Harmonic Superspace Gaugeon Formalism for the ABJM Theory

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Abstract
In this paper we will analyse the ABJM theory in harmonic superspace. The harmonic superspace variables will be parameterized by the coset $SU(2)/U(1)$ and thus will have manifest $\mathcal{N} = 3$ supersymmetry. We will study the quantum gauge transformations and the BRST transformations of this theory in gaugeon formalism. We will use this BRST symmetry to project out the physical sub-space from the total Hilbert space. We will also show that the evolution of the $S$-matrix is unitary for this ABJM theory in harmonic superspace.

Key Words: Harmonic Superspace, BRST, Gaugeon, ABJM Theory.

1 Introduction
The harmonic superspace is parametrized by the coset $SU(2)/U(1)$ and thus has $\mathcal{N} = 2$ supersymmetry in four dimensions [1]-[2], and $\mathcal{N} = 3$ supersymmetry in three dimensions [3]-[5]. A superconformal Chern-Simons-matter theory called the ABJM theory has already been studied in this Harmonic superspace [6]. The ABJM theory that is thought to capture the dynamics of multiple $M2$-branes. It has manifest $\mathcal{N} = 6$ supersymmetry which is expected to get enhanced to $\mathcal{N} = 8$ supersymmetry [7]-[11]. The ABJM theory is invariant under the gauge group $U(N)_k \times U(N)_{-k}$ [12]. As the ABJM theory has a gauge symmetry associated with it, we have to fix a gauge in order to quantization it. The gauge fixing condition can be incorporated at a quantum level by adding ghost and gauge fixing terms to the original classical Lagrangian density. However, the Fock space in one gauge is different from that in other gauge because the Fock space in any gauge is not wide enough to realize the quantum gauge freedom. This problem is solved by the gaugeon formalism [13]-[17]. In this formalism the quantum gauge transformation are accounted for by introducing a set of extra fields called gaugeon fields. As the ABJM theory has gauge symmetry associated with it, so it can be analysed in the gaugeon formalism. This has already been done for ABJM theory in $\mathcal{N} = 1$ superspace formalism [18]. In this paper we will do it for ABJM theory in $\mathcal{N} = 3$ harmonic superspace. It may be noted that unlike the ABJM theory in $\mathcal{N} = 1$ superspace, the ABJM theory in $\mathcal{N} = 3$ harmonic superspace contains no explicit superspace potential term. We will
also study the extended BRST symmetry of this model and use it to show that the $S$-matrix for the ABJM theory in $N = 3$ harmonic superspace is unitarity. It may be noted that similar results could be obtained by using conventional BRST transformations along with the Yokoyama’s subsidiary condition [19]-[20].

The BRST symmetry for ordinary Chern-Simons theory has been thoroughly studied [21]-[22]. In fact, the BRST symmetry of $N = 1$ Chern-Simons theory has also been analysed [23, 24]. The BRST of the Chern-Simons theory is similar to the BRST symmetry of Yang-Mills theories. This is because the structure of the gauge fixing term and the ghost terms is similar in both these theories. The BRST symmetry for gauge theories has also been studied in the background field method. In the background field method, all the fields in a theory are shifted. The BRST symmetry of these shifted fields can be analysed by the use of Batalin-Vilkovisky formalism [25]-[31]. In this formalism the modified BRST symmetry arise due to the invariance of a theory under the original BRST transformations along with these shift transformation. This have been done for the ordinary Yang-Mills theories and the ordinary Chern-Simons theories [32]-[38]. Furthermore, as the BRST transformations mix the fermionic and bosonic coordinates, they can be viewed as supersymmetric transformations. The BRST transformations of gauge theories have been expressed in the extended superspace formalism by adding new Grassmann coordinates [39]-[40].

The main use of the BRST symmetry is to project out the physical sub-space from the total Hilbert space of the theory. Thus, in this paper we will analyse the BRST symmetry for the ABJM theory and use it to project out the physical sub-space from the total Hilbert space of the theory.

