NON-LOCALITY AND GRAVITY-INDUCED CP VIOLATION

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We consider a space-time dependent relative phase between the right- and left-handed spinors and show that it results in a violation of locality in the presence of gravity once the demand of parity covariance is dropped. This violation of locality is such that it readily interprets itself as a gravity-induced CP violation, and at the same time confirms an earlier remark by Wigner that a representation space carries more information than a wave equation. This happens, as Kirchbach has noted, because while the dimensionality of an irreducible representation space does not depend upon the concrete realization of the symmetry generators, Noether currents (Dirac, versus Majorana, versus the CP violating construct presented here) do. The gravity-induced CP violation provides a dynamical reason on how a neutron star carrying its baryon and lepton numbers can collapse into a black hole and lose information on the latter characteristics.

1. Introduction and historical background

The uncertainty relation $\Delta x \Delta p_x \sim \hbar$ is a direct consequence of the fundamental commutator $[x, p_x] = i\hbar$ between the position and momentum of a particle. Its role in the foundations of physics can hardly be overemphasized. Yet, as the century that began with this profound change in our understanding of Nature reaches towards its end, we are beginning to realize hints for further deeper changes. It is becoming increasingly clear that in the presence of gravitation the fundamental uncertainty relations, and hence the fundamental commutators, must be modified. For instance, in the context of string theories $\Delta x \Delta p_x \sim \hbar$ gets replaced by

$$\Delta x \Delta p_x \sim \hbar \left[ 1 + \left( \lambda_P \Delta p_x / \hbar \right)^2 \right], \quad (1)$$

with $\lambda_P$ of the order of the Planck length $\sqrt{\hbar G/c^3}$. However, as emphasised by Witten, a proper theoretical framework for the extra term in the uncertainty relation has not yet emerged.

Once modifications of the uncertainty relations are taken seriously, the question naturally arises if such can also occur within the context of the existing point-particle quantum field theoretic framework, in four-dimensional space-time, with minimal changes. Modifications to the uncertainty relations of the type considered in clearly imply that the canonical bosonic commutators, and the fermionic anticommutators, in being mathematical expressions of the locality of the underlying quantum field theory, have to change to incorporate non-locality.

Our answer to the question posed in the opening remark of the preceding paragraph is a well-formulated and clear ‘yes.’ The fundamental modifications to the

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structure of quantum field theory can be induced by gravity and occur via a non-locality that resides in a certain space-time derived fields. The origin of the presented modifications goes beyond simply considering a given flat-space-time field in a curved background. The precise nature of these statements shall become clear in due course.

We here show that an allowed relative phase between the right- and left-handed spinors that may appear in covering the full Lorentz group without demanding parity covariance, can serve as a source for non-locality in the presence of gravity. The violation of locality produced in this way is such that it readily interprets itself as a gravity-induced CP violation, and at the same time confirms an earlier remark by Wigner that a representation space carries more information than a wave equation.

A few historical remarks are now in order. Even before the experimental discovery of CP violation, the possibility was considered by Good that gravitation may induce CP violation and he used the then-existing circumstances to study the gravitational behavior of antimatter. Since then the idea of gravity-induced CP violation has been repeatedly considered. The Good framework is built upon the Morrison-Gold conjecture presented in their celebrated 1957 Gravity Research Foundation essay that particles and antiparticles may carry opposite gravitational masses. Note that the appearance of different particle–antiparticle gravitational masses within the gravity-induced CP violation scenario of Good suggests violation of the principle of equivalence. In a recent work Chardin argued that gravity-induced CP violation provides a parameter free explanation of the CP violation observed in the neutral Kaon system in the Chardin argument no violation of the equivalence principle occurs. Instead, gravitational repulsion naturally emerges in the context of wormholes and the Kerr geometry. Independently, Fischbach et al. have arrived at a similar conclusion.

A further argument in favor of gravitation as a source of CP violation can be found in Hawking’s work where black holes are shown to posses a thermodynamic entropy and consequently lead to T- (and CP-, in a CPT preserving framework) violating processes.

In this paper we show how to drop the Morrison-Gold conjecture in a non–trivial way and obtain a gravity-induced CP violation within a CPT covariant framework. Since the principle of equivalence is no longer violated within the present scheme, the proposed gravity-induced CP violation is fundamentally different from the Morrison-Gold framework. Since no reference is made to wormholes and Kerr geometry, the presented framework has greater generality than the Chardin-Fischbach scheme.

