Capacity Region of MISO Broadcast Channel for SWIPT

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Abstract

This paper studies a multiple-input single-output (MISO) broadcast channel (BC) featuring simultaneous wireless information and power transfer (SWIPT), where a multi-antenna access point (AP) delivers both information and energy via radio signals to multiple single-antenna receivers simultaneously, and each receiver implements either information decoding (ID) or energy harvesting (EH). We characterize the capacity region for ID receivers under given energy requirements for EH receivers, by solving a sequence of weighted sum-rate (WSR) maximization (WSRMax) problems subject to a maximum sum-power constraint for the AP, and a set of minimum harvested power constraints for individual EH receivers. The problem corresponds to a new form of WSRMax problem in MISO-BC with combined maximum and minimum linear transmit covariance constraints (MaxLTCCs and MinLTCCs), which has not been addressed in the literature and is challenging to solve. By extending the general BC-multiple access channel (MAC) duality [11], which is only applicable to WSRMax problems with MaxLTCCs, we propose an efficient algorithm to solve this problem globally optimally. Numerical results are presented to validate our proposed algorithm.

I. INTRODUCTION

Far-field wireless energy transfer (WET) enabled by radio-frequency (RF) signals has been recognized as a potentially viable way to power energy constrained wireless networks, in which nodes may not be easily rechargeable. On the other hand, RF signals have also been widely used as a vehicle for transporting information. To enable a dual use of RF signals, simultaneous wireless information and power transfer (SWIPT) is becoming an interesting new area of research [1]–[10].

The practical implementation of SWIPT is limited by the severe path loss and fading of wireless channels, and multi-antenna processing is an appealing solution to improve the efficiency of both information and energy transfer. Recently, there have been a handful of papers on studying the multi-antenna SWIPT system under various setups [3]–[10]. In particular, multi-antenna broadcast channel (BC) for SWIPT has been investigated in [3]–[6]. The authors in [3] first characterized the rate-energy (R-E) tradeoff for a simplified multiple-input-multiple-output (MIMO) BC with two either separated or co-located receivers implementing information decoding (ID) and energy harvesting (EH), respectively. The study in [3] was then extended to the case with imperfect channel state information (CSI) at the transmitter [4]. Moreover,
[5] and [6] studied the multiple-input-single-output (MISO) BC for SWIPT with multiple separated and
colocated ID and EH receivers, respectively. However, all these prior works on multi-antenna BC consider
low-complexity linear precoding/beamforming for SWIPT, which is in general sub-optimal. Therefore, the
fundamental limits on information and the energy transfer in general multi-antenna BCs for SWIPT remain
unknown.

This paper studies a MISO-BC for SWIPT, where a multi-antenna access-point (AP) delivers both wire-
less information and energy to multiple receivers. Given the fact that the existing RF front-end for wireless
EH are not yet able to be used for ID directly and vice versa [1], we consider that each receiver implements
either ID or EH separately. In particular, pseudo-random sequences that are known to and therefore can be
cancelled at each ID receiver is used as the energy signals, and the information-theoretically optimal dirty
paper coding (DPC) is employed for the information transmission. To characterize the capacity region
for the ID receivers while ensuring given energy requirements for EH receivers, we solve a sequence of
weighted sum-rate (WSR) maximization (WSRMax) problems for all ID receivers subject to a maximum
sum-power constraint for the AP, and a set of minimum harvested power constraints for individual EH
receivers. The corresponds to a new form of WSRMax problem with combined maximum and minimum
linear transmit covariance constraints (MaxLTCCs and MinLTCCs).

It should be noted that the studied WSRMax problem differs from the celebrated capacity region
characterization problem for MISO-BC under a set of MaxLTCCs [11], due to the newly introduced
MinLTCCs that arise from the minimum harvested power constraints for the EH receivers. As a result,
our considered WSRMax problem is a new one to which the general BC-MAC duality in [11] does not directly apply. To solve this problem globally optimally, we propose an efficient
algorithm by extending the general BC-MAC duality and applying the ellipsoid method. To the best of
our knowledge, our approach is novel and has not been studied in the literature. It is shown that at the
optimal solution, the energy signals should be in the null space of all ID receivers’ channels. Numerical
results are provided to validate our proposed algorithm.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider a MISO-BC for SWIPT with an AP delivering both information and energy to multiple
receivers over a single frequency band, where each receiver implements either ID or EH. In this system,
there are $K_I \geq 1$ ID receivers and $K_E \geq 1$ EH receivers, denoted by the sets $\mathcal{K}_I = \{1, \cdots, K_I\}$ and
\( \mathcal{K}_I = \{1, \cdots, K_E\} \), respectively. It is assumed that all ID and EH receivers are each equipped with one single antenna, whereas, the AP is equipped with \( N > 1 \) antennas.

We assume a quasi-static channel model, and denote \( h_i \in \mathbb{C}^{N \times 1} \) and \( g_j \in \mathbb{C}^{N \times 1} \) as the channel vectors from the AP to ID receiver \( i \in \mathcal{K}_I \) and EH receiver \( j \in \mathcal{K}_E \), respectively. The AP is assumed to perfectly know the instantaneous values of \( h_i \)'s and \( g_j \)'s, while each ID receiver knows its own instantaneous channel.

Without loss of generality, the AP transmits \( K_I \) independent information signals, i.e., \( x_i \in \mathbb{C}^{N \times 1}, \forall i \in \mathcal{K}_I \), one for each ID receiver, and one common energy signal, i.e., \( x_E \in \mathbb{C}^{N \times 1} \), for all the EH receivers. Thus, the AP transmits the \( N \)-dimensional complex baseband signal

\[
    x = \sum_{i \in \mathcal{K}_I} x_i + x_E. \tag{1}
\]

For information signals, we consider Gaussian signalling, and thus \( x_i \)'s are independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian (CSCG) vectors with zero mean and covariance matrix \( S_i \triangleq \mathbb{E}[x_i x_i^H], i \in \mathcal{K}_I \). Here, the superscript \( H \) denotes the conjugate transpose and \( \mathbb{E}[\cdot] \) denotes the statistical expectation. For the energy signal, since \( x_E \) does not carry any information, it can be implemented with a pseudo-random sequence that mimics a stationary \( N \)-dimensional random process with zero mean and covariance matrix \( S_E \). Suppose that the maximum sum-power at the AP is denoted by \( P_{\text{sum}} > 0 \). Then we have \( \mathbb{E}[x^H x] = \text{Tr} (\sum_{i \in \mathcal{K}_I} S_i + S_E) \leq P_{\text{sum}} \) with \( \text{Tr}(X) \) denoting the trace of a square matrix \( X \).

