METAL ABUNDANCE IN THE SOLAR INTERIOR
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Received 1997 June 19; accepted 1997 December 9

ABSTRACT
It is shown that the metal abundance in the solar interior is constrained by the current solar neutrino experiments under the assumption of neutrino conversion in the Sun due to neutrino oscillation via the Mikheyev-Smirnov-Wolfenstein (MSW) effect, the most widely accepted solution to the solar neutrino problem. The metal abundance strongly affects the CNO neutrino flux, and an excessive increase of the CNO flux is incompatible with the region allowed by the solar neutrino experiments in neutrino-oscillation parameter space. The result shows that the metal abundance in the interior should be within the range 0.5–1.4 times that in the surface, supporting the idea that the Sun formed by a contraction of a gas cloud with an almost homogeneous composition.

Subject headings: Sun: abundances — Sun: interior

1. INTRODUCTION
Whether the metal abundance measured in the stellar surface correctly represents the value in the stellar core is a nontrivial question. In computations of the evolution of stars it is generally assumed that the initial metal abundance of the core is identical to the present-day abundance of the surface, for which spectroscopic information is available for heavy elements. This agrees with the simple picture that stars formed by a contraction of a molecular cloud of uniform composition.

In principle, however, the metallicities of the core and the surface can be different, unless the whole star is fully convective as in low-mass \((M < 0.4 \, M_\odot)\), main-sequence stars (e.g., Iben 1967). An extreme example is Jupiter, where the core consists mostly of silicates, while the surface is dominated by hydrogen and helium with a small admixture of heavier elements. It may well be that the origin of Jupiter-like stars is completely different from that of ordinary stars (e.g., Podolak, Hubbard, & Pollack 1993); nevertheless, it would not be absurd to imagine that rocks are taken into the core at the time of gas contraction and that the core has a metallicity higher than that of the surface since the birth of the star, for which the radiative transport dominates. There is also an effect that heavy metals tend to sink by diffusion toward the core, while the envelope becomes hydrogen rich. A calculation shows that such an effect does exist in the Sun—as evidenced from helioseismology—although the amount is small (Bahcall et al. 1997).

The main reason that one assumes homogeneous composition throughout the star is basically a lack of information on the metal abundance in the stellar interior. In this paper we consider the problem of whether the Sun might offer a possibility of studying the metal abundance in the deep interior or in the core through the solar neutrino observations that probe the core region of the Sun. This problem, however, is not very simple because of the famous solar neutrino problem: the observed neutrino flux is smaller by a factor of 2–4 than is predicted with the standard model of the Sun (Bahcall 1989). The most elegant and widely accepted solution to this problem is that electron neutrinos emitted in the nuclear reactions in the Sun are converted by neutrino oscillation into muon or tau neutrinos that are sterile in the nuclear detector or have a much smaller cross section for scattering off electrons, as advocated by Mikheyev & Smirnov (1986a; referred to as the MSW effect).

At first glance, this solution appears to make things so flexible that almost any amount of the neutrino flux before the oscillation effect is experimentally allowed, if one tunes the neutrino mass and mixing parameters in some appropriate way. Indeed, forgetting about all knowledge on the nuclear reaction cross sections, even the case that almost 100% of the solar energy is generated by the CNO cycle is not excluded solely by the solar neutrino experiment (Bahcall, Fukugita, & Krasnov 1996).

This, however, is too extreme. If we adopt the knowledge of nuclear reactions within the range allowed by the current experiment, the freedom is not that large. For instance, the amount of \(^{12}\text{B}\) must be in the range between \(\frac{1}{2}\) and 2 times the value that the standard solar model predicts, and the core temperature can be determined to within 5% of the standard solar model value (Hata & Langacker 1997, hereafter HL97) in order to satisfy the current solar neutrino experiments, whatever neutrino parameters one takes. In this paper we study to what extent the current solar neutrino experiments constrain the metal abundance in the Sun, allowing for MSW neutrino oscillation (or neutrino conversion).

The increase of the metal abundance obviously promotes the CNO cycle. It also increases opacity, which modifies the core temperature. Therefore, it increases the relative importance of the neutrino flux from the CNO cycle significantly. This increment of the CNO neutrino flux must be canceled by the increasing importance of the suppression factor coming from the neutrino oscillation, in order to maintain consistency with the solar neutrino experiments. Insofar as this works, a larger metal abundance in the solar interior is allowed. If the metal abundance is increased more, however, we no longer have solutions that satisfy all the solar neutrino experiments in a consistent way. This is the logic that we are going to explore in this study.