To keep the physics transparent our thesis shall be presented in flat space-time, and gravitation considered in the weak field limit a la Sakurai (see pp. 126-129), on the one side, and in the spirit of experimental work on gravitationally induced quantum interference, on the other side. A further reason for this approach resides in the fact that if one confines oneself to a purely general relativistic framework, certain physically observable quantum mechanical phases can become non-observable. In essence, this means that the general relativistic description of gravitation may not be considered complete in the quantum realm — see section 2.4 for further details.

2. A space-time origin of non-locality

The thesis that an origin of CP violation is to be found at the level of the representations of the Lorentz group, and that in the presence of gravity this violation is deeply connected with the space-time metric shall be made in three parts. In the first part an unsuspected space-time dependent relative phase between the right- and left-handed spinors is introduced. In the second part it is discovered that this phase is deeply connected with the C, P, and T properties of the $(1/2, 0) \oplus (0, 1/2)$
representation space, and that despite CPT-covariance it carries in it an essential element of non-locality. In the third part a physical interpretation is put forward in which the space-time dependent relative phase induces deviations of the metric tensor from its flat space-time form. As a result a gravity-induced CP violating structure emerges which has profound consequences for astrophysical and cosmological processes.\(^{a}\) This thesis spans sections 2.1 to 2.3. Section 2.4 then provides a parenthetic argument regarding the observability of constant gravitational potentials in the context of an earlier work of Kenyon.\(^{21}\)

2.1. **New Relative phases between the right- and left-handed spinors**

To present our thesis let us begin by recalling that in the recent generalization of the Case-McLennan reformulation (see Refs. \([31-34]\)) of the Majorana field the relative phase (to be denoted by \(\zeta\) here) between the right- and left-handed spinors plays an important physical role:

\[
\lambda(p^\mu) \equiv \left( \zeta \Theta_{[j]} \right) \phi^*_L(p^\mu) / \phi^*_L(p^\mu), \quad \rho(p^\mu) \equiv \left( \zeta \Theta_{[j]} \right)^* \phi^*_R(p^\mu) / \phi^*_R(p^\mu) . \tag{2}\]

Indeed, for fermion fields these phases must take on the values \(\pm i\) to ensure that the spinors of the \((j, 0) \oplus (0, j)\) representation are self/anti-self charge conjugate, i.e., they are of the extended Majorana type. It is this phase, equal to \(\pm i\), that emerges as the intrinsic parity of the Majorana particles in the original analysis of Racah.\(^{35}\) On the other hand, for Dirac spinors, within the wisdom of the 1960s\(^{36}\) it was argued by Ryder in his recent textbook on quantum field theory that (see Ref. \([37]\), p. 44),

*Now when a particle is at rest, one cannot define its spin as either left- or right-handed, so*

\[
\phi_R(\vec{0}) = \phi_L(\vec{0}) . \tag{3}\]

Here \(\vec{0}\) represents the vanishing three momentum of the particle at rest, while its four momentum is represented by \(\vec{p}^\mu \equiv \{m, \vec{0}\}\).

We now make the crucial observation of this paper. While the above quoted argument remains valid in the classical domain, it does not contain the full physics allowed by the relativistic quantum-mechanical framework.\(^{37}\) In the quantum realm it must be generalized to read:

*The full exploitation of the relativistic quantum-mechanical framework allows the equality expressed by Eq. (3) up to a phase,*

\[
\phi_R(\vec{p}^\mu) = \pm \exp \left[ \pm i \phi(x) \right] \phi_L(\vec{p}^\mu) . \tag{4}\]

The phase, \(\phi(x)\), may have a space-time dependence. While the existence of this phase is permitted by the quantum mechanical framework, its space-time dependence is a requirement of relativity – the arguments for this dual requirement are similar to the standard textbook arguments that are found for the introduction of space-time dependent “local” gauge transformation (see Ref. \([37]\), Sec. 3.3).

\(^{a}\) In this, but not the following, section we shall set \(\hbar\) and \(c\) equal unity.

