Generalized source multipole moments of dynamical horizons in binary black hole mergers

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In this work, we uncover new features in the source multipole moments of the dynamical horizons in binary black hole mergers using numerical relativity. We show that they encode detailed information about the dynamics of the binary black hole system. Owing to the deformations, the dynamical horizons of the two black holes are found to acquire multipole moments that vanish when the horizons are isolated. Out of these, the dominant moment is found to be the quadrupole moment. The dominant quadrupole multipole moment is also shown to be strongly correlated with the gravitational field of the system at null infinity. Therefore the gravitational waves carried away from the system contain information about the geometrical structure of the black holes in the strong-field regime. We also find that, in the post-merger phase, the multipolar structure of the outer common dynamical horizon of the system is strongly correlated with that of the individual horizons just before the merger. The outer common horizon then settles down to equilibrium as suggested by the decay of its multipole moments gained by the system through the inspiral phase.

Introduction: Numerous binary black hole mergers have been observed to date starting with the first detection in 2015 [1], [2,7]. The parameters of these binary systems, including the masses and spins of the individual black holes, can be inferred from the observed data [8]. These observations have been used to extract information about the overall dynamics of the binary black hole system. The observations have so far been found to be consistent with standard general relativity [9,11].

The dynamics of gravitational radiation in the far field regime, far away from the horizons of the merging black holes is fairly well understood. In particular, the multipole moments of gravitational radiation are relevant here [12,16]. In the strong field regime, the dynamical horizon of black holes are located inside the event horizons and are outside of the domain of outer communications. In recent times, advances in numerical relativity have paved way for understanding the dynamics of the binary black hole system in the strong field regime, particularly of the dynamical horizons of black holes. There have been some studies in the past directed towards understanding of the dynamics of the horizons, but mostly of post-merger axisymmetric scenarios like head-on collisions [17,19]. However understanding of the dynamics of the horizon geometry in the inspiral, merger and ringdown phase of generic non-axisymmetric dynamical scenarios, like that of a binary black hole merger, is still in its infancy. Some basic properties of dynamical horizons in binary black hole merger scenario was studied in [20].

In binary black hole systems, the horizons of the black holes are dynamical, tidally interacting with each other and absorbing gravitational radiation. In such a system, we may thus expect the general features of the dynamics of the system, such as the orbital phasing, relative location of the black holes, etc, to be imprinted on the geometry of the dynamical horizons. In this work, we find evidence precisely for this. In a previous study involving dynamical horizons in binary black hole scenarios, the infalling radiation at the dynamical horizons was found to be strongly correlated with the news of the outgoing gravitational radiation from the system [21]. In another recent study, the axisymmetric deformations of the dynamical horizons in binary black hole scenario were studied and were used to quantify the strong field tidal deformability of black holes [22]. In this work, we also add to the understanding of the strong field dynamics of black holes in general relativity by studying the evolution of the source multipole moments of dynamical horizons for the first time in a binary black hole scenario. We study and show that not just the infalling radiation at the horizon, but the geometry of the gravitational field in the strong field regime i.e. that of the dynamical horizon itself is strongly correlated with the spacetime geometry at null infinity. Thus, although future directed causal curves from the dynamical horizons cannot reach $I^+$, one can potentially use these results to discern the multipolar structure of the horizon geometry using information at $I^+$. We also find that the evolution of the multipole moments of the system can be described by a generic expansion in terms of the distance of separation between the black holes. This was previously reported in [22,23] and here, we extend it to more general non-axisymmetric deformations of the dynamical horizons here.

The main objects of study are the generalized source multipole moments of the dynamical horizons and the multipolar structure of the gravitational radiation at $I^+$, which we will briefly describe.

Basic notions: Like in the previous study [21], we will be discussing physics at two surfaces: (a) future null infinity $I^+$, the end point of future null-geodesics which escape to infinity [13,24], and (b) a dynamical horizon $H$, [25,26] obtained by a time evolution of marginally trapped surfaces. Future null infinity $I^+$ is an invariantly defined null surface where outgoing null geodesics
end and a dynamical horizon is located inside the event horizon, which marks the boundary of a trapped spacetime region. These surfaces share some similarities. Both $\mathcal{I}^+$ and $\mathcal{H}$ are one-way membranes and exact flux formulae hold at both surfaces.

