Tidal radii of main sequence stars - III. Partial disruptions

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ABSTRACT

In this paper, the third in this series, we continue our study of tidal disruption events of main-sequence stars by a non-spinning \(10^6 \, M_\odot\) supermassive black hole. Here we focus on the outcomes of partial disruptions. As the encounter becomes weaker, the debris mass is increasingly concentrated near the outer edges of the energy distribution. As a result, the mass fallback rate can deviate substantially from a \(t^{-5/3}\) power-law, becoming more like a single peak with a tail declining as \(t^{-p}\) with \(p \approx 2 - 5\). Surviving remnants are spun-up in the prograde direction and are hotter than MS stars of the same mass. Their specific orbital energy is \(\approx 10^{-3}\times\) that of the debris (but of either sign with respect to the black hole potential) while their specific angular momentum is close to that of the original star. Even for strong encounters, remnants have speeds at infinity relative to the black hole potential \(\lesssim 300 \, \text{km s}^{-1}\), so they are unable to travel far out into the galactic bulge. Remnants bound to the black hole can possibly go through a second tidal disruption event.

Keywords: black hole physics – gravitation – hydrodynamics – galaxies:nuclei – stars: stellar dynamics

1. INTRODUCTION

Supermassive black holes (SMBHs) exert a significant tidal gravity on stars when their separation becomes comparable to or shorter than the “tidal radius”. Only if the star passes inside the physical tidal radius \(R_t\) is it fully disrupted; otherwise, if its pericenter \(r_p \gtrsim R_t\), it is partially disrupted and loses only a fraction of its mass. In both cases, roughly half of the mass removed from the star is bound to the black hole. When the bound debris returns to the vicinity of the BH, it may produce a luminous flare.

This is the third paper in a series of four whose aim is to study quantitatively the key properties of tidal disruption events (TDEs) as a function of stellar mass \(M_*\): the physical tidal radius \(R_t\), the largest pericenter distance at which stars are fully disrupted; the energy distribution of stellar debris and the fallback rate; the relation between pericenter distance (when \(> R_t\)) and the remaining mass of partially disrupted stars; and the properties of the stellar remnants of partial disruptions.

To achieve these goals, we have performed a suite of fully relativistic simulations of TDEs in which main-sequence (MS) stars are tidally perturbed by a non-spinning \(10^6 \, M_\odot\) BH. We adopt as the stars’ initial structures the radial profiles of MS stars evolved to half their main-sequence lifetimes using the stellar evolution code MESA, doing so for eight different masses from \(0.15 \, M_\odot\) to \(10 \, M_\odot\).

In the first paper of this series (Ryu et al. 2019a, Paper 1 hereafter), we presented \(R_t\) and the characteristic energy width of stellar debris as functions of stellar mass. We also introduced a semi-analytic model for \(R_t\) and a functional relation between the remnant mass and the pericenter distance for \(r_p > R_t\). We discussed the principal observational implications of our results focusing on the mass-dependence of: the mass return rate and timescale, the rate of complete disrup-
tion events; and the properties of unbound material and remnants. In Ryu et al. (2019b) (Paper 2), we provided a detailed description of our numerical methods and presented detailed results for full disruptions. We showed that full disruptions are different from the conventional approximation in two ways. First, complete disruptions are not instantaneous. Full tidal disruptions continue until the star’s center-of-mass has reached \( r \gtrsim 10 r_t \) where \( r_t = (M_{BH}/M_\star)^{1/3} \) is the traditional estimate of the tidal distance. Second, the debris energy range for higher-mass stars \( (M_\star \geq 1 \, M_\odot) \) is roughly two times wider than the traditional prediction, and the edges of the energy distribution for higher-mass stars are more gradual than for lower-mass stars, resulting in a higher and broader peak return rate and shorter delay time than given by the traditional prediction (Paper 2).

In this paper, we focus on the outcomes (surviving remnants and stellar debris) of partial disruptions. We provide a short overview of our simulation setup in Section 2. In Section 3, we present the distribution of energy and the fallback rate of stellar debris (Section 3.1). Then we analyze the properties of the surviving remnants (Section 3.2): the mass of surviving remnants for different degrees of partial disruption (Section 3.2.1); the specific orbital energy of the remnants (Section 3.2.2); remnant spin (Section 3.2.3) and remnant internal structure (Section 3.2.4). We discuss the future fate of partially disrupted stars in Section 4. Finally, we conclude with a summary of our findings in Section 5.

Throughout this paper, symbols with the subscript \( \star \), such as \( R_\star \) (stellar radius) and \( M_\star \) (stellar mass), always refer to the properties of the star at the beginning of the tidal encounter. All masses are measured in units of \( M_\odot \) and stellar radii in units of \( R_\odot \).

2. SIMULATIONS

Using the fully general relativistic hydrodynamics code HARM3D (Gammie et al. 2003; Noble et al. 2006, 2009), we simulate the disruption of MS stars by the tidal gravity of a non-spinning \( 10^6 \, M_\odot \) BH. For the initial density profiles of the stars, we adopt MS stellar models evolved using the stellar evolution code MESA (Paxton et al. 2011) to halfway through their main-sequence lifetimes. We consider eight different stellar masses from 0.15 to 10 (see Table 1). All begin with solar abundances. We calculate the star’s self-gravity by a (Newtonian) Poisson solver in a frame comoving with the star defined by a tetrad transformation so that its metric is exactly Minkowski at the center-of-mass, and differs only slightly from Minkowski elsewhere in
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Figure 2. $dM/dE$ for partial TDEs with $M_{\text{rem}}/M_\star \simeq 40 - 60\%$ (left panel) and $\gtrsim 90\%$ (right panel). We normalize the distribution with $M_\star/\Delta \epsilon$, where $\Delta \epsilon = GM_{\text{BH}}R_t/r_\star^2$. The integrated area under each curve is therefore the fractional mass of the stellar debris ($1.0 - M_{\text{rem}}/M_\star$). $M_{\text{rem}}/M_\star$ is given in Table 1. The diagonal dotted line in each panel represents $dM/dE \propto e^{-k(E)/\Delta \epsilon}$ with $k = 4.0$ (left panel) and 7.5 (right panel).

