Mass-derivative relations for leptogenesis

Tomáš Blažek, Peter Maták

Department of Theoretical Physics, Comenius University in Bratislava, Mlynská dolina, 84248 Bratislava, Slovak Republic

Abstract
In this work, a diagrammatic representation of thermal mass effects is derived from the $S$-matrix unitarity both in the classical and quantum Boltzmann equations. Within the example of the seesaw type-I leptogenesis, we discuss the connection of the Higgs thermal mass and the cancelations of infrared divergences in zero- and finite-temperature calculations of the right-handed neutrino decay and scattering processes.

1 Introduction

Quantum thermal corrections to reaction rates of the seesaw type-I leptogenesis at higher perturbative orders have been investigated in both the real-time [1–5] and imaginary-time [6–8] formalisms. A more complete treatment of the zero-temperature amplitudes can be found in Ref. [9], where interference between connected and disconnected amplitudes was taken into account. Here we fill in the gap between the two approaches. Completing the zero-temperature reaction rates at a given perturbative order we show that some of the contributions can be interpreted in terms of the finite-temperature field theory.

The $S$-matrix unitarity and Cutkosky cutting rules are often used to describe $CP$ violation effects, or, via Kinoshita-Lee-Nauenberg theorem [10–12], the infrared finiteness in particles’ interactions. Models of matter asymmetry generation in the early universe often contain massless particles leading to infrared divergences in reaction rates at higher perturbative orders. Here we focus on a simple extension of the Standard Model containing heavy right-handed neutrinos $N_i$ coupled to the Standard Model left-handed leptons and Higgs doublet via Yukawa interaction

$$\mathcal{L} \supset -\frac{1}{2} M_i \tilde{N}_i N_i - (\gamma_{iq} \tilde{N}_i P_L l_q H + \text{H.c.})$$

(1)

with $M_i$ denoting the right-handed neutrino Majorana masses. The Higgs particles and standard model leptons are considered with zero nonthermal mass. When higher-order corrections due to the top Yukawa coupling [13–15], or gauge interactions [16] are included, some authors put the Higgs thermal mass in by hand to obtain infrared-finite results. More recently, it has been argued by the Kinoshita-Lee-Nauenberg theorem that $2 \leftrightarrow 3$ processes, in which connected and disconnected amplitudes interfere, may be used to cancel infrared divergences in the $2 \leftrightarrow 2$ scatterings [9]. In our work, the diagrammatic representation developed in Refs. [19,20] is used to study the connection of the two approaches to thermal masses and infrared finiteness at a given perturbative order. It is shown that some of the $2 \leftrightarrow 3$ contributions are directly related to the Higgs thermal mass effects in the $N_i \rightarrow lH$ decay kinematics. The mass-derivative formula has originally been derived for the propagator in thermal field theory in Ref. [21]. Although this paper is not recent, here, for the first time, we derive similar type of relations for the reaction rates in the classical Boltzmann equation. As the next step, we confirm their validity for quantum thermal-corrected rates. The results are important as they are applicable to other models containing forward diagrams similar to those considered in this paper.

The article is structured as follows. In Sect. 2 we consider the simplest infrared-finite example of the mass-derivative relation, originating from the Higgs self-coupling correction to the $N_i \rightarrow lH$ decay rate. There the classical thermal mass is obtained from the Maxwell–Boltzmann statistics. Section 3 deals with the inclusion of quantum thermal corrections using the cylindrical diagrammatic representation of Ref. [20]. Finally, in Sect. 4, the top Yukawa corrections,
Concerning the assumption of the right-handed neutrinos being in kinetic equilibrium, see Ref. [22] for a detailed discussion.

