Capacity Scaling for MIMO Two-Way Relaying

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Abstract

A multiple input multiple output (MIMO) two-way relay channel is considered, where two sources want to exchange messages with each other using multiple relay nodes, and both the sources and relay nodes are equipped with multiple antennas. Both the sources are assumed to have equal number of antennas and have perfect channel state information (CSI) for all the channels of the MIMO two-way relay channel, whereas, each relay node is either assumed to have CSI for its transmit and receive channel (the coherent case) or no CSI for any of the channels (the non-coherent case). The main results in this paper are on the scaling behavior of the capacity region of the MIMO two-way relay channel with increasing number of relay nodes. In the coherent case, the capacity region of the MIMO two-way relay channel is shown to scale linearly with the number of antennas at source nodes and logarithmically with the number of relay nodes. In the non-coherent case, the capacity region is shown to scale linearly with the number of antennas at the source nodes and logarithmically with the signal to noise ratio.

I. INTRODUCTION

Relay channels are the most basic building block for cooperative and multihop communication in wireless networks. In a relay channel, one or more nodes, without data of their own to transmit, help a source destination pair communicate. The origins of the relay channel - as a three terminal communication channel - go back to Van der Meulen [1]. Despite the passage of time, the capacity of even the most
basic relay channels is still unknown. Nonetheless, bounds derived in [1], [2] show that using a relay, it is possible to increase the reliable rate of data transfer between the source and the destination.

Motivated by the capacity improvements obtained by using multiple antennas at the source and the destination for point-to-point channels [29], recently, there has been a significant research focus on finding the capacity of the multiple input multiple output (MIMO) relay channel, where the source, the destination, and the relay may have multiple antennas [3], [10], [11]. The capacity of the MIMO relay channel was first studied in [3], [13], where upper and lower bounds on the capacity of the MIMO relay channel are derived for the deterministic and the Gaussian fading channel. Improved lower bounds for the MIMO relay channel with Gaussian fading channel were provided by [10], where message splitting and superposition coding are used at the transmitter to improve the bounds provided in [3]. In [3], [10] only full-duplex relays (can transmit and receive at the same time) were considered. Upper and lower bounds on the capacity for the more practical Gaussian MIMO relay channel with half-duplex relays, where the relays cannot transmit and receive at the same time, were developed in [11]. The bounds in [3], [10], [11] indicate that with relays there is a potential capacity gain to be leveraged by using multiple antennas.

In [1]–[3], [10], [11] only a single source destination pair is considered with a single relay node. For a practical wireless network setting, where there are multiple source destination pairs, the concept of cooperative communication has been recently proposed [4]–[7], where different users in the network cooperate by taking turns relaying each others data. Thanks to the spatial separation between users, cooperation between users provides a means to obtain and exploit spatial diversity gain, called cooperative diversity gain, which increases the achievable data rate between each source and its destination. Several different protocols have been proposed to exploit the cooperative diversity gain, e.g. amplify and forward (AF) [4]–[7], [13], decode and forward (DF) [15], [18], with half-duplex [14], and full-duplex assumptions [16].

Prior work on the relay channel mostly considers one-way communication, i.e. a source wants to send data to a destination. In most networks, however, the destination also has some data to send to the source, e.g. packet acknowledgements from the destination to the source, downlink and uplink in cellular networks. Consequently, there has been interest in the two-way relay channel, where the bidirectional nature of communication is taken into account [21]–[24]. The two-way relay channel was studied in [22], where upper and lower bounds on the capacity region were derived for a general discrete memoryless channel.

The MIMO two-way relay channel was introduced in [21], where two terminals $T_1$ and $T_2$ want to
exchange information with each other through a single relay node as shown in Fig. 1 and both $T_1, T_2$ and the relay node is equipped with multiple antennas. It was assumed in [21] that each node can only work in half-duplex mode and there is no direct path between $T_1$ and $T_2$. The communication protocol proposed in [21] for the MIMO two-way relay channel is as follows. In the first time slot, both $T_1$ and $T_2$ transmit simultaneously and the relay node receives the superposition of the signals transmitted by $T_1$ and $T_2$. In the next time slot, the relay node transmits an amplified version of the signal, received in the last time slot, to both $T_1$ and $T_2$, subject to a power constraint. Since both $T_1$ and $T_2$ know what they transmitted in the last time slot, both can remove the effect of their own signal from the received signal, to decode the other terminal’s message. Thus, the MIMO two-way relay channel facilitates simultaneous communication between $T_1$ and $T_2$ without creating any self interference. This idea is reminiscent of network coding [28], though note that here the coding is done in analog domain rather than in digital domain. The MIMO two-way relay channel is also known by several other names in the literature, namely, bidirectional MIMO relay channel [24] and is also a special case of analog network coding [28].

In prior work, achievable rate region (region enclosed by the rates achievable on the $T_1 \rightarrow T_2$ and $T_2 \rightarrow T_1$ links, simultaneously) expressions were derived for the Gaussian half-duplex MIMO two-way relay channel (fading coefficients as well as additive noise is Gaussian distributed) using AF [21] and DF [23], [24] at the relay node. A main conclusion derived in prior work [21], [23], [24], is that it is possible to remove the $\frac{1}{2}$ rate loss factor in spectral efficiency due to the half-duplex assumption on the relay node. To the best of our knowledge, none of the achievable rate region expressions for the MIMO two-way relay channel meet the best known upper bounds [25] and therefore the capacity region of the MIMO two-way relay channel is unknown.

In this paper we consider a MIMO two-way relay channel with multiple relay nodes, deriving upper and lower bounds on its capacity region with different channel state information (CSI) assumptions. We show that the upper and lower bounds are only a constant term away, as the number of relays $K$ grows
large, \( K \to \infty \) with probability 1. Thus, we characterize the scaling behavior of the capacity region of the MIMO two-way relay channel as the number of relay nodes grow large. Our approach is similar to the asymptotic (in the number of relays) capacity formulation of [12], [19], [20].

Our system model and the key assumptions are as follows. We assume that two terminals \( T_1 \) and \( T_2 \) want to communicate with each other via \( K \) relay nodes. None of the relays have any data of their own and only facilitate communication between \( T_1 \) and \( T_2 \). Both \( T_1 \) and \( T_2 \) are equipped with \( M \) antennas, while all the \( K \) relays have \( N \) antennas each. We consider a two-phase communication protocol, where in the any given time slot, for the first \( \alpha, \alpha \in [0,1] \) fraction of the time slot, both \( T_1 \) and \( T_2 \) transmit simultaneously and all the relays receive. In the rest \( 1 - \alpha \) fraction of the time slot, all the relays simultaneously transmit and both \( T_1 \) and \( T_2 \) receive the signal transmitted by all relays. We assume that there is no direct path between \( T_1 \) and \( T_2 \) and that \( T_1, T_2 \) and all the nodes \((T_1, T_2 \) and all relay nodes) can only operate in half-duplex mode. No direct path assumption is reasonable for the case when relay nodes are used for coverage improvement and the signal strength on the direct path is very weak. The half-duplex assumption is made since full-duplex nodes are difficult to realize in practice. We assume that both \( T_1 \) and \( T_2 \) have perfect CSI for all the channels of the MIMO two-way relay channel in the receive mode. This could be enabled through a combination of channel reciprocity and feedback, however, we do not explore the practicalities of this assumption in this work. We consider two different assumptions about the availability of CSI at each relay node. First we consider the case when each relay is assumed to have perfect CSI for its own transmit and receive channel states, which is denoted the \textit{coherent} MIMO two-way relay channel. Second we consider the case where the relays are assumed to have no CSI for any of their channel states, which is denoted the \textit{non-coherent} MIMO two-way relay channel.

Under similar assumptions, capacity scaling results have been found in [19] and [20] for the one-way relay channel (when \( T_2 \) has no data for \( T_1 \)). With a single antenna at both \( T_1, T_2 \) and each relay node, it is shown in [19] that the capacity of the one-way relay channel scales logarithmically in the number of relay nodes, as the number of relay nodes grow large. The capacity scaling result of [19] was extended in [20] to the case where the source and the destination are equipped with \( M \) antennas and the all the relay nodes are equipped with \( N \) antennas and it was shown that there is a \( M \) fold increase in the capacity compared to the single antenna nodes [19].

The main results in this paper are on the capacity scaling laws for the MIMO two-way relay channel. For the coherent MIMO two-way relay channel, the capacity region is given by the convex hull of

\[
R_{12} \leq \frac{M}{2} \log K + O(1) \\
R_{21} \leq \frac{M}{2} \log K + O(1)
\]
with probability 1 as $K \to \infty$, where $R_{12}$ and $R_{21}$ is the rate of information transfer from $T_1 \to T_2$ and $T_2 \to T_1$, and we use the notation $u(x) = O(v(x))$ if $\frac{|u(x)|}{v(x)}$ remains bounded, as $x \to \infty$. For this result, the upper bound on the capacity region is obtained for all $\alpha \in [0, 1]$ and an achievable strategy with $\alpha = \frac{1}{2}$ is proposed to achieve the upper bound within a $O(1)$ term.

For the non-coherent MIMO two-way relay channel, for a fixed $\alpha = \frac{1}{2}$, i.e. $T_1$ and $T_2$ transmit and receive for same amount of time, the capacity region is given by the convex hull of

$$R_{12} \leq \frac{M}{2} \log P_R + O(1)$$
$$R_{21} \leq \frac{M}{2} \log P_R + O(1)$$

with probability 1 as $K \to \infty$, and $P_R$ is the sum of the power available at each relay.