Figure 1 shows the evolution of the density distribution of a $1 M_\odot$ star when it is partially disrupted as it traverses an orbit with $r_p = 0.55 r_t = 1.16 R_\star$. Note how it begins to stretch shortly before reaching pericenter, but continues to lose mass until it swings out to $\gtrsim 10 r_t$.

3. RESULTS

Partial tidal disruptions produce two distinct products: a remnant and gaseous debris. The debris resembles that of full disruptions in the sense that roughly half is unbound and half is bound to the black hole. The bound debris can return to the black hole, generating a bright flare. On the other hand, there is a remnant, of course, only in a partial disruption.

3.1. Stellar debris - Distribution of specific energy and fallback rate

The most observationally-significant property of the debris is its energy distribution $dM/dE$. This quantity determines the fallback rate of bound debris and the ejection speeds of unbound debris. In the left panel of Figure 2, we show $dM/dE$ for the stellar debris produced by severe partial disruptions. By “severe”, we mean events in which the remnant mass $M_{\text{rem}}/M_\star \simeq 40 - 60\%$. These events have pericenters not much greater than $R_t$ ($r_p/R_t \simeq 1.2$). The right panel of Figure 2 shows $dM/dE$ for “weak” partial disruptions, those in which $M_{\text{rem}}/M_\star \gtrsim 90\%$ and $r_p/R_t \simeq 1.5 - 2.0$. Because our sample was bimodal in terms of mass-loss (only 3 of our 32 cases had fractional mass-loss between 10\% and 40\%), these two extremes comprise most of the cases we studied.

the problem volume. Full details of our procedures are given in Paper 2.

For each stellar mass, we performed a suite of simulations for TDEs with various pericenter distances $r_p/r_t(\equiv \psi)$. The largest pericenter studied was chosen so that mass lost from the star was several percent of the star’s initial mass. We distinguish full from partial disruptions by three conditions:

1. Lack of any approximately-spherical bound structure.

2. Monotonic (as a function of time) decrease in the maximum pressure of the stellar debris.

3. Monotonic decrease in the mass within the computational box. The mass remaining in the box for complete disruption falls with increasing distance from the BH $\propto r^{-\alpha}$ with $\alpha \simeq 1.5 - 2.0$, whereas for partial disruptions the remaining mass eventually becomes constant, which signifies a persistent self-gravitating object.

Events violating any one of these conditions we deem “partial”; in all cases, if one is violated, all are.

We estimate the physical tidal radius $R_t$ as the mean of the largest $r_p$ yielding a full disruption and the smallest $r_p$ producing a partial disruption. As shown in Paper 1, $R_t/r_t \simeq 1 - 1.4$ for low-mass stars ($0.15 \leq M_\star \leq 0.5$); falls rapidly between $M_\star \gtrsim 0.5$ and 1.0; and is roughly constant at $\simeq 0.45$ for high-mass stars ($M_\star \geq 1$). As a result, for stars with $0.15 \leq M_\star \leq 3$, all orbits with $r_p \gtrsim 27 r_g$ lead to at most partial disruption. Here, $r_g$ refers to the gravitational radius of the BH.
Lacy et al. (1982) first pointed out that there is a characteristic scale for the energy of tidal disruption debris,

\[ \Delta \epsilon \sim \frac{GM_{\text{BH}} R_*}{r_t^2}, \]  

(1)

and the distribution \( dM/dE \) should be roughly symmetric around \( E = 0 \). As we showed in Paper 2, explicit calculations find that the distribution is, indeed, very symmetric, but the magnitude of the energy is correct only at the order of magnitude level. The characteristic spread in energy \( \Delta E \), defined as the energy width containing 90% of the total mass, is \( \simeq 0.8 \Delta \epsilon \) for low-mass stars (\( 0.15 \leq M_* \leq 0.5 \)), but jumps to \( \simeq 1.5 \Delta \epsilon \) for \( M_* = 1 \) and rises to almost 2 for higher-mass stars. For all masses, \( dM/dE \) has local maxima at \( E \approx \pm \Delta E \), but drops smoothly toward \( E \approx 0 \), where there is a local minimum whose value is only \( \simeq 2/3 \) that found at the maxima. In low-mass stars, \( dM/dE \) plummets for \( |E| > \Delta E \); in high-mass stars, it falls exponentially toward larger \( |E| \), but on a scale \( \approx \Delta \epsilon/3 \), so that there can be a noticeable amount of mass in the wings.

As shown in Figure 2, some of these characteristics are replicated in partial disruptions, but with the notable contrasts that the local minimum near \( E = 0 \) is much deeper, and \( \Delta E \) is a function of \( r_t/R_* \) as well as of \( M_* \). Not too surprisingly, in severe partial disruptions \( \Delta E \) is consistently close to its value in full disruptions. However, it drops by a factor \( \approx 2 \) going from severe disruptions to weak ones. Severe disruptions also resemble full disruptions in that \( dM/dE \) for high-mass stars, but not low-mass stars, has exponential wings. These differ, however, in that they are somewhat steeper: \( dM/dE \propto e^{-4|E|/\Delta \epsilon} \) rather than \( \propto e^{-3|E|/\Delta \epsilon} \).

In weaker partial disruptions, the exponential wings decline more rapidly, on scales a factor \( \sim 2 \) shorter than in the severe cases.

The greatest contrast between partial disruptions and full disruptions is in the depth of the central minimum. The factor \( \simeq 2/3 \) between \( dM/dE(E = 0) \) and \( dM/dE(E = \Delta E) \) for full disruptions becomes a factor \( \sim 10^{-2} \) for partial disruptions. The very deep central minimum results in nearly all the debris mass being concentrated near \( E \approx \pm \Delta E \).

