SUPERSYMMETRIC ELECTROWEAK
RENORMALIZATION OF THE $Z$-WIDTH IN THE MSSM
(II)

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ABSTRACT

We address the computation of $\Gamma_Z$ and of the intriguing quantity $R_b$ in the MSSM including full treatment of the Higgs sector. Contrary to previous partial approaches, and due to the possible relevance of the result to the fate of the MSSM, we perform a complete calculation, without approximations. For a pseudoscalar Higgs mass $m_{A^0} > 70\, GeV$ and CDF limits on $m_t$, the bounds on $R_b$ at $1\sigma$ level leave no room to the MSSM to solve the “$R_b$ crisis” for any combination of the parameters, not even admitting the possibility of a light chargino and a light stop of $O(50)\, GeV$; however, for $m_t$ not restricted by CDF, a “tangential” solution exists in the window $2 < \tan \beta < 10$ with a light chargino and stop. In contrast, for a pseudoscalar mass $40\, GeV \lesssim m_{A^0} < 60\, GeV$ and CDF limits on $m_t$, the “$R_b$ crisis” can be solved in a comfortable way, for any SUSY spectrum above the phenomenological bounds, provided $\tan \beta \gtrsim m_t/m_b$. Our general conclusion is that, if there is a “$R_b$ crisis” at all, its solution within the MSSM has to do more with the peculiar structure of the SUSY Higgs sector rather than with the spectrum of genuine supersymmetric particles. In view of the range predicted for $m_{A^0}$, LEP 200 should be able to definitely settle down this question.

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Discovering Supersymmetry (SUSY) would be a fact of paramount importance both theoretically and phenomenologically in the world of elementary particle physics. In the past ten to fifteen years, a lot of effort has been directed to settle down the question of whether SUSY is real or not, and although the question remains unanswered the quest still goes on and on, even with renewed interest, especially with the advent of LEP 100, its subsequent planned upgrading to LEP 200, and also spurred by the prospect of new and more powerful machines in the future: LHC, $e^+e^-(500\text{ GeV})$, .... Whether the finding of SUSY particles—if real at all—will have to await physical production in those big collider experiments, or perhaps some hints of existence might creep earlier through non-negligible quantum effects on physical observables, is not clear for the moment, so we better keep on exploring both possibilities. Here we shall exploit one example of the second possibility, which may help to shed light on SUSY physics through $Z$-decay dynamics. Indeed, at present the cleanest and most accessible laboratory to test possible manifestations of SUSY is LEP. In Part I, whose notation and definitions we shall adopt hereafter, we have studied systematically the potential size of the full virtual contributions to the width of the $Z$ boson, $\Gamma_Z$, from the plethora of “genuine” ($R$-odd) supersymmetric particles of the MSSM; namely, from sleptons, squarks, charginos and neutralinos, with the result that for not too heavy sparticle masses (i.e. not heavier than the electroweak scale), they could be of the order or even larger than the pure SM electroweak corrections—though opposite in sign in most cases. This warns us of the possibility that there could be a remarkable cancellation between the two contributions, and even an overcompensation of the (electroweak) SM corrections by genuine SUSY effects. It also suggests to try to better appreciate these features in particular decay channels, such as e.g. in the partial width of $Z \rightarrow b\bar{b}$, where the genuine SUSY contributions are maximal. However, to definitely assess whether this could be the case or not, we have to take into account also the additional contributions from the Higgs sector of the MSSM. As in Part I, we identify the SM with a “Reference Standard Model” (RSM) in which the Higgs mass is set equal to the mass of the lightest $CP$-even Higgs scalar, $h^0$, of the MSSM. Since in a certain limit ($m_{A^0} \rightarrow \infty$, see later on) the couplings of $h^0$ to fermions and gauge boson are identical to those of the SM Higgs, we may easily subtract out the RSM contribution from the MSSM. In this way the total additional correction from the MSSM with respect to the RSM is given by eq.(6) of Part I, viz.

$$\delta \Gamma_{Z}^{MSSM} = \delta \Gamma_{Z}^{H} + \delta \Gamma_{Z}^{SUSY} ,$$

where $\delta \Gamma_{Z}^{SUSY}$ has been considered in detail in that reference, whereas $\delta \Gamma_{Z}^{H}$ will be taken into account in the present study. Moreover, in this note we shall also consider the full MSSM contribution in a context where mixing effects in the third squark family are
included in the evaluation of $\delta \Gamma^{SUSY}_Z$. These effects were not considered in Part I, since we treated (conservatively) all squarks generations alike. In the present case, however, we will distinguish between the first two generations and the last generation, where mixing effects are most likely to arise. This will prove useful to emphasize the conclusion, obtained from previous studies \cite{6, 7, 8}, that the virtual SUSY corrections could help in reducing the disagreement between theory and experiment for the conflicting ratio \cite{9}

$$R_b^{\text{exp}} = \frac{\Gamma_b}{\Gamma_h} \equiv \frac{\Gamma(Z \to b\bar{b})}{\Gamma(Z \to \text{hadrons})} = 0.2208 \pm 0.0024. \quad (2)$$

