Abstract

The standard method for observationally confirming the existence of a predicted finite topology of the universe involves searching for the repetition of the same finite or extended source in different directions. However, serious problems are encountered when studying both types of sources, finite and extended. In studying a finite source, such as a star, galaxy, or quasar, the problem of its evolution must be dealt with. The same source seen in different directions is observed at different distances (i.e., ages), making its identification extremely difficult. Studying extended sources, such as circles-in-the-sky (CIS) within the cosmic microwave background (CMB), is also problematic since it requires an unrealistic zero width last scattering surface (LSS) to produce the predicted identical temperatures on the circles. It is shown here that these temperatures are no longer identical when a realistic finite width LSS is taken into account. A new method for studying the topology of the universe which avoids the above mentioned problems in studying both finite and extended sources is suggested here. It consists of searching for increased temperature fluctuations in the regions of the intersections of the LSS with the faces of the fundamental polyhedron of the finite topology. Appreciably greater fluctuations are predicted in these regions. A worked-out example is given.

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I. INTRODUCTION

Cosmic Topology aims to answer the questions as to whether the universe is finite or infinite and the nature of its shape. In the light of existing observational data, the three homogeneous and isotropic solutions of Einstein’s equations are the best approximations with which to describe our universe. Each solution gives a possible local geometry of space with constant curvature: $k = 0, +1, \text{ or } -1$, described by the Friedmann-Lemaître-Robertson-Walker (FLRW) cosmological model. Although the FLRW solutions describe the local geometry of space, they do not fix its topology. In general, the topology of the spatial sections of these solutions have been taken to be trivial. For example, the case where $k = 0$ is usually taken to be the infinite Euclidean space $E^3$.

In non-trivial topologies, identical copies of a fundamental polyhedron (FP) which recovers these spaces are observed. The FP is the real space in which we live. An illusion of multiple copies of the FP and repeated images of its sources arises from the identification of pairs of its faces [1]. Because light from a source travels in all directions, crossing the faces of the FP, the illusion of observing the same source as well as multiple copies of the FP from different directions is created.

Until now, the methods used to look for signs of non-trivial topologies have been based on the presence of multiple images of the same finite or extended source. These methods were developed by analyzing temperature distributions in the cosmic microwave background (CMB) radiation (for extended sources) [2, 3, 4] or the three-dimensional distributions of cosmic sources (for finite sources) [1, 5, 6, 7]. In recent years, several of these methods have been extended to the three geometries with constant curvature, $k = 0, +1, -1$, and their respective non-trivial topologies [8, 9, 10, 11].

One of the methods suggested for trying to detect non-trivial topologies in the CMB was the circles-in-the-sky (CIS) method. This method is based on searching for circles in the CMB with identical temperature distributions [2]. In a universe with a non-trivial topology, the LSS is seen to be repeated in each copy of the FP which is observed. If the dimensions of the FP observed are less than or on the order of the LSS, copies of the LSS will intersect with one another. These intersections will then produce matched circle pairs, where the temperature distributions along the circles are predicted to be identical. The pattern of matched circles will vary according to the topology. However, as noted above, the observed multiple copies of the FP are, in fact, actually an illusion due to the fact that light travels in all directions, crossing the faces of the FP.

Recently, cosmic topology aroused great interest, when Luminet et al. [13] suggested that the WMAP data indicate that we live in a universe with Poincaré dodecahedron space. However, it was subsequently argued by Cornish et al. [14] that, when applied to the WMAP data, the CIS analysis excludes the Poincaré dodecahedron space.

An essential condition is imposed on the LSS by the CIS method for studying topology, namely, that the transition of an opaque to a transparent universe occurs instantaneously. Therefore, the shell, from where the observed photons came, should be infinitesimally thin [12].

