An M Dwarf’s Chromosphere, Corona, and Wind Connection via Nonlinear Alfvén Waves

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Abstract

An M dwarf’s atmosphere is expected to be highly magnetized. The magnetic energy can be responsible for heating the stellar chromosphere and corona and driving the stellar wind. The nonlinear propagation of Alfvén waves is a promising mechanism for both heating the stellar atmosphere and driving the stellar wind. Based on this Alfvén wave scenario, we carried out a 1D compressive magnetohydrodynamic simulation to reproduce the stellar atmospheres and winds of TRAPPIST-1, Proxima Centauri, YZ CMi, AD Leo, AX Mic, and the Sun. The nonlinear propagation of Alfvén waves from the stellar photosphere to the chromosphere, corona, and interplanetary space is directly resolved in our study. The simulation result particularly shows that the slow shock generated through the nonlinear mode coupling of Alfvén waves is crucially involved in both the dynamics of the stellar chromosphere (stellar spicule) and stellar wind acceleration. Our parameter survey further revealed the following general trends of the physical quantities of the stellar atmosphere and wind: (1) The M dwarf coronae tend to be cooler and denser than the solar corona. (2) The M dwarf stellar winds can be characterized by a relatively faster velocity and much smaller mass-loss rate compared to those of the solar wind. The physical mechanisms behind these tendencies are clarified in this paper, where the stronger stratification of the M dwarf’s atmosphere and relatively smaller Alfvén wave energy input from the M dwarf’s photosphere are remarkable.

Unified Astronomy Thesaurus concepts: Stellar winds (1636); Stellar coronae (305); Stellar chromospheres (230); M dwarf stars (982); Alfvén waves (23); Magnetohydrodynamics (1964)

1. Introduction

The M-type main-sequence stars (M dwarfs) have a highly magnetized atmosphere. The magnetic energy generated in their convection zone emerges into the outer layer and contributes to heating the chromosphere and corona. The high-energy radiation from this hot plasma is the manifestation of the stellar magnetic activity and observed in the X-ray (Giampapa et al. 1996; Fleming et al. 2000, 2003; Ness et al. 2004; Williams et al. 2014), ultraviolet (UV) (Stelzer et al. 2013; France et al. 2016), optical lines such as Hα and Ca II (Gizis et al. 2000; Walkowicz & Hawley 2009), and the radio band (Guedel & Benz 1993; Berger et al. 2010). The multilayered and spectroscopic observations have paid particular attention to the M dwarf’s flare and subsequent dynamics of the stellar atmosphere (Houdebine et al. 1990; Honda et al. 2018; Vida et al. 2019; Namekata et al. 2020; Maehara et al. 2021).

The M dwarf’s magnetic activities have been particularly discussed with the focus on their impact on the planetary atmosphere. The planets orbiting M dwarfs are favorable targets for extrasolar habitable worlds (Kasting et al. 1993; Scalo et al. 2007; Tarter et al. 2007; Kaltenegger & Traub 2009; Seager 2013; Kopparapu et al. 2017). Their upper atmospheres are exposed to the high-energy radiation in the UV to X-ray range from the stellar atmosphere (Tian 2009; Lammer et al. 2012; Tian & Ida 2015; Owen & Mohanty 2016) and affected by the stellar wind (Vidotto et al. 2011, 2014; Cohen et al. 2014, 2015; Garraffo et al. 2016, 2017; Dong et al. 2017, 2018; Alvarado-Gómez et al. 2020). The resultant mass loss from the planet’s atmosphere determines its evolution, especially for lower-mass planets.

Therefore, it is important for studies about the exoplanets or astrobiology to realize the underlying physics for the structure of stellar atmosphere and wind. Several numerical magnetohydrodynamics (MHD) modelings have been employed to explore the interplanetary environment around an M dwarf, but partly due to the lack of observational constraints, the theoretical predictions are not well established. For instance, the mass-loss rate of the stellar wind from TRAPPIST-1 (M8) is estimated to be $\sim 4.1 \times 10^{-15} M_{\odot} \text{yr}^{-1}$ by the global 3D MHD simulation of Dong et al. (2018) but $3 \times 10^{-14} M_{\odot} \text{yr}^{-1}$ by Garraffo et al. (2017). The simulated mass-loss rate of EV Lac (M3.5) by Cohen et al. (2014); $3 \times 10^{-14} M_{\odot} \text{yr}^{-1}$ is 4 orders of magnitude higher than that estimated by Cranmer & Saar (2011). Because these stellar wind modelings are sensitive to the inner boundary condition that represents the energy injection from the star to interplanetary space (Boro Saikia et al. 2020; Mesquita & Vidotto 2020), it is required to consider the connection between the stellar atmosphere and wind in a more self-consistent manner. The lower atmosphere of an M dwarf is characterized by a lower temperature, stronger stratification, higher density, smaller convective motion, and stronger magnetic field compared to the Sun (Reid & Hawley 2005). Therefore, in order to discuss the diversity and universality of the stellar atmosphere and wind, it is inevitable to consider such properties unique to the M dwarf lower atmospheres.

In addition to the connection between the stellar atmosphere and wind, the dynamics related to the nonlinear Alfvén wave is another important ingredient for the modeling of the stellar atmosphere and wind (Hollweg 1986; Velli 1993). The nonlinear propagation of Alfvén waves is a promising mechanism for both heating the stellar atmosphere and driving the stellar wind. Alfvén waves transfer the magnetic energy efficiently in the
magnetized plasma. The various nonlinear processes of Alfvén waves are responsible for the energy conversion from the magnetic energy to the kinetic or thermal energy of the background media (Alfvén 1947; Osterbrock 1961; Coleman 1968; Belcher & MacGregor 1976; Heinemann & Olbert 1980; Heyvaerts & Priest 1983). Owing to the high-resolution MHD simulations, it is found that, while the atmosphere and wind are maintained by the energy and momentum transfer by Alfvén waves, their propagation is affected by the dynamics of the atmosphere, such as spicules (Hollweg et al. 1982; Kudoh & Shibata 1999; Matsumoto & Shibata 2010) and stellar wind (Suzuki & Inutsuka 2005; Matsumoto & Suzuki 2012; Suzuki et al. 2013; Matsumoto & Suzuki 2014; Shoda et al. 2018, 2019, 2020; Suzuki 2018; Matsumoto 2021). These studies highlight the importance of resolving the relatively small-scale dynamics associated with Alfvén wave propagation, as well as reproducing the global structure of the stellar atmosphere and wind.

In this paper, therefore, we extend our recent solar atmosphere and wind model (Sakaue & Shibata 2020) to the M dwarf atmosphere and wind. By carrying out the 1D time-dependent MHD simulations, the nonlinear propagation of Alfvén waves in the nonsteady stellar atmosphere and wind is calculated from the M dwarf’s photosphere to the chromosphere, corona, and interplanetary space. Part of the simulation results in this paper is also discussed in our previous paper (Sakaue & Shibata 2021), in which we briefly summarized the similarities and differences in the structures of the reproduced stellar atmosphere and wind among the Sun and M dwarfs. The present paper mainly focuses on the development of our semiempirical method to estimate the stellar atmosphere and wind parameters (e.g., coronal temperature, wind velocity, and mass-loss rate) based on the simulation results.

2. Numerical Setting

2.1. Basic Equations

The nonlinear propagation of Alfvén waves in the time-dependent stellar atmosphere and wind is simulated by using 1D MHD equations based on the axial symmetry assumption of the magnetic flux tube. The surface of the axisymmetric flux tube is defined by the poloidal and toroidal axes, which are noted in this study with \( x \) and \( \phi \) (Figure 1(a)). The basic equations are written as follows:

\[
\frac{\partial \rho}{\partial t} + \frac{1}{A} \frac{\partial}{\partial x} (\rho v_x A) = 0, \tag{1}
\]

\[
\frac{\partial}{\partial t} \left( \frac{p}{\gamma - 1} + \frac{1}{2} \rho v^2 + \frac{B^2}{8\pi} \right) + \frac{1}{A} \frac{\partial}{\partial x} \left[ A \left( \frac{\gamma p}{\gamma - 1} + \rho v^2 + \frac{B^2}{8\pi} \right) v_x - \frac{B_x}{4\pi} v_\phi \right] = \rho v_x \frac{\partial}{\partial x} \left( \frac{GM_x}{r} \right) - \frac{1}{A} \frac{\partial}{\partial x} (AF_x) - Q_{\text{rad}}, \tag{2}
\]

\[
\frac{\partial (\rho v_x)}{\partial t} + \frac{\partial p}{\partial x} + \frac{1}{A} \frac{\partial}{\partial x} \left( \frac{\rho v_x^2 + B_x^2}{8\pi} A \right) - \rho v_x^2 \frac{\partial \ln A}{\partial x} - \frac{\rho}{A} \frac{\partial}{\partial x} \left( \frac{GM_x}{r} \right) = 0, \tag{3}
\]

\[
\frac{\partial (\rho v_\phi)}{\partial t} + \frac{1}{A} \frac{\partial}{\partial x} \left( A \sqrt{A} \left( \rho v_x v_\phi - \frac{B_x B_\phi}{4\pi} \right) \right) = 0, \tag{4}
\]

\[
\frac{\partial B_x}{\partial t} + \frac{1}{A} \frac{\partial}{\partial x} \left( \sqrt{A} (v_x B_\phi - v_\phi B_x) \right) = 0, \tag{5}
\]

\[B_x A = \text{const.}, \tag{6}\]
Table 1
Parameters of Stars

| Spectral Type | G2 | M0 | M3.5 | M5 | M5.5 | M8 |
|---------------|----|----|------|----|------|----|
| $T_{\text{eff}}$ (K) | 5770 | 3800 | 3473 | 3280 | 3042 | 2559 |
| $\log_{10} g$ (cm s$^{-2}$) | 4.44 | 4.77 | 4.79 | 4.91 | 5.21 | 5.21 |
| $r_e/r_\odot$ | 1 | 0.51 | 0.46 | 0.32 | 0.15 | 0.12 |
| $M_*/M_\odot$ | 1 | 0.60 | 0.47 | 0.31 | 0.12 | 0.08 |
| $H_p$ (km) | 134 | 35.5 | 29.4 | 20.8 | 9.44 | 6.93 |
| $v_{\text{esc}}$ (km s$^{-1}$) | 618 | 647 | 624 | 602 | 569 | 511 |
| $\mu_{\phi}$ (cm$^2$ g$^{-1}$) | 2.6 | 14 | 19 | 27 | 69 | 190 |
| $B_p$ (G) | 1560 | 2722 | 2936 | 3395 | 5139 | 7313 |
| $v_{\text{esc}}$ (mHz) | 1.6 | 2.1 | 2.4 | 2.6 | 3.1 | 4.0 |
| Typical star | Sun | AX Mic | AD Leo | YZ CMi | Proxima Centauri | TRAPPIST-1 |

Notes. The effective temperature ($T_{\text{eff}}$), surface gravity ($\log_{10} g$), stellar radius ($r_e/r_\odot$), stellar mass ($M_*/M_\odot$), pressure scale height of the photosphere ($H_p$), surface escape velocity ($v_{\text{esc}}$), and the physical quantities of the photosphere, including mass density ($\rho_{\phi}$), Rosseland opacity ($\kappa_{\phi}$), mean molecular weight ($\mu_{\phi}$), magnetic field strength ($B_p$), mixing length parameter ($\alpha_{\text{MLT}}$), convective velocity ($v_{\text{conv}}$), and acoustic cutoff frequency ($v_{\text{ac}}$).

Above the photosphere, the magnetic flux tube expands exponentially so that the magnetic pressure inside the flux tube balances with the ambient plasma pressure decreasing with the scale height $H_p = R_p T_{\text{eff}}/(\mu_{\phi} g)$. The filling factor $f$ in this layer is expected to be $f_{\text{atm}}(r) = f_{\phi} \exp(r/r_p H_p(1 - r/r_p))$, where $f_{\phi}$ is the filling factor of the photosphere. In the lower atmosphere, where $r = r_p + h$ ($h \ll r_p$), we obtain $f_{\text{atm}}(h) = f_{\phi} e^{h/(2H_p)}$. This exponential expansion of the flux tube stops at some height where it merges with the neighboring flux tube. Above this height, hereafter the merging height $H_m$, the magnetic pressure dominates the plasma pressure, and the flux tube extends vertically with the constant cross section. The poloidal magnetic field strength is almost constant above $h = H_m$ through the upper chromosphere and coronal base; thus, $B = B_{\phi} e^{h/(2H_p)}$.

