Gravimetric Radar:Gravity-Based Detection of a Point-Mass Moving in a Static Background

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This paper discusses a novel approach for detecting moving massive objects based on the time variation that these objects produce in the local gravitational field measured by several detectors. Such an approach may provide a viable method for detecting stealth aircraft, UAVs, cruise, and ballistic missiles. By inverting a set of nonlinear algebraic equations, it is possible to use the time variation in the gravitational fields to compute the mass, position, and velocity of one or more moving objects. The approach is essentially a gravity-based form of triangulation. Based on order-of-magnitude calculations, we estimate that under realistic scenarios, this approach will be feasible if it is possible to design gravimetric devices that are four to five order of magnitude more sensitive than current devices. To achieve such a level of sensitivity, we suggest designing detectors that exploit a quantum-mechanical effect known as gravity-induced quantum interference. Furthermore, even if we have a perfect detector, it will be necessary to determine the magnitude of various atmospheric disturbances and other sources of noise.

Keywords: Gravity, gravity-induced quantum interference, stealth, detector, signal processing

I. THE PROBLEM OF MASS DETECTION

Since its introduction during World War II, radar (radio detection and ranging) has been the standard method for the detection of moving objects. The future efficacy of radar for military applications is being called into question, as many countries have become interested in developing weapons systems that employ various technologies to evade radar detection. Currently, only the United States has radar-avoiding, or “stealth,” aircraft in service, namely the F-22 Raptor and the B-2 Spirit (the first stealth-capable fighter-bomber, the F-117 Nighthawk, is no longer in service, while the F-35 Joint Strike Fighter is still under development). However, other countries are currently working on developing stealth-capable aircraft. Notably, Russia is currently developing the T-50 fighter jet as a rival to the F-22, with a planned operational deployment sometime around 2015.

The acquisition and development of stealth technologies by rivals of the United States presents a potential threat to the United States and her interests. Thus, in the near future, it will be necessary to develop new, stealth-resistant, methods for detecting aircraft and other moving objects, such as UAVs, cruise and ballistic missiles. Such detection methods will both provide warning time to prepare for an impending attack, as well as to identify the location of a threat, which may then be destroyed with appropriate counter-measures.

A variety of detection methods other than traditional radar-based methods are possible. First of all, some stealth technology works by reflecting the incoming radar beam away from the source. While the source radar cannot detect the incoming object, using multiple station radar allows for the reception of this diverted radar signal, which may
then be used to determine the location of the aircraft via triangulation. The problem with this approach is that it will not work for stealth aircraft that achieve their radar-evading capability by absorbing the incoming radar beam, using so-called radar-absorbing materials, or RAM. Another approach relies on detecting the heat signature of an aircraft using infrared sensors. Once again, the problem with this approach is that modern stealth aircraft are generally designed to minimize their heat signatures, making this approach problematic. A third approach involves using high resolution optical imaging to directly observe the moving object in the visible spectrum. While this approach may be viable for the time being, at least for detection of aircraft during daytime, there are currently research efforts underway to design “cloaking” devices that can bend light around an object and make the object appear transparent. Such devices are based on “meta-materials,” whose optical properties can be controlled by appropriate design of their internal structure. Finally, a fourth approach involves detecting the sound made by an approaching aircraft, UAV, or missile. If the object is subsonic, then this approach is feasible, as the sound wave generated by the object arrives at the detector before the object itself. However, for supersonic objects, the sound wave will arrive at the detector after the object, rendering this approach useless.

Here, we propose an alternative method for the detection of moving objects. This method exploits the fact that all massive objects generate a gravitational field, and that a moving object will lead to a time-varying gravitational field that can be measured at various points. By measuring this time-varying field at a sufficient number of points, it is possible to obtain the mass, position, and velocity of the object by solving a system of nonlinear algebraic equations. This approach has an advantage over other detection methods, in that, because it is impossible to hide or shield a gravitational field, this method should be much more difficult, if not impossible, to counter, than other methods.

There are two main drawbacks and one limitation with this method. The first drawback is that it requires the ability to detect gravitational fields that are four to five orders of magnitude weaker than what is possible with current gravimetric devices. The second drawback is that, even with a perfect detector, the gravitational signal generated by the moving object of interest may be masked by effects such as atmospheric disturbances and clutter due to the random motion of various other objects (e.g., cars, animals, etc.).

A potential limitation of this method is that it may not work well for ships and submarines that displace a mass of water equal to their own mass. The reason for this is that a variation in the gravitational field is generated by variations in the mass density distribution. If a moving ship or submarine simply displaces an equal mass of water, then the mass distribution may not change sufficiently to lead to a detectable signal. Thus, this mass detection method, if feasible, is likely only to be applicable to massive objects that travel on the ground or in the air.

