Implementation of residual nucleus de-excitations associated with proton decays in $^{12}$C based on the GENIE and TALYS generators

Hang Hu,$^1$‡ Wan-Lei Guo,$^2$§ Jun Su,$^3$ Wei Wang,$^{1,3}$† and Cenxi Yuan$^3$¶

$^1$School of Physics, Sun Yat-Sen University, Guangzhou, 510275, China
$^2$Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China
$^3$Sino-French Institute of Nuclear Engineering and Technology, Sun Yat-Sen University, Zhuhai, 519000, China

(Dated: August 27, 2021)

We implement the de-excitation processes of residual nuclei associated with proton decays in $^{12}$C based on the GENIE and TALYS generators. To derive the reasonable excitation energy spectra of residual nuclei, $^{11}$B, $^{10}$B and $^{10}$Be, the default GENIE nucleon decay generator is modified in terms of the Spectral Function nuclear model. Then we use the TALYS code to estimate the de-excitation processes of residual nuclei. The TALYS calculations are partly consistent with the experimental data. The de-excitation neutron emission is more crucial than other particle emissions for liquid scintillator detectors due to the neutron identification capability. We find that the predicted branching ratio of neutron emission can well account for the current experimental measurements.

I. INTRODUCTION

The proton decay is a generic prediction in the Grand Unified Theories (GUTs) [1, 2], since these theories put quarks and leptons into the same multiplet of the GUT gauge group. The GUTs can naturally unify the strong, weak, and electromagnetic interactions into a single underlying force at a very high energy scale. The proton decay is a crucial test of the GUTs. Although many searches have been performed over multiple decades, no experimental evidence to date for proton decay has been found [3]. The Super-Kamiokande experiment gives the best limits to the proton lifetime for most proton decay modes. In the future, three types of detectors will continue to look for the proton decay [4]: Water Cherenkov detectors [5], liquid scintillator detectors [6, 7] and liquid Argon TPC detectors [8]. Here we focus on the large liquid scintillator detectors for future proton decay searches.

The liquid scintillator (LS) as the detection medium in the past neutrino experiments has achieved great successes in [9, 11]. Now the next generation large LS detectors JUNO [6, 7] and LENA [12] are constructing in China and proposed in Europe, respectively. As a calorimeter, the LS detector has some specific advantages, such as its good energy resolution and a very low energy threshold, the LS experiment can measure the visible energy of daughter particles from the proton decay very well, which depends on the initial energy and momentum of the decayed proton. For the bound proton decay, one should consider the following nuclear effects [17], Fermi motion, binding energy, and nucleon-nucleon (NN) correlation, to describe the initial state of the decayed proton. On the other hand, the final state interaction (FSI) and de-excitation processes of the residual nucleus associated with the proton decay may also change the visible energy and simultaneously produce secondary neutrons. These secondary neutrons will reduce the ability to suppress the atmospheric neutrino background through the neutron identification. To model these nuclear effects, a proton decay Monte Carlo (MC) generator is necessary for LS experiments to search for the proton decay.

GENIE is a neutrino MC generator and widely used by many neutrino experiments [18]. The recent versions of GENIE can simulate more than 60 kinds of nucleon decay modes. In GENIE, the Bodek-Ritchie nuclear model [19] with short-range NN correlation is used for all processes, since it is simple and applicable across a broad range of target atoms and neutrino energies. For the proton decay in $^{12}$C, the residual nucleus $^{11}$B in GENIE would be left in the ground state. Namely, the GENIE generator does not consider the de-excitation of the residual nucleus. In fact, the Super-Kamiokande experiment has used $\gamma$-ray emissions from the de-excitation processes of $^{15}$N [20] to search for $p \rightarrow \bar{\nu}K^+$ in $^{16}$O [17]. For LS experiments, the neutron emission of residual nucleus is more crucial than other particle emissions. The emitted neutrons from the s-hole state of $^{11}$C have been used by the KamLAND experiment to identify the neutron invisible decay mode in $^{12}$C [15, 21]. In addition, the initial state proton energy in $^{12}$C is limited to be a very narrow range of 910.0 MeV $\leq E_p \leq$ 922.4 MeV, which does not match with the measured s-shell and p-shell binding energies from the $^{12}$C(e,e$p$)$^{11}$B reaction [22]. In this paper, we modify the GENIE nucleon decay generator in terms of the Spectral Function (SF) nuclear model [23] to account for the nuclear effects, and then use the TALYS code [24].
to implement the de-excitation processes of the residual nucleus. This paper is organized as follows. In Sec. II, we present the GENIE modifications, and then show the energy and momentum distributions of bound protons in the default and modified GENIE. Sec. III describes the de-excitation processes of residual nuclei $^{11}$B, $^{10}$B and $^{10}$Be based on the TALYS software and their excitation energy spectra. In Sec. IV, we compare the predicted results with the experimental measurements. Finally, a summary will be given in Sec. V.

