The break of shielding current at pulsed field magnetization of a superconducting annulus (experiment and model simulation)

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Abstract

During the pulsed field magnetization of a high-\(T_c\) annulus in liquid nitrogen the shielding current drops abruptly, providing rapid penetration of the magnetic flux into the hole of the superconductor. After the break of current the trapped field in the hole is small and negative although the body of the annulus remains highly magnetized. In the present work the current breaking effect is investigated both experimentally and numerically. The influence of the pulse parameter on the shielding current evolution during the break is researched. A simple model for the qualitative description of this process is proposed. The model shows the development of a high resistive channel with temperature near to \(T_c\). The appearance of this hot channel leads to the rapid reduction of the shielding current and presents a new scenario of flux jump at high temperature.

Keywords: pulsed field magnetization, bulk superconductors, flux dynamics, flux jumps

(Some figures may appear in colour only in the online journal)

Introduction

Hard superconductors of type-II have thermomagnetic instability at low temperatures. This instability manifests as avalanche-like penetrations of the magnetic field inside the superconductor. These are so-called flux jumps, appearing in the isothermal conditions, at any low perturbation. The existing theory of thermomagnetic instability at low temperature describes the experimental results well enough [1, and references in 2, 3]. The criteria of instability shows square root dependence of the instability field \(H_b\) on the heat capacity \(C : H_b \sim C^{1/2}\). According to this, thermomagnetic instability in high-temperature superconductors (HTSs) is not observed above 40 K due to a large heat capacity, which is confirmed by multiple experiments [4]. At high temperatures, at the conditions of isothermal magnetization of a superconductor hollow cylinder or annulus (a ring with rectangular cross section), an inhomogeneous field is formed inside the body according to the critical state model, so \(dH/dr \approx J_c\) according to the Bean model. The superconducting current shields the interior until it fills the entire cross section of the annulus. On further magnetization the circulating current does not change its value, maintaining the difference between the external \((H_{ex})\) and internal field \((H_{in})\) : \((H_{ex}-H_{in})/(R_{out}-R_{in}) = J_c\), where \(R_{in}\) and \(R_{out}\) are the internal and outer radii of the hollow cylinder, respectively.

In the case of pulsed field magnetization of a superconducting annulus made of a melt grown HTS, a break of shielding current (BSC) followed by a flux jump inside the annulus hole were observed at \(T = 78\) K [5]. The dynamics of these flux jumps differs from classical thermomagnetic instability. A BSC appears only in the case of the high rate of external magnetic field increasing. In general, the external field ramp rate \(dH/dr\) should be high enough that the generated power is larger than the heat flow into the surrounding liquid nitrogen. In this case, the BSC is initiated by the ‘weak spot’ of the superconductor. Weak spots exist due to azimuthal anisotropy of critical current in the crystal (in the
(a)–(b) plane [6], or due to technological reasons during the HTS sample manufacturing. The increase in the resistance on this part of the annulus leads to the reduction of the dissipated power in the rest of the sample. As a result, the magnetic flux penetrates into the annulus through the narrow high resistive region. The size of this resistive region can be determined from the ratio of average and local temperature. For the annulus with an external diameter of 30 mm the flux penetration region occupied the sector with an angle of less than 10 degrees. The penetration time of magnetic flux was 0.3–0.5 ms [5].

Investigations of the BSC effect in a bulk HTS annulus are of interest for the realization of pulsed magnetized devices such as hybrid magnets [7], which can be used in portable electron spin resonance and nuclear magnetic resonance apparatus. The field drops observed at pulsed field magnetization of the hybrid magnet can be associated with partial circular current destruction.

In the present work the conditions of a BSC were investigated. For the qualitative description of this process we carried out a model calculation of pulse magnetization of a thin superconducting ring.

**Experimental technique and results**

For the experiments a melt grown annulus YBCO was used. The inner and outer diameters were $R_{\text{out}} = 52$ and $R_{\text{in}} = 28$ mm and the thickness was 11 mm. The annulus was installed between two magnetization coils. A photo of the HTS annulus, Rogowski coil and magnetization coil is presented in figure 1. The pulsed field magnetization technique and measurements of the circulated shielding current are described in [5]. The experiments were performed under liquid nitrogen temperature. After each pulse magnetization the superconductor was heated above $T_c$.

