Quantum Optomechanics with Single Atom

Yue Chang, H. Ian, and C. P. Sun
Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing, 100190, China

The recently increasing explorations for cavity optomechanical coupling assisted by a single atom or an atomic ensemble have opened an experimentally accessible fashion to interface quantum optics and nano (micro) -mechanical systems. In this paper, we study in details such composite quantum dynamics of photon, phonon and atoms, specified by the triple coupling, which only exists in this triple hybrid system: The cavity QED system with a movable end mirror. We exactly diagonalize the Hamiltonian of the triple hybrid system under the parametric resonance condition. We find that, with the rotating-wave approximation, the hybrid system is modeled by a generalized spin-orbit coupling where the orbital angular momentum operator is defined through a Jordan-Schwinger realization with two bosonic modes, corresponding to the mirror oscillation and the single mode photon of the cavity. In the quasi-classical limit of very large angular momentum, this system will behave like a standard cavity-QED system described by the Jaynes-Cummings model as the angular momentum operators are transformed to bosonic operators of a single mode. We test this observation with an experimentally accessible system with the atom in the cavity with a moving mirror.

PACS numbers: 42.50.Tx, 03.67.Bg, 32.80.Qk, 85.85.+j

I. INTRODUCTION

Nowadays, there are an increasing number of researches on cavity optomechanical systems assisted by atoms or atomic ensembles \[ 1 \leq 2 \]. Such hybrid systems show a convergence between quantum optics and nano (micro) -mechanical systems. In this context, a Fabry-Perot cavity is applied to form an approximate standing light wave while one of the mirrors at the end of the cavity is allowed to oscillate. Furthermore, the novel quantum natures are discovered in this composite system by placing an atomic ensemble confined in a gas chamber inside the cavity. With the helps of the atomic ensemble, theoretical explorations have been made to show the possibilities to create not only the entanglement of the cavity field and a macroscopical object \[ 4 \leq 5 \leq 6 \leq 7 \], i.e., a mirror, but also the entanglement of atom-light-mirror \[ 1 \leq 8 \]. In this paper, we will consider this atom assisted optomechanical (AAOPM) system with the strong coupling of a single atom to the photon field inside the cavity, which is modified by the moving end mirror of the cavity (see Fig. 1). We find that a three body coupling term of photon, phonon and atom will play the crucial role in the quantum dynamics of the triple system.

The triple system we will refer to contains a two-level system interacting with a single mode electromagnetic field inside the cavity with an oscillating mirror. Due to the vibration of the cavity length, which is conventionally thought to induce the light pressure term \[ 8 \leq 9 \leq 10 \], we illustrate that the vibrating length can also result in the triple coupling term of photon-atom-mirror. Under a certain condition concerning the frequencies and the coupling strength in the so-called parametric resonance, we can exactly diagonalize the Hamiltonian to obtain the eigenstates and the eigenvalues. Here, the eigenstate is the direct product of the mirror’s state and two body hybrid system of the cavity field and a macroscopical object \[ 1 \leq 4 \leq 5 \leq 6 \leq 7 \], i.e., a mirror, but also the entanglement of atom-light-mirror \[ 1 \leq 8 \]. In this paper, we will consider this atom assisted optomechanical (AAOPM) system with the strong coupling of a single atom to the photon field inside the cavity, which is modified by the moving end mirror of the cavity (see Fig. 1). We find that a three body coupling term of photon, phonon and atom will play the crucial role in the quantum dynamics of the triple system.

![FIG. 1: The schematic of the atom assisted optomechanical system. It contains an optical cavity ended with a fixed mirror A and a slightly moving mirror B which is attached in a spring. Inside the cavity there is a two level atom.](image-url)
two dressed states. We find that, though the decoherence factor depends on the initial state of the mirror, the LE has nothing to do with this initial coherent state. This means the mirror’s initial position only affect the decoherence factor in the form of a phase factor.

The above condition for exact solution seems too special to be realized, thus we consider a more general case with the rotating-wave approximation, which only requires a set of matching frequencies rather than the coupling strength, among the atom, the cavity field and the mirror. In this case, our model is reduced to the generalized spin-orbit coupling model where the spin is referred to as the internal energy level of the atom, while the orbit is depicted by the quasi-orbital angular momentum defined by two bosonic modes (the mirror oscillation and the single-mode photon of the cavity) through the Jordan-Schwinger representation.

It is well known that the quantum system with a large angular momentum $L$ can be regarded as a classical rotor when the angular momentum approaches infinity in the classical limit $\text{[12]}$. This is because the component variations $\Delta J_x$, $\Delta J_y$ and $\Delta J_z$ become vanishingly small in comparison with the large angular momentum $L$. When the angular momentum is large enough, but not infinite, the ladder operators of any large angular momentum can behave as the creation and annihilation operators of a single-mode boson $\text{[13, 14]}$. This point can be seen from the Holstein-Primakoff transformation straightforwardly. This case is named the quasi-classical limit and its significance in many-body physics can be understood as the low-energy excitation above the ordered ground states. This quasi-classical reduction of large angular momentum has been extensively studied and applied in quantum storage $\text{[15, 16, 17]}$. Here, we study this quasi-classical reduction by referring to the experimentally accessible parameters in triple hybrid system. In this triple system, when the frequency of the electromagnetic field almost matches the eigenfrequency of the energy level spacing of the atom plus the mirror oscillation, the photon and the mirror oscillation are coupled to form a composite object, which exactly behaves as an angular momentum. Then our triple system is modeled by the generalized spin-orbit coupling model to demonstrate the similarities and differences to that of the J-C model. In Sec. VII, we summarize our results.

