Long wavelength spatial oscillations of high frequency current noise in 1D electron systems

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Finite frequency current noise is studied theoretically for a 1D electron system in presence of a scatterer. In contrast to zero frequency shot noise, finite frequency noise shows spatial oscillations at high frequencies with wavelength $\pi v_F/\omega$. Band curvature leads to a decay of the amplitude of the noise oscillations as one moves away from the scatterer, superimposed by a beat. Furthermore, Coulomb interaction reduces the amplitude and modifies the wavelength of the oscillations, which we inspect in the framework of the Luttinger liquid (LL) model. The oscillatory noise contributions are only suppressed altogether when the LL interaction parameter $g \to 0$.

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I. INTRODUCTION

Current noise in mesoscopic systems has become an attractive research area, because it contains information that cannot be extracted from the current-voltage characteristics alone\textsuperscript{7}. Most of the theoretical discussions about current noise in one-dimensional (1D) systems focus on the zero frequency shot noise, because it may provide insights into quantization of charge and statistics of charge carriers in a quantum wire. In earlier theoretical work non interacting (Fermi liquid) 1D systems were analyzed in the spirit of the Landauer-Büttiker scattering matrix formalism\textsuperscript{2,3,4,5}. Short wavelength spatial oscillations of shot noise close to an impurity in the quantum wire were investigated by Gramespacher and Büttiker\textsuperscript{6} based on the assumption of non interacting charge carriers. More recently, shot noise of strongly correlated electron systems has been addressed using different algebraic techniques, such as for instance bosonization\textsuperscript{7}, the thermodynamic Bethe ansatz\textsuperscript{8,9}, and renormalization\textsuperscript{8}. It has been shown by different groups\textsuperscript{7,8,9,10,11} that due to the coupling of a strongly correlated (Luttinger liquid) electron system to noninteracting (Fermi liquid) electron reservoirs, the ideal Poisson shot noise is not renormalized by the interaction, as might have been expected from earlier theoretical work\textsuperscript{7,8,9}.

Most of the theoretical investigations on current noise calculate the finite frequency noise at the impurity site, so that spatial oscillations remain unnoticed. However, Lesovik\textsuperscript{12} has calculated the current-current correlator $\langle I(t,x)I(t',x') \rangle$, and therefore the integrand of the current noise given by Eq. (6) below, in the most general form ($t \neq t'$, $x \neq x'$), but he has only analyzed this correlator for the case of a clean quantum wire, where spatial oscillations of finite frequency current noise do not exist. How averaging over space affects the shot noise of a two- and three-terminal 1D system was investigated by Gavish, Levinson, and Imry\textsuperscript{13}. Furthermore, the wave behavior of a high-frequency current through a double-barrier tunneling structure was studied numerically by Cai, Hu, and Lax\textsuperscript{14}. Although most of these works have investigated certain aspects of the spatial structure of the current noise, a discussion of the long wavelength spatial dependence of finite frequency noise is lacking. This will be done in this article.

The spatial dependence of the noise becomes important in the high frequency regime, because, apart from the length of the quantum wire, $v_F/\omega$, where $\omega$ is the noise frequency and $v_F$ the Fermi velocity, provides an additional length scale, which affects current noise measurements. Only for $\omega \to 0$ charge conservation implies a noise strength independent of the position, where it is measured. We will show that once $\omega$ is finite, the current noise exhibits long wavelength spatial oscillations, which depend on the noise frequency, the Fermi velocity, and the distance between the point of measurement and the impurity in the quantum wire. We first address non interacting systems with linear dispersion, where the origin of the spatial oscillations is easily seen. However, these noise oscillations are affected by band curvature and Coulomb interaction. A finite band curvature calculation shows that the spatial oscillations decay away from the impurity site and show a beating behavior. Coulomb interaction, which we take into account by describing the quantum wire as a Luttinger liquid (LL), suppresses the amplitude of the noise oscillations by a factor $g$ and modifies its wavelength. Here, $g$ is the LL interaction parameter ($g < 1$ for repulsive interactions).

