Coulomb and nuclear components of the breakup, their interference and effect on the fusion process

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Abstract. We discuss reaction mechanisms involving weakly bound nuclei, at near barrier energies, and the couplings between different reaction channels. This paper may be thought as a brief description of state of the art of this field, particularly on breakup reactions and their influence on the fusion cross section. Recent experimental and theoretical results are presented, including the interference between Coulomb and nuclear components of the breakup and the systematics so far reached on the static effects due to the characteristic of weakly bound nuclei, especially halo-nuclei and the dynamic effects of the breakup coupling on the fusion cross section.

1-Introduction

Several reaction mechanisms may occur when weakly bound nuclei are involved, at energies near the Coulomb barrier, apart from the usual processes which are present when tightly bound nuclei interact (inelastic excitations, direct transfer of nucleons or clusters of nucleons, fusion). If at least one of the colliding nuclei (usually the projectile) has small breakup threshold energy, typically smaller than 3 MeV, this nucleus may breakup in the field of the partner nucleus and different characteristic processes may occur, such as sequential complete fusion, when all fragments fuse, incomplete fusion (ICF) - when some but not all fragments fuse and non-capture breakup (NCBU) - when neither fragment fuses. Complete fusion (CF) is the sum of sequential and direct complete fusion. Total fusion (TF) is the sum of CF and ICF.

It has been observed that the optical potential in the elastic scattering of weakly bound nuclei does not have the usual energy dependence of tightly bound nuclei, namely the threshold anomaly, corresponding to the decrease of the imaginary potential when the bombarding energy decreases towards the barrier energy, but rather a different behaviour that was named breakup threshold anomaly (BTA) \cite{1}. The BTA occurs because the breakup cross section is still important at sub-barrier energies and it is possible to observe that the imaginary potential may even increase at energies close to the...
barrier, because breakup produces repulsive polarization potentials [2-4]. Theoretically, the most suitable calculations involving breakup are the so-called CDCC (continuum discretized coupled channel) calculations, since the breakup feeds states in the continuum. It has been shown that if one wants to describe the behaviour of elastic scattering angular distributions of weakly bound nuclei, it is essential that continuum-continuum couplings are included in the CDCC calculations [3,5].

These and other special features of reactions involving weakly bound nuclei, both stable and radioactive, and particularly the very specific characteristics of halo-nuclei, made this field a subject of great interest, both theoretically and experimentally, in the last years. A comprehensive review report has been published on this field [6]. Recently it has been observed experimentally [7] that at sub-barrier energies the breakup following direct transfer of nucleons of stable weakly bound nuclei (\(^6\)Li, \(^7\)Li, \(^7\)Be) predominates over the direct breakup of those nuclei.

Among the most important questions on this subject, one finds: (i) Does the breakup enhance or suppress the CF and or the TF cross sections? (ii) What is the relative importance of the Coulomb and nuclear components of the breakup? Do they interfere? (iii) How large are the NCBU cross sections, compared with the fusion cross sections? The answers to these questions depend on the energy regime (above or below the barrier), the target mass or charge and if the projectile has halo-characteristics.

### 2-The non-capture breakup

The measurement of NCBU cross section is a very difficult task. It requires very accurate exclusive experiments with coincidences between the fragments and then to convert the events in integrated cross sections. A clear identification of the processes, including sequential breakup (breakup following transfer) is possible through the Q-values of the reactions. [7]. However, if one is interested in the investigation of the effect of breakup on the fusion cross section, it is of fundamental importance to have indications on the time scale of the breakup. If the breakup occurs when the projectile approaches the target, what is called prompt breakup, it may affect fusion. Otherwise, if the breakup occurs when the projectile is already far from the target and moving away from it, what is called delayed breakup, the process cannot affect fusion. There are different forms of delayed breakup, those corresponding to the population of a long-lived resonance before the breakup and the breakup following a direct transfer of nucleons. In the prompt breakup, the relative energy between the fragments usually is large, whereas for delayed it is small. The relative energy may be determined in some experiments where the trajectory of the fragments can be determined [7].

