Integrated Modelling and Static Experiment of a Novel Tandem Ducted Fan Flying Robot

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Abstract. This paper puts forward a novel tandem ducted fan flying robot which has nice passivity and compact structure aiming at operating in complicated and dangerous environment. The structure of its ducted coaxial twin-rotor with reverse rotation is unconventional and make its aerodynamic characteristics much more complex. To solve the problem, the modelling method of the traditional open rotors with the blade element theory and the momentum theory is extended to the ducted coaxial twin-rotor structure, and the integrated dynamic model of the ducted fan flying robot is established. After the modelling, a static lift experiment has been carried out, and the aerodynamic model of the ducted coaxial twin-rotor structure is modified based on the test results.

1. Introduction

Nowadays, autonomous unmanned flying robots have become hotspots of production and research [1]. On the one hand, the highly developed autonomous flight control technology reduces the difficulty of operating the flying robot and reduces its operating costs for production; on the other hand, more and more researchers choose unmanned flying robots as the research object of advanced technology because of their superior performance.

With the rapid development of society and technology, people's expectations for unmanned flying robots are getting higher and higher. For example, one might expect an unmanned aerial vehicle to enter an indoor environment, or to replace or assist related personnel in other more complex work environments [2-3]. This makes the operating environment of unmanned flying robots more and more challenging. At present, relatively mature unmanned flying robots use lift structures of fixed-wings or open rotors. The fixed-wing unmanned aerial robot is suitable for relatively long-term high-speed conditions, but it cannot hover, which makes it less functional in complex environments. Open-rotor unmanned aerial robots, such as helicopters, rotorcraft and multi-axis aircraft, are the most common and widely used unmanned flying robots on the market. However, the innate structure of the open rotor greatly reduces the adaptability of this type of flying robot to complex operating environments, and the relatively poor safety has become a major limitation of this type of flying robot in actual working conditions.

In order to improve the applicability of unmanned flying robots in complex operating environments, ducted fan unmanned aerial robots have become a better choice. Compared with the traditional open-rotor flying robot, the ducted fan flying robot has many unique advantages, including good safety,
higher load capacity, and smaller structure size under the same conditions [4]. These advantages make the ducted fan flying robot capable of operating in an unknown, complex, and dangerous environment.

The multi-domain vehicle research group of Beijing Institute of Technology has been exploring for a long time in the research of ducted fan flying robots. The development and iteration of prototypes have also been carried out on the structure of the ducted fan flying robot. Whether the versatile robot can obtain good work quality, in addition to the advanced controller structure and algorithm, has an important relationship with the robot's own configuration and layout.

Here we introduce a novel tandem ducted fan flying robot shown in figure 1 which has nice passivity and compact structure aiming at operating in complicated environment and narrow space. It is mainly composed of a lift system, rudders, landing gears, and a control system. The lift system consists of two ducted fans with coaxial rotors which rotate in opposite directions in the same duct. The pitch of the ducted fan flying robot is achieved by the speed difference of the front and rear ducted rotors. The roll is achieved by swinging the rudders. The yaw is achieved by the speed difference of the two coaxial rotors in the same duct.

![Figure 1. The novel tandem ducted fan flying robot](image-url)

The research object of this paper is the ducted coaxial twin-rotor structure. At present, the aerodynamic model research of the ducted coaxial twin-rotor structure is not rich enough. In the paper [5], the aerodynamic model of the open coaxial countersrotating rotors is analyzed by using the blade element momentum theory (BEMT). In the paper [6], the Froude conservation theory is applied to calculate the aerodynamic characteristics of the ducted single-rotor structure. In this paper, the methods in the above articles are combined and improved, and the BEMT modelling method is extended to the ducted coaxial twin-rotor structure.

