Spin evolution of long-period X-ray pulsars

N.R. Ikhsanov, Y.S. Likh and N.G. Beskrovnaya

Pulkovo Observatory, Pulkovskoe Shosse 65, Saint-Petersburg 196140, Russia

Abstract. Spin evolution of X-ray pulsars in High Mass X-ray Binaries (HMXBs) is discussed under various assumptions about the geometry and physical parameters of the accretion flow. The torque applied to the neutron star from the accretion flow and equilibrium period of the pulsars are evaluated. We show that the observed spin evolution of the pulsars can be explained in terms of a scenario in which the neutron star accretes material from a magnetized stellar wind.

Keywords: Accretion and accretion disks, X-ray binaries, neutron star, pulsars, magnetic field

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1. INTRODUCTION

Long-period X-ray pulsars constitute a subclass of X-ray sources displaying regular pulsations with periods in the range from a few tens to a few thousand seconds in their X-ray emission. This subclass now includes more than 40 objects [1, 2]. The majority of them are identified with X-ray binary systems containing an early-type star and a strongly magnetized neutron star. The X-ray emission of these systems is accretion-driven, i.e. is produced due to accretion of matter onto the neutron star surface in the region of its magnetic poles. Regular intensity variations of radiation emitted from these regions occur with a period equal to the spin period of the neutron star whose magnetic axis is inclined to its rotational axis (see [3] and references therein).

The periods of long-period X-ray pulsars are not constant. They undergo, as a rule, chaotic variations over timescales from several days to several years while the average period does not change significantly (see Fig. 1). Such a behavior is explained by the exchange of angular momentum between a star and surrounding material which is governed by the equation [3]

\[ 2\pi I \dot{\nu} = K_{su} - K_{sd}. \]  

(1)

Here \( I \) is the moment of inertia of a neutron star, and \( \nu = 1/P_s \) is the frequency of its axial rotation with the period \( P_s \). The right side of the equation is the difference between the spin-up, \( K_{su} \), and spin-down, \( K_{sd} \), torques applied to the star from the accretion flow. Evaluation of these torques is the key problem in the modeling of the pulsar spin evolution. The spin period of a neutron star at which a balance between spin-up and spin-down torques is achieved \( (K_{su} = K_{sd}) \), is called the equilibrium spin period, \( P_{eq} \).
Until recently the spin evolution of long-period X-ray pulsars has been modeled exclusively under the assumption that the mass transfer beyond the magnetospheric boundary of a neutron star proceeds through either a Keplerian disk or a quasi-spherical flow. In particular, this approach has led to the conclusion that the absolute value of the spin-down torque is limited by inequality $|K_{sd}| \leq |K_{sd}^{(0)}|$, where \[4, 5, 6\]

$$|K_{sd}^{(0)}| = k_1 \mathcal{M} \omega_s r_A^2.$$  \hspace{1cm} (2)

The accretion rate onto the surface of a neutron star of the radius $R_{ns}$ and mass $M_{ns}$, which appears in this equation, $\dot{M} = L_X R_{ns} / GM_{ns}$, is evaluated from the X-ray luminosity of the pulsar, $L_X$. Here $\omega_s = 2\pi \nu$ is the angular velocity of the neutron star and $k_1$ is a dimensionless parameter of an order of unity. The magnetospheric radius of a neutron star within this approach coincides with the canonical Alfven radius, \[r_A = \left( \frac{\mu^2}{\mathcal{M} (2GM_{ns})^{1/2}} \right)^{2/7},\]  \hspace{1cm} (3)

at which the magnetic pressure due to dipole magnetic field of the neutron star, $\mu^2/(2\pi r^3)$, reaches the ram pressure of free-falling gas, $E_{ram}(r) = \rho(r) v_{ff}^2(r)$. The density and velocity of the gas in the spherical accretion flow at a given radius are determined as $\rho(r) = \dot{M} / [4\pi r^2 v_{ff}(r)]$ and $v_{ff}(r) = (2GM_{ns}/r)^{1/2}$, respectively, and the dipole magnetic moment of a neutron star, $\mu = (1/2)B_{*} R_{ns}^3$, can be written in terms of the surface magnetic field, $B_{*}$.

Studies of the most fully investigated long-period X-ray pulsars (see Table 1) have shown, however, that their observed spin-down rates, $|\dot{\nu}_{sd}|$, occasionally (see Table 2) significantly exceed the maximum possible value, $|\dot{\nu}_{sd}^{(0)}| = |K_{sd}^{(0)}|/(2\pi I)$, predicted within the canonical models (see Table 3). This discrepancy between theoretical and empirical data has been stated many times in the literature starting from 1975 (see, e.g., \[4, 7, 8, 9, 10\]). It can be avoided in the frame of traditional accretion scenarios only under assumption that the surface magnetic field of these neutron stars is $10^{14} - 10^{15}$ G which exceeds by two orders of magnitude the value inferred from observations of cyclotron line in their X-ray spectra.

