Minimal supersymmetric SU(5) theory and proton decay: where do we stand?

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Abstract. We review the situation regarding $d = 5$ proton decay in the minimal supersymmetric SU(5) GUT. The minimal theory is defined as the theory with the minimal matter and Higgs content all the way up to the Planck scale; of course, this allows for the possible presence of Planck induced physics. It can be said that either higher dimensional operators must be present or/and some fine-tuning of $O(1\%)$ of the Higgs mass must be tolerated in order to save the theory.

1. Introduction

There are two popular scenarios for the solution of the hierarchy problem. One is large extra dimensions, which for two such new ones may be as large as a fraction of a mm \cite{1, 2, 3}. In this case the field-theory cutoff ($\Lambda_F$) must be low and experiments demand: $\Lambda_F > (10 - 100)$ TeV. Clearly, one then must fine-tune (somewhat) the Higgs mass, since

$$m_h^2 \approx m_0^2 + \frac{y_t^2}{16\pi^2}\Lambda_F^2 \approx \text{few } 100\text{ GeV}^2.$$ \hspace{1cm} (1)

We believe this is acceptable; compared to the fine-tuning problem when $\Lambda_F$ is pushed to $M_{Pl}$ (or $M_{GUT}$), this is negligible. What is missing in this program is some serious physical reason to have $\Lambda_F$ so low.

Another scenario is the low-energy supersymmetry, where $\Lambda_F$ gets traded for $\Lambda_{SUSY}$ (here defined as the mass difference between particles and superparticles of the MSSM). In this $\Lambda_{SUSY}$ can be as low as a few hundred GeV and, strictly speaking, no fine-tuning whatsoever is needed. On the other hand, low-energy supersymmetry with $\Lambda_{SUSY} \approx (1 - 10)$ TeV leads to the unification of gauge couplings (predicted before experiment! \cite{4, 5, 6, 7}) and through radiative symmetry breaking \cite{8} can explain the Higgs mechanism, i.e. the negative Higgs mass squared. This motivates us to focus on this scenario.

The minimal supersymmetric grand unified theory is based on SU(5) symmetry \cite{9} and in its minimal version contains three generations of quarks and leptons (and their partners) and the 24-dimensional ($\Sigma$) and 5-dimensional ($\Phi$ and $\bar{\Phi}$) Higgs supermultiplets. In the

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renormalizable case there is a problem: one has the same Yukawa couplings for the down quarks ($Y_D$) and charged leptons ($Y_E$), thus

$$m_D = m'_E .$$

(2)

This works very well for $b$ and $\tau$, and fails progressively for the second and first generation. Thus the first dilemma: to change the theory or not? A minimalist refuses to do so, and can still invoke the higher dimensional operators suppressed by $\langle \Sigma \rangle / M_{Pl}$, where $\langle \Sigma \rangle \approx M_{GUT}$. Strictly speaking, this is still the minimal theory, in a sense of its structure at the scale $\approx M_{GUT}$, where it should be defined. In the same manner we speak of the standard model (SM) at $E \approx M_Z$, and the non-vanishing of the neutrino mass does not require changing the structure of the SM, but simply invoking a higher dimensional operator:

$$m'_\nu \approx \frac{\langle \Phi \rangle^2}{M} ,$$

(3)

where $M$ is some new scale ($M \gg M_W$), for example the mass of the right-handed neutrino in the see-saw scenario [10].

Now, supersymmetric GUTs in general, and the SU(5) theory in particular, lead to quite fast proton decay through $d = 5$ operators generated by the superheavy coloured triplet Higgs supermultiplets ($T$ and $\bar{T}$), with masses $m_T \approx M_{GUT}$ [11, 12] (more later on $m_T$). The burning issue in recent years was whether the minimal supersymmetric SU(5) theory is already ruled out on this basis [13], especially after it was found out that the RRRR operators play a crucial role [14] (in the context of SO(10) this was shown before in [15]), and it was finally argued last year that this was indeed true [16]. The trouble is that one must be very careful in the definition of the minimal theory and it is worth reconsidering all the issues that enter in this question and which may save the theory. We wish to carefully discuss here all the subtleties involved, since to us this is one of the major problems of grand unification. After all, there is no predictive theory beyond SU(5) and we should be absolutely sure before we rule out the only predictive theory we have. We will see that the presence of higher-dimensional operators and the lack of knowledge of sfermion masses and mixings may be sufficient to make SU(5) be still in accord with experiment [17]. If we require, though, absolute naturalness (no fine-tuning at all) and a complete desert between $M_Z$ and $M_{GUT}$, the theory is ruled out. We feel however that the above assumptions are too drastic and do not allow for the probe of the principle of unification.

