Abstract

We have examined the physics and the experimental feasibility at the AGS of various kaon decay processes in which the polarization of a muon in the final state is measured. Valuable information on CP violation, the CKM matrix or new physics can be obtained with these measurements and therefore they are well motivated. In particular, models of non-standard CP violation that produce the baryon asymmetry of the universe could also produce effects observable in these measurements. Limits from measurements such as the neutron and electron electric dipole moment, and $\tilde{\epsilon}$ in neutral kaon decays do not eliminate all of these models. We have made a more detailed examination of the measurement of the out of the plane muon polarization in $K^+ \rightarrow \mu^+ \pi^0 \nu$ decays. With our current knowledge of the AGS kaon beams and detector techniques it is possible to measure this polarization with an error approaching $\sim 10^{-4}$. Such an experiment would be well justified since the sensitivity is well beyond the current direct experimental limit ($5.3 \times 10^{-3}$) and the projected sensitivity ($< 10^{-3}$) of the currently running experiment at KEK in Japan.

Introduction

We have examined the possibility of measuring various muon decay asymmetries that are sensitive to P, T or CP symmetries; these are tabulated in Table 1. Experimentally, CP
violation has only been observed in the neutral kaon system so far. Although a theoretical description of the CP-violation in the neutral kaon system exists through the complex phase in the Standard Model CKM matrix, part or all of these phases could be consequences of deeper causes that have so far eluded experiments. Over the last decade experiments at FNAL and CERN directed towards the measurement of the direct $K_0^0 \to \pi\pi$ transition or $\varepsilon^\prime/\varepsilon$ have been inconclusive in revealing the true nature of CP-violation. Over the next decade ambitious efforts towards understanding CP-violation and the CKM matrix elements are planned with new $\varepsilon^\prime$ experiments and B-factories. The importance of these efforts is undeniable, yet it must also be important to investigate the possibility that some or all of the CP-violation comes from effects outside the minimal Standard Model, particularly the CKM matrix.

The CPT invariance of local quantum field theories requires that CP violation is equivalent to T-violation. Therefore, it would be particularly interesting to look for direct violation of T-invariance outside the neutral kaon system.

It should also be noted that CP-violation is required to generate the observed baryon asymmetry of the universe, and it is now accepted that the CP-violation embodied in the CKM matrix does not have sufficient strength for this purpose [1]. Physics beyond the Standard Model that could generate the baryon asymmetry can also generate CP or T violating muon polarizations in the kaon decay modes in Table 1.

### Table 1: The decay modes and the polarization asymmetries or correlations of interest.

| Decay | Correlations | Symmetries tested |
|-------|--------------|-------------------|
| (1) $K^+ \to \pi^0 \mu^+\nu$ | $\vec{s}_\mu \cdot (\vec{p}_\mu \times \vec{p}_\pi)$ | T |
| (2) $K^+ \to \mu^+\nu\gamma$ | $\vec{s}_\mu \cdot (\vec{p}_\mu \times \vec{p}_\gamma)$ | T |
| (3) $K_L \to \mu^+\mu^-$ | $\vec{s}_\mu \cdot \vec{p}_\mu$ | P, CP |
| (4) $K^+ \to \pi^+\mu^+\mu^-$ | $\vec{s}_\mu \cdot \vec{p}_\mu$ | P |
| (5) | $\vec{s}_\mu \cdot (\vec{p}_{\mu^+} \times \vec{p}_{\mu^-})$ | T |
| (6) | $(\vec{s}_\mu \cdot \vec{p}_\mu)\vec{s}_\mu \cdot (\vec{p}_{\mu^+} \times \vec{p}_{\mu^-})$ | P, T |
Table 2: The decay modes and asymmetries discussed by the working group. The rest of the columns are: the known branching ratio, the estimated Standard Model value, the value due to final state interactions, the maximum possible value allowed by non-standard physics, and the theoretical reference.

