Optically injected quantum dot lasers: impact of nonlinear carrier lifetimes on frequency-locking dynamics

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Abstract. Carrier scattering is known to crucially affect the dynamics of quantum dot (QD) laser devices. We show that the dynamic properties of a QD laser under optical injection are also affected by Coulomb scattering processes and can be optimized by band structure engineering. The nonlinear dynamics of optically injected QD lasers is numerically analyzed as a function of microscopically calculated scattering lifetimes. These lifetimes alter the turn-on damping of the solitary QD laser as well as the complex bifurcation scenarios of the laser under optical injection. Furthermore, we find a pump current sensitivity of the frequency-locking range, which is directly related to the nonlinearity of the carrier lifetimes.

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1. Introduction

Self-organized semiconductor quantum dot (QD) structures have turned out to be very promising for applications in semiconductor lasers [1]. An important feature of QD structures is the discrete energy levels that form the laser transition leading to unique properties such as very low threshold current, low linewidth enhancement factor and large temperature stability [1]. Further, these lasers show highly damped turn-on dynamics [2] and, related to that, a low sensitivity to optical feedback [3, 4]. For the case of optical injection, where a laser is unidirectionally coupled to an injecting master laser, QD lasers show smaller chaotic regions and less complicated trajectories compared to other laser devices [5]. The general features of optically injected semiconductor lasers have already been studied intensively (see [6] for a review). Based on experimental studies on QD lasers [7–9], simple rate equation approaches have been used to model their optical response under optical injection and to investigate the stability regions and bifurcation scenarios [9, 10]. Further models based on the standard rate equation approach [11] that take into account the gain material and excited state filling typical of QD-based devices by an effective gain compression parameter have been proposed [12]. They have been proven to successfully model the modulation response of quantum-dash lasers subject to optical injection [13]. In these models the gain compression parameter is obtained by fitting the nonlinear dependence of the relaxation oscillation frequency on the output intensity [14], which permits the modelling of non-constant $\alpha$-factors that depend nonlinearly on the pump current [15].

The aim of this paper is to focus on the distinguishing properties of QD lasers and to understand the interplay between epitaxial structure and optical properties by linking the dynamics of the QD laser under injection directly to the carrier exchange dynamics between the QD and QW states without the need for fit parameters.

We extend our microscopically based QD laser rate equation model, which treats electrons and holes separately (see [16, 17] for details), to the investigation of QD laser dynamics under optical injection and include the dynamics of the phase of the laser light. When subject to an injected field, the output frequency of the QD laser can be affected and simulations show...
Figure 1. (a) Scheme of the optically injected QD laser. QDs (light blue pyramids) are surrounded by a QW. (b) Energy diagram of the band structure across a QD. $h\nu$ labels the ground state (GS) lasing energy. $\Delta E_e$ and $\Delta E_h$ mark the distance of the GS from the QW band edge for electrons and holes, respectively. $\Delta_e$ and $\Delta_h$ denote the distance to the bottom of the QD.

stable, pulsating and chaotic lasing behavior, connected by a variety of bifurcations. Here, our simulations agree well with experimental data for QD [9] and quantum-dash lasers [18]. Furthermore, we compare the dynamics of different QD lasers under optical injection by implementing different energetic band structures into the calculation of the microscopic carrier–carrier scattering rates. The influence of the scattering rates on the turn-on behavior and the relaxation oscillations of a solitary QD laser has been studied in [19]. In this paper, we focus on their impact on the locking behavior and show how band structure engineering can lead to optimized operation conditions of optically injected QD lasers by shifting critical bifurcation points as a function of the carrier lifetimes. Our results obtained by direct numerical integration are supported by path continuation techniques. Further, we predict a current dependence of the locking region that can be explained by the non-constant, carrier density-dependent carrier lifetimes.

This paper is organized as follows. In section 2 we present our QD laser rate equation model. In section 3 simulations based on the rate equation model are discussed, and in section 4 the impact of the pump current on the dynamics of an optically injected QD laser is investigated. Those effects are traced in section 5 to the impact of carrier lifetimes on the turn-on dynamics and therewith on the laser locking behavior.

2. Quantum dot (QD) laser model

The band structure of the QDs embedded in a quantum well (QW) laser structure is shown in figure 1. The crucial parameters are the confinement energies $\Delta E_e$ and $\Delta E_h$ that mark the energy differences between the QD ground state (GS) and the band edge of the surrounding QW for electrons and holes, respectively. Our QD laser model is based on the model described in [16, 17], which has shown good quantitative agreement with experiments regarding the turn-on behavior and the modulation response of QD lasers [17].

We extend this model very similarly to the one for optical feedback in [4], but include optical injection instead of optical feedback. Our ansatz for the optical injection is consistent
with approaches made in [6, 20]. Eliminating the laser’s steady-state frequency \( \nu_L \) (which refers to an optical wavelength of \( \lambda = 1.3 \, \mu \text{m} \)), the field equation for the normalized slowly varying amplitude \( E \) of the electric field \( E = \frac{1}{2}(\mathcal{E}e^{2\pi \nu t} + \text{c.c.)} \) is given by

\[
\dot{E}(t) = \frac{(1 + i\alpha)}{2} \left[ g(\rho_e + \rho_h - 1) - 2\kappa \right] E(t) + |E_{\text{inj}}(t)|e^{i2\pi \Delta \nu_{\text{inj}}t} + F_c(t).
\]  

(1)

Here 2\( \kappa \) are the optical intensity losses and \( g = 2\tilde{W}Z_{a}^{\text{QD}} \) is the linear gain coefficient for the processes of induced emission and absorption. The linewidth enhancement factor \( \alpha \) is chosen to be constant although it is noted that it may vary slightly with the pump current [21]. The gain coefficient is proportional firstly to the Einstein coefficient of induced emission \( \tilde{W} \) that measures the coherent interaction between the two-level system and the laser mode, and secondly to the number \( Z_{a}^{\text{QD}} \) of lasing QDs inside the waveguide (the factor 2 is due to spin degeneracy). The number of lasing QDs \( Z_{a}^{\text{QD}} \) is given by \( Z_{a}^{\text{QD}} = a_{L}A N_{a}^{\text{QD}} \) where \( a_{L} \) is the number of self-organized QD layers, \( A \) is the in-plane area of the QW and \( N_{a}^{\text{QD}} \) is the density per unit area of the active QDs. As a result of the size distribution and material composition fluctuations of the QDs, the gain spectrum is inhomogeneously broadened, and only a subgroup (density \( N_{a}^{\text{QD}} \)) of all QDs (\( N^{\text{QD}} \)) matches the mode energies for lasing. \( \rho_e \) and \( \rho_h \) denote the electron and hole occupation probabilities in the confined QD levels, respectively, and \( E_{\text{n}} \) the input detuning. The complex Gaussian white noise term \( F_c(t) \) models spontaneous emission. The time \( \tau_{\text{in}} \) for one round trip of the light in the cavity of length \( L \) is given by \( \tau_{\text{in}} = 2L/\sqrt{\varepsilon_{bg}} \) with background permittivity \( \varepsilon_{bg} \).