The article is organized as follows. In Chapter II we describe the general model of a quantum wire with an impurity. Then, in Chapter III we calculate the finite frequency noise with special emphasis on the noise oscillations. Chapter IV treats the influence of band curvature, and Chapter V the effect of Coulomb interaction on the spatial oscillations. Finally, we summarize our results and point out open problems in Chapter VI. Throughout this article we formally treat spin-polarized (single channel) quantum wires, but our findings apply generally to the case of electrons with spin. For non interacting electrons, the effect of spin on the noise is trivial, leading to an overall factor 2. The results in Chapter VI for electrons with Coulomb interaction, are also only trivially affected by the inclusion of spin, again leading to an overall factor

\[ \text{Eq. (6)} \]
II. GENERAL MODEL

Non interacting electrons of mass \( m \) moving along a quantum wire with a \( \delta \)-scatterer at \( x = 0 \) are described by the Schrödinger equation

\[
\left( \frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} + \frac{\hbar^2}{m} \Lambda \delta(x) \right) \psi(x) = E \psi(x),
\]

(1)

where the scattering potential is \( V(x) = (\hbar^2 \Lambda/m) \delta(x) \) with a scattering strength \( \Lambda \). For all positive energies \( E_k = \hbar^2 k^2/2m \), the eigenstates \( (k > 0) \)

\[
\psi_k(x) = \frac{1}{\sqrt{L}} \left\{ \begin{array}{ll} \psi_{k+} & (x < 0) \\ \psi_{k-} & (x > 0) \end{array} \right\},
\]

(2)

where \( L \) is the length of the quantum wire, describing a wave incident from the left, commonly called left-mover, that is partially transmitted and partially reflected and solves Eq. (1) with a transmission amplitude

\[
t_k = \frac{1}{1 + i \Lambda/k}.
\]

Likewise, there is a solution \( (k > 0) \)

\[
\phi_k(x) = \psi_k(-x)
\]

descrribing a wave incident from the right, a so called left-mover eigenstate. We apply a voltage \( U \) to the ends of the quantum wire by connecting them adiabatically to external leads with chemical potentials \( \mu_L = E_F + eU \) and \( \mu_R = E_F \), where \( E_F \) is the Fermi energy. Consequently, the states \( \psi_k(x) \) are filled up to \( k = (k_F^2 + 2meU/\hbar^2)^{1/2} \), while the states \( \phi_k(x) \) are only filled up to \( k = k_F \). If we assume a rather small voltage \( U \) (compared to \( E_F \)), the transmission \( (t_k) \) and reflection amplitudes \( (r_k) \) do not vary much in the \( k \)-range \( [k_F, (k_F^2 + 2meU/\hbar^2)^{1/2}] \) and can effectively be replaced by constant values at the Fermi edge, \( t_{k_F} \equiv t \) and \( r_{k_F} \equiv r \).

In the second quantized form, the eigenstates \( \psi_k(x) \) allow to expand the electron field operator

\[
\Psi(t, x) = \sum_{k > 0} (a_k \psi_k(x) + b_k \phi_k(x)) e^{-(i/\hbar)E_k t}
\]

(4)

where the operators \( a^\dagger_k \) (\( b^\dagger_k \)) and \( a_k \) (\( b_k \)) are Fermi creation and annihilation operators obeying the usual anticommutation relations. Then, in the grand canonical ensemble

\[
\rho = \frac{e^{-\beta \sum_k (E_k - \mu_L) a^\dagger_k a_k + (E_k - \mu_R) b^\dagger_k b_k}}{\text{tr} \left\{ e^{-\beta \sum_k (E_k - \mu_L) a^\dagger_k a_k + (E_k - \mu_R) b^\dagger_k b_k} \right\}}
\]

the expectation values of products of creation and annihilation operators read

\[
\langle a^\dagger_k a_{k'} \rangle = f(k_1; U) \delta_{k_1 k_2},
\]

\[
\langle a^\dagger_k b_{k'} \rangle = 0,
\]

\[
\langle b^\dagger_k b_{k'} \rangle = f(k_1; 0) \delta_{k_1 k_2}
\]

with the standard Fermi distribution function

\[
f(k; U) = \frac{1}{1 + \exp[\beta(E_k - (E_F + eU))]},
\]

where \( \beta^{-1} = k_B T \). For future reference, we now briefly discuss the calculation of the current through a 1D non interacting quantum wire. Inserting the expansion (4) into the standard expression for the quantum mechanical current operator \( I(t, x) \) yields

\[
I(t, x) = \frac{e}{2mi} \sum_{k, k'} e^{(i/\hbar)(E_k - E_{k'}) t} \times \left( a^\dagger_k \psi_k^* \psi_{k'} + b^\dagger_k \phi_k \phi_{k'}^* \right),
\]

