Indirect absorption in germanium quantum wells

R. K. Schaevitz,1 D. S. Ly-Gagnon,2 J. E. Roth,3 E. H. Edwards,1 and D. A. B. Miller1,a
1Ginzton Labs, Stanford University, Stanford, California 94305, USA
2Intel, Inc., Santa Clara, California 95054, USA
3Aurrion, Inc., Goleta, California 93117, USA

(Received 5 August 2011; accepted 1 September 2011; published online 21 September 2011)

Germanium has become a promising material for creating CMOS-compatible optoelectronic devices, such as modulators and detectors employing the Franz-Keldysh effect (FKE) or the quantum-confined Stark effect (QCSE), which meet strict energy and density requirements for future interconnects. To improve Ge-based modulator design, it is important to understand the contributions to the insertion loss (IL). With indirect absorption being the primary component of IL, we have experimentally determined the strength of this loss and compared it with theoretical models. For the first time, we have used the more sensitive photocurrent measurements for determining the effective absorption coefficient in our Ge/SiGe quantum well material employing QCSE. This measurement technique enables measurement of the absorption coefficient over four orders of magnitude. We find good agreement between our thin Ge quantum wells and the bulk material parameters and theoretical models. Similar to bulk Ge, we find that the 27.7 meV LA phonon is dominant in these quantum confined structures and that the electroabsorption profile can be predicted using the model presented by Frova, Phys. Rev., 145 (1966).

I. INTRODUCTION

Optical interconnects have the potential to alleviate the interconnect bottleneck by meeting future density and energy requirements.1,2 However, in order to replace electrical interconnects with optical solutions, we need to meet strict off-chip and on-chip power requirements for the optical output devices (e.g. a modulator with an off-chip laser), which one author predicts to be as little as <30 fJ/bit (off-chip) and <7 fJ/bit (on-chip).3 To meet these targets by 2022, we need to employ very strong optoelectronic mechanisms, such as the Franz-Keldysh effect (FKE)4,5 or the quantum-confined Stark effect (QCSE).6 Typically, these strong mechanisms have only been demonstrated in direct band gap group III-V materials. III-V materials, however, present a particular challenge for integrating them with CMOS chips as they act as dopants in Si.

Recently, however, the Ge and SiGe material systems have shown great promise by demonstrating both FKE and QCSE, despite being indirect band gap materials. In 2008, Ge showed initial promise in using FKE for modulation with only 50 fJ/bit of energy consumption.6 Since then, Ge-based FKE modulation has achieved 12.5 Gb/s modulation, a 3 dB bandwidth of 30 GHz with only 100 fJ/bit of energy consumption in an integrated Si waveguide architecture.7 In 2005, QCSE was first demonstrated in Ge/SiGe quantum wells8,9 with initial results showing the possibility of modulation.10,11 While it has not yet been tested under digital modulation to such high speeds,12,13 power consumption has already met 2022 off-chip energy targets of <20 fJ/bit through tight integration with a Si waveguide.13 Ge-based optoelectronic modulators employing FKE and QCSE
show great promise in meeting the strict energy requirements with high speed operation needed to alleviate the future interconnect bottleneck.

While Ge-based modulators have the potential to alleviate the interconnect bottleneck with strong electroabsorption mechanisms and tight CMOS integration, they do have an added insertion loss over typical III-V modulators. This additional loss is largely due to indirect absorption and it is important to fully understand this mechanism in detail to optimize future modulator designs in Ge or GeSi. While bulk Ge and Si have been extensively studied with respect to their indirect absorption,\textsuperscript{14–20} how this absorption changes in a quantum well (QW) heterostructures (for QCSE devices) has not yet been similarly investigated. Only one published paper shows the presence of the longitudinal acoustic (LA) phonon in Ge/SiGe quantum wells with the same energy as that of the bulk material.\textsuperscript{21}

In this paper, we present experimental results for Ge indirect absorption using a more sensitive photocurrent measurement technique. This technique enables us to measure effective absorption coefficient over 4 orders of magnitude, from \(>10^3\) cm\(^{-1}\) to a few tenths of a cm\(^{-1}\), and across a broad wavelength range, from 1200 nm to \(\sim\)1950 nm, with one sample. Typical transmission measurements used to determine such a range of absorption coefficients would need multiple samples with various thicknesses (thick samples for small coefficients and thin samples for large coefficients). Very thick samples are especially difficult for quantum well growth. The ability to experimentally determine using photocurrent a large range of absorption coefficients over a broad wavelength spectrum allows a detailed quantitative comparison with theoretical models and gives empirical absorption data for device design. Because the technique is sensitive only to those absorption mechanisms that give rise to photocurrent, other background loss mechanisms that do not generate free carriers are experimentally eliminated from the measurements. We compare these results with theoretical models and find good agreement with the model presented by Frova, \textit{et al.},\textsuperscript{19} with a 27.7 meV phonon energy consistent with bulk Ge.\textsuperscript{15} By validating the model for indirect absorption out to very long wavelengths, we can then deduce the effect of indirect absorption at wavelengths shorter than 1600 nm (where other absorption mechanisms are present). Understanding of the relative contribution of indirect absorption to insertion loss of possible Ge/SiGe QCSE electroabsorption based modulators is critical in future device design. Detailed understanding of the strength of the indirect absorption tail is also important for the design of any electro-optic devices based on the refractive index changes that are associated with the electroabsorption since such absorption will limit the usable length of waveguide electro-optic devices or the Q of resonator structures. We will first present the experimental setup prior to the theory of modeling indirect absorption for our Ge/SiGe QWs. We will then compare our theory with the experimental results followed by a discussion and a conclusion.

