Mathematical model of concrete strain diagram under heating

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\textbf{Abstract.} New regulatory documents have been adopted and they have significant differences. Firstly, this is a new concept of calculation, a nonlinear deformation model, and secondly, new approaches to the assessment of reliability, loads and impacts, methods of construction. The calculation consists of two stages. The first begins with the determination of the concrete class at temperature of 20°C. At the second stage, the calculation is carried out at high temperatures according to Eurocode 2 part 1-2. Comparison of the "stress - strain" diagram of concrete class at compression and temperature of 20°C by two formulae showed their big gap. There was a need to improve the mathematical model of the ratio of stress-strain of the concrete under elevated temperatures. The method of determination of maximum strain based on energy approach has been developed in the work and allowed to formulate the adjusted dependence of the limit strain on temperature, dependence of maximum strain on temperature, values of the parameters of stress-strain diagram.

1. Introduction

At present, the regulatory framework of Ukraine has been changed to European standards. The process of harmonization with the Eurocodes has been completed, new national regulatory documents of the DNB V.2.6-98:2009 (National Building Standards) and DSTU-N B EN 1992-1-2:2012 (National Standards of Ukraine), which have replaced the SNiP 2.03.01-84* (Civil Design Codes), have been developed and put into practice. However, a large section of "Concrete and Reinforced Concrete Structures Intended to Work under Elevated and High Temperatures" turned out to be omitted in the Eurocodes, for which SNiP building standards and rules were developed.

The new regulation documents have significant differences. Firstly, this is a new concept of calculation - a nonlinear stress-strain model, and secondly, new approaches to the assessment of reliability, loads and impacts, methods of construction.

2. Analysis of recent research

On account of the implementation of DNB V.2.6-98:2009, the methodology and principles of calculating the load-bearing capacity of reinforced concrete structures have significantly changed compared to the calculation of SNiP 2.03.01-84*: the model of calculation for the destructive forces, which is based on the principle of ductile fractures. According to this principle, the strength of the normal section is determined by its stress-strain state at the fracture stage, when the stresses in the reinforcement and concrete reach the limit values simultaneously. The model of calculation for the destructive forces is replaced by the model of the deformation method, according to which the criterion for the appearance of limit state in the normal section is accepted as the reaching by deformations of
compressed concrete or stretched reinforcement of the limit values of the relative strains $\varepsilon_{cu}$ and $\varepsilon_{ck}$ on the respective diagrams of their state (Figure 1 and Figure 2).

In general, to determine the load-bearing capacity, curvature, displacement, redistribution of forces in statically indeterminate structures, it is necessary to start from the stress-strain state of reinforced concrete sections, determined on the basis of the use of a non-linear diagram of stress-strain state (Figure 1).

The compressive strength of concrete is determined by the concrete classes C, which are related to the characteristic cube strength with assured probability of 95%.

The strength classes in these standards are based on the characteristic cubic strength determined on the 28th day with a maximum value of C50/60 and with a statistical reliability of 0.95. The accumulation of sufficient data for the standardization of the database DBN V.2.6-98:2009 on the physical and mechanical characteristics of concrete classes C55/70-C100/115 as for using them in the practice of design and construction is possible providing the implementation of experimental studies performed by the basic scientific and technical organization activities in accordance with the program approved by the profile ministry.

Figure 1. Diagram of concrete stress-strain

Figure 2. Diagrams of tensile stress-strain for conventional reinforcing steel that has (a) and does not have (b) physical yield line
In certain cases (e.g. prestress), it is appropriate to determine the compressive strength of concrete before or after 28 days on the basis of samples test being stored under conditions other than those determined in DSTU B V.2.7-214:2009.

The tensile strength of concrete is based on the highest tensile stresses and can be determined by its strength class. The elasticity modulus of concrete depends on the elasticity moduli of its components. The corresponding values of the modulus of elasticity $E_{cm}$, the magnitude of the tangent of the secant angle between $\sigma_c = 0$ and $0.3f_{cm}$ for concrete on silicate (quartz) aggregates, are shown in Table 1, for carbonate aggregates (from limestone and sandstone), the values of the modulus of elasticity, respectively by 10% and 30%. For basalt aggregate, the $E_{cm}$ value should be increased by 20%. The value $0.3f_{cm}$ for determining $E_{cm}$ is approximate.

The Poisson's ratio can be assumed 0.2 at stress levels not exceeding 0.5 $f_{cm}$ for crack-free concrete and zero for cracked concrete.

In the absence of more accurate data, the coefficient of linear thermal expansion can be taken as equal $1\times10^{-5}$ °C$^{-1}$.

3. Results of theoretical research

3.1. Stress-strain dependence for nonlinear calculations of structures

In general, to determine the bearing capacity, curvature, displacement, redistribution of efforts in statically indeterminate structures it is necessary to take into account the stress-strain state of reinforced concrete sections defined on the basis of nonlinear stress-strain diagram. The relationship between $\sigma_c$ and $\varepsilon_c$ is shown in Figure 1 for short-term loading, which is described by equations of type (2) or (3).