### 2 Thermal mass effects in zero-temperature calculations

At the leading perturbative order, the momentum-integrated Boltzmann equation for the right-handed neutrino number density, in the universe expanding at the Hubble rate $H$, can be written as

$$\dot{n}_{N_i} + 3Hn_{N_i} = -\gamma_{N_i \rightarrow iH} - \gamma_{N_i \rightarrow lH} + \gamma_{H \rightarrow N_i} + \gamma_{H \rightarrow N_i}.$$  

(2)

The small circle indicates the decay and inverse decay reaction rates calculated using zero-temperature quantum field theory and classical Maxwell–Boltzmann phase-space densities denoted $\dot{f}$. We shall discuss the inclusion of quantum statistics in the next section. At temperatures of the order of the right-handed neutrino mass, Higgs particles are effectively massless. However, for a while, let us consider the reaction rate with finite Higgs mass. Then

$$\gamma_{N_i \rightarrow iH} = \frac{1}{4\pi} (\mathcal{M}^2/2) M_i^2 \left(1 - \frac{m_i^2}{M_i^2}\right)^2 \int [dp_{N_i}][\dot{f}]_{N_i}$$  

(3)

where $[dp_{N_i}] = d^3p_{N_i}/((2\pi)^32E_{N_i})$. Throughout this work, kinetic equilibrium is assumed for all particle species, with the right-handed neutrinos out of chemical equilibrium.$^3$

We will further denote the particles’ four-momenta by a subscript (such as $p_{N_i}$), while the four-momentum of the internal Higgs lines will be labeled simply by $k$. The energy dependence of the phase-space densities will only be explicit for the internal Higgs particles or whenever necessary.

Now, one may ask if any correction to the right-hand side of Eq. 2 occurs when the first-order in the Higgs self-coupling from $\mathcal{L} \supset -\lambda (H^\dagger H)^2$ is taken into account. The positive answer is rooted in the fact that any contribution to the reaction rate can be obtained from the thermal average of a square of amplitude and thus by cutting a forward diagram. Instead of the standard Cutkosky cuts, we use the $S$-matrix unitarity condition directly for $S = 1 + iT$, simplifying the treatment of on-shell intermediate states. From

$$1 - iT = (1 + iT)^{-1} \rightarrow iT = IT - (IT)^2 + \cdots$$  

(4)

we receive for the amplitude squared the relation [19]

$$|T_{fi}|^2 = -IT_{fi}IT_{fi} + \sum_n IT_{in}IT_{nf}IT_{fi} - \cdots$$  

(5)

Therefore, each contribution to a reaction rate for a particular process can be obtained from a forward diagram with one or more on-shell intermediate states included with appropriate sign according to Eq. (5). The simplest nontrivial forward diagrams at the $\mathcal{O}(\mathcal{M}^2)$ order, containing $N_i$ in the initial state, can be seen in Fig. 1. Cutting the diagram in Fig. 1a then leads to

$$\gamma_{N_i H \rightarrow HH} = -\int [dp_{N_i}][\dot{f}]_{N_i}$$  

(6)

that represents the rate of the $N_i H \rightarrow lH H$ reaction. Each of the on-shell cuts indicated by the dotted lines corresponds to the phase-space integration with a four-momentum conservation delta function. Thus, for the cut in the middle of the first term in Eq. (6) we have

$$\int [dp_{[k'}}] [dP_{[k']}] (2\pi)^4 \delta^{(4)} (P_{N_i} + P_{H} - P_{l} - k - k')$$  

(7)

while the standalone Higgs line in the disconnected part brings $(2\pi)^32E_{H}\delta^{(3)} (P_{H} - k')$, such that the $[dP_{[k']}]$ integration can be done immediately putting $P_{H} = k'$. Furthermore, the phase-space integration with circled densities of the initial state particles,

$$\int [dp_{N_i}] [dP_{H}] [\dot{f}]_{N_i} [\dot{f}_H}$$  

(8)

has to be included in Eq. (6). The summation over the discrete degrees of freedom is always implicit. In the first and second terms in Eq. (6), the uncuted Higgs propagators come with the same four-momentum $k$ as the on-shell Higgs in the

