Understanding the newly observed $\Xi^0_c$ states through their decays

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Inspired by the newly observed $\Xi^0_c$ states by the LHCb Collaboration, we investigate the OZI-allowed two-body strong decays of the $\lambda$-mode 1P wave $\Xi^0_c$ states within the chiral quark model. Our results indicate that: (i) the newly observed states $\Xi_c(2923)^0$ and $\Xi_c(2939)^0$ are good candidates of the $\lambda$-mode 1P wave $\Xi^0_c$ states with the spin-parity $J^P = 3/2^-$, namely $[\bar{P}_{3/2}^+ \Xi]$ and $[\bar{P}_{3/2}^+ \Xi]$, respectively. (ii) The another newly observed state $\Xi_c(2965)^0$ mostly corresponds to the $\lambda$-mode 1P-wave $\Xi^0_c$ state with the spin-parity $J^P = 5/2^-$, namely $[\bar{P}_{5/2}^+ \Xi]$. (iii) For the two $\lambda$-mode $J^P = 1/2^-$ mixed states, the $|P_{3/2}^+\Xi\rangle$ is a narrow state with a width of $\Gamma \sim 15$ MeV and mainly decays into $\Xi\pi$; while the $|P_{3/2}^+\Xi\rangle$ state has a width of $\Gamma \sim 52$ MeV and dominantly decays into $\Xi\pi$ and $\Lambda K$ channels. If the broad structure around 2880 MeV observed at LHCb arises from the new $\Xi^0_c$ state, this state is very likely to be the $|P_{3/2}^+\Xi\rangle$ state.

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1. INTRODUCTION

As an important kind of singly heavy baryons, the charmed-strange baryon $\Xi^0_c$ spectrum belonging to the flavor sextet $6_F$ plays a crucial role in perfecting baryon spectra. Although there are some discussions for the $\Xi^0_c$ states in theory during the past several decades [1–4], no obvious progress of the observations for the $\Xi^0_c$ state has been achieved in experiments [5]. In the $\Xi^0_c$ spectrum, only the two ground states $\Xi^0_c$ with $J^P = 1/2^+$ and $\Xi^0_c(2645)$ with $J^P = 3/2^+$ (1S wave) have been established. So far, the low-lying P-wave $\Xi^0_c$ states predicted in the quark model are still missing. It should be mentioned that in 2007 a structure $\Xi_c(2930)^0$ was observed by BaBar in the $\Lambda_c^+K^-$ mass spectrum in $B^+\rightarrow K^-\Lambda_c^+\Lambda_c^-$ process [6]. Later, the $\Xi_c(2930)^0$ was confirmed by the Belle Collaboration in the same decay process [7], while its charged partner $\Xi_c(2930)^+$ was also observed in $K^0\Lambda^+_c$ final state in the reaction $B^0\rightarrow K^0\Lambda^+_c\Lambda^+_c$ [8]. In addition, another structure $\Xi_c(2970)^0$ was first observed by BaBar in the $\Sigma_c(2455)^0K^0_S$ decay mode [9]. This structure was also observed in both $\Xi^0_c\pi^+$ [10] and $\Xi^0_c(2645)^0\pi^-$ [11] final states at Belle. The structures $\Xi_c(2930)^0$ and $\Xi_c(2970)^0$ may be some signals of the missing P-wave $\Xi^0_c$ states first observed in experiments.

Very recently, the LHCb Collaboration observed three new states $\Xi_c(2923)^0$, $\Xi_c(2939)^0$, and $\Xi_c(2965)^0$ in the $\Lambda^+_cK^-$ mass spectrum with a large significance [12], and their masses and natural widths are determined to be

\[
\begin{align*}
 m[\Xi_c(2923)^0] &= 2923.04 \pm 0.59 \text{ MeV}, \\
 \Gamma[\Xi_c(2923)^0] &= 7.1 \pm 2.6 \text{ MeV}, \\
 m[\Xi_c(2939)^0] &= 2938.55 \pm 0.52 \text{ MeV}, \\
 \Gamma[\Xi_c(2939)^0] &= 10.2 \pm 1.9 \text{ MeV}, \\
 m[\Xi_c(2965)^0] &= 2964.88 \pm 0.54 \text{ MeV}, \\
 \Gamma[\Xi_c(2965)^0] &= 14.1 \pm 2.2 \text{ MeV}.
\end{align*}
\]

As pointed out in Ref. [12], the $\Xi_c(2930)^0$ observed in $B\rightarrow K^-\Lambda^+_c\Lambda^+_c$ process [6, 7] might be due to the overlap of the two new narrower states, $\Xi_c(2923)^0$ and $\Xi_c(2939)^0$. Stimulated by these newly observed states, recently some groups have discussed their nature. In Ref. [13], these three newly observed states $\Xi_c(2923)^0$, $\Xi_c(2939)^0$, and $\Xi_c(2965)^0$ are suggested to be assigned as the P-wave $\Xi^0_c$ states. In Ref. [14], $\Xi_c(2923)^0$ and $\Xi_c(2939)^0$ may be good candidates of the P-wave states with $J^P = 3/2^-$ and $5/2^-$ states and $\Xi_c(2965)^0$ can be assigned as $\Xi^0_c(2S)$ state. To understand the nature of these newly observed states and clarify whether they can be identified as the $\Xi^0_c(1P)$ and $\Xi^0_c(2S)$ state or not, more theoretical analysis is urgently needed.