Recently, however, doubts have been pronounced on a feasibility of an assumption about a presence of super-strong magnetic fields on the surface of long-period X-ray pulsars \[11, 12\]. Investigation of the long-period X-ray pulsar GX 301-2 has revealed that a controversy between theoretical predictions and observational data arises only when one attempts to explain the episodes of its rapid breaking. The magnetic field of the neutron star estimated from observations of its rapid spin-up episodes agrees with the value obtained through observations of the cyclotron feature in the X-ray spectra of this source (see \[11\] and references therein). An attempt to describe the episode of rapid spin-down of the neutron star SPX 1062 within the conventional model leads to a more paradoxical result: The Alfven radius of the neutron star turns out to exceed its corotation radius, $r_{cor} = (GM_{ns}/\omega_s^2)^{1/3}$, that excludes a possibility of stationary matter accretion onto its surface and obscures the nature of this X-ray source \[12\].
These results indicate that the spin-down torque exerted on a neutron star by the accretion flow is likely to exceed $|K_{sd}^{(0)}|$, and underestimation of this parameter may be connected with undue simplification of commonly used accretion scenarios. Most doubt is cast upon a validity of an assumption that the magnetic field in the material captured by a neutron star from the wind of its massive companion is negligibly weak. At early stages of the accretion theory development this notion was grounded on a common belief about a relatively weak magnetization of the massive early-type stars. In contrast to solar-type stars and late-type dwarfs, these objects do not contain sub-photospheric convective zones in which the magnetic field can be amplified in the dynamo process. Spectropolarimetric observations of the last decade have shown, however, that the large-scale magnetic fields of O/B stars can reach hundreds and even a thousand gauss [13, 14, 15]. Magnetic pressure in the stellar wind of such a star remains comparable to the thermal pressure of the outflowing plasma even at large distances from the star [16, 17] and can essentially affect the pattern of matter flow inside Bondi radius, $r_G = 2GM_{ns}/v_{rel}^2$, of its degenerate companion moving with velocity $v_{rel}$ relative to the stellar wind.

In this paper we estimate the spin-down torque applied to a neutron star from the accretion flow in the frame of a model problem on a sphere rotation in viscous medium [3]. We show that for the parameters of interest the torque is increasing with the sphere’s radius decreasing as $K_{sd} \propto r^{-3/2}$. Its estimate (see Eq. 12) corresponds to previously obtained solution, $K_{sd}^{(0)}$, for $r_m = r_A$ and significantly exceeds this value if the radius at which the flow enters the stellar magnetic field is less than the Alfven radius, $r_m < r_A$. In particular, the observed spin-down rate of the long-period X-ray pulsars within our approach can be obtained if $r_m \sim (0.1 - 0.5) r_A$ (see Table 4). This conclusion makes
it difficult to model the accretion picture in terms of Keplerian disk or quasi-spherical flow (see Section 2) and force us to consider alternative scenarios. One of them is the model of accretion from the magnetized wind, first suggested by Shvartsman [18] and further elaborated by Bisnovatyi-Kogan and Ruzmaikin [19, 20]. Within this approach, a neutron star accretes material from a non-Keplerian magnetic slab surrounding its magnetosphere. The radial motion of the matter inside the slab occurs in the diffusion regime on the annihilation time scale of the intrinsic magnetic field of the accretion flow. We show that the magnetospheric radius of a neutron star under these conditions can be significantly less than the canonical Alfven radius and close to the value inferred from observed spin-down rates of long-period X-ray pulsars (Section 3). Equilibrium period of a neutron star expected in different accretion scenarios (Keplerian disk, quasi-spherical flow and magnetic slab) is estimated in Section 4. We briefly discuss basic conclusions of the paper in Section 5.

2. SPIN-DOWN TORQUE

Following Lipunov [3], the spin-down torque exerted on a neutron star by the accretion flow can be estimated in the frame of model problem in which a sphere of radius \( r_m \) is rotating with angular velocity \( \omega_s \) in a medium with viscosity \( \nu_t \). The absolute value of
spin-down torque in this case is evaluated as

$$|K_{sd}| = v_t S_{eff} \rho_0 v_\phi,$$  \hspace{1cm} (4)$$

where $S_{eff}$ is the effective area of interaction between the sphere and accretion flow of density $\rho_0 = \rho(r_m)$. A parameter

$$v_\phi = \omega_r r_m \left(1 - \frac{\Omega_0}{\omega_k}\right),$$  \hspace{1cm} (5)$$

coming into this expression defines the azimuthal velocity component of the magnetospheric boundary relative to the surrounding gas rotating with angular velocity $\Omega_0 = \Omega(r_m)$ (assuming for simplicity $\Omega || \omega_k$). Combining these expressions we get the spin-down torque in a form

$$K_{sd} = r_m \times F,$$  \hspace{1cm} (6)$$

corresponding to the definition of this parameter as the cross product of force exerted on a sphere by surrounding gas,

$$|F| = v_t S_{eff} \rho_0 \omega_k \left(1 - \frac{\Omega_0}{\omega_k}\right),$$  \hspace{1cm} (7)$$

and the lever arm, $r_m$.

