Wilson Line Integrals in the Unparticle Action

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We consider the unparticle action that is made gauge invariant by inclusion of an open Wilson line factor. In deriving vertexes from such an action it has been customary to use a form of differentiating the Wilson line originally proposed by Mandelstam. Using a simple example, we show that the Mandelstam derivative is mathematically inconsistent. We show that there are two ways to define differentiation of the Wilson line. The mathematically consistent method is to differentiate the explicit dependence of the line on the endpoint. The other method is a functional derivative and corresponds in a limiting case to the Mandelstam derivative. We also show that the only path that can be used in the Wilson line integral that leaves the unparticle action both Poincare and scale invariant is the straight line.

1. INTRODUCTION

The scalar bosonic unparticle action introduced by Georgi [1], [2] has been modified by Terning et al [3], [4] to include gauge field interactions. The resulting bosonic unparticle action is, apart from constants,

\[ I = \int d^4x d^4y \Phi_u^\dagger(x) K(x, y) W_\Lambda(x, y) \Phi_u(y) \]  

where the \( K \) is determined as the inverse of the propagator,

\[ K(x, y) = \left[ -\left( \partial_\mu \partial^\mu \right)_x - i\varepsilon \right]^{2-d_u} \delta^4(x-y) \]  

and the \( W_\Lambda \) is a path ordered Wilson line, introduced to make the action gauge invariant:

\[ W_\Lambda(x, y) = P \exp \left[ -ig \int_y^x A_\alpha(\zeta) d\zeta^\alpha \right] \]  

The symbol \( \Lambda \) indicates the particular path chosen between \( x \) and \( y \). The symbol \( P \) indicates the path ordering, from \( y \) on the right to \( x \) on the left. Choosing a path amounts to finding a vector valued function of the position vectors \( x, y \) and of a parameter \( \lambda, \zeta^\alpha(x, y, \lambda) \), such that

\[ \zeta^\alpha(x, y, 0) = y^\alpha \]  
\[ \zeta^\alpha(x, y, 1) = x^\alpha \]  

The integral in Eq. (1) cannot be evaluated without an explicit choice of the function \( \zeta \). In calculating Eq. (1) it can be necessary to find the derivatives of the Wilson line. In much of the current literature [3], [4], [5], [6], the Mandelstam condition [7] is used, which defines the derivative as

\[ \frac{\partial}{\partial x^\nu} W_\zeta(x, y) = -igA_\nu(x) W_\zeta(x, y) \]  

There has been some controversy recently as to the validity of the Mandelstam condition [8], [9], [10]. In the following we will show in Section 2, using a simple example, that the use of the Mandelstam derivative in integrals similar to that in the unparticle action is indeed mathematically inconsistent.

In Section 3 we will give the appropriate form of Wilson line derivative to be used in the unparticle action. It takes into account the effect on the path \( \Lambda \) of the displacement of \( x \) to \( \delta x \). In Section 4 we show that the Mandelstam derivative is a special case of a functional derivative of the path function \( \zeta \). It could conceivably be of use in some context, but not in evaluating the unparticle action.

In Section 5 we show that the requirement that the unparticle action be both Lorentz and scale invariant requires that the path be the straight line between the points \( x \) and \( y \).
2. THE MANDELSTAM DERIVATIVE

We consider in this section a simplified form of Eq. (1) that can be evaluated either by ordinary integration, or by partial integration. The partial integration involves differentiating a Wilson line which we may do using either ordinary derivatives or Mandelstam derivatives. We will show that using the ordinary derivative in the partial integration gives the same result as does ordinary integration, but using the Mandelstam derivative does not.

