A New General Relativistic Cosmology

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In this work, we outline a new general relativistic cosmology. In this cosmology, the universe originates in the infinite past from sparsely distributed neutral matter and ends in the infinite future as a hot, relativistic plasma. The spatial distribution of matter on the “initial” hyper-surface is arbitrary. Hence, observed structures can arise in this cosmology from suitable “initial” density contrast. The red-shifts of different objects in this cosmology are indicative of their different states of collapse and need not possess any correlation to their distance from the observer. Further, the microwave background radiation arises in this cosmology as thermalized radiation from all the radiating matter in the universe. This cosmology predicts that the temperature of the microwave background increases with time. Thus, any conclusive evidence that the temperature of the Microwave Background Radiation was more in the past can falsify this cosmology.

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I. INTRODUCTION

In the 3+1 formulation [1] of General Relativity, source matter or energy data can be specified on an “initial” spacelike hyper-surface and, that data can be evolved using the Einstein field equations. Different four-dimensional spacetime geometries are obtainable for different initial data.

When matter or energy is present over all of the initial hyper-surface, we obtain a “cosmological” situation or a “cosmological” spacetime.

A famous is the case of maximally symmetric, homogeneous and isotropic, Friedmann-Lemaître-Robertson-Walker (FLRW) spacetime of the Big Bang Cosmology (BBC) [2, 3, 4].

Now, to fix ideas, recall that the Newtonian Law of Gravitation is a statement of the force of attraction between two mass points in space. The presence of other mass-points does not alter the statement of this Law of Gravitation. Moreover, redistribution of mass-points does not affect the statement of this Law of Gravitation.

General Relativity prescribes a spacetime geometry for a Law of Gravitation. In analogy with the Newtonian case, we therefore seek a spacetime that “does not change” its geometrical or physical characters when matter or energy is differently distributed on its initial hyper-surface. Moreover, the requirement that the addition of “more” matter or energy does not change the geometrical characters of this spacetime means that it is also a “cosmological” spacetime.

But, we note that the FLRW spacetime does change its geometrical characters when matter is differently distributed, say, in some inhomogeneous way, on its initial hyper-surface.

Therefore, we consider here a Petrov-type D, cosmological spacetime which “does not change” its geometrical characters under redistribution of matter on its initial hyper-surface and explore the cosmology implied by it.

II. SPACETIME METRIC

Consider the spacetime metric:

\[ ds^2 = -X^2Y^2Z^2dt^2 + \gamma^2X^2Y^2Z^2B^2dx^2 \\
+ \gamma^2X^2Y^2Z^2C^2dy^2 \\
+ \gamma^2X^2Y^2Z^2D^2dz^2 \]  

(1)

where \( X \equiv X(x), Y \equiv Y(y), Z \equiv Z(z), B \equiv B(t), C \equiv C(t), D \equiv D(t) \) and \( \gamma \) is a constant. Also, \( X' = dX/dx, Y = dY/dy \) and \( Z = dZ/dz \).

Singularities and Degeneracies of (1)

There are, in general, two types of singularities of the metric (1). The first type is when any of the temporal functions \( B, C, D \) is vanishing and the second type is when any one of the spatial functions \( X, Y, Z \) is vanishing. Singularities of the second type constitute a part of the initial data, singular initial data, for (1). On the other hand, vanishing of the temporal functions is a singular hyper-surface of (1).

The locations for which the spatial derivatives vanish are, however, coordinate singularities. The curvature invariants of (1) do not blow up at such locations. Therefore, before such coordinate singu-
larities are reached, we may transform coordinates to other suitable ones.

There are also obvious degenerate metric situations when any of the spatial functions is infinite for some range of the coordinates.

In what follows, we shall assume, unless stated explicitly, that there are no singular initial-data and that there are no degenerate situations for the metric (1).

The metric (1) admits three hyper-surface orthogonal spatial homothetic Killing vectors (HKVs)

\[
\begin{align*}
X &= (0, X, 0, 0) \\
Y &= (0, 0, -Y, 0) \\
Z &= (0, 0, 0, -Z)
\end{align*}
\]

(2)

(3)

(4)

Now, the existence of three HKVs is equivalent to two Killing vectors (KVs) and one HKV. Then, the metric (1) can be expressed in a form that displays the existence of KVs explicitly. However, we will use the form (1) in this paper.

The spacetime of (1) is required, by definition, to be locally flat at all of its points. In general, this will require some conditions on the metric functions \(X, Y, Z\).

