SOLAR FLOWS AND THEIR EFFECT ON FREQUENCIES OF ACOUSTIC MODES

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ABSTRACT

We have calculated the effects of large-scale solar flows, such as the meridional circulation, giant convection cells, and solar rotation on the helioseismic splitting coefficients using quasi-degenerate perturbation theory (QDPT). Our investigation reveals that the effect of poloidal flows like the large-scale meridional circulation are difficult to detect in observational data of the global acoustic modes since the frequency shifts are much less than the errors. However, signatures of large-scale convective flows may be detected if their amplitude is sufficiently large by looking for frequency shifts due to nearly degenerate modes coupled by convection. In this comprehensive study, we attempt to put limits on the magnitude of flow velocities in giant cells by comparing the splitting coefficients obtained from the QDPT treatment with observational data.

Key words: Sun: helioseismology – Sun: interior

1. INTRODUCTION

Solar convection is believed to be organized in a variety of spatial and temporal scales ranging from granules (size \(\sim 1\) Mm, 0.2 hr lifetime), mesogranules (size \(\sim 10\) Mm, 3 hr lifetime), supergranules (size \(\sim 30\) Mm, 1 day lifetime) to the giant cells (size \(\geq 100\) Mm, one month lifetime). The power spectra of convective velocities show distinct peaks representing mesogranules and supergranules but no distinct features at wavenumbers representative of mesogranules or giant cells (Wang 1989; Chou et al. 1991; Straus et al. 1992; Straus & Bonaccini 1997; Hathaway et al. 2000). Numerical simulations of solar convection routinely show the existence of mesogranules and giant cells (Miesch et al. 2000, 2008). In this study, we will only concentrate on the theoretical frequency shifts in global surface gravity and acoustic modes, denoted \(p\) - and \(f\)-modes, respectively due to interaction with large-scale flows, namely the giant cells, the meridional circulation, and rotation. The meridional circulation, believed to play an important role in magnetic flux transport by solar dynamo models (Choudhuri et al. 1995; Dikpati & Charbonneau 1999; Chatterjee et al. 2004), was observed at the solar surface by Doppler measurements of photospheric lines (Duvall 1979; LaBonte & Howard 1982; Komm et al. 1993; Hathaway 1996) and later by using techniques of ring diagram and time-distance helioseismology (Giles et al. 1997; Basu et al. 1999; González Hernández et al. 1999). These studies can only measure the flow velocity in the near surface layers. This flow having a maximum surface velocity of 30 m s\(^{-1}\) is apparently poleward all the way from the equator to the poles at the Solar surface. Conservation of mass requires existence of a return flow advecting mass from the poles to the equator somewhere below the surface. The dynamo models assume that the return flow occurs near the base of the convection zone, but there is no direct evidence about where the return flow is located. It is believed that the only hope of detecting such a return flow is by analyzing the effect of the meridional circulation on the global helioseismic acoustic modes (Roth & Stix 2008). Their investigation using quasi-degenerate perturbation theory (QDPT) showed that theoretical \(p\)-mode frequency shifts due to meridional circula-

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to study such long-lived and large-scale features more reliably. Beck et al. (1998) were able to detect giant cells at the solar surface with large aspect ratio (~4) and velocities ~10 m s$^{-1}$ using the MDI data. Hathaway et al. (2000) used spherical harmonic spectra from full disk measurements to detect long-lived power at $l \leq 64$. This means that the spatial extent of the features were $\sim 2\pi R_{\odot}/\sqrt{l(l+1)} \geq 70$ Mm. Later Ulrich (2001) identified these long-lived features not as giant cells but higher order components of the torsional oscillation. Interestingly, it was earlier proposed by Snodgrass & Wilson (1987) that torsional components of the torsional oscillation. Interestingly, it was pointed out by Roth et al. (2002) that these long-lived features not as giant cells but higher order components of the torsional oscillation. Interestingly, it was pointed out by Roth et al. (2002) that torsional oscillations occur due to modulation of differential rotation by the Coriolis force arising from meridional motions of giant cells.

The elusiveness of the giant cell features in the surface velocities lead modelers to speculate about the subsurface nature of the giant cell motions (Latour et al. 1981). Roth & Stix (1999, 2003) used QDPT to calculate the effect of giant cells on $p$-modes and claimed that giant cells could be found by modeling the asymmetries and line broadening in the solar power spectrum. They claimed that finite line width of the multiplets would limit the detection of the frequency splittings to vertical velocity amplitude of 100 m s$^{-1}$ or larger. However, Roth et al. (2002) claimed that giant cells could be detected with current inversion methods of global helioseismology as long as they exceed an amplitude of 10 m s$^{-1}$. A usual inversion procedure for rotation neglects giant cells and assumes that the odd splitting coefficients arise only from rotation. If there are additional contributions to these coefficients, rotation inversions would not give correct results. According to Roth et al. (2002), the effect of giant cells would appear as distortions in the rotation inversion if their velocities are $\geq 10$ m s$^{-1}$. However, using the same technique of QDPT, we find that with the present global helioseismic data we can put an upper limit of 50–100 m s$^{-1}$ on the vertical velocity associated with giant cell flows. This is mainly because our calculations show much smaller effects from giant cells.

This paper is organized as follows. In Section 2, we define zonal and poloidal flows inside the Sun, while Section 3 discusses the QDPT, its differences from degenerate theory as well as the details of our calculations. We present our theoretical results and compare them with present $f$- and $p$-mode data from both Global Oscillation Network Group (GONG) and SOHO/MDI instruments in Section 4. Finally conclusions are described in Section 5.

2. ZONAL AND POLOIDAL SOLAR FLOWS

Following earlier works (Lavely & Ritzwoller 1992; Roth et al. 2002), we express the velocity field in terms of spherical harmonics. For completeness, we give the expression here again:

$$v(r, \theta, \phi) = \text{Re}[u_i'(r)Y_0'(\theta, \phi)\hat{r} + v_i'(r)\nabla_\theta Y_0'(\theta, \phi) - w_i'(r)\hat{r} \times \nabla_\phi Y_0'(\theta, \phi)].$$ (1)

The quantities $u_i'$, $v_i'$, and $w_i'$ determine the radial profiles of the flows, and $\nabla_\theta$ is the horizontal gradient operator. The Re refers to using only the real part of the spherical harmonics as in

$$\text{Re}[Y_0'(\theta, \phi)] = \begin{cases} \{Y_0'^{-1}(\theta, \phi) + Y_0'(\theta, \phi) \}/2 & \text{if } t \text{ is even}, \\
\{Y_0'^{-1}(\theta, \phi) - Y_0'(\theta, \phi) \}/2 & \text{if } t \text{ is odd}. \end{cases}$$ (2)

The first two terms in Equation (1) define the poloidal component of the flow whereas the last term is the toroidal component.