We take as the gauge field $A$ the electromagnetic vector potential in the presence of a uniform magnetic field $B$:

$$\vec{A} = -\frac{1}{2} \vec{r} \times \vec{B}$$  \hspace{1cm} (6)

The field is abelian, so no path ordering is required. For the path we will take the straight line between $x$ and $y$, parametrised as

$$\vec{r}(\lambda) = \lambda \vec{x} + (1 - \lambda) \vec{y}$$ \hspace{1cm} (7)

Then

$$d\vec{r} = (\vec{x} - \vec{y}) \, d\lambda$$ \hspace{1cm} (8)

and

$$\int_{y}^{x} \vec{A} \cdot d\vec{r} = \int_{0}^{1} - \frac{1}{2} \frac{\lambda \vec{x} + (1 - \lambda) \vec{y}}{2} \times \vec{B} \cdot (\vec{x} - \vec{y}) \, d\lambda$$

$$= -\frac{1}{2} \vec{x} \times \vec{y} \cdot \vec{B}$$  \hspace{1cm} (9)

Then the Wilson line is

$$W(x, y) = \exp \left[ i g \frac{1}{2} \vec{x} \times \vec{y} \cdot \vec{B} \right]$$  \hspace{1cm} (10)

One way to take the derivative of this is to differentiate the explicit $x$ dependence, then

$$\vec{\nabla}_x W(x, y) = \vec{\nabla}_x \exp \left[ i g \frac{1}{2} \vec{x} \times \vec{y} \cdot \vec{B} \right]$$

$$= ig \frac{1}{2} \vec{x} \times \vec{B} \exp \left[ i g \frac{1}{2} \vec{x} \times \vec{y} \cdot \vec{B} \right]$$  \hspace{1cm} (11)

It is also possible to use the Mandelstam derivative $\vec{D}_x^M$, defined as

$$\vec{D}_x^M W(x, y) = -i g \vec{A}(\vec{x}) \exp \left[ -i g \int_{y}^{x} \vec{A}(\vec{r}) \cdot d\vec{r} \right]$$ \hspace{1cm} (12)

Then

$$\vec{D}_x^M W(x, y) = ig \frac{1}{2} \vec{x} \times \vec{B} \exp \left[ i g \frac{1}{2} \vec{x} \times \vec{y} \cdot \vec{B} \right]$$  \hspace{1cm} (13)

We take $B$ to be normal to a plane, and consider the integral over the plane,

$$I(a, b) = \int d^2x d^2y \Psi_a(x) W(x, y) \Phi_b(y)$$  \hspace{1cm} (14)
Where

\[
\Psi_a(x) = \nabla_x^2 \exp \left[-\frac{a x^2}{2}\right]
\]
\[
\Phi_b(y) = \exp \left[-\frac{b y^2}{2}\right]
\]

(15)

We evaluate this integral three ways:

1. A simple straightforward integration, without integrating by parts. We call the result \(I(a, b)\).
2. Integrating by parts, so that the derivatives act on \(W(a, b)\), but differentiating the explicit dependence of \(W(a, b)\) on \(x\), using the operator \(\nabla_x^2\). We call this integral \(I_{\nabla}(a, b)\).
3. Integrating by parts, but using the Mandelstam operator \(\vec{D}_{x}^{M}\) when the derivative acts on \(W(a, b)\). We call the result \(I_{\vec{D}}(a, b)\).

The result is

\[
I(a, b) = I_{\nabla}(a, b) = -2 \frac{(\pi gB)^2}{a \left(b + \frac{(gB)^2}{4a}\right)^2}
\]

(16)

but

\[
I_{\vec{D}}(a, b) = -2 \frac{(\pi gB)^2 b}{a^2 \left(b + \frac{(gB)^2}{4a}\right)^2}
\]

(17)

In the general case, when \(a \neq b\), we have

\[
I_{\vec{D}}(a, b) \neq I(a, b)
\]

(18)

It should be obvious that \(I(a, b)\) is the correct value of the integral. Integrating by parts should not change the value of an integral. We have to conclude that the Mandelstam derivative is not the appropriate way to differentiate the Wilson line in a non-local action integral.

