Role of the experimental filter in obtaining the Arrhenius plot in multi-fragmentation reactions.

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(March 30, 2022)
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I. ABSTRACT

Recently it has been argued that the linear relation between the transverse energy and the apparent probability to emit a fragment proves that the total system is in thermal equilibrium. It is shown, for the specific reaction Xe+Sn at 50 A.MeV, that the same behavior is obtained in the context of Quantum Molecular Dynamical without invoking the idea of equilibrium. The linear dependence is shown to be a detector effect.
The process of multiple emission of intermediate mass fragments (I.M.F), so-called multi-fragmentation, is one of the central issues in heavy ion reactions at intermediate energies: Is the multi-fragmentation a thermal or a dynamical process or a mixing of both? Some recent papers try to demonstrate that production of I.M.F. can be essentially attributed to dynamical effects \[1,2\], at the same time other authors claim that multi-fragmentation is a thermal process \[4,5\]. Recently it has been conjectured by L.Moretto that the observation of regularities in I.M.F production rates, viewed as functions of ”temperature” have the form of an Arrhenius plot \[6,7\]. This finding, well known from chemical process in equilibrium, is for the authors \[6,7\] a proof that multi-fragmentation is indeed a thermal process.

The analysis is based on the fact that the probability \(p\) of one-fragment emission is independent of the emission of the other fragments, so the probability \(p\) for the elementary emission of I.M.F is related to the binomial law:

\[
P_m^n(p) = \frac{m!}{n!(m-n)!} p^n (1-p)^{(m-n)}
\]

(1)

where \(m\) is the number of chances to emit a fragment by the system and \(n\) is the multiplicity of fragment.

The process is considered as thermal, if \(\log(1/p)\) versus the inverse temperature \((1/T)\) is linear. In the ref. \[3\] it is assumed that the transverse energy \((E_t = \sum_i E_i \sin^2(\theta_i))\), where \(E_i\) is the kinetic energy of all charged particles, and \(\theta_i\) the polar angle) is proportional to the excitation energy \(E^*\) which in the Fermi gas model is proportional to the temperature squared \((E^* \propto T^2)\), so that the square root of \(E_t\) is proportional to the temperature \((\sqrt{E_t} \propto T)\).

Various point of this analysis can be critisized:

- the transverse energy contains in part the transverse energy of the I.M.F. and because of that, the multiplicity \(n\) and the transverse energy are correlated \[9,10,8\]. In reference \[7\] and in some recent publications \[11,12\] the authors claim that the Arrhenius law seen is not due to such correlations.

- It is based on the hypothesis that the detection efficiency, for a 4\(\pi\) detector, is independent of the reaction mechanism i.e of the impact parameter and on the total multiplicity.

- All the fragments are not emitted by a unique source. In this case it is difficult to believe that a unique temperature can be achieved for all the sources.

The aim of this paper is to show that the Arrhenius law can be obtained with a model without the idea of thermalization and also to demonstrate the effect of the detector efficiency. In the first part, dynamical calculations on the total range of impact parameters is used, then the relation between \(\log(1/p)\) and \(E_t\) is plotted with events filtered by the response of one detector. In the second part a simplified filter is used in order to explain the role of the detector in obtaining the Arrhenius law.

The Quantum-Molecular Dynamic model (QMD) \[13\] shows a good agreement with experimental heavy-ions reaction data at high incident energies. We have compared the
data of the Xe+Sn reaction at 50 A.MeV obtained with the INDRA detector [15] with the dynamical QMD code [14]. For this we have calculated 63000 events with an impact parameter between 0 and 12 fm in order to cover the complete reaction cross section. We find a very good agreement for a large part of the reaction cross section both for kinematical (velocity of I.M.F. and light particles...etc.), and statistical variables (fragments multiplicity, charge distribution...etc.) [4]. It was thus concluded that the reaction mechanism process is mainly binary and that the IMF’s seem to be produced dynamically and not by thermalized sources. The necessary assumption to observe the Arrhenius law is normally, in these simulations, not satisfied.