2. Integrated modelling

2.1. Rigid-body Kinematic and dynamic model

The coordinate systems of the flying robot are shown in figure 1. $O-x_y_z$ is the earth coordinate system $C_e$. $C_b-x_b y_b z_b$ is the body coordinate system $C_b$ and its origin is located at the center of gravity of the flying robot. $O_h-x_h y_h z_h$ is the hub coordinate system and it is based on the rotation center of the rotor. $O_r-x_r y_r$ is the rudder coordinate system and its origin is located at the swing center of the main rudder. The hub coordinate system and the rudder coordinate system are fixedly
connected to $C_b$, and move along with $C_b$ with respect to $C_e$. $C_e$ can be converted to $C_b$ by the
standard Euler rotation and translation. The transformation matrix from $C_e$ to $C_b$ is

$$
R_e^b = \begin{bmatrix}
\cos \theta \cos \psi & \cos \theta \sin \psi & -\sin \theta \\
\sin \theta \cos \psi \sin \phi - \sin \psi \cos \phi & \sin \theta \sin \psi \sin \phi + \cos \psi \cos \phi & \cos \theta \sin \phi \\
\sin \theta \cos \psi \cos \phi + \sin \psi \sin \phi & \sin \theta \sin \psi \cos \phi - \cos \psi \sin \phi & \cos \theta \cos \phi
\end{bmatrix}
$$ (1)

We define the position and attitude vector of the flying robot in $C_e$ as

$$
P = [x \ y \ z]^T \\
\Phi = [\phi \ \theta \ \psi]^T
$$ (2)

and the velocity and angular velocity vector of the flying robot in $C_b$ as

$$
V = [U \ V \ W]^T \\
\Omega = [p \ q \ r]^T
$$ (3)

The kinematic model of the flying robot can be described as

$$
\begin{bmatrix}
V \\
\Omega
\end{bmatrix} = W\Phi
$$ (4)

where $W$ is given as

$$
W = \begin{bmatrix}
1 & 0 & -\sin \theta \\
0 & \cos \phi & \cos \theta \sin \phi \\
0 & -\sin \phi & \cos \theta \cos \phi
\end{bmatrix}
$$ (5)

The dynamic model of the flying robot can be described by Newton-Euler equations as

$$
\begin{bmatrix}
\dot{V} = -\Omega \times V + \frac{1}{m}F \\
\dot{\Omega} = I^{-1}(M - \Omega \times I \times \Omega)
\end{bmatrix}
$$ (6)

where $m$ and $I$ are the total mass and inertia, $F$ and $M$ are the resultant force and moment of
the flying robot. $F$ includes the forces generated by the rotors and rudders, additional lift of the duct, and
the gravity. The moments generated by rotors, rudders and ducts constitute $M$.

2.2. Duct thrust model
The structures of two ducts of the flying robot are identical, and the front duct is selected as the
research object. The modelling process of the rear duct is the same as that of the front duct, so its
derivation process is not described. The aerodynamic characteristics of the ducted rotors are very
complicated. In order to simplify the modelling and derivation process, we first give the assumption
that the whole structure works in the axial flow condition, and the airflow is the ideal fluid, and the
airflow state does not change with time [7-8].

2.2.1. Duct inlet lift model. Apply the momentum theorem to the gas at the inlet region of the duct:

$$
F_{in} + P_{in} S_{in} - P_{dr1} S_1 = \rho V_{in} S_{in} - \rho V_{dr1} S_{dr1}
$$ (7)

where $S_{in}$ and $S_1$ are the area of the duct entrance and the upper rotor disc respectively, $P_{in}$ and $V_{in}$ are
the air pressure and airflow velocity at the duct entrance, $P_{dr1}$ is the air pressure at the upper surface
of the upper rotor disc, $\rho$ is the air density, and $V_i$ is the induced velocity of the air at the upper rotor. Now we define the equivalent inlet ratio $h_i$ and outlet ratio $h_e$ of the ducted rotors as

$$
\begin{align*}
    h_i &= \frac{S_{in}}{S_1} \\
    h_e &= \frac{S_{out}}{S_2}
\end{align*}
$$

where $S_{out}$ and $S_2$ are the area of the duct outlet and the lower rotor disc respectively. Selecting the duct inlet and upper surface of the upper rotor disc as the control surface, the Bernoulli equation is

$$
P_{in} + \frac{1}{2} \rho V_{in}^2 = P_{dual} + \frac{1}{2} \rho V_{i}^2
$$

Including the pressure outside the duct, the net lift of the duct inlet is

$$
F_{in1} = F_{in} + P_{in}(S_{in} - S_1) = \frac{1}{2} \rho S_1 \left( \frac{h_i}{h} - 1 \right)^2 V_{i}^2
$$

