An Energy Approach to the Calculation of Forces Acting on Solid Bodies in Ferrofluids

A. S. Ivanov

Institute of Continuous Media Mechanics, Ural Branch, Russian Academy of Sciences, Perm, 614013 Russia
e-mail: lesnichiy@icmm.ru

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Abstract—The main advantages of the energy approach to solving the problem of determining magnetic forces acting on solid bodies immersed into magnetized ferrofluids (FFs) are shown. Characteristic disadvantages of the standard approach to the calculation of magnetic forces using the Bernoulli equation for FFs and an equation for the magnetic pressure jump at the interface are considered. A review of works devoted to the study of forces acting on solid bodies immersed in magnetized FFs is presented. This literature review convincingly demonstrates the need for and potential advantage of using the energy approach to these problems, since the analytical expressions significantly depend on the body shape and obtaining the final numerical results is complicated by the error of magnetic field calculation at the “solid body—FF” interface where the normal component of induction and the tangential component of the magnetic field exhibit a discontinuity. In contrast, the energy approach allows using the standard functions of program packages for determining thermodynamic potentials. The choice of a thermodynamic potential correctly describing experimental data is discussed. The method of magnetic energy determination is justified by the problem setting and verified by comparison of the results of several numerical solutions obtained using the open software package FEMM for FFs obeying a nonlinear magnetization law. This analysis was previously performed neither experimentally nor theoretically in view of the commonly accepted use of simplifying assumptions (approximations of weak and strong magnetic fields or a noninductive approximation). Here, the energy approach to determining forces acting on solid bodies in FFs has been justified by pairwise comparison of the results obtained in the framework of this approach to the data of laboratory experiment and the results of standard calculations.

Keywords: ferrofluid, floating of bodies, magnetic field, finite element method, FEMM, thermodynamic potential

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1. INTRODUCTION

Solid bodies floating in magnetic ferrofluids (FFs) have been studied since the second half of 1960s. The first works in this field [1–3] were devoted to a unique phenomenon of permanent magnet levitation in a container with a FF. The floating of a solid magnet having density $\rho_{sb}$ immersed in a FF of lower density $\rho_{ff} \leq \rho_{sb}$ takes place due to the repulsion of its poles from the container walls representing interface between the magnetic and nonmagnetic media. According to an original approach to the description of forces acting on bodies immersed in FFs, which has been formulated by Rosensweig [2, 3] and is still used at present without significant modifications, the buoyancy force is calculated as integral of the pressure determined using the Bernoulli equation for the given FF over the body surface [1, 4].

Thus, for a point with coordinates $(r, \varphi, z)$ in FF where the magnetic field strength is $H$, the pressure $p$ is given by the following formula:

$$p = p_0 - \rho_{ff} g (z - z_0) + \mu_0 \int_0^\eta M(H_i) dH_i,$$

where $p_0 = \rho_0 (r_0, \varphi_0, z_0)$ is the constant pressure at some reference point (e.g., the atmospheric pressure at the point on a flat horizontal surface of the FF), $g = (0, 0, -g)$ is the gravitational acceleration, $\mu_0 = 4\pi \times 10^{-7}$ H/m is the magnetic constant, subscript $i$ denotes the integration variable, and $M(H)$ is a
nonlinear dependence of colloid magnetization $M$ on the magnetic field. The curve of FF magnetization $M(H)$ is a standard characteristic that can be measured in experiment. In addition to pressure (1), it is also necessary to take into account a specific (for the given FF) jump of magnetic pressure on the body surface (solid/fluid interface). Thus, the final expression for the force acting upon a solid body immersed in a magnetized FF takes the following form [4]:

$$F = m_{sb}g - \oint_S \rho dS - \mu_0 \oint_S \left( M_n^2/2 \right) dS = - \left( \rho_{sb} - \rho_{ff} \right) V_{sb}g - F_1 - F_{II}, \quad (2)$$

