MATTER INDUCED NEUTRINO DECAY:
NEW CANDIDATE FOR THE SOLUTION TO THE SOLAR
NEUTRINO PROBLEM

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Abstract

Effects of the dense matter can induce the neutrino decay even if neutrinos are stable in vacuum. This is due to coherent interactions with matter which lead to energy level-splitting between the neutrino and antineutrino states and thereby provide available phase space for the majoron emission. We show that the matter induced decay can be a plausible candidate for the explanation of solar neutrino deficit, provided that there exist new flavour-changing weak range interactions of neutrino with matter constituents, and constant of \( \tau \)-neutrino coupling to the majoron is sufficiently large. This mechanism naturally implies the hierarchy between the ratios \( Z = \frac{\text{measured signal}}{\text{SSM expectation}} \) for Ga-Ge, \( \nu_e e \) scattering and Cl-Ar experiments - \( Z_{\text{Ga}} > Z_K > Z_{\text{Cl}} \). The important feature of the matter induced decay is the prediction of solar antineutrino flux with substantially degraded energy spectrum compared to solar neutrinos. This scenario can be unambiguously tested by future solar neutrino detectors BOREXINO/BOREX and SNO.
1. Solar Neutrino Problem (SNP). The discrepancy between the neutrino flux predicted by the Standard Solar Model (SSM) \[1, 2\] and the observed flux, is the main open issue in neutrino physics. The ratio $Z$ of the observed signal to that is expected from the SSM is different for different solar neutrino experiments. Namely, the Homestake Cl-Ar experiment gives

$$Z_{Cl} = 0.28 \pm 0.04$$

(1)

The results of Kamiokande II and Kamiokande III are \[3\]:

$$Z_{KII} = 0.46 \pm 0.05 \pm 0.06, \quad Z_{KIII} = 0.56 \pm 0.07 \pm 0.06$$

(2)

The combined result of both implies $Z_K = 0.49 \pm 0.05 \pm 0.06$. Finally, the results of the Ga-Ge radiochemical experiments GALLEX \[6\] and SAGE \[7\] are

$$Z_{Ga}^{GALLEX} = 0.63 \pm 0.14 \pm 0.06, \quad Z_{Ga}^{SAGE} = 0.44 \pm 0.15 \pm 0.11$$

(3)

with the combined result $Z_{Ga} = 0.54 \pm 0.11$ (all the above data are given with $1\sigma$ error due to experiment and do not include "3\sigma" theoretical uncertainties of the SSM \[2\]). Focusing on the central values and neglecting experimental uncertainties, the following hierarchy of the data is obeyed:

$$Z_{Ga} > Z_K > Z_{Cl}$$

(4)

This points out that the SNP cannot be explained by a non-standard temperature of the solar core, since this would imply $Z_{Cl} > Z_K \[3\].

The non-standard particle physics solutions require generally the violation of lepton number or lepton flavour, that leads to new neutrino properties and new physical phenomena. Namely, neutrinos produced in the sun can be converted into neutrinos of different flavour and/or helicity while they propagate to the earth. The popular explanations of the solar neutrino deficit through the neutrino oscillations in vacuum\[8, 9\] or in matter \[10, 11\] and spin-flavour transitions \[12, 13\] are still waiting for being confirmed (or excluded) by new data from future facilities.

2. The SNP solution through neutrino decay in vacuum. The idea that the deficit of solar neutrinos can be due to their decay during the flight from sun to earth was suggested long time ago \[15\]. Since the fast radiative decay is excluded both from particle physics and astrophysical arguments,

\footnote{The averaged vacuum oscillations $\nu_e \rightarrow \nu_\mu, \nu_\tau$ predict, in contradiction with the data, the equal signal for two radiochemical experiments: $Z_{Ga} = Z_{Cl} > 0.33$, and also $Z_K = Z_{Cl} + 0.15(1 - Z_{Cl}) > Z_{Ga}$. However, the vacuum oscillations in "Just So" regime \[\text{\footnotesize{1}}\] still can be regarded as a plausible candidate for the SNP solution \[\text{\footnotesize{14}}\].}
one has to consider fast invisible decay modes of the neutrino, e.g. with the emission of majoron, the Goldstone boson related to spontaneous violation of the global lepton number symmetry $U(1)_{B-L}$. From the viewpoint of the majoron model building the possibility of neutrino decay during the flight time $t \simeq 500$ s implies the following two conditions:

(i) *Sufficiently strong $\nu$-majoron couplings* ($h > 10^{-4}$). This in turn requires very low scale of the lepton number violation ($\eta_{BL} < 10$ keV). The most familiar candidate for such a low $\eta_{BL}$, the triplet majoron model [16], has been ruled out by LEP data on $Z$-boson invisible width, whereas the ”seesaw” type singlet majoron [17] generally implies $\eta_{BL} > 100$ GeV and therefore is extremely weakly coupled to neutrinos. However, a variety of new singlet majoron models can be considered [18, 19] in which the scale $\eta_{BL}$ can be sufficiently low as to provide coupling constants in the needed range.

