Adaptive fuzzy practical tracking control for flexible-joint robots via command filter design

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Abstract
This paper investigates the issue of finite-time tracking control for flexible-joint robots. In the design scheme, the unknown continuous function is identified by a fuzzy system. By introducing the command filter technique, "explosion of complexity" problem which arises from repeated differentiation of virtual controllers is avoided. Meanwhile, errors resulting from the first-order filters can be reduced with the introduced compensation signal. Besides, the proposed method ensures that the tracking performance could be achieved within a limited time. Eventually, the simulation is given to demonstrate the effectiveness of the proposed scheme.

Keywords
Flexible-joint (FJ) robots, adaptive tracking control, command filter backstepping, finite-time control

Introduction
Compared to the rigid-joint robots, flexible-joint (FJ) robots have many advantages of high performance, such as light mass, small size, and low energy consumption. They have been widely studied in the past decades.1–3 Therefore, the research on tracking control of FJ robots also is of great significance.4–8 For example, based on the singular perturbation method, Kim and Croft8 realize the full-state tracking control of FJ robots. With the aid of the tan-type barrier Lyapunov function, Sun et al.9 propose an adaptive tracking controller for FJ robot systems with full-state constraints. In fact, the FJ robot system is a typical under-actuated system, and the research about this kind of system can refer to the wheeled inverted pendulum system,10 the crane system,11,12 and so on.

The backstepping method is used in many of the above papers; however, one of its disadvantages is that it requires repeated derivatives, which can result in “explosion of complexity” problem and increase the complexity of controllers, especially for the system with a higher dimension. Although the above troubles were handled by the dynamic surface control (DSC) method in previous works,13–15 errors arising from the filters are not solved and the quality of the controller is also greatly reduced in this way. Another way is to apply the command filter technique to the backstepping design, by which the first problem can be successfully avoided. By introducing the compensated signal, the drawback of the DSC can be overcome (see the work by Farrell et al.,20 Hu and Zhang,21 and Niu et al.22). Considering the uncertain nonlinear systems with actuator faults, Li23 developed a fault-tolerant control scheme by the aid of command filter design. For the switched nonlinear systems in Hou and Tong,24 the issue of output feedback control is addressed with the command filter backstepping technique. For nonlinear systems with saturation input, the finite-time tracking control problem with command filter is investigated in Yu et al.25

As we know, due to the ability to deal with structural uncertainty, the adaptive control method is widely employed to address uncertain nonlinear systems. With the quality of approximating unknown function, fuzzy logic systems (FLS) play a crucial part in handling the unknown items needed in control design. Therefore, the successful application of FLS in adaptive control can properly avoid burdensome computations and significantly improve the control performance of systems; many results have been obtained.26–30 With the help of Nussbaum-type function in Sun et al.,27 an adaptive fuzzy control method is proposed for the nonlinear systems with unknown control directions. For the high-

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order stochastic nonlinear systems, Sun et al.\textsuperscript{28} consider the issue of reduced adaptive fuzzy control. Similarly, the unknown and uncertainty problems in this paper are also addressed by the adaptive fuzzy control scheme.

Inspired by the above works, this paper studies the problem of finite-time tracking control for FJ robots and develops an adaptive fuzzy control algorithm with the help of the command filter technique. The main contributions are summarized as follows:

1. Compared with the design in Sun et al.,\textsuperscript{9} the explosion of complexity problem is avoided by applying the command filter technique to the backstepping design. Thus, the computing burden is also reduced to some extent. With the aid of compensated signals, the errors resulting from the utilization of DSC in Liu and Wu\textsuperscript{15} can be removed.

2. Different from the existing schemes that can only guarantee infinite-time stability, this paper considers the convergence rate of tracking error and makes full use of the finite-time stability criterion to design an adaptive fuzzy controller, which ensures that the tracking error can achieve practical finite stable.

**System description and preliminaries**

The dynamic model of an n-link FJ robot can be expressed as

\[
M(q)\ddot{q} + C(q, \dot{q}) + G(q) + F(\dot{q}) + Kq = Kq_m \tag{1}
\]

\[
J\ddot{q}_m + B\dot{q}_m + K(q_m - q) = u \tag{2}
\]

in which \( q, \dot{q}, \ddot{q} \in \mathbb{R}^n \) represent the link position, velocity, and acceleration vectors, respectively. \( M(q) \in \mathbb{R}^{n\times n} \) stands for the inertia matrix that is symmetric and positive definite, \( C(q, \dot{q}) \in \mathbb{R}^n \) is the Coriolis and centripetal forces, \( G(q) \in \mathbb{R}^n \) represents the gravity vector, and \( F(\dot{q}) \in \mathbb{R}^n \) denotes the friction term. \( q_m, \dot{q}_m, \ddot{q}_m \in \mathbb{R}^n \) represent the rotor angular position, velocity, and acceleration vectors, respectively. \( K, J, B \in \mathbb{R}^{n\times n} \) are constant positive definite diagonal matrices and denote the joint flexibility, the actuator inertia, and the natural damping term, respectively. \( u \in \mathbb{R}^n \) is the torque input at each actuator.

