GRAVITATIONAL ENERGY OF CONICAL DEFECTS

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Abstract

The energy density $\varepsilon_g$ of asymptotically flat gravitational fields can be calculated from a simple expression involving the trace of the torsion tensor. The integral of this energy density over the whole space yields the ADM energy. Such energy expression can be justified within the framework of the teleparallel equivalent of general relativity, which is an alternative geometrical formulation of Einstein’s general relativity. In this letter we apply $\varepsilon_g$ to the evaluation of the energy per unit length of a class of conical defects of topological nature, which include disclinations and dislocations (in the terminology of crystallography). Disclinations correspond to cosmic strings, and for a spacetime endowed with only such a defect we obtain precisely the well known expression of energy per unit length. However for a pure spacetime dislocation the total gravitational energy is zero.

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I. Introduction

It is believed that phase transitions in the early universe can give rise to topological defects, which can lead to very important cosmological consequences\cite{1}. In order to understand the formation of galaxies and cluster of galaxies it has been suggested that these structures have evolved from the gravitational instability of small density fluctuations. One of the major unresolved cosmological problems is the origin of these initial density fluctuations. One possibility is that the latter are due to cosmic strings\cite{2, 3}. For this reason cosmic strings have been widely studied in the literature.

In the terminology of crystallography\cite{4, 5} cosmic strings correspond to disclinations, which is one possible defect in a crystal. To our knowledge, other common crystal defects like dislocations have not yet been considered in the various cosmological models. From a geometrical point of view, disclinations and dislocations are conical singularities in a flat, four dimensional Lorentzian spacetime, i.e., they can be described by a metric which is flat away from the $r = 0$ axis, but with a coordinate singularity that cannot be removed. Tod\cite{6} has recently considered these defects in a unified fashion. He generalized an argument due to Vickers\cite{7}, according to which a conical singularity like a cosmic string can be interpreted in terms of a $\delta$-function of curvature supported in the origin of the $(x, y)$ plane, say. Tod extended this idea to conical singularities of the dislocation type and argued likewise that instead of interpreting the latter as defects in an otherwise flat Minkowski spacetime, one can alternatively consider a flat Minkowski spacetime endowed with a delta function of torsion as a source in the $r = 0$ axis.

In this paper we will calculate the gravitational energy of the field configuration considered by Tod, which includes altogether disclinations and dislocations. We will obtain the energy per unit length along the defect axis $r = 0$. For a metric field which describes a
disclination only the energy per unit length turns out to be exactly the same of the cosmic string. Together with other previous calculations of energy of black hole configurations, this result supports the validity of the present energy expression.

For the dislocations the result is remarkable. The energy per unit length of these defects is zero, when the integration extends over the whole three dimensional space. This result may be of significance for cosmology. It may be conjectured that a physical, cosmological disclination needs a very low energy to be formed, and hence should play an important role in phase transitions in the early universe.

The difficulty in obtaining an expression for localized energy density in the framework of the Hilbert-Einstein action integral has led to the widespread belief that the gravitational energy cannot be localized. We do not share this idea. The energy density of the gravitational field can be naturally obtained from the Hamiltonian formulation of the teleparallel equivalent of general relativity (TEGR) \cite{9}. The TEGR is an alternative geometrical formulation of Einstein’s general relativity. The gravitational field in the TEGR is described by the tetrad field, and its dynamics is dictated by Einstein’s equations. Therefore this is not an alternative theory of general relativity. The gravitational energy density for asymptotically flat geometries has been presented and justified in ref.\cite{10}.

In section II we present the mathematical preliminaries of the TEGR, its Hamiltonian formulation and the expression of the energy for an arbitrary asymptotically flat spacetime. We also calculate the energy inside a surface of constant radius \( r_o \) for both the Schwarzschild and the Kerr solutions. In section III we present the calculation of the energy of conical defects.

II. The TEGR in Hamiltonian form

Notation: spacetime indices \( \mu, \nu, \ldots \) and local Lorentz indices \( a, b, \ldots \) run from 0 to 3.
In the 3+1 decomposition latin indices from the middle of the alphabet indicate space indices according to $\mu = 0, i, a = (0), (i)$. The tetrad field $e^a{}_{\mu}$ and the spin connection $\omega_{\mu ab}$ yield the usual definitions of the torsion and curvature tensors: $R^a{}_{b\mu\nu} = \partial_\mu \omega^a_{\nu b} + \omega^a_{\mu c} \omega^c_{\nu b} - \ldots$, $T^a{}_{\mu\nu} = \partial_\mu e^a{}_{\nu} + \omega^a_{\mu b} e^b{}_{\nu} - \ldots$. The flat spacetime metric is fixed by $\eta(0)(0) = -1$.

