ABSTRACT
Thermocline is of great significance to marine scientific research. Multi-autonomous underwater vehicles (AUV) have great advantages over single autonomous underwater vehicle in ocean observation. This paper describes a control method for multi-agent formation. Based on this method, a strategy of multi-AUV formation for thermocline tracking is proposed. This paper firstly analyzes the stability of the formation control method based on the virtual body and artificial potential method (VBAP), and verifies the feasibility of this control method on the target-tracking problem by tracking a curved surface in space. Whereafter, in this paper, a thermocline tracking strategy based on vertical temperature gradient is proposed. Then the simulation experiment is designed based on the above control method and temperature data. The experimental results express that the multi-AUV formation can always keep working between the upper and lower boundary of thermocline, and tracks the thermocline effectively.

CCS Concepts
Information systems → Process control systems

Keywords
Artificial potential field; Multiple autonomous underwater vehicles (AUV); Thermocline tracking; Virtual leader

1. INTRODUCTION
Thermocline as an important sea phenomenon has very important significance to the Marine environment of physical, chemical and biological research [1]-[4]. Using multiple autonomous underwater vehicle (AUV) to track the thermocline, in order to get the distribution of temperature, density, salinity in the thermocline is very significant [5]-[9].

Currently, the mainstream method of tracking ocean thermocline is to judge the upper and lower boundary of thermocline, and then let the AUV within the scope during operation. In order to determine the upper and lower boundary of thermocline, the common method includes the vertical gradient method, curvature extremum method and Quasi-step function method [10]-[13], etc. In 2009, MIT has been used single underwater vehicle to track the thermocline. During the trial, the average temperature gradient was regarded as the threshold for thermocline tracking. The location where the temperature gradient is more than the average temperature gradient is considered as a thermocline. On this account, it can determine the upper and lower boundary of the thermocline [14]. In 2010, Yanwu Zhang, etc. used Tethys AUV to track the thermocline in Monterey Bay. By determining the location of temperature gradient extremum, and on this basis, a certain distance is extended upwards and downward as the upper and lower boundary of the thermocline [15]. In China, Wu Wei [16] and others had made a detailed comparison and discussion on several methods for determining thermocline boundary based on the measured CTD data in the South China Sea, and discussed the key problems under different situations of deep water and shallow water. On the whole, the research on thermocline tracking, thermocline detection and boundary judgment is more mature for a single autonomous underwater vehicle. However, the multi AUV formation has a great advantage over a single AUV to track the thermocline. Multi AUV formation is more robust and has wider detection range than a single AUV. In addition, each AUV of the multi AUV formation has small volume and high fault-tolerance [17].

At present, there are not many studies on thermocline tracking for multi AUV formation. However, many scholars have studied the control method for the multi autonomous underwater vehicles. In literature [18], proposed a combination of virtual body and artificial potential (VBAP) method to control multi-agent formation. In addition, they used multi-glider formation to achieve adaptive sampling in Monterey Bay. Shaowei Zhang [19], China Academy of Sciences, Shenyang Institute of automation established model for the real-time estimation of ocean feature based on the extended Kalman filter method. In addition, they designed the strategy and control law of multiple underwater vehicles formation’s center to track the oceanic characteristic contour, and set up the simulation platform for tracking and observing the ocean characteristic field of multi underwater vehicles.

The thermocline tracking is of great significance for marine scientific research and the multi AUV formation has a great advantage in the thermocline tracking. In this paper, the formation’s control method based on virtual body and artificial potential method is applied to track thermocline. Then, a multi AUV formation thermocline tracking method is designed. The
VBAP method is used to control the movement of multi AUV formation. In vertical direction, we calculate whether the vertical temperature gradient is in thermocline, to ensure that the multi AUV formation operates in thermocline. Compared with previous thermocline tracking methods, this paper realizes the effective tracking of thermocline by multi AUV formation, and ensures that multi AUV formation always operates between the upper and lower boundary of thermocline. When judging thermocline, according to the real-time vertical gradient value, it has strong autonomy compared with the previous peak-gradient method, and the real-time computation is small.