\(^{b}\) In Eq. (2), \(\Theta_{[j]}\) is the Wigner time-reversal operator, \(\Theta_{[j]} \hat{J} \Theta_{[j]}^{-1} = -\hat{J}\). It is a consequence of this property that if \(\phi_L(p^\mu)\) transforms as a \((0, j)\) co-spinor under Lorentz boosts, the construct \(\Theta_{[j]} \phi^*_L(p^\mu)\) transforms as a \((j, 0)\) spinor, i.e. as a right-handed spinor. Similarly, if \(\phi_R(p^\mu)\) transforms as a \((j, 0)\) spinor, then the construct \(\Theta_{[j]} \phi^*_R(p^\mu)\) transforms as a \((0, j)\) co-spinor, i.e., as a left-handed spinor. The notation \(\hat{J}\) stands for the usual \((2j + 1) \times (2j + 1)\) spin-\(j\) matrices.
The first ± sign on the right-hand side of Eq. (4) is introduced to span the full $(j, 0) \oplus (0, j)$ representation space. In the second ± sign of Eq. (4), the plus sign corresponds to the particles (the ‘positive energy solution’) and the negative sign to the anti-particles (the ‘negative energy solution’) – see below.

It is this observation, we shall argue, that contains the answer—in-affirmative to the question posed in Sec. 1 above. In its physical essence, this observation parallels the ideas of Weyl, \(^39\) and Yang and Mills.\(^40\) What follows is a mathematical exercise to distill the physical content of the generalization contained in Eq. (4).

If the phase \(\phi(x)\) has to carry a physical meaning, its space-time dependence must be associated with some dynamical property. The phase \(\phi(x)\) may be interpreted either as a Higgs-like field\(^41\), or, as done in Sec. 2.3 below, as the deviation the metric tensor in the weak–field limit from its flat space-time form \(\eta_{\mu\nu} = \text{diag. (1, } -1, -1, -1\}.\) Whether the Higgs field is some manifestation of gravitation in disguise remains a pregnant possibility.

The right- and left-handed spinors, without reference to a wave equation or a Lagrangian, Lorentz transform as:

\[
\phi_R(p^\mu) = \exp \left( + \vec{J} \cdot \vec{\varphi} \right) \phi_R(\hat{p}^\mu), \quad \phi_L(p^\mu) = \exp \left( - \vec{J} \cdot \vec{\varphi} \right) \phi_L(\hat{p}^\mu).
\]  

Here, \(p^\mu\) represents the boosted energy-momentum vector \((E, \vec{p})\). The boost parameter, \(\vec{\varphi}\), is defined as

\[
cosh(\varphi) = \gamma = (1 - v^2)^{-1/2} = \frac{E}{m}, \quad \sinh(\varphi) = v\gamma = \frac{|\vec{p}|}{m},
\]  

with \(\vec{\varphi} = \vec{p} / |\vec{p}|\), and \(\vec{v}\) stands for the velocity of the particle.

The considerations so far are generally true for any spin. To present our thesis it suffices to confine to spin one-half (however, note that an extension to higher spins may contain important physics).

### 2.2. Non-locality for spin \(1/2\)

For spin one-half, when Eqs. (4) and (5) are coupled \textit{a la} Ryder (see Ref. \[^37\], Sec. 2.3), we obtain on setting \(\vec{J} = \vec{\sigma}/2\),

\[
- \zeta^{-1}(x) \phi_R(p^\mu) + \left( \frac{E + \vec{\sigma} \cdot \vec{p}}{m} \right) \phi_L(p^\mu) = 0,
\]

\[
\left( \frac{E - \vec{\sigma} \cdot \vec{p}}{m} \right) \phi_R(p^\mu) - \zeta(x) \phi_L(p^\mu) = 0.
\]

In Eqs. (7) and (8), \(\zeta(x)\) stands for the phase factor \(\pm \exp \left[ \pm i \phi(x) \right]\). Now we introduce the \((1/2, 0) \oplus (0, 1/2)\) Weyl-representation spinor [\textit{cf.} Weyl-representation extended Majorana spinors of Eq. (2)],

\[
\psi(p^\mu) = \left( \begin{array}{c} \phi_R(p^\mu) \\ \phi_L(p^\mu) \end{array} \right),
\]

with \(\phi_R(p^\mu)\) and \(\phi_L(p^\mu)\) related by Eq. (4). Thus, note is to be immediately taken that in case of the arbitrary phases, \(\phi(x)\), the spinors (4), are not identical to the

\(^4\)Equation (4) owes its origins to discussions with C. Burgard when we both were at Texas A&M University in the early nineties. It is my pleasure to have this opportunity to acknowledge this with thanks.
standard Dirac spinors. By the end of this paper the reader shall infer that these
spinors become Dirac spinors when $\phi(x)$ acquires a space-time independent value
that is an integral multiple of $2\pi$.