We remark that metal abundance may not be so tightly constrained from helioseismology information. Helioseismology is sensitive to the sound velocity, the change of which reads approximately \(\Delta c_s/c_s \approx \frac{3}{2}(\Delta T/T - \Delta \mu/\mu)\), where \(T\) is temperature and \(\mu\) is mean molecular weight,
\[ \mu^{-1} = 2X + 0.75Y + 0.58Z(12^{c}C) + \cdots \]

A cancellation takes place between temperature and mean molecular weight.

Although the present best solar model is known to give \( c_i \) as accurate as 0.2\% (Bahcall et al. 1997), and the 50\% change in \( Z \) might be detected if it changes with satisfying \( \Delta Z = -\Delta X \), the presence of \( Y \) complicates the situation. Hence, it is not obvious how a strong constraint can be derived on the metallicity independent of \( X \) and \( Y \).

2. CALCULATION

We take the standard solar model of Bahcall and collaborators as our fiducial (Bahcall & Ulrich 1988; Bahcall & Pinsonneault 1992; Bahcall & Pinsonneault 1995, hereafter BP95) and consider a small departure from their best model. The energy of the Sun is generated from the \( pp \) chain (98\%) and the CNO cycle (2\%). Neutrinos are produced in \( pp \rightarrow d \nu, \; pep \rightarrow d \nu, \; e^+ \rightarrow \nu \rightarrow 7^Li + \nu \), and \( 8^B \rightarrow 3^He + e + \nu \) from the \( pp \) chain, and in beta decay of \( ^{13}N, \; ^{15}O, \; \) and \( ^{17}F \) in the CNO cycle. The experimental information comes from (1) the long-running Homestake experiment with \( 37Cl \), which is sensitive to both high-energy neutrinos (\( 8^B \) neutrinos) and intermediate-energy neutrinos (\( ^{12}Be, \; pep \), and CNO neutrinos; Cleveland et al. 1998); (2) water Cerenkov experiments at Kamiokande and Superkamiokande measuring only for high-energy neutrinos (Fukuda et al. 1996; Totsuka et al. 1997); and (3) gallium experiments—the Gallium Experiment at Gran Sasso (GALLEX; Hampel et al. 1996) and the Soviet-American Gallium Experiment (SAGE; Abdurashitov et al. 1996)—that are very sensitive to low-energy neutrinos (\( pp \) neutrinos). The problem is that the detection rate is smaller than predicted, by factors \( 3.7 \pm 0.6, \; 2.6 \pm 0.5 \), and \( 2.0 \pm 0.2 \), respectively. Furthermore, this specific energy-dependent suppression pattern makes the explanation of the problem by modifying the solar model highly unlikely, leaving the neutrino-oscillation explanation as the most attractive possibility (Bahcall & Bethe 1990; Fukugita & Yanagida 1991).

Indeed, this energy-dependent suppression is very naturally understood by considering the conversion of electron neutrinos into other types of neutrinos inside the Sun by neutrino oscillation (Mikheyev & Smirnov 1986a). For the neutrino flux given by the standard solar model, the current solar neutrino experiments allow two small parameter regions that are located in the two-parameter space, neutrino mass-squared difference \( \Delta m^2 = m^2_\nu - m^2_\nu \) (\( i = \mu \) or \( \tau \)) and the intrinsic mixing angle (\( \theta \)) between the two relevant neutrinos: one is called the small-angle solution, in which intermediate-energy neutrinos are suppressed, and the other the large-angle solution, for which the suppression of neutrino fluxes is almost energy independent. The most up-to-date calculations are found in Bahcall & Krastev (1996) and in HL97.

Additional information can be obtained from an upper limit on the possible flux variation between the day and nighttime (day-night effect; Mikheyev & Smirnov 1986b; see also Bahcall & Krastev 1997 and references therein). For some specific neutrino parameter range, the converted muon neutrinos are changed back to electron neutrinos during the propagation through the Earth, causing an increase of the neutrino-capture rate in night in the Kamiokande and Superkamiokande detectors. The absence of this effect down to a 2\% level (Fukuda et al. 1996) excludes a substantial part of the parameter regions in which we are interested.

We repeat the neutrino-propagation calculation, allowing for a variation in the metal abundance of the Sun. We use the scaling law of Bahcall & Ulrich (1988), which has given the explicit metallicity dependence for each component of the neutrino flux. The range of the model explicitly studied covers about \( \pm 50\% \) around the value of metallicity determined for the solar surface. The dependence outside this range is a simple extrapolation with a power law. Although this calculation is rather old, we expect that the gross metallicity dependence does not differ from what one could obtain from the more modern BP95 calculation. We impose a luminosity constraint, so that the luminosity that would change because of a change in the metal abundance is renormalized to today’s luminosity of the Sun. Specifically, we study the model at fixed luminosity.

