Parity doubling of baryons in a chiral approach with three flavors

Chihiro Sasaki\textsuperscript{a}

\textsuperscript{a}Institute of Theoretical Physics, University of Wroclaw, PL-50204, Wroclaw, Poland

Abstract

We formulate a set of mass relations for the baryon octet and decuplet with positive and negative parity in terms of the order parameter of QCD chiral symmetry. The Gell-Mann–Okubo mass formula and Gell-Mann’s equal spacing rule hold manifestly in this approach. Thermal masses of the baryons are calculated in the mean field approximation for various pion masses, and the results are compared with the recent lattice studies. A general trend of the nucleon, \(\Delta\) and \(\Omega\) parity-doublers seen in the available lattice data can be understood qualitatively. Expected mass modifications of other strange baryons are also given with the physical and heavier pion masses.

Key words: Parity doubling, Chiral symmetry breaking

1 Introduction

Modifications of hadron properties in a hot/dense medium have been explored as one of the key issues in the context of QCD phase transition expected in heavy-ion collisions and in the interior of compact stars [1]. As chiral symmetry becomes restored, the hadron spectra with opposite parity are expected to be degenerate. Yet, it remains unclear to what extent they would influence over bulk thermodynamics and experimental observables.

Recently, the first systematic study of thermal masses of the octet and decuplet baryons with positive and negative parity has been carried out in \(N_f = 2 + 1\) flavored lattice QCD [2]. The temperature dependence of the nucleon, \(\Delta\) and \(\Omega\) masses were extracted from temporal correlators, and they obviously exhibit the parity doubling structure. The ground-state mass with positive parity is rather stable against temperature, whereas the mass of the negative parity partner drops substantially toward the chiral crossover temperature. Although the simulations in [2] have been performed with a relatively large pion mass,
$m_\pi \sim 400$ MeV, this is a clear signature of the partial restoration of chiral symmetry in the baryonic sector.

In chiral approaches, a non-vanishing nucleon mass which stays finite in chiral restored phase is introduced via so-called mirror assignment of chirality in the parity doublet model \[3,4,5\]. The model has been applied to hot and dense baryonic matter and neutron stars \[6,7,8,9,10,11,12,13,14,15,16,17\] as well as the phenomenology in vacuum \[18,19,20,21\].

The two-flavored physics with parity doubling has been rather extensively studied, whereas the studies with three flavors remain quite limited. In particular, a systematic study of the in-medium masses of the octet and decuplet states is still missing. In this paper, we start with the general SU(3) Lagrangian and deduce a complete set of the mass relations in the parity doubling scenario, in a manifestly consistent manner with the celebrated Gell-Mann–Okubo mass formula and Gell-Mann’s equal spacing rule. We also study the thermal behavior of the baryon masses in a self-consistent chiral approach under the mean field approximation. For qualitative comparison to the lattice data \[2\], we demonstrate the calculations with the physical and heavier pion masses.

2 Octet and decuplet baryons

Introducing an octet $g_8$ and a singlet $g_1$ coupling constants, the general SU(3) interaction Lagrangian with a meson field $\Phi$ is given by \[22,23\]

$$L_{BB\Phi} = -\sqrt{2}g_8 \alpha \text{tr} \left[ B [\Phi, B] \right]$$
$$- \sqrt{2}g_8 (1 - \alpha) \left( \text{tr} \left[ B \{\Phi, B\} \right] - \frac{2}{3} \text{tr} \left[ B B \right] \text{tr} [\Phi] \right)$$
$$- \frac{g_1}{\sqrt{3}} \text{tr} \left[ B B \right] \text{tr} [\Phi],$$  \(1\)

where $\alpha$ is known as the $F/(F+D)$ ratio. Masses of the baryon octet are generated when an octet of scalar fields get condensed \[1\], and all of them depend on the light-quark $\sigma_q$ and the strange-quark condensates $\sigma_s$. As suggested in \[24\], there exists a special set of the parameters, $(\alpha, g_1) = (1, \sqrt{6}g_8)$, which leads to the nucleon mass depending only on the $\sigma_q$.

