Low energy kaon-hyperon interaction

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I. INTRODUCTION

Until today a subject that is very interesting and remains not very well studied is the low energy hyperon interactions. Despite the fact that experimental data for many hyperon processes are not available (as for example the $K\Lambda$ and $K\Omega$ interactions) and that by the theoretical side they are not fully described, this kind of interaction is a fundamental element for several physical systems of interest.

In the study of the hypernuclei structure [11]-[13], the knowledge of the nucleon-hyperon and hyperon-hyperon interactions is an essential aspect. In order to understand these interactions, and to determine the potentials of interest, an accurate understanding of the meson-hyperon interactions is needed.

Another system where hyperon interactions are required is in the study of the hyperon stars. After the proposal of the hypothesis where hyperons could be produced inside neutron stars at high densities, many models have been proposed, as for example in [5]-[8], and the effect of the presence of hyperons in the equations of state, and consequently in the determination of the star masses have been studied. The indeterminations in the nucleon-hyperon and hyperon-hyperon potentials cause difficulties in the understanding of these stars.

In high energy physics this kind of interaction is very important also. When studying hyperon polarization, produced in proton-nucleus and nucleus-nucleus collisions [9]-[18], in [19]-[21] the final interactions of the hyperons and antihyperons with the produced pions is a central ingredient in order to explain the final polarizations. As it has been shown, the effect of the hyperon interactions with the surrounding hot medium, composed predominantly of pions, is very important. The observed differences between the polarizations of hyperons and antihyperons are very difficult to be explained in another way. The effect of the final kaon-hyperon interactions has not been considered yet, and it may cause corrections in the final polarization. For this reason, this work is very important and this effect must be investigated. Recent results from RHIC [22] and even the hyperons produced in the LHC may be studied in a similar form, and in order to obtain accurate results these interactions must be considered.

For these reasons, this work will be devoted to the study of the kaon-hyperon ($KY$) and antikaon-hyperon ($K\bar{Y}$) interactions. This work may be considered as a continuation of the study proposed in [23]-[25], where the pion-hyperon interactions have been described with a model based in effective chiral Lagrangians where the exchange of mesons and baryons has been taken into account. In this model [23], the resonances dominate many channels of the reactions as it may be seen in the results. This behavior may be considered as an experimental feature, fact that is similar to what happens in the low energy pion-nucleon interactions, where the isospin $3/2$ and spin $3/2$ channel is dominated by the $\Delta^{++}$ resonance. Comparison with the data from the HyperCP experiment [26], [27] shows a very good accord with the results obtained for the $\pi\Lambda$ scattering in [24]. So, the work that will be shown in this paper is based on the ideas presented in this model.

This paper will present the following content: in Sec. II, the basic formalism will be shown. In Sec. III, the kaon-lambda ($K\Lambda$) interactions will be studied, in Sec. IV, the antikaon-lambda ($K\bar{\Lambda}$), and in Sec. V, the kaon-sigma will be shown. In Sec. VI we will present the antikaon-sigma interactions ($K\bar{\Sigma}$) and finally, the discussions and conclusions in Sec. VII. In the Appendix, some expressions of interest will be presented.

II. THE METHOD

In order to study the $KY$ and $K\bar{Y}$ interactions, we will use a model proposed with the purpose of studying the low energy pion-hyperon interactions [23]-[25], that is based in an analogy with models successfully used to describe the $\pi N$ interactions considering chiral effective Lagrangians. These interactions are very well studied, as for example in [28]-[31], both theoretically, where many models have been proposed, and experimentally, with a large amount of experimental data available. A basic characteristic of this system is the dominance of reso-
nances in the scattering amplitudes at low energies. The \( \Delta^{++} \), for example, dominates the cross section of the \( \pi^+p \) scattering at low energies. As this particle has spin 3/2 and isospin 3/2, it may be introduced in the theory by considering a Lagrangian in the form of eq. (2). In the study of the pion-hyperon interactions [23], a similar behavior has been observed, so we expect that in the KY interactions it also occurs.

In this section we will present the basic formalism that will be used to study the kaon-hyperon interactions (that is the same one worked out in the study the pion-hyperon interactions [24]) and how the observables may be obtained. In the method that will be followed in this work, some important characteristics of the interacting particles will be implemented, the spin, the isospin, and the masses of each one of them. These characteristics determine which Lagrangian have to be used in order to build the model.

For example, in [28], the Lagrangians considered to study the \( \pi N \) scattering are given by

\[
\mathcal{L}_{\pi NN} = \frac{g}{2m} (\gamma_\mu \gamma_5 \vec{r} N) \cdot \partial^\mu \vec{\phi} ,
\]

(1)

\[
\mathcal{L}_{\pi N\Delta} = g_\Delta \left\{ \mathcal{N} \left[ g_{\mu\nu} - \left( Z + \frac{1}{2} \gamma_\mu \gamma_\nu \right) \hat{M} \mathcal{N} \right] \cdot \partial^\nu \vec{\phi} \right\} ,
\]

(2)

\[
\mathcal{L}_{\pi N\pi} = \frac{g_0}{2} \left\{ \mathcal{N} \left[ \gamma_\mu \vec{r} N \right] \cdot \partial^\nu \vec{\phi} + \frac{g_0}{2} \left\{ \mathcal{N} \left( \frac{\mu_n - \mu_\pi}{4m} \right) i \sigma_{\mu\nu} \vec{r} N \right\} \times \left( \partial^\nu \vec{\phi} - \partial^\nu \vec{\phi} \right) \right\} ,
\]

