On the red-giant luminosity bump

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ABSTRACT

The increase in luminosity as a star evolves on the red-giant branch is interrupted briefly when the hydrogen-burning shell reaches the vicinity of the composition discontinuity left behind from the first convective dredge-up. The non-monotonic variation of luminosity causes an accumulation of stars, known as the ‘bump’, in the distribution of stars in the colour-magnitude diagrams of stellar clusters, which has substantial diagnostic potential. Here I present numerical results on this behaviour and discuss the physical reason for the luminosity variation, with the goal of strengthening the understanding of origin of the phenomenon and hence of its diagnostic potential.

Key words: Stars: evolution – stars: interiors

1 INTRODUCTION

During the evolution up the red-giant branch models of low-mass stars show a brief phase of decreasing luminosity (cf. Fig. 1). This was early (Thomas 1967; Iben 1968) associated with the approach of the hydrogen-burning shell to the discontinuity in the hydrogen abundance left behind by the first dredge-up, where the convective envelope reaches into parts of the star earlier affected by hydrogen fusion during main-sequence evolution. As a result of this luminosity variation the star obviously spends a little longer in the relevant region of the Hertzsprung-Russell diagram. In stellar clusters this corresponds to an accumulation of stars in that region, leading to a ‘bump’ in the luminosity distribution (Iben 1968; King et al. 1985); consequently the phenomenon is known as the ‘red-giant bump’.\textsuperscript{1} The location in luminosity of the bump and the excess number of stars are clearly closely related to a specific phase in stellar evolution, making it an interesting diagnostics for the properties of the clusters and/or the stellar modelling (e.g., Fusi Pecci et al. 1994; Cassisi & Salaris 1997; Bono et al. 2001; Riello et al. 2003; Nataf 2014). A particularly interesting aspect is the effect of overshoot from the convective envelope, which obviously increases the depth of the composition discontinuity and hence decreases the luminosity at which the bump is found (e.g., Alongi et al. 1991; Angelou et al. 2013). For a detailed overview of the diagnostic potential, and further references, see Salaris et al. (2003).

The study of red giants has recently received an enormous boost through the asteroseismic analyses made possible by the photometric observations from the CoRoT (Baglin et al. 2009) and Kepler (Borucki et al. 2010) space missions, motivating further investigations of the properties of the bump. An example is the signature brought about in the observed frequencies by buoyancy glitches, which appears to be related to the evolutionary location of the star relative to the bump (Cunha et al. 2015).

The detailed reason for the variation in luminosity has been subject of some discussion. It has been associated with the direct effect of the increase in the hydrogen abundance in the hydrogen-burning shell as it crosses the composition discontinuity, possibly involving departure from thermal equilibrium (e.g., King et al. 1985; Salaris et al. 2002; Riello et al. 2003; Gai & Tang 2015). However, in a careful

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\textsuperscript{1} Not to be confused with the ‘red-giant clump’ consisting of stars in the core helium burning phase.
presentation of the evolution in the vicinity of the bump
Sweigart et al. (1990) noted that since the decrease in lu-
minosity happens before the hydrogen-burning shell reaches
the composition discontinuity the effect cannot be due to the
increase in the fuel available to the nuclear reactions; instead
they pointed to the increase in the opacity just outside the
shell due to the increased hydrogen abundance in the enve-
lope. A perhaps more likely interpretation was obtained by
Refsdal & Weigert (1970) (see also Kippenhahn & Weigert
1990) who carried out a homology analysis of the properties
of the region just outside the burning shell. They noted a
strong increase in the luminosity with the mean molecular
weight $\mu$ and pointed out that the increase in the hydrogen
abundance outside the discontinuity caused a decrease in $\mu$
and hence in luminosity.

Here I illustrate the properties of the evolution in the
vicinity of the bump in a little more detail, based on a series
of evolution sequences of varying mass. In addition, I carry
out a simplified analysis of the structure of the model just
above the hydrogen-burning shell which essentially confirms
the importance of the variation in mean molecular weight
as the driving force behind the bump.

2 PROPERTIES OF THE BUMP
The models considered here follow the computations of
Jiang & Christensen-Dalsgaard (2014). Briefly, they use the
ASTEC code (Christensen-Dalsgaard 2008), neglecting dif-
fusion and settling and with no overshoot from the con-
vective envelope and possible convective core. The models
have an initial hydrogen abundance $X_0 = 0.72$ and a heavy-
element abundance $Z = 0.02$; convection is treated with
the Böhm-Vitense (1958) mixing-length formulation with a
mixing-length parameter of 1.8.