We consider the information-theoretical optimal DPC for the information transmission, for which the causal interference can be pre-cancelled at the transmitter. To be more specific, consider the encoding order as \( \pi(1), \ldots, \pi(K_I) \), i.e., the information signal for ID receiver \( \pi(1) \) is encoded first, that for \( \pi(2) \) is encoded second, and so on, where \( \pi \) denotes some desired permutation over \( \mathcal{K}_I \). In this case, for any ID receiver \( \pi(i) \), the causal interference due to ID receivers \( \pi(1), \ldots, \pi(i - 1) \) can be canceled via DPC at the AP. Moreover, since the energy signal is pseudo-random, whose waveform can be assumed to be known at both the AP and each ID receiver prior to the transmission, their resulting interference can be efficiently cancelled via DPC at the AP or an extra interference cancellation process at each ID receiver.

As a result, the effective received signal for ID receiver \( \pi(i) \) is expressed as

\[
    y_{\pi(i)} = h_{\pi(i)}^H x_{\pi(i)} + \sum_{k=i+1}^{K_I} h_{\pi(k)}^H x_{\pi(k)} + z_i, \forall i \in \mathcal{K}_I \tag{2}
\]
where \( z_i \sim \mathcal{CN}(0, \sigma^2) \) denotes the additive white Gaussian noise (AWGN) at the \( i \)th ID receiver with noise power being \( \sigma^2 \). With Gaussian signalling employed, the achievable rate region for ID receivers, defined as the rate-tuples for all ID receivers (in bps/Hz) with given information covariance matrices \( \{ S_i \} \), is thus given by

\[
C_{BC}(\{ S_i \}, \{ h_i \}) = \bigcup_{\pi \in \Pi} \left\{ r \in \mathbb{R}^{K_I^+} : \right. \\
\alpha \leq \log_2 \left( \frac{\sigma^2 + h_{\pi(i)}^H \left( \sum_{k=1}^{K_I} S_{\pi(k)} h_{\pi(k)} \right)}{\sigma^2 + h_{\pi(i)}^H \left( \sum_{k=1}^{K_I} S_{\pi(k)} h_{\pi(k)} \right)} \right) \left. \right\}
\]

where \( \Pi \) is the collection of all possible permutations over \( K_I^+ \), and \( r = [r_1, \ldots, r_{K_I}]^T \) denotes the vector of achievable rates for all ID receivers with the superscript \( T \) denoting the transpose.

On the other hand, consider the WET. Due to the broadcast property of wireless channels, the energy carried by all information and energy signals can be harvested at each EH receiver. As a result, the harvested power for the \( j \)th EH receiver, denoted by \( Q_j \), can be expressed as

\[
Q_j = E[|g_j^H x|^2] = \zeta \text{Tr} \left( \sum_{i \in K_E} S_i + S_E \right) G_j, \forall j \in K_E
\]

where \( 0 < \zeta \leq 1 \) denotes the energy harvesting efficiency and \( G_j \triangleq g_j g_j^H, \forall j \in K_E \). Since \( \zeta \) is a constant, we normalize it as \( \zeta = 1 \) for simplicity unless otherwise specified.

Now, we are ready to present the optimization problem of interest. To characterize the boundary points of the capacity region for the MISO-BC with SWIPT, we maximize the WSR of all ID receivers subject to minimum harvested power constraints at individual EH receivers, as well as the maximum sum-power constraint for the AP. By denoting the minimum harvested power requirement at EH receiver \( j \in K_E \) as \( E_j > 0 \), the WSRMax problem is formulated as

\[
(P1) : \max_{\{ S_i \}, r, S_E} \sum_{i \in K_I} \alpha_i r_i \\
\text{s.t.} \quad r \in C_{BC}(\{ S_i \}, \{ h_i \}) \\
\text{Tr} \left( \sum_{i \in K_I} S_i + S_E \right) G_j \geq E_j, \forall j \in K_E \\
\text{Tr} \left( \sum_{i \in K_I} S_i + S_E \right) \leq P_{\text{sum}} \\
S_E \succeq 0, S_i \succeq 0, \forall i \in K_I
\]

where \( \alpha_i > 0 \) denotes a given rate weight for ID receiver \( i \in K_I \), and \( S_E \succeq 0 \) means that \( S_E \) is positive semi-definite. Note that by solving problem (P1) via exhausting all possible \( \{ \alpha_i \} \)'s, the whole capacity
region can then be characterized. Let $\mathcal{D}$ denote the set containing all admissible information covariance matrices $\{S_i\}$ and all achievable rates $\{r_i\}$, specified by the constraints in (6) and (9). It is then observed that (P1) is non-convex due to the non-convexity of $\mathcal{D}$, and thus the globally optimal solution of (P1) is difficult to obtain in general. One commonly adopted approach to deal with non-convex WSRMax problem for the multi-antenna BC is to use the BC-MAC duality to transform it into an equivalent convex WSRMax problem for a dual MAC [11]. However, the existing BC-MAC duality is only applicable for the case of involving only MaxLTCCs with information signals only. In contrast, (P1) has both a MaxLTCC in (8) and a set of MinLTCCs in (7) as well as an extra energy covariance matrix $S_E$. As a result, solving problem (P1) is not a trivial exercise.

Prior to solving problem (P1), we first have to check its feasibility. It can be observed that (P1) is feasible if and only if its feasibility is guaranteed by ignoring all the ID receivers, i.e., setting $S_i = 0$ and $r_i = 0, \forall i \in K_I$. Thus, the feasibility of (P1) can be verified by solving the following problem:

$$\begin{align*}
&\text{find } S_E \\
&\text{s.t. } \text{Tr}[S_E G_j] \geq E_j, \forall j \in K_E \\
&\quad \text{Tr}(S_E) \leq P_{\text{sum}}, S_E \succeq 0. \quad (10)
\end{align*}$$

Since problem (10) is a convex semi-definite program (SDP), it can be solved by standard convex optimization techniques such as the interior point method [13]. In the rest of this paper, we only focus on the case that (P1) is feasible.

