Chronology protection in stationary 3D spacetimes

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Abstract. We address chronology protection in stationary, rotationally symmetric spacetimes in 2+1 dimensional gravity, focusing on the case of negative cosmological constant. We show that, if such spacetimes contain closed timelike curves, they belong to one of the following two classes: (i) bad asymptotics: the boundary itself contains closed timelike curves, or (ii) unphysical stress tensor in the bulk: the matter stress tensor violates the null energy condition. We illustrate these properties in a class of examples involving rotating dust in anti-de Sitter space and comment on the use of the AdS/CFT correspondence to study chronology protection.

1. Introduction
The question whether and how the laws of physics prevent the construction, in principle, of time machines is a fascinating one which goes to the heart of our understanding of spacetime geometry and quantum physics. Hawking’s chronology protection conjecture [1] states that the laws of physics prevent the formation of closed timelike curves (CTCs) that would allow one to travel to one’s past (see [2, 3] for reviews and further references). At the classical level, evidence for the conjecture comes from the fact that spacetimes with a compactly generated chronology horizon require matter sources that violate the null energy condition (NEC).

Some of the simplest and most well-known examples of spacetimes with closed timelike curves don’t have a compactly generated chronology horizon and are therefore not included in Hawking’s original argument. This is the case for ‘eternal’ time-machines, where the space-time is stationary and every CTC is a member of a one-parameter family of time-translated CTCs. Examples include the two earliest known spacetimes with CTCs: the Van Stockum [4] and Gödel [5] solutions. Here we will address the issue of chronology protection in stationary spacetimes in the simplest context: we consider the case of 2+1 dimensional gravity and consider stationary spacetimes which are also rotationally symmetric. For zero cosmological constant, chronology protection in such spacetimes was addressed by Menotti and Seminara [6] and was related to violations of the NEC. Here we shall focus instead the case of negative cosmological constant, since we are ultimately interested in the application of the AdS/CFT correspondence [7] to chronology protection. We will comment more on the application of AdS/CFT to chronology protection in section 5.

Our main result will be to show that such spacetimes belong to one of the following categories:

(i) the bad asymptotics category: the spacetime has closed timelike curves on the boundary.
(ii) the unphysical stress tensor category: the matter stress in the bulk violates the NEC.

Recall that the NEC requires $n^i T_{ij} n^j \geq 0$ for every null vector $n^i$. 

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The behavior of the local lightcones in a constant time slice in two classes of time
machines. The black curve is the is the metric component $g_{\phi\phi}$ (see (2) below).

The behavior of the local lightcones in these two categories of spacetimes is illustrated in figure
1. We will also analyze the implications of violations of the NEC for the conformal weights in
the dual CFT, and discuss several examples. We refer to [8] for technical details and a more
complete list of references.

2. Chronology protection for stationary rotationally invariant metrics
We consider 2+1-dimensional gravity in the presence of a cosmological constant. Einstein’s
equations are

$$R_{ij} = 8\pi G (T_{ij} - T g_{ij}) + 2\Lambda g_{ij}. \quad (1)$$

We restrict attention to stationary, rotationally symmetric metrics. We can choose coordinates
$t, \rho, \phi$ such that the metric takes the form (see e.g. [9])

$$ds^2 = d\rho^2 + g_{tt} dt^2 + 2 g_{t\phi} dtd\phi + g_{\phi\phi} d\phi^2. \quad (2)$$

where $\phi$ is identified modulo $2\pi$ and $g_{tt}, g_{t\phi}, g_{\phi\phi}$ are functions of $\rho$ that satisfy

$$g_{tt} < 0 \quad (3)$$

$$-\det g = g_{\phi\phi}^2 - g_{tt} g_{\phi\phi} > 0. \quad (4)$$

We start by expressing Einstein’s equations (1) in a null triad $(e^+, e^-, e^\rho)$:

$$e^\pm = \frac{e^{\mp B}}{\sqrt{2}} \left[ \sqrt{1 - g_{tt} dt - \frac{g_{t\phi} \pm \sqrt{-g}}{\sqrt{-g}} d\phi} \right] \quad (5)$$

$$e^\rho = d\rho. \quad (6)$$

such that the metric takes the form

$$ds^2 = -2e^+ e^- + (e^\rho)^2. \quad (7)$$

The function $B(\rho)$ parameterizes the freedom to locally Lorentz boost the triad; our argument
will be simplified by making a convenient gauge choice for this function.

