The cosmological constant and the size of the Physical Universe

Enrique Gaztañaga
Institute of Space Sciences (ICE, CSIC), 08193 Barcelona, Spain
Institut d’Estudis Espacials de Catalunya (IEEC), 08034 Barcelona, Spain
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The cosmological constant \( \Lambda \) is a free parameter in Einstein’s equations of gravity. We propose to fix its value with a boundary condition: test particles should be free when outside causal contact, e.g. at infinity. Under this condition, constant vacuum energy does not gravitate and there is no cosmic acceleration for an infinitely large and uniform Universe. The observed acceleration requires either a large Universe with evolving Dark Energy (DE) and equation of state \( \omega > -1 \) (\( \omega \leq -1 \) violates our condition) or a finite causal boundary (that we call the Physical Universe) without DE. The former can’t explain why \( \Omega_\Lambda \simeq 2.3\Omega_m \) today, something that comes naturally with a finite Physical Universe. This boundary condition, combined with the anomalous lack of correlations observed above 60 degrees in the CMB predicts \( \Omega_\Lambda \simeq 0.70 \) for a flat universe, with independence of any other measurements. This provides new clues and evidence for inflation.

I. INTRODUCTION

One of the most striking changes to Newton’s gravity proposed by Einstein is that energy gravitates. Scientists have since been wondering if vacuum energy \( \rho_{\text{vac}} \) (vacuum fluctuations, zero-point fluctuations, quantum vacuum, dark energy or aether) could also gravitate. Current measurements of the cosmic acceleration (see e.g. [1-3] and references therein) point to a model with \( \Lambda \), that we refer to as \( \Lambda \text{CDM} \). Even while the accuracy and precision of measurements have greatly improved, the mean values of cosmological parameters have remained similar for well over a decade (see e.g. [1-3] and references therein). Both \( \rho_{\text{vac}} \) and \( \Lambda \) produce cosmic acceleration but we can only measure the combination:

\[
\rho_\Lambda \equiv \frac{\Lambda}{8\pi G} + \rho_{\text{vac}}. \tag{1}
\]

The measured \( \rho_\Lambda \) is extremely small compared to what we expect from particle physics for \( \rho_{\text{vac}} \). Moreover, the value agrees, to within a factor of two, with the matter energy density today: \( \rho_\Lambda \simeq 2.3\rho_m \). This is a remarkable coincidence as the matter density changes very rapidly with the expansion, while \( \rho_\Lambda \) remains constant. Possible solutions are: I) \( \Lambda = 0 \), II) \( \rho_{\text{vac}} = 0 \) or III) a cancellation between them (for a review see [4,5] and references therein). In option I) the observed \( \rho_\Lambda \) originates only from vacuum or dark energy (DE) [9] and references therein]. In quantum field theory (QFT) the vacuum energy is not physical (\( \rho_{\text{vac}} = \infty \)) and observations only depend on energy differences. If this is also true for gravity, only \( \Lambda \) contributes to \( \rho_\Lambda \). This is option II), which include modified gravity models [10,11] and references therein]. Option III) comes naturally with the boundary condition proposed here.

In this paper we take \( \Lambda \) to be a fundamental part of the gravitational interactions. To understand the role of \( \Lambda \) consider first the implications in classical physics. This corresponds to adding a Hooke term, i.e. proportional to distance, to the gravitational acceleration:

\[
\vec{g} = -\left( \frac{Gm}{r^3} - \frac{\Lambda}{3} \right) \vec{r} \tag{2}
\]

or equivalently a Poisson equation \( \nabla^2 \phi = 4\pi G\rho - \Lambda \). This is the only possible combination that has a key property required for gravity: a spherical mass shell of arbitrary density produces a gravitational field which is identical to a point source of equal mass in its center [14]. Thus this generalization of Newton’s law is consistent with the symmetries of gravity, as already noticed by Newton and other scientist [15]. Here, in addition, we require that test particles should be free when outside causal contact. For example, if we request \( \vec{g} \to 0 \) for \( r \to \infty \) in the equation above, we obtain \( \Lambda = 0 \). While a finite boundary at \( r \to r_\infty \) results in \( \Lambda = 4\pi G\rho (r < r_\infty) \), which, as we will see, it is related to the coincidence problem.

