SOLVING PURE INTEGER PROGRAMMING PROBLEMS WITHOUT USING GOMORIAN CONSTRAINT BY USING CMI METHOD

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Abstract

The objective of this paper is to solve pure integer programming problems without using Gomorian constraints. In this, CMI method is used for solving linear programming problems instead of simplex method. In CMI method, there is no need to calculate net evaluations, which is essential and mandatory in pre-existing methods. By discarding the calculation of net evaluations, the iterations in the procedure gets reduced or remains at most equal in number. After getting a non-integer value in final CMI table, here we use a reduction technique instead of adding Gomorian constraint to get the integer solution directly. The main advantage of using this reduction technique is to avoid using, any additional constraints and the Dual simplex method for getting an integer solution. With the elimination of the above processes, the integer solutions are arrived very easily. Hence this new approach of pure integer programming problem ensures time conservation at various levels in deriving the optimal solutions. This proposed method is illustrated with examples.

Keywords: CMI Method, LPP, IPP, Optimal Solution, Reduction technique.

I. Introduction

Optimization is an actual and real world tool to solve the problems in various fields like engineering and science. In many real life problems we get the fractional values, but may not be meaningful or don’t make sense all the time. If the desired results are to be an integer, the fractional values are to be rounded off to the nearest integer without deviating the end result by satisfying the constraints. However it is very difficult to round off the value without violating any of the constraints. All the above incurred complications can be avoided if these optimization problems are solved with pure integer programming problems. Hence the pure integer programming problems associated with solving linear models, all the variables are restricted to be an integer. The main reason for using integer variables while...
modelling problems as a linear program problem is to represent the integer variables as quantities that can only be integers. Solving these pure integer programming problems is usually quite complicated to solve than LPP problems. To reduce the complexity of solving LPP and IPP, (i) Kalpana Lokhande; Pranay. Khobragade and W. Khobragade [III] introduced an Alternative approach to simplex method. (ii) P. Pandian and M. Jayalakshmi [IV] introduced a new approach for solving a class of pure integer programming problems. With reference from the above article [III], S. Cynthia Margaret Indrani & Dr.N. Srinivasan [VI] developed CMI method for the solution of linear formulating problem and the concept is integrated in this paper and the reduction technique is developed from the second article. Hence this new approach reduces the complexity in pure integer programming problems and the concepts are entirely different from the above first two referred articles.

II. Proposed Algorithm

To Find the Solution of Any Pure IPP by CMI Method, the Algorithm is as Follows,

Step 1: Reformulate the given LPP into a standard maximisation problem and then determine the optimum solution by using CMI method for solving linear formulating problems which is given in reference [VI]

Step 2: Check the optimum solution, whether it is an integer or a non-integer. If an integer solution is obtained then the resultant optimum solution is the integer solution for the given IPP problem. If the decision variable does not appear in the optimum table then, the corresponding value of that variable should be considered as zero for the given IPP problem. If in case any one of the solution is not an integer then go to the next step.

Step 3: From the optimum table, we have to reformulate the equations by using the body matrix with their corresponding values of nx_B. In these equations split up the values of x_B as an integer ± fractional part where the fractional part must be 0<x<1. The following 3 cases may arise while splitting up the values of x_B

Case (i): If the value of x_B is an integer then no need to split up and can consider as it is

Case (ii): If the value of x_B is a non-integer and its decimal point lies in the interval 0.0 to 0.6 between any two integers, then that non-integer value should be splitted in to left integer ± fractional part, else the right integer ± fractional part

Case (iii): If in case of two equations when x_B value has same mid interval between any two integers then one value should be splitted into left integer ± fractional part and the other value should be splitted into right integer ± fractional part. In Case of three equations, the third equation will be right integer ± fractional part.
Step 4: Reduction Technique

In step-3, consider only integer values and discard the fractional parts as they small values which will not impact the result in finding the optimum solution by using the following reduction formulae.

