A Semi-Linear Approximation of the First-Order Marcum $Q$-function with Application to Predictor Antenna Systems

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Abstract—First-order Marcum $Q$-function is observed in various problem formulations. However, it is not an easy-to-handle function. For this reason, in this paper, we first present a semi-linear approximation of the Marcum $Q$-function. Our proposed approximation is useful because it simplifies, e.g., various integral calculations including the Marcum $Q$-function as well as different operations such as parameter optimization. Then, as an example of interest, we apply our proposed approximation approach to the performance analysis of predictor antenna (PA) systems. Here, the PA system is referred to as a system with two sets of antennas on the roof of a vehicle. Then, the PA positioned in the front of the vehicle can be used to improve the channel state estimation for data transmission of the receive antenna that is aligned behind the PA. Considering spatial mismatch due to the mobility, we derive closed-form expressions for the instantaneous and average throughput as well as the throughput-optimized rate allocation. As we show, our proposed approximation scheme enables us to analyze PA systems with high accuracy. Moreover, our results show that rate adaptation can improve the performance of PA systems with different levels of spatial mismatch.

Index Terms—Backhaul, channel state information (CSI), integrated access and backhaul (IAB), linear approximation, Marcum $Q$-function, mobility, mobile relay, outage probability, predictor antenna, rate adaptation, spatial correlation, throughput.

I. INTRODUCTION

The first-order Marcum $Q$-function, as defined in [1] Eq. (1)

$$ Q_1(\alpha, \beta) = \int_{\beta}^{\infty} x e^{-\frac{x^2}{2}} I_0(x\alpha)dx, $$

where $\alpha, \beta \geq 0$ and $I_n(x) = (\frac{x}{2})^n \sum_{i=0}^{\infty} \frac{(\frac{x}{2})^{2i}}{i! (i+n+1)!}$ is the $n$-order modified Bessel function of the first kind, and $\Gamma(z) = \int_{0}^{\infty} x^{z-1}e^{-x}dx$ represents the Gamma function. Reviewing the literature, the Marcum $Q$-function has appeared in many areas such as statistics/signal detection [2], and in the performance analysis of different setups such as temporally correlated channels [3], spatial correlated channels [4], free-space optical (FSO) links [5], relay networks [6], as well as cognitive radio and radar systems [7]–[24]. However, in these applications, the presence of the Marcum $Q$-function makes the mathematical analysis challenging, because it is difficult to manipulate with no closed-form expressions especially when it appears in parameter optimizations and integral calculations.

For this reason, several methods have been developed in [1], [25]–[36] to bound/approximate the Marcum $Q$-function. For example, [25], [26] have proposed modified forms of the function, while [27], [28] have derived exponential-type bounds which are good for the bit error rate analysis at high signal-to-noise ratios (SNRs). Other types of bounds are expressed by, e.g., error function [33] and Bessel functions [34]–[36]. Some alternative methods have been also proposed in [1], [29]–[32]. Although each of these approximation/bounding techniques are fairly tight for their considered problem formulation, they are still based on difficult functions, or have complicated summation/integration formations, which may be not easy to deal with in, e.g., integral calculations and parameter optimizations.

In this paper, we first propose a simple semi-linear approximation of the first-order Marcum $Q$-function (Lemma 1 Corollaries [1][3]). As we show through various examples, the developed linearization technique is tight for a broad range of parameter settings. More importantly, the proposed approach simplifies various integral calculations and derivations and, consequently, allows us to express different expectation- and optimization-based operations in closed-form (Lemmas [2][4]).

To demonstrate the usefulness of the proposed approximation technique in communication systems, we analyze the performance of predictor antenna (PA) systems which is of our current interest. Here, the PA system is referred to as a setup with two (sets of) antennas on the roof of a vehicle. The PA positioned in the front of a vehicle can be used to improve the channel state estimation for downlink data reception at the receive antenna (RA) on the vehicle that is aligned behind the PA [37]–[45]. The feasibility of such setups, which are of interest particular in public transport systems such as trains and buses, but potentially also for the more design-constrained cars, have been previously shown through experimental tests [37]–[40], and different works have analyzed their system performance from different perspectives [41]–[45].

Among the challenges of the PA system is the spatial mismatch. If the RA does not arrive in the same position as the PA, the actual channel for the RA would not be
identical to the one experienced by the PA before. Such inaccurate channel state information (CSI) estimation will affect the system performance considerably at moderate/high speeds [38], [45]. In this paper, we address this problem by implementing adaptive rate allocation. In our proposed setup, the instantaneous CSI provided by the PA is used to adapt the data rate of the signals sent to the RA from the base station (BS). The problem is cast in the form of throughput maximization. Particularly, we use our developed approximation approach to derive closed-form expressions for the instantaneous and average throughput as well as the optimal rate allocation maximizing the throughput (Lemma 1). Moreover, we study the effect of different parameters such as the antennas distance, the vehicle speed, and the processing delay of the BS on the performance of PA setups.

Our paper is different from the state-of-the-art literature because the proposed semi-linear approximation of the first-order Marcum $Q$-function and the derived closed-form expressions for the considered integrals have not been presented by, e.g., [1]–[45]. Also, as opposed to [37]–[44], we perform analytical evaluations on the system performance with CSIT (T: at the transmitter)-based rate optimization to mitigate the effect of the spatial mismatch. Moreover, compared to our preliminary results in [45], this paper develops the semi-linear approximation method for the Marcum $Q$-function and uses our proposed approximation method to analyze the performance of the PA system. Also, we perform deep analysis of the effect of various parameters, such as imperfect CSIT feedback schemes, and processing delay of the BS on the system performance.

The simulation and the analytical results indicate that the proposed semi-linear approximation is useful for the mathematical analysis of different Marcum $Q$-function-based problem formulations. Particularly, our approximation method enables us to represent different Marcum $Q$-function-based integrations and optimizations in closed-form. Considering the PA system, our derived analytical results show that adaptive rate allocation can considerably improve the performance of the PA system in the presence of spatial mismatch. Finally, with different levels of channel estimation, our results show that there exists an optimal speed for the vehicle optimizing the throughput/outage probability, and the system performance is sensitive to the vehicle speed/processing delay as speed moves away from its optimal value.

