Holographic Penta and Hepta Quark State in Confining Gauge Theories

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Abstract

We study a new embedding solutions of D5 brane in an asymptotic AdS$_5 \times S^5$ space-time, which is dual to a confining SU($N_c$) gauge theory. The D5 brane is wrapped on $S^5$ as in the case of the vertex of holographic baryon. However, the solution given here is different from the usual baryon vertex in the point that it couples to $k$-anti-quarks and $N_c+k$ quarks on the opposite two points of $S^5$, the north and south poles, respectively. The total quark number of this state is preserved as $N_c$ when minus one is assigned to anti-quark, then it forms a color singlet like the baryon. However, this includes anti-quarks and quarks, whose number is larger than that of the baryon. When we set as $N_c = 3$, we find the so called penta and hepta-quark states. We study the dynamical properties of these states by solving the vertex and string configurations for such states. The mass spectra of these states and the tension of the stretched vertex are estimated, and they are compared with that of the baryon.

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1 Introduction

In the context of string/gauge theory correspondence [1, 2, 3], the baryon has been studied as a system of fundamental strings (F-strings) and D5 branes wrapped on $S^5$ in AdS$_5 \times S^5$ space-time [4, 5, 6, 7, 8, 11, 12, 13, 14]. They are dual to the quarks and the baryon vertex respectively. The F-strings dissolve as the $U(1)$ flux in the D5 brane, and they flow out as separated $N_c$ free strings from the cusp(s) on the D5 brane.

This idea has been recently studied furthermore [19] along the approach given in [6]-[11], and also extended to the finite temperature case [20, 21, 24]. We could find complicated structures of the D5 brane embedded in the background dual to a confining gauge theory. Especially, a new configuration has been found and proposed as the baryonium, which is constructed of $k$-quarks, $k$-anti-quarks and the vertex (D5 brane) [22, 23]. In this case, the vertex is described by the polar angle ($\theta$) on $S^5$ in the range $0 < \theta_0 \leq \theta \leq \pi$, and the D5 brane solution is two valued for $\theta$.

In the case of the baryon, the D5 brane covers the range $0 \leq \theta \leq \pi$. The embedded solution has cusps at the pole points, $\theta = 0$ and $\pi$, where the quarks are attached. Their total number is $N_c$ to form a color singlet. The partition of the quark numbers on the two poles is determined by a constant parameter $\nu$ ($0 \leq \nu \leq 1$), which is obtained as an integration constant of the equation of motion for the D5 brane vertex. Namely they are separated as $N_c \nu$ and $N_c(1 - \nu)$. These numbers should be quantized as integers physically.

Here, we propose a new configuration which is given by extending the parameter $\nu$ to the negative region $\nu < 0$. This extension is possible since the parameter $\nu$ is an arbitrary integration constant in solving the equations of motion. Then we find a new hadron configuration, which has $N_c|\nu|$ anti-quarks at one cusp and $N_c(1 + |\nu|)$ quarks on the other one. While the total number of the quarks is preserved as $N_c$ when the anti-quark number is assigned as minus, but this configuration is clearly different from the usual baryon.

It is possible to have the same quark and anti-quark numbers in different configurations which are obtained by attaching some pairs of quark and anti-quark on the same cusp point of $S^5$. However such pairs would be removed by considering the pair annihilation of the quark and anti-quark on the D5 brane. When we remove the anti-quark by this rule, we find configurations, in which the quarks and the anti-quarks are separated to the opposite side of $S^5$ each other as stated above. In this case, the anti-quarks cannot be removed any more by the rule of pair annihilation since they are separated.

Here we concentrate on these new solutions of negative $\nu$. As a simple example, consider the case of $N_c = 3$ or equivalently $SU(3)_c$, then we find a configuration of one anti-quark at one cusp and four quarks on the other one. The corresponding configuration is shown in the Fig.1. This might be considered as the candidate of the
penta-quark [29]. In this case, the anti-quark is separated from the quarks on the opposite poles of the D5 brane, then they are stable against the pair annihilation as stated above. In the Fig. 1 such possible configurations, the penta and hepta quark states, are shown for $N_c = 3$ case. However, we find that there is no stable solution for the penta quark, and the stable states are found only for the case $N_c \nu \leq -2$. The stability is assured by the balance conditions of the force at each cusp. For penta quark, the condition at the lower cusp-point is not satisfied as shown below.

The energy and the configuration of those exotic states depend on the configuration of the state. It is determined by the boundary conditions of the equations of motion, which are called as no force conditions. So, varying the boundary conditions, the relation between the vertex energy and the distance of the two cusps is examined. Then we could estimate the tension of the vertex, and we could find a minimum energy configuration of the hepta quark. Its ratio to the baryon mass is given by $7/3$, which is equivalent to the one of the quark number.

In Section 2 we give our model and D5 brane action with non-trivial $U(1)$ gauge field. And the equations of motion for D5 brane are given, and various kinds of solutions are explained. In section 3 the quarks and the vertex with negative $\nu$ are studied by solving the equations of motion with no force conditions. In the section 4, the stable hepta quark states are studied numerically for $N_c = 3$. In the section 5, the new baryonic states are examined in other holographic models for the confining gauge theories, and we find a common property to forbid the penta quark. And in the final section, we summarize our results and discuss related problem.

Fig. 1: The configuration of penta and hepta-quark states with a D5 brane vertex for $N_c = 3$ theory.
2 Model

2.1 Bulk background

We start from 10d IIB model retaining the dilaton $\Phi$, axion $\chi$ and self-dual five form field strength $F_{(5)}$ with the following action,

$$S = \frac{1}{2\kappa^2} \int d^{10}x \sqrt{-g} \left( R - \frac{1}{2} (\partial \Phi)^2 + \frac{1}{2} e^{2\Phi} (\partial \chi)^2 + \frac{1}{4 \cdot 5!} F_{(5)}^2 \right),$$  \hspace{1cm} (1)

where the axion is Wick rotated to obtain the solution given in [15]. We notice that the axion $\chi$ corresponds to the source of D(-1) brane and it is Wick rotated in the supergravity action. This is necessary to preserve the supersymmetry. Under the Freund-Rubin ansatz for $F_{(5)}$, $F_{\mu_1 \ldots \mu_5} = -\sqrt{\Lambda}/2 \epsilon_{\mu_1 \ldots \mu_5}$ [15, 16, 17], and for the 10d metric as $M_5 \times S^5$ or $ds^2 = g_{mn} dx^m dx^n + g_{ij} dx^i dx^j$, the equations of motion are solved. Here $m, n = 0 \sim 4, i, j = 5 \sim 9$ and $\Lambda$ denotes a constant.