This paper contains the basic theory underlying a gravity-based approach for mass detection. We will discuss various ways to deal with anticipated drawbacks of this method. In particular, we will propose an initial set of studies to determine whether the method is at all feasible. Therefore, the present work has the nature of an entire research program. A full journal version with details of the relevant physical background may be found in [9].

II. GRAVITY-BASED DETECTION OF MOVING OBJECTS: THEORY

According to Newton’s Universal Law of Gravitation, two point-objects of mass \( m_1 \) and \( m_2 \) interact through a gravitational force of magnitude \( F = \frac{Gm_1m_2}{r^2} \), where \( r \) is the distance between the objects, and \( G \) is the gravitational constant, which in SI units is \( 6.67300 \times 10^{-11}[\text{meters}^3\text{kg}^{-1}\text{s}^{-2}] \). The force is purely attractive, so that it is directed along the line connecting the two objects.

For more than two point masses, the gravitational force acting on a given mass is simply the sum of all the forces due to the interactions with each of the other masses. Generalizing to a mass distribution, we obtain that the gravitational
field at each detector, then we require at least 3

\[ g(\vec{x}, t) = G \int_{R^3} d^3 \vec{x}' \rho(\vec{x}', t)(\vec{x}' - \vec{x}) / ||\vec{x}' - \vec{x}||^3 \]  

where \( g(\vec{x}, t) \) is the gravitational field at the point \( \vec{x} \) at the time \( t \), and \( \rho \) denotes the mass distribution function. Note that we include an explicit time dependence into our formula, since our mass distribution may be time-dependent, which will then generate a time-varying gravitational field.

Now, we may partition the mass distribution into a “background,” consisting of the Earth and atmosphere, and the “object,” consisting of the object, or objects, to be detected. We assume that the time-variation of the mass distribution is due to the object component, and so we may write,

\[ g(\vec{x}, t) = g_{\text{background}}(\vec{x}) + g_{\text{object}}(\vec{x}, t) \]

Differentiating both sides of the equation with respect to time, we obtain that,

\[ \frac{\partial g}{\partial t} = \frac{\partial g_{\text{object}}}{\partial t} \]

This equation implies that the time-variation of the local gravitational field is due to the object alone. For a single moving point-object of mass \( M \), located at \((x, y, z)\), and, assuming a detector located at the coordinates \((x_i, y_i, z_i)\), we have,

\[
\begin{align*}
g_{\text{object}, x}(x_i, y_i, z_i, t) &= GM \frac{x - x_i}{[(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2]^{3/2}} \\
g_{\text{object}, y}(x_i, y_i, z_i, t) &= GM \frac{y - y_i}{[(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2]^{3/2}} \\
g_{\text{object}, z}(x_i, y_i, z_i, t) &= GM \frac{z - z_i}{[(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2]^{3/2}}
\end{align*}
\]

Differentiating with respect to time, we obtain,

\[
\begin{align*}
\dot{g}_x(x_i, y_i, z_i, t) &= GM \frac{-2(x - x_i)^2 p_x + (y - y_i)^2 p_y + (z - z_i)^2 p_z - 3(x - x_i)(y - y_i)p_y - 3(x - x_i)(z - z_i)p_z}{[(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2]^{5/2}} \\
\dot{g}_y(x_i, y_i, z_i, t) &= GM \frac{-2(y - y_i)^2 p_y + (z - z_i)^2 p_z + (x - x_i)^2 p_x - 3(y - y_i)(z - z_i)p_z - 3(y - y_i)(x - x_i)p_x}{[(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2]^{5/2}} \\
\dot{g}_z(x_i, y_i, z_i, t) &= GM \frac{-2(z - z_i)^2 p_z + (x - x_i)^2 p_x + (y - y_i)^2 p_y - 3(z - z_i)(x - x_i)p_x - 3(z - z_i)(y - y_i)p_y}{[(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2]^{5/2}}
\end{align*}
\]

where \( \vec{p} = (p_x, p_y, p_z) := (M\dot{x}, M\dot{y}, M\dot{z}) \) is the momentum of the object.

Note that, if we know the position and momentum coordinates of the object, then we can obtain the velocity by differentiating the position, and from there we can compute the mass. Therefore, the motion of the object is completely characterized by six coordinates. Since each detector provides three pieces of information about the object, namely, the time variation of the gravitational field along each of the coordinate axes, with at least two detectors it is possible to solve a system of six nonlinear algebraic equations and determine the position and momentum of the object. As we will discuss later in this paper, for certain technical reasons we may choose to only make use of the \( x \) and \( y \) components of the gravitational field, in which case at least three detectors will be necessary.