This paper is organized as follows. In Section II, we present our proposed semi-linear approximation of the first-order Marcum $Q$-function, and derive closed-form solutions for some integrals of interest. Section III deals with the application of the approximation in the PA system, deriving closed-form expressions for the optimal rate adaptation, the instantaneous throughput maximization. Particularly, we use our developed approximation approach to derive closed-form expressions for the optimal rate adaptation, the instantaneous and average throughput as well as the optimal rate allocation maximizing the throughput (Lemma 1). Moreover, we study the effect of different parameters such as the antennas distance, the vehicle speed, and the processing delay of the BS on the performance of PA setups.

II. APPROXIMATION OF THE FIRST-ORDER MARCUM $Q$-FUNCTION

In this section, we present our semi-linear approximation of the cumulative distribution function (CDF) in the form of $y(\alpha, \beta) = 1 - Q_1(\alpha, \beta)$. The idea of this proposed approximation is to use one point and its corresponding slope in that point to create a line approximating the CDF. The approximation method is summarized in Lemma 1 as follows.

**Lemma 1.** The CDF of the form $y(\alpha, \beta) = 1 - Q_1(\alpha, \beta)$ can be semi-linearly approximated as $Y(\alpha, \beta) \simeq Z(\alpha, \beta)$ where

$$Z(\alpha, \beta) = \begin{cases} 0, & \beta < c_1 \\ \frac{\alpha + \sqrt{\alpha^2 + 2}}{2} \left( \frac{\alpha + \sqrt{\alpha^2 + 2}}{2} \right)^2 \times I_0 \left( \frac{\alpha + \sqrt{\alpha^2 + 2}}{2} \right) - 1, & \beta = c_1 \\ 1 - Q_1 \left( \alpha, \frac{\alpha + \sqrt{\alpha^2 + 2}}{2} \right), & c_1 \leq \beta \leq c_2 \\ 1, & \beta > c_2, \end{cases}$$

with

$$c_1(\alpha) = \max \left( 0, \frac{\alpha + \sqrt{\alpha^2 + 2}}{2} + \frac{Q_1 \left( \alpha, \frac{\alpha + \sqrt{\alpha^2 + 2}}{2} \right)}{\alpha + \sqrt{\alpha^2 + 2} e^{-\frac{1}{2} \left( \alpha^2 + \frac{(\alpha + \sqrt{\alpha^2 + 2})^2}{2} \right) I_0 \left( \frac{\alpha + \sqrt{\alpha^2 + 2}}{2} \right)}}, \frac{Q_1 \left( \alpha, \frac{\alpha + \sqrt{\alpha^2 + 2}}{2} \right)}{\alpha + \sqrt{\alpha^2 + 2} e^{-\frac{1}{2} \left( \alpha^2 + \frac{(\alpha + \sqrt{\alpha^2 + 2})^2}{2} \right) I_0 \left( \frac{\alpha + \sqrt{\alpha^2 + 2}}{2} \right)}}, \frac{Q_1 \left( \alpha, \frac{\alpha + \sqrt{\alpha^2 + 2}}{2} \right)}{\alpha + \sqrt{\alpha^2 + 2} e^{-\frac{1}{2} \left( \alpha^2 + \frac{(\alpha + \sqrt{\alpha^2 + 2})^2}{2} \right) I_0 \left( \frac{\alpha + \sqrt{\alpha^2 + 2}}{2} \right)}} \right),$$

$$c_2(\alpha) = \frac{\alpha + \sqrt{\alpha^2 + 2}}{2} + \frac{Q_1 \left( \alpha, \frac{\alpha + \sqrt{\alpha^2 + 2}}{2} \right)}{\alpha + \sqrt{\alpha^2 + 2} e^{-\frac{1}{2} \left( \alpha^2 + \frac{(\alpha + \sqrt{\alpha^2 + 2})^2}{2} \right) I_0 \left( \frac{\alpha + \sqrt{\alpha^2 + 2}}{2} \right)}}.$$

**Proof.** We aim to approximate the CDF in the range $y \in [0, 1]$ by

$$y - y_0 = m(x - x_0),$$

where $C = (x_0, y_0)$ is a point on the CDF curve and $m$ is the slope at point $C$ of $y(\alpha, \beta)$. Then, the parts of the line outside this region are replaced by $y = 0$ and $y = 1$ (see Fig. 1).

To obtain a good approximation of the CDF, we select the point $C$ by solving

$$x = \arg x \left\{ \frac{\partial^2 (1 - Q_1(\alpha, x))}{\partial x^2} = 0 \right\}.$$  \hspace{1cm} (6)

Using the derivative of the first-order Marcum $Q$-function with respect to $x$ [46, Eq. (2)]

$$\frac{\partial Q_1(\alpha, x)}{\partial x} = -xe^{-\frac{x^2 + \alpha^2}{2}} I_0(\alpha x),$$

(6) is equivalent to

$$x = \arg x \left\{ \frac{\partial \left(x e^{-\frac{x^2 + \alpha^2}{2}} I_0(\alpha x)\right)}{\partial x} = 0 \right\}.$$  \hspace{1cm} (8)
Proof. Using the approximation $I_0(x) \simeq \frac{e^x}{\sqrt{2\pi x}}$ [47, Eq. (9.7.1)] for moderate/large values of $x$ and writing
\[
\frac{\partial}{\partial x} \left( \frac{x}{2\pi} e^{-\frac{(x-\alpha)^2}{2}} \right) = 0
\]
we obtain
\[
x = \frac{\alpha + \sqrt{\alpha^2 + 2}}{2}, \quad \text{since } x \geq 0.
\]
In this way, we find the point
\[
C = \left( \frac{\alpha + \sqrt{\alpha^2 + 2}}{2}, 1 - Q_1 \left( \frac{\alpha + \sqrt{\alpha^2 + 2}}{2} \right) \right). \tag{11}
\]
To calculate the slope $m$ at the point $C$, we plug (10) into (7) leading to
\[
m = \frac{\alpha + \sqrt{\alpha^2 + 2}}{2} \times e^{-\frac{1}{2} \left( \frac{\sqrt{\alpha^2 + 2}}{2} \right)^2} I_0 \left( \frac{\alpha + \sqrt{\alpha^2 + 2}}{2} \right).	ag{12}
\]
Finally, using (3), (10) and (12), the CDF $y(\alpha, \beta) = 1 - Q_1(\alpha, \beta)$ can be approximated as in (2). Note that, because the CDF is limited to the range [0, 1], the boundaries $c_1$ and $c_2$ in (2) are obtained by setting $y = 0$ and $y = 1$ which leads to the semi-linear approximation as given in (2).