The extension to the parity doublet picture is carried out following \[18\]. The

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\[1\] For details of the scalar vacuum expectation matrix staying invariant via a non-linear transformation, see Appendix of \[23\].
chiral invariant mass $m_0$ of two fermions $\psi_1$ and $\psi_2$ is introduced as
\[ \mathcal{L}_{\text{inv}} = m_0 \text{tr} \left[ \bar{\psi}_1 \gamma_5 \psi_2 - \bar{\psi}_2 \gamma_5 \psi_1 \right]. \] (2)

In the physical basis, the nucleon masses with positive and negative parity are found as
\[ m_{N\pm} = \sqrt{\alpha_N^2 \sigma_q^2 + m_0^2 \mp \beta_N \sigma_q}, \] (3)
where $\alpha_N$ and $\beta_N$ are the nucleon coupling constants to the scalar mesons. The decuplet baryons are introduced as the Rarita-Schwinger fields, and the delta masses are given in a similar fashion [4,5]:
\[ m_{\Delta\pm} = \sqrt{\alpha_{\Delta}^2 \sigma_q^2 + m_0^2 \mp \beta_{\Delta} \sigma_q}, \] (4)
with constants $\alpha_{\Delta}, \beta_{\Delta}$ and $m_0$.

In extending the above to the other octet and decuplet baryons with a chiral invariant mass, we encounter a problem. Because of the nonlinear $\sigma$ dependence in Eq. (3), the Gell-Mann–Okubo mass relation for the baryon octet,
\[ \frac{3}{4} m_\Lambda + \frac{1}{4} m_\Sigma - \frac{1}{2} (m_N + m_\Xi) = 0, \] (5)
is violated, unless the exact SU(3) limit is taken. Another non-trivial relation for the decuplet baryon, Gell-Mann’s equal spacing rule,
\[ m_{\Sigma^*} - m_\Delta = m_{\Xi^*} - m_{\Sigma^*} = m_\Omega - m_{\Xi^*}, \] (6)
is not satisfied either. We note that, without $m_0$, the two relations, (5) and (6), hold for any $\alpha, g_1$ and $g_8$ when the Lagrangian (1) is used.

Therefore, we shall adopt the following mass relations for the octet parity doublers
\[ m_{N\pm} = (a_N \mp b_N) 3 \sigma_q + m_0, \]
\[ m_{\Sigma\pm} = (a_N \mp b_N) \left( 2 \sigma_q + \sqrt{2} \sigma_s \right) + m_0 + m_1, \]
\[ m_{\Lambda\pm} = (a_N \mp b_N) \left( 2 \sigma_q + \sqrt{2} \sigma_s \right) + m_0 + m_3, \]
\[ m_{\Xi\pm} = (a_N \mp b_N) \left( \sigma_q + 2 \sqrt{2} \sigma_s \right) + m_0 + m_2, \] (7)

The invariant mass for the octet state can be different from that for the decuplet. In this work, we take a common value for simplicity. In fact, the two invariant masses are found to be rather close in the recent lattice study [2]. See also the discussion in the Conclusions section.

Therefore, the parameterization for the baryon parity doublers given in [14,9] does not reproduce the low-energy relations (5) and (6).
where three parameters $m_{1,2,3}$ are introduced in order to generate a mass difference between the $\Sigma$ and $\Lambda$ due to the explicit chiral symmetry breaking. They are related via the Gell-Mann–Okubo relation (5) as

$$m_1 = 2m_2 - 3m_3.$$  

(8)

The decuplet parity doublers follow

$$m_{\Delta \pm} = (a_\Delta \mp b_\Delta) 3\sigma_q + m_0,$$

$$m_{\Sigma^* \pm} = (a_\Delta \mp b_\Delta) \left(2\sigma_q + \sqrt{2}\sigma_s\right) + m_0 + m_s,$$

$$m_{\Xi^* \pm} = (a_\Delta \mp b_\Delta) \left(\sigma_q + 2\sqrt{2}\sigma_s\right) + m_0 + 2m_s,$$

$$m_{\Omega \pm} = (a_\Delta \mp b_\Delta) 3\sqrt{2}\sigma_s + m_0 + 3m_s,$$

(9)

where the terms including $m_s$ are added for relatively strong explicit symmetry-breaking in the strange-quark sector in such a way that all is consistent with the equal spacing rule (6). We will use the current strange-quark mass $m_s = 0.1$ GeV [26]. One readily sees that the low-energy relations (5) and (6) are now satisfied even in the presence of explicit SU(3) breaking.