The flux tube expands superradiantly again in the extended corona such that the interplanetary space is filled with the open flux tube. This expansion occurs around the height (i.e., loop height $H_l$) wherein the magnetic pressure of the closed loop significantly decreases. The functional form of the filling factor $f(r)$ is suggested by Kopp & Holzer (1976). Based on these considerations, the profile of the filling factor $f(r)$ is determined as follows:

$$f_{\text{atm}}(r) = f_{\text{m}} \tanh \left[ \frac{f_{\phi}}{f_{\text{m}} \exp \left( \frac{r}{2H_p} - 1 \frac{r}{r_p} \right) - 1} \right] ,$$

(9)

where $f_{\text{m}} = f_{\phi} B_{\phi}/B$.

$$f_{\text{wind}}(r) = \frac{e^{(r-r_p)H_l}/\sigma + f_{\text{m}}(1 - f_{\phi}) e^{-(H_l/r_p)}}{e^{(r-r_p)H_l}/\sigma + 1} ,$$

(10)
\[ f(r) = f_{\text{atm}}(r) + \frac{1}{2}(\max[f_{\text{wind}}(r), f_{\text{atm}}] - f_{\text{atm}}(r)) \times \left\{ 1 + \tanh\left(\frac{r - r_\ast - H_I}{H_I}\right) \right\}, \quad (11) \]

\[ f(r) = f_{\text{ph}} + (1 - f_{\text{ph}}) \frac{\hat{f}(r) - \hat{f}(r_\ast)}{1 - \hat{f}(r_\ast)}. \quad (12) \]

The \( H_I \) and \( \sigma_I \) in Equation (10) are set to 0.1\( r_\ast \). The meanings of the parameters used in the above definition are summarized in Figure 1(b). We discuss the effect of varying \( f_{\text{ph}} \) in Appendix A and hereafter focus on the simulation results with the fixed \( f_{\text{ph}} \) at 1/1600. When \( f_{\text{ph}} = 1/1600 \), the total open magnetic flux on the solar surface (\( \Phi_{\text{open}} = 4\pi r_\ast^2 B_{\text{ph}} \)) is \( 5.9 \times 10^{22} \text{ Mx} \), and \( \Phi_{\text{open}}/(10^{22} \text{ Mx}) = 2.7, 2.4, 1.4, 0.41, \) and 0.38 for M0, M3.5, M5, M5.5, and M8 dwarfs, respectively. The configuration of the magnetic flux tube with \( f_{\text{ph}} = 1/1600 \) is depicted in Figure 2. We carried out the parameter survey of \( B/B_{\text{ph}} \) and used \( \ln(B/B_{\text{ph}}) = -3, -4, -5, -6 \).

2.3. Heat Conduction and Radiative Cooling

The equation of state is \( p = \rho R_T/\mu(T) \). The radiative cooling \( Q_{\text{rad}} \) is considered as the empirical formula, which is composed of three distinct terms, namely, the photospheric radiation \( Q_{\text{ph}} \), chromospheric radiation \( Q_{\text{ch}} \), and coronal radiation \( Q_{\text{co}} \). Here

\[
\mu(T) = \begin{cases} 
\mu_{\text{ph}}(1 - \chi(T)/2) & (T > T_{\text{eff}}) \\
\mu_{\text{ph}} + (\mu_{\text{ph}} - \mu_{\text{ch}}) T - T_{\text{eff}}/T_{\text{eff}} & (T_{\text{eff}} < T < T_{\text{eff}}) \\
\mu_{\text{ch}} & (T < T_{\text{eff}}) 
\end{cases} 
\]

\[ (13) \]

where \( \chi(T) \) is the ionization degree as a function of temperature, which is calculated by referring to Carlsson & Leenaarts (2012):

\[ Q_{\text{rad}} = (1 - \xi_1)(1 - \xi_2)Q_{\text{ph}} + \xi_1(1 - \xi_2)Q_{\text{ch}} + \xi_2Q_{\text{co}}. \quad (14) \]

\[ \xi_1 = \frac{1}{2} \left[ 1 + \tanh\left(\frac{r - r_*}{H_{\text{ph}} - 3}\right) \right], \quad (15) \]

\[ \xi_2 = \exp\left(-4 \times 10^{-20} \int_0^r n_{\text{H}_1} dr'\right). \quad (16) \]

where \( n_{\text{H}_1} \) is the neutral hydrogen number density; i.e., \( n_{\text{H}_1} = (1 - \chi(T))\rho/m_p \), where \( m_p \) is the proton mass. Here

\[ Q_{\text{ph}} = 4\rho \sigma_{\text{SB}} T^4 \max\left(\frac{T^4}{T_{\text{ref}}^4} - 1, -e^{-(r - r_\ast)^2/H_{\text{ph}}^2}\right). \quad (17) \]

\[ Q_{\text{ch}} = 4.9 \times 10^9 \text{ erg s}^{-1} \text{ s}^{-1} \rho, \quad Q_{\text{co}} = \chi(T)n^2\Lambda(T). \quad (19) \]

Here \( \sigma_{\text{SB}} \) is the Stefan–Boltzmann constant; \( n \) is the number density of neutral or ionized hydrogen, i.e., \( n = \rho/m_p \); \( \Lambda(T) \) is the radiative loss function for the optically thin plasma; and \( Q_{\text{ch}} \) and \( \Lambda(T) \) are the same functions as used in Hori et al. (1997), which are always positive. The negative \( Q_{\text{ph}} \) in Equation (17) represents the radiative heating, and it is allowed only where \( e^{-(r - r_\ast)^2/H_{\text{ph}}^2} \sim 1 \).

The heat conductive flux is composed of the collisional and collisionless terms:

\[ F_c = -\kappa(T) \frac{\partial T}{\partial x}, \quad (20) \]

\[ \kappa(T) = q\kappa_{\text{coll}} + (1 - q)\kappa_{\text{sat}}, \quad (21) \]

\[ q = \max(0, \min(1, 1 - 0.5\kappa_{\text{coll}}/\kappa_{\text{sat}})), \quad (22) \]

\[ \kappa_{\text{sat}} = \frac{3}{2}\nu_{\text{e,thr}} \frac{r}{T}. \quad (23) \]

Here \( \kappa_{\text{coll}}(T) \) is adopted from Nagai (1980), who considered the effect of partial ionization in lower temperatures, while it agrees with the Spitzer–Härm heat conductivity \( \kappa_0 T^{8/2} \) (Spitzer & Härm 1953; \( \kappa_0 = 10^{-6} \text{ in cgs units} \)) when \( T > 10^6 \text{ K} \). Also, \( \kappa_{\text{sat}} \) represents the saturation of the heat flux due to the collisionless effect (Parker 1964; Bale et al. 2013), and \( \nu_{\text{e,thr}} \) is
the thermal speed of the electron. The above expression of \( \kappa_{\text{sat}} \) means that the transition of heat conductivity from \( \kappa_{\text{coll}} \) to \( \kappa_{\text{sat}} \) occurs around \( r \sim \lambda_{c,\text{mfp}} (\lambda_{c,\text{mfp}} \text{ is the electron mean free path}) \), and the heat flux is limited to \( \frac{3}{2} \alpha_{p} v_{\text{th}} \) in the distance where \( T \sim r^{-\alpha} \) (\( \alpha = 0.2 \)–0.4 for wind faster than 500 km s\(^{-1}\); Marsch et al. 1989). Based on the above heat conductivity, the heat conduction is solved by the super-time-stepping method (Meyer et al. 2012, 2014).

### 2.4. Initial and Boundary Conditions

We set the static atmosphere with a temperature of \( 10^{4} \text{ K} \) as the initial state. The toroidal velocity \( v_{\phi} \) on the bottom boundary represents the convective motion on the stellar photosphere. We define it by the frequency-dependent fluctuation with the following power spectrum, where \( v_{\text{ph}} \) is the free parameter corresponding to the amplitude of the convective velocity. The phase offsets of fluctuation are randomly assigned,

\[
v_{\text{ph}}^2 \propto \int_{v_{\text{min}}}^{v_{\text{max}}} \nu^{-1} d\nu, \tag{24}
\]

where we assumed that \( v_{\text{min}} = v_{\text{min}}, v_{\text{max}} = v_{\text{ac}} \) (see Table 1 for the \( v_{\text{ac}} \) of each star) and that \( v_{\text{min}} \) and \( v_{\text{max}} \) are set to 30 minutes and 20 s, respectively. The amplitude of fluctuation \( v_{\text{ph}} \) is a subject of survey in this study. Based on the convection theory (Bohn 1984; Ulmschneider 1986; Samadi et al. 2013), we estimate the fiducial convective velocity \( v_{\text{conv}} \) on the stellar photosphere based on the following scaling:

\[
v_{\text{conv}}^3 \propto \frac{\alpha_{\text{MLT}} T_{\text{eff}}^4}{\rho_{\text{ph}}}, \tag{25}
\]

where \( \alpha_{\text{MLT}} \) is the mixing length parameter we derived by referring to Ludwig et al. (1999, 2002) and Magic et al. (2015). The determined \( \alpha_{\text{MLT}} \) and \( v_{\text{conv}} \) for each star are summarized in Table 1. For the M dwarf atmospheres and winds, we carried out the simulations with \( v_{\text{ph}}/v_{\text{conv}} = 1.0, 1.4, 2.0, \) and 3.0 in each case of \( \text{ln}(B/B_{\text{ph}}) = -4, -5, -6 \) and \( v_{\text{ph}}/v_{\text{conv}} = 1 \) in the case of \( \text{ln}(B/B_{\text{ph}}) = -3 \). These values of \( v_{\text{ph}}/v_{\text{conv}} \) for solar and M dwarf atmospheres and wind simulations are summarized in Table 2.

Table 2: \( v_{\text{ph}}/v_{\text{conv}} \) Used in Our Parameter Survey

| \( \text{ln}(B/B_{\text{ph}}) \) | -3   | -4, -5, -6 |
|--------------------------|------|------------|
| Sun                      | 1.0  | 0.33, 0.67, 1.0, 2.0 |
| M dwarfs                 | 1.0  | 1.0, 1.4, 2.0, 3.0 |

To excite the purely outward Alfvén waves on the bottom boundary, the toroidal magnetic field \( B_{\phi} \) is determined by \( B_{\phi} = -\sqrt{4\pi \rho} v_{\phi} \). That means that Elsässer variables \( \zeta_{\text{out}} = \psi_{\phi} - B_{\phi}/\sqrt{4\pi \rho}, z_{\text{in}} = \psi_{\phi} + B_{\phi}/\sqrt{4\pi \rho} \) on the bottom boundary satisfy the condition that \( \zeta_{\text{out}} = 2\psi_{\phi} \) and \( z_{\text{in}} = 0 \). The longitudinal velocity component \( v_{\phi} \) on the bottom boundary is the same fluctuation as the \( v_{\phi} \) of the photosphere.

The upper boundary is treated as the free boundary. It corresponds to 100 \( r_{*} \), and 19,200 grids are placed non-uniformly in between. The numerical scheme is based on the HLLD Riemann solver (Miyoshi & Kusano 2005) with the second-order MUSCL interpolation and the third-order TVD Runge–Kutta method (Shu & Osher 1988).

### 3. Results

#### 3.1. Solar and Stellar Winds

Figure 3 shows the snapshots of solar and stellar wind profiles about the velocity, temperature, mass density, and temporally averaged profiles of Alfvén wave amplitude. The left, middle, and right columns correspond to the results of the solar wind and the stellar winds of M3.5 and M8 dwarfs, respectively. The results shown here are obtained by setting \( v_{\text{ph}}/v_{\text{conv}} = 1 \) and \( \text{ln}(B/B_{\phi}) = -5 \). The red dotted lines show the temporally averaged physical quantities of the stellar wind, while the black dotted lines correspond to that of the solar wind as a function of \( (r-r_{*})/r_{*} \). As shown in Figure 3, the typical late M dwarf’s stellar wind is faster than the solar wind and characterized by the smaller Alfvén wave amplitude in the lower corona, \( r < r_{*} \).

