The patterns’ changes study in the proportion of simultaneously stopped cars in the city’s road network

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Abstract. The article presents the results of the theoretical and experimental studies of the patterns’ changes in the proportion of simultaneously stopped cars in the city’s road network. Ergodicity of the traffic flow, i.e. it is shown that the studied parameters measured by a test car for a sufficiently long period of time are equal to the average indicators of many cars located in a given area of the road network in the same time period.

1. Introduction
The traffic flows’ mathematical modeling use is of great importance in the large cities’ transport systems design [1, 2, 3]. One of the tools for solving this kind of problem is to analyze the patterns of changing the proportion of the simultaneously stopped cars on the road network. However, ergodicity issues in such models require special consideration [4, 5, 6]. In the processes’ analysis, the measurement values of which is limited to one implementation of the process, the proof of the ergodic hypothesis is important. The ergodicity hypothesis of the process assumes that the average over the observations set for a set of the same type objects at the same time is equal to the average over time when measuring the parameters of the same object over a period of time.
For the traffic flow, we can say that the process is ergodic if the studied parameters measured by a test car for a sufficiently long period of time will be equal to the average of many cars located in a given area of the road network in the same time period [7, 8, 9]. In fact, when proving ergodicity, the average studied parameters values of a certain set of test cars are usually analyzed.

2. Goals, Tasks, Methods of Study
The change in the proportion of the simultaneously stopped cars in the network was estimated both using the experimental data and on the basis of information obtained during modeling. The experimental studies were carried out in the road network of Rostov-on-Don. With the simultaneous movement of test cars, their motion parameters were recorded and, in particular, the proportion of the simultaneously stopped cars was determined. The change of this parameter within one hour for 11 test cars is shown in Fig. 1.
Figure 1. Change in the proportion of the simultaneously stopped cars in the road network

Considering the given in Fig. 1 data on the change in the proportion of the simultaneously stopped cars in time $f_s$ we can conclude that this parameter describes a random process that develops in time according to the certain probabilistic laws [10]. In fact, we are dealing with a time series $f_{sl}(t)$, which depends on time $t$, as from a parameter. With this in mind, it is very important to study the patterns of change $f_s$, to make a selection and statistical evaluation of the time series identification models.

The condition that the average over the set of observations for a set of objects of the same type at the same time is equal to the average over time when measuring the parameters of the same object over a period of time can be expressed by the following equations. The aggregate average for the objects of the same type at the same time

$$\bar{x}_M = \int_{-\infty}^{\infty} x \ p(x,t) \ dx$$  \hspace{1cm} (1)

where $p(x,t)$ is the random probability distribution density $x$.

The average time is determined by the equation

$$\bar{x}_T = \lim_{T \to \infty} \frac{1}{T} \int_{-T}^{T} x(t) \ dt$$  \hspace{1cm} (2)

Since one of the main requirements for the random process properties a in the analysis of time series is the requirement of its normality, the laws of change were studied $f_s$ under various conditions. In this analysis it was found that in a wide range of measurement discreteness, averaging $f_s$ over various time intervals, the normal share distribution of the simultaneously stopped cars is maintained. This invariance is maintained even with a minimum sample size of 30 measurements.

One hour was used as a control time period, during which the characteristics of traffic flows were measured, including the simultaneously stopped cars’ shares and the ratio of the specific parking time to the specific travel time. The results of the experimental studies show that for this observation duration (one hour) the average values of the simultaneously stopped cars’ share $f_s$ and the ratio of parking time to travel time $t_s/t$ match for the test car and the entire cars sum-total. The normality of these parameters’ distribution is also preserved. The comparative data is shown in Fig. 2, 3.
To predict the changes sequence $f_S$ in time, we use the statistical methods for analyzing the time series. When analyzing the time series, important information is provided by the autocorrelation functions [11, 12].

The autocorrelation and private autocorrelation functions of the changing process the proportion of simultaneously stopped cars are shown in Fig. 4.

The autocorrelation functions’ properties analysis makes it possible to establish the type of model that can be used to describe this time series. In practice, mixed models of autoregression - the moving average - have found the greatest application. The autocorrelation sequence of this process with one autoregression parameter is determined as follows
\[
R(0) = \frac{1 + \Theta_1^2 - 2\gamma_1 \Theta_1}{1 - \gamma_1^2} \sigma^2 ; \\
R(1) = \frac{(1 - \gamma(\Theta_1)(\gamma_1 - \Theta_1))}{1 - \gamma_1^2} \sigma^2 ; \\
R(t) = \gamma_1 R(t-1), \quad t \geq 2 ,
\]

where \( R \) – is the autocorrelation function; \\
\( \gamma \) - is the autoregressive model parameter; \\
\( \Theta \) - defines the moving average model parameter; \\
\( \sigma^2 \) – is the white noise variance.

