State reduction and energy extraction from black holes

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Abstract

I show that attempts to detect Hawking quanta would reduce the quantum state to one containing ultra-energetic incoming particles; couplings of these to other systems would extract ultra-high energies from the gravitational collapse. As the collapse proceeds, these energies grow exponentially, rapidly become trans-Planckian, and quantum-gravitational effects must enter.

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Energy, in quantum theory, is measured by the Hamiltonian operator. Since this operator generates temporal evolution, energy will be conserved if the operator remains constant. But if the quantum system passes through a time-dependent potential, the Hamiltonian will not be constant, and energy may be exchanged between the quantum system and the potential. Two states, whose energy-contents (precisely, resolutions into energy eigenstates) differ from each other but little at one time may evolve to have very different energy-contents at another.

This has consequences for quantum measurements. The measurement of an operator may alter the state in a way which seems mild (that is, does not involve very energetic excitations) at one time, but more substantial at another. Thus the measurement — or the attendant reduction of the state vector — may induce a further exchange of energy between the quantum system and the potential.

These apparently reasonable considerations can lead to startling conclusions if the passage through the time-dependent region induces gross enough changes.
in the Hamiltonian. In this letter, I shall examine what happens in an extreme case, that of quantum fields passing through a gravitationally collapsing space–time — the system famously investigated by Hawking [1,2]. I shall use Hawking’s model, but shall be led to substantially different conclusions from his.

In this case, the time-dependence is the gravitational collapse, and the change in the Hamiltonian involves both a squeezing transformation (which leads to particle creation) and a red-shift of field modes passing close to the incipient black hole. We shall see that when measurements of Hawking quanta are made, the state reduces to one in which quanta which are the blue-shifted ingoing precursors of the Hawking quanta are present. Couplings of these precursors to other systems will result, when Hawking quanta are measured, in the extraction of these blue-shifted energies from the collapsing space–time.

When a black hole forms, these red- and blue-shifts increase exponentially quickly [3,4]. This means that, as the collapse proceeds, the magnitudes of the energies which might be extracted also grow exponentially. We thus have a period in which reduction effects allow larger and larger energies to be drawn from the collapsing system.

But in short order we face a more fundamental issue. After a sufficient number of e-folding times have passed, the magnitudes of the energies of the exchanged quanta have passed the Planck scale. At this point, attempts to detect Hawking quanta will inevitably generate Planck-scale, quantum-gravitational, precursors. Couplings of the field to other systems mean that these precursors will mediate Planck-scale, quantum-gravitational, extractions of energy from the collapsing object. As we have no theory of quantum gravity, we can say nothing about what will happen from this point on.

The picture that is drawn here is thus very different from the one drawn by Hawking. In Hawking’s analysis, the black hole approaches a quasistationary state in which it can be well-described as a classical object subject to small quantum corrections resulting in the emission of a weak flux of thermal radiation. Here we find that, when the collapse begins, there is a transient period during which any Hawking quanta which might be produced are entangled with ultra-high energy excitations of coupled systems, so that the detection of Hawking quanta would force ultra-high energy exchanges. However, this period does not last long. After a number of e-foldings, these ultra-high energies have passed the Planck scale, and the entire theory (which was predicated on the neglect of quantum gravity) has broken down. This result is exciting, for it means that the black hole becomes an essentially quantum-gravitational object. As we shall see, this quantum-gravitational character is not confined to the event horizon, but influences a neighborhood of the hole, the probabilities of the effects falling off as a power of the distance.
Fig. 1. A diagram of a black-hole space–time, suppressing angular variables. The left-hand edge is the spatial origin. Time increases upwards, and lines at 45° represent the paths of radial light rays. The scale has been distorted so that the entire space–time, and some ideal points at infinity, can be represented. The region occupied by the collapsing matter is shaded. The event horizon is the dashed line; the black hole itself is the set of points at and above this. The dotted line represents a radial light ray beginning at a point on past null infinity, moving radially inwards and passing through the spatial origin (where, in the diagram, it appears to reflect from the left-hand edge), and escaping to a point on future null infinity.

Related arguments, not involving reduction of the state vector but leading to the same conclusions, are presented elsewhere [5].

Terminology and conventions. In what follows, the distinction between coupling of quantum systems and measurement will be important. Two systems couple if there are mixing terms in the total Hamiltonian. However, we shall reserve the term measurement for the action which reduces the quantum state to an eigenstate of an observable. We shall use natural units throughout, so that energy, temperature and frequency are equivalent concepts.