III. Optimal Solution

In this section, we present the optimal solution to problem (P1) by transforming it into a series of equivalent WSRMax sub-problems with a single MaxLTCC and accordingly solving these sub-problems via the BC-MAC duality. Specifically, we first define the following auxiliary function $g(\{\lambda_j\})$ as

$$g(\{\lambda_j\}) = \max_{\{S_i\}.r.S_E} \sum_{i \in K_I} \alpha_i r_i \quad (11)$$

$$\text{s.t. } r \in C_{BC}(\{S_i\}, \{h_i\}) \quad (12)$$

$$\sum_{i \in K_I} \text{Tr}(AS_i) + \text{Tr}(AS_E) \leq P_A \quad (13)$$

$$S_E \succeq 0, S_i \succeq 0, \forall i \in K_I \quad (14)$$

1The MaxLTCC can be expressed as $\text{Tr}(SQ) \leq P$, where $S$ is the transmit covariance matrix, $Q$ is a given positive semi-definite matrix, and $P$ is a prescribed power constraints. Note that our defined MaxLTCC is the same as the general LTCC (GLTCC) in [11].

2The MinLTCC can be similarly defined as $\text{Tr}(SQ) \geq P$. 
where $\lambda_j \geq 0, j \in \{0\} \cup K_E$ are auxiliary variables, $A = \lambda_0 I - \sum_{j \in K_E} \lambda_j G_j$ and $P_A = \lambda_0 P_{\text{sum}} - \sum_{j \in K_E} \lambda_j E_j$. Since any feasible solution to problem (P1) is also feasible to (11) but not vice versa, $g(\{\lambda_j\})$ serves as an upper bound on the optimal value of (P1) for any $\{\lambda_j \geq 0\}$. We then define the following problem by minimizing $g(\{\lambda_j\})$ over $\{\lambda_j \geq 0\}$:

$$
(P2): \min_{\{\lambda_j \geq 0\}} g(\{\lambda_j\}).
$$

In general the optimal value of problem (P2) also serves as an upper bound on that of (P1). However, as will be rigorously shown later (see Lemma 3.4), this upper bound is indeed tight. As a result, we will solve (P1) by equivalently solving problem (P2). In the following, we first solve problem (11) to obtain $g(\{\lambda_j\})$ under any given $\{\lambda_j \geq 0\}$, based on which the strong duality between problems (P1) and (P2) is then proved. Next, we solve problem (P2) to obtain the optimal $\{\lambda_j\}$, and finally, we construct the optimal solution to (P1) based on that to (P2).

A. Solving problem (11) to obtain $g(\{\lambda_j\})$

To start, we present some important properties for problem (11) in the following lemma.

**Lemma 3.1:** In order for problem (11) to be feasible and $g(\{\lambda_j\})$ to have an upper-bounded value, i.e., $g(\{\lambda_j\}) < +\infty$, the following conditions must be satisfied:

1) $A$ is positive semi-definite, i.e., $A \succeq 0$.

2) The null space of $A$ lies in the null space of $H \triangleq \sum_{i \in K_E} h_i h_i^H \in \mathbb{C}^{N \times N}$, i.e., $\text{Null}(A) \subseteq \text{Null}(H)$, where $\text{Null}(A) \triangleq \{ x \in \mathbb{C}^{N \times 1} : Ax = 0 \}$.

3) $P_A \geq 0$.

**Proof:** See Appendix A.

From Lemma 3.1 it is sufficient for us to solve (11) with $A \succeq 0$, $\text{Null}(A) \subseteq \text{Null}(H)$ and $P_A \geq 0$.

Suppose that $\text{rank}(A) = m$ with $\text{rank}(H) \leq m \leq N$. Then, the singular value decomposition (SVD) of $A$ can be expressed as

$$
A = [U_1, U_2] \Lambda [U_1, U_2]^H
$$

where $U_1 \in \mathbb{C}^{N \times m}$ and $U_2 \in \mathbb{C}^{N \times (N-m)}$ consist of the first $m$ and the last $N - m$ left singular vectors of $A$, which correspond to the non-zero and zero singular values in $\Lambda$, respectively. Therefore, the vectors in $U_1$ and $U_2$ form the orthogonal basis for the range and null space of $A$, respectively. Then we have the optimal $\bar{S}_E$ for problem (11) as follows.
Lemma 3.2: The optimal energy covariance matrix to problem (11) is expressed as

$$\bar{S}_E = U_2 \bar{E} U_2^H$$  \hspace{1cm} (17)$$

where $\bar{E} \in \mathbb{C}^{(N-m) \times (N-m)}$ can be any positive semi-definite matrix. That is, any $\bar{S}_E \succeq 0$ satisfying $A \bar{S}_E = 0$ is optimal to problem (11).

Proof: See Appendix B.

Lemma 3.2 intuitively shows that the optimal energy covariance matrix $\bar{S}_E$ of problem (11) lies in the null space of $A$. By using this result, problem (11) can thus be simplified as

$$\underset{\mathcal{K}_I}{\text{Max.}} \{ S_i \} r \sum_{i \in \mathcal{K}_I} \alpha_i r_i$$

s.t. $r \in \mathcal{C}_{BC} (\{ S_i \}, \{ h_i \})$

$$\sum_{i \in \mathcal{K}_I} \text{Tr}(A S_i) \leq P_A$$

$$S_i \succeq 0, \forall i \in \mathcal{K}_I.$$  \hspace{1cm} (18)$$

Now, it remains to solve (18) to obtain the optimal information covariance matrices $\{ \bar{S}_i \}$. Note that problem (18) corresponds to a WSRMax problem in MISO-BC under a single MaxLTCC. For the special case with $A$ of full rank, this problem has been solved by the general BC-MAC duality \cite{11}. To handle the general case of $A$ being rank deficient, which has not been addressed in the literature, we present the following lemma.

Lemma 3.3: The optimal information covariance matrices, i.e., $\{ \bar{S}_i \}$, to problem (18) can be expressed as

$$\bar{S}_i = U_1 \bar{B}_i U_1^H + U_1 \bar{C}_i U_2^H + U_2 \bar{C}_i^H U_1^H$$

$$+ U_2 \bar{D}_i U_2^H, \forall i \in \mathcal{K}_I$$  \hspace{1cm} (19)$$

where $\bar{B}_i \in \mathbb{C}^{m \times m}$ can be obtained by solving problem (20) as follows, $\bar{C}_i \in \mathbb{C}^{m \times (N-m)}$ and $\bar{D}_i \in \mathbb{C}^{(N-m) \times (N-m)}$ can be any matrices with appropriate dimensions such that $\bar{S}_i \succeq 0$.

$$\underset{\mathcal{K}_I}{\text{Max.}} \{ B_i \} r \sum_{i \in \mathcal{K}_I} \alpha_i r_i$$

s.t. $r \in \mathcal{C}_{BC} (\{ B_i \}, \{ \hat{h}_i \})$

$$\sum_{i \in \mathcal{K}_I} \text{Tr}(A B_i) \leq P_A$$

$$B_i \succeq 0, \forall i \in \mathcal{K}_I.$$  \hspace{1cm} (20)$$
where $\hat{h}_i = U_1^H h_i \in \mathbb{C}^{m \times 1}, \forall i \in K_I$ and $\hat{A} = U_1^H A U_1 \in \mathbb{C}^{m \times m}$. Note that $\hat{A}$ is of full rank, and thus problem (20) can be solved by the general BC-MAC duality as in [11].