The effective area of interaction within approximation of azimuthal symmetry of the accretion flow can be expressed as

$$S_{eff} = 4\pi r_m h_z(r_m),$$  \hspace{1cm} (8)$$

where

$$h_z(r_m) = \frac{r_m^2 c_s^2(r_m)}{GM_{ns}}$$

is the height of homogeneous atmosphere which determines the scale of accretion flow in the direction, perpendicular to the plane of its rotation. Here $c_s(r_m)$ is the sound speed in the surrounding gas. The case of $c_s = v_k(r_m)$, where $v_k(r_m) = (GM_{ns}/r_m)^{1/2}$ is the Keplerian velocity at the magnetospheric boundary, corresponds to the spherical geometry of the accretion flow in which the effective area of interaction reaches the maximum value $4\pi r_m^2$.

Plasma density at the magnetospheric boundary in general case can be estimated from the balance of magnetic pressure due to the neutron star dipole field, $\mu^2/(2\pi r_m^6)$, and the gas pressure, $\rho_0 c_s^2(r_m)$. This leads us to expression

$$\rho_0 = \frac{\mu^2}{2\pi r_m^6 c_s^2(r_m)},$$  \hspace{1cm} (10)$$

Putting Eqs. (8–10) into (4), we find

$$|K_{sd}| = v_t \left(\frac{\mu^2}{r_m^2 GM_{ns}}\right) \left(1 - \frac{\Omega_0}{\omega_k}\right).$$  \hspace{1cm} (11)$$
Finally, adopting the turbulent viscosity of the medium, $\nu_t = k_t \ell_t v_t$, and taking into account that the scale and velocity of the turbulent motions at the magnetospheric boundary under conditions of interest are limited by the inequalities $\ell_t \leq r_m$ and $v_t \leq v_k(r_m)$, we derive the expression

$$|K_{sd}| = k_t \frac{\mu^2}{(r_m r_{\text{cor}})^{3/2}} \left(1 - \frac{\Omega_0}{\omega_k}\right),$$  \hspace{1cm} (12)

where $0 < k_t < 1$ is the dimensionless parameter.

The Eq. (12) is a generalized prescription of the spin-down torque applied to the neutron star from the accretion flow. The previous estimate of this parameter, $|K_{sd}^{(0)}|$, is a particular case of this expression which can be obtained substituting $r_m = r_A$ provided the angular velocity of the accretion flow at the boundary is zero, $\Omega_0 = 0$. If the turbulent velocity in the accretion flow at the magnetospheric boundary does not exceed $\omega_k r_m$, we come to the expression

$$|K_{sd}^{(t)}| = |K_{sd}^{(0)}| \left(\frac{\omega_k r_m}{v_k(r_m)}\right) = k_t \frac{\mu^2}{r_{\text{cor}}^3},$$  \hspace{1cm} (13)

presented earlier in [21, 22].

Thus, our evaluation of the spin-down torque agrees with previous estimates of this parameter obtained under assumption that the neutron star accretes material from a Keplerian disk or turbulent quasi-spherical flow. However, it indicates that the magnitude of the spin-down torque can be significantly larger in the frame of accretion scenario in which the magnetospheric radius of the neutron star is smaller than the canonical Alfven radius. In particular, the observed spin-down rates of the long-period X-ray pulsars (see Table 2) can be explained using Eq. (12) provided $r_m \leq r_0$, where

$$r_0 = \frac{1}{r_{\text{cor}}} \left(\frac{k_t \mu^2}{2\pi I\nu_{sd}}\right)^{2/3}.$$  \hspace{1cm} (14)

The estimates of $r_0$, calculated for the parameters of the most fully studied long-period X-ray pulsars and $k_t = 1$ are significantly smaller than the Alfven radii of these objects (see Table 4) and, hence, are less than the magnetospheric radii of neutron stars expected in case of accretion from a Keplerian disk or quasi-spherical flow. This circumstance force us to address alternative accretion models, one of which considers accretion from a magnetized stellar wind.

### 3. ACCRETION FROM A NON-KEPLERIAN MAGNETIC SLAB

As was first shown by Shvartsman [18], the distance at which the spherical accretion flow with magnetic field $B_f$ can approach the compact star in the free-fall regime is defined by the expression

$$R_{sh} = \beta_0^{-2/3} \left(\frac{c_s(r_G)}{v_{\text{rel}}}\right)^{4/3} r_G.$$  \hspace{1cm} (15)
Here $\beta_0 = E_{\text{th}}(r_G)/E_m(r_G)$ is the ratio of the thermal pressure, $E_{\text{th}} = \rho c_s^2$, to the magnetic pressure, $E_m = B_t^2/8\pi$, in the material captured by a compact star from the wind of its companion at the Bondi radius, and $c_s$ is the sound speed in the accretion flow. At the radius $R_{\text{sh}}$, a so called Shvartsman radius, the flow is decelerated by its own magnetic field and the process of accretion is switched to the diffusion regime. Studies of the matter flow inside the Shvartsman radius presented by Bisnovatyi-Kogan and Ruzmaikin [19, 20], have shown that in the region $r < R_{\text{sh}}$ the spherical flow is transformed into the magnetic slab with small angular momentum. The material of the slab is confined by the intrinsic magnetic field of the flow and is moving towards the compact star as the field dissipation proceeds. Numerical simulations of the magnetized accretion flow onto a black hole are presented in [23].

