Chiral symmetry-breaking corrections to strong decays of $D_{s0}^*(2317)$ and $D_{s1}'(2460)$ in $HH\chi PT$

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Abstract

The two narrow mesons $D_{s0}^*(2317)$ and $D_{s1}'(2460)$, assigned to be the $(0^+, 1^+)$ chiral partners of the $D_s$ and $D_s^*$ doublet in heavy quark spin symmetry, are studied within the framework of heavy hadron chiral perturbation theory. Using the available experimental data of the nonstrange partners, the chiral symmetry-breaking coupling constants in the next-to-leading order in $1/\Lambda_\chi$ are estimated by minimizing $\chi^2$. Through $\eta - \pi^0$ mixing we calculate their single-pion decay widths, which are consistent with the experimental constraints and comparable with other theoretical predictions. The chiral-symmetry corrections to the decay widths are significant. Implications and predictions for the corresponding beauty mesons are provided.

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I. INTRODUCTION

In recent years, many heavy mesons with open charm or hidden charm were discovered, which contribute to the revival of hadron spectroscopy (for recent reviews, see Ref. [1]). Two outstanding examples of these mesons are the narrow mesons $D_{s0}^*(2317)$ and $D_{s1}^*(2460)$, observed in the final states $D_s^+\pi^0$ and $D_s^{*+}\pi^0$ [2], which are naturally assigned as the $0^+, 1^+$ c\bar{s} mesons. But it is attracting that their measured masses and widths do not match the predictions from potential-based quark models, unexpectedly [3]. Since their discoveries, there have been lots of experimental investigations of these two narrow resonances [4-6]. Meanwhile, many theoretical papers are dedicated to the understanding of their underlying structures. Proposed schemes include the $(0^+, 1^+)$ chiral partners of the ($D_s, D_s^*$) doublet in heavy quark effective theory (HQET) [7-14], conventional c\bar{s} states [12-18], $DK$ molecules [19-21], four-quark states [22], and $D\pi$ atoms [23].

Because masses of these two states are lower than the $DK$ and $D^*K$ thresholds respectively, their strong decays are isospin violating and occur through two steps: $D_{s0}^*(2317) \rightarrow D_s + \eta \rightarrow D_s + \pi^0$ and $D_{s1}^*(2460) \rightarrow D_s^* + \eta \rightarrow D_s^* + \pi^0$. Many discussions of their strong and radiative decays, and the decays into them from the beauty mesons have been presented [8-15, 19, 24-26]. Moreover, the branching ratios of their strong and radiative decays were measured quite accurately by Belle Collaboration [4] and Babar Collaboration [6]. However, the strong decay widths from various approaches differ significantly [14, 15, 24-26]. Moreover, decay widths of their observed non-strange partners cannot be well fitted by just leading order contributions [7, 11]. To resolve this discrepancy, a more careful calculation of their strong decay widths will be very helpful.

In this work, we assume the two states as the $(0^+, 1^+)$ chiral partners of the ($D_s, D_s^*$) doublet in HQET, and consider chiral symmetry-breaking corrections to the strong decays of $D_{s0}^*(2317)$ and $D_{s1}^*(2460)$, within the framework of heavy hadron chiral perturbation theory (HH\chi PT) [27], which is a combination of HQET and chiral perturbation theory. In Section II we incorporate the doublets into the effective heavy hadron chiral Lagrangian, which is written out to terms of next-to-leading order in $1/\Lambda_\chi$. In Section III we discuss single-pion decays of charmed heavy mesons and the corresponding beauty ones in the heavy quark spin-flavor symmetry. Numerical results are discussed in Section IV. This section also includes a brief summary.
II. THE CHIRAL LAGRANGIAN

The strong decays of excited heavy-light mesons involve the emission of soft pions and kaons, and hence it is useful to analyze these interactions with the help of chiral perturbation theory \[27\]. The octet of light pseudoscalar mesons is introduced through the definition

\[
\Sigma = \xi = \exp\left(\frac{2i}{f_\pi} M / f_\pi \right),
\]

where

\[
M = \pi_0 + \frac{1}{\sqrt{6}} \eta \pi + K + \bar{K} - \frac{1}{\sqrt{2}} \eta \pi + \frac{1}{\sqrt{6}} \eta K
\]

(1)

As to the heavy-light mesons, they are customarily cataloged by the total angular momentum of the light degrees of freedom \(s_p \lambda\) (\(p\) denotes the parity), which is a good quantum number because of heavy quark spin symmetry in the heavy quark limit \(m_Q \to \infty\).