(3)

where \( N, \Delta, \vec{\phi}, \vec{\phi} \) are the nucleon, delta, pion, and rho fields with masses \( m, m_\Delta, m_\pi, \) and \( m_\rho, \) respectively, \( \mu_\rho \) and \( \mu_\pi \) are the proton and neutron magnetic moments \( \mathcal{M}, \hat{M} \) and \( \vec{r} \) are the isospin recombination matrices, and \( Z \) is a parameter representing the magnetic possibility of the off-shell-\( \Delta \) having spin 1/2. The parameters \( g, g_\Delta \) and \( g_0 \) are the coupling constants. In [23] similar Lagrangians have been used to study the pion-hyperon interactions, and in this work the same procedure will be adopted. So, in the following sections these Lagrangians will be adapted to the kaon-hyperon systems.

Calculating the diagrams, considering the interactions described by the Lagrangians above for an arbitrary process, the scattering amplitudes may be written in the form

\[
T^{\beta\alpha}_{\pi N} = \sum_I T^I \langle \beta | P_I | \alpha \rangle = \sum_I T^I P_I^{\beta\alpha} ,
\]

(5)

that is a sum over all the \( I \) isospin states where \( P_I \) is a projection operator, the indices \( \alpha \) and \( \beta \) are relative to the initial and final isospin states of the \( \pi, \) and \( T^I \) is an amplitude for a given isospin state that may be written as

\[
T^I = \pi(\vec{p}) \left[ A^I + \frac{1}{2} (k + k') B^I \right] u(\vec{p}) ,
\]

(6)

where \( u(\vec{p}) \) is a spinor representing the initial baryon, incoming with four-momentum \( p_\mu. \) The final baryon has a spinor \( \pi(\vec{p}'), \) four-momentum \( p'_\mu, \) and \( k_\mu \) and \( k'_\mu \) are the incoming and outgoing meson four-momenta. The amplitudes \( A^I \) and \( B^I \) are calculated from the diagrams. So, if these amplitudes are determined, the \( T^I \) amplitudes may be obtained and then we will be able to compute the observables of interest.

The scattering matrix for an isospin state is given by the expression

\[
M^I = \frac{T^I}{8\pi\sqrt{s}} ,
\]

(7)

which may be decomposed into the spin-non-flip and spin-flip amplitudes \( f^I(k,\theta) \) and \( g^I(k,\theta), \) defined in terms of the momentum \( k = |\vec{k}| \) and \( x = \cos \theta, \) \( \theta \) the scattering angle, as

\[
M^I = f^I(k, x) + g^I(k, x) \hat{\sigma} \cdot \hat{n} ,
\]

(8)

where \( \hat{n} \) is a vector normal to the scattering plane, and may be expanded in terms of the partial-wave amplitudes \( a_{l\pm} \) with

\[
f^I(k, x) = \sum_{l=0}^{\infty} \left( (l + 1) a_{l+} \right) (k) + l a_{l\pm}(k) \right) P_l(x) ,
\]

(9)

\[
g^I(k, x) = i \sum_{l=1}^{\infty} \left[ a_{l-} - a_{l+} \right] P_l^{(1)}(x) .
\]

(10)

These amplitudes may be calculated using the Legendre polynomials orthogonality relations

\[
a_{l\pm}(k) = \frac{1}{2} \int_{-1}^{1} \left[ P_l(x) f^I_1(k, x) + P_{l\pm}(x) f^I_2(k, x) \right] dx ,
\]

(11)

with

\[
f^I_1(k, x) = \frac{(E + m)}{8\pi\sqrt{s}} \left[ A^I + (\sqrt{s} - m) B^I \right] ,
\]

(12)

\[
f^I_2(k, x) = \frac{(E - m)}{8\pi\sqrt{s}} \left[ -A^I + (\sqrt{s} + m) B^I \right] ,
\]

(13)

where \( E \) is the baryon energy in the center-of-mass frame and \( \sqrt{s} \) is given by a Mandelstam variable (see the Appendix). At low energies the \( S \) \( (l=0) \) and \( P \) \( (l=1) \) waves dominate the scattering amplitudes, and for higher values of \( l \) the amplitudes are much smaller (almost negligible), so they may be considered as small corrections.

Calculating the amplitudes at the tree-level, the results obtained will be real, and then violate the unitarity of the
S matrix. As it is usually done, we may reinterpret these results as elements of the K reaction matrix \[23\, 25\] and then obtain unitarized amplitudes

\[ a_{i\pm}^U = \frac{a_{i\pm}}{1 - ika_{i\pm}}. \] (14)

The differential cross sections may be calculated using the previous results

\[ \frac{d\sigma}{d\Omega} = |f|^2 + |g|^2, \] (15)

and integrating this expression over the solid angle we obtain the total cross sections

\[ \sigma_T = 4\pi \sum l \left[ (l + 1)|a_{i+}^U|^2 + l|a_{i-}^U|^2 \right]. \] (16)

The phase shifts are given by

\[ \delta_{l\pm} = \tan^{-1}(ka_{l\pm}), \] (17)

and finally, the polarization,

\[ \bar{\beta} = -2 \frac{Im(f^*g)}{|f|^2 + |g|^2} \hat{n}. \] (18)