Let us recall the basic structure of a space–time modeling the collapse of an isolated spherically symmetric system of mass $M$ (Fig. 1). We may introduce null coordinates $u$ and $v$, the retarded and advanced times (so $u \simeq t - r$ and $v \simeq t + r$ near future and past null infinity, respectively). The mapping of surfaces of constant phase, that is, the geometric-optics approximation to propagation, will be important. For a spherically symmetric null hypersurface arriving at retarded time $u$ in the future, let $v(u)$ be the advanced time in the past at which it originated. That a black hole forms means that there is a limiting value $v(+\infty)$, which is the advanced time of formation of the hole. (The event horizon is $u = +\infty$.) Note that if a spherically symmetric wave has period $du$ in the future, its period in the past would be $dv = v'(u) \, du$, so $v'(u)$ is the red-shift suffered by a spherically symmetric wave passing through the space–time. According to gravitational collapse theory [3,4], when a black
These exponentially changing red-shifts will play a central role.

Now let $\phi$ be a massless linear Bose field in the space–time. In the distant past, the field may be resolved into a sum of normalized field modes $p_j$ times annihilation ($a^j$) or creation ($a_j^*$) operators:

$$\phi = \sum_j \left( p_j a^j + p^j a_j^* \right).$$

Similarly in the distant future, for an appropriate set of field modes $f_{j'}$ and operators $b^{j'}$, $b_{j'}^*$,

$$\phi = \sum_{j'} \left( f_{j'} b^{j'} + f^{j'} b_{j'}^* \right).$$

The linearity of the field equation implies linear relations between the field modes (and inversely the operators):

$$f_{j'} = \sum_j \left( \alpha_{j'j} p_j + \beta_{j'j} p^j \right), \quad a^j = \sum_{j'} \left( \alpha_{ij} b^{j'} + \beta_{ij}^{j'} b_{j'}^* \right)$$

where the factors $\alpha_{j'j}$, $\beta_{j'j}$ are known as Bogoliubov coefficients. Note that it is the $\beta_{j'j}$ coefficients which mix creation and annihilation terms; these coefficients are generally responsible for changes in particle number.

The equations (4) define an invertible transformation between the field operators in the distant past and the distant future. In each of these regimes, there is a well-defined vacuum (the in-vacuum, characterized by $a^j |0_p\rangle = 0$, and the out-vacuum, by $b^{j'} |0_{\bar{f}}\rangle = 0$) and well-defined creation and annihilation operators. Any physical state may be expressed equally well as an in-state (given by a sum of $a^j_+ \text{creation operators acting on } |0_p\rangle$) or an out-state (given by a sum of $b_{j'}^\ast \text{operators on } |0_{\bar{f}}\rangle$). We may call these alternative expressions of the state the past and future presentations.

Suppose the field is initially in the in-vacuum state $|0_p\rangle$. To understand what this state would look like if examined at late times, that is, its future presentation, we rewrite the characterizing equation in terms of the operators in the future:

$$a^j |0_p\rangle = \sum_{j'} \left( \alpha_{j'j} b^{j'} + \beta_{j'j}^{j'} b_{j'}^* \right) |0_p\rangle = 0.$$
It is easily verified that the solution of this is

$$|0_p\rangle = \text{(normalization)} \exp\left[(1/2) \sum_{j',k'} Q^{j'k'} b^*_j b^*_k |0_f\rangleight],$$  \hspace{1cm} (6)$$

where $Q^{j'k'} = -\sum_j (\alpha^{-1})^{j'} j \beta^{j'k'}$. Here the various powers of $Q^{j'k'} b^*_j b^*_k$ create pairs, quadruples, and so on, of particles. Thus the past vacuum appears in the future to have a superposition of various particle numbers: passage of the field through the potential has resulted in the creation of particles. Again, note that it is the possibility $\beta_{j'j} \neq 0$ which allows this production.

In the case of gravitational collapse, the Bogoliubov coefficients were studied by Hawking. He found that the primary contributions to $\beta_{j'j}$ came from specific classes of modes, which we shall denote $j = j_H, j' = j'_H$, the characteristic ingoing and outgoing Hawking modes. The modes $j'_H$ have frequencies $\sim T_H = (8\pi M)^{-1}$ (the Hawking temperature), and are maximally compatible with the spherical symmetry. The modes $j_H$ are the ingoing precursors to $j'_H$. They are concentrated just before the limiting surface $v(+\infty)$, the advanced time of formation of the hole. They are again maximally compatible with the spherical symmetry, but their frequencies are much higher on account of the red-shift caused by propagation through the gravitationally-collapsing region: their frequencies are $\sim T_H/v'(u) \simeq T_H \exp +u/(4M)$.