In the TEGR the tetrad field $e^a{}_{\mu}$ and the spin connection $\omega_{\mu ab}$ are completely independent field variables. The latter is enforced to satisfy the condition of zero curvature. The Lagrangian density in empty spacetime is given by

\[
L(e, \omega, \lambda) = -k e \left( \frac{1}{4} T^{abc} T_{abc} + \frac{1}{2} T^{abc} T_{bac} - T^a T_a \right) + e \lambda^{ab\mu\nu} R_{ab\mu\nu}(\omega). \tag{1}
\]

where $k = \frac{1}{16\pi G}$, $G$ is the gravitational constant; $e = \det(e^a{}_{\mu})$, $\lambda^{ab\mu\nu}$ are Lagrange multipliers and $T_a$ is the trace of the torsion tensor defined by $T_a = T^b{}_{ba}$.

The equivalence of the TEGR with Einstein’s general relativity is based on the identity

\[
e R(e, \omega) = e R(e) + e \left( \frac{1}{4} T^{abc} T_{abc} + \frac{1}{2} T^{abc} T_{bac} - T^a T_a \right) - 2 \partial_\mu (e T^\mu), \tag{2}
\]

which is obtained by just substituting the arbitrary spin connection $\omega_{\mu ab} = \partial_\nu \omega_{\nu ab}(e) + K_{\mu ab}$ in the scalar curvature tensor $R(e, \omega)$ in the left hand side; $\omega_{\mu ab}(e)$ is the Levi-Civita connection and $K_{\mu ab} = \frac{1}{2} e^\lambda_a \epsilon_{b\nu} (T_{\lambda\mu\nu} + T_{\nu\lambda\mu} - T_{\mu\nu\lambda})$ is the contorsion tensor. The vanishing of $R^a{}_{b\mu\nu}(\omega)$, which is one of the field equations derived from (1), implies the equivalence of the scalar curvature $R(e)$, constructed out of $e^a{}_{\mu}$ only, and the quadratic combination of the torsion tensor. It also ensures that the field equation arising from the variation of $L$ with respect to $e^a{}_{\mu}$ is strictly equivalent to Einstein’s equations in tetrad form. Let $\frac{\delta L}{\delta e^a{}_{\mu}} = 0$ denote the field equation satisfied by $e^a{}_{\mu}$. It can be shown by explicit
calculations that
\[
\frac{\delta L}{\delta e^a\mu} = \frac{1}{2}\{R_{a\mu} - \frac{1}{2}e_{a\mu}R(e)\}.
\]
(we refer the reader to refs. [8, 9] for additional details).

It is important to note that for asymptotically flat spacetimes the total divergence in (2) does not contribute to the action integral. Therefore the latter does not require additional surface terms, as it is invariant under coordinate transformations that preserve the asymptotic structure of the field quantities [10].

The Hamiltonian formulation of the TEGR can be successfully implemented if we fix the gauge \(\omega_{0ab} = 0\) from the outset, since in this case the constraints (to be shown below) constitute a first class set [8]. The condition \(\omega_{0ab} = 0\) is achieved by breaking the local Lorentz symmetry of (1). We still make use of the residual time independent gauge symmetry to fix the usual time gauge condition \(e_{(k)}^0 = e_{(0)i} = 0\). Because of \(\omega_{0ab} = 0\), \(H\) does not depend on \(P^{kab}\), the momentum canonically conjugated to \(\omega_{kab}\). Therefore arbitrary variations of \(L = p\dot{q} - H\) with respect to \(P^{kab}\) yields \(\dot{\omega}_{kab} = 0\). Thus in view of \(\omega_{0ab} = 0\), \(\omega_{kab}\) drops out from our considerations. The above gauge fixing can be understood as the fixation of a global reference frame.