An important task to achieve is to determine the coupling constant for each resonance that will be considered. We will adopt the same procedure considered in \[23\], comparing the amplitude obtained in the calculations with the relativistic Breit-Wigner expression, that is determined in terms of experimental quantities

\[ \delta_{l\pm} = \tan^{-1}\left[ \frac{\Gamma_0 \left( \frac{m}{m_r} \right)^{2J + 1}}{2(m_r - \sqrt{3})} \right], \] (19)

where \( \Gamma_0 \) is the width, \( k_0 = |\vec{k}_0| \) is the momentum at the peak of the resonance in the center-of-mass system, \( m_r \) is its mass and \( J \) the angular momentum, considering the data from \[29\]. We will consider the coupling constant that better fits this expression in each case.

In the following sections we will apply this formalism in the study of the reactions of interest.

### III. KAON-LAMDA INTERACTION

Since the \( \Lambda \) hyperon has isospin 0, the scattering amplitude for the \( K\Lambda \) interaction will have the form

\[ T_{K\Lambda} = \bar{u}(\vec{p}) \left[ A(k, \theta) + \left( \frac{\vec{k} + \vec{k}'}{2} \right) B(k, \theta) \right] u(\vec{p}), \] (20)

with the variables defined in section II. Comparing this expression with \[24\], we have a simple result, \( F_{1/2} = 1 \), as the kaon has isospin 1/2, and just one isospin amplitude.

In FIG. 1 we show the diagrams and the particles considered to formulate the \( K\Lambda \) interaction. The particles considered for each diagram are shown in Tab. I.

![FIG. 1. Diagrams for the \( K\Lambda \) interaction](image)

| \( J^* \) | \( I \) | Mass (MeV) |
|---|---|---|
| \( N \) | 1/2\(^+\) | 1/2 | 938 |
| \( N(1650) \) | 1/2\(^-\) | 1/2 | 1950 |
| \( N(1710) \) | 1/2\(^-\) | 1/2 | 1710 |
| \( \Xi^*(1875) \) | 3/2\(^-\) | 1/2 | 1875 |
| \( \Xi^*(1900) \) | 3/2\(^-\) | 1/2 | 1900 |
| \( \Xi^* (1820) \) | 3/2\(^-\) | 1/2 | 1820 |

TABLE I. Particles considered in the \( K\Lambda \) interaction

For the calculation of the contribution of particles with spin-1/2 (\( N \) and \( \Xi \)) in the intermediate state (FIG. 1a and c), the Lagrangian of interaction is (considering the necessary adaptations from eq. (1))

\[ \mathcal{L}_{K\Lambda B} = \frac{g_{K\Lambda B}}{2m_\Lambda} (\bar{B} \gamma_\mu \gamma_5 \Lambda) \partial^\mu \phi + \text{H.c.}, \] (21)

where \( \phi \) represents the kaon field, \( B \) the intermediate baryon field, with mass \( m_B \), and \( \Lambda \), the hyperon field, with mass \( m_\Lambda \).

Calculating the Feynman diagrams and comparing with eq. (20) we find the amplitudes for the \( N \) (spin-1/2) particles contribution

\[ A_N = \frac{g_{K\Lambda N}^2}{4m_\Lambda^2} \left( m_N + m_\Lambda \right) \left( \frac{s - m_\Lambda^2}{s - m_N^2} \right), \] (22)

\[ B_N = -\frac{g_{K\Lambda N}^2}{4m_\Lambda^2} \left[ \frac{2m_\Lambda(m_N + m_\Lambda) + s - m_\Lambda^2}{s - m_N^2} \right], \] (23)

and for the \( \Xi \) (spin-1/2) hyperon in the crossed diagram (FIG. 1c) the contribution is

\[ A_\Xi = \frac{g_{K\Lambda \Xi}^2}{4m_\Lambda^2} (m_\Xi + m_\Lambda) \left( \frac{u - m_\Lambda^2}{u - m_\Xi^2} \right), \] (24)

\[ B_\Xi = \frac{g_{K\Lambda \Xi}^2}{4m_\Lambda^2} \left[ \frac{2m_\Lambda(m_\Lambda + m_\Xi) + u - m_\Lambda^2}{u - m_\Xi^2} \right], \] (25)

where \( u \) is a Mandelstam variable, defined in the appendix and \( g_{K\Lambda N(\Xi)} \) are the coupling constants.
In a similar way, we adapted the interaction Lagrangian \( \mathcal{L}_{\Lambda K N^*} \) for the exchange of spin-3/2 resonances, shown in FIG. 1, and d

\[
\mathcal{L}_{\Lambda K N^*} = g_{\Lambda K N^*} \left\{ B^\mu [g_{\mu
u} - (Z + \frac{1}{2}) \gamma_\mu \gamma_\nu] A \right\} \partial^\nu \phi + \text{H.c.} \tag{26}
\]