A possibility for such a scenario to be realized in case of accretion onto a neutron star has been recently discussed in [16, 11]. In particular, formation of the magnetic slab has been shown to occur under the condition $r_A < R_{\text{sh}}$, which is satisfied provided the relative velocity of the neutron star with respect to the wind of its component is $v_{\text{rel}} < v_{\text{ma}}$, where

$$v_{\text{ma}} \simeq 460 \beta_0^{-1/5} \mu_30^{-6/35} \mu_15^{3/35} m^{12/35} \left( \frac{c_s(r_G)}{10\,\text{km s}^{-1}} \right)^{2/5} \text{km s}^{-1}. \quad (16)$$

The magnetospheric radius of the neutron star in this case ranges as $r_{\text{ma}} \leq r_m \leq r_A$, where

$$r_{\text{ma}} = \left( \frac{c m_p^2}{16 \sqrt{2} e k_B} \right)^{2/13} \alpha_B^{2/13} \mu_30^{2/13} (GM_{\text{ns}})^{1/13} \quad (17)$$

is a solution of the system of equations

$$\begin{cases}
\frac{\mu^2}{2\pi r_m^6} = \rho(r_m)c_s^2(r_m)

\dot{M}_\text{in}(r_m) = \frac{L_X R_{\text{ns}}}{GM_{\text{ns}}}

\dot{M}_\text{in}(r_m) = 2\pi r_m \delta_m \rho_0 v_{\text{ff}}(r_m)

\delta_m = [\alpha_B t_{\text{ff}}(r_m) D_B(r_m)]^{1/2}
\end{cases} \quad (18)$$

Here $\dot{M}_\text{in}$ is the rate of plasma penetration from the slab into the neutron star magnetic field, $\delta_m$ is the diffusion layer thickness at the magnetospheric boundary,

$$D_B = \frac{ck_BT_i(r_m)}{16eB(r_m)} \quad (19)$$

is the Bohm diffusion coefficient and $\alpha_B < 1$ is a dimensionless parameter. Finally, $k_B$ is the Boltzmann constant and $T_i(r_m)$ is the ion temperature in the magnetopause plasma [24].
Table 3. The ratio of expected to observed spin-down rate

| Name          | $\dot{\nu}_{sd}^{(t)}/\dot{\nu}_{sd}^{obs}$ | $\dot{\nu}_{sd}^{(0)}/\dot{\nu}_{sd}^{obs}$ | $\dot{\nu}_{sd}^{(sl)}/\dot{\nu}_{sd}^{obs}$ |
|---------------|---------------------------------|---------------------------------|---------------------------------|
| OAO 1657–415  | 0.02                            | 0.06                            | 2.1                             |
| Vela X–1      | 0.003                           | 0.08                            | 2.7                             |
| 4U 1907+09    | 0.005                           | 0.24                            | 7.2                             |
| 4U 1538–52    | 0.001                           | 0.04                            | 1.3                             |
| GX 301–2      | 0.003                           | 0.24                            | 8.4                             |
| X Persei      | 0.005                           | 0.09                            | 2.7                             |

$\dot{\nu}_{sd}^{(t)} = \frac{K_{sd}^{(t)}}{2\pi I}$,  $\dot{\nu}_{sd}^{(0)} = \frac{K_{sd}^{(0)}}{2\pi I}$,  $\dot{\nu}_{sd}^{(sl)} = \frac{K_{sd}^{(sl)}}{2\pi I}$

The first equation of this system reflects that the magnetic pressure due to the stellar field balances the accretion flow pressure at the magnetospheric boundary. The second one is the continuity equation. It shows that in the process of stationary accretion, the rate of plasma penetration from the slab into the magnetosphere is equal to the accretion rate onto the stellar surface. The value of $\dot{M}_{in}$ is to a large extent determined by an effective diffusion coefficient which, in its turn, depends on the stability of the magnetospheric boundary. If it is unstable with respect to interchange instabilities (such as Raleigh-Tailor and/or Kelvin-Helmholtz), the rate of accretion flow penetration into the magnetosphere at the radius $r_A$ is comparable to the rate of mass transfer between the system components [25, 26, 27]. If the interchange instabilities of the magnetospheric boundary are suppressed (e.g. by the magnetic field shear, see [28]), the plasma entry into the field is governed by anomalous (Bohm) diffusion. The rate of plasma outflow from the slab to the magnetosphere at the radius $r_A$ in this situation is substantially smaller than the mass transfer rate between the system components derived from the observed X-ray luminosity of a pulsar [29]. The rate of plasma inflow to the magnetospheric boundary in this case exceeds the rate of plasma penetration into the stellar magnetic field. This results in accumulation of matter at the inner radius of the slab and increase of external pressure on the stellar magnetic field. According to the first and the third equations of system (18), this leads to magnetospheric radius decreasing, while the rate of plasma entry into the stellar magnetic field increases as $\dot{M}_{in} \propto r_m^{-13/4}$. The accretion process gets into a steady state as the magnetospheric radius reaches the value $r_{max}$, at which the rate of plasma diffusion into the stellar magnetic field attains the rate of mass transfer between the system components evaluated from the observed X-ray luminosity of a pulsar.

