The Calculation Of The Normal Height Of The State Geodetic Network Point Based On The Local Quasigeoid Model

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Abstract. The data obtained during GPS observations is the basis for the creation of local quasigeoid models and obtaining differences in the heights of normal and ellipsoid surfaces of WGS-84. The network of satellite base stations located in the Rostov region is used as points of the state geodetic network (SGN). The gravimetric model is used as a model of the quasigeoid height (QH). The model of the quasigeoid is calculated and developed on the basis of gravimetric data. Five base stations located within the city of Rostov-on-Don are selected to obtain the normal heights of SGN points. The above points are recalculated from the local system to the WGS-84 system. The equation of the plane, approximating the local quasigeoid is constructed. The gravimetric heights of the quasigeoid are obtained by substituting the coordinate values of each point. The calculation of the normal heights of points is made after an estimation of the model accuracy.

1. Introduction

The method of high-precision satellite leveling, currently used in the engineering and geodetic works for the research and construction of real estate, in many types of work has replaced the traditional method of geometric leveling. A certain model of QH should be used in postprocessing the results of satellite leveling. Doctor of Technical Sciences Mayorov Andrei Nikolaevich distinguishes 2 types of QH: gravimetric and geometric (method and type of source information on which the QH models are based).

The geometric method of geodesy is the basis for creating a model of geometric QH. The essence of the method consists in creating the equation of the surface of the first order (plane) from the available data (the plane rectangular coordinates X, Y of the reference points of the SGN, their ellipsoidal and normal heights). The approximation to the surface KG and the subsequent ellipsoidal altitudes to normal altitudes at defined points of the satellite geodetic network (SGN) are carried out using this equation. More than three SGN points are used to obtain the equation of the plane, hence, excessive information is obtained. The possibility of obtaining approximation errors and estimating their magnitude appears due to data redundancy. The accuracy of geometric QH is determined by the accuracy of measuring the difference in ellipsoidal heights, the accuracy of determining the normal heights of the points of the SGN, and should not exceed 30-50 mm.

The physical method of geodesy is the basis for creating a model of gravimetric QH. The physical method of geodesy is based on the theory of M. S. Molodensky. His theory is used in the creation of local digital models (DM) and global models of gravimetric QH over a level (reference) ellipsoid.

Local available models of gravimetric QH are built, as a rule, in small territories.
Greater attention should be paid to the global DM of the QH, created by foreign researchers in the study of the figure of the Earth and its external gravitational field. The DM of the Earth's gravitational field (EGF) EGM2008 was published in 2008 on the website of the National Geospatial Information Agency of the USA. It allows translating information about the EGF anomalies into gravimetric QH, which are used in the implementation of the satellite leveling method in practice. Studies conducted in Russia and abroad have shown that the accuracy of gravimetric QH according to the data of the DM of the EGF EGM2008 is 50-100 mm for flat terrain.

The actuality of the conversion of geodetic heights to the reference surface of a quasigeoid is shown in the articles [1-3].

Analysis of the transformation of ellipsoidal heights obtained from GNSS observations to normal absolute heights is presented in [4-8].

The dependence between geometric and orthometric heights is presented in the articles [9-12]. Instead of immeasurable true orthometric heights, Helmert's orthometric heights are used.

The use of the test of the Mobile Automated Astronomical System in the form of a local network is shown in the articles [13-15].

The method that makes it possible to estimate the height difference between a quasigeoid and a geoid around the world is presented in [16-18].

The analysis of the simulation of a local quasigeoid based on geophysical data is presented in [19-20].

2. The creation of the quasigeoid model

The local quasigeoid model for obtaining the difference in heights between normal and ellipsoidal surfaces of WGS-84 is developed on the basis of data obtained by GPS observations at all points. The values of the parameters of the transition coordinates are determined by mathematical formulas and regional additional exact coefficients that express the differences between the ellipsoidal surface of the global coordinate system WGS-84 and the ellipsoidal surface of Krassovsky.

WGS-84 is an astronomo-geodesic-gravimetric reference system inscribed in the figure of the Earth. Any such system is characterized by the establishment of certain parameters. These parameters in the WGS-84 frame of reference include:

- the geocentric rectangular coordinate system with the origin at the point of the geometric center of the Earth mass;
- the mathematical basis for which the shape of the ellipsoid of rotation with specific geometric and physical quantities is adopted;
- the gravitational model of the Earth with the values determined on a specific date.

For the transition from the local coordinate system to the WGS-84 system, only three components of the origin shift must be defined and therefore only one control point has been captured using GPS. The formula of the inverse Helmert transform to determine the three parameters of the origin shift is as follows:

\[
\begin{bmatrix}
\Delta X \\
\Delta Y \\
\Delta Z
\end{bmatrix}
= \begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}_{WGS-84}
- \begin{bmatrix}
X \\
Y \\
Z
\end{bmatrix}_{La}
\]

where \(\Delta X, \Delta Y, \Delta Z\) – increments of coordinates;

\((X, Y, Z)_{WGS-84}\) – coordinates in the system WGS-84;

\((X, Y, Z)_{La}\) – coordinates in the local available system.

