The roots of scalar-tensor theory: an approximate history

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Abstract

Why are there no fundamental scalar fields actually observed in physics today? Scalars are the simplest fields, but once we go beyond Galilean-Newtonian physics they appear only in speculations, as possible determinants of the gravitational constants in the so-called Scalar-Tensor theories in non-quantum physics, and as Higgs particles, dilatons, etc., in quantum physics. Actually, scalar fields have had a long and controversial life in gravity theories, with a history of deaths and resurrections. The first gravity theory of scientific interest was developed by Newton, using a scalar potential field. After developing special relativity into which electromagnetism fit so nicely, it was natural for Einstein and others to consider the possibility of incorporating gravity into special relativity as a scalar theory. Of course, in its original form this effort was not successful, but it did help in pointing the way to standard Einstein general relativistic theory of gravity as a metric field. However, the original investigation of a scalar field did reinforce Einstein’s interest in Mach’s principle, suggesting an influence of gravity on inertial mass. Also, five dimensional unified field theories as studied by Fierz, Jordan, and others suggested a spacetime scalar field that might well provide a “varying” gravitational constant. Even Einstein and Bergmann were briefly interested in such possibilities. However, Dicke, motivated by Dirac’s numbers and Mach’s principle, provided the major driving force for theoretical and experimental investigations of such a possibility. While later experimentation seems to indicate that if such a scalar exists its influence on solar system size interactions is negligible, other reincarnations of a scalar-tensor formulation have been proposed in the contexts of dilatons in string theory and inflatons in cosmology. This paper presents a brief overview of this history. A recent, and much more thorough, study of the subject of scalar-tensor theories can be found in the book by Fujii and Maeda, [1].

1 Introduction

We begin by briefly reviewing the role of scalars and scalar fields in physics. Before Einstein, the basic relativity principle in Galilean-Newtonian physics required invariance in form of the laws of physics under transformations of the Galilean group. Restricting ourselves to Newtonian gravity and Maxwell’s electromagnetism in this context, we can easily find examples of scalars, such as mass, electric charge, energy, etc., under the static (excluding velocity transformations) affine subgroup of the full Galilean group. However, when we allow constant velocity transformations, the notion of scalars becomes a little

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less obvious. For example, under the (constant) rotation group spatial intervals are clearly scalars, but this is not the case for non-trivial velocity transformations, for which the spatial distance between two events at the origin of one frame is zero as measured in that frame, but not zero in another. Similar considerations apply to speed, and thus kinetic energy. Clearly, the kinetic energy of a particle is not a scalar under non-trivial velocity transformations. Similarly, when we try to understand Maxwell’s electromagnetism in terms of a “scalar” and a “vector” potential, we find ourselves not able to formulate a consistent theory invariant under constant velocity transformations, and must rely on some fixed rest frame such as the ether. Of course, these considerations are precisely those that led from Galilean to Einsteinian special relativity, and a formulation of Maxwell’s electromagnetism in terms of a four-vector potential, with the complete elimination of any scalar component of the electromagnetic potential. The next step, from special to general relativity describes gravity in terms of a tensor, not a scalar, field.

Thus, while scalars (constants) naturally abound, fundamental, i.e., not derived, scalar fields are only hypothetical to date. In cosmology, pure Einstein theory uses only a 2-tensor, while in quantum theory, the observed “particles,” such as quarks, leptons, are represented by fermi spinors and the “gauge forces” are carried by boson vector fields.

- As of 2004, fundamental scalars appear only as hypothetical, as yet unobserved, fields related possibly to the gravitational or cosmological “constants,” dark energy, inflatons, dilatons, or Higgs fields.

In other words, nature seems to abhor using fields which have the same value in all reference frames. This is surely a curious fact.

2 Special Relativistic gravity

In the early days of special relativity, Einstein’s first successful field theory was a special relativistic re-formulation of Maxwell’s electromagnetic theory. Newtonian mechanics could be reformulated in terms of force as a four-vector, \( F^\alpha \), leading to a fully Lorentz covariant theory of mechanics describing motion parameterized by proper time, \( x^\alpha(\tau) \)

\[
\frac{d}{d\tau}(m_i \frac{dx^\alpha}{d\tau}) = F^\alpha. \tag{1}
\]

where \( m_i \) is a constant inertial mass, and of course,

\[
\eta_{\alpha\beta} \frac{dx^\alpha}{d\tau} \frac{dx^\beta}{d\tau} = -1. \tag{2}
\]

The constancy of \( m_i \) and the consistency of (1) and (2) then require that the four-force be four-orthogonal to the velocity,

\[
\eta_{\alpha\beta} \frac{dx^\alpha}{d\tau} F^\beta = 0. \tag{3}
\]
This is clearly satisfied identically for the electromagnetic four-force, \( \mathcal{F}^\alpha = F^{\alpha \beta} \eta_{\beta \gamma} \frac{dx^\gamma}{dt} \)

But what of a gravitational field theory?

