Investigating and improving introductory physics students’ understanding of symmetry and Gauss’s law

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Received 15 August 2017, revised 6 September 2017
Accepted for publication 18 September 2017
Published 7 December 2017

Abstract
We discuss an investigation of student difficulties with symmetry and Gauss’s law and how the research on students’ difficulties was used as a guide to develop a tutorial related to these topics to help students in the calculus-based introductory physics courses learn these concepts. During the development of the tutorial, we interviewed students individually at various stages of development and administered written tests in the free-response and multiple-choice formats on these concepts to learn about common student difficulties. We also obtained feedback from physics instructors who teach introductory physics courses regularly in which these concepts were covered. The students in several ‘equivalent’ sections worked on the tutorial after traditional lecture-based instruction. We discuss the performance of students on the written pre-test (administered after lecture-based instruction in relevant concepts) and post-test given after students worked on the tutorial. We find that on the pre-test, all sections of the course performed comparably regardless of the instructor. Also, on average, student performance on the post-test after working on the tutorial is significantly better than on the pre-test after lecture-based instruction. We also compare the post-test performance of introductory students in sections of the course in which the tutorial was used versus not used and find that sections in which students engaged with the tutorial outperformed those in which students did not engage with it.

Keywords: physics education research, Gauss’s law, electric flux, tutorial, student difficulties
Introduction

Investigations of students’ common conceptual difficulties with symmetry and Gauss’s law are important for designing instructional strategies to reduce them and also to improve students’ problem solving skills [1–8]. Several prior studies have focused on the difficulties that introductory physics students have with electricity and magnetism concepts and strategies that may help students learn the concepts better [9–34]. Learning to reason whether Gauss’s law can be exploited in a particular situation to determine the electric field, without having to evaluate complicated integrals, can provide an excellent context for helping students develop a good grasp of symmetry considerations and their reasoning skills. Unfortunately, students often memorise a collection of formulas for the magnitude of the electric field for various geometries, without paying attention to symmetry considerations. Many students have difficulty identifying situations in which Gauss’s law is useful and overgeneralise results obtained for highly symmetric charge distributions to situations in which they are not applicable. Most textbooks do not sufficiently emphasise the symmetry considerations or the chain of reasoning required to determine if Gauss’s law is useful for finding the electric field in a given situation. Distinguishing between the electric field and flux is often difficult for students. Choosing appropriate Gaussian surfaces to calculate the electric field using Gauss’s law when sufficient symmetry exists is also challenging. Many students confuse the symmetry of the charge distribution with the symmetry of the object on which the charges were embedded.

Here we discuss an investigation of the conceptual difficulties that college students in a traditionally taught calculus-based introductory physics course have with Gauss’s law and how the research on student difficulties was used as a guide to develop and evaluate a research-validated tutorial to help students learn to recognise the symmetry of the charge distribution, determine whether Gauss’s law can be used to find the electric field and if so, apply Gauss’s law. Since these concepts are challenging, research-validated learning tools can help students develop a more coherent knowledge structure.

Gauss’s law helps relate the net electric flux through a closed surface to the net charge enclosed by the surface:

$$\Phi = \frac{Q_{\text{enclosed}}}{\varepsilon_0}.$$  \hspace{1cm} (1)

This means:

• Knowing the net electric flux through a closed surface, we can find the net charge inside it.
• Knowing the net charge inside a closed surface, we can find the net electric flux through it.

In general, Gauss’s law cannot be used to easily find the electric field $\vec{E}$ at a point. Only in situations in which the charge distribution has a very high symmetry can we find the field $|\vec{E}|$ from the net flux gotten from Gauss’s law. There are only three types of symmetries (spherical, cylindrical, and planar) for which Gauss’s law can easily be used to determine the field from the information about the flux. However, students find these concepts challenging and need help in learning to identify when these symmetries are present. The net flux through a closed surface is as follows:

$$\Phi = \oint \vec{E} \cdot d\vec{A} = \oint |\vec{E}| \cos \theta |d\vec{A}|,$$  \hspace{1cm} (2)
where $\theta$ is the angle between the field $\vec{E}$ and infinitesimal area vector $d\vec{A}$ (outward normal). The net flux $\Phi$ over a Gaussian surface can be exploited to determine the field magnitude $|\vec{E}|$ at an arbitrary point $P$ on the surface easily only if the following conditions are met:

- We can determine the direction of $\vec{E}$ with respect to the area vector everywhere on the closed surface by using symmetry (only $\theta = 0^\circ$, $180^\circ$ or $\pm 90^\circ$ angles are associated with very high symmetry).
- In some cases, we can divide the closed surface into sub-sections (for each sub-section, the flux can be computed, e.g., side and two caps of a cylinder) such that one of the following is true:
  1. $|\vec{E}|$ is the same everywhere on the sub-section by the symmetry of the charge distribution.
  2. $|\vec{E}|$ and the area vector (outward normal to the surface) are perpendicular ($\theta = 90^\circ$) so that there is no flux through that sub-section.

Thus, to determine if the information about the net flux through a closed surface can be used to determine $|\vec{E}|$ at a point $P$, we may choose a Gaussian (closed imaginary) surface such that:

- It contains the point $P$ where we want to determine $|\vec{E}|$.
- $|\vec{E}| \cos \theta$ is known (by symmetry) to have a constant value on each sub-section of the surface so that it can be pulled out of the flux integral in equation (2). Then, $\int |d\vec{A}| = \text{total area of the sub-section of the surface}$.

Below, we start with the methodology for the investigation of students’ conceptual difficulties with symmetry and Gauss’s law and categorise the common student difficulties found. We then discuss the development and validation of the tutorial, including how research on student difficulties was used as a guide in the development process. Finally, we compare the pre-test and post-test data for students who worked on the tutorial versus those who did not in different sections of a calculus-based introductory physics course at the University of Pittsburgh (Pitt). The findings suggest that the tutorial was effective in improving students’ understanding of symmetry and Gauss’s law.

**Methodology for the investigation of conceptual difficulties**

We note that before working on the Gauss’s law tutorial, students worked on a tutorial that was designed to help students learn about the electric field and superposition principle [34]. The tutorial on symmetry and Gauss’s law discussed here was designed to help students learn to exploit Gauss’s law to calculate the electric field at a point due to a given charge distribution if high symmetry exists. The methodology for the development and evaluation of the Gauss’s law tutorial started with an investigation of the common conceptual difficulties that introductory physics students have with these concepts and was analogous to the methodology discussed in detail elsewhere for another tutorial to help students learn about the electric field and superposition principle [34]. Therefore, we do not repeat it in entire detail here [34] although we reproduce parts of the methodology for clarity. These difficulties were investigated by administering free-response and multiple-choice questions on in-class quizzes to college students in several sections of a second semester introductory calculus-based physics course after traditional lecture-based instruction on Gauss’s law concepts. Students in these courses were mainly engineering, chemistry, mathematics, and physics majors. We also conducted individual interviews with a subset of students using a think-aloud protocol to
better understand the rationale for student responses before, during, and after the development of different versions of the tutorial and the corresponding pre-test and post-test. During the semi-structured interviews, introductory students were asked to verbalise their thought processes while they answered the questions. They first read the questions and reasoned about them without interruptions except that they were prompted to think aloud if they were quiet for a long time. After students had finished answering a particular question to the best of their ability, we asked them to further clarify and elaborate issues that they had not clearly addressed on their own.

