Superlight inverse Doppler effect

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It has long been thought¹ that the inverse Doppler frequency shift of light¹⁴⁻¹⁸ is impossible in homogeneous systems with a positive refractive index. Here we break this long-held tenet by predicting a previously unconsidered Doppler effect of light inside a radiation cone, the so-called Vavilov–Cherenkov cone, under specific circumstances. It has been known from the classic work of Ginzburg and Frank that a superlight (that is, superluminal) normal Doppler effect¹⁰⁻¹² appears inside the Vavilov–Cherenkov cone if the velocity of the source v is larger than the phase velocity of light vₚ. By further developing their theory, we discover that an inverse Doppler frequency shift will arise if v > 2vₚ. We denote this as the superlight inverse Doppler effect. Moreover, we show that the superlight inverse Doppler effect can be spatially separated from the other Doppler effects by using highly squeezed polaritons (such as graphene plasmons), which may facilitate the experimental observation.

The Doppler effect, as a well-known phenomenon of motion-induced frequency shift, is one of the most fundamental mechanisms in physics and has vast applications in fields as varied as weather and aircraft radars, satellite global positioning systems, blood flow measurement in fetal vessels, laser vibrometry and the detection of extrasolar planets¹. It occurs whenever the source and the observer move relative to each other. For the conventional Doppler frequency shift, the received frequency is higher (lower) compared to the emitted frequency during the approach (recession). In 1843, Christian Doppler propounded the conventional Doppler effect, first in relation to sound, and then to light¹. Since then, it is believed that the counter-intuitive inverse Doppler frequency shift of light, where the sign of the frequency shift is opposite to that of the conventional Doppler frequency shift, cannot occur in homogeneous systems with a positive refractive index. In 1968, Victor Veselago predicted the inverse Doppler effect (with the inverse Doppler frequency shift) in homogeneous systems with a negative refractive index⁶⁻⁹, for their capability of hosting the unusual waves (including surface plasmons) whose phase and group velocities are antiparallel. Moreover, the inverse Doppler effect has been predicted and demonstrated in new scenarios that rely on strongly inhomogeneous systems (without a negative refractive index), using various periodic structures¹⁰⁻¹³ and interactions with shock waves¹⁰⁻¹³.

Here we find that it is possible to create the inverse Doppler frequency shift inside the Vavilov–Cherenkov cone in homogeneous systems with a positive refractive index, as long as the radiation source moves with a velocity v larger than twice the phase velocity vₚ of light, that is, v > 2vₚ. We denote this new phenomenon as the superlight inverse Doppler effect, which breaks the above century-old belief. The superlight inverse Doppler effect is different from the superlight normal Doppler effect¹⁴⁻¹⁸, which, first studied by Ginzburg and Frank in 1947, emerges when v > vₚ. While Ginzburg and Frank’s theory predicted that the frequency of the emitted photons transits from positive to negative when crossing the Vavilov–Cherenkov cone, we reveal that the Doppler effects inside the Vavilov–Cherenkov cone can be divided into two categories: the superlight normal and superlight inverse Doppler effects. The predicted superlight inverse Doppler effect circumvents the requirement for a negative-index homogeneous system or an inhomogeneous system, to create the inverse Doppler frequency shift. Moreover, Ginzburg and Frank’s theory focused on the sign of change in the internal energy of the source during its radiation; in particular, the change in the internal energy of the source is positive (negative) inside (outside) the Vavilov–Cherenkov cone. We find that the change in the internal energy of the source, when larger (smaller) than the energy of the emitted photon, leads to the superlight inverse (superlight normal) Doppler effect. This also indicates that the major part of the change in the kinetic energy of the source turns into a positive increase in the internal energy of the source (the energy of the emitted photon) for the superlight inverse (superlight normal) Doppler effect. Therefore, our finding further develops Ginzburg and Frank’s theory of the superlight normal Doppler effect.

