Day-to-day traffic dynamics: laboratory-like experiment on route choice and route switching in a simple network with limited feedback information

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Abstract

An experimental analysis of day-to-day route choice dynamics in a simple three-route network is described in this paper. A laboratory-like experiment involving thirty subjects was conducted over a 50-period time span. Participants acted non-cooperatively and at each period received only feedback information on travel times of chosen routes. The results indicate that the day-to-day route choice process was characterized by a high degree of volatility, and that User Equilibrium, even though reached occasionally, did not persist as a steady state of the network. However, the detection of statistically significant trends underlying the evolution of network state descriptors suggests the possibility that equilibrium conditions might have been approached over an appropriately extended time horizon. Individual route switching frequency was found to be significantly correlated to average experienced travel time, while both variables did not exhibit any significant relationship to personal characteristics of participants. Finally, observations collected in the experiment were used to derive estimates of behavioral parameters of a deterministic process model of day-to-day route choice dynamics.

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Keywords: Traffic networks; route choice; route switching; day-to-day dynamics; experimental analysis.

1. Introduction and purpose of the study

The analysis of day-to-day route choice dynamics has emerged in the last three decades as one of the most interesting and challenging areas of research within the field of transport network modeling. Understanding and forecasting this type of dynamics is now believed to be a necessary step towards enhancing the realism of traffic

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assignment models as well as an essential tool for improving the design and operation of Advanced Traveler Information Systems (ATIS). The day-to-day dynamic approach to the analysis of route choice represents a significant departure from the traditional equilibrium paradigm, whose limitations have been emphasized by several authors.

Broadly speaking, the study of day-to-day route choice dynamics can be approached either from a theoretical or from an experimental standpoint. The first is concerned with the formulation of models representing the evolution of network state variables (flows and costs) over successive time periods (typically days), either as a deterministic or stochastic process, in continuous or discrete time (e.g. Horowitz, 1984; Smith, 1984; Cascetta, 1989; Cantarella & Cascetta, 1995; Bie & Lo, 2010). Emphasis in these studies is on analyzing the convergence of appropriately defined dynamic processes to different kinds of attractors, as well as the stability properties of network equilibria. The second approach aims at observing directly route choice behavior of travelers over time, sometimes in combination with departure time choice. Due to the practical difficulty of collecting real-world observations of the above choices over extended time periods, laboratory-like experiments have been the preferred way of performing such empirical analyses (see the literature review in Section 2). While in this type of studies the choice situation is hypothetical but the subjects involved are real individuals, a further alternative is to employ a simulation model to analyze the day-to-day evolution of traffic states resulting from the interaction of drivers' behavior and network performance; see, for example, Nakayama et al. (1999), Nakayama and Kitamura (2000).

Regardless of the adopted approach, scenarios with or without information supplied by external sources (e.g. ATIS) can be considered. In the former case, descriptive information provided to users could be purely historical (e.g. a record of past travel times on chosen and non-chosen routes) or predictive (forecasts of travel times with reference to future periods). In addition, ATIS may typically provide drivers with recommendations on "best" routes to follow (prescriptive information). On the other hand, when information from outside sources is not available, a key issue is the analysis of how drivers learn about network performance (travel times in particular) based on previous travel experiences, a process often studied using theories from cognitive psychology (for example reinforcement learning). Network knowledge acquired through this kind of process is often referred to as experiential information.

The present study describes and analyzes a laboratory-like experiment on repeated route choice in a simple network, addressing in particular the case in which drivers do not have access to any information other than their own experienced travel times. This is accomplished by providing participants with feedback information which is limited to the performance of the route chosen in the most recent repetition of the experiment. The paper is organized as follows. A brief review of previous experimental research on day-to-day route choice dynamics is provided in Section 2. The main features of the experimental setup are described in Section 3. Statistical analyses performed on the experimental data in order to identify significant correlations and temporal trends of the relevant variables are presented and discussed in Section 4. Section 5 describes the implementation of a deterministic process model of day-to-day route choice dynamics on the experimental network and presents the results of the estimation of the relevant parameters based on the collected choice observations. Concluding remarks are outlined in Section 6 together with possible developments of the study.

2. Literature Review

Experimental studies dealing with the day-to-day dynamics of the interaction between travel choices (in particular departure time and route decisions) and congestion experienced by network users have been carried out by several authors in the last three decades.

