Higgs-Dilaton Cosmology: an effective field theory approach

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The Higgs-Dilaton cosmological model is able to describe simultaneously an inflationary expansion in the early Universe and a dark energy dominated stage responsible for the present day acceleration. It also leads to a non-trivial relation between the spectral tilt of scalar perturbations $n_s$ and the dark energy equation of state $\omega$. We study the self-consistency of this model from an effective field theory point of view. Taking into account the influence of the dynamical background fields, we determine the effective cut-off of the theory, which turns out to be parametrically larger than all the relevant energy scales from inflation to the present epoch. We finally formulate the set of assumptions needed to estimate the amplitude of the quantum corrections in a systematic way and show that the connection between $n_s$ and $\omega$ remains unaltered if these assumptions are satisfied.

I. INTRODUCTION

The shortcomings of the hot big bang model can be solved in an elegant way if we assume that the Universe underwent an inflationary period in its early stages. The easiest way for this paradigm to be realized is by a scalar field slowly rolling down towards the minimum of its potential [1].

As discussed in Ref. [2], inflation does not necessarily require the existence of a new degree of freedom. The role of the inflaton can be played by the Standard Model (SM) Higgs field with its mass lying in the interval where the SM can be considered a consistent effective field theory up to the inflationary scale. More precisely, if the Higgs boson is non-minimally coupled to gravity and the value of the corresponding coupling constant $\xi_h$ is sufficiently large, the model is able to provide a successful inflationary period followed by a graceful exit to the standard hot Big Bang theory [3, 4]. The implications of this scenario have been extensively studied in the literature [5–22]. Earlier studies of non-minimally coupled scalar fields in the context of inflation can be also found in Refs. [23–25].

When the Higgs inflation model described above is rewritten in the so-called Einstein frame, where the gravity part takes the usual Einstein-Hilbert form, it becomes essentially non-polynomial and thus non-renormalizable, even if the gravity part is dropped off. Therefore, it should be understood as an effective field theory valid only up to a certain “cut-off” scale. One should distinguish between two different definitions of the “cut-off”. Quite often the cut-off of the theory is understood as the energy at which the tree level unitarity in high-energy scattering processes is violated. A second definition of the cut-off is the energy associated to the onset of new physics. As it was recently stressed in Ref. [26], the breaking of tree level unitarity does not imply the appearance of new physics or extra degrees of freedom right above the corresponding energy scale; it just signals that the perturbation theory in terms of low-energy variables breaks down. For the case of Higgs inflation, the tree-level scattering amplitudes above the electroweak vacuum appear to hit the perturbative unitarity bound at energies $\Lambda \sim M_P/\xi_h$ [8, 9, 16, 17]. Whether the theory requires an ultraviolet completion at these energies or simply enters into the non-perturbative strong-coupling regime with onset of new physics at higher energies (which could be as large as the Planck scale) is still an open question. Nevertheless, the Higgs inflation scenario is self-consistent. As shown in Ref. [27] (see also [28]), the beginning of the strong coupling regime (i.e. the cut-off scale according to the first definition which will be used in this article) depends on the dynamical expectation value of the Higgs field, which makes the theory weakly coupled for all the relevant energy scales in the evolution of the Universe. In other words, the SM with a large non-minimal coupling of the Higgs field to gravity represents a viable effective theory for the description of inflation, reheating, and the hot Big Bang theory.

The Higgs inflation scenario can be easily incorporated into a larger framework, the Higgs-Dilaton model [29, 30]. The key element of this extension is scale-invariance (SI). No dimensional parameters such as masses are allowed to appear in the action. All the scales are instead induced by the spontaneous breaking of SI.
the introduction of a new scalar degree of freedom, the dilaton, which becomes the Goldstone boson of the broken symmetry and remains exactly massless. The coupling of the dilaton field to matter is weak and takes place only through derivative couplings, not contradicting therefore any 5th force experimental bounds \[31\].

Although the dilatation symmetry described above forbids the introduction of a cosmological constant term, the ever-present cosmological constant problem reappears associated to the fine-tuning of the dilaton self-interaction \[29\]. However, if the dilaton self-coupling $\beta$ is chosen to be zero (or required to vanish due to some yet unknown reason), a slight modification of general relativity (GR), known as Unimodular Gravity (UG), provides a dynamical dark energy (DE) stage in good agreement with observations. The scale-invariant UG gives rise to a “run-away” potential for the dilaton \[29\], which plays the role of a quintessence field. The strength of such a potential is determined by an integration constant $\Lambda_0$ that appears in the Einstein equations of motion due to the unimodular constraint $\hat{g} = -1$ on the metric determinant. The common origin of the inflationary and DE dominated stages in Higgs-Dilaton inflation allowed to derive extra bounds on the initial inflationary conditions \[4\] as well as potentially testable relations between the early and late Universe observables \[30\].

Some of the properties of the Higgs-Dilaton model described above were previously noted in the literature. The first attempt to formulate a viable SI theory non-minimally coupled to gravity was done by Fujii in Ref. \[32\], although without establishing any connection to the SM Higgs. The role of dilatation symmetry in cosmology was first considered by Wetterich in Refs. \[33\, 34\]. In these seminal papers, the dynamical dark energy, associated with the dilaton field, appears as a consequence of the dilatation anomaly and is related to the breaking of SI by quantum effects. The present paper has a number of formal analogies and similarities regarding the cosmological consequences for the late Universe with Refs. \[33\, 34\]. At the same time, our approach to the source of dark energy is different from the one adopted in Refs. \[33\, 34\], as we assume that SI is an exact (but spontaneously broken) symmetry at the quantum level, leading therefore to a massless dilaton. In Ref. \[33\, 34\], both the cases of exact and explicitly broken dilatation symmetry were considered. Our theory with exact dilatation symmetry is different from that of \[33\, 34\] in two essential aspects. First, in our work the Higgs field of the SM has non-minimal coupling to gravity (it is absent in Ref. \[33\, 34\]), which is important for the early Universe and leads to Higgs inflation. Second, the unimodular character of gravity (as opposed to standard general relativity used in \[33\, 34\]) leads to an automatic and very particular type of dilatation symmetry breaking, which results in dynamical dark energy due to the dilaton field (absent in \[33\, 34\] for the case of exact scale invariance).

Our purpose here is to study, following the approach of Ref. \[27\], the self-consistency of the Higgs-Dilaton model by adopting an effective field theory point of view. We will estimate the field-dependent cut-offs associated to the different interactions among scalars fields, gravity, vector bosons and fermions. We will identify the lowest cut-off as a function of the background fields and show that its value is higher than the typical energy scales describing the Universe during its different epochs. The issue concerning quantum corrections generated by the loop expansion is also addressed. Since the model is non-renormalizable, an infinite number of counter-terms must be added in order to absorb the divergences. It is important to stress at this point that, in the lack of a quantum theory for gravity, the details of the regularization scheme to be used cannot be univocally fixed. This means that the predictions of the model will be sensitive to the assumptions about the UV-completion of the theory (corresponding to different regularization prescriptions). We will adopt a “minimal setup” that keeps intact the exact and approximative symmetries of the classical action and does not introduce any extra degrees of freedom. Within this approach, the relations connecting the inflationary and the dark energy domination periods hold even in the presence of quantum corrections.

The structure of the paper is as follows. In Section \[II\] we briefly review the Higgs-Dilaton model. In Section \[III\] we calculate the cut-off of the theory in the Jordan frame and compare it with the other relevant energy scales in the evolution of the Universe. In Section \[IV\] we propose a “minimal setup” which removes all the divergences and discuss the sensitivity of the cosmological observables to radiative corrections. Section \[V\] contains the conclusions.

\section{Higgs-Dilaton Cosmology}

We start by reviewing the main results of Refs. \[29\, 30\], where the Higgs-Dilaton model was proposed and studied in detail. The two main ingredients of the theory are outlined below. The first one is the invariance of the SM action under global scale transformations, which leads to the absence of any dimensional parameters or scales. Denoting by $\Phi(x)$ the field content of the theory in a metric $g_{\mu\nu}(x)$, these transformations can be written as:

\begin{equation}
\tag{2.1}
g_{\mu\nu}(x) \rightarrow g_{\mu\nu}(\sigma x) \quad , \quad \Phi(x) \rightarrow a^4 \Phi(\sigma x) ,
\end{equation}

\footnote{The fine-tuning needed to reproduce the present dark energy abundance is transferred into the initial inflationary conditions for the fields at the beginning of inflation.}

\footnote{For a theory invariant under all diffeomorphisms, this is equivalent to $g_{\mu\nu}(x) \rightarrow a^{-2} g_{\mu\nu}(x) \quad , \quad \Phi(x) \rightarrow a^4 \Phi(x)$.}
with $\sigma^{d_D}$ the so-called scaling dimension and $\sigma$ an arbitrary constant. In order to achieve invariance under these transformations, we let the masses and dimensional couplings in the theory to be dynamically induced by a field. The simplest choice would be to use the SM Higgs, already present in the theory. Note however that this option is clearly incompatible with the experiment. As discussed in Refs. [24], the excitations of the Higgs field in this case become massless and completely decoupled from the SM particles.

The next simplest possibility is to introduce a new scalar singlet under the SM gauge group. We will refer to it as the dilaton $\chi$. The coupling between the new field and the SM particles, with the exception of the Higgs boson, is forbidden by quantum numbers. The corresponding Lagrangian is given by

$$\mathcal{L} = \frac{1}{2} (2 \xi_h \phi^* \phi + \xi_{h} \chi^2) R + \mathcal{L}_{\text{SM}[\lambda \to 0]} - \frac{1}{2} g^{\mu\nu} \partial_{\mu} \chi \partial_{\nu} \chi - V(\phi, \chi),$$

(2.2)

where $\phi$ is the SM Higgs field doublet and $\xi_h \sim 10^{-4} - 10^5$, $\xi_{h} \sim 10^{-3}$, are respectively the non-minimal couplings of the Higgs and dilaton fields to gravity [30]. The term $\mathcal{L}_{\text{SM}[\lambda \to 0]}$ is the SM Lagrangian without the Higgs potential, which in the present scale-invariant theory becomes

$$V(\phi, \chi) = \lambda \left( \phi^* \phi - \frac{\alpha}{2\lambda} \chi^2 \right)^2 + \beta \chi^4,$$

(2.3)

with $\lambda$ the self-coupling of the Higgs field.