For conceptual demonstration, we begin with the derivation of various Doppler effects of light. Consider that a radiation source (such as a point source with a dipole moment of P(τ', t')) moves in a system with a positive refractive index n (n > 0) and has a natural angular frequency of ωn (ωn > 0) in the moving source frame. After applying the Lorentz transformation¹⁰,¹¹, the frequency and wavevector in the two different frames (that is, the laboratory frame and the moving source frame) can be directly linked through the Lorentz transformation; that is,

\[
\begin{align*}
\vec{k} &= \frac{\vec{k}'}{\omega/c} = \frac{\vec{\alpha}}{\omega - \gamma \vec{\beta} r} \\
\vec{\alpha} &= \vec{\alpha}' = \vec{\alpha}'(\vec{k}' = \vec{k}' + \vec{k}''),
\end{align*}
\]

In equation (1), \(\vec{k} = \vec{k}' + \vec{k}''\) (\(\vec{k}' = \vec{k}' + \vec{k}''\)) and \(\omega (\omega' = \omega)\) are the wavevector and the frequency in the laboratory frame (the moving source frame), respectively; \(\vec{\gamma} = \vec{\gamma} + \vec{\gamma} v\) is the velocity of the source, with its normalized form being \(\vec{\beta} = \vec{\gamma}/c\); \(\gamma = (1 - \beta^2)^{-1/2}\) is the Lorentz factor; finally, we use the definition \(\vec{\alpha} = \vec{I} + (\gamma - 1)\vec{Z}\), with \(\vec{I}\) being the unity dyad. From the Lorentz transformation,
In equation (2), $k_z \in (-\infty, +\infty)$, and the positive (negative) value of $k_z$ represents the generated waves propagating along the $+z$ ($-z$) direction in the source static frame. Since $\omega = \gamma \omega_0$ is proportional to $k_z$, one also has $\omega \propto (\omega_0, +\infty)$, and $\omega < 0$ $(\omega > 0)$ when $k_z < -\omega_0/c (k_z > -\omega_0/c)$. By combining equations (2) and (3), $k_z = v \gamma \omega_0 / c + \gamma k_z^\prime$, where $\gamma$ is the Lorentz factor. The angles of $\theta_v$ and $\theta_{\text{SIDE}}$ are the opening angles of the Vavilov–Cherenkov cone and of the cone in which the superlight normal Doppler effect occurs, respectively. The value of $|\omega|$ goes to infinity at the angle of $\theta_v$ and is equal to $\omega_0 / \gamma$ at the angle of $\theta_{\text{SIDE}}$ (that is, no Doppler shift).

$$\omega = \gamma \omega_0 + \nu k_z^\prime$$  \hspace{1cm} (2)

$$k_z = \gamma \frac{\nu \omega_0}{c} + \gamma k_z^\prime$$  \hspace{1cm} (3)

In equation (2), $k_z^\prime \in (-\infty, +\infty)$, and the positive (negative) value of $k_z^\prime$ represents the generated waves propagating along the $+z$ ($-z$) direction in the source static frame. Since $\omega = \gamma \omega_0$ is proportional to $k_z^\prime$, one also has $\omega \propto (\omega_0, +\infty)$, and $\omega < 0$ $(\omega > 0)$ when $k_z^\prime < -\omega_0/c (k_z^\prime > -\omega_0/c)$. By combining equations (2) and (3), $k_z^\prime = \gamma \omega_0 / c + \gamma k_z^\prime$, where $\gamma$ is the Lorentz factor. The angles of $\theta_v$ and $\theta_{\text{SIDE}}$ are the opening angles of the Vavilov–Cherenkov cone and of the cone in which the superlight normal Doppler effect occurs, respectively. The value of $|\omega|$ goes to infinity at the angle of $\theta_v$ and is equal to $\omega_0 / \gamma$ at the angle of $\theta_{\text{SIDE}}$ (that is, no Doppler shift).

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The radiation source (represented by the red dot) moves along the \( \hat{z} \) direction in a homogeneous system with a constant positive refractive index \( n > 2.5 \) for illustration. The multiple phase fronts, illustrated by circular lines, are equally distributed in time. \( \theta \) is the angle between \( \vec{F} \) and \( \vec{z} \) (or \( \vec{F'} \)); when \( \theta = 0^\circ \), \( \vec{F} \) and \( \vec{z} \) are in the same direction. When \( v = 0 \), there is no Doppler effect. When \( v < c/n \), the conventional Doppler effect exists at \( \theta = 0^\circ \). When \( c/n < v < 2c/n \), the superlight normal Doppler effect appears at \( \theta = 0^\circ \). When \( v < 2c/n \), the distance between successive wavefronts at \( \theta = 0^\circ \) (labelled in each panel) is reduced and the waves bunch together. This leads to \( |u| > \omega_0/y \) at \( \theta = 0^\circ \) in b,c,d. When \( v > 2c/n \), the distance between successive wavefronts at \( \theta = 0^\circ \) is enlarged and the waves spread out. This leads to the superlight inverse Doppler effect (\( |u| < \omega_0/y \) at \( \theta = 0^\circ \)) in d. The Lorentz factor \( \gamma \) is different in each panel.

