Modulated nematic structures induced by chirality and steric polarization

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(Dated: November 24, 2015)

What kind of one-dimensional modulated nematic structures (ODMNS) can form nonchiral and chiral bent-core and dimeric materials? Here, using Landau-deGennes theory of nematics, extended to account for molecular steric polarization, we study a possibility of formation of ODMNS, both in nonchiral and intrinsically chiral liquid crystalline materials. Besides nematic and cholesteric phases, we find four bulk ODMNS for nonchiral materials, two of which have not been reported so far. These new structures are longitudinal (N_{LP}) and transverse (N_{TP}) periodic waves where the polarization field being periodic in one dimension stays parallel and perpendicular, respectively, to the wave vector. The other two phases have all characteristic features of the twist-bend nematic phase (N_{TB}) and the splay-bend nematic phase (N_{SB}), but their fine structure appears more complex than that considered so far. The presence of molecular chirality converts nonchiral N_{TP} and N_{SB} into new N_{TB} phases. Interestingly, the nonchiral N_{LP} phase can stay stable even in the presence of intrinsic molecular chirality. Exemplary phase diagrams provide further insights into the relative stability of these new modulated nematic structures.

Until very recently only four classes of nematics were recognized: (i) uniaxial and (ii) biaxial nematics for nonchiral liquid crystalline materials and (iii) cholesteric and (iv) blue phases for chiral liquid crystals. The most surprising recent discovery is the identification of the fifth. In this new phase, found in liquid crystalline systems of chemically achiral dimers, bent-core mesogens, and their hybrids, the molecules are arranged to form a helical superstructure with nanoscale periodicity. This periodicity is about two orders of magnitude shorter than typically found in cholesteric and blue phases of ordinary chiral materials. As the molecular centers of mass are distributed randomly in space the structure belongs to the nematic class. It coined the name twist-bend nematic phase (N_{TB}). Contrary to cholesterics, the director in N_{TB} is not perpendicular to the helix axis, but precesses on a cone, with the helical axis parallel to the cone’s axis. As formation of N_{TB} does not require any presence of molecular chirality experimentally coexisting domains of opposite chirality are observed. So far the N_{TB} phase is stabilized below uniaxial nematic phase (N) as a result of first order N – N_{TB} phase transition on the temperature scale. Hence, we observe a fundamentally new phenomenon namely the spontaneous chiral symmetry breaking within the nematic class of materials.

Very recently, chiral asymmetric dimers were also studied. Here the intrinsic molecular chirality is an extra factor giving rise to overall a sequence of up to seven distinct nematic phases. As the constituent molecules are intrinsically chiral the highest-temperature phase is the cholesteric phase (N^*) or a blue phase. Stable phases observed at lower temperatures are variants of N_{TB} with pitch which is larger than one in achiral N_{TB}.

The issue of stable N_{TB} and N_{SB} structures has been addressed at theoretical level in two important papers. With the aid of symmetry arguments, supplemented by second-order elasticity theory of the director field, Meyer and later on Dozov, have analyzed some consequences of spontaneous local bend or splay deformations of the nematic director on the polar organization of the molecules - the so called flexoelectric effect. While the cholesterics can fill the space with homogeneous twist it appears that the corresponding bend state should always be associated either with some twist or splay. According to this picture the uniform nematic phase would then become unstable to the formation of a modulated phase, which could be either N_{TB} or N_{SB}. A prerequisite to such behaviour would be the sign change of the (effective) bend Frank elastic constant, K_3. Indeed, K_3 determined experimentally in N_{LP} is anomalously small as the transition to N_{TB} is approached.

Recently, Shamid et al. have developed Landau theory and lattice simulations of polar order and director bend deformations, correlating flexoelectricity, negative K_3 and stability of N_{TB} and N_{SB} phases. They predicted a second-order phase transition from high-temperature N phase to low-temperature N_{TB} or N_{SB}. At the transition, the effective K_3 changes sign and the corresponding structure develops modulated polar order, averaging to zero globally. All phases are assumed uniaxial and described entirely using director and polarization fields.