In order for this theory to be phenomenologically viable, we demand the existence of a symmetry-breaking ground state with non-vanishing background expectation value for both the dilaton $(\bar{\chi})$ and the Higgs field in the unitary gauge $(\bar{h})$. This is given by

$$\bar{h}^2 = \frac{\alpha}{\lambda} \chi^2 + \frac{\xi_h}{\lambda} R, \quad \text{with} \quad R = \frac{4\beta \lambda}{\lambda \xi_h + \alpha \xi_{h}} \chi^2.$$

(2.4)

All the physical scales are proportional to the non-zero background value of the dilaton field. For instance, the SM Higgs mass is given by

$$m_{H}^2 = 2\alpha M_{P}^2 \left( \frac{1+6\xi_h}{1+6\xi_{h}} + \frac{\alpha}{\lambda} (1+6\xi_h) \chi^2 \right) + O(\beta),$$

(2.5)

with $M_{P}^2 \equiv \xi_h \bar{h}^2 + \xi_{h} \bar{\chi}^2 \propto \chi^2$ the effective Planck scale in the Jordan frame. The same happens with the effective cosmological constant

$$\Lambda = \frac{1}{4} M_{P}^2 R = \frac{\beta M_{P}^2}{(\xi_{h} + \frac{\alpha}{\lambda} \xi_h)^2 + 4\frac{\beta}{\lambda} \xi_{h}^2},$$

(2.6)

which depending on the value of the dilaton self-coupling $\beta$, gives rise to a flat ($\beta = 0$), deSitter ($\beta > 0$) or anti-deSitter ($\beta < 0$) spacetime. It is important to notice however that physical observables, corresponding to dimensionless ratios between scales or masses, are independent of the particular value of the background field $\bar{\chi}$. In order to reproduce the ratio between the different energy scales, the parameters of the model must be properly fine-tuned. As shown in Eq. (2.5), the difference between the electroweak and the Planck scale is encoded in the parameter $\alpha \sim 10^{-35} \ll 1$. Similarly, the hierarchy between the cosmological constant and the electroweak scale, cf. Eq. (2.6), implies $\beta \ll \alpha$. The smallness of these parameters, together with the tiny value of the non-minimal coupling $\xi_{h}$, gives rise to an approximate shift symmetry for the dilaton field at the classical level, $\chi \to \chi + \text{const}$. As we will show in Section IV, this fact will have important consequences for the analysis of the quantum effects.

The second ingredient of the Higgs-Dilaton cosmological model is the replacement of GR by Unimodular Gravity, which is just a particular case of the set of theories invariant under transverse diffeomorphisms. These theories generically contain an extra scalar degree of freedom on top of the massless graviton (for a general discussion see for instance Ref. [36] and references therein). In UG the number of dynamical components of the metric is effectively reduced to the standard value by requiring the metric determinant $\bar{g}$ to take some fixed constant value, conventionally $|\bar{g}| = 1$. As shown in Ref. [29], the equations of motion of a theory subject to that constraint

$$\mathcal{L}_{UG} = \mathcal{L}[\bar{g}_{\mu\nu}, \partial_{\mu} \bar{g}_{\nu\alpha}, \Phi, \partial \Phi],$$

(2.7)

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3 If $\bar{\chi} = 0$ the Higgs field is massless, and if $\bar{h} = 0$ there is no electroweak symmetry breaking.

4 Note that the alternative choice $\xi_h \gg 1$ is not compatible with CMB observations, cf. Eq. (2.24) and Fig. [3].
coincide with those obtained from a diffeomorphism invariant theory with modified action
\[
\frac{\mathcal{L}}{\sqrt{-g}} = \mathcal{L}[g_{\mu\nu}, \partial g_{\mu\nu}, \Phi, \partial \Phi] + \Lambda_0 .
\] (2.8)
Note that, from the point of view of UG, the parameter \(\Lambda_0\) is just a conserved quantity associated to the unimodular constraint and it should not be understood as a cosmological constant.

Since the two formulations are completely equivalent\(^5\) we will stick to the diffeomorphism invariant language. Expressing the theory resulting from the combination of the above ideas in the unitary gauge \(\varphi^T = (0, h/\sqrt{2})\) we get
\[
\frac{\mathcal{L}}{\sqrt{-g}} = \frac{1}{2}(\xi h^2 + \xi \chi^2) R - \frac{1}{2}(\partial h)^2 - \frac{1}{2}(\partial \chi)^2 - U(h, \chi) ,
\] (2.9)
where the potential includes now the UG integration constant \(\Lambda_0\)
\[
U(h, \chi) = V(h, \chi) + \Lambda_0 = \frac{\lambda}{4} \left(h^2 - \frac{\alpha}{2} \chi^2 \right)^2 + \beta \chi^4 + \Lambda_0 .
\] (2.10)
Notice that the Lagrangian given by Eqs. (2.9) and (2.10) bears a clear resemblance with the models studied in Ref. 33\,\,34. In particular, it coincides (up to the non-minimal coupling of the Higgs field to gravity) with the Brans-Dicke theory with cosmological constant studied in 33. However, the interpretation of the \(\Lambda_0\) term is different. In our case this constant is not a fundamental parameter associated with the anomalous breaking of SI \,\,34, but an automatic result of UG.

The phenomenological consequences of Eq. (2.9) are more easily discussed in the Einstein frame. Let us then perform a conformal redefinition of the metric \(g_{\mu\nu} = \Omega^2 g_{\mu\nu}\) with conformal factor \(\Omega^2 = M_F^{-2}(\xi h^2 + \xi \chi^2)\). Using the standard relations \,\,37
\[
\sqrt{-g} = \Omega^{-4} \sqrt{g} \quad \text{and} \quad R = \Omega^2 \left( \hat{R} + 6 \hat{\Box} \log \Omega - 6 \hat{g}^{\mu\nu} \partial_\mu \log \Omega \partial_\nu \log \Omega \right) ,
\] (2.11)
we get
\[
\frac{\mathcal{L}}{\sqrt{-g}} = \frac{M_F^2}{2} \hat{R} - \frac{1}{2} \hat{K}(h, \chi) - \hat{U}(h, \chi) ,
\] (2.12)
where
\[
\hat{U}(h, \chi) = \frac{U(h, \chi)}{\Omega^4} = \frac{M_F^4}{(\xi h^2 + \xi \chi^2)^2} \left[ \frac{\lambda}{4} \left(h^2 - \frac{\alpha}{2} \chi^2 \right)^2 + \beta \chi^4 + \Lambda_0 \right] ,
\] (2.13)
is the potential (2.10) in the new frame. The non-canonical kinetic term in Eq. (2.12) can be written as
\[
\hat{K}(h, \chi) = \kappa_{ij} \hat{g}^{\mu\nu} \partial_\mu \Phi^i \partial_\nu \Phi^j ,
\] (2.14)
where the quantity
\[
\kappa_{ij} = \frac{1}{\Omega^2} \left( \delta_{ij} + \frac{3}{2} M_F^2 \frac{\partial_i \Omega^2 \partial_j \Omega^2}{\Omega^2} \right) ,
\] (2.15)
can be interpreted as the metric in the two-dimensional field space \((\Phi^1, \Phi^2) = (h, \chi)\) in the Einstein-frame. Note that, unlike the simplest Higgs inflationary scenario 2, Eq. (2.14) cannot be recast in canonical form by field redefinitions. In fact, the Gaussian curvature associated to (2.14) does not identically vanish unless \(\xi_h = \xi_\chi\), which, as shown in Ref. 30, is not consistent with observations. Nevertheless, it is possible to write the kinetic term in a quite simple diagonal form. As shown in Ref. 30, the whole inflationary period takes place inside a field space domain in which the contribution of the integration constant \(\Lambda_0\) is completely negligible. We will refer to this domain as the “scale invariant region” and assume that it is maintained even when the radiative corrections are taken into account (cf.

\(^5\) As usual, there are some subtleties related to the quantum formulation of (unimodular) gravity. However, these will not play any role in the further developments. The interested reader is referred to the discussion in Ref. 38 and references therein.
Section IV. In this case, the dilatational Noether’s current in the slow-roll approximation, \((1 + 6\xi_h)h^2 + (1 + 6\xi_h)\chi^2\), is approximately conserved, which suggests the definition of the set of variables

\[
\rho = \frac{M_P}{2} \log \left[ \frac{(1 + 6\xi_h)h^2 + (1 + 6\xi_h)\chi^2}{M_P^2} \right], \quad \tan \theta = \sqrt{1 + 6\xi_h^2} \frac{h}{1 + 6\xi_h \chi}.
\]

(2.16)

The physical interpretation of these variables is straightforward. They are simply adequately rescaled polar variables in the \((h, \chi)\) plane. Expressed in terms of \(\rho\) and \(\theta\), the kinetic term (2.14) turns out to be

\[
\tilde{K} = \left(\frac{1 + 6\xi_h}{\xi_h}\right) \frac{1}{\sin^2 \theta + \varsigma \cos^2 \theta} (\partial \rho)^2 + \frac{M_P^2}{\xi_h} \frac{\varsigma}{\cos^2 \theta} (\tan^2 \theta + \varsigma)^2 (\partial \theta)^2,
\]

(2.17)

with

\[
\eta = \frac{\xi_\chi}{\xi_h} \quad \text{and} \quad \varsigma = \frac{(1 + 6\xi_h)\xi_\chi}{(1 + 6\xi_h)\xi_h}.
\]

(2.18)

The potential (2.13) is naturally divided into a scale-invariant part, depending only on the \(\theta\) field, and a scale-breaking part, proportional to \(\Lambda_0\) and depending on both \(\theta\) and \(\rho\). These are respectively given by

\[
\tilde{U}(\theta) = \frac{\lambda M_P^4}{4\xi_h^2} \left(\frac{\sin^2 \theta}{\sin^2 \theta + \varsigma \cos^2 \theta}\right)^2, \quad \tilde{U}_{\Lambda_0}(\rho, \theta) = \frac{\Lambda_0}{\xi_h^2} \left(1 + 6\xi_h\right)^2 \frac{e^{-4\rho/M_P}}{(\sin^2 \theta + \varsigma \cos^2 \theta)^2},
\]

(2.19)

where we have safely neglected the contribution of \(\alpha\) and \(\beta\) in Eq. (2.13). Note that the non-minimal couplings of the fields to gravity with \(\Lambda_0 > 0\) naturally generate a “run-away” potential for the physical dilaton, similar to those considered in the pioneering works on quintessence [33, 34, 38].

