Optimal control in the New Keynesian model with monetary and fiscal policy interactions

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Abstract. Dynamics of the New Keynesian model in continuous time with the Rotemberg pricing mechanism is considered within a framework of an optimal control problem. Various regimes of monetary and fiscal policy ('active' and 'passive') can lead to unstable dynamics in the economy. Parameters of the Taylor rules for both monetary and fiscal policies determine conditions for local equilibrium determinacy. Mapping out the ranges of the Taylor coefficient values where local determinacy cannot be obtained allows to control the economic system by controlling these parameters.

1. Introduction

In the modern macroeconomics, New Keynesian (NK) models of business cycles are essential tools for analyzing how monetary and fiscal policies can affect the economy. These models have been actively applied since the 1970s (see, e.g. [1-6]) and have a range of advantages. First, the stylized behavior rules of consumers, firms and other economic agents are derived in NK models from microfoundations: for every agent the optimization problem is constructed and solved subject to given constraints. Second, NK models can include realistic assumptions about financial frictions, price and wage stickiness, and information constraints. These assumptions make the effects of the monetary policy non-neutral. Third, the system derived from microfoundations is considered as an aggregated process that describes the dynamics of main macroeconomics variables (GDP, inflation, aggregate consumption, etc.).

The New Keynesian framework models can be constructed in discrete or continuous time. Recently, researches started to be more interested in NK models in continuous time due to their “analytical tractability” [4-8]. Continuous time models can be analyzed through a developed set of mathematical tools including mathematical optimal control theory and stochastic differential equations approach. Thus, the researchers get an opportunity to analyze the dynamics presented in NK models in more detail and to find some non-trivial effects, for example, periodic, quasiperiodic and chaotic solutions in case of local equilibrium indeterminacy [9-13].
One of the problems raised in the studies of NK models relates to the interaction of monetary and fiscal policies. In the literature there is a certain asymmetry: monetary policy has been analyzed in more detail than fiscal aspects. However, in papers [14-20] the importance of studying the policies in combination is shown. Moreover, new effects can be found in case of their interactions. Switching one policy from active to passive regime and vice versa can lead to qualitative changes in the behavior of the economic system.

Our research is focused on the analysis of the New Keynesian model in continuous time based on the system presented by C. Leith and L. Von Thadden [15]. The system describes the equilibrium in an economy where agents’ lives are limited in time, which therefore makes the state debt a net asset of consumers. In our model, there are four macroeconomic agents, namely a representative consumer, a representative producer, the government and the Central Bank (CB). The government and the CB are responsible for fiscal and monetary policies, respectively, which can be implemented in active or passive regimes. The analysis of policy regimes’ interaction is focused on the investigation of their combination that ensures the local equilibrium determinacy. In case the equilibrium is not locally determinate, the model dynamic behavior can exhibit features ranging from continued unpredictability to global indeterminacy.

We analyze the regimes of policies in terms of their influence on system dynamics. Taylor rules for monetary and fiscal policy can be considered as a mechanism of controlling agents’ behavior, as they affect the sign and magnitude of reaction to the inflation and the total debt value by the CB and the government. It is demonstrated in our analysis that if the monetary policy is active and thus stabilizing the inflation, the local equilibrium is not always determinate. If the monetary policy is passive and does not stabilize the inflation, the local equilibrium can be determinate depending on the Taylor coefficient and the fiscal policy regime. Therefore, different combinations of monetary and fiscal policies’ regimes may lead to local equilibrium determinacy as well as local equilibrium indeterminacy.

2. Model construction as a problem of optimal control theory
We consider a model presented in [15]. While our core structure of the economy is similar to [15], we incorporate two assumptions that are different from theirs. First, we change the price setting mechanism from Calvo (where at each period of time every firm faces constant probability of changing prices) to Rotemberg [21] (every firm changes prices at each period of time, but it has to pay adjustment costs). In the first approximation two mechanisms lead to equivalent results. However, application of the Rotemberg price setting mechanism makes the analysis of nonlinear system easier due to lower dimensionality of the resulting dynamic system. Second, the model incorporates the assumption of finite lifetime for consumers. They are faced with probability of death that is constant over lifetime and across consumers. In this case the standard Ricardian equivalence is broken, since the consumers with a certain probability will not pay future taxes issued to retire the government debt that pays interest to them today. As a result, the consumers consider a debt as a net asset, and fiscal policy influences the consumers’ behavior. This leads to creation of an additional channel for fiscal and monetary policy interaction, and to nontrivial dynamic effects.

