Analysis of a M/M/1 Queueing System with Two-Phase, N-Policy, Server Failure and Second Optional Batch Service with Customers impatient behaviour

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Abstract: In this paper an M/M/1 queueing system with server vacation, Startup, breakdowns and second optional service with impatient customer behavior is considered. All the customers will be given unique type of individual service in the first phase and then second phase service will be provided on option of the customer. We obtain some important system characteristics, such as the number of customers in the system, the probability that the server is idle, busy and broken down states, the expected waiting time in the system. Sensitivity analysis is also conducted with various parameters on system’s performance measures.

Keywords: Two-phase, Vacation, N-Policy, Optional service and Server Breakdown.

1. Introduction

In the Present scenario of globalization the queuing models have gained a lot of significance. Some of the areas where queueing models have valuable applications are traffic flow (vehicles, communications, people), Scheduling (patients in hospitals, jobs n machines, programs on a computer), and facility design (banks, post offices, food courts). The current work deal with the analysis of a M/M/1 Queueing System with two phases of service, N-Policy, Server Failure, Customers’ impatient behavior where customers may opt a second service in addition to the first essential service.

Vacation queueing models have attracted great attention of researchers and became an active research area. Such models have got wide applications in computer, communication, production etc. Miller [11] was the first to study an M/G/1 queueing system where the server is unavailable called as vacation for some random length of time. To optimize the length of the vacation period that can minimize the cost by deciding proper rule for switching the server on and off, various types of control policies like N-Policy, (M,N)-policy, T-policy, (N,T)-policy, min (N,T)-policy, D-policy, (p,T)-policy, Q-policy are generated among which more research is done on N-Policy when compared with other control policies. Yadin and Naor [16] were first to study the technique of N-policy and have obtained the optimal value of the queue size at which to start on a single server.
Krishna and Lee [9] have first studied two-phase queueing system by considering the exhaustive service for the M/M/1 queueing system with and without gating and derived the sojourn time distribution and its mean for an arbitrary customer. Vasanta Kumar et al. [15] studied the optimal control policy of two-phase, N-policy M^N/M/1 and M^N/E_1 queueing systems with server startup and breakdowns. Madan [10] has first studied second optional service in an M/G/1 queueing system in which the distributions of first essential and second optional service times follow General and Exponential respectively. He also mentioned some important applications in day-to-day life conditions. His work is being extended by many other researchers where their work includes inclusion of N-Policy, breakdowns and optimization. Jau-chung-ke et al. [7] derived some important performance measures to optimize a finite capacity M/M/1 queueing model with F-policy where some customers may request a second service in addition to the first essential service.

In many real systems, the server may meet unpredictable breakdowns or any other interruptions. For instance, in manufacturing systems the machine may suddenly breakdown due to machine or job related problems which leads a period of unavailable time until it is repaired. Study of the effect of breakdowns and repairs is important as this affects many system performance measures. Therefore, queueing models with server breakdowns are most popular models which have attracted extensive researcher attention over the past fifty years. Rama Devi et al. [14] have studied the impact of server breakdowns in two phase queueing systems and also derived optimum cost with N-Policy.

These days customers are busy units. Customers willingness to receive service when they enter the system must be treated with great attention. A customer is said to be impatient if he tends to join the queue only when a short wait is expected and tends to remain in the line if his wait has been sufficiently small. Impatience generally takes three forms. Haight [4] was the first who introduced concept of customer impatience in the queueing theory and still people are exploiting this concept in various applications. Haight has analyzed the queue where the individual customer upon arrival measures the queue by its length. Recently, Altman and Yechiali [3] have proved that customers become annoyed only when the server will be on vacation by making a comprehensive study on some queueing models such as M/M/1, M/G/1 and M/M/c queue with server vacations and customer impatience. Adan et al. [5] have worked on queueing models with vacations and synchronized reneging.

