Application of Response Surface Method in Modification of Virtual Material Model

Fangyuan Cui¹,*, Dengxin Hua¹,*, Yan Li¹,a, Pengyang Li¹,c and Mingshun Yang¹, d

¹School of Mechanical and Precision Instrument Engineering, Xi’an University of Technology, Shaanxi, China

*Corresponding author e-mail: cfyxaut@126.com, *dengxinhua@xaut.edu.cn, aJyxy-ly@xaut.edu.cn, cJyxy-ly@xaut.edu.cn, dJyxy-ly@xaut.edu.cn

Abstract. Virtual material model is an effective equivalent modelling technique to simplify and characterize the stiffness weakening effect of mechanical joints. In order to further improve the accuracy of virtual material model, the density and thickness parameters of virtual material model were optimized and modified by response surface method. The advantage of this method is that under the condition of maintaining the original surface characteristics of the joint, the micro-characteristics of the joint under certain pre-tightening conditions can be quantitatively characterized macroscopically, which makes the analysis results more close to the actual situation. The natural frequencies of the virtual material model before and after modification are compared with the experimental natural frequencies, and the validity of the method and the modified model are verified.

1. Introduction
Mechanical structures are usually assembled together by joints, which causes a significant difference between the mechanical properties of joints and those of the mating substructures [1]. The virtual material model proposed by Tian [2] has been widely investigated from theory and experiment. Based on this model, Guo [3] proposed a method of modeling a taper joint and identifying its parameters. Hui [4] presented an experimental method to calculate the virtual material parameters.

All the mentioned methods of parameter identification are focused on the elastic modulus and Poisson’s ratio of the virtual material model. However, the density and thickness parameters of this model have never been updated. In this paper, the two parameters of the virtual material model are optimized and modified by response surface method. And the validity of the modified model has been verified.

2. Statement of the problem
In this virtual material model, the average density of two contact substructures is regarded as the equivalent density of the virtual material, and the thickness of virtual material layer is set as a constant of 1 mm.

The statement of the problem is as follows:
1. The thickness of asperities on the interface is a microscopic parameter which is not easy to measure accurately. Besides, this parameter will change with the different processing methods and lubrication conditions.

2. With the increase of normal load on the joint interface, there will be a certain degree of deformation in the bulk substrate [5], which is not considered in the available methods.

3. The physical, chemical and mechanical properties of the asperities will vary with different processing methods and lubrication conditions, which means that it is unreasonable to take the average density as the equivalent density of the virtual material model.

For such a joint interface with special material properties, its density and thickness are related to the density, surface roughness and normal pressure of the contact bodies. Therefore, the mathematical model of the two parameters can be revised as, respectively

\[
\rho = \rho (P_n, \rho_1, \rho_2, R_{a1}, R_{a2}) \tag{1}
\]

\[
h = h (P_n, h_1, h_2, R_{a1}, R_{a2}) \tag{2}
\]

3. Model modification

3.1. Model updating principle

The general idea is to achieve the minimization of the discrepancies between the experimental and FE natural frequencies jointed by the objective function, as shown in equation (3). Then the response surface method is adopted for arrangement of samples calculation and the two design parameters are updated by the multi-objective genetic algorithm (MOGA).

\[
\text{Min } F = \sum_i \left(1 - \frac{f_i^{\text{ex}}}{f_i^{\text{fe}}} \right)^2 
\]

3.2. Response surface modeling for modified parameters

The FE model of the test specimen is shown in Fig. 1. The tightening torque of each bolt is controlled by a pre-tightening wrench of 30 N·m. The first six natural frequencies before model modification are obtained by FE modal analysis. The simulation results are tabulated in Table 1. The measured natural frequencies [6] are also tabulated in Table 1.

![Figure 1. The FE model of the test specimen](image)
Table 1. The measured and simulated results

| T (N·m) | Natural frequencies/ Hz | 1st (Hz) | 2nd (Hz) | 3rd (Hz) | 4th (Hz) | 5th (Hz) | 6th (Hz) |
|---------|-------------------------|----------|----------|----------|----------|----------|----------|
| 30      | Test                    | 354.54   | 377.89   | 620.31   | 1600.53  | 1610.66  | 1760.12  |
|         | simulated               | 326.08   | 408.27   | 594.14   | 1516.3   | 1642     | 1747.9   |

Taking the two parameters, i.e. the density and thickness of the virtual material model, as the input values and the first six FE natural frequencies as the output values, respectively, the response surface model can be constructed. The obtained 17 groups of design points are shown in Table 2.