The solution is obtained under the ansatz,

$$\chi = -e^{-\Phi} + \chi_0,$$

where $\chi_0$ is an arbitrary constant. This ansatz is necessary to obtain supersymmetric solutions, and

$$ds^2_{10} = G_{MN} dX^M dX^N = e^{\Phi/2} \left\{ \frac{r^2}{R^2} A^2(r) \left( -dt^2 + \sum_{i=1}^3 (dx^i)^2 \right) + \frac{R^2}{r^2} dr^2 + R^2 d\Omega_5^2 \right\}. \hspace{1cm} (3)$$

Then, the supersymmetric solution is obtained as

$$e^\Phi = 1 + \frac{q}{r^4}, \quad A = 1,$$  \hspace{1cm} (4)

where $M, N = 0 \sim 9, R = \sqrt{\Lambda}/2 = (4\pi N)^{1/4}$ and $N_c$ denotes the number of D3 branes. The parameter $q$ represents the vacuum expectation value (VEV) of gauge fields condensate [17]. In this configuration, the four dimensional boundary represents the $\mathcal{N}=2$ SYM theory. In this model, we find quark confinement in the sense that we find a linear rising potential between quark and anti-quark with the tension $\sqrt{q}/R^2$ [16, 17].

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1 The five dimensional $M_5$ part of the solution is obtained by solving the following reduced 5d action,

$$S = \frac{1}{2\kappa_{(5)}^2} \int d^5x \sqrt{-g} \left( R_{(5)} + 3\Lambda - \frac{1}{2} (\partial \Phi)^2 + \frac{1}{2} e^{2\Phi} (\partial \chi)^2 \right),$$

which is written in the string frame and taking $\alpha' = g_s = 1$. 

3
Then, we solve the embedding equations of D5 brane and fundamental string in the following 10D background,

$$ds^{2}_{10} = e^{\Phi/2} \left( \frac{r^2}{R^2} \eta_{\mu \nu} dx^\mu dx^\nu + \frac{R^2}{r^2} dr^2 + R^2 d\Omega_{5}^{2} \right),$$

where $e^{\Phi}$ is given above. We notice the equivalence of this metric in the Einstein frame with the one of the AdS$_5 \times S^5$.

### 2.2 D5 brane

The baryon is constructed from the vertex and $N$ fundamental strings and the vertex is given by the D5 brane wrapped on the $S^5$ of the above metric. The $N$ fundamental strings terminate on this vertex and they are dissolved in it as $U(1)$ flux. The D5 brane action is thus written by the Dirac-Born-Infeld (DBI) plus WZW term

$$S_{D5} = -T_5 \int d^6 \xi \sqrt{-\det \left( g_{ab} + \tilde{F}_{ab} \right)} + T_5 \int d^6 \xi \tilde{A}(1) \wedge \mathcal{G}(5),$$

where $\tilde{F}_{ab} = 2\pi\alpha' F_{ab}$ and $T_5 = 1/(g_s(2\pi)^5 l_s^6)$ is the brane tension. And $\mathcal{G}(5)$ represents the induced five form field strength, which is obtained from the bulk five form,

$$G_{(5)} \equiv dC_{(4)} = 4R^4 \left( \text{vol}(S^5) d\theta_1 \wedge \ldots \wedge d\theta_5 - \frac{r^3}{R^8} dt \wedge \ldots \wedge dx_3 \wedge dr \right),$$

where $\text{vol}(S^5) \equiv \sin^4 \theta_1 \text{vol}(S^4) \equiv \sin^4 \theta_1 \sin^3 \theta_2 \sin^2 \theta_3 \sin \theta_4$.

The D5 brane is embedded in the world volume $\xi^a = (t, \theta, \theta_2, \ldots, \theta_5)$, where $(\theta_2, \ldots, \theta_5)$ are the $S^4$ part with the volume, $\Omega_4 = 8\pi^2/3$, where we set as $\theta_1 = \theta$. Restrict our attention to $SO(5)$ symmetric configurations of the form $r(\theta), x(\theta)$, and $A_t(\theta)$ (with all other fields set to zero). Then the above action is written as

$$S = T_5 \Omega_4 R^4 \int dt \ d\theta \{-\sin^4 \theta \sqrt{e^{\Phi} (r^2 + r'^2 + (r/R)^4 x'^2) - \tilde{F}_{t\theta}^2 - \tilde{F}_{t}\theta D},$$

where the WZW term is rewritten by partial integration with respect to $\theta$. The factor $D(\theta)$ is therefore defined by

$$\partial_\theta D = -4 \sin^4 \theta ,$$

then it is solved as

$$D(\nu, \theta) \equiv \left[ \frac{3}{2} (\nu \pi - \theta) + \frac{3}{2} \sin \theta \cos \theta + \sin^3 \theta \cos \theta \right],$$

---

2 Hereafter we denote $N_c$ as $N$ for simplicity.
where the integration constant $\nu$ plays an important role to determine the configuration of the D5 brane, and its meaning is explained below.

Before considering $\nu$, we rewrite the above action by eliminating $\tilde{F}_{t\theta}$ in terms of the Legendre transformation. From (8), the equation of motion for $\tilde{A}_t$ is obtained as

$$
\partial_{\theta} \left( \frac{\sin^4 \theta \tilde{F}_{t\theta}}{\sqrt{e^\Phi (r^2 + r'^2 + (r/R)^4 x'^2) - \tilde{F}_{t\theta}^2}} - D \right) = 0 ,
$$

(11)

where $r' = \partial_\theta r(\theta)$ and $x' = \partial_\theta x(\theta)$. This equation is solved as

$$
D = \frac{\sin^4 \theta \tilde{F}_{t\theta}}{\sqrt{e^\Phi (r^2 + r'^2 + (r/R)^4 x'^2) - \tilde{F}_{t\theta}^2}} + c_1.
$$

(12)

Here a new $\theta$-independent constant ($c_1$) seems to be appeared. However, this is absorbed by the constant $\nu$ given in Eq. (10), then we set this constant to be zero without any ambiguity. From the viewpoint of $U(1)$ gauge theory, we can call this $D$ as displacement, and is related to $\tilde{F}_{t\theta}$ by the above equation with $c_1 = 0$.

Then, the action is rewritten by the Legendre transformation with respect to the field $\tilde{A}_t$, and we obtain an energy functional written by the embedding coordinate only

$$
U = \frac{N}{3 \pi^2 \alpha'} \int d\theta \ e^{\Phi/2} \sqrt{r^2 + r'^2 + (r/R)^4 x'^2} \sqrt{V_{\nu}(\theta)} .
$$

(13)

where we used $T_3 \Omega_4 R^4 = N/(3 \pi^2 \alpha')$. Then, $r(\theta)$ and $x(\theta)$ are obtained by the variation of this energy function $U$ for the embedding solutions of the D5 brane. Before solving the equations of motion, we give comments on the equations obtained from (8).

**Equations of motion for $r(\theta)$ and $x(\theta)$ from Eq.(8)**

The vertex part of the penta-quark state is given by solving the D5 brane action (8) or its energy function (13). The equations of the latter case are useful. Then they have been used in [7, 19, 20, 22] and are shown in the Appendix. We use them here also.