3. EXPLICIT DIFFERENTIATION

One way of differentiating the Wilson line is to differentiate with respect to one of the two endpoints \(x\) or \(y\), keeping the other endpoint fixed, and also keeping the function \(\zeta\) fixed. Writing

\[
W_{\Lambda}(x, y) = P \exp \left[-ig \int_{0}^{1} A_\alpha(\zeta) \frac{d\zeta_\alpha}{d\lambda} d\lambda \right]
\]

(19)

Defining

\[
W_{\Lambda}(x, \zeta(\lambda)) = P \exp \left[-ig \int_{\lambda}^{1} A_\alpha(\zeta) \frac{d\zeta_\alpha}{d\lambda'} d\lambda' \right]
\]

(20)

\[
W_{\Lambda}(\zeta(\lambda), y) = P \exp \left[-ig \int_{0}^{\lambda} A_\alpha(\zeta) \frac{d\zeta_\alpha}{d\lambda'} d\lambda' \right]
\]

(21)

We have
\[
\frac{\partial W_{\Lambda}(x, y)}{\partial x^\mu} = -ig \int_0^1 W_{\Lambda}(x, \zeta(\lambda)) \frac{\partial}{\partial x^\mu} \left( A_\alpha(\zeta) \frac{d\zeta^\alpha}{d\lambda} \right) W_{\Lambda}(\zeta(\lambda), y) d\lambda
\]

(22)

Using
\[
\frac{\partial}{\partial x^\mu} \left( A_\alpha(\zeta) \frac{d\zeta^\alpha}{d\lambda} \right) = \frac{\partial A_\alpha(\zeta)}{\partial x^\mu} \frac{d\zeta^\alpha}{d\lambda} + A_\alpha(\zeta) \frac{d}{d\lambda} \frac{d\zeta^\alpha}{d\lambda}
\]

(23)

we get, integrating by parts,
\[
\int_0^1 W_{\Lambda}(x, \zeta(\lambda)) A_\alpha(\zeta) \frac{d\zeta^\alpha}{d\lambda} W_{\Lambda}(\zeta(\lambda), y) d\lambda = W_{\Lambda}(x, \zeta(\lambda)) A_\alpha(\zeta) \frac{d\zeta^\alpha}{d\lambda} W_{\Lambda}(\zeta(\lambda), y) |^1_0
\]

\[
- \int_0^1 W_{\Lambda}(x, \zeta(\lambda)) \left\{ \frac{\partial A_\alpha(\zeta)}{\partial \zeta^\beta} + ig [A_\beta(\zeta), A_\alpha(\zeta)] \right\} \frac{d\zeta^\beta}{d\lambda} \frac{d\zeta^\alpha}{d\lambda} W_{\Lambda}(\zeta(\lambda), y) d\lambda
\]

(24)

In differentiating by x we are holding y fixed, so Eqs. (4) imply that
\[
\frac{\partial \zeta^\gamma(x, y, 1)}{\partial x^\mu} = \frac{\partial x^\gamma}{\partial x^\mu} = \delta^\gamma_\mu
\]

\[
\frac{\partial \zeta^\gamma(x, y, 0)}{\partial x^\mu} = \frac{\partial y^\gamma}{\partial x^\mu} = 0
\]

(25)

Also using
\[
W_{\Lambda}(x, \zeta(1)) = W_{\Lambda}(x, x) = 1
\]
\[
W_{\Lambda}(\zeta(1), y) = W_{\Lambda}(x, y)
\]

(26)

and combining Eqs. (22), (24) and (24), we get
\[
\frac{\partial}{\partial x^\mu} W_{\Lambda}(x, y) = -ig A_\mu(x) W_{\Lambda}(x, y) + ig \int_0^1 W_{\Lambda}(x, \zeta(\lambda)) F_{\beta\alpha}(\zeta(\lambda)) \frac{d\zeta^\alpha}{d\lambda} \frac{d\zeta^\beta}{d\lambda} W_{\Lambda}(\zeta(\lambda), y) d\lambda
\]

(27)

where \( F_{\beta\alpha} \) is the field strength:
\[
F_{\beta\alpha} = \partial_\beta A_\alpha - \partial_\alpha A_\beta + ig [A_\beta, A_\alpha]
\]

(28)

For a continuous non-null path, the second term on the right hand side of Eq. (27) can only be eliminated if the field strength is zero.