(5)

where the operator \( \hat{\nabla} \) is defined by \( A \hat{\nabla} B = A(\nabla B) - (\nabla A)B \). Note, that in Eq. (5) and in the rest of the article all \( \psi_k \)'s and \( \phi_k \)'s depend explicitly on \( x \). Then, the expectation value of the current becomes

\[
\langle I \rangle = \frac{e}{2mi} \sum_k \times \left( \langle a^\dagger_k a_k \rangle \hat{\nabla} \psi_k + \langle b^\dagger_k b_k \rangle \phi_k \phi_k^* \right).
\]

(6)

If we now consider the quantum wire, say, on the right side of the impurity \( (x > 0) \) we find

\[
\psi_k \hat{\nabla} \psi_k = \frac{i 2k}{L} |\psi|^2 = \frac{i 2k}{L} \mathcal{T},
\]

\[
\phi_k^* \hat{\nabla} \phi_k = - \frac{i 2k}{L} (1 - |r|^2) = - \frac{i 2k}{L} (1 - \mathcal{R}).
\]

Here, \( \mathcal{T} = |t|^2 \) is the transmission coefficient of the current flow through the barrier and \( \mathcal{R} = |r|^2 = 1 - \mathcal{T} \) the reflection coefficient. Going over to the continuum limit \( (L \to \infty) \) and changing variables \( k \to E_k = \hbar^2 k^2/2m \), the current reads

\[
\langle I \rangle = \frac{e^2}{\hbar} \mathcal{T} \int dE_k (f(E_k; U) - f(E_k; 0))
\]

\[
= \frac{e^2}{\hbar} \mathcal{T} U.
\]

(7)

This is the well-known Landauer formula for the current through a quantum wire. Without inelastic scattering in the wire, Eq. (7) holds for finite, but moderate temperatures and applied voltages \( (k_B T, eU \ll E_F) \).
III. NOISE OSCILLATIONS

The finite frequency current noise at position $x$ of the quantum wire is defined by the expression

$$P(\omega, x) = \int dt e^{i\omega t} \left\langle \{ \Delta I(t, x), \Delta I(0, x) \} \right\rangle_+, \quad (8)$$

where $\Delta I(t, x) = I(t, x) - \langle I \rangle$ is the current fluctuation operator. Without loss of generality, we assume $\omega > 0$ and $x > 0$. Inserting the current operator [15] into the definition of the current noise [16], we find

$$P(\omega, x) = \frac{e^2}{4m\hbar L} \sum_{k = 0}^1 \frac{1}{|k_{\omega}|} \times$$

$$\left\{ f^{a\alpha}(k, \omega) \left( \psi_{k_{\omega}}^* \hat{\nabla} \psi_{k_{\omega}} \right) \left( \psi_{k_{\omega}}^* \hat{\nabla} \psi_{k_{\omega}} \right) + f^{b\alpha}(k, \omega) \left( \phi_{k_{\omega}}^* \hat{\nabla} \phi_{k_{\omega}} \right) \left( \phi_{k_{\omega}}^* \hat{\nabla} \phi_{k_{\omega}} \right) \right\}$$

$$+ \text{h.c.} , \quad (9)$$

where the $t$-integration in Eq. [8] has already been carried out, and the symbol $\langle \ldots \rangle_c$ represents the connected expectation value. Exploiting Wick’s theorem, see e.g. Ref. [17] we can only pair $k_1 = k_4 = (k_2^2 - 2m\omega/\hbar)^{1/2}$ and $k_2 = k_3$. To simplify our notation, we set $k_2 = k$ and define $k_\omega \equiv (k^2 - 2m\omega/\hbar)^{1/2}$. Then, the resulting expression for the current noise reads

$$P(\omega, x) = \frac{e^2}{4m\hbar L} \sum_{k = 0}^1 \frac{1}{|k_{\omega}|} \times$$

$$\left\{ f^{a\alpha}(k, \omega) \left( \psi_{k_{\omega}}^* \hat{\nabla} \psi_{k_{\omega}} \right) \left( \psi_{k_{\omega}}^* \hat{\nabla} \psi_{k_{\omega}} \right) + f^{b\beta}(k, \omega) \left( \phi_{k_{\omega}}^* \hat{\nabla} \phi_{k_{\omega}} \right) \left( \phi_{k_{\omega}}^* \hat{\nabla} \phi_{k_{\omega}} \right) \right\}$$