Particles separated by distances larger than the comoving Hubble radius \( d_H(t) = c/[a(t)H(t)] \) can’t communicate at time \( t \). Distances larger than the horizon

\[
\eta(a) = c \int_0^t \frac{dt}{a(t)} = c \int_0^a d\ln(a) dH(a), \tag{3}
\]

have never communicated. We know from the cosmic microwave background (CMB) that the Universe was very homogeneous on scales that were not causally connected. This either means that the initial conditions where smooth to start with or that there is a mechanism like inflation [16-18] which inflates causally connected regions outside the Hubble radius. This allows the full observable Universe to originate from a very small causally connected homogeneous patch, which here we call the Physical Universe \( \chi_5 \). During inflation, \( d_H \) decreases which freezes out communication on scales larger than the horizon \( \eta(a_i) = d_H(t_i) \) when inflation begins, at \( a_i = a(t_i) \). Here we propose to identify \( \chi_5 \simeq \eta(a_i) \). When inflation ends, radiation from reheating makes \( d_H \) grow again. When \( \chi_5 \) re-enters causal contact, we will see that the Universe starts another inflationary epoch so that the \( \chi_5 \) keeps frozen. Thus, causality can only play a role for comoving scales \( \chi < \chi_5 \). The Physical Universe \( \chi_5 \) is therefore fixed and is the same for all times, while the horizon \( \eta \) and \( d_H \) change with time. Fig.1 illustrates this situation.

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II. FIXING THE VALUE OF \( \Lambda \)

The symmetries of Einstein’s gravitational field equations allow a cosmological constant \( \Lambda \) (19):

\[
R_{\mu\nu} + \Lambda g_{\mu\nu} = -8\pi G (T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T), \tag{4}
\]

For a homogeneous and isotropic perfect relativistic fluid with total density \( \rho \) and pressure total \( p \):

\[
T_{\mu\nu} = \rho u_{\mu} u_{\nu} + p g_{\mu\nu}, \tag{5}
\]

Our Universe is filled with matter \( \rho_m \) (with equation of state \( p_m = 0 \)), radiation \( \rho_r \) \( (p_r = \rho_r/3) \) and vacuum energy \( \rho_{\text{vac}} \) \( (\rho_{\text{vac}} = -\rho_{\text{vac}}) \), so that \( \rho = \rho_m + \rho_r + \rho_{\text{vac}} \) and \( p = p_m + p_r + p_{\text{vac}} \).

In the weak field limit, \( g_{00} \simeq -(1+2\phi) \), where \( \phi \) is the Newtonian potential (see eg [19]). The field equations become a covariant generalization of Poisson equation:

\[
R_{00} \simeq -\partial^\mu \partial_\mu \phi = -4\pi G (\rho + 3p) + \Lambda, \tag{6}
\]

where \( g_{\mu\nu} = -\partial^\mu \phi \partial_\nu \phi = -4\pi G (\rho + 3p) + \Lambda \).

\[
\Phi = \int_{\partial M} dx_{\mu} g_{\mu\nu} = -\int_{M} d^4x [4\pi G (\rho + 3p) - \Lambda], \tag{7}
\]

where \( d^4x \) and \( dx_{\mu} \) are the invariant 4-D volume element and normal surface element of \( \partial M \).

