Quantum voltage oscillations observed on segments of an inhomogeneous superconducting loop

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The theoretical prediction published in Phys. Rev. B 64, 012505 (2001) is verified. In accordance with this prediction a dc voltage oscillating with magnetic field is observed on segments of an inhomogeneous loop in the temperature region close to the superconducting transition. The temperature dependence of the amplitude of the voltage oscillation implies that a transformation of the energy of an external electrical noise to the dc power is observed.

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According to the universally recognized explanation [1] of the Little-Parks (LP) experiment [2] the resistance oscillations are observed [3] because of the flux quantization [2,4]. Because of the quantization

\[ \int_l dl v_s = \frac{\pi h}{m} (n - \frac{\Phi}{\Phi_0}) \]  

the circulation of the velocity of superconducting pairs \(v_s\) cannot be equal zero when the magnetic flux \(\Phi\) contained within a loop is not divisible by the flux quantum \(\Phi_0 = \pi h c / b\) [1]. Therefore the energy of superconducting state increases and as consequence the \(T_s\) decreases when \(\Phi \neq n \Phi_0\), \(\Delta T_s \propto -v_s^2 \propto -(n - \Phi/\Phi_0)^2\) [1]. The resistance increases at \(\Phi \neq n \Phi_0\) because of the \(T_s\) decrease: \(\Delta R = -(dR(T_s - T)/dT)\Delta T_s \propto (dR/dT)v_s^2\) [1].

Without any external current the \(v_s\) value is proportional to the superconducting screening current \(v_s \propto I_{sc} = s I_{sc} = s 2 e n_s v_s = s 2 e n_s v_s < n_s^{-1} >^{-1} (\pi h/mL)(n - \Phi/\Phi_0)\). The spatial average \(< n_s^{-1} > = I_{long}^{-1} \int_{long} dl n_s^{-1}\) of the value \(n_s^{-1}\) inverse of the density of superconducting pairs \(n_s\) is handy to use because the \(I_{sc}\) value should be constant along the loop in the stationary state. The magnetic flux \(L_{sc}\) induced by the screening current \(I_{sc}\) is small \(L_{sc} \ll \Phi_0\) at \(T \approx T_c\) (when the \(n_s\) value is small) and therefore \(\Phi = B S + L_{sc} \approx B S\) [1]. Here \(B\) is the magnetic induction induced by an external magnet; \(S\) is the area of the loop.

Because of the thermal fluctuation the \(< n_s^{-1} >^{-1}\) and \(I_{sc}\) values oscillate strongly in time at \(T \approx T_c\). The \(n_s\) can be also an random integer number but in the majority of cases this integer number is corresponded to minimum possible value \(v_s^2 \propto (n - \Phi/\Phi_0)^2\). Therefore the time average \((n - \Phi/\Phi_0) = I_{long}^{-1} \int_{long} dt (n - \Phi/\Phi_0) \approx (n - \Phi/\Phi_0)_{min}\) when \(\Phi\) is not close to \((n + 0.5) \Phi_0\). This is corroborated by the comparison of the theoretical dependence \(\Delta T \propto -(n - \Phi/\Phi_0)^2_{min}\) [1] with the experimental data for \(\Delta T_c(\Phi)\) (see for example Fig.4 in [3]).

Thus, according to the LP experiment the persistent current

\[ I_{p.c.} = I_{sc} = s 2 e < n_s^{-1} >^{-1} \frac{\pi h}{mL} (n - \Phi/\Phi_0)_{min} \]  

i.e. a current with a direct component \(I_{sc} = \int_{long} dt I_{sc}\) flows along the loop at a constant magnetic flux, \(\Phi \neq n \Phi_0\) and \(\Phi \neq (n + 0.5) \Phi_0\), and a nonzero resistance along the loop \(R_t > 0\) in spite of the Ohm’s law \(R_t I_{sc} = \int_j dl E = -(1/c) d\Phi/dt\).

In order to explain the existence of the persistent current at zero Faraday’s voltage and nonzero resistance the quantum force is proposed to introduce in the paper [5]. It is predicted also in [5] that not only the persistent current but also a persistent voltage can be observed on segments of an inhomogeneous superconducting loop. The value and sign of the persistent voltage as well as of the persistent current should depend in a periodic way on a magnetic flux \(\Phi\). Such voltage oscillations \(V_{os}(\Phi/\Phi_0)\) without any external current was predicted first in [6]. These oscillations can be observed in the temperature region close to \(T_c\) [5,6].