We consider $\mathcal{H}$ and $\mathcal{I}^+$ to be foliated by 2-surfaces $S$ of spherical topology, with an intrinsic Riemannian metric $g_{ab}$. For the former, we obtain a marginally trapped surface and for the latter, we approximate it by large coordinate spheres in the numerical domain. For every cross section $\mathcal{S}$, we assign outgoing and ingoing directions. Denote the outgoing future directed null vector normal to $S$ by $n^+$, and the ingoing null normal as $n^-$ satisfying $n^+ \cdot n^- = -1$. Let $x$ be a complex null vector tangent to $S$ satisfying $x \cdot \bar{x} = 1$ (the overbar denotes complex conjugation), and $n^+ \cdot x = n^- \cdot x = 0$.

In the weak-field regime, spacetime geometry is completely described by the Weyl tensor $C_{abcd}$. In particular, outgoing transverse radiation is described by the Weyl tensor component [27]

$$
\Psi_4 = C_{abcd}n^a\bar{x}^b n^c\bar{x}^d.
$$

$\Psi_4$ can be expanded in spin-weighted spherical harmonics $-2Y_{\ell,m}$ of spin weight $-2$ [28]. Let $\Psi^{(\ell,m)}_4$ be the mode component with $\ell \geq 2$ and $-m \leq \ell \leq m$. The $(\ell, m)$ component of the News function $N^{(\ell,m)}$ and its polarizations $N_{+,x}$ are defined as [13]

$$
N^{(\ell,m)}(u) = N^{(\ell,m)}_+ + iN^{(\ell,m)}_x = \int_{-\infty}^u \Psi^{(\ell,m)}_4 du.
$$

The outgoing energy flux is related to the integral of $|N|^2$ over all angles. In a numerical spacetime it is in principle possible to extract $\Psi_4$ going out all the way to $\mathcal{I}^+$ [29] to reduce systematic errors. We shall however follow the common approach of calculating $\Psi_4$ on a sphere at a finite radial coordinate $r$ and the integral in the previous equation is over time instead of the retarded time coordinate $u$. The lower limit in the integral is not $-\infty$ but the earliest time available in the simulation. The News function is then a function of time at a fixed value of $r$, starting from the earliest time available in the simulation. A further time integration of $N$ yields the gravitational wave strain.

On the black hole, the basic object here is a marginally outer trapped surface (MOTS), again denoted by $\mathcal{S}$. This is a closed space-like 2-surface with vanishing outgoing expansion $\Theta_+$:

$$
\Theta_+ = \bar{q}^{ab} \nabla_a n_b^+ = 0.
$$

Its scalar curvature is denoted by $\bar{\mathcal{R}}$, the two-metric by $\bar{q}_{ab}$, and its extrinsic curvature by $\bar{K}_{ab}$. The shear of the dynamical horizon is denoted by $\sigma = \epsilon^{ab} \epsilon^{cd} \nabla_a n_b^+ \nabla_c n_d^+$. These are two sets of numbers (the mass and angular momentum multipoles: $M_{\ell m}, J_{\ell m}$), although defined on a foliation of the three-dimensional dynamical horizon, can be used to reconstruct the horizon geometry in a gauge invariant manner. Thus the geometry of a dynamical horizon is characterized by its source multipole moments. These moments were first defined for isolated horizons [30] and extended to axisymmetric [31] and non-axisymmetric dynamical horizons [32]. Due to these reasons, these multipole moments can be used to study the intrinsic geometry of dynamical horizons. They have been used in predictions of the anti-kick in binary black hole mergers [33], in the study of the no-hair conjecture in general astrophysical environments [34], and for studying tidal deformations of black holes [22, 23, 34]. In [22], the axisymmetric tidal deformations of dynamical horizons were studied by using the source multipole moments of the dynamical horizon in binary black hole scenarios.

We now describe how to compute these moments briefly. First, a preferred coordinate system on the leaves $S$ of the dynamical horizon is constructed using an appropriately defined axial vector field $\varphi^a$ (analogous to the axial Killing vector field on an isolated Kerr horizon). We use the method of Killing transport to compute the axial field. Once the axial field is defined, an invariant coordinate $\zeta$ (analogous to the polar coordinate variable $\cos(\theta)$ of the Boyer-Lindquist coordinates) can be defined. Further details can be found in [30, 31]. Using these, one can define the mass multipole moments as:

$$
M_{\ell m} = \frac{M_S \bar{R}_{\ell}}{8\pi} \sqrt{(\ell + 1)(\ell - |m|)!} \int_{\mathcal{S}} \bar{R} P^m_{\ell}(\zeta) e^{-i m \phi} \kappa^2 S,
$$

