Light storing in a medium of atoms in the tripod configuration

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Light propagation and storing in a medium of atoms in the tripod configuration driven by two control pulses are investigated theoretically in terms of two polaritons and numerically. It is shown that a magnetic field switched on at the pulse storage stage changes the phase relations between the atomic coherences due to the stored pulse, which leads to an essential modification of the released pulse. Quantitative relations concerning the released pulse and the coherences are given. A general situation when the two control fields are not proportional at the pulse release stage is also examined. It is shown that in both cases a single dark state polariton is not sufficient to account for the pulse evolution, which is connected with the fact that a part of the signal remains in the medium after the release stage.

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I. INTRODUCTION

A propagation of a weak laser pulse in atomic media driven by laser control pulses has recently been a subject of intensive investigations due to both the problem’s fundamental character in quantum optics as well as possible future applications. In particular a significant pulse slowdown or its storing in the medium have been achieved experimentally and interpreted theoretically [1, 2, 3, 4, 5, 6]. This was first done by controlling in time the electromagnetically induced transparency (EIT) [7] in a medium of atoms in the so-called lambda configuration including three active levels: the initial ground state is coupled by a signal pulse with an upper state which is coupled by the control field with another low-lying state. Switching the control field off after the signal has entered the medium results in light storing in the form of an atomic coherence between the two lower states. Switching the control field on after some time leads to a release of the trapped pulse. Light propagation, storing and release can be described in terms of a dark-state polariton, being a joint excitation (quasiparticle) of the atom+field system [8].

An interesting question, also from the practical point of view, concerns a possibility of modifying the pulse at the storing stage, which would enable one to process the information conveyed by the light. This would require influencing the atomic coherence in which the pulse has been written. The first experimental attempt was to additionally switch on a magnetic field in the direction of the pulse propagation [8]. This led to a controlled change of the phase of the coherence corresponding to the stored pulse and as a consequence to the respective modification of the overall phase of the released pulse: the latter was made to interfere with a reference pulse (which passed by the medium) and the interference pattern was indeed observed.

Extending the atom+field system to include four active states gives new possibilities to control light propagation and storing. Coupling any of the lower states of the lambda system with a fourth state allows one to temporarily prevent the signal from being released or to release a signal of a different frequency than that of the stored signal [10]. On the other hand, in the double lambda configuration (with two upper states) it is possible, after fulfilling a certain condition, to make the medium transparent simultaneously for two pulses and, after storing them in the form of a single atomic coherence between the two lower states, one can turn the coherence in a controlled way into two pulses of different frequencies [11, 12, 13, 14].

Another atomic four-level system, which however has not been exploited in the context of light storing, is that in a tripod configuration, i.e. one including three low-lying states coupled with a single upper state [15, 16]. Light propagation in the generalized case of EIT and its slowdown in such an atomic medium have been studied in Refs. [17, 18]; however those works do not concern light storing. Note that, contrary to the case of lambda systems, for the tripod configuration a single pulse is stored in the form of two atomic coherences. This enriches the dynamics and open new possibilities of a coherent control of light pulses. In particular light releasing in the case of a tripod system may be performed at a few stages and in a more flexible way than for a double lambda system. One also obtains new possibilities of processing the stored light by changing the relations between the two atomic coherences at the storing stage using an additional interaction; this is more elaborate than manipulating only a single coherence as in Ref. [8].

In the present paper we theoretically investigate light propagation and storing in a medium of atoms in the tripod configuration. In the next section we first present the system of the Maxwell-Bloch equations and then we discuss their approximate solutions in terms of two polaritons. An evolution due to an additional interaction at the storage stage, which does not cause transitions but changes the phases of the coherences, cannot be described in terms of the polaritons but it creates new initial conditions for the polaritons’ evolution at the release stage. For proportional control pulses we give simple analytical predictions of the height of the released pulse and for the values of the coherences which will remain trapped in the medium. Section III includes a numerical illustration of the theoretical results. In particular we verify the predictions of the previous section in the case in which the additional interaction is due to a constant magnetic field parallel to the direction of propagation. We also demonstrate the pulse evolution if at the release stage the two control pulses are not switched on simultaneously.

