Study on Calculation of Liquid Level And Storage of Tanks for LNG-fueled Vessels

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Abstract. As the ongoing development of the application of LNG as a clean energy in waterborne transport industry, the fleet scale of LNG-fueled vessels enlarged and the safety operation has attracted more attention in the industry. Especially the accurate detection of liquid level of LNG tanks is regarded as an important issue to ensure a safe and stable operation of LNG-fueled ships and a key parameter to keep the proper functioning of marine fuel storage system, supply system and safety control system. At present, detection of LNG tank liquid level mainly adopts differential pressure detection method. Liquid level condition could be found from the liquid level reference tables. However in practice, since LNG-fueled vessels are generally not in a stationary state, liquid state within the LNG tanks will constantly change, the detection of storage of tanks only by reference to the tables will cause deviation to some extent. By analyzing the temperature under different pressure, the effects of temperature change on density and volume integration calculation, a method of calculating the liquid level and storage of LNG tanks is put forward making the calculation of liquid level and actual storage of LNG tanks more accurately and providing a more reliable basis for the calculation of energy consumption level and operation economy for LNG-fueled vessels.

1. Introductions
Liquefied natural gas (LNG) is a kind of high quality clean energy fuel, and it is widely used in ships. Storage tank is the medium for storing LNG fuel. LNG is an ultra-low temperature liquid. When it is stored, there is a gas-liquid two-phase state, and the volume expansion of LNG is about 600 times after gasification. When the temperature rises, the pressure will increase dramatically and is easy to make the tank in a dangerous state. Accurate measurement of the liquid level in the tank has become a security problem that can not be ignored. At the same time, by measuring the real-time liquid level of the tank, the energy consumption level of the ship can be calculated, so as to provide the basis for the ship to grasp the economic operation of the ship.

The existing research on the LNG tank level detection using differential pressure detection, reference level control table check the liquid level\cite{1-2}. In practice, due to LNG power ship in a non-stationary state, the LNG storage tank of liquid state will change constantly, the storage tank storage only through the reference level table will have a certain degree of error. In this paper, the relationship between LNG tank height and volume, LNG density and temperature, LNG pressure and temperature, and LNG tank level and pressure difference are analyzed. puts forward a method to calculate the liquid level and reserves of LNG dynamic storage tank, and solves this problem well.

Liquefied Natural Gas (LNG) is widely applied in transport industry due to it is clean and of high quality. As the formulation of ‘Emission Standards for Pollutants from Shipbuilding Industry’, ship emission control zones have been set up in the waters of the Pearl River Delta, Yangtze River Delta and
Bohai Bay to control ship emissions of sulfur oxides, nitrogen oxides and particulate matter and improve environment and air quality of coastal and river area especially in port cities, the advantages of LNG as a shipping fuel are widely accepted and promoted.

2. The Storage of LNG for LNG-fueled Vessels

2.1. The Structure of LNG Tanks
The structure of tanks for LNG-fueled vessels is double metal cylindrical tanks, including container, hull and holder, insulating layer, piping and valve, instrument and security accessories[3]. Type-C horizontal standard ellipse head structure is generally adopted. Inside container and its piping are made of austenite stainless steel (S30408) and the design of main pressure parts are strictly compliance with GB24511-2009, the hull is as well as made of austenite stainless steel (S30408) according to ‘Rules for Natural Gas Fueled Ships’ issued by China Classification Society (CCS) in 2013. The tank settings include upper and lower inflow liquid, outflow liquid, BOG, self-pressurization, overflow, pressure gage, liquid indicator and thermometer, on-site or remote pipeline monitoring. The thermal insulation performance directly affects the container, in other words, good insulation is a key factor to ensure the storage for a long time. Tanks generally use vacuum powder and high vacuum multiple wraps so far now. LNG in tanks is stored in saturation liquid, the temperature inside tank below 163°C and the top pressure is around 11 bar[4-6].

2.2. Monitoring System
Since liquefied natural gas stored in tanks of LNG-fueled vessels is low temperature, flammable and explosive, a monitoring system to ensure normal operation and crew safety is necessary. The system presents status parameter such as liquid level and pressure on the spot. At the same time a monitoring instrument with the function of remote transmission to provide status parameter for chain control of safety control system is required.