The inflationary period of the expansion of the Universe takes place for field values \(\xi_h h^2 \gg \xi_\chi \chi^2\). From the definition of the angular variable \(\theta\) in Eq. (2.16), this corresponds to \(\tan^2 \theta \gg \eta\). In that limit, we can neglect the \(\eta\) term in the kinetic term (2.17) and perform an extra field redefinition

\[
r = \gamma^{-1} \rho \quad \text{and} \quad |\phi'| = \phi_0 - \frac{M_P}{a} \tanh^{-1} \left[ \sqrt{1 - \varsigma \cos \theta} \right],
\]

(2.20)

where

\[
\gamma = \sqrt{\frac{\xi_\chi}{1 + 6\xi_h}} \quad \text{and} \quad a = \sqrt{\frac{(1 - \varsigma)}{\varsigma}}.
\]

(2.21)

The variable \(\phi'\) is periodic and defined in the compact interval \(\phi' \in [-\phi_0, \phi_0]\), with \(\phi_0 = M_P/a \tanh^{-1} \left[ \sqrt{1 - \varsigma} \right]\) the value of the field at the beginning of inflation. In terms of these variables the Lagrangian (2.12) takes a very simple form

\[
\mathcal{L} = \frac{M_P^2}{2} \tilde{R} - \frac{\varsigma \cosh^2[a\phi/M_P]}{2} (\partial r)^2 - \frac{1}{2} (\partial \phi)^2 - \tilde{U}(\phi) - \tilde{U}_{\Lambda_0}(r, \phi),
\]

(2.22)

with \(\phi = \phi_0 - |\phi'|\). The potential (2.19) becomes

\[
\tilde{U}(\phi) = \frac{\lambda M_P^4}{4\xi_h^2(1 - \varsigma)^2} \left(1 + \varsigma \cosh^2[a\phi/M_P]\right)^2, \quad \tilde{U}_{\Lambda_0}(r, \phi) = \frac{\Lambda_0}{\gamma^2} \varsigma^2 \cosh^4[a\phi/M_P] e^{-4\gamma r/M_P},
\]

(2.23)

whose scale-invariant part \(\tilde{U}(\phi)\) resembles the potential of the simplest Higgs inflationary scenario [2], cf. Fig. 1. The analytical expressions for the amplitude and the spectral tilt of scalar perturbations at order \(O(\xi_\chi, 1/\xi_h, 1/N^*)\) can be easily calculated to obtain [30]

\[
P_c(k_0) \simeq \frac{\lambda \sinh^2[4\xi_\chi N^*]}{1152\pi^2 \xi_\chi^2 \xi_h^2}, \quad n_s(k_0) \simeq 1 - 8\xi_\chi \coth(4\xi_\chi N^*),
\]

(2.24)

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6 Strictly speaking, the condition \(\tan^2 \theta \gg \eta\) holds beyond the inflationary region \(\xi_h h^2 \gg \xi_\chi \chi^2\) and includes also the reheating stage.

7 Note that the definition of the angular variable \(\phi\) used in this work is slightly different from that appearing in Ref. [33]. The new parametrization makes explicit the symmetry of the potential and shifts its minimum to make it coincide with that in Higgs-inflation.
where $N^*$ denotes the number of e-folds between the moment at which the pivot scale $k_0/a_0 = 0.002$ Mpc$^{-1}$ exited the horizon and the end of inflation. Note that for $1 < 4\xi\chi N^* \ll 4N^*$, the expression for the tilt simplifies and becomes linear in $\xi\chi$.

$$n_s(k_0) \simeq 1 - 8\xi\chi . \quad (2.25)$$

An interesting cosmological phenomenology arises with the peculiar choice $\beta = 0$. In this case, the DE dominated period in the late Universe depends only on the dilaton field $\rho$, which give rise to an intriguing relation between the inflationary and DE domination periods. Let us start by noticing that around the minimum of the potential the value of $\theta$ is very close to zero. In that limit, $\tan^2 \theta \ll \eta$, which prevents the use of the field redefinition $\phi^2$. The appropriate redefinitions needed to diagonalize the kinetic term (2.17) in this case turn out to be

$$r = \gamma^{-1} \rho \quad \text{and} \quad \phi' \simeq \frac{M_P}{2\sqrt{\xi\chi}} \theta . \quad (2.26)$$

Using Eqs. (2.17) and (2.19) it is straightforward to show that the part of the theory associated to the Higgs field $\phi$ simplifies to the SM one. The resulting scale-invariance breaking potential for the dilaton is still of the “run-away” type

$$\tilde{U}_{\Lambda_0}(r) = \frac{\Lambda_0}{\gamma^4} e^{-4\gamma r/M_P} , \quad (2.27)$$

making it suitable for playing the role of quintessence. Let us assume that $\tilde{U}_{\Lambda_0}$ is negligible during the radiation and matter dominated stages but responsible for the present accelerated expansion of the Universe. In that case, it is possible to write the following relation between the equation of state parameter $\omega_r$ of the $r$ field and its relative abundance $\Omega_r$ [30]

$$1 + \omega_r = \frac{16\gamma^2}{3} \left[ \frac{1}{\sqrt{\Omega_r}} - \frac{1}{2} \left( \frac{1}{\Omega_r} - 1 \right) \log \frac{1 + \sqrt{\Omega_r}}{1 - \sqrt{\Omega_r}} \right]^2 . \quad (2.28)$$

$^8$ Some arguments in favour of the $\beta = 0$ case can be found in Ref. [30][32][39].
For the present DE density $\Omega_{DE} = \Omega_r \simeq 0.74$, the above expression yields

$$1 + \omega_{DE} = \frac{8}{3} \frac{\xi_x}{1 + 6\xi_x}.$$ (2.29)

Comparing Eqs. (2.25) and (2.29), it follows that the deviation of the scalar tilt $n_s$ from the scale-invariant one is proportional to the deviation of the DE equation of state from a cosmological constant$^9$ and makes the Higgs-Dilaton scenario rather unique. We will be back to this point in Section IV where we will show that the consistency relation (2.30) still holds even in the presence of quantum corrections computed within the “minimal setup”.

### III. THE DYNAMICAL CUT-OFF SCALE

Following Ref. [27], we now turn to the determination of the energy domain where the Higgs-Dilaton model can be considered as a predictive effective field theory. This domain is bounded from above by the field-dependent cut-off $\Lambda(\Phi)$, i.e. the energy where perturbative tree-level unitarity is violated [41]. At energies above that scale, the theory becomes strongly-coupled and the standard perturbative methods fail. In order to determine this (background dependent) energy scale, two related methods, listed below, can be used.

1. Expand the generic fields of the theory around their background values
   
   $$\Phi(x, t) = \bar{\Phi} + \delta\Phi(x, t),$$
   
   such that all kind of higher-dimensional non-renormalizable operators
   
   $$c_n \frac{O_n(\delta\Phi)}{[\Lambda(\bar{\Phi})]^{n-4}},$$
   
   with $c_n \sim O(1)$ appear in the resulting action. These operators are suppressed by appropriate powers of the field-dependent coefficient $\Lambda(\bar{\Phi})$, which can be identified as the cut-off of the theory. This procedure gives us only a lower estimate of the cut-off, since it does not take into account the possible cancelations that might occur between the different scattering diagrams.

2. Calculate at which energy each of the N-particle scattering amplitudes hit the unitarity bound. The cut-off will then be the lowest of these scales.

In what follows we will apply these two methods to determine the effective cut-off of the theory. We will start by applying the method (1) to compute the cut-off associated with the gravitational and scalar interactions. The cut-off associated to the gauge and fermionic sectors will be obtained via the method (2).