4. THE FUNCTIONAL DERIVATIVE

Mandelstam however takes for the derivative only the first term on the right hand side of Eq. (27):
\[
\frac{\partial}{\partial x^\nu} W_{\Lambda}(x, y) = -ig A_\nu(x) W_{\Lambda}(x, y)
\]

(29)

The argument for this result is essentially the idea that the derivative can be taken by defining it as
\[
\frac{\partial}{\partial x^\nu} W_{\Lambda}(x, y) = \lim_{\Delta x \to 0} \frac{W_{\Lambda}(x + \Delta x, y) - W_{\Lambda}(x, y)}{\Delta x^\nu}
\]

(30)
where $\Lambda'$ is a path that is identical with $\Lambda$ up to the point $x$, where it goes off to $x + \Delta x$. This is a change in the form of the path function $\zeta$. We will show that it can be expressed in terms of a functional derivative.

Consider the vector tangent to the path defined as

$$\eta^\alpha (\lambda) = \frac{d\zeta^\alpha (\lambda)}{d\lambda}$$

(31)

We can define a functional derivative with respect to this vector:

$$\frac{\delta\eta^\alpha (\lambda)}{\delta\eta^\beta (\lambda')} = \delta^\alpha_\beta \delta (\lambda - \lambda')$$

(32)

now the path function can be written as

$$\zeta^\alpha (\lambda) = y^\alpha + \int_0^\lambda \eta^\alpha (\lambda') d\lambda'$$

$$= x^\alpha - \int_\lambda^1 \eta^\alpha (\lambda') d\lambda'$$

(33)

which leads to

$$\frac{\delta\zeta^\alpha (\lambda)}{\delta\eta^\beta (\lambda_0)} \bigg|_y = +\delta^\alpha_\beta \theta (\lambda - \lambda_0)$$

$$\frac{\delta\zeta^\alpha (\lambda)}{\delta\eta^\beta (\lambda_0)} \bigg|_x = -\delta^\alpha_\beta \theta (\lambda_0 - \lambda)$$

(34)

The functional derivative of the Wilson line, holding $y$ fixed, is

$$\frac{\delta W_\Lambda (x, y)}{\delta\eta^\beta (\lambda)} \bigg|_y = -ig \int_0^1 W_\Lambda (x, \zeta (\lambda')) \frac{\delta}{\delta\eta^\beta (\lambda)} (A_\alpha (\zeta (\lambda')) \eta^\alpha (\lambda')) \bigg|_y W_\Lambda (\zeta (\lambda'), y) d\lambda'$$

(35)

Expanding,

$$\frac{\delta}{\delta\eta^\beta (\lambda)} (A_\alpha (\zeta (\lambda')) \eta^\alpha (\lambda')) \bigg|_y = \frac{\partial A_\alpha (\zeta (\lambda'))}{\partial\zeta^\gamma (\lambda')} \frac{\delta\zeta^\alpha (\lambda)}{\delta\eta^\gamma (\lambda')} \bigg|_y \eta^\alpha (\lambda') + A_\alpha (\zeta (\lambda')) \frac{\delta\eta^\alpha (\lambda')}{\delta\eta^\beta (\lambda)}$$

$$= \frac{\partial A_\alpha (\zeta (\lambda'))}{\partial\zeta^\gamma (\lambda')} \delta^\beta_\gamma (\lambda' - \lambda) \eta^\alpha (\lambda') + A_\alpha (\zeta (\lambda')) \delta^\beta_\gamma (\lambda - \lambda')$$

(36)