$$+ \text{h.c.} , \quad (10)$$

where we have introduced the short-hand notation

$$f^{a\alpha}(k, \omega) = f^{a\alpha}(k_{\omega}) (1 - f^{b\beta}(k)) f^{b\beta}(k)$$

with $f^a = f(k; U)$ and $f^b = f(k; 0)$. In the remaining part of this section we assume a linear dispersion relation, which is justified for instance in single wall carbon nanotubes [18] and is also appropriate to cleaved edge overgrowth quantum wires [19] for small applied voltages $U$ and low temperatures $T$. In that case, the energy $E_k$ of the right-mover eigenstates may be replaced by $\hbar v_F(k - k_F)$ and analogously the energy of the left-mover eigenstates by $-\hbar v_F(k - k_F)$. We then find for the finite frequency current noise ($x > 0$) at zero temperature

$$P(\omega, x) = \frac{\hbar^2 e^2 v_F}{(2m)^2} \times$$

$$\left\{ \frac{e^2}{\pi |\omega|} \left( \frac{2\omega x}{v_F} \right)^2 \left( \frac{\omega}{v_F} \right)^2 \right\}.$$
As a consequence of the fluctuation-dissipation theorem\textsuperscript{22}, $P_0(\omega, x)$ is related to the real (dissipative) part of the frequency-dependent conductance by

$$P_0(\omega, x) = 2\hbar|\omega| \text{Re}[G(\omega, x)] .$$  

(13)

Therefore, $\text{Re}[G(\omega, x)]$ is an oscillating function of frequency with a period $\pi v_F / x$ and shows peaks with a maximum height of $2e^2 / \hbar$, only truly reached for $R = 1$. These findings are in agreement with the analysis by Blanter, Hekking, and Büttiker of the dynamic conductance of an interacting quantum wire capacitively coupled to a gate\textsuperscript{23}.

The appearance of the $\cos(2\omega x / v_F)$-term in Eq. (12) is the main result of this chapter. Note, that the noise oscillations with the long wavelength $v_F / \pi \omega$ have to be distinguished from the Friedel-type noise oscillations with the short wavelength $\pi / k_F$ discussed in Ref. \textsuperscript{17}. Evidently, for a very strong backscatterer ($R \approx 1$) the oscillating part of Eq. (12) is most pronounced. Therefore, to further analyze the noise oscillations, we will focus on the situation of a half-open quantum wire ($R = 1$ at the boundary) in the following. Then, the shot noise contribution of Eq. (12) vanishes and the equilibrium finite frequency current noise is given by

$$P(\omega, x) = 2 \frac{e^2}{h} |\omega| \left[ 1 - \cos \left( \frac{2 \omega x}{v_F} \right) \right] .$$

(14)

At the point $x$ of measurement of the current noise the incoming and the reflected current noise waves interfere, leading to a suppression or an enhancement of the measured noise depending on the time $2x / v_F$ an electron needs to return to the position $x$ after a reflection at $x = 0$. Remarkably, the noise oscillations at zero temperature in Eq. (14) do not decay away from the open boundary – a situation certainly due to idealizations. To see if and how these noise oscillations survive in more realistic systems we explicitly treat finite band curvature and intrinsic electron-electron interaction in the next two chapters.

**IV. EFFECT OF BAND CURVATURE**

Band curvature of the dispersion relation of electrons induces a decay of the noise oscillations away from the scatterer. To investigate how fast these noise oscillations decay, we go back to Eq. (15), but keep only track of the non-vanishing terms for a half-open quantum wire, where $R = 1$ ($T = 0$). Again we neglect all $(2k_F x)$-oscillating contributions. Then, we find the following expression for the current noise at zero temperature with $k_F^x = (k_F^2 - 2m\omega / \hbar)^{1/2}$ and $k_\omega = (k^2 - 2m\omega / \hbar)^{1/2}$.

$$P(\omega, x) = \frac{\hbar e^2}{m} \int \frac{dk}{2\pi} k \left[ 1 - \cos \left( 2 (k_\omega - k) x \right) \right] .$$

(15)