The SM prediction, including the variation with the top quark mass within the allowed range of $\delta M_W$ and $\Delta r$, is \cite{10}

$$R_b^{\text{SM}} = 0.2158 \pm 0.0013. \quad (3)$$

If one further incorporates the recently claimed CDF result ($m_t = 174 \pm 10^{+13}_{-12}$ GeV \cite{12}) for the top quark mass it reads \cite{10}

$$R_b^{\text{SM}}|_{CDF} = 0.2160 \pm 0.0006. \quad (4)$$

It is well-known \cite{11} that the SM result decreases with $m_t^2$, due to an overcompensation of the propagator correction by a large, negative, vertex contribution to the $b\bar{b}$ mode

$$\nabla_{V_b} = \delta \rho_{V_b} - 8|Q_b|s^2 \frac{v_b a_b}{v_b^2 + a_b^2} \delta \kappa_{V_b} = - \frac{4}{3} \frac{1}{1 + v_b^2/a_b^2} \Delta \rho \simeq -1.5 \Delta \rho, \quad (5)$$

where the dominant part of $\Delta \rho$ in the SM is

$$\Delta \rho^t = \frac{3 G_F m_t^2}{8 \pi^2 \sqrt{2}}. \quad (6)$$

In comparing theory and experiment we shall consider the two SM results, eqs.(3)-(4), separately \footnote{The announced “evidence” on the top quark mass \cite{12} is for the moment not absolutely compelling and we should be open-minded to all possible eventualities.}. In either case the discrepancy with the experimental data is statistically significant: the SM prediction is $\sim 2\sigma$ below the experimental result \footnote{Although we are aware of the controversy over the measurement of $R_b^{\text{exp}}$ in connection to $b$-tagging and its anticorrelation to $c$-tagging \cite{13}, the matter is not settled at all. Thus we shall take the point of view that there is a “$R_b$ crisis” in the SM and explore its consequences in the MSSM.}. Furthermore, the rather large preferred CDF value for the top quark mass just goes in the opposite direction to reconcile theory with experiment. Fortunately, there is some hope to improve things in the framework of the MSSM, where to start with the fits to $m_t$ lead to a lighter central value $m_t = 162 \pm 9$ GeV \cite{14} compatible with the CDF errors, whereas in the SM the central value increases by as much as about 20 GeV, i.e. closer to the central CDF
mass. We shall therefore take advantage in our analysis of the different values of \( m_t \) at our disposal and in particular of the favourable one corresponding to the MSSM fit.

Notice that the ratio \( R_b \) is insensitive to \( \alpha_s \). Moreover, since it is also essentially independent of the Higgs mass in the SM, we can simply identify the above theoretical predictions on \( R_b^{SM} \) with the RSM result, \( R_b^{RSM} \). Denoting by \( \delta R_b^{MSSM} \) the radiative correction induced on \( R_b^{RSM} \) by the quantum effects \( (1) \) on the various partial widths, we have

\[
R_b^{MSSM} = R_b^{RSM} + \delta R_b^{MSSM},
\]

where in an obvious notation

\[
\delta R_b^{MSSM} = \delta R_b^{SUSY} + \delta R_b^H = R_b^{RSM} \left( \frac{\delta \Gamma_b^{MSSM}}{\Gamma_b^{RSM}} - \frac{\delta \Gamma_h^{MSSM}}{\Gamma_h^{RSM}} \right) .
\]

Therefore, the question arises on whether the extra contributions from the MSSM with respect to the RSM–the MSSM being at present the most predictive framework for physics beyond the SM–can solve or at least soften this conflict between theory and experiment. We feel that this issue is important enough for the present and future credibility of the MSSM to deserve detailed studies from different points of view. In particular we reconsider it within the context of our fully fledged computation of electroweak SUSY one-loop corrections to \( \Gamma_Z \) presented in Part I. In our approach we extend former calculations \( [3, 4, 8] \) by including the full MSSM corrections, not only to the partial width \( \Gamma(Z \rightarrow b\bar{b}) \) but also to all quark channels contributing to \( \Gamma(Z \rightarrow \text{hadrons}) \). We treat the Higgs sector of the MSSM at the one-loop level. Furthermore, our calculation is not just a leading order calculation projecting specific contributions from \( m_t \)-dependent and/or large \( \tan\beta \) Yukawa couplings, but an exact one-loop calculation including both gauge and Yukawa couplings on equal footing and for arbitrary values of \( \tan\beta \). This will be necessary to find out a reduced interval of allowed values for \( \tan\beta \) where to cure or at least to alleviate the above discrepancy. For completeness, we also include for each \( q\bar{q} \) channel the contribution from the terms \( \nabla_{U,Q}^{SUSY} \) on eq.(23) of Part I, which in particular involve the full \( \Delta r^{MSSM} \).

These contributions do not completely cancel in the ratio \( R_b \).