The significance of this exponential growth in the precursors’ frequencies has been a matter of debate [6]. On one hand, the frequencies involved quickly surpass the Planck scale and thus make the neglect of quantum gravity problematic. (This is the trans-Planckian problem.) On the other, while ultra-high frequency modes are indeed implicated in the past, these modes are unpopulated (the corresponding particle numbers are zero, since the initial state is vacuum). Thus many have felt that perhaps with a correct understanding of the physics one could somehow bypass the awkward use of ultra-high frequencies. But we shall find that measurement processes can populate these ultra-energetic modes and transfer exponentially increasing energies out of the gravitationally collapsing system.  

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1 A few comments on one main line of attack on the trans-Planckian problem are in order. Jacobson, Unruh et al. [7,8,9,10] aim to resolve the trans-Planckian problem by substituting non-standard rules for propagation of the fields. Their goal is to “insulate” Hawking’s result from trans-Planckian physics, and this is done by radically altering the actual mechanism that is used to get that result. At the moment, work on these approaches is ad-hoc and preliminary, and the degree to which they can be said to eliminate the trans-Planckian problem not entirely clear [6]. If, however, one of these non-standard approaches were to prove correct (that is, not only self-consistent but the way the world works), then the present analysis, which relies heavily on the original Hawking model, would have to be
While an exact mathematical treatment in terms of the Bogoliubov coefficients is possible, in order to present the main ideas without distracting technicalities we shall just consider the ingoing modes $j_H$ and the outgoing modes $j'_H$. In this approximation, the Bogoliubov transformation becomes a combination of the effect of the red-shift and a squeezing transformation. It red-shifts, and mixes positive and negative frequencies, but does not convert one frequency into a range of frequencies:

$$a^{jn} \simeq \alpha^{jn} b^{jn}_H + \beta^{jn} b^{*n}_H.$$  (7)

From now on, we shall work only with these modes, and so we shall drop the postscripts $j = j_H, j' = j'_H$. Thus the (relevant) Bogoliubov coefficients will be denoted simply $\alpha$ and $\beta$, and the in-vacuum is

$$|0_p\rangle \simeq \text{(normalization)} \exp\left[(1/2)Qb^{*}b^{*}\right] |0_f\rangle.$$  (8)

Hawking found $|\alpha| \sim |\beta|$, so $|Q| = |\beta/\alpha| \sim 1$.

Now let us suppose that the number of particles in the Hawking modes is actually measured, that is, the operator $n(b) = b^{*}b$ is measured. Then the state reduces to an eigenstate of the operator, with the probability of having eigenvalue $n$ determined by the state vector, eqn. (8). (Since the modes $j'_H$ correspond to only a few characteristic periods, and the characteristic time for emission is typically somewhat greater than this [6], in fact the most likely value for $n$ is zero.) Of course, the reduced eigenstate, in the future presentation, is simply $|n_f\rangle = (n!)^{-1/2}(b^{*})^{n}|0_f\rangle$, the state with $n$ Hawking quanta outgoing.

We may also consider the past presentation of this reduced state. A routine calculation gives

$$|n_f\rangle \simeq \text{(normalization)} H_n((2\pi/\beta)^{-1/2}a^{*}) \exp\left[(1/2)((\beta/\alpha)(a^{*})^{2}\right] |0_p\rangle,$$  (9)

where $H_n$ is the $n^{th}$ Hermite polynomial. Since $a^{*}$ creates ultra-energetic particles, we see that the past presentation of the reduced state is a superposition of ultra-energetic components. This will be the case even if no Hawking quanta are detected (that is, $n = 0$).

The upshot of the discussion so far is that the measurement of the particle-content of field modes of moderate frequencies in the future would lead to a reduction of the state vector which, in terms of the operators defined in the past, would contain very energetic particles. One might think that, given that reconsidered.
Fig. 2. Effect of measuring the number of ultra-high frequency quanta absorbed by a photographic plate, after the number of Hawking quanta has been measured. The patterned area (shaped <) represents the region of space–time occupied by the measured Hawking quanta and their precursors. The forking line represents two possible trajectories of the plate, according to whether it is found to have absorbed the ultra-energetic precursors (and their energy–momentum) or not. (Before measurement of the plate, its state is a superposition of such trajectories.) There is a finite probability that ultra-high energy–momenta will be transferred to the plate.

the measurement is made in the future, this past presentation of the state was irrelevant or simply a mathematical way of speaking without direct physical significance. But this is not so. Once the measurement of particle number in the future has been made, the actual state of the system is the reduced state (until a subsequent measurement is made). This means that any device which had in fact been present in the past and coupled to high-frequency modes would have done so.

