Probing Anomalous Top-Couplings at Polarized Linear Collider

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ABSTRACT

The energy spectra of the lepton(s) in $e\bar{e} \rightarrow t\bar{t} \rightarrow \ell^{\pm}X/\ell^{+}\ell^{-}X'$ at next linear colliders (NLC) are analyzed a model-independent way for arbitrary longitudinal beam polarizations as a general test of possible anomalous top-quark couplings.

*Talk at the fourth International Workshops on Linear Colliders (LCWS99), April 28 - May 5, 1999, Sitges, Barcelona, Spain.
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1. Introduction

A lot of data have been accumulated on the top-quark since its discovery. However it is still an open question whether its interactions obey the standard scheme like all the other fermions or there exists some new-physics contribution to its couplings. The top quark decays immediately after being produced because of its huge mass. Therefore the decay process is not influenced by any hadronization effects and consequently its decay products are expected to carry valuable information on the top properties.

Next linear colliders (NLC) of $e\bar{e}$ will give us fruitful data on the top through $e\bar{e} \rightarrow t\bar{t}$. In particular the energy spectra of the lepton(s) produced in its semileptonic decay(s) turn out to be a useful analyzer of the top-quark couplings [1]. Indeed many authors have worked on this subject (see the reference list of Ref.[2]), and we also have tackled them over the past several years. Here I would like to show some of the results of our latest model-independent analyses [2] via arbitrary longitudinal beam polarizations, where we have assumed the most general anomalous couplings both in the production and decay vertices in contrast to most of the existing works.

2. Framework

We can represent the most general $t\bar{t}$ couplings to the photon and $Z$ boson as

$$\Gamma_{vrt}^\mu = \frac{g}{2} \bar{u}(p_t) \left[ \gamma^\mu \{ A_v + \delta A_v - (B_v + \delta B_v)\gamma_5 \} + \frac{(p_t - \bar{p}_t)^\mu}{2m_t}(\delta C_v - \delta D_v\gamma_5) \right] v(p_\bar{t}) \ (1)$$

in the $m_e = 0$ limit, where $g$ denotes the $SU(2)$ gauge coupling constant, $v = \gamma, Z$, and

$$A_\gamma = \frac{4}{3} \sin \theta_W, \quad B_\gamma = 0, \quad A_Z = \frac{v_t}{2 \cos \theta_W}, \quad B_Z = \frac{1}{2 \cos \theta_W}$$

with $v_t \equiv 1 - (8/3) \sin^2 \theta_W$. Among the above form factors, $\delta A_{\gamma,Z}$, $\delta B_{\gamma,Z}$, $\delta C_{\gamma,Z}$ and $\delta D_{\gamma,Z}$ are parameterizing $CP$-conserving and $CP$-violating non-standard interactions, respectively.

On the other hand, we adopted the following parameterization of the $Wtb$ vertex
suitable for the $t \to W^+ b$ and $\ell \to W^- \bar{b}$ decays:

$$\Gamma_{Wtb}^\mu = - \frac{g}{\sqrt{2}} \bar{u}(p_b) \left[ \gamma_\mu (f_1^L P_L + f_1^R P_R) - \frac{i\sigma^\mu\nu k_\nu}{M_W} (f_2^L P_L + f_2^R P_R) \right] u(p_t),$$

(2)

$$\bar{\Gamma}_{Wtb}^\mu = - \frac{g}{\sqrt{2}} \bar{v}(p_\ell) \left[ \gamma_\mu (\bar{f}_1^L P_L + \bar{f}_1^R P_R) - \frac{i\sigma^\mu\nu k_\nu}{M_W} (\bar{f}_2^L P_L + \bar{f}_2^R P_R) \right] v(p_\ell),$$

(3)

where $k$ is the momentum of $W$ and $P_{L/R} = (1 \mp \gamma^5)/2$. It is worth to mention that the form factors for top and anti-top satisfy the following relations [4]:

$$f_1^{L,R} = \mp \bar{f}_1^{L,R}, \quad f_2^{L,R} = \pm \bar{f}_2^{R,L},$$

(4)

where upper (lower) signs are those for $CP$-conserving (-violating) contributions.