II. THE BOUND PROTONS IN $^{12}$C

For a given proton decay mode, the energy and momentum distributions of final states only depend on those of decayed protons if we ignore the FSI and de-excitation processes of residual nucleus. The free proton from Hydrogen has an energy of $m_p = 938.27$ MeV and the zero momentum. Compared to the free proton, the energy and momentum of bound protons in the Carbon nucleus will be influenced by the nuclear effects [17], including the nuclear binding energy, the Fermi motion and the Nucleon-Nucleon correlations. In GENIE (version 3.0.2), the Bodek-Ritchie relativistic Fermi gas nuclear model [15] with short-range NN correlation is used for all processes [18]. In this case, the off-shell energy $E_p$ of bound protons in $^{12}$C is given by

$$E_p = M_{^{12}C} - \sqrt{M_{^{11}B}^2 + k_1^2},$$

(1)

where $M_{^{12}C}$ ($M_{^{11}B}$) denotes the nuclear mass of $^{12}$C ($^{11}$B). In Fig. 1 we plot the momentum $k_1$ distribution of bound protons, where the tail $k_1 > 221$ MeV comes from the short-range NN correlation. By use of Eq. 1, one can easily calculate $E_p$ as shown in the bottom panel of Fig. 1. It is found that $E_p$ has a very narrow range of $910.0$ MeV $\leq E_p \leq 922.4$ MeV. Meanwhile, the $E_p$ shape doesn’t display the nuclear shell structure, which is not consistent with the measured s-shell and p-shell binding energies from the $^{12}$C$(e,e'p)^{11}$B experiment [22]. In addition, the residual nucleus $^{11}$B associated with proton decays in $^{12}$C is assumed to be the ground state, which means $^{11}$B does not emit any particles through the de-excitation processes.

To solve the above two problems, the off-shell energy $E_p$ of bound protons in $^{12}$C is modified by

$$E_p = M_{^{12}C} - \sqrt{M_{^{11}B}^2 + k_1^2},$$

(2)

where $M_{^{11}B} = M_{^{11}B} + E_x$ and $E_x$ is the excited energy of the residual nucleus $^{11}$B. Note that Eq. 2 is usually used in the analysis of electron/neutrino scattering from a nucleon bound in a nucleus [23, 26]. Due to $k_1 \ll M_{^{11}B}$, we may simplify Eq. 2 and derive

$$E_p \approx m_p - (S^p + E_x + \frac{k_1^2}{2M_{^{11}B}}) = m_p - E_R,$$

(3)

where $S^p = M_{^{11}B} + m_p - M_{^{12}C} = 15.96$ MeV is the proton separation energy and $E_R = S^p + E_x + \frac{k_1^2}{2M_{^{11}B}}$ is defined as the removal energy [23, 26]. $E_R$ can also be expressed by $E_R = E_B + \frac{k_1^2}{2M_{^{11}B}}$ and $E_B = S^p + E_x$ is the binding energy. In order to describe the initial state of decayed protons, we adopt the Spectral Function (SF) nuclear model [24], which provides a 2-dimensional distribution of momentum $k_1$ and removal energy $E_R$ for bound protons in $^{12}$C. Note that the SF model is also an optional nuclear model in GENIE. The SF nuclear model can well describe the neutrino-nucleus reactions [28, 25] and could be further improved with continued experimental investigations. Based on the GENIE SF model and the modified $E_p$ in Eq. 3, we plot the $k_1$ and $E_p$ distributions in Fig. 1. It is clear that the $E_p$ distribution shows the shell structure. The left and right peaks refer to the s-shell and p-shell protons, respectively.
With the help of $k_1$ and $E_x$, one can easily calculate the invariant mass $m_{inv}$ of bound protons in $^{12}$C. Compared with the modified GENIE, the default GENIE has a more narrow range as shown in Fig. 2. For the proton decay to be possible, the invariant mass of the initial proton has to be larger than the sum of masses of final products, namely $m_{inv} > \sum m_f$. Otherwise, this decay will be kinematically forbidden. Based on the invariant mass spectra in Fig. 2 we calculate the forbidden ratios for all possible two-body proton decay modes, including an antilepton and a meson. As listed in Table I the modified GENIE has a greater forbidden ratio than the default GENIE for every proton decay mode. This is because that $m_{inv}$ has a long tail below 760 MeV in the modified GENIE case. It is worthwhile to stress that significant differences between the default and modified GENIE can be found for some proton decay modes of $\sum m_f$ close to the proton mass.