To make the discussion more concrete, we will consider a device which we will refer to as a “photographic plate” (sensitive to the ultra-high frequency, $j_H$-mode, quanta) following a trajectory from the distant past to the distant future, passing through the region potentially occupied by the $j_H$ quanta. However, whether the device actually is a plate or not (or whether a real plate could be constructed to respond to the frequencies in question) will not be important. What will matter is that the “plate” couples to the operator $n(a) = a^*a$.

Now suppose we first measure $n(b)$ and then examine the plate. When we measure $n(b)$ the state reduces to one with a superposition of ultra-high frequency quanta. The plate will couple to those, and so, when it is examined, it will with positive probability have recorded the passage of the ultra-energetic $j_H$ modes. And the energy–momentum of the $j_H$ modes will have been transmitted to the plate. (See Fig. 2.)
It should be noted that the plate does not have to be near the collapsing object for this effect to occur, for the coupling with the $j_H$ modes will occur in a neighborhood of the advanced time of formation of the hole, and the plate will inevitably pass through this neighborhood. It is true that the probability for any given plate to detect such a mode falls as the radius at which the plate passes through this neighborhood increases, for the strength of the mode would fall off like a power of $r$. However, given that the plate does detect a quantum, the energy of that quantum is essentially (apart from red-shift effects if the plate is very near the collapsing object) independent of the plate’s position. Thus ultra-high energy-momentum transfers to the plate are possible, no matter how great its distance from the hole.

One can make the argument more symmetric by imagining two photographic plates, one sensitive to the moderate-frequency, $j'_H$-mode, Hawking quanta, and the other to the ultra-high frequency, $j_H$-mode, quanta. Of course, these plates are really just ways of visualizing devices coupling to the number operators $n(b) = b^*b$ and $n(a) = a^*a$, respectively. We have seen that if first the Hawking plate is examined, and then the other, the latter will be found with positive probability to have recorded the passage of ultra-energetic quanta. On the other hand, examining the plates in the other sequence would lead to nothing remarkable. Examining first the $n(a)$ plate, one would find no quanta to have been recorded, since the state was the in-vacuum. Then examining the Hawking-sensitive plate one might or might not find Hawking quanta. This order-dependence is a vivid consequence of the fact that $n(a)$ and $n(b)$ do not commute.

There is another, important, perspective. The coupling of the $n(a)$ plate to the field results in a coupling of the ultra-energetic $j_H$ modes with the plate. Since the Hawking ($j'_H$) modes are coupled to the $j_H$ modes, this means that the Hawking modes couple with the plate. However, since each number state of the Hawking modes corresponds to a superposition of number states of the $j_H$ modes, the number-states of the Hawking modes are entangled with states of the plate corresponding to ultra-energetic disturbances. Observing a Hawking mode therefore forces the plate into a superposition of ultra-excited states.

Once this point of view is appreciated, the conventional Hawking out-state (that is, the future presentation of the state $|0_p\rangle$, eqs. (6,8)) appears as a highly non-generic state. It is a superposition of different out-particle numbers with coefficients arranged with a cunning precision in such a way that the couplings to the ultra-energetic modes in the plate exactly cancel. These cancellations can be upset by moderate perturbations of the out-state, as when the number of Hawking quanta is observed. We therefore expect that in general couplings of the quantum field to other systems would result in ultra-energetic entanglements of those systems.
One natural concern about the thought-experiment presented here is that it has the flavor of altering the past (since ultra-energetic precursors, not present initially, are created). When we examine this carefully, however, we shall see that the place for concern seems not so much with the thought-experiment or its analysis, but with the use of conventional quantum field theory to describe physics beyond a certain stage in a gravitationally-collapsing space–time. And this conclusion is precisely the main point of the paper.

Before examining the deeper aspects of this issue, it is probably well to address a more superficial argument against the analysis here which, while motivated by alteration-of-history concerns, is not really well-founded. The argument might be put like this: “The initial state is the in-vacuum; this is a basic datum for the problem; altering this datum corresponds to doing a different problem. In other words, the thought-experiment seems to be changing the rules in the middle of the game.” This sort of argument really amounts to denying the possibility of state-vector reduction as an actual physical process. It is therefore not really an argument which is tenable within conventional quantum theory.

Now let us take up the alteration-of-history concerns directly. We first point out that there are good general reasons for believing that the analysis of the thought-experiment presented here will not give rise to causal paradoxes.