Proof: See Appendix C

By combining Lemmas 3.2 and 3.3 we have obtained the optimal solution to (11).

Remark 3.1: Note that if $A$ is of full rank, i.e., $m = N$, then we have $U_2 = 0$. In this case, it is evident that the optimal solution to (11) is unique and can be expressed as

$$\bar{S}_i = U_1 \bar{B}_i U_1^H , \forall i \in K_I \quad \text{and} \quad \bar{S}_E = 0 \quad (21)$$

However, if $A$ is rank deficient, i.e., $m < N$, then it holds that $U_2 \neq 0$. In this case, there exist infinite sets of optimal solution $\{\{\bar{S}_i\}, \bar{S}_E\}$ based on Lemmas 3.2 and 3.3 and as a result the optimal solution to problem (11) is not unique. For simplicity, we employ the specific optimal solution in (21) to solve (11) for obtaining $g(\{\lambda_j\})$.

B. Solving problem (P2)

Then, we solve problem (P2) to find the optimal $\{\lambda_j\}$, to maximize $g(\{\lambda_j\})$. The strong duality between (P1) and (P2) can be proven rigorously, and is made explicit in the following lemma.

Lemma 3.4: The optimal value of problem (P1) is equal to that of problem (P2).

Proof: See Appendix D

Next, we proceed to solve (P2). Since $g(\{\lambda_j\})$ is upper bounded only when the conditions in Lemma 3.1 are satisfied, we can rewrite (P2) as follows by explicitly adding these constraints.

$$\begin{align*}
(P3): \quad \min_{\{\lambda_j \geq 0\}} \quad & g(\{\lambda_j\}) \\
\text{s.t.} \quad & \text{Null}(A) \subseteq \text{Null}(H) \\
& \lambda_0 I - \sum_{j=1}^{K_E} \lambda_j G_j \succeq 0 \\
& \lambda_0 P_{\text{sum}} - \sum_{j=1}^{K_E} \lambda_j E_j \geq 0.
\end{align*} \quad (22) \quad (23) \quad (24) \quad (25)$$

Note that for problem (P3), the objective function $g(\{\lambda_j\})$ is not necessarily differentiable. Nonetheless, we have the following lemma.

Lemma 3.5: For the function $g(\{\lambda_j\})$ at any two non-negative points $[\lambda_0, \ldots, \lambda_{K_E}]$ and $[\bar{\lambda}_0, \ldots, \bar{\lambda}_{K_E}]$, we have
where $\tilde{S}_i = \sum_{j \in K_E} \tilde{S}_i$ with $\{ \tilde{S}_i \}$ being the optimal solution of problem (P1) given $\lambda_j = \bar{\lambda}_j, j = 0, 1, \cdots, K_E$, and $c \geq 0$ is a constant.

**Proof:** The proof is similar to that of [11, Proposition 6], and is thus omitted for brevity.

Lemma 3.5 ensures that at any point $[\bar{\lambda}_0, \bar{\lambda}_1, \cdots, \bar{\lambda}_{K_E}]$, the optimal point that minimizes $g(\{ \lambda_j \})$ cannot belong to the set of points $[\lambda_0, \lambda_1, \cdots, \lambda_{K_E}]$ with

$$
\left[ P_{\text{sum}} - \text{Tr}(\tilde{S}_i), \text{Tr}(\tilde{S}_i G_1) - E_1, \cdots, \text{Tr}(\tilde{S}_i G_{K_E}) - E_{K_E} \right] \cdot \left[ [\bar{\lambda}_0, \bar{\lambda}_1, \cdots, \bar{\lambda}_{K_E}] - [\lambda_0, \lambda_1, \cdots, \lambda_{K_E}] \right]^T > 0
$$

and thus this set should be eliminated when searching for the optimal $\{ \lambda_j \}$. This property motivates us to use the ellipsoid method [14] to solve problem (P3). In order to successfully implement the ellipsoid method, we need to further obtain the sub-gradients for the constraints Null($A$) $\subseteq$ Null($H$) in (23) and $\lambda_0 I - \sum_{j=1}^{K_E} \lambda_j G_j \succeq 0$ in (24), which are shown in the following two lemmas.

**Lemma 3.6:** The constraint in (23) is equivalent to the following linear constraints

$$
f_l(\{ \lambda_j \}) \triangleq -\lambda_0 + \sum_{j=1}^{K_E} \lambda_j |v_l^H g_j|^2 < 0, \forall l \leq t
$$

where $t$ denotes the rank of matrix $H$, and $v_l, l = 1, \cdots, t$, denote the $t$ left singular vectors of $H$ corresponding to its non-zero singular values. As a result, the sub-gradient of $f_l(\{ \lambda_j \})$ at given $\{ \lambda_j \}$ can be expressed as $[-1, |v_l^H g_1|^2, \cdots, |v_l^H g_{K_E}|^2]^T, l = 1, \cdots, t$.

**Proof:** See Appendix E

**Lemma 3.7:** Define $F(\{ \lambda_j \}) = -\lambda_0 I + \sum_{j=1}^{K_E} \lambda_j G_j$. Then the constraint in (24) is equivalent to $F(\{ \lambda_j \}) \succeq 0$. Let $z$ denote the dominant eigenvector of $F(\{ \lambda_j \})$, i.e., $z = \arg \max_{\|z\|=1} z^H F(\{ \lambda_j \}) z$. Then, the sub-gradient of $F(\{ \lambda_j \})$ at given $\{ \lambda_j \}$ is $[-\|z\|^2, z^H G_1 z, \cdots, z^H G_{K_E} z]^T$.

**Proof:** See Appendix E

With Lemma 3.5 and the sub-gradients in Lemmas 3.6 and 3.7 at hand, we can successively solve problem (P2) by applying the ellipsoid method to update $\{ \lambda_j \}$ towards the optimal solution $\{ \lambda_j^* \}$.