The spin-down torque applied to the neutron star due to interaction between its dipole field and the magnetic slab at the magnetospheric boundary is limited by the inequality $K_{sd} \leq K_{sd}^{(sl)}$, where [30]

$$K_{sd}^{(sl)} = k_t \frac{\mu^2}{(r_{ma}r_{cor})^{3/2}} \left( 1 - \frac{\Omega_0}{\omega_s} \right).$$

The maximum spin-down rate of the neutron star expected within this scenario can be
TABLE 4. Magnetospheric radii calculated using Eq. (14) and observed spin-down rates of the pulsars

| Name         | $r_0 (\nu^{\text{obs}}_sd)$, cm | $r_A (L_x)$, cm | $r_0 / r_A$ |
|--------------|---------------------------------|-----------------|------------|
| OAO 1657–415 | $1.4 \times 10^8$               | $6.8 \times 10^8$ | 0.20       |
| Vela X–1     | $1.3 \times 10^8$               | $5.6 \times 10^8$ | 0.24       |
| 4U 1907+09   | $2.9 \times 10^8$               | $6.0 \times 10^8$ | 0.48       |
| 4U 1538–52   | $9.5 \times 10^7$               | $6.3 \times 10^8$ | 0.15       |
| GX 301–2     | $2.6 \times 10^8$               | $5.5 \times 10^8$ | 0.48       |
| X Persei     | $4.6 \times 10^8$               | $1.8 \times 10^9$ | 0.25       |

evaluated, taking $\Omega_0 \ll \omega_s$ and solving inequality $|\dot{\nu}_{sd}| \leq |K^{(sl)}_{sd}|/(2\pi I)$. As a result one finds $|\dot{\nu}_{sd}| \leq |\dot{\nu}^{(sl)}_{sd}|$, where

$$|\dot{\nu}^{(sl)}_{sd}| = \frac{k_{t} \mu^2}{2\pi I (r_{ma}r_{cor})^{3/2}}.$$

The ratio of the maximum possible spin-down rate expected within different accretion scenarios to the observed rate for a number of the best studied long-period X-ray pulsars are presented in Table 3. The values are derived for the following neutron star parameters: radius $R_{ns} = 10$ km, moment of inertia $I = 10^{45}$ g cm$^2$ and dipole magnetic moment $\mu = (1/2)B_{\text{CRSF}}R_{ns}^3$, where $B_{\text{CRSF}}$ is the magnetic field strength on the stellar surface estimated through observations of the cyclotron line in the spectrum of a corresponding pulsar. Normalization of parameter $\alpha_B = 0.25$ has been chosen by analogy with solar wind penetration into the Earth magnetic field [31]. As clearly seen from Table 3, the spin-down rates expected for both a spherical accretion, $\dot{\nu}^{(t)}_{sd} = K^{(t)}_{sd} / (2\pi I)$ and an accretion from a hot turbulent envelope, $\dot{\nu}^{(0)}_{sd} = K^{(0)}_{sd} / (2\pi I)$, are significantly smaller than the observed values, while the spin-down rates calculated for the case of accretion from a magnetic slab exceeds the observed values. Observational results in this case can be fitted adopting the efficiency parameter value in the range $k_t \sim 0.1 - 0.8$.

4. EQUILIBRIUM PERIOD

According to Eq. (1), a spin period of an accreting neutron star evolves towards the equilibrium period, $P_{eq}$, defined through the balance of spin-up and spin-down torques, exerted on a star from the accretion flow. The value of this period depends on the parameters of both a binary system and its components as well as on the accretion flow properties, i.e. its geometry and physical conditions in the accreting material.
4.1. Accretion from a Keplerian disk