| Asym. Mode | Branch. Fraction | Standard Model | Final State Int. | Non-SM value | Ref. |
|------------|-----------------|----------------|------------------|--------------|-----|
| (1) $K^+ \rightarrow \pi^0\mu^+\nu$ | 0.032 | 0.0 | $\sim 10^{-6}$ | $\leq 10^{-3}$ | 2 |
| (2) $K^+ \rightarrow \mu^+\nu\gamma$ | $5 \times 10^{-3}$ | 0.0 | $\sim 10^{-3}$ | $\leq 10^{-3}$ | 14 |
| (3) $K_L \rightarrow \mu^+\mu^-$ | $7 \times 10^{-9}$ | $\sim 10^{-4}$ | 0.0 | $\leq 10^{-2}$ | 17, 18 |
| (4) $K^+ \rightarrow \pi^+\mu^+\mu^-$ | $5 \times 10^{-8}$ | $\sim 10^{-2}$ | – | – | 21–27 |
| (5) | 0.0 | $\sim 10^{-3}$ | $\sim 10^{-3}$ | 28, 23 |
| (6) | $\sim 6 \times 10^{-2}$ | $\sim 0.0$ | $\sim 0.1$ | 28, 23 |

$K^+ \rightarrow \pi^0\mu^+\nu$

The transverse or out of plane muon polarization in this decay has recently been analyzed by many authors [2, 3, 23]. The out of plane polarization is expected to be zero to first order in the Standard Model because of the absence of the CKM phase in the decay amplitude. It has been shown that any arbitrary models involving effective V or A interactions cannot produce this type of polarization. Therefore, the existence of a non-zero value of this polarization will be a definite signature of new physics beyond these models. In particular, some multi-Higgs and leptoquark models could produce such a polarization. In multi-Higgs models a charged Higgs particle mediates an effective scalar interaction that interferes with the Standard Model decay amplitude; in such models the polarization could be as large as $10^{-3}$ without conflicting with other experimental constraints including the measurements of the neutron electric dipole moment and the branching fraction for $B \rightarrow X\tau\nu$, or $b \rightarrow s\gamma$ [4–5]. Irreducible backgrounds, i.e., final state interactions (FSI), to the out of plane polarization in this decay are expected to be small ($\sim 10^{-6}$) and therefore can be ignored [3].

The best previous experimental limits were obtained almost 15 years ago with both neutral [7] and charged kaons [8] at the BNL-AGS. The experiment with $K^+$ decays produced a measurement of the transverse polarization, $P_T^{\mu} = 0.0031 \pm 0.0053$. The combination of both experiments could be interpreted as a limit on the imaginary part of the ratio of the
hadron form factors, \( \text{Im}\xi = \text{Im}(f_-/f_+) = -0.01 \pm 0.019 \). This limit is mostly independent of theoretical models and the experimental acceptance. By using the approximate formula, \( P_T^\mu \approx 0.183 \times \text{Im}\xi \), one may reinterpret the above measure of \( \text{Im}\xi \) as a combined limit on the polarization, \( P_T^\mu \approx -0.00185 \pm 0.0036 \). This 1980 era measurement was based on \( 1.2 \times 10^7 K^0_L \) and \( 2.1 \times 10^7 K^+ \) decays to \( \mu^+\pi\nu \) and was limited by statistics and backgrounds.

Currently an experiment is in progress at the KEK-PS, E246 \cite{9}, to measure \( P_T^\mu \) with a new technique of using a stopping \( K^+ \) beam and measuring the muon decay direction without spin precession. They expect to reach a sensitivity of \( 9 \times 10^{-4} \ (\text{Im}\xi < 4 \times 10^{-3}) \) with \( 1.8 \times 10^7 \) events. The experiment will try to minimize systematics by using the cylindrical symmetry of the apparatus and by using the backward-forward \( \pi^0 \) symmetries of the decay at rest. The disadvantage of the stopping technique, however, is the low \( K^+ \) stopping rate. Nevertheless, the results of this experiment will be very valuable to future experiments.

