Formation of Energy Features of the Image based on Wavelet Transform

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² The research was carried out within the framework of the state task No 2.1724.2017/4.6

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Abstract. In this paper the new approach of image analysis using energy characteristics of wavelet transform is proposed. We consider a model based on the wavelet transform, which makes it possible to form features characterizing the energy estimates of the image points and reflecting their significance. To form a model of energy characteristics, a procedure is performed based on taking into account the dependencies between the wavelet coefficients of various levels. This approach based on image representation using of energy features models can be used for contours detection, salient points detection and texture features extraction. Energy features models can be used as the basis to detect and recognize the object in various systems of information processing and control, such as systems of unmanned aerial vehicle control, distant exploration of the Earth systems, access control systems for protected objects etc.

1. Introduction
Computer vision systems are currently actively introduced into different spheres of human activity. These systems are based on various video information analyzing and processing technologies. Computer vision is based on the image processing procedures, which, in general, provide the solution of the following tasks:
1. Image acquisition;
2. Image transformation;
3. Features extraction;
4. Analysis of features.

The first task is to generate or enter images into the system in the form of special data structures, for example, it may be an array of scalar or vector quantities. The solution of the second task provides transformation of the image into the given form by changing their color and geometric features. For example, it may be noise elimination, rotation on the plane, reduction in size, changing the contrast. Attribute highlighting means that the image is represented by the set of characteristics which are significant from the point of view of the functioning of the system. The last task is to determine the semantic content of the image. For example, it may be forming information about objects in the images, their parameters and relationships.
The efficiency of the computer vision tools largely depends on the image representation models. The classification of image representation models exists, which includes classes of low, middle and high image presentation models [1]. The low level of image representation models class includes functional, probabilistic and hierarchical models.

Functional models describe the image by using some functions. The example of such model is the image description through the function of spatial coordinates. The arguments of this function are scalar for binary and gray-scale images or vector for color and multi-spectral images values. Probabilistic models describe images like probability processes. In this case density functions and statistic moments (mean, variance etc.) are used to describe the image. Hierarchical models represent images like a set of images of different scales. Gaussian pyramid of images is the example of the hierarchical model.

The middle level of image representation uses the specification of their specific characteristics. In this case, contour models, regions of interests, models of points of interest and models of structural elements are widely used. Construction of the image model is performed in two stages. On the first stage image segmentation is executed, when detection and marking of contours, regions of interest, points of interest, structural elements are performed. On the second stage image description is constructed as the set of feature vectors, characterizing relevant elements.

High level of image representation is based on models of explicit and implicit knowledge use. The example of the model based on implicit knowledge use is the model of patterns of images. It means that knowledge about objects is contained in the patterns of the objects. The set of rules to interpret contained in the image information is used in explicit description.

Most of methods which are used in practice to detect and recognize objects are based on implicit knowledge use models implementation. In this case the image description is based on using color, texture, shape and structural characteristics [2]. Such parameters as a color histogram [3], the vector of color connectivity [4], a color correlogram [5], color moments [6], a dominant color descriptor [7], statistical texture features [2], local binary patterns [8], spectral characteristics [9], Tamura features [10] are widely used. Features area circularity or rectangularity, a perimeter, a square, major axes orientation [2, 11], the presence in the images of certain objects and their relative position may complement this list [2].

The choice of the image characteristics impacts on the image model used in the analysis algorithms. For example, it takes place in procedures for detection and recognition objects in images. At the same time, it is necessary to take into account the conditions of system functioning in which these algorithms will be implemented. For example, real-time processing, real-time decision-making, limitations on computing resources, interference on the means of recording, processing, storage and transmission of images may be such specific conditions.