In order to obtain rate equations the complex stochastic differential equation (equation (1)) for the electric field, i.e. given by \( \mathcal{E} = \sqrt{n_{ph}A} \exp(i\Phi) \), is transformed by an Ito transformation [22, 23] into two real stochastic differential equations for the photon density \( n_{ph} \) and the phase \( \Phi \) (for details, see [6]). Since stochastic effects are beyond the scope of this paper, the noise terms are neglected in the following and only the deterministic spontaneous emission rate \( R_{sp} \) remains in the rate equations (2). The final nonlinear, coupled and six-variable rate equation system including also the electron and hole occupation probabilities in the QDs, \( \rho_e \) and \( \rho_h \), and the electron and hole densities in the QW, \( w_e \) and \( w_h \), respectively, is given by the following equations:

\[
\dot{n}_{ph} = n_{ph} \left[ 2\tilde{W}Z_{a}^{\text{QD}}(\rho_e + \rho_h - 1) - 2\kappa \right] + \frac{\beta}{A} 2Z_{a}^{\text{QD}} R_{sp}(\rho_e, \rho_h) + \frac{2K}{\tau_{\text{in}}} \sqrt{n_{ph}} n_{ph}^{0} \cos(\Phi - 2\pi \Delta \nu_{\text{inj}}t),
\]

(2a)

\[
\dot{\Phi} = \frac{\alpha}{2} \left[ 2\tilde{W}Z_{a}^{\text{QD}}(\rho_e + \rho_h - 1) - 2\kappa \right] + \frac{K}{\tau_{\text{in}}} \sqrt{n_{ph}} n_{ph}^{0} \sin(\Phi - 2\pi \Delta \nu_{\text{inj}}t),
\]

(2b)

\[
\dot{\rho}_e = -\tilde{W} A(\rho_e + \rho_h - 1)n_{ph} - R_{sp}(\rho_e, \rho_h) + S_{e}^{\text{in}}(w_e, w_h)(1 - \rho_e) - S_{e}^{\text{out}}(w_e, w_h) \rho_e,
\]

(2c)

\[
\dot{\rho}_h = -\tilde{W} A(\rho_e + \rho_h - 1)n_{ph} - R_{sp}(\rho_e, \rho_h) + S_{h}^{\text{in}}(w_e, w_h)(1 - \rho_h) - S_{h}^{\text{out}}(w_e, w_h) \rho_h,
\]

(2d)
for three different QD structures. The different structures are

\[ \hat{w}_c = \frac{j}{e_0} - 2N^{QD} \left[ S^\text{in}_c(w_c, w_h)(1 - \rho_e) - S^\text{out}_c(w_c, w_h)\rho_e \right] - \tilde{R}_{sp}, \]  

\[ \hat{w}_h = \frac{j}{e_0} - 2N^{QD} \left[ S^\text{in}_h(w_c, w_h)(1 - \rho_h) - S^\text{out}_h(w_c, w_h)\rho_h \right] - \tilde{R}_{sp}. \]  

Here \( n^0_{ph} \) denotes the steady-state photon density without injection (\( K = 0 \)). It is introduced to normalize the injected photon density. This normalization is explained by the definition of the injection strength \( K \):

\[ K = \sqrt{\frac{T_{\text{inj}}n_{\text{inj}}}{n^0_{ph}}}. \]  

As in equation (1) \( \hat{W} \) is the Einstein coefficient for the coherent interaction and \( Z^{QD} \) is the number of active QDs inside the waveguide. The spontaneous emission from one QD is taken into account by \( R_{sp}(\rho_e, \rho_h) = W\rho_e\rho_h \), where \( W \) is the Einstein coefficient for spontaneous emission resulting from the incoherent interaction of the QD with all resonator modes. Please note that the coefficients \( \hat{W} \) and \( W \) differ by three orders of magnitude; see the appendix for details of their derivation. \( \beta \) is the spontaneous emission factor, measuring the probability that a spontaneously emitted photon is emitted into the lasing mode. The in- and out-scattering rates for electrons and holes between QD and QW, \( S^\text{in}_c, S^\text{out}_c \) and \( S^\text{in}_h, S^\text{out}_h \) as depicted in figure 1, are calculated microscopically from Coulomb interaction and are connected by the detailed balance relation derived in [16, 19]:

\[ S^\text{out}_c(w_c, w_h) = S^\text{in}_c(w_c, w_h)e^{-\Delta E_c/e_kT} \left[ e^{\frac{e_kT}{e_{kT}}} - 1 \right]^{-1}, \]

\[ S^\text{out}_h(w_c, w_h) = S^\text{in}_h(w_c, w_h)e^{-\Delta E_h/e_kT} \left[ e^{\frac{e_kT}{e_{kT}}} - 1 \right]^{-1}. \]

Here \( \Delta E_c = E_c^{QW} - E_c^{QD} \) and \( \Delta E_h = E_h^{QD} - E_h^{QW} \) are the energy differences between the QD levels \( E_c^{QD} \) and \( E_h^{QD} \) and the band edges of the QW \( E_c^{QW} \) and \( E_h^{QW} \) for electrons and holes, respectively. The carrier degeneracy concentrations are given by \( D_{c/h}kT \), where \( D_{c/h} = m_{c/h}/(\pi\hbar^2) \) are the 2D densities of state in the QW with the effective masses \( m_{c/h} \) (see also figure 1). \( T \) is the temperature and \( k \) is the Boltzmann constant. The current density \( j \) is injected into the QW and is normalized by the elementary charge \( e_0 \). \( N^{QD} \) is the total QD density given by surface imaging techniques, which includes all non-lasing QDs that are present in the layer due to inhomogeneous broadening. The factor two in equations (2e) and (2f) accounts for spin degeneracy of the QD levels. The spontaneous emission in the QW is incorporated by \( \tilde{R}_{sp}(w_c, w_h) = B^S w_c w_h \), where \( B^S \) is the band–band recombination coefficient.

