The SAS Gamma-Ray Spectrometer

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Abstract

A new type of compact high-resolution high-sensitivity gamma-ray spectrometer for short-pulse intense gamma-rays (250 keV – 50 MeV) has been developed by combining the principles of scintillators and attenuation spectrometers. The first prototype of this scintillator attenuation spectrometer (SAS) was tested successfully in Trident laser experiments at LANL. Later versions have been used extensively in the Texas Petawatt laser experiments in Austin TX, and more recently in OMEGA-EP laser experiments at LLE, Rochester, NY. The SAS is particularly useful for high repetition rate laser applications. Here we give a concise description of the design principles, capabilities and sample preliminary results of the SAS.

1. Introduction

Conventional gamma-ray spectrometers use the “single photon counting” method by employing scintillators (NaI, CsI, BGO, Midgall et al 2013, Lowdon et al 2019, Wender 1983) coupled to photo-multiplier tubes (PMT, Polyakov 2013), or solid-state (e.g. HPGe) detectors (Pehl 1977). Each gamma-ray deposits all of its energy in the scintillator or solid-state detector, converting its energy into optical photons and/or photoelectrons, which are then amplified for electronic current readout. The gamma-ray spectrum is built up one photon at a time by measuring the total energy deposited by each gamma-ray. In this approach consecutive gamma-rays arriving at the detector must be separated in time longer than the scintillation plus electronic readout time, typically > nanoseconds. Otherwise multiple low-energy gamma-rays cannot be distinguished from a single high-energy gamma-ray. Hence single-photon-counting gamma-ray spectrometers cannot be used in short-pulse intense laser experiments, where a large number of gamma-rays arrive in ps to sub-ps time scales, much shorter than the scintillation or electronic readout time.

