Sensitivity of deexcitation energies of superdeformed secondary minima to the density dependence of symmetry energy with the relativistic mean-field theory

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Abstract

The relationship between deexcitation energies of superdeformed secondary minima relative to ground states and the density dependence of the symmetry energy is investigated for heavy nuclei using the relativistic mean field (RMF) model. It is shown that the deexcitation energies of superdeformed secondary minima are sensitive to differences in the symmetry energy that are mimicked by the isoscalar-ivector coupling included in the model. With deliberate investigations on a few Hg isotopes that have data of deexcitation energies, we find that the description for the deexcitation energies can be improved due to the softening of the symmetry energy. Further, we have investigated deexcitation energies of odd-odd heavy nuclei that are nearly independent of pairing correlations, and have discussed the possible extraction of the constraint on the density dependence of the symmetry energy with the measurement of deexcitation energies of these nuclei.

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1 Introduction

The nuclear symmetry energy plays an important role in astrophysics [1, 2, 3, 4], the structure of neutron- or proton-rich nuclei, and the reaction dynamics of heavy-ion collisions, see, e.g., Refs. [5, 6, 7]. However, the density dependence of the symmetry energy is still poorly known, for instance, see Ref. [7]. Recently, considerable progress has been made in constraining the
density dependence of the symmetry energy using data from heavy-ion reactions [8, 9, 10, 11, 12]. On the other hand, it is promising to constrain the density dependence of the symmetry energy at subsaturation densities by accurately measuring the neutron skin thickness in heavy nuclei. In the past, Horowitz et. al. proposed to measure the neutron radius of $^{208}$Pb by virtue of the parity-violating electron scattering on the neutrons that promises a 1% accuracy [2, 5, 13, 14]. While the precision measurement is still in progress, it is valuable to explore whether some other structural properties of finite nuclei are sensitive to differences in the symmetry energy.

As one knows, the neutron skin thickness in heavy nuclei depends sensitively on the density dependence of the symmetry energy. The sensitive probe to differences in the symmetry energy may thus possibly exist in systems that undergo a relative variation of the proton and neutron matter distributions. This relative variation can follow from the collective excitation or deexcitation between the ground state and superdeformed secondary minimum (SSM) in heavy nuclei with appreciable neutron excesses. Especially, the large relative variation can be expected for nuclei in the mass region $A \sim 190$ where the prolate superdeformation of the secondary minima usually occurs with the oblate ground states [15, 16, 17, 18]. In deed, quite different deexcitation energies of the SSM for nuclei in the region $A \sim 190$ were predicted by various models that usually diversify the symmetry energies, see Ref. [19] and references therein. Since in obtaining the deexcitation energy the isoscalar ingredients of the ground state and SSM cancel largely, the variation of the deexcitation energy can be mainly attributed to the uncertainty of the symmetry energy that is controlled by the isovector potential. However, a direct relationship between the deexcitation energy and the density dependence of the symmetry energy is not available in the literature. In this work, it is thus meaningful to establish this relationship. It is the aim of this work to constrain the density dependence of the symmetry energy in the relativistic mean-field (RMF) model through deexcitation energies measured and to be measured for nuclei in the region $A \sim 190$. We note that the constraint on differences in the symmetry energy has been investigated using properties of the isovector giant and pigmy dipole resonances [20, 21, 22, 23]. Apart from these dynamical resonances, the SSM features a static structure. Indeed, this is an attempt to constrain the density dependence of the symmetry energy using the information of atomic masses, since the deexcitation energy is the difference between masses of the SSM and ground state.