\section*{II. EXPERIMENTAL SETUP}

Photocurrent measurements, one of the more sensitive methods of determining interband absorption, were completed using a tunable optical parametric oscillator (OPO) light source to access the wavelength range of 1200 nm to \(\sim\)1950 nm. In this section, we will describe both the experimental setup as well as the test material.

Using the optical signal from a tunable OPO system (Opotek Opolette 355 II) that is spectrally filtered to remove sub-750 nm wavelengths (the idler beam of the doubly resonant OPO cavity), we performed spectral absorption measurements from 1200 nm to \(\sim\)1950 nm. The optical signal is spatially filtered, strongly attenuated with neutral density filters, and focused on the sample to \(\sim\)10 \(\mu\)m diameter spot size with a reverse Cassegrain reflective objective. Such an objective, being reflective, is non-dispersive over this broad wavelength range, so the focus position is independent of wavelength. Under reverse bias voltages ranging from 2.5 V to 22.5 V using an external DC power supply (HP6612C), photocurrent was extracted and amplified using a Stanford Research Systems (SRS570) low-noise current amplifier. In order to avoid changes in the electrical bandwidth with gain in the SRS570 amplifier, the transconductance of the amplifier was fixed at 20 \(\mu\)A/V throughout the experiment. The signal was then integrated over a period of 6 \(\mu\)s to coincide with the OPO pulses using a Stanford Research Systems (SRS250) boxcar integrator. The measurements were performed under low power such that the photocurrent showed a linear response to optical power. In order to
calibrate and normalize the curves to the optical power, the output of the OPO was measured before the focusing objective and all the neutral density filters were calibrated individually using both the OPO over the entire wavelength range and a continuous wave light source from 1380 nm to 1480 nm (Agilent 8164A). The Cassegrain objective was calibrated using the continuous wave light source and the experimental results agreed well with the manufacturer’s provided spectral loss curve. The final absolute spectra were scaled using d. c. photocurrent measurements with external efficiency maximized on a parameter analyzer (HP4145B) with the low-noise continuous wave laser source swept in steps of 10 nm from 1380 nm to 1480 nm. The final spectra (taken in 1 nm increments) were smoothed through averaging over 11 nm of wavelength as well as time-averaging to reduce the effects of noise from the OPO source.

The sample tested was a 100 μm × 100 μm square mesa with 60 periods of Ge/SiGe QWs in an intrinsic region. Starting with a Si(100) wafer, a virtual substrate (VS) of B-doped Si_{0.1}Ge_{0.9} was grown beyond the critical thickness at 500°C. Subsequent growth and anneals (~800°C) help to prevent the defects from propagating into the active device material. Following the B-doped Si_{0.1}Ge_{0.9} VS of ~1 μm of thickness, a thin ~50 nm intrinsic spacer layer of the same composition is inserted prior to and following the growth of 60 periods of ~14 nm pure Ge wells and ~35 nm of Si_{0.15}Ge_{0.85} barriers. The entire structure is capped by an As-doped Si_{0.1}Ge_{0.9} layer. The final thickness of the epitaxial growth was measured at 5.3 μm, in which the intrinsic region was ~3.2 μm as determined by capacitance-voltage measurements. The p- and n-doping were determined from similarly grown material by SIMS to be ~2 × 10^{17} cm^{-3} and ~1 × 10^{18} cm^{-3}, respectively. The final epitaxial growth was dry-etched to form the mesa and a silicon nitride anti-reflection coating was deposited to ~λ/4 thickness (with the wavelength centered around 1500 nm) on both the substrate and the top of the epitaxy to reduce effects from surface reflections. Contacts were formed on the top and bottom of the mesa using Ti/Au.

