Employment of vacation queueing models for maintenance of engineering systems of residential buildings

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Abstract We consider two main functions for servicing the housing stock-scheduled prophylactic inspection and repair of technical objects and the elimination of sudden malfunctions of technical equipment. An approach to analysis based on vacation queueing systems is proposed. We introduce two vacation queueing models in which during vacations the team realizes the prophylactic inspection and repair of technical objects. In the first model during vacation the elimination of sudden malfunctions of technical equipment stops and the second model has a working vacation scheme. For each model the characteristics which determine the quality of the work of the server (team), as well as the boundaries of the change of parameters at which the team copes with the work from the standpoint of a particular criterion are defined.

1. Introduction

Maintenance of residential buildings is a set of measures that ensure the highest reliability of all elements and systems of a building (see, for example, [1]-[5]). The main element of the technical operation of residential buildings is a system of scheduled prophylactic inspection and repairs. However even with its rational organization, there is always a positive probability of failure of building elements. Therefore, we consider two main functions for servicing the housing stock - scheduled prophylactic inspection and repair of technical objects and the elimination of sudden malfunctions of technical equipment. In this paper we propose an approach to analysis of technical operation of residential buildings, based on vacation queueing models. A vacation queueing system is one in which a server may become unavailable for a random period of the time from a primary service center. The time a way from a primary service center is called a vacation. There are various types of behavior of the server on the vacation period. In classical models the server completely stops service or is switched off when he is on a vacation. Servi and Finn [6] introduced the working vacation scheme, in which the server works in different rate rather than complexly stopping service during a vacation. We consider two vacation models, in which during vacations the team realizes the prophylactic inspection and repair of technical objects. In the first model during vacation the elimination of sudden malfunctions of technical equipment stops and the second model has the working vacation scheme.
The simplest vacation models for the technical operations of residential buildings were proposed in the articles [7,8]. The models considered in this paper generate obtained there results to the case of working vacation scheme and arbitrary distributed of the emergency repair times for breakdowns.

2. Methods: mathematical models based on queueing theory

Suppose that the management company (MC) of housing and communal services (HCS) has one or more teams of specialists to ensure the functioning of technical equipment’s (heat supply, water supply, ventilation, etc) of residential buildings. These teams have two main tasks – elimination sudden equipment failures and conducting prophylactic inspections and repairs to ensure the necessary level of reliability of the relevant technical systems.

To simplicity, this article assumes that there is only one work team, i.e. one device in the queueing system. The transition to the multichannel case has no fundamental obstacles, but is quite complicated technically.

We consider two vacation queueing models $M_1$ and $M_2$. For the both models we make the following assumptions.

- Intervals $\{a_n\}_{n=1}^{\infty}$ between emergency repair requests are independent exponentially distributed random variables with rate $\lambda$, i.e. $P(a_n > x) = e^{-\lambda x}$. This means that the flow of customers incoming to the service system is Poisson (see, for example, [9]) and mathematical expectation $E a_n = \frac{1}{\lambda}$.

- Emergency repair times for breakdowns are independent random variables with distribution function $B(x) = P(\xi_n \leq x)$, $b = E \xi_n$, $b_2 = E \xi_n^2$, $\beta(s) = E e^{-s \xi_n}$, ($Re s \geq 0$).

- There is one service team that can be busy either with repairing sudden breakdowns, or with prophylactic inspection or equipment repair. This means that there is one server at the service system.

- The team may begin prophylactic inspection and repair only if there are no request for emergency service. We will call such requests (customers or requirements) of the first type.

At moments when the team is exempted from the customers of the first type, the team proceeds to a prophylactic inspection and repair of the next facility, which lasts a random time $\eta$, which is called a vacation in queueing theory. Distribution function of the vacation period is $G(x) = P(\eta \leq x)$ and $g(s) = \int_0^\infty e^{-sx}dG(x)$. We assume that $G(x) = 1 - e^{\mu x}$ for the model $M_2$. Also denote $\bar{\eta} = E \eta$, so that $\bar{\eta} = \frac{1}{\mu}$ for the model $M_2$.

For the model $M_1$, we assume that customers of the first type are not served during vacation periods. For the model $M_2$, we have working vacation scheme. Namely, during vacation periods customers of the first type are served by unique server and service times have an exponential distribution with rate $\nu$.