The ++ and −− components of Einstein’s equations (1) read

$$R_{\pm\pm} = 8\pi GT_{\pm\pm}. \quad (8)$$

We now choose the boost function $B$ to satisfy

$$B' = \frac{g_{tt} g_{t\phi} - g_{t\phi} g_{tt}}{2g_{tt} \sqrt{-g}} \quad (9)$$
In view of (4) this differential equation is regular and we are justified in making this choice. The advantage of this gauge is that the Ricci components \( R_{\pm\pm} \) become total divergences

\[
R_{\pm\pm} = \nabla_i v_\pm^i
\]

where

\[
v_\pm^i \equiv e_\pm^j \nabla_j e_\pm^i - e_\pm^i \nabla_j e_\pm^j.
\]

The ++ and -- components of Einstein’s equations (8) then become simply

\[
\nabla_i v_\pm^i = 8\pi G T_{\pm\pm}
\]

which, since everything depends only on \( \rho \), can be written as

\[
\partial_\rho (\sqrt{-g} v_\pm^\rho) = 8\pi G \sqrt{-g} T_{\pm\pm}.
\]

The left hand side is a total derivative while the right-hand side is positive when the NEC holds. This identity will be our main tool in what follows. For later use, we record here also the explicit expression for \( \sqrt{-g} v_\pm^\rho \):

\[
\sqrt{-g} v_\pm^\rho = \frac{e^{\pm 2B}}{4\sqrt{-g}} \left( g_{\rho\phi} g_\rho^\phi - g_\phi^\phi g_\rho^\rho + 2 g_\rho^\phi \frac{\sqrt{-g}}{g_\rho^\rho} g_\rho^\phi (g_\rho^\rho - g^\rho_\rho g_\phi^\phi) \right).
\]

Now we will focus on spacetimes which contain CTCs. As was shown in [6], this implies that \( g_{\phi\phi} \) must become negative for some values of \( \rho \), so that for these values the azimuthal circles are timelike. We will refer to these timelike \( \phi \)-circles as azimuthal closed timelike curves (ACTCs).

Now suppose that the spacetime doesn’t belong to the category (i) discussed in the Introduction. This implies that there are no ACTCs for \( \rho \) sufficiently large, and hence there must be a radius \( \rho_+ \) where \( g_{\phi\phi} \) has a zero and changes sign from negative to positive. We will further assume that \( g_{\phi\phi} \) has a second zero at a radius \( \rho_- < \rho_+ \) where it changes sign from positive to negative. This assumption can be made without much loss of generality for the following reason. For spacetimes with a regular axis of symmetry, there must always be such a \( \rho_- \) since \( g_{\phi\phi} \) is positive in the vicinity of the axis, as one can see by choosing local inertial coordinates on the axis (see [10] for a rigorous proof). If there is no axis of symmetry and no radius \( \rho_- \), the metric has a wormhole-like behavior with a second asymptotic region for \( \rho \to -\infty \) where \( g_{\phi\phi} \) is negative, and hence would belong to the category (i) contrary to our assumption.

Summarized, we have the following behavior in the interval \([\rho_-, \rho_+]\) (see also Figure 2):

\[
g_{\phi\phi}(\rho_-) = g_{\phi\phi}(\rho_+) = 0
\]

\[
g_{\phi\phi}(\rho) \leq 0 \quad \text{for } \rho_- \leq \rho \leq \rho_+
\]

\[
g_{\phi\phi}(\rho_-) < 0
\]

\[
g_{\phi\phi}(\rho_+) > 0
\]

This behavior implies some properties of \( g_{t\phi} \) which we will need later. First of all, \( g_{t\phi} \) cannot be zero in \( \rho_- \) or \( \rho_+ \) because of (4). Furthermore, \( g_{t\phi} \) has no zeroes in the interval \([\rho_-, \rho_+]\), because otherwise the metric would not have Minkowski signature. So we can consistently define

\[
\sigma \equiv \text{sign } g_{t\phi}(\rho_-) = \text{sign } g_{t\phi}(\rho_+)
\]

This gauge choice corresponds to setting to zero the spin connection coefficient \( \omega_{\rho-\pm} \).
Now, when we evaluate the quantity $\sqrt{-g}v^\sigma_\rho$ in $\rho_\pm$ using (14), we see that only the first term is nonzero:

$$\sqrt{-g}v^\sigma_\rho(\rho_\pm) = e^{2\sigma_B}/4\sqrt{-g} g_{\phi\phi}'(\rho_\pm).$$