The carrier lifetimes \( \tau_e \) and \( \tau_h \) that result from Coulomb scattering between QDs and QW are defined by the nonlinear scattering rates as \( \tau_e = 1/(S^\text{in}_c + S^\text{out}_c) \) and \( \tau_h = 1/(S^\text{in}_h + S^\text{out}_h) \). It is crucial to note that these lifetimes are not constant but depend on the carrier densities in the surrounding QW and thus on the injected pump current. Their pump-dependent steady-state values are shown in figure 2 for three different QD structures. The different structures are modeled by using three different sets of confinement energies between QD and QW (see table 2 for details of the parameters). By controlling the growth mode during epitaxy it is possible to create QDs with different sizes and compositions. As such the reference rates plotted in
Figure 2. Nonlinear steady-state carrier lifetimes $\tau_e$ and $\tau_h$ for electrons and holes, respectively, resulting from the scattering rates calculated microscopically for three different band structures of the QD–QW system (see table 2). These simulations are carried out without optical injection ($K = 0$).

Table 1. Numerical parameters used in the simulation unless stated otherwise.

| Symbol | Value | Symbol | Value | Symbol | Value |
|--------|-------|--------|-------|--------|-------|
| $W$    | $0.7 \text{ ns}^{-1}$ | $A$    | $4 \times 10^{-6} \text{ cm}^2$ | $T$    | 300 K |
| $\tilde{W}$ | $0.11 \mu\text{s}^{-1}$ | $N^\text{QD}_0$ | $0.3 \times 10^{10} \text{ cm}^{-2}$ | $L$    | 1 mm  |
| $2\kappa$ | $0.1 \text{ ps}^{-1}$ | $N^\text{QD}$     | $1 \times 10^{11} \text{ cm}^{-2}$ | $\varepsilon_{\text{bg}}$ | 14.2  |
| $\beta$ | $2.2 \times 10^{-3}$ | $B^S$    | $540 \text{ ns}^{-1} \text{ nm}^2$ | $\tau_{\text{in}}$ | 24 ps |
| $a_L$  | 15 | $j_{\text{th}}$ | $4.2 \times e_0 \times 10^{24} \text{ C m}^{-2} \text{ s}^{-1}$ | $j$    | $3.5 j_{\text{th}}$ |
| $Z^\text{QD}_a$ | $1.8 \times 10^6$ | $m_e (m_h)$ | $0.043 m_0 (0.45 m_0)$ | $\lambda$ | 1.3 $\mu\text{m}$ |
| $g$    | $3.9 \times 10^{11} \text{ s}^{-1}$ | $\Delta E_e (\Delta E_h)$ | $210 \text{ meV (50 meV)}$ | $\nu_L$ | 230 THz |

Figure 2(b) result from square-based pyramidal QDs with a base length of 18 nm and a ratio between the effective masses of holes and electrons of 10 (as used in [17]). Differences in the effective masses (e.g. obtained by changing the QD composition) will show up in a different ratio between electron and hole confinement energies (see [19]). Figure 2(a) depicts the case of large confinement energies that are similar for electrons and holes, i.e. $\Delta E_e \approx \Delta E_h$, resulting in long Auger scattering lifetimes. Increasing the size of the dots leads to smaller confinement energies (shallow dot with smaller energetic distance between the GS and the QW transition) and thus to fast scattering rates (figure 2(c)). By comparing the injection properties of the QD laser for these three different cases, we are able to show the crucial impact of the growth mode and thus the QD size on the laser dynamics.

In sections 3 and 4 the reference lifetimes (figure 2(b)) are implemented, while in section 5 the dynamics of the three different structures are compared. The calculation of the scattering rates has been described thoroughly in [16, 24] and the values implemented for the simulations can be found in the appendix. All other numerical parameters that were used in the simulation are shown in table 1.
Figure 3. Time series of the photon density $n_{ph}$ for (a) $K = 0.39$ and detunings of $\Delta \nu_{inj} = -4.1 \text{GHz}$ (solid) and $\Delta \nu_{inj} = -7 \text{GHz}$ (red dashed) and (b) $K = 0.21$ with detunings $\Delta \nu_{inj} = -1.8 \text{GHz}$ (solid) and $\Delta \nu_{inj} = -1.7 \text{GHz}$ (red dashed). The injection started at $t = 5 \text{ns}$. Parameters as in table 1 with $\alpha = 0.9$ and reference scattering rates of figure 2(b).

3. Dynamics of the optically injected QD laser

3.1. One-parameter bifurcation diagrams

In this section, the dynamics of the optically injected QD laser modeled by equations (2) is discussed. Outside a certain detuning range around $\Delta \nu_{inj} = 0$, the so-called locking tongue, the QD laser under optical injection shows complex oscillatory behavior with increasing injection strength $K$. To illustrate the dynamics we simulated time series of the photon density for different input detuning frequencies. As an example of the complex dynamics, figure 3(a) depicts the results for constant injection strength $K = 0.39$, where sustained multipulse emission is found at $\Delta \nu_{inj} = -4.1 \text{GHz}$ (solid line) and regular pulsations are seen at $\Delta \nu_{inj} = -7 \text{GHz}$ (dashed line). On the other hand, figure 3(b) depicts the results for $K = 0.21$ for negative detunings $\Delta \nu_{inj} = -1.7 \text{GHz}$ and $\Delta \nu_{inj} = -1.8 \text{GHz}$ showing multipulses with different pulse shapes and a pulse repetition frequency that is sensitive to the detuning (compare the solid and dashed lines in figure 3(b)). By plotting the extremal points of the time series shown in figure 3 as a function of the input detuning $\Delta \nu_{inj}$, as was done in figures 4(a)–(d) for $\alpha = 0.9$, one-parameter bifurcation diagrams are formed. In these diagrams a single point indicates continuous wave (cw) operation, while two points (maxima and minima) indicate periodic pulsing, marked by light shading. In figures 4(a)–(d), bifurcation diagrams of the photon density are shown as a function of the detuning for different injection strengths. For all these figures, cw operation is found for zero detuning. In figures 4(a) and (b), saddle-node (SN) bifurcations can be found at the edges of the range of cw operation (locking range marked with dashed blue vertical lines in figure 4). These SN bifurcation points are characterized by an abrupt change from cw operation to periodic pulsating light output. In fact, a saddle and a stable node collide on a cycle, disappear and leave a stable limit cycle (periodic pulsing). The period of the oscillations goes to infinity at the bifurcation point and follows an inverse square root law while the amplitude is constant. This global bifurcation is called SN infinite period or

