Evidence for Ground- and Excited-State Efimov Trimmers in a Three-State Fermi Gas

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We observe enhanced three-body recombination in an ultracold three-component $^6$Li Fermi gas with large but unequal scattering lengths attributable to an excited Efimov trimer state near the three-atom scattering threshold. We find excellent agreement between the measured three-body recombination rate and the recombination rate calculated in the zero-range approximation where the only free parameters are the Efimov parameters $\kappa_s$ and $\eta_s$. The value of $\kappa_s$ determined by the location of the Efimov resonance we observe at 895 G also predicts the locations of loss features previously observed near 130 and 500 G [1, 2] suggesting that all three features are associated with universal Efimov trimer states. We also report on the first realization of a quantum degenerate three-state Fermi gas with approximate SU(3) symmetry.

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A landmark result of few-body physics is Efimov’s solution to the quantum mechanical three-body problem for low-energy, identical particles with a large $s$-wave scattering length $a$ [3, 4]. Being independent of the particular character of the two-body interaction, Efimov’s predictions are universal. At resonance ($a \rightarrow \pm \infty$), an infinite series of arbitrarily shallow three-body bound states (Efimov trimers) exist with a geometric spectrum: $E_n = -(e^{2\pi/s_0})^{-n} \hbar^2 \kappa_s^2/m$ (for $n = 0, 1, 2, \ldots$). Here, $s_0 \approx 1.00624$ is a universal constant, $m$ is the particle’s mass, and the entire spectrum is fixed by a single three-body parameter $\kappa_s$ which is determined by short-range interactions [3]. For large but finite $a$, the spectrum of Efimov trimer states are universal functions of $a$ and $\kappa_s$. Further, all low-energy scattering observables display a log-periodic dependence on $a$ and the collision energy [3]. These startling predictions ignited a search for Efimov physics in a range of physical systems [6, 7].

The first experimental evidence for Efimov trimers was obtained by observing both a resonant enhancement and interference minimum of three-body recombination in an ultracold gas of bosonic Cs near a Feshbach resonance [8]. The Efimov effect in bosonic systems near an isolated Feshbach resonance is now becoming well understood following the observation of atom-dimer resonances [9], Efimov resonances between bosons with different masses [10] and tests of universal log-periodic scaling [11, 12].

The Efimov effect is also expected to occur for three fermions in distinguishable states [(1), (2), and (3)] if at least two of the three pairwise scattering lengths $a_{ij}$ ($a_{12}$, $a_{23}$, and $a_{13}$) are larger than the range of interaction (which, for atoms, is the van der Waals length $\ell_{vdW} = \sqrt{mC_6/\hbar^2}$) [3]. For a Fermi gas of $^6$Li atoms in the three lowest spin states, investigated here, all three scattering lengths can be resonantly enhanced by three overlapping Feshbach resonances (see Fig 1 (a)) [13] allowing for a study of the Efimov effect in a Fermi system which is deeply in the universal regime $|a_{ij}| \gg \ell_{vdW}$. For this $^6$Li system, the scattering lengths are typically unequal, and any number of them can be positive. This gives rise to an Efimov spectrum with a rich structure which is only now becoming understood theoretically [14, 15]. Entirely new phenomena related to the Efimov effect have been predicted in this case (e.g. interference minima in the rate of atom-dimer exchange reactions) [15].

