Strong lensing probability in TeVeS theory

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Abstract. We recalculate the strong lensing probability as a function of the image separation in TeVeS (tensor-vector-scalar) cosmology, which is a relativistic version of MOND (Modified Newtonian Dynamics). The lens is modeled by the Hernquist profile. We assume an open cosmology with \( b = 0.04 \) and \( \Omega = 0.5 \) and three different kinds of interpolating functions. Two different galaxy stellar mass functions (GSMF) are adopted: PHJ (Panteleeva-Haines-Jimenez, 2004) determined from SDSS data release one and Fontana (Fontana et al., 2006) from GOODS-MUSIC catalog. We compare our results with both the predicted probabilities for lenses by Singular Isothermal Sphere (SIS) galaxy halos in LCDM (Lambda cold dark matter) with Schechter velocity function, and the observational results of the well defined combined sample of Cosmology All-Sky Survey (CLASS) and Jodrell Bank/Very Large Array Astrometric Survey (JVAS). It turns out that the interpolating function \( f(x) = x(1 + x) \) combined with Fontana GSMF matches the results from CLASS/JVAS quite well.

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1. Introduction

The standard LCDM cosmology is very successful in explaining the cosmic microwave background (CMB, see, e.g., [78]), baryonic acoustic oscillation (BAO, see, e.g., [27]), gravitational lensing (see, e.g., [35]) and large scale structure (LSS) formation. However, LCDM faces some fundamental difficulties. From the observational point of view, the challenges to LCDM arise from smaller scales. For example, the theory cannot explain Tully-Fisher law and the Freeman law [25, 80]. The most difficult ones are the satellites problem and cusps problem. The most key problems are, of course, the unknown nature of Dark Matter (DM) and Dark Energy (DE). Before CDM particles are detected in the lab, science should remain open to the prospect that DM (and for the similar reasons, DE) phenomena may have some deep underlying reason in new physics.

There are several proposals for resolving DM and DE problems by modifying Newtonian gravity or general relativity (GR) rather than resorting to some exotic kinds of matter or energy. MOND [48] was originally proposed to explain the observed asymptotically at rotation curves of galaxies without DM, however, it was noticed that MOND can also explain Tully-Fisher law and Freeman law [46, 47]. It is believed that MOND is successful at galactic scales [84, 88] (but see [37] for satellites problem). The challenges to MOND arise from clusters of galaxies [71], in which, some kind of dark matter, possibly some massive neutrinos with the mass of 2eV, is also needed to explain the dynamics of galaxies [4]. MOND and its relativistic version, TeVeS [6], are only concerned with DM, remain DE as it is. By adding a \( f(R) \) term in Einstein-Hilbert Lagrangian, where \( R \) is the Ricci scalar, the so called \( f(R) \) gravity theory can account for DE [1, 12, 13, 57, 58, 76, 77, 82]. Another interesting theory is Modified Gravity (MOG) [50], it is a fully relativistic theory of gravitation that is derived from a relativistic action principle involving scalar, tensor and vector fields. MOG has been used successfully to account for galaxy clusters [5], the rotation curves of galaxies (similar to MOND) [10], velocity dispersions of satellite galaxies [51], globular clusters [52] and Bullet Cluster [11], all without resorting to DM. Most recently, MOG is used to investigate some cosmological observations (CMB, galaxy mass power spectrum and supernova), and it is found that MOG provides good fits to data without DM and DE [53].

Any modification to traditional gravity theory must be tested with observational experiments. Gravitational lensing provides a powerful probe to test gravity theory [75, 83]. It is well known that, in standard cosmology (LCDM), when galaxies are modeled by a SIS and galaxy clusters are modeled by a Navarro-Frenk-White (NFW) profile, the predicted strong lensing probabilities can match the results of CLASS/JVAS quite well [15, 17, 18, 20, 24, 35, 53, 60, 62, 63, 64, 65, 66, 67].