Then Eq. (35) becomes

$$\frac{\delta W_\Lambda (x, y)}{\delta\eta^\beta (\lambda)} \bigg|_y = -ig \int_\lambda^1 W_\Lambda (x, \zeta (\lambda')) \frac{\partial A_\alpha (\zeta (\lambda'))}{\partial\zeta^\gamma (\lambda')} \eta^\alpha (\lambda') W_\Lambda (\zeta (\lambda'), y) d\lambda'$$

$$-ig W_\Lambda (x, \zeta (\lambda)) A_\beta (\zeta (\lambda')) W_\Lambda (\zeta (\lambda'), y)$$

(37)

If we go to the limit, $\lambda \to 1$, the first term on the right hand side of Eq. (37) vanishes, and we are left with

$$\frac{\delta W_\Lambda (x, y)}{\delta\eta^\beta (1)} \bigg|_y = -ig A_\beta (x) W_\Lambda (x, y)$$

(38)

and similarly,
\[
\frac{\delta W_\Lambda (x, y)}{\delta \eta^\alpha (0)} \bigg|_x = +igW_\Lambda (x, y) A_\beta (y)
\] (39)

These are very similar to the Mandelstam conditions. It is tempting therefore to define a kind of end point derivative as

\[
\begin{align*}
\frac{DW_\Lambda (x, y)}{Dx^\mu} &= \frac{\delta W_\Lambda (x, y)}{\delta \eta^\mu (1)} \\
\frac{DW_\Lambda (x, y)}{Dy^\mu} &= \frac{\delta W_\Lambda (x, y)}{\delta \eta^\mu (0)}
\end{align*}
\] (40)

The problem arises in identifying this \( D/Dx^\mu \) with the \( \partial/\partial x^\mu \) that appears in Eq. (5). The operation \( \partial/\partial x^\mu \) is simple differentiation of the explicit dependence on \( x \). It is hard to see how it could involve a change in the function that defines that explicit dependence. The operator \( D/Dx^\mu \) is the limit of a functional derivative. It is defined only on functionals of the path defining function \( \zeta^\alpha (\lambda) \). It could not be applied to the unparticle field \( \Phi_u (x) \).

**5. SCALE INvariance and the STRAIGHT LINE**

With no gauge fields, the Bosonic unparticle action is, apart from an overall constant:

\[
I_u = \int d^4x d^4y \Phi_u^\dagger (x) K (x-y) \Phi_u (y)
\] (41)

where

\[
K (x-y) = \int \frac{d^4k}{(2\pi)^4} \left( k^2 - i\epsilon \right)^{2-d_u} e^{-ik \cdot (x-y)}
\] (42)

\( K \) is a Poincare invariant function, and with

\[
\begin{align*}
U (\Lambda, b) \Phi_u (x) U^\dagger (\Lambda, b) &= \Phi_u (\Lambda x + b)
\end{align*}
\] (43)

the action is invariant under the Poincare group.

\( I_u \) is also scale invariant. With the scale transformation defined as

\[
U (a) \Phi_u (x) U^\dagger (a) = a^{d_u} \Phi_u (ax)
\] (44)

We get

\[
U (a) I_u U^\dagger (a) = a^{2d_u} \int d^4x d^4y \Phi_u^\dagger (ax) K (x-y) \Phi_u (ay)
\] (45)

Letting

\[
\begin{align*}
x \to x'/a \\
y \to y'/a
\end{align*}
\] (46)

and using

\[
K \left( \frac{z}{a} \right) = a^{8-2d_u} K (z)
\] (47)
we get

\[ U(a) I_u U^\dagger(a) = \int d^4 x' d^4 y' \Phi_u^\dagger(x') K(x' - y') \Phi_u(y') = I_u \quad (48) \]

To make the action gauge invariant we can include a Wilson line as in Eq. (1), involving an integral over a path \( \Lambda \). We will show here that the requirement that \( I_u \) be both Poincare and scale invariant implies that the path \( \Lambda \) can only be the straight line connecting \( x \) and \( y \).

