E_{\alpha,1}(-\tilde{D}_P(k) \tau^\alpha) \]

\[- \frac{\tilde{b}}{\tilde{a}(d - \chi)} \frac{1}{2\pi} \int_{-\infty}^{\infty} \text{d}k \int_{-\infty}^{\infty} \text{d}\xi' T(\xi', 0) e^{ik(\xi' - \xi')} \]

\[\times \left[ dE_{\alpha,1}(-\tilde{D}_P(k) \tau^\alpha) - \chi E_{\alpha,1}(-\tilde{D}_T(k) \tau^\alpha) \right],
\]

(17)

\[
T(\xi, \tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \text{d}k \int_{-\infty}^{\infty} \text{d}\xi' e^{ik(\xi' - \xi)} T(\xi', 0) E_{\alpha,1}(-\tilde{D}_T(k) \tau^\alpha) \]

\[- \frac{\tilde{b}}{\tilde{c}_P \rho_0} \frac{1}{2\pi} \int_{-\infty}^{\infty} \text{d}k \int_{-\infty}^{\infty} \text{d}k' \int_{-\infty}^{\infty} \text{d}\xi' \int_{-\infty}^{\infty} \text{d}\xi'' \frac{e^{ik(\xi' - \xi'')} e^{ik(\xi - \xi'')}}{\tilde{D}_T(k) - \tilde{D}_P(k')} T(\xi', 0) P(\xi'', 0) \xi \]

\[\times \tilde{D}_P(k') E_{\alpha,1}(-\tilde{D}_P(k') \tau^\alpha) - \tilde{D}_T(k) E_{\alpha,1}(-\tilde{D}_T(k) \tau^\alpha) \]

(18)

Here

\[E_{\alpha,1}(z^\alpha) = \sum_{k=0}^{\infty} (-1)^n \frac{z^{\alpha n}}{\Gamma(\alpha n + 1)}\]

is the Mittag-Leffler function,

\[d = \left( \frac{t^2_0}{t_0} \right)^{-1} \frac{k}{\mu \alpha}, \quad \chi = \left( \frac{t^2_0}{t_0} \right)^{-1} \frac{\lambda}{c_P \rho}, \quad \tilde{D}_P(k) = |k|^\beta, \quad \tilde{D}_T(k) = \chi |k|^\beta.
\]

Obtained equations allow to study a non-isothermal filtration processes for different initial conditions. The qualitative difference of the pressure distribution with the account of non-local effects is that asymptotic behavior has power character, in contrast to the exponential behavior of the traditional pressure distribution. In non-isothermal filtration process, thermal characteristics of the layer affect pressure distribution unlike the isothermal filtration process.

Moreover equation (18) depends on the rates of fractional order derivatives in time and coordinate and leads to fundamentally new solutions and other interpretation of experimental
data. Particularly, obtained solutions allow to study the affect of the hydraulic fracturing process on the temperature field. When initial values $P(\xi, 0) = P_0\delta(\xi)$, $T(\xi, 0) = T_0$ (fracturing), solutions of (17, 18) take the form:

$$P(\xi, \tau) = \frac{P_0}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dk dk' e^{ik\xi} \frac{dE_{\alpha,1}(-d|k'|^\beta \tau^\alpha) - \tilde{b} \tilde{T}_0}{a(d-\chi)} \frac{P_0 T_0}{2\pi(p-\chi)} \frac{dE_{\alpha,1}(-d|k'|^\beta \tau^\alpha) - \chi E_{\alpha,1}(-\chi|k'|^\beta \tau^\alpha)}{\chi|k|^\beta - d|k'|^\beta} \times$$

$$\times \int_{-\infty}^{\infty} dk dE_{\alpha,1}(-d|k'|^\beta \tau^\alpha) - \chi E_{\alpha,1}(-\chi|k'|^\beta \tau^\alpha)),$$  

$$T(\xi, \tau) = \frac{T_0}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dk dk' e^{ik\xi} \frac{dE_{\alpha,1}(-d|k'|^\beta \tau^\alpha) - \chi E_{\alpha,1}(-\chi|k'|^\beta \tau^\alpha)}{\chi|k|^\beta - d|k'|^\beta} \times$$

$$\times \int_{-\infty}^{\infty} dk dk' \frac{e^{ik\xi}}{\chi|k|^\beta - d|k'|^\beta} (d|k'|^\beta E_{\alpha,1}(-d|k'|^\beta \tau^\alpha) - \chi|k|^\beta E_{\alpha,1}(-\chi|k'|^\beta \tau^\alpha)).$$