In Figure 3, we show the fallback rate for the two partial disruption cases, calculated using the energy distributions shown in Figure 2 and the expression for the fallback rate (Rees 1988; Phinney 1989).

\[
\dot{M}_{\text{fb}} = \left( \frac{M_*}{3P_{\Delta \epsilon}} \right) \left( \frac{dM/M_*}{d\epsilon/2\Delta \epsilon} \right) \left( \frac{t}{P_{\Delta \epsilon}} \right)^{-5/3},
\]

(2)

where \( P_{\Delta \epsilon} = (\pi/\sqrt{2})GM_{\text{BH}}\Delta \epsilon^{-3/2} \) is the orbital period for orbital energy \( -\Delta \epsilon \). The most noticeable feature is greater deviations from the \( t^{-5/3} \) power-law for weaker tidal encounters. The different decline rate results from the very small amount of mass with \( E \approx 0 \). Going from severe partial disruptions (left panel) to events with minor mass-loss (right panel), we see that the fallback rate after the peak departs more dramatically for weaker events. In those cases, rather than a sharp rise and a power-law fall, the fallback rate exhibits only a rather narrow peak. For low-mass stars, the post-peak fallback rate decreases \( \propto t^{-p} \) with \( p \approx 2 - 2.7 \) in severe partial disruptions, but even more steeply, \( p \approx 5 \), in weak disruptions. The \( M_* = 0.3 \) star behaves a bit differently; its fallback rate diminishes more gradually. For high-

Figure 3. The fallback rate \( \dot{M}_{\text{fb}} \) for partial TDEs using the energy distribution in Figure 2. We normalize the time \( t \) by the orbital period \( P_{\Delta \epsilon} \) and the fallback rate \( \dot{M}_{\text{fb}} \) by \( \dot{M}_\star = M_\star/(3P_{\Delta \epsilon}) \). The diagonal solid lines show the power-law \( t^{-p} \) with \( p = 8/3 \) (left panel) and \( p = 5 \) (right panel). The fractional mass of the debris bound to the BH is \( \simeq 0.5(1 - M_{\text{rem}}/M_\star) \), and \( M_{\text{rem}}/M_\star \) is given in Table 1.
mass stars, $p$ increases from $\simeq 2.7$ to 3 as the mass-loss becomes smaller. The time of maximum fallback rate is more delayed and the peak is sharper for low-mass stars than high-mass stars, due to the different widths of the peaks in $dM/dE$.

These results conflict with the claim of Coughlin & Nixon (2019) that the post-peak slope $p$ for partial disruptions asymptotes to $\simeq 9/4$ independent of $M_{\text{rem}}$, owing to a continuous gravitational influence of the remnant on the debris marginally bound to the BH. Several methodological contrasts may account for this disagreement. Whereas we use a full 3-dimensional hydrodynamic simulation to describe the complex geometry of the tidal streams and remnant, Coughlin & Nixon (2019) use a 1-dimensional analytic model in which both the debris streams and the remnant move exclusively in the radial direction with respect to the black hole. This assumption has the consequences that the gravitational force exerted by the remnant on a gas parcel is purely radial, and its magnitude is determined by the difference between their distances from the black hole. It also implies that the work done by the remnant on the fluid elements does not reflect any obliquity between the direction of motion of the fluid and the direction between it and the remnant. Finally, whereas we compute the self-gravity of both the mass in the stellar remnant and the debris contained within a large box around the remnant ($17 \, R_\ast \times 9 \, R_\ast \times 10 \, R_\ast$), Coughlin & Nixon (2019) ignore the self-gravity of the debris. Our approach accurately calculates the work done on the fluid by the remnant while it remains within the simulation box: because the total amount of work is dominated by the portion done while the fluid element is nearest the remnant, our box is large enough to account for the majority of this effect.

Golightly et al. (2019) presented one example of a partial TDE simulated using the SPH code PHANTOM in which the fallback rate exhibited a late-time slope $\simeq -9/4$. In this case, the initial stellar structure was a MESA model of a main sequence 3 $M_\odot$ star halfway through its lifetime, i.e., identical to one of the cases we considered. The pericenter for this encounter, $r_p = 0.33 \, R_t$, was, however, smaller than $R_t$ as determined by our simulations ($\simeq 0.4 - 0.45 \, R_t$). It is possible that they found only a partial disruption, but perhaps a rather strong one, because they employed Newtonian rather than relativistic gravity, even though this pericenter is only $27 \, R_g$.

Goicovic et al. (2019) also studied the shape of the debris energy distribution for a $M_\ast = 1 \, M_\odot$ star whose initial mass profile was taken from MESA data. Comparing their $\beta = 1.6$ and $\beta = 1.1$ cases with our $\psi = 0.65$ and $\psi = 1.0$ runs, we find good consistency. Similarly to ours, the $dM/dE$ distribution in their Figure 4 shows the appearance of the wings near the outer boundaries, being more conspicuous for weaker encounters. Given the consistency in $dM/dE$, it is not surprising to find similar fallback rates as well. Both studies show a lower and earlier peak of $\dot{M}_\text{fb}/\dot{M}_0$ for weaker encounters.

3.2. Surviving remnants

3.2.1. Mass

Figure 4 shows the fractional remnant mass $M_{\text{rem}}/M_\ast$ as a function of $r_p$. When low-mass stars have $r_p \gtrsim 1.5 \, R_t$, even though they are tidally deformed near the pericenter, they recover their (quasi-) spherical structures without a significant loss of mass ($\lesssim 10\%$). For high-mass stars, such weak mass-loss occurs for $r_p \gtrsim 1.8 \, R_t$.

We find that a simple functional form,

$$\frac{M_{\text{rem}}}{M_\ast} = 1.0 - \left( \frac{r_p}{R_t} \right)^{-3.0}, \quad (3)$$

captures the key features of the pericenter-dependence of $M_{\text{rem}}/M_\ast$. In fact, by coincidence, it reproduces the curve for $M_\ast = 3$ almost exactly. Guillochon &
Ramirez-Ruiz (2013) also provides fitting formulae for the remnant mass of polytropic stars with $\gamma = 4/3$ and $5/3$ ($1.0 - C_\gamma$ in their Appendix), as a function of $r_t/r_p$. Their formulae for these two values of $\gamma$ run along the envelope of the remnant mass curves shown in Figure 4: the curve for $\gamma = 5/3$ lies slightly above that for $M_* = 0.15$, while the curve for $\gamma = 4/3$ is close to the that for $M_* = 1$.

In Paper 1, we introduced a semi-analytic model which predicts the functional relation between the remnant mass and pericenter distance. By combining Equation 3 and the remnant mass predicted using the semi-analytic model (Equation 9 in Paper 1), we found a direct, albeit approximate, link between three dimensionless spatial scales, e.g., $R_t/r_p(=\Psi)$, $r_p/r_t(=\psi \geq \Psi)$ and the radius $R$ inside the original star such that $M(\leq R) = M_{\text{rem}}$:

$$
\frac{R}{R_*} \approx 0.47\left(\psi^3 - \Psi^3\right)^{1/3}.
$$

Thus, with a model for the star’s initial mass profile and knowledge of $R_t$, the remnant mass can be predicted easily for any pericenter larger than the physical tidal radius.