Eqs. (7) and (8) can be cast into the compact form,

$$ (\gamma^\mu p_\mu - \xi(x) m) \psi(p^\mu) = 0, \quad (10) $$

where $\gamma^\mu$ are the Dirac's gamma matrices in the Weyl representation (see Ref. [37],
Eq. 2.92), and $\xi(x)$ is a $4 \times 4$ matrix,

$$ \xi(x) = \begin{pmatrix} \zeta^{-1}(x) I_2 & 0_2 \\ 0_2 & \zeta(x) I_2 \end{pmatrix}. \quad (11) $$

We use the notation $I_n$ for the $n \times n$ unit matrix, while $0_n$ stands for a
$n \times n$ zero-matrix.

Several observations are immediately in order.

(a) Equation (10) has been derived from the most basic space-time arguments.
In particular, no Lagrangian needed to be postulated. A Lagrangian may be con-
structed for field theoretic purposes which yields Eq. (10). The he-
riticity of this Lagrangian is assured because, apart from the known properties of
$\gamma^\mu$, we also have

$$ \gamma^0 \xi(x) \gamma^0 = \xi(x'), \quad (12) $$

where $x'$ is the parity transformed $x$. Tentatively assuming $\zeta(x)$ to be an even
function of $x$ (without a priori justification), this requirement reduces to the con-
dition $\zeta^{-1}(x) = \zeta(x)$, and implies $\zeta = \pm 1$. In terms of the phase, this translates
to a space-time independent $\phi = 0$, or more generally an integral multiple of $2\pi$
- cf. Ref. [41]. Further, the phase $\pm i$ encountered in the case of the extended
Majorana spinors is an exact counterpart of the $\pm 1$ for the Dirac spinors. For
Dirac spinors it refers to the relative parities of fermions and anti-fermions. For
the extended Majorana spinors this phase is the parity of the self/anti-self charge
conjugate particles. The relative phase of $\pm i$ appearing for Majorana fields is deter-
mined by the requirement of self/anti-self conjugacy under the operation of charge
conjugation, $C$. For the Dirac fields the phase of $\pm 1$ follows from the requirement
that the respective spinors be eigenstates of the charge operator, $Q$. The physical
origin for the different intrinsic parities of the Dirac particle and anti–particle, as
well as of the extended Majorana spinors, arises from the fact that the latter are
eigenstates of the charge conjugation operator, while the former are eigenstates of the
charge operator – and that $C$ and $Q$ do not commute, $[C, Q] \neq 0$. The require-
ment of parity covariance collapses the phase field $\zeta(x)$ to the constant value of $\pm 1$
throughout the space-time. This means that the measurement of parity collapses
the relative phase factor between the right- and left-handed spinors to the constant
eigenphases $\pm 1$. Here is the first hint that the demand of parity covariance is related
to the locality. If the demand for parity covariance is dropped the resulting theory
may become non-local (with the understanding that this non-locality is of a similar
origin as that involved in the “collapse of a wave function”). In particular,

$$ \psi[p^\mu, \phi(x) \to 0, 2\pi, \cdots] \to \psi(p^\mu)_{\text{Dirac}}, \quad (13) $$

(c) We find

$$ \det |\gamma^\mu p_\mu - \xi m| = (\vec{p}^2 + m^2 - E^2)^2. \quad (14) $$
The independence of \( \text{Det} \left[ \gamma^\mu p_\mu - \xi m \right] \) on the phase \( \phi(x) \) enables one to give on the classical level a physical interpretation of the CP-violating fields in terms of particles and antiparticles, i.e., in terms of positive- and negative-energy solutions of Eq. (10) with \( E = \pm \sqrt{\vec{p}^2 + m^2} \).

(d) Because

$$
\exp \left[ i \gamma^5 \phi(x) \right] \left( \begin{array}{c} \phi_R (p^\mu) \\ \phi_L (p^\mu) \end{array} \right) = \exp \left[ i \phi(x) \right] \left( \begin{array}{c} \phi_R (p^\mu) \\ \exp \left[ -i 2 \phi(x) \right] \phi_L (p^\mu) \end{array} \right)
$$

the chiral transformation introduces a relative phase between the right- and left-handed spinors. However, it is important to note that Eq. (10) differs from the chirally transformed Dirac equation.