Then, the Einstein tensor has the components

\[
G_{tt} = -\frac{1}{\gamma^2 B^2} - \frac{1}{\gamma^2 C^2} - \frac{1}{\gamma^2 D^2} + \dot{B}\dot{C} \frac{\dot{B}}{BC} + \dot{B}\dot{D} \frac{\dot{B}}{BD} + \dot{C}\dot{D} \frac{\dot{C}}{CD}
\]

\[
G_{xx} = \frac{\gamma^2 B^2 X^2}{X^2} \left[ -\frac{\dot{C}}{C} - \frac{\dot{B}}{D} - \frac{\dot{C}}{CD} \right]
\]

\[
G_{yy} = \frac{\gamma^2 C^2 Y^2}{Y^2} \left[ -\frac{\dot{B}}{B} - \frac{\dot{D}}{D} - \frac{\dot{B}D}{BD} \right]
\]

\[
G_{zz} = \frac{\gamma^2 D^2 Z^2}{Z^2} \left[ -\frac{\dot{B}}{B} - \frac{\dot{C}}{C} - \frac{\dot{B}C}{BC} \right]
\]

\[
G_{tx} = \frac{\dot{B}X}{BX} \\
G_{ty} = \frac{\dot{C}Y}{CY} \\
G_{tz} = \frac{\dot{D}Z}{DZ}
\]

(5)

(6)

(7)

(8)

(9)

(10)

\[
G_{xy} = \frac{2}{\gamma^2} \frac{\dot{Y}}{X^2} \\
G_{xz} = \frac{2}{\gamma^2} \frac{\dot{Z}}{XZ}
\]

(11)

(12)

where an overhead dot is used to denote a time-derivative.

Now, let the energy-momentum tensor be:

\[
T_{ab} = (\rho + p + \mathcal{E}) U_a U_b + (p + \mathcal{E}) g_{ab} + q_a U_b + q_b U_a + \Pi_{ab}
\]

(13)

where \(\rho\) is the energy density, \(p\) is the isotropic pressure, \(q_a\) is the energy-flux four-vector, \(\mathcal{E}\) is the bulk-viscous pressure, \(\Pi_{ab}\) is the anisotropic stress tensor etc. Note also that \(q_a U^a = 0\) and that \(\Pi_{ab} U^b = \Pi_{a}^a = 0\). Thus, \(\Pi_{ab}\) is symmetric, spatial and trace-free.

Now, from the Einstein field equations, with \(8\pi G/c^4 = 1\), it follows that \(\rho = G_{tt}/X^2 Y^2 Z^2\). The isotropic pressure is obtainable from the combination of (1), (2) and (3). The energy-fluxes \(q_x\), \(q_y\) and \(q_z\) are related to corresponding components of the Einstein tensor in (1), (4) and the anisotropic stresses are related to corresponding components of the Einstein tensor in (1), (5) and (6). All the spatial functions \(X, Y, Z\) are not determined by the field equations and, hence, are arbitrary.

Then, the “initial hyper-surface” is given by \(\dot{B} = \dot{C} = \dot{D} = 0\). Now, the energy-fluxes are vanishing for the initial hyper-surface. The anisotropic stresses get determined from the spatial functions \(X, Y\) and \(Z\) as a part of the initial data.

Now, to be able to explicitly solve the corresponding ordinary differential equations, we require physical information about matter or energy. To provide for the required information of “physical” nature is a non-trivial task in general relativity just as it is for the Newtonian gravity (4). The details of these considerations are, of course, beyond the scope of the present paper.

Also, the non-vanishing components of the Weyl tensor for (1) are:

\[
C_{ttxx} = \frac{B^2 \gamma^2 X^2 Y^2 Z^2}{6} F(t) \quad C_{tyy} = \frac{C^2 \gamma^2 X^2 Y^2 Z^2}{6} G(t)
\]

(14)

(15)

(16)

(17)

(18)
\[ C_{yiz} = -\frac{C^2D^2\gamma^4X^2\tilde{Y}^2\tilde{Z}^2}{6} F(t) \]  

where

\[ F(t) = \frac{\ddot{C}}{C} + \frac{\ddot{D}}{D} - 2\frac{\dot{B}}{B} - 2\frac{\dot{C}\dot{D}}{CD} + \frac{B\dot{C}}{BC} + \frac{\dot{B}\dot{D}}{BD} \]  \hspace{1cm} (20)

\[ G(t) = \frac{\ddot{D}}{D} + \frac{\ddot{B}}{B} - 2\frac{\dot{C}}{C} + \frac{\dot{C}\dot{D}}{CD} + \frac{B\dot{C}}{BC} - 2\frac{\dot{B}\dot{D}}{BD} \]  \hspace{1cm} (21)

\[ H(t) = \frac{\ddot{B}}{B} + \frac{\ddot{C}}{C} - 2\frac{\dot{D}}{D} + \frac{\dot{C}\dot{D}}{CD} + \frac{\dot{B}\dot{D}}{BD} - 2\frac{B\dot{C}}{BC} \]  \hspace{1cm} (22)

Now, for (1), it can be verified that the non-vanishing Newman-Penrose (NP) complex scalars \([\Psi_0, \Psi_4]\) are \(\Psi_0 = \Psi_4\) and \(\Psi_2\).