2 Harmonic superspace

In this paper we use the harmonic variables $u^\pm$ subjected to the constraints

$$u^{+i}u_i^- = 1, \quad u^{+i}u_i^+ = u^{-i}u_i^- = 0. \quad (1)$$

These harmonic variable parameterize the coset $SU(2)/U(1)$. So, this superspace is parameterized by the following coordinates

$$z = (x^{ab}, \theta_a^{++}, \theta_a^{--}, \theta_a^0, u^\pm_i), \quad (2)$$

where $\theta^{\pm}_a = \theta_a^{ij}u^j_i$ and $\theta^0_a = \theta_a^{ij}u^j_i u_i^j$. Now we can define the following derivatives

$$D^{++} = \partial^{++} + 2i\theta^{++a}\theta^{0b}\partial_{ab}^A + \theta^{++a}\frac{\partial}{\partial \theta^{0a}} + 2\theta^{0a}\frac{\partial}{\partial \theta^{--a}},$$

$$D^{--} = \partial^{--} - 2i\theta^{--a}\theta^{0b}\partial_{ab}^A + \theta^{--a}\frac{\partial}{\partial \theta^{0a}} + 2\theta^{0a}\frac{\partial}{\partial \theta^{++a}},$$

$$D^0 = \partial^0 + 2\theta^{++a}\frac{\partial}{\partial \theta^{++a}} - 2\theta^{--a}\frac{\partial}{\partial \theta^{--a}},$$

and

$$D_a^{--} = \frac{\partial}{\partial \theta^{++a}} + 2i\theta^{--b}\partial_{ab}^A, \quad D_a^0 = \frac{1}{2} \frac{\partial}{\partial \theta^{0a}} + i\theta^{0b}\partial_{ab}^A,$$

$$D_a^{++} = \frac{\partial}{\partial \theta^{--a}}, \quad (3)$$

$$2$$
where
\[ \partial^{++} = u^+_i \frac{\partial}{\partial u^+_i}, \quad \partial^{--} = u^-_i \frac{\partial}{\partial u^-_i}, \quad \partial^0 = u^+_i \frac{\partial}{\partial u^+_i} - u^-_i \frac{\partial}{\partial u^-_i}. \]

(5)

These derivatives satisfy
\[
\{ D^{++}_a, D^{--}_b \} = 2i \partial_{ab}^A, \quad \{ D^0_a, D^0_b \} = -i \partial_{ab}^A,
\]
\[
[D^{++}_a, D^{--}_a] = 2 D^0_a, \quad [D^0_a, D^{--}_a] = \pm 2 D^{++}_a,
\]
\[
\partial^0 = [\partial^{++}, \partial^{--}], \quad [D^{++}, D^{--}] = D^0.
\]

(6)

The analytic superfields \( \Phi_A = \Phi_A(\zeta_A) \) are independent of the \( \theta^-_a \), thus defined by \( D^{++}_a \Phi_A = 0 \). The analytic subspace is parametrized by the following coordinates
\[
\zeta_A = (x^{ab}_A, \theta^{++}_a, \theta^0_a, u^\pm_i),
\]

(7)

where
\[
x^{ab}_A = (\gamma_m)^{ab} x^m_A = x^{ab} + i(\theta^{++}_a \theta^{--}_b + \theta^{++}_b \theta^{--}_a).
\]

(8)

The generators of the supersymmetry are given by
\[
Q^{++}_a = u^+_i u^+_j Q^{ij}_a, \quad Q^{--}_a = u^-_i u^-_j Q^{ij}_a, \quad Q^0_a = u^+_i u^-_j Q^{ij}_a,
\]

(9)

where
\[
Q^{ij}_a = \frac{\partial}{\partial \theta^{ij}_a} - \theta^{ij}_a \partial_{ab}.
\]

(10)

It is convenient to denote the superspace measure in this superspace as
\[
d^9z = \frac{1}{16} d^3x (D^{++})^2 (D^{--})^2 (D^0)^2,
\]
\[
d^{(4)} = \frac{1}{4} d^3x_A du (D^{--})^2 (D^0)^2.
\]

(11)

A conjugation in the \( \mathcal{N} = 3 \) harmonic superspace is defined by
\[
(\bar{u}^{\pm}_i) = u^{\mp}_i, \quad (\bar{x}^m_A) = x^m_A,
\]
\[
(\bar{\theta}^{\pm}_a) = \theta^{\mp}_a, \quad (\bar{\theta}^0_a) = \theta^0_a.
\]

(12)

It is squared to \(-1\) on the harmonics and to \(1\) on \( x^m_A \) and Grassmann coordinates. So, the analytic superspace measure is real \( d^9\zeta^{(-4)} = d\zeta^{(-4)} \) and the full superspace measure is imaginary \( d^9\bar{z} = -d^9z \).
3 ABJM Theory