The result for $P(\omega, x) / P_0$ (with $P_0 = 2(e^2 / \hbar) |\omega|$) derived in the previous section, see Eq. (14), only depends on a single dimensionless parameter, namely $\omega x / v_F$, in Eq. (15) a second dimensionless parameter arises. This is $E_0 = \hbar \omega / E_F$, where $E_F = \hbar^2 k_F^x / 2m$. In the limit $E_0 \to 0$ with $\omega x / v_F$ fixed, the full result \textsuperscript{15} reduces to Eq. (14), the result with linear dispersion. A simple expansion of Eq. (15) in powers of $E_0$ does not capture the qualitative behavior of the full solution \textsuperscript{15}, characterized by a decay and a beat of the noise oscillations as a function of $\omega x / v_F$ (see Fig. 1). Similarly, an asymptotic expansion for large $\omega x / v_F$, which could explain the decay, does not give any information about the interesting parameter regime $\omega x / v_F \in [0, 2\pi]$. To further analyze the behavior of Eq. (15), we have expanded the lower integrand in powers of $2m\omega / \hbar k_F^x$ and $2m\omega / \hbar k^2$, respectively. This procedure is reasonable for $\hbar \omega \ll E_F$, because the integration variable $k$ only takes values close to $k_F$, $k \in [k_F^x, k_F]$, and the integrand of Eq. (15) is smooth for $k$ near $k_F$. Then, we can do the integration in Eq. (15) easily and find

$$P(\omega, x) = \frac{P_0}{\hbar \omega x} \left( 1 - \frac{2v_F \sin \left( \frac{E_0 \omega x}{2v_F} \right)}{E_0 \omega x} \cos \left( \frac{2\omega x}{v_F} \right) \right).$$

(16)

Evidently, Eqs. (15) and (16) both reduce to Eq. (14) in the limit $E_0 \to 0$ with $\omega x / v_F$ fixed. Furthermore, Eqs. (15) and (16) show qualitatively the same oscillating behavior superimposed by a beat as a function of $\omega x / v_F$. This is illustrated in Fig. 1, where the noise deduced from Eqs. (14), (15), and (16) is plotted versus $\omega x / v_F$. A comparison of the full solution (15) and our simple model (16) tells us that the amplitude of the oscillations of $P(\omega, x)$ decays at least like $v_F / E_0 \omega x$ away from the scatterer and reaches asymptotically the value $P_0$. Additionally, we also see that the phase of the noise oscillations is modified by band curvature effects.

**V. EFFECT OF COULOMB INTERACTION**

Now, we turn to the effect of Coulomb interaction on the noise oscillations. A half-open strongly interacting 1D electron system can most easily be described in the framework of open boundary bosonization\textsuperscript{24}. Following closely the notation of Fabrizio and Gogolin\textsuperscript{24}, we set $\hbar \equiv 1$ in this section, except for the final result. Then, the full Hamiltonian of the system reads

$$H = H_0 + H_I$$

(17)

with the kinetic contribution

$$H_0 = \int_{-L}^0 dx \psi^\dagger(x) e(-i\partial_x) \psi(x)$$

(18)
The Gogolin have bosonized the fermionic operators in To simplify the full Hamiltonian (17), Fabrizio and ψ and left moving fields x right moving fields only, living in the parameter space boundary bosonization can be expressed in terms of e.g. P FIG. 1: Finite frequency current noise P(ω, x) in units of P_0 = 2(ε^2/h)h[ω] as a function of ωx/v_F for a half-open quantum wire. The (black) solid line shows the result [15] corresponding to a 1D electron system with a quadratic dispersion relation, the (blue) dot-dashed line illustrates the approximate result [16] for E_0 ≡ hω/E_F = 0.1, and the (red) dashed line corresponds to Eq. (14), the current noise of an electron system with linear dispersion. It is clearly visible that a finite band curvature modifies the amplitude and the phase of the noise oscillations.

and the electron-electron interaction contribution

\[ H_I = \frac{1}{2} \int dx dy \psi^\dagger(x)\psi^\dagger(y)U(x-y)\psi(y)\psi(x) , \]  

where ε(k) is the dispersion relation of the 1D band, and ψ(x) is the electron annihilation operator obeying the open boundary conditions

\[ ψ(-L) = ψ(0) = 0 . \]  

To simplify the full Hamiltonian [17], Fabrizio and Gogolin have bosonized the fermionic operators in Eqs. [13] and [19] assuming a linear dispersion relation. The ψ operator can be written down in terms of right and left moving fields

\[ ψ(x) = e^{ikFx}\psi_R(x) + e^{-ikFx}\psi_L(x) . \]  