Here $\phi$ is an affine coordinate on the integral of the vector field $\varphi^a$, $P^m_{\ell}$ are the associated Legendre polynomials corresponding to the eigenfunctions of the Laplacian on $\mathcal{S}$, and $P^m_{\ell}$ their derivatives. In this notation, $M_{00}$ is the mass $M_S$ of the slices $\mathcal{S}$ of the dynamical horizon, and $J_{10}$ is its angular momentum $J_S$. $n!$ denotes the factorial of an integer $n$. These moments are defined for all positive $\ell$, and for each $\ell$, the azimuthal mode number takes integer values ranging from $(-\ell, \ell)$ i.e. $|m| < \ell$. It is to be noted that modes for which $m \neq 0$ are complex in general and we denote their absolute magnitude by $|M_{\ell m}|$.

We compute and study the evolution of the mass dipole ($\ell = 1$) and the quadrupole ($\ell = 2$) moments, $|m| \leq \ell$ moments of the individual dynamical horizons of the black holes during the inspiral phase, and that of the common horizon in the post-merger phase, and their relationship with the dynamics of the binary black hole system.

**The numerical simulations**: Our numerical simulations are performed using the publicly available Einstein Toolkit framework [35, 36]. The initial data is generated based on the puncture approach [37, 38], which is then evolved through BSSNOK formulation [39, 40] using the 1 + log slicing and T-driver shift conditions. Gravitational waveforms are extracted [42] on coordinate spheres.
at various radii between 100\(M\) to 500\(M\). The computational grid set-up is based on the multipatch approach using Llama [43] and Carpet modules, along with adaptive mesh refinement (AMR). The various horizons are located using the method described in [44,45]. General quasi-local physical quantities are computed on the horizons following [31,46]. The framework to compute the generalized multipole moments in Eq. (4) does not exist in the QuasiLocalMeasures thorn of the Einstein Toolkit. They were computed in post-processing using numerical relativity data.

We consider non-spinning binary black hole systems with varying mass-ratio \(q = M_2/M_1\), where \(M_{1,2}\) are the component horizon masses (\(M_1 \geq M_2\)). We use the GW150914 parameter file available from [47] as our template. For each of the simulations, as input parameters we provide initial separation between the two punctures \(D\), mass ratio \(q\) and the radial and azimuthal linear momenta \(p_r, p_\phi\) respectively, while keeping the total physical horizon masses \(M = M_1 + M_2 = 1\) fixed in our units. Parameters are listed in Table I. We compute the corresponding initial locations, the \(x, y, z\) components of linear momentum for both black holes, and grid refinement levels, etc., before generating the initial data and evolving it. We chose 2 non-spinning cases with mass-ratios \(q = 0.6, 0.7\) for the purposes of this study, based on the initial parameters listed in [18,49]. For computing the quasi-local quantities, we use a grid of size (36, 74) on each of the dynamical horizons. This grid resolution allows us to safely study multipole moments of up to \(\ell = 2\). Our simulations agree very well with the catalog simulations [50], with merger time discrepancies less than a few percent. The results presented here are general features that were seen across the simulations \(q = 1, 0.6, \) and 0.7 described in Tab. I. The simulation We present the results using \(q = 0.6\). The outer common horizon of the configuration \(q = 0.6\) appears at at \(t = 1656.045 M\), which we designate as merger time. When the common horizon is found, it has an areal radius of \(R_c = 1.708\). 3D visualizations were performed using VisIt [51].

