Cover Letter
This manuscript is related with our previous submissions: JMSE-D-20-04600, JMSE-D-20-05225, JMSE-D-20-05213.

The central idea
The quarter-wavelength model is a famous model well-known to material scientists in the field of microwave absorption. It is widely used and has been accepted without question. But in this work, we show that the model is incorrect.

The methods
The methods used to obtain our conclusions are simple and straightforward and only involve physics at college level. Our results are confirmed from different perspectives including numerical verification from experimental data easily available and equations well-known to material scientists, and one data set is provided as supplementary material.

The significance
The significance of this work is that it revealed the fact, along with the discussion of the quarter-wavelength model, that the reflection loss parameter used to characterize material and the related impedance matching, which are the fundamental basis of thousands of reports published, have no scientific basis.

Our work shows that there is a general practice of using established theory believing that it must be correct because it has been used for many years by many researchers. However, such theories need to be constantly checked, particularly when they proved to be inconsistent with published experimental data and fundamental physical principles. For example, there are many published data showing that reflection loss $RL$ has not achieved its minimum value when $|Z_{in}| = |Z_0|$, but researchers have still adhered to the impedance matching theory that $RL$ achieves its minimum value when $|Z_{in}| = |Z_0|$.

Reliability of the results
The published conclusions concerned are commonly accepted for long but our opposite conclusions are obtained from rigorous analysis, fundamental principles at college level established centuries ago, and consistent from different analyses.

Sincerely,

Ying Liu

Appendix
JMSE-D-20-04600

Dear Dr. Liu,

I have followed up with the Deputy Editor-in-Chief, responsible for Replies and Comments, and he has again reviewed the situation. The decision not to publish the Comment and the Reply still stands and is final.
However, since there is quite a lot of disputation between both parties, could I suggest that you can rectify this by seeking to publish your own Journal Manuscript in either JMSE or another Journal of your choosing. This would give you the opportunity to lay out your own theories and verify them via experiment. Obviously, if you choose to submit to JMSE, your submission will be treated as a regular paper and be subject to our standard Journal review process.

Thank you very much.

Best regards,

Pravin

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Pravin Selvakumar (Mr.)
JEO Assistant
Journals Editorial Office (JEO)
The problems in the quarter-wavelength model and impedance matching theory in analysising microwave absorption material

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Abstract

It is shown here that many concepts in current mainstream microwave absorption theory are used inappropriately. Reflection loss $RL$ has been used to characterize microwave absorption from material instead of film and the results have been rationalized by impedance matching theory. The quarter-wavelength model states that the reflection of microwaves with wavelength $\lambda$ from a film is minimized if the thickness of the film is $m\lambda/4$ where $m$ is an odd integer. But we show here that the model is wrong because the phase effects from interfaces have been overlooked. $RL$ is an innate property only for metal-backed film. Impedance matching theory is developed from transmission-line theory for scattering parameter $s_{11}$ but cannot be generalized to $RL$.

Key words

Microwave absorption; wave addition; phase effect of interface; material; impedance matching; quarter-wavelength model.

1. Introduction

In current microwave absorption theory, reflection loss $RL$ has been used to characterize absorption from material.1, 2, 3, 4, 5, 6 The results obtained have been rationalized by using the impedance matching theory (IM).1 - 4, 7, 8 In order to search for the minimum value of $RL$, the quarter-wavelength model (QWM) has been used.1, 5, 6, 8, 9 Based on IM, $\Delta$ parameter10, 11, 12, 13, 14, 15 have been devised and widely used, but this parameter...
cannot be a validation of IM since it is developed from the wrong theory thus is problematical itself.

RL is defined as the reflection coefficient or the scattering parameter $s_{11}$ for metal-backed film (MBF) shown by Fig. 1.

![Diagram of metal-backed film](image)

Fig. 1 A metal-backed film with thickness $d$. $i$ and $r$ represent the incident and reflected beams, respectively. Microwaves are reflected back-and-forth in the film and $f_M$, $b_M$, and $t$ represent the total forward, backward, and transmitted beams, respectively. $b$ is the sum of beams $r$ and $t$. $\varepsilon_0$ and $\mu_0$ are permittivity and permeability for open space. $\varepsilon_r$ and $\mu_r$ are the relative permittivity and permeability for the film, and the prime and double prime respectively indicate the real and imaginary parts of a complex number. Superscripts $-$ and $+$ have been used to distinguish positions before and after a specific location.

In units of dB, $RL$(dB) is used and defined as

$$\begin{align*}
RL(dB) &= 20 \log_{10} |RL| \\
&= 20 \log_{10} \left| \frac{V_k(x_i)}{V_i(x_i)} \right| \\
&= 20 \log_{10} \left| \frac{R_M(x_i) - e^{-\frac{4\pi\nu}{c}\varepsilon\mu d}}{1 - R_M(x_i)e^{-\frac{4\pi\nu}{c}\varepsilon\mu d}} \right| \tag{1}
\end{align*}$$

$V_k(x)$ is the voltage of beam $k$ at position $x$. $\nu$ is frequency and $c$ is the velocity of microwaves in vacuum. $R_M(x)$ is the reflection coefficient for an interface at $x$. The other symbols are given in Fig. 1. A proof of Eq. 1 is provided as Eqs. S8 – S10 in the Supplementary Materials.

For a definite MBF, the value of $RL$ is not affected by the strength of the incident microwaves. When $|RL| = 0$ or $RL$(dB) = $-\infty$, all the energy of the microwaves incident to the film is totally dissipated by the film and none has leaked back into the open space. When $|RL| = 1$ or $RL$(dB) = 0, all this energy has returned into the open space and none of it has been dissipated by the film. Thus, $RL$ is a useful parameter to characterize microwave absorbing ability from MBF. It is demonstrated in this work that $RL$ is a function of film thickness $d$ which is not an innate property of material, and should not be used to characterize material.

It has been pointed out previously\textsuperscript{16} that when $RL$ is used to characterize material, this leads to erroneous conclusions, such as that less microwaves would be absorbed if the microwaves traveled a longer distance in a material. To get around this difficulty but retaining $RL$ to characterize material, the theory of IM has been introduced but this theory is inadequate. IM has been developed in transmission-line theory based on the fact that $s_{11}$(dB) for film without metal-back (NMBF) achieves its minima when the absolute value of the maximum or minimum of the input impedance $|Z_{in}|$ of the film is close to the characteristic impedance $Z_0$ of...
the open space, or when \(|Z_0 - Z_0|\) is close to zero. It is demonstrated in this work that IM cannot be applied to \(RL(dB)\) or be generalized to MBF.

QWM has also been used to search for the minima of RL. The model states that reflection of microwaves from a film is minimized if the thickness of the film is \(m \lambda/4\) where \(m\) is an odd integer. It is argued that the incident \(i\) and backward \(b\) beams,\(^{15, 17, 18, 19, 20, 21, 22}\) or the beams \(r\) and \(t^{23, 24}\) in Fig. 1 are out of phase by \(\pi\) if the thickness of the film is \(m \lambda/4\). However, this claim takes no account of the phase effects from the interfaces. “Strict proof” of the model exists\(^{6}\) but there are serious problems with the proof.

The methods used herein are simple and straightforward. By introducing the phase effects of interfaces in a film, the problems with the QWM have been identified and the results then verified rigorously from well-known formulae and confirmed from experimental data. Along with the discussions, all the above issues concerning IM and RL have been addressed.

2. The problems with the quarter-wavelength model

2.1 Wave addition and phase effects from interfaces

There are two key points in this discussion. Firstly, the phase difference required to cancelate beams \(r\) and \(t\) in Fig. 1 and secondly the phase effects of the interfaces at \(x_1\) and \(x_1 + d\) which need to be applied with wave addition to obtain the minima of \(RL(dB)\) for MBF and \(s_{11}(dB)\) for NMBF.

\(Z_M\) and \(Z_0\) are the characteristic impedances of film and open space, respectively.

\[
Z_M = \sqrt{\frac{\mu_0 \mu_r}{\varepsilon_0 \varepsilon_r}} = Z_0, \quad \left|\frac{\mu_r}{\varepsilon_r}\right| e^{i \psi} \tag{2}
\]

\(\psi\) is the phase of \(Z_M\) which is usually very small for microwave absorbing material. The reflection coefficient \(R_M(x_1^-)\) for the interface at \(x_1^-\) in Fig. 1 is given by

\[
R_M(x_1^-) = \frac{V_i(x_1^-)}{V_i(x_1^-)} = \frac{Z_M - Z_0}{Z_M + Z_0} = \left|\frac{Z_M - Z_0}{Z_M + Z_0}\right| e^{i \phi} \tag{3}
\]

\(\phi\) is the phase of \(R_M(x_1^-)\). At \(x_1^+\)

\[
R_M(x_1^+) = \frac{Z_0 - Z_M}{Z_M + Z_0} = \frac{Z_M - Z_0}{Z_M + Z_0} e^{i \pi} = \left|\frac{Z_M - Z_0}{Z_M + Z_0}\right| e^{i (\pi + \phi)} \tag{4}
\]

Since

\[
Z_M - Z_0 = \left|Z_M\right| \cos \psi_1 - Z_0 + j \left|Z_M\right| \sin \psi_1 \tag{5}
\]

and

\[
Z_0 - Z_M = (Z_0 - \left|Z_M\right| \cos \psi_1) - j \left|Z_M\right| \sin \psi_1 = -(Z_M - Z_0) = (Z_M - Z_0) e^{i \pi} \tag{6}
\]

It will be noted that there is a phase difference of \(\pi\) represented by Eq. 5 and 6. Phase differences for the incident and the reflected beams at interfaces have already been included in Eq. 3 and 4. Equation 7 is applicable for interfaces in the film equivalent to that shown in Fig 1 but with the metal-back removed.
\[ R_M(x^-) = -R_M(x^+) = R_M(x^+) e^{i\pi} \]
\[ = R_M(x_1 + d^-) = R_M(x_1 + d^+) e^{i\pi} \] (7)

For MBF, only \( R_M(x_1 + d^-) = -1 \) needs to be considered at \( x_1 + d \). The phase difference of \( \pi + \varphi \), or approximately \( \pi \) since \( \psi_1 \) is small, also applies to reflected beams between films with \( Z_M > Z_0 \) and \( Z_M < Z_0 \).