The above approach is in principle readily generalizable to the detection of an arbitrary number of \( N \) point-objects. Since each object is characterized by six parameters, \( N \) objects are characterized by \( 6N \) parameters. If we use all the components of the gravitational field at each detector, then since each detector provides us with three measured parameters, we require at least \( 2N \) detectors. If instead we only use the \( x \) and \( y \) components of the gravitational field at each detector, then we require at least \( 3N \) detectors. In this paper, since we are considering 1 point-object, and we will use all the components of the gravitational field at each detector, we will analyze the case of 2 detectors.
III. NUMERICAL SOLUTION OF THE SINGLE OBJECT CASE

We will provide exact details of the solution of Equations (5) for the case of a single object and a single object in Sections VII A and VII B. Here we give an overview of these methods for the convenience of the reader.

We may solve the nonlinear system of equations (5) using Newton-Raphson Iteration, or Newton’s Method. Because the object is moving along a continuous path, denoted \( \vec{x}(t) \), then, if we know the position and momentum of the object at some time \( t \), we may use this information as the initial guess for determining the position and momentum of the object at time \( t + \Delta t \). If \( \Delta t \) is sufficiently small, then the object’s position and momentum will have changed by a sufficiently small amount that this initial guess will converge to the object’s position and momentum at time \( t + \Delta t \).

Continuing in this way, the object’s position and momentum at one time may be used as the initial guess to obtain the object’s position and momentum at some time in the near future. The result is that Newton’s Method may be used to readily track the movement of the object. However, this requires some initial position and momentum for the object, i.e., it is necessary to first acquire the object. With object, or target, acquisition, we do not have any kind of a priori information on the object that can be used as a good initial guess for Newton’s Method. Thus, target acquisition is much more difficult than target tracking, because the initial guess is far likelier to significantly deviate from the true position and momentum of the object. To get around this problem, and to enable target acquisition with a fairly "bad" initial guess, we use a method that we call Newton-Raphson Iteration with Solution Deformation, which is discussed in Section VII B.

We place detectors at \((\pm d/2, 0, 0)\) and \((0, \pm d/2, 0)\), where \( d \) denotes the detector spacing. The reason for this is that we have found that target acquisition using our modified Newton’s Method only occurs if the target is located within a region defined by the line connecting a given detector pair. For the pair of detectors located at \((\pm d/2, 0, 0)\), this region is defined by \( \{ (x, y, z) : |y| \leq |x| \} \) which is illustrated in Figure 1. Therefore, in order to obtain target acquisition independently of the initial target location, we also place detectors at \((0, \pm d/2, 0)\), which has a region of convergence given by \( \{ (x, y, z) : |y| > |x| \} \), which is also indicated in Figure 1.

For each pair of detectors, we consider two initial guesses, corresponding to the two sectors defining the region of convergence for the detector pair. For the pair of detectors at \((\pm d/2, 0, 0)\), we consider the initial guesses \((x_1, x_2, x_3, p_1, p_2, p_3) = (x_0, 0, 0, -p_0, 0, 0), (-x_0, 0, 0, p_0, 0, 0)\), where the first initial guess is used for objects in the set...
\{(x, y, z) : x > 0, |y| \leq x\} while the second initial guess is used for objects in the set \{(x, y, z) : x < 0, |y| \leq -x\}.

Similarly, for the pair of detectors at \((0, \pm d/2, 0)\), we consider the initial guesses \((0, x_0, 0, 0, -p_0, 0)\) and \((0, -x_0, 0, 0, p_0, 0)\). Note that this leads to a set of four initial guesses with which to acquire the target, at two guesses per pair of detectors. The algorithm cycles through the four initial guesses and associated detector pairs until the target is acquired.

If the acquisition fails after going through the entire object track, the algorithm stops and indicates that this is the case. Otherwise, the procedure begins to track the object using the detector pair with which the object was acquired, and indicates the time at which the target was acquired. Target tracking continues until the program cycles through the entire input object track, or until target acquisition is lost. At this point, our proposed scheme attempts to re-acquire the target using the target acquisition component of our algorithm. If target re-acquisition is achieved, the algorithm indicates when this happened, and continues with target tracking. Target re-acquisition and tracking continues as necessary until the procedure has run through the entire object track.