Results: Inspiral. We first discuss the relative strengths of the various multipole moments \(|M_{\ell m}|\). For both the black holes of all the simulations, amongst all the moments at \(\ell = 1, 2\) multipolar order, the multipole moment \(M_{22}\) was found to have the largest and monotonically increasing amplitude, followed by \(M_{20}\) and \(M_{1\pm 1}\) (note that for an isolated Kerr horizon, only some of the even \(\ell, m = 0\) mass moments are non-zero). This can be seen in the movie [52], where the evolution of the 2D-Ricci scalars of the dynamical horizons in the inspiral phase have been visualized. In Fig. 1 a snapshot of the movie is presented. The multipolar deformations of the horizon geometries are mutual and are dependent on the location of the black holes in the binary system. The quadrupolar pattern can be clearly seen in the movie. The non-axisymmetric multipole moments (i.e., \(m \neq 0\)) of the individual dynamical horizons of the black holes are oscillatory in nature. It was found that the multipole moments \(M_{22}\) of the two black holes were in-phase with each other while the moments \(M_{1\pm 1}\) differed by a phase of \(\pi\) radians. Secondly, we found the multipole moments of dynamical horizons of the two black holes to be strongly correlated with each other. Furthermore, the dominant multipole moment \(M_{22}\) of the dynamical horizons is found to be strongly correlated with the dominant \((\ell = 2, m = 2)\) mode of the gravitational wave strain extracted at a very large distance \(r = 100M\) from the system. The movie [52] aids in the visualization of some of these results. Thus, the source multipole moments are strongly correlated with the multipole moments of the gravitational field at null infinity [13,16]. In Fig. 3 we plot the time derivative of the multipole moments \(M_{11}\) and \(M_{22}\) vs. the News of the gravitational waves emitted from the system (Eq. 2), suitably normalised and aligned in time and phase. The timeshift was found to be 101.3\(M\), approximately consistent with the light travel time corresponding to the extraction radius for the news. It was found that the quadrupole mass moment \(M_{22}\) encodes accurate information about the phasing of the gravitational waveform from the system whereas the dipole moment \(M_{1\pm 1}\) reflects the orbital phasing of the system. These multipole moments can be therefore used to extract information of the binary black hole system like their masses, velocities, orbital angular momentum, etc. To demonstrate this, using the multipole moments and a standard least squares figure of merit, the parameters of the binary system could be estimated using the multipole moment \(M_{22}\). For this, we used a template bank of gravitational wave strain in the mass-ratio, chirp-mass parameter space constructed using the well known phenomenological waveform model IMRPhenomPv2. The parameters of the binary system could be estimated quite accurately (with an error of 0.12\% in the mass ratio and 0.01\% in the chirp-mass of the binary system).

The evolution of the strengths of the multipole moments were also found to be generic. By means of maximizing a least-squares figure of merit, we found that the evolution of the multipole moments of both the black holes, and across the two simulations, can be described

| \(q\)  | \(D/M\) | \(p_r/M\) | \(p_\phi/M\) |
|------|--------|--------|--------|
| 1    | 9.5332 | 0      | 0.09932|
| 0.6  | 11.5   | -5.46e-04 | 0.08206|
| 0.7  | 12.0   | -5.07e-04 | 0.08246|

Table I. Initial parameters for non-spinning binary black holes with quasi-circular orbits. \(q = M_2/M_1\) is mass ratio, \(D\) is the initial separation between the two holes, \(p_r\) and \(p_\phi\) are radial and azimuthal linear momenta respectively.
FIG. 1. The deformation of the dynamical horizons of the black holes can be directly visualized in terms of the 2-Ricci scalar \( \tilde{R} \) of their two-dimensional slices \( S \). Here the 2-Ricci scalars of \( S \) are visualized for the \( q = 0.4 \) system when the black holes are at a separation of \( d \approx 7.5M \) (about 66% of the initial separation), approximately 5 orbits after the start of the simulation, as shown by the thick green line in the waveform plot below the figure. The total number of orbits before the merger is around 9 and the corresponding complete waveform cycles in the simulation are shown by the thin red line. The more massive black hole \( BH_1 \) is on the left. The values of the Ricci scalar are shown on the color bars to the left of each black hole. A movie for \( q = 0 \) can be viewed here [52]. This movie shows that the deformation of the horizon geometry is mutual for both the horizons and the Ricci scalar distribution patterns face each other at all points on the orbit. The dominant quadrupolar structure can be seen, which is reflected in the numerical values of the strengths of the multipole moments in Fig. 2.

FIG. 2. The time evolution of the real part of the multipole moments \( \ell = 1, \ell = 2 \) \(|m| < \ell \). Here, the values below \( 10^{-8} \) are below machine precision. The time of crossing of the light ring of the system is denoted as a dotted line in magenta.

by a generic tidal expansion of the form:

\[
\frac{|M_{l|m}|}{M_H^{l+1}} = \sum_{i=3}^{\infty} \frac{a_i}{d^i}
\]  