From equation (7), the occurrence condition for the superlight inverse Doppler effect is \( v > 2c/n \). We note that this condition is numerically similar to the threshold for quantum friction between two uncharged polarizable moving slabs—that is, \( |v_1 - v_2| > c \left( \frac{1}{\gamma} + \frac{1}{\gamma'} \right) \) (see equation (20) in ref.21), where \( v_1(n_1) \) and \( v_2(n_2) \) are the moving velocities (the refractive indices) of the two slabs; especially when \( n_1 = n_2 = n_0 \), one has \( |v_1 - v_2| > 2c/n \). In addition, since the condition of \( n_0 \cos \theta = 1 \) is possible only for \( \theta < 90^\circ \), one always has \( \theta < 90^\circ \) for the superlight normal and superlight inverse Doppler effects (see Fig. 1).

Figure 2 schematically shows the difference between the different Doppler effects in the time domain, marking multiple wavefronts that are equally distributed in phase. In the forward direction \( (\theta = 0^\circ) \); that is, \( \vec{F} \) and \( \vec{z} \) are in the same direction), both the conventional and superlight normal Doppler effects (that is, when \( v < 2c/n \)) have the distance between successive wavefronts reduced, so the waves bunch together (Fig. 2b,c); in contrast, the superlight inverse Doppler effect (that is, when \( v > 2c/n \)) has the distance between successive wavefronts enlarged, so the waves spread out (Fig. 2d). This shows a clear difference between the superlight inverse Doppler effect and the other two Doppler effects. Figure 2 also highlights a clear difference between the conventional Doppler effect and the two superlight Doppler effects: at \( \theta = 0^\circ \), for the conventional Doppler effect (that is, when \( v < c/n \)), the observer first receives the waveform emitted at an earlier time (Fig. 2b); however, for the superlight normal and superlight inverse Doppler effects (that is, when \( v > c/n \)), the observer first receives the waveform emitted at a later time (Fig. 2c–d), corresponding to \( \omega < 0 \) in equations (6) and (7).

Since the predicted superlight inverse Doppler effect requires two times the Cherenkov threshold (that is, \( v > 2c/n ) \), it is experimentally challenging to achieve for light waves. However, the high effective refractive index of highly squeezed polaritons (such as those in graphene)24 may enable the superlight inverse Doppler effect to occur with a relatively small value of \( v \). As an example, Fig. 3 uses the surface plasmon polaritons in graphene for conceptual demonstration. Assume that the source is a dipole with \( \omega_0/2\pi = 10 \) THz, moving parallel to a graphene monolayer. The graphene has a chemical potential of 0.15 eV and a relaxation time of 0.3 ps. Then graphene plasmons have an effective refractive index \( n = \omega_0/\gamma_0 = 19 \) at \( \omega = 10 \) THz, where \( \gamma_0 = \omega_0/c = (1/19) \), and thus there is only the conventional and superlight normal Doppler effects (see the dashed lines in Fig. 3). When \( v = 0.3c \), one has \( v > 2\gamma_0 \). This way, the superlight inverse Doppler effect also emerges, as shown by the solid lines in Fig. 3 that cross between the two regimes.

The spatial overlap between the forward/backward-propagating graphene plasmons (both can have a smaller frequency than that of the source; see Fig. 2d) makes it difficult to detect the superlight inverse Doppler effect. Next we show in Fig. 4 that it is possible to convert the propagation of excited graphene plasmons from forward/backward to right/left, which can further facilitate potential experimental demonstrations in both the near field and far field (simply by introducing a scatter in the right region).

