Pairing mechanism in the ferromagnetic superconductor UCoGe

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Superconductivity is a unique manifestation of quantum mechanics on a macroscopic scale, and one of the rare examples of many-body phenomena that can be explained by predictive, quantitative theories. The superconducting ground state is described as a condensate of Cooper pairs, and a major challenge has been to understand which mechanisms could lead to a bound state between two electrons, despite the large Coulomb repulsion. An even bigger challenge is to identify experimentally this pairing mechanism, notably in unconventional superconductors dominated by strong electronic correlations, like in high-Tc cuprates, iron pnictides or heavy-fermion compounds. Here we show that in the ferromagnetic superconductor UCoGe, the field dependence of the pairing strength influences dramatically its macroscopic properties like the superconducting upper critical field, in a way that can be quantitatively understood. This provides a simple demonstration of the dominant role of ferromagnetic spin fluctuations in the pairing mechanism.

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In the so-called conventional superconductors, the pairing interaction has been identified as arising from the coupling between electrons and the lattice, which can overcome the Coulomb repulsion because of retardation effects. The extensive knowledge of the electron–phonon interaction in conventional metals, has allowed a complete description of the pairing mechanism and its signatures on the superconducting properties, already back in the sixties. However, even with these powerful theoretical developments, predicting from first principles a precise value of the superconducting critical temperature \( T_c \) remains a formidable task. This is due to the exponential dependence of \( T_c \) on the strength of the pairing interaction, and to the difficulties in computing the screened retarded Coulomb repulsion potential. As a result, the most convincing demonstration that the pairing arises from the electron–phonon coupling in conventional superconductors does not come from the predicted values of \( T_c \); it comes from measurements and analysis of phonon anomalies in the tunnelling spectra, which have been quantitatively explained by the Eliashberg theory.

In strongly correlated electron systems, where the pairing mechanism may arise from magnetic or other exotic degrees of freedom, the situation is much more complicated. No equivalence of the Eliashberg theory can describe accurately the normal state of these systems. In most cases, there is not even a clear separation between electronic quasiparticles (if they exist), and excitations mediating the pairing (usually over-damped, instead of being well defined modes as for phonons), as they both originate from the same electronic degrees of freedom.

Thus for these unconventional superconductors, the main method to get some insight on the pairing mechanism remains to force a change of its strength, and to monitor the corresponding change of \( T_c \). This is typically what is done in conventional superconductors with the isotope effect, where the change of \( T_c \) with ion masses can be related to the change of phonon frequencies. In strongly correlated electron systems, one can use pressure or doping to tune electronic instabilities and provoke the appearance of a superconducting phase. This was first realized with heavy fermion superconductors, revealing that superconductivity appears frequently at the verge of a magnetic instability. Similar behaviour has been identified in organic, cuprates and iron pnictides superconductors. In heavy fermion systems, the problem has also been tackled by comparisons between the pairing strength and normal state quantities, when they are changed under pressure or magnetic field.

In this study, we investigate the pairing strength of the ferromagnetic superconductor UCoGe. Macroscopic homogeneous coexistence of ferromagnetism and superconductivity has been clearly established in three uranium based systems: UGe\(_2\) (ref. 14) under pressure, URhGe\(_{15}\) and UCoGe\(_{10}\) at ambient pressure. The last two, URhGe and UCoGe, show similar properties in many ways. They have the same orthorhombic crystal structure (space group Pnma), and are both weak itinerant ferromagnets, with a strong uniaxial anisotropy along the easy magnetization axis (c-axis). However, their upper critical fields \( (H_{c2}) \) present some remarkable differences. We will show that these differences, and some anomalous properties of \( H_{c2} \) in UCoGe, can be understood within a general theoretical framework describing superconductivity arising from ferromagnetic fluctuations.