- **How does gravity fit into special relativity?**

First, recall Newton’s formulation in terms of a Galilean scalar gravitational potential field:

\[
\nabla^2 \phi = \frac{\kappa}{2} \rho_{ag},
\]

(4)

where \( \rho_{ag} \) is mass density, with the \( ag \) subscript indicating that here the mass is acting as a source for gravity. Also, \( \kappa \equiv 8\pi G \), and \( G \) the usual Newton constant. Using Galilean three-vector notation,

\[
E_g = -\nabla \phi,
\]

(5)

the equations of motion become

\[
\frac{d}{dt}(m_i \frac{d\mathbf{r}}{dt}) = m_{pg} E_g.
\]

(6)

Here the \( i \) subscript indicates inertial mass, while \( pg \) corresponds to passive gravitational mass. Of course, it was and is common to simply assume

\[
\frac{m_{ga}}{m_{gp}} = 1,
\]

(7)

and

\[
\frac{m_{gp}}{m_i} = 1.
\]

(8)

It is fairly easy to give an argument that momentum conservation requires the satisfaction of (7). On the other hand, (8) is less trivial, and corresponds to the operationally significant

- **Weak principle of equivalence:** *Gravitational acceleration at a given point is independent of mass.*

So, as was common around 1900, let us temporarily assume

\[
m_{ag} = m_{pg} = m_i = m = \text{constant}.
\]

(9)

Finally, before leaving pre-Einsteinian gravity, we note that this potential, \( \phi \), has units of velocity squared, so that in the standard relativistic choice used in this paper, \( c = 1 \), \( \phi \) is dimensionless.

So, how do Einstein and his colleagues attack the problem of integrating Newtonian gravity into special relativity? Fortunately there are excellent, easily readable, accounts of this process, [2], [5]. What might seem to be the most natural way to proceed? Simply assume that gravity in special relativity will be described by a 4-scalar, \( \phi \), satisfying

\[
\Box^2 \phi = \frac{\kappa}{2} \rho,
\]

(10)
\[ F_\alpha^\beta = -m\phi^\alpha, \]  
(11) as equation of motion. However, \( (3) \), applied to \( (11) \) results in

\[ \frac{dx^\alpha}{d\tau} \frac{\partial \phi}{\partial x^\alpha} = \frac{d\phi}{d\tau} = 0. \]  
(12)

In other words, if we use \( (11) \) and \( (3) \) the potential must constant along the path of every particle, so the gravitational force must necessarily be zero on every particle! Clearly, something is wrong here.

Consider the problem from the viewpoint of an action. A point particle with path \( z^\mu(\tau) \), and density, \( \delta^4(x^\nu - z^\nu(\tau)) \). Here \( \tau \) is proper time, so

\[ \eta_{\mu\nu} \frac{dx^\mu}{d\tau} \frac{dx^\nu}{d\tau} = -1. \]  
(13)

As a guide, look at the electromagnetic equations, field and particle motion, as derived from particle, field, and interaction parts,

\[ A_p + A_{em} + A_I = -\int \left( \int m \sqrt{-\dot{z}^\mu \dot{z}_\mu} \delta^4(x^\mu - z^\mu(\tau))d\tau \right) d^4x \]
\[ -\frac{1}{16\pi} \int (A_{\mu,\nu} - A_{\nu,\mu})(A^{\mu,\nu} - A^{\nu,\mu})d^4x + \]
\[ q \int \left( \int \dot{z}^\mu(\tau) A^\mu I(x^\nu - z^\nu(\tau))d\tau \right) d^4x. \]  
(14)

Now consider a scalar gravitational modification of such a formalism,

\[ A_p + A_\phi + A_I = -\int \left( \int m \sqrt{-\dot{z}^\mu \dot{z}_\mu} \delta^4(x^\mu - z^\mu(\tau))d\tau \right) d^4x \]
\[ -\frac{1}{8\pi} \int \phi_{,\mu} \phi_{,\mu} d^4x \]
\[ -\int \phi \left( \int m \sqrt{-\dot{z}^\mu \dot{z}_\mu} \delta^4(x^\mu - z^\mu(\tau))d\tau \right) d^4x. \]  
(15)

The field variation of this action results in \( (10) \) with \( \rho(x^\mu) = m \int \delta^4(x^\mu - z^\mu(\tau))d\tau \) in the conformal gauge, \( (13) \). However, the variation over the particle’s variables, \( z^\mu(\tau), \dot{z}^\mu(\tau) \) results in something quite new, namely,

\[ \frac{d}{d\tau}(m(1 + \phi) \dot{z}^\mu(\tau)) = -m\phi_{,\mu}. \]  
(16)

When the four-vector equations of motion \( (10) \) are expressed in terms of local coordinate time, it is clear that local coordinate acceleration of a particle will depend not only on the the particle’s kinetic energy, but also on a modified inertial mass, \( m(1 + \phi) \), thus violating the equal acceleration principle, WEP. Neglecting the \( \frac{d}{d\tau} \phi \) term, the usual expansion of the left side of \( (10) \) into local coordinate expressions gives

\[ \frac{d^2r}{dt^2} \sim -(1 - v^2) \nabla \phi. \]  
(17)
Thus, the gravitational acceleration would depend on the velocity, so spinning bodies would have smaller accelerations in a gravitational field than non-spinning identical ones, hot bodies than cold, etc. Of course, this effect was too small to be noticed by early 20th century technology, but naturally Einstein was disturbed by the dependence of gravitational acceleration on internal structure of the falling body occurring in this initial attempt to “relativize” gravity.