The goal of design is to construct the adaptive tracking controller which can guarantee that the link position \( q \) tracks the target signal \( q_d \) in a finite time and all signals in the closed-loop system remain bounded, where \( q_d \in \mathbb{C}^1, \dot{q}_d \) and \( \ddot{q}_d \) are bounded.

Let \( x_1 = q, x_2 = \dot{q}, x_3 = q_m, \) and \( x_4 = \dot{q}_m \), then equations (1) and (2) can be converted into

\[
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= M^{-1}(x_1)Kx_3 + M^{-1}(x_1) [-C(x_1, x_2) - G(x_1) - F(x_2) - Kx_1] \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= J^{-1}u + J^{-1}[-Bx_4 - K(x_3 - x_1)]
\end{aligned}
\tag{3}
\]

In what follows, for simplicity, we note

\[
\begin{aligned}
g_1 &= I \\
g_2 &= M^{-1}(x_1)K \\
g_3 &= I \\
g_4 &= J^{-1}F \\
f_1 &= M^{-1}(x_1)[-C(x_1, x_2) - G(x_1) - F(x_2) - Kx_1] \\
f_2 &= J^{-1}[-Bx_4 - K(x_3 - x_1)]
\end{aligned}
\tag{4}
\]

Hence, equation (3) can be rewritten as

\[
\begin{aligned}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= g_2x_3 + f_2 \\
\dot{x}_3 &= x_4 \\
\dot{x}_4 &= g_4u + f_4
\end{aligned}
\tag{5}
\]

Next, we will introduce some definitions and lemmas that are useful to achieve control objective.

**Definition 1.** For the nonlinear system \( \dot{x} = f(x(t)) \), the equilibrium \( x = 0 \) is practical finite-time stable, if for any initial condition \( x(0) \in x_0 \) there exists a constant \( \varepsilon > 0 \) and the settling time \( T(\varepsilon, x_0) < \infty \) such that\textsuperscript{31}

\[
\|x(t)\| < \varepsilon, \forall t > T
\]

**Lemma 1.** For any real numbers \( \alpha, \beta > 0, 0 < p, \mu < 1, 0 < \gamma < \infty \), if there exists the function \( V(x) \) satisfying\textsuperscript{23}

\[
\dot{V}(x) \leq -\alpha V(x) - \beta V^p(x) + \gamma
\]

then the trajectory of \( \dot{x} = f(x(t)) \) is practical finite-time stable, and the settling time \( T \) satisfies

\[
T \leq \frac{1}{\alpha(1 - p)} \ln \frac{\alpha V^{1-p}x(0) + \mu \beta}{\mu \beta}
\]

**Lemma 2.** For any constant \( b_i > 0 \) \((i = 1, 2, 3)\) and real variables \( x, y \), one has\textsuperscript{32}

\[
|x|^{b_1} |y|^{b_2} \leq \frac{b_1}{b_2} |x|^{b_3} |y|^{b_4} + \frac{b_2}{b_1} |x|^{b_3} |y|^{b_4} + b_2
\]

FLS will be used to estimate the unknown continuous function in the subsequent design process.
IF-THEN Rules: If $x_1$ is $F_1^i$ and ... and $x_n$ is $F_n^i$, then $y$ is $G^i$, $i = 1, ..., n$.

FLS can be expressed as

$$y(x) = \frac{\sum_{i=1}^{N} \Phi_i \prod_{j=1}^{n} \mu_{F_j}(x_j)}{\sum_{i=1}^{N} \left( \prod_{j=1}^{n} \mu_{F_j}(x_j) \right)}$$

(6)

Let

$$w_i(x) = \frac{\prod_{j=1}^{n} \mu_{F_j}(x_j)}{\sum_{i=1}^{N} \prod_{j=1}^{n} \mu_{F_j}(x_j)}$$

$$W(x) = [w_1(x), w_2(x), \ldots, w_N(x)]^T, \quad \Phi = [\Phi_1, \Phi_2, \ldots, \Phi_N]^T$$

then we have

$$y(x) = \Phi^T W(x)$$

(7)

Lemma 3. For any $\epsilon > 0$ and a continuous function $h(x)$ defined on a compact set $U$, there is an FLS $\Phi^T W(x)$ satisfying

$$\sup_{x \in U} |h(x) - \Phi^T W(x)| \leq \epsilon$$

(8)

Finite-time controller

This section is devoted to the design procedure of the command filtered controller.