Under the above gauge fixing the canonical action integral obtained from (1) becomes [8]

\[
A_{TL} = \int d^4x\{\Pi^{(j)k}\dot{e}_{(j)k} - H\},
\]
where

\[
H = NC + N^iC_i + \Sigma_{mn}\Pi^{mn} + \frac{1}{8\pi G}\partial_k(NeT^k) + \partial_k(\Pi^{jk}N_j).
\]
\[ C = \partial_j (2k e T^j) - k e \Sigma^{kij} T_{kij} - \frac{1}{4k e} (\Pi^{ij} \Pi_{ji} - \frac{1}{2} \Pi^2) , \] (5)

\[ C_k = - e_{(j)k} \partial_i \Pi^{(j)i} - \Pi^{(j)\nu} T_{(j)\nu k} , \] (6)

with \( e = \text{det}(e_{(j)k}) \) and \( T^i = g^{ik} \epsilon^{(j)\nu} T_{(j)\nu k} \), \( T_{(j)\nu k} = \partial_{(j)k} - \partial_k e^{(j)\nu} \). We remark that (3) and (4) are invariant under \textit{global} SO(3) and general coordinate transformations.

We assume in this section the asymptotic behaviour
\[ e_{(j)k} \approx \eta_{jk} + \frac{1}{2} h_{jk} \left( \frac{1}{r} \right) \] for \( r \to \infty \).

In view of the relation
\[ \frac{1}{8\pi G} \int d^3 x \partial_j (e T^j) = \frac{1}{16\pi G} \int_S dS_k (\partial_k h_{ik} - \partial_k h_{ii}) \equiv E_{\text{ADM}} \] (7)

where the surface integral is evaluated for \( r \to \infty \), we note that the integral form of the Hamiltonian constraint \( C = 0 \) may be rewritten as
\[ \int d^3 x \left\{ k e \Sigma^{kij} T_{kij} + \frac{1}{4k e} (\Pi^{ij} \Pi_{ji} - \frac{1}{2} \Pi^2) \right\} = E_{\text{ADM}} . \] (8)

The integration is over the whole three dimensional space. Given that \( \partial_j (e T^j) \) is a scalar density, from (7) and (8) we define the gravitational energy density enclosed by a volume \( V \) of the space as
\[ E_g = \frac{1}{8\pi G} \int_V d^3 x \partial_j (e T^j) . \] (9)

It must be noted that this expression is also invariant under global SO(3) transformations.

We will briefly recall two applications of \( E_g \). Let us initially consider a spherically symmetric geometry and fix the triads \( e_{(k)i} \) as
\[
\epsilon_{(k)i} = \begin{pmatrix}
e^\lambda \sin \theta \cos \phi & r \cos \theta \cos \phi & -r \sin \theta \sin \phi \\
e^\lambda \sin \theta \sin \phi & r \cos \theta \sin \phi & r \sin \theta \cos \phi \\
e^\lambda \cos \theta & -r \sin \theta & 0
\end{pmatrix}
\]  \tag{10}

\( (k) \) is the line index and \( i \) is the column index. The function \( \lambda(r) \) is determined by

\[
e^{-2\lambda} = 1 - \frac{2mG}{r}
\]

The one form \( e^{(k)} = e^{(k)}_r dr + e^{(k)}_\theta d\theta + e^{(k)}_\phi d\phi \) yields

\[
e^{(k)} \epsilon_{(k)} = e^{2\lambda} dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2 .
\]

Therefore the triads given by (10) represent the spatial section of the Schwarzschild solution. We can easily calculate \( \varepsilon_g = \frac{1}{8\pi G} \partial_i (e T^i) \) associated to (10). We obtain

\[
\varepsilon_g = \frac{1}{G} \frac{\partial}{\partial r} [r(1 - e^{-\lambda})] , \tag{11}
\]

The energy inside a spherical surface of arbitrary radius \( r_o \) can be calculated from (11). It is given by

\[
E_g = r_o \left\{ 1 - \left(1 - \frac{2mG}{r_o} \right) \right\} . \tag{12}
\]