2. Materials and methods

2.1. Image wavelet transform

Nowadays, methods based on the wavelet transform are actively being developed to solve the problem of image analysis [12]. The wavelet transformation in generally can be expressed as it follows:

$$Wf (u, s) = \frac{1}{s^{D/2}} \psi^*(\frac{x-u}{s}) f(x) dx,$$

where $Wf$ is the result of wavelet transform; $f$ is the initial function; $\psi^*$ is complex conjugation of shifted and scaled function $\psi$ that has a zero average value, the center at the zero point and the unit normal; $D$ is the dimension of the signal; $u$ is $D$-dimensional vector of shift parameters; $s$ is the parameter of the scale [13].

The wavelet transform resolves the signal into basis functions:
Decomposition (2) allows revealing signal features in the local area determined by shift parameters according to some scale determined by the parameter of the scale. As the two-dimensional signal accords to the image, so the following values should be used in equality (2): $D = 2$, $x = (x_1, x_2)^T$, $u = (u_1, u_2)^T$.

Wavelet transformations represented by expression (1) are continuous. Various effective feature descriptions of images are built on their basis. For example, they may be contour and textural [14-17] features. However, in practice, using of them implies the approximate nature of the results and relatively slow computational procedure.

For digital signals, which include digital images, a multiple discrete orthogonal-scale wavelet transform can be applied. They are free of these shortcomings. They are based on using a discrete function $f(x)$ that describes the signal by the sum (3).

$$f(x) = f_a(x) + f_d(x),$$

where $f_a(x)$ is the approximating part of the initial function and $f_d(x)$ is the detailing part of the initial function. The process of transformation of the function $f(x)$ can be expressed in iterative manner that defines it at different levels of decomposition:

$$f(x) = f^{J}_{a}(x),$$

$$f^{j}_{a}(x) = f^{j-1}_{a}(x) + f^{j-1}_{d}(x),$$

where $J$ is the number of decomposition levels; $j = J, \ldots, j_0 + 1$. The result of a discrete wavelet transforms looks like the set of approximation coefficients $\{a_{j,l}\}$ and the set of detailing coefficients $\{d_{j,l}\}$. The scale wavelet transform for the one-dimensional function $f(x)$ allows getting the following representation:

$$f(x) = \sum_{l=0}^{2^{j_0}-1} a_{j_0,l} \varphi_{j_0,l}(x) + \sum_{j=j_0}^{J} \sum_{l=0}^{2^{j}-1} d_{j,l} \psi_{j,l}(x),$$

where $\varphi_{j,l}(x)$ is the copy of the scaling function $\varphi(x)$, which is shifted on the axis of abscissas and extended on the axis of ordinates, and $\psi_{j,l}(x)$ is the copy of the wavelet function $\psi(x)$, which is shifted on the axis of abscissas and extended on the axis of ordinates. Scaling function $\varphi(x)$ and wavelet function $\psi(x)$ define the wavelet basis. Copies of these functions that used to decompose the source signal function into a series, are determined by means (7) and (8).

$$\varphi_{j,l}(x) = \sqrt{2^{j-l}} \varphi(2^{j-l} x - l),$$

$$\psi_{j,l}(x) = \sqrt{2^{j-l}} \psi(2^{j-l} x - l).$$

One of the advantages of scale wavelet transformations is separability. This means that the wavelet transform of single-channel images (e.g. grayscale images) can be performed in two stages. At first, transform is executed for rows, and then transform is executed for columns or vice versa. Results of transformation at the $j$-th level are grouped into the matrix of approximation coefficients $[LL_{j,m,n}]$, the matrix of detailing horizontal coefficients $[HL_{j,m,n}]$, the matrix of vertical coefficients $[HL_{j,m,n}]$ and the matrix of diagonal coefficients $[HH_{j,m,n}]$, where $m, n = 0, \ldots, 2^j - 1$. Example of the wavelet transform is shown in the figure 1. For multiband images, such as color images, each channel is transformed separately.
2.2. Energy features model

To analyze images on the base of discrete orthonormal wavelet transform energy characteristics is effectively used. They are constructed on the basis of the following equality, which is analogue of the well-known Parseval equality (9).