The above mass relations (7) and (9) can be deduced from Eqs. (3) and (4) by assuming $\sigma_{q,s} \ll m_0$. As discussed in the Introduction, our main attention will be put to the mass modifications near chiral symmetry restoration, where in-medium condensates are certainly smaller than their vacuum values. In fact, its order of magnitude extracted from the lattice results [2] is compatible to the vacuum nucleon mass with positive parity. Thus, the limiting case $\sigma_{q,s} \ll m_0$ is well justified even at very low temperature, and the mass relations are now fully consistent with the model-independent low-energy theorems (3) and (6) [4]. We note that the approximated expression (7) leads to a quite similar behavior in the thermal nucleon masses to that obtained from the original non-linear form with respect to the condensate, Eq. (3). In particular, temperature dependence of the mass difference of the nucleon parity-doublers is identical. Thus, the two expressions describe the physics equally well in the range of temperature of interest. This encourages us to apply the same scheme to the strange baryons.

We emphasize that the value taken from the lattice study [2] is not the conclusive number. However, such a large $m_0$ is actually consistent with the earlier

4 At $T \sim 0$, the negative-parity nucleon can be integrated out, so that the resultant Lagrangian includes only the positive-parity nucleon and mesons. Non-linear realization of chiral symmetry allows the fermion mass-operator $\bar{\psi}\psi$. When one introduces a term $m_0\bar{N}N$ on top of the meson-nucleon interaction in the Lagrangian, one finds the entire nucleon mass in the form of $m_N = c_1\sigma_q + m_0$ with a certain constant $c_1$. 

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model studies applied to the vacuum and to nuclear matter \[19,7\], where the $m_0$ values were determined to optimize the known phenomenological properties of the systems. The smallest value found in the available papers is of order $\Lambda_{\text{QCD}}$ \[3\]. Thus, the expansion in the ratio $\sigma_q/m_0 \sim 92 \text{ MeV}/\Lambda_{\text{QCD}} < 1$ is still adequate. Although the thermal profiles of the baryon masses will certainly change according to how large the $m_0$ is, the normalized mass difference is unaffected, as emphasized.

The parameters in Eqs. (7) and (9) at zero temperature are determined as in Table \[1\] where the following input was used $^5$: $m_{N^+} = 0.939 \text{ GeV}$, $m_{N^-} = 1.535 \text{ GeV}$, $m_{\Sigma^+_i} = 1.193 \text{ GeV}$, $m_{\Xi^+_i} = 1.318 \text{ GeV}$, $m_{\Delta^+_i} = 1.232 \text{ GeV}$, $m_{\Delta^-_i} = 1.710 \text{ GeV}$, $m_{\Sigma^+_*} = 1.383 \text{ GeV}$, and the pion and kaon decay constants $f_\pi = \sigma_q = 92.4 \text{ MeV}$, $f_K = (f_\pi + \sqrt{2}\sigma_s)/2 = 113 \text{ MeV}$ \[26\]. This leads to the masses of the remaining octet states,

$$m_{\Lambda^+_i} = 1.11 \text{ GeV}, \quad m_{\Lambda^-_i} = 1.79 \text{ GeV},$$
$$m_{\Sigma^-_i} = 1.88 \text{ GeV}, \quad m_{\Xi^-_i} = 2.09 \text{ GeV},$$

and of the other decuplet states,

$$m_{\Sigma^+_*} = 1.93 \text{ GeV},$$
$$m_{\Xi^+_*} = 1.53 \text{ GeV}, \quad m_{\Xi^-_*} = 2.15 \text{ GeV},$$
$$m_{\Omega^+_i} = 1.69 \text{ GeV}, \quad m_{\Omega^-_i} = 2.38 \text{ GeV}. \quad (11)$$

These masses of the positive-parity states are in quite good agreement with the PDG values. Masses, spin and parity of the above negative-parity states are not fully confirmed in experiments, thus they are excluded in the PDG Summary Table.