Figure 2 shows the evolution of current $I(t)$ in the annulus and the external magnetizing field $\mu_0 H(t)$. Left axis—$I(t)$, right axis—$\mu_0 H(t)$. Right side—schematic distribution of the magnetic field axial component $B_z(r)$ at points (1, 2, 3).

The sharp drop in the current at $\tau_m$ is the BSC. The observed picture of the BSC is similar to that in [5]. The discrepancy in the parameters of the BSC are related to the size of the annulus (inductance $L$) and the value of the critical current. It is seen that with increasing $H_a$ (curves (a), (b), and (c)) the BSC happens earlier ($\tau_m$ is reduced) and the maximal current value $I_m$ increases. $I_m$ versus $H_a$ is shown in the inset to figure 2. At low amplitudes the field in the hole is shielded by the induced current. Therefore, in this range $I_m$ depends on $H_a$ linearly and flux jump is absent. Above a certain ‘critical’ value of the magnetic field amplitude ($\mu_0 H_a > 0.9$ T) the BSC is observed and the magnetic flux abruptly penetrates into the annulus (flux jump). In this region the $I_m$
weakly increases with $\mu_0 H_a$. In our case $dH/dt \approx (\pi H_a/\tau_p)$ for most of the pulse. Therefore, in the inset to figure 2 at high amplitudes (in the case of the BSC) the presented dependence $I_m$ versus $\mu_0 H_a$ is in fact the dependence of $I_m(dH/dt)$. This increasing dependence of $I_m(dH/dt)$ corresponds to the increasing dependence of $H_0(dH/dt)$ as $I_m \sim H_0$, where $H_0$ is the external field at which the flux jump appears.

To show the determining role of $dH/dt$ in the BSC effect an experiment with equal $dH/dt$ was performed. In figure 3 the evolutions of the shielding currents for two magnetizing pulses are shown. The magnetizing pulses have the same ratio of the amplitude $H_a$ to the pulse duration $\tau_p$, resulting in nearly the same $dH/dt$ for a long time. One can see that at the equal ramp rates, the shielding current evolutions practically coincide near the BSC.

The right-hand side of figure 3 shows schematically the radial distribution of the magnetic field axial component $B_z$ measured in [5] at different moments of time (marked by numbers 1, 2, 3).

By analogy with [5] the $I(t)$ dependence at the BSC depicted in figure 2 allows the estimation of the size of the overheated annulus region. The drop in current was approximated by an exponential function with characteristic time $\tau_I = I/R_{\text{flow}}$, where $R_{\text{flow}}$ is the flux flow resistance. The annulus inductance $L = 2.95 \times 10^{-8}$ H was calculated from [8] (for a rough estimation $L = \mu_0 R \ln(\beta R/(\alpha + r)) / 0.5$) can be used, $R = (R_{\text{in}} + R_{\text{out}}) / 2$, $r = R_{\text{out}} - R_{\text{in}}$, $a$ is the thickness), the flux flow resistance evaluated from the characteristic time was $R_{\text{flow}} = 50 \mu\Omega$.

At different pulses $I(t)$, presented in figures 2(a)–(c), the dumping times $\tau_I$ are practically equal. For this reason we suggest very weak dependence of $R_{\text{flow}}$ on $dH/dt$. The exponential approximation of the current drop $I(t) = I_0 \exp(-R_{\text{flow}} t / L)$ is rather a rough approximation. First of all, it is seen that shielding current does not fall to the zero value, being supported by the rising magnetic field. Second, $R_{\text{flow}} = \rho_{\text{flow}} / S$ and the resistivity depends on the current voltage characteristic (CVC). During the current drop $\rho_{\text{flow}}$ increases (approaching $\rho_a$) following the increase in $T$. On the other hand the width of channel I decreases.

The heat dissipated during the BSC is $q_I = \int I^2 R_{\text{flow}} dt \approx 5.1$. If this energy was distributed over the whole volume of the annulus, the average temperature increase would not exceed $(\Delta T) \approx 0.35$ K. On the other hand the six-fold drop in the shielding current at the BSC corresponds to local overheating $\Delta T \approx 9$ K. The volume of the annulus occupied by the moving fluid during the BSC is $\Omega_{\text{flow}} = q_I / (C \Delta T)_{\text{C}}$, where $C$ is the volumetric heat capacity [9]. In this case the width of the resistive hot region can be estimated as a sector with an angle of $\delta \varphi = 2\pi (\Delta T) / \Delta T \approx 15^\circ$.