(ii) *Existence of majoron off-diagonal couplings*. As it was emphasized in [20], the neutrino decay scenario cannot be realized in simple majoron models, in which global $U(1)_{B-L}$ symmetry acts on all lepton families in the same way. In order to achieve the existence of majoron tree-level off-diagonal couplings between neutrino mass eigenstates one has to complicate the theory. Namely, different lepton flavours should be distinguished by different charges of the global $U(1)$ symmetry (in which case the ”virgin” idea of the $U(1)_{B-L}$ symmetry is actually lost), or different lepton number symmetries ($U(1)_{e} \otimes U(1)_{\mu} \otimes U(1)_{\tau}$) should be invoked [21].

The simplest decay scenario with negligible neutrino mixing, i.e. the case $\nu_e \rightarrow \nu_x + \chi$ with decay length adjusted to the sun-earth distance [16, 22], is completely excluded by the $\bar{\nu}_e$ pulse observation from SN1987A. However, this does not rule out the scenario with large neutrino mixing [23, 24]. Its implications were investigated to a full extent in [25, 26]. It was shown that this scenario can reconcile the Davis and Kamiokande data, leading to $Z_{Cl} < Z_{K}$. However, this scenario implies $Z_{Ga} < Z_{Cl}$ due to the energy dependence of the decay probability in vacuum which suppresses more the low energy neutrinos. Thus, this mechanism is disfavoured by the GALLEX data even if not excluded yet.

3. Matter Induced Decay (MID). As it was shown in [23], the effects of dense matter can induce the neutrino decay with majoron emission even in the case of simplest majoron model, when neutrinos are stable in vacuum. The point is that the coherent interactions with medium lead to the energy splitting between the $\nu$ and $\bar{\nu}$ states providing available phase space for the

\footnote{Another implication of these models can be observable neutrinoless $2\beta$ decay with majoron emission.}
emission of the majoron. Subsequently the implications of neutrino decay in matter were studied in a number of papers [27, 28, 29, 30].

Let us remind the main features of the MID with majoron emission. For the simplicity we consider the case of two neutrino flavours $\nu_e, \nu_x$ ($x = \mu, \tau$), which are defined as left-handed Weyl spinors: $\nu_e = \nu_{eL}$, $\nu_x = \nu_{xL}$ (the antineutrino states have opposite chirality: $\bar{\nu}_e = \bar{\nu}_{eR} = C\bar{\nu}_{eL}$, $\bar{\nu}_x = \bar{\nu}_{xR} = C\bar{\nu}_{xL}$, where C is the matrix of charge conjugation). In the majoron picture the neutrino masses and mixing arise from the Yukawa couplings to some complex scalar field $\sigma$ with non-zero vacuum expectation value $\langle \sigma \rangle = \eta_{BL}/\sqrt{2}$, which spontaneously violates the lepton number:

$$\sigma = \frac{1}{\sqrt{2}}(\eta_{BL} + \rho)e^{i\chi}$$

where $\rho$ is a Higgs scalar with a mass $\sim \eta_{BL}$ and $\chi$ is a massless majoron. In the flavour basis the Lagrangian of neutrino interaction with the majoron has the form

$$L = (\bar{\nu}_e, \bar{\nu}_x) \begin{pmatrix} h_{ee} & h_{ex} \\ h_{xe} & h_{xx} \end{pmatrix} \frac{i}{2} \chi \begin{pmatrix} \bar{\nu}_e \\ \bar{\nu}_x \end{pmatrix} + h.c. \tag{6}$$

and the neutrino mass matrix reads as

$$\hat{M} = \begin{pmatrix} m_{ee} & m_{ex} \\ m_{xe} & m_{xx} \end{pmatrix} = \eta_{BL} \begin{pmatrix} h_{ee} & h_{ex} \\ h_{xe} & h_{xx} \end{pmatrix} \tag{7}$$