Calculating the amplitude for the exchange of a spin-3/2 \( N^* \) (FIG. 1), we have

\[
A_{N^*} = \frac{g_{\Lambda K N^*}}{6} \left[ \frac{2A + 3(m_{N^*} + m_{N^*})t}{m_{N^*} - s} + a_0 \right],
\]

\[
B_{N^*} = \frac{g_{\Lambda K N^*}}{6} \left[ \frac{2B + 3t}{m_{N^*} - s} - b_0 \right],
\]

where

\[
\hat{A} = 3(m_{N^*} + m_{N^*})(q_{N^*})^2 \\
+ (m_{N^*} - m_{N^*})(E_{N^*} + m_{N^*})^2 \tag{29}
\]

\[
\hat{B} = 3(q_{N^*})^2 - (E_{N^*} + m_{N^*})^2 \tag{30}
\]

\[
a_0 = -\frac{(m_{N^*} + m_{N^*})}{m_{N^*}^2} \left( 2m_{N^*} + m_{N^*}m_{N^*} \right) \\
- \frac{1}{m_{N^*}^2} \left[ (m_{N^*} - m_{N^*})(m_{N^*} + m_{N^*})Z \\
+ (2m_{N^*} + m_{N^*})Z^2 \right] \left[ s - m_{N^*}^2 \right],
\]

\[
b_0 = \frac{8}{m_{N^*}^2} \left[ \left( m_{N^*}^2 + m_{N^*}m_{N^*} - m_{N^*}^2 \right) Z \\
+ (2m_{N^*}m_{N^*} + m_{N^*}^2)Z^2 \right] \left( m_{N^*} + m_{N^*} \right)^2 \\
+ \frac{4Z^2}{m_{N^*}^2} \left[ s - m_{N^*}^2 \right] .
\]

For the spin-3/2 \( \Xi^* \) resonance (FIG. 1), the amplitudes are

\[
A_{\Xi^*} = \frac{g_{\Lambda K \Xi^*}}{6} \left[ \frac{2A^\prime + 3(m_{\Xi^*} + m_{\Xi^*})t}{m_{\Xi^*}^2 - u} \\
+ c_0 + c_z(u - m_{\Xi^*}^2) \right],
\]

\[
B_{\Xi^*} = \frac{g_{\Lambda K \Xi^*}}{6} \left[ - \frac{2B^\prime + 3t}{m_{\Xi^*}^2 - u} \\
+ d_0 + d_z(u - m_{\Xi^*}^2) \right],
\]

where

\[
c_0 = -\frac{(m_{N^*} + m_{N^*})}{m_{\Xi^*}^2} \left( 2m_{\Xi^*}^2 + m_{N^*}m_{N^*} \right) \\
- m_{N^*}^2 + 2m_{N^*}^2 \right),
\]

\[
c_z = \frac{4}{m_{\Xi^*}^2} \left[ (m_{\Xi^*} + m_{N^*})Z \\
+ (2m_{\Xi^*} + m_{N^*})Z^2 \right] .
\]

\[
d_0 = \frac{8}{m_{\Xi^*}^2} \left[ (m_{\Lambda} + m_{\Xi^*} - m_{K})Z \\
+ (2m_{\Xi^*} + m_{\Lambda})Z^2 \right] \\
+ \frac{(m_{\Lambda} + m_{\Xi^*})^2}{m_{\Xi^*}^2} ,
\]

\[
d_z = \frac{4Z^2}{m_{\Xi^*}^2} ,
\]

where \( t, q_{N^*}, E_{N^*} \) are defined in the appendix and \( m_K, m_{N^*} \) are the kaon and the \( N^* \) masses respectively. For \( A^\prime \) and \( B^\prime \) we change the \( N^* \) parameters, inserting the \( \Xi^* \) ones in eqs. (29) and (30). \( g_{\Lambda K N^*}(\Xi^*) \) are the coupling constants.

For the last diagram, FIG. 1, the scalar \( \sigma \) meson exchange, a parametrization of the amplitude has been considered \[23]-\[25]\:

\[
A_\sigma = a + bt , \tag{39}
\]

\[
B_\sigma = b , \tag{40}
\]

with \( a = 1.05/m_{\sigma}^{-1} \), \( b = -0.8m_{\sigma}^{-2} \) and the pion mass \( m_{\pi} \). Some discussions about this term may be found in \cite{24}-\cite{26}.

The parameters considered in the \( KA \) interaction are shown in Tab. I the masses are taken from \cite{36}.

| Parameter | Value |
|-----------|-------|
| \( m_{\pi} \) | 140 MeV |
| \( m_K \) | 496 MeV |
| \( m_{\Lambda} \) | 1116 MeV |
| \( Z \) | -0.5 |
| \( g_{\Lambda K N} \) | 11.50 |
| \( g_{\Lambda K N}(1650) \) | 10.7 GeV^{-1} |
| \( g_{\Lambda K N}(1710) \) | 5.2 GeV^{-1} |
| \( g_{\Lambda K N^*}(1875) \) | 0.53 GeV^{-1} |
| \( g_{\Lambda K N^*}(1900) \) | 2.6 GeV^{-1} |
| \( g_{\Lambda K \Xi^*} \) | 0.2 |
| \( g_{\Lambda K \Xi^*}(1820) \) | 1.8 GeV^{-1} |

**TABLE II.** Parameters for the \( KA \) interaction

The coupling constants \( g_{\Lambda K N} \) and \( g_{\Lambda K \Xi^*} \) are determined using \( SU(3) \) and the ones of the \( \Lambda \) with the resonances, by using the Breit-Wigner expression eq. 19, as described above, in the same way it has been done in \cite{23}.