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Figure 4. (a)–(d) One-parameter bifurcation diagrams showing the maxima and minima of the photon density $n_{\text{ph}}$ as a function of the input detuning $\Delta \nu_{\text{inj}}$. (e)–(h) Output detuning $\nu_{\text{inj}} - \bar{\nu}$ in terms of input detuning $\Delta \nu_{\text{inj}} = \nu_{\text{inj}} - \nu_{L}$. The injection strengths are (a, e) $K = 0.1$, (b, f) $K = 0.21$, (c, g) $K = 0.39$ and (d, h) $K = 0.52$. SN indicates saddle-node (SN) bifurcations. The blue dashed lines indicate the input detuning region where cw operation is found, the light shading marks periodic pulsing. Parameters as in table 1, $\alpha = 0.9$ and reference scattering rates of figure 2(b). Movies 1–3, available from stacks.iop.org/NJP/14/053018/mmedia, show $\nu_{\text{inj}} - \bar{\nu}$ versus $\Delta \nu_{\text{inj}}$ for increasing $K$, $n_{\text{ph}}$ versus $\Delta \nu_{\text{inj}}$ for increasing $K$ and $n_{\text{ph}}$ versus $K$ for increasing $\Delta \nu_{\text{inj}}$, respectively.

SN on an invariant cycle bifurcation and has already been studied in QW lasers under optical injection [25], as well as in QW lasers with saturable absorbers [26] and in the framework of delay equations [27]. The time series of the pulsating photon density observed behind the SN point are plotted in figure 3(b). The small local maximum found in between the spikes is caused by oscillations around the reminiscent ghost of the formally stable fixed point that changes the shape of the limit cycle. In the bifurcation diagrams, e.g. figure 4(b), this ghost causes additional lines behind the SN point that suddenly disappear beyond a certain detuning. In this region close to the SN line the system is excitable and noise-induced dynamics, such as irregular spiking, can be observed [5, 10].

For higher injection strengths $K$ the situation changes and the bifurcation diagrams plotted in figures 4(c) and (d) show supercritical Hopf bifurcations at the positive side of the locking range. This means that at these points a stable limit cycle is born. In contrast to the SN point the period of the oscillations is constant, while the amplitude is detuning dependent and goes to zero at the Hopf point. For $K = 0.39$ this Hopf bifurcation occurs at $\Delta \nu_{\text{inj}} = 2.94$ GHz (figure 4(c)) and for $K = 0.52$ it occurs at $\Delta \nu_{\text{inj}} = 3.56$ GHz (figure 4(d)). With increasing positive detuning the limit cycle that is born in the Hopf bifurcation increases in size until it touches the point.
of zero intensity where the locking behavior changes (discussed below). Behind this point the amplitude of the limit cycle starts decreasing.

A period doubling (PD) bifurcation that leads to period-2 oscillations can be observed for $K = 0.39$ at $\Delta v_{inj} = -5.98 \, \text{GHz}$ and for $K = 0.52$ at $\Delta v_{inj} = -6.57 \, \text{GHz}$ (while decreasing $\Delta v_{inj}$) as can be seen in figures 4(c) and (d). Another PD bifurcation to a period-4 orbit can be seen for $K = 0.39$ at $\Delta v_{inj} = -4.38 \, \text{GHz}$. This leads to complex multipulses similar to those shown in figure 3(a). Note also that with increasing injection strength (figures 4(a)–(d)) the amplitude of the oscillations increases as well. For a complete overview of the bifurcation diagrams as a function of $K$ and $\Delta v_{inj}$, see the movies in the supplementary material (available from stacks.iop.org/NJP/14/053018/mmedia) of this publication. Possibly chaotic emission behavior can be seen for $K = 0.52$ around $\Delta v_{inj} = -4 \, \text{GHz}$ and $\Delta v_{inj} = -5 \, \text{GHz}$ (figure 4(d)).

Another important feature of an optically injected laser is the frequency locking, meaning that the output frequency $\nu$ of the laser can be entrained to the frequency $\nu_{inj}$ of the injected laser ($\nu - \nu_{inj} = 0$). Here $\nu_{inj}$ is called output detuning. In order to extract the output detuning frequency, the momentary frequency $\nu(t) = \frac{1}{2\pi} \Phi(t)$ is defined, which is constant for the stable locking case. Outside the stable locking regime an average output frequency $\bar{\nu} = \frac{1}{2\pi} \langle \Phi \rangle$, is defined, which is determined by averaging over a long time series. The point of interest here is the following: for which input detuning $\Delta v_{inj} = \nu_{inj} - \nu_L$ and which injection strengths $K$ is the output frequency of the laser locked? Figures 4(e)–(h) answer this question by plotting the output detuning $\nu_{inj} - \bar{\nu}$ as a function of the input detuning $\Delta v_{inj}$. By comparing these figures with the bifurcation diagrams in figures 4(a)–(d), it is obvious that the laser is frequency locked for the range where it shows cw operation, while it is unlocked behind the SN bifurcation. This type of locking is usually referred to as Adler’s locking [28]. However, immediately beyond the Hopf bifurcation point the mean frequency of the emission on the limit cycle is still locked (see figure 4(h) for $\Delta v_{inj} \approx 4 \, \text{GHz}$) and the phase, although oscillating, is still bounded. Recently, this phenomenon was discussed for two coupled oscillators in [29]. The type of locking transition, found at the Hopf bifurcation, is also called undamping of relaxation oscillations (see [30]). The transition to an unbounded phase happens where the minima of the photon density reach zero while the output detuning performs a sudden jump (see figures 4(c) and (g) for $\Delta v_{inj} = 4.5 \, \text{GHz}$). An explanation can be given by looking at the complex $E$-field. Before the minima of the photon density reach zero, the projection of the $E$-field to the complex plane is a cycle which does not include or touch the origin (thus the phase is bounded). When the minima of the photon density reach zero the projection of the $E$-field crosses the origin and with increasing positive input detuning surrounds the origin. This leads to a constantly increasing phase and thus to an emission at a different frequency. Details of that phenomenon have been discussed in [6]. Several bifurcation diagrams of local extrema of the photon density as a function of the input detuning as well as diagrams of output detuning over input detuning are concatenated to a film, which is available online in the supplementary material at stacks.iop.org/NJP/14/053018/mmedia.

