Sources and gain in photonic random media

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In this chapter (1+1)D transverse localization of electromagnetic radiation at microwave frequencies is studied by two-dimensional spatial scans. Since the longitudinal direction can be mapped onto time, our experiments provide unique snapshots of the build-up of localized waves. The ensemble averaged evolution of the wave functions is in great agreement with numerical calculations. The excitation of the system with a source in its close vicinity leads to oscillatory behavior of the wave functions. These oscillations are explained in terms of a beating between the system's eigenstates.

3.1 Transverse localization

Recent years witnessed a renaissance in experimental studies on Anderson localization. This phenomenon, introduced in Sec. 1.2.5, originally described the absence of diffusion of electrons in random lattices due to interference [61]. Since Anderson localization is in essence a wave phenomenon, physicists have successfully extended the scope of localization studies to electromagnetic waves [117–120], ultrasound [69], and matter waves [121–123].

Similar to other phase transition phenomena, dimensionality plays an important role. For $d \leq 2$, all states are localized, whereas for $d = 3$ a phase transition from diffusive to localized behavior occurs at a critical scattering strength [62]. In the special case of transverse localization, formulated by De Raedt et al. [124], one dimension is designed not to be disordered, whereas disorder is introduced in the other dimension(s). As a consequence, waves spread out in the disorder-free dimension, but are confined in the other dimensions. Waves are always localized in the transverse directions as long as the transverse system length $L$ is larger than the localization length $\xi$. In the paraxial limit transverse localization is described by an equation which closely resembles the time-dependent Schrödinger
equation [124] where \( z \) plays the role of time

\[
\frac{\partial \psi}{\partial z} = \frac{1}{2kn_0} H \psi,
\]

with \( \psi \) the wave field and \( k \) the vacuum wave number. The effective index of refraction is given by \( n_0^2 = L^{-1} \int_L n^2(x)dx \). The Hamiltonian is defined as

\[
H = \frac{\partial^2}{\partial x^2} + k^2[n^2(x) - n_0^2].
\]

Effectively, transverse localization reduces the number of spatial coordinates in the system: the coordinate along which the sample is extruded can be seen as the time-axis in the time-dependent Schrödinger equation. Stationary transverse localization experiments could thus provide a unique insight into how a localized wave develops over time. Studying and understanding this intriguing aspect of transverse localization experimentally is the central topic of this chapter.

Pivotal experiments on weakly scattering disordered photonic lattices [34, 125, 126] have focused on the observation of localized wave functions after a certain fixed propagation distance and the effect of nonlinearity on the transverse localization length. Both theoretical and experimental studies have revealed interesting dynamical properties of the periodically kicked quantum rotator which bears close resemblance to Anderson localization [127, 128], suggesting that studying the dynamics of localization itself is important. The unique property of transverse localization experiments that enables us to map one spatial dimension onto time is ideally suited for this purpose.

The experimental apparatus used to study transverse localization with microwaves is discussed in Sec. 3.2. Ever since localization was introduced for classical waves, the issue of absorption has been the subject of immense discussions and various opinions [117, 129]. Tackling this issue has shown to be unavoidable in any experiment [66, 67, 130–132]. Our measurements are performed on samples consisting of scattering bars placed parallel to each other in an open system. In such samples out-of-plane scattering plays the role of dissipation and we therefore analyze what the impact of these losses is on our experiment. The size of the nylon bars is small compared to the wavelength and therefore guided modes are leaky in \( x \)-direction. As a consequence our system cannot be analyzed in terms of a coupled set of discrete Schrödinger equations [125]. The experimental results on disordered samples are presented in Sec. 3.3. Instead of probing the intensity at the end of our samples, our experiment allows us to study the evolution of the extent of the waves as a function of propagation distance. In Sec. 3.4, the experimental results are compared with numerical solutions to the 2D Schrödinger-like equation. Interesting non-stationary behavior of single wave functions is observed. These oscillations are analyzed by decomposing these functions into the system’s eigenstates semi-analytically. We discuss how the source of radiation that is used to excite the system influences the dynamics of the wave functions.