The strategy we use for deriving the capacity region of the MIMO two-way relay channel as $K \to \infty$ is to obtain an upper bound using the cut-set bound [30] and then derive an achievable rate region that approaches the upper bound. For the coherent case, we propose the following achievable strategy. Both $T_1$ and $T_2$ transmit $M$ independent data streams from their $M$ antennas. Each relay node using its CSI, does match filtering for the channels experienced by the $M$ data streams from $T_1 \to T_2$ and $T_2 \to T_1$, simultaneously, and all the $M$ streams from $T_1$ are decoded jointly at $T_2$ and vice versa. We show that this strategy achieves the capacity region upper bound within a $O(1)$ term without any cooperation between $T_1$ and $T_2$. For the non-coherent case, we propose an achievable strategy where both $T_1$ and $T_2$ transmit $M$ independent data streams from their $M$ antennas. Since none of the relays have any CSI in this case, we propose an AF achievable strategy where each relay transmits a scaled version of received signal subject to its power constraint, similar to [20]. With this strategy, as $K \to \infty$, the channel between $T_1 \to T_2$ and $T_2 \to T_1$ converges to an $M \times M$ matrix with independent and identically distributed entries that are Gaussian distributed and we show that the achievable rate region provided by this AF strategy is within a $O(1)$ term of the upper bound in the high signal-to-noise ratio (SNR) regime.

From an analytical perspective our work is closely related to [20], which only deals with MIMO one-way relay channel. We summarize the key differences and improvements of the proposed work compared to the MIMO one-way relay channel capacity scaling result of [20] as follows.

- We assume a sum power constraint across all the relays, which is a generalization of the individual power constraint considered in [20]. An individual power constraint might seem more reasonable from a practical point of view. We show that even with a sum power constraint, however, the upper bound on the capacity region of both the coherent and non-coherent MIMO two-way relay channel can be achieved by allocating equal power to all relay nodes. Thus, with a sum power constraint, as
$K \to \infty$, the total power transmitted by all relay nodes remains bounded as opposed to [20], where it is unbounded. The optimal power allocation is similar to [20] from a practical perspective, since each relay node is required to transmit the same amount of power.

- We upper bound the capacity of the coherent MIMO two-way relay channel over all possible two-phase protocols, i.e. over arbitrary $\alpha$, while in [20] an upper bound is derived only for $\alpha = \frac{1}{2}$.

- Our achievable AF strategy for the coherent MIMO two-way relay channel allows all the relays to help all the data streams going from $T_1$ and $T_2$ and $T_2$ to $T_1$ as opposed to [20] where only $K/M$ relays are allowed to help each data stream. Moreover, in our AF strategy joint decoding is performed at both the receivers in contrast to [20], where each data stream is decoded by a single receive antenna treating all other streams as interference. Due to both these advantages, our AF strategy provides with better achievable rate regions compared to [20] for any finite $K$ and a better $O(1)$ term as $K \to \infty$.

- For the non-coherent MIMO two-way relay channel, we derive an upper and lower bound for $\alpha = \frac{1}{2}$, which differs by only a constant term at high SNR, while in [20] only an achievable AF strategy is provided without any upper bound for $\alpha = \frac{1}{2}$.

Our results show that with the MIMO two-way relay channel there is a improvement in the capacity scaling by a factor of 2, compared to MIMO one-way relay channel [20], for both the coherent and the non-coherent case. We show that with the MIMO two-way relay channel, both $T_1$ and $T_2$ can simultaneously communicate with each other at a rate which is equal to the maximum rate at which $T_1$ can communicate to $T_2$ if $T_2$ was silent. Therefore as $K \to \infty$, the MIMO two-way relay channel is shown to create two interference free parallel channels, one for $T_1 \to T_2$ and another for $T_2 \to T_1$, where on each channel a rate given by the maximum possible rate at which $T_1$ can communicate to $T_2$ link if $T_2$ was silent (one-way communication [20]) is achievable.

Organization: The rest of the paper is organized as follows. In Section II we describe the MIMO two-way relay channel system model, the protocol under consideration and the key assumptions. In Section III we derive an upper bound on the capacity of the coherent MIMO two-way relay channel. In Section IV by using a simple combining operation at the relays, we derive the asymptotic achievable rate region for the coherent MIMO two-way relay channel and show that it is possible to achieve the upper bound on the capacity region of the coherent MIMO two-way relay channel within a $O(1)$ term. Section V summarizes and discusses the implication of the coherent MIMO two-way relay channel capacity region. For the non-coherent MIMO two-way relay channel, in Section VI-A we derive an upper bound on the achievable rate region. Section VI-B gives a result on asymptotic achievable rate region for the non-
coherent MIMO two-way relay channel using AF strategy at relays. We draw some final conclusions in Section VII.

**Notation:** The following notation is used in this paper. The superscripts $T, \ast$ represent the transpose and transpose conjugate. $M$ denotes a matrix, $m$ a vector and $m_i$ the $i^{th}$ element of $m$. For a matrix $M = [m_1 \ m_2 \ \ldots \ m_n]$ by $\text{vec}(M)$ we mean $[m_1^T \ m_2^T \ \ldots \ m_n^T]^T$. $\det(M)$ and $\text{tr}(M)$ denotes the determinant and trace of matrix $A$, respectively. $\mathbb{E}_x(f(x))$ denotes the expectation of function $f$ with respect to $x$. $\| \cdot \|$ denotes the usual Euclidean norm of a vector. $I_m$ is a $m \times m$ identity matrix. $|X|$ is the cardinality of set $X$. We use the usual notation for $u(x) = O(v(x))$ if $\frac{|u(x)|}{|v(x)|}$ remains bounded, as $x \to \infty$. A circularly symmetric complex Gaussian random variable with zero mean and variance $\sigma$ is denoted by $x \sim \mathcal{CN}(0, \sigma)$ and $x|y \sim \mathcal{CN}(0, \sigma)$ denotes that given $y$, $x$ is a circularly symmetric complex Gaussian random variable with zero mean and variance $\sigma$. The variance of a random variable $a$ is denoted by $\text{var}(a)$. $\mathbb{C}^{MN}$ denotes the set of $M \times N$ matrices with complex entries. $x_n \xrightarrow{w.p.1} y$ denotes that the sequence of random variables $x_n$ converge to a random variable $y$ with probability 1. We use $a = b$ to denote equality with probability 1 i.e. $\text{Prob.}(a = b) = 1$ and $\leq_{w.p.1}$ is defined similarly. $I(x; y)$ denotes the mutual information between $x$ and $y$ and $h(x)$ the differential entropy of $x$ [30]. To define a variable we use the symbol $:=$.

**II. SYSTEM AND CHANNEL MODEL**

In this section we describe the MIMO two-way relay channel communication protocol, followed by signal and channel models. Consider a wireless network where there are two terminals $T_1$ and $T_2$ who want to exchange information via $K$ relays, as shown in Fig. 2. The $K$ relays do not have any data of their own and only help $T_1$ and $T_2$ communicate. We assume that there is no direct path between $T_1$ and $T_2$ and that they can communicate only through the $K$ relays. This is a realistic assumption when relaying is used for coverage improvement in cellular systems, since at the cell edge the signal to noise ratio is extremely low for the direct path. In ad-hoc networks, this occurs when two terminals want to communicate, but are out of each other’s transmission range.

We assume that both the terminals $T_1$ and $T_2$ have $M$ antennas while all the $K$ relays each have $N$ antennas. The terminals $T_1, T_2$ and all the relays operate in half-duplex mode i.e. cannot transmit and receive at the same time. The communication protocol is summarized as follows [21]. In any given time slot, for the first $\alpha$ fraction of time, called the transmit phase, both $T_1$ and $T_2$ are scheduled to transmit and all the relays receive a superposition of the signals transmitted from $T_1$ and $T_2$. In the rest $(1 - \alpha)$ fraction of the time slot, called the receive phase, all the relays are scheduled to transmit simultaneously and both the terminals receive.
A. Channel and Signal Model

In this paper we assume that all the channels are frequency flat slow fading block fading channels, where in a block of time duration $T_c$ (called the coherence time), the channel coefficients remain constant and change independently from block to block. We assume that $T_c$ is more that the duration of the time slot used by $T_1$ and $T_2$ to communicate with each other as described before. As shown in Fig. 3 let the forward channel between $T_1$ and the $k^{th}$ relay be $H_k = [h_{1k} \ h_{2k} \ \ldots \ h_{Mk}]$ and the backward channel between $k^{th}$ relay and $T_1$ be $H_k^{(r)} = [h_{1k}^{(r)} \ h_{2k}^{(r)} \ \ldots \ h_{Mk}^{(r)}]$. Similarly let the forward channel between the $k^{th}$ relay and $T_2$ be $G_k = [g_{1k} \ g_{2k} \ \ldots \ g_{Mk}]$ and the backward channel between $T_2$ and the $k^{th}$ relay be $G_k^{(r)} = [g_{1k}^{(r)} \ g_{2k}^{(r)} \ \ldots \ g_{Mk}^{(r)}]$. We assume that $H_k, G_k \in \mathbb{C}^{N \times M}$, $H_k^{(r)}, G_k \in \mathbb{C}^{M \times N}$ with independent and identically distributed (i.i.d.) $\mathcal{N}(0, 1)$ entries to keep the analysis simple and tractable. The ideas
presented in this paper, however, apply to a broad class of channel distributions.

In the transmit phase, the $N \times 1$ received signal at the $k^{th}$ relay is given by
\begin{equation}
    r_k = \sqrt{\frac{PE_k}{M}} H_k x + \sqrt{\frac{PF_k}{M}} G_k^{(r)} u + n_k
\end{equation}
where $x$ and $u$ are the $M \times 1$ signals transmitted from $T_1$ and $T_2$ to be decoded at $T_2$ and $T_1$ respectively, with $\mathbb{E}\{x^*x\} = \mathbb{E}\{u^*u\} = M$, $P$ is the power transmitted by $T_1$ and $T_2$ and $E_k$ and $F_k$ are the path loss and shadowing parameters from $T_1$ and $T_2$ to the $k^{th}$ relay, respectively. The noise $n_k$ is a spatio-temporal white complex Gaussian random vector independent across relays, with $\mathbb{E}(n_k n_k^*) = \sigma^2 I_N$.