Obviously, the matrix (6) becomes diagonal together with (7). Thus the majoron couplings with neutrino eigenstates $\nu_1 = c\nu_e + s\nu_x$, $\nu_2 = -s\nu_e + c\nu_x$, with masses $m_1$ and $m_2$ respectively, are the following:

$$L = (\bar{\nu}_1, \bar{\nu}_2) \begin{pmatrix} h_1 \\ h_2 \end{pmatrix} \frac{i}{2} \chi \begin{pmatrix} \bar{\nu}_1 \\ \bar{\nu}_2 \end{pmatrix} + h.c. \tag{8}$$

where

$$c = \cos\theta, \quad s = \sin\theta, \quad \tan 2\theta = \frac{2h_{ex}}{h_{xx} - h_{ee}}$$

$$h_1 = c^2h_{ee} + s^2h_{xx} + 2cs_h_{ex}, \quad h_2 = c^2h_{xx} + s^2h_{ee} - 2cs_h_{ex} \tag{9}$$

As far as there are no off-diagonal tree-level $\nu$-majoron couplings, the heavier neutrino mass eigenstate cannot decay into the lighter one with majoron emission. Thereby, in the simplest majoron models the neutrinos are stable in vacuum [20].

However, the presence of matter will induce the neutrino decay even in the simplest majoron model. Indeed, the neutrino propagation in matter is
described by the following Schrödinger equation:

\[ i \frac{d}{dt} \begin{pmatrix} \nu_e \\ \nu_x \end{pmatrix} = \hat{H}_\nu \begin{pmatrix} \nu_e \\ \nu_x \end{pmatrix} \]  \hspace{1cm} (10)

where the Hamiltonian reads:

\[ \hat{H}_\nu = \begin{pmatrix} V_e + \left( c^2 m_1^2 + s^2 m_2^2 \right)/2E & -cs(m_2^2 - m_1^2)/2E \\ -cs(m_2^2 - m_1^2)/2E & V_x + \left( c^2 m_2^2 + s^2 m_1^2 \right)/2E \end{pmatrix}. \]  \hspace{1cm} (11)

where \( E \) is the neutrino energy and \( V_e \) and \( V_x \) are the matter induced potentials of the current eigenstates \( \nu_e \) and \( \nu_x \) respectively:

\[ V_{e,x} = \sqrt{2} G_F \frac{\rho}{m_N} v_{e,x}; \quad v_e = Y_e - \frac{Y_n}{2} = 1 - \frac{3}{2} Y_n, \quad v_x = -\frac{Y_n}{2}. \]  \hspace{1cm} (12)

Here \( G_F \) is the Fermi constant, \( \rho \) is the matter density, \( m_N \) is the nucleon mass, and \( Y_{e,n} \) are the number of electrons and neutrons per nucleon (\( Y_e = 1 - Y_n \) for the electrically neutral matter). The evolution of antineutrino states is described by the matrix \( \hat{H}_{\nu} \) of form analogous to (11) but with potentials of opposite sign: \( V_{\tilde{e},\tilde{x}} = -V_{e,x} \). Therefore, the \( \nu - \tilde{\nu} \) level-splitting appears that provides non-zero phase space for certain transitions between neutrino and antineutrino matter eigenstates with majoron emission. Clearly, the vacuum (mass) eigenstates do not coincide with the matter eigenstates. Moreover, in this case of light enough neutrinos \( (m^2 \ll V E) \) the latter are essentially the flavour eigenstates \( \nu_e, \nu_x \) (and \( \tilde{\nu}_e, \tilde{\nu}_x \) for antineutrinos), so that the \( \nu - \tilde{\nu} \) transition matrix is given by (6) and \( \hat{H}_\nu = -\hat{H}_{\nu} \). Therefore, the majoron transitions can be flavour diagonal as well as flavour changing (but necessarily with helicity-flipping). The corresponding decay widths were calculated in [23]:

\[ \Gamma_{ee} = \frac{h^2}{16\pi} 2V_e, \quad \Gamma_{e\tilde{x}} = \frac{h^2}{16\pi} (V_e + V_x), \quad \Gamma_{x\tilde{x}} = \frac{h^2}{16\pi} 2V_x \]  \hspace{1cm} (13)

where \( \Gamma_{e\tilde{x}} \equiv \Gamma(\nu_e \to \tilde{\nu}_e + \chi) \) etc. Negative width means that the corresponding \( \nu \) and \( \tilde{\nu} \) states must be interchanged (e.g. since \( V_x < 0 \), \( \tilde{\nu}_x \) decays into \( \nu_x \)). These decay widths do not depend on neutrino energy, because the increase of the phase space for a fast moving neutrino \( (\propto EV) \) is cancelled by time dilatation effect \( (\propto 1/E) \). The energy distribution of the secondary states does not depend on matter potentials [27]:

\[ W(E, E') = \frac{1}{\Gamma(E)} \frac{d\Gamma(E, E)}{dE'} = 2 \frac{E - E'}{E^2} \]  \hspace{1cm} (14)
where E’ is the energy of the secondary antineutrino. Therefore, the secondary \( \bar{\nu} \) is strongly degraded - in average 2/3 of the initial neutrino energy E is taken away by the majoron.