2.2.2. Duct outlet resistance model. The derivation of the duct outlet resistance is similar to that of the duct inlet lift, so the net resistance of the duct outlet is obtained as

$$
F_{out1} = F_{out} - P_{out}(S_{out} - S_2) = -\frac{1}{2} \rho S_2 \left( \frac{h_e}{h} - 1 \right)^2 (V_{i} + V_{r2})^2
$$

where $V_{r2}$ is the induced velocity of the air at the lower rotor.

2.2.3. Thrust of the front duct. Combining the inlet lift and outlet resistance of the duct, we get the thrust of the front duct

$$
F_{d1} = F_{in1} + F_{out1}
$$

2.3. Coaxial twin-rotor model

2.3.1. Upper rotor model. The momentum theorem model of the upper rotor characterizes the rotor’s lift as the pressure difference generated by the pressure jump between the upper and lower surfaces of the rotor. Therefore, it is only need to obtain the upper and lower surface pressure of the rotor to obtain the rotor lift. The pressure difference between the upper and lower surfaces of the upper rotor is obtained by using Bernoulli’s equation and mass conservation theory:

$$
\Delta P_{up1} = \frac{1}{2} \rho V_{i}^2
$$

In order to facilitate subsequent derivation, we define the differential element $dx_i$ of the blade distributed along the blade extension in the plane of the rotor disc, as shown in Figure 2.
Taking $dx_h$ as the calculation object, the lift of the blade element ring is

$$dF_i = \Delta P_{up} dS_i = \pi \rho V_i^2 x_i dx_h$$  \hspace{1cm} (14)$$

Here we define the equivalent radius $r_1$ and $r_2$ of the rotor and the inflow ratio $\lambda_1$, $\lambda_2$ and $\lambda$:

$$\begin{align*}
  r_1 &= \frac{x_h}{R_1}, r_2 = \frac{x_h}{R_2} \\
  \lambda_1 &= \frac{V_{in} + V_{out}}{\omega_1 R_1}, \lambda_2 = \frac{V_{in} + V_{out}}{\omega_2 R_2}, \lambda &= \frac{V_{in}}{\omega_1 R_2} \\
\end{align*}$$  \hspace{1cm} (15)$$

where $R_1$ and $R_2$ are the radius of the upper and lower rotor, $\omega_1$ and $\omega_2$ are the rotating speed of the upper and lower rotor. The dimensionless lift of the blade element ring can be re-expressed as

$$dC_{F1} = \frac{dF_i}{\pi \rho R_1^2 (\omega_1 R_1}^2 = \lambda_1^2 r_i dr_i$$  \hspace{1cm} (16)$$

The dimensionless lift and torque of the upper rotor can also be calculated by using the blade element theory:

$$\begin{align*}
  dC_{F1} &= \frac{1}{2} \sigma \left[ C_{l1i} (\theta_1 r_i^2 - \lambda_1 r_i) - C_{l1i} \lambda r_i \right] dr_i \\
  dC_{M1} &= \frac{1}{2} \sigma \left[ C_{l1i} r_i^2 + C_{l1i} \lambda (\theta_1 r_i - \lambda_1) \right] r_i dr_i \\
\end{align*}$$  \hspace{1cm} (17)$$

where $C_{l1}$ is the drag coefficient, $C_{l1i}$ is the slope of the lift characteristic curve, $\theta_1$ is the mounting angle of the upper rotor, and $\sigma$ is the rotor solidity. Now we derive the aerodynamic model of the upper rotor based on the momentum theorem, and obtain a set of relationship between rotor lift and inflow ratio as expressed in equation (16). Another set of relationship between rotor lift and inflow ratio based on the blade element theory is shown in equation (17). Combining these two sets of relationships, we get a general solution for the equivalent inflow ratio of the upper rotor disc:

$$\lambda_1 = \frac{1}{4} \sigma C_{l1i} \left[ \left( 1 + \frac{8 \theta_1 r_i}{\sigma C_{l1i}} \right)^{1/2} - 1 \right]$$  \hspace{1cm} (18)$$

