The article proposes a method of determining the patterns of a distribution of stresses and strains in the array. It allows to determine the depth and degree of compaction of a particular type of technology and to make rational selection of it for specific conditions. Also, the data obtained in the article make it possible to select the right equipment for tillage.

**Keywords:** machine-tractor unit, wheels, soil compaction.

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**Formulation of the problem.** One of the current problems that attracts the attention of scientists is the protection of the soil from over-compaction by working bodies and wheels working in the fields of machines. It is known that soil compaction has a strong influence on its porosity, and hence on humidity and water permeability, gas exchange and aeration, temperature and thermal conductivity, as well as many other physical, chemical and mechanical properties of the soil. The pressure arising under the wheels and tracks of the moving vehicles reduces the porosity of the substrate, pushes the moisture into the lateral layers, and thus causes it to redistribute, causing uneven saturation of the soil with water. Compacted soil has less water permeability, so it absorbs water less when artificial moistening. It is also known that dense soil layers conduct heat more intensively. Increased thermal conductivity causes a twofold effect: first, the earth warms faster and deeper when it is warm in the atmosphere, and at the same time cools down more rapidly as the outside temperature drops. In general, soil compaction is a negative phenomenon because it degrades the main indicator - the fertility of the earth. Therefore, the study of mechanical phenomena in soil, flowing under the weight of technical means of processing, is one of the most important problems that occur in agricultural production.

**Formulation of the task of the article.** The purpose of the work is the correct choice of the theory of determining the laws of distribution of stresses and strains in the array, which is quite a difficult task because the soil is a specific material that is very different from the conventional model.

**Analysis of the literature.** It should take into account the following features:

1. Soil is characterized by an uneven distribution in its volume of crystalline and amorphous qualities, so it can be considered homogeneous and isotropic very conditionally.
2. Due to the moisture in the soil, plastic shifts, creep and residual deformation occur even at relatively low pressure under wheels tractors.
3. Soil does not resist tensile loads and hardly retains its shape even under the influence of mass forces, and the dependence between stresses and strains does not follow a linear pattern.
4. In addition to the instantaneous deformation that occurs during the application of forces, the soil also deforms in time, exhibiting creep properties, which is especially dangerous in the construction of buildings and structures.
5. Changing the volume of soil in the field of plastic deformation follows the law of elasticity [4, p. 155].

For these reasons, it is practically impossible to create a theory of mechanical soil calculation, covering all of the above features, although attempts to make progress in this direction are not stopped.

However, the theoretical science of soil mechanics is increasingly complicated, and practical confirmation of experience leaves much to be desired. In this regard, the most rational approach from our point of view would be to choose a method from among the proven ones, both in its simplicity and in practice.

**Main research material.** The complexity of solving the problem is due to many circumstances. First, different plants respond differently to soil compaction due to the diversity of the root system, individual predisposition to the nature and condition of the soil, climatic conditions and other conditions. In addition, the variety of machinery used determines the different types of mechanical impact on the soil. For example, a wheel alone can exert simply static pressure or difficulty destroy the base on various types of movement - without drifting, with drifting, etc. At the same time, much depends on the composition and condition of the soil itself.

In this paper an attempt is made to evaluate the influence of agricultural machinery motors on soil compaction and to establish the relationship between stresses and compaction.

Nowadays, in addition to classical soil mechanics based on the assumption of a linear relationship between forces and strains, other methods are applied, such as the principle of conditional calculations, the principle of limit states, the theory of reliability [1, p. 148 - 148]. The problems of soil mechanics can be solved at present on the basis of the theory of elasticity, the theory of plasticity and creep, hydromechanics, mathematical physics, fracture mechanics. Numerical solution methods are widely used. Based on the behavior of soil under load, and also given the small pressure on the soil from vehicles (compared, for example, with foundations), in the practical calculations in the phase of soil compaction can use the theory of linearly deformed environment [1, p. 149], according to which the deformation changes...
in proportion to the stress and the soil behaves as elastic material, obeying the Hooke law. Therefore, in this case, you can use a mathematical apparatus of the theory of elasticity. As stated, soil can be considered as a continuous, homogeneous and isotropic medium with only some approximation, so attempts have been made to create more accurate mechanical soil models, such as rheological and others, but for known reasons they have not been used in solving real problems. Therefore, there is every reason to dwell on the principle of linearly deformed environment.

Another important factor in solving this problem is the choice of the calculation scheme. The interaction of the wheel with the soil surface is complex. First, it is a dynamic task: the soil beneath the wheel settles and there are compressive stresses, with the soil surface is complex. For known reasons they have not been used in solving real problems. Therefore, there is every reason to dwell on the principle of linearly deformed environment.

