Violation of Einstein’s Equivalence Principle on Gravitational Wave Event GW150914 Associated with GBM Transient GW150914-GBM

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In this paper, we test Einstein’s equivalence principle (EEP) about gravitational wave transient GW150914 associated with weak electromagnetic signal GW150914-GBM observed by Gamma-ray Burst Monitor (GBM). We study the differences of parameterized post Newtonian (PPN) parameter γ between different particles in LIGO and GBM including the initial outburst gravitons and photons, the gravitons with frequency from 35 to 250 Hz and the photons in hard X-ray emission between 1 KeV and 10 MeV. The calculations are performed on the possible localizations of GW150914 which contains a best fit localization with 68% statistical uncertainty region over 9000 square degrees and the 11 sky locations with 5° apart on LIGO localization arc being visible to GBM. The results show that the violation of EEP, at least on the order of magnitude, is quite insensitive to the location of source. The initial outburst gravitons give us a very tight constrain on the violation of EEP and the accuracy is up to at least $10^{-10}$ with millisecond order time delay. The EEP violation between gravitons and photons is also up to the order of $10^{-8}$ which is about 3 orders of magnitude tighter than recent PeV neutrino and gamma ray photons from blazar flares, and also is 5 orders of magnitude tighter than previous result of MeV neutrinos from SN1987A. Else, the minimal violation of EEP maybe give us some indicates about the puzzle of GW150914’s localization.

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I. INTRODUCTION

Recently one gravitational wave (GW) event is observed by LIGO and Virgo collaborations on September 14, 2015 at 09:50:45 UTC \textsuperscript{1,2,3}. These transients are called G184098 initially in GCN Circulars, and later are given the name GW150914. The luminosity distance of source is $410^{+160}_{-180} \text{ Mpc}$ corresponding to a redshift $z = 0.09^{+0.03}_{-0.04}$ assuming the standard cosmology. The reconstructed waveform of GW150914 is consistent with the signal from a binary black hole (BH) merger in which the initial BH masses are $36^{+4}_{-2} \text{ M}_\odot$ and $29^{+4}_{-2} \text{ M}_\odot$ and the final BH mass is $62^{+4}_{-4} \text{ M}_\odot$ with $3.0^{+0.5}_{-0.5} \text{ M}_\odot \text{ c}^2$ radiation of gravitational waves. The signal of GW150914 sweeps upwards in frequency from 35 to 250 Hz in duration 200ms with about 10 cycles. Later LIGO releases four sky maps including the prompt cWB and LIB localizations, the rapid BAYESTAR localization and the final localization from LALInference in Ref.\textsuperscript{4} where the progress status of possible electromagnetic (EM) counterpart emission also reported as well as the GW-EM follow-up program. Though the rapid determination of likely sky localization of GW150914 is presented, the precise localization for GW150914 is ill-constrained. Once the relativistic jet forms in the source, one may observe a prompt short gamma ray burst (SGRB) lasting on the order of one second or less, possible followed by X-ray, optical and radio afterglows of hours-days duration \textsuperscript{5,6}. In the case of a stellar mass binary black hole (BBH), the current consensus is that no significant EM counterpart emission is expected, except for those in highly improbable environments pervaded by large ambient magnetic fields or baryon densities. Later V. Connaughton et.al. reported that a weak transient source above 50 KeV, 0.4 s after GW150914, was detected by GBM on the Fermi Gamma-ray Space Telescope \textsuperscript{7}. It is possible one GW-triggered EM observation about GW150914 with a false alarm probability of 0.0022 by searching 30 seconds of GBM data before and after the LIGO coalescence time for a plausible counterpart with duration between 0.256s and 2s. An all sky search of the GBM data revealed one candidate transient called GW150914-GBM which is identified a hard transient about 0.4 s after the reported LIGO burst trigger time and lasted for about 1.024 second. Its luminosity is $1.8^{+1.7}_{-1.0} \times 10^{49} \text{ ergs}^{-1}$ in hard X-ray emission between 1 KeV and 10 MeV.