Define the tracking error as

$$z_1 = x_1 - q_d$$
$$z_i = x_i - \tilde{\alpha}_i$$

(9)

(10)

where $\tilde{\alpha}_i \in R^n$ is the output of command filter with respect to $\alpha_i$, which is defined as

$$\tilde{\alpha}_i = \hat{\alpha}_i, \quad \hat{\alpha}_i(0) = \alpha_i(0), \quad i = 2, 3, 4$$

with $\epsilon_i > 0$ being a designed constant.

To remove the filtering errors arising from the command filters, we employ the compensating signal $\xi_i$

$$\dot{\xi}_1 = -k_1 \xi_1 + (\alpha_2 - \alpha_3) + \xi_2 - L_1 \text{sgn}(\xi_1)$$
$$\dot{\xi}_2 = -k_2 \xi_2 + g_2(\alpha_3 - \alpha_4) - \xi_1 + g_2 \dot{\xi}_3 - L_2 \text{sgn}(\xi_2)$$
$$\dot{\xi}_3 = -k_3 \xi_3 + (\alpha_4 - \alpha_5) - g_2 \xi_2 + \xi_4 - L_3 \text{sgn}(\xi_3)$$
$$\dot{\xi}_4 = -k_4 \xi_4 + g_4(\alpha_5 - \alpha_3) - \xi_3 - L_4 \text{sgn}(\xi_4)$$

(11)

where $k_i > 0$, $\xi_i \in R^n$, $\xi_i(0) = 0$, $L_i = \text{diag}(l_{i1}, l_{i2}, \ldots, l_{in}) \in R^{n \times n}$, $l_i = [l_{i1}, l_{i2}, \ldots, l_{in}]^T \in R^n$, $l_{ij} > 0$, $i = 1, ..., 4$, $j = 1, ..., n$, and $\text{sgn}(\xi_i) = [\text{sgn}(\xi_{i1}), \text{sgn}(\xi_{i2}), \ldots, \text{sgn}(\xi_{in})]^T \in R^n$. Furthermore, define the compensating tracking error as

$$\chi_i = z_i - \xi_i, \quad i = 1, ..., 4$$

(12)

Before providing the following detailed design, we need to define a constant

$$\theta = \max\{\|\Phi_i\|^2\}, \quad i = 1, 2$$

and $\hat{\theta}$ is the estimation of $\theta$.

Step 1. Taking the derivative of $\chi_1$ yields

$$\dot{\chi}_1 = \dot{z}_1 - \dot{\xi}_1$$
$$= x_2 - \dot{q}_d - \dot{\xi}_1$$
$$= z_2 + (\alpha_2 - \alpha_3) + \alpha_2 - \dot{q}_d - \dot{\xi}_1$$
$$= \chi_2 + \alpha_2 - \dot{q}_d + k_1 z_1 - \dot{\xi}_1 + L_1 \text{sgn}(\xi_1)$$

Define

$$V_1 = \frac{1}{2} \chi_1^T \chi_1$$

(14)

then we have

$$\dot{V}_1 = \chi_1^T \chi_2 + \chi_1^T (\alpha_2 - \dot{q}_d + k_1 z_1)$$
$$= -k_1 \chi_1^T \chi_1 + \chi_1^T \chi_2 + \chi_1^T (\alpha_2 - \dot{q}_d + k_1 z_1)$$
$$= \chi_1^T \chi_2 + \chi_1^T (\alpha_2 - \dot{q}_d + k_1 z_1)$$
$$= \chi_1^T \chi_2 + \chi_1^T L_1 \text{sgn}(\xi_1)$$

(15)

By the aid of Young’s inequality, one has

$$\chi_1^T L_1 \text{sgn}(\xi_1) \leq \frac{1}{2} \chi_1^T \chi_1 + \frac{1}{2} \| L_1 \|^2 \| L_1 \|$$

(16)