The \( \zeta(x) \) for the “particle” \( u \)-spinors is, \( \zeta_\mu(x) = + \exp \left[ + i \phi(x) \right] \), and for the antiparticle \( v \)-spinors it is, \( \bar{\zeta}_\mu(x) = - \exp \left[ - i \phi(x) \right] \). We thus have (in the usual notation), using Eqs. (4), (3), and (1):

$$
\begin{align*}
{u_{+1/2}}(p^\mu) &= A \begin{pmatrix} E_+ \\ p_+ e^{i \phi(x)} \\ -E_- e^{-i \phi(x)} \\ p_+ e^{i \phi(x)} \end{pmatrix}, \\
{u_{-1/2}}(p^\mu) &= A \begin{pmatrix} p_- \\ -E_+ e^{-i \phi(x)} \\ E_- e^{i \phi(x)} \\ p_- e^{i \phi(x)} \end{pmatrix}, \\
{v_{+1/2}}(p^\mu) &= A \begin{pmatrix} E_+ \\ p_+ e^{i \phi(x)} \\ -E_- e^{-i \phi(x)} \\ p_+ e^{i \phi(x)} \end{pmatrix}, \\
{v_{-1/2}}(p^\mu) &= A \begin{pmatrix} p_- \\ -E_+ e^{-i \phi(x)} \\ E_- e^{i \phi(x)} \\ p_- e^{i \phi(x)} \end{pmatrix},
\end{align*}
$$

with \( A = \left[ 2(m + E) \cos(\phi(x)) \right]^{-1/2} \), \( E_\pm = E \pm m \pm p_z \), and \( p_\pm = p_x \pm i p_y \). These spinors have the standard norm:

$$
\overline{\psi}(p^\mu) u_\sigma(p^\mu) = + 2 m \delta_{\sigma\sigma'} \text{ and } \overline{\psi}(p^\mu) v_\sigma(p^\mu) = - 2 m \delta_{\sigma\sigma'}.
$$

As usual, \( \overline{\psi}(p^\mu) = \psi^\dagger(p^\mu) \gamma^0 \).

It is to be explicitly noted that \( \psi(p^\mu) \) contains an implicit \( x \) dependence via Eq. (4). This fact requires extra care in obtaining configuration-space representation. However, in the remainder of this essay we are only concerned in obtaining consequences for an experimental region over which \( \phi(x) \) is essentially constant. Under these circumstances Eq. (10) yields the “CP violating Dirac equation” postulated by Funakubo et al. in the cosmological context.

Now to study the structure of the theory as regards locality, the field operator has to be constructed first:

$$
\Psi(x) = \int \frac{d^3k}{(2\pi)^3} \sum_{\sigma=+1/2,-1/2} \left[ b_\sigma(k^\mu) u_\sigma(k^\mu) \exp(-ik_\mu x^\mu) + d_\sigma(k^\mu) v_\sigma(k^\mu) \exp(ik_\mu x^\mu) \right].
$$

In writing \( \Psi(x) \) we assume, for the moment and as already indicated, that we are only interested in application to an experiment confined to a region of space-time over which \( \phi(x) \) is essentially constant.

\( \text{d} \) In order to account for the translational invariance of the arguments – see Ref. [51], the algebra of the Lorentz group has to be extended to incorporate the generators of the space-time translations.
For the particle interpretation of the theory, the following equal-time anticommutators are assumed to be satisfied,

\[
\{ b_\sigma(k^\mu), b_{\sigma'}^{\dagger}(k') \} = \{ d_\sigma(k^\mu), d_{\sigma'}^{\dagger}(k') \} = \frac{(2 \pi)^3 k_0}{m} \delta^3(\vec{k} - \vec{k}') \delta_{\sigma \sigma'},
\]

with the remaining four anticommutators being zero. With these definitions the non-locality of the theory becomes manifest on calculating the anticommutator \( \{ \Psi_\sigma(\vec{x}, t), \Psi_\sigma^\dagger(\vec{x'}, t) \} \). Exploiting the easily derivable identities,

\[
\sum_\sigma u_\sigma(p^\mu) \tau_\sigma(p^\mu) = \frac{1}{\cos(\phi(x))} (\gamma_\mu p^\mu + \xi_u^{-1}(x) m), \\
\sum_\sigma v_\sigma(p^\mu) \tau_\sigma(p^\mu) = \frac{1}{\cos(\phi(x))} (\gamma_\mu p^\mu + \xi_v^{-1}(x) m),
\]

and exploiting the standard textbook techniques (see, e.g., Ref. [37], Sec. 4.3), one is led to

\[
\{ \Psi_\sigma(\vec{x}, t), \Psi_\sigma^\dagger(\vec{x'}, t) \} = \frac{2m}{\cos(\phi(x))} (\delta_{ij} + O_{ij}) \delta^3(\vec{x} - \vec{x'}) .
\]

