Numerical modelling of heat and mass transfer in a gas pipe conducting system: Application to methane gas

Serge Samuel Bell1*, René Oum Lissouck1,2 and Rachel Raissa Ngono Mvondo1,3

1The University Institute of Wood Technology, the University of Yaoundé I, P. O. Box 306, Mbalmayo, Cameroon.
2Laboratory of Civil Engineering and Mechanics, National Advanced School of Engineering, The University of Yaoundé I, Cameroon.
3Laboratory L3E: Energy, Water and Environment, National Advanced School of Engineering, The University of Yaoundé I, P. O. Box 8390, Yaoundé, Cameroon.

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Methane is the principal component of natural gas. In order to be liquefied, it has to pass through a compression station where its temperature is lowered. An experimental investigation of the temperature drop behaviour of methane gas has been carried out. The test section is a horizontal circular tube with an inner diameter of 6 mm, an outer diameter of 8 mm and a length of 1500 mm. The physical parameters of the gas are explicit. Extensive measurements of the temperature along the pipe were realized by considering an air temperature of 26°C. A numerical approach of the heat and mass transfer problem was carried out thanks to the 4th ordered Runge-Kutta method. Numerical results showed a good agreement with experimental data.

Key words: Drop, natural gas, temperature, Runge-Kutta, simulation.

INTRODUCTION

The global energy demand per annum is heading towards nearly $10^{21}$ J for a forecasted population of about 7500 millions in 2050 (Mishra and Govindarajan, 2020). The industrial production of liquefied natural gas (LNG) contributes to meet that important demand. Design and manufacturing such LNG facilities need a good monitoring of the flow and heat transfer process of the gas involved. In that perspective, converting natural gas into the liquid state can ease the problems of storage and transport.

Methane is the principal component of natural gas. It has a large share of global energy consumption and its infrastructure is widespread. It can be used as an energy carrier (secondary energy source) for all primary renewable energy sources.

In addition, hydrocarbons belong to the group of natural refrigerants and they are environment friendly (Chen et al., 2012). With the smallest molecular weight and the minimum boiling point at standard atmospheric pressure among all hydrocarbons, methane is an often-used component in composing mixture refrigerants for the low-temperature Joule-Thomson refrigerator (Gong et al., 2004). In order to be liquefied, the methane gas has to pass through a compression station where it is compressed and by expansion its temperature is lowered. The product in its liquid form is then pumped at very low temperature for transportation and consumption purposes. Nevertheless, during the pumping process,
internal friction between the walls of the pipe and the fluid generally occurs and results in decreasing the pressure inside the system. As a consequence, the fluid may not move from one point to another of the pipe duct. In other words, the mechanical energy is no more converted into hydraulic energy. The phenomena is called “pressure drop”. Two main causes that may explain this are generally considered. The first type deals with the resistance to flow offered by straight pipe section while the second one refers to bends, fittings, and other elements present in the pipe system (Mishra et al., 2020). In both cases, a good monitoring of the pumping power will require at any moment a good knowledge of several parameters like the temperature, the density and the viscosity of the fluid any time at each point within the pipe duct. Such parameters are important since they are correlated with the aptitude of the fluid to flow. They can also vary with the climatic atmosphere in which the pipe duct is placed.

