Abstract

At a surface between electromagnetic media the Maxwell equations allow either the usual boundary conditions, or exactly one alternative: continuity of $E_\perp$, $H_\perp$, $D_\parallel$, $B_\parallel$. These ‘flipped’ conditions on the top and bottom surfaces of an FQH layer capture all its known static electromagnetic properties and so may be considered a deduction from microscopic quantum theory, yet are unobtainable in any realistic, purely classical model. An unrealistic model with free magnetic monopole currents illustrates this.

At the microscopic level, the FQH system is a laboratory for the particle concept. Identifying quasiparticles in terms of kinematics or in terms of asymptotic states gives two different perspectives. The kinematics exhibits exclusion rules similar to those in exactly solvable models for quantum systems in one space dimension. An effort here to tie some of these aspects together may be useful as a foundation for a future more comprehensive assessment of the roles and limitations of quasiparticles.
Both the original [1] and the fractional quantum Hall effects [2] are striking illustrations of ways in which quantum phenomena can violate classical intuition. In particular, the fractional effect [FQHE] is remarkable for many reasons, not least that in a very short time it found its ‘standard model’ in the composite-fermion picture [3], which unifies Laughlin’s original description of simple Hall fractions \( \nu = \frac{1}{2p+1} \) with a host of other observed phenomena in the fractional Hall domain, as well as earlier understanding [5] of the integer quantum Hall effect. Perhaps it is because progress in microscopic theory has been so rapid that a familiar stage from previous studies of macroscopic systems – phenomenological description in terms of an electromagnetic medium – appears to have been skipped. Another possible reason is that the thickness of the layer (\( O(500\,\text{Å}) \)) is small enough that attempting a macroscopic description might have been viewed as a risky enterprise.

One purpose of the present work is to show that indeed there is a unique, internally consistent variation on usual macroscopic descriptions of electromagnetic media which does fit the FQHE perfectly. This description cannot be reproduced by any realistic classical model, although a model involving copious supplies of freely moving magnetic monopoles (objects yet to be observed) can account for most of the desired features, thus underlining the exotic character of the FQHE. Another feature with roots in classical physics which remains essential in quantum considerations is the concept of a particle. Quasiparticle excitations are used to describe phenomena for FQH just as for many other systems, but there are a number of subtleties about how and to what extent a description like one associated with familiar elementary particles remains valid in this setting. The second goal of this work is tying together some relevant observations scattered in the literature, as a way of laying a foundation for a comprehensive discussion of quasiparticles, their nature and their fragility.

Jain noted [3, 6] that the FQHE can be described as combining a familiar property, renormalization of local quasiparticle charge by polarization of the medium, with an en-
tirely novel property extending even beyond the fractional Hall plateaux, renormalization of the perpendicular magnetic field and tangential electric field inside the medium with respect to values in the external regions immediately adjacent to the surfaces of the Hall layer. Such multiplicative renormalization of external electromagnetic field components in turn was a development of the notion that in the medium there is gauge coupling not only to the electromagnetic field but also to a Chern-Simons field whose strength is proportional to the electron density [7, 8], yielding an additive renormalization of the effective field. These results for FQHE may be summarized by the statements:

1) The Gauss-law charge of a quasiparticle, measured by its electric field far away, is $e^\ast$, where the ratio $e/e^\ast = 2pn \pm 1$ is related to the Hall fraction $\nu = \frac{n}{2pn \pm 1}$.

2) Quasiparticles move in the presence of an electromagnetic field as if they carried electric charge $e^\ast$ – or equally well as if they carried a charge $e$, but in fields $\mathbf{B}_\perp^\ast = e^\ast \mathbf{B}_\perp/e$ and $\mathbf{E}_\parallel^\ast = e^\ast \mathbf{E}_\parallel/e$.

Further, it is well accepted that

3) If an electron were gently inserted into the surface of a Hall layer, it would break up into $2pn \pm 1$ quasiparticles [9].

A naive effort to describe the Hall layer as a conventional dielectric medium fails at once. To take the most elementary aspect, far from a charge located in a thin dielectric layer there is no renormalization of that charge by the dielectric constant of the medium, because the surface charge which together with the local charge makes a total $e$ is itself localized quite close to the particle. By the same token, property 2) also does not hold. Is there any other option? The usual boundary conditions for the electromagnetic fields at a surface between two different media are continuity of $\mathbf{D}_\perp, \mathbf{B}_\perp, \mathbf{E}_\parallel, \mathbf{H}_\parallel$. These conditions follow directly from a classical model of the medium as a collection of small electric and magnetic dipoles whose orientation and/or dipole moment magnitude is affected by the applied electric or magnetic field.