### Specific energy - bound or unbound

In this section, we focus on the specific energies of surviving remnants to see whether or not they are bound to the BH. We consider the question of whether they are bound to the galaxy’s bulge separately. As a prologue to this topic, it is useful to lay out the hierarchy of orbital energy scales in this problem. The most useful unit for this hierarchy is the specific kinetic energy of stars in the region of the galaxy from which the disrupted stars are drawn, i.e., $(1/2)\sigma^2$, where $\sigma$ is the 3-dimensional velocity dispersion. In terms of this unit, the initial orbital energy of stars in our simulations counting only the black hole’s contribution to the gravitational potential is very small, $\sim -10^{-3}(\sigma^2/2)$, which, in relativistic terms, is a specific energy $\sim -10^{-10}c^2$.

In this sense, one might think of our stars as having energy very close to the mean of the bulge stars’ energies. On the other hand, the magnitude of the typical remnant’s specific energy is relatively large, $\sim 1 - 10$. Because the typical remnant energy changes by an amount greater than the actual energy with which stars begin the event, we can approximate the remnant’s final energy as its actual energy with respect to the BH potential. Moreover, because it is also several times larger than the potential associated with the stars of the inner galaxy, it is appropriate to label remnants with positive final energy as “unbound” with respect to the innermost portion of the galaxy. However, we must also emphasize that “large” is a relative term. Although the remnants’ energies are large compared to the kinetic energy of bulge stars, they are tiny compared to the magnitude of the debris energy, whether bound or unbound—they are $\sim 10^{-3}$ on that scale.

It is a good approximation to suppose that the BH potential dominates the entire region through which bound remnants travel because nearly all their apocenters ($\sim 0.05 - 1$ pc, Table 1) are smaller than the BH’s radius of influence ($\sim 1 - 10$ pc; see Section 4.1 for further discussion of this point).

With only one exception, the semimajor axes of the bound remnants in our sample range from $a \approx 0.03 - 0.5$ pc, and therefore have periods between $\approx 400$ and $\approx 40,000$ yr. Their eccentricities are exceedingly close to 1, mostly with $|1 - e| \sim 10^{-5}$. With specific energies similar in magnitude to those of the bound remnants, the unbound remnants have ejection speeds $v_{\text{ejec}} \approx 100 - 330$ km s$^{-1}$. There is also one case ($M_* = 0.7, r_p/r_t = 0.9$) that is intermediate in the sense that it is bound, but only weakly, having $a \approx 1.7$ pc and $P \approx 0.2$ Myr. The comparative rarity of remnants whose net energy is very close to zero is likely due to the small associated phase space.

We indicate in Figure 4 using circle markers which remnants are bound (filled circles) or unbound (unfilled circles). For low-mass stars, the unbound remnants are associated with the most severe partial disruptions whereas relatively weak encounters yield bound remnants. However, for high-mass stars, even some severe partial disruptions yield bound remnants. Because the specific angular momentum of a remnant (either bound or unbound) is essentially identical to the specific angular momentum of the original star, its pericenter is very nearly unchanged.

A similar study was reported by Manukian et al. (2013). Using Newtonian hydrodynamics simulations of tidal disruption of polytropic stars with $\gamma = 4/3$ they determined the orbital energies of remnants at a time $\approx 100(R_t^3/GM_\star)$ after pericenter passage. Contrary to what we found, all their surviving remnants were, in our language, unbound, and their ejection speeds were considerably greater than ours. For example, in the case of stars with $M_* = 1$ (for which a $\gamma = 4/3$ polytrope is a reasonable approximation), the ejection speed for their remnants ranged from $\approx 100$ km/s (for $r_p/r_t = 1$) to $\approx 600$ km/s (for $r_p/r_t = 0.55$). By contrast, the remnants of our $M_* = 1$ simulations with $0.5 \leq r_p/r_t \leq 1$ were all bound, and the greatest ejection speed we found for any other case was $\approx 330$ km/s. It is unclear how to account for these differing results; the difference be-
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between relativistic and Newtonian tidal forces might play a part.

We summarize the properties of the remnants in Table 1.

3.2.3. Spin

All surviving remnants are spun-up in the prograde direction as they are tidally torqued near the pericenter (Rees 1988; Goicovic et al. 2019). They are approximately oblate spheroids in shape, with the minor axis perpendicular to the orbital plane. One noticeable feature in common to all our remnants is that their angular frequencies increase outwards. As an example, we present in the left panel of Figure 5 the angular frequency \( \bar{\Omega}(R_t) \), an azimuthal average over cells at the same cylindrical radius from an axis through the remnant’s center of mass perpendicular to the orbital