Calculation of the shielding current in the thin ring

For a qualitative description of the BSC process we considered the pulse magnetization of a thin superconducting ring. It was supposed that:

1. the ring has a small cross section, and the current is distributed homogeneously;
2. the diffusion time of the magnetic field is small;
3. the adiabatic approximation is valid, because the real characteristic time of the cooling of the sample in liquid nitrogen $\tau_\text{q} = 10–20$ s [10] is three orders of magnitude higher than the pulse duration ($\tau_p = 35$ ms) and four orders of magnitude higher than the duration of a flux jump.

The evolution of the current in a thin ring with an inductance $L$ and resistance $r$ is described by the well-known equation

$$LdI/dt = -Ir + \varepsilon.$$  

Here $\varepsilon = -S dH/dt$ is emf induced by the magnetizing field. In the case of the superconductor the calculations are complicated by the fact that the resistance $r = U/I$ is a non-linear function of the current, which we approximated by the power-law current CVC. This CVC is ordinarily used for simulations of HTS electrodynamics [11]. In the result we have:

$$LdI/dt = -U_0(I/I_c)^N + \varepsilon.$$  

Normally, for bulk HTSS the exponent $N$ is in range from 6 [12] to 21 [13]. In our experiment exponent $N$ is estimated from the dependence $I_m(H_a)$ presented in the inset to figure 2. At pulsed magnetization, the shielding current drops due to heating, which in turn depends on the CVC. Formally, as one can see from (1), at $I = I_m$, $dI/dt = 0$. This results in $U_0(I/I_c)^N = \varepsilon$. The local high resistive state appears when the current in the ‘weak spot’ exceeds the critical value. In this place the heat is predominantly dissipated. The conditions for the BSC can be formulated as a dissipation of the heating energy $q = \int I dU/dt$ in the weak spot, which is enough to raise the temperature to $T_c$. The maximum current $I_m$ after which the BSC happens, one can estimate from the condition $I_m U \approx q/I_c$. The relationship between the maximal current and the time to the current break $(I_m)^{N+1} \times 1/I_c$ can be obtained using $U = U_0(I/I_c)^N$. This allows the evaluation of $N$ by fitting the dependence $I_m(H_a)$ shown in the inset of figure 2. For the different annuli investigated in this work $N$ was in the range 8–10.

In our calculations we assumed $N = 9$, as defined above. For the primary calculations $U_0 = 0.126 \times 10^{-5}$ V $(E = 10^{-5}$ V m$^{-1}$) and $I_{\text{c0}} = 13$ kA ($J_{\text{c0}} = 10$ kA cm$^{-2}$) were set. These $E$ and $J_{\text{c0}}$ values are typical for the electric field and current densities in melt grown HTSS. In the final stage of the calculation, the critical current value $I_{\text{c0}}$ was adjusted to improve the agreement with the experiment.

The ‘weak spot’ results in inhomogeneous azimuthal distribution of the critical current in the annuli $I_c = I_c(\varphi)$. We placed the reference point $\varphi = 0$ in the center of the weak spot and assumed $I_c(\varphi) = I_c(0)(1 - \alpha \exp(-\varphi^2 / D^2))$. Here $\alpha$ is the ‘depth’ of the inhomogeneity of critical current in the center of the weak spot, and $D$ the ‘width’ of the inhomogeneity (dispersion). Considering the linear decrease of the
critical current with temperature one can write:

\[
I_C(\varphi) = I_{C0} \left(1 - \alpha \exp\left(-\frac{\varphi^2}{D^2}\right)\right) \left(1 - \frac{T(\varphi) - T_0}{T_C - T_0}\right)
\]

(3)

where \(I_{C0}\) is the critical current at initial temperature \(T_0 = 77.5\) K. Due to the inhomogeneity of \(I_c\) the resistance depends on the angle \(\varphi\). In this case equation (2) should be written as

\[
L \frac{dI(t)}{dt} = -\frac{U_0}{2\pi} \int_{0}^{\pi} \frac{1}{I_C^2(\varphi)} \, d\varphi + \varepsilon.
\]

(4)

This equation is complicated by the fact that \(I_c\) depends on the temperature, which is determined by the generated power related to the CVC of the superconductor and the shielding current value.