In the early phases of red-giant evolution the mass in
the convective envelope increases to a maximum and subse-
quently decreases. The convective envelope is assumed to
be fully mixed, leading to the first dredge-up of nuclear-
processed material, down to the maximum extent of the
convective envelope where the mass fraction at its base is
$q_{\text{dis}} = m_{\text{dis}}/M$, $m_{\text{dis}}$ being the mass inside the discon-
tinuity and $M$ the total mass of the star. Owing to the neglect
of diffusion and settling the composition at the base of the
mixed region is discontinuous. Figure 2 shows the hydrogen
abundance as a function of fractional mass in the $1 M_{\odot}$ se-
quence, at the times of the maximum extent in mass of the
convective envelope, the maximum luminosity in the bump,
and the time where the location, at a fractional mass $q_\epsilon$, of
the hydrogen-burning shell reaches the composition discon-
tinuity; here $q_\epsilon$ is defined by the maximum in the energy-
generation rate. The latter time corresponds closely to the
minimum in luminosity in the bump.

The evolution with time in the luminosity in the vicinity
of the bump is illustrated in Fig. 3. The lower panel shows
the separation in mass fraction between the location of the composit-
on discontinuity and the hydrogen-burning shell. In both panels the
dashed and dot-dashed lines, as in Fig. 2 mark the luminosity
maximum and minimum respectively.

The properties of the bump depend substantially on
stellar mass (see also Gai & Tang 2015). This is illustrated
in Fig. 4 showing the ratio between the maximum and min-
maximum luminosity in the bump. As shown by the lower panel, this is very strongly correlated with the step $\Delta X$ in hydrogen abundance at the discontinuity. Note that the red-giant bump only appears for stars of mass below around $2.2 \, M_\odot$. For more massive stars helium ignition occurs before the hydrogen-burning shell reaches the composition discontinuity. In such stars the bump can be identified during the helium-burning phase (e.g., Cunha et al. 2015).

The variation of $\Delta X$ with mass arises from a somewhat complex interaction between the extent of the convectively mixed regions and main-sequence nuclear burning. Figure 5 shows the variation of the mass fraction $q_{\mathrm{dis}}$ at the discontinuity and the maximum extent $q_{c, \mathrm{max}}$ of the convective core, as functions of stellar mass. With increasing stellar mass the maximum penetration in mass of the convective envelope increases, and hence $q_{\mathrm{dis}}$ decreases, monotonically. At the lowest masses this causes a slight increase in $\Delta X$ with mass (cf. Fig. 3), but above around $1 \, M_\odot$ the decrease in $q_{\mathrm{dis}}$ is more than compensated by the increasing central concentration of hydrogen burning as the CNO cycle becomes more important. This is illustrated in Fig. 6 which shows the hydrogen-abundance profile at the end of central hydrogen burning, for various stellar masses. For masses higher than around $1.2 \, M_\odot$ the models have a convective core whose maximal extent $q_{c, \mathrm{max}}$ increases rapidly with mass. When $q_{c, \mathrm{max}}$ crosses $q_{\mathrm{dis}}$, around $1.9 \, M_\odot$, the steep hydrogen profile left behind by the retreating convective core leads to a rapid increase in $\Delta X$ and hence in the amplitude of the bump.

I note in passing that, even though the phase associated with the bump is relatively brief, it does not lead to any substantial departure from thermal equilibrium; such departure was implied, e.g., by Riello et al. (2003). In the $1 \, M_\odot$ sequence, for example, the contribution to the total luminosity from gravitational effects peaks at a value slightly less than 1 per cent around the time of minimum luminosity during the bump. In general, the dominant gravitational effect appears to be associated with the region of shell burning, even far away from the bump.
3 ANALYSIS OF THE BUMP

According to the shell homology analysis by Refsdal & Weigert [1970] the temperature \( T \) at homologous points scales as

\[
T \propto \frac{\mu m_{\text{shell}}}{r_{\text{shell}}},
\]

where \( \mu \) is the mean molecular weight, and \( m_{\text{shell}} \) and \( r_{\text{shell}} \) are the mass and radius at the hydrogen-burning shell source. The increasing mass of the helium core causes an increase in temperature and hence in luminosity, owing to the strong temperature sensitivity of the energy generation rate and efficiency of the energy transport. This assumes a constant mean molecular weight in the region considered, although Refsdal & Weigert [1970] noted the potential effect of the decrease in the average \( \mu \), leading to a decrease in luminosity, as the shell source approaches the composition discontinuity from the dredge-up. Here I analyse in more detail the effect of the discontinuity on the structure of this region.

As did Refsdal & Weigert [1970] I assume the ideal gas law, so that pressure \( p \), density \( \rho \) and \( T \) are related by

\[
p = \frac{k_b \mu T}{\mu m_a}, \tag{2}
\]

where \( k_b \) is Boltzmann’s constant and \( m_a \) is the atomic mass unit. From the equations of hydrostatic equilibrium and radiative energy transport we obtain

\[
\frac{dT^4}{dp} = \frac{3\pi L}{4\pi \alpha G m}, \tag{3}
\]

where \( \kappa \) is the opacity, \( \alpha \) the radiation density constant, \( c \) the speed of light, \( G \) the gravitational constant and \( m \) the mass inside the given point. I consider the region between the shell source and the base of the convective envelope, where the temperature gradient changes to being nearly adiabatic. For simplicity I neglect \( T_{\text{bcz}} \) and \( p_{\text{bcz}} \) compared with the values around the discontinuity, to obtain

\[
T = B p^\gamma, \tag{4}
\]

where \( \gamma = (a + 1)/(4 - b) \) and

\[
B = \left( \frac{A}{4\gamma} \right)^{1/(4-b)}. \tag{5}
\]

