COSMOLOGY AND MODELS OF
SUPERSYMMETRY BREAKING
IN STRING THEORY

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ABSTRACT

Supersymmetry breaking in string theory is expected to occur when moduli fields acquire
non-trivial expectation values. In the early universe these fields start out displaced from
their final destinations. I present some recent ideas about the cosmological evolution of the
dilaton modulus field on the way to its vacuum expectation value.

∗) Contribution to the proceedings of SUSY94’ Workshop,
May 14-171994, Ann Arbor, Michigan.
Supersymmetry (SUSY) breaking in string theory is expected to occur when certain gravitationally interacting fields, moduli, obtain vacuum expectation values (VEV’s) [1]. These VEV’s may break SUSY spontaneously. When considering the cosmological time evolution of these fields an obvious question arises [2]. The fields, because of thermal effects and quantum fluctuations, start out displaced from their final destinations. How do they get to their VEV’s? Is there a reasonable dynamical evolution that brought them there? It is not always possible to ignore these questions, even by specifying very special initial conditions, because thermal fluctuations and quantum fluctuations tend to smear the initial conditions. A partial list of previous work on the subject is given in [3]. The most interesting modulus field seems to be the dilaton field, \( \phi \), whose expectation value determines the string coupling parameter, \( g_{\text{string}} \sim \langle e^\phi \rangle \). The question seems to arise for other moduli as well, for example, in the class of models of the no-scale type [4].

A suggestion as to what the cosmological evolution of the dilaton modulus field may be at the earlier stages was put forward recently [5, 6]. I combine these suggestions with ideas about the evolution in the later stages as suggested in [7, 2, 8, 9].

i) Accelerated Inflation
The first, and best understood, stage of the evolution starts when the dilaton is far away from its value today, deep in the weak-coupling region (\( \phi \ll -1 \)). The Hubble parameter, \( H \), is small. The evolution in this epoch is determined by the vacuum solution of the string dilaton-gravity equations of motion [10]. After a period whose length is determined by the initial conditions, but has to be at least one second (a very long time!) to solve the horizon and flatness problems, a strong curvature phase is reached.

ii) “Branch Change”
After the long period of accelerated inflation, the universe is much larger than at the beginning. Curvatures and kinetic energies are of the order of the string curvature and energy. The correct dynamical description of this phase should, therefore, be stringy in nature. If the value of the dilaton is small throughout this stage of evolution, dynamics can be described by classical string theory in terms of a two-dimensional conformal field theory. This stage is not yet well understood. At the moment, the only existing examples are not quite realistic [11, 12]. In this epoch, a branch change, or a phase transition, from the accelerated expansion phase into what will eventually become a phase of decelerated expansion, occurs. This epoch lasts a short period and may be identified with the “Big-Bang”. A large amount of radiation is generated in this epoch. After some time the universe cools down enough and the next stage follows. The value which the dilaton takes at the end of this epoch is important and determines many aspects of the later evolution.

iii) Radiation Domination
Some time after the “branch change” event, the universe cools down and may be described accurately, again, by means of string dilaton-gravity effective theory. Now, however, radiation and matter are important factors determining the dynamical evolution. The dilaton remains approximately at the value that it had at the end of the “branch change” epoch. The universe evolves as a regular Friedman-Robertson-Walker (FRW) radiation-dominated
universe.

iv) Dilaton Roll and Coherent Oscillations
After a while the universe cools down further and the dilaton potential becomes important. The dilaton starts to roll on its potential. To end up at a non-trivial minimum the dilaton has to start this phase in the basin of attraction of the minimum. As explained in [2], for non-perturbative potentials that are expected to induce SUSY breaking the “basin of attraction” of their minimum is quite small. Trapping the dilaton in a minimum remains a challenge for models of SUSY breaking. If the dilaton gets into the basin of attraction of a non-trivial minimum it coherently oscillates around the minimum, producing radiation in various forms, and the universe reheats. Some aspects of this last stage were discussed in [8, 9, 13].