For the mass spectrum of the $\Xi^0_c$ baryon, there exist many calculations with various models and effective theories in the literature [15–25]. We collect the predicted masses in Table 1. From this table, it is found that the masses of the three $\Xi^0_c$ states observed by the LHCb Collaboration [12] are in the predicted region of the $\lambda$-mode 1P wave $\Xi^0_c$ excitations. Here, "$\lambda$-mode" denotes one orbital excitation in a Jacobi coordinate between the light quarks and the heavy c quark. Moreover, the possibility as $2S$ excitations cannot be excluded absolutely based simply on the predicted masses. In addition, it should be emphasized that in the light of the equal spacing rule [26, 27], the $\Xi_c(2923)^0$, $\Xi_c(2939)^0$ and $\Xi_c(2965)^0$ states probably correspond to their flavour multiplets $\Omega_c(3050)^0$, $\Omega_c(3065)^0$ and $\Omega_c(3090)^0$, respectively. Meanwhile, based on our previous work [28], $\Omega_c(3050)^0$ and $\Omega_c(3065)^0$ could be assigned to be two $J^P = 3/2^-$ states, $[1^+P_{3/2}^+\Xi]$ and $[1^+P_{3/2}^+\Xi]$, respectively; $\Omega_c(3090)^0$ very likely corresponds to the $J^P = 5/2^-$ state $[1^+P_{5/2}^-\Xi]$. To this extent, $\Xi_c(2923)^0$ and $\Xi_c(2939)^0$ are

\[\ldots\]
likely to have $J^P = 3/2^-$, and $\Xi_c(2965)^0$ has $J^P = 5/2^-$. Besides mass spectra, the radiative and strong decay properties are crucial in pinning down the inner structures of a state. Before the LHCb’s measurement [12], there are also some discussions of the radiative and strong decay properties of the $1P$ wave $\Xi_c$ states [21, 25, 28–36].

In our previous works [28, 35], the decay properties of the $1P$ wave $\Xi_c$ states were estimated with a chiral quark model, the mostly predicted decay widths of the $\lambda$-mode $1P$ wave $\Xi_c$ states were about a few dozen MeV, which were roughly consistent with the LHCb’s measurement [12]. In the present work, by combining the newest data we further analyze the strong decay properties of the $\lambda$-mode $1P$ wave $\Xi_c$ states with the chiral quark model, and attempt to put forward views on the inner structures of the three $\Xi_c^0$ states observed by the LHCb Collaboration [12].

This paper is organized as follows. In Sec. II we give a brief introduction of the strong decay model. We discuss the strong decays of the low-lying $\lambda$-mode $1P$ wave $\Xi_c$ states in Sec. III and summarize our results in Sec. IV.

II. THE MODEL

In this work we apply the chiral quark model [37] to study the strong decay properties. Within the chiral quark model, the effective low energy quark-pseudoscalar-meson coupling in the SU(3) flavor basis at tree level is adopted as [37]

$$H_m = \sum_j \frac{1}{f_m} \bar{\psi}_j \gamma_\mu \gamma_5 \psi_j \hat{\tau} \cdot \hat{\sigma}^\mu \phi_m,$$  

(1)

where $f_m$ is the pseudoscalar meson decay constant; $\psi_j$ represents the $j$th quark field in a baryon, and $\phi_m$ denotes the pseudoscalar meson octet. To match the nonrelativistic harmonic oscillator spatial wave function in our calculations, one should adopt a nonrelativistic form of the quark-pseudoscalar coupling [38–40]

$$H_m^{nr} = \sum_j \frac{\omega_m}{E_j + M_j} \sigma_j \cdot P_j + \frac{\omega_m}{E_j + M_j} \sigma_j \cdot P_j - \sigma_j \cdot q + \frac{\omega_m}{2\mu_q} \sigma_j \cdot P'_j \Gamma e^{-i\mathbf{q} \cdot \mathbf{r}},$$  

(2)

where ($\omega_m$, $\mathbf{q}$) denote the energy and three-vector momentum of the final light pseudoscalar meson; ($E_j$, $M_j$, $\mathbf{P}_j$) and ($E_f$, $M_f$, $\mathbf{P}_f$) are the energy, mass and three-vector momentum of the initial and final baryons, respectively. The $\mathbf{P}'_j = (\mathbf{P}_j - (m_j/M_j)\mathbf{P}_{c.m.})$ stands for the internal momentum of the $j$th quark in the baryon rest frame; $\sigma_j$ stands for the Pauli spin vector on the $j$th quark; $\mu_q$ represents the reduced mass expressed as $1/\mu_q = 1/m_j + 1/m_j$.