The evolution of the temperature in each element of the superconducting ring is determined by the Joule–Lenz law. The temperature increment is defined by \(dT = dt \cdot IU/\text{C}\), where \(C\) is the heat capacity. To solve this equation we used the implicit numerical method of Cauchy–Euler [14].

**Procedure of the calculation**

The ring was divided into \(M = 1000\) identical elements. Every element \(\varphi_i = i2\pi/M\) is the sector of ring with angle interval \(d\varphi = 2\pi/M\). We assume that within \(d\varphi\) the parameters of the superconductor are unvarying. The current, temperature and voltage (from the CVC) for elements \(\varphi_i\) were calculated systematically for each moment of time \(t_j = j \cdot dt\). The time step \(dt = 1 \mu s\) was chosen from the condition that it is small enough in comparison with the time of the BSC.

We introduce two-dimension arrays: temperature \(\{T_{ij}\} = T(\varphi_i, t_j)\), critical current \(\{I_{cij}\} = I_c(\varphi_i, t_j)\), and voltages \(\{U_{ij}\} = U(\varphi, t_j) = 1/M \sum U_0 I_{Nij}^N(\varphi)\), which correspond to ring element \(\varphi_i\) in the moment of time \(t_j\). In addition, we introduce a one-dimensional array \(\{I_j\}\), which corresponds to the current value in the ring at moment \(t_j\).

To find the current at the moment \(t_j\) equation (4) was solved by the implicit Cauchy–Euler method. This method, as it turned out, is more precise and stable for solving (4) than the explicit method [14]. In this method the difference (in this case, of the current) is written as half the sum of the adjacent values of the function on the right-hand side of equation (4). The increment in the current was calculated under the assumption that during the time interval \(\Delta t = 1 \mu s\) the temperature increment (and corresponding \(I_j\)) gives the contribution of the second order. So,

\[
I_{j+1} - I_j = \frac{dt}{2L} \left(-U_0 I_{Nij+1} \sum \frac{1}{I_c^2(\varphi, t_j)} + E(t_{j+1})\right) - \left(-U_0 I_{Nij} \sum \frac{1}{I_c^2(\varphi, t_j)} + E(t_j)\right).
\]

(5)

Equation (5) is an algebraic equation of the \(N\)th degree of the current \(I_{j+1}\). This one was solved numerically, by the Newton method. From the set of solutions only a positive real root was selected.

After the calculation of the current \(I_{j+1}\) the dissipated energy in each element \(\varphi_i\) and the temperature of each element was calculated from the relation \(T_{ij+1} = T_{ij} + \Delta T_{ij+1}\), where \(\Delta T_{ij+1} = U_{ij+1} I_{j+1} \Delta t/C\). At \(t = 0\) the current is absent \((I_0 = 0)\). Using the value \(T_{ij+1}\) the critical current \(I_{cij+1}\) was calculated using the factor of the anisotropy from equation (3). Using the value \(I_{cij+1}\) the procedure of the calculation was repeated for the next current increment (equation (5)).

When the temperature approaches \(T_c\) the critical current \(I_c\) tends to zero, so the CVC \(U \propto (1/I_c)^5\) has a divergence near \(T_c\). In this region the power law for the CVC turns into Ohm’s law. This fact complicates the algorithm of the calculation. The flux flow resistance was estimated in [5]. It was shown that the resistivity of a superconductor does not exceed half the resistivity in a normal state \((\rho_{flow} < \rho_n/2)\). So we defined:

- if \(r_{ij} < r_n/2\), then \(U_{ij} = U_0(I_{Nij}/I_{cij})\),
- if \(r_{ij} > r_n/2\), then \(U_{ij} = I_{ij} r_{ij}/2\).