$$
\sum_{k=0}^{N-1} \sum_{l=0}^{N-1} f_{k,l}^2 = \sum_{m=0}^{2^J-1} \sum_{n=0}^{2^J-1} LL^2_{j_0,m,n} + \sum_{j=j_0}^{J-1} \sum_{m=0}^{2^j-1} \sum_{n=0}^{2^j-1} LH^2_{j,m,n} + \sum_{j=j_0}^{J-1} \sum_{m=0}^{2^j-1} \sum_{n=0}^{2^j-1} HL^2_{j,m,n} + \sum_{j=j_0}^{J-1} \sum_{m=0}^{2^j-1} \sum_{n=0}^{2^j-1} HH^2_{j,m,n},
$$

(9)

where $f_{k,l}$ is the values of the discrete function that describes the image as a matrix $N \times N$; $LL_{j,m,n}$, $LH_{j,m,n}$, $HL_{j,m,n}$, $HH_{j,m,n}$ are coefficients of the wavelet transform.

![Wavelet transform](image)

**Figure 1.** Wavelet transform.

It is possible to get energetic estimation from (9) for each point of the image using the following algorithm.

1. Execute transform until the level $j_0$.
2. Assume:

$$
w^2_{j_0,m,n} = LL^2_{j_0,m,n};
$$

(10)

3. Consequently for $j = j_0, \ldots, J - 1$, where $J = \log_2 N$, $m = 0, 1, \ldots, 2^{j+1} - 1$, $n = 0, 1, \ldots, 2^{j+1} - 1$ execute energy estimation:

$$
w^2_{j,m,n} = \frac{1}{4} w^2_{j-1,m,n} + LH^2_{j,m/2,n/2} + HL^2_{j,m/2,n/2} + HH^2_{j,m/2,n/2}.
$$

(11)

The calculated values retain the energy equality according to expression (12).
where \( w_{k,j}^2 = w_{j-1,k,j}^2 \). All of values of sets \( \{w_{k,j}^2\}_k=0^N \) or \( \{w_{k,j}^2\}_k=0^N \) can be a weight corresponding to the pixel, which evaluates its contribution to the total energy of the image.

The algorithm described above can be presented in a more generally by introducing tuning coefficients as it follows.

1. Execute transform until the level \( j_0 \);
2. Assume:
   \[
   w_{j-1,m,n}^2 = K'_{j_0} \cdot LL_{j_0,m,n}^2; \tag{13}
   \]
3. Consequently for \( j = j_0, \ldots, J - 1 \), where \( J = \log_2 N \), \( m = 0, 1, \ldots, 2^{j_1} - 1 \), \( n = 0, 1, \ldots, 2^{j_1} - 1 \) execute energy estimation:
   \[
   w_{j,m,n}^2 = K'_{j} w_{j-1,m,n}^2 + K''_{j} \times \left[ LH_{j,m/2,n/2}^2 + HL_{j,m/2,n/2}^2 + HH_{j,m/2,n/2}^2 \right]. \tag{14}
   \]

This approach allows obtaining different modifications of the procedure for executing energy features using additional heuristics. In addition, it allows describing the algorithm based not only on orthonormal wavelet transformations. For example, in the case of an orthogonal (non-orthonormal) Haar-transform of images according values are evaluated (15) – (18).

\[
LL_{j-1,m,n} = \frac{LL_{j,2m,2n} + LL_{j,2m+1,2n} + LL_{j,2m,2n+1} + LL_{j,2m+1,2n+1}}{4}, \tag{15}
\]
\[
LH_{j-1,m,n} = \frac{LL_{j,2m,2n} + LL_{j,2m+1,2n} - LL_{j,2m,2n+1} - LL_{j,2m+1,2n+1}}{4}, \tag{16}
\]
\[
HL_{j-1,m,n} = \frac{LL_{j,2m,2n} - LL_{j,2m+1,2n} + LL_{j,2m,2n+1} - LL_{j,2m+1,2n+1}}{4}, \tag{17}
\]
\[
HH_{j-1,m,n} = \frac{LL_{j,2m,2n} - LL_{j,2m+1,2n} - LL_{j,2m,2n+1} + LL_{j,2m+1,2n+1}}{4}. \tag{18}
\]

To achieve energy equality for the orthogonal Haar-transform (15) – (18), the following steps must be performed.