The magnetosphere of a neutron star accreting matter from a Keplerian disk contains both open and closed field lines. Angular velocity of the Keplerian disk material at the radius \( r < r_{\text{cor}} \) exceeds the spin angular velocity of the star, \( \omega_s \). Therefore, the interaction between the stellar magnetic field and the disk in this spatial region leads to spin-up of the star. Interaction of the magnetic field with outer parts of the disk located beyond the corotation radius \( r > r_{\text{cor}} \) results in deceleration of stellar rotation since the angular velocity of the matter in this part of the Keplerian disk is smaller than the angular velocity of the star. Equilibrium period of the star in this case can be evaluated from the equation \( |K_{su}^{(Kd)}| = |K_{sd}^{(Kd)}| \), where \[32\]

\[
|K_{su}^{(Kd)}| = \mathcal{M} (GM_{\text{ns}} r_{A})^{1/2}
\]  

is the spin-up torque, which is the product of specific angular momentum of the matter located at the magnetospheric boundary, \( \sim r_{A}^{2} \omega_k (r_{A}) \), and the rate of accretion of this matter onto the stellar surface, whereas \[21\]

\[
|K_{sd}^{(Kd)}| = \frac{\mu^2}{r_{\text{cor}}^3}
\]  

is the spin-down torque arisen due to interaction between the stellar field lines and remote parts of the disk. Here \( \omega_k = \left[ r^3 / (GM_{\text{ns}}) \right]^{1/2} \) is the Keplerian angular velocity. Solving this equation for \( P_s \), one finds \[3, 33\]

\[
P_{\text{eq}}^{(Kd)} \simeq 17 \mu_{30}^{-6/7} \mathcal{M}_{15}^{-3/7} m^{-5/7} \text{ s}.
\]  

Let us note that the value of \( P_{\text{eq}}^{(Kd)} \), calculated for the parameters of long-period X-ray pulsars is significantly smaller than the observed spin periods of these objects. This finding inevitably casts doubt on the presence of a Keplerian disk in these sources. Moreover, numerical simulations \[34\] have shown that expected spin evolution of long-period X-ray pulsars in this case differs from observed. The formation of a Keplerian accretion disk around a slowly rotating neutron star leads to monotonous decrease of its spin period at a rather high rate. For example, the typical spin-up time-scale of the neutron star GX 301–2 within this scenario is only 10 years \[11\]. That is why the geometry of accretion flow in long-period X-ray pulsars is usually considered in quasi-spherical approximation.

4.2. Quasi-spherical accretion

Magnetosphere of the neutron star undergoing accretion from a quasi-spherical flow or a hot turbulent envelope is closed and does not contain open field lines. Interaction between the neutron star magnetic field and accretion flow in this case occurs in the local region called magnetopause which is situated at the magnetospheric boundary of
the neutron star. Hence, the neutron star can spin down only provided \( \omega_I(r_A) < \omega_s \), where

\[
\omega_I(r_A) = \xi \omega_0 \left( \frac{r_G}{r_A} \right)^2
\]  

(25)
is the angular velocity in the quasi-spherical flow at the magnetospheric boundary, \( \omega_0 = \omega_I(r_G) \) is the angular velocity of matter captured by the star from the wind of its companion at the Bondi radius, and \( \xi \) is the dimensionless parameter accounting for dissipation of angular momentum in the quasi-spherical flow. Otherwise, as first noted by Bisnovatyi-Kogan [35], interaction between the magnetosphere and material at its boundary will result in spin-up of the star. This means that evaluating the equilibrium period of the star in the case of quasi-spherical accretion one should account for not only the absolute value of torques applied to the neutron star but also for their sign.

Specific angular momentum of matter with angular velocity \( \omega_I(r_A) \) at the magnetospheric boundary is \( r_A^2 \omega_I(r_A) \). The torque due to infall of this matter from the magnetospheric boundary onto the stellar surface at the rate \( \dot{M} \) can be evaluated \([36,37]\) as

\[
K_{\text{sp}}^{(sp)} = \xi \dot{M} r_G^2 \omega_0
\]

(26)
and causes spin-up of the star.

The torque due to viscous tensions in the quasi-spherical flow at the magnetospheric boundary in general case can be evaluated putting \( r_m = r_A \) into the Eq. (12) and taking into account that the turbulent velocity at the magnetospheric boundary, \( v_t(r_A) \), in this expression is normalized on its maximum value, a free-fall velocity, \( v_{ff}(r_A) \). This yields

\[
K_{\text{visc}} = \dot{M} \omega_s r_A^2 \left( 1 - \frac{\omega_I(r_A)}{\omega_s} \right) \left( \frac{v_t(r_A)}{v_{ff}(r_A)} \right).
\]

(27)

In case of accretion from a non-rotating \( (\omega_I(r_A) = 0) \) hot turbulent \( (v_t = v_{ff}(r_A)) \) envelope (scenario of a so called subsonic propeller \([38,39,40]\)), this expression is reduced to the canonical value of the maximum possible spin-down torque which is realized in the picture of quasi-spherical accretion, \( K_{\text{sd}}^{(0)} \). In the scenario of spherical (Bondi) accretion \( (\omega_I(r_A) = 0, \text{accretion from a free-falling flow}) \), in which the turbulent velocity does not exceed the linear velocity of matter in the magnetopause, \( v_t \leq \omega_s r_A \), the value of spin-down torque, \( K_{\text{visc}} \), turns out to be equal to \( \mu^2/r_{\text{cor}}^3 \), which is consistent with the result, previously obtained by Lipunov [22].