#### A. Cut-off in the scalar-gravity sector

We choose to work in the original Jordan frame where the Higgs and dilaton fields are non-minimally coupled to gravity$^{11}$. Expanding these fields around a static background$^{12}$

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}, \quad \chi = \bar{\chi} + \delta\chi, \quad h = \bar{h} + \delta h,$$

(3.3)
we obtain the following kinetic term for the quadratic Lagrangian of the gravity and scalar sectors

\[ \mathcal{K}^{G+S}_2 = \frac{\xi_\chi \chi^2 + \xi_h \tilde{h}^2}{8} (\delta g^{\mu \nu} \Box g_{\mu \nu} + 2 \partial_\mu \delta g^{\mu \nu} \partial^\rho g_{\mu \rho} - 2 \partial_\mu \delta g^{\mu \nu} \partial_{\nu} g - \delta g \Box g) \]

(3.4)

The leading higher-order non-renormalizable operators obtained in this way are given by

\[ \xi_\chi (\delta \chi)^2 \Box g , \quad \xi_h (\delta h)^2 \Box g . \]

(3.5)

Note that these operators are written in terms of quantum excitations with non-diagonal kinetic terms. In order to properly identify the cut-off of the theory, we should determine the normal modes that diagonalize the quadratic Lagrangian (3.4). After doing that, and using the equations of motion to eliminate artificial degrees of freedom, we find that the metric perturbations depend on the scalar fields perturbations, a fact that is implicit in the Lagrangian (3.4). The gravitational part of the above action can be recast into canonical form in terms of a new metric perturbation \( \delta g_{\mu \nu} \) given by

\[ \delta g_{\mu \nu} = \frac{1}{\sqrt{\xi_\chi \chi^2 + \xi_h \tilde{h}^2}} \left[ (\xi_\chi \chi^2 + \xi_h \tilde{h}^2) \delta g_{\mu \nu} + 2 \delta g_{\mu \nu} (\xi_\chi \chi \delta \chi + \xi_h \tilde{h} \delta h) \right] . \]

(3.6)

The cut-off scale associated to purely gravitational interactions becomes in this way the effective Planck scale in the Jordan frame

\[ \Lambda_P^J = \xi_\chi \chi^2 + \xi_h \tilde{h}^2 . \]

(3.7)

The remaining non-diagonal kinetic term for the scalar perturbations \( (\delta \Phi^1, \delta \Phi^2) = (\delta h, \delta \chi) \) is given in compact matrix notation by

\[ \mathcal{K}_2^S = -\frac{1}{2} \tilde{\kappa}^J_{ij} \partial_\mu \delta \Phi^i \partial^\mu \delta \Phi^j , \]

(3.8)

where \( \tilde{\kappa}^J_{ij} = \Omega^2 \kappa^J_{ij} \) is the Jordan frame analogue of Eq. (2.15) and depends only on the background values of the fields, i.e.

\[ \tilde{\kappa}^J_{ij} = \frac{1}{\xi_\chi \chi^2 + \xi_h \tilde{h}^2} \begin{pmatrix} \xi_\chi \chi^2 (1 + 6 \xi_\chi) + \xi_h \tilde{h}^2 & 6 \xi_\chi \chi \xi_h \tilde{h} \\ 6 \xi_\chi \chi \xi_h \tilde{h} & \xi_\chi \chi^2 + \xi_h \tilde{h}^2 (1 + 6 \xi_\chi) \end{pmatrix} . \]

(3.9)

In order to diagonalize the above expression we make use of the following set of variables

\[ \delta \tilde{\chi} = \sqrt{\frac{\xi_\chi \chi^2 (1 + 6 \xi_\chi) + \xi_h \tilde{h}^2 (1 + 6 \xi_\chi)}{(\xi_\chi \chi^2 + \xi_h \tilde{h}^2)(\xi_\chi \chi^2 + \xi_h \tilde{h}^2)}} (\xi_\chi \chi \delta \chi + \xi_h \tilde{h} \delta h) , \]

\[ \delta \tilde{h} = \frac{1}{\sqrt{\xi_\chi \chi^2 + \xi_h \tilde{h}^2}} (-\xi_h \tilde{h} \delta \chi + \xi_\chi \chi \delta h) . \]

(3.10)

Note here that this is precisely the change of variables (up to an appropriate rescaling with the conformal factor \( \Omega \)) needed to diagonalize the kinetic terms for the scalar perturbations in the Einstein frame. To see this, it is enough to start from Eq. (2.14) and expand the fields around their background values \( \Phi^i \to \Phi^i + \delta \Phi^i \). Keeping the terms with the lowest power in the excitations, \( \tilde{K} = \tilde{\kappa}^E_{ij} \partial_\mu \delta \Phi^i \partial^\mu \delta \Phi^j + \mathcal{O}(\delta \Phi^3) \), it is straightforward to show that the previous expression can be diagonalized in terms of

\[ \delta \tilde{\chi} = \tilde{\Omega}^{-1} \frac{\xi_\chi \chi^2 (1 + 6 \xi_\chi) + \xi_h \tilde{h}^2 (1 + 6 \xi_\chi)}{(\xi_\chi \chi^2 + \xi_h \tilde{h}^2)(\xi_\chi \chi^2 + \xi_h \tilde{h}^2)} (\xi_\chi \chi \delta \chi + \xi_h \tilde{h} \delta h) , \]

\[ \delta \tilde{h} = \tilde{\Omega}^{-1} \frac{1}{\sqrt{\xi_\chi \chi^2 + \xi_h \tilde{h}^2}} (-\xi_h \tilde{h} \delta \chi + \xi_\chi \chi \delta h) . \]

(3.11)

Written in terms of the canonically normalized variables (3.6) and (3.10) these operators read

\[ \frac{1}{\Lambda_1} (\delta \tilde{h})^2 \Box g , \quad \frac{1}{\Lambda_2} (\delta \tilde{\chi})^2 \Box g , \quad \frac{1}{\Lambda_3} (\delta \tilde{\chi}) (\delta \tilde{h}) \Box g . \]

(3.12)
where the different cut-off scales are given by

\[
\Lambda_1 = \frac{\xi^2 h^2 + \xi^2 h h^2}{\xi^2 h^2 + \xi^2 h^2},
\]  
(3.13)

\[
\Lambda_2 = \frac{(\xi^2 h^2 + \xi^2 h h^2)(\xi^2 h^2 + \xi^2 h h^2)}{\xi^2 h^2 + \xi^2 h^2},
\]  
(3.14)

\[
\Lambda_3 = \frac{(\xi^2 h^2 + \xi^2 h h^2)(\xi^2 h^2 + \xi^2 h h^2)}{\xi^2 h^2 + \xi^2 h^2}.
\]  
(3.15)

The effective cut-off of the scalar theory at a given value of the background fields will be the lowest of the previous scales. We will be back to this point in Section \textit{III C}.

\section*{B. Cut-off in the gauge and fermionic sectors}

Let us now move to the cut-off associated with the gauge sector. Since we are working in the unitary gauge for the SM fields, it is sufficient to look at the tree-level scattering of non-abelian vector fields with longitudinal polarization. It is well known that in the SM the “good” high energy behaviour of these processes is the result of cancellations that occur when we take into account the interactions of the gauge bosons with the excitations \(\delta h\) of the Higgs field\textsuperscript{13} [12, 13].

In our case, even though purely gauge interactions remain unchanged, the graphs involving the Higgs field excitations are modified due to the non-canonical kinetic term. This changes the pattern of the cancellations that occur in the standard Higgs mechanism, altering therefore the asymptotic behaviour of these processes. As a result, the energy scale where this part of the theory becomes strongly coupled becomes lower.

To illustrate how this happens, let us consider the \(W_LW_L \to W_LW_L\) scattering in the \(s\)-channel. The relevant part of the Lagrangian is

\[ g m_W W^+ W^{-\mu} \delta h, \]  
(3.16)

where \(m_W \sim g \bar{h}\). After diagonalizing the kinetic term for the scalar fields with the change of variables (3.10), the above expression becomes

\[ g' m_W W^+ W^{-\mu} \delta h + g'' m_W W^+ W^{-\mu} \delta \bar{X}, \]  
(3.17)

where the effective coupling constants \(g'\) and \(g''\) are given by

\[ g' = \frac{\xi X \bar{X}}{\sqrt{\xi^2 \bar{X}^2 + \xi^2 \bar{X} h^2}}, \quad g'' = \frac{\xi h \bar{h}}{\sqrt{\xi^2 \bar{X}^2 + \xi^2 \bar{X} h^2}} \sqrt{\xi^2 \bar{X}^2 + \xi^2 \bar{X} h^2} \left(\xi X \bar{X} + \xi X \bar{X} + \xi X \bar{X} + \xi h \bar{h} \right), \]  
(3.18)

From the requirement of tree unitarity of the \(S\)-matrix, it is straightforward to show that the scattering amplitude of this interaction hits the perturbative unitarity bound at energies

\[ \Lambda_G \approx \sqrt{\xi^2 \bar{X}^2(1 + \xi X \bar{X} + \xi h \bar{h} \bar{h} \bar{h} \bar{h} \bar{h} \bar{h} \bar{h})}. \]  
(3.19)

It is interesting to compare the previous expression with the results for the gauge cut-off of the simplest Higgs inflationary model [27]. In order to do that, let us consider two limiting cases: the inflationary/high-energy period corresponding to field values \(\xi X \bar{X} \ll \xi X \bar{X} \) and the low-energy regime at which \(\xi X \bar{X} \gg \xi X \bar{X} \). In these two cases, the above expression simplifies to

\[ \Lambda_G \approx \begin{cases} \frac{\bar{h}}{\xi_X} & \text{for } \xi X \bar{X} \ll \xi X \bar{X}, \\ \sqrt{\xi_X} & \text{for } \xi X \bar{X} \gg \xi X \bar{X}, \end{cases} \]  
(3.20)

\textsuperscript{13} In the absence of the Higgs field, the scattering amplitudes grow as the square of the center-of-mass energy, due to the momenta dependence of the longitudinal vectors \(\sim g'/m_W\).
in agreement with the Higgs inflation model.

To identify the cut-off of the fermionic part of the Higgs-Dilaton model, we consider the chirality non-conserving process $f f \rightarrow W_L W_L$. This interaction receives contributions from diagrams with $\gamma$ and $Z$ exchange ($s$-channel) and from a diagram with fermion exchange ($t$-channel). In the asymptotic high-energy limit, the total amplitude of these graphs grows linearly with the energy at the center of mass. Once again, the $s$-channel diagram including the Higgs excitations unitarizes the associated amplitude \[44–46\]. Following therefore the same steps as in the calculation of the gauge cut-off, we find that this part of the theory enters into the strong-coupling regime at energies

$$\Lambda_F \simeq y^{-1} \xi h \bar{h}^2 (1 + 6\xi h) + \xi h \bar{h}^2 (1 + 6\xi h)\xi h, \quad (3.21)$$

where $y$ is the Yukawa coupling constant. The above cut-off is higher than that of the SM gauge interactions \[3.19\] during the whole evolution of the Universe.

