Quantum Resistive Transition
in type II Superconductors under Magnetic Field

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It is shown that, within a Ginzburg-Landau (GL) formalism, the superconducting fluctuation is insulating at zero temperature even if the fluctuation dynamics is metallic (dissipative). Based on this result, the low temperature behavior of the $H_{c2}$-line and the resistivity curves near a zero temperature transition are discussed. In particular, it is pointed out that the neglect of quantum fluctuations in data analysis of the dc resistivity may lead to an underestimation of the $H_{c2}$ values near zero temperature.
It is well understood now that thermal fluctuations of the superconducting order parameter $\psi$ in a type II superconductor destroy the second order transition at the mean field $H_{c2}$ line and describe the resistive broadenings in the resulting vortex liquid (VL) regime below the $H_{c2}$ line. Since the $H_{c2}$ line, more or less, approaches zero temperature ($T=0$) and, at such high fields, the fluctuations in the VL regime will be safely described within the lowest Landau level (LL) where the phase of $\psi$ cannot be separated from its amplitude, one needs to take account of quantum fluctuations of $\psi$ within a Ginzburg Landau (GL) model appropriate in such low temperatures and high fields. To our best knowledge, quantum fluctuation effects on the dc linear transport have not been studied in such a high field GL region. At present, there are many experimental situations closely related to this theoretical subject.

In this letter, we point out that the GL superconducting fluctuation in a disordered phase at $T=0$ is insulating, even when its dynamics is metallic (dissipative), and possibly, for arbitrary dynamics. Namely, irrespective of characters of the quantum fluctuations in static phenomena, the dc diagonal conductivity $\sigma_{xx}$ in a $T=0$ disordered phase does not have any fluctuation enhancement $\sigma_{s,xx}$ even below the $H_{c2}$-line. This is in contrast to the case with purely thermal fluctuations where fluctuation effects on static quantities are consistent with those on dc linear dissipations. Therefore, in the (quantum) VL regime at nonzero temperatures where the quantum fluctuations significantly mask the reduction of the thermal fluctuation accompanying cooling, the dc diagonal resistivity below the $H_{c2}$-line has to (again) show a broadening which becomes more remarkable upon cooling. Consequently, in the VL regime at such low temperatures, the simple vortex dynamics based on a force balance is not applicable even qualitatively.

First, we present the derivation of the $T=0$ result $\sigma_{s,xx} = \sigma_{xx} - \sigma_{n,xx}$ (the normal conductivity) = 0, and its relevance to real experiments will be discussed at the end of the paper. As is seen below, the analysis leading to this result is, in a sense, transparent mathematically. For simplicity, we consider the usual time-dependent GL equation of $\psi$ (in the dimensionless unit) even at $T=0$ and in strong fields; $(\gamma + i\gamma')\partial\psi/\partial \tau = -[\varepsilon \psi + (-i\partial - A)^2 \psi + g|\psi|^2\psi] (+$ noise terms), where the constants $\gamma$ and $g$ are positive, and $A$ is the gauge field. Alternatively, we can work in the following Euclidean action (with $\hbar = 1$)

$$S = \int_r \left[ \beta^{-1} \sum_\omega (\gamma|\omega| + i\gamma'\omega)|\tilde{\psi}_\omega|^2 + \int_0^\beta d\tau \left( \varepsilon|\psi|^2 + |(-i\partial - A)\psi|^2 + \frac{g}{2}|\psi|^4 \right) \right], \quad (1)$$