In a parallel vein, von Laue was looking into the models of internal stress in extended bodies and found what we now call the four dimensional stress-energy tensor, with $T^{00}$ identified with energy density, and $T^{ij} = p^{ij}$ the components of the spatial stresses on the body. However, the application of a Lorentz velocity transformation to such a tensor would mix the purely spatial stress components into the energy density, so that the energy density of a moving body would depend on its internal stress. Thus, these internal stress components should contribute to the gravitational mass.

It might have been something along these lines that motivated Einstein in 1907 to discount the appropriateness of a scalar special relativistic theory of gravitation because it did not allow “...the inert mass of a body to depend on the gravitational potential.”[3] A related critique was formulated by Abraham, [2].

Actually, Nordström, [4] suggested that the inertial mass might depend on $\phi$,

$$m = m_0 \exp \phi, \quad (18)$$

or for a weak field

$$m = m_0 (1 + \phi). \quad (19)$$

In fact, (16) is related to Nordström’s (18), with $\exp(\phi) \approx 1 + \phi$, to first order in $\phi$. Most importantly, the resulting equation of motion, (16), is consistent with (13), as well as the suggested field and force equations, (10) and (11).

On the other hand, why associate $\phi$ with the mass? Why not associate it with the metric, replacing

$$d\tau^2 = (dt^2 - dx^2 - dy^2 - dz^2), \quad (20)$$

with

$$d\tau^2 = \exp(2\phi)(dt^2 - dx^2 - dy^2 - dz^2). \quad (21)$$

This is the direction taken by Einstein leading to his full general relativistic field equations using a 2-tensor, the metric as the potential. In the spirit of this paper this early scalar form for metric gravity, with the scalar appearing as a metric “dilation” was very notable.

Historically, however, after its brief, but passing, appearance in a Nordström model, (21), there seemed to be no room in physics for a scalar field. But Nordström’s suggestion (18), led Einstein to further pursue a **Mach’s Principle** in the sense of having inertial mass depend on the gravitational interaction of all of the other masses in the universe. We will briefly return to this later.

Of course, in parallel to relativity theory, quantum theory was being developed, and scalar fields again appear in the context of the Klein-Gordon equation. In turn, this equation, and its corresponding scalar field were replaced by Dirac equations. As we now
understand observed quantum physics, elementary particles are fermions, satisfying Dirac equations, while forces correspond to gauge fields which, while bosons, are spacetime vectors rather than scalars. When we go beyond present day observation, however, scalar fields may indeed return, as Higgs bosons, dilatons, etc. We will mention these later.

As of the beginning of the 21st century, fundamental scalar fields exist only as hypothetical structures in physics, such as outlined in the following:

- **Hypothetical non-quantum scalar fields**
  - Scalar-tensor fields, such as the JBD determinant of $G$, 
  - Inflatons, scalar field to give rise to observed anomalies of cosmological expansion,

- **Hypothetical quantum scalar fields**
  - Higgs particle, quantum scalar field providing mass by interactions with massless particles.
  - Dilatons, etc., quantum fields appearing in superstring and M theory.

### 3 The First Searches for Unified Field Theories

The hunt for a theory unifying gravity and electromagnetism began in the very earliest days of Einstein’s general relativity. For our purpose, the most significant was the 5-dimensional versions associated with the names of Kaluza and Klein. Applequist et al. have prepared a convenient review and reprints of original papers on the subject.

Briefly, KK theories enlarge the dimension of spacetime by one, so that the metric has a form

$$\gamma_{AB} = \begin{pmatrix} V^2 & V^2 A_\beta \\ V^2 A_\alpha & g_{\alpha\beta} + V^2 A_\alpha A_\beta \end{pmatrix}. \quad (22)$$

By restricting the five dimensional transformation group appropriately, $A_\alpha$ appear as the components of a spacetime 4-vector, with $V$ a spacetime scalar\(^2\). Furthermore, these transformations could also account for electromagnetic gauge transformations. Formally, this unification of spacetime and gauge transformations made this sort of formalism highly attractive, although the unobserved extra dimension was generally regarded as an obstacle to serious consideration of the model. Also, there was the question of the appearance of the unwanted scalar field, which Kaluza in 1921 described as “noch ungedeutet.”

But what are the field equations? By apparently natural extension of the four dimensional Einstein equations, consider

$$\delta \int d^5x \mathcal{R} \sqrt{|g^{(5)}|} = 0. \quad (23)$$

\(^2\)Jordan et al. [9], were able to characterize these transformations as those of a projective group.
These lead to spacetime four dimensional equations as well as 4 spacetime-fifth dimension equations and a single 5-5 equation, involving only derivatives of $V = g_{55}$. The spacetime equations are

$$ R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = \frac{V^2}{2} (F_{\alpha\mu} F^{\mu}_{\beta} + \frac{\eta_{\alpha\beta}}{4} F_{\mu\nu} F^{\mu\nu}) - \left( \frac{V_{,\alpha;\beta}}{V} - \frac{\eta_{\alpha\beta}}{V^2} \Delta V \right), \quad (24) $$

with $F_{\alpha\beta}$ the electromagnetic components derived from the potentials $A_\alpha$ as usual. These are the standard Einstein equations with electromagnetic stress tensor source, if we identify $V^2$ with 4 times the usual Newtonian gravitational constant, $G$. However, in these equations $V$ may not be constant and its derivatives also contribute.

- Equations (24) are the first hint of a varying gravitational constant.