One of the earliest studies on this subject is the paper by Mahmassani et al. (1986), who performed a controlled experiment with actual commuters choosing their departure times in a simulated traffic corridor over a period of 24 days. Even though route choice was not investigated, this study provides many interesting insights
regarding general properties of the interaction between individual behavior and collective outcomes in a traffic system. In the particular experiment described by Mahmassani and co-workers, a steady state of the system was reached in a relatively short time (21 days).

Iida et al. (1992) conducted a laboratory-like experiment in which a group of 40 commuters were asked to select one of two alternative routes joining an origin and a destination, and to repeat this choice for twenty days. More specifically, the experiment involved both route travel time forecasting and route choice by participants, and two different scenarios were simulated with respect to the type of information available to users (only actual travel time of last chosen route or a complete record of predicted and actual travel times of chosen routes for each day). Unlike the study by Mahmassani and co-workers, the findings of this study do not support the notion of equilibrium, because convergence to a steady state was not observed in either information scenario, although the amplitude of system state oscillations was larger when only the actual travel time of the last chosen route was provided to participants. The study also tried to identify different types of travelers with respect to the frequency of route switching.

Mahmassani and Liu (1999) investigated day-to-day commuters’ behavior in response to Advanced Traveler Information Systems (ATIS) using an interactive travel simulator. They considered departure time and pre-trip route selection as well as en-route switching decisions over a period of five days, and used the data collected in the experiment to estimate a model based on the notion of "indifference bands". The findings of the study indicate, among other things, that the availability of real-time information tends to increase the frequency of route switching (both pre-trip and en-route) as compared to situations without real-time information.

Selten et al. (2007) performed an experimental analysis of repeated route choice in a simple hypothetical two-road scenario within a game-theoretical framework. Two different ways of providing travel time feedback information to participants were implemented: in the first, at each repetition subjects received information only on the travel time of the route chosen in the preceding period, while in the second the value of travel time of the non-chosen alternative ("foregone payoff") was revealed as well. In both scenarios it was found that, despite the very large number of simulated periods, route flows did not converge to their User Equilibrium (UE) values, but showed persistent fluctuations of significant amplitude. Nevertheless, the average values of route flows over the entire duration of experimental sessions turned out to be very close to UE. In their experiment, Selten and co-workers also found evidence of different "response modes" characterizing the route switching decisions. Two modes are defined in terms of the decision to change/confirm the route chosen in response to a "bad"/"good" payoff, or the opposite. Both modes were detected in the experimental observations, and the authors suggest that these different attitudes may be, at least in part, the result of a context-specific learning process.

The role of learning in the context of day-to-day route choice dynamics under travel information was investigated by Bogers et al. (2007). Their study focused on two specific types of learning, known in cognitive psychology as reinforcement and belief-based learning, and on the effect of memory in the formation of perceived travel costs. Based on data collected through a travel simulator in a large experiment involving 40 repetitions of choices from a three-route set, they estimated a Mixed Multinomial Logit model and concluded, in particular, that the most recent experience contributes about 20% to the formation of perceived route travel times. Their results also suggest that reinforcement learning and habit formation can play a significant role in day-to-day route choice.

Lu et al. (2011) studied the effects of information on repeated route choice in a simple hypothetical network through an interactive laboratory-like experiment. In particular, they investigated how en-route real-time information and ex-post information on foregone payoffs could affect route flows, total system travel time and number of route switches in an uncertain environment characterized by the occurrence of random network disruptions such as incidents.

Tawfik et al. (2010) used a driving simulator to carry out a controlled experiment on repeated choice in a simple two-route network involving fifty participants. They identified four different patterns of evolution of individual route choices over time, and suggested that drivers' route choice behavior is likely to be affected by
socio-demographic factors and driving experience. In a subsequent paper, Tawfik et al. (2011) used the same experimental data to estimate several models of route switching behavior.

Albert et al. (2011) explored the effect of personality traits on repeated route choice behavior. They performed a laboratory experiment in which 54 participants had to choose one of two routes in a simple real-world network and repeat their choice for fifty trials, receiving ex-post information only on the travel time of the chosen route. Based on the experimental observations, they used logistic regression to estimate a route choice model and a route switching model, and found, in particular, that geographic ability and sensation seeking characteristics of individuals had a significant impact on the propensity to switch route. Moreover, a negative relationship between the participants' familiarity with the trip and their frequency of route switching was detected.