The purpose of this Letter is to investigate how nematics can self-organize into ODMNS for nonchiral and intrinsically chiral V-shaped molecules, using first-principles, symmetry-based generalization of Landau-deGennes theory of nematics. We assume that the second-rank 3 × 3 traceless and symmetric alignment tensor field, Q(r), is the primary order parameter accounting for nematic order. It permits that locally a system is described by a tripod of orthonormal directors \{\hat{n}, \hat{l}, \hat{m}\}.
and corresponding eigenvectors \( \{\lambda_n, \lambda_l, \lambda_m\} \). Identifying the full biaxial field \( \mathbf{Q} \) with primary order parameter of nematics, rather than its \( n \)-part only [12], should also clarify whether biaxiality \( (\lambda_n \neq \lambda_l \neq \lambda_m) \) is relevant for \( N_{TB} \), for we know that chiral nematic phases of at least intrinsically chiral mesogens are all biaxial [13]. Important theoretical questions are thus about structure characterization of ODMNS, both for non-chiral and intrinsically chiral materials [6, 10].

For the modeling of spontaneous chiral symmetry breaking as observed in \( N_{TB} \) the \( \mathbf{Q} \)-tensor alone is not sufficient. In the lowest order scenario we need, in addition to \( \mathbf{Q} \), at least one secondary order parameter, which can be either a first-rank (polar) field, \( \mathbf{P}(\mathbf{r}) \) [10], or a third-rank tensor field \( \mathbf{T}(\mathbf{r}) \), invariant with respect to tetrahedral point group symmetry [17–23]. The difference between these scenarios would be the polar order for \( \mathbf{Q} \) and \( \mathbf{P} \) couplings [10] and lack of polarity but the presence of nonlinear dielectric tensor for \( \mathbf{Q} \) and \( \mathbf{T} \). Here we focus on one-dimensional modulated structures as induced by \( \mathbf{Q} \) and \( \mathbf{P} \). We look systematically into extended Landau-deGennes-Ginzburg (LdG) free energy expansion and discuss the role played by various symmetry-allowed, lowest-order couplings. The essential ordering mechanism towards one-dimensional modulated structures will then be identified with the help of bifurcation and numerical analyses.

Before we proceed further it is important to realize that the polar field, \( \mathbf{P} \), does not need to be of electrostatic origin. Bend-core molecules are primarily polar due to their “V” shape while bimesogens acquire steric polarization in their conformational states. Such steric dipoles are present even in the absence of electrostatic dipoles. In a densely packed environment, we expect that these entropic, excluded volume interactions, rather than charge moments, define the local order, such as \( \mathbf{P} \). Recently, Greco and Farrarini have studied systems composed of crescent-shaped molecules interacting through a purely repulsive potential providing strong support for entropy-driven \( N \rightarrow N_{TB} \) transition [24].

We start by introducing the minimal coupling, LdG free energy per volume, constructed as a power-series expansion in \( \mathbf{Q} \) and \( \mathbf{P} \), and their derivatives. It can be decomposed as

\[
F = \sum_{i=2}^{4} F_i = \frac{1}{V} \sum_{i=2}^{4} \int \left( f_{iQ} + f_{iP} + f_{iQP} \right) d^3r, \tag{1}
\]

where \( f_{iX} \) are the free energy densities constructed out of the order parameters \( \{X\} \) and contributing to \( F_i \) in \( i \)-th order. By taking suitable units of energy, length, \( \mathbf{Q} \) and \( \mathbf{P} \) and disregarding electric field, magnetic field and surface terms the general form of \( f_{iQ} \) up to fourth-order in \( \mathbf{Q} \) can be written as

\[
\begin{align*}
f_{2Q} & = \frac{1}{4} \left[ t_Q \operatorname{Tr}(\mathbf{Q}^2) + (\nabla \otimes \mathbf{Q}) \cdot (\nabla \otimes \mathbf{Q}) \right. \\
& \quad + \rho (\nabla \cdot \mathbf{Q}) \cdot (\nabla \cdot \mathbf{Q}) - 2k_0 \mathbf{Q} \cdot (\nabla \times \mathbf{Q}) \left. \right] \\
f_{3Q} + f_{4Q} & = -\sqrt{6} B \operatorname{Tr}(\mathbf{Q}^3) + \operatorname{Tr}(\mathbf{Q}^4)^2, \tag{3}
\end{align*}
\]

where, as usual, \( t_Q \) is the reduced temperature associated with \( \mathbf{Q} \); \( \kappa \) is the reduced chirality, proportional to the wave vector of the cholesteric phase and \( \rho \) is the relative elastic constant. Likewise, the polar parts \( f_{iP} \) are

\[
\begin{align*}
f_{2P} + f_{3P} + f_{4P} & = \frac{1}{4} \left[ t_p \mathbf{P}^2 + (\nabla \otimes \mathbf{P}) \cdot (\nabla \otimes \mathbf{P}) \right. \\
& \quad + a_c(\nabla \cdot \mathbf{P})^2 - 2k_0 \mathbf{P} \cdot (\nabla \times \mathbf{P}) \left. \right] + a_d(\mathbf{P}^2)^2 \tag{4}
\end{align*}
\]