The isospin operator $I_j$ associated with $\pi$ and $K$ mesons is given by

$$I_j = \begin{cases} \frac{1}{2} [a_j(u, u) a_j(u) - a_j(d) a_j(d)] & \text{for } \pi^0, \\ a_j(u, u) a_j(d) & \text{for } \pi^-, \\ a_j(u, u) a_j(s) & \text{for } K^-. \end{cases}$$  

Here, $a_j(u, d)$ and $a_j(u, d, s)$ are the creation and annihilation operator for the $u$, $d$ and $u$, $d$, $s$ quarks on $j$th quark, respectively.

Then the partial decay width for the emission of a light pseudoscalar meson in a hadron strong decay can be calculated with [41, 42]

$$\Gamma = \left( \frac{\delta}{f_m} \right)^2 \left( \frac{E_f + M_f}{4\pi M_f} \right) \left( \frac{1}{2J_f + 1} \right) \sum_f |M_{f, J_f}|^2,$$  

(4)

where $\delta$ is a global parameter accounting for the strength of the quark-meson couplings; $J_c$ and $J_f$ are the third components of the total angular momenta of the initial and final baryons, respectively; $M_{f, J_f}$ denotes the transition amplitude.

With momentum $\mathbf{q}$ of the final light pseudoscalar meson increasing, the relativistic effect should be significant [43]. To partly remedy the inadequacy of the nonrelativistic wave function as the momentum $q$ increases, a commonly used Lorentz boost factor $\gamma_f$ is introduced into the decay amplitudes [42, 44–46]

$$M(\mathbf{q}) \rightarrow \gamma_f M(\gamma_f \mathbf{q}),$$  

(5)

where $\gamma_f \equiv M_f/E_f$. In most decays, the corrections from the Lorentz boost are not drastic and the nonrelativistic prescription is reasonable.

The model parameters have been well determined in previous works [28, 35], and we collect them in Table II. In the calculations the masses of the final baryons and mesons are taken from the PDG [5] and collected in Table II as well. The harmonic oscillator space-wave functions $\Psi_n l m = R_n Y_l m$ are adopted to describe the spatial wave function of the initial and final baryons, and the harmonic oscillator parameter $\alpha_\rho$ in the wave functions for $ds/uds$ system is taken as $\alpha_\rho = 420$ MeV. Another harmonic oscillator parameter $\alpha_\lambda$ can be related to $\alpha_\rho$ with the relation $\alpha_\lambda = [3m_c/(2m_q + m_\lambda)]^{1/3} \alpha_\rho$, where $m_q$ denotes the light quark mass.

III. RESULTS AND ANALYSIS

The masses of the three $\Xi_c^0$ states newly observed by the LHCb Collaboration [12] are in the predicted mass region of the $\lambda$-mode $1P$ wave $\Xi_c$ states (see Table I). To clarify the possibility and further investigate their inner structures, we conduct a systematic study of the strong decay properties for the $\lambda$-mode $1P$ wave $\Xi_c$ states within the framework of a chiral quark model. Our results and theoretical predictions are presented as follows.

A. 1P states with $J^P = 1/2^-$

In the $\Xi_c$ family, there are two $\lambda$-mode $J^P = 1/2^-$ states $|1^2P_11/2^-\rangle$ and $|1^3P_11/2^-\rangle$. Their masses are predicted to be about $M = (2830 \pm 2940$) MeV (see Table I), and their OZI-allowed two-body strong decay channels are $\Xi_c \pi$, $\Xi'_c \pi$, 

TABLE I: The mass spectrum of Ξ’ belonging to 6f_7 up to the 1P-wave states in various models and effective theories. The Ξ’ states are denoted by |N^{2S+1}L_JJ'\rangle in the LS coupling scheme. The unit of mass is MeV in the table.

| \( |N^{2S+1}L_JJ'\rangle \) | Ref. [15] | Ref. [23] | Ref. [22] | Ref. [24] | Ref. [16] | Ref. [17] | Observed state |
|-------------------|----------|----------|----------|----------|----------|----------|----------------|
| \( |0^2S^+\rangle \) | 2579     | 2592     | 2579     | 2578     | 2579     | 2594     | \( \Xi^* \)       |
| \( |1^4S^+_\frac{1}{2}\rangle \) | 2649     | 2650     | 2649     | 2654     | 2649     | 2649     | \( \Xi^* \)       |
| \( |1^2P^+_\frac{1}{2}\rangle \) | 2839     | 2859     | 2854     | 2928     | 2936     | \( \Xi \)           |
| \( |1^2P^+_\frac{3}{2}\rangle \) | 2921     | 2871     | 2935     | 2931     | \( \Xi \)           |
| \( |1^4P^+_\frac{1}{2}\rangle \) | 2900     | \( \cdots \)  | 2936     | 2934     | \( \Xi \)           |
| \( |1^4P^+_\frac{3}{2}\rangle \) | 2932     | \( \cdots \)  | 2912     | 2900     | \( \Xi \)           |
| \( |1^4P^+_\frac{5}{2}\rangle \) | 2927     | 2905     | 2929     | 2921     | 2929     | 2895     | \( \Xi \)           |

TABLE II: The parameters and final hadron’ masses [5] used in this work. The unit is MeV except the parameter \( \delta \), which is a dimensionless quantity. \( \Xi' \) denotes \( \Xi \) (2645) in the table.