**Student difficulties**

As described in [34], ‘we note that students in the sections of the course in which they engaged with the tutorial as a pedagogical tool were less likely to have the difficulties after working on the tutorial (on the post-test) than on the pre-test (after traditional lecture-based instruction only). For reference, the entire pre-test and post-test for the tutorial are included in the appendix. Also, the difficulties on the pre-test and post-test were similar in nature for both the tutorial sections and ‘equivalent’ comparison group consisting of students in a section which did not work on the tutorial. The main difference between these groups was that the tutorial group students were significantly less likely to have the difficulties after working on the tutorial (quantitative data comparing the performance of these two groups are discussed later in the tutorial evaluation section’).

All of the questions on the pre-/post-tests require understanding of symmetry and Gauss’s law. In particular, exploiting Gauss’s law to find the electric field requires a high level of symmetry of the charge distribution and also requires that students are able to come up with a suitable Gaussian surface in order to find the electric field for the given charge distribution. Our investigation shows that introductory physics students had great difficulties with these issues. As discussed in [34], ‘we first discuss data on the common student difficulties found without separating the performance of the tutorial group and comparison (non-tutorial) group on the pre-/post-tests. Later, in the tutorial evaluation section, we will present quantitative data related to how students performed in the tutorial and non-tutorial groups. In order to determine these categories, two researchers separately proposed the categories for the difficulties. Most of their categories were overlapping, and were kept. A few categories that were not on both researchers’ lists were discussed and some of those categories were included after discussion.’ We note that these categories of student difficulties discussed below are not mutually exclusive. For example, even though some difficulties in determining the Gaussian surface are due to the lack of recognition of the symmetry requirement for the charge distribution, these concepts are discussed both in the categories involving the difficulty in determining the Gaussian surfaces and lack of recognition of the symmetry requirement for using Gauss’s law.

**Difficulty with the principle of superposition**

In the questions students were asked, they must contemplate if the electric field at a point can be found easily using Gauss’s law. The questions probe the extent to which students can discern the underlying symmetry of the charge distribution. We find that the earlier difficulty with the superposition principle in the context of determining the net electric field due to a discrete charge distribution (after using Coulomb’s law to find the field due to each point charge) persists even when students encounter the superposition principle in the context of problems with continuous charge distributions. The performance of many students was
closely tied with their conceptual and procedural facility with the principle of superposition. The conceptual difficulties with the superposition principle include situations in which students did not have a clear understanding of the fact that the superposition principle is useful for discerning the underlying symmetry of the electric field due to the continuous charge distributions they were provided. For example, many students incorrectly claimed that the electric field inside a hollow non-conducting object of any shape (e.g., a finite cylinder with charge uniformly distributed on its surface) is zero and explained that the net electric field in a closed hollow region always works out to be zero. Many students also drew incorrect Gaussian surfaces (discussed later in detail) for exploiting Gauss’s law to find the electric field partly because they failed to account for the symmetry of the charge distribution and what the symmetry implies for the net electric field at various points using the superposition principle. These students did not have a sufficient understanding of the superposition principle to make conceptual inferences about the net electric field for a given charge distribution. The students who lacked procedural facility with the superposition principle generally knew that it can be used to find the net electric field due to a given charge distribution, but they did not know how to correctly use this principle in the given situations.

Confusing the electric flux with electric field

Students often had difficulty distinguishing between the electric field and the electric flux. While the electric field is a vector which is defined at each point, the electric flux is a scalar quantity which is defined through a surface and is a measure of the total number of field lines passing through the surface area. However, since the electric field and electric flux are related, students often had difficulty differentiating between the two and lacked a functional understanding of the relation between the two. It was difficult for many students to understand, e.g., that no net electric flux through a closed surface does not imply that there cannot be a non-zero electric field at each point on the surface.

The difficulty in differentiating and relating the electric field and electric flux was manifested, e.g., in response to questions (1) and (2) on the pre-test (see the appendix). Responses to these questions suggest that many students struggled to interpret the statement of Gauss’s law that relates the net flux through a closed surface to the net charge enclosed. They had difficulty differentiating between the electric flux through a closed surface and the electric field at a point on that surface. For example, on question (1) in the pre-test, many students chose option (I) (or options (I) and (III)) and thereby claimed incorrectly that only those surfaces can be used to determine the net electric flux through them because the other surfaces did not have a suitable symmetry to find the flux. Interviews suggest that one common reason for this mistake was the confusion between the electric flux and electric field.

Confusion between an open and closed surface with regard to Gauss’s law

Students were sometimes unsure about the distinction between an open and a closed surface and that Gauss’s law is only applicable for closed surfaces. For example, the pre-test questions (1) and (2) at least partly assess student understanding of this distinction. In response to question (1), some students claimed that the flux due to an infinitely long line of charge (with uniform linear charge density $\lambda$) is $\lambda = L/\varepsilon_0$ for the two-dimensional square sheet. Interviews suggest that some students incorrectly thought that Gauss’s law applies to any symmetrical surface even if it is not closed (e.g., to a square or circular sheet).
**Confusion between the underlying symmetry of a charge distribution versus the symmetry of a Gaussian surface that can be useful to find the electric field for a given charge distribution**

Many students had difficulty realising that it is the symmetry of the charge distribution that is important for determining whether Gauss’s law can easily be applied to calculate the electric field at a point due to that charge distribution. For example, in question (2) on the post-test (see the appendix), students were asked to select the hollow non-conducting objects with a uniform surface charge density for which they could easily use Gauss’s law and symmetry considerations to find the electric field at any point due to the charge on the object. In response to this question, some students incorrectly claimed that one can use Gauss’s law to find the field at a point easily due to a cube or a finite cylinder with uniform surface charge. In the interviews, students with these types of responses sometime recalled using Gauss’s law to find the electric field for these types of charge distributions. More probing showed that students with these types of responses were either confusing the fact that those surfaces can be used as Gaussian surfaces for appropriate charge distributions (e.g., a finite cylinder can be used as a Gaussian surface for both cylindrical and planar charge symmetries and a cube when appropriately oriented can be used as a Gaussian surface for planar charge symmetry, e.g., for an infinite sheet with uniform charge), or the fact that for an infinite uniformly charged cylinder, it is possible to exploit Gauss’s law to find the field easily. It appears that some of these students were confused between the underlying symmetry of a charge distribution versus the symmetry of a Gaussian surface that can be useful to find the field for a given charge distribution. Interviews suggest that they had not thought carefully about the symmetry of the charge distribution and the principle of superposition and its implications for finding the electric field due to a given charge distribution. They were applying memorised knowledge to incorrect situations, since the correct applicability condition for what they remembered was forgotten.