System (1) is constructed as a solution for agents’ optimizing problems (representative consumer and representative producer) and given debt dynamics:

\[
C_t = (r - \rho)C_t + f^M \pi_t C_t - \xi(\xi + \rho)h_t,
\]

\[
\dot{h}_t = (r - \rho)h_t + f^M \pi_t h_t + g - \tau + f^F b_t,
\]

\[
\pi_t = \frac{\varepsilon - 1}{\varepsilon} \left( 1 - \frac{1}{\varepsilon - 1} C_t^{\gamma + \rho} \right) \left( \rho + \xi(\xi + \rho) \frac{b_t}{C_t} \right) \pi_t,
\]

where $C_t$ is the aggregate consumption in period $t$; $h_t$ is the value of debt in period $t$; $\pi_t$ is the inflation rate in period $t$; $r$ is the target level of real interest rate; $g$ is the fixed level of government consumption;
τ is the target level of lump-sum taxes; b is the target level of debt; ρ, ξ, ε, θ, φ, R are parameters of the model.

2.1. The consumer problem and dynamics of consumption
The consumption dynamics is derived by solving optimization problem of a representative consumer. Controlling the consumption level and supplying labor on labor market, the agent maximizes its welfare (2) limited by budget constraint (3)

\[ U_t^j = \ln C_t^j - \left( \frac{N_t^j}{1 + \varphi} \right)^{1+\varphi}, \]

\[ b_t^j = (i_t + \xi - \pi_t) b_t^j + \frac{W_t N_t^j}{P_t} - \tau_t - C_t^j + \Omega_t^j, \]

where \( C_t^j \) is real consumption of consumer \( j \) in period \( t \); \( N_t^j \) is labor supply of consumer \( j \) in period \( t \); \( b_t^j \) is real government debt of consumer \( j \) in period \( t \); \( i_t \) is nominal interest rate; \( W_t \) is nominal wage in period \( t \); \( \tau_t \) is real lump-sum taxes in period \( t \); \( P_t \) is price level in period \( t \); \( \Omega_t \) is level of transfers from the government to consumers in period \( t \).

2.2. The producer problem and dynamics of inflation
The inflation dynamics is derived by solving optimization problem of a representative producer. Controlling the price level under the conditions of monopolistic competition, the agent maximizes its profit net of transaction costs (according to Rotemberg price setting mechanism) (4) constrained by price dynamics (5):

\[ \Pi_t^i(p) = p_t^i \left( \frac{p_t^i}{p_t} \right)^{-\epsilon} Y_t - W_t \left( \frac{p_t^i}{p_t} \right)^{-\epsilon} Y_t - \frac{\theta}{2} \left( \frac{\dot{p}_t^i}{p_t} \right)^2, \]

\[ \dot{\pi}_t = \frac{\dot{p}_t^i}{p_t}, \]

where \( p_t^i \left( \frac{p_t^i}{p_t} \right)^{-\epsilon} Y_t \) is demand for firm \( i \) product in period \( t \); \( W_t \left( \frac{p_t^i}{p_t} \right)^{-\epsilon} Y_t \) is labor costs of firm \( i \) in period \( t \); \( \theta \left( \frac{\dot{p}_t^i}{p_t} \right)^2 \) is transaction costs because of price setting mechanism in period \( t \); \( \pi_t \) is inflation rate in period \( t \).