To further simplify the calculation, considering different ranges of $\alpha$, the approximation (2) can be simplified as stated in the following corollaries.

**Corollary 1.** For moderate/large values of $\alpha$, we have $y(\alpha, \beta) \simeq \tilde{Z}(\alpha, \beta)$ where
\[
\text{if } \beta < \frac{-1}{\alpha e^{-\alpha^2} I_0(\alpha^2)} + \frac{1}{2} \left( 1 - e^{-\alpha^2} I_0(\alpha^2) \right),
\]
\[
\tilde{Z}(\alpha, \beta) \simeq \begin{cases} 
0, & \text{if } \beta < \frac{-1}{\alpha e^{-\alpha^2} I_0(\alpha^2)} + \frac{1}{2} \left( 1 - e^{-\alpha^2} I_0(\alpha^2) \right), \\
\alpha e^{-\alpha^2} I_0(\alpha^2)(\beta - \alpha) + \frac{1}{2} \left( 1 - e^{-\alpha^2} I_0(\alpha^2) \right), & \text{if } \beta \leq \frac{-1}{\alpha e^{-\alpha^2} I_0(\alpha^2)} + \frac{1}{2} \left( 1 - e^{-\alpha^2} I_0(\alpha^2) \right), \\
\frac{1}{\alpha e^{-\alpha^2} I_0(\alpha^2)} + \frac{1}{2} \left( 1 - e^{-\alpha^2} I_0(\alpha^2) \right), & \text{if } \beta > \frac{-1}{\alpha e^{-\alpha^2} I_0(\alpha^2)} + \frac{1}{2} \left( 1 - e^{-\alpha^2} I_0(\alpha^2) \right). 
\end{cases}
\]

**Proof.** Using (10) for moderate/large values of $\alpha$, we have
\[
x \simeq \alpha
\]
and
\[
\tilde{c}_1 = \frac{-1}{2} \left( 1 - e^{-\alpha^2} I_0(\alpha^2) \right) + \alpha, \quad \text{and} \quad \tilde{c}_2 = 1 - \frac{1}{2} \left( 1 - e^{-\alpha^2} I_0(\alpha^2) \right) + \alpha.
\]

which leads to (13). Note that in (13) we have used the fact that [48, Eq. (A-3.2)]
\[
Q_1(\alpha, \beta) = \frac{1}{2} \left( 1 + e^{-\alpha^2} I_0(\alpha^2) \right). \tag{16}
\]

**Corollary 2.** For small values of $\alpha$, we have $y(\alpha, \beta) \simeq \tilde{Z}(\alpha, \beta)$ with
\[
\tilde{Z}(\alpha, \beta) \simeq \begin{cases} 
0, & \text{if } \beta < \hat{c}_1, \\
\alpha e^{-\alpha^2} I_0(\alpha^2)(\beta - \alpha) + \frac{1}{2} \left( 1 - e^{-\alpha^2} I_0(\alpha^2) \right), & \text{if } \hat{c}_1 \leq \beta \leq \hat{c}_2, \\
1, & \text{if } \beta > \hat{c}_2.
\end{cases}
\]

with $\hat{c}_1$ and $\hat{c}_2$ given in (18) and (19), respectively.

**Proof.** Using (10) for small values of $\alpha$, we have $x \simeq \alpha + \sqrt{2\alpha}$, which leads to
\[
\hat{c}_1 = \frac{1}{\alpha e^{-\alpha^2} I_0(\alpha^2)} + \alpha, \quad \text{and} \quad \hat{c}_2 = \frac{1}{\alpha e^{-\alpha^2} I_0(\alpha^2)} + \alpha.
\]

Corollary 3. Equation (13) can be further simplified as
\[
\tilde{Z}(\alpha, \beta) \simeq \begin{cases} 
0, & \text{if } \beta < \hat{c}_1, \\
\frac{1}{\sqrt{2\pi}} (\beta - \alpha) + \frac{1}{2} \left( 1 - \frac{1}{\sqrt{2\pi} \alpha^2} \right), & \text{if } \hat{c}_1 \leq \beta \leq \hat{c}_2, \\
1, & \text{if } \beta > \hat{c}_2.
\end{cases}
\]

with $\hat{c}_1$ and $\hat{c}_2$ given by (20) and (21), respectively.

**Proof.** For moderate/large values of $\alpha$, using the approximation $I_0(x) \simeq \frac{e^x}{\sqrt{2\pi x}}$ for (13) leads to (20) where
\[
\hat{c}_1 = \frac{-\sqrt{2\pi} \beta - \alpha}{2} + \frac{1}{2} \left( 1 - \frac{1}{\sqrt{2\pi} \alpha^2} \right), \quad \text{and} \quad \hat{c}_2 = \frac{\sqrt{2\pi} - \sqrt{2\pi} \alpha^2}{2} + \alpha.
\]

To illustrate these semi-linear approximations, Fig. 1 shows the CDF $y(\alpha, \beta) = 1 - Q_1(\alpha, \beta)$ for both small and large values of $\alpha$, and compares the exact CDF with the approximation schemes of Lemma 1 and Corollaries 1-3. From Fig. 1 we can observe that Lemma 1 is tight for a broad range of $\alpha$ and moderate values of $\beta$. Moreover, the tightness is
where \( \Gamma \) (gamma function \[47, Eq. 6.5.1\].