In FIG. 2 we show our results for the total elastic cross section and the phase shifts as functions of the kaon momentum \( k \), defined in the center-of-mass frame. Figure FIG. 3 shows the angular distributions and the polarizations as functions of \( x = \cos \theta \) and \( k \).

Observing the figure we can note that the resonances, and in special the \( N(1650) \) contribution, dominate the total cross section when \( k \sim 150 \) MeV, as it was expected. At higher energies, the other resonances also have an important effect. The polarization oscilates for \( k < 150 \) MeV, but as the momentum increases, it becomes negative.
IV. ANTIKAON-LAMBDA INTERACTION

The $\bar{K}\Lambda$ interactions may be studied exactly in the same way as it has been done in the last section for the $K\Lambda$ interactions. Now we have the contributions presented in FIG. 4, where the Lagrangians take into account the $N$, $\Xi$, $\Lambda$ and $\phi'$ (representing the antikaon) fields

$$L_{\Lambda KB} = g_{\Lambda KB} \frac{1}{2m_\Lambda} \left[ \bar{B} \gamma_\mu \gamma_5 \Lambda \partial^\mu \phi' \right],$$

$$L_{\Lambda K B^*} = g_{\Lambda KB^*} \left\{ \bar{B} \gamma_\mu \left[ g_{\mu\nu} - (Z + \frac{1}{2}) \gamma_\mu \gamma_\nu \right] \Lambda \right\} \partial^\nu \phi' .$$

The parameters considered are given before, $m_K = m_K$, and for the crossed diagrams in FIG. 4c and d we have considered only $N(938)$ and $N^*(1900)$, that are the most important processes. The amplitudes (39) and (40) have been calculated and the results are shown in Figures 5 and 6.

V. KAON-SIGMA INTERACTION

In this case the interacting particles have isospin 1/2 and 1 ($K$ and $\Sigma$ respectively). So, we have two possible total isospin states, 1/2 and 3/2, which allow also the exchange of $\Delta$ particles.

The scattering amplitude has the general form

$$T_{K\Sigma}^{3\alpha} = \bar{u}(\vec{p}') \left\{ \left[ A^+ + \left( \frac{\vec{k} + \vec{k}'}{2} \right) B^+ \right] \delta^{3\alpha} + \left[ A^- + \left( \frac{\vec{k} + \vec{k}'}{2} \right) B^- \right] i e^{3\alpha \tau_3} \right\} u(\vec{p}) ,$$

and the considered projection operators are

$$P_\frac{1}{2}^{3\alpha} = \frac{1}{3} \delta^{3\alpha} + \frac{i}{3} e^{3\alpha \tau_3} ,$$

$$P_\frac{3}{2}^{3\alpha} = \frac{2}{3} \delta^{3\alpha} - \frac{i}{3} e^{3\alpha \tau_3} .$$
where the indices $\alpha$ and $\beta$ are relative to the initial and final isospin states of the $\Sigma$.

The contributing diagrams are shown in FIG. 7 and the considered particles in Tab. III. The Lagrangians (1), (2) now become,

$$L_{\Sigma KB} = \frac{g_{\Sigma KB}^2}{2m_\Sigma} \left( B\gamma_\mu \gamma_5 \vec{\tau} \cdot \vec{\Sigma} \right) \partial^\mu \phi , \quad (46)$$

$$L_{\Sigma KB^*} = g_{\Sigma KB^*} \left\{ \frac{B^\mu}{\sqrt{2}} \left[ g_{\mu\nu} - \frac{1}{2} \gamma_\mu \gamma_\nu \right] \vec{Q} \cdot \vec{\Sigma} \right\} \partial^\nu \phi , \quad (47)$$

where $\vec{Q}$ is the $\vec{M}$ matrix for $\Delta$ ($I = 3/2$) or $\vec{\tau}$ matrix for the $N^*$ and $\Xi^*$ ($I = 1/2$).

The resulting amplitudes for nucleons in the intermediate state (FIG. 7a) are

$$A_N^+ = \frac{g_{\Sigma KN}^2}{4m_\Sigma^2} \left( m_N + m_\Sigma \right) \left( s - m_\Sigma^2 \right) \left( s - m_N^2 \right) , \quad (48)$$

$$B_N^+ = -\frac{g_{\Sigma KN}^2}{4m_\Sigma^2} \left[ \frac{2m_\Sigma(m_\Sigma + m_N) + s - m_\Sigma^2}{s - m_N^2} \right] , \quad (49)$$

$$A_N^- = \frac{g_{\Sigma KN}^2}{4m_\Sigma^2} \left( m_N + m_\Sigma \right) \left( s - m_\Sigma^2 \right) \left( s - m_N^2 \right) , \quad (50)$$

$$B_N^- = -\frac{g_{\Sigma KN}^2}{4m_\Sigma^2} \left[ \frac{2m_\Sigma(m_\Sigma + m_N) + s - m_\Sigma^2}{s - m_N^2} \right] , \quad (51)$$

and for the $\Xi$ exchange in the diagram [Si]

$$A_\Xi^+ = \frac{g_{\Sigma K\Xi}^2}{4m_\Sigma^2} \left( m_\Xi + m_\Sigma \right) \left( u - m_\Xi^2 \right) , \quad (52)$$

$$B_\Xi^+ = \frac{g_{\Sigma K\Xi}^2}{4m_\Sigma^2} \left[ \frac{2m_\Sigma(m_\Sigma + m_\Xi) + u - m_\Sigma^2}{u - m_\Xi^2} \right] , \quad (53)$$

| $J^*$ | $I$ | Mass (MeV) |
|-------|-----|------------|
| $N$   | $1/2^+$ | 1/2 | 938 |
| $N(1710)$ | $1/2^+$ | 1/2 | 1710 |
| $N^*(1875)$ | $3/2^+$ | 1/2 | 1875 |
| $N^*(1900)$ | $3/2^+$ | 3/2 | 1900 |
| $\Delta$ | $3/2^+$ | 1/2 | 1920 |
| $\Xi$ | $1/2^+$ | 1/2 | 1320 |
| $\Xi^*(1820)$ | $3/2^+$ | 1/2 | 1820 |

