Heavy $\bar{Q}Q$ free energy from hadronic states

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Abstract

Within the spirit of the Hadron Resonance Gas model, we study a representation of the heavy $\bar{Q}Q$ free energy at temperatures below the phase transition in terms of the string and heavy-light hadrons. We discuss the string breaking phenomenon and the relevance of avoided crossings between the fundamental string and the hadron spectrum. Good agreement with lattice data is achieved.

Keywords: finite temperature, QCD thermodynamics, heavy quarks, chiral quark models, Polyakov loop

1. Introduction

A fruitful approach to study the confined phase of QCD is the Hadron Resonance Gas (HRG) model, in which the equation of state is obtained by assuming a gas of non-interacting massive stable and point-like particles, which are taken as the conventional hadrons [1]. This model has arbitrated the discrepancies between the different collaborations in the lattice community [2].

The effort to study the thermodynamics of QCD has led also to important advances in the study of the interquark forces at finite temperature. The vacuum expectation value of the Polyakov loop is related to the propagator of a static quark, and it has been used as an order parameter for the confinement/deconfinement transition of color charges [3]. The free energy of a heavy $\bar{Q}Q$ pair in a thermal medium, which can be computed from the correlation function of Polyakov loops, has important phenomenological consequences in the study of quarkonia physics in heavy ion experiments [4]. In line with the HRG approach we have proposed recently [5, 6] a hadronic representation for the vacuum expectation value of the Polyakov loop which has been quite successful in describing the available lattice data up to $T \approx 180$ MeV [5, 7]. In this communication we represent the heavy $\bar{Q}Q$ free energy in terms of the HRG, and compare with recent lattice simulations.

2. Quark potential and string breaking

One of the most studied quantities in QCD is the static energy between heavy sources, such as heavy quark systems [8] and also for more exotic states like glueballs [9, 10]. We will present in this section the most relevant properties of this interaction which will be used later in the study of the heavy $\bar{Q}Q$ free energy.

2.1. Static energies in QCD

Inspired by perturbation theory to second order [11] the static interaction between heavy sources $A$ and $B$ in QCD is often modeled implementing Casimir scaling,

$$V_{AB}(r) = \frac{\lambda_A \cdot \lambda_B}{r} \left[ \frac{\alpha_S}{r} - kr \right],$$

where $\lambda$ is the SU($N_c$) color group generator corresponding to the representation of the source. The Coulomb-like term accounts for the perturbative behavior from one-gluon exchange at short distances, while the confining linearly growing term accounts for the energy of the string connecting the two heavy quarks. Casimir scaling implies a relation between the fundamental $\bar{Q}Q \equiv 3 \otimes \bar{3}$ and adjoint $GG \equiv 8 \otimes 8$ color sources given by $V_{\bar{Q}Q}(r) = \frac{3}{8} V_{GG}(r)$, and holds on the lattice for $N_c = 3$ [12] and for heavy sources in the fundamental representation $\bar{Q}Q \equiv 3 \otimes \bar{3}$ at $N_c = 3, 5, 7$ [13]. The $N_c$...
independence of the $V_{Q\bar Q}$ potential has also been established. For $N_c = 3$ one gets

$$V_{Q\bar Q}(r) = \sigma r - \frac{4\alpha_s}{3r},$$

(2)

where $\alpha_s \approx \pi/16$ is the effective coupling and $\sigma \equiv 4\pi/3 \approx (0.42 \text{ GeV})^2$ is the string tension.

2.2. String breaking

Using this potential, one can study the transition between a $Q\bar Q$ system and a meson-antimeson system. When the separation between the two heavy quarks increases, the energy of the system rises linearly due to the string tension term. Eventually this system becomes unstable at a critical distance $r_c$ when a light $\bar q q$ is created from the vacuum, so that a heavy-light meson-antimeson $MM \equiv (Q\bar q)(\bar Qq)$ channel opens.

Neglecting the meson-antimeson interaction, the energies of these channels are

$$E_{Q\bar Q}(r) = m_q + m_{\bar q} + V_{Q\bar Q}(r),$$

$$V_{(Q\bar q)(\bar Qq)}(r) = \Delta_{Q\bar q} + \Delta_{\bar Qq} \equiv 2\Delta,$$

(3) (4)

where $\Delta_{Q\bar q} = \text{Im} m_{Q\bar q} - m_q$ is the mass of the heavy-light meson (the mass of the heavy quark itself $m_q$ being subtracted). Using these formulas, one can estimate the string breaking to be $r_c \approx 4M_0/\sigma \sim 1.2 \text{ fm}$ [14, 15], where $M_0$ is the constituent quark mass, which is in fair agreement with lattice results [16].