However, it is consistent with the Maxwell equations to interchange simultaneously...
the roles of $E$ and $D$, $B$ and $H$, imposing continuity of $E_\perp, H_\perp, D_\parallel, B_\parallel$. Note that just interchange of one pair would not be consistent. An easy way to see this is that such mixed boundary conditions would violate the duality-rotation symmetry of the Maxwell equations (discussed more below) under which $E \rightarrow H$ and $D \rightarrow B$. Thus, there is a unique alternative to consider as a possible description of the FQHE. Let us test it.

Assume the dielectric constant $\epsilon$ and magnetic permeability $\mu$ are given by $\epsilon = \mu^{-1} = e/e^*$. Then continuity of $E_\perp$ at the top and bottom surfaces assures that the total electric flux coming out of a charge placed in the layer corresponds to a charge $e^*$, not $e$, thus confirming 1). Continuity of $H_\perp$ yields a perpendicular magnetic field inside the layer of magnitude $\mu B$, and continuity of $D_\parallel$ implies a parallel electric field inside of magnitude $E/\epsilon$, confirming 2). If an electron descends into the Hall layer, total local charge $e$ must be conserved, but inside the layer each individual charge is $e^*$. In a conventional insulator, the electron would leave part of its charge on the surface, but in our context, where distributing the residual charge in the surface of the layer is not possible, the only way to achieve local implementation of the conservation law is by generation of $2pn \pm 1$ quasiparticles. Thus, even fact 3) follows from this novel set of constitutive relations. Note that, although the treatment here is not explicitly quantum-mechanical, local conservation of electric charge requires quantization of $\epsilon$ at integer values, as otherwise conservation of charge through breakup would not be possible \[9\]. The fact that only odd integers are allowed follows because at the core of each quasiparticle must be a fermion, and by a superselection rule an odd number of fermions may not turn into an even number \[10\].

In other condensed-matter settings, quasiparticles often are described as dressed electrons, meaning they have renormalized local charge, but unrenormalized Aharonov-Bohm or Lorentz-force charge $e$, i.e., in a specified electromagnetic field a quasiparticle experiences exactly the same influence as would a free electron in the same field configuration. Clearly that assumption is intrinsic in the above deductions also. However, this leads to a potentially disturbing result, that penetration of an electron into the layer, with result-
ing breakup, does not conserve Aharonov-Bohm charge. For a gauge shift $\lambda$ nominally uniform throughout space, this result can be made consistent with usual considerations of gauge invariance if we insist that inside the layer the corresponding shift is $\lambda^* = \lambda/\epsilon$.

Now this definition is nothing more than a convenient bookkeeping choice, but if we go on to consider the spatial variation of $\lambda$ in a direction parallel to the Hall layer, then we are forced by the boundary conditions to insist that this variation, which is related to the tangential part of the vector potential $A$, must be modified by the factor $\epsilon^{-1}$ inside the layer compared to outside. The reason is that in $A_0 = 0$ gauge the time derivative of $A$ gives the electric field $E$, and therefore the tangential component of $A$ must be renormalized exactly as the tangential component of $E$. Thus, the only consistent choice in this scheme is one in which an otherwise continuous $\lambda$ defined everywhere is renormalized to $\lambda^*$ inside the layer. The same conclusion follows if we pay attention to the discontinuity in $B_\perp$, emphasizing the necessity already discussed of interchanging both magnetic and electric field boundary conditions together to assure mathematical consistency of the description.

The consequence of $\lambda$ renormalization is that $\sum \lambda_i e_i$ for all charges must be time independent when an electron passes into the layer, making non-conservation of AB charge not only consistent but actually required in this context.

It is interesting to ask if there might be a strictly classical model which would exhibit all or some of these features. As electric charge should be continuously adjustable in such a model, we would not expect the quantization of $e/e^*$ which characterizes FQHE, and therefore would not expect breakup as an electron enters the Hall medium. As we have seen already, that implies a failure of local conservation of electric charge. Recognizing these limitations, let us see whether a model reproducing features 1) and 2) is possible. For a classical medium, there is no escape from the usual boundary conditions, but one may describe electric charge in an exotic way: Consider a solenoid carrying not ordinary electric current but rather magnetic-monopole current. Then one end of the solenoid looks
very much like an electric charge. Suppose the solenoid is very long (so that interactions involving the other end of the solenoid may be ignored) and lies in a direction parallel to the Hall layer. We may call the nearby end an ‘artificial’ charge.

