Quasiparticle generation efficiency in superconducting thin films

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Abstract
Thin-film superconductors with thickness ∼30–500 nm are used as non-equilibrium quantum detectors for photons, phonons or more exotic particles. One of the most basic questions in determining their limiting sensitivity is the efficiency with which the quanta of interest couple to the detected quasiparticles. As low temperature superconducting resonators, thin films are attractive candidates for producing quantum-sensitive arrayable sensors and the readout uses an additional microwave probe. We have calculated the quasiparticle generation efficiency $\eta_s$ for low energy photons in a representative, clean thin-film superconductor (Al) operating well below its superconducting transition temperature as a function of film thickness, within the framework of the coupled kinetic equations described by Chang and Scalapino (1978 J. Low Temp. Phys. 31 1–32). We have also included the effect of a lower frequency probe. We show that phonon loss from the thin film reduces $\eta_s$ by as much as 40% compared to earlier models that considered relatively thick films or infinite volumes. We also show that the presence of the probe and signal enhances the generation efficiency slightly. We conclude that the ultimate limiting noise equivalent power of this class of detector is determined by the thin-film geometry.

Keywords: non-equilibrium superconductivity, kinetic inductance detectors, noise equivalent power

(Some figures may appear in colour only in the online journal)

1. Introduction
Superconductive detectors have revolutionized experimental astrophysics. Many of these detectors exploit Cooper pair-breaking in a thin-film low transition temperature superconductor operating at low reduced temperatures $T/T_c \approx 0.1$ ($T$ is the temperature and $T_c$ is the superconducting transition temperature, $T_c \sim 1$ K). These detectors rely on non-equilibrium effects, but to the best of our knowledge no detailed microscopic description exists of the efficiency with which the excess quasiparticles are created in a thin-film superconductor of thickness ∼30–500 nm. This problem is very relevant not just for kinetic inductance detectors (KIDs) [1, 2], but also for superconducting tunnel junction detectors [3], transition edge sensors [4], and quantum capacitance detectors [5]. All of these devices can be fabricated by photolithography usually on a relatively thick substrate such as Si or sapphire that is held at $T$ and that functions as a heat bath. KIDs are thin-film superconducting resonators that can be configured as ultra-sensitive detectors of signal photons across the electromagnetic spectrum. KIDs are typically read out with a microwave probe with photons of energy $h \nu_p \sim 0.05 \Delta$, where $\nu_p$ is the probe frequency, $2 \Delta$ is the low temperature superconducting energy gap and $h$ is Planck’s constant. In this instance, an understanding of the combined effects of the signal and the probe is clearly important. In [6] we described a detailed microscopic calculation of the spectrum of the non-equilibrium quasiparticles and phonons in a superconducting resonator operating at $T/T_c = 0.1$ considering only a probe. In [7] that model was compared with precise experimental measurements of the temperature and power dependence of the behavior of ultra-sensitive
Al resonators, finding good agreement between model and measurement.