More generally, the $Q\bar Q$ state can decay into any of the many excited states of the meson spectrum after $\bar q q$ pair creation from the vacuum, i.e. $\gamma^{\text{(exc.)}}_{Q\bar q}(\bar Qq) = \Delta^{(n)} + \Delta^{(m)}$, or to the baryon spectrum, as long as they have the same quantum numbers as the $Q\bar Q$ system.

2.3. Avoided crossings

In the picture presented above we have assumed a diabatic crossing structure in which the states $Q\bar Q$ and $MM$ are degenerate at the point $r = r_c$. However, the existence of a transition potential $V_{Q\bar Q\rightarrow MM}(r)$ between the two states, implies the matrix interaction

$$V(r) = \begin{pmatrix} V_{Q\bar Q}(r) & V_{Q\bar Q\rightarrow MM}(r) \\ V_{Q\bar Q\rightarrow MM}(r) & V_{MM}(r) \end{pmatrix}. $$

(5)

After diagonalization, the finite energy of the non-diagonal $Q\bar Q \rightarrow MM$ interaction lifts the degeneracy, a feature called level repulsion, and there emerges the picture of adiabatic avoided crossing shown in Fig. 1 [14, 15], a phenomenon familiar from molecular physics in the Born-Oppenheimer approximation [17].

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2The mechanism of decaying into baryons implies the creation of two pairs of light quarks $\bar q q$, leading to the formation of two heavy-light baryons with one heavy quark.

3This normalization of the free energy differs by a factor $N_c^2$ from other conventions found in the literature.
where \( \Delta_q = \Delta_{q0}^{(n)} = \Delta_{q0}^{(s)} \), which constitutes a hadronic representation of the heavy \( \bar{Q}Q \) free energy \([14, 15]\). In Fig. 2(A) we display the lattice results of \([25]\) for the heavy \( \bar{Q}Q \) free energy along with the calculation from Eq. (3). We use the spectrum of heavy-light mesons and baryons with a charm quark and no strangeness, \( N_f = 2 \), from the Isgur model \([26]\) up to \( \Delta = 3.19 \) GeV. We have taken \( \sigma = (0.40 \text{ GeV})^2 \) and \( \alpha_S = \pi/16 \). We find an excellent agreement for the lowest temperatures and for all distances.

The Polyakov loop expectation value can be computed from the large separation limit of the correlator between Polyakov loops. From Eq. (3) one has

\[
L(T) := \lim_{r \to \infty} e^{-F_{\bar{Q}Q}(CT)/(2T)} = \frac{1}{2} \sum_n e^{-\Delta_n/T}. \tag{9}
\]

This is precisely the hadronic representation of the Polyakov loop introduced in Ref. \([3]\), which was shown to provide a good agreement with lattice data for \( T \lesssim 180 \) MeV. This means that the spectrum from the Isgur model already saturates the sum rules at these temperatures. It was shown in \([27]\) that Eq. (9) can also be obtained in chiral quark models coupled to the Polyakov loop, when one advocates the local and quantum nature of the Polyakov loop \([28, 29]\).

A different exercise is to extract from the finite temperature lattice data the zero temperature heavy \( \bar{Q}Q \) potential. By using Eqs. (3) and (9), one gets

\[
V_{\bar{Q}Q}(r) = -T \log \left[ e^{-F_{\bar{Q}Q}(r/T, T)/T} - e^{-F_{\bar{Q}Q}(\infty, T)/T} \right]. \tag{10}
\]

The terms in the argument of the log-function are known from lattice data and they are in general \( T \)-dependent, so that this is a nontrivial check. The result for \( V_{\bar{Q}Q}(r) \) is plotted in Fig. 2(B). This quantity turns out to be \( T \)-independent with a high accuracy, and it is remarkably close to the \( \bar{Q}Q \) Cornell potential, Eq. (2), giving a strong indication of the validity of Eq. (3).

### 3.2. Avoided crossing and heavy \( \bar{Q}Q \) free energy

The lattice data of Ref. \([25]\) are computed for 2 light flavors with a pion mass of \( m_\pi \approx 770 \) MeV, and assumes the zero temperature \( \bar{Q}Q \) potential behavior of the free energy at short distances. The recent study of Ref. \([30]\) uses \( N_f = 2 + 1 \) with physical quark masses, and releases the short distance assumption in the renormalization procedure. The lattice results of \([30]\) are displayed in Fig. 3(A) along with our \( N_f = 2 + 1 \) calculation assuming the hadronic representation of Eq. (8). The most striking observation is that a quite high value of the string tension has to be assumed, \( \sigma = (0.55 \text{ GeV})^2 \), in order to reproduce the lattice data of \([30]\), while \( \sigma = (0.40 \text{ GeV})^2 \) worked for the data of \([25]\). The same conclusion is reached by extracting the zero temperature \( \bar{Q}Q \) potential by using Eq. (10), as shown in Fig. 3(B).