Now we can construct the Lagrangian density for ABJM theory in this deformed superspace using $V_L^{++}$ and $V_R^{++}$, which are defined by

$$V_L^{++} = u_i^+ u_j^+ V_L^{ij},$$
$$V_R^{++} = u_i^+ u_j^+ V_R^{ij},$$

(13)

where $V_L^{ij}$ and $V_R^{ij}$ are fields transforming under the gauge group $U(N)_k$ and $U(N)_{-k}$, respectively. We also define matter fields $q^+$ and $\bar{q}^+$, which transform under the bifundamental representation of the group $U(N)_k \times U(N)_{-k}$. The Lagrangian density for the ABJM theory can now be written as

$$\mathcal{L}_{ABJM} = \mathcal{L}_{CS,k}[V_L^{++}] + \mathcal{L}_{CS,-k}[V_R^{++}] + \mathcal{L}_M[q^+, \bar{q}^+],$$

(14)

where

$$\mathcal{L}_{CS,k}[V_L^{++}] = \frac{ik}{4\pi} \text{tr} \sum_{n=2}^{\infty} \frac{(-1)^n}{n} \int d^6\theta du_1 \ldots du_n H_L^{++},$$
$$\mathcal{L}_{CS,-k}[V_R^{++}] = -\frac{ik}{4\pi} \text{tr} \sum_{n=2}^{\infty} \frac{(-1)^n}{n} \int d^6\theta du_1 \ldots du_n H_R^{++},$$
$$\mathcal{L}_M[q^+, \bar{q}^+] = tr \int d(\zeta(-4)q^+ \nabla^{++}q^+),$$

(15)

and

$$H_L^{++} = \frac{V^{++}(z, u_1)_L V^{++}(z, u_2)_L \ldots V^{++}(z, u_n)_L}{(u_1^+ u_2^+) \ldots (u_n^+ u_1^+)}$$
$$H_R^{++} = \frac{V^{++}(z, u_1)_R V^{++}(z, u_2)_R \ldots V^{++}(z, u_n)_R}{(u_1^+ u_2^+) \ldots (u_n^+ u_1^+)}$$
$$\nabla^{++}q^+ = D^{++} q^+ + V_L^{++} q^+ - q^+ V_R^{++},$$
$$\nabla^{++}\bar{q}^+ = D^{++} \bar{q}^+ - \bar{q}^+ V_L^{++} + V_R^{++} \bar{q}^+.$$  

(16)

The covariant derivatives for the matter fields are given by

$$\nabla^{++}q^+ = D^{++} q^+ + V_L^{++} q^+ - q^+ V_R^{++},$$
$$\nabla^{++}\bar{q}^+ = D^{++} \bar{q}^+ - \bar{q}^+ V_L^{++} + V_R^{++} \bar{q}^+.$$  

(17)

It is useful to define $V_L^{--}$ and $V_R^{--}$ as

$$V_L^{--} = \sum_{n=1}^{\infty} (-1)^n \int du_1 \ldots du_n E_L^{++},$$
$$V_R^{--} = \sum_{n=1}^{\infty} (-1)^n \int du_1 \ldots du_n E_R^{++},$$

(18)

where

$$E_L^{++} = \frac{V^{++}(z, u_1)_L V^{++}(z, u_2)_L \ldots V^{++}(z, u_n)_L}{(u_1^+ u_2^+) \ldots (u_n^+ u_1^+)}$$
$$E_R^{++} = \frac{V^{++}(z, u_1)_R V^{++}(z, u_2)_R \ldots V^{++}(z, u_n)_R}{(u_1^+ u_2^+) \ldots (u_n^+ u_1^+)}.$$  

(19)
It is also useful to define $W^{++}_L$ and $W^{++}_R$ as

$$W^{++}_L = -\frac{1}{4}D^{++}aD^{++}aV^{--}_L - 1,$$
$$W^{++}_R = -\frac{1}{4}D^{++}aD^{++}aV^{--}_R. \tag{20}$$

