Borromean nuclei, be them halo or not, are quite common and their study has been intensive. The three-body systems have the property that any one of their two-body subsystems is unbound. The halo-type Borromean nuclei are of special interest as they are unstable and their radii are quite large compared to neighboring stable nuclei. Typical cases are \( {^1\text{H}}_2 \) and \( {^6\text{He}}_2 \). The cores, \( ^4\text{He} \) and \( ^9\text{Li} \) are bound but the three two-body subsystems are not: \( ^5\text{He} \) and \( ^{10}\text{Li} \) and the \( nn \) system (\( n \) represents a neutron). Borromean excited states of stable nuclei also exist. A well known example is the Hoyle resonance in \( ^{12}\text{C} \) at an excitation energy of about 6.8 MeV. This resonance, of paramount importance in stellar nucleosynthesis, is a cluster of three \( ^4\text{He} \) nuclei, with the unbound \( ^8\text{Be} \) as two-body subsystems.

An important issue that has to be addressed is how the halo Borromean nuclei are formed as more neutrons are added to a given stable nucleus. In order to answer this question we have made a survey of the isotopes of several nuclei in the proton \( p \)-shell region as well as the fluorine isotopes. We have discovered that the halo develops in the Borromean nuclei in a gradual fashion. Further, to reach the halo Borromean nucleus the system, two neutrons down, acquires a new configuration which in several cases is a halo-type as well. This new type of halo nuclei has the feature that only one of its two-body subsystems, the di-neutron, is unbound. The other two subsystems are weakly bound. Another feature of this configuration is that the one neutron separation energy, \( E_n \), is smaller than that of the two-neutron, \( E_{2n} \), in contrast to the Borromean halo nuclei where \( E_n > E_{2n} \). Of course the situation \( E_n < E_{2n} \) that prevails in the new “doorway” configuration is shared by the other normal isotopes. In what follows we shall use the available information about the isotopes studied here contained in the Nuclear Wallet Cards (Sixth edition, 2000). Note that no Borromean isotope exists in oxygen (see, however, Ref. [1]).

As an example we consider the boron isotopes: \( A = 8, 9, 10, 11, 12, 13, 14, 15, 17, 19 \). Both \( ^{17}\text{B} \) and \( ^{19}\text{B} \) would be Borromean halo nuclei \( (^{19}\text{B} \) more so). The isotope \( ^{15}\text{B} \) is the doorway configuration. The halo radius is 5.15 fm while nuclear radius is 2.91 fm, to be compared to 2.50 fm of normal \( A = 15 \) nucleus. The details of our radius calculation are given below. We have identified five candidates for Samba halo nuclei \( ^{22}\text{B} \) of the doorway type. They are \( ^{12}\text{Be}, ^{15}\text{B}, ^{20}\text{C}, ^{23}\text{N} \) and \( ^{27}\text{F} \). The name Samba partly is inspired by the work of Robicheaux [2] who exploited yet another type of halo three-body systems, such as the hypertriton \( ^3\text{H}_1 \), where only one of the two-body subsystems is tightly bound, and called such systems Tango halo systems. Whereas, the Tango halo nucleus has a bound two-body subsystem that has to move in a rather “harmonic” fashion in the presence of the core, the Samba nucleus (two of the two-body subsystems are tightly bound) is distinguished by the fact that the motion of the two-neutron subsystem can be a rather more “agitated” and the three-body system remains bound.

We have calculated the reduced dipole transition strengths \( B(E1) \) and the radii of these five candidates for Samba halo nuclei. The \( B(E1) \)'s were calculated using a simple cluster model usually employed to get a rough estimate [4]. This model treats the two neutrons as a cluster that vibrates against the core. A Yukawa type wave function is used to describe the ground state and a plane wave is employed for the final continuum state in the calculation of the dipole matrix element. The model allows for the derivation of a simple analytical formula for the dipole strength distribution, \( dB(E1)/dE^* \). The dipole distribution is an important quantity in the study of exotic nuclei that is usually measured through the electromagnetic dissociation of these fragile systems.
in the field of a heavy target such as $^{208}$Pb. Integrating $dB(E1)/dE^*$ over $E^*$ gives the $B(E1)$ value. The cluster model gives a simple formula for this