2. MULTI AUV FORMATION CONTROL METHOD

The formation’s control method based on virtual bodies and artificial potential method is used to determine the motion rules of each agent (including virtual leaders) in formation by artificial potential field [20]. The following figure is the distribution of multi AUV formation and virtual bodies in the three-dimensional space.

![Figure 1. Multi AUV formation and virtual leader in three-dimensional space](image)

In Figure 1, the spatial coordinates of the three AUV are expressed as \( x_i \) \((i = 1, 2, 3)\), the spatial coordinates of the virtual leaders are expressed as \( \bar{b}_i \). The position coordinates of the \( l \) th virtual leader is \( \bar{b}_l(t) = R(t)\bar{b} + \vec{r}(t) \), and \( \bar{b} = \bar{b}_0(t_0) - \vec{r}(t_0) \). \( t_0 \) is the starting time. \( \vec{r} \) is the central coordinates of formation, and is located in the center of all virtual leaders in the formation. When there is only one virtual leader, it is \( \vec{r}(t) = \bar{b}_0(t) \). \( \theta \) is the angle between the formation’s course and the X-axis. \( R(t) \) is a deflection matrix of formation.

\[
R(t) = \begin{pmatrix}
\cos \theta(t) & -\sin \theta(t) \\
\sin \theta(t) & \cos \theta(t)
\end{pmatrix}
\]

By the VBAP algorithm, the multi AUV formation will follow the virtual leader. Next, the VBAP algorithm for multi AUV formation is derived in detail.

2.1 The Potential Energy between AUVs

When calculating the potential energy between AUV, the potential energy between \( i \) th AUV and \( j \) th AUV can be expressed as \( V_i \left(x_i, \alpha_i, d_{o}, d_{i}\right) \). And \( x \) is the distance between AUVs. \( \alpha \) is a coefficient that can adjust the size of the gradient. \( d_{o} \) is the expected distance between AUVs. If \( x = d_{o} \), \( V_i \) is the global minimum. \( d_{i} \) is the maximum distance between AUVs. If the distance between AUVs is larger than \( d_{i} \), the potential energy is zero. The expression of \( V_i \left(x_i, \alpha_i, d_{o}, d_{i}\right) \) is as follows:

\[
V_i = \alpha_i \left( \frac{1}{3} x - d_{o} \right) \left( x - \frac{1}{3} d_{o} + d_{o} \right) 0 < x \leq d_{i}
\]

\[
0 \quad \quad x > d_{i}
\]

So the gradient of the potential energy \( V_i \) to the \( i \) th AUV is:

\[
-\nabla_i V_i = -f_i(x_i) \hat{r}_i = f_i(x_i) \hat{x}_i
\]

In equation (2):

\[
\hat{x}_i = \frac{\hat{x} - \hat{x}_i}{x_i}
\]

In the above equation, \( \hat{x} \) is the spatial coordinates of AUV. \( \hat{x}_i \) represents the unit vector of the \( i \) th AUV pointing to the \( j \) th AUV. In addition:

\[
f_i(x_i) = \alpha_i \left( x_i - \frac{d_{i}}{x_i} \right) 0 < x_i \leq d_{i}
\]

\[
0 \quad \quad x_i > d_{i}
\]

It can be found that \( f_i(x_i) \) is discontinuous while \( x_i = d_{i} \). Thus, the force of each AUV is discontinuous. There is potential risks to the system. For example, if \( x_i \) becomes greater than \( d_{i} \) from less than \( d_{i} \), \( f_i(x_i) \) becomes zero from a positive number suddenly. It can cause vibration of the system. To avoid this situation, a continuous function \( \beta(x_i) \) with a range of \([0, 1]\) is introduced.

\[
\beta(x_i) = \begin{cases}
1 & x \leq a \\
\sin^2 \left( \frac{\pi}{2} \frac{x}{a-b} \right) & a < x \leq b \\
0 & x > b
\end{cases}
\]