Note that \(\Psi_4 \neq 0\). Now, consider a NP-tetrad rotation of Class II, with complex parameter \(b\), that leaves the NP-vector \(n\) unchanged, and demand that the new value, \(\Psi_4^{(1)}\), of \(\Psi_0\) is zero. Then, we have:

\[ \Psi_4 b^4 + 6\Psi_2 b_2 \Psi_2 + \Psi_4 = 0 \]  \hspace{1cm} (23)

This equation has two distinct double-roots. Thus, the spacetime of (1) is of Petrov-type D.

It is well-known that Penrose [1] is led to the Weyl hypothesis on the basis of thermodynamical considerations, in particular, those related to the thermodynamic arrow of time. On the basis of these considerations, we may consider the Weyl tensor to be “some” sort of measure of the entropy in the spacetime at any given epoch.

Now, note that the Weyl tensor, [14] - [19], blows up at the singular hyper-surface of (1) but is “vanishing” at the “initial” hyper-surface for non-singular and non-degenerate data since \(B = C = D = 0\) for the “initial” hyper-surface.

Then, we note that the entropy at the “initial” hyper-surface of (1) is “zero” while that at its singular hyper-surface is “infinite”.

This behavior of the Weyl tensor of (1) is in conformity with Penrose’s Weyl curvature hypothesis [1]. Thus, the spacetime of (1) has the “right” kind of thermodynamic arrow of time in it.

**Other features of the metric (1)**

The spatial distribution of matter is arbitrary for (1) and the evolution of non-singular and non-degenerate “initial data” is completely governed by only the temporal functions in (1).

Therefore, changing matter distribution on the initial hyper-surface does not change the geometrical properties of (1) for non-singular and non-degenerate data. Thus, (1) is the metric of the spacetime that we have been looking for.

Now, non-gravitational processes are included, in general relativity, via the energy-momentum tensor. Non-gravitational processes primarily determine the relation of density and pressure of matter - the equation of state. Matter properties, as are applicable at any given stage of evolution of matter, such as an equation of state of matter, the radiative characteristics of matter etc. determine the temporal functions of (1).

For physically realizable spacetime, matter must pass through different “physical” stages of evolution, namely, from dust to matter with pressure and radiation, to matter with exothermic nuclear reactions etc. This is the causal and, hence, physical, development of matter data. The spacetime of (1) therefore describes physical development of matter data.

Then, the initial hyper-surface, \(t \to -\infty\), has sparsely distributed matter for which the comoving velocity of matter is vanishing. The \(x, y, z\) co-moving velocities are respectively \(\dot{B}, \dot{C}, \dot{D}\) and, hence, \(\dot{B} = \dot{C} = \dot{D} = 0\) on the initial hyper-surface of (1).

Therefore, the temporal evolution in (1) for the non-singular and non-degenerate data leads only to a temporal singularity in the future.

Clearly, since the spatial distribution of matter is arbitrary for (1), the present structures can evolve out of some suitable density distribution.

The matter 4-velocity is

\[ u^a = \frac{1}{X Y Z \sqrt{\Delta}} (1, V^x, V^y, V^z) \]  \hspace{1cm} (24)

where \(V^x = dx/dt\), \(V^y = dy/dt\), \(V^z = dz/dt\) and

\[ \Delta = 1 - \gamma^2 \left[ \frac{X^2 B^2 V^x^2}{X^2} + \frac{Y^2 C^2 V^y^2}{Y^2} + \frac{Z^2 D^2 V^z^2}{Z^2} \right] \]  \hspace{1cm} (25)

Now, it can be inferred that the velocity of matter with respect to a co-moving observer is the speed of light at the singular hyper-surface of (1). At the singular hyper-surface of (1), \(\Delta = 1\). Then, a co-moving observer also moves with the speed of light at the singular hyper-surface.

After all, matter everywhere should become relativistic as different mass condensates continue to grow (due to accretion onto them) to influence the entire spacetime to become relativistic everywhere.
This is happening asymptotically for infinite comoving time.

Consequently, in the cosmology of (1), the universe begins in the infinite past as “cold” but ends as a soup of high energy plasma and radiation in the infinite future. This is evidently consistent with the behavior of the Weyl tensor as a measure of the entropy.