---

$^3$ Concerning the assumption of the right-handed neutrinos being in kinetic equilibrium, see Ref. [22] for a detailed discussion.
intermediate state, leading to a singular expression. A similar situation has been encountered in different processes in Refs. [9,12], using standard Cutkosky rules. Here, to cure the singularity, we employ

\[ \frac{i}{k^2 + i\epsilon} = \text{P.V.} \frac{i}{k^2} + \pi \delta(k^2) \]

(diagrammatically expressed as [19])

\[ \begin{array}{c}
\begin{array}{c}
\text{P.V.} \\
H & H
\end{array}
\end{array} + \frac{1}{2} \begin{array}{c}
\begin{array}{c}
\text{P.V.} \\
H & H
\end{array}
\end{array} \]

(10)

to replace the Higgs internal lines. Then the contribution of the last double-cut diagram in Eq. (6), containing the square of the delta function, is completely canceled by what is obtained from the first two diagrams when using Eq. (10).

To evaluate the reaction rate \( \dot{\gamma}_{Ni,H \rightarrow 1HH} \), we need to integrate over the intermediate-state phase-space. For that purpose, we use the identity [21]

\[ 2\delta_+(k^2)\text{P.V.} \frac{1}{k^2} = -\frac{1}{(k^0 + |k|)^2} \frac{\partial \delta(k^0 - |k|)}{\partial k^0}. \]

(11)

where \( \delta_+(k^2) = \theta(k^0)\delta(k^2) \). Summation over the spin, isospin, and flavor then gives

\[ \dot{\gamma}_{Ni,H \rightarrow 1HH} = -\frac{3}{\pi} \lambda \langle \mathcal{O} \rangle_{ii} \int [dp_{Ni}] [dp_H] \hat{f}_{Ni} \hat{f}_H. \]

(12)

Strangely enough, the expression in Eq. (12) is negative. This would mean that even though the right-handed neutrino \( Ni \) appears in the initial state and not in the final state, its abundance is increased in this process. Instead, we rather suggest taking this reaction rate as a mere correction to the \( Ni \rightarrow 1H \) decay, the process already included in Eq. (2) at the leading order. To see that this is indeed the case, let us consider the Higgs thermal mass effect due to its self-interaction in a thermal medium, defined as

\[ m^2_{H,\lambda}(T) = 12\lambda \int [dp_H] \hat{f}_H = \frac{3}{\pi^2} \lambda T^2, \]

(13)

where, however, the usual Bose–Einstein density has been replaced by \( \hat{f}_H = \exp \{-E_H/T\} \) to treat the reaction rates consistently. Otherwise, a more common result of an uncircled thermal mass \( m^2_{H,\lambda}(T) = \frac{1}{2} \lambda T^2 \) would be obtained [1]. Taking Eqs. (3) and (12) into consideration, we can easily check that

\[ \frac{i}{k^2 + i\epsilon} \rightarrow \frac{i}{k^2 + i\epsilon} + 2\pi \sum_{w=1}^{\infty} \left[ \hat{f}_H(k^0) \right]^w \delta_+(k^2). \]

(15)
In thermal equilibrium, \( \dot{f}_H = \exp\left\{ -k^0/T \right\} \) and the sum in Eq. (15) leads to Bose–Einstein distribution

\[
f_H(k^0) = \sum_{w=1}^{\infty} [\dot{f}_H(k^0)]^w = \frac{1}{\exp \{ k^0/T \} - 1} \tag{16}\]

reproducing the positive frequency part of the thermal propagator.\(^4\) To account for thermal corrections to the reaction rate in Eq. (12) in a consistent way, we have explained in full detail the procedure in Ref. [20]. In short, we start with the diagram with the lowest winding numbers for all particles (the diagram in Fig. 2a) and consider all possible cuttings (such as in Eq. (6)). Then in each term, we include \( f_{Ni} \) and \( f_H \), the Fermi–Dirac and Bose–Einstein distributions, respectively, for the initial state particles. Factors \( f_H \) result from summation of the geometric series as in Eq. (16). The geometric series stands for the contributions of cut forward diagrams with all possible winding numbers. Similarly, the on-shell intermediate states (indicated by the dotted lines in Eq. (6)) receive factors \( 1 - f_l \) and \( 1 + f_H(k^0) \).