In many applications one has to derive transient solutions in Queueing models. Transient analysis is dependent on time, it uses different analysis algorithms, control options with different convergence related issues and different initialization parameters. Many methods are available to solve the transient state of equations. A time-dependent solution for the number in a single-server queueing system with Poisson arrivals and exponential service times is derived in a direct way by P. R. Parthasarathy [12]. Jacob M.J. and Madhusoodanan T.P. [6] examined the behaviour of the infinite capacity M/G/1 model with batch arrivals and server vacations in transient state. Dong-Yuh Yang and Ying-Yi Wu [2] presented Transient Behavior Analysis of a Finite Capacity Queue with Working Breakdowns and Server Vacations. Kalidass et al. [8] have presented the transient behaviour of an M/M/1 multiple vacation queue and the possibilities of catastrophes. Sudhesh and Francis Raj [13] have derived the time dependent system size probabilities for an M/M/1 model with working vacation. Recently, Ammar [1] has studied the transient solution of a M/M/1 queue with multiple vacations and customers impatience.

However, to the best of our knowledge, there is no literature which takes time dependent probabilities for M/M/1 queueing systems with N-Policy, second optional service, server failure and reneging. This motivates us to present the current work. The main objective of this paper is to calculate various system parameters with numerical illustrations by using Runge-Kutta method of order 4.

This paper is systematized in V sections. Section II details the mathematical model and includes the of the analysis carried out in this paper are:

i. To establish the Transient state equations and obtain the Transient state probability distribution of the number of customers in the system in each state.
ii. To derive values for the expected number of customers in the system when the server is in different states
iii. To carry out sensitivity analysis on the System performance measures for various system parameters through numerical experiments.

2. The system and assumptions
We consider the M/M/1 queueing model with N-policy, unreliable server, customer’s impatience and second service on optional basis in Transient state with the following assumptions:

1. Customers are assumed to arrive according to Poisson process with mean arrival rate $\lambda$. Customers will get the service in the order in which they arrive. The customers who arrive during the first phase service are also allowed to join the queue which is in service.

2. Customers are given service such that Individual service times are assumed to be exponentially distributed with mean $1/\mu$ in exhaustive manner and batch service times in the second phase are also exponentially distributed with an average of $1/\beta$.

3. The server provides first essential service to all existing customers in individual manner. Then it proceeds to the second phase and if there is a batch of customers of at least b, it provides batch service. Then returns to first phase and continues the cycle. The probability to opt second phase is p.

4. Whenever the system becomes empty, the server goes on vacation. As soon as the total number of arrivals in the queue reaches or exceeds the pre-determined threshold N, the server is turned on and is temporarily unavailable for the waiting customers. The server startup time follows exponential distribution with mean $1/\theta$. As soon as the server finishes startup, it starts serving the waiting customers.

5. The breakdowns are generated by Poisson process with rates $\xi_1$ for the first phase of service and $\beta_1$ for the second phase of service. When the server fails it is immediately repaired at a repair rate $\xi_2$ in first phase and $\beta_2$ in second phase, where the repair times are exponentially distributed. After repair the server immediately resumes the concerned service.

6. Customers are assumed to be annoyed and renege the system. The probability that $i^{th}$ customer will renege is $(i-1)^*\alpha$

Notations
We use the following notations to represent transient probabilities for the system to be in various modes:

- $p_{i0}(t)$ = p(i customers in the system when server is in Vacation); $i = 1, 2, \ldots, N - 1$
- $p_{i1}(t)$ = p(i customers in the system when server is in start-up mode); $i = N, N + 1, \ldots, S$
- $p_{i2}(t)$ = p(i customers in the system when server is doing first essential service); $i = 0, 1, \ldots, S$
- $p_{i3}(t)$ = p(i customers in the system when server is broken down during first essential service); $i = 0, 1, \ldots, S$
- $p_{ik}(t)$ = p(i customers in the system when server is doing second optional service); $i = 1, 2, \ldots, S - b, k = b, b + 1, \ldots, S$
- $p_{ik}(t)$ = p(i customers in the system when server is broken down during second optional service); $i = 1, 2, \ldots, S - b, k = b, b + 1, \ldots, S$