More recently, Ben-Elia et al. (2013) investigated the impact of the accuracy of information provided by ATIS on repeated route choice using a travel simulator. Based on a panel of 36 individuals performing twenty repetitions of a route choice task and receiving descriptive, prescriptive and experiential feedback information, they concluded that decreasing information accuracy tends to reduce drivers' compliance with ATIS advice and to promote risk-averse route choice behavior.

3. Description of the experiment

A very simple interactive experiment on repeated route choice dynamics was carried out in this study. The main features of the experimental setup are as follows:

• thirty participants, all holding a valid driver license, were selected so as to ensure sufficient variability with respect to some basic personal characteristics (e.g. age, gender, years of driving experience);
• only route choice was available to participants (no departure time choice);
• route choices were made in a congested network consisting of three links connecting the same origin-destination pair;
• the links were intended to represent three routes having significantly different geometric and functional characteristics (central urban road, arterial road, freeway-type city bypass);
• at each period (day) of the experiment, individual route choices were aggregated into link flows and used to compute the corresponding travel times by means of simple BPR-type performance functions;
• the different characteristics of the available routes were reflected by different numerical values of free-flow travel time, capacity and functional coefficients adopted in the respective performance functions;
• the experiment was conducted over a total of 50 periods, each intended to correspond to one day. In order to relate the experimental situation to the real-world experience of participants, the latter were asked to make their route choice twice a day and five days per week (for a total of five weeks). In this respect, we claim that our approach appears to be more realistic than that of other laboratory-like experiments reported in the literature, in which subjects were often asked to perform the route choice task a very high number of times in a period of a few hours or less;
• at the end of each period, each participant was informed about the value of travel time prevailing on the link he/she had chosen (computed as a function of link flows through the adopted performance functions);
• no information was provided on the travel times of the non-chosen alternatives;
• at the beginning of the experiment, the values of free-flow travel times were communicated to the participants, together with general indications about the type of infrastructure corresponding to each link, while the values of link capacities were not revealed to participants;
• the participants were not aware of the total number of subjects involved in the experiment;
• in the survey presentation, the participants were instructed to avoid any form of cooperation or communication with others and any competitive attitude during the experiment;
• based on the total number of travelers and on the assumed travel time functions, deterministic User Equilibrium (UE) and System-Optimum (S-O) flows were computed, to be used as reference states in the subsequent analysis of experimental results.

4. Statistical analysis of experimental results

The main results of the experiment are presented and analyzed in this section, considering in particular route choice and route switching behavior and their evolution over time. More specifically, the following research questions are addressed in our analyses:
- convergence (or lack of it) of route flows to the respective values predicted by the UE traffic assignment model;
- detection of statistically significant trends in the temporal evolution of aggregate descriptors of network performance and switching behavior over the time span of the experiment;
- search for statistically significant correlations between the above descriptors;
- investigation of possible relationships between these descriptors and personal characteristics of experiment participants.

The detailed findings of the study with reference to the above questions are presented in the following subsections. While the analyses reported in this section are entirely based on a descriptive statistical approach, the results obtained from the implementation of a deterministic process model of day-to-day route choice dynamics estimated on the basis of the available experimental observations are presented in Section 5.

4.1. Day-to-day evolution of route choices and network performance

In order to answer the first question of the above outline, the initial task was to compute the values of route flows and mean network travel time corresponding to the UE state. The System-Optimum values of the same variables were also computed as a further reference point for the analysis. These calculations were performed based on the given origin-destination travel demand (equal to the total number of participants) and the adopted BPR travel time functions; the results are presented in Table 1. We observe that, in the specific case under consideration, the UE and S-O solutions are very similar in terms of both flows and mean network travel time.

