Schmidt-like coherent mode decomposition and spatial intensity correlations of thermal light

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Abstract. We experimentally study the properties of coherent mode decomposition for the intensity correlation function of quasi-thermal light. We use the technique of spatial mode selection developed for studying the transverse entanglement of photon pairs, and show that it can be extended to characterize classical spatial correlations. Our results demonstrate the existence of a unique, for a given thermal source, basis of coherent modes, correlated in a way much resembling the Schmidt modes of spatially entangled photons.

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1. Introduction

Entanglement in spatial degrees of freedom of quantum light, such as pairs of photons generated in spontaneous parametric down-conversion (SPDC), is currently an object of active research.

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These studies are motivated by applications in quantum information science, where the infinite-dimensional Hilbert space of the spatial states of photons offers attractive capabilities for high-dimensional quantum state engineering. The effective dimensionality of accessible state space is closely related to the degree of spatial entanglement for bipartite states, making its experimental quantification an important task. One of the most successful approaches makes use of a Schmidt decomposition. As stated by Law and Eberly [1] in their seminal work, a unique basis of coherent modes \{\ket{u_k}, \ket{v_k}\} exists, such that a spatial state of biphoton pairs \(\ket{\Psi_{12}} = \int dx_1 dx_2 \Psi(x_1, x_2) a^\dagger(x_1) a^\dagger(x_2) \ket{\text{vac}}\) can be decomposed as \(\ket{\Psi} = \sum_k \sqrt{\lambda_k} \ket{u_k} \ket{v_k}\), with these so-called Schmidt modes being eigenvectors of reduced single-particle density matrices and \(\lambda_k\) the corresponding eigenvalues. A remarkable feature of this decomposition is its single-sum form, implying perfect one-to-one correlations between Schmidt modes. These correlations have been studied in several recent works [2–4], and the number of significant terms in the decomposition is routinely used as an entanglement quantifier [1, 5–12].

A natural question is can a similar approach be used to study spatial correlations of other origin, for example correlations in spatially multi-mode classical light? The form of Schmidt decomposition, and the way in which explicit expressions for Schmidt modes are derived in the case of SPDC biphotons [1, 13], resembles the well-known notion of coherent mode decomposition for spatial correlation functions in classical statistical optics [14–16]. Here we show that this similarity does not reside only in the form of mathematical equations, but in underlying physics as well. Namely, for a given source of light with quasi-thermal statistics there exists a unique physically distinguished set of coherent modes, correlated in much the same way as Schmidt modes of SPDC. Similar experimental settings would provide similar correlations, with only a quantitative difference in visibility. The situation here resembles that with ghost imaging, which was for some time believed to be possible only due to entanglement [17] and thus only with quantum light. It is, however, well understood now that spatial correlations required to obtain ghost images should not necessarily be of quantum origin, and a purely classical thermal light may be used as well [18, 19]. In this work we try to push this analogy further and show that a framework of Schmidt decomposition may also be generalized to classical states of light.

This paper is organized as follows. In section 2 we review some properties of coherent mode decomposition for a Gaussian Schell-model of quasi-thermal light and derive the expressions for measured correlation functions, experimental realization is described in section 3, and we end with a discussion of the results and the conclusion in section 4.

2. Coherent mode decomposition for quasi-thermal light

The coherence properties of classical light are described by the coherence function \(G^{(1)}(\mathbf{r}_1, \mathbf{r}_2, \omega) = \langle E^*(\mathbf{r}_1, \omega) E(\mathbf{r}_2, \omega) \rangle\), where we neglect the polarization and use scalar complex amplitudes \(E(\mathbf{r}, \omega)\) to describe the electromagnetic field. In the following, we will consider the monochromatic case and omit the frequency dependence for simplicity. One of the results of classical coherence theory states that it can be decomposed in series of coherent mode functions as

\[
G^{(1)}(\mathbf{r}_1, \mathbf{r}_2) = \sum_n \lambda_n \phi_n^*(\mathbf{r}_1) \phi_n(\mathbf{r}_2),
\]
where $\phi_n(r)$ are eigenfunctions of an integral operator with a kernel $G^{(1)}(r_1, r_2)$ and $\lambda_n$ are corresponding eigenvalues [16].