EEP plays an important role in general relativity (GR), which is well supported by many ground-based and space-based experiments such as the Eötvös experiment, test of local Lorentz invariance, clock experiment and so on. Tests of GR at the post-Newtonian level have reached high precision. The status of the test of GR as well as EEP are well reviewed both in the aspects of experiments and theories by Will in Ref.\textsuperscript{8}. Under the frameworks of PPN, the violation of EEP is conveniently given out because that the space curvature produced by mass can be described through PPN parameters such as $\gamma$. The present of space curvature leads to an increase
in travel time of particles like photon between any two given points as compared to the value in flat space. Measuring this increase time delay using the propagation of radar signals in solar system is first proposed by Shapiro [8], well known as radar ranging. The first high accuracy result observed by Viking spacecraft about the time delay shows that $\gamma = 1.000 \pm 0.002$ with estimated accuracy of 0.1% [10]. Hereafter, various astronomical transient as cosmological and astrophysical probes to test EEP, such as Supernova [11, 12], Gamma Ray Bursts (GRBs) [13], Fast Radio Bursts (FRBs) [14, 15], blazar flares [17, 18] and so on. The time delay caused by the space curvature of gravitational field. The arrived time delays are given [3, 11, 12] by

$$\Delta t_{gra} = \frac{\gamma_1 - \gamma_2}{c^3} \int_{r_o}^{r_s} U(r)dr,$$

where $c = 3 \times 10^8 m/s$ is the speed of massless photons or gravitons, $r_o$ denotes the position of emission source and $r_s$ is the position of observer. Half of the delay is caused by the warping of space by the gravitational field, the other half can be thought of as a result of gravitational redshift [11, 14]. $U(r)$ is the gravitational potential which has three parts. One is the potential of Milky Way galaxy $U_{mw}$, one is the host galaxy $U_{host}$ and the intergalactic potential $U_{IG}$ between our galaxy and the host galaxy. The EEP violation could be expressed by the different of PPN parameter $\gamma$ of test particles. According to the formulas presented by Refs. [14, 17], the different of $\gamma$ can be written as

$$\gamma_1 - \gamma_2 \leq \Delta t_{obs} \frac{c^3}{GM_{mw} \log(d/b)},$$

where $G = 6.67 \times 10^{-11} m^3 kg^{-1} s^{-2}$ is the gravitational constant, $M_{mw}$ is the mass of Milky Way galaxy and it is estimated as $6 \times 10^{11} M_{\odot}$ [19, 20]. $d$ is the distance from the transient to Earth. $b$ is the impact parameter of the light rays relative to the center of the galaxy. The impact parameter $b$ is given by

$$b^2 = r_G^2 \left[ 1 - \left( \sin \delta_S \sin \delta_G + \cos \delta_S \cos \delta_G \cos(\beta_S - \beta_G) \right)^2 \right],$$

where the cosmic source is in the direction of R.A. = $\beta_S$, Dec. = $\delta_S$. The distance from Sun to galaxy center is $r_G = 8.3 kpc$. The galaxy center is in the coordinates of ($\beta_G = 17^h 45^m 40.0^s$, $\delta_G = -29^o 00' 28.1''$) the equatorial coordinate system J2000.0 [21]. In order to facilitate our calculations we divide the signals of GW150914 and GW150914-GBM into four types. The first is initial signal arrived at L1 and H1 with time $\Delta t_{GW1}$. The second is initial signal arrived at LIGO and GBM with time $\Delta t_{GBM1}$. The third is the swept upward signal with frequency from 35 to 250 Hz with duration time $\Delta t_{GW2}$ in LIGO. The fourth is weak transient above 50 KeV with lasting time $\Delta t_{GBM2}$ in GBM.

This paper is organized as follows. In section II, we calculate EEP violation between the initial GW signal of GW150914 arrived at Livingston and Hanford observatories with $\Delta t_{GW1}$, and the initial GW signal of GW150914 arrived at LIGO and initial EM signal of GW150914-GBM arrived at GBM with $\Delta t_{GBM1}$. In section III, we calculate EEP violation about the duration of GW150914 with $\Delta t_{GW2}$ in LIGO and the duration of GW150914-GBM with $\Delta t_{GBM2}$ in GBM. Section IV is conclusion.