Table 1. Calculated values of UE and S-O state descriptors

|                  | Route 1 | Route 2 | Route 3 | Network     |
|------------------|---------|---------|---------|-------------|
| UE flow          | 8.55    | 9.99    | 11.46   | 30.00 (total) |
| S-O flow         | 8.11    | 9.87    | 12.02   | 30.00 (total) |
| UE travel time (min.) | 53.89  | 53.89   | 53.89   | 53.89 (mean)   |
| S-O travel time (min.) | 44.15  | 51.99   | 60.19   | 53.15 (mean)   |

The day-to-day evolution of route flows over the 50 repetitions of the experiment is shown in Figure 1, together with the corresponding evolution of mean network travel time. All descriptors are seen to exhibit wide oscillations that persist over the entire duration of the experiment, although it is possible to identify a few intervals in which the system enjoys a condition of relative stability in terms of mean travel time. Even though the total duration of the experiment is rather limited, our findings are similar to those emerging from studies involving a much larger number of trials (e.g. Selten et al., 2007). It is interesting to observe that UE conditions are approximately reached on a few occasions, but do not persist as a steady state.
Basic descriptive statistics computed on the values of route flows and mean network travel time over the entire time span of the experiment are shown in Table 2. It is seen that, despite the high volatility characterizing all network state descriptors, the time-average values of route flows are very close to their UE (and S-O) values, again in agreement with the findings of Selten et al. (2007). The same is not true, however, for the time-average value of mean network travel time, which is about 53% (56%) higher than its UE (S-O) counterpart.

Table 2. Average, standard deviation and coefficient of variation of route flows and mean network travel time over the experiment time span

| Route 1 flow | Route 2 flow | Route 3 flow | Mean network travel time (min.) |
|--------------|--------------|--------------|---------------------------------|
| Average      | 8.14         | 9.94         | 11.92                           | 82.65                           |
| Standard Deviation | 2.30         | 2.50         | 2.63                           | 30.76                            |
| Coefficient of Variation | 0.283       | 0.252        | 0.221                           | 0.372                           |

Fig. 1. Day-to-day evolution of route flows and mean network travel time
Given that the descriptive analysis presented so far does not reveal any apparent evolution toward a state of equilibrium, it is interesting to test the experimental data for the existence of a statistically significant temporal trend which could indicate a structural tendency of the system to approach equilibrium conditions in the longer term. It seems meaningful to perform such a test on the day-to-day series of mean network travel time, both because it is an aggregate descriptor of the network state (unlike route flows), and because, as indicated by the previous analysis, its experimental time average differs considerably from the theoretical equilibrium value (again unlike that of route flows).

Among the available techniques that can be employed to detect statistically significant trends in time series data, a nonparametric method known as Daniels' test on moving averages (Daniels, 1950) was chosen for use in the present study (see Appendix A for details). The results of this test for different numbers of periods used in the calculation of the moving average are shown in Table 3. Assuming a 5% threshold for the significance level of the test, it is seen that the null hypothesis of no trend is rejected when the moving average is computed over 5, 7 or 9 consecutive time periods. The non significant result obtained using \( k = 3 \) indicates that, in this case, the smoothing effect is not sufficient to reveal the underlying trend of the time series. Note that the sign of Spearman's rank correlation coefficient (\( \rho \)) is always negative, suggesting a tendency of mean network travel time to decrease towards the UE/S-O value.

| \( k \) | \( N \) | \( \rho \) | \( N-2 \) | Student's \( t \) | Significance (one tail) |
|---|---|---|---|---|---|
| 3 | 48 | -0.1902 | 46 | -1.313985 | 0.097683 |
| 5 | 46 | -0.2596 | 44 | -1.783124 | 0.040733 |
| 7 | 44 | -0.3125 | 42 | -2.132763 | 0.019417 |
| 9 | 42 | -0.3623 | 40 | -2.458407 | 0.009191 |

4.2. Day-to-day evolution of route switching

Route switching is the elementary mechanism that determines the day-to-day dynamics of the system. According to Wardrop's (1952) first principle, in an equilibrated network route switching activity should be absent, because all users are satisfied with their current travel time and thus have no incentive to attempt any change of route. In this subsection, route switching is analyzed in terms of both its day-to-day evolution and its variability across the individuals participating in the experiment.

Figure 2 illustrates daily route switching activity in aggregate terms (i.e. considering all participants), by showing its frequency distribution and its evolution over the 50 repetitions of the experiment, while Table 4 provides the basic descriptive statistics characterizing the distribution of daily switches.