$^5$ A different assignment is possible; one can chose e.g. $N(1650)$ as the negative-parity partner of the lowest nucleon. Such a variation in the assignment does not yield any significant difference in the bulk equation of state, fluctuations and correlations \[25\].
3 Effective masses in hot matter

The mass modifications will be brought by the quark condensates $\sigma_q$ and $\sigma_s$ in a medium. To quantify those effects, we take the standard linear sigma model Lagrangian with three flavors:

$$\mathcal{L}_L = \bar{q} (i \partial - g T^a (\sigma^a + i \gamma_5 \pi^a)) q + \text{tr} \left[ \partial_\mu \Sigma^\dagger \cdot \partial^\mu \Sigma \right] - V_L(\Sigma),$$

where the potential, including $U(1)_A$ breaking effects, is

$$V_L = m^2 \text{tr} \left[ \Sigma^\dagger \Sigma \right] + \lambda_1 \left( \text{tr} \left[ \Sigma^\dagger \Sigma \right] \right)^2 + \lambda_2 \text{tr} \left[ \left( \Sigma^\dagger \Sigma \right)^2 \right] - c \left( \text{det} \Sigma + \text{det} \Sigma^\dagger \right) - \text{tr} \left[ h \left( \Sigma + \Sigma^\dagger \right) \right],$$

with the chiral field $\Sigma = T^a \Sigma^a = T^a (\sigma^a + i \pi^a)$ as a $3 \times 3$ complex matrix in terms of the scalar $\sigma^a$ and the pseudoscalar $\pi^a$ states. The last term with $h = T^a h^a$ breaks the chiral symmetry explicitly.

For thermodynamic calculations, we employ the mean field approximation. We also assume that there is the SU(2) isospin symmetry in the up and down quark sector. This leads to $\sigma_0$ and $\sigma_8$ as non-vanishing condensates, which contain both strange and non-strange components. The pure non-strange and strange parts are obtained through the following rearrangement,

$$\begin{pmatrix} \sigma_q \\ \sigma_s \end{pmatrix} = \frac{1}{\sqrt{3}} \begin{pmatrix} \sqrt{2} & 1 \\ 1 & -\sqrt{2} \end{pmatrix} \begin{pmatrix} \sigma_0 \\ \sigma_8 \end{pmatrix}. \quad (14)$$

In this basis, the effective quark masses read

$$M_q = \frac{g}{2} \sigma_q, \quad M_s = \frac{g}{\sqrt{2}} \sigma_s. \quad (15)$$

The explicit symmetry breaking terms are related with the pion and kaon masses as

$$h_q = f_\pi m_\pi^2, \quad h_s = \sqrt{2} f_K m_K^2 - \frac{f_\pi m_\pi^2}{\sqrt{2}}. \quad (16)$$

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6 Since Eqs. (7) and (9) are model-independent at tree level, they are also true in a quark-meson model as considered in this section. The in-medium condensates can be computed in any alternative approaches.
The entire thermodynamic potential is given by

\[ \Omega = \Omega_q + V_L, \]  

(17)

with the thermal-quark contribution

\[ \Omega_q = 6T \sum_{f=u,d,s} \int \frac{d^3p}{(2\pi)^3} \left[ \ln (1 - n_f) + \ln (1 - \bar{n}_f) \right], \]  

(18)

with the Fermi-Dirac distribution functions, \( n_f, \bar{n}_f = 1 / (1 + e^{(E_f - \mu_f)/T}) \), and the quasi-quark energies, \( E_f = \sqrt{p^2 + M_f^2} \). By minimizing the thermodynamic potential, the two mean-fields are determined self-consistently at a given \( T \) and \( \mu_f \) via \( \frac{\partial \Omega}{\partial \sigma_q} = \frac{\partial \Omega}{\partial \sigma_s} = 0 \). In this work, we consider thermodynamics at \( \mu_f = 0 \), and use the model parameters fixed in the vacuum \[27\], summarized in Table 2.