Of course, we can obtain the same results when we start from (8). Here we comment on the relation between the equations obtained from (8) and the one from (13). The equation for $A_t$ is already given above, then we write other equations for $r(\theta)$ and $x(\theta)$, they are given as follows

$$
\sin^4 \theta \partial_r \left( e^{\Phi/2} \sqrt{K(0)} - \partial_{\theta} \left( \sin^4 \theta e^{\Phi/2} r' \right) \right) = 0 ,
$$

---

3 $U$ is obtained by a Legendre transformation of $L$, which is defined as $S = \int dt L$, as $U = \frac{\partial L}{\partial F_{t\theta}} \tilde{F}_{t\theta} - L$. Then equations of motion of (8) provides the same solutions of the one of $U$.

4 Here $\alpha'$ is retained for reminding the physical dimension.
\[
\partial_{\theta} \left( \sin^4 \theta \frac{r^4 e^{\Phi/2}}{R^4 \sqrt{\bar{K}(0)}} x' \right) = 0, \quad (15)
\]

where
\[
\bar{K}(0) = \left( r^2 + (r')^2 + \frac{r^4}{R^4} (x')^2 - e^{-\Phi} \frac{F_{t\theta}^2}{2} \right). \quad (16)
\]

The latter equation of (15) is solved as
\[
\sin^4 \theta \frac{r^4 e^{\Phi/2}}{R^4 \sqrt{\bar{K}(0)}} x' = h, \quad (17)
\]

where \(h\) is a constant representing a conserved quantity due to the translation invariance in the direction of \(x\) in \(U\). This is identified with the one used in our previous paper by the same notation \([22]\). Actually, the parameter \(h\) given in the above equations (15) is equivalent with the one of equation (71) in the Appendix. Using this \(h\) and \(A_t\) solved above, we obtain the equation for \(r(\theta)\). Then we obtain the solutions by solving this equation, however, it will be solved numerically since it has very complicated form.

It is easy to prove the equivalence of the solutions of the above equations (15) and the one given in \([22]\) in more efficient way as given in the Appendix.

**Meaning of \(\nu\) and various solutions**

The above equations are common to all the vertex configurations of baryon \([19, 7]\), baryonium \([22, 23]\) and new baryonic stats. They are discriminated by the value of \(\nu\) and the range of variable \(\theta\).

(i) **Baryon:** \(0 \leq \theta \leq \pi, \ 0 \leq \nu \leq 1\)

First, we consider the value of \(\nu\) defined in the range of \(0 \leq \nu \leq 1\). This case corresponds to the baryon. Using (13), we review the meaning of the integration constant \(\nu\) in (10). The solution has two cusps at \(r(\theta = \pi)\) and \(r(0)\), namely at poles on \(S^5, \ \theta = 0\) and \(\theta = \pi\). At these points, \(r' = \partial_{\theta} r \to \infty\) and \(x' \simeq 0\) for \(q \to 0\) as shown in \([19, 7]\). Then the configuration near these positions represents the bundle of the fundamental strings, and their numbers at the cusps are estimated as follows. At \(\theta = \pi\) and for \(q \to 0\ (\Phi \to 0)\), we obtain the following approximate formula
\[
U \simeq \frac{N}{3 \pi^2 \alpha'} \int dr \frac{3}{2} (1 - \nu) \pi = \frac{N}{2 \pi \alpha'} (1 - \nu) \int dr. \quad (18)
\]

And similarly, we obtain the following at \(\theta = 0\),
\[
U \simeq \frac{N}{2 \pi \alpha'} \nu \int dr. \quad (19)
\]

Since \(\frac{1}{2 \pi \alpha'} \int dr\) represents the bundle of a fundamental string, the total number of fundamental strings is given by \(N\), which are separated to \(N(1 - \nu)\) and \(N\nu\) to each
cusp point. The meaning of $\nu$ is then the ratio of this partition, so it must be defined as $0 \leq \nu (\equiv k/N) \leq 1$, where $k(\leq N)$ is an integer. Then the total number of the flux is counted as $N$ when we sum up the one of the two cusps at $\theta = 0$ and $\theta = \pi$.

Next, we give a comment on the orientation of the flux of $U(1)$. It is characterized by the sign of the displacement, $D$. The function $U$ given by (13) is common to positive and negative $D$. However, at the cusps, $D$ is given as

$$D(\nu, 0) = \frac{3}{2} \nu \pi, \quad D(\nu, \pi) = \frac{3}{2}(\nu - 1) \pi$$

and we can see its sign. For $0 < \nu < 1$, we find $D(\nu, 0) > 0$ and $D(\nu, \pi) < 0$. This means that the $U(1)$ flux comes into $\theta = 0$ then opposite oriented flux does into $\theta = \pi$. Then we find totally $N$ flux at the baryon vertex. The anti-baryon is then obtained by the change of $D \rightarrow -D$.

(ii) Baryonium: $\theta_0 \leq \theta \leq \pi$

In general, two possible flux numbers are assigned as $\pm N(1 - \nu)$ and $\pm N\nu$ at each cusp. For the split baryon vertex, which extends between the cusps at $\theta = \theta_0$, $\pi$, and the flux with the number $\pm N\nu$ and $\pm N(1 - \nu)$ are assigned at each cusps. Then we find $\pm N$ quark number since the baryon must be a color singlet. However, we found new solutions, which extend between the cusps at the same point $\theta(= \theta_0, \pi)$ as shown in [22, 23]. In this case, for the solution with two cusps at $\theta = \pi$, we must choose the flux-combination as $\pm N(1 - \nu)$ and $\mp N(1 - \nu)$. And for the one with the cusps at $\theta = \theta_0$, the flux should be assigned as $\pm N\nu$ and $\mp N\nu$. Then the total flux or the quark number is zero in both cases, and these solutions have been assigned as the baryonium states.

(iii) New Baryonic States $0 \leq \theta \leq \pi$, $\nu < 0$ (or $1 < \nu$);

New states are obtained when $\nu$ is set as negative value, $\nu < 0$. In this case, $N|\nu|$ flux comes in $\theta = 0$ and $N(1 + |\nu|)$ flux comes out from $\theta = \pi$. This configuration represents a bound state of $N|\nu|$ anti-quarks, $N(1 + |\nu|)$ quarks and the D5 brane as a color singlet. As a special case, we can consider a state with $N|\nu| = 1$ and $N(1 + |\nu|) = 4$ for $N = 3$. This state is formed with four quarks and an anti-quark. And this state is called as penta-quarks. By interchanging the positions ($\theta = 0$ and $\pi$) of the attached $N|\nu|$ anti-quarks and the $N|\nu|$ anti-quarks, we find the configurations in the case of $1 < \nu$. It is very interesting to study whether such a state exists or doesn’t. Here we call all the states of $\nu < 0$ and $1 < \nu$ as new baryonic states as a whole. These states have not been studied up to now in the holographic model. Since the states of $\nu < 0$ and $1 < \nu$ are symmetric, we study the case of $\nu < 0$ hereafter.
3 New Baryonic States