II. THEORY

Consider a four-level atomic model in the tripod configuration (see Fig. 1) with a single upper state $a$ and three lower states $b$ (initially populated), $c$ and $d$. The lower states are coupled with the upper state by three co-propagating fields: a weak signal field 1 of the envelope $\rho_{bc}$ and the frequency $\omega_1 = E_a - E_b + \Delta_1$ and two stronger control fields 2 and 3 of the envelopes $\rho_{ac}$ and $\rho_{bd}$ and frequencies $\omega_2 = E_a - E_c + \Delta_2$ and $\omega_3 = E_a - E_d + \Delta_3$. Similarly as in earlier papers dealing with similar subjects, propagation effects for the control fields will be neglected. The density matrix $\rho$ fulfills the von Neumann equation completed with phenomenological relaxation terms describing relaxation within the system. If we transform-off the terms rapidly oscillating in time and make the rotating wave approximation we obtain the following equations for the density matrix $\sigma$: $\sigma_{ab} = \rho_{ab} \exp[i\phi_1]$, $\sigma_{ac} = \rho_{ac} \exp[i\phi_2]$, $\sigma_{ad} = \rho_{ad} \exp[i\phi_3]$, $\sigma_{bc} = \rho_{bc} \exp[-i(\phi_1 - \phi_2)]$, $\sigma_{bd} = \rho_{bd} \exp[-i(\phi_1 - \phi_3)]$, $\sigma_{cd} = \rho_{cd} \exp[-i(\phi_2 - \phi_3)]$, $\sigma_{ii} = \rho_{ii}$, where $\phi_j \equiv \omega_j t - k_j z$.
In the above equations $\Gamma_{aj}$ is the transition rate from $a$ to $j$ due to the spontaneous emission, $\gamma = \Gamma_{ab} + \Gamma_{ac} + \Gamma_{ad}$, relaxation is due solely to the latter process, $\Omega_1(z,t) = \frac{1}{\hbar}d_{ba}\epsilon_1(z,t)$, $\Omega_2(t) = \frac{1}{\hbar}d_{ac}\epsilon_2(t)$, $\Omega_3(t) = \frac{1}{\hbar}d_{ad}\epsilon_3(t)$ are the Rabi frequencies corresponding to the particular couplings, $d_{ij}$ being the dipole transition moments.

The above Bloch equations are accompanied by the Maxwell propagation equation for the field 1, which reads in the slowly varying envelope approximation

$$\left(\frac{\partial}{\partial t} + c \frac{\partial}{\partial z}\right) \Omega_1 = -i\kappa^2 \sigma_{ba},$$

where $\kappa^2 = \frac{N|d_{ba}|^2\omega_0}{2\epsilon_0}$, $N$ being the atom density and $\epsilon_0$ the vacuum electric permittivity.

As in earlier papers, a simplified discussion can be performed in the perturbative approximation with respect to the signal field 1 (but not with respect to control fields), in the resonance conditions, with relaxation neglected, in the adiabatic approximation, i.e. with $\dot{\sigma}_{ab} = 0$. The Maxwell-Bloch equations are then reduced to the form

$$\Omega_1 = -\Omega_2 \sigma_{bc} - \Omega_3 \sigma_{bd},$$

$$\frac{\partial}{\partial t} \dot{\sigma}_{bc} = \frac{1}{\Omega_2} \Omega_1,$$

$$\left(\frac{\partial}{\partial t} + c \frac{\partial}{\partial z}\right) \Omega_1 = \kappa^2 \frac{1}{\Omega_2} \dot{\sigma}_{bc}.$$ (3)

It is convenient to perform an analysis of the solutions in terms of the dark state polariton

$$\Psi = \exp(-i\chi)[\cos\theta \Omega_1 - \kappa \sin\theta (\cos\phi \exp(i\chi_2)\sigma_{bc} + \sin\phi \exp(i\chi_3)\sigma_{bd})],$$

and another polariton

$$Z = [\sin\phi \exp(-i\chi_3)\sigma_{bc} - \cos\phi \exp(-i\chi_2)\sigma_{bd}].$$ (5)

We have set $\Omega^2 \equiv |\Omega_2|^2 + |\Omega_3|^2$, $\tan\phi = |\Omega_3|/|\Omega_2|$, $\chi_{2,3} = \arg(\Omega_{2,3})$, $\tan\theta \equiv \kappa/\Omega$. The phase $\chi$ is defined by the relation

$$\chi = \sin^2\theta (\cos^2\phi \chi_2 + \sin^2\phi \chi_3).$$ (6)

Eq. (4) is a natural generalization of the form of dark state polaritons used for single and double lambda systems, in our case of a single signal and two atomic coherences. The phase factors have been singled out to admit complex control fields (note that this variation of the problem has not been discussed before even for a simple lambda system).