By the poloidal component, we imply the meridional and non-zonal toroidal flows (average over $\phi$ direction is zero) e.g., (1)

The meridional circulation which carries mass poleward near the surface and sinks near the poles and (2) the giant convection cells, respectively. These flows are also called large-scale flows to distinguish them from other small-scale flows like the turbulent eddies which are of the size smaller than the typical scale of global modes used in helioseismology. In the presence of only the poloidal flow ($u_i' = 0$), we can apply the equation of mass conservation $\nabla.(\rho_0 v) = 0$ to get a relation between $u_i'(r)$ and $v_i'(r)$ e.g.,

$$v_i'(r) = \frac{1}{r} \frac{\partial}{\partial r} \left[ \rho_0 r^2 u_i'(r) \right] / s(s+1).$$ (3)

Here, $\rho_0(r)$ is the density in a spherically symmetric solar model. So now it only remains to choose $u_i'(r)$ appropriately and $v_i'(r)$ will be determined by Equation (3). We choose the radial profile of $u_i'(r)$ as given by Equation (19) of Roth et al. (2002):

$$u_i'(r) = \begin{cases} \frac{4(R_{\text{top}} - r)(r - r_b)}{(R_{\text{top}} - r_b)^2} & \text{if } r_b \leq r \leq R_{\text{top}}, \\
0 & \text{otherwise}. \end{cases}$$ (4)

Here, $r_b$ and $R_{\text{top}}$ define the boundaries of region where the flow is confined. The tangential component $v_i'$ also vanishes outside these boundaries. Further, because of very small density scale height near the solar surface, $v_i'$ increases very rapidly near the top boundary and its maximum value depends on the choice of $R_{\text{top}}$. For the meridional flow, such a behavior has not been seen in the actual observed profile (e.g., Basu et al. 1999). In most of our computations, we use $r_b = 0.7 R_{\odot}$, the approximate position of the base of the convection zone, while $R_{\text{top}}$ is taken to be the top of the solar model used in this work, which is around 1.0017 $R_{\odot}$.

The radial profile of the vertical velocity and the horizontal velocity for the upper boundary at $R_{\text{top}} = R_{\odot}$ is shown in Figure 1. We wish to point out that Roth et al. (2002) have used an upper boundary at $R_{\text{top}} = 0.99 R_{\odot}$ in Equation (4). This avoids the region with small density scale height, and hence the maximum value of $v_i'(r)$ is smaller. In this work, we have used a standard solar model with the OPAL equation of state (Rogers...
of accuracy which is set by $\epsilon$ and (2) the selection rules of the perturbation operator.

In the NRSSM, the frequencies are independent of the azimuthal-order $m$, and this degeneracy is lifted by departures from spherical symmetry due to rotation or magnetic field. Helioseismic data traditionally give the splitting coefficients for all modes that are detected. These coefficients are defined by (e.g., Ritzwoller & Lavely 1991)

$$\omega_{nlm} = \omega_{nl} + \sum_q a_q^{(nl)} p_q^f(m),$$

where $\omega_{nl}$ is the mean frequency of the multiplet, $p_q^f(m)$ are the orthogonal polynomials of degree $q$, and $a_q^{(nl)}$'s are the so-called splitting coefficients.

The equations of motion for a mode $k$ with eigenfrequency $\omega_k$ for a NRSSM and a model perturbed by addition of differential rotation and/or large-scale flow can be respectively represented by

$$L_0 s_k = -\rho_0 \omega_k^2 s_k,$$

$$L_0 s_k' + L_1 s_k' = -\rho_0 \omega_k'^2 s_k',$$

where $\omega_k'$ is the perturbed frequency and $s_k'$ given by Equation (5) is the perturbed eigenfunction. Taking scalar product with $s_j$ in Equation (8) and using the notation $H_{jk'} = -\int s_j^\dagger L s_k' dV$ and the definition $L^\dagger s_k = -2i\omega_{ref}\rho_0 (\nabla \cdot \nabla) s_k$, we obtain the matrix eigenvalue equation,

$$\sum_{k \in K} \{ H_{jk'}(\omega_k'^2 - \omega_{ref}^2)\} a_k = (\omega_k'^2 - \omega_{ref}^2) a_j,$$

with eigenvalue $\lambda = (\omega_k'^2 - \omega_{ref}^2)$ and eigenvector $X_j \equiv \{a_j\}$. Here, $\omega_{ref}$ is a reference frequency which approximates $\omega_k'$. In this work, we use $\omega_{ref} = \omega_k$, the frequency of the mode being perturbed. It will be clear from Section 3.2 that the perturbation matrix $[H_{jk';nl}]$ is Hermitian for both differential rotation and poloidal flows. For example, if we consider only two modes with frequencies $\omega_1$ and $\omega_2$, then QDPT gives the following coupling matrix for the mode $\omega_2$:

$$\begin{bmatrix} H_{11} - \Delta & H_{12} \\ H_{21} & H_{22} \end{bmatrix},$$

with $\Delta = \omega_2^2 - \omega_1^2$. We are interested in the eigenvalues $\lambda$ of $[H_{ij}]$ corresponding to perturbation in $\omega_2$ which may be easily shown to be

$$\lambda \sim H_{22} - \frac{|H_{12}|^2}{H_{11} - H_{22} - \Delta}.$$ 

Let $\omega_2'$ be the modified frequency of the mode which initially has a frequency $\omega_2$. Then from Equation (11) we have

$$\omega_2' \sim \left[\omega_2^2 + H_{22} - \frac{|H_{12}|^2}{H_{11} - H_{22} - \Delta}\right]^\frac{1}{2}.$$ 