The probability that the neutrino will undergo MID passing a medium with varying density does not depend on its energy. The flux of \( \nu_e \) survived the MID at the distance R from the origin, is [27]

\[
\Phi_e(R) = \Phi_0^e \exp \left( -\frac{1}{8d_0} \sum_x h_{ex}^2 d_{ex}^{\text{eff}} \right),
\]

where \( d_0 = \sqrt{2\pi G_F^{-1} m_N} \approx 1.6 \cdot 10^9 \text{ g/cm}^2 \) is the refraction width and

\[
d_{ex}^{\text{eff}} = \int_0^R \rho(r) [v_e(r) + v_x(r)] dr, \quad x = e, \mu, \tau.
\]

According to eqs. (12) we have:

\[
d_{ee}^{\text{eff}} = 2d^{\text{eff}} - 3d_n^{\text{eff}}, \quad d_{ex}^{\text{eff}} = d^{\text{eff}} - 2d_n^{\text{eff}} \quad (x = \mu, \tau)
\]

where

\[
d^{\text{eff}} = \int_0^R \rho(r) dr, \quad d_n^{\text{eff}} = \int_0^R Y_n(r) \rho(r) dr
\]

are the matter effective widths traversed by neutrinos.

The properties of the MID are drastically different from the properties of decay in vacuum. First, the MID of neutrino occurs into the state of opposite helicity, i.e. antineutrino state, whereas in vacuum both the helicity conserving and helicity flipping modes have comparable decay width. Second, in matter both flavour-changing \( \nu_e \rightarrow \bar{\nu}_\mu,\tau + \chi \) and flavour conserving \( \nu_e \rightarrow \bar{\nu}_e + \chi \) decays are possible. Third, the MID exhibits unusual dependence of neutrino lifetime on its energy in laboratory frame, which does not depend or even decreases with energy, quite opposite to the case of decay in vacuum when the slow particles decay faster. Finally, the neutrino lifetime versus the decay in matter is effectively determined by the matter width passed by neutrinos.

Recalling that for the solar medium \( Y_n \leq 0.33 \), the solar neutrinos can decay in both channels, \( \nu_e \rightarrow \bar{\nu}_e + \chi \) and \( \nu_e \rightarrow \bar{\nu}_x + \chi \) \( (x = \mu, \tau) \). Let us calculate the mean matter widths \( < d^{\text{eff}} > \) for the solar neutrinos coming from different sources \( (^8B, ^7Be, pp) \), averaged with their production distribution in the sun and different directions of flight:

\[
< d^{\text{eff}} > = \frac{1}{4\pi} \int_V \int_{\hat{n}} d\hat{r}d\hat{n} Q(\hat{r}) \tilde{d}_{\text{eff}}(\hat{r}, \hat{n})
\]

where \( d\hat{r} \) is the elementary volume, \( d\hat{n} \) is the elementary solid angle, \( Q(\hat{r}) \) are the distributions of production rates for the neutrinos from different sources.
and \( \tilde{d}^{eff} \) is the matter width passed by neutrino created in the volume \( d\vec{r} \) and flying in the direction \( d\vec{n} \). The above expression is easily simplified due to spherical symmetry of the sun - \( Q(\vec{r}) = \frac{1}{4\pi r^2}Q(r) \):

\[
< d^{eff} > = \int_0^R \int_0^1 drd(cos\theta')Q(r)\tilde{d}^{eff}(r\sin\theta')
\]

(20)

were \( \theta' \) is the angle between \( \vec{r} \) and \( \vec{n} \), and

\[
\tilde{d}^{eff}(r\sin\theta') = \int_0^R dl\rho(z), \quad z = \sqrt{r^2\sin^2\theta' + l^2}
\]

(21)

