The geometry ability of junior high school students in Karanganyar based on the Hoffer’s theory

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Abstract. Geometry ability is the aspect which underlay students to solve the geometry problems. However, some studies suggests the difficulty students when learning geometry. This leads to the ability of the geometri students difficult to develop. There are five the geometry ability based the Hoffer’s theory, namely visual, verbal, drawing, logical, and applied. These five aspects are basic geometry ability to be mastered by Junior High School students level. This study aimed to describe the students’ geometry ability according to the Hoffer’s theory. The participants of this study are six students from 9th grade in State Junior High School 1 Jaten at Karanganyar that consisted of three categories, namely higher ability, moderate ability, and lower ability students. The data collection methods used are geometry test and in-depth interview and than analyzed using triangulation. The result of the study showed that the ability of those three categories is different. Each of the students' geometry ability can be described as follows. (1) On visual skill, higher ability and moderate ability students could mention the elements of the geometrical shapes correctly based on its shapes obtained. However, lower ability students were unable to mention it specifically; (2) On verbal skill, moderate ability students were able to link the relationship among shapes based on the characteristics correctly, despite that the higher ability and lower ability seemed to have difficulty; (3) On drawing skill, higher ability students could construct the shapes based on the relationship among shapes well, but moderate ability and lower ability students continually faced difficulty; (4) On logical skill, both higher ability, and moderate ability students were able to determine the formula of a particular geometrical shape based on the relationship among the elements of the shape well, while the lower ability students were unable to; (5) On applied skill, higher ability, and moderate ability students could apply the concept of geometry into the problem-solving question, but seemed to have difficulty with the calculation. Besides that, lower ability students could not ultimately implement the concept of geometry.

1. Introduction
The scope of school mathematics consists of algebra, geometry, probability and statistics, trigonometry, and calculus. Specifically, on the Junior High School level, it focuses more on the number, geometry, probability and statistics, also algebra. Geometry is a branch of mathematics concerned with a point, straight line, plane figures, space, spatial figures, and the relation between them [1]. Since it has various geometrical concepts, the geometry has a special place in the mathematics curriculum [2]. The variety of the geometrical concepts like shape and size also geometry system links with the numerical thinking as the base of mathematics. It causes the geometry learning becomes a complex learning of mathematics. In resolving a geometrical problem, it needs a clear
concept to be able to apply the geometry abilities. They are visualizing, recognizing kinds of two-dimensional figures and geometries, describing and portraying geometrical images, labeling points, identifying the geometrical similarities and differences. Furthermore, it needs a thinking skill in applying concept and way to solve a geometry problem. Therefore, the geometry is one of the hardest to be understood and the most hated-lesson by the students [3]. The geometry is the material in mathematics that is considered as difficult material by the students [4]. However, geometry still becomes the part of learning mathematics that the students need to be learned.

There are several reasons why it is important to study geometry [5]. First, geometry helps human to have a whole appreciation of their world. Second, the geometrical exploration can help the students' development of problem-solving skill. Third, it plays a significant role in the other mathematics fields. Fourth, geometry is used by a lot of people on their daily basis. Fifth, it is full of interesting challenges. It also covers our life aspects. Therefore, it is essential to explore the shape, line, angle, and figure that link with the students’ daily life [6]. It is supported by some of the statements that geometry is a part of mathematics which related to student’s life because almost all of the objects around them are geometry [7], and the study of geometry contributes to helping students develop the skills of visualization, critical thinking, intuition, problem-solving, deductive reasoning, logical argument and proof [8]. It makes the geometry as an important part of mathematics.

Considering the importance of learning geometry, therefore students need some basic skills in learning it. Commonly, they are: 1) the ability to analyse the character and nature of geometrical shape—whether it is two or three dimensional—also able to make a mathematical statement regarding to the relation among geometries; 2) the ability to specifically determine the position of a point and portray the spatial relationship with the other system; 3) apply the transformation and use it symmetrically to analyze the mathematical situation; 4) use the visualizing skill, spatial reasoning and geometry modelling to resolve a problem [9].

These abilities can be packed into five basic geometrical learning skills. The five skills include the visual, verbal, drawing, logic, and applied skills [10]. Five basic capabilities of geometry was conceived by Hoffer, so called Hoffer’s theory. The visual skill covers the ability to recognize the variety of geometries, identify the elements of geometry, classify the geometry based on its nature, and compile the information based on the visual observation, also visualize the geometry models or examples on the deductive mathematical system. The verbal skill includes the ability to identify the variety of geometries according to its names, visualize the geometry based on the verbal description, give the explanation of the geometry and its characteristics, state the correct definition of geometry, explain the relationship between geometries, recognize the logical structure from a verbal problem, also form questions in general and abstract. The drawing skill involves create the sketch based on its label, make the picture according to its verbal definition, draw geometry that has a certain connection with the given pictures, make the geometry parts, and sketch along with labeling these parts. The logic skill includes identify the similarities and differences of geometries, classify the geometries as well as which definition they are included according to its nature, develop the logic proof, also explain the relation among geometries. The applied skill includes the ability to recognize the geometry according to its physical model, make a geometry model based on its real object, apply the nature of the geometrical model, also develop and apply it on the problem-solving [10].