**Difficulty in realising that a finite cylinder with uniform surface charge density does not have a sufficiently symmetric charge distribution to find the electric field easily using Gauss’s law**

Gauss’s law is useful for finding the electric field easily only when there is a high level of symmetry to simplify the surface integral involved in calculating the electric flux in terms of the electric field and the area vector. Although students are taught that spherical, cylindrical and planar symmetries of the charge distribution are the only types of symmetries for which Gauss’s law can be exploited easily to find the electric field, students have difficulty in figuring out when these symmetries exist. For example, the pre-test question (6) (see the appendix) was very difficult for many students and they had difficulty discerning if there is sufficient symmetry to exploit Gauss’s law to find the electric field easily. In particular, on question (6) in the pre-test, students were asked to evaluate the validity of a statement which states that to find the magnitude of the electric field at a point due to uniformly distributed charge on the surface of a finite cylinder of length L, one can imagine a Gaussian cylinder that passes through the point and is concentric with the charged cylinder and use Gauss’s law without any complicated integrals. A majority of students incorrectly agreed with this statement after lecture-based instruction. These students typically claimed that the finite cylinder with a uniform charge on the surface had sufficient cylindrical symmetry so that Gauss’s law can be exploited to easily find the electric field due to the uniform charge on the surface of the finite cylinder. Moreover, those who did not agree with the statement often provided incorrect reasoning. They did not realise that, unlike an infinite cylinder with uniform surface charge, for a finite cylinder with uniform surface charge, the electric field magnitude at a fixed distance from the axis of the cylinder varies so that Gauss’s law cannot
easily be used without doing complicated integrals to find the electric field. The following are two examples of typical student responses manifesting this type of difficulty:

’I agree with him. The use of Gauss’s law can help find the electric field without the use of any complicated integral because this is a typical cylindrical problem’.

’Agree. For the integral to find electric field $\oint E\cdot dA$ could be solved using the cylinder for $dA$ just becomes the dimensions of the cylinder $2\pi rL$. Those getting rid of any integrals in an easy manner’.

Difficulties in internalising that a cube with uniform surface charge does not have sufficient symmetry required to find the electric field using Gauss’s law

After traditional lecture-based instruction, a majority of students struggled to identify situations in which Gauss’s law may be useful to easily find the electric field and overgeneralized results obtained for a highly symmetric charge distribution to situations in which they are not applicable. For example, many students who did not engage with the tutorial claimed that a cube with charge uniformly distributed over its surface (e.g., on post-test question (6)) is symmetric enough to use Gauss’s law to easily find the electric field at a point. One common incorrect response on post-test question (6) was that to determine the electric field at a point outside of a cube with uniform charge on its surface, one can choose a cube as the Gaussian surface that is concentric with this cube (and oriented so that its faces are parallel to the faces of the actual cube) and passes through the point. For example, one student stated, ‘Yes, I would use a big cube since it is the same shape.’ Students with this type of response often incorrectly assumed that the electric field on the cubic Gaussian surface they chose was the same everywhere. In interview situations, when students with this type of response were asked to find the magnitude of the electric field at point P in post-test question (6), some of them tried to solve for the electric field by dividing the net charge on the surface of the cube by the area of the cubic Gaussian surface. There were also students (especially among those who did not engage with the tutorial) who thought that on the post-test question (6), it is possible to use a sphere or a cylinder as a Gaussian surface. Some claimed that any closed surface can be used as a Gaussian surface for finding the electric field due to a cube with uniform surface charge. Thus, the fact that a very high level of symmetry of the charge distribution is required was ignored. Some of the interviewed students noted that if the charge was ‘uniformly’ distributed on an object, Gauss’s law can be used to find the electric field easily.

Difficulty in determining a Gaussian surface to find the electric field at a point due to a symmetric charge distribution

In exploiting symmetry and using Gauss’s law to find the electric field at a point due to a highly symmetric charge distribution, students should draw an appropriate hypothetical Gaussian surface consistent with the symmetry of the charge distribution through the point where the field is desired. Many students struggled to determine an appropriate Gaussian surface for cases in which there is sufficient symmetry to use Gauss’s law to find the electric field easily. For example, on the pre-test questions (3) and (4), students were provided a solid uniformly charged non-conducting sphere (see the appendix for the question) and asked to draw a Gaussian surface that could be used to find the electric field at points B and A (outside and within the sphere, respectively). They were also asked to explain whether the same
Gaussian surface can be used to find the electric field for both points A and B. The difficulty in drawing the Gaussian surface was evident on the pre-test questions (3) and (4) (see the appendix for the question). To determine the electric field at a point easily using Gauss’s law, students should know that the Gaussian surface should satisfy the following conditions: (1) The point where we want to determine the electric field should be on the Gaussian surface. (2) By Gauss’s law, \( \oint \vec{E} \cdot d\vec{A} = \frac{Q_{\text{enclosed}}}{\varepsilon_0} \) \( \cos \theta \) should have a constant value on each sub-section of the Gaussian surface so that it can be pulled out of the flux integral. Then, \( \int |d\vec{A}| = \text{total area of the sub-section of the surface} \). On the pre-test questions (3) and (4), since the charge distribution is spherically symmetric, any point on a Gaussian sphere that is concentric with the uniformly charged sphere will have the same magnitude of the electric field. Also, because of the symmetry of the charge distribution, the direction of the electric field at any point on the concentric Gaussian sphere is perpendicular to the area vector, so that \( \cos \theta = 1 \). So for points A and B on the pre-test questions (3) and (4), the Gaussian surface should be two different concentric spheres on which those points lie. After traditional lecture-based instruction, very few students drew the correct Gaussian surface for both of the points.

Student responses to the pre-test questions (3) and (4) suggest that many students struggled to find an appropriate Gaussian surface for the given charge distribution. The most common mistake was drawing a Gaussian sphere to calculate the electric field at points A and B with point A or B at the centre (see figure 1(a)). These students did not realise that in order to determine the electric field at a point due to a highly symmetric charge distribution using Gauss’s law, they should draw the Gaussian surface in such a way that the point where they have to find the electric field is on the surface. There were also students who incorrectly chose a cylinder or a cube as the Gaussian surface for the pre-test question (4) and placed the points where the electric field is to be calculated at the centre of the cylinder or the cube (see figure 1(b)). On the other hand, some students placed the points where the electric field is to be calculated on the surface of the Gaussian surface they drew (e.g., at the centre of a cylindrical cap or the face of a cube as in figure 1(c)). Some students also incorrectly claimed that the same Gaussian surface can be used for both points A and B. For example, on the pre-test question (4), some students claimed that they would use a Gaussian sphere that passes through both points A and B as the Gaussian surface (see figure 1(d)) while other students claimed that their Gaussian surface would be a cylinder with points A and B at the centre of each cylindrical cap (see figure 1(e)).

Below, we provide specific examples from actual student responses of these difficulties shown schematically in figure 1. As noted, many students did not realise that the point where they were supposed to find the electric field due to the charge distribution should be on the Gaussian surface and incorrectly situated the point where the electric field was to be determined at the centre of a Gaussian sphere or at the centre on the axis of a Gaussian cylinder they drew. For example, figure 2 shows that on the pre-test question (3), one student claimed, ‘A + B \( \rightarrow \) cylinder because you only have to account for the circular edges to find the electric field. The same shape can be used (to find the electric field at both points A and B) because the electric field magnitude will be the same no matter what shape is used.’ The students with these types of responses had not understood the role of symmetry in exploiting Gauss’s law to find the electric field. Other students claimed that points A and B in the pre-test question (3) should be at the centre of a Gaussian sphere, or a circle (which is not even a Gaussian surface). In fact, as noted earlier, some students incorrectly assumed that if the charge is distributed on a surface with a particular symmetry, the Gaussian surface should have the same symmetry and should be drawn with the point where the electric field is to be calculated at the centre of the Gaussian surface. For example, for a cube with uniform surface
charge (e.g., post-test question (6) in the appendix), students with this misconception incorrectly claimed that the Gaussian surface should be cubic with the point where they have to find the field at the centre of the Gaussian cube.
Another common misconception regarding the pre-test question (3) was that in order to find the electric field at two points A and B, e.g., inside and outside the sphere due to a uniform sphere of charge, there should be a single Gaussian surface drawn through both points A and B. For example, the Gaussian surface that some students drew for the pre-test question (3) to find the field at points A and B due to a solid uniformly charged non-conducting sphere was a finite cylinder with both points A and B at the caps of the same cylinder. Figure 3 is a typical response to the pre-test question (3) of a student with this type of difficulty who stated, ‘I would draw a cylinder for A and use it for B as well’. The directions of the electric field at points A and B in figure 3 suggest that the student does not take into account the symmetry of the charge distribution in determining the direction of the field at each of those points. We note that in figure 3, the student also situated both points A and B where the electric field is to be calculated at the caps of his Gaussian cylinder, which was also common amongst students in general. Other students also used Gaussian surfaces with incorrect symmetry to find the field at points A and B on the pre-test question (3) as shown in figure 4.