I proceed to describe in more detail the different phases of cosmological evolution. To describe the first phase, look for solutions of the effective string equations of motion in which the metric is of the isotropic, FRW type with vanishing spatial curvature

\[ ds^2 = -dt^2 + a^2(t)dx_idx^i \]
\[ \phi = \phi(t) \] (1)

The Hubble parameter, \( H \), is related to the scale factor, \( a \) in the usual way, \( H \equiv \frac{\dot{a}}{a} \). Some algebra leads to three independent first order equations for the dilaton and \( H \) [10]. The original dilaton equation is a consequence of these equations, which read

\[ \dot{H} = \pm H \sqrt{3H^2 + U + e^\phi \rho - \frac{1}{2} U'} + \frac{1}{2} e^\phi p \] (2a)
\[ \dot{\phi} = 3H \pm \sqrt{3H^2 + U + e^\phi \rho} \] (2b)
\[ \dot{\rho} + 3H(\rho + p) = 0 \] (2c)

Some sources in the form of an ideal fluid were included [10, 15] as well. The \((\pm)\) signifies that either a \((+)\) or \((-)\) is chosen for both equations simultaneously. The solutions to the equations (2a-2c) belong to two branches, according to which sign is chosen. The \((+)\) branch has some unusual properties. In the absence of any potential or sources the solution for \( \{H, \phi\} \) is given by

\[ H^{(+)} = \pm \frac{1}{\sqrt{3}} \frac{1}{t - t_0} \]
\[ \phi^{(+)} = \phi_0 + (\pm \sqrt{3} - 1) \ln(t_0 - t) \quad , \quad t < t_0 \] (3)

\(^1\)The situation for non-vanishing spatial curvature is discussed in [14].
This solution describes either accelerated inflationary expansion and evolution from a cold, flat and weakly coupled universe towards a hot, curved and strongly coupled one or accelerated contraction and evolution towards weak coupling. In general, the effects of a potential and sources on this branch are quite mild. The dilaton of this branch flies through potential minima. As can be seen from Eq.(2b), if \( H > 0 \), it is impossible that \( \dot{\phi} = 0 \). Inflation, in this solution, is driven by the dilaton’s kinetic energy, thanks to the negative value \((-1)\) of the BD \( \omega \) parameter.\(^2\) To solve the flatness and horizon problems, at least 60 e-folds of inflation are necessary.\(^3\) Setting \( t_0 = 0 \), one sees that \( t_{\text{initial}}/t_{\text{final}} \gtrsim 10^{43} \). As we will see, this phase ends when the curvature becomes strong. This happens at \( t_{\text{final}}/t_{\text{string}} \approx -1 \). Therefore the minimal duration of this stage is \( 10^{43} t_{\text{string}} \approx 1 \) second. The dilaton displacement during that time is \( \sqrt{\alpha'} (\phi_{\text{final}} - \phi_{\text{initial}}) > (\sqrt{3} + 1) \sqrt{3} \cdot 60 \sqrt{\alpha'} \gtrsim 300 \sqrt{\alpha'} \).

The last two stages of evolution are described by the \((-\)) branch. This is a regular, negative feedback, branch. The solution \( \{H(\cdot), \phi(\cdot)\} \) in the absence of potential or sources is simply given by

\[
H(\cdot) = \pm \frac{1}{\sqrt{3} t - t_0}, \\
\phi(\cdot) = \phi_0 + (\pm \sqrt{3} - 1) \ln(t - t_0), \quad t > t_0
\]

