Trainspotting: Extraction and Analysis of Traffic and Topologies of Transportation Networks

Maciej Kurant, Patrick Thiran
(Dated: March 31, 2022)

The knowledge of real-life traffic pattern is crucial for good understanding and analysis of transportation systems. This data is quite rare. In this paper we propose an algorithm for extracting both the real physical topology and the network of traffic flows from timetables of public mass transportation systems. We apply this algorithm to timetables of three large transportation networks. This makes us to make a systematic comparison between three different approaches to construct a graph representation of a transportation network; the resulting graphs are fundamentally different. We also find that the real-life traffic pattern is very heterogenous, both in space and traffic flow intensities.

PACS numbers: 89.75.Hc, 89.75.Fb, 89.40.Bb

I. INTRODUCTION

In the recent years, studies of transportation networks have drawn a substantial amount of attention in the physics community. The graphs derived from the physical infrastructure of such networks were analyzed on the examples of a power grid [1, 2], a railway network [3, 4], road networks [5, 6, 7, 8, 9], pipeline network [10] or urban mass transportation systems [11, 12, 13, 14]. These studies, in turn, neglect the underlying physical structure of the network and the traffic flows in it, providing us with the available connections and their times. Therefore the most important nodes and edges from a topological point of view might not necessarily carry the most traffic. In [21] we show that in typical transportation networks the correlation between the real load and the betweenness is very low. Therefore it is essential for some applications to know the real traffic pattern. Consequently, the networks of traffic flows were studied separately, see the example of flows of people within a city [22] and commuting traffic flows between different cities [23]. These studies, in turn, neglect the underlying physical topology, making the analysis incomplete. For instance, it is impossible to detect the most loaded physical edges, which might have a crucial meaning for the resilience of the system. A comprehensive view of the system often requires to analyze both layers (physical and traffic) together.

Unfortunately, the data sets including both physical topology and traffic flows are rather sparse, and difficult to get. In this paper we propose an approach to extract the physical structure and the network of traffic flows from timetables. Timetables of trains, buses, trams, metros and other means of mass transportation (henceforth called vehicles) are publicly available. They provide us with the available connections and their times. Timetables also contain the information about the physical structure of the network and the traffic flows in it, but, as we show later, they often require a nontrivial pre-processing to be revealed.

II. SPACES AND THE DIFFICULTY OF THE PROBLEM

In order to position our contribution in the range of works in the field, we begin with a systematic definition of the topology of transportation systems. The set of stations is defined by the set of all stations (train stations, bus stops, etc). It is not obvious, however, what should be interpreted as an edge. Its choice depends on what we want to be reflected by the topology of the physical graph. In the literature there are essentially three approaches that define three different ‘spaces’: here we call them ‘space–of–changes’, ‘space–of–stops’ and ‘space–of–stations’.

In space–of–changes, two stations are considered to be connected by a link when there is at least one vehicle that stops at both stations. In other words, all stations used by a single vehicle are fully interconnected and form a clique. This approach neglects the physical distance between the stations. Instead, in the resulting topology, the length of a shortest path between two arbitrary stations $A$ and $B$ is the number of changes of mean of transportation one needs to get from $A$ to $B$ [34]. This approach was used in [34]; in the latter the authors used the term space $P$.

In space–of–stops, two stations are connected if they are two consecutive stops on a route of at least one vehicle [12]. Here the length of a shortest path between two stations is the minimal number of stops one needs to make. Note that the number of stations traversed on the way might be larger, because the vehicles do not necessary stop on all of them.
FIG. 1: (Color online) An illustration of the transportation network topology in three spaces. (a) The routes of three vehicles. The route of Line 2 passes through node C on the way from B to D, but the vehicle does not stop there. (b) The topology in space–of–changes. Each route results in a clique. An edge is indicated by two colors, when it originates from two routes, but is merged into a single link. (c) The topology in space–of–stops. The “shortcut” B-D is a legitimate edge in this space. (d) The topology in space–of–stations. This graph reflects the topology of the real-life infrastructure.