**Difficulty in drawing appropriate Gaussian surfaces due to confusion between the electric field and electric flux**

Earlier we discussed that some students had difficulty, e.g., with the pre-test question (1) because they confused the electric flux for the field and incorrectly claimed that the electric flux through a closed surface can be found using Gauss’s law only if the surface has a high level of symmetry (sometimes students also were confused about the symmetry of the charge distribution versus the shape of a Gaussian surface). Written responses and interviews also suggest that some students incorrectly thought that the shape of the Gaussian surface does not matter for determining the electric field at a point using Gauss’s law due to the confusion between the electric field and the flux. Figure 5(a) shows that in response to the pre-test question (3), in order to find the electric field due to a uniformly charged non-conducting solid sphere at point A inside and point B outside the sphere, one student wrote, ‘You could contain the entire system in a single Gaussian surface such as a cylinder or cube. What is important is that you perform calculation for electric flux based only on the charge inside your Gaussian surface. Personally, I would choose a sphere to encompass each point because point A is already in the configuration + it would simplify your intuitive approach’. This student incorrectly drew the direction of the electric field to be horizontal and pointing to the right. Figures 5(b) and (c) show the responses of two additional students who thought that the shape
of the Gaussian surface is not important for determining the electric field due to the charge distribution. In response to the same question, another student claimed, ‘The shape does not matter. The electric field can be calculated regardless of the shape because the flux lines will enter and leave any shape’. Yet another student claimed that for point A, ‘you could choose pretty much anything that is 3D solid object so I chose a cube’ and for point B, ‘I chose a cylinder to show that because its (sic) on the face of the cylinder, outside the sphere’. This student claimed that the electric field at point A would be similar to the field due to a point charge and the field at point B would be zero. These types of responses suggest, among other things, confusion between the field and flux. Gauss’s law is indeed valid for a closed surface of any shape and the knowledge of the charge enclosed inside the closed surface is sufficient to yield information about the electric flux. However, information about the flux through a Gaussian surface does not yield information about the electric field at a point unless the charge distribution is symmetric and the Gaussian surface is chosen carefully.

We note that the difficulties in drawing the Gaussian surface on the post-test questions (3) and (4) in the context of cylindrical symmetry of charge (see the appendix) were very similar in spirit to the pre-test questions (3) and (4) for the spherically symmetric charge distribution discussed above. For example, in the post-test question (3) (see the appendix),
many students had difficulty with the fact that to find the electric field due to an infinitely long uniformly charged non-conducting solid cylinder, the point where they have to find the electric field should be on the Gaussian surface chosen. In the tutorial group, the percentages of students who had such a difficulty on the post-test questions (3) and (4) was significantly less than on the pre-test questions (3) and (4).

**Difficulty in selecting an appropriate Gaussian surface to find the electric field using Gauss’s law due to an infinite uniform line or sheet of charge**

On question (2) of the pre-test (see the appendix), students had to choose the Gaussian surfaces that would help them determine the electric field at point P readily due to an infinite uniform line of charge. All of the alternative choices in the multiple-choice question were selected with each answer choice selected by about equal number of students. Students were often unsure about the symmetry that was relevant for making appropriate decisions. In fact, in response to these questions, some students claimed that a sphere is an appropriate Gaussian surface to find the field for a uniformly charged infinite line. Their preference for choosing a
sphere as a Gaussian surface regardless of whether it is appropriate indicates that they did not have a deep understanding of the symmetry considerations and why the determination of the Gaussian surface should be done with care to ensure that Gauss’s law can easily be used to find the electric field for a given charge distribution. Interviews suggest that students had some idea that symmetry was relevant but they did not deeply understand the relation between the choice of the Gaussian surface and the symmetry of the charge distribution. Some students thought that the sphere was the ‘most’ symmetric shape so it can be used as a Gaussian surface. Some interviewed students who selected option (c) claimed that the magnitude of the electric field due to the infinite line of uniform charge must be the same at every point on the Gaussian cube as well.

Similarly, on question (1) in the post-test, students had to identify the appropriate Gaussian surfaces that would make it easy to use Gauss’s law to calculate the electric field due to an infinite sheet of charge. On this question, students must realise that for an infinite sheet of charge, the electric field is uniform and the direction is perpendicular to the sheet. If we put the sheet symmetrically half way between the top and bottom surface of a cube or a finite closed cylinder, the direction of the field is parallel to the area vector of some surfaces and perpendicular to the area vector of other surfaces. Thus, the electric field can be easily calculated by using Gauss’s law. However, if we choose a sphere as a Gaussian surface, the direction of the electric field is not parallel or perpendicular everywhere to the area vector for the sphere. Thus, a sphere cannot be used as a Gaussian surface to calculate the electric field due to an infinite uniform sheet of charge easily. Although for this infinite uniform sheet of charge, the calculation with a Gaussian sphere is not easy because the area vector and the field make different angles for different infinitesimal areas on the sphere, some students claimed that a spherical Gaussian surface will work because a sphere is ‘very symmetric’.

**Incorrectly assuming that there must be a charge present at the point where the electric field is desired**

When working on the symmetry and Gauss’s law tutorial in a one-on-one interview situation, some students claimed that if they have to find the electric field at a point A, there must be a charge present at that point. For example, in the pre-test question (5), students were asked to find the direction of the electric field at points A and B due to a solid non-conducting sphere with uniform surface charge. In response to this question, some students drew electric field lines coming out of points A and B as though there was a point charge located at each point and that was the only charge that was contributing to the electric field at that point. A typical response from a student who made such a claim is shown in figure 6. On the other hand, some students assumed that there were two charges with equal magnitude but opposite signs present at points A and B and they drew the electric field lines from A to B as one would draw for a dipole because they were asked for the electric field at both point A (which was inside the uniform sphere of charge) and point B (which was outside). Moreover, some students claimed that the electric field at point B is zero because there is no charge at point B (since point B is outside the uniformly charged sphere).