On the contrary, in recent publication, A. Wieloch et al. found the linear dependance of \( \log(1/p) \) versus \( 1/\sqrt{E_t} \) in the data of Xe+Sn at 50 A.MeV. We have therefore used the QMD events to construct an Arrhenius plot. All the events were filtered with the code simulating the total response (energy threshold, geometrical aperture, calibration and identification as in the experiment) of the INDRA detector [15].

For each event we calculate the total transverse energy \( (E_t) \) and for each bin in \( E_t \) we extract the value of \( p \) and \( m \) from the prescription of ref. [6] with the following formulae :

\[
p = 1 - \frac{\sigma^2}{\langle N_{IMF} \rangle}, \tag{2}
\]

\[
m = \frac{\langle N_{IMF} \rangle}{p}, \tag{3}
\]

where \( N_{IMF} \) is the mean value, and \( \sigma^2 \) the variance of the I.M.F multiplicity distribution.

For the determination of \( p \) and \( m \) it is clear that only ”emitted fragments” must be taken into account and that the residual nucleus must be excluded. In every such analysis, special care is therefore taken to treat the detection of projectile and target residue. In ref [6] due to experimental thresholds the projectile and target residues are never detected and the charge of I.M.F’s is taken to be between 3 and 20 (essentially to eliminate the fission process, certainly always present in this reaction). In ref. [8] they do not take into account the projectile residue as an I.M.F even if its charge is in this interval \( (3 \leq Z \leq 20) \) and they assume that the target residue is always lost due to the energy thresholds. In the present case, we take as IMF’s all fragments with a charge greater than 3, and because of the binary character of the reaction we do not take into account, in our I.M.F multiplicity, the projectile residue that it is always detected and, as is it done in the reference [8], we assume that the target residue is supposed to be undetected. This correspond, in this case, to eliminate one fragment from each filtered event.

The results of \( \log(1/p) \) as a function of \( 1/\sqrt{E_t} \) (see figure 1) looks like the linear Arrhenius law if we do not take into account the high energy point for which the variance has no signification, and the first low energies points for which the statistic is poor (due to the cross section for the low impact parameters). In the same way, \( m \) is rather constant as it is expected. This entails two questions: first, if the linear Arrhenius plot is a characteristic of a thermal process why is it found in a non-thermodynamical calculation? Second, what is exactly the bias of the detection system and of the reaction mechanism in obtaining this behavior?
In order to understand the detection effects, an analysis without filtering the events was performed. In this case, the efficiency is 100% and only the IMF’s are taken into account: the projectile and target residues are systematically removed event by event due to the binary character of the event on the total range of impact parameter.

Figure 2 (see full circle) shows that, with the unfiltered events, the linear aspect of the Arrhenius plot is not observed anymore. The deviations occur essentially for the semi-peripheral events with \(400 < E_t (MeV) < 600\) which correspond to impact parameter in the range of \(4.5 < b(fm) < 7.5\) for this reaction in the QMD model.

With respect to figure 1, this different behaviour can only come from a detector effect. Indeed, for the most peripheral collisions, the quasi-projectile is always detected while the quasi-target energy is below the detector threshold. It is therefore normal, taking into account the behaviour of the detector, to subtract, in the \(E_t < 400\) region, two fragments (i.e. the quasi-projectile and the quasi-target) from the distribution of the multiplicity of fragments.

For the more central collisions, the quasi-target has enough kinetic energy to be detected. Therefore, to have considered that it was not detected in constructing figure 1, corresponds to having subtracted not two but only one fragment.

In the intermediate region, the number of fragments subtracted is varied smoothly, and statistically, between the two limiting values.

The experimental construction of the \(\log(1/p)\) versus \(E_t\) curve is totally equivalent. Indeed, restricting IMFs to have charges between 3 and 20 and subtracting only one fragment if it is the ”largest” is equivalent to:

- In the low transverse energy to subtract two fragments to the total multiplicity, the quasi-projectile whose charge is greater than 20 and the quasi-target because it is not detected.