**Remark 3.2:** Although we cannot prove the convexity of problem (P2), the convergence of the ellipsoid method can be ensured as explained in the following. The Lagrangian function of problem (P1) can be written as
\[
\sum_{i=1}^{K_I} \alpha_i r_i + \sum_{j=1}^{K_E} \left[ \theta_j \left( \text{Tr} \left( \sum_{i \in K_I} S_i + S_E \right) G_j \right) - E_j \right] - \theta_0 \left( \text{Tr} \left( \sum_{i \in K_I} S_i + S_E \right) - P_{\text{sum}} \right)
\]

(29)

where \( \theta_0 \) and \( \{ \theta_j \}_{j=1}^{K_E} \) are the Lagrange multipliers with respect to the constraints in (8) and (7), respectively. The Lagrangian function of problem (11) can be written as

\[
\sum_{i=1}^{K_I} \alpha_i r_i - \beta \left[ \lambda_0 \text{Tr} \left( \sum_{i \in K_I} S_i + S_E \right) \right] - \sum_{j=1}^{K_E} \lambda_j \text{Tr} \left( \sum_{i \in K_I} S_i + S_E \right) G_j - \lambda_0 P_{\text{sum}} + \sum_{j=1}^{K_E} \lambda_j E_j
\]

(30)

where \( \beta \) is the Lagrange variable associated with the constraint of (13). By observing (29) and (30), we can see that the two Lagrangian functions are identical to each other if we choose \( \theta_j = \beta \lambda_j, j = 0, 1, \ldots, K_E \). Thus, the auxiliary variables \( \{ \lambda_j \} \) can be viewed as the scaled (by a factor of \( 1/\beta \)) Lagrange dual variables of problem (P1). Correspondingly, \( g(\{ \lambda_j \}) \) is related to the dual function of problem (P1), which is known to be convex. However, since the optimal dual solution for \( \beta \) in problem (11) varies with \( \{ \lambda_j \} \), \( g(\{ \lambda_j \}) \) is not necessarily a convex function. Nevertheless, the above relationship reveals that in Lemma 3.5, the vector \( P_{\text{sum}} - \text{Tr}(\tilde{S}_I), \text{Tr}(\tilde{S}_I G_1) - E_1, \ldots, \text{Tr}(\tilde{S}_I G_{K_E}) - E_{K_E} \) is indeed the exact sub-gradient for the convex dual function of problem (P1), given the fact that \( \text{Tr}(A\tilde{S}_E) = 0 \) from Lemma 3.2. Thus, the convergence of the ellipsoid method based on this sub-gradient is guaranteed.

C. Finding a Primal Optimal Solution to (P1)

So far, we have obtained the optimal solution to (P2), i.e., \( \{ \lambda_j^* \} \), as well as the corresponding optimal solution to (11) given in (21). According to Remark 3.7, if \( A^* \triangleq \lambda_0^* I - \sum_{j=1}^{K_E} \lambda_j^* G_j \) is of full rank, (21) is the unique solution to (11), which is thus optimal to (P1). However, if \( A^* \) is not of full rank, (21) is not the unique solution to (11), and thus may not meet the minimum harvested power constraints in (7). In the latter case, we need to find one feasible (thus optimal) solution of (P1), denoted by \( \{ \{ S_i^* \}, S_E^* \} \), from all the optimal solutions of (11) given in (17) and (19) with \( \{ \lambda_j^* \} \).

Denote the SVD of \( A^* \) as \( [U_1^*, U_2^*] \Lambda^* [U_1^*, U_2^*]^H \). Then following (17) and (19), we can write the information and energy covariance matrices as

\[
S_i = U_1^* B_i^* (U_1^*)^H + U_1^* C_i (U_2^*)^H + U_2^* C_i^H (U_1^*)^H
+ U_2^* D_i (U_2^*)^H, \forall i \in K_I
\]

(31)

\[
S_E = U_2^* E (U_2^*)^H
\]

(32)
where $B_i^*$ is obtained by solving (20) with $\{\lambda_j^*\}$. Therefore, it remains to find the feasible and thus optimal $\{C_i\}, \{D_i\}$ and $E$ such that the minimum harvested power constraints in (P1) are all satisfied. Since $r_i^*$ does not depend on the choice of $\{C_i\}, \{D_i\}$ and $E$, $\forall i \in K_{I}$, finding the primal optimal solution corresponds to solving a feasibility problem only involving the constraints in (8) and (9). Note that in general, there can be more than one feasible solutions to such a feasibility problem. Among them, we are interested in the solution with low-rank information covariance matrices in order to minimize the decoding complexity at the ID receiver. Therefore, we propose to minimize the sum of the ranks of all information covariance matrices, i.e., $\sum_{i \in K_{I}} \text{rank}(S_i)$. However, the rank function is not convex. By using the convex approximation of the rank function [12], we solve the following problem to find a desirable solution.

\[(P4): \quad \min_{\{C_i\}, \{D_i\}, E} \sum_{i \in K_{I}} \text{Tr}(S_i) \quad \text{(33)}\]
\[\text{s.t.} \quad \text{Tr} \left( \sum_{i \in K_{I}} S_i + S_E \right) G_j \geq E_j, \forall j \in K_{E} \quad \text{(34)}\]
\[\text{Tr} \left( \sum_{i \in K_{I}} S_i + S_E \right) \leq P_{\text{sum}} \quad \text{(35)}\]
\[S_E \succeq 0, S_i \succeq 0, \forall i \in K_{I} \quad \text{(36)}\]

where $S_i$ and $S_E$ are given in (31) and (32), respectively. As a result, the primal optimal solution to (P1) is finally obtained. By combining the procedures in Sections III-A, III-B and III-C the overall algorithm for solving problem (P1) is summarized in Table I.

It is worth pointing out that in general there exist three cases for the optimal solution of (P1) obtained by the algorithm in Table I. For convenience, we denote $S_{I}^* = \sum_{i \in K_{I}} S_i^*$. 

1) $S_{I}^* = 0$ and $S_{E}^* \succeq 0$: in this case, no information can be transferred without violating the minimum harvested power constraints. This situation can only occur when the channels of all the ID receivers are orthogonal to those of all the EH receivers, and the harvested power constraints are met with equality. Note that under practical channel setup, this case may never occur.

2) $S_{I}^* \succeq 0$ and $S_{E}^* = 0$: in this case, no dedicated energy signal is required. It is because that the energy harvested from the information signals at each EH receiver is sufficient to satisfy the harvested power constraints. One situation for this case to occur is that if $H$ is full rank, then $A^*$ is also full rank such that the unique optimal solution to problem (11) (and thus optimal to (P1)) is $S_i^* = U_i^* B_i^* (U_i^*)^H, \forall i \in K_{I}$, and $S_{E}^* = 0$ from Remark 3.1.