3.2. Two-parameter bifurcation diagrams and path continuation

The parameter dependence of the dynamics of the QD laser with optical injection can be visualized in two-parameter bifurcation diagrams. The results obtained so far by direct integration are summarized as contour plots in figures 5(a) and (b), for $\alpha = 0.9$ and $\alpha = 3.2$, New Journal of Physics 14 (2012) 053018 (http://www.njp.org/)
Figure 5. Two-parameter bifurcation diagram for (a) $\alpha = 0.9$ and (b) $\alpha = 3.2$. The color code marks the number of extremal values, i.e. the number of maxima and minima of the photon density $n_{\text{ph}}$. The light yellow area marks the average frequency locking with steady-state photon density, i.e. cw-lasing. The yellow–orange hatched area shows oscillating photon density (with one minimum and one maximum) but with a locked average frequency. Blue, black and white solid lines indicate the PD, Hopf and SN bifurcation lines, respectively. Parameters as in table 1 and reference scattering rates of figure 2(b).

respectively. The number of extrema found in the times series is used as a color code (for periodic waveforms, this corresponds to the number of local extrema per period).

The region of frequency-locked cw operation appears yellow. For increasing injection strength $K$ this locking range grows. The yellow–orange hatched area indicates the mean frequency locking where the photon density oscillates on a limit cycle with bounded phase. The transition to an unbounded phase (indicated by ‘phase unbounding’ in figure 5) found at the upper edge of the hatched region (where $n_{\text{ph}} \approx 0$) is not a bifurcation, but it appears as a numerical artifact due to division by zero. The orange region corresponds to periodic pulsations that show one maximum and one minimum, while more complex time series are found for parameters chosen within the dark-colored and the white regions in figure 5.

In order to obtain a clear picture of the bifurcations, we have performed path continuation with the software tool matcont [31]. The results are plotted as solid and dashed lines superimposed upon the data from direct numerical integration of the dynamical equations in figure 5. SN, PD and Hopf bifurcation lines are indicated by white, blue and black lines, respectively. First, the dynamics for a small $\alpha$-factor, i.e. for a typical QD laser, as depicted in figure 5(a), is discussed. It can be seen that the cusp of the hatched area is a codimension-two SN–Hopf-point where a Hopf and a SN line intersect. At this point the Hopf bifurcation undergoes a change from subcritical in the lower part (dashed line) to supercritical in the upper part (solid line) and becomes thus detectable in direct numerical integrations. Furthermore, the path continuation gives proof that the elliptically shaped red region (four extrema per period) on the negative detuning side is bordered by a PD bifurcation. In this region, we distinguish PD
lines, where a formerly stable limit cycle undergoes a PD bifurcation (solid line), from those where an unstable saddle-cycle with one Floquet multiplier larger than unity undergoes a PD bifurcation (thin dashed lines). PD lines of saddle-cycles having two or more Floquet multiplies larger than unity are not plotted. For positive detuning only PD lines involving stable limit cycles are plotted in figure 5(a). We find two large PD loops, one for positive detuning and one for negative detuning. For the lower PD loops also secondary PD bifurcations giving birth to period-4 limit cycles are plotted (solid and dashed blue lines). Stable period-4 limit cycles as detected by direct numerical integration are marked by dark violet areas (eight extrema per period). White regions inside the PD loops indicate more complex dynamics, i.e. more than eight extrema. Within the large lower PD loop, we find multistability between period-2 and different period-4 orbits. A description of the full bifurcation scenario is beyond the scope of this paper. Here we plot only PD bifurcations that enclose the main regions of more complex dynamics detected by direct numerical integrations.

The narrow sickle-shaped darker regions found for small $K$ close to the SN lines for positive and negative detunings (regions with three (gray) and four (red) extrema) indicate the appearance of oscillations around the ghost of the formally stable fixed point. In agreement with the path continuation these are not PD bifurcations but period-1 oscillations. Only the shape of the limit cycle changes, resulting in additional extrema (see figure 3(b) for a possible time series). As noted before, the system is excitable near these regions.

Figure 5(a) depicts the situation for a small $\alpha$-factor as commonly predicted for QD lasers. In figure 5(b) a larger $\alpha$-factor is studied, which is more typical of QW lasers. As expected the locking range shrinks as the Hopf bifurcation line bends towards zero input detuning. Thus, pulsating behavior is found also for $\Delta v_{\text{inj}} = 0$ at high injection strength. Close to the codimension-2 SN–Hopf-point (cusp of the hatched area), the laser shows complex bifurcations leading to chaotic emission. However, the complex dynamics found for QW devices within the homoclinic teeth on the negative detuning side of the locking range [25] is absent. Nevertheless, large regions of multistability between the simple cw solution and solutions with periodically modulated intensity, i.e. limit cycles, are found within the yellow area of frequency locking. The data shown in figure 5 were obtained by up-sweeping the value of the injection strength $K$ and thus show parts of the stable limit cycle operation inside the yellow area of cw operation. Down-sweep instead reveals only stable cw operation up to the SN line for negative detunings. In the experiment, up-sweeping and down-sweeping indicate the direction in which the injection strength is changed. For example, starting with zero injection and increasing it slowly can lead to different dynamic scenarios from starting with high injection strength and decreasing it slowly. In the simulations this is done by choosing the last value of the time series as initial conditions for the next simulation.