The table of Bahcall & Ulrich (1988) shows that the CNO neutrino fluxes are indeed the most sensitive to metal abundance: the power \( \gamma \) of the flux \( \phi \sim (Z/X)^\gamma \) is 1.86 for the \( 13N \) neutrino, 2.03 for the \( 15O \) neutrino, and 2.09 for the \( 17F \) neutrino. This high power is caused by a multiplicative effect due to the increase of the abundance of catalyzing \( 13C \) and the increase of opacity that makes the core temperature higher. In spite of its sharp temperature dependence, the effect on the \( 8B \) neutrino flux is smaller (\( \gamma \sim 1.27 \)) than for the CNO neutrinos.

As for the fiducial flux, we use the value of the BP95 calculation with metal diffusion effect taken into account:

\[ \phi(B) = (6.6 \pm 1.1) \times 10^6 \, \text{cm}^{-2} \, \text{s}^{-1}, \; 9.3 \pm 1.3 \, \text{SNU} \text{ for captures with } 37Cl \text{ and } 137 + 8 \, \text{SNU for captures with } 71Ga. \]

We take \( 2.55 \pm 0.14 \pm 0.14 \) SNU for the Homestake experiment (Cleveland et al. 1998), \( 2.80 \pm 0.19 \pm 0.33 \times 10^6 \, \text{cm}^{-2} \, \text{s}^{-1} \) for Kamiokande (Fukuda et al. 1996), \( 2.51 \pm 0.13 \pm 0.18 \times 10^6 \, \text{cm}^{-2} \, \text{s}^{-1} \) for Superkamiokande [combined \( 2.586 \pm 0.195 \times 10^6 \, \text{cm}^{-2} \, \text{s}^{-1} \); Totsuka 1997], and \( 69.5 \pm 6.7 \) SNU for combined SAGE and GALLEX experiments. The absence of the day-night effect is also imposed on our data analysis. In our actual calculation we use the nighttime flux data divided into five bins, according to the cosine of the angle from the Sun (Fukuda et al. 1996).

The data are then fitted with the three free parameters, \( \Delta m^2, \; \sin^2 2\theta, \; \) and \( Z/X; \) we then calculate a likelihood function that takes account of experimental errors of both neutrino reaction rates and those arising from uncertainties of solar models as given by BP95, in the same way as done in HL97.

The resulting probability distribution is displayed in Figure 1, taking \( Z \) as a parameter. The range allowed at 95\% confidence level (CL) is

\[ 0.4 < Z/Z_{\text{surface}} < 1.4, \]  

where \( Z_{\text{surface}} = 0.0175 \) (Grevesse & Noels 1993), using the standard solar model value \( X = 0.71 \). The region outside this range is excluded even if we assume the flux reduction due to neutrino oscillation. That is, the metal abundance in the solar interior cannot be much different from that in the surface. The allowed range is, of course, much larger than the change of the metal abundance induced by the diffusion effect, which is about 15\% in \( Z \) (BP95). We remark that the range given in equation (1) is in the range where explicit solar model studies are made by Bahcall & Ulrich (1988), and the behavior regarding the variation of \( Z \) is well...
studied; so, in hindsight, we need not use the power law of the $Z$ dependence out to the range where its behavior is not well established.

We also display (Fig. 2) the allowed region for the two other parameters, $\Delta m^2$ and $\sin^2 2\theta$, overlaid on the corresponding figure for the standard case (i.e., $Z = Z_{\text{surface}}$, allowing for errors in the value at the surface of 6.1%). The contour is a parameter range corresponding to 95% CL. Compared with the contour of the standard model case, the allowed region is elongated horizontally for the small-angle solution or vertically for the large-angle solution. Most of the elongated parts (a part in the left-hand side of the small-angle solution and one in the upper part of the large-angle solution) correspond to the case with metallicity lower than the default value. For larger metallicity, a small change of the neutrino parameters can accommodate the increased neutrino fluxes. Nevertheless, there is a sharp cutoff against the increase of metallicity, beyond which appropriate neutrino parameters do not exist to make the flux consistent with the three solar neutrino experiments. In our calculation, the constraint from the absence of the day-night effect serves to squeeze the high-metallicity end in the large-angle solution, but it plays little role in the small-angle solution.