III. THEORY

To simulate the electroabsorption spectra of our Ge/SiGe QWs in the pin diode, we must include both the direct (QCSE) and indirect components. Simulation of QCSE is a combination of the tunneling resonance method for the quantum-confined levels structure, a 2D Sommerfeld model for excitonic enhancement of the above band gap absorption, and a variational method on an effective 1S exciton for excitonic effects below the bandgap. This model, which we call the Simple Quantum Well Electroabsorption calculator, or SQWEAC,22 fully simulates the QCSE portion of the electroabsorption spectra in Ge/SiGe quantum wells. The detail of the QCSE portion of SQWEAC is published elsewhere.22 In this section, we will specifically discuss the modeling of the indirect absorption used in that SQWEAC. To model the indirect absorption in Ge/SiGe quantum wells, we need to understand the overall band structure (specifically the band gaps and potentially any type-I confinement), the type of phonons present and the temperature of measurement. In this section, we will first discuss the calculation for the overall band structure in the Ge/SiGe material system following the model-solid theory.23–25 We will then present the models used to predict absorption spectra in bulk Ge and Si materials followed by a discussion of the dominant phonons present in these materials.

A. Band structure

We use the model-solid theory presented by Van de Walle and Martin23,24 to calculate the indirect band gaps and band alignments of Ge/SiGe QWs on a tensile strained VS.22 The effectiveness of this method was verified by Busby, et al., for calculating the band alignments of the direct, L and Δ-valleys in a Ge/Si_{0.2}Ge_{0.8} quantum well system grown on a graded Si_{0.15}Ge_{0.85} relaxed substrate.25 For ease of later discussion, we will reprint the equations and constants used to calculate the band gaps and band offsets.23–25

The calculations presented here are all at 0K for epitaxial Si_{1-x}Ge_{x} on a relaxed Si_{1-x}Ge_{x} substrate, which can later be shifted to room temperature using the Varshni relationship presented in Eq. (1) and the related parameters for Si and Ge in Table I. The overall equation for the band lineup
TABLE I. Varshni Parameters for the relevant band gaps of Si and Ge. $\alpha$ is in units of $10^{-4}$ eV/K², $\beta$ is in K and $E_{g,0K}$ is in eV.

|       | $E_{g,0K}$ | $\alpha$ | $\beta$ | $E_{g,0K}$ | $\alpha$ | $\beta$ |
|-------|------------|----------|---------|------------|----------|---------|
| L     | 0.744$^{26}$ | 4.774$^{27}$ | 235$^{27}$ | 2.176$^{28}$ | 4.774$^{29}$ | 235$^{29}$ |
| $\Delta$ | 1.105$^{28}$ | 4.730$^{29}$ | 636$^{29}$ | 1.170$^{30}$ | 4.730$^{27}$ | 636$^{27}$ |
| $E_{g,dir}$ | 0.892$^{31}$ | 7.25$^{31}$ | 433$^{31}$ | 4.185$^{32}$ | 7.25$^{29}$ | 433$^{29}$ |

The experimentally determined value from x-ray diffraction (XRD) analysis for the thermally induced strain in the virtual substrate of $\sim$0.18% tensile strain was used to calculate the in-plane strain ($\epsilon_{||}$) for each subsequent material grown pseudomorphically to the substrate. The in-plane lattice constant ($a_\parallel$) as established by the strained virtual substrate determines the strain in the layers

$$E_g(T) = E_g(T = 0) - \frac{\alpha T^2}{T + \beta}$$

$$E_{L,\Delta}^e(x, y) = E_{v,avg}(x, y) + \frac{1}{3} \Delta_0(x)$$

$$E_{g,strained}^e(x, y) = E_{g,\Delta}^e(x, y) + E_{h,\Delta}^e(x, y)$$

$$E_{L,\Delta}^u(x, y) = 0$$

where $E_{L,\Delta}^e$ are the unstrained bulk band gaps given in Eqs. (4) and (5). The average valence band, $E_{v,avg}$ is given in Eq. (6). The split-off energy, $\Delta_0$, is linearly interpolated from the Ge and Si values given in Table II. The hydrostatic shift, $E_{L,\Delta}^h$ is given in Eq. (7) and the uniaxial splitting is given in Eqs. (8)-(10) for the $L$, $\Delta_\perp$ ($\Delta_2$) and $\Delta_\parallel$ ($\Delta_4$) bands.

$$E_g(x) = 2.01 - 1.27x$$

$$E_{g}(x) = 1.155 - 0.43x + 0.206x^2$$

$$E_{v,avg}(x, y) = (0.47 - 0.06y)(x - y)$$

$$E_{h,\Delta}^e(x, y) = (a_{e,\Delta}^e - a_e^e) \cdot (2\epsilon_{||} + \epsilon_{\perp})$$

$$E_{u,\Delta}^e(x, y) = \frac{2}{3} \Xi_u^\Delta \cdot (\epsilon_{\perp} - \epsilon_{||})$$

$$E_{u,\Delta}^e(x, y) = -\frac{1}{3} \Xi_u^\Delta \cdot (\epsilon_{\perp} - \epsilon_{||})$$

$$E_{u}(x, y) = 0$$

where $\epsilon_{||}$ and $\epsilon_{\perp}$ are the strain components along the parallel and perpendicular growth planes, respectively, and have the relation given by Eq. (11), and $a_{e,\Delta}^e$ and $\Xi$ are the deformation potentials linearly interpolated between the Si and Ge values given in Table II.