The small phase effects from \( Z_M \) shown in Eqs. 8 and 9 also need to be considered for transmitted beams from interfaces at \( x_1^- \) and \( x_1^+ \). The transmission coefficients \( \gamma_M \) at interfaces for NMBF are given in Eqs. 8 and 9.

\[ \gamma_M(x_1^+) = \frac{2Z_M}{Z_M + Z_0} \]
\[ = 2 \left| \frac{Z_M}{Z_M + Z_0} \right| e^{i\phi_1} = 2 \left| \frac{Z_M}{Z_M + Z_0} \right| e^{i(\psi_1 - \psi)} \]
\[ = \gamma_M(x_1 + d^+) = 1 + R_M(x^-) \] (8)

\[ \gamma_M(x_1^-) = \frac{2Z_0}{Z_M + Z_0} = 2 \left| \frac{Z_0}{Z_M + Z_0} \right| e^{-i\phi} \]
\[ = \gamma_M(x_1 + d^-) = 1 - R_M(x^-) \] (9)

\( \phi \) is the phase for \( (Z_M + Z_0) \). Beams \( f_M \), \( b_M \), and \( t \) in the film shown by Fig. 1 are the resultant of many beams. When the phase effects expressed in Eqs. 3, 4, 7 - 9 are considered for each beam in MBF and NMBF, precise positions for the minima of \( S_{11}(\text{dB}) \) and \( RL(\text{dB}) \) can be obtained and the flaw in QWM can then be revealed. This analysis leads to the results obtained under 11 different conditions and summarized in Table 1.
Table 1: Relationships between reflection coefficients (|RL| or |s₁₁|) and thickness (d) of films with respect to the quarter-wavelength model. (n = integer; m = odd integer)

| Row | Film | Conditions | Properties | Validation of the model |
|-----|------|------------|------------|-------------------------|
| 1   | NMBF characterized by | | | × |
|     | | \( \varepsilon_r \neq \mu_r \) | Minima of | \(|s_{11}| \) occur near where \( |Z_0(x_1)| \) is the closest to \(|Z_0| \) | × |
|     | | \( \varepsilon_r^* \neq \mu_r^* \) | at \( d = n\lambda/2 \) | | |
|     | | \( \varepsilon_r^* = \mu_r^* = 0 \) | | | |
|     | | \( |s_{11}| \) at \( d = m\lambda/4 \) | | | |
| 2   | MBF characterized by | | | | |
|     | | \( \varepsilon_r = \mu_r \) | Minimum: \(|s_{11}| = 0 \) | | |
|     | | \( |Z_0'| = |Z_0| \) | when \( d = 0 \) | | |
|     | | \( \varepsilon_r^* = \mu_r^* = 0 \) | Maximum: \(|s_{11}| = |R_M| \) | when \( d = \infty \) | | |
| 3   | MBF characterized by | | | | |
|     | | \( \varepsilon_r = \mu_r \) | Minimum: \(|s_{11}| = 0 \) | | |
|     | | \( |Z_0'| = |Z_0| \) | when \( d = 0 \) | | |
| 4   | MBF characterized by | | | | |
|     | | \( \varepsilon_r = \mu_r \) | Minimum at \( Z_{in}(x_1) = |Z_0| \) | | |
|     | | \( |Z_0'| = |Z_0| \) | | | |
| 5   | MBF characterized by | | | | |
|     | | \( \varepsilon_r = \mu_r \) | Minimum at \( Z_{in}(x_1) = |Z_0| \) | | |
|     | | \( |Z_0'| = |Z_0| \) | | | |
| 6   | MBF characterized by | | | | |
|     | | \( \varepsilon_r = \mu_r \) | Minimum: \(|s_{11}| = 0 \) | | |
|     | | \( |Z_0'| = |Z_0| \) | when \( d = 0 \) | | |
| 7   | MBF characterized by | | | | |
|     | | \( \varepsilon_r = \mu_r \) | Minimum at \( Z_{in}(x_1) = |Z_0| \) | | |
|     | | \( |Z_0'| = |Z_0| \) | | | |
| 8   | MBF characterized by | | | | |
|     | | \( \varepsilon_r = \mu_r \) | Minimum at \( Z_{in}(x_1) = |Z_0| \) | | |
|     | | \( |Z_0'| = |Z_0| \) | | | |
| 9   | MBF characterized by | | | | |
|     | | \( \varepsilon_r = \mu_r \) | Minimum at \( Z_{in}(x_1) = |Z_0| \) | | |
|     | | \( |Z_0'| = |Z_0| \) | | | |
| 10  | MBF characterized by | | | | |
|     | | \( \varepsilon_r = \mu_r \) | Minimum at \( Z_{in}(x_1) = |Z_0| \) | | |
|     | | \( |Z_0'| = |Z_0| \) | | | |
| 11  | MBF characterized by | | | | |
|     | | \( \varepsilon_r = \mu_r \) | Minimum at \( Z_{in}(x_1) = |Z_0| \) | | |
|     | | \( |Z_0'| = |Z_0| \) | | | |

Rows 1-4 in Table describe results for NMBF which are characterized by |s₁₁|, while rows 5-11 describe MBF characterized by |RL|. Of these 11 distinct examples, only that represented by row 6 conforms to QWM, i.e., the minima of |RL| occur at \( d = -m\lambda/4 \) only for MBF when \( Z_M < |Z_0| \). For all the other cases QWM fails.

Rows 1, 5, 6 provide the most general examples, and in particular rows 1 and 5 represent cases contrary to QWM in that the minima of |s₁₁(dB)| or |RL(dB)| are achieved at \( d = -n\lambda/2 \). All the other rows show results for specific cases. Rows 2 and 7 represent films that do not dissipate microwaves (\( \varepsilon_r^* = \mu_r^* \)) and their results signify that the small phase \( \psi_1 \) in other cases is important for microwave absorption; rows 3 and 8 are examples where the interface disappears (\( \varepsilon_r = \mu_r \)); rows 4, 9 - 11 provide instances of resistive films where the inductance and capacitance can be neglected (\( \varepsilon_r^* = \mu_r^* = 0 \)). While a detailed analysis is omitted, a brief outline is given in the Supplementary Materials. As shown in the next section, the results shown by Table 1 from wave addition theory can be confirmed quantitatively using experimental data and formulae well-known to microwave absorption researchers.
2.2 Verification

The results of Table 1 have been obtained using wave addition theory which involves the phase effects of the interfaces. It can be seen with real numbers from Eq. 2 that $Z_M > Z_0$ when $\mu_r > \varepsilon_r$ and $Z_M < Z_0$ when $\mu_r < \varepsilon_r$. Omitting the phase in $Z_M$ simplifies wave addition theory though it will be problematical for microwave absorption. In this section, the results shown in Table 1 are verified theoretically from concrete calculations based on experiments or well accepted formulae. The precise phase effects are already included in the expressions and the data though these are omitted in the qualitative discussion for simplicity where “approximately” or “near” are often used, which will be stand by “≈”. We now consider each row in Table 1 in turn.

Row 1: The values of $s_{11}$(dB) for Cu@ZnFe$_2$O$_4$ in Fig. 2 are calculated by Eq. 10 with the measured $\varepsilon_r$ and $\mu_r$ at specified frequencies. Published experimental reports present data using Eq. 10, 12, and 13 for $s_{11}$ and Eq. 1, 17 and 18 for $RL$.

$$s_{11} = 20 \log_{10} \left| s_{11} \right|$$

$$= 20 \log_{10} \left( \frac{R_M(x^+_i)[1 - e^{-4j/\nu}]^{\sqrt{\varepsilon_r \mu_r}}}}{1 - R_M(x^-_i)e^{-4j/\nu}} \right)$$

(a) $\varepsilon_r = 1.33 - 0.031j$, $\mu_r = 2.50 - 0.0017j$

$\nu = 16.64$ GHz, $\lambda = 9.89$ mm
Fig. 2 The verification of row 1. Values of $\varepsilon_r$ and $\mu_r$ are measured using the composite Cu@ZnFe$_2$O$_4$ at the frequency indicated, except in the bold black curve in (b) which represents $|Z_m(x_1)/Z_0|$ with $\varepsilon''_r = \mu''_r = 0$ for the example in row 2. $s_{11}(\text{dB})$ is calculated from Eq. 10 and $Z_m(x_1)/Z_0$ from Eq. 13, using the measured $\varepsilon_r$ and $\mu_r$. Whether $Z_M > Z_0$ (a) or $Z_M < Z_0$ (b), the minima of $|s_{11}|$ occur at $-n\lambda/2$ which is contrary to QWM. The same conclusion can be obtained using the data measured at every frequency in the supplementary data file and can be generalized to published experimental data.

The requirements of Eqs. 3, 4, 7 - 9 used in wave addition theory have already been considered in the derivations of Eq. 10 and 13. It can now be seen that

$$e^{-j\pi \sqrt{\frac{\varepsilon_r}{c}} d} e^{-4\pi \frac{\text{Im} \sqrt{\frac{\varepsilon_r}{c}}}{c} d} = e^{-4\pi \frac{\text{Im} \sqrt{\frac{\varepsilon_r}{c}}}{c} d} e^{-j\pi \sqrt{\frac{\mu_r}{c}} d}$$

(11)

where Re(x) and Im(x) are the real and imaginary parts of x, respectively. From Eqs. 10 and 11, the minima of $s_{11}(\text{dB})$ occur near $\exp(-j\alpha d) = 1$ where $d$ is $-n\lambda/2$ when beams r and t in Fig. 1 are out of phase by exactly $\pi$. The result conforms to that represented in row 1 and is contrary to QWM.

The global minimum of $|s_{11}| = 0$ or $s_{11}(\text{dB}) = -\infty$ occurs at $d = 0$ which is logical because when $d = 0$ there is no film, all the microwaves pass straight through and are not reflected. As shown by Eq. 10 and Fig. 2, $s_{11}(\text{dB})$ is a decayed oscillation, since $\alpha_r$ in Eq. 10 or 11 is not zero, i.e., the oscillation amplitudes of $s_{11}(\text{dB})$ is attenuated by the microwave dissipating power. $s_{11}(\text{dB})$ approaches $R_d(x_r)(\text{dB})$ of -16.13 dB in Fig. 2a or of -13.82 dB in Fig. 2b when $d$ approaches $\infty$, which signifies that other minima of $s_{11}(\text{dB})$ can never reach $-\infty$.

