Hadronic matrix elements of proton decay on the lattice

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Abstract. We report on our on-going project to calculate proton decay matrix elements using domain-wall fermions on the lattice. By summarizing the history of the proton decay calculation on the lattice, we reveal the systematic errors of those calculations. Then we discuss our approach to tackle those uncertainties and show our preliminary results on the matrix elements.

Nucleon decay is one of the most important aspect that any (SUSY) GUT model has. At low energy dimension-six operators are dominant contribution to the proton decay while higher dimensional operators are suppressed by the inverse power of the heavy mass ($M_X$). The dimension-six operators consist of three quark and one lepton fields. While the lepton part is treated trivially, the matrix element of the three-quark part

\[ \mathcal{O}_{R/L;L}^B \equiv \epsilon^{ijk} (u^T C P_{R/L} d^j) P_L u^k \]  

between the initial proton and final pion (K or $\eta$ meson) states receives a highly non-perturbative contribution from QCD, which we want to tackle in this study. The matrix element has a tensor structure [1],

\[ \langle \pi; \vec{p}| \mathcal{O}_{R/L;L}^B | p; \vec{k} \rangle = P_L [W_0 - i q W_q] u_p, \]  

where $q = k - p$ is the momentum transfer, $u_p$ is the proton spinor. The relevant form factor $W_0$ is what we need since the $q$ is practically zero by the on-shell condition of the outgoing lepton.

The lattice gauge theory gives the first principle computational ground for the hadronic quantities like this matrix element of proton decay. In the first two calculations on the lattice [2, 3], the tree level chiral perturbation theory [4] was used to evaluate $W_0$ from the low energy constant $\alpha$ and $\beta$,

\[ \alpha P_L u_p \equiv \langle 0 | \mathcal{O}_{R/L}^B | p \rangle, \quad \beta P_L u_p \equiv \langle 0 | \mathcal{O}_{L;L}^B | p \rangle, \]  

which are calculated on the lattice. This method is sometimes called the indirect method. Few years ago, JLQCD published on their large simulation of the nucleon decay matrix

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element \[\Pi\], where they employed both direct and indirect methods. The direct method was first used by the authors of ref. [5]. However, the treatment of the form factors was improper, which led a large discrepancy in the results from direct and indirect methods. Once the direct method is treated properly, JLQCD [1] found the discrepancy not so large, yet, 30 – 40% in most of the cases. Their results of the matrix elements are 3–5 times larger than those from a model calculation commonly used, which pushes down the theoretical estimate of the life time of the proton, and makes much severe constraint on the GUT models.

The existing calculations are all done with the Wilson fermion at a single lattice spacing. The Wilson fermion has an \(O(a)\) discretization error, where \(a\) is the lattice spacing. There are two sources of error propagating to the matrix elements. One is the measurement of the matrix element in lattice unit. The other is the estimate of the lattice scale \(a^{-1}\). Even for the state of the art calculation by JLQCD, the systematic error of the \(a^{-1}\) is as much as 30% \(^2\). Of course there should be a scaling violation for \(W_0\), \(\alpha\) and \(\beta\), too, which could diminish the overall violation by compensating that from the scale. But it is unknown until it is studied. Also the Wilson fermion breaks chiral symmetry explicitly. Thus, the applicability of the chiral perturbation theory at a finite lattice spacing is not guaranteed. One has to take the continuum limit of quantities of interest.

The second problem is that up to now the operator renormalization has been done by one-loop (tadpole-improved) perturbation theory. This should be improved by employing a non-perturbative technique \(^3\).

Finally the calculations are all done in the quenched approximation, where all quark loop effects are neglected. This approximation is commonly used in the lattice calculation as the unquenched simulation is much more expensive. One has to check how large is the effect of quenching by doing the unquenched simulation.

We use the domain-wall fermions [10, 11, 12] in our simulation. This fermion discretization has almost exact chiral symmetry and exact flavor symmetry. Hence there practically is no mixing of the operators with different chiral structure, making the data cleaner. Moreover, there is no \(O(a)\) discretization error. This second point has been demonstrated in the simulation results for the hadron spectrum [13] and the kaon \(B\) parameter [14, 15]. The chiral symmetry can be further improved dramatically by improving the gauge action [16, 13]. We use DBW2 gauge action which reduce the residual chiral symmetry breaking by factor 100 from that for the Wilson gauge action at a typical lattice spacing [13].

\(^2\) The dimensionless quantity, the product of the Sommer scale [6, 7] and the \(\rho\) mass \(m_\rho\), is about 30% off from its continuum limit [8] at the simulation point of JLQCD. Note that the decay width is proportional to the square of \(W_0\) (dimension two), or \(\alpha\) and \(\beta\) (dimension three).

\(^3\) For the recent summary of the non-perturbative renormalization on the lattice, see [9].
We use the $16^3 \times 32$ lattice with $a^{-1} \simeq 1.3$ GeV \(^4\). The direct method uses the ratio of the three- and two-point functions

$$R(t) \equiv \frac{\langle J_\pi(t_1) \mathcal{O}_{R/L,L}(t) \bar{J}_p(t_0) \rangle}{\langle J_\pi(t_1) \mathcal{O}_p^2(t) \rangle \langle J_p(t) \bar{J}_p(t_0) \rangle} \sqrt{Z_\pi Z_p},$$

where the proton and pion interpolating fields are located at $t_0 = 6$ and $t_1 = 24$ respectively. Momentum $\pm \vec{p}$ with $\vec{p}a = (1,0,0)\pi/8$ or $(1,1,0)\pi/8$ is injected to the pion and the operator in the three point function, as well as in the pion two point function in the denominator. $\sqrt{Z_\pi}$ and $\sqrt{Z_p}$ are overlap of $J_\pi$ and $J_p$ to the corresponding pion and proton states, which is estimated from the fit of two point functions.

Figure 1 shows ratio at a parameter point with the particular projection and subtraction to get $W_0$, which is taken from the fit to the plateau. In addition to the data shown in the ref. [17], we have further performed the calculation for the non-degenerate quark mass $m_1, m_2$ in the final pseudoscalar state, where the initial proton state is made up of quarks with $m_1$. Then we get $W_0$ as a function of $m_1$, $m_2$, and $q^2$. The chiral perturbation [1] helps to fit $W_0$ to get to the physical point. The results for various decay amplitudes are shown in Fig. 2. We are assuming the SU(2) symmetry for the $u$ and $d$ quarks. There are other possible matrix elements, but they can be calculated with the matrix elements in the figure when the SU(2) symmetry is intact. We are yet to have the renormalization factor for the operators by a non-perturbative renormalization. Preliminary value using the perturbative estimate of the renormalization factor [18] is listed in ref. [17].

The results of the indirect method are also shown in Fig. 2. The direct and indirect calculations give consistent results within the error, in contrast to the result of JLQCD. However, this could be caused by larger statistical error in our calculation. We need to have more statistics to judge it. Nevertheless, the relative size of the matrix element in our calculation for each decay mode is similar to that obtained by JLQCD.

\(^4\) The more precise description of our simulation is given in ref. [17].
We have investigated the proton decay matrix elements at a lattice cut off of $a^{-1} \simeq 1.3$ GeV with the domain-wall fermion in the quenched approximation. The direct and indirect methods give consistent results within our precision. The non-perturbative renormalization program [19, 20] is underway to get the continuum matrix elements. Also we are performing the two flavor dynamical domain-wall fermion simulation, which will give us an idea of the size of the quenching error.

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