(20)

Using (18) we see that $\sqrt{-g}v^\sigma_\rho$ changes sign between $\rho_-$ and $\rho_+$, so there must be a radius $\rho_c$ where it has a zero (see Figure 2):

$$\sqrt{-g}v^\sigma_\rho(\rho_c) = 0.$$  

(21)

Now we can integrate (13) between $\rho_c$ and $\rho_+$ in (18) to get our main identity

$$8\pi G \int_{\rho_c}^{\rho_+} d\rho \sqrt{-g}T_{\sigma\sigma} = e^{2\sigma_B}/4\sqrt{-g} g_{\phi\phi}'(\rho_+) < 0.$$  

(22)

We then see that the null energy condition must be violated on average in the interval $[\rho_c, \rho_+]$ and the spacetime belongs to the category (ii) of the Introduction.

3. CTCs and conformal weights

One expects intuitively that the violation of the NEC in the CTC region represents a negative contribution to the total energy of the system. In this section we will make this more precise in the case of negative cosmological constant: we will see that the CTC region represents a negative contribution to either $L_0$ or $\bar{L}_0$.

First we evaluate the asymptotic values of the quantities $\sqrt{-g}v^\sigma_\rho$ in the cases where the matter stress tensor vanishes sufficiently fast for $\rho \to \infty$. The metric then asymptotically approaches a vacuum solution which is stationary and rotationally invariant and is characterized by two integration constants which can be identified with the mass and angular momentum [11]. Asymptotically, the metric takes the Fefferman-Graham [12] form (where $\Lambda = -1/L^2$)

$$ds^2 = d\rho^2 + e^{2\rho/L}/2 (-dt^2 + L^2 d\phi^2) + M/2 dt^2 - Jdt d\phi + M/2 L^2 d\phi^2 + O(e^{-2\rho/L}).$$  

(23)

The equation (9) near the boundary implies that $B$ goes to a constant

$$B = B_\infty + O(e^{-2\rho/L}).$$  

(24)

Similarly, integrating (13) between $\rho_-$ and $\rho_c$ one shows that the NEC is also violated on average in the interval $[\rho_-, \rho_c]$.
Since we have not yet specified the integration constant in (9), we can fix it by imposing the boundary condition
\[ B_\infty = \lim_{\rho \to \infty} B(\rho) = 0. \] (25)

For the boundary behavior of \( \sqrt{-g}v_\pm \) we then find
\[ \sqrt{-g}v_\pm^\rho = \frac{1}{2} \left( M \pm \frac{J}{L} \right) + \mathcal{O}(e^{-2\rho/L}). \] (26)

Hence the asymptotic values of \( \sqrt{-g}v^\rho_+ \) and \( \sqrt{-g}v^\rho_- \) measure essentially the left- and right-moving conformal weights.

We now use this to give a convenient expression for the conformal weight for spaces with CTCs in the bulk. Integrating (13) between \( \rho_c \) and infinity and using (26) we get, e.g. for \( \sigma = +1 \):
\[ \frac{24}{c} L_0 = 2 \int_{\rho_c}^{\rho_+} d\rho \sqrt{-g}T_{++} + 2 \int_\rho^\infty d\rho \sqrt{-g}T_{++} + 1 \] (27)

For \( \sigma = -1 \) we have to replace \( L_0 \) with \( \bar{L}_0 \) and \( T_{++} \) with \( T_{--} \) in this expression. The first term on the RHS comes from the CTC region and is negative as we argued already in (22). In this precise sense, the CTC region represents a negative contribution to either \( L_0 \) (when \( \sigma = 1 \)) or to \( \bar{L}_0 \) (when \( \sigma = -1 \)).

4. Examples: rotating dust solutions in AdS
We will now illustrate the above properties in some examples. We will find a simple class of analytic solutions involving rotating dust in AdS. These generalize the three-dimensional Gödel solution and can be engineered to have CTCs. In this section we set \( \Lambda = -1/4 \) and consider metrics of the form
\[ ds^2 = d\rho^2 - dt^2 + 2l dt d\phi + (l'^2 - l^2) d\phi^2 \] (28)

for some function \( l(\rho) \). Since the determinant of the metric is \( |l'| \), \( l' \) cannot vanish (except on the symmetry axis if there is one), and hence it must have the same sign everywhere.

