Edge states protected by chiral symmetry in disordered photonic graphene

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We experimentally investigate the impact of uncorrelated composite and structural disorder in photonic graphene. We find that in case of structural disorder not only chiral symmetry, but also the vanishing of the density of states at zero energy is preserved. This is in contrast to composite disorder, where chiral symmetry as well as the vanishing of the density of states are destroyed. Our observations are experimentally proven by exciting edge states at the bearded edge in disordered photonic graphene.

Graphene, a honeycomb patterned monolayer of carbon atoms, has been celebrated as a new wonder material since its first experimental realization in 2004 \cite{Novoselov2004}. Especially due to its unique properties such as, e.g., very high thermal conductivity, high charge-carrier mobility and very high mechanical stability, this material is a candidate for many different future electronic and photonic devices. Due to its special properties, graphene offers a platform for addressing a number of questions of a fundamental physical nature. Indeed, novel phenomena such as the (fractional) quantum Hall effect \cite{deGennes1980,Laughlin1981}, strong suppression of quantum interference \cite{Kane1997}, and Klein tunnelling \cite{Klein1929} have been shown to exist in this material. The graphene lattice has a two-member basis, composed of shifted hexagonal lattices, and can be therefore described in a tight-binding model containing two bands. The bands meet at conical intersections (called Dirac points) at zero energy, where electrons have a dispersion relation akin to that of the Dirac equation. Importantly, the density-of-states (DOS) near those points goes to zero. If only nearest-neighbor atoms interact, atoms of one sublattice interact merely with the other sublattice and not their own. Such systems possess chiral symmetry \cite{Wen1988,Hasan2010}, from which two important properties follow: (1) any state with nonzero energy \(E\) must have a chiral partner with energy \(-E\) and (2) any state with zero energy cannot be altered by perturbations respecting the chiral symmetry.

Due to the fundamental difference of Dirac-like dispersion and Schrödinger-like dispersion, a full understanding of the impact of disorder on graphene has been recently extensively investigated, but a consensus has not been fully reached \cite{CastroNeto2009,Geim2009}. Disorder that is introduced into the system may be categorized in two different ways: composite disorder (also called on-diagonal disorder), which corresponds to impurities in the lattice, and structural disorder (also called off-diagonal disorder), which corresponds to random displacements of the atoms in the lattice. Composite disorder destroys the chirality and therefore the Dirac spectrum with the associated vanishing DOS at zero energy \cite{CastroNeto2009,Geim2009,Barlas2013}. Structural disorder preserves the chiral symmetry of the honeycomb lattice \cite{CastroNeto2009}. However, how this affects transport properties and the DOS in the Dirac region of the spectrum is still an open question.

In this work, we experimentally probe the impact of composite and structural disorder in a graphene-like lattice, and demonstrate the physical effects for both cases, chiral symmetry breaking and preserving. We present strong experimental evidence that under structural disorder - which preserves the chiral symmetry - the vanishing DOS at the Dirac point remains. To this end we use optical honeycomb waveguide lattices - photonic graphene - which constitute an exceptional system for probing the unique graphene geometry \cite{Novoselov2005,Novoselov2008}. We use an optical system since probing the effects of disorder in atomic graphene is an essentially impossible task due to the difficulty of engineering graphene samples with controllable amounts of disorder, types thereof, and correlations therein. In our system, we employ zero-energy edge states at the bearded edge of the lattice \cite{Kane2005} which turn out to be well suited for experimentally analyzing the spectrum at the zero-energy region. We find that these edge states are destroyed under composite disorder, but survive structural disorder. This suggests that the vanishing of the DOS at zero energy is preserved in the latter case. Note that such edge states were experimentally demonstrated only recently in photonic graphene \cite{Goblot2012} as in electronic graphene an expanded bearded edge was not realized so far due to stability problems \cite{Peres2012}. A very interesting feature of such edge states was demonstrated lately: In \cite{Robert2012,Robert2013} it was shown that edge states experience a topological transformation when a linear strain is applied to the lattice. In Fig. (a), a sketch of a honeycomb photonic lattice with all basic edge types is depicted. A micrograph of the front facet of one fabricated sample is shown in Fig. (b) with the bearded edge at the bottom.