At a frequency of 16.64 GHz, the minima of $s_{11}(\text{dB})$ in Fig. 2a occur at $d = 4.94, 9.88, 14.82, \text{and} 19.76$ mm (0.50, 1.00, 1.50, and 2.00$\lambda$). At a frequency of 6.61 GHz, the minimum of $s_{11}(\text{dB})$ in Fig. 2b occurs at $d = 11.04$ mm = 0.48$\lambda$, while the maxima occur at 4.90 and 16.76 mm (0.21 and 0.72$\lambda$). In both Figs. 2a and b, the minima of $|s_{11}|$ occur at $d = -n\lambda/2$ and the maxima at $d = m\lambda/4$. The same result for both cases is originated from the fact that $R_d(x_r) > 0$ in (a) when $Z_M > Z_0$ by Eq. 3 and $R_d(x_r) < 0$ in (b) when $Z_M < Z_0$ while as shown by rows 5 and 6 the different results of the two cases for MBF are because Eqs 1 and 18 are different from Eqs.
10 and 13. From Eq. 10 the sign of $R_{\text{ld}}(x_1)$ does not affect the value of $s_{11}(\text{dB})$. The results are consistent with those presented in row 1 but out of step with the prediction of $d = m\lambda/4$ for the minima of $s_{11}(\text{dB})$ from QWM. $s_{11}$ is usually expressed by two equations as

$$s_{11} = \frac{Z_m(x_1) - Z_0}{Z_m(x_1) + Z_0}$$

$$Z_m(x_1) = \frac{V_{1i}(x_1) + V_{lt}(x_1)}{I_{1i}(x_1) - I_{lt}(x_1)} = Z_0 \frac{V_{1i}(x_1) + V_{lt}(x_1)}{V_{1i}(x_1) - V_{lt}(x_1)}$$

$$= Z_M \frac{1 - R_M(x_1')e^{-j\frac{\sqrt{\epsilon\mu\pi}}{c}d}}{1 + R_M(x_1')e^{-j\frac{\sqrt{\epsilon\mu\pi}}{c}d}}$$

$|Z_{\text{a}(x_1)}/Z_0|$ in Fig. 2 is obtained from Eq. 13 and can be understood along with Eqs. 2 and 3. From Fig. 2 and Eq. 13, it can be shown that $|Z_{\text{a}(x_1)}|$ is a decaying periodic function approaching $|Z_0|$ when $d \to \infty$. $|Z_{\text{a}/Z_0}|$ is 1.37 in Fig. 2a and 0.73 in Fig. 2b.

As shown by Fig. 2a, the minima of $|Z_{\text{a}(x_1)}/Z_0|$ occur at $d = 4.96, 9.90, 14.84,$ and $19.80\,\text{mm}$ $(0.50, 1.00, 1.50, \text{and} 2.00\lambda)$. These positions are at $d = -n\lambda/2$ where $\exp(-j\alpha_d) = 1$. Each minimum of $|Z_{\text{a}(x_1)}|$ signifies a position where $|Z_{\text{a}(x_1)}|$ is closest to $|Z_0|$ or where $|Z_{\text{a}(x_1)} - Z_0|$ is closest to 0.

From Eq. 12 the minimum positions for $|Z_{\text{a}(x_1)}|$ in Fig. 2a are correlated with the minima of $|s_{11}|$ or $s_{11}(\text{dB})$. The result reveals that the minima of $|Z_{\text{a}(x_1)}|$ occur near the positions where the phase difference of beams $t$ and $r$ in Fig. 1 is effectively $\pi$ or $m\pi$, otherwise $|Z_{\text{a}(x_1)}| = |Z_0|$ cannot lead to the minimum of $s_{11}(\text{dB})$ which has already shown in Fig. 2b at $d = 1.66\,\text{mm}$. The global minimum of $|Z_{\text{a}(x_1)}|$ occurs at $d = 0$ and $|s_{11}| = 0$ or $s_{11}(\text{dB}) = -\infty$ when $Z_{\text{a}(x_1)} = Z_0$. The amplitudes of the minima of $|Z_{\text{a}(x_1)}/Z_0|$ decay as $d$ increases which signifies that the minimum $|Z_{\text{a}(x_1)}|$ deviates more from $|Z_0|$. Thus, $s_{11}(\text{dB}) = -\infty$ can never be obtained when $d \neq 0$ even though the phase requirement for the minimum of $s_{11}(\text{dB})$ has been met at the minima of $|Z_{\text{a}(x_1)}|$.

As shown by Fig. 2b, a maximum of $|Z_{\text{a}(x_1)}/Z_0|$ occur at $d = 12.36\,\text{mm} = 0.53\lambda$ and conforms to condition that $d = -n\lambda/2$ where $\exp(-j\alpha_d) = -1$, and its minima at 6.66 and 18.14 mm $(0.29$ and $0.78\lambda)$ which are close to $d = m\lambda/4$ where $\exp(-j\alpha_d) = -1$. The maxima of $|Z_{\text{a}(x_1)}|$ correlate with the minima of $s_{11}(\text{dB})$ not just because the maximum of $|Z_{\text{a}(x_1)}|$ is the closest to $|Z_0|$ or where the $|Z_{\text{a}(x_1)} - Z_0|$ is the closest to 0 since what is important is the phase requirement.

The concept of IM as developed in transmission-line theory specifies that $s_{11}(\text{dB})$ achieves its minima when the maxima or minima of $|Z_{\text{a}(x_1)}|$ approaches $|Z_0|$ or the minima of $|Z_{\text{a}(x_1)} - Z_0|$ approaches zero. As shown by Fig. 2, this concept fits quite well for the minima values of $s_{11}(\text{dB})$. However, this IM theory can only be applied to $s_{11}$. It should be noted that at these maxima or minima of $|Z_{\text{a}(x_1)}|$, not only $|Z_{\text{a}(x_1)}|$ does approach $|Z_0|$, but also these maxima or minima also occur at positions which meet the phase requirement of the minima of $s_{11}(\text{dB})$, i.e., beam $r$ and $t$ are out of phase exactly by effectively $\pi$, i.e. $m\pi$. It should be pointed out that there are positions where $Z_{\text{a}(x_1)}/Z_0 = 1$ or $|Z_{\text{a}(x_1)}| = |Z_0|$ such as those when the value of $d = -1.66\,\text{mm}$ in Fig. 2b. However, this position does not correspond to minima of $s_{11}(\text{dB})$ because the phase requirement has not been met. Unfortunately, IM is inappropriately generalized from $s_{11}(\text{dB})$ to $RL(\text{dB})$. It will cause more serious problems for $RL(\text{dB})$ if only emphasizing the condition that $|Z_{\text{a}(x_1)}|$ approaches $|Z_0|$ but ignoring the condition for phase difference. The subject will be discussed later.

These contradictions to the QWM should be readily apparent from experimental data but have remained unrecognized over the years. We have provided a file obtained from the measurement of Cu@ZnFe2O4 provided in the Supplementary Material. Each entry in the file contains a pair of $\omega$ and $\mu$ values measured at
a particular frequency. The above results apply to each entry.

Row 2: When both $\varepsilon^\prime$ and $\mu^\prime$ are zero, the film does not dissipate microwaves. Thus, the oscillations of $|s_{11}|$ and $|Z_{0}(x)|$ are similar to those represented by row 1 but without decay, as shown by the bold line in Fig. 2b for $|Z_{0}(x)|$ where both $\varepsilon^\prime$ and $\mu^\prime$ have been set to zero. The bold line is indeed oscillating without decay. The results for $s_{11}(\text{dB})$ are similar and can be readily inferred from Fig. 2. $|s_{11}|$ and $|Z_{0}(x)|$ oscillate asymmetrically around $|R_M|$ and $|Z_0|$, respectively. From Eqs. 3, 8, 9, and 13, it is clear that $|Z_{0}(x)|$ oscillates between $|Z_0|$ and $|Z_0|^2/|Z_0|$. With $Z_M > Z_0$, $|Z_{0}(x)|$ achieves its minima when $|Z_{0}(x)| = |Z_0|$ and its maxima when $|Z_{0}(x)| = |Z_0|^2/|Z_0| > Z_M$. However, with $Z_M < Z_0$, $|Z_{0}(x)|$ achieves its maxima when $|Z_{0}(x)| = |Z_0|$ and its minima when $|Z_{0}(x)| = |Z_0|^2/|Z_0| < Z_M$. For both cases, $|Z_{0}(x)| = |Z_0|$ occurs at exactly $d = n\lambda/2$ where $s_{11}(\text{dB}) = -\infty$. These maximum and minimum values of $|Z_{0}(x)|$ are unaffected by the value of $d$ because $\varepsilon^\prime$ and $\mu^\prime$ are real numbers and $\alpha_d$ in Eq. 11 is zero. IM applies at $|Z_{0}(x)| = |Z_0|$ here but the film does not dissipate microwave since $\varepsilon^\prime\prime = \mu^\prime\prime = 0$ while IM has been developed in field material for characterize microwave absorption. The numerical verification is included as Fig. S2 in the Supplementary Materials. When $Z_M = Z_0$, both the maxima and the minima of $|Z_{0}(x)|$ become $|Z_0|$ which signify that $|Z_{0}(x)|$ is a constant, $s_{11}(\text{dB}) = -\infty$ at any value of $d$ and the case reduced to that represented by row 3.

It should be noted that when $s_{11}(\text{dB}) = -\infty$, this condition does not characterize the microwave dissipation power of the film with $\varepsilon^\prime\prime = \mu^\prime\prime = 0$. When $d = n\lambda/2$ or $\exp(2\pi d/\lambda) = \pm 1$, the energy of the microwaves has not leaked back to the open space from interface at $x_j$, but it is so leaked from interface at $x_j + d$ and $|s_{11}| = 1$ according to Eq. 14. The result here illustrates that $s_{11}(\text{dB})$ is a parameter faithfully characterizing film rather than material.

$$s_{21}(x_i + d) = \frac{[1 - R_M^2(x_i)] e^{-2\pi i d / \lambda}}{1 - R_M^2(x_i) e^{-2\pi i d / \lambda}}$$

The expression of $|Z_{0}(x)|$ can be obtained from Eq. 13 as

$$|Z_{0}(x)| = |Z_M| \sqrt{\left(e^{2\alpha_d} - |R_M(x_i)|^2 e^{-2\alpha_d}\right)^2 + 4 |R_M(x_i)|^2 \sin^2(4\pi \frac{d}{\lambda} + \varphi)}$$

$$= |Z_M| \left[ e^{2\alpha_d} - |R_M(x_i)|^2 e^{-2\alpha_d}\right]^2 + 2 |R_M(x_i)|^2 \left[ \cos(4\pi \frac{d}{\lambda} + \varphi) + 1 \right]$$

$$|Z_{0}(x)|$$ is not just a function of $\alpha_d$. The phase of $R_M(x_i)$ in Eq. 3 and the value of $\alpha_d$ in Eq. 15 affect positions of the minima and maxima positions of $|Z_{0}(x)|$. From Eq. 15, $|Z_{0}(x)| = |Z_0|$ occurs exactly at $d = n\lambda/2$ when $\varepsilon^\prime\prime = \mu^\prime\prime = 0$ since both the phase of $R_M(x_i)$ and the value of $\alpha_d$ are zero (the bold line in Fig. 2b). The value of $s_{11}(\text{dB})$ is $-\infty$ exactly at $d = n\lambda/2$ when both $\alpha_d$ and $\varphi$ are zero from Eq. 16 which is obtained from Eq. 10.

$$s_{11}(x_i) = |R_M(x_i)| \sqrt{1 + e^{-2\alpha_d} - 2 e^{-\alpha_d} \cos(\alpha_d d)}$$

However, when $\varepsilon^\prime\prime$ or $\mu^\prime\prime$ is not zero, the phase of $R_M(x_i)$ and the value of $\alpha_d$ will shift the minima and maxima positions of $|Z_{0}(x)|$ away from $d = n\lambda/2$ and the related minima of $s_{11}(\text{dB})$ are also affected. The shift in $|Z_{0}(x)|/|Z_0|$ is apparent in the dotted line in Fig. 2b. Not just the minima values of $s_{11}(\text{dB})$ are different from $-\infty$ dB, but so are their minima positions. For example, as shown by the bold line in Fig. 2b and from Eq. 13, a maximum of $|Z_{0}(x)| = |Z_0|$ is achieved at $d = 0$ when $\varepsilon^\prime\prime = \mu^\prime\prime = 0$. But this maximum has shifted to the right
shown by the dotted line when $\epsilon_r = 0.017$ and $\mu_r = 0.75$ which ensures that $\alpha d \neq 0$ and the phase of $R_{s1}(x_1)$ deviates from zero. These changes in $|Z_{in}(x_1)|$ will also affect the position and the value of the minimum of $s_{11}(d)$. 