The integral (23) is approximately given by

\[
G = \left[ \begin{array}{c}
\bar{F}_1(c) - \bar{F}_1(b) + \bar{F}_2(\max(c, \theta_2)) - \bar{F}_2(c) \\
\bar{F}_2(\max(c, \theta_2)) - \bar{F}_2(c) \\
0
\end{array} \right],
\]

(25)

with \( \theta_2 > \theta_1 \geq 0 \), which does not have a closed-form expression for different values of \( m, \alpha, \). This integral is interesting as it is often used to analyse the expected performance of outage-limited systems, e.g., \[7\], \[27\], \[32\], \[49\]. Then, using Lemma \(1\) \( T(\alpha, m, \alpha, \theta) \) can be approximated in closed-form as follows.

**Lemma 3.** The integral (25) is approximately given by

\[
T(\alpha, m, \alpha, \theta_1, \theta_2) \simeq
\begin{cases}
\bar{F}_1(\theta_2) - \bar{F}_1(\theta_1), & \text{if } 0 < \theta_1 < \theta_2 < c_1 \\
\bar{F}_1(c_1) - \bar{F}_1(b) + \bar{F}_2(\max(c_2, \theta_2)) - \bar{F}_2(c_1), & \text{if } \theta_1 < c_1, \theta_2 \geq c_1 \\
\bar{F}_2(\max(c_2, \theta_2)) - \bar{F}_2(c_1), & \text{if } \theta_1 > c_1 \\
0, & \text{if } \theta_1 > c_2,
\end{cases}
\]

(26)

where \( c_1 \) and \( c_2 \) are given by \[3\] and \[4\], respectively. Moreover,

\[
\bar{F}_1(x) = \frac{1}{m} \left( -e^{\frac{x}{m}} E_1 \left( mx + \frac{m}{a} \right) - e^{-mx} \log(ax + 1) \right),
\]

(27)

and

\[
\bar{F}_2(x) = \left( mn_2 - an_2 - amn_1 \right) e^{\frac{m(ax + 1)}{a}} E_1 \left( \frac{m(ax + 1)}{a} \right) - a \left( mn_2 x + n_2 + mn_1 \right) e^{\frac{m(ax + 1)}{a}} E_1 \left( \frac{m(ax + 1)}{a} \right) - a \left( mn_2 x + n_2 + mn_1 \right)
\]

(28)

with

\[
n_1 = 1 + \frac{\alpha + \sqrt{\alpha^2 + 2}}{2} e^{-\frac{1}{2} \left( \alpha^2 + \left( \frac{\alpha + \sqrt{\alpha^2 + 2}}{2} \right)^2 \right)} \times
\]

\[
I_0 \left( \frac{\alpha + \sqrt{\alpha^2 + 2}}{2} \right) \times \frac{\alpha + \sqrt{\alpha^2 + 2}}{2} - 1 + Q_1 \left( \alpha, \frac{\alpha + \sqrt{\alpha^2 + 2}}{2} \right)
\]

(29)

and

\[
n_2 = -\frac{\alpha + \sqrt{\alpha^2 + 2}}{2} e^{-\frac{1}{2} \left( \alpha^2 + \left( \frac{\alpha + \sqrt{\alpha^2 + 2}}{2} \right)^2 \right)} \times
\]

\[
I_0 \left( \frac{\alpha + \sqrt{\alpha^2 + 2}}{2} \right) \times \frac{\alpha + \sqrt{\alpha^2 + 2}}{2} - 1 + Q_1 \left( \alpha, \frac{\alpha + \sqrt{\alpha^2 + 2}}{2} \right)
\]

(30)

In \[27\] and \[28\], \( E_1(x) = \int_0^\infty e^{-x} \frac{t}{t} \, dt \) is the Exponential Integral function \[47, p. 228, (5.1.1)\].
with one can follow the same procedure in (25) to approximate (31)

one can follow the same procedure in (25) to approximate (31) as

\[ T(\alpha, 0, a, \theta_1, \theta_2) \simeq \begin{cases} 
F_3(\theta_2) - F_3(\theta_1), & \text{if } 0 \leq \theta_1 < \theta_2 < c_1 \\
F_3(c_1) - F_3(\theta_1) + F_4(\max(c_2, \theta_2)) - F_4(\theta_1), & \text{if } \theta_1 < c_1, \theta_2 \geq c_1 \\
F_4(\max(c_2, \theta_2)) - F_4(c_1), & \text{if } \theta_1 > c_1 \\
0, & \text{if } \theta_1 > c_2,
\end{cases} \tag{32} \]

with \( c_1 \) and \( c_2 \) given by (3) and (4), respectively. Also,

\[ F_3 = \frac{(ax + 1)(\log(ax + 1) - 1)}{a}, \tag{33} \]

and

\[ F_4 = \frac{n_2 \left(2a^2x^2 - 2\log(ax + 1) - a^2x^2 + 2ax\right)}{4a^2} + \frac{n_1(ax + 1)(\log(ax + 1) - 1)}{a}, \tag{34} \]

where \( n_1 \) and \( n_2 \) are given by (29) and (30), respectively.

In Figs. 2 and 3, we evaluate the tightness of the approximations in Lemmas 2 and 3, for different values of \( m, n, \rho, \alpha, a \) and \( a \). From the figures, it can be observed that the approximation schemes of Lemmas 2 and 3 are very tight for different parameter settings, while our proposed semi-linear approximation makes it possible to represent the integrals in closed-form. In this way, although the approximation (2) is not tight at the tails of the CDF, it gives tight approximation results when it appears in different integrals. Also, as we show in Section III, the semi-linear approximation scheme is efficient in optimization problems involving the Marcum \( Q \)-function.

Finally, to tightly approximate the Marcum \( Q \)-function at the tails, which are the range of interest in, e.g., error probability analysis, one can use the approximation schemes of \([27], [28]\).}

III. APPLICATIONS IN PA SYSTEMS

In Section II, we showed how the proposed approximation scheme enables us to derive closed-form expressions for a broad range of integrals, as required in various expectation-based calculations, e.g., \([7], [16]–[18], [27], [32], [49]\). On other hand, the Marcum \( Q \)-function may also appear in optimization problems, e.g., \([19], \text{eq. (8)}, [20], \text{eq. (9)}, [21], \text{eq. (10)}, [22], \text{eq. (10)}, [23], \text{eq. (15)}, [24], \text{eq. (22)}\). For this reason, in this section, we provide an example of using our proposed semi-linear approximation in an optimization problem for the PA systems.