If DPT were used then the last term involving the off-diagonal term $H_{12}$ would not be present, and hence this term gives the correction arising from using the QDPT. It is clear that this correction can be large if the two modes are nearly degenerate.
If $|H_{12}| \approx |\Delta|$, then the frequency shift due to use of QDPT would be of order of $\Delta \omega_2$, which is comparable to the difference $\omega_2 - \omega_1$. On the other hand, if $|H_{12}| \ll |\Delta|$, then the frequency shift would be much less. For a typical p-mode, the spacing $\omega_2 - \omega_1$ is of order of a few $\mu$Hz. For a rotation velocity of $2000 \text{ m s}^{-1}$, the frequency shift is of the order of $450 \text{ m nHz}$, which would define the magnitude of diagonal terms. Thus, for meridional flow with an averaged velocity of $20 \text{ m s}^{-1}$, we may expect the matrix elements $H_{12}/\omega_2 \sim 4.5 \text{ nHz}$. However, in this case the diagonal elements are zero, and we need the off-diagonal element which involves cross product of two different eigenfunctions in the integration (cf., Equation (15)), and (for the same velocity) this integral would be much more than order of magnitude smaller than that in the diagonal term for rotation. Thus, the magnitude of $H_{12}$ would be an order of magnitude less, giving a frequency shift of order of $0.2 \Delta \omega_2/\omega_1$ nHz, taking the maximum value of $m$. For $l \approx 100$, this would give a shift of order of a few nHz. This is much smaller than that obtained by Roth & Stix (2008).

In the following section, we shall give the expression for the perturbation matrix elements $|H_{12}^{m_m}|$ due to rotation as well as poloidal flows to be used for solving the eigenvalue Equation (9).

3.2. Calculating the Perturbation Matrix

The Wigner–Eckart theorem (Equation (5.4.1) of Edmonds 1960) states that the general matrix element of any tensor perturbation operator can be expanded in terms of Wigner 3j symbols whose coefficients of expansion are independent of azimuthal-order $m$ and $m'$:

$$H_{n,n',\ell_1}^{m_m}(1) = (-1)^m \begin{pmatrix} l' & s & l \\ -m' & t & m \end{pmatrix} (n'l'||L_{n'}|nl).$$

The coefficient of the Wigner 3j symbol in Equation (13) due to coupling by the differential rotation is given by

$$\langle n'l'||L_{n'}|nl \rangle = 8\pi \omega_{ref}(1 - (-1)^{s+l+*}) \times \gamma_l\gamma_{l'} \int_0^{R_o} \gamma_s u_s \rho r^2 dr T_s(r),$$

where

$$T_s(r) = -\frac{1}{r} \left[ \xi' \eta + \eta' \xi - \xi \eta \right] \left[ \frac{1}{2} \eta' \eta [l(l+1) + l'(l'+1) - s(s+1)] \right] \times \Gamma_l \Gamma_{l'} \left( \begin{array}{ccc} l' & s & l \\ -1 & 0 & 1 \end{array} \right).$$

The selection rules for the Wigner 3j symbol to be non-zero are:

1. $p/2 \geq \max(l, l', s)$, where $p = l + l' + s$,
2. $m - m' + t = 0$.

The selection rule for non-trivial reduced matrix elements due to differential rotation is given by

3. $l + l' + s$ must be odd.

This means that the $n_{\ell s}$ component of differential rotation couples modes with $l = l, l \pm 2$. The set of modes that constitute the eigenstate $K$ must follow selection rules (1), (2), and (3) and the quasi-degenerate condition $|\omega_{n\ell m} - \omega_{ref}| < 100 \text{ \muHz}$. The selection rule for reduced matrix elements due to large-scale poloidal flow to be non-zero is

4. $l + l' + s$ must be even.

In this case, the set of modes in the eigenstate $K$ must follow selection rules (1), (2), and (4) and the quasi-degenerate condition as before. Note that the matrix elements for rotation are real and symmetric, whereas those for poloidal flow are imaginary and antisymmetric. The perturbation matrix $[H_{n,n',\ell_1}^{m_m}]$ is Hermitian for both cases and will have real eigenvalues.

4. RESULTS

4.1. Differential Rotation

We show in this section that QD treatment would give non-zero even coefficients $a_{2q}$ even in the absence of the centrifugal term, and their effect on $a_{2q+1}$ is indeed small as argued by Lavelle & Ritzwoller (1992). The determination of even splitting coefficients due to differential rotation is important as this may be used to remove the effect of rotation from the observed splitting coefficients to isolate the effect of the magnetic field and other large-scale flows in the observed splitting coefficients. However, if DPT is used, only the odd coefficients $a_{2q+1}$ are non-zero if we neglect the centrifugal force which is of second order in $\Omega$. Further, in this approximation only, the north–south symmetric component of rotation gives non-zero frequency shifts. Lavelle & Ritzwoller (1992) applied the QDPT to differential rotation to find that for intermediate $l$ modes ($l \sim 50$) the quasi-degenerate coupling will have little effect on the modal frequencies and can be ignored for all practical purposes.

It is easy to verify from Equation (13) that for pure rotation ($v = \Omega(r, \theta) r \sin \theta \phi$) the symmetric matrix elements are odd functions of $m$. Hence, from Equations (6) and (11), one can
infer that the perturbed frequency will have both odd and even splitting coefficients. It can be easily shown that there will be no additional effect in QDPT due to component \(w^0\) of rotation velocity as the off-diagonal elements in the resulting matrix would vanish. Thus, the leading effect of rotation arises from \(w^0\) term. We calculate this contribution by using

\[
w^0(r) = \begin{cases} 34.9 \text{ m s}^{-1} & \text{if } r \geq 0.7 \text{ R}_{\odot} \\ 0 & \text{otherwise.} \end{cases} \quad (19)
\]

This value is chosen such that the DPT gives a value of \(a_3\) close to the observed value for most modes trapped in the convection zone. The QDP treatment modifies the odd splitting coefficient \(l_{a_1}\) by up to 0.4 nHz (see Figure 2(a)) and \(l_{a_2}\) by up to 0.2 nHz, which are a couple of orders of magnitude smaller than the typical errors in these coefficients. In Figure 2(b), we plot the splitting coefficient \(l_{a_2}\) for all the modes with frequency less than 4.5 mHz and \(l \geq 10\), against \(r'_f/R_s\), the lower turning point of the mode.

The calculation has been done using an equation similar to Equation (11) but for all the modes in the neighborhood of radius 100 \(\mu\)Hz about the central mode. The mathematical details of the calculation has been discussed in Section 3.2 (see Equations (14) and (15)). The maximum value of \(l_{a_2}\) is 30 nHz which may be significant depending upon the magnitude of the large-scale flow or magnetic field perturbations. Note that the effect of centrifugal force on \(l_{a_2}\) also happens to be of the same order (see Figure 5 of Antia et al. 2000). We have emphasized earlier that in order to spot the signatures of the large-scale flows and magnetic fields, we need to remove the effect of rotation on \(a_2\).