Relay $k$ processes its incoming signal to transmit a $N \times 1$ signal $\sqrt{\gamma_k} t_k$ (with $\mathbb{E}\{t_k^*t_k\} = 1$) in the receive phase so that the transmitted power is $\gamma_k$. We assume a power constraint of $P$ at both $T_1$ and $T_2$ and a sum power constraint of $P_R$ across all the relays, i.e. $\sum_{k=1}^K \gamma_k \leq P_R$. The $M \times 1$ received signal $v$ and $y$ at terminal $T_1$ and $T_2$ respectively in the receive phase, are given by
\begin{equation}
    v = \sum_{k=1}^K \sqrt{\gamma_k Q_k} H_k^{(r)} t_k + w
\end{equation}
\begin{equation}
    y = \sum_{k=1}^K \sqrt{\gamma_k P_k} G_k t_k + z
\end{equation}
where $\gamma_k$ is the power transmitted by the $k^{th}$ relay, $Q_k, P_k$ are the path loss and shadowing parameters from the $k^{th}$ relay to $T_1$ and $T_2$, respectively, while $w$ and $z$ are $M \times 1$ spatio-temporal white complex Gaussian noise vectors with $\mathbb{E}(ww^*) = \mathbb{E}(zz^*) = \sigma^2 I_M$.

The path loss and shadowing effect parameters $E_k, F_k$ and $Q_k \forall k$ for the link between $T_1 \rightarrow T_2$ and $T_2 \rightarrow T_1$, are assumed to be independent and identically distributed (i.i.d.) random variables, strictly positive, bounded and remain constant over the entire time period of interest.

Throughout this paper we assume that both $T_1$ and $T_2$ perfectly know $\{H_k, H_k^{(r)}, G_k, G_k^{(r)}\} \forall k$, $k = 1, 2, \ldots, K$ in the receive mode. To be precise, in the receive phase (i.e. when $T_1$ and $T_2$ receive signal from all the relays), $T_1$ and $T_2$ both know $\{H_k, G_k\}$ and $\{H_k^{(r)}, G_k^{(r)}\}$ $\forall k$, $k = 1, 2, \ldots, K$. We also assume that no transmit CSI is available at $T_1$ and $T_2$, i.e. in the transmit phase $T_1$ and $T_2$ have no information about what the realization of $H_k$ and $G_k$ is going to be when it transmits its signal to all the relays in the transmit phase, respectively.

In this paper we consider two different assumptions about the CSI at the relays. The first case we consider is the coherent MIMO two-way relay channel, where all the relays have CSI in the transmit as well as the receive phases. For the coherent MIMO two-way relay channel, in the transmit phase the $k^{th}$ relay knows the realization of $H_k, G_k^{(r)}$ and in the receive phase it knows the realization of $G_k, H_k^{(r)}$. 
which could be achieved through channel reciprocity or feedback. We also consider the non-coherent MIMO two-way relay channel where we assume that no CSI is available at any relay.

III. UPPER BOUND ON THE CAPACITY REGION OF THE COHERENT MIMO TWO-WAY RELAY CHANNEL

The main result in this section is an upper bound on the rate $R_{12}$ and $R_{21}$ of reliable transmission from $T_1$ to $T_2$ and from $T_2$ to $T_1$, given by the next Theorem.

**Theorem 1:** The capacity region of the coherent MIMO two-way relay channel is upper bounded by

$$\lim_{K \to \infty} R_{12} \leq \frac{M}{2} \log K + O(1)$$

$$\lim_{K \to \infty} R_{21} \leq \frac{M}{2} \log K + O(1).$$

**Outline of the Proof:** We start by first separating $T_1$ and then $T_2$ from the network and apply the cut-set bound [30] to upper bound the rate of information transfer between $T_1 \to T_2$ and $T_2 \to T_1$, respectively. Using the cut-set bound, we first show that the maximum rate of information transfer from $T_1 \to T_2$ ($T_2 \to T_1$) is upper bounded by the maximum rate of information transfer between $T_1$ ($T_2$) and relays 1 to $K$ (the broadcast cut) and also by the maximum rate of information transfer between relays 1 to $K$ and $T_2$ ($T_1$) (the multiple access cut), Fig. 4 and Fig. 5. Then we use the capacity result from Section 4.1 [29] to upper bound the maximum rate through the broadcast cut for the case when CSI is only available at the receiver (all relays) and all the relays collaborate to decode the information. Similarly, for the multiple access cut as shown in Fig. 5 we upper bound the maximum rate at which all relays can communicate to $T_2$ and $T_1$ by using the capacity result from Section 3.1 [29], when CSI is known both at all the relays and at $T_1$ and $T_2$ and all the relays collaborate to transmit the information.
Remark 1: For the broadcast cut, the upper bound on the capacity of the MIMO one-way relay channel can be found in [20] which also trivially serves as an upper bound on $R_{12}$ and $R_{21}$. It is easy to identify, however, that there is a gap in the proof of Theorem 1 [20]. In Theorem 1 [20], it is argued that

$$I(s; r_1, r_2, \ldots, r_K | t_1, t_2, \ldots, t_K) = I(s; r_1, r_2, \ldots, r_K)$$

where $t_k$ is a function of $r_k$, $k = 1, 2, \ldots, K$, to which a counterexample can be easily found. Thus, we do not use result of [20] directly and attempt a different proof which is quite similar to the one given in [20], but closes the gap.

The formal proof is as follows.

**Proof:** Throughout this proof we assume that both $T_1$ and $T_2$ perfectly know $H_k, H_k^{(r)}, G_k, G_k^{(r)}, E_k, F_k, Q_k,$ and $P_k$ for $k = 1, 2, \ldots, K$, in the receive phase, and the $k^{th}$ relay knows $H_k, G_k^{(r)}, E_k, F_k$ in the transmit phase and $H_k^{(r)}, G_k, Q_k, P_k$ in the receive phase. For notational simplicity, we do not include $H_k, G_k^{(r)}, E_k, F_k, H_k^{(r)}, G_k, Q_k, P_k$ in the mutual information expressions. We clearly point out, though, whenever their knowledge is used to derive the upper bound.

**Broadcast cut** - To prove the upper bound we make use of the cut-set bound (Section 14.10 [30]). Separating the terminal $T_1$ from the rest of the network and applying the cut-set bound on the broadcast cut as shown in Fig. 4,

$$R_{12} \leq I(x; r_1, r_2, \ldots, r_K, y | t_1, t_2, \ldots, t_K, u).$$

(4)

Applying the cut-set bound while separating the terminal $T_2$,

$$R_{21} \leq I(u; r_1, r_2, \ldots, r_K, v | t_1, t_2, \ldots, t_K, x)$$

(5)

for some joint distribution $p(x, t_1, t_2, \ldots, t_K, u)$. By definition of mutual information [30]

$$I(x; r_1, r_2, \ldots, r_K, y | t_1, t_2, \ldots, t_K, u) = I(x; r_1, r_2, \ldots, r_K | t_1, t_2, \ldots, t_K, u) + I(x; y | r_1, r_2, \ldots, r_K, t_1, t_2, \ldots, t_K, u).$$

Expanding the mutual information in terms of differential entropy,

$$I(x; r_1, r_2, \ldots, r_K | t_1, t_2, \ldots, t_K, u) = h(x | t_1, t_2, \ldots, t_K, u) - h(x | r_1, r_2, \ldots, r_K, t_1, t_2, \ldots, t_K, u).$$

Since conditioning can only reduce entropy [30],

$$I(x; r_1, r_2, \ldots, r_K | t_1, t_2, \ldots, t_K, u) \leq h(x | u) - h(x | r_1, r_2, \ldots, r_K, t_1, t_2, \ldots, t_K, u).$$
Note that \( t_1, t_2, \ldots, t_K \) is a function of \( r_1, r_2, \ldots, r_K \), which implies
\[
I(x; r_1, r_2, \ldots, r_K | t_1, t_2, \ldots, t_K, u) \leq h(x | u) - h(x | r_1, r_2, \ldots, r_K, u)
\]
and hence
\[
I(x; r_1, r_2, \ldots, r_K | t_1, t_2, \ldots, t_K, u) \leq I(x; r_1, r_2, \ldots, r_K | u).
\]

From (3), with knowledge of \( P_k \) and \( G_k \), \( \forall k \) at terminal \( T_2 \),
\[
I(x; y | r_1, r_2, \ldots, r_K, t_1, t_2, \ldots, t_K, u) = I(x, z)
\]
where \( z \) is the AWGN noise. Since \( x \) and \( z \) are independent, \( I(x, z) = 0 \), and therefore
\[
I(x; r_1, r_2, \ldots, r_K, y | t_1, t_2, \ldots, t_K, u) \leq I(x; r_1, r_2, \ldots, r_K | u).
\]

Note that
\[
I(x; r_1, r_2, \ldots, r_K | u) = I \left( x; \frac{r_1}{\sqrt{K}}, \frac{r_2}{\sqrt{K}}, \ldots, \frac{r_K}{\sqrt{K}} | u \right)
\]
\[
= h \left( \frac{r_1}{\sqrt{K}}, \frac{r_2}{\sqrt{K}}, \ldots, \frac{r_K}{\sqrt{K}} | u \right) - h \left( \frac{r_1}{\sqrt{K}}, \frac{r_2}{\sqrt{K}}, \ldots, \frac{r_K}{\sqrt{K}} | x; u \right). \tag{6}
\]

Next we evaluate (6) using (1). From (1),
\[
r_k = \sqrt{\frac{P E_k}{M}} H_k x + \sqrt{\frac{P F_k}{M}} G_k^{(r)} u + n_k.
\]

Now with knowledge of \( F_k \) and \( G_k^{(r)} \) at each relay
\[
h \left( \frac{r_1}{\sqrt{K}}, \frac{r_2}{\sqrt{K}}, \ldots, \frac{r_K}{\sqrt{K}} | u \right) = h \left( \sqrt{\frac{P E_1}{K M}} H_1 x + \frac{n_1}{\sqrt{K}}, \sqrt{\frac{P E_2}{K M}} H_2 x + \frac{n_2}{\sqrt{K}}, \ldots, \sqrt{\frac{P E_K}{K M}} H_K x + \frac{n_K}{\sqrt{K}} | u \right).
\]