3. DISCUSSION

We have shown that the solar neutrino experiments put a strong constraint on the metal abundance in the interior of the Sun, even if we allow the neutrino oscillation due to the matter effect. Namely, the possibility of neutrino oscillation does not lend us much freedom to increase our estimate of metallicity inside the Sun. The metallicity in the interior of the Sun should not be larger than the surface value by more than 40%, or 0.15 dex in $[\text{Fe/H}]$. This is good news for the people working on stellar evolution calculations, since we expect that the Sun is not a special case; the same probably applies to more general cases, justifying the standard assumption that stars are formed by contraction of a homogeneous gas sphere (Hayashi 1966).

On the other hand, the errors in the current neutrino experiments and uncertainties in nuclear reaction cross sections still allow the possibility that the metal abundance in the solar interior is slightly larger (or smaller) than on the surface. Accepting this uncertainty, energy generation from the CNO cycle may take any value from 0.4% to 4% of the total, which is compared to 1.8% for the standard value (see Fig. 3). The $^{13}\text{N}$ or $^{15}\text{O}$ neutrino fluxes may vary from $1 \times 10^8$ to $12 \times 10^8 \text{cm}^{-2} \text{s}^{-1}$ within this range.

One might think that the $^{13}\text{N}$ or $^{15}\text{O}$ neutrino flux (the endpoint energies 1.199 or 1.732 MeV, respectively) itself can give us a useful indicator for the metallicity in the solar interior because of its sensitivity to the carbon abundance. One may prepare a detector with a detection threshold set just above the energy of $^7\text{Be}$ neutrinos (0.862 MeV). Once a new detector that can measure $^7\text{Be}$ neutrinos (e.g., one with a liquid scintillator measuring for $\nu e \rightarrow \nu e$) is constructed (see the BOREXINO experiment; Arpesella 1992), this will not be difficult, since the CNO neutrino flux is 100 times higher than the $^8\text{B}$ neutrino flux in the standard solar model. The oscillation effect, however, makes the situation somewhat subtle. For the allowed regions of neutrino parameters, we expect the MSW suppression factor for the $^{13}\text{N}$ neutrino flux to be 0.1–0.5 (0.4–0.7) for the small-angle (large-angle) solution for a given metallicity. Unless this large uncertainty arising from neutrino oscillation is reduced, the $^{13}\text{N}$ flux does not give us useful information on the metallicity.

We find that the most effective way to reduce this uncertainty is to measure the gallium capture rate as precisely as possible. If the error attached to the gallium capture rate is
reduced, the region shrinks in the vertical (or horizontal) direction for the small-angle (large-angle) solution; i.e., the error in the suppression factor is reduced. A $^8$B neutrino flux measurement via the neutral current interaction planned at Sudbury Neutrino Observatory, when combined with a better gallium experiment, serves to further reduce the error in the suppression factor for a given metallicity (although this experiment alone is not very effective for our purpose). For instance, if we achieve an error in the gallium experiment as small as $\pm 2$ SNU around the current best value and measure the $^8$B neutrino neutral current reaction to a 10% accuracy, we would obtain $0.85 < Z/Z_{surface} < 1.2$ at 95% CL if the small-angle solution is correct or $0.9 < Z/Z_{surface} < 1.1$ if the large-angle solution is correct (see Fig. 4).

We have argued that solar neutrino experiments have given unique information on the metals in the solar interior, which is not accessible by other means. Assuming that the Sun is not special, this removes our worry that metal abundance in the solar interior might be different from that in the surface. For example, this can be an issue in studying metallicity dependence of the Cepheid period-luminosity relation, for which the metal effect on the color, which is affected by the surface metal abundance, to a large degree cancels out that on the luminosity, which is affected by metal abundance in the core of the star (Stothers 1988). If metallicity were different in these two places, the large cancellation would no longer take place, resulting in a much larger metallicity dependence in the Cepheid period-luminosity relation. The present analysis, if our result applies to star formation from molecular clouds in general, implies that the metallicity difference, at least, is not a likely possibility to account for a large metallicity dependence of the Cepheid period-luminosity relation suggested recently (Gould 1994; Sasselov et al. 1997; Sekiguchi & Fukugita 1998). We have also noted that one could reduce the error in $Z/Z_{surface}$ to a level of 10%-20% with further improvement in some specific solar neutrino experiments. At this level, the effect of metal diffusion could be seen with the solar neutrino experiments, opening the possibility of carrying out a cross check with the result from helioseismology.

We would like to thank John Bahcall, Plamen Krastev, and Bohdan Paczyński for very stimulating discussions, and John Bahcall for valuable comments on the preliminary draft. M. F. wishes to acknowledge support from the Fuji Xerox Corporation. N. H. is supported by National Science Foundation contract NSF PHY 95-13835.

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