A. Problem Formulation

Vehicle communication is one of the most important use cases in 5G. Here, the main focus is to provide efficient and reliable connections to cars and public transports, e.g., buses and trains. CSIT plays an important role in achieving these goals, since the data transmission efficiency can be improved by updating the transmission parameters relative to the instantaneous channel state. However, the typical CSIT acquisition systems, which are mostly designed for (semi)static channels, may not work well for high-speed vehicles. This is because, depending on the vehicle speed, the position of the antennas may change quickly and the channel information becomes inaccurate. To overcome this issue, \([37]–[42]\) propose...
Fig. 4. A PA system with mismatch problem. Here, \( \hat{h} \) is the channel between the BS and the PA while \( h \) refers to the BS-RA link. The vehicle is moving with speed \( v \) and the antenna separation is \( d_a \). The red arrow indicates the spatial mismatch, i.e., when the RA does not reach at the same point as the PA when sending pilots. Also, \( d_m \) is the moving distance of the vehicle which is affected by the processing delay \( \delta \) of the BS.

the PA setup as shown in Fig. 3 [37] With a PA setup, which is of interest in Vehicle-to-everything (V2X) communications as well as integrated access and backhauling [50], two antennas are deployed on the top of the vehicle. The first antenna, the PA, estimates the channel and sends feedback to the BS at time \( t \). Then, the BS uses the CSIT provided by the PA to communicate with a second antenna, which we refer to as RA, at time \( t + \delta \), where \( \delta \) is the processing time at the BS. In this way, BS can use the CSIT acquired from the PA and perform various CSIT-based transmission schemes, e.g., [37], [42].

We assume that the vehicle moves through a stationary electromagnetic standing wave pattern. Thus, if the RA reaches exactly the same position as the position of the PA when sending the pilots, it will experience the same channel and the CSIT will be perfect. However, if the RA does not reach the same spatial point as the PA, due to, e.g., the BS processing delay is not equal to the time that we need until the RA reaches the same point as the PA, the RA may receive the data in a place different from the one in which the PA was sending the pilots. Such spatial mismatch may lead to CSIT inaccuracy, which will affect the system performance considerably. Thus, we need adaptive schemes to compensate for it.

Considering downlink transmission in the BS-RA link, the received signal is given by

\[
Y = \sqrt{P}hX + Z.
\]  

(35)

Here, \( P \) represents the transmit power, \( X \) is the input message with unit variance, and \( h \) is the fading coefficient between the BS and the RA. Also, \( Z \sim \mathcal{CN}(0, 1) \) denotes the independent and identically distributed (IID) complex Gaussian noise added at the receiver.

We denote the channel coefficient of the PA-BS uplink as \( h \).

Also, we define \( d \) as the effective distance between the place where the PA estimates the channel at time \( t \), and the place where the RA reaches at time \( t + \delta \). As can be seen in Fig. 4, \( d \) can be calculated as

\[
d = |d_a - d_m| = |d_a - v\delta|,
\]  

(36)

where \( d_m \) is the moving distance of the vehicle during time interval \( \delta \), and \( v \) is the velocity of the vehicle. Also, \( d_a \) is the antenna separation between the PA and the RA. In conjunction to \( \hat{h} \), here, we assume \( d \) can be calculated by the BS.

Using the classical Jake’s correlation model [51, p. 2642] by assuming uniform angular spectrum, the channel coefficient of the BS-RA downlink can be modeled as

\[
h = \sqrt{1 - \sigma^2} \hat{h} + \sigma q,
\]  

(37)

Here, \( q \sim \mathcal{CN}(0, 1) \) which is independent of the known channel value \( \hat{h} \sim \mathcal{CN}(0, 1) \), and \( \sigma \) is a function of the effective distance \( d \) as

\[
\sigma = \sqrt{\frac{\phi_2^2 - \phi_1^2}{\phi_1^2}} = \sqrt{\frac{\phi_2^2 - \phi_1^2}{\phi_1^2}}.
\]  

(38)

Here, \( \phi_1 = \Phi_{1/2, 1/2} \) and \( \phi_2 = \Phi_{1/2, 2} \), where \( \Phi \) is from Jake’s model [51, p. 2642]

\[
[\hat{h}] = \Phi^{1/2} \mathbf{H}_e,
\]  

(39)

In (39), \( \mathbf{H}_e \) has independent circularly-symmetric zero-mean complex Gaussian entries with unit variance, and \( \Phi \) is the channel correlation matrix with the \((i, j)\)-th entry given by

\[
\Phi_{i,j} = J_0((i - j) \cdot 2\pi d/\lambda) \forall i, j.
\]  

(40)

Here, \( J_n(x) = (\frac{2}{\pi})^n \sum_{i=0}^{\infty} \frac{(\frac{x}{2})^{2i}}{(i!)(n+i+1)!} \) represents the \( n \)-th order Bessel function of the first kind. Moreover, \( \lambda \) denotes the carrier wavelength, i.e., \( \lambda = c/f_c \) where \( c \) is the speed of light and \( f_c \) is the carrier frequency.

From (37), for a given \( \hat{h} \) and \( \sigma \neq 0 \), \(|h|\) follows a Rician distribution, i.e., the probability density function (PDF) of \(|h|\) is given by

\[
f_{|h|}(x) = \frac{2x}{\sigma^2} e^{-\frac{x^2 + \tilde{\sigma}^2}{2\sigma^2}} I_0 \left( \frac{2x\sqrt{\tilde{\sigma}}}{\sigma^2} \right),
\]  

(41)

where \( \tilde{\sigma} = |\hat{h}|^2 \). Let us define the channel gain between BS-RA as \( g = |h|^2 \). Then, the PDF of \( f_{g|\tilde{g}} \) is given by

\[
f_{g|\tilde{g}}(x) = \frac{1}{\sigma^2} e^{-\frac{x + \tilde{\sigma}}{2\sigma^2}} I_0 \left( \frac{2\sqrt{x\tilde{\sigma}}}{\sigma^2} \right),
\]  

(42)

which is non-central Chi-squared distributed with the CDF containing the first-order Marcum Q-function as

\[
F_{g|\tilde{g}}(x) = 1 - Q_1 \left( \frac{2\tilde{\sigma}}{\sigma^2}, \sqrt{\frac{2x}{\sigma^2}} \right).
\]  

(43)

B. Analytical Results on Rate Adaptation Using the Semi-Linear Approximation of the First-order Marcum Q-Function

We assume that \( d_a, \delta \), and \( \tilde{\sigma} \) are known by the BS. It can be seen from (42) that \( f_{g|\tilde{g}}(x) \) is a function of \( v \). For a given \( v \), the distribution of \( g \) is known by the BS, and a rate adaption scheme can be performed to improve the system performance.