A. Causal Boundary condition

We require next that a test particle should be free outside causal contact, e.g. at infinity. Thus \( g_{\mu\nu} = 0 \) and \( \Phi = 0 \) at causal boundary \( \partial M_\chi \). This fixes \( \Lambda \):

\[
\Phi = 0 \Rightarrow \frac{\Lambda}{8\pi G} = \bar{\rho}_5 = \frac{1}{2M_5} \int_{M_5} d^4x (\rho + 3p), \tag{8}
\]

where \( M_5 \) is the volume inside the lightcone to the surface \( \partial M_\chi \), which is defined by the radial comoving coordinate \( \chi_5 \) (see Fig.2). Thus, our boundary condition results in a simple relation between \( G \) and \( \Lambda \). As we approach the boundary \( \partial M_4 \) there is no gravitational field and therefore there is no energy associated with it. Because of energy conservation, the \( \Lambda \) term has to be constant and so does \( T_{\mu\nu} \). An observer situated close to the causal boundary will find a similar solution, but could measure different values for \( \rho \) and \( p \), depending on the initial conditions. Nearby regions are connected which creates a smooth background across disconnected regions with an infrared cutoff in homogeneities at \( \chi_5 \).

B. Vacuum Energy does not gravitate

We can re-write Eq.8 as:

\[
\frac{\Lambda}{8\pi G} = \bar{\rho}_5 = \frac{\rho_m(\bar{\chi}_5)}{2} + \rho_r(\bar{\chi}_5) - \rho_{\text{vac}} \equiv \bar{\rho}_5 - \rho_{\text{vac}}, \tag{9}
\]

where \( \bar{\rho}_5 \) is the matter and radiation contribution in the integral of Eq.8. The values of \( \rho_m \) and \( \rho_r \) evolve with space-time, so that \( \bar{\rho}_5 \) is the average contribution inside the volume \( M_5 \), while the vacuum density contribution is constant. As pointed out in Eq.10, \( \rho_{\text{vac}} \) has the same effect in Eq.9 as \( \Lambda \), and the only observable is:

\[
\rho_\Lambda = \frac{\Lambda}{8\pi G} + \rho_{\text{vac}} = \bar{\rho}_5, \tag{10}
\]

where in the last equality we have used our causality condition in Eq.9. So we see how vacuum energy cancels out and can not change the observed value of \( \rho_\Lambda \), even for \( \rho_{\text{vac}} = \infty \), as predict by QFT. If vacuum energy suffers a phase transition or changes in some other way, as is believed to have happened during inflation, then this cancellation will not necessarily happen and \( \rho_{\text{vac}} \) could contribute to the effective value of \( \rho_\Lambda \). If the vacuum changes between the time we measure \( \rho_\Lambda \) and the time the horizon reaches \( \chi_5 \), with a change \( \Delta \rho_{\text{vac}} \), we can re-write the above equation as:

\[
\rho_\Lambda = \bar{\rho}_5 + \Delta \rho_{\text{vac}}, \tag{11}
\]

Thus, because of the causal boundary condition the observed \( \rho_\Lambda \) is only sensitive to changes in \( \rho_{\text{vac}} \) and not to the absolute value of \( \rho_{\text{vac}} \). In the limit of a Physical Universe that is infinitely large we have \( \bar{\rho}_5 \Rightarrow 0 \) and the only way we can have cosmic acceleration for \( \Delta \rho_{\text{vac}} \neq 0 \), which results in an effective dark energy equation of state \( \omega \neq -1 \).
C. Effective Dark Energy (DE)

The general case considered here is:

\[
\begin{align*}
\rho_{DE}(a) &= \rho_vac + \rho_{DE} a^{-3(1+\omega)} \quad (12) \\
\rho_{DE}(a) &= -\rho_vac + \omega \rho_{DE} a^{-3(1+\omega)},
\end{align*}
\]

where only one component of DE is evolving. We then have from Eq\([8]\) and Eq\([1]\):

\[
\rho_{\Lambda} = \rho_5 + \rho_{DE} \left[ 1 + \frac{1 + 3\omega}{2} \hat{a}_5^{-3(1+\omega)} \right].
\]

