A Fast Algorithm of Direct Position Determination Using TDOA and FDOA

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Abstract. Weiss’s Direct position determination (WDPD) method compensate the received signals of the M stations to the transmitted signals by the Delay-Doppler operator, and Delay-Doppler operator is computed M times to compensate the time delay and Doppler shift for every search point, and the dimension of the Delay-Doppler operator is related to the size of data points N. And Delay-Doppler operator is equal to four matrices multiplying continuously, whose size is N×N, and matrix multiplication needs lots of time and memory, limiting the size of N. This paper deduces the DPD that don’t need to compensate to the transmitted signals. In practice, any one of M stations maybe is the reference station, and the signal of the reference station can compensate to the receiving signals of other stations by Delay-Doppler operator, and the objective function of the DPD is given. Through the DPD, Delay Doppler operator only is calculated M-1 times for every search point, and reduce the computation complexity, especially M is small. And for DPD algorithm, there are a lot of redundant computations in the Delay Doppler operator, so the fast algorithm of DPD (FDPD) is proposed. The experimental results verify that DPD reduce the computation complexity and FDPD saves the computation time and maintain the performance of the DPD algorithm simultaneously.

1. Introduction

In passive localization, the two-step approach is widely applied in radar, sonar, remote sensing and microphone arrays. But the time differences of arrival (TDOA) and the frequency difference of arrival (FDOA) among spatially separated stations are measured firstly, in the second step, the emitter position may be estimated by the classical iterative process or other ways [1]. However, the two-step approach usually performs badly when the emitter signal is narrow bandwidth or short observation time. A novel one-step location method, WDPD, has been proposed by Weiss in [2] for its high positioning accuracy and advantages over two-step approach. And WDPD quickly attracts hundreds of scholars and researchers’ attention[3]. The performance of WDPD is researched and compare with the two-step approach[4-5]. Following upon, the WDPD is extended to multiple emitters localization [6]. In [7], the WDPD method is applying wideband sources localization based on the time delay and Doppler according to the maximum likelihood estimation (MLE), and a closed-form of the Cramer-
Rao lower bound is deduced. Recently, WDPD also is applied in tracking of a moving emitter using receivers salted on stationary or moving platforms is considered[8-9]. What’s more, location is applied in artificial intelligence (AI), such as location of moving and unmoving people behind obstacles and so on[10-11]. So WDPD will be extended to AI in the future. The semidefinite programming method also can be used in location [12-14], transforming the nonconvex MLE problem into a convex optimization problem.

The remainder of this paper is organized as follows. In this paper, firstly, we describe the location problem in Section 2. Then we present the DPD that don’t need to compensate to the transmitted signals in Section 3, and the object function is given. In Section 4, the fast version of DPD algorithm is presented. Computation complexity of FDPD, DPD and WDPD are compared in Section 5. The experimental results verify that FDPD saves the computation time and maintain the performance of the DPD simultaneously in Section 6. A conclusion is given in Section 7.

2. Problem Description
Any one of the receiver is the reference receiver, reference receiver position vectors is $B_0 = (x_0, y_0, z_0)^T$ and speed vectors is $\dot{B}_0 = (v_{x_0}, v_{y_0}, v_{z_0})^T$, the other receiver position is $B_i = (x_i, y_i, z_i)^T$ and the speed is $\dot{B}_i = (v_{x_i}, v_{y_i}, v_{z_i})^T$, $i = 1, 2, \ldots, M - 1$, they are known. The location of the emitter is $P = (x, y, z)^T$ and the speed is $\dot{P} = (v_x, v_y, v_z)^T$, when the emitter is a fixed target, $\dot{P} = (0, 0, 0)^T$. A set of $M$ TDOA $\tau_i$ between receiver pair $i$ and $0$.

$$\tau_i = \frac{\|B_i - P\|}{c} - \frac{\|B_0 - P\|}{c}$$

The constant $c = 3 \times 10^8 \text{m/s}$ denotes the propagation velocity of radio signal. $\|\|$ denotes Euclidean norm.