Figure 4 shows the mass-loss rates of the solar and stellar winds as a function of the Alfvén wave energy flux of the photosphere. The squares, triangles, circles, and asterisks correspond to the results obtained by setting \( \text{ln}(B/B_{\phi}) = -6, -5, -4, \) and -3, respectively. Table 3 summarizes the mass-loss rates in the case of \( v_{\text{ph}}/v_{\text{conv}} = 1 \) and shows that the wind’s mass-loss rate of M dwarfs is generally much smaller than the solar wind value. Another remarkable feature is that \( M \) as a function of \( v_{\text{ph}} \) is dependent on \( B/B_{\phi} \). When \( \text{ln}(B/B_{\phi}) = -6 \) (square symbols), the \( M \) of the solar wind and the early M dwarf’s wind are less dependent on \( v_{\text{ph}} \). This phenomenon was well investigated by Sakaue & Shibata (2020), who showed that the magnetic energy cannot be transferred by Alfvén waves across the chromosphere when the nonlinearity of the Alfvén waves is extremely high.

#### 3.2. Solar and Stellar Spicules

Figure 5 gives time-slice diagrams of mass density in the lower atmospheres of (a) Sun, (b) M3.5, and (c) M8 dwarfs. The simulation results shown here are obtained by setting \( v_{\text{ph}}/v_{\text{conv}} = 1 \) and \( \text{ln}(B/B_{\phi}) = -3 \). The green solid lines in Figure 5 represent the contour lines of \( T = 4 \times 10^{4} \text{ K} \) (temperature of the top of the chromosphere). The repetitive vertical motions of the top of the chromosphere correspond to the dynamics of the spicule. The onset of upward motion of the spicule usually corresponds to the collision of the chromospheric slow shock with the transition region. The trajectories of the chromospheric slow shocks are clearly seen, especially in panel (c) of Figure 5. An example of them is indicated by the red dotted line.

The green horizontal dotted lines represent the median height of the transition region (\( H_{t} \)). It is notable that the normalized transition region height by pressure scale height of the photosphere (\( H_{t}/H_{\text{ph}} \)) is highest in the M8 dwarf, while the \( H_{t}/H_{\text{ph}} \) of the M3.5 dwarf is smaller than that of the Sun. The \( H_{t}/H_{\text{ph}} \) of various M dwarfs and the Sun are plotted in Figure 6 as a function of \( \rho_{p}/\rho_{p_{\text{e}}} \), where \( \rho_{p} \) is the plasma pressure at the stellar transition region. The simulation results shown here are obtained by setting \( v_{\text{ph}}/v_{\text{conv}} = 1 \) and \( \text{ln}(B/B_{\phi}) = -3, -4, -5, -6 \). As shown in Figure 6, \( H_{t}/H_{\text{ph}} \) are well described by a negative power-law function of
$p_{tr}/p_{ph}$ (black line), except for three cases corresponding to the solar transition region (red circles). These results lie within a red rectangle, which represents the range of observed plasma pressure in the spicule (Alissandrakis et al. 2018; Shimojo et al. 2020) and that of the observed spicule height (Pereira et al. 2012; Zhang et al. 2012). Note that the observed spicule height means the maximum altitude that the transition region reaches, which is higher than the median height of the transition region indicated by green dotted lines in Figure 5.

Because a higher transition region is associated with lower plasma pressure at a transition region in the stratified atmosphere, an anticorrelation between $p_{tr}/p_{ph}$ and $H_{tr}/H_{ph}$ is naturally expected. The height of the solar transition region is, on the other hand, significantly higher than the negative power-law function for M dwarfs. This difference seems to be due to the stronger shocks in the solar chromosphere compared to those in M dwarf chromospheres. That means that even though the $p_{tr}/p_{ph}$ of the Sun is comparable to that of M0 or M3.5 dwarfs, higher spicules can be driven by the stronger shocks in the solar chromosphere. As seen in Table 1, the Mach number of the convective motion of the photosphere ($v_{conv}/c_{s,ph}$) increases with the increasing effective temperature ($T_{eff}$). The higher the $v_{conv}/c_{s,ph}$, the higher the nonlinearity of Alfvén waves in the chromosphere, which results in a stronger chromospheric shock.

**Figure 3.** Snapshots of solar and stellar wind profiles about the velocity, temperature, mass density, and temporally averaged profiles of Alfvén wave amplitude. The left, middle, and right columns correspond to the results of the solar wind and the stellar winds of M3.5 and M8 dwarfs, respectively. The results shown here are obtained by setting $v_{ph}/v_{conv} = 1$ and $ln(B/B_{ph}) = -5$. The red dotted lines show the temporally averaged physical quantities of the stellar wind, while the black dotted lines correspond to that of the solar wind as a function of $(r - r_\odot)/r_\odot$. The Astrophysical Journal, 919:29 (23pp), 2021 September 20 Sakaue & Shibata
4. General Trends of Physical Quantities of the Stellar Atmosphere and Wind

A numerical parameter survey reveals the general trends of various characteristics of the stellar atmosphere and wind, including the wind velocity ($v_{\text{wind}}$), mass-loss rate ($\dot{M}$), coronal temperature ($T_{\text{co}}$), and plasma pressure at the transition region ($p_{\text{tr}}$). The differences and similarities in each of these parameters among the Sun and M dwarfs are described in this section. In the following, the coronal parameters are indicated with the subscript $\text{co}$ and represent those at $r = 1.1 r_*$. The parameters at the transition region are indicated with the subscript tr. The transition region is defined as the height with a temperature of $T_{\text{tr}} = 4 \times 10^4$ K. Because the transition region violently repeats the upward and downward motion (Figure 5), the physical quantities at the transition region should be defined as the temporally averaged values along the trajectory of the position of $T_{\text{tr}} = 4 \times 10^4$ K.

4.1. Stellar Wind Acceleration

To discuss the acceleration of the stellar wind, we pay attention to the poloidal component of the equation of motion:

$$\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + \frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{1}{\rho} \frac{\partial}{\partial x} \left( \frac{B_y^2}{8\pi} \right) - v_y^2 \frac{\partial \ln \sqrt{A}}{\partial x} + \frac{B_y^2}{4\pi\rho} \frac{\partial \ln \sqrt{A}}{\partial x} - \frac{\partial}{\partial x} \left( \frac{GM_*}{r} \right) = 0. \quad (26)$$

After the temporal average and the integral from $r = r_*$ to $r$ of both sides of Equation (26), the Bernoulli integral (Parker 1963; Mestel 1999) for the stellar wind velocity is obtained as
follows:

\[
v^2_r(r) = -2 \int_{r_c}^{r} \left( \frac{\partial \nu_v}{\partial r} \right) dx + \Delta_p^r + \Delta_{\rho_b}^r + \Delta_r^T + \Delta_s^r + \Delta_g^r,
\]

where the first term on the right-hand side is negligible when the stellar wind is in a quasi-steady state, and \( \langle \cdot \rangle \) means the temporal average. The other terms are defined as follows:

\[
\Delta_p^r = -2 \int_{r_c}^{r} \left( \frac{1}{\rho} \frac{\partial \rho}{\partial x} \right) dx,
\]

\[
\Delta_{\rho_b}^r = -2 \int_{r_c}^{r} \left( \frac{1}{\rho} \frac{\partial \rho}{\partial x} \right) dx,
\]

\[
\Delta_r^T = 2 \int_{r_c}^{r} \left( \frac{v_s}{2} \frac{\partial \ln \sqrt{A}}{\partial x} \right) dx,
\]

\[
\Delta_s^r = -v_{\text{esc}}^s \left( 1 - \frac{r}{r_s} \right),
\]

where \( v_{\text{esc}} = \sqrt{2GM_*/r_*} \). Equation (27) is confirmed in Figure 7, which shows the simulation result of the stellar wind of the M3.5 dwarf with \( v_{\text{ph}}/v_{\text{conv}} = 1 \) and \( \ln(B/\mu_{\text{ph}}) = -6 \). The black solid line corresponds to \( \Delta_p^r + \Delta_s^r + \Delta_{\rho_b}^r + \Delta_r^T + \Delta_g^r \), which agrees well with \( v_s^r \) (thick gray line), as indicated by Equation (27). It is most remarkable in Figure 7 that the stellar wind is mainly accelerated by the plasma...
The background plasma pressure gradient \( \Delta'_p \) is much less effective in producing the simulated stellar wind velocity (red dotted line in Figure 7):

\[
\Delta'_p = -2 \int_{r_1}^{r_2} \frac{1}{\rho} \frac{\partial \rho}{\partial x} dx.
\]

Figure 8(a) shows the comparison between the temporal average of the plasma pressure gradient \(-\langle \rho^{-1} \partial_x p \rangle\) (black line) with the plasma pressure gradient calculated from the temporal average of the mass density and plasma pressure \(-\langle \rho \rangle^{-1} \partial_x \langle p \rangle\) (red line). Beyond \( r \sim 2r_* \), we can see that \(-\langle \rho^{-1} \partial_x p \rangle\) surpasses \(-\langle \rho \rangle^{-1} \partial_x \langle p \rangle\) and becomes comparable to the centrifugal force (blue line). Figure 8(b) is the time-slice diagram of the plasma pressure gradient after subtracting the gravitational acceleration. The dashed and dashed-dotted lines correspond to the typical characteristics of slow- and fast-mode waves, respectively. This figure shows that the signatures with a stronger plasma pressure gradient propagate at slow-mode speed through \( r \sim 2r_* \) to \( 10r_* \). Therefore, it is concluded that the numerous slow shocks excited around these distances lead to larger \(-\langle \rho^{-1} \partial_x p \rangle\) than \(-\langle \rho \rangle^{-1} \partial_x \langle p \rangle\) and significantly contribute to the stellar wind acceleration.

Because the slow shock in the stellar wind is excited by nonlinear Alfvén waves, a faster wind velocity is expected when Alfvén waves are more amplified in the stellar wind. The resultant strong correlation between wind velocity \((v_{\text{wind}})\) and
the maximum amplitude of the Alfvén waves in the stellar wind ($v_{\phi,\text{max}}$) is shown in Figure 9.

4.2. Coronal Temperature $T_{co}$

The coronal temperature ($T_{co}$) is determined by the balance between the heat conduction flux and transmitted Poynting flux into the corona, according to the following energy conservation law in the quasi-steady stellar wind:

$$\frac{\partial}{\partial x}[A(F_{A} + F_{v}) + F_{g} + F_{e} + F_{\text{rad}}] = 0,$$

where $F_{A}, F_{g}, F_{e},$ and $F_{\text{rad}}$ are the Poynting flux carried by the Alfvén wave, gravitational energy flux, heat conduction flux, and radiative loss flux, respectively:

$$F_{A} = -\frac{B_{0}^{2}(\nu_{A}v_{\phi0})}{4\pi},$$

$$F_{g} = -\langle \rho v_{e}^{2}G_{M}\rangle/r,$$

$$F_{\text{rad}} = \frac{1}{A}\int_{-\infty}^{x}A(Q_{\text{rad}})dx.$$

The $F_{v}$ is the sum of the enthalpy flux $F_{\text{ent}},$ kinetic energy flux $F_{\text{kin}},$ and the Poynting flux advected with the stellar wind. Namely,

$$F_{\text{ent}} = \langle \gamma p v_{e}/(\gamma - 1) \rangle,$$

$$F_{\text{kin}} = \langle \rho v^{2}v_{e}/2 \rangle,$$

$$F_{v} = F_{\text{ent}} + F_{\text{kin}} + \langle B_{0}^{2}v_{e}/(4\pi) \rangle.$$

Equation (34) means that there is an integral constant $L_{\text{total}} = F_{\text{total}}A$ corresponding to the conservation of total energy flux,

$$L_{A} + L(v_{e}) + L_{g} + L_{e} + L_{\text{rad}} = L_{\text{total}},$$

where $L_{A} = F_{A}A,$ $L(v_{e}) = F(v_{e})A,$ $L_{g} = F_{g}A,$ $L_{e} = F_{e}A,$ and $L_{\text{rad}} = F_{\text{rad}}A.$ Figure 10 shows the profile of each term in Equation (41) in the case of the M3.5 dwarf with

\[\ln(B_{0}/B_{\text{ph}}) = -6 \quad \text{and} \quad v_{\phi0}/v_{\text{conv}} = 1.\]

The energy fluxes are normalized by $F_{\text{mass}}GM_{*}/r_{*}$, where $F_{\text{mass}}$ is the mass flux and $F_{\text{mass}}GM_{*}/r_{*} \approx 5 \times 10^{3}\gamma^{2}/(r^{2})$ erg cm$^{-2}$ s$^{-1}$. Here $F_{A}, F_{e}, F_{g}, F_{\text{rad}}$, and $F_{v}$ are the Alfvén wave energy flux, kinetic energy flux, enthalpy flux, gravitational energy flux, heat conduction flux, and sum of the enthalpy, kinetic energy, and Poynting fluxes advected with the stellar wind. The $L_{\text{total}}$ is the integral constant in Equation (41).