We certainly stick to the requirement of minimality allowing for no change of the theory all the way to the Planck scale (string scale, ...). Also, we make no assumptions of the Yukawa couplings of the heavy particles in $\Sigma$. Specifically, we allow for the colour octet and weak triplet in $\Sigma$ to have arbitrary masses, since the theory cannot predict them. We discuss this below in detail.

In short, here we review the predictions of the minimal supersymmetric SU(5) theory. We allow for arbitrary sfermion masses and mixings, keeping of course flavour violation in accord with experiment, and we allow for small ($\approx 1\%$) amount of fine-tuning. We focus mainly on the issue of proton decay while requiring as many as possible superpartners detectable at LHC. It is this requirement rather than the extreme naturalness that should make low-energy supersymmetry interesting to experimentalists and phenomenologists (at least in our opinion). With this in mind, our conclusion is that the minimal SU(5) theory is still in accord with experiment, but the situation is quite tight.

Let us now systematically discuss all the issues involved in predicting the $d = 5$ proton decay amplitude:

(i) the determination of the GUT scale and the masses ($m_T$) of the heavy triplets $T$ and $\bar{T}$ responsible for $d = 5$ proton decay. Specifically, we allow for arbitrary triplic couplings of
the heavy fields in $\Sigma$ and use higher dimensional terms as a possible source of their masses [18, 19]. It will turn out that $m_T$ may go up naturally by a factor of thirty, which would increase the proton lifetime by a factor of $10^3$.

(ii) The impact of higher dimensional operators on fermion masses and the couplings of $T$ and $\bar{T}$ with fermionic supermultiplets [20, 21, 22, 23]. If we keep the minimal theory intact, this is a must, since without these operators fermion masses cannot be reproduced.

(iii) The freedom in sfermion masses and sfermion and fermion mixings. Here one must be quite careful, though, in keeping flavour violation in neutral currents (FCNC) in accord with experiments (for a review see for example [24]).

Now, for generic values of the parameters of the theory we will see that SU(5) would be ruled out. But this, however, can be said also of FCNC in low-energy supersymmetry with generic soft terms. Instead, we should let experiment decide the values of the parameters of the theory. In short, with arbitrary parameters we will see that because of (i) to (iii) SU(5) theory is not ruled out yet.

2. $M_{\text{GUT}}$ and $m_T$: uncertainties

The superpotential for the heavy sector is (up to terms $1/M_{\text{Pl}}$)

$$W = m T r \Sigma^2 + \lambda T r \Sigma^3 + a \frac{(T r \Sigma^2)^2}{M_{\text{Pl}}} + b \frac{T r \Sigma^4}{M_{\text{Pl}}}.$$  \hspace{1cm} (4)

Of course, if $\lambda \approx O(1)$, we ignore higher-dimensional terms. However, in supersymmetry $\lambda$ is a Yukawa-type coupling, i.e. self-renormalizable. For small $\lambda$ ($\lambda \ll M_{\text{GUT}}/M_{\text{Pl}}$), the opposite becomes true and $a$ and $b$ terms dominate. In this case, it is a simple exercise to show that

$$m_3 = 4m_8,$$  \hspace{1cm} (5)

where $m_3$ and $m_8$ are the masses of the weak triplet and color octet in $\Sigma$. In the renormalizable case $m_3 = m_8$.