The object track is input as a series of waypoints, specifying the location of the object at various time intervals. In the computer implementation of our proposed methodology, the user specifies a certain number of waypoints, and provides the \((x, y, z)\) coordinates for each waypoint. The user also specifies a time interval \(\Delta t\) and a time interval number \(\text{Num}_{\Delta t}\), so that the total time between waypoints is \(T_{\text{WayPoint}} = \text{Num}_{\Delta t} \times \Delta t\). We assume a constant velocity between waypoints, so that the successive locations of the waypoints, as well as the value of \(T_{\text{WayPoint}}\) may be readily used to compute the velocity of the object. Combined with the user-input object mass, this allows us to compute the momentum of the object between waypoints.

IV. SIMULATIONS

We consider three scenarios: In the first case, an object starts at some distance \(R\), with some altitude \(h\), and heads directly for the origin at a constant velocity \(v\). The initial coordinates of the object are given by \((R\cos \theta, R\sin \theta, h)\), and the velocity vector of the object is \((-v\cos \theta, -v\sin \theta, 0)\). This flight path corresponds to a “raid” scenario on a target at the origin. The results of our target acquisition and tracking algorithm are shown in Figures 2 and 3 (parameters are provided in the figure captions). In the second case, an object first moves in a straight line toward a point near the origin, and then moves away, in a ”zigzag” flight pattern, corresponding to a ”reconnaissance” profile. The results of our target acquisition and tracking algorithm is shown in Figures 4 and 5 (parameters are provided in the figure captions).

Finally, in the third case, we consider an object that moves in a square or diamond flight pattern around the origin with constant speed. The results of our target acquisition and tracking algorithm is shown in Figures 6 and 7 (parameters are provided in the captions).

V. GRAVITY-INDUCED QUANTUM INTERFERENCE

In order for gravity-based detection to emerge as a practical method for detecting moving objects, it will be necessary to develop devices that can detect gravitational fields several orders of magnitude weaker than what is possible with current instruments. An accessible discussion about about quantum interference may be found in [2]. To illustrate, suppose we wish to detect an object with a mass of 100[metric tonnes] = \(10^5\)[kg], at a distance of 100[km] flying at a speed of \(10^3\)[km/hr] \(\equiv 278\)[meters/s] (these parameters are based on those from an aircraft such as the B-2 Spirit). Assuming that this object is headed directly toward the detector, such an object will lead to a local fluctuation in the gravitational field of approximately \(7 \times 10^{-16}\) [meters/s²], and a time variation in the local gravitational field of
FIG. 2: Comparison of actual object tracks (solid line) versus computed object track (dots) for the “raid” scenario. Object mass = 100, with two waypoints, at (70, 70, 10) and (0, 0, 10). We took a value of $\Delta t = 0.01$ and $T_{\text{WayPoint}} = 10$, giving an object velocity of 990, and a momentum of 99,000. We took a detector spacing $d = 1$, and initial guess parameters $x_0 = 50$, $p_0 = 10,000$. We used 100 sub-intervals for the solution deformation implementation of the target acquisition algorithm. The target was immediately acquired at its initial position, but target acquisition was lost at $t = 0.08$.

![Graph of object tracks](image)

FIG. 3: Comparison of actual object track (solid line) versus computed object track (dots) for the “raid” scenario. All parameters are identical as for Figure 2 except that now we took $\Delta t = 0.001$ and $Num_{\Delta t} = 100$. Here target acquisition was immediate and never lost.

![Graph of object tracks](image)

approximately $4 \times 10^{-18}[\text{meters/s}^3]$. The most sensitive gravimetric device to date is the superconducting gravimeter, which is capable of measuring changes of $10^{-11}[\text{meters/s}^2]$ in the local gravitational field [7]. This is about four orders of magnitude larger than what is required to detect a moving object with the characteristics given above. Clearly, then, to make our gravity-based detection method practical, we will need to develop gravimetric devices that are far more sensitive than what is currently available. One possible approach for the development of a gravimetric device with
FIG. 4: Comparison of actual object track (solid line) versus computed object track (dots) for the “reconnaissance” profile. Object mass = 100, with three waypoints, at (70, 70, 10), (10, 0, 10), and (70, −70, 10). We took a value of $\Delta t = 0.01$ and Num$_{\Delta t} = 10$. We took a detector spacing of $d = 1$, and initial guess parameters $x_0 = 50$, $p_0 = 10,000$. We used 100 sub-intervals for the solution deformation implementation of the target acquisition algorithm. The target was immediately acquired at its initial position. Target acquisition was lost at $t = 0.09$, but re-acquired at $t = 0.12$.

