ADVANCED BURNING STAGES AND FATE OF 8–10 M\(_\odot\) STARS

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Received 2013 February 19; accepted 2013 June 4; published 2013 July 17

ABSTRACT

The stellar mass range 8 \(< M/M_\odot \leq 12\) corresponds to the most massive asymptotic giant branch (AGB) stars and the most numerous massive stars. It is host to a variety of supernova (SN) progenitors and is therefore very important for galactic chemical evolution and stellar population studies. In this paper, we study the transition from super-AGB (SAGB) star to massive star and find that a propagating neon–oxygen-burning shell is common to both the most massive electron capture supernova (EC-SN) progenitors and the lowest mass iron-core-collapse supernova (FeCCSN) progenitors. Of the models that ignite neon-burning off-center, the 9.5 M\(_\odot\) star would evolve to an FeCCSN after the neon-burning shell propagates to the center, as in previous studies. The neon-burning shell in the 8.8 M\(_\odot\) model, however, fails to reach the center as the URCA process and an extended (0.6 M\(_\odot\)) region of low \(Y_e\) (0.48) in the outer part of the core begin to dominate the late evolution; the model evolves to an EC-SN. This is the first study to follow the most massive EC-SN progenitors to collapse, representing an evolutionary path to EC-SN in addition to that from SAGB stars undergoing thermal pulses (TPs). We also present models of an 8.75 M\(_\odot\) SAGB star through its entire TP phase until electron captures on \(^{20}\)Ne begin at its center and of a 12 M\(_\odot\) star up to the iron core collapse. We discuss key uncertainties and how the different pathways to collapse affect the pre-SN structure. Finally, we compare our results to the observed neutron star mass distribution.

Key words: nuclear reactions, nucleosynthesis, abundances – stars: AGB and post-AGB – stars: evolution – stars: neutron – supernovae: general

Online-only material: color figures

1. INTRODUCTION

As helium fuel is exhausted at their center, stars with initial masses \(M \gtrsim 1 M_\odot\) develop cores consisting of mostly carbon and oxygen (CO). These CO cores become partially degenerate in stars with \(M \lesssim 9 M_\odot\) before the threshold temperature for carbon ignition can be reached at the center. Neutrino processes cause a temperature inversion in the core and if the star is massive enough \((6 \lesssim M/M_\odot \lesssim 9)\), carbon ignites off-center and proceeds to burn inward (see, e.g., Nomoto 1984; Garcia-Berro et al. 1997; Siess 2007). After the core has been processed by carbon burning, it consists of mostly oxygen and neon (ONE) in a degenerate configuration. Stars experiencing the off-center ignition of carbon to form a degenerate ONE core are known as super-AGB (SAGB) stars.

Efforts to better understand the evolution of SAGB stars through numerical modeling are ongoing (see, e.g., Siess 2010; Doherty et al. 2010) and it is now computationally possible to follow several thousands of thermal pulses (TPs) in order to explore the complex evolution that can be compared with observations. The shortcomings of hydrostatic one-dimensional modeling of this phase were recently briefly discussed by Lau et al. (2012).

8–12 M\(_\odot\) stars are of crucial importance to galactic chemical evolution and stellar population studies. SAGB stars are at the lower end of this mass range, whose massive envelopes enrich the interstellar medium. At the upper end of this mass range are the most abundant of the massive stars (because the initial mass function (IMF) is bottom heavy). These massive stars produce violent explosions in their deaths, producing and expelling heavier elements. The statistical contribution of stars in this mass range to supernovae (SNe) and their remnants is well reflected in the derived progenitor mass distribution of M31 (Jennings et al. 2012). The authors found the IMF of M31 to be well reflected in the derived progenitor mass distribution of M31 (Jennings et al. 2012). The authors found the IMF of M31 to be

\[ M/M_\odot \lesssim 12 \]

\[ M \gtrsim 1 M_\odot \]

\[ M \lesssim 9 M_\odot \]

\[ 6 \lesssim M/M_\odot \lesssim 9 \]

\[ Y_e \approx 0.48 \]

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calculations so far only predict a contribution to the lighter nuclei, Zn and Zr for example. Increasing evidence suggests that there may be weak and main components to the r-process (Hansen et al. 2012), of which EC-SN events could contribute the weak component.

Stars in this mass range may hold the key to explaining observations of sub-luminous type II-P SN with low $^{56}$Ni ejecta (Smartt 2009). Interestingly, there is a non-monotonic feature of the progenitor star mass—luminosity relation in this mass range, where the occurrence of deep second dredge-up (2DUP) boosts the luminosity of SAGB stars. The luminosity of SAGB stars can then come close to exceeding even that of a 15 $M_\odot$ star (see, e.g., Eldridge et al. 2007).

Nomoto (1987, case 2.2) and (1984, case 2.4) has provided the canonical pre-SN structures for EC-SN simulations (Kitaura et al. 2006; Fischer et al. 2010; and for the resulting nucleosynthesis see Wanajo et al. 2009, 2011). The model from Nomoto (1984) dubbed case 2.6 was not followed any further than the ignition of off-center neon- and oxygen-burning shells. Subsequently, Nomoto & Hashimoto (1988) followed the propagation of the neon-burning shell in a helium star of 3.0 $M_\odot$ (case 3.0) to the stellar center, concluding that the star would produce an Fe core before collapsing. Timmes & Woosley (1992) and Timmes et al. (1994) studied in detail the properties of nuclear flames in degenerate compositions of C + O and O + Ne + Mg. In these studies it was proposed that, should neon- and oxygen-burning ignite off-center in the core of a star significantly far from the center, then it may compete with the contraction of the center to determine its fate—EC-SN or iron-core-collapse supernova (FeCCSN). Currently, there are no progenitor models for this additional path to EC-SN. We call these failed massive stars and present them for the first time in this paper. The subtle differences between the pre-SN evolution of these progenitors and SAGB progenitors could affect the explosion (Gutierrez et al. 1996). More recently, Eldridge & Tout (2004) reported that for the most massive SAGB stars, an ONe core with $M > M_{\text{Ch}}$ is produced before the completion of the 2DUP, and subsequently its mass is reduced to $M_{\text{Ch}}$ by the ignition of a carbon-burning convective shell. It is unclear from this study what then happens to these stellar models as the evolution was not calculated any further and a limited nuclear reaction network was used for the calculation.