At the one loop level, the RGE’s for the gauge couplings are

$$\alpha_1^{-1}(M_Z) = \alpha_U^{-1} + \frac{1}{2\pi} \left( -\frac{5}{2} \ln \frac{\Lambda_{\text{SUSY}}}{M_Z} + \frac{33}{5} \ln \frac{M_{\text{GUT}}}{M_Z} + \frac{2}{5} \ln \frac{m_T}{m_3} \right),$$  \hspace{1cm} (6)

$$\alpha_2^{-1}(M_Z) = \alpha_U^{-1} + \frac{1}{2\pi} \left( -\frac{25}{6} \ln \frac{\Lambda_{\text{SUSY}}}{M_Z} + \ln \frac{M_{\text{GUT}}}{M_Z} + 2 \ln \frac{m_T}{m_3} \right),$$  \hspace{1cm} (7)

$$\alpha_3^{-1}(M_Z) = \alpha_U^{-1} + \frac{1}{2\pi} \left( -4 \ln \frac{\Lambda_{\text{SUSY}}}{M_Z} - 3 \ln \frac{m_8}{M_Z} + \ln \frac{M_{\text{GUT}}}{m_3} \right).$$  \hspace{1cm} (8)

Here we take for simplicity $M_{\text{GUT}} = M_{X,Y} =$ superheavy gauge bosons masses, while at the one-loop level we could as well take $\Lambda_{\text{SUSY}} = M_Z$. From (6)-(8) we obtain

$$2\pi \left( 3\alpha_1^{-1} - 2\alpha_3^{-1} - \alpha_1^{-1} \right) = - \frac{2}{5} \ln \frac{\Lambda_{\text{SUSY}}}{M_Z} + \frac{12}{5} \ln \frac{m_T}{M_Z} + 6 \ln \frac{m_8}{m_3},$$  \hspace{1cm} (9)

$$2\pi \left( 5\alpha_1^{-1} - 3\alpha_2^{-1} - 2\alpha_3^{-1} \right) = 8 \ln \frac{\Lambda_{\text{SUSY}}}{M_Z} + 36 \ln \left( \sqrt{m_3 m_8 M_{\text{GUT}}^2} \right)^{1/3}.$$  \hspace{1cm} (10)

This gives
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\[ m_T = m^0_T \left( \frac{m_3}{m_8} \right)^{5/2}, \]

\[ M_{\text{GUT}} = M^0_{\text{GUT}} \left( \frac{M^0_{\text{GUT}}}{2m_8} \right)^{1/2}. \]

(11)

Since, in the case (5) is valid, \( m_8 \approx M^2_{\text{GUT}}/M_{\text{Pl}} \), we can also write

\[ M_{\text{GUT}} \approx \left[ \left( M^0_{\text{GUT}} \right)^3 M_{\text{Pl}} \right]^{1/4}. \]

(12)

(13)

In the above equations the superscript \(^0\) denotes the values in the case \( m_3 = m_8 \). From (5) we get

\[ m_T = 32 m^0_T, \quad M_{\text{GUT}} \approx 10 M^0_{\text{GUT}}. \]

(14)

Now, \( M^0_{\text{GUT}} \approx 10^{16} \text{ GeV} \) and it was shown last year [16] that \( M_T > 7 \times 10^{16} \text{ GeV} \) is sufficiently large to be in accord with the newest data on proton decay. On the other hand, since

\[ m^0_T < 3 \times 10^{15}\text{GeV}, \]

(15)

from (14) we see that \( m_3 = 4m_8 \) is enough to save the theory. Obviously, an improvement of the measurement of \( \tau_p \) is badly needed. It is noteworthy that in this case the usual \( d = 6 \) proton decay becomes out of reach: \( \tau_p(d = 6) > 10^{38} \text{ yrs}. \)

3. Higher dimensional operators and fermion masses and mixings

In the minimal SU(5) theory at the renormalizable level we have the Yukawa coupling relations at \( M_{\text{GUT}} \)

\[ Y_U = Y^T_U, \quad Y_E = Y_D, \]

(16)

where in the supersymmetric standard model language the Yukawa sector can be written as

\[ W_Y = HQ^TY_Uu^c + \bar{H}Q^TY_D\bar{d}^c + \bar{H}e^cY_EL + \frac{1}{2}TQ^T\Delta Q + Tu^cB\bar{e}c + \bar{T}Q^TCL + \bar{T}u^cD\bar{d}^c. \]