By integrating the above from $x_{tr}$ to $x_{co}$ and neglecting $T_{tr}/T_{co} \ll 1$, we obtain

$$T_{co}^{7/2} \approx \frac{\gamma}{2\kappa_{0}} \int_{x_{tr}}^{x_{co}} dx \frac{L_{x}}{A}.$$

When we assume that $L_{x}$ is almost constant at $L_{x,co}$ from $x_{tr}$ to $x_{co}$, as suggested by Figure 10, $T_{co}$ is estimated as

$$T_{co} \approx \left[\frac{7L_{x,co}^{2}}{2\kappa_{0}}\right]^{2/7},$$

where $l_{B,co}$ represents the spatial scale of the expanding magnetic flux tube,

$$l_{B,co} = \int_{x_{tr}}^{x_{co}} dx \frac{A_{co}}{A}.$$
A similar scaling relation has been considered to discuss the temperature of the quiescent or flaring coronal loop (Rosner et al. 1978; Yokoyama & Shibata 1998).

Figure 11 shows the relation between $F_{A,co}$ and $T_{co}$. As a result of the tight correlation between $F_{A,co}$ and $|F_{c,co}|$, a power-law relation similar to Equation (45) can be seen in Figure 11. It is also shown that the $T_{co}$ of the late M dwarfs (especially the M5.5 and M8 dwarfs) is systematically lower than the solar coronal temperature with respect to a given $F_{A,co}$. Equation (45) suggests that this difference originates in the much smaller spatial scale of the magnetic flux tube ($l_{B,co}$) of the late M dwarfs, compared to the Sun. The dotted lines in Figure 11 represent $y = [7x/(2\mu_0)]^{2/7}$ with $l = (10^{-1}, 10^{-2}, 10^{-3})r_G$.

4.3. Plasma Pressure at Transition Region $p_\alpha$

The coronal temperature ($T_{co}$) is one of the fundamental parameters determining the plasma pressure at the transition region ($p_\alpha$). Rosner et al. (1978) pointed out that the energy balance along the coronal loop between the heat conduction flux and the radiative energy loss leads to the power-law relation among the coronal loop temperature ($T_{loop}$), plasma pressure ($p_{loop}$), and loop length ($l$), i.e., $T_{loop} \approx 1.4 \times 10^{7}(p_{loop})^{1/3}$ in cgs units (RTV scaling). Although their assumption of constant pressure along the coronal loop is not straightforwardly applicable to our case of an open flux tube, a similar power-law relation between $T_{co}$ and $p_\alpha$ is seen in our simulation results. Figure 12 shows the relation between $T_{co}$ and $p_\alpha$ obtained from our simulation. It is seen that the $p_\alpha$ of the late M dwarfs (especially the M5.5 and M8 dwarfs) is systematically higher than the solar value with respect to a given $T_{co}$. The dotted lines represent RTV scaling; $y = (x/1.4 \times 10^{3})^{2}/l$, with $l = (1, 10^{-1}, 10^{-2})r_G$.

5. Semiempirical Method to Predict the Characteristics of the Stellar Atmosphere and Wind

The present study aims at establishing the empirical formulae to estimate the physical quantities of an M dwarf's atmosphere and wind, including the wind velocity ($v_{wind}$), mass-loss rate ($\dot{M}$), coronal temperature ($T_{co}$), and plasma pressure at the transition region ($p_\alpha$).

5.1. Stellar Wind Velocity versus Plasma $\beta$ of the Stellar Wind

Based on the discussion in Section 4.1, $v_{wind}$ (wind velocity at $r = 100r_*$) is expressed as

$$v_{wind}^2 = \Delta_p + \Delta_{p_\alpha} + \Delta_c + \Delta_t + \Delta_\phi,$$

(47)

where $\Delta_p = \Delta_p^{100r_*}$, $\Delta_{p_\alpha} = \Delta_{p_\alpha}^{100r_*}$, $\Delta_c = \Delta_c^{100r_*}$, $\Delta_t = \Delta_t^{100r_*}$, and $\Delta_\phi = \Delta_\phi^{100r_*}$ (see Equations (28)–(32)).

The maximum amplitude of the Alfvén waves in the stellar wind ($v_{max}$) characterizes the above terms as follows:

$$\Delta_p + \Delta_c = a_{1.1}v_{max}^{k_{1.1}},$$

(48)

$$\Delta_t = a_{1.2}v_{max}^{k_{1.2}},$$

(49)

$$\Delta_{p_\alpha} = a_{1.3}v_{max}^{k_{1.3}},$$

(50)

$$\Delta_\phi = a_{1.4}v_{max}^{k_{1.4}},$$

(51)

where $v_{max} = v_{max}/(300 \text{ km s}^{-1})$, $\Delta_c = \Delta_c/(319^2 \text{ (km s}^{-1})^2)$, and $\Delta_\phi = \Delta_\phi/(319^2 \text{ (km s}^{-1})^2)$. The coefficients ($a_{1.1}$, $a_{1.2}$, $a_{1.3}$, $a_{1.4}$) and power-law indices ($k_{1.1}$, $k_{1.2}$, $k_{1.3}$, $k_{1.4}$) are determined by regression analyses about our simulation results,

$$a_{1.1} = 653^2, \quad a_{1.2} = 585^2,$$

$$a_{1.3} = 666^2, \quad a_{1.4} = 472^2,$$

in units of (km s$^{-1}$)$^2$.

$$k_{1.1} = 2.31, \quad k_{1.2} = 2.04,$$

$$k_{1.3} = 2.12, \quad k_{1.4} = 0.682.$$
and $\beta_{\max}$ (Figure 13), where $V_{A,\max}$ and $\beta_{\max}$ are the Alfvén speed and plasma $\beta$ at the distance where the Alfvén wave amplitude reaches a maximum:

$$\frac{\nu_{\max}}{V_{A,\max}} = 0.133 \left(\frac{\beta_{\max}}{10^{-2}}\right)^{0.378}.$$  \hspace{1cm} (52)

The positive correlation between the nonlinearity of the Alfvén waves and the plasma $\beta$ of background media has been discussed in terms of decay instability (Sagdeev & Galeev 1969; Derby 1978). It is well known that circularly polarized Alfvén waves with a frequency $\nu$ are susceptible to the decay instability in the low-$\beta$ plasma, whose growth rate $\gamma$ is approximately expressed with $\gamma^2/\nu^2 = \frac{1}{4} \eta^2 \beta^{-1/2}$. Here $\eta = v_{\perp}/V_A$ is the nonlinearity of the circularly polarized Alfvén waves with an amplitude of $v_{\perp}$. This suggests that the Alfvén wave propagation is drastically destabilized within the timescale comparable to its wave period, when $\eta \gtrsim 2/\beta^{1/4}$. Although the power-law index of 1/4 is smaller than that of Equation (52), it is generally expected that larger maximum nonlinearity of the Alfvén waves is possible in a higher-$\beta$ stellar wind.

Because $\beta_{\max} \propto c_{s,\max}^2 / V_{A,\max}^2 \propto T_{\max} / V_{A,\max}^2$, $\left(\beta_{\max} \propto \beta^{-1/2}\right)$ is the sound speed at the distance where the Alfvén wave amplitude reaches a maximum) and $T_{\max} \sim 10^6$ K regardless of the stars, it is expected that $V_{A,\max} \propto \beta^{-1/2}$. Our simulation results show

$$V_{A,\max} = 2.15 \times 10^3 \text{ km s}^{-1} \left(\frac{\beta_{\max}}{10^{-2}}\right)^{-0.549},$$  \hspace{1cm} (53)

and consequently, we obtain

$$\nu_{\max} = 286 \text{ km s}^{-1} \left(\frac{\beta_{\max}}{10^{-2}}\right)^{-0.171}.$$

Equations (47) and (54) determine $v_{\text{wind}}$ as a function of $\beta_{\max}$. In particular, a lower $\beta_{\max}$ leads to a faster $v_{\text{wind}}$.

**Figure 13.** Maximum nonlinearity of Alfvén waves in the stellar wind vs. plasma $\beta$ at the distance where the amplitude of the Alfvén waves reaches the maximum.

Because larger-amplitude Alfvén waves can propagate in the stellar wind.

### 5.2. Coronal Temperature and Mass-loss Rate

We expect that $T_{\text{co}}$ (the coronal temperature at $r = 1.1 r_s$) is expressed as a function of $F_{A,\text{co}}$ (transmitted Alfvén wave energy flux into the corona at $r = 1.1 r_s$) through the energy conservation law (Equations (41) and (45)). Here the energy conservation law is simplified as

$$-L_{c,\text{co}} \approx L_{A,\text{co}} - (L_{\text{kin,wind}} - L_{E,\text{co}}).$$  \hspace{1cm} (55)

Note that the radiative energy loss, which is neglected in the above, can be involved with energy conservation when the open flux tube filling factor ($f_{\text{ph}}$) is much smaller than that used in this study (1/1600). The details are described in Appendix A.

Because

$$L_{\text{kin,wind}} = \dot{M} v_{\text{wind}}^2 / 2,$$

and

$$L_{E,\text{co}} = -\dot{M} v_{\text{esc}}^2 / 2 = -M G M_s / r_s,$$

we obtain the following by dividing both sides of Equation (55) by $L_{A,\text{co}}$:

$$\alpha_c / \Lambda = 1 - \alpha_{\text{wind/\Lambda}} \left(1 + \frac{v_{\text{esc}}^2}{v_{\text{wind}}^2}\right),$$

where $\alpha_c / \Lambda$ and $\alpha_{\text{wind/\Lambda}}$ are the energy conversion efficiencies from the Alfvén wave energy flux ($L_{A,\text{co}}$) to the heat conduction flux ($L_{c,\text{co}}$) and the wind’s kinetic energy flux ($L_{\text{kin,wind}}$), respectively. That means

$$\alpha_c / \Lambda = -L_{c,\text{co}} / L_{A,\text{co}}.$$  \hspace{1cm} (59)
and

$$\alpha_{\text{wind}/A} = L_{\text{kin}, \text{wind}}/L_{A, \text{co}}.$$  \hfill (60)

Hereafter, we assume that $\alpha_{\text{wind}/A}$ is independent of stars, chromospheric magnetic field strengths, and energy inputs from the photosphere; namely, $\alpha_{\text{wind}/A} = 0.442 \pm 0.166$. This assumption is confirmed, at least in our parameter survey (Figure 14), and enables us to solve the degeneracy between the coronal temperature ($T_{\text{co}}$) and mass-loss rate ($\dot{M}$) in the energy conservation law. That means, by using $\alpha_{\text{wind}/A}$, we can express $T_{\text{co}}$ and $\dot{M}$ as functions of $F_{A, \text{co}}$ as follows:

$$T_{\text{co}} = a_2 \left[ 1 - \alpha_{\text{wind}/A} \left( 1 + \frac{v_{\text{esc}}^2}{v_{\text{wind}}^2} \right) \right] F_{A, \text{co}} = f_2,$$  \hfill (61)

where $f_{A, \text{co}} = F_{A, \text{co}}/(10^5 \text{ erg cm}^{-2} \text{ s}^{-1})$, $\dot{L}_{\text{co}} = \dot{L}_{\text{co}}/v_{\text{co}}$, and $a_2$ and $k_2$ are determined by our simulation results: $a_2 = 1.62 \times 10^6$, $k_2 = 0.256$ (see Equation (45)).

$$\dot{M} = 2\alpha_{\text{wind}/A} \frac{L_{A, \text{co}}}{v_{\text{wind}}}.$$

(62)

It should be noted that, however, $\alpha_{\text{wind}/A}$ possibly depends on the filling factor of the open flux tube ($f_{\text{ph}}$), which is beyond our present parameter survey. In addition, the $C_{\text{ph}}/A$ of Equation (58) is often quenched to zero when $v_{\text{wind}} < v_{\text{esc}}$, which means the approximation for Equations (42) and (58) becomes invalid. In the following analysis, therefore, we assume a monotonic increase in $|L_{\text{co}}|$ with $L_{A, \text{co}}$, i.e., $\partial \ln \alpha_{C/A}/\partial \ln L_{A, \text{co}} > -1$, to avoid this problem.