### 3.2 Experimental methods

Figure 3.1 shows a sketch of the experimental apparatus. Samples were fabricated by placing nylon bars (3 mm × 10 mm × 1000 mm) on top of an oxygen free copper plate (500 mm × 1000 mm). These nylon bars (\( n = 1.73 \) at 10 GHz [133]) are the scatterers in our system. Disorder was introduced into the system by varying the spacing between the nylon bars.
3.2. Experimental methods

The spacings were chosen randomly from a Poissonian distribution with a mean of 10 mm, and a minimum of 1 mm. Introducing Poissonian disorder makes sure that the presence of stop band effects is negligible [134]. Styrofoam spacers ensured parallel alignment of the nylon bars. The samples were studied by measuring the microwave transmission spectrum around 10 GHz using a vector network analyzer (Rhode and Schwartz ZVA 67). Two coax-to-waveguide adapters functioned as antennas. The detection antenna was scanned over the sample by using a stepper motor (Newport ESP 301) and a home-built scanning stage. A typical two-dimensional scan (40 cm × 40 cm with ~4000 spatial measurement points and 200 spectral points) took approximately 2.5 hours. The excitation antenna was aligned along the z-axis, whereas the detection was aligned along the y-axis.

The end facets of both antennas are placed in the near-field of the sample. This measurement scheme has two advantages. First, the source radiates over a large range of solid angles that is caused by diffraction of waves at the end facet of the waveguide adapter. In contrast to plane wave excitation, such a source has the capability of exciting modes with different $k$-vectors at once. Second, the evanescent waves that are due to wave guiding in the nylon bars can be detected.

3.2.1 Measurement characterization

To test our experimental apparatus and to investigate the photonic strength of the nylon bars, periodic structures were used. An ordered sample was fabricated with a lattice spacing of 20 mm. The transmission spectrum of such a Bragg stack [135, 136] was measured in a waveguide configuration by putting a second copper plate on top of the sample and by placing the two antennas opposite to each other perpendicular to the position of the bars. The spectrum of an empty waveguide was taken as a reference. Figure 3.2(a) shows that the transmission of the Bragg stack drops significantly around 7, 13, and 20 GHz. In 1D, the spectral and angular positions of the stopbands in a periodic structure can be calculated analytically [136]. The calculated stopbands at normal incidence are shown by the gray shaded areas in the spectrum of Fig. 3.2(a). The minima in transmission correspond with the position of the calculated stopbands, from which we conclude our sample is photonic and the refractive index of the nylon bars is in agreement with the used literature value.
Figure 3.2: (a) Measured transmission spectrum for a Bragg-stack of nylon bars and air layers. The small cartoon illustrates the position of the two antennas with respect to the alignment of the nylon bars. The gray shaded areas indicate the calculated stop bands. The structure consists of alternating air \((n = 1, d = 17 \text{ mm})\) and nylon \((n = 1.73 \text{ and } d = 3 \text{ mm})\) layers. (b) Calculated band diagram for TM and TE polarized waves. Black dashed line: frequency of spatial scan. (c) False color image of the measured amplitude distribution above the surface of the Bragg stack at 12.2 GHz. White lines: angular position of the TM stop band. White scale bar: 100 mm.

The complete band diagram, Fig. 3.2(b), shows that TM and TE polarized waves behave differently when the angle of incidence becomes larger. For TE modes propagation along the bars (corresponding to an incident angle of 90°) is impossible for frequencies larger than \(\sim 10 \text{ GHz}\), whereas for TM modes propagation along the bars is allowed until \(\sim 15 \text{ GHz}\). Moreover, around 11 GHz waves are allowed to propagate in nearly all directions for TM modes. The two-dimensional scanning scheme enables us to study the full band diagram experimentally. Figure 3.2(c) provides the distribution of the field amplitude obtained by a two-dimensional spatial scan above the surface of the Bragg stack at 12.2 GHz. The data was normalized for every row to enhance the visibility of the wave function far away from the source. Most of the measured radiation propagates in forward direction parallel to the orientation of the bars. For a small range of angles the radiation is suppressed. These angles correspond with the calculated stopbands of the TM modes. Thus, the orientation of our detection antenna is such that it primarily picks up the evanescent waves of the TM mode, which is a consequence of the perpendicular alignment of the detection and excitation antenna.

The impact of losses

Our “open” experimental configuration requires that we analyze the role of out-of-plane scattering in our experiment. In Fig. 3.3(a), the integrated transmitted intensity between the two antennas is plotted versus propagation distance for a 1D photonic crystal with 2 cm lattice spacing at 9.2 GHz and 12.2 GHz. For the measurement at 12.2 GHz, the integrated intensity drops by about 50% in the first 10 cm. After 300 mm of propagation the integrated intensity flattens off. A similar trend is observed for the data at 9.2 GHz. The first 100 mm witnesses a drop in intensity of a factor 4, but from that point onwards the integrated intensity hardly decreases anymore. These trends indicate that contributions from out-of-plane scattering are foremost present close to the excitation antenna. Far away from the excitation antenna, the main contribution of the detected fields stems from the evanescent waves surrounding the nylon bars. These waves experience very low loss and therefore the integrated intensity hardly decreases anymore.