### Results

**Bulk determination of \( H_{c2} \) in UCoGe with thermal conductivity.**

UCoGe has a Curie temperature \( (T_{\text{Curie}}) \) of about 2.5 K and a bulk superconducting transition at \( T_{\text{sc}} \approx 0.5 \) K (ref. 16). For clean, well-oriented samples, \( H_{c2} \) along the b-axis shows an S-shape behaviour, which is reminiscent of the re-entrant superconducting phase observed in URhGe for the same field direction; this suggests that in both cases, superconductivity is enhanced under magnetic field along the b-axis. However, there are other features of \( H_{c2} \) which are unique in UCoGe, and very puzzling. At first, according to previous resistivity measurements, \( H_{c2} \) is strongly anisotropic: it is \( \gtrsim 20 \) times higher along the two transverse directions (a,b) than along the easy magnetization axis. In addition to this strong anisotropy, the angular dependence of \( H_{c2}(0) \) in the (a,c) plane is extremely sharp near the a-axis, at odds with the usual elliptical behaviour. Strong anisotropy in \( H_{c2} \) is commonly observed in low dimensional superconductors, like organics\(^{25}\), or high-Tc cuprates\(^{23,24}\). However, the transport properties\(^{21,25}\) and the almost spherical Fermi surface pocket observed in Shubnikov-de Haas measurements\(^{25}\) suggest that the electronic structure of UCoGe is essentially isotropic in the normal phase.

In addition, the temperature dependence of \( H_{c2} \) in UCoGe is marked by unusual positive curvatures along the three crystallographic directions\(^{18,26}\). These features call on confirmation by a bulk sensitive probe: the resistive superconducting transition under field occurs when vortex pinning is strong enough for the measurement current. So it has been found that in some organic superconductors, and in most cuprates, the resistive transition under field does not determine \( H_{c2} \). Instead, it measures an irreversibility line, that might separate a vortex liquid and a vortex solid phase. This line can be much lower than \( H_{c2} \), and it displays a strong upward curvature. Owing to the similarities of \( H_{c2} \) in UCoGe and in these systems, we have used thermal conductivity as a probe (as in refs 27,29), to obtain the bulk \( H_{c2} \) along the three crystallographic directions in UCoGe (see Fig. 1a,b for a zoom on the c-axis).

Figure 1b shows that both bulk and resistive measurements along the c-axis display the same upward (positive) curvature, with the absence of saturation down to the lowest temperatures (10 mK). At very low field, the bulk \( H_{c2} \) is lower than the resistive determination, but the agreement is much better at higher field, particularly for the criterium of zero resistance. This complies with the usual sensitivity of the resistive transition to filamentary superconducting paths, rapidly suppressed under field. Altogether, the anisotropy and the anomalous temperature dependence of \( H_{c2} \) in UCoGe reported in previous resistivity studies\(^{18}\), are well confirmed by our bulk measurements, which excludes an explanation of these anomalies by an irreversibility line.

A specific feature of UCoGe is the possible role of ferromagnetic fluctuations in the pairing mechanism, and their rapid suppression by magnetic fields applied along the c-axis. This suppression has been previously demonstrated by NMR studies, which revealed strong Ising-type longitudinal magnetic fluctuations in UCoGe, with a very anisotropic response to an applied magnetic field\(^{21}\). It has also been confirmed by electrical and thermal transport\(^{30}\). A first empirical model for the effects of a field dependence of the superconducting pairing strength has been derived from the NMR response\(^{21,31}\). It could qualitatively account for the \( H_{c2} \) anisotropy between the c and a-axis, but it predicts a vanishing slope of \( H_{c2} \) along the c-axis, which is not observed (Fig. 1b). References 17,32 proposed an earlier theoretical approach based on a general and well defined framework. It had predicted that when pairing is mediated by ferromagnetic spin fluctuations, the field dependence of the magnetization or of the Curie temperature drives the field dependence of the superconducting coupling strength. In the following sections we discuss this idea of a field-dependent pairing mechanism, probed by the field dependence of normal-
state properties, with emphasis on the field direction $\mathbf{H}/c$. To perform quantitative comparison to theoretical predictions\textsuperscript{17}, we measured specific heat, magnetization and ac susceptibility for this field direction, on the same single crystal. The raw data are presented in Methods Fig. 7a and in Supplementary Figs 2 and 3.