(i) For 2 variables:
\[ \sum_{j=1}^{2} a_{ij} x_j = bi \text{ Where } i=1, 2. \]

\[ P = \text{Minimum of } \{ \frac{b_i}{a_{ij}}, i=1, 2 \}, j = 1, 2. \]

Which implies \( x_j \in \{0, 1, 2 \ldots P\}, j = 1, 2. \)

and \( x'_j = \text{Minimum of } \{ \frac{b_i-a_{i1}x_j}{a_{i2}} \} , i=1, 2. \)

(ii) For 3 variables:
\[ \sum_{j=1}^{3} a_{ij} x_j = bi \text{ Where } i=1, 2, 3 \]

\[ P = \text{Minimum of } \{ \frac{b_i}{a_{ij}}, i=1, 2, 3 \}, j = 1, 2. \]

which implies \( x_j \in \{0,1,2, \ldots \ldots P\}, j = 1,2,3. \)

After finding the value of \( x_j \), reposition the variable in the equations from LHS to RHS and substitute the value of \( P \) for \( x_j \) in the equations. Hence with this the 3 variable reduces to 2 variable equations and then we need to find the value of the remaining

Variables \( x'_j \) and \( x''_j \) by using 2 variables reduction formulae.

Step 5: Substitute the values of \( x_j(P) \), \( x'_j \) & \( x''_j \) obtained from the reduction technique in the given objective function of pure IPP and find out the maximum value of \( Z \). The obtained solution is called the optimum solution.

Step 6: Check the constraints with the optimum solution and verify whether it is satisfies or not. If it is not satisfied then consider the before value of \( P \) from \( x_j \) and again verify in the constraints. This procedure is repeated till the constraints are satisfied with the optimum solution.

III. Illustrated Examples

Example –I

Solve the Integer Programming Problem

Maximize \( Z = 7x_1 + 9x_2 \)

Subject to the constraints

\[ -x_1 + 3x_2 \leq 6 \]

\[ 7x_1 + x_2 \leq 35, \]

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Where \( x_1, x_2 \geq 0 \) and are Integers

**Solution:**

**Step-I**

Convert the given problem into Standard LPP by introducing non-negative slack.

\[
\text{Maximize } Z = 7x_1 + 9x_2 + 0s_1 + 0s_2
\]

Subject to the constraints

\[
\begin{align*}
-x_1 + 3x_2 + s_1 &= 6 \\
7x_1 + x_2 + s_2 &= 35
\end{align*}
\]

where \( x_1, x_2, s_1, s_2 \geq 0 \)

| \( C_j \) | \( s_1 \) | \( s_2 \) | \( z_1 \) | \( z_2 \) | \( s_0 \) |
|---|---|---|---|---|---|
| \( x_1 \) | \( x_2 \) | \( s_1 \) | \( s_2 \) |
| 7 | 9 | 0 | 0 |
| 0 | \( s_1 \rightarrow 6 \) | -1 | (3) | 1 | 0 |
| 0 | \( s_2 \rightarrow 35 \) | 7 | 1 | 0 | 1 |

In initial iteration the variable \( x_2 \) column is having greatest coefficient in the objective function. So select that column as an entering column. The minimum value is -1 which is in \( s_1 \) row. Hence that corresponding row is a leaving row. The Pivotal element is (3).

**First Iteration:**

By using Step 6 in CMI Method for solving linear formulating problem which is given in Ref [VI], the initial iteration becomes

| \( C_j \) | \( s_1 \) | \( s_2 \) | \( z_1 \) | \( z_2 \) | \( s_0 \) |
|---|---|---|---|---|---|
| \( x_1 \) | \( x_2 \) | \( s_1 \) | \( s_2 \) |
| 7 | 9 | 0 | 0 |
| 9 | \( x_2 \rightarrow 2 \) | -1/3 | 1 | 1/3 | 0 |
| 0 | \( s_2 \rightarrow 33 \) | (22/3) | 0 | -1/3 | 1 |

In the above iteration the maximum value is 22/3 which occurs in \( x_1 \) column, therefore it is an entering column. The minimum value is -1/3 which occurs in both \( x_2 \) and \( s_2 \) rows. Here tie break occurs, hence by using the condition (iii) of in step-5 Ref [VI], \( s_2 \) is a leaving row. The Pivotal element is (22/3).
Second Iteration: By using Step 6 in CMI Method for solving linear formulating problem which is given in Ref [VI], the first iteration becomes

| Cn | BASIS | x₁ | x₂ | s₁ | s₂ |
|----|-------|----|----|----|----|
| 9  | x₂    | 7/2| 0  | 1  | 7/22| 1/22|
| 7  | x₁    | 9/2| 1  | 0  | -1/22| 3/22|