TABLE III. Resonances of the $K\Sigma$ interaction
\[ A^-_\Xi = \frac{g_{\Sigma K \Xi}^2}{4m^2_{\Sigma}} (m_\Xi + m_\Sigma) \left(\frac{u - m^2_{\Sigma}}{u - m^2_{\Xi}}\right), \quad (54) \]
\[ B^-_\Xi = -\frac{g_{\Sigma K \Xi}^2}{4m^2_{\Sigma}} \left[2m_{\Sigma}(m_\Xi + m_\Sigma) + u - m^2_{\Xi}\right], \quad (55) \]

Figure 7 gives
\[ A^+_{N^*} = \frac{g_{\Sigma K N^*}^2}{6} \left[2\hat{A} + 3(m_\Sigma + m_{N^*})t\right] \frac{m^2_{N^*} - s}{m^2_{N^*} - u} + a_0, \quad (56) \]
\[ B^+_{N^*} = \frac{g_{\Sigma K N^*}^2}{6} \left[2\hat{B} + 3t\right] \frac{m^2_{N^*} - s}{m^2_{N^*} - u} - b_0, \quad (57) \]
\[ A^-_{N^*} = \frac{g_{\Sigma K N^*}^2}{6} \left[2\hat{A} + 3(m_\Sigma + m_{N^*})t\right] \frac{m^2_{N^*} - s}{m^2_{N^*} - u} + a_0, \quad (58) \]
\[ B^-_{N^*} = \frac{g_{\Sigma K N^*}^2}{6} \left[2\hat{B} + 3t\right] \frac{m^2_{N^*} - s}{m^2_{N^*} - u} - b_0, \quad (59) \]

and for the crossed diagram shown in FIG. 7, where a \( \Xi^* \) exchange is taken into account,
\[ A^+_{\Xi^*} = \frac{g_{\Sigma K \Xi^*}^2}{6} \left[2\hat{A} + 3(m_\Xi + m_{\Xi^*})t\right] \frac{m^2_{\Xi^*} - u}{m^2_{\Xi^*} - u}
+c_0 + c_\Xi(u - m^2_{\Xi}) \right], \quad (60) \]
\[ B^+_{\Xi^*} = \frac{g_{\Sigma K \Xi^*}^2}{6} \left[2\hat{B} + 3t\right] \frac{m^2_{\Xi^*} - u}{m^2_{\Xi^*} - u} - d_\Xi(u - m^2_{\Xi^*}) \right], \quad (61) \]
\[ A^-_{\Xi^*} = -\frac{g_{\Sigma K \Xi^*}^2}{6} \left[2\hat{A} + 3(m_\Xi + m_{\Xi^*})t\right] \frac{m^2_{\Xi^*} - u}{m^2_{\Xi^*} - u}
+c_0 + c_\Xi(u - m^2_{\Xi}) \right], \quad (62) \]
\[ B^-_{\Xi^*} = \frac{g_{\Sigma K \Xi^*}^2}{6} \left[2\hat{B} + 3t\right] \frac{m^2_{\Xi^*} - u}{m^2_{\Xi^*} - u} - d_\Xi(u - m^2_{\Xi^*}) \right], \quad (63) \]

where the expressions for \( \hat{A}, \hat{B}, \hat{A}', \hat{B}', b_0, c_0, d_0, c_\Xi \) and \( d_\Xi \) are the same ones presented in Sec. III, but replacing the \( \Lambda \) hyperon for the \( \Sigma \) hyperon.

For the spin-isospin-3/2 \( \Delta \) resonance in FIG. 7, we have the amplitudes
\[ A^+_\Delta = \frac{g_{\Sigma K \Delta}^2}{9} \left[2\hat{A}' + 3(m_\Sigma + m_{\Delta})t\right] \frac{m^2_{\Delta} - s}{m^2_{\Delta} - s} + a_0', \quad (64) \]
\[ B^+_\Delta = \frac{g_{\Sigma K \Delta}^2}{9} \left[2\hat{B}' + 3t\right] \frac{m^2_{\Delta} - s}{m^2_{\Delta} - s} - b_0', \quad (65) \]
\[ A^-_\Delta = \frac{g_{\Sigma K \Delta}^2}{9} \left[2\hat{A}' + 3(m_\Sigma + m_{\Delta})t\right] \frac{m^2_{\Delta} - s}{m^2_{\Delta} - s} + a_0', \quad (66) \]
\[ B^-_\Delta = \frac{g_{\Sigma K \Delta}^2}{9} \left[2\hat{B}' + 3t\right] \frac{m^2_{\Delta} - s}{m^2_{\Delta} - s} - b_0', \quad (67) \]

where the expressions for \( \hat{A}', \hat{B}', a_0' \) and \( b_0' \) are given in (29), (30), (31) and (32) replacing \( \Lambda \) for \( \Sigma \) and \( N^* \) for \( \Delta \).