At present, filter-stack attenuation spectrometer (FSS), made up of a series of high-Z filters arranged in tandem (Chen C.D. et al 2009) or configured as step wedges combined with radiographic films or image plates (IP), has been the main diagnostic to measure continuum gamma-ray spectrum in high-energy-density (HED) and laser experiments. However, FSS allows only a small number of energy channels (typically < 20), and covers energies only up to ~ 6 MeV. For gamma-rays with energy ≥ ~5 MeV, the mass attenuation coefficient reverses its decline due to photoelectric effect and rises with increasing energy due to pair production (Heitler 1954). Hence above ~ 5 MeV the attenuation length can no longer provide an unambiguous measure of the gamma-ray energy, even though detailed Monte Carlo modeling can provide some constraints on the spectral shape up to ~ 10 MeV. Other techniques that have been attempted to measure the energy of short-pulse intense gamma-rays include nuclear activation thresholds (Leemans et al. 2001) and Forward Compton scattering (FCS, Morgan et al. 1991, Kojima et al 2014). However, none of these techniques have been able to provide high-resolution high-sensitivity spectroscopy of intense gamma-rays emitted in short-pulse laser and HED experiments.
Over the past decade, a collaboration between Rice University and the medical imaging group at the MD Anderson Cancer Center (MDACC) in Houston, has developed a new type of gamma-ray spectrometer, which we will call scintillation attenuation spectrometer (SAS). The basic idea is to image the 2-dimensional (2D) scintillation light pattern emitted by a finely pixelated (with ~mm-sized pixels) scintillator matrix when it is irradiated sideways by a narrow collimated beam of gamma-rays. Since the 2D energy deposition pattern in the scintillator block varies with incident gamma-ray energy, the high-resolution 2D scintillation light patterns can in principle be used to reconstruct the incident gamma-ray spectrum, provided the emerging scintillation light profile is sufficiently bright and faithfully reproduce the local energy deposition profile of the gamma-rays. For this technique to work well, a large number of conditions must be met. Some of the most important requirements are listed here. (a) Each pixel must internally reflect 100% of scintillation photons emitted in that pixel except at the front surface, so that the light emerging from the front surface of that pixel faithfully represents the energy deposited only in that pixel. (b) Each pixel must be as narrow as possible in order to maximize the spatial resolution and minimize the internal absorption of the scintillation light. (c) The scintillation material should have the highest light output for each MeV of absorbed gamma-ray energy. (d) Scintillator material of the highest-Z and highest-density should be used so that the gamma-ray is attenuated efficiently, in order to make the detector as compact as possible. Guided by these four principles and the optimization of the many other conflicting requirements, we created a working prototype of the SAS after many trials and errors, using mm-sized Ce-doped LYSO scintillator crystals. Each pixel was manually coated with a special low-Z ultra-thin reflector optimized for 420 nm photons. The cutting, polishing, coating and gluing processes are highly labor intensive and required many specialized equipment developed at MDACC. Ce-doped LYSO was the scintillator of choice because of its high-Z, high-density (7.8 gm/cc) and high light output (50000 420nm blue photons per MeV absorbed) and it is non-hydroscopic. The first SAS prototype, consisting of a 24 x 36 matrix of 1.5mm x1.5mm x 10mm pixelated LYSO(Ce) crystals was successfully demonstrated in the summer of 2015 in a high-intensity Trident short-pulse laser experiment at LANL. Despite the low quality of this first image (Fig.1) due to the crude CCD camera and light leakage from the container box, this proof-of-principle experiment demonstrated the utility and functionality of the SAS in short-pulse laser experiments. This groundbreaking experiment at Trident demonstrated several important properties which provided confidence in the successful construction of a high-resolution compact SAS for laser, fusion and many other HED gamma-ray applications. (a) The LYSO crystals produced abundant light that can be easily imaged using conventional CCD cameras without intensification or cryogenics. (b) The gamma-rays penetrated all 36 longitudinal LYSO pixels. This means that a larger crystal matrix incorporating more pixels will still produce sufficient light output from all pixels. (c) The 2D light patterns produced by different incident gamma-ray energies were clearly distinguishable. (d) The intense EMP and neutron flux in the experiment did not affect the SAS performance.
We note that pixelated scintillators coupled to PMTs have been widely used for high-resolution gamma-ray imaging in many fields, such as PET cameras in medical imaging, and in high-energy physics experiments (Yamamoto et al 2018). However their use for spectroscopic measurements of short-pulse intense gamma-rays have not been systematically exploited. Recently, Belm et al (2018) explored the use of segmented blocks of NaI and CsI scintillators to measure gamma-ray spectrum in LWFA laser experiments. However, these detectors were designed for gamma-rays with higher energies (100 MeV-GeV range). Because large scintillator blocks of NaI and CsI (> cm) must be used, they result in a large (~meter-sized) detector, low light intensity and low spectral resolution. Typically only a simple model spectrum with a few input parameters can be meaningfully constrained using iterative Monte Carlo simulations, by matching the observed light profiles with GEANT4 (Allison et al 2016) model predictions (Belm et al 2018). To our knowledge no previous pixelated scintillator spectrometers have been designed to provide the first-principles reconstruction of completely unknown incident gamma-ray spectrum with high spectral resolution and high fidelity. This is the goal of the SAS.

2. Design, Sample Data and Calibration of SAS

After the successful proof-of-principle demonstration at the Trident laser experiment in 2015, we constructed 36x48 (Fig.2) and 36x60 LYSO matrix blocks and used them in our Texas Petawatt (TPW) laser experiments in 2016 and 2018. The SAS was also used successfully in OMEGA-EP experiments in 2020. A 36x60 LYSO matrix block, measuring only 6 cm × 8 cm, can fully capture the scintillation light pattern of gamma-rays up to 50 MeV. We have upgraded the CCD camera to a high-sensitivity high-speed non-cryogenic camera with wide-field non-distorting lens and a CCD chip optimized for 420 nm light, plus remote control and data-link. The entire SAS apparatus is housed in a light-tight black box approximately the size of a shoe box (Fig.3), with thick (> 2 inches) lead shielding all around and a 3 mm - 6 mm pinhole for the collimation of incident gamma-rays, which irradiate the matrix block along the central long axis. A tiny hole in the SAS housing is used for alignment of the matrix block central long axis with the laser target. In some high neutron flux experiments, additional neutron-shielding material is used to protect the front end of the housing around the LYSO block. After using the SAS in many TPW shots, we have found that the CCD camera itself is insensitive to EMPs. However, the cable connecting the CCD camera to the control computer (located far from the target chamber) must still be protected with ferrite coils.
to avoid upsetting the computer. Our current CCD camera is capable of taking repeated 1ms exposures. Hence the SAS is ideal for use in high-repetition-rate laser experiments.