The goal of this paper is thus to present evolutionary models and progenitor structures for both SAGB and failed massive stars and discuss the possible impact of this additional channel on EC-SNe. The structure of this paper is as follows. In Section 2, the input physics for the models is described; Section 3 contains detailed description of the evolution of our models and their fate. The neon–oxygen shell burning in the 8.8 and 9.5 $M_\odot$ (Sections 3.3.1 and 3.3.2) and the progenitor structures of our models (Section 3.4) are also presented. Section 4 discusses the key uncertainties in modeling stars in the 8–12 $M_\odot$ mass range and finally, we summarize our results and conclude in Section 5.

2. METHODOLOGY AND MODELS

We calculated stellar models with initial masses of 8.2, 8.7, 8.75, 8.8, 8.9, 9.5, and 12.0 $M_\odot$ with the Modules for Experiments in Stellar Astrophysics (MESA) stellar evolution code (Paxton et al. 2011), revision 3709. We calculated non-rotating models from the pre-main sequence (pre-MS) assuming a uniform initial composition with a metal fraction of $Z = 0.014$ and elemental abundances taken from Asplund et al. (2004).

In MESA, convective mixing is treated as a time-dependent, diffusive process with a diffusion coefficient, $D_{\text{MLT}}$. See Paxton et al. (2011) for the implementation details of standard mixing length treatment. The mixing length parameter is chosen as $\alpha_{\text{MLT}} = 1.73$ from fitting of the parameters of the Sun. During the entire evolution sequence, we assume the Schwarzschild criterion for convection with the exception of the late stages of the 8.75 $M_\odot$ and 8.8 $M_\odot$ models (when electron captures begin to dominate the evolution of the core) where we employ the Ledoux criterion (Miyaji & Nomoto 1987). Mixing at convective boundaries is treated with an exponentially decaying diffusion coefficient (Freytag et al. 1996; Herwig 2000) of the form

$$D = D_0 \exp\left(-\frac{2z}{f_{\text{CBM}} \lambda_{P,0}}\right),$$  

where $D_0$ is the diffusion coefficient, taken equal to the mixing length diffusion coefficient value ($D_{\text{MLT}}$) at a distance $f_{\text{CBM}} \lambda_{P,0}$ inside the convection zone from the Schwarzschild boundary. At this location, the pressure scale height is $\lambda_{P,0}$, while $\lambda_{P,S}$ is the pressure scale height at the Schwarzschild boundary. This is because the value of $D_0$ drops sharply toward zero at the Schwarzschild boundary. $D$ is the diffusion coefficient as a function of distance $z$ from this location and $f_{\text{CBM}}$ is a free parameter, for which we assume the value of 0.014 at all convective boundaries except for at the base of convective shells burning nuclear fuel, for which we assume a stricter value of $f_{\text{CBM}} = 0.005$. Such a reduced efficiency of convective boundary mixing at the bottom of shell-flash convection zones is indicated from both He shell-flash convection in asymptotic giant branch (AGB) stars (Herwig 2005) as well as nova shell flashes (Denissenkov et al. 2013). During the silicon-burning stage of the 12 $M_\odot$ model, no convective boundary mixing is assumed ($f_{\text{CBM}} = 0$). Future three-dimensional simulations are required to constrain the behavior of convective boundary mixing during these late stages.

MESA solves the coupled stellar structure, nuclear burning, and abundance mixing equations simultaneously. In cases where the burning timescale is much longer than the mixing timescale, as, for example, during core H-burning on the main sequence, then MESA’s coupled calculation and an operator-split calculation will agree. In cases where the nuclear burning timescale is similar or shorter compared to the mixing timescale, the coupled method provides consistent abundance profiles in convection zones, whereas operator-split calculations require a special treatment for chemical species with short nuclear timescales and smaller time steps. Note that in exceptional cases where the energy release by simultaneous burning and mixing is so large that the approximations of MLT are violated, then all one-dimensional methods become inaccurate and three-dimensional hydrodynamic simulations are necessary (e.g., Herwig et al. 2011).

We trace the nuclear energy production and composition evolution with a network of 114 isotopes from $^1$H to $^{60}$Co including the NeNa cycle, URCA processes, alpha chains, and electron captures by $^{24}$Mg, $^{24}$Na, $^{20}$Ne, and $^{20}$F along with their inverses. Figure 1 shows the detail of the network. Such a large network is required to follow both nucleosynthesis and energy production in these models. For example, $^{30}$Si and $^{34}$S are the main products of O-burning in the lowest mass massive stars as opposed to $^{28}$Si and $^{32}$S in more massive stars owing to higher degeneracies and thus higher electron capture rates (see, e.g., Thielemann & Arnett 1985). In stars with degenerate cores close to the Chandrasekhar limit, accurately calculating the electron fraction, $Y_e$,
is very important because only a slight reduction in $Y_e$ can cause significant contraction. Further isotopes are included implicitly to account for non-negligible reaction channels, for example, $^{44}\text{Ti}(\alpha, p)^{47}\text{V}(p, \gamma)^{48}\text{Cr}$ is included though we do not explicitly calculate the abundance of $^{47}\text{V}$. These implicit isotopes can be seen in Figure 1 where there is an arrow junction on an unshaded isotope. For the 8.2, 8.7, and 8.75 $M_\odot$ models that become SAGB stars, a network optimized for the AGB phase, including 37 isotopes and the relevant nuclear processes listed above, was employed from the time of completion of 2DUP. During the silicon-burning stage of the 12 $M_\odot$ model, we employ the simplified 21-isotope network, approx21.net, that is available in the MESA code. It is common for simplifications to the nuclear reaction network to be made in order to efficiently deal with the many high rates of forward and reverse reactions. Weak reaction rates and associated neutrino-loss rates are those of Fuller et al. (1985), Takahara et al. (1989), Oda et al. (1994), Langanke & Martínez-Pinedo (2000), and, as will be discussed in Sections 3.3 and 4.1, Toki et al. (2013). Assumed mass-loss rates comprise that of Reimers (1975, $\eta = 0.5$) for the red giant branch (RGB) phase and Blöcker (1995, $\eta = 0.05$) during the AGB phase.

3. EVOLUTION AND FATES

In this section, the evolution and fate of the models are described in the following order. In Section 3.1, the early evolution of the models from the main sequence to the end of carbon burning is briefly outlined. Sections 3.2 and 3.3 then describe in detail the late evolution of the SAGB and massive star models, respectively, wherein Sections 3.3.1 and 3.3.2 describe the behavior of neon–oxygen-burning shells in the 8.8 and 9.5 $M_\odot$ models. Lastly, in Section 3.4 the progenitor structures of our models are described, comparing both between models calculated for this study and with other progenitor models currently published in this mass range (Nomoto 1987; Woosley et al. 2002).