This paper is devoted to explore the strong lensing statistics in TeVeS theory. As an alternative to LCDM cosmology, TeVeS cosmology has received much attention in the recent literature, in particular in the aspect of gravitational lensing [2, 23, 53]. For reviews see [1, 12]. Before TeVeS, strong gravitational lensing in the MOND regime could only be manipulated by extrapolating non-relativistic dynamics [55, 54], in which the deflection angle is only half the value in TeVeS [91]. In TeVeS theory, it is now established that, for galaxy clusters, both weak and strong lensing need extra DM to explain observations [5, 54, 29, 19], possibly neutrinos with the mass of 2eV, like the dynamics of galaxies. The situation is better for galaxies, as will be shown in
where $\alpha$ is the critical mass density, and the corresponding angular diameter distance therefore is

$$D(\alpha_1;\alpha_2) = \frac{1}{1+\alpha_2} f_\alpha(\alpha_1;\alpha_2).$$

In our previous paper [22], as a first try to calculate the strong lensing probability as a function of the image-separation in TeVeS cosmology, we assumed a complex cosmology with $b = 1 = 0.04$ and the simplest interpolating function $(x) = \min(1; x)$. In this paper, we assume an open cosmology with $b = 0.04$ and $x = 0.5$ and three different kinds of interpolating functions. As a form of mass function, in addition to the PHJ GSF [56] used in our previous paper, we also adopt a redshift-dependent Fontana GSF [51]. Further more, the amplification bias is calculated based on the total magnification of the outer two brighter images rather than the magnification of the second bright image of the three images as did in our previous work [22].

2. TeVeS cosmology and deflection angle

Gravitational lensing can be used to test TeVeS in two aspects. First, in the non-relativistic and spherical limit, TeVeS reduces to MOND. The deflection angle of the light ray passing through the lensing object can be calculated in MONDian regime (this will be discussed later). Second, the distances between the source, the lens and the observer are cosmological and thus depend on the geometry and evolution properties of the background universe. As argued by Bekenstein [6, 89], the scalar field, which is used to produce a MONDian gravitational acceleration in non-relativistic limit, contributes negligibly to Hubble expansion. According to the cosmological principle, the physical metric takes the usual Friedman-Robertson-Walker (FRW) form in TeVeS [5],

$$d^2 = \hat{c} dt^2 + a(t)^2 [d^2 + f_K(\theta)(d^2 + \sin^2 d^2)],$$

where $c$ is the speed of light, $a(t)$ is the scale factor and

$$f_K(\theta) = \begin{cases} K^{-1/2} \sinh(K^{-1/2}) & (K > 0) \\ K^{-1/2} \sin(K^{-1/2}) & (K = 0) \\ 0 & (K < 0) \end{cases}$$

As in general relativity (GR), we define the cosmological parameters:

$$\begin{align*}
\frac{b}{\Omega_{\text{crit}}(0)} = \frac{3H_0^2}{H^2} = & \frac{K\hat{c}}{H_0} \\
\frac{b}{\Omega_{\text{crit}}(0)} = & \frac{3H_0^2}{H^2} = & \frac{K\hat{c}}{H_0}
\end{align*}$$

where $b$ is the mean baryonic matter density in the universe at present time $t_0$ (redshift $z_0 = 0$), $\Omega_{\text{crit}}(0) = 3H_0^2 = (8 G) = 2.78 \times 10^{-11} M_p c^{-3}$ is the present critical density, and $H_0 = 100 km s^{-1} M_p c^{-1}$ is the Hubble constant. We choose $a(t_0) = 1$. Since $d = c dz = H(\bar{z})$, the proper distance from the observer to an object at redshift $\bar{z}$ is $D(\bar{z}) = c \int_{0}^{\bar{z}} [(1+z)H(z)]^{1/2} dz$, where the Hubble parameter at redshift $\bar{z}$ is (known as Friedmann’s equation)

$$\frac{\dot{a}}{a} = H_0 \frac{p}{b(1+z)^3} + \frac{K}{(1+z)^2} + \frac{K\hat{c}}{H_0}.$$
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In TeVeS, the lensing equation has the same form as in general relativity (GR), and for a spherically symmetric density profile \[89\]

\[
\frac{\partial}{\partial z} \left( \frac{D_{LS}}{D_S} \right) = \frac{4b}{c^2r} \frac{d}{dr} \left( r^2 \right); \tag{7}
\]

where \(a=\frac{b}{D_L}\) and \((b)\) are the source position angle, image position angle and deflection angle, respectively; \(b\) is the impact parameter; \(D_L\), \(D_S\) and \(D_{LS}\) are the angular diameter distances from the observer to the lens, to the source and from the lens to the source, respectively; \(g(r) = \frac{d}{dr}(r^2)\) is the actual gravitational acceleration [here \((r)\) is the spherical gravitational potential of the lensing galaxy and \(l\) is the light path]. It is well known that the stellar component of an elliptical galaxy can be well modeled by a Hernquist profile