| \( M_\star \) | \( r_p/r_\star \) | \( r_p/R_t \) | \( M_{\text{rem}}/M_\star \) | \( B/U \) | \( \log_{10}(|\bar{E}|/\Delta \epsilon) \) | \( v_{\text{ejec}} [\text{km s}^{-1}] \) | \( \log_{10}(1 - \bar{e}) \) | \( a [\text{pc}] \) | \( P [10^3 \text{ yr}] \) |
|---|---|---|---|---|---|---|---|---|---|
| 0.15 | 2.00 | 1.38 | 0.14 | 0.99 | B | -2.9 | - | -4.6 | 0.058 | 1.3 |
| 0.3 | 1.50 | 1.24 | 0.13 | 0.87 | B | -3.2 | - | -4.9 | 0.11 | 3.4 |
| 0.4 | 1.60 | 1.10 | 0.08 | 0.59 | U | -3.3 | 174 | - | - | - |
| 0.5 | 1.50 | 1.03 | 0.05 | 0.39 | U | -2.9 | 295 | - | - | - |
| 0.7 | 1.80 | 1.44 | 0.28 | 0.92 | B | -3.4 | - | -5.0 | 0.18 | 7.3 |
| 1.0 | 1.50 | 1.20 | 0.17 | 0.56 | U | -3.8 | 94 | - | - | - |
| 1.10 | 1.12 | 0.11 | 0.36 | U | -3.2 | 180 | - | - | - |
| 1.20 | 1.04 | 0.06 | 0.18 | U | -3.7 | 110 | - | - | - |
| 1.30 | 1.71 | 0.49 | 0.97 | B | -3.2 | - | -4.7 | 0.11 | 3.6 |
| 1.40 | 1.43 | 0.41 | 0.81 | B | -3.8 | - | -5.4 | 0.50 | 33 |
| 1.50 | 1.14 | 0.22 | 0.43 | U | -3.8 | 93 | - | - | - |
| 1.60 | 1.05 | 0.13 | 0.24 | U | -3.4 | 150 | - | - | - |
| 1.80 | 2.22 | 0.67 | 0.96 | B | -3.1 | - | -4.6 | 0.11 | 3.5 |
| 2.00 | 1.91 | 0.91 | 0.91 | B | -2.9 | - | -4.6 | 0.087 | 2.4 |
| 2.5 | 1.33 | 0.49 | 0.49 | B | -4.2 | - | -6.0 | 1.7 | 200 |
| 0.80 | 1.19 | 0.21 | 0.30 | U | -2.6 | 322 | - | - | - |
| 0.70 | 1.04 | 0.06 | 0.09 | U | -2.7 | 299 | - | - | - |
| 1.00 | 2.11 | 0.91 | 0.91 | B | -2.9 | - | -4.6 | 0.087 | 2.4 |
| 0.65 | 1.37 | 0.48 | 0.48 | B | -2.6 | - | -4.5 | 0.047 | 0.97 |
| 0.55 | 1.16 | 0.23 | 0.22 | B | -2.4 | - | -4.4 | 0.028 | 0.44 |
| 0.50 | 1.05 | 0.08 | 0.08 | B | -2.9 | - | -4.9 | 0.088 | 2.5 |
| 0.85 | 2.00 | 2.85 | 0.95 | B | -2.8 | - | -4.4 | 0.083 | 2.2 |
| 0.60 | 1.61 | 2.02 | 0.67 | B | -3.6 | - | -5.4 | 0.56 | 40 |
| 0.50 | 1.18 | 1.18 | 0.39 | B | -2.9 | - | -4.7 | 0.10 | 3.0 |
| 0.45 | 1.06 | 0.53 | 0.17 | U | -2.3 | * | * | * | * |
| 0.85 | 2.00 | 9.19 | 0.91 | B | -3.0 | - | -4.4 | 0.13 | 4.5 |
| 0.60 | 1.41 | 5.87 | 0.58 | B | -2.9 | - | -4.4 | 0.097 | 2.8 |
| 0.50 | 1.18 | 3.52 | 0.35 | B | -2.9 | - | -4.6 | 0.11 | 3.6 |
| 0.45 | 1.06 | 1.28 | 0.12 | U | -1.7 | * | * | * | * |

Table 1. The properties of partial disruption remnants. In the left-hand columns, we list the original mass of our model stars \( M_\star \), the ratios of \( r_p \) to \( r_\star \) and \( R_t \), the remnant mass \( M_{\text{rem}} \), the mass fraction \( M_{\text{rem}}/M_\star \), the sign of the mass-weighted specific energy \( \bar{E} \) (B: \( \bar{E} < 0 \) and U: \( \bar{E} > 0 \)) and the magnitude of the average specific energy in units of \( \Delta \epsilon \). The right-hand four columns give orbital parameters for the remnants: for unbound stars, only the ejection velocity \( v_{\text{ejec}} \); for bound stars, the eccentricity \( \bar{e} \), semimajor axis \( a \) and orbital period \( P \). Note that we do not show \( v_{\text{ejec}} \) for the \( M_\star = 3 \) and 10 cases’ most severe disruptions. This is because even at \( r \simeq 20 - 30 \) \( r_\star \), they had not settled into an approximate steady state; in addition, their mean specific energy was so different from that of the initial star’s that the remnant was offset far enough from the center of the simulation box that some of its mass was no longer inside the box. We exclude these two cases from the analysis of the unbound population in the text.
plane, at three different heights. The star in this simulation began with mass $M_\star = 1$, passed through a pericenter $r_p = 0.65 \, r_t$, and emerges from the event with $M_{rem} \approx 0.48$. The angular frequency $\Omega$ at each height increases outwards until it reaches a maximum at $R \approx 0.8 - 1.0$. The maximum frequency at the equator is around $25 - 30\%$ of the equatorial break-up angular frequency $\Omega_{bk}$, defined as $\Omega_{bk}(R) = \sqrt{GM_{en}(R)/R^3}$. Here $M_{en}(R)$ is the enclosed mass inside cylindrical radius $R$ on the equatorial plane. The rotational velocity $v_\phi$ (right panel) therefore rises steeply at small radius and then $\propto R$ for $R \gtrsim 0.6 \, R_\odot$. Its maximum is $\approx 100 - 120 \, \text{km s}^{-1}$.

We find a general trend that, for fixed fractional mass loss, the more massive the initial star, the closer its remnant comes to break-up rotation. This trend is illustrated in Figure 6. For this figure, we consider partially disrupted stars with $M_{rem}/M_\star \approx 0.5$, corresponding to $r_p/R_\odot \approx 1.1 - 1.3$. That the high-mass stars reach higher fractions of the break-up rotation rate than the lower-mass stars can be explained simply. To zeroth order, when a star passes through pericenter, tidal forces torque it so that its outer layers rotate at roughly the local orbital frequency. But the local orbital frequency is, by definition, about the same as the vibrational frequency when the distance from the black hole is similar to $r_t$. By the same token, the break-up rotational frequency is similar to the vibrational frequency. Consequently, $\Omega/\Omega_{bk} \approx (r_p/R_\odot)^{-3/2}$.

### 3.2.4. Internal structure

Due to tidal heating and distortion of the stellar density distribution, the remnants are out of thermal equilibrium. The central specific entropy of a remnant star is generally higher than that of a MS star of the same mass by tens of percent to factors of a few for severe disruptions. Figure 7 shows the entropy profiles of the star shown in Figure 5 and that of a MS star (derived...
Figure 7. The specific entropy $P/\rho^\Gamma$ ($\Gamma = 5/3$, in cgs units) of the same remnant star shown in Figure 5 ($M_\star = 1$, $M_{\text{rem}} \simeq 0.48$, $\psi = 0.65$ and $r \simeq 23 \ r_\odot$). The black curves represent the entropy profile in the equatorial plane (solid) and along the $z$–axis (dashed). The red dashed curve indicates the entropy profile of the MESA-MS analog. The radius $R$ on the $x$–axis is normalized by the radius $R_{\star, 99\%}$ for 99% of the remnant mass. The vertical magenta solid line indicates the radius of the original 1 $M_\odot$ MS star, $R_\star$.