In a superconductor each absorbed signal photon with energy $h\nu_s \geq 2\Delta$ breaks a pair ($\nu_s$ is the signal frequency). Probably the most important consideration in calculating the detection sensitivity of these photons in any thin-film superconducting detector is $\eta_s$, the average fraction of the photon energy that creates low energy quasiparticles $E \sim \Delta$, where $E$ is the quasiparticle energy. We distinguish the primary spectrum of quasiparticles generated by the signal from the driven quasistatic population that is established as the primary spectrum relaxes temporally and energetically. Absorbed photons create a spectrum of excess primary quasiparticles that relaxes to energies $E \sim \Delta$ by emitting phonons on a timescale $\tau_{\text{cascade}} \sim 0.1\text{–}10$ ns [8, 9], determined by the quasiparticle–phonon scattering time at $E = 3\Delta$. $\tau_{\text{cascade}}$ is much shorter than the effective loss time from the film of the excess energy contained in the quasistatic distribution by $2\Delta$-phonon loss. This time is determined by the effective quasiparticle recombination time of the excess $\tau_{\text{ee}}^\text{eff}$ provided otherwise relatively slow direct quasiparticle loss-mechanisms such as out-diffusion or tunneling can be ignored. For $T/T_c \sim 0.1$ and for low detected power, $\tau_{\text{ee}}^\text{eff} \sim ms$ even in a thin Al film [7]. Since $\tau_{\text{ee}}^\text{eff} \gg \tau_{\text{cascade}}$, the low energy quasiparticle population determines the detector response. During the energy relaxation, pair-breaking by the emitted phonons occurs provided $\Omega \geq 2\Delta$ (\(\Omega\) is the phonon energy), and this increases the quasistatic population near $\Delta$, although phonon loss is also possible with characteristic time $\tau_l$. At low temperature and low phonon energies $\Omega \sim 2\Delta$, the pair-breaking time $\tau_{pb}(\Omega) \sim \tau_{0}^{\phi}$, where $\tau_{0}^{\phi}$ is the characteristic phonon lifetime [10]. We assume that $\tau_l$ is independent of $\Omega$ and is determined by the film thickness and the coupling to the substrate [11]. For thin films $\tau_l$ is comparable with, or even less than, $\tau_{pb}$. Phonon loss means that energy is lost from any finite thickness film before the quasistatic population is established.

Up to this point we have ignored the effect of the electron–electron interaction in the energy down-conversion. Figure 1 shows the energy dependence of the normal-state scattering rates due to the electron–phonon ($e$--$\phi$) interaction $\tau_{\text{ee}}^{-1}$ [10], the clean-limit electron–electron ($e$--$e$) rate $\tau_{\text{ee}}^{-1}$, which is also valid for disordered films at high energies [8, 12], and the $e$--$e$ rate including the effect of disorder ($\tau_{\text{ee}}(D))^{-1}$ [13, 14], where $D$ denotes the diffusion coefficient.

The calculations assume an Al thickness $d = 35$ nm with resistivity $\rho = 8 \times 10^{-9}$ $\Omega$ m, typical of a clean Al film on Al$_2$O$_3$ [7], which is representative of the thinnest films modeled here. For $E > 200\Delta$, $(\tau_{\text{ee}}^{-1})^{-1}$ is cut-off at the Debye energy $\Omega_D$. At the highest energies $E \sim 10^4 \Delta$, $e$--$e$ scattering becomes the main energy relaxation mechanism. At lower energies ($E \sim 25\Delta$), disorder increases the $e$--$e$ rate in this instance. For $E \sim 1.2\Delta$ we see that the $e$--$e$ and $e$--$\phi$ rates again become equal. For $1.2\Delta < E < 10^4\Delta$ the $e$--$\phi$ interaction is the principal relaxation mechanism. A detailed description of the energy dependence of the disorder-enhanced $e$--$e$ rate in a superconductor at low $T/T_c$, including the effect of the energy gap, seems to be lacking although for $E = \Delta$ the $e$--$e$ rate is further reduced compared to $e$--$\phi$ becoming negligible [15].

![Figure 1](image-url)  
**Figure 1.** Energy dependence of the relaxation rates in a clean, thin Al film in the normal state: (red) solid line, electron–phonon scattering; (black) dashed line, clean-limit electron–electron scattering; and (blue) dash-dot line, the electron–electron scattering time including the effect of disorder. The calculations are for a 35 nm Al film with $\rho = 8 \times 10^{-9}$ $\Omega$ m. $\Delta$ is the low temperature energy gap.

We note also that the energy scale of interest determining the relative importance of low energy $e$--$e$ compared to $e$--$\phi$ scattering in the relaxation is not $\Delta$ but rather $3\Delta$: below this energy pair-breaking is forbidden for both. For the thicker Al films discussed below $D$ is enhanced in clean films so that the effect of disorder is again reduced. For these reasons we ignore $e$--$e$ relaxation for all energy scales, temperatures and film parameters considered. Extrapolation of our results to other low-$T_c$ superconductors should thus be carried out with caution, particularly for higher resistivity or very thin films.