3.1. Evolution to the End of Carbon Burning

The evolution of all the models in the Hertzsprung–Russell diagram and the $\rho_c–T_c$ plane are shown in Figures 2 and 3, respectively. Carbon is ignited centrally in all but the 8.2 $M_\odot$ model, in which it is ignited at a mass coordinate of 0.15 $M_\odot$ away from the center and the C-burning front propagates to the
center (see Figure 4(a)) in the manner of a canonical SAGB flame (Nomoto 1984).

Following the core He-burning stage, core contraction is accompanied by an expansion of the envelope seen in Figure 4 as a deepening of the base of the convective envelope in mass. Core contraction and the related envelope expansion continue until they are halted by the ignition of carbon. After the exhaustion of carbon in the center, carbon burning proceeds in shells and from this point onward the behavior of the envelope begins to diverge across the 8–12 $M_\odot$ mass range.

In our models with $M \lesssim 8.8 M_\odot$, the timescale for expansion of the H-envelope is comparable to the evolutionary timescale. Owing to the higher degree of degeneracy in the core, the envelope in the 8.2 $M_\odot$ model has time to engulf almost the entire helium shell, whereas in the 8.7, 8.75, and 8.8 $M_\odot$ models gravo-thermal energy release induces convection in the helium shell that merges with the envelope, referred to as dredge-out (Iben et al. 1997; Siess 2007). In the 8.8 $M_\odot$ model, as much as 0.8 $M_\odot$ of He-rich material is mixed into the envelope. Aside from the huge increase in the amount of helium that now resides at the surface following this deep mixing event, there are many other observable quantities resulting from dredge-out. In particular, the dredge-out is accompanied by a large increase in luminosity, inducing luminosities at the pre-SN stage larger than in the 12 $M_\odot$ model as shown in Figure 2 (see also Eldridge & Tout 2004).

In the 12 $M_\odot$ model, the evolution of the core is accelerated by plasma neutrino-energy losses whereas the envelope expands on a thermal timescale. As a result, the convective envelope remains unaltered after carbon burning. With decreasing initial mass, the core is more degenerate and compact following carbon burning and thus contraction is slower. This provides further energy and time for the expansion of the envelope, as can be seen at log$_{10}(T_e^*/\text{yr}) \approx 4$–3 in Figures 4(a)–(e).

3.2. Late Evolution of the 8.2, 8.7, and 8.75 $M_\odot$
(Super-AGB Models)

The 8.2 $M_\odot$, 8.7 $M_\odot$, and 8.75 $M_\odot$ models develop cores with masses that fall short of the critical mass for neon ignition (see Section 3.3) following 2DUP ($M_{\text{CO}} = 1.2670, 1.3509, \text{and } 1.3621 M_\odot$, respectively), developing thin (of the order of $10^{-5}$–$10^{-4} M_\odot$) He shells that soon develop a recurrent thermal instability producing transient He-fueled convection zones (TPs). The 8.2 $M_\odot$ star expels its envelope to become an ONe white dwarf (WD). It is uncertain whether the 8.7 $M_\odot$ star would produce an ONe WD like the 8.2 $M_\odot$ star, or whether its core would reach the critical central density for electron captures on $^{24}\text{Mg}$, $\rho \approx 10^6$ g cm$^{-3}$, before the envelope is lost. We have modeled the 8.75 $M_\odot$ star through the entire TP-SAGB phase (about 2.6 $\times$ 10$^6$ time steps) including the URCA process and electron captures by $^{24}\text{Mg}$ and $^{20}\text{Ne}$ (see Figure 3). It becomes an EC-SN.

The outcome of these models is highly sensitive to the mass–loss prescription on the SAGB and the rate at which the core grows (Poelarends et al. 2008). We have modeled the TP-SAGB phase of the 8.7 $M_\odot$ star for about 240 pulses, at which point $\rho_e = 10^9$ g cm$^{-3}$. Though still far from $\rho_{\text{crit}}(^{24}\text{Mg} + e^-)$, the central density has exceeded the thresholds for both major URCA process reactions, accelerating the contraction of the core toward $\rho_{\text{crit}}(^{24}\text{Mg} + e^-)$. Due to this acceleration in contraction and comparison with literature (Nomoto 1984, 1987; Ritossa et al. 1999; Poelarends et al. 2008), the most probable outcome for the 8.7 $M_\odot$ model is an EC-SN.

In order to maintain numerical stability in the 8.75 $M_\odot$ model, after the depletion of $^{24}\text{Mg}$ at the center by electron captures, the input physics assumptions were modified. First, the effects of mass loss were excluded from the calculation and second the surface was relocated to a region where the optical depth is an order of magnitude greater than that at the photosphere (which is where the surface had previously been defined). Choosing to set the boundary at a larger optical depth is one way to deal with the inappropriate way we are simulating the final stages of these massive SAGB envelopes. In a one-dimensional code (and probably in the real star), large pulsations occur signaling an increasing instability of the envelope which may lead to enhanced mass loss or even ejection phases, such as the super-wind. These issues have been alluded to
recently by Lau et al. (2012). Choosing the photosphere to be at a larger optical depth indeed lets the star be hotter and smaller, and the mass loss calculated from the stellar parameters, if it were still included, will not be the same as for the default photosphere parameters. Through this treatment, the details of the envelope evolution are increasingly inaccurate from this point. When these changes were made, the remaining envelope mass was $4.48 \, M_\odot$ and the central density $\rho_c = 4.67 \times 10^9 \, g \, cm^{-3}$. For further discussion of numerical instabilities and their physical interpretation, we refer the reader to Wagenhuber & Weiss (1994) and Lau et al. (2012). A simple calculation involving the mass of the envelope at the first TP of the $8.75 \, M_\odot$ model (see Table 1) and the time spent on the TP-SAGB yields a critical mass-loss rate of $M_{\text{crit}} = 6.75 \times 10^{-4} \, M_\odot \, yr^{-1}$. That is to say, a mass-loss rate higher than $M_{\text{crit}}$ would have reduced the star to an ONe WD before it could produce an EC-SN. This critical mass-loss rate is within the wide realms applied to SAGB stars (see Poelarends et al. 2008, and references therein).