Differential pressure method is the most common way to measure the liquid level of low temperature liquid in tanks. Gas derived through the bottom and top of the tank (see A, B tubes in Figure 1, pressure in tube A is for bottom pressure, pressure in tube B is for top pressure), the liquid level height inside the tank is confirmed on the spot through the liquid indicator which measures the pressure difference from tube A and B (the pressure difference is the static pressure made by the liquid column inside the tank). The pressure difference made by the liquid level could also be confirmed through transmitter sending the signal to the central control room participating control interlock of the tank or system.

![Figure 1](image_url)

**Figure 1** Liquid level detection of pressure in the tank

3. Calculation Parameters of Liquid Storage Capacity for Tanks

3.1. The Relationship Between Height and Volume of Liquid Level
Tank volume could be divided into two parts for easy calculation, see figure 2 the straight pipes and heads, two of which (standard elliptical heads) could be regarded as one ellipsoid[1]. Conditions are as follows:
3.1.1. Calculation the Ellipsoid Liquid Volume

Figure.2 Standard Elliptical Heads Type-C Horizontal Tank
R——Inside tank radius
L——Inside tank straight pipe length (including the head straight side)
H——Inside tank liquid height

Ellipsoid 3D Coordinate System
Ellipsoid equation is \( \frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \) of which \( a = R \); \( b = R \); \( c = R \).
Shadow in XY plane is ellipse model, a cross section coordinate system is established as follows:

Figure.4 XY plane coordinate system
Ellipse equation is \( \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \) of which \( a = R \); \( b = R \)
When liquid level height is \( H \), \( y = H - R \)
Plug it into equation to get H point: \( x = \sqrt{\frac{R^2 - (H - R)^2}{2}} \)
Shadow in YZ plane is round model, cross section coordination system is established as follows:

Figure.5 YZ plane coordinate system
Round equation is \( \frac{y^2}{b^2} + \frac{x^2}{c^2} = 1 \) of which \( b = R; c = R \)

When liquid level height is \( H \), \( y = H - R \)

Plug it into equation to get \( H \) point: \( z = \sqrt{R^2 - (H - R)^2} \).

When liquid level in \( XZ \) coordinate system is \( H \), shadow of liquid level is ellipse, of which major semi-axis \( A = z \) and minor semi-axis \( B = x \).

According to the formula of ellipse area: \( S_H = \pi AB \), the area of liquid level where the height is \( H \) is:

\[
S_1 = \frac{\pi R^2 - (H - R)^2}{2}
\]

Therefore when the liquid level height is \( H \) and \( y = R - H \), volume of ellipsoid liquid is:

\[
V_1 = \int_{-R}^{R-H} S_1 \, dy = \frac{\pi}{2} \left( RH^2 - \frac{1}{3}H^2 \right);
\]

3.1.2. Calculation the Cylinder Liquid Volume. When liquid height is \( H \), shadow of liquid level in \( YZ \) plane is as figure 5. Shadow in \( XZ \) plane is rectangle, of which length is \( L \), width is \( z = \sqrt{R^2 - (H - R)^2} \).

Area of liquid level is \( S_2 = L \sqrt{R^2 - (H - R)^2} \)

When liquid level of cylinder is \( H \), \( y = R - H \), the cylinder volume is:

\[
V_2 = \int_{-R}^{R-H} S_2 \, dy = L \left[ R^2 \cos^{-1} \frac{R-H}{R} - \sqrt{R^2 - (R - H)^2} (R - H) \right].
\]

3.1.3. Total Volume of Liquid in Tanks. When liquid height is \( H \), the total volume of liquid is the sum of the above two parts:

\[
V_{\text{liquid}} = L \left[ R^2 \cos^{-1} \frac{R-H}{R} - \sqrt{R^2 - (R - H)^2} (R - H) \right] + \frac{\pi}{2} \left( RH^2 - \frac{1}{3}H^2 \right).
\]

The gross capacity of tank is

\[
V_{\text{total}} = \pi LR^2 + 2\pi R^3 - \frac{2}{3}\pi R^2.
\]

Unit: \( V \) -- m³; \( R, H \) -- m.