$$B(E1) = \frac{3e^2}{16\pi\mu} \left( \frac{2Z}{A} \right)^2 \frac{1}{E_{2n}},$$  \hfill (1)

where $\mu$ is the reduced mass of the core and the $2n$ cluster. The factor $2Z/A$ corresponds to the number of neutrons in the halo, 2; the charge of the core, Z and the mass number of the whole halo nucleus, A. Note that the two neutron separation energy is inversely proportional to the average distance between the core center and the three-body center-of-mass (CM), $\sqrt{\langle r_{\text{CM}}^2 \rangle}$, which is used as the healing distance of the cluster model wave function. For, e.g., $^{12}$Be we find for $B(E1)$ the value 0.043 $e^2$fm$^2$ (another calculation gives 0.05-0.06 $e^2$fm$^2$\textsuperscript{[11]}) to be compared with 0.051(13) $e^2$fm$^2$ for the experimental value \textsuperscript{[12]} and to 0.61 $e^2$fm$^2$ for the well developed halo in the Borromean nucleus $^6$He. Thus one should be able to get reasonably reliable and appreciable dissociation cross section for this Samba nucleus (the yield or production cross section of these nuclei should be larger than those of the Borromean halo nuclei \textsuperscript{[14]}, rendering the experiment quite feasible). It is worth mentioning here that for Tango halo nuclei, where the halo is a pair of proton-neutron, the B(E1) is similar to Eq. \textsuperscript{[11]} except for the energy $E_{2n}$, which is replaced by $E_{pn}$ and the factor $(2Z/A)^2$ which is replaced by $[(A - 2Z)/A]^2$.

This will render the $B(E1)$ for Tango halo nuclei a factor $[(A - 2Z)/2Z]^2$ smaller than that of a corresponding Samba or Borromean nucleus if $E_{2n}$ is maintained equal to $E_{pn}$.

In figure \textsuperscript{[11]} we show the halo radii of our candidates for two-neutron halo nuclei as a function of the isospin projection $T_z = (N - Z)/2$. These radii can be estimated from the neutron-CM root mean square radius $\langle r_{\text{CM}}^2 \rangle$ estimated as $\rho \sqrt{m_nS_3}$ (see Refs. \textsuperscript{[1]} and \textsuperscript{[2]}). The quantity rho is adimensional. For our purposes $\rho$ is obtained calculating the average $(R_{(14\text{Be})} + R_{(17\text{B})})/2$ and associating this to squareroot $\sqrt{\langle r_{\text{CM}}^2 \rangle}$ above. Here $R_{(14\text{Be})} = 3.74$ fm and $R_{(17\text{B})} = 3.8$ fm, from Ref. \textsuperscript{[2]}, $S_3$ is the three-body weight with respect to the binding energy of the neutron in the two-body subsystem, $S_3 = E_{2n}(A) - E_n(A - 1)$ for Samba nuclei, where $E_n(A - 1)$ is the neutron separation energy in the bound $A - 1$ system, and $S_3 = E_{2n}$ for Borromean nuclei. With this value of $\rho$ we calculate all the radii shown in the table and figure. The full symbols (see Table \textsuperscript{[1]} are the radii of Samba type halo nuclei calculated exactly within the three-body model described below and the open symbols are the radii of the Borromean halo nuclei calculated as above with $\rho$ set equal 1.35. Both radii appear divided by the radii calculated assuming a normal nature of the isotopes, namely, $R_0 = 1.013A^{1/3}$ fm (the factor 1.013 fm was taken to reproduce the experimental radius of $^{12}$C of R= 2.32 fm \textsuperscript{[13]}).

Conspicuous in the figure is lack of indication of one-neutron halo nuclei whose radii are quite large and deviant from $R_0$. These nuclei, such as $^{11}$Be, $^{13}$C and others have been shown to have a well developed one-neutron halo. We did not show these halo nuclei as the thrust of our work here is on three-body, two-neutron halo nuclei. Also absent in the figure is the, what would be, Borromean halo nucleus following the Samba one, $^{23}$N. We are tempted to predict that such a nucleus, $^{25}$N, may exist, though we have no information about $E_n$, $E_{2n}$ and its life time. From the above considerations and we can clearly rule out $^{20}$C as a Samba halo nucleus and accordingly $^{22}$C as a Borromean halo nucleus.