II. VIOLATION OF EEP BETWEEN INITIAL GRAVITON-GRAVITON AND INITIAL GRAVITON-PHOTON

In this section, we study the difference of PPN parameter $\gamma$ between initial particles outburst in the GW event associated with EM counterpart. According to the report of LIGO [1], the GW event GW150914 arrived first at LIGO Livingston (L1) and $6.9^{+0.5}_{-0.3} ms$ later at Hanford (H1). The initial gravitons with arrived time delay $\Delta t_{GW1} = 6.9^{+0.5}_{-0.3} ms$ between L1 and H1 are most likely to be simultaneously radiation particles when the stellar mass compact binary coalescences (CBCs) is triggered. Initial gravitons propagate in the same gravitational space including the host galaxy, intergalactic and our Milky Way galaxy. If the EEP is right, initial gravitons will arrive the Earth in the same time. The difference of initial burst time would be reached the minimum value. Hence, in GW event GW150914, the initial gravitons arrived between L1 and H1 can provide a more accurate test to the violation of EEP. However, in the early work about Supernova [11, 12], GRBs [13], FRBs [14, 15], the difference of initial burst time take more uncertainty about the results. Meanwhile, Fermi GBM Observations reported that the localization of GW150914-GBM has a best fit position (BFP) to the hard model spectrum and the source position in celestial equatorial coordinate is (RA = 57 deg, Dec = -22 deg) with a 65% statistical uncertainty region over 9000 square degrees ($\sigma = 54^o$) [7]. If GW150914 is associated with GW150914-GBM, we can get a more accurate upper limit of the different of PPN parameter $\gamma$ about the gravitons emitted simultaneously in the GW event GW150914 based on the localization of Fermi GBM Observations,

$$|\gamma_{H1}(gra) - \gamma_{L1}(gra)| \leq 2.13^{+0.12}_{-0.06} \times 10^{-10}.$$ (4)

Besides providing BFP, Fermi GBM Observations also divide the LIGO arc into 11 positions with 5° apart, 10 on the southern portion, one in the north, excluding the parts of the arc that were occulted to Fermi. The celestial equatorial coordinates of the 11 positions are listed in Ref. [7] which also shows the probability of the LIGO source location lying near each position based on the LIGO location map. It is comparable to the accuracy with that GBM could localize a weak triggered transient source using the standard localization techniques. According to these 11 possible sky localizations in LIGO
localization map for GW150914, we calculate the different of PPN parameter $\gamma$ with uncertainties of distance or time. The results of $\Delta \gamma$ about these 11 positions are illustrated in Fig. 1. It illustrate that the upper limit of $\Delta \gamma$ reaches the order of $10^{-10}$ which is two orders of magnitude tighter than the result $10^{-8}$ of recent FRB 110220 [14] and FRB 150418 [15]. The average value of $\Delta \gamma$ for 11 possible sky localizations is $2.16 \times 10^{-10}$. On the side of distance uncertainty $410^{+160}_{-180}$ Mpc, the maximum positive deviation $1.23 \times 10^{-11}$ with 5.8% accuracy, and the maximum negative deviation $6.4 \times 10^{-12}$ with 3% accuracy. On the side of time delay uncertainty $6.9_{-0.4}^{+0.5}$ ms, the maximum positive deviation $1.57 \times 10^{-11}$ with 7.3% accuracy, and the maximum negative deviation $1.25 \times 10^{-11}$ with 5.8% accuracy. Hence according to the uncertain provided by LIGO [11] we can find the deviation on PPN parameter $\Delta \gamma$ from the distance is more lower than that from the time delay. The result of EEP violation is more easily influenced by the uncertain of time delay. Interestingly, from Fig. 1 one can easy read that the order of magnitude of the up limit of $\Delta \gamma$ about these 11 positions are all kept in the same order of $10^{-10}$. In order to further confirm this point, we calculate the whole LIGO arc with continuous angle change. According to the GW sky maps with the 90% credible level provided by cWB, LIB, BSTR and LAL-Inf of LIGO, we obtain the contour profile of up limit of $\Delta \gamma$ variation with R.A. and Dec., i.e. $RA = [0, 180^\circ]$ and $Dec = [-75^\circ, 0]$. The results are illustrated in Fig. 2 where the maximal value is $2.166 \times 10^{-10}$ and the minimum value is $1.706 \times 10^{-10}$. With further assumptions, if GR is right the positions of ripples with minimal violation of EEP are potential for discovery the source of GW150914.