The relationship between LNG density and temperature

Density of LNG depends on its constitution, generally is 430–470Kg/m³, the higher the methane content, the lower the density, and the variation gradient is 1.35 Kgm⁻³/°C. A similar calculation of Low temperature fluid density (kg/m³) and temperature (thermodynamics temperature K) referred in ‘Low Temperature Engineering Technology’[7] is:

\[
\rho = AB^{-(1-T/T_c)^n}.
\]

A, B, n is regression coefficient of compounds

\( A = 0.15998, B = 0.2881, n = 0.277 \)

\( T_c \) -- critical temperature; \( T_c = 190.58 \)

Since the design pressure of LNG is generally 1.2MPA, actually the storage temperature is between 110K and 150K. The saturated vapor pressure will exceed the design pressure if more than 150K. Pressure will discharge through safety valve when overpressure occurs. Therefore the article adopts the temperature range from 110K to 150K for calculation (As shown in Table 1).
| T   | Calculation Density | Physical Property Density[7] | Difference | T   | Calculation Density | Physical Property Density | Difference |
|-----|---------------------|------------------------------|------------|-----|---------------------|------------------------------|------------|
| 110 | 426.4465            | 424.89                       | 1.556477   | 129 | 397.4657            | 395.86                       | 1.6057     |
| 111 | 424.0929            | 422.53                       | 1.562918   | 130 | 395.8324            | 394.23                       | 1.6023     |
| 112 | 423.5556            | 422                          | 1.555622   | 131 | 394.1862            | 392.58                       | 1.6062     |
| 113 | 422.0977            | 420.53                       | 1.5677     | 132 | 392.527             | 390.93                       | 1.5969     |
| 115 | 419.156             | 417.58                       | 1.575988   | 133 | 390.8542            | 389.26                       | 1.5942     |
| 116 | 417.6718            | 416.1                        | 1.571828   | 134 | 389.1676            | 387.57                       | 1.5975     |
| 117 | 416.1785            | 414.6                        | 1.578542   | 135 | 387.4667            | 385.87                       | 1.5966     |
| 118 | 414.6759            | 413.09                       | 1.585929   | 136 | 385.7511            | 384.16                       | 1.5910     |
| 119 | 413.1638            | 411.57                       | 1.593778   | 137 | 384.0204            | 382.43                       | 1.5903     |
| 120 | 411.6419            | 410.05                       | 1.591874   | 138 | 382.2741            | 380.69                       | 1.5840     |
| 121 | 410.11              | 408.51                       | 1.599993   | 139 | 380.5117            | 378.93                       | 1.5816     |
| 122 | 408.5679            | 406.97                       | 1.5979     | 140 | 378.7327            | 377.15                       | 1.5826     |
| 123 | 407.0154            | 405.41                       | 1.605354   | 142 | 375.1228            | 373.54                       | 1.5828     |
| 124 | 405.4521            | 403.85                       | 1.602103   | 144 | 371.4399            | 369.85                       | 1.5898     |
| 125 | 403.8779            | 402.27                       | 1.607886   | 146 | 367.6786            | 366.08                       | 1.5985     |
| 126 | 402.2924            | 400.69                       | 1.602431   | 148 | 363.8332            | 362.22                       | 1.6132     |
| 127 | 400.6955            | 399.09                       | 1.605454   | 150 | 359.8973            | 358.26                       | 1.6373     |
| 128 | 399.0867            | 397.48                       | 1.606662   | 152 | 355.8635            | 354.19                       | 1.6734     |

There are some deviations between the calculation density and physical property density, the average difference is 1.597479, and the revised formula 3 is:

\[ \rho_{\text{liquid}} = AB^{-1-(1-T/T_c)^n} - 1.597479. \]  (4)

Natural gas in gas space remains saturated state could be treated as an ideal gas, according to the gas state equation: \( PV=nRT \) \(\quad n=m/M \)

\[ \rho_{\text{gas}} = \frac{m}{V} = \frac{PM}{RT} \]  (5)

P: unit of state pressure is pa, T is for thermodynamic temperature K, R is for gas constant 8.314J·mol⁻¹·K⁻¹, M is for methane molar mass 16.414g/mol.
3.2. The Relationship of LNG Pressure and Temperature

When LNG liquid stored in tanks, the gas space turns into a saturated vapor state. Saturated vapor pressure and temperature have a certain relationship described by Antoine equation[2].

