HEAVY-TO-LIGHT MESON TRANSITIONS IN QCD

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Abstract

I discuss QCD sum rules determinations of the form factors governing the decay $B \to \pi(\rho)\ell\nu$. For some of these form factors the computed dependence on the momentum transferred does not agree with the expectation from the nearest pole dominance hypothesis. Relations are observed among the form factors, that seem to be compatible with equations recently derived by B.Stech. The measurement of a number of color suppressed nonleptonic B decay rates could shed light on the accuracy of the calculation of these form factors and on the factorization approximation.

Talk presented at the
6th International Symposium on Heavy Flavour Physics
Pisa, Italy, June 6 - 10, 1995
1. Form factors of heavy-to-light meson transitions

The exclusive semileptonic $B$ decays to $\pi$ and $\rho$ play a prime role in the measurement of $V_{ub}$. Let us consider, for example, the spectrum of $\bar{B}^0 \to \pi^+ \mu^- \bar{\nu}$:

$$\frac{d\Gamma(\bar{B}^0 \to \pi^+ \mu^- \bar{\nu})}{dq^2} = \frac{G_F^2}{24\pi^3} |V_{ub}|^2 |F_1^{B\pi}(q^2)|^2 |\vec{p}_\pi(q^2)|^3$$

(1)

where $\vec{p}_\pi(q^2)$ is the pion three-momentum (at fixed $q^2$) in the $B$ meson rest frame. It is clear that a measurement of $\text{eq.}(5)$ is the Callan-Treiman relation valid in the chiral limit.

It is related to $F_V$ where

$$\text{eq.}(4)$$

describes the dominance of the $B$ mass limit at the point of zero recoil ($q^2 = 0$). For example, when $m_b \to \infty$ the following relations can be worked out for $F_1$ and $F_0$:

$$F_1^{B\pi}(q^{2}_{\text{max}}) \simeq \frac{g}{2f_\pi} \frac{\hat{F}}{\Delta + M_\pi} \sqrt{M_B}$$

(4)

$$F_0^{B\pi}(q^{2}_{\text{max}}) \simeq - \frac{\hat{F}}{f_\pi} \frac{1}{\sqrt{M_B}} + \mathcal{O}(M_\pi);$$

(5)

eq.(4) describes the dominance of the $B^*$ pole for $F_1$ at zero recoil (in the limit $m_b \to \infty$) $\hat{F}$ is related to $f_B$ and $f_{B^*}$, $g$ is the rescaled $B^*B\pi$ strong coupling, and $\Delta = M_{B^*} - M_B$; eq.(5) is the Callan-Treiman relation valid in the chiral limit.

The above scaling relations, that could be used, e.g., to relate $B \to \pi\ell\nu$ to $D \to \pi\ell\nu$ at zero recoil, are not sufficient to describe the form factors in the physical range of
transferred momentum; therefore, a dynamical calculation based on QCD is required for
$F_i(q^2)$, $V(q^2)$ and $A_i(q^2)$.

The method of QCD sum rules \[2\] is a fully relativistic field-theoretical approach in-
corporating fundamental features of QCD, such as perturbative asymptotic freedom and
nonperturbative quark and gluon condensation. This method allows us, by analyzing
three-point correlators of quark currents, to compute the form factors from zero to quite
large values of $q^2$; in this respect, the method complements lattice QCD, where B meson
form factors, extrapolated from the charm quark mass, are computed in the region near
$q^2_{\text{max}}$ \[3, 4, 5\].

Several QCD sum rules calculations of $F_1^{B\pi}$ can be found in the literature \[6\]; a calculation
of both $F_1$ and $F_0$ has been performed in ref.\[7\] in the limit $m_b \to \infty$. The result
for $F_1^{B\pi}(q^2)$, depicted in fig.1 and common to other QCD sum rules analyses, supports
the simple pole model: $F_1^{B\pi}(q^2) = [0.24 \pm 0.02]/(1 - \frac{q^2}{M_{B^*}^2})$; on the other hand $F_0^{B\pi}(q^2)$
increases slowly with $q^2$. The feature of $F_0$ of being nearly independent of $q^2$ has been
confirmed by a calculation, at finite $m_b$, in the channel $B \to K$ (fig.2a) \[8\].

Fig.1: Form factors $F_1^{B\pi}$ (continuous line) and $F_0^{B\pi}$ (dashed line).

The computed $q^2$ dependence of $F_0^{B\pi}(q^2)$ must be compared with the expectation based
on the hypothesis of the dominance of the nearest singularity in the $t-$ channel, assumed
in a number of models \[9\]: the nearest pole contributing to $F_0^{B\pi(BK)}$ is the $0^+ b\bar{u}$ ($b\bar{s}$) state
with mass in a range near 6 GeV (in the BSW model the value $M_{(b\bar{s})}(0^+) = 5.89 \text{ GeV}$
is used); on the other hand, a fit of the obtained $F_0^{B\pi}(q^2)$ and $F_0^{BK}(q^2)$ to a simple pole formula can be performed provided that $M_P \geq 7 - 7.5 \text{ GeV}$.