| State | Mass (MeV) | State | Mass (MeV) |
|-------|------------|-------|------------|
| \( \Xi^* \) | 2467.9     | \( \pi \) | 139.6      |
| \( \Xi' \) | 2470.9     | \( \eta' \) | 135.0      |
| \( \Xi^* \) | 2577.4     | \( K' \) | 493.8      |
| \( \Xi^{*0} \) | 2578.8     | \( \Lambda^* \) | 2286.5    |
| \( \Xi^{*+} \) | 2645.5     | \( \Xi^{**} \) | 2646.3    |

Parameter  \( \delta = 0.557 \), \( f_K = 132 \), \( f_{\pi} = 160 \), \( \alpha_q = 420 \), \( m_q = 330 \), \( m_{\pi} = 450 \), \( m_{\pi} = 1480 \).

\( \Xi' \pi \) and \( \Lambda^* K' \). Considering the uncertainties of the predicted masses, we plot the partial decay widths as functions of the masses of the states \( \Xi'|1^2P^+_1/2^-\rangle \) and \( \Xi'|1^4P^+_1/2^-\rangle \) in Fig. 1. Their decay properties remain relatively stable within the mass range of \( (2830 - 2940) \) MeV.

We notice that for the singly heavy flavour quark systems, proper consideration of the heavy quark symmetry is necessary [47]. Namely, the physical states with the spin-parity \( J'^* = 1/2^- \) are very likely to be mixed states between \( |1^2P^+_1/2^-\rangle \) and \( |1^4P^+_1/2^-\rangle \) by the following mixing scheme,

\[
\begin{pmatrix}
|1^2P^+_1/2^-\rangle_1 \\
|1^4P^+_1/2^-\rangle_2
\end{pmatrix} = \begin{pmatrix} \cos \phi & \sin \phi \\ -\sin \phi & \cos \phi \end{pmatrix} \begin{pmatrix}
|1^2P^+_1/2^-\rangle \\
|1^4P^+_1/2^-\rangle
\end{pmatrix}.
\]

The mixing angle \( \phi \) may range from \( \phi = 0^\circ \) to that of heavy quark symmetry limit \( \phi = 35^\circ \). In our previous work [28, 48], we obtained that the state \( \Omega_0(3000) \) could be explained as the mixed state \( |1P^+_1/2^-\rangle_1 \) with a mixing angle \( \phi \simeq 24^\circ \). As the same flavour multiplet, the mixing angle in the \( \Xi' \) family should be comparable with that in the \( \Omega_0 \) family. Meanwhile, according to the equal spacing rule [26, 27], the mass of the mixed state \( |1P^+_1/2^-\rangle_1 \) in the \( \Xi' \) family is lighter about \( \sim 120 \) MeV than that of state in the \( \Omega_0 \) family, namely \( M_{\Xi'}|1P^+_1/2^-\rangle_1 \approx 2880 \) MeV. Thus according to the mixing scheme defined in Eq. (6), in Fig. 2 we plot the strong decay width of the \( |1P^+_1/2^-\rangle_1 \) state as a function of the mixing angle \( \phi \) in the range of \( (0^\circ, 35^\circ) \) by fixing the mass at \( M = 2880 \) MeV.

It is found that with the mixing angle increasing, the partial decay widths of the \( \Lambda^* K \) and \( \Xi' \pi \) modes for \( \Xi'|1P^+_1/2^-\rangle_1 \) are rapidly suppressed and the \( \Xi' \pi \) decay channel almost saturates...
as well. It is found that the total decay width. Taking a possible mixing angle $\phi \approx 24^\circ$ constrained by the $\Omega_c(3000)$ state, we find the $\Xi'_c|1P_1 \ 1/2^+\rangle_1$ state has a narrow width of $\Gamma \sim 15.0$ MeV, and the branching fraction of the dominant decay channel $\Xi'_c \pi$ is

$$\mathcal{B}[\Xi'_c|1P_1 \ 1/2^+\rangle_1 \rightarrow \Xi'_c \pi] \approx 85\%.$$  

The decay rate of $\Xi'_c|1P_1 \ 1/2^+\rangle_1$ into $\Lambda_cK$ strongly depends on the mixing angle. If taking a slightly larger mixing $\phi \approx 28^\circ$ than that ($\phi \approx 24^\circ$) determined by $\Omega_c(3000)$, one finds the decay rate into the $\Lambda_cK$ channel is nearly zero. Thus, the $\Xi'_c|1P_1 \ 1/2^+\rangle_1$ may be hardly observed in the $\Lambda_cK$ final state.

Furthermore, the predicted decay width of the mixed state $\Xi'_c|1P_1 \ 1/2^+\rangle_1$ seems to be comparable with the decay width of the three $\Xi'_c$ states observed by the LHCb Collaboration [12]. However, it should be kept in mind that the mass of $\Xi'_c|1P_1 \ 1/2^+\rangle_1$ should be around $\sim 2880$ MeV considering a reasonable explanation for the properties of $\Omega_c(3000)$. Thus, most likely the three $\Xi'_c$ states as the state $\Xi'_c|1P_1 \ 1/2^+\rangle_1$ is excluded. Considering the predicted width of $\Xi'_c|1P_1 \ 1/2^+\rangle_1$ being narrow, this state might be observed in the $\Xi'_c \pi$ channel when enough data are accumulated in experiments.