A number of calculations exist of $\eta_s$ for high energy photons $h\nu_s \gg 2\Delta$ and $\Omega_D$ that have considered infinite superconducting volumes, finding $\eta_s \sim 0.57$--0.6 for Al [8], Nb [16] and Sn [17]. Hjømmer et al [18] calculated quasiparticle creation efficiencies in thin-film Al–Ta bilayers taking account of the modification of the quasiparticle density of states due to the proximity effect but ignored loss of pair-breaking phonons. Zehnder [19] calculated $\eta_s$ in a number of thin-film superconductors with thickness $d = 500$ nm at $T = 0.5$ K including quasiparticle diffusion and phonon loss. $\eta_s$ was determined from the number of quasiparticles remaining at time $t \sim 10$ ns when the initial energy down-conversion was considered complete, giving $\eta_s \sim 0.7$ for Al.

Here, we consider the regime $90 < \nu_s < 450$ GHz. To date no work has calculated $\eta_s$ for these signal photon energies at low $T/T_c$, or the technologically important range of film thicknesses considered here including $2\Delta$-phonon loss. This frequency range is particularly relevant for mm and sub-mm astronomy. We have also included a lower frequency probe. We followed Chang and Scalapino [20] to solve the coupled kinetic equations describing the quasiparticle and phonon populations. Our approach explicitly includes the contribution of all phonon branches because it relies on the measured Eliashberg function $\alpha^2F(\Omega)$ in the calculation of the characteristic times [10] and the sum over the three branches is essential to conserve energy [6].
2. The effect of a pair-breaking signal

The coupled kinetic equations described in [20] were solved using Newton–Raphson iteration to find the non-equilibrium quasiparticle and phonon energy distributions \( f(E) \) and \( n(\Omega) \). Details of the scheme are given in [6]. The absorbed powers per unit volume from the signal \( P_s \) and probe \( P_p \) are assumed to be spatially uniform. We ignore changes in \( \Delta \) due to \( P_s \) and \( P_p \). In [7] we found that changes in \( \Delta \) were very small, \( \ll 0.001 \Delta \) for typical experimental \( P_p \). The effect of \( P_s \) is to introduce an additional drive term [21] into equation (2) of [6] for the quasiparticle distribution function \( \delta f(E)/\delta t \)

\[
K_s(E, v_s) = K_p(E, v_s) + 2\rho(E', \Delta) \left[ 1 - \frac{\Delta^2}{EE'} \right] \times \left[ 1 - f(E) - f(E') \right],
\]

\( E' = h\nu_s - E \) and the prefactor \( B_s \) is calculated with \( B_s = P_s/4N(0) \int_0^\infty E p(E)K_s(E, v_s) dE \), ensuring that the absorption of \( P_s \) conserves energy. A prefactor for the probe power absorption \( B_p \) can be similarly defined [6]. \( N(0) \) is the single-spin electronic density of states. Some solutions require calculation of differences between distributions. To ensure numerical accuracy we increased the precision requirements in the code so that the errors in the power flow between the quasiparticles and thin-film phonons and then the heat bath phonons (see [6] for details) were converged to better than \( 2 \times 10^{-6} \) and likewise the iterated solutions for \( B_s \) and \( B_p \). We used a quadratic density of states \( \rho(E, \Delta) = \text{Re} ((E + iy)^2/(E + iy)^2 - \Delta^2)^{1/2} \). The factor \( \gamma \) takes account of the broadening of the peak in \( \rho \) near \( E = \Delta \) due to lifetime effects or film inhomogeneity [1]. The choice \( \gamma = 1.125 \times 10^{-3} \Delta \) minimizes the difference between the thermal quasiparticle number density \( N_T \) calculated by summing over the discretized distributions (where we used a 1 \( \mu \)eV grid) compared to numerical integration of the functions \( N_T = 4N(0) \int_{-\infty}^{\infty} \rho(E, \Delta) f(E, T) dE \), where \( f(E, T) = 1/(1 + \exp(E/k_BT)) \) is the Fermi–Dirac function and \( k_B \) is Boltzmann’s constant. We used the parameters of a thin Al film as in [6]: \( \Delta = 180 \) \( \mu \)eV, \( T_c \approx 1.17 \) K, \( N(0) = 1.74 \times 10^4 \text{ eV}^{-1} \text{ cm}^{-3} \), characteristic quasiparticle time \( \tau_0 = 438 \) ns [10], \( \tau_0^{\phi} = 0.26 \) ns and \( T_c/\tau_0 = 0.1 \). This ratio of \( \tau_0/\tau_0^{\phi} \) means that our numerical solutions conserve energy; they are not independent variables. Equation (11) of [6] gives the overall parameter dependences. The value we use for \( \tau_0 \) has given a good account of the temperature dependence of the generation–recombination noise measured in clean, thin Al films [7, 22], in which the effect of phonon trapping should be small (we estimate \( \tau_1/\tau_0^{\phi} \sim 0.5 \) in this case). A number of previous authors have used \( \tau_0 \sim 100 \) ns [23], although this value seems inconsistent with the more recent measurements.