A set of $M$ FDOA $f_d$ between receiver pair $i$ and $0$.

$$f_d = \frac{f_c}{c} \left[ \frac{(\dot{B}_i - \dot{P})^T (B_i - P) - (\dot{B}_0 - \dot{P})^T (B_0 - P)}{\|B_i - P\|} \right]$$

$(\cdot)^T$ stands for the vector transpose, $f_c$ is carrier frequency. There is an assumption that the time difference $\tau_i$ can be considered constant during the observation gap, so $\tau_i \ll T$ (observation time $T$).

It is generally known that the emitter position can be estimated by TDOA and FDOA. However, this approach usually performs badly when the emitter signal is narrow bandwidth or short observation time. And high positioning accuracy is necessary by direct position determination method.

3. DPD
The emitter signal is a narrowband signal with a bandwidth $B$. And sampling frequency is $f_s$. The observation time is $T$, and the signal length is $N = f_s T$ (sampling points). Therefore, the receiving signal of the reference receiver denotes $x_0 \in \mathbb{R}^{N \times 1}$, other receiving signal is $x_i \in \mathbb{R}^{N \times 1}$.

$$x_i = \mu D x_0 + n_i$$

$n_i \in \mathbb{R}^{N \times 1}$ is Complex Gaussian White Noise (CWGN) with mean of 0 and variance of $\sigma^2$ ($\sigma^2$ unknown), and is denoted as $n_i$ satisfying $CN(0_N, \sigma^2 I_N)$, $0_N \in \mathbb{R}^{N \times 1}$ is full zero vector, $I_N \in \mathbb{R}^{N \times N}$ is unit matrix, and the signal and noise are independent. $\mu$ stands for the ratio of an unknown complex scalar between receiver pair $i$ and 0, representing the channel effect (attenuation). Delay-Doppler
operator $D_i = D(r_i, f_{d_i}) \in R^{N_x \times N}$ is with time delay and Doppler shift between receiver pair $i$ and 0. Delay-Doppler operator $D(r, f_d)$ is given by

$$D(r, f_d) = D_N(f_d/f_s) W^H D_N(−r f_s/N) W$$  \hspace{1cm} (4)

($\cdot^H$ is vector conjugate transposition. $D_N(x) = \text{diag}(e^{p \pi (0)^{i}}, \ldots, e^{p \pi (N−1)^{i}}) \in R^{N_x \times N}$ is the diagonal matrix, $p = \sqrt{−1}$ is the imaginary unit and $W \in R^{N_x \times N}$ is the Discrete Fourier Transform (DFT) matrix. The element of the $i$th row $j$th column is given by

$$[W]_{m,n} = \frac{1}{N} e^{-\frac{\pi}{N}(m-n)^{2}}$$  \hspace{1cm} (5)

Notice that $W$ is the unitary matrix (satisfying $W^H W = I_N$). $D_N(x)$ is the diagonal matrix, and the elements on the diagonal line are all exponential functions.

According to (6), the Delay Doppler operator is the unitary matrix.

$$D^H(r, f_d) D(r, f_d) = I_N$$  \hspace{1cm} (6)

According to the deterministic signal model, signal and noise are independent. So $x_i$ satisfies Joint complex Gaussian distribution. The Joint conditional probability density function is

$$p(X) = \prod_{i=1}^{M+1} p(x_i)$$  \hspace{1cm} (7)

$$p(x_i) = \frac{1}{\pi^N |R_N|} \exp\left(-\left(x_i - \mu_i D_i x_0\right)^H R_{−1}^{-1} \left(x_i - \mu_i D_i x_0\right)\right)$$  \hspace{1cm} (8)

We define

$$X = \left[(x_1)^{T}, \ldots, (x_{M+1})^{T}\right]^{T} \in R^{(M+1) \times N}$$  \hspace{1cm} (9)