**Incorrectly assuming that a non-conductor completely shields the inside from the electric field due to outside charges**

When students were asked to determine the net electric field at points inside of a hollow non-conducting object with charge uniformly distributed on its surface, many students claimed that the electric field inside the hollow non-conducting object is zero everywhere. The shape of the object was not limited to a sphere. This idea was also applied by some students to
objects such as a cylinder, a cube or of any other shape. In other words, students with these types of responses had difficulty with the electric field inside hollow non-conducting objects of different shapes with the charges on their surface or charges outside. Many students claimed that such a material will ‘isolate’ or ‘screen’ the inside completely from the electric field due to outside charges. These students were confused about the role of a conductor versus insulator in shielding the inside from the electric field produced by the external charges. For example, some students claimed that the electric field inside a non-conducting hollow cube with charge uniformly distributed on its surface will be zero everywhere. Interviews suggest that these students often thought that a hollow region inside an object is always shielded from the charges on the surface or charges outside regardless of whether the object is a conductor or non-conductor. This notion of shielding was maintained by some of the interviewed students even when they were reminded by the interviewer that the object on which the charges are distributed is non-conducting and there are charges outside. Some students incorrectly claimed that the net electric field due to any charge outside of the hollow region must work out to be zero everywhere inside the hollow region of any shape regardless of whether the hollow region was bounded by a conductor or an insulator. Some students drew spherical or cubic Gaussian surfaces inside a hollow cube made of non-conducting material and argued that because there is no charge enclosed, the electric field will be zero everywhere according to Gauss’s law. For example, one student incorrectly claimed that he always been amazed at how Gauss’s law can be used to prove that the electric field in the hollow region inside a closed object is always zero everywhere, a result that appears to be counterintuitive to him.

**Difficulty visualising in three dimensions**

Some students had great difficulty visualising in three dimensions. They did not distinguish between two and three dimensional surfaces, e.g., some of them used the words sphere and circle interchangeably or used the words cube and square interchangeably. Some students incorrectly claimed that the Gaussian surface for a spherical charge distribution is a circle although a circle is not a closed surface (and hence cannot be a Gaussian surface). In response to question (2) on the pre-test, one interviewed student argued for a long time that the distances of all points on the surface of the Gaussian sphere must be the same from the axis of the infinitely long line (with uniform charge) similar to the distance of all points on a coaxial Gaussian cylinder. It was clear from the discussion that the student had difficulty in

![Figure 6. A sample student response on the pre-test question (5) regarding the direction of the electric field at points A and B due to a uniformly charged non-conducting solid sphere.](image-url)
visualising how different points on the surface of the Gaussian sphere with its diameter coinciding with the infinite line of charge can have different distances from the axis of the cylinder. Visualising this situation became more difficult because the student kept confusing what was relevant for the problem with the fact that all points on the sphere are at the same distance from the centre of the sphere. On the post-test question (3), some interviewed students had difficulty visualising that the cross-section and side-view provided were both for the same infinite non-conducting cylinder with uniform charge. Some referred to the cross-sectional view as a sphere or a circle and when they drew the Gaussian surfaces at points A and B in the cross-sectional view in the post-test question (4) that looked like a sphere or a circle, they called it a sphere or a circle (instead of calling it a cross-section of a cylinder).

Methodology for the tutorial development and administration

The difficulties described in the previous section indicate that many students struggle with symmetry and Gauss’s law concepts. Therefore, we developed a guided inquiry-based tutorial using the research on students’ conceptual difficulties as a guide. As students work through the guided learning sequences in the tutorial, they are asked to predict what should happen in a particular situation. Then, the tutorial provides scaffolding support and feedback and strives to help them build a robust knowledge structure.

As described in [34], ‘The development of the research-validated guided inquiry-based tutorial was carried out with the following core issues in mind: (1) the tutorial must build on students’ prior knowledge so it is important to investigate the difficulties students have related to relevant concepts before the development of the tutorial, (2) the tutorial must create an active learning environment where students get an opportunity to build a good knowledge structure in which there is less room for misconceptions, (3) the tutorial must provide scaffolding support, guidance and feedback to students and opportunity to organise, reconstruct, and extend their knowledge. The preliminary version of the tutorial and the corresponding pre-/post-tests not only used research on student difficulties as a guide but also a cognitive task analysis of the underlying concepts from an expert perspective. The cognitive task analysis from the perspective of an expert involves making a fine-grained flow chart of the concepts involved in solving a specific class of problems. Such analysis can help identify some stumbling blocks for students. However, investigation of students’ difficulties using written tests and interviews was critical for developing the tutorial because theoretical analysis from the perspective of an expert of the difficulties students have with Gauss’s law often does not capture all the difficulties students actually have with relevant concepts’.

During the development of the tutorial, in individual interviews, we administered the different versions of the pre-test, tutorial and post-test to some introductory physics students, who were asked to talk aloud while working on them. After each individual interview with a particular version of the tutorial (along with the administration of the pre-test and post-test), modifications were made based upon the feedback obtained from the interviewed students. For example, if students got stuck at a particular point and could not make progress from one question to the next with the scaffolding already provided, suitable modifications were made to the tutorial. Thus, the administration of the tutorial to students individually was useful to ensure that the guided approach was effective and the questions were unambiguously interpreted. The tutorial was also iterated several times with four physics instructors who had taught introductory electricity and magnetism courses for their feedback, and modified after each round of feedback. These individual administrations helped fine-tune the tutorial and
improve its organisation and flow. The iteration of the tutorial with instructors several times ensured that the content and wording of the questions were appropriate. Modifications were made based upon their feedback. When we found that the tutorial was working well in individual administration and the post-test performance was significantly improved compared to the pre-test performance, it was administered in the classes. In summary as described in [34], ‘the development of the tutorial went through a cyclic, iterative process which included the following stages before the in-class implementation: (1) development of the preliminary version based on a cognitive task analysis from an expert perspective of the underlying knowledge and research on student difficulties; (2) implementation and evaluation of the tutorial by administering it individually to students and obtaining feedback from faculty members who teach the introductory courses; (3) determining its impact on student learning and assessing what difficulties were not adequately addressed by the tutorial; (4) refinements and modifications based on the feedback from the implementation and evaluation’.

For in-class administration of the tutorial in several sections of the course, which were otherwise taught primarily using lecture and traditional recitations (in which the teaching assistants (TAs) answered homework questions from students who were typically engineering, chemistry, mathematics and physics majors), the pre-test and then tutorial were administered after traditional lecture-based instruction in relevant concepts. Students in these sections of the introductory physics course worked on the tutorial in groups of two or three. The number of matched students (these students were present when both pre-test and post-test were administered) in the three sections of the introductory course in which the tutorial and both pre-/post-tests were administered ranged from 59 to 83.

As discussed in [34], ‘The pre-test was administered individually after traditional instruction right before students worked on the tutorial in groups. Although the pre-test and post-test accompanying the tutorial assess the same concepts, the same test was not used as both the pre-test and post-test to minimise the effect of pre-test on post-test. Also, the pre-test was neither returned to the students nor was it discussed by the instructors with the students. Since not all students completed the tutorial during the class, they were asked to complete it as part of their homework assignment. At the beginning of the next class, students were given an opportunity to ask for clarification on any issue related to the part of the tutorial they completed at home and then after they submitted the tutorial, they were administered the corresponding post-test individually. Both the pre-test and post-test counted for a quiz grade for the course. While the pre-test, which was graded for completeness, was not returned, the post-test was returned after grading. In all of the sections of the course in which the tutorial was administered, the total time devoted to symmetry and Gauss’s law is not significantly different from what the instructors in other sections in which the tutorial was not used allocated to this material. The main difference between the tutorial and the non-tutorial sections of the course is that fewer solved examples were presented by the instructor during lecture in the tutorial sections (students worked on many problems themselves in the tutorial).