\[
g(r) = \frac{M_0}{2\pi(r^2+l^2)^{3/2}}; \tag{8}\]

with the mass interior to \(r\) as

\[
M(r) = \frac{\pi^2M_0}{(r^2+l^2)^2}; \tag{9}\]

where \(M_0 = \int_0^{R_1} 4\pi r^2(r)dr\) is the total mass and \(l\) is the scale length. The corresponding Newtonian acceleration is \(g_N(r) = GM(r)r^2 = GM_0(r^2+l^2)^{1/2}\). According to MOND \[48,71,72\], the actual acceleration \(g(r)\) is related to Newtonian acceleration by

\[
g(r) = \frac{a_0}{g_N(r)} = g_{\text{H}}(r); \tag{10}\]

where \((x)\) is the interpolating function and has the properties

\[
(x) = \begin{cases} 
\frac{x}{x^2+1} & \text{for } x > 1 \\
1 & \text{for } x = 1 
\end{cases}; \tag{11}\]

and \(a_0 = 1.2 \times 10^8\) cm s\(^{-2}\) is the critical acceleration below which gravitational law transits from Newtonian regime to MONDian regime. The concrete form of \((x)\) function should be determined by observational data (e.g., the rotation curves of spiral galaxies) and expected by a reasonable scalar field theory (e.g., TeVeS). The "standard" function one usually takes is \((x) = x = 1 + x^2\), which is well to the rotation curves of most galaxies. Unfortunately, if the MOND effect is produced by a scalar field (such as TeVeS), the "standard" \((x)\) function turns out to be multivalued \[90\]. On the other hand, a \(\sin^2 x\) function \((x) = x = (1 + x)\) suggested by Famaey & Binney \[28\] is observational data better than the "standard" function and is consistent with a scalar field relativistic extension of MOND \[90,73\].

In order to explore a broad class of modified gravity models, Zhao and Tian \[92\] proposed a parameterized modification function

\[
\frac{1}{g(a_0)} = \frac{g}{g_N} = 1 + \frac{a_0}{g_N} \frac{kn}{1 + \#}; \tag{12}\]

in which, MOND gravity corresponds to \(k = 1\). Substituting equation \[12\] with \(k = 1\), we have

\[
(g = a_0) = 1 + \frac{a_0}{g(a_0)} \frac{kn}{1 + \#}; \tag{13}\]
which can be easily solved to obtain the usual form of the function for MOND \[92\]

\[
(x) = x \frac{1}{2} + \frac{1}{4} + x^n ; \quad x = \frac{a_0}{g_N}.
\]  
(14)

It is easy to verify that the "simple" and "standard" function are approximated with high accuracy by equation \[14\] with \( n = 3 = 2 \) and \( n = 3 \), respectively \[92\]. The requirement for a physical and monotonic function limits the parameter \( n \) to the range of \( 1.5 \leq n \leq 2.3 \). In this paper, we consider three cases: \( n = 1.5, 2.0 \) and 3.0.

Since the MONDian gravitational acceleration \( g \) is explicitly expressed in terms of the Newtonian acceleration \( g_N \), it is very convenient to use equation \[13\] to calculate the deflection angle

\[
(b) = \frac{4}{\pi} \frac{Z}{Z_1} \int_0^r \frac{g(r) b}{r} dr
\]

\[
= \frac{4}{\pi} \frac{Z}{Z_1} \left[ \int_0^r \frac{g(r) b}{r} dr \right] [1 + \left( \frac{a_0}{g_N} \right)^{n-2} \int_{r_n}^r \frac{b}{r} \left( \frac{r}{r+n} \right)^n dr]
\]

(15)

By using \( r = b^2 \), \( 1 + (b)^2 \) and \( b = b_1 \), we have

\[
(\theta) = 0.207 h \frac{Z}{Z_1} \frac{D_L}{D_0} \int_0^r \frac{g(r) b}{r} dr
\]

\[
= \frac{4}{\pi} \frac{Z}{Z_1} \left[ \int_0^r \frac{g(r) b}{r} dr \right] [1 + \left( \frac{a_0}{g_N} \right)^{n-2} \int_{r_n}^r \frac{b}{r} \left( \frac{r}{r+n} \right)^n dr]
\]

where \( M = M_0 = M_\gamma \) and \( M_\gamma = 7.5 \times 10^8 h^{-2} M_\odot \) is the characteristic mass of galaxies \[66\], and

\[
\frac{a_0}{g_N} = 2.38 \frac{D_L}{D_0} \frac{2}{2} \frac{M_0}{M} \frac{1 + x^2 + 0.05}{x} h_0 \frac{r_n^2}{D_L}
\]

(17)

In equations \[15\] and \[17\], the image position angle \( \theta \) and the scale length \( r_n \) are in units of arcsecond (\( ^{\prime} \)) and kpc, respectively.