**Analysis.** To get an idea about the effect of a field dependence of the pairing strength on $H_{c2}$, a simple back-of-the-envelop calculation can be useful. For a superconductor in a weak coupling scheme, $T_{sc}$ is given by:

$$T_{sc} \sim \Omega \exp\left( -\frac{1}{\lambda - \mu^*} \right)$$  \hspace{1cm} (1)

$\Omega$ is the characteristic frequency of the pairing interaction (proportional to the Debye temperature for electron-phonon coupling), $\lambda$ is the pairing strength, which can now vary under field ($\lambda = \lambda(H)$), and $\mu^*$ is the Coulomb repulsion parameter (usually of order 0.1–0.15). Like all heavy fermion
superconductors, UCoGe is in the clean limit. According to standard Ginzburg-Landau theory, for a clean single-band superconducting system dominated by the orbital limitation, the upper critical field near $T_{c}$ is given by:

$$H_{c2}^{orb} \sim \frac{\Phi_{0}}{2\pi R(T)} \sim \frac{\Phi_{0}}{0.1(b/v_{F})} T_{c} (T_{c} - T)$$

(2)

where we used the Bardeen–Cooper–Schrieffer (BCS) expressions for the superconducting coherence length $\langle \xi(T) \rangle = 0.7 \xi_{0} (1 - T/T_{c})^{-1/2}$, $\xi_{0} = 0.18 b / \tau_{F}$, and $\langle v_{F} \rangle$ is the average Fermi velocity perpendicular to the field.

Taking account of the additional field dependence of $\lambda(H)$, the initial slope of $H_{c2}$ ($dH_{c2}/dT|_{T=T_{c}}$) can be calculated from equation (2):

$$\left. \frac{dH_{c2}}{dT} \right|_{T=T_{c}} = \left( \frac{dT}{dH_{c2}} \right)_{T=T_{c}} \frac{dT_{c}}{d\lambda} \frac{d\lambda}{dH}$$

(3)

According to equation (3), $dH_{c2}/dT|_{T=T_{c}}$ is determined both by the usual orbital limit (first term on the right side), and by an additional term $(dT/dH_{c2})$ arising from the field dependence of $\lambda$. The idea for UCoGe is that this second term could be dominant: a large negative $\frac{dT}{dH_{c2}}$ would lead to a much reduced initial slope, so that the anisotropy of $H_{c2}$ could just reflect that of the field suppression of the ferromagnetic fluctuations. Another consequence of this dominant term is that the orbital effect would play little role in the temperature dependence of $H_{c2}$, opening new routes to explain the data of Fig. 1.

To study quantitatively this hypothesis of a field-dependent pairing strength, we choose an new angle of attack: instead of searching for a model that can reproduce $H_{c2}(T)$, we extract from the experimental data of Fig. 1a the field and direction dependence of $\lambda$ required to reproduce them. For this, we calculate $H_{c2}$ for a series of fixed values of the pairing strength $\lambda$, with the help of a simple strong-coupling model for the upper critical field described in ref. 9 (details in the Methods). The parameters of this model are the same as those of equations (1 and 2), but $T_{c}$ and $H_{c2}$ are now calculated from a microscopic model, which also includes the effective mass $m^{*}$ (or equivalently, the Fermi velocity) renormalization by the pairing interactions derived from

Figure 3 | Analysis of the field dependence of the pairing strength for $H//c$ in UCoGe. Filled circles: from the experimental $H_{c2}$ curve. Open circles: from specific heat measurements. Lines: from equation (6), based on magnetization measurements performed on the same sample: solid line, for the optimized value of $\xi_{mag} k_{F} = 3.2$; dash-dotted line for $\xi_{mag} k_{F} = 1$ (localized magnetism). The low field regime of $\lambda(H)$ for $H//b$ is also presented. Squares: $\lambda$ from $H_{c2}$. Dotted line: prediction from equation (6) based on the measured field variation of $T_{c}$.

Figure 4 | Analysis of $H_{c2}$ of URhGe. (a) closed symbols, $\lambda(H)$ in URhGe along b-c-axis. Data along b extracted from $H_{c2}$, data along c extracted from the specific heat of ref. 44 (see Methods). Solid line: $\lambda(H)$ predicted by theory in ref. 17 with $\xi_{mag} k_{F} \sim 1$. Open symbols: $\lambda(H)$ in UCoGe (same as in Fig. 2a) for comparison. (b) Solid line: $H_{c2}/c$ in URhGe based on $\lambda(H)$ obtained from the specific heat (see Fig. 4a). Broken line: for comparison, $H_{c2}$ with $\lambda$ fixed at $\lambda(0)$ (40% larger). Experimental data are from ref. 35. Circles: $H//c$. Squares: $H//b$, Diamonds: $H//a$. Note the much weaker anisotropy of $H_{c2}$ in URhGe compared with UCoGe (displayed on Fig. 1a).
Eliashberg equations\(^1\)\(^{33}\):