In contrast to the $8.8 \, M_\odot$ model, which is discussed in Section 3.3, there is no significant $Y_e$ reduction in the outer core, since there was no Ne–O flash. Instead, the contraction is driven by the steady growth of the core during each TP and the contraction is slower and heating competes with neutrino losses so that the core resumes cooling until electron captures by $^{24}\text{Mg}$ are activated (see Figure 3). The difference can again be seen following the depletion of $^{24}\text{Mg}$ at the center of both models, where the $8.8 \, M_\odot$ model continues to heat while the $8.75 \, M_\odot$ model again cools down. This difference in temperature between the center of the $8.8 \, M_\odot$ and $8.75 \, M_\odot$ models is important when considering the next phase of their evolution—electron captures by $^{20}\text{Ne}$.

3.3. Late Evolution of the 8.8, 9.5, and 12.0 $M_\odot$ (Massive Star) Models

The mass of the CO core, $M_{\text{CO}}$, continues to grow for the entire lifetime of the secondary C-burning shells in all models due to helium shell burning. Previous studies (see Nomoto 1984, and references therein) show that the core mass limit for neon ignition is very close to $1.37 \, M_\odot$, which our models confirm. Indeed, in all models with initial mass greater than $8.8 \, M_\odot$, a CO core develops, with a mass that exceeds the limit for neon ignition, $M_{\text{CO}}(8.8 \, M_\odot, 9.5 \, M_\odot, 12.0 \, M_\odot) = 1.3696, 1.4925, 1.8860 \, M_\odot$.

A temperature inversion develops in the core following the extinction of carbon burning in both the $8.8 \, M_\odot$ and $9.5 \, M_\odot$ models. The neutrino emission processes that remove energy from the core are (over)compensated by heating from gravitational contraction in more massive stars. However in these lower-mass stars the onset of partial degeneracy moderates the rate of contraction and hence neutrino losses dominate, cooling the central region. As a result, the ignition of neon in the $8.8$ and $9.5 \, M_\odot$ models takes place off-center, at mass coordinates of $0.93 \, M_\odot$ and $0.40 \, M_\odot$, respectively. This result confirms the work of Nomoto (1984) (case 2.6), but diverges from that of Eldridge & Tout (2004), which we will discuss later. In both models, the temperature in the neon-burning shell becomes high enough to also ignite $^{16}\text{O} + ^{16}\text{O}$. As we mention in Section 2, owing to the high densities in the cores of these stars,
the products of neon and oxygen burning are more neutron-rich than in more massive stars. This results in an electron fraction in the shell of as low as \(Y_e \approx 0.48\) (see Section 3.4 and Figure 5). Such low \(Y_e\) causes the adiabatic contraction in the following way. If the temperature is high during the flash, the flashing outer layer expands and exerts lower pressure (less weight) on the central region (as can be seen in Figure 3 labeled “Ne-flash,” \(\rho_e\) decreases due to the almost adiabatic expansion of the central region). However, when the flashed region has cooled down by neutrino emission following the extinction of nuclear burning, the outer layer shrinks and exerts more weight on the core, which is less able to provide support than before the flash because there are fewer electrons available to contribute to the degeneracy pressure. The center then reaches higher densities, and hence temperatures, than before. As mentioned above, for this reason the reduction in \(Y_e\) is important for cores so close to \(M_{Ch}\).

As illustrated in Figure 4(e), following the neon shell flashes the 9.5 \(M_\odot\) model recurrently ignites neon and oxygen burning in shells at successively lower mass coordinates that eventually reach the center, following which Si-burning is ignited off-center. Although neon (and oxygen) burning in the 8.8 \(M_\odot\) model begins as a flash and later propagates toward the center, the evolution of the 8.8 \(M_\odot\) model diverges from that of the 9.5 \(M_\odot\) star when its center reaches the conditions necessary for the first URCA process pair to become significant (whereas the 9.5 \(M_\odot\) model avoids such dense conditions). More details of the neon and oxygen shell burning episodes are discussed in Sections 3.3.1 and 3.3.2.

The CO core (or equivalently He-free core) in the 8.8 \(M_\odot\) model at the time of neon ignition is 1.36964 \(M_\odot\), very close to \(M_{Ch}\), while that of the 9.5 \(M_\odot\) model is 1.49246 \(M_\odot\) (see Table 1). Under these conditions, the 8.8 \(M_\odot\) model experiences a much more marked contraction due to the reduction in \(Y_e\). The central density at this time is as high as \(3.43 \times 10^9\) g cm\(^{-3}\), which is exceedingly close to the threshold density for \(^{27}\text{Al}(e^-\nu)^{27}\text{Mg}\). Although there is no cooling effect from the \(A = 27\) pair because the decay channels are blocked, the further removal of electrons from the core causes contraction toward the threshold densities of the second and third URCA pairs (\(A = 25\) and \(A = 23\), respectively). The cooling effect supplied by the \(A = 25\) URCA pair (and later the \(A = 23\) pair, shown in Figure 10) allows for a small amount of contraction but again it is the associated change in the electron fraction that enables the largest contraction when the core is so close to the Chandrasekhar limit \((M_{Ch} \propto Y_e^2)\). The core of the 8.8 \(M_\odot\) model continuously contracts until the center reaches the critical density for electron captures by \(^{24}\text{Mg}\), quickly followed by further contraction to the critical density for those by \(^{20}\text{Ne}\) (see Figure 3).

### Table 1

Summary of Model Properties

|          | 8.2 \(M_\odot\) | 8.7 \(M_\odot\) | 8.75 \(M_\odot\) | 8.8 \(M_\odot\) | 9.5 \(M_\odot\) | 12.0 \(M_\odot\) |
|----------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| \(M_{\text{He}}/M_\odot\) | 0.15 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| \(M_{\text{Ne}}/M_\odot\) | ... | ... | ... | 0.93 | 0.42 | 0.00 |
| \(T_{\text{Ne}}/\text{ GK}\) | ... | ... | ... | 1.318 | 1.311 | 1.324 |
| \(\rho_{\text{Ne}}\) | ... | ... | ... | 46.0 | 15.2 | 5.6 |
| \(R_{\text{Ne}}^c/\text{ g cm}^{-3}\) | ... | ... | ... | 3.343 \times 10^8 | 7.396 \times 10^7 | 1.730 \times 10^7 |
| \(M_{\text{tot}}/M_\odot\) | 7.299 | 7.910 | 8.572 | 8.544 | 9.189 | 11.338 |
| \(M_{\text{env}}/M_\odot\) | 6.031 | 6.559 | 7.210 | 7.174 | 6.702 | 8.023 |
| \(M_{\text{tot}}/M_\odot\) | 1.26721 | 1.35092 | 1.36230 | 1.36967 | 2.48733 | 3.31580 |
| \(M_{\text{env}}/M_\odot\) | 1.26695 | 1.35086 | 1.36227 | 1.36964 | 1.49246 | 1.88602 |
| Remnant | ONe WD | ONe WD/NS | NS | NS | NS | NS |
| SN type | ... | ... | EC-SN (IIP) | EC-SN (IIP) | EC-SN (IIP) | CC-SN (IIP) |