Clearly, for electric dipoles \( \nabla \times \mathbf{P} \) would vanishes in [15] while the \( a_c \)-term should be replaced by direct interactions between charge distributions. However, for purely steric dipoles, associated with excluded volume interactions [24] these terms are all present. In particular, for intrinsically chiral materials that develop steric polar order the chiral parameters \( \kappa, \kappa_P \) and \( \kappa_{QP} \) are all nonzero.

General, orientational properties of polar biaxial liquid crystals, characterized by \( \mathbf{Q} \) and \( \mathbf{P} \), were analyzed in [14, 23]. In particular the flexopolarization couplings were identified and classified. Assuming deformations to appear only in a quadratic part of the free energy and neglecting surface terms the lowest-order cross-coupling terms \( f_{iQP} \) are

\[
\begin{align*}
f_{2QP} & = -\frac{1}{4} \left[ \epsilon_\rho \mathbf{P} \cdot (\nabla \cdot \mathbf{Q}) + 2k_0 \mathbf{P} \cdot (\nabla \times \mathbf{Q}) \right] \tag{5} \\
f_{3QP} & = -\lambda P_0 \mathbf{Q} \mathbf{P}_3 \\
f_{4QP} & = \lambda_1 \mathbf{P} \mathbf{Q}^2 \mathbf{P}_3 + \lambda_2 \mathbf{P}^2 \mathbf{T}(\mathbf{Q})^2. \tag{7}
\end{align*}
\]

The LdG expansion [21] is the minimal coupling theory for systems described in terms of \( \mathbf{P} \) and \( \mathbf{Q} \), where \( \mathbf{P} \) is of steric origin. Our objective is to identify possible ODMNS that minimize \( F(\mathbf{Q}(\mathbf{r}), \mathbf{P}(\mathbf{r})) \) for arbitrary \( t_Q \) and \( t_P > 0 \), and for fixed values of the material parameters. By taking \( t_P > 0 \) we assume from the start that \( \mathbf{P} \) is secondary order parameter. A brief account of what to expect from such theory has already been presented long ago \( [10] \) where we indicated on a possibility of flexopolarization-induced periodic, one-dimensional structures. A more quantitative analysis of modulated nematic structures that can be driven by (flexo-)polarization is found in [26, 28]. In their theory Alexander and Yeomans [26] showed that applying an electric field to a sample with a large flexoelectric response can stabilize \( N_{SB} \) and a flexoelectric blue phase. Shamid, Allender and Selinger [27], on the other hand, have taken a simpler version of the expansion [10] and showed that the system can stabilize a polar analogue of chiral blue phases if polar order is allowed to be induced spontaneously.

In this letter we study inhomogeneous nematics with inhomogeneity propagating in one spatial direction, both for nonchiral and intrinsically chiral materials. We show that modulated phases, identified so far as a result of polar coupling, do not exhaust all possibilities that the theory [14] allows for. Our ultimate goal will be to look into fine structure of these phases and clarify the role played by biaxiality. In order to address these issues
we explore the bifurcation theory supplemented by numerical minimization and identify global minima of $F$, Eq. (11), within the ODMNS family, leaving a more complex issue of stable blue phases to our future studies.