The paper is organized as follows. In Section 2, we briefly introduce the formalism of the deformed RMF model for finite nuclei. Results on the SSM and ground states of finite nuclei, especially the deexcitation energies are presented in Section 3. A summary is finally given in Section 4.
2 Formalism

The model lagrangian is written as:

\[
\mathcal{L} = \bar{\psi} [i\gamma_\mu \partial^\mu - M_N + g_\sigma \sigma - g_\omega \gamma_\mu \omega^\mu - g_\rho \tau_3 b^\mu_0 - e \frac{1}{2} (1 + \tau_3) \gamma_\mu A^\mu] \psi \\
- \frac{1}{4} F^{\mu \nu}_{\mu \nu} + \frac{1}{2} m_\omega^2 \omega^\mu \omega^\mu - \frac{1}{4} B^{\mu \nu} B_{\mu \nu} + \frac{1}{2} m_\rho^2 b^\mu_0 b^{\mu}_0 - \frac{1}{4} A^{\mu \nu} A_{\mu \nu} \\
+ \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma - m_\sigma^2 \sigma^2) + U(\sigma, \omega^\mu, b^\mu_0),
\]

where \( \psi, \sigma, \omega, \) and \( b_0 \) are the fields of the nucleon, scalar, vector, and neutral isovector-vector, with their masses \( M_N, m_\sigma, m_\omega, \) and \( m_\rho, \) respectively. \( A_\mu \) is the photon field. \( g_i(i = \sigma, \omega, \rho) \) are the corresponding meson-nucleon couplings. \( F^{\mu \nu}, B^{\mu \nu} \) and \( A^{\mu \nu} \) are the strength tensors of \( \omega \) and \( \rho \) mesons, and photon, respectively

\[
F^{\mu \nu} = \partial_\mu \omega_\nu - \partial_\nu \omega_\mu, \quad B^{\mu \nu} = \partial_\mu b_0^\nu - \partial_\nu b_0^\mu, \quad A^{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu.
\]

The self-interacting terms of \( \sigma, \omega \) mesons and the isoscalar-isovector coupling are given generally as

\[
U(\sigma, \omega^\mu, b^\mu_0) = - \frac{1}{3} g_2 \sigma^3 - \frac{1}{4} g_3 \sigma^4 + \frac{1}{4} c_3 (\omega_\mu \omega^\mu)^2, \\
+ 4 g_\rho^2 g_\omega^2 A_\nu \omega_\mu \omega^\nu b^\mu_0 b^{\mu}_0.
\]

Here, the isoscalar-isovector coupling term is introduced to modify the density dependence of the symmetry energy. In fact, the symmetry energy can be modified by many theoretical factors, for instance, the isoscalar-isovector coupling terms[2], the isovector-scalar mesons, the density dependent coupling constants[24], and the model chirality constraint [25]. In the past, the density dependence of the symmetry energy had been extensively explored through the inclusion of the isoscalar-isovector coupling terms [2, 3, 5, 13] in the RMF theory, and this allowed one modify the neutron skin of heavy nuclei without compromising the success in reproducing a variety of ground-state properties [2, 5]. In this work, the isoscalar-isovector coupling term is thus included in the deformed RMF model to modify the density dependence of the symmetry energy.

Using the Euler-Lagrangian equation, the Dirac equation of motion in RMF is written as

\[
[-i\alpha \cdot \nabla + \beta M^*_N + g_\omega \omega_0(\mathbf{r}) + g_\rho \tau_3 b_0(\mathbf{r}) + e \frac{1}{2} (1 + \tau_3) A_0(\mathbf{r})] \psi_\alpha(\mathbf{r}) = E_\alpha \psi_\alpha(\mathbf{r}),
\]

with \( M^*_N = M_N - g_\sigma \sigma(\mathbf{r}) \) and \( E_\alpha \) being the nucleon eigen energy. For simplicity, the isospin subscript for the \( \rho \)-meson field is omitted hereafter. For the mesons and photon, the equations of motion are given as

\[
(\Delta - m_\sigma^2) \phi(\mathbf{r}) = - s_\phi(\mathbf{r})
\]
where for the photon, $m_\phi = 0$, and

$$s_\phi(r) = \begin{cases} 
    g_\sigma \rho_s(r) - g_2 \sigma^2(r) - g_3 \sigma^3(r), & \sigma, \\
    g_\omega \rho_B(r) - c_3 \omega_0^3 - 8g_\omega^2 \rho_A \omega_0 \omega_0(r) b_0^2(r), & \omega, \\
    g_\rho \rho_3(r) - 8g_\rho^2 \rho_A \rho_0 \omega_0^2(r), & \rho, \\
    e \rho_c(r), & \text{photon.}
\end{cases}$$