**Notes.**

a Mass coordinate of carbon ignition.

b Mass coordinate of neon ignition.

c Temperature at locus of neon ignition.

d Central degeneracy at time of neon ignition.

e Central density at time of neon ignition.

f Total mass at time of first thermal pulse or neon ignition.

g Envelope mass at time of first thermal pulse or neon ignition.

h Helium core mass (H-free core mass) at time of first thermal pulse or neon ignition.

i Carbon–oxygen core mass (He-free core mass) at time of first thermal pulse or neon ignition.

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**Figure 5.** Radial profiles of the electron fraction, \(Y_e\), in the progenitor structure of the 8.8 \(M_\odot\) model and the 8.75 \(M_\odot\) model after central \(^{24}\text{Mg}\) depletion. The silicon-rich shell of the 8.8 \(M_\odot\) model, where the material has been processed by the neon–oxygen shell flashes, displays a severely reduced electron fraction, reflected by high ratios of \(^{28}\text{Si}/^{29}\text{Si} and \(^{32}\text{S}/^{32}\text{S}.\n
(A color version of this figure is available in the online journal.)
There is a significant discrepancy between the URCA process trajectories of our models and those of Ritossa et al. (1999). This is due to the undersampling of weak reaction rates for the URCA process that we employ in the MESA code (Oda et al. 1994). In Section 4.1, we discuss the implications of this undersampling and show that, using new well-sampled weak rates (Toki et al. 2013), the URCA process central trajectory of Ritossa et al. (1999) is qualitatively reproduced in the 8.8 $M_\odot$ case. The difference between the Oda et al. (1994) compilation and the newly calculated Toki et al. (2013) rates that are available in our calculations for the $A = 25$ pair is shown in Figure 6.

This central evolution is significantly different from that for the 8.75 $M_\odot$ model, which is described in Section 3.2. The energy release from both the rapid contraction and the $\gamma$-decays from electron capture products raises the temperature high enough to ignite neon and oxygen in quick succession. We have followed from the resulting oxygen deflagration onward with the AGILE-BOLTZTRAN hydrodynamics code and can confirm that the model results in core collapse (T. Fischer 2013, in preparation). Although Eldridge & Tout (2004) report the same fate for their 10 $M_\odot$ model in which a limited network was used, there is no neon shell flash following the completion of the 2DUP. In these models, neon burning was found to take place at the edge of the core during the last carbon shell flash, reducing the core mass to $M_{cb}$ (Eldridge 2005; J. J. Eldridge, 2013, private communication). Subsequently, the core contracted directly to central densities of about $\log_{10}(\rho Y_e/\text{g cm}^{-3}) = 9.8$ (roughly the critical density for electron captures by $^{20}\text{Ne}$ to start) with no further neon shell flashes, though electron captures were not included in the nuclear reaction network. Neon-burning reaction rates were artificially limited to prevent numerical problems and a low spatial resolution was used. We believe these two caveats to be the reason that the neon–oxygen shell flashes we find to occur in such stars were not present in these earlier models.

In this work, we were able to follow the evolution all the way to oxygen deflagration by using a very large network of 114 nuclei including all the relevant fusion and weak reactions. Our models thus highlight the importance of neon-shell burning in determining the path to collapse.

As mentioned above, the 9.5 $M_\odot$ model starts silicon-burning off-center in a shell that later propagates toward the center. This is another example of the continuous transition toward massive stars, in which all the burning stages begin centrally. Although we have not evolved this model to its conclusion, we expect that silicon burning will migrate to the center, producing an iron core, and that it will finally collapse as an FeCCSN. Such a low-mass progenitor will make for interesting explosion simulations (Müller et al. 2012). The 12 $M_\odot$ is the canonical massive star in our grid, igniting C-, Ne-, O-, and Si-burning centrally (see Figure 4(f)). It eventually collapses, and would produce a type-II FeCCSN.

### 3.3.1. Neon–Oxygen Flashes

As briefly mentioned above, following the extinction of the final carbon-burning shell, a degeneracy neutrino-induced temperature inversion arises in the core in a similar way to the temperature inversion in SAGB stars. This causes the ignition of carbon to take place away from the center. Neon is thus ignited off-center at mass coordinates of 0.93 $M_\odot$ and 0.40 $M_\odot$ for the 8.8 $M_\odot$ and 9.5 $M_\odot$ models, respectively. Some of the important model properties are given in Table 1 at this time.

At the point of Ne-shell ignition, the density profile of the two stars is very different (see Figure 7). While the 8.8 $M_\odot$ model is structured more like an SAGB star due to the previous dredge-out episode, the 9.5 $M_\odot$ model resembles more a massive star, with a distinct He-shell and C-shell still present.

When neon and oxygen is first ignited, fuel is abundant and a convective shell quickly develops. The sharp increase in energy production briefly halts the contraction of the core and causes the center to expand and cool (see Figure 3), while convection brings in fresh fuel to be burnt at the base of the shell. Conduction at the base of the shell is slow and so when the fuel is depleted the shell is extinguished and the core contracts. This contraction continues until the temperature becomes high enough where neon and oxygen is abundant, re-igniting the nuclear burning and producing a new convective shell. After a few flashes, the region previously engulfed by the shell convective shell as it extended radially outward has become heavily depleted in Ne and O and so the closest fuel is in the direction of the center. At this point a new regime, the Ne–O flame, is begun.

### 3.3.2. Neon–Oxygen Flame

In this section, and throughout the remainder of the manuscript, it should be noted that we use the term flame to describe the inward propagation of a nuclear burning shell, driven...
deflagration, and collapse, respectively. The 9.5 M⊙ model, except that the core is more degenerate in the 8.8 M⊙ star. Electron conduction is therefore much more efficient and the localized effect of heat generation due to contraction and any subsequent nuclear burning is instead diluted across the core. This smoothing of the temperature profile across the core prevents the region directly below the previously ONe-burning shells from reaching temperatures in excess of the Ne-burning threshold. Instead of a flame developing as in the 9.5 M⊙ star, the core contraction, driven by the neutron-rich composition in the NeO shell, causes local heating much further from the center where the degeneracy is lower, and a new neon- and oxygen-burning shell ignites (where the fuel is still abundant) above the outermost extent of the previous ONe shells.