$$\epsilon_{\perp} = -\frac{2}{C_{12}} \frac{C_{11}}{C_{12}} \epsilon_{||}$$

The experimentally determined value from x-ray diffraction (XRD) analysis for the thermally induced strain in the virtual substrate of $\sim$0.18% tensile strain was used to calculate the in-plane strain ($\epsilon_{||}$) for each subsequent material grown pseudomorphically to the substrate. The in-plane lattice constant ($a_{||}$) as established by the strained virtual substrate determines the strain in the layers...
TABLE II. Ge and Si elastic constants, spin orbit splitting and deformation potentials

|                          | Silicon | Germanium |
|--------------------------|---------|-----------|
| $C_{11}$ (10^7 N cm$^{-2}$) | 1.675   | 1.315     |
| $C_{12}$ (10^7 N cm$^{-2}$) | 0.650   | 0.494     |
| $\Delta \delta$ (eV)     | 0.044   | 0.296     |
| $\Delta_0^L - \Delta_0$ (eV) | -3.12   | -2.78     |
| $\Delta_0^H - \Delta_0$ (eV) | 1.72    | 1.31      |
| $\Xi_1$ (eV)              | 8.7     | 9.42      |

(barrier and well) grown pseudomorphic to the virtual substrate. The in-plane strain is calculated with $\epsilon_{||,\text{epi}} = a_{\text{||,sub}} / a_{\text{epi, bulk}} - 1$, where the in-plane lattice constant (which sets the in-plane strain for pseudomorphically grown layers) is determined by Vegard’s Law in Eq. (12).

$$a_{\text{Si}, \text{Ge}} = a_{\text{Si}} + 0.01992x + 0.0002733x^2 \text{Å}$$ (12)

Following calculation of the $L$, $\Delta_1$ ($\Delta_2$) and $\Delta_1$ ($\Delta_4$) band gaps, the splitting of the light hole (LH) and heavy hole (HH) bands must also included. The band gaps calculated thus far are from the bottom of the conduction band to the topmost valence band, which is either the LH or the HH depending on whether the material has tensile or compressive strain, respectively. To calculate the two transitions from conduction to LH and HH, the splitting energies of the LH and HH as deduced by QCSE spectra (simulated in SQWEAC) are added to the relevant band gap. The indirect masses associated with the conduction bands are calculated according to the model presented by Rieger, while the LH and HH masses are calculated in the same way as presented in the SQWEAC model.

B. Phonons and the indirect absorption model

Modeling of the band gaps, offsets and masses is an important component to understanding the indirect absorption in Ge/SiGe quantum wells. However, we must also have a good understanding of the phonons present, or at least those that dominate the absorption spectra, as well as the appropriate model to use. In this section, we will first present a few models used to approximate the indirect absorption spectra, including the model by Frova, et al., which we use to successfully simulate our experimental data. We will then discuss the dominant phonons in bulk Ge and Si as well as the limited experimental evidence for the 27.7 meV longitudinal acoustic (LA) phonon in a Ge/SiGe QW heterostructure.

1. Modeling indirect absorption

There are three main models that have been effectively used to approximate the indirect absorption spectra in Ge and Si bulk materials: a one-phonon model, a multiple-phonon model and an electric-field dependent model. The one-phonon model (Eq. (13)) or the electric-field dependent model (Eq. (14)) can use either an averaged phonon energy (one energy value that weights the relative strength of each phonon present) or a single phonon energy for the dominant phonon present in the system. In both of these models, the resulting indirect absorption has the characteristic quadratic dependence on the energy difference between the photon energy on one hand and the band gap energy plus or minus a phonon energy on the other hand. The main difference between these two equations is that the latter also includes the very slight electric field dependence (the limit of this equation as the electric field, $F$, approaches 0 is similar in form to the one-phonon equation).

$$\alpha = A \left[ \frac{1}{1 - e^{-\frac{h\nu - E_{G, \text{ind}} - k\Theta}{k\nu}}} \right]^2$$
and $\alpha'$ refer to the Airy and Airy derivative functions, field dependence gave the best agreement with our experimental results.\(^{19}\) In the next section, quantum number of the hydrogenic exciton and can be taken as 1).

The masses in Eq. (15) are those of the relevant bands, such as the LH or HH for the valence band and $L_e, \Delta_{\perp} (\Delta_2)$ or $\Delta_{||} (\Delta_4)$ for the conduction band.