(13)

where \(\hat{a}_5\) is some mean value of \(a\) in the past light-cone of \(a_5\) in Eq\([8]\). This reduces to \(\rho_{\Lambda} = \rho_5\) for \(\omega = -1\), which indicates that we need a finite \(\chi_5\) to explain cosmic acceleration. For \(a_5 \rightarrow \infty\) we have \(\rho_5 \rightarrow 0\) because \(\rho_m(a)\) and \(\rho_r(a)\) tend to zero as we increase \(a_5\). The same happens with \(\hat{a}_5^{-3(1+\omega)}\) for \(\omega > -1\), so that:

\[
\rho_{\Lambda} = \rho_{DE} \quad \text{for} \quad a_5 \rightarrow \infty \quad \& \quad \omega > -1. \quad (14)
\]

So evolving DE could produce the observed cosmic acceleration in an infinitely large Universe. This solution does not explain why \(\rho\Lambda = \rho_{DE} \simeq 2.3\rho_m\). The original motivation to introduce DE was to understand why the vacuum energy \(\rho_vac\) can be as small as the measured \(\rho\Lambda\) \([6,8]\). The causal boundary condition shows that \(\rho_vac\) does not contribute to \(\rho_{\Lambda}\), which removes the motivation to have DE. So we will explore here a different way to get \(\Omega_{\Lambda} \simeq 0.70\) without DE, i.e. for \(\omega = -1\).

III. THE SIZE OF THE PHYSICAL UNIVERSE

We assume in this section that vacuum energy is constant after inflation (\(\omega = -1\)). In this case Eq\([8]\) gives:

\[
\rho_{\Lambda} = \rho_5 = \frac{1}{2M_{pl}} \int_{M_i} d^4x (\rho_m + 2\rho_r). \quad (15)
\]

From Eq\([8]\) the horizon after inflation is:

\[
\chi(a) = \eta(a) - \eta(a_e)
\]

(16)

where \(a_e\) represents the end of inflation. We then have \(\chi_5 = \chi(a_5) = \eta(a_5)\) where \(a_5\) is the time when the causal boundary enters the horizon after inflation and \(a_1\) the beginning of inflation. Fig\([2]\) illustrate this. We calculate \(\rho_5\) in Eq\([15]\) by integrating within the light-cone of \(\chi_5\):

\[
\rho_5 = \frac{\int_{0}^{\chi_5} d\chi \chi^2 a^3 \rho_m a^{-3} + 2\rho_r a^{-4}}{2 \int_{0}^{\chi_5} d\chi \chi^2 a^3}, \quad (17)
\]

where \(a = a(\chi)\) in Eq\([16]\). For \(H(a)\) we use:

\[
H^2(a) = H_0^2 \left( \Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_{\Lambda} \right), \quad (18)
\]

with \(\Omega \equiv \rho/\rho_c, \rho_c = 3H^2_{0}/8\pi G, \Omega_r = 4.2 \times 10^{-5}\) \([9]\) and flat Universe \(\Omega_m = 1 - \Omega_{\Lambda} - \Omega_r\). We use Eq\([17]\) to solve \(\Omega_{\Lambda} = \Omega_5\) numerically for \(\Omega_{\Lambda} = 0.69 \pm 0.01\) \([10]\):

\[
\chi_5 = (3.149 \pm 0.004) \frac{c}{H_0}, \quad (19)
\]

\[
a_5 = 0.933 \pm 0.004. \quad (20)
\]

This scale factor corresponds to an age:

\[
t_5 = (0.887 \pm 0.004) \frac{1}{H_0} \simeq 12.5 \text{Gyr}, \quad (21)
\]

compared to \(t_{age} \simeq 0.955/H_0\) today, i.e. about \(\Delta t \simeq 0.84\) Gyr into our past. We can’t observe this boundary \(\chi_5\) today (see Fig\([2]\)) but we will be able to observe it in the future and in our past (see section III.B).