One may predict the whole direct breakup (prompt + delayed) by performing reliable CDCC calculations. This was done [8, 9] for the \(^6\)Li projectile on four different targets, from \(^59\)Co to \(^208\)Pb. By reliable calculation we mean that no free parameter was used, only predictions using potentials which agree with available elastic scattering angular distributions. In what follows we give some details of our CDCC calculations.

The wave function with total angular momentum \(J\) and z-projection \(M\) can be schematically written as

\[
\Psi^{\alpha}(\hat{r}R) = \sum_{i} \frac{F^{\dagger}_{i}(\hat{R})}{R} \Psi^{\alpha}(\hat{r}) ,
\]

where \(i\) stands for the set of the quantum numbers \(\{\epsilon_i, l_i, j_i, L_i\}\), being the centroid bin energy, the \(\alpha\) -d angular momentum, \(j_i = l_i + s\) (where \(s\) is the spin of the deuteron) and the \(^6\)Li – target angular momentum, respectively. \(\hat{r}\) represents the internal coordinate of the \(^6\)Li projectile; \(\hat{R}\) is the projectile-target separation vector; \(\hat{r}\) stands for its angular degrees of freedom. \(\Psi^{\alpha}(\hat{r})\) is the tensor product of the internal wave function of the projectile and the angular part of the projectile-target relative motion wave function. Inserting this wave function in the usual Shrödinger equation, and after some manipulations one obtains the following set of coupled channels equations

\[
[T_\alpha + U^{\dagger}_{\alpha}(\hat{R}) - E_\alpha]F_{\alpha}(\hat{R}) = - \sum_{\alpha'} \sum_{i} \int U^{\dagger}_{\alpha i}(\hat{R})F^{\dagger}_{i}(\hat{R}) .
\]
The index \( i = 0 \) stands for the elastic channel, while \( i > 0 \) are associated with the continuum bin states. Within the cluster model approximation, the interaction of the \(^4\)Li with the targets is split into two parts

\[
V(\mathbf{R}) = V_{\sigma}(\mathbf{R}) + V_{\alpha}(\mathbf{R}),
\]

(3)

corresponding to the \( \alpha \)-target and deuteron-target interactions, respectively. The matrix-elements in expression (2) are given by

\[
U_{ij}(\mathbf{R}) = \int d^3 R \frac{d^3 r}{r^4} r^2 Y_i^m(\mathbf{R}) V(\mathbf{R}) Y_j^m(\mathbf{R}).
\]

(4)

The calculations were performed by switching on and off the Coulomb and nuclear components of the potential of the equation (3), to investigate separately the nuclear and Coulomb components of the breakup. The calculations have shown that for large scattering angles, corresponding to short range scattering, the nuclear breakup component may predominate over the Coulomb component, and that at energies above the barrier of light systems, the integrated nuclear component may also predominate. Very interesting results \[8, 9\] are the observation that there is a strong destructive interference between the two breakup components, and that the Coulomb component is larger than the total breakup at energies below the barrier and, for heavy targets, even for some energies slightly above the barrier. Otmar et al. and Hussein et al. \[8, 9\] were able to show that the nuclear breakup component cross section increases linearly with \( A_t^{1/3} \), for the same \( E_{c.m.} / V_B \) energy, where \( A_t \) and \( V_B \) are the target mass and height of the Coulomb barrier, respectively. For the Coulomb component of the breakup, the cross section increases linearly with the \( Z_t \). The calculated integrated total breakup cross section was found to be larger than the experimental fusion cross section at sub-barrier energies for medium mass and heavy targets, and smaller than fusion cross section at energies above the barrier. This may be understood because at sub-barrier energies it is required the tunneling of the barrier to occur fusion, whereas breakup is a direct process and no tunneling is required.