(17)

Also, in the minimal renormalizable model (at \( M_{\text{GUT}} \))

\[ \mathcal{A} = \mathcal{B} = Y_U = Y^T_U, \quad \mathcal{C} = \mathcal{D} = Y_D = Y_E. \]

(18)

The fact that \( \mathcal{A} = \mathcal{B} = Y_U, \mathcal{C} = Y_E, \mathcal{D} = Y_D \), is simply a statement of SU(5) symmetry. On the other hand \( Y_U = Y^T_U \) and \( Y_D = Y_E \) result from the SU(4) Pati-Salam like symmetry left unbroken by \( \langle H \rangle \) and \( \langle \bar{H} \rangle \). Under this symmetry \( d^c \leftrightarrow e, u \leftrightarrow u^c, d \leftrightarrow e^c \). Of course, this symmetry is broken by \( \langle \Sigma^\alpha \rangle \neq \langle \Sigma^4 \rangle \), where \( \alpha = 1, 2, 3 \); this becomes relevant when we include higher dimensional operators suppressed by \( \langle \Sigma \rangle /M_{\text{Pl}} \). The Yukawa couplings are readily diagonalized through

\[ U^TY_Uu = Y^d_U, \quad D^TY_D\bar{d}_c = Y^d_D, \quad E^T_cY_EE = Y^d_E. \]

(19)
where \( X (X_c) \) are unitary matrices that rotate \( x (x^c) \) fermions from the flavour to the mass basis and \( Y^d_5 \) stand for the diagonal Yukawa matrices. Similarly, unitary matrices \( \tilde{X} (\tilde{X}_c) \) diagonalize \( \tilde{x} (\tilde{x}^c) \) sfermions. The only low energy information we have is

\[
U^T D = V_{CKM} , \quad N^T E = V_l ,
\]

where \( V_l \) is the lepton analog of \( V_{CKM} \) (\( N \) rotates left-handed neutrinos). From (16) we have

\[
U_c = U , \quad E_c = D , \quad E = D_c ,
\]

in the minimal renormalizable model at \( M_{GUT} \).

The minimal renormalizable theory, as is well known, fails badly. Relations \( m_s = m_{\mu} , m_d = m_e \) at the GUT scale are simply wrong, while at the same time \( m_b = m_\tau \) can be considered a great success of the theory. We can imagine many ways out, but the simplest and the most suggestive is to include \( 1/M_{Pl} \) suppressed operators which are likely to be present; after all, these operators should be more important for the first two generations where the theory fails, and they require no change in the structure of the theory. This is analogous to a long ago speculated possibility that in the SM the neutrino mass is not zero, but of order \( 1/M \), where \( M \) would correspond to some new physics.

The idea of higher dimensional operators has been pursued in the past and applied to the proton decay issue in the case of specific mass textures, but never in a systematic manner. The main point is that when one corrects the relations (16), one will also affect the heavy triplet couplings to the heavy triplets. The idea of higher dimensional operators has been pursued in the past and applied to the proton decay issue in the case of specific mass textures, but never in a systematic manner. The main point is that when one corrects the relations (16), one will also affect the heavy triplet couplings to the heavy triplets.

To see this, let us consider for example all the relevant couplings up to order \( 1/M_{Pl} \) (\( i, j, k, l, m, n \) are SU(5) indices, \( a, b = 1, 2, 3 \) are generation indices):

\[
W_Y = \epsilon_{ijklm} \left( 10^{ij} f_{ab} 10^{kl} \Phi^m + 10^{ij} f_{1ab} 10^{kl} \frac{\Sigma^m}{M_{Pl}} \Phi^n + 10^{ij} f_{2ab} 10^{kn} \Phi^l \frac{\Sigma^m}{M_{Pl}} \right) \\
+ \Phi_i 10^{ij} g_{ab} \tilde{\epsilon}_{bj} + \Phi_i \frac{\Sigma^i}{M_{Pl}} 10^{jk} g_{1ab} \tilde{\epsilon}_{bk} + \Phi_i 10^{ij} g_{2ab} \frac{\Sigma^k}{M_{Pl}} \tilde{\epsilon}_{bk} ,
\]