The only two vectors in the integral are \( x^\mu \) and \( y^\mu \). Lorentz invariance then implies that \( \zeta^\mu(x, y, \lambda) \) must be of the form

\[ \zeta^\mu(x, y, \lambda) = f(u, v, w, \lambda) x^\mu + g(u, v, w, \lambda) y^\mu \quad (49) \]

where

\[
\begin{align*}
  u &= x \cdot x \\
  v &= x \cdot y \\
  w &= y \cdot y
\end{align*}
\]

and

\[
\begin{align*}
  f(u, v, w, 0) &= 0, & f(u, v, w, 1) &= 1 \\
  g(u, v, w, 0) &= 1, & g(u, v, w, 1) &= 0
\end{align*}
\]

We take \( A_\mu(z) \) to be a zero mass field, which can be shown to scale as

\[ U(a) A_\mu(z) U^\dagger(a) = a A_\mu(az) \quad (52) \]

with dimension 1. The line integral now scales as

\[ U(a) \int_0^1 d\lambda A_\mu(\zeta(x, y, \lambda)) \frac{d\zeta^\mu(x, y, \lambda)}{d\lambda} U^\dagger(a) = a \int_0^1 d\lambda A_\mu(a \zeta(x, y, \lambda)) \frac{d\zeta^\mu(x, y, \lambda)}{d\lambda} \quad (53) \]

Making the coordinate change of Eq. (46), we see that scale invariance of the line integral requires that

\[ a \zeta^\mu\left(\frac{x'}{a}, \frac{y'}{a}, \lambda\right) = \zeta^\mu(x', y', \lambda) \quad (54) \]

The form given for \( \zeta^\mu(x, y, \lambda) \) in Eq. (49) then implies that

\[
\begin{align*}
  f(a^{-2} u', a^{-2} v', a^{-2} w', \lambda) &= f(u', v', w', \lambda) \\
  g(a^{-2} u', a^{-2} v', a^{-2} w', \lambda) &= g(u', v', w', \lambda)
\end{align*}
\]

Differentiating with respect to \( a^{-2} \) at \( a = 1 \) then gives

\[
\begin{align*}
  \left(\frac{u}{\partial u} + \frac{v}{\partial v} + \frac{w}{\partial w}\right) f(u, v, w, \lambda) &= 0 \\
  \left(\frac{u}{\partial u} + \frac{v}{\partial v} + \frac{w}{\partial w}\right) g(u, v, w, \lambda) &= 0
\end{align*}
\]

We now look at the consequences of translational invariance. Under translation through a vector \( b^\mu \), the line integral becomes
\[ U(1, b) \int_0^1 A_\mu(x, y, \lambda) \frac{d\zeta(\mu)}{d\lambda} U^\dagger(1, b) d\lambda = \int_0^1 A_\mu(x, y, \lambda + b) \frac{d\zeta(\mu)}{d\lambda} d\lambda \] (57)

The ungauged action of Eq. (41) under such a transformation is restored to its original form by the change of variables:

\[
\begin{align*}
x^\mu &\to x'^\mu - b^\mu \\
y^\mu &\to y'^\mu - b^\mu
\end{align*}
\] (58)

The line integral then goes into its original form if and only if

\[
\begin{align*}
f \left( u' - 2x' \cdot b + b^2, v' - x' \cdot b - y' \cdot b + b^2, w' - 2y' \cdot b + b^2, \lambda \right) (x'^\mu - b^\mu) + \\
g \left( u' - 2x' \cdot b + b^2, v' - x' \cdot b - y' \cdot b + b^2, w' - 2y' \cdot b + b^2, \lambda \right) (y'^\mu - b^\mu) \\
= f \left( u', v', w', \lambda \right) x'^\mu + g \left( u', v', w', \lambda \right) y'^\mu - b^\mu
\end{align*}
\] (59)