In this paper, only two doublets corresponding to \(s_p \lambda = \frac{1}{2} - \frac{1}{2}, \frac{1}{2} + \frac{1}{2}\) are discussed, which can be respectively represented by the superfields

\[
H_a = 1 + \frac{\nu}{2} \left[ \gamma^\mu - \gamma^5 \right] (a = u, d, s, \text{a light flavor index})
\]

and

\[
S_a = 1 + \frac{\nu}{2} \left[ \gamma^5 \right] (a = u, d, s, \text{a light flavor index})
\]

Considering heavy quark spin-flavor symmetry and light quark chiral symmetry, an effective Lagrangian responsible for the strong decay \(S \to HM\) (\(M\) is a light pseudoscalar meson) can be written with these superfields. The leading order contribution in \(1/\Lambda_\chi\) and \(1/m_Q\) is

\[
\mathcal{L}_{mix} = h Tr[\bar{H}_b S_a A_{ab} \gamma_5] + h.c.
\]

(2)

According to Refs.\[28, 29\], the corresponding chiral symmetry-breaking corrections to the Lagrangian Eq. (2) to next-to-leading order in \(1/\Lambda_\chi\) read

\[
\mathcal{L}_{mix}^{sb} = \frac{1}{\Lambda_\chi} \left\{ \kappa_1 Tr[(\bar{H} S A \gamma_5)_{ab} (m^2_q)_{ba}] + \kappa_2 Tr[(\bar{H} S A \gamma_5)_{aa} (m^2_q)_{bb}] 
+ \kappa_3 Tr[\bar{H}_a S_b A_{bc} \gamma_5 (m^2_q)_{cb}] + \kappa_4 Tr[\bar{H}_c S_a A_{bc} \gamma_5 (m^2_q)_{ab}] 
+ \kappa_5 Tr[\bar{H}_a S_b i \nu \cdot D_{bc} A_{ca} \gamma_5] + \kappa_6 Tr[\bar{H}_a S_b j \nu \cdot A_{ca} \gamma_5] \right\} + h.c.
\]

(3)

Meanwhile, the effective Lagrangian responsible for \(\eta - \pi^0\) mixing, through which the pionic decays of \(D_{s0}^{*}(2317)\) and \(D_{s1}^{*}(2460)\) occur, can be described by the isospin violating piece in the chiral Lagrangian

\[
\mathcal{L}_{\eta - \pi^0} = \frac{m^2_{\pi} f^2_{\pi}}{4(m_u + m_d)} Tr[\pi^0 \Sigma + \Sigma^\dagger m_q] = \frac{m^2_{\pi}(m_u - m_d)}{\sqrt{3}(m_u + m_d)} \pi^0 \eta + \cdots.
\]

(4)
Here, $H_a = \gamma^0 H_a \gamma^0$, and $D^\mu_{ab} = \delta_{ab} \partial^\mu - V^\mu_{ab}$. $V_\mu = 1/2(\xi^\dagger \partial_\mu \xi + \xi \partial_\mu \xi^\dagger)$ and $A_\mu = i/2(\xi^\dagger \partial_\mu \xi - \xi \partial_\mu \xi^\dagger)$ are the light meson vector and axial currents, containing an even number and an odd number of pseudoscalar fields, respectively. $D^\mu_{ab} A^\nu_{bc} = \partial^\mu A^\nu_{ac} + [V^\mu, A^\nu]_{ac}$ and $\Lambda_\chi = 4\pi f_\pi$. The $3 \times 3$ matrix $m_q = \text{diag}(m_u, m_d, m_s)$, and $m^\xi_q = \xi m_q \xi + \xi^\dagger m_q \xi^\dagger$.