### 4 Dirac’s numbers

Meanwhile, Dirac [8], building on the work of Eddington and Milne, became intrigued by apparently “coincidental” approximate equality between important physical quantities expressed in dimension free manner. Atomic scales can be deduced from $\hbar, e$ and $m_p$, say the mass of the proton. Then an atomic time(distance) scale is supplied by $T_a = R_a = e^2/m_a$. On the other hand we have the age(distance scale) and the mass of the universe, $T_u = R_u, M_u$ as cosmological scales. Finally, we have the gravitational constant, $\kappa$. The resulting dimensionless quantities could be approximately grouped into powers of the incredibly large number, $10^{40}$,

$$ \alpha = \frac{e^2}{\hbar} \approx 10^9, $$
$$ T_u / T_a \approx 10^{40}, $$
$$ T_a / \kappa \approx 10^{40}, $$
$$ M_u / m_p \approx 10^{80}. \quad (25) $$

For our purposes, the combination of these equations in the following form is most important

$$ \frac{1}{\kappa} \approx M_u / R_u. \quad (26) $$

### 5 Scalar-Tensor Theories

Perhaps the earliest work in this direction was pursued independently by Jordan in Germany and Einstein and Bergmann in the US beginning in the late 1930’s. Of course, this work proceeded under all of the terrible constraints of the second world war and the resulting isolation of the two groups. Actually, Einstein and Bergmann apparently decided not to proceed with the variable gravitational constant idea to the point of publication. Bergmann [11] reviews these parallel efforts in his paper, “Unified field theory with fifteen field variables,” from which we now quote:
In the spring of 1946, Professor W. Pauli turned over to the author of this paper galleys of a paper by P. Jordan entitled “Gravitationstheorie mit veränderlicher Gravitationszahl”, which was to have appeared in the Physikalische Zeitschrift sometime in 1945, but which was, of course, never published because the Phys. Zeitschrift in the meantime ceased publication. In this paper, Jordan attempted to generalize Kaluza’s five dimensional unified field theory by retaining $g_{55}$ as a fifteenth field variable. Professor Einstein and the present author had worked on that same idea several years earlier, but had finally rejected it and not published that abortive attempt. The fact that another worker in this field has proposed the same idea, and independently, is an indication of its inherent plausibility. Therefore, it seemed worthwhile to review these attempts to “vary the constant of gravitation” and to discuss the possibilities inherent in geometries of this kind.

Thus, independently of Einstein and Bergmann in the USA, Pascual Jordan and his colleagues in Germany [9] began an extensive look at Kaluza-Klein theories with special concern for the possibility that the new five-dimensional metric component, a spacetime scalar, might play the role of a varying gravitational “constant,” as suggested by Dirac’s [26]. Certainly the resulting four-dimensional form of the field equations can interpreted this way. However, Jordan and his colleagues went beyond the 5-dimensional origins of this scalar and proposed purely four dimensional field equations involving a scalar field related to Newton’s constant. Later Brans and Dicke [12] independently arrived at similar point. However, for Brans and Dicke, Mach’s ideas on inertial induction, that the total mass distribution in the universe should determine local inertial properties, were of prime concern. In fact, Sciama [13] had earlier proposed a model theory of inertial induction.

Sciama’s work provided a theoretical model in which inertial forces felt during acceleration of a reference frame relative to the “fixed stars,” are of gravitational origin. From this assumption, Dicke argued that Mach’s principle would manifest itself in having the ratio of inertial to gravitational mass depend on the average distribution of mass in the universe. That is,

**Dicke’s form of Mach’s Principle:** The gravitational constant, $\kappa$, should be a function of the mass distribution in the universe.

Because of Dirac’s large number hypothesis in the form

$$\frac{1}{\kappa} \approx M/R,$$  \hspace{1cm} (27)

it seems that the reciprocal of the gravitational constant will likely be the field quantity. In other words, $1/\kappa$ itself might be a field variable and satisfy a field equation with mass as a source, something like

$$\Box \frac{1}{\kappa} = \rho.$$  \hspace{1cm} (28)
So, introduce a scalar field, $\phi$, which will play the role, at least locally and approximately, of the reciprocal Newtonian gravitational constant, $\kappa$.

The usual Lagrangian for Einstein theory including matter contains $\kappa$ directly multiplying the matter contributions. Keeping the field directly coupled to matter would then inevitably lead to changes in the local behavior of matter, the local equations of motion, as a result of variations in $\phi$. So, in order to incorporate a Mach’s principle by way of a variable gravitational “constant,” we need to look at further modifications in the form of the general relativistic action. Begin with the standard Einstein action as

$$\delta \int d^4x \sqrt{-g}(R + \kappa L_m) = 0,$$

(29)

where $L_m$ is the “usual” matter Lagrangian, a priori derived from some particular classical or quantum model. Equation (29) is clearly not enough since it provides no field equation for the new field, $\kappa$.

Before proceeding, we need to review some aspects of the famous (or infamous) “principle of equivalence.” As usual we neglect tidal forces, extended bodies, etc., in these idealized models. Dicke often pointed out that we need to distinguish two forms:

- **WEP.** One form asserts that all bodies at the same spacetime point in a given gravitational field will undergo the same acceleration. We will refer to this as the “weak” equivalence principle, WEP. As it stands, this does not exclude possible effects of gravity other than acceleration.

- **SEP.** A stronger statement, which actually is important to Einstein’s general relativistic theory of gravity, is that the only influence of gravity is through the metric, and can thus (apart from tidal effects) be locally, approximately transformed away, by going to an appropriately accelerated reference frame. This is the “strong” principle, SEP.