The expression for the other width \( < d^{eff}_n > \) is completely the same with \( \rho \rightarrow \rho Y_n \) in eq. (21). Taking the distribution functions \( Q(r) \) for the neutrinos from different sources (\(^8B, ^7Be, pp\)) as they are tabulated in [1], we have (in g/cm\(^2\)):

\[
< d^{eff} > = 1.38 \cdot 10^{12}, \quad < d^{eff}_n > = 0.29 \cdot 10^{12} \quad (^8B: E = 0 - 15 MeV)
\]

\[
< d^{eff} > = 1.26 \cdot 10^{12}, \quad < d^{eff}_n > = 0.25 \cdot 10^{12} \quad (^7Be: E = 0.861 MeV)
\]

\[
< d^{eff} > = 1.08 \cdot 10^{12}, \quad < d^{eff}_n > = 0.20 \cdot 10^{12} \quad (pp: E = 0 - 0.42 MeV)
\]

(22)

These differences are due to the fact that high energy neutrinos are mostly produced in the deeper and more dense solar core and thereby have to pass larger matter width before leaving the sun.

Therefore, if the solar neutrino deficit can be related with MID, then the relation \( Z_{Ga} > Z_{Cl} \) is expected naturally. This is due to the fact that the solar \( pp \) neutrinos (which do not contribute to the Chlorine experiment, but are responsible for about 55% of the signal in Gallium experiments) pass about 20-30% less effective width compared to the Boron neutrinos and thereby have less chance to undergo MID.

However, in the case of Hamiltonian (11), due to only standard interactions (i.e. neutrino scattering off the particles with Z and W boson exchange), the effect of MID with majoron emission cannot provide a solution to the SNP due to the strong existing bounds on the \( \nu \)-majoron coupling costants:

\[
h_{ee} < 3 \cdot 10^{-4} \quad [32],
\]

\[
\sum_x h_{xx}^2 \leq 4.5 \cdot 10^{-5} \quad \sum_x h_{\mu x}^2 \leq 5.4 \cdot 10^{-4} \quad (x = e, \mu, \tau, ... ) \quad [33]
\]

(23)

Moreover, these constraints allow \( \bar{\nu}_e \) signal originated by the solar neutrino decay to be at most at the borderline of detectability even for the future large volume detectors like Super-Kamiokande or BOREX [31] - only few percents
of solar neutrinos can undergo MID and, moreover, the energy spectrum of secondary antineutrinos is strongly degraded. In the case when neutrino masses are not negligible, the decay probabilities become even less [31].

However, the $\nu_\tau$-majoron coupling constant $h_{\tau\tau}$ is not really restricted by any laboratory constraints. It can be as large as $O(10^{-1})$, providing very fast decay $\bar{\nu}_\tau \rightarrow \nu_\tau + \chi$ in the solar medium. This cannot solve SNP by the simple reason that the solar neutrinos are not $\bar{\nu}_\tau$. However, the presence of neutrino non-standard weak range interactions with matter constituents can drastically change the situation. In the next section we shall show how to take advantage of this fact.

3. MID due to non-standard neutrino interactions. Indeed, the MID scenario can be more appealing if possible non-standard neutrino interactions with the matter particles are included. Generally, such interactions emerge inevitably in the context of singlet majoron models [18] that potentially can provide the $\nu$-majoron coupling constants in a strong regime relevant for MID. These models utilize new charged (or coloured+charged) scalars with masses within 100 GeV range. Their exchange, after proper Fierz trasformation, effectively provides the new channels of the neutrino neutral vector current scattering with quarks and charged leptons, which effectively contribute to the neutrino potentials in unpolarized medium (see also majoronless models of refs.[34, 35, 36]). These non-standard interactions generally can be flavour-conserving as well as flavour changing.

Let us analyse the possible impact of such interactions in the presence of enough strong $\nu_\tau$ - majoron coupling. Taking into account that the $\mu$-neutrino cannot be relevant due to the strong bound (23) on $\nu_\mu$-majoron coupling constants, we omit for the simplicity the $\nu_\mu$ state and assume that $\nu_x = \nu_\tau$. Let us consider, as an example, only neutrino elastic scattering off d-quarks (e.g. in the context of ”coloured” Zee model in ref. [18]) due to the following NC interactions:

$$L_{\text{eff}} = - \sum_{\alpha, \beta} \sqrt{2} G^d_{\alpha\beta} (\bar{\nu}_\alpha L \gamma_\mu \nu_\beta L) (\bar{d} \gamma_\mu d + \xi_{\alpha\beta} \bar{d} \gamma_\mu \gamma^5 d), \quad (\alpha, \beta = e, \tau)$$ (24)