The multiple-phonon model has a very different form near the emission and absorption energies for each phonon. Macfarlane, et al., found that there were two classes of phonons that behaved differently around their “turn-on” energies (within ~10 binding energies).\(^{15}\) A symmetry-allowed phonon had an initial $(\Delta E)^{1/2}$ component, while the forbidden one had a $(\Delta E)^{3/2}$ component before the $(\Delta E)^2$ shape dominated the spectra. These characteristic “knees” and the mathematical deconvolution of the two components are readily seen in Macfarlane’s publication.\(^{15}\) These two types of absorption spectra are given by Eqs. (18) and (19) for the allowed and forbidden transitions, respectively.\(^{20}\) While these two transitions have different shape upon “turn-on,” they also have very different dependence on temperature (when excluding the temperature dependence due to the phonon population as determined by Bose-Einstein statistics). The allowed transitions, with coefficient $A_a$, have no temperature dependence, while the forbidden (which are weaker in magnitude), with coefficient $A_f$, increase with higher temperatures.

$$\alpha_a = \frac{A_a}{\hbar \omega} \left( \hbar \omega - E_{g,ind} + \frac{R_e}{n^2} \pm k\Theta \right)^{1/2}$$  \hspace{1cm} (18)

$$\alpha_f = \frac{A_f}{\hbar \omega} \left( \hbar \omega - E_{g,ind} + \frac{R_e}{n^2} \pm k\Theta \right)^{3/2}$$  \hspace{1cm} (19)

where $R_e$ is the Rydberg energy and $n$ is unspecified by the authors (but is likely the principal quantum number of the hydrogenic exciton and can be taken as 1).

In our study, we found that the model presented by Frova, et al., in Eq. (14) with the electric field dependence gave the best agreement with our experimental results.\(^{19}\) In the next section,
TABLE III. Ge and Si associated phonon energies (meV) and equivalent temperatures (K) calculated using $kT$, where $k$ is Boltzmann’s constant.

|     | TA  | LA  | TO  | LO  |
|-----|-----|-----|-----|-----|
| Ge  | 7.8 | 320 | 30.0| 350 |
|     | 90  | 36.0| 420 |
| Si  | 18.3| 670 | 90.5| 122.4|
|     | 212 | 1050| 1420|

we will discuss in more detail the phonons present and whether they are forbidden or allowed for Si and Ge.

2. Phonons

There are typically two types of phonons relating to absorption, which can be categorized based on the consequences of the symmetry of the relevant unit cell wavefunction at the conduction band minima of interest: the transition allowed and the transition forbidden. The four phonons that can have either allowed or forbidden transitions are the transverse and longitudinal acoustic (TA, LA) and the transverse and longitudinal optic (TO, LO). In silicon, all four phonons are allowed, while in germanium, only the LA and TO phonons have symmetry allowed transitions. The experimentally determined energy values ($k\theta$) as well as their equivalent temperatures ($\theta$) for both Si and Ge are given in Table III.

When modeling with the one-phonon equations (Eqs. (13) and (14)), we can use either the dominant phonon or the weighted average phonon energy. For Ge, the dominant phonon is the LA phonon with energy 27.7 meV while the averaged phonon has a slightly lower energy of $\sim$23 meV. The nominally forbidden TA phonon, with only 7.8 meV, also contributes increasingly to the absorption spectra at elevated temperatures (such as room temperature), thus reducing the weighted average phonon. While the TO phonon is symmetry allowed (with energy 30.0 meV), its contribution is much weaker in comparison to the LA (and TA) phonon. The forbidden LO phonon with its comparatively larger energy of 36.0 meV is difficult to resolve even in much more sensitive experiments. Similarly to Ge, the LA phonon for Si is dominant with energy 57.7 meV and a corresponding weighted average phonon energy of 51.7 meV (or 47.4 meV). For the multiple-phonon model given in Eqs. (18) and (19), all phonons can be included with the appropriate $A_a$ and $A_f$ coefficients.

While it is important to understand the indirect absorption contributions for bulk pure Ge and pure Si materials, our Ge/SiGe quantum well heterostructure also has composite SiGe material. Braunstein, et al., provides significant experimental data using the one-phonon model for many SiGe compositions and measurement temperatures. They successfully modeled over 20 compositions of SiGe ranging from pure Ge to pure Si over a number of temperatures from 78K to 295K. An important result of this extensive study was that the variation in phonon energy (or equivalent temperature) had an S-like shape with change in composition. At low Si concentrations (<30%), the phonon energy was relatively constant at the Ge average value. At high Si concentrations (>70%), the phonon energy was similar to the Si average value. However, between 30% and 70% Si, there was a clear weighted average phonon energy dependence on composition.