These metrics satisfy the Einstein equations with a pressureless, rotating dust source:
\[ T^{ab} = Ru^a u^b. \] (29)

where the energy density \( R \) is given in terms of \( l \) as
\[ R = 1 - \frac{l''}{l}. \] (30)

The velocity vector is \( u = \partial_t \), so this coordinate system is comoving with the dust. Within this class of metrics we can construct analytic solutions with the desired behavior of \( g_{\phi\phi} \) by finding a suitable function \( l \).

4.1. The Gödel universe
The simplest example of this kind is obtained by taking the energy density of the dust to be constant. This is achieved by taking
\[ l = \mu \left( 1 - \cosh \frac{\rho}{\sqrt{\mu}} \right). \] (31)

with \( \mu \) a constant greater than one. This leads to the energy density \( R = \frac{\mu - 1}{\mu} \).
This solution is known as the 3D Gödel universe [13], because for the particular value \( \mu = 2 \) it describes the nontrivial three-dimensional part of Gödel’s original solution [5]. For \( \mu = 1 \) one recovers the AdS metric in global coordinates. The \( g_{\phi\phi} \) component of the metric behaves as follows (see Figure 3). At the radius

\[
\rho_{\text{max}} = \sqrt{\mu} \arccosh \left( \frac{\mu}{\mu - 1} \right)
\]  

(32)

it reaches a maximum beyond which it decreases with \( \rho \). At the radius

\[
\rho_{\text{CTC}} = \sqrt{\mu} \arccosh \left( \frac{\mu + 1}{\mu - 1} \right)
\]  

(33)

g_{\phi\phi} \) is zero; beyond this radius it becomes negative and there are ACTCs. This is an example of the ‘bad asymptotics’ category (i): the NEC is satisfied everywhere but the ACTCs persist to the boundary.

4.2. A Gödel dust ball

One way of obtaining an asymptotically AdS solution is to take a finite ball of dust with constant density \( R \) up to some radius \( \rho_0 \). The solution for \( \rho \geq \rho_0 \) is then a vacuum solution determined by solving the Israel matching conditions [14] at the edge of the dust ball \( \rho = \rho_0 \). This is the example considered in [15, 16] which we will here expand on and clarify. When solving the matching problem without a thin shell of matter on the edge of the dust ball, one finds that matching onto an asymptotically AdS metric is only possible for radii \( \rho_0 \leq \rho_{\text{max}} \) defined in (32).

Since we are interested in matchings with \( \rho_0 \geq \rho_{\text{ctc}} \) we will include a thin shell of matter in the matching problem. In particular, we will consider the matched configuration

\[
l_+ = \cosh \frac{\rho_0}{\sqrt{\mu}} (\mu - 1 + \cosh(\rho - \rho_0)) - \sqrt{\mu} \sinh \frac{\rho_0}{\sqrt{\mu}} \sinh(\rho - \rho_0) - \mu \quad \text{for } \rho \geq \rho_0.
\]  

(35)

For this configuration, the function \( l \) and the metric are continuous, but \( l' \) and hence also the extrinsic curvature\(^4\) \( K_{ij} \) change sign across the matching surface. From the Israel matching matching condition \( K_{ij} = - (T_{ij} - T g_{ij}) \) we deduce that our configuration corresponds to the singular source

\[
T_{ij} = 2(K_{ij}^- - K_{ij}^- g_{ij}).
\]  

(36)

The behavior of various quantities in this example are plotted in Figure 4. For \( \rho_0 > \rho_{\text{ctc}} \),

\(^4\) In our coordinate system one has \( K_{ij} = \frac{1}{2} g_{ij}' \).
the matched configuration has ACTCs in the bulk$^5$. Our argument of section 2 tells us that the null energy condition should be violated in the CTC region. It is well-known that the stress tensor of the interior Gödel space doesn’t violate any energy conditions, and we shall presently see that the NEC violation comes from the thin shell contribution at the edge. To verify our main identity (22), we should put $\rho_c = \rho_0$ and include the singular shell contribution to the stress tensor. One then obtains

$$4\pi G\sqrt{-g}T_{++}^\text{a} (\rho_0) = \frac{e^{2B}}{4\sqrt{-g}} g_{tt} g^{\phi\phi}_\text{r} (\rho_+) < 0. \quad (37)$$

One can check that this equality holds in our example and hence the NEC is violated.