$$F_1 = \mu_0 \oint_S \left( M(H) dH \right) n dS, \quad (3)$$

$$F_{II} = \mu_0 \oint_S \left( M_n^2/2 \right) n dS, \quad (4)$$

where $V_{sb}$ is the body volume, $n$ is the vector of normal to the body surface, and $M_n = M n$ is the normal component of magnetization. In Eq. (2), the first term describes the Archimedean force, and terms $F_1$ and $F_{II}$ represent ponderomotive forces, which introduce certain difficulties in the analytical investigation since they are expressed via integrals of a complex nonlinear multiparametric function that is specific for each particular FF. The particular magnetization curve depends on the dispersion composition of ferrocolloid [5, 6], that is, on parameters of the colloidal particle size distribution and on the volume fraction $\varphi$ of the magnetic phase (concentration of magnetic nanoparticles). Since Eqs. (1)–(4) were obtained three decades before the exact theory of magnetogranulometric analysis of polydispersed FFs [5, 6] was formulated, the correct description of the variation was approximated (for simplification) by considering two limiting cases of a weak and a strong magnetic field, as determined by the corresponding dimensionless Langevin parameter

$$\xi = \mu_0 m H / (k_b T),$$

defined as the ratio of the magnetic energy of a colloidal particle possessing average magnetic moment $m$ to its thermal energy $k_b T$, where $k_b$ is the Boltzmann constant and $T$ is the absolute temperature. In case of FFs of the “magnetite–oleic acid–kerosene” type obtained by the standard chemical deposition method [4], the average magnetic moment of colloidal particles is $m \sim 10^{-19}$ A m$^2$ and the magnetic field is considered as weak ($\xi \ll 1$) for $H \sim 10^3$ A/m and as strong ($\xi \gg 1$) for $H \sim 10^5$ A/m. Both these approximations rather unsatisfactorily describe real laboratory experiments, since the characteristic magnetic fields obtained under laboratory conditions with solenoids and permanent magnets usually amount to $H \sim 10^4$ A/m. Nevertheless, the rough approximations of weak and strong fields are still in use because they allow a complicated nonlinear magnetization law $M(H)$ to be replaced by a linear function with a single parameter determined from experimental measurements:

$$M = \chi_0 H, \quad \mu = 1 + \chi_0 \gg 1 \quad \text{for} \quad \xi \ll 1,$$

$$M = M_s, \quad \mu = 1 + \chi = 1 \quad \text{for} \quad \xi \gg 1, \quad (5)$$

where $\chi_0$ is the initial magnetic susceptibility, $\mu$ is the magnetic permeability, $M_s$ is the saturation magnetization, and $\chi$ is the current magnetic susceptibility of the given FF. Both approximations are based on a simplifying assumption that $\chi = \text{const}$. Using this approach, it is possible to obtain simple analytical expressions for forces (see Eqs. (3) and (4)) and qualitatively estimate these by the order of magnitude.

The ideas of pioneering works [2, 3] were further developed in [7–10]. Cebers et al. [7, 8] obtained an analytical solution of the problem in an unusual (not practically encountered) setting, where a cylindrical magnet magnetized perpendicular to its principal axis is levitating in a horizontal cylindrical cavity filled with FF obeying a linear magnetization law ($M = \chi_0 H$). The same problem in a noninductive approximation was solved in [9]. This approximation is a very popular simplified approach to the calculation of magnetic fields and forces in FFs under assumption that $\chi \ll 1$ and $\mu = 1$, which is justified in two cases. First, in the case of the aforementioned strong external magnetic fields $H_0$ capable of magnetizing FFs up to saturation ($M = M_s, \chi \ll 1, \mu = 1$) and second, in low-concentration FFs ($\varphi \ll 1, \chi = M/H \ll 1, \mu = 1$) such as used in biomedical practice. The first case is applicable (with some reservations) to the problem of permanent magnet levitation in FFs. For example, estimation of the accuracy of the noninductive
approach [9] showed that the approximated solution satisfactorily agreed with the exact one [7, 8] only provided that \( \chi < 0.25 \). At the same time, e.g., for \( \chi = 1 \), a noninductive solution deviated from the exact one by 20\%, the difference for \( \chi = 1.5 \) exceeded 30\% and kept monotonically growing with increasing \( \chi \).