In space–of–stations, two stations are connected only if they are physically directly connected (with no station in between). This reflects the topology of the real-life infrastructure. Here, the length of a shortest path between two stations is the minimal number of stations one has to traverse (stopping or not). This approach was used in \textsuperscript{4, 10, 11, 14}.

In Fig. 1 we give an illustration of the three spaces. It is easy to see that the graph in space–of–stations is a subgraph of the graph in space–of–stops, which in turn is a subgraph of the graph in space–of–changes.

The topologies in space–of–changes and space–of–stops can be directly obtained from timetables. In space–of–changes, for each vehicle, we fully connect all stations it stops at. Then we simplify the resulting graph by deleting multi-edges. In space–of–stops, we connect every two consecutive stops in routes of vehicles. As shown in Fig. 1, the topology in space–of–stops can have shortcut links that do not exist in the real-life infrastructure. These shortcuts should be eliminated in the space–of–stations topology, which makes it more challenging to obtain. To the best of our knowledge, the only work on extracting the real physical structure (the topology in space–of–stations) from timetables was done in the PhD dissertation of Annegret Lebers \textsuperscript{24}. The proposed solution first obtains the physical graph in space–of–stations. Next, specific structures in the initial physical graph, called edge bundles, are detected. The Hamilton paths within these bundles should indicate the real (non-shortcut) edges. Unfortunately, the bundle recognition problem turned out to be NP-complete. The heuristics proposed in \textsuperscript{24} result in a correct real/shortcut classification of 80\% of edges in the studied graphs. The approach we propose in this paper is based on simple observations that were omitted in \textsuperscript{24}. This results in a much simpler and more effective algorithm.

III. RELATED WORK

Timetables have been used as a data source for a network construction in \textsuperscript{3, 13}. However, the topologies obtained in these works were either in space–of–changes or in space–of–stops; neither of them reflected the real-life infrastructure. Moreover, the real traffic patterns were not considered in these studies. This is understandable, because it is difficult to interpret a traffic flow in spaces of changes and stops. Does the “traffic” on a shortcut link have any physical meaning? We know that this traffic actually traverses other non-shortcut links that exist in reality. In contrast, in space–of–stations, the traffic flows have clear, unambiguous and natural interpretation.

Another class of networks that can be constructed with the help of timetables are airport networks \textsuperscript{6, 25, 26, 27}. There, the nodes are the airports, and edges are the flight connections. The weight of an edge reflects the traffic on this connection, which can be approximated by the number of flights that use it during one week. In this case, both the topology and the traffic information are explicitly given by timetables. This is because the routes of planes are not constrained to any physical infrastructure, as opposed to roads for cars or rail-tracks for trains. So there are no “real” links and “shortcut” links. In a sense all links are real, and the topologies in space–of–stops and in space–of–stations actually coincide.

Inferring the space–of–stations topology from timetables becomes simple also in another special case, where the vehicles stop at each station they traverse (e.g.,
in many subway networks). This naturally eliminates the shortcuts, making the topologies in space–of–stops and stations identical. This is not true in a general case, with both local and express vehicles.

In the reminder of this paper, we introduce necessary notation in Section IV. Next, in Section V, we give an algorithm that extracts the real physical structure (a topology in space–of–stations) and the network of traffic flows from timetables. In Section VI, we test our algorithm on timetables of three large transportation networks at three different scales: city, country, and continent. We also analyze the resulting physical topologies and compare them with those obtained by alternative approaches. Finally, in Section VII, we conclude the paper.

IV. NOTATION

A. Two layers

We follow the two-layer framework introduced in [21]. The lower-layer topology is called a physical graph \( G^\phi = (V^\phi, E^\phi) \), and the upper-layer topology is called a logical graph \( G^\lambda = (V^\lambda, E^\lambda) \). We assume that the sets of nodes at both layers are identical, i.e., \( V^\phi = V^\lambda \), but as a general rule, we keep the indexes \( \phi \) and \( \lambda \) to make the description unambiguous. Let \( N = |V^\phi| = |V^\lambda| \) be the number of nodes. Every logical edge \( e^\lambda = (u^\lambda, v^\lambda) \) is mapped on the physical graph as a path \( M(e^\lambda) \subset G^\phi \) connecting the nodes \( u^\phi \) and \( v^\phi \), corresponding to \( u^\lambda \) and \( v^\lambda \). (A path is defined by the sequence of nodes it traverses.) The set of paths corresponding to all logical edges is called a mapping \( M(E^\lambda) \) of the logical topology on the physical topology.