\[
\begin{align*}
m^* &= m_{\text{band}} (1 + \lambda) \\
\nu_F &= \frac{v_{\text{band}}^{\parallel \text{ax}}}{1 + \lambda}
\end{align*}
\]  

where \(m_{\text{band}}\) (\(v_{\text{band}}^{\parallel \text{ax}}\)) is the mass (Fermi velocity) of the quasiparticles (band Fermi velocity), renormalized by all the interactions apart from the pairing interactions. Then we compare our experimental data to the series of calculated \(H_{c2}\) (inset in Fig. 2a) and extract \(\lambda(H)\) needed to reproduce \(H_{c2}\) in the three directions (Fig. 2a). The two major assumptions used in this process are that UCoGe in the normal phase is isotropic, and that the influence of \(p\)-wave pairing on \(H_{c2}\) is approximated by a calculation for \(s\)-wave superconductors, with no paramagnetic limitation. For the first point, it assumes average Fermi velocities \(\langle v_F^i \rangle\) along each axis \(i\), so that the \(H_{c2}\) anisotropy arises only from the difference in \(\lambda(H)\) between \(a\), \(b\), and \(c\)-axis. For the second point, some anisotropy and small differences in the temperature dependence of \(H_{c2}\) could also come from the exact form of the \(p\)-wave order parameter\(^34\), as discussed in URhGe\(^35\). These effects are neglected here, as in UCoGe, anisotropy originating from superconducting gap nodes has not been detected so far, even on the best available samples\(^36\).

As seen in Fig. 2a, for fields along the \(a\)-axis, \(\lambda\) is essentially constant up to 5 T with only a slight increase of order 1.5% induced by the visible upward curvature in Fig. 1a. For \(H/\mathbf{b}\), the pronounced S-shape of \(H_{c2}\) gives rise to a monotonous \(~20\%\) increase of \(\lambda\) up to 15 T. The most striking feature is the sharp decrease of \(\lambda\) for \(H/\mathbf{c}\), of nearly 30%, in a narrow field range between 0 and 0.5 T. Figure 2b zooms on the data for \(H/\mathbf{c}\), and the series of calculated \(H_{c2}\) lines: they are almost vertical, meaning that \(H_{c2}\) is almost entirely controlled by \(T_{c2}(\lambda(H))\), not by the orbital limitation. This justifies \textit{a posteriori} the assumption of isotropic \(\langle v_{F}^{2} \rangle\) in zero field.

The determination of \(\lambda(H)\) from \(H_{c2}\) depends quantitatively on the choice of the zero field value \(\lambda(0)\). It has been fixed by comparison with the variation of the Sommerfeld coefficient \(\gamma\) of the specific heat \((C_p/\gamma = C_p/T)\) for \(H/\mathbf{c}\). Indeed, the renormalization of the effective mass (equation (4)) leads to a renormalization of \(\gamma \propto m^* \propto (1 + \lambda)\). So the field variation of \(\lambda\) reported in Fig. 2a should be reflected in a field variation of \(\gamma\) as:

\[
\lambda(H) = \frac{\gamma(H)/\gamma(0)}{1 + \lambda(0)} - 1
\]

Figure 3 shows that, for the value \(\lambda(0) = 0.57\), a satisfactory agreement is reached between \(\lambda(H)\) obtained from the strong-coupling analysis of \(H_{c2}\), and \(\lambda(H)\) deduced from our data of \(\gamma(H)\) according to equation (5) (see Methods Fig. 7b). In both cases, there is an initial linear-decrease of \(\lambda\) at low field, as opposed to the \(\sqrt{H}\) dependence of \(d\lambda/dH\) proposed in the pioneering work ref. 21. Experimentally, because of the torque exerted by the spontaneous magnetization under perpendicular field orientation, there are still no measurements of \(\gamma\) under fields along \(b\) or \(a\)-axis. However, an increase of \(\lambda\) for \(H/\mathbf{b}\) is qualitatively expected from the coefficient of the inelastic contribution to the resistivity \((AT^2)\) term, which shows a maximum near 14 T (ref. 18).