And Noise covariance matrix is $R_N = \mathbb{E}\{n n^H\} \in R^{N_x \times N}$. $n_i$ satisfies $CN(0_N, \sigma_i^2 I_N)$, we can rewrite (8) as

$$p(X) \propto \exp\left(-\frac{1}{\sigma^2} \sum_{i=1}^{M+1} \|x_i - \mu_i D_i x_0\|^2\right)$$  \hspace{1cm} (10)

And $A = \frac{1}{\left(\pi \sigma^2\right)^{M+1} N} \cdot$

First, $\mu_i$ needs to be estimated. From (11), we can know that MLE is equivalent to the least square estimator in Gaussian noise. $\mu_i$ is considered unknown, the MLE of $\mu_i$ satisfies by minimizing $\|x_i - \mu_i D_i x_0\|^2$.

So, the MLE of $\mu_i$ is given by

$$\mu_i = \frac{x_i^H \left(D_i(p)^H x_i\right)}{\|x_0\|^2}$$  \hspace{1cm} (11)

$D_i(p)$ is the estimation of $D_i$. According to (8), Delay-Doppler operator is a unitary matrix and its Euclidean norms are unitary invariant norms.
\[ p(X) = A \exp \left( -\frac{1}{\sigma^2} \sum_{i=1}^{M-1} \| x_i - \hat{x}_{i0} \|^2 \right) \]  

(12)

\[ \hat{x}_{i0} \] is defined by

\[ \hat{x}_{i0} = (D_i(p)x_0)^H \frac{x_i}{\| x_i \|^2} D_i(p)x_0 \]  

(13)

\[ \hat{x}_{i0} \] can be regarded as the MLE of the receiving signal of the \( i \)th receiver through compensation or removal of time delay and Doppler shift. Directly using waveform fitting, the objective function is transformed to minimum \( \sum_{i=1}^{M-1} \| x_i - \hat{x}_{i0} \|^2 \), and is shown as

\[ \arg \min_p \sum_{i=1}^{M-1} \| x_i - \hat{x}_{i0} \|^2 \]  

(14)

The emitter location is corresponding to the search point to make objective function minimize.

4. FDPD

Delay-Doppler operator of the DPD and WDPD is in time domain. In other words, the receiving signal is transformed into frequency domain for time delay compensation, and then transformed into time domain for frequency shift compensation. However, the dimension of Delay-Doppler operator of the DPD algorithm is related to the number of data points \( N \). Delay-Doppler operator is equal to four matrices multiplying continuously, whose size are \( N \times N \). Matrix multiplication needs a large amount of calculation, limiting the size of \( N \) and the application of direct position determination method. Aiming at this problem, the fast version of DPD is proposed.

\[ D_n(x) \] is a diagonal matrix, so we transform matrix multiplication into two vectors directly dot multiplication. Fast Fourier Transform (FFT) is used to transform the receiving signal into frequency domain to compensate the delay, and then Inverse Fast Fourier Transform (IFFT) is used to transform the receiving signal compensated time delay into time domain to compensate the frequency shift. Finally, the receiving signal compensated time delay and frequency shift still be in time domain. The vector \( x_n \in R^{N \times 1} \) is transformed into the frequency domain by FFT and the size of the vector still is \( N \times 1 \), \( D_n(-\tau, f_s / N) \) is a diagonal matrix of \( N \times N \), and the diagonal elements are taken out to form a vector \( U_n(-\tau, f_s / N) \).

\[ U_n(x) = \begin{bmatrix} 1 & e^{2\pi i x} & \ldots & e^{2\pi i (N-1) x} \end{bmatrix} \] is defined. By two vectors point multiplication can be used to compensate time-delay in frequency domain. Similarly, the receiving signal compensated time-delay transform into the time domain by IFFT. \( D_n(f_a / f_s) \) is a diagonal matrix and its size is \( N \times N \), and the diagonal elements of \( D_n(f_a / f_s) \) are extracted directly to form a vector \( U_n(f_a / f_s) \). The frequency shift can be compensated by the two vectors point multiplication in the time domain.