Consider a circularly polarized source, which moves along the \( +\hat{z} \) direction and has a dipole moment of \( \mathbf{P} = (\mathbf{r}' ; t') = \text{Re} (\xi \mathbf{\hat{P}} + \eta \mathbf{\hat{P}}^*) e^{-i\omega t'} \delta(\mathbf{r}) \) with \( \mathbf{p}_\perp = p_\perp = 1 \) in the source static frame. Such circularly polarized sources with \( v = 0 \) have been widely exploited for asymmetric excitation and can be formed by the interaction between the circularly polarized light and the designed defect25. Similarly, the circularly polarized moving dipole with \( v \neq 0 \) (or with \( v > 0.1c \) in Fig. 4) can be effectively created by the interaction between a circularly polarized moving optical spot26 and a designed defect (such as a long nanowire deposited above graphene, or a slit in the substrate of graphene); the optical spot moving with a tunable velocity, for example, can be achieved by illuminating the designed defect with an obliquely incident optical pulse, which is focused by a cylindrical lens to a line24.

Figure 4a shows the distribution of the emitted plasmonic field in the graphene plane in the time domain; see the dynamics of the emitted plasmons in Supplementary Movie. When \( v = 0.3c \), two asymmetric caustics27,28 are formed in the regions \( x < 0 \) (the left side of the source) and \( x > 0 \) (the right side of the source), respectively. Since the caustic frequency is close to the frequency of the wave component that dominates the plasmonic emission at each caustic27,
Fig. 3 | Superlight inverse Doppler effect of graphene plasmons. A dipole with $\omega_0/2\pi = 10$ THz moves parallel to a graphene monolayer surrounded by air. When $v = 0.1c$, only the conventional and superlight normal Doppler effects exist. When $v = 0.3c$, the superlight inverse Doppler effect also occurs. The chemical potential of graphene is $0.15$ eV and the relaxation time is 0.3 ps. Here, and in the figure below, the frequency dispersion of graphene plasmons is considered and the effective refractive index of graphene plasmons is $n = 19$ at 10 THz. The superlight inverse Doppler effect thus appears only when $v > 2c/n = 2c/19$.

it can be used to determine the type of Doppler effect that dominates each region. Due to the asymmetry of the two caustics in Fig. 4a, regions $x < 0$ and $x > 0$ have different caustic frequencies. The caustic frequency is calculated from $\omega_{\text{caustic}} = \omega_0/v$ (ref. 1), where $\varphi = k_{\text{spp}} + k_{\text{caustic}}$ (see Supplementary Information).

Figure 4b,c shows the two caustic frequencies $|\omega_{\text{caustic}}|$ and the propagation angles $\theta_{\text{caustic}}$ (the angle between $k_{\text{spp}}$ and $\varphi$ at each caustic frequency) for the regions $x < 0$ and $x > 0$ as a function of $v$, respectively. One can see that the region $x < 0$ is dominated by the conventional Doppler effect with $\omega_{\text{caustic}} < \omega_0/v$, independent of the value of $v$. In contrast, the region $x > 0$ is dominated by the superlight normal Doppler effect with $|\omega_{\text{caustic}}| > \omega_0/v$ when $v < 0.15c$, and, importantly, becomes dominated by the superlight inverse Doppler effect with $|\omega_{\text{caustic}}| < \omega_0/v$ when $v > 0.15c$. It should be emphasized that, when $v > 0.15c$, the excited plasmon in region $x < 0$ (having the conventional Doppler effect) propagates along the backward ($-\hat{z}$) direction with $\theta_{\text{caustic}} > 90^\circ$; in contrast, the excited plasmon in region $x > 0$ (dominated by the superlight inverse Doppler effect) propagates along the forward ($+\hat{z}$) direction with $\theta_{\text{caustic}} < 90^\circ$ (see Fig. 4a and Supplementary Movie for an example).

Perhaps even more important is the vision our findings emphasize: that the analogous phenomenon of the superlight inverse Doppler effect will exist in virtually any wave system in nature, including classical wave systems such as acoustic waves and surface waves, as well as quantum wave systems such as the Dirac equation; all of which have shown the occurrence of the Doppler effect, and therefore can now also support the analogous phenomenon of the superlight inverse Doppler effect. For example, the superlight normal Doppler effect has been used to construct the fibre-optical analogue of Hawking radiation$^{29,30}$. It is thus intriguing to note the potential emergence of the superlight inverse Doppler effect in such a system, when the pulse (establishing an effective moving medium) moves with a velocity at least two times larger than the phase velocity of some ultraviolet modes$^{29}$.