The possibility of the persistent voltage predicted in [5] is obvious from the analogy with a conventional inhomogeneous loop with a current \(I_{sc}\) induced by Faraday’s voltage \(I_{sc} = R_t^{-1} \int_j dl E = -(1/c) d\Phi/dt\). According to the Ohm’s law \(\rho \Phi_{sc} = E = -\nabla V - (1/c) dA/dt = -\nabla V - (1/c) d\Phi/dt\) the potential difference

\[ V = \langle \rho >_{ls} - \langle \rho >_{l} \rangle I_s I_{sc} \]  

should be observed on a segment \(l_s\) of an inhomogeneous loop at \(j_{sc} \neq 0\) if the average resistivity along this segment \(< \rho >_{ls} = \int_l dl \rho / l_s\) differs from the one along the loop \(< \rho >_l = \int_l dl \rho / l\).

The object of the present work is an experimental verification of the theoretical prediction [5] and of the analogy with a conventional loop. According to both the prediction [5] and the analogy (3) the voltage oscillations \(V_{os}(\Phi/\Phi_0)\) without any external current can be observed in an inhomogeneous loop where \(< \rho >_{ls} - < \rho >_{l} \neq 0\) and should not observed in a homogeneous one where \(< \rho >_{ls} - < \rho >_{l} = 0\). In order to investigate the influence of the heterogeneity of loop segments we made both symmetrical and asymmetric loops in each investigated structure (see Fig.1). Because of the additional potential contacts \(V_3\) the higher and lower segments of the lower loop (on Fig.1) can have a different resistance at \(T \approx T_c\) when \(\Phi \neq n \Phi_0\), whereas the upper and lower segments of the higher loop should have the same resistance if any
accidental heterogeneity is absent.

We used the mesoscopic Al structures, one of them is shown on Fig.1. These microstructures are prepared using an electron lithograph developed on the basis of a JEOL-840A electron scanning microscop. An electron beam of the lithograph was controlled by a PC, equipped with a software package for proximity effect correction “PROXY”. The exposition was made at 25 kV and 30 pA. The resist was developed in MIBK: IPA = 1: 5, followed by the thermal deposition of a high-purity Al film 60 nm and lift-off in acetone. The substrates are Si wafers. The measurements are performed in a standard helium-4 cryostat allowing us to vary the temperature down to 1.22 K. The applied perpendicular magnetic field, which is produced by a superconducting coil, never exceeded 35 Oe. The voltage variations down to 0.05 µV could be detected. In order to diminish an influence of an external electric noise resistances were used as cold filters.

We have investigated the dependencies of the dc voltage $V$ on the magnetic flux $\Phi \approx BS$ of some round loops with a diameter $2r = 1, 2$ and $4 \mu m$ and a linewidth $w = 0.2$ and $0.4 \mu m$ at the dc measuring current $I_m$ and different temperature close to $T_c$. The sheet resistance of the loops was equal approximately $0.5 \Omega/\mu m$ at $4.2 K$, the resistance ratio $R(300K)/R(4.2K) \approx 2$ and the midpoint of the superconducting resistive transition $T_c \approx 1.24 K$. All loops exhibited the anomalous features of the resistive dependencies on temperature and magnetic field which was before observed on mesoscopic Al structures in some works [3,7].

The results of our measurements Fig.2-4 corroborate the theoretical prediction [5] and the analogy with a conventional loop (3). We observe the conventional LP oscillations of the resistance [2] at the symmetrical loops Fig.2. This result repeats the observations made before in many works and is not new result. In accordance with the prediction [5] and the analogy with a conventional loop (3) the voltage measured at the $V_1$ contacts equals zero at the measuring current $I_m = 0$ and the oscillations with the amplitude $\Delta V = \Delta R_m(\Phi/\Phi_0, T/T_c(I_m))I_m$ are observed only at $I_m \neq 0$ Fig.2. The LP oscillations are observed against a background of an anomalous behaviour, the downfall before the disappearance of the LP oscillation Fig.2. Such anomalies were observed also on other our loops and in other works [7].

FIG. 2. The voltage oscillations measured on the $V_1$ contacts of the symmetrical loop with $2r = 4 \mu m$ and $w = 0.2 \mu m$ at different $I_m$ values between the $I_1$ contacts: 1 - $I_m = 0.000 \mu A$; 2 - $I_m = 1.83 \mu A$; 3 - $I_m = 2.10 \mu A$; 4 - $I_m = 2.66 \mu A$; 5 - $I_m = 3.01 \mu A$. $T = 1.23 K$ is corresponded to the bottom of the resistive transition.

In contrast to the symmetrical loops we observe new phenomenon, which was not published before anywhere, at the voltage measurement on the contacts $V_2$ and $V_3$ of the asymmetric loop. We observe no resistance but voltage oscillations: $V \approx V_{os}(\Phi/\Phi_0) + R_{nos}I_m$ Figs.3,4. The greatest amplitude $\Delta V$ of the voltage oscillations are observed at $I_m = 0$ and the $\Delta V$ value does not increase with the $I_m$ Fig.3 in contrast to the symmetrical loop Fig.2. The voltage oscillations at a high $I_m$ are observed against a background of an anomalous behaviour, the negative magnetoresistance $R_{nos}$ Fig.3, like qualitatively the one observed in the symmetrical loop Fig.2.