where 10 and 5 are the fermionic supermultiplet representations. After taking the SU(5) vev \( \langle \Sigma \rangle = \sigma \text{diag}(2, 2, 2, -3, -3) \) we get at \( M_{GUT} \)

\[
Y_U = 4 \left( f + f^T \right) - 12 \frac{\sigma}{M_{Pl}} \left( f_1 + f_1^T \right) - 2 \frac{\sigma}{M_{Pl}} \left( 4 f_2 - f_2^T \right) , \\
A = 4 \left( f + f^T \right) + 8 \frac{\sigma}{M_{Pl}} \left( f_1 + f_1^T \right) + 2 \frac{\sigma}{M_{Pl}} \left( f_2 + f_2^T \right) , \\
B = 4 \left( f + f^T \right) + 8 \frac{\sigma}{M_{Pl}} \left( f_1 + f_1^T \right) + 4 \frac{\sigma}{M_{Pl}} \left( 3 f_2 - 2 f_2^T \right) , \\
Y_D = -g + 3 \frac{\sigma}{M_{Pl}} g_1 - 2 \frac{\sigma}{M_{Pl}} g_2 , \\
Y_E = -g + 3 \frac{\sigma}{M_{Pl}} g_1 + 3 \frac{\sigma}{M_{Pl}} g_2 , \\
C = -g - 2 \frac{\sigma}{M_{Pl}} g_1 + 3 \frac{\sigma}{M_{Pl}} g_2 , \\
D = -g - 2 \frac{\sigma}{M_{Pl}} g_1 - 2 \frac{\sigma}{M_{Pl}} g_2 .
\]

In the limit \( M_{Pl} \to \infty \) we recover the old relations, but for finite \( \sigma/M_{Pl} \approx 10^{-3} - 10^{-2} \) one can correct the relations between Yukawas and at the same time have some freedom for the couplings to the heavy triplets.
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Clearly, due to SU(5) breaking through \( \langle \Sigma \rangle \), the \( T, \bar{T} \) couplings are different from the \( H, \bar{H} \) couplings. However, under the SU(4) symmetry discussed before \( A \leftrightarrow \bar{B}, C \leftrightarrow \bar{D} \) and \( Y_U \leftrightarrow \bar{Y}_U^T \). Only the terms that probe \( \langle \Sigma_{\alpha}^u \rangle - \langle \Sigma_{\alpha}^d \rangle \) can spoil that; this is why \( f_1 \) and \( g_1 \) still keep \( Y_U = Y_U^T, A = B \) and \( C = D \).

Notice that worrying about proton decay in a theory with \( Y_D = Y_E \) may not be reasonable. Namely, suppose that you wish to preserve this relation; you can do that easily in eq. (23) if you take \( g_2 = 0 \). It is amusing that you can set \( C = D = 0 \) by choosing \( g + 2(\sigma/M_{Pl})g_1 = 0 \) (for the third generation this requires a not too large \( \tan \beta \) and/or \( M_{Pl} \), which could be \( M_{string} \), below \( 10^{18} \text{ GeV} \)). In this way you can simply decouple the heavy triplets \( T \) and \( \bar{T} \) (an old idea of Dvali [20]) and get rid of the \( d = 5 \) proton decay. Obviously we do not wish to advocate this. After all, the idea of introducing higher dimensional operators, while preserving the SU(5) minimality structure all the way to proton decay. Obviously we do not wish to advocate this. After all, the idea of introducing higher dimensional operators, while preserving the SU(5) minimality structure all the way to proton decay, is precisely to avoid \( Y_D = Y_E \) for the first two generations. Notice further that with \( Y_D \neq Y_E \) you can not set both \( C \) and \( D \) to vanish. You may be tempted to make \( A \) and \( B \) vanish, but that cannot work for the large top Yukawa coupling.

Although for realistic Yukawas the matrices \( A, B, C \), and \( D \) are not completely arbitrary, there is a new freedom not present if (18) are valid: the fermion mixing matrices defined by (19) are no more related by (21), but can be chosen freely, as long as (21) are satisfied. This freedom could further diminish the decay amplitude.