It has been found from Doppler measurements that there exists a hemispheric asymmetry in surface rotation having an angular dependence \(\partial Y^2\) (e.g., Hathaway et al. 1996). This component has an amplitude of \(w^0 = -7.8 \pm 0.3 \text{ m s}^{-1}\), which indicates that the southern hemisphere was rotating slightly faster than the northern hemisphere during the period of the data. We have also calculated the splitting coefficient \(l_{a_2}\) due to north–south asymmetry assuming that it is constant throughout the convection zone, and they happen to be small with \(l_{a_2} \lesssim 0.1 \text{ nHz}\).

4.2. Meridional Circulation

In this section, we investigate theoretical frequency shifts due to meridional circulation. We start with \(s = 2, t = 0\), which is the dominant component in the observed meridional flow near the surface. In our analysis, we have included all \(f\)- and \(p\)-modes with frequency less than 4.5 mHz and \(l \geq 10\). We consider a meridional circulation with one cell in each hemisphere and maximum horizontal velocity at the surface equal to 30 m s\(^{-1}\) which is consistent with the Doppler measurements and ring diagram analysis. This gives \(u_0 = 9 \text{ m s}^{-1}\) in Equation (4).

From Equations (17) and (18), we see that the coupling matrix for poloidal flows is Hermitian with zero diagonal elements. This means that degenerate perturbation treatment cannot be applied to calculate the frequency shifts, and it becomes essential to use the QDPT, unlike for rotation where QDPT just provides a second-order correction. In the presence of zero diagonal elements of the matrix \([\mathcal{H}_0]\), Equation (11) for the frequency shift for coupling between two modes simplifies to

\[
\delta \nu = \frac{\omega'_1 - \omega_2}{2\pi} \sim \frac{\mathcal{H}^2_{12}}{4\pi \omega_2 \Delta}. \quad (20)
\]

It may be noted that the sign of the frequency shift for the central multiplet with \(\omega_{ref}\) depends on the sign of \(\Delta = \omega_2^2 - \omega_1^2\), where \(\omega_1\) is the nearest mode to the central frequency. If there are many modes with frequency close to frequency of the central mode, then we can expect these contributions to be added. In this case, if the sign of frequency differences is not the same, the terms will partially cancel each other. If for some pair of modes which satisfy the selection rules, the frequency difference is very small, then their frequencies would be shifted significantly.

In Figure 3, we show an example of the frequency shift \(\delta \nu(m)\) of the multiplet \((n, l) = (1, 292)\), which was also considered by Roth & Stix (2008). The Wigner 3j symbol with \(t = 0\) (implying \(m' = m\) in Equation (13)) is an odd function of \(m\) and so \(\delta \nu(m) \propto H^2_{12}\), is symmetric about \(m = 0\) for all the multiplets. Another important point to note from Equation (20) is that \(\delta \nu \propto u_0^3\). This has also been verified by numerical calculations. The maximum shift we get for this multiplet from
our calculations is $-8.5 \text{ nHz}$ in contrast to Figure 2 of Roth & Stix (2008) where they obtain a maximum shift of $0.1 \text{ \mu Hz}$. On comparison with their Equation (15), we find a difference of a factor of 2 in the first term in the expression of $\Delta \nu$. Even after using their version of Equation (15) in our calculations, we were not able to reproduce the large shifts reported by them. The reason for this discrepancy is not clear. One possibility is the normalization of velocity amplitude. The velocity decreases rapidly with height, and if the normalization is applied at a higher level, then the entire velocity profile will be scaled up. In these calculations, we have normalized $u_0$ by comparing it with the surface velocity. We find that the peak value of $v'(r_s)$ in Figure 1 occurs a little below the surface. For instance, for a flow with $s = 2$ and a velocity of $30 \text{ m s}^{-1}$ near $r = R_⊙$, this peak in $v'(r_s)$ happens to be $72 \text{ m s}^{-1}$ at $r = 0.996 R_⊙$. However, in reality such a rapid increase in $v_s$ has not been found from local helioseismology which can probe up to a depth of $0.96 \text{ R}_⊙$. Hence, in our opinion the theoretical frequency shift calculated using such a radial profile of $v'(r_s)$ is an upper limit to the actual shift. Ideally, we should normalize $u_0$ to get the maximum value of the horizontal velocity to be $30 \text{ m s}^{-1}$. That will bring down the splitting coefficients by a factor of 4 or more. On the other hand, if we normalize the velocity further up then for the same value of velocity the calculated splitting coefficients would go up. However, that is not realistic as in that case the maximum velocity would be much larger than the observed value. In Figures 4(a) and (b), we plot the frequency shifts averaged over $m$ for each multiplet, whereas Figure 4(c) shows the splitting coefficient $I_2$ calculated for all the multiplets.

We consider only even values of $s$ so that the meridional flow does not have cross-equatorial components. As we increase $s$, selection rules allow more and more multiplets in a radius of $100 \text{ \mu Hz}$ to couple, and there are many more instances of “near degeneracy,” so that the $\Delta \nu \propto 1/\Delta$ in Equation (20) for some multiplets becomes quite large. For example, the multiplets (17, 56) and (16, 64) which have a frequency difference of $-0.06 \text{ \mu Hz}$ can be coupled by the meridional flow with $s = 8$ to give $I_2 \sim 480 \text{ nHz}$. In these calculations, we have used $u_0 = 100 \text{ m s}^{-1}$. In Figure 3, we have plotted the frequency shifts for the pair of multiplets (15, 34) and (14, 40) also considered by Roth et al. (2002), coupled by the same flow. The result for all multiplets with frequency less than $4.5 \text{ \text{ mHz}}$ and $l \geq 10$ is shown in Figures 5(a) and (b). It is to be noted from Figure 5(b) that only some multiplets have $I_2$ larger than the errors in $I_2$ for those modes in observational data. It is only these multiplets which we may hope to detect in the observations. In this case, we have taken the errors from the GONG data set centered at 2002 November 19. We present comparisons with GONG as well as MDI data in Section 4.4. Apart from the nearly degenerate modes the splitting coefficients of other modes are larger than that found for $s = 2$ with a similar value of horizontal velocity near the surface. It may be noted that for large values of $s$, the maximum value of horizontal velocity would be quite different from $v'(r_s)$ because of the additional factor arising from the gradient of spherical harmonics (cf. Equation (1)). In the case, where we normalize $u_0$ such that $v'(r_s) = 30 \text{ m s}^{-1}$, we also obtain an increase with $s$ in the splitting coefficients as reported by Roth & Stix (2008). This increase is not seen if the maximum value of the horizontal velocity near the surface is normalized to $30 \text{ m s}^{-1}$. Of course, observations near the solar surface do not show such large magnitudes for these higher order components of meridional flow. If realistic amplitudes are used then the effect will be negligible.

