A magneto-optic modulator with unit quantum efficiency

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We propose a device for the reversible and quiet conversion of microwave photons to optical sideband photons, that can reach 100\% quantum efficiency. The device is based on an erbium doped crystal placed in both an optical and microwave resonator. We show that efficient conversion can be achieved so long as the product of the optical and microwave cooperativity factors can be made large. We argue achieving this regime is feasible with current technology and we discuss a possible implementation.

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In recent years there has been spectacular progress in the development of devices based on superconducting qubits \textsuperscript{3–8}. Superconducting qubits are small superconducting circuits incorporating Josephson junctions, and possessing discrete energy levels connected by microwave transitions. Electronic manipulation of these qubits is very fast with quantum logic operations taking just nanoseconds. Achievable coherence times are much longer - 20\,\mu s - and rapidly improving \textsuperscript{3, 10}, enabling many logic operations to be carried out before the qubits decohere. However there are two problems that hinder the application of superconducting qubits: the inability to send quantum states over long distances; and the lack of a long term memory. These two problems have spawned the new field of ‘hybrid quantum systems’, where the coupling of superconducting qubits to a wide range of other physical systems, such as spin systems \textsuperscript{11} and nano-mechanical systems \textsuperscript{12}, are being investigated.

Rather than coupling directly to the superconducting qubits, our approach is to take the microwave photons, which couple naturally to the superconducting qubits, and convert these into optical photons. The quantum memories that are available for light \textsuperscript{13–16} could then be used, as could optical fibers for the long distance transmission of quantum states. The photons produced will have a wavelength near 1540 nm, where optical fibers have their lowest loss and quantum states have been sent a number of kilometers.

There has been an impressive report \textsuperscript{17} of reversible and efficient but noisy conversion of microwave photons to optical sideband photons using a microwave and an optical resonator both coupled to the same a nano-mechanical oscillator. The advantage of our proposed scheme is that we only require temperatures cold enough to freeze out microwave frequency excitations rather than the very low temperatures required to freeze out the mechanical resonances. The narrow mechanical resonance also restricts their conversion bandwidth.

There have been a number of investigations of cavity QED using rare earth ion dopants, using both microwave \textsuperscript{21–23} and optical \textsuperscript{24, 25} transitions. Here we propose building on this work by placing an erbium doped crystal in both a microwave and optical resonator.

The setup we consider is shown diagrammatically in FIG. 1. In this setup a collection of three level atoms interact with an optical cavity mode (frequency $\omega_a$), a microwave cavity mode (frequency $\omega_b$) and a coherent driving field (frequency $\omega_{\Omega}$). The coupling strengths for the $k$-th atom to the microwave and optical resonators are $g_{\mu,k}$ and $g_{o,k}$ respectively and the coherent driving field has Rabi frequency $\Omega_k$. We will use \textit{a} to denote the lowering operator for the optical resonator field and \textit{b} to denote the lowering operator for the microwave resonator field.

This leads to the following Hamiltonian for our atoms-
The Hamiltonian in Eq. (2) has three terms. The first two are due to the off resonant atoms pulling the resonant frequencies of the two cavities. We will ignore these terms as they can easily be compensated for by tuning the two resonators. The third term is a linear coupling between the two modes with strength which we shall denote by $S$. If we make the assumption $\delta_{o,k}\delta_{\mu,k} \gg |\Omega_k|^2$, which is required for the adiabatic elimination of the atoms, then $S$ becomes

$$ S = \sum_{k} \frac{\Omega_k g_{\mu,k}g_{\mu,k}^*}{\delta_{o,k}\delta_{\mu,k}} $$

(3)

To see how this interaction can efficiently convert between microwave and optical fields, we consider the two resonators interacting with their input and output modes [28].

$$\dot{a} = -iSb - \frac{\kappa_a}{2}a - \sqrt{\kappa_a}\eta_{in}(t)$$

$$\dot{b} = -iS^*a - \frac{\kappa_b}{2}b - \sqrt{\kappa_b}\eta_{in}(t)$$

(4)

Here the $\kappa$ are the decay rates for the two cavities. Fourier transforming this and using the input output relations [28] gives.