Note that a full calculation of the strong decay should contain, in addition to the chiral symmetry-breaking corrections, the heavy quark symmetry breaking corrections in $1/m_Q$ as discussed in Ref. [30]. However, there are too many unknown couplings to be determined in comparison with the data available, thus weakening the effectiveness of our numerical method. Additionally, recent lattice QCD studies [31, 32] of the strong couplings of heavy mesons indicated that these $1/m_Q$ corrections seem not to be significant but pointed out the importance of controlling chiral corrections, based on which chiral loop corrections to strong decays of non-strange charmed mesons of $S$ doublet has been studied [29]. Therefore, we ignore the heavy quark symmetry corrections in our calculation.

### III. SINGLE-PION DECAY OF EXCITED HEAVY MESONS

Using the Lagrangian given in Section II, the formula of the single-pion decays $S_a \to H_b \pi^i$ ($a, b = u, d, s$ and $i = 1, 2, \ldots, 8$) are

$$\Gamma(P'_{1a} \to P_b^* \pi^i) = \frac{1}{8\pi} \frac{M_{P'_{1a}}}{M_{P_b^*}} E^2_{\pi^i} |\vec{P}_{\pi^i}| \theta^2_{ab} F^2,$$  
$$\Gamma(P_{0a}^* \to P_b \pi^i) = \frac{1}{8\pi} \frac{M_{P_{0a}^*}}{M_{P_b^*}} E^2_{\pi^i} |\vec{P}_{\pi^i}| \theta^2_{ab} F^2,$$  

where the function $F$ is defined as

$$F = \frac{2h}{f_\pi} \lambda^{i}_{ab} + \frac{4\kappa_1}{\Lambda_\chi f_\pi} \lambda^{i}_{ac}(m_q)_{cb} + \frac{4\kappa_2}{\Lambda_\chi f_\pi} \lambda^{i}_{ab}(m_q)_{cc} + \frac{4\kappa_3}{\Lambda_\chi f_\pi} \lambda^{i}_{cd}(m_q)_{dc} \delta_{ab} + \frac{4\kappa_4}{\Lambda_\chi f_\pi} \lambda^{i}_{cd}(m_q)_{ac} - \frac{2\kappa'_5}{\Lambda_\chi f_\pi} \lambda^{i}_{ab} E_{\pi^i}.$$  

$\theta_{ab} = \theta$ for $ab = 33$ and 1 for others, $\kappa'_5 = \kappa_5 + \kappa_6$, and $\lambda^i$ is the corresponding coefficient matrix of $\pi^i$ in the definition of $\mathcal{M} = \pi^i \lambda^i$. The $\eta - \pi^0$ mixing angle is

$$\theta = \frac{\sqrt{3}}{4} \frac{m_d - m_u}{m_s - (m_u + m_d)/2},$$

accounting for the isospin violation.

The observed heavy-light mesons of $S$ doublet are $D^0_{b0}(2400)$, $D^0_{b0}^\pm(2400)$, $D^0_{c1}(2430)$, $D^*_{s0}(2317)$ and $D_{s1}^*(2460)$, as shown in Table II. It can be seen that the decay rates of $D^0_{s0}^+$ and $D^1_{s1}$ mesons are decoupled.
TABLE I: Measured masses and widths of the observed $S$ doublet heavy-light mesons with open charm. All the results are from the PDG [33], and the quoted bounds are at 95% CL.

| $cq(q = u, d)$ | Mass (MeV) | $\Gamma$ (MeV) | $c\bar{s}$ | Mass (MeV) | $\Gamma$ (MeV) |
|---------------|------------|----------------|------------|------------|----------------|
| $D_{0}^0(2400)$ | 2318 ± 29  | 267 ± 40       |            |            |                |
| $D_{0}^+(2400)$ | 2403 ± 14 ± 35 | 283 ± 24 ± 34 | $D_{s0}^+(2317)$ | 2317.8 ± 0.6 | < 3.8         |
| $D_{1}^0(2430)$ | 2427 ± 26 ± 25 | 384 ± 107 ± 74 | $D_{s1}^+(2460)$ | 2459.6 ± 0.6 | < 3.5         |