For the \( \sigma \) exchange (FIG. 7) the parametrization from eqs. (39) and (40) will be considered.

Thus, to calculate the observables for each reaction we use (44) and (45), resulting in the amplitudes
\[ A^+ = A^+ + 2A^-, \quad (68) \]
\[ B^+ = B^+ + 2B^-, \quad (69) \]
\[ A^- = A^+ - A^-, \quad (70) \]
\[ B^- = B^+ - B^-, \quad (71) \]

and the parameters are shown in Tabs. IV and V.

| \( m_\Sigma \) | 1190MeV |
| \( g_{\Sigma K N} \) | 6.9 |
| \( g_{\Sigma K N(1710)} \) | 8.4GeV\(^{-1} \) |
| \( g_{\Sigma K N^*(1875)} \) | 0.7GeV\(^{-1} \) |
| \( g_{\Sigma K N^*(1900)} \) | 1.3GeV\(^{-1} \) |
| \( g_{\Sigma K \Delta} \) | 1.7GeV\(^{-1} \) |
| \( g_{\Sigma K \Xi} \) | 13.4 |
| \( g_{\Sigma K \Xi^*}(1820) \) | 1.8GeV\(^{-1} \) |

**TABLE IV. Parameters for the \( K\Sigma \) interaction**

To determine the coupling constants \( g_{\Sigma K N}, g_{\Sigma K \Xi} \) and the ones with resonances we take into account the same arguments presented in Sec. III.

Using the isospin formalism for the elastic and the charge exchange scattering, we can determine the amplitudes for the reactions (that we name \( C_i \), for simplicity)
\[ \langle \Sigma^+ K^0 | T | \Sigma^+ K^0 \rangle = \langle \Sigma^- K^0 | T | \Sigma^- K^0 \rangle \]
\[ = T^+_\frac{1}{2} \equiv C_1, \quad (72) \]
\[ \langle \Sigma^+ K^0 | T | \Sigma^0 K^0 \rangle = \langle \Sigma^- K^+ | T | \Sigma^- K^+ \rangle \]
\[ = \frac{1}{3} T^+_{\frac{1}{2}} + \frac{2}{3} T^-_{\frac{1}{2}} \equiv C_2, \quad (73) \]
\[ \langle \Sigma^0 K^0 | T | \Sigma^0 K^0 \rangle = \langle \Sigma^0 K^+ | T | \Sigma^0 K^+ \rangle \]
\[ \langle \Sigma^0 K^0 \mid T \Sigma^- K^+ \rangle = \langle \Sigma^+ K^0 \mid T \Sigma^0 K^+ \rangle = \langle \Sigma^- K^+ \mid T \Sigma^0 K^0 \rangle = \langle \Sigma^0 K^+ \mid T \Sigma^+ K^0 \rangle \]
\[ = \frac{\sqrt{2}}{3} (T_2 - T_4) \equiv C_4 , \] (74)

and with these amplitudes we can calculate all observables of interest. The total elastic cross sections and the phase shifts as functions of the kaon momentum are shown in FIG. 8. Figures 9 and 10 show the angular distributions and the polarizations.

VI. ANTIKAON-SIGMA INTERACTION

In this case, we will proceed in the same way as we have done in the last section for the \( K\Sigma \) interaction. The diagrams to be considered are shown in FIG. 11.

The Lagrangians are very similar to the ones used to study the \( K\Sigma \) interaction, (46), (47), replacing \( N \) and \( N^* \) for \( \Xi \) and \( \Xi^* \). Then, if these changes are implemented, we may use the same amplitudes given by (48)-(63), (68)-(71).

In this case we have the following reactions
\[ \langle K^0 \Sigma^+ \mid T \bar{K}^0 \Sigma^+ \rangle = \langle K^- \Sigma^- \mid T \bar{K}^- \Sigma^- \rangle \]
\[ = T_4 \equiv D_1 , \] (76)

\[ \langle \Sigma^+ K^- \mid T \Sigma^+ K^- \rangle = \langle \Sigma^- K^0 \mid T \Sigma^- K^0 \rangle \]
\[ = \frac{1}{3} T_2 + \frac{2}{3} T_2 \equiv D_2 , \] (77)

\[ \langle \Sigma^0 \bar{K}^0 \mid T \Sigma^0 \bar{K}^0 \rangle = \langle \Sigma^0 K^- \mid T \Sigma^0 K^- \rangle \]
\[ = \frac{2}{3} T_2 + \frac{1}{3} T_2 \equiv D_3 , \] (78)

\[ \langle \Sigma^0 K^- \mid T \Sigma^- \bar{K}^0 \rangle = \langle \Sigma^+ K^{-} \mid T \Sigma^0 \bar{K}^0 \rangle \]
\[ = \langle \Sigma^- \bar{K}^0 \mid T \Sigma^0 K^- \rangle = \langle \Sigma^0 \bar{K}^0 \mid T \Sigma^+ K^- \rangle \]
\[ = \langle \Sigma^- \bar{K}^0 \mid T \Sigma^0 K^- \rangle = \langle \Sigma^0 \bar{K}^0 \mid T \Sigma^+ K^- \rangle \]
\[ = \frac{\sqrt{2}}{3} (T_2 - T_4) \equiv D_4 . \] (79)

For diagram 11d the resonance to be considered is \( N^*(1900) \). Using the parameters given in Tab. IV we have obtained the results for the \( \bar{K}\Sigma \) scattering shown in Figures 12, 13 and 14.