### 4.3. Giant Convection Cells

In this section, we calculate the splittings due to poloidal flows with angular dependences that depend on longitude, $\phi$. We consider the flow given by spherical harmonics $Y_n^m(\theta, \phi)$ and $Y_n^m(\theta, \phi)$ (also called sectoral rolls or banana rolls). This calculation is little more involved than that in Section 4.2 as a non-zero $t$ allows coupling between different $m$ and $m'$ in Equation (13). Hence, it becomes important to take into account the effect of rotational splitting on $\Delta$ before calculating the effect of these kind of poloidal flows. It is easy to see from the properties of the Wigner $3j$ symbols that the $s = 8, t = 8$ flow couples the mode $(n, l, m)$ with $(n', l', m \pm 8)$. Hence, the difference of the square of frequencies $\Delta$ is no longer independent of $m$ unlike in Section 4.2.

Let us consider the perturbation in the frequency of the multiplet (15, 34). The nearest multiplet to this happens to be...
Figure 6. (a) Solid line gives the frequency shift because of interaction between modes (15, 34, m) and (14, 40, m − 8) due to the flow with an angular dependence $Y_8^0(\theta, \phi)$. The dashed line is the frequency shift due to interaction between modes (15, 34, m) and (14, 40, m + 8). The total shift in frequency for the mode (15, 34) is the sum of the solid and the dashed lines. The frequency shift for the mode (14, 40) is given by the dotted line. (b) $\Delta m, m \rightarrow m − 8/2\nu$ (solid line) for the coupling (15, 34, m) $\leftrightarrow$ (14, 40, m − 8); and $\Delta m, m + 8/2\nu$ (dashed line) for the coupling (15, 34, m) $\leftrightarrow$ (14, 40, m + 8). These values are in $\mu$Hz.

Figure 7. (a) Solid line gives the frequency shift because of interaction between modes (18, 61, m) and (17, 69, m − 8) due to the flow with an angular dependence $Y_8^0(\theta, \phi)$. The dashed line is the frequency shift due to interaction between modes (18, 61, m) and (17, 69, m + 8). The total shift in frequency for the mode (18, 61) is the sum of the solid and the dashed lines. The frequency shift for the mode (17, 69) is given by the dotted line. (b) $\Delta m, m \rightarrow m − 8/2\nu$ (solid line) for the coupling (18, 61, m) $\leftrightarrow$ (17, 69, m − 8); and $\Delta m, m + 8/2\nu$ (dashed line) for the coupling (18, 61, m) $\leftrightarrow$ (17, 69, m + 8). These values are in $\mu$Hz.

Figure 8. Asymmetric shift in frequency of the multiplet (a) (15, 34) (solid line) upon coupling with neighboring multiplets (14, 40) and (16, 28) in the presence of large-scale flow $s = 8$, $t = 4$. The dashed-dotted line is the shift for the mode (14, 40) due to coupling with (15, 34) and (16, 58). (b) Same as (a) but for (18, 61) (solid line) upon coupling with neighboring multiplets (17, 69) and (19, 55) in the presence of large-scale flow $s = 8$, $t = 4$. The dashed-dotted line is the corresponding shift in (17, 69) due to coupling with two nearest neighbors.

(14, 40) with a frequency difference of 4.12 $\mu$Hz, and the next one is (16, 28) with the frequency difference 19.01 $\mu$Hz. This is the same mode shown by Figure 3 of Roth et al. (2002). We use the rotational splittings calculated from temporally averaged rotation rate as inferred from the GONG data to calculate $\Delta$. The result of our calculation including only the nearest multiplet (14, 40) has been shown in Figure 6. The asymmetric frequency shift shown by the solid line in Figure 6(a) denotes the change in the frequency of the mode (15, 34, m) due to coupling with the mode (14, 40, m − 8), whereas the dashed line denotes the change due to coupling with (14, 40, m + 8). As shown in Equation (20), a crucial component of the shift comes due to the difference of the squared frequencies $\Delta_{m, m'}$ of the modes coupling according to the QDPT condition and the selection rules. The corresponding $\Delta_{m, m−8}$ and $\Delta_{m, m+8}$ are plotted in Figure 6(b). The total shift is given by the sum of the solid and dashed curves. The couplings within a multiplet, i.e., (\(n, l, m\)) $\leftrightarrow$ (\(n, l, m \pm 8\)) happen to be zero because of the antisymmetry of the matrix elements. The values of splitting coefficients $\lambda_1$ and $\lambda_2$ for the mode (15, 34) are calculated to be −12.2 nHz and −5.7 nHz and that for (14, 40) are 11.1 nHz and 5.3 nHz, respectively.

We repeat this calculation for all modes with frequency less than 4.5 mHz and find that some of the multiplets have quite high values of $a_2$ (see Figure 9(b)). For example, the multiplet (18, 61) which couples with the nearest multiplet (17, 69) with a frequency difference of −3.4 $\mu$Hz. In Figure 7(a), we have plotted $\delta \nu(m)$ versus $m$ for the coupling (18, 61, m) $\leftrightarrow$ (17, 69, m − 8) which seems to have a discontinuity at $m = −22$. This is due to the fact that $\Delta_{m, m−8}$ for this coupling also changes sign at $m = −22$. Note that two modes (18, 61, m) and (17, 69, m − 8) having a mean frequency difference of 4.12 $\mu$Hz become “nearly” degenerate at $m = −22$. This jump presumably gives rise to high values of splitting coefficients. The other coupling (18, 61, m) $\leftrightarrow$ (17, 69, m + 8) is smooth and so is the $\Delta_{m, m+8}$ as shown in Figure 7(b) by the dashed line. We also calculate the total shift experienced by the multiplet (18, 61) while coupling with three nearest modes (17, 69), (19, 55), (19, 53) and find no significant difference with Figure 7(a). We have also shown a calculation for the asymmetric shifts for the multiplet (14, 34) and (14, 40) coupled due to a flow varying as $Y_8^0(\theta, \phi)$ in Figure 8.