This is exactly the expression found by Brown and York in their analysis of quasi-local gravitational energy. They define a general expression for quasi-local energy as minus the proper time rate of change of the Hilbert-Einstein action (with surface terms included), in analogy with the classical Hamilton-Jacobi equation which expresses the energy of a classical solution as minus the time rate of change of the action. The application of their procedure to the Schwarzschild solution yields (12). Note that when \( r_o \to \infty \) we find \( E = m \).
The definition (9) for the gravitational energy can also be successfully applied to the Kerr black hole\cite{12}. In terms of Boyer and Lindquist coordinates\cite{13} \((t, r, \theta, \phi)\) the spatial section of the Kerr metric is given by

\[ds^2 = \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 + \frac{\Sigma^2}{\rho^2} \sin^2 \theta d\phi^2\]  

with the following definitions:

\[\Delta = r^2 - 2mr + a^2,\]

\[\rho^2 = r^2 + a^2 \cos^2 \theta,\]

\[\Sigma^2 = (r^2 + a^2)^2 - \Delta a^2 \sin^2 \theta.\]

\(a\) is the specific angular momentum defined by \(a = \frac{J}{m}\). The triads appropriate to the three-metric above are given by

\[e_{(k)i} = \begin{pmatrix} \frac{\rho}{\sqrt{\Delta}} \sin \theta \cos \phi & \rho \cos \theta \cos \phi & -\frac{\Sigma}{\rho} \sin \theta \sin \phi \\ \frac{\rho}{\sqrt{\Delta}} \sin \theta \sin \phi & \rho \cos \theta \sin \phi & \frac{\Sigma}{\rho} \sin \theta \cos \phi \\ \frac{\rho}{\sqrt{\Delta}} \cos \theta & -\rho \sin \theta & 0 \end{pmatrix}\]  

(14)

Indeed, defining again the one-form \(e^{(k)} = e^{(k)}_r dr + e^{(k)}_\theta d\theta + e^{(k)}_\phi d\phi\) we easily find that \(e^{(k)} e_{(k)} = ds^2\) given by (13).

There is another set of triads that yields the Kerr solution, namely, the set which is diagonal and whose entries are given by the square roots of \(g_{ii}\). This set is not appropriate for our purposes, and the reason can be understood even in the simple case of flat space-time. In the limit when both \(a\) and \(m\) go to zero (14) describes flat space: the curvature
and the torsion tensor vanish in this case. However, for the diagonal set of triads (again requiring the vanishing of $a$ and $m$),

$$e^{(r)} = dr, \ e^{(\theta)} = r \, d\theta, \ e^{(\phi)} = r \, \sin\theta \, d\phi,$$

some components of the torsion tensor do not vanish, $T_{(2)12} = 1$, $T_{(3)13} = \sin\theta$, and $E_g$ calculated out of the diagonal set above diverges when integrated over the whole space. Moreover, it is the flat space form of (14), i.e., when $m = a = 0$, that can be brought to a diagonal form in cartesian coordinates, and not the diagonal form above in spherical coordinates. Thus the asymptotic behaviour $e_{(j)k} \approx \eta_{jk} + \frac{1}{2} h(\frac{1}{r})$ when $r \to \infty$ can only be achieved by means of (14).

In ref.[14] we have obtained the expression of the energy contained within a surface of constant radius $r = r_o$:

$$E_g = \frac{1}{4} \int_0^\pi d\theta \sin\theta \left\{ \rho + \frac{\Sigma}{\rho} - \sqrt{\Delta} \left( 2r^2 + a^2 - a^2 \sin^2 \theta (r - m) \right) \right\}_{r = r_o}. \quad (15)$$

In the limit of slow rotation, namely, when $\frac{a}{r_o} \ll 1$ all integrals in the expression above can be calculated, and $E_g$ finally reads

$$E_g = r_o \left( 1 - \sqrt{1 - \frac{2m}{r_o} + \frac{a^2}{r_o^2}} \right) + \frac{a^2}{6r_o} \left[ 2 + \frac{2m}{r_o} + \left( 1 + \frac{2m}{r_o} \right) \sqrt{1 - \frac{2m}{r_o} + \frac{a^2}{r_o^2}} \right]. \quad (16)$$

This is exactly the expression found by Martinez[15] who approached the same problem by means of Brown and York’s procedure. However the present approach is more general than that of ref.[15]. The energy given by (15) can be calculated by means of numerical integration for any value of $a$. On the other hand Brown and York’s procedure requires the embedding of an arbitrary two dimensional surface of the Kerr type in the reference
space $E^3$, a construction which is not possible in general \[^{13}\] (the evaluation of the energy in ref.\[^{13}\] is only possible in the limit $\frac{a}{r_o} << 1$).