The Transient state equations governing the system size probabilities at any arbitrary point of time are given by the following Differential Equations:
\[ \frac{dP_{0,0}(t)}{dt} = -\lambda P_{0,0}(t) + \mu (1-p)P_{1,0}(t) + \beta P_{0,k}(t); b \leq K \leq S \]  
\[ \frac{dP_{1,0}^{(2)}(t)}{dt} = -\lambda + (i-1)\alpha P_{1,0}^{(1)}(t) + \lambda P_{i-1,0}(t) + \beta P_{1,k}(t) + \mu (1-p)P_{i+1,0}(t); 1 \leq i \leq N - 1, b \leq K \leq S \]  
\[ \frac{dP_{0,k}(t)}{dt} = -\lambda + (N-1)\alpha + \theta P_{0,0}(t) + \lambda P_{N-1,0}(t) \]  
\[ \frac{dP_{i,0}(t)}{dt} = -\lambda + (i-1)\alpha P_{i,0}^{(2)}(t) + \lambda P_{i-1,0}(t); N + 1 \leq i \leq S - 1 \]  
\[ \frac{dP_{S,0}(t)}{dt} = -\lambda + (S-1)\alpha + \theta P_{S,0}(t) + \lambda P_{S-1,0}(t) \]  
\[ \frac{dP_{i,k}(t)}{dt} = -\lambda + \mu + \xi_1 + (i-1)\alpha P_{i,k}^{(3)}(t) + (1-p)P_{i+1,0}(t) + \lambda P_{i-1,k}(t) + \beta P_{i,k}(t) + \xi P_{1,0}(t); 1 \leq i \leq N - 1; b \leq K \leq S - 1 \]  
\[ \frac{dP_{S,k}(t)}{dt} = -\lambda + \mu + \xi_1 + (i-1)\alpha P_{S,0}^{(2)}(t) + \lambda P_{S-1,0}(t) + \xi P_{S,0}(t); 1 \leq i \leq S - 1 \]  
\[ \frac{dP_{S,k}^{(2)}(t)}{dt} = -\lambda + \mu + \xi_1 + (i-1)\alpha P_{S,0}^{(2)}(t) + \lambda P_{S-1,0}(t) + \xi P_{k,0}(t) \]  
\[ \frac{dP_{i,k}^{(3)}(t)}{dt} = -\lambda + \mu + \xi_1 + (i-1)\alpha P_{i,k}^{(3)}(t) + (1-p)P_{i+1,0}(t) + \lambda P_{i-1,k}(t) + \beta P_{i,k}(t) + \xi P_{1,0}(t); 1 \leq i \leq S - 1 \]  
3. Performance measures

Some performance measures are calculated to predict the system behaviour using the probabilities obtained through Runge-Kutta method:

1. \( P(\text{Server being Idle at time } t) = I(t) = \sum P_{i,0}^{(1)}(t) + \sum P_{1,0}^{(2)}(t) \)
2. \( P(\text{Server being busy(working) at time } t) = S(t) = \sum P_{i,0}^{(3)}(t) + \sum P_{i,k}^{(5)}(t) \)
3. \( P(\text{Server being broken down at time } t) = B(t) = \sum P_{i,0}^{(4)}(t) + \sum P_{i,k}^{(6)}(t) \)
4. \( L(t) = \text{Expected length of customers in the system at time } t = \sum_{n=0}^{S} n \cdot P_{n} \)
5. Waiting time in the system at time \( t = W(t) = \frac{L(t)}{\lambda \ast (1 - p_{\max \text{customers}}(t))} \)
4. Numerical results

MATLAB software is used to develop the computational program to find out system performance measures by giving numeric values to all the parameters. And also the effect of various parameters on the system performance measures is studied. The effect of different parameters in the system on performance measures is summarized in Tables 1-8.

In all numerical computations, the model parameters are taken as 
\[
\begin{align*}
N = 4, s = 6, \lambda = 4, \mu = .9, \beta = .6, \theta = 1, b = 2, p = .005, \beta_1 = .001, \beta_2 = .002, \\
\xi_1 = .002, \xi_2 = .003, T = 2 \text{ and } h = .5
\end{align*}
\]

Table 1: The effect of variation of parameter $\lambda$ on probabilities that the system will be idle($I(t)$), busy($S(t)$), broken down ($B(t)$) modes and mean length($L(t)$) and average waiting time($W(t)$) in the system are detailed in the following table:

| Parameters | Performance measures/Time | 0 | 1 | 2 | 3 | 4 | 5 |
|------------|----------------------------|---|---|---|---|---|---|
| $\lambda = 0.4$ | $I(t)$ | 1 | 1 | 0.999862 | 0.999972 | 1 | 0.999982 | 0.999982 | 1 | 0.999992 | 1 | 0.999998 | 1 |
| $\lambda = 0.42$ | $S(t)$ | 0 | 0 | 0.000137 | 0.000829 | 0.002761 | 0.006693 |
| $\lambda = 0.44$ | $B(t)$ | 0 | 0 | 0 | 0 | 5.03E-07 | 5.81E-06 | 3.08E-05 |
| $\lambda = 0.46$ | $L(t)$ | 0 | 0 | 0 | 0 | 5.9E-07 | 5.5E-07 | 5.0E-07 |
| $\lambda = 0.48$ | $W(t)$ | 0 | 1.001066 | 2.014069 | 3.057142 | 4.142851 | 5.271443 |

