THE CHANCES OF PRECISION ENHANCE FOR ULTRASONIC IMAGING

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Abstract. The results of ultrasonic imaging with the aid of an algorithm with the virtual rays is presented in this paper. The signal associated with the virtual rays is calculated as an arithmetical mean value of the signals of the rays surrounding the virtual one. Developed algorithm was tested on synthetic free noise data then polluted synthetic data in order to move for the real measurements. Conclusions about the imaging with new algorithm are not obvious. In same cases the significant improvement was achieved but in some not.

Keywords: ultrasound tomography, inverse problems, singular value decomposition

1. Additional, virtual ray between the real sensors – synthetic data case

One of the possibilities of enhancing the number of observations is to introduce an additional (virtual) rays between the real ones. In such a case the signal which belongs to the virtual rays will not be taken as a reference image. In some cases the significant improvement was achieved but in some not.

In this part of numerical experiments, one can say, that the introduction of virtual rays is able to improve the quality of the images. As was mentioned already such an information are called the virtual information, as the signal is not measured. They are only in our imagination and the virtual signals are calculated on the base of the real measurements. The virtual signals are calculated as an arithmetical mean value of the surrounding rays’ measurement.

Numerical experiment will be carried out in two steps. First, we will test the developed algorithm for the synthetic noise free data and next the noisy data. The second part of the experiment will be carried out for real measured data. The measured data for different configurations, were obtained with the aid of the sonic tomograph design by NETRIX R&D company [8]. The reconstructions treated as the reference images were carried out with the PICUS 3 Sonic Tomograph software [5].
It is worth to notice that the additional, virtual rays cause that the density of the rays inside the region is really high.

In order to be as close as possible to the real laboratory experiment the two closely placed inside objects were selected. Those objects are separated by 4 or 2 pixels as it is shown in Fig. 3. Such an example gives us a chance to investigate the proximity effect.

As a reference image (see Fig. 4) it was selected an image achieved without of additional virtual rays with 1% noisy data for the full fan ray after median filtering.

The number of 32 sensors restrict the number of measurements which for the full fan ray is $32 \times 31 = 992$. For the spatial resolution $64 \times 64$ pixels make 4096 unknowns excluding forbidden pixels visible in the Fig. 3 and Fig. 5 as a blue or black subarea respectively.

So, the imaging problem is reduced to the solution of a generalized (underdetermined) algebraic system of equations. Because the number of observations is less over four times than the number of unknowns, so the system of equations is deeply underdetermined.

The authors experience says that the best results could be achieved when the number of unknowns is not more than two times bigger than the number of observations. In spite of that, the solution by the FOCUSS function [4] gives acceptable results as it is presented in Fig. 4.

Adding the virtual rays, the number of observations rise to $32 \times (31+30) = 1952$, which means two times with respect of the case without of virtual rays. Reducing the ratio of unknowns to measurements by adding the virtual rays causes that the imaging produces much better results as it is shown in Fig. 5. Also, in this case the system of algebraic equations was solved by FOCUSS function and filtered by median filter [20].

Comparing the image in the Fig. 4 without of virtual rays with the image in Fig. 5 with the virtual rays one can justifiably say that in this particular case much better results was achieved. Such a conclusion is valid for pollution free data so far.

For the farther comparison, Fig. 6 shows the image which was achieved by left sided multiplication of a transposed coefficient matrix as well as the right-hand side vector and an application of SVD decomposition in order to get the solution. Using the left multiplication, the underdetermined system of equations became the square one. However, bed conditioned and what is more, very often rank deficient, so the solution is only possible by the decomposition method (SVD) [9].

As before, in a similar way the image was gain for synthetic noise free data using the full fan ray and filtered with the median filter. It is visible by comparison of the images in Fig. 5 and Fig. 6, the results are very similar, and is difficult to say which one is better.

Before we will pass to the real data measured in the NETRIX laboratory the algorithm was tested with the synthetic noisy data using the same object with the same obstacles inside. Results are presented in the following figures.

In Fig. 7 for the noisy synthetic data (1% of the noise) two images are presented for the separated object by four and by two pixels. Such an image maybe not ideal one could be compared with the image without the virtual rays (Fig. 4). Now for the noisy data it is not so obvious which algorithm with or without of virtual
rays is better. One thing is obvious. The virtual rays helped to see the gap between the separated obstacles. The gap is deliberately very narrow to see the influence of the proximity effect.

Fig. 7. Images for the noisy synthetic observations

From the other side the background of the image without of virtual rays is calmer what is an obvious advantage.