The polariton $Z$ (Eq. (5)) is the superposition of the atomic coherences orthogonal to that of Eq. (4).

The two new variables satisfy the equations

$$\frac{\partial}{\partial t} \Psi + c \cos^2\theta \frac{\partial}{\partial z} \Psi = \exp[i(\chi_2 + \chi_3 - \chi)] \tan^2\theta \cos\theta[\dot{\phi} + i \sin\phi \cos\phi(\chi_3 - \chi_2)] \Omega Z,$$

$$\frac{\partial}{\partial t} Z = [-i (\cos^2\phi \chi_2 + \sin^2\phi \chi_3)] Z + \exp[i(\chi - \chi_2 - \chi_3)] \cos\theta[\dot{\phi} + i \sin\phi \cos\phi(\chi_3 - \chi_2)] \frac{1}{\Omega} \Psi.$$ (7)
where $\Psi$ has the shape given by Eq. (19) and it has moved from the initial position by $t \Delta t$. At the time $Z = 0$ initially, the evolution can be described by a single dark state polariton, similarly as in the case of a Λ system with a properly chosen control field $\Omega$: $\Psi(z,t) = \Psi(z - \int_0^1 c \cos^2 \theta(t) dt, 0)$. On the other hand if $\phi \neq 0$ or if the coherences at the storing stage have been somehow changed so that $Z \neq 0$, the further evolution must be described in terms of both polaritons and in general not all the excitation will leave the medium.

Let us now admit the simplest possible manipulation performed on the pulse at the storage stage: we modify the phases of the coherences $\sigma_{bc}$ and $\sigma_{bd}$ by switching on an interaction $U$ which is diagonal in the basis $(a, b, c, d)$. The levels will be then shifted and the two coherences will acquire additional phase shifts. Assume that a pulse has been stopped by a simultaneous switch-off of the two control fields and the coherences at the time $t_0$ have the values $\sigma_{bc}^0(z)$ and $\sigma_{bd}^0(z)$. Then the interaction $U$ is switched on. At this stage the evolution cannot be simply described in terms of polaritons, because the resonance is spoilt due to the level shifts. Assume for simplicity that the additional interaction has modified only the phase of $\sigma_{bc}$. For $t > t_0$ the interaction $U$ has been switched off but before switching off the control fields 2 and 3 on, the coherences are $\sigma_{bc}(z,t_1) = \sigma_{bc}^0(z) \exp(i\delta)$ and $\sigma_{bd}(z,t_1) = \sigma_{bd}^0(z)$, $\delta$ being the pulse area corresponding to $U$. Those values of the coherences are in general not suitable for the evolution to be described by a dark state polariton only; in particular it is not so even if the control fields which will release the pulse are the replicas of those used to store it. However, each of the two coherences can be decomposed into two parts, one of which (marked by ’) will contribute to the dark state polariton and the other (marked by ”) to the excitation $Z$.

The following equations will be satisfied during the pulse release stage ($t > t_1$), again with $\phi = 0$,

$$\sigma_{bd}^0(z) = \tan \phi \sigma_{bc}^0(z),$$

$$\sigma_{bd}(z,t) = \tan \phi \sigma_{bc}(z,t) - \sigma_{bc}(z,t) \exp(i\delta),$$

$$\sigma_{bc}(z,t) = \sigma_{bc}^0(z,t) + \sigma_{bc}''(z,t),$$

$$\sigma_{bd}(z,t) = \sigma_{bd}^0(z,t) + \sigma_{bd}''(z,t),$$

$$\cos \phi \sigma_{bc}''(z,t) + \sin \phi \sigma_{bd}''(z,t) = 0,$$

$$\sin \phi \sigma_{bc}''(z,t) - \cos \phi \sigma_{bd}''(z,t) = 0.$$  

The solutions $\sigma''$ of the above equations are

$$\sigma_{bc}''(z,t) = \sin^2 \phi \exp(i\delta) - 1 \sigma_{bc}^0(z) = \text{const.},$$

$$\sigma_{bd}''(z,t) = -\cos^2 \phi \exp(i\delta) - 1 \sigma_{bd}^0(z) = \text{const.}.$$  