Correspondingly, decay widths of these observed $S$ doublet charmed mesons to $H$ doublet are

\[
\Gamma(D_{0}^0 \rightarrow D^{+}\pi^{-}) = \frac{1}{8\pi M_{D_{0}^0}} \left| E_{\pi^{-}} \right|^2 \left| \vec{p}_{\pi^{-}} \right|^2 \left( \frac{2h}{f_{\pi}} + \frac{4m_{d}}{\Lambda_{\chi}} \kappa_{1} \right) + \frac{4(m_{u} + m_{d} + m_{s})}{\Lambda_{\chi}} \kappa_{2} - \frac{2E_{\pi^{-}}}{\Lambda_{\chi}} \kappa_{3} \kappa_{4}^{\prime 2},
\]

(7a)

\[
\Gamma(D_{0}^{0\ast} \rightarrow D^{0\ast}\pi^{0}) = \frac{1}{8\pi M_{D_{0}^{0\ast}} E_{\pi^{0}} |\vec{p}_{\pi^{0}}|^{2}} \left( \frac{2h}{f_{\pi}} + \frac{4m_{d}}{\Lambda_{\chi}} \kappa_{1} \right) + \frac{4(m_{u} + m_{d} + m_{s})}{\Lambda_{\chi}} \kappa_{2} - \frac{2E_{\pi^{0}}}{\Lambda_{\chi}} \kappa_{3} \kappa_{4}^{\prime 2},
\]

(7b)

\[
\Gamma(D_{1}^{0\ast} \rightarrow D^{+}\pi^{0}) = \frac{1}{16\pi M_{D_{1}^{0\ast}} E_{\pi^{0}} |\vec{p}_{\pi^{0}}|^{2}} \left( \frac{2h}{f_{\pi}} + \frac{4m_{d}}{\Lambda_{\chi}} \kappa_{1} \right) + \frac{4(m_{u} + m_{d} + m_{s})}{\Lambda_{\chi}} \kappa_{2} - \frac{2E_{\pi^{0}}}{\Lambda_{\chi}} \kappa_{3} \kappa_{4}^{\prime 2},
\]

(7c)

As to the $D_{s0}^+(2317)$ and $D_{s1}^+(2460)$, they decay to related $H$ doublet via $\eta$ meson. The
decay widths are

\[ \Gamma(D_{s0}^+(2317) \to D_s^+\eta \to D_s^+\pi^0) = \frac{1}{12\pi} \frac{M_{D_{s0}^+}^2}{M_{D_{s0}^+}^2} E_\pi^2 |f_\pi|^2 |2h| f_\pi \frac{4m_s}{\Lambda_f \eta} \kappa_1 + \frac{4(m_u + m_d + m_s)}{\Lambda_f \eta} \kappa_2 \]

\[ + \frac{2(2m_s - m_u - m_d)}{\Lambda_f \eta} \kappa_3 + \frac{4m_s}{\Lambda_f \eta} \kappa_4 - \frac{2E_\pi^0 |\kappa_5'|^2}{\Lambda_f \eta} \] \quad (8a)

\[ \Gamma(D_{s1}^+(2460) \to D_s^+\eta \to D_s^+\pi^0) = \frac{1}{12\pi} \frac{M_{D_{s1}^+}^2}{M_{D_{s1}^+}^2} E_\pi^2 |f_\pi|^2 |2h| f_\pi \frac{4m_s}{\Lambda_f \eta} \kappa_1 + \frac{4(m_u + m_d + m_s)}{\Lambda_f \eta} \kappa_2 \]

\[ + \frac{2(2m_s - m_u - m_d)}{\Lambda_f \eta} \kappa_3 + \frac{4m_s}{\Lambda_f \eta} \kappa_4 - \frac{2E_\pi^0 |\kappa_5'|^2}{\Lambda_f \eta} \] \quad (8b)

In calculations above, the normalization relations for annihilation operators \( P_a, P_{a\mu}, P_{0a}, P'_{1a\mu} \) are

\[ \langle 0 | P_a | Q\bar{q}(0^-) \rangle = \sqrt{M_H}, \quad \langle 0 | P_{a\mu}^* | Q\bar{q}(1^-) \rangle = \varepsilon_\mu \sqrt{M_H} \]

\[ \langle 0 | P_{0a}^* | Q\bar{q}(0^+) \rangle = \sqrt{M_S}, \quad \langle 0 | P_{1a\mu}^* | Q\bar{q}(1^+) \rangle = \varepsilon_\mu \sqrt{M_S} \]