Since conditioning can only decrease entropy,
\[
h \left( \frac{r_1}{\sqrt{K}}, \frac{r_2}{\sqrt{K}}, \ldots, \frac{r_K}{\sqrt{K}} | u \right) \leq h \left( \sqrt{\frac{P E_1}{K M}} H_1 x + \frac{n_1}{\sqrt{K}}, \sqrt{\frac{P E_2}{K M}} H_2 x + \frac{n_2}{\sqrt{K}}, \ldots, \sqrt{\frac{P E_K}{K M}} H_K x + \frac{n_K}{\sqrt{K}} | u \right).
\]

With perfect knowledge of \( E_k, F_k \) and \( H_k, G_k^{(r)} \) at each relay
\[
h \left( \frac{r_1}{\sqrt{K}}, \frac{r_2}{\sqrt{K}}, \ldots, \frac{r_K}{\sqrt{K}} | x, u \right) = h \left( \frac{n_1}{\sqrt{K}}, \frac{n_2}{\sqrt{K}}, \ldots, \frac{n_K}{\sqrt{K}} \right)
\]
and using (6) it follows that
\[
I(x; r_1, r_2, \ldots, r_K | u) \leq h \left( \sqrt{\frac{P E_1}{K M}} H_1 x + \frac{n_1}{\sqrt{K}}, \sqrt{\frac{P E_2}{K M}} H_2 x + \frac{n_2}{\sqrt{K}}, \ldots, \sqrt{\frac{P E_K}{K M}} H_K x + \frac{n_K}{\sqrt{K}} \right)
\]
\[
- h \left( \frac{n_1}{\sqrt{K}}, \frac{n_2}{\sqrt{K}}, \ldots, \frac{n_K}{\sqrt{K}} \right).
\]
Using the capacity result from Section 4.1 [29] when CSI is only known at the receiver, the R.H.S. can be upper bounded by

$$\log \det \left( I_M + \frac{1}{\sigma^2} \sum_{k=1}^{K} \frac{P E_k}{K M} H_k^* H_k \right)$$

and the maximum is achieved when \( x \) is circularly symmetric complex Gaussian with \( \mathbb{E}(x x^*) = I_M \).

Thus, it follows that

$$I(x; r_1, r_2, \ldots, r_K | u) \leq \log \det \left( I_M + \frac{1}{\sigma^2} \sum_{k=1}^{K} \frac{P E_k}{K M} H_k^* H_k \right).$$

(7)

Similarly, by interchanging the roles of \( x \) and \( u \) and replacing \( E_k \) with \( F_k \) and \( H_k \) with \( G_k \),

$$I(u; r_1, r_2, \ldots, r_K | x) \leq \log \det \left( I_M + \frac{1}{\sigma^2} \sum_{k=1}^{K} \frac{P F_k}{K M} G_k^* G_k \right).$$

(8)

Using the strong law of large numbers

$$\lim_{K \to \infty} \frac{1}{K} \sum_{k=1}^{K} \frac{P E_k}{M} H_k^* H_k \xrightarrow{w.p.1} \frac{P}{M} \mathbb{E}\{E_k H_k^* H_k\}$$

and

$$\lim_{K \to \infty} \frac{1}{K} \sum_{k=1}^{K} \frac{P F_k}{M} G_k^* G_k \xrightarrow{w.p.1} \frac{P}{M} \mathbb{E}\{F_k G_k^* G_k\}.$$ 

Since \( \mathbb{E}\{H_k^* H_k\} = \mathbb{E}\{G_k^* G_k\} = NI_M \) and let \( \mathbb{E}\{E_k\} = \mathbb{E}\{F_k\} = \mu \), using (4), (5), (7), (8) and the fact that the sources \( T_1 \) and \( T_2 \) transmit only for \( \alpha \) fraction of the time in each time slot, it follows that

$$\lim_{K \to \infty} R_{12} \leq \alpha M \log \left( 1 + \frac{K N P \mu}{M \sigma^2} \right)$$

(9)

and

$$\lim_{K \to \infty} R_{21} \leq \alpha M \log \left( 1 + \frac{K N P \mu}{M \sigma^2} \right).$$

(10)

Since \( M, N, P, \mu \) and \( \sigma^2 \) are finite integers, as \( K \to \infty \)

$$\lim_{K \to \infty} R_{12} \leq \alpha M \log(K) + \mathcal{O}(1)$$

(11)

and

$$\lim_{K \to \infty} R_{21} \leq \alpha M \log(K) + \mathcal{O}(1).$$

(12)

**Multiple access cut** - Again by using the cut-set bound, we bound the maximum rate of information transfer \( R_{12} \) \((R_{21}) \) from \( T_1 \rightarrow T_2 \) \((T_2 \rightarrow T_1) \) by the maximum rate of information transfer across the multiple access cut as shown in Fig. 5. Using the cut-set bound, \( R_{12} \) and \( R_{21} \) are bounded by

$$R_{12} \leq I(x, t_1, t_2, \ldots, t_K; y | u)$$

(13)
By definition of mutual information
\[
I(x, t_1, t_2, \ldots, t_K; y | u) = h(y | u) - h(y | t_1, t_2, \ldots, t_K, u) \\
+ h(y | t_1, t_2, \ldots, t_K, x, u) - h(y | t_1, t_2, \ldots, t_K, x, u).
\]
Note that given $t_1, t_2, \ldots, t_K$, $y$ is independent of $x$ and $u$, thus
\[
h(y | t_1, t_2, \ldots, t_K, x, u) = h(y | t_1, t_2, \ldots, t_K, u) = h(y | t_1, t_2, \ldots, t_K).
\]
Therefore
\[
I(x, t_1, t_2, \ldots, t_K; y | u) = h(y | u) - h(y | t_1, t_2, \ldots, t_K).
\]
Since conditioning can only reduce entropy,
\[
I(x, t_1, t_2, \ldots, t_K; y | u) \leq h(y) - h(y | t_1, t_2, \ldots, t_K) \\
= I(t_1, t_2, \ldots, t_K; y).
\]
Hence from (13),
\[
R_{12} \leq I(t_1, t_2, \ldots, t_K; y). \tag{15}
\]
Following similar steps
\[
R_{21} \leq I(t_1, t_2, \ldots, t_K; v). \tag{16}
\]
Clearly $R_{12}, R_{21}$ are bounded by the maximum rate of information across the multiple access cut Fig. 5.

Next, we compute $I(t_1, t_2, \ldots, t_K; y)$. Recall From (3), that the received signal $y$ at $T_2$ is
\[
y = \sum_{k=1}^{K} \sqrt{\gamma_k P_k G_k t_k} + z.
\]
Note that

\[ I(t_1, t_2, \ldots, t_K; y) = I\left(t_1, t_2, \ldots, t_K; \frac{y}{\sqrt{K}}\right). \]

Dividing \(y\) by \(\sqrt{K}\), the scaled signal is

\[ \frac{y}{\sqrt{K}} = \frac{1}{\sqrt{K}} \sum_{k=1}^{K} \sqrt{\gamma_k P_k G_k t_k} + \frac{z}{\sqrt{K}}. \]

This can also be written as

\[ \frac{y}{\sqrt{K}} = \frac{1}{\sqrt{K}} \left[ \sqrt{P_1 G_1} \sqrt{P_2 G_2} \ldots \sqrt{P_K G_K} \right] \left[ \sqrt{\gamma_1 t_1} \sqrt{\gamma_2 t_2} \ldots \sqrt{\gamma_K t_K} \right]^T + \frac{z}{\sqrt{K}}. \]

Note that \(\Phi\) is a \(M \times NK\) matrix. Now assuming that all the relays know \(G_k \forall k\) (allowing cooperation among all relays), with total power available across all relays bounded by \(P_R\), from Section 3.1 [29],

\[ I(t_1, t_2, \ldots, t_K; \frac{y}{\sqrt{K}}) \leq \min_{\{NK,M\}} \sum_{l=1}^{\min\{NK,M\}} \max \left\{ 0, \log \left( \frac{K \lambda_l \nu}{\sigma^2} \right) \right\} \]

(17)

where \(\lambda_l, l = 1, 2, \ldots, \min\{NK,M\}\) are the eigen values of \(\Phi \Phi^*\) matrix and \(\nu\) is chosen such that

\[ \min_{\{NK,M\}} \sum_{l=1}^{\min\{NK,M\}} \max \left\{ 0, \nu - \frac{1}{\lambda_l} \right\} = P_R. \]

By definition, \(\Phi \Phi^* = \frac{1}{K} \sum_{k=1}^{K} P_k G_k G_k^*\). From the strong law of large numbers

\[ \lim_{K \to \infty} \frac{1}{K} \sum_{k=1}^{K} P_k G_k G_k^* \overset{w.p.1}{\to} \mathbb{E}\{P_k G_k G_k^*\} = \mathbb{E}\{P_k\} \mathbb{E}\{G_k G_k^*\} = \mu N I_M \]

since \(\mathbb{E}\{G_k G_k^*\} = NI_M\) and \(\mu := \mathbb{E}\{P_k\}\). Therefore, it follows that

\[ \lambda_i = N \mu \ \forall \ i = 1, 2, \ldots, M. \]

which implies

\[ \nu = \left( \frac{P_R}{M} + \frac{1}{N \mu} \right) \]

and

\[ I\left(t_1, t_2, \ldots, t_K; \frac{y}{\sqrt{K}}\right) \leq \sum_{l=1}^{M} \log \left( \frac{KN \rho}{\sigma^2} \left( \frac{P_R}{M} + \frac{1}{N \mu} \right) \right). \]

Since \(M, N, P_R, \sigma^2\) and \(\mu\) are all finite,

\[ \lim_{K \to \infty} I\left(t_1, t_2, \ldots, t_K; \frac{y}{\sqrt{K}}\right) \leq M \log K + O(1). \]