VII. DISCUSSION AND RESULTS

In this work the low energy \( K\Lambda, \bar{K}\Lambda, K\Sigma \) and \( \bar{K}\Sigma \) interactions have been studied considering a model based on effective Lagrangians where mesons, baryons and
baryonic resonances have been taken into account. The coupling constants have been determined and then the $S$ and $P$ phase shifts, cross sections and polarizations have been calculated and shown in the figures of the previous sections. As it was expected, for many channels the resonances dominated the cross sections, and for this reason, we believe in the formulation of the proposed model at low energies ($k < 0.4$ GeV). In [23] a similar behavior has been observed, and the predictions of the model, when compared with the HyperCP data, showed to be very accurate.

For the $K\Lambda$ scattering, at the $\Omega$ hyperon mass ($m_\Omega = 1672$ MeV), $\delta_{P_1} = 2.71^\circ$, $\delta_{P_3} = 2.90^\circ$, $\delta_{D_3} =$
FIG. 11. Diagrams for the $\bar{K}\Sigma$ interaction

FIG. 12. Total Cross Section and Phase Shifts of the $\bar{K}\Sigma$ scattering

$-0.0008^\circ$ and $\delta_{D_5} = -0.0001^\circ$. These strong phases may be used in a possible search of CP violation in the $\Omega \to \bar{K}\Lambda$ decay, in addition to the weak CP violating phases, in the same way it has been done in [25] (even considering that for other similar decays, no CP violation has been observed [20], [27]).

In the study of high energy hyperon polarization, produced in proton-nucleus and in heavy ion collisions, if we consider the polarizations obtained in the final-state interactions, the processes studied in this work may have some effect in the final polarization of the $\Lambda$, $\bar{\Lambda}$, $\Sigma$ and...
FIG. 14. Polarization in the $K\Sigma$ scattering

$\Sigma$ hyperons produced in these reactions. In special, in some reactions of Fig. 14 considerable polarization may be observed, and some signs of this fact probably may be observed in the $\Sigma$ and in the $\bar{\Sigma}$ polarizations. Probably this effect is smaller than the one obtained when considering the $\pi Y$ interactions $^{19}$-$^{21}$, but as these interactions ($KY$) provide polarizations of different signs, it is possible to obtain some differences in the final results. And certainly, a more accurate final result will be obtained.

These reactions are also important in the determination of nucleon-hyperon and hyperon-hyperon potentials, as they are subprocesses of these interactions, and as it has been discussed before, these interactions have a fundamental importance in the structure of the hypernuclei and in the hyperon stars.

It must be pointed that the study of the $\Xi$ hyperons and other related reactions have been left for future works.

So, as it has been shown, the study presented in this work is very important for many physical systems of interest, and for this reason, must be continued and improved in future works.

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IX. APPENDIX

Considering a process where $p$ and $p'$ are the initial and final hyperon four-momenta, $k$ and $k'$ are the initial and final meson four-momenta, the Mandelstam variables are given by

$$s = (p+k)^2 = (p'+k')^2 = m^2 + m^2_R + 2Ek_0 - 2\vec{k} \cdot \vec{p}, \quad (80)$$

$$u = (p' - k)^2 = (p - k')^2 = m^2 + m^2_R - 2Ek_0 - 2\vec{k'} \cdot \vec{p'} , \quad (81)$$

$$t = (p - p')^2 = (k - k')^2 = 2|\vec{k}|^2 x - 2|\vec{k'}|^2 . \quad (82)$$

In the center-of-mass frame, the energies will be defined as

$$k_0 = k'_0 = \sqrt{|\vec{k}|^2 + m^2_K} , \quad (83)$$

$$E = E' = \sqrt{|\vec{k'}|^2 + m^2} , \quad (84)$$

and the total momentum is null

$$\vec{p} + \vec{k} = \vec{p'} + \vec{k'} = 0 . \quad (85)$$

We also define the variable

$$x = \cos \theta , \quad (86)$$

where $\theta$ is the scattering angle. Other variables of interest are

$$\nu_r = \frac{m^2_z - m^2 - k \cdot k'}{2m} , \quad (87)$$
\[ \nu = \frac{s - u}{4m} = \frac{2Ek_0 + |\vec{k}|^2 + |\vec{k}'|^2}{2m}, \]  
\[ k\cdot k' = m_K^2 + |\vec{k}'|^2 - |\vec{k}|^2 x = k_0^2 - |\vec{k}'|^2 x, \]

where \( m, m_r \) and \( m_K \) are the hyperon mass, the resonance mass and the kaon mass, respectively.

For the energy and the 3-momentum of the intermediary particles we also have the relations

\[ (E_{B^*} \mp m_\Lambda) = \frac{(m_{B^*} \pm m_\Lambda)^2 - m_K^2}{2m_{B^*}}. \]

where \( E_{B^*} \) and \( \vec{q}_{B^*} \) are the energy and the momentum of intermediary baryon \( B^* \) in the center-of-mass frame, respectively.

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