Although the ratio \( (2) \) is practically independent of the Higgs mass in the SM, it turns out that the additional Higgs contributions in the MSSM could play an important role, due to enhanced Yukawa couplings. To this aim, as already advertised, a first step is called for; namely, the computation of the quantity \( \delta \Gamma_Z^H \) on eq.(4). Some comments on previous work in this direction are in order. The Higgs vertex corrections for the \( b\bar{b} \)-channel were first computed in Refs. \( [13, 16] \). In the former, extreme values of the Yukawa couplings were used and the small oblique contributions were neglected; in the latter, the non-oblique corrections were considered in detail in the general unconstrained two-Higgs-doublet-model (2HDM) for the \( b\bar{b} \) and \( \tau^+\tau^- \) modes and the universal part was dealt
with using a large mass splitting approximation. (There are some disagreement in the numerical results between these two references.) We have nonetheless redone ourselves the entire calculation without any of the aforementioned approximations, neither in the treatment of the universal nor in that of the non-universal parts. We perfectly agree with the numerical results of ref.\[16\] for the general 2HDM, but, as noted, use is made of the (one-loop) mass relations in the Higgs sector of the MSSM [17]. In this way we may assess the relative importance of $\delta\Gamma_H^Z$ as a part of the total radiative shift (II). The leading one-loop effects on the Higgs sector can be extracted from the general formulae of Ref.[17] and one finds the following mass spectrum:

$$m_{H\pm}^2 = m_{A_0}^2 + M_W^2 - \frac{1}{4} \omega_t \frac{M_W^2}{m_t^2},$$

$$m_{H^{0,a_0}}^2 = \frac{1}{\omega_t} \left[ (m_{A_0}^2 + M_Z^2 + \omega_t \pm \left( (m_{A_0}^2 + M_Z^2)^2 + \omega_t^2 \right. \right. - 4 m_{A_0}^2 M_Z^2 \cos^2 2\beta + 2 \omega_t \cos 2\beta (m_{A_0}^2 - M_Z^2) \right]^{1/2},$$

(9)

where $m_{A_0}$ is the pseudoscalar mass [4] and

$$\omega_t = \frac{2 N_C \alpha}{4 \pi s^2 M_W^2 \sin^2 \beta} \log \left( \frac{M_{SUSY}^2}{m_t^2} \right),$$

(10)

with $M_{SUSY}^2 = m_{\tilde{t}_L} m_{\tilde{t}_R}$. For $\omega_t = 0$, the tree-level relations of the MSSM Higgs sector are recovered. To subtract the RSM effects one just notes that in the limit $m_{A_0} \to \infty$, $h^0$ behaves like the SM Higgs [4].

The indispensable formulae for the radiative corrections in the on-shell scheme are given in Part I and the computational details are displayed in Ref.[18], so we jump right away to the final numerical results. For the present analysis we present all our results in the framework of Model I as defined in Part I. The reason is simply that the SUSY spectrum from Model II has no chance to solve by itself the “$R_b$ crisis”, since the corresponding sparticles are too heavy (see, however, later on). To start with, we display for completeness [5] in Figs.1a-1b the quantity $\delta M_W^H$, i.e. the additional Higgs corrections to $M_W$ with respect to the RSM both for the tree-level and for the one-loop Higgs sector, where in the latter case we have taken $M_{SUSY} = 1 TeV$. This allows direct comparison of the Higgs effects with the genuine SUSY corrections $\delta M_W^{SUSY}$, whose study we have presented in Ref.[19]. In particular, note that there is a large negative correction (Fig.1b) for small values of $\tan \beta$ and of $m_{A_0}$ as compared to the tree-level correction (Fig.1a). For sfermions and charginos of $O(100)$ GeV, this correction could compensate in part the positive genuine SUSY effect from the sparticle spectrum of Model II, though it represents only a small fraction of the total SUSY correction in Model I (cf. Figs.1,2 of

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\[\text{\textsuperscript{5}}\text{See also the parallel study of ref.20.}\]
Ref. [19]) As a matter of fact, the one-loop Higgs sector gives, unlike the tree-level case, a correction to $M_W$ which is mostly negative and non-negligible for $m_{A^0} < 100 \text{GeV}$. Thus the one-loop relations (9) may help to distinguish between the radiative corrections from the Higgs sectors of the MSSM and of the SM.

The extra effects from the one-loop relations (9), although potentially important for $M_W$, have a limited influence on the corrections to the partial widths of the $Z$ into fermions. They have essentially negligible repercussion on the propagator corrections, which were already very small. Notwithstanding, for the $b\bar{b}$ channel, they may in some cases noticeably shift the non-oblique corrections, which are overwhelming with respect to the oblique contributions. In general the Higgs effects can be important only for those channels where enhanced Yukawa couplings may be involved (cf. eqs.(32),(33) of Part I). Thus we plot on Figs.2a-2b the full quantity (11) for the $b\bar{b}$, $\tau^+\tau^-$ and $\nu_\tau\bar{\nu}_\tau$ channels. The plots include all sorts of oblique and non-oblique effects from sparticles and Higgses. In particular, the SUSY vertex contributions to the $b\bar{b}$ channel were already considered in Refs.[7] and [8] in the Yukawa coupling approximation. We have checked that in this limit we are in good agreement with the numerical plots provided by the latter reference both for charginos and for neutralinos. For non-extreme values of $\tan \beta$, the gauge parts of the SUSY contributions are non-negligible in front of the Yukawa couplings and have to be included too. Remarkably enough, the intermediate $\tan \beta$ region will be essential to the analysis of $R_b$ in the MSSM for large $m_{A^0}$, as will be shown below. The differences introduced by the MSSM Higgses can be appreciated on comparing Figs.2a-2b of this paper with Figs.4a-4b of Part I. Indeed, for pseudoscalar masses in the range $20 \text{GeV} < m_{A^0} < 60 \text{GeV}$ there is a substantial additional, positive, correction for high values of $\tan \beta$, especially for the $b\bar{b}$ mode. For heavier Higgs masses and/or lower values of $\tan \beta$, the correction becomes negative. Asymptotically in $m_{A^0}$ (and very slowly), the $\delta \Gamma^{Z-}_H$-effect in the three decay modes goes away, as we would expect of any MSSM contribution entailing a departure with respect to the RSM. To be precise, in that limit the total vertex Higgs correction in the MSSM should boil down to the corresponding RSM contribution, which is negligible. The existence of the “positiveness region” $20 \text{GeV} < m_{A^0} < 60 \text{GeV}$ was already noticed in Ref.[16] in the context of the general 2HDM. However, while in that framework the large, positive, contributions correspond to a peculiar choice of the free parameters of the model, in the MSSM the wellcome effects appear automatically from the constrained structure of the SUSY Higgs potential. Thus the “$R_b$ crisis” can naturally be solved in the MSSM within the “positiveness region”, as we shall show explicitly. In