In the field of transportation networks the undirected, unweighted physical graph \( G^\phi \) captures the topology of the physical infrastructure (i.e., in space–of–stations), and the weighted logical graph \( G^\lambda \) reflects the undirected traffic flows. Every logical edge \( e^\lambda \) is created by connecting the first and the last node of the corresponding traffic flow, and by assigning a weight \( w(e^\lambda) \) that represents the intensity of this flow. The mapping \( M(e^\lambda) \) of the edge \( e^\lambda \) is the path taken by this flow.

B. Timetable data

We take a list of all vehicles departing in the system within some period (e.g., one weekday). Denote by \( R = \{r_i\}_{i=1..|R|} \) the list of routes followed by these vehicles, where \(|R|\) is the total number of vehicles. A route \( r_i \) of \( i \)th vehicle is defined by the list of nodes it traverses. Note that since there are usually more vehicles (than one) following the same path on one day, some of the routes may be identical.

V. ALGORITHM

The algorithm has three phases. In the first one, initialization, based on the set of routes \( R \), we create the set of nodes \( V^\phi = V^\lambda \) and the physical topology \( G^\phi_{\text{stop}} = (V^\phi, E^\phi_{\text{stop}}) \) in space–of–stops. In the second, main phase, the sets \( R \) and \( E^\phi_{\text{stop}} \) are iteratively refined by detecting and erasing the shortcut links in the physical graph \( G^\phi_{\text{stop}} \), resulting in the physical topology \( G^\phi_{\text{stat}} = (V^\phi, E^\phi_{\text{stat}}) \) in space–of–stations. Finally, in the third phase, we group the vehicles with identical routes, and obtain the logical graph \( G^\lambda \) and the mapping \( M(E^\lambda) \) of the logical edges on the physical graph \( G^\phi_{\text{stat}} \). We describe below each phase separately.

A. Phase 1 - initialization

In this phase we interpret every two consecutive nodes in any route \( r_i \in R \) as directly connected. Consequently, we connect these nodes with a link, which can be written as

\[
E^\phi_{\text{stop}} = \bigcup_{i=1..|R|} E(r_i)
\]

where \( E(r_i) \) is the set of all pairs of adjacent nodes in \( r_i \) (i.e., all edges in \( r_i \)). This results in the physical topology \( G^\phi_{\text{stop}} = (V^\phi, E^\phi_{\text{stop}}) \) in space–of–stops.

B. Phase 2 - deleting shortcuts

In this phase, at each iteration, we detect a shortcut in the set of physical edges, delete it, and update all routes \( r_i \) that use this shortcut. Denote by \( e^\phi_{(1)}, e^\phi_{(2)} \) the two end-nodes of \( e^\phi \), and by \( \text{Rev}(P_{e^\phi}) \) the reversed version of \( P_{e^\phi} \) (the sequence from the last node to the first one). The algorithm is as follows:

1. \( E^\phi_{\text{stat}} := E^\phi_{\text{stop}} \)
2. Find a tuple \((e^\phi, r_i)\) such that \( e^\phi \) is a shortcut for \( r_i \) and \( e^\phi_{(1)} \in r_i \) and \( e^\phi_{(2)} \in r_i \) and \( e^\phi \notin E(r_i) \).
3. IF no \((e^\phi, r_i)\) found THEN RETURN \( E^\phi_{\text{stat}} \) and \( R \).
4. \( P_{e^\phi} := \text{subpath of } r_i \text{ from } e^\phi_{(1)} \text{ to } e^\phi_{(2)} \)
5. FOR all \( r_j \in R \) DO:
   - IF \((e^\phi_{(1)}, e^\phi_{(2)}) \in r_j \) THEN replace it with \( P_{e^\phi} \)
   - IF \((e^\phi_{(2)}, e^\phi_{(1)}) \in r_j \) THEN replace it with \( \text{Rev}(P_{e^\phi}) \)
6. \( E^\phi_{\text{stat}} := E^\phi_{\text{stat}} \setminus \{e^\phi\} \)
7. GOTO 2
In Step 2, we look for a physical link that is a shortcut. We declare a physical link $e^\phi$ to be a shortcut, if there exists a route $r_i \in R$, such that $e^\phi$ connects two nonconsecutive nodes in $r_i$. For example, in Fig. 4, $e^\phi = \{B, D\}$ is a shortcut because it connects two not neighboring nodes in the route $r_1$ of Line 1. If no physical edge can be declared a shortcut, the algorithm quits in Step 3, returning $E^\phi_{stat}$ and $R$. Otherwise, in Step 4, we find the path $P_{e^\phi}$ that this shortcut should take. In Fig. 3, this path is $P_{e^\phi} = (B, C, D)$. In Step 5, we update the set of routes $R$ by replacing every shortcut link $e^\phi$ in every route using it with the corresponding path $P_{e^\phi}$. In our example, the updated route of Line 2 becomes $r_2 = (A, B, C, D, E)$. It is thus identical to the route of Line 1. Finally, in Step 6 we delete the shortcut $e^\phi$ from the physical graph. We iterate these steps until no shortcut is found (Step 2). The resulting physical graph $G^\phi_{stat} = (V^\phi, E^\phi_{stat}) \subset G^\phi_{stop}$, is a graph in space–of–stations.

C. Phase 3 - grouping the same routes together

Finally, based on the list $R$ of routes updated in phase 2, we find groups of vehicles that follow the same path (in any direction). Each such group defines one edge $e^\lambda$ in the logical graph; $e^\lambda$ connects the first and the last node of the route. The number of vehicles that follow this route becomes the weight $w(e^\lambda)$ of the logical edge $e^\lambda$; the route itself becomes the mapping $M(e^\lambda)$ of $e^\lambda$ on the physical graph.

Denote by $r_{i(first)}$, $r_{i(last)}$, the first and the last nodes in $r_i$, and by $E(M(e^\lambda))$ the set of all physical edges in the mapping of $e^\lambda$. Now, Phase 3 can be stated as follows:

1. $E^\lambda = \emptyset$, $M = \emptyset$
2. FOR $i = 1$ TO $|R|$ DO:
   • $e^\lambda_i = \{r_{i(first)}, r_{i(last)}\}$
   • IF $e^\lambda_i \in E^\lambda$ THEN $w(e^\lambda_i) := w(e^\lambda_i) + 1$  
     ELSE $E^\lambda = E^\lambda \cup \{e^\lambda_i\}$, $M(e^\lambda_i) = r_i$, $w(e^\lambda_i) = 1$
3. $E^\phi_{stat} = \bigcup_{e^\lambda \in E^\lambda} E(M(e^\lambda))$

In the example in Fig. 4 after phase 2 the routes of Line 1 and Line 2 become identical; therefore in phase 3 they are grouped together defining a logical edge $e^\lambda = \{A, E\}$ with the weight $w(e^\lambda) = 2$ and the mapping $M(e^\lambda) = (A, B, C, D, E)$. A second logical edge is $e^\lambda = \{F, H\}$ with $w(e^\lambda) = 1$ and $M(e^\lambda) = (F, B, G, H)$.

D. Accuracy of the algorithm

There are potential sources of mistakes and inaccuracies in our approach. First, the links that we delete as being shortcuts, might actually exist in reality. However, a comparison of the results of our algorithm with the real maps (see Section VI) reveals very few differences, which means that this source of failures occurs very rarely in real data sets.

