Signal transmission on lossy lines as a dissipative quantum state propagation.

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The transmission of electric signals on a coupled line with distributed RLC-parameters is considered as a propagation of a dissipative quasi particle. A calculation technique is developed, alternative to the one, accepted for lumped lines. The relativistic wave equation for the transient response is deduced following the common Ohm-low-type considerations. The exact expressions for the Green function, for information transfer velocity and for time delay are obtained on this base. The fundamental restrictions on the measurement accuracy of the time delay are pointed out. The obtained results are naturally generalized for the multilevel networks of the arbitrary dimension.

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I. INTRODUCTION

The electric signals transmission on distributed RLC-lines will be solved analytically. Our goals are deformation of a signal shape and time delay. These questions are usually considered by the calculation technique, made for lumped lines [1-5]. On the other hand, in the works [6-9] people build models, based on the Maxwell- or Schrödinger-like equations.

The physical approach, developed here, is based on the introduction of electric impulses on the lossy line as a dissipative quasi particle. The space-time propagation of it is described by the Klein-Gordon equation - the equation of motion of free relativistic scalars (see, for example, [10]). The parameters of an electric signal have a direct particle-like meaning. For example, the time decay rate plays the role of a negative mass square.

II. EQUATIONS, SOLUTIONS. INFORMATION TRANSFER

The lossy interconnect line can be introduced as consisting of a row of infinitesimally small RLC-cells. Each cell of length $dl$ has a local resistance $dR$, inductance $dL$ and capacitance $dC$ :

$$
dR = r(l)dl, \quad dL = \ell(l)dl, \quad dC = c(l)dl,
$$

(1)
determined by the linear densities as:

$$
r(l) = dR/dl, \quad \ell(l) = dL/dl, \quad c(l) = dC/dl.
$$

(2)

The target voltage of cell $U_C = U_{out}(t)$ is expressed through entrance one - $U_{in}(t)$ by the condition:

$$
U_{in} = U_L + U_R + U_{out},
$$

(3)

where:

$$
U_L = dL \frac{\partial J}{\partial t}, \quad U_R = dRJ, \quad \frac{\partial J}{\partial l} = c \frac{\partial U}{\partial t}.
$$

(4)

So, the voltage $U(l, t)$, as a function of time and distance, obeys the equation:

$$
\frac{c \ell}{t^2} \frac{\partial^2 U}{\partial l^2} + r \frac{\partial U}{\partial t} = 0.
$$

(5)

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This equation is known as the one-dimensional telegraph equation and describes the wave distribution with damping. For the case $l = 0$ it has been treated in [1]. The space variable can be normalized by replacing $l$ with $x = l/v$. Then, using the notations: $m = r/2l$, $v^2 = 1/c^2$, the function $\Phi(x, t)$ is introduced: $U(x, t) = \exp(-mt)\Phi(x, t)$. It obeys the equation:

$$\frac{\partial^2 \Phi}{\partial t^2} - \frac{\partial^2 \Phi}{\partial x^2} - m^2 \Phi = 0. \tag{6}$$

This is the Klein-Gordon equation* with the negative mass square - so-called "tachyons". Earlier we dealt with such an exotic object in a real situation [10]. This representation looks like the motion of the tachyon quasi particle in the constant external dissipative field.

The basis solution of the main equation (5) with the wave vector $k$

$$\varphi(\omega_0|x,t) = e^{-mt-i(\omega_0 t-kx)}, \quad \omega_0(k) = \sqrt{k^2 - m^2} \tag{7}$$

is a plane wave, damped in time at the rate $m$, identical for all frequencies. A macro-packet - $U(x,t)$ can be built as a linear superposition of basis states:

$$U(x,t) = \int d\omega_0 U(\omega_0)\varphi(\omega_0|x,t). \tag{8}$$

The wave $\varphi(\omega_0|\ x, t)$ is a not a stationary state, but its energy loss in time as the square of amplitude:

$$\dot{\omega}\varphi(\omega_0|x,t) = \omega(t)\varphi(\omega_0|x,t) = e^{-2mt}\omega_0\varphi(\omega_0|x,t). \tag{9}$$