Due to the boundary conditions [20] these fields are not independent, but satisfy by their definition [24]

\[ ψ_L(x) = -ψ_R(-x) . \]  

Therefore, all electron operators in the framework of open boundary bosonization can be expressed in terms of e.g. right moving fields only, living in the parameter space x ∈ [−L, L]. In the usual way, the interacting electron field ψ_R(x, t) can be bosonized with the help of a free boson field φ(x, t)

\[ ψ_R(x, t) = \frac{1}{\sqrt{2πa_0}} e^{iφ(x,t)/\sqrt{a_0}} . \]  

Here, a_0 is the lattice constant of the underlying lattice model and g is the LL interaction parameter, which is related to the Fourier transform \( U(k → 0) \) of the interaction potential \( U(x − y) \) in Eq. (19). Then, \( 0 < g < 1 \) for repulsive interactions and \( g → 1 \) in the noninteracting limit. In the next step, the field φ(x, t) may be expanded in terms of boson creation \( b^\dagger_q \) and annihilation \( b_q \) operators, yielding

\[ ϕ_R(x, t) = \sum_{q > 0} \sqrt{\frac{π}{qL}} \left[ e^{iq(x-vt)}b_q + e^{-iq(x-vt)}b^\dagger_q \right] e^{-a_0q/2} . \]  

In that representation the momentum of the system takes quantized values \( q = πn/L \). With the approximations of linear dispersion and short range interactions made, the bosonic Hamiltonian of the full interacting fermion systems, Eq. (17), takes a very simple form [22]

\[ H_b = v \sum_{q > 0} q b^\dagger_q b_q , \]  

where \( v = v_F/g \) is the renormalized sound velocity.

In terms of the bosonic field the electron density of right- and left-moving excitations reads

\[ ρ_R/L(x, t) = \frac{k_F}{π} + \frac{1}{2π\sqrt{g}} \partial_x φ(\pm x, t) . \]  

This allows us to write a bosonic form of the current operator

\[ I(x, t) = \frac{e_v F}{2π\sqrt{g}} (\partial_x φ(x, t) - \partial_x φ(-x, t)) , \]  

which we combine with Eq. (8) to compute the current noise

\[ P(ω, x) = \left( \frac{e_v F}{2π\sqrt{g}} \right)^2 \int dt e^{iωt} \times \]  

\[ \partial_x \left\{ ϕ(x, t) - ϕ(-x, t), ϕ(x, 0) - ϕ(-x, 0) \right\} _+ . \]  

To proceed, we need to calculate the bosonic correlation function with respect to the groundstate of the Hamiltonian [23]. At zero temperature, a straightforward calculation for large system size L yields

\[ ϕ(x, t)ϕ(x’, 0) = -ln \left( \frac{iπ}{T} (x - x’ - vt + iα_0) \right) . \]  

In the limit \( L → \infty \) this may be inserted into the expression [24] for the current noise to give

\[ P(ω, x) = -\frac{e_v^2}{2π^2} \int dt e^{iωt} \left( \frac{2}{(t - iα_0/v)^2} - \frac{1}{(2x/v - t + iα_0/v)^2} \right) \]  

\[ \left( \frac{2x/v + t - iα_0/v)^2} \right) . \]
Doing the integral and then sending \( a_0 \to 0 \), the final result reads

\[
P(\omega, x) = 2g^2 \frac{e^2}{\hbar} \omega |x| \left( 1 - \cos \left( \frac{2\omega g|x|}{v_F} \right) \right),
\]

where we have re-introduced \( \hbar \). A comparison of Eqs. \( \text{[24]} \), \( \text{[25]} \), and \( \text{[28]} \) shows, that unlike band curvature short range electron-electron interaction with a linear dispersion does not lead to a decay of the noise oscillations, but instead suppresses the amplitude by a factor \( g \) and increases the wavelength of the oscillations by a factor \( 1/g \). This shows that the noise oscillations are also present in an interacting quantum wire with finite \( g \). Only for very strong interactions \( (g \to 0) \) the noise oscillations will vanish. That the noise oscillations survive in the presence of Coulomb interaction can be understood as a consequence of the lack of strict electroneutrality in a LL. In the presence of gates that screen the long range part of the Coulomb interaction, only the part \( (1-g^2)Q \) of a charge \( Q \) on the quantum wire is screened internally, the remaining part \( g^2Q \) being compensated by the nearby gate. Therefore, if \( g > 0 \), which is the typical experimental situation, the quantum wire can become charged and exhibit the spatial noise oscillations \( \text{[28]} \).