Then the dimensioned lift and torque of the upper rotor can be calculated by

$$\begin{align*}
  F_1 &= \pi \rho R_1^2 (\omega_1 R_1}^2 \int_{r_1}^{r_2} \frac{1}{2} \sigma \left[ C_{l1i} (\theta_1 r_i^2 - \lambda_1 r_i) - C_{l1i} \lambda r_i \right] dr_i \\
  M_1 &= \pi \rho R_1^3 (\omega_1 R_1}^3 \int_{r_1}^{r_2} \frac{1}{2} \sigma \left[ C_{l1i} r_i^2 + C_{l1i} \lambda (\theta_1 r_i - \lambda_1) \right] r_i dr_i \\
\end{align*}$$  \hspace{1cm} (19)$$
2.3.2. lower rotor model. The derivation of the lift of the lower rotor based on the momentum theorem is similar to that of the upper rotor. The difference is that the lower rotor works in the downward wash airflow of the upper rotor. Therefore, it is necessary to count the air inflow velocity when calculating the lift of the lower rotor. So the dimensionless lift of the blade element ring of the lower rotor is

\[
dC_{f2} = \frac{dF_{l2}}{\pi \rho R_s^2 (\alpha_2 R_s)^2} = \left( \frac{1}{h_2^2} - \lambda_2^2 \right) r_2 dr_2
\]

Similarly, considering the downward wash airflow of the upper rotor, the dimensionless lift and torque of the lower rotor calculated by the blade element theory are

\[
\begin{align*}
\{ dC_{f2} &= \frac{1}{2} \sigma \left[ C_{l_{21}} (\theta_2 r_2^3 - \lambda_2 r_2) - C_{d2} \lambda_2 r_2 \right] dr_2 \\
\{ dC_{m2} &= \frac{1}{2} \sigma \left[ C_{d2} r_2^2 + C_{l_{21}} (\theta_2 r_2 - \lambda_2) \right] dr_2
\end{align*}
\]

Combining equation (20) and (21), the solution for the inflow ratio of the lower rotor is given as

\[
\lambda_2 = \frac{1}{4} h_2^2 \sigma c_{l_{21}} \left\{ \left[ \frac{4 \lambda_2}{h_2 \sigma c_{l_{21}}} \right]^2 + \frac{8 \theta_2 r_2}{h_2 \sigma c_{l_{21}}} + 1 \right\}^{1/2} - 1
\]

Then the dimensioned lift and torque of the lower rotor can be calculated by

\[
\begin{align*}
F_z &= \pi \rho R_s^2 (\alpha_2 R_s)^2 \int_0^r \frac{1}{2} \sigma \left[ C_{l_{21}} (\theta_2 r_2^3 - \lambda_2 r_2) - C_{d2} \lambda_2 r_2 \right] r_2 dr_2 \\
M_z &= \pi \rho R_s^2 (\alpha_2 R_s)^2 \int_0^r \frac{1}{2} \sigma \left[ C_{d2} r_2^2 + C_{l_{21}} (\theta_2 r_2 - \lambda_2) \right] r_2 dr_2
\end{align*}
\]

where \( C_{d2} \) is the drag coefficient, \( c_{l_{21}} \) is the slope of the lift characteristic curve, \( \theta_1 \) is the mounting angle of the lower rotor.