The asymmetric shift due to these couplings gives rise to non-zero odd splitting coefficients in addition to the even coefficients. Since rotation inversions neglect this contribution, the result may not be correct if the giant cells have significant amplitudes. The effect of giant cells on rotation inversions can be estimated by performing rotation inversion using odd splitting coefficients from these calculations. The resulting rotation rate should be subtracted from the actual rotation inversions. It is likely that if these multiplets are used for helioseismic inversion using the rotation kernel, they will give rise to distortion in the inverted rotation profile. The effect on the odd splitting coefficients, $a_1$.
using the same smoothing as used for inverting real data (e.g., modes for inversions using a regularized least squares method
July 23, (6) 2007 September 18 and (7) 2008 July 2. For 29, (2) 1997 June 18, (3) 2000 September 18, (4) 2002 November 19, (5) 2004 with the date about which the 108-day GONG month is centered: (1) 1995 June Notes.

Data set centered about 2002 November (see Table 1).

Figure 9. (a) $a_1/\sigma_1$ and (b) $a_2/\sigma_2$ and (c) $a_3/\sigma_3$ as a function of lower turning point radius $r_l$ for the $Y^2(\theta, \phi)$ kind of flow with $u_0 = 100$ m s$^{-1}$. The $\sigma_{1,2,3}$ are the errors in the corresponding observational splitting coefficients from the GONG data set centered about 2002 November (see Table 1).

![Figure 9](image_url)

| Data set | $a_1/\sigma_1$ | $a_2/\sigma_2$ | $a_3/\sigma_3$ | CL | $a_1/\sigma_1$ | $a_2/\sigma_2$ | $a_3/\sigma_3$ | CL |
|---------|---------------|---------------|---------------|----|---------------|---------------|---------------|----|
| 1       | 14.7          | 16.9          | 22.6          | 1.3 | 3.7           | 14.8          | 21.9          | 1.5 |
| 2       | -8.7          | 14.9          | 22.2          | 1.5 | -12.4         | 14.0          | 21.3          | 1.5 |
| 3       | -23.4         | 13.2          | 28.1          | 2.1 | 4.7           | 11.7          | 24.8          | 2.1 |
| 4       | -24.5         | 14.6          | 21.7          | 1.5 | -21.8         | 14.3          | 22.6          | 1.6 |
| 5       | -30.2         | 13.8          | 29.7          | 2.1 | -17.7         | 12.3          | 24.2          | 2.0 |
| 6       | -1.3          | 14.5          | 22.3          | 1.5 | -12.0         | 14.3          | 21.8          | 1.5 |
| 7       | -30.2         | 13.8          | 29.7          | 2.1 | -25.2         | 12.0          | 23.9          | 1.9 |
| 8       | -28.3         | 14.2          | 22.0          | 1.5 | -5.8          | 13.9          | 22.0          | 1.5 |
| 9       | -5.3          | 13.1          | 27.3          | 2.0 | -4.4          | 11.8          | 24.1          | 2.0 |
| 10      | -0.4          | 12.3          | 22.3          | 1.8 | -22.1         | 12.8          | 22.2          | 1.7 |
| 11      | -0.6          | 10.7          | 28.1          | 2.6 | -4.5          | 10.4          | 26.0          | 2.5 |
| 12      | -13.2         | 14.2          | 22.2          | 1.6 | -27.0         | 13.5          | 21.8          | 1.6 |
| 13      | -27.4         | 12.7          | 28.9          | 2.2 | -0.6          | 11.4          | 24.6          | 2.2 |

Table 1: Comparison of Observed $l\sigma_2^h$ with Theory for Flows with an Angular Dependence $Y^2(\theta, \phi)$

| Data set | $l\sigma_2^h < -10$ nHz | $l\sigma_2^h > 10$ nHz |
|----------|--------------------------|------------------------|
| 1        | 17.7 / 16.9 / -22.6 / 1.3 | 3.7 / 14.8 / 21.9 / 1.5 |
| 2        | -8.7 / 14.9 / -22.2 / 1.5 | -12.4 / 14.0 / 21.3 / 1.5 |
| 3        | -23.4 / 13.2 / -28.1 / 2.1 | 4.7 / 11.7 / 24.8 / 2.1 |
| 4        | -24.5 / 14.6 / -21.7 / 1.5 | -21.8 / 14.3 / 22.6 / 1.6 |
| 5        | -30.2 / 13.8 / -29.7 / 2.1 | -17.7 / 12.3 / 24.2 / 2.0 |
| 6        | -1.3 / 14.5 / -22.3 / 1.5 | -12.0 / 14.3 / 21.8 / 1.5 |
| 7        | -30.2 / 13.8 / -29.7 / 2.1 | -25.2 / 12.0 / 23.9 / 1.9 |
| 8        | -28.3 / 14.2 / -22.0 / 1.5 | -5.8 / 13.9 / 22.0 / 1.5 |
| 9        | -5.3 / 13.1 / -27.3 / 2.0 | -4.4 / 11.8 / 24.1 / 2.0 |
| 10       | -0.4 / 12.3 / -22.3 / 1.8 | -22.1 / 12.8 / 22.2 / 1.7 |
| 11       | -0.6 / 10.7 / -28.1 / 2.6 | -4.5 / 10.4 / 26.0 / 2.5 |
| 12       | -13.2 / 14.2 / -22.2 / 1.6 | -27.0 / 13.5 / 21.8 / 1.6 |
| 13       | -27.4 / 12.7 / -28.9 / 2.2 | -0.6 / 11.4 / 24.6 / 2.2 |

Notes. The numbers in the GONG data set have the following correspondence with the date about which the 108-day GONG month is centered: (1) 1995 June 29, (2) 1997 June 18, (3) 2000 September 18, (4) 2002 November 19, (5) 2004 July 23, (6) 2007 September 18 and (7) 2008 July 2. For $t = 4, u_0 = 100$ m s$^{-1}$ whereas for $t = 8, u_0 = 50$ m s$^{-1}$. Note that for each $t$ the group I of multiplets correspond to $a_2^h < -10$ nHz whereas the group II is for $a_2^h > 10$ nHz. CL is the confidence level on the upper limit of $a_0$.