Here $\rho_s$, $\rho_B$, $\rho_3$ and $\rho_c$ are the scalar, vector, isovector and charge densities, respectively. The total binding energy is written as

$$E_{\text{total}} = E_N + E_\sigma + E_\omega_0 + E_b_0 + E_c + E_{\text{CM}}$$

$$= \sum_\alpha (E_\alpha - M_\alpha) - \frac{1}{2} \int d^3r [g_\sigma \sigma(r) \rho_s(r) + \frac{1}{3} g_2 \sigma^3(r) + \frac{1}{2} g_3 \sigma^4(r)]$$

$$+ \frac{1}{2} \int d^3r [g_\omega \omega_0(r) \rho_B(r) + \frac{c_3}{2} \omega_0^3(r)]$$

$$+ \frac{1}{2} g_\rho \int d^3r b_0(r) [\rho_3(r) + 8g_\rho^2 \rho_A \omega_0^2(r) \rho_0(r)]$$

$$+ \frac{1}{2} e \int d^3r A_0(r) \rho_c(r) - \frac{3}{4} A^{1/3}.$$  \hspace{1cm} (7)

In practical calculations, we also include the BCS pairing interaction using the constant pairing gaps which are obtained from the prescription of Möller and Nix [26]: $\Delta_n = 4.8/N^{1/3}$, $\Delta_p = 4.8/Z^{1/3}$ with N and Z the neutron and proton numbers, respectively. Here, the cut-off $82A^{-1/3}$ MeV above the nucleon chemical potentials is used to normalize the pairing energy. We note that the BCS description for nucleon pairing is not as popular as the Bogoliubov approach, e.g., see Refs. [27, 28] and references therein. However, in this work we do not pursue a complete description of nucleon pairings but investigate the relative change in binding energies of the ground state and SSM with respect to the density dependence of the symmetry energy. For both ground states and SSM, the numbers of oscillator shells $N_F = N_B = 18$ are used in the basis expansion. The solution of the coupled Dirac and meson equations can be obtained in an iterative procedure that can be easily found in the literature [29], and it is not reiterated here.

3 Results and discussions

In this work, we make analyses based on the calculations with the RMF parameter set NL3* [30] that is an improved version of the parameter set NL3 [31]. Also, the isoscalar-isovector coupling is included to modify the density dependence of the symmetry energy. This new parameter set has been successfully tested by the properties of the ground states and collective excitations of finite nuclei. Based on this parameter set, here we investigate the deexcitation energies relative to the ground states for Hg and Au isotopes, while attention will be paid to the sensitivity of the deexcitation energy to differences in the symmetry energy.
Prior to practical calculations, it is useful to write here the explicit expression of the symmetry energy in the RMF models

\[ E_{\text{sym}} = \frac{1}{2} \left( \frac{g_\rho}{m_\rho^*} \right)^2 \rho_B + \frac{k_F^2}{6E_F^*} = \frac{1}{2\delta} g_\rho b_0 + \frac{k_F^2}{6E_F^*}, \]

where \( m_\rho^* \) is the \( \rho \)-meson effective mass with \( m_\rho^* = \sqrt{m_\rho^2 + 8\Lambda v (g_\omega g_\rho \omega_0)^2} \), \( \delta \) is the isospin asymmetry with \( \delta = \rho_3/\rho_B \), and \( E_F^* \) is the Fermi energy. The first term is the potential part of the symmetry energy, and the second term is the kinetic part. The modification to the symmetry energy is dictated by the potential part through the isoscalar-isovector coupling.

