Dynamic analysis of ISD suspension based on mechanical impedance

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Abstract. The development of inerter has brought rapid development of vehicle suspension’s configuration design. More and more inerter vibration isolation networks have been proposed by scholars. As vibration isolation networks became more complex, traditional dynamic model calculations became difficult. So, a mechanical impedance dynamic model was proposed in this paper, which can be applied to the suspension of different vibration isolation networks. At the same time, the influence of the inerter on the frequency of the three-element parallel ISD suspension was studied, which provides a basis for the selection of suspension parameters.

1. Introduction
Inerter is a new type of mechanical element with two free endpoints, which has similar mechanical characteristics to the mass element, but its characteristics of free ends are different from the mass element [1, 2]. It is precisely because it has two free ends, it promotes the development of electromechanical analogy. The "Inerter-Spring-Damper" vibration isolation network has been developed, which provides a new direction for the development of the suspension system configuration [3].

In 2015, Zan Hao analyzed the influence of inerter on the vibration isolation system and found that the introduction of inerter can change the vibration characteristics of the vibration isolation system [4]. In 2016, during the dynamic simulation of a three-element parallel ISD suspension, Du Fu found that the main frequency of the suspension changed with the change of the inerter coefficient [5]. In 2018, Yang Xiaofeng conducted a systematic study on the influence of the inerter coefficient on the frequency characteristics of the ISD suspension system, and found that the inerter coefficient does have an effect on the main frequency of the suspension [6].

According to the above research, this paper proposes a general model of suspension impedance dynamics based on the principle of mechanical impedance in the new electromechanical similarity theory, and uses this model to analyze the performance of a three-element parallel ISD suspension system to verify the correctness of the model. At the same time, the influence of inerter on the performance of the suspension system is discussed, which lays the foundation for the subsequent research on the design of the suspension network that introduces multiple inerter.

2. Electromechanical analogy
In the study of the "asymmetric relationship" of the traditional electromechanical comparison, Smith. M. C. proposed a concept of an inertial device with two free ends and named it inerter [1]. The appearance of inerter made the traditional electromechanical correspondence relationship no longer limited to the condition of the capacitor ground. It realized the strict correspondence between the ISD
vibration isolation network and the RLC circuit. It provided convenience for the application of mature circuit research methods and electrical network theory in the design of mechanical vibration isolation networks.

In the new electromechanical analogy: the inerter’s coefficient $b$ of the inerter corresponds to the capacitance $C$ of the capacitor with free ends, the damper’s coefficient $c$ corresponds to the inverse $1/R$ of the resistance, and the spring’s stiffness $k$ corresponds to the inverse $1/L$ of the inductance. Formed a new electromechanical analogy based on these three correspondences (Table 1).

| Mechanical element | Electrical components |
|--------------------|----------------------|
| $f$ --- $b$ --- $v_2$ | $i$ --- $C$ --- $u_1$ |
| Inerter $f = b \frac{d(v_2 - v_1)}{dt}$ | Capacitor $i = C \frac{d(u_2 - u_1)}{dt}$ |
| $f$ --- $k$ --- $v_1$ | $i$ --- $L$ --- $u_1$ |
| Spring $\frac{df}{dt} = k(v_2 - v_1)$ | Inductor $\frac{di}{dt} = \frac{1}{L}(u_2 - u_1)$ |
| $f$ --- $c$ --- $v_1$ | $i$ --- $R$ --- $u_1$ |
| Damper $f = c(v_2 - v_1)$ | Resistor $i = \frac{1}{R}(v_2 - v_1)$ |

2.1. Mechanical impedance
In the circuit system, the impedance of capacitors, resistors, and inductors can be used to describe the electrical characteristics of a circuit network. In the mechanical system, the concepts of mechanical impedance and admittance were proposed to describe the mechanical characteristics of mechanical networks based on the theory of electromechanical analogy.

Mechanical impedance was defined as the ratio of the simple harmonic excitation of a linear steady-state stable system to the complex ratio or amplitude of the steady-state response it causes. Made the system’s excitation force be $f(t)$ and the steady-state response be $x(t)$, then the mechanical impedance $Z(s)$ and mechanical admittance $H(s)$ of the system were as follows:

$$Z(s) = \frac{L \int_{t_0}^{t} f(t) e^{-st} dt}{L \int_{t_0}^{t} x(t) e^{-st} dt} = \frac{F(s)}{X(s)}$$  \hspace{1cm} (1)$$

$$H(s) = \frac{L \int_{t_0}^{t} x(t) e^{-st} dt}{L \int_{t_0}^{t} f(t) e^{-st} dt} = \frac{X(s)}{F(s)}$$  \hspace{1cm} (2)$$

It can be seen from Equations (1) and (2) that the mechanical impedance of the system was the ratio of the excitation force $f(t)$ to the Laplace transform with a steady-state response of $x(t)$, and the mechanical admittance was its inverse.

