NON-EXISTENCE OF GRAVITATIONAL WAVES
THE STAGES OF THE THEORETICAL DISCOVERY
(1917-2003)

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ABSTRACT. A short history of the theoretical discovery that the gravitational waves of general relativity do not have a physical reality.

Introduction
I shall recall here the main stages of an important theoretical discovery: the general theory of relativity (GR) does not allow the physical existence of gravitational waves. The solutions of the Einstein field equations which have a wave character describe only formal undulations, quite destitute of a physical reality.

1917
In this year Tullio Levi-Civita published a very fundamental memoir “On the analytic expression that must be given to the gravitational tensor in Einstein’s theory” [1]. The conclusion of the paper is straightforward: the Einstein field equations tell us that when the mass tensor $T_{jk}$ vanishes the same occurrence must happen to the gravitational tensor $(1/\kappa) \left( R_{jk} - \frac{1}{2} g_{jk} R \right)$. “This fact entails total lack of stresses, of energy flow, and also of a simple localisation of energy”.

This result has an unquestionable logical soundness, as it was finally admitted by Einstein himself. Of course, it implies the rejection of the various pseudo (false) energy tensors of the gravitational field proposed by Einstein and by other authors: a false tensor cannot have a true physical meaning!

Einstein objected that in such a way the total energy-momentum of a closed system would always be equal to zero – and this fact would not imply the further existence of the system under whatever form. However, from the standpoint of the coherence of the formalism, Levi-Civita – and Lorentz – [11] were undoubtedly right. It is indeed sufficient to remember that in the action principle of any physical theory referred to general coordinates, the coefficients of the variations $\delta h^{jk}$ of the metric tensor $h^{jk}$ are the components, say $E_{jk}$, of the energy tensor of the considered field. But in GR this property is just possessed, in vacuo, by the tensor $R_{jk} - \frac{1}{2} g_{jk} R$.

The gravitational waves, as objects without a true energy-momentum, are only ghost undulations.
1930

In this year Tullio Levi-Civita published an original study on the characteristic hypersurfaces of Einstein field equations \[2\].

He discovered that the functions \( z(x), \quad [x \equiv (x^0, x^1, x^2, x^3)] \), of the characteristic hypersurfaces \( z(x) = 0 \) of Einstein field equations are solutions of the Hamiltonian equation

\[
H := \frac{1}{2} g^{jk}(x) \frac{\partial z(x)}{\partial x^j} \frac{\partial z(x)}{\partial x^k} = 0
\]

According to Levi-Civita, the equation \( z(x) = 0 \) gives the law of motion of an electromagnetic wave front – or of the wave front of any field, capable of transmitting signals, different from the gravitational field. This interpretation is quite obvious when \( g^{jk}(x) \) has a non-undulatory form. If \( g^{jk}(x) \) has a wavy form, there is no reason to repudiate the above interpretation because the undulatory character of \( g^{jk}(x) \) depends on the chosen system of co-ordinates \[3\].

Remark that Levi-Civita’s conception is the reasonable extension of that valid for the null lines of special relativity. Thus, also GR contains the basic law of geometric optics – and independently of Maxwell equations.

1953

By means of perturbative computations, Scheidegger \[4\] could affirm that “...having explicitly shown that all the radiation terms [of the gravitational field] whatsoever can be destroyed by coordinate transformations, one observes that the terms that have been found by straightforward calculations must be entirely due to the particular choice of the coordinate system. Thus there is no radiation damping of gravitational motion”. But no damping means no emission of gravitational waves.

1960

In 1960 it was published an interesting book by Infeld and Plebanski \[5\]. At pages 200 and 201 we read: “...it is hardly possible to connect any physical meaning with the flux of energy and momentum tensor defined with the help of the pseudo-energy-momentum tensor. Indeed, the [gravitational] radiation can be annihilated by a proper choice of the coordinate system. On the other hand, if we use a coordinate system in which the flux of energy may exist, then it can be made whatsoever we like by the addition of proper harmonic functions ... – In the linear theory we were faced with the choice between the retarded and advanced potential. Here in the theory of gravitation the choice is not so simple. Using the approximation procedure, we are faced with the choice between single and double jumps. We can speak only about [gravitational] radiation in the case of single jumps. However, its existence or non-existence or its value will depend upon the choice of arbitrary harmonic functions.” We see that also Infeld and Plebanski were quite sceptical about the real existence of the gravitational waves.
1998-2003

In these years I have published several proofs of the non-existence of the GW’s; they are exact, non-perturbative proofs \[6\]. I recall here only two demonstrations, which are particularly simple.

i) let us assume that at a given instant \( t \) of its motion a given point mass \( M \) begins to send forth a GW, and let us suppose that we know the kinematical characteristics of the motion between \( t \) and \( t + |d|t| \). Then, we can reproduce these characteristics in a purely gravitational motion of \( M \) in a suitable “external”, “rigid” gravitational field, within a time interval equal to \( |d|t| \), conveniently chosen. But in this case the mass \( M \) moves along a geodesic – and therefore it cannot emit any gravitational radiation: indeed, the geodesic motions are “free” motions; they are the analogues of the rectilinear and uniform motions of an electric charge of the customary Maxwell-Lorentz theory.