II. MODELING THE TRIPLE COUPLING OF ATOM-PHOTON-MIRROR

In this section, we study an experimentally accessible AAOPM system, as illustrated in Fig. 1. This system consists of three parts: an atom, the photons inside a cavity, and a movable end mirror. We show that such a hybrid system can be modeled by a spin-orbit coupling system, where the orbital angular momenta are realized by the phonon of the mirror dressed by the photon of the light field inside the cavity.

The single mode electric field inside the cavity along the $x$-axis is quantized as

$$E(x_0, t) = \varepsilon a \sin kx_0 + h.c.,$$  \hspace{1cm} (1)

where $x_0$ is the position of the atomic center of mass, and $\varepsilon = \sqrt{\omega_0/\varepsilon_0 V}$. The frequency of the cavity field is dependent of the cavity length, $\omega_0 = k = 2\pi/l_0$; $\varepsilon_0$ is the dielectric constant in vacuum; $l_0$ and $V$ are the length and the volume of the cavity, respectively. Here, we omit the polarization of the field, but this will not affect our final conclusion for a practical system. When the length of the cavity slightly changes from $l_0$ to $l_0 + x$ due to the mirror’s displacement $x$ (see Fig. 2), the electric field becomes

$$E(x_0, t) \approx \varepsilon a \sin kx_0 - (\eta' x_0 a + h.c.),$$  \hspace{1cm} (2)

where $\eta' = (\sin kx_0 + kx_0 \cos kx_0) \varepsilon / l_0$. Then, the total Hamiltonian reads

$$H = \omega_0 a^\dagger a + \omega_m b^\dagger b - \xi (b + b^\dagger) a^\dagger a + \omega_e S_z + \left[ ga S_+ + \eta (b + b^\dagger) a S_+ + h.c. \right].$$  \hspace{1cm} (3)

FIG. 2: Schematic of the origin of the triple coupling. When the length of the cavity oscillates, besides the light pressure term, the displacement of the mirror will affect the coupling strength between the atom and the light. The triple coupling term appears with this.
where $a^\dagger$ ($b^\dagger$) is the creation operator of the oscillating mode of the cavity field (mirror); $S_z$, $S_+$ and $S_-$ are the spin operators, which represent the transitions among the atomic inner states. $g = -\mu e \sin kx_0$ is the atomic-position-dependent coupling strength between the cavity field and the atom, where $\mu$ is the electric-dipole transition matrix element. $\xi = \omega_0/\sqrt{2 M \omega_M}$, where $\omega_M$ is the frequency of the mirror’s oscillation and $M$ is the mass of the mirror, describes the light pressure, while $\eta = \eta \mu/\sqrt{2 M \omega_M}$ denotes the coupling strength of “three body” due to the vibration of the mirror. With some experimentally feasible parameters, there exists the situation where $\eta$ is on the same order of magnitude of $\xi$, for which the strong coupling region is reached (e.g., $g = \omega_0 = 10^{15} Hz$ and $kx_0 = \pi/2$, then $\eta = -\xi$). In this case, the triple coupling term

$$V_T = \eta \langle b + b^\dagger \rangle a \vert e \rangle \langle g \vert + h.c. \quad (4)$$

should not be neglected. In the following discussion, we will focus on this strong coupling case.

It follows from the Hamiltonian in Eq. (3) that, when $\sin kx_0 \to 0$, i.e., $g \to 0$, the J-C type interaction $g a S_z + h.c.$ vanishes, but the three-body interaction remains. Near the photon-phonon resonance case where $\omega_c + \omega_M - \omega_0 \approx 0$, the rotating-wave approximation reduces the Hamiltonian to

$$H_{\text{RWA}} = \omega_0 a^\dagger a + \omega_M b^\dagger b + \omega_c S_z + \eta \langle b^\dagger a S_+ + h.c. \rangle. \quad (5)$$

In the following discussions, we invoke the Jordan-Schwinger representation of the SO(3) group [18]

$$L_+ = a^\dagger b, \quad L_- = ab^\dagger, \quad L_z = \frac{1}{2} (a^\dagger a - b^\dagger b), \quad (6)$$

where the commutation relations of the angular momenta

$$[L_+, L_-] = 2L_z, \quad [L_z, L_+] = L_+, \quad [L_z, L_-] = -L_- \quad (7)$$

are satisfied due to the generic commutation relations between the $a$ and $b$ bosons. Then, the Hamiltonian can be rewritten as

$$H_{\text{RWA}} = \Omega N + \kappa L_z + \omega_c S_z + \eta (L_- S_+ + L_+ S_-), \quad (8)$$

where $N = a^\dagger a + b^\dagger b$, $\Omega = (\omega_0 + \omega_M)/2$, and $\kappa = \omega_0 - \omega_M$.

We remark that the three body interaction in the AAOPM system can be approximately modeled with the x-y coupling

$$\eta (L_- S_+ + L_+ S_-) = 2\eta (L_z S_x + L_y S_y). \quad (9)$$

This is a kind of “spin-orbit coupling” referred to as Paschen-Back effect [19]. The “orbital” angular momentum defined by $L_\pm$ and $L_z$ essentially results from the joint excitation of photon and phonon. Physically, this excitation can be understood as a kind of effective mechanical oscillation of the mirror, which is dressed by the single mode photon. Such kinds of dressed bosons just satisfy the angular momentum algebra.