2.2. Mechanical impedance of three components in a suspension system
The suspension system was a classic mechanical vibration isolation network. According to the new electromechanical analogy, the new suspension system included three types of components: springs, dampers, and inerteres. For different problems, the response of the suspension system can be
displacement, speed, and acceleration. There were three different forms of corresponding mechanical impedance: displacement impedance, velocity impedance, and acceleration impedance.

According to the definition of mechanical impedance, it was easy to deduce the impedance expressions of the three components in different forms, as shown in Table 2.

**Table 2.** Mechanical impedance of three types of mechanical components.

| Component | Displacement impedance | Velocity impedance | Acceleration impedance |
|-----------|------------------------|--------------------|-----------------------|
| Spring    | $k$                    | $k/s$              | $k/s^2$               |
| Damper    | $cs$                   | $c$                | $c/s$                |
| Inerter   | $bs^2$                 | $bs$               | $b$                   |

In the circuit system, resistors, inductors, and capacitors can be connected in series and parallel. Analogous to circuit systems, the connection of mechanical systems also had two forms of series and parallel and the Literature [7] analyzed it in detail and came to the following conclusions:

1. The inverse of the speed impedance of a series system consisting of any mechanical element is equal to the algebraic sum of the inverse of the impedance of each element.

   $$Z^{-1} = \sum_{i=1}^{n} \frac{1}{Z_i} = \sum_{i=1}^{n} Z_i^{-1} \quad (3)$$

2. The speed impedance of a parallel system composed of any mechanical element is the algebraic sum of the speed impedance of each element.

   $$Z = \sum_{i=1}^{n} Z_i \quad (4)$$

According to the substitution relationship between displacement impedance, velocity impedance, and acceleration impedance. It can be known that the above conclusions were also applicable to the case of displacement impedance and acceleration impedance.

3. **ISD suspension dynamics modeling**

According to the mechanical impedance characteristics of the $n$-element parallel system described above. The two-mass three-element ISD suspension dynamic model can be reduced to an impedance dynamic model shown in Figure 1. In this figure, the overhanging mass and the hanging mass were represented by $m_1$ and $m_2$. The wheel stiffness was represented by $k$. The $k$, $c$, and $b$ were the displacement impedances of the spring, damper, and inerter. $Y(s)$ was the total impedance of the isolation network. Assuming the road excitation was $q$, the vertical displacements of the suspended mass and suspended mass under this excitation were $z_1$ and $z_2$ [8, 9].

![Figure 1. 1/4 vehicle suspension impedance dynamics model.](image-url)
Based on the above conditions, the dynamic equation of the suspension dual-mass system can be listed according to Newton's second law:

\[
\begin{align*}
& m_s s^2 Z_s(s) + Y(s)[Z_s(s) - Z_i(s)] = 0 \\
& m_s s^2 Z_i(s) + Y(s)[Z_i(s) - Z_s(s)] + k[Z_i(s) - Q(s)] = 0
\end{align*}
\]

(5)

Where \( Y(s) = bs^2 + cs + k \).

The following result can be obtained from Equation (5).

\[
G_1(s) = \frac{Z_i(s)}{Z_i(s)} = \frac{Y(s)}{m_s s^2 + Y(s)}
\]

(6)

\[
G_2(s) = \frac{Z_s(s)}{Q(s)} = \frac{[m_s s^2 + Y(s) + k][m_s s^2 + Y(s)] - Y^2(s)}{[m_s s^2 + Y(s) + k][m_s s^2 + Y(s)] - Y^2(s)}
\]

(7)

\[
G_3(s) = \frac{Z_s(s)}{Q(s)} = \frac{Y(s) - k}{[m_s s^2 + Y(s) + k][m_s s^2 + Y(s)] - Y^2(s)}
\]

(8)

In the analysis of 1/4 suspension system, ACC was usually used to characterize passenger comfort; DLT was used to characterize the safety and handling stability of the wheels and SWS limited the stroke of the wheel up and down. Therefore, the above three indicators were often used to evaluate the performance of the suspension system.