The equilibrium period of the star accreting material from a quasi-spherical flow can be found solving equation \( K_{\text{sp}}^{(sp)} = K_{\text{visc}} \) for \( P_s \) and normalizing the angular velocity of the matter captured by the neutron star at the Bondi radius on the orbital angular velocity, \( \omega_0 = 2\pi/P_{\text{orb}} \), (here \( P_{\text{orb}} \) is the orbital period of the system) as

\[
P_{\text{eq}}^{(sp)} = \frac{P_{\text{orb}} r_A^2}{\xi r_G^3} \left( \frac{v_t(r_A)}{v_{ff}(r_A) + v_t(r_A)} \right).
\]

(28)
The value of this period calculated under assumption $v_t(r_A) = v_{ff}(r_A)$ for the parameters of the long-period pulsar GX 301-2,

$$P_{eq}^{(sp)} \simeq 13 \times \xi^{-1}_{0.2} B_{12}^{8/7} L_{37}^{-4/7} m^{-12/7} R_6^{20/7} \left( \frac{P_{orb}}{41.5 \text{ d}} \right) \left( \frac{v_{rel}}{400 \text{ cm s}^{-1}} \right)^4,$$

is substantially smaller than the observed value (see Table 1). Here the parameter $\xi_{0.2} = \xi/0.2$ is normalized on its average value obtained in numerical simulations of quasi-spherical accretion without accounting for the magnetic field of the accretion flow [41], $L_{37} = L_X/10^{37} \text{ erg s}^{-1}$, $R_6$ is the neutron star radius in units of $10^6 \text{ cm}$, and the velocity of the star in the frame of surrounding gas is normalized on the upper limit of the stellar wind velocity of its massive counterpart inferred from observations [42]. Let us note that assumption about a higher stellar wind velocity encounters problems while explaining high X-ray luminosity of this source (expected mass capture rate by the neutron star under this assumption turns out to be significantly smaller than the accretion rate onto its surface). As a consequence, the modeling of long-period pulsars in terms of quasi-spherical accretion is also rather complicated.

### 4.3. Accretion from a magnetic slab

The torque applied to a neutron star undergoing accretion of matter onto its surface from a magnetic slab can be presented (see above) as the difference of two components. The first of them is caused by accretion of matter with angular momentum onto the stellar surface,

$$K_{su}^{(sl)} = M \Omega_0 \varpi^2,$$

and leads to spin-up of the star. The angular velocity of matter at the inner radius of the slab, $\Omega_0$, can be approximated taking into account that magnetic field of the accretion flow does not strongly influence the accretion process beyond the Shvartsman radius. This implies that mass accretion in the region $R_{sh} < r < R_G$ occurs in the quasi-spherical regime. In this case the angular velocity of material at the Shvartsman radius can be estimated using Eq. (25) as

$$\omega_f(R_{sh}) = \xi \omega_0 \left( \frac{r_G}{R_{sh}} \right)^2.$$

The matter infall inside Shvartsman radius is fully controlled by the magnetic field of the accretion flow and proceeds in the diffusion regime. This circumstance essentially complicates the problem about the evolution of angular momentum in the magnetic slab, i.e. in the region $r_m < r < R_{sh}$. A particular situation which is analyzed in this paper is the case of solid-body rotation of the slab. In the frame of this hypothesis the angular velocity of matter at the inner radius of the slab is adopted equal to that at the Shvartsman radius, $\omega_f(R_{sh})$.

The torque component applied to the star by the magnetic slab, $K_{sd}^{(sl)}$, describes interaction between the stellar magnetic field and the material located at the magnetospheric...
boundary and is determined by Eq. (20). The equilibrium period of the neutron star in this case can be evaluated solving equation $K_{\text{sl}}(s_l) = K_{\text{sl}}(s_d)$, which implies

$$P_{\text{eq}}^{(\text{sl})} = \frac{k_t \mu^2 c_s^{8/3}(r_G)P_{\text{orb}}}{\xi \beta_0^{4/3}v_{\text{rel}}^{8/3}\left(\mathcal{M}r_{\text{ma}}^{7/2}(GM_{\text{ns}})^{1/2} + k_t \mu^2\right)}. \quad (32)$$

Solving this equation for $v_{\text{rel}}$

$$v_{\text{rel}} = \frac{k_t^{3/8} \beta_0^{1/2} \mu^{3/4} c_s(r_G)P_{\text{orb}}^{3/8}}{\xi^{3/8}\beta_0^{1/2}P_{\text{s}}^{3/8}\left(\mathcal{M}r_{\text{ma}}^{7/2}(GM_{\text{ns}})^{1/2} + k_t \mu^2\right)^{3/8}}, \quad (33)$$

and adopting $\xi = 0.2$, $c_s(r_G) = 10\,\text{km s}^{-1}$ and $\beta_0 = 1$, we find that the average spin period measured in the long-period X-ray pulsar GX 301–2 can be fitted in the scenario of accretion from a magnetic slab provided the neutron star velocity relative to the wind of its massive counterpart is $v_{\text{rel}} \sim 450k_t^{3/8} \, \text{km s}^{-1}$, which is in good agreement with its estimate inferred from observations [42].