### III. Exact Solutions for Parametric Resonance

From Eq. (3), obviously the particle number operator $N = a^\dagger a + S_z$ is conserved, i.e., $[N, H] = 0$. Then, in the subspace $\{ n_a + 1, g \}, \{ n_a, e \}$, where $\{ n_a, g (e) \}$ denotes that the photon is prepared in a Fock state $| n_a \rangle$ while the atom in the ground (excited) state. In the following we consider the situation in which the eigen-equation of the Hamiltonian in Eq. (3) can be solved exactly. To this end, we first show that the Hamiltonian is formally expanded as follows:

$$H_{n_a} = h \langle b, b^\dagger \rangle + H'_{n_a}, \quad (10)$$

where

$$h \langle b, b^\dagger \rangle = \omega_c + \omega_0 n_a + \omega_M b^\dagger b - \xi n_a \langle b + b^\dagger \rangle \quad (11)$$

and

$$H'_{n_a} = \sqrt{n_a + 1} \left( \frac{\Delta - \xi \langle b + b^\dagger \rangle}{\sqrt{n_a + 1}} \right) g - \eta \langle b + b^\dagger \rangle \quad (12)$$

In Eq. (12), $\Delta = \omega_0 - \omega_c$ is the atom-photon detuning. The above argument shows that in the subspace the total system can be reduced to a spin-boson model defined by Eqs. (10-12).

Now let us temporarily leave the above concrete system to consider a more general spin-boson model with the Hamiltonian

$$H_{\text{SP}} = h \langle b, b^\dagger \rangle + W \langle S; b, b^\dagger \rangle, \quad (13)$$

where $h \langle b, b^\dagger \rangle$ is a function that only depends on the boson model and $W \langle S; b, b^\dagger \rangle$ is spin-dependent. There is a seemingly trivial proposition for this spin-boson system: If the coupling can be factorized as

$$W \langle S; b, b^\dagger \rangle = f \langle b, b^\dagger \rangle M \langle S \rangle, \quad (14)$$

with $f \langle b, b^\dagger \rangle (M \langle S \rangle)$ depending on boson (spin) only, then $H_{\text{SP}}$ can be exactly diagonalized through the diagonalizations of the two pure boson systems with branch Hamiltonians

$$H^{(+, -)}_{\text{SP}} = h \langle b, b^\dagger \rangle + \lambda_{(+, -)} f \langle b, b^\dagger \rangle, \quad (15)$$

where $\lambda_+$ and $\lambda_-$ are the eigenvalues of the C-number coefficient matrix $M \langle S \rangle$ in the basis $\{ \{ + \}, \{ - \} \}$. The proof of this proposition is rather straightforward and we only need to diagonalize $M \langle S \rangle$ first.

Next we return to the concrete example.

We explore when $H'_{n_a}$ can be factorized to $f \langle b, b^\dagger \rangle M_{n_a}$, where $f \langle b, b^\dagger \rangle$ is a function of $b$ and $b^\dagger$, and $M_{n_a}$ is a C-number matrix. Actually when the detuning $\Delta$ and the three coupling coefficients have the relation

$$g \xi = \Delta \eta, \quad (16)$$
which we call the parametric resonance, the Hamiltonian indeed becomes the form of Eq. (14):

\[ H_{na} = \hbar \left( b \right) b^\dagger + f \left( b \right) b^\dagger M_{na}, \]  

where

\[ f \left( b \right) = \Delta - \xi \left( b + b^\dagger \right), \]  

and

\[ M_{na} = \left( \frac{1}{\sqrt{n_a + 1} \eta / \xi} \sqrt{n_a + 1} / \xi \right). \]  

Thus, to diagonalize the Hamiltonian \( H_{na} \), all we need to do is to diagonalize the matrix \( M_{na} \) and the left quadratic part formed by \( b \) and \( b^\dagger \). Then, the eigenvalues of \( H \) are obtained as

\[ E_{j,n_a,n_b} = \omega_0 \left( n_a + \frac{1}{2} \right) + \frac{1}{2} \omega_e + \left( - \right)^j \frac{1}{2} R_{na} \Delta \]

\[ + n_b \omega_M - \alpha_{jn_a}^2, \]  

for \( j = 1, 2 \), where

\[ R_{na} = \sqrt{1 + 4 \eta^2 \left( n_a + 1 \right) / \xi^2}, \]  

and

\[ \alpha_{jn_a} = \frac{\eta}{2 \omega_M} \left( 2n_a + \left( - \right)^j R_{na} + 1 \right). \]  