These five basic skills have a connection among each other. For example, students will able to apply a concept of geometry to their daily life problem if they are able to develop their geometrical logical thinking and capable to explain it verbally. This is supported by a statement: the verbal and visual skills have to develop the basic logical reasoning or logic skill [3]. Furthermore, those basic geometrical skills are important and need to be balanced with the development of problem-solving which needs an integration of the skills [10]. The broad and well-structured of geometrical knowledge also can help the students to solve the problem with a better problem-solving skill on their daily basis [11]. Therefore, these five geometrical basic skills are the whole unit that the students must have at the middle school level to resolve geometry problems.
In fact, not all of the students at middle school level have these five basic skills in geometry. This is proved by the result of some research that previously has been conducted. A study on grade VIII of junior high school students showed that only 64% of 52 students who knew that a rectangle was also a parallelogram. As much 50% of the students did not like the problem-proofing, also they were better in resolving the visual geometrical problem compared to the verbal one [12]. Another fact also showed that students, in general, did not have a good skill about the natures owned by each triangle type. This caused the lack of the ability to classify the triangle objects, in which the visual and the logic skills were still low [13]. Generally, the students’ knowledge of an example and a non-example from a triangle concept is limited on what is given by the teacher on the teaching and learning process. Students merely know that the equilateral, isosceles, and right triangles concept can be modeled into various shapes [13]; in this case, their logic skill is still weak. The factor to these weaknesses is due to each student has a different thinking skill. The teaching and learning process that still focuses more on memorizing, not on understanding the geometry concept, also becomes one of the factors. Avidenced by the existence of the research revealed that most students learn geometry based on memorizing [14]. In reality, the students’ geometrical skill is closely related to the learning process that suits with their thinking skill. The learning of geometry which makes the students have those skills will help them to understand the geometry concept well. For instance, they will know the natures of a cylinder or other solids if they are able to correctly draw and explain it verbally. Therefore, it needs a further investigation on how far the students' geometrical skills as the result of learning it. The identification of basic geometry skills of the students is required to be an alternative of the teachers' knowledge in conducting the teaching and learning of mathematics, especially on the solids geometry. It also becomes a reference in choosing the suitable learning model and media for the students.

2. Methods
This study is a qualitative study in the form of case study. The purpose of the study is to describe the junior high school students’ basic geometry skills in details based on Hoffer’s theory. A qualitative study is a research that produces descriptive data in a written or spoken form from the object of the study. The data collecting methods is through the geometry test and in-depth interview. The test is used to find the students' basic geometry skills according to Hoffer's theory. The test is conducted by provided seven questions related to curved-face three-dimensional objects. The in-depth interview is done in a semi-structured interview. These methods are to gather details information related to students’ basic geometrical skills. Further, the result from the students’ answers is analyzed based on the five basic geometrical skills according to Hoffer’s theory. They include the visual, verbal, draw, logic, and applied skills. The participants of the study are six students of 9th grade in State Junior High School 1 Jaten, Karanganyar. They are chosen based on the high, moderate, and low learning skills.

3. Results and Discussion
This study is started by grouping the students into several categories based on their learning skill. They are high, moderate, and low. Next, they are given the geometry skills test. Then, 2 students are taken from each category. From these students answer, the researchers analyze the basic geometry skills according to Hoffer’s theory. To get a details information needed, the researchers conduct an in-depth interview regarding the student's test result. From each category, the findings are as follows.

3.1. High Learning Skill Category
The data analysis of the student's answer, students in this category have a good visual skill. It is shown from their answer that they can identify the cylinder elements from its nets. They are also able to gather the related information of the cylinder based on their visual observation. On their verbal skill, the students in this category are still not capable to explain the relation among rectangular, hexagonal, polyhedron, and also prisms. The students are indicated not able to recognize the logical structure from these solids. Based on the in-depth interview, turns out the students are puzzled in answering the test. These are the students’ answers from the high learning skill category.
Figure 1. the test result of the first student in high learning skill category

Figure 2. the test result of the second student in high learning skill category

Based on the Figure 1 and Figure 2, the students are able to show their drawing skill when they are given the task to draw the cylinder nets. The students in this category are also able to show their logic skill. It shows on their answer that they could find the cylinder volume formula based on the prism volume which is the base time the height of the prism. However, based on the answer, they are still mistakenly wrote the base of area formula in the form of circle plane. In the in-depth interview, the students state that they forget the circle area formula. This shows that they still memorize the formula, not understand the concept. At this point, actually, students are already able to show their applied skill. However, due to they are still mistakenly write the base formula in a circle form, they are still having the problem in getting the answer.