\footnote{The structure of the fermion-sfermion-chargino/neutralino coupling, including both gauge and Yukawa couplings in a general mass-eigenstate basis, is given e.g. in eqs.(18)-(19) of Ref.[24]. Detailed plots in $(\mu, M)$-space accounting for the full corrections are provided in Ref.[18].}
fact, part of this region ($m_{A^0} \gtrsim 40 \text{GeV}$) has not yet been convincingly excluded by experiment \cite{2,21} and we shall take advantage of this fact in our analysis.

Aiming at a closer study of the ratio $R_b$ in the MSSM, we plot in Fig.3 contour lines of $\delta \Gamma^H(Z \to b\bar{b})$ in the ($m_{A^0}, \tan \beta$)-plane. The extremely slow decoupling of the negative contribution to $\delta \Gamma^H(Z \to b\bar{b})$ for $m_{A^0} \to \infty$ is also manifest here. It is worth noticing from Fig.3 that the large, positive, genuine SUSY contribution from $\delta \Gamma^{SUSY}_Z$ in the $\tan \beta < 1$ region (which we remarked in Part I) turns out to be cancelled and even overridden by the big, negative, contributions from $\delta \Gamma^H_Z$ over a wide range of $m_{A^0}$. The upshot is that the total MSSM correction \cite{2} in the $\tan \beta < 1$ region is negative, contrary to naive expectations from the analysis of $\delta \Gamma^{SUSY}_Z$ alone. Treating the Higgs sector at 1-loop gives differences in $\delta \Gamma^H(Z \to b\bar{b})$ which can be of order of $-1 \text{MeV}$ with respect to the corrections from the Higgs sector at tree-level. The extra correction basically comes from the vertices, only in the region around $m_{A^0} = 90 \text{GeV}$ and for large $\tan \beta$. These differences, small as they are, are of the same order of magnitude—and opposite in sign—to the typical genuine SUSY effects on the leptonic modes (cf. Fig 4b of Part I), and therefore they could result in some cancellation at the level of the total quantum correction to $\Gamma_Z$. In general we find that the one-loop effects on the Higgs masses have little impact on $\Gamma_Z$.

Next we analyze numerically, and in a systematic way, the possible solutions to the “$R_b$ crisis” both in the “intermediate Higgs mass range” ($40 \text{GeV} \lesssim m_{A^0} \lesssim 70 \text{GeV}$) and in the “heavy Higgs mass range” ($m_{A^0} > 70 \text{GeV}$). We start from the latter, which has already been addressed in the literature from a different approach \cite{3}. Here the MSSM might find itself in deep water and we have to struggle a lot more to rescue it from wreckage. Simple inspection of Fig.3, combined with Figs.2,4 of Part I, suggests that if the “$R_b$ crisis” has any chance to be solved in this range it has to be handled in the “Higgs desert” $2 \lesssim \tan \beta \lesssim 40$, where indeed the negative effects from Higgses are practically non-existent, say less than $1 \text{MeV}$ (in absolute value), as compared to the typical SUSY corrections to the $q\bar{q}$ channels. Above and below a well-defined band in the ($m_{A^0}, \tan \beta$)-plane, the Higgs corrections are negative definite, sizeable enough, and thus responsible for a lower as well as for an upper limit on $\tan \beta$. It is one of the main purposes of this work to show that the effective range of admissible values for $\tan \beta$ can still be drastically reduced.

When analyzing the extra positive corrections from SUSY to $R_b^{RSM}$ in the heavy Higgs mass range, we immediately realize that the main effect comes from the non-oblique contributions $\delta \Gamma^{SUSY}(Z \to b\bar{b})$ from the genuine SUSY sector. This contribution becomes relevant provided one of the chargino and stop masses is light enough. In this respect, let us insist on the possibility, not yet ruled out experimentally in a compelling way, that the
lightest R-odd partner of the top quark, \( \tilde{t}_1 \), could be much lighter than the other squarks (even \( < M_Z/2 \) \[21, 22, 23\]). Thus since heavy stops are disfavoured in this case, it follows that the parameter \( M_{SUSY} \) on eq.(10) is much smaller than 1 TeV and hence the one-loop relations are indistinguishable from the tree-level ones.