In Eq. (23), \( O_{ij} \) is purely off-diagonal,

\[
O_{ij} = i \frac{m}{k_0} \left( \begin{array}{cc} 0 & I_2 \\ -I_2 & 0 \end{array} \right) \sin(\phi(x)).
\]

The non-locality completely resides in \( O_{ij} \). It vanishes when (a) the phase \( \phi(x) \) is an exact integral multiple of \( \pi \), and/or (b) the particle is massless, or \( m/k_0 \) approaches zero. Further, the non-locality manifests in the spinorial space (i.e., the spinorial indices) and not in the configuration space (i.e., the \( \vec{x} \) space).\(^8\)

\[\text{2.3. Identifying the non-locality with gravity — a conjecture}\]

We are now faced with the problem of identifying the phase field \( \phi(x) \) with a physical field. Towards this end we shall put forward our own arguments, and find that supportive formal arguments already exist in literature.\(^{24}\)

As a final movement forward in our thesis, we take note again that the requirement of parity conservation collapses the phase \( \zeta(x) \) to \( \pm 1 \) to obtain a Dirac field throughout the space-time. It is tempting to suggest the phase \( \pm 1 \) to be somehow related to the two outstanding facts: (a) The metric of space-time known in the absence of gravitation is \( (+1, -1, -1, -1) \),\(^4\) (b) Only in one time, and three space dimensions, one finds equal numbers of generators of rotation and boost. This is due to this circumstance that the universal covering of the Lorentz group is essentially given by the chiral group \( SU_R(2) \otimes SU_L(2) \) (see Ref. [17] for further discussion). For that reason introducing relative intrinsic parities within the particle-antiparticle pair gets possible. In flat space-time the relative intrinsic parities of the particle-antiparticle pair and the signature of the space-time are deeply intertwined. We,

\[^8\]It is to be parenthetically noted that the particle interpretation contained in Eqs. (20) may breakdown for \( \sin(\phi(x)) \sim 1 \) in which case Eqs. (21) would need to contain a non-trivial \( \phi(x) \) dependence. This observation is dictated by the fact that for \( \sin(\phi(x)) \ll 1 \), Eq. (22) contains \( \phi(x) \) to the first order while Eqs. (21) contain no \( \phi(x) \) dependence, etc.

\[^4\]It is of no physical relevance to take the space-time metric as \( \text{diag.}(-1, +1, +1, +1) \). Parallel with this observation stands the fact only the relative intrinsic parties are of physical significance.
therefore, suspect that the space-time dependence of $\zeta(x)$ is closely related to the metric of space-time in the presence of the gravitating sources.

Further, in Ref. \[1\] we argued that in the quantum realm gravitation must introduce an “in-principle unavoidable” non-local element. Specifically, this non-local element should appear via modification of the commutativity, or anti-commutativity, of the fields. Similar conclusions have been arrived at in the context of string theories, leading to modification of the fundamental uncertainty relations as briefly discussed in Sec. 1.

Given these observations, we tentatively put forward the conjecture of identifying the discovered non-locality in the $(1/2,0) \oplus (0,1/2)$ representation space with gravity. Our conjecture consists of the proposal that the indicated non-locality should not be used to fix the phase $\phi(x)$ to the integral multiple of $\pi$, or $2\pi$ for the conventional Dirac limit, in order to recover locality. Instead, we propose that in the weak gravitational environments one has to approximate the relative phase by

$$\xi(x) \approx \pm [1 \pm i\phi(x)],$$

(25)

and identify $\phi(x)$ with $2GM/c^2r$ (up to a factor of the order of unity perhaps). Here, $r$ refers to coordinate distance of the region of experimental environment from a gravitational source of mass $M$. Recalling that Majorana particles carry imaginary intrinsic parity, while the Dirac particles possess real relative intrinsic parity, the appearance of $i$ in $\xi(x) \approx \pm [1 \pm i\phi(x)]$ is interpreted as a direct indication of the deviation of the particle’s intrinsic C and P properties from the purely Dirac type, and towards the Majorana type (in the limit $\phi(x) \to 0$, $\xi(x)$ is immediately seen to be the relative intrinsic parity).