Higher-order correlation functions, such as the intensity correlation function $G^{(2)}(r_1, r_2) = \langle I(r_1)I(r_2) \rangle$, may be accessed via Hanbury Brown–Twiss (HBT) interferometry, which is exactly the experimental setting commonly used for studying the spatial entanglement of photon pairs. In the biphoton case, the fourth-order correlation function $G^{(2)}(r_1, r_2)$ is measured by counting coincidences between photocounts of two single-photon detectors positioned at points $r_1$ and $r_2$, respectively: $R_c \propto \langle a^\dagger(r_1)a^\dagger(r_2)a(r_1)a(r_2) \rangle = |\Psi(r_1, r_2)|^2$ with $a(r)$ being the photon annihilation operator and $\Psi(r_1, r_2)$ the biphoton amplitude. A direct analogue of coherent mode decomposition (1) for the biphoton amplitude is exactly the Schmidt decomposition. To see whether this analogy goes beyond simply similar mathematical expressions, let us consider the behavior of a classical light field in the HBT scheme with spatial mode filters in the arms of the interferometer. Such filters are usually composed of phase holograms followed by single-mode fibers (SMFs) and bucket detectors as shown in figure 1. The main feature of Schmidt decomposition—its single-sum form—manifests itself in the absence of coincidences between photocounts in such a setup when appropriate mode filters are used to select orthogonal Schmidt modes in different arms of the setup [2]. We will show below that similar effects may be observed with classical quasi-thermal light.

For quasi-thermal light, the intensity correlation function $G^{(2)}(r_1, r_2)$ can be expressed in terms of the second-order correlation function $G^{(1)}(r_1, r_2)$ using the Siegert relation:

$$G^{(2)}(r_1, r_2) = \langle I(r_1)\rangle\langle I(r_2) \rangle + |G^{(1)}(r_1, r_2)|^2.$$  \hspace{1cm} (2)

If the spatial mode filters in the two arms of the HBT scheme are described by impulse response functions $h_1(r_1, r_1')$ and $h_2(r_2, r_2')$, where coordinate $r$ corresponds to the source plane and coordinate $r'$ to the detection plane, the transformed intensity correlation function takes the form [18, 20]

$$G^{(2)}(r_1', r_2') = \langle \tilde{I}(r_1') \rangle\langle \tilde{I}(r_2') \rangle + \int dr_1 \int dr_2 h_1^*(r_1, r_1')h_2(r_2, r_2') G^{(1)}(r_1, r_2)^2,$$  \hspace{1cm} (3)

where $\tilde{I}(r_{1,2}) = |\int dr_{1,2} h_{1,2}(r_{1,2}, r_{1,2}')E(r_{1,2})|^2$ is the intensity distribution in the detection plane. One can always choose the mode filters to satisfy the relation:

$$\int dr_1 h_1^{(m)}(r_1, r_1')\phi_n(r_1) \propto e^{-\frac{w_s^2}{4\tau^2}} \delta_{nm},$$  \hspace{1cm} (4)

$$\int dr_2 h_2^{(k)}(r_2, r_2')\phi_n(r_2) \propto e^{-\frac{w_s^2}{4\tau^2}} \delta_{nk},$$

where $w_s$ is the waist of the fundamental Gaussian mode of the fiber. This choice of propagators corresponds to ‘projection’ on modes $\phi_m(r_1)$ and $\phi_k(r_2)$; we will call the transformation corresponding to such propagators an eigenmode filter. Substituting (1) and (4) into (3) and taking into account that bucket detectors placed after the SMF integrate the signal over the whole detection plane, we obtain the following expression for coincidence to single counts ratio:

$$\frac{R_c^{(m,k)}}{R_s^{(m)}} \propto \delta^{(2)}_{m,k} = 1 + \delta_{mk},$$  \hspace{1cm} (5)
where

\[ G^{(2)}_{(m,k)} = \frac{\int d\mathbf{r}_1'd\mathbf{r}_2'G^{(2)}(\mathbf{r}_1', \mathbf{r}_2')}{\int d\mathbf{r}_1'(I(\mathbf{r}_1'))\int d\mathbf{r}_2'(I(\mathbf{r}_2'))} \]  