The virtual controller $\alpha_2$ is designed as

$$\alpha_2 = \dot{q}_d - k_1 z_1 - c_1 (\chi_1^T \chi_1)^{p-1}$$

(17)

with a known constant $c_1 > 0$. By plugging equations (16) and (17) into equation (15), we have

$$\dot{V}_1 \leq - \left( k_1 - \frac{1}{2} \right) \chi_1^T \chi_1 - c_1 (\chi_1^T \chi_1)^p + \chi_1^T \chi_2 + \frac{1}{2} \| L_1 \|^2 \| L_1 \|$$

(18)

Step 2. The time derivative of $\chi_2$ is

$$\dot{\chi}_2 = \dot{z}_2 - \dot{\xi}_2$$
$$= g_2 x_3 + f_2 - \dot{\alpha}_2 - \dot{\xi}_2$$
$$= g_2 x_3 + g_2 (\alpha_3 - \alpha_4) + g_2 \alpha_3 + f_2 - \dot{\alpha}_2 - \dot{\xi}_2$$
$$= g_2 x_3 + g_2 (\alpha_3 - \alpha_4) + f_2 - \dot{\alpha}_2 - k_2 z_2 - k_2 \chi_2$$
$$+ \dot{\xi}_1 + L_2 \text{sgn}(\xi_2)$$

(19)

Choose

$$V_2 = V_1 + \frac{1}{2} \chi_2^T \chi_2 + \frac{1}{2} \theta^2$$

(20)

where $\dot{\theta} = \theta - \dot{\theta}$, and $r > 0$ is a known constant. Then, we could get
\( \dot{V}_2 = V_1 + x_2^T g_2 x_3 + g_2 \alpha_3 + f_2 - \dot{\alpha}_2 + k_2 z_2 = k_2 x_2 + k_2 x_2 \)
\[ + \xi_1 + L_{s2} sgn(\xi_2) - \frac{1}{r} \dot{\theta} \theta \]
\[ \leq - \left( k_1 - \frac{1}{2} \right) x_1^T x_1 - c_1 (\chi_1^T x_1)^p + \chi_2^T g_2 x_3 \]
\[ + \frac{1}{2} \eta_i^2 + \frac{1}{r} \dot{\theta} \theta \]
\[ + \chi_2^T (z_1 + g_2 \alpha_3 - \dot{\alpha}_2 + k_2 z_2) + \| x_2^T \| \| f_2 \| \]
\[ - k_2 x_2^2 + \chi_2^T L_{s2} sgn(\xi_2) \]
\[ (21) \]

The following inequality is similar to equation (16)
\[ \chi_2^T L_{s2} sgn(\xi_2) \leq \frac{1}{2} \chi_2^T x_2^2 + \frac{1}{2} \eta_i^2 \]
\[ (22) \]

In view of Lemma 3, \( \| f_2 \| \) is estimated by following FLS. For any \( \varepsilon_i > 0 \)
\[ \| f_2 \| = \Phi_T W_1(x_i) + \delta_i(x_i), \]
\[ \| \delta_i(x_i) \| \leq \varepsilon_i, \]
where \( \varepsilon_i(x_i) \) is the approximation error. With the completion of squares, it is obtained that
\[ \| x_2^T \| \| f_2 \| = \| x_2^T \| \Phi_T W_1(x_i) + \| x_2^T \| \delta_i(x_i) \]
\[ \leq \frac{\theta x_2^T x_1 W_1^T W_1}{2a_i^2} + \frac{x_2^T x_2}{2} + \frac{a_i^2}{2} + \frac{\varepsilon_i^2}{2} \]
\[ (23) \]

Substituting equations (22) and (23) into equation (21) produces
\[ \dot{V}_2 \leq - \frac{1}{2} \sum_{i=1}^{2} \left( k_i - \frac{1}{2} \right) x_i^T x_i - c_1 (\chi_i^T x_i)^p + \chi_2^T g_2 x_3 \]
\[ + \frac{1}{2} \sum_{i=1}^{2} \eta_i^2 + \frac{1}{r} \dot{\theta} \theta \]
\[ + \chi_2^T (z_1 + g_2 \alpha_3 - \dot{\alpha}_2 + k_2 z_2) + \frac{\theta x_2^T x_1 W_1^T W_1}{2a_i^2} \]
\[ + \frac{x_2^T x_2}{2} + \frac{a_i^2}{2} + \frac{\varepsilon_i^2}{2} \]
\[ (24) \]