The above programme is realized in practice by expanding $Q(r)$ and $P(r)$ into plane waves of definite helicity: $Q(r) = \sum_k \sum_{m=-2}^2 Q_m(k) \exp(i \mathbf{k} \cdot \mathbf{r}) M_{m,k}^{|2|}$, $P(r) = \sum_k \sum_{m=-1}^1 P_m(k) \exp(i \mathbf{k} \cdot \mathbf{r}) M_{m,k}^{|1|}$. Here $k$ are wave-vectors, $P_m(k)$ and $Q_m(k)$ are the variational parameters in the free energy expansion, and $M_{m,k}^{|1|} = \mp \frac{1}{\sqrt{2}} (\xi \pm i \eta)$, $M_{m,k}^{|2|} = \frac{1}{2} (\xi \pm i \eta) \otimes (\xi \pm i \eta)$, $M_{\pm 1,k}^{|1|} = \mp \frac{1}{\sqrt{2}} (\xi \pm i \eta) \otimes k \otimes (\xi \pm i \eta)$, and finally $M_{\pm 1,k}^{|2|} = \frac{1}{\sqrt{6}} (3k \otimes \kappa - 1)$ are the spin $L = 1/2$ spherical tensors represented in an orthonormal, right-handed local coordinate system $\{\xi, \eta, \kappa\}$ with $k$ as quantization axis. Two amplitudes of opposite helicity out of $\{Q_m(k), P_m(k)\}$ can be taken real due to invariance of $P$ and $Q$ with respect to uniform translations in 3D and global rotations about the $k$-axis. We choose $\Im Q_{\pm 1} = 0$. The reality condition: $\{Q(r) = Q(r)^*, P(r) = P(r)^*\}$ additionally implies that $Q_m(-k) = (-i)^m Q_m(k)^*$ and $P_m(-k) = (-i)^{m+1} P_m(k)^*$. The selection of $k$ and relevant amplitudes $\{Q_m(k), P_m(k)\}$ is fixed by the bifurcation analysis [23, 30] while their numerical values are found by the subsequent minimization of $F$.

In the vicinity of the isotropic phase the dominant contributions to $F$, Eq. (11), comes from $F_2$ which, for the ODMNS structures, is

$$F_2 = \sum_n \sum_{m=-2}^2 \left\{ A_m(|n|k)|Q_m(n)|^2 + (1 - \delta_m, 4) \left[ B_m(|n|k)|P_m(n)|^2 + \frac{1}{2} C_m(|n|k) (Q_m(n) P_m^*(n) - Q_m^*(n) P_m(n)) \right] \right\},$$  

where

$$A_m(|n|k) = \frac{t_0 + n^2 k^2 + \rho (1 - m^2) k^2}{4},$$

$$B_m(|n|k) = \frac{t_0 + n^2 k^2 + a_c (1 - m^2) k^2}{4} - 2\kappa P_m |n|k / 4,$$

$$C_m(|n|k) = -\frac{1}{4} \left( e \rho \sqrt{4 - m^2} |n|k + \sqrt{2} \kappa Q_P mn^2 k^2 \right).$$

Here $k$ is replaced by $nk$, $n = 0, \pm 1, \ldots$, $P_m(k)$ by $P_m(n)$ and $Q_m(k)$ by $Q_m(n)$; $\delta_{ij}$ is the Kronecker delta. Setting $\partial F / \partial Q_m(n) = \partial F / \partial P_m(n) = \partial F / \partial k = 0$ determines the equilibrium value of the amplitudes and $k$-vector for given material parameters. Since explicit dependence on $k$ appears only in second-order contributions, we have $\partial F / \partial k = \partial F_2 / \partial k = 0$. Note that the $F_2$-terms linear in $k$ promote modulated structures, among which are ODMNS. As it turns out an interesting class of ODMNS can already be identified by studying a simpler model where $\kappa_P = \kappa_{QP} = \lambda_1 = \lambda_2 = 0$. In what follows we shall consider this simpler case leaving analysis of the full model, along with the issue of stable blue phases, to our forthcoming publications. In addition we take $t_P > 0$ and $\kappa > 0$. For thermodynamic stability it is also necessary that $\rho > -\frac{3}{4}$ and $1 + a_c > 0$. Additionally, $a_4$ must be positive if $\lambda \neq 0$.

We shall now proceed by analyzing the case of $\lambda = a_4 = 0$ and later discuss the effect of $\lambda \neq 0$. For the first-mentioned case the polarization field appears only in $F_2$ and, hence, the condition $\partial F \partial P_m(n) = 0$ can be solved for $P_m(n)$ given fixed $Q_m(n)$. It yields

$$P_m(n) = \frac{-1}{2B(m, |n|k)} Q_m(n), \quad m = 0, \pm 1.$$  

(10)

Substituting (10) back to $F$ we obtain the effective free energy that still has to be minimized with respect to $Q_m(n)$. Only $F_2$ is modified by this substitution. It reads

$$F_{2, eff} = \frac{1}{2} \int V \left( F_{2, eff}(Q, \partial Q) d^3 r \right)$$  

(11)

Note that the leading elastic part of $F_{2, eff}$ can again be cast in form [24], but with $\rho$ being replaced by $\rho - \frac{\sigma^2}{4}$. Since $(\nabla \cdot Q)^2$ vanishes for twist deformations [25] the flexopolarization must induce splay-bend instability for $\rho - \frac{\sigma^2}{4} \leq -\frac{3}{4}$. However, the $e \rho$ term alone cannot bring about spontaneous chiral symmetry breaking. For that we need sufficiently large $|\lambda|$.