5. Comparison of computational complexity between FDPD , DPD and WDPD

The number of data points of receiving signals in each slot is \( N \), the computation complexity of each Delay-Doppler operator is \( N^3 \) (the size of matrix is \( N \times N \), so computation complexity of two matrices with same size \( N \times N \) is \( N^3 \) by matrix multiplication). WDPD that the receiving signal of each station returning to the transmitting signal is proposed by Weiss, so for every search point, Delay-Doppler operator is computed \( M \) times. DPD algorithm proposed in this paper only computes the \( M-1 \) times Delay-Doppler operator for every search point, which reduces the computation complexity. On this basis, the FDPD algorithm is proposed and the computation complexity by utilizing the characteristics of diagonal matrix. The computation complexity of each Delay-Doppler operator is \( N \log N \) by FDPD, because computation complexity of a vector (whose size is \( N \)) by FFT or IFFT is \( N \log N \).
6. Simulation

6.1. Scenario

\( M=2 \) is the number of stations and two aircrafts in the air receive signals from fixed emitter on the earth, so \( z=0 \) and the emitter is a fixed target, so \( \mathbf{P}=(0,0,0)^T \).

The problem about fixed emitter on the earth is a two-dimensional search. The position of the fixed emitter is \( \mathbf{P}=(50\text{km},30\text{km},0\text{km})^T \). The two stations fly parallel to the Y axis and their positions and speeds are shown in Table 1. And the station 1 is the reference station. The baseline length \( D \) is 20km, and the distance \( R \) is 58.3km between the baseline center and emitter. Search area is \((0\text{km},100\text{km})\times(0\text{km},100\text{km})\), the search step is 2.5km×2.5km. The carrier frequency of emitter is 2 GHz, and the bandwidth \( B \) is 200KHz. The signal observation time is 4ms, and sample frequency is 400KHz.

In all the simulation, the root mean square error (RMSE) of position estimation is used as the metrics of measurement, which is defined by

\[
\text{RMSE} = \sqrt{\frac{1}{L} \sum_{j=1}^{L} \left| \mathbf{P}_j - \mathbf{P} \right|^2}
\]  

(15)

Where \( L=1000 \) is the number of Monte Carlo trails, \( \mathbf{P}_j \) is the emitter position estimate in the \( j \)th trial.

| Station 1 | x(km)  | y(km)  | z(km)  | Vx(m/s) | Vy(m/s) | Vz(m/s) |
|-----------|--------|--------|--------|---------|---------|---------|
| Station 2 | 10.000 | 0.000  | 10.000 | 0.000   | 200.000 | 0.000   |

6.2. Result

To analyse the performance of WDPD, DPD and FDPD, we compare the correlation spectrum (correlation between true position and estimated position of emitter) of three algorithms at the same Signal-to-noise ratio (SNR) and SNR=20dB. The peak of correlation spectrum is corresponding to the emitter position. According to Figure 1, it’s obvious that the correlation spectrums of FDPD are sharp. Combined with Figure 1, it verifies that DPD reduce the computation complexity and FDPD saves the computation time and maintain the performance of the DPD algorithm simultaneously. The performance of WDPD, DPD and FDPD is analysed at different SNRs. As the Figure 2 shown, the positioning error of three algorithms steady decline with the SNR increasing. And the positioning accuracy of three algorithms is similar at the same SNR.
7. Conclusion
In the paper, we present the DPD that don’t compensate to the transmitted signals and the fast algorithm of DPD is also proposed, reducing the computation complexity. The experimental results verify that FDPD saves the computation time and maintain the performance of the DPD algorithm simultaneously. FDPD don’t limit the size of N and DPD’s application in other occasions, and FDPD makes direct position determination method available for other application, such as AI, anti-interference, tracking and position prediction.

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