The frequency-width of a packet: $\Delta\omega = \omega - \omega_0$ also falls down in time as $e^{-2mt}$. The packet does not spread in time, but agglomerates in frequencies. The wave $\varphi(\omega_0|\ x, t)$ has a phase velocity:

$$v_{ph} = \omega_0/k = \omega_0/\sqrt{\omega_0^2 + m^2}. \tag{10}$$

But the information transfer velocity - the group velocity of a packet, defined as: $u = \partial\omega_0/\partial k$, for the dispersion law (7) results:

$$v_{gr} = \sqrt{\omega_0^2 + m^2}/\omega_0 = k/\omega_0. \tag{11}$$

On the first view this velocity (in the natural units: $v_{gr} = vk/\omega_0$) is higher than the light speed in the wire: $v_{gr} > v$. But the packet decays in time as $e^{-mt}$. In a short time $dt$ it loses the part: $dx_2 \approx \frac{\omega_0}{v} dt$ from the forward edge, which amplitude becomes $e^{-1}$ of initial. This part must be subtracted from the whole packet relocation: $dx_1 = v_{gr} dt$. So, the effective speed of the packet can be approximated as:

$$u \approx v_{gr} - \frac{\pi m}{\omega_0} = \left(\sqrt{\omega_0^2 + m^2 - \pi m}\right)/\omega_0 < 1. \tag{12}$$

The treatment of electric impulses on the lossy line as a quasi particle gives a physical restriction on the measurement accuracy of the time delay. Following the uncertainty relations, the measurement of the time-slice $\Delta t$ is accompanied with the energy variation: $\Delta\omega_0 \gtrsim 1/\Delta t$. At the signal frequency $\omega_0$ the delay time $\Delta t$ can be measured up to accuracy: $\Delta\delta \gtrsim 1/\omega_0$. This inequality gives a minimal error, physically permissible by the measurements of time-slices. This restriction has a fundamental character, and does not depend on the chosen approach.

### III. GREEN-FUNCTION. INTEGRAL REPRESENTATION

The Green function for the equation (5) can be built from the solutions (7) as:

$$G(x,t) = \frac{e^{-mt}}{(2\pi)^2} \int_{-\infty}^{\infty} d\omega \int_{-\infty}^{\infty} dk \frac{\exp[-i(\omega t-kx)]}{-m^2-\omega^2+k^2}. \tag{13}$$
The integrand has two poles at $\omega = \omega_{1/2}$, which form two complex resonance frequencies:

$$\omega_{1/2}(k) = -mi \pm \omega_0(k).$$

At $k < m$ frequency $\omega_0$ becomes imaginary- the wave does not propagate. But for all $-\infty < k < \infty$ the both poles lay below the real $\omega$-axis. To build a retarded Green function, the integration contour can be kept on the real $\omega$-axis. At $t > 0$ the contour closes in the low $\omega$-half-plane and contains all possible positions of poles. The retarded Green function is the sum of the residues in the both poles $\omega_{1/2}$:

$$G^{ret}(x, t) = e^{-mt} \Theta(t) \int_{-\infty}^{\infty} \frac{dk}{2\pi} e^{ikx},$$

where: $\Theta(t)$- is a Heaviside function. The real range of the integration in (15) is restricted by the condition: $|k| \geq m$. Such “damped tachyon” Green function in the coordinate representation is the analytical continuation of the scalar one into the area $m^2 \to -m^2$. Using the integrals, calculated in [10], this function can be expressed as:

$$G^{ret}(x, t) = -\Theta(t)\Theta(\lambda)e^{-mt}I_0(m\sqrt{\lambda}),$$

where: $\lambda = t^2 - x^2$, and $I_0(x)$- modified Bessel - function.