FIG. 5: Comparison of actual object track (solid line) versus computed object track (dots) for the “reconnaissance” profile. All parameters are identical as for Figure 4 except that now we took $\Delta t = 0.001$ and Num$_{\Delta t} = 100$. Here target acquisition was immediate and never lost.

the required sensitivity relies on a quantum-mechanical effect known as \textit{gravity-induced quantum interference}.

Gravity-induced quantum interference is an interference phenomenon that occurs when a particle interferes with itself after traveling along two paths with differing potential energies in a gravitational field. If a particle is introduced into a waveguide as illustrated in Figure 4 then the particle may either travel first along the bottom path (path $AB$) and then up toward the interference region (path $BC$), or first along path $AD$ and then toward the interference region (path $DC$).
FIG. 6: Comparison of actual object track (solid line) versus computed object track (dots) for a square flight profile. Object mass = 100, with five waypoints, at (70, 70, 10), (−70, 70, 10), (−70, −70, 10), (70, −70, 10), and (70, 70, 10). We took a value of \( \Delta t = 0.01 \) and \( \text{Num}_{\Delta t} = 10 \). We took a detector spacing of \( d = 1 \), and initial guess parameters \( x_0 = 50 \), \( p_0 = 10,000 \). We used 100 sub-intervals for the solution deformation implementation of the target acquisition algorithm. The target was immediately acquired at its initial position, and tracked over the entire time of the object track.

Because of the potential energy difference between paths \( AB \) and \( DC \), the classical momentum of a particle moving along the two paths differs, resulting in an accumulated phase difference. The result is that, when the particle interferes with itself in the interference region at point \( C \), the probability density is characterized by an interference pattern of length \( L = h \nu/(ma\Delta x) \), with a time-variation given by \( \dot{L} = -L^2 m \Delta x \dot{a}/(h \nu) \). Here, \( h = 6.626068 \times 10^{-34} \) [meters^2 kg/s] is Planck’s constant, \( \nu \) is the classical velocity of the particle moving through the detector, \( m \) is the particle mass, \( a \) is the gravitational acceleration along the width of the detector (paths \( AD, BC \)), and \( \Delta x \) is the height of the detector. Now, taking \( \Delta x = 1 \) [meter], and using parameters obtained from our sample point object modeled after the B-2 Spirit, we obtain that \( L = 10^{18} (\nu/m) \) [meters] (taking into account the units of the other factors as well), and \( \dot{L} = 5.5 \times 10^{15} (m/\nu) \) [meters/s]. If \( m/\nu = 10^{18} \) [kgs/meters], we obtain that \( L \equiv 1 \) [meter], and \( \dot{L} \equiv 5.5 \) [mm/s]. These parameters are within readily measurable limits, so that, if we can design a gravity-based interferometer that produces an interference effect characterized by these parameters, then it should be possible to design a gravimetric device with the necessary sensitivity.

One approach for obtaining the desired mass to velocity ratio is to take an object with a mass of \( 10^6 \) [amu] moving at 1.7 [mm/sec]. Such an object has a mass comparable to that of a virus. While it may seem counter-intuitive or difficult to exploit quantum interference effects with such massive objects, it should be noted that the double-slit diffraction experiment has been pushed considerably beyond electrons to small molecules. There is currently active research on creating quantum superpositions of even more massive objects such as viruses, and perhaps even as large as 1 millimeter. Of course, it is necessary to cool such objects to near absolute zero in order to ensure that they are in their ground quantum state, thereby preventing a phenomenon known as decoherence from destroying quantum superposition effects. Furthermore, in order to exploit these effects to create a usable device, it will be necessary to create a steady particle beam out of these comparatively massive objects. Clearly then, while the design of ultra-sensitive gravimeters based on gravity-induced quantum interference is not necessarily an idea based in science fiction, it is nevertheless at the outer edge of our current technological capabilities. A second possibility would be
FIG. 7: Comparison of actual object track (solid line) versus computed object track (dots) for a diamond flight profile. Object mass = 100, with five waypoints, at (100, 0, 10), (0, 100, 10), (−100, 0, 10), (0, −100, 10), and (100, 0, 10). We took a value of $\Delta t = 0.01$ and Num$\Delta t = 10$. We took a detector spacing of $d = 1$, and initial guess parameters $x_0 = 50$, $p_0 = 10,000$. We used 100 sub-intervals for the solution deformation implementation of the target acquisition algorithm. The target was immediately acquired at its initial position, and tracked over the entire time of the object track.