It is observed that, on average, every day about one out of three participants has changed his/her route choice; moreover, the persistence of substantial switching activity over the entire time span of the experiment confirms the prevalence of disequilibrium conditions already noted in the analysis of route flows and mean network travel time. When trying to interpret this result, one should consider that route switching was the only means available to participants to improve their knowledge about travel times of alternative routes, given that information on foregone payoffs was not provided in our experiment.

Since the simple observation of the time series of daily route switches is not in itself sufficient to identify a possible structural trend in the evolution of this descriptor, a Daniels' test on moving averages was performed as already done for mean network travel time; any hypothetical long-term evolution of the system towards equilibrium conditions should cause the number of daily switches to decrease and approach zero in the long run.
The results shown in Table 5 strongly support the rejection of the null hypothesis of no trend for all tested values of \( k \), and the trend appears to be more robust than in the case of mean network travel time, as suggested by the values of Spearman's rank correlation coefficient and by the levels of significance. The trend is, again, characterized by a negative sign, confirming the conjecture that the network might have evolved toward equilibrium conditions over an extended time horizon. Note also that here, unlike the case of Table 3, the effect of \( k \) is not monotonic in the tested range of its values, as the best results are obtained for \( k = 7 \).

![Fig. 2. Aggregate number of daily route switches: frequency distribution and day-to-day evolution](image)

Table 4. Average, standard deviation and coefficient of variation of number of daily route switches

| Number of daily route switches | Average | Standard Deviation | Coefficient of Variation |
|-------------------------------|---------|--------------------|--------------------------|
| 10.69                         | 3.28    | 0.307              |

Table 5. Results of Daniels' test on moving averages of number of daily route switches (see Appendix A for the meaning of symbols)

| \( k \) | \( N \) | \( \rho \) | \( N-2 \) | Student's \( t \) | Significance (one tail) |
|---------|---------|---------|---------|-----------------|------------------------|
| 3       | 47      | –0.4408 | 45      | –3.294253       | 0.000964               |
| 5       | 45      | –0.4867 | 43      | –3.653664       | 0.000349               |
| 7       | 43      | –0.5157 | 41      | –3.853789       | 0.000201               |
| 9       | 41      | –0.5039 | 39      | –3.642967       | 0.000392               |
The number of individual route switches over the entire time span of the experiment was considered next. Basic descriptive statistics and the frequency distribution of this variable are shown in Table 6 and Figure 3, respectively. Individual switching behavior is characterized by a considerable degree of variability across participants, as indicated by the high value of the coefficient of variation. It can be seen that, on average, each participant has made a change of route approximately every three days.

| Number of individual route switches | Average | Standard Deviation | Coefficient of Variation |
|-----------------------------------|---------|--------------------|--------------------------|
| Number of individual route switches | 17.47   | 11.77              | 0.674                    |

Fig. 3. Frequency distribution of individual route switches over the entire experiment

4.3. Relationship between route switching and experienced travel times

Starting from the basic idea that, in a day-to-day dynamic context, route switching decisions are likely to be strongly motivated by users' satisfaction or dissatisfaction with their current and previous travel experiences (mainly in terms of travel times), the search for simple empirical relationships measuring this correlation in the experiment under analysis was undertaken next. As stated in the introductory remarks of Section 4, while the analyses described here are based on a statistical approach, and thus have an essentially descriptive character, a
A deterministic process model of day-to-day route choice dynamics dealing explicitly with aspects such as travelers’ learning, memory and habit is presented in Section 5.

More specifically, our analysis investigates the relationship between the following two variables: the total number of route switches performed by each participant during the entire experiment (see Figure 3 and Table 6), and the average travel time experienced by each participant during the entire experiment. Thus, a total of 30 observations are available for each of these variables; note, in particular, that the second is computed for each individual as an average over 50 time periods.

As a preliminary step in this analysis, a Chi-Square test on a $3 \times 2$ cross-tabulation based on intervals of values of the above variables was carried out in order to test the null hypothesis of independence between them. The results of this test led to the rejection of the null hypothesis at the 1.9% significance level, thus indicating the existence of a statistically significant relationship between number of individual route switches and average individual travel time.