Since the symmetry energy is not well constrained at saturation density, some average of the symmetry energy at saturation density and the surface energy is needed according to the constraint from the binding energy of nuclei. For a given \( \Lambda_v \), we follow Ref. [2] to readjust the \( \rho NN \) coupling constant \( g_\rho \) so as to keep the symmetry energy unchanged at \( k_F = 1.15 \text{ fm}^{-1} \). In doing so, the symmetry energy is softened by the isoscalar-isovector coupling, as shown in Fig. 1. As a result, it was found that the neutron skin thickness relies sensitively on the \( \Lambda_v \), while the total binding energy of \(^{208}\text{Pb}\) just changes by a few MeV with the \( \Lambda_v \) of interest [2]. The small change in the total binding energy results from two cancellation mechanisms. The first mechanism is inherent in the RMF theory due to the cancellation between the big scalar attraction provided by the \( \sigma \) meson and the big vector repulsion provided by the \( \omega \) meson [32]. Though the isoscalar-isovector coupling modifies predominantly the isovector potential, its influence can be passed on to the isoscalar potential through modified nucleon matter distributions. Here, the \( \sigma \) field energy changes with respect to \( \Lambda_v \) coherently with the \( \omega \) field energy but with an opposite sign. The second mechanism can be understood according to the virial theorem, which can be written in the non-relativistic form as:

\[ <\psi_i|\frac{\mathbf{p}^2}{2M_N}|\psi_i> = \frac{1}{2} <\psi_i|\mathbf{r} \cdot \nabla V(\mathbf{r})|\psi_i> \]

where \( V(\mathbf{r}) \) is the nuclear potential. As shown in Fig. 1, the symmetry energy is modified oppositely below and above the fixed point (\( k_F = 1.15\text{ fm}^{-1} \)), and this is ensured by the similar modification to the isovector potential. Due to the strong coupling between protons and neutrons, modifications in the isoscalar potential are similar, albeit small. According to the virial theorem, the modification to the nucleon kinetic energy is associated with the gradient of the potential. The opposite modifications to the potential give rise to opposite increments for the gradient of the potential. Thus, the cancellation between opposite increments leads to just a small change to the nucleon kinetic energy. Eventually, the change in the total binding energy is small with respect to \( \Lambda_v \). Nevertheless, since the empirical binding energies of finite nuclei were firstly reproduced quite accurately by the best-fit models, the variation of the total binding energy, albeit small, can not simply be used to constrain the density dependence of the symmetry energy.

If the variation of the binding energy is expected to be informative to the density dependence of the symmetry energy, it should be the relative variation of the binding energies of
two systems that clearly differ by an isovector ingredient. In this work, we are thus interested in the deexcitation energy of the SSM relative to the ground state. However, since the isoscalar-iso-vector coupling modifies not only the isovector potential but also the isoscalar potential, an efficient probe to the density dependence of the symmetry energy should be able to separate clearly the modification to the isovector potential from that to the isoscalar potential. In the following, it is necessary to exhibit such a separation in obtaining the deexcitation energy. In order to analyze the potentials obtained with the deformed RMF code conveniently in one dimension, it is necessary to perform the multipole expansion as

$$U(r) = \sqrt{4\pi} \sum_{L=0,2,4,\ldots} U_L(r)Y_{L0}(\theta),$$

(10)

where $Y_{L0}$ is the spherical harmonic function. The deexcitation energy, which is the energy difference between the secondary minimum (s.m.) and the ground state (g.s.), is associated with the corresponding difference of the potentials

$$\Delta U_L^{iso}(r) = U_L^{iso, s.m.}(r) - U_L^{iso, g.s.}(r),$$

(11)

where the superscript $iso$ represents the isoscalar (IS) or isovector (IV). Here, the isoscalar and isovector potentials are given as $U_L^{IS}(r) = g_\omega \omega(r) - g_\sigma \sigma(r)$ and $U_L^{IV}(r) = g_\rho b_0(r)$, respectively.