3-Effect of breakup on the fusion cross section

Now we turn to the discussion of fusion cross section and the effect of the breakup on the fusion mechanism. The first point that has to be clear when one talks about enhancement or suppression of fusion of weakly bound nuclei owing to the breakup is in relation to what one is talking about. One may compare data with theoretical predictions or compare data between weakly bound and tightly bound systems. However, another very important point to make is to disentangle between two different effects, the static and the dynamic. The former is due to the longer tail of the optical potential owing to the weakly bound nucleons, especially if the nucleus is of halo-type. The latter is due to the strong coupling between the elastic channel and the continuum states representing the breakup channel. When one compares experimental fusion cross sections with theoretical predictions, the difference between them should be the “ingredients” missing in the theoretical calculations. So, if the calculations consider only a single channel, without any couplings and furthermore, standard densities for the weakly bound nuclei, the difference between fusion data and theory corresponds to all static plus dynamic effects. However, standard densities for weakly bound nuclei, especially halo-nuclei, are not realistic, since those nuclei have much more diffuse densities than the tightly bound ones. Figure 1 shows fusion cross section data for the \(^4\)He + \(^{209}\)Bi system [10] and two theoretical curves. In both calculations there are no couplings and the double folding Sao Paulo potential [11] was used. The dashed curve is the result if one uses as the neutron density of the neutron-halo \(^4\)He a “standard” density corresponding to the density of \(^4\)He scaled to consider the ratio of the number of neutrons (4 and 2). Therefore, the difference between fusion data and the dashed curve corresponds to all static plus dynamic effects on the fusion cross section. The full curve is the result when one considers the realistic proton and neutron densities of \(^4\)He, a neutron-halo nucleus, which produces a smaller Coulomb barrier height [12]. The difference between the fusion data and the full curve corresponds to all dynamic effects or all coupling effects not considered in the theory. So, the difference between the full and dashed curves represents the static effects of the neutron halo-characteristics of \(^4\)He. One can
observe from figure 1 that this effect is of one magnitude order at sub-barrier energies, which can be considered as a large effect, although smaller than the low lying coupling effects of highly deformed nuclei, like $^{154}$Sm, at sub-barrier energies [13-15]. We believe that the static effect of halo-nuclei, enhancing the sub-barrier fusion cross section is well understood at the present. Figure 1 also shows that the static effects also enhance the fusion cross section at energies above the Coulomb barrier.

### 4- Fusion cross section reduction method

If one wants to compare fusion cross section data of different systems, including tightly and weakly bound ones, in the same figure, of course one can not compare cross sections versus $E_{\text{c.m.}}$, since the gross dependences of sizes and charges, as well as diffused densities, leading to different barriers, should be removed before the comparison is made. The most widely “reduction” methods are to divide, in the vertical axis, the cross section by $R_B^2$, where $R_B$ is the position of the Coulomb barrier (related to the size of the system) and to divide $E_{\text{c.m.}}$ by $V_B$ or to use $E_{\text{c.m.}} - V_B$, in the horizontal axis. Gomes et al [16] suggested an improved and alternative method to deal with weakly bound systems. The conclusions of the comparisons between fusion cross sections of different systems were shown to change depending on the reduction procedure used [17], even for very similar systems. Furthermore, it was shown that none of the above mentioned methods fully eliminates the static effects [17, 18]. Canto et al. [17, 18] proposed the use of dimensionless quantities, which appropriately eliminates static effects, as a procedure to investigate dynamic effects on the fusion cross sections due to the breakup couplings. Furthermore, the proposed method allows reaching a systematic understanding of this subject, since it allows the comparison of any kind of system in the same graphic. This method uses a benchmark curve, called the Universal Fusion Function (UFF), given by $F_0(x) = \ln [1 + \exp (2\pi x)]$, where $x = (E-V_B) / \hbar \omega$ and $F(x) = (2 E_{\text{c.m.}} / \pi R_B^2 \hbar \omega) \sigma_{\text{fus}}$. $\hbar \omega$ is related to the barrier curvature, $\sigma_{\text{fus}}$ is the fusion cross section and $F(x)$ is called fusion function. Since this method is inspired by the Wong formula [19], which is not valid for light systems at sub-barrier energies and one is not usually interested in the well known dynamic effects on fusion owing to inelastic excitation couplings, the experimental fusion function has to be renormalized by using appropriated coupled channel calculations, in order to be compared with the UFF curve. Details of this procedure may be found in refs. [17, 18]. The differences between the renormalized experimental fusion functions and the UFF curve are dynamic effects due to the channels left out of the coupled channel calculations, in this case, breakup and transfer reactions. In our calculations, the bare potential used was the double folding Sao Paulo potential [11]. The method proposed by Canto et al. [17] was later extended for the analysis of total reaction cross section [20].