### 5.3. Plasma \( \beta \) of the Stellar Wind

In Section 5.1, we express $v_{\text{wind}}$ as a function of $\beta_{\phi, \text{max}}$. Here $\beta_{\phi, \text{max}}$ is the plasma $\beta$ at the distance (hereafter $r_{\phi, \text{max}}$) where the amplitude of the Alfvén waves reaches the maximum. In this section, we thus consider how $\beta_{\phi, \text{max}}$ is determined. The physical quantities with the subscript $\phi, \text{max}$ mean those at $r = r_{\phi, \text{max}}$ in the following.

The plasma $\beta$ of the stellar wind is expressed as

$$\beta = \frac{8\pi p}{B_s^2} = \frac{8\pi (\gamma - 1) L_{\text{ent}}/(4\pi f_{\text{ph}}^2)}{\gamma v_s (B_{\text{ph}, \text{ph}}/f_{\text{ph}})^2} = \left( \frac{r}{r_s} \right)^2,$$

(63)

where $L_{\text{ent}} = 4\pi f_{\text{ph}}^2 \gamma p v_s/(\gamma - 1)$ is the enthalpy luminosity. In the isothermal stellar wind, $L_{\text{ent}} = \gamma R_s M T/\mu (\gamma - 1)$ is almost constant, and then $\beta_{\phi, \text{max}} \propto (r_{\phi, \text{max}}/r_s)^2 \nu_s/\nu_{\phi, \text{max}}$. In the following, we aim at expressing $\beta_{\phi, \text{max}}$ with several integral constants, such as $M$, $B_{\text{ph}, \text{ph}}$, and $v_{\text{wind}}$, and coronal parameters, such as $T_{\text{co}}$ and $L_{A, \text{co}}$.

First, the $r/r_s$ in Equation (63) can be related to the ratios of mass density, Alfvén wave energy luminosity ($L_A \approx \sqrt{4\pi \rho B_{\text{ph}, \text{ph}}^2 r_s^2 v_s^3}$), and Alfvén wave nonlinearity ($\eta = v_s/\nu_{\phi}$). In fact, by using $B_{\text{ph}, \text{ph}}^2 = \text{const}$., we have $L_A \propto \sqrt{\rho v_s^3}$ and $\nu_{\phi} \propto \eta/\sqrt{\nu_{\phi} f_{\phi}}$, and then

$$\frac{L_A}{L_{A, \text{co}}} = \frac{\rho (\eta/\eta_{\text{co}})^2 (f_{\text{ph}} r_s^2)^2}{\nu_{\phi}^2},$$

(64)

where $f_{\text{co}}$ is the filling factor of the open flux tube at $r = r_{\text{co}} = 1.1 r_s$.

The $\sqrt{\rho_{\text{co}}/\rho}$ in Equation (64) is inconvenient for later discussion and rewritten as follows:

$$\sqrt{\rho_{\text{co}}/\rho} = \frac{M_A}{L_{A, \text{co}}} = \frac{v_{s}}{v_{\text{co}}},$$

(65)

Note that $M_A$ is the Alfvén Mach number of the stellar wind, which is expressed as $M_A = v_s/\sqrt{\nu_{\phi} f_{\phi}}$.

By substituting the above into Equation (64), we have

$$\left( \frac{r}{r_{\text{co}}} \right)^4 = \frac{L_{A, \text{co}}}{L_A} \frac{\eta_{\text{co}}^2 M_{A, \text{co}}}{v_{\text{co}}} \frac{v_{s}}{f_{\text{co}}},$$

(66)

The $\eta_{\text{co}}^2 M_{A, \text{co}}$ in the above can be further rewritten as follows:

$$\eta_{\text{co}}^2 M_{A, \text{co}} = \frac{v_{\text{co}}^2}{V_{\text{A, co}}^2} = \frac{1}{V_{\text{A, co}}^2} = \frac{L_{A, \text{co}}}{4\pi \rho_{\text{co}} B_{\text{ph}, \text{ph}}^2 r_{\text{co}}^2} = \frac{B_{\text{ph}, \text{ph}}^2}{V_{\text{A, co}}^2} \frac{v_{\text{co}}^2}{f_{\text{co}}} \frac{r_{\text{co}}^2}{r_{\phi, \text{max}}^2},$$

(67)

$$M_{A, \text{co}} = \frac{V_{\text{A, co}}}{L_{A, \text{co}}} = \frac{\nu_{\phi}^2}{V_{\text{A, co}}^2} = \frac{M/4\pi \rho_{\text{co}}}{B_{\text{ph}, \text{ph}}^2 r_{\text{co}}^2}.$$  \hfill (68)

These lead to

$$\eta_{\text{co}}^2 M_{A, \text{co}} = \frac{1}{(B_{\text{ph}, \text{ph}}/f_{\phi})^2} \left( \frac{r_{\phi, \text{max}}}{r_s} \right)^4 \frac{L_{A, \text{co}}}{L_A} \frac{M}{r_{\text{co}}^2} = \frac{M}{r_{\text{co}}^2}.$$  \hfill (69)

By substituting the above into Equation (66),

$$\left( \frac{r}{r_s} \right)^4 = \frac{\nu_{\phi}^2}{V_{\text{A, co}}^2} \frac{r_{\phi, \text{max}}}{r_s} \frac{M}{L_A}.$$  \hfill (70)

Combining the above and Equation (63) leads to

$$\beta = \frac{2 R_s}{\mu} \sqrt{\frac{M}{L_A}} \frac{\eta_{\phi, \text{max}}^2}{2 \nu_s} \frac{\nu_s}{V_{\text{A, co}}},$$

(71)

and $\beta_{\phi, \text{max}}$ is obtained by using $L_A = L_{A, \phi, \text{max}}, T = T_{\phi, \text{max}} \approx T_{\text{co}}, \eta = \eta_{\phi, \text{max}}, v_{\phi} = v_{\phi, \text{max}}$, and $v_s = v_{s, \phi, \text{max}}$.

Meanwhile, we can expect the following three relations.

First, $\sqrt{M/L_{A, \phi, \text{max}} \propto 1/v_{\text{wind}}}$ because $L_{A, \phi, \text{max}} \propto L_{A, \text{co}}$ and Equation (62). Our parameter survey shows $L_{A, \phi, \text{max}} L_{A, \text{co}} = 0.193 \pm 0.087$. Second, $\eta_{\phi, \text{max}}$ and $v_{\phi, \text{max}}$ are written as the power-law functions of $\beta_{\phi, \text{max}}$ (Equations 52 and 54). Third, $v_{s, \phi, \text{max}} \propto v_{\phi, \text{max}}$ because $v_{s, \phi, \text{max}}$ is determined by $v_{\phi, \text{max}}$ in the Alfvén wave-driven wind. Our parameter survey shows $v_{s, \phi, \text{max}}/v_{\phi, \text{max}} = 1.35 \pm 0.27$. 

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Based on these considerations, we assume the following power-law relation between $\beta_{\phi,\text{max}}$ and $T_{\phi}/v_{\text{wind}}$:

$$\beta_{\phi,\text{max}} = a_3 \left( \frac{T_{\phi}}{v_{\text{wind}}} \right)^{k_3},$$  \hspace{1cm} (72)

where $T_{\phi} = T_{\phi}/10^6$ K, $v_{\text{wind}} = v_{\text{wind}}/600$ km s$^{-1}$, and $a_3$ and $k_3$ are determined by our simulation results: $a_3 = 2.09 \times 10^{-2}$ and $k_3 = 1.85$ (Figure 15).

It is notable that $\beta_{\phi,\text{max}}$ is not explicitly dependent on the stellar magnetic field strength ($B_{\phi,\text{ph}}$). Although the plasma $\beta$ at a given distance is negatively correlated with $B_{\phi,\text{ph}}$ (Equation (63)), the distance where the amplitude of the Alfvén waves reaches the maximum ($v_{\phi,\text{max}}$) is positively correlated with $B_{\phi,\text{ph}}$ (Equation (70)) because the nonlinearity of the Alfvén waves at a given distance tends to be smaller when $B_{\phi,\text{ph}}$ becomes larger (Equation (69)). These effects of varying $B_{\phi,\text{ph}}$ cancel each other out in Equation (72).

### 5.4. Transmissivity of Alfvén Waves into the Stellar Corona

From Sections 5.1 to 5.3, we describe the relations among $v_{\text{wind}}$, $T_{\phi}$, and $\beta_{\phi,\text{max}}$. In particular, Equations (47), (61), and (72) can be used to estimate $v_{\phi}$ and $T_{\phi}$ for a given parameter set of $v_{\phi,\text{esc}}$, $B_{\phi,\text{co}}$, and $F_{A,\phi}$.

The $F_{A,\phi}$ is the transmitted Alfvén wave energy flux into the corona and related to Alfvén wave energy input from the photosphere ($L_{A,\phi,\text{ph}}$). Here

$$L_{A,\phi,\text{in}} = 4\pi r^2 F_{A,\phi,\text{in}}$$  \hspace{1cm} (73)

corresponds to the luminosity of outward/inward Alfvén waves, where $F_{A,\phi,\text{in}}$ is the Poynting flux associated with the outward/inward Alfvén wave,

$$F_{A,\phi,\text{in}} = \frac{1}{4} \rho c^2 l_{A,\phi,\text{in}},$$  \hspace{1cm} (74)

where

$$z_{\phi,\text{out}} = v_\phi - \frac{B_\phi}{\sqrt{4\pi \rho}},$$  \hspace{1cm} (75)

and

$$z_{\phi,\text{in}} = \frac{B_\phi}{\sqrt{4\pi \rho}}.$$  \hspace{1cm} (76)

We discuss the transmissivity of Alfvén waves from the stellar photosphere to the corona by defining it as $\alpha_{\phi,\text{co,ph}} = L_{A,\phi,\text{co}}/L_{A,\phi,\text{out}}$ in the following.