This ABJM theory is invariant under the following $\mathcal{N} = 3$ supersymmetric transformations,

$$\delta q^+ = i\epsilon^a_\Lambda\tilde{\nabla}^a q^+, \quad \delta \bar{q}^+ = i\epsilon^a_\Lambda\tilde{\nabla}^a \bar{q}^+, \quad \delta V^{++}_L = \frac{8\pi}{k}e^a_\Lambda \theta^{a}_0 q^+ q^+, \quad \delta V^{++}_R = \frac{8\pi}{k}e^a_\Lambda \theta^{a}_0 \bar{q}^+ q^+. \tag{21}$$

where

$$\tilde{\nabla}^a q^+ = \nabla^a q^+ + \theta^{a}_a (W^{++}_L q^+ - q^+ W^{++}_R), \quad \tilde{\nabla}^a \bar{q}^+ = D^a q^+ + V^a_0 q^+ - q^+ V^a_0,$$
$$V^a_0 = -\frac{1}{2}D^{++}aV^{--}_L, \quad V^a_0 = -\frac{1}{2}D^{++}aV^{--}_R. \tag{22}$$

Thus, apart from the original manifest $\mathcal{N} = 3$ supersymmetry, this model has additional $\mathcal{N} = 3$ supersymmetry, $\delta_q \mathcal{L}_{ABJM} = 0$. So, this ABJM theory has $\mathcal{N} = 6$ supersymmetry.

## 4 Gaugeon Formalism

In the gaugeon formalism extra fields are added to account for the quantum gauge transformations. Thus, apart from the gauge fixing and ghost terms a gaugeon term is also added to the original classical Lagrangian density. In this section we will analyse the ABJM theory in gaugeon formalism. This ABJM theory is invariant under the infinitesimal gauge transformations, $\delta \mathcal{L}_{ABJM} = 0$,.

$$\delta q^+ = \Lambda_L q^+ - q^+ \Lambda_R, \quad \delta \bar{q}^+ = \Lambda_R \bar{q}^+ - \bar{q}^+ \Lambda_L, \quad \delta V^{++}_L = \nabla^{++} \Lambda_L, \quad \delta V^{++}_R = \nabla^{++} \Lambda_R. \tag{23}$$

where

$$\nabla^{++} \Lambda_L = -D^{++} \Lambda_L - [V^{++}_L, \Lambda_L], \quad \nabla^{++} \Lambda_R = -D^{++} \Lambda_R - [V^{++}_R, \Lambda_R]. \tag{24}$$

Due to the invariance of the ABJM theory under these infinitesimal gauge transformations, we can not quantize it without fixing a gauge. So, we choose the following gauge fixing conditions,

$$D^{++} V^{++}_L = 0, \quad D^{++} V^{++}_R = 0. \tag{25}$$
To incorporate these gauge fixing conditions at the quantum level we have to add the corresponding gauge fixing and ghost terms to the original classical Lagrangian density. In order to incorporate the quantum gauge transformations we have to also add a gaugeon term to it. Thus, the total Lagrangian density is given by the sum of the original classical Lagrangian density $L_{ABJM}$, the gauge fixing term $L_{gf}$, the ghost term $L_{gh}$, and the gaugeon term $L_{go}$,

$$ L_t = L_{ABJM} + L_{gf} + L_{gh} + L_{go}, $$

where

$$ L_{gf} = \int d\zeta (-4) \text{tr} \left[ b_L (D^{++} V_L^{++}) + \frac{\alpha}{2} b_L^2 - \frac{\alpha}{2} \tilde{b}_R^2 \right. $$

$$ - b_R (D^{++} V_R^{++}) \left| \right. , $$

$$ L_{gh} = \int d\zeta (-4) \text{tr} \left[ \tau L D^{++} \nabla^c c_L - \tau R D^{++} \nabla^c c_R \right] , $$

$$ L_{go} = \int d\zeta (-4) \text{tr} \left[ D^{++} \tau_L D^{++} y_L + \frac{1}{2} (\tau_L + \alpha b_L)^2 $$

$$ - D^{++} \tau_R D^a k - D^{++} \tau_R D^{++} y_R $$

$$ - \frac{1}{2} (\tau_R + \alpha b_R)^2 + D^{++} \tau_R D^a k_R \right] . $$

Now we can analyse the quantum gauge transformations by varying the gauge fixing parameter $\alpha$. So, we transform $\alpha$ as

$$ \delta qg \alpha = \tau \alpha . $$

Under this transformation the matter fields transform as,

$$ \delta qg q = i(\tau \alpha q_L - q \tau \alpha R), \quad \delta qg \bar{q} = i(\tau \alpha \bar{q}_L - \bar{q} \tau \alpha R). $$