3.2. Moderate Learning Skill Category
The data analysis of the student's answer, the students in this category also have a good visual skill as shown in Figure 3 and Figure 4. It is shown from their answer that can identify the cylinder elements from its nets. They are also able to gather the cylinder elements information well based on their visual observation. The students may hold the visualization and the verbal definitic visual prototype which classifying geometric figures.

On their verbal skill, the students in this category are still capable enough to explain the relation among rectangular, hexagonal, polyhedron, and also prisms. On their answer, they write the relation among these solids is equally prism but has different elements. They are also able to create a cylinder form. In their in-depth interview, the students are also able to recognize the logical structure from those solids. However, when they are given the test to draw the nets, the students are not capable enough to draw the prism nets. But, they are able to draw well on the cylinder nets. This shows that their drawing skill still lacks on the recognizing the logical structure of rectangle prism to a cylinder. The same as the students in high learning skill category, the moderate learning skill students are also able to show their logical skill. It can be seen on their answer that they are able to find the formula of the cylinder volume based on the prism volume which is a base time the height of the prism. Nevertheless, one of the students in this group still mistakenly writes the formula of the base area. This makes the student gets the wrong answer. Yet, in the in-depth interview, the student could answer the question correctly, which the formula is $\pi r^2$. She also could explain her incorrect answer verbally. This indicates that this student has known the concept well and able to show her applied skill. She is just mistakenly in writing it.
3.3. Low Learning Skill Category

Based on the data analysis of the students answer as shown in Figure 5 and Figure 6, the students in this category still not reach the good visual skill. It shows in their answer that they could not identify the cylinder elements from its nets, even though according to their drawing skill they are able to portray it correctly. Moreover, the students answer, they state that the cylinder did not have sides. This shows that they are not capable to gather the cylinder details information well based on their visual observation. The following figures are the answer of the students in this category.
On their verbal skill, the students in this group could not write the relation among rectangular, hexagonal, polyhedron, and also prisms. They are also could not explain anything on their answer sheet. In the in-depth interview, the students also could not recognize the logical structure of the presented solids. It shows that they do not have a good verbal skill. When they are given the drawing nets test, they are also not capable to draw it well on the rectangular prism. However, they could draw it well on the cylinder nets. It shows that their drawing skill is still not able to recognize the logical structure from the prism to prisms well. In contrast with the students in high and moderate categories, the students in this group are also could not show their logical skill. It could be seen in their answer that mistakenly written on the prism volume formula. This makes the students unable to solve the problem related to the cylinder volume concept application on the test. According to the in-depth interview, the students tend to confuse and say that they do not know. It shows their applied skill is still lacking. Based on the data analysis of the students answer as well as in-depth interview related to their geometry basic skills, the conclusions are shown in Table 1.

Table 1. The Basic capabilities of geometry based on students’ ability

| Category | High Learning Skill | Moderate Learning Skill | Low Learning Skill |
|----------|---------------------|-------------------------|-------------------|
| Visual skill | Able to identify the cylinder elements based on its nets, also able to gather the information related to the cylinder elements well based on their visual observation. | Able to identify the cylinder elements based on its nets, also able to gather the information related to the cylinder elements well based on their visual observation. | Unable to identify the cylinder elements based on its nets, also unable to gather the information related to the cylinder elements based on their visual observation. |
| Verbal skill | Not capable enough to write the relation among rectangular, hexagonal, polyhedron, and also prisms. Still not able to recognize the logical structure of the presented solids. | Capable enough to write the relation among rectangular, hexagonal, polyhedron, and also prisms. Able to recognize the logical structure of the presented solids. | Unable to explain anything on their answer sheet. Could not recognize the logical structure of the presented solids. |
| Drawing skill | Able to show the drawing skill well. | Not capable enough to draw the rectangle prism nets, but able to draw the cylinder nets well. | Not capable enough to draw the rectangle prism nets, but able to draw the cylinder nets well. |
| Logical skill | Capable enough to show the logical skill, even though still mistakenly write the base area formula. | Capable enough to show the logical skill, even though still mistakenly write the base of area formula. However, in depth interview, can explain it correctly; shows the ability in logical skill. | Not capable to show the logical skill. |
| Applied skill | Able to show the applied skill, but still wrongly write the formula that leads to mistakenly get the answer. | Able to understand the concept skill and show the applied skill but mistakenly write the answer. | Unable to apply the cylinder concept into the presented problem. |

4. Conclusion

There is a difference between the geometry of the basic capabilities of the students ability of high, moderate and low. This difference is seen in some basic capabilities of geometry, such as on high ability students can master visual, drawing, logical and applied skill. Students with moderate ability are able to achieve visual, verbal, logical and applied skill, whereas low ability students on just the visual
skill can achieve alone. This of course can be a material consideration for the teacher to determine appropriate action in the learning process in order to enhance the ability of basic geometry in students.

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