The past — as an unalterable historical record — consists of those classical data which may be known to an observer together with the results of quantum measurements already made. In this paper, what is altered is the quantum state, and that alteration is by the conventional rules of quantum theory. So there is no change in the historical record, and the analysis of the thought-experiment should be internally consistent against causality paradoxes, at least to the extent that conventional quantum theory is. (This characterization of the historical record will be adequate for the purposes here, but it becomes problematic when explicitly quantum-gravitational effects, for instance superpositions of causal structures, must be considered.)

So there are good general reasons for thinking the analysis of the thought-experiment is internally consistent. We shall learn more, however, when we look more particularly at specific concerns and their resolutions, although this also requires more extensive discussion. Let us consider what happens, in the thought-experiment, as the gravitational collapse proceeds. The energies which might be exchanged grow larger and larger. Is there a point at which these energies become so large that they affect the known distribution of energy–momentum, and so the known gravitational field? Would this not be an alteration of history?

In answer to this, let us first recall that the neglect of quantum-gravitational
effects must become invalid by the time the energy-exchanges approach the Planck scale. Whatever happens beyond this point is at present a matter of speculation. For energy-exchanges below the Planck scale, we must clarify the sense in which the pre-existing energy–momentum they compete with is “known.” Is it a classical known quantity, or one measured quantum-theoretically?

If, prior to the thought-experiment, the stress–energy is adequately modeled as a classical quantity, but the reduction effects a significant enough change that this modeling no longer is valid, then one does indeed have an alteration-of-history paradox as defined above. However, the possible resolutions to this would seem to be that either there must be new laws of quantum physics forbidding the sorts of energy-exchanges in the thought experiments, or that the stress–energy, and hence presumably the space–time geometry, takes on an essentially quantum character. Either (or both) of these resolutions would be in keeping with the main point of this paper, that conventional quantum field theory must break down in a gravitationally-collapsing space–time.

If, on the other hand, even prior to the thought-experiment, the stress–energy must be taken to be a quantum operator, then there is no alteration-of-history phenomenon as defined above, since one is simply measuring a sequence of quantum operators (the stress–energy, and the number operators $n(a)$ and $n(b)$). Again, since the stress–energy is the source for Einstein’s equation, a quantum stress–energy implies a quantum gravitational field and a deeper, quantum-gravitational, treatment of the entire question is really required. Again, this is in line with the main argument here, that quantum gravity must be considered in the quantum physics of gravitational collapse.

Some final comments are in order.

First, although the discussion has been phrased in terms of “photographic plates,” there is nothing very special about these. It was not even important that these responded precisely to the number operator $n(a) = a^*a$. What was really relevant was that the devices coupled to the ultra-energetic field modes $j_H$. Any such device would be affected by a measurement of the number of Hawking quanta, because such a measurement will reduce the state to the form $(9)$, a distribution of the ultra-energetic quanta. If any coupling to these modes is present, it will be implicated when attempts to measure the number of Hawking quanta are made.\(^2\)

\(^2\) In a practical sense it would be important to know what sorts of devices can couple to the ultra-energetic field modes, and with what efficiencies. Even before the Planck regime, however, the physics in question becomes speculative, as we do not really know the behaviors of quantum field theories at extreme energies. Here, we are mainly interested in questions of principle, so any coupling, leading to a positive probability of ultra-energetic effects, is significant.
Second, I have used the term “ultra-energetic” to characterize the implicated ingoing modes and energy-transfers. However, this term must be considered bland in face of the exponentially fast growth of these energies (eq. (1)). The e-folding time is typically short. (It is $4M$, the light-crossing time of the hole, $\simeq 2.0 \times 10^{-5} \text{s}$ for a solar-mass hole.) This means that the energies of the precursors, and the energy-transfers to the plates, rapidly pass the Planck scale. At that point, the theory — quantum field theory in curved space–time, neglecting quantum gravity — has entirely broken down.

This is the main lesson. When couplings between the quantum field and other systems are allowed, observation of Hawking quanta will, before long in the gravitational collapse process, have resulted in Planck-scale energy-transfers and therefore it will be impossible to analyze the physics without taking quantum gravity into account.

The results here imply a more severe breakdown of the conventional theory of black-hole radiation than had been contemplated. Previously, there had been some concerns about the theory’s reliance on trans-Planckian modes, but these seemed to be only concerns about virtual, vacuum-fluctuation, processes. And while the neglect of possible quantum-gravitational corrections was considered (e.g. refs. [11,12]), the primary speculations were that manifestly quantum-gravitational, Planck-scale, effects would be confined to the hole itself and leave only secondary imprints, at conventional physical scales, on the Hawking process at macroscopic distances. We have seen here, though, that real Planck-scale effects at significant distances are possible, and thus the trans-Planckian problem and manifest quantum-gravitational consequences must be faced directly.

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