The voltage oscillations $V_{os}(\Phi/\Phi_0)$ observed at $I_m = 0$ Figs.3,4 correspond with the LP experiment (2) and the analogy with a conventional loop (3). According to (2)
and (3)

\[ V_{os}(\Phi/\Phi_0) = (\langle \rho \rangle_{r_1} - \langle \rho \rangle_{r_1}) l_s j_{p.c.}(\Phi/\Phi_0) \]  

(4)

should be observed when loop segments have different resistivity \(< \rho \rangle_{r_1} \neq < \rho \rangle_{r_1}. The relation (4) describes enough well the voltage oscillations observed at \( I_m = 0 \) Figs.3,4. The \( V_{os}(\Phi/\Phi_0) \) oscillations Figs.3,4 and the LP oscillations Fig.2 have the same period and are observed in the same temperature region near \( T_c \) where \( j_{p.c.} \neq 0 \) and \(< \rho \rangle_{r_1} = R_l/l > 0 \). The magnetic field regions, where they are observed, are also close. The oscillations on Fig.2 are observed in more wide magnetic field region than on Figs.3,4 because the width of the wire defining the loop in the first case \( w = 0.2 \mu m \) is smaller than in the second case \( w = 0.4 \mu m \). In any real case only some oscillations are observed because a high magnetic field breaks down the superconductivity, i.e \( j_{p.c.} \), in the wire defining the loop. According to (1) \( v_s = (\pi \hbar/m)B_r/2 \) along the loop and \( v_s = (\pi \hbar/m)B_w/2 \) along the boundaries of the wire at \( n = 0 \). Therefore a limited number of oscillations \( \propto 2r/w \) are observed.

![Graph](image)

**FIG. 3.** The voltage oscillation measured on the \( V_2 \) contacts of the asymmetric loop with \( 2r = 4 \mu m \) and \( w = 0.4 \mu m \) at different value of the measuring current between the \( I_2 \) contacts: 1 - \( I_m = 0.000 \mu A \); 2 - \( I_m = 0.29 \mu A \); 3 - \( I_m = 0.65 \mu A \); 4 - \( I_m = 0.93 \mu A \); 5 - \( I_m = 1.29 \mu A \); 6 - \( I_m = 1.79 \mu A \); 7 - \( I_m = 2.06 \mu A \); 8 - \( I_m = 2.82 \mu A \); 9 - \( I_m = 3.34 \mu A \); 10 - \( I_m = 3.85 \mu A \). \( T = 1.231K \) corresponded to the bottom of the resistive transition.

According to (4) the amplitude of the voltage oscillations \( \Delta V \) is proportional to the lengths of the segment \( l_s \). The \( \Delta V \) values observed between \( V_3 \) and between \( V_2 \) differ approximately in 6 times Fig.4 whereas the \( l_s \) values between these contacts differ in 3 times only. This means that the \( (\langle \rho \rangle_{r_1} - \langle \rho \rangle_{r_1}) \) value in the first case is smaller than in the second. Such difference ought be expected because both additional and main contacts should influence on superconducting state of loop segments.

Using the analogy with a conventional loop (4) we can evaluate the lower limit of the persistent current \( I_{p.c.} = s j_{p.c.} \) inducing the voltage oscillations with the amplitude \( \Delta V \approx 1 \mu V \) observed on the \( V_2 \) contacts Figs.3,4. According to (4) \( V_{os} = 0.5(R_{ls} - R_{ls})I_{p.c.} \) on these contacts, where \( R_{ls} \) and \( R_{ls} \) are the resistances of the higher and lower segments at the temperature of measurement. Because \( 0 < R_{ls} < R_{ls} \) the \( I_{p.c.} \) value should exceed \( 2V_{os}/R_{ls} \), where \( R_{sn} \) is the resistance of the higher and lower segments in the normal state. The oscillations Figs.3,4 are observed on segment with \( R_{sn} \approx 5 \Omega \). Consequently the amplitude of the persistent current (2) inducing these oscillations \( \Delta I_{p.c.} \geq 0.4 \mu A \).

![Graph](image)

**FIG. 4.** Oscillation of the voltage measured on the \( V_2 \) contacts (upper curve) and on the \( V_3 \) contacts (lower curve) of the asymmetric loop with \( 2r = 4 \mu m \) and \( w = 0.4 \mu m \). \( I_m = 0 \). \( T = 1.231K \) corresponded to the bottom of the resistive transition.