Five-parameter Antoine equation:

\[ \log P = A + \frac{B}{T} + C \log T + DT + ET^2. \]  

(6)

\[ A = 14.6667, \ B = -5.7097 \times 10^2, \ C = -3.3373, \ D = 2.1999 \times 10^{-9}, \ E = 1.3096 \times 10^{-5}. \]

Simplified equation: \[ \log P = A - \frac{B}{T+C} \cdot \]  

From the change rules of temperature range of two equations in the coordinate system from 110K to 150K, there have no great difference. The relationship could be described by simplified equation which can be concluded that:

\[ T = \frac{B}{A-\log(750P)} - C. \]  

(7)

\[ A = 6.69561, \ B = 405.420, C = 267.777. \]

![Figure 6](image_url)

**Figure 6** Five-parameter Antoine equation and Simplified equation

From the results indicated in Figure 6 and the physical properties, the results almost the same. The calculation result from Formula 7 could be treated as an important parameter basis to calculate for different state of LNG.

![Figure 7](image_url)

**Figure 7** Relations between saturation pressure and temperature calculated by physical properties actual data and simplified equation

From the results indicated in Figure 6 and the physical properties, the results almost the same. The calculation result from Formula 7 could be treated as an important parameter basis to calculate for different state of LNG.
3.3. The Relationship Between Liquid Level and Differential Pressure of LNG Tanks

LNG tanks generally use differential pressure level meter to measure differential pressure, the volume or quality of LNG is confirmed through the liquid level comparison table provided by manufacturers (As shown in Table 2). Differential pressure level meter is an instrument which calculates the liquid level (pressure difference) by measuring the pressure difference from two different points in tanks. The comparison data is a calculation result of physical property parameters of water in a stationary state. However LNG constantly changes in tanks in the course of usage, the table is likely to produce a certain deviation in practice.

| Liquid indicator Water (mmWC) | Volume (L) | Liquid indicator Water (mmWC) | Volume (L) | Liquid indicator Water (mmWC) | Volume (L) |
|------------------------------|------------|------------------------------|------------|------------------------------|------------|
| 0                            | 0          | 120                          | 471        | 300                          | 1808.3     |
| 20                           | 30.5       | 140                          | 595        | 320                          | 1916.2     |
| 40                           | 87.6       | 160                          | 727.2      | 340                          | 2114.2     |
| 60                           | 162.9      | 180                          | 866        | 360                          | 2213.5     |
| 80                           | 253.3      | 200                          | 1010.3     | 380                          | 2406       |
| 100                          | 356.6      | 220                          | 1158.5     | 400                          | 2493.2     |
| 120                          | 471        | 240                          | 1309.2     | 420                          | 2672       |
| 140                          | 595        | 260                          | 1461.1     | 440                          | 2744.2     |
| 160                          | 727.2      | 280                          | 1612.7     | 458                          | 2844       |

Table 2. Comparison table of tank liquid level in one manufacturer

Indicating value made by differential pressure level meter is the pressure difference produced by liquid level height $H$ from two monitoring points. $P = \rho g H$

The unit of indicating value $P$ is KPA, the liquid level $H$ (unit m) is:

$$H = \frac{1000P}{\rho g}$$

Combined with Formula 1 and Formula 4, tank pressure $P$ and density $\rho$ confirmed by Formula 4 after dynamic changes, a relatively accurate level height could be real-time calculated.

$$H = \frac{102P}{AB - (1 - 177e^{-9}) - 1.597479}$$

4. The Practical Application of Tanks for LNG-fueled Vessels

4.1. The Actual Storage Capacity of LNG Tanks

Real-time sensors monitoring tank pressure and liquid level are generally equipped in LNG tanks. However, due to the structure features of tanks and temperature sensors, the actual storage temperature of LNG inner tanks could not be effectively monitored in time. While the storage temperature could be speculated according to tank pressure using the thermodynamic formulas as stated earlier (assuming the tank in vapor-liquid equilibrium saturation condition), and the density of natural gas from liquid to gas in different pressure condition could be confirmed. Parameters in different pressure conditions could be further calculated.