**Fig. 2** (above left) Image of the pixelated LYSO matrix block used in the SAS, consisting of 36 x 48 1.5mm x 1.5mm x 10mm LYSO pixels. The entire block measures 6 cm H x 8 cm W x 1 cm D. Collimated gamma-rays irradiate the crystal block from the center of the left edge.

**Fig. 3** (above right) Sketch of the SAS layout with overall dimensions. Not shown are the external and internal lead shields with 3mm – 6mm pinholes used to collimate the incident gamma-ray beam. The lens of the CCD camera is positioned at 18 cm from the scintillator block to provide the best image of the entire 36 x 48 crystal.

Fig. 4 shows a typical SAS raw image taken during the 2016 TPW experimental campaign, using 6mm pinhole collimators. Fig. 5 shows raw SAS images at two different detector angles from a single 2018 TPW shot using 3mm pinhole collimators. It demonstrates that the gamma-rays emitted at the target normal (TN) direction is much brighter and harder than those emitted at 90° from target normal.

**Fig. 4** SAS raw image of LYSO scintillation light of Shot 10026 from 2016 TPW experiments. Gamma-rays entered from a 6mm-diameter pinhole on the left. This image shows over 400 bright pixels, which in principle can be deconvolved into a gamma-ray spectrum of up to 200 energy channels.
Fig. 5 Two SAS raw images from a 2018 TPW shot using 3 mm pinholes. The left image comes from a SAS located at ~ 90° from target normal. The right image comes from another SAS located at target normal. These raw images clearly demonstrate that the gamma-rays emitted at target normal are brighter and harder than those emitted at ~ 90° from TN. The CCD camera used to obtain the left picture is 50% more sensitive than the one used to obtain the right picture. Hence the background of the left picture appears higher even though the intrinsic signal is weaker.

Fig. 6 Calibration of the SAS using a $^{137}$Cs source of known intensity. (a) is raw SAS image, (b) is GEANT4 simulated image, (c) is the SAS longitudinal light profile compared to the corresponding GEANT4 simulated profile, (d) is the SAS transverse light profile compared to the corresponding GEANT4 simulated profile. The agreements between experimental data and GEANT4 predictions are excellent. This validates the GEANT4 simulation results at least for low-energy gamma-rays.
We have carefully calibrated the SAS response in the laboratory using gamma-ray emitting isotopes such as $^{137}$Cs, $^{22}$Na and $^{207}$Bi. Fig.6 shows the SAS image for $^{137}$Cs (0.67 MeV) compared to the GEANT4 (Allison et al 2016) simulated image (cf. Sec.3 below) using a detailed model of the LYSO matrix block, including all the material lying between the pixels (glue, reflectors etc). We see that the SAS data agrees very well with GEANT4 predictions. This gives us confidence in the validity of our GEANT4 model simulations of the SAS response discussed in the next section.

3. GEANT4 Modeling of LYSO Light Profiles

We use GEANT4 (Allison et al 2016) to model the response of our LYSO matrix block to incident gamma-rays. GEANT4 is the industry-standard particle physics code which incorporates almost all relevant physics of gamma-ray interaction with matter. GEANT4 employs the Monte Carlo method, which creates and tracks each particle (photons, electrons, positrons) after emission and scattering events, and eliminates the particle after an absorption event. Interactions are based on their energy-angle-dependent mean-free-paths. Below we summarize the model inputs.