The gravity-induced CP-violating effects vary from gravitational environment to gravitational environment. So, while these CP-violating effects are expected to be large in the vicinity of neutron stars and the early universe, they are tiny in the terrestrial environment. The exact magnitude of these CP-violating effects shall depend on the specific context, but it is expected to depend on the combination $(mc^2/E) \sin (2GM/c^2r)$. Whether or not this CP violation is energy-independent, shall depend upon whether or not a derivative coupling is considered. In this context it is to be noted that in experiments (E-82 and E-425) where the Kaon beam was not horizontal, but entered the ground at an angle to the horizontal, there remains an “anomalous energy-dependence of the Kaon parameters;” a dependence that can be further checked by new and carefully planned experiments as argued by Fischbach and Talmadge.

The $\beta$ decay processes $n \to p + e^- + \bar{\nu}_e$ and $\pi \to \eta + e^- + \bar{\nu}_e$ which appear conjugated under the CP transformation, can allow in the presence of gravity-induced CP violation, processes like: $\pi \to p + e^- + \bar{\nu}_e$ and $n \to \eta + e^- + \bar{\nu}_e$, where the baryon number is no longer conserved. Similarly, in the corresponding inverse $\beta$ decays, gravity-induced CP violation would lead to lepton-number violating nuclear reactions. It is to be noted that gravitationally induced neutrino oscillations already respect lepton flavor oscillations (see Ref. \[47\], and last Ref. of \[29\]). In addition, the indicated CP-violating nuclear reactions generate anti-matter in the matter-rich environment. Depending on the exact size of these gravity-induced CP violations, this last observation could provide an efficient mechanism for converting a part of the neutron star into gamma rays and neutrinos in a manner recently suggested by Pen, Loeb, and Turok.

Now, once baryon number non–conservation is addressed, one may consider a linear superposition of a fermion and an antifermion in analogy to the Kaon sys-

\[8\] cf. Secs. 12.5 and 23.6 of Ref. \[13\].

\[9\] Here we have taken liberty of making the speed of light explicit, and we do not worry about whether the source is spherical or not, and whether it rotates, etc. Our interest is essentially qualitative in that regard.
Referring to Eqs. (25) and (15) one immediately infers that the gravitational phase carried by the fermion is opposite to that of the antifermion without invoking antigravity. This suggests that the apparent success of the Chardin-Fischbach antigravity framework, in explaining the observed CP violation for the Kaon system, may lie (once one goes to the underlying quark level) in the italicised observation above.

If our framework is realized in nature, then the gravity-induced CP violation provides the dynamical reason for how a baryon and lepton number carrying neutron star collapses into a black hole and loses information on the baryonic and leptonic characteristics. The gravity-induced CP violating nuclear reactions may have important consequences for the collapse of a black hole into a space-time singularity. They may indeed prevent the formation of such a singularity. In addition, the cosmologically observed matter-antimatter asymmetry would also owe its origins to the gravity-induced CP violation.

2.4. Argument Regarding the Observability of Constant Gravitational Potentials

The above conjecture requires us to comment on what contributions are to be considered in \( \phi(x) \). The answer is all possible contributions. The local galactic cluster, known as the Great attractor, embeds us in a dimensionless gravitational potential \( \phi_{GA}/c^2 \sim -3 \times 10^{-5} \), see Ref. [21]. Whereas, the same quantity for Earth is \( \sim -7 \times 10^{-10} \), and it is \( \sim -2 \times 10^{-6} \) for the Sun. For experiments performed in the vicinity of Earth, or the Solar system, one finds

\[
\vec{\nabla} \phi_{GA} \ll \vec{\nabla} \phi_{\{\text{Sun, Earth}\}},
\]

\[
\phi_{GA} \gg \phi_{\{\text{Sun, Earth}\}},
\]

with \( \phi_{\{\text{Sun, Earth}\}} \) standing for the gravitational potentials of the Sun, or Earth, at the experimental site. As a result, for most classical experiments (such as orbits of Moon, or planets) the essentially constant gravitational potential \( \phi_{GA}/c^2 \) has no physical consequence. Other examples of constant gravitational potentials that arise in general relativity are known under the name of homoids.