The virtual controller \( \alpha_3 \) is designed as
\[ \alpha_3 = g_2 \]
\[ \left[ -z_1 + \dot{\alpha}_2 - k_2 z_2 - c_2 x_2 (\chi_2^T x_2)^{p-1} - \frac{\theta x_2^T x_1 W_1^T W_1}{2a_i^2} - \frac{x_2}{2} \right] \]
\[ (25) \]

with \( c_2 > 0 \) being a known constant. By plugging equation (25) into equation (24), one can get
\[ \dot{V}_2 \leq - \frac{1}{2} \sum_{i=1}^{2} \left( k_i - \frac{1}{2} \right) x_i^T x_i - \frac{1}{2} \sum_{i=1}^{2} c_i (\chi_i^T x_i)^p + \chi_2^T g_2 x_3 \]
\[ + \frac{1}{2} \sum_{i=1}^{2} \eta_i^2 + \frac{a_i^2}{2} + \frac{\varepsilon_i^2}{2} + \theta \left( \frac{x_2^T x_1 W_1^T W_1}{2a_i^2} - \frac{1}{r} \right) \]
\[ (26) \]

Step 3. From equation (12), the derivative of \( x_3 \) gives
\[ \dot{x}_3 = \dot{z}_3 - \dot{\xi}_3 \]
\[ = z_4 - \dot{\alpha}_3 - \dot{\xi}_3 \]
\[ = z_4 + (\dot{\alpha}_4 - \dot{\alpha}_3) + \dot{\alpha}_4 - \dot{\alpha}_3 - \dot{\xi}_3 \]
\[ = x_4 + \alpha_4 - \dot{\alpha}_3 + k_3 z_3 - k_3 x_3 + g_2^T \xi_2 + L_3 sgn(\xi_3) \]
\[ (27) \]

Choose
\[ V_3 = V_2 + \frac{1}{2} \chi_3^T x_3 \]
\[ (28) \]

The derivation of equation (28) is presented as
\[ \dot{V}_3 = \dot{V}_2 + \chi_3^T [x_4 + \alpha_4 - \dot{\alpha}_3 + k_3 z_3 - k_3 x_3 + g_2^T \xi_2 + L_3 sgn(\xi_3)] \]
\[ \leq - \sum_{i=1}^{3} \left( k_i - \frac{1}{2} \right) x_i^T x_i - \sum_{i=1}^{3} c_i (\chi_i^T x_i)^p + \chi_3^T x_3 \]
\[ + \frac{1}{2} \sum_{i=1}^{3} \eta_i^2 + \frac{a_i^2}{2} + \frac{\varepsilon_i^2}{2} + \theta \left( \frac{x_3^T x_3 W_1^T W_1}{2a_i^2} - \frac{1}{r} \right) \]
\[ + k_3 x_3^T x_3 + \chi_3^T L_3 sgn(\xi_3) \]
\[ (29) \]

Similar to equation (16), we obtain
\[ \chi_3^T L_3 sgn(\xi_3) \leq \frac{1}{2} \chi_3^T x_3 + \frac{1}{2} \eta_i^2 \]
\[ (30) \]

Design the virtual controller \( \alpha_4 \) as
\[ \alpha_4 = - g_2 z_2 + \dot{\alpha}_4 - k_3 z_3 - c_3 x_3 (\chi_3^T x_3)^{p-1} \]
\[ (31) \]

where \( c_3 > 0 \) is a known constant. By substituting equations (30) and (31) into equation (29), we have
\[ \dot{V}_3 \leq - \sum_{i=1}^{3} \left( k_i - \frac{1}{2} \right) x_i^T x_i - \sum_{i=1}^{3} c_i (\chi_i^T x_i)^p + \chi_3^T x_3 \]
\[ + \frac{1}{2} \sum_{i=1}^{3} \eta_i^2 + \frac{a_i^2}{2} + \frac{\varepsilon_i^2}{2} + \theta \left( \frac{x_3^T x_3 W_1^T W_1}{2a_i^2} - \frac{1}{r} \right) \]
\[ (32) \]

Step 4. From equation (12), we have
\[ \dot{x}_4 = \dot{z}_4 - \dot{\xi}_4 \]
\[ = \dot{x}_4 - \dot{\alpha}_4 - \dot{\xi}_4 \]
\[ = g_4 \alpha_5 + f_4 - \dot{\alpha}_4 + k_4 z_4 - k_4 x_4 + \xi_3 + L_4 sgn(\xi_4) \]
\[ (33) \]

Choose the Lyapunov function as
\[ V_4 = V_3 + \frac{1}{2} \chi_4^T x_4 \]
\[ (34) \]