For the initial beam-polarization we used the following convention:

$$P_{e^-} = + \left[ N(e^-, +1) - N(e^-, -1) \right] / \left[ N(e^-, +1) + N(e^-, -1) \right],$$

(5)

$$P_{e^+} = - \left[ N(e^+, +1) - N(e^+, -1) \right] / \left[ N(e^+, +1) + N(e^+, -1) \right],$$

(6)

where $N(e^{-(+), h})$ is the number of $e^-(e^+)$ with helicity $h$ in each beam.

### 3. Lepton-energy spectra

After some calculations, we arrived at the normalized single distribution, which we express as

$$\frac{1}{\sigma} \frac{d\sigma}{dx} = \sum_{i=1}^3 c_i^\pm f_i(x).$$

(7)

Here $\pm$ corresponds to $\ell^\pm$ and the variable $x$ is defined from the top velocity $\beta$ and the lepton energy $E_\ell$, both in the $e\bar{e}$ c.m. frame, as [3]

$$x \equiv \frac{2E_\ell}{m_t} \left( \frac{1 - \beta}{1 + \beta} \right)^{1/2}.$$  

The coefficients $c_i^\pm$ on the right-hand side are given by

$$c_i^+ = 1,$$

$$c_2^+ = a_1 \delta D_V^{(*)} - a_2 \left[ \delta D_A^{(*)} - \text{Re}(G_1^{(*)}) \right] + a_3 \text{Re}(\delta D_{VA}^{(*)}) \mp \xi^{(*)},$$

$$c_3^+ = \text{Re}(\bar{f}_2^R), \quad c_3^- = \text{Re}(\bar{f}_2^L),$$
where $\delta D_{V,A,V}^{(*)}$ and $G_1^{(*)}$ in $c_2^+$ are combinations of the SM and non-SM form factors in eq.(4) but without $\delta D_v$ while $\xi^{(*)}$ is one including $\delta D_v$. This means $\xi^{(*)}$ is a parameter to express CP violation in the $t\bar{v}$ couplings, and that is why the signs of the $\xi^{(*)}$ terms for $\ell^+$ and $\ell^-$ are opposite to each other. On the other hand, the coefficients $a_{1,2,3}$ consist of the SM parameters only, and $f_{1,2,3}(x)$ are analytic functions calculated in the SM.

Similarly, the normalized double lepton-energy spectrum is given by the following formula:

$$ \frac{1}{\sigma} \frac{d^2\sigma}{dx d\bar{x}} = \sum_{i=1}^{6} c_i f_i(x, \bar{x}), $$

where $x$ and $\bar{x}$ are for $\ell^+$ and $\ell^-$ respectively,

$$ c_1 = 1, \quad c_2 = \xi^{(*)}, \quad c_3 = \frac{1}{2} \text{Re}(f_2^R - \bar{f}_2^L), $$

$$ c_4 = a'_1 \delta D_{V}^{(*)} + a'_2 \delta D_{A}^{(*)} + a'_3 \text{Re}(G_1^{(*)}), $$

$$ c_5 = a_1 \delta D_{V}^{(*)} - a_2 [\delta D_{A}^{(*)} - \text{Re}(G_1^{(*)})] + a_3 \text{Re}(\delta D_{VA}^{(*)}), $$

$$ c_6 = \frac{1}{2} \text{Re}(f_2^R + \bar{f}_2^L), $$

and again $a'_{1,2,3}$ are combinations of the SM parameters and $f_i(x, \bar{x})$ are analytic functions derived in the SM.