**Theoretical framework.** Several theoretical studies have been proposed to describe the emergence of superconductivity in a ferromagnetic background\(^{17,37-42}\). All these models suggest a pairing mechanism based on spin-spin interactions proportional to the dynamical spin susceptibility. In ref. 17 (extended in the more recent ref. 32), a field dependence of the pairing strength is predicted from a general framework of the ferromagnetic state (Landau approach), valid both in the
are then fitted with a function field range, equally for normalized to its zero field value. The same data for URhGe on the same Sommerfeld coefficient associated Sommerfeld coefficient (see Methods Fig. 6).

Figure 7 | Specific heat in UCoGe and URhGe. (a) Temperature dependence of \( C_p/T \) of UCoGe, for \( H/c \). For magnetic fields close to 0 T, a small increase in \( C_p/T \) occurs when \( T \) approaches \( T_{Curie} \). The \( C_p/T \) curves are then fitted with a function \( C_p/T = \gamma + \Delta \exp(-T_0/T) \) to obtain the Sommerfeld coefficient \( \gamma \). (b) Field dependence of the Sommerfeld coefficient \( \gamma \) in UCoGe (magenta circles) for \( H/c \) normalized to its zero field value. The same data for URhGe\(^{34} \) on the same field range, equally for \( H/c \), are presented for comparison, with the blue squares.

Figure 8 | Strong coupling calculation of \( H_{c2} \) in URhGe. Blue squares: data of \( H_{c2}//b \) in URhGe from ref. 45. Full lines: series of calculations of \( H_{c2} \) with fixed values of the strong coupling constant \( \lambda \), which changes from 0.7 to 1.5 by steps of 0.05. \( \Omega \) and \( \nu^\text{and} \) have been adjusted so as to match the experimental \( T_{Curie} \) and initial slope for \( \lambda = 0.75 \) and \( \nu^* \sim 0.1 \). Note that if \( \lambda \) had, in zero field, the value it reaches at \( H_b (\lambda(H_b) \sim 1.4) \), \( T_{Curie} \) would be of order 0.67 K instead of 0.26 K.

Reference 32 gives expressions for the susceptibilities \( \chi(T, M, q) \), with a wave vector \( q \) dependence arising from the exchange terms in the Landau free energy. The pairing strength for a \( p \)-wave order parameter (called \( \'q^* \) instead of \( \lambda^* \) in ref. 32) is calculated in a form which can be cast as:

\[
\lambda(H) = \lambda(0) \left( 1 + \frac{a^2}{(\Theta + a^2)^2} \right)
\]

(6)

with \( a = \frac{\xi_{mag}}{k_F} \) a numerical parameter (\( \xi_{mag} \) the magnetic coherence length associated to the ferromagnetic order and \( k_F \) the Fermi wave vector), and the factor \( \Theta \) given by

\[
\Theta(H//c) = \frac{1}{2} \left( \frac{3 M_c^2}{M_0^2} - 1 \right)
\]

(7)

\[
\Theta(H//c) = \frac{T_{Curie}(H) - T_{Curie}(0)}{T_{Curie}(0)}
\]

\( M_c \) (\( M_0 \)) is the (spontaneous) magnetization along the \( c \)-axis. This simple form is valid in a one-band approximation\(^{17} \): two-band effects arise automatically for \( p \)-wave - equal spin pairing states\(^{43} \), but in practice they change very little the results, notably for \( H/c \) (see Supplementary Notes 1 and 2).

Equations (6 and 7) show that this general framework for ferromagnetic superconductors predicts a decrease of the pairing strength with field along the easy axis, controlled by the field dependence of the magnetization. Our magnetization data yield an excellent agreement between equation (6) and \( \lambda(H) \) deduced from \( H_{c2} \) and \( \gamma(H) \) for a value of \( \xi_{mag}k_F \sim 3.2 \) (Fig. 3). The large value of \( \xi_{mag}k_F \) \( \sim 10 \) damps the strong increase of \( M/M_0 \) (+ 60% from 0 to 0.5 T); otherwise, the predicted theoretical suppression of \( \lambda \) would be much larger (see dash-dotted line for \( \xi_{mag}k_F \sim 1 \) in Fig. 3). This large value of \( \xi_{mag} \) points to the itinerant nature of the ferromagnetism in this compound: for a localized magnetic system, one would expect \( \xi_{mag} \) of the order of interatomic distances, so \( \xi_{mag}k_F \sim 1 \).