Using MESA with the same mass as a function of distance $R$ from the star’s center of mass. Note that the entropy on the equatorial plane is an average of cells located at the same cylindrical radius from the center of mass. At larger radii (the outer 10% of mass), the specific entropy rises much farther above that of the MS mass-analog. The entropy gradient along the rotation axis is much steeper than the entropy gradient in the equatorial plane. Although the main sequence analog has a well-defined photosphere with a very sharp entropy gradient, as expected, these remnants have much more extended atmospheres.

In Figure 8, we compare the density distributions of this star and its main-sequence counterpart. The top panel of Figure 8 shows its density profile both in the equatorial plane and along the $z$–axis; line colors and line types are the same as in Figure 7. The density on the equatorial plane is calculated in the same way as the entropy in that plane. The middle and bottom panels depict 2–dimensional snapshots of the star’s density in the $x$–$y$ and $x$–$z$ planes, respectively.

The most noticeable feature in Figure 8 is the star’s oblate spheroidal shape. The density drops outward more rapidly along the $z$–direction than on the equatorial plane. In comparison with the density profile of the ordinary MS star, the core density of the surviving star is relatively low and the density outside the core falls

Figure 8. The density $\rho$ of the same remnant star in Figure 5 ($M_\star = 1$, $M_{\text{rem}} \simeq 0.48$, $\psi = 0.65$ and $r \simeq 23 \ r_\odot$). The spatial scales ($R$, $x$, $y$) are normalized to $R_{\star, 99\%}$, the radius containing 99% of the remnant mass. (Top panel) The density on the equatorial plane is shown by the black solid curve and along the $z$–axis by the black dashed curve. The red dashed curve depicts the density profile of a MESA-MS analog. The vertical magenta solid line indicates the original $R_\star$. (Middle and bottom panels) 2–dimensional density maps of the star in the equatorial plane ($x$–$y$) and in the vertical plane ($x$–$z$), respectively. The solid (larger) magenta circle delineates $R_\star$ and the red (smaller) dashed line the radius for 99% of the mass of the MESA-MS star.
outward much more gradually. We find other surviving remnants share qualitatively similar trends. Typically, the ratio of the core density of partially disrupted stars to that of ordinary MS stars of the same mass is around 0.03 – 0.8, with the ratio increasing from small to large $r_p/r_t$.

4. DISCUSSION - THE FATE OF THE STELLAR REMNANTS

Due to slightly asymmetric mass-loss, remnants whose parent stars had exactly zero energy with respect to the BH have a small, but non-zero, orbital energy per unit mass after their tidal encounters. In real events, the initial stellar orbital energy can also be slightly non-zero, but the magnitude of the surviving remnants’ energy is sufficiently larger than the initial energy that the latter can be neglected (Section 3.2.2). The orbits of the remnants can then be conveniently divided into two classes according to the sign of their energy those with positive energy are “unbound”, and those with negative energy are “bound”.

4.1. Unbound population

The ejection velocities of the unbound remnants we simulated range from $90 – 330$ km s$^{-1}$. Extrapolating from the bulge dispersion data of galaxies with central BHs slightly more massive than $10^6 M_\odot$, we find that the dispersions of galaxy bulges containing BHs with $M_{BH} \simeq 10^6$ is of order $\sigma = 60 – 90$ km s$^{-1}$ (e.g., Woo et al. 2013; Kormendy & Ho 2013; Graham 2016). Our unbound remnants can therefore easily escape the radius of influence, $r_{\text{inf}} = G M_{BH}/\sigma^2 \simeq 0.5$ pc($M_{BH}/10^6$)($\sigma/90$ km s$^{-1}$)$^{-2}$, of the central BH. Nonetheless, if the potential beyond the sphere of influence is logarithmic, the remnants are likely to reach a turning point $r_{\text{max}}$ at only a few $r_{\text{inf}}$, i.e., $r_{\text{max}} \simeq \Lambda r_{\text{inf}}$ with $\Lambda = e^{(v_{\text{ejec}}/2\sigma)^2} \simeq 1 – 10$. Such a turning point would be well within the bulge region.

As the angular momentum of a remnant is much smaller than the value corresponding to a circular orbit at its semimajor axis, the pericenter distance is determined almost purely by the angular momentum. If it is unchanged during the time spent near apocenter, any such remnant will return to the black hole with the same pericenter as the original stellar orbit, raising the prospect of a second tidal interaction.

To estimate how large these perturbations may be, as a crude approximation, we compare the travel time for a partial disruption remnant to reach its turning point with the time required for weak stellar encounters to alter their original specific angular momentum $L_0$ by a factor of order unity. The travel time is $t_{\text{travel}} = r_{\text{max}}/v_{\text{ejec}} \simeq 10^5(r_{\text{max}}/10$ pc)($v_{\text{ejec}}/90$ km s$^{-1}$)$^{-1}$ yr. On the other hand, the evolution time for remnant angular momentum is $t_L \simeq (L_0/L_\Lambda)^3 t_\tau$ (Merritt 2013) where $L_\tau$ refers to the specific angular momentum change during a collisional relaxation time $t_\tau$; by definition, $L_\tau \simeq \sigma r_{\text{max}}$. Using the relation $t_\tau \simeq 0.1(N/\ln N)t_{\text{cross}}$ (Binney & Tremaine 1987), where $N$ is the number of stars within the region the test-particle star travels through, $t_{\text{cross}} = r_{\text{max}}/\sigma$, $r_{\text{max}} = \Lambda r_{\text{inf}}$, and $L_0 \simeq \sqrt{2GM_{BH} r_{\text{inf}}}$, we find that the ratio between the two characteristic times at $r_{\text{max}} > r_{\text{inf}}$ is,

$$\frac{t_L}{t_{\text{travel}}} \approx 2 \left( \frac{0.1 N}{\ln N} \right) \left( \frac{v_{\text{ejec}}}{\sigma} \right) \left( \frac{r_{\text{inf}} r_\tau}{r_{\text{max}}^2} \right),$$

$$\approx 10^{-2} \left( \frac{v_{\text{ejec}}}{\sigma} \right) \left( \frac{M_{BH}}{10^6} \right)^{4/3} \left( \frac{r_{\text{max}}}{5 \text{ pc}} \right)^{-1}.$$ (5)

For this estimate, we also assumed the mass of background stars is $1 M_\odot$, giving $N(< r_{\text{max}}) \simeq 2 M_{BH} \Lambda$ for a logarithmic potential.