4. Parameter determination

In order to study how precisely we can determine the coefficients $c_i$ in eqs.(7,8) when we have $N$ corresponding events, we used the optimal observable procedure \[5\]. According to its prescription, we can deduce $c_i$ from the spectra (which we express as $\Sigma(\phi)$, where $\phi$ means $x$ or $(x, \bar{x})$) with statistical uncertainty

$$ \Delta c_i = \sqrt{X_{ii}/N}, $$

where $X_{ij}$ is the inverse matrix of

$$ M_{ij} = \int d\phi \frac{f_i(\phi)f_j(\phi)}{\Sigma(\phi)}. $$
From the theoretical point of view, perfectly-polarized beams \((P_e^+ = P_e^- = \pm 1)\) are the most attractive. However, those are difficult to achieve in practice, especially for the positron beam. So we discussed the following two cases:

1. \(P_e^+ = 0\) vs \(P_e^- = 0, \pm 0.5, \pm 0.8\) and \(\pm 1\),

2. \(P_e^+ = P_e^- (\equiv P_e) = 0, \pm 0.5, \pm 0.8\) and \(\pm 1\).

In the analyses we assumed \(\epsilon_\ell = 0.6\) as the lepton-tagging efficiency and \(L = 100 \text{ fb}^{-1}\) as the integrated luminosity.

Now let me show the main feature of the results focusing on the single spectrum: we found both \(\Delta c_{2,3}^\pm\) become smallest for \(P_e^- = -1/P_e = -1\). We can thereby conclude immediately that the best precision is obtained at \(c_3\) measurements for these polarizations. However we have to be a bit more careful for \(c_2\) measurements. This is because \(c_2\) themselves vary depending on the polarization. Therefore we should discuss the statistical significance \(N_{SD} \equiv |c_2^\pm|/\Delta c_2^\pm\) inevitably instead of statistical errors only. For this purpose we considered the following two sets of the couplings as an example:

(a) \(\text{Re}(\delta A_{\gamma,Z}) = \text{Re}(\delta B_{\gamma,Z}) = \text{Re}(\delta C_{\gamma,Z}) = \text{Re}(\delta D_{\gamma,Z}) = 0.1\),

(b) \(\text{Re}(\delta A_{\gamma}) = \text{Re}(\delta B_{\gamma}) = \text{Re}(\delta C_{\gamma}) = \text{Re}(\delta D_{\gamma}) = 0.1,\)
\[\text{Re}(\delta A_Z) = \text{Re}(\delta B_Z) = \text{Re}(\delta C_Z) = \text{Re}(\delta D_Z) = -0.1.\]

As a result, we found that the use of negatively-polarized beam(s) is not always optimal: for the parameter set (a) a good precision in \(c_2^+\) measurements is obtained when \(P_e < 0\), but even in this case the precision in \(c_2^-\) measurements becomes better for \(P_e > 0\) or even \(P_e = 0\). Moreover in case (b) both \(c_2^+\) and \(c_2^-\) get the highest precision for \(P_e = +1\). Therefore one should carefully adjust optimal polarization to test any given model.

In any case one can conclude that (as far as the coefficient sets discussed here are concerned) appropriate beam polarization(s) provides measurements of \(c_{2,3}^\pm\) at least at 2\(\sigma\) and 3\(\sigma\) level for \(P_e^+ = 0\) and \(P_e^+ \neq 0\), respectively except for \(c_2^-\) in case (a), where \(|c_2^-|\) becomes tiny due to an accidental cancellation.
We reached a similar conclusion for the double spectrum too: It depends on the structure of tested models what polarization(s) is best to study $t\bar{t}v$-couplings. I skip showing the details here, however, for want of space.

5. Summary

Next-generation linear colliders of $e^+e^-$, NLC, are expected to work as the cleanest facilities for studying top-quark interactions. There, we will be able to perform detailed tests of the top-quark couplings to the vector bosons and either confirm the SM simple generation-repetition pattern or discover some non-standard interactions. In this talk, I have shown main points of our latest model-independent analyses of the single- and the double-leptonic energy spectra for arbitrary longitudinal beam polarizations [2].

We found (i) the use of longitudinal beams could be very effective in order to increase precision of the determination of non-SM couplings. However (ii) optimal polarization depends on the model of new physics under consideration. Therefore polarization of the initial beams should be carefully adjusted for each tested model.

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