3) $S_{I}^* \succeq 0$ and $S_{E}^* \succeq 0$: in this case, dedicated energy signal is required to guarantee the harvested
power constraints while maximizing the WSR. Note that, given the strong duality between (P1) and (P2) as well as Lemma 3.2, the dedicated energy signal should be orthogonal to the MISO channels of all the ID receivers, such that the extra processing of pre-canceling the energy signal at each ID receiver is not needed.

It is also worth pointing out that the obtained optimal information and energy covariance matrices can be of higher rank in general. However, an interesting observation from our extensive simulation trials is that, Algorithm 1 always returns rank-one information covariance matrices due to the approximated rank minimization employed in (P4). It is difficult for us to prove such existence of optimal rank-one information covariance matrices, which will be an interesting open problem worthy of future investigation.

**TABLE I**

| Algorithm 1: Algorithm for Solving Problem (P1) |
|------------------------------------------------|
| 1) Initialize $\lambda_j \geq 0, j = 0, 1, \cdots, K_E$. |
| 2) Repeat: |
| a) Obtain $\{\bar{S}_i\}$ by solving problem (20) with given $\{\lambda_j\}$. |
| b) Compute the sub-gradients of $g(\{\lambda_j\})$ and the constraints in (23), (24) and (25), and update $\{\lambda_j\}$ accordingly using the ellipsoid method [13]. |
| 3) Until $\{\lambda_j\}$ converges within a prescribed accuracy. |
| 4) Set $\lambda_j^* = \lambda_j, \forall j \in K_E$. If $A^*$ is not of full rank, then obtain the optimal covariance matrices for information and energy transfer by solving problem (P4). |

**IV. Numerical Examples**

In this section, we provide numerical examples to validate our results. We set $P_{\text{sum}} = 5 \text{ Watt(W)}$, $\sigma^2 = -50 \text{ dBm}$, $N = 5$, $K_E = 3$ and $K_I = 2$. It is assumed that the path loss from the AP to all EH receivers is 30 dB corresponding to an equal distance of 1 meter, and that to all ID receivers is 70 dB at an equal distance of 20 meters. The channel vector $h_i$’s and $g_j$’s are randomly generated from i.i.d. Rayleigh fading. For the purpose of initial study, we only consider fixed information weights for ID receivers and will show the whole capacity region in the journal version of this paper. Specifically, we set $\alpha_i = 1, \forall i \in K_I$, and thus consider the achievable sum-rate of all ID receivers. We also set the harvested power constraints of all the EH receivers identical for simplicity, i.e., $E_j = E, \forall j \in K_E$.

For comparison, we consider two benchmark algorithms, which separately design the information and energy signals.
• **Separate information and energy signals design with power allocations (SIEDw/PA):** In this algorithm, the total transmit power $P_{\text{sum}}$ is divided into two parts: $P_I$ and $P_{\text{sum}} - P_I$ ($0 \leq P_I \leq P_{\text{sum}}$), which are exclusively allocated for information and energy signals, respectively. The transmit information and energy covariance matrices are separately designed with correspondingly allocated transmit power, which correspond to one WSRMax problem in MISO-BC under a single sum-power constraint and one feasibility problem similar as (10), respectively. The optimal power allocation strategy can be found via a bisection search, such that the WSR is maximized and the harvested power constraints are all satisfied.

• **Energy harvesting oriented separate information and energy signals design (EHSIED):** In this algorithm, the energy signals are first designed to ensure all the harvested power requirements at EH receivers with minimum power consumption, and the remaining power is then allocated for information signals to maximize the WSR.

Fig. 1 compares the sum-rate obtained by the optimal algorithm and two benchmark algorithms versus different EH constraint values of $E$. It is first observed that the optimal algorithm outperforms both the two suboptimal algorithms, and the performance gap increases as $E$ increases. This observation validate our theoretical results and the effectiveness of joint information and energy signals design. Note that all the three algorithms achieve the same WSR when $E = 0$, which is the maximum sum-rate achievable without harvested energy constraint. Second, we observe that the optimal algorithm and the SIEDw/PA have the same performance when $E$ is small. This is because that when $E$ is sufficiently small, the information signals obtained by maximizing the sum-rate exclusively are sufficient to guarantee the harvested power constraints at each EH receiver. However, as $E$ increases, the information transfer needs to be compromised for energy transfer, such that the optimal directions of the information signals are shifted from those obtained by maximizing the sum-rate exclusively. Finally, by comparing SIEDw/PA and EHSIED, it is observed that SIEDw/PA outperforms EHSIED over the entire range of values of $E$. As $E$ increases, SIEDw/PA converges to EHSIED, since most of the power is allocated for energy transfer as the harvested power constraints become stringent.

V. CONCLUSION

In this paper, we study a MISO-BC for SWIPT, where a multi-antenna AP delivers information and energy simultaneously to multiple single-antenna receivers. We characterize the capacity region for the ID
receivers by maximizing their WSR subject to the sum-power constraint at the AP and a set of minimum harvested power constraints at EH receivers. This problem corresponds to a new type of WSRMax problem for MISO-BC with combined MaxLTCC and MinLTCCs, for which a new optimal algorithm is proposed by extending the BC-MAC duality and applying the ellipsoid method. The proposed algorithm provides useful insights on solving general WSRMax problems with both MaxLTCCs and MinLTCCs, and the established capacity region provides a performance upper bound on all practically implementable precoding/beamforming algorithms for SWIPT in MISO-BC.

APPENDIX A

PROOF OF LEMMA 3.1

The first two conditions of Lemma 3.1 can be proved by contradiction. For convenience, we define $S_I \triangleq \sum_{i \in \mathcal{K}_I} S_i$, i.e., sum of all information covariance matrices. Furthermore, it is sufficient to only consider the case that $S_I$ can be expressed as

$$S_I = \sum_{n=1}^{N} \mu_n u_n u_n^H$$

(37)

where $u_n \in \mathbb{C}^{N \times 1}$ is the $n$th eigenvector of $A$, i.e., $[u_1, \cdots, u_N] = [U_1, U_2]$ from (16), and $\mu_n$ is an arbitrary real number, $n = 1, \cdots, N$. As a result, $\sum_{i \in \mathcal{K}_I} \text{Tr}(A S_i) = \text{Tr}(A S_I)$ can be expressed as $\sum_{n=1}^{N} \mu_n^2 u_n^H A u_n$.