2.3. Debt dynamics and monetary and fiscal policy rules
The government and the CB are the agents which influence the economy by setting fiscal and monetary policy rules. Depending on the regimes of policies, the behavior of the system and conditions of local equilibrium determinacy can be changed. Moreover, the government is responsible for issuing the bonds and creating the debt. The debt dynamics is determined by equation (6), where new bonds issuance (\( \dot{b}_t \)) plus tax revenue (\( \tau_t \)) are the sources of government revenue which is spent for the interest on the existing bonds (\( r_t b_t \)) and the governmental consumption (\( g_t \)):

\[ \dot{b}_t = r_t b_t + g_t - \tau_t. \]
In order to determine the level of lump-sum taxes, the government applies the Taylor rule \([22]\) for identifying the level of taxes in the period \(t\) \((7)\). The rule reacts to deviation of the debt from its target level:

\[
\tau_t = \tau + f^F (h_t - b),
\]

where \(\tau\) is the target level of lump-sum taxes; \(b\) is the target level of debt; \(f^F\) is the Taylor coefficient for fiscal policy. The government chooses active or passive regime of fiscal policy by controlling the Taylor coefficient \(f^F\). According to the logic presented in \([17]\), passive regime means \(f^F > r\), where the government stabilizes the debt by collecting more taxes in order to finance increasing debt. Active regime of fiscal policy \((f^F < r)\) means that the government does not assure a source for financing increasing debt. This leads to the situation where the government disregards the budget balance.

The CB uses real interest rate as an instrument to control economic conditions. According to the Taylor rule \((8)\), real interest rate depends on the deviation of the target level of inflation:

\[
r_t = r + f^M (\pi_t - \pi),
\]

where \(r\) is the target level of real interest rate, \(\pi\) is the target level of inflation, and \(f^M\) is the Taylor coefficient for monetary policy.

Controlling the Taylor coefficient, the CB chooses an active or a passive monetary policy regime. Active policy \((f^M > 0)\) means that the CB stabilizes inflation by increasing the real interest rate in response to positive deviation of inflation from its target level. Passive policy implies \((f^M < 0)\) that the CB has a weaker reaction to positive deviation of inflation from its target level and allows real interest rate to fall, which increases the inflationary pressure on the economy. As a result, when choosing passive regime, the monetary authority cannot control the inflation level in the economy.

### 3. Local equilibrium determinacy

#### 3.1. Local equilibrium determinacy analysis

Local dynamics around the stationary point depends significantly on the regimes of both monetary and fiscal policies, and is nontrivially affected by their interaction. We derive analytical conditions for the existence of local equilibrium determinacy depending on the parameter system and Taylor coefficients \((7)\) and \((8)\).

Steady state levels for bonds, consumption, and inflation in system \((1)\) are the following:

\[
C_{ss} = \frac{\xi (\xi + \rho) (\tau - g)}{(r - \rho) r},
\]

\[
b_{ss} = \frac{(\tau - g)}{r},
\]

\[
\pi_{ss} = 0.
\]

They are derived from the assumption of zero inflation in the steady state \((11)\). Moreover, we have chosen the parameter \(R\) as follows:

\[
R = \frac{\varepsilon}{\varepsilon - 1} C_{ss}^{1+\phi}. \tag{12}
\]

This value makes the dynamics of inflation \((3^{rd} \text{ line in } \text{system } (1))\) equal zero in the steady state by making the difference \(1 - \frac{\varepsilon}{\varepsilon - 1} \cdot \frac{1}{R} C_{1+\phi}\) equal to zero. From the economic point of view, \(R\) means gross subsidy that the government gives to the wages paid by the firms. In case when \(C_{ss}\) \((9)\) is equal to 1, the value of the subsidy is set such that at the stationary point the firms would set prices to be equal to their marginal costs, thus achieving the efficiency of the stationary allocation. Depending on the
parameters that define consumption in the steady state, the gross subsidy may be more or less than \( \frac{e}{e-1} \).