We need a relationship between the scale length \( r_n \) and the mass \( M \), which could be determined by observational data. First, the scale length is related to the effective (or half-light) radius \( R_e \) of a luminous galaxy by \( r_n = R_e = 1.8 \) \[34\]. It has long been recognized that there exists a correlation between \( R_e \) and the mean surface brightness \( h_0 \) interior to \( R_e \), which is \( R_e \) \( h_0 \) \( 0.4 \). Since the luminosity interior to \( R_e \) (half-light) is \( L_e = L = 2 \pi h_0 R_e \) \( h_0 \) \( 0.2 \), one immediately deduces \( R_e / L = 1 / h_0 \). Second, we need to know the mass-to-light ratio \( M = L / \mu \) for elliptical galaxies. The observed data gives \( \mu = 0.35 \) \[81\]; according to MOND, however, we should have \( \mu = 0 \) \[72\]. In any case we have

\[
L / M = 1 + (1 + \mu) - 1 \geq 0.35 \frac{L}{M}
\]

(18)

Therefore, the scale length should be related to the stellar mass of a galaxy by \( r_n = A_M \frac{1.26}{1 + \mu} \), and the coefficient \( A \) should be further determined by observational data. Without a well defined sample at our disposal, we use the galaxy lenses which have an observed effective radius \( R_e \) (and thus \( r_n \)) in the CASTLES survey \[53\], which are listed in Table 2 of \[89\]. The fitted formulae for \( r_n \) are

\[
r_n = \frac{0.72 M}{M_\gamma} \text{ Kpc; for } \mu = 0 \text{,}
\]

\[
r_n = \frac{1.24 M}{M_\gamma} \text{ Kpc; for } \mu = 0.35
\]

(19)

In later calculations, except indicated, we use the fitted formula of \( r_n \) for \( \mu = 0 \) as required by MOND.
3. Galaxy stellar mass function

In LCDM cosmology, mass function of virialized CDM halos can be obtained in two independent ways. One is via the generalized Press-Schechter (PS) theory, the other is via Schechter luminosity function. In TeVeS, however, the PS-like theory does not exist. Fortunately, the stellar mass function of galaxies is available in the literature, including the one constrained by the most recent data [31, 66].

Before giving the galaxy stellar mass functions (GSMF) appeared in the most recent literature, it is helpful to derive a GSMF directly from the Schechter luminosity function and mass-to-light ratio. The Schechter luminosity function is

\[
(L) = \frac{\gamma}{L_\gamma} \exp \left( \frac{L}{L_\gamma} \right) \frac{dL}{L_\gamma};
\]

For \(L = L_\gamma = (M - M_\gamma)^{3(1 + \gamma)}\) implied by equation (13), we have a GSMF

\[
(M) = \frac{\gamma}{1 + \gamma} \frac{M}{M_\gamma} \exp \left( \frac{M}{M_\gamma} \right) \frac{dM}{M_\gamma};
\]

While the average number density of galaxies \(\gamma\), the slope at low mass end \(\gamma\), and the slope of mass-to-light ratio \(\gamma\) may be easily found from the published observational data or assumptions, the characteristic stellar mass of galaxies \(M_\gamma\) can be derived from

\[
\rho_{\text{lin}} = \rho_{\text{lin}}\text{crit}(0) = \int_0^\gamma M(M)dM;
\]

where \(\rho_{\text{lin}}\) is the luminous baryonic matter density (note that \(\rho_{\text{lin}}\) b). The characteristic mass \(M_\gamma\) is

\[
M_\gamma = \frac{\rho_{\text{lin}}\text{crit}(0)}{\gamma (1 + \gamma)};
\]

For example, for \((\gamma; \rho_{\text{lin}}) = (0.014; 0.003): 1.11; 0.003; 0.35)\) from [41], \(M_\gamma = 6.56 \times 10^3 h_0^3 M_\odot\); for the same parameters except that \(p = 0.2\) (MOND), \(M_\gamma = 5.56 \times 10^3 h_0^3 M_\odot\).