Now, if \( d\tau_{CM} \) is a small time duration for a co-moving observer and if \( d\tau_{RF} \) is the corresponding time duration for the observer in the rest frame of matter, then we have

\[
d\tau_{CM} = \frac{d\tau_{RF}}{\sqrt{\Delta}} \tag{26}
\]

From (26), we also get the red-shift formula

\[
\nu_{CM} = \nu_{RF} \sqrt{\Delta} \quad z = \frac{\nu_{RF}}{\nu_{CM}} = \frac{1}{\sqrt{\Delta}} \tag{27}
\]

in the spacetime of (1) where \( \nu_{CM} \) is the frequency of a photon in the co-moving frame, \( \nu_{RF} \) is the frequency in the rest frame and \( z \) is the total red-shift of a photon.

Then, the red-shift, (27), depends on \( \nu_{CM} \) which determine the spatial density of matter and, of course, on the temporal functions in (1). In the absence of expansion in (1), the entire red-shift is indicative of the state of collapse of matter at given co-moving time.

Now, we have sufficient characteristics of the spacetime of (1) to discuss its cosmological implications to which we now turn to.

III. COSMOLOGICAL IMPLICATIONS

Firstly, the universe of (1) need not be expanding, rather, different matter condensates in it could simply be contracting and collapsing onto more massive condensates.

This is true unless, of course, “matter or energy” properties are such as to lead to positive expansion, \( \Theta \), for (1). But, no matter condensate in (1) is then collapsing in all dimensions. There must be, at least, one direction in which matter must expand sufficiently rapidly for, say, \( \dot{B}/B \) to dominate over other negative terms in \( \Theta \).

Now, it has been aptly emphasized that the red-shift of an object should have an “intrinsic” component, over and above that due to expansion. Then, for (1), the red-shift, (27), has both the contributions, from expansion, if any, and from density characteristics of the object.

Note also that the entire red-shift is “intrinsic” to the object or is “gravitational of origin” if the expansion of the universe is non-existent for the spacetime of (1).

However, correlation of the red-shift with distance could also arise from the initial density distribution if distant objects are more collapsed in it. In this connection, we note that the observational basis for the Hubble Law of expansion of the universe has also been called into question from time to time.

For (1), two neighboring objects, physically connected with luminous bridges, can be in different states of gravitational collapse and, hence, can show different red-shifts. There is therefore a natural explanation in the present cosmology for the discordant or anomalous red-shifts of quasars and galaxies that Arp has been observing in quasar-galaxy associations. The famous such case is that of NGC 7619 and its companion.

An important issue is that of the observed Microwave Background Radiation (MBR). The early universe of (1) is cold since matter is pressureless and radiation-less close to the initial hyper-surface. Thus, the MBR can only arise as thermalized radiation emitted by all the radiating matter in the universe of (1).

Then, it is remarkable that a spacetime that does not change its geometrical properties under redistribution of mass on its initial hyper-surface leads to only such an explanation of the MBR. We also note here that the energy density in the microwave, \( \rho_{mbr} \), is the same as that found in the starlight of our own galaxy. This is suggestive of the importance of thermalization of light from radiating matter in the universe.

However, we note that all the earlier considerations of radiation by warm dust producing the observed MBR consider an expanding universe. As a result, these considerations face the problem of energy supply needed to heat the dust for large red-shifts in an expanding universe.

Another objection to the explanation of MBR as re-emission of radiation from small-sized dust is that the spectrum of radiation from such dust is unlikely to be black-body while the spectrum of MBR is closely that of a black-body at \( \sim 2.75^\circ \text{K} \). Small dust particles are inefficient radiators at long wavelengths.

However, sufficiently large (\( \sim 10-100 \mu m \)) sized dust particles would thermalize the stellar radiation. But, for expanding universe models, a prohibitively huge total dust mass is obtained. That is, there is the need for large amounts of dust, almost comparable to the density of heavy elements in the universe. Such an era is not available for most cosmological models.

Note, however, that the present cosmological model based on (1) begins with an era of entire...
matter as being dust. Consequently, for this model the above objections against the explanation of MBR as thermalized emission by dust need to be re-examined in details.

Now, since the entire matter in the present cosmology is expected to attain the speed of light in the asymptotic future, we also expect that the temperature of MBR should increase with time. This is, therefore, a definite and falsifiable prediction of this cosmology.

Any conclusive evidence that the temperature of MBR was more in the past would also falsify this cosmology and, with it, the premise that we need to seek a spacetime that “does not change” its geometrical characters when masses are differently distributed on its initial hyper-surface.

As a separate remark, we note that, following the works of Ellis and Sciama [12], there is an interpretation [13] of Mach’s principle, namely that there should be no source-free contributions to the metric or that there should be no source-free Weyl tensor for a Machian spacetime.

We, therefore, note that the vacuum is a degenerate case for [1] and, hence, the metric [1] has no source-free contributions. Further, the spacetime of [1] does not possess the source-free Weyl tensor. The spacetime of [1] is, then, also a Machian spacetime in this sense.

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