An action of the form in (29) with variable $\kappa$, will clearly change the geodesic equation for test particles, thus, possibly the WEP, and even mass conservation. As Dicke noted, the Eötvös experiment verifies the WEP (but not the SEP), so, in the 1960’s, we would like to at least modify (29) to agree with the WEP. To ensure the geodesic equations for point particles we isolate $\kappa$ from matter in the original (29) by dividing by it,

$$\delta \int d^4x \sqrt{-g}(\phi R + L_m) = 0,$$

(30)

where we have replaced $\kappa \rightarrow 1/\phi$. However, we should note the following. While we seem to have saved the geodesic equations for test particles, the motion of composite bodies is more complex. It turns out that the coupling of a new, universal field, $\phi$ directly to the gravitational field gives rise to potentially observable effects for the motion of matter configurations to which gravitational energy contributes significantly. This is now known as the “Dicke-Nordtvedt” effect and has been investigated in the earth-moon system with the lunar laser reflector, leading to possible violations of even the WEP for
extended masses. These possibilities were not considered in the early days. So, let us proceed to see what follows from (29). We need field equations for \( \phi \) so some action for this new field must be supplied,

\[
\delta \int d^4x \sqrt{-g}(\phi R + L_m + L_\phi) = 0. \tag{31}
\]

We must note, that by allowing a new, scalar, field, we are opening the door to other consequences. Since gravity is universally coupled to all physics, the direct coupling of \( \phi \) to geometry, \( \phi R \), means that \( \phi \) is universally coupled in some sense.

**Consequences of (31):** *We are allowing for a possible violation of the SEP, since gravity, the universal interaction of mass, can influence local physics, not only through geometry, but also by changing the local universally coupled \( \phi \), thus changing internal gravitational structure.*

The usual requirement that the field equations be second order leads to

\[
L_\phi = L(\phi, \phi, \mu). \tag{32}
\]

Apart from this, there seem to be few \textit{a priori} restrictions on \( L_\phi \). The standard choice for a scalar field,

\[
L_\phi = -\omega \phi, \mu \phi, \nu g^{\mu\nu}, \tag{33}
\]

results in a wave equation for \( \phi \) with \( R \) as source seems natural. However, the coupling constant \( \omega \) would itself then need to have the same dimensions as the gravitational \( \kappa \) that the new field is to replace! But, one of the motivations for extending Einstein’s theory is to eliminate the dimensional constant, \( \kappa \). So we require that any new coupling constant appear as dimensionless. An obvious natural minimal choice is

\[
L_\phi = -\omega \phi, \mu \phi, \nu g^{\mu\nu}/\phi, \tag{34}
\]

in which the field \( \phi \) has dimensions of inverse gravitational constant,

\[
[\phi] = [\kappa^{-1}]. \tag{35}
\]

In fact, we will soon see, (41), that this results in field equations suggestive of (28). The form (34) leads to an action which is often referred to as the “Jordan-Brans-Dicke,” JBD, action,

\[
\delta \int d^4x \sqrt{-g}(\phi R + L_m - \frac{\omega}{\phi} \phi, \mu \phi, \nu g^{\mu\nu}) = 0. \tag{36}
\]

The variational principle, with standard topological and surface term assumptions, results in

\[
\delta_m \int dx^4 \sqrt{-g}L_m = 0, \tag{37}
\]

\[
\phi S_{\alpha\beta} = T_{(m)\alpha\beta} + \phi_{,\alpha;\beta} - g_{\alpha\beta} \Box \phi + \frac{\omega}{\phi} (\phi_{,\alpha} \phi_{,\beta} - \frac{1}{2} g_{\alpha\beta} \phi, \lambda \phi, \lambda), \tag{38}
\]
\[ \omega \left( \frac{\partial}{\partial \phi} \frac{\delta}{\phi} - \frac{\phi \Lambda \phi^4}{\phi^2} \right) = -R. \]  

(39)

The first of these, (37), is the standard variational principle for matter, leading to the same expression for matter motion in terms of the metric. It thus (apparently) satisfies the weak equivalence principle. For example test particles, (37), follow geodesics. However, if the matter is extended, not a point particle, this may no longer be true, even in standard general relativity. However, for scalar-tensor model, there is a second order interaction of matter through the scalar-metric coupling. This thus gives rise to violations of the weak equivalence principle. In other words, extended bodies of different mass may have different gravitational accelerations at the same point in a gravitational field. Of course, we do have the standard energy tensor for matter and resulting matter conservation laws. This is a result of the choice because of the free standing \( L_m \) in (36), not directly coupled to \( \phi \),

\[ T_{(m)\alpha \beta} = 0. \]  

(40)

We can couple \( \phi \) directly to matter by taking the trace of (38), solving for \( R \). The result is another form for (39),

\[ \Box \phi = \frac{1}{(2\omega + 3)} T_{(m)}, \]  

(41)

in which \( T_{(m)} \) is the trace of the ordinary matter tensor. It should be noted that traceless matter, such as null electromagnetic fields, do not directly couple to \( \phi \).