Beyond few-body physics, superfluid phenomena in degenerate three-component Fermi gases have recently been a subject of intense theoretical interest [16, 17, 18, 19, 20]. For fermions with $a_{12} \approx a_{23} \approx a_{13} < 0$, there is a competition between trimer formation and several types of Cooper pairing [19, 20], breached pair and unbreached pair phases [18], as well as a unique interplay between mag-
three-body recombination was observed near fields of 130 and 500 G despite its questionable applicability near the atom scattering threshold. We measure the three-body recombination rate as the scattering lengths (\(a\)) and measured the three-body recombination rate for fields between 842 and 1500 G against decay via three-body recombination [1, 2]. These component Fermi gases open a window of opportunity against decay to deeply bound dimer states. We produce the first degenerate three-state Fermi gas in a harmonic trap, where the energetic atom and molecule from each spin state can occur in addition to one-body loss due to the decay of each of the possible two-state mixtures of the three states is consistent with one-body loss depending on the bias field \(B\). Starting from an incoherent 50/50 mixture of atoms in states |1⟩ and |2⟩, we create a three-state mixture with equal population in states |1⟩, |2⟩ and |3⟩ by simultaneously applying radio-frequency (RF) magnetic fields which drive |1⟩ - |2⟩ and |2⟩ - |3⟩ transitions at the field \(B\). Each of the RF fields has a resonant Rabi frequency \(\Omega/2\pi \approx 10\ kHz\), is broadened to a width of 10 kHz and applied for a variable time (chosen to be at least twice the observed decoherence time) between 10 and 100 ms depending on the bias field.

We have confirmed (for the fields studied here) that the decay of each of the possible two-state mixtures of the three states is consistent with one-body loss due to background gas collisions. One-body loss for these experiments gives a 1/e lifetime \(1/\Gamma = 2.8\ s\). In the three-state mixture, three-body recombination involving one atom from each spin state can occur in addition to one-body loss. In this case, the number of atoms in each of the equally populated spin states \(N(t)\) evolves according to \(N/N = -\Gamma - L_3(n^2)\) where \(L_3\) is the recombination rate constant and \(n^2\) is the average value of the squared density per spin state. For a thermal distribution at temperature \(T\) in a harmonic trap, \(N(t)\) evolves according to

\[
\frac{1}{N} \frac{dN}{dt} = -\Gamma - \frac{L_3}{k_B T}\left(\frac{2\pi m \tilde{v}^2}{k_B T}\right)^{3/2} N^2,
\]

where \(\tilde{v} = (\nu_x \nu_y \nu_z)^{1/3}\). Empirically, the temperature of the gas remains approximately constant. This is consistent with the fact that the energetic atom and molecule
produced in a recombination event have a mean free path much larger than the sample size and exit the cloud without depositing energy. Our measurements are consistent with a model that includes a rise in temperature due to "anti-evaporation." In the inset the solid (dashed) line shows the fit to a model which does not include (includes) anti-evaporation.

To determine the recombination rate constant \( L_3 \), we measure for each field \( B \) the number and temperature of the trapped atoms as a function of time by \textit{in situ} absorption imaging. We show an example of data recorded at a particular field value in the inset to Fig. 2. The number evolution at each field is fit with an analytic solution to Eqn. 1 using \( L_3 \), the initial number \( N_0 \) and temperature \( T \) as free parameters. The recombination rate constant for fields between 834 and 1500 G is shown in Fig. 2 for data sets taken with traps A and B. The triangles (circles) correspond to trap A (B) and give \( L_3 \) for a cloud at a temperature \( \lesssim 30 \text{ nK} \) and an initial peak density per spin state \( n_0 \approx 5 \times 10^{10} \text{ atoms/cm}^3 \) \((n_0 \approx 5 \times 10^{10} \text{ atoms/cm}^3)\). The error bars give the statistical error from the fit and the uncertainty in the trap frequencies added in quadrature. A systematic uncertainty of \( \pm 60\% \), which arises from our uncertainty in the absolute atom number \( \pm 30\% \), is not included in these error bars. As we vary the field, the scattering lengths are tuned by broad Feshbach resonances at 690, 811, and 834 G and the observed value of \( L_3 \) changes by several orders of magnitude. In addition to a smooth variation of \( L_3 \), resonant three-body recombination is observed near 900 G in both data sets.