The other mixed state $|1P_4 \ 1/2^+\rangle_2$ should be a relatively broad state with a width much larger than the three newly observed $\Xi'_c$ states. At this moment we investigate its width range with $M=2880$ MeV in Fig. 2 as well. From the figure, the total decay width is about $\Gamma \sim (38 - 50)$ MeV with the mixing angle varying in range of $\left(0^\circ, 35^\circ\right)$. Taking a possible mixing angle $\phi = 24^\circ$ constrained by the $\Omega_c(3000)$ state, we find that the mixed state $\Xi'_c|1P_4 \ 1/2^+\rangle_2$ has a width of $\Gamma \sim 52$ MeV, and dominantly decays into the $\Xi' \pi$ and $\Lambda_cK$ final states with comparable branching fractions

$$\mathcal{B}[\Xi'_c|1P_4 \ 1/2^+\rangle_2 \rightarrow \Xi' \pi] \approx 42\%,$$

$$\mathcal{B}[\Xi'_c|1P_4 \ 1/2^+\rangle_2 \rightarrow \Lambda_cK] \approx 57\%.$$

Both the mass and decay width of this mixed state are inconsistent with the observations of the three newly observed $\Xi'_c$ states, thus, the three $\Xi'_c$ states as the state $\Xi'_c|1P_1 \ 1/2^+\rangle_2$ should be excluded. Since $\Xi'_c|1P_4 \ 1/2^+\rangle_2$ is not a very broad state, it might be observed in the $\Xi'_c \pi$ and $\Lambda_cK$ channels.

It should be remarked that according to the LHCb measurement [12], the $\Lambda_cK$ mass spectrum shows a broad structure around 2880 MeV, which might be due to the presence of additional new $\Xi'_c$ states. Combining the predicted decay properties of $\Xi'_c|1P_1 \ 1/2^+\rangle_2$ in this work, if the broad structure in the region around $M \sim 2880$ MeV arises from a new $\Xi'_c$ state, this state is very likely to be the $\Xi'_c|1P_1 \ 1/2^+\rangle_2$ state. Moreover, the possibility of the broad structure arising from the overlapping of $\Xi'_c|1P_1 \ 1/2^+\rangle_1$ and $\Xi'_c|1P_1 \ 1/2^+\rangle_2$ cannot be ruled out as well.

### B. $1P$ states with $J^P = 3/2^-$

There are two $\lambda$-mode $J^P = 3/2^-$ states $\Xi'_c|1^2P_1 \ 3/2^-\rangle$ and $\Xi'_c|1^4P_3 \ 3/2^-\rangle$. The predicted masses of these two states are listed in Table I. From the table, it is seen that predicted masses are about $\sim 2930$ MeV, which are comparable with the masses of $\Xi'_c(2923)$ and $\Xi'_c(2939)$ measured by the LHCb Collaboration [12]. As the good candidates of $\Xi'_c(2923)$ and $\Xi'_c(2939)$, it is essential to study the decay properties of the $\Xi'_c|1^2P_1 \ 3/2^-\rangle$ and $\Xi'_c|1^4P_3 \ 3/2^-\rangle$.

We plot the decay properties of the $\Xi'_c|1^2P_1 \ 3/2^-\rangle$ and $\Xi'_c|1^4P_3 \ 3/2^-\rangle$ as functions of their masses in the range of $M = (2860 - 2970)$ MeV in Fig. 1 as well. It is found that the total decay width of $\Xi'_c|1^2P_1 \ 3/2^-\rangle$ is about $\Gamma \sim (7 - 28)$ MeV and that of $\Xi'_c|1^4P_3 \ 3/2^-\rangle$ is $\Gamma \sim (7 - 18)$ MeV with masses increasing in the range what we considered. The dominant decay mode of $\Xi'_c|1^2P_1 \ 3/2^-\rangle$ is $\Xi'_c \pi$, while $\Xi'_c|1^4P_3 \ 3/2^-\rangle$ mainly decays into $\Xi'_c \pi$ channel.

The predicted decay widths of $\Xi'_c|1^2P_1 \ 3/2^-\rangle$ and $\Xi'_c|1^4P_3 \ 3/2^-\rangle$ are also comparable with the observed values of $\Xi'_c(2923)$ and $\Xi'_c(2939)$ within the uncertainties. Meanwhile, with the similar masses, $|1^2P_1 \ 3/2^-\rangle$ is slightly broader than $|1^4P_3 \ 3/2^-\rangle$. By combining the width order $\Gamma[\Xi'_c(2923)] > \Gamma[\Xi'_c(2939)]$ from experiments and our theoretical predictions, we may conclude that $\Xi'_c(2939)$ and $\Xi'_c(2923)$ prefer to the $\Xi'_c|1^2P_1 \ 3/2^-\rangle$ and $\Xi'_c|1^4P_3 \ 3/2^-\rangle$ states, respectively. In addition, according to the equal spacing rule [26, 27], the $\Xi'_c(2923)$ and $\Xi'_c(2939)$ states probably correspond to their flavour multiplets $\Omega_c(3050)$ and $\Omega_c(3065)$, respectively. In our previous work [28, 48], $\Omega_c(3050)$ could be assigned to be $|1^2P_1 \ 3/2^-\rangle$ and $\Omega_c(3065)$ was assigned to be $|1^4P_3 \ 3/2^-\rangle$. To this extend, the $\Xi'_c(2923)$ and $\Xi'_c(2939)$ states assigned to be $|1^4P_3 \ 3/2^-\rangle$ and $|1^2P_1 \ 3/2^-\rangle$, respectively, is reasonable as well.