Following [19], we analyze the conditions of the existence of local equilibrium determinacy in system (1). We consider the characteristic equation for system (1) has the form

\[
\chi(\lambda) = J^{ss} - \lambda J = \lambda^3 + p_1 \lambda^2 + p_2 \lambda + p_3 = 0, \tag{13}
\]

where \( J^{ss} \) is the Jacobian matrix of system (1); \( p_i \) (i = 1,2,3) is the coefficients of characteristic polynomial (13). The Jacobian matrix \( J^{ss} \) evaluated subject to (12) at the steady state (equilibrium) point \((C_{ss}, b_{ss}, \pi_{ss})\) is expressed as follows:

\[
J^{ss}_0 = \begin{pmatrix}
    r - \rho & -\xi(\xi + \rho) & f^M \frac{\xi (\xi + \rho)(\tau - g)}{(r - \rho)r} \\
    0 & r - f^F & f^M \frac{\tau - g}{r} \\
    -\frac{(e - 1)(1 + \varphi)(r - \rho)\theta \xi(\xi + \rho)(\tau - g)}{\theta} & 0 & r
\end{pmatrix}. \tag{14}
\]

Accordingly, the coefficients of the characteristic polynomial \( p_i^{ss} \) (i = 1,2,3) of the Jacobian matrix (14) in the equilibrium point \((C_{ss}, b_{ss}, \pi_{ss})\) have the following form:

\[
p_1^{ss} = f^F + \rho - 3r, \tag{15}
\]

\[
p_2^{ss} = \frac{(1 + \varphi)(e - 1)}{\theta} f^M + \left(\rho - 2r\right)f^F + \left(3r^2 - 2\rho\right), \tag{16}
\]

\[
p_3^{ss} = r(r - \rho)\left(f^F - r\right) + \frac{(1 + \varphi)(e - 1)}{\theta} \left(\rho - 2r + f^F\right)f^M. \tag{17}
\]

It is known that \(-p_3^{ss} = \det J^{ss}\) is equal to the product of the roots. A necessary condition for local equilibrium determinacy can be provided with \( p_3^{ss} > 0 \). In this case, it means that the signs of the roots may be “+++” or “----”. Due to the fact that the system has two “jump” variables, consumption and inflation, and one “non-jump” (predetermined) variable, bonds, the equilibrium can only be locally determinate when the system possesses two roots with a positive real part.

The Routh-Hurwitz criterion on stability applies necessary and sufficient conditions for the real parts of all the roots to be negative if

\[
p_1^{ss} > 0, p_2^{ss} > 0, p_3^{ss} > 0, ds^{ss} = p_1^{ss} p_2^{ss} - p_3^{ss} > 0. \tag{18}
\]

In case the Routh-Hurwitz criterion is not satisfied, we identify the pattern of the real parts of the roots as “+++”. Consequently, the equilibrium is locally determinate if

\[
p_3^{ss} > 0 \text{ and } \{ p_1^{ss} < 0 \text{ or } p_2^{ss} < 0 \text{ or } ds^{ss} < 0 \}. \tag{19}
\]

Following [6, 23-25] and taking into consideration the assumption that consumption in steady state is close to 1, we have chosen the following values of parameters:

\[
r = 0.0401; \quad \rho = 0.04; \quad \xi = 0.005; \quad e = 10; \quad \theta = 100; \quad \varphi = 1/2; \quad \tau = 0.168; \quad g = 0.15. \tag{20}
\]

These values of parameters lead to \( C_{ss} = 1.0099 \) and \( R = 1.1278 \). Taking into account (15)-(17) and (18)-(20), we express the conditions on local equilibrium determinacy depending on \( f^M, f^F \):

\[
f^F < 0.0803, \tag{21}
\]

\[
f^M < 0.2977778 f^F - 0.0119706, \tag{22}
\]

\[
f^M > -0.0000297(f^F - 0.0401)/ (f^F - 0.0402), \tag{23}
\]
\[ f^M > -7.4258797(f^F)^2 + 0.8940759 f^F - 0.0239413. \] (24)

Therefore, the equilibrium is locally determinate if the intersect of the set determined by (23) with the sets defined by (21), (22), or (24), is not empty.