In equation (5), if \( b = d_{i}, a \in \left(d_{o}, d_{i}\right) \), the force of each AUV can be expressed as:

\[
f_i(x_i) = \alpha_i \beta(x_i) \left( x_i - \frac{d_{o}}{x_i} \right), x_i > 0
\]
2.2 The Potential Energy between AUVs and Virtual Leaders

When calculating the potential energy between each AUV and the virtual leaders, the potential energy between the \( i \) th AUV and the \( l \) th virtual leader can be expressed as \( V^C_h(c_i, \alpha_i, h_i, h_l) \). And:

\[
V_h^C = \begin{cases} 
\alpha_b \left( \frac{1}{3} h_d^3 - h_d h_l + \frac{1}{3} h_l^3 \right) & 0 < h_d \leq h_l \\
0 & h_d > h_l 
\end{cases} \tag{7}
\]

In equation (7), \( h_d \) is the distance between \( i \) th AUV and \( l \) th virtual leader. \( \alpha_b \) is a coefficient that can adjust the size of the gradient. \( h_l \) is the expected distance between AUVs and virtual leaders. If \( h_d = h_l \), \( V_h^C \) is the global minimum. \( h_l \) is the maximum distance between AUVs. If the distance between AUVs is larger than \( h_l \), the potential energy is zero. Therefore, the gradient of the potential energy \( V_h^C \) to the \( j \) th AUV is:

\[
-\nabla_{h^C} V_h^C = -f_h^C(h_l) \hat{h}_i = f_h^C(h_l) \hat{h}_i 
\]

In equation (8),

\[
\hat{h}_i = \frac{x_i - \hat{x}_i}{h_i} 
\]

And \( x_i \) is the spatial coordinates of \( i \) th AUV. \( \hat{x}_i \) is the spatial coordinates of \( l \) th virtual leader. \( \hat{h}_i \) represents the unit vector of the \( i \) th AUV pointing to the \( l \) th virtual leader. In addition,

\[
f_h^C(h_l) = \alpha_b \beta(h_l) (h_l - \frac{h_l}{h_l}) 
\]

In equation (10), the form of \( \beta(h_l) \) is the same as equation (5), and \( b = h_l, a \in (h_l, h_l) \).

2.3 Design and Stability Analysis of Multi AUV Formation System

Suppose that the multi AUV formation consists of \( m \) AUVs and \( n \) virtual leaders. In the VBAP algorithm, the control force for each agent can be regarded as the gradient of the resultant force of the agent minus the sum of the total potential energy of the multi AUV formation, so that the total potential energy of the system is the smallest. Then, the state equation of the \( l \) th AUV can be expressed as:

\[
\dot{x}_i = -\sum_{i=1}^m \sum_{j=1}^n \nabla_x V_h^C(h_l) - \sum_{i=1}^n \sum_{i=1}^n \nabla_x V_j(x_i) - \dot{f}_i 
\]

The Lyapunov equation of the system is as follows:

\[
\Phi(\chi,s) = \frac{1}{2} \sum_{i=1}^m \left\| \dot{x}_i \right\|^2 + \sum_{j=1}^n \left( -f_i(x_j) + 2 \sum_{i=1}^n V_h^C(h_l) \right) 
\]

In equation (12), \( \chi = (x_1^T, x_2^T, \ldots, x_n^T) \). Because \( \Phi = -\sum_{i=1}^m \dot{x}_i \dot{\chi}_i \), if \( \dot{\chi}_i = 0, \dot{\chi}_i > 0 \), \( \Phi \) is negative definite. If and only if \( \dot{\chi}_i = 0 \), \( \Phi \) is equal to zero. And \( \Phi(\chi,s) \geq 0 \), so the system is stable.