IV. NUMERICAL RESULTS

In the numerical evaluation, from Ref. [33], the quark masses are \( m_u = 2.3\text{MeV}, \) \( m_d = 4.8\text{MeV}, \) \( m_s = 95\text{MeV} \) and \( f_\pi = 130.4\text{MeV} \). The suppression factor is \( \theta \simeq 0.01 \) [34].

As for the leading-order coupling constant \( h \), the weighted average is obtained as \( h = 0.56 \pm 0.04 \) under the same theoretical framework [7], using the experimental data of the doublet \( (D_0^0(2400), D_1^+(2430)) \) and considering only the leading-order contributions in Eq. (7). This result nicely agrees with the QCD sum rule outcome [35] and with the lattice QCD determination [36]. We thus quote it directly here, and estimate \( 1/\Lambda_\chi \) corrections to this leading-order coupling constant in the following.

Next, we numerically fit the six \( 1/\Lambda_\chi \) chiral symmetry-breaking coupling constants using the available data. However, with the three available widths of the \( S \) doublet in Table I and the fact that \( \kappa_5 \) and \( \kappa_6 \) enter in a fixed linear combination \( \kappa_5 + \kappa_6 \), we are still left with two arbitrary parameters. Thus we estimate them by minimizing \( \chi^2 \) following the approach by M. Di Pierro and E. Eichten [3]. Here, considering that the three widths of the \( S \) doublet vary relatively largely, the \( \chi^2 \) function is defined as

\[ \chi^2 = \sum_{i=1}^{3} \frac{(\Gamma_{\text{theo}}^{(i)} - \Gamma_{\text{exp}}^{(i)})^2}{(\delta \Gamma_{\text{exp}}^{(i)})^2} \]

where \( \Gamma_{\text{exp}}^{(i)} \) and \( \delta \Gamma_{\text{exp}}^{(i)} \) are the experimental widths and uncertainties of \( D_0^0(2400), D_1^+(2400) \)
TABLE II: Results of the six $1/\Lambda_\chi$ parameters by minimizing $\chi^2$ (only central values)

| $h$ | $\kappa_1$ | $\kappa_2$ | $\kappa_3$ | $\kappa_4$ | $\kappa_5 + \kappa_6$ |
|-----|-----------|-----------|-----------|-----------|----------------|
| 0.56 | 0.88 | 0.52 | 0.89 | 0.85 | 0.28 |

and $D^0_1(2430)$; $\Gamma_{\text{theo}}^{(i)}$ is the numerical value corresponding to a set of given symmetry-breaking coupling constants.

Before minimizing the $\chi^2$, we should at first ensure that the leading order contributions are dominant over the chiral symmetry-breaking corrections, physically. Thus we can get the upper bounds of these parameters:

$$|\kappa_1| < 4.83, \quad |\kappa_2| < 4.50, \quad |\kappa_3| < 5.02,$$

$$|\kappa_4| < 4.83, \quad |\kappa'_5| < 1.70.$$ (10)

We then minimize $\chi^2$ in its five dimensional domain within these bounds. It is found that the minimum of $\chi^2$ depends both on the starting parameters $\kappa^0_i$s within these bounds, and on the masses of involved particles, especially $D^+_{0^0}, D^+_{0^+}$ and $D'_1$, varying within the experimental uncertainties. Therefore, we repeat the procedure of minimization with different sets of starting parameters $\kappa^0_i$s and the masses of involved particles until we are confident that we have found the absolute minimum. Finally, we work out the six $1/\Lambda_\chi$ parameters, as shown in Table III and the value of $\chi^2$ is 0.71, satisfying the demand of $\chi^2/2 < 1$. The major contribution to the value of $\chi^2$ comes from the discrepancy between decay rates of $D^+_{0^+}$ and $D'_1$ mesons.

