Experimental filtering of two-, four-, and six-photon singlets from single PDC source

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(Dated: March 13, 2009)

Invariant entangled states remain unchanged under simultaneous identical unitary transformations of all their subsystems. We experimentally generate and characterize such invariant two-, four-, and six-photon polarization entangled states. This is done only with a suitable filtering procedure of multiple emissions of entangled photon pairs from a single source, without any interferometric overlaps. We get the desired states utilizing bosonic emission enhancement due to indistinguishability. The setup is very stable, and gives high interference contrasts. Thus, the process is a very likely candidate for various photonic demonstrations of quantum information protocols.

PACS numbers: 03.67.Hk, 03.67.Dd, 03.67.-a.

Entanglement is an essential tool in many quantum information tasks. Entangled states of two qubits proved to be useful in various quantum communication protocols like quantum teleportation, quantum dense coding, and quantum cryptography. They are the essence of the first versions of Bell’s theorem[1]. However, the expansion of quantum information science has now reached a state in which many schemes are involving multiparty processes, and could require multiqubit entanglement.

There is an interesting series of multiqubit states, \( |\Psi^-_k\rangle \), where \( k = 2, 4, 6 \) or more. They are invariant under actions consisting of identical unitary transformations of each qubit \( \mathcal{U} \cdot |\Psi^-_k\rangle = |\Psi^-_k\rangle \), where \( \mathcal{U}^\otimes k = U \otimes \ldots \otimes U \) denotes a tensor product of \( k \) identical unitary operators \( U \). The property protects the states against collective noise. The states are useful e.g. for communication of quantum information between observers who do not share a common reference frame \( \mathfrak{R} \): any realignment of the receiver’s reference frame corresponds to an application of the same transformation to each of the sent qubits. The states \( |\Psi^-_k\rangle \) can also be used for secure quantum multiparty cryptographic protocols, such as the multi-party secret sharing protocol [10, 11].

We generate correlations which characterize the six-photon \( |\Psi^-_6\rangle \) entangled state. This is done in a six-photon interference experiment. A six-photon interference was reported recently in [12]. To obtain graphs states the authors of ref. [12] used three pulse-pumped parametric down-conversion (PDC) crystals, and interferometric overlaps, to entangle independently emitted pairs (each from a different crystal) with each other. Schemes of this kind are generalizations of those of ref. [13]. However, the overlaps make the scheme fragile.

In our experiment, by pulse pumping just one crystal and extracting the right order process via suitable filtering and beamsplitting (the method of [14]), we observe simultaneously effects attributable to the multi-photon invariant entangled states \( |\Psi^-_2\rangle \), \( |\Psi^-_4\rangle \), and \( |\Psi^-_6\rangle \). The setup has no overlaps and therefore no interferometric alignment is needed. It is strongly robust, and the output is of high fidelity with respect to the theoretical states \( |\Psi^-_k\rangle \).

A simple quantum optical description of two phase matched modes, of the multiphoton state that results out of a single pulse acting on a type-II PDC crystal, can be put as

\[
C \sum_{n=0}^{\infty} \frac{1}{n!} \left[ -i\alpha (a_{0H}^{+} b_{OV}^{+} - a_{0V}^{+} b_{OH}^{+}) \right]^{n} |0\rangle \tag{1}
\]

The symbol \( a_{0H}^{+} \) (\( b_{0V}^{+} \)) denotes a creation operator for one horizontal, \( H \), (vertical, \( V \)) photon in mode \( a_0 \) (\( b_0 \)), etc. \( C \) is a normalization constant, the coupling parameter \( \alpha \) is a function of pump power, non-linearity and length of the crystal. This is a good approximation of the actual state, provided one collects the photons under conditions that allow full indistinguishability between separate two-photon emissions [14].