VI. CONCLUSIONS

To summarize, first, we have studied the finite frequency current noise of a non interacting quantum wire with an impurity. In the equilibrium part of the finite frequency current noise, which is related to the real part of the dynamic conductance by means of the fluctuation-dissipation theorem, we have found long wavelength spatial oscillations, which can be explained by simple quantum mechanical wave dynamics. Those oscillations are absent in the shot noise limit \( (\omega \to 0) \) as well as the ballistic limit \( (T \to 1) \). The oscillations are most pronounced for a half-open quantum wire, where the boundary can be regarded as a very strong backscatterer. We have shown that finite band curvature leads to a moderate decay of the noise oscillations. The strength of this decay is proportional to \( E_0 \equiv \hbar \omega / E_F \), where small \( E_0 \) means weak decay. Furthermore, we have discussed the noise oscillations in strongly interacting half-open 1D systems using the LL model. We have found that the amplitude of the spatial oscillations decreases with increasing electron-electron interaction, while the wavelength of the oscillations is enhanced by the LL interaction parameter. Finally, we remark that all results for the current noise derived here were obtained in the limit \( L \to \infty \) of a very long quantum wire and thus only hold for frequencies \( \omega \) above \( v_F/L \). To treat frequencies below \( v_F/L \), a finite \( L \) calculation has to be carried out. Then, in particular for the case of interacting electrons in the quantum wire, the (non interacting) external reservoirs must explicitly be taken into account.

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1. Ya. M. Blanter and M. Büttiker, Phys. Rep. 336, 1 (2000).
2. G. B. Lesovik, JETP Lett. 49, 592 (1989).
3. M. Büttiker, Phys. Rev. Lett. 65, 2901 (1990).
4. M. Büttiker, Phys. Rev. B 45, 3807 (1992).
5. T. Martin and R. Landauer, Phys. Rev. B 45, 1742 (1992).
6. T. Gramespacher and M. Büttiker, Phys. Rev. Lett. 81, 2763 (1998); Phys. Rev. B 60, 2375 (1999).
7. C. L. Kane and M. P. A. Fisher, Phys. Rev. Lett. 72, 724 (1994).
8. P. Fendley, A. W. W. Ludwig, and H. Saleur, Phys. Rev. Lett. 75, 2196 (1995).
9. C. de C. Chamon, D. E. Freed, and X. G. Wen, Phys. Rev. B 53, 4033 (1996).
10. V. V. Ponomarenko and N. Nagaosa, Phys. Rev. B 60, 16865 (1999).
11. B. Trauzettel, R. Egger, and H. Grabert, Phys. Rev. Lett. 88, 116401 (2002).
12. G. B. Lesovik, JETP Lett. 70, 208 (1999).
13. U. Gavish, Y. Levinson, and Y. Imry, Phys. Rev. Lett. 87, 216807 (2001).
14. W. Cai, P. Hu, and M. Lax, Phys. Rev. B 44, 3336 (1991).
15. F. D. M. Haldane, J. Phys. C 14, 2585 (1981).
16. D. S. Datta, Electronic Transport in Mesoscopic Systems (Cambridge University Press, Cambridge, England, 1995).
17. G. D. Mahan, Many-Particle Physics (Plenum, New York, 1990).
18. S. G. Lemay et al., Nature 412, 617 (2001).
19. O. M. Auslaender et al., Science 295, 825 (2002).
20. J. Friedel, Nuovo Cim. Suppl. 7, 287 (1958).
21. S. R. E. Yang, Solid State Comm. 81, 375 (1992).
22. M. Büttiker, A. Pretre, and H. Thomas, Phys. Rev. Lett. 70, 4114 (1993); Phys. Rev. B 54, 8130 (1996).
23. Ya. M. Blanter, F. W. J. Hekking, and M. Büttiker, Phys. Rev. Lett. 81, 1925 (1998).
24. M. Fabrizio and A. O. Gogolin, Phys. Rev. B 51, 17827 (1995).
25. R. Egger and H. Grabert, Phys. Rev. Lett. 79, 3463 (1997).
26. D.L. Maskov and M. Stone, Phys. Rev. B 52, R5539 (1995); V.V. Ponomarenko, ibid. 52, R8606 (1995); I. Safi and H.J. Schulz, ibid. 52, R17040 (1995).