The “doorway” aspect of the Samba halo nuclei is quite evident especially in the circles, squares and stars. They always precedes the final Borromean halo nucleus in the chain of isotopes. It would be indeed very interesting to perform Coulomb dissociation experiment on, e.g. $^{23}$N, and $^{27}$F to investigate the dipole strength distribution and also the longitudinal momentum distribution to assess the halo nature of these Samba systems. The nucleus $^{12}$Be has a rather subtle shell model structure with the ground state containing $(1s_1/2)^2$, $(1p_{1/2})^2$ and possibly $(1d_5/2)^2$ configurations, making it less likely to be a clear cut Samba type halo nucleus.

In Table \textsuperscript{[1]} we have collected several physical quantities of the following Samba type nuclei (treated as a $n - n - c$ three-body system): $^{12}$Be, $^{15}$B, $^{20}$C, $^{23}$N and $^{27}$F. The lower frame shows the results for the core radius $R_{\text{core}} =$
\[ R \sim \sqrt{\frac{2}{A}(\langle \tau_{n-CM}^2 \rangle + \frac{A - 2}{A}R_{\text{core}}^2)} \quad (2) \]

\( B(E1,0 \to 1) \) (eq. 1), and the half-life, \( T_{1/2} \), of the nuclei.

| nucleus | \( E_n \) (MeV) | \( E_{2n} \) (MeV) | \( \sqrt{\langle \tau_{n-CM}^2 \rangle} \) (fm) | \( R_0 \) (fm) |
|---------|----------------|----------------|--------------------------------|----------------|
| \(^{12}\text{Be}\) | 0.504 | 3.669 | 4.81 | 2.75 |
| \(^{13}\text{B}\) | 0.973 | 3.734 | 5.15 | 2.96 |
| \(^{20}\text{C}\) | 0.191 | 3.462 | 4.00 | 3.26 |
| \(^{23}\text{N}\) | 1.200 | 3.672 | 6.07 | 3.41 |
| \(^{27}\text{F}\) | 1.041 | 2.412 | 8.94 | 3.60 |

| nucleus | \( R_{\text{core}} \) (fm) | \( R \) (fm) | \( B(E1,0 \to 1) \) (\( e^2\text{fm}^2 \)) | \( T_{1/2} \) |
|---------|----------------|----------------|--------------------------------|----------------|
| \(^{12}\text{Be}\) | 2.18 | 2.80 | 0.043 | 21.3 ms |
| \(^{13}\text{B}\) | 2.38 | 2.91 | 0.037 | 9.87 ms |
| \(^{20}\text{C}\) | 2.66 | 2.82 | 0.013 | 14 ms |
| \(^{23}\text{N}\) | 2.80 | 3.22 | 0.024 | 37.7 s |
| \(^{27}\text{F}\) | 2.96 | 3.75 | 0.051 | >200 ns |

TABLE I: Physical quantities of the Samba halo nuclei given in the first column. The second and third columns are, respectively, the \( n-c \) and the \( n-n-c \) energies used to calculate the \( n-CM \) root mean-square radii, \( \sqrt{\langle \tau_{n-CM}^2 \rangle} \). \( R_0 \) is the nucleus radius and \( R_{\text{core}} \) is the core radius, \( R \) (eq. 2) is the average between \( R_0 \) and \( \sqrt{\langle \tau_{n-CM}^2 \rangle} \). \( B(E1,0 \to 1) \) is given by eq. 1. \( T_{1/2} \) is the nucleus half-life.

The calculated radii of the Samba nuclei in the three-body model are a bit larger than the measured one (e.g. \[^{12}\text{Be}\], \( R_{\text{exp}} = 2.59 \pm 0.06 \) fm see Ref. 13). We trace this small discrepancy to the neglect of Pauli blocking effect which tends to make the nuclear potential between the core and the neutrons less attractive at short distances. In order to not change the three-body binding energy the system needs to shrink a little to better feel the nuclear attraction.