Figure 7 shows the opacity profiles following the extinction of the last neon–oxygen flash and at a later time in each model. Although electron conduction dominates the heat transfer in both cases, it is more efficient (lower κ) by a factor of about three in the 8.8 M⊙ model’s early flame and by a factor of more than 10 later, meaning that any energy production from subsequent radiative neon–oxygen burning or contraction is diluted across the majority of the core. In contrast, the higher conductive opacities in the 9.5 M⊙ model allow for the nuclear and compressional energy to take effect much more locally, heating the underlying shell of material to ignition temperatures and causing the development of a nuclear flame. These two contrasting paths are further illustrated in Figure 9, which shows clearly a flame front developing in the 9.5 M⊙ model and the dilution of heat across the core of the 8.8 M⊙ model. The effects of spatial resolution on flame development and energy transport are discussed in Section 4.3.

In summary, the propagation of the flame in the 8.8 M⊙ model is more difficult because of the lower opacity. Furthermore, the combined effect of electron captures in the center and the low Ye in the shell due to Ne- and O-burning leads to the core contraction on a shorter timescale than the evolution of the flame.

3.4. Progenitor Structure

The structure of the progenitor star, in terms of density and electron-fraction profiles of the stellar core, has a strong impact on the timescale at which the later SN explosion may develop as well as on the explosion energetics. Core-collapse SN explosions are related to the revival of the stalled shock wave, which forms when the contracting core reaches normal nuclear matter density and bounces back. For massive iron-core stars, the structure of the core at the onset of contraction is determined by the mass enclosed inside the carbon shell. In general, a sharp density gradient separating iron-core and silicon layer results in a strong acceleration of the bounce shock at the onset of shock revival early after core bounce on a timescale of only few 100 ms. Progenitors with a shallower density gradient suffer from a more extended mass accretion period after core bounce, during which the standing bounce shock oscillates, driven by neutrino-energy deposition behind and mass accretion from above. This results in a delayed onset of shock revival by several 100 ms and more energetic explosions due to the larger heat deposition behind the shock via neutrinos before shock expansion. For a recent review of the connection between progenitor structure and recent axially symmetric SN explosion models, see Janka et al. (2012).
(a) 8.8 $M_\odot$ early flame

(b) 9.5 $M_\odot$ early flame

(c) 8.8 $M_\odot$ flame failure

(d) 9.5 $M_\odot$ late flame

Figure 8. Radial profiles with respect to mass coordinate of the radiative ($\kappa_{\text{rad}}$), conductive ($\kappa_{\text{ec}}$), and total ($\kappa_{\text{tot}} = [1/\kappa_{\text{rad}} + 1/\kappa_{\text{ec}}]^{-1}$) opacities following the extinction of the final neon–oxygen convective flash episode. The heat transport in both stars is dominated by conduction (lower $\kappa$), however a stable nuclear flame only develops in the 9.5 $M_\odot$ model by virtue of its higher total opacity allowing for heating to take effect on a much more local scale than in the 8.8 $M_\odot$ model.

(A color version of this figure is available in the online journal.)

In addition to the standard iron-core progenitors commonly explored in core-collapse SN studies, we provide a selection of new models of lower zero-age main-sequence mass that belong to the SAGB class as well as to low-mass massive stars. Therefore, in Figure 7, we compare the structures of our SAGB model (8.75 $M_\odot$) after central $^{24}\text{Mg}$ depletion, EC-SN progenitor (8.8 $M_\odot$, failed massive star) at ignition of oxygen deflagration, low-mass massive star (9.5 $M_\odot$) at the point of neon-shell ignition, and standard iron-core progenitor (12 $M_\odot$) at the onset of core contraction. Note that the 9.5 $M_\odot$ progenitor is not then as evolved as the other models and hence its central density is still lower than those of the other models. It is therefore only used as a reference case. The major difference between the low-mass (8.75 and 8.8 $M_\odot$) and the more massive iron-core progenitors is the very steep density gradient separating the core and the envelope. There the density drops about 16 orders of magnitude, from about $10^8$ to $10^{-8}$ g cm$^{-3}$.

Distinguishing the 8.75 $M_\odot$ and 8.8 $M_\odot$ progenitor structures becomes clearer when inspecting the density profiles with respect to radius (Figure 7(b)). The bulge from log$_{10}(R/km) \approx 3.2$ to 3.8 that features in the 8.8 $M_\odot$ structure but is absent in the 8.75 $M_\odot$ structure, is a carbon-burning shell. One would expect that, since the 8.8 $M_\odot$ model experienced several neon–oxygen flashes, the structure within the core should be significantly different from that of the SAGB model. Aside from the abundance profiles showing a large region in which the composition is dominated by Si-group isotopes, the most striking difference is in the electron fraction, $Y_e$, which is shown in Figure 5.

In Figure 7, we have included the progenitor structures of the Nomoto (1987; SAGB-like) 8.8 $M_\odot$ and the Woosley et al. (2002) 12 $M_\odot$ models for comparison. The Nomoto (1987) structure is at a later evolutionary stage compared to our models. A fraction of the core has already been burnt to nuclear statistical equilibrium (NSE) composition, but the core structure is qualitatively similar to our 8.75 $M_\odot$ SAGB model. It is also clear from Figure 7, bottom panel, that there are differences in the structure of the Nomoto (1987; SAGB-like) model and our 8.8 $M_\odot$ (failed massive star) model, where there is a CO-rich
layer at the edge of the core. As discussed previously, there is a neutron-rich layer in our 8.8 $M_\odot$ model where the Ne–O shell flash consumed previously that is not a feature of the Nomoto (1987) model. There is a clear clustering of the SAGB EC-SN progenitor structures and the CCSN progenitor structures in the density profiles as a function of radius (Figure 7, bottom panel), while the 8.8 $M_\odot$ model lies in-between.