It can be concluded that...
\[ \dot{V}_4 = \dot{V}_3 + \chi_4^T g_4 \alpha_5 + f_4 - \dot{\alpha}_4 + k_4 z_4 - k_4 x_4 + \xi_3 + L_4 \text{sgn}(\xi_4) \]

\[ \leq - \frac{3}{2} \sum_{i=1}^{3} (k_i - \frac{1}{2}) \chi_i^T x_i - \frac{3}{2} \sum_{i=1}^{3} c_i(\chi_i^T x_i)^p + \sum_{i=1}^{3} \left( \frac{a_i^2}{2} + \frac{e_i^2}{2} \right) + \dot{\theta} \left( \frac{\chi_3^T \chi_2 W_1^T W_1}{2 a_i^2} - \frac{1}{r \dot{\theta}} \right) + \chi_4^T (z_3 + g_4 \alpha_5 - \dot{\alpha}_4 + k_4 z_4) + \|x_4^T\| \|f_4\| - k_4 x_4^T x_4 + \chi_4^T L_4 \text{sgn}(\xi_4) \]

(35)

By the aid of equation (8), \( \|f_4\| \) can be approximated by following FLS

\[ \|f_4\| = \Phi_2^T W_2(X_2) + \delta_2(X_2) \]

\[ |\delta_2(X_2)| \leq e_2 \]

where \( e_2 \) is an arbitrary positive constant and \( \delta_2(X_2) \) is the approximation error. Based on the completion of squares, it is concluded that

\[ \|x_4^T\| \|f_4\| = \|x_4^T\| \|\Phi_2 W_2(X_2)\| + \|x_4^T\| |\delta_2(X_2)| \]

\[ \leq \frac{\theta \chi_3^T \chi_4 W_2^T W_2}{2 a_i^2} + \frac{\chi_4^T x_4}{2} + \frac{a_i^2}{2} + \frac{e_i^2}{2} \]

(36)

Similar to equation (16), we obtain

\[ \chi_4^T L_4 \text{sgn}(\xi_4) \leq \frac{1}{r} \chi_4^T x_4 + \frac{1}{2} \mu^2_4 \]

(37)

Substituting equations (36) and (37) into equation (35) yields

\[ \dot{V}_4 \leq - \sum_{i=1}^{3} \left( k_i - \frac{1}{2} \right) \chi_i^T x_i - \sum_{i=1}^{3} c_i(\chi_i^T x_i)^p + \sum_{i=1}^{3} \frac{1}{r \dot{\theta}} \dot{\theta} \]

\[ + \sum_{i=1}^{3} \left( \frac{a_i^2}{2} + \frac{e_i^2}{2} \right) + \chi_4^T (z_3 + g_4 \alpha_5 - \dot{\alpha}_4 + k_4 z_4) + \frac{\theta \chi_3^T \chi_4 W_2^T W_2}{2 a_i^2} + \frac{\chi_4^T x_4}{2} \]

(38)

Design the actual controller \( u \) and the adaptive law \( \dot{\theta} \) as

\[ u = \alpha_5 = g_4^{-1} \]

\[ \begin{bmatrix} \dot{z}_3 + \dot{\alpha}_4 - k_4 z_4 - c_4 x_4 \dot{\alpha}_4 \end{bmatrix} \]

\[ \dot{\theta} = \left( \frac{\chi_3^T \chi_2 W_1^T W_1}{2 a_i^2} + \frac{\chi_4^T x_4^T W_2^T W_2}{2 a_i^2} \right) - \frac{\chi_4^T x_4}{2} \]

(39)

(40)

where \( c_4 > 0 \) and \( q > 0 \) are known constants.

Substituting equations (39) and (40) into equation (38), we obtain

\[ \dot{V}_4 \leq - \sum_{i=1}^{3} \left( k_i - \frac{1}{2} \right) \chi_i^T x_i - \sum_{i=1}^{3} c_i(\chi_i^T x_i)^p + \sum_{i=1}^{3} \frac{1}{r \dot{\theta}} \dot{\theta} \]

\[ + \sum_{i=1}^{3} \left( \frac{a_i^2}{2} + \frac{e_i^2}{2} \right) + \frac{q}{r \dot{\theta}} \dot{\theta} \]

(41)

\[ \textbf{Stability analysis} \]

\textbf{Theorem 1.} Consider the nonlinear system (equation (3)), under the virtual controller (equations (17), (25), and (31)), the actual controller (equation (39)), and the adaptive law (equation (40)), the tracking error is practical finite stable and all signals in the resulting system are bounded.