and all other fields transform as,

$$ \delta qg \nabla^{++} = \tau \nabla^{++} (\alpha y_L), \quad \delta qg \nabla^{++} = \tau \nabla^{++} (\alpha y_R), $$

$$ \delta qg \tau_L = \tau \tau_L, \quad \delta qg \tau_R = \tau \tau_R, $$

$$ \delta qg c_L = [\tau c_L, \alpha y_L] + \tau \alpha k_L, \quad \delta qg c_R = [\tau c_R, \alpha y_R] + \tau \alpha k_R, $$

$$ \delta qg \bar{y}_L = \bar{\tau} \tau_L, \quad \delta qg \bar{y}_R = \bar{\tau} \tau_R, $$

$$ \delta qg b_L = [\tau b_L, \alpha y_L] - [\tau \tau_L, \alpha k_L], \quad \delta qg b_R = [\tau b_R, \alpha y_R] - [\tau \tau_R, \alpha k], $$

$$ \delta qg y_L = \delta qg k_L = 0, \quad \delta qg y_R = \delta qg k_R = 0 . $$

The total Lagrangian density is invariant under these quantum transformations, $\delta qg L_t = 0$. Thus, by adding a gaugeon term we have derived the quantum gauge transformation for the ABJM theory in harmonic superspace.

### 5 BRST Symmetry

The ABJM theory was initially invariant under gauge transformations. But, after fixing a gauge this gauge invariance was broken. However, the sum of the
original Lagrangian density, gauge fixing term, the ghost term and the gaugeon term is invariant under a new symmetry called the BRST symmetry. The BRST transformations of the matter fields are given by

\[ s q = i(c_L q - q c_R), \quad s \bar{q} = i(c_R \bar{q} - \bar{q} c_L). \] (31)

and the BRST transformations of all other fields are given by

\[ s V^{++} = \nabla^{++} c_L, \quad s V^{++} = \nabla^{++} c_R, \]
\[ s c_L = -\frac{1}{2} \{c_L, c_L\}, \quad s \bar{c}_R = b_R, \]
\[ s \bar{c}_L = b_R, \quad s c_R = -\frac{1}{2} \{c_R, \bar{c}_R\}, \]
\[ s y_L = k_L, \quad s y_R = k_R, \]
\[ s \bar{y}_L = -\bar{y}_L, \quad s \bar{y}_R = -\bar{y}_R, \]
\[ s b_L = s k_L = s \bar{y}_L = 0, \quad s b_R = s k_R = s \bar{y}_R = 0. \] (32)

These BRST transformations are nilpotent, \( s^2 = 0 \), and they also commute with the quantum gauge transformations, \( \delta_{qg} s = s \delta_{qg} \). The total Lagrangian density is invariant under these BRST transformations, \( s L_t = 0 \), so we can obtain a conserved BRST charge \( Q_B \). This conserved charge commutes with the Hamiltonian of the theory and generates the BRST transformations. We can also use this change to project out the physical state and show that the \( S \)-matrix is unitarity. The states that are annihilated by \( Q_B \) are defined to be the physical states,

\[ Q_B|\phi_p\rangle = 0. \] (33)

The inner product of physical states, which are obtained by the action of \( Q_B \) on unphysical states \( |\phi_{up}\rangle \), vanishes with all physical states,

\[ \langle \phi_p|Q_B|\phi_{up}\rangle = 0. \] (34)

This is because \( Q_B \) is the generator of the BRST transformations and these transformations are nilpotent, and so we have

\[ Q_B^2|\phi\rangle = 0. \] (35)

Thus, the relevant physical states actually are those that are not obtained by the action of \( Q_B \) on any other state. Now we can write a \( S \)-matrix element as

\[ \langle \phi_{pa, out}|\phi_{ph, in}\rangle = \langle \phi_{pa}|S^* S|\phi_{ph}\rangle, \] (36)

where the asymptotic physical states are given by

\[ |\phi_{pa, out}\rangle = |\phi_{pa}, t \to \infty\rangle, \]
\[ |\phi_{ph, in}\rangle = |\phi_{ph}, t \to -\infty\rangle. \] (37)