III. DE-EXCITATION PROCESSES OF RESIDUAL NUCLEI

As mentioned above, the residual nucleus $^{11}$B is always left in the ground state, and the de-excitation process is not taken into account in the default GENIE. In fact, the $s$-shell proton decay in $^{12}$C will leave a $s$-hole state of $^{11}$B, which de-excites by emitting $n, p, d, t$ and $\alpha$ particles. The previous experiments have measured some two-body de-excitation modes for the excitation energy range of 16 MeV $\leq E_x \leq 35$ MeV \[24,30\]. For the higher $E_x$ region or three-body de-excitation modes, we do not know the corresponding branching ratio of every de-excitation mode from the $s$-hole state of $^{11}$B. In addition to $^{11}$B, the daughter particles of proton decays may interact with the spectator nucleons, and knock one or more of them out of $^{11}$B. Due to the FSI, the residual nucleus $^{11}$B may be converted to $^{10}$B or $^{10}$Be, and they are left in many particle-hole states. In order to determine the de-excitation processes of $^{11}$B, $^{10}$B and $^{10}$Be, the statistical-model calculation is need. Here we shall calculate the excitation energy spectra of three kinds of residual nuclei based on the modified GENIE, and then use the TALYS code to estimate branching ratios of their de-excitation modes. TALYS is a nuclear reaction program, and is extensively used for both basic and applied science \[24,31,32\]. In Sec. IV we shall compare the predicted branching ratios with the experimental measurements \[24,30\].

### A. De-excitation processes of $^{11}$B states

In the modified GENIE, the excitation energy $E_x$ can be derived from $E_x = E_R - S^p - \frac{k^2}{2M_{^{11}B}}$. On the other hand, one can also use the invariant mass and static mass of the residual nucleus $^{11}$B to calculate $E_x$. As shown in Fig. 3 the $E_x$ spectrum of $^{11}$B states has two peaks. The left peak (0.0 MeV) and right peak (20.2 MeV) correspond to the $p$-shell and $s$-shell proton decays in $^{12}$C, respectively. Due to the NN nuclear correlation, the excitation energy $E_x$ may be up to 300 MeV.

In a simple shell model of $^{12}$C, four protons occupy the $p_3/2$ orbit, and two protons occupy the $s_1/2$ orbit. Due to pairing effects, a pair of protons may be in the $p_1/2$ shell in about 40% of the times \[21\]. In any case, 33.33% of $^{11}$B states are left in a highly excited state because of a $s_1/2$ proton decay. For the case of two protons in the $p_1/2$
shell, there is a one-third chance that the decayed protons come from the $p_{3/2}$ shell. Therefore, 13.33\% of $^{11}\text{B}$ states are excited with medium excitation energy. For the other 53.33\% possibility, the residual nucleus $^{11}\text{B}$ is assumed to be the ground state. Based on the $E_x$ distribution in Fig. 3, 53.33\%, 13.33\%, and 33.33\% possibilities correspond to $E_x \leq 6.1$ MeV, $6.1 < E_x < 15.5$ MeV, and $E_x \geq 15.5$ MeV ranges, respectively. For simplicity, we require that $^{11}\text{B}$ states with $E_x \leq 6.1$ MeV do not de-excite, $^{11}\text{B}$ states with $6.1 < E_x < 15.5$ MeV will emit a $\gamma$ with the energy of $E_x$.