We shall now seek for ODMNS that can be stabilized as result of a phase transition from the isotropic phase. The general method is to analyse the nonlinear equations $\partial F / \partial Q_m(n) = \partial F / \partial P_m(n) = 0$ for the amplitudes $Q_m(n)$ and $P_m(n)$ using the bifurcation analysis (BA) [23, 30]. We apply this method to identify the leading amplitudes in the expansion of $Q$ and $P$ close to a phase transition from the isotropic phase (I), where $Q_m(n) = P_m(n) = 0$, to an ordered ODMNS. The procedure is straightforward for $\lambda = a_4 = 0$. In the zeroth-order of BA the amplitudes $Q_m(n)$ are close to their isotropic values and, hence, governed by the $F_{2, eff}$ part of $F$. Since $F_{2, eff}$ is in its diagonal form we can identify five different ODMNS, each characterized by the single $Q_m(1)$ mode of helicity $m$, that bifurcates from 1 at temperatures $t_Q = t_m$, where $t_m$ equals to $t_Q$ at which the coefficient in front of $|Q_m(1)|^2$ in Eq. (11) vanishes.

The corresponding wavevectors, $k_m$, are determined from $\partial F_2 / \partial k = 0$ with $P_m(n)$ given by Eq. (10). The maximal of the temperatures $t_m$, represents a potential transition temperature from 1 to ODMNS for a continuous phase
transition and spinodal for a first order phase transition. Explicitly, the \( m = \pm 2 \) modes bifurcate from \( I \) when \( t_Q = t_{\pm 2} = \kappa^2 \) and \( k_{\pm 2} = \pm \kappa \). The condensation of \( m = \pm 1 \) modes occurs when \( t_{\pm 1} \) and \( k_{\pm 1} \) satisfy the implicit relations \( t_{\pm 1} = \frac{k_{\pm 1}(\sqrt{c_1^2 + 4(c_1 + t_P)T_B})}{4(c_1 + t_P)} \)

and \( \kappa = \frac{k_{\pm 1}(\sqrt{c_1^2 + 4(c_1 + t_P)T_B})}{4(c_1 + t_P)} \). They can be resolved for non-chiral materials (\( \kappa = 0 \)) giving \( t_{\pm 1} = \frac{1}{\sqrt{1}} \left( e_{2p}^2 - 4(\rho + 2)e_{2p}^2 + 4(\rho + 2)t_P \right) \) and \( k_{\pm 1} = \frac{\sqrt{e_{2p}^2 - 2/2(2p + 3)e_{2p}^2 + 2(2p + 3)t_P}}{4(\rho + 2)} \).

Finally, \( t_0 = \frac{\sqrt{2e_{2p}^2 - 2t_P}}{4(\rho + 2)} \), for \( 0 < t_P < \frac{e_{2p}^2}{4(\rho + 2)} \). We should mention that the bifurcation from \( I \) to \( N \) takes place at \( t_Q = 0 \). First-order BA, consistent with modelling of the \( N^* \) phase, allows to identify the next to leading amplitudes \( Q_m(n) \), \( n = 0, 1 \) of ODMNS that couple to those given above through \( F_3 \) and \( F_4 \). A subsequent minimization of \( F \) either with respect to so identified trial states or with respect to all amplitudes \( |n| \leq 1 \) gives (consistently) six different ODMNS, shown in Fig. 1.

Our numerical minimization is carried out for the exemplary sets of material parameters. The corresponding phase diagrams are shown in Figs. 2. The bifurcation temperatures from the isotropic phase are also plotted as dashed lines. All phase transitions involved are at least weakly first order although with increasing \( t_Q \) and decreasing \( t_P \) the difference between the bifurcation- and transition temperatures becomes numerically negligible.