Using wave addition theory, beams $r$ and $t$ in Fig. 1 with the metal-back removed are out of phase by $\pi$ at

d = n\lambda/2 when $\epsilon_r = \mu_r = 0$. When $\epsilon_r = \mu_r$, the phase of $R_M(x_1)$ will affect the phase of beam $r$ as described by Eq. 3. Thus, the values of $d$ must be adjusted from $n\lambda/2$ so that beam $r$ is still out of phase by $\pi$ to provide the minima of $s_{11}(d)$. The phase effects from interfaces expressed by Eqs. 7 - 9 also affect the phase of beam $t$ which in turn affects the adjustment of $d$. The different amplitudes of each beam in $t$ also make a contribution. All these effects have been included in Eq. 15 for $|Z_{in}(x_1)|$ and Eq. 16 for $|s_{11}(x_1)|$.

Row 3: $Z_M = Z_0$ when $\epsilon_r = \mu_r$ and the interfaces at $x_1$ and $x_1 + d$ disappear thus there are no returning beams $r$ and $t$ in Fig. 1, and as a result $|s_{11}| = 0$ or $s_{11}(d) = -\infty$. The result can be predicted from Eqs. 3 and 10, or from Eq. 16 since $R_M(x_1) = 0$ when $Z_M = Z_0$. Equations 12 and 13 also leads the same result since $Z_{in}(x_1) = Z_M = Z_0$ when $\epsilon_r = \mu_r$.

Row 4: When $\epsilon_r = \mu_r = 0$, these conditions represent a resistive film where the inductance and capacitance are not important. Here $|s_{11}| = 0$ or $s_{11}(d) = -\infty$ when $d = 0$ since there is no film and all the microwaves pass through. $|s_{11}|$ approaches $|R_M(x_1)|$ when $d$ approaches infinity where the only returning beam is $r$ from the interface at $x_1$ and no returning beam from the interface at $x_1 + d$ of the NMBF.

From Eq. 10, $s_{11}(d) = -\infty$ when $d = 0$ which is true for all cases in rows 1- 4. Also it should be noted that $|s_{11}| = |R_M(x_1)|$ when $d = \infty$.

From Eqs. 8, 9, and 13, $|Z_{in}(x_1)| = |Z_0|$ when $d = 0$. As $d$ approaches infinity, $|Z_{in}(x_1)| = |Z_M/Z_0|$, which is 2 when $|Z_M| > |Z_0|$ and 0.5 when $|Z_M| < |Z_0|$ in Fig. 3. ($\epsilon_r + j\mu_r)d$ becomes a real number when $\epsilon_r = \mu_r = 0$. Thus, either $|s_{11}|$ or $|Z_{in}(x_1)|$ is no longer a periodic function. $|Z_{in}(x_1)|$ will change monotonically as $d$ increases. When $|Z_M| > |Z_0|$, $Z_{in}(x_1)$ increases from $|Z_0|$ to $|Z_M|$. When $|Z_M| < |Z_0|$, $Z_{in}(x_1)$ decreases from $|Z_0|$ to $|Z_M|$. $Z_{in}(x_1)$ deviates more from $|Z_0|$ in both cases and from Eq. 12, $s_{11}(d)$ increases and approaches the $R_M(d)$ with the value of - 9.54. This result signifies that the further the microwaves travel in the material, the more intense will be the backward beam $b$ that emerges from the film. (Fig. 3) Thus, $s_{11}$ is a parameter faithfully characterizing film rather than material.

This result of $|s_{11}|$ for NMBF is equivalent to that of $R_M(x_1)$ which is the $s_{11}$ for interface. From Eq. 3, the intensity of the reflected beam $r$ from the interface at $x_1$ will be weak when $Z_M$ is nearly equal to $Z_0$, but will become stronger when $Z_M$ differs more from $Z_0$.

The voltages before and after a node between a wire and a resistor are equal. But the resistances of the wire and the resistor are not equal. Reflecting current at a node is used in transmission line theory to solve the problem that the currents at both sides of the node are not equal with the Ohm’s law.

The analyses for microwave and conventional circuits are essentially the same except the former involves high frequency. A node in circuit is equivalent to an interface. When the resistance of a resistor in circuit is zero, the current is infinity which signifies that all current goes through the node without reflection; Similarly, when $Z_M = Z_0$ at an interface, all the incident microwaves penetrate the interface without reflection, thus from Eq. 3 $R_M = 0$. As the resistance increases, more current is reflected at the node and less passes through, so that the current of the circuit is reduced; as $Z_M$ increases from $Z_0$, reflection at interface is increased and less microwave transmitted, thus, $R_M$ is increased. When the resistance increases to infinity, the current is zero so that none passes through the node and all incoming current is reflected back at the node to cancel the original current; when $Z_M$ increases to infinity, all the microwaves are reflected with no transmission and $R_M = 1$ from Eq. 3.
still needed to ensure continuity. The reflected beams r and bM from the interfaces at x1 and x1 + d are opposite in signs from Eqs. 3, 7 - 9. However, beam bM reflected from the interface at x1 + d is no longer a periodic function of x and the increase in d cannot change the sign of it at x1'. Thus, beams r and t at x1 are always opposite in sign which is unrelated to the value of d. As d increases, the intensity of beam bM decreases at x1', thus, beam t at x1' becomes weaker and thus s_{11}(dB) is increased. When d becomes infinity, there will be no returning beam t and only beam r exists, thus, s_{11}(dB) = R_M(x_1')(dB). When Z_M increase to infinity, there will be no transmitted beam f_M and s_{11}(dB) = R_M(x_1')(dB).

![Graph showing s_{11}(dB) vs d/mm](image)

**Fig. 3** The special case when \( \varepsilon_r' = \mu_r' = 0 \). The values of s_{11}(dB) for Z_M > Z_0 and Z_M < Z_0 are coincident with each other. The QWM fails in this case.

**Rows 5 and 6:** The conclusions in rows 5 and 6 are obtained using wave addition theory and the phase effects of the interfaces. The values of RL(dB) in Figure 4 are calculated from Eq. 1. The minima of RL(dB) shown by Fig. 4a for Z_M > Z_0 occur at 4.52, 14.94, 22.38 mm (0.50, 1.00, 1.50\( \lambda \)). From Eq. 3, \( R_M(x_1') > 0 \) when Z_M > Z_0. From Eq. 1, \( |RL| \) reaches its minima when exp(-j\( \alpha d \)) is ~1. From Eq. 11 exp(-j\( \alpha d \)) = 1 when \( d = n\lambda/2 \). The result from Eq. 1 conforms to the prediction in row 5 from wave addition. The result that \( d = -n\lambda/2 \) is contradictory to the QWM.

The minima of RL(dB) shown by Fig. 4b for Z_M < Z_0 occur at 9.06 and 26.8 mm (0.25 and 0.75\( \lambda \)). \( R_M(x_1') \) < 0 when Z_M < Z_0. \( |RL| \) reaches its minima near exp(-j\( \alpha d \)) = -1 or \( d = -m\lambda/4 \). This conclusion shown in row 6 is the only example where the minima of \( |RL| \) occur at \( d = -m\lambda/4 \) conforming with QWM.
Figure 4 Metal-backed film from the composite Cu@ZnFe$_2$O$_4$ with measured ε and µ at specified frequency. (a) shows examples contrary to QWM. (b) is the only one conforms to QWM. The values of $RL$(dB) and $|Z_{in}(x)|$ are calculated from using Eqs. 1 and 18, respectively. In published reports $RL$(dB) and $|Z_{in}(x)|$ are commonly obtained from Eqs. 1 and 18 using experimental values of ε and µ.$^{1-26}$

This numerical verification shows that the phase effects of interfaces have been included in Eq. 1. The
RL are also expressed by two equations familiar to material scientists.

\[ RL = \frac{Z_m(x_1) - Z_0}{Z_m(x_1) + Z_0} \]  

(17)

\[ Z_m(x_1) = Z_M \frac{1 - e^{-4j\pi\sqrt{\varepsilon\mu/\lambda}d}}{1 + e^{-4j\pi\sqrt{\varepsilon\mu/\lambda}d}} \]  

(18)

In Fig. 4a, the minima of both \(|Z_m(x_1)|\) and \(|\tanh(2\pi\nu(\varepsilon\mu)^{1/2}/c)|\) occur at 7.46, 9.50, 14.90, 22.36 mm (0.50\(\lambda\), 1.00\(\lambda\), 1.50\(\lambda\), 1.99\(\lambda\)) conforming to \(d = -n\lambda/2\). For MBF from Eq. 18, \(|\tanh(2\pi\nu(\varepsilon\mu)^{1/2}/c)|\) oscillates around 1 and \(|Z_m(x_1)|\) oscillates around \(|Z_0|\); both are decayed and asymmetric oscillations. The minima of \(|\tanh(2\pi\nu(\varepsilon\mu)^{1/2}/c)|\) are less than 1 and their positions are near \(d = n\lambda/2\) where \(\exp(-j\alpha d) = 1\). Thus, from Eq. 18 the minima of \(|Z_m(x_1)|\) is less than \(|Z_0|\).

When \(Z_0 > Z_0\) (Fig. 4a), the minima of \(|Z_m(x_1)|\) approaches \(Z_0\) as \(d\) increases. However, the fact that the minima of \(RL(dB)\) also occur near the minima positions of \(|Z_m(x_1)|\) does not provide sufficient evidence to validate IM even though it seems true from Eq. 17 that the minima positions of \(RL(dB)\) at \(d = -n\lambda/2\) are near those of the minima of \(|Z_m(x_1)| \| Z_0|\) which are the nearest to zero as shown by Fig. 4a. When the positions deviate from those for the minima of \(|Z_m(x_1)|\), \(|Z_m(x_1)| = |Z_0|\) has been reached at \(d = 1.40, 6.04, 8.86, 13.50, 16.30, 20.96,\) and 23.76 mm, but at these positions the minima of \(RL(dB)\) have not been reached and \(|Z_m(x_1)| - |Z_0|\) deviates more from zero. Thus, the fact that the minima of both \(RL(dB)\) and \(|Z_m(x_1)|\) occur at \(d = -n\lambda/2\) is not because of IM but because beams r and t in Fig. 1 are out of phase by \(\pi\). There are many reports published with data similar to those shown in Fig. 4 but IM theory has never been questioned. It worth noting that both \(|Z_m(x_1)| - 1\) and \(|Z_m(x_1)| + 1\) are greater than \(|Z_m(x_1)|\) at the minimum positions of \(|Z_m(x_1)|\) in Fig. 4a because \(Z_m(x_1)\) is near 0. The fact shows that IM theory is problematical since in the theory only the effect of \(|Z_m(x_1)| - 1\) in Eq. 17 is taken into account.