Wilson and Prober have also observed unexpectedly long lifetimes in 200 nm Al films (estimating \( \tau_0 \) to be even longer than the value used here), and suggested that the observation resulted from an anomalously long \( \tau_1 \) [24]. Interestingly, the longer \( \tau_0 \) seems to be associated with those measurements that have implemented stringent experimental procedures to minimize the effect of stray light from higher temperature stages in cryogenic systems; note that the typical photon energies emitted by a 4 K source significantly exceed \( 2\Delta \) in Al. Where we used assumed \( h\nu_p = 16 \) \( \mu \)eV (\( \nu_p = 3.88 \) GHz).

3. Calculating \( \eta_s \)

Consider \( m \), the average number of driven quasistatic quasiparticles generated by each absorbed photon. Signal photons interact with rate \( \Gamma_\phi = P_p/h\nu_\phi \) and each photon creates two primary quasiparticles. These quickly relax in energy generating the driven quasiparticulate population with rate \( \Gamma_s = m\Gamma_\phi \). Assuming that all of the excess quasiparticles have \( E = \Delta \) then \( \eta_s = m\Delta/h\nu_s = \Gamma_s/\Gamma_p \). We use a modified set of Rothwarf–Taylor rate equations [25] to find \( \Gamma_s \). With \( \Gamma_p \) the generation rate of quasistatic quasiparticles due to the probe, the number density of quasiparticles and \( N_{2\Delta} \) the number density of \( 2\Delta \)-phonons

\[
\frac{dN}{dt} = \Gamma_s + \Gamma_p - RN_2^s + 2\beta N_{2\Delta},
\]

\[
\frac{dN_{2\Delta}}{dt} = \frac{RN_2^s}{2} - \beta N_{2\Delta} - \frac{N_{2\Delta} - N_2^s}{\tau_1}.
\]

Here, \( R \) and \( \beta \) are the recombination and pair-breaking coefficients respectively and \( N_{2\Delta} \) is the thermal density of \( 2\Delta \)-phonons. We assume that \( \Gamma_s \) and \( \Gamma_p \) are independent. With \( \Gamma_s = 0 \), equations (2) and (3) can be solved by first also setting \( \Gamma_p = 0 \) so that in steady-state, \( dN/\tau_1 = dN_{2\Delta}/\tau_1 = 0 \), giving \( RN_2^s/2 = \beta N_{2\Delta}^T \). This leads to \( \Gamma_p = R(N_2^p - N_2^s)/(\beta \tau_1 + 1) \), where \( N_p \) is the total number density of quasiparticles with the probe. For the additional signal \( \Gamma_s \) we find

\[
\Gamma_s = R(N_2^s - N_2^0) \frac{1}{\beta \tau_1 + 1},
\]

and with \( \beta = 1/\tau_{pb} \)

\[
\eta_s = \frac{R \Delta(N_2^s - N_2^0)}{P_p} \frac{1}{\tau_1/\tau_{pb} + 1}.
\]