In order to compare our data to zero-temperature calculations of the recombination rate, we require that the temperature be in the threshold regime and that the data is unaffected by the unitarity limit [24]. As argued above, the threshold regime should be obtained for the 30 nK (180 nK) gas for fields above 875 G (960 G). For a gas at a low but nonzero temperature \( T \), the recombination rate constant for three distinguishable fermions with equal mass is unitarity limited to a maximum value \( L_{3\text{max}} = \alpha/(k_B T)^2 \) where \( \alpha = \sqrt{108\pi^2 \hbar^2 / m^3} \) [24, 26]. For a thermal gas with \( T = 30 \text{ nK} \) \((T = 180 \text{ nK})\), \( L_{3\text{max}} = 8 \times 10^{-18} \text{ cm}^6/\text{s} \) \((L_{3\text{max}} = 2 \times 10^{-19} \text{ cm}^6/\text{s})\). The measured values of \( L_3 \) are \( \lesssim (1/10) L_{3\text{max}} \) for the 30 nK \((180 \text{ nK})\) data for fields above 907 G \((975 \text{ G})\). These subsets of the data should be in excellent agreement with zero-temperature calculations of the recombination rate.

Recently, Braaten and co-workers calculated the three-body recombination rate constant in the zero-range approximation for three fermions in distinguishable spin states at threshold [14]. By numerically solving a generalization of the Skorniakov–Ter–Martirosian (STM) equation, they calculate the recombination rate constant for the case \( a_{12}, a_{23}, a_{13} < 0 \). The only inputs to this calculation are \( a_{12}, a_{23} \) and \( a_{13} \) (known for \( ^6\text{Li} \)) and the two three-body parameters, \( \kappa_\gamma \) and \( \eta_\gamma \), which respectively fix the spectrum and linewidth of the Efimov trimer states. For equal scattering lengths, the STM equation reduces to that for identical bosons and, in this case, an analytic expression for \( L_3 \) is known [14].

\[
L_3 = \frac{16\pi^2 C \sinh(2\eta_\gamma)}{\sin^2[D |a| \kappa_\gamma] + \sinh^2 \eta_\gamma} \frac{\hbar a^4}{m} \quad \text{(for } a_{ij} = a < 0) \tag{2}\]

Here the constants \( C = 29.62(1) \) and \( D = 0.6642(2) \) have been determined from numerical calculations [14]. The analytic expression above exhibits resonant enhancement at values \( a = a^- = -\left(\pi/n^2 \right) m D\kappa_\gamma^{-1} \) for integer \( n \) which occur when Efimov trimer states cross the three-atom threshold. For \( a_{12} \neq a_{23} \neq a_{13} \), as in our experiment, the STM equations must be solved numerically [27] to accurately determine \( L_3 \).

Since \( |a_{12}|, |a_{23}|, |a_{13}| \gg \ell_{\text{vdW}} \) and \( a_{12}, a_{23}, a_{13} < 0 \) for fields above 834 G, the data shown in Fig. 2 should be well described by the zero-range calculation of \( L_3 \) in Ref. [14] and will allow the first determination of the three-body parameters for this three-component \( ^6\text{Li} \) gas in the high-field regime. The best fit (solid line) to subsets of the 30 nK data \((B \geq 907 \text{ G})\) and the 180 nK data \((B \geq 975 \text{ G})\) (as described above) is shown in Fig. 2. The only free parameters in this fit are \( \kappa_\gamma \) and \( \eta_\gamma \). From the fit we obtain \( \kappa_\gamma = 6.9(2) \times 10^{-3} a_0^{-1} \) and \( \eta_\gamma = 0.016^{+0.006}_{-0.010} \) where the uncertainties – combined statistical and systematic – indicate one standard deviation. The zero-temperature recombination rate for these three-body parameters calculated for fields between 840 and 1550 G is shown as the dotted line in Fig. 2. In this model, the resonant peak in the recombination rate occurs at 895 G.
where an Efimov trimer crosses the three-atom scattering threshold. The measured three-body recombination rate for the 30 nK data also peaks near 895 G, though at a significantly smaller magnitude due to the unitarity limit. The dashed line shows a “unitarized” recombination rate given by \( (1/L_3 + 1/L_\text{sat})^{-1} \) to account for the observed saturation of the recombination rate in the thermal gas to a value \( L_\text{sat} \approx L_\text{max}/3 \).