First, second, and third order terms in the expansion in eq. (1), correspond to an emission of two, four, and six photons, respectively, into two spatial modes. These terms can be re-interpreted as the following superpositions of photon number states:

\[
|1H_{a_0}, 1V_{b_0}\rangle - |1V_{a_0}, 1H_{b_0}\rangle \tag{2}
\]
\[
|2H_{a_0}, 2V_{b_0}\rangle - |1H_{a_0}, 1V_{a_0}, 1V_{b_0}, 1H_{b_0}\rangle + |2V_{a_0}, 2H_{b_0}\rangle \tag{3}
\]
\[
|3H_{a_0}, 3V_{b_0}\rangle - |2H_{a_0}, 1V_{a_0}, 2V_{b_0}, 1H_{b_0}\rangle \tag{4}
\]

where e.g. \( 2H_{a_0} \) and \( 3H_{a_0} \) denote two and three horizontally polarized photons in mode \( a_0 \), respectively, etc. The second and third order PDC is intrinsically different than simple products of two and three entangled pairs. Due to the bosonic nature of photons, emissions of completely indistinguishable photons are more likely, than the ones giving birth to photons with orthogonal polarization.
We report a joint observation, in one setup, of the correlations of the invariant two, four and six-photon polarization entangled states given by the following superpositions: $|\Psi^-_2 \rangle = \frac{1}{\sqrt{2}}(|HV \rangle - |VH \rangle)$,

$$|\Psi^-_4 \rangle = \frac{2}{\sqrt{3}}|GHZ^+ \rangle - \frac{1}{\sqrt{3}}|EPR \rangle |EPR \rangle, \quad (5)$$

and

$$|\Psi^-_6 \rangle = \frac{1}{\sqrt{2}}|GHZ^- \rangle + \frac{1}{2}(|W\rangle_3 |W\rangle_3 - |W\rangle_3 |W\rangle_3). \quad (6)$$
The states in the superpositions are given by:

$$|GHZ^+_i \rangle = \frac{1}{\sqrt{2}}(|HHVV \rangle + |VVHH \rangle)/\sqrt{2},$$

$$|GHZ^-_i \rangle = (|HHHV \rangle - |VVHH \rangle),$$

and

$$|EPR \rangle = \frac{1}{\sqrt{2}}(|HV \rangle + |VH \rangle).$$

Finally

$$|W\rangle_3 = \frac{1}{\sqrt{3}}(|HHV \rangle + |HVV \rangle + |VHH \rangle).$$

The ket $|W\rangle$ is the spin-flipped $|W\rangle$. The states (5) are obtained out of different orders of the PDC emission (figure 1), by selecting specific double, quadruple and six-fold coincidences.

In our setup we use a frequency-doubled Ti:Sapphire laser (80 MHz repetition rate, 140 fs pulse length) yielding UV pulses with a central wavelength at 390 nm and an average power of 1300 mW. The pump beam is focused to a 160 μm waist in a 2 mm thick BBO (β-barium borate) crystal. Half wave plates and two 1 mm thick BBO crystals are used for compensation of longitudinal and transversal walk-offs. The third order emission of non-collinear type-II PDC is then coupled to single mode fibers (SMF), defining the two spatial modes at the crossings of the two frequency degenerated PDC emission cones. Leaving the fibers the down-conversion light passes narrow band ($\Delta \lambda = 3$ nm) interference filters (F) and is split into six spatial modes ($a, b, c, d, e, f$) by ordinary 50%-50% beam splitters (BS), followed by birefringent optics (to compensate phase shifts in the BS’s). Due to the short pulses, narrow band filters, and single mode fibers the down-converted photons are temporally, spectrally, and spatially indistinguishable [14], see Fig. 1. The polarization is being kept by passive fiber polarization controllers. Polarization analysis is implemented by a half wave plate (HWP), a quarter wave plate (QWP), and a polarizing beam splitter (PBS) in each mode. The outputs of the PBS’s are lead to single photon silicon avalanche photo diodes (APD) through multi mode fibers. The APD’s electronic responses, following photo detections, are being counted by a multi-channel coincidence counter.