In Fig. 4b, the maxima of both \(|Z_m(x_1)|\) and \(|\tanh(2\pi\nu(\varepsilon\mu)^{1/2}/c)|\) occur at 8.92 and 26.76 mm (0.25 and 0.752,\(\lambda\)) conforming to \(d = m\lambda/4\) where \(\exp(-j\alpha d) = -1\). The phase requirement for cancelling beams r and t when \(Z_M < Z_0\) is satisfied at \(d = -m\lambda/4\). The maxima of \(|\tanh(2\pi\nu(\varepsilon\mu)^{1/2}/c)|\) are greater than 1 and the maxima of \(|Z_m(x_1)|\) are greater than \(|Z_0|\). It is worth to note in Fig. 4b that the minima of \(RL(dB)\) occur near the positions of the maxima of \(|Z_m(x_1)|\) where \(|Z_m(x_1)|\) deviate the most from \(|Z_0|\), but \(RL(dB)\) has not achieved its minima at \(d = 5.56, 12.30,\) and 23.44 mm where \(|Z_m(x_1)| = |Z_0|\).

The results from Fig. 4a apply to every entry of data when the frequency is higher than 8.14 GHz and the results from Fig. 4b apply to every entry of data when the frequency is lower than 7.97 GHz in the file provided in the Supplementary Material. The result is natural since \(Z_M > Z_0\) at high frequency and \(Z_M < Z_0\) at low frequency for magnetic material. Because the data acquisition step is \(\Delta f = 170\) MHz, the exact frequency at which \(Z_M - Z_0\) changes sign cannot be established.

It is claimed in IM that \(RL(dB)\) reaches its minima when \(|Z_m(x_1)/Z_0| = 1\) and 18, 20 - 22, 24 However, as shown by Fig. 4, this is not true because the phase requirement must also be satisfied.

Indeed, using the IM concept developed from transmission line theory illustrated in Fig. 2, the minima of \(s_{11}(dB)\) is indeed achieved at the maximum or minimum positions of \(|Z_m(x_1)|\) where it approaches \(|Z_0|\) or where \(|Z_m(x_1) - Z_0\) approaches zero. But the concept has been wrongly generalized to \(|RL|\) for MBF since Eqs. 17 and
18 superficially look similar to Eqs. 12 and 13. In fact, the most important condition for the minimum positions of \( s_{11}(dB) \) and \( RL(dB) \) is the phase condition for the cancelling of beams r and t at the positions indicated by \( d = -n\lambda/2 \) in Figs. 2 and 4a or \( d = -m\lambda/4 \) in Fig. 4b.

The IM theory cannot be generalized from \( s_{11}(dB) \) to \( RL(dB) \). In Fig. 4a, the value \( RL(dB) \) becomes its minima when \( |Z_{in}(x_1)| \) approaches zero where \( |Z_{in}(x_1)| \) deviates the most from \( |Z_0| \). In Fig. 4b, \( RL(dB) \) becomes its minima when \( |Z_{in}(x_1)| \) approaches its maximum values. In other word, contrary to IM, \( RL(dB) \) achieves its minima when \( |Z_{in}(x_1)| \) deviates the most from \( |Z_0| \) and \( |Z_{in}(x_1) - Z_0| \) deviates the most from 0. The result signifies that \( Z_{in}(x_1) \) and \( Z_0 \) are in phase with each other is not the same as that beams r and t are out of phase with each other. Thus, to minimize \( RL(dB) \), it is not sufficient to just consider the numerator \( |Z_{in}(x_1) - Z_0| \) in Eq. 17 and ignore the effect of the denominator \( |Z_{in}(x_1) + Z_0| \) on \( RL \).

From Eqs. 11 and 18, we obtain

\[
Z_{in}(x_1) = Z_M^j \frac{1 - e^{-2\sigma_d} + 2j e^{-\sigma_d} \sin(\alpha_j d)}{1 + e^{-2\sigma_d} + 2e^{-\sigma_d} \cos(\alpha_j d)}
\]

\[
= |Z_M^j| \sqrt{\left(1 - e^{-2\sigma_d}\right)^2 + 4e^{-2\sigma_d} \sin^2(\alpha_j d)} \left(1 - e^{-2\sigma_d}\right)^2 + 2e^{-\sigma_d} \left[\cos(\alpha_j d) + 1\right]}
\]

\[
\tan \theta = \frac{2e^{-\sigma_d} \sin(\alpha_j d)}{1 - e^{-2\sigma_d}}
\]

Equation 19 is related to Eq. 15 if \( R_M(x_1) = 1 \) and \( \varphi = 0 \). From Eq. 19, the maxima and minima of \( Z_{in}(x_1) \) occur exactly at \( d = n\pi/2 \) if \( \alpha_d = 0 \). In their so called ‘strict proof’ Zhang et al. derived an equation from Eq. 20 for the deviation of \( d \) from \( m\lambda/4 \) for the minima positions of \( RL(dB) \). But when using Eq. 20, \( \theta \) was replaced wrongly by \( \psi_1 \) from Eq. 2 and \( \alpha_d d \) was replaced wrongly by \( \alpha_j d \) so invalidating their proof. Although the deviations of the minima positions of \( RL(dB) \) from \( d = m\pi/4 \) or \( n\pi/2 \) are related to Eq. 19, the most relevant equation should be Eq. 21 obtained from Eq. 1. All the phase effects from the interfaces on the deviations have been included in Eq. 21.

\[
|RL(x_1)| = \frac{R_M^2(x_1) \cos(2\varphi) + e^{-2\sigma_d} - 2\left[R_M^2(x_1)\right]e^{\alpha_d} \cos(\alpha_d d + \varphi)}{1 + R_M^2(x_1) e^{-2\sigma_d} - 2\left[R_M^2(x_1)\right] e^{\alpha_d} \cos(\alpha_d d - \varphi)}
\]

Row 7: When both \( \varepsilon'' \) and \( \mu'' \) are zero, the film does not dissipate microwaves. For metal-backed film, all those part of beam i that enters the film will finally return to the open space if the film does not dissipate microwaves and the amplitude of beam b will be the same as that of beam i. As a consequence, \( |RL| = 1 \) or \( RL(dB) = 0 \). It is proved by Eq. S7 in the Supplementary Material that \( |RL| \) is irrelevant to the phases of beams i and b. The numerical verification from Eqs. 1 and 18 is presented by Fig. 5a.
Fig. 5 $RL$ calculated from Eq. 1 and $Z_{in}(x)$ from Eq. 18 for MBF with $\varepsilon'' = \mu'' = 0$ (a) and $\varepsilon = \mu$ (b).

When both $\varepsilon$ and $\mu$ are real, from Eq. 1 we obtain
\[ |RL| = \sqrt{\frac{R_M^2 - 2R_M \cos(4\pi \nu d \sqrt{\varepsilon_r \mu_r}) + 1}{1 - 2R_M \cos(4\pi \nu \sqrt{\varepsilon_r \mu_r} d) + R_M^2}} = 1 \]  
(22)

Equation 22 can also be obtained from Eq. 21 when both \( \alpha_P \) and \( \varphi \) are zero. From Eq. 2 and 18, we obtain

\[ Z_{in}(x_1) = Z_0 \sqrt{\frac{\mu_r}{\varepsilon_r} \frac{j \sin(4\pi \nu \sqrt{\varepsilon_r \mu_r} d)}{1 + \cos(4\pi \nu \sqrt{\varepsilon_r \mu_r} d)}} \]

(23)

Equation 23 can also be obtained from Eq. 19 when both \( \alpha_P \) and \( \psi_1 \) are zero. Since \( Z_{in}(x_1) \) is an imaginary number by Eq. 23 and \( Z_0 \) is a real number, \( |Z_{in}(x_1) - Z_0| = |Z_{in}(x_1) + Z_0| \) as shown by Fig. 5a. Thus, \( |RL| = 1 \) from Eq. 17. The result shows that the effect of \( |Z_{in}(x_1) + Z_0| \) on \( |RL| \) cannot be neglected which invalidates the argument from IM.

Row 8: From IM theory, all the incident microwaves penetrate into the film when \( Z_M = Z_0 \) if \( \varepsilon_r = \mu_r \) (Eq. 2). Thus, it is suggested by IM that the minima of \( RL(dB) \) could be achieved when \( \varepsilon_r = \mu_r \). However, such a condition leads to a case violating the QWM and the minima of \( RL(dB) \) cannot be achieved.

\[ Z_M = Z_0 \] when \( \varepsilon_r = \mu_r \) (Eq. 2), and the interface at \( x_1 \) disappears so that there is only a forward beam and a backward beam from the interface at \( x_1 + d \) in Fig. 1. This makes \( RL(dB) \) correlate inversely with \( d \), i.e., the thicker the film, the smaller the value of \( RL(dB) \). Since \( R_M(x_1) = 0 \) from Eq. 3, we obtain Eq. 24 from Eq. 1 or 21.

\[ RL(dB) = -80 \frac{\pi (\log_{10} \varepsilon_r) \varepsilon_r}{c} |d| \]

(24)

\( RL(dB) \) in Fig. 5b can be drawn from Eq. 1 or 24 while \( Z_{in}(x_1) \) is drawn from Eq. 18 which shows that both QWM and IM theory cannot be correctly applied to \( |RL| \) since the value of \( |Z_{in}(x_1) - 1| \) is not directly related to that of \( RL(x_1') \).

The proof of QWM published by Zhang et al. is based on the condition that \( \psi_1 \) in Eq. 2 is zero but this can only be achieved when \( \varepsilon_r = \mu_r \) or when both \( \varepsilon_r \) and \( \mu_r \) are real numbers. As shown in rows 7 and 8, no minimum of \( RL(dB) \) can be found for either condition, the proof is flawed.