(5)

where \( M_H \) is the mass of the black hole that is being discussed, and \( d \) is a measure of distance of separation between the holes. In particular, the dipole moment was found to be well described by the above expansion that includes terms up to the order of \( 1/d^4 \) and the quadrupole moment required terms up to the order \( 1/d^6 \) (see Fig. 4). These are consistent with the results of [22]. These moments can therefore be used to study the tidal deformability of black holes in a manner described there. Apart from the real and imaginary parts of the moment, its magnitude also has oscillatory behaviour as shown in Fig. 4. These oscillations are decaying with time, and exist in the multipole moments of both the dynamical horizons. Isolating these oscillations in the data up to \( t = 1000M \), and we found that they can be described by a superposition of power-law damped sinusoids of the
with power law indices $\gamma = 1.47$ and 2.29 respectively. The time periods of oscillations of these modes were found to be at $T_1 = 161.56M$ and $T_2 = 124.71M$ respectively. It is worth noting that the latter is close to half the average orbital time period in the domain considered. These values are expected to be dependent on the mass-ratio the system. As this feature was observed in both the simulations $q = 0.6, 0.7$, we are led to speculate if these correspond to dynamic tides or quasi-normal modes of the tidally coupled dynamical horizons excited in the inspiral phase, or are mere numerical artefacts.

Results: Plunge. The multipole moments were found to display two distinct behaviours in the pre-merger phase Fig. 5. While the majority of the portion of the evolution of these moments reflected the adiabatic, quasi-circular dynamics of the system, their behaviour change to a steeper increase in strength with the seizure of oscillations closer to the merger. We found that this epoch is very close to the time of crossing of the light ring of the binary black hole system. The light ring of the system is defined as the last circular (unstable) photon orbit. We estimate the light ring radius of the system using the adiabatic resummed 1PN hamiltonian [53]. It was found that the sharp change in behaviour occurs when the black holes cross the light ring of the system.

Some of the multipole moments which were zero (below machine precision) for the majority of the inspiral phase, were found to gain in magnitude with oscillatory behaviour as the black holes approached each other. Thus, during these last moments of the merger of the individual horizons of the black holes and before the common horizon appears, the various multipole moments of the horizons grow to comparable strengths as the dynamical horizons strongly deform under each other’s influences.

A distinctive feature was observed for $q = 1$ case. The $\ell = 2, m = 2$ moment, although the strongest, does not monotonically increase in strength closer to the merger. This may be due to the enhanced symmetry of the equal mass system.

Owing to these and the previous results in [21], we can conclude that the multipole moments of a dynamical horizon is strongly correlated with the dynamics of the system. It is also strongly correlated with and complements the gravitational radiation falling at the dynamical horizon (described the shear of its outgoing null normal), and the outgoing gravitational radiation emitted from the system.

Results: Post-merger. Extending the correlations of the multipole moments with the outgoing gravitational waves further, in the post-merger phase, we analyze the multipole moments of the common horizon, immediately after its formation.

As the two black holes cross the light ring of the system, common envelopes surrounding the individual black hole horizons form. The outer common horizon is a dynamical horizon of the remnant that settles down to the Kerr isolated horizon, which is spinning and highly distorted when formed. Thus, one expects the multipole moments of the common horizon formed to be different from that of an isolated Kerr horizon. The dynamical horizon then proceeds to equilibrium by absorbing radiation and losing ‘hairs’, i.e. the multipole moments are expected to decay to the isolated Kerr values. Fig. 6 shows graphically that this is indeed the case. Here the time evolution of the mass multipole moments of the common horizon are plotted. The multipole moments display quasi-normal behaviour. The damping rate of the strengths of the moments $|M_{\ell m}|$ were found to be close to the theoretical estimate of a Kerr black hole with the same mass and spin as that of the remnant. E.g, damping rate of $M_{22}$ was found to be consistent with the $n = 0, \ell = 2, m = 2$ mode with a deviation of 2.74%. Since the remnant black hole is spinning, the coordinate system on the dynamical horizon can also rotate along with it and the real part of the quasi-normal frequen-
FIG. 4. Left: The fit of the $\ell = 1, m = \pm 1$ multipole moments to the relation in Eq. [5] with two terms at the third and fourth order in the separation $d$, i.e. $1/d^3$ and $1/d^4$. Center: the fit of the $\ell = 2, m = \pm 2$ multipole moments to the relation in Eq. [5] with terms up to fourth, fifth, and sixth order in $d$. Right: the isolated oscillations of the overall amplitude of the multipole moment $|M_{22}|$ seen in the figure at the center.

FIG. 5. The time evolution of some of the $\ell = 1 m = 1$ (top), $\ell = 2, m = 2$ (bottom) multipole moments. The $\ell = 1, m = \pm$ moments are identical. The red line shows the temporal location of the light ring $r \approx 2.856$. The close-up of these plots within these show that the change in the growth of the multipole moments is consistent with the time of light ring crossing of the system.