and $a_3$ is shown in Figures 9(a) and (c) in terms of observational errors in those multiplets. We use the theoretically calculated $a_1$ and $a_3$ to perform a 1.5d helioseismic rotation inversion using the formulation due to Ritzwoller & Lavely (1991). We use 1.5d inversion as we have calculated only two splitting coefficients, $a_1$ and $a_3$. For this purpose, we use only those modes which are present in a GONG data set and use the errors in those modes for inversions using a regularized least squares method using the same smoothing as used for inverting real data (e.g., Antia et al. 1998). The results are shown in Figure 10, which shows the resulting $w_1^2(r)$ and $w_2^2(r)$ in terms of the estimated errors. Because of the presence of modes with large $a_1, a_3$ with $r_l \approx 0.7 R_\odot$, there are some oscillations in this region, but their amplitudes are less than the estimated errors. Thus, the effect on rotation inversion may not be significant even when the giant cells have velocity of 100 m s$^{-1}$. With increasing velocity, the effect would be larger, and it may be possible to detect this effect as it will give an oscillatory signal in rotation inversion at these depths. This has been pointed out by Roth et al. (2002). They find a much larger effect as their estimate of frequency shift is generally larger than what we find. In real data, the effect may not be as dramatic as shown by Figure 8 of Roth et al. (2002) since many of the modes with large shifts in $a_{2q+1}$ would yield large residuals in inversions, and any reasonable inversion would eliminate such modes. In our calculation, we found residuals going to about $4 \sigma$ level and did not eliminate any modes. Thus, it is difficult to put any limits on magnitude of flows using inversion results.

The even coefficient $a_2/\sigma_2$ is also shown in Figure 9(b). If we find some multiplets with $0.6 < r_l/R_\odot < 0.8$ having magnitudes larger than the errors in observations as in Figure 9(b), we should be able to throw light on the flow velocities in giant cells. We shall look for such features in Section 4.4.

4.4. Comparison with Observed Data

We use seven different data sets from the GONG observations, covering the late descending part of cycle 22 to end of cycle 23, i.e., from 1995 June to 2008 November. We choose only those modes for which $a_1$ is available in the data sets. Since the different data sets can have different set of modes we repeat the calculation for all these data sets. Most of the contribution to observed splitting coefficients appear to come from near surface effects, while we are interested in modes with turning point close to the base of the convection zone. Thus, we first separate the surface effects in the data by fitting a cubic spline in terms of frequency to $I_{nl}a_{2q}/I_{nl}Q_{1k}$ versus frequency. Here, $I_{nl}$ is the mode inertia of the mode (e.g., Christensen-Dalsgaard 2002), while $I_{nl}Q_{1k}$ is the mode inertia for the $l = 0$ mode at the same
negative values of the running mean tends to average modes with positive and to error bars. The main problem with this approach is that region 0 considering comparing the size of the valley and the hump in the get an idea of the magnitude of giant cell flows, one can on 2002 November 19 and 2007 September 18, respectively. To averaging for the GONG 108-day averaged data sets centered

\[
Y_{l,2}^{2}(\theta, \phi) \quad \text{flow;} \\
Y_{l,2}^{3}(\theta, \phi) \quad \text{flow;} \\
Y_{l,2}^{4}(\theta, \phi) \quad \text{flow.}
\]

The solid line with errorbars show the observed data points. To avoid congestion only a few representative errorbars are shown. frequency, while \( Q_{ab} \) is a geometric factor as defined by Antia et al. (2001). After subtraction of the surface effect from the observed splitting coefficients, all the multiplets are sorted in the order of increasing \( r_{l} \) and then an error weighted 100 point average is applied. In Figures 11(a) and (b), we show the result of the averaging for the GONG 108-day averaged data sets centered on 2002 November 19 and 2007 September 18, respectively. To get an idea of the magnitude of the giant cell flows, one can consider comparing the size of the valley and the hump in the region 0.6 < \( r_{l}/R_{\odot} \) < 0.8 in the theoretical curve with that in the observation. It is difficult to make out if the expected signature is present in the observed data sets. In most cases, no significant hump is seen in this region, though in some cases like in Figure 11(a) there is some hint of hump which is comparable to error bars. The main problem with this approach is that the running mean tends to average modes with positive and negative values of \( a_{2} \), thus reducing the significance of the results. Thus, it may be better to separate out these modes to look for the signal.

We have said earlier that only way to put an upper limit on the flow velocity is to look for signatures of nearly degenerate modes in the data. In order to do that we divide the multiplets from the theoretical calculations in two groups: one for which \( la_{2} > 10 \text{ nHz} \) and the other with \( la_{2} < -10 \text{ nHz} \). Amongst 3700 theoretically evaluated multiplets, we find only about 250 multiplets satisfying either criteria. Then, we further search for these multiplets in the GONG data sets and perform an averaging over the available multiplets to get an estimate of \( a_{2} \) from theory and error \( \sigma_{2} \) in \( a_{2} \) from the observations. Only about 10–12 of the 250 multiplets are usually found to be present in the GONG data sets. We find that for all the data sets, \( |la_{2}| \sim 22 \text{ nHz} \) for theoretically calculated coefficients, whereas the error in \( la_{2} \) (in observed set) is \( \sim 14 \text{ nHz} \) for the \( Y_{l,2}^{2}(\theta, \phi) \) flow with \( u_{0} = 100 \text{ m s}^{-1} \). The corresponding values for the \( Y_{l,2}^{3}(\theta, \phi) \) flow with a \( u_{0} = 50 \text{ m s}^{-1} \) are 28 nHz and 13 nHz, respectively. Since the observed data do not show these values (in both > 10 nHz and < -10 nHz sets) we can only put an upper limit on velocities in such flows. For this purpose we compare the \( a_{2} \) values from theory with observational errors to calculate the confidence level of the upper limit. So we can say that up to a 1.5\( \sigma \) level we can rule out the \( Y_{l,2}^{2}(\theta, \phi) \) dependent flow with \( u_{0} > 100 \text{ m s}^{-1} \), and the existence of the \( Y_{l,2}^{3}(\theta, \phi) \) flow with \( u_{0} > 50 \text{ m s}^{-1} \) can be eliminated with a confidence level of \( 2\sigma \). We have also performed some calculations with \( Y_{l,2}^{3}(\theta, \phi) \) angular dependence and can say with a 1.5\( \sigma \) confidence level that such flows also cannot have \( u_{0} > 70 \text{ m s}^{-1} \). Our results for multiplets in all the seven GONG data sets, divided into two groups according to the theoretical value of \( la_{2} \), are represented in Table 1. MDI data also give similar results. By looking at detailed peak profiles of these abnormal modes in observed data, it may be possible to put more stringent limits on velocity of such flows.