We note that since the tutorial was administered during the regular class time, the instructor and the TA were present during these tutorial sessions to ensure smooth facilitation. Typically, students worked in small groups and they were asked to raise their hands for questions and clarifications. Once the instructor/TA knew that a group of students was making good progress, that group was invited to help other groups in the vicinity with similar questions. Thus, students not only worked in small groups discussing issues, but some of them also had an opportunity to help students in the other groups’.
An overview of the tutorial

The entire tutorial can be found at http://per-central.org/items/detail.cfm?ID=12620. The tutorial uses a guided inquiry-based approach to learning to help students determine, e.g., whether sufficient symmetry exists to exploit Gauss’s law to calculate the electric field for a given charge distribution. Then, students learn to select the appropriate Gaussian surfaces that would aid in using Gauss’s law to find the electric field. Finally, they use Gauss’s law to calculate the electric field in the cases with a high level of symmetry. The first part of the tutorial helps students recognise the symmetry of the charge distribution and the Gaussian surface that is useful to calculate the electric field using Gauss’s law for a given charge distribution. Then students are provided guidance and support to determine appropriate Gaussian surfaces to find the electric field due to an infinite cylinder or line of uniform charge. After students learn how to find the field for an infinite cylinder with uniform charge, they learn how to choose an appropriate Gaussian surface to find the electric field due to an infinite sheet with uniformly distributed charge and how to find the field due to this highly symmetric charge distribution.

In order to help students internalise the symmetry of a charge distribution and determine the Gaussian surface that is useful for finding the electric field using Gauss’s law easily, students first consider four hollow objects made with non-polarisable non-conducting material of different shapes (cube, sphere, finite cylinder and irregularly shaped) as shown in Figure 7. Each of these objects is electrically isolated and has the same charge uniformly distributed over its surface. The students are first asked to imagine a spherical Gaussian surface for each of the objects and find the electric flux through the surface. Then they are asked whether they can easily find the electric field at a point on the Gaussian surface using Gauss’s law (without being provided help). Then, they are provided guidance and asked to consider the validity of statements from two hypothetical students in a conversation in which the students have different opinions. For example, student 1 states that the magnitude of the electric field at a point on the spherical Gaussian surface can only be found for the uniformly charged sphere while student 2 claims that the cube and the finite cylinder are also symmetric shapes so that the electric field at a point on the spherical Gaussian surface can be found by using Gauss’s law in those cases as well. Then student 1 gives a specific example of a uniformly charged cube to explain that the magnitude of the electric field or the angle between the electric field and the area vector on the spherical Gaussian surface are not the same everywhere so that Gauss’s law cannot be used easily to find the magnitude of the field because there is not enough symmetry in the charge distribution.

After students are guided to make sense of the situations in which the spherical Gaussian surface can be used to find the electric field using Gauss’s law, they are asked to replace the spherical Gaussian surface with a cubic Gaussian surface and asked similar questions about...
the electric flux and field. By working through these guided inquiry-based learning sequences, students realise that a cubic Gaussian surface does not work even for a cube with uniform charge for finding the electric field due to that charge distribution. They learn that a cube with charge uniformly distributed on it does not have a sufficiently symmetric charge distribution to guarantee that $|\vec{E}| \cos \theta$ will be the same on any Gaussian surface (or some faces of a Gaussian surface) and so they cannot find the electric field due to the uniformly charged cube easily from the information about the net electric flux through any Gaussian surface.

Moreover, the guided inquiry-based sequence guides students to learn that only the interior of a conductor is shielded from the effects of charges outside or from the charges on its surface. They also learn that surface charges on a conductor can redistribute to make the net electric field zero everywhere inside due to the charges but for hollow insulating objects, the electric field in the hollow region need not be zero. This type of scaffolding support is necessary in light of the fact that after traditional lecture-based instruction, many students had the misconception that the field inside should be zero for objects of any shape even if the object is made with non-conducting material.

The guided inquiry-based approach in the tutorial also helps students determine the electric field due to an infinite non-conducting hollow cylinder (or line) with uniform surface charge since students struggled with this situation after lecture-based instruction. In particular, it was challenging for them to determine the Gaussian surface that works for finding the electric field in this situation. The guided approach in the tutorial introduces students to an infinitely long, hollow non-conducting cylinder with a uniform surface charge and asks them to find the field at a point A which is outside of the cylinder as shown in figure 8.

The students are guided to draw the cross-sectional view and side-view of an imaginary cylindrical surface with a length $L$ and without caps, which includes point A and on which the magnitude of the field is the same at EVERY point. They are also asked to indicate the direction of the field at the points marked A, B, and C and the angle between the area vector (outward normal) and the direction of electric field. Then, students are asked to agree or disagree with statements by hypothetical students in a conversation and explain their reasoning, which guides them to learn that an open cylinder cannot be used as a Gaussian surface but a closed cylinder with two caps symmetrically situated with respect to the charge distribution works well. Students are guided to make sense of the fact that for a closed cylinder with caps, the area vector on the caps is perpendicular to the field everywhere (so the flux through the caps is zero), so it is possible to calculate the magnitude of the field easily. The validity of each person’s statement(s) in another conversation between hypothetical people
guides students to reflect upon the fact that the Gaussian surface for an infinite uniformly charged cylinder does not have to be infinite and that the electric field only depends on the charge per unit length and the length of the Gaussian cylinder will cancel out in the end when we find the electric field magnitude due to this charge distribution. After choosing the correct Gaussian surface, students are asked to find the charge $Q$ enclosed by the Gaussian cylinder of length $L$ and to determine the magnitude of the electric field by using $\Phi E = \frac{Q_{\text{enclosed}}}{\varepsilon_0} = |\vec{E}| \cos \theta \int |d\vec{A}| = |\vec{E}| A \cos \theta$. The tutorial also guides students to consider the field at a point inside of the cylinder. By working through the guided inquiry-based learning sequences, students learn that they can determine the field inside the uniformly charged cylinder using a similar procedure to the one they used for calculating the field outside of the charged cylinder but the field inside is zero because there is no net charge enclosed. At the end of this part, students learn that a finite coaxial cylinder is useful as a Gaussian surface not only for an infinite uniformly charged cylinder but also for an infinite line with uniform linear charge density since they both have cylindrical symmetry. The tutorial also focuses on helping students learn that one cannot use Gauss’s law to find the electric field if the charge distribution is not uniform on the infinite cylinder because the electric field magnitude is not the same at every point on the side of the Gaussian cylinder (also, the infinitesimal area vector is not necessarily perpendicular to the electric field vector on the caps of the Gaussian cylinder).

In one guided inquiry-based sequence, students are asked to determine the electric field due to an infinite sheet of charge. In that sequence, students consider a horizontal infinite non-conducting sheet of charge with uniform charge density (see figure 9) and two closed Gaussian surfaces: a cylinder and a rectangular box. Each Gaussian surface is symmetrically situated with half of each surface above and half below the infinite sheet of charge and with their axes of symmetry perpendicular to the sheet as shown.

This guided sequence helps students learn that the electric field due to the infinite sheet with uniform charge is perpendicular to the sheet (and the field is in the opposite directions on the two sides of the sheet) and there is no electric flux through either the cylindrical side or through the vertical faces of the box chosen as the Gaussian surface. They learn to make sense of the fact that any Gaussian surface that is flat on the top and bottom with sides perpendicular to the sheet will work for finding the field due to the charged sheet. Students are then guided to learn to use Gauss’s law to calculate the electric field at a point due to the charged sheet and they learn that the magnitude of the electric field due to this infinite sheet with uniform charge is the same everywhere. After these guided sequences, students are also asked to consider whether the electric field will be different if we use a cylinder with a different area of the caps or a cubic box with different surface area of the sides. They are also asked to contemplate...
whether the distance of the point where they are finding the electric field from the infinite sheet matters in what they find for the magnitude of the field and whether is it possible to find the electric field due to a finite uniform sheet of charge using Gauss’s law easily.