$$\tilde{a}_{out}(\omega) = \frac{4iS\sqrt{\kappa_a\kappa_b}}{4|S|^2 + (\kappa_a - 2i\omega)(\kappa_b - 2i\omega)}\tilde{b}_{in}(\omega)$$

$$+ \frac{4|S|^2 - (\kappa_a - 2i\omega)(\kappa_b - 2i\omega)}{4|S|^2 + (\kappa_a - 2i\omega)(\kappa_b - 2i\omega)}\tilde{b}_{in}(\omega)$$

(5)

$$\tilde{b}_{out}(\omega) = \frac{4iS^*\sqrt{\kappa_a\kappa_b}}{4|S|^2 + (\kappa_a - 2i\omega)(\kappa_b - 2i\omega)}\tilde{a}_{in}(\omega)$$

$$+ \frac{4|S|^2 - (\kappa_a - 2i\omega)(\kappa_b + 2i\omega)}{4|S|^2 + (\kappa_a - 2i\omega)(\kappa_b + 2i\omega)}\tilde{a}_{in}(\omega)$$

The first terms in the right-hand side of Eqns. (5) give photon conversion between the microwave and optical fields and the second terms give a reflected signal. One can define a number-conversion efficiency

$$\eta(\omega) = \left| \frac{4iS\sqrt{\kappa_a\kappa_b}}{4|S|^2 + (\kappa_a - 2i\omega)(\kappa_b - 2i\omega)} \right|^2$$

(6)

In the impedance matched case where $|S|^2 = \kappa_a\kappa_b$, we have the desired result that on resonance the input microwave field is mapped completely onto the output optical field and vice-versa. The bandwidth that this works over is given by the geometric mean of the two cavity linewidths. Fig. 3 shows the efficiency as a function of the detuning from resonance and $R$, which is defined by $R = 2|S|/\sqrt{\kappa_a\kappa_b}$.

Our situation is now completely analogous to two optical cavities share a partially transmissive end mirror, see Fig. 4.

To get a feeling for what is required to reach impedance matching let us first assume that the $g$, $\Omega$, $\delta$, $\Delta$ parameters are real and the same for each atom. By doing this
we are ignoring for the moment the problems of phase matching and are assuming that all the atoms sit in a maximum of both the optical microwave fields. With this assumption we can write our impedance matching condition 2|S| = \sqrt{\kappa_a \kappa_b} as

$$\sqrt{\frac{N g_a^2}{\kappa_a \delta_\mu} \times \frac{N g_b^2}{\kappa_b \delta_o} \times 2\Omega} = 1$$

(7)

Obviously we can easily reduce the left hand side by turning down the classical drive field and therefore reducing \( \Omega \). The challenge is to get the left hand side up one.

In order that the cavity is detuned from the microwave resonance the \( \delta_o \) needs to be bigger than the inhomogeneous linewidth of the spin transition. This means that the first term in Eq. (7) is bounded above by the microwave cooperativity factor. For an analogous reason the second term is bounded above by the optical cooperativity factor. The third term is bounded above by one due to the conditions for adiabatic elimination. From this one can see that the essential challenge for efficient upconversion is getting the microwave and optical cooperativity factors much larger than one. This should be feasible in light of recent research in rare earth cavity QED. Using microwave resonators and spin transitions there are reports close to [21] and achieving [22] [23] strong coupling. For optical cavity QED people have strived, but not yet achieved strong coupling. For optical cavity QED people have

\[ \text{FIG. 3: Conversion efficiency } \eta(\omega) \text{ for different } R. \]

\[ \text{FIG. 4: Two optical cavities that are coupled by sharing a partially transmissive end mirror. If the reflectivity of this end face mirror is tuned appropriately then, on resonance, the three mirrors will have 100% transmission.} \]

\[ a_{\text{in}}(t) \quad a(t) \quad b(t) \quad b_{\text{out}}(t) \]

\[ \kappa_a \quad S \quad \kappa_b \]

\[ a_{\text{out}}(t) \quad b_{\text{in}}(t) \]

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\[ \text{the microwave cooperativity factor. For an analogous rea-} \]

\[ \text{son, the two stable isotopes of Er have spin zero and ground state splitting of} \]

\[ \text{the order of 5 GHz is possible with an applied magnetic} \]

\[ \text{field} \]

\[ \text{Unfortunately nobody to our knowledge has made a } \]

\[ \text{A system using the full 5 GHz splitting. It may or} \]

\[ \text{may not be possible to find an excited state level to} \]

\[ \text{complete the } \Lambda \text{ system. Unfortunately, while the full} \]