A new experiment has been designed at the BNL-AGS to perform this measurement with an error on the polarization approaching \( 10^{-4} \) \cite{11}. The design is based on the 1980 experiment. The main improvement in the experiment will be the 2 GeV/c separated charged kaon beam decaying in flight. The separated beam will reduce background counting rates in the detector per accepted event. The other improvements will be higher acceptance and analyzing power with a larger apparatus and a more finely divided polarimeter. Unlike the 1980 design the apparatus will also have better overall event reconstruction using tracking chambers and the larger calorimeter. There is a possibility of improving the polarimeter design significantly using liquid scintillator mixtures that retain muon polarization \cite{12}. The experiment will collect approximately 550 events per AGS pulse per 3.6 seconds. Thus the statistical accuracy of the polarization measurement in a 2000 hr \( (2 \times 10^6 \) pulses) run will be:

\[
\delta P_T \approx \frac{1.2 \sqrt{2.1} \cdot \sqrt{0.35(2 \times 10^6 \cdot 550)}}{2.35} \approx 1.3 \times 10^{-4}
\]

where \( \sqrt{1.2} \), \( \sqrt{2} \), \( 0.35 \), are dilution factors in the analyzing power due to backgrounds, the precession magnetic field, and the muon decay, respectively. The sensitivity to \( \text{Im}\xi \) is given by

\[
\delta \text{Im}\xi \approx \delta P_T / 0.2 \approx 7 \times 10^{-4}
\]

where 0.2 is a kinematic factor that includes the acceptance in the Dalitz plot and the orientation of the decay in the center of mass. With such high statistical power, systematic issues will become the main concern. The cylindrical symmetry of the apparatus and the precession technique (see \cite{8}) will cancel most systematic errors to first order. Nevertheless,
the second order systematics will require some new techniques. The experiment will collect a large sample of data including $K^+ \rightarrow \mu^+\nu$, $K^+ \rightarrow \pi^+\pi^0$, and $K^+ \rightarrow \pi^0\mu\nu$ events in different parts of the decay phase space. The muon decay asymmetries from these various ensembles of events can be measured to understand the detector systematics to very high accuracy.

\[ K^+ \rightarrow \mu^+\nu\gamma \]

The T violating out of plane polarization of the muon in this decay is related to the same in $K^+ \rightarrow \pi^0\mu^+\nu$ decay. The former can be caused by an effective pseudo-scalar interaction, while the latter by an effective scalar interaction. Therefore searches for T violation in both decay modes are complementary [14]. The T violating polarization could be $\sim 10^{-3}$ without violating other experimental bounds. It is estimated [15] that the electro-magnetic FSI for this interaction can induce an out of plane muon polarization of the same order of magnitude. An accurate theoretical calculation will be needed to subtract the FSI from any observation. On the other hand, this FSI induced effect could be considered a useful calibration point for the apparatus that will also be used for the new $K^+ \rightarrow \pi^0\mu^+\nu$ experiment.

The proposed new experiment is optimized to study muon polarization in $K^+ \rightarrow \mu^+\pi^0\nu$ decays. Nevertheless, we have investigated the feasibility of measuring T-violation in $K^+ \rightarrow \mu^+\nu\gamma$. The event selection and analysis of $K^+ \rightarrow \mu^+\nu\gamma$ will be very similar to $K^+ \rightarrow \mu^+\pi^0\nu$ events except that events containing more than 1 photon will be vetoed to reject background from $K^+ \rightarrow \mu^+\pi^0\nu$, $K^+ \rightarrow \pi^+\pi^0$, and $K^+ \rightarrow \pi^+\pi^0\pi^0$ events. Further background rejection will be achieved by matching the measured muon range in the polarimeter with the muon energy from a constrained fit to the photon momentum, the muon direction, and the known kaon momentum. We expect to collect $\sim 100$ events per AGS pulse. However, the signal to background ratio with our current design will be about 0.3, making it difficult to reach sensitivities of 0.001 for the polarization. Two improvements to the detector will reduce the backgrounds further: If the decay volume can be surrounded by photon veto counters with a veto threshold of 10 MeV to detect the low energy photons from $\pi^0$ decays, the background level can be reduced to about 10%. Secondly, if the calorimeter resolution can be improved (we have assumed $\sigma(E)/E \sim 8%/\sqrt{E}$) then the muon range match can be made narrower, thus separating the signal and background better.
The longitudinal muon polarization in this decay violates CP invariance. This decay amplitude is known to be dominated by the two photon intermediate state. Interference of this amplitude with some other flavor changing neutral scalar interaction could produce a non-zero longitudinal polarization. Within the Standard Model such an interaction could take place through second order loop diagrams involving the Higgs particle. However, direct constraints on the top quark and Higgs masses make the value of the polarization within the Standard Model quite small, \(|P_L(K_L \to \mu^+\mu^-)| \sim 7 \times 10^{-4}\) \[17\]. Such a polarization could also arise in non-standard models that introduce new flavor changing neutral scalars. For example, Wolfenstein and Liu \[18\] have suggested that in two Higgs doublet models such a polarization could be as large as 0.10 without violating the bounds from the neutron electric dipole moment, \(m_{K_L} - m_{K_S}\), \(\epsilon\), and \(\epsilon'\).