The radii in Table I were calculated using the three-body formalism ofRefs. 11, 10. In these references, subtracted Faddeev equations for the three-body system \( n-n-c \) are used. The subtraction energy which is required in the model is taken to be \( \mu_{(3)}^2 \) \((E = -\mu_{(3)}^2)\) is an arbitrary subtraction point where the \( T \)-matrix (from this point a small \( t \) will be used when we refer to the two-body \( t \)-matrix and a capital one when we refer to the three-body \( T \)-matrix), \( T(-\mu_{(3)}^2) \), should be known. A more detailed description about our subtraction method can be found in Refs. 17. The motivation behind the subtraction is the use in the model of a delta function potential for the \( nn \) interaction which yields divergent two-body \( t \)-matrix at large momenta (short distances). Thus the Faddeev equations for the three-body system must be appropriately subtracted. The \( n-c \) interaction is also taken to be of a zero range. Therefore, in this model the only physical scales used as input are directly related to observables: the \( nn \) scattering length, the energies \( E_n \) and \( E_{2n} \). It is worth mentioning that the subtracted three-body equations are obtained through an elimination procedure that involves the renormalization of the delta function interaction strength so that the large momentum divergence alluded to above is removed. In a nutshell, one takes the two-body matrix \( t(E) \) for the delta interaction, \( \delta(\vec{r} - \vec{r}') \), at an energy, \( -\mu_{(3)}^2 \), and identify it with the renormalized coupling strength \( \lambda R(-\mu_{(3)}^2) \). The delta potential \( t \)-matrix at any energy can be obtained through usual Lippmann-Schwinger manipulations as

\[ t_R(E)^{-1} = \lambda_R(-\mu_{(3)}^2)^{-1} + 2\pi^2(\mu_{(3)} + i\hbar). \quad (3) \]

where the index \( R \) will refer to a renormalized \( t \)- or \( T \)-matrix.

The three-body \( T \)-matrix can be handled in a similar fashion with the above subtracted two-body matrix. The subtracted three-body \( T \)-matrix can be written as

\[ T_R(E) = T_R(-\mu_{(3)}^2) + T_R(-\mu_{(3)}^2) [G_0(E) - G_0(-\mu_{(3)}^2)] T_R(E). \quad (4) \]

It is a simple matter to show that \( T_R \) above is independent on the value of the subtraction energy \( -\mu_{(3)}^2 \), namely \( dT_R(E)/d\mu_{(3)}^2 = 0 \). This is clear by construction. Formally, this independence of \( T(E) \) on \( \mu_{(3)}^2 \) can be shown by resorting to the identity, \( dT(E)/d\mu_{(3)}^2 = -T(E)G_0^2(E)T(E) \), which is clearly also valid for \( dT(-\mu_{(3)}^2)/d\mu_{(3)}^2 \). Thus we have, using Eq. 4,

\[ \frac{dT_R(E)}{d\mu_{(3)}^2} = 2\mu_{(3)}^2 \frac{dT_R(E)}{d\mu_{(3)}^2} = 0. \quad (5) \]

The second relation in the above formula is a Callan-Symanzik type, commonly used in renormalization procedure of field theories.

Armed with the above invariance of \( T_R(E) \) with regards to the subtraction energy and the removal of the ultraviolet divergence of the delta potential two-body \( t \)-matrix, the three-body \( T \)-matrix can be used to construct the corresponding wave function needed to calculate matrix elements among which is the root mean-square radius of a given nucleus. The mean-square distances \( \langle \tau_{n-CM}^2 \rangle \) and \( \langle \tau_{n-CM}^2 \rangle \) and the coupled subtracted integral equations for the Faddeev spectator components are obtained from expressions given in Refs. 11, 12.

Before ending, we should mention what is expected of the Samba halo nuclei when they are used to induce reactions on heavy targets. In particular, at very low energies (in the vicinity of the Coulomb barrier) we expect that the Samba halo nucleus to fuse with, say, \(^{208}\text{Pb}\), with a probability which is not so much affected by the breakup coupling, and may exhibit a halo enhancement, contrary to what was found in the \(^{6}\text{He} + \(^{238}\text{U}\) studied in Ref. 13.
In further contrast to $^6$He, the Samba nucleus would exhibit a one-neutron transfer process competing with the two-neutron one. The one-neutron transfer cross section could be larger than the two-neutron one, depending on the extension of the corresponding configuration in the Samba halo nuclei. A specific Samba nucleus that we suggest to investigate experimentally is $^{23}$N which has a life time of 37.7 s and a halo radius of 6.07 fm. The enhancement of the fusion, sought for in vain in Borromean nuclei \cite{18,19}, may come into light with the Samba nuclei.

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