The states $|\Psi^-_k \rangle (k = 2, 4, 6)$ exhibit perfect two, four, and six qubit correlations. The correlation function is defined as an expectation value of the product of local polarization “Pauli” observables. If one limits the measurement to the local observables $\cos \theta \sigma_z^{(l)} + \sin \theta \sigma_x^{(l)}$ (with eigenvectors $\sqrt{1/2}(|L\rangle_{l} \pm e^{i \theta} |R\rangle_{l})$ and eigenvalues $\pm 1$), the measurements correspond to linear polarization analysis in each spatial mode ($l = a, b, c, d, e, f$). In such a case the quantum prediction for the two photon (in modes b and d) correlation function reads:

$$E(\theta_a, \theta_b, \theta_d, \theta_e) = \frac{2}{3} \cos(\theta_a + \theta_b - \theta_d - \theta_e) + \frac{1}{3} \cos(\theta_a - \theta_b) \cos(\theta_e - \theta_d). \quad (7)$$

FIG. 1: Experimental setup for generating and analyzing the six-photon polarization-entangled state. The six photons are created in third order PDC processes in a 2 mm thick BBO pumped by UV pulses. The intersections of the two cones obtained in non-collinear type-II PDC are coupled to single mode fibers (SMF) wound in polarization controllers. Narrow band interference filters (F) ($\Delta \lambda = 3$ nm) serve to remove spectral distinguishability. The coupled spatial modes are divided into three modes each by 50%-50% beam splitters (BS). Each mode can be polarization analyzed using half wave plates (HWP) and a polarizing beam splitter (PBS). Simultaneous detection of six photons (two single photon detectors for each mode) are being recorded by a twelve channel coincidence counter.
Finally for the six photon events one has

\[
E(\theta_a, \theta_b, \theta_c, \theta_d, \theta_e, \theta_f) = \\
-\frac{1}{2} \cos(\theta_a + \theta_b + \theta_c - \theta_d - \theta_e - \theta_f) \\
-\frac{1}{18} \sum \cos(\theta_a \pm \theta_b \pm \theta_c \pm \theta_d \pm \theta_e \pm \theta_f), \quad (8)
\]

where \(\sum\) is a sum over all possible sign sequences which contain only two positive signs, with the sign sequence in the first term, proportional to \(\frac{1}{18}\), excluded. Due to the invariance, the correlation functions for all measurements around any single great circle of the Bloch sphere look the same.

Fig. 2 shows three experimentally observed two-photon correlation functions, \(E(\theta_i, \theta_j)\), where \((l = d, e, f)\). The setting \(\theta_1\) is varied, while the other analyzer is fixed at \(\theta_1 = \theta_m = \theta_2 = \pi/2\). This corresponds to diagonal/antidiagonal, \(D/A\), linear polarization analysis. A sinusoidal least-square fit was made to the data. The average visibility, defined here, as the average amplitude of the three fits, is \(V_2 = 0.962\% \pm 0.003\%\).

Fig. 3 shows how six experimentally observed four photon correlation functions \(E(\theta_i, \theta_j, \theta_m, \theta_n)\) depend on \(\theta_3\). The other analyzers were fixed at \(\theta_1 = \pi/2\), where \((l = a, c)\), \((m = d, e)\), and \((n = e, f)\). The average value of the six visibilities is \(V_4 = 0.9189\% \pm 0.0049\%\).

Finally, Fig. 4 shows similar data for the experimentally observed six photon correlation function \(E(\theta_i, \theta_{a,c,d}, \theta_{b,c,d}, \theta_{e,f})\). Again \(\theta_b\) was varied with the other five analyzers fixed at \(\theta_1 = \pi/2\) where \((l = a, c, d, e, f)\). The value on the visibility is \(V = 83.79\% \pm 2.98\%\).

In Table I we present all the experimentally obtained two-, four-, and six- photon visibilities.