Here, \( n_a \) \( n_b \) represents the quantum number of the photons (phonons). Correspondingly, the eigenstates of the AAOPM system are \( \left| j_{n_a} \otimes n_b \right\rangle_{jn_a} \), where the photon dressed state

\[ \left| 1_{n_a} \right\rangle = \cos \theta_{n_a} \left| n_a + 1, g \right\rangle + \sin \theta_{n_a} \left| n_a, e \right\rangle, \]  

and

\[ \left| 2_{n_a} \right\rangle = - \sin \theta_{n_a} \left| n_a + 1, g \right\rangle + \cos \theta_{n_a} \left| n_a, e \right\rangle, \]  

are defined by the mixing angle \( \theta_{n_a} \):

\[ \tan \theta_{n_a} = \frac{2 \eta \sqrt{n_a + 1}}{\xi - \xi R_{na}}, \]  

while the mirror’s states \( \left| n_b \right\rangle \) are

\[ \left| n_b \right\rangle_{jn_a} = \left( b^\dagger - \alpha_{jn_a} \right)^n_{n_b} D_b \left( \alpha_{jn_a} \right) \left| 0 \right\rangle, \]  

with the displacement operator \( D_b \left( \alpha_{jn_a} \right) = \exp \left[ \alpha_{jn_a} \left( b^\dagger - b \right) \right]. \)

If \( \xi = \eta = 0 \), the atom-assisted optomechanical coupling model goes back to the J-C model, and the eigenvalues, together with the eigenstates, degenerate to that of the J-C model. However, due to the vibration of the mirror, there emerge fruitful results in our model due to the complex three-body coupling. First, we examine the realization of the parametric resonance condition, in experiments. Substituting the experimental feasible parameters into the parametric resonance condition, we know that it is easily satisfied if the detuning \( \Delta \) is adjusted properly to adapt to different positions of the atom. In experiments, \( \xi \) and \( \eta \) can reach \( 10^5 \)Hz, while \( g \) is on the order of \( 10^{15} \)Hz, thus \( \Delta \) can be on any order that lies on the atomic position. In the special case when \( \sin kx_0 = 0 \), i.e., \( g = 0 \), \( \Delta = 0 \) is sufficient to meet the condition in Eq. (10).

It is observed from Eq. (20) that the terms in the second line obviously differ from the eigenvalues of the J-C model. The first term is the mirror’s eigenvalue, and the second one (without the sign) is expanded as

\[ \frac{\xi^2 n_a^2}{\omega_M^2} + \frac{\xi^2}{4 \omega_M^2} \left( 4n_a + (-)^j R_{na} + 1 \right) \left( (-)^j R_{na} + 1 \right), \]  

where the first term \( \xi^2 n_a^2 / \omega_M \) describes the energy of the light pressure term \( \xi \left( b + b^\dagger \right) a^\dagger a \) [6, 21], but the left terms are induced by the atom-assisted optomechanical coupling.

### IV. CONDITIONAL DYNAMICS FOR DECOHERENCE

In this section we demonstrate the parametric resonance will lead to a conditional dynamics with respect to two superpositions of atomic inner states \( \left\{ \left| + \right\rangle, \left| - \right\rangle \right\} \) for the spin part. Obviously this is a non-demolition Hamiltonian. From the above argument about the exact solvability of the AAOPM system, we can find that the operator-valued Hamiltonian matrix

\[ H = \begin{bmatrix} H^{(+)}_{SP} & 0 \\ 0 & H^{(-)}_{SP} \end{bmatrix} \]  

is diagonalized with respect to the basis \( \left\{ \left| + \right\rangle, \left| - \right\rangle \right\} \) for the spin part. Obviously this is a non-demolition Hamiltonian with respect to the basis vectors \( \left| + \right\rangle \) and \( \left| - \right\rangle \), and thus results in the corresponding decoherence. Driven by this non-demolition Hamiltonian, the factorized initial state for the cavity QED system

\[ \left| \psi \left( 0 \right) \right\rangle = \left( C_+ \left| + \right\rangle + C_- \left| - \right\rangle \right) \otimes \left| \varphi \right\rangle \]  

will evolve into an entanglement state

\[ \left| \psi \left( t \right) \right\rangle = C_+ \left| + \right\rangle \left| \varphi_+ \left( t \right) \right\rangle + C_- \left| - \right\rangle \left| \varphi_- \left( t \right) \right\rangle, \]  

where

\[ \left| \varphi_\pm \left( t \right) \right\rangle = e^{-iH_{SP}^{(+)} t} \left| \varphi \right\rangle, \]  

and the extent of decoherence due to this quantum entanglement is characterized by the so-called decoherence factor

\[ D\left( t \right) = \left\langle \varphi_+ \left( t \right) \left| \varphi_- \left( t \right) \right\rangle \right\rangle = Tr \left( \rho \left( 0 \right) e^{iH_{SP}^{(+)} t} e^{-iH_{SP}^{(-)} t} \right), \]  

(31)
and its norm square \( L(t) = |D(t)|^2 \) is the so-called Loschmidt echo [22].

In the context of quantum chaos, the Loschmidt echo characterizes the sensitivity of evolution of quantum system in comparison with the butterfly effect in classical chaos. Starting from the same initial state, the quantum system is separately driven by two slight different Hamiltonians. Quantum chaos is implied by the much larger differences in the two corresponding final states; namely, their overlap (Loschmidt echo) vanishes to illustrate the dynamical sensitivity of the quantum chaos system.

Next we return to the concrete system.