As \( Q_B \) commute with the Hamiltonian, the time evolution of any physical state will also be annihilated by \( Q_B \),

\[ Q_B S|\phi_{ph}\rangle = 0. \] (38)
This implies that the states $S|\phi_{pb}\rangle$ must be a linear combination of physical states $|\phi_{p,i}\rangle$. Thus, we can write
\[
\langle \phi_{p_a}|S|\phi_{pb}\rangle = \sum_i \langle \phi_{p_a}|S|\phi_{p,i}\rangle \langle \phi_{p,i}|S|\phi_{pb}\rangle.
\] (39)

Now as the full $S$-matrix is unitary this relation implies that the $S$-matrix restricted to physical sub-space is also unitarity.

6 Conclusion

In this paper we analysed the ABJM theory in $\mathcal{N} = 3$ harmonic superspace formalism. The harmonic superspace variables used were parameterized by the coset $SU(2)/U(1)$ and thus had manifest $\mathcal{N} = 3$ supersymmetry. The ABJM theory expressed in these variable had manifest $\mathcal{N} = 6$ supersymmetry. We analysed the quantum gauge transformations of this theory in gaugeon formalism. The gauge fixed ABJM theory in the gaugeon formalism had extra fields called the gaugeon fields. This made the Hilbert space of the theory large enough to consider quantum gauge transformations. We also analysed the BRST transformations of this theory and used them to show that the unitarity of the $S$-matrix. These result could also have been obtained by using a conventional BRST symmetry along with the Yokoyama’s subsidiary condition. It is well know that for gauge theories there is symmetry dual to the BRST symmetry. This symmetry is called the anti-BRST symmetry. It will be interesting to include the anti-BRST transformations in the present analysis. It is expected that the results obtained will again be the same as obtained by using a conventional anti-BRST symmetry along with the Yokoyama’s subsidiary condition.

Chern-Simons theories also have important condense matter applications. This is because the fractional quantum hall effect is generated by Chern-Simons theories [41]-[44]. Fractional quantum hall effect is a property of a collective state in which electrons bind magnetic flux lines to make new quasi-particles. These quasi-particles have a fractional elementary charge. Thus, the fractional quantum Hall effect is based on the concept of statistical transmutation. In two dimensions, fermions can be described as charged bosons carrying an odd integer number of flux quanta. The fractional quantum hall effect is generated by Chern-Simons theories fields coupled to these bosons. If these electrons are analyzed in a combined external and statistical magnetic field, then at special values of the filling fraction the statistical field cancels the external field, in the mean field sense. At these values of the filling fraction and the system is described as a gas of bosons feeling no net magnetic field. Thus, these bosons condense into a homogeneous ground state. This model also describes the existence of vortex and anti-vortex excitations.

Lately, supersymmetric generalisation of the fractional quantum Hall effect has also been investigated [45]-[48]. The physical properties of the topological excitations in the supersymmetric quantum Hall liquid have also been discussed in a dual supersymmetric Chern-Simons theory [49]. Furthermore, the fractional quantum Hall effect is closely related to noncommutativity of the spacetime [50]-[53]. Thus, the results of this paper can have interesting condensed matter applications. This is because we can analyse the supersymmetric fractional quantum Hall effect in the gaugeon formalism see if it generates some new physics. In
fact, we can also include different deformations of the Harmonic superspace and then analyses this deformed ABJM theory in the gaugeon formalism. These deformations are expected to change the behavior of fractional condensates and thus have important consequences for the transport properties in the quantum hall system. Recently, supersymmetric Chern-Simons theory has defined to be been used to study fractional quantum Hall effect via holography [54]. In fact, the ABJM theory has been used to study various interesting examples of $AdS_4/CFT_3$ correspondence [55]-[59]. It will be interesting to analyse similar effects in the harmonic superspace in gaugeon formalism.

It may be noted that $\mathcal{N} = 1$ supersymmetric Chern-Simons theory super-symmetry coupled to parity-preserving matter fields has also been analysed using the Parkes-Siegel formulation [60]. In the Parkes-Siegel formulation the dimensional reduction from $(2 + 2)$ to $(2 + 1)$ of massive Abelian $\mathcal{N} = 1$ super-$QED$ coupled to a self-dual super-multiplet produces simple supersymmetric extension of the $\tau_3 QED$ coupled to Chern-Simons theory. It will be interesting to derive similar result in the harmonic superspace.

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