Bearing in mind that only vector currents are relevant for the coherent neutrino scattering off unpolarized medium, we define 3 new parameters which represent the ratios of the new amplitudes to the standard one

$$\varepsilon_{e,\tau} = A^{VNC}(\nu_{e,\tau} d \rightarrow \nu_{e,\tau} d)/A^{W} = G^d_{ee,\tau \tau}/G_F$$ (25)

It was shown in refs. [33, 34] that such non-standard interactions of neutrinos, for a certain region of corresponding coupling constants, can effectively induce resonant neutrino conversion in solar medium even in the absence of neutrino mass terms and thereby solve the SNP.
The laboratory bounds on the new coupling constants are rather weak. E.g. from $\nu_e$ scattering we have limits on $\varepsilon = \sqrt{\varepsilon_e^2 + \varepsilon_{e\tau}^2}$ parameter: $-2.73 < \varepsilon < 0.81$ (without fixing axial-vector coupling $\xi$) $-1.10 < \varepsilon < 0.64$ ($V + A$ coupling, $\xi = 1$) and $-0.14 < \varepsilon < 0.15$ ($V - A$ coupling, $\xi = -1$) \cite{38}, whereas there is no reliable limit on the $\varepsilon_{\tau}$.

Let us assume also that neutrino mass terms are negligible. Then the Hamiltonian of neutrino evolution in matter takes the form:

$$
\hat{H}_\nu = \frac{\sqrt{2} G_F \rho}{m_N} \begin{pmatrix}
\nu_e & \nu_{e\tau} \\
\nu_{e\tau} & \nu_{\tau}
\end{pmatrix}
$$

where, recalling that $Y_d = Y_e + 2Y_n = 1 + Y_n$, we have:

$$
v_e = 1 - \frac{3}{2} Y_n + \varepsilon_e (1+Y_n), \quad v_{e\tau} = \varepsilon_{e\tau} (1+Y_n), \quad v_{\tau} = -Y_n/2 + \varepsilon_{\tau} (1+Y_n) \quad (28)
$$

(For the matter antineutrino states $\hat{H}_{\bar{\nu}}$ is just distinguished by opposite sign.) Obviously, due to the new flavour changing interactions an effective mixing appears between neutrino matter eigenstates $\nu_{1m} = c_m \nu_e + s_m \nu_{\tau}$ and $\nu_{2m} = -s_m \nu_e + c_m \nu_{\tau}$:

$$
c_m = \cos \theta_m, \quad s_m = \sin \theta_m, \quad \tan 2\theta_m = \frac{2\varepsilon_{e\tau}}{(\varepsilon_{\tau} - \varepsilon_e) - \frac{1-Y_n}{1+Y_n}} \quad (29)
$$

The mixing angle between antineutrino matter eigenstates $\bar{\nu}_{1m}$ and $\bar{\nu}_{2m}$ is the same. For the Hamiltonian eigenvalues we find

$$
V_{1,2} = \sqrt{2} G_F \frac{\rho}{m_N} v_{1,2} \quad (30)
$$

$$
v_{1,2} = \frac{1}{2} \left( v_e + v_{\tau} \mp \frac{|v_{\tau} - v_e|}{v_{\tau} - v_e} \sqrt{(v_{\tau} - v_e)^2 + 4v_{e\tau}^2} \right).
$$

and for antineutrinos $V_{1,2} = -V_{1,2}$. Then the transition matrix between $\nu - \bar{\nu}$ matter eigenstates becomes

$$
(\bar{\nu}_{1m} \bar{\nu}_{2m}) \begin{pmatrix}
h_{11} & h_{12} \\
h_{12} & h_{22}
\end{pmatrix} \frac{i}{2} \chi \begin{pmatrix}
\bar{\nu}_{1m} \\
\bar{\nu}_{2m}
\end{pmatrix} \quad (31)
$$

Where

$$
h_{11} = s_m^2 h_{\tau\tau}, \quad h_{12} = -c_m s_m h_{\tau\tau}, \quad h_{22} = c_m^2 h_{\tau\tau} \quad (32)
$$