Fortunately, the parameters in equation (22) have been determined by recent observational data. By detemining non-parametrically the stellar mass functions of 96545 galaxies from the Sloan Digital Sky Survey Data (SDSS) release one, Panter, Heavens and Jimenez [66] (PHJ, hereafter) give the GSMF [22]

\[
(M) dM = \frac{\gamma}{M_\gamma} \exp \left( \frac{M}{M_\gamma} \right) \frac{dM}{M_\gamma};
\]

where, we use \(\rho_{\text{lin}}\) to denote the comoving number density of galaxies with mass between \(M\) and \(M + dM\), and

\[
\gamma = (7.8 \pm 0.1) \times 10^3 h_0^3 M_\odot; \quad \gamma - 1.159 \pm 0.008; \quad M_\gamma = (7.54 \pm 0.09) \times 10^3 h_0^3 M_\odot.
\]

Most recently, in order to study the assembly of massive galaxies in the high redshift Universe, Fontana et al. [31](Fontana, hereafter) used the GOODS-MUSIC catalog.
to measure the evolution of the GSMF and of the resulting stellar mass density up to redshift $z = 4$. The GSMF they obtained is

$$ (M \gamma z; z) dM = \gamma (z) \frac{M}{M \gamma (z)} \exp \frac{M}{M \gamma (z)} dM, \quad (26) $$

where

$$ \gamma (z) = n_0 (1 + z)^{\frac{1}{2}}; n_0 = 0.0035; n_1 = 2.20; 0.18; $$

$$ \gamma (z) = \gamma_0 + \gamma_1 z; \quad \gamma_0 = 1.18; \gamma_1 = 0.082; 0.033; $$

$$ M \gamma (z) = 10^{M_0 + M_1 z + M_2 z^2} h^2 M; $$

$$ M_0 = 11.16; M_1 = 0.17; 0.05; M_2 = 0.87; 0.01. $$

It would be interesting to compare PHJ and Fontana GSFs to the mass function of galaxies in LCDM cosmology when the galactic halos are modeled by SIS. The comoving number density of galactic halos with velocity dispersion between $v$ and $v + dv$ [49, 22] is

$$ (v) dv = \gamma \frac{v}{v_\gamma} \exp \frac{v}{v_\gamma} \frac{v}{v_\gamma} dv. $$

Figure 1. Comoving number density for PHJ (solid), Fontana (dotted) and SIS halos (dash). Since Fontana mass function depends on redshift, four cases with redshift $z = 0.0; 0.5; 1.0; 2.0$ are displayed. For comparison, we normalize the three mass functions to the same value of characteristic mass $M = 7.64 \times 10^{10} h^{-2} M_\odot$. 
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For comparison, we need to transform equation (23) from velocity dispersion to halo mass $M$

$$M = 4 \int_0^{r_{200}} \rho_{\text{sis}}(r) r^2 dr = \frac{800}{3} r_{200}^3 \rho_{\text{cr}}(z);$$

(29)

where $r_{200}$ is the virial radius of a galactic halo within which the average mass density is 200 times the critical density of the Universe $\rho_{\text{cr}}(z)$. Substituting the well-known expression $\rho_{\text{sis}}(r) = \frac{v^2}{2Gr^2}$ into equation (29), it is easy to nd

$$M(z) = \frac{6.58 \times 10^6 V}{\text{km s}^{-1}} \frac{\rho_m}{\rho_{\text{cr}}(z)} \left[ (1+z)^3 + \frac{\Omega_c (1+z)^3}{1+\Omega_c} \right]^{1/2} \frac{M}{h^2};$$

(30)

where $\rho_m$ is the matter density parameter (including dark and baryonic components) [42]. Equation (30) means that at any redshift $z$ we should have $M/v^2$, or for our purpose, another form

$$\frac{M}{M_z} = \frac{v}{v_z}^3;$$

(31)

we thus have the galaxy mass function for SIS halos

$$\frac{M}{M_z} \sim \frac{M}{M_z} \left[ \frac{\rho_m}{\rho_{\text{cr}}(z)} \right]^{3/2} \exp \left[ \frac{M}{M_z} \left( \frac{\rho_m}{\rho_{\text{cr}}(z)} \right)^{3/2} \right];$$

(32)

We plot PHJ and Fontana GGMFs in figure 1 together with the galaxy mass function for SIS halos (comoving number density). For SIS halos, we use $(\gamma,\omega,\rho) = (0.064 h^2 Mpc^{-3}; 1.0; 0.1)$ [14]. For comparison, we normalize the three mass functions to the same value of characteristic mass $M_z = 7.64 \times 10^9 h^{-2} M$. Note that, for Fontana GGMF, the comoving number density of galaxies decreases with increasing redshift, as expected [22].