Dutkowski (2008) conducted experimentations of pressure drops in mini channels with use of water and air as working fluids. Similar investigations were carried out by Saha and Celata (2016). Thome et al. (2008) presented a review on flow boiling of hydrocarbons, in which experimental studies of two-phase pressure drops of hydrocarbons were addressed and the prediction methods described. However, except for some related studies about propane and higher hydrocarbons, it is rarely available to find such measured frictional pressure drop data of methane in open published literature, as well as its natural gas mixtures (Chen et al., 2012). As a response to this lack of knowledge, Chen et al. (2012) carried out an experimental investigation of two-phase frictional pressure drop behavior of methane with saturation pressure ranging from 0.3 to 0.6 MPa. The test section was a horizontal circular tube with an inner diameter of 6 mm and a length of 1860 mm. Two-phase pressure drops over the vapor quality ranging from 0 to 0.35 and mass flux ranging from 110 to 350 kg/m²s were obtained. Results highlight influences of saturation pressure and mass flux on the frictional pressure drop. Nevertheless, their stability with time was not analyzed. Many researches were subsequently conducted. With the aim of developing economically viable heat sinks, Saha and Celata (2016) analyzed the problem of higher pressure drop by comparing two phases and one single phase flows. These authors applied experimental and analytical approaches on micro-channels. There is a need to confirm such results on full size channels in order to identify potential sources responsible for large pressure drops. Roachand and Bell (1989) analyzed the fundamentals of fluid mechanics necessary to understand and determine pressure losses in a system using a U-type evacuated tube solar collector model with 52 collector units and conducted analytical pressure drop calculation in fluid entrance and fluid existing due to parameters like the inclination of the surface as well as the length from the storage tank to collector arrays. Nevertheless, variation of these results with time was not analyzed.

The objective of this study is to determine the temperature of the methane gas at any time and at a given position along a pipe duct, during the flowing process, by using a numerical method. The study will take into account important aspects like the ambient weather conditions (cool or heat for instance) and the physio-chemical characteristics of the conductive pipe which may influence the flowing process in the pipe.

EXPERIMENTS

The apparatus was designed to study the flow boiling heat transfer and pressure drop characteristics of pure and mixed-refrigerants in the low temperature range mixed loss in the temperature differences along the pipe of length L = 1.5 m. Figure 1 illustrates the schematic diagram of the apparatus. The circulation of the fluid under test is driven by a self-designed magnetic pump with a capacity of between 2 â· 6 m³/h, which is controlled by a speed regulator to achieve the required fluid flow rate (1 m/s). A mass flow meter with a pronounced uncertainty of 0.1% is installed after the circulation pump to measure the mass flow rate. The electrical preheater and the main water heater are fixed before the inlet of the test section to obtain the required vapor quality (x). A 2500-mm long circular copper tube with an inner diameter d_i of 6 mm and an outer diameter d_o of 8 mm was installed after the preheater and the main heater. The absolute roughness of copper ε = 0.002 mm, the relative roughness of the surface of the inner wall of the tube is Ra = ε/d_i = 0.00033 mm, and the fluid flow is laminar (Re < 2000). To eliminate the U-bend effect on the flow, the first 50 cm of the copper tube (before the test section) was not included in the test section. A differential pressure sensor, which covers the test section with the length of 1860 mm, was fixed to obtain experimental pressure drop data. The preheater and the main heater were powered by the direct-current (DC) electric heating wire coil. Two quartz glass tubes with the inner diameter of 6 mm were installed for flow pattern visualization. A cryogenic refrigerator based on the mixed-gases Joule-Thomson refrigeration cycle was built to provide the required cooling capacity at temperatures ranging from 110 to around 240 K (Gong et al., 2004). Heaters and the test tubes at each interval Ax (Figure 1a) were well insulated in a vacuum vessel. All signals of temperature, pressure, pressure drop and mass flow rate were collected by a data acquisition system and transmitted to a computer.

Numerical modeling

Mathematical model

Heat transfer is non-stationary, as the temperature varies with space and time. The heat consumption during the exchange process in a gas pipe may be explained by three phenomena: thermal conductivity, convection heat exchange, and thermal radiation (Anderson et al., 1990). The above heat transfer processes are different in quality and quantity. The process of flow in a variable volume of gas significantly changes the cooling conditions. Equations 1, 2 and 3 respectively present the mathematical model of conduction, convection and radiation.

\[ \alpha \frac{\partial T(x,t)}{\partial t} = \lambda \frac{\partial^2 T(x,t)}{\partial x^2} + \rho c \frac{\partial \rho}{\partial t} \frac{\partial T(x,t)}{\partial x} \]