The iron-core progenitors have extended high-density silicon as well oxygen and carbon layers above the core. These result in a shallower transition from iron core to helium envelope. The density decreases steadily stepwise according to the different composition interfaces (see Figure 7, top panel). Moreover, different evolutionary tracks for the 8.75 and 8.8 $M_\odot$ progenitor cores lead to low-mass cores of only about 1.376 $M_\odot$, which is significantly lower than for the 12 $M_\odot$ model of 1.89 $M_\odot$ (see Table 1). Note that the 12 $M_\odot$ iron-core results are in qualitative agreement with those of the KEPLER code (Woosley et al. 2002) and, as a function of radius, match very well. The reason for the discrepancy between the two as a function of mass is the difference in assumption for convective overshooting, which has led to the production of larger cores in the MESA model. We are currently working on a code comparison study of MESA, KEPLER, and the Geneva stellar evolution code (Hirschi et al. 2004) for the evolution, explosion, and nucleosynthesis of massive stars in order to quantify some of the related uncertainties. We expect that the resulting steep density gradient at the edge of the core of our EC-SN progenitor models will accelerate the SN shock on a short timescale after core bounce, producing a weak explosion with little $^{56}$Ni ejecta. Such an explosion should produce qualitatively similar results as obtained for the 8.8 $M_\odot$ progenitor from Nomoto (1987; for details about EC-SN explosions see Kitaura et al. 2006; Janka et al. 2008; Fischer et al. 2010). The split between weaker, more rapid EC-SN explosions and stronger, slower FeCCSN explosions is a possible explanation for the observed bimodality in the spin period and orbital eccentricity of X-ray binaries, although it is not clear how this is manifested (Knigge et al. 2011).

4. KEY NUCLEAR AND MODELING UNCERTAINTIES

In this section, we discuss the main modeling uncertainties affecting the study of stars in the transition mass range. We propose some solutions and suggest ways in which future studies could improve on our models in order to quantify and minimize these uncertainties.

4.1. Weak Reaction Rates and the URCA Process

During the very late stages of the 8.75 and 8.8 $M_\odot$ stars electron captures on $^{23}$Mg and $^{23}$Na cool the central regions while those on $^{27}$Al provide little contribution due to the low abundance of fuel and the Pauli blocking of $^{27}$Mg $\rightarrow$ $^{27}$Al + $^8$Be $+ \bar{\nu}_{e}$. (A color version of this figure is available in the online journal.)
density the oxygen deflagration is ignited. We should want to know that density so that it can be determined whether nuclear energy release from burning the core to NSE composition is high enough to exceed the gravitational binding energy of the core and thus lead to its explosion (Gutierrez et al. 1996). Otherwise, the core would collapse to a neutron star following its core and thus lead to its explosion (Gutierrez et al. 1996).

There are still more shortcomings of calculations involving electron capture rates that are poorly resolved in the $\rho-T$ plane. For example, the vast majority of widely used rate tables for sd-shell nuclei possess a grid spacing of 1 dex in $\rho Y_e$. As an example one of these crucial reactions, $^{24}\text{Mg} + e^-$, jumps by about 20 orders of magnitude from $\log^{10}(\rho Y_e/\text{g cm}^{-3}) = 9.0$ to 10.0 at the temperature of interest ($T \approx 0.4\ \text{GK}$). This is not only a problem for resolving the rate at the threshold density, because at lower densities the rate, $\lambda/s^{-1}$, is significantly underestimated through linear interpolation of $\log^{10}(\lambda/s^{-1})$ (see Figure 11).

It is clear from Figure 3 that the central evolution of the 8.75 and 8.8 $M_\odot$ models is dominated by weak reactions. The onset of the URCA process disrupts the propagation of the neon–oxygen flame and aids the central contraction. In fact, the same is true for all EC-SN progenitors and thus it is imperative to treat the URCA process as accurately as possible to best predict the fate of 8–12 $M_\odot$ stars. Toki et al. (2013) have produced well resolved ($\Delta \log^{10}(\rho Y_e/\text{g cm}^{-3}) = 0.02$ and $\Delta \log^{10}(T/\text{K}) = 0.05$) reaction and neutrino-loss rates for the $A = 23, 25$, and 27 URCA pairs under the conditions $7.0 \leq \log^{10}(T/\text{K}) \leq 9.2$ and $8.0 \leq \log^{10}(\rho Y_e/\text{g cm}^{-3}) \leq 9.2$. The differences between these new rates and those of Oda et al. (1994) is shown in Figure 6 for the $A = 25$ pair at $T_9 = 0.4$. The impacts of these new, well-resolved rates compared to those of Oda et al. (1994) are shown in Figure 12. Not only is the cooling effect more pronounced, the reaction thresholds are more clearly identifiable and occur at higher densities than with the rates of Oda et al. (1994). The cooling effect is also more pronounced.

Because the rates are so sensitive to density, any form of interpolation cannot properly represent the physical situation without some input from knowledge of the nuclear physics. This is why several groups employ an interpolation of effective log $ft$ values (Fuller et al. 1985). An effective log $ft$ value for a reaction is related to its raw rate by the relationship in Equation (2),

$$ft = \frac{\phi}{\lambda} \ln 2,$$

where $\phi$ is the ground-state to ground-state phase space integral. The aim is to produce a quantity that varies smoothly with $T$ and $\rho$ from which the raw rate may be obtained within a stellar evolution calculation by approximation of the phase space integral at the desired conditions. This method is relatively robust for those weak rates for which ground-state to ground-state transitions dominate. However, this is not the case for the reactions of interest in EC-SN progenitors. The change in $Y_e$ is not the only important facet of the electron captures; they also possess a strong heating effect due to the $\gamma$-decay following transitions to excited states of the daughter nuclei. Hence, this demonstrates the importance of excited states when we attempt to normalize the reaction rate using simplifications or approximations.

Therefore, we conclude that there are two possible sets of desired quantities, either grids of weak reaction rates for sd-shell nuclei that are appropriately resolved through the threshold density or log $ft$ values that incorporate all important transitions in the normalization of the rate. There are contributions from many states of the parent and daughter nuclei for these reactions and to perform phase space integral routines within a stellar evolution code to account for this could be exceptionally inefficient. It is also important to use $\beta^-$-decay and neutrino-loss rates calculated with the same physics and grid resolution to ensure consistency when we look at the impact of the URCA process on the evolution. The most up-to-date rates should also include the effects of Coulomb screening, which has been shown to increase the threshold density for electron captures (Gutierrez et al. 1996). An increase in the threshold density of $^{20}\text{Ne}(e^-, \nu)^{20}\text{F}$ would cause the oxygen deflagration to ignite under denser conditions in the SAGB progenitors. However in the failed massive star case the center is approaching the ignition temperatures of Ne and O adiabatically, and so the
oxygen deflagration could be ignite before $^{20}$Ne + $e^-$ becomes significant if there were an increase in the threshold density.