4. lensing probability

Usually, lensing cross section de ned in the lens plane with image separations larger than $\Delta_x > D_{\text{cr}} \left[ \frac{M}{M_z} \right]$, where $\Delta_x$ is the Heaviside step function and $D_{\text{cr}}$ is the caustic radius within which sources are multiply imaged. This is true only when $\frac{M}{M_z}$ is approximately constant within $D_{\text{cr}}$, and the eect of the ux density ratio $q_x$ between the outer two brighter and fainter images can be ignored. Generally this is not true, readers are referred to [22] for details. We introduce a source position quantity $q_x$ determined by

$$\left( \frac{d}{d \Delta_x} \right)_{\Delta_x > 0} = q_x \left( \frac{d}{d \Delta_x} \right)_{\Delta_x < 0} < 0;$$

(33)

where $\Delta_x = 0$ is the absolute value of which is the Einstein radius, and $D_{\text{cr}}$ is determined by $d = 0$ for $\Delta_x < 0$. Equation (33) means that when $q_x < 0$, the u density ratio would be larger than $q_x$, which is the upper limit of a well-de ned sam ple. There, the source position should be within $q_x$ according to the sample selection criterion. For example, in the CLASS/JVAS sample, $q_x = 10$.

The ampli cation bias should be considered in lensing probability calculations. For the source QSOs having a power-law u distribution with slope $\sim = 2d$ in the CLASS/JVAS survey), the ampli cation bias is $B(\sim) = \sim^{-1} [65]$, where, in this paper,

$$\sim = \left( \frac{d}{d \Delta_x} \right)_{\Delta_x < 0} + \left( \frac{d}{d \Delta_x} \right)_{\Delta_x > 0}$$

(34)
is the total magnification of the outer two brighter images. In our previous work [22], however, the amplification bias is calculated based on the magnification of the second brightest in age of the three images.

Therefore, the lensing cross section with age-separation larger than and linearity ratio less than \( q \), and combined with the amplification bias \( B(\tau) \) is [15,19,22]

\[
\begin{align*}
P(\tau < q) &= \frac{Z V}{Z V} \int_{0}^{\infty} dz \left( 1 + \frac{z}{r_{e}} \right)^{n} \
&= \frac{Z V}{Z V} \int_{0}^{\infty} dz \left( 1 + \frac{z}{r_{e}} \right)^{n} \
&= \frac{Z V}{Z V} \int_{0}^{\infty} dz \left( 1 + \frac{z}{r_{e}} \right)^{n} \
&= \frac{Z V}{Z V} \int_{0}^{\infty} dz \left( 1 + \frac{z}{r_{e}} \right)^{n},
\end{align*}
\]

where \( \tau \) is the source position at which a lens produces the image separation \( \rho \), \( n \) is the separation of the two images which are just on the Einstein ring, and \( q \) is the upper-limit of the separation above which the density ratio of the two images will be greater than \( q \).

The lensing probability with age separation larger than and linearity ratio less than \( q \), in TeVeS cosmology, for the source QSO at mean redshift \( z_{s} = 1.27 \) lensed by foreground elliptical stellar galaxies is [19,20,21,22]

\[
P(\tau < q) = \frac{Z V}{Z V} \int_{0}^{\infty} dz \left( 1 + \frac{z}{r_{e}} \right)^{n} \left( 1 + \frac{z}{r_{e}} \right)^{n}.
\]