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\[ \text{the excited state spin Hamiltonian is not. All of the other stable iso-} \]

\[ \text{topes of Er have spin zero and ground state splitting of} \]

\[ \text{the order of 5 GHz is possible with an applied magnetic field} \]

\[ \text{field} \]

\[ \text{Unlike the hyperfine structure, the fine structure is well understood} \]

\[ \text{For this reason, and also the potential problems that stray light might cause to superconducting cavities and the convenience of being able to tune the microwave transition frequency with a magnetic field, we will focus here on non-superconducting cavities made out of copper and use Zeeman split fine structure in the spin zero isotopes. This obviously limits the available microwave } Q \text{-factors but we are still confident that high efficiency operation is possible, as we discuss below.} \]

\[ \text{Taking into account spatial variation and inhomogeneous broadening of the atoms, we write our impedance matching parameter } R \text{ as } R = \Omega a F \sqrt{Q_a Q_b}, \text{ where } \Omega \text{ is the peak Rabi frequency,} \]

\[ \alpha \equiv \sqrt{\frac{\mu_0}{\hbar^2 \varepsilon_0}} d_{31} \mu_{21} \rho \int_{0.5\sigma_\mu}^{\infty} \frac{D_\mu(\delta_\mu)}{\delta_\mu} d\delta_\mu \int_{0.5\sigma_o}^{\infty} \frac{D_o(\delta_o)}{\delta_o} d\delta_o \]

(8)

\[ \text{is a crystal dependent parameter,} \]

\[ F \equiv \frac{1}{\sqrt{V_p V_o}} \left| \int_V \chi(r) \psi(r) \phi(r) d^3r \right| \]

(9)

\[ \text{is the “filling factor” and } Q_a \text{ and } Q_b \text{ are quality factors for the optical and microwave resonators respectively.} \]

\[ \text{Here } d_{31} \text{ is the electric dipole moment for the } 1 \leftrightarrow 3 \text{ transition, } \mu_{21} \text{ is the magnetic dipole moment for the} \]

\[ \text{single dopant [24, 25]. In the many atom regime it is much easier to achieve strong coupling because the penalty you pay, which is the square root of the ratio of inhomogeneous to homogeneous broadening, is much smaller than the benefit you get, which is the square root of the number of dopants. This is especially the case in systems where erbium replaces yttrium where the inhomogeneous broadening tends to be small.} \]

\[ \text{There are two important issues which this simplified analysis doesn’t take into account: firstly, whether the spectroscopy of the erbium dopants allows a level scheme like we have illustrated in Fig. 2 and secondly, how close can one get to the perfect overlap between an optical and a microwave mode, especially given their vastly different wavelengths.} \]

\[ \text{With respect to the spectroscopy of erbium, one of the attractive features is the hyperfine structure for } ^{167}\text{Er. In the case of Er:YSO (Er:Y}_2\text{SiO}_5 \text{) the ground electronic state is split over approximately 5 GHz [29]. Zero field splitting is desirable because the } Q \text{-factor of superconducting resonators drops dramatically in a magnetic field [30]. Unfortunately nobody to our knowledge has made a } \Lambda \text{ system using the full 5 GHz splitting. It may or may not be possible to find an excited state level to complete the } \Lambda \text{ system. Unfortunately, while the full ground state spin Hamiltonian is known [29], the excited state spin Hamiltonian is not. All of the other stable isotopes of Er have spin zero and ground state splitting of the order of 5 GHz is possible with an applied magnetic field } \Omega(100 \text{ mT). Unlike the hyperfine structure, the fine structure is well understood [31]. For this reason, and also the potential problems that stray light might cause to superconducting cavities and the convenience of being able to tune the microwave transition frequency with a magnetic field, we will focus here on non-superconducting cavities made out of copper and use Zeeman split fine structure in the spin zero isotopes. This obviously limits the available microwave } Q \text{-factors but we are still confident that high efficiency operation is possible, as we discuss below.} \]

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\[ \text{is a crystal dependent parameter,} \]

\[ F \equiv \frac{1}{\sqrt{V_p V_o}} \left| \int_V \chi(r) \psi(r) \phi(r) d^3r \right| \]