Conclusion: in case of synthetic data virtualization of rays has some sense. However, their influence on the quality of image, particularly for the noisy data is a little disappointed. The more vital will be behaviour of the proposed algorithm for the real data which will be presented in the next sections of this paper.

2. Additional, virtual ray between the real sensors – measured data case

Research of influence of virtual rays on the quality of imaging using the synthetic noise free data allow for a bit of optimism. It is not so optimistic as we applied the noisy data. That is why the next step with real life laboratory data will be investigated. Then, could be answered the main question if the virtual rays help improve the quality of the sonic images or do not. The three following cases will be considered.

The first case: three objects – excitation frequency 48 kHz

The measuring set up for the first case is shown in Fig. 8. The arrangement consists of three bottles filled out with an air. One of the bottles is placed in the geometrical centre of the region, where the sensitivity is the smallest. The frequency of excitation is 48 kHz.

Fig. 8. Setup for ultrasonic measurements with the aid of NETRIX tomograph

The Phantom of the region is presented in Fig. 9 and the image reconstruction with the aid of software of the Tomograph PICUS 3 [5] is shown in Fig. 10. This image would be treated as a reference image. It is easy to notice that the obstacle in the centre of the region has the worse representation in the reference picture. The images obtained with the aid of the algorithm with additional virtual rays are presented in the following figures. Images were obtained by three different methods which were described above, for two different spatial resolution: 32×32 pixels and 64×64 pixels.

Fig. 11. Imaging of the three objects based on laboratory measurements

For the spatial resolution 32×32 pixels in Fig. 11a without of virtual rays and Fig. 11b with virtual rays for different number of singular values were constructed the trial solutions. In Fig. 11c the solution was achieved by left sided multiplication by the coefficients matrix transposition (A'A, where A' means A\textsuperscript{T} in MATLAB nomenclature [20]). The differences in the images it is difficult to distinguish. Similarly, to the reference image, the central obstacle is not distinctly represented.

Fig. 12. Influence of the beam width on the imagining

The influence of the wideness of the fan ray on the quality of the image is presented in Fig. 12. The image for the whole fan ray is shown in Fig. 12a. For a narrower fan ray without one sensor on both side of the transmitter is shown in Fig. 12b and slightly narrower the fan ray without of two sensors on both sides of the transmitter in Fig. 12c. As one can see from those images the narrower ray produces slightly better results.

It could be explained by the following fact. The adjacent sensors have the measurements with the highest relative error due to their shortest distance between them. If such measurements would be excluded than the quality of data increase resulting with nicer images.
Increasing the spatial resolution do not enhance the quality of the images, as could be observed in Fig. 13. For the same number of observations, the number of unknowns become higher, leading to far worse underdetermination of algebraic system of equations. The Fig. 13a shows the image obtained by underdetermined system solution. One can see that something happened in the second quarter of the region. A slightly better result was achieved by the solution of the square system of equations for the first 400 singular values and then for 928. As one can see increasing more than two times the number of singular values does not help much (consult Fig. 13c).

The only way to enhance the quality of an image is increasing the number of observations by generating non-existent virtual rays.

The second case: four objects – excitation frequency 48 kHz

As a second case the four internal objects located as it is shown in Fig. 14 has been considered. The next Fig. 15 illustrates the reconstruction results. As before the object placed in the centre of the region has the weakest representation. This image will be a reference one in our experiment.

Reconstruction of the four distributed objects are presented in Fig. 16. In Fig. 16a and Fig. 16b the spatial resolution was 32×32 pixels but in Fig. 16c increased up to 64×64. For 32 sensors the lower spatial resolution guarantee overdetermination of the system of algebraic equations. One has got slightly more observations than the unknowns for such spatial resolution. The forbidden pixels inside of the square region which lay outside the circular region are not treated as unknown values. In Fig. 16a image was reconstructed without of virtual rays but in Fig. 16b with the virtual rays. It is very hard to judge which one is better.

The authors would like to believe that the second one, because it poses more tranquil background. Increasing the spatial resolution to 64×64 pixels the image deteriorates as it was in previous case (Fig. 13a). Explanation of this phenomenon remain the same.

So again, raises the question if for such a number of sensors better resolution is justified as it leads for worse imaging results?

In this case the left side multiplication by the transposition of the coefficient matrix are able to improve a little bit the image but under condition that the number of singular values for trial solution would be properly chosen. In Fig. 17 we can observe the distribution of singular values. At a first glance 500 singular values seems to be the correct one. But in the range of 400 till 500 singular values the curve goes down rapidly. We have to remember that the vertical axis is in a logarithmic scale, so within this range the singular values decreasing significantly. The best results were achieved not for 500 but for 150 singular values (see Fig. 18).