FIG. 8: A particle moving along path $ADC$ experiences a phase shift relative to the particle moving path $ABC$, due to a difference in potential energy between paths $DC$ and $AB$, given by $ma\Delta x$, where $a$ denotes the local acceleration due to the local gravitational field.

to use what is known as a coherent matter wave, or “matter laser,” generated from a Bose-Einstein condensate. A Bose-Einstein condensate, or BEC, is a phase of matter whereby all of the particles are in the ground energy state. Because the particles of a BEC are in the same quantum state, the BEC exhibits strong quantum superposition effects at macroscopic scales.

A BEC fluid cannot be treated using classical fluid mechanics, rather, a purely quantum-mechanical approach must be adopted. In analogy with coherent light that is used to create a laser, researchers are interested in using BECs to create particle beams that are essentially coherent matter waves, i.e., a “matter laser.” Such a matter laser could form the “working fluid” of our proposed gravimetric device. In particular, in analogy to laser physics, the interference of two individual Bose-Einstein condensate wavefunctions demonstrates multiparticle interference with matter waves;
see [1, 2, 6] and the references therein. This body of work demonstrates that Bose condensed molecules are “laser like”: they are coherent and show long-range correlations. Indeed, the first BEC was achieved with Rubidium atoms in 1995 [1], cooled to 170[nK]. Rubidium has an atomic weight of approximately 85[au], so that, in order to achieve the desired mass to velocity ratio for our device as described above, we require a particle velocity of $1.4 \times 10^{-7}$[m/s]. Using the de Broglie formula, this corresponds to a particle wavelength of $\lambda = h/(mv) = 3.4$[cm]. Since this corresponds to the ground state of a single-particle wavefunction, the condensate would have to be created in a box with a length on the order of 1–10[cm].

In SI units, the critical temperature for condensate formation is given by,

$$T_c = 2.67 \times 10^{-45} n^{2/3}/m$$

where $n$ is the particle density and $m$ is the mass per particle. A condensate temperature of 1[$\mu$ K] then requires a particle density of $n = 4 \times 10^{20}$[particles/m$^3$] = 4 $\times$ 10$^{14}$[particles/cm$^3$]. For a condensate temperature of 1[n K], which is on the order of the lowest achievable temperature to date, a particle density on the order of 10$^9$[particles/cm$^3$] is required. Given the required dimensions of the container in which our BEC is to be created, this means that it will be necessary to create a BEC with on the order of a minimum of 10$^9$ particles. Given that the largest BECs to date have been achieved using on the order of 10$^6$ particles [6], it is clear that atom cooling and BEC technologies will have to be developed some more before our proposed gravimetric device is feasible.

Further, it has recently been experimentally demonstrated [6] that it is possible to split gaseous Bose-Einstein condensate into two coherent condensates by deforming an optical well where the condensate was trapped. Experiments analogous to what is envisioned for the proposed device have been performed and the two condensates were brought together at which point a matter-wave interference pattern was observed. The coherent condensates were separated for a duration of 5[ms] by 13[$\mu$m] and by 80[$\mu$m] in [6, 8]. The spatial scales in these experiments have been several orders of magnitude smaller than the scales envisioned for the proposed gravimeter. However, cryogenic technology exists, and is developing fast, that is expected to make the necessary space scales feasible. The temperatures that are needed to be maintained along the path of the beam are of the order of 1[$\mu$K] while the present-day record for achievable low temperatures is below 1[nK], i.e., the record is more than two orders of magnitude lower than the temperature that a gravitometer will require. This is very encouraging although the needed conditions will have to be maintained for the substantial length that the matter wave would need to traverse.

Finally, it should be emphasized that each gravimetric device will actually consist of up to three interferometers, which will separately measure the gravitational field in the $x$, $y$, and $z$ directions, respectively. The one potential issue with using the $z$ direction is that the gravitational field in this direction is already relatively strong, since it is the direction of the Earth’s gravitational field. A BEC or virus-based detector aligned along this field may be too sensitive, and could essentially be overwhelmed by the strength of the signal (i.e., the interference pattern will be characterized by parameters that will make it difficult to measure). Using an alternative working fluid, such as a neutron beam, for this direction, will allow one to measure the gravitational field in the $z$ direction, however, it will not have the required sensitivity to measure the extremely weak fluctuations generated by a moving object. As a result, it may be preferable to only measure the time-variation in the local gravitational fields in the $x$ and $y$ directions. Then, instead of requiring two detectors per moving object, it will be necessary to use three detectors.