Figure 16(a) shows the relation between $\alpha_{\phi,\text{co,ph}}$ and $L_{A,\phi,\text{in}}/L_{A,\phi,\text{out}}$. The latter represents the fraction of Alfvén wave energy reflected back to the stellar photosphere. As seen in this panel, while a larger reflection rate than $\sim 0.5$ is negatively correlated with the transmissivity of the Alfvén waves, the correlation is reversed when the reflection rate is smaller than $\sim 0.5$. This positive correlation between the transmissivity and reflection rate of Alfvén waves suggests that the Alfvén wave energy is attenuated mainly due to the wave dissipation, rather than reflection. To characterize this wave dissipation, we introduce the Alfvén travel time from the
photosphere to the corona ($\tau_{A,co}$) as

$$\tau_{A,co} = \int_{r_c}^{1.1r_c} \frac{1}{B_t/\sqrt{4\pi\rho}} \frac{dx}{dr}. \quad (77)$$

Figure 16(b) shows the correlation between $\alpha_{co/ph}$ and the nondimensionalized Alfvén travel time with respect to the typical wave frequency $\nu_A$, where we take the acoustic cutoff frequency of the photosphere ($\nu_{ac}$ in Table 1) for $\nu_A$. Figure 16(b) looks like Figure 16(a), but they are flipped left to right. When $\tau_{A,co/\nu_{ac}} \geq 1$, the chromosphere is too thick for Alfvén waves to transmit into the corona. The resultant transmissivity of the Alfvén waves practically follows

$$\alpha_{co/ph} \approx a_{4,1} \left( \frac{\tau_{A,co/\nu_{ac}}}{a_{4,2}} \right)^{k_4}, \quad (78)$$

where $a_{4,1} = 2.41 \times 10^{-2}$, $a_{4,2} = 1.04$, and

$$k_4 = \begin{cases} 1.25 & (\tau_{A,co/\nu_{ac}} < a_{4,2}) \\ -1.10 & (\tau_{A,co/\nu_{ac}} > a_{4,2}). \end{cases} \quad (79)$$

It should be noted that $\tau_{A,co}$ is approximated by the Alfvén travel time up to the merging height ($H_m$, the height at which $B_t = B$). This is because the integrand in Equation (77) decreases exponentially with increasing Alfvén speed above $H_m$. That means we can expect that $\tau_{A,co/\nu_{ac}} \approx (x_m/\nabla_x)\nu_{ac}$, where

$$x_m = \int_{r_c}^{r_c+H_m} \frac{dx}{dr},$$

$$\nabla_x = \frac{1}{x_m} \int_{r_c}^{r_c+H_m} B_t \frac{dx}{dr}. \quad (80)$$

Our parameter survey shows $\tau_{A,co/\nu_{ac}} \approx 1.96(x_m\nu_{ac}/\nabla_x)^{-1.17}$. Furthermore, $x_m\nu_{ac}$ is a factor of $c_{i,ph}$, depending on $B$, and $\nabla_x \propto c_{i,ph}$ in this study. The $\nabla_x$ is also influenced by the Alfvén wave amplitude of the photosphere ($\nu_{ph}$) through the varying density profile below $H_m$. These considerations lead to the following practical fit:

$$\tau_{A,co/\nu_{ac}} \approx a_5 \tilde{g}_s \left( \frac{B}{B_{ph}} \right)^{-k_{5,1}} \left( \frac{\nu_{ph}}{c_{i,ph}} \right)^{k_{5,2}} \gamma^{k_{5,3}}, \quad (82)$$

where $\tilde{g}_s = g_s/10^5$ cm s$^{-2}$, $a_5 = 0.921$, $k_{5,1} = 0.240$, $k_{5,2} = 0.408$, and $k_{5,3} = 0.697$.

Equation (82) shows that the weaker the magnetic field is, the larger the $\tau_{A,co/\nu_{ac}}$ and the more significant the wave dissipation is in the lower atmosphere. When $\tau_{A,co/\nu_{ac}} \leq 1$, on the other hand, Alfvén waves transmit into the corona on a shorter timescale than their typical wave period. In this case, the wave dissipation is relatively negligible compared to the wave reflection, resulting in a negative correlation between the reflection rate and transmissivity of Alfvén waves.

### 5.5. Plasma Pressure at the Transition Region

The plasma pressure at the transition region ($p_{tr}$) is determined so that the radiative cooling approximately balances with the heat conduction heating,

$$n_u^2 \Lambda(T_{tr}) \approx |\text{div} F|_{tr}, \quad (83)$$

where $n = \rho/m_p$ and $\Lambda(T)$ represents the radiative loss function for the optically thin plasma. By substituting $n_u = p_u/(2k_B T_{tr})$,

![Figure 17. Relation between plasma pressure at the transition region $p_{tr}$ and the heating rate due to the heat conduction flux $|\text{div} F|_{tr}$, showing a strong positive correlation results from the energy balance between radiative cooling and heat conduction heating at the transition region.](image)
By substituting Equation (61) into Equation (88) and using Equation (86), we obtain
\[
p_{tr} = 9.00 \times 10^{-3} \text{ erg cm}^{-3} \times T_{B, tr}^{-0.294} \left( \frac{A_{co}}{A_{tr}} \right)^{0.529} \left( \frac{\rho_{co}}{\rho_{tr}} \right)^{0.273},
\]
where \( \rho_{co}/A \) is defined by Equation (58) and \( F_{A,co} = F_{A,co}/10^5 \) erg cm\(^{-2}\) s\(^{-1}\).

The coronal mass density \( \rho_{co} \) is immediately obtained from Equation (90). By assuming a hydrostatic atmosphere with a constant gravitation acceleration of \( g_{tr} \), the pressure at \( r = 1.1r_s \) \( (\rho_{co}) \) is estimated as \( \rho_{co} \approx e^{-T_{tr}/T_{eff}}p_{tr} \), where
\[
T_{tr} = 0.1r_s g_{tr}/R_g \approx 1.15 \times 10^6 K \left( \frac{M_s/r_s}{M_o/\rho_o} \right).
\]

The ratio of \( T_{tr}/T_{co} \) is identical to 0.1\( r_s H_{pc,co} \), where \( H_{pc,co} = R_g T_{co}/(\mu g_{tr}) \) is the pressure scale height in the corona. By using Equation (90) and \( \rho_{co} \equiv \rho_{pc}/R_g T_{co} \), we obtain
\[
\rho_{co} = 3.33 \times 10^{-17} \text{ g cm}^{-3} \times e^{-T_{co}/T_{eff}}T_{B, tr}^{-0.550} \left( \frac{A_{co}}{A_{tr}} \right)^{0.273} \left( \frac{\rho_{co}}{\rho_{tr}} \right)^{0.273}.
\]

As suggested in Equation (61), the coronal temperature in M dwarfs with smaller \( T_{B, tr} \) is cooler than that of the Sun for a given \( F_{A,co} \). This is because the temperature profile in the corona with a smaller \( T_{B, tr} \) is characterized by a larger temperature gradient. In this case, the hotter coronal temperature is not required for the heat conduction flux from the corona to balance with the transmitted Alfvén wave energy flux into the corona.

On the other hand, \( p_{tr} \) is less dependent on \( l_{B, tr} \) or \( l_{B, co} \) for a given \( F_{A,co} \). This is because the cooler coronal temperature as a result of a smaller \( l_{B, co} \) cancels out the tendency that a smaller \( l_{B, tr} \) leads to a larger temperature gradient around the transition region. The simulated \( p_{co} \) of the M dwarf is, on the other hand, relatively higher than the solar value (e.g., Figure 17). This is partly because the \( F_{A,co} \) of the M dwarf tends to be larger than that of the Sun due to the higher transmissivity of the Alfvén waves across the stellar chromosphere. It should be noted that the plasma pressure of the photosphere \( (p_{ph}) \) has a larger increase with decreasing \( T_{eff} \) compared to the increase in \( p_{tr} \). As a result, \( p_{tr}/p_{ph} \) decreases with decreasing \( T_{eff} \), leading to a more extended chromosphere associated with higher \( H_{co}/H_{ph} \) (Figure 6).

5.6. \( T_{eff}-L_{A, ph} \) Diagrams for M Dwarf Atmospheres and Winds

On the basis of the discussion in Section 5.4, we can calculate the transmitted Alfvén wave energy flux into the corona \( (F_{A,co}) \) by specifying the Alfvén wave energy input from the photosphere \( (L_{A, ph}) \); the basic parameters of the stars, such as \( r_s, M_s \), and \( T_{eff} \); and the parameters of the open flux tube configuration, such as \( B_{ph}, B_r \), and \( l_{B, co} \). The obtained \( F_{A,co} \) is necessary to predict the stellar coronal temperature \( (T_{co}) \), stellar wind velocity \( (v_{wind}) \), and mass-loss rate \( (M) \), according to Sections 5.1–5.3. Furthermore, the plasma pressure at the transition region \( \rho_{co} \) and coronal mass density \( \rho_{co} \) can be derived by using \( T_{co} \) (Section 5.5).

Therefore, when we limit our interest to the main-sequence stars’ atmospheres and winds so that the parameters of \( r_s, M_s, B_{ph}, B_r \), and \( l_{B, co} \) can be roughly expressed as the functions of \( T_{eff} \), it is possible to estimate these physical quantities for a given \( T_{eff} \) and \( L_{A, ph} \). We developed a python code called AWSAWS to calculate \( v_{wind}, T_{co}, M, p_{co} \), and \( \rho_{co} \), as functions of \( T_{eff} \) and \( L_{A, ph} \). The code is provided on the first author’s website.

The resultant \( T_{eff}-L_{A, ph} \) diagrams for \( v_{wind}, T_{co}, M, p_{co} \), and \( \rho_{co} \) are shown in Figure 18. Note that we assume \( T_{eff}=T_{tr} \) and \( r_s-M_s \), relations of the main-sequence stars as explained in Appendix B. The \( B/B_{ph} = e^{-4} \) is also assumed in order to obtain Figure 18. The chromospheric magnetic field strength \( (B) \) can have an influence on the physical quantities in Figure 18 by a factor of a few through the transmissivity of the Alfvén wave energy from the photosphere to the corona (Section 5.4, especially Equation (82)). Even by changing \( B \), however, \( T_{eff} - L_{A, ph} \) diagrams are almost identical to those of Figure 18. Therefore, our discussion about the trends of the physical quantities based on Figure 18 is applicable to the case where \( B/B_{ph} = e^{-4} \).

In Figure 18, the thick dashed line corresponds to the fiducial \( L_{A, ph} \) as a function of \( T_{eff} \), which is the case where the Alfvén wave amplitude of the photosphere \( (v_{ph}) \) is equal to the convective velocity \( (v_{c conv}) \). The thin dashed-dotted line corresponds to the largest \( L_{A, ph} \) obtained by assuming that the convective velocity reaches the sound speed of the photosphere \( (v_{ph} = c_s, ph) \). The thin dashed line represents \( L_{A, ph} \) as a function of \( T_{eff} \), which results in \( v_{wind} = v_{esc} \).

Along the thick dashed line, it is seen that \( v_{wind} \) and \( T_{co} \) are faster and cooler with decreasing \( T_{eff} \), and that the mass-loss rate \( M \) of the M dwarf winds is much smaller than the solar wind’s value. The differences in \( p_{co} \) and \( \rho_{co} \) are less remarkable, but they systematically increase with decreasing \( T_{eff} \), as discussed in Section 5.5.

Figure 19 summarizes the causal relations among the varying \( v_{wind}, T_{co}, M, p_{co} \), and \( \rho_{co} \) with respect to decreases in \( L_{A, ph}/(4\pi r_s^2) \) and \( r_s(T_{eff}) \). Here \( L_{A, ph}/(4\pi r_s^2) \) and \( r_s(T_{eff}) \) correspond to the vertical and horizontal axes of the panels in Figure 18, respectively. As depicted with the thick arrows in Figure 19, the decrease in \( L_{A, ph}/(4\pi r_s^2) \) causes cooler \( T_{co} \) faster \( v_{wind} \) and smaller \( M/(4\pi r_s^2) \). More in detail, the smaller \( L_{A, ph}/(4\pi r_s^2) \) is, the smaller \( L_{c, ph}/(4\pi r_s^2) \) and \( L_{kin, wind}/(4\pi r_s^2) \) are. This energy partitioning from \( L_{A, ph} \) to \( L_{co} \) or \( L_{kin, wind} \) is represented by \( O_{wind}/A \) (Section 5.2 and Appendix A).

Smaller \( L_{co} \) tends to be associated with cooler \( T_{co} \) (Equation (45)), which leads to lower \( \beta_{max} \) (Equation (72)). Here \( \beta_{max} \) is amplified when \( \beta_{max} \) is lower (Equation (54)), which is responsible for greater \( (\Delta_p+\Delta_c) \) and faster \( v_{wind} \) (Section 5.1). Smaller \( L_{kin, wind}/(4\pi r_s^2) \) and \( v_{wind} \) drive smaller \( M/(4\pi r_s^2) \) (Equation (62)). On the other hand, smaller \( r_s \) (or cooler \( T_{eff} \)) is associated with smaller \( l_{B, co} \) and \( l_{B, tr} \). The former leads to cooler \( T_{co} \) as well as smaller \( L_{A, ph}/(4\pi r_s^2) \) (Equation (45)). The effects of smaller \( l_{B, tr} \) and cooler \( T_{co} \) on \( p_{co} \) cancel each other out (Equation (88)), but smaller \( l_{B, co} \) tends to enhance \( p_{co} \) in general. As a result, cooler \( T_{co} \) and larger \( p_{co} \) lead to larger \( \rho_{co} \).

