Simplified analysis about horizontal displacement of deep soil under tunnel excavation

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Abstract. Most of the domestic scholars focus on the study about the law of the soil settlement caused by subway tunnel excavation, however, studies on the law of horizontal displacement are lacking. And it is difficult to obtain the horizontal displacement data of any depth in the project. At present, there are many formulas for calculating the settlement of soil layers. In terms of integral solutions of Mindlin classic elastic theory, stochastic medium theory, source-sink theory, the Peck empirical formula is relatively simple, and also has a strong applicability at home. Considering the incompressibility of rock and soil mass, based on the principle of plane strain, the calculation formula of the horizontal displacement of the soil along the cross section of the tunnel was derived by using the Peck settlement formula. The applicability of the formula is verified by comparing with the existing engineering cases, a simple and rapid analytical method for predicting the horizontal displacement is presented.

1. Introduction
Twenty-first Century is a new era of underground space, major cities have set off a boom in the construction of the subway. The internal force distribution of surrounding soil caused by subway excavation will bring harm to the environment. At present, the environmental damage caused by the vertical displacement of subway excavation is more familiar. It is easy to monitor vertical displacement of the surface in the project. But most of the tunnel inevitably neighbors underground engineering facilities, underground pipeline, the existing subway tunnel, pile foundation, etc. However, studies on the law of horizontal displacement in deep soil are lacking. The horizontal displacement of a certain position in the soil layer is measured by the inclinometer tube [1-2]. However, due to the human factors and the complexity of the construction site, the accuracy of data acquisition is limited. In addition, when the traditional analytical solution is used to determine the displacement of the soil, the pressure distribution around the tunnel is assumed to be uniform [3-6]. For the deep buried tunnel, the error is smaller. In fact, the stress of the upper and lower sides of the tunnel is distributed evenly, but the left and right sides of the tunnel is trapezoidal distribution [7]. For shallow tunnels, especially, it cannot be simplified easily as uniform pressure. Therefore, the obtained approximate solution of the deep buried tunnel under the assumption of uniform pressure, which is not suitable for shallow tunnel, and also lack of simple and practical engineering prediction methods.

Foreign scholars, Mair[8], Attewell[9], Moh[10], Dyer[11], respectively, through the experimental study and field measurements found: In terms of representative sand and cohesive soils, there are similar characteristics in settlement curves between surface and underground. The settlement curve of
underground soil is also in accordance with the law of Gauss distribution curve. That is to say, it is only necessary to adjust the coefficient of settlement tank widths.

2. **Vertical displacement of surface**

There are a lot of calculation principles about vertical displacement of the surface. In this paper, the traditional Peck\([12]\) theory is used to calculate:

\[
S(y, z) = S_{\text{max}} \exp\left(-\frac{y^2}{2l^2}\right) = \frac{V_l}{2.5i} \exp\left(-\frac{y^2}{2l^2}\right)
\]

(1)

\(S(y, z)\): vertical displacement of any point on the ground;

\(S_{\text{max}}\): the maximum vertical displacement corresponding to \(Z=0\);

\(i\): the coefficient of settlement tank widths;

\(V_l\): the volume of ground loss.

3. **Vertical displacement of soil depth**

The relationship between \(i(z)\) and \(k_z\) is obtained by model test and measured data in the literature\([8]\):

\[
i(z) = k_z (H - z)
\]

(2)

\[
k_z = 0.5 + \frac{0.175z}{H - z}
\]

(3)

\(z\) increases, \(k_z\) also increases. It indicates that the settlement area of any deep soil layer is larger than assumed. In Formula (3), \(z=0, k_z=0.5\), which is inconsistent with the results of Burland and O’Reilly. \(k_z\) is not a single fixed value, but a numerical range for different soil layers. Given this situation, The literature\([7]\) adopts Xuan Han’s normalized research findings, take:

\[
k_z = \frac{1 - \alpha(z/H)}{1 - \alpha} k (0 < \alpha < 1)
\]

(4)

In different engineering cases, \(\alpha\) has different values, cohesive soil \(\alpha = 0.65\), sand \(\alpha = 0.5\), \(\alpha\) needs to be analyzed according to the monitoring data, so the applicability is limited.