As explained above, the new baryonic state is obtained by solving the equations of the system of D5 brane, quarks and anti-quarks. The equations of motion of this system are given by the following energy function,

\[ U_{\text{total}} = U_{D5} + U_{F-q} + U_{F-\text{anti-q}} \]  

(21)

where

\[ U_{D5} = \frac{N}{3\pi^2\alpha'} \int_0^\pi d\theta \ e^{\Phi/2} \sqrt{r^2 + r'^2 + (r/R)^4 x_r^2 \sqrt{V_\nu(\theta)}} \]  

(22)

\[ U_{F-\text{anti-q}} = \sum_{i=1}^{N|\nu|} \frac{1}{2\pi\alpha'} \int_{r^{(i)}(\theta=0)}^{r^{\text{max}}(i)} dr^{(i)} \ e^{\Phi/2} \sqrt{1 + (r^{(i)}/R)^4 (x_r^{(i)})^2} \]  

(23)

\[ U_{F-q} = \sum_{i=1}^{N(1+|\nu|)} \frac{1}{2\pi\alpha'} \int_{r^{(i)}(\theta=\pi)}^{r^{\text{max}}(i)} dr^{(i)} \ e^{\Phi/2} \sqrt{1 + (r^{(i)}/R)^4 (x_r^{(i)})^2} \]  

(24)

where \( x_r = \partial_r x \). And \( r^{\text{max}} \) denotes the cutoff position of the F-strings, namely the one of D7 brane. Each quark or anti-quark string is discriminated by its superscript \((i)\).

In solving the equations, we notice the following point for the case of negative \( \nu \). In general, \( V_\nu(\theta) \) has a minimum at \( \theta_c \) which is given as a solution of

\[ \pi\nu = \theta_c - \sin \theta_c \cos \theta_c , \]  

(25)

and we find the minimum value as \( V_\nu(\theta_c) = \sin^6(\theta_c) \). However the point \( \theta_c \) is now negative since \( \nu < 0 \). Then this point is out of the \( S^5 \). This implies that the electric field in the \( S^5 \) does not vanish at any point of \( \theta \), which is restricted as \( 0 \leq \theta \leq \pi \). So the shape of the embedded D5 brane is largely restricted compared to the baryon.

**No-force condition**

In the present case, the D5 brane has cusps at the two poles on the \( S^5 \). At these points, the tensions of the D5 brane appear toward smearing the cusp shape. On the other hand, the force coming from this tension of D5 brane could be balanced by the tension working in the opposite direction when fundamental strings are added at these cusp points. The condition to balance both forces are called as no-force conditions and they are studied in considering baryons [19, 20, 30]. Also in the present case, we must add quarks and anti-quarks at the cusps of the brane to see the full configuration of the penta quark.

The no-force conditions are obtained by considering the boundary terms of the equations of motion of the system of D5 brane and F-strings given above. At the cusp \( \theta = 0 (\theta = \pi) \), we give the balance of the tensions between the brane and the \(|\nu|N\).
\((1 + |\nu|)N\) fundamental strings corresponding to the anti-quarks (quarks). They are given for \(\theta = 0\) as,

\[
N|\nu|\frac{r'}{\sqrt{r^2 + r'^2 + (r/R)^4x'^2}} = \sum_{i=1}^{N|\nu|} \frac{r_x^{(i)}}{\sqrt{(r_x^{(i)})^2 + (r/R)^4}}.
\]

(26)

for \(r\) direction and

\[
N|\nu|\frac{x'(r/R)^2}{\sqrt{r^2 + r'^2 + (r/R)^4x'^2}} = \sum_{i=1}^{N|\nu|} \frac{x_x^{(i)}(r/R)^2}{\sqrt{1 + (r/R)^4(x_x^{(i)})^2}}.
\]

(27)

for \(x\) direction. For the boundary \(\theta = \pi\), we obtain the conditions with the same form by replacing \(|\nu|N\) by \((1 + |\nu|)N\) in the above equations. Here the factor \((r/R)^2\) in (26), (27) can be removed by dividing the left and the right hand sides, but it is retained for the purpose given in the following analysis.

In this expression, the above conditions are represented by the following two dimensional vectors in the \(x - r\) plane,

\[
\vec{\tau}_v = \left( \frac{x'(r/R)^2}{\sqrt{r^2 + r'^2 + (r/R)^4x'^2}}, \frac{r'}{\sqrt{r^2 + r'^2 + (r/R)^4x'^2}} \right),
\]

(28)

\[
\vec{\tau}_s^{(i)} = \left( \frac{x_x^{(i)}(r/R)^2}{\sqrt{1 + (r/R)^4(x_x^{(i)})^2}}, \frac{r_x^{(i)}}{\sqrt{(r_x^{(i)})^2 + (r/R)^4}} \right),
\]

(29)

for the vertex (\(\vec{\tau}_v\)) and strings (\(\vec{\tau}_s\)), respectively. Here we notice

\[
|\vec{\tau}_v| \leq |\vec{\tau}_s^{(i)}| = 1,
\]

(30)

where the equality is satisfied for \(r = 0\). This is important due to the reason given below. Then the above no-force conditions are written as

\[
N|\nu|\vec{\tau}_v = \sum_{i=1}^{N|\nu|} \vec{\tau}_s^{(i)}.
\]

(31)

As a special case, consider the configuration of \(|\nu|N = 1\). In this case, one fundamental string, which corresponds to an anti-quark, is connected at \(\theta = 0\) to the brane. For the case of \(N = 3\), on the side of \(\theta = \pi\), four F-strings are attached corresponding to the four quarks. Then this configuration corresponds just to the so called penta-quark state. Considering the above condition at \(\theta = 0\), we obtain \(|\vec{\tau}_v| = |\vec{\tau}_s|\) then

\[
r^2 = 0.
\]

(32)

This implies that the point of \(\theta = 0\) must be on at \(r = 0\) for the fundamental quark string. However this configuration needs infinite energy due to the theory is in the
confinement phase \[17\]. Then this configuration, the penta quark state, cannot be realized.

In general, it is possible to satisfy the equality (31) for plural F-strings for the right hand side under the inequality (30). Then possible configurations, which belong to the bound state of anti-quarks and quarks, are obtained for

\[ |\nu|N \geq 2, \]

which means the number of the anti-quarks is larger than two, so the total number is larger than \( N + 3 \). It is then larger than six for \( N = 3 \). We give some example of these configurations below.

This condition is applied for the case of baryon, and we find for \( N = 3 \) that the baryon configuration is obtained only for \( \nu = 0 \) and 1. In other words, the three quarks ends only on the one cusp, namely there is no configurations with separated distribution of quarks.

### 4 Numerical solutions for \(|\nu|N = 2\)

Here we restrict to the case of \( N = 3 \) for the numerical calculations. In this case, the penta quarks are unstable as seen above, then we study here the configuration of the seven (hepta)-quark states, which is the next simple multi-quarks state. We solve the equation of motion of the system consisting of the D5 brane vertex and strings corresponding to five quarks and two anti-quarks given by (21) with the no force condition.