Electric-magnetic duality tells us that monopoles respond to $H$ and $D$ in the way that electric charges respond to $E$ and $B$. An early example of this conclusion came in the elegant energy-conservation argument of Kittel and Manoliu that a monopole interacting with a ferromagnet cannot respond to $B$. Another way to see this is to require that the Maxwell equations in a medium should exhibit duality symmetry. In that case, the usual boundary conditions evidently are consistent with duality only if the field correspondences are as indicated. To complete the correspondence, we must identify a free monopole as an unrenormalized source of $B$ and $E$ in just the way that a free electric charge is an unrenormalized source of $D$ and $H$. These interchanges mean that a medium which is dielectric for ordinary charges and diamagnetic for artificial magnetic monopoles (i.e., ends of ordinary solenoids) is ‘para-electric’ (antiscreening) for artificial charges and paramagnetic for true magnetic monopoles. For this reason, classical considerations of the constitutive relations already imply that renormalizations of true magnetic and electric charges must be reciprocal to each other, making renormalization consistent with Dirac’s quantum condition on the product of electric and magnetic charge. This issue was a source of confusion at an earlier stage in quantum-theoretical studies of monopoles, but in that context was clarified long ago.

The above considerations imply that we want to allow our artificial charges to move in a layer with usual dielectric constant $\epsilon' = \mu'^{-1} = e^*/e < 1$, i.e., a para-electric and paramagnetic medium for ordinary charges. This means that when the solenoid is submerged in the Hall layer it will have a $D$ flux through its end smaller by a factor $e^*/e$ than when the solenoid is outside the layer, and hence the effective charge of an isolated quasiparticle will be $e^*$, just as required. It is easy to check that the other requirements also are obeyed. Of course, a para-electric medium is itself peculiar, but perhaps one
may be forgiven for assuming such a system in a hypothetical world where solenoids of magnetic current are available!

As mentioned, the above description implies that when the solenoid descends into the layer its electric flux is bleached, with flux lines disappearing all along the length of the solenoid simultaneously. That makes the descent of a charge into the layer, and its resulting attenuation, a highly nonlocal process. Of course, this classical model is extraordinarily different from the actual quantum system, but the nontriviality of the conversion from ‘electron’ to ‘quasiparticle’ is found again. For entrance of an actual electron into the Hall layer, electric charge is conserved but particle number is not, while the classical model conserves particle (i.e., solenoid end) number, but lets charge leak to the other end of the solenoid. The model is less flexible than the constitutive relations plus boundary conditions, because it would give the same peculiar properties at a second boundary between two non-FQHE layers, contrary to experience. Nevertheless, the model may be helpful for visualizing the notion of flipped boundary conditions, and understanding why they could never occur for conventional classical systems.

So far we have seen how the constitutive relations and surface boundary conditions reproduce known results, but one might wonder if they can give any new information. One obvious question to address is whether the Hall layer exhibits new trapped modes, which in turn might be detected by scattering experiments. For electromagnetic waves, as mentioned just above, the Hall layer is equivalent by duality to a para-electric, paramagnetic layer. Such a system has no trapped modes, because critical internal reflection requires refractive index greater than unity inside, and in this case we have (relative) refractive index exactly equal to unity. Furthermore, the fact that the layer must be quite thin on the wavelength scale relevant to the FQHE means that even reflection of external waves must be strongly suppressed. Besides possible optical modes, one might wonder if there could be a longitudinal mode, but any such mode must have finite mass because of the incompressibility of the FQH ground state in a specified perpendicular magnetic field.
Of course, for compressible states, as exist near $\nu = \frac{1}{2}$, there can be longitudinal modes, and consideration of these has led to the suggestion that there may be modifications of the simple composite-fermion Fermi surface expected if these modes were ignored. Recent studies have focused on these longitudinal ‘plasmon’ modes to obtain remarkably detailed analytic results for both the compressible regime and the incompressible regime (where the plasmon has an effective mass).

Of course, the defining property of the FQH layer is the Hall effect. How is that incorporated into this formulation? If we label the direction normal to the layer as $\hat{z}$, then in view of the boundary conditions the Hall current should be given by

$$J = \nu \frac{e^2}{h} f(z) \hat{z} \times D.$$ 

Here the integral of $f(z)$ over the thickness $t$ of the layer is normalized to unity, so that $f$, which reflects the (rigid) distribution of the contributing electron wave function in the $z$ direction, scales inversely with $t$. For one example, associated with a plane electromagnetic wave propagating parallel to the layer (with its $B$ perpendicular to the layer), there will be an induced oscillating longitudinal electric current, and by local charge conservation an associated oscillating electric charge density. These currents should be at most weak sources of electromagnetic radiation, because they are longitudinal except at edges and surfaces of the sample. On the other hand, an incoming wave with normal incidence would induce strong dipole radiation from the sample as a whole, having polarization rotated by $\pi/2$ with respect to that of the incident wave.