(6)

is the normalized second-order correlation function. It is clear from (5) that spatial correlations between appropriately chosen coherent modes of thermal light show the same correlation features as Schmidt modes of entangled photons—namely, they are only pairwise correlated. The difference turns out to be rather quantitative than qualitative, similar to the situation with thermal and quantum ghost imaging—the achievable visibility defined as

\[ V = \frac{g^{(2)}_{(m,m)} - g^{(2)}_{(m,n)}}{g^{(2)}_{(m,m)} + g^{(2)}_{(m,n)}}. \]

∀m ≠ n cannot exceed 1/3 for thermal light, while it can reach values arbitrarily close to unity for biphotons.

A simple expression (5) is only valid for eigenmode filters; any other mode filters \(h_{1,2}^{(m,k)}(\mathbf{r}_{1,2}, \mathbf{r}'_{1,2}) = f_{m,k}(\mathbf{r}_{1,2})f_{m,k}(\mathbf{r}'_{1,2})\) will give rise to a non-unity value of the normalized intensity correlation function for \(m \neq k\). Indeed, the expression for \(g^{(2)}_{(m,k)}\) in the case of arbitrary filters has the following form:

\[ g^{(2)}_{(m,k)} = 1 + \sum_{i,j} \lambda_i \lambda_j (\alpha^{(m)}_i \alpha^{(k)}_j^*) - \alpha^{(m)}_i \alpha^{(k)}_j^* \sum_{i,j} \lambda_i |\alpha^{(m)}_i|^2 \sum_{i,j} \lambda_j |\alpha^{(k)}_j|^2, \]

(7)

where \(\alpha^{(m)}_i = \int d\mathbf{r} f_m(\mathbf{r}) \phi_i(\mathbf{r})\) are coefficients in expansion of mode functions \(f_m(\mathbf{r})\) in the basis of coherent eigenmodes. This expression reduces to (5) only if \(\alpha^{(m)}_i = \delta_{mi}\), which is the case of eigenmode filters. This fact is a unique and, as we will show below, experimentally verifiable property of coherent eigenmodes for thermal light.

Let us now derive the explicit expressions for coherent modes constituting decomposition (1) for a simple model of partially spatially coherent thermal light. Following [14] we describe light from the quasi-thermal source with a Gaussian Schell-model for the second-order coherence function:

\[ G^{(1)}(x_1, x_2) = \sqrt{I(x_1)I(x_2)} \mu(x_1 - x_2), \]

(8)

where spectral intensity distribution and degree of spatial coherence are taken in the form

\[ I(x_1, x_2) = A \exp[-x_1^2/2\sigma^2], \quad \mu(x_1 - x_2) = \exp[-(x_1 - x_2)^2/2\sigma^2] \]

(9)

with \(x_1\) and \(x_2\) being the transverse coordinates, \(A\) a positive normalization constant, \(\sigma\) beam waist and \(\sigma_\mu\) its coherence radius. For the sake of simplicity, we will first consider a one-dimensional source; generalization to an experimentally relevant two-dimensional case is straightforward.

In the case of partially spatial coherent light described by a Gaussian Schell-model, one can find the eigenfunctions in a closed analytical form [14, 16]:

\[ \phi_n(x) = \left(\frac{2c}{\pi}\right)^{1/4} \frac{1}{\sqrt{2^n n!}} H_n(x \sqrt{2c}) \exp(-cx^2), \]

(10)

\[ \lambda_n = A \left(\frac{\pi}{a+b+c}\right)^{1/2} \left(\frac{b}{a+b+c}\right)^n, \]

(11)

where

\[ a = \frac{1}{4\sigma^2}, \quad b = \frac{1}{4\sigma^2}, \quad c = \sqrt{a^2 + 2ab} \]

(12)
and $H_n(x)$ are Hermite polynomials. Eigenvalues in this coherent mode decomposition are determined by the ratio of coherence radius to beam waist $\beta = \frac{\sigma_\mu}{\sigma_I}$:

$$\frac{\lambda_n}{\lambda_0} = \left(\frac{1}{\beta^2/2 + 1 + \beta[(\beta/2)^2 + 1]^{1/2}}\right)^n.$$  \hspace{1cm} (13)