C. Comparison with the energy scales in the early and late Universe

In this section we compare the cut-offs found above with the characteristic energy scales in the different periods during the evolution of the Universe. If the typical momenta involved in the different processes are sufficiently small, the theory will remain in the weak coupling limit, making the Higgs-Dilaton scenario self-consistent.

Let us start by considering the inflationary period, characterized by $\xi h \bar{h}^2 \gg \xi \bar{\chi}^2$. As shown in Fig. 2, the lowest cut-off in this region is the one associated with the gauge interactions $\Lambda_G$. The typical momenta of the scalar perturbations produced during inflation are of the order of the Hubble parameter at that time. This quantity can be easily estimated in the Einstein frame, where it is basically determined by the energy stored in the inflationary potential \[2.23\]. We obtain $H \sim \sqrt{\lambda M_P / \xi_h}$. When transformed to the Jordan frame ($H = \Omega H$) this quantity becomes $H \sim \sqrt{\xi_h \xi_h}$, which is significantly below the cut-off scale $\Lambda_G$ in that region. The same conclusion is obtained for the
total energy density, which turns out to be much smaller than $\Lambda_G^4$. Moreover, the cut-off $\Lambda_G$ exceeds the masses of all particles in the Higgs background, allowing a self-consistent estimate of radiative corrections (cf. Section IV).

After the end of inflation, the field $\phi$ starts to oscillate around the minimum of the potential with a decreasing amplitude, due to the expansion of the Universe and particle production. This amplitude varies between $M_0/\sqrt{\xi_h}$ and $M_0/\xi_h$, where $M_0 = \sqrt{\xi_h} \lambda$ is the asymptotic Planck scale in the low energy regime. As shown in Fig. 1 the curvature of the Higgs-Dilaton potential around the minimum coincides (up to $\mathcal{O}(\xi_h)$ corrections) with that of the Higgs-inflation scenario. All the relevant physical scales, including the effective gauge and fermion masses, agree, up to small corrections, with those in Higgs-inflation [17]. This allows us to directly apply the results of [14, 48] to the Higgs-Dilaton scenario. According to these works, the typical momenta of the gauge bosons produced at the minimum of the potential in the Einstein frame is of order $k \sim (m_A/M)^{2/3} M$, with $m_A$ the mass of the gauge bosons in the Einstein frame and $M = \sqrt{\lambda/3} M_P/\xi_h$ the curvature of the potential around the minimum. After transforming to the Jordan frame we obtain $k \sim (m_A^4 M_P^{-2})^{1/6} \Lambda_G$, with $g$ the weak coupling constant. The typical momentum of the created gauge bosons is therefore parametrically below the gauge cut-off scale $(A)$ in that region.

At the end of the reheating period, $\xi_h \chi^2 \gg \xi_h \hbar^2$, the system settles down to the minimum of the potential $U(\phi)$, cf. Eq. (2.23). In that region the effective Planck mass coincides with the value $M_0$. The cut-off scale becomes $\Lambda_1 \simeq \sqrt{\xi_h} \chi/\xi_h \simeq M_P/\xi_h$. This value is much higher than the electroweak scale $m_H^2 \sim 2\alpha/\xi_h M_P$ (cf. Eq. (2.5)), where all the physical processes take place. We conclude therefore that perturbative unitarity is maintained for all the relevant processes during the whole evolution of the Universe.

**IV. QUANTUM CORRECTIONS**

In this section we concentrate on the radiative corrections to the inflationary potential and on their influence on the predictions of the model.

Our strategy is as follows. We regularize the quantum theory in such a way that all multi-loop diagrams are finite, whereas the exact symmetries of the chosen classical action (gauge, diffeomorphisms and scale invariance) remain intact. Moreover, we will require the regularization to respect the approximate shift symmetry of the dilaton field in the Jordan frame, cf. Section II. Then we add to the classical action an infinite number of counter-terms (including the finite parts as well) which remove all the divergences from the theory and do not spoil the exact and approximate symmetries of the classical action. Since the theory is not renormalizable, these counter-terms will have a different structure from that of the classical action. In particular, terms that are non-analytic with respect to the Higgs and dilaton fields will appear [49]. They can be considered as higher-dimensional operators, suppressed by the field-dependent cut-offs. For consistency with the analysis made earlier in this work, we demand these cut-offs to exceed those found in Section III.

An example of the subtraction procedure which satisfies all the requirements formulated above has been constructed in Ref. [39] (see also earlier discussion in [30]). It is based on dimensional regularization in which the ’t Hooft-Veltman normalization point $\mu$ is replaced by some combination of the scalar fields with an appropriate dimension, $\mu^2 \rightarrow F(\chi, h)$ (we underline that we use the Jordan frame here for all definitions). The infinite part of the counter-terms is defined as in $\mathcal{MS}$ prescription, i.e. by subtracting the pole terms in $c$, where the dimensionality of space-time is $D = 4 - 2c$. The finite part of the counter-terms has the same operator structure as the infinite part, including the parametric dependence on the coupling constants.

Although the requirement of the structure of higher-dimensional operators, formulated in the previous paragraphs puts important constraints on the function $F(\chi, h)$, its precise form is not completely determined [39, 39, 52], and the physical results do depend on the choice of $F(\chi, h)$. This somewhat mysterious fact from the point of view of uniquely defined classical theory [2.2] becomes clear if we recall that we are dealing with a non-renormalizable theory. The quantization of this kind of theories requires the choice of a particular classical action together with a set of subtraction rules. The ambiguity in the choice of the field-dependent normalization point $F(\chi, h)$ simply reflects our ignorance about the proper set of rules. Different subtractions prescriptions applied to the same classical action do produce unequal results. Sometimes this ambiguity is formulated as a dependence of quantum theory on the choice of conformally related frames in scalar-tensor theories [51]. The use of the same quantization rules in different frames would lead to quantum theories with different choices of $F(\chi, h)$.

Among the many possibilities, the simplest and most natural choice is to identify the normalization point in the Jordan frame with the gravitational cut-off [3.7],

$$\mu_I^2 \propto \xi_h \chi^2 + \xi_h \hbar^2,$$

(4.1)

which corresponds to the scale-invariant prescription of Ref. [39]. In the Einstein frame the previous choice becomes
standard (field-independent)
\[ \tilde{\mu}_I^2 \propto M_P^2. \]  (4.2)

A second possibility is to choose the scale-invariant direction along the dilaton field, i.e.
\[ \mu_{II}^2 \propto \xi \chi^2. \]  (4.3)

When transformed to the Einstein frame it becomes
\[ \tilde{\mu}_{II}^2 \propto \frac{\xi \chi^2 M_P^2}{\xi \chi^2 + \xi h^2}, \]  (4.4)

and coincides with the prescription II of Ref. [10] at the end of inflation.

In what follows we will use this “minimal setup” for the analysis of the radiative corrections. It will be more convenient to work in the Einstein frame, where the coupling to gravity is minimal and all non-linearities are moved to the matter sector. The total action in the Einstein frame naturally divides into an Einstein-Hilbert (EH) part, a purely scalar piece involving only the Higgs and dilaton (HD) fields and a part corresponding to the chiral SM (CH) without the radial mode of the Higgs boson [10, 53, 54]
\[ S = S_{EH} + S_{HD} + S_{CH}. \]  (4.5)

In the next section we estimate the contribution of the scalar sector to the effective inflationary potential, postponing the study of the chiral SM to Section IV B. All the computations will be performed in flat spacetime, since the inclusion of gravity does not modify the results\[14\].

A. Scalar contribution to the effective inflationary potential

Let us start by reminding that the initial value of the dilaton field has to be sufficiently large to keep its present contribution to DE at the appropriate observational level [30]. The latter fact allows us to neglect the exponentially suppressed contributions to the effective action stemming from \( \tilde{U}_\Lambda_0 \) in Eq. (2.23). As a result, the remaining corrections due to the dilaton field will emerge from its non-canonical kinetic term, whereas all the radiative corrections due to the Higgs field will emerge from the inflationary potential.

The construction of the effective action for the scalar sector of the theory is most easily done in the following way: expand the action (2.22) near the constant background of the dilaton and the Higgs fields and drop the linear terms in perturbations. After that, compute all the vacuum diagrams to account for the potential-type corrections and all the diagrams with external legs to account for the kinetic-type corrections.

1. Dilaton contribution

Let us consider first the quantum corrections to the dilaton itself. Since our subtraction procedure respects the symmetries of the classical action (in particular scale invariance, corresponding to the shift symmetry of the dilaton field \( r \) in the Einstein frame), no potential terms for the dilaton can be generated. Thus, the loop expansion can only create two types of contributions, both stemming from its kinetic term. The first type are corrections to the propagator of the field, and as we will show below they are effectively controlled by \( (m_H/M_P)^k \), with \( m_H^2 \equiv -\tilde{U}''(\phi) \) and \( k \) the number of loops under consideration. The second type are operators with more derivatives of the field suppressed by appropriate powers of the scalar cut-off \( M_P \). One should bear in mind that the appearance of these operators in the effective action is expected and consistent. As discussed in the previous section, their presence does not affect the dynamics of the model, since the scalar cut-off is much larger than the characteristic momenta of the particles involved in all physical processes throughout the whole history of the Universe.