Since all the rows and columns are ignored and the optimum solution is obtained as

\[ x₁ = 9/2, \quad x₂ = 7/2 \quad \& \quad \text{Max } Z = 63 \]

Step-2:

Here the obtained optimum solution is not an integer hence move down to step-3

Step-3: Reformulate the equations from the optimum table

\[ x₁ + 0x₂ = 9/2 \quad (4+1/2) \]
\[ 0x₁ + x₂ = 7/2 \quad (3+1/2) \]

According to case (ii) in step-3 in the above algorithm, 9/2 and 7/2 were splitted into left integer + fractional part

Step-4: In the above step, consider only integer and discard the fractional part for finding the optimum solution for the given pure IPP by using reduction formula

\[ x₁ + 0x₂ = 4 \]
\[ 0x₁ + x₂ = 3 \]

\[ P = \text{Minimum of \{4/1, 4/0, 3/0, 3/1\}} = 3 \text{ which is in coefficient of } x₂ \]

Therefore \( x₂ \epsilon \{0,1,2,3\} \)

And \( x₁ = \text{Minimum of \{4−0x₂ \over 1}, \quad 3−x₂ \over 0 \} = 4 \)

Hence \( x₁ = 4, x₂ = 3 \)
Step-5: Substitute the above values in the objective function of given IPP, we get

\[ x_1 = 4, \ x_2 = 3 \ \& \ Z = 55 \]

Hence the optimal solution is satisfied with constraints.

Example -2: Solve the Integer Programming Problem

Maximize \( Z = x_1 + x_2 \)

Subject to the constraints

\[ 2x_1 + 5x_2 \leq 16 \]
\[ 6x_1 + 5x_2 \leq 30 \]

Where \( x_1, x_2 \geq 0 \) and are integers

Step-I

Convert the given LPP into a standard maximisation problem and then determine the optimum solution by using CMI method for solving linear formulating problems which is given as reference [VI]

| \( C_i \) | \( 1 \) | \( 1 \) | \( 0 \) | \( 0 \) |
|-----------|--------|--------|--------|--------|
| \( C_B \) | \( x_1 \) | \( x_2 \) | \( s_1 \) | \( s_2 \) |
| 0 | \( s_1 \) | 16 | (2) | 5 | 1 | 0 |
| 0 | \( s_2 \) | 30 | 6 | 5 | 0 | 1 |
| 1 | \( x_1 \) | 8 | 1 | 5/2 | 1/2 | 0 |
| 0 | \( s_2 \) | -18 | 0 | -10 | -3 | 1 |
| 1 | \( x_1 \) | 7/2 | 1 | 0 | -1/4 | 1/4 |
| 1 | \( x_2 \) | 9/5 | 0 | 1 | 3/10 | -1/10 |

Since all the rows and columns are ignored, the optimum solution is obtained as

\[ x_1 = 7/2, \ x_2 = 9/5 \ \& \ Max \ Z = 53/10 \]

Step-2: Here the obtained optimum solution is not an integer hence move down to step-3
Step-3: Reformulate the equations from the optimum table

\[ x_1 + 0x_2 = 7/2 \ (3+1/2) \]
\[ 0x_1 + x_2 = 9/5 \ (2-1/5) \]

According to case (ii) in step-3 in the above algorithm, 7/2 is splitted into left integer + fractional part and 9/5 is splitted into right integer – fractional part.

Step-4: In the above step, consider only the integer and discard the fractional part for finding the optimum solution for the given pure IPP by using reduction formulae

\[ x_1 + 0x_2 = 3 \]
\[ 0x_1 + x_2 = 2 \]

\[ P = \text{Minimum of } \{3/1, 3/0, 2/0, 2/1\} = 2 \text{ which is in coefficient of } x_2 \]

Therefore \( x_2 \in \{0,1,2\} \)

And \( x_1 = \text{Minimum of } \{\frac{3-0x_2}{1}, \frac{2-x_2}{0}\} = 3 \)

Hence \( x_1 = 3, x_2 = 2 \)

Step-5: Substitute the above values in the objective function of given IPP, we get

\[ x_1 = 3, x_2 = 2 \& Z = 5 \]

Hence the optimal solution is satisfied with constraints.