A. Inflation and the coincidence problem

Eq\([15]\) indicates that when the Physical Universe re-enters the Horizon the expansion becomes dominated by \(\rho_{\Lambda}\). This is because \(\rho_m(a_5) < \rho_5 = \rho_{\Lambda}\), as density decreases with the expansion. This results in another inflationary epoch at \(a = a_5\) which keeps the Physical Universe frozen (see Fig\([1]\)). We can now recast the coincidence problem (why \(\rho_{\Lambda} \simeq 2.3\rho_m\)?) into a new question: why we live at a time which is close to \(a_5\)? Looking at Fig\([1]\) we can see that the best time to host observers is a time close to \(a_5\) as the Universe is dominated by \(\rho_m\) (so
there are galaxies) and the Hubble radius is the largest. There is nothing special about this coincidence.

The reason why $\chi_8 \sim 3c/H_0$ and not some other value could reside in the details of inflation: when inflation begins $a_i$ and ends $a_e$ (see Fig[1]). This recasts the coincidence problem into an opportunity to better understand inflation and the origin of homogeneity. We propose to identify $\chi_8 = \eta(a_i)$ with the comoving horizon before inflation begins at time $t_i$, $H_i = H(t_i)$ or $a_i = a(t_i)$:

$$a_i H_i = c \chi_8^{-1} \simeq (0.321 \pm 0.004) H_0$$

(22)

The Hubble rate during inflation $H_I$ is proportional to the energy of inflation. During reheating this energy is converted into radiation: $H_I^2 \simeq \Omega_\Lambda H_0^2 a_i^{-4}$, with $a_e \equiv e^{N} a_i$. We can combine with Eq(22) to find:

$$a_i \chi_8 = \frac{H_i}{H_I} e^{-2N} \Omega_r^{1/2} (\chi_8^2 H_0/c) \simeq 4 \times 10^{8} l_{\text{planck}}$$

(23)

where for the second equality we have used the canonical value of $N \simeq 60$ and $H_i \simeq H_I$, which also yields $a_i \simeq 1.56 \times 10^{-53}$ and $H_I \sim 10^{10}$ GeV. The condition $a_i \chi_8 > l_{\text{planck}}$ requires $N < 70$, close to the value found in [25].

B. Implications for CMB

The (look-back) comoving distance to the surface of last scattering $a_s \simeq 9.2 \times 10^{-4}$ [11] is $\chi_{CMB} = \eta(1) - \eta(a_s) \simeq 3.145 \frac{\chi_8}{a_s}$. This is shown as the horizontal dashed line in the bottom of Fig[2]. This is just slightly smaller than our estimate of the scale when the Physical Universe re-enters the Horizon. Thus, we would expect to see no correlations in the CMB on angular scales $\theta > \theta_8$, where:

$$\theta_8 \equiv 2 \arcsin \frac{\chi_8/2}{\chi_{CMB}} \simeq 60.1 \text{ deg.}$$

(24)

The lack of structure seen in the CMB on these large scales is one of the well known anomalies in the CMB data (e.g. see [20] [21] and references therein). Fig[3] (from [22]) shows a comparison of the measured CMB temperature correlations (points with error-bars) with the $\Lambda$CDM prediction for an infinite Universe (continuous line). There is a very clear discrepancy, which [20] estimates to happen in only 0.025 per cent of the realizations of the infinite $\Lambda$CDM model. If we suppress the large scale modes above $\simeq 60$ deg in the $\Lambda$CDM simulations, the agreement is much better (shaded red area in Fig[3]).