VI. A STUDY OF THREE REAL-LIFE NETWORKS

In this section we apply our algorithm to extract the data from the timetables of three examples of transportation networks, with sizes ranging from city to continent. As an example of a city, we take the mass transportation system (buses, trams and metros) of Warsaw (WA), Poland; its timetables are available at [28]. At a country level, we study the railway network of Switzerland (CH). Finally, we investigate the railway network formed by major trains and stations in most countries of central Europe (EU) [11]. The timetables of both CH and EU networks are available at [29]. The basic parameters of the data sets and of the resulting graphs can be found in Table I.

This section is organized as follows. First, we focus on a particular data set in order to study the performance of our algorithm. Next, we analyze and compare the physical graphs originating from all three data sets in each of the considered spaces. Finally, we focus our attention on the logical graphs and traffic flows extracted by our algorithm.

A. An example: The railway network of Switzerland (CH)

As an illustration, let us consider more closely the railway network of Switzerland (CH). According to our timetable, on a typical weekday there are $|R| = 6957$ different trains that follow $|E^\lambda| = 505$ different routes (usually there is more than one train following the same route during one day). Our data contains $N = 1613$ stations in Switzerland, together with their physical coordinates. In Fig. 5 we present the graphs obtained from this data set. The physical graphs in the three spaces are shown in Figs. 6abc. The graph in space–of–stations was obtained with the help of the algorithm introduced.
FIG. 2: The railway network in Switzerland (CH). (a,b,c) Physical graphs in space–of–changes, stops and stations, respectively. (d) The real map of the rail tracks in Switzerland. (e) The logical graph. Every edge connects the first and the last station of a particular train route; its weight reflects the number of trains following this route in any direction.
Traffic stops 8 50.9

Physical graph

| Dataset        | Area [km²] | N    | | | | | | | | |
|----------------|------------|------|---|---|---|---|---|---|---|---|
| WA (Warsaw)    | 480        | 1533 | 25,995 | 221 | changes 78437 | 102.3 | 4 | 2.3 | 0.6829 |
| CH (Switzerland) | 41'300   | 1613 | 6'957 | 505 | changes 19827 | 24.6 | 8 | 3.6 | 0.9095 |
| EU (Europe)    | 2'081’000 | 4853 | 60'775 | 6703 | changes 88329 | 36.4 | 8 | 3.7 | 0.7347 |

TABLE I: The studied datasets. “Area” is the surface occupied by the region covered by the network. N is the number of nodes (stations/stops). | | is the total number of vehicles departing in the network during one weekday. | E | is the number of edges in the logical graph (number traffic flows); it is much smaller than | R | , because the vehicles following the same route are grouped together in phase 3 of our algorithm. All the remaining parameters are computed for the physical graphs | G | : | E | is the number of edges, | k | is the average node degree, | d | stands for the diameter, | l | is the average shortest path length, and | c | is the clustering coefficient.

in the previous section. The number of vertices is the same in all three spaces. The number of edges in space–of–changes, | E | = 19827, is much larger than in the other two spaces. Although at first sight the physical graphs in space–of–stations and in space–of–stops look comparable, the latter has a number of (nonexisting in reality) shortcut links. For a visual verification of correctness of our algorithm, we show in Fig. 2d the real map of the Swiss railway system; we observe only minor differences between (c) and (d). Finally, in Fig. 2d, we present the logical graph that reflects the traffic flows in the network. This graph is very heterogenous both in the weights of edges and in the layout of traffic.

B. The physical graph in three spaces

How does the choice of space affect the topology? We study in this section the physical graphs in the three spaces with respect to the basic metrics often used in the analysis of complex networks.

1. Diameter | d | , and average shortest path length | l | |

The average shortest path length | l | is computed over the lengths of shortest paths between all pairs of vertices. The diameter | d | is the longest of all shortest path lengths. These parameters are usually closely related.

The diameters and average shortest path lengths of the graphs in space–of–stations are large, and scale roughly as \( \sqrt{N} \) with the number of nodes | N | . This is typical of many planar, lattice-like infrastructure networks embedded in a two dimensional space.