Meanwhile, according to the report of Fermi GBM Observations [12] there was a weak transient GW150914-GBM above 50 KeV at the time of GW event GW150914 and maybe was the EM counterparts to GW150914 with a false alarm probability of 0.0022. Its localization was consistent with the direction of GW150914. The combined LIGO and GBM observations reduced the 90% confidence interval on sky location from 601 to 199 square degrees. This weak transient GW150914-GBM lasting about 1.024 s was observed 0.4 s after GW150914 was detected and did not appear connected with other known before astrophysical, solar, terrestrial, or magnetospheric activities. The duration and spectrum of GW150914-GBM showed it was a weak short GRB. Hence, the delay between LIGO and GBM, i.e. $\Delta t_{GBM1} = 0.4$s, can be treated as the arrived time delay between beginning outburst gravitons and photons simultaneously when CBCs was triggered. According to the coordinate of BFP, we can get a up limit of the different of PPN parameter $\gamma$ between the initial gravitons and photons emitted out simultaneously when GW event GW150914 is triggered,

$$|\gamma_{LIGO}(gra) - \gamma_{GBM}(pho)| < 1.24^{+0.07}_{-0.04} \times 10^{-8},$$ (5)

The upper bound of $\Delta \gamma$ between the initial graviton in LIGO and the initial photon in GBM of 11 possible sky localizations are drawn by dark yellow circle points illustrated in Fig. 1. Like the case of $\Delta t_{GW1}$, they are all in the same order $10^{-8}$ of magnitude. Their average value is $1.25 \times 10^{-8}$ with the maximum positive deviation 5.7% and the maximum minus deviation 3.0%.

III. VIOLATION OF EEP BY THE DURATION OF GW150914 AND GW150914-GBM

In this section, we analyze the difference of PPN parameter $\gamma$ between different frequencies or energies in the completed GW and EM signals. From the frequency-dependent delay of GW150914 (see Fig. 1 of Ref. [11]), one can identify the arrival time delay $\Delta t_{GW2} \sim 0.45$s for graviton ranging in frequency from about 35 to 250 Hz. $\Delta t_{GW2}$ is the signal duration time of GW150914 detected by LIGO, which sweeps upwards in frequency from 35 to 250 Hz. If we assume the gravitons with the frequencies from 35 Hz to 250 Hz are emitted out simultaneously in GW event GW150914, the we can get a up limit of the different of PPN parameter $\gamma$ by BFP,

$$|\gamma(35Hz) - \gamma(250Hz)| < 1.39^{+0.11}_{-0.04} \times 10^{-8},$$ (6)

where the time delay $\Delta t_{GW2}$ is the dispersion in arrival times for the gravitons ranging in frequencies from 35 to 250 Hz are observed to be less than 0.45s. The results for