In literature [18], the formation is kept stable by introducing a feedback control speed \( \dot{s} \), such as equation (13). In equation (13), \( v_0 \) is the expected speed. \( \chi = (x_1^T, x_2^T, \ldots, x_n^T) \).

\[
\dot{s} = \min \left[ v_0, \Phi(\chi,s) + \frac{-\left( \frac{\partial \Phi}{\partial \chi} \right)^T \dot{\chi}}{\delta + \left( \frac{\partial \Phi}{\partial \chi} \right)} \right] \tag{13}
\]

In addition:

\[
\Phi(\chi,s) \leq \frac{\Phi_0}{2} \tag{14}
\]

Then, the central coordinate of formation \( \hat{r}(s) \) is \( \dot{\chi}_i = \frac{d \hat{r}}{ds} \dot{s} \). The deflection matrix of formation is \( R(s) = \frac{d \hat{r}}{ds} \dot{s} \). The coordinate of the \( i \) th virtual leader is \( \hat{b}_i = R(s) \hat{r} + \hat{v} \).

3. THERMOCLINE TRACKING BASED ON MULTI AUV FORMATION

3.1 The Methods of Thermocline Tracking or Body

When we use multi AUV formation to track thermocline, we first need to determine the upper and lower boundary of thermocline, and then track thermocline according to the algorithm for thermocline tracking.

At present, the main methods of thermocline detection include vertical gradient method, optimal segmentation method and peak-gradient method [21]. Vertical gradient method is widely used in the calculation of thermocline indicative characteristics [22]. The vertical gradient method calculates the temperature gradient in the vertical direction firstly, and then expands the distance from the top to the bottom artificially. It is considered as the upper and lower bounds of the thermocline. According to specifications for oceanographic survey (GB/T 12763.9---2007), this paper takes 0.2°C/m as the threshold.
In the process of tracking thermocline, we first use the temperature data obtained by AUVs to calculate "the first three temperature gradient values", "the current temperature gradient value" and "the next temperature gradient value". The equation for calculating the temperature gradient is as follows.

\[ T_{grad} = \frac{(T - T_{pre})}{(depth - depth_{pre})} \]  

(15)

In equation (15), \( T_{grad} \) is the calculated temperature gradient. \( T \) is the temperature of current depth. \( T_{pre} \) is the temperature of the previous depth. \( depth - depth_{pre} \) is the change of depth.

And the meanings of “the previous temperature gradient value”, “the current temperature gradient value” and “the next temperature gradient value” are as shown in Figure 2.

1. If “the current temperature gradient value” (negative) is not more than the threshold and “the first three temperature gradient values” are more than the threshold and “the next temperature gradient value” is less than the threshold, we update the value of \( z \) to \(-1m/s\).

2. If “the current temperature gradient value” (negative) is not more than the threshold and “the first three temperature gradient values” are less than the threshold and “the next temperature gradient value” is more than the threshold, we update the value of \( z \) to \(1m/s\).

3.2 The Simulation Experiment of Multi AUV Formations for Thermocline Tracking

Using the temperature data obtained by AUV in a process of dive, the simulation experiment for the above algorithm is carried out. It is worth noting that the temperature data is not equal-depth interval. Therefore, the temperature data is linearly interpolated every 0.5 meters to obtain the data at equal-depth interval. The result is shown in Figure 3.

As we can see from Figure 3, the temperature varies greatly in range of 15 to 25 meters. In addition, the temperature gradient can be obtained according to the equation (15). Because there are many noises in the temperature data obtained by AUV, this paper applies five-point three-times filtering method to process these data. Finally, we get the relationship between the temperature gradient and depth as shown in Figure 4. As we can be seen from Figure 4, there is a strong thermocline in this region.

Because the temperature data is only in a vertical direction, if we want to verify the thermocline-tracking algorithm proposed in this paper, it needs to be extended to the three-dimensional space. In order to test the performance of the algorithm better, we set that the temperature of the same depth varies greatly. Figure 5 shows the distribution of temperature gradient in a 10km range on a longitudinal profile.
boundary of the thermocline is changing. This can better test the tracking performance of the above algorithm.

The speeds in the vertical direction of each AUV and virtual leader, \( v \), in equation (15), are updated according to the thermocline-tracking algorithm mentioned in section 3.1.