Fig. 2 displays the difference of potentials between the SSM and ground state for $^{192,194}$Hg as a function of radius. In Fig. 2, only the two most important components $U_0(r)$ and $U_2(r)$
Figure 2: Difference of the potentials between the SSM and ground state of $^{192,194}$Hg as a function of the radius for various $\Lambda_v$. The calculation is performed within the NL3*. The left panel is for the isoscalar potential and the right panel for the isovector one. The result for $^{192}$Hg is displaced downwards by 2 MeV for clarity.

that are respectively responsible for the spherical matter distribution and quadrupole deformation are drawn, while the difference between the higher-L components of the potentials is becoming clearly smaller and much less sensitive to the $\Lambda_v$. It is shown in Fig. 2 that the difference of the isoscalar potential is almost independent of the isoscalar-isovector coupling $\Lambda_v$. On the other hand, the difference of the isovector potential varies significantly with the $\Lambda_v$ in the surface region. Especially, a large variation against the $\Lambda_v$ is seen for the isovector potential difference $\Delta U_2(r)$, associated with the large oblate-prolate shape difference between the ground state and SSM in $^{192}$Hg and $^{194}$Hg. These naturally exhibit a clear separation of the relative variation of the isovector potentials from that of the isoscalar potentials. In particular, the relative variation of isoscalar portions of the two states with respect to the $\Lambda_v$ nearly cancels out.

Since the isoscalar-isovector coupling modifies just slightly the nucleon potentials, it can be treated as a small residue interaction. This rules out the occurrence of large coherent changes in nuclear potentials of the ground and metastable states while with slight relative changes between various states, and thus the small change in total binding energies can be determined by that of nuclear potentials with rather stable nuclear structures. Consequently, the shift of the deexcitation energy caused by the isoscalar-isovector coupling can be obtained from the small relative change in nuclear potentials between the ground state and SSM. Due
to the nearly exact cancellation between the variations of isoscalar potentials in the SSM and ground state, the variation of the deexcitation energy results predominantly from the modification to the isovector potential caused by the isoscalar-isovector coupling. Besides from the potential part, the kinetic energy and the nonlinear term of the isoscalar-isovector coupling also directly cause the variation of the deexcitation energy, but they can after all be determined from the nuclear potential, for instance, according to Eqs.(5) and (9). Eventually, this builds up a direct relationship between the uncertainty of the deexcitation energy and the modification to the symmetry energy.

Table 1: Properties of the ground state and SSM for $^{194}$Hg within the NL3* with respect to $\Lambda_v$. The binding energy per nucleon ($B/A$), charge radius ($r_c$), neutron skin thickness ($r_p - r_n$), and quadrupole deformation parameter $\beta$ are listed. The properties of the SSM are denoted by the subscript $SD$. Energies are in unit of MeV and radii in unit of fm.

| $\Lambda_v$ | $g_p$ | $B/A$ | $r_c$ | $r_n - r_p$ | $\beta$ | $(B/A)_{SD}$ | $(r_n - r_p)_{SD}$ | $\beta_{SD}$ |
|------------|-------|-------|-------|-------------|---------|--------------|-----------------|--------------|
| 0.000      | 4.5748| 7.913 | 5.460 | 0.213       | -0.146  | 7.885        | 0.182           | 0.625        |
| 0.010      | 4.9005| 7.928 | 5.461 | 0.189       | -0.145  | 7.898        | 0.159           | 0.622        |
| 0.020      | 5.3074| 7.940 | 5.464 | 0.165       | -0.144  | 7.907        | 0.137           | 0.618        |
| 0.030      | 5.8360| 7.948 | 5.468 | 0.141       | -0.142  | 7.914        | 0.115           | 0.614        |