5. CONCLUSIONS

Results presented in this paper give evidence that the intrinsic magnetic field of the matter captured by the neutron star from the wind of its massive companion can essentially influence the accretion process in the long-period X-ray pulsars. Traditional accretion scenarios (from a Keplerian disk or a quasi-spherical flow) can be applied in this case only provided $R_{\text{sh}} < r_A$. Under this condition the magnetic field of the accretion flow can be neglected. Otherwise ($r_A < R_{\text{sh}}$), we come to a qualitatively new accretion picture. Formation of a non-Keplerian magnetic slab expected in this case force us to reconsider the interaction of the accretion flow with the stellar magnetic field. This leads to new evaluation of the magnetospheric radius (Eq. 17), torques exerted on the neutron star by the accretion flow (Eq. 20) and equilibrium period of a pulsar (Eq. 32). Numerical estimates of the spin-down rate of a neutron star and its equilibrium period expected within this approach are consistent with those derived from observations without additional assumption about significant deviation of the neutron star parameters from canonical values.

It should be noted that difficulties arising in the modeling of X-ray pulsars in the frame of Bondi accretion were mentioned by Arons and Lea [25] at the dawn of the age of the accretion theory. They have shown that magnetospheric radius of the neutron star forming in the process of interaction between its dipole magnetic field and free-falling material cannot be smaller than the canonical Alfvén radius $r_A$. In this case the magnetosphere does not contain open field lines and forms two cusp points located at the magnetic axis (see Fig. 2 from [25] and Eq. 5 from [43]). Plasma entry into the magnetic field of the star in this situation occurs in the direction perpendicular to the magnetic lines of force, and in general case proceeds via diffusion. The rate of both the classical (Coulomb) and anomalous (Bohm) diffusion of the spherical flow into the
magnetic field evaluated by Arons and Lea turned out to be a few orders of magnitude less than the accretion rate onto the stellar surface estimated from X-ray luminosity of the pulsars (see also [29]). This result has given evidence that either there exists a process which makes it possible for the accretion flow to enter the stellar magnetic field at the rate, substantially exceeding the rate of diffusion, or the structure of accretion flow in X-ray pulsars deviates from that realized in Bondi scenario. A rapid entry of the accreting matter into the stellar field could occur if the magnetospheric boundary were unstable with respect to interchange-type instabilities (in particular, Raleigh-Tailor and/or Kelvin-Helmholtz instabilities). Such a situation is possible, however, only in the high-luminosity pulsars ($L_X > 3 \times 10^{36} \text{erg s}^{-1}$ [25, 26]) with periods in excess of 1000 s [28]. Otherwise, the interchange instabilities of the magnetospheric boundary are suppressed. The overwhelming majority of long-period X-ray pulsars known nowadays do not satisfy these criteria that makes very difficult to describe these objects within the scenario suggested by Arons and Lea.

Similar obstacles are met when one considers accretion from a Keplerian disk. Relatively low efficiency of gas diffusion from the disk into the stellar magnetic field and stability of the magnetospheric boundary were mentioned in [44, 45, 46]. The main problem of this approach is, however, a relatively small value of the neutron star equilibrium period, $P_{\text{eq}}^{(Kd)}$. Bearing in mind that in case of accretion from a Keplerian disk the time scale of pulsar evolution to the equilibrium period constitutes only fractions of percent of the X-ray source lifetime [34], a probability to discover a long-period X-ray pulsar proves to be negligibly small. At the same time, the abundance of X-ray pulsars with periods $P_s \gg P_{\text{eq}}^{(Kd)}$ significantly exceeds the number of short-period pulsars.

Account for the intrinsic magnetic field of the accretion flow in modeling of mass transfer between the components of high-mass binaries with wind-fed X-ray pulsars greatly facilitate solving above mentioned problems. High rate of the flow entry into the stellar magnetic field in this case is reached due to increase of neither the effective area of interaction (as in case of interchange instabilities of the magnetospheric boundary) nor the diffusion time (as in the model of Ghosh and Lamb [47]), but rather due to generation of large density gradient at the magnetospheric boundary. The process of gas entry into the stellar magnetic field in this case is caused by the same mechanisms as in the solar wind penetration into the Earth magnetosphere (e.g. drift-dissipative instabilities of the magnetopause [31, 24]) and can be treated in terms of anomalous (Bohm) diffusion. The expected spin evolution of pulsars undergoing accretion from a magnetic slab is consistent with observed evolution of long-period X-ray pulsars. This gives us compelling reasons to believe that further development of the approach involving accretion from a magnetized stellar wind is very promising with regard to constructing the model of energy release in the accretion-powered sources.

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Spin evolution of long-period X-ray pulsars September 13, 2018 16