Now, to look at the possible satisfaction of Dirac’s (27), consider a weak field model situation with a static spherical shell of mass \( M \), radius \( R \) and otherwise empty universe this equation. The result is

\[ \phi \approx \phi_\infty + \frac{1}{4\pi(2\omega + 3)} \frac{M}{R}. \]  

(42)

Dividing equation (38) by \( \phi \) results in an equation in which the “ordinary” matter tensor, \( T_{(m)\alpha \beta} \) is divided by \( \phi \), which thus can be identified with the local reciprocal gravitational constant. Also, of course, the \( \phi \) contributes its own stress energy matter tensor to the right side of (38). If \( \phi_\infty \) is set zero as a default asymptotic condition, then (42) is seen to be consistent with the Dirac coincidence, (27). A natural approximation to (41) is to consider the effect of local matter over some background \( \phi_0 \) equal to the present observed value,

\[ \phi \approx \phi_0 + \frac{1}{4\pi(2\omega + 3)} \sum_{\text{local m}} \frac{m}{r}. \]  

(43)

This can be regarded as an extension of Dirac’s (27).

In equation (38), \( T_{(m)\alpha \beta} \) are the components of the stress-energy tensor for matter derived from the matter Lagrangian \( L_m \) in the standard fashion. Grouping this term with the \( \phi \) ones, results in an interpretation of

\[ S_{\alpha \beta} = \frac{1}{(\phi)}(T_{(m)\alpha \beta} + T_{(\phi)\alpha \beta}), \]  

(44)
as the total source for the Einstein tensor in (38). So, \(1/\phi\) does indeed act as a generalized gravitational “constant”, with both ordinary matter and the field \(\phi\) itself serving as sources for the metric. Actually the the \(\phi\) term on the right hand side of (38), together with (11) results in two occurrences of the matter tensor as a source. Thus there could be some argument for renormalizing \(1/\phi\) as the “gravitational constant” multiplying ordinary matter as it contributes to the Einstein tensor.

Pascual Jordan and his collaborators were the earliest serious investigators of equations of this sort. Most of the work by Jordan and his group is summarized in Jordan’s book, [9]. See also a more recent review by Schücking [10]. In addition to surveying the projective UFT’s motivation, Jordan’s book contains thorough studies of the static, spherically symmetric generalizations (the Heckmann solutions) of the Schwarzschild solution as well as cosmological solutions and other topics.

Equation (36) brings to mind actions obtained by conformal changes of the metric. So, it is natural to look at the action of the local “conformal group” on the representations of the theory. Replace the metric, \(g_{\mu\nu} \rightarrow \bar{g}_{\mu\nu} = \psi g_{\mu\nu}\). Discarding the surface (topological) part, (36) becomes

\[
\delta \int d^4x \sqrt{-\bar{g}} \left[ \frac{\phi}{\psi} \bar{R} + \frac{3\phi}{2} \frac{\nabla\psi}{\psi^3} |\nabla\psi|^2 - 3 \nabla\psi \cdot \nabla\phi/\psi^2 + L_m/\psi^2 - \frac{\omega}{\phi^2} |\nabla\phi|^2 \right] = 0. \quad (45)
\]

If \(\psi\) is chosen to be \(\phi\), (45) becomes

\[
\delta \int d^4x \sqrt{-\bar{g}} \left[ \bar{R} - \left( \omega + \frac{3}{2} \right) |\nabla\alpha|^2 + e^{-2\alpha} L_m(\bar{g}) \right] = 0, \quad (46)
\]

where \(\phi = e^\alpha\). It is easy to see that this variational principle is just the Einstein one for a massless scalar field (dimensionless), \(\alpha\), but universally coupled to all other matter through the \(e^{-2\alpha}\) factor. These conformal rescalings of the metric constitute the “metric gauge group.” Thus (46) is an expression of the theory in the “Einstein gauge,” as opposed to the original (36), the “Jordan” gauge. But there is more to the conformal scaling than merely the formal expression of the equations. Most significantly, the universal coupling of \(\alpha\) to all matter in (45) or (46) means that, in this metric, test particles will not follow geodesics, nor have conserved inertial mass, etc., in the Einstein gauge. In other words, conservation laws derived from the matter tensor depend on the construction of that tensor from the function multiplying \(\sqrt{-g}\) in the action, (45) or (46). Choosing the Einstein metric in (46) as the “physical” metric leads to significant and observable violations of mass conservation and the WEP.

In the 1960’s and 1970’s, Bob Dicke was a leading influence in the push to experimentally test general relativity in Einstein’s original form as well as alternatives such scalar-tensor generalizations [14]. In fact, the explosion of interest in relativity and gravitational theories and tests was prompted at least in part by the presence of theoretically viable alternatives to standard Einstein theory, and Dicke’s energetic promotion of them. Also NASA was coming of age and searching for space related experiments of fundamental importance. The important bridge between theory and experiment in gravitational theories was developed by Thorne, Nordtvedt, Will and others [15]. Their work
provided rigorous underpinnings for the operational significance of various theories, especially in solar system context. An important tool is the parameterized post Newtonian (PPN) formalism which provides theoretical standard for expressing the predictions of relativistic gravitational theories in terms which can be directly related to experimental observations.