In order to model an optical honeycomb waveguide lattice, we employ the tight-binding-equations

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\begin{equation}
 i\partial_z \psi_n(z) = - \sum_{<m>} c_{n,m} \psi_m - \kappa_n \psi_n(z) \equiv H_{TB} \psi_n(z),
\end{equation}
\]

where the summation is taken only over nearest-neighbor sites. Here, \(\psi_n\) is the amplitude of the guided mode in the
nth waveguide, $z$ is the propagation direction, $c_{n,m}$ is the coupling constant between the nth and mth waveguide, and $\kappa$ denotes the propagation constant. Evidently, this can be written using a tight-binding Hamiltonian matrix $H_{TB}$, which is identical to that of electronic graphene. The only difference of both systems is the evolution coordinate: whereas in electronic graphene the wave function evolves in time, in the optical system the light propagation occurs along $z$. Hence, the temporal dynamics of electrons in graphene is emulated by light diffraction in the optical waveguide system. In order to achieve time-independent equations for the eigenvalues $\beta$, one substitutes the plane wave solutions $\psi(z) = \psi e^{\beta z}$ into Eq. (1), which results in an eigenvalue equation for the eigenmodes of the system:

$$-\beta \psi = H_{TB} \psi. \quad (2)$$

For periodic boundary conditions, this equation allows the determination of the band structure of a honeycomb lattice, where $\beta$ is a function of the transverse Bloch wave numbers $k = (k_x, k_y)$, as shown in Fig. 1(c). Indeed, there is no bandgap, as both bands intersect at six singular points. In the vicinity of these Dirac points, the Hamiltonian can be approximated by the relativistic Dirac Hamiltonian $H_D \sim k \cdot \sigma$ with $\sigma$ being a vector of the Pauli matrices, which is an analogue to the Dirac equation for massless relativistic particles [24]. This is the central characteristic of the graphene structure and one of the main reasons for many exotic properties of this material. A further key feature of graphene is the observation that at the Dirac points the DOS vanishes. In Fig. 2(a) we show all eigenvalues $\beta$ in ascending order of a system, where periodic boundary conditions are applied in all directions. The analytic expression of the DOS - which is inversely proportional to the slope of the eigenvalue function shown in Fig. 2(a) - is plotted in Fig. 2(b) and exhibits a clear drop at $\beta = 0$. The question arises how stable the vanishing of the DOS is under the influence of disorder.

We start our analysis using simulations and examine a photonic graphene structure under the influence of both different kinds of disorder: composite disorder (which manifests in random depth of the lattice sites) and structural disorder (which results by a random shift of the lattice sites around their average position). Note that in our optical system, we can realize both disorders independently of each other. Moreover, the structural disorder is fully uncorrelated, in contrast to electronic graphene, where a random shift of the atoms always influences the neighboring sites. We implement the structural disorder in Eq. (1) by $c = c_0 + \Delta c \cdot \xi$, where $c_0$ is the mean coupling constant, $\Delta c$ defines the strength of the disorder in units of the coupling constant, and $\xi$ is randomly distributed in the sequence $[-1,1]$. Composite disorder is realized by a random distribution of the propagation constants: $\kappa = \Delta \kappa \cdot \xi$, where $\Delta \kappa$ defines the strength of the disorder in units of the propagation constant. Additionally, we assume periodic boundary conditions in the direction perpendicular to the zigzag and bearded edges. In the upper row of Fig. 2 we plot the eigenvalues in ascending order for no disorder (Fig. 2(a), with a coupling constant $c_0 = 1 \text{ cm}^{-1}$), structural disorder (Fig. 2(c), $\Delta c = 8 \text{ cm}^{-1}$, $c_0 = 5 \text{ cm}^{-1}$, and fixed propagation constant $\kappa_0 = 0$) and composite disorder (Fig. 2(e), $\Delta \kappa = 8 \text{ cm}^{-1}$, $\kappa_0 = 0$, and fixed coupling constant $c_0 = 1 \text{ cm}^{-1}$). In the lower row of Fig. 2 we plot the according DOS. In order to take into account the statistical nature of disorder [25], in both disorder cases we averaged over 5000 realizations, each containing 750 waveguides. As expected for graphene, the eigenvalue spectrum of the completely ordered system shows a DOS equal to zero at $\beta = 0$ (the Dirac points). In case of composite disorder the picture completely changes - the DOS is not longer zero at $\beta = 0$. Indeed, with structural disorder one still observes a vanishing DOS at $\beta = 0$. Therefore, the chiral symmetry has caused the preservation of the Dirac-like DOS even for very strong disorder.