For the highly excited $^{11}\text{B}$ states, we use the TALYS (version 1.95) code [24] to estimate the de-excitation processes based on the spectrum of $E_x \geq 15.5$ MeV. The de-excitation modes, corresponding thresholds, and branching ratios are listed in Table 1. The dominant de-excitation modes, including $n + ^{10}\text{B}$, $\gamma + ^{11}\text{B}$, $n + p + ^{9}\text{Be}$, $n + d + ^{7}\text{Be}$, $n + p + d + ^{7}\text{Be}$, $n + p + d + ^{7}\text{He}$, and $n + p + t + ^{9}\text{Li}$, contribute at a branching ratio of less than 0.5\% have been classified as the other modes. About 70.3\% of highly excited $^{11}\text{B}$ states can produce one or more neutrons through their de-excitation processes. Most of these neutrons will give a 2.2 MeV $\gamma$-ray from the neutron capture in LS detectors.

### Table II. The de-excitation modes, corresponding thresholds and branching ratios from the residual nucleus $^{11}\text{B}$ in the case of a $s_{1/2}$ proton decay.

| De-excitation Mode | Threshold (MeV) | Branching ratio (%) |
|--------------------|----------------|---------------------|
| $\gamma + ^{11}\text{B}$ | 0 | 10.6 |
| $n + ^{10}\text{B}$ | 11.5 | 9.4 |
| $p + ^{10}\text{Be}$ | 11.2 | 3.3 |
| $d + ^{7}\text{Be}$ | 15.8 | 4.6 |
| $t + ^{7}\text{Be}$ | 11.2 | 2.1 |
| $\alpha + ^{7}\text{Li}$ | 8.7 | 2.6 |
| $2n + ^{7}\text{B}$ | 19.9 | 5.3 |
| $n + p + ^{9}\text{Be}$ | 18.0 | 11.7 |
| $n + d + ^{8}\text{Be}$ | 17.5 | 13.3 |
| $2d + ^{7}\text{Li}$ | 32.5 | 0.6 |
| $n + ^{3}\text{He} + ^{7}\text{Li}$ | 29.2 | 0.5 |
| $n + \alpha + ^{6}\text{Li}$ | 15.9 | 10.9 |
| $d + \alpha + ^{7}\text{He}$ | 18.1 | 1.3 |
| $2n + p + ^{8}\text{Be}$ | 19.7 | 8.6 |
| $n + 2p + ^{8}\text{Li}$ | 34.9 | 0.9 |
| $2n + d + ^{7}\text{Be}$ | 36.4 | 0.9 |
| $n + p + d + ^{7}\text{Be}$ | 34.7 | 2.2 |
| $n + p + t + ^{9}\text{Li}$ | 35.7 | 0.7 |
| $2n + ^{4}\text{He} + ^{6}\text{Li}$ | 29.2 | 0.7 |
| $n + p + \alpha + ^{9}\text{He}$ | 20.3 | 3.0 |
| $2n + 2p + ^{7}\text{Li}$ | 37.0 | 0.9 |
| $2n + p + d + ^{6}\text{Li}$ | 42.0 | 0.8 |
| other modes | - | 4.8 |

### B. De-excitation processes of $^{10}\text{B}$ and $^{10}\text{Be}$ states

In the above subsection, we only consider the de-excitation processes of the residual nucleus $^{11}\text{B}$. Other residual nuclei can also be produced through the FSI. Due to the strong interaction, the daughter mesons from the proton decay may interact with spectator nucleons before escaping from the residual nucleus surface. In this case, the FSI can knock the struck nucleon out of $^{11}\text{B}$, and the residual nucleus is converted to the $^{10}\text{B}$ or $^{10}\text{Be}$ nucleus. The multiple FSI can result in other residual nuclei, such as $^{9}\text{B}$, $^{9}\text{Be}$, and $^{8}\text{Li}$, etc. Here we focus on the de-excitation processes of $^{10}\text{B}$ and $^{10}\text{Be}$ states.

In the default GENIE, the spectator nucleon is on-shell and its energy is given by

$$E_N = \sqrt{m_{N}^2 + k_2^2},$$

where $m_N$ is the nucleon mass and the momentum $k_2$ is same with the red line in the top panel of Fig. 1. With the help of Eqs. 1 and 4, one can calculate the invariant masses of $^{10}\text{B}$ and $^{10}\text{Be}$ through