As the wheel rolls, the soil underneath it deforms in all directions, so a volumetric stress state, characterized by nine magnitudes of stress, occurs at an arbitrary point in the array. Considering that in the final state along the strip the soil is not deformable, the problem can be reduced to a simple solution.

We distinguish the element of the deformed strip of unit thickness by two sections perpendicular to its longitudinal axis, and by placing the origin (point O) in the center of the distributed load on the free surface of the half-plane. Direct the Oz axis along the strip, the Ox axis deep into the array, and the Oy axis along the free half-plane surface.

Then the single width strip can be considered as a semi-infinite plane with a uniformly distributed load q along the width of the strip 2a, (fig. 1).

There is a common solution to this problem [3, p.102]:

\[ \sigma_x = \frac{q}{2\pi} \left[ 2(\alpha_2 - \alpha_1) + \sin 2\alpha_2 - \sin 2\alpha_1 \right] \]

\[ \sigma_y = \frac{q}{2\pi} \left[ 2(\alpha_2 - \alpha_1) - \sin 2\alpha_2 + \sin 2\alpha_1 \right] \]

\[ \tau_{xy} = \frac{q}{\pi} \left[ \sin^2 \alpha_2 - \sin^2 \alpha_1 \right] \]

It can be written in the Cartesian coordinate system according to the designations in fig. 1:

\[ \sigma_x = \frac{q}{\pi} \left[ \arcsin \left( \frac{y-a}{\sqrt{x^2 + (x-a)^2}} \right) - \arcsin \left( \frac{y+a}{\sqrt{x^2 + (x+a)^2}} \right) \right] \]

\[ \sigma_y = \frac{q}{\pi} \left[ \arcsin \left( \frac{y-a}{\sqrt{x^2 + (y-a)^2}} \right) - \arcsin \left( \frac{y+a}{\sqrt{x^2 + (y+a)^2}} \right) \right] \]

\[ \tau_{xy} = \frac{q}{\pi} \left[ \frac{(y-a)^2}{x^2 + (y-a)^2} - \frac{(y+a)^2}{x^2 + (y+a)^2} \right] \]

Provided that the deformation is in the direction of the axis Oz:

\[ \varepsilon_z = \frac{\sigma_z}{E} = \frac{\sigma_z - \mu(\sigma_x + \sigma_y)}{E} = 0 , \]

there are third normal stresses acting in sections perpendicular to the axis Oz:

\[ \sigma_z = \mu(\sigma_x + \sigma_y) = \frac{2\mu q}{\pi} \left[ \arcsin \left( \frac{y-a}{\sqrt{x^2 + (x-a)^2}} \right) - \arcsin \left( \frac{y+a}{\sqrt{x^2 + (x+a)^2}} \right) \right] \]
where $\mu$ - Poisson's ratio; $E$ - modulus of longitudinal elasticity.

Since in planes perpendicular to the Oz axis, there is no shear force and $\tau_{xz} = \tau_{yz} = 0$, then by virtue of the law of the parity of tangent stresses $\tau_{xx} = \tau_{yy} = 0$; $\tau_{xy} = \tau_{xz} = 0$.

Thus, all nine components of the stress tensor are defined.

If, in calculating the stresses, the coordinates of the points are expressed in terms of $a$, then after reducing the results by $q$, we obtain the ratio of the values of the stresses $\sigma_x/q; \sigma_y/q; \sigma_z/q; \tau_{xy}/q$. They will be common to all load cases, regardless of value $a$ or $q$. In Fig. 2 shows the soil stress matrix for the right side of the half-space symmetrical about the Ox axis. When calculating stresses, the value of $\mu = 0.25$ was accepted as the most suitable for black earth soils; in addition, the minus sign, which is characteristic of all stresses under conditions of comprehensive compression, is rejected.

The table in Fig. 2 is a universal matrix of relative stresses, using which it is possible to determine all stresses at an arbitrary point in the array, regardless of the width of the wheel $2a$. It is enough to multiply the number in the corresponding cell of the matrix by the intensity $q$ - in this case the specific pressure of the wheels of tractors, cars or combines [2, p.54 - 57]. From the table it follows that the maximum modulus of voltage extends to a considerable depth and at $x = 3a$ is still about 40% of its maximum value. As expected, maximum soil compression occurs at the center of load. The vertical and horizontal stresses here are equal to the specific pressure $q$. The tangent stresses in the vertical and horizontal segments at all points on the Ox line are zero, so there will be major stresses. On the free surface of the soil at $y > |a|$ all components of the voltage are zero. Under special conditions, points $A$ and $B$ due to pressure breakage result in a displacement of the soil under the die. It is at these points that plastic deformations arise and at a certain value the loaded soil goes into a plastic state, which with increasing pressure spreads deep into the array along the so-called sliding lines. In fact, this transition should be smooth, since the wheel usually has a rounding.
In the horizontal direction, as the ordinates grow, the stresses decrease rapidly and already at $y = 2a$ make up 1 - 5\% of their maximum values. The involved stresses have extremes that can be determined from the conditions:

$$
\frac{d\tau_{xy}}{dy} = 0; \quad \frac{d\tau_{yx}}{dy} = 0.
$$

The relative change in volume (relative volume deformation) at an arbitrary point is

$$
\theta = \frac{1}{E} \left( \sigma_x + \sigma_y + \sigma_z \right) / E.
$$

If $\mu = 0.25$, $E = 10$ MPa, calculate the values for the points with $x$ and $y$ coordinates within the range 0-3a. The results of the calculations are summarized in the table in Fig. 3, where the value has the dimension $[1/\text{MPa}]$. After multiplying it by $q$, we get the true value at the corresponding point. Increased from compression relative density will be:

$$
\xi = \frac{p}{p_0} = \frac{1}{1 - \theta},
$$

when $p_0$ – density of soil in stress.

**Conclusions.** The developed method makes it possible to quickly determine the depth and degree of soil compaction from a particular type of machinery and to justify the choice of the machine, providing the compaction within the necessary limits, acceptable for a particular type of crop and nature of the soil. Example 1. Let us determine the relative change in soil density under the trace of the MAZ-500 A car wheel at a depth and from the surface. From the table [2, p. 57] we find $q = 0.35$ MPa, and according to Fig. 3 for the $x = a$ relation $\theta / q = 0.41$. After that $\theta = 0.35 \times 0.063 = 0.022$ and

$$
\xi = 1 / (1 - 0.022) = 1.022.
$$

Increasing the density by 10 - 12\% will cause a decrease in the yield of potatoes by 8\%, and the yield of winter wheat may decrease by 20 - 40\% [2, c. 9 - 12].

The results of the calculations for the accepted values of the soil elastic constants do not differ with the data available in the literature. For example, the average soil pressure at the depth 3a under the wheels of the K-700 tractor according to the literature data [2, p. 35] is 0.07 MPa. According to the proposed method, the average static pressure at depth 3a (Fig. 2) is equal to:

$$
\frac{0.818 + 0.182 + 0.250}{3} \times 0.105 = 0.044 \text{ MPa}.
$$

Here, $q = 0.105$ MPa is the specific pressure on the soil from the front wheels of the K-700 tractor [2, p. 55]. Taking into account the dynamic coefficient, which is equal to 1.32 for the K-700 tractor at a speed of 12 km / h [2, p. 53], the average pressure will be 0.058 MPa. The relative change in volume with the previously accepted mechanical characteristics of the soil $\theta = 0.375$, and the relative increase in its density will be 16\%.

**References**

1. Zotsenko M, Kovalenko V (2003): Engineering geology. Soil mechanics, foundations and foundations: a textbook. Poltava, PNTU, 554.
2. Kravchenko V (1986): Soil compaction by machines. Alma-Ata, Science, 96.
3. Samus V (1982): Fundamentals of the theory of elasticity and plasticity. Moskow, Higher School, 264.
4. Teregulov I (1984): Resistance of Materials and Fundamentals of Theory of Elasticity and Plasticity. Moskow, Higher School, 472.
Основні напруження в ґрунті під колесами працюючої техніки

У статті запропоновано метод визначення закономірностей розподілу напруження і деформацій у масиві ґрунту. Це дозволяє визначити глибину і ступінь ущільнення технології певного типу і зробити раціональний вибір для певних умов обробітку ґрунту. Також дані, отримані в статті, дозволяють правильно підібрати техніку для обробки ґрунту. Однією з актуальних проблем, яка привертає увагу вчених, є захист ґрунту від надмірного ущільнення колесами машин, які працюють на полі. Відомо, що ущільнення ґрунту впливає на його пористість і, отже, на вологість і водопроникність, газообмін і аерацию, температуру і теплопровідність, а також на багато інших фізичних, хімічних і механічних властивостей. Тиск, що виникає під колесами і гусеницями рухомих транспортних засобів, зменшує пористість масиву ґрунту, вищує вологу в бічні шари і, таким чином, перерозподіляє її, викликаючи нерівномірне насичення ґрунту водою. Підвищення теплопровідності викликає двоякий ефект: по-перше, земля нагрівається швидше і глибше, коли в атмосфері тепло, і в той же час швидше охолоджується при падінні температури зовнішнього повітря. В цілому, ущільнення ґрунту є негативним явищем, оскільки воно погіршує основний показник - родючість землі. Тому вивчення механічних явищ в ґрунті, що протікає під вагою технічних засобів, є однією з найважливіших проблем, що виникають в сільськогосподарському виробництві.

Ключові слова: машино-тракторний агрегат, колісні рушії, ущільнення ґрунту.

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