We have compared the observed visibilities with theoretical predictions. \[17\]. To estimate maximal predictable visibilities one can use a less simplified description of the two-photon state emitted by SPDC event, and replace in

| \(k\) | Modes | Visibility |
|---|---|---|
| 2 | \(b, d\) | 0.962 ± 0.004 |
| 2 | \(b, c\) | 0.963 ± 0.006 |
| 2 | \(b, f\) | 0.962 ± 0.004 |
| 4 | \(a, b, d, e\) | 0.919 ± 0.014 |
| 4 | \(a, b, d, f\) | 0.918 ± 0.011 |
| 4 | \(a, b, e, f\) | 0.919 ± 0.014 |
| 4 | \(b, c, d, f\) | 0.918 ± 0.009 |
| 4 | \(b, c, e, f\) | 0.920 ± 0.012 |
| 6 | \(a, b, c, d, e, f\) | 0.838 ± 0.030 |

FIG. 2: Two-photon polarization correlation function. Modes \(a, d, e\) and \(f\) are analyzed in \(D/A\) basis and mode \(b\) analysis basis is rotated around the equator of the Bloch sphere \((\sigma_z \cos(\theta_3) + \sigma_x \sin(\theta_3))\). The solid lines show sinusoidal fits to the experimental data with a average visibility of \(V_2 = 0.962\% \pm 0.003\%\).

FIG. 3: Four-photon polarization correlation functions of \(|\Psi^+\rangle\). Modes \(a, c, d, e\) and \(f\) are analyzed in the \(+/-/-\) and mode \(b\) analysis basis is rotated around the equator of the Bloch sphere \((\sigma_z \cos(\theta_3) + \sigma_x \sin(\theta_3))\). The figures correspond to different implementations of the state, using different modes \((abde, abdf, bcde, bcdf\) and \(becf)\). The solid lines show sinusoidal fits to the experimental data. The solid lines show sinusoidal fits to the experimental data with a average visibility of \(V_4 = 0.9189\% \pm 0.0049\%\).
introduced in [18]. It guarantees that N-qubit state is of the "experimentally friendly" entanglement indicators. With just a part of our data one can use the simplest one maximal theoretical visibility as a function of the ratio $V_{\text{cess}}$: $f_{\text{f}}$ $f_{\text{g}}$ profiles, and $a$ to the experimental data with a visibility of 83.79% ± 2.98%.

This approach is rich enough to take into account the frequency phase matching conditions. The creation operation basis is varied around the equator of the Bloch sphere $\omega$. The high visibility has the following consequences. The six qubit state that we observed is invariant with respect to simultaneous identical (unitary) transformations of all qubits. This makes it particularly useful for multiparty quantum communication and general quantum computation tasks: it is very robust against deformations in transfer. Since we use just one source, we avoid alignment problems, and thus the setup is very stable. We would like to note that the interference contrast is high enough for our source to be used in two-, four-, and six party demonstrations of quantum communication protocols [20].

In summary, we have experimentally demonstrated that a suitable filtering procedure applied to a triple emission from a single pulsed source of polarization entangled photons leads to two, four and six-photon, high visibility, interference due to entanglement, observable in a single setup. We utilize the bosonic emission enhancement occurring in the emission of three photon-pairs in PDC, thus the process is not entirely spontaneous. The six qubit state that we observed is invariant with respect to simultaneous identical (unitary) transformations of all qubits. This makes it particularly useful for multiparty quantum communication and general quantum computation tasks: it is very robust against deformations in transfer. Since we use just one source, we avoid alignment problems, and thus the setup is very stable. We would like to note that the interference contrast is high enough for our source to be used in two-, four-, and six party demonstrations of quantum reduction of communication complexity in some joint computational tasks, and for secret sharing, as well as in many other quantum informational technologies.

This work was supported by Swedish Research Council (VR). M.Z. was supported by Wenner-Gren Foundations and by the EU programme QAP (Qubit Applications, No. 015858).

FIG. 4: Six-photon polarization correlation function. Modes a, c, d and f are analyzed in D/A basis and mode b analysis basis is varied around the equator of the Bloch sphere $(\sigma_0 \cos(\theta_b) + \sigma_0 \sin(\theta_b))$. The solid line shows a sinusoidal fit to the experimental data with a visibility of 83.79% ± 2.98%.