Moreover, since the relays transmit only for \((1 - \alpha)\) fraction of time in any given time slot,

\[ \lim_{K \to \infty} R_{12} \leq (1 - \alpha) I\left(t_1, t_2, \ldots, t_K; \frac{y}{\sqrt{K}}\right) \leq (1 - \alpha) M \log K + O(1). \]

(18)
Similarly, we can derive a bound for \( R_{21} \) by using (2) and (16),

\[
\lim_{K \to \infty} R_{21} \leq (1 - \alpha) I \left( t_1, t_2, \ldots, t_K; \frac{v}{\sqrt{K}} \right) \leq (1 - \alpha) M \log K + O(1).
\]  

(19)

Combining (11), (12), (18) and (19)

\[
\lim_{K \to \infty} R_{12} \leq \min_{w.p.1} \{ \alpha, 1 - \alpha \} M \log K + O(1)
\]

\[
\lim_{K \to \infty} R_{21} \leq \min_{w.p.1} \{ \alpha, 1 - \alpha \} M \log K + O(1).
\]

Since \( \alpha \in [0, 1] \), \( \min \{ \alpha, 1 - \alpha \} \leq \frac{1}{2} \), therefore

\[
\lim_{K \to \infty} R_{12} \leq \frac{M}{2} \log K + O(1)
\]

\[
\lim_{K \to \infty} R_{21} \leq \frac{M}{2} \log K + O(1).
\]

Discussion: In Theorem 1, we obtained upper bounds on \( R_{12} \) and \( R_{21} \) by using cut-set bound on the broadcast cut (Fig. 4) and the multiple access cut (Fig. 5). For the broadcast cut, the upper bound corresponds to the case when the transmitter \( T_1 \) or \( T_2 \) has no CSI while all the relays collaborate to decode the message sent by \( T_1 \) or \( T_2 \) with perfect CSI, while the upper bound in the multiple access cut corresponds to the case when all the relays collaborate to transmit data to \( T_1 \) or \( T_2 \) using all their \( NK \) antennas with transmit CSI available at all relays. An important point to note is that the upper bound obtained in Theorem 1 is for any arbitrary \( \alpha \), which implies that the upper bound is valid for all two-phased MIMO two-way relay channel protocols and not for only \( \alpha = 1/2 \) as is the case in [20].

In the next section we illustrate a simple amplify and forward (AF) strategy whose achievable rate is a constant term away from the upper bound.

IV. LOWER BOUND ON THE CAPACITY REGION OF THE COHERENT MIMO TWO-WAY RELAY CHANNEL

In this section we propose an AF strategy to achieve the upper bound obtained in Theorem 1 on the capacity region of the MIMO two-way relay channel within a constant term. The motivation to consider AF is because with DF, at each relay, to decode \( T_1 \)'s message \( T_2 \)'s message is treated as interference and vice-versa, which implies that the achievable rate region with DF is same as that of the achievable rate region for the multiple access channel [30]. Since the achievable rate region of the multiple access channel is strictly less than the upper bound derived in Theorem 1 one cannot hope to achieve the upper bound given by Theorem 1 using DF protocol. With AF, however, each relay processes the received signal using its CSI and transmits it to \( T_1 \) and \( T_2 \) in the receive phase without any decoding. Since both
$T_1$ and $T_2$ know what they transmit, (i.e. $T_1$ knows $x$ and $T_2$ knows $u$) with perfect receive CSI, both $T_1$ and $T_2$ can cancel the contribution of their own transmitted signal from the received signal and decode other terminal’s message without any self interference.

Before discussing the MIMO two-way relay channel with multiple relays, let us first consider the case of a MIMO one-way relay channel (i.e. $T_2$ has no data for $T_1$) with only one relay. For this case, the optimal AF strategy to maximize mutual information at the destination is to multiply $V_2^T D_k U_1^*$ to the signal at the relay, where the singular value decomposition of $H_1$ is $U^*_1 D_1 V_1^*$ and $G_1$ is $U^*_2 D_2 V_2^*$ and $D$ is a diagonal matrix whose entries are chosen by waterfilling [31]. Finding the optimal AF strategy for the multiple relay case is a non-trivial problem and has not been found to the best of our knowledge. Moreover, the two-way nature of our problem makes it even more difficult to find the optimal AF strategy.

To obtain a lower bound on the achievable rates for the MIMO two-way relay channel we propose a dual channel matching AF strategy in which relay $k$ multiplies $\sqrt{\beta_k} (G_k^* H_k^* + H_k^{(r)*} G_k^{(r)*})$ to the received signal and forwards it to $T_1$ and $T_2$, where $\beta_k$ is the normalization constant to satisfy the power constraint. In this AF strategy, each relay tries to match both the channels which the data streams from $T_1$ to $T_2$ and $T_2$ to $T_1$ experience. In dual channel matching, the complex conjugates of the channels are used directly rather than the unitary matrices from the SVD of the channels [31]. This modification makes it easier to compute the achievable rates for the MIMO two-way relay channel.

Together with dual channel matching we restrict the signal transmitted from $T_1$ and $T_2$, $x$ and $u$, respectively, to be circularly symmetric complex Gaussian with covariance matrix $E\{xx^*\} = E\{uu^*\} = Q$ with $tr (Q) = M$ (to meet the power constraint) to obtain an achievable rate region for the coherent MIMO two-way relay channel. Moreover, we use $\alpha = \frac{1}{2}$ i.e. $T_1$ and $T_2$ transmit and receive for same amount of time. The achievable rates $R_{12}$ and $R_{21}$ using the above described AF strategy are given by the following Theorem.

**Theorem 2:** For the coherent MIMO two-way relay channel, the achievable rates are given by

$$\lim_{K \to \infty} R_{12} = \frac{M}{2} \log (K) + O(1) \quad w.p.1$$

$$\lim_{K \to \infty} R_{21} = \frac{M}{2} \log (K) + O(1) \quad w.p.1$$

with no cooperation required between $T_1$ and $T_2$.

**Proof:** From [1], the received signal at the $k^{th}$ relay is given by

$$r_k = \sqrt{\frac{P E_k}{M}} H_k^* x + \sqrt{\frac{P F_k}{M}} G_k^{(r)*} u + n_k. \quad (20)$$
Using dual channel matching, at relay $k$ the transmitted signal $t_k$ is given by
\[ t_k = \left( G_k^* H_k^* + H_k^{(r)*} G_k^{(r)*} \right) r_k \]  
where $\beta_k$ is to ensure that $\mathbb{E} \{ t_k^* t_k \} = 1$. The received signal at $T_2$ is given by
\[ y = \sum_{k=1}^{K} \sqrt{\gamma_k P_k} G_k t_k + z. \]  
(22)

Expanding (22) using (20) and (21)
\[ y = \sum_{k=1}^{K} \sqrt{\gamma_k P_k} G_k \left( G_k^* H_k^* + H_k^{(r)*} G_k^{(r)*} \right) H_k x + \sum_{k=1}^{K} \sqrt{\gamma_k P_k P_{E_k}} M \beta_k \left( G_k^* H_k^* + H_k^{(r)*} G_k^{(r)*} \right) G_k^{(r)*} u 
+ \sum_{k=1}^{K} \sqrt{\gamma_k P_k \beta_k} G_k \left( G_k^* H_k^* + H_k^{(r)*} G_k^{(r)*} \right) n_k + z. \]


\[ y = \sum_{k=1}^{K} \sqrt{\gamma_k P_k} G_k \left( G_k^* H_k^* + H_k^{(r)*} G_k^{(r)*} \right) H_k x + \sum_{k=1}^{K} \sqrt{\gamma_k P_k P_{E_k}} M \beta_k \left( G_k^* H_k^* + H_k^{(r)*} G_k^{(r)*} \right) G_k^{(r)*} u \]

Since $u$ and the channel coefficients $H_k, G_k, H_k^{(r)}, G_k^{(r)}$ are known at $T_2$, $\forall k$, the second term can be removed from the received signal at $T_2$. Moreover, as described before $x$ is circularly symmetric complex Gaussian vector with covariance matrix $Q$, thus the achievable rate for $T_1$ to $T_2$ link is [29]
\[ R_{12} = \frac{1}{2} \log \det \left( I_M + \frac{AQA^*}{\sum_{k=1}^{K} B_k B_k^* + M I_M} \right), \]

since $\mathbb{E} \{ n_k n_k^* \} = \mathbb{E} \{ z z^* \} = I_M$, $\forall k$. Using $Q = \frac{1}{M^2} I$ and dividing the numerator and denominator by $K^2$,
\[ R_{12} = \frac{1}{2} \log \det \left( I_M + \frac{M K}{K} \frac{A\hat{A}^*}{\sum_{k=1}^{K} B_k B_k^* + I_M} \right). \]

Note that as $K \to \infty$, the contribution from $\frac{1}{K} I_M$ can be neglected and it follows that
\[ R_{12} = \frac{1}{2} \log \det \left( I_M + \frac{K}{M} \frac{A\hat{A}^*}{\sum_{k=1}^{K} B_k B_k^*} \right). \]