\( N \) and \( N_p \) were calculated by numerically integrating solutions of the coupled kinetic equations. We used equation (A9) of Chang and Scalapino [26] to define a recombination rate \( R_{CS} \). We find that setting \( R = 2R_{CS} \) ensures that the population-averaged recombination time \( \langle \tau_{rp} \rangle = 1/RN \) in thermal equilibrium \( (N = N_T) \) is the same calculated using either [26] or [10]. We calculate \( \tau_{pb} \) for \( f(E) \) and \( n(\Omega) \) using equation (A10) of [26]. Writing equation (4) in terms of the excess number densities \( N_{2\Delta}^s \) due to the signal and \( N_{2\Delta}^p \) due to the probe alone so that \( N = N_{2\Delta}^s + N_{2\Delta}^p + N_T \) and \( N_p = N_{2\Delta}^p + N_T \) the effective recombination time \( \tau_{re} = N_{2\Delta}^s + N_{2\Delta}^p \). This can be calculated for any combination of the magnitudes of \( N_{2\Delta}^s, N_{2\Delta}^p \) and \( N_T \). In the calculations reported we consider signal and probe powers relevant to ultra-sensitive KIDs for astronomical applications [7] so that \( N, N_p \gg N_T \) for all cases of \( P_p, P_s \) studied.
the quasiparticle distribution with $T/T_c = 0.1$: full red line, probe power only; dashed blue line, with additional signal $P_s/p_p = 0.01$. The signal photon energy $h\nu_s = 5.1\Delta$ and $\tau_1/\tau_0^\phi = 1$. The inset shows the contribution to the number drive $K_s\rho(E/\Delta)$ for the signal normalized so that each absorbed photon produces two primary quasiparticles per second.

4. Results

Figure 2 shows $f(E)$ for $P_p = 20$ aW $\mu m^{-3}$ as the solid curve and the additional effect of $P_s/p_p = 0.01$ (dashed blue curve) with $h\nu_s = 5.1\Delta$. The inset shows the contribution to the number drive $K_s\rho(E/\Delta)$ for the signal normalized so that each absorbed photon produces two quasiparticles per unit time. The double peak arises because $K_s\rho(E,\Delta)$ involves the product of final state densities $\rho(E,\Delta)\rho(E')$, which is symmetric with respect to the final state energies. The main figure shows that $f(E)$ for the probe alone has multiply peaked structure at $E \sim \Delta$ due to absorption of the probe photons by the large density of quasiparticles near $\Delta$. At energy $E = 3\Delta$ there is a step in $f(E)$ corresponding to reabsorption of $2\Delta$-phonons by the driven quasiparticles which also exhibits peaks associated with multiple photon absorption from the probe. A smaller feature at $E = 3\Delta - h\nu_p$ is visible which arises from stimulated emission.

The dashed curve showing $f(E)$ with $P_s$ has similar structure at low energies but shows a step at $E = h\nu_s - \Delta$. The curvature $f(E)$ below this primary peak arises from the energy dependence of the quasiparticle scattering and recombinations rates. The peak also has a smaller ‘satellite’ at $E = h\nu_s - \Delta + h\nu_p$ as multiple photon processes involving the signal and probe occur. A further similar feature is evident at $E = h\nu_s + \Delta$.

Figure 3 shows the change in contributions to the power flow to the heat bath $\delta P(\Omega)_{\phi-b} = P(\Omega)^\phi_{\phi-b} - P(\Omega)^b_{\phi-b}$, where $P(\Omega)^\phi_{\phi-b}$ is the contribution to the phonon–bath power flow with signal and probe, and $P(\Omega)^b_{\phi-b}$ that for the probe alone. At low phonon energies $\Omega < 0.3\Delta$, $\delta P(\Omega)_{\phi-b}$ is increased due to pair-breaking. At energies $0.3 < \Omega < 0.5\Delta$ the net flow is negative. The first effect arises as the signal itself has a sharply peaked structure near the gap. The reduction arises from the blocking of final states for the scattering of higher energy probe-generated quasiparticles towards the gap. At higher phonon energies there is a significant change in $\delta P(\Omega)_{\phi-b}$ due to phonons $\Omega \gtrsim 2\Delta$. The spectrum also shows a broad low background contribution at all phonon energies $\Omega \leq (h\nu_s - 2\Delta)$ generated as the primary spectrum scatters to energies $E \sim \Delta$ and at higher $\Omega$ from the highest energy quasiparticles shown in figure 2.