Example -3:

Solve the Integer Programming Problem

Maximize \( Z = 4x_1 + 6x_2 + 2x_3 \)

Subject to the constraints

\[ 4x_1 - 4x_2 \leq 5 \]
\[ -x_1 + 6x_2 \leq 5 \]
\[ -x_1 + x_2 + x_3 \leq 5 \] Where \( x_1, x_2, x_3 \geq 0 \) and are integers

Step-I

Convert the given LPP into a standard maximisation problem and then determine the optimum solution by using CMI method for solving linear formulating problems which is given as reference [VI]
Since all the rows and columns are ignored, the optimum solution is obtained as 

$$x_1 = \frac{5}{2} \hspace{1cm} x_2 = \frac{5}{4} \hspace{1cm} x_3 = \frac{25}{4}$$

Max $Z = 30$

**Step-2:** In the above table, the obtained optimum solution is not an integer hence move down to step-3

**Step-3:** Reformulate the equations from the optimum table

$$x_1 + 0x_2 + 0x_3 = \frac{5}{2}$$
$$0x_1 + x_2 + 0x_3 = \frac{5}{4}$$
$$0x_1 + 0x_2 + x_3 = \frac{25}{4}$$

According to case (ii) in step-3 in the above algorithm, $\frac{5}{2}$, $\frac{5}{4}$ & $\frac{25}{4}$ was splitted into left integer + fractional part.

**Step-4:** In the above step, consider only integer and discard the fractional part for finding the optimum solution for the given pure IPP by using reduction formulae

$$x_1 + 0x_2 + 0x_3 = 2$$
$$0x_1 + x_2 + 0x_3 = 1$$
0x₁ + 0x₂ + x₃ = 6

P = Minimum of {2/1, 2/0, 2/0, 1/0, 1/1, 1/0, 6/0, 6/0, 6/1} = 1 which is in coefficient of x₂

Therefore x₂ ∈ {0,1}

x₁ + 0x₃ = 2 − 0x₂
0x₁ + 0x₃ = 1 − x₂
0x₁ + x₃ = 6 − 0x₂

If x₂ = 1, then the 2nd equation is meaningless and it is reduced to 2 variables

x₁ + 0x₃ = 2
0x₁ + x₃ = 6

P=Minimum of {2/1, 2/0, 6/0, 6/1} = 2 which is in coefficient of x₁

Therefore x₁ ∈ {0,1,2}

And x₃ = Minimum of { 2−x₁ 0 , 6−0x₁ 1 } = 6

Hence x₃ = 6, x₂ = 1, x₁ = 2

Step-5: Substitute the above values in the objective function of given IPP, we get

x₃ = 6, x₂ = 1, x₁ = 2 & Z = 26

Hence the optimal solution is satisfied with constraints.

IV. Results and Discussions

| EXAMPLE No | IPP Method (Existing Method) | CMI method (New Approach) |
|------------|-----------------------------|----------------------------|
| 1          | Z=55                        | Z=55                       |
| 2          | Z=5                         | Z=5                        |
| 3          | Z=26                        | Z=26                       |

The solutions found using the CMI method are same as found with all the other existing ones and it is also noted that, no additional constraints or dual simplex methods are used. In using the pre-existing methods, 2 additional constraints and 4 iterations in dual simplex are to be used for the same problem worked out above. Hence by adopting the CMI method in which reduction technique is used to make the problem simpler to solve with less number of iterations and also the calculations here are much easier with the elimination of Gomorian constraint.

V. Conclusion

The CMI method used to find solution to the pure Integer programming problem eliminates the usage of Gomorian constraint with which the problem gets simpler. By avoiding the usage of net evaluation and by using reduction technique,
time saving is achieved. In CMI pure integer programming problem there is no need to introduce additional constraints and dual simplex in order to get the integer solution. Hence this is an efficient and an effective method in easing the complications in finding the solution. Hopefully by adopting this method the users will feel the comfort in solving the pure integer programming problem much easier than expected.

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