6. Discussion

6.1. Mass-loss Rate of the M Dwarf Stellar Wind

The simulated mass-loss rates \( (M) \) of M dwarf stellar winds are much smaller than the solar wind’s value (Table 3). This is mainly the result of the smaller surface areas of M dwarfs and

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3. https://www.kwasan.kyoto-u.ac.jp/~E{sakaue}/awsaws/awsaws.py
their relatively faster winds. In addition, our wind’s mass-loss rates of M dwarfs are typically smaller than reported by the previous global stellar wind modelings. The $M$ of an M8-type star in this study is no more than $6.9 \times 10^{-17} M_\odot$ yr$^{-1}$, while Garraffo et al. (2017) and Dong et al. (2018) showed $3 \times 10^{-14}$ and $4.1 \times 10^{-15} M_\odot$ yr$^{-1}$, respectively, for TRAPPIST-1 (M8). The $M$ of Proxima Centauri (M5.5) by Garraffo et al. (2016) and EV Lac (M3.5) by Cohen et al. (2014) are $1.5 \times 10^{-14}$ and $3 \times 10^{-14} M_\odot$ yr$^{-1}$, respectively, which is 10–100 times higher than reproduced in our simulation. Because the inner boundary condition is arbitrarily determined in these 3D simulations (Sokolov et al. 2013; van der Holst et al. 2014), the Alfvén wave energy flux on their inner boundary is likely to be inappropriately high for M dwarfs. Furthermore, because the spatial resolution of these modelings is too low to discuss the crucial role of compressible waves (slow shock) in the stellar wind acceleration, it is probable that they may not reproduce our simulation result.

Observational measurements of stellar wind have recently made remarkable progress, especially in M dwarfs. In order to quantify the stellar wind’s properties observationally, Wood et al. (2001, 2002, 2005a, 2005b, 2021) investigated the absorption signatures in stellar Lyα spectra that originate in the “neutral hydrogen wall” around the astrospheres. By assuming that the stellar wind’s velocity is constant regardless of the stars, they succeeded in estimating the mass-loss rates ($M$) of several nearby stars. Another method to estimate $M$ was developed by Bourrier et al. (2016), Vidotto & Bourrier (2017), and Villarreal D’Angelo et al. (2021), who deduced an $M$ of GJ 436 (M2.5) around $(0.45–2.5) \times 10^{-15} M_\odot$ yr$^{-1}$ by analyzing the transmission spectra of Lyα of GJ 436 b (a warm Neptune).

The number of observations of M dwarf $M$ drastically increased owing to Wood et al. (2021). The published $M$ of M and G, K dwarfs are plotted in an $r_s$–$L_{\alpha,ph}$ diagram for $M$ (Figure 20), which is obtained by converting the $T_{\text{eff}}$–$L_{\alpha,ph}$ diagram for $M$ in Figure 18 with the relation between $r_s$ and $T_{\text{eff}}$ for the main-sequence stars (Appendix B).

Figure 20 shows the required Alfvén wave energy flux on the stellar photosphere ($L_{\alpha,ph}/(4\pi r_e^2)$) to realize the observed $M$. The red filled circle of GJ 436, for example, lies in the
vicinity of the thick black dashed line, indicating that the Alfvén wave energy flux driven by the surface convective motion is almost adequate to reproduce the observed $M$ of GJ 436. On the other hand, the observed $M$ of EV Lac or YZ CMi could require a nearly 50 or 5000 times larger Alfvén wave energy flux than expected from the scenario of Alfvén wave generation by surface convective motion. In particular, YZ CMi lies in the gray area, which means that the Alfvén wave energy flux driven by the surface convective motion is not adequate to reproduce the observed $M$.
wave-driven stellar wind model discussed here cannot account for the observed \( M \), even if the velocity of convective motion exceeds the sound speed of the stellar photosphere. There are many data points suggesting that a much larger Alfvén wave energy flux is required than expected, similar to EV Lac and YZ CMi. This discrepancy between the observed and predicted \( M \) might be resolved by investigating the contribution of coronal mass ejection to the total mass-loss rate or the origin of Alfvén wave energy flux on the stellar surface.

6.2. Properties of the Stellar Transition Region

Although the detailed prediction of the stellar spectrum from the simulated stellar atmosphere is beyond our scope, some implications are obtained to be compared with the observational studies. One of them is the plasma pressure at the stellar transition region \( p_{\text{tr}} \), which has been investigated by spectral analyses of optical or UV lines as one of the fundamental parameters to constrain the semiempirical model atmosphere (Linsky 2017). Another is the radiative loss from the optically thin plasma around the transition region \( F_{\text{rad, tr}} \), which we define by using \( Q_{\text{rad}} \) in our simulation as follows:

\[
F_{\text{rad, tr}} = \frac{1}{4\pi r_{*}^{2}} \int_{r}^{\infty} 4\pi r^{2} Q_{\text{rad}} dr, \tag{93}
\]

where we assume that the transition region’s emission is uniform over the whole stellar disk. Figure 21 shows the \( Q_{\text{rad}} \) of the solar and stellar atmospheres as a function of \( p/g_{*} \).

Figure 22 shows the relation between \( p_{\text{tr}}^{2}/g_{*} \) and \( F_{\text{rad, tr}} \) obtained from our simulation. They are expected to be proportional to each other when \( Q_{\text{rad}}/p^{2} \approx \Lambda(T)/(2k_{B}T)^{2} \) steeply drops from the transition region to the corona. In Figure 22, we also show the observed stellar Ly\( \alpha \) flux \( (F_{\text{Ly} \alpha}) \) and \( m_{\text{tr}} g_{*} \), which is obtained from the spectral analyses or emission line diagnostics. The Ly\( \alpha \) line is formed from the upper chromosphere to the transition region (Fontenla et al. 2016) and is generally the brightest and most dominant emission line in the far-UV spectra of late-type stars (Landsman & Simon 1993; France et al. 2013). According to Houdéine et al. (1995), Ly\( \alpha \) fluxes of M dwarfs are almost proportional to the column mass density at the transition region. The \( F_{\text{Ly} \alpha} \) of the M dwarfs in Figure 22 is reported by Byrne & Doyle (1989; for YZ CMi and AX Mic), Wood et al. (2005b; for AD Leo, EV Lac, and Proxima Centauri), Younghblood et al. (2016; for GJ 436, GJ 581, GJ 667C, GJ 832, GJ 876, and GJ 1214; they are indicated together by “Y16”) and Bourrier et al. (2017; for TRAPPIST-1). The \( F_{\text{Ly} \alpha} \) from the solar AR, QR, and CH is estimated based on Fontenla et al. (1988, 1999), Curdt et al. (2008), and Tian et al. (2009). The \( p_{\text{tr}} \) of the M dwarfs is reported by Giampapa et al. (1982; for YZ CMi), Mauas & Falchi (1994; for AD Leo), Fontenla et al. (2016; for GJ 832), Peacock et al. (2019a; for TRAPPIST-1), and Peacock et al. (2019b; for GJ 436, and GJ 832, indicated by “P19”). The \( p_{\text{tr}} \) in the solar CH and QR is cited from Maxson & Vaiana (1977), while the \( p_{\text{tr}} \) in the solar AR is cited from Yashiro & Shibata (2001). The dashed-dotted line represents a fitted line obtained by assuming the proportional relation between \( p_{\text{tr}}^{2}/g_{*} \) and \( F_{\text{rad, tr}} \) \((\gamma = 1.93 \times 10^{12})\).
Lac, and Proxima Centauri), Youngblood et al. (2016; for GJ 176, GJ 436, GJ 581, GJ 667C, GJ 832, GJ 876, and GJ 1214; they are indicated together by “Y16” in Figure 22) and Bourrier et al. (2017; for TRAPPIST-1). Note that the M dwarfs reported by Youngblood et al. (2016) are the targets of the Measurements of the Ultraviolet Spectral Characteristics of Low-mass Exoplanetary Systems (MUSCLES) Treasury Survey (France et al. 2016), which are optically inactive and known to be planet-hosting stars. We also show the $F_{\text{Ly}}$ from the solar active region (AR), quiet region (QR), and CH (Fontenla et al. 1988, 1999; Curdt et al. 2008; Tian et al. 2009). Note that we converted the observed intensity of $\text{Ly}_{\alpha}$ ($I_{\text{Ly}_{\alpha}}$) to $F_{\text{Ly}_{\alpha}}$ by assuming $F_{\text{Ly}_{\alpha}} = \pi I_{\text{Ly}_{\alpha}}$. The observed $F_{\text{Ly}_{\alpha}}$ is much larger than the $F_{\text{rad,tr}}$ for flare stars such as AD Leo, as expected because these stars are believed to be largely covered by ARs (Linsky et al. 1982; Saar & Linsky 1985). On the other hand, $F_{\text{Ly}_{\alpha}}$ is comparable to $F_{\text{rad,tr}}$ for moderately active stars such as TRAPPIST-1 and MUSCLES targets.

The plasma pressures at the transition region ($P_{\text{t}}$) in Figure 22 are reported by Giampapa et al. (1982; for YZ CMi), Mauas & Falchi (1994; for AD Leo), Fontenla et al. (2016; for GJ 832), Peacock et al. (2019a; for TRAPPIST-1), and Peacock et al. (2019b; for GJ 176, GJ 436, and GJ 832, included in the MUSCLES targets and indicated by “P19” in Figure 22). The $P_{\text{t}}$ in the solar CH and QR is cited from Maxson & Vaiana (1977), while the $P_{\text{t}}$ in the solar AR is cited from Yashiro & Shibata (2001).

The simulated values of $P_{\text{t}}$ are significantly lower than those of flare stars but comparable to those of moderately active stars, similar to the trend seen in the comparison between $F_{\text{rad,tr}}$ and $F_{\text{Ly}_{\alpha}}$. On the other hand, it should be noted that the plasma pressure at the stellar transition region is sometimes not well constrained from the semiempirical modeling of the stellar atmosphere. Indeed, the plasma pressure at the transition region of GJ 832 reported by Peacock et al. (2019b) is 2 orders of magnitude lower than that by Fontenla et al. (2016). The atmospheric modeling from the spectral analysis possibly depends on the model of microturbulent velocity (Jevremović et al. 2000), the partial frequency redistribution effect, and ionization mechanisms due to the coronal back-heating. Our model of the M dwarf’s chromosphere and transition region also ignores the detailed radiative transfer and partial ionization effects; therefore, much more effort is required to develop the realistic atmosphere model for M dwarfs.

We finally present the predicted relation between X-ray flux ($F_X$) and the wind’s mass-loss rate ($\dot{M}$) based on this study (colored curves in Figure 23). The observed relations between them are provided by Wood et al. (2021; symbols in Figure 23). To derive these prediction curves, we assume that $F_X$ is represented by $F_{\text{rad,tr}}$ (Equation (93)) and that $F_{\text{rad,tr}} = 1.90 \times 10^2 P_{\text{t}}^2 / g_s$ (Figure 22). Because Figure 18 shows that both $\dot{M}$ and $P_{\text{t}}$ are expressed as the functions of $(T_{\text{eff}}, L_{\text{A,ph}})$, the relation between $F_{\text{rad,tr}}$ and $\dot{M}$ is obtained for each $T_{\text{eff}}$ with $L_{\text{A,ph}}$ as an auxiliary variable.

The prediction curves in Figure 23 suggest the tight correlation between $F_{\text{rad,tr}}$ and $\dot{M}$, around which the data points scatter. However, regarding the correlation between them, it is often pointed out that X-ray radiation from an active star originates in the plasma confined in the closed coronal loops, and that, even in the case of the Sun, the X-ray flux varies by an order of magnitude during its activity cycle (Cohen 2011).