Figure 4 shows calculations of $\eta_s$ as a function of $h\nu_s$ for five values of $\tau_1/\tau_0^\phi$. The calculation used $P_s = 0.2$ aW $\mu m^{-3}$ and $P_p = 20$ aW $\mu m^{-3}$. For $2\Delta \leq h\nu_s \leq 4\Delta$, $\eta_s$ reduces monotonically and is independent of the phonon loss time. In this regime the high energy primary quasiparticle peak is created at $\Delta \leq E \leq 3\Delta$ and phonons emitted in scattering are unable to break pairs. At higher signal energies $4\Delta \leq h\nu_s \leq 6\Delta$ the efficiency tends to increase again and the increase depends on $\tau_1/\tau_0^\phi$. Pair-breaking enhances $\eta_s$ and the enhancement depends on the probability of pair-breaking compared to other phonon losses. At higher energies multiple pair-breaking is necessary to create the low energy steady state distribution, but multiple phonon loss also occurs. The overall effect is a reduction in $\eta_s$ as $h\nu_s$ increases for finite $\tau_1/\tau_0^\phi$. We note that $\eta_s \rightarrow 1$ as $h\nu_s \rightarrow 2\Delta$ for all $\tau_1/\tau_0^\phi$ so that equations (2)–(5) and our definition of $R$ self-consistently conserve energy.

Figure 5 shows $\eta_s$ for $h\nu_s = 3$ and $5\Delta$ as a function of $P_p$ for two values of the probe power. For $P_p = 0$ the generation efficiency is constant over five orders of magnitude of absorbed signal powers. For $P_p = 20$ aW $\mu m^{-3}$ and for low signal power, $\eta_s$ is slightly enhanced. We discuss this in section 5. Figure 6 shows the distribution-averaged values of $\tau_c$ associated with the calculations shown in figure 5 (note that equation (5) does not involve $\tau_c$ explicitly), while figure 7 shows the $\tau_{pb}$ that is directly used in these calculations. Considering figure 6 for the small signal regime $P_s \ll P_p$, $N_p$ determines $\tau_c$. For the large signal regime $P_s \gg P_p$, $\tau_c$ is independent of $P_p$ because $N_p^ex$ determines $\tau_c$. We see that for fixed $h\nu_s$, $\tau_c$ changes by nearly three orders of magnitude whilst $\eta_s$ shown in figure 5 is constant for all $P_s$. (Very close
Figure 4. Number generation efficiency $\eta_s$ as a function of $h\nu_s/\Delta$ for five values of $\tau_l/\tau_0^\phi$.

Figure 5. The number generation efficiency $\eta_s$ for $h\nu_s = 3$ and $5\Delta$ with $P_s = 0$ and $20\text{ aW}\mu\text{m}^{-3}$ with $\tau_l/\tau_0^\phi = 1$.

inspection of the results for $P_p = 0$ would show that $\eta_s$ varies for the range of calculated $P_s$ by about $\pm 0.03\%$ which we consider acceptable given the numerical precision used and the discretization of the distributions.) Also, the $\tau_r$ for the two signal photon energies differ, but the generation rate of excess quasiparticles depends on $h\nu_s$, as does the fraction of power lost before the quasistatic driven distributions are created. We would note that equation (5) correctly takes into account these underlying changes in $\tau_r$ due to signal and probe.