In Table 1, we tabulate as an example calculated quantities of the ground state and SSM of $^{194}$Hg with respect to the isoscalar-isovector coupling constant $\Lambda_v$. We see that except for the neutron skin thickness all the properties of $^{194}$Hg depend just slightly on the $\Lambda_v$. With the inclusion of the isoscalar-isovector coupling, the neutron skin thickness reduces appreciably. These are consistent with the early findings for heavy nuclei in the literature [2, 5]. It is seen in Table 1 that the SSM has a different neutron skin thickness from that of the ground state. This is due to the excitation of protons and neutrons in the vicinity of Fermi surfaces to different shells. Specifically, in $^{194}$Hg neutrons with the configuration $2f_{5/2}^21h_{9/2}$ that are below the major shell $N = 126$ in the ground state are excited to intruder orbitals that originate from spherical orbitals $2g_{9/2}1i_{11/2}$ above the major shell $N = 126$. The neutron occupation in intruder orbitals forms the important configuration $2g_{9/2}^11i_{11/2}^2$ for the superdeformation in the SSM. For protons, similar excitation to intruder orbitals occurs from the shell $50 < Z < 82$ in the ground state up to the shell $Z > 82$ in the SSM. Besides the characteristic excitation between the major shells, the excitation in subshells also contributes to the formation of the superdeformation in the SSM. Thus, the creation of interior holes and occupation of exterior orbitals at different major shells for protons and neutrons results in an appreciably different neutron skin thickness in the SSM from that in the ground state. As a result, the SSM differs from the ground state by an isovector ingredient from which the sensitivity of the deexcitation energy to the isoscalar-isovector coupling originates. For other Hg isotopes, the results are similar to those for $^{194}$Hg, and are not given here.

In Table 2, we list the deexcitation energies relative to the respective ground state for
Table 2: Deexcitation energies (MeV) of the SSM relative to ground states for a few Hg isotopes with respect to the $\Lambda_v$. The experimental values for $^{192,194}$Hg and $^{191}$Hg are taken from Refs. [19, 33], respectively.

| Model | $\Lambda_v$ | $^{194}$Hg | $^{192}$Hg | $^{191}$Hg |
|-------|-------------|-------------|-------------|-------------|
| NL3*  | 0.000       | 5.52        | 4.03        | 3.59        |
|       | 0.010       | 5.81        | 4.40        | 3.94        |
|       | 0.020       | 6.27        | 4.73        | 4.26        |
|       | 0.030       | 6.60        | 5.03        | 4.44        |
| Expt. |             | 6.0         | 5.3         | 4.6         |

$^{194}$Hg, $^{192}$Hg and $^{191}$Hg that have experimental data [19, 33]. We may observe from Table 2 and Fig. 1 that the deexcitation energies are sensitive to differences in the symmetry energy. Comparing with the experimental data, it is seen that the description for the deexcitation energies can be largely improved by including the isoscalar-isovector coupling. With $\Lambda_v = 0.03$, the deexcitation energies for $^{194}$Hg, $^{192}$Hg and $^{191}$Hg agree fairly well with the data within acceptable accuracy. This confirms the softening of the symmetry energy.

Table 3: Deexcitation energies for Hg isotopes recalculated with the slightly modified $m_\sigma$ with respect to the $\Lambda_v$. As compared to $g_\rho$ in Table 1, it is just slightly modified to keep the symmetry energy unchanged at $k_F = 1.15$fm$^{-1}$. Ground-state binding energies per nucleon $B/A$ are listed for $^{194}$Hg.

| $\Lambda_v$ | $m_\sigma$ (MeV) | $g_\rho$ | $^{194}$Hg | $^{192}$Hg | $^{191}$Hg | $B/A$ |
|-------------|------------------|----------|-------------|-------------|-------------|-------|
| 0.000       | 502.5742         | 4.5748   | 5.51        | 4.06        | 3.59        | 7.913 |
| 0.010       | 502.6050         | 4.9006   | 5.63        | 4.37        | 3.85        | 7.912 |
| 0.020       | 502.6310         | 5.3076   | 6.19        | 4.62        | 4.29        | 7.913 |
| 0.030       | 502.6465         | 5.8363   | 6.54        | 4.99        | 4.46        | 7.913 |