Table 2: The effect of variation of parameter $\mu$ on probabilities that the system will be idle($I(t)$), busy($S(t)$), broken down ($B(t)$) modes and mean length($L(t)$) and average waiting time($W(t)$) in the system are detailed in the following table:

| Parameter | Performance measures/Time | 0 | 1 | 2 | 3 | 4 | 5 |
|------------|----------------------------|---|---|---|---|---|---|
| $\mu = 0.9$ | $I(t)$ | 1 | 1 | 0.999862 | 0.999972 | 0.999982 | 0.999992 | 1 | 0.999998 | 1 | 0.999999 | 1 | 0.999999 | 1 |
| $\mu = 0.92$ | $B(t)$ | 0 | 0 | 0.000137 | 0.000829 | 0.002761 | 0.006693 |
| $\mu = 0.94$ | $L(t)$ | 0 | 0 | 0 | 0 | 5.03E-07 | 5.81E-06 | 3.08E-05 |
| $\mu = 0.96$ | $W(t)$ | 0 | 1.001066 | 2.014069 | 3.057142 | 4.142851 | 5.271443 |

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Table 3: The effect of variation of parameter $/\beta_2010$ on probabilities that the system will be idle $I(t)$, busy $S(t)$, broken down $B(t)$ modes and mean length $L(t)$ and average waiting time $W(t)$ in the system are detailed in the following table:

| Parameters | Performance measures/Time | 0 | 1 | 2 | 3 | 4 | 5 |
|------------|---------------------------|---|---|---|---|---|---|
| $\mu = .92$ |                           |   |   |   |   |   |   |
| $L(t)$     | 0                         | 0.4 | 0.799939 | 1.199269 | 1.596411 | 1.988442 |
| $W(t)$     | 0                         | 1.001066 | 2.014069 | 3.057142 | 4.142851 | 5.271443 |
| $I(t)$     | 1                         | 1   | 0.999862 | 0.999139 | 0.997055 | 0.992686 |
| $B(t)$     | 0                         | 0   | 0.000137 | 0.000828 | 0.002755 | 0.006672 |
| $T(t)$     | 0                         | 0   | 0       | 5.62E-07 | 6.47E-06 | 3.40E-05 |
| $L(t)$     | 0                         | 0.4 | 0.799937 | 1.199247 | 1.596303 | 1.988106 |
| $W(t)$     | 0                         | 1.001066 | 2.014064 | 3.057086 | 4.142582 | 5.270604 |
| $\mu = .94$ |                           |   |   |   |   |   |   |
| $L(t)$     | 0                         | 0.4 | 0.799937 | 1.199224 | 1.596196 | 1.987774 |
| $W(t)$     | 0                         | 1.001066 | 2.014059 | 3.057029 | 4.142314 | 5.269773 |

Table 4: The effect of variation of parameter $\theta_2016$ on probabilities that the system will be idle $I(t)$, busy $S(t)$, broken down $B(t)$ modes and mean length $L(t)$ and average waiting time $W(t)$ in the system are detailed in the following table:

| Parameters | Performance measures/Time | 0 | 1 | 2 | 3 | 4 | 5 |
|------------|---------------------------|---|---|---|---|---|---|
| $\beta = .6$ |                           |   |   |   |   |   |   |
| $I(t)$     | 1                         | 1   | 0.999862 | 0.999139 | 0.997055 | 0.9926797 |
| $B(t)$     | 0                         | 0   | 0.000137 | 0.000829 | 0.002761 | 0.0066931 |
| $T(t)$     | 0                         | 0   | 0       | 5.03E-07 | 5.81E-06 | 3.08E-05 |
| $L(t)$     | 0                         | 0.4 | 0.799939 | 1.199269 | 1.596411 | 1.988442 |
| $W(t)$     | 0                         | 1.0011 | 2.0140 | 3.0571 | 4.142851 | 5.271443 |
| $\beta = .62$ |                          |   |   |   |   |   |   |
| $I(t)$     | 1                         | 1   | 0.999862 | 0.999149 | 0.997149 | 0.9926805 |
| $S(t)$     | 0                         | 0   | 0.000137 | 0.00083 | 0.00276 | 0.006693 |
| $B(t)$     | 0                         | 0   | 0       | 4.95E-07 | 5.69E-06 | 3.00E-05 |
| $L(t)$     | 0                         | 0.4 | 0.799939 | 1.199269 | 1.596411 | 1.988442 |
| $W(t)$     | 0                         | 1.0011 | 2.0141 | 3.0571 | 4.142851 | 5.271443 |
| $\beta = .64$ |                          |   |   |   |   |   |   |
| $I(t)$     | 1                         | 1   | 0.999862 | 0.999139 | 0.997055 | 0.992681 |
| $S(t)$     | 0                         | 0   | 0.000137 | 0.00083 | 0.00276 | 0.0067 |
| $B(t)$     | 0                         | 0   | 0       | 4.87E-07 | 5.58E-06 | 2.92E-05 |
| $L(t)$     | 0                         | 0.4 | 0.799939 | 1.199269 | 1.596411 | 1.988442 |
| $W(t)$     | 0                         | 1.001066 | 2.014069 | 3.057142 | 4.142851 | 5.271443 |

Table 3: The effect of variation of parameter $\beta_2010$ on probabilities that the system will be idle $I(t)$, busy $S(t)$, broken down $B(t)$ modes and mean length $L(t)$ and average waiting time $W(t)$ in the system are detailed in the following table:

| Parameters | Performance measures/Time | 0 | 1 | 2 | 3 | 4 | 5 |
|------------|---------------------------|---|---|---|---|---|---|
| $\mu = .92$ |                           |   |   |   |   |   |   |
| $L(t)$     | 0                         | 0.4 | 0.799939 | 1.199269 | 1.596411 | 1.988442 |
| $W(t)$     | 0                         | 1.001066 | 2.014069 | 3.057142 | 4.142851 | 5.271443 |
| $I(t)$     | 1                         | 1   | 0.999862 | 0.999139 | 0.997055 | 0.992686 |
| $B(t)$     | 0                         | 0   | 0.000137 | 0.000828 | 0.002755 | 0.006672 |
| $T(t)$     | 0                         | 0   | 0       | 5.62E-07 | 6.47E-06 | 3.40E-05 |
| $L(t)$     | 0                         | 0.4 | 0.799937 | 1.199247 | 1.596303 | 1.988106 |
| $W(t)$     | 0                         | 1.001066 | 2.014064 | 3.057086 | 4.142582 | 5.270604 |
| $\mu = .94$ |                           |   |   |   |   |   |   |
| $L(t)$     | 0                         | 0.4 | 0.799937 | 1.199224 | 1.596196 | 1.987774 |
| $W(t)$     | 0                         | 1.001066 | 2.014059 | 3.057029 | 4.142314 | 5.269773 |

Table 4: The effect of variation of parameter $\theta_2016$ on probabilities that the system will be idle $I(t)$, busy $S(t)$, broken down $B(t)$ modes and mean length $L(t)$ and average waiting time $W(t)$ in the system are detailed in the following table:

| Parameters | Performance measures/Time | 0 | 1 | 2 | 3 | 4 | 5 |
|------------|---------------------------|---|---|---|---|---|---|
| $\Theta=1$ |                           |   |   |   |   |   |   |
| $I(t)$     | 1                         | 1   | 0.999862 | 0.999139 | 0.997055 | 0.992679 |
| $S(t)$     | 0                         | 0   | 0.000137 | 0.000829 | 0.002761 | 0.0066931 |