Thus we see that no “mechanism” exists for the generation of gravitational waves – the above restriction to motions of mass points is conceptually inessential. All the solutions of the Einsteinnian field equations having an undulatory character do not describe physical waves \[7\].

ii) As it is well known, in GR only the concepts and the results that are independent of the choice of the system of general co-ordinates have a physical meaning. Consider a solution of the Einstein field equations which has – in a given co-ordinate system – a wavy character. Through a finite sequence of co-ordinate transformations, endowed with convenient undulatory properties, the primary undulating character of our solution can be completely destroyed. Thus, this character is only a property of the original co-ordinate system, and therefore it has no physical meaning. (According to a metropolitan legend, Bondi et al. – cf., e.g., H. Bondi, F.A.E. Pirani and I. Robinson, Proc.Roy.Soc., A251 (1959) 219 – would have proved the existence of a class of privileged frames insofar as the GW’s are concerned. Now, their “proof” is fully destitute of logical rigour, it is the mere expression of a desire. See at p.55 of my book quoted in \[6\].)

I remark further that the propagation velocity of any metric tensor depends on the reference system: with a suitable choice of general co-ordinates, this velocity can take any value between zero and infinite.

APPENDIX

On the linear approximation of GR

If we restrict ourselves to the linear approximation of GR – as the experimentalists generally do –, which has Minkowski spacetime as its substrate, the physical existence of the GW’s seems, at first sight, a theoretical possibility. But the energy-momentum of such GW’s has a tensor character only under Lorentz transformations, not under general transformations. Therefore, it is always possible to find – and we remain, of course, in the ambit of the linearized version of GR – a general system of co-ordinates for which the above energy-momentum is equal to zero.

In 1944 Weyl published a remarkable article entitled “How far can one get with a linear field theory of gravitation in flat space-time?” \[8\]. He remarked, in particular, that Einstein’s theory of weak gravitational fields (i.e.,
the linear approximation of GR) resembles very closely Maxwell’s theory of the e.m. fields, and satisfies a principle of gauge invariance involving four arbitrary functions, but its gravitational field exerts no force on matter, i.e. it remains “a powerless shadow”. From the standpoint of the exact GR, this is as it should be, because “the gravitational force arises only when one continues the approximation beyond the linear stage”. Clearly, Weyl alludes here a fundamental result of the EIH-method \[9\]. Thus, we find another argument – and a strong argument – against the physical adequacy of the linearized version of GR insofar as the question of the GW’s is concerned.

I am very grateful to Prof. A. Gsponer, who has called my attention to Weyl’s paper.

PARERGON

On the PSR 1913+16

The overwhelming majority of the astrophysicists believe that the time decrease of the revolution period of the binary radiopulsar PSR 1913+16 gives an experimental (indirect) proof of the physical reality of the gravitational radiation. As a matter of fact, the perturbative quadrupole formula gives a decrease of the revolution period which agrees very well with the observational data.

I emphasize the following points. i) In the exact theory the quadrupole formula loses any meaning because the hypothesized gravitational waves do not have a true energy; therefore, the true mechanical energy which is lost during the revolution motion ought to transform itself into the pseudo (false) energy of the hypothetical gravitational radiation: the energy account does not balance. ii) Many observational astrophysicists know that realistic explanations of the decrease of the revolution period are quite possible: for instance, viscous losses of the pulsar companion give a decrease of the same order of magnitude of that given by the alleged emission of gravitational waves. iii) The empirical success of a given theory – or of a given computation – is not an absolute guaranty of its conceptual adequacy: for instance, the Ptolemaic theory of cycles and epicycles explained very well the planetary orbits (with the only exception of Mercury’s).

The serious scientists should abstain from wishful thinking.

“The king is naked! – cried a child”.

(From an Andersen’s tale)

References

[1] T. Levi-Civita, Rend. Acc. Lincei, 26 (1917) 381. An English translation in arXiv:physics/9906004 (June 2nd, 1999). See also H.A. Lorentz, Amst. Versl., 25 (1916) 468.
[2] T. Levi-Civita, Rend. Acc. Lincei, 11 (s.6a) (1930) 3 and 113.
[3] A. Loinger, arXiv:astro-ph/9906058 (June 3rd, 1999). Idem, Spacetime & Substance, in course of publication; title of this Note: “Again on the non-existence of GW’s and of BH’s”; also in arXiv:physics/0311075 v1 (Nov. 16th, 2003).
[4] A.E. Scheidegger, Revs. Mod. Phys., 25 (1953) 451.
[5] L. Infeld and J. Plebanski, Motion and relativity, (Pergamon Press, Oxford, etc.) 1960, see in particular Chapt.VI.
[6] A. Loinger, Nuovo Cimento B, 115 (2000) 679; Idem, Spacetime & Substance, 3 (2002) 129; Idem, ibidem 3 (2002) 145; Idem, On Black Holes and Gravitational Waves, (La Goliardica Pavese, Pavia) 2002, Part II. See also Idem, arXiv:gr-qc, arXiv:astro-ph, arXiv:physics (1998-2003).

[7] For a bibliography on the GW’s see, e.g., the review article by B.F. Schutz, Class. Quantum Grav., 16 (1999) A131. This paper is a summa of all the current (and erroneous) opinions about the GW’s.

[8] H. Weyl, Amer. J. Math., 66 (1944) 591.

[9] Cf. e.g. L. Infeld, Acta Phys. Polonica, 13 (1954) 187 and the book by Infeld and Plebanski cited in [5].