In other words, we take forward diagrams with all possible windings of the external and internal lines, cut them according to Eq. (5), apply Eq. (9) to the uncut Higgs lines in each particular case, and add a power of circled density for each initial state particle. The summation of cut diagrams then leads to a geometric series of circled densities as in Eq. (16), and we obtain

\[
\gamma_{Ni \rightarrow 1H} = 12\lambda \int [dp_{Ni}] [dp_H] f_{Ni} f_H
\times \int d^3k \mathcal{F}(k^0, k) \frac{2\delta_+(k^2)P_V}{k^2} \tag{17}\]

where

\[
\mathcal{F}(k^0, k) = \frac{1}{4\pi^2} (\gamma^+ \gamma)_ii (M^2_i - k^2) \delta_+[(p_{Ni} - k^2)]
\times [1 + f_H(k^0)][1 - f_l(E_{Ni} - k^0)] \tag{18}\]

Then, using Eq. (11) and replacing \( \dot{f}_H \) by \( f_H \) in Eq. (13), we can write

\[
\gamma_{Ni \rightarrow 1H} = m^2_{H,\lambda}(T) \int [dp_{Ni}] f_{Ni}
\times \int d^3k \frac{\partial}{\partial k^0} \bigg|_{k^0=|k|} \frac{\mathcal{F}(k^0, k)}{(k^0 + |k|)^2} \tag{19}\]

\(^4\) In a general nonequilibrium case, it may seem unclear what the classical density \( \dot{f}_H \) is, while \( f_H \) corresponds to the mean occupation number of a particular single-particle state in the Fock space. Nevertheless, for bosonic particles, one may formally define \( \dot{f}_H = f_H/(1 + f_H) \) and proceed along the same lines [20].
that, in the case of massless quarks, contains an infrared divergence when \( Q \) and \( t \) momenta are collinear. To regularize it, we follow the literature [9] and introduce a small mass \( m \) to these particles. For the reaction rate, we obtain

\[
\dot{\gamma}_{N_i Q \rightarrow H^0} = \left. \frac{1}{4\pi^3} \int \frac{d^4 \mathbf{p}_N}{(2\pi)^4} \frac{d^4 \mathbf{p}_Q}{(2\pi)^4} \delta^4(\mathbf{p}_N + \mathbf{p}_Q - \mathbf{p}_i - \mathbf{p}_t) \right|_{\mathbf{p}_N = \mathbf{p}_Q = 0} \times 2 p_{N_i} \cdot p_i \left. \frac{2 p_{Q_i} \cdot p_i}{(p_{Q_i} - p_i)^2} \right|_{\mathbf{p}_N = \mathbf{p}_Q = 0} \]

where the integration over the \( l \) and \( t \) phase-space can be performed easily using the Lorentz invariance, leading to

\[
\frac{1}{4\pi^3} \left[ \frac{1}{2} + \frac{s}{M_i^2} - \frac{M_i^2}{s - M_i^2} \left( \ln \frac{M_i^2}{s - M_i^2} + \ln \frac{m}{\sqrt{s}} \right) \right]
\]

with \( s = (p_{N_i} + p_Q)^2 \). The logarithm \( \ln m/\sqrt{s} \) represents the infrared divergence that should be canceled by the contribution of other reactions.