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\[ \text{is the “filling factor” and } Q_a \text{ and } Q_b \text{ are quality factors for the optical and microwave resonators respectively.} \]

\[ \text{Here } d_{31} \text{ is the electric dipole moment for the } 1 \leftrightarrow 3 \text{ transition, } \mu_{21} \text{ is the magnetic dipole moment for the} \]
$1 \leftrightarrow 2$ transition, and $D_\mu(\delta_\mu)$ and $D_\sigma(\delta_\sigma)$ are inhomogeneous broadening distributions for the microwave and optical transitions respectively, which we shall assume to be Gaussian with standard deviations $\sigma_\mu$ and $\sigma_\sigma$. The lower bounds on the integrals in $\alpha$ are chosen not $-\infty$ to avoid the problems at $\delta = 0$ due to the breakdown of the adiabatic approximation. We use $\rho$ to denote the number density of the Er ions within the crystal and $V_c$ for the crystal volume. The mode function for the RF magnetic field of the resonator is denoted by $\chi(r)$, and $\psi(r)$ and $\phi(r)$ denote the optical modes for $a$ and the classical driving field respectively.

One geometry that could be used is a whispering gallery mode optical resonator in combination with an transmission line microwave resonator, similar to the most efficient electro-optic modulators demonstrated [20]. It should be noted that the feasibility is independent of the size of the system and only dependent on the filling factor. This is because as the volume of the cavities go up the $g$ factors go down with $\sqrt{N}$ but the many atom cooperativity factor scales as $\sqrt{N}$ so the two affects cancel each other out. For this reason and ease of construction we here consider a shielded loop-gap resonator [32], and a Fabry-Perot resonator as shown in Fig. 5. The magnetic field of the loop-gap resonator is concentrated in and reasonably uniform over the middle hole, the proposed mode diameter for the Fabry Pérot resonator is large at 1 mm but not unprecedented [33]. We have made a number of microwave resonators similar to Fig. 5 and have achieved $Q_b > 2000$. The value for $F$ for these resonators was calculated using FDTD (finite difference time domain) solutions for the microwave resonator and paraxial optics for the optical resonator, giving $F = 0.0084$.

To calculate the $\alpha$ for our resonators we take $D_\mu$, to have a standard deviation of 1 MHz and a mean of $3\sigma_\mu$, and $D_\sigma$ to have a standard deviation of 500 MHz and a mean of $3\sigma_\sigma$. For Er:YSO we have that $d_{31} = 2.13 \times 10^{-32}$ Cm and for the $\left| -15/2 \right> \rightarrow \left| 15/2 \right>$ spin transition we have that $\mu_21 \approx 15\mu_B$, where $\mu_B$ is the Bohr magneton. We assume that the crystal is a 0.001% doped Er:YSO cylinder that fills the small hole of our loop-gap resonator. We then obtain that $\alpha = 2.86 \times 10^{-10}$ s. We take $\Omega = 10$ MHz, which ensures that $\Omega^2 < \delta_\mu \delta_\sigma$ as required for the adiabatic approximation.

A contour plot of $R$ versus $F$ and $Q_aQ_b$ provides a means to visualize the feasibility of achieving complete photon conversion and is shown in Fig. 6. Complete photon conversion is achievable in Fig. 6. Complete photon conversion is achievable in the red region where $R > 1$.

Quality factors of $Q_a \gtrsim 10^8$ are obtainable for Fabry-Perot resonators out of YSO at 606 nm [30]. Taking $Q_a = 10^7$ and $Q_b = 2000$ gives $R = 3.4$ and therefore our resonator design is theoretically capable of achieving complete photon conversion. It should be pointed out that there is room for improvement in our parameters. For example, isotopically pure erbium doped yttrium lithium fluoride has yielded 16 MHz optical inhomogeneous linewidths [35] rather than the 500 MHz used here. If the spectroscopy of erbium allows operation in zero magnetic field, and thus the use of superconducting resonators, then many orders of magnitude improvement in the microwave Q-factor would also be possible.

In conclusion we propose using an erbium doped crystal in both an optical and microwave resonator to achieve complete photon conversion between microwave and optical fields. We present a theoretical analysis of a proposed...
design that should be in the reach of current technology. The analysis shows that our design is capable of achieving complete photon conversion.

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