IV. DELAYS. LEADING TRANSFER MODE

The representation (19) can be applied to the macro-packet (8). The amplitude of a signal, propagating in the positive (negative-) $x$-direction- $U_r(U_l)$, is equal:

$$U_r(x, \tau) = e^{-mx}u_0(t - x) + x \int_0^\tau d\lambda e^{-m\sqrt{\lambda + x^2}*} I_1(m\sqrt{\lambda})u_0(t - \sqrt{\lambda + x^2}), x < \tau.$$
at small frequencies: \( \omega_m \to 0, u_0 = const \), the asymptotic of the Green-function, also corresponding to the DC propagation, has a view:

\[
G_0^{rel}(x, t) \approx -\frac{te^{-mt}}{\pi} \sin \frac{mx}{x}, \quad \omega_0 \to 0.
\]  

(22)

For the constant input signal \( u_0 \) equation for the delay time in the point \( x \) is:

\[
\frac{te^{-mt}}{\pi} \left[ \left| \sin \frac{mx}{x^2} - \frac{m \cos \frac{mx}{x^2}}{x^2} \right| \left( (t + 1/m)e^{-mt} - 1/m \right) \right] = b
\]  

(23)

V. REFLECTIONS

For the finite RLC-lines, length \( \bar{l} \) ( \( \bar{x} = l/v \) ), reflections occur. The wave packet is now a superposition of incoming and reflected waves. The resulting voltage with \( N_r \)-reflections is:

\[
U_{ref}(x, t) = \sum_{s=0}^{N_r} \left[ \Gamma^2 U_s(x + 2s \bar{x}, t) + \Gamma^{2s+1} U_l(2s \bar{x} - x, t) \right],
\]  

(24)

where \( \Gamma \) is the reflection coefficient. The flying signal decays on the length: \( \bar{l}_d \sim v/m \). For the line of length \( \bar{l} \), the reflection number is the ratio: \( N_r \approx \bar{l}_d/\bar{l} \). Equation: \( U_{ref}(x, \delta) = bU_{max} \) gives now the signal delay time \( \delta \).

In the modern interconnection RLC-lines, for example, at the parameters values as in \([4,5]\): \( r = 37.8 \Omega/cm, \ k = 3.28e - 13F/cm, \ Z_0 = \sqrt{c/e} = 266.5\Omega \) and the length of the line: \( \bar{l} = 3.6cm \), we have: \( \bar{l} = 2.3e - 8Hz/cm, \) the decay rate: \( m = 8.1e + 08Hz \). For the signal frequency \( \nu \approx 3GHz \) the decay length is: \( \bar{l}_d \approx 14.1cm \) - and, at least: \( N_r \approx 4 \) reflections are observable.

VI. MULTILEVEL NETWORKS

For generalizing the obtained results on the multilevel network, the network can be represented as the \( N \)-component space formed from \( N \) RLC-lines. An orthogonal basis will be put on the directions of RLC-lines: \( \{ e \} = \{ \bar{e}_1, \bar{e}_2\ldots \bar{e}_N \}, \quad \bar{e}_i, \bar{e}_k = \delta^{ik} \). The voltage impulse, flowing through the network, becomes now a vector in this space. For example, for the network, consisting of three lines, the voltage vector is: \( \bar{U}(l, t) = [U_1(l, t), U_2(l, t), U_3(l, t)] \). The vector \( \bar{U} \) obeys the equation (5) in the matrix form [5]:

\[
\hat{\mathcal{L}} \bar{U} \frac{\partial^2 \bar{U}}{\partial t^2} - \frac{\partial^2 \bar{U}}{\partial l^2} + r \hat{\mathcal{C}} \frac{\partial \bar{U}}{\partial t} = 0,
\]  

(25)

where the matrices \( \hat{\mathcal{C}} \) and \( \hat{\mathcal{L}} \), are:

\[
\hat{\mathcal{C}} = \begin{bmatrix}
2c_{grd} + c_m & -c_m & 0 \\
-c_m & 2c_{grd} + 2c_m & -c_m \\
0 & -c_m & 2c_{grd} + c_m \\
\end{bmatrix},
\]  