Figure 7 shows that the distribution-averaged $\tau_{pb} \sim \tau_0^\phi$, which would often be assumed, but moreover $\tau_{pb} < \tau_0^\phi$. For $T/T_c = 0.1$, $\tau_{pb}(\Omega) \leq \tau_{pb}(2\Delta)$, $\tau_{pb}(\Omega)$ scales approximately as $1/\Omega$ in thermal equilibrium [10]; hence the slight reduction when $\tau_{pb}$ is calculated for the non-equilibrium distributions. We find that the variation of $\tau_{pb}$ as a function of the drive (both probe and signal) arises from the detailed spectra of the 2$\Delta$-phonons for each case and is (to first-order) independent of the quasiparticle spectrum. It is possible to define an effective phonon temperature $T_{2\Delta}^{\text{eff}}$ that accounts for the total number of 2$\Delta$-phonons. This approach accounts for the calculated $\tau_{pb}$ to within 1%, but not for the detailed behavior as a function of $P_s$. In the presence of the probe and signal the probe determines $\tau_{pb}$ if $P_p \gg P_s$, and in this case we find that $T_{2\Delta}^{\text{eff}}$ more closely accounts for the calculated $\tau_{pb}$. We would emphasize here that the full calculation of $\tau_{pb}$ is necessary to find that $\eta_s$ is independent of the power $P_s$.

5. Discussion and conclusions

We have presented calculations of the quasiparticle generation efficiency $\eta_s$ for a pair-breaking signal in thin Al films at $T/T_c = 0.1$ with photon energies in the range $2\Delta \leq h\nu_s \leq 10\Delta$, $90 \leq \nu_s \leq 450\text{ GHz}$. We have also investigated the effect of including a probe with power and frequency typical of those used in low-noise KID readout. The calculated detailed spectra show the effects of multiple interactions of the probe and the signal in the driven $f(E)$ with structure, for example, at $E = h(\nu_s + \nu_p)$. Our results demonstrate the importance of phonon loss on the quasiparticle creation efficiency. For thick films, $\tau_l/\tau_0^\phi = 8$, our calculations are in general agreement with earlier work for much higher signal energies, in calculations that ignore 2$\Delta$-phonon loss, showing $\eta_s \simeq 0.59$. For resonators, thinner films would tend to be used since these
maximize the kinetic inductance fraction of the response [1], but these have limited the detection sensitivity of thin-film superconductors. The limiting noise equivalent power of a thin-film detector is determined by generation–recombination noise [6, 24, 27] and is given by \( \text{NEP} = 2\Delta \sqrt{V/N} / \tau_{\text{ef}} / \eta \), where \( V \) is the volume of the film and \( \eta \) is the overall detection efficiency. \( \eta \) is the product of all detection efficiencies (including the coupling efficiency), but \( \eta \) shown in figure 4 determines the limiting efficiency in the thin-film case. The present work shows that the best-possible coupled NEPs are even higher than the case \( \eta = 0.59 \) shown there for much of the mm and sub-mm spectrum in thin superconducting films. In deriving equation (5) we assumed that all quasiparticles have energy \( \Delta \). It is possible to take account of the energy distribution of the excess quasiparticles in the derivation and this would increase our calculated \( \eta \) by about 4%, but for consistency with earlier work we have assumed \( E = \Delta \) for all of the excess.

We identify a coupling between the signal and the probe that enhances \( \eta \) by about 2%. This may be the effect described by Gulian and van Vechten [28], who suggested that for low \( P_s \) multiple probe photon absorption by the higher energy primary peak of figure 2 (inset) occurs and some fraction of these quasiparticles are driven to energies \( E \geq 3\Delta \). By contrast, figure 2 suggests that \( 2\Delta \)-phonon reabsorption occurs to enhance \( \eta \). As the signal power increases there is a slight reduction in \( \eta \) because the relevant quantity is the fraction of quasiparticles in the photon peak driven above the pair-breaking threshold. The fraction reduces because the probe power is fixed and the probe generates (most of) the excess \( 2\Delta \)-phonons. In future work we intend to extend this work to consider other low temperature superconductors, to investigate the detection linearity of a resonator with the driven distributions and also to consider the probe power levels that optimize detector NEPs.

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