Methods

Methods, including statements of data availability and any associated accession codes and references, are available at https://doi.org/10.1038/s41567-018-0209-6.
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**Author contributions**

All authors contributed extensively to the work presented in this paper. X.L., X.S. and B.Z. conceived the research. X.S. and X.L. performed the calculation. I.K., J.D.J., M.S., F.G., Z.Y. and B.Z. contributed insight and discussion on the results. X.L., X.S., B.Z., I.K., J.D.J. and M.S. wrote the paper. B.Z., I.K., J.D.J. and M.S. supervised the project.

**Competing interests**

The authors declare no competing interests.

**Additional information**

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Methods

Surface conductivity of graphene. The Drude model is adopted to characterize the surface conductivity of graphene in the studied frequency range of ~2–18 THz. The Drude model is sufficient, with no need to consider the nonlocal effect and the interband contribution, because the studied frequency range is far smaller than the frequency corresponding to twice the Fermi energy (that is, $2\mu_c = 0.3$ eV translates to ~72 THz; see Supplementary Information). We have considered the frequency dispersion of graphene plasmons for all of the results in Figs. 3 and 4.

Doppler effects of light derived from the conservation laws of energy and momentum. Here we derive the various Doppler effects from the conservation laws of energy and momentum. Consider that a radiation source (such as a dipole) has a natural angular frequency of $\omega_0$ in the moving source frame and moves in a system with a positive refractive index $n$; this basic set-up is the same as that in the main text. When the momentum of the emitted photon $\Delta P = n\hbar\omega_0/c$ is trivial in comparison with that of the source in the laboratory frame, the conservation law of momentum during radiation requires $\Delta E = n\hbar\omega_0/c$ (see Fig. 1), the change in the internal energy of the source is $\Delta U = \hbar\omega_0/c$ (refs 14–17); that is,

$$\Delta E = n\hbar\omega_0/c \cos \theta$$  \hspace{1cm} (9)

where $\theta$ is the angle between the velocity $\vec{v} = 2\vec{v}$ of the source and the wavenumber of the emitted photon. In the laboratory frame, $\Delta E$ is determined by the energy of the emitted photon $\hbar\omega_0$ and the change in the internal energy of the source $\Delta U = \hbar\omega_0/c$ (refs 14–17); that is,

$$\Delta E = \hbar\omega_0 \pm \hbar\omega_0/c \gamma$$  \hspace{1cm} (10)

where $\gamma = (1-\nu^2/c^2)^{-1/2}$ is the Lorentz factor. In equation (10), as emphasized by Ginzburg and Frank \cite{14–17}, the minus sign should be adopted if the system passes from an upper energy level to a lower one during the radiation (that is, if the energy of the emitted photon is supplied from both the internal energy of the source and the kinetic energy); the plus sign should be used if, when emitting the photon, the system becomes excited (that is, if the kinetic energy supplies both the energy of the emitted photon and the positive increase in the internal energy of the source). From equations (9) and (10), one obtains

$$\frac{n\hbar\omega_0/c}{\gamma} \cos \theta = \omega_0 \pm \omega_0/c \gamma$$  \hspace{1cm} (11)

To satisfy equation (11), the minus sign should be adopted when $n\hbar\omega_0/c \cos \theta < 1$ and the plus sign should be adopted when $n\hbar\omega_0/c \cos \theta > 1$. Namely, equation (11) is exactly the same as equation (4) in the main text.

In addition, from equation (11), the change in the internal energy of the source can be expressed as

$$|\Delta U| = \left| \hbar\omega_0 / \left( \frac{n\hbar\omega_0/c}{\gamma} \cos \theta - 1 \right) \right|$$  \hspace{1cm} (12)

From equation (12) and the above analysis, the positive increase in the internal energy of the source is larger than the energy of the emitted photon for the case of $n\hbar\omega_0/c \cos \theta > 2$ (that is, leading to the superlight inverse Doppler effect), while is less than the energy of the emitted photon for the case of $1 < n\hbar\omega_0/c \cos \theta < 2$ (that is, leading to the superlight normal Doppler effect). This indicates that the major part of the change in the kinetic energy of the source turns into the positive increase in the internal energy of the source for the superlight inverse Doppler effect, and turns into the energy of the emitted photon for the superlight normal Doppler effect.

Data availability. The data that support the plots within this paper and other findings of this study are available from the corresponding author upon reasonable request.