3.1 Condensates

In-medium condensates \( \sigma_q \) and \( \sigma_s \) at finite temperature are shown in Fig. 1 (left). It is clearly seen that the melting strange condensate is delayed because of the stronger explicit symmetry breaking with the strange quark. Nevertheless, the \( \sigma_s \) exhibits an abrupt, milder than the \( \sigma_q \) though, change near the crossover temperature \( T_c \), which is driven by the light flavor chiral dynamics. For comparison, we also show the corresponding lattice date taken from \[28\] where the thermodynamic quantities were calculated in the physical pion and kaon masses. The light-quark condensate follows more or less the model result, whereas the strange-quark condensate shows a rather mild behavior but the trend seen in the model calculation stays. We will not make any extrapolation to higher temperature using the model since the validity of this sort of hadronic models is questionable.

In the recent lattice study \[2\], their simulations were carried out with a relatively large up- and down-quark masses and the physical strange quark mass, leading to \( m_\pi \sim 400 \) MeV. Thus, it is constructive to study the condensates...
Fig. 1. Thermal expectation values of the mean fields, $\sigma_q$ and $\sigma_s$, calculated in the chiral model for the physical $m_\pi$ and $m_K$ (left) and for heavier meson masses with a quadratic $r = (m_\pi/m_K)^2 = 0.64$ and a linear $r = m_\pi/m_K = 0.8$ ratios (right). The pseudo-critical temperatures fixed from the chiral susceptibility are $T_c = 151$ MeV for the physical $r$, $T_c = 203$ MeV for $r = 0.64$ and $T_c = 272$ MeV for $r = 0.8$, respectively. On the left-hand figure, the corresponding lattice data for the physical pion and kaon masses [28] are given for comparison.

also for a heavier quark mass. To this end, we introduce the mass ratio,

$$r = \left(\frac{m_\pi}{m_K}\right)^2,$$

which scales like the quark mass ratio $m_q/m_s$ guided by the Gell-Mann–Oakes–Renner (GOR) relation. The physical value is $r_{\text{phys}} = 0.077$ with $m_\pi = 138$ MeV and $m_K = 496$ MeV [26]. Since this is considerably smaller than the above-mentioned lattice setup $r_{\text{lat}} \sim 0.64$, the quadratic scaling imposed by the GOR relation might be violated in the system with a very massive $m_\pi$.

We therefore examine the quark-mass dependence assuming a linear scaling, $r = m_\pi/m_K$, as well, by replacing $m_\pi^2$ with $r m_K$ in Eq. (16). The results are summarized in Fig. 1 (right). It is found that a somewhat stronger deviation from the quadratic scaling is seen in the $\sigma_s$ than in the $\sigma_q$, but the difference is rather minor. Thus, in the following, we will consider only the quadratic case.

Note that the Gell-Mann–Okubo mass formula (5) and Gell-Mann’s equal spacing rule (6) hold at any temperature.

3.2 Baryon octet

By substituting the obtained in-medium condensates into the mass relations (7) and (9), all the baryon masses are now obtained. We emphasize that chiral symmetry restoration does not dictate directly how the masses of the nucleon and baryon resonances should go. What is required for the mass spectra is that the parity partners become degenerate. Namely, the mass difference be-
The thermal mass difference of the baryon octet is presented in Fig. 2. For comparison, we also show the masses calculated with the thermal profiles $\sigma_q, s$ shown in Fig. 1 (left). The $\delta m_N$ evolves with the thermal $\sigma_q$ and drops substantially toward $T_c$. It agrees well with the result calculated with the lattice $\sigma_q$. The trend becomes milder due to stronger explicit symmetry breaking when the pion mass is increased. One sees a fairly good agreement with the $\delta m_N$ from lattice QCD with heavier pion [2].