To demonstrate explicitly what we described above, let us consider some of the associated diagrams. Following the ideas of Ref. [39], we perform the computations in dimensional regularization in \( D = 4 - 2\epsilon \) dimensions. We avoid therefore the use of other regularizations schemes, such as cut-off regularization, where the scale invariance of

\[14\] We recall that, in the Einstein frame, the coupling among SM particles and gravity is minimal.
the theory is badly broken at tree level. The magnitude of the corrections in dimensional regularization is of the order of the masses of the particles running in the loops, or in the case of the massless dilaton, its momentum. The structure of the corrections can be therefore guessed by simple power-counting and it becomes apparent already at the one-loop order. We get

\[ c_{d_{1,1}}(\bar{\phi}) \left( \frac{1}{\ell} + f \right) \left( \frac{m_H}{M_P} \right)^2 (\partial r)^2 , \]

\[ = c_{d_{2,2}}(\bar{\phi}) \left[ \left( \frac{1}{\ell} + f' \right) \left( \frac{m_H}{M_P} \right)^2 + d \left( \frac{\partial}{M_P} \right)^2 \right] (\partial r)^2 , \]

where the Higgs and dilaton fields are represented by solid and dashed lines respectively. To keep the expressions as compact as possible we set \( \frac{1}{\ell} = \frac{1}{\epsilon} - \gamma + \log 4\pi \) and denoted by \( f \) and \( f' \) the finite parts of the diagrams, whose values depend on the normalization point \( \mu \). The higher-derivative operator in the second diagram is included for completion, but turns out to vanish accidentally in this particular case. Numerical factors are absorbed into the background-dependent coefficients \( c_{d_{k,V}}(\bar{\phi}) \), which depend on the particular diagram \( d_i \) under consideration, the number of loops \( k \) and the number of vertices \( V \). Their values are always smaller than unity, and vary slightly with the background value \( \bar{\phi} \). Their specific form of is presented in the Appendix B.

In two-loops the situation is somehow similar. The divergent (and finite) part of the corrections (consider for example the diagrams presented in Fig. 3) is proportional to

\[ c_{d_{k,V}}(\bar{\phi}) \left[ \left( \frac{m_H}{M_P} \right)^4 + \left( \frac{m_H}{M_P} \right)^2 \left( \frac{\partial}{M_P} \right)^2 + \left( \frac{\partial}{M_P} \right)^4 \right] (\partial r)^2 , \quad V \leq 4 . \quad (4.6) \]

It is not difficult to convince oneself that this happens in the higher order diagrams as well. The structure of the corrections is therefore proportional to

\[ c_{d_{k,V}}(\bar{\phi}) \left[ \left( \frac{m_H}{M_P} \right)^{2k} + \left( \frac{m_H}{M_P} \right)^{2k-2} \left( \frac{\partial}{M_P} \right)^2 + \ldots + \left( \frac{m_H}{M_P} \right)^2 \left( \frac{\partial}{M_P} \right)^{2k-2} + \left( \frac{\partial}{M_P} \right)^{2k} \right] (\partial r)^2 , \quad (4.7) \]

up to \( \mathcal{O}(1) \) numerical factors. Notice that some operators involving higher derivatives were already present at lower orders, but they reappear with extra suppression factors \( (m_H/M_P)^2 \) on top of the scalar cut-off \( M_P \). The corrections from diagrams with gauge bosons and fermions running inside the loops are given also by (4.7), by consistently replacing \( m_H \) by the mass of the particle considered.

FIG. 3. Some of the two-loop diagrams for the dilaton.

2. Higgs contribution

We now turn to the corrections to the Higgs field. Once again we consider first the potential-type contributions. The situation now is more complicated, since the effective potential for the Higgs field \( \phi \) will be modified by terms

\[ ^{15} \text{Similar arguments about the artifacts created by regularization methods that explicitly break scale invariance can be found for instance in Ref. 55.} \]

\[ ^{16} \text{We introduce the index} \quad d_i \text{to distinguish between the diagrams with the same number of vertices but different combinations of hyperbolic functions that appear in higher loops.} \]
stemming from the scale-invariant part of the tree-level potential \([2.23]\) as well as from the non-canonical kinetic term of the dilaton field \(r\), with the latter starting from the second order in perturbation theory.

Let us start by considering the contributions due to the tree-level potential. To keep the notation as simple as possible, we express the scale-invariant part of the potential \([2.23]\) in the following compact form

\[
\tilde{U}(\phi) = \lambda U_0 \left( u_0 + \sum_{n=1}^{\infty} u_n \cosh[2n\alpha\phi/M_P] \right), \quad U_0 = \frac{M_P^4}{4\xi_h^2(1-\zeta)^2},
\]  

(4.8)

where, for completion, we have explicitly recovered the \(\alpha\) and \(\beta\) dependence and defined

\[
u = c^2 - c\sigma + \frac{3\alpha^2}{8} + \frac{3\beta'}{2}, \quad u_1 = \frac{c^2}{2} - c\sigma - 2\beta', \quad u_2 = \frac{c^2}{8} + \frac{3\beta'}{2},
\]  

(4.9)

with

\[
c = 1 + \alpha \frac{1 + 6\xi_h}{\lambda} \frac{1 + 6\xi_h}{\lambda}, \quad \sigma = \zeta + \alpha \frac{1 + 6\xi_h}{\lambda} \frac{1 + 6\xi_h}{\lambda}, \quad \beta' \equiv \frac{\beta}{\lambda} \left( \frac{1 + 6\xi_h}{1 + 6\xi_h} \right)^2.
\]  

(4.10)

Expanding the field around its background value \(\bar{\phi}\), we get

\[
\tilde{U}(\bar{\phi} + \delta\phi) = \lambda U_0 \sum_{n=1}^{\infty} \sum_{l=0}^{\infty} c_{n,l} \cosh[2n\alpha\bar{\phi}/M_P] \left( \frac{2l+2n\alpha\bar{\phi}}{M_P} \right)^l \left( \frac{a\delta\phi}{M_P} \right)^{2l+2n\alpha\bar{\phi}} \cosh[2l\delta\phi/M_P] \cosh[2n\alpha\bar{\phi}/M_P],
\]

(4.11)

where \(c_{n,l}\) and \(d_{n,l}\) account for numerical coefficients and combinatorial factors. Since the theory is non-renormalizable, the perturbative expansion creates terms which do not have the same background dependence of the original potential. Up to numerical factors, the contributions turn out to be of the form\(^{17}\)

\[
\frac{\lambda^{i+j} M_P^4}{4\xi_h^2(1-\zeta)^2} \left( \frac{1}{\epsilon} \right)^{f_{i,j}} \sum_{n,m} u_n^i u_m^j \cosh[2n\alpha\bar{\phi}/M_P] \sinh[2n\alpha\bar{\phi}/M_P] \cosh[2m\alpha\bar{\phi}/M_P] \sinh[2m\alpha\bar{\phi}/M_P],
\]

(4.12)

where \(f_{i,j}\) denotes the (finite) integration constant, and \(g(1/\epsilon)\) is a function of the divergent terms. Note that if we set \(\beta = 0\), we make sure that terms which contribute to the cosmological constant \([2.6]\) will not be generated by the loop expansion.

By inspection of the structure of divergences, we can see that the leading corrections are those appearing with the lowest power in \(\zeta\). To gain insight on their contribution, we calculate the finite part of Eq. (4.12) for the maximal value of the hyperbolic functions. This corresponds to \(\phi_{\text{max}} = \phi_0 \equiv M_P/a \tanh^{-1} \sqrt{1-\zeta}\). We get

\[
\frac{\lambda^{i+j} M_P^4}{4\xi_h^2(1-\zeta)^2} \left( \frac{1}{\epsilon} \right)^{f_{i,j}} \sum_{n,m} u_n^i u_m^j \cosh[2n\alpha\bar{\phi}/M_P] \sinh[2n\alpha\bar{\phi}/M_P] \cosh[2m\alpha\bar{\phi}/M_P] \sinh[2m\alpha\bar{\phi}/M_P] \bar{\phi} = \phi_{\text{max}} \sim \left( \frac{\lambda \zeta}{4\xi_h^2} \right)^{i+j} f_{i,j},
\]

(4.13)

which makes the corrections coming from the order \(i+j+1\) negligible compared to the ones from \(i+j\) order. In the last step we have simply set \(c = 1, \sigma = \zeta\), which, given the small value of the parameter \(\alpha\) appearing in Eq. (4.10), constitutes a very good approximation.

As we mentioned earlier, potential-type corrections to the Higgs field are also generated from diagrams associated to the kinetic term of the dilaton \(r\), starting from two loops. This happens because the first order vacuum diagrams with dilaton running in the loop, vanish. If we consider higher loop diagrams, like those in Fig. 4(b) but without momenta in the external legs, we see that even though the background dependence of the corrections is complicated due to the non-canonically normalized dilaton that runs inside the loops, their contributions to the effective action are of the same order as those in Eq. (4.13).

We now turn to the kinetic-type corrections to the Higgs field. By that we mean corrections to the propagator, as well as terms with more derivatives of the field suppressed by the scalar cut-off. The first type of contributions come

\(^{17}\) To maintain the expressions as compact as possible we decided not to express the result in terms of \(m_H/M_P\).
FIG. 4. Characteristic diagrams produced by the non-canonical kinetic term of the dilaton field $r$. Solid and dashed lines represent the Higgs and dilaton fields respectively. The first one-loop diagram presented in (a) vanishes in dimensional regularization due to the massless character of the dilaton field. On the other hand, the second diagram gives rise to higher derivative terms of the Higgs field. In (b) we consider two and three loop diagrams which, apart from generating higher dimensional operators, contribute to the effective potential once we amputate them.

only from the scale-invariant part of the potential given by (4.8), when the momenta associated to the external legs are considered. It is not difficult to show that these are precisely of the same form as those in (4.12). The second type of contributions, i.e. the higher dimensional operators, are generated both from the Higgs potential at higher loops, as well as from the non-vanishing diagrams associated to the non-canonical kinetic term of the dilaton. The terms we get are proportional to

\[ \frac{\partial^2}{M_P^2} (\partial \phi)^2, \quad \frac{\partial^4}{M_P^4} (\partial \phi)^2 \ldots, \]

and they can be safely neglected for the typical momenta involved in the different epochs of the evolution of the Universe.

Before moving on, we would like to comment on the appearance of mixing terms with derivatives of the fields. These manifest themselves when we consider diagrams with both fields in the external legs. They are higher dimensional operators, and it can be shown that they appear suppressed by the scalar cut-off of the theory, as before.