![Figure 1: Comparison of experimental fusion cross section of the neutron-halo $^6$He nucleus with two theoretical predictions. See text for details.](image-url)
Figure 2 illustrates, with four systems, the systematics obtained for the dynamic effects of breakup plus transfer channels on the total fusion of stable weakly bound nuclei, complete fusion of stable weakly bound nuclei, total fusion of neutron-halo nuclei and total fusion of proton-halo nuclei. The figure on the left is in logarithmic scale, more suitable to analyze the sub-barrier energy region and the figure on the right, in linear scale, is more appropriate to analyze the effects at energies above the barrier. Total fusion of several stable tightly and weakly bound systems, represented in figure 2 by the $^9$Be + $^{208}$Pb system [21], from very light to heavy targets, coincide with the UFF at energies above the barrier. This means that there is no dynamic effect of breakup plus transfer on the total fusion cross section at this energy regime. At energies below the barrier, some small enhancement may be observed. For the complete fusion of stable weakly bound nuclei, represented in figure 2 by the $^9$Be + $^{208}$Pb system [21], some suppression of the order of 30% is found, at energies above the barrier, for several systems, with target masses larger than around 90, since there are no CF cross section data available for light systems. It is very interesting to observe that for each weakly bound projectile, the CF suppression factor is independent of the target mass [22]. An analytical relation of the suppression factor with the breakup threshold energy was derived by Wang et al. [22]. However, a physical explanation for that expression is still missing. For the fusion of neutron-halo nuclei, represented in figure 2 by the $^6$He + $^{209}$Bi system [10], suppression above the barrier and some enhancement below the barrier is also observed for most of the available data. However, a quantitative estimation of the suppression factor has not been achieved, due to the lack of a larger number of systems investigated and the large error bars in the fusion cross sections of radioactive projectiles. Finally, for fusion with proton-halo nuclei, there are only two reported systems, for $^8$Be + $^{58}$Ni [23] and $^8$Be + $^{28}$Si [24], with contradictory results. Fusion data by Pakou et al. [24] follow the same systematics as for neutron-halo systems, whereas Aguilerá’s data [23], shown in figure 2, presents a very unusual enhancement at energies above the barrier. Rangel et al. [25] suggested that the fusion cross section for the $^8$B + $^{58}$Ni system may be overestimated, because the protons detected and considered by the authors to originate from fusion evaporation have also contributions from protons from breakup.

![Figure 2: Renormalized experimental fusion function for some selected systems. The curve is the benchmark UFF curve. See text for details.](image)

An interesting question is why the dynamic effect of breakup plus transfer on the fusion cross section is to suppress fusion at energies above the barrier and to enhance fusion below the barrier. Gomes et al. [26] suggested an explanation based on the energy dependent optical model and dynamic polarization potentials (DPP). Calculations for direct breakup [2-4] and for quasi-elastic barrier distributions [27, 28] show that the direct breakup produces repulsive DPP, owing to the couplings among continuum breakup states (continuum-continuum couplings), which increases the barrier height and suppress fusion. On the other hand, recent experimental evidences mentioned at the beginning of this paper [7] show that breakup of stable weakly bound nuclei triggered by nucleon transfer
predominates over the direct breakup, at sub-barrier energies. So, the polarization potentials for each one should be evaluated separately and the results summed. Thus, the suppression of CF above the Coulomb barrier should result from the predominance of the DPP associated with direct breakup, whereas transfer and transfer followed by breakup, both producing attractive DPP, predominates at sub-barrier energies, especially for neutron-halo nuclei. The suppression above the barrier can also be explained by the BTA already mentioned.

Finally, we would like to mention the apparent contradiction between the observed facts that the effect of the breakup on the suppression of the complete fusion cross section does not depend on the target mass (or charge), but the breakup cross section increases with the target mass and charge. As mentioned at the beginning of this paper, only prompt breakup may affect fusion, and the calculated breakup contains both prompt and delayed breakups. So, the conclusion is that the prompt breakup cross section may not depend on the target mass or charge.

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