From (38), it appears that the equations of scalar-tensor theory approach those of standard Einstein theory as $\phi$ approaches a constant. From (42) this would seem to occur in the limit of large $\omega$. Of course, this equation is just an approximation to a solution of (41) for an asymptotically empty universe, with $\phi \to 0$ as boundary condition. Actually, these comments obscure the need for rigorous analysis for the action (36) as $\omega \to \infty$. This is not surprising since the limiting dependence of solutions of field equations on parameters in these equations is in general a complex problem with all of the subtleties associated with the topology of a space of functions. However, it is true that

**Approach to standard Einstein:** In the realm of solar system experiments, the predictions of a theory of the form (36) approach those of standard Einstein theory as $\omega \to \infty$.

So, tests of such theories are often expressed as providing lower limits for $\omega$. For more details, see [16].

As the experimental data on solar system gravitational measurements come in, new limits on the value of the parameter $\omega$ have become so large as to make the predictions of this theory essentially equivalent to those of standard Einstein theory. In other words, from solar system experimentation it seems that scalar-tensor modifications of standard Einstein theory would necessarily differ insignificantly from the standard.

Gravitational radiation provides another arena for experimental studies of gravitation. In 1975 the Hulse-Taylor binary pulsar decay data[17] showed that gravitational radiation can provide another tool for testing gravitational theories. More recently, part of the justification for the LIGO gravitational radiation study is to provide further comparison of standard Einstein to alternative theories[18].

In spite of the apparently unpromising solar system experimental results, it turns out that universally coupled, thus gravitational, scalar fields continue to play important roles in contemporary physics. David Kaiser[19] has given a review of this topic, comparing JBD and Higgs fields, for example. We will briefly consider some of these possibilities in the following sections.

6 Dilatons

As discussed in the introduction, it is surprising that scalar fields do not seem to occur naturally in special relativistic, pre-quantum physics. However, from the earliest days of quantum theory, scalar quantum fields were prevalent, first as the pre-relativistic Schrödinger wave function, then as the Klein-Gordon boson field, providing an early,
but later discarded, model of a “meson”. Of course, Dirac’s spinor took over as the basic field for “permanent” particles as fermions, with force-field carriers such as photons, being bosons. Of course, the photon field is a vector, not a scalar. However, investigations of internal spaces for particle symmetries directly involve gauge theories of force fields. In this model the internal symmetry spaces for families of fields have interesting transformation properties from the internal gauge group viewpoint but are nonetheless spacetime scalars. Some of the earliest are the $SO(N)$ bosons of the dual model, the Nambu-Goldstone bosons and the famous Higgs fields. Of course, the motivations for considering quantum scalar fields is certainly very different from those leading to the scalar field in scalar-tensor theories. Nevertheless, certain forms of the quantum formalism, and perhaps its macroscopic manifestations may turn out to be not too different from the classical scalar fields. Such comparisons may be most evident in the context of cosmological quantum particle models. We begin with the quantum origin of “dilatons.”

The late 1960’s and early 1970’s saw the birth of quantum dual models, which eventually led to string theory and later superstring theory. These theoretical models quite naturally lead to a scalar field referred to as a “dilaton.” This field couples directly to the trace of the two-dimensional string stress tensor. This coupling breaks the Weyl conformal symmetry of the string. Since conformal metric transformations are dilations, we arrive at the word “dilaton.” The dilaton turns out to be what is needed to balance the quantum anomalies of this tensor by way of beta functionals of this tensor. In this analysis, the Einstein equations for the enveloping spacetime metric are “derived” as the beta functions. This rather involved arguments is discussed in the first volume of the book which thus provides useful description of the origin and role of dilatons. Here we only briefly summarize the argument in the following.

Start with a string action as a natural generalization of a point particle action. Given a background metric, $g_{\alpha \beta}$, an obvious choice is

$$S_1 = -\frac{1}{4\pi\alpha'} \int d^2 \sigma \sqrt{|h|} h^{ab} \partial_a X^\alpha \partial_b X^\beta g_{\alpha \beta}(X^c),$$

(47)

with internal coordinate area $d^2 \sigma$, internal string metric, $h_{ab}$, $a,b... = 1,2$, and $\alpha'$ a tension related coupling parameter. Comparing $S_1$ to a relativistic point particle action, we see the need for an intrinsic surface metric, $h_{ab}$ for the string that is not present for point particle. Now, assume that the derived physics should be independent of the internal parameterization, that is the choice of string metric. However, any two-dimensional metric is conformally flat (but only locally, in general!),

$$h_{ab} = \phi \eta_{ab},$$

(48)

with constant $\eta_{ab}$. So the surface element appearing in (47) reduces to the flat one,

$$d^2 \sigma \sqrt{|h|} h^{ab} = d^2 \sigma \eta^{ab}.$$
In addition to $S_1$, other terms have been proposed. One of these makes use of the string geometry through its curvature scalar,

$$\chi = \frac{1}{4\pi} \int d^2\sigma \sqrt{|h|} R^{(2)}. \tag{50}$$

Of course, one of the earliest discoveries relating geometry and topology was that this integral depends only on the topology of the string surface, and not the particular geometry. In fact, (50) defines the first Chern class for two dimensions. The value for $\chi$ is the Euler number of the surface, and cannot be a dynamical variable. However, dynamics can be restored by modifying the form of (50) by adding to (50) a scalar field factor, the “dilaton,” $\Phi$, giving