Since the kinetic-type operators do not modify the dynamics, we will consider only potential-type corrections to estimate the change in the tree-level predictions of the model. At one-loop, the contribution of the scalar sector to the inflationary potential becomes

\[ \Delta \tilde{U}_{HD} \simeq \frac{U_0}{64\pi^2} \frac{\lambda a^4}{\xi^2(1-\varsigma)^2} \left( \frac{1}{\zeta} + f_{2,0} \right) \left[ \frac{2 + \cosh[4a\phi/M_P]}{2} + \mathcal{O}(\varsigma^2) \right], \]

where we just kept the leading contribution in $\varsigma$. The finite part $f_{2,0}$ in the previous expression is given by

\[ f_{2,0} = \frac{3}{2} - \log \left[ \frac{-U''(\phi)}{\mu^2} \right] = \frac{3}{2} - \log \left[ \frac{\lambda a^2 M_P^2}{\xi^2(1-\varsigma)^2 \mu^2} \left( \varsigma \cosh[2a\phi/M_P] + \mathcal{O}(\varsigma^2) \right) \right]. \]

If we adopt the $\overline{MS}$ scheme, the remaining (logarithmic) corrections will be suppressed by an overall factor $\mathcal{O}(10^{-15})$ (apart from different powers of $\varsigma$) with respect to the tree-level potential (4.8). The quantum contribution of the scalar sector to the effective inflationary potential is therefore completely negligible and thus hardly sensitive to the particular choice of the renormalization point $\mu$. This allows us to approximate the value of $\phi$ at the end of inflation by its classical value $\phi_f \simeq M_P/a \tanh^{-1} \left[ \sqrt{1-\varsigma \cos(2 \times 3^{1/4} \sqrt{\xi})} \right]$, and compute analytically the spectral tilt $n_s$ of primordial scalar perturbations, which turns out to be

\[ n_s(k_0) - 1 \simeq -8\xi + \frac{\lambda \xi^2}{96\pi^2 \xi^2} f_{2,0}, \quad \text{for} \quad 1 \lesssim 4\xi N^* \ll 4N^*. \]

We see therefore that the correction to the tree-level result is controlled by the effective self-coupling of the Higgs field in the Einstein frame $\lambda/\xi^2$. The small value of this parameter makes the scalar radiative contribution completely negligible and thus hardly modify the consistency relation (2.30). Note however that there might be still a significant contribution to the inflationary potential coming from the SM particles, especially from those with a large coupling to the Higgs field. The study of this effect is the purpose of the next section.
The action for the SM fields during the inflationary stage is similar to that appearing in Higgs inflation \[10\] and takes the form of a chiral SM with a nearly decoupled Higgs field. Its contribution to the effective potential can be analyzed by the methods presented in Ref. \[10\]. The one-loop contribution during inflation reads

\[
\Delta U_1 = \frac{6m_W^4}{64\pi^2} \left( \log \frac{m_W^2}{\mu^2} - \frac{5}{6} \right) + \frac{3m_Z^2}{64\pi^2} \left( \log \frac{m_Z^2}{\mu^2} - \frac{5}{6} \right) - \frac{3m_t^4}{16\pi^2} \left( \log \frac{m_t^2}{\mu^2} - \frac{3}{2} \right)
\]

(4.18)

where \(m_W^2 = g^2 h^2/2\), \(m_Z^2 = g^2 h^2/2 \cos^2 \theta_W\) and \(m_t^2 = y_t^2 h^2/2\) stand for the effective \(W, Z\) and top quark masses in the Jordan frame. The choice of the \(\mu\) parameter here defines the renormalization prescription, as described in the beginning of Sec. \textbf{IV}. To retain the possibility to use the RG equations to run between the electroweak and inflationary scales we will write \(\mu^2 = \frac{\mu^2}{M_p^2} F(h, \chi)\). Here the function \(F(h, \chi)\) corresponds to the choice of the renormalization prescription and leads to different physical results, while the parameter \(\hat{\mu}\) plays the role of the usual choice of momentum scale in the RG approach and should disappear in the final result. The conformal transformation to the Einstein frame \(\Delta U_1 = \Delta U_1 / \Omega^4\) acts only on the coefficients of the logarithmic terms in (4.18), leaving their arguments completely unchanged. We obtain therefore

\[
\Delta \tilde{U}_1 = \frac{6\tilde{m}_W^4}{64\pi^2} \left( \log \frac{\tilde{m}_W^2}{\hat{\mu}^2 F(h, \chi)/M_p^2} - \frac{5}{6} \right) + \frac{3\tilde{m}_Z^2}{64\pi^2} \left( \log \frac{\tilde{m}_Z^2}{\hat{\mu}^2 F(h, \chi)/M_p^2} - \frac{5}{6} \right) - \frac{3\tilde{m}_t^4}{16\pi^2} \left( \log \frac{\tilde{m}_t^2}{\hat{\mu}^2 F(h, \chi)/M_p^2} - \frac{3}{2} \right)
\]

(4.19)

where the Einstein-frame masses \(\tilde{m}_W^2\) are proportional to the effective vacuum expectation value of the Higgs field in the Einstein frame\footnote{We neglect the contribution \(4.18\) associated to the scalar sector, which, as shown in the previous section, turns out to be very small.}, which is a slowly varying function during inflation,

\[
v^2(\phi) = \frac{h^2}{\Omega^2} = \frac{M_p^2}{\xi_h (1 - \varsigma)} \left( 1 - \varsigma \cosh^2 \frac{a\phi}{M_p} \right).
\]

(4.20)

This fact allows us to completely factor out the \(\phi\) dependence in front of the logarithms in Eq. (4.19) and perform the analysis below as if \(v\) was a constant, \(v \simeq M_p / \sqrt{\xi_h}\).

Note that the explicit dependence on the ’t Hooft-Veltman normalization point \(\mu\) in Eq. (4.18) is spurious and is compensated by the running of the coupling constants \(\lambda(\tilde{\mu}), \xi_h(\tilde{\mu})\) in the tree level part of the potential (see \[10\]). Once the RG running of the couplings is fixed, it is convenient to choose the value of \(\tilde{\mu}\) in such a way that the logarithmic contribution (4.19) for each given value \(\phi\) of the Higgs field, is minimized, \(\tilde{\mu}^2 \simeq \frac{h^2}{2} \frac{M_p^2}{F(h, \chi)/M_p^2}\). In that case, the RG enhanced (RGE) inflationary potential becomes

\[
\tilde{U}_{\text{RGE}}(\phi) = \frac{\lambda(\tilde{\mu}(\phi))}{4} \frac{M_p^4}{\xi_h^2(\tilde{\mu}(\phi))(1 - \varsigma^2)} \left( 1 - \varsigma \cosh^2 \frac{a\phi}{M_p} \right)^2,
\]

(4.21)

which in fact suffices for practical purposes, with the corrections form the 1-loop logarithms being rather small.

As discussed at the beginning of Section \textbf{IV}, the different choices of \(\mu\) correspond to different subtraction rules and produce different results. In what follows we will consider the two most natural choices. The first one is associated to the scale invariant prescription (4.1). The RG enhancement of the potential in this case dictates

\[
\tilde{\mu}_1^2(\phi) = \frac{y_t^2}{2} \frac{M_p^2 h^2}{\xi_h^2 + \xi_{\chi}^2} = \frac{y_t^2}{2} v^2(\phi) \left( 1 - \varsigma \right),
\]

(4.22)

which is nothing else than the effective top mass in the Einstein frame. With this choice, the change in the shape of the potential is very small, given the insignificant variation of \(v^2(\phi)\) during inflation. The change in the inflationary observables \(r\) and \(s\) is therefore expected to be completely negligible. The second possibility that we will consider is associated to the prescription (4.3). In this case the optimal choice of \(\tilde{\mu}\) is

\[
\tilde{\mu}_{\text{II}}^2(\phi) = \frac{y_t^2}{2} \frac{M_p^2 h^2}{\xi_{\chi}^2} = \frac{y_t^2}{2} v^2(\phi) \frac{1 - \varsigma}{\varsigma \sinh^2 (a\phi / M_p)},
\]

(4.23)
which, at the end of inflation, coincides with the effective top mass in the Jordan frame. This corresponds to the prescription II in Ref. [10]. Note that contrary to the previous case, this choice strongly depends on the value of the $\phi$ field and noticeable contributions to the inflationary parameters are expected.

The calculation proceeds now along the same lines as those in Ref. [10], using the tree level RG enhanced potential and the one loop correction. The addition of the two loop effective potential does not significantly modify the result. The numerical outcome for the two prescriptions is shown in Fig. 5. As expected, the inflationary observables computed
with the first prescription coincide with the tree level result. The only effect of the quantum corrections is setting a minimal value for the Higgs mass. This turns out to be \(m_H > m_{\text{min}}\), with \(m_{\text{min}} \approx 129.5 \pm 5\) GeV (for details on the latest calculations of this value see Ref. \[57, 58\]). After the end of inflation and preheating, the system is outside the scale-invariant region and the fields settle down to the minimum of the potential. From the expansion of the potential around the background, it is clear that all the contributions to the effective action will be again suppressed by powers of the exponent \(e^{-\gamma r/M_P}\), in addition to powers of \(M_P\), not affecting therefore the predictions of the model concerning the DE equation of state \([2.29]\). Taking into account the above results, we conclude that the quantum corrections computed with the prescription I do not modify the classical consistency relation \([2.30]\) characterizing Higgs-Dilaton cosmology. On the other hand, the inflationary observables computed using the prescription II clearly differ from the tree level result, especially for Higgs masses close to the critical value \(m_{\text{min}}\) at large \(\xi_h\). Note that in this prescription, the recent observation of a light Higgs-like state \[59, 60\], together with the present bounds on the spectral tilt \(n_s\) \[61\], further restrain the allowed \(\xi_h\) interval.