Rows 9 - 11: It is a resistive film when both the real parts of \( \varepsilon_r \) and \( \mu_r \) are zero which means that inductance and capacitance are not important. \( Z_{in}(x_1) \) is real and no longer a periodic function. \( Z_{in}(x_1) = 0 \) when \( d = 0 \). For real function, the IM theory applies since the effect of \( |Z_{in}(x_1) - 1| \) on the value of \( |RL| \) is enough and the effect of \( |Z_{in}(x_1) + 1| \) can be omitted in qualitative analysis. \( \tanh[2\pi \nu d(\varepsilon_r \mu_r)] \) increases from 0 to 1 as \( d \) increases from 0 to infinity. Thus from Eqs. 17 and 18, if \( Z_M > Z_0 \), \( RL(dB) \) reaches \( -\infty \) when \( Z_{in}(x_1) \) increases to \( Z_0 \). When \( Z_{in}(x_1) \) increases further from \( Z_0 \), \( RL(dB) \) increases (Fig. 6a). On the other hand, if \( Z_M < Z_0 \), \( |RL| \) decreases when \( Z_{in}(x_1) \) increases toward \( Z_0 \) (Fig. 6b). If \( Z_M = Z_0 \), the result reduces to that represented by row 8 (Fig. 5b) except that \( Z_{in}(x_1) \) is no longer periodic (Fig. 6c).
\( \varepsilon_r = -0.2j, \mu_r = 0.8j \)

\( \nu = 12.73 \text{ GHz} \)

\[ |Z_{\text{in}}(x_1)/|Z_0 + 1| \]

\[ |Z_{\text{in}}(x_1)/|Z_0 - 1| \]

\[ |Z_{\text{in}}(x_1)/|Z_0| \]

\( d/\text{mm} \)

\( RL(\text{dB}) \)

\( -80 \)

\( -60 \)

\( -40 \)

\( -20 \)

\( 0 \)

\( RL(\text{dB}) \)

\( -10 \)

\( 0 \)

\( 5 \)

\( 10 \)

\( 15 \)

\( 20 \)

\( 25 \)

(a) \( \varepsilon_r = -0.2j, \mu_r = 0.8j \)

\( \nu = 12.73 \text{ GHz} \)

\[ |Z_{\text{in}}(x_1)/|Z_0 + 1| \]

\[ |Z_{\text{in}}(x_1)/|Z_0 - 1| \]

\[ |Z_{\text{in}}(x_1)/|Z_0| \]

\( d/\text{mm} \)

\( RL(\text{dB}) \)

\( -80 \)

\( -60 \)

\( -40 \)

\( -20 \)

\( 0 \)

\( RL(\text{dB}) \)

\( -10 \)

\( 0 \)

\( 5 \)

\( 10 \)

\( 15 \)

\( 20 \)

\( 25 \)

(b) \( \varepsilon_r = -0.8j, \mu_r = 0.2j \)

\( \nu = 12.73 \text{ GHz} \)
Fig. 6 The numerical verifications from Eqs. 1 and 18 for the results shown in rows 9 -11.

Equation 25 can be obtained from Eq. 1.

\[
RL(x_i^-) = R_M(x_i^-) - \frac{[1 - R_M^2(x_i^-)]e^{4\pi\sqrt{\epsilon_\mu/\lambda}}}{1 - R_M^2(x_i^-)e^{4\pi\sqrt{\epsilon_\mu/\lambda}}} \quad (25)
\]

On the right side of Eq. 25, the first term \(R_M(x_i^-)\) is the contribution from the interface at \(x_i\), while the second term is the contribution from the interface at \(x_i + d\).

A derivation of Eq 25 is provided as S8 – S11 in the Supplementary Materials.

3. Conclusions

By considering the phase effects from interfaces in a film, the flaws in QWM have been found and a complete solution has been obtained for numerous different conditions which are summarized in Table 1. The results have been verified from experimental data and from well-known formulae. Along with the analysis, it is proved that the concept of IM developed from transmission line theory for \(s_{11}(dB)\) cannot be applied to \(RL(dB)\) with MBF. Although the minima of \(s_{11}(dB)\) are achieved when \(|Z_{in}(x_i)|\) is the closest to \(|Z_0|\) or \(|Z_{in}(x_i) - Z_0|\) is the closest to zero as shown by Figs. 2 and S2, it has proved that the more relevant reason is not IM but because beams r and t in Fig. 1 are out of phase by \(\pi\). This result has been demonstrated by Figs. 2b and 4 that there is not a minimum of \(s_{11}(dB)\) and \(RL(dB)\) when \(|Z_{in}(x_i)| = |Z_0|\) where the phase requirement is not satisfied. It is shown by Figs. 4 and 5 that the IM is wrong because the effect of \(|Z_{in}(x_i) + Z_0|\) on the value of \(RL(dB)\) cannot be overlooked in microwave absorption. The effect cannot be ignored since \(Z_{in}(x_i)\) is a complex number. It is shown by Figs. 3, 6a and 6b that IM is valid only when \(Z_{in}(x_i)\) is a real function since the periodicity of \(|Z_{in}(x_i)|\) has been lost when \(Z_{in}(x_i)\) is a real function. \(s_{11}(dB)\) and \(RL(dB)\) have been shown to be parameters which faithfully characterize device rather than material. As shown by Figs. 2, 3, 4, 6a, \(s_{11}(dB)\) and \(RL(dB)\)
can be larger when the microwaves travel longer in a material.

Our work shows that there is a general practice of using established theory believing that it must be correct because it has been used for many years by many researchers. However, such theories need to be constantly checked, particularly when they are inconsistent with published experimental data and fundamental physical principles.

Supplementary Material

The procedures of wave addition for obtaining the results of Table 1 are provided in the Supplementary Material.

Data showing the flaws in QWM can be easily obtained from experiment. At the guidance of theoretical analysis from this work, we have identified the data for many compounds and one of them is presented as Supplementary Material. Each entry is a $\varepsilon$ and $\mu$ pair measured at different frequency. $RL$(dB) presented in published experimental reports are calculated from Eqs. 17 and 18 which is consistent to Eq. 1 in this work. Figs. 2 and 4 in this work are based on the data. The value of $RL$(dB) presented in this work are obtained with the same method of published reports. The only difference is that the conclusions obtained in this work is completely different from those published.

The published conclusions are commonly accepted for long but our conclusions are obtained from rigorous analysis, fundamental principles at college level established centuries ago, and consistent from different analysis.

Authors' Contributions

The authors share the same views and contribute equally.

Conflicts of interest

Conflict of academic interest is certainly involved since the views presented in this work challenge the dominant concepts and methods in the field.

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Supplementary Materials

The problems in the quarter-wavelength model and impedance matching theory in analysising microwave absorption material

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1 Wave addition and the minima of $RL$ and $S_{11}$

The quarter-wavelength model is widely used as a general model to find the minima of reflection loss $RL$ (dB) of microwaves.1, 2, 3, 4, 5, 6, 7, 8 In the model it is claimed that the reflection coefficient of a film is minimized when the thickness $d$ of the film is $m\lambda/4$ where $m$ is an odd integer. The model is based on the reasoning that beams $r$ and $t$ in Fig. 1 are out of phase by $\pi$ when $d = m\lambda/4$.9, 10 What is wrong with the model is that the phase effects of interfaces have been overlooked.

We have investigated the quarter-wavelength model by the principle of wave addition in which the phase effects of interfaces have been included. All the $\varepsilon_r$ and $\mu_r$ pairs measured from Cu@ZnFe2O4 at different frequencies conform to the conclusions obtained in this work. The data have been provided in the Supplementary Materials and the conclusions can be applied to data from films of any compounds.

The parameter $S_{11}$ (dB) is referenced to film shown by Fig. S1a while $RL$ (dB) is referenced to film shown by Fig. S1b. It should be noted that these parameters can only be correctly used as reflection coefficients when there are no incident beam from the opposite side of the relevant film as shown by Fig. S1.
Fig. S1 Microwaves with wavelength $\lambda$ are reflected back-and-forth within the film. The film is an expansion of Fig. 1 for film without metal-back (a) and with metal-back (b). Beams $f_1$, $f_2$, $f_3$, etc. and beams $b_1$, $b_2$, $b_3$, etc. result in the total forward $f_M$ and backward $b_M$ beams in the film, respectively. Beams $t_1$, $t_2$, $t_3$, etc. result in the total transmitted beam $t$. Beam $b$ is the resultant of beams $r$ and $t$.

1.1 Wave addition with phase effects for film without metal-back

Prime ′ and double prime ″ are used to indicate the real and imaginary parts of a complex number, respectively. In microwave absorption material, $\varepsilon$″ and $\mu$″ is usually very small and thus their contributions to various phase effects are small even though their contributions to microwave absorption cannot be neglected because the back-and-forth reflections within a film.
1.1.1 For film with $Z_M' > Z_0$

$s_{11}(dB)$ is used to characterize the film without metal-back as shown by Fig. S1a. The results shown in rows 1 and 2 of Table for $Z_M' > Z_0$ can be obtained by wave addition with the phase effects of the interfaces included, i.e., $s_{11}(dB)$ achieves its minima at $d = -n\lambda/2$ where beam r and t in Fig. S1a are out of phase by exactly $m\pi$, and its maxima at $d = -m\lambda/4$ where beam r and t are exactly in phase with each other. $n$ is any integer. The result is contrary to that predicted by the quarter-wavelength model and the analysis is as shown below.

From Eq. 3 or S1 it can be seen that at position $x_i^-$, the phase difference between beams r and i is $\phi$.

$$R_M(x_i^-) = \frac{V_i(x_i^-)}{V_r(x_i^-)} = \frac{Z_M - Z_0}{Z_M + Z_0} = \frac{Z_M - Z_0}{Z_M + Z_0} e^{i\phi} \quad (S1)$$

The phase contribution from $R_M(x_i)$ is usually small for microwave absorption materials. So, beam r is approximately in phase with beam i.

Also, at $x_i^-$ beam $t_1$ is out of phase by $m\pi$ or effectively $\pi$ with beam r if $d = -n\lambda/2$ since:

The phase difference between beam $f_1$ at $x_i^+$ and beam $i$ at $x_i^-$ is $\psi - \phi$ as shown by Eq. 8 or S2 which defines the transmission coefficient $\gamma_M(x_i^+)$ of interface at $x_i$. The phase contribution from $\gamma_M(x_i)$ is small. So, beam $f_1$ at $x_i^+$ is approximately in phase with beam i at $x_i^-$.  

$$r(x_i^+) = \frac{V_{f_1}(x_i^+)}{V_i(x_i^-)} = \frac{2Z_M}{Z_M + Z_0}$$

$$= \frac{2[Z_M e^{i\phi}]}{Z_M + Z_0} = 2 \frac{Z_M}{Z_M + Z_0} e^{i(\psi - \phi)} \quad (S2)$$

At $x_i^+ + d^-$ beam $b_1$ is out of phase by $\pi + \phi$, or $\sim \pi$ with beam $f_1$ shown by Eq. 7 or S3 which defines the reflection coefficient $R_M(x_i + d^-)$ of interface at $x_i$ for film without metal-back.

$$R_M(x_i + d^-) = \frac{V_{b_1}(x_i + d^-)}{V_{f_1}(x_i + d^-)} = \frac{Z_0 - Z_M}{Z_0 + Z_M} = R_M(x_i^-) e^{i\phi} = |R_M(x_i^-)| e^{i(\pi + \phi)} \quad (S3)$$

The phase difference between beam $b_1$ at $x_i^-$ and beam $b_1$ at $x_i^+$ is $-\phi$ or $\sim 0$ shown by Eq. 9 or S4 which provides the transmission coefficient $r(x_i^-)$ of interface at $x_i$.

$$r(x_i^-) = \frac{V_{f_1}(x_i^-)}{V_{b_1}(x_i^+)} = \frac{2Z_0}{Z_M + Z_0}$$

$$= 2 \frac{Z_0}{Z_M + Z_0} e^{-i\phi} = 1 - R_M(x_i^-) \quad (S4)$$

Only an optical distance of $-n\lambda/2$ can result in a phase difference of exactly $m\pi$ between beams r and t1. A phase difference of $m\pi$ is effectively $\pi$ since m is an odd number. If $d = m\lambda/4$ as specified in the quarter-wavelength model, then beam t1 would be approximately in phase with beam r at $x_i^-$, which
is near the maximum of $s_{11}(\text{dB})$.