Consider the time evolution of the system when the initial state is as follows:

\[
\psi(0) = \sum_{j,n_a} \lambda_{jn_a} |jn_a\rangle \otimes |\beta\rangle_b, \tag{32}
\]

where \(|\beta\rangle_b\) is a coherent state of phonon that satisfies

\[
b |\beta\rangle_b = \beta |\beta\rangle_b, \tag{33}
\]

\(|jn_a\rangle, j = 1, 2,\) is the dressed state mentioned in last section, and \(\lambda_{jn_a}\) is the weight of each dressed state. Therefore, at time \(t\) the wave function of the total system is

\[
\psi(t) = \sum_{j,n_a} \lambda_{jn_a} |jn_a\rangle \otimes e^{-\beta C_{jn_a} t} \left[(\beta - \alpha_{jn_a}) e^{-j\omega_M t} + \alpha_{jn_a}\right]_b
\]

where

\[
C_{jn_a} = \frac{1}{2} \omega_\epsilon + \omega_0 \left(n_a + \frac{1}{2}\right) + \frac{(-j)^2}{2} R_n a \Delta
- \frac{\xi^2}{4\omega_M} \left(2n_a + (-j)^2 R_n + 1\right)^2. \tag{35}
\]

The mirror’s motion will result in the collapse of the decoherence, with the Loschmidt echo (LE) being

\[
LE_{jn_a,m_a}^{j_1,j_2} = \left| b \langle \beta | e^{iH_j z_{jm_a} t} e^{-iH_{j_21} m_a t} |\beta\rangle_b\right|^2
= \exp\left[2 \left(\Delta_{jn_a,m_a}^{j_1,j_2}\right)^2 (\cos \omega_M t - 1)\right], \tag{36}
\]

where

\[
\Delta_{jn_a,m_a}^{j_1,j_2} = \alpha_{jn_a} - \alpha_{jm_a}. \tag{37}
\]

We notice that \(\beta\) does not play any role in the LE. Note that the mirror’s initial coherent state evolves to another coherent state, and \(\beta\) only determine the initial position of the center of the wavepacket \((x|\beta\rangle_b\). Thus the overlap of the two wavepackets \(e^{iH_j z_{jm_a} t} |\beta\rangle_b\) and \(e^{-iH_{j_21} m_a t} |\beta\rangle_b\) is independent of \(\beta\). Physically, this fact shows that the decoherence of the cavity-QED system is irrelevant to the phonon excitations of the mirror if its wavepacket is Gaussian.

In Fig. 3, for different photon-phonon excitations, we plot the time evolution of the Loschmidt echo with the parameters set to \(\omega_0 = 10^{10}\text{Hz}, \eta = 10\omega_0/\sqrt{2M\omega_M} = 10\xi, \delta_0 = 1\mu\text{m}, M = 10^{-10}\text{kg}, \omega_M = 10^{10}\text{Hz}\). From Fig. 3, we see that all the curves have the same period, i.e., \(2\pi/\omega_M\), and the larger the difference of the photon number \(|n_a - m_a|\) is, the larger the amplitudes of the curve are. We remark that the large photon number means a classical electromagnetic field, and thus the quantum decoherence of the atomic inner states reflects the classical transition of optical field from the quantum regime.

V. GENERALIZED SPIN-ORBIT COUPLING IN COMPARISON WITH JAYNES-CUMMINGS MODEL

In Sec. II, we have derived the generalized spin-orbit coupling model under the rotating-wave approximation, which can be rewritten as

\[
H_{\text{RWA}} = \Omega N + \kappa L_z + \omega_\epsilon S_z + 2\eta \left(\vec{L} \cdot \vec{S} - L_z S_z\right). \tag{38}
\]

Here, \(\kappa\) and \(\omega_\epsilon\) characterize the coupling of the angular momentum and the spin to the external field, respectively; \(2\eta\) is the coupling strength of the spin-orbit.

This model can be studied by exactly diagonalizing the model Hamiltonian in Eq. 38 within its invariant subspace spanned by

\[
|l, m, e\rangle = |l, m\rangle \otimes |\uparrow\rangle \tag{39}
\]

and

\[
|l, m + 1, g\rangle = |l, m + 1\rangle \otimes |\downarrow\rangle. \tag{40}
\]

Here, \(|l, m\rangle\) is the standard angular momentum basis, while \(|\uparrow\rangle\) and \(|\downarrow\rangle\) denote the spin-up and spin-down vectors, respectively. In this basis, the spin-orbit coupling Hamiltonian in Eq. 38 is reduced to a quasi-diagonal matrix with an \(2 \times 2\) block

\[
H_{\text{RWA}} = 2\Omega l + 2\eta \begin{pmatrix} m + \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & gm - \delta \end{pmatrix}, \tag{41}
\]
where $\Delta = \omega_c - \delta$, and
\[
g_{lm} = 2\eta \sqrt{l(l+1) - m(m+1)}. \tag{42}
\]
Then it can be further diagonalized to obtain the eigenvectors
\[
|\pm lm\rangle = \cos \theta_{lm} |l, m, e\rangle + \sin \theta_{lm} |l, m+1, g\rangle, \tag{43}
\]
and
\[
|\mp lm\rangle = -\sin \theta_{lm} |l, m, e\rangle + \cos \theta_{lm} |l, m+1, g\rangle, \tag{44}
\]
with the corresponding eigenvalues
\[
E_{\pm lm} = 2\Omega l + \left(m + \frac{1}{2}\right) \kappa \pm \frac{1}{2} R_{lm}, \tag{45}
\]
where $\tan \theta_{lm} = g_{lm}/(\delta + R_{lm})$ and $R_{lm} = \sqrt{g_{lm}^2 + \delta^2}$.