It should also be emphasized that, in the case of nonmagnetic bodies immersed in FFs, the noninductive approximation not only gives quantitatively incorrect results, but also qualitatively (conceptually) contradicts the setting of the problem as such. Indeed, demagnetizing field \( \mathbf{H}_{df} \) of the FF occurring in homogeneous external magnetic field \( \mathbf{H}_0 \) is the only cause of appearance of the levitation forces, whereas the noninductive approximation implies paradoxical neglect of this demagnetizing field based on the assumption that \( \mathbf{H}_{df} \ll \mathbf{H}_0 \).

Analytical investigations of the 2000s [11, 12] proposed an exact solution of the Laplace equation for a magnetic field in the problems of a spherical magnet or paramagnetic body floating in a spherical vessel filled with magnetic fluid in the presence of homogeneous external magnetic field \( \mathbf{H}_0 \). In the framework of the noninductive approximation, results [11, 12] obtained for the problems with spherical symmetry were also generalized to the case of an ellipsoidal vessel and considered in application to two limiting cases (infinite cylinder and infinite flat layer). Approximate formulas were obtained for the forces acting upon floating bodies. These results were significant, but admitted quantitative comparison with real laboratory experiments only in cases where the experimental conditions obeyed criteria of the noninductive approximation. Studies [11, 12] were followed by a series of works devoted to various principles of driving the mechanical motion of solid bodies by variable magnetic fields (see, e.g., [13]). These works were also based on the noninductive approximation and had an applied rather than a fundamental character. There also are many other works (see, e.g., reviews [14, 15]) related with the problem of solid bodies floating in magnetic-fluid devices (accelerometers, slope sensors, differential manometers, separators). However, despite numerous applications, these works did not suggest any new ideas for the fundamental science and only used approximate expressions obtained in the 1960–1980s. In other words, the only correct approach to calculations of forces acting on solid bodies immersed in FFs is still based on numerical solution of the system of Maxwell equations with a material equation for \( M(H) \) and substitution of the obtained numerical fields \( \mathbf{B}, \mathbf{H} \) into Eqs. (2)–(4).

Calculations using formulas (2)–(4) encounter, as a rule, some difficulties related to the determination of fields \( \mathbf{B}, \mathbf{H} \) immediately at the “solid body—FF” interface. These difficulties arise because the available open software packages solve the magnetostatic problem by the finite-element method for vector potential \( \mathbf{A} \):

\[
\text{curl} \left( \frac{1}{\mu_0 \mu} \text{curl} \mathbf{A} \right) = \mathbf{J},
\]

where \( \mathbf{J} \) is the vector of electric current density. By virtue of the well-known feature of this method, the differential operation of \( \mathbf{B} = \text{curl} \mathbf{A} \) at the interface leads to large computational errors. It is usually suggested [16, 17] to get around this difficulty by replacing calculations over the body surface with calculations over an auxiliary virtual surface (eggshell) spaced from the real one by certain distance exceeding the minimum sum of several periods of the computational grid or sometimes (in case of the automatic choice performed in the open software package) by a distance comparable with a characteristic size of the body. The validity of this approach was mathematically justified for a constant magnetic permeability of the medium \( \mu = \text{const} \). The force determined for the body surface is exactly the same as that calculated for a virtual surface occurring outside at an arbitrary distance from the body (the grid size is not important since the result is analytical and determined by properties of the Maxwell tensor). However, in view of the limiting condition \( \mu = \text{const} \), this approach is inapplicable in the case of a body immersed in FF, since inhomogeneous field \( \mathbf{H} \) in the vicinity of this body produces inhomogeneous magnetization of the FF and, hence, \( \mu \) is not constant. This problem is usually solved in the following way: true values of integrals (3) and (4) are found by extrapolation after multiply repeated calculations over imbedded virtual surfaces closely spaced both from each other and from the body under consideration [18, 19].