In short, correct mass relations add more uncertainty to the proton decay amplitude and it may be worthwhile to perform a more quantitative analysis.

4. Sfermion and fermion masses and mixings and their impact on \( \tau_p \)

Last, but not least, the \( d = 5 \) proton decay is generated through Yukawa couplings, and thus fermion and sfermion masses and mixings play an important role. Unfortunately, we know nothing about supersymmetry breaking, except that we must satisfy the experimental limits on FCNC. Ideally we wish to keep all the sparticles below TeV, but then FCNC becomes a serious issue, although still under control for carefully chosen mixings. Furthermore, if we ignore both (i) and (ii) the limits from proton decay can be used to rule out the minimal SU(5) theory. In all honesty, we do not know what that means, for this theory is obviously already ruled out by the wrong fermion relations.

Now, the FCNC are mainly a problem for the first two generations; a popular approach is to assume the first two generations of sfermions heavy \( (m \approx 10 \text{ TeV}) \), the so called decoupling regime [23, 24, 27]. In this case it is enough that the third generation of sfermions does not have large mixings with both of the first two generations of sfermions.

One can also worry about the naturalness [28, 29, 30, 31]. Through the large top Yukawa couplings, the formula (11) becomes here \( (i = 1, 2, 3) \) (for large \( \tan \beta \) there are similar contributions of (s)bottom and (s)tau)

\[
m_h^2 \approx m_0^2 + \frac{g_2^2}{16\pi^2} \left[ (\bar{U}^\dagger U)_{i3} (U^\dagger \bar{U})_{3i} \bar{m}_i^2 + (\bar{U}^\dagger U)_{i3} (U^\dagger \bar{U})_{3i} \bar{m}_i^2 \right],
\]

where \( \bar{m}_i \) and \( \bar{m}_i^c \) are left-handed and right-handed squark masses. Here and in the following we ignore the left-right mixing proportional to the small ratio \( M_Z/m_{3/2} \); in fact, as long as \( \tan \beta > 10 \), the LR mixing can be safely put even to zero without contradicting the experimental constraints on the Higgs mass [32].

Now, for \( \bar{m}_a \approx \bar{m}_a^c \approx 10 \text{ TeV} \) \( (a = 1, 2) \) in the decoupling regime, large \( (\bar{U}^\dagger U)_{i3} \) or \( (\bar{U}^\dagger U)_{i3} \) would imply a small amount of fine-tuning \( (\approx 1\%) \) in (24). Hereafter, we accept that. No fine-tuning whatsoever, although appealing, to us seems exaggerated; after all it would eliminate large extra dimensions as a solution to the hierarchy problem.
Strictly speaking, one could then ask why not simply push \( \tilde{m}_3 \) and \( \tilde{m}_\tilde{\nu}_3 \) all the way up to 10 TeV and be safe? A sensible point, but as we said before, we wish to have as many as possible superpartners below TeV and thus hopefully detectable at LHC. In other words, all the gauginos and Higgsinos and the third generation of sfermions are assumed to have masses lower or equal TeV, we only take \( \tilde{m}_{1,2} \approx 10 \) TeV or so.

In this case, we need to worry only about the third generation of sfermions. We also assume light gauginos and Higgsinos, \( m \approx 100 \) GeV. We have recently performed a detailed analysis of all \( d = 5 \) proton decay amplitudes neglecting the mixing of left and right sfermions \([17]\). An interesting question is: can the contribution of the third generation of sfermions be made arbitrary small? Remarkably enough, the answer is yes, i.e. it can be set even to zero, or, even easier, it can be small enough to keep \( \tau_p \) above the experimental limit.