From the spectrum structure of the AAOPM system described above, we demonstrate that the triple coupling system can realize the entanglement between an orbital angular momentum and a spin. The $z$-component of the total angular momentum is conserved. Thus, while the orbital angular momentum is flipped from down (up) to up (down), the spin will make a reverse flip.

Now we can consider the quasi-classical limit of the above spin-orbit coupling model for $l$ large enough with low excitation. Obviously, the above basis vector in this limit becomes a Fock state, i.e., $|n\rangle$, where $n = l + m$, and $n/l \ll 1$; while $\tan \theta_{lm} \rightarrow \tan \theta_n = g_n/(\delta + R_n)$. Correspondingly, the eigenstates become the dressed states in the usual J-C model with the eigenvalues
\[
E_{\pm lm} \rightarrow E_{\pm n} = \left(n + \frac{1}{2}\right) \kappa \pm \frac{1}{2} R_n, \quad i = 1, 2, \tag{46}
\]
where $g_n = 2\eta \sqrt{2l(l+1)}$ and $R_n = \sqrt{g_n^2 + \delta^2}$.

We remark that the system we considered in the limit above can also be described by the J-C model
\[
H_{J-C} = \kappa a^\dagger a + \omega_c S_z + \eta \sqrt{2l} \left(aS_x + a^\dagger S_-\right). \tag{47}
\]
The correspondence between the Fock state $|n\rangle$ and the standard angular momentum basis is
\[
|n\rangle \leftrightarrow |l, l - n\rangle. \tag{48}
\]

Actually, the above equivalence of spin-orbit coupling model and J-C interaction can be found directly by considering the Holstein-Primakoff mapping
\[
L^+ = a^\dagger \sqrt{2l - a^\dagger a}, \quad L^- = L^+_\dagger, \quad L_z = a^\dagger a - l \tag{49}
\]
in the large $l$ limit.

Next we come back to the practical physics of the AAOPM system. Then the joint states $|l, m, e(g)\rangle = |l, m\rangle \otimes |1, (1)\rangle$ is re-expressed in terms of the two Fock states $|l + m\rangle_a$ and $|l - m\rangle_b$ as
\[
|l, m, e(g)\rangle = |l + m\rangle_a |l - m\rangle_b |e(g)\rangle. \tag{50}
\]
Here $|l + m\rangle_a$ and $|l - m\rangle_b$ represent the states with $l + m$ photons and $l - m$ phonons of the mirror’s vibration mode respectively.

In Fig. 4 and Fig. 5, with different values of $l$ and $m$, we plot $E_{\pm lm}$ as the functions of $\omega_c$. Here, we take the physical parameters as $\omega_0 = 1.9 \times 10^{15}$Hz, $\eta = \omega_0/l\sqrt{2M\omega_M}$, $l_0 = 1\mu$m, $M = 10^{-10}$kg, $\omega_M = 10^9$Hz.

As shown in the figures 4 and 5, when $l$ is fixed, the spectrum diagram of the AAOPM system looks quite like that of the J-C model’s [23]. However, in general, its number of energy levels are much more than that in the J-C model’s within the same energy range. Furthermore, we can see from the several lowest levels that there exist small differences under different values of $l$, such as $E_{+1, -1}$ vs. $E_{+1000, -1000}$, $E_{-, 1, 0}$ vs. $E_{-, 1000, -999}$ and so on. Accordingly, the larger the value of $l$ (the orbital angular momentum) is, the closer the spectrum is to the corresponding ones in the J-C model. For evidence, we have plotted the curves with a much more larger range of the variable $\omega_c$ that is not valid in our rotating wave approximation that requires $|\omega_c - \omega_0| \ll \omega_M$. 

FIG. 4: Schematic of the relation of the eigenvalues of the Atom-Photon-Mirror coupling system with the spacing frequency of the two-level atom. Here we take $m$=-1, 0, 1 when $l$=1. Each real line represents $\varepsilon_{+lm}$ while the dashed with the same color represents $\varepsilon_{-lm}$.

FIG. 5: Schematic of the lowest 5 levels when $l$=1000.
VI. QUASI-CLASSICAL DYNAMICS VIA JAYNES-CUMMINGS MODEL

In the above sections, we have shown the similarity between the triple hybrid system and the J-C model in their energy spectra. Now we continue to consider this similarity in quantum dynamics, which is referred to as the so-called quasi-classical one as we make the analysis for very large angular momentum.

We consider generally the system described by the Hamiltonian Eq. (38) is initially prepared in the state represented by the density matrix

\[ \rho (0) = \sum_{ijkl} \lambda_{ijkl} |\psi_{ij}(0)\rangle \langle \psi_{kl}(0)|, \]  

where the joint states \( |\psi_{ij}(0)\rangle = |\alpha\rangle_a |\beta\rangle_b |e\rangle \) denote the initial factorized structure of the triple system. According to the similarity between the generalized L-S coupling system and the triple coupling system we mentioned in the last section, the density matrix for the time evolution reads

\[ \rho (t) = \sum_{ijkl} \lambda_{ijkl} |\psi_{ij}(t)\rangle \langle \psi_{kl}(t)|, \]  

where the branch wavefunctions

\[ |\psi_{ij}(t)\rangle = e^{-i\omega_{ij}t} \left[ \lambda_{ij}^c (t) |i\rangle_a |j\rangle_b |e\rangle - \lambda_{ij}^p (t) |i+1\rangle_a |j-1\rangle_b |g\rangle \right], \]  

