STABILITY ANALYSIS OF HYBRID ENERGY STORAGE BASED ON
SUPERCAPACITOR AND BATTERY

The aim of the work is to analyze the stability of the battery-supercapacitor hybrid storage of power supply for resistance micro-welding equipment, considering the possible variation of the system parameters and taking into account parallel series resistance of the circuit components. Methodology. The sufficient accurate mathematical model of the hybrid energy storage system to stability analysis has been obtained by the state-space average method. According to the state-space averaging method, PWM switching converters are described by separate circuit topologies for each switching period. The system of differential equations for each time interval has been derived by use of the Kirchhoff rules. The small-signal model transfer function of the SEPIC converter has been obtained by applying the Laplace transform to linear state equations averaged over one switching cycle. Finally, the Nyquist stability criterion has been considered to evaluate the stability of the proposed energy storage system. Results. Bode diagrams of an open-loop system for different values of the duty cycle, average load current, and input voltage have been obtained by using MATLAB software. The gain margin ranges from 14.6 dB to 26.4 dB and the phase margin ranges from 45.4 degrees to 54.8 degrees. From these results, it is obvious that the proposed system meets the stability criteria regardless of the aforementioned parameter fluctuations. Originality. The high-efficiency energy storage system for micro resistance welding technology has been proposed. Developing of the energy storage system according to the battery semi-active hybrid topology enables to control the Li-ion battery discharge current within the maximum allowable value. SEPIC converter utilization ensures the high-efficient operation of the power supply despite the battery charge state. Moreover, this topology allows implementing series and parallel configuration of both batteries and supercapacitors to obtain the required value of voltage and current. Practical significance. The mathematical model of the SEPIC converter has been developed by applying the state-space averaging technique for parameter variation, such as duty cycle and the average load current, the input voltage has been performed by using Nyquist criteria. References 10, 8, 1, figures 8.

Key words: hybrid energy storage, SEPIC converter, stability analysis, state-space average method, micro resistance welding.

Introduction. The vast majority of portable electronic devices have a complex nonlinear nature of power consumption. Power supplies for such devices must provide average and peak load powers, provide acceptable weight and size and high energy efficiency [1]. It is common to use different types of batteries as accumulators for portable systems. However, a significant peak load current that exceeds the average battery current can significantly reduce their service life. The use of combinations of batteries and supercapacitors can be an effective solution to such problems [1].

Depending on the configuration of storages and load, there are three main topologies of hybrid energy storage systems: passive, semi-active and active topologies [2]. Each of them is widely used in the field of electric transport, Microgrid technology, renewable energy systems [1-3]. Also, the use of combined capacitive storages is a promising area in the field of resistance micro-welding [4].

Resistance micro-welding is an effective technology for obtaining integral joints which is widely used in the modern process of manufacturing electronic equipment. Welding technology is realized by heating the parts due to the flow of electric current of large amplitude through the place of their contact. The current amplitude usually varies from hundreds to thousands of amperes and depends on the shape and material of the welded parts [5].

The power consumption of welding machines has a specific character, namely the consumption of significant power by short pulses compared to the pauses between...
them. These features of energy consumption can be the cause of the negative impact of welding machines on the industrial network. To counteract this effect, the power supplies of welding machines are developed according to the topology with intermediate energy storage (Energy Storage Topology). Usually sources built on such a topology can be divided into three main functional blocks:

- Charger provides better electromagnetic compatibility with the network and regulates the energy consumption for charging the intermediate storage.
- Intermediate capacitive storage provides the required energy during the welding cycle.
- Generator of welding pulses provides high accuracy of regulation of parameters of pulses of welding current [5].

Energy for the charge of such storages is consumed from the network uniformly, almost without causing a negative impact on it [5]. Combinations of supercapacitors, batteries and electrolytic capacitors can be used as intermediate energy storage for Energy Storage Topology [4, 5].

However, regardless of the field of use, energy efficiency, sustainability as well as weight and size are key parameters in the development of systems based on combined energy storage. The presence of a large-capacity energy storage device and nonlinear load in the DC-DC converter can adversely affect the stability of its operation. The instability of the system can manifest itself in the form of bifurcations, chaotic and quasi-periodic modes of operation [6]. Therefore, minimizing the likelihood of such phenomena is a critical task to prevent power supply system failure and reducing the rate of degradation of the characteristics of batteries and supercapacitors.