**VI. SIGNAL PROCESSING**

Clearly, it is essential to determine whether or not our method for detecting moving objects will work in the presence of various disturbances that can introduce noise into the gravitational signal at the detectors. There are two major
sources of noise that will need to be filtered out from the gravimetric signal. The first major source of noise is due to moving objects that are not of interest, such as civilian traffic and the movement of local wildlife. The second is noise due to atmospheric disturbances. This includes weather phenomena such as wind, precipitation, clouds, and even simple convection patterns. There are several complementary approaches to dealing with these sources of noise, all of which will need to be considered in any feasibility study. First of all, a moving object with a flight profile that would make it of interest for detection will likely generate a gravimetric signal that has distinct characteristics from the sources of noise described above. Determining the defining features of this signal, along with the defining features of the various sources of noise described above, will allow us to design appropriate digital filters for extracting the gravimetric signal generated by the moving object itself. In particular, for certain types of moving objects, such as civilian traffic, it may be possible to detect individual cars and airplanes using optical or radar trackers. This could be done within a certain radius of the detectors where their signal could be expected to significantly distort the gravimetric signal generated by the object to be detected. By accounting for the gravitational field generated by these objects, it will be possible to filter out the contribution that these objects make to the gravimetric signal at the detectors. We will also explore general distributed models of civilian traffic and wildlife, which is not directly detectable by other means, that could affect the gravimetric signal, especially if the detectors are placed next to major cities. Such civilian traffic should generate a relatively constant signal with low frequency noise. In fact, one should expect relative constancy of motion and slow speeds for many typical non-military sources. Filtering out the effect of such additional background sources from a typical military target should be achievable quite effectively using standard bandpass filtering techniques in the frequency domain. Next, it should be noted that, for the case of general atmospheric disturbances, the overall effect on the gravimetric signal is expected to be relatively weak. The reason for this is that the gravitational field is determined entirely by the mass distribution, and not by the velocity field associated with the distribution. Therefore, even if there are moving masses of air and precipitation, the overall change in the mass distribution may be sufficiently small as to have a comparatively small effect on the gravitational field.

The precise contribution of disturbances such as weather fronts and wind gusts, albeit small, is expected to be more challenging. In order to model such disturbances, we will need to work with fluid dynamic models that describe atmospheric phenomena. For our purposes, such models do not need to be overly sophisticated, as our goal is not to predict specific weather patterns, but rather to characterize the gravimetric signal generated by such phenomena, in order to develop an appropriate filter to detect and remove them. An appropriate class of such models has been proposed in the meteorology literature based on the movements of weather fronts and using the theory of mass transport \[3\]. These models are very easy to implement on computer \[4\], and could allow us to make the necessary differentiation of the signature of a moving (compressible) mass of air as opposed to that of a rigid object such as an aircraft. We anticipate the spectral content of these two signals to be significantly different, thereby allowing statistical analysis and the relevant filtering techniques to be brought to bear on the problem.

VII. DISCUSSION OF NUMERICAL METHODS

We review some of the numerical methods that were employed in our simulations of Section IV.

A. Newton-Raphson Iteration

The goal behind Newton-Raphson Iteration, or simply Newton’s Method, is to solve a nonlinear system of equations \( \vec{F}(\vec{a}) = \vec{x} \). The idea behind the method is to choose an initial guess, denoted \( \vec{x}_0 \), that is reasonably close to the actual
solution, which we denote by $\vec{x}_{sol}$. If this is the case, then we may make a first-order approximation and write,

$$\vec{a} = \vec{F}(\vec{x}_{sol}) = \vec{F}(\vec{x}_0) + D\vec{F}(\vec{x}_0)(\vec{x}_{sol} - \vec{x}_0),$$

and so we may solve for $\vec{x}_{sol}$ by solving the linear system $D\vec{F}(\vec{x}_0)(\vec{x}_{sol} - \vec{x}_0) = \vec{a} - \vec{F}(\vec{x}_0)$.

In reality, however, the value obtained for $\vec{x}_{sol}$ using the above procedure will not give the actual value for $\vec{x}_{sol}$, but some other value, denoted $\vec{x}_1$. The reason for this is that the first-order Taylor expansion is not exact. Nevertheless, if $\vec{x}_0$ is close enough to $\vec{x}_{sol}$ so that the first-order Taylor expansion is sufficiently accurate, then at the very least $\vec{x}_1$ will be closer to the true value of $\vec{x}_{sol}$ than $\vec{x}_0$. This means that $\vec{x}_1$ may be used as an improved initial guess for Eq. [6], which should then generate an improved estimate for $\vec{x}_{sol}$, denoted $\vec{x}_2$. Continuing in this way, we generate a sequence $\{\vec{x}_n\}$ of points, converging to $\vec{x}_{sol}$, that are related to each other by the recursion relation,

$$D\vec{F}(\vec{x}_n)(\vec{x}_{n+1} - \vec{x}_n) = \vec{a} - \vec{F}(\vec{x}_n).$$