In this simple but experimentally relevant model, the eigenmodes turn out to be familiar Hermite–Gaussian (HG) modes, which were also found to be Schmidt modes of SPDC biphotons under a double-Gaussian approximation for the biphoton amplitude [2, 3, 13]. We note that similar decomposition may be carried out in polar coordinates giving rise to coherent modes with orbital angular momentum (OAM) [21–23]. The spectrum of OAM eigenvalues in this decomposition was measured experimentally in [24] with the interferometric technique.

3. Experimental realization

The experimental setup is shown in figure 1. We used a well-known technique to create a pseudothermal light source with controlled spatial coherence [25]. In this scheme, a beam of a He–Ne laser with wavelength $\lambda = 632.8$ nm is scattered on a slowly rotating grounded-glass disc. It is important to make the characteristic time of variation of the scattering pattern larger than the detection time $\tau_{\text{det}} \sim 2$ ns, defined by the coincidence circuit time window. The disc was located at the focus of an effective lens consisting of two microscope objectives O1 and O2, forming a collimated beam. The spatial coherence properties of the beam are controlled by iris apertures S and S2, determining the coherence radius $\sigma_\mu$ and the beam waist $\sigma_I$, respectively. The detection part of the setup is an HBT scheme, consisting of a 50/50 non-polarizing beam splitter; spatial mode filters—a reflective liquid-crystal-on-silicon (LCoS) spatial light modulator (SLM) in the transmitted channel and a glass step mask in the reflected one, followed by SMF1,2 placed in the focal planes of 8× microscope objectives C1,2; electrical signals from two single-photon detectors D1,2 (Perkin–Elmer) were fed to a coincidence circuit. The model of SLM used was limited to $0.8\pi$ phase shifts only, forcing us to use double reflection as shown in figure 1. Polarizers P2,3 were used to specify the correct polarization required for operation of the polarization-sensitive SLM matrix and to remove the undiffracted components, while polarizer P1 was used to control the overall intensity.

The propagator for each arm of the HBT scheme described above is

$$h(r, r') \propto \exp\left[-\frac{(r')^2}{w_f^2}\right] \times \int \mathrm{d}r'' \exp\left(-i\frac{rr''k}{f}\right) \times \exp[i \arg(H_n(\sqrt{2c}x)H_n(\sqrt{2c}y))] \times \exp\left[-\frac{(r'' - r_i)^2}{w_f^2}\right].$$  \hspace{1cm} (14)

where $k = 2\pi/\lambda$ is the wave number, $f$ the focal length of the microscope objectives $C_{1,2}$, $w_f$ the waist of the fundamental gaussian mode of the fiber and $r_i$ the fiber tip displacement in the focal plane. This propagator defines the detection modes, which for our purposes should be matched with the eigenmodes given by (10). The mode matching is accomplished by choosing the focal length $f = \sqrt{\frac{k w_f^2}{4c}}$, in this case the detection mode waist coincides with the waists of the eigenmodes:

$$\frac{k^2 w_f^2}{4f^2} = c = \frac{1}{4\sigma_f^2} \sqrt{1 + \frac{2}{\beta^2}}.$$
The mode filters were tested with a coherent single-mode laser beam. The sizes and positions of the phase masks relative to the beam were determined experimentally following the operational criterion of minimizing the counting rate for higher-order modes in the central position of the SMF fiber.

Satisfying the condition \( k^2 w^2 = c \) is essential for obtaining the desired matching of the detected modes and the eigenmodes of the coherence function. In the actual experiment the detection mode waist was fixed, while the mode matching was done by changing the parameters of the input beam. The widths of the apertures S and \( S_2 \) determine the coherence radius \( \sigma_\mu \) and the beam waist \( \sigma_I \), respectively, and thus define the eigenmode width \( c \) through (12). One can see from (7) that the value of the normalized intensity coherence function \( g_{(m,0)}(r_f = 0) = 1 \) for \( m \neq 0 \) only if the mode-matching condition is satisfied; otherwise it is always larger than 1. This fact was used as an operational criterion of good mode matching. The corresponding beam waist and coherence radius were measured as \( \sigma_I = (2.3 \pm 0.1) \text{ mm} \) and \( \sigma_\mu = (0.57 \pm 0.02) \text{ mm} \), corresponding to the value of \( \beta = 0.24 \pm 0.02 \).