with $\psi(\tau) = \beta^{-1} \sum_\omega \tilde{\psi}_\omega e^{-i\omega\tau}$ and $Z = \text{Tr}_\psi \exp(-S)$. Here $\omega$ is Matsubara frequency, and $\tau$ the imaginary time. Extending our proof to more realistic models is straightforward and, together with quantum fluctuation effects on statics and detailed calculations relevant to resistivity data, will be given elsewhere. Below, fluctuation contributions to be absorbed into the normal conductivity and unrelated to the GL description are assumed to be negligible, because they have nothing to do with the vortex flow behavior existing away from the quantum VL regime and hence, should not be essential to the ordering even at $T=0$. Further, we assume, as usual, that a resummation scheme of Feynman graphs, such as the perturbative renormalization group, can accurately describe fluctuation effects on the linear responses. Then, the dissipative $\psi$-dynamics is indispensible to obtaining...
nonzero superconducting contributions to conductivities existing at nonzero temperatures. Hence, one needs knowledge of the dissipative quantum fluctuations at \( T = 0 \) when focusing on nonzero temperatures at which a localization effect on the \( \psi \)-dynamics may still be negligible.

At nonzero fields, the order parameter field \( \psi \) is Landau-quantized: \( \tilde{\psi}_\omega = \sum_{n,K} \phi_{n,K}(\omega) \)
\( u_{n,K}(r) \), where \( n \) is the LL index, and other quantum numbers are specified by \( K \), and \( u_{n,K} \) is the LL eigen-basis. Correspondingly, we have the renormalized fluctuation propagators \( G_{n,K}(\omega) = |\phi_{n,K}(\omega)|^2 \). Hereafter, the index \( K \) playing no role in the following discussions need not be denoted below. We can assume the following form of \( G_n \) in the VL regime at \( T = 0 \)
\[
G_n(\omega) = \frac{1}{\gamma|\omega| + i\gamma \omega + d_n} = \int_{-\infty}^{\infty} dx \frac{\rho_n(x)}{x - i\gamma \omega},
\]
where \( \rho_n(x) = |a_n|^2 x/(\pi|x - ia_n d_n|^2) \) with \( a_n = 1/(1 - i\gamma_n/\gamma) \). By using this spectral representation (following the second equality) in each order of the perturbation series and noting that \( \rho_n(x) \) is real, one can verify that there have no self energy corrections to the dissipative term \( \sim |\omega| \) in (1). Hereafter we set \( \gamma = 1 \) below. The mass term in the static part \( d_n^{-1} \) of \( G_n \) is always renormalized to a positive value in the VL regime, and the coefficient of the ‘Hall’ term \( \sim i\omega \) is also renormalized and generally becomes \( n \)-dependent. For simplicity, we neglect other self energy corrections leading to higher order terms in \( \omega \) which, one can recognize, does not change the result of this work. Due to the vanishing of the superfluid rigidity at zero wavevector in any direction, the superconducting (fluctuation) part \( \sigma_{s,ij} \) of dc conductivity tensor can be found from the Kubo formula. In the case with a current \((|x| \neq 0)\) perpendicular to the magnetic field \((||z| \neq 0)\), its diagonal \((\sigma_{s,xx})\) and Hall \((\sigma_{s,xy})\) components are generally given by
\[
P(0) - P(\Omega) = |\Omega| \sigma_{s,xx} + i\Omega \sigma_{s,xy} + O(\Omega^2),
\]
where \( P(\Omega) = T \sum (n+1)(n'+1) \langle \phi_{n+1}^*(\omega+\Omega) \phi_n(\omega) \phi_{n'}^*(\omega') \phi_{n'+1}(\omega'+\Omega) \rangle \) is a polarization function (We do not need the details of the coefficient of \( P(\Omega) \) below ), and the general property \( G_n(-\omega) = G_n^*(\omega) \) was used. Hereafter, we can assume \( \Omega > 0 \), and, we only have to see the \( O(\Omega) \) term of the real part of \( P(\Omega) \), denoted by \( \partial_\Omega \text{Re} P |_{\Omega=0} \) below, in order to find \( \sigma_{s,xx} \). First, let us calculate explicitly the Hartree term \( P_H(\Omega) = T \sum (n+1) G_n(\omega) G_{n+1}(\omega + \Omega) \) (i.e., the Aslamasov-Larkin graph) with no vertex corrections. Using the spectral representation of (2), one obtains for arbitrary \( T \)
\[
P_H(\Omega) = \sum_n (n+1) f_{H,n}(\Omega) = \sum_n (n+1) T \sum_\omega \frac{p_{n,n+1}(\omega,\Omega) + p_{n+1,n}^*(\omega,\Omega)}{2},
\]
where
\[
p_{n,n'}(\omega,\Omega) = \frac{a_n}{|\omega| + a_n d_n + \Omega} \left( \frac{a_{n'}}{|\omega| + a_n d_n'} + \frac{a_{n'}^* \Omega}{(a nd_n + a_{n'} d_n' + \Omega)(|\omega| + a_{n'} d_n')} \right).
\]
The zero field case of (4) was derived previously by Tsuzuki\(^8\) focusing only on nonzero temperatures. The \( \omega = 0 \) term of \( \partial_\Omega \text{Re} P_H |_{\Omega=0} \) directly gives the Hartree result in the thermal
case, and, deep in the thermal VL regime, their real and imaginary parts can give, respectively, the diagonal and Hall vortex flow conductivities. Hereafter we focus on $T=0$. One can verify from (4) that the $\partial_\Omega \Re P_{0|0}$ at $T=0$ is precisely zero, implying $\sigma_{s,xx}=0$ up to the Hartree level. The corresponding imaginary part of (4) (i.e., $\sigma_{s,xy}$ at $T=0$), which is nonvanishing in general, will be discussed elsewhere. We note that higher order terms in $|\Omega|$ (such as $O(|\Omega|^3)$ one) are nonvanishing in the real part of (4). To understand this, we rewrite (4) into another form following from the $\omega$-integral:

$$\Re f_{H,n}(\Omega) = - \int_0^\infty dx \int_0^\infty dy \left( \rho_n(x)\rho_{n+1}(-y) + \rho_n(-x)\rho_{n+1}(y) \right) \frac{x+y}{(x+y)^2 + \Omega^2}. \quad (5)$$

Due to the power counting of the parameters $x$ and $y$, one finds that, if (5) is expanded in powers of $\Omega^2$, its $O(\Omega^{2p})$ terms have divergent integrals for vanishing $x$ and $y$ if $p \geq 2$, implying nonanalytic appearances of $|\Omega|^{2p+1}$ ($p \geq 1$) terms in (5). Namely, only the $\partial_\Omega \Re P_{0|0}$, in a sense, trivially vanishes.

Actually, the vanishing of $\partial_\Omega \Re P_{0}$ (i.e., of $\sigma_{s,xx}$) can be seen more simply in arbitrary graph of $P(\Omega)$ in the following way\textsuperscript{6}. First, let us apply the identity $\delta \sum j \omega_j s = \int_s \exp(i\sum j \omega_j s)$ to the frequency conservation at all bare vertices (including the current vertices) appearing in each graph, and divide $G_n(\omega)$ into its real and imaginary parts, where the real (imaginary) part is even (odd) with respect to $\omega$ due to the property $G_n(-\omega) = G_n^*(\omega)$. Then, taking account of the fact that arbitrary Feynman graph of $P(\Omega)$ includes even number of $G$’s, it is easy to find that, after performing all frequency-integrals in each graph, the $O(\Omega^{2p+1})$ terms of the resulting expression becomes pure imaginary. However, it is not clear from this discussion whether a nonanalytic origin, in the sense of (5), of $\partial_\Omega \Re P_{0}$ is also impossible or not. In order to examine this, let us apply similar discussions to that for (5) to lower order graphs in perturbation series. For instance, the corresponding expression to (5) resulting from the lowest (second) order vertex corrections becomes

$$\delta \Re P_2(\Omega) \sim \int \prod_{i=1}^4 dx_i \sum_{j=1}^4 x_j \frac{\sum_{j=1}^4 x_j}{(\sum_{j=1}^4 x_j)^2 + \Omega^2}. \quad (6)$$