![Autocorrelation and private autocorrelation process functions](image)

**Figure 4.** Autocorrelation and private autocorrelation process functions of changing the simultaneously stopped cars share in the network

The analysis of these equations shows that with one parameter of the autoregressive model, the autocorrelation function of the process should decrease exponentially. Turning to the autocorrelation function form in Fig. 4, we see that its values decrease exponentially, therefore the change in the proportion of the simultaneously stopped cars is described by a mixed model with one autoregressive parameter. In addition, the equations (3) make it possible to determine the autoregression parameter sign. If the autocorrelation function decreases without oscillating, then the autoregressive model parameter should have a positive sign.

The partial autocorrelation sequence of the mixed autoregression process - the moving average depends only on the first value of the autocorrelation function. Private autocorrelation process sequence of changing the simultaneously stopped cars’ proportion, shown in Fig. 4, fully complies with these criteria. In addition to the first value of the sample autocorrelation function, all other values are insignificant.

Thus, according to the classification criteria, it is possible to identify the process of changing the simultaneously stopped cars’ share in time, as a mixed process of a moving average and autoregression with one autoregression parameter and one moving average parameter.
After determining the model type of the time series, to describe the change in the simultaneously stopped cars’ proportion, it is necessary to calculate the model coefficients. The general view of a mixed model of autoregression - moving average is represented by the following equation

\[ x(t) = \gamma_1 x(t-1) + \ldots + \gamma_p x(t-p) + a(t) - \Theta_1 f(t-1) - \ldots - \Theta_q a(t-q) \quad \text{and} \]

\[ (4) \]

where \( \gamma \) - is an autoregressive parameter;
\( \Theta \) - defines the moving average parameters;
\( a \) - defines white noise.

Since in this case a mixed model of autoregression is identified - a first-order moving average, then based on the equation (4), we can obtain a specific form of this model

\[ x(t) = \gamma_1 x(t-1) + a(t) - \Theta_1 a(t-1) \quad \text{and} \]

\[ (5) \]

As a result of statistical processing of the time series characterizing the change in the share of simultaneously stopped cars, a mixed autoregression model is obtained - a moving average with the following parameter values:

\[ f_s(t) = 0.263 + 0.822 f_s(t-1) + a(t) + 0.089 a(t-1) \quad \text{and} \]

\[ (6) \]

The standard errors of the model parameters are:
- for a free component - 0.0163;
- for the autoregressive parameters - 0.0252;
- for the moving average parameter - 0.042.

For all parameters of the model, standard errors are much smaller than the parameters themselves and this indicates the reliability of the model. In addition, the autoregressive parameter significantly exceeds the moving average parameter and this is a sign of model stability.

To assess the adequacy of the model, it is necessary to analyze the residuals, which show the discrepancy between the actual and calculated values of the simultaneously stopped cars’ share. Important information in assessing the adequacy of time series models can be obtained by studying the residues’ distribution, analyzing the autocorrelation and particular autocorrelation functions. Figures 5 and 6 show the distribution of residues and these autocorrelation functions constructed for the difference between the experimental and calculated values \( f_s \) by the model (6).

The remnants of an adequate time series model have a normal distribution with a zero mean. The autocorrelation and partial autocorrelation functions of the residues should be insignificant and not indicate the presence of any trends.

All three graphs are proof of the resulting time series model’s adequacy (6). The remains of the series have a normal distribution with the average practically no different from zero. All the values of the autocorrelation and particular autocorrelation functions are insignificant. Therefore, the model for changing the simultaneously stopped cars’ share in time is adequate and can be used to predict the values of this parameter.
**Figure 5.** Normal distribution of residues between the experimental and calculated values $f_S$

**Figure 6.** Autocorrelation and private autocorrelation functions of the changes residuals in the simultaneously stopped cars’ proportion in the network

**Summary**

The results of forecasting the change in the simultaneously stopped cars’ share in the network are shown in Fig. 7. Considering the process of changing the values $f_S$ within an hour with a resolution of 5 sec, a high degree of adequacy with experimental data can be noted.
Figure 7. Predicted share of simultaneously stopped cars

The average value of the simultaneously stopped cars’ share during the experiment is 0.2732, and according to the simulation data - it is 0.2719. The corresponding standard deviation values are 0.1352 and 0.1143. Such a high degree of correspondence between the experimental and forecast data makes it possible to apply the obtained time series model (6) for solving the scientific and practical problems, including for predicting the changes in traffic conditions according to the data of “floating” cars [2].

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