This estimate implies that gravitational encounters are very likely to result in changes of the unbound remnants’ angular momenta large enough to alter their pericenter distances. Because $r_p \propto L^2$ for these highly-eccentric orbits, the resulting change of $r_p$ should be $\propto t_{\text{travel}}/t_L$. Thus, for these unbound remnants, the pericenter upon return is likely to be considerably larger than the value of $r_\tau$ of the returning remnant. It is also possible for their angular momenta to be affected by other mechanisms, e.g. scattering by giant molecular clouds (Perets et al. 2007) or torques due to non-spherical galactic potentials (Merritt & Poon 2004). These remnants, although on unclosed orbits, will nonetheless return to the galactic center close to the BH, but their pericenters are likely to be altered enough so as not to produce interesting tidal encounters.

4.2. Bound population

Our entire sample of bound remnants (except for one that is exceptionally weakly bound) has eccentricities less than unity by $\sim 10^{-4} – 10^{-5}$, semi-major axes $a \sim 0.03 – 0.5$ pc, and orbital periods $P \sim 400 – 40000$ yr. Although it is likely that our sample does not span the full range of possibilities, these numbers may be taken as indicative of the typical magnitudes for events with $M_{BH} \sim 10^6$.

These bound remnants are also subject to stellar encounters, but within the black hole sphere of influence. For this case, we can not use the same expression for $t_L$, used above as it is derived for remnants whose motions are dominated by the potential from surrounding stars while, within $r_{\text{inf}}$, the BH potential dominates. The typical velocity of stars at $r_{\text{max}} = 2a < r_{\text{inf}}$ is...
roughly $\sigma \simeq \sqrt{GM_{BH}/2a}$. This leads to a relaxation time $t_r = 0.1(M_{BH}/m)^2/[N \ln(M_{BH}/m)] t_{cross}$, where $m$ is the mean mass per star. With $r_{max} = 2a$ and $t_{travel} = P/2$,

$$\frac{t_L}{t_{travel}} \simeq \frac{0.1 \times 2^{3/2}}{\pi} \left( \frac{M_{BH}/m}{N \ln(M_{BH}/m)} \right) \left( \frac{r_t}{a} \right),$$

$$\simeq 2 \times 10^{-2} \left( \frac{N}{2 \times 10^6} \right)^{-1} \left( \frac{\ln(M_{BH}/m)}{13.8} \right)^{-1},$$

$$\times m^{-2} \left( \frac{M_{BH}}{10^6} \right)^{7/3} \left( \frac{a}{0.5 \text{ pc}} \right)^{-1},$$

where we have scaled to values appropriate to the one of the longer semi-major axes in our sample. The apocenter distance for such a semi-major axis is comparable to $r_{inf}$ for $M_{BH} = 10^6$, within which, by definition, $N(< r_{inf}) \simeq 2 \times 10^6$. However, the value of $N$ in this expression also depends on $a$. If the stellar density $\rho_\star \propto r^{-n}$, $N(< r) \propto r^{3-n}$. The ratio $t_L/t_{travel}$ then scales $\propto a^{n-4}$. Therefore, for a density profile near the BH with $n < 4$, $t_L/t_{travel}$ increases as $a$ decreases, possibly becoming larger than unity at a sufficiently small $a$ (e.g., for $n = 7/4$, the steady-state solution of Bahcall & Wolf (1976), the ratio becomes larger than unity at $a \lesssim 0.07$–$0.08$ pc). This means that for bound remnants with sufficiently small semimajor axes, the pericenter upon return remains unchanged from its value during the first passage. Because our sample includes some remnants with semimajor axes as small as $\simeq 0.03$ pc, a fraction of the bound remnant population will return with pericenters either the same as during their first passage, or enlarged by only a little.

### 4.3. A second tidal disruption?

Whether a significant tidal disruption event takes place at the next pericenter passage depends on how the (possibly larger) pericenter compares to the star’s new tidal radius. If the remnant returns to the main sequence before returning to the vicinity of the black hole, its smaller mass would imply a smaller size and a smaller $r_t$ while its new pericenter is likely to be at least as large as in the original event. Significant disruption would probably not occur.

However, return to the main sequence in time for the next return to pericenter may be problematic. These remnants are extended by both extra heat and more rapid rotation. For the example shown in Figure 8, the radius of the remnant star is close to the radius of the original star, while the ratio $(M_{BH}/M_\star)^{1/3}$ is greater by 28%. Thus, if there is too little time for it to cool before the next pericenter passage, its $r_t$ might be comparable to the one applicable to the original event, while the relevant $\mathcal{R}_t$ would, of course, require further simulation to determine. If the pericenter has been increased by scattering by less than a factor $\sim 2$, a significant tidal event might take place.

Whether thermal relaxation can be completed by the time the remnant returns to periastron depends upon the ratio of the cooling time to the orbital period. The photon diffusion time from the center of a star to its edge is

$$t_{th} \simeq \kappa_c \rho_c R_c^2/c,$$

$$\simeq 2 \times 10^4 \text{ yr} \left( \frac{\kappa_c}{10 \kappa_c} \right) \left( \frac{\rho_c}{10^2 \text{ g cm}^{-3}} \right) \left( \frac{R_c}{0.1} \right)^2,$$

where $\kappa_c$ is the core opacity, $\rho_c$ is the core density and $R_c$ is the radial length scale of the core. The Thomson opacity is $\kappa_e$. In the conditions of our stellar remnants ($\rho_c \sim 1 - 10^2$ g cm$^{-3}$, core temperature $T_c \sim 10^6 - 10^7$ K), $\kappa_c/\kappa_e \sim 1 - 10^2$ (Hayashi et al. 1962). Comparing this time to the orbital periods shown in Table 1 demonstrates that the more tightly bound remnants ($P < t_{th}$) would return back to the BH without significant changes in their internal structures. These are also the remnants likely to suffer the least increase in orbital pericenter due to scattering with background stars. Thus, for both reasons, the more tightly bound remnants have the greatest probability of going through a second TDE.