Now, we clarify the concern whether the variation of deexcitation energies can be disguised by the much larger change in the ground-state energy, which is about 7 MeV for the $\Lambda_v$ of interest, as estimated in Table 1. To do this, we just need to observe the variation of the deexcitation energy in the case of the constant ground-state energy. It is possible to reduce or even eliminate the variation of the ground-state binding energy by suitably choosing the fixed point which can affect the extent of cancellations, whereas this is numerically complicated. Alternatively, the elimination of the variation in the ground-state energy can actually be fulfilled by slightly readjusting the meson-nucleon coupling constants or meson masses. Without priority, here we realize it by slightly readjusting the $\sigma$ meson mass $m_\sigma$. The recalculated results are given in Table 3. As seen in Table 3, the variation of the total ground-state binding energy is nearly eliminated by just modifying the $m_\sigma$ up to 0.07 MeV. With this slight readjustment of the $m_\sigma$, the modification to the incompressibility is just
about 0.8 MeV. Noticeably, the variation of deexcitation energies is almost unchanged, as compared with that given in Table 2. Definitely, the sensitivity of deexcitation energies to differences in the symmetry energy is irrelevant to the variation of the ground-state energy. The underlying physics for this is attributed to the fact that the variation of the deexcitation energy is conditioned predominantly on the modification to the isovector potential that changes the density dependence of the symmetry energy, as analyzed for results shown in Fig. 2.

On the other hand, the difference between deexcitation energies of $^{194}$Hg and $^{192}$Hg is still large (about 1.5 MeV), as compared to the experimental value 0.7 MeV, and is not reduced by including the isoscalar-isovector coupling. It means that the accurate deexcitation energies for $^{192}$Hg and $^{194}$Hg both can not be obtained with the same $\Lambda_v$. It is seen in Table 2 that the difference between the deexcitation energies for $^{194}$Hg and $^{192}$Hg is not sensitive to the $\Lambda_v$. This is because the isospin asymmetries $\delta$ in $^{194}$Hg and $^{192}$Hg are not very different. Consequently, the considerable reduction of the difference between the deexcitation energies for $^{194}$Hg and $^{192}$Hg should resort to the alteration of the isoscalar potential. For instance, this difference is reduced to be 0.5 MeV with the RMF parameter set TM1 [34] whose isoscalar potential is moderately different from that with the NL3* due to the inclusion of the $\omega$-meson self-interaction. However, with the $\omega$-meson self-interaction, the softened vector potential leads to small deexcitation energies and much shallower potential wells of the SSM. As deexcitation energies for $^{192}$Hg and $^{194}$Hg are both much smaller than the data within the TM1, the isoscalar-isovector coupling can be included to partly compensate this discrepancy, whereas an overall compensation seems yet to be available in the model that features the $\omega$-meson self-interaction.