$$S_2 = \frac{1}{4\pi} \int d^2\sigma \sqrt{|h|} \Phi(X^c) R^{(2)}. \tag{51}$$

Classically this term breaks the conformal invariance. However, perhaps surprisingly, it is precisely this term which can restore conformal invariance after quantization. When the action $S = S_1 + S_2$ is quantized, conformal invariance is broken (an anomaly) unless the external fields satisfy three equations. This argument is described in detail in GSW, volume 1, page 180. Here we drop the $B_{\alpha\beta}$ for simplicity and get (in the magical string dimension 26!) Einstein-like equations,

$$0 = R_{\alpha\beta} - 2\Phi_{,\alpha;\beta}, \tag{52}$$

$$0 = 4\Phi_{,\alpha} \Phi^{,\alpha} - 4\Phi^{,\alpha}_{;\alpha} + R. \tag{53}$$

This “derivation” of the Einstein equations from string theory was one of the attractive features of string theory. Recall, however, that this required the introduction of a dilaton, spacetime scalar, field to break conformal invariance, which is later restored only if Einstein-like equations are satisfied.

Now, without regard for their string theory origins, field equations can be derived from an “effective action,”

$$\delta \int d^D X e^{-2\Phi} (R - 4\Phi_{,\alpha} \Phi^{,\alpha}) = 0. \tag{54}$$

Of course, this action is nothing but a special case of the vacuum scalar-tensor one, (36), with $-2\Phi = \ln \phi$, and $\omega = 1$. While the motivation and physics of the scalar field in the classical, pre-quantum, scalar-tensor theories is vastly different from the dilaton scalar field, it is difficult not to notice the close parallel between the universally coupled scalar of the old scalar-tensor theories and the new dilaton.

### 7 Inflatons

We will not attempt to review the rapidly expanding field of rapidly expanding (accelerating) cosmological models, but end this paper with a few comments about the early days of inflationary cosmology.
Standard general relativity has long been known to have difficulties in its application to observed cosmological facts. For example, standard general relativity requires that the initial big bang conditions be fantastically fine-tuned in order to result in the universe as we now see it some $10^{11}$ years later. See for example Peebles [21], Linde [22]. Look at the standard Robertson-Walker isotropic homogeneous metric model,

$$ds^2 = -dt^2 + R(t)^2d\sigma_i^2,$$

where the three-space metric, $d\sigma_i^2$, is hyperbolic, flat, or spherical depending on whether $\epsilon$ is -1, 0 or +1. The Einstein equations result in

$$\left(\frac{\dot{R}}{R}\right)^2 = \frac{\kappa \rho}{3} + \frac{\epsilon}{R(t)^2} + \frac{\Lambda}{3}.$$

Defining the Hubble variable as usual, this can be rewritten,

$$1 = \Omega + \epsilon \Omega_R + \Omega_{\Lambda},$$

where

$$\Omega \equiv \frac{\kappa \rho}{3H^2},$$

$$\Omega_R \equiv \frac{1}{(RH)^2},$$

and

$$\Omega_{\Lambda} \equiv \frac{\Lambda}{3H^2}.$$

As of the 1980’s these three quantities were measured to be each in the ballpark of one. In fact,

$$\Omega(\text{now}) \approx \frac{\kappa M}{R} \approx 10^0,$$

which is one of Dirac’s large number coincidences which was so instrumental in leading to the scalar-tensor theories. However, if we stick to standard GR, not a scalar-tensor variation, $\kappa$ is constant, [61] is valid only now, and takes this value now only if the universe evolves from very finely tuned earlier values. For example, in the present matter dominated era the equation of state leads to

$$\rho R^3 = M \approx \text{const},$$

whereas in an earlier radiation dominated state

$$\rho R^4 \approx \text{const}.$$

An analysis of the time evolution of these quantities in standard general relativity under drastically different regimes show that an extremely small variation of the values of the $\Omega$’s at early times would result in drastically different values now. But this is not the
only conceptual problem. For example, there are questions of how the universe could have homogenized itself from random early data (the “horizon” problem), and others, [21, 22]

Guth [23] pointed out that this myriad of difficulties could be at least partially resolved if the early stages of evolution were “inflationary,” that is

\[ R(t) = R(0)e^{Ht}, \] (64)

with constant \( H \). Such a model is consistent with [16] for \( \rho = \epsilon = 0, \Lambda \neq 0 \). Of course, this is not consistent with present data, so something other than a cosmological constant is needed. One way to achieve it is to introduce a new massless scalar field, the “inflaton,” \( \phi \), with Lagrangian density,

\[ \mathcal{L} = g^{\alpha \beta} \dot{\phi}_\alpha \dot{\phi}_\beta - V(\phi). \] (65)

This field contributes an effective mass density and pressure given by

\[ \rho_\phi = \dot{\phi}^2 / 2 + V, \quad p_\phi = \dot{\phi}^2 / 2 - V. \] (66)

The introduction of \( \phi \) and its potential, \( V \), can be used to resolve at least some, but certainly not all, of the problems discussed above. In some models, this inflaton has a dilaton-like nature, in others it is reminiscent of the \( \phi \) in the old scalar-tensor theories.

The problems of scalar-tensor field theories with solar-system sized observations require that \( \omega \) be very large. However, this restriction need not diminish the significance of the inflaton field in earlier cosmological contexts.

Of course, as of the beginning of the 21st century, cosmological observations and theory have expanded well past these early inflationary models, but we will stop here, and remind the reader that universally coupled, thus gravitational, scalar fields are still active players in contemporary theoretical physics. So perhaps we can say that the scalar field is still alive and active, if not always well, in current gravity research.

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