![FIG. 1. The up limit of $\Delta \gamma$ of the 11 possible positions coming from the directions along the LIGO arc. The error bars come from the uncertain of luminosity distance and from left to right it corresponds to the case of 570 Mpc and 230 Mpc, respectively. The blue down triangle point denotes the time delay $\Delta t_{GW1}$ between H1 and L1. The dark yellow circle point denotes the time delay $\Delta t_{GBM1}$ between LIGO and GBM. The purple square point denotes the time delay $\Delta t_{GW2}$ of GW signals with the frequencies from 35 Hz to 250 Hz. The red up triangle point denotes the time delay $\Delta t_{GBM2}$ of hard X-ray emission from 1 KeV to 10 MeV.](image-url)
11 possible sky localizations are also listed in Fig. 1. The average value of $\Delta\gamma$ is $1.41\times10^{-8}$ with 5.7% maximum positive deviation and 3.0% maximum minus deviation. We also notice that there are two works [22, 23] involved the tests of EEP about GW event GW150914. However, they are all calculate the inspiral part of the signal with the tests of EEP about GW event GW150914. However, these works depend on the time of inspiral $\Delta t_{GW2}$ with the other observations in Fig. 2. The limit obtained with $\Delta t_{GW1}$ is about one orders of magnitude tighter than the previous minimal up limits obtained with GHz photons from FRB/GRB 100704A. This result is also two orders of magnitude lower than recent limits obtained with GHz photons from FRB110220 [14]. On the other hand, the result from $\Delta t_{GBM1}$, i.e. the time delay between graviton and photon, is three orders of magnitude lower than recent limits obtained with PeV neutrino and gamma-ray photons from observations of the blazar PKS B1424-418 [17]. This result is up to five orders of magnitude lower than the previous limits, i.e. $\gamma_\gamma - \gamma_\nu \lesssim 3.4 \times 10^{-3}$, obtained with neutrino and photon from SN1987A [11, 12]. Hence, the observations of GW150914 and GW150914-GBM offer us a more accuracy constraint on the equivalence principle between the graviton and the graviton or the graviton and the photon.

IV. CONCLUSION

In this paper, we use the possible localizations from GW150914 sky map associated with GW150914-GBM to present the constraints on EEP violation. The results obtained via fours various types time delays involved gravitons and photons are insensitive to the localization of source, at least on the order of magnitude. It is interesting that the initial burst of gravitons arrived at detectors H1 and L1 with millisecond time delay would offer us a very high accuracy results about the test of EEP violation. Its up limit of $\Delta\gamma$ is high to the order of magnitude $10^{-10}$, which is almost the lowest result about the PPN parameter $\gamma$ given by recent astronomical transients [13–18]. Comparing with the results obtained by the photons in FRBs and GRBs, the result of initial gravitons is at least two orders of magnitude lower than recent FRB 110220 [14] and FRB 150418 [15].

For other three results, i.e. between initial graviton in LIGO and initial photon in GBM, between gravitons with 35Hz and 250 Hz, between photons with 1 KeV and 10 MeV, they have the same magnitude of $\Delta\gamma$ in the order of $10^{-7}$. Comparing with previous constrains on EEP through photons and neutrinos within PPN framework, the case of photons and gravitons will give us a more tighter constrains. The result of between photons...
FIG. 3. The up limit of $\Delta t$ from GW event GW150914 against previous results of FRB/GRB. The red circle point denotes that of GW150914 about four kinds time delays: $\Delta t_{GW1}$, $\Delta t_{GBM1}$, $\Delta t_{GW2}$ and $\Delta t_{GBM2}$ where the best fit of the localization of GW150914-GBM provided by Fermi GBM is used. The error bars comes from the uncertain of luminosity distance $410^{\pm 160}$kpc provided by LIGO. Else, only the initial gravitons arrived at H1 and L1 need to consider the time uncertain of $6.9^{+0.5}_{-0.4}$ms. The blue diamond point denotes previous results came from the observations of FRB and GRB [13–15].

and gravitons is up to an accuracy of $10^{-8}$ which is three orders of magnitude tighter than recent constraint given by PeV neutrinos from blazar flares [17], and also is five orders of magnitude tighter than the results given by MeV neutrinos from supernova 1987A [11]. Although the calculations [22, 23] involved the time delay about GW150914, the possible locations provided by GBM and the nontrivial initial outburst gravitons with millisecond order time delay are not considered. Here we use the possible localization of GW150914 associated with GW150914-GBM and give a more accurate constraint on EEP violation via the united GW and its EM counterparts. Most of all, the observed time delay of GW150914 is $6.9^{+0.5}_{-0.4}$ ms between the Livingston and Hanford observatories. If we treat the delay is caused by space curvature, we can through it give a more precision limit on the violation of EEP. Meanwhile, after the analysis of continuous angular distribution our results maybe give some hints in the orientation of GW150914 which maybe occur at the positions near minimal violation of EEP.

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