$$M_{inv} = \sqrt{M_{12C} - E_p - E_N + E_h}^2 - (k_1 + k_2)^2. \quad (5)$$

Since Eq. 4 overestimates the energy of spectator nucleon, the default GENIE uses a fixed binding energy $E_b = 25$ MeV to offset the reduction of residual nucleus energy. Then the excitation energy spectra of $^{10}\text{B}$ and $^{10}\text{Be}$ states can be easily obtained from $E_x = M_{inv} - M_R$ as shown in Fig. 1, where $M_R$ is the corresponding mass in the ground state. It is clear that the average excitation energy $\langle E_x \rangle$ is negative in the default GENIE case.
In order to solve the $\langle E_x \rangle < 0$ problem, we modify the spectator nucleon energy with

$$E_N = m_N - E_R,$$

where the removal energy $E_R$ is given by the SF nuclear model [23]. Here we assume spectator nucleons in $^{11}$B have the same SF distribution with protons in $^{12}$C. In this case, the invariant masses of the residual nuclei $^{10}$B and $^{10}$Be can be derived from

$$M_{inv} = \sqrt{(M_{12C} - E_p - E_N)^2 - (k_1 + k_2)^2}.$$  \hspace{1cm} (7)

Using $E_x = M_{inv} - M_R$, we plot the $E_x$ spectra of $^{10}$B and $^{10}$Be states in Fig. 4. The difference between the two curves is very small.

About 27% $^{10}$B or $^{10}$Be states will be assumed to be the ground state because of $E_x < 0$. Then we use the TALYS code to estimate the de-excitation modes of $^{10}$B and $^{10}$Be states based on the $E_x > 0$ distributions. The branching ratio of each de-excitation mode from the residual nuclei $^{10}$B and $^{10}$Be have been listed in Table III and Table IV respectively. About 27% $^{10}$B and 30% $^{10}$Be states can produce one or more neutrons in their de-excitation processes.

### IV. COMPARISON WITH EXPERIMENTAL DATA

In order to study the properties of proton-hole states in $^{11}$B, the quasifree $^{12}$C($p$, $2p$)$^{11}$B [29, 30] and $^{12}$C($e$, $e'p$)$^{11}$B [22] reactions have been extensively investigated. Yosoi et al. measured the excitation energy spectrum of the proton-hole states in $^{11}$B through the quasifree $^{12}$C($p$, $2p$)$^{11}$B reaction [29]. As shown in Fig. 5, the predicted $E_x$ curve (red line) from Fig. 5 can well

| De-excitation Mode | Threshold (MeV) | Branching ratio (%) |
|-------------------|----------------|---------------------|
| ground state      | -              | 27                  |
| $\gamma + ^{10}$B | 0              | 61.6                |
| $n + ^{7}$B       | 6.8            | 19                  |
| $d + ^{8}$Li      | 21.5           | 12                  |
| $t + ^{7}$Li      | 17.2           | 1.1                 |
| $2n + ^{8}$Be     | 8.5            | 9.4                 |
| $n + p + ^{6}$Li  | 23.7           | 2.3                 |
| $n + d + ^{7}$Li  | 23.5           | 5.0                 |
| $n + t + ^{6}$Li  | 24.5           | 3.2                 |
| $d + t + ^{5}$He  | 26.7           | 0.5                 |
| $n + \alpha + ^{5}$He | 9.1           | 2.4                 |
| $2n + p + ^{7}$Li | 25.7           | 1.7                 |
| $2n + d + ^{6}$Li | 30.8           | 1.3                 |
| $n + 2d + ^{5}$He | 33.0           | 0.7                 |
| $n + p + t + ^{7}$He | 28.9       | 0.9                 |
| $3n + p + ^{4}$Li | 33.0           | 0.8                 |
| other modes       | -              | 3.3                 |
describe the measured one (black dash-dotted line) in the region of 15 MeV $\leq E_x \leq 40$ MeV. For the convenience of following discussions, we have scaled the measured and predicted spectra to match the experimental data in Ref. 30. For $E_x < 15$ MeV, the predicted spectrum is inconsistent with the experimental measurement, and displays a continuous distribution as shown in Fig. 5. It can only account for several discrete peaks in Fig. 1a of Ref. 29. This is because that we use the continuous spectral function to calculate the excitation energy.