**Point Vertex and Lowest Mass State:**

We consider firstly the simple vertex configuration, which is observed as a point in our real three dimensional space. This configuration is obtained by solving (21) by setting as \( x = \text{constant} \). Of course, we can choose at any point of \( x \) where the vertex sits, we choose \( x = 0 \) for simplicity. Then the equation of motion for \( r(\theta) \) is given as

\[ \partial_\theta \left( \frac{r'}{\sqrt{r^2 + (r')^2}} \sqrt{V_\nu(\theta)} \right) - \left( 1 + \frac{r}{2} \partial_r \Phi \right) \frac{1}{\sqrt{r^2 + (r')^2}} \sqrt{V_\nu(\theta)} = 0. \]  

(34)

The vertex extends from \( r(0)(>0) \) to \( r(\pi) \) with \( \theta \), and the strings of the quarks and anti-quarks are attached at \( r(\pi) \) and \( r(0) \) respectively for negative \( \nu(= -2/3) \).

As for the no-force conditions, (26), (27), they are represented by noticing \( N|\nu| = 2 \) as follows,

\[ \frac{2r'}{\sqrt{r^2 + r'^2}} = \sum_{i=1}^{2} \frac{r_i^{(j)}}{\sqrt{(r^{(j)}_i)^2 + (r/R)^4}}, \]  

(35)
Fig. 2: Typical hepta-quark states for point D5 vertex. D5 brane, quarks and anti-quarks are respectively represented by the bold line at $x = 0$, red curves and blue curves. Here, we take $N = 3$, $q = 0.3$, $\nu = -2/3$, $r(\theta = 0) = 1.54$ and the cutoff $r_{\text{max}} = 20$. In the left figure, two cusps stretch from $r(0) = 1.54$ to $r_{\text{max}}$ and five quarks flux emerge at the point $r_{\text{max}}$ and the total energy $U_{\text{tot}}$ is given by $U_{\text{tot}} = 142.0$. In the right one, two cusps are almost localized at $r(\theta = 0) = r(\theta = \pi)$ and $U_{\text{tot}} = 144.8$. In the right, note that it seems that there are only two quarks but we just gather three quarks and two quarks together into one curve respectively.
Fig. 3: The lowest energy configuration of point vertex. Here we take $N = 3$, $q = 0.3$, $\nu = -2/3$, $r(0) = r(\pi) = 20$, so each cusp is at the boundary. We find this configuration has the lowest energy $U_{\text{tot}} = 140.7$. The configuration is given in $x-r$ plane (left) and in $\theta-r-x$ three dimensional space (right). In the right figure, the arrows indicate the orientation of the $U(1)$ flux in the D5 brane. And, at the point $(s)$ two anti-quarks (five quarks) couple to the brane.

for $r$ direction and

$$0 = \sum_{i=1}^{2} \frac{x_r^{(i)}}{\sqrt{1 + \left(r/R\right)^4(x_r^{(i)})^2}},$$

(36)

for $x$ direction. In this case, the condition in the $x$ direction is given only by the F-strings.

Here we introduce the position of the flavor brane at $r = r_{\text{max}}$. This plays a role of a cut off of the coordinate $r$ to obtain finite energy of the states with strings. Namely the quark strings here are connecting the D5 brane and the D7 brane. In order to give the explicit hepta quark solutions mentioned above, the equations of motion are solved numerically since it would be impossible to solve them analytically. After solving them, we show two explicit configurations in the Fig. [2]. In the right hand configuration, two cusps of the vertex are near at $r = 1.54$. Namely the vertex starts from one cusp at $r(\theta = 0) = 1.54$ and goes down slightly in the $r$-direction toward $r_{\text{min}}$, then goes up to the other cusp at $r(\theta = \pi) = 1.54$. This configuration has rather large energy $U$. It is possible to obtain the configuration with smaller energy than that of this by pushing up the one cusp point $r(\theta = \pi) > 1.54$ keeping the other side cusp position, $r(\theta = 0) = 1.54$. Then we find the left configuration of the Fig. [2] as a limit of this deformation. This configuration has smaller energy than the one of the right one, because the energy of strings is larger than the one of the D5 brane in which the same number of strings are absorbed. Namely, the five quark strings of the right configuration
are absorbed by the D5 brane, which stretches from $r = 1.54$ to $r = r_{\text{max}}$.

Due to the same reason, we could find the minimum energy configuration by pushing the other cusp up to $r = r_{\text{max}}$. Actually we can see this fact by the numerical estimation of the energy by varying $r(0)$, the results are shown in the Fig. 4. Then the lowest energy configuration is shown in the Fig. 3 and its energy is given as $U_{\text{tot}} = 140.7$ for $N = 3$, $q = 0.3$, $\nu = -2/3$, $r(0) = r(\pi) = 20 = r_{\text{max}}$. Then the lowest energy of hepta-quark state is obtained by the D5 brane only. Namely, all quark-strings are absorbed in the D5 brane in the case of the lowest energy state.

We should notice the following point for the above solution with lowest energy, which should be expected to be stable. From the left figure of Fig. 3, one may suspect an instability due to the pair annihilation of quarks and anti-quarks on the top of the vertex. However, such a possibility is avoided since they are separated into the opposite poles, $n$ and $s$, of $S^5$ as shown in the right-figure, which is given in the $x - r - \theta$ three dimensional space. The distance between $n$ and $s$ is about $2r_m$ in the $r$ direction.

Another apprehension is the instability due to the tachyon which appears when a D brane and an anti-D brane are facing with a small distance [26]. The left figure looks like a similar configuration since the oppositely oriented D brane parts are facing at the same point of $x$. But, as seen from the right figure, it shows a five dimensional sphere with the radius which varies with $\theta$, and it is not bending as being misunderstood from the left figure. Thus there is no reason to consider the tachyonic instability.

The situation is similar to the case of the baryon ($\nu = 0, 1$). The minimum energy of the baryon is also obtained by the configuration of D5 brane only, which extends from $r_{\text{max}}$ to $r_{\text{min}}$ and absorbs three quark strings for $N = 3$ case. And its mass is about $60/R$ for $r_{\text{max}} = 20$ [19] for $N = 3$. On the other hand, in the present case, the
seven quark mass is about $141/R$ (see Fig. 4), which is about 2.4 times of the lowest baryon mass. Then the ratio $m_{\text{hepta}}/m_B$ is well approximated by the quark number ratio, $7/3 \sim 2.3$. This implies that the vertex energy is proportional to the quark number $n_q$, and it might be written as

$$m = m_q n_q,$$

where $m_q$ represents the effective quark mass. Here we can approximate as $m_q \propto r_{\text{max}} - r_{\text{min}}$. We notice the above formula is satisfied when $r_{\text{min}}$ is preserved as a constant for any state of $n_q$. In this case, it suggests a quark counting rule of masses of baryonic states. In the right of Fig. 4, we show this relation by adding the baryonium state with the quark number $n_q = 4$, and its mass is obtained as $80/R$. We expect that this rule would hold for other hadronic states. In the below, we show that the rule also holds for the ones of split vertex.