With the mode-matching conditions satisfied, the spatial mode filters in both arms of the setup act as projectors on the eigenmodes of coherent mode decomposition (1). In this case, the single count rate of a detector in one of the arms is proportional to the partial intensity of a constituent mode selected by the filter:

\[
I_{(m)} \propto \int d\mathbf{r}' \left| \int d\mathbf{r} h_{(m)}^{(1)}(\mathbf{r}, \mathbf{r}', E(\mathbf{r})) \right|^2 \propto \lambda_m. \tag{15}
\]

It provides a way of measuring the eigenvalues in (1) experimentally. The distribution of eigenvalues, normalized to \( \lambda_0 \), is shown in figure 2. We find excellent agreement with theoretical predictions obtained from (13) using the independently measured value \( \beta = 0.24 \pm 0.02 \).

The spatial shape of modes (10) selected by the corresponding mask in the transmitted channel is revealed in the dependence of intensity correlation function on fiber displacement in the reflected channel. Indeed, if a phase mask corresponding to the \( m \)th eigenmode is installed...
Figure 2. Experimentally measured eigenvalues of coherent mode decomposition. (a) Two-dimensional distribution of normalized eigenvalues $\lambda_{mn}/\lambda_{00}$ (indexes $m$ and $n$ correspond to HG$_{m0}$ and HG$_{0n}$ modes, respectively). (b) One-dimensional section ($n = 0$) of the distribution. Blue bars are experimental data and black circles are values calculated from (13) using the independently measured value $\beta = 0.24 \pm 0.02$. Error bars correspond to error in the measured value of $\beta$ and error bars for experimental points are smaller than point markers.

in the transmitted channel with no mask in the reflected channel, the general expression (7) for a normalized intensity correlation function reduces to the following form:

$$g^{(2)}_{(m,0)}(r_f) = 1 + \frac{\lambda_m |\alpha_m(r_f)|^2}{\sum_k \lambda_k |\alpha_k(r_f)|^2},$$

(16)

where $r_f$ is the position of the SMF in the reflected channel and $\alpha_k(r_f) = \int dr \phi_k(r)\phi_0(r-r_f)$.

Experimental data for HG$_{m0}$ modes are shown in figure 3(b). It is clear that the selected modes are indeed orthogonal in the sense that $g^{(2)}_{(m,0)}(0) = 1$, $\forall m \neq 0$, consistently with (5). Dependence on the fiber displacement is also in qualitative agreement with (16). However, for large displacements we observe anomalously high values of $g^{(2)}_{(m,0)}(r_f)$ for large $m$. This is most probably an artifact of our method for mode selection based on using phase-only step masks, inevitably causing higher-order mode contributions and compromising mode purity for large displacements. Figure 3(a) shows the results of numerical simulations for the imperfect masks, showing reasonable agreement with experimental data.

For zero displacement and low-order modes, we find the detected modes to be close to Hermite–Gaussian eigenmodes, and thus they are pairwise correlated, as illustrated by figure 4. We expect $g^{(2)}_{(m,n)}(r_f = 0)$ to behave in accordance with expression (5), i.e. to be essentially unity for modes of different order and larger than unity only for modes of the same order. Experimental results of figure 4 confirm this statement, although some amount of anomalous correlation is observed for modes of high order. These correlations are more pronounced in figure 4(b), which may be partially explained by the much poorer quality of the glass phase masks used in the reflected channel. Nevertheless, one can clearly observe the good agreement with the expected correlation properties of quasi-Schmidt coherent modes.

The main source of discrepancies between our experimental results and theoretical expectations for ideal projective measurements in a Schmidt-like basis is the use of non-ideal mode filters. Technical limitations of the SLM used do not allow us to use it for amplitude
Figure 3. Spatial structure of eigenmodes: dependence of normalized $g^{(2)}_{(m,0)}$ function on fiber displacement in the reflected channel. (a) Numerical simulation and (b) experimental results. Unity values for zero displacement correspond to the absence of correlations for modes of different order, predicted by theory.