This solution describes decelerated expansion \((H > 0, \dot{H} < 0)\) or decelerated contraction \((H > 0, \dot{H} < 0)\) depending on the initial sign of \( H \). Correspondingly, the evolution is towards strong or weak coupling, respectively. In the presence of appropriate sources, as in Eqs.(2), and when the dilaton potential, \( U \), is negligible the \((-\)) branch solution becomes an ordinary FRW expanding universe with constant dilaton (see also \([7]\)). The radiation-dominated phase occurs when

\[
e^\phi \rho \gg U(\phi) \\
e^\phi \rho \gg U'(\phi) \\
p = \frac{1}{3} \rho
\]

Then, as a straightforward calculation shows, the solution of Eqs.(2) is

\[
a = \sqrt{t}, \\
H = \frac{1}{2t} \\
\phi = \phi_E \\
\rho = \frac{3}{2} e^{-\phi_E} \frac{1}{a^4}
\]

\(^2\)See \([16]\) for a discussion of these issues in a general Brans-Dicke theory.

\(^3\)There may be subtleties associated with late time evolution which may change the number 60 by some amount.
where $\phi_E$ is a constant. During the branch change phase a large amount of radiation is expected to be produced. A rough estimate would be $\rho \approx H_{\text{BranchChange}}^4$, and therefore the conditions (5) are expected to hold.

The branch change phase marks the transition from accelerated inflation to decelerated expansion. The transition between inflationary evolution and a FRW expanding universe is usually referred to as “graceful exit” from inflation and it is a well known problem to be faced by any model of inflation. The possibility of graceful exit from accelerated inflation is closely related, in our setup, to the question of whether the two branches can be smoothly connected to one another. In [6] the possibility of having a branch change while the curvature is still small was examined. The conclusion was that this is not possible and that, therefore, if a branch change does occur, it has to be during a strong (string scale) curvature era.

It is not possible to continue and study the evolution using the effective dilaton-gravity theory because the whole field-theoretic framework breaks down. If the dilaton VEV is still small then classical string dynamics should be a good approximation. The best is therefore to describe this phase using a 2-d conformal field theory. An example of a conformal field theory that realizes a branch change between two dual branches (similar, but not the same as the (+) and (−) branches) has been constructed [11, 12], and should be regarded as an existence proof showing that such a transition can in fact occur during a high curvature era. The branch change phase is followed smoothly by a radiation-dominated phase described by the (−) branch as explained previously.

![Figure 1](image_url)

**Figure 1.** A typical expected dilaton potential. The wavy lines at $\phi = \phi_L$ and $\phi = \phi_R$ mark the limits of the basin of attraction of the minimum of the potential. The region of large negative values of $\phi$ is the weak coupling region. The units on the vertical axis are arbitrary.

The evolution during the fourth phase depends in a stronger way on the details of the particular model of supersymmetry breaking. It is in this phase that cosmological evolution and models of supersymmetry breaking are most closely related. For a summary of relevant properties of the dilaton and its interaction during this phase see, for example [17]. A typical
expected dilaton potential is shown in figure 1.

The exit value of the dilaton, $\phi_E$, determines its subsequent evolution. If $\phi_E < \phi_L$ or $\phi_E > \phi_R$, the dilaton will continue to roll on its potential towards weak coupling \[2\]. After a while the evolution will be described by the vacuum (−) branch. This is definitely not a description of our universe today. If the exit value of the dilaton, $\phi_E$, is within the basin of attraction of the minimum, marked by the two wavy lines in figure 1, the dilaton will start to coherently oscillate around the minimum and finally settle down in its minimum. As it oscillates around the minimum of the potential the dilaton reheats the universe to a temperature, $T_{RH}$, determined by the dilaton potential and interactions. Different aspects of the coherent oscillation phase have been considered in \[3, 8, 13\]. It is clear that the deeper and wider basin of attraction near the potential minimum, the better the dilaton chances of getting trapped there. The only suggestion, to date, of how to explain the trapping of $\phi$ can be found in \[18\].

Acknowledgement
This talk is based in part on joint work with Paul Steinhardt and with Gabriele Veneziano. I would like to thank Gabriele Veneziano for discussions.

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