The dielectric response function of any medium should depend on frequency and wavenumber. Provided $\epsilon$ and $\mu$ approach unity as the frequency rises above some critical value, the unique description proposed here should remain in agreement with observation. In particular, it should be possible to describe the full frequency and wavenumber dependence of $\epsilon$, including a peak in Raman response found for the $\nu = 1/3$ state.

Another aspect which deserves attention is the boundary conditions on the edge surfaces of the FQH layer, which of course are quite narrow compared to the top and bottom
surfaces. At an edge one must have (partially) the traditional conditions. This is immediately clear for \( D_\perp \) and \( B_\perp \) and for the components of \( H_\parallel \) and \( E_\parallel \) lying perpendicular to the Hall plane. On the other hand, consistency with conditions on the top and bottom surfaces also imply that the components of \( D_\parallel \) and \( B_\parallel \) lying in the Hall plane should be continuous. At first sight these hybrid conditions might seem unsatisfactory. However, because the strong external magnetic field perpendicular to the layer determines a special direction, these requirements are no more peculiar than the familiar appearance of a complicated dielectric tensor in some anisotropic medium.

If anything, it is remarkable that in this case the electric and magnetic susceptibilities are just scalars, with the anisotropy entirely described by the boundary conditions. One might ask whether the hybrid boundary conditions might lead to some interesting behavior near the edge of the sample. It seems plausible that the answer might be affirmative. Continuity of \( D_\perp \) suggests that a single electron entering might proceed as a quasiparticle into the medium, leaving the remainder of the charge to populate edge states, and thus mimicking – on the edge only – the induced charge found on the surfaces of a traditional insulator. Whether this possibility gives any useful insight about edge states remains to be seen.

Even though the emphasis above has been on a macroscopic description, we were compelled to focus on quasiparticles, as the objects which act as sources of the fields described, fields which in turn influence the motion of the particles. Therefore it may be worth noting that the term ‘quasiparticle’ is used in two related but distinct senses. The one of primary interest here is as an excitation which is stable in isolation, having well-defined properties determining its (weak) long-range interactions with other quasiparticles. The related meaning is as an entity appearing in an effective dynamics divided into a ‘kinetic’ part for which these quasiparticles move without interaction, and an ‘interaction’ part which is weak enough to be treated by perturbative techniques.

Composite fermions are quasiparticles in this second sense, and therefore are spin-half
particles with intrinsic electric charge equal to that of an electron, and ordinary Fermi-
Dirac statistics. However, the weak residual-interaction part of the dynamics implies that
the phenomenological quasiparticles associated with the first meaning have fractional local
charge [4], exhibit a dynamically induced long-range interaction [18] associated with the
concept of ‘fractional statistics’ [19, 20], and possess a well-defined fractional localized
contribution to the spin [6, 21]. Failure to distinguish between these two meanings of a
single term can easily lead to confusion. This discussion brings up a subtle discontinuity
between the macroscopic and microscopic descriptions. Composite fermions, being just
strongly-correlated electrons, have conserved Aharonov-Bohm charge $e$ just like electrons.
Thus in the composite-fermion picture the addition of an electron not only produces an
increase of AB charge for the several quasiparticles produced, but also a decrease in the
number of composite fermions, and hence the AB charge, of the ground state on which the
quasiparticle excitations are built [6]. Because this decrease is spread uniformly over the
entire Hall layer, it represents an intrinsically nonlocal aspect of the electron ‘splitting’,
and by the same token becomes unobservable even in principle once the thermodynamic
limit holds.