We can identify the Efimov trimer that crosses threshold at 895 G with the first excited state of the Efimov spectrum. The binding energy of this Efimov trimer, in the limit of infinite scattering lengths, is \( E_0 = e^{-2\pi/\hbar^2}h^2\kappa_s^2/m = h \times 55(3) \text{kHz} \). In this limit, the ground state Efimov trimer has a binding energy a factor of \( e^{2\pi/\hbar^2} \approx 515 \) larger, \( E_0 = h \times 28(2) \text{MHz} \). This is the lowest state with a binding energy smaller than the van der Waals energy scale \( E_{\text{vdW}} = h^2/m\ell_w^3 \). If it were the case that \( a_{13} = a_{23} = a_{12} = a < 0 \), the first excited Efimov trimer would cross threshold at \( a = a_{13} = -5.0(1) \times 10^3 a_0 \) and the adjacent trimer states would cross threshold at values of \( a \) larger or smaller than a factor of \( e^{\gamma} \approx 22.7 \). In reality, an Efimov trimer of the \( ^6\text{Li} \) system crosses threshold at 895 G where \( a_{12} = -8584 a_0 \), \( a_{23} = -5702 a_0 \) and \( a_{13} = -2893 a_0 \). We identify this trimer as the first excited Efimov trimer since \( a_{13} \) falls within this range of \( a_{ij} \) values. In the high-field region \( (B > 608 \text{G}) \), the ground state Efimov trimer asymptotes to a binding energy \( E_0(a_{ij} = -2140 a_0) = h \times 24(2) \text{MHz} \) with a width \( \gamma = (2\pi) 1.5^{0.6}_0 \text{MHz} \) (corresponding to a \( 100^{180}_24 \text{nK} \) lifetime) as calculated from expressions given in Ref. [2].

One aspect of Efimov physics that can now be tested with ultracold atom experiments is the extent to which the universal results can be applied when scattering lengths are tuned across polar or across nodes. For the \( ^6\text{Li} \) gas studied here, there are two universal regions \((122 \text{G} < B < 485 \text{G} \text{ and } B > 608 \text{G}) \) where the zero-range approximation should be applicable, separated by a non-universal region where the scattering lengths become smaller than \( 2\ell_{\text{vdW}} \). We find that the value \( \kappa_s = 6.9(2) \times 10^{-3} a_0^{-1} \), determined above for the high-field region, is close to the value \( \kappa_s = 6.56 \times 10^{-3} a_0^{-1} \) previously determined from the position of the narrow loss feature in the low-field region near 130 G [14]. Indeed, the recombination rate calculated in the zero-range approximation using \( \kappa_s = 6.9(2) \times 10^{-3} a_0^{-1} \) predicts Efimov resonances at 125(3) and 499(2) G in reasonable agreement with observations [11, 2]. These resonances occur when the ground state Efimov trimer crosses threshold. The value of \( \eta_3 \) is nearly 1 order of magnitude larger in the low field region [14] than in the high-field regime. The significant change in \( \eta_3 \) is likely due to the dramatic difference in the binding energy of the deep dimer states for the two regions [23].

Finally, we note that despite the large three-body recombination rate that occurs in this system, we can produce quantum degenerate three-component Fermi gases in the high-field regime where the three scattering lengths are large, negative and approximately equal. Starting from a degenerate two-state mixture of \(^6\text{Li} \) atoms at \( B = 1500 \text{G} \) in trap B, we prepare a three-state mixture by applying RF pulses as described above. After equilibration, the three-state mixture has \( N = (6)(2) \times 10^4 \) atoms per spin state, a temperature \( T = 50(10) \text{nK} \), a Fermi temperature \( T_F = 180(20) \text{nK} \) and a degeneracy temperature \( T/T_F = 0.28(6) \). The difference in mean field energies is more than one order of magnitude smaller than any other energy scale in the system making it useful for future investigations of three-component Fermi gases with SU(3) symmetry [16, 17, 18, 19, 20].

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