III. Conical Defects

The calculations of the previous section support the correctness of expression (9) for the energy of the gravitational field. In ref.\[^{9}\] expression (9) was justified in the framework of asymptotically flat gravitational fields. The action integral of the TEGR for compact spacetimes differs from the one for asymptotically flat geometries by a surface (boundary) term and consequently the two Hamiltonian densities also differ by a surface term. However the Hamiltonian constraint is the same for both kinds of geometries (except, of course, for the possible presence of additional terms; had we added a cosmological constant to the Lagrangian density, such a term would also appear in the Hamiltonian constraint).

We have seen from (8) that the integral form of the Hamiltonian constraint equation may be written as $C = H - E_{ADM} = 0$ for asymptotically flat spacetimes. We will assume here that this is a general feature of gravitational theories, namely, we will assume that for an arbitrary geometry the Hamiltonian constraint equation may be written in integral form as $C = H - E = 0$. Therefore we will tentatively evaluate the energy of simple well known geometries which are not asymptotically flat by means of (9). In the following we will evaluate the latter for the class of geometries considered by Tod\[^{6}\]. Such a class of geometries is particularly suitable for our purposes since the energy per unit length of a cosmic string is a priori known.

We will consider conical singularities along the $z$ axis, in an otherwise flat Minkowski spacetime, described by the metric
where $\alpha$, $\beta$ and $\gamma$ are real constants and $\phi$ runs from 0 to $2\pi$. The metric is everywhere flat except at the axis $r = 0$. For $\alpha = \gamma = 0$ and $\beta < 1$ the metric describes a cosmic string type singularity. Thus $\beta$ parametrizes a disclination. As shown in ref.[6], $\alpha$ and $\gamma$ parametrize dislocations, in the terminology used in crystallography. The metric as given above has also been considered by Gal’tsov and Letelier[16]. The spatial section of (17) reads

\[
g_{ij} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \delta^2 & \gamma \\ 0 & \gamma & 1 \end{pmatrix}
\] (18)

where $\delta^2 \equiv \beta^2 r^2 + \gamma^2 - \alpha^2$. The corresponding triads are given by

\[
e_{(k)j} = \begin{pmatrix} \cos \phi & -\delta \sin \phi & -\frac{\gamma}{\delta} \sin \phi \\ \sin \phi & \delta \cos \phi & \frac{\gamma}{\delta} \cos \phi \\ 0 & 0 & \sqrt{1 - \delta^2/\delta^2} \end{pmatrix}
\] (19)

Recall that $(k)$ and $j$ are the line and column index, respectively. Note that if we make $\alpha = \gamma = 0$, $\beta = 1$ (19) can be brought to a diagonal form in cartesian coordinates by means of a coordinate transformation. Initially we evaluate the components of the torsion tensor:

\[
T_{(1)12} = \frac{\partial}{\partial r}(-\delta \sin \phi) - \frac{\partial}{\partial \phi}(\cos \phi) = (1 - \delta')\sin \phi
\]

\[
T_{(1)13} = \frac{\partial}{\partial r}\left(-\frac{\gamma}{\delta} \sin \phi\right) - \frac{\partial}{\partial z}(\cos \phi) = \frac{\gamma}{\delta^2} \delta' \sin \phi
\]
\[ T_{(1)23} = \frac{\partial}{\partial \phi} \left( -\frac{\gamma}{\delta} \sin \phi \right) - \frac{\partial}{\partial z} \left( -\delta \sin \phi \right) = -\frac{\gamma}{\delta} \cos \phi \]

\[ T_{(2)12} = -(1 - \delta') \cos \phi \]

\[ T_{(2)13} = -\frac{\gamma}{\delta^2} \delta' \cos \phi \]

\[ T_{(2)23} = -\frac{\gamma}{\delta} \sin \phi \]

\[ T_{(3)12} = 0 \]

\[ T_{(3)13} = \frac{\delta'}{\delta} \frac{\gamma^2}{\sqrt{1 - \frac{\gamma^2}{\delta^2}}} \]

\[ T_{(3)23} = 0 \]

where the prime denotes differentiation with respect to \( r \). We wish to evaluate (9) and for this purpose we need the expression of \( T^i \). After a long but simple calculation we obtain

\[ T^1 = \frac{1}{\delta} (1 - \delta') - \frac{\gamma^2}{\delta^2 - \gamma^2} \frac{\delta'}{\delta} , \]

\[ T^2 = T^3 = 0 . \]