With a deviation of 2.73% from the expected theoretical estimate. The strengths of the moments $M_{1 \pm 1}$ and $M_{22}$, which are expected to go to zero, decay to the order of $10^{-4}$. We suspect that this is due to systematic errors arising from the rotation of the coordinate system on the common horizon, and its limited grid resolution. As expected, the multipole moment $M_{10}$ is practically zero, whereas the moment $M_{2-2}$ falls below the machine precision.

What decides the deformed state of the common horizon once it is formed? The initial configuration of the parent black holes that led to the merger is expected to decide the deformed state of the common horizon once it is formed. Interestingly, we found that the relative strengths of the multipole moments of the individual horizons of the black holes just before the formation of the common horizon are similar to that of the common horizon when it is formed (more so for the horizon of the more massive black hole). This can be seen in Fig. 6. Thus the deformed state of the individual horizons just before the common horizon appears plays a role in the setting up of initial conditions of the common horizon for the post-merger dynamics. The common-horizon thus formed roughly inherits the multipolar structure of the horizon geometry of the black holes at the end of the inspiral phase and then loses them as it settles down to equilibrium in the post-merger phase.

Conclusions: The horizons of black holes in a binary environment are not isolated. The horizon geometry of the black holes are distorted due to the mutual tidal interactions, details of the strong field dynamics of the system, and the absorption of radiation falling on the horizons of the black holes. The source multipole moments of the dynamical horizon captures this information. In a binary black hole merger, they encode accurate information about the dynamics of the system, allowing one to estimate its parameters. These moments show a distinctive behaviour towards the late inspiral phase, as the black holes plunge and is consistent with the crossing of the light ring by the system. The evolution of the
strengths of the generalized source multipole moments were also found to have a universal behaviour: they can be described by a series expansion in inverse distance of separation of the black holes. This can be used to study the tidal deformability of the dynamical horizons in the strong field regime.

These correlations allow one to study the strong field regime using gravitational wave observations. Another result is that the source multipole moments, infalling radiation at the dynamical horizons and the gravitational waves at null infinity emitted by the system are all intercorrelated. This is analogous to the multipolar formula for the generation of gravitational waves due to changing source multipole moments of a system. These correlations also connects the tidal deformability of dynamical horizons with gravitational wave observations.

We also may expect these correlations to extend to all multipolar orders, which will be studied in a future work. Thus, in principle, they allow one to discern the multipolar structure of the horizon in fully dynamical scenarios. Understanding these strong correlation across mass ratios, one can in principle reconstruct the horizon geometry using gravitational wave observations. They also can be used to test and understand further important aspects of the no-hair conjecture and yet unknown nonstandard structure of gravity in the strong field regime.

These results urge us to state the following conjecture: In a dynamical scenario involving binary black holes, the source multipole moments associated with charges of the dynamical horizon will be correlated with the multipolar structure of the Bondi flux of outgoing gravitational radiation from the system received at future null infinity $T^+$.

Potential applications of these results are numerous. The estimation of non-axisymmetric tidal deformability coefficients, can be studied. In the modelling of gravitational waveforms from binary black hole systems (e.g. in the Post-Newtonian and effective one-body approach), the deformation of the dynamical horizons are not considered. As shown here, the dynamical horizons are strongly deformed in the late inspiral phase and can provide important corrections to the late-inspiral, plunge waveforms. The results can also help in understanding quantitatively how the dynamical horizons of black-holes acquire gravitational hair in fully dynamical scenarios. These can be used to study aspects of the no-hair conjecture in more general astrophysical environments and a means to probe the hairs of black holes using gravitational wave observations.

The results presented here can be interpreted in the following way. In a binary black hole scenario, the individual dynamical horizon geometries of black holes gain a structure i.e., "gravitational hairs", away from the isolated Kerr geometries in the inspiral phase though their tidal interactions and absorption of gravitational radiation, only to loose them in post-merger dynamics of the common horizon by absorption of complementing radiation.

On the numerical side, the computation of multipole moments require a choice of an axial vector field on the horizon. In this work, this has been achieved by using the method of Killing transport. Numerically, more accurate description of the multipolar dynamics of the dynamical horizons, especially in the late inspiral phase, require a better choice of the axial vector field. Studies can also include spinning binary black hole configurations.

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