In order to determine the functional form and the strength of this relationship, several models (both linear and nonlinear) were fitted to the experimental observations using ordinary least squares regression. The best results were obtained using the following power function:

$$AITT = 69.313 NIS^{0.0657}$$

where $AITT$ denotes average individual travel time and $NIS$ denotes number of individual route switches. Figure 4 shows a graphical illustration of the above regression model (linearized by taking natural logarithms of both variables), while Table 7 provides the values of the associated regression statistics.

![Graphical illustration of the above regression model](image)

### Table 7. Regression statistics for the log-linearized model relating average individual travel time to number of individual route switches (Number of observations = 30; Coefficient of determination $R^2 = 0.2578$)

|          | Coefficient | Standard Error | t statistic | Significance |
|----------|-------------|----------------|-------------|--------------|
| Constant | 4.2386      | 0.0571         | 74.1998     | 0.0000       |
| Ln(NIS)  | 0.0657      | 0.0210         | 3.1189      | 0.0042       |
The results show that the relationship being investigated is weak but statistically significant: despite the rather low value of the coefficient of determination ($R^2$), the $t$ statistic associated with the NIS regressor indicates a highly significant effect of this variable upon $AITT$. The interpretation of this regression model, however, is not straightforward, in the sense that two opposite meanings in terms of cause and effect are possible. If NIS is thought to be the cause and $AITT$ the effect, the conclusion of our analysis would be that individual routing strategies involving a high frequency of day-to-day switches are, on average, unsuccessful inasmuch as they lead to large values of travel time; if the opposite view is taken, then it would appear that users experiencing high travel times tend to change route more frequently as a way to ameliorate their negative performance.

4.4. Testing for the effect of personal characteristics of experiment participants

The final part of the statistical analysis was devoted to investigating possible effects of personal characteristics of experiment participants on the two variables considered in the previous analyses (NIS and $AITT$). More specifically, the characteristics of interest are gender, age and prevailing type of trip-making in real life (commuter vs. non commuter); see Table 8.

The statistical technique known as two-sample $t$ test was adopted in order to detect any differences of NIS and $AITT$ in relation to the above characteristics, the null hypothesis being that there are no differences between the mean values of the relevant variables for the sub-populations from which the two samples are extracted. The results of these tests, which are presented in Tables 9 and 10 for NIS and $AITT$ respectively, do not reveal any statistically significant difference of the two variables in relation to the three personal characteristics. Thus, the null hypothesis cannot be rejected in any of the considered cases. It is, of course, possible that these results are, at least in part, attributable to the fairly small sizes of the samples.

Table 8. Sample sizes and sample means of NIS and $AITT$ by gender, age and prevailing type of trip-making

|                    | Female | Male | Age 18-26 | Age 27-60 | Non commuter | Commuter |
|--------------------|--------|------|-----------|-----------|--------------|----------|
| Sample sizes       | 11     | 19   | 16        | 14        | 13           | 17       |
| Mean NIS           | 15.55  | 18.58| 19.44     | 15.21     | 14.77        | 19.53    |
| Mean $AITT$ (min.)| 84.22  | 81.75| 84.04     | 81.07     | 83.25        | 82.20    |

Table 9. Results of two-sample $t$ tests for effect of gender, age and prevailing type of trip-making on NIS

|                      | Female vs. Male | Age 18-26 vs. Age 27-60 | Non commuter vs. Commuter |
|----------------------|-----------------|--------------------------|---------------------------|
| $t$ statistic        | – 0.6738        | 0.9798                   | – 1.1017                  |
| Significance (two-tailed) | 0.5059        | 0.3356                   | 0.2800                    |

Table 10. Results of two-sample $t$ tests for effect of gender, age and prevailing type of trip-making on $AITT$

|                      | Female vs. Male | Age 18-26 vs. Age 27-60 | Non commuter vs. Commuter |
|----------------------|-----------------|--------------------------|---------------------------|
| $t$ statistic        | 0.7800          | 0.9757                   | 0.3382                    |
| Significance (two-tailed) | 0.4419        | 0.3376                   | 0.7377                    |
5. Deterministic Process model of day-to-day route choice dynamics

In order to provide further insights regarding the behavior of travelers in the simple situation under analysis, a Deterministic Process (DP) model of day-to-day route choice dynamics was implemented and estimated using the observations collected during the experiment. More specifically, we considered a deterministic, discrete-time dynamic process of route choice (as described, for example, in Cantarella, 1993), whose main elements are a route choice model, a cost updating model and a flow updating model.