For a given instantaneous value of \( \tilde{g} \), the data is transmitted with instantaneous rate \( R_{g|\tilde{g}} \) bits-per-channel-use (bpcu). If the instantaneous channel gain realization supports the transmitted
data rate $R_{\hat{g}}$, i.e., $\log(1 + gP) \geq R_{\hat{g}}$, the data can be successfully decoded. Otherwise, outage occurs. Hence, the outage probability in each time slot is

$$\Pr(\text{outage}|\hat{g}) = F_{g|\hat{g}}\left(\frac{e^{R_{\hat{g}}} - 1}{P}\right).$$

(44)

Also, the instantaneous throughput for a given $\hat{g}$ is

$$\eta_{\hat{g}}(R_{\hat{g}}) = R_{\hat{g}}(1 - \Pr(\log(1 + gP) < R_{\hat{g}})), \quad (45)$$

and the optimal rate adaptation maximizing the instantaneous throughput is obtained by

$$R_{\hat{g}}^{opt} = \arg\max_{R_{\hat{g}} \geq 0}\left\{ (1 - \Pr(\log(1 + gP) < R_{\hat{g}})) R_{\hat{g}} \right\}$$

$$= \arg\max_{R_{\hat{g}} \geq 0}\left\{ (1 - F_{g|\hat{g}}\left(\frac{e^{R_{\hat{g}}} - 1}{P}\right)) R_{\hat{g}} \right\}$$

$$= \arg\max_{R_{\hat{g}} \geq 0}\left\{ Q_1\left(\frac{2\hat{g}}{\sigma^2}; \frac{2(e^{R_{\hat{g}}} - 1)}{P\sigma^2}\right) R_{\hat{g}} \right\}, \quad (46)$$

where the last equality comes from (43).

Using the derivatives of the Marcum $Q$-function, (46) does not have a closed-form solution. For this reason, Lemma 4 uses the semi-linear approximation scheme of Lemma 1 and Corollaries 13 to find the optimal data rate maximizing the instantaneous throughput.

**Lemma 4.** For a given channel realization $\hat{g}$, the throughput-optimized rate allocation is approximately given by (49) where $W(\cdot)$ denotes the Lambert $W$-function.

**Proof.** The approximation results of Lemma 1 and Corollaries 13 can be generalized by $y(\alpha, \beta) \simeq \tilde{Z}_{\text{general}}(\alpha, \beta)$ where

$$\tilde{Z}_{\text{general}}(\alpha, \beta) \simeq \begin{cases} 0, & \text{if } \beta < c_1(\alpha) \\ o_1(\alpha)(\beta - o_2(\alpha)) + o_3, & \text{if } c_1(\alpha) \leq \beta \leq c_2(\alpha) \\ 1, & \text{if } \beta > c_2(\alpha). \end{cases}$$

(47)

$\alpha, i = 1, 2, 3,$ are given by (2), (13), (17), or (20) depending on if we use Lemma 1 or Corollaries 13. In this way, (45) is approximated as

$$\eta_{\hat{g}} \simeq R_{\hat{g}}(1 - o_1(\alpha)\beta + o_1(\alpha)o_2(\alpha) - o_3(\alpha)), \quad (48)$$

where $\alpha = \sqrt{\frac{2\hat{g}}{\sigma^2}}$. To simplify the equation, we omit $\alpha$ in the following since it is a constant for given $\hat{g}, \sigma$. Then, setting the derivative of (48) equal to zero, we obtain

$$R_{\hat{g}}^{opt} \simeq \arg_{R_{\hat{g}} \geq 0}\left\{ (1 + o_1o_2 - o_3 - o_1) \left(\frac{(R_{\hat{g}} + 2)e^{R_{\hat{g}}/2} - 2}{2P\sigma^2(e^{R_{\hat{g}}/2} - 1)}\right) = 0 \right\}$$

$$\simeq \arg_{R_{\hat{g}} \geq 0}\left\{ \frac{(R_{\hat{g}}/2 + 1)e^{R_{\hat{g}}/2} + 1}{2\sigma^2} \frac{(1 + o_1o_2 - o_3)e\sqrt{2P\sigma^2}}{2\sigma^2} - 1 \right\}. \quad (49)$$

Here, (a) comes from $e^{R_{\hat{g}}/2} - 1 \simeq e^{R_{\hat{g}}} / 2$ and $e^{R_{\hat{g}}} + 2 \simeq (R_{\hat{g}}/2 + 1)e^{R_{\hat{g}}}$ which are appropriate at moderate/high values of $R_{\hat{g}}$. Also, (b) is obtained by the definition of the Lambert $W$-function $xe^x = y \iff x = W(y)[52]$.

Finally, the expected throughput, averaged over multiple time slots, is obtained by $\eta = \mathbb{E}\{\eta_{\hat{g}}(R_{\hat{g}}^{opt})\}$ with expectation over $\hat{g}$.

Using (49) and the approximation [53, Thm. 2.1] $W(x) \simeq \log(x) - \log\log(x), x \geq 0,$

(50)

we obtain

$$R_{\hat{g}}^{opt} \simeq 2 \log\left(\frac{(1 + o_1o_2 - o_3)e\sqrt{2P\sigma^2}}{2\sigma^2} - 1\right)$$

$$- 2 \log\left(\frac{(1 + o_1o_2 - o_3)e\sqrt{2P\sigma^2}}{2\sigma^2} - 1\right) \quad (51)$$

which implies as the transmit power increases, the optimal instantaneous rate increases with the square root of the transmit power (approximately) logarithmically.