After traditional lecture-based instruction in Gauss’s law, many students had the misconception that the electric field inside a hollow object of any shape is zero if there are no charges inside. Therefore, the tutorial helps students make sense of the fact that only a conductor is shielded from the charges on its surface because surface charges can redistribute to make the net electric field zero everywhere inside and for insulating objects, the electric field in the hollow region need not be zero if the charge distribution does not have sufficient symmetry. At the end of the tutorial, students reflect upon the cases with spherical, cylindrical and planar symmetries in which there is sufficiently high symmetry of the ‘right kind’ to use the information about the net flux to find the electric field easily.

Assessment: performance of the tutorial and comparison groups

As described in [34], ‘The pre-test and post-test (see the appendix) were graded by two individuals based upon an agreed rubric developed together, and the inter-rater reliability was better than 85%. The grading rubric scores each answer as correct or incorrect (based upon whether the student responses were conceptually correct or not) and if there was an explanation required, student responses for that part of the question were graded on the following scale: full point for correct explanation, zero point for incorrect or no explanation and half point for partially correct. Table 1 shows the average pre-/post-test scores on each question and the overall score for three sections of the course in which the tutorial was administered. In the fourth section of the course, the post-test was returned without photocopying them so we only have data on student performance for that tutorial class on the standardised conceptual test [32] which was administered at the end of the semester (performance on this test is discussed at the end of this section).’ As shown in table 1, an additional question was included in the pre-test for sections 1 and 3 after analysis of data for section 2. Table 1 shows that the average performance was significantly better on the post-test compared to the pre-test for the tutorial group. The differences in the performance of different sections of the course on the pre-/post-test in table 1 may partly be due to the differences in student samples, instructor/TA differences or the manner in which the tutorial was administered.

Table 2 shows the pre-/post-test data from a comparison group which consists of a section of the course in which students did not work on the tutorial. The pre-test was given to students in the comparison group (non-tutorial group) immediately after relevant instruction similar to the tutorial group but post-test was given in the following week as part of the weekly recitation quizzes after students had the opportunity to complete all the homework problems on those topics.

The results of a t-test that compares the performance of the tutorial group and comparison group (non-tutorial group) on pre-/post-tests in tables 1 and 2 show that regardless of which section of the course students belong to (i.e., whether they belonged to the tutorial or comparison groups), their performance on the pre-test was poor after traditional lecture-based instruction (the averages for tutorial and non-tutorial classes are not statistically significantly different with a p value of 0.52 on the t-test). On the other hand, students in the comparison group performed significantly worse on the post-test than those in the sections of the course in which students engaged with the tutorial (p value < 0.0001 on the t-test comparing the post-test averages for these two groups).
Table 1. Average percentage scores obtained on individual questions and overall on the pre-/post-tests (matched students who took both pre-/post-tests). The pre-test was administered after traditional lecture-based instruction but before the tutorial. As shown in the table, fewer questions were given in the pre-test and post-test for section 2. The symbol N refers to the matched number of students in a given section for the pre-test/post-test and 'total' refers to the total average percentage score including all questions on a pre-test or post-test administered to a given section. The relative weights for all questions for sections 1 and 3 were identical (pre-test questions: 10%, 10%, 20%, 20%, 20%, 20% and post-test questions: 20%, 10%, 20%, 10%, 20%, 20%). The relative weights for the pre-test and post-test questions for section 2 were 20%, 20%, 30%, 30% and 30%, 30%, 30%, 10%, respectively.

| Section | N  | 1  | 2  | 3  | 4  | 5  | 6  | Pre-total | 1  | 2  | 3  | 4  | 5  | 6  | Post-total |
|---------|----|----|----|----|----|----|----|-----------|----|----|----|----|----|----|------------|
| 1       | 64 | 41%| 19%| 53%| 54%| 43%| 3%| 37%       | 86%| 84%| 92%| 95%| 96%| 86%| 90%        |
| 2       | 51 | 30%| 8% | 57%| 29%|-- | --| 31%       | 88%| 91%| 88%| 54%|-- | --| 87%        |
| 3       | 65 | 29%| 22%| 58%| 58%| 41%| 18%| 40%       | 83%| 91%| 96%| 92%| 91%| 92%| 91%        |
Table 2. Average percentage scores obtained on individual questions of the pre-/post-tests and total scores for the section taught via traditional lecture in which the tutorial was not used but the pre-test was administered immediately after lecture-based instruction and the post-test the following week as a recitation quiz after the students had turned in their homework on these topics. The weights given to different questions are the same as the corresponding weights for sections 1 and 3 of the tutorial group.

| N  | 1   | 2   | 3   | 4   | 5   | 6   | Pre-total | 1   | 2   | 3   | 4   | 5   | 6   | Post-total |
|----|-----|-----|-----|-----|-----|-----|-----------|-----|-----|-----|-----|-----|-----|-----------|
| 69 | 33% | 13% | 52% | 36% | 46% | 12% | 34%       | 45% | 59% | 51% | 51% | 45% | 26% | 43%       |
We also assessed the effectiveness of the tutorial by separately analysing the performance of students who performed differently on the pre-test after traditional lecture-based instruction. In particular, students in the tutorial and comparison (non-tutorial) groups were divided into three sub-groups based on their pre-test scores. In this analysis also, we only considered students who took both the pre-test and post-test (matched pairs). Table 3 shows the performance of students on the pre-/post-tests partitioned into three sub-groups based upon the pre-test performance. As can be seen from table 3, the tutorial generally helped students in all pre-test ranges (with significant improvement on post-test) including those who performed poorly on the pre-test.

Table 4 shows the performance of students in the comparison group on the pre-/post-tests partitioned into three sub-groups based upon the pre-test performance for students who did not work on the tutorial. N denotes the total number of students in each pre-test range, who took both the pre-/post-tests. For students in the high pre-test range, sometimes there are too few students for a statistical interpretation.

![Table 3](image1)

| Pre-test range (%) | N  | Pre | Post |
|-------------------|----|-----|------|
| All               | 180| 36% | 90%  |
| 0%–34%            | 87 | 16% | 86%  |
| 34%–67%           | 74 | 49% | 92%  |
| 67%–100%          | 19 | 76% | 95%  |

![Table 4](image2)

| Pre-test range (%) | N  | Pre | Post |
|-------------------|----|-----|------|
| All               | 69 | 34% | 43%  |
| 0%–34%            | 40 | 18% | 36%  |
| 34%–67%           | 23 | 51% | 50%  |
| 67%–100%          | 6  | 77% | 65%  |

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In order to evaluate retention of learning by different student populations at the end of the semester on these concepts related to symmetry and Gauss’s law, we administered a 25 question multiple-choice standardised conceptual test [32] in which 13 questions focus on the concepts covered in the tutorial and analysed performance of different groups. Table 5 shows the average scores on the 13 questions on the concepts relevant for the tutorial on the standardised survey administered to different student populations. In table 5, N refers to the total number of students in each group. The introductory physics group without tutorial consists of two sections of the course, one of which was the section for which the pre-test and post-test data are available. Also, four classes are included in the ‘with tutorial’ group in table 5, out of which three of the classes are the ones for which the pre-test and post-test data are available (see table 1 for pre-/post-test data). The group ‘Honors Students’ in table 5 consists of students who were in a separate section of the introductory course in which more challenging quantitative problems were assigned to students but the instructional approach
was primarily lecture-only. In all undergraduate courses, this cumulative test was administered after instruction in these concepts in that course except in the upper-level E&M course, in which it was given both before and after instruction in that course (since students had earlier instruction in these concepts at the introductory level). The last row of the table lists the average performance of the tutorial group as well as other groups on all of the 13 questions combined.