- In the high transverse energy region, the quasi-projectile and the quasi-target are, in most cases, detected with a charge less than 25 (see the velocity distribution in the laboratory of the two heaviest fragments in the Xe+Sn reaction [16]. The elimination of only the heaviest fragment is equivalent to subtracting only one fragment.

- For the intermediate energy region, a progressive transition between these two cases is the probable scenario. We have filtered QMD events with the prescription summarized in table II and the results are shown on figure 2 (see open circle).
TABLE I. Number of fragments taken into account for the different domain in $E_t$. For the range $400 < E_t (MeV) < 600$ we have taken $N_{\text{fragment}} - 1$ for $E_t = 400 MeV$ and $N_{\text{fragment}} - 2$ for $E_t = 600 MeV$, and a statically linear prescription in between.

| $E_t$ (MeV) | 0   | 400 | 400 | 600 | 600 | $\infty$ |
|-------------|-----|-----|-----|-----|-----|----------|
| $b$ (fm)    | 12  | 7.5 | 7.5 | 4.5 | 4.5 | 0        |
| $N_{IMF}$   | $N_{\text{fragment}} - 2$ | $N_{\text{fragment}} - [1; 2]$ | $N_{\text{fragment}} - 1$ |

The function $\log(1/p)$ versus $1/\sqrt{E_t}$ become smoother, i.e the simple filter used reduces the effects and the results look like the "Arrhenius law". It seems that the filter cancels the former effects in the region of $E_t$. Of course, the real effects of the $4\pi$ detector is more complex, but we can conclude that an important part of the linear aspect of the Arrhenius law is drastically modified by the detection system.

As there is a strong correlation between $E_t$ and $b$ (see figure 3) it is useful to perform the same analysis for the impact parameter instead of $E_t$. From figure 3, we extract on average, that $E_T \propto (\alpha b + b_0)$ with $b_0 = b_{\text{max}}$ and $\alpha < 0$ then $1/\sqrt{E_t} \propto 1/\sqrt{b_0 - \alpha b}$.

Figure 4 shows the results, as a function of $1/\sqrt{b_0 - \alpha b}$ with the same prescription as explained in table I. We can see the same trends as in figure 4. In conclusion, the linear behavior can be due to geometrical effects related to the impact parameter.

In this paper we demonstrated that the linear aspect of Arrhenius plot in the multi-fragmentation regime depends strongly on the acceptance of the detection system. A recent publication [17] shows also the effect of the detection system. The fact that, we can find the linear behaviour with a non-thermal process could not be a justification of thermal nature of multi-fragment emission process. Furthermore, this can be due to the geometrical effect in the reaction including all the impact parameter.

Acknowledgement: we thank E. Plagnol for stimulating discussions.
Figure Caption

Figure 1: Top panel elementary probability $1/p$ versus $1/\sqrt{E_t}$, bottom panel number of attempt $m$ versus $1/\sqrt{E_t}$. The plain line correspond to the INDRA results for Xe + Sn at 50 A.MeV [8], is presented here like "guide the eye" because the x axis $E_t$ is not totally reproduced by QMD code.

Figure 2: Right panel $1/p$ versus $1/\sqrt{E_t}$, left panel $m$ versus $1/\sqrt{E_t}$. The full circle represent the results for QMD with $N_{IMF} = N_{frag} - QP - QT$. The open circle represent the results for QMD with the prescription explain in table I.

Figure 3: Correlation between the impact parameter and the transverse energy in the QMD model for the reaction Xe+Sn at 50 A.MeV. Dashed line, the linear fit between $b$ versus $E_t$ used in figure 2 see text.

Figure 4: Arrhenius plot for QMD events. The x axis is $1/\sqrt{b_{max} - b}$ (see text).
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FIG. 1.
FIG. 2.
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