Figs. 2 show new ODMNS structures predicted by the model for \( \kappa = 0 \). These flexopolarization-induced nonchiral structures are referred to as \( N_{LP} \) and \( N_{BP} \). In \( N_{BP} \) the polarization vector, Fig. 1, is always perpendicular to \( \mathbf{k} \) and \( \mathbf{m} \), and \( \Re Q_{-1}(1) = -\Re Q_{-1}(1) \). The homogeneous nematic background \( \Re Q_{0}(0) \neq 0 \) makes \( N_{BP} \) locally biaxial with biaxiality modulated along \( \mathbf{k} \). The \( N_{LP} \) phase is constructed out of \( m = 0 \) modes. Here \( \mathbf{n} \) and \( \mathbf{P} \) are always parallel to \( \mathbf{k} \). The phase is uniaxial and periodically changes between prolate and oblate. In order to obtain the transition from \( I \) to \( N_{LP} \) the \( a_c \) term, \( (\text{Eq.} \[8\]) \), should be small positive or negative. In the \( N_{SB} \) phase an inhomogeneous biaxial modulation is also generated. Here \( \mathbf{P} \) is periodically modulated in the \( \{\mathbf{n}, \mathbf{l}\} \)-plane of splay-bend deformations. The phase diagrams presented in Figs. 3 show changes induced by intrinsic molecular chirality for \( \lambda = 0 \). Clearly, \( \kappa \neq 0 \) results in replacing \( N \) by \( N^* \), where \( \mathbf{P} = 0 \) and \( \mathbf{n} \) rotates about \( \mathbf{k} \) and remains everywhere perpendicular to \( \mathbf{k} \). Changes also concern \( N_{TP} \) and \( N_{SB} \). \( N_{TP} \) transforms into \( N_{TB} \), Fig. 1 which is biaxial with \( \mathbf{n} \) precessing on the cone about \( \mathbf{k} \) and \( \mathbf{P} \) parallel to \( \mathbf{m} \) and perpendicular to \( \mathbf{k} \). The \( N_{SB} \) phase is also biaxial, where two out of the three directors possess twist deformations similar to the ones modelled in [28].

\( \mathbf{P} \) in this phase is a linear combination of all three directors. Interestingly, the nonchiral \( N_{LP} \) phase can become stable even in intrinsically chiral materials, Fig. 3.

The \( N_{TB} \) phase can be stabilized not only for \( \kappa \neq 0 \), but primarily when \( \lambda \neq 0 \) \( (a_c > 0) \). In order to obtain the \( N_{TB} \) phase for nonchiral materials the sign of \( \lambda \) must be consistent with the sign of \( e_P \) and \( |\lambda| \) should exceed a threshold value. For example, if we take parameters of Fig. 2 and \( a_c = 1 \), the \( N_{TB} \) phase becomes stable for \( \lambda < -0.4 \). Calculations carried out for \( \lambda = -1/2 \) show rich sequence of phase transitions:

\( I \leftrightarrow (N_{TP}, N_{SB}, N) \leftrightarrow N_{TB} \), where phases in parentheses are optional. Although the \( N_{TB} \) phases obtained for \( (a) \{ \kappa \neq 0, \lambda = 0 \} \) and \( (b) \{ \kappa = 0, \lambda \neq 0 \} \) have the same symmetry, in the first case the structure of helic-
FIG. 3: (Color online) Phase diagram for $\rho = 1$, $\kappa = 0$, $e_P = -3$, $B = \frac{1}{\sqrt{6}}$ and $a_c = -\frac{1}{4}$. For further details see caption to Fig. 2.

FIG. 4: (Color online) Phase diagram for $\rho = 1$, $\kappa = \frac{1}{10}$, $e_P = -4$, $B = \frac{1}{\sqrt{6}}$ and $a_c = 2$. Here all modulated phases are chiral. For further details see caption to Fig. 2.

ity $m = \text{Sign}(\kappa)$ minimizes $F$, while for the case (b) structures of opposite helicities $m = \pm 1$ are of the same free energy, which is known as ambidextrous chirality. The cases (a) and (b) differ quantitatively, i.e., in periodicities and biaxiality parameter $[15]$.

In conclusion, LdG phenomenological theory of nematics extended to account for molecular steric polarization, stabilizes four bulk ODMNS for nonchiral materials, two of which: $N_{LP}$ and $N_{TP}$ have not been reported so far. The presence of molecular chirality converts $N_{TP}$ into $N_{TB}$ and $N_{SB}$ into new chiral $N_{SB}^*$, but the nonchiral $N_{LP}$ phase can stay stable even in the presence of intrinsic molecular chirality.

Acknowledgments

The authors thank Misha Osipov for stimulating discussions. This work was supported by the Grant No. DEC-2013/11/B/ST3/04247 of the National Science Centre in Poland.

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