The background is calm, and it is easy to detect the trace of the internal objects, however with a small offset. The problems with the central object remained.

As a general remark in case of measurements is that one has to select the trial solution with a reasonable condition number. It is very hard to define the “reasonable” condition number. It depends on the case considered. By the numerical experiments the authors think that it is rather the dozens but definitely not the thousands.
For the four bottles case the condition number for jump of the singular values is \( \frac{d(1)}{d(496)} = 1231.7 \) (consult the Fig. 17). For such a number of singular values the trial solution produces a low-quality image. That is why finally only 150 singular values were selected what gives the condition number equal only 10.58. The results are visible in Fig. 18.

Comparing this result with the reference image (Fig. 15) there is no the central object and also the object placed vertically over the central one has also a very week representation.

**The third case: four objects – excitation frequency 400 kHz**

As the last experiment the four a smaller than in previous cases, bottles filled in air for excitation of 400 kHz was selected.

In the Fig. 21a the image from the solution of under-determined system of equations is shown but the Fig. 21b and Fig. 21c presents the images from the SVD solution for different number of singular values achieved after left side multiplication by transposition of the coefficient matrix.

It is worth to notice that one of the internal objects in the reference image is weaker than the rest ones. For our method as it is seen in Fig. 21, this one is hardly visible at all.

Enlarging the number of singular values does not improve the image (Fig. 22a). So far only the median filtering was applied. But if the image was treated by adaptive Wiener filter [20] we can observe an improvement of the image. Now all four internal objects are visible (see Fig. 22b).

![Fig. 19. Four objects location inside the region filled with the water](image)

![Fig. 20. Reconstruction with the aid of PICUS 3 software [5]](image)

The configuration of internal obstacles is presented in Fig. 19. In spite of that the smaller object is more difficult to identify the reference image is very good. Again, the question is if the proposed algorithm with the virtual rays would be able to produce reasonable image equally good as the reference one.

This time the images are presented in the highest resolution 64\times64 pixels. In the Fig. 21 the upper row is showing the raw images but the lower row images after median filtering.

![Fig. 21. Images for the virtual rays and four objects from the Fig. 19: upper row the raw images but the lower row images after 2D median filtering a) FOCUSS b) SVD \( A'A - d(100) \) c) SVD \( A'A - d(300) \)](image)

![Fig. 22. Image for the four objects from the Fig. 19: a) SVD \( A'A - d(500) \), b) FOCUSS filtered with the aid of the adaptive Wiener method [20]](image)

**3. Conclusion**

In this paper a new method for sonic imaging with virtual rays was presented. Algorithm tested on synthetic noise free data shows significant improvement when the virtual rays were engaged.

However noised data reveal sensitivity of the new algorithm on the noisy data. Merely 1% noise was able to distort significantly the image.

That was not good perspective for the real application of the algorithm. So, the most important was the behaviour of the proposed algorithm in the second part of experiment, with the real data.

The results are not as obvious and not unambiguous as one could expect. For some experiment, improvement could be visible but for the other rather not.

That is why, according the authors opinion, this algorithm based on a very strong simplifying assumptions like for example not taking into account reflecting signals, has reached the end of its ability.

The further sonic imaging improvement could be reached due to relaxing some of the strongest simplifying assumptions.

It will depend on the ability of the measurement, if we would be able to measure the reflecting signals inside the region. Such an ability allows to move from the transition mode to the reflecting mode. Authors believe that it helps to get much more precise images.

**References**

[1] Bartušek K., Fišla P., Mikulka J.: Numerical Modeling of Magnetic Field Deformation as Related to Susceptibility Measured with an MR System. Radioengineering 17(2)2008, 113–118.

[2] Bartušek K., Drexler P., Fišla P. et al.: Magnetoinductive Lens for Experimental Mid-field MR Tomograph. Progress in Electromagnetics Research Symposium Location Cambridge 2010, 1047–1050.

[3] Dušek J., Hladký D., Mikulka J.: Electrical Impedance Tomography Methods and Algorithms Processed with a GPU. PIERS Proceedings (Spring) 2017, 1710–1714.

[4] Goroditskny I.F., George J.S., Rao B.D.: Neuromagnetic source imaging with FOCUSS: a recursive weighted minimum norm algorithm. Clinical Neurophysiology 95/1995, 231–231.

[5] Göcke L.: PICUS Sonic Tomograph Software Manual, Argus Electronic GmbH, Erich-Schlesinger-Straße 49d, 18059 Rostock, Germany, 2017, www.argus-electronic.de.
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