Qualitatively, our results agree well with results for QD lasers that are modeled with equal and constant lifetimes for electrons and holes [5, 10]. However, in our model the lifetimes are not fitted parameters but nonlinear functions of the carrier density in the reservoir that are given by the band structure. Thus changes concerning the operation point of the device can be studied without re-adjusting parameters. The next section of the paper will explore the impact of different pump currents on the dynamics of the QD laser with injection.

4. Impact of the pump current on the dynamics

The pump current affects many characteristics of the QD laser, e.g. with increasing pump current the photon density and the electron and hole densities in the QD and QW increase. This in turn
Figure 6. Two-parameter bifurcation diagram for $\alpha = 0.9$ for injected current densities of (a) $j = 2.1j_\text{th}$ and (b) $j = 4.9j_\text{th}$. The light yellow area indicates the locking region at cw operation and the yellow–orange hatched area indicates the region of frequency locking of a limit cycle. The color code indicates the number of extrema per period of the photon density. The SN, PD and Hopf bifurcation lines are indicated. Parameters as in table 1 and reference scattering rates of figure 2(b).

influences the scattering rates and therefore the turn-on behavior of the QD laser (see [19]). The analytic relation between the damping of the turn-on dynamics and the value of the scattering lifetimes is derived in [32]. In this section, we want to focus on the impact of the pump current on the frequency locking of the optically injected QD laser.

Figure 6 shows the two-parameter bifurcation diagrams for current densities $j = 2.1j_\text{th}$ and $j = 4.9j_\text{th}$. As in figure 5, where $j = 3.5j_\text{th}$, the range of frequency locking at cw operation is marked by the yellow area, while the number of maxima observed in the time series is color coded. The figure shows that the codimension-2 SN–Hopf-point, at which the subcritical Hopf bifurcation changes to a supercritical one, shifts towards higher injection strengths $K$ and higher input detunings $\Delta \nu_\text{inj}$ with increasing pump current. For $j = 2.1j_\text{th}$ (figure 6(a)) the Hopf line starts at $K = 0.14$ (for $j = 3.5j_\text{th}$ at $K = 0.22$) and for a pump current of $j = 4.9j_\text{th}$, the Hopf line starts at $K = 0.27$ (figure 6(b)). Along with the increasing pump current, the region where the frequency is locked at cw operation grows in the region of positive input detuning. For the negative detuning region, the different pump currents show no influence on the frequency-locked region but the occurrence of the PD bifurcation shifts to higher $K$ values. Furthermore, the regions where the oscillations around the ghost of the fixed point are found next to the SN bifurcation shrink in size.

Note that, if the lifetimes $\tau_e$ and $\tau_h$ are chosen to be constant, the locking range is invariant under changes of the pump current. Thus the changes in the locking behavior with the pump current can be attributed to the nonlinearity of the carrier lifetimes, which makes them a crucial ingredient for the quantitative modeling of injected QD lasers. High current densities lead to high electron and hole densities $w_e$ and $w_h$ in the QW, which lead to short carrier

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Table 2. Energy differences $\Delta E_e$ and $\Delta E_h$ between QW and QD GS for the three different scattering rates used for the simulations (second and third columns), threshold current density $j_{th}$ compared to the threshold current of the reference rates ($j_{th-ref}$) (fourth column) and carrier lifetimes for electrons ($\tau_e$) and holes ($\tau_h$) at steady state without injection ($K = 0$) and with pump current $j = 3.5 j_{th}$ (fifth and sixth columns).

| Data set | $\Delta E_e$ (meV) | $\Delta E_h$ (meV) | $j_{th}$ | $\tau_e$ (ns) | $\tau_h$ (ns) |
|----------|------------------|------------------|---------|--------------|--------------|
| Slow     | 140              | 120              | 0.28 $j_{th-ref}$ | 0.395        | 0.129        |
| Reference| 210              | 50               | 1.00 $j_{th-ref}$ | 0.071        | 0.005        |
| Fast     | 74               | 40               | 2.14 $j_{th-ref}$ | 0.021        | 0.01         |

lifetimes of electrons and holes, $\tau_e$ and $\tau_h$, respectively. Predictions of certain laser dynamics at given experimental operation parameters are only possible if the nonlinear dependence of the scattering rates on the QW carrier densities is incorporated into the model.

5. Impact of carrier lifetimes on the dynamics

Many characteristics of the QD laser dynamics can be traced back to the carrier lifetimes in the QD [16, 32]. In the previous section, we have already pointed out their importance for modeling current-dependent injection properties. Now we wish to discuss different QD structures and their impact on the frequency locking behavior. Different carrier lifetimes can be implemented in the simulations by using scattering rates that have been calculated for different QD sizes and thus different band structures. Figure 2 displays the dependence of the electron and hole scattering lifetimes on the pump current for three different structures. In table 2, the confinement energies (see figure 1(b)) of the corresponding QD–QW band structures are given as well as their corresponding carrier lifetimes in the QD at steady state for a pump current of $j = 3.5 j_{th}$. Note that the threshold current density also changes with the scattering rates. So far, only the reference rates (figure 2(b)) have been used for the simulations.