(26)

\[
\hat{\mathcal{L}} = \begin{bmatrix}
\mathcal{L}_{11} & \mathcal{L}_{12} & \mathcal{L}_{13} \\
\mathcal{L}_{12} & \mathcal{L}_{22} & \mathcal{L}_{23} \\
\mathcal{L}_{13} & \mathcal{L}_{23} & \mathcal{L}_{33} \\
\end{bmatrix},
\]  

(27)

c_{grd} - line-to-ground capacitance, \( c_m \)- line-to-line capacitance; and \( \mathcal{L}_{ik} \)-self- and mutual between-conductor-inductances. The non-diagonal components of tensors describe interline transmission. It was pointed out in [5], the product of the matrices \( \hat{\mathcal{C}} \) and \( \hat{\mathcal{L}} \) is proportional to the unit matrix: \( \hat{\mathcal{C}} \hat{\mathcal{L}} = \frac{1}{\nu} \hat{\mathbf{I}} \). It is natural as voltage obeys the inhomogeneous Maxwell equation, containing, as well as (6), D’Alamber operator. Then the space variable \( l \) can again be normalized by: \( x = l/v \) and the particle mass (which is now a tensor) can be introduced: \( \hat{m} = \frac{m^2}{\nu^2} \hat{\mathcal{C}} = \frac{\nu}{\bar{l}} \hat{\mathcal{L}}^{-1} \). The multilevel network with distributed RLC-parameters appears as an anisotropic solid medium. The situation with the
quasi particle parameters, depending on direction, is here well known. By substitution: 
\[ \overrightarrow{U}(x, t) = \exp(-\hat{m}t) \overrightarrow{\Phi}(x, t) \]
the matrix analog of the equation (6) for \( \overrightarrow{\Phi} \) can be obtained:

\[ \frac{\partial^2 \overrightarrow{\Phi}}{\partial t^2} - \frac{\partial^2 \overrightarrow{\Phi}}{\partial x^2} - \hat{m}^2 \overrightarrow{\Phi} = 0. \]  

(28)

The vectors \( \overrightarrow{U}(x, t) \) and \( \overrightarrow{\Phi}(x, t) \) in general case are not parallel, so as the decay operator \( \exp(-\hat{m}t) \) mix’s the components. It seems that the last term in (28) describes the self-interaction of the field \( \overrightarrow{\Phi} \). But the numerical tensor \( \hat{m} \), generally, can be diagonalized and thus the field components will be split. With it the Green function will also be diagonalized. Then, the proper directions of the tensor \( \hat{m} \): \( \{\epsilon\} = \{\overrightarrow{\epsilon}_1, \overrightarrow{\epsilon}_2, \overrightarrow{\epsilon}_3\} \) can be chosen as the new basis in the network-space. The same quasi-free propagation occurs along the each of new orts as in the case of the single RLC-line:

\[ U_i(x, t) = \exp(-m_i t)\Phi_i(x, t). \]  

(29)

The decay rates \( (m_1, m_2, m_3) \) are different for each ort of this basis. Now the further reasons and results of the work can be reproduced for multilevel networks. The Green function has the same form as (15):

\[ \overrightarrow{U}_{(m)}(x, t) = \int_0^\tau dt G^{ret}(m_i | x, t - t') \overrightarrow{U}_{(in)}(t'), \]  

(30)

where \( \overrightarrow{U}_{(in)}(t) \) is incoming voltage vector, in the new basis \( \{\epsilon\} \). The traveling voltage, propagating in the real space on the RLC-lines, becomes the projection of \( \overrightarrow{U}_{(m)} \), calculated by (30), on the corresponding unit vector of the old basis \( \{\epsilon\} \).

VII. CONCLUSION

The suggested representation for the transient response of the distributed RLC-line has allowed to obtain the exact formulas for the Green function and time delay. The simple description of signals propagation by the frequency characteristics in the LTM-approximation is obtained. The natural generalization on the multilevel networks of arbitrary dimension is made. The physical approach can be helpful at the multilevel networks design and in the quantum information.

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*Indeed, it is necessary to multiply the equation by factor \( \hbar^2/c^2 \), but in the field theory the system of unit is adopted, where: \( \hbar=1, \ c=1 \).
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