itinerant and localized limits. It has been derived in the weak coupling limit for superconductivity: comparing this theory to our experimental results implicitly assumes that the same field dependence is preserved in the intermediate coupling regime, and shows up in normal state quantities like \( \gamma(H) \). Another assumption is that the expression of the Landau free energy, valid at temperatures close to \( T_{Curie} \), is still valid near \( T_{Curie} \). This hypothesis is supported by our magnetization (\( M \)) data: at 0.5 K, they are straight lines on an Arrott plot (see Methods Fig. 6).
For $H$ along $a$ and $b$-axis, the field dependence of $\lambda$ in equations (6 and 7) is related to the change of $T_{\text{Curie}}$, which has been determined experimentally in ref. 18. In the same field range (0–1 T), there is essentially no detectable change of $C_{24}$. For example, the emergence of a soft magnon mode has been proposed in another work to account for the re-entrant superconducting phase in $\text{UCoGe}$, and most likely in $\text{URhGe}$, the pairing mechanism does arise from ferromagnetic spin-fluctuations. The demonstration has been possible because of the particularly strong variation of the pairing strength with magnetic field in $\text{UCoGe}$, controlling the abnormal temperature dependence of $H_{c2}$ (along the easy axis) and its enormous anisotropy. These robust experimental features of the superconducting phase are quantitatively related to the field variation of normal state properties, and they are explained naturally with a general theoretical framework, without a need for a precise microscopic description of the complex ferromagnetic background. This leads to a solid and rare identification of an unconventional pairing mechanism among strongly correlated electron systems.

### Methods

**Sample.** $\text{UCoGe}$ crystallizes in an orthorhombic structure, and the $c$-axis is the easy magnetization axis, the $a$ and $b$-axis are the hard and intermediate axis, respectively. High-quality single crystals were grown by the Czochralski method in a tetra-arc furnace and further annealed: we have chosen a sample with a modest residual resistivity ratio (RRR > 16), in order to have a good determination of $T_m$ by thermal transport. Indeed for large RRR samples, the suppression of inelastic scattering at low temperatures leads to a large increase of the thermal transport ($\kappa$), which masks the onset of the superconducting transition. By contrast, when the RRR is not as good, inelastic process do not dominate over elastic scattering, deviations from the Wiedemann-Franz law at $T_m$ are smaller, and the occurrence of the superconducting transition is marked by a clear kink in $\kappa(T)$.

**Measurement of $H_{c2}$ by thermal transport.** Thermal conductivity measurements were performed with the standard one-heater-two-thermometers method. The resistive heater and two thermometers were connected to the sample with 15 μm gold wires, spot-welded on the sample and glued with silver paste on the thermometer side. The temperature rise along the sample was set to be around 3% for all the measurements. The same gold-wire contacts were used for the four-wire ac-resistivity measurements, which permits to compare measurements from the two probes with exactly the same geometrical factor. For magnetic fields along $c$-axis, both thermal conductivity and resistivity were measured in a dilution refrigerator which can be cooled down to 10 mK. For magnetic fields along $a$ and $b$ axes, the measurements were performed in another dilution refrigerator down to 150 mK, in magnetic fields up to 15 T. The fact that $H_{c2}$ in $\text{UCoGe}$ is extremely sensitive to any field component along the $c$-axis, makes precise field orientation a crucial issue: we are equipped with two piezo-goniometers (with perpendicular rotating axes) and one piezo-rotator (from Attocube). This allowed us to orient the field direction with a deviation less than 0.05° in situ, by following the angular dependence of resistivity in the width of the superconducting transition.

Supplementary Fig. 1 shows some of the thermal conductivity ($\kappa$) measurements in $\text{UCoGe}$ for magnetic fields along the $a$-axis. For improved control and precision on the determination of the superconducting transition, we used a fitting procedure of the data, based on the following considerations:

- the thermal conductivity of metals is the sum of contributions from electrons (quasiparticles), phonons and in some cases, other bosonic contributions like magnons or more generally, magnetic fluctuations:

$$\kappa = \kappa_{\text{elec}} + \kappa_{\text{ph}} + \kappa_{\text{magnons}} + \ldots$$

Figure 9 shows the adjustment of $\lambda_0$ in $\text{URhGe}$. Red circles: specific heat Sommerfeld coefficient for $H/b$ from ref. 44, normalized to its value at zero field. Dark blue squares: ratio $(1 + \lambda(H))/(1 + \lambda(0))$ deduced from the calculations of $H_{c2}$ presented in Fig. 8 for $\lambda(0) = 0.75$. Light blue squares: same ratio, but with the calculations of $H_{c2}$ for $\lambda(0) = 0.6$. It shows that the amplitude of variation of the specific heat coefficient is a very selective criterion for the determination of $\lambda(0)$ in this system. Inset: Corresponding variation of $\lambda$, for $\lambda(0) = 0.75$.