2.4. Rudder model
There are two sets of rudders below the front duct, each of which consists of a main rudder and two auxiliary rudders. The two sets of rudders are linked and their deflection angles are the same. For the rudders under the front duct, the lift and resistance can be obtained as follows:

\[
\begin{align*}
F_{r1} &= \frac{1}{h_2^2} \rho \theta_1 \left[ C_{m1} S_m (\omega_2 R_s \lambda_2)^2 + 2 C_{dr2} S_m (\omega_2 R_s \lambda_2)^2 \right] \\
T_{r1} &= \frac{1}{h_2^2} \rho \left[ C_{dr1} S_m (\omega_2 R_s \lambda_2^2)^2 + 2 C_{dr2} S_m (\omega_2 R_s \lambda_2)^2 \right]
\end{align*}
\]

where \( \theta_1 \) is the rudder deflection angle, \( C_{m1} \) and \( C_{dr1} \) are lift coefficient and drag coefficient for the main rudder, \( C_{m2} \) and \( C_{dr2} \) are lift coefficient and drag coefficient for the auxiliary rudder, \( S_m \) and \( S_m \) are the windward area of the main rudder and auxiliary rudder.

3. Static experiment and model correction
It is worth noting that we have simplified the aerodynamics of the structure during the modelling. In addition, the ducted rotors are pneumatically special, and there are very complicated aerodynamic interferences between the two rotors in the same duct. These aerodynamic disturbances are often difficult to accurately describe by mathematical methods. These factors have brought some errors to aerodynamic modelling of the ducted rotors [9]. Therefore, we designed and carried out the static
experiment of the ducted twin-rotor, introduced the aerodynamic interference factor, and corrected the aerodynamic weight of the duct and the upper and lower rotor.

The static test system is mainly composed of test ducts, test rotors, fixed base, sensors and data acquisition systems, as shown in figure 3. Each of the ducted rotor rotation centers is equipped with a photoelectric encoder and a two-component balance to measure the real-time rotating speed, lift and torque of each rotor. On the outer wall symmetrical with the center of the duct, two lift sensors are installed to measure the additional lift generated on the duct. The data acquisition system includes amplifiers, capture cards and electronic governors. The amplifiers output a voltage change signal of the two-component balance and the lift sensor, and perform the corresponding amplitude gain for subsequent processing. The electronic governors are used to collect the real-time voltage and current of each motor. The capture cards are used to collect the voltage and current signals mentioned above and calculate the final results which we want to collect.

By analyzing the data collected by the static test, we correct the lift of the upper rotor and lower rotor and the duct thrust calculated from our model:

$$
F_{1u} = c_{F1}F_{1d}
\quad F_{2u} = c_{F2}F_{2d}
\quad F_{d1} = c_{Fdl}F_{d1}
$$

(25)

where $c_{F1}$, $c_{F2}$ and $c_{Fdl}$ are the correction coefficients. Figure 4 shows the comparison of the theoretical thrusts of the upper and lower rotor and the duct of the modified ducted twin-rotor model and static test results. It can be obtained in figure 4 that we get the ideal correction effect of the model when the speed of the ducted rotor is less than 5000 rpm. When the rotor speed is above 5000 rpm, even if it is corrected, there is still some error between the theoretical calculation results and the test results. This is because the performance of each pneumatic component in the test is not only affected by its own structure but also by the motor. When the rotor speed is too high, the output power and current of the motor increases, and the temperature rises obviously, which would have an impact on the test results. Considering that the motor of the ducted fan flying robot mainly works at 4000~5000rpm, the model correction is considered to be effective.
4. Conclusion
This paper proposes a novel ducted fan flying robot whose structure is compact aiming at operating in complicated environment and narrow space. Considering that the structure of its ducted coaxial twin-rotor with reverse rotation is unconventional, the modelling method of the traditional open rotors with the blade element theory and the momentum theory is extended to the ducted coaxial twin-rotor structure, and the integrated dynamic model of the ducted fan flying robot has been founded. After the modelling, we designed and carried out the static experiment of the aerodynamic structure of the ducted twin-rotor, introduced the aerodynamic interference factor, and corrected the aerodynamic weight of the duct and the upper and lower rotor. At last we obtain the ideal correction effect and get a more accurate model of the ducted fan flying robot. Our future work will focus on control system designing and corresponding flight experiments of the ducted fan flying robot.

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