4.2. Uncertainties Due to Convection

Still one of the largest uncertainties in any one-dimensional stellar evolution calculation is the treatment of convection. Extra mixing at convective boundaries may explain many observed phenomena, for example, the s-process abundance patterns in AGB stars, and hence we include such mixing in our models. Due to the turbulent and advective nature of convection, it is physically plausible to infer some extra mixing across the boundary between convective and radiative layers but without the benefit of three-dimensional hydrodynamical simulations of the physical conditions it is difficult to quantify its extent. We use the term convective boundary mixing rather than overshooting for the advanced evolution phases of the deep stellar interior, such as convective shells. This is because the term overshooting suggests a physical picture in which coherent convective structures or blobs cross the Schwarzschild boundary before they notice the reversal of buoyancy acceleration. However, in the deep interior other hydrodynamic instabilities, such as Kelvin–Helmholtz or internal gravity wave induced turbulence dominate mixing at the convective boundary. Largely, the effect of including convective boundary mixing is to shift the transition masses due to increased core sizes. However it is intuitive to hypothesize that increased amounts of extra mixing below the ONe-burning shells would have a crucial effect on their inward propagation. To test this, we assumed extra mixing below the convective ONe-burning shells to behave as an exponentially decaying diffusion process as outlined in Equation (1) with $f_{\text{flame}} = 0.005$ (our original assumption), 0.014, 0.028, and 0.100 (extreme), where $f_{\text{flame}}$ is the value of the parameter $f$ in Equation (1) at the base of the ONe-burning shell. (A color version of this figure is available in the online journal.)

Figure 13. Central density–temperature evolution of the 8.8 $M_\odot$ model showing the differences created when we assumed $f_{\text{flame}} = 0.005$ (our original assumption), 0.014, 0.028, and 0.100 (extreme), where $f_{\text{flame}}$ is the value of the parameter $f$ in Equation (1) at the base of the ONe-burning shell.

The development and propagation of a nuclear flame front is highly sensitive to the spatial resolution due to the thin flame width. Figure 9 showed the evolution of the $T$-profile at the time of flame development/propagation in both models. Each red dot represents a mesh point in the calculation. It can be seen that the model possesses a spatial resolution much finer than the width of the flame front, however in the transition at the base of the flame it is evident that there is a less than desirable resolution very early on in its development.

To examine the effect of spatial resolution on the outcome of the 8.8 $M_\odot$ model, we increased the resolution of the model 10-fold and then 20-fold at the base of the Ne + O-burning convection zone, from before the ignition of the Ne + O flame. In a second test, we increased the resolution 10-fold in the regions where energy production from $^{16}$O($\alpha$, $\gamma$)$^{20}$Ne, $^{20}$Ne($\alpha$, $\gamma$)$^{24}$Mg, $^{16}$O($^{16}$O, $\gamma$)$^{32}$S, $^{16}$O($^{16}$O, $p$)$^{31}$P, or $^{4}$He($^{16}$O, $\alpha$)$^{20}$Si became significant (greater than $10^{4}$erg g$^{-1}$s$^{-1}$). Neither of the enhanced resolutions at the base of the convective shell alter the outcome of the 8.8 $M_\odot$ model (EC-SN), and nor does the re-meshing based on energy production. This demonstrates that our results concerning Ne–O flame propagation are robust and not due to an underresolved flame front.

5. DISCUSSION AND CONCLUDING REMARKS

We have begun to explore in detail the transition mass between SAGB stars and massive stars. Using the MESA code, we were able to model stars across the transition (AGB, SAGB, EC-SNe progenitors, and massive stars) with a consistent set of input physics, while current published stellar evolution calculations limit themselves to either massive stars or SAGB stars.

We were able to follow the evolution of the entire star from pre-MS up to the ignition of an oxygen deflagration for the
the critical density is already reached for $^{24}\text{Mg}$ + which, during this phase, is shorter than the central contraction timescale and the envelope and the TP-SAGB phase, and the $^8\text{Be}$ case is the first EC-SN progenitor model published including the second neon flash event in the $8.8\ M^\odot$ model. Using the AGILE-BOLTZTRAN hydrodynamics code, we confirmed the $8.8\ M^\odot$ model to result in core collapse—an EC-SN (T. Fischer 2013, in preparation). Our models confirm the no-flame assumption adopted purely for the purpose of testing the robustness of our models.

For this reason, we stress the importance of further investigation into the initial mass range between $8.8\ M^\odot$ and $9.5\ M^\odot$ from our two models, an initial mass range of only $0.7\ M^\odot$ contains about 15% of all single stars with the potential to give birth to a NS (assuming a Salpeter IMF and that single stars in the mass range $8.5 \lesssim M/M^\odot \lesssim 20$ produce neutron stars in their deaths). For this reason, we stress the importance of further investigation into the initial mass range between $8.8\ M^\odot$ and $9.5\ M^\odot$. From examination of these two models in our set, there may be an interesting correlation between the propagation of the neon–oxygen flame and the URCA process.

If both failed massive stars and SAGB stars have the potential to produce EC-SNe then the EC-SN channel is wider than we think at present. It is our intention to produce EC-SN progenitor models from both SAGB stars and failed massive stars for several metallicities. Detailed SN simulations with our models and including full nucleosynthesis will help constrain what observational features and nucleosynthesis we can expect from EC-SNe.

The research leading to these results has received funding from the European Research Council under the European Union’s Seventh Framework Programme (FP2007-2013)/ERC Grant Agreement No. 306901. NuGrid acknowledges significant support from NSF grants PHY 02-16783 and PHY 09-22648 (Joint Institute for Nuclear Astrophysics, JINA). R.H. thanks the Eurocore project Eurogenesis for support. K.N., R.H., and S.J. acknowledge support from the World Premier International Research Center Initiative (WPI Initiative), MEXT, Japan. T.F. acknowledges support from the Swiss National Science Foundation under project no. PBBS2-133378 and HIC for FAIR. B.P.’s research has been supported by the National Science Foundation under grants PHY 11-25915 and AST 11-09174. M.G.B.’s research was carried out under the auspices of the National Nuclear Security Administration of the U.S. Department of Energy at Los Alamos National Laboratory under contract No. DE-AC52-06NA25396.
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