Theoretically the role of homogeneous dissipation or out-of-plane scattering can be analyzed by adding a term \(-i\alpha\) to \(H\) in definition (3.2) creating an effective Hamiltonian
3.3 Results on disordered samples

Figure 3.3: (a) Integrated transmitted intensity versus propagation distance measured for a 1D photonic crystal at 12.2 GHz (black dots) and at 9.2 GHz (gray dots). The data for 9.2 GHz has been scaled down with a factor 10. (b) Calculated mode amplitude profile versus transverse distance for different values of the loss coefficient $\alpha$ after 350 mm of propagation. Red: $\alpha = 0 \text{ cm}^{-2}$. Blue: $\alpha = 0.5 \text{ cm}^{-2}$. Black: $\alpha = 1 \text{ cm}^{-2}$. (b) Calculated integrated output versus loss coefficient on a semilog scale.

Figure 3.4: Experimentally determined amplitude distribution for (a) an ordered and (b) a disordered sample at 9.2 GHz. Every row is normalized independently. The scale bar denotes 100 mm.

that also describes losses. The amplitude profile is calculated after 350 mm of propagation for a single realization of disorder for various values of $\alpha$. To calculate the amplitude, partial differential Eq. (3.1) is rewritten as a set of ordinary differential equations in $z$ by using the method of lines [137]. After separating the real and imaginary part of $\psi$, we use MatLab to solve the equation numerically by means of a Runge-Kutta algorithm. The $xz$-plane is discretized in 600×200 steps. Figure 3.3(b) plots the amplitude profile for different values of $\alpha$. For higher values of the absorption coefficient, the amplitude decreases, but the wave function shape does not alter. We conclude from these curves that absorption merely scales the wave intensity thereby supporting our experimental approach of studying transverse localization in the possible presence of out-of-plane losses. In Fig. 3.3(c), the integrated output is plotted versus the absorption coefficient. The output intensity clearly attenuates exponentially, confirming that homogeneous dissipation only introduces an exponential scaling [130].

3.3 Results on disordered samples

The propagation of waves within an ordered and a disordered sample is shown in Fig. 3.4. The excitation frequency was set at 9.2 GHz. At this frequency the angular position
Probing the dynamics of Anderson localization through spatial mapping

Figure 3.5: (a) Participation ratio versus propagation distance at 9.2 GHz for an ordered sample (blue) and a disordered ensemble (red). Red line: calculation for an ensemble of 100 disordered samples. Blue line: linear fit. (b) Transverse ensemble averaged amplitude distribution for different propagation distances. Red line: exponential fit.

of the stop gap hardly affects the diffraction from the excitation antenna. The data was normalized for every row in the $xz$-plane to enhance the visibility of the wave function far away from the source. In the ordered sample, Fig. 3.4(a), waves spread out ballistically as a function of propagation distance. However, for the disordered sample, Fig. 3.4(b), the wave propagation is strikingly different: the wave initially spreads out, but at a certain stage stays confined to a bounded region. In contrast to the amplitude distribution in the ordered sample, the amplitude in the disordered sample is not symmetric around the source position. These type of two-dimensional spatial scans provide us with exceptional data for analyzing transverse localization in unprecedented detail. In this section, we start by studying ensemble averaged data and then focus on single realizations.

### 3.3.1 Ensemble averaged data

In order to quantify the transverse confinement of wave intensity as function of propagation distance, the inverse participation length (IPL) \[138\] is calculated. The IPL for a one-dimensional intensity distribution $I(x)$ is defined as

$$P(z) = \frac{\int I^2(x,z)dx}{(\int I(x,z)dx)^2}$$  \hspace{1cm} (3.3)

and has a unit of inverse length. The IPL is inversely proportional to the spread of the wave function: a homogeneously extended wave spread out over the entire sample length $L$ leads to an IPL of $1/L$. To obtain a reliable value for the spread of wave functions, the ensemble averaged intensity profiles were determined by averaging over 20 realizations of disorder.