Suppose that $A \not\succeq 0$, i.e., at least one of the eigenvalues of $A$ is negative, and $g(\lambda_j)$ has an upper bounded value, i.e., $g(\lambda_j) < +\infty$. Without loss of generality, we assume that $u_k$ is one of the eigenvectors associated with the negative eigenvalues of $A$. Then, it follows that $u_k^H A u_k < 0$, and
\[ \mu_k^2 A_{uk} \rightarrow -\infty \text{ as } \mu_k^2 \text{ approaches } +\infty. \] Therefore, it is easy to verify that by choosing \( S_I \) based on (37) with \( |\mu_k| \) being large enough, \( |\mu_i|, \forall i \neq k \) can be set to be arbitrary large such that we can achieve arbitrary large WSR for ID receivers without violating (13), which results in \( g(\{\lambda_j\}) = +\infty \). Consequently, \( A \) has to be positive semi-definite. Since similar arguments can be used to verify the second condition of Lemma 3.1, the details are omitted for brevity.

Next, we prove the third condition of Lemma 3.1. Given \( A \) being positive semi-definite, it has a positive semi-definite square root, i.e., \( A = A^{1/2}A^{1/2} \). Therefore, \( \text{Tr}(AS_I) \) and \( \text{Tr}(AS_E) \) can be expressed as \( \text{Tr}(A^{1/2}S_IA^{1/2}) \) and \( \text{Tr}(A^{1/2}S_EA^{1/2}) \), respectively. Since both \( A^{1/2}S_IA^{1/2} \) and \( A^{1/2}S_EA^{1/2} \) are positive semi-definite, it follows that \( \text{Tr}(A^{1/2}S_IA^{1/2}) \geq 0 \) and \( \text{Tr}(A^{1/2}S_EA^{1/2}) \geq 0 \). Lemma 3.1 is thus proved.

**APPENDIX B**

**PROOF OF LEMMA 3.2**

From the proof of Lemma 3.1 in Appendix A, \( \sum_{i \in K} \text{Tr}(AS_I) \geq 0 \) and \( \text{Tr}(AS_E) \geq 0 \). Given the fact that only \( \{S_i\} \) is related to the information transfer, any solution to problem (11) with \( \text{Tr}(AS_E) > 0 \) reduces the transmit power allocated to the information transfer and is thus suboptimal. Therefore, the optimal energy covariance matrix needs to satisfy \( \text{Tr}(A\bar{S}_E) = 0 \) equivalently \( A\bar{S}_E = 0 \), which means \( \bar{S}_E \) lies in the null space of \( A \). According to (16), the vectors in \( U_2 \) form the orthogonal basis for the null space of \( A \). Therefore, \( \bar{S}_E \) in general can be expressed as \( \bar{S}_E = U_2 \hat{E}U_2^H \), where \( \hat{E} \in \mathbb{C}^{(N-m)\times(N-m)} \) is any positive semi-definite matrix. Lemma 3.2 is thus proved.

**APPENDIX C**

**PROOF OF LEMMA 3.3**

Without loss of generality, \( S_i \) can be expressed as
\[
S_i = [U_1, U_2] \begin{bmatrix} B_i & C_i \\ C_i^H & D_i \end{bmatrix} [U_1, U_2]^H
= U_1B_iU_1^H + U_1C_iU_2^H + U_2C_i^HU_1^H + U_2D_iU_2^H
\] (38)
(39)
where \( B_i \in \mathbb{C}^{m \times m}, D_i \in \mathbb{C}^{(N-m)\times(N-m)} \) and \( C_i \in \mathbb{C}^{m \times (N-m)}, \forall i \in K_I \). Note that \( B_i = B_i^H \) and \( D_i = D_i^H \). Since \( U_2 \) lies in the null space of \( A \) (from Lemma 3.2) and consequently in the null space of \( H \) (from Lemma 3.1), it is observed that \( r_i \) and \( \sum_{i \in K_I} \text{Tr}(AS_i) \) do not depend on \( C_i \) and \( D_i, \forall i \in K_I \). Consequently, it is optimal to set \( C_i = 0 \) and \( D_i = 0, \forall i \in K_I \), and accordingly problem (18) with given \( \{\lambda_j\} \) can be further simplified as (20) given in Lemma 3.3 where \( \hat{A} \) is a full rank diagonal matrix. With
\( \hat{A} \) being full rank, problem (20) can be solved by the general BC-MAC duality as in \([11]\), which results in unique rank-one solution, i.e., \( U_1 \bar{B}_i U_1^H, i \in K_I \). Lemma 3.3 is thus proved.

**APPENDIX D**

**PROOF OF LEMMA 3.4**

Since the encoding order of the BC is the reverse of the decoding order of its dual MAC, which can be obtained from Section III-A while solving problem (20) and is assumed to be in accordance with the ID receiver index, the problem (18) can now be written explicitly as

\[
\max_{\{S_i\}} r \sum_{i=1}^{K_I} \alpha_i r_i \\
\text{s.t. } \sum_{i \in K_I} \text{Tr}(AS_i) \leq P_A \\
S_i \succeq 0, \forall i \in K_I
\]

(40)

where \( r_i \) is given by

\[
r_i = \log \left( \frac{\sigma^2 + h_i^H \left( \sum_{k=i}^{K_I} S_k \right) h_i}{\sigma^2 + h_i^H \left( \sum_{k=i+1}^{K_I} S_k \right) h_i} \right). \tag{41}
\]

The KKT optimality conditions of problem (40) are given by

\[
\frac{\partial}{\partial S_i} \sum_{i=1}^{K_I} r_i = \omega A + \Psi_i, \forall i \in K_I \\
\omega \left[ \sum_{i \in K_I} \text{Tr}(AS_i) - P_A \right] = 0 \\
\text{Tr}(\Psi_i S_i) = 0, \forall i \in K_I \tag{42}
\]

where \( \omega \geq 0 \) and \( \Psi_i \succeq 0, \forall i \in K_I \) are the Lagrange multipliers associated with \( \sum_{i \in K_I} \text{Tr}(AS_i) \leq P_A \) and \( S_i \succeq 0, \forall i \in K_I \), respectively.

This lemma can be proven by first showing that the duality gap between problem (40) and its Lagrange dual problem is zero, and the KKT conditions given in (42) are sufficient for a solution to be optimal for problem (40). Since the proofs are similar that of \([11\) Proposition 2] and \([11\) Proposition 3], they are omitted for brevity. To complete the proof, we need further show that the optimal value of problem (18) is equal to that of (P1) with fixed encoding order, the details of which is given as follows.