\textbf{Proof.} According to \( \dot{\theta} = \theta - \hat{\theta} \), one has

\[ \frac{q}{r \dot{\theta}} \dot{\theta} \leq - \frac{q}{2 r} \dot{\theta}^2 + \frac{q}{2 r} \theta^2 \]

(42)

Substituting equation (42) into equation (41) produces

\[ \dot{V}_4 \leq - \sum_{i=1}^{3} \left( k_i - \frac{1}{2} \right) \chi_i^T x_i - \sum_{i=1}^{3} c_i(\chi_i^T x_i)^p + \sum_{i=1}^{3} \frac{1}{r \dot{\theta}} \dot{\theta} \]

\[ + \sum_{i=1}^{3} \left( \frac{a_i^2}{2} + \frac{e_i^2}{2} \right) - \frac{q}{2 r} \dot{\theta}^2 + \frac{q}{2 r} \theta^2 \]

\[ = - \sum_{i=1}^{3} \left( k_i - \frac{1}{2} \right) \chi_i^T x_i - \sum_{i=1}^{3} c_i(\chi_i^T x_i)^p + \sum_{i=1}^{3} \frac{1}{r \dot{\theta}} \dot{\theta} \]

\[ + \sum_{i=1}^{3} \left( \frac{a_i^2}{2} + \frac{e_i^2}{2} \right) - \frac{q}{2 r} \dot{\theta}^2 + \frac{q}{2 r} \theta^2 - \frac{q}{2 r} \theta^p \]

(43)

Using Lemma 2 to the term \( q(\dot{\theta}^2/2r)^{\frac{p}{2}} \), the following inequality holds

\[ q \left( \frac{\dot{\theta}^2}{2r} \right)^{\frac{p}{2}} \leq \sqrt{q_1} q \frac{\dot{\theta}^2}{2r} + q(1 - p) \left( \frac{p}{q_1} \right)^{\frac{p}{2}} \]

(44)

with \( 0 < q_1 < 1 \). By substituting equation (44) into equation (43), it can be proved that
\[ V_4 \leq - \sum_{i=1}^{4} \left( k_i - \frac{1}{2} \right) x_i^p + \sum_{i=1}^{4} c_i (x_i^p) + \sum_{i=1}^{4} \frac{1}{2} \dot{q}_i^2 \\
\leq - \sum_{i=1}^{4} k_i \xi^p_i + \sum_{i=1}^{4} \frac{\xi^p_i}{2} \left| g(\alpha_i + \alpha_i + 1) - L \text{sgn}(\xi_i) \right| \\
= - \sum_{i=1}^{4} k_i \xi^p_i - \sum_{i=1}^{4} \left| \xi^p_i \right| \left| g(\alpha_i + \alpha_i + 1) - L \text{sgn}(\xi_i) \right| \\
\leq - \sum_{i=1}^{4} k_i \xi^p_i - \sum_{i=1}^{4} \left| \xi^p_i \right| \left( \left| L \text{sgn}(\xi_i) \right| - \left| g(\alpha_i + \alpha_i + 1) \right| \right) \\
(47) \]

According to the lemma in Farrell et al., \[ ||\alpha_i + 1 - \alpha_i + 1|| \leq \eta \] can be obtained in the fixed time \( T_2 \) with a known constant \( \eta \). Hence, we obtain the following conclusion by choosing a suitable matrix \( L_i \):

\[ \dot{V}_3 \leq - \sum_{i=1}^{4} k_i \xi^p_i - \sum_{i=1}^{4} \left| \xi^p_i \right| \left( ||L \text{sgn}(\xi_i)|| - ||g(\alpha_i + \alpha_i + 1)|| \right) \]

\[ \leq - k_m V_3 - k_a V_3^2 + \gamma \]

(48)

where \( k_m = 2 \min \{ k_i \} \) and \( k_a = \sqrt{2} \min \{ ||L \text{sgn}(\xi_i)|| - ||g|| \eta \} \).

Based on Lemma 1, we know that \( \xi_i \) can converge to the origin in a finite time \( T_3 \). It can be concluded that \( z_i \) is practical finite stable within the fixed time \( T = T_1 + T_2 + T_3 \).