Figure 3.5(a) shows how the inverse of the IPL develops with increasing propagation distance for both the ordered sample and the ensemble of disordered samples at 9.2 GHz. In agreement with the qualitative picture we obtained from Fig. 3.4, we see that the extent of the wave intensity given by the inverse of the IPL increases linearly for the ordered sample. For the disordered ensemble on the other hand the IPL flattens off after a certain propagation distance. This settling of the IPL to a finite value constitutes a direct experimental observation of the spatial evolution of transversely localized waves.
3.3. Results on disordered samples

Besides a different evolution of the waves’ extent, the eventual spatial shape of the ensemble averaged wave function changes while propagating through the sample. In contrast to Gaussian shaped extended wave functions, ensemble averaged localized wave functions obtain exponential tails. Figure 3.5(b) shows the ensemble averaged wave function profile for three different propagation distances on a semi-logarithmic scale. Close to the excitation source, the ensemble averaged wave function is strongly peaked in the transverse dimension. For longer propagation distances, the intensity in the wings of the wave function increases and the peak becomes less pronounced. The ensemble averaged wave function quenches once the IPL saturates. Its shape is well described by an exponential. However we note that the decay in amplitude is too little to make conclusive statements on its shape. From an exponential fit to the data we find a characteristic length of 192 ± 6 mm. This length might come as a surprise since in the previous chapter we calculated localization lengths for the same type of structures that were on the order of 100 mm. The deviation between this experimental value and this transfer matrix result is due to the excitation of the sample by a source in the near-field. Analyzing the system by one k-vector as done in the transfer matrix model is then invalid. In that sense, the characteristic length found by fitting the data in Fig. 3.5(b) does not represent the localization length, but rather a localization length averaged over several k-vectors. In the next sections, we will see how the near-field excitation leads to another unexpected result.

3.3.2 Single realizations of disorder

After having studied these ensemble averaged properties of our system, we now aim to understand the propagation of waves for single realizations of disorder. To map a sample completely, two-dimensional scans were taken for different positions of the excitation

Figure 3.6: (a) Experimental and (b) numerically calculated plots of the normalized wave function intensity in transverse direction after 365 mm of propagation along the z-direction for different positions of the excitation antenna at 9.2 GHz. The white lines indicate the position of the excitation antenna. The dashed box marks an anti-diagonal wave profile. Scale bar denotes 100 mm. (c)-(e) show calculations using mode decomposition for the area marked with the dashed box in (a) for 180, 365, and 1950 mm of propagation respectively. Beating of eigenmodes can result in (c) circular, (d) anti-diagonal, or (e) diagonal patterns.
antenna. This measurement procedure allows us to construct excitation maps.

In Fig. 3.6(a), the spatial profile for 17 different excitation positions in one sample after 365 mm of propagation is plotted. Based on Fig. 3.5, this distance ensures we are looking at wave functions of which the IPL is saturated. The individually measured spatial profiles are strongly dependent on the position of the excitation antenna. To a large extent the detected radiation follows the position of the excitation antenna as indicated by the white diagonal. The excitation map displays a high degree of symmetry along the diagonal. Naively, one might expect for a localizing sample clearly isolated regions of higher intensity that are independent on the position of excitation. Such patterns would appear as vertical stripes in Fig. 3.6(a) and represent the localized modes of the sample. However, much to our surprise, the spatial patterns of these isolated regions along the transverse dimension $x$ are dependent on the excitation position. In fact, for the measurement shown in Fig. 3.6, some patterns appear to be anti-diagonal.

### 3.4 Model

In order to build a basis for understanding the ensemble averaged data and the remarkable excitation dependence of localized wave functions in single realizations of disorder, the system is analyzed numerically and semi-analytically. Numerically, the system is solved by rewriting the Schrödinger like Eq. (3.1) as a set of ordinary differential equations and using a Runge-Kutte routine, as explained in Sec. 3.2. The initial wave at $z = 0$ is modeled as a Gaussian with a width of 1.15 cm given by the aperture of the excitation antenna. To compare the numerical calculation with experiment, we convolved the intensity of the calculated wave function with the aperture of the detection antenna.

In Fig. 3.5(a), the mean of the participation length for an ensemble of 100 realizations of disorder was shown. This theoretical value for the waves’ extent falls within the standard deviation of the experimentally determined values. Motivated by the experimentally observed and unforeseen excitation dependence of the wave functions, we also calculated the excitation-detection patterns. In Fig. 3.6(b), it is shown that the position and shape of these patterns roughly correspond with the measurements. The anti-diagonal shapes are also clearly present in our numerical calculation, indicating that they are not caused by spurious effects such as mode perturbation due to the proximity of the receiver antenna.