In view of this, one may wonder whether this effect in the string tension could be effectively due to the existence of an appreciable mixing between the string and the excited states in the lattice calculation of \([30]\). To investigate this possibility, we take a simple model consisting of only two coupled states, the \( \bar{Q}Q \) pair (string state) and the lightest meson-antimeson state:

\[
V(r) = \left( -\frac{\Delta r}{\sigma T(r)} + \frac{W(r)}{2\Delta} \right). \tag{11}
\]

As a first estimate, we assume for the mixing a function of the form

\[
W(r) = W_0 e^{-mr}. \tag{12}
\]

From a fit of the lowest temperature lattice data of \([30]\) for the heavy \( \bar{Q}Q \) free energy, \( T = 150 \) MeV, one gets

\[
W_0 = 1.03(50) \text{ GeV}, \quad m = 0.85(38) \text{ GeV}, \quad \sigma = (0.42(10) \text{ GeV})^2, \tag{13}
\]

with \( \chi^2/\text{dof} = 0.036 \). We have taken \( 2\Delta = 0.944 \) GeV corresponding to the lowest heavy-light meson state of the Isgur model. The result is displayed in Fig. 4 where one can see that the mixing mimics a larger value of the string tension at distances \( r \lesssim 0.5 \) fm. The behavior of the first excited state at short distances is in qualitative agreement with lattice simulations, see e.g. Ref. \([16]\).

This result leads to the problem of the identification of the parameter \( m \) with the mass of some particle. At first sight it would be tempting to assume that \( m \) should be the pion mass, however, the value we get for \( m \) is heavier by a factor \( \approx 2\pi \).

The model dependence of this result can be relaxed by not assuming any functional form for the mixing \( W(r) \). To do so, for each point of the lattice data \([30]\) at a given temperature we obtain the value of \( W \) that allows the two states model, Eq. (11), to reproduce the free energy at that separation. The result for \( W(r) \) computed in this way is displayed in Fig. 5. A result quite similar to Eqs. (12)-(13) is obtained for \( T \lesssim 200 \text{ MeV} \). The \( T \)-independence of \( W(r) \) goes in favor of the reliability of the model of Eq. (11). For temperatures above 150 MeV, one would need to include a huge amount of states to reproduce the Polyakov loop. Even after including all the available states of the Isgur model, the sum rule for the Polyakov loop cannot be saturated for temperatures above 180 MeV \([5, 31]\). For this reason, and to avoid a source of error in \( W(r) \) coming from the
the lowest temperature extracted from the lattice data. This is not necessary for the lowest temperature \( T = 150 \text{ MeV} \), as the sum rule is saturated in this case.

4. Discussion and outlook

We have studied a natural extension of the HRG model to the heavy \( Q\bar{Q} \) free energy. This quantity has been computed in the confined phase of QCD from a string and the HRG model with heavy-light hadrons, and compared with existing lattice computations \[25, 30\] giving an excellent agreement for \( T \leq 200 \text{ MeV} \).

The relevance of string breaking at large distances has been discussed as well. Finally, we have investigated the possible role played by the avoided crossings between the string and the spectrum of hadrons.

Our analysis of the avoided crossings leads to possible contradicting results when analyzing the lattice data for the heavy \( Q\bar{Q} \) free energy from two different groups. While the interaction between the string state and meson-antimeson states seems to be negligible in the data from \[23\], a noticeable effect appears in the analysis of the more recent data of \[30\]. This could be well due to the different approaches for quark masses assumed in these references, or the different renormalization prescriptions. So, more accurate lattice data and better agreement between different groups would be desirable on the one hand. On the other hand, our analysis can be improved either by including some subleading effects like string excitations or extending the two states...
mixing model with the inclusion of more states in the spectrum. Work along these lines is in progress.

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Figure 4: With solid lines, string state and first meson-antimeson state as a function distance using the model of Eq. (11) with \( W(r) = W_0 e^{-mr} \). The parameters are obtained from a best fit to the lattice data of the heavy \( \bar{Q}Q \) free energy of Ref. [28] at the lowest available temperature, \( T = 150 \text{ MeV} \). The dashed lines represent the result with this model taking \( \sigma = (0.42 \text{GeV})^2 \) and no maxing, \( W(r) = 0 \). The dot-dashed line (black) is the result from the Cornell potential, \( V(r) = - \frac{1}{12} + \sigma r + C \) with \( \sigma = (0.55 \text{GeV})^2 \) and \( C = -0.3 \text{GeV} \).

Figure 5: The transition potential \( W \) of Eq. (11) as a function of the \( \bar{Q}Q \) separation. The dots reproduce the lattice data of the heavy \( \bar{Q}Q \) free energy of Ref. [28] at different separations and temperatures assuming \( \sigma = (0.42 \text{GeV})^2 \). The dashed line corresponds to the best fit obtained by assuming the functional form \( W(r) = W_0 e^{-mr} \).

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