| Quest# | Without tutorial Honors students | Upper-level undergrad | With tutorial | Graduate students |
|--------|----------------------------------|-----------------------|--------------|------------------|
|        | N = 135                          | N = 182               | N = 33       | N = 278          | N = 33         |
| 5      | 82                               | 77                    | 70           | 68               | 84             | 88             |
| 6      | 60                               | 55                    | 82           | 82               | 69             | 76             |
| 10     | 41                               | 46                    | 45           | 50               | 56             | 70             |
| 11     | 46                               | 25                    | 24           | 68               | 72             | 94             |
| 12     | 45                               | 35                    | 43           | 53               | 69             | 79             |
| 13     | 3                                | 19                    | 18           | 14               | 26             | 36             |
| 15     | 23                               | 26                    | 36           | 25               | 52             | 61             |
| 16     | 13                               | 14                    | 9            | 36               | 45             | 52             |
| 17     | 15                               | 25                    | 27           | 7                | 45             | 45             |
| 18     | 40                               | 28                    | 58           | 39               | 63             | 73             |
| 21     | 20                               | 43                    | 36           | 53               | 34             | 88             |
| 22     | 21                               | 30                    | 27           | 50               | 46             | 73             |
| 24     | 22                               | 32                    | 18           | 61               | 38             | 55             |
| Avg    | 33                               | 35                    | 38           | 47               | 54             | 68             |

Summary

We described an investigation of introductory physics students’ difficulties with symmetry and Gauss’s law and how that research was used as a guide in the development and evaluation of a research-validated tutorial on these concepts to help students develop a functional understanding of when and how Gauss’s law can be used to find the electric field due to a given charge distribution. The tutorial uses a guided inquiry-based approach to learning and involved an iterative process of development and evaluation. The common misconceptions...
are explicitly brought out and students are provided guidance and support to build on their prior knowledge and develop a good knowledge structure of these concepts so that there is less room for misconceptions. The guided inquiry-based tutorial strives to help students learn that only for highly symmetric charge distribution can Gauss’s law be used to find the electric field and learn to determine the appropriate Gaussian surface and calculate the electric field for a given charge distribution.

The final version of the tutorial was administered in several sections of a college calculus-based physics course after lecture-based instruction in the symmetry and Gauss’s law concepts. The student performance on the pre-test administered before this tutorial on the symmetry and Gauss’s law (but after traditional lecture-based instruction on those concepts) was compared with their performance on the post-test administered after the tutorial in three sections of the introductory physics course. The student performance in the sections in which students worked on the tutorial was also compared with other section in which students only learned about symmetry and Gauss’s law concepts via lecture-based instruction. The data from the pre-/post-tests suggest that the tutorial was effective in helping students’ learn these concepts. The guided inquiry-based learning sequences in the tutorial that build on each other were central to ensuring that students were actively engaged in the learning process and obtained appropriate immediate feedback and scaffolding support as needed. The fact that the instructor and the TA played the role of facilitators and ensured that the student groups were making good progress with the guided inquiry-based learning sequences must have also played an important role in helping students learn these challenging concepts.

Acknowledgments

We are grateful to Z Isvan for help in grading and tabulating the pre-/post-test data. We thank F Reif, P Reilly, R P Devaty and J Levy for helpful discussions and also thank the faculty and students who helped with this research.

Appendix. Pre-/post-tests

The following information was provided for both the pre-/post-tests.

Assume all insulators (non-conductors) are non-polarisable.

Pre: Setup for the next two questions

Shown in figure A1 are four hollow imaginary surfaces coaxial with an infinitely long line of charge (with uniform linear charge density \( \lambda \)):

(I) A closed cylinder of length \( L \).
(II) A sphere of diameter \( L \).
(III) A closed cubic box with side \( L \).
A two-dimensional square sheet with side \( L \). The line of charge is perpendicular to the plane of the sheet.

(1) List all of the above surfaces through which the net electric flux is \( \Phi = \lambda L / \varepsilon_0 \)
(A) (I) only
(B) (I) and (II) only
(C) (I) and (III) only
(D) (I), (II), and (III) only
(E) (I), (II), (III), and (IV)

(2) List all of the above surfaces that can be used as Gaussian surfaces to easily find the electric field magnitude (due to the infinite line of charge) at a point P shown on the surface using Gauss’ law:
(A) (I) only
(B) (I) and (II) only
(C) (I) and (III) only
(D) (I), (II), and (III) only
(E) (I), (II), (III), and (IV)

(3) The diagram in figure A2 shows the cross-sectional view of a solid uniformly charged non-conducting sphere. To find the magnitudes of the electric field at points A (inside the sphere) and B (outside the sphere) using Gauss’s law, what kind of Gaussian surface would you choose for each? Explain your reasoning. Can the same Gaussian surface be used for both points A and B? Explain.

(4) Draw cross sections of your Gaussian surfaces for each of the points A and B on the cross-sectional view in figure A2.

(5) Show the directions of the electric field at points A and B in figure A2 due to the solid non-conducting sphere. Explain your reasoning.

(6) Consider the following statement from Harry: ‘To find the magnitude of the electric field at point P due to uniformly distributed charge on the surface of a finite cylinder of length \( L \), we should imagine a Gaussian cylinder that passes through point P and is coaxial with the charged cylinder. Then, Gauss’ law can help us find the electric field without any complicated integrals’. Explain why you agree or disagree with him.

Post: (1) Circle all of the closed imaginary surfaces in figure A3 which can be used as Gaussian surfaces to determine the magnitude of the electric field due to an infinite uniform sheet of charge. For those surfaces you circled, indicate in words or drawing how you would orient the infinite sheet of charge with respect to each of the Gaussian surfaces. Explain your reasoning for each case.

(2) Consider hollow non-conducting objects with a uniform surface charge density in figure A4. Circle all of the objects listed in figure A4 for which we could easily use Gauss’
law and symmetry considerations to find the electric field at any point due to the charge on the object. Explain your reasoning for each case.

(a) Cube, (b) closed cylinder of finite length \( L \), (c) sphere

(3) The diagram in figure A5 shows the cross-sectional and side views of a solid non-conducting cylinder which is infinitely long and uniformly charged (with negative charge uniformly distributed throughout its volume). To find the magnitude of the electric field at points A (inside the cylinder) and B (outside the cylinder) using Gauss’s law, what kind of Gaussian surface would you choose for each? Explain your reasoning. Can the same Gaussian surface be used for both points A and B? Explain.

(4) Draw cross sections of your Gaussian surfaces for each point A and B both on the cross-sectional and side views in figure A5.

(5) Show the directions of the electric field at points A and B due to the solid non-conducting infinitely long cylinder with uniform charge in figure A5 on both views. Explain your reasoning.
Shown in figure A6 is a hollow thin-walled non-conducting cube with a net charge $+Q$ uniformly distributed on its surface. The sides of the cube are 1 m long. Point P is located outside the cube at a distance of 2 m from its centre. Can you use Gauss’ law to easily find the magnitude of the electric field at point P? If so, explain what kind of Gaussian surface you would choose. If not, explain why it is not easy.

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