Using equal power allocation $\gamma_k = \frac{P_k}{K}$ to satisfy the total power constraint of $P_R$ across all relays
\[ I(x; y) = \log \det \left( I_M + \frac{K}{M} \frac{A\hat{A}^*}{\sum_{k=1}^{K} B_k B_k^*} \right), \]

where $\hat{A} = \sum_{k=1}^{K} \sqrt{\frac{P_k P_{E_k}}{M \beta_k}} G_k \left( G_k^* H_k^* + H_k^{(r)*} G_k^{(r)*} \right) H_k$ and $\hat{B}_k = \sqrt{\frac{P_k \beta_k}{M}} G_k \left( G_k^* H_k^* + H_k^{(r)*} G_k^{(r)*} \right)$. Using the strong law of large numbers,
\[ \lim_{K \to \infty} \frac{\hat{A}}{K} \overset{w.p.1}{\to} \sqrt{\frac{P}{M}} \kappa M^2 I_M \]

and
\[ \lim_{K \to \infty} \frac{\hat{A}^*}{K} \overset{w.p.1}{\to} \sqrt{\frac{P}{M}} \kappa M^2 I_M \]
since $\mathbb{E} \{ G_k G_k^* \} = \mathbb{E} \{ H_k^* H_k \} = M I_M \forall \ k, \ \mathbb{E} \{ G_k (r)^* \} = \mathbb{E} \{ H_k^* (r)^* \} = 0 I_M \forall \ k$ and $P_k, E_k, \beta_k$ are i.i.d. with $\kappa = \mathbb{E} \left\{ \sqrt{P_k E_k} \beta_k \right\}$. Moreover, from the strong law of large numbers

$$
\lim_{K \to \infty} \frac{1}{K} \sum_{k=1}^{K} \hat{B}_k \hat{B}_k^* \xrightarrow{w.p.1} \mathbb{E} \{ \hat{B}_k \hat{B}_k^* \} = \theta I_M
$$

for some finite $\theta$, since $\hat{B}_k \hat{B}_k^*$ are i.i.d. for each $k$ and each entry of $\hat{B}_k$ has finite variance. Thus, using these approximations,

$$
\lim_{K \to \infty} R_{12} = \frac{M}{2} \log \left( \frac{\det(I_M + \frac{K P \kappa^2 M^2}{\theta} I_M)}{1 + \frac{K P \kappa^2 M^2}{\theta}} \right).
$$

Since $M, P, \kappa$ and $\theta$ are finite, as $K \to \infty$,

$$
\lim_{K \to \infty} R_{21} = \frac{M}{2} \log K + O(1),
$$

and similarly

$$
\lim_{K \to \infty} R_{21} = \frac{M}{2} \log K + O(1).
$$

**Discussion:** Theorem 2 shows that the achievable rate in each direction $T_1 \to T_2$ or $T_2 \to T_1$ with the coherent MIMO two-way relay channel using dual channel matching at each relay is given by $\frac{M}{2} \log (K) + O(1)$ as $K \to \infty$. More importantly, Theorem 2 also shows that both $T_1$ and $T_2$ can simultaneously transmit at rate $\frac{M}{2} \log (K) + O(1)$, without affecting each other’s data rate and without requiring any cooperation between themselves. The result can be interpreted as follows. With dual channel matching, the transmitted signals from $T_1$ and $T_2$ are coherently added by all relays and the equivalent array gain of $\log K$ is obtained at both the receivers $T_1$ and $T_2$ for all the $M$ data streams transmitted from $T_2$ and $T_1$. Moreover, with perfect channel knowledge, $T_1$ and $T_2$ can cancel the self interference their own transmitted signals create, which enable $T_1$ and $T_2$ to simultaneously achieve the rate of $\frac{M}{2} \log (K) + O(1)$, without requiring any cooperation.

Recall that for the MIMO one-way relay channel an AF strategy was proposed in [20] to achieve the upper bound within a constant term. With the AF strategy of [20], $M$ independent data streams are transmitted from $T_1$ and all the relays are divided into $M$ sets with each set helping a particular data stream and independent decoding of data streams is employed at the receiver. Compared to the AF strategy of [20], with dual channel matching all relays participate in transmission of all data streams from $T_1$ to $T_2$ and $T_2$ to $T_1$ and thus provides a better achievable rate region. Moreover, joint decoding of data
Fig. 6. Asymptotic Capacity of the coherent MIMO two way relaying

streams at respective receivers with dual channel matching removes the adverse effect of inter-stream interference which is caused due to independent decoding of different data streams in [20]. Thus it is clear that dual channel matching improves the achievable rate regions as compared to the AF strategy of [20].

V. COHERENT MIMO TWO-WAY RELAY CHANNEL CAPACITY REGION

Combining the results from Section III and Section IV we establish the following characterization of the scaling behavior of the capacity region of the coherent MIMO two-way relay channel.

Theorem 3: Neglecting the $O(1)$ term, the capacity region of the coherent MIMO two-way relay channel is given by the convex hull of

$$\lim_{K \to \infty} R_{12} = \frac{M}{2} \log(K)$$

$$\lim_{K \to \infty} R_{21} = \frac{M}{2} \log(K)$$

where $R_1$ and $R_2$ are the rate of information transfer between $T_1 \to T_2$ and $T_2 \to T_1$, respectively.

Proof: Follows from Theorem 1 and 2.

Discussion: The capacity region of the coherent MIMO two-way relay channel is illustrated in Fig. 6. Combining Theorems 1 and 2, the sum capacity (sum of $R_{12}$ and $R_{21}$) of the coherent MIMO two-way relay channel is given by $M \log(K) + O(1)$, as $K \to \infty$, which is exactly double of the capacity
achievable in each direction $T_1 \rightarrow T_2$ or $T_2 \rightarrow T_1$ [20]. The $O(1)$ term in the upper and lower bound can in general be different and hence we characterize the exact capacity up to a $O(1)$ term. An important implication of Theorem 2 is that with enough relays, the dual channel matching strategy is optimal in the sense of achieving the right capacity scaling. Therefore, neglecting the $O(1)$ term, what this result shows is that with the coherent MIMO two-way relay channel, one can communicate at rate $\frac{M_2}{2} \log K$ from $T_1 \rightarrow T_2$ while simultaneously communicating at rate $\frac{M_2}{2} \log K$ from $T_2 \rightarrow T_1$.

Recall that we obtained the lower bound in Theorem 2 by fixing $\alpha = \frac{1}{2}$ i.e. $T_1$ and $T_2$ transmit and receive for equal amount of time. Since this lower bound is only a $O(1)$ term away from the upper bound, allocating equal amount of time for the transmit and the receive phase is optimal for the coherent MIMO two-way relay channel.

From Theorem 2, it is also clear that the upper bound on the capacity region of the MIMO two-way relay channel is achievable within a $O(1)$ term without any cooperation between $T_1$ and $T_2$. This is significant since the upper bound is for some joint encoding between $T_1$ and $T_2$. This is made possible because with channel knowledge, both $T_1$ and $T_2$ are able to cancel off the self interference.

Compared to the asymptotic capacity result for MIMO one-way relay channel [20], our results show that with the coherent MIMO two-way relay channel one can remove the $\frac{1}{2}$ rate loss factor on the capacity, which comes from the half-duplex assumption on the terminals and relays. Therefore with the coherent MIMO two-way relay channel it is possible to can achieve unidirectional full-duplex performance with half-duplex terminals.

To compute the capacity of the coherent MIMO two-way relay channel, we assumed that CSI was available at both $T_1$, $T_2$ and each relay stage, which is a very strict requirement to meet in practice. To do dual channel matching for the coherent MIMO two-way relay channel, the $k^{th}$ relay needs to know the realization of $H_k$ and $G_k^{(r)}$ of the transmit phase and the realization of $G_k$ and $H_k^{(r)}$ of the receive phase. To cancel the self-interference and to detect the incoming signal in the receive phase, both the terminals $T_1$ and $T_2$ need to know the realization of $\{H_k, G_k\}$ for the transmit phase and $\{H_k^{(r)}, G_k^{(r)}\}$ for the receive phase. In practice, this is a very strict and challenging requirement, but by sending training sequence and using standard channel estimation techniques together with intelligent channel information feedback algorithms, all the nodes can learn the required receive channel coefficients with good enough accuracy.

For example, by sending training sequences from $T_1$ and $T_2$, the $k^{th}$ relay can learn $H_k$ and $G_k^{(r)}$ in the transmit phase (when $T_1$ and $T_2$ transmit signals to all the relays). Learning the $G_k$ and $H_k^{(r)}$ realization at the $k^{th}$ relay for the receive phase (when all the relays transmit and both $T_1$ and $T_2$ receive)
is more challenging. In time-division duplex system, however, by employing calibration at transmitter and receiver, the forward and backward channel can be assumed to be reciprocal in which case the realization of $H_k^{(r)}$ and $G_k$ for the receive phase is approximately equal to the realization of $H_k^T$ and $G_k^{(r)T}$ for the transmit phase. Instead if a frequency-division duplex (FDD) system is used, assuming block fading channel, $G_k$ and $H_k^{(r)}$ can be learnt at each relay for receive phase, by feeding back the information about $G_k$ and $H_k^{(r)}$ from $T_1$ and $T_2$ in the transmit phase, learnt in the last receive phase at $T_1$ and $T_2$.

To decode the incoming signal and to cancel the self interference, $T_1$ and $T_2$ needs to know the realization of $G_k, H_k$ of the transmit phase and the realization of $G_k^{(r)}, H_k^{(r)}$ of the receive phase. By sending training sequences from all the relays to $T_1$ and $T_2$, $T_1$ and $T_2$ can learn the realization of $H_k^{(r)}, G_k$ of the receive phase, respectively. At the start of receive phase each relay knows the realization of $G_k, H_k, G_k^{(r)}, H_k^{(r)}$, therefore if each relay transmits quantized channel information about $G_k, H_k, G_k^{(r)}, H_k^{(r)}$ using strategies such as Grassmannian codebook [32] etc. to $T_1$ and $T_2$, both $T_1$ and $T_2$ can learn the required CSI in the receive phase.

VI. NON-COHERENT MIMO TWO-WAY RELAY CHANNEL

In the last section we derived the scaling behavior of the capacity region of the MIMO two-way relay channel when both $T_1$ and $T_2$ have receive CSI while all the relays have perfect transmit and receive CSI. It is well known, however, that acquiring accurate CSI in a real-time communication is a challenging problem (large overhead and complexity) and guaranteeing near perfect CSI is almost impossible in practice. Therefore in this section we study the scaling behavior of the capacity region of the MIMO two-way relay channel when CSI is only available at $T_1$ and $T_2$ in the receive phase and no CSI is available at any of the relays. Furthermore, for this case we fix $\alpha = \frac{1}{2}$, i.e. $T_1$ and $T_2$ transmit and receive for equal amount of time (transmit phase is equal to receive phase).