This different $q^2$ behavior of $F_1^{B\pi}(q^2)$ and $F_0^{B\pi}(q^2)$ has been observed also in lattice QCD \cite{1,2}. Moreover, a different functional dependence is expected if one considers that the scaling laws with the heavy mass in eqs.(4,5) are compatible with the relation $F_1(0) = F_0(0)$ if the $q^2$ dependence is, e.g., of the type: $F_i(q^2) = F_i(0)/(1 - \frac{q^2}{M_i^2})^{n_i}$, with $n_1 = n_0 + 1$.

Deviations from the single pole model have been observed also for the axial form factors $A_1^{B\rho}$ and $A_2^{B\rho}$, that turn out to be rather flat in $q^2$ (see the first article in ref.\cite{3}); on the other hand, $V^{B\rho}$ can be fitted with a polar formula, the pole given by $B^*$. As for the last form factor in eq.(3), $A_0$, the calculation both in the channels $B \to \rho$ and $B \to K^*$ \cite{8,11} shows that it also increases like a pole, with the pole mass compatible with the mass of $B$ (or $B_s$) as expected by the nearest-resonance dominance hypothesis (fig.2b).

Interesting enough, the relation $A_0(0) \simeq F_0(0)$ is obtained.

To summarize the results from QCD Sum rules analyses, the following scenario emerges for the transitions $B \to \pi, \rho$ ( $B \to K, K^*$): $F_1$, $V$ and $A_0$ following a polar dependence in $q^2$, $F_0$, $A_1$ and $A_2$ rather flat in $q^2$, $A_0(0) = F_0(0) = F_1(0)$. It is worth reminding that such results are obtained after an involved analytic and numerical analysis, independent for each one of the above form factors.

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Fig.2: Form factors $F_0^{BK}$ (a) and $A_0^{BK^*}$ (b).

\footnote{1 A steeper increase of $V$ compared to $A_1$ is obtained also in ref.\cite{10}, but the slopes are different with respect to Ball’s results in \cite{6}.}
One could wonder whether QCD sum rules results suggest the existence of relations among the form factors governing the transitions of heavy mesons to light mesons. For semileptonic decays where both the initial and the final meson contains one heavy quark, such relations can be derived in the limit \( m_Q \to \infty \): they connect the six form factors as in (2,3) to the Isgur-Wise function \([12]\) incorporating the nonperturbative dynamics of the light degrees of freedom. It is intriguing that relations among heavy-to-light form factors have been obtained in a constituent quark model by B.Stech \([13]\), assuming that the spectator particle retains its momentum and spin before the hadronization, and that in the rest frame of the hadron the constituent quarks have the off-shell energy close to the constituent mass. Under these hypotheses the following equations can be written for \( B \to \pi, \rho \) form factors:

\[
F_0(q^2) = \left( 1 - \frac{q^2}{m_B^2 - m_\pi^2} \right) F_1(q^2) \tag{6}
\]

\[
V(q^2) = \left( 1 + \frac{m_\rho}{m_B} \right) F_1(q^2) \tag{7}
\]

\[
A_1(q^2) = \frac{1 + \frac{m_2^2}{m_B^2}}{1 + \frac{m_2^2}{m_B^2}} \left( 1 - \frac{q^2}{m_B^2 + m_\rho^2} \right) F_1(q^2) \tag{8}
\]

\[
A_2(q^2) = \left( 1 + \frac{m_\rho}{m_B} \right) \left( 1 - \frac{2m_\rho/(m_B + m_\rho)}{1 - q^2/(m_B + m_\rho)^2} \right) F_1(q^2) \tag{9}
\]

\[
A_0(q^2) = F_1(q^2). \tag{10}
\]

The above relations are very similar to the relations holding for heavy-to-heavy transitions, e.g. \( B \to D, D^* \). QCD sum rules results seem to confirm them. As a matter of fact, for a polar dependence of \( F_1(q^2) \) the above relations suggest that both \( F_0 \) and \( A_1 \) should be nearly constant in \( q^2 \), and that \( A_0 \) should be equal to \( F_1 \).

Of course, more work is needed to put equations (6-10) on the same theoretical grounds of the relations among the form factors of heavy-to-heavy transitions.