Choose the Lyapunov function as

\[ V = V_3 + \frac{1}{2} \gamma \left( \frac{\gamma}{1 - \mu} \right)^3, \quad 0 < \mu < 1 \]

and \( \|x_i\| \) is bounded in a fixed time \( T_1 \). Because \( z_i = x_i + \xi_i \), we can conclude that \( z_i \) is convergent in a fixed time if \( \xi_i \) is bounded. Next, we will deal with this problem. Construct the Lyapunov function

\[ V_3 = \sum_{i=1}^{4} \frac{1}{2} \xi^p_i, \]

(46)

It is concluded that

\[ \dot{V}_3 = - k_i \xi^p_i + \xi_i g(\alpha_i + \alpha_i + 1) - \xi^p_i L \text{sgn}(\xi_i) \\
- k_i \xi^p_i + \xi_i g(\alpha_i + \alpha_i + 1) - \xi^p_i L \text{sgn}(\xi_i) \\
- k_i \xi^p_i + \xi_i g(\alpha_i + \alpha_i + 1) - \xi^p_i L \text{sgn}(\xi_i) \\
- k_i \xi^p_i + \xi_i g(\alpha_i + \alpha_i + 1) - \xi^p_i L \text{sgn}(\xi_i) \\
= - \sum_{i=1}^{4} k_i \xi^p_i + \sum_{i=1}^{4} \xi_i g(\alpha_i + \alpha_i + 1) - \sum_{i=1}^{4} \xi^p_i L \text{sgn}(\xi_i) \\
= - \sum_{i=1}^{4} k_i \xi^p_i + \sum_{i=1}^{4} \xi_i g(\alpha_i + \alpha_i + 1) - \sum_{i=1}^{4} \xi^p_i L \text{sgn}(\xi_i) \]

Simulation example

To examine the efficiency of the proposed approach, we will carry out a simulation study for the single-link FJ manipulator

\[ m \ddot{q} + g q + F(q) + Kq = Kq_m \]

\[ J \ddot{q}_m + B \dot{q}_m + K(q_m - \dot{q}) = \dot{u} \]

(49)

where \( q, q_m \in R \). Let \( m = 1 \text{ kg}, l = 1 \text{ m}, \ J = 1 \text{ kg\cdot m}^2, \ g = 10 \text{ m/s}^2, F(q) = \theta_1 \sin(q), \ K = 1, \) and \( B = \theta_2, \) where \( \theta_1 \) and \( \theta_2 \) are unknown parameters, \( g = 1/2 \sin t \) is the expected trajectory. The following equations are the fuzzy membership functions required in the simulation

\[ \mu_{\theta_1} = e^{-0.5(x + 1.5)^2}, \quad \mu_{\theta_2} = e^{-0.5(x + 1.5)^2}, \quad \mu_{\theta_3} = e^{-0.5(x + 0.5)^2}, \]

\[ \mu_{\theta_4} = e^{-0.5(x - 0.5)^2}, \quad \mu_{\theta_5} = e^{-0.5(x - 1)^2}, \quad \mu_{\theta_6} = e^{-0.5(x - 1.5)^2}. \]

To achieve the control objective, we construct the virtual and actual controllers as equations (17), (25),
Figure 1. The trajectories of $q$ and $q_d$.

Figure 2. The trajectory of $q - q_d$.

Figure 3. The trajectories of states $\dot{q}, q_m$, and $\dot{q}_m$. 
(31), and (39) according to the method previously designed. Figures 1–5 show the simulation results under the initial conditions $q(0) = 0.01$, $\dot{q}(0) = 0$, $q_m(0) = 0.01$, and $\dot{q}_m(0) = 0$. Figure 1 expresses the trajectories of the position state $q$ and reference signal $q_d$. The trajectory of $q - q_d$ is shown in Figure 2, which indicates that $q - q_d$ can converge to a small neighborhood of zero in the finite time. That is, position state $q$ can follow target signal $q_d$ within a limited time. Figures 3–5 are the trajectories of states $\dot{q}$, $q_m$, $\dot{q}_m$, input $u$, and adaptive law $\dot{\theta}$, respectively. It is found that states $\dot{q}$, $q_m$, $\dot{q}_m$, input $u$, and adaptive law $\dot{\theta}$ are bounded. As a result, the proposed method can achieve the control objective.

Conclusion

In this paper, the proposed scheme settles the issue of finite-time tracking control for FJ robots. With the help of the command filtered technology, both explosion of complexity and singularity problems in the standard backstepping design are avoided. By the aid of the finite-time control technique, the tracking error can achieve convergence quickly. The effectiveness of the proposed scheme is illustrated by simulation results. In this research direction, how to apply command filtered technology to under-actuated mechanical systems is meaningful work, such as the crane system.34–36

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