Further, we notice that the deexcitation energy is nearly of the same magnitude as the pairing energy. The correct deexcitation energy of some nuclei may be obtained with the inclusion of the isospin-dependent pairing interactions. For instance, using different pairing gaps and strengths in Ref. [35], the deexcitation energy of $^{194}$Hg with the NL3 was reproduced in nice agreement with the experimental value. On the other hand, the difference of the deexcitation energies between $^{192}$Hg and $^{194}$Hg was still 0.8 MeV larger than the experimental value [35]. If this prescription is applied to resolve the discrepancy of the deexcitation energy with the experiments for other isotopes, it will impose stringent constraints on pairing strengths. On the other hand, the symmetry energy in finite nuclei can successfully be extracted regardless of the pairing interaction [36]. This implies that the observable response to the symmetry energy would be rather independent of the pairing interaction. Specifically, we find that the deexcitation energies of the SSM in odd-odd nuclei in the region $A \sim 190$ just depend very weakly on the pairing interaction, while the sensitive dependence on the isoscalar-isovector coupling does not alter at all. The former is because the pairing interaction in odd-odd nuclei is substantially suppressed by the Pauli blocking and the nucleonic current that breaks the double degeneracy. If the pairing interaction turns off while reviving the nucleonic current and the contribution of the $\pi$ meson in these odd-odd nuclei [37], the
deexcitation energy is just changed by about 0.2 MeV. Therefore, no appreciable effect on deexcitation energies can be observed by modifying pairing strengths in these odd-odd nuclei. In Fig. 3, we display as examples the deexcitation energies for a few odd-odd Au isotopes as a function of the neutron skin thickness for $^{208}$Pb. Since the neutron skin thickness in $^{208}$Pb depends almost linearly on the $\Lambda_v$ \cite{2, 5}, the sensitive dependence on the $\Lambda_v$ can be observed for deexcitation energies of odd-odd Au isotopes. We can thus expect that the measurement of the deexcitation energy of these odd-odd isotopes can be useful not only for constraining the density dependence of the symmetry energy but also clarifying this theoretical uncertainty.

Next, let us discuss more the factors that are necessary for the description of the deexcitation energy. First of all, since the SSM is a result of the collective excitation that undergoes a significant isovector change, the appropriate isovector potential of the model is necessary. We find from Table 2 and 3 that the description for deexcitation energies of various isotopes is improved by including the isoscalar-isovector coupling. In other words, the inclusion of the isoscalar-isovector coupling improves the isovector potential of the model. Secondly, we note that the isoscalar potential is also an important ingredient to carry out correct deexcitation energies. Especially, the difference between the deexcitation energies for the neighboring even-even isotopes is tightly associated with the appropriate fit to the isoscalar potential. The present study indicates that an overall description for the deexcitation energies and difference between them needs to reconstruct the model appropriately with the isovector potential that
brings out a softened symmetry energy.

At last, we should stress that though the present work is performed based on a specific parametrization (NL3*), the conclusion that the deexcitation energy is sensitive to differences in the symmetry energy is rather free of this specific RMF model. We have examined this with many other RMF models. The current situation is that most best-fit models have been constructed regardless of the detail of the density dependence of the symmetry energy. Thus, the agreement with data of deexcitation energies may play a significant role in reconstructing models with the appropriate density dependence of the symmetry energy. Moreover, we have noted that the treatment of the nucleon pairing with the BCS theory is comparatively simple as compared with the Bogoliubov approach. This would affect the accuracy of the correlation between deexcitation energies of even nuclei and the density dependence of the symmetry energy. One can expect the improvement with the relativistic Hartree-Bogoliubov models. However, this is beyond the capacity and scope of this preliminary effort.

4 Summary

In summary, we have studied the relationship between the deexcitation energies of the SSM in heavy nuclei and the density dependence of the symmetry energy. The investigation is based on the RMF model (NL3*) with the isoscalar-isovector coupling included to modify the density dependence of the symmetry energy. It is found that the uncertainty of the deexcitation energy originates almost uniquely from the modification to the isovector potential induced by the isoscalar-isovector coupling. The deexcitation energies can thus serve as a theoretical probe to the density dependence of the symmetry energy. As a result, we find that the theoretical estimates of the deexcitation energies of Hg isotopes can be improved by the inclusion of the isoscalar-isovector coupling that softens the symmetry energy. To separate the effect of the symmetry energy on deexcitation energies from that of pairing correlations, we propose to measure the deexcitation energies of odd-odd heavy nuclei in the region $A \sim 190$ such as Au isotopes that are nearly independent of pairing correlations. This can be used to constrain the isovector content of the model and thus the density dependence of the symmetry energy.

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