2. Tests of factorization for color suppressed B decays

Semileptonic form factors are useful not only to predict semileptonic BR’s, but also to compute nonleptonic two-body decay rates if the factorization approximation is adopted. A dependence of \( F_1 \) on the mass of the final particle has to be taken into account since there is no spin symmetry in the final state.
In particular, for color suppressed transitions $B \to K^{(*)} J/\Psi$ and $B \to K^{(*)} \eta_c$, only the heavy-to-light form factors are needed. The decays $B \to K^{(*)} J/\Psi$ have been analyzed in [15] to constrain the semileptonic $B \to K^*, K$ form factors using data on the longitudinal polarization of the final particles in the decay $B \to K^* J/\Psi$: $\rho_L = \Gamma(B \to K^* J/\Psi)_{LL}/\Gamma(B \to K^* J/\Psi) = 0.84 \pm 0.06 \pm 0.08$, and on the ratio $R_{J/\Psi} = \Gamma(B \to K^* J/\Psi)/\Gamma(B \to K J/\Psi) = 1.64 \pm 0.34$ [14]. In the same spirit, the decays $B \to K^{(*)} \eta_c$ are interesting since they could help in testing the factorization scheme and the accuracy of the computed hadronic quantities [8].

To predict the decay rates $B \to K^{(*)} \eta_c$, besides $F_{BK}^{0K}(q^2)$ and $A_{BK}^{*0}(q^2)$ we need the leptonic constant $f_{\eta_c}$. Together with $f_{\eta'_c}, f_{J/\Psi}$ can be obtained by QCD sum rules considering the two-point function

$$ \psi_5(q) = i \int d^4x \, e^{iqx} < 0|T(\partial^\mu A_\mu(x)\partial^\nu A_\nu(0))|0 >$$

(11) 

$$(\partial^\mu A_\mu(x) = 2m_c : \bar{c}(x)i\gamma_5 c(x) : )$$

that is known in perturbative QCD to two-loop order, including also the leading $D = 4$ non-perturbative term in the Operator Product Expansion. Exploiting two different types of QCD sum rules, viz. Hilbert transforms at $Q^2 = 0$, and Laplace transforms, we get [8]:

$$ f_{\eta_c} \simeq 301 - 326 \text{ MeV}, \quad f_{\eta'_c} \simeq 231 - 255 \text{ MeV}, \quad (12) $$

$$ f_{\eta_c} \simeq 292 - 310 \text{ MeV}, \quad f_{\eta'_c} \simeq 247 - 269 \text{ MeV}, \quad (13) $$

respectively. These results have been obtained by varying the parameters in the ranges dictated by the gluon condensate and quark-mass analyses, and using $m_c = 1.46 \pm 0.07$ GeV, $\Lambda_{QCD} = 200 - 300$ MeV, with the constraint that $M_{\eta_c}$ and $M_{\eta'_c} = 3595 \pm 5$ MeV are correctly reproduced by the sum rules. Combining the predictions from the Hilbert and Laplace method we obtain:

$$ f_{\eta_c} = 309 \pm 17 \text{ MeV}, \quad f_{\eta'_c} = 250 \pm 19 \text{ MeV}, \quad \frac{f_{\eta'_c}}{f_{\eta_c}} = 0.8 \pm 0.1, \quad (14) $$

$$ \frac{f_{\eta_c}}{f_{J/\Psi}} = 0.81 \pm 0.05, \quad \frac{f_{\eta'_c}}{f_{\Psi'}} = 0.88 \pm 0.08. \quad (15) $$

In (15) the experimental values: $f_{J/\Psi} = 384 \pm 14 \text{ MeV}$ and $f_{\Psi'} = 282 \pm 14 \text{ MeV}$ have been used. In the constituent quark model the leptonic constants of the charmonium system can be expressed in terms of the $c\bar{c}$ wave function at the origin $\Psi(0)$:

$$ f^2_{\eta_c} = 48 \frac{m_c^2}{M_{\eta_c}^3} |\Psi(0)|^2, \quad f^2_{J/\Psi} = 12 \frac{1}{M_{J/\Psi}} |\Psi(0)|^2 ; \quad (16) $$

$$ 5 $$
therefore, the ratio $f_{\eta_c}/f_{J/\Psi}$ can be predicted in terms of the meson masses and of the charm quark mass:

$$\frac{f_{\eta_c}}{f_{J/\Psi}} = 2m_c \left(\frac{M_{J/\Psi}}{M_{\eta_c}^3}\right)^{\frac{1}{2}} = 0.97 \pm 0.03; \tag{17}$$

the deviations from the outcome of QCD sum rules, at the level of $15 - 20\%$ for $\eta_c$, $J/\Psi$ and $5 - 8\%$ for the radial excitations $\eta'_c$, $\Psi'$, can be attributed to relativistic and radiative corrections to the constituent quark model formula.