**Split Vertex and its Tension**

![Fig. 5: Typical split vertex hepta-quarks for various $r(\theta = \pi)$ at $N = 3$, $q = 0.3$, $R = 1$ and $r(\theta = 0) = 1.54$. In this case, D5 brane (bold curve) also stretches to $x$-direction. Two curves correspond to the point vertex case in Fig. 2. In the left one, the configuration has $L = 1.05$ and $U_{\text{tot}} = 142.6$. In the right one, the configuration has $L = 1.47$ and $U_{\text{tot}} = 145.5$.](image)

Next, we consider higher energy state by extending the vertex in the our three dimensional space, say $x$ direction. Such a configuration is called as the “split vertex”
Fig. 6: The upper curve shows total energy of split hepta-quarks and the lower curve shows the vertex energy of split hepta-quarks in the case of $R = 1$, $q = 0.3$, $r(0) = 1.54$ and $r(\pi) = 20$. $L$ represents the length of the split of D5 brane, namely $L = |x(\theta = 0) - x(\theta = \pi)|$. The point A and B correspond to the energy of the left of Fig. 2 ($L = 0$) and Fig. 5 ($L = 1$) respectively.

of D5 brane similar to the baryon case [19]. Here we concentrate on the tension of this extended D5 vertex.

For this configuration, the no force conditions are given by (26), (27) and we solve the equations of motion (21) or (71)∼(73) in Appendix without fixing $x(\theta)$. Therefore, unlike the case of point vertex, split vertex solutions have a configuration of length of vertex separation $L$ defined by $L = |x(\theta = 0) - x(\theta = \pi)|$.

In the Fig. 5, it is shown two typical examples of the split hepta-quark state which satisfy no force conditions and the equation of motion, and each figure corresponds to the ones of point vertex in Fig. 2. The left one of Fig. 5 represents split vertex configuration of hepta-quarks which starts from the cusp $(x(0), r(0)) = (0, 1.54)$ where two anti-quarks couple, to the cusp $(x(\pi), r(\pi)) = (1.05, 20)$ where five quarks couple. We can say the similar statement about the right one of Fig. 5.

We can say the similar statement about the right one of Fig. 5.

We find that the value of the total energy is higher than that of point vertex by the configuration of vertex length $L$. Then the energy difference would be about

$$
\Delta U \simeq \tau_{D5} L
$$

where $\tau_{D5}$ denotes the tension of the D5 brane and is given below. In the Fig. 6, we show the relation of the total energy and vertex energy as a function of $L$ for $r(\pi) = 20$ and a fixed $r(0) = 1.54$. The total energy increases with the vertex separation $L$, and its minimum is given at $L = 0$, namely for the point vertex as expected.

The two curves are almost parallel, then the slope of them represents the tension of the vertex only. It seems to converge to $2\sqrt{q}/R^2$, which is twice of one fundamental string. Actually, in numerical analyses of Fig. 6, at $L = 5$ the one for upper curve is $2.18\sqrt{q}/R^2$, while its value of lower one is $2.21\sqrt{q}/R^2$. Moreover, at $L = 400$ the both slope is equal to $2.01\sqrt{q}/R^2$. Therefore, we can confirm that in the limit $L \to \infty$, the
slope of the energy seems to converge to $2\sqrt{q}/R^2$ as mentioned above.

We know that the tension of D5 brane at each cusp is estimated by the one of the string times their number at the cusp point. In this case, the number of anti-quarks of cusp at $\theta = 0$ is two, then the tension obtained above represents the brane tension at $\theta = 0$. The reason that the point $\theta = 0$ of the brane is pulled is that the tension at $\theta = \pi$ is larger than the one at $\theta = 0$ due to the number of the strings attached at the cusp. This implies that the point with the smallest tension in the brane is stretched when we pull the brane.

5 Penta-quark in other confining theories

In the previous sections, we show that the five-quark state, which is made of one anti-quark and $N + 1$ quarks, cannot be constructed in our model given by Eqs.(3) and (4), and we need two anti-quarks are needed at least to form a penta-quark like exotic state. Here we show that this conclusion is common to other holographic confining gauge theories through two typical examples.

5.1 Non-Susy solution

For, the non-supersymmetric case, the solution is obtained as [17],

$$ds_{10}^2 = e^{\Phi_{NS}/2} \left( \frac{r^2}{R^2} A^2(r) \eta_{\mu\nu} dx^\mu dx^\nu + \frac{R^2}{r^2} dr^2 + R^2 d\Omega_5^2 \right),$$

where

$$A(r) = \left(1 - \left(\frac{r_0}{r}\right)^8\right)^{1/4}, \quad e^{\Phi_{NS}} = \left(\frac{(r/r_0)^4 + 1}{(r/r_0)^4 - 1}\right)^{\sqrt{3}/2}.$$  \hspace{1cm} (39)

This configuration has a singularity at $r = r_0$. So we cannot extend our analysis to near the singularity where higher curvature contributions are important. This theory provides confinement and chiral symmetry breaking at zero temperature. The confinement is realized due to the gauge condensate, which is proportional to $r_0^4$ in the present case. The chiral symmetry breaking means that the massless quark has the non-zero chiral condensate in this theory. Therefore, a dynamical quark mass would be generated for a massless quark. This point is different from the supersymmetric case.

As in the supersymmetric case, the D5 brane action (DBI term plus WZW term) is written as

$$S_{NS} = T_5 \Omega_4 R^4 \int dt d\theta \sin^4 \theta \left(-\sqrt{G(\theta)} - \tilde{F}^2_{\theta\theta} + 4\tilde{A}_t(t)\right),$$

$$G(\theta) = e^{\Phi_{NS}} A(r)^2(r^2 + r'^2 + (r/R)^4 A(r)^2 x'^2).$$  \hspace{1cm} (41)

$$\tilde{F}^2_{\theta\theta} = \frac{\tilde{F}^2_{\theta \theta}}{\sin^4 \theta} = \frac{\tilde{F}^2_{\theta \theta}}{\sin^4 \theta}.$$  \hspace{1cm} (42)
From the equation of motion for $\tilde{A}_t$, we obtain
\[
\partial_\theta \left( \frac{\sin \theta^4 \tilde{F}_{t\theta}}{\sqrt{G(\theta) - \tilde{F}_{t\theta}^2}} \right) = -4 \sin^4 \theta = \partial_\theta D, \tag{43}
\]
where $D$ is the same one given in (9) and (10). The action is thus rewritten by eliminating the gauge field in terms of the Legendre transformation as follows,
\[
U_{\text{NS}} = \frac{N}{3\pi^2 \alpha'} \int d\theta \sqrt{G(\theta)} V_\nu(\theta), \quad V_\nu(\theta) = D(\nu, \theta)^2 + \sin^8 \theta. \tag{44}
\]