We plot in Figure 2 and Figure 3 the numerical results of the lensing probability according to equation (34). In TeVeS (solid lines), we assume an open cosmology with \( \Omega_{b} = 0.04 \) and \( \Omega_{c} = 0.5 \), as implied by fitting to a high-z Type Ia supernova luminosity modulus [59]. The lensing galaxy is modeled by Hernquist profile with length scale \( r_{h} = 0.72(M_{b} - M_{0})^{1/2} \) Kpc for constant mass-to-light ratio as required by MOND (see equation (19)). The interpolating functions with three cases \( n = 1, 2, 3 \) are considered (top-down) according to equation (13). In order to investigate the effects of MOND on strong lensing, we also calculated the probabilities (dotted lines) with no model calculation to gravitational theory (i.e., in GR) and without dark matter (i.e., lensing galaxy is modeled by Hernquist profile). In this case, two types of the tidal form factor with masses \( m = 0.3 \) and \( m = 0.5 \) are considered (top-down) in equation (13). In TeVeS and GR (with no dark matter), we adopt the GSF as the mass function \( m(\tau) \), with \( m = PHJ \) in Figure 2 and \( m = Fontana \) in Figure 3. As did in our previous work [22], we re-calculate the lensing probability with age separation larger than and linearity ratio less than \( q \), in a LCDM cosmology \( \Omega_{b} = 0.03 \) and \( \Omega_{c} = 0.7 \), for the source QSO at mean redshift \( z_{s} = 1.27 \) lensed by foreground SIS modeled galaxy halos [4,43,49]:

\[
P_{\text{SIS}}(\tau < q) = \frac{Z V}{Z V} \int_{0}^{\infty} dz \left( 1 + \frac{z}{r_{e}} \right)^{n} \left( 1 + \frac{z}{r_{e}} \right)^{n} \left( 1 + \frac{z}{r_{e}} \right)^{n},
\]

\[
\text{SIS}(\tau) = 16 \left( \frac{v}{c} \right)^{4} \frac{D_{L} D_{L}}{D_{S}}.
\]
is the lensing cross section,\[ v = 4 \pi 10^4 \frac{C}{V^2} \frac{S}{D_S} \frac{D_L}{D_{LS}} \] (39)
is the minimum velocity for lenses to produce an image separation \( \Delta r \) and \( \Delta \) is the amplification bias. We adopt \( (\gamma; \nu; \Delta \gamma; \Delta \nu) = (0.0064h^3\text{Mpc}^2; 198\text{km s}^{-1}; 1.0; 4.2) \) for early-type galaxies from [14]. A subset of 8958 sources from the combined JVAS/CLASS survey form a well-defined statistical sample containing 13 multiply imaged sources (lens system a) suitable for analysis of the lens statistics [58,21]. The observed lensing probabilities can be easily calculated [18,19,21] by \( P_{\text{obs}}(\Delta r > \Delta r) = N(\Delta r > \Delta r)/8958 \), where \( N(\Delta r > \Delta r) \) is the number of lenses with separation greater than \( \Delta r \) in 13 lenses. For comparison, the observational probability \( P_{\text{obs}}(\Delta r > \Delta r) \) for the survey results of CLASS/JVAS is also shown (thick histogram). It would be helpful for us to figure out differences among models to summarize the values of the probabilities \( P(\Delta r > \Delta r = 0.3) \) in the Table 1.
5. Discussion and conclusions

We have calculated the lensing probability with image separation larger than a given value in an open, TeVeS cosmology. The results are sensitive to the interpolating function \((x)\) and mass function \((M; z)\). For a given GSMF (PHJ in Figure 2 and Fontana in Figure 3), the lensing probability decreases with increasing value of \(n\) (given in equation (14)). Obviously, for PHJ GSMF (Figure 2), the lensing probabilities calculated in TeVeS (solid lines for three cases of interpolating functions) are too large at small lensing image separations compared with the results of CLASS/JVAS. This unreasonable result is further confirmed, when we note that, even the lensing probabilities in GR cosmology (with no DM, dotted lines) are much larger than that in LCDM cosmology (dashed line) at small image separations. Actually, however, this result can be easily explained: at small mass-end (corresponding to small in age separation), the comoving number density for PHJ mf is much larger than that for SEX halos (Figure 4), which results in the corresponding lensing probabilities according to equation (15). This is why in our previous work [22], we calculated the amplification bias based on the magnification of the second bright image rather than the total magnification of the two images considered. According to the resolution of CLASS/JVAS, however, it is difficult to resolve the two images for small image separations. Therefore, in this paper, we calculate the amplification bias based on the total magnification of the outer two brighter images, as usually done in the literature.