The route choice model was specified as a simple Multinomial Logit (MNL) model with dispersion parameter denoted by $\theta$; it should be remarked that in our experiment the three alternative routes do not exhibit any overlap, and therefore the MNL can be properly applied in the case under consideration.

The cost updating model represents the mechanism by which network users develop their forecasts of travel costs based on previous experiences and possibly on information provided by outside sources (note that in the present study only the former element is relevant). Such a model is usually specified as an exponential smoothing filter:

$$C_t = \beta C_{t-1} + (1-\beta) K_{t-1}$$

where $t$ is the time period index, $C_t$ represents the users' forecast of route costs for period $t$, $K$ is the vector of route costs experienced in period ($t-1$), expressed as a function of route flows in the same period, and $\beta$ is a parameter representing the weight of the most recent experience in the formation of travel cost forecasts. Note that $\beta$ represents in an aggregate and time-independent fashion the effect of user memory and learning in the context of the route choice process; in a more general model, however, this parameter could depend on both time period and user type/class.

A formally similar expression is commonly adopted for the flow updating model:

$$F_t = \alpha H(C_t) + (1-\alpha) F_{t-1}$$

where, in addition to the above definitions, $H(C)$ is the vector of route flows induced by $C_t$ according to the specified route choice model, and $\alpha$ is a parameter representing the fraction of users who reconsider their previous day’s route choice (in short, the route choice reviewing rate); it is intended to model in an aggregate way the effect of user habit and inertia in the route choice process and is, again, assumed to be time-independent and common to all users for simplicity.

The cost and flow updating models together define a deterministic process in the state space of route costs and flows, describing the evolution of the system over successive time periods. Depending on the values of $\alpha$, $\beta$ and other model parameters, this process may converge to a fixed point (Stochastic User Equilibrium) or to different kinds of attractor, such as $k$-periodic, quasi-periodic or chaotic (see, for example, Cantarella, 1993).

As observed by Cantarella (2013), the calibration of $\alpha$ and $\beta$ is still an open issue, because empirically supported estimates of these parameters are rarely found in the literature, and thus extensive experimentation in laboratory as well as real-world contexts is needed in order to fill this gap. The results reported in this section, although based on a simple situation and on a limited amount of data, should be considered as a contribution in this direction.

In the present study, the joint estimation of $\alpha$, $\beta$ and $\theta$ was carried out by minimizing the distance between the route flows computed using the above described DP model and the corresponding experimental values. Intuitively, the task of such an estimation process is to find the parameter values that produce the best fit of the time series of modeled route flows to their observed counterparts. In particular, the optimal values of $\alpha$, $\beta$ and $\theta$ were found by means of an exhaustive grid search through the three-dimensional parameter space, using the sum of the squared differences between modeled and observed flows as a performance metric.
Estimation results are presented in Table 11, which also shows the standard deviation of the perceived travel times, whose value, for the MNL model, is inversely related to parameter $\theta$ according to the following expression:

$$ \sigma = \frac{1.2825}{\theta} \quad (4) $$

where $\sigma$ represents the standard deviation of perceived travel times.

Table 11. Estimated values of parameters of DP model of day-to-day route choice dynamics

| $\alpha$ (route choice reviewing rate) | $\beta$ (weight of most recent travel experience) | $\theta$ (dispersion parameter of MNL model) | $\sigma$ (standard deviation of perceived travel times) (min.) |
|----------------------------------------|-----------------------------------------------|------------------------------------------|-------------------------------------------------|
| 0.3803                                 | 0.8005                                        | 0.0628                                   | 20.42                                           |

It should be observed that a factor contributing to the relatively high value of $\beta$ is probably the type of feedback provided to experiment participants, in which emphasis was placed on travel times experienced in the most recent period (even though this does not exclude that some subjects may have kept a full record of travel times over the entire experiment).

Fig. 5. Modeled vs. observed flows on routes 1 and 2 over the time span of the experiment
A comparison of modeled and observed flows on routes 1 and 2 over the entire time span of the experiment is shown in Figure 5; note that, because of demand conservation, the state of the network can be fully described by specifying the values of only two (out of three) route flows.