Clearly, an efficient computer code facing a systematic exploration of positive contributions to \( \delta R_b^{SUSY} \) (more specifically, contributions capable of restoring the quantity \( \delta \) within 1\( \sigma \) of the experimental result) is what is needed. The scatter plot method of Ref.[6] is one example. However, in that reference, \( \tan\beta \) and \( m_{A^0} \) were definitely fixed at just a couple of values and only the rest of the parameters were varied. In our case, we use a simple and straightforward “lattice” method in which we include both of them as additional parameter axes. This will prove very useful to explore the range of allowed values for \( \tan\beta \) and \( m_{A^0} \). Thus we first set up “seed intervals” over all SUSY parameter axes and endow them with a reasonably fine subdivision in order to generate a sufficiently large number of candidate points (above \( 10^8 \)). Any of such points is defined by a 8-tuple

\[
(\tan\beta, m_{A^0}, M, \mu, m_{\tilde{\nu}}, m_{\tilde{u}}, m_{\tilde{b}}, M_{LR}).
\]

(11)

Here \( m_{\tilde{\nu}} \) is the sneutrino mass, which enters through the oblique corrections, and \( m_{\tilde{u}} \) stands for the common mass of the \( T^3 = +1/2 \) squark components of the two first generations. However, as announced, we shall treat the third squark generation (\( \tilde{t}, \tilde{b} \)) apart. In particular, we consider the effect of L-R mixing for the \( \tilde{t} \) squarks and parametrize the stop mass matrix in the usual way [1]

\[
M^2_{\tilde{t}} = \left( \begin{array}{ccc}
M^2_{b_L} + m_t^2 + \cos 2\beta(\frac{1}{2} - \frac{2}{3}s^2) M^2_Z & m_t M_{LR} & m_t M_{LR} \\
m_t M_{LR} & M^2_{t_R} + m_t^2 + \frac{2}{3} \cos 2\beta s^2 M^2_Z & M^2_{t_R} + m_t^2 + \frac{2}{3} \cos 2\beta s^2 M^2_Z
\end{array} \right),
\]

(12)

where we have used the fact that \( SU(2)_L \)-gauge invariance requires \( M_{t_L} = M_{b_L} \) and thus the first entry of this matrix can be written in terms of the sbottom mass \( m_{\tilde{b}_L} \) (cf.eq.(25) of Part I), already included in (11). To illustrate the effect of the mixing it will suffice to choose the soft SUSY-breaking mass \( M_{t_R} \) in such a way that the two diagonal entries of \( M^2_{\tilde{t}} \) are equal– the mixing angle is thus fixed at \( \pi/4 \)– and the remaining free parameter, \( M_{LR} \), is just the last component of the 8-tuple (11). For the mixing parameter, however, we have the proviso

\[
M_{LR} \leq 3 m_{\tilde{b}}, \tag{13}
\]

which roughly corresponds to a well-known necessary, though not sufficient, condition to avoid false vacua, i.e. to guarantee that the \( SU(3)_c \times U(1)_{em} \) minimum is the absolute one [25]. With all the parameters of the 8-tuple defined, the ranges that have been effectively
explored for each one of them are the following:

\[
\begin{align*}
0.7 < \tan \beta < 70 & \quad \quad 40 \text{GeV} < m_{A^0} < 150 \text{GeV} \\
0 < M < 250 \text{GeV} & \quad \quad -200 \text{GeV} < \mu < 200 \text{GeV} \\
50 < m_{\tilde{g}} < 500 \text{GeV} & \quad \quad 90 < m_{\tilde{u}} < 500 \text{GeV} \\
90 < m_{\tilde{b}} < 500 \text{GeV} & \quad \quad |M_{LR}| < 3m_{\tilde{b}}.
\end{align*}
\]

The final intervals recorded here are sufficiently stable against progressive stretching. As a matter of fact, they are the result of a number of consecutive widenings of original, narrower, seed intervals until clear stabilization was achieved.

Collecting the previous conditions, all SUSY masses are well determined within the framework of Model I. Obviously, our analysis has unequal sensitivity to the 8 parameters in (11), and this has been taken into account in the number and distribution of points assigned to the various axes. Furthermore, in order to proceed in an efficient way (i.e. without wasting a lot of CPU time on obviously sterile points) we first select (“flag 1”) a subset of points (11) that give rise to at least one light chargino, one light neutralino and one light stop. For example, a typical setting would be to require (using the notation of Part I) that there exists at least one index triad \((i, \alpha, a)\) such that

\[
48 \text{GeV} < M_{\Psi^\pm_i} < 60 \text{GeV}; \quad M_{\Psi^0_\alpha} > 20 \text{GeV}; \quad 45 \text{GeV} < m_{\tilde{t}_a} < 60 \text{GeV}. \tag{15}
\]

Finding flag-1-successful points is trivial and very little time consuming. However, once they are found, the points enter the full computer flow evaluating the radiative corrections and a massive numerical analysis is required to ascertain, among the many combinations of SUSY parameters that passed flag 1 (several millions), those combinations (“flag 2”) that fall within the experimental 1\(\sigma\) range for \(R^{\text{MSSM}}_b\) both for the results (4) and (3) corresponding, respectively, to plugging or unplugging the CDF limits on \(m_t\). Points that successfully pass the two flags are to be called “admissible points”. Whenever one such point is found, our code projects the corresponding value of \(\tan \beta\) and in this way we are able to generate a range of admissible values for this parameter, if any. In particular, using this procedure for \(m_t\) within CDF limits no point was found for stop masses in the range (15). Only for \(m_\tilde{t}\) well below 45 GeV a small set of admissible points was detected at the single value \(\tan \beta = 4\). Unavoidably one is forced to go beyond 1\(\sigma\) to be able to generate admissible points for \(m_{\tilde{t}} > 45 \text{GeV}\); for example, the range \(2 < \tan \beta < 11\) is allowed at 1.25\(\sigma\).