The main experimental difficulty in this measurement is the small branching fraction of the decay, \(7 \times 10^{-9}\). Therefore much effort must be put into separating these events from background before polarization analysis can be performed. Experiment E871 \[19\] has collected the largest number of these events so far; they expect to have \(\sim 10000\) events at the end of the 1996 running period with little background. The experiment is optimized to look for \(K_L \to \mu^\pm e^\mp\). We have made a rough estimate that if the experiment were optimized for \(K_L \to \mu^\pm\mu^-\) and the beam intensity were increased E871 could collect about 20000 events in two years of running. With appropriate upgrades to the marble muon range detector will allow approximately 50% of the muon decays to be analyzed. Y. Kuno has suggested that a polarimeter made with liquid scintillator could help this measurement by improving the analyzing power and lowering the cost of the polarimeter. Thus aside from kinematic factors the polarization could be measured with the following error:

\[
\delta P \approx \frac{\sqrt{2}}{0.3\sqrt{10000}} \approx 0.05
\]

where \(\sqrt{2}\) and 0.3 are factors due to the precession magnetic field in the polarimeter and the muon decay analyzing power, respectively. We have not calculated the kinematic dilution factors that could arise from the orientation of the decay in the center of mass of the kaon.
\[ K^+ \rightarrow \pi^+ \mu^+ \mu^- \]

This decay has recently been experimentally observed and measured to have the branching ratio of \(5 \times 10^{-8}\) \[20\]. The decay has a very rich structure which could lead to important measurements: Table 1 shows three different asymmetries that could be interesting to measure. The decay has recently been analyzed quite extensively \[21\]–\[28\]. The several different processes that govern the decay are as follows: one photon intermediate state, two photon intermediate state, short distance graphs of “Z-penguin” and “W-box”, and potential contributions from extensions to the Higgs sector. The interference of these graphs leads to various polarization effects. Although the theoretical analysis in the literature does not seem to be complete – in particular, strong interaction corrections and electro-magnetic final state effects – there is a consensus on the following:

The CP conserving longitudinal polarization (asymmetry (4) from Table 1) of the \(\mu^+\) is sensitive to the Standard Model Wolfenstein parameter \(\rho\). The value of this polarization within the Standard Model is estimated to be \(\sim 0.01\) and depends on the experimentally accepted phase space region. There is a small but non-negligible contribution to this polarization from the long distance 2 photon graph which cannot be calculated accurately at this time.

The T violating out of plane polarization (asymmetry (5) in Table 1) is expected to be very small within the Standard Model and the final state interaction correction to this polarization is expected to \(\sim 10^{-3}\). T violating spin correlations that involve both \(\mu^+\) and \(\mu^-\) polarizations (asymmetry (6)) are expected to have much smaller final state interaction corrections and are theoretically clean. Such asymmetries have substantial T violating contributions from the CKM matrix; they are expected to be \(\sim 0.06\) in some parts of the decay phase space. A good measurement would be sensitive to both the top quark mass and the CKM parameter \(\eta\). It could also be sensitive to non-standard model physics in the same manner as \(K_L \rightarrow \mu^+ \mu^-\).