An alternative analysis of the system in terms of eigenstates rather than the previous “brute force” numerical calculation has the advantage of reducing the problem’s complexity. When $\alpha = 0$, the solutions to Eq. (3.1) can be written as a linear combination of the Hamiltonian’s eigenstates $u_n(x)$: $\psi(x, z) = \sum_n c_n u_n(x) \exp(-i\lambda_n z)$, where $\lambda_n$ is the eigenvalue.
belonging to eigenstate $u_n$ and $c_n$ is the $n$'th expansion coefficient given by $c_n = \langle \psi | u_n \rangle$. The eigenstates and eigenvalues are calculated by diagonalizing a $598 \times 598$ matrix. The diagonal of the matrix contains the potential $k^2 [n^2(x) - n_0^2]$ and the derivative in $x$ is approximated by using central differences creating a tridiagonal matrix when assuming absorbing boundary conditions. In principle, diagonalizing a $N \times N$ matrix results in $N$ eigenvalues and eigenstates. However, most of these eigenvectors contain too high spatial frequencies, $k_n > k$, that are not excitable in our system. Figure 3.7 shows three calculated eigenstates for one of the disordered samples used in experiment. By calculating the Fourier spectrum of these eigenstates, one can analyze whether the states are excitable. As a result of this Fourier analysis, we end up with only 30 eigenstates that obey the relation $k_n \leq k$ for $f = 9.2$ GHz. This number could have been anticipated, since the total number of modes in our 1D system is given by $2L f / c \approx 30$. The number of modes that are excited depends on the source used in experiment.

The small number of modes in the system can lead to observable beatings of the system’s eigenstates in our two-dimensional spatial scans. A prerequisite for the occurrence of such beating effects is that more than one mode is excited in the first place. In our experiment this condition is fulfilled, because we use a spatially confined source radiating in all directions in close proximity to the sample. Alternatively, defects or scattering particles inside the structure provide a different method for exciting modes with different $k$-vectors.

Figure 3.8(a) shows a clear example of beating behavior of the wave intensity in experiment. The image of the scan has been rotated by 90 degrees with respect to the spatial scans shown in Fig. 3.4. A decomposition into the system eigenstates for this particular sample reveals just two eigenstates contribute significantly to the wave function as shown in Fig. 3.8(c). Using only these two eigenstates and their corresponding eigenvalues, we
calculated the \( z \)-development of the wave function in Fig. 3.8(b). To compare experiment with theory, the center of mass was calculated for all \( z \)-positions as shown in Fig. 3.8(d). The calculated oscillations are quantitatively similar to those observed in experiment.

In general, the number of significantly contributing eigenvectors is often higher than two, which makes the beatings less visible. Yet, the anti-diagonal patterns shown in Fig. 3.6(a) and (b) are another observable consequence of the beating between the system's eigenstates. Depending on the accumulated phase during propagation, these anti-diagonal excitation-detection patterns can in fact become circular or diagonal as shown in Fig. 3.6(c-e). The patterns are to a large extent point-symmetric which originates from a flip in sign of the expansion coefficients when the excitation antenna crosses the central position of the beating oscillation.

### 3.5 Conclusion and discussion

We have measured how electromagnetic wave functions develop over time in localizing samples by carrying out a (1+1)D transverse localization experiment. Because of the limited number of modes in our system, excitation by a source in the near-field can be described as a superposition of a few of the system's eigenstates. The different eigenvalues and simultaneous excitation of these eigenstates lead to observable beatings in wave functions.

Out-of-plane scattering was used as an experimental analog of energy dissipation. The ensemble averaged extent of the wave profiles is in quantitative agreement with calculations from a numerical solution to a Schrödinger type of equation. By introducing homogeneous dissipation into this model, we deduce that dissipation is of no influence to the occurrence of transverse localization except for an exponential attenuation.

Since the transverse localization scheme allows for measuring snapshots of wave functions in time, it is a very convenient tool for studying the effect of different forms of disorder on wave propagation as put forward by recent work on photonic quasicrystals [126, 139]. Our work on transverse localization and dissipation suggests that transverse localization can also be an excellent platform for studying the influence of perturbations and partial incoherence on localization [140]. Transverse localization experiments require samples to be ordered in one dimension. Introducing scatterers at random positions inside the sample might reveal how strict the condition of a well aligned system is in experiment. A single scatterer excites modes with widely varying \( k \)-vectors probably leading to beating effects even in the case of plane wave excitation. It would be interesting to study when a description in terms of eigenstates of the transverse localization Hamiltonian breaks down.