At the time $t_1$ the solutions $\sigma'$, the dark state polariton and the $Z$ excitation are

$$\sigma_{bc}'(z,t_1) = \sigma_{bc}^0(z) \cos^2 \phi \exp(i\delta) + \sin^2 \phi,$$

$$\sigma_{bd}'(z,t_1) = \sigma_{bd}^0(z) \cos^2 \phi \exp(i\delta) + \sin^2 \phi.$$  

$$\Psi(z,t_1) = -\kappa(\cos \phi \sigma_{bc}'(z,t_1) + \sin \phi \sigma_{bd}'(z,t_1)),$$

$$Z(z,t_1) = \sin \phi \sigma_{bc}''(z,t_1) - \cos \phi \sigma_{bd}''(z,t_1).$$  

Note that for $t > t_1$ $Z(t) = Z(t_1)$. This means that the part of the atomic excitation constituting the polariton $Z$ does not leave the medium and preserves its shape, while the other part is turned back into the pulse and leaves the medium. The height and phase of the leaving pulse can be obtained from the equation

$$\Omega_1(z,t) = \Psi(z,t) \cos \theta(t),$$  

where $\Psi$ has the shape given by Eq. (19) and it has moved from the initial position by $\int_0^1 c \cos^2 \theta(t) dt$. 

In the situation when the phases $\chi_{2,3}$ of the control fields are constant and equal to zero the equations simplify to the form

$$\frac{\partial}{\partial t} \Psi + c \cos^2 \theta \frac{\partial}{\partial z} \Psi = \tan^2 \theta \phi \Omega Z,$$

$$\frac{\partial}{\partial t} Z = -\cos \theta \frac{1}{\Omega} \Psi.$$  

When $\phi = \text{const.}$, which means that the control fields change at the same rate, the two equations are decoupled.
III. NUMERICAL EXAMPLES

In this section we demonstrate numerical results of the pulse storing and release in two cases in which the polariton $Z$ is engaged: (1) the case of both control fields being switched on and off simultaneously at the same rate with the interaction $U$ present at the storage stage and (2) the case of a time delay between the instants of the switch-on of the control fields. We have numerically solved the Maxwell-Bloch equations (Eqs (1) and Eq. (2)) for the tripod system in the moving reference frame ($\xi = z$, $\tau = t - z/c$). Resonance has been assumed for all three transitions. The upper state $a$ has a width of $3\Gamma$ due to a spontaneous emission of rate $\Gamma = 4 \times 10^{-10}$ a.u. (2.6 MHz) to all three lower states. The relaxation rates for the coherences have been taken half of that of the upper state (as in the spontaneous emission). The entering signal pulse $\Omega_1$, shaped as a sine square, had the maximum value of 0.025$\Gamma$ while the control fields, taken real and equal, both switched on and off as a hyperbolic tangent, had maximum values of 5 $\Gamma$, which was enough to make our probe transparent. The length of the probe was of order of 1 cm. The initial width of the signal pulse was 2.4 $\mu$s, while the time of light storing was of order of 10 $\mu$s. The atom density was $N = 2 \times 10^{12}$ cm$^{-3}$.

The first case concerned changing the phases of the coherences at the storing stage. Its realization, for the levels engaged in the transitions being Zeeman sublevels of given quantum numbers $F$ and $M$, may be carried out by switching on a magnetic field $B$ parallel to the propagation direction. Each of the levels is shifted by $\Delta E = g_F B M/2$, $g_f$ being the Lande factor. The phases of the states $a - d$ and thus the phases of the coherences $\sigma_{bc}$ and $\sigma_{bd}$ evolve according to Eqs (1), with $\Omega_{1,2,3} = 0$ and with modified detunings. If, for example, in the case of $^{87}$Rb, the numbers $(F,M)$ are $(2,-1)$ for the state $b$, $(2,0)$ for $a$, $(2,1)$ for $c$ and $(1,1)$ for $d$, $\Delta E_b = -B/4$, $\Delta E_c = B/4$, $\Delta E_d = -B/4$. This means that $\sigma_{bd}$ will remain unchanged while $\sigma_{bc}$ will obtain an extra phase $\delta = -B\tau/2$, where $\tau$ is the duration of the magnetic field. We have assumed rectangular pulses of the magnetic field of duration of 2.4 $\mu$s and of the value of the magnetic induction up to $3 \times 10^{-5}$ T. The two control pulses were identical during the signal storing and release.