For the purpose of modeling our experimental data, we found that using the dominant LA phonon in Ge gave the best results with the electric-field dependent Eq. (14). With experimental evidence showing that the Ge material gives the dominant phonon energy for SiGe compositions below 30% Si, we have included a constant phonon energy of 27.7 meV (equivalent to the bulk Ge LA phonon) for each type of material present in our test sample, giving us good fit with our experimental data. Our assumption for the phonon energy compares well with the only experimental evidence published on Ge/SiGe QWs. Bonfanti, et al., found only the presence of a $\sim$28 meV phonon in photoluminescence spectroscopy of a similar Ge/SiGe QWs. This phonon energy was independent of well width, which ranged from very small $\sim$5 nm to very larger $\sim$23 nm. Bonfanti also found that the indirect L-band, with its type-I offset of $\sim$100 meV, provided some confinement
to the electrons (thus increasing the overall band gap). This confinement was clearly more prevalent with the smaller wells (with no electric field).

In the next section, we will provide our experimental results using photocurrent measurements and compare them with the model presented here using the 27.7 meV phonon and Eq. (14) for the shape of the indirect absorption spectra as well as model-solid theory for calculation of the relevant band gaps. We will then discuss the validity of using this model in the Ge/SiGe QW material system compared with other models.

IV. EXPERIMENTAL RESULTS AND DISCUSSION

Scaling the OPO measurements of the 60 QW sample with d. c. photocurrent from a continuous wave source, we measured the absolute electroabsorption profile for a number of electric fields and wavelengths ranging from 1.2 μm to ~1.95 μm. Beyond 1.9 μm in wavelength, the data extrapolated was close to the noise threshold, especially at higher electric fields. The results for two electric fields in which the full wavelength spectrum of photocurrent was obtained are presented in Fig. 1.
for the percentage of light absorbed as well as the effective absorption coefficient (over the total intrinsic thickness, which is mainly well and barrier material). An experimentally determined value of $K = 5.5 \times 10^{10}$ with units of $[m^{4/3}/(s^{4/3}kg^{7/3})] \times \text{cm}^{-1}$ for Eq. (14) was used to fit the spectra and gives an absorption coefficient in units of $\text{cm}^{-1}$. (Note that our effective absorption coefficients here are based on the total thickness of well and barrier material). Given the characteristic quadratic dependence of indirect absorption on the energy difference, it is useful to plot the square root of absorption to compare experiment and theory. The resulting linear component is presented in Fig. 2(a), which shows good agreement between theory and experiment.

For the direct band gap QCSE spectra, we can make a few important observations. The first is that the exciton enhanced direct transitions appear to have a $\sim 15\%$ reduction in peak height from the theoretically predicted value given by SQWEAC. The theoretically predicted value is based on the nominal 10 nm QWs in contrast to the $\sim 14$ nm well widths simulated here. The wider well widths likely have a reduction in the excitonic enhancements than is predicted by the model (as previously discussed). The second observation is that despite calculating the effective absorption coefficient over the entire intrinsic region (in which the SiGe material minimally absorbs in comparison to the pure Ge wells), the magnitude of absorption is still on the same order as bulk Ge for the direct band
gap. Thus, the enhancement in direct band gap absorption due to quantum confinement and excitonic effects is still quite large.

While the QCSE model\textsuperscript{22} and Eq. (14)\textsuperscript{19} effectively describe the electroabsorption spectra in the Ge/SiGe quantum wells, there is a region between 1470 nm and 1570 nm with a slight deviation of theory from experiment. The slight deviation could be due to other absorption effects in the material, such as defects, or possibly some tail on the exciton absorption peak. While we have some deviation, the one-phonon Macfarlane model\textsuperscript{14} (Eq. (13)) shows a much larger discrepancy for the same phonon contributions and band gap as seen in Fig. 2(b). Additionally, the slope of the linear portion in this figure does not approximate the experimental data as well as the Frova model. Using a more averaged phonon value of 23.3 meV (as derived by Braunstein for bulk Ge) yields slightly different results, but the larger discrepancy still remains.

Comparing our results with the low temperature photoluminescence results by Bonfanti, et al.,\textsuperscript{21} we are able to model our data using the same LA phonon value present in their experimental data.\textsuperscript{21} However, the confinement increase in the band gap is not as prevalent in our experimental results, which is likely due to the application of electric field and our wide 14 nm well width. The increase in overall transition energy shifts the linear portions (when plotted as the square root of absorption) towards shorter wavelengths (or higher energy). This effect results in the model not effectively approximating the experimental data (evident in Fig. 3). When removing the confinement energy from the transition energy for the L-valley to HH-valley and shifting their results to room temperature using the Varshni relationship, our experimental band gap agrees very well with their results.