We can also evaluate in order of value the persistent current inducing the LP oscillations Fig.2. The persistent current at \( \Phi = (n + 0.5)\Phi_0 \) induces a resistance change which equals in order of value the one induced by an increase of the measuring current \( \Delta I_m \approx 0.5 \mu A \) Fig.2. One should assume that \( \Delta R_m/\Delta I_m \) and \( \Delta R_m/\Delta I_{p.c.} \) are close in order of value. According to this estimation the persistent current inducing the LP oscillations Fig.2 can induce the voltage oscillations at \( I_m = 0 \) Figs.3,4.

Thus, the existance of the persistent current (2) and the analogy with a conventional loop (3) allow to explain enough well the voltage oscillations observed at \( I_m = 0 \) Figs.3,4. But this does not mean that our result shown on Figs.3,4 is not new in essence. There is an important difference between the conventional current \( I_{sc} = R_l(-1/c)d\Phi/dt \) induced by the Faraday’s voltage \((-1/c)d\Phi/dt \) and the persistent current at \( d\Phi/dt = 0 \). In the first case the current \( I_{sc} \) and the electric field
\[ E = -\nabla V - (1/c)dA/dt = -\nabla V - (1/c)\Phi/dt \]

have the same direction in all segments, i.e. each segment \( l_s \) is a load in which the power \( \int dl \nabla V \) is dissipated. In the second case the persistent current and the electric field \( \vec{E} = -\nabla V \) should have opposite directions in one of the segments because \( \int dl \nabla V = 0 \), i.e. this segment is a dc power source \( W = V_{os}I_{p.c.} \) at \( V_{os} \neq 0 \).

Already the classical LP experiment is evidence of the dc power source because an energy dissipation with power \( R_l I_{p.c.}^2 \) takes place at \( R_i > 0 \) and \( I_{p.c.} \neq 0 \). The conventional current at \( R_i > 0 \) is maintained by the quantum force \([5]\). According to \([5]\) the quantum force as well as the Faraday’s voltage is distributed uniformly among the loop. This theoretical result is corroborated by our observation of the voltage oscillations \([3,4]\). Only a load in which the power \( I_ldl \) is induced by a switching \( \Phi \) of the loop between superconducting states with different connectivity, i.e. between the states with \( <n_s^{-1}> = 0 \) and \( <n_s^{-1}> \neq 0 \) and its value is proportional to the average frequency of the switching \( \Delta V \propto \omega_{sw} \). This switching at \( T \approx T_c \) can be induced by both thermal fluctuations and an external electric noise. Therefore the chaotic energy of thermal fluctuations or an external electric noise is transformed in the dc power \( W = V_{os}I_{p.c.} \) according to \([5]\).

The amplitude \( \Delta V \) of the voltage oscillation induced by thermal fluctuations should have a maximum value near \( T_c \) because the frequency \( \omega_{sw} \) of the switching induced by the fluctuations is maximum near \( T_c \). We observed the oscillations in the temperature region corresponds to the bottom of the resistive transition Fig.5.

The temperature dependence \( \Delta V(T) \) Fig.5 shows that the voltage oscillations Fig.3,4 are induced rather by a high-frequency external electric noise. The frequency \( \omega_{sw} \) of the switching induced by the electric noise can faintly decrease with the temperature lowering and the \( \Delta V \) value should increase because \( I_{p.c.} \propto T_c - T \).

We assume that the dc voltage induced by the fluctuations is smaller than the value which we can detected. The limit value of the power which can be induced by the thermal fluctuations \((k_B T)^2/h \approx 10^{-12} Wt\) \([5]\). We detected the power down to \( 10^{-14} Wt \). But in our case the power induced by the fluctuations can be much lower than the theoretical limit and lower than we can detected. We observed in some samples voltage oscillations at the temperature corresponded to the top of the resistive transition where any electric noise should not influence on the voltage value. But this observation cannot be considered as an evidence of the power induced by the fluctuation because it is no enough reliable.

Although our experimental resources did not allow to detect the dc voltage induced by thermal fluctuations our result is new in essence. It can not be explained by conventional rectification of an external electric noise which was observed in superconductors. Such rectification is explained by an asymmetry of the sample but it can not explain the voltage oscillations Figs.3,4. Only a reasonable and natural explanation of these oscillations is the relation \((4)\) followed from \([5]\). In this relation \( j_{p.c.} \propto (n - \Phi/\Phi_0)_{\min} \) because of the quantization and \( <n_0> \geq 0 \) because of reiterated switching of \( l_s \) in the state with \( \rho > 0 \).

In conclusion, we have observed voltage oscillations measured on segments of an inhomogeneous loop at zero external direct current. This result corroborates the theoretical prediction \([5]\) according to which such voltage oscillations can be induced by thermal fluctuations or an external electric noise. We can detect the dc voltage induced only by an external electric noise.

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