The high visibility has the following consequences. With just a part of our data one can use the simplest one of the "experimentally friendly" entanglement indicators introduced in [18]. It guarantees that N-qubit state is entangled, if the norm of the N-particle correlation tensor is higher than 1. For the $|\Psi_6^-\rangle$ we take just $T_{xx}$, $T_{yy}$, $T_{zz}$, for $|\Psi_4^-\rangle$ again just $T_{xxxx}$, $T_{yyyy}$, $T_{zzzz}$, and for $|\Psi_6^-\rangle$ the components $T_{xxxxxx}$, $T_{yyyyyy}$, and $T_{zzzzzz}$. With our data we have obtained the 2.785 ± 0.007, 2.517 ± 0.011, and 2.29 ± 0.14, respectively for each of the case. The entanglement threshold is violated by of 242, 133, and 9.3 standard deviations. Additionally, according to the criteria given in [19] the state cannot be described by a local realistic model, if the sum of squares of two out of the listed components is above 1. This is again achieved by our data: for four particles we get $T_{xxxx}^2 + T_{yyyy}^2 = 1.646 ± 0.009$ (exceeding 1 by 74.8 standard deviations) and for six particles we have $T_{xxxxxxx}^2 + T_{yyyyyyy}^2 = 1.52±0.11$ (exceeding 1 by 4.5 standard deviations). Thus the state can be directly (that is without still enhancing its fidelity) utilized in classical threshold beating communication complexity protocols [20].

\[ V_{\text{cess}}(\omega) = \frac{\int d\omega_0 \int d\omega_1 \int d\omega_2 f(\omega_1) f(\omega_2) g(\omega_0) e^{i\omega t}}{\Delta(\omega_0 - \omega_1 - \omega_2) (a_{0H}^\dagger (\omega_1) b_{0V} (\omega_2) - a_{0V}^\dagger (\omega_1) b_{0H} (\omega_2))}. \]

\[ V_{\text{cess}}(\omega) = \exp[-(\omega - \omega_0)^2/(2\sigma_f^2)] \text{ and } g(\omega) = \exp[-(\omega - \omega_0)^2/(2\sigma_p^2)] \]

This corresponds to $V_4 = 0.93$ and $V_6 = 0.90$. The actual measured values of visibility for two, four, and six photon interference are very close to the predicted ones, see table 1. Thus, the fact that our setup use only filtering and beamsplitting, has interferometric advantages. In other words the obtained four and six particles visibilities are almost as high as one get for the ratio $r_{\text{exp}}$. The high visibility has the following consequences. With just a part of our data one can use the simplest one of the "experimentally friendly" entanglement indicators introduced in [18]. It guarantees that N-qubit state is entangled, if the norm of the N-particle correlation tensor is higher than 1. For the $|\Psi_2^-\rangle$ we take just $T_{xx}$, $T_{yy}$, $T_{zz}$, for $|\Psi_4^-\rangle$ again just $T_{xxxx}$, $T_{yyyy}$, $T_{zzzz}$, and for $|\Psi_6^-\rangle$ the components $T_{xxxxxx}$, $T_{yyyyyy}$, and $T_{zzzzzz}$. With our data we have obtained the 2.785 ± 0.007, 2.517 ± 0.011, and 2.29 ± 0.14, respectively for each of the case. The entanglement threshold is violated by of 242, 133, and 9.3 standard deviations. Additionally, according to the criteria given in [19] the state cannot be described by a local realistic model, if the sum of squares of two out of the listed components is above 1. This is again achieved by our data: for four particles we get $T_{xxxx}^2 + T_{yyyy}^2 = 1.646 ± 0.009$ (exceeding 1 by 74.8 standard deviations) and for six particles we have $T_{xxxxxxx}^2 + T_{yyyyyyy}^2 = 1.52±0.11$ (exceeding 1 by 4.5 standard deviations). Thus the state can be directly (that is without still enhancing its fidelity) utilized in classical threshold beating communication complexity protocols [20].

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