V. CONCLUSION

For the first time, we have used very sensitive photocurrent absorption measurements to resolve the indirect (and direct) electroabsorption profile in Ge/SiGe quantum wells. We show good agreement between theory and experiment in both the band gaps and the dominant LA phonon present. The ability to model the indirect absorption effectively using the electric-field dependent model presented by Frova, et al.,\textsuperscript{21} allows us to accurately determine the expected insertion loss for devices, such as waveguide or surface-normal modulators, made in this material system employing FKE or QCSE. Future modulator designs can use this model to effectively design modulators that have minimized background absorption and thus optimized extinction ratios and insertion losses.
ACKNOWLEDGMENTS

This material is based upon work supported, in part, under Agreement HR0011-08-09-0001 between DARPA and Oracle. The views expressed are those of the author and do not reflect the official policy or position of the Department of Defense or the U.S. Government. The authors acknowledge the financial support of the Interconnect Focus Center, one of the six research centers funded under the Focus Center Research Program, a Semiconductor Research Corporation program. The authors also acknowledge the financial support of the Intel Fellowship Program, Stanford Graduate Fellowship, and the National Science Foundation Graduate Research Fellowship.

1 D. A. B. Miller, “Device requirements for optical interconnects to silicon chips,” Proc. IEEE, 97, 1166–1185 (2009).
2 M. Hilbert and P. López, “The world’s technological capacity to store, communicate, and compute information,” Science, 332, 60–65 (2011).
3 W. Franz, “Influence of an electric field on an optical absorption edge,” Z. Naturforsch, 13a, 484 (1958).
4 L. V. Keldysh, “The effect of a strong electric field on the optical properties of insulating crystals,” Zh. Eksp. Teor. Fiz., 34, 1138 (1958).
5 D. A. B. Miller, D. S. Chemla, T. C. Damen, A. C. Gossard, W. Wiegmann, T. H. Wood, and C. A. Burrus, “Band-edge electroabsorption in quantum-well structures: The quantum-confined Stark effect,” Phys. Rev. Lett., 53, 2173–2176 (1984).
6 J. Liu, M. Beals, A. Pomerene, S. Bernardis, R. Sun, J. Cheng, L. C. Kimerling, and J. Michel, “Waveguide-integrated, ultralow-energy GeSi electro-absorption modulators,” Nature Photonics, 2, 433–437 (2008).
7 Y. Luo, J. Simons, J. Costa, I. Shubin, W. Chen, B. Frans, M. Robinson, R. Shafiha, S. Liao, N.-N. Feng, X. Zheng, G. Li, J. Yao, H. Thacker, M. Asghari, K. Goossen, K. Raj, A. V. Krishnamoorthy, and J. E. Cunningham, “Experimental studies of the Franz-Keldysh effect in CVD grown GeSi epi on SOI,” Proc. of SPIE, 7944 (2011).
8 Y.-H. Kuo, Y. K. Lee, S. R. Y. Ge, J. E. Roth, T. I. Kamins, D. A. B. Miller, and J. S. Harris, “Strong quantum-confined Stark effect in germanium quantum-well structures on silicon,” Nature, 437, 1334–1336 (2005).
9 Y.-H. Kuo, Y. K. Lee, Y. Ge, S. Ren, J. E. Roth, T. I. Kamins, D. A. B. Miller, and J. S. Harris, “Quantum-confined Stark effect in Ge/Ge quantum wells on Si for optical modulators,” IEEE J. Selected Topics Quant. Electron., 12, 1503–1513 (2006).
10 J. E. Roth, O. Fidaner, R. K. Schaevitz, Y.-H. Kuo, T. I. Kamins, J. S. Harris, and D. A. B. Miller, “Optical modulator on silicon employing germanium quantum wells,” Opt. Express, 15, 5851–5857 (2007).
11 J. E. Roth, O. Fidaner, E. H. Edwards, R. K. Schaevitz, Y.-H. Kuo, N. C. Helman, T. I. Kamins, J. S. Harris, and D. A. B. Miller, “C-band side-entry Ge quantum-well electroabsorption modulator on SOI operating at 1 V swing,” Electron. Lett., 44, 49–50 (2008).
12 Y. Rong, Y. Ge, Y. Hao, M. Fiorentino, M. R. T. Tan, T. I. Kamins, T. J. Ochalski, G. Huyet, and J. S. Harris, “Quantum-confined Stark effect in Ge/Ge quantum wells on Si,” IEEE J. Selected Topics Quant. Electron., 16, 85–92 (2010).
13 S. Ren, Y. Rong, S. A. Claussen, R. K. Schaevitz, T. I. Kamins, J. S. Harris, and D. A. B. Miller, “A GeSi/Ge quantum well waveguide modulator monolithically integrated with SOI waveguides,” in IEEE Group IV Photonics (London, England, 2011) p. WA3.