For the calculations of the BSC the following parameters were used:

- the azimuthal inhomogeneity \(D = 0.1, \alpha = 0.1\),
- the magnetization pulse amplitude \(\mu_0 H_s = 1.38\) T, pulse duration \(\tau_p = 35\) ms,
- the external field was approximated by function \(\mu_0 H_s = \mu_0 H_s \sin(\pi t/\tau_p)\), \(t < \tau_p\),
- the normal resistance of the crystal YBCO at 92 K was \(\rho_n = 100 \mu \Omega\) cm,
- the volumetric heat capacity \(C = 0.75\) J K\(^{-1}\) cm\(^{-3}\) [9].
The calculated shielding current \( I(t) \) was compared with the experimental curve and then the initial value of the critical current of the CVC was corrected. The value \( I_{0} \) was changed to make the calculated \( I(t) \) as close as possible to the experimental curve. The result of the calculation of the CVC with corrected \( I_{0} = 7.0 \text{ kA} \) is shown by the dashed curve in the figure 4. In the initial part of the curve the critical current is high and the induced current completely screens the external field. With further increase in the shielding current the right-hand side of equation (2) becomes zero and the shielding current approaches the maximal value.

Since the current \( I_{j} \) is common for all elements of the ring, the generated heat (and temperature increment) is maximal in the \( r^i \)th elements where the critical current value \( I_{d,ij} \) is minimal. The temperature in these elements increases and this leads to further decrease in the \( I_{c} \) value. As a consequence, the heat dissipation increases there. Thereby, the increase in the local temperature and the corresponding drop in \( I_{c} \) develops in the ring’s sector with reduced value of the critical current.

In the inset to figure 4 the azimuthal profiles of the temperature \( T(\phi) \) at different moments in time are shown. The numbers 1–4 on the curve \( I(t) \) mark the time for which corresponding \( T(\phi) \) profiles (1–4) were calculated. It is seen that before the BSC (curves 1 and 2), the temperature profile is relatively flat. During the BSC (curves 3 and 4) the temperature precipitously rises up to the \( T_{e} \), which is accompanied by a current decrease by an order of magnitude.

After the BSC the distribution \( T(\phi) \) has a narrow peak in the vicinity of the inhomogeneity and the flat part far away from the inhomogeneity. The temperature in the maximum is higher than the liquid nitrogen temperature by 12 K. The high resistive region (the character overheat width of the channel) is about 12°. In this experiment, as mention above, the temperature jump is \( \Delta T_{j} \approx 9 \text{ K} \), and the width of the channel is \( \Delta \phi \approx 15^\circ \).

We analyzed the influence of the parameters \( \alpha \) and \( D \) from (3) on the temperature peak. It turned out that the increase of \( \alpha \) from 0.01 to 0.2 does not affect the temperature distribution much in the vicinity of the peak, but it lowers the flat part (baseline \( T(\phi) \)) by \( \approx 2.5 \text{ K} \). At these conditions the BSC appears earlier (\( \tau_{m} \) decreases). This result is obvious because in the case of small inhomogeneity of the critical current (small \( \alpha \)) the local overheating develops slower and the average heating is higher. The calculations also showed that the size of the inhomogeneity \( D \) does not affect the width of the channel \( \Delta \phi \). Therefore, increasing \( D \) by three orders of magnitude (from 0.01 to 10) the width of the channel increases by a factor of two (from 10° to 23°).

This model also demonstrates that increasing the pulse amplitude leads to a slight increase in the shielding current maximum and reduction of \( \tau_{m} \). This is in good agreement with the experimental results shown in figure 2. Using the model calculation one can find that changing the \( N \) value from 10 to 20 changes the following: \( I_{m} \) decreases by a factor of two (from \( \tau_{m} = 4 \text{ ms} \) to \( \tau_{m} = 2 \text{ ms} \)) and \( I_{m} \) decreases by a factor of 1.6 (from \( I_{m} = 16.5 \text{ kA} \) to \( I_{m} = 10 \text{ kA} \)).

**Conclusion**

During pulsed magnetization of multiply connected superconductors, such as rings or annuli, the local heating in the region with lowest \( I_{c} \) creates a narrow channel of a high resistive state. This leads to the break of the shielding current and to a jump in the magnetic flux into a hole of the superconductor.

It was shown that the rate of field variation \( dH/dt \) is a crucial parameter for the BSC. For equal \( dH/dt \) the shielding current evolutions practically coincide. Increase in \( dH/dt \) does not change the channel parameters (width) but reduces the time to jump \( \tau_{m} \) and slightly increases the value of maximal shielding current \( I_{m} \). The same results can be obtained using the proposed simple numerical model of a superconducting ring with azimuthal inhomogeneity. The shielding current evolution in the model is in good agreement with the results of the experiments.

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