![Geometry of the LYSO matrix plus lead shields used in GEANT4 simulations](image)

*Fig.7 Geometry of the LYSO matrix plus lead shields used in GEANT4 simulations. The bottom panel depicts the depth of each pixel. The lead collimator on the right is only modeled for the $^{137}$Cs calibration experiment of Fig.6.*

**Geometry and Material Input**

The entire LYSO scintillator block is modeled as an array of 36 x 1 x 48 crystal pixels coated with an optically reflective plastic wrap of thickness $=0.01667/2$ cm. Each pixel is a LYSO block fully coated with plastic wrap, with a combined dimension of 0.16667cm x 1.0cm x 0.16667cm since the LYSO crystal is 0.15cm thick, while the wrap is 0.01667/2 cm thick on each side. The scintillator medium is Lu$_{1.9}$Y$_{0.1}$SiO$_5$ doped with 0.5% Ce. The plastic wrap is C$_5$H$_8$O$_2$ with a density of 1.18 g/cm$^3$. 
Fig. 8a (left) Sample GEANT4 gamma-ray tracks for the $^{137}$Cs calibration experiment. Fig. 8b (right) Sample GEANT4 gamma-ray tracks for a 6mm diameter cylindrical beam incident along the longitudinal central axis of the matrix block.

**Gamma Ray Source Input**

For the calibration experiment of Fig.6, a hemispheric Cs-137 source of 5 million particles at 0.662 MeV is injected and filtered by a lead collimator of 6cm x 0.6cm x 1 cm with a 3mm pinhole. For the distant collimated gamma-ray source, a mono-energetic cylindrical beam of 1 million gamma-ray photons is injected. The diameter of the cylindrical beam is 6 mm. To produce a detector response matrix (DRM) relevant to our applications, 200 uniformly spaced gamma-ray energies are simulated, ranging from 0.25 MeV to 50 MeV, with 0.25 MeV intervals. To reduce the running time, the energy deposition in each pixel is computed instead of the scintillation light output, because in the GEANT4 scintillation process, the number of optical photons emitted is directly proportional to local energy deposition. A root file containing the 2D histogram of energy deposition values in all (36 x 48) pixels is generated for each incident gamma-ray energy.

Fig.9 shows GEANT4-simulated LYSO scintillation light patterns of various monoenergetic incident gamma-rays to highlight the gradual change of the light pattern with gamma-ray energy. The gradual transition from the “candle light” pattern (left) of 0.5 MeV gamma-rays to the “tear drop” pattern (right) of 50 MeV gamma-rays is caused by the domination of photoelectric effect in the former case, and pair production in the latter case. Compton electrons dominate in the middle panel for 5 MeV gamma-rays. For 50 MeV gamma-rays, it takes up to 60 longitudinal pixels to fully capture their light output.

**Fig.9** GEANT4-simulated scintillation light patterns for different monoenergetic gamma-ray energies. The color is in log scale.
4. Construction of The Detector Response Matrix

For all gamma-ray spectrometers, the first step to invert or extract the incident gamma-ray spectrum from the detector signal is to build up a detector response matrix (DRM) using Monte Carlo simulations described in Sec.3, which maps the incident gamma-ray energies to the light output of all pixels. For our SAS spectrometer, only gamma-rays up to 50 MeV are effectively captured, hence we inject monoenergetic gamma-rays from 0.25 MeV to 50 MeV at 0.25 MeV intervals, using $10^6$ Monte Carlo particles per run. This allows us to construct a 200x200 DRM, which maps 200 gamma-ray energy channels onto the scintillation light output of 200 “effective” SAS pixels (Fig.10). As Fig.9 shows, almost all scintillation light is concentrated in the first 5 longitudinal columns adjacent to the central axis. Hence we order the pixel numbers as 1-48 for the first column, 49-96 for the second column, 97-144 for the third column, 145-192 for the fourth column. Since the fifth column is already very faint, we combine the 48 pixels into 8 “large” pixels by averaging the light of every 6 pixels. We also average the light from corresponding pixels of the left half and right half of the matrix to get better statistics. This gives us the light output of 200 “effective pixels”. We emphasize that a 200 x 200 DRM is optimal only for gamma-rays up to 50 MeV. To accommodate gamma-rays > 50 MeV, we will need more longitudinal pixels and more energy channels, resulting in a larger DRM. Moreover, a square DRM is only required if we want to directly compute the inverse matrix. The inverse matrix is useful only if the DRM is highly non-singular so that the incident gamma-ray spectrum can be directly obtained by multiplying the light output profile with the inverse matrix. As we discuss in Section 5 below, the SAS DRM is “almost singular”, meaning that any tiny noise in the light data will be amplified by the inverse matrix to dominate the true signal and give unphysical results. In practice, a rectangular matrix, with more pixels than energy channels (i.e. an over-determined system of linear equations), will lead to more stable and reliable inversion solutions. These are discussed in the next section.
**Fig.10 (a)** Labeling of the pixel number starting from the center line of the 36 x 48 LYSO matrix: pixels 1 – 48 denote first column, pixels 49-96 denote second column etc. However, in the fifth column we combine five LYSO pixels into 1 pixel due to the faintness of the light output. Gamma-rays enter the matrix from the bottom center. Left and right halves of the matrix are assumed to be reflection symmetric, so that their light outputs are averaged to compute the DRM. (b) 3D contour plot of 200 x 200 DRM obtained by 200 GEANT4 simulations. We assume that the scintillation light output of each LYSO pixel is proportional to the total energy deposited inside that pixel. The folded accordion pattern of the DRM is due to the way we order the pixel numbers in Fig.10 (a). (c) 200-pixel light intensity profiles corresponding to the three pictures of Fig.9.