Suppose that the MC wants to organize the maintenance of buildings so that the average number of calls $\bar{q}$ for urgent repairs in the system does not exceed a certain $\delta > 0$ and the average number $\bar{n}(T)$ of facilities preventive inspection and repair of which is completed in time $T$ is not less than $N(T)$, i.e.

$$\bar{q} < \delta, \quad \bar{n}(T) \geq N(T). \quad (1)$$

3. Results: formulars for operational characteristics

For the model $M_i$ ($i = 1,2$) let $T_n^{(i)}$ be the moment when the nth vacation starts and $T_n^{(i)} = T_{n+1}^{(i)} - T_n^{(i)}$ ($n = 0,1,2,...$). Then $\{T_n^{(i)}\}_{n=1}^{\infty}$ – sequence of independent identically distributed random variables. One may easy obtain from results for a queueing system $M|G|1$ (see, for example [9]) the following formula for the mathematical expectation

$$\bar{T}_i = E \bar{T}_n^{(i)} = \frac{\lambda \bar{n} (1 - \rho) + \rho \nu^{(i)}}{\lambda (1 - \rho)} \quad (i = 1,2). \quad (2)$$
For the model $M_1$ here $\bar{\eta}_1 = E\eta_n^{(1)}$ is the average of the vacation duration, $Y_1^{(1)}(t) = EY_n\left(\eta_n^{(1)}\right)$ and $Y_n\left(\eta_n^{(1)}\right)$ – the number of the first type customers in the system at the end of the nth vacation period, $\rho = \lambda b$.

We introduce a random process $q(t)$ which is the number of emergency calls (the first type customers) at time $t$ in the system $M_1$. From queueing theory (see, for example [9,10]) it follows that there are limits

$$\lim_{t \to \infty} P(q(t) = j) = P_j^{(1)} \quad (j = 0, 1, 2, \ldots)$$

and $P_j^{(1)} > 0, \sum_{j=0}^{\infty} P_j^{(1)} = 1$ if and only if $\rho = \lambda b < 1$. Further, we assume that stability condition $\rho < 1$ is satisfied. We define the generating functions $P_i(z) = \sum_{j=0}^{\infty} z^j P_j^{(i)}$ for the limit distribution of $q_i(t) (i = 1, 2)$. Then the avaverage number of emergency calls $\bar{q}_i = P_i'(1)$ in the system $M_i$. Using well-known results from queueing theory [9] one can obtain the following formula for $P_i(z)$

$$P_i(z) = \lim_{t \to \infty} E z^{q_i(t)} = \frac{(1 - \rho)(1 - z)\beta(\lambda - \lambda z)}{\beta(\lambda - \lambda z) - z} \cdot \frac{1 - g(\lambda - \lambda z)}{\lambda \bar{\eta}_i (1 - z)}. \quad (3)$$

Differentiating $P_i(z)$ with respect to $z$ and assuming $z = 1$ we obtain from (3) the average value $\bar{q}_i$ of the number of emergency calls

$$\bar{q}_i = E q_i = \rho + \frac{\lambda^2 b_i}{2(1 - \rho)} + \frac{\lambda E(\eta^{(1)}_i)^2}{2\bar{\eta}_i}. \quad (4)$$

Since $Y_1^{(1)} = \lambda \bar{\eta}_1$ for the model $M_1$ it follows from (2) that

$$\bar{t}_1 = \frac{\bar{\eta}_1}{1 - \rho}. \quad (5)$$

Now consider the model $M_2$. We define $Y_n^{(2)}(t)$ as the number of emergency calls which are situated in the system $M_2$ at time $T_n^{(2)} + t$. Then $Y_n^{(2)}(t)$ is the number of customers in the classical queueing system $M|\mu|1$ at time $t$. This system has a Poisson input flow with rate $\lambda$ exponentially distributed service time with parameter $\mu$ and initial condition $Y_n^{(2)}(0) = 0$. It is well known [9] that

$$P(z, s) = \int_0^\infty e^{-st} E z^{Y_n^{(2)}(t)} dt = \frac{1}{\alpha_1(s) - z(1 - \alpha_2(s))} \quad (Re \ s \geq 0). \quad (6)$$