On the other hand, in the CDF-unrestricted case (3), we find admissible points already at 1\(\sigma\) in the range \(2 < \tan \beta < 10\). All of them reach the experimental bounds for \(R^{\text{MSSM}}_b\) “tangentially” from below, as shown in Fig.4. In this figure, which is rather laborious to compute, we plot the maximal contributions to \(R^{\text{MSSM}}_b\) as a function of \(\tan \beta\) when all the parameters of the 8-tuple (11) are varied with \(m_{A^0}\) in the heavy Higgs mass range.
It should be mentioned that the alleged “tangential solutions”, which are exclusively associated to light charginos and stops, are compatible with the experimental bound on the total width (cf. eq.(2) of Part I), as we have checked explicitly. These solutions are obtained (automatically by our code) by picking points very close to the boundary of the allowed region in the \((M, \mu)\)-plane for each \(\tan \beta\). Indeed, near the boundary, the total SUSY contribution to the \(Z\)-width is minimum (cf. Fig.1 of Part I) while at the same time \(R_{b}^{MSSM}\) is maximum. The reason for this ambivalent behaviour is that the large, negative, self-energy corrections from the “ino” sector near that boundary practically cancel in the difference \((8)\) while the positive SUSY vertex corrections to \(\Gamma(Z \to b \bar{b})\) are maximum.

Finally, we face systematically the computation of \(R_{b}^{MSSM}\) in the intermediate Higgs mass range \(40 \text{ GeV} \lesssim m_{A} \lesssim 70 \text{ GeV}\) for \(m_{t}\) within CDF limits. Here we wish to show that the “\(R_{b}\) crisis” may comfortably be solved in the MSSM for any SUSY spectrum above the current phenomenological bounds (hence without resorting to too light stops and charginos), provided \(\tan \beta\) is large enough. To this end we first plot in Fig.5a the quantity \(R_{b}^{RSM} + \delta R_{b}^{H}\) versus \(\tan \beta\) for various pseudoscalar masses in the aforementioned range. This situation corresponds to \(R_{b}^{MSSM}\) with a SUSY spectrum fully decoupled (cf. eq.\((8)\)). Since in the range under consideration the Higgs contribution to the \(b \bar{b}\) mode is large, we have to be careful in dealing with \(\delta R_{b}^{H}\) by at the same time keeping an eye on the corrections to the total width. Thus in computing Fig.5a we have imposed the condition that the total width, given by

\[
\Gamma_{Z}^{MSSM} = \Gamma_{Z}^{RSM} + \delta \Gamma_{Z}^{MSSM},
\]

should not exceed to 1\(\sigma\) the experimental value \(\Gamma_{Z}^{exp}\) (cf. eq.(2) of Part I) with all the errors (experimental and theoretical) added in quadrature. As a result of this bound, all the curves in Fig.5a are cut off at some point (some of them beyond the range explicitly shown) before exiting the allowed experimental band for \(R_{b}^{exp}\) at 1\(\sigma\). In spite of the \(\Gamma_{Z}\) bound, it is clear from Fig.5a that a well defined solution to the “\(R_{b}\) crisis” exists in the shaded area. Therefore, the lower bound \(\tan \beta > \sim 36\) ensues.

Next we repeat a similar analysis when switching on the sparticle spectrum. Here the computation is more difficult since we have to perform a systematic exploration of the parameter space \((11)\), e.g. using the lattice method described above. Another complication is that we have to separate the case of “light charginos” (defined as those within the mass interval assumed on eq.\((15)\)) from the case of “heavy charginos” (> 60 GeV). The reason for the separate treatment is that in the light chargino case, as already noticed when discussing the “tangential solutions”, the global SUSY contribution to the \(Z\)-width is minimum and so the analysis of \(R_{b}^{MSSM}\) is not interfered by the bound on \(\Gamma_{Z}^{exp}\). On the other hand, in the heavy chargino case the large negative self-energy cor-
rections disappear, hence the SUSY contribution to $\Gamma_{Z}^{\text{MSSM}}$ is boosted up significantly (cf. Fig.1 of Part I) and thereupon the analysis of $R_b^{\text{MSSM}}$ becomes severely restricted by $\Gamma_{Z}^{\text{exp}}$. In this letter we shall limit ourselves to display the results corresponding to a heavy chargino case: specifically in the intriguing situation where they cannot be pair produced at LEP 200, i.e. $M_{\Psi^\pm} \simeq 100\,\text{GeV}$. Although we shall briefly comment on the analysis of $R_b$ for light charginos, and also for “intermediately heavy” charginos (viz. $60\,\text{GeV} < M_{\Psi^\pm} < 100\,\text{GeV}$), we shall defer a detailed exposition of these results for Ref.[18]. The resulting curves for heavy charginos are shown in Fig.5b. In this figure, whose numerical computation is highly CPU-time-demanding, we project the maximum contribution to $R_b^{\text{MSSM}}$ as a function of $\tan\beta$ when varying all the parameters of the 8-tuple (11) (except $m_{A_0}$, which is fixed for each curve) with the condition that the full sparticle spectrum generated lies just out of the possibilities of pair production at LEP 200. In practice this means that we required for sfermions and charginos