### C. On the Effect of Imperfect Channel Estimation

In Section III-B, we assumed perfect channel estimation at the BS. Here, we follow the similar approach as in, e.g., [54], to add the effect of estimation error of $h$ as an independent additive Gaussian variable whose variance is given by the accuracy of channel estimation.

Let us define $\hat{h}$ as the estimate of $h$ at the BS. Then, we further develop our channel model [37] as

$$\hat{h} = \kappa h + \sqrt{1 - \kappa^2} z, \quad (52)$$

for each time slot, where $z \sim \mathcal{CN}(0, 1)$ is a Gaussian noise which is uncorrelated with $H_k$. Also, $\kappa$ is a known correlation factor which represents estimation error of $\hat{h}$ by $\kappa = \frac{\mathbb{E}(|h|^2)}{\mathbb{E}(|h^2|)}$. Substituting (52) into (37), we have

$$h = \kappa \sqrt{1 - \sigma^2} z + \kappa \sigma q + \sqrt{1 - \kappa^2} z. \quad (53)$$

Then, because $\kappa \sigma q + \sqrt{1 - \kappa^2} z$ is equivalent to a new Gaussian variable $w \sim \mathcal{CN}(0, (\kappa\sigma)^2 z + 1 - \kappa^2)$, we can follow the same procedure as in (46)-(49) to analyze the system performance with imperfect channel estimation of the PA (see Figs. 5-6 for more discussions).
D. Simulation Results

In this part, we study the performance of the PA system and verify the tightness of the approximation scheme of Lemma 4. Particularly, we present the average throughput and the outage probability of the PA setup for different vehicle speeds/channel estimation errors. As an ultimate upper bound for the proposed rate adaptation scheme, we consider a genie-aided setup where we assume that the BS has perfect CSIT of the BS-RA link without uncertainty/outage probability. Then, as a lower-bound of the system performance, we consider the cases with no CSIT/rate adaptation as shown in Fig. 5. In the simulations, we set \( f_c = 2.68 \text{ GHz} \) and \( d_a = 1.5\lambda \). Finally, each point in the figures is obtained by averaging the system performance over \( 1 \times 10^5 \) channel realizations.

In Fig. 5, we show the expected throughput \( \eta \) in different cases for a broad range of signal-to-noise ratios (SNRs). Here, because the noise has unit variance, we define the SNR as \( 10 \log_{10} P \). Also, we set \( v = 114 \text{ Km/h} \) in Fig. 5 as defined in (36). The analytical results obtained by Lemma 4 and Corollary 2 i.e., the approximation of (46), are also presented. We have also checked the approximation result of Lemma 4 while using Lemma 1/Corollaries 1-3. Then, because the results are similar as those presented in Fig. 5, they are not included to the figure. Moreover, the figure shows the results with no CSIT/rate adaptation as a benchmark. Finally, Fig. 6 studies the expected throughput \( \eta \) for different values of estimation error variance \( \kappa \) with SNR = 10, 19, 25 dB, in the case of partial CSIT. Also, the figure evaluates the tightness of the approximation results obtained by Lemma 4. Here, we set \( v = 114.5 \text{ Km/h} \) and \( \delta = 5 \text{ ms} \).

Setting SNR = 23 dB and \( v = 120, 150 \text{ Km/h} \) in Fig. 7 we study the effect of the processing delay \( \delta \) on the throughput. Finally, the outage probability is evaluated in Fig. 8 where the results are presented for different speeds with SNR = 10 dB, in the case of partial CSIT. Also, we present the outage probability for \( \delta = 5.35 \text{ ms} \) and \( \delta = 4.68 \text{ ms} \) in Fig. 8.

From the figures, we can conclude the following points:

- The approximation scheme of Lemma 4 is tight for a broad range of parameter settings (Figs. 5-6). Thus, the throughput-optimized rate allocation can be well approximated by (49), and the semi-linear approximation of Lemma 1/Corollaries 1-3 is a good approach to study the considered optimization problem.
- With deployment of the PA, remarkable throughput gain is achieved especially in moderate/high SNRs (Fig. 5). Also, the throughput decreases when the estimation error is considered, i.e., \( \kappa \) decreases. Finally, as can be seen in Figs. 5-6 with rate adaptation, and without optimizing the processing delay/vehicle speed, the effect of estimation error on expected throughput is small unless for large values of \( \kappa \).
- As it can be seen in Figs. 7 and 8 for different channel estimation errors, there are optimal values for the vehicle speed and the BS processing delay optimizing the system throughput and outage probability. Note that the presence of the optimal speed/processing delay can be proved via (36) as well. Finally, the optimal value of the vehicle speed, in terms of throughput/outage probability, decreases with the processing delay. However, the optimal vehicle speed/processing delay, in terms of throughput/outage probability, is almost insensitive to the channel estimation error.
- With perfect channel estimation, the throughput/outage probability is sensitive to the speed variation, if we move away from the optimal speed (Figs. 7 and 8). However, the sensitivity to the speed/processing delay variation decreases as the channel estimation error increases, i.e., \( \kappa \) decreases (Figs. 7 and 8). Finally, considering Figs. 7 and 8, we will study the effect of the processing delay

\[
\delta
\]

on the throughput.

\[
\kappa
\]

Finally, the outage probability is evaluated in Fig. 8, where \( \delta = 5 \text{ ms} \). Both the exact values from simulation as well as the analytical approximations from Lemma 4 are presented.

### Fig. 5. Expected throughput \( \eta \) in different cases, \( v = 114 \text{ Km/h}, \kappa = 1 \), and \( \delta = 5 \text{ ms} \). Both the exact values from simulation as well as the analytical approximations from Lemma 4 are presented.

### Fig. 6. Expected throughput \( \eta \) for different estimation errors \( \kappa \) with SNR = 10, 19, 25 dB in the case of partial CSIT, exact and approximation, \( v = 114.5 \text{ Km/h}, \) and \( \delta = 5 \text{ ms} \). Both the exact values from simulation as well as the analytical approximations from Lemma 4 are presented.
Problem formulations. Particularly, as an application of interest, we used the proposed approximation to analyze the performance of PA setups using rate adaptation. As we showed, with different levels of channel estimation error/processing delay, adaptive rate allocation can effectively compensate for the spatial mismatch problem, and improve the throughput/outage probability of PA networks. It is expected that increasing the number of RA antennas will improve the performance of the PA system considerably.