Here $\alpha_{1,2}(s) = \frac{s + \lambda + \sqrt{(s + \lambda + \mu)^2 - 4s\mu}}{2\lambda}$. Since $E z^{Y_n^{(2)}(\eta_n^{(2)})} = P(z, \mu)$ then

$$Y_1^{(2)} = E Y_n^{(2)}(\eta_n^{(2)}) = (\alpha_1(\mu) - 1)^{-1}. \quad (7)$$

Hence for model $M_2$ we have from (2)

$$\bar{t}_2 = E Y_n^{(2)} = \frac{1}{\mu} + \rho \frac{\lambda(1 - \rho)(\alpha_1(\mu) - 1)}{\lambda(1 - \rho)(\alpha_1(\mu) - 1) + \rho\mu}. \quad (7)$$

The stationary distribution of $q_2(t)$ is given by formula

$$P_2(z) = \lim_{t \to \infty} E z^{q_2(t)} = \frac{(1 - \rho)(\alpha_1(\mu) - 1)}{\lambda(1 - \rho)(\alpha_1(\mu) - 1) + \rho\mu} \cdot \frac{\beta(\lambda - \lambda z)(\lambda\alpha_1(\mu) - 1) - \mu z + \mu(\lambda - \lambda z) - \lambda(\alpha_1(\mu) - 1)}{\left(\alpha_1(\mu) - 1\right)^2(\beta(\lambda - \lambda z) - z)}. \quad (8)$$
The proof is enough complicated and we omit it here. Differentiating $P_2(z)$ with respect to $z$ and putting $= 1$, we have from (10)

$$\bar{q}_2 = E q_2 = \frac{\mu}{\lambda(1 - \rho)(\alpha_1(\mu) - 1)} + \rho \mu \left( \rho + \frac{\lambda^2 b_2}{2(1 - \rho)} + \frac{(1 - \rho) \lambda}{\mu \alpha_1(\mu) - 1} \right).$$  \hspace{1cm} (9)

4. Discussion

Suppose that the MC wants to organize the maintenance of buildings so that conditions (1) will be fulfilled. From the renewal theorem [11] we obtain that the average number $\bar{n}(T)$ of completed vacations for large $T$ has asymptotics $\bar{n}(T) \sim \frac{T}{\tau}$.

When $N(T) = \gamma T$ for the model $M_1$ conditions (1) have the form

$$\rho + \frac{\lambda^2 b_2}{2(1 - \rho)} + \frac{\lambda E(\eta^{(1)})^2}{2\eta_1} \leq \delta, \quad \frac{1 - \rho}{\eta_1} \geq \gamma.$$ \hspace{1cm} (10)

If the system parameters $\lambda, b, b_2, E(\eta^{(2)})^2, E\eta^{(1)}$ satisfy the inequalities (10), then we can assume that the team satisfactory copes with the tasks. If even one of the inequalities is not satisfied, managing actions should be taken.

We assume that model $M_2$ instead of the Model $M_1$ is employed. Then we have the inequalities

$$\bar{q}_2 \leq \delta, \quad \bar{\tau}_2 \leq \gamma^{-1}$$ \hspace{1cm} (11)

where $\bar{q}_2$ and $\bar{\tau}_2$ are defined by (11) and (9) respectively. We note that $\bar{q}_2$ and $\bar{\tau}_2$ are decreasing functions of $\nu$ and for $\nu = 0$

$$\bar{q}_2 = \bar{q}_1, \quad \bar{\tau}_2 = \bar{\tau}_1.$$  

When $\nu \to \infty$ the following asymptotics take place

$$\bar{q}_2 \sim \frac{\mu}{(1 - \rho)\nu} \left( \overline{q}_1 - \rho \frac{\lambda}{\mu} \right), \quad \bar{\tau}_2 \sim \frac{\rho}{\nu(1 - \rho)}.$$  

Therefore, there exist $\nu_0$ such that for model $M_2$ with $\nu = \nu_0$ inequalities (11) are fulfilled. So MC may guarantee the feasibility conditions (11) providing the service of emergency calls with rate $\nu_0$ during vacation period.

5. Summary

In the present paper we propose two mathematical models with the help of which MC may solve two main tasks – eliminating sudden equipment failures and conducting prophylactic inspections and repairs to ensure the necessary level of reliability of the relevant technical systems. Our analysis relies on queueing theory, which is part of the probability theory that emerged from applications and widely used in solving applied problems in various fields.

6. References

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