for \( \omega_{ij} = \omega_0 i + \omega_M j + \delta/2 \), are determined by the time dependent parameters defined by

\[ \lambda_{ij}^c (t) = \cos \frac{\Omega_{ij} t}{2} - i \cos 2\phi_{ij} \sin \frac{\Omega_{ij} t}{2}, \]  

\[ \lambda_{ij}^p (t) = i \sin 2\phi_{ij} \sin \frac{\Omega_{ij} t}{2}, \]  

\[ \tan \phi_{ij} = \frac{2\eta \sqrt{j (i+1)}}{\omega_e - \delta + \Omega_{ij}}, \]  

\[ \Omega_{ij} = \sqrt{4\eta^2 [j (i+1)] + (\omega_e - \delta)^2}. \]  

It is noticed that the above Eqs. (53,54) have forms similar to that for the J-C model in the limit that \( i/j \ll 1 \). We remark that this limit means a low excitation of the mirror's vibration mode and one atom in the excited (ground) state. Therefore the probability \( p_n(t) \) that \( n \) photons are measured is

\[ p_n(t) = \sum_j \lambda_{nj;j} \left( \lambda_{nj}^p (t) + \cos^2 2\phi_{nj} s_{nj}^a (t) \right) \]

\[ + |\lambda_{nj;j-1}^p|^2 \sin^2 2\phi_{nj} s_{nj}^a (t), \]  

where \( c_{nj}(t) = \cos \Omega_{nj} t/2, \) and \( s_{nj}(t) = \sin \Omega_{nj} t/2 \).

Another important quantity is the population inversion \( W(t) \) depending on the probability amplitudes \( \lambda_{ij}^c (t) \) and \( \lambda_{ij}^p (t) \) as

\[ W(t) = \sum_{i,j} \lambda_{ij}^c (t)^2 - \lambda_{ij+1,j-1}^p (t)^2 \]

\[ = \sum_{i,j} \lambda_{ij}^c [c_{ij}^2 (t) + \cos 4\phi_{ij} s_{ij}^a (t)]. \]  

Note that if the initial state of the mirror is the vacuum state, i.e., \( \lambda_{ijkl} \propto \delta_{00} \delta_{00} \), then it follows from Eqs. (58) and (59) that \( p_n(t) = \sum_j \lambda_{nj;j} \) and \( W(t) = 1 \), both of which are time independent no matter which state the light field is initially in. This result can be explained as follows: with the rotating-wave approximation, we only hold the slowly varying terms in the original Hamiltonian to obtain effective Hamiltonian. These terms make a transition from the atom's upper level state \( |e\rangle \) to the ground state \( |g\rangle \), together with a decrement of the quanta of the mirror’s vibration mode and an increment of the photon number, or vice versa. Thus when initially the mirror is in vacuum and the atom is in the excited state, the total state can not evolve to the “dressed state”, but only stays in the initial state accompanied by a dynamical phase factor.

The above obtained results are very similar to that of the J-C model: \( p_n(t) \) and \( W(t) \) also contain many Rabi oscillations with various frequencies, and in different initial states, \( p_n(t) \) and \( W(t) \) behave differently.

Next we consider that the mirror is initially in a thermal state, while the photon field is in one of several different states: a thermal state, a Fock state \( |n_0\rangle_a \) and a coherent state \( |\alpha\rangle_a \). We obtain different initial parameters as follows:

\[ \lambda_{\text{thermal}}^{ij} = \frac{e^{\beta \omega_0} - 1}{e^{\beta \omega_0 (j+1)} - 1} \frac{e^{\beta \omega_M (j+1)} - 1}{e^{\beta \omega_M (j+1)} + 1}, \]  

\[ \lambda_{\text{Fock}}^{ij} = \delta_{n_0} \frac{e^{\beta \omega_M} - 1}{e^{\beta \omega_M (j+1)}}, \]  

\[ \lambda_{\text{coherent}}^{ij} = \exp \left( -|\alpha|^2 \right) \frac{e^{2\beta \omega_a}}{t!} \frac{e^{\beta \omega_M} - 1}{e^{\beta \omega_M (j+1)}}, \]  

where \( \beta = 1/k_B T, \) \( T \) is the temperature.

In Figs. 6, 7 and 8, we plot the evolution of \( W(t) \) in the three initial states mentioned above with the parameters
we studied the physically intrinsic relation between the
generalized L-S coupling system and the J-C model, i.e.,

\[ W(t) \]

\[ t \ (10^{-4} \text{s}) \]

FIG. 6: Time evolution of the population inversion \( W(t) \) for
an initially thermal state for both the cavity field and the
mirror.

\[ T = 1 \text{K}, M = 10^{-10} \text{kg}, l_0 = 1 \mu\text{m}, \omega_0 = \eta = 1.9 \times 10^{15} \text{Hz}, \]
\[ \omega_e = \omega_0 - 0.999\omega_M, \omega_M = 10^9 \text{Hz}, n_0 = 10, \alpha = 10. \]

It can be seen from the figures that in each case, as time
increases, collapses and revivals appear cyclically, but the
time duration in which each collapse and each revival take
place differs from each other because of different \( \lambda_{ij;ij} \)
that represents the weight of the Rabi oscillation with
fixed frequency. This behavior of collapse and revival
of inversion is repeated with increasing time, with the
amplitude of Rabi oscillations decreased and the time
duration in which the revival takes place increased and
ultimately overlapping with the earlier revival.