However, it is also possible to use an alternative energy approach to solving problems of this kind, which is free of the aforementioned difficulties and drawbacks and has the following evident advantages. First, the body energy is considered as integral over the volume and, hence, the accuracy of its calculation is less prone to the aforementioned errors related to the discontinuity of \( \mathbf{B}, \mathbf{H} \) functions on the body surface. Second, electromagnetic energy in the software packages employed is realized in the form of preset functions, which are much simpler to use than formulas (3) and (4), since the latter expressions depend on the body shape.
In the framework of the energy approach, the force acting upon a body immersed in the FF is expressed as the gradient of magnetic potential energy $W$:

$$ F = -\nabla W. \quad (7) $$

Results of calculations using formula (7) depend on two principally important circumstances. The first circumstance consists in that there are two approaches to determining the value of $W$ [20]:

$$ W_e = \int_V \left( \int_0^B H(B) dB \right) dV, \quad (8) $$

$$ W_{coe} = \int_V \left( \int_0^H B(H) dH \right) dV. \quad (9) $$

Here, $W_e$ is the energy of magnetic field and $W_{coe}$ (not given a special name in the Russian nomenclature) is sometimes referred to as “coenergy.” The internal energy $U_0$ and free energy $F_0$ defined by formulas (8) and/or (9) have different physical meaning. The change of thermodynamic potentials as $dW_e$ (8) represents the work performed by magnetic field on the system at constant potential of the magnetic field, that is, when the field in a laboratory experiment is generated by superconducting magnets. On the other hand, the changes in $dU_0$ or $dF_0$ as calculated via $dW_{coe}$ (9) specify the work performed on the system with constant sources of magnetic field, i.e., when the field in a laboratory experiment is generated by direct currents (solenoids and electric magnets connected to electric current sources). The latter variant is used in laboratory investigations much more frequently than the former variant. For this reason, many researchers used to write Eq. (7) without any explanations in the following form:

$$ F = -\nabla W_{coe}. \quad (10) $$

It should be emphasized that the aforementioned simplifications (5) principally do not distinguish Eqs. (8) and (9), since the induction $B$ in a linear system is directly proportional to field $H$ and hence $BdH = HdB$. This implies that the numerical verification of Eq. (7) with various values of (8) and (9) has a certain independent meaning, which is given special attention in the present work. The second circumstance concerning the applicability of the energy approach to the description of forces and its equivalence to Eqs. (2)–(4) is related to establishing the boundaries of space $V$ for the integration in expressions (8) and (9).

In concluding the introduction section, the purpose of this investigation can be formulated as using straightforward calculations to answer the question: which one of the two thermodynamic potentials, (8) or (9), and which region of space $V$ should be employed in the energy approach to the description of magnetic forces (2)–(4) really measured in the laboratory experiment.

2. PRACTICAL

For exactly answering the above questions, it is necessary to compare the magnetic forces as calculated by formulas (2)–(4) to two values of the force (7) obtained using the energy approach with energies (8) and (9), and compare these numerical data to the results of laboratory experiment [21] performed with the same purpose of studying forces acting on a nonmagnetic body immersed in the FF. The experiment [21] consisted in determining the electromagnetic forces from changes in the weight of a body immersed in a vessel with the FF. In the absence of field $H_0$, the body weight is determined entirely by the gravitational force and Archimedean buoyancy. Upon switching on the external field $H_0$, a magnetic component is added to the resultant force and the initial weight changes. This change is exactly the electromagnetic force to be determined.

The geometry and conditions of the computational problem exactly corresponded to parameters of the experimental setup [21] depicted in Fig. 1. In this configuration, tin ball $I$ with radius $R = 3.79\, \text{mm}$ was immersed on suspension wire into cylindrical glass vessel $2$ with diameter $D = 26.7\, \text{mm}$ and height $h = 13.65\, \text{mm}$ containing the magnetic fluid. The vessel with ball was mounted on flat horizontal platform $3$ that was rigidly fixed with a rod on the column of cathetometer $4$, while being mobile in the vertical direction. The position of the platform with the vessel inside a solenoid could be smoothly controlled in height, so that the mobile platform ensured vertical shift of the vessel with FF relative to the ball at rest on the suspension wire. The weight of the nonmagnetic ball was measured by high-precision analytical bal-
The value of the magnetic force acting on the ball during its shift relative to the center of vessel was determined as the difference of ball weights at this point in the presence of the applied magnetic field and in the zero field: \([m(z) - m_0(z)]g\). At the center of vessel, the probing body weight does not change at any value of the external magnetic field, \(m(0)g = m_0(0)g\), because the top and bottom lids (interfaces) act on the ball with equal force in opposite directions.