IV. CONCLUSION

We derived a simple semi-linear approximation method for the first-order Marcum Q-function, as one of the functions of interest in different problem formulations of wireless networks. As we showed through various analysis, while the proposed approximation is not tight at the tails of the function, it is useful in different optimization- and expectation-based problem formulations. Particularly, as an application of interest, we used the proposed approximation to analyze the performance of PA setups using rate adaptation. As we showed, with different levels of channel estimation error/processing delay, adaptive rate allocation can effectively compensate for the spatial mismatch problem, and improve the throughput/outage probability of PA networks. It is expected that increasing the number of RA antennas will improve the performance of the PA system considerably.

APPENDIX A
Proof of Lemma

Using Corollary 3, we have

\[
\begin{align*}
G(\alpha, \rho) & \approx \int_{\rho}^{\rho'} e^{-n x} \times (1 - Q_1(\alpha, x)) \, dx + \\
& \int_{\rho'}^{\infty} e^{-n x} \times \frac{1}{\sqrt{2\pi\alpha}} \left(1 - \frac{1}{\sqrt{2\pi\alpha\rho}}\right) \, dx,
\end{align*}
\]

Then, for \( \rho \geq \bar{\rho}' \), we obtain

\[
\int_{\rho}^{\infty} e^{-n x} \times x^m \times (1 - Q_1(\alpha, x)) \, dx \\
\approx \int_{\rho}^{\infty} e^{-n x} \times x^m \, dx = \Gamma(m + 1, n\rho)n^{-m-1},
\]

while for \( \rho \leq \bar{\rho}' \), we have

\[
\int_{\rho}^{\infty} e^{-n x} \times x^m \times (1 - Q_1(\alpha, x)) \, dx \\
\approx \int_{\max(\bar{\rho}', \rho)}^{\infty} \left[ \frac{1}{\sqrt{2\pi\alpha}} \left(1 - \frac{1}{\sqrt{2\pi\alpha\rho}}\right) \right] \times \\
\times e^{-n x} \times x^m \, dx + \int_{\bar{\rho}'}^{\infty} e^{-n x} \times x^m \, dx
\]

\[
\approx \Gamma(m + 1, n\bar{\rho}_2)n^{-m-1} + \\
\left[ \frac{-\alpha}{\sqrt{2\pi}} + 0.5 \times \left(1 - \frac{1}{\sqrt{2\pi\alpha\rho}}\right) \right] \times n^{-m-1} \times
\]

\[
(\Gamma(m + 1, n\max(\bar{\rho}', \rho)) - \Gamma(m + 1, n\bar{\rho}_2)) + \\
(\Gamma(m + 2, n\max(\bar{\rho}', \rho)) - \Gamma(m + 2, n\bar{\rho}_2)) \frac{n^{-m-2}}{\sqrt{\pi}},
\]

Note that (c) and (e) come from (20) while (d) and (f) use the fact that \( \Gamma(s, x) \rightarrow 0 \) as \( x \rightarrow \infty \).

APPENDIX B
Proof of Lemma

Using Lemma 1, the integral (25) can be approximated as

1) for \( \theta_2 > \theta_1 > \theta_2 \), \( T(\alpha, m, a, \theta_1, \theta_2) \approx 0 \),
2) for \( c_1 < \theta_1 \leq c_2, \theta_2 > \theta_1 \),

\[
T(\alpha, m, a, \theta_1, \theta_2) = \int_{\theta_1}^{c_2} (n_2 x + n_1) e^{-mx} \log(1 + ax) dx,
\]

and

\[
\mathcal{F}_2(x) = \int (n_2 x + n_1) e^{-mx} \log(1 + ax) dx
\]

(i) \(-\frac{(mn_2 x + n_2 + mn_1) e^{-mx} \log(1 + ax)}{m^2} - \frac{a(-mn_2 x - n_2 - mn_1) e^{-mx}}{m^2} dx\)

(j) \(-\frac{(mn_2 x + n_2 + mn_1) e^{-mx} \log(1 + ax)}{m^2}

\]

\[
= -\frac{1}{a} \left( e^m (mn_2 x + n_2 + mn_1) E_1 \left( \frac{mx + m}{a} \right) + \frac{n_2 e^{-mx}}{a} - \frac{e^m n_2 (mx + \frac{m}{a})}{a} E_1 \left( \frac{mx + \frac{m}{a}}{a} \right) + C, \right.
\]

\[
= \mathcal{F}_1(c_1) - \mathcal{F}_1(\theta_1) + \mathcal{F}_2(\theta_2) - \mathcal{F}_2(c_1),
\]

(59)

with \( n_1 \) and \( n_2 \) being functions of the constant \( \alpha \) and given by (29) and (30), respectively. Here, we assume \( \theta_1 < c_1 \) and \( c_2 > \theta_2 > c_1 \) for simplicity. The other cases can be proved with the same procedure. Moreover, the functions \( \mathcal{F}_1(x) \) and \( \mathcal{F}_2(x) \) are obtained by

\[
\mathcal{F}_1(x) = \int e^{-mx} \log(1 + ax) dx
\]

(g) \(-\frac{e^{-mx} \log(ax + 1)}{m} - \frac{a e^{-mx}}{m(ax + 1)} dx\)

(h) \(-\frac{1}{m} \left( -e^m E_1 \left( \frac{mx + m}{a} \right) - e^{-mx} \log(ax + 1) \right) + C, \)

(60)

Then, consider case 3 where

\[
T(\alpha, m, a, \theta_1, \theta_2) \simeq \int_{\theta_1}^{c_1} e^{-mx} \log(1 + ax) dx + \int_{c_1}^{\theta_2} (n_2 x + n_1) e^{-mx} \log(1 + ax) dx
\]

\[
= \mathcal{F}_1(c_1) - \mathcal{F}_1(\theta_1) + \mathcal{F}_2(\theta_2) - \mathcal{F}_2(c_1),
\]

(59)

where (g), (i) and (j) come from partial integration and some manipulations. Also, (h) and (k) use [55, p. 195]

\[
\int E_1(u) du = u E_1(u) - e^{-u}.
\]

(62)

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