Different behavior of the $\sigma_s$ from the $\sigma_q$ comes in to the states including strangeness, $\Sigma, \Lambda$ and $\Xi$. There is a clear contrast to the $\delta m_N$: The mass difference $\delta m$ of the strange baryons is reduced when the ratio $r$ is increased, despite a stronger explicit symmetry breaking. This is because the underlying flavor symmetry turns into an SU(3) when the ratio $r$ approaches unity, which is seen already in Fig. 1.
3.3 Baryon decuplet

The mass modifications of the baryon decuplet in Fig. 3 have a quite similar trend to those of the baryon octet.

We see a rather strong deviation in the $\delta m_{\Omega}$ with $r = 0.64$ from the corresponding lattice points. An approximate SU(3) structure for larger $r \sim 1$ supports this tendency to some extent. Yet, there remains a qualitative difference. The actual mass may not follow the linear dependence as in Eq. (9). It requires more realistic treatment of the in-medium strange baryons to resolve this discrepancy. One missing piece in the current hadronic approach is a mechanism of deconfinement. It is interesting to see in a more microscopic model how much the onset of deconfinement affects the hadronic quantities slightly below $T_c$.

4 Conclusions

We have formulated a complete set of mass relations for the baryon octet and decuplet with positive and negative parity in a chiral approach. The celebrated Gell-Mann–Okubo mass formula and Gell-Mann’s equal spacing rule are now manifest. We have also demonstrated the thermodynamic calculations with several pion masses in the mean field approximation, and have shown thermal
baryon masses in terms of the approximate order parameter of QCD chiral symmetry.

For the physical pion and kaon masses, the mass splitting $\delta m$ between the positive and negative parity states crucially depends on their strange-quark content, which is clearly seen in the numbers the $\sigma_s$ in the mass relations. The $\delta m_N$ and $\delta m_\Delta$ exhibit an abrupt drop near the chiral crossover temperature $T_c$, whereas the $\delta m$ of strange baryons drops rather slowly. The size of the $\delta m$ grows gradually with strangeness.

For a qualitative comparison to the recent lattice results [2], we have studied the thermodynamics with a larger $m_\pi$-to-$m_K$ ratio $r$ than its physical value. Given $r$ comparable to the lattice setup of [2], it is clearly seen that the $\sigma_s$ does not differ much from the $\sigma_s$ because of the pion mass rather closer to the kaon mass. Thus, the underlying flavor symmetry is an approximate SU(3). Consequently, the $\delta ms$ of the strange baryons are more reduced for $r$ closer to unity (the exact SU(3) limit). The $\delta ms$ of $\Sigma, \Lambda, \Xi, \Sigma^*$ and $\Xi^*$ are to be compared with future lattice results when available.

One of the immediate questions is how we see such thermal modifications of the baryons masses in bulk thermodynamic quantities and observables. These resonances get broadened because of the medium effects, and in fact the importance of the resonance widths have been studied in the context of the fluctuations of conserved charges and the pion distributions in heavy-ion collisions [29,30,31]. Since the width broadening and the mass modification should be linked, a more realistic next-step would be in line with the Greens function method with a proper extension of the work done in [32].

The lattice results [2] also show that the chiral-invariant mass for the nucleon is quite close to that of the $\Delta$ state. In general, they can differ due to e.g., the spin-spin interaction. These survival masses should be saturated by the condensate of the chiral-even operators, in particular, the gluon condensate is a promising major contributor [10,13]. In hadronic approach, the gluon condensate is introduced as a dilaton associated with the conformal symmetry breaking. When it is applied to a hot/dense medium, the vacuum expectation value (VEV) of the dilaton gives the non-vanishing value of $m_0$ at a given temperature or density. Under the mean field approximation, however, the VEV is found not to change much up to the chiral crossover, reflecting the fact that the sigma boson is much lighter than the glueball [13]. Therefore, as the first approximation, a frozen dilaton, leading to a constant $m_0$, is fairly acceptable. A nearly-constant $m_0$ may imply that it is dominated by the color-magnetic gluons, rather than the color-electric component which drives deconfinement. Since this issue is ultimately linked to the fundamental question, what the origin of mass is, further studies at finite temperature and density will shed more light on the dynamical generation of the hadron masses.
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