In recent years, a large number of studies have focused on methods for assessing the stability of DC-DC converters [6] and, in particular, power supplies based on combinations of capacitive storages [7]. For example, in [6] a detailed review of various methods for assessing the stability of systems based on DC-DC converters is presented, the features of application, advantages and disadvantages of these methods, as well as examples of analysis of system stability are given. Various mathematical models are also analyzed, including discrete and time-continuous models of DC-DC converters used to investigate the stability according to different criteria. Stability analysis and hierarchical control of systems based on combined capacitive energy storage devices for Microgrid are considered in [8].

Despite a number of advantages of control systems for combined capacitive storages proposed in the mentioned works [7, 8], the task of stability research needs special attention for systems used in resistance micro-welding technology as such equipment has higher reliability requirements.

Therefore, the goal of the paper is the stability analysis of a hybrid energy storage of power supply for a contact micro-welding machine.

Mathematical model of the combined energy storage system. The generalized block diagram of the power supply for the resistance micro-welding machine is built on a topology with intermediate energy storage and shown in Fig. 1.

The charger consumes energy from the industrial network and provides the required value and shape of the charging current. In addition, it is necessary to provide galvanic isolation between the network and the load and the correction of the power factor. High-capacity electrolytic capacitors, various types of batteries, supercapacitors and combinations of the above-mentioned storages can be used as a storage device. The pulse generator in the figure is shown in the form of two cells, but to ensure the welding current of the required shape and amplitude in the load, N cells connected in parallel to the combined storage are used. The step-down converter (BUCK is marked in Fig. 1) acts as one such a cell [4, 5].

In Fig. 2 the proposed system of energy storage on the basis of semi-active topology of the battery is presented. SEPIC was chosen as a DC-DC converter to control the energy distribution between the battery and the supercapacitor. The main advantage of the semi-active battery topology is the consumption of DC from the battery with a low level of pulsations despite the fluctuations of the load current. This feature allows to significantly increase the performance of the battery in a sharp increase in load current [2]. SEPIC converter is selected as an auxiliary one because the basic requirements are met: DC current consumption from the battery; output current regulation; wide range of output voltage regulation. Such an adjusting characteristic is necessary for Li-ion battery, because the voltage of a fully charged battery is approximately 4.2 V and gradually decreases to 2.5 V. At the same time for efficient operation of the output generator of welding pulses, powered by the supercapacitor, its input voltage must maintained at 2.7 V.
The main source of static losses in low-power circuits and with a relatively large average value of the output current is the resistance of semiconductor switches in the conduction state. To increase the energy efficiency of the proposed system, the Schottky diode, which is commonly used in the SEPIC topology, is replaced by a MOSFET transistor, because the voltage drop on the open channel of such transistors \( U_{\text{VTDS,on}} = 0.3 \text{ mV} \ldots 0.7 \text{ V} \) at nominal values of switching current is simultaneously a direct voltage drop of the Schottky diode \( U_{\text{F}} = 0.3 \ldots 1.5 \text{ V} \). However, it should be noted that with increasing frequency, the dynamic losses of the transistor increase due to recharging of the parasitic capacitances [5].

To obtain a sufficiently accurate for the analysis of the stability of the mathematical model the method of averaging state variables is used [9]. To simplify the analysis, the system can be represented as two separate circuits, for time intervals when the key is closed \([0, dT]\) and open \([dT; T]\). Parameter \( d \) is the pulse filling factor that determines the conduction intervals of the keys of the PWM-controlled converters. For SEPIC, the minimum and maximum value of \( d \) depending on the input voltage level is determined by expressions (1) and (2), respectively:

\[
\begin{align*}
\text{d}_{\text{min}} &= \frac{U_{\text{out}} + U_{f}}{U_{\text{in}_{\text{max}}} + U_{\text{out}} + U_{f}}, \quad (1) \\
\text{d}_{\text{max}} &= \frac{U_{\text{out}} + U_{f}}{U_{\text{in}_{\text{min}}} + U_{\text{out}} + U_{f}}, \quad (2)
\end{align*}
\]

where \( U_{\text{out}} \) is the output voltage; \( U_{f} \) is the direct voltage drop on the closed key \( S_{2} \); \( U_{\text{in}_{\text{min}}} \) is the minimum value of the input voltage; \( U_{\text{in}_{\text{max}}} \) is the maximum value of the input voltage.

The energy for welding is consumed by short pulses with much longer pauses between them and at a certain interval can be considered as a pulse load with a period of \( T_{w} \). Thus, the average current consumption for one welding cycle \([0, T_{w}]\) can be defined as [2]:

The paper considers a quasi-steady state mode when the battery and the supercapacitor are charged to the nominal value. The value of the maximum frequency for analysis is selected so that the phase margin for the converter does not exceed 50°, which is within generally accepted standards. The proposed model is valid for this type of converters at frequencies up to 150 kHz, because it does not take into account the dynamic losses of semiconductor elements. MOSFET transistors \( V T_{1} \) and \( V T_{2} \) have been replaced by ideal switches \( S_{1} \) and \( S_{2} \), the resistance in the closed state of which is infinitely small, and in the open state it is infinitely large. Idealized diagrams of voltage and current of the converter showing the operation of the converter for the switching period are presented in Fig. 3.