It can be proved similarly that at $x_1^-$ each beam of $t_2$, $t_3$, etc. is out of phase by $\sim m\pi$ with beam $r$ if $d$ is $\sim n\lambda/2$ since:

At $x_1^+$ beam $f_2$ is out of phase by $-\pi$ with beam $b_1$ by Eq. 4, 7 or S5 for the reflection coefficient $R_M(x_1^+)$ of interface at $x_1^+$.

$$R_M(x_1^+) = \frac{V_{f_2}(x_1^+)}{V_{b_1}(x_1^+)}$$

$$= \frac{Z_0 - Z_M}{Z_M + Z_0} = R_M(x_1^- e^{i\pi}) = |R_M(x_1^-)| e^{i(\pi + \varphi)} \quad (\text{S5})$$

With similar analysis, it is easy to prove that each beam of $t_2$, $t_3$, etc. is out of phase by $\sim m\pi$ with beam $r$ and thus beam $t$ is exactly out of phase with beam $r$ by effectively $\pi$ when $d$ is $\sim n\lambda/2$. Therefore, the minimum of $s_{11}(\text{dB})$ is achieved at $d = -n\lambda/2$ but not at $d = -m\lambda/4$ as specified in the quarter-wavelength model.

1.1.2 For film with $Z_M' < Z_0$

When $Z_M' < Z_0$ beam $r$ and $i$ in Fig. S1a are out of phase by $-\pi$, and beams $b_1$ and $f_1$ are approximately in phase with each other. Following the same argument, beams $t$ and $r$ will be out of phase exactly by $\pi$ if $d$ is $-n\lambda/2$.

The results for $Z_M' > Z_0$ and $Z_M' < Z_0$ for film without metal-back have been presented by rows 1 and 2 in Table 1. All the data measured at different frequencies for Cu@ZnFe$_2$O$_4$ in the data file provided give consistent results to those shown in row 1 in Table 1. Examples have been given in Fig. 2 in the main text.

1.2 Wave addition with phase effects for metal-backed film

1.2.1 For film with $Z_M' > Z_0$

Row 5 in Table 1 provides the result from wave addition for metal-backed film with $Z_M' > Z_0$, in which $RL(\text{dB})$ achieves its minima at $-n\lambda/2$ where beam $r$ and $t$ in Fig. S1b are out of phase by exactly $m\pi$, and its maxima at $-m\lambda/4$ where beam $r$ and $t$ in Fig. S1b are exactly in phase with each other. The result is contrary to that predicted by the quarter-wavelength model as shown below.

At position $x_1^-$ beam $r$ is approximately in phase with beam $i$.

Also, at $x_1^+$ beam $t_1$ is out of phase by $-\pi$ with beam $r$ if $d$ is $-n\lambda/2$ since:

Beam $f_1$ at $x_1^+$ is approximately in phase with beam $i$ at $x_1^+$.

At $x_1 + d$ beam $b_1$ is out of phase by $\pi$ with beam $f_1$ as shown by Eq. S6 which defines the reflection coefficient $R_M(x_1 + d)$ of interface at $x_1 + d$ for metal-backed film.
In Eq. S6 the characteristic impedance $Z_{metal}$ for metal is zero.

At $x_1^-$ beam t1 at $x_1^-$ is approximately in phase with beam b1 at $x_1^-$. The optical distance of $n\lambda/2$ will result in a phase difference of $m\pi$ or an effectively $\pi$ between beams f1 and b1 at $x_1^-$. If $d = m\lambda/4$ as specified in the quarter-wavelength model, then beam t1 would be approximately in phase with beam r at $x_1^-$. Only if $d$ is $-n\lambda/2$ can beam t1 be out of phase by effectively $\pi$ with beam r at $x_1^-$. It can be proved similarly that at $x_1^-$ each beam of t2, t3, etc. is out of phase by effectively $\pi$ with beam r if $d$ is $-n\lambda/2$ since:

At $x_1^-$ beam f2 is out of phase by $\sim\pi$ with beam b1. Similarly, it is easy to prove that each beam of t2, t3, etc. is out of phase by $m\pi$ with beam r and thus beam t is out of phase with beam r by effectively $\pi$ when $d = -n\lambda/2$. Therefore, the minimum of $RL$ is achieved at $d = -n\lambda/2$ but not at $d = m\lambda/4$ as specified in the quarter-wavelength model.

At high frequency for magnetic materials, $Z_{M} > Z_0$. The data measure for Cu@ZnFe$_2$O$_4$ at frequencies higher than 8.14 GHz are consistent with this result which is represented by row 5.

### 1.2.2 For film with $Z_M' < Z_0$

When $Z_M' < Z_0$, beam r and i in Fig. S1b are out of phase by $\sim\pi$, and beams b1 and f1 are out of phase by $\pi$ with each other. Following the same argument, beams t and r are out of phase by exactly $\pi$ if $d$ is $-m\lambda/4$. This is the only result conforms to the quarter-wavelength model.

At low frequency, $Z_M' < Z_0$. The data for Cu@ZnFe$_2$O$_4$ at frequencies lower than 7.97 GHz conform to this result which is represented by row 6. All the data measured at different frequencies for Cu@ZnFe$_2$O$_4$ give consistent results as that shown by row 1.

### 1.3 The positions of minimum reflection loss and phase effect

For the quarter-wavelength mode, it is usually claimed that minima of $RL$ occur when the phase difference between the incident and reflected beams is $\pi$. In fact, return loss $L_R$ for metal-backed film is the negative of reflection loss $RL$ in units of dB and is defined as

$$L_R(dB) = 10 \log_{10} \frac{P_i}{P_b} = 20 \log_{10} \left( \frac{|V_i(x_i^-)|}{|V_b(x_i^-)|} \right) = -RL(dB)$$

where $P_i$ and $P_b$ are the powers of beam i and b in Fig. 1, respectively. But their values are determined by the maximum voltage amplitudes $V_i(x_i^-)$ and $V_b(x_i^-)$ and are independent of the phases of the two beams. The local maximum dissipation powers are characterized by the maxima of $L_R$ or the minima of $RL$ which should be realized by reducing beam b rather than beam $(i + b)$. 

$$R_M(x_i + d^-) = \frac{V_{b1}(x_i + d^-)}{V_{f1}(x_i + d^-)}$$

$$= \frac{Z_{Metal} - Z_M}{Z_{Metal} + Z_M} = -1 = e^{i\pi}$$

(S6)
2. Evaluating $s_{11}$ and $Z_{in}(x_1)$ when $\varepsilon'' = \mu''$

$s_{11}(\text{dB})$ and $Z_{in}(x_1)$ are periodic functions similar to Fig. 2 but without decays when $\varepsilon'' = \mu''$. The related Figures have been omitted in the main text but included here as Fig. S2, $s_{11}(\text{dB})$ achieves its minimum $-\infty$ exactly at $d = n\lambda/2$ where $Z_{in}(x_1) = Z_0$. The values of the minima of $|Z_{in}(x_1)|$ are $|Z_0|$ when $|Z_M| > |Z_0|$ while those maxima of $|Z_{in}(x_1)|$ are $|Z_0|$ when $|Z_M| < |Z_0|$.
Fig. S2 Numerical verification for the conclusion represented by row 2. $s_{11}$(dB) is calculated from Eq. 1 and $Z_{in}(x_1)$ is calculated from Eq. 13 with the values of $\varepsilon_r$ and $\mu_r$ indicated. $Z_m/Z_0 = 1.37$ and $R_{in}(x_1) = -16.13$ dB in (a) and $Z_m/Z_0 = 0.68$ and $R_{in}(x_1) = -14.37$ dB in (b).

3. A derivation of Eq. 1 and 25

It shows here that the phase effects of interfaces have already been included in the derivation of Eq. 1. From Fig. A1b for MBF, the reflection from an interface at $x_i$ is given by

$$V(r, x_i^+) = R_{x_i^+}(x_i^+)V(i, x_i^-)$$

(S8)

The reflection from the interface at $x_i + d$ is therefore given by
\[ V(t, x_i^-) = V(t1, x_i^-) + V(t2, x_i^-) + V(t3, x_i^-) + \ldots + \]
\[ = V(i, x_i^-)\left[ \gamma_M(x_i^-)(-1)^2 \gamma_M(x_i^-)e^{-2j\pi \sqrt{\frac{\mu}{c} d}} \right. \]
\[ + \gamma_M(x_i^-) R_M(x_i^-)(-1)^2 \gamma_M(x_i^-)e^{-2j\pi \sqrt{\frac{\mu}{c} d}} \]
\[ + \left. \gamma_M(x_i^-) R_M^2(x_i^-)(-1)^3 \gamma_M(x_i^-)e^{-2j\pi \sqrt{\frac{\mu}{c} d}} \right] \quad \text{(S9)} \]
\[ = -V(i, x_i^-)[1 - R_M^2(x_i^-)]e^{-4j\pi \sqrt{\frac{\mu}{c} d}} \{ 1 + R_M(x_i^-)e^{-4j\pi \sqrt{\frac{\mu}{c} d}} \}
\[ + [R_M(x_i^-)e^{-4j\pi \sqrt{\frac{\mu}{c} d}}]^2 + \ldots \]}
\[ = -V(i, x_i^-) \frac{[1 - R_M^2(x_i^-)]e^{-4j\pi \sqrt{\frac{\mu}{c} d}}}{1 - R_M(x_i^-)e^{-4j\pi \sqrt{\frac{\mu}{c} d}}} \]

From Eq. S9, Eq. 1 can be obtained as Eq. S10.

\[ RL(x_i^-) = \frac{V(b, x_i^-)}{V(i, x_i^-)} \]
\[ = \frac{V(t, x_i^-)}{V(i, x_i^-)} + \frac{V(t, x_i^-)}{V(i, x_i^-)} \]
\[ = R_M(x_i^-) - \frac{[1 - R_M^2(x_i^-)]e^{-4j\pi \sqrt{\frac{\mu}{c} d}}}{1 - R_M(x_i^-)e^{-4j\pi \sqrt{\frac{\mu}{c} d}}} \quad \text{(S10)} \]