The magnetostatic problem corresponding to experiment [21] was numerically solved for the axisymmetric setting in cylindrical coordinates. The computational region comprised a spherical body at the center, a cylindrical vessel with FF, and a solenoid powered from a DC source. Dimensions \((R, D, h)\) of all objects and the solenoid were taken from experimental data. In view of the cylindrical symmetry, the three-dimensional (3D) problem admitted simplification to a 2D setting, which allowed its numerical solution using the open FEMM software package, the possibilities of which are restricted to flat and axisymmetric 2D systems.

Boundaries of the computational region consisted of a vertical line coinciding with the axis of symmetry \((z\) axis of cylindrical coordinates) and a semicircular arc with radius \(R\). Boundary conditions on the semicircular arc and vertical line were set as the open and symmetric conditions, respectively, for vector potential \(A\) [22].

Since expressions (3) and (4) were naturally not available as functions incorporated into FEMM program package, they were preliminarily written in the form of analytical expressions with allowance for the symmetry of the solid body. After some analytical transformations, the \(z\)-components of forces (3) and (4) acquire the following form:

\[
F_{iz} = 2\pi\mu_0 R^3 \int_{0}^{\theta_0} \int_{0}^{\phi_0} M(\hat{H}(r, z)) \hat{H}(r, z) \cos(\theta) \sin(\theta) d\theta d\phi,
\]

\[
F_{Hz} = \pi\mu_0 R^2 \int_{0}^{\theta_0} \int_{0}^{\phi_0} M_\pi^2(r, z) \cos(\theta) \sin(\theta) d\theta d\phi,
\]

(11)

where \(\theta\) is the polar angle. Calculations using Eqs. (11) were performed with allowance for the real physical properties of materials. The magnetization curve \(M(H)\) (Fig. 2) measured in experiment was loaded into FEMM package in a tabulated form as \(B(H) = \mu_0(M(H) + H)\).

The working program for FEMM package represented a script written in the Lua language, which contained consistently described geometry of the problem, properties of materials, initial conditions, and the main computational cycle. The initial conditions implied that the vessel with FF occurred at the top (with the tin ball touching the bottom). The computational cycle included the following sequence of operations: cleaning of the computational grid created in the preceding cycle; parallel shift of the vessel with FF along axis \(z\) by 0.1 mm downward (with the ball considered immobile); formation of the new computational grid; and calculation of surface integrals (3) and (4) for five concentric spherical surfaces with radii

\[
R_k = R + kdR \quad (k = 1, \ldots, 5),
\]
where increment \(dR\) amounted to one and a half step of the computational grid. The obtained values of forces (3) and (4) were recorded in the memory file and the main computational cycle was continued until the center of the vessel with FF sank below the origin of the coordinates. The need in multiply repeated calculations of integrals (11) for the extrapolation of their true values was explained in the introduction section.

The second stage of computational program also reproduced the geometry of the problem, and used it to determine (instead of forces (11)) the values of magnetic energy (8) and (9) separately for each region of space (solid body, FF, ambient air). Note that expressions (8) and (9) exist in FEMM in the form of incorporated functions, which is a significant advantage since the simplicity of expressions (11) is only related to the spherical geometry of nonmagnetic bodies. The values of energies \(W_{\varepsilon}(z)\) and \(W_{\text{coe}}(z)\) calculated with a 0.1-mm interval were interpolated using the corresponding polynomials in Wolfram Mathematica and then differentiated in the same package according to Eq. (7). The obtained values of forces \(F(z)\) were compared to those calculated by formulas (11) and with experimental data [21].

It should be pointed out that, to achieve the goal of the study, numerical simulations were performed for several values of the external magnetic field \(H_0\) and it was checked that the model FF with a given magnetization law \(M(H)\) would definitely obey neither conditions (5) nor the noninductive approximation.

3. RESULTS OF CALCULATIONS

Figures 3 and 4 present the results of calculations performed for an external magnetic field of \(H_0 = 20\) kA/m. The first stage (Fig. 3) was devoted to checking formulas for the direct determination of electromagnetic forces (3) and (4). Results presented in Fig. 3 are important for two reasons. First, the data show good accuracy of formulas (3) and (4) which is sufficient for the description of experimental measurements. Second, there is excellent coincidence of the calculated and experimentally measured values of forces, which confirms the correctness of magnetic field calculations because both field strength \(H\) and induction \(B\) enter immediately into formulas (3) and (4).