B. Newton-Raphson Iteration with Solution Deformation

Very often, the difficulty with obtaining convergence using Newton’s Method stems from an inability to pick a good initial guess. We illustrate one approach for dealing with this problem: Suppose we wish to solve the nonlinear system of equations $\vec{F}(\vec{x}) = \vec{a}$, and we are given some initial guess $\vec{x}_0$. This initial guess will not generate a sequence $\{\vec{x}_n\}$ that converges to the desired solution, nevertheless, this initial guess is essentially as good as any other, since the system of equations is such that it is difficult to choose a good initial guess. The idea is to therefore start with the given initial guess $\vec{x}_0$ and see if it is possible to reach the desired solution to the system of equations. We begin by defining $\vec{a}_0 = \vec{F}(\vec{x}_0)$, so that $\vec{a}_0$ is the value of the function evaluated at the initial guess. We then define a continuous curve $\vec{a}(s)$ with the properties that $\vec{a}(0) = \vec{a}_0$ and $\vec{a}(1) = \vec{a}$. Thus, as $s$ ranges from 0 to 1, $\vec{a}$ goes from $\vec{a}_0$ to $\vec{a}$. We now choose some positive integer $M$, and for $m = 0, \ldots, M$, we define $\vec{a}_m = \vec{a}(m/M)$. Note that this generates a sequence $\vec{a}_0, \vec{a}_1, \ldots, \vec{a}_M = \vec{a}$. By continuity, the distance between successive values of $\vec{a}_m$ decreases as $M$ increases.

So suppose we are able to obtain the solution, denoted $\vec{x}_m$, to the nonlinear system of equations $\vec{F}(\vec{x}) = \vec{a}_m$. For large $M$, the difference between $\vec{a}_m$ and $\vec{a}_{m+1}$ should be fairly small, so that $\vec{x}_{m+1}$ should be fairly close to the solution to the nonlinear system of equations $\vec{F}(\vec{x}) = \vec{a}_{m+1}$. Therefore, if we use $\vec{x}_m$ as the initial guess to the solution for $\vec{F}(\vec{x}) = \vec{a}_{m+1}$, then Newton’s Method is fairly likely to converge in such a case.

In this way, starting from the initial guess $\vec{x}_0$, which is the solution to $\vec{F}(\vec{x}) = \vec{a}_0$, we obtain from Newton’s Method a sequence of points $\vec{x}_0, \vec{x}_1, \ldots, \vec{x}_M$, where $\vec{F}(\vec{x}_m) = \vec{a}_m$. At each step, where Newton’s Method is used to generate $\vec{x}_{m+1}$ from $\vec{x}_m$, convergence is likely, because $\vec{a}_m$ and $\vec{a}_{m+1}$ are sufficiently close that $\vec{x}_m$ and $\vec{x}_{m+1}$ are close enough as well for Newton’s Method to converge using $\vec{x}_m$ as the initial guess. Note that this approach does not attempt to solve the original system $\vec{F}(\vec{x}) = \vec{a}$. Rather, it starts with a solution vector $\vec{a}_0$ for which $\vec{x}_0$ is the solution to $\vec{F}(\vec{x}) = \vec{a}_0$, and then continuously deforms $\vec{a}_0$ into $\vec{a}$. As a result, we term this method Newton-Raphson Iteration with Solution Deformation. Finally, although many deformations of $\vec{a}_0$ into $\vec{a}$ are possible, here we employ a linear deformation, given by $\vec{a}(s) = (1 - s)\vec{a}_0 + s\vec{a}$, where $s \in [0, 1]$, in our simulations.

VIII. CONCLUSIONS AND FUTURE RESEARCH

This paper describes a possible scheme for the construction of a gravimetric radar. The goal is the detection of large, fast moving objects, within a reasonable range, through the extraction of signals from gravimetric data. This will have direct applications to stealth technology. At present, feasibility of such a device hinges on the development
of sensors that are four orders of magnitude better than existing technology. It is envisioned that such a device will be based on gravity-induced quantum interferometry and the use of Bose-Einstein condensates in the form of particle beams with relatively massive, ultra-cold particles. The functionality and reliability of the gravimetric radar will rely critically on a substantial signal processing component to account and mediate the effects of known disturbances produced by other large moving objects or weather fronts.

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