Fixing the mass of $\Xi'_c|1^4P_3 \ 3/2^-\rangle$ with $M = 2923$ MeV, we
TABLE III: The decay properties of the $P$-wave $\Xi_c$ states compared with the observations. $\Gamma_{\text{tot}}^{\text{th}}$ presents the total decay width calculated in the present work, while $\Gamma_{\text{tot}}^{\exp}$ presents the total width obtained from the LHCb experiment [12]. The units of mass and width are MeV in the table.

| State | Mass | $\Gamma[\Xi_c]$ | $\Gamma[\Xi_c^\prime]$ | $\Gamma[\Xi_c]$ | $\Gamma[\Lambda_cK]$ | $\Gamma_{\text{tot}}^{\text{th}}$ | $\Gamma_{\text{tot}}^{\exp}$ | Possible assignment |
|-------|------|----------------|---------------------|----------------|-----------------|----------------|----------------|------------------|
| $|P_\perp \frac{1}{2}\rangle$ | 2880 | 0.86 | 12.9 | 0.18 | 1.17 | 15.1 | $\cdots$ | $\cdots$ |
| $|P_\perp \frac{1}{2}\rangle$ | 2880 | 21.7 | 0.51 | 0.01 | 29.6 | 51.8 | $\cdots$ | $\cdots$ |
| $|P_\perp \frac{1}{2}\rangle$ | 2923 | 1.74 | 0.15 | 10.7 | 0.48 | 13.1 | $7.1 \pm 0.8 \pm 1.8$ | $\Xi_c(2923)_0^0$ |
| $|P_\perp \frac{1}{2}\rangle$ | 2939 | 10.2 | 3.80 | 2.46 | 3.74 | 20.2 | $10.2 \pm 0.8 \pm 1.1$ | $\Xi_c(2939)_0^0$ |
| $|P_\perp \frac{1}{2}\rangle$ | 2965 | 15.5 | 1.64 | 3.57 | 5.43 | 26.1 | $14.1 \pm 0.9 \pm 1.3$ | $\Xi_c(2965)_0^0$ |

obtain

$$\Gamma_{\text{tot}}[\Xi_c[14P_3/2^-]] \approx 13 \text{ MeV}.$$ \hspace{1cm} (10)

The predicted branching fraction of the dominant decay mode $\Xi_c^-$ is

$$\mathcal{B}[\Xi_c[14P_3/2^-] \to \Xi_c^-] \approx 82\%.$$ \hspace{1cm} (11)

Meanwhile, the decay rates of $\Xi_c[14P_3/2^-]$ into $\Xi_c^-$ and $\Lambda_cK$ are considerable, and the predicted branching fractions are

$$\mathcal{B}[\Xi_c[14P_3/2^-] \to \Xi_c^-]) \approx 13\%,$$ \hspace{1cm} (12)

$$\mathcal{B}[\Xi_c[14P_3/2^-] \to \Lambda_cK] \approx 4\%.$$ \hspace{1cm} (13)

The sizeable branching fraction for $\Xi_c[14P_3/2^-]$ into $\Lambda_cK$ is consistent with the nature of $\Xi_c(2923)_0^0$, which is observed in the $\Lambda_cK$ channel. If $\Xi_c(2923)_0^0$ corresponds to the state $\Xi_c[14P_3/2^-]$ indeed, the $\Xi_c^-$ and $\Lambda_cK$ may also be measured in future experiments due to their large partial branching fractions.

In the same way, we fix the mass of $\Xi_c[12P_3/2^-]$ with $M = 2939$ MeV. Then, the total decay width is predicted to be

$$\Gamma_{\text{tot}}[\Xi_c[12P_3/2^-]] \approx 20 \text{ MeV}.$$ \hspace{1cm} (14)

This state mainly decays into $\Xi_c^-$ channel with the branching fraction

$$\mathcal{B}[\Xi_c[12P_3/2^-] \to \Xi_c^-] \approx 51\%.$$ \hspace{1cm} (15)

The partial decay widths of the other three decay channels $\Xi_c^0, \Xi_c^{-},$ and $\Lambda_cK$ are comparable. The partial decay ratios are

$$\frac{\Gamma[\Xi_c[12P_3/2^-] \to \Lambda_cK]}{\Gamma[\Xi_c[12P_3/2^-] \to \Xi_c^-]} \approx 0.98,$$ \hspace{1cm} (16)

$$\frac{\Gamma[\Xi_c[12P_3/2^-] \to \Xi_c^-]}{\Gamma[\Xi_c[12P_3/2^-] \to \Xi_c^0]} \approx 0.65.$$ \hspace{1cm} (17)

If $\Xi_c(2939)_0^0$ observed in the $\Lambda_cK$ channel corresponds to the state $\Xi_c[14P_3/2^-]$ indeed, the comparable partial decay widths indicate this state may be established in the $\Xi_c^-$ and $\Xi_c^0$ decay channels as well as in future experiments.