Fig. 2. Simplified circuit of combined capacitive energy storage (a); linearized equivalent circuit of the converter in the interval \([0; dT]\) (b); linearized equivalent circuit of the converter on the interval \([dT; T]\) (c)

Fig. 3. SEPIC converter voltage and current idealized diagrams for the switching period
\[ i_{av}(t) = \frac{1}{T_w} \int_0^{T_w} i_{load}(t) \, dt = D_u i_{\text{max}} + (1 - D_u) i_{\text{min}} = I_{av}, \quad (3) \]

where \( i_{load} \) is the current consumed by the generator of welding pulses; \( i_{\text{max}} \) is the amplitude of welding current; \( i_{\text{min}} \) is the the minimum value of welding current (equal to zero); \( D_u \) is the pulse filling factor, a fixed value determined by the technological features of the welding cycle.

Since the switching period of switches \( S_1 \) and \( S_2 \) is much smaller than the duration of one welding cycle \( T_w \), the load current in the period \([0; T]\) will be constant and will be determined by the average current \( I_{av} \) for one welding cycle.

The mathematical model is based on differential equations compiled for each linear substitution circuit. In circles with a variable structure, the systems of differential equations for linear circuits for different intervals are compiled independently of each other. Thus, the average model of the system for one switching cycle can be described by the following system of differential equations:

\[
\begin{align*}
    X &= (dA_1 + (1 - d)A_2) \cdot X + (dB_1 + (1 - d)B_2) \cdot U; \\
    Y &= (dC_1 + (1 - d)C_2) \cdot X + (dE_1 + (1 - d)E_2) \cdot U,
\end{align*}
\]

where \( X \) is the vector of state variables; \( A_1 \) and \( A_2 \) are the matrices of coefficients for state variables for each linear substitution circuit; \( U \) is the vector-column of external action; \( B_1 \) and \( B_2 \) are the matrices of coefficients for the elements of external action for each linear substitution circuit; \( Y \) is the vector-column of initial values; \( C_1 \) and \( C_2 \) are the matrices of relationship of initial quantities with state variables for each linear substitution circuit; \( E_1 \) and \( E_2 \) are the matrices of the relationship between the initial quantities and the vector of external action for each linear substitution circuit.

The system of equations can be represented as the sum of the system of algebraic equations (5) for the constant component and the system of differential equations (6) for the variable component:

\[
\begin{align*}
    X' &= A'B U; \\
    Y &= -CA'B U + E U.
\end{align*}
\]

After applying the Laplace transform, the system of differential equations for the variable component takes the form:

\[
\begin{align*}
    \dot{x}(s) &= \left[ C(sI - A)^{-1} B \right] \left[ u(s) \right] \quad (6) \\
    \dot{y}(s) &= \left[ C(sI - A)^{-1} B + E \right] \left[ u(s) \right].
\end{align*}
\]

where \( B_d = (A_1 - A_2)X + (B_1 - B_2)U \) and \( E_d = (C_1 - C_2)X + (E_1 - E_2)U \).

The solution of the system of equations (6) gives the transfer characteristic of the converter in the small deviations mode:

\[ G_{ds}(s) = C(sI - A)^{-1} B_d + E_d. \]

Based on the above equations, the analysis of the proposed topology is performed. The equation of state in matrix form for the operation interval \([0, dT]\) is obtained on the basis of Kirchhoff laws (8). The equation of the initial values in matrix form for the operation interval \([0, dT]\) is defined as (9). Similarly, the equation of state (10) and the equation of the initial values (11) in matrix form for the operation interval \([dT, T]\) are obtained:
The transfer characteristic of the converter in the mode of small deviations is obtained analytically on the basis of the solution of the generalized system of differential equations for both intervals and has the form:

\[
G_{dv}(s) = \frac{b_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0}{a_4 s^4 + a_3 s^3 + a_2 s^2 + a_1 s + a_0},
\]

where \(a_0, a_1, a_2, a_3, a_4\) are the coefficients of the denominator of the transfer characteristic; \(b_0, b_1, b_2, b_3, b_4\) are coefficients of the numerator of the transfer characteristic.