5. Gamma-Ray Spectrum Inversion of SAS Images

Because the 200x200 SAS DRM is almost-singular with a condition number $> 10^7$, direct inversion using the inverse DRM matrix is impractical. Over the past few years, we have explored a large variety of regularization and forward folding methods. At present, we have settled on a hybrid algorithm based on a combination of the Non-Negative Truncated Singular Value Decomposition method (TSVDNN with 5 – 6 terms) and the Parametrized Forwarded Folding (PFF) method, which seem to give the best results. When this hybrid algorithm is applied to model input spectrum with both broad spectral features and narrow spectral lines, it gives reliable stable solutions. When this algorithm is applied to actual SAS data from TPW experiments, it also appears to produce robust stable results. We have also determined that in most cases, the best solutions are obtained by consolidating the 200 energy channels into 100 channels, so that the reduced DRM becomes a 200 x 100 rectangular matrix. This reduced DRM still provides us with 0.5 MeV spectral resolution for the inverted spectrum up to 50 MeV.

Here we provide a short introduction to the hybrid algorithm. The TSVDNN is based on truncated singular value decomposition, with a constraint added to the solution space to accept only positive spectrum (Verkruysse et al 2005). Singular value decomposition decomposes the solution to the inversion problem into vectors with associated coefficients related to the singular values (Yagle 2005). The vectors are ordered such that the first terms with the largest singular values are least susceptible to data noise. As a result, the truncation of higher terms most susceptible to noise leads to a stable solution. In our case keeping only the first 5 – 6 terms give the best results. However, we find that such brute force truncation frequently gives us spectrum with negative values. Through experiments with simulated data, we realize that the negative values are likely cancelled by the truncated terms in the case of perfect data without noise, while the spectrum becomes dominated by noise if we add back those terms for real data with noise. To circumvent this difficulty, we apply an iteration criterion in which we zero out all the negative values in the TSVD series during each iteration, when we solve for the coefficients of the decomposition vectors. It turns out that this TSVDNN algorithm gives rather good results in reproducing the position of the peaks and general shape of the input spectrum. Only when we have a very narrow peak in the input spectrum, the TSVDNN solution starts to deviate from the true shape of the peak and returns a wider peak. Nevertheless, the position of the peak is still well captured. We can then resort to our physical knowledge about the truth width of the peaks (e.g. narrow nuclear lines). We can improve on the TSVDNN result with what we call a Parametrized Forward Folding (PFF) technique. We use as the model for data-fitting reasonable analytic trial functions which are based on the TSVDNN solution. The best-fit parameters are then determined
by least square fitting to the original light profile data using the Levenberg-Marquardt algorithm, rather than fitting the TSVDNN inversion solution.