In Ref. 30, Panin et al. measured three de-excitation modes of $^{11}$B states from the $^{12}$C(p, 2p)$^{11}$B reaction, namely $^{11}$B $\rightarrow$ n$+^{10}$B, $^{11}$B $\rightarrow$ d$+^{9}$Be and $^{11}$B $\rightarrow$ $\alpha+^{7}$Li. The $^{11}$B $\rightarrow$ n$+^{10}$B (pink triangle) and total (blue square) spectra have been plotted in Fig. 5. For comparison, we use the TALYS code to calculate branching ratios of the three de-excitation modes in terms of the $E_x$ distribution between 16-35 MeV. The predicted $^{11}$B $\rightarrow$ n$+^{10}$B (pink line) and total (blue line) spectra have been shown in Fig. 5. For 16 MeV $\leq E_x \leq 27$ MeV, the predicted spectra basically agree with the experimental data. However, they are much smaller than the measured spectra for the $E_x > 27$ MeV region.

In addition to the excitation energy $E_x$, Ref. 29 has also measured the branching ratios of charged particle emissions from the s–hole state in $^{11}$B as shown in Fig. 6. The hatched areas indicate two-body de-excitation modes of $^{11}$B states. The blank parts come from contributions of three-body and sequential de-excitation modes. The experimental data of three two-body de-excitation modes in Fig. 5 have been converted into the branching ratios of two-body n and d/$\alpha$ emissions with the help of the measured $E_x$ spectrum, where the n branching ratio is most crucial for LS experiments. In Ref. 29, the authors have also provided the results of a statistical-model calculation from the CASCADE code 33. In Fig. 6, the red and blue branching ratios denote the TALYS and CASCADE results, respectively. In the TALYS calculation, we only take the $E_x$ distribution between 16-35 MeV, since the measured results in Ref. 29 are based on this excitation energy range. Note that branching ratios in Table 1 correspond to the larger range of $E_x \geq 15.5$ MeV. It is found that TALYS and CASCADE give similar results for all particle emissions. The predicted total branching ratios of both t and $\alpha$ de-excitation modes are much smaller than the measured results. For the combination of d and $\alpha$ emissions, the TALYS calculation can well account for the measured branching ratio from Ref. 29, but can’t explain the two-body branching ratio from Ref. 30. For two-body de-excitation modes, the predicted branching ratio of each particle emission is basically consistent with the experimental data except for the t emission, especially the key n emission.

V. SUMMARY

In summary, we have investigated the de-excitation processes of the residual nuclei associated with proton decays in $^{12}$C based on the GENIE and TALYS generators. Since the residual nucleus $^{11}$B would be left in the ground state, the default GENIE nucleon decay generator has been modified in terms of the SF nuclear
model. After deriving the excitation energy spectra of $^{11}$B, $^{13}$B and $^{10}$Be, we use the TALYS code to implement their de-excitation processes. For the $s$-hole state in $^{11}$B, we compare the predicted results with the experimental measurements. It is found that the predicted branching ratios of de-excitation modes are partly consistent with the experimental data. For $n, p, d$ and $\alpha$ emissions, the predicted branching ratios of two-body de-excitation modes can well explain the measurements, especially the key $n$ emission. For LS experiments, the total $n$ branching ratio from the residual nucleus $^{11}$B is the most crucial quantity due to the neutron capture. However, it has not been measured up to now. We hope that future experiments can measure the total branching ratio including one or more neutrons from de-excitation processes of the residual nucleus $^{11}$B.

ACKNOWLEDGMENTS

We are grateful to Jie Cheng, Xianguo Lu, Yufeng Li, Yuhang Guo, Benda Xu and Aiqiang Zhang for their helpful discussions. This work is supported in part by the National Nature Science Foundation of China (NSFC) under Grants No. 11575201, No. 11675273 and No. 11775316, and the Strategic Priority Research Program of the Chinese Academy of Sciences under Grant No. XDA10010100.
lani, K. Boretzky and C. Caesar, et al, Phys. Lett. B 753, 204-210 (2016) doi:10.1016/j.physletb.2015.11.082

[31] J. Cheng, Y. F. Li, L. J. Wen and S. Zhou, Phys. Rev. D 103, no.5, 053001 (2021) doi:10.1103/PhysRevD.103.053001 arXiv:2008.04633 [hep-ph].

[32] J. Cheng, Y. F. Li, H. Q. Lu and L. J. Wen, Phys. Rev. D 103, no.5, 053002 (2021) doi:10.1103/PhysRevD.103.053002 arXiv:2009.04085 [hep-ex].

[33] F. Puhlhofer, Nucl. Phys. A 280, 267-284 (1977) doi:10.1016/0375-9474(77)90308-6; M. N. Harakeh, Extended Version of Code CASCADE, (1983) (unpublished)