Note that the temperature is so low that
\[ \exp (\beta\omega_0) - 1) / \exp [\beta\omega_0 (i + 1)] \rightarrow \delta_{i0}, \]
and thus the case described in Fig. 6 reflects the phenomenon
that the atomic transition between the upper and the
lower levels can happen even when the light field is
initially prepared in the vacuum state. This is obviously
a purely quantum effect to prove the role of vacuum.

In Fig. 7 we observe that even if the cavity field in a
Fock state, the collapse and revival appear explicitly.
This case differs from the J-C model based collapse and
revival phenomenon, in which the evolution of inversion
is just a cosine curve when the field in a Fock state.

VII. CONCLUSION AND REMARKS

We have shown a AAOPM coupling in the triple hybrid
system composing of atoms, a cavity field and a movable
mirror and discovered that under parametric resonance
condition, this complicated model can be solved exactly.
Furthermore, we have demonstrated that this triple hy-
brid system can be modeled by generalized L-S coupling
under the rotating-wave approximation. It is shown that
the composite object formed by the cavity-field-dressed
mirror acts like an orbital angular momentum. Then

when the orbital angular momentum is large enough, the
former is quite like the latter. Same to the generalized
L-S coupling system, in quasi-classical limit, the ladder
operators behave as the bosonic operators and thus the
large angular momentum can be regarded as “excitons”
in the low excitation limit. We also investigated some
characteristic properties of the J-C model in our triple
hybrid system and discovered their similarities and dif-
ferences.

When preparing this paper we find a paper [24], where
we studied the physically intrinsic relation between the
generalized L-S coupling system and the J-C model, i.e.,

ACKNOWLEDGMENTS

C. P. Sun acknowledges supports by the NSFC with
Grants No. 10474104, No. 60433050, and No. 10704023,
NFRPCNo. 2006CB921205 and 2005CB724508.
[1] C. Genes, D. Vitali, and P. Tombesi, Phys. Rev. A 77, 050307 (2008).
[2] D. Meiser and P. Meystre, Phys. Rev. A 73, 033417 (2006).
[3] H. Ian, Z. R. Gong, Yu-xi Liu, C. P. Sun and Franco Nori, Phys. Rev. A 78, 013824 (2008).
[4] D. Vitali, S. Gigan, A. Ferreira, H. R. Böhm, P. Tombesi, A. Guerreiro, V. Vedral, A. Zeilinger, and M. Aspelmeyer, Phys. Rev. Lett. 98, 030405 (2007).
[5] W. Marshall, C. Simon, R. Penrose, and D. Bouwmeester, Phys. Rev. Lett. 91, 130401 (2003).
[6] S. Bose, K. Jacobs, and P. L. Knight, Phys. Rev. A 59, 3204 - 3210 (1999).
[7] S. Mancini and P. Tombesi, Phys. Rev. A 49, 4055 (1994).
[8] S. Mancini, V. Giovannetti, D. Vitali, and P. Tombesi, Phys. Rev. Lett. 88, 120401 (2002).
[9] Z. R. Gong, H. Ian, Yu-xi Liu, C. P. Sun, Franco Nori, arXiv:0805.4102 (2008).
[10] M. Bhattacharya and P. Meystre, Phys. Rev. Lett. 99, 073601 (2007).
[11] P. Zhang, X. F. Liu, and C. P. Sun, Phys. Rev. A 66, 042104 (2002).
[12] C. P. Sun, Phys. Rev. A 48, 898 (1993).
[13] Y. X. Liu, N. Imoto, Ş. K. özdemir, G. R. Jin, and C. P. Sun, Phys. Rev. A 65, 023805 (2002).
[14] G. R. Jin, P. Zhang, Yu-xi Liu, and C. P. Sun, Phys. Rev. B 68, 134301 (2003).
[15] C. P. Sun, Y. Li and X. F. Liu, Phys. Rev. Lett. 91, 147903 (2003).
[16] Y. Li and C. P. Sun, Phys. Rev. A 69, 051802 (2004).
[17] L. He, Y. X. Liu, S. Yi, C. P. Sun, and F. Nori, Phys. Rev. A 75, 063818 (2007).
[18] J. J. Sakurai, Modern Quantum Mechanics (Addison-Wesley, Reading, Ma, 1994).
[19] L. D. Landau and E. M. Lifshitz, Quantum Mechanics (Non-relativistic Theory) (Butterworth-Heinemann, Oxford, 1977) Third Edition.
[20] D. Meiser and P. Meystre, Phys. Rev. A 74, 065801 (2006).
[21] S. Bose, K. Jacobs, and P. L. Knight, Phys. Rev. A 56, 4175 - 4186 (1997).
[22] H. T. Quan, Z. Song, X. F. Liu, P. Zanardi, and C. P. Sun, Phys. Rev. Lett. 96, 140604 (2006).
[23] M. Orszag, Quantum Optics: Including Noise Reduction, Trapped Ions, Quantum Trajectories and Decoherence (Springer-Verlag, Berlin Heidelberg, 2000).
[24] X. X. Yi, H. Y. Sun, and L. C. Wang, arXiv:0807.2703 (2008).