Complex Term Elimination of Time-Shift Signal Based on Frequency Domain Conjugate Pair Method and MATLAB

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Abstract. This paper mainly describes that a signal displacement in time domain and DFT transform will produce complex exponential terms in frequency domain due to time shift. By constructing conjugate symmetry structure in frequency domain, the complex terms can fluctuate back and forth in order of 10^{-16} magnitude, which is called minimal complex terms. The real functions in MATLAB can eliminate 10^{-16} magnitude terms. The minimal complex term makes the real sequence approximate to the real sequence after translation in time domain.

1. Introduction
With the continuous development of information technology, digital signal processing technology is also developing, but there are still many inconveniences in practical application. For example, the time shift of DFT transform is often used in signal processing, but the nature of DFT transform [1-3] can understand that frequency will produce a complex exponential term, which will bring a lot of inconvenience to the subsequent signal processing. The approximation error and the approximation parameters will be generated when the system is constructed, which increases the uncertainty of system analysis. Therefore, in many cases, it is hoped that the real sequence can still be obtained after the signal is time-shifted. This paper will achieve the goal of time-shifted still real sequence by forming conjugate symmetry [4] in frequency domain and eliminating the complex term through real function [5] in MATLAB.

2. The problem of making real number sequence still be real number sequence after translation in time domain is introduced and analyzed

2.1. Problem Orientation
According to the nature of DFT, the sequence displaced in time domain is a complex sequence. In fact, we sometimes hope that the sequence displaced is a real sequence.

Theoretical basis:
\[
\begin{align*}
\text{DFT}[x(n-m)] &= W_N^{km}X(k) = X(k)e^{-j2\pi\frac{mk}{N}},
\text{DFT}[x(n+m)] &= W_N^{km}X(k) = X(k)e^{j2\pi\frac{mk}{N}}.
\end{align*}
\]

(1)

\(m\) is an integer, multiplying \(X(k)\) by \(W_N^{km}\) destroys the conjugate symmetry of the original \(X(k)\), so that the latter \(x(n-m)\) of IFFT is not guaranteed to be a real sequence.

### 2.2. Problem analysis

Next, the article will analyze it theoretically. (1) Setting the displacement of a phase angle to the displacement of a sample point

In digital signal processing, sinusoidal signal is often expressed as \(x(n) = A\cos(2\pi f_0 n/ f_s + \theta)\).

According to the sampling theorem [6-7], \(f_s\) is the sampling frequency, \(f_0\) is the frequency of sinusoidal signal, \(A\) is the amplitude of signal, \(\theta\) is the initial phase angle of signal. The initial phase angle is equal to the displacement of a time. The displacement in the time domain is calculated by the sample points, and the initial angle corresponds to a displacement.

The signal may be further expressed as follows:

\[
\begin{align*}
x(n) &= A\cos(2\pi f_0 n/ f_s + \theta) = A\cos[2\pi f_0 (n+d)/ f_s],
\theta &= 2\pi f_0 d / f_s,
d &= \theta f_s / 2\pi f_0.
\end{align*}
\]

(2) The relationship between displacement \(d\) and \(W_N^{km}\) [2]

To restore the signal to zero initial phase angle is equivalent to setting a function \(y(n) = x(n-d)\) the recombination formula (1) can be obtained:

\[
\begin{align*}
Y(k) &= X(k)W_N^{dk},
W_N^{dk} &= \exp(-j2\pi dk / N).
\end{align*}
\]

To displace by formula (1), \(m\) must be an integer, and \(d\) given by us is not an integer. Next, an example is given.

Let's start with an example. assume \(x(n) = A\cos(2\pi f_0 n/ f_s + \theta)\), Among them, \(f_s\) sampling frequency is 2000 Hz. Signal frequency \(f_0\) is 100 Hz, \(\theta\) is the initial phase angle of the signal - pi/3, \(n\) is the sample point value, and the signal length \(N\) is 40. It is hoped that the initial phase angle of the signal will be 0 by displacement.

\(A=1.00, \theta=-1.047198\)

Since the initial angle is negative and the waveform needs to be shifted to the left, the Fig after FFT transformation is shown in Fig 1.
Figure 1. Cosine signal translation diagram

The first image in Fig 1 is the original image after signal FFT transformation, while the initial phase of the second displacement FFT transformation is still not zero.

3. Solutions to the problem

A Conjugate Symmetry Structure in Frequency Domain [8-9]

$X(k)$ is multiplied by rotation factor $W_N^{dk}$ to form a conjugate symmetric spectrum [9] to ensure that $x(n)$ is a real sequence. The specific steps are as follows:

1. After passing $x(n)$ through FFT, it becomes $X(k)$.
2. $X(k)$ multiplied by rotation factor $W_N^{dk}$, $Y(k) = X(k) \cdot W_N^{dk}$;
3. $Y(k)$ only takes the positive frequency part, and the negative frequency part is calculated to form $Y(k)$;

$Y(k)$ is obtained by IFFT and $y(n)$ is obtained by IFFT, The real part of $y(n)$ is the sequence after the displacement we need.

Fig. 2 is obtained by constructing a conjugate symmetric structure in frequency domain. A=1.00  Theta=0.000000

Figure 2. Frequency Domain Constituted Conjugate Pair Post FFT Transform
From Fig 2, it is not difficult to see that by forming conjugate pairs in frequency domain, the complex terms can be controlled in the order of 10^-16 magnitude, which achieves the purpose of weakening the influence of the complex terms on the whole signal, but the complex terms have not been completely eliminated. Therefore, in order to achieve the purpose that the real sequence is still a real sequence after translation in time domain, it is necessary to eliminate the poles of 10^-16 magnitude with the help of real function in MATLAB [5]. The small complex term makes the real sequence approximate to the real sequence after translation in the time domain.

4. The conclusion

In practical engineering applications, it is necessary to understand the initial signal in the process of real processing and to determine many details of signal processing according to the specific processing requirements. If you want to get more stable and accurate results after the system and reduce the computational complexity, you can consider the method in the article. By forming conjugate pairs in frequency domain, you can weaken them. For the purpose of affecting the whole signal by complex terms and eliminating the minimum complex terms of 10^-16 orders of magnitude with the help of real function in MATLAB, the real sequence can still be approximated to real sequence after translation in time domain, so as to meet the target requirements of signal processing.

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