The solution of this equation should be matched to the temperature \( T_{\text{bcz}} \) and the pressure \( p_{\text{bcz}} \) at the base of the convective envelope, where the temperature gradient changes to being nearly adiabatic. For simplicity I neglect \( T_{\text{bcz}} \) and \( p_{\text{bcz}} \) compared with the values around the discontinuity, to obtain

\[
T = B p^\gamma, \tag{5}
\]

where \( \gamma = (a + 1)/(4 - b) \) and

\[
B = \left( \frac{A}{4\gamma} \right)^{1/(4-b)}. \tag{5}
\]

I take \( \mu = \mu_a \) above the discontinuity and \( \mu = \mu_b \) below the discontinuity; in the latter case I neglect the small variation, due to the gradient in the hydrogen abundance, in the relatively thin region considered, so that both \( \mu_a \) and \( \mu_b \) are taken to be constant. Furthermore I assume that there is a point \( r_0 \) with \( r_{\text{shell}} \ll r_0 \ll r_{\text{bcz}} \), where \( r_{\text{bcz}} \) is the distance of the base of the convective envelope from the centre, such that equation (2) is valid for \( z > 1/r_0 \). Integrating equation (2) downwards from \( z = 1/r_0 \) yields, for \( r < r_{\text{dis}} \),

\[
T = \left( \frac{\gamma G m_{\text{shell}}}{k_B} \right) \left( \int_1^{1/r_{\text{dis}}} \frac{\mu_a \, dz}{r_{\text{dis}}} + \int_1^{1/r_0} \frac{\mu_b \, dz}{r_{\text{dis}}} \right), \tag{6}
\]

where I neglected \( T(r_0) \) and \( 1/r_0 \) in the second line; here \( r_{\text{dis}} \) is the distance of the discontinuity from the centre. Thus, with \( r \approx r_{\text{shell}} \) we recover the homology scaling in equation (4) but with the correction factor

\[
\phi = 1 - \left( r_{\text{shell}}/r_{\text{dis}} \right) [1 - (\mu_a/\mu_b)], \tag{7}
\]

which becomes significant as \( r_{\text{shell}} \) approaches \( r_{\text{dis}} \), reducing \( T \) and hence the luminosity.

To illustrate the properties of the correction induced by the discontinuity, the upper panel in Fig. 4 shows \( r_{\text{dis}} \) and \( r_{\text{shell}} \), while the lower panel shows the corresponding correction factor, in the 1 M\(_\odot\) model. It is obvious that the general shape of the correction factor matches the reduction of \( L \) shown in Fig. 4, keeping in mind that the energy generation and transport depend on temperature to a high power. A quantitative comparison with the model properties shows substantial departures from the simple expressions; this is true both of the homology scaling in equation (4) well below the bump and the solution in equation (6) in the vicinity of the bump. This is likely a result of the rather drastic approximations made in the derivation in the latter equation, particularly the neglect of quantities at the base of the convective envelope. Even so I find it plausible, as implied

\[2\] On the other hand, the scaling in equation (4) becomes increasingly well satisfied for more luminous models on the red-giant branch.
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Figure 7. The top panel shows the distance \( r_{\text{dis}} \) of the discontinuity from the centre (solid) and the radius \( r_{\text{shell}} \) of the hydrogen-burning shell (dashed), as a function of age, in a 1 M_\odot evolution sequence. The bottom panel shows the corresponding correction factor \( \phi \) (cf. equation 11). After the minimum in luminosity \( r_{\text{dis}} \) is replaced by \( r_{\text{shell}} \), and \( \phi \) is set to \( \mu_a/\mu_b \).

The results presented in the present note relate the luminosity excursion in the red-giant bump directly to the jump in mean molecular weight at the dredge-up composition discontinuity, as a consequence of its effect on the hydrostatic structure of the region immediately above the hydrogen-burning shell. The amplitude of the luminosity excursion, and hence the over-density of stars in this region of the Hertzsprung-Russell diagram, is closely correlated with the magnitude \( \Delta X \) of the jump in the hydrogen abundance, which in turn depends on the main-sequence evolution of the star; this includes, in a rather narrow range of stellar masses, the maximum extent of the convective core. From this insight, possibly enhanced through more careful analysis, one may hope to get a better feel for the diagnostic potential of the observed bumps.

Here I have considered the simple case of models neglecting diffusion and settling and with no overshoot from the convective envelope. Further investigations of how such effects might affect the bump would clearly be interesting. This includes studies of the effect on the observed oscillation frequencies through the glitch in the buoyancy frequency (e.g., Cunha et al. 2012). Convective overshoot and turbulent diffusive mixing near the base of the convective envelope will certainly influence the location and shape of the composition structure resulting from the dredge-up, which may well have effects that can be inferred from asteroseismic analyses and which may also be detectable in the luminosity-distribution bump for stellar clusters.

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