We first consider a fixed encoding order for problem (P1) termed as problem (P1F), given by the optimal encoding order for problem (P2), which has been assumed to be the same as the ID receiver index order. Under this encoding order, the information rate for ID receiver \( i \) is given in (41).
Note that the optimal solution of problem (P1F) is a lower bound on the optimal solution of problem (P1). The KKT conditions of problem (P1F) can be written as

\[
\frac{\partial \sum_{i=1}^{K_I} r_i}{\partial S_i} = \theta_0 I - \sum_{j=1}^{K_E} \theta_j G_j + \Omega_i, \forall i \in K_I
\]  

(43)

\[
\frac{\partial \sum_{i=1}^{K_I} r_i}{\partial S_E} = \theta_0 I - \sum_{j=1}^{K_E} \theta_j G_j + \Omega_E
\]  

(44)

\[
\theta_j (\text{Tr} [(S_I + S_E)G_j] - E_j) = 0, \forall j \in K_E
\]  

(45)

\[
\theta_0 (\text{Tr} [S_I + S_E] - P_{\text{sum}}) = 0
\]  

(46)

where \( \{\theta_j\}_{j=1}^{K_E}, \theta_0, \{\Omega_i\} \) and \( \Omega_E \) are the Lagrange multipliers with respect to the constraints in (7), (8) and (9), respectively. For convenience, we define \( S_I = \sum_{i \in K_I} S_i \). When the optimal solution of problem (P1F) is achieved, we assume that the corresponding optimal primal and dual solutions are \( \tilde{S}_I, \tilde{S}_E, \{\tilde{\theta}_j\}_{j=1}^{K_E}, \tilde{\theta}_0, \{\tilde{\Omega}_i\} \) and \( \tilde{\Omega}_E \).

We now write the KKT conditions of problem (18) with \( \lambda_0 = \tilde{\theta}_0 \) and \( \lambda_j = \tilde{\theta}_j, \forall j \), as follows:

\[
\frac{\partial \sum_{i=1}^{K_I} r_i}{\partial S_i} = \omega \left( \tilde{\theta}_0 I - \sum_{j=1}^{K_E} \tilde{\theta}_j G_j \right) + \Psi_i, \forall i \in K_I
\]

\[
\omega \left[ \tilde{\theta}_0 \text{Tr} (S_I) - \sum_{j=1}^{K_E} \tilde{\theta}_j \text{Tr} (S_I G_j) - \tilde{\theta}_0 P_{\text{sum}} + \sum_{j=1}^{K_E} \tilde{\theta}_j E_j \right] = 0
\]  

(47)

If we choose \( S_I = \tilde{S}_I + \tilde{S}_E \tilde{A} \), where \( \tilde{S}_E \tilde{A} = \tilde{U}_1 \tilde{U}_1^H \tilde{S}_E \tilde{U}_1 \tilde{U}_1^H \) and \( \tilde{U}_1 \) consists of the orthogonal basis defining the range of \( \tilde{A} = \tilde{\theta}_0 I - \sum_{j=1}^{K_E} \tilde{\theta}_j G_j \) similar as that in (16), \( \omega = 1 \), and \( \Psi_i = \tilde{\Omega}_i, \forall i \in K_I \), the KKT conditions in (47) are satisfied. According to the fact that the duality gap between problem (40) and its Lagrange dual problem is zero, \( \tilde{S}_I + \tilde{S}_E \tilde{A} \) is optimal for problem (18). Therefore, the optimal value of problem (18) with \( \lambda_0 = \tilde{\theta}_0 \) and \( \lambda_j = \tilde{\theta}_j, \forall j \), is equal to the optimal value of problem (P1F). Therefore, the optimal value of problem (P1F), which is a lower bound on the optimal value of problem (P1), meets the optimal value of problem (18) with \( \lambda_0 = \tilde{\theta}_0 \) and \( \lambda_j = \tilde{\theta}_j, \forall j \), which is an upper bound on the optimal value of problem (P1). The above results also imply that the minimum value of \( g(\{\lambda_j\}) \) over \( \{\lambda_j\} \) is achieved when \( \lambda_0 = \tilde{\theta}_0 \) and \( \lambda_j = \tilde{\theta}_j, \forall j \). The proof thus follows.

Appendix E

Proof of Lemma 3.6 and Lemma 3.7

We start with proving Lemma 3.6. It is first observed that the condition \( \text{Null}(A) \subseteq \text{Null}(H) \) is equivalent to that \( v_i \notin \text{Null}(A), \forall i \leq t \), where \( t \) denotes the rank of matrix \( H \), and \( v_i, i = 1, \ldots, t \),
denote the left singular vectors of $H$ corresponding to its non-zero singular values. Furthermore, given $A \succeq 0$, the condition $v_i \notin \text{Null}(A), \forall i \leq t$, can be further expressed as $v_i^H Av_i > 0, \forall i \leq t$. The proof thus follows.

Next, we proceed to prove Lemma 3.7. For the purpose of illustration, we define $F(\lambda) = -\lambda_0 I + \sum_{j=1}^{K_E} \lambda_j G_j$, where $\lambda = [\lambda_0, \cdots, \lambda_{K_E}]^T$. Then the constraint in (24) is equivalent to $F(\lambda) \preceq 0$. First, the semi-definite constraint $F(\lambda) \preceq 0$ can be equivalently expressed as a scalar inequality constraint as

$$f(\lambda) \triangleq \lambda_{\text{max}}(F(\lambda)) \leq 0$$

(48)

where $\lambda_{\text{max}}(A(\lambda))$ denotes the largest eigenvalue of $A(\lambda)$. Thus, the latter constraint can be equivalently written as

$$f(\lambda) = \max_{\|z\|^2=1} z^H F(\lambda) z \leq 0$$

(49)

Given a query point $\lambda_1 = [\lambda_{0,1}, \cdots, \lambda_{K_E,1}]^T$, we can find the normalized eigenvector $z_1$ of $F(\lambda_1)$ corresponding to $\lambda_{\text{max}}(F(\lambda_1))$. Consequently, we can determine the value of the scalar constraint at a query point as $f(\lambda_1) = z_1^H F(\lambda_1) z_1 = \lambda_{\text{max}}(F(\lambda_1))$. To obtain a subgradient, we make the following considerations

$$f(\lambda) - f(\lambda_1) = \max_{\|z\|^2=1} z^H F(\lambda) z - z_1^H F(\lambda_1) z_1$$

(50)

$$\geq z_1^H [F(\lambda_1) - F(\lambda)] z_1$$

(51)

$$= \|z_1\|^2 (\lambda_0 - \lambda_{0,1}) - \sum_{j=1}^{K_E} (z_1^H G_j z_1) (\lambda_j - \lambda_{j,1}).$$

(52)

where the last equality follows from the affine structure of the semi-definite constraint. Lemma 3.7 thus follows.

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