$$m_{\tilde{l}^\pm}, m_{\tilde{q}^a}, M_{\Psi^\pm} \gtrsim 100\,\text{GeV}. \quad (17)$$

Again the restriction from $\Gamma_{Z}^{\text{exp}}$ was imposed on the corresponding theoretical results (16).

Two novel features emerge as compared to Fig.5a, namely:

a) A solution to the “$R_b$ crisis” exists in the shaded area of Fig.5b, but this time for $\tan\beta \gtrsim 32$, i.e. starting about 4 units below the case with the Higgses alone;

b) The upper cut-off from $\Gamma_{Z}^{\text{exp}}$ on the solution curves of Fig.5b is so stringent that $m_{A_0}$ and $\tan\beta$ become strongly correlated; for instance, one reads from Fig.5b that for $m_{A_0} = 50\,\text{GeV}$ the only admissible values for $\tan\beta$ lie in the narrow window $54 \lesssim \tan\beta \lesssim 58$.

We point out that for intermediately heavy charginos the correlation between $m_{A_0}$ and $\tan\beta$ is even higher than in Fig.5b, due to the severe bound from $\Gamma_{Z}^{\text{exp}}$ on the large vertex contributions to the $b\bar{b}$ mode. Quite in contrast, in the light chargino region, the correlation disappears and the lower bound on $\tan\beta$ diminishes 12 units with respect to the previous case, i.e. $\tan\beta > 20$ [18], which is still remarkably high. In the other extreme, namely for heavier and heavier sparticle spectrum, one reaches asymptotically the situation in Fig.5a, where it is worth noticing that it corresponds in good approximation to the one expected for Model II. In fact, remember that this model is characterized by a rather heavy SUSY spectrum and it is closely related to the class of MSSM’s with radiatively induced breaking of the gauge symmetry [1]. From this point of view the solution to the “$R_b$ crisis” in the intermediate Higgs mass regime is theoretically preferred to the “tangential solution” obtained for heavy Higgses in Model I. Finally, we mention that we have detected only small differences in the previous results in the case where the superpartners of the top quark are very heavy ($\simeq 1\,\text{TeV}$), that is to say, we have veri-
fied that the one-loop relations (9) do not alter significantly the shape of the large Higgs contributions to the $b\bar{b}$ mode in the “positiveness region”. Completion of our numerical search for admissible points (11) in all the cases described above took several hundred hours of direct CPU time in an IBM(RISC/6000) and an “α”-computer (DEC 3000/300 AXP).

To summarize, we have studied the full set of MSSM corrections (1) to the partial widths of the $Z$ boson into fermions in the context of phenomenological and supergravity inspired models. In particular, we have specialized our general framework to find out regions of parameter space where the MSSM could help to cure an apparent discrepancy between $R_b^{\exp}$ and $R_b^{SM}$ - the alleged “$R_b$ crisis” in the SM. Although further, and more robust, experimental information is needed before jumping into conclusions, the following considerations may tentatively be put forward in the meanwhile:

i) In the heavy Higgs mass range ($m_{A^0} > 70 \text{ GeV}$), we basically agree with the early results of Refs. [6, 7, 8], in the sense that both a light stop and a light chargino of $\mathcal{O}(50) \text{ GeV}$ are needed to try to rescue the MSSM from the impasse. However, in the light of an extended multiparametric one-loop analysis of both $R_b$ and $\Gamma_Z$ we enlarge the scope of the conclusions as follows:

ii) On general grounds we may state that for small statistical fluctuations around the numbers (2)-(4), in the heavy Higgs mass range the experimental result $R_b^{\exp}$ can only be approached “tangentially” (from below) by the MSSM. In particular, for $m_t$ within CDF limits, we find very unlikely that the MSSM could account for $R_b^{\exp}$ at $1\sigma$.

iii) If we, instead, base the previous analysis on the CDF-unrestricted case, eq.(3), and the top quark mass happens to be around the central value of the MSSM fit (specifically $m_t = 160 \text{ GeV}$), we find admissible points already at $1\sigma$ in the interval $2 < \tan \beta < 10$, and only in this interval.

iv) As far as the intermediate Higgs mass range is concerned, our main conclusion is that the Higgs sector of the MSSM could by itself comfortably solve the “$R_b$ crisis” in the “positiveness region” $40 \text{ GeV} \lesssim m_{A^0} < 60 \text{ GeV}$. A solution also exists in this region if we superimpose on the Higgs contribution any SUSY spectrum above the present phenomenological bounds. However, if the charginos lie in the intermediate range $60 \text{ GeV} \lesssim M_{\Psi^\pm} \lesssim 100 \text{ GeV}$ and $m_t$ is bound within CDF limits, then, the previous “comfortable solution” is traded for a “cut-off solution” (which in a sense is also “tangential”). A characteristic feature of this solution is that the parameters $m_{A^0}$ and $\tan \beta$ become so correlated that once we are given one of them the other gets “predicted” within only a small margin. In general, for $m_{A^0}$ in the intermediate mass range, the solution space always projects onto a segment of $\tan \beta$ starting approximately at the suggestive value $\tan \beta = m_t/m_b \simeq 35$, which is still far below the perturbative limit $\tan \beta \lesssim 70$. 