### Analysis of the Stability of the Combined Energy Storage System

The control system of the SEPIC converter is presented in the form of a block diagram in Fig. 4, where the main links of the control system are replaced by their transfer characteristics.

![Block Diagram of SEPIC Converter Control System](image)

**Fig. 4.** Generalized structure of the SEPIC converter control system for combined capacitive energy storage

Transfer characteristic of the open system for the analysis of influence of change of parameters (filling factor, average load current and input voltage) on the stability of the system is determined as:

\[
H(s) = G_C(s) \cdot F_M \cdot G_{dv}(s),
\]

where \(G_C(s)\) is the transfer characteristic of the PI regulator, \(F_M\) is the gain of the PWM comparator, \(G_{dv}(s)\) is the SEPIC transfer characteristic in small deviation mode.

The transfer characteristic \(G_C(s)\) of the SEPIC converter used to control the energy distribution between the elements of the combined capacitive storage device is obtained analytically on the basis of the equations presented in the previous section.

The main function of the regulator is to ensure the required control accuracy and margin for phase and amplitude in accordance with the Nyquist criterion of stability. PI (proportional-integral) regulator supplemented by a low-pass filter is selected as a regulator. This type of regulators is widely used in industry due to its simple design, low cost and simple tuning algorithm. The PI regulator eliminates forced oscillations and static error, the transfer characteristic of which is as follows [10]:

\[
G_C(s) = \frac{K_C(1+T_C s)}{T_C s(1+T_f s)},
\]

where \(K_C\) is the gain; \(T_C\) is the integration time constant; \(T_f\) is the filtration time constant.

The gain of the PWM comparator \(F_M\) is determined by the amplitude of the sawtooth signal and has the following form:

\[
F_M = \frac{1}{U_M},
\]

where \(U_M\) is the amplitude of sawtooth voltage.

Table 1 shows the main parameters of the SEPIC converter and PI regulator components, as well as the initial data of the combined energy storage system used for stability analysis.

| Output parameters | Component parameters |
|-------------------|----------------------|
| \(U_{in}\), V | L_1, L_2, \(\mu\)H | 10 |
| \(U_{out}\), V | C_1, \(\mu\)F | 820 |
| \(I_{in}\), A | C_{SC}, F | 350 |
| \(\gamma\) | 0.4; 0.5; 0.6 | |
| \(U_{ref}\), V | C_2, pF | 100 |
| \(R_1, R_2\), k\(\Omega\) | R_{C1}, R_{C2}, m\(\Omega\) | 10 |

The solution of the averaged system of differential equations and the logarithmic amplitude-phase frequency characteristics (LAPFC) of an open system under different conditions was obtained using the MATLAB software package.

Figure 5 shows the LAPFC of the open system for different values of the load current at the nominal parameters of the circuit components, the input voltage \(U_{in} = 3.7\) V and the pulse filling factor \(d = 0.5\). The diagrams show that the control system provides a margin for the phase from \(45.4^\circ\) to \(54.8^\circ\) and for the amplitude from \(14.6\) dB to \(26.4\) dB; when the load current changes the system remains stable.

![Bode Diagram](image)

**Fig. 5.** LAPFC of the open system for different values of the load current

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LAPFCs of the system when changing the pulse filling factor and nominal input voltage $U_i = 3.7$ V, current loads $I_{out} = 10$ A are shown in Fig. 6. The system is stable at different values of the pulse filling factor. Similarly, the stability of the system is affected by the change in input voltage at $d = 0.5$ and $I_{out} = 10$ A (Fig. 7). All other system parameters remain unchanged in all three cases.

The reaction of the system to the influence in the form of a single step function is shown in Fig. 8. Because the system has a supercapacitor of large capacitance, the duration of the transient is in milliseconds. In order to counteract this effect, the supercapacitor can be represented as a voltage source, because the voltage on it during one switching period is almost unchanged.

**Conclusions.**

An energy storage device based on a combination of a supercapacitor and a battery for a power supply developed according to the topology of intermediate energy storage used for resistance micro-welding technology is proposed. A semi-active battery topology and a SEPIC converter have been selected for energy distribution between the storages, which allows to provide the battery discharge with the rated current and rated voltage on the supercapacitor regardless of the battery charge level.

A mathematical model of the converter which takes into account the parasitic resistances of the circuit components is obtained. The Nyquist criterion is used to study the stability of the proposed control link. As a result of the analysis the area of stability of system at variation of key parameters of system is defined. The presented topology is stable when changing the pulse filling factor, load current and input voltage in a wide range.

Further work will be devoted to the practical verification of the obtained results using the physical model of the proposed combined storage.

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