Once again the main experimental difficulty will be in selecting the rare \(K^+ \rightarrow \pi^+ \mu^+ \mu^-\) decays from background. The main background is \(K^+ \rightarrow \pi^+ \pi^+ \pi^-\) decays in which the charged pions are misidentified as muons. This background must be suppressed in the trigger and the analysis. Experiment E865 \[29\] at the AGS is currently the best apparatus to perform this measurement. The experiment is, however, optimized for a search for \(K^+ \rightarrow \pi^+ \mu^+ e^-\), and therefore will require some reconfiguration. In particular, the muon range finder will have to be changed to stop more muons and analyze the polarization. A rough estimate of the sensitivity can be made based on the number of \(K^+\) in the decay region of the experiment,
per AGS spill per 3.6 sec, the geometric acceptance for $K^+ \rightarrow \pi^+\mu^+\mu^-$, 0.1, and the efficiency for muon decay detection of about 0.5. The longitudinal polarization could be measured in a 2000 hr run ($2 \times 10^6$ spills) with an error of about:

$$\delta P_L \approx \frac{\sqrt{2}}{0.3(2 \times 10^6 \cdot 10^7 \cdot 5 \times 10^{-8} \cdot 0.1 \cdot 0.5)^{\frac{1}{2}}} \approx 0.02$$

(2)

where $\sqrt{2}$ and 0.3 are factors due to the precession magnetic field, and the muon decay analyzing power, respectively. Clearly, measuring asymmetries that require analyzing both $\mu^+$ and $\mu^-$ polarizations will be very difficult with current technology since $\mu^-$ decays have a much lower analyzing power due to muon capture into atomic orbits around nuclei in the polarimeter.

Table 1 contains a summary of the various polarization asymmetries discussed by the working group. Table 2 contains the approximate estimated values of the asymmetries within the Standard Model and outside the Standard Model. Some of the numbers from the various references have been adjusted to account for the new knowledge of the top quark mass (174 GeV/$c^2$). In the case of $K_L \rightarrow \mu^+\mu^-$ and $K^+ \rightarrow \pi^+\mu^+\mu^-$ the theoretical estimates for the maximum possible non-standard contributions to $T$ violation do not agree; here the mean value of various estimates is chosen.

**Conclusion**

Muon polarization from kaon decays have a rich phenomenology. In the case of $K_L \rightarrow \mu^+\mu^-$ and $K^+ \rightarrow \pi^+\mu^+\mu^-$ new measurements could lead to important constraints on the Standard Model CKM parameters, in particular the Wolfenstein parameters $\rho$ and $\eta$. It is, however, difficult to reach the level of sensitivity needed to measure these parameters well with current technology. Nevertheless, the experimental difficulties should be compared to the difficulties facing the rare kaon decay measurement of $K_L \rightarrow \pi^0\nu\bar{\nu}$, which is sensitive to the same physics.

As shown in Tables 1 and 2 for many cases limits on the muon polarization will probe new physics beyond the Standard Model. In particular, the polarization will be sensitive to the physics of a more complicated Higgs sector or leptoquarks that could give rise to CP or T violation outside the Standard Model. The other source of CP violation needed for baryogenesis could be the motivation for such searches.

We have examined the measurement of the out of plane muon polarization in $K^+ \rightarrow \mu^+\pi^0\nu$ decays in more detail. Such a measurement will not be sensitive to the Standard
Model CP violation physics. Nevertheless, the measurement can be performed with sensitivity approaching $\delta P \sim 10^{-4}$, which is well beyond both the current direct limit of $\sim 5.3 \times 10^{-3}$ and indirect limit of $\sim 10^{-3}$ from other experimental constraints. Although the electric dipole moments of the neutron and electron are considered more favorably for $T$ violation outside the Standard Model they do not cover the entire spectrum of models. At the moment the measurement of $T$ violating polarization in $K^+ \rightarrow \mu^+\pi^0\nu$ decays is well justified and should be considered complementary to other efforts in understanding CP violation.

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