![Figure 23](image_url)  
**Figure 23.** Predicted relation between X-ray flux ($F_X$) and the wind’s mass-loss rate ($\dot{M}$) based on this study (colored curves). The circles and triangles correspond to the observed ($F_X$, $\dot{M}$) of M and G, K dwarfs, respectively, where the $M$ of the stars indicated by open symbols is constrained only by an upper limit. The dotted lines indicate the pairs of binaries. The numbers assigned to the symbols are the ID numbers of the stars used in Wood et al. (2021), and the corresponding star names are listed in Figure 20. Here $y \propto x^{0.77}$ is a fitted line derived by Wood et al. (2021).

7. Summary

We summarize the conclusions about the differences and similarities in the stellar chromospheres, coronae, and winds among the Sun and M dwarfs. These findings are obtained by analyzing the results of a parameter survey based on the 1D MHD numerical simulations.

(i) Regardless of the Sun or M dwarfs, the nonlinear propagation of Alfvén waves is responsible for driving the stellar spicule, heating the stellar atmosphere, and accelerating the stellar wind (Section 3).

The following items (ii)–(iv) are the arguments for a given transmitted Alfvén wave energy flux into the corona ($F_{\text{A,co}}$).

(ii) The M dwarf’s corona tends to be cooler and denser than the solar corona (Sections 4.2, 4.3). The shorter spatial scale of the coronal magnetic field of the M dwarf results in this tendency.

(iii) The M dwarf’s stellar winds are relatively faster than the solar wind (Section 4.1). The lower plasma $\beta$ of the M dwarf’s stellar wind is a suitable environment for Alfvén wave amplification, which leads to the generation of a stronger slow shock and further acceleration of the stellar wind (Section 5.1).

(iv) The mass-loss rates of M dwarf stellar winds are much smaller than those of the solar wind because of (1) the much smaller stellar surface area, (2) the constant energy conversion efficiency from the Alfvén wave energy flux in the corona to the kinetic energy flux of the stellar wind ($\alpha_{\text{wind/A}}$ in Section 5.2), and (3) the relatively faster stellar wind.

(v) The transmissivity of the Alfvén wave energy flux from the photosphere to the corona ($\alpha_{\text{co/ph}}$) is determined mainly by the Alfvén travel time between them ($T_{\text{A,co}}$ in Section 5.4). Here $\alpha_{\text{co/ph}}$ is negatively correlated with $T_{\text{A,co}}$ when $T_{\text{A,co}} \gtrsim \nu_{\text{ac}}^{-1}$ ($\nu_{\text{ac}}$ is the acoustic cutoff frequency), while the correlation is reversed when $T_{\text{A,co}} \lesssim \nu_{\text{ac}}^{-1}$ (Figure 16). The stronger chromospheric
magnetic field ($\mathbf{B}$) reduces $T_{\text{A,co}}$ (Equation (82)) and affects the transmissivity of the Alfvén waves.

(vi) We developed the semiempirical formulae to estimate $v_{\text{wind}}$, $T_{\text{co}}$, $M$, $p_{\text{co}}$, and $\rho_{\text{co}}$ from a given combination of $L_{A,\text{ph}}$ and $T_{\text{eff}}$ (Figure 18). Comparison of them to the observations suggests that the observed $M$ of EV Lac or YZ CMi could require a nearly 50 or 5000 times larger Alfvén wave energy flux than expected from the scenario of Alfvén wave generation by surface convective motion (Section 6.1).

8. Future Works

The proposed semiempirical formulae would contribute to studies of the dynamics of a planetary atmosphere or magnetosphere (Khodachenko et al. 2007; Lammer et al. 2007; Linsky 2019), the evolution of stellar rotation (Pantolmos & Matt 2017; Shoda et al. 2020), or the atmosphere structures of main-sequence stars (Wood et al. 2021). It would be helpful as an initial guess to constrain the physical quantities of the stellar atmosphere and wind in more unified way.

The present numerical scheme is based on a 1D approximation (axisymmetry assumption), by which the global magnetic field configuration and its interaction with stellar wind flow cannot be addressed. It is likely that the other simplifications in this study limit the applicability of our results. The partial ionization and collisionless effects are involved with the dissipation of Alfvén waves in the stellar chromosphere and interplanetary space, respectively, but not considered in this study. Alfvén wave turbulence is also an important process for heating the stellar atmosphere and wind.

As for our interpretation of the simulation results, it should be kept in mind that there are many heuristic relations adopted in our semiempirical formulae; e.g., the simplified energy conservation law ($L_{\text{A,co}} \approx L_{\text{kin,wind}} - L_{\text{rad,co}} - L_{\text{q,co}}$; Equation (42)), the constant energy conversion efficiency ($\alpha_{\text{wind}}/A = L_{\text{kin,wind}}/L_{\text{A,co}} = \text{const.}$), and the relation between the maximum nonlinearity of the Alfvén waves in the stellar wind and the plasma $\beta$ of the stellar wind (Equation (52)). The efforts for validation or generalization of these assumptions will lead to a more unified understanding of the stellar atmosphere and wind physics from the Sun to M dwarfs.

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Appendix A
Effect of Varying the Filling Factor of the Open Flux Tube

While we discuss the simulation results with the fixed $f_{\text{ph}}$ at 1/1600, where $f_{\text{ph}}$ is the filling factor of the open magnetic flux tube, there are many observational and theoretical studies that point out the crucial role of $f_{\text{ph}}$ in determining the solar wind velocity ($v_{\text{wind}}$), especially the positive correlation between $f_{\text{ph}}$ and $v_{\text{wind}}$ (Wang & Sheeley 1990; Arge & Pizzo 2000; Suzuki & Inutsuka 2006; Cranmer et al. 2007; Tokumaru et al. 2017).

Moreover, the $f_{\text{ph}}$ of stars could generally vary within a wide range, depending on the rotation period, convective turnover time, Rossby number, amplitude of the magnetic activity cycle, and so on. Even in the case of the Sun, $f_{\text{ph}}$ can vary by an order of magnitude (Wang et al. 2000; Cranmer & Saar 2011). Although the filling factor of the magnetically active region on the stellar surface has been investigated by spectroscopic observations, it is still challenging to distinguish the filling factor associated with open flux tubes from the total filling factor (See et al. 2019).

To check the applicability of our semiempirical method, we performed the additional parameter survey about the stellar atmosphere and wind of the M3.5 dwarf by varying $f_{\text{ph}}$ from 1/32 to 1/400, 1/1600, 1/6400, 1/32,000, 1/64,000, and 1/80,000. The other parameters, such as the chromospheric magnetic field strength ($\mathbf{B}$) and Alfvén wave amplitude of the photosphere ($v_{\text{ph}}$), are set to $\mathbf{B} = B_{\text{ph}} e^{-5}$ and $v_{\text{ph}} = v_{\text{conv}}$, respectively.

Figure 24(a) shows the stellar wind velocity at $r = 100 a$ ($v_{\text{wind}}$) as a function of $f_{\text{ph}}$. They are positively correlated with each other, as found in the previous studies. Our semiempirical method is, on the other hand, not useful to understand the physical mechanism leading to this relation. This is because the coefficients and power-law indices appearing in the semiempirical formulae probably depend on $f_{\text{ph}}$. One of the remarkable examples is the dependence of $\alpha_{\text{wind}}/A$ on $f_{\text{ph}}$, where $\alpha_{\text{wind}}/A$ is the energy conversion efficiency from the transmitted Alfvén wave energy in the corona to the kinetic energy of the stellar wind; i.e., $\alpha_{\text{wind}}/A = L_{\text{kin,wind}}/L_{A,co}$. The filled circles in Figure 24(b) show $\alpha_{\text{wind}}/A$ as a function of $f_{\text{ph}}$. As shown in this panel, $\alpha_{\text{wind}}/A$ steeply drops from $\sim 0.4$ to $\sim 10^{-2}$ with decreasing $f_{\text{ph}}$. This phenomenon is probably related to the rapid increase in the radiative energy loss with decreasing $f_{\text{ph}}$. In Figure 24(b), the squares show the energy conversion efficiency from the transmitted Alfvén wave energy in the corona to the radiative energy loss; i.e., $\alpha_{\text{rad}}/A = -L_{\text{rad,co}}/L_{A,co}$. Because the flux tube with the smaller $f_{\text{ph}}$ is characterized by a larger expansion ratio of $A/A_{\text{tr}}$ ($A_{\text{tr}} \leq A \leq A_{\text{co}}$) and smaller $l_{B,\text{tr}}$ (Equation (86)), the plasma pressure at the transition region (Equation (90)) and coronal mass density (Equation (92)) increases for a given transmitted Alfvén wave energy flux into the corona, so that the radiative energy loss is enhanced. As a result, the approximated energy conservation law (Equation (42)) becomes invalid in the case of a smaller $f_{\text{ph}}$, which requires further improvement of our semiempirical method to more comprehensively understand the mechanisms of heating stellar chromospheres and coronae and driving the stellar winds.

We note that the stellar atmosphere and wind structures would similarly depend on $H_{\text{p}}$, the parameter of the typical closed-loop height. Several radio observations suggest that the closed-loop system of the M dwarf extends to a few times the
stellar radii (Benz et al. 1995; Davis et al. 2021), while $H_f = 0.1r_e$ in this study. If we adopt $H_f = a \times r_e$ straightforwardly based on these observations, the stellar wind velocity will increase, and the mass-loss rate will decrease. This is because a larger $H_f$ represents the stronger magnetic field in the distance and leads to a shorter spatial scale of the expanding flux tube ($l_{B,\text{co}}$ in Equation (46)). The subsequent phenomena caused by a shorter $l_{B,\text{co}}$ are depicted in Figure 19. The stellar wind velocity ($v_{\text{wind}}$) is further accelerated by more largely amplified Alfvén waves. Because $H_f$ does not so much affect the wind’s kinetic energy luminosity ($l_{\text{kin},\text{wind}}$), a faster $v_{\text{wind}}$ results in a smaller mass-loss rate.

Appendix B
Main-sequence Star’s Radius and Mass

To obtain Figure 18, we assume the $T_{\text{eff}}$-$r_e$ and $r_e$-$M_*$ relations of main-sequence stars by referring to Boyajian et al. (2012) and Rabus et al. (2019). The adopted relation of $T_{\text{eff}}$-$r_e$ is as follows:

$$
\frac{r_e}{r_\odot} = \max \left[ -0.367 + 1.041 \left( \frac{T_{\text{eff}}}{T_{\text{eff},\odot}} \right), -8.133 + 29.389 \left( \frac{T_{\text{eff}}}{T_{\text{eff},\odot}} \right) - 32.8468 \left( \frac{T_{\text{eff}}}{T_{\text{eff},\odot}} \right)^2 + 12.4474 \left( \frac{T_{\text{eff}}}{T_{\text{eff},\odot}} \right)^3, \left( \frac{T_{\text{eff}}}{T_{\text{eff},\odot}} \right)^{1.866} \right] H(\frac{T_{\text{eff}}}{T_{\text{eff},\odot}} - 4000),
$$

(B1)

where $H(x)$ is the Heaviside step function; i.e., $H(x) = 0$ for $x < 0$ and $H(x) = 1$ for $x \geq 0$. The obtained $r_e(T_{\text{eff}})$ is smoothed with a boxcar average of 500 K.

The relation of $r_e$-$M_*$ is defined as follows:

$$
\frac{M_*}{M_\odot} = \max \left[ x_M, \left( \frac{r_e}{r_\odot} \right)^{0.1091} H(\frac{r_e}{r_\odot} - 0.5r_\odot) \right],
$$

(B2)

where $x_M$ is the solution of the following equation for a given $r_e/r_\odot$:

$$
\frac{r_e}{r_\odot} = 0.013 + 1.238 x_M - 1.13 x_M^2 + 1.21 x_M^3.
$$

(B3)

We used the ÆSOPUS opacity table published by Marigo & Aringer (2009) to estimate physical quantities such as mass density and plasma pressure on the photosphere for stars with $T_{\text{eff}} > 4000$ K.

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