3.2. Graphical analysis

Figure 1 demonstrates the areas where the conditions (21), (22), and (24) are satisfied (upper left panel — condition (21), upper right panel — condition (22), lower left panel — condition (24)). Corresponding areas are shaded grey. In the lower right panel, the conditions (21), (22), (24) are combined together. It can be shown that at any point of the graph, the condition (23) intersects with at least one other condition (21), (22) or (24). As a result, the key point in finding equilibrium determinacy is to analyze the condition (23).

Figure 1. The areas of conditions satisfaction.

Figure 2 demonstrates the area where the condition (23) is satisfied. It is shown that if monetary policy is active (\( f^M > 0 \)), local equilibrium determinacy is achieved not depending on fiscal policy regime. The result of active monetary policy regime always leading to equilibrium determinacy was previously reported in [14, 19-20].

Figure 2. The area of condition (23) satisfaction.
However, it is necessary to take into consideration that the function that defines the condition (23) is not linear. Moreover, it has a gap point in arguments \( f^F \neq 0.0402 \) and should be analyzed separately within two subranges of \( f^F \). For this purpose, we analyze the condition (23) within different intervals.

Analyzing the areas where the condition (23) is not satisfied (Figure 3), we conclude that it is possible to fail to achieve local equilibrium determinacy while the monetary policy is active (central and right panels of the Figure 3), as well as to have local equilibrium determinacy while the monetary policy is passive (left and right panels of the Figure 3). These possibilities can be realized in case the Taylor coefficient \( f^M \) is close to zero.

![Figure 3. The area of condition (23) satisfaction in another scale (red horizontal line is an asymptote of function (23); red vertical line demonstrates \( f^F = r = 0.0401 \).](image)

In the left panel of the Figure 3, it is shown that if the Taylor coefficient of the monetary policy rule is around \(-0.0000298 < f^M < -0.0000297\), passive monetary policy and active fiscal policy can ensure local equilibrium determinacy. The central panel of the Figure 3 demonstrates that it is possible to achieve local equilibrium determinacy when the monetary and fiscal policies are both passive. In this case the Taylor coefficients should be around \(-0.0002 < f^M < 0\) and \(0.0401 < f^F < 0.0402\). Moreover, the central panel shows that active monetary policy does not necessarily lead to local equilibrium determinacy. For example, if the Taylor coefficients are in the intervals \(0 < f^M < 0.00002\) and \(0 < f^F < 0.0402\), local equilibrium indeterminacy can occur. The right panel demonstrates the existence of local equilibrium determinacy in case of passive monetary and fiscal policies, when \(f^F > 0.0402\).

4. Conclusion
We consider a New Keynesian model with Rotemberg price setting mechanism and finitely living consumers. We analyze the Taylor coefficients related to the rules for monetary and fiscal policies in terms of their influence on local equilibrium determinacy. We determine the range of the Taylor rule parameters that lead to the equilibrium being locally determinate and indeterminate. Finding a range of parameters with different policies regimes makes it possible to propose new approaches for stabilizing the macroeconomic system and to further investigate and describe the irregular dynamics of the model. Our main results are as follows. First, local equilibrium indeterminacy can be achieved when the monetary policy is active (especially, when the Taylor coefficient of the monetary policy rule is close to zero and \(0 < f^F < r\)). This means that active monetary policy does not guarantee the uniqueness of the equilibrium solution. Second, there is a range of the Taylor rule coefficients corresponding to the monetary and fiscal policies, being both passive, that lead to local equilibrium determinacy. Third, when
the Taylor coefficient of the monetary policy rule is close to zero, switching fiscal policy from active to passive regime can influence the equilibrium determinacy of system (1). Finding a range of parameters within different policies regimes that lead to the equilibrium indeterminacy influences the investigation of an undetermined dynamics in the model. That leads to an unpredictable behavior of agents that may cause complex, perhaps chaotic, behavior of the system. Therefore, controlling parameters of the policies' rules, the government and the CB can influence the existence of local equilibrium determinacy and the dynamics of the system. This field needs to be better investigated under the application of control theory.

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