1. Execute the orthogonal Haar-transform until the level \( j_0 \);
2. Assume:
   \[
   w_{j-1,m,n}^2 = 2^j \cdot LL_{j_0,m,n}^2; \tag{19}
   \]
3. Consequently for \( j = j_0, \ldots, J - 1 \), where \( J = \log_2 N \), \( m = 0, 1, \ldots, 2^{j_1} - 1 \), \( n = 0, 1, \ldots, 2^{j_1} - 1 \) execute energy estimation:
   \[
   w_{j,m,n}^2 = \frac{1}{4} w_{j-1,m,n}^2 + 2^{(j-j)} \left[ LH_{j,m/2,n/2}^2 + HL_{j,m/2,n/2}^2 + HH_{j,m/2,n/2}^2 \right]. \tag{20}
   \]

As the result of the described approach it is possible to obtain images in which energy weights will be associated with pixels instead of values of intensities.
3. Results and discussion

3.1. Edge detection

As the example, the task of edge detection in the image is considered below. It is assumed that the weights of the boundary points should have greater values than the weights of the internal points of the regions in the proposed solution based on formation of energy features. The value of the average brightness of the image is not significant. To get boundaries, perform the image transformation specified by the expressions (13) and (14), where

\[
K'_{j-1} = 0, K'_{j_0} = K'_{j_0+1} = \ldots = K'_{j-1} = \frac{1}{4}, K''_{j_0} = 1, K''_{j+1} = 2K''_j, j = j_0, \ldots, J - 2. \tag{21}
\]

Examples of edge detection in test images are shown in figure 2. The obtained weight images can be further processed to form contours, connect them and describe. This approach can be used as the basis to detect and recognize the object in various systems of information processing and control, such as systems of unmanned aerial vehicle control, distant exploration of the Earth systems, access control systems for protected objects etc.

![Figure 2. Edge detection.](image)
3.2. Texture analysis

The proposed energetic model can also be used to improve the resistance to noise in textured image analysis, for example, using statistical characteristics which allow describing the texture as smooth, coarse, grainy etc [18]. Statistical texture features of the image are calculated by the histogram of its intensity, which is considered as a random variable. The following statistical characteristics are simple and, at the same time, effective:

- mean (the first raw moment) that characterizes the average intensity of the image;
- variance (the second central moment) that characterizes the scattering intensity on the image (the standard deviation, whose value is the square root of the variance, is often used instead of the variance);
- the third central moment that characterizes symmetry distribution of intensity about mean (skewness coefficient is used often as the symmetry characteristic);
- the fourth central moment that characterizes the sharpness of the peak of intensity distribution (the kurtosis is often used as the sharpness of the peak of intensity distribution);
- statistical moments of higher orders, for example, fifth and sixth central moments, which allow to provide even more detailed textural description of images.

Statistical moments are computed according to the following formulas (22) and (23).

\[ m = \sum_{i=1}^{L} I_i p(I_i), \]  
\[ \mu_n = \sum_{i=1}^{L} (I_i - m)^n p(I_i), \]

where \( m \) is mean; \( \mu_n \) is the central moment of the nth order; \( L \) is the number of levels of intensity in the image; \( I_i \) is the value of intensity of the level \( i \) and \( p(I_i) \) estimates the probability of intensity of the level \( i \) that is calculated from intensity histogram.

Skewness \( A_s \) and kurtosis \( E_s \) are calculated by formulas (24) and (25).

\[ A_s = \frac{\mu_3}{\sigma^3}, \]  
\[ E_s = \frac{\mu_4}{\sigma^4} - 3, \]

where \( \sigma \) is standard deviation.