Differentiating Eq. (59) with respect to \( b^\alpha \) at \( b = 0 \) yields, after dropping the primes,

\[
\begin{align*}
x^\mu \left( 2x_\alpha \partial_u + 2y_\alpha \partial_w + \left( x_\alpha + y_\alpha \right) \partial_v \right) f + y^\mu \left( 2x_\alpha \partial_u + 2y_\alpha \partial_w + \left( x_\alpha + y_\alpha \right) \partial_v \right) g \\
+ \left( f + g \right) \delta_\alpha^3 - \delta_\alpha^3 = 0
\end{align*}
\] (60)

The tensors \( x^\mu x_\alpha, y^\mu y_\alpha, x^\mu x_\alpha, y^\mu y_\alpha, \delta_\alpha^3 \) are linearly independent. In Eq. (60) their coefficients must each add up to zero. This yields

\[
\begin{align*}
(2\partial_u + \partial_v) f &= 0 \\
(2\partial_w + \partial_v) f &= 0 \\
(2\partial_u + \partial_v) g &= 0 \\
(2\partial_w + \partial_v) g &= 0
\end{align*}
\] (61)

and

\[
f + g = 1
\] (62)

Eqs. (61a and b) imply that

\[
\partial_u f = \partial_w f
\] (63)

Then using this and Eq. (61a) in Eq. (56a) we have

\[
(u - 2v + w) \partial_u f = (x - y)^2 \partial_u f = 0
\] (64)

With \( x \neq y \) we get \( f \) independent of \( u \), then Eqs. (61) and (63) imply that \( f \) is also independent of \( v \) and \( w \), therefore it depends only on \( \lambda \). Similarly, we find that \( g \) also depends only on \( \lambda \). Defining a new line parameter,

\[
\lambda' = f(\lambda)
\] (65)

we see, using Eq. (62) and Eq. (49), that

\[
\zeta^\mu(x, y, \lambda) = \lambda' x^\mu + (1 - \lambda') y^\mu
\] (66)

the equation for a straight line connecting \( x \) and \( y \).
6. CONCLUSION

We have shown that the Mandelstam derivative of the open Wilson line leads to mathematically inconsistent results in the unparticle action. It is not the same as the ordinary derivative, but it is a special case of a functional derivative of the Wilson line. We give explicit expressions for the ordinary derivative of the Wilson line and also for the general functional derivative. Lastly, we have shown that the combination of Poincare and scale invariance for the gauged unparticle action require that the path in the Wilson line integral be a straight line.

The interaction of an unparticle and a gauge field with a straight line Wilson integral has been investigated in Ref. [9]. It leads to rather complicated expressions for the unparticle-gauge field vertexes. In Ref. [11] however it was shown that it is possible to construct a gauge invariant unparticle action without the use of the Wilson line integral.

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[1] H. Georgi, Unparticle Physics, arXiv:hep-ph/0703260.
[2] H. Georgi, Another Odd Thing About Unparticle Physics, arXiv:0704.2457 [hep-ph].
[3] G. Cacciapaglia, G. Marandella and J. Terning, Colored Unparticles, arXiv:0708.0005 [hep-ph].
[4] J. Terning, Gauging Nonlocal Lagrangians, Phys. Rev. D 44 (1991) 887.
[5] Y. Liao Effects of Unparticles on Running of Gauge Couplings arXiv:0708.3327 [hep-ph], Eur. Phys. J. C. (in press)
[6] Y. Liao Some Issues in a Gauge Model of Unparticles arXiv:0804.4033 [hep-ph]
[7] S. Mandelstam, Quantum Electrodynamics without Potentials, Ann. Phys. 19 (1962) 1.
[8] J. Galloway, D. Martin and D. Stancato, Comments on “Gauge Fields and Unparticles”, arXiv:0802.0313 [hep-th]
[9] A. L. Licht, Gauge Fields and Unparticles, arXiv:0801.0892 [hep-th]
[10] A. L. Licht The Mandelstam-Terning Line Integral in Unparticle Physics arXiv:0802.4310 [hep-th]
[11] A. L. Licht, Operator Coupling of Gauge Fields and Unparticles, arXiv:0801.1148 [hep-th]