**No-force condition**

As in the supersymmetric case, we can find the no-force condition between the system of D5 brane and (anti-)quarks. The total energy of the system is represented as follows,
\[
U_{\text{total}} = U_{\text{NS}} + U_{F-q} + U_{F-\text{anti-q}}, \tag{45}
\]
where
\[
U_{F-q} = \sum_{i=1}^{N(1+|\nu|)} \frac{1}{2\pi \alpha'} \int_{r^{(i)}(\theta=\pi)}^{r^{(i)}(\theta=0)} dr^{(i)} A(r^{(i)}) e^{\Phi^{(i)}_{\text{NS}}/2} \sqrt{1 + \left( \frac{r^{(i)}}{R} \right)^4 A(r^{(i)})^2 x_r^{(i)}}, \tag{46}
\]
\[
U_{F-\text{anti-q}} = \sum_{i=1}^{N|\nu|} \frac{1}{2\pi \alpha'} \int_{r^{(i)}(\theta=\pi)}^{r^{(i)}(\theta=0)} dr^{(i)} A(r^{(i)}) e^{\Phi^{(i)}_{\text{NS}}/2} \sqrt{1 + \left( \frac{r^{(i)}}{R} \right)^4 A(r^{(i)})^2 x_r^{(i)}}. \tag{47}
\]

At the cusp $\theta = 0(\theta = \pi)$, we give the balance of the tensions between the D5 brane and the $|\nu|N((1 + |\nu|)N)$ F-strings corresponding to the anti-quarks (quarks). Then, the no-force conditions are given for $\theta = 0$ as for $r$ direction,
\[
N|\nu| \frac{r'}{\sqrt{r^2 + r'^2 + (r/R)^4 A(r)^2 x'^2}} = \sum_{i=1}^{N|\nu|} \frac{r^{(i)}_x}{\sqrt{r^{(i)}_x^2 + (r/R)^4 A(r)^2}}, \tag{48}
\]
For $x$ direction as,
\[
N|\nu| \frac{x'}{\sqrt{r^2 + r'^2 + (r/R)^4 A(r)^2 x'^2}} = \sum_{i=1}^{N|\nu|} \frac{x^{(i)}_r}{\sqrt{1 + (r/R)^4 A(r)^2 x^{(i)}_r}}. \tag{49}
\]

At the boundary $\theta = \pi$, we obtain the condition with the same form by replacing $N|\nu|$ by $N(1 + |\nu|)$ in the above conditions.

As a special case of baryon, let consider the configuration of $|\nu|N = 1$, $N = 3$ and $\nu < 0$, namely penta-quark. By considering the above condition at $\theta = 0$, we obtain
\[
r^2 = 0. \tag{50}
\]
In the present case, we must consider the region of $r$ as $r \geq r_0 > 0$. Thus, the penta-quark cannot be realized by this baryon model in the non-supersymmetric and confining theory because of the existence of the singularity at $r = r_0$. But the hepta-quark state ($N|\nu| = 2$) is also allowed for this model. And other states with $N|\nu| > 2$ is also possible.
5.2 Type IIA D4/D4 Model

In type IIA model, the baryon vertex is given by D4 brane, which wraps on $S^4$ in the background of stacked D4 \[11, 28\]. In this model, the 10d background metric is given by

$$\begin{align*}
  ds_{10}^2 &= \left(\frac{r}{R}\right)^{3/2} \left[ f(r) d\tau^2 + \eta_{\mu\nu} dx^\mu dx^\nu \right] + \left(\frac{R}{r}\right)^{3/2} f(r)^{-1} dr^2 + R^{3/2} r^{1/2} d\Omega_4^2, \\
  f(r) &= 1 - r_h^3/r^3, \quad R^3 = \pi g_s N l_s^3, \quad r_h = \frac{16\pi^2}{9} R^3 T^2, \tag{51}
\end{align*}$$

And, the world-volume action of D4 brane is given as

$$S = -T_4 \int d^5 \xi e^{-\tilde{\phi}} \sqrt{-\det(g + \tilde{F})} + T_4 \int A_{(1)} \wedge G_{(4)}, \tag{53}$$

$$e^{\tilde{\phi}} = \left(\frac{r}{R}\right)^{3/4}, \tag{54}$$

where $T_4 = 1/(g_s(2\pi)^4 l_s^5)$, and the world-volume gauge field $A_{(1)}$ couples to the dual of the background four-form field strength $G_{(4)}$.

Using the explicit background (52) one can rewrite the action in the form

$$S = -T_4 \Omega_3 R^3 \int dt d\theta \left[ \sin^3 \theta \sqrt{G_{(4)} - \tilde{F}_{i\theta}^2 + \tilde{F}_{i\theta} D_{(4)}^2} \right], \tag{55}$$

$$G_{(4)} = r^2 + f(r)^{-1} r'^2 + (r/R)^3 (x'^2 + f(r) \tau'^2), \tag{56}$$

where $\Omega_3 = 2\pi^2$. In the above expression (56), we added the freedom $\tau(\theta)$. However, we do not solve the classical equations of motion for it in the present paper. The displacement $D_{(4)}$ now satisfy the equation

$$\partial_\theta D_{(4)} = -3 \sin^3 \theta,$$

and is consequently given by

$$D_{(4)} = 3 \cos \theta - \cos^3 \theta - 2 + 4\nu. \tag{57}$$

The constant of integration has been written again in terms of a parameter $\nu < 0$.

$\tilde{F}_{i\theta}$ and $D_{(4)}$ is related by the equation of motion for $\tilde{A}_i$ as,

$$\tilde{F}_{i\theta} = \frac{D_{(4)} \sin^3 \theta}{\sqrt{D_{(4)}^2 + \sin^6 \theta}} \sqrt{G_{(4)}}. \tag{58}$$

Then the action is rewritten as,

$$S = -T_4 \Omega_3 R^3 \int dt d\theta \sqrt{D_{(4)}^2 V_{\nu}}, \tag{59}$$

$$V_{\nu} = D_{(4)}^2 + \sin^6 \theta. \tag{60}$$
No-force condition

No-force condition in this model is obtained as similar way in D5-model. The system is represented by the following total energy,

\[ U_{\text{total}} = \frac{N}{8\pi\alpha'} \int_0^\pi d\theta \sqrt{G(4)} \sqrt{V_{\nu}(\theta)} + U_{\text{F-}q} + U_{\text{F-anti-}q}, \]

where

\[ U_{\text{F-anti-}q} = \sum_{i=1}^{N|\nu|} \frac{1}{2\pi\alpha'} \int_{r(i)(\theta=0)}^{r_{\max}(i)} dr(i) \sqrt{D_s^{(i)}}, \]

\[ U_{\text{F-}q} = \sum_{i=1}^{N(1+|\nu|)} \frac{1}{2\pi\alpha'} \int_{r(i)(\theta=\pi)}^{r_{\max}(i)} dr(i) \sqrt{D_s^{(i)}}, \]

\[ D_s^{(i)} = f(r^{(i)})^{-1} + (r^{(i)}/R)^3(x^{(i)}_r)^2 + f(r^{(i)})(\tau^{(i)}_r)^2, \]

and \( x_r = \partial_r x \).