The three panels in figure 7 show time series of the photon density for three different QD lasers that have been modeled by using the sets of carrier lifetimes given in figure 2. The upper panel (figures 7(a)–(c)), middle panel (figures 7(d)–(f)) and lower panel (figures 7(g)–(i)) show the dynamics for the slow rates, reference rates and fast rates, respectively. In the plots the optical injection starts at $t = 5$ ns after the laser has been turned on. The most important difference in the turn-on dynamics (without injection) of the three lasers is the damping of the relaxation oscillations, which is related to changes of the carrier lifetimes. For the slow rates the damping is small as for a class B laser, i.e. a typical QW semiconductor laser (figures 7(a)–(c)). For the reference rates the relaxation oscillations are strongly suppressed, which is typical of QD lasers, but one can still observe several oscillations (figures 7(d)–(f)). For the fast rates the relaxation oscillations are overdamped as for a class A laser, i.e. a typical gas laser (figures 7(g)–(i)). This dependence was analytically described [32]; however, the different rates also alter the behavior of the laser with optical injection. This can be seen by comparing QD lasers with different sets of carrier lifetimes for various detuning frequencies. In figure 7, the dynamics has been compared for three different detuning frequencies $\Delta v_{inj} = 2.3$ GHz (left column in figure 7), $\Delta v_{inj} = -0.5$ GHz (central column in figure 7) and $\Delta v_{inj} = -2.5$ GHz (right column in figure 7).
In agreement with the results regarding the modulation response of QW lasers [33, 34], the relaxation oscillations of the laser become less damped under injection at positive detunings inside the locking range (figure 7(d)), while damping and frequency of the relaxation oscillations increase inside the locking range for negative detunings (figure 7(e)). Furthermore, the different carrier lifetimes influence the size of the locking range in the $(\Delta \nu_{\text{inj}}, K)$-plane, as can be seen by comparing the yellow areas in figures 8(a) and (b). For lower turn-on damping (larger carrier lifetime, slow rates) the codimension-2 SN–Hopf-point at the positive detuning side of the frequency locked region shifts to lower $K$ and the size of the yellow–orange hatched area indicating mean frequency locking on a limit cycle increases.

A second crucial effect of the lifetimes on the bifurcation diagram can be found at the negative detuning side of the locking range. For large lifetimes and thus small damping, the range of sustained multipulse emission with complex pulse shape (e.g. shown in figure 7(c)) is large and the lower PD bifurcation line is already found for small $K$ values (figure 8(a)). Further chaotic transients and complex bifurcations are found outside the locking range for negative detunings. Instead, the bifurcation diagram plotted in figure 8(b) for the strongly damped QD laser shows a large range of frequency locked cw operation. Outside this region only period-1 oscillations (see figure 7(i) for the time series of the photon density) are found up to $K = 0.35$. A very small area with a more complex pulse shape is found near the SN–Hopf-point and for
Figure 8. Two-parameter bifurcation diagram for the slow scattering rates (a) and fast scattering rates (b) introduced in figure 2. Parameters as in table 1 and $\alpha = 0.9$.

For $K > 0.5$ a small region surrounded by a PD bifurcation is found at the negative detuning side. It is interesting to note that the period-2 oscillations within the PD area have a bounded phase and thus show mean frequency locking as indicated by hatching. Thus we conclude that a stronger damping of the turn-on dynamics leads to a reduction of the areas with complex dynamics and enlarges the range of stable frequency locked cw operation. This relation between turn-on damping and locking behavior should be interesting for experimentalists as the knowledge of the $\alpha$-factor and the small-signal modulation response should be sufficient to predict and optimize the locking behavior of a laser.

We recall that changing the current density in section 4 (see figure 6) also leads to changes in the locking behavior that have a similar tendency as found for the lifetime variations. Long carrier lifetimes have a similar effect as low current density, and fast carrier lifetimes have a similar effect as high current density. Both can be traced back to the carrier lifetimes, $\tau_e$ and $\tau_h$, respectively, and thereby to the damping of the turn-on dynamics. Increasing the damping shifts the beginning of the codimension-2 SN–Hopf-point to higher injection strength and leads to a shrinkage of the areas with complex dynamics found outside the locking range for negative detuning. The removal of the multipulse excitability was also found for class A lasers [5]. These lasers can be approximately described by a single rate equation for the complex electric field and thus, as a two-variable system, cannot show chaos.

6. Conclusion

Using microscopic carrier–carrier scattering rates as the input for electron and hole lifetimes in the QD, we have modeled an optically injected QD laser and described its nonlinear dynamics as a function of injection strength and input detuning. We have identified several changes in the locking behavior as well as in the bifurcation scenarios that are connected to variations in the carrier lifetimes of electrons and holes and thereby to the damping of the turn-on dynamics.
of the QD laser. Most importantly, we found that short carrier lifetimes that are of the order of 10 ps lead to strongly damped relaxation oscillations, which in turn lead to large regions of stable cw operation. Sensitivity of the locking range to changes in the pump current is also found and attributed to changes in the pump current-dependent carrier lifetimes. Thus it is crucial to incorporate the nonlinear dependence of the scattering rates on the carrier density into the surrounding reservoir in order to obtain quantitative results on the dynamics of an injected QD laser, i.e. the range of stable injection locked operation.

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Appendix A. Scattering rates

The calculation of the carrier–carrier scattering rates implemented in the above simulations is described in detail in [16, 24]. The results of the microscopic simulations have been fitted with the following functions to enable their use with path continuation software and to allow the reader to follow the calculations. Using the following definition of normalized carrier densities in the reservoir:

\[ W_e = w_e/(2N^{QD}) \quad \text{and} \quad W_h = w_h/(2N^{QD}), \]  
(A.1)

we have implemented the following functions for the in-scattering rates.

\[
S_{e}^{in}(W_e, W_h) = W \cdot (\tanh(a_e W_e + b_e)) \cdot (c_{e,1} W_e + c_{e,2} W_e^2 + c_{e,3} W_e^3 + c_{e,4} W_e^4 + d_{e,1} W_h + d_{e,2} W_h^2 + d_{e,3} W_h^3 + d_{e,4} W_h^4),
\]

\[
S_{h}^{in}(W_e, W_h) = W \cdot (\tanh(a_h W_e + b_h)) \cdot (c_{h,1} W_e + c_{h,2} W_e^2 + c_{h,3} W_e^3 + c_{h,4} W_e^4 + d_{h,1} W_h + d_{h,2} W_h^2 + d_{h,3} W_h^3 + d_{h,4} W_h^4).
\]

The parameter values of the coefficients are given in table A.1 for the reference rates, in table A.2 for the fast rates and in table A.3 for the slow rates. Due to the principle of detailed balance, the
Table A.2. Parameters for the fast rates from table 2.