The value of smoothness \( R \), homogeneity \( U \) and entropy \( H \) are also used:

\[ R = 1 - \frac{1}{1 + \sigma^2}, \]  
\[ U = \sum_{i=1}^{L} p^2(I_i), \]  
\[ H = \sum_{i=1}^{L} p(I_i) \log_a \frac{1}{p(I_i)}. \]

Here square of the standard deviation \( \sigma^2 \) is the variance \( \mu_2 \) and to calculate the value of entropy binary, decimal or natural logarithms can be used. In general case any base of logarithm \( a \) can be chosen. Smoothness \( R \) takes value nearly to 0 if intensity of pixels is approximately equal. \( R \) takes value nearly to 1 if significant scattering value of intensity of pixels takes place. Homogeneity \( U \) will have maximal
value in case of the similar intensities in image and decrease if their differences grow. Entropy $H$ is maximal if distribution intensities is uniformed and minimal if image is homogeneous.

Table 1 shows values of considered features that are calculated for the texture images $D1 – D6$ from well-known base Brodatz which are shown on the figure 3 (images are given in size $512 \times 512$). Intensities of pixels of the images have values of range from 0 to 255, i.e. the number of intensity levels in images is equal 256.

In practice it is necessary to take into account the effect of noise. The values of the textural features of noisy versions of the images are shown in table 2 as the example. In this case normally distributed noise with zero mean and standard deviation equal to one-tenth of the maximum intensity is considered. The absolute values of the resulting errors of the texture features are shown in table 3.

It is possible to increase the resistance to noise through use of energetic weights of image points. The table 4 shows the root-mean-square differences between the original images and their noisy versions with the noise parameters given above. It can be seen that the RMSE for the images in the original form is greater than the RMSE for the images represented by the energy features. The notation $I$ and $W$ are used in the table 4 to denote the image attributes of the pixels which are the intensity and weight respectively. The procedure on the basis of expressions (15) – (20) was used to evaluate energetic weights.

![Figure 3. Texture images: D1 (a), D2 (b), D3 (c), D4 (d), D5 (e), D6 (f).](image-url)
Table 1. Texture features of images.

| Image | $m$   | $\sigma$ | $A_s$ | $E_s$ | $R$   | $U$  | $H$   |
|-------|-------|----------|-------|-------|-------|------|-------|
| D1    | 164.915 | 43.565   | -1.364 | 2.133 | 0.9994 | 0.0088 | 4.958 |
| D2    | 143.126 | 58.889   | -0.274 | -1.096 | 0.9997 | 0.0055 | 5.299 |
| D3    | 163.174 | 64.207   | -0.866 | -0.528 | 0.9997 | 0.0085 | 5.136 |
| D4    | 118.156 | 56.707   | 0.223  | -0.931 | 0.9996 | 0.0049 | 5.374 |
| D5    | 136.208 | 62.489   | -0.206 | -1.181 | 0.9997 | 0.0049 | 5.379 |
| D6    | 51.673  | 39.301   | 2.248  | 5.281  | 0.9993 | 0.0178 | 4.512 |

Table 2. Texture features of noisy images.

| Image | $M$   | $\sigma$ | $A_s$ | $E_s$ | $R$   | $U$  | $H$   |
|-------|-------|----------|-------|-------|-------|------|-------|
| D1    | 164.385 | 49.606   | -0.886 | 0.997 | 0.9995 | 0.0065 | 5.201 |
| D2    | 142.601 | 63.342   | -0.213 | -0.891 | 0.9997 | 0.0046 | 5.446 |
| D3    | 162.515 | 67.877   | -0.705 | -0.531 | 0.9997 | 0.0061 | 5.348 |
| D4    | 117.777 | 61.329   | 0.191  | -0.767 | 0.9997 | 0.0047 | 5.436 |
| D5    | 135.757 | 66.548   | -0.156 | -0.986 | 0.9997 | 0.0045 | 5.468 |
| D6    | 52.221  | 45.011   | 1.537  | 2.291  | 0.9995 | 0.0146 | 4.816 |