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v) If the pseudoscalar Higgs is heavy enough, the upper bound derived on \( \tan \beta \) in the heavy Higgs mass region gives little hope for the recent \( t - b - \tau \) Yukawa coupling \( SO(10) \) unification models, which tend to favour very large values for that parameter. However, if the pseudoscalar Higgs is intermediately heavy, then, these models are definitely the favoured ones from the point of view of \( R_b \).

vi) Of the two frameworks that we have explored for the sparticle spectrum (Models I and II), only the more phenomenological one (Model I) could solve—and only “tangentially”—the “\( R_b \) crisis” both in the heavy and in the intermediate Higgs mass region. Model II, instead, has no chance unless a Higgs in the intermediate mass region is invoked, in which case the solution would be comfortable (not “tangential”). Thus, surprisingly enough, Model II, which is theoretically more sounded (in the sense that it is closely related to SUSY GUT’s) could be, in our opinion, the most natural and appealing scenario in spite of being initially rejected due to its rather heavy sfermion spectrum.

In short, we are tempted to believe that a possible solution to the “\( R_b \) crisis” within the MSSM has to do more with the Higgs sector of the model than with its spectrum of genuine SUSY particles\(^8\). Thus, if there is a “\( R_b \) crisis” at all, LEP 200 should be able to discover a supersymmetric Higgs, otherwise the MSSM could be in trouble. We became aware of a preprint by J.D. Wells, C. Kolda and G. L.Kane (UM-TH-94-23) and a preprint by J.E. Kim and G.T. Park (SNUTP 94-66) where similar questions are addressed from a different point of view.

Acknowledgements: One of us (JS) is thankful to Wolfgang Hollik for useful discussions and gratefully acknowledges the hospitality at the Institut für Theoretische Physik der Universität Karlsruhe during a visit. He is also grateful to M. Martínez for helpful conversations on the experimental status of \( R_b \). This work has been partially supported by CICYT under project No. AEN93-0474. The work of DG has also been financed by a grant of the Comissionat per a Universitats i Recerca, Generalitat de Catalunya.

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\(^8\)A place where one could find the reverse situation, i.e. potentially large effects from the genuine supersymmetric part of the MSSM while at the same time rather handicapped effects from the Higgses, is in the physics of the top quark decay, as shown in Ref.[24]. The large effects (comparable to QCD) arise for big \( \tan \beta \), even admitting heavy squarks and moderately heavy charginos, i.e. a situation compatible with a possible MSSM solution to the “\( R_b \) crisis” in the intermediate Higgs mass region.
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Figure Captions

- **Fig.1** (a) Additional corrections $\delta M_W^H$ from the tree-level Higgs sector of the MSSM as a function of the pseudoscalar mass; (b) As in case (a), but for the one-loop Higgs sector with $M_{SUSY} = 1\,\text{TeV}$.

- **Fig.2** (a) Full correction $\delta \Gamma^M_{Z,\text{MSSM}}$ to the $b\bar{b}$ mode as a function of $\tan \beta$, for different pseudoscalar masses and the same spectrum as in Fig.4 of Part I; (b) As in case (a), but for the $\tau^+\tau^-$ and $\nu_\tau\bar{\nu}_\tau$ modes.

- **Fig.3** Contour plots of $\delta \Gamma^H(Z \rightarrow b\bar{b})$ in the ($m_{A^0}, \tan \beta$)-plane.

- **Fig.4** Best “tangential solution” in the heavy Higgs mass region and for the CDF-unrestricted case. The top quark mass is $160\,\text{GeV}$ and the sfermion spectrum is from Model I under the optimizing conditions [13]. The shaded area starts at $R_b^{exp}$ at 1$sigma$.

- **Fig.5** (a) The “comfortable solution” for various pseudoscalar masses (in GeV) in the intermediate Higgs mass region and for a very heavy SUSY spectrum; (b) The “cut-off solution” for the same pseudoscalar masses as before but for a SUSY spectrum just above the LEP 200 discovery range. In both cases the shaded area starts at $R_b^{exp}$ at 1$sigma$. 

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This figure "fig1-1.png" is available in "png" format from:

http://arxiv.org/ps/hep-ph/9410311v5
Fig. 1

(a) \text{tree}

\begin{align*}
(a) & \tan \beta = 0.7 \\
(b) & \tan \beta = 2.0 \\
(c) & \tan \beta = 3.5 \\
(d) & \tan \beta = 10. \\
(e) & \tan \beta = 70.
\end{align*}

(b) 1-loop (M_{SUSY} = 1 TeV)
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Fig. 2
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Fig. 5