3.2. Calculated compressive and tensile strength of concrete

The value of the calculated compressive strength of concrete is defined as:

$$f_{cd} = \alpha_{cc} f_{ck} / \gamma_c,$$  

where $\gamma_c$ - safety factor for concrete, $\alpha_{cc}$ - factor taking into account the impact on the strength of concrete compressive duration of actions and adverse effects caused by means of applying the load.

The value of $\alpha_{cc}$ can vary from 0.8 to 1.0. Recommended value $\alpha_{cc} = 1.0$.

The first stage begins with the determination of their bearing capacity at a normal temperature of 20°C, i.e. when using Eurocode 2 EN 1992-1-1:2005 or DBN V.2.6-98:2009. In these standards, it is proposed to use two equations to describe the relationship between $\sigma_c$ and $\varepsilon_c$ for short-term axial loading.

The equation (2) that is used in Eurocode 2 EN 1992-1-1:2005:

$$\frac{\sigma_c}{f_{cm}} = \frac{k\eta - \eta^2}{1+(k-2)\eta}$$  

where $\eta = -\varepsilon_c / \varepsilon_{c1}$, $\varepsilon_{c1}$ is strain at maximum stresses according to Table 1.

$k = 1.05E_{cm}\varepsilon_{c1}/f_{cm}$ ($f_{cm}$ - according to Table 1) and the equation (3) in the form of a fifth degree polynomial, which is based on the results of numerous experimental studies of the National Scientific Research Institute, which statistical processing allowed to offer a more complete regulatory framework.

$$\sigma_c = f_{(ck),(cd)} \sum_{k=1}^5 a_k \eta^k.$$  

Usage limits are defined for both formulae:

$$0 < |\varepsilon_c| < |\varepsilon_{c1}|,$$

where $\varepsilon_{c1}$ is nominal limit strain of concrete.

In the work it is also shown that the graphs according to DBN B.2.6 - 98:2009 and EN 1992-1-1 practically coincide on the ascending branch. In both regulations, the classes of concrete, the values of the relative strain of concrete compression $\varepsilon_{c1}$ at maximum stresses $f_c$, the nominal marginal strain of
concrete $\varepsilon_{cul}$ (the value of relative marginal strain of concrete compression) and the average value of the initial modulus of elasticity of concrete $E_{cm}$ (GPa) are given. The data in Tables 1 and 2 are sufficient to calculate reinforced concrete structures at 20°C.

The characteristic strength values and the corresponding mechanical properties of the concrete required for the design are given in Tables 1 and 2.

**Table 1.** Concrete strength classes according to Eurocode 2 EN 1992-1-1:2005

| $f_{ck,cube}$ (MPa) | 15 | 20 | 25 | 30 | 37 | 45 | 50 | 55 | 60 | 67 | 75 | 85 | 95 | 105 |
|---------------------|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| $\varepsilon_{cl}$ (‰) | 1.8 | 1.9 | 2.0 | 2.1 | 2.2 | 2.25 | 2.3 | 2.4 | 2.45 | 2.5 | 2.6 | 2.7 | 2.8 | 2.8 |
| $\varepsilon_{cul}$ (‰) | 3.5 | 3.5 | 3.5 | 3.5 | 3.5 | 3.5 | 3.5 | 3.5 | 3.2 | 3.0 | 2.8 | 2.8 | 2.8 | 2.8 |
| $E_{cm}$ (GPa) | 27 | 29 | 30 | 31 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 41 | 42 | 44 |

**Table 2.** Concrete strength classes according to DBN V.2.6-98:2009

| $f_{ck,cube}$ (MPa) | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45 | 50 | 55 | 60 |
|---------------------|----|----|----|----|----|----|----|----|----|----|----|
| $E_{cl}$ (‰) | 1.56 | 1.58 | 1.62 | 1.65 | 1.69 | 1.72 | 1.76 | 1.80 | 1.84 | 1.87 | 1.91 |
| $\varepsilon_{cul}$ (‰) | 3.37 | 3.70 | 3.59 | 3.44 | 3.28 | 3.10 | 2.93 | 2.72 | 2.57 | 2.43 | 2.29 |
| $E_{cm}$ (GPa) | 18 | 23 | 27 | 30 | 32.5 | 34.5 | 36 | 37.5 | 39 | 39.5 | 40 |

Thus, using basic Eurocode 2 EN 1992-1-1:2005 or national building codes of Ukraine DBN B.2.6-98:2009, which takes into account the basic provisions (principles) of Eurocode 2 EN 1992-1-1:2005, calculations of reinforced concrete structures are made at 20°C. The classes of concrete strength, compression strain $\varepsilon_{cl}$ at maximum stresses $f_{cm}$, nominal concrete strain limit $\varepsilon_{cul}$, average value of the initial modulus of elasticity of concrete $E_{cm}$ and the complete stress-strain diagram of concrete are determined [1].