However, we caution that a more careful calculation of the remnant’s cooling is necessary to determine what happens when it next passes through pericenter. The evolution of the remnant star’s rotation may also influence its fate. Angular momentum may be lost through magnetic braking (e.g. Fricke & Kippenhahn 1972); it may also be mixed inward from the outer $\sim 10$% of the star’s mass where it initially resides by any of a variety of processes (Maeder & Meynet 2000). Because only a minority of the remnants’ mass rotates rapidly, evolution in the star’s rotation may be a next-order correction to the effect of cooling.

### 5. SUMMARY

In this paper, the third in this series, we continue our study of tidal disruption events of main-sequence stars, focusing on the properties of partial disruptions. Our results are based upon a suite of fully general relativistic simulations in which the stars’ initial states are described by realistic main-sequence models. We examined tidal disruption events for eight different stellar masses, from $M_\star = 0.15$ to $M_\star = 10$ with a fixed black hole mass ($10^6 M_\odot$).
We find that the energy distribution $dM/dE$ of the stellar debris created from partial disruptions is different from the one that arises in full disruptions, with the contrast growing for weaker encounters. For full disruptions, the characteristic energy width $\Delta E$ of the stellar debris for low-mass stars is $\simeq 0.8 \Delta \epsilon$, while that for high-mass stars can be as large as $\simeq 2 \Delta \epsilon$, where $\Delta \epsilon$ is the traditional order of magnitude estimate for this width. The energy distribution $dM/dE$ for all masses has a local minimum near $E \simeq 0$ and “shoulders” near the outer boundaries, with a contrast between the two $\simeq 1.5$ (Paper 2). On the other hand, for partial disruptions, most of the mass of the stellar debris is concentrated near the shoulders, with little mass near $E \simeq 0$: the contrast is $\sim 10$ for strong disruptions, in which a large fraction of the stellar mass is lost, and it increases to $\sim 100 - 1000$ for weaker disruptions. Although the outer edges of the distribution are quite sharp for low-mass stars subjected to either partial or full disruption, there can be significant tails for high-mass stars. These become progressively steeper for weaker partial disruptions. Because there is so little mass near $E \simeq 0$, late-time fallback is suppressed, and the overall shape of the fallback rate becomes more like a single peak as mass loss becomes smaller, with a tail declining as $t^{-p}$ with $p \simeq 2 - 5$ very unlike the consistent $p = -5/3$ for full disruptions.

Another product of partial disruptions is surviving remnants. We have found a simple analytic expression linking the ratio between the stellar orbit’s pericenter and the physical tidal radius for that stellar mass to the ratio between the remnant mass and the original stellar mass (see equation 3). The remnants retain around 50% of the original mass at $r_p/R_t \simeq 1.2 - 1.5$, while the mass loss becomes less than 10% at $r_p/R_t \gtrsim 1.5 - 1.8$.

Due to tidal heating, surviving remnants are out of thermal equilibrium and tend to be larger in size than a MS star of the same mass. They are also rapidly-rotating, reaching angular frequencies near break-up in the outer layers of the remnants left by events causing substantial mass-loss from initially massive stars. The rapid rotation makes those oblate spheroids.

The change in specific orbital energy of partially-disrupted stars is quite small compared to the spread in energy of the debris: $\simeq 10^{-3} \Delta \epsilon$ (see Table 1), but it can be of either sign. Particularly for low-mass stars, weaker encounters lead to remnants that lose orbital energy and therefore remain within the sphere of influence of the black hole, while the strongest encounters can create remnants able to travel some distance out into the galaxy’s bulge. For high-mass stars, most partial disruptions lead to bound remnants, except for those that are nearly strong enough to cause total disruption.

When a stellar remnant, whether bound to the black hole or able to travel out into the bulge, reaches its orbital apocenter, weak gravitational interactions with buldge stars can alter its angular momentum. The change can be large compared to the remnant’s original angular momentum when the remnant goes as far as the stellar bulge, or even the outer portion of the black hole’s sphere of influence, but if the remnant’s apocenter is smaller than the black hole’s sphere of influence, the change can be comparable to the original angular momentum or even less. When the increase in specific angular momentum is relatively small, the remnant may become a victim of another TDE if its cooling time is longer than its orbital period. Because the more tightly-bound remnants also confined to within the black holes’ sphere of influence have substantially shorter orbital periods than those able to reach the bulge, their prospects for a second tidal event are further enhanced.

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Software: matplotlib (Hunter 2007); MESA (Paxton et al. 2011); HARM3D (Gammie et al. 2003; Noble et al. 2006, 2009);

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51x90}Coughlin, E. R., & Nixon, C. J. 2019, arXiv e-prints, Bahcall, J. N., & Wolf, R. A. 1976, ApJ, 209, 214 can be of either sign. Particularly for low-mass stars, in energy of the debris: $\simeq 0.8 \Delta \epsilon$, while that for high-mass stars can be as large as $\simeq 2 \Delta \epsilon$, where $\Delta \epsilon$ is the traditional order of magnitude estimate for this width. The energy distribution $dM/dE$ for all masses has a local minimum near $E \simeq 0$ and “shoulders” near the outer boundaries, with a contrast between the two $\simeq 1.5$ (Paper 2). On the other hand, for partial disruptions, most of the mass of the stellar debris is concentrated near the shoulders, with little mass near $E \simeq 0$: the contrast is $\sim 10$ for strong disruptions, in which a large fraction of the stellar mass is lost, and it increases to $\sim 100 - 1000$ for weaker disruptions. Although the outer edges of the distribution are quite sharp for low-mass stars subjected to either partial or full disruption, there can be significant tails for high-mass stars. These become progressively steeper for weaker partial disruptions. Because there is so little mass near $E \simeq 0$, late-time fallback is suppressed, and the overall shape of the fallback rate becomes more like a single peak as mass loss becomes smaller, with a tail declining as $t^{-p}$ with $p \simeq 2 - 5$ very unlike the consistent $p = -5/3$ for full disruptions.

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