4.1 Infrared finiteness and the Kinoshita-Lee-Nauenberg theorem

The diagram in Fig. 3a may also be cut in analogy to what we have seen in our first example in Sect. 2, obtaining

\[
\dot{\gamma}_{N_i Q \rightarrow H^0} = \left. \frac{1}{4\pi^3} \int \frac{d^4 \mathbf{p}_N}{(2\pi)^4} \frac{d^4 \mathbf{p}_Q}{(2\pi)^4} \delta^4(\mathbf{p}_N + \mathbf{p}_Q - \mathbf{p}_i - \mathbf{p}_t) \right|_{\mathbf{p}_N = \mathbf{p}_Q = 0} \times 2 p_{N_i} \cdot p_i \left. \frac{2 p_{Q_i} \cdot p_i}{(p_{Q_i} - p_i)^2} \right|_{\mathbf{p}_N = \mathbf{p}_Q = 0} \]

Again, using Eq. (9) in the first two terms, we obtain [9]

\[
\dot{\gamma}_{N_i Q \rightarrow H^0} = \frac{1}{4\pi^3} \int \frac{d^4 \mathbf{p}_N}{(2\pi)^4} \frac{d^4 \mathbf{p}_Q}{(2\pi)^4} \delta^4(\mathbf{p}_N + \mathbf{p}_Q - \mathbf{p}_i - \mathbf{p}_t) \times \frac{1}{4\pi^3} \int d^4 \mathbf{k} \delta^4((M_i - k_i)^2 - |\mathbf{k}|^2) \times (M_i^2 - k_i^2) \times \mathcal{G}_+(k, p_Q) \frac{2\delta^4(k_2) P \cdot V}{k^2} \]

where we define

\[
\mathcal{G}_\pm(k, p_Q) = \frac{2k \cdot p_Q \pm m^2}{2k \cdot p_Q \pm k^2}.
\]

Note that in Eq. (27), \( p_Q \) is considered in the \( N_i \) rest frame. Using Eq. (11), the integration over \( k \) gives

\[
\frac{1}{4\pi} \left[ 1 - \frac{s}{M_i^2} - \ln \frac{M_i\sqrt{s}}{s - M_i^2} + \ln \frac{m}{s} \right]
\]

and we may immediately see that the sum of the rates in Eqs. (24) and (27) is infrared finite. The divergent logarithms in Eqs. (25) and (29) completely cancel, as discussed in Ref. [9] using standard Cutkosky rules. This cancellation is typical for zero-temperature calculations and is based on the Kinoshita-Lee-Nauenberg theorem [10,11]. It is a consequence of the stronger version of this theorem, introduced in Ref. [12], that for a forward diagram, the sum of all its cuts is infrared finite up to a forward scattering contribution. In Ref. [9], however, the diagrams with crossed quark legs, such as in Fig. 3b, c, were not considered. For us, these are crucial, as, without them, we would not obtain the correct form of the mass-derivative relation in this case.

4.2 Infrared finiteness and 2 \( \rightarrow \) 3 processes at zero and finite temperature

We want to focus on the thermal mass effects and their diagrammatic representation. To understand the role of the 2 \( \rightarrow \) 3 reaction rates in this respect, let us first investigate the \( \mathcal{O}(\mathcal{M}_T^2) \) medium contribution to the scalar self-energy. At this order, the zero-temperature self-energy of the Higgs field is equal to the diagram in Fig. 4a times the imaginary unit. In a thermal medium, the on-shell parts of the \( Q \) and \( t \) propagators are modified. We expand the phase-space densities as

\[
f_Q = \sum_{w=1}^{\infty} (-1)^{w-1} (\hat{f}_Q)^w \text{ for } \hat{f}_Q = \frac{f_Q}{1 - f_Q}
\]