Fig. 2 presents the shape of the signal pulse at the end of the sample (1 cm) after it has been stored and released. Its height is a function of the area $\delta$ of the magnetic pulse. For $\delta$ being a multiple of $2\pi$ the magnetic field does not change the coherence $\sigma_{bc}$, so the analysis may be performed in terms of the single dark state polariton $\Psi$ and the released pulse has the same height as the stored one. For other values of $\delta$ the release stage may be again discussed in terms of uncoupled polaritons but with new initial conditions. The coherences are split into two parts. The absolute value of the part of both $\sigma_{bc}$ and $\sigma_{bd}$, which will enter $\Psi$, is $\sqrt{1 - \sin^2 2\phi \sin^2 \delta/2}$ times smaller than its original value (cf. Eq. (17) and Eq. (18)) and so will be the height of the released pulse compared with that of the stored one. Indeed, one can see in Fig. 2 that for $\phi = \pi/4$ the heights vary as $\cos \delta$. Fig. 3 shows the spatial distribution of the corresponding $Z$ polariton at the end of the evolution, i.e. after the released pulse has left the medium. Its height varies as $\sin \delta/2$, in agreement with Eqs (15), (16) and (20). These numerical results confirm our quantitative predictions of the values of the heights and phases of both the released pulse and that part of the excitation which will stay inside the medium unless it is destroyed by relaxation processes absent from our model.

The other case examined in this paper, in which the polariton $Z$ is engaged, is that in which the two control pulses are not proportional at the release stage. In the situation of Figs 4 and 5 the control fields were proportional at the storage stage but the field 3 preceded the field 2 by 3.6 $\mu$s at the release stage: due to $\phi \neq 0$ this leads to the production of nonzero $Z$ (cf. Eqs (8)). Fig. 4 shows the space-time evolution of the signal pulse. One can see the entering pulse at the left hand side. After the storage stage the signal is released in two portions. The first portion is released from the single coherence $\sigma_{bd}$ by the control pulse 3, similarly as in a simple lambda system. Then the other control pulse 2 causes a release of the signal stored in the coherence $\sigma_{bc}$. However, the latter process occurs at the presence of both control fields, which causes a difference of the two parts of the signal, as concerns they heights and initial velocities. The first part is released with a zero initial velocity while the second part has a nonzero velocity from the very beginning (see Fig. 4): after the two control fields attain their final values the velocities become equal: the structures in the figure become parallel for large $\tau$. To obtain a full symmetry one should switch the first control field off before switching the second one on. Then one would obtain two identical released pulses shifted in time, as in two independent simple lambda systems. Fig. 5 shows the time evolution of the polariton $Z$. One can see that it is equal to zero at the storing stage, when $\phi = 0$. Later it becomes almost equal to $\sigma_{bc}$ when the first pulse is generated from the coherence $\sigma_{bd}$. Finally due to a coupling of the two coherences there occurs their alignment, so that they have the same absolute value and opposite signs. This polariton does not leave medium at all. The proportions of the heights of the signal pulses at the particular stages of release can be regulated by changing the heights of the control pulses. However, it seems that there are no simple formulas which would allow for quantitative predictions, as in the first case discussed in this section.
IV. CONCLUSIONS

Light propagation and storing in a medium of atoms in the tripod configuration has been examined numerically and discussed in terms of polaritons. The signal was trapped in the form of two atomic coherences. In the very special situation in which the two control pulses were proportional the pulse storing and release were equivalent to those for a single lambda system with a properly chosen control field. Two particular situations were investigated in detail in which the usual dark state polariton was not sufficient to describe the process and another polariton had to be invoked. Both in the case in which the medium with a pulse stored inside was subject to an interaction with a magnetic field as well as in the case in which the two control pulses were not proportional, a part of the excitation remained trapped also after the release phase. In the former case we have given simple relations which allow one to predict the height of the released pulse and the shape of the trapped excitation. We thus have a rather simple means of processing information carried by a pulse: not only can it be stored in the medium or made inaccessible in a reversible way, but also the atomic excitation in which it has been written, can be split into parts, of which the signal can be read independently when required.

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Figures

FIG. 1: Color online. Atomic levels in the tripod configuration.

FIG. 2: Color online. The time dependence of the released pulse at the end of the sample as a function of the magnetic pulse area.

FIG. 3: Color online. The spatial distribution of the Z polariton after the released pulse has left the sample as a function of the magnetic pulse area.

FIG. 4: Color online. The space-time course of the pulse trapping and release in the case of the releasing control fields shifted in time.

FIG. 5: Color online. The evolution of the polariton Z corresponding to the pulse of Fig. 3.
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