However, in 1990 Kenyon [21] emphasised the observability of constant gravitational potentials in the context of gravitationally induced CP violation. This suggestion was strongly questioned by Nieto and Goldman in their classic 1991 Physics Report. In particular, Nieto and Goldman, following the canonical wisdom, objected that no independent experimental means are available to measure absolute gravitational potentials. These authors, however, apparently failed to realize that weak field limits of classical gravity and any theory of quantum gravity have different behaviour with respect to the gravitational potential. While the classical weak-field limit contains gradient of the gravitational potential, the quantum weak-field limit contains the gravitational potential itself. On these grounds it was shown in Ref. [30] that the general relativistic description of gravitation turns out to be incomplete. This incompleteness, it was further argued in the previous work, allows for independent experiments to measure the constant gravitational potentials.

Further, it is one of the fundamental assumptions of Einstein’s theory of gravitation that the freely falling frames are independent of a frame’s location. This independence has been termed local position invariance, LPI, by Mann [24]. The incompleteness argument of Ref. [30] shows that LPI is violated by the existence of homoid potentials. As an example, this happens because a freely falling frame outside the homoid cavity (inside of which the gravitational potential is constant, and varies outside with radial distance from center of the cavity) is not identical to a freely falling frame inside the cavity. The violation of LPI is also manifest in the weak-field limit that all quantum theories of gravitation must satisfy, and it leads
to physical observability of the homoid, or homoid-like, gravitational potentials. Thus the Nieto-Goldman objection to the Kenyon argument is overcome by the LPI violation contained in the incompleteness of Einstein’s theory of gravitation.

3. Conclusion
It is explicitly confirmed, following a remark by Wigner, that a representation space carries more information than a wave equation. Both the celebrated Dirac equation and the CP-violating Eq. \( \text{Eq. (10)} \) belong to the \((1/2, 0) \oplus (0, 1/2)\) representation space, and yet both carry different C, P, and T properties. Similarly, the extended Majorana spinors and the Dirac spinors belong to the \((1/2, 0) \oplus (0, 1/2)\) representation space and yet each set has different physical properties and describe different physics. The core of the physics is contained in the choice of the C, P, and T properties of the spanning basis spinors. This situation is paralleled in the \((u, d, c, s)\)-flavor space of the quarks as is immediately seen from a recent work of Kirchbach.\(^{51}\) There the difference in the choice for the flavor symmetry generators (Gell-Mann’s versus Weyl’s) is observable and is revealed through the \(\eta-N\) coupling constant. The underlying reason for this is that while the dimensionality of an irreducible representation space does not depend upon the concrete realization of the symmetry generators, the Noether currents (Dirac, versus Majorana, versus the CP violating construct, in space-time; and Gell-Mann versus Weyl in the flavor space) do. Taking the \((1/2, 0) \oplus (0, 1/2)\) representation space as a study case, we see that in going from pure right- and left-handed \((1/2, 0)\) and \((0, 1/2)\) spinors to their direct sum \((1/2, 0) \oplus (0, 1/2)\), one transports information about the C, P, and T properties of the representation space as well as about the properties of the theory with respect to locality. Having derived the CP-violating wave equation for spin one-half, one could have demanded locality, and could have recovered Dirac equation. However, we have taken a different path to proceed, and conjecture the non-locality in quantum field theory to originate in a specific manner from gravity. From that one predicts that there exists a CP violation that varies from gravitational environment to gravitational environment, remaining small in the terrestrial environment, and becoming significantly large in the vicinity of neutron stars and the early universe.

In summary, we have presented a thesis on an origin of CP violation that lies at the level of the representations of the Lorentz group, and is related to the space-time metric in the presence of a gravitational source. A CP violation that depends on the gravitational environment via the factor \(GM/c^2r\) shall have dramatic astrophysical and cosmological implications. Especially, because this factor can vary from about 0.2 for the surface of a 1.4 solar mass neutron star down to roughly \(10^{-9}\) for Earth’s surface. The last number can, however, increase to roughly \(10^{-5}\) if the contribution from the great attractor is taken into account (as it must be). Gravity induced CP violation shall alter the equation of state for nuclear matter in intense gravitational fields and hence the fate of neutron stars, supernovae, and may be an important physics factor in the cosmic gamma ray bursts.

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