The transfer function of ACC for road excitation \( \dot{q} \) was as follows:

\[
H_1(s) = \frac{z_2}{\dot{q}} = \frac{s^2 Z_s(s)}{s \cdot Q(s)} = \frac{s \cdot k_i \cdot Y(s)}{[m_s s^2 + Y(s) + k][m_s s^2 + Y(s)] - Y^2(s)}
\]

(9)

The transfer function of DLT for road excitation \( \dot{q} \) was as follows:

\[
H_2(s) = \frac{k_i (z_i - \dot{q})}{\dot{q}} = k_i \left( \frac{Z_s(s) - Q(s)}{s \cdot Q(s)} \right) = \frac{k_i}{s} \left( \frac{[m_s s^2 + Y(s) + k][m_s s^2 + Y(s)] - Y^2(s)}{[m_s s^2 + Y(s) + k][m_s s^2 + Y(s)] - Y^2(s)} - 1 \right)
\]

(10)

The transfer function of SWS for road excitation \( \dot{q} \) was as follows:

\[
H_3(s) = \frac{z_2 - z_i}{\dot{q}} = \frac{Z_2(s) - Z_i(s)}{s \cdot Q(s)} = \frac{-m_s s \cdot k}{[m_s s^2 + Y(s) + k][m_s s^2 + Y(s)] - Y^2(s)}
\]

(11)

Generally, ACC and DLT were used as the objective function for suspension system optimization, and SWS was used as a constraint condition to prevent the wheel's dynamic stroke from being too large and frequently hitting the limit block. \( H_1(s) \), \( H_2(s) \) and \( H_3(s) \) represent the transfer functions of three evaluation indexes obtained from a three-element parallel ISD suspension system. For different suspension systems, it can be analyzed by changing the impedance \( Y(s) \) of the isolation network. Therefore, Equations (9) – (11) can be used as a general model of suspension dynamics.

4. Influence of inerter’s coefficient on vibration peak of three-element parallel ISD suspension

Because the inerter was a component that had two free ends that replace the mass element. Its virtual mass had a certain effect on the overhanging mass and the hanging mass, which affected the suspension offset frequency and then caused the suspension vibration peak to change [6, 10]. Made the suspension model shown in Figure 1 perform undamped free vibration, which means \( c = 0, q = 0 \), then the equation of motion was built as the following form:

\[
\begin{align*}
&m_2 \ddot{z}_2 + b(\dot{z}_2 - \dot{z}_i) + k(z_2 - z_i) = 0 \\
&m_1 \ddot{z}_1 + b(\dot{z}_1 - \dot{z}_i) + k(z_1 - z_2) + k_i z_i = 0
\end{align*}
\]

(12)

Made \( m_1 \) and \( m_2 \) did simple harmonic excitation with the same frequency and phase. The system can obtain the following results without damping vibration:

\[
\begin{align*}
&-(m_1 \omega^2 + b \omega^2 - k) z_{20} + (b \omega^2 - k) z_{10} = 0 \\
&-(m_2 \omega^2 + b \omega^2 - k_i) z_{10} + (b \omega^2 - k) z_{20} = 0
\end{align*}
\]

(13)
Solving characteristic equations

\[ (m_1m_2 + bm_1 + bm_2)\omega^2 - (m_k + m_k + m_k + k)b\omega^2 + kk = 0 \]  

(14)

Made the \( A = m_1m_2 + bm_1 + bm_2 \), \( B = m_k + m_k + m_k + k \). We can get the following formula.

\[ \omega_{1,2}^2 = \frac{B \pm \sqrt{B^2 - 4Ak}}{2A} \]  

(15)

Based on the above calculations and analysis with the parameters of a certain vehicle suspension structure in Table 3, the influence of the inertia coefficient of the three-element parallel ISD suspension system on the vibration peak \( \omega_1 \) and \( \omega_2 \) can be simulated and analyzed. The results are shown in Figure 2, which showed the first and second vibration peaks’ frequency changed with the inertia coefficient \( b \). When the \( b \) had increased, the frequency of vibration peak had decreased, and the impact on the second peak is greater than the impact on the first peak.

Table 3. Parameter list of a certain vehicle suspension structure.

| Parameters       | Value   |
|------------------|---------|
| Overhanging mass | 2.774   |
| Hanging mass     | 289     |
| Wheel stiffness  | 6,000,000 |
| Spring stiffness | 61,600  |
| Damper coefficient | 6,800 |
| Inerter coefficient | 2,200 |

Figure 2. Effect of inertia coefficient \( b \) on the main suspension peak.

According to ISO 2631, in the vertical direction, the human sensitive frequency range was 4 ~ 12.5 Hz. It can be seen from Figure 2 that the frequency of the first peak was always < 4 Hz, which is not within the human sensitive range. When \( b > 900 \) kg, the frequency of the second peak entered the human sensitive range, which caused the performance of the suspension system to deteriorate.

5. Conclusions

This paper uses the principle of mechanical impedance to establish a suspension impedance dynamic model, and analyzes the effect of the inerter on the three-element parallel ISD suspension. The following conclusions are obtained:

(1) The impedance dynamic model established in this paper is used to analyze the suspension characteristics. It is applicable to different suspension vibration isolation networks and has certain versatility.
(2) Increasing inertia coefficient $b$ has a greater impact on the second frequency of vibration peak. When designing the suspension, the parameters of the inertial container can be determined according to the human frequency sensitivity range combined with the influence of the inertia coefficient on the main frequency.

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