**Discussion.**

Apart from the high-field behaviour along the transverse directions, the theoretical framework in ref. 32 successfully explains the positive curvature of $H_{c2}$ in $\text{UCoGe}$ for $H/c$, as well as the anomalous anisotropy between $c$-axis and the $(a,b)$ plane. This theory includes no element specific to $\text{UCoGe}$, and should apply equally to $\text{URhGe}$. The validity of our claims for $\text{UCoGe}$ is challenged by the usual negative curvatures and the small anisotropy of $H_{c2}$ in $\text{URhGe}$.

A major difference between the two systems is that in $\text{URhGe}$, $M_s/M_0$ has a much weaker field dependence for $H/c$ than in $\text{UCoGe}$ (~3% instead of 60% in $\text{UCoGe}$, at 0.5 T). This is the key element to understand, according to equation (6), the smaller suppression of $\lambda$ for $H/c$. Quantitatively, we can take advantage of the specific heat data available for $H/b$ in this system, to determine the value of $\lambda(0) \sim 0.75$ from comparison to $H_{c2}$ (ref. 45) (see Methods Fig. 9). Once $\lambda(0)$ is fixed, the specific heat data for $H$ along the $c$-axis determines how $\lambda$ changes with field according to equation (5). It compares successfully to the theoretical predictions of equation (6), and the magnetization data along the $c$-axis, in a wide field range, for $\gamma_{\text{mag}} \approx \kappa_F$ ~ 1 (see Fig. 4a). This value of $\gamma_{\text{mag}} \approx \kappa_F$ ~ 1 indicates that magnetism in $\text{URhGe}$ lies close to the localized limit, in accordance with the larger ordered moment and the net spin rotation induced by fields along the $b$-axis. Figure 4a compares $\lambda(H)$ along $c$ and $b$-axis for $\text{URhGe}$ and $\text{UCoGe}$ clearly, apart from the sharp maximum in $\text{UCoGe}$ at 12 T corresponding to the moment rotation, the difference between the two systems is just quantitative.
For our sample with $B_{eff} \sim 16$, we can apply the Wiedemann-Franz law to estimate the electronic contribution to the thermal conductivity from the resistivity data ($\rho$) in the normal phase. With $L_T = 2.44 \times 10^{-2}$ W K$^{-2}$ (the Lorentz number):

$$\frac{\kappa_{\text{elec}} - \rho}{T} = L_T \rho$$

(9)

The ratio of the thermal conductivity to the normal-phase electronic contribution to the thermal conductivity, is calculated as:

$$\frac{\kappa}{\kappa_{\text{elec}}} = \frac{\kappa_{\text{elec}} - \rho}{L_T \rho} + \frac{\kappa_{\text{ph}} + \text{other}}{L_T \rho} + \frac{\kappa_{\text{vol}}}{L_T \rho}$$

(10)

The first term of equation (10) is the normalized electronic contribution, $\kappa_{\text{elec}}/\kappa_{\text{elec}}$, which should equal 1 in the normal phase and decrease with temperature in the superconducting phase. The second term contains the effects of the other contributions (phonons, magnons, and so on), $\kappa_{\text{ph}}/\kappa_{\text{elec}}$, which should in principle exhibit little change in the neighborhood of the superconducting transition.

The temperature dependence of the ratio in equation (10) can be fitted as the sum of two functions: for the other contributions, we use a polynomial of order 3 that extrapolates to zero at $T = 0$. For the contribution due to the phonon part, we use a piecewise function $f(T)$, which equals 1 for $T > T_c$ and decreases linearly with $T$ below $T_c$. The superconducting transition temperature $T_c$ is a non-linear parameter of the fit. In practice, in order to improve the fit quality, we have introduced a (quadratic in temperature) smearing of the transition around $T_c$, which is estimated for each field.