To obtain the total value of $L_0$ we can use our expression (27)

$$\frac{24}{c} L_0 = 1 + T_{++}^\text{a} = 1 - \left( \mu - (\mu - 1) \cosh \frac{\rho_0}{\sqrt{\mu}} \right)^2. \quad (38)$$

From (33) we see that the current example has the intriguing property that the total $L_0$ becomes negative precisely when the spacetime contains CTCs, i.e. when $\rho_0 = \rho_{\text{ctc}}$. From the point of view of AdS/CFT, such spacetimes are unphysical because unitarity of the dual CFT forbids negative values of $L_0$. Since we saw that our example involved a tuned source on the matching surface, a natural question to ask is whether this behavior is generic. In the following example we will see that the answer is in the negative: there exist spacetimes with localized ACTCs which have positive conformal weights.

### 4.3. A smooth solution with CTCs in the bulk

In this example we display a simple class of smooth solutions which are asymptotically AdS and have localized CTCs. We take the function $l$ to be of the form

$$\ln l = \rho - a^2 e^{-\rho} + \frac{b - 2a^4}{4} e^{-2\rho} + (c_1 + c_2 \rho + c_3 \rho^2) e^{-3\rho}. \quad (39)$$

The first three terms guarantee that the metric has asymptotic AdS behavior (23) with the constants $a, b$ related to the total mass and angular momentum. The coefficients $c_i$ in the fourth term can then be chosen such that $g_{\phi\phi}$ has two zeroes for small $\rho$.

$^5$ In fact, these were the matched solutions considered in [16] although the presence of the singular source was overlooked there.
Figure 5. The smooth example with ACTCs in the bulk discussed in the text. The blue curve is $g_{\phi\phi}$, the red one is $\sqrt{-g}u^\rho_+$ and the purple one is $8\pi G\sqrt{-g}T_{++}$ (the latter two are scaled down by a factor 15).

As an illustrative example, we take $a^2 = b = \frac{1}{2}$, $c_1 = -\frac{1}{2}$, $c_2 = 3$, $c_3 = -10$. One finds that the corresponding metric has positive conformal weights:

$$\frac{24}{c}L_0 = \frac{1}{2}, \quad \frac{24}{c}\bar{L}_0 = \frac{3}{4}$$

This metric has CTCs between $\rho_- \simeq 0.246$ and $\rho_+ \simeq 0.638$. The behavior of various quantities is illustrated in figure 5. We see that the NEC is indeed violated in the ACTC region. In verifying the main identity (22) one finds that the radius $\rho_c$ defined in (21) is $\rho_c \simeq 0.412$, and

$$\int_{\rho_c}^{\rho+} d\rho \sqrt{-g}T_{++} = \frac{e^{2B}}{4\sqrt{-g}} g_{\phi\phi}(\rho_+) \simeq -3.526$$

We can also verify numerically the expression for $L_0$ in (27): the first term on the RHS, coming from the ACTC region, contributes approximately -7.052 and the second term, coming from outside the ACTC region, contributes approximately 6.552, leading indeed to $\frac{24}{c}L_0 = \frac{1}{2}$. We see that in this example, while the ACTC region contributes negatively to $L_0$, the total value of $L_0$ is still positive.

5. Outlook

In this work we showed that simple time machines in asymptotically AdS spaces necessarily violate the NEC, and that the NEC violating region contributes negatively to the total conformal weight in the dual CFT. It would be very interesting to bring the AdS/CFT correspondence to bear on the issue of chronology protection and pinpoint the pathologies of spacetimes with CTCs.

A first open question is whether CTCs in the bulk are always linked to violation of unitarity in the dual CFT. In some examples, such as the Gödel dust ball example of section 4.2, this is obviously the case: the violation of the NEC is so severe that the negative contribution to (27) outweighs the positive one and the total value of $L_0$ is negative. In other examples, such as the one discussed in section 4.3, there are CTCs in the bulk while $L_0$ remains positive. This does not guarantee however that unitarity is respected. In such examples, one has to turn on a variety of terms in the metric which are subleading at large $\rho$ and have the effect of driving $g_{\phi\phi}$ negative at small values of $\rho$ (in our example 4.3, these are the terms parameterized by
$c_1, c_2, c_3$). These subleading terms are expected to encode one-point functions of other operators in the CFT than the stress tensor\(^6\), and hence also to be constrained by unitarity. It would be interesting to explore these constraints further.