| Coefficient | Value         | Coefficient | Value         |
|-------------|---------------|-------------|---------------|
| $a_e$       | $-1.73923 \times 10^{-5}$ | $a_h$       | $1.34424 \times 10^{-5}$ |
| $b_e$       | $-1.30964 \times 10^{-6}$ | $b_h$       | $-3.35376 \times 10^{-4}$ |
| $c_{e,1}$   | $-707.607.0$  | $c_{h,1}$   | $-626.353$    |
| $c_{e,2}$   | $106.490.0$   | $c_{h,2}$   | $-307.788$    |
| $c_{e,3}$   | $-8788.67$    | $c_{h,3}$   | $356.116$     |
| $c_{e,4}$   | $287.055$     | $c_{h,4}$   | $-17.8843$    |
| $d_{e,1}$   | $51.376.6$    | $d_{h,1}$   | $-13.087.5$   |
| $d_{e,2}$   | $7347.35$     | $d_{h,2}$   | $-15153.0$    |
| $d_{e,3}$   | $-663.515$    | $d_{h,3}$   | $1066.2$      |
| $d_{e,4}$   | $17.169$      | $d_{h,4}$   | $-55.0711$    |

Table A.3. Parameters for the slow rates from table 2.

| Coefficient | Value         | Coefficient | Value         |
|-------------|---------------|-------------|---------------|
| $a_e$       | $-2.6612 \times 10^{-5}$ | $a_h$       | $1.94259 \times 10^{-5}$ |
| $b_e$       | $-1.6475 \times 10^{-6}$ | $b_h$       | $-4.74478 \times 10^{-4}$ |
| $c_{e,1}$   | $-363.381$    | $c_{h,1}$   | $-3601.34$    |
| $c_{e,2}$   | $50519.5$     | $c_{h,2}$   | $-15193.1$    |
| $c_{e,3}$   | $-4290.71$    | $c_{h,3}$   | $1441.14$     |
| $c_{e,4}$   | $146.18$      | $c_{h,4}$   | $-47.7236$    |
| $d_{e,1}$   | $69984.1$     | $d_{h,1}$   | $-19129.2$    |
| $d_{e,2}$   | $-74.2397$    | $d_{h,2}$   | $-5584.61$    |
| $d_{e,3}$   | $-86.6277$    | $d_{h,3}$   | $435.245$     |
| $d_{e,4}$   | $1.65736$     | $d_{h,4}$   | $-27.6885$    |

out-scattering rates can be calculated from the in-scattering rates with the help of equations (4) and (5) for electrons and holes, respectively [16, 19].

Appendix B. Derivation of the photon rate equation

The coherent interaction between a two-level system (e.g. QD) and a light mode can be described by semiconductor Bloch equations [24, 35, 36]. Eliminating the fast microscopic polarization $p$ of one QD (the probability amplitude for an optical transition) by assuming $\dot{p} = 0$ leads to a quasi-static relation between $p$ and the slowly varying complex amplitude $E$ of the electric field. By further assuming equal energy for light mode and level spacing, this relation reads

$$p = -i \frac{\mu E T_2}{\hbar} (\rho_e + \rho_h - 1). \tag{B.1}$$

Here $\mu$ is the associated dipole moment of the optical transition and $T_2$ is the lifetime of the microscopic polarization defining the homogeneous linewidth $\hbar / T_2$ of the levels. The term $(\rho_e + \rho_h - 1)$ describes the inversion of the two-level system with the electron and hole
occupation probabilities in the QDs, \( \rho_e \) and \( \rho_h \). For the derivation of the photon rate equation, we start with the reduced field equation for the electric field [37] without damping,

\[
\dot{E} = \frac{i \omega L}{2 \epsilon_0 \epsilon_{bg}} \Gamma P,
\]

where \( \epsilon_0 \) is the vacuum permittivity, \( \epsilon_{bg} \) is the background dielectric constant and \( \omega_L \) is the transition frequency of the two-level system. Using the total macroscopic polarization inside one QW layer given by \( P = \frac{2}{\hbar \omega} N_d^{QD} \mu \rho \) and the optical confinement factor \( \Gamma \) perpendicular to the direction of light propagation \( \Gamma = \frac{a_L h^{QW}}{h^w} \) (height \( h^w \) of the waveguide, height \( h^{QW} \) of the QW layers that contain the self-organized QDs and the number of QW layers \( a_L \)), we arrive at the following field equation:

\[
\dot{E} = \frac{2 |\mu|^2 \omega L a L N_d^{QD} \epsilon_0 \epsilon_{bg}}{2 \epsilon_0 \epsilon_{bg} h^w} (\rho_e + \rho_h - 1) E = Z_d^{QD} \dot{W} (\rho_e + \rho_h - 1) E \tag{B.3}
\]

with the Einstein coefficient of induced emission

\[
\dot{W} = \frac{|\mu|^2 \omega L T_2}{\epsilon_0 \epsilon_{bg} \hbar V^w}, \tag{B.4}
\]

which measures the strength of the coherent interaction. \( \dot{W} \) depends on the volume of the optical waveguide \( V^w = A \cdot h^w \) and on the width of the optical transition \( \hbar / T_2 \). The number \( Z_d^{QD} = a_L N_d^{QD} \) is the number of active QDs inside the waveguide. Rewriting equation (B.3) for the photon density per unit area \( n_{ph} = |E|^2 h^{*} v_{ph} \) gives

\[
\dot{n}_{ph} = 2 Z_d^{QD} \dot{W} (\rho_e + \rho_h - 1) n_{ph}. \tag{B.5}
\]

In previous publications [4, 16, 17, 19], a different value for \( \dot{W} \) was implemented because the QD volume \( V^{act} \) was used instead of the optical waveguide volume \( V^w \) in equation (B.4). As a result the geometric optical confinement factor \( \Gamma_g = \frac{V^{act}}{V^w} \) appeared in the equation for the photon density per unit area equation (B.5), which was somewhat misleading. Because the differing \( \dot{W} \) was also used in the carrier equations, those simulations yield a rescaled photon density. Thus, the values for \( n_{ph} \) given in [4, 16, 17, 19] need to be multiplied by \( 6.6 \times 10^3 \) to yield the real photon density in the cavity. Related to that, also the coefficient for spontaneous emission is scaled in those papers and the unscaled value should be \( \beta = 2.2 \times 10^{-3} \).

The Einstein coefficient for the spontaneous emission can be derived by calculating the incoherent interaction of the two-level system with all resonator modes in the framework of second quantization [37]. It give \( W = \frac{|\mu|^2 \sqrt{\pi \epsilon} (\frac{\omega_k}{c})^3}{3 \pi \epsilon o h} \).

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