We now turn to the estimation of the strong decay rates of $D^*_{s0}(2317)$ and $D^*_{s1}(2460)$, the numerical results are showed in Table III. Our results are consistent with the experimental constraints and comparable with other theoretical works in literature. It is shown that both $D^*_{s0}(2317)$ and $D^*_{s1}(2460)$ are quite narrow and the chiral symmetry-breaking corrections are significant, compared to the leading-order contributions in Ref. [14], which is mainly because that $m_s$ is relatively large as can be seen from our width formulae.

As to the excited beauty heavy-light mesons, no candidate of the $S$ doublet is observed. However, using the masses predicted by P. Colangelo et al. in Ref. [7] within the same framework, we can obtain the single-pion decay widths of these mesons, which have the same formula as the charmed ones for the reason of heavy quark spin symmetry. It is worth
TABLE III: Strong decay rates of $D_{s0}^*(2317)$ to $D_s\pi^0$ and $D_{s1}'(2460)$ to $D^*_s\pi^0$ (in KeV)

|            | [14] | [15] | [24] | [25] | [26] | ours |
|------------|------|------|------|------|------|------|
| $D_{s0}^*(2317)$ | 21.5 | 34 − 44 | 7 ± 1 | ≈ 10 | 16 | 40 ± 0.2 |
| $D_{s1}'(2460)$  | 21.5 | 35 − 51 | 7 ± 1 | ≈ 10 | 32 | 40 ± 0.2 |

TABLE IV: Strong decay widths of $B_0^*$ and $B_1'$ (in MeV), $B_{s0}^*$ and $B_{s1}'$ (in KeV)

|            | Mass[7] | Γ[7] | Γ[14] | Γ [37] | Γ[38] | Γ[39] | Γ(ours) |
|------------|---------|------|-------|---------|-------|-------|---------|
| $B_0^*(0^+)$ | 5708.2 ± 22 | 269 ± 58 | - | - | 87 | - | 302 ± 43 |
| $B_1'(1^+)$  | 5753.3 ± 31 | 268 ± 70 | - | - | 93 | - | 301 ± 60 |
| $B_{s0}^*(0^+)$ | 5706.6 ± 1.2 | - | 21.5 | 13.6 ± 5.6 | 1.6 | 6.8 − 30.7 | 41 ± 0.2 |
| $B_{s1}'(1^+)$ | 5765.6 ± 1.2 | - | 21.5 | 13.8 ± 3.6 | 1.9 | 5.7 − 20.7 | 45 ± 0.2 |

remarking that the masses of $B_{s0}^*$ and $B_{s1}'$ predicted are below the $BK$ and $BK^*$ thresholds, and therefore these two mesons are expected to be very narrow, with main decays into $B_s\pi^0$ and $B_s^*\pi^0$. Further considering the uncertainties of the masses predicted, the OZI allowed decay channels $B_{s0}^* \rightarrow B\bar{K}$ and $B_{s1}' \rightarrow B^*\bar{K}$ are discussed in Ref. [37]. The numerical results are shown in Table IV.

In summary, we have investigated the strong decays of the exotic states $D_{s0}^*(2317)$ and $D_{s1}'(2460)$, within the framework of $HH\chi PT$. Considering the chiral symmetry-breaking effects, the effective heavy hadron chiral Lagrangian up to terms of next-to-leading order in $1/\Lambda_\chi$ is given. Single-pion decay widths of charmed heavy mesons and the corresponding beauty ones with the heavy quark spin-flavor symmetry are calculated.

Using the existing experimental data of the non-strange partners of $D_{s0}^*(2317)$ and $D_{s1}'(2460)$, the coupling constants are estimated by minimizing $\chi^2$. Numerical results show that our results are consistent with the experimental constraints and comparable with the other theoretical works in literature. And the chiral symmetry-breaking corrections of $b\bar{u}$ and $b\bar{d}$ are small compared to the leading order contributions of Ref. [7], while these of $b\bar{s}$ are significant, which is mainly because that $m_s$ is relatively large, while $m_u$ and $m_d$ are small as can be seen from our width formulae. The confirmation of such predictions is expected in the very near future by experiments at the CERN LHC and the hadron B factories.
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