Tests of factorization can be performed by analyzing ratios of decay widths, such as $B \to K^{(*)}\eta_c$ and $B \to K^{(*)}\eta'_c$, where the dependence on the Wilson coefficients in the effective hamiltonian governing the decays, and on other weak parameters drops out. Let us consider, e.g., the ratio:

$$\tilde{R}_K = \frac{\Gamma(B^{-} \to K^{-}\eta'_c)}{\Gamma(B^{-} \to K^{-}\eta_c)} = 0.771 \left(\frac{f_{\eta'_c}}{f_{\eta_c}}\right)^2 \left(\frac{F_0(M_{\eta'_c}^2)}{F_0(M_{\eta_c}^2)}\right)^2 = 0.60 \pm 0.15. \tag{18}$$

The interesting point is that, because of the flat shape of $F_0(\eta)/F_0(\eta'_c)$ (fig.2a), $\tilde{R}_K$ mainly depends on the ratio of the leptonic constants, so that in factorization approximation a measurement of $\tilde{R}_K$ would provide us with interesting information on $f_{\eta'_c}/f_{\eta_c}$; and complement our knowledge of the $c\bar{c}$ wavefunction. The analogous ratio for the decays into $K^*$ is given by

$$\tilde{R}_{K^*} = \frac{\Gamma(B^{-} \to K^{*-}\eta'_c)}{\Gamma(B^{-} \to K^{*-}\eta_c)} = 0.381 \left(\frac{f_{\eta'_c}}{f_{\eta_c}}\right)^2 \left(\frac{A_0(M_{\eta'_c}^2)}{A_0(M_{\eta_c}^2)}\right)^2 = 0.381 \left(\frac{f_{\eta'_c}}{f_{\eta_c}}\right)^2 (1.4 \pm 0.2)^2. \tag{19}$$

Here, the ratio of the form factors deviates from unity due to the $q^2$-dependence of $A_0$ (fig.2b). The prediction from (19) would be: $\tilde{R}_{K^*} = 0.45 \pm 0.16$. Moreover, the quantity $\sqrt{\tilde{R}_{K^*}/\tilde{R}_K}$ is sensitive to the $q^2$-dependence of the ratio $A_0/F_0$:

$$1.42 \sqrt{\frac{\tilde{R}_{K^*}}{\tilde{R}_K}} = \left(\frac{A_0(M_{\eta'_c}^2)}{A_0(M_{\eta_c}^2)}\right)/\left(\frac{F_0(M_{\eta'_c}^2)}{F_0(M_{\eta_c}^2)}\right). \tag{20}$$

We also get:

$$R_{\eta_c} = \frac{\Gamma(B^{-} \to K^{*-}\eta_c)}{\Gamma(B^{-} \to K^{-}\eta_c)} = 0.373 \left(\frac{A_0(M_{\eta_c}^2)}{F_0(M_{\eta_c}^2)}\right)^2 = 0.73 \pm 0.13 \tag{21}$$

and $R_{\eta'_c} = 0.56 \pm 0.12$ for the analogous ratio $R_{\eta'_c}$. Finally, the ratio:

$$R_K = \frac{\Gamma(B^{-} \to K^{-} J/\Psi)}{\Gamma(B^{-} \to K^{-} J/\Psi)} = 2.519 \left(\frac{f_{\eta_c}}{f_{J/\Psi}}\right)^2 \left(\frac{F_0(M_{\eta_c}^2)}{F_1(M_{J/\Psi}^2)}\right)^2 \tag{22}$$
can be predicted using the simple pole model for $F_1^{BK}$. We obtain: $R_K = 0.94 \pm 0.25$, and, for $R_K' = \frac{\Gamma(B^- \to K^−J/Ψ)}{\Gamma(B^- \to K^−Ψ)}$: $R_K' = 1.61 \pm 0.53$. Using the CLEOII measurements [14]: $\mathcal{B}(B^- \to K^-J/Ψ) = (0.11 \pm 0.01 \pm 0.01) \times 10^{-2}$ and $\mathcal{B}(B^- \to K^-Ψ) = (0.06 \pm 0.02 \pm 0.01) \times 10^{-2}$ we expect: $\mathcal{B}(B^- \to K^-η_c) = (0.11 \pm 0.03) \times 10^{-2}$ and $\mathcal{B}(B^- \to K^-η'_c) = (0.10 \pm 0.05) \times 10^{-2}$, that should be within reach of present experimental facilities. The measurement of some of the above decay rates could shed more light on the problem of factorization, which is a basic assumption in the present analysis of heavy meson nonleptonic decays.

Acknowledgments. It is a pleasure to thank F.De Fazio, C.A.Dominguez, G.Nardulli, N.Paver and P.Santorelli for their collaboration on the subjects discussed here.

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