(contributions from $h_{ee}, h_{e\tau}$ are neglected because of the strong limits of eqs.(23)). The key point now is that if the matter mixing angle $\theta_m$ is not very
the large coupling constant $h_{\tau\tau}$ can propagate to every entry of the majoron transition matrix (31) between $\nu$ and $\tilde{\nu}$ matter eigenstates. Thus, the transitions $\nu_i \rightarrow \tilde{\nu}_j + \chi$ or $\tilde{\nu}_i \rightarrow \nu_j + \chi$ are possible if are allowed by positive phase space. (We remind that in matter only helicity-flipping decays are relevant for very light neutrinos.) Since in the solar medium the $\nu_e = c_m \nu_{1m} - s_m \nu_{2m}$ state is produced, we are interested only in decays of neutrino mass eigenstates $\nu_i \rightarrow \tilde{\nu}_j + \chi (i, j = 1, 2)$, for which the widths are:

$$\Gamma_{ij} = \frac{h_{ij}^2}{16\pi} (V_i + V_j) \Theta(V_i + V_j) \tag{33}$$

where the $\Theta$-function remarks that the decay occurs only when the relevant phase space is positive. In order to discuss the implication of this scenario for the solar neutrino problem, we need the expression of the decay probability. As was mentioned above, the energy independence implies that the decay probability in a medium with varying density is essentially determined by the effective matter width passed by neutrinos. This is not absolutely exact in our case, since the effective coupling constants $h_{ij}$ are also variable during neutrino propagation in the sun. Apart from this, the new non-standard interactions (24) will also contribute to matter effective widths. Moreover, now neutrino mixing angle also varies during propagation. Even in absence of decay, this implies the possibility of matter-induced oscillations (up to resonant conversion [35, 36]) which should be taken into account. As a result of both matter oscillation and decay effects, the initial $\nu_e$ flux is converted into $\nu_e$ and $\nu_\tau$ fluxes having the initial energy spectrum (due to the oscillation effect) and $\tilde{\nu}_e$ and $\tilde{\nu}_\tau$ fluxes strongly degraded in the energy spectrum (due to the MID).

Using the fact that the decay probabilities do not depend on the neutrino energy, we give now the exact analytical expressions for the expected fluxes of $\nu_e, \nu_\tau, \tilde{\nu}_e$ and $\tilde{\nu}_\tau$ at the earth:

$$\Phi_{\nu_e}(E) = \Phi_{SSM}(E) \int_0^R \int_0^1 dr d(\cos\theta') c_m^2 (r) Q(r) e^{-P_1(t,R)}$$

$$\Phi_{\nu_\tau}(E) = \Phi_{SSM}(E) \int_0^R \int_0^1 dr d(\cos\theta') s_m^2 (r) Q(r) e^{-P_2(t,R)}$$

4In fact, it is expected to be reasonably large, if the flavour-changing interaction has the strength comparable to Fermi constant, i.e. $\varepsilon_{ee} \sim 1$.

5In order not to interfere with resonant neutrino conversion and, thereby, not to provide over-suppression of solar neutrino flux, we should exclude the interval of non-standard interactions which implies the existence of resonance at sufficiently large densities. According to ref. [34], the interval $\varepsilon_\tau - \varepsilon_e = 0.5 \div 0.75$ is relevant for MSW conversion (namely, at the lower limit fully adiabatic conversion happens, at the upper one moderate non-adiabatic regime occurs).
\[
\Phi_{\bar{\nu}_e}(E) = \int_0^R \int_0^1 dr d(cos\theta') Q(r) \left[ c_m^2(r) B_{11}(t) + s_m^2(r) B_{21}(t) \right] \Phi(E)
\]
\[
\Phi_{\bar{\nu}_\tau}(E) = \int_0^R \int_0^1 dr d(cos\theta') Q(r) \left[ c_m^2(r) B_{12}(t) + s_m^2(r) B_{22}(t) \right] \Phi(E)
\]
(34)

where
\[
\Phi(E) = 2 \int_{E_{\text{end}}}^{E_{\text{end}}} dE' \frac{E' - E}{E'^2} \Phi_{\text{SSM}}(E'), \quad t = r \sin\theta',
\]
\[
P_{ij}(t, l) = \int_0^l dl' \Gamma_{ij}(z), \quad B_{ij}(t) = \int_0^R dl' \frac{\Gamma_{ij}^2(z)}{\Gamma_i(z)} e^{-P_i(t, l')}, \quad z = \sqrt{t^2 + l'^2},
\]
\[
P_i = \sum_j P_{ij}, \quad \Gamma_i = \sum_j \Gamma_{ij} \quad (i, j = 1, 2)
\]

and \(\Phi_{\text{SSM}}(E)\) is the differential flux of \(\nu_e\)'s as expected from the SSM [1].