Where:
$T_r(x,t)$ is the pipe temperature; $t$ is the time; $\alpha_r$ is heat transfer coefficient; $\rho$ is the density of the gas; $c$ is the specific heat capacity.

$$-\frac{\lambda}{4} \frac{d^2 T_r}{dx^2} + \alpha_r \pi d_t L (T_g - T_r) = 0$$

(2)

Where: $\lambda$ is the thermal conductivity of the gas; $T_g$ is the gas temperature; $d_t$ is the inner diameter of the pipe; $L$ is the length of the pipe; $\alpha_r$ is heat transfer coefficient ...

$$\frac{dT_g}{dt} = \frac{\lambda}{\rho c V} (\sigma (T_g^4 - T_r^4) + \alpha_r (T_g - T_r))$$

(3)

Where, $A_t$ is area of the pipe; $V_t$ is the volume of the pipe; $T_g$ is air temperature; $\sigma$ is the Stefan-Boltzman constant.

In this paper, the physical parameters of the gas were set as follows: $\lambda = 80.2 \text{Br/m/(m} \cdot \text{K);$ $\rho = 7870 \text{kg/m}^3$ ; $c = 447 \text{K} \cdot \text{kg}.$ These values led to $\alpha_r = 2.28 \times 10^{-4} \text{m}^2/\text{s}.$ Solving the previous equations may require the use of numerical methods. In that perspective, discretization which is a process of transferring continuous functions, models, variables, and equations into discrete counterparts may play a key role. The gas line is represented by a cross section of a cylindrical tube of constant section of length $L$ (Figure 2) and is divided into separate sections ($\Delta x$). Each section corresponds to a system of energy savings.

The second step was the formulation of the discretized differential Equations 1, 2 and 3. The total area and the total volume of the pipe were expressed as follows (Equations 4 and 5):

$$A_t = \frac{1}{2} \pi (2L (D_t + d_t) + (D_t^2 - d_t^2))$$

(4)

$$V_t = \frac{\pi (D_t^2 - d_t^2)}{4} x$$

(5)

Where $D_t$ is the outer diameter of the pipe.

We assumed that the gas temperature $T_g$ and the pipe temperature $T_r$ at each interval $[x_{i-1}, x_i]$ are linear functions and the air temperature $T_a$ which was set to 27°C (300K) in this study may correspond to summer or spring climates. While considering that $\Delta x$ is the length of the interval $[x_{i-1}, x_i]$, we may write (Equations 6, 7, 8 and 9):

$$\frac{dT_g}{dt} = \frac{\lambda}{\rho c (\Delta x)^2} \left(T_{g,i-1}(t) - 2T_{g,i}(t) + T_{g,i+1}(t)\right)$$

(6)

$$\frac{d^2 T_g}{dx^2} = \frac{\lambda}{(\Delta x)^2} \left(T_{g,i-1} - 2T_{g,i} + T_{g,i+1}\right)$$

(7)

$$\int_{x_{i-1}}^{x_i} (T_r - T_g) \, dx = \frac{\Delta x}{2} \left[(T_{g,i+1} + T_{r,i+1}) - (T_{g,i} + T_{r,i})\right]$$

(8)

$$\int_{x_{i-1}}^{x_i} (297^4 - T_r^4) \, dx = \Delta x \left[297^4 \left(1 - \frac{5}{5(T_{r,i+1} + T_{r,i})}\right)\right]$$

(9)

By substitution in Equation 1, the discretized form of the initial equations are (Equations 10 and 11):

$$\frac{dT_r}{dt} = \frac{\lambda}{\rho c (\Delta x)^2} \left(T_{r,i-1}(t) - 2T_{r,i}(t) + T_{r,i+1}(t)\right)$$