On every horizontal section, the state equation of each AUV is shown in equation (11). In equation (11), \( \dot{x} \) and \( \ddot{x} \) only include X-axis and Y-axis components now. The state equation of the X-axis and Y-axis of the \( i \) th AUV is as follows.

\[
\dot{x}_{i,x} = -f_i^x(h_i)\ddot{h}_i - \sum_{j \neq i, j \neq 0} f_j(x_i)\dot{x}_j - \alpha_i x_{i,x},
\]

\[
\dot{x}_{i,y} = -f_i^y(h_i)\ddot{h}_i - \sum_{j \neq i, j \neq 0} f_j(x_i)\dot{x}_j - \alpha_i x_{i,y},
\]

In the above equation, set \( i = 1, 2, 3 \), \( \alpha_i = 2 \). When calculating \( f_i(x) \) and \( f_i^x(h_i) \), the \( a \) and \( b \) of \( \beta - function \) in equation (6) and (10) is 90 and 100. And set \( \alpha_h = \alpha_v = 10 \).

The initial positions for each AUV are \((\sqrt{5},1,0)\), \((0,0,0)\), \((\sqrt{5},-1,0)\). The initial speed for all AUVs is zero. The initial course angle of multi AUV formation is 0 deg. Moreover, the initial coordinate of the virtual leader is \( \hat{b} = \left( \frac{2}{5}\sqrt{5},0,0 \right) \). And the initial coordinate of multi AUV formation’s center is \( \hat{r} = \hat{b} \). The unit of the above position data corresponds to the unit of the coordinate axis in Figure 1. In addition, we set \( \Phi_0 = 1.6 \), \( \delta = 0.01 \).

The simulation area is 10km x 2km x 40m (length x width x depth). Moreover, in the progress of simulation experiment, the multi AUV formation does not deflect, that is \( \frac{db}{ds} = 0 \).

According to the above analysis and initial values, a group of differential equations is obtained.

Then, the model can be solved by using “ode45” algorithm. Through the simulation, the trajectory of multi AUV formation, as shown in Figure 6, is obtained. Figure 6 shows the trajectory of the multi AUV formation for thermocline tracking. We can see the trend of formation changing with the change of upper and lower boundary of thermocline.

In order to show the change of formation better, Figure 7 drew the trajectory of “AUV 1” on a longitudinal profile.

From Figure 7, we can see that AUV begins to dive from the surface of the sea. When the lower boundary of the thermocline was detected, it begins to rise. After detecting the upper boundary of the thermocline, it begins to dive again. Repeating several cycles in this way. When arrived at 2km, the whole trajectory of AUV is starting to rise. In addition, when arrived at 3km, the whole trajectory of AUV is starting to decrease. Compared with the upper and lower boundary of the thermocline, the trajectory of the AUV is distributed in the thermocline completely. Moreover, at 6km and 8km, the position of the upper and lower boundary of the thermocline is decreased and rise suddenly. Compared with the trajectory of the AUV, it can be seen that the position which began to float become descend at 6km and the position which began to dive become rise. In general, the multi AUV formation tracks the thermocline well.

4. CONCLUSION

This paper studies the control method for multi-agent formation based on virtual body and artificial potential method. Based on this, we propose a method about the multi AUV formation for thermocline tracking. This method considers the characteristics of the thermocline, and ensures the multi AUV formation in the thermocline during the operation. The stability of multi AUV formation control method is analyzed in this paper. Based on the stability analysis of VBAP method and the validation of this method for target tracking problem, this method can be used for the multi AUV formation to track the thermocline. And then, this paper proposes a strategy for multi AUV formation to track thermocline. Finally, a simulation experiment is designed based on the temperature data obtained by AUV. The experiment results verify the effectiveness of the above strategy. More importantly, it is more autonomous than the peak-gradient method in the literature [11] while judging the thermocline. And the response is timely because of the small amount of calculation.

5. ACKNOWLEDGMENTS

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