The solution is the following. In our paper \([17]\) we give a typical set of constraints need to make the proton decay small \((a, b = 1, 2)\):

\[
(\tilde{U}^T D)_{3a} \approx 0 \quad , \quad (\tilde{D}^T D)_{3a} \approx 0 \quad , \quad (\tilde{E}^T E)_{3a} \approx 0 \quad , \quad (\tilde{N}^T E)_{3a} \approx 0 \quad , \quad (\tilde{D}^T D)_{3a} \approx 0 \quad , \quad (\tilde{E}^T E)_{3a} \approx 0 \quad , \quad (\tilde{U}^T Y^T D)_{3a} \approx 0 \quad ,
\]

and

\[
A_0 = \epsilon_{ab}(D^T C \tilde{N})_{a3}(\tilde{U}^T A D)_{3b} \approx 0 \quad .
\]

The constraints (25) can clearly be satisfied exactly by the sfermion mixing matrices at 1 GeV. It is reassuring that the sfermionic sector does not break strongly SU(2). This is consistent with the SU(2) invariance of the soft masses, which dominate the total sfermion masses. The last constraint, eq. (26), can be satisfied in the approximation \( C = Y_D = Y_E \), which is true in the minimal renormalizable model, but at \( M_{GUT} \), not at 1 GeV. The relation \( C = Y_D = Y_E \) is however not stable under running. To get an idea of how big this contribution is at the electroweak scale, one can take the approximation that the Yukawas do not run. In the leading order in small Yukawas (except for \( y_t \)) one gets

\[
A_0 \approx y_t y_t V_{33}^* V_{23} V_{32} V_{21} \left[ 1 - \left( \frac{M_Z}{M_{GUT}} \right)^{b_3^2/16\pi^2} \right] x_1^{-1/33} x_2^{-3} x_3^{4/3},
\]

where \( V \) is the CKM matrix and \( x_i = \alpha_i(M_Z)/\alpha_U \). There is only one non-vanishing diagram (the rest vanishes due to (25)) and it is proportional to \( V_{13} A_0 \): fortunately, this seems to be small enough. On top of this, in the amplitude the combination (26) gets multiplied with a combination of neutralino soft masses \( m_{\tilde{\nu}_3} \) and \( m_\tilde{j} \), which can be fine-tuned to an arbitrary small (or even zero) value. And, of course, we must keep in mind (i) and (ii), which tell us that \( m_T \) can be large and \( A \) and/or \( C \) completely different than \( Y_U, Y_E \).

5. Constraints from FCNC

Although there is no realistic theory of sfermion soft terms, there are low energy constraints on sfermion mixings. These come from the flavour changing neutral currents phenomena: \( \mu \rightarrow e\gamma, b \rightarrow s\gamma, B \rightarrow \tilde{B}, K \rightarrow \tilde{K} \), etc. It is easy to see, that the combinations which appear in (25) are exactly the ones that appear in these flavour changing processes. So they automatically take care also of these low-energy experimental data. The only flavour changing processes that could get sizeable contributions are the ones which involve up type sfermions like for example \( D \rightarrow \tilde{D} \) or \( c \rightarrow u\gamma \). These are not constrained by (25), but at the same time
are not very much constrained by the low-energy experiments, so they do not represent a real issue at this stage.

Constraints \((25)\) are not unique. One can find other relations between sfermion and fermion mixing matrices that make the proton decay amplitude zero or small. However, typically, these solutions can be dangerous for FCNC processes, since they do not automatically cancel their contributions. So one has to analyze the FCNC processes case by case. At the present day status (or ignorance) of proton decay and FCNC experiments we believe that this is premature.

6. Summary

We have emphasized here three major sources of uncertainties in estimating the \(d = 5\) proton decay. These are, in no particular order: (i) the ignorance of the masses of the color octet and weak triplet supermultiplets in the adjoint Higgs; this can easily increase the proton lifetime by a factor of thousand or so; (ii) higher dimensional operator correction of fermion masses; more difficult to quantify, but not necessarily less important; (iii) the ignorance of sparticle masses and mixings; although somewhat artificial, this possibility alone is enough to keep \(\tau_p \geq (\tau_p)_{\text{exp}}\).

In short, there is at least \(10^3\) uncertainty in predicting \(\tau_p\), and possibly as large as \(10^4\) or bigger. Since none of the points (i), (ii), (iii) requires any change of the structure of the theory, the minimal supersymmetric SU(5) GUT is still in accord with all the experimental constraints. It is true, though, that the parameter space is becoming small and improvement in \((\tau_p)_{\text{exp}}\) is badly needed.

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