The graphs in space–of–stops have about 10 – 15% more edges than their counterparts in space–of–stations. The difference is not large, and one could possibly expect similar values of the diameter and the average shortest path length. However, these 10 – 15% edges are fundamentally different from typical edges in space–of–stations; they are shortcut links. It was shown in | 30 | that the diameter of a graph is very sensitive to the existence of shortcuts. Even a relatively small number of shortcuts can dramatically bring down the diameter and the average shortest path length. We observe this phenomenon in our graphs. For instance, in the EU data set, the diameter drops about four times, from | d | = 184 in space–of–stations to 48 in space–of–stops. Similarly, the average shortest path length drops by roughly the same factor. Therefore, the shortcut edges, although not very numerous, play a very important role and make the graphs in space–of–stations very different from those in space–of–stations.

This effect is not so strongly pronounced in the WA data set. The underlying reason is the relatively short length of shortcuts (usually 2 hops), which was shown to affect the diameter only to a small extent | 31 | .

Finally, the graphs in space–of–changes have very small diameters and average shortest path lengths. This is mainly because of their high density (number of edges).

2. Node degree | k | |

The node degree distributions in all three spaces are plotted in a semi-logarithmic scale in Fig. 3abc. Additionally, for space–of–stops, we plot the degree distributions in a log-log scale (Fig. 3b), because it is not obvious which fit is better, exponential or power law (it was also pointed out in | 13 | ). For the other two spaces we observe a clear linear trend indicating the exponential behavior. This was expected in space–of–stations, because the degree distribution of many infrastructure networks was shown to be narrow (here one decade) and to decay exponentially (see e.g., power lines in | 32 | ). In space–of–stations the vast majority of nodes have degree equal to two, indicating long segments of stations without junctions.
We have studied the clustering coefficients $c$

The clustering coefficient of topologies in space–of–changes and stops is much higher than in space–of–stations. As in the case of the graph diameter, here again the shortcut links turn out to be very important in the topology.

### C. Traffic flows and the logical graph

Now we turn our attention to the traffic that flows in our networks. We extracted this scarce data with the help of the algorithm introduced in this paper. As we argued before, the interpretation of traffic flowing through networks in space–of–changes and stops is rather cumbersome. Therefore we restrict our analysis to the traffic flows traversing the physical graph in space–of–stations.

In Fig. 4 we compare the lengths of traffic flows before and after application of our algorithm. A new traffic flow can be either equal in length to the original one (if no shortcut was detected on its path), or longer. We observe that for all three data sets, there is a significant number of flows that become longer. In some cases this increase in length is by as much as 10 times. Generally, the longer the original flow is, the less extended it gets during a run of our algorithm. This is expected, because a long flow in a timetable usually corresponds to a local train that stops at all stations (i.e., uses no shortcuts).

In Fig. 5 we present basic distributions measured for logical graphs in the three data sets. Recall that the edges in a logical graph reflect the traffic flows. Therefore, the node degree $k^\lambda$ is the number of different connections starting/ending at the corresponding station (Fig. 5a). The strength $s^\lambda$ of a node is the sum of the weights of neighboring edges [25]; here it is the number of all connections starting/ending at this station (Fig. 5b). Finally, the weight $w(e^\lambda)$ of a logical edge is the traffic flow intensity (Fig. 5c).

All three distributions are heavily right-skewed meaning that there is a small number of nodes/edges with very high values of the observed parameter. We conclude that the real-life traffic patterns are very heterogenous, both in space (node degree and strength) and traffic flow intensities. This was shown in [21] to be the reason of high unpredictability of load distribution in transportation networks.

### VII. CONCLUSIONS

The knowledge of real-life traffic pattern is crucial in the analysis of transportation systems. This data is usually much more difficult to get than the pure topology of a network. In this paper we have proposed an algorithm for extracting both the physical topology and the network of traffic flows from timetables of public mass transportation systems. We have applied our algorithm...
FIG. 5: Properties of logical graphs. (a) Node degree distribution. Many nodes are isolated - they represent intermediate stations on which no train starts or terminates its journey. The isolated nodes we represent here as having “degree” equal to 0.1. (b) Node strength distribution. (c) Edge weight (traffic flow intensities) distribution. All data are log-binned and plotted in a log-log scale.