where the contributions leading to (if any) nonanalytic $O(|\Omega|^{2p+1})$ terms were merely considered here, and the property at small $x$ of the spectral function, $\rho_n(x) \sim x$, characteristic of the Ohmic dissipation in $\psi$-dynamics, was used. Due to the fact that (6) does not have any divergence for vanishing $x_j$’s up to $O(\Omega^4)$, even an $O(|\Omega|^3)$ term does not follow from (6). Since this discussion is based on the power counting and hence systematic, other higher order terms of the vertex correction cannot bring any nonanalytic appearance of $\partial_\Omega \Re P_{0}$. Therefore, $\sigma_{s,xx}=0$ in general in a $T=0$ disordered phase.

This result is essentially independent of the size of the field and trivially holds for the conductivity $\sigma_{zz}$ parallel to the field. Further, a similar analysis can also be used for fluctuation corrections to the mean field vortex flow conductivity $\sigma_{MF}$ in the vortex lattice at $T=0$ with phase long ranged order and hence, $\sigma_{s,xx}$ at $T=0$ jumps discontinuously at the melting transition field $H_m(0)$ from $\sigma_{MF}$ to zero (in the disordered case\textsuperscript{7}, from $\infty$ to zero). The resulting picture on the resistive transition in 2D case near $T=0$ is sketched in Fig.1, where it is, for simplicity, assumed that the (extrapolated) normal resistance $R_n$ and $H_{c2}$
are temperature-independent and that the Hall coefficient is negligibly small. Reflecting
the above $T = 0$ result on $\sigma_{s,xx}$, the resistivity above $H_m(0)$ may show a quasi-reentrant
behavior at nonzero temperatures, of which the presence has been often speculated\textsuperscript{9} within
a phase-only model for granular systems in $H = 0$. However, an insulating behavior of
$\sigma_{n,xx}$ may interrupt observations of such a behavior of $\sigma_{xx}$ in experiments.

The present result on superconducting fluctuations tends to support the suggestions\textsuperscript{2},
based on resistivity data, of an upwardly curved $H_{c2}$-line at low $T$: It is not difficult
to imagine that, in general, such an $H_{c2}$-determination in the temperature range with
significant quantum fluctuations tends to result in an (incorrectly) underestimated $H_{c2}$-
value because, according to our result, a vestige of the (correct) $H_{c2}$-line should gradually
disappear from dc resistive data as $T \to 0$. A rapid variation of the dc resistivity in
the quantum regime should, in the clean limit, occur near the true (melting) transition,
while the $H_{c2}$-line should be reflected, even when $T \to 0$, in the magnetization\textsuperscript{6}. Such an
increase of $H_{c2}$ near $T = 0$ becomes another origin of covering the quasi-reentrant behavior
in experimental data. Nevertheless, the data in Ref.\textsuperscript{2} at lowest temperatures have shown
broadenings becoming remarkable upon cooling, which can be explained\textsuperscript{6} based on the
present result. We note that a broadening in real systems with randomness at low $T$ cannot
be explained without quantum superconducting fluctuations, which provide interactions
among the order parameter fields in the quantum VL regime. Any approach\textsuperscript{10} of such
phenomena based on the mean field theory is theoretically invalid. The strongly disordered
case\textsuperscript{3} will be discussed elsewhere\textsuperscript{6}. We merely note, together with the importance of
dissipation mentioned in the introduction, that there are no reasons why the scenario
based on a phase-only model\textsuperscript{7} is applicable in strong field regime where the amplitude
fluctuation of the order parameter is qualitatively quite different from that in low field
regime.

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Figure Caption

Fig.1 Schematic temperature variations of 2D resistance curves at low $T$ and at each field $H$, drawn by assuming the normal resistance $R_n$ and the $H_{c2}$ line to be $T$-independent. The dotted (solid) curves are for $H_{c2}(0) \geq H > H_m(0)$ ($H < H_m(0)$).