The geodetic coordinates of the same balanced points and the main sizes of ellipsoids located in two systems in the Rostov region are used in the systems to obtain the transition parameters.
The level of accuracy of the transition parameters depends on the number of points contained in the two systems and the features of their location.

Initial indicators are also included in GPS-observations within the framework of the works carried out in the Rostov region, and are intended for the creation of networks of alignment of heights of the first order and the second order. Ellipsoidal geodetic elevations in the WGS-84 system are also determined based on observations, with the exception of the normal height in the Baltic system of 1977.

The results of the data obtained by gravimetric and the aforementioned measurements served as the basis for creating a local quasigeoid model in order to obtain differences in altitudes of normal and ellipsoidal surfaces of WGS-84.

3. The calculation of the normal height of SGN point

The network of satellite base stations located on the territory of the Rostov region is used as points of SGN in the scientific work, and as a model of QH – gravimetric model, because for the implementation of the geometric model is not enough source data (catalog of normal heights of SGN points).

Five base stations located within the city of Rostov-on-Don are selected for calculating the normal heights of the points of the SGN. The initial information on these points is taken from the site https://geobridge.ru/maps [2], namely the name of the point, the ellipsoidal (geodesic) height \( H \) and the geodetic coordinates of the point (latitude \( B \) and longitude \( L \)) in the WGS-84 coordinate system. Angular coordinates are recalculated using a special program in linear plane rectangular coordinates \( X \) and \( Y \). Example of calculating the rectangular coordinates by geodesic:

Initial data:

\[
\text{Latitude} = 47^{\circ}14'26.52000045776367'' \\
\text{Longitude} = 39^{\circ}35'36.95999908447266''
\]

Calculation results:

\[
X = 5234269.519627487 \\
Y = 44944.37574798123
\]

The equation of the plane approximating the local quasigeoid is obtained by means of another special program. This equation has the form:

\[
QH = 0,0000170771X - 0,0000034157Y - 85,4210440331
\]

where the numerical values are the coordinate coefficients and the free term of the equation, respectively.

This equation shows the dependence between the gravimetric QH and the plane rectangular coordinates of the point. Substituting the values of the coordinates of each of the points, their gravimetric heights of the quasigeoid are obtained. It is necessary to find the arithmetic mean of all five results to estimate this local QH model. This value is 3,794 m. Then the deviation of the QH from the arithmetic mean is determined and the root-mean-square error in constructing the model is obtained from the well-known formula from the error theory:
\[ \mu = \frac{\sqrt{\sum P^2}}{(n-1)} \]  

where \( n \) – number of points.

The mean square deviation (MSD) is 0.064 m, that is 64 mm, which corresponds to the accepted accuracy of gravimetric QH. After evaluating the accuracy of the model, the normal heights of the points are determined as the difference in ellipsoidal height and QH:

\[ N_{\text{norm}} = N_{\text{el}} - \text{QH} \]

An example of the calculation is given in Table 1.

**Table 1.** The calculation of the normal height of the SGN points.

| Point name                        | Ellipsoidal height Nel, m | Coordinates, m | Gravimetric QH, m | The deviation of QH from the mean, V, m | Normal height Nnor, m |
|----------------------------------|---------------------------|----------------|-------------------|----------------------------------------|----------------------|
| South geodetic network - Rostov1 | 89.310                    | 5233287.578    | 58142986          | -0.045                                 | 85.561               |
| South geodetic network – Rostov2 | 105.630                   | 5240004.946    | 49671510          | 0.099                                  | 101.737              |
| SmartNet-ROST                    | 104.267                   | 5234269.52     | 44944376          | 0.017                                  | 100.456              |
| HIVE - RNDN                      | 33.275                    | 5231275.265    | 53511467          | -0.063                                 | 29.544               |
| HIVE - RNDN                      | 96.360                    | 5234619.326    | 54874826          | -0.010                                 | 92.576               |

| 3,794                             | 0,064                     |

**4. Conclusion**

The quasigeoid model was computed and developed on the basis of gravimetric data. Five base stations located within the city of Rostov-on-Don were selected to obtain the normal heights of SGN points. The above points were recalculated from the local system to the WGS-84 system. The equation of the plane, approximating the local quasigeoid is constructed. The gravimetric heights of the quasigeoid were obtained by substituting the coordinate values of each point. The MSD is 0.064 m, which corresponds to the accepted accuracy of gravimetric QH. The calculation of the normal heights of points was made after an estimation of the model accuracy. Gravimetric QH is 3,794 m, the deviation of QH from the mean is 0.064 m.
5. References

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