As can be seen from the solutions (19, 20), in the case of a non-isothermal filtering and fracturing causes an additional change in temperature and pressure as compared with the case of isothermal filtration. The account of non-isothermal filtration lowers the pressure, and non-locality on the spatial coordinate leads to the power-law dependence of pressure, that means an increase in the area of hydraulic impact.

The solution allows us to consider a number of others interesting problems arising from the sharp increase in temperature and pressure up to an appearance of extreme states. For example, in the case of the initial conditions $P(\xi, 0) = P_0\delta(\xi)$, $T(\xi, 0) = T_0\delta(\xi)$, which corresponds to a dramatic change in temperature and pressure (explosion), we obtain:

$$P(\xi, \tau) = \frac{P_0}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dk dk' e^{ik\xi} \frac{dE_{\alpha,1}(-d|k'|^\beta \tau^\alpha) - \tilde{b} \tilde{T}_0}{a(d-\chi)} \frac{P_0 T_0}{2\pi(p-\chi)} \frac{dE_{\alpha,1}(-d|k'|^\beta \tau^\alpha) - \chi E_{\alpha,1}(-\chi|k'|^\beta \tau^\alpha)}{\chi|k|^\beta - d|k'|^\beta} \times$$

$$\times \int_{-\infty}^{\infty} dk dE_{\alpha,1}(-d|k'|^\beta \tau^\alpha) - \chi E_{\alpha,1}(-\chi|k'|^\beta \tau^\alpha)),$$  

$$T(\xi, \tau) = \frac{T_0}{2\pi} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} dk dk' e^{ik\xi} \frac{dE_{\alpha,1}(-d|k'|^\beta \tau^\alpha) - \chi E_{\alpha,1}(-\chi|k'|^\beta \tau^\alpha)}{\chi|k|^\beta - d|k'|^\beta} \times$$

$$\times \int_{-\infty}^{\infty} dk dk' \frac{e^{ik\xi}}{\chi|k|^\beta - d|k'|^\beta} (d|k'|^\beta E_{\alpha,1}(-d|k'|^\beta \tau^\alpha) - \chi|k|^\beta E_{\alpha,1}(-\chi|k'|^\beta \tau^\alpha)).$$

In this case, both the pressure and the temperature changes depend on the difference $(d - \chi)$, and for this temperature dependence is non-linear.
Figure 2. Well dimensionless temperature change dynamics depending on dimensionless time for different values of the fractional derivative rate (solid curve for $\alpha = 1$, dashed curve for $\alpha = 0.8$).

Figures 1 and 2 represent diagrams of solutions (21) and (22) for the pressure and temperature distribution in a well depending on time. Calculations are performed for a fixed point of the layer radius with initial pressure $P(\xi, 0) = 10.1$ and temperature $T(\xi, 0) = 28$. As it can be observed, due to the transition to fractional derivatives, pressure and temperature decrease faster. Such dynamics is inherent in fractal medium.

4. Conclusion
The proposed mathematical model, as well as the calculations carried out, will allow a more accurate study of quantitative characteristics of thermohydrodynamic layers having a fractal structure with high-viscous oil. And also it can be used at oil deposits research.

The description of the non-isothermal filtration process on the base of differential equations of fractional order provides the natural account for the space and time non-locality of properties of the matter.

Differential equations in derivatives of fractional order have wide class of solutions, which depends on the rates of derivatives of fractional order. Using different values of these parameters leads to a set of solutions from which we can choose the solution that closely matches the actual filtration process including the case of explosion or shock compression.

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