For example, for the concrete strength class 30 according to Eurocode 2 EN 1992-1-1 concrete strain at maximum stresses $f_c = 30$ MPa equal to $\varepsilon_{cl} = 2.1$ ‰, the nominal limit strain of concrete $\varepsilon_{cul} = 3.5$ ‰, the average value of the initial modulus of elasticity of concrete $E_{cm} = 31$ GPa, the complete diagram of the "stress - strain" of concrete is determined by the formula (3.1).

The concrete strain for concrete strength class C30/35 at maximum stresses $f_{cm} = 35$ MPa is equal to $\varepsilon_{cl} = 1.72$ ‰, the nominal limit concrete strain $\varepsilon_{cul} = 3.1$ ‰, the average value of the initial modulus of the basic parameters of the strain diagram of compressed concrete at elevated temperatures of concrete elasticity $E_{cm} = 34.5$ GPa [2, 3], the complete diagram of the "stress - strain" of concrete is determined by formula (2) according to DBN V.2.6-98:2009.

Another task of the mathematical model of the concrete strain diagram when heated is Eurocode EN 1992-1-2:2004 and examples of application of heat-resistant concrete in the elements of structures of thermal units [4]. Instead of a real fire, we use the Eurocode table.
Table 3. The values of basic parameters of the strain diagram of compressed concrete at elevated temperatures

| Concrete temperature $\theta$, °C | Silicate aggregate | Carbonate aggregate |
|-----------------------------------|--------------------|--------------------|
|                                   | $f_c,0/f_{ck}$ | $\varepsilon_{c1,0}$ | $\varepsilon_{c1,0}$ | $f_c,0/f_{ck}$ | $\varepsilon_{c1,0}$ | $\varepsilon_{c1,0}$ |
| 20                                | 1.00              | 0.0025             | 0.02              | 1.00              | 0.0025             | 0.02              |
| 100                               | 1.00              | 0.004              | 0.0225            | 1.00              | 0.004              | 0.0225            |
| 200                               | 0.95              | 0.0055             | 0.025             | 0.97              | 0.0055             | 0.025             |
| 300                               | 0.85              | 0.007              | 0.0275            | 0.91              | 0.007              | 0.0275            |
| 400                               | 0.75              | 0.01               | 0.03              | 0.85              | 0.01               | 0.03              |
| 500                                | 0.60              | 0.015              | 0.0325            | 0.74              | 0.015              | 0.0325            |
| 600                                | 0.45              | 0.025              | 0.035             | 0.60              | 0.025              | 0.035             |
| 700                                | 0.30              | 0.025              | 0.0375            | 0.43              | 0.025              | 0.0375            |
| 800                                | 0.15              | 0.025              | 0.040             | 0.27              | 0.025              | 0.04              |
| 900                                | 0.08              | 0.025              | 0.0425            | 0.15              | 0.025              | 0.0425            |
| 1000                               | 0.04              | 0.025              | 0.045             | 0.06              | 0.025              | 0.045             |
| 1100                               | 0.01              | 0.025              | 0.0475            | 0.02              | 0.025              | 0.0475            |
| 1200                               | 0.00              | —                  | —                 | 0.00              | —                  | —                 |

The method of determination of $\varepsilon_{c1,0}$, on the basis of the energy approach [6-8] has allowed us to formulate the adjusted dependence of the limit strain on the temperature, the dependence of the maximum strain on the temperature, and the values of the parameters of the stress-strain diagram [9-10]. According to these data, the stress-strain diagrams of concrete when compressed and heated are calculated using the formulae of the first stage according to Eurocode EN 1992-1-2:2004.

4. Conclusions
The main purpose of the proposed mathematical model is to facilitate the development of a technical system, such as the creation of DBN B.2.6-98:2009 concrete and reinforced concrete structures, designed to work under the impact of elevated and high temperatures. Moreover, in the limited 8 pages it is difficult to solve this problem. At the same time, a large section of “Concrete and Reinforced Concrete Structures Intended to Work under Elevated and High Temperatures” turned out to be omitted in the Eurocodes, for which building standards and SNiP 2.03.04-84* rules were developed. It was submitted to develop a regulatory instrument to replace the SNiP 2.03.04-84* with the change of status for DBN B.2.6-98:2009. The purpose of the project is to create a new national regulatory instrument, based on the modern achievements of science, machinery and technology, advanced domestic and foreign experience in design and construction, and at the same time uses theoretical and experimental studies of the updated version of SNiP 2.03.04-84*. This regulatory instrument is extremely necessary for the development of national production of Ukraine.

The dependence of “stress-strain” for nonlinear calculations of structures has been developed. The compressive and tensile strength of concrete has been calculated. The method of determination of maximum strain on the basis of energy approach has been developed. The stress-strain diagrams of compressed and heated concrete have been calculated according to Eurocode EN 1992-1-2:2004.
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