Since the first stage confirmed the correctness of the calculation of the magnetic field by the FEMM software package, the subsequent stage was devoted to calculating the same magnetic forces in the framework of energy approach (7). For this purpose, energies \(W_{\varepsilon}\) and \(W_{\text{coe}}\) were calculated by formulas (8) and (9), respectively, in three regions of computational space (tin ball, cylindrical vessel with FF, ambient air). The tin volume and ambient air give the equality \(W_{\varepsilon} = W_{\text{coe}}\) because these media are paramagnetic with constant \(\mu\) values. In contrast, the FF volume has \(W_{\varepsilon} \neq W_{\text{coe}}\) and this fact accounts for the difference of curves in Figs. 4a and 4b.

In this way, three values of \(W_{\varepsilon}\) (8) and one value of \(W_{\text{coe}}\) (9) were calculated for each magnetic field \(H_0\) and the tables of \(W(z)\) obtained were processed in Wolfram Mathematica by the algorithm described in the preceding section. Results of \(W(z)\) processing had the form of functions \(F(z)\) analogous to those pre-
sented in Fig. 4. As can be seen from these plots, only the use of \( W_{\text{coe}}(z) \) in the entire computational region provides a correct description of the acting forces (curve 4 in Fig. 4b). On one hand, this result is quite natural because, according to the problem setting, the magnetic field is created by a DC solenoid (rather than by superconducting magnets) and, hence, thermodynamic potentials of the internal \( (U_0) \) and free \( (F_0) \) energies should be determined using \( W_{\text{coe}} \). On the other hand, the need for taking into account the entire computational space is a somewhat unexpected result. Indeed, at the beginning of this study, the main assumption was that the magnetic force acting on a nonmagnetic body and really measured in laboratory experiment was determined by the “coenergy” \( (9) \) of the body, i.e., it was suggested that the integral in Eq. (9) should be calculated over the body (tin ball) volume. However, numerical results show that it is necessary to consider the entire “solid body–FF–air” system interacting by means of electromagnetic forces. Motion of the solid body leads to modification of the magnetic field not only inside the body, but in the FF and surrounding space as well, which (by virtue of Newton’s third law) produces a reverse action on the body. This action has a magnetic nature, but is manifested as a mechanical force that can be measured by an analytical balance in a laboratory experiment.

4. CONCLUSIONS

This paper presents a thorough review of works devoted to studying forces acting upon solid bodies immersed in magnetized FFs. This review gives convincing evidence for the need and potential advantage
of using the energy approach to solving these tasks since the expressions for forces (3) and (4) significantly depend on the body shape, while obtaining the final numerical results using Eqs. (3) and (4) is complicated by the considerable error of calculating fields $\mathbf{H}$ and $\mathbf{B}$ at the “solid body–FF” interface because the normal component of $\mathbf{H}$ and tangential component of $\mathbf{B}$ at this interface exhibit a discontinuity. In contrast, the energy approach in this case allows us to use standard functions of software packages for determining thermodynamic potentials (8) and (9). The difference between Eqs. (8) and (9) is determined by setting of the problem and can only be revealed for a medium obeying a nonlinear magnetization law. This analysis was previously performed neither experimentally nor theoretically in view of the commonly accepted use of simplifying assumptions (approximations of weak and strong magnetic fields or noninductive approximation).

The energy approach to determining forces that act on nonmagnetic bodies in FFs has been justified by pairwise comparison of resultant magnetic forces calculated by formulas (2)–(4) and (7) for energies (8) and (9), respectively. Results of these calculations were compared to the data of laboratory experiment [21]. The calculations were repeated for several values of the external magnetic field (15, 25, and 30 kA/m). All results were analogous to those presented in Fig. 4. These investigations showed that the magnetic force acting upon a solid body immersed in the FF was determined by energy (9) of the entire system “solid body–FF–ambient air,” rather than of the solid body alone. The explanation consists in that motion of the immersed body leads to a modification of the magnetic field not only inside the body but also in the FF and ambient air as well. The modified field, in turn (by virtue of Newton’s third law), produces the reaction of equal magnitude and opposite direction on the body. This mechanical force has a magnetic character and is really measured in a laboratory experiment.

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