Fig.11 illustrates the process of inverting the light profile created by a known analytic model gamma-ray spectrum. The input spectrum consists of a low-energy exponential plus a narrow line at 4.5 MeV plus a broad Gaussian bump centered at 20 MeV (Fig.11(a)). 5% random noise is added to the simulated light profile data (Fig.11(b)) before it is inverted. We first use TSVDNN with 6 terms to obtain a “regularized” spectrum. This “regularized” inversion spectrum (Fig.11(c)) is obtained with no knowledge of the shape or functional form of the input spectrum. Hence it is a “first principles” solution, without any knowledge of the incident spectrum. We then follow up with PFF optimization, using an analytic function based on the TSVDNN spectrum. The final best-fit PFF solution (Fig.11(d)) is in good agreement with the original input spectrum. We have performed other similar exercises using different analytic input spectra, all with satisfactory results. These exercises give us confidence in applying this algorithm to extract gamma-ray spectrum from experimental data obtained by the SAS.

(a) Model input gamma-ray spectrum consisting of an exponential plus a narrow (1-bin) line plus a broad Gaussian “bump”. The x-axis is in units of 0.5 MeV. This model spectrum generates the light profile of Fig.11(b) black curve when 5% random noise is added. The x-axis labels are the pixel numbers based on Fig.10(a). (c) Best-fit TSVD-inverted spectrum derived from the black curve of Fig.11(b). (d) Best-fit PFF spectrum based on the spectral parameters of the TSVD spectrum of Fig.11(c). The light profiles corresponding to the TSVD spectrum (red) and the PFF spectrum (green) are included in Fig.11(b) for comparison.
Fig. 12 illustrates the process of extracting the incident gamma-ray spectrum from the SAS image of Fig. 4. We first digitize the raw image into 36x48 uniform pixels, remove the nonlinearity in the CCD camera response, interpolate and extrapolate over the defective or saturated pixels, and subtract the background. The reprocessed light profile data (Fig. 12a) is first inverted using the TSVDNN algorithm without any assumption or knowledge of the incident gamma-ray spectrum. This solution is shown in Fig. 12b. An analytic model based on the TSVDNN spectrum of Fig. 12b is then used as the trial function in the PFF algorithm. The final best-fit PFF solution is given in Fig. 12c. In Fig. 12d we compare the light profiles of Fig. 12(a) with those derived from the spectra of Fig. 12b and Fig. 12c. Fig. 12c shows that the best fit gamma-ray spectrum consists of a low-energy exponential component plus a broad bump centered at ~17 MeV. The low-energy exponential spectrum is consistent with former results obtained from our FSS spectrometer, but the broad bump at ~17 MeV is a new spectral feature which could not have been discovered using previous spectrometers. We detected such 15-20 MeV gamma-ray bumps in many of our TPW shots.

In general, we find that the gamma-ray spectrum inverted from the SAS images using the TSVDNN+PFF algorithm is rather robust and stable, even when we vary the background noise level substantially. Uncertainty in the digitization and reprocessing of the light profile data is
inevitable. We believe that the key to the stability and reliability of our inverted gamma-ray spectra is the large number of bright pixels in the light profiles, which tightly constrains the inverted spectrum. This is especially true when we reduce the DRM to 200x100 so that the inversion solution is nominally “over-determined”.

6. Discussion and Summary

In this paper we have presented the basic concept, design and some preliminary results of a new type of gamma-ray spectrometer called the SAS, which combines the physical principles of attenuation and scintillation by a finely pixelated matrix. The key innovation is the use of mm-scale pixelated LYSO(Ce) crystals with 100% reflective coating for every pixel to form a large matrix, that can fully capture and localize gamma-ray energy deposition patterns up to 50 MeV in a compact volume. The abundant light output of LYSO crystals provides high signal-to-noise images with a large number of bright pixels even at low gamma-ray fluence, and the high-speed CCD camera can capture scintillation light profiles with millisecond exposures, ideal for high-repetition rate laser applications. We have demonstrated that the SAS can provide reliable spectrum with 100 energy channels for gamma-rays up to 50 MeV. Expansion of the LYSO matrix to more pixels (e.g. 36x72, 36x84) should be able to extend the gamma-ray spectrum to 100 MeV with many more channels. If necessary, we can also increase the spectral resolution by using smaller pixels. Inverted spectra obtained from our past TPW, Trident and Omega-EP experiments will be published in future papers.

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