14 G. G. Macfarlane and V. Roberts, “Infrared absorption of germanium near the lattice edge,” Phys. Rev., 97, 1714–1716 (1955).
15 G. G. Macfarlane, T. P. McLean, J. E. Quarrington, and V. Roberts, “Fine structure in the absorption-edge spectrum of Ge,” Phys. Rev., 108, 1377–1383 (1957).
16 G. G. Macfarlane, T. P. McLean, J. E. Quarrington, and V. Roberts, “Fine structure in the absorption-edge spectrum of Si,” Phys. Rev., 111, 1245–1254 (1958).
17 R. Braunstein, A. R. Moore, and F. Herman, “Intrinsic optical absorption in germanium-silicon alloys,” Phys. Rev., 109, 695–710 (1958).
18 G. G. Macfarlane, T. P. McLean, J. E. Quarrington, and V. Roberts, “Exciton and phonon effects in the absorption spectra of germanium and silicon,” J. Phys. Chem. Solids, 8, 388–392 (1959).
19 A. Frova, P. Handler, F. A. Germano, and D. E. Aspnes, “Electro-absorption effects and the band edges of silicon and germanium,” Phys. Rev. 145, 575–583 (1966).
20 T. Nishino, M. Takeda, and Y. Hamakawa, “Indirect exciton absorption in germanium,” J. Phys. Soc. Jap, 37, 1016–1023 (1974).
21 M. Bonfanti, E. Grilli, M. Guzzi, M. Virgilio, G. Grosso, D. Chrestina, G. Isella, H. von Känel, and A. Neels, “Optical transitions in Ge/SiGe multiple quantum wells with Ge-rich barriers,” Phys. Rev. B, 78, 041407 (2008).
22 R. K. Schaevitz, E. H. Edwards, J. E. Roth, E. Fei, Y. Rong, P. Wahl, T. I. Kamins, J. S. Harris, and D. A. B. Miller, “Simple electroabsorption calculator for designing 1310nm and 1550nm modulators in germanium quantum wells,” (Aug. 1, 2011), submitted for publication
23 C. G. Van de Walle and R. M. Martin, “Theoretical calculations of heterojunction discontinuities in the Si/Ge system,” Phys. Rev. B, 34, 5621–5634 (1986).
24 C. G. Van de Walle, “Band lineups and deformation potentials in the model-solid theory,” Phys. Rev. B, 39, 1871–1883 (1989).
25 Y. Busby, M. D. Seta, G. Capellini, F. Evangelisti, M. Ortolani, M. Virgilio, G. Grosso, G. Pizzi, P. Calvani, S. Lupi, M. Nardone, G. Nicotra, and C. Spinella, “Near- and far-infrared absorption and electronic structure of Ge-SiGe multiple quantum wells,” Phys. Rev. B, 82 (2010).
26 S. Zwerdling, B. Lax, L. M. Roth, and K. J. Button, “Exciton and magneto-absorption of the direct and indirect transitions in germanium,” Phys. Rev., 114, 80–89 (1959).
27 C. D. Thurmond, “The standard thermodynamic functions for the formation of electrons and holes in Ge, Si, GaAs, and GaP,” J. Electrochem Soc., 122, 1133–1141 (1975).
28 S. Krishnamurthy, A. Sher, and A.-B. Chen, “Generalized Brook’s formula and the electron mobility in Si, Ge1−x alloys,” Appl. Phys. Lett., 47, 160–162 (1985).
29 L. Yang, J. R. Watling, R. C. W. Wilkins, M. Boriçi, J. R. Barker, A. Asenov, and S. Roy, “Si/SiGe heterostructure parameters for device simulations,” Semicond. Sci. Technol., 19, 1174–1182 (2004).
30 W. Bludau, A. Onton, and W. Heinke, “Temperature dependence of the band gap of silicon,” J. Appl. Phys., 45, 1846–1848 (1974).
31 P. A. Dafesh and K. L. Wang, “Temperature dependence of the E0 transitions in bulk Ge and a Ge-rich (Si)m/(Ge)n superlattice,” Phys. Rev. B, 45, 1712–1718 (1992).
32 D. E. Aspnes and A. A. Studna, “Direct observation of the E0 and E0 + Δ0 transitions in silicon,” Solid State Commun., 11, 1375–1378 (1972).
33 R. K. Schaevitz, J. E. Roth, S. Ren, O. Fidaner, and D. A. B. Miller, “Material properties of Si-Ge/Ga quantum wells,” IEEE J. Selected Topics Quant. Electron., 14, 1082–1089 (2008).
34 M. M. Rieger and P. Vogl, “Electronic-band parameters in strained Si1−xGex alloys on Si1−yGe y substrates,” Phys. Rev. B, 48, 276–287 (1993).
35 L. H. Hall, J. Bardeen, and F. J. Blatt, “Infrared absorption spectrum of germanium,” Phys. Rev., 95, 559–560 (1954).
36 G. G. Macfarlane and V. Roberts, “Infrared absorption of silicon near the lattice edge,” Phys. Rev., 98, 1865–1866 (1955).
37 R. J. Elliot, “Intensity of optical absorption by excitons,” Phys. Rev., 108, 1384–1389 (1957).