Table 2. Samples of the design points

| NO. | h/mm | / kg/m³ | f₁ (Hz) | f₂ (Hz) | f₃ (Hz) | f₄ (Hz) | f₅ (Hz) | f₆ (Hz) | F     |
|-----|------|---------|---------|---------|---------|---------|---------|---------|-------|
| 1   | 0.595| 43673   | 357.50  | 470.02  | 676.37  | 1668.35 | 1708.78 | 1961.27 | 0.09  |
| 2   | 0.09 | 43673   | 425.88  | 613.62  | 856.18  | 1828.24 | 1963.59 | 2381.64 | 0.77  |
| 3   | 0.3425 | 43673   | 385.34  | 528.05  | 750.77  | 1765.39 | 1797.09 | 2143.28 | 0.28  |
| 4   | 1.1  | 43673   | 317.34  | 394.15  | 575.87  | 1476.70 | 1609.56 | 1700.16 | 0.03  |
| 5   | 0.8475 | 43673   | 335.52  | 427.31  | 620.21  | 1563.84 | 1657.08 | 1817.34 | 0.02  |
| 6   | 0.595| 6606    | 357.50  | 471.35  | 676.93  | 1668.48 | 1708.91 | 1961.37 | 0.09  |
| 7   | 0.595| 25139.5 | 357.50  | 470.68  | 676.65  | 1668.42 | 1708.85 | 1961.32 | 0.09  |
| 8   | 0.595| 25139.5 | 357.49  | 470.68  | 676.65  | 1668.42 | 1708.85 | 1961.32 | 0.09  |
| 9   | 0.595| 62206.5 | 357.50  | 469.37  | 676.08  | 1668.42 | 1708.92 | 1961.23 | 0.09  |
| 10  | 0.09 | 6606    | 425.88  | 613.82  | 856.26  | 1828.26 | 1963.60 | 2381.65 | 0.77  |
| 11  | 0.3425 | 25139.5 | 385.34  | 528.44  | 750.93  | 1765.42 | 1797.11 | 2143.29 | 0.28  |
| 12  | 1.1  | 6606    | 317.34  | 396.41  | 576.88  | 1477.14 | 1609.93 | 1700.45 | 0.03  |
| 13  | 0.09 | 25139.5 | 335.52  | 428.22  | 620.61  | 1563.97 | 1657.20 | 1817.43 | 0.02  |
| 14  | 0.09 | 25139.5 | 335.52  | 428.22  | 620.61  | 1563.97 | 1657.20 | 1817.43 | 0.02  |
| 15  | 0.3425 | 62206.5 | 385.34  | 527.66  | 750.61  | 1765.36 | 1797.07 | 2143.26 | 0.28  |
| 16  | 1.1  | 80740   | 317.33  | 391.93  | 574.88  | 1476.26 | 1609.19 | 1699.88 | 0.02  |
| 17  | 0.8475 | 62206.5 | 335.52  | 426.41  | 619.82  | 1563.70 | 1656.97 | 1817.25 | 0.02  |

The constructed 3D response surface diagram is shown in Fig. 2. Apparent nonlinearity between the objective function and the thickness of the virtual material can be observed from this figure. However, the objective function and density are almost linearly inversely proportional. The sensitivity of Fig. 3 shows that the thickness has the greatest impact on the objective function, while the density has little impact on the objective function.
Objective function $F$

Local Sensitivity
Thickness $h$
Density $r$

Figure 2. The 3D response surface

Figure 3. The sensitivity relationships

On the basis of the established response surface model, multi-objective genetic algorithm (MOGA) is used to solve the parameters. Fig. 4 shows the convergence process of the objective function. It can be seen that the objective function converges after seven iterations where 50 design points are evaluated in each iteration.
3.3. Comparison of Results with Other Researchers

Table 3 shows the comparison of natural frequencies from the test and those of before and after the FE model updating. The comparison results show that the modified virtual material parameters can describe the dynamic characteristics of the system more accurately to a certain extent.

| Mode | Test natural frequencies(Hz) | FE natural frequencies(Hz) | Error 1(%) | Error 2(%) |
|------|-----------------------------|---------------------------|------------|------------|
|      | Before updating             | After updating            |            |            |
| 1    | 354.54                      | 326.08                    | -8.03      | 327.2      | -7.71      |
| 2    | 377.89                      | 408.27                    | 8.04       | 413.83     | 9.51       |
| 3    | 620.31                      | 594.14                    | -4.22      | 600.53     | -3.19      |
| 4    | 1600.53                     | 1516.3                    | -5.26      | 1524.3     | -4.76      |
| 5    | 1610.66                     | 1642                      | 1.95       | 1636.1     | 1.58       |
| 6    | 1760.12                     | 1747.9                    | -0.69      | 1763.6     | 0.20       |
| Total error |                          |                            | 28.19      | 26.95      |

4. Conclusion

The modified model can describe the dynamic characteristics of the system more accurately to some extent. However, because of the dry friction contact surface used in this case study, the comparative advantages are not obvious. Application of this method in oiled joint interface can be explored in the future.

Acknowledgments

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