We can also predict $\Omega_\Lambda$ from the lack of CMB correlations. From Fig[3] we estimate $\theta_8 = 62.2 \pm 4.0$ deg to find (using Eq(24) and Eq(17)):

$$\theta_8 = 62.2 \pm 4.0 \text{ deg.} \Rightarrow \Omega_\Lambda = 0.63 \pm 0.11$$

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Note that the above estimate does not take into account the foreground ISW and lensing effects [23] [24], which will typically reduce $\theta_8$ slightly. The value most used in the literature, $\theta_8 \simeq 60$ deg., corresponds to $\Omega_\Lambda \simeq 0.70$.

Note that there are temperature differences on scales larger $\theta_8$, but they are not correlated, as expected in causality disconnected regions. Nearby regions are connected which creates a smooth background across disconnected regions.

IV. DISCUSSION AND CONCLUSIONS

$\Lambda$CDM assumes that $\rho$ is constant everywhere at a fixed comoving time. This breaks causality unless there is inflation, where a tiny homogeneous and causally connected patch, the Physical Universe $\chi_8$, was inflated to be very large today. Regions larger than $\chi_8$ are out of causal contact. Here we require that test particles become free as we approach $\chi_8$. This leads to Eq(8) which is the main result in this paper.

If we ignore the vacuum for now, this condition requires: $\Lambda = 8\pi G \rho_\Lambda$, where $\rho_\Lambda$ is the matter and radiation inside $\chi_8$ (Eq(15)). For an infinite Universe ($a_8 \rightarrow \infty$) we have $\rho_8 \Rightarrow 0$ which requires $\Lambda \Rightarrow 0$. This is also what we find in classical gravity with a $\Lambda$ term, because Hooke’s term diverges at infinity (see Eq(2)). So the fact that $\rho_\Lambda \neq 0$ indicates that the $\chi_8$ may not be infinite. If we add vacuum $\rho_{vac}$, we have that $\rho_\Lambda = \Lambda/8\pi G + \rho_{vac} = \rho_8^{\gamma}$ is independent of $\rho_{vac}$. Thus, whether the Physical Universe is finite or not, $\rho_{vac}$ can not gravitate.
For constant vacuum ($\omega = -1$), we find $\chi_5 \simeq 3.15 c/H_0$ for $\Omega_5 = \Omega_\Lambda \simeq 0.69$. We can also estimate $\chi_5$ as $c/(a_5 H_5)$ when inflation begins, see Eq.[22]. After inflation $\chi_5$ freezes out until it re-enters causality at $a_5 \simeq 0.93$, close to now ($a = 1$). This starts a new inflation (as $\rho_\Lambda = \rho_5 > \rho_m$) which keeps the Physical Universe frozen. Thus a finite $\chi_5$ explains why $\rho_\Lambda \simeq 2.3 \rho_m$, and looking at Fig.[1] we argued that $a_5$ is the best time for observers like us to exist.

For $\omega = -1$, the measured value of $\Omega_\Lambda \simeq 0.70$ predicts that CMB temperature should not be correlated above $\theta > \theta_5 \simeq 60$ deg, a prediction that matches observations (see Fig.[3]). One can also reverse the argument and use the lack of CMB correlations above $\theta_5 \simeq 60$ deg, to predict the size of $\chi_5 \simeq 2\chi_{CMB} \sin(\theta_5/2)$. Together with condition $\rho_5 = \rho_\Lambda$, this provides a prediction of $\Omega_\Lambda \simeq 0.70$, which is independent of other measurements for $\Omega_\Lambda$.

If DE exists, we have shown that only the evolving component of DE is observable. A universe with $\omega < -1$ violates our causality condition. In the limit of an infinite Universe with $\omega > -1$, we find that $\rho_\Lambda = \rho_{DE}$ (see Eq.[14]). But DE gives no clue as to why $\rho_{DE} \simeq 2.3 \rho_m$ and can not explain the lack of CMB correlations for $\theta > 60$ deg. We apply Occam’s razor to argue that there is no need for DE: measurements of cosmic acceleration and CMB can be explained by the finite size of our Physical Universe.

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