The subtle limitations on the applicability of the quasiparticle concept illustrated by
this discussion are part of a pattern which is getting increasing recognition. The descrip-
tion of quasiparticle dynamics tends to involve constraints on allowed configurations which
are more complex than the simple Bose-Einstein or Fermi-Dirac statistics [22]. Especially
notable, and relevant to our case, are generalized exclusion rules found in exactly soluble
quantum problems in one space dimension, where adding a quasiparticle or quasihole to
a configuration modifies the number of available hole and particle states, and not just
the ones which have been emptied or filled [23]. A pertinent example is the resonance
observed in Raman scattering for the $\nu = \frac{1}{3}$ system [17], which one would like to describe
as a quasiparticle-quasihole pair. The most obvious excitation at small spatial momen-
tum transfer is one in which the created quasiparticle sits geometrically on top of the hole
formed by the excitation. However, because all the states under consideration lie in the lowest Landau level in the external magnetic field, Kohn’s theorem [24] forbids this pure dipole configuration to occur, and thus gives directly a constraint on which particle-hole configurations can form. It seems possible that more detailed examination of the allowed excitations will show a strong correspondence with what has been found in exactly solvable models, but perhaps also some significant differences, as it is by no means obvious that the FQH dynamics constitute an exactly solvable system.

We may conclude that the long-wavelength physics of the FQHE is captured by introducing the familiar constitutive relations for the dielectric and diamagnetic response of the Hall layer, except that the conventional continuity conditions on the electromagnetic fields at the top and bottom surfaces are interchanged between perpendicular and parallel components of the fields. No simple classical system duplicates this (although an unrealistic model with artificial electric charges comes close). Thus, the flipped surface conditions express in macroscopic terms the quantum ‘magic’ of the fractional quantum Hall effect. At the same time, phenomena associated with excitations, including addition of an electron to the system as well as bosonic excitations which evoke description in terms of particle-hole pairs, give a fascinating glimpse for a real system of ways in which the particle concept begins to fray when it is pushed very hard. Work is underway on exploring this phenomenon, and its connections with behaviors for exactly soluble models in one space dimension [25].

This work was supported in part by the National Science Foundation. I thank Jainendra Jain and Barry McCoy for discussions, and Steven Kivelson for emphasizing some time ago that a conventional electromagnetic medium fails to describe the fractional quantum Hall effect.
References

[1] K. v. Klitzing, G. Dorda, and M. Pepper *Phys Rev Lett* **45** 494 (1980).

[2] D.C. Tsui, H.L. Stormer, and A.C. Gossard *Phys Rev Lett* **48** 1559 (1982).

[3] J.K. Jain *Phys Rev Lett* **63** 199 (1989).

[4] R. B. Laughlin *Phys Rev Lett* **50** 1395 (1983).

[5] R.B. Laughlin *Phys Rev B* **23** 5632 (1981).

[6] A.S. Goldhaber and J.K. Jain *Phys Lett* **199A** 267 (1995).

[7] S.-C. Zhang, H. Hansson, and S. Kivelson *Phys Rev Lett* **62** 82 (1989).

[8] A. Lopez and E. Fradkin *Phys Rev B* **44** 5246 (1991).

[9] W.P. Su *Phys. Rev. B* **34** 1031 (1990).

[10] G.C. Wick, A.S. Wightman, and E.P.Wigner *Phys Rev* **88** 101 (1952).

[11] C. Kittel and A. Manoliu *Phys. Rev. B* **15** 333 (1977).

[12] P.A.M. Dirac *Proc R Soc London, Ser A* **133** (1931).

[13] J. Schwinger *Phys Rev* **151** 1048; 1055 (1966).

[14] S. Coleman *The Unity of Fundamental Interactions*, edited by A. Zichichi (Plenum, New York, 1983) p 21.

[15] B.I Halperin in Perspectives in the Quantum Hall Effect Eds S. Das Sarma and A. Pinczuk (Wiley, New York 1997) 225.

[16] R. Shankar and G. Murthy, *Phys Rev Lett* **79** 4437 (1997).
[17] A. Pinczuk in Perspectives in the Quantum Hall Effect Eds S. Das Sarma and A. Pinczuk (Wiley, New York 1997) 307.

[18] B.I. Halperin Phys Rev Lett 52 1583 (1984).

[19] J.M. Leinaas and J. Myrheim, Nuovo Cimento 37B 1 (1977).

[20] F. Wilczek Phys Rev Lett 49 957 (1982).

[21] T. Einarsson, S. L. Sondhi, S. M. Girvin, and D. P. Arovas Nucl Phys B441 515 (1995).

[22] F.D.M. Haldane Phys Rev Lett 67 937 1991).

[23] R. Kedem, B.M. McCoy and E. Melzer in Recent Progress in Statistical Mechanics and Quantum Field Theory Eds P. Bouwknegt et al. (World Scientific, Singapore 1995) 195.

[24] W. Kohn Phys Rev 123 1242 (1961).

[25] A.S. Goldhaber, J.K. Jain, B.M. McCoy, and K. Park (in preparation).