Summary: For a commutative ring $R$ and an ADE Dynkin quiver $Q$, we prove that the multiplicative preprojective algebra of Crawley-Boevey and Shaw, with parameter $q = 1$, is isomorphic to the (additive) preprojective algebra as $R$-algebras if and only if the bad primes for $Q - 2$ in type $D$, $2$ and $3$ and for $Q = E_6, E_7$ and $2, 3$ and $5$ for $Q = E_8$ are invertible in $R$. We construct an explicit isomorphism over $\mathbb{Z}[1/2]$ in type $D$, over $\mathbb{Z}[1/2, 1/3]$ for $Q = E_6, E_7$ and over $\mathbb{Z}[1/2, 1/3, 1/5]$ for $Q = E_8$. Conversely, if some bad prime is not invertible in $R$, we show that the additive and multiplicative preprojective algebras differ in zeroth Hochschild homology, and hence are not isomorphic. In fact, one only needs the vanishing of certain classes in zeroth Hochschild homology of the multiplicative preprojective algebra, utilizing a rigidification argument for isomorphisms that may be of independent interest. In the setting of Ginzburg dg-algebras, our obstructions are new in type $E$ and give a more elementary proof of the negative result of Etgü-Lekili [5, Theorem 13] in type $D$. Moreover, the zeroth Hochschild homology of the multiplicative preprojective algebra, computed in Section 4, can be interpreted as the space of unobstructed deformations of the multiplicative Ginzburg dg-algebra by Van den Bergh duality. Finally, we observe that the multiplicative preprojective algebra is not symmetric Frobenius if $Q \neq A_1$, a departure from the additive preprojective algebra in characteristic 2 for $Q = D_{2n}$, $n \geq 2$ and $Q = E_7, E_8$.

MSC:

16G20 Representations of quivers and partially ordered sets
16E40 (Co)homology of rings and associative algebras (e.g., Hochschild, cyclic, dihedral, etc.)
16S38 Rings arising from noncommutative algebraic geometry
16E45 Differential graded algebras and applications (associative algebraic aspects)

Keywords:

preprojective algebra; Hochschild homology; Ginzburg dg algebra; Frobenius structure

Full Text: DOI

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