V. CONCLUSIONS

The purpose of this paper was to study the self-consistency of the Higgs-Dilaton cosmological model. We determined the field-dependent UV cut-offs and studied their evolution in the different epochs throughout the history of the Universe. We showed that the cut-off value is higher than the relevant energy scales in the different periods, making the model a viable effective field theory describing inflation, reheating, and late-time acceleration of the Universe. Since the theory is non-renormalizable, the loop expansion creates an infinite number of divergences, something that may challenge the classical predictions of the Higgs-Dilaton model. We argued that this is not the case if the UV-completion of the theory respects scale-invariance and the approximate shift symmetry for the dilaton field.

We computed within this framework the effective inflationary potential in the one-loop approximation and concluded that the dominant contribution comes from the chiral SM sector of the theory. We used two different regularizations prescriptions consistent with the symmetries of the model. In the “SI-prescription” of Ref. \[39\], with a field-dependent normalization point proportional to the effective Planck scale in the Jordan frame, the effective potential turns out to coincide with the tree level one. This leaves practically intact the consistency relation \([2.30]\) which connects the inflationary spectral tilt to the deviation of the DE equation of state from a cosmological constant. This relation is however modified if the normalization point is chosen only along the dilaton’s direction, especially for Higgs masses near the critical value \(m_{\text{min}} \approx 129.5 \pm 5\) GeV, which is amazingly close to the mass of the recently observed Higgs-like particle at the LHC \[59, 60\]. In the lack of a Planck scale UV completion, the proper choice of the normalization point \(\mu\) can only be elucidated by improving the precision of the cosmological and particle physics observables.

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Appendix A: Einstein frame cut-offs

In this appendix we briefly discuss the computation of the effective cut-off in the Einstein frame. As before, the cut-off is understood as the energy at which perturbative unitarity is violated and not necessarily as the onset of new physics. As shown in Eq. \([2.12]\), the gravitational part of the action in the transformed frame takes the usual Einstein-Hilbert form, which allows us to directly identify the gravitational cut-off with the reduced Planck mass \(M_P\). The cut-off associated to the gauge sector can be also easily determined by looking at the scattering of gauge bosons with longitudinal polarization. Since the kinetic terms for the gauge fields are invariant under the conformal rescaling, the only modification comes through their coupling to the Higgs field \(h\). The interaction under consideration can be schematically written as

\[
g^2 h^2 W_\mu^+ W^-_\mu \rightarrow g^2 \frac{h^2}{\Omega^2} \tilde{W}_\mu^+ \tilde{W}^-_\mu .
\]  

where we have rescaled the gauge boson fields in the Einstein frame with the corresponding conformal weight, \(\tilde{W}^\pm = W^\pm/\Omega\). Expanding \([A1]\) around the background value of the Higgs field, \(h \rightarrow h + \delta h\), we find the following interaction

\[
W_\mu^+ W^-_\mu - \frac{1}{18} \gamma r/W^2 + \text{terms suppressed by } e^{-\gamma r/M_P}.
\]
where mass $m_W \sim g \hat{h}$ is the mass of the $W$ bosons in the Jordan frame and the conformal factor $\tilde{\Omega}$ depends now on the background values of the Higgs and dilaton fields. Taking into account the canonically normalized perturbations of the Higgs field (2.11), together with the unitarity of the S-matrix, we find that the cut-off scale associated to the gauge sector is given by

$$\Lambda_G \simeq \tilde{\Omega}^{-1} \sqrt{\xi_h \hat{h}^2 (1 + 6 \xi_h) + \xi_h \hat{h}^2 (1 + 6 \xi_h)}.$$  \hspace{1cm} (A3)

For the two limiting cases discussed in Section III B the previous expression becomes

$$\Lambda_G \simeq \begin{cases} \frac{M_P}{\sqrt{\xi_h}} & \text{for } \xi_h \hat{h}^2 \ll \xi_h \hat{h}^2, \\ \frac{M_P}{\xi_h} & \text{for } \xi_h \hat{h}^2 \gg \xi_h \hat{h}^2. \end{cases}$$ \hspace{1cm} (A4)

where we have identified $\sqrt{\xi_h} = M_P$. As expected, the gauge cut-off in the Einstein frame is nothing else that the conformal rescaling of the Jordan frame cut-off, $\Lambda_G = \Lambda_G / \tilde{\Omega}$.

The computation of the scalar cut-off in the Einstein frame is more complicated than in the single field case 27. Although all the non-linearities of the initial frame are moved to the matter sector of the theory, the existence of non-minimal couplings to gravity give rise to a non-trivial kinetic mixing for the scalar fields in the Einstein frame (cf. Eq. (2.14)). This fact substantially complicates the treatment of the problem in terms of the original ($h, \chi$) variables, especially in the high energy region. Therefore, in order to compute the scalar cut-off at large energies, we choose to recast the kinetic terms (2.14) in a diagonal form by means of the angular variables defined in (2.20). Expanding the resulting inflationary potential (cf. Eq. (A11)) around the background value of the Higgs field $\phi$ we obtain a series of terms of the form (cf. Eq. (4.11))

$$c_{n,l} \cosh[2an\delta \phi / M_P] \left( \frac{a \delta \phi}{M_P} \right)^{2l} + d_{n,l} \sinh[2an\delta \phi / M_P] \left( \frac{a \delta \phi}{M_P} \right)^{2l+1}. \hspace{1cm} (A5)$$

The scalar cut-off during inflation and reheating can be directly read from the previous expression. Note however that a direct comparison of the previous result with those obtained in the Jordan frame is only possible in some limiting cases. The angular perturbation $\delta \phi$ depends on both of the original field perturbations and only coincides with the Higgs perturbation $\delta h$ in the very high energy regime. Indeed, at the beginning of inflation the angular dependence on the background field in Eq. (A5) becomes negligible. We are left therefore with a series of higher order operators suppressed by the reduced Planck mass $M_P$, which coincides with the conformally transformed Jordan frame cut-off in the corresponding regime, $\Lambda \simeq \Lambda / \tilde{\Omega} \simeq \sqrt{\xi_h} h / \Omega$.

The determination of the scalar cut-off in the low-energy regime, $\xi_h h^2 \ll \xi_h h^2$, is also non-trivial, since the field redefinition (2.20) is no longer applicable. Fortunately, the kinetic mixing between the Higgs and dilaton fields can be neglected at low energies and Eq. (2.14) simplifies to

$$\hat{K}(\chi, h) \simeq (\partial \chi)^2 + \left( 1 + \frac{\xi_h^2 h^2}{M_P^2} \right) (\partial h)^2,$$ \hspace{1cm} (A6)

where again we identified $\sqrt{\xi_h} = M_P$. The kinetic term for the Higgs field can be recast into canonical form in terms of

$$\hat{h} = h \left( 1 + \frac{\xi_h^2 h^2}{M_P^2} + \ldots \right) = h \left( 1 + \sum_{n=1} c_n \left( \frac{\xi_h^2 h^2}{M_P^2} \right)^n \right),$$ \hspace{1cm} (A7)

where $c_n$ are numerical factors. Inverting the above relation and plugging it to the potential in this limit

$$\hat{U}(\hat{h}) \simeq \frac{\lambda}{4} \hat{h}^4,$$ \hspace{1cm} (A8)

we see that the cut-off is proportional to $M_P / \xi_h$, in agreement with the Jordan frame result 22.

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20 Equivalently we could consider higher order terms arising from the non-canonical kinetic term of the dilaton.

21 The background value of the field $\phi$ is very close to zero. Remember that $\phi$ is defined as $\phi = \phi_0 - [\phi^0]$.

22 Notice that in the low energy regime the conformal factor is approximately equal to one.
Appendix B: Feynman rules for the dilaton

In this appendix we gather the Feynman rules as well as the expressions for the coefficients appearing in the one-loop diagrams in Section IV A. We denote with a dashed (solid) line the dilaton (Higgs) and perform the calculations in normalizing the kinetic term for the dilaton, we find the following Feynman rules stemming from its kinetic term

\[ \frac{a}{M_P} \tanh \left( \frac{a \phi}{M_P} \right), \quad \frac{a^2}{2M_P^2} \left( 1 + \tanh^2 \left( \frac{a \phi}{M_P} \right) \right). \]

Using the above expression, we can calculate the coefficients appearing in the different diagrams. Let us start by considering the simplest diagram \( d_1 \). We obtain

\[ = c_{1,1}^d(\phi) \left( \frac{1}{\epsilon} + f \right) \left( \frac{m_H}{M_P} \right)^2 (\partial r)^2, \]

with \( 1/\epsilon = 1/\epsilon - \gamma + \log 4\pi, \) and

\[ c_{1,1}^d(\phi) = \frac{a^2}{64\pi^2} \left( 1 + \tanh^2 \left( \frac{a \phi}{M_P} \right) \right), \quad f = - \log \left( \frac{m_H^2}{\mu^2} \right). \] \hspace{1cm} (B1)

Let us move to the more complicated diagram \( d_2 \). We find

\[ = c_{1,2}^d(\phi) \left[ \left( \frac{1}{\epsilon} + f' \right) \left( \frac{m_H}{M_P} \right)^2 + d \left( \frac{\partial}{M_P} \right)^2 \right] (\partial r)^2, \]

where

\[ c_{1,2}^d(\phi) = \frac{a^2}{16\pi^2} \tanh^2 \left( \frac{a \phi}{M_P} \right), \quad f' = \frac{1}{2} - \log \left( \frac{m_H^2}{\mu^2} \right) \quad \text{and} \quad d = 0. \] \hspace{1cm} (B2)

Note that in this particular diagram, the coefficient \( d \) is coincidentally zero. As we argued in Section IV A this kind of terms are expected to appear by simple power-counting arguments in higher-loop diagrams. We see that in both diagrams, for the maximal value of the hyperbolic tangent, the corrections are suppressed by loop factors as well as powers of \( M_P \).

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