to three large transportation networks. This enabled us to make a systematic comparison between three different approaches (or “spaces”) to construct a graph representation of a transportation network. The resulting physical topologies are very different. In particular, the seemingly similar graphs in space-of-stops and in space-of-stations, turn out to be very different in terms of basic graph-theory metrics such as diameter, average shortest path length, clustering coefficient and node degree distribution. This is due to the existence of shortcut links in space-of-stops. Our algorithm detects and eliminates these shortcuts, and extracts the topology in space-of-stations. Only this graph reflects the real-life physical infrastructure that is used by the traffic flows, gets congested or can be prone to failures or susceptible to attacks. In contrast, the edges in space-of-changes and in space-of-stations are somewhat “virtual,” and the notion of traffic in these graphs is unclear, if at all makes any sense. What is important, the results are consistent across three different scales of the studied networks (city, country, continent).

This work has several possible directions for the future. For instance, the knowledge of real traffic pattern allows us to revisit the error and attack tolerance of transportation systems, which might look completely different when focussing on traffic instead of on topology. Another direction would be to exploit additional information available in some timetables. For instance, in our data sets CH and EU, we also know the geographical coordinates of the nodes. They fall therefore in the category of spatial networks that have been recently intensively studied [4, 6, 9, 34, 35, 36]. In particular, we think that incorporating the real traffic pattern in the models can help understanding the processes that govern the evolution of spatial networks.

Finally, we note that the data will be soon available at 37.

The work presented in this paper was financially supported by grant DICS 1830 of the Hasler Foundation, Bern, Switzerland.

[1] D. J. Watts and S. H. Strogatz. Collective dynamics of “small-world” networks. Nature, 393:440–442, 1998.
[2] Reka Albert, Istvan Albert, and Gary L. Nakarado. Structural vulnerability of the north american power grid. Phys. Rev. E, 69:025103(R), 2004.
[3] Parongama Sen, Subinay Dasgupta, Arnab Chatterjee, P. A. Sreram, G. Mukherjee, and S. S. Manna. Small-world properties of the Indian railway network. Phys. Rev. E, 67:036106, 2003.
[4] Michael T. Gastner and M. E. J. Newman. Shape and efficiency in spatial distribution networks. J. Stat. Mech., (P01015), January 2006.
[5] Sergio Porta, Paolo Crucitti, and Vito Latora. The network analysis of urban streets: A dual approach. cond-mat/0411247, 2004.
[6] Michael T. Gastner and M. E. J. Newman. The spatial structure of networks. Eur. Phys. J. B, 49:247–252, 2006.
[7] M. Rosvall, A. Trusina, P. Minnhagen, and K. Sneppen. Networks and cities: An information perspective. Phys. Rev. Lett., 94:028701, 2005.
[8] Sergio Porta, Paolo Crucitti, and Vito Latora. The network analysis of urban streets: A primal approach. physics/0506009, 2005.
[9] Alessio Cardillo, Salvatore Scellato, Vito Latora, and Sergio Porta. Structural properties of planar graphs of urban street patterns. physics/0510162, 2005.
[10] V. Latora and M. Marchiori. Efficient behavior of small-world networks. Phys. Rev. Lett., 87:198701, 2001.
[11] V. Latora and M. Marchiori. Is the boston subway a small-world network? Physica A, 314:109, 2002.
[12] Katherine A. Seaton and Lisa M. Hackett. Stations, trains and small-world networks. Physica A, 339:635, 2004.
[13] J. Sienkiewicz and J. A. Holyst. Statistical analysis of
[10] Hamilton path is a path that passes through every vertex of a graph exactly once.

[40] In this sense, a graph in space–of–changes is closely related to the dual interpretation of urban road networks [3, 4, 38], where streets (of a given name) map to nodes, and intersections between streets map to links between the nodes. In a transportation network in space–of–changes, the length of a shortest path is the number of changes of mean of transportation, whereas the length of a shortest path in a dual graph of a city is the number of changes of streets on the way from the starting point to destination.

[41] In the EU data set, Paris has originally several station s that are not directly connected between each other. Following the approach in [3], we merged them into one common node.