Figure 4. Mode cross-correlations: (a) $g^{(2)}_{(m,0)} - 1$ values measured with HG$_{m0}$ modes selected in the transmitted arm and the HG$_{00}$ mode in the reflected one; (b) $g^{(2)}_{(m,1)} - 1$ values, corresponding to HG$_{m0}$ modes filtered in the transmitted arm, and the HG$_{01}$ mode selected in the reflected arm.

modulation, which is necessary for perfect transformations of Hermite–Gaussian modes. Nevertheless, our results are very similar to what is expected for exact Hermite–Gaussian filters.

4. Discussion and conclusion

In conclusion, we have investigated spatial intensity correlations in partially spatially coherent thermal light using the technique of projective measurements originally developed to study the spatial entanglement of photon pairs. In particular, our detection scheme is similar to those used in experiments on orbital angular momentum entanglement [27] and essentially the same as was used by authors to study approximate Schmidt decomposition for SPDC biphotons [2]. We have found much similarity in the properties of coherent mode decomposition for the intensity correlation function of classical thermal light and Schmidt decomposition for biphoton
amplitude. Although the expressions for eigenmodes and eigenvalues are different, they are still governed by a single experimental parameter $\beta = \sigma_\mu / \sigma_I$—the ratio of coherence radius to the width of intensity distribution. A similar quantity for entangled photons is known as the Fedorov ratio $R = \Delta x_c / \Delta x_s$—the ratio of widths of coincidence (conditional) and single count (single particle) distributions in either the coordinate or momentum space of photons [28]. This quantity is shown to be equal to Schmidt number $K$, which is the effective number of nonzero eigenvalues $\lambda_i$ in the Schmidt decomposition, defined as $K = \sum_i \lambda_i^{-2}$. Thus for a pure two-photon state it can serve as a measure of spatial entanglement. It is clear that this quantity has a purely classical analogue—optical etendue [6]. Results of this work show that the analogy between spatial entanglement and classical spatial correlations goes further. In purely operational terms the qualitative similarity is almost complete, with the difference being only quantitative—excess of $g^{(2)}$ over unity is limited to 1, while it can be unlimited in the case of entangled photons.

One can consider the described experiments as a special case of thermal light ghost interference/imaging with mode-sensitive detection. In this context, our work adds new arguments to the discussion of quantum and classical features of spatial correlations underlying this method. Coherent mode decomposition for the intensity correlation function inherits most of the properties of its ‘quantum’ counterpart. Most importantly, it defines a naturally preferred set of modes that are both $\delta$-correlated and have maximal partial intensities. Recently, an essentially similar concept of ‘optical eigenmodes’ was introduced by De Luca et al [29] in the context of compressive imaging. These authors used coherent mode decomposition to reconstruct the optical field passing through a complex amplitude and/or phase mask, and argued that coherent eigenmodes are the best choice of modes to fulfill this task. Our work demonstrates that in the case of spatially incoherent light with strong intensity correlations (such as quasi-thermal light), eigenmodes also exhibit stronger correlations than any other coherent spatial mode. This means that they are to be used for a ‘ghost’ generalization of eigenmode imaging. Our experiment is a proof-of-principle one, and treats only the simplest case of a freely propagating Gaussian Schell-model beam; that is why eigenmodes are Hermite–Gaussian in our case. For a more complex field distribution, the eigenmodes may be different but the general framework stays the same, making our experiments an important step toward compressive ghost imaging with thermal light.

From a more fundamental point of view, the spatial mode-filtering technique used in this type of experiment may serve as an additional tool to study the differences between classical and non-classical spatial correlations. The differences here may go further than obvious reduction of visibility. The statistics of mode-filtered light is determined by both the initial photon statistics and the spatial structure of detection modes relative to the (quasi)-Schmidt eigenmodes of the input field. The presented technique may contribute to a deeper understanding of what is classical and what is really quantum in spatial correlations of multi-mode light.

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