At the cusp \( \theta = 0 \) (\( \theta = \pi \)), we give the balance of the tensions between the brane and the \( |\nu|N \ (1+|\nu|)N \) F-strings corresponding to the anti-quarks (quarks). They are given for \( \theta = 0 \) as,

\[ N|\nu| \sqrt{G(4)} \frac{r'}{\sqrt{D_s^{(i)}}} = \sum_{i=1}^{N|\nu|} \frac{r^{(i)}_x}{\sqrt{D_s^{(i)}}}, \]

for \( r \) direction

\[ N|\nu| \sqrt{G(4)} \frac{x'}{\sqrt{D_s^{(i)}}} = \sum_{i=1}^{N|\nu|} \frac{x^{(i)}_r}{\sqrt{D_s^{(i)}}}, \]

for \( x \) direction

\[ N|\nu| \sqrt{G(4)} \frac{\tau'}{\sqrt{D_s^{(i)}}} = \sum_{i=1}^{N|\nu|} \frac{\tau^{(i)}_x}{\sqrt{D_s^{(i)}}}, \]

and for \( \tau \) direction. At the boundary \( \theta = \pi \), we obtain the conditions with the same form by replacing \( |\nu|N \) by \( (1+|\nu|)N \) in the above equations.

These no-force conditions imply \( r^2 = 0 \) as in D5-model when \( |\nu| = 1 \). However, \( r \) is restricted to the region \( r \geq r_h > 0 \). Thus the penta quark state is forbidden as in D5-model when \( |\nu| = 1 \). But the hepta-quark state (\( N|\nu| = 2 \)) is allowed. And other states with \( N|\nu| > 2 \) is also possible.
6 Summary and Discussion

New type of holographic baryonic-states are studied by solving the system of the D5 brane and F-strings. They are embedded in a 10D background, which is dual to a confining gauge theory. The D5 brane wraps on $S^5$ and couples to the self-dual five form field strength formed by the $N$ stacked D3 branes. And they are dissolved into the D5 brane as the $U(1)$ electric field. This electric field could come out from two poles on the $S^5$ and they are replaced outside of D5 brane by the F-strings corresponding to quarks. For the usual baryon, $N$ quarks ($N$ number of fluxes) go out from the cusps.

The embedded solution is characterized by an integral constant $\nu$, which determines the distribution of the outgoing fluxes from the D5 brane and also the orientation of the flux. Choosing negative $\nu$, we find negative number ($-N|\nu|$) of flux at one pole, and positive number of flux on the other pole is given by $N(1 + |\nu|)$, which is larger than the total number of the quarks given by $N$. However, these configuration has the same total flux number with the one of the baryon since they are constructed with anti-quarks and quarks.

The quarks and the anti-quarks are bounded at the opposite side of the two cusps of the vertex separately, so they cannot vanish through the pair annihilation of quark and anti-quark on the vertex since the two cusps on D5 brane are separated in the 10d space-time.

Simple example of such a state is considered in the case of $N = 3$ (like QCD) and $\nu = -1/3$. This corresponds to the so-called penta-quark state ($4Q + 1\bar{Q}$), but this state is not realized because of the no-force conditions for $\bar{Q}$. In the case of baryon, this implies that the allowed configurations are obtained only for $\nu = 0$ and $1$.

On the other hand, the state of $\nu = -2/3$ has $2\bar{Q}$ on one cusp of the vertex, then the no-force condition could be satisfied and this hepta-quark state ($5Q + 2\bar{Q}$) is allowed. The states with larger number of anti-quarks on the one cusp of the vertex are possible, but they have larger mass. Here we studied the configurations and the masses of the hepta-quark states by fixing as $N = 3$ for simplicity. We find the lowest mass is proportional to the number of quarks, namely, the mass of the hepta quark is about $7/3$ times of the baryon mass.

The excitation of the state is realized by extending the quark strings or the D5 vertex. In the case of the former, the energy would be needed proportional to the number of the extending strings times a string tension, which is here given by $\sqrt{q}/R^2$. For the extension of the D5 brane, we observed that its tension is about $2\sqrt{q}/R^2$ for hepta quark. This implies that the extending part of the vertex is near the cusp point where two anti-quarks couple to.

We also could assure that the qualitative situation obtained for the supersymmetric background is also seen in the other confining theories considered here, namely for Non-Susy D5 theory and Type IIA D4/D4 model.
Finally we briefly comment on the other possible model to form the penta-quark by introducing two D5 and one anti-D5 branes. The two D5 branes are combined to the anti-D5 brane by two anti-quark strings, and four quarks and one anti-quark are connected to the two D5 branes and one anti-D5 brane, respectively. It would be an interesting problem to estimate its mass and compare it with the one of the heptaquark state obtained here. This would be done in the next step of our work.

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Appendix; Equations of motion with parameter $s$

Here we show another formulation of solving the equations of motion derived from (13) used in [11, 19]. Firstly, rewrite (13) in terms of a general world-volume parameter $s$ defined by functions $\theta = \theta(s), r = r(s), x = x(s)$ as:

$$U = \frac{N}{3\pi^2\alpha'} \int ds \ e^{\Phi/2} \sqrt{r^2 \dot{\theta}^2 + \dot{r}^2 + (r/R)^4 \dot{x}^2} \sqrt{V_\nu(\theta)},$$

(68)

where dots denote derivatives with respect to $s$. Then the momenta conjugate to $r, \theta$ and $x$ are given as

$$p_r = \dot{r} \Delta, \quad p_\theta = r^2 \dot{\theta} \Delta, \quad p_x = (r/R)^4 \dot{x} \Delta, \quad \Delta = e^{\Phi/2} \sqrt{V_\nu(\theta) \sqrt{r^2 \dot{\theta}^2 + \dot{r}^2 + (r/R)^4 \dot{x}^2}}.$$  

(69)

Since the Hamiltonian that follows from the action (68) vanishes identically due to reparametrization invariance in $s$. Then we consider the following identity

$$2\tilde{H} = p_r^2 + \frac{p_\theta^2}{r^2} + \frac{R^4}{r^4} p_x^2 - (V_\nu(\theta)) e^\Phi = 0.$$  

(70)

Regarding this constraint as a new Hamiltonian, we obtain the following canonical equations of motion,

$$\dot{r} = p_r, \quad \dot{p}_r = 2 \frac{p_\theta^2}{r^3} R^4 + \frac{p_\theta^2}{r^3} + \frac{1}{2} (V_\nu(\theta)) e^\Phi \partial_r \Phi, \quad (71)$$

$$\dot{\theta} = \frac{p_\theta}{r^2}, \quad \dot{p}_\theta = -6 \sin^4 \theta (\pi \nu - \theta + \sin \theta \cos \theta) e^\Phi, \quad (72)$$

$$\dot{x} = \frac{R^4}{r^4} p_x, \quad \dot{p}_x = 0 \quad (73)$$

The initial conditions should be chosen such that $\tilde{H} = 0$. By solving these equations, we could find the same solutions given above.
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