The literature\([13]\) points out:

\[
i(z) = i(0)(1 - z/H)^n
\]

(5)

In the calculation of the depth of the soil settlement trough, the literature\([14]\) take use of Xinliang Jiang’s research findings. \(n\) takes a constant value of 0.3, but the literature\([15]\) study shows that, cohesive soil: \(n=0.35 \sim 0.85\). The results of 22 measured data show that, \(i(0)\) is also affected by soil properties. For cohesive soil, there is a linear relationship between \(i(0)\) and \(R + H \tan (45 - \varphi/2)\).

\[
i(0) = m[R + H \tan (45 - \varphi/2)]
\]

(6)

\(\varphi\) soil frictional angle; \(m=0.45 \sim 0.50\).

4. **Lateral horizontal displacement of soil depth**

The vertical displacement of the soil layer will be caused by the excavation unit, and the corresponding horizontal displacement of the soil will be produced. It is assumed that the rock mass is incompressible and continuous in macroscopic\([11]\), namely:

\[
\frac{\partial s(y, z)}{\partial y} + \frac{\partial u(y, z)}{\partial z} = 0
\]

(7)
Solving differential equations under boundary conditions: \( x = 0, u(y,z) = 0 \)

\[
\begin{align*}
i(z) &= m \left[ R + H \tan \left( \frac{45 - \varphi}{2} \right) \right] \left( 1 - \frac{z}{H} \right)^n \\
u(y,z) &= -\int \left( \frac{\partial s(y,z)}{\partial y} \right) dz + c \\
&= -\int \frac{V_z}{2.5i(z)} \exp \left\{-\frac{y^2}{2[i(z)]^2}\right\} dz + c
\end{align*}
\]

\( u(y,z), \ i(z) \): The horizontal displacement of y direction at z position and the corresponding coefficient of settlement tank widths;

\( R, H \): Tunnel radius and buried depth.

Formula (9) calculation is simple, and the parameters of the formula include two aspects: the tunnel itself and the surrounding soil. When the parameters, the section of the tunnel, buried depth, reduced volume of soil and the soil properties, etc., are known, the result of calculation by integral is just a simple expression of coordinate \( y, z \). Compiling a simple loop can achieve the final solution, therefore, it can avoid the cockamamie programming to calculate the horizontal displacement of soil by using the random medium theory.

5. Example analysis

In this paper, citing two engineering cases of the literature [3], which are London Heathrow fast track tunnel project and Thailand Bangkok sewage tunnel project, to verify the applicability of the formula (9), the results are shown as Fig.1:

![Fig.1. Horizontal displacement of the soil at 4 m from the center line of Bangkok tunnel](image_url)
The measured value is basically between the research method of this paper and the literature solution \[7\]. The calculated results of this method are slightly larger than the measured values. The change curve is consistent with the measured value and the existing research results. Parameter analysis: In the process of numerical calculation, when m and n are fixed, changing the value of φ, the results show that the horizontal displacement has little change; when m and φ are fixed, changing the value of n, the horizontal displacement is larger. In any case, for cohesive soil: \[m=0.475, n=0.5\sim0.6,\] it can meet the requirements of calculation. The parameters of m and n are small, as well as the calculation error.

6. Conclusion
In this paper, based on the incompressibility of soil layer, used calculation principle of vertical displacement of soil, the horizontal displacement of the soil at any point in the soil layer is obtained. The study results show that: The method chosen in this paper can meet the requirements of engineering accuracy. Likewise, it can also provide a simple and rapid method for calculating the horizontal displacement of any point in the soil.

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