Table 3. Errors of texture features.

| Image | $\Delta m$ | $\Delta \sigma$ | $\Delta A_s$ | $\Delta E_s$ | $\Delta R$ | $\Delta U$ | $\Delta H$ |
|-------|------------|------------------|--------------|--------------|------------|------------|-----------|
| D1    | 0.53       | 6.04             | 0.48         | 1.136        | 0.0001     | 0.0023     | 0.243     |
| D2    | 0.53       | 4.45             | 0.06         | 0.205        | 0.0000     | 0.0009     | 0.147     |
| D3    | 0.66       | 3.67             | 0.16         | 0.003        | 0.0000     | 0.0024     | 0.212     |
| D4    | 0.38       | 4.62             | 0.03         | 0.164        | 0.0001     | 0.0002     | 0.062     |
| D5    | 0.45       | 4.06             | 0.05         | 0.195        | 0.0000     | 0.0004     | 0.089     |
| D6    | 0.55       | 5.71             | 0.71         | 2.361        | 0.0002     | 0.0032     | 0.304     |

Table 4. RMSE for original and noisy images.

| Image | D1       | D2       | D3       | D4       | D5       | D6       |
|-------|----------|----------|----------|----------|----------|----------|
| I     | 12595.4  | 12526.3  | 12299.6  | 12548.6  | 12473.8  | 11914.9  |
| W     | 4060.9   | 4413.1   | 4938.1   | 6154.9   | 4489.2   | 8403.5   |

3.3. Key points of images

One of the actively being developed approaches for image analysis in computer vision systems is based on using of singular points or key points. The image analysis techniques based on the key points involve appropriate detectors and descriptors. The detector is used to find singular points on the images and the descriptor is used to describe them. Currently there are many detectors and descriptors.
used. For example, they are the Moravec detector, the Harris detector, SUSAN, SIFT, SURF, FAST, BRIEF, ORB, GLOH, FREAK and BRISK.

The energy model can also be used to construct the image key point detector. Search of key points on the basis of this model is following.

1. Transform the image into a square view where the number of rows and columns is a multiple of 2.
2. Calculate the weight of the points in the image.
3. Set a threshold.
4. Determine the points with weights not less than the threshold value.
5. Select a specified number from the found points, which in this case are the key points.

The given algorithm allows analyzing the position of points with the highest energy weights of the wavelet transform on the image. Change of scale and shift do not affect the nature of energy distribution in a wide range. Therefore, the considered algorithm is invariant relative to the scale and shift. However, they are not invariant relative to turn. Direct use of the obtained weights in its vicinity is impossible because their values depend on the representation scale. However, it is possible to estimate the distribution of the weights. Figure 4 shows the result of highlighting, describing and matching the key points of two images obtained using the described approach (images are taken from the site http://sipi.usc.edu/database/).

![Figure 4. Key points matching.](image)

4. Conclusion

It should be noted in conclusion, that the described approach to forming and using energy features of images based on wavelet transform can be applied to analyze images in different systems based on the methods and tools of computer vision [19 – 21].

This approach based on the procedure of transform original image to a form in which different points will have different weights, characterizing their contribution to the overall energy of the image. Since the image points of different scales are interrelated, it is possible to obtain characteristics that take into account the significance of points on all considered scales. This reasoning can be used as a basis for the procedure of detecting and analyzing contours in an image. If we consider the distribution of the weights of image points, we can obtain a description of the texture that characterizes its different regions. In addition to contours and regions, the analysis of key points is often used. In this
case, by key points we mean the points with the largest weights in the neighborhoods of the given dimensions.

Thus this approach can be used for contours detection, salient points detection and texture features extraction in the images.

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