Moreover, assigning the $\Xi_c(2923)_0^0$ and $\Xi_c(2939)_0^0$ to $\Xi_c[14P_3/2^-]$ and $\Xi_c[12P_3/2^-]$, respectively, we notice that the predicted total decay width ratio

$$R_1 = \frac{\Gamma_{\text{tot}}[\Xi_c[14P_3/2^-]]}{\Gamma_{\text{tot}}[\Xi_c[12P_3/2^-]]} \approx 0.65,$$ \hspace{1cm} (18)

highly agrees with the observed central value $R_1^{\exp} = \frac{\Gamma_{\text{tot}}[\Xi_c(2923)_0^0]}{\Gamma_{\text{tot}}[\Xi_c(2939)_0^0]} \approx 0.70$. 

C. $1P$ states with $J^P = 5/2^-$

There is only one $\lambda$-mode $J^P = 5/2^-$ state $[14P_5/2^-]$. The predicted mass of this state is listed in Table I. In terms of the predicted mass, the possibility of the three newly observed $\Xi_c^0$ states taken as the state $\Xi_c[14P_5/2^-]$ cannot be excluded. To investigate the effects of the uncertainties of the mass on the decay properties of the $\Xi_c[14P_5/2^-]$ state, we show the variation of the partial decay width with the change of mass in Fig. 1. From the figure, the variation curve between the partial decay width and the mass for this state is similar to that for $\Xi_c(12P_3/2^-)$. The dominant decay mode for $\Xi_c[14P_5/2^-]$ is $\Xi_c^-$. Meanwhile, the partial decay width of the $\Lambda_cK$ mode is sizeable, and becomes more and more significant with the mass increasing in the region of $(2860 - 2970)$ MeV.

The predicted decay properties of $\Xi_c[14P_5/2^-]$ are consistent with the observations of $\Xi_c(2965)_0^0$. Meanwhile, considering the equal spacing rule [26, 27], $\Xi_c(2965)_0^0$ most likely corresponds to its flavour multiplet $\Omega_c(3090)$, which was assigned to the $[14P_5/2^-]$ state according to our previous study [28]. Thus, it is reasonable to assign $\Xi_c(2965)_0^0$ as the $\Xi_c[14P_5/2^-]$ state.

According to our calculations, we get

$$\Gamma_{\text{tot}}[\Xi_c[14P_5/2^-]] \approx 26.1 \text{ MeV}$$ \hspace{1cm} (19)

with a mass of $M = 2965$ MeV (see Table III). The dominant decay mode is $\Xi_c^-$ with the branching fraction

$$\mathcal{B}[\Xi_c[14P_5/2^-] \to \Xi_c^-] \approx 59\%.$$ \hspace{1cm} (20)

The decay rate of $\Xi_c[14P_5/2^-]$ into the $\Lambda_cK$ channel is significant as well, and the predicted branching ratio is

$$\mathcal{B}[\Xi_c[14P_5/2^-] \to \Lambda_cK] \approx 21\%.$$ \hspace{1cm} (21)

The sizeable branching fraction of $\Xi_c[14P_5/2^-]$ into $\Lambda_cK$ is consistent with the observation of the $\Xi_c(2965)_0^0$ signal in the $\Lambda_cK$ decay channel.

In addition, the total decay width ratios among the $\Xi_c[14P_3/2^-], \Xi_c[12P_3/2^-],$ and $\Xi_c[14P_5/2^-]$ states are predicted to be

$$R_2 = \frac{\Gamma_{\text{tot}}[\Xi_c[14P_3/2^-]]}{\Gamma_{\text{tot}}[\Xi_c[14P_5/2^-]]} \approx 0.50,$$ \hspace{1cm} (22)

$$R_3 = \frac{\Gamma_{\text{tot}}[\Xi_c[12P_3/2^-]]}{\Gamma_{\text{tot}}[\Xi_c[14P_5/2^-]]} \approx 0.77.$$ \hspace{1cm} (23)
The predicted ratios are good consistent with the experimental central values ($R_2^{\text{exp}} = \frac{\Gamma_{\text{total}}(2923)}{\Gamma_{\text{total}}(2965)} = 0.50$ and $R_1^{\text{exp}} = \frac{\Gamma_{\text{total}}(2939)}{\Gamma_{\text{total}}(2965)} = 0.77$) of those among the three states $\Xi_c(2923)^0$, $\Xi_c(2939)^0$, and $\Xi_c(2965)^0$, respectively.