(10)

$$-\frac{\lambda}{4} \frac{d^2 T_r}{dx^2} + \alpha_r \pi d_t L (T_g - T_r) = 0$$

(11)
Figure 2. (a) Discretization of the pipe and boundary conditions of temperature (b) Thermal transfer between the pipe and the atmosphere, and (c) Gas movement along the pipe.
The boundary conditions of the problem are presented in Figure 1. The initial condition corresponds to:

\[ T = 0 \text{ s}, \quad x = 50 \text{ cm}, \quad T(x, t) = 700 \text{ K} \]

\( \forall t > 0, \ x > 50 \text{ cm}, \ T(x, t) = T \)

\[ t = 180 \text{ s}, \ x = L = 150 \text{ cm}, \ T(x, t) = 300 \text{ K}, \ x = 25 \text{ cm}, \] that is, the difference between two points of numerical and experimental temperature measurements, \( x_1 \) and \( x_2 \) of the gas pipe.

The solution of the differential equation system is written as

\[ T' = \begin{bmatrix} T_{g1} \\ T_{g2} \\ T_{g3} \\ \vdots \\ T_{g(k-2)} \\ T_{g(k-1)} \\ T_gk \end{bmatrix} = \frac{\alpha_T}{(\Delta x)^2} \begin{bmatrix} 1 & -2 & 0 & 0 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & -2 \end{bmatrix} \begin{bmatrix} T_1 \\ 0 \\ 0 \\ \vdots \\ 0 \\ T_k \\ T_g \end{bmatrix} \]

**Numerical solving of the discretized problem**

Among the various numerical solving methods available in the literature are the Runge-Kutta methods (Chauhan V et al., 2019). In numerical analysis, the Runge-Kutta methods are a family of implicit and explicit iterative methods. They are used in temporal discretization for the approximate solutions of ordinary differential equations. Among them, we have used the classic Runge-Kutta method, which is a one-step method with multiple stages, and the number of stages determining the order of the method. The number of stages was set to 4 in this study, corresponding to the RK4 method. A complete description of the RK4 method is provided by Wang and Weile (2011). The numerical solutions concerning the distribution of the gas temperature in the pipe were presented as follows (Equation 12):

\[ \begin{bmatrix} T_g \end{bmatrix} = M[T] + [S] \]

Where:

\[ \begin{bmatrix} T_g \end{bmatrix} \] is the column vector of the gas distribution along the various sections of the pipe; \( [T] \) is the column vector corresponding to the boundary conditions and \( M \) is the matrix of system of equations (Ciarlet, 1999). These parameters are specified in Equation 13.

**RESULTS AND DISCUSSION**

Figure 3 illustrates the experimental and numerical distribution of the gas temperature in the pipe with a constant ambient air temperature of 27°C. The simulations results make it possible to confirm experimental results with a good accuracy since the gap between these approaches is not significant. In general, there is a considerable and rapid decrease in the initial temperature as the interval \( \Delta x \) increases (Figure 4). The temperature of the gases leaving the source drops considerably from 700°C to a thermodynamic equilibrium value of 300°C (Figure 4). The duration of the gas temperature was estimated at 180 s. The speed of the drop can be described according to two main steps. The first step is characterized by a very high speed from 0 to 80 s. At that time, the temperature is 350°C (50% of drop
Figure 4. Plot of distribution of the gas temperature with space and time.
in the gas temperature). Indeed, when the heat exchange process is initiated, the gradient of temperature between the gas and ambient air is very high. As a consequence, the intensity of the exchanges between the gas and the external medium is the highest. In the second step, the speed of the drop is practically constant, corresponding to a slight decrease in the temperature evolution from 350 to 300°C.

**Conclusion**

The objective of this study is to determine the temperature of the methane gas at any time and at a given position along a pipe duct, during the flowing process, by using a numerical method. Experimental investigations were conducted in a pipe and led to extensive measures of temperature. Simulations based on the 4th ordered Runge-Kutta method were carried out. Numerical results showed a good agreement with experimental results. The selection and monitoring of the characteristics of the pipes are key factors which may aid in reducing the temperature drops and operating costs in the gas pipes.

**CONFLICT OF INTERESTS**

The authors have not declared any conflict of interests.

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