**Figure 11.** \( la_{2} \) as a function of turning point radius \( r_{l} \) after averaging over 100 nearest points in \( r_{l} \) for GONG data set centered around (a) 2002 November 19 and (b) 2007 September 18. The dotted line correspond to the \( Y_{l,2}^{2}(\theta, \phi) \) flow; dashed to \( Y_{l,2}^{3}(\theta, \phi) \); thick solid to \( Y_{l,2}^{4}(\theta, \phi) \). The solid line with errorbars show the observed data points. To avoid congestion only a few representative errorbars are shown.

5. CONCLUSIONS

In this study, we have calculated the effect of differential rotation, the meridional circulation and the giant cell flows on \( p \)-mode frequencies using QDPT. For toroidal flows like differential rotation, the QDPT provides only a second-order correction to the eigenfrequencies, whereas for the poloidal flows it becomes necessary to use the quasi-degenerate treatment since the diagonal terms of the perturbation matrix vanishes. We agree with Lavelly & Ritzwoller (1992) that the effect of rotation on the odd coefficients is negligible and hence using the degenerate theory is sufficient. However, the even splitting coefficients \( a_{2} \) are about one-half of the errors in observations and they might not be negligible while calculating the \( a_{2} \) for other effects like the magnetic field. This contribution is comparable to the effect of centrifugal term.

The frequency shift due to N–S antisymmetric component of rotation rate vanishes in the degenerate perturbation treatment, and it is necessary to use QDPT. We find that with realistic magnitude of velocities in the antisymmetric component, the computed splitting coefficients are very small and this effect may be neglected.

For a meridional circulation with maximum horizontal velocity at the surface of 30 m s\(^{-1}\) and one cell per hemisphere, we find that \( la_{2} \sim 8.5 \text{ nHz} \) for multiplets with turning points \( r_{l} \) near the surface. Because of our choice of velocity field, the horizontal velocity increases rapidly with depth near the surface. This behavior is not seen in the Sun. In that case, it may be more meaningful to choose a velocity profile with a maximum value of 30 m s\(^{-1}\), which occurs a little below the surface. In that case, the splitting coefficients would need to be reduced by a factor of 4 or more, making them even smaller. The mean frequency shift due to the meridional flow is found to be up to 12 nHz, which is much smaller than what is found by Roth & Stix (2008). The reason for this discrepancy is not clear. This could be due to differences in where the velocity is normalized to a specified value.

In any case, \( la_{2} \) is very small for multiplets with \( r_{l}/R_{\odot} < 0.9 \) coupled with single-celled meridional circulation. This value is less than roughly one-seventh of the errors in the observed splitting coefficients. We also do not find any change with increasing the number of cells in the radial direction or by changing the depth of penetration of the meridional circulation while keeping the surface amplitude constant. This makes it impossible to comment on the return flow believed to be present at the base of the convection zone. When we increase the number of cells per hemisphere, the selection rules allow
more multiplets to couple with each other. In the process, some of the nearly degenerate multiplets combine to give rather large values of $\Delta \omega_2$. Interestingly, most of these nearly degenerate modes have turning points in the range $0.6 R_\odot - 0.8 R_\odot$. These purely meridional flows give frequency shifts which are symmetric about $m = 0$ and only give non-zero values for even splitting coefficients. The observed amplitude of these higher order components near the surface is rather small and with such amplitudes the effect is expected to be small. It appears that in general the magnitude of splitting coefficients due to $u_0 = 9$ m s$^{-1}$ with $s = 2$ is smaller than that with $u_0 = 100$ m s$^{-1}$ and $s = 8$, even after excluding modes with nearly degenerate frequencies. This could be due to a larger maximum horizontal velocity as well as larger radial velocity (for $s = 8$) or some geometric factors arising in the calculations.

In addition to this, we have also calculated the asymmetric frequency shifts caused due to giant convection cells with angular flow profiles given by $Y_{\ell}^{\pm}(\theta, \phi)$ and $Y_{\ell}^{\mp}(\theta, \phi)$. Our results differ significantly from earlier works of Roth et al. (2002) and Roth & Stix (2008), who find the effect of poloidal flows on $p$-mode frequencies to be an order of magnitude larger than what we find. The asymmetric shifts also contribute to the theoretically calculated odd coefficients. We have performed a 1.5 day rotation inversion on these coefficients to detect any discernible feature in the inverted profile. However, since the magnitude of the features in the inverted profile is smaller than the inversion errors we conclude that giant cells with $u_0 \leq 100$ m s$^{-1}$ do not have much effect on the rotation inversion.

Finally, we have analyzed some of the GONG data sets covering solar cycle 23 to look for possible evidence of giant cells. We do not find any signal of these flows in selected modes with large splitting coefficients, and we can put upper limits on velocities of such cellular flows. From our analysis one can say that the existence of giant convection cells of $u_0 = 40$ m s$^{-1}$ can be ruled out with a confidence level of 1.5$\sigma$, whereas cells with an angular dependence $Y_{\ell}^{\pm}(\theta, \phi)$ cannot have vertical velocities $>50$ m s$^{-1}$ with a confidence level of 2$\sigma$. It is important to remember here that the GONG data sets are averaged over 108 days, while MDI data sets over 72 days. This may be somewhat larger than the lifetimes of giant cells and hence the signal may be averaged out. We can look at shorter data sets, but in that case the errors would be larger. Further, the convective cells are not expected to have smooth velocity profiles of the form used by us over the entire solar surface. This would also reduce the contribution to splitting coefficients.

Finally, we would like to point out that while the DPT can be easily adopted for inversion because of the linearity of effect and the form of perturbation which can be represented by appropriate kernels. In contrast, the effect of QDPT is nonlinear in velocity and further is mainly governed by frequency differences between nearly degenerate modes, rather than the matrix elements which depend on velocity profile. Hence, it is difficult to use these shifts in inversions. For example, the main effect of flow with $s = 8$ is felt in modes with turning points close to $0.7 R_\odot$, which happens to be close to the lower limit of the region in which our flows were localized. But we have verified that this is just a coincidence by performing calculations with different lower boundary. The same set of modes is affected irrespective of lower boundary, because it is these modes that have small frequency differences. This is only determined by the value of $s$ which determines the range of $l$ values that are coupled. Thus, various types of perturbations with the same coupling will give large splittings in these modes, and it is difficult to identify the actual source of perturbation.

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