On the other hand, if we adopt another most recent mass function, Fontana
Table 1. The predicted values of the lensing probabilities $P_\text{obs} (> x = 0.3^0) = 0.3^0)$ for all models and $P_\text{obs} (> x = 0.3^0) = 0.3^0)$ for CLASS/JVAS.

|         | TeVeS | GR  | LCDM | CLASS/JVAS |
|---------|-------|-----|------|------------|
| PHJ     | n=1.5 | -2.460 | -2.513 | -2.552 | -2.601 | -2.691 | -2.848 | -2.838 |
| Fontana | n=3.0 | -2.873 | -2.926 | -2.965 | -3.021 | -3.112 | -2.848 | -2.838 |

GSMF (Figure 3), we nd that the predicted lensing probabilities in an open TeVeS cosmology with the simple interpolating function $(x) = x = (1 + x)$ [ie., $n = 3$ in equation (12) match the observational data of CLASS/JVAS quite well. Similarly, this is reasonable when we note that the comoving number density of galaxies for PHJ GSMF is much higher than that for Fontana GSMF at all mass-samples (Figure 1). Clearly, the standard interpolating function $(x) = x = 1 + x^2$ [ie., $n = 3$ in equation (13) is ruled out due to its too low lensing rates at small in age separations. Interestingly, this conclusion is in agreement with the most recent result of Sanders and Noordermeer (73), who constrained the interpolating function with the rotation curves of early-type disc galaxies.

In our calculations for deflection angle in TeVeS cosmology, we have adopted the value of the critical acceleration $a_\text{cr}$ and modeled the lensing galaxies with the Hernquist profile, and the only free choice is the interpolating function $(x)$. We note that the PHJ GSMF includes all types of galaxies, whereas the model for SIS halos includes only early-type galaxies, this can partly explain the relatively low abundance of SIS halos compared with PHJ GSMF in Figure 1. On the other hand, Fontana GSMF, like PHJ GSMF, also includes all types of galaxies, but its value is close to (when $z = 0$) or lower (for high $z$) than the model for SIS halos. Therefore, the major uncertainty for lensing probability arises from the GSMF, which is independent of any gravitational theory and should be determined by observational data. Can we conclude from Figure 2 and Figure 3 that Fontana GSMF is preferred and PHJ GSMF is ruled out? Recall that, in LCDM cosmology, there are also uncertainties for the mass function derived from the luminosity function, the equation (29) (10, 16, 49, 61). Actually, the parameters we adopted in equation (29) $(\gamma; v_\text{f}; -\gamma) = (0.006h^3\text{Mpc}^{-3}; 198\text{km s}^{-1}; 1.045)$ (14), are selected so that the predicted lensing probabilities $P_\text{obs} (> 0.3^0)$ can exactly match the observed value $P_\text{obs} (> 0.3^0)$, i.e., $P_\text{obs} (> 0.3^0) = P_\text{obs} (> 0.3^0)$, see Table 1. The most recent parameters derived from SDSS DR3 (24) is $(\gamma; v_\text{f}; -\gamma) = (0.006h^3\text{Mpc}^{-3}; 161\text{km s}^{-1}; 2.32; 2.67)$, however, this will not affect our results. One can see clearly from Figure 2 that, at larger in age separations, the predicted lensing probabilities (dashed line) are well below the observed values, i.e., $P_\text{obs} (> 0.3^0) < P_\text{obs} (> 0.3^0)$. As a matter of fact, in strong lensing statistics, one usually compares the predicted cumulative lensing probability at the age separation of $a_\text{cr} = 0.3^0$, and regards the under-estimates at larger age-separations to be unimportant. We note, however, that there is an in exon at $a_\text{cr} = 1.16^0$ for $P_\text{obs} (> 0.3^0)$ calculated from the well-defined sample of CLASS/JVAS (thick histogram), and there are no physical interpretations for the at part of the line when $a_\text{cr} < 1.16^0$. Although the sample of CLASS/JVAS is well defined, this should not include each detail such as the in exon, and other observations, like (74), would provide more information at $a_\text{cr} < 1.16^0$. So we can reasonably guess that the correct
obervational data should avoid the in exon, and the trend of the rising probabilities with smaller and smaller age-separations should continue inward at $< 146^\circ$. In this sense, the predicted lensing probabilities in TeVeS cosmology, with a PH J G SM F and the \"standard\" interpolating function, match the observational data quite well as shown in Figure 2.

We also note that, in Figure 3, the lensing probabilities in GR cosmology (with no DM, dotted lines) are much lower than the observational data. This implies that, as an alternative to CDM, MOND can sufficiently account for the strong lensing observations.

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