Together with \( e = \sqrt{\delta^2 - \gamma^2} \), the energy density can now be easily obtained and integrated. The energy \( E_g \) contained within a cylindrical region with length \( L \) and radius \( r_o \) is given by
\[ E_g = \frac{L}{4} \sqrt{1 - \frac{\gamma^2}{\delta^2}} \left\{ 1 - \left( \frac{\delta^2}{\delta^2 - \gamma^2} \right) \frac{\beta^2 r_o}{\delta} \right\} \]  

(20).

We will consider next the three individual situations in which the metrics are parametrized by only one of the parameters.

I. \( \alpha = \gamma = 0 \)

The metric parametrized by \( \beta \) only describes a disclination. Expression (20) reduces to

\[ E_g = \frac{L}{4} (1 - \beta) \]  

(21).

This is precisely the energy per unit length for a cosmic string. We note that \( E_g \) above does not depend on the radius of integration \( r_o \). Therefore we may conclude that the whole energy is concentrated along the defect axis \( r = 0 \).

II. \( \alpha = 0, \beta = 1 \)

In this case we have a simple dislocation parametrized by \( \gamma \). We are mostly interested in the value of \( E_g \) for very large values of \( r_o \). From (20) we obtain

\[ E_g \approx -\frac{\gamma^2}{8} \frac{L}{r_o^2} \]  

(22).

Therefore in the limit when both \( L \) and \( r_o \) go to infinity, namely, when the integration is performed over the whole three dimensional space, \( E_g \) vanishes. Thus the total energy of in the disclocation is zero. However, in the limit \( r_o \to 0 \) we find \( E_g \to -\frac{L}{4} \).

III. \( \beta = 1, \gamma = 0 \)
For this metric expression (20) reduces to

\[ E_g = \frac{L}{4} \left( 1 - \frac{1}{\sqrt{1 - \frac{\alpha^2}{r_o^2}}} \right). \]

For large values of \( r_o \) we obtain

\[ E_g \approx -\frac{\alpha^2}{8} \frac{L}{r_o^2}. \] (23)

Therefore the total energy corresponding to this disclination is also zero. In the limit \( r_o \to \alpha \) we observe that \( E_g \to -\infty \).

We observe that whereas for a disclination the whole energy per unit length is concentrated along the defect axis, for both types of dislocations the energy is distributed over the whole three dimensional space.

**IV. Comments**

By means of expression (9) for the energy of the gravitational field we have obtained the correct value of the energy per unit length of a cosmic string. This fact is a good indication that the Hamiltonian constraint may be generically written as \( C = H - E = 0 \), which is the assumption we made in this paper. It also supports the conjecture that expression (9) might have a universal character, since it yields the expected values of energy for totally distinct spacetimes, namely, Kerr-type and conical spacetimes.

From the results of section III we conclude that dislocations are more likely to appear as a result of cosmological phase transitions in the early universe than disclinations (cosmic strings). The energy of an actual, physical dislocation might be nonvanishing, but anyhow we would expect it to be very small. The situation here is very much similar to what happens in a real crystal. It is well known that disclinations require too much energy to
be formed in crystals, whereas dislocations are more favourable defects since they require much less energy (see, for instance, sections 6.3.2 and 6.5 of ref.[5] for a discussion as to why the energy cost for a disclination in crystal is prohibitively high). In order to understand the vanishing of the total gravitational energy for dislocations, let us consider the metric for which $\alpha = 0, \beta = 1$ (case II). As discussed by Tod[6], this metric can be transformed into a flat metric if we define a new coordinate $Z = z + Z_o \frac{\phi}{2\pi} = z + \gamma \phi$. The associated Burgers vector is determined by $Z_o = 2\pi \gamma$. We know, however, that the energy of an actual dislocation in a crystal depends not only on the Burgers vector but also on an extra quantity, the rigidity modulus $\mu$. Such quantity is not given in (17). This fact might explain why the total energy of the dislocations above is zero. Disclinations and dislocations are concepts used in the deformation models of crystals and metals. We conclude from our analysis that dislocations of type considered here might be as well useful concepts in cosmological models.

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