and similarly for \( t \) and the antiquark distributions. These circled densities may be understood as auxiliary functions that allow us to rewrite thermal-corrected rates as an infinite series.
of the zero-temperature ones \cite{20}. To the first order in circled densities, the medium part of the self-energy denoted \( \Pi_T \) comes from the sum of the diagrams in Fig. 4b–e where each external quark or antiquark is considered with the corresponding density integrated over the phase-space.\(^5\) Assuming all quarks and antiquarks are in thermal equilibrium, for a four-momentum \( q \), each circled density has the same form \( \tilde{f}_q = \exp \left\{ -q^0/T \right\} \) and

\[
\tilde{\Pi}_T(k^0, k) = 6 \mathcal{A}_T^2 \int [d\mathbf{q}] \tilde{f}_q [\mathcal{A}_+(k, q) + \mathcal{A}_-(k, q)].
\]  

(31)

In analogy to the standard definition in Ref. \cite{2}, we introduce the thermal Higgs mass

\[
m_H^2(T) = \tilde{\Pi}_T(k^0, k)|_{q^0=|k|} = 12 \mathcal{A}_T^2 \int [d\mathbf{q}] \tilde{f}_q.
\]  

(32)

For the thermal part of the wave-function renormalization factor of the Higgs field, we have\(^6\)

\[
\delta \tilde{Z}_{H,\mathcal{A}}(T) = \frac{1}{2|k|} \frac{\partial}{\partial k^0} \bigg|_{k^0=|k|} \text{Re} \tilde{\Pi}_T(k^0, k)
\]  

(33)

that multiplies the \( N_i \rightarrow IH \) decay rate leading to

\[
\delta \dot{\gamma}_{N_i \rightarrow IH} = -6 \mathcal{A}_T^2 (\mathcal{A}_+^\dagger \mathcal{A}_-) \frac{1}{4\pi^2} \int d^3k \delta \left( |k| - \frac{M_i}{2} \right)^2 \left( k^0 |k| \right) \tilde{f}_N \tilde{f}_q 
\]

where the expression in the second row equals \( 1/\pi \). Furthermore, the sum of all \( 2 \rightarrow 3 \) reaction rates can be evaluated similarly to Eq. (27) with \( \mathcal{A}_+ (k, p_Q) \) replaced by \( 2\mathcal{A}_+(k, q) + 2\mathcal{A}_-(k, q) \) coming from Fig. 3a–d, respectively. Unlike the integration in Eq. (27), they together lead to an infrared-finite result and, remarkably, we obtain

\[
\dot{\gamma}_{N_i \rightarrow IH} + \dot{\gamma}_{N_i \rightarrow IH} + \delta \dot{\gamma}_{N_i \rightarrow IH} = m_H^2(\mathcal{A}_T^2) \left. \frac{\partial}{\partial m_H^2} \right|_{m_H^2=0} \dot{\gamma}_{N_i \rightarrow IH} + \delta \dot{\gamma}_{N_i \rightarrow IH} 
\]  

(35)

Here the two terms on the right-hand side are of equal size and represent both the effect of thermal mass and the thermal part of the wave-function renormalization of the Higgs field. The left-hand side corresponds to zero-temperature reaction rates obtained from specific cuttings of the forward diagrams in Fig. 3. As in Ref. [9], we have seen the cancelations between the \( 2 \rightarrow 2 \) and \( 2 \rightarrow 3 \) processes. Therefore, even for the reaction rates calculated using the Maxwell–Boltzmann distributions and zero-temperature Feynman rules, the thermal mass effects, in terms of the expansion of Eq. (30), have to be included.

Finally, one may avoid the expansion in Eq. (30) and work with uncircled quantum distributions corresponding to the mean occupation numbers in the Fock space. Then, the thermal mass and wave-function renormalization will appear exactly as in Refs. [1,2]. Here we note that they can be represented by the finite-temperature \( 2 \rightarrow 3 \) rates obtained from cutting the forward diagrams with all possible windings of internal and external lines. That corresponds to the inclusion of the appropriate statistical factors, as it is in Eq. (18). Still, the form of the mass-derivative relation in Eq. (35) is preserved even for uncircled quantities.