Therefore, provided that \(h_{\tau\tau} \approx 10^{-1}\) and mixing in matter is large (\(\varepsilon_{e\tau} \sim 1\)), the MID can be relevant for the SNP. Moreover, it can naturally explain the origin of the hierarchy (4) between the signals of different experiments. The effective matter widths given by eqs. (22) provide some numerical insight of why the lower energy neutrinos (solar \(pp\) neutrinos) are less depleted due to the MID compared to the higher energy ones (\(^7\)Be and \(^8\)B neutrinos), which in turn explains why we observe \(Z_{\text{Ga}} > Z_{K, \text{Cl}}\). On the other hand, some difference between \(Z_K\) and \(Z_{\text{Cl}}\) can be achieved due to neutral current contributions from \(\nu_\tau\) and \(\bar{\nu}_{e, \tau}\) to Kamiokande events. It is clear, however, that the difference between Kamiokande and Homestake signals cannot be very large:
\[
Z_K - Z_{\text{Cl}} < 0.15(1 - Z_{\text{Cl}})
\]
(35)

Where the upper bound actually corresponds to the limit when the contribution of the MID mechanism is not relevant and the SNP solution is due to massless neutrino oscillation in matter due to new flavour-changing interactions [35, 36]. However, taking into account the possible effects of \(\tau\)-neutrino mass, the energy dependence of decay probabilities can be achieved, which is necessary to split more Kamiokande and Homestake signals. In general case the magnitudes of the fluxes (34) depend on many parameters, as are \(\varepsilon_e, \varepsilon_\tau, \varepsilon_{e\tau}, h_{\tau\tau}\) and, possibly, \(m_{\nu_\tau}\), so that the detailed quantitative study with selection of the parameter range relevant for the SNP solution deserves special numerical computations and will be presented elsewhere.

One of the remarkable effects of the neutrino matter induced decay is the appearance of the solar antineutrino flux. According to eqs. (34) substantial portion (up to 25 per cents) of solar neutrinos can be transformed into \(\bar{\nu}_e\)'s. Due to the substantial energy degradation there is no contradiction with
limits on \( \bar{\nu}_e \) flux from Kamiokande [37] and LSD [38] data. However, this \( \bar{\nu}_e \)-signal can be detected in free proton rich detectors as are BOREXINO or BOREX through the inverse \( \beta \) decay \( \bar{\nu}_e + p \rightarrow n + e^+ \). E.g., for 100 t of fiducial volume of BOREXINO with positron energy threshold \( E_+ = 3.7 \) MeV one can expect up to 35 events per year, whereas background due to nearby nuclear power reactors is about 3-4 events per year [39]. (For the comparison, in [31] we have shown that the upper limit on the possible \( \bar{\nu}_e \)-signal in the absence of new flavour-changing interactions can hardly exceed the level of background).

On the other hand, the strong energy degradation of \( \bar{\nu}_e \)'s can discriminate the neutrino decay from the alternative \( \bar{\nu}_e \)-signal provided by hybrid models of neutrino oscillation and spin-flavour precession [40], in which case the \( \bar{\nu}_e \) spectrum should not be significantly altered as compared to the initial solar \( \nu_e \) spectrum.

For testing the MID solution to the SNP is also important to measure neutral current signal from any \( \nu_x \) states in which the missing solar neutrinos could be transformed. This can be done by new detectors like SNO [41] and BOREX [42]. Obviously, for any mechanism of solar neutrino conversion \( \nu_e \rightarrow \nu_x \), not changing the initial neutrino energy (like oscillation or magnetic moment transition into active neutrino or antineutrino states \( \nu_x \)), one has to expect the following sum rule:

\[
\Phi_{\nu_e}(E) + \sum_x \Phi_{\nu_x}(E) = \Phi_{SSM}(E) \quad (36)
\]

for any energy \( E \). As for the neutrino decay scenarios, the energy degradation of secondary neutrinos implies, that l.h.s. of the eq. (36) should be less than \( \Phi_{SSM}(E) \) for the high energy part of neutrino spectrum and larger for the low energy fraction (the latter, however, is rather difficult to observe experimentally). Taking into account that the SNP solution through the neutrino decay in vacuum is strongly disfavoured by combined data of all experiments under operation, the observation of such a ”particle number non-conservation” could strongly point out the MID solution. Clearly, in the case of oscillation or spin-flavour precession into sterile states one also expects that \( \Phi_{\nu_e}(E) + \sum_x \Phi_{\nu_x}(E) < \Phi_{SSM}(E) \), but now this inequality will be respected for every part of the spectrum.

Thus, as we see, MID can provide well testable solution to the SNP.

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