6. Conclusions

The study described in this paper provides an additional piece of empirical evidence regarding day-to-day route choice dynamics in the absence of information provided by external sources. In our experiment, thirty individuals repeatedly selected in a non-cooperative manner one of three available routes over a 50-period time span, and at each period they were provided only with feedback information on the travel time of the most recently chosen route.

Regarding the key issue of the attainability of equilibrium conditions, our results indicate that the dynamics of this simple system did not lead to a stable UE state within the specified duration of the experiment. However, the detection of statistically significant trends underlying the evolution of network state descriptors (average travel time and total number of daily route switches) suggests the possibility that equilibrium conditions might have been approached over an appropriately extended time horizon. These findings, like those of several previous studies documented in the literature, indicate that the circumstance that road networks operate in or close to equilibrium conditions should not be assumed a priori.

Route switching has been analyzed in this study in terms of both its day-to-day evolution and its variability across the individuals participating in the experiment. A statistically significant, though not strong, nonlinear relationship between the total number of individual route changes and the average value of individual travel time during the entire experiment has been estimated based on the empirical data.

Moreover, tests of the possible effects of personal characteristics of experiment participants on the two above variables did not reveal any statistically significant differences related to gender, age and prevailing type of trip-making in real life (commuter vs. non commuter). This should not be considered as a conclusive result, since it may be, at least in part, attributable to the fairly small sizes of the sub-samples obtained by breaking down the panel of experiment participants according to the selected characteristics. Therefore, this issue will be the subject of additional investigation in future developments of this research.

Finally, the implementation of a deterministic process model of day-to-day route choice dynamics on the experimental network allowed us to derive empirically based estimates of three key parameters, namely the route choice reviewing rate, the weight of the most recent experience in the formation of users' cost forecasts and the dispersion parameter of the underlying Logit model. These results appear valuable inasmuch as experimental values of the first two parameters are hard to find in the published literature.

The research described in this paper could be extended in several directions. First, more sophisticated time-series analysis techniques could be applied to the data collected in this experiment. Second, a larger-scale experiment could be carried out by employing a higher number of participants and by extending the time horizon. Note, however, that an excessive duration of the trials might induce in some subjects a "fatigue" effect, potentially reducing the reliability and "truthfulness" of their behavior in the experiment. Third, an alternative approach to providing feedback to participants could be implemented by informing them, at the end of each period, about travel times experienced on all available routes (including those not chosen). Last, conditions of non-recurrent congestion could be introduced into the experimental setting, for example by simulating the occurrence of random incidents.

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### Appendix A. Detecting statistically significant trends in time series data

In this appendix a general description of Daniels' test on moving averages for detecting trends in time series data is presented. Essentially, this technique uses Spearman's rank correlation coefficient to test the null hypothesis that the variable under analysis is independent of time against the alternative hypothesis that a statistically significant trend exists in the series of observations. The test can be one-sided or two-sided, depending on whether the expected sign of the trend (increasing or decreasing) is specified or not.
Starting from a series of observations of a given variable over time (where \( t \) denotes the generic time period), the MA(\( t, k \)) series of \( k \)-period moving averages is determined first. Each element of this series is computed as the average of observations at times \( t, t-1, \ldots, t-k+1 \), and is centered \((k-1)/2\) periods behind \( t \) (where \( k \) is an odd integer, with \( k \geq 3 \)). For each element \( i \) of the series of moving averages, the difference \( d_i \) between its rank and the corresponding time index is computed. Based on these values, the Spearman's rank correlation coefficient (\( \rho \)) is determined according to the following expression:

\[
\rho = 1 - \frac{6 \sum d_i^2}{N(N^2-1)}
\]

where \( N \) represents the total number of values of the series of moving averages. Note that a negative sign of \( \rho \) indicates a negative correlation between time and the variable under examination, i.e. a decreasing trend of the time series.

Finally, the value of \( \rho \) is used to test the null hypothesis that the variable under analysis is independent of time; this test can be performed by computing the Student's \( t \) statistic:

\[
t_{(N-2)} = \rho \sqrt{\frac{N-2}{1-\rho^2}}
\]

If the calculated value of \( t \) is larger than the critical value with \((N-2)\) degrees of freedom, then the null hypothesis of no trend is rejected. A one-sided test should be used only when the alternative hypothesis is defined in terms of a specific sign of the trend (increasing or decreasing) of the time series.