Finally, it should be mentioned that the known $\Xi_c(2970)^0$ was observed in the $\Sigma_c(2455)^0K_0^0$ [9], $\Xi_c^+\pi^-$ [10], and $\Xi_c(2645)^+\pi^-$ [10, 11] decay modes may correspond two different resonances with a very similar mass. The $\Xi_c(2970)^0$ observed in the $\Sigma_c(2455)^0K_0^0$ [9] final state cannot be considered as the same state of $\Xi_c(2965)^0$ although they have a very similar mass, because the $\Sigma_c(2455)^0K_0^0$ mode of $\Xi_c(2965)^0$ is forbidden as the $\Xi'_c[1^+P_3/2^-]$ state. However, one cannot exclude the $\Xi_c(2965)^0$ resonance as the same resonance observed in the $\Xi_c^+\pi^-$ [10], and $\Xi_c(2645)^+\pi^-$ [10, 11] final states. The $\Xi_c(2970)^0$ observed in the $\Sigma_c(2455)^0K_0^0$ final state may be explained with the $\rho$-mode $1P$ wave $\Xi'_c$ states [35], or the first positive parity excitations of the $\Xi_c$ [29, 31].

IV. SUMMARY

In this paper, we carry out a systematic study of the OZI allowed two-body strong decays of the $\lambda$-mode $1P$ wave $\Xi'_c$ states in the framework of a chiral quark model. Combining our theoretical predictions and the experimental observations, we give possible interpretations for the three new states $\Xi_c(2923)^0$, $\Xi_c(2939)^0$, and $\Xi_c(2965)^0$ observed by the LHCb Collaboration.

Our theoretical results show that the newly observed states $\Xi_c(2923)^0$ and $\Xi_c(2939)^0$ are most likely to be explained as the $\lambda$-mode $1P$ wave $\Xi'_c$ states with spin-parity $J^P = 3/2^-$, namely $\Xi'_c[1^+P_3/2^-]$ and $\Xi'_c[1^+P_3/2^-]$, respectively. The $\Xi_c(2923)^0$ and $\Xi_c(2939)^0$ may be flavour partners of $\Omega_c(3050)^0$ and $\Omega_c(3065)^0$, respectively. Meanwhile, if the arrangements in this work are correct, then the dominant decay mode of $\Xi_c(2923)^0$ is $\Xi'_c\pi$ and that of $\Xi_c(2939)^0$ is $\Xi_c\pi$. This can be tested in future experiments.

The another newly observed state $\Xi_c(2965)^0$ may correspond to the $\lambda$-mode $1P$-wave $\Xi'_c$ state with spin-parity $J^P = 5/2^-$, namely $\Xi'_c[1^2P_5/2^-]$. The $\Xi'_c(2965)^0$ may be a flavour partner of $\Omega_c(3090)^0$. Besides the $\Lambda_cK$ decay channel, the decay rate of $\Xi'_c[1^2P_5/2^-]$ into $\Xi_c\pi$ is significant as well, and the predicted branching ratio is about 59%. The large branching fraction indicates that this state may be reconstructed in the $\Xi_c\pi$ decay channel as well.

There are strong configuration mixings in the $J^P = 1/2^-$ $\lambda$-mode states. The mixed state $\Xi'_c[1P_1 1/2^-1]$ might be a flavour partner of $\Omega_c(3000)^0$. This $J^P = 1/2^-$ mixed state may have a mass of $M \sim 2880$ MeV and a narrow width of $\Gamma \sim 15$ MeV. The dominant decay mode of $1P_1 1/2^-1$ is $\Xi'_c\pi$ with a branching fraction of $\sim 85\%$. The decay rate into $\Lambda_cK$ is strongly suppressed due to the heavy quark symmetry. The $\Xi'_c[1P_1 1/2^-1]$ may be observed in the $\Xi_c\pi$ final state.

The other $J^P = 1/2^-$ mixed state $\Xi'_c[1P_1 1/2^-2]$ has a relatively broad width of $\Gamma \sim 48$ MeV, which is about 3 times larger than that of $\Xi'_c[1P_1 1/2^-1]$. The $\Xi'_c[1P_1 1/2^-2]$ mainly decays into $\Xi_c\pi$ and $\Lambda_cK$ channels with branching ratios $45\%$ and $54\%$, respectively. Considering the mass and decay properties, if the broad structure in the $\Lambda_cK$ mass spectrum around $M \sim 2880$ MeV observed by the LHCb Collaboration arises from a new $\Xi'_c$ state, this state is very likely to be the $\Xi'_c[1P_1 1/2^-2]$ state. Moreover, the possibility of the broad structure arising from the overlapping of $\Xi'_c[1P_1 1/2^-1]$ and $\Xi'_c[1P_1 1/2^-2]$ cannot be ruled out as well.

Finally it should be mentioned that combining our previous study [49] of the newly observed $\Omega_c$ states at LHCb [50], we find that the $\Xi_c(2923)^0$, $\Xi_c(2939)^0$, and $\Xi_c(2965)^0$ may be flavour partners of $\Omega_b(6330)$, $\Omega_b(6340)$, and $\Omega_b(6350)$, respectively. The missing mixed state $\Xi'_c[1P_1 1/2^-1]$ may be a flavour partner of $\Omega_b(6316)$.

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