For the non-coherent MIMO two-way relay channel, we first upper bound the achievable rates from $T_1 \rightarrow T_2$ and $T_2 \rightarrow T_1$ using the cut-set bound for the multiple access cut. Then using a simple AF strategy at each relay, we compute the achievable rates from $T_1 \rightarrow T_2$ and $T_2 \rightarrow T_1$ which are shown to be within a $O(1)$ term from the upper bound in the high signal to noise (SNR) regime, thereby characterizing high SNR capacity.

A. Upper Bound on The Capacity Region of The Non-Coherent Two-Way Relay Channel

As proved in the section[III] the rate of information transfer from $T_1 \rightarrow T_2$ ($T_2 \rightarrow T_1$) is upper bounded by the rate of information transfer between all-relays put together and $T_2(T_1)$ (multiple access cut). We evaluate this upper bound in the following Theorem, when CSI is not available at any of the relay.
Theorem 4: In the high-SNR regime (large $P_R$), the rates $R_{12}$ and $R_{21}$ from $T_1 \rightarrow T_2$ and $T_2 \rightarrow T_1$ for the non-coherent MIMO two-way relay channel are upper bounded by

$$\lim_{K \to \infty} R_{12} \leq \frac{M}{2} \log (P_R) + O(1)$$

$$\lim_{K \to \infty} R_{21} \leq \frac{M}{2} \log (P_R) + O(1),$$

where $P_R$ is the total power constraint across all relays.

Proof: Using the multiple access cut-set bound from (15) and (16), we have

$$R_{12} \leq I(t_1, t_2, \ldots, t_K; y)$$

and

$$R_{21} \leq I(t_1, t_2, \ldots, t_K; v),$$

for some joint distribution $p(t_1, t_2, \ldots, t_K)$ and with no CSI at any relay. Recall from (3) that the received signal $y$ is given by

$$y = \sum_{k=1}^{K} \sqrt{\gamma_k} P_k G_k t_k + z$$

with power constraint $\sum_{k=1}^{K} \gamma_k \leq P_R$. Using the capacity result from Section 4.1 [29] for no transmit CSI

$$I(t_1, t_2, \ldots, t_K; y) \leq \log \det \left( I_M + \frac{\Sigma Q \Sigma^*}{\sigma^2} \right)$$

where

$$\Sigma = [\sqrt{P_1} G_1 \sqrt{P_2} G_2 \ldots \sqrt{P_K} G_K] \in \mathbb{C}^{M \times NK}$$

and $Q$ is the covariance matrix of

$$[\sqrt{\gamma_1} t_1 \sqrt{\gamma_2} t_2 \ldots \sqrt{\gamma_K} t_K]^T \in \mathbb{C}^{NK \times 1}$$

with $[t_1 t_2 \ldots t_K]^T$ circularly symmetric complex Gaussian and equivalent power constraint of $tr(Q) \leq P_R$ and the maximum is achieved when $Q = \frac{P_R}{NK} I_{NK \times NK}$. Therefore, using $Q = \frac{P_R}{NK} I_{NK \times NK}$

$$R_{12} \leq \log \det \left( I_M + \frac{P_R}{NK \sigma^2} \sum_{k=1}^{K} P_k G_k G_k^* \right).$$

From the strong law of large numbers,

$$\lim_{K \to \infty} \frac{1}{K} \sum_{k=1}^{K} P_k G_k G_k^* \overset{w.p.1}{\to} \mathbb{E} \{ P_k G_k G_k^* \} = \mathbb{E} \{ P_k \} \mathbb{E} \{ G_k G_k^* \}. $$

Since $\mathbb{E} \{ G_k G_k^* \} = NI_M$ and let $\mu := \mathbb{E} \{ P_k \}$,

$$R_{12} \leq \log \det \left( I_M + \frac{P_R \mu}{\sigma^2} I_M \right).$$
Since $\mu$ and $\sigma^2$ are finite, for large $P_R$

$$R_{12} \leq M \log P_R + \mathcal{O}(1). \quad \text{w.p.} 1$$

Since $T_1$ and $T_2$ transmit only for half the time ($\alpha = \frac{1}{2}$) in any given time slot

$$R_{12} \leq \frac{M}{2} \log P_R + \mathcal{O}(1). \quad \text{w.p.} 1$$

Similarly, it can be shown that

$$R_{21} \leq \frac{M}{2} \log P_R + \mathcal{O}(1). \quad \text{w.p.} 1$$

B. Lower Bound on The Capacity Region of The Non-coherent MIMO Two-Way Relay Channel

In this subsection we compute achievable rates $R_{12}$ and $R_{21}$ for the non-coherent MIMO two-way relay channel using a simple AF strategy at each relay. The strategy is the following: with no CSI at any relay, each relay just normalizes the received signal to meet its power constraint and retransmits it in the receive phase. With CSI available at each destination $T_1$ ($T_2$), self interference generated by $T_1$ ($T_2$) is removed from the received signal and the equivalent channel between $T_1 \rightarrow T_2$ ($T_2 \rightarrow T_1$) for the non-coherent MIMO two-way relay channel is given by $\sum_{k=1}^{K} H_k G_k \left( \sum_{k=1}^{K} H_k^{(r)} G_k^{(r)} \right)$. As $K \to \infty$, this channel is shown to behave as i.i.d. MIMO Gaussian channel. Then by using the capacity results from [29], we lower bound the capacity region of the non-coherent MIMO two-way relay channel. We show that with approximately same power used at $T_1$ ($T_2$) and all relays (i.e. $P \approx P_R$), the lower bound meets the upper bound in the high SNR regime (high $P$).

The following Theorem gives the expressions for achievable $R_{12}$ and $R_{21}$ pair, when each relay uses AF.

**Theorem 5:** In the high SNR regime, the achievable rate region for the non-coherent MIMO two-way relay channel using AF strategy at each relay, is given by

$$\lim_{K \to \infty} R_{12} \quad \text{w.p.} 1 = \frac{M}{2} \log (P_R) + \mathcal{O}(1)$$

$$\lim_{K \to \infty} R_{21} \quad \text{w.p.} 1 = \frac{M}{2} \log (P_R) + \mathcal{O}(1).$$

**Proof:** Recall from (1) that the received signal at each relay is given by

$$r_k = \sqrt{\frac{P_E k}{M}} H_k x + \sqrt{\frac{P F_k}{M}} G_k^{(r)} u + n_k$$

(23)

Therefore the average received signal plus noise power at each relay is given by $N(P(E_k + F_k) + \sigma^2)$. We assume that the $k^{th}$ relay knows the average received signal plus noise power $N(P(E_k + F_k) + \sigma^2)$
and transmits $t_k = \left( \frac{1}{N(P(E_k + F_k) + \sigma^2)} \right)^\frac{1}{2} r_k$ to ensure that $E\{t_k^*t_k\} = 1$. With this normalization, from (2) and (3), the received signal at terminal $T_1$ and $T_2$ is given by $v$ and $y$, respectively, where

$$v = \sum_{k=1}^{K} \sqrt{\frac{\gamma_k Q_k}{N(P(E_k + F_k) + \sigma^2)}} H_k^{(r)} r_k + w$$

$$y = \sum_{k=1}^{K} \sqrt{\frac{\gamma_k P_k}{N(P(E_k + F_k) + \sigma^2)}} G_k r_k + z.$$ 

Substituting for $r_k$ from (1) in the above equation

$$y = \sum_{k=1}^{K} \sqrt{\frac{\gamma_k P_k E_k}{N M(P(E_k + F_k) + \sigma^2)}} G_k H_k x$$

$$+ \sum_{k=1}^{K} \sqrt{\frac{\gamma_k P_k F_k}{N M(P(E_k + F_k) + \sigma^2)}} G_k G_k^{(r)} u$$

$$+ \sum_{k=1}^{K} \sqrt{\frac{\gamma_k P_k}{N(P(E_k + F_k) + \sigma^2)}} G_k n_k + z.$$ 

Since $T_2$ knows $u$ and has perfect CSI, it can cancel the self interference. Removing the self interference from $y$ and dividing both sides by $\sqrt{K}$,

$$y' = \frac{1}{\sqrt{K}} \sum_{k=1}^{K} \sqrt{\frac{\gamma_k P_k E_k}{N M(P(E_k + F_k) + \sigma^2)}} G_k H_k x$$

$$+ \frac{1}{\sqrt{K}} \sum_{k=1}^{K} \sqrt{\frac{\gamma_k P_k}{N(P(E_k + F_k) + \sigma^2)}} G_k n_k + \frac{1}{\sqrt{K}} z.$$ 

Similarly $T_1$ knows $x$ and also has perfect CSI, therefore it can also remove the self interference. Removing the self interference from $v$ and dividing both sides by $\sqrt{K}$

$$v' = \frac{1}{\sqrt{K}} \sum_{k=1}^{K} \sqrt{\frac{\gamma_k P_k E_k}{N M(P(E_k + F_k) + \sigma^2)}} H_k^{(r)} G_k^{(r)} u$$

$$+ \frac{1}{\sqrt{K}} \sum_{k=1}^{K} \sqrt{\frac{\gamma_k Q_k}{N(P(E_k + F_k) + \sigma^2)}} H_k^{(r)} n_k + \frac{1}{\sqrt{K}} w.$$ 

As $K \to \infty$, it can be shown that (Theorem 3 [20])

$$A_{i,j} \sim CN \left( 0, \frac{1}{K} \sum_{k=1}^{K} \mathbb{E} \left\{ \frac{\gamma_k P_k E_k}{M(P(E_k + F_k) + \sigma^2)} \right\} \right)$$
\[ C_{i,j} \sim \mathcal{CN} \left( 0, \frac{1}{K} \sum_{k=1}^{K} \mathbb{E} \left\{ \frac{\gamma_k P Q_k F_k}{M (P (E_k + F_k) + \sigma^2)} \right\} \right) \]

\[ b_i \sim \mathcal{CN} \left( 0, \frac{\sigma^2}{K} \left( \sum_{k=1}^{K} \mathbb{E} \left\{ \frac{\gamma_k P_k}{(P (E_k + F_k) + \sigma^2)} \right\} + 1 \right) \right) \]

\[ d_i \sim \mathcal{CN} \left( 0, \frac{\sigma^2}{K} \left( \sum_{k=1}^{K} \mathbb{E} \left\{ \frac{\gamma_k Q_k}{(P (E_k + F_k) + \sigma^2)} \right\} + 1 \right) \right) \]

and

\[ R_A \xrightarrow{w.p.1} \frac{1}{K} \sum_{k=1}^{K} \mathbb{E} \left\{ \frac{\gamma_k P P_k E_k}{M (P (E_k + F_k) + \sigma^2)} \right\} I_{M^2} \]

\[ R_C \xrightarrow{w.p.1} \frac{1}{K} \sum_{k=1}^{K} \mathbb{E} \left\{ \frac{\gamma_k P Q_k F_k}{M (P (E_k + F_k) + \sigma^2)} \right\} I_{M^2} \]

where \( A_{i,j}, C_{i,j} \) denotes \( i^{th} \) row and \( j^{th} \) column entry of \( A \) and \( C \) respectively and \( b_i, d_i \) denotes the \( i^{th} \) element of \( b \) and \( d \) respectively, \( R_A = \mathbb{E}\{a a^*\} \) where \( a = \text{vec}(A) \) and \( R_C = \mathbb{E}\{c c^*\} \) where \( c = \text{vec}(C) \).