To further elaborate our results, in Fig. 3 we plot the simulated averaged DOS at $\beta = 0$ for increasing composite and structural disorder, respectively. The small fluctuations are due to the finite number of realizations, which were taken for the averaging process. With increasing composite disorder the DOS at $\beta = 0$ is increasing, so that the zero DOS disappears. In contrast, for increasing structural disorder the DOS at $\beta = 0$ stays constantly at zero since the chirality is preserved, even for very high disorder strengths. We attribute this behavior to the intrinsically different impact of composite and structural disorder on the distribution of eigenvalues [26, 27]. For structural disorder the eigenvalues appear in pairs with identical absolute values and opposite sign, such that they are symmetrically distributed around $\beta = 0$, which is not the case for composite disorder. This is a direct result of chiral symmetry breaking.

To test the conjectures based on the numerical sim-
FIG. 2. Eigenvalues and DOS plotted for three different cases: no disorder, structural and composite disorder.

FIG. 3. Dependence of the DOS at $\beta = 0$ on the strength of disorder plotted for structural and composite disorder.

FIG. 4. Simulation of the light distribution after $z = 7L_c$ when a broad Gaussian beam is launched at the armchair edge. The averaging was taken over 200 realizations. The edge state in the case of structural disorder propagates in a stable fashion, whereas it is destroyed for composite disorder.
beam. Without disorder the edge state does not spread into the lattice. This is because it exists at the same energy as the Dirac points where the DOS is zero, so that there are no states the edge state could couple to. If structural disorder (in the coupling coefficients) is implemented the edge state is still present and can be observed even after a propagation length of $z = 7L_c$ with the coupling length $L_c = \frac{\pi}{c}$ and $c$ as the mean coupling constant. In contrast, for composite disorder the edge state is destroyed and the light spreads into the array as it is no longer localized at the edge. Note that in all cases, averaging was performed over 200 random realizations.

In order to experimentally demonstrate the described effects, we fabricate a photonic graphene waveguide array using the femtosecond direct-writing technology [28]. The structural disorder is realized by randomly shifting the positions of the waveguides in the transverse directions with respect to their average positions by $\pm 1 \mu m$ for a disorder of $\Delta c = 0.45 \text{ cm}^{-1}$, and $\pm 2 \mu m$ for a disorder of $\Delta c = 1.1 \text{ cm}^{-1}$, while the mean nearest-neighbor distance was $20 \mu m$. The composite disorder was realized by changing the translation velocity of the writing beam during fabrication. The mean velocity for all realizations was $80 \text{ mm} / \text{min}$ with a random shift of $\pm 10 \text{ mm} / \text{min}$ and $\pm 15 \text{ mm} / \text{min}$ for $\Delta \kappa = 2.5 \text{ cm}^{-1}$ and $\Delta \kappa = 4.5 \text{ cm}^{-1}$, respectively. For the investigation of the light propagation a broad elliptical beam with its main axis along the edge was launched into the bearded edge waveguides and the output facet of the sample was imaged onto a CCD camera. For each disorder level, we averaged over 20 realizations, and the resulting averaged output distributions are shown in Fig. [3]. It is clearly seen that for increasing composite disorder the edge state couples to the bulk modes and spreads into the lattice. Only for sufficiently high disorder Anderson localization takes place.

In conclusion, we theoretically and experimentally analyzed the impact of composite and structural disorder on the DOS of photonic graphene. Our results are general and hold also for electronic graphene, where such controlled investigations of disorder are impossible. We found that for composite disorder, which breaks the chiral symmetry of the lattice, the DOS does not vanish at $\beta = 0$. However, for structural disorder we could experimentally demonstrate that edge states still exist, which implies that the DOS vanishes at $\beta = 0$ even for strong disorder, so that chirality prevails. Therefore, all properties of the graphene structure that arise from chirality and the vanishing DOS at $\beta = 0$ do not break down for structural disorder. Our work may provide further insight into certain properties of graphene that are strongly affected by disorder such as, e.g., the value of the minimal conductivity. Furthermore, our photonic system offers a unique platform for investigating the influence of predetermined disorder on the electronic transport properties of graphene.

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