When length is $L$, tank pressure of which radius is $R$ is $P$, the differential pressure is $p$, all parameters as follows:

- Quality of natural gas in tanks: $M = M_{\text{liquid}} + M_{\text{gas}} = \rho_{\text{liquid}} V_{\text{liquid}} + \rho_{\text{gas}} V_{\text{gas}}$

- The volume of natural gas in standard condition: $V_{\text{standard}} = \frac{M}{\rho_{\text{standard}}} \rho_{\text{standard}} = 0.7167 kg/m^3$

Calculating the actual storage capacity of LNG tanks and converting into gas volume in standard condition provide a data basis for calculating the energy consumption, instead of relying on flow meter to calculate gas consumption.
4.2. Calculation of Parameters in Practice

Loading Limit (LL) and Filling Limit (FL) are introduced in the IGF Code. LL is the ratio of the maximum permissible liquid volume to the fuel tank loadable capacity. FL is the ratio of the maximum liquid volume in the fuel tank to the total fuel tank volume when the liquid fuel reaches the reference temperature. The two concepts raise strict requirements on the limits of practical use of storage tanks. Due to the variation conditions of LNG and the difference of filling gas sources, parameters as mentioned before will change to some extent, which requires a clear formula to provide a reference for monitoring system and energy management system of tanks.

The reference temperature as mentioned above is the corresponding temperature of fuel steam pressure under pressure relief valve setting pressure. The pressure relief setting of Type-C tank should not exceed 1.2MPA for inland LNG-fueled vessels. The corresponding temperature in pressure saturation condition according to physical properties and formula mentioned above is -120°C, the density of LNG is 353.8kg/m³. IGF Code and other domestic rules ask for 98% and 95% on Filling Limit respectively. The article adopts 98% FL under the consideration of the practical usage of Type-C tanks. Therefore the actual LL is confirmed as follows:

\[ LL = FL \frac{\rho_R}{\rho_L} = \frac{347}{\rho_L} \]  

(10)

\( \rho_R \) is the relative density of fuel under the reference temperature, that is 353.8 kg / m³. \( \rho_L \) is the relative density of fuel under loading temperature, that is the density of LNG filing tanks which could be required from the filling party or confirmed according to the Formula 4 and 7 mentioned above.

5. Conclusion

LNG-fueled vessels have been widely employed in China and the technical level of hardware equipment is sufficient to meet the requirements of existing domestic vessels. However, unlike other traditional vessels using diesel as the shipping fuel, LNG-fueled vessels make stricter requirements on security and monitoring aspects due to its characteristics. The calculation of parameters mentioned in the article provides a reference for real-time monitoring of LNG tanks and the entire bunkering and energy management system. However, LNG is a compound with various components, parameters and formula involved in the article on the basis of methane physical properties. A deviation may occur in practice which requires to adjust promptly.

6. Reference

[1] Yanling Wang, Ming Li. Establishment of Corresponding Relationship between Level and Volume of Horizontal Tank with Elliptical Head. Journal of qiqihar university, March 2002;
[2] Fan Yang. Measurement method of liquid level in LNG tank[J]. Tianjin chemical. 2014.4.27(4):54-55.
[3] Weiming Zhou, Rongshun Wang, Yumei Shi. Advances and Prospects of Transportable Pressure Vessel Standards and Codes[J]. Pressure Vessel Technology2004.20(1):1-7.
[4] Huasheng Zou, Chunlai Huang. Analysis of rolling phenomena in the storage and transportation of liquefied natural gas[J]. oil and gas storage and transportation. 2006,25(11):13-15.
[5] Rui Yang. Risk analysis of liquefied natural gas and its safety protection scheme[J]. chemical management, 2018.8:57.
[6] Ruozhu Liu. Research on risk assessment and control for LNG storage system[D]. East of China university of science and technology.2011.
[7] Guobang Chen, Rui Bao, Yonghua Huang. Cryogenic engineering technology (data volume). Chemical industry press, 2005.12