This shows that the channel matrices \( A, C \) and the noise vectors \( b, d \) are i.i.d. Gaussian, therefore using results from Section 4.1 [29] with only receive CSI and no transmit CSI, the achievable rate \( R_{12} (R_{21}) \) of the \( T_1 \rightarrow T_2 \) \( (T_2 \rightarrow T_1) \) link for \( \alpha = \frac{1}{2} \), is given by

\[
\lim_{K \to \infty} R_{12} = \frac{1}{2} \mathbb{E}_{H_w} \left\{ \log \det \left( I_M + \frac{\rho_1}{M} H_w H_w^* \right) \right\}
\]

\[
\lim_{K \to \infty} R_{21} = \frac{1}{2} \mathbb{E}_{H_w} \left\{ \log \det \left( I_M + \frac{\rho_2}{M} H_w H_w^* \right) \right\}
\]

where \( H_w \) is an \( M \times M \) matrix with i.i.d. \( \mathcal{CN}(0, 1) \) entries and

\[
\rho_1 = \frac{1}{K} \sum_{k=1}^{K} \mathbb{E} \left\{ \frac{\gamma_k P P_k E_k}{(P (E_k + F_k) + \sigma^2)} \right\} + 1 \right) \right),
\]

\[
\rho_2 = \frac{1}{K} \sum_{k=1}^{K} \mathbb{E} \left\{ \frac{\gamma_k P Q_k F_k}{(P (E_k + F_k) + \sigma^2)} \right\} + 1 \right) \right).
\]

Note that \( \rho_1 \) and \( \rho_2 \) are effective SNRs. Denoting \( \mu = \mathbb{E}\{E_k\} = \mathbb{E}\{F_k\} = \mathbb{E}\{P_k\} = \mathbb{E}\{Q_k\} \forall k, \) and

\[
\frac{1}{\eta} = \mathbb{E}\left\{ \frac{E_k}{(P (E_k + F_k) + \sigma^2)} \right\} = \mathbb{E}\left\{ \frac{F_k}{(P (E_k + F_k) + \sigma^2)} \right\} \forall k,
\]

\[
\rho_1 = \rho_2 = \frac{\frac{P \mu}{\eta} \sum_{k=1}^{K} \gamma_k}{\sigma^2 \left( \frac{1}{\eta} \sum_{k=1}^{K} \gamma_k + 1 \right)}.
\]

Since the relay power is constrained by \( \sum_{k=1}^{K} \gamma_k = P_R \)

\[
\rho_1 = \rho_2 = \frac{P P_R \mu}{\sigma^2 (P R \eta_2 + \eta)}.
\]
Choosing $P \approx P_R$,
\[ \rho_1 = \rho_2 \approx \frac{P_R}{\sigma^2} \]
since $E_k, F_k, P_k, Q_k \forall k$ are bounded. Therefore
\[
\lim_{K \to \infty} R_{12} = \lim_{K \to \infty} R_{21} = \mathbb{E}_{H_w} \left\{ \log \det \left( I_M + \frac{P_R}{M\sigma^2} H_w H_w^* \right) \right\}.
\]
In high SNR regime $P \approx P_R \gg 1$, from [29], it follows that
\[
\begin{align*}
\lim_{K \to \infty} R_{12} &= \mathbb{w.p.1} \frac{M}{2} \log (P_R) + O(1) \\
\lim_{K \to \infty} R_{21} &= \mathbb{w.p.1} \frac{M}{2} \log (P_R) + O(1).
\end{align*}
\]

**Discussion:** In this section, we first obtained an upper bound on the capacity region of the non-coherent MIMO two-way relay channel using multiple access cut-set bound when CSI is only known at $T_1$ and $T_2$. Then with the help of a simple AF strategy we provided a lower bound which is a $O(1)$ term away from the upper bound in the high SNR regime. We find that, contrary to the coherent case, with the non-coherent MIMO two-way relay channel, as the number of relay nodes grow large, the capacity region expression is independent of the number of relays and no coherent combining gain (array gain) is available when there is no CSI at any relay. Similar to the coherent case, however, it turns out that even in the non-coherent case both $T_1$ and $T_2$ can simultaneously transmit at a rate which is equal to the maximum possible rate at which they could have transmitted when there is no data flowing in the opposite direction. Therefore, the non-coherent MIMO two-way relay channel creates two orthogonal channels, one from $T_1 \to T_2$ and another from $T_1 \to T_2$ with rate $\frac{M}{2} \log P_R$ achievable on each link simultaneously, thereby removing the $\frac{1}{2}$ rate loss factor because of half-duplex nodes.

The lower bound provided by Theorem 5 shows that the achievable rate for the non-coherent MIMO two-way relay channel is same as the capacity of a point to point $M \times M$ i.i.d. Gaussian channel with receive SNR $P_R$, with perfect CSI at receiver and no CSI at transmitter and where $\frac{1}{2}$ factor is due to the half-duplex requirement. This result is quite intuitive, since with absence of CSI at the relays, as $K \to \infty$ the equivalent channel between $T_1 \to T_2 \ (T_2 \to T_1)$ converges to an $M \times M$ i.i.d. Gaussian channel and therefore the result follows from [29].

Compared to (Theorem 3 [20]), this result shows that with the non-coherent MIMO two-way relay channel it is possible to remove the $\frac{1}{2}$ rate loss factor due to the half-duplex constraint and can achieve the same rate as promised by Theorem 3 [20] (for unidirectional communication), in each direction $T_1 \to T_2$ and $T_2 \to T_1$. This is again due to the fact that, with perfect CSI both $T_1$ and $T_2$ can cancel the
self interference terms their own transmitted signals generate and hence the received signal at \( T_2 \) (\( T_1 \)) when \( T_2 \) is also sending information is equivalent to the received signal at \( T_2 \) in [20], where there is no communication happening on \( T_2 \rightarrow T_1 \) link. Therefore there is a two-fold increase in achievable rate with the non-coherent MIMO two-way relay channel in comparison to [20].

VII. CONCLUSION

In this paper we developed capacity scaling laws for the MIMO two-way relay channel under coherent and non-coherent assumptions. First we upper bounded the capacity region of the coherent MIMO two-way relay channel using the broadcast and multiple access cut-set bound. Then we proposed a dual channel matching strategy to obtain an achievable rate region for the coherent MIMO two-way relay channel. The achievable rate region was shown to be a \( O(1) \) term away from the upper bound, as \( K \rightarrow \infty \). Hence we characterized the coherent MIMO two-way relay channel capacity region within a \( O(1) \) term as \( K \rightarrow \infty \).

The dual channel matching strategy we proposed for the coherent MIMO two-way relay channel is a decentralized strategy, where each relay node does not cooperate with any other relay node and only uses its CSI to coherently match the channels which the streams from \( T_1 \) and \( T_2 \) experience. An interesting outcome of our analysis is that the dual channel matching strategy, which requires no cooperation between relays, achieves the capacity region upper bound which allows for full cooperation between relays, within a \( O(1) \) term. Thus, dual channel matching not only simplifies the practical protocol design, but also achieves capacity region upper bound within a \( O(1) \) term.

For the coherent MIMO two-way relay channel, there is a strict requirement that all the nodes need to know perfect CSI, which in practice can be quite challenging and resource consuming. Therefore we also considered the case when only \( T_1 \) and \( T_2 \) have perfect receive CSI and none of the relays have any CSI, which is referred to as the non-coherent MIMO two-way relay channel. For this case we upper bounded the capacity region using only the multiple access cut-set bound and fixing \( \alpha = \frac{1}{2} \) (i.e. \( T_1 \), \( T_2 \) and all the relays transmit for equal amount of time in each time slot). Then with the help of a simple AF strategy, we showed that in the high SNR regime the upper bound is achievable within a \( O(1) \) term, and hence characterize high SNR capacity region of the non-coherent MIMO two-way relay channel. Thus, we showed that a very simple AF strategy which transmits the power normalized version of the received signal is an optimal strategy. The intuition behind this result is that by using AF with no CSI at any relay, the effective channel from the source to destination \( T_1 \rightarrow T_2 \) or \( T_2 \rightarrow T_1 \) converges to an \( M \times M \) i.i.d. MIMO Gaussian channel as \( K \rightarrow \infty \), which is similar to the effective channel considered...
for the capacity region upper bound (using the multiple access cut). The upper and lower bound differ by $O(1)$ term because of the forwarded noise from each relay node.

Compared to [19], [20], our capacity scaling results for the coherent and non-coherent MIMO two-way relay channel shows that with the MIMO two-way relay channel there is a two-fold increase in the capacity than unidirectional communication with large number of relays. Hence, the MIMO two-way relay channel helps in improving the spectral efficiency and unidirectional full-duplex performance while using half-duplex terminals.

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