For temperatures below $T_c$, the decrease of $\kappa_{\text{elec}}/\kappa_{\text{elec}}$ is because of the increase of the phonon part, while above $T_c$, it is because of the suppression of the electronic part due to the transition.

**Strong coupling calculation of $H_2$.** The strong-coupling calculations for the upper critical field are based on a theoretical model described in ref. 9, where the Eliashberg equations are solved for a system of electrons and phonons with an Einstein spectrum. The calculations were performed in the clean limit, in which the electron-phonon coupling is in the strong-coupling limit. The orbital limitation is controlled by an averaged Fermi velocity ($v_F$) perpendicular to the applied field. The superconducting critical temperature $T_c$ is monitored by the strong-coupling parameter $\lambda$, the screened Coulomb repulsion parametrized by $\mu'$, and an average frequency $\omega_F$ for the pairing mechanism (close to the Debye frequency). The computation of $\mu'$ remains a difficult problem even for simple metals, but is expected to be between 0.1 and 0.15. In our calculation, $\mu'$ was fixed at 0.1, and was considered to be independent of the magnetic field. The value of $\lambda$ at zero field was estimated by comparing the experimental $H_2$ with specific heat data, as explained in the text. The parameter $\omega_F$ was then adjusted to give the right $T_c$ at zero field. The averaged Fermi velocity $v_F$ was adjusted to match the slope at $T_c$ of the experimental $H_2$ curve, for field along the a-axis: in UCoGe, along this hard magnetization axis, the field dependence of $\lambda$ should be very small (experimentally $\lambda$ remains almost unchanged for field H/a up to 5T). When the strong-coupling pairing strength $\lambda$ is varied, the renormalization of the Fermi velocity $v_F$ was taken into account, through equation (4). ($\nu_{\text{ba}}$) was considered to be field independent.

**Determination of $\lambda(0)$ in URhGe.** As explained in the main paper, the effects of the suppression of $\lambda$ under field in URhGe for H/a are much weaker, so the same method as used for UCoGe is not applicable. However, contrary to UCoGe, the specific heat has been estimated from magnetization measurements using Maxwell relations for the three directions. At the field $H_a$ parallel to b, the superconducting temperature is maximum, the specific heat coefficient is increased by more than 30%. If we believe that this increase of $\kappa_T$ is because of the reinforcement of the pairing mechanism, this puts severe constraints on the value of $\lambda(0)$, as $\lambda(0) = \frac{\gamma - 1}{\gamma - 1} \lambda(0)$. So we used the data along the b-axis in URhGe, to fix the value of $\lambda(0)$. Figure 8 shows the calculations used to derive $\gamma(T)$ for the optimal value $\lambda(0) = 0.75$, and Fig. 9 shows the comparison with the specific heat data, as well as the sensitivity to the value of $\lambda(0)$. It shows that the maximum value of the specific heat coefficient gives a selective criterion to fix $\lambda(0)$. With $\lambda(0) = 0.75$, $\lambda(T)$ increases almost by a factor 2, up to 1.4 at $H_a$ (see inset of Fig. 9) that puts URhGe in the strong-coupling limit, and it is a natural consequence of the large increase of the specific heat coefficient for H/b when it is attributed to the field variation of the pairing mechanism. The values of $\lambda$ along the b and c direction have been derived, as for UCoGe, using a field independent average frequency for the pairing mechanism $\omega_F = 5.3$ K, adjusted to match the experimental $H_c(0)$ at zero field, with a screened Coulomb repulsion parameter $\mu' = 0.1$. The renormalized Fermi velocity $\lambda(0)$ at $H_a$ is $3.1000$ m/s for H/b, and $3.7000$ m/s for H/c. However, the real values might be different, as a calculation for a p-wave state also includes an anisotropy coming from the gap anisotropy, which has not been taken into account in these calculations.

**Data availability.** The data that support the findings of this study are available from the corresponding author on reasonable request.

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Author contributions

D.A. prepared the crystals, J.H. and M.T. performed the first measurements of thermal and electrical transport, which were extended to high field by B.W., who also performed the specific heat measurements. C.P. did the magnetization and ac susceptibility measurements. Data analysis and interpretation were done by B.W., G.B. and J.-P.B. who supervised the work. The paper was written by B.W., and J.-P.B. with inputs from all authors.

Additional information

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