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Table 5: The effect of variation of parameter $\beta_1$ on probabilities that the system will be idle ($I(t)$), busy ($S(t)$), broken down ($B(t)$) modes and mean length ($L(t)$) and average waiting time ($W(t)$) in the system are detailed in the following table:

| Parameters | Performance measures/Time | 0 | 1 | 2 | 3 | 4 | 5 |
|------------|---------------------------|---|---|---|---|---|---|
| $\beta_1 = 0.001$ | $I(t)$ | 1 | 1 | 0.999862 | 9.99E-01 | 9.97E-01 | 9.93E-01 |
| | $S(t)$ | 0 | 0 | 0.000137 | 0.000829 | 0.002761 | 0.006693 |
| | $B(t)$ | 0 | 0 | 0 | 5.03E-07 | 5.81E-06 | 3.08E-05 |
| | $L(t)$ | 0 | 0.4 | 0.799939 | 1.199269 | 1.596411 | 1.988442 |
| | $W(t)$ | 0 | 1.001066 | 2.014069 | 3.057142 | 4.142851 | 5.2714431 |
| $\beta_1 = 0.0012$ | $I(t)$ | 1 | 1 | 0.999886 | 9.99E-01 | 9.97E-01 | 9.93E-01 |
| | $S(t)$ | 0 | 0 | 0.000121 | 0.000811 | 0.002671 | 0.006621 |
| | $B(t)$ | 0 | 0 | 0 | 5.03E-07 | 5.81E-06 | 3.08E-05 |
| | $L(t)$ | 0 | 0.4 | 0.799939 | 1.199269 | 1.596411 | 1.988442 |
| | $W(t)$ | 0 | 1.001066 | 2.014069 | 3.057142 | 4.142851 | 5.2714431 |
| $\beta_1 = 0.0014$ | $I(t)$ | 1 | 1 | 0.999886 | 9.99E-01 | 9.97E-01 | 9.93E-01 |
| | $S(t)$ | 0 | 0 | 0.000121 | 0.000811 | 0.002671 | 0.006621 |
| | $B(t)$ | 0 | 0 | 0 | 5.03E-07 | 5.81E-06 | 3.08E-05 |
| | $L(t)$ | 0 | 0.4 | 0.799939 | 1.199269 | 1.596411 | 1.988442 |
| | $W(t)$ | 0 | 1.001066 | 2.014069 | 3.057142 | 4.142851 | 5.2714431 |

Table 6: The effect of variation of parameter $\beta_2$ on probabilities that the system will be idle ($I(t)$), busy ($S(t)$), broken down ($B(t)$) modes and mean length ($L(t)$) and average waiting time ($W(t)$) in the system are detailed in the following table:

| Parameters | Performance measures/Time | 0 | 1 | 2 | 3 | 4 | 5 |
|------------|---------------------------|---|---|---|---|---|---|
| $\beta_2 = 0.001$ | $I(t)$ | 1 | 1 | 0.999862 | 9.99E-01 | 9.97E-01 | 9.93E-01 |
| | $S(t)$ | 0 | 0 | 0.000137 | 0.000829 | 0.002761 | 0.006693 |
| | $B(t)$ | 0 | 0 | 0 | 5.03E-07 | 5.81E-06 | 3.08E-05 |
| | $L(t)$ | 0 | 0.4 | 0.799939 | 1.199269 | 1.596411 | 1.988442 |
| | $W(t)$ | 0 | 1.001066 | 2.014069 | 3.057142 | 4.142851 | 5.2714431 |
| $\beta_2 = 0.0012$ | $I(t)$ | 1 | 1 | 0.999886 | 9.99E-01 | 9.97E-01 | 9.93E-01 |
| | $S(t)$ | 0 | 0 | 0.000121 | 0.000811 | 0.002671 | 0.006621 |
| | $B(t)$ | 0 | 0 | 0 | 5.03E-07 | 5.81E-06 | 3.08E-05 |
| | $L(t)$ | 0 | 0.4 | 0.799939 | 1.199269 | 1.596411 | 1.988442 |
| | $W(t)$ | 0 | 1.001066 | 2.014069 | 3.057142 | 4.142851 | 5.2714431 |
| $\beta_2 = 0.0014$ | $I(t)$ | 1 | 1 | 0.999886 | 9.99E-01 | 9.97E-01 | 9.93E-01 |
| | $S(t)$ | 0 | 0 | 0.000121 | 0.000811 | 0.002671 | 0.006621 |
| | $B(t)$ | 0 | 0 | 0 | 5.03E-07 | 5.81E-06 | 3.08E-05 |
| | $L(t)$ | 0 | 0.4 | 0.799939 | 1.199269 | 1.596411 | 1.988442 |
| | $W(t)$ | 0 | 1.001066 | 2.014069 | 3.057142 | 4.142851 | 5.2714431 |
Table 7: The effect of variation of parameter $\beta_2$ on probabilities that the system will be idle ($I(t)$), busy ($S(t)$), broken down ($B(t)$) modes and mean length ($L(t)$) and average waiting time ($W(t)$) in the system are detailed in the following table:

| Parameters | Performance measures/Time | 0   | 1     | 2     | 3     | 4     | 5     |
|------------|---------------------------|-----|-------|-------|-------|-------|-------|
| $\beta_2 = 0.002$ | $I(t)$                  | 1   | 0.999862 | 9.99E-01 | 9.97E-01 | 9.93E-01 |
|             | $S(t)$                  | 0   | 0.000137 | 0.000829 | 0.002761 | 0.006693 |
|             | $B(t)$                  | 0   | 0      | 5.03E-07 | 5.81E-06 | 3.08E-05 |
|             | $L(t)$                  | 0   | 0.4    | 0.799939 | 1.199269 | 1.596411 | 1.988442 |
|             | $W(t)$                  | 0   | 1.001066 | 2.014069 | 3.057142 | 4.142851 | 5.271443 |
| $\beta_2 = 0.022$ | $I(t)$                  | 1   | 0.999864 | 0.998973 | 9.98E-01 | 9.94E-01 |
|             | $S(t)$                  | 0   | 0      | 0.000122 | 0.000828 | 0.002824 | 0.006662 |
|             | $B(t)$                  | 0   | 0      | 4.27E-07 | 5.83E-06 | 3.04E-05 |
|             | $L(t)$                  | 0   | 0.3912 | 0.8416 | 1.2897 | 1.64564 | 2.0823 |
|             | $W(t)$                  | 0   | 1.00122 | 2.01534 | 3.064008 | 4.158123 | 5.29691 |
| $\beta_2 = 0.024$ | $I(t)$                  | 1   | 0.99987 | 0.99989 | 9.99989 | 9.9999 |
|             | $S(t)$                  | 0   | 0      | 0.00012 | 0.000826 | 0.002761 | 0.006693 |
|             | $B(t)$                  | 0   | 0      | 4.06E-07 | 5.82E-06 | 4.06E-05 |
|             | $L(t)$                  | 0   | 0.39981 | 0.841911 | 1.1213 | 1.69782 | 2.19431 |
|             | $W(t)$                  | 0   | 1.00112 | 2.01486 | 3.04467 | 4.00002 | 5.22234 |

Table 8: The effect of variation of parameter $\xi_1$ on probabilities that the system will be idle ($I(t)$), busy ($S(t)$), broken down ($B(t)$) modes and mean length ($L(t)$) and average waiting time ($W(t)$) in the system are detailed in the following table:

| Parameters | Performance measures/Time | 0   | 1     | 2     | 3     | 4     | 5     |
|------------|---------------------------|-----|-------|-------|-------|-------|-------|
| $\xi_1 = 0.002$ | $I(t)$                  | 1   | 0.999862 | 0.999139 | 0.997055 | 0.99267 |
|             | $S(t)$                  | 0   | 0.000137 | 0.000829 | 0.002761 | 0.006693 |
|             | $B(t)$                  | 0   | 0      | 5.03E-07 | 5.81E-06 | 3.08E-05 |
|             | $L(t)$                  | 0   | 0.4    | 0.799939 | 1.199269 | 1.596411 | 1.988442 |
|             | $W(t)$                  | 0   | 1.001066 | 2.014069 | 3.057142 | 4.142851 | 5.271443 |
| $\xi_1 = 0.022$ | $I(t)$                  | 1   | 0.99987 | 0.99989 | 0.99989 | 0.9999 |
|             | $S(t)$                  | 0   | 0      | 0.00012 | 0.000826 | 0.002761 | 0.006693 |
|             | $B(t)$                  | 0   | 0      | 4.27E-07 | 5.83E-06 | 3.04E-05 |
|             | $L(t)$                  | 0   | 0.3912 | 0.8416 | 1.2897 | 1.64564 | 2.0823 |
|             | $W(t)$                  | 0   | 1.00122 | 2.01534 | 3.064008 | 4.158123 | 5.29691 |
| $\xi_1 = 0.024$ | $I(t)$                  | 1   | 0.99987 | 0.99989 | 0.99989 | 0.9999 |
|             | $S(t)$                  | 0   | 0      | 0.00012 | 0.000826 | 0.002761 | 0.006693 |
|             | $B(t)$                  | 0   | 0      | 4.06E-07 | 5.82E-06 | 4.06E-05 |
|             | $L(t)$                  | 0   | 0.39981 | 0.841911 | 1.1213 | 1.69782 | 2.19431 |
|             | $W(t)$                  | 0   | 1.00112 | 2.01486 | 3.04467 | 4.00002 | 5.22234 |
are detailed in the following table:

| Parameters | Performance measures/Time | 0  | 1  | 2  | 3  | 4  | 5  |
|------------|---------------------------|----|----|----|----|----|----|
| $\xi_2 = 0.033$ | I(t) | 1  | 1  | 0.999862 | 0.999139 | 0.997055 | 0.99267 |  
|   | S(t) | 0  | 0  | 0.000137 | 0.000829 | 0.002761 | 0.006693113 |
|   | B(t) | 0  | 0  | 0  | 5.03E-07 | 5.81E-06 | 3.08E-05 |
|   | L(t) | 0  | 0.4 | 0.799939 | 1.199269 | 1.596411 | 1.98844 |
|   | W(t) | 0  | 1.001066 | 2.014069 | 3.057142 | 4.142851 | 5.2714431 |
| $\xi_2 = 0.032$ | I(t) | 1  | 1  | 0.99862 | 0.99914 | 0.99831 | 0.99412 |
|   | S(t) | 0  | 0  | 0.00015 | 0.000836 | 0.002813 | 0.006802 |
|   | B(t) | 0  | 0  | 0  | 5.05E-07 | 6.23E-06 | 3.098E-05 |
|   | L(t) | 0  | 0.4 | 0.799928 | 1.199154 | 1.595895 | 1.98692 |
|   | W(t) | 0  | 1.001066 | 2.014042 | 3.056854 | 4.141559 | 5.267624 |
| $\xi_2 = 0.034$ | I(t) | 1  | 1  | 0.998623 | 0.999142 | 0.99832 | 0.99469 |
|   | S(t) | 0  | 0  | 0.000181 | 0.001064 | 0.00346 | 0.008205 |
|   | B(t) | 0  | 0  | 0  | 5.11E-07 | 6.24E-06 | 3.12E-05 |
|   | L(t) | 0  | 0.423 | 0.6214 | 1.199142 | 1.5953 | 1.967 |
|   | W(t) | 0  | 1.001064 | 2.01400 | 3.05652 | 4.0204 | 5.2643 |

From Tables 1-8 we observe that

- As $\lambda, \theta_1$ and $\beta_1$ are increasing, length and waiting times are increasing.
- As $\mu, \beta, \xi_2$ and $\beta_2$ are increasing, length and waiting times are slightly decreasing.

5. Conclusion

In this model, we have calculated system performance measures by giving numeric values to the parameters using MATLAB software. Sensitivity analysis is done for different values of parameters to illustrate the validity of the proposed model. One can extend this work by incorporating cost function and derive the optimum value of $N$ that can minimize the cost function.

References

[1] E Ramesh Kumar, Y.Praby Loit (2016), A Study on Vacation Bulk Queueing Model with Setup time and server timeout. *IJCMS*, 5(12):81-89

[2] J.R. Murray, W.D. Kelton (1988): The transient behaviour of the $M/E_k/2$ queue and steady-state simulation, *Comput. Ops. Res.*, 15, N 4, 357-367.

[3] K. R Baker(1973), A note on operating policies for the queue $M/M/1$ with exponential start-up. *INFOR*, 11, 71–72

[4] L.W. Miller(1964), Alternating priorities in multi-class queue. *Ph. D. Dissertation*, Cornell University, Ithaca, New York

[5] M.Yadin, M. and P. Naor(1963). Queueing Systems with a removable service station. *Operational Research Quarterly*, 14, 4, 393–405.

[6] Oliver C. Ibe (2015), $M/G/1$ Vacation Queueing Systems with Server Timeout, American *Journal*, 5:77-88.

[7] V.Vasanta Kumar and K. Chandan(2008a). Cost Analysis of a Two-Phase MX/EK/1 Queueing System with N-policy, *OPSEARSCH*, 45(2), 155-174.
[8] Y. Levy and U. Yechiali, (1975), Utilization of idle time in an M/G/1 queueing system, *Management Science*, **22**, 202 – 211