From Eq. S10, Eq. 25 can be obtained.

\[ RL(x_i^-) = \]
\[ = R_M(x_i^-) - \frac{[1 - R_M^2(x_i^-)]e^{-4j\pi \sqrt{\frac{\mu}{c} d}}}{1 - R_M(x_i^-)e^{-4j\pi \sqrt{\frac{\mu}{c} d}}} \]
\[ = R_M(x_i^-) - \frac{[1 - R_M^2(x_i^-)]e^{-4j\pi \sqrt{\frac{\mu}{c} d}}}{1 - R_M(x_i^-)e^{-4j\pi \sqrt{\frac{\mu}{c} d}}} \quad \text{(S11)} \]
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Transmission Line And Free Space Method 15.0.15051201
Agilent Tec E5071C MY461120 A.09.62

Ch 1
Ports: 1 & 2
Measurements: Reflection/Transmission Mu and Epsilon
Sample Hc: Coax/TEM
Sample Hc: 0.000000 mm
Distance tc: 36.890000 mm
Sample Th: 2.080000 mm
IFBW: 30.000000 Hz
Power: -5.000000 dBm
Date: Tuesday, November 06, 2018 13:48:29

| Frequency (GHz) | e' | e'' | u' | u'' |
|----------------|----|-----|----|-----|
| 1E+09          | 2.6363 | 0.006 | 1.2187 | 0.0645 |
| 1.17E+09       | 2.63 | -0.0027 | 1.2083 | 0.0734 |
| 1.34E+09       | 2.6208 | -0.0008 | 1.2024 | 0.0683 |
| 1.51E+09       | 2.6194 | -0.0049 | 1.1864 | 0.0701 |
| 1.68E+09       | 2.6099 | -0.0122 | 1.1764 | 0.0719 |
| 1.85E+09       | 2.5931 | -0.0166 | 1.1721 | 0.0737 |
| 2.02E+09       | 2.575 | -0.0242 | 1.1708 | 0.0761 |
| 2.19E+09       | 2.5509 | -0.0232 | 1.1767 | 0.0725 |
| 2.36E+09       | 2.5322 | -0.0314 | 1.1795 | 0.077 |
| 2.53E+09       | 2.5132 | -0.032 | 1.1852 | 0.0753 |
| 2.7E+09        | 2.5071 | -0.0384 | 1.1778 | 0.0794 |
| 2.87E+09       | 2.5005 | -0.0358 | 1.1749 | 0.0758 |
| 3.04E+09       | 2.5025 | -0.0376 | 1.1655 | 0.0764 |
| 3.21E+09       | 2.5008 | -0.0357 | 1.1656 | 0.0747 |
| 3.38E+09       | 2.5002 | -0.0341 | 1.1704 | 0.0757 |
| 3.55E+09       | 2.4989 | -0.0324 | 1.18 | 0.0758 |
| 3.72E+09       | 2.5061 | -0.0273 | 1.1862 | 0.0734 |
| 3.89E+09       | 2.5244 | -0.0241 | 1.1821 | 0.0722 |
| 4.06E+09       | 2.5451 | -0.0184 | 1.1796 | 0.0704 |
| 4.23E+09       | 2.5715 | -0.0145 | 1.1725 | 0.0714 |
| 4.4E+09        | 2.5924 | -0.0044 | 1.1759 | 0.0694 |
| 4.57E+09       | 2.6107 | -0.0004 | 1.182 | 0.0706 |
| 4.74E+09       | 2.6209 | 0.0075 | 1.1953 | 0.0716 |
| 4.91E+09       | 2.6344 | 0.0115 | 1.2067 | 0.0761 |
| 5.08E+09       | 2.6477 | 0.0183 | 1.2184 | 0.0793 |
| 5.25E+09       | 2.6689 | 0.022 | 1.2268 | 0.085 |
| 5.42E+09       | 2.6872 | 0.0289 | 1.2365 | 0.0938 |
| 5.59E+09       | 2.7001 | 0.0319 | 1.2587 | 0.1103 |
| 5.76E+09       | 2.7077 | 0.0346 | 1.2886 | 0.1346 |
| 5.93E+09       | 2.7108 | 0.0393 | 1.3321 | 0.17 |
| 6.1E+09        | 2.7154 | 0.0354 | 1.3778 | 0.2316 |
| 6.27E+09       | 2.7227 | 0.0365 | 1.4252 | 0.332 |
| 6.44E+09       | 2.7449 | 0.0241 | 1.4239 | 0.5104 |
| 6.61E+09       | 2.7775 | 0.0166 | 1.2799 | 0.7495 |
| 6.78E+09       | 2.8254 | 0.0118 | 0.9044 | 0.8367 |
| 6.95E+09       | 2.8687 | 0.0364 | 0.6251 | 0.6029 |
| 7.12E+09       | 2.8894 | 0.0547 | 0.5978 | 0.3556 |
| 7.29E+09       | 2.9053 | 0.076 | 0.6576 | 0.2076 |
| 7.46E+09       | 2.9238 | 0.0905 | 0.7214 | 0.1275 |
| 7.63E+09       | 2.955 | 0.1066 | 0.7692 | 0.0842 |
| 7.8E+09        | 2.9791 | 0.1216 | 0.8064 | 0.0566 |
| 7.97E+09       | 3.0058 | 0.1308 | 0.8284 | 0.0427 |
| 8.14E+09       | 0.8532 | 0.0289 | 3.0123 | 0.1479 |
| 8.31E+09       | 0.8743 | 0.0253 | 3.0149 | 0.146 |
| x          | y    | z    | a       | b       |
|------------|------|------|---------|---------|
| 8.48E+09   | 0.8983 | 0.014 | 3.0004  | 0.1599  |
| 8.65E+09   | 0.9166 | 0.014 | 2.9933  | 0.148   |
| 8.82E+09   | 0.9268 | 0.0053 | 2.9935  | 0.1626  |
| 8.99E+09   | 0.9347 | -0.0002 | 2.9858  | 0.1477  |
| 9.16E+09   | 0.9363 | -0.0018 | 2.9566  | 0.1324  |
| 9.33E+09   | 0.9439 | -0.0005 | 2.9476  | 0.1128  |
| 9.5E+09    | 0.9483 | -0.0092 | 2.8896  | 0.1164  |
| 9.67E+09   | 0.9702 | 0.014 | 2.8958  | 0.0854  |
| 1.02E+10   | 0.9771 | -0.003 | 2.8672  | 0.0531  |
| 1.04E+10   | 0.9944 | 0.0023 | 2.8298  | 0.0874  |
| 1.05E+10   | 0.9791 | 0.0031 | 2.8391  | 0.0183  |
| 1.07E+10   | 1.0128 | -0.0288 | 2.7785  | 0.0795  |
| 1.09E+10   | 1.0053 | -0.0006 | 2.7794  | -0.0094 |
| 1.1E+10    | 1.0327 | -0.008 | 2.7481  | 0.0684  |
| 1.12E+10   | 1.0527 | 0.0265 | 2.7284  | -0.0239 |
| 1.14E+10   | 1.04  | 0.0027 | 2.7313  | 0.0887  |
| 1.15E+10   | 1.0404 | 0.0791 | 2.6724  | -0.0278 |
| 1.17E+10   | 1.0329 | 0.0041 | 2.6545  | 0.1011  |
| 1.19E+10   | 1.0206 | 0.0761 | 2.6046  | -0.0403 |
| 1.21E+10   | 1.0423 | 0.0034 | 2.5238  | 0.0771  |
| 1.22E+10   | 1.032 | 0.0526 | 2.4815  | -0.0438 |
| 1.24E+10   | 1.068 | 0.0136 | 2.4174  | 0.0373  |
| 1.26E+10   | 1.0478 | 0.0438 | 2.3862  | -0.0326 |
| 1.27E+10   | 1.0634 | 0.0164 | 2.3475  | 0.0048  |
| 1.29E+10   | 1.054 | 0.0224 | 2.298   | -0.0329 |
| 1.31E+10   | 1.0619 | 0.0075 | 2.2673  | -0.0216 |
| 1.32E+10   | 1.0847 | 0.0121 | 2.2118  | -0.0356 |
| 1.34E+10   | 1.0945 | 0.0055 | 2.1946  | -0.0353 |
| 1.36E+10   | 1.1242 | 0.0037 | 2.1557  | -0.0434 |
| 1.38E+10   | 1.1243 | 0.0064 | 2.1636  | -0.0496 |
| 1.39E+10   | 1.1342 | 0.0055 | 2.1632  | -0.0614 |
| 1.41E+10   | 1.1249 | 0.0252 | 2.1782  | -0.0809 |
| 1.43E+10   | 1.1189 | 0.0175 | 2.2164  | -0.0791 |
| 1.44E+10   | 1.1013 | 0.0457 | 2.2576  | -0.1132 |
| 1.46E+10   | 1.1192 | 0.0289 | 2.2696  | -0.0931 |
| 1.48E+10   | 1.1185 | 0.0423 | 2.3259  | -0.1134 |
| 1.49E+10   | 1.1284 | 0.0351 | 2.3676  | -0.0899 |
| 1.51E+10   | 1.1278 | 0.0328 | 2.4236  | -0.0883 |
| 1.53E+10   | 1.122 | 0.0259 | 2.4677  | -0.053  |
| 1.55E+10   | 1.1271 | 0.0287 | 2.485  | -0.0452 |
| 1.56E+10   | 1.1219 | 0.0192 | 2.5004  | -0.026  |
| 1.58E+10   | 1.1472 | 0.0163 | 2.4782  | -0.0208 |
| 1.6E+10    | 1.1631 | 0.0101 | 2.4756  | -0.012  |
| 1.61E+10   | 1.1976 | 0.0112 | 2.4584  | -0.0108 |
| 1.63E+10   | 1.2219 | 0.0132 | 2.4742  | -0.0065 |
| 1.65E+10   | 1.2644 | 0.0158 | 2.4785  | -0.0004 |
| 1.66E+10   | 1.3302 | 0.0312 | 2.4967  | 0.0017  |
| 1.68E+10   | 1.506 | 0.09 | 2.4756  | 0.0019  |
| 1.7E+10    | 2.0676 | 0.946 | 2.2979  | -0.234  |
| 1.72E+10   | 0.3631 | 0.4099 | 2.8855  | -0.0602 |
| 1.73E+10   | 0.7503 | 0.1056 | 2.8084  | 0.0317  |
| 1.75E+10   | 0.8798 | 0.0678 | 2.8397  | 0.0644  |
| 1.77E+10   | 0.9428 | 0.0796 | 2.899  | 0.0906  |
| 1.78E+10   | 0.9179 | 0.138 | 2.9677  | 0.1093  |
| 1.8E+10    | 0.8497 | 0.0972 | 3.0034  | 0.0865  |