5 Conclusions

We have introduced a diagrammatic representation of the thermal Higgs mass in the \( N_i \rightarrow IH \) reaction rate within the model of the seesaw type-I lepton genesis. It has been shown that along with \( 2 \rightarrow 2 \) scatterings, the thermal mass effects in the right-handed neutrino decay (of the same perturbative order) can and should be implemented even within the calculations based on the zero-temperature Feynman rules. The latter enters via the infrared-finite sum of \( 2 \rightarrow 3 \) reaction rates, although individual contributions diverge. Thus we identify an alternative cancelation of infrared divergences in addition to the one based on the Kinoshita-Lee-Nauenberg theorem used in Ref. [9].

Acknowledgements We thank our colleague Fedor Simkovic for his longterm support, and Viktor Zaujec for carefully reading the manuscript. The authors were supported by the Slovak Ministry of Education Contract No. 0243/2021.

Data Availability Statement This manuscript has no associated data or the data will not be deposited. [Authors’ comment: The work is purely theoretical, and thus there are no data.]

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you
give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article’s Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article’s Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit http://creativecommons.org/licenses/by/4.0/.

Funded by SCOAP3.

References

1. G. Giudice, A. Notari, M. Raidal, A. Riotto, A. Strumia, Nucl. Phys. B 685, 89 (2004)
2. A. Salvio, P. Lodone, A. Strumia, J. High Energy Phys. 2011, 116 (2011). [Erratum: arXiv:1106.2814v4]
3. S. Biondini, N. Brambilla, M.A. Escobedo, A. Vairo, J. High Energy Phys. 2013, 28 (2013)
4. S. Biondini, N. Brambilla, M.A. Escobedo, A. Vairo, J. High Energy Phys. 2016, 191 (2016)
5. S. Biondini, N. Brambilla, A. Vairo, J. High Energy Phys. 2016, 126 (2016)
6. M. Laine, Y. Schröder, J. High Energy Phys. 2012, 68 (2012)
7. D. Bödeker, M. Sangel, J. Cosmol. Astropart. Phys. 2017, 052 (2017)
8. D. Bödeker, D. Schröder, J. Cosmol. Astropart. Phys. 2020, 033 (2020)
9. J. Racker, J. High Energy Phys. 02, 042 (2019)
10. T. Kinoshita, J. Math. Phys. 3, 650 (1962)
11. T.D. Lee, M. Nauenberg, Phys. Rev. 133, B1549 (1964)
12. C. Frye, H. Hannesdottir, N. Paul, M.D. Schwartz, K. Yan, Phys. Rev. D 99, 056015 (2019)
13. E. Nardi, J. Racker, E. Roulet, J. High Energy Phys. 2007, 090 (2007)
14. A. Pilaftsis, T.E. Underwood, Nucl. Phys. B 692, 303 (2004)
15. A. Pilaftsis, T.E.J. Underwood, Phys. Rev. D 72, 113001 (2005)
16. C.S. Fong, M. Gonzalez-Garcia, J. Racker, Phys. Lett. B 697, 463 (2011)
17. C.P. Kießig, M. Plümacher, AIP Conf. Proc. 1200, 999 (2010)
18. C.P. Kießig, M. Plümacher, M.H. Thoma, J. Phys. Conf. Ser. 259, 012079 (2010)
19. T. Blažek, P. Maták, Phys. Rev. D 103, L091302 (2021)
20. T. Blažek, P. Maták, Eur. Phys. J. C 81, 1050 (2021)
21. Y. Fujimoto, H. Matsumoto, H. Umezawa, I. Ojima, Phys. Rev. D 30, 1400 (1984)
22. F. Hahn-Woermle, M. Plümacher, Y. Wong, J. Cosmol. Astropart. Phys. 2009, 028 (2009)
23. S.M.H. Wong, Phys. Rev. D 64, 025007 (2001)