Another possible role of AdS/CFT could be to further justify the NEC at the classical level: it is obeyed in most systems but there are exceptions [18]. In the AdS/CFT correspondence, the classical gravity limit describes a certain large $N$ limit of the dual CFT. As a consequence, gravity systems that admit CFT duals are constrained already at the classical level by properties of the dual CFT such as unitarity. It seems promising in this regard that the NEC has been shown to be related to the c-theorem (and hence ultimately to unitarity) in the dual CFT [19].

And finally, since AdS/CFT allows one to address quantum corrections in the bulk systematically by taking into account $1/N$ corrections in the boundary theory, it could shed light on the poorly understood issue of chronology protection in quantum gravity.

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[1] S. W. Hawking, “The Chronology protection conjecture,” Phys. Rev. D 46, 603 (1992).
[2] K. S. Thorne, “Closed timelike curves,” Prepared for 13th Conference on General Relativity and Gravitation (GR-13), Cordoba, Argentina, 29 Jun - 4 Jul 1992.
[3] M. Visser, “The Quantum physics of chronology protection,” [gr-qc/0204022].
[4] W. J. van Stockum, “The gravitational field of a distribution of particles rotating about an axis of symmetry,” Proc. Roy. Soc. Edinburgh 57, 135 (1937).
[5] K. Godel, “An Example of a new type of cosmological solutions of Einstein’s field equations of gravitation,” Rev. Mod. Phys. 21, 447 (1949).
[6] P. Menotti and D. Seminara, “Closed Timelike Curves And The Energy Condition In (2+1)-Dimensional Gravity,” Phys. Lett. B 301, 25 (1993) [Erratum-ibid. B 307, 404 (1993)] [arXiv:hep-th/921078].
[7] J. M. Maldacena, “The Large N limit of superconformal field theories and supergravity,” Adv. Theor. Math. Phys. 2, 231 (1998) [Int. J. Theor. Phys. 38, 1113 (1999)] [arXiv:hep-th/971120].
[8] J. Raeymaekers, “Chronology protection in stationary three-dimensional spacetimes,” [arXiv:1106.5098 [hep-th]].
[9] S. Deser and B. Laurent, “Stationary Axisymmetric Solutions of Three-Dimensional Einstein Gravity,” Gen. Rel. Grav. 18, 617 (1986).
[10] M. Mars and J. M. M. Senovilla, “Axial symmetry and conformal Killing vectors,” Class. Quant. Grav. 10, 1633 (1993) [arXiv:gr-qc/9201045].
[11] M. Banados, C. Teitelboim and J. Zanelli, “The Black hole in three-dimensional space-time,” Phys. Rev. Lett. 69, 1849 (1992) [arXiv:hep-th/9204099].
[12] C. Fefferman and C.R. Graham, “Conformal Invariants,” in Elie Cartan et les Mathematiques daujourdhui (Asterisque, 1985) 95.
[13] M. Rooman and P. Spindel, “Goedel metric as a squashed anti-de Sitter geometry,” Class. Quant. Grav. 15, 3241 (1998) [arXiv:gr-qc/9804027].
[14] W. Israel, “Singular hypersurfaces and thin shells in general relativity,” Nuovo Cim. B 44S10 (1966) 1 [Erratum-ibid. B 48 (1967 NUCIA,B44.1.1966) 463].
[15] M. Lubo, M. Rooman and P. Spindel, “(2+1)-dimensional stars,” Phys. Rev. D 59, 044012 (1999) [arXiv:gr-qc/9806104].
[16] J. Raeymaekers, D. Van den Bleeken and B. Veronnecke, “Relating chronology protection and unitarity through holography,” JHEP 1004, 021 (2010) [arXiv:0911.3893 [hep-th]].
[17] A. Campoleoni, S. Fredenhagen, S. Pfenninger and S. Theisen, “Asymptotic symmetries of three-dimensional gravity coupled to higher-spin fields,” JHEP 1011, 007 (2010) [arXiv:1008.4744 [hep-th]].
[18] C. Barcelo, M. Visser, “Twilight for the energy conditions?,” Int. J. Mod. Phys. D11, 1553-1560 (2002). [gr-qc/0205066].
[19] D. Z. Freedman, S. S. Gubser, K. Pilch and N. P. Warner, “Renormalization group flows from holography supersymmetry and a c theorem,” Adv. Theor. Math. Phys. 3 (1999) 363 [arXiv:hep-th/9904017].

For example, in the case of 3D gravity coupled to higher spins [17], these subleading terms encode the one-point functions of primaries of conformal weight greater than two in the dual CFT.