However, decoupling of the photons in the early universe was, in fact, not an instantaneous event, as assumed in the CIS method, but took place in a redshift interval $\Delta z_{\text{LSS}}$. The probability that an observed photon was last scattered in the redshift interval $\Delta z_{\text{LSS}}$ around an average $z_{\text{LSS}}$ can be approximated by a Gaussian distribution with a mean value $z_{\text{LSS}}$ and a width $\Delta z_{\text{LSS}}$. A redshift interval $\Delta z_{\text{LSS}} \sim 80$, for example, corresponds to a proper
length $\Delta l \sim 15\Omega_0^{-1/2}$ Mpc and an angle $\Delta l \sim 8'\Omega_0^{1/2}h$, where $h = H_0/100$ km s$^{-1}$ Mpc$^{-1}$, $H_0$ is the Hubble constant, and $\Omega_0$ is the density parameter [15]. The density parameter for the recent WMAP data is $\Omega_0 = 1.02 \pm 0.02$.

The problem with observing multiple images of a single source, such as a star, galaxy, or quasar, is the time evolution of the source. For example, if the dimension of the FP is on the order of the horizon, the difference in the ages of the same source is on the order of the age of the universe. Thus, an observed source at redshift $z \sim 5$ may have an age $\sim 10$ million years, while the same source observed at $z \sim 0$ may have an age $\sim 10$ billion years. Obviously, the two images will not look the same. Consequently, observing multiple images of a single source, such as a star, galaxy or a quasar, must take into account the extremely difficult problem of identifying the source, which is evolving in time.

Instead of trying to observe the same source from different directions, where time evolution must be taken into account, or searching for identical CBR temperatures on separate circles, where a zero, unrealistic width LSS ($\Delta z_{\text{LSS}} = 0$) is assumed, as in the CIS method, we suggest observing the regions where the LSS intersects the face of the FP, in which we predict an appreciable increase in the temperature fluctuations. The new method suggested here avoids the problems mentioned above. It is based on a realistic finite width of the LSS and does not involve the problem of time evolution. In Section III, we describe the method and give a worked-out example. Conclusions and discussion are given in Section III.

II. TOPOLOGY OF THE UNIVERSE WITH A FINITE WIDTH LSS

Let us observe the CMB in a direction where the LSS intersects the face of the FP at the mean redshift $z_{\text{LSS}}$, which has a width $\Delta z_{\text{LSS}}$. We define the plasma of the LSS between us and $z_{\text{LSS}}$ (i.e., between the redshifts $z_{\text{MIN}} = z_{\text{LSS}} - (\Delta z_{\text{LSS}})/2$ and $z_{\text{LSS}}$) as $P_1$. The temperature $T_1$ of $P_1$ is different from $T_2$, the temperature of the plasma $P_2$ between $z_{\text{LSS}}$ and $z_{\text{MAX}}$ (i.e., between the redshifts $z_{\text{LSS}}$ and $z_{\text{MAX}} = z_{\text{LSS}} + (\Delta z_{\text{LSS}})/2$). This difference in temperature is due to density fluctuations in the primordial plasma. Since $z_{\text{LSS}}$ is at the face of the FP, both $P_1$ and $P_2$ are again observed in the opposite direction in the FP, where $P_2$ is now observed before $P_1$. Clearly, the temperature fluctuations created by $P_1$ and $P_2$ are not independent since the same plasmas are observed in both directions.

We now observe the CMB in two different directions in the case where the LSS is within the FP and does not intersect its faces. Here, the temperature fluctuations are independent since the plasma observed in one direction is not repeated in the other direction.

The average temperature fluctuations observed in case I, where the LSS intersects the faces of the FP, will be compared with those in case II, where the faces of the FP are not intersected by the LSS. From the following calculations, it is seen that the temperature fluctuations in the first case are appreciably greater than those in the second.

In case I, an observer looking at one side of the FP in the direction of face A measures the intensity of the radiation from $P_1$ and $P_2$ in terms of an effective temperature $T_{12}$, which is a weighted mean of the temperatures of $P_1(T_1)$ and $P_2(T_2)$:

$$T_{12} = g_{\text{NEAR}} T_1 + g_{\text{FAR}} T_2,$$

where $g_{\text{NEAR}}$ is the probability that an observed photon came from the near plasma, from $z_{\text{MIN}}$ to $z_{\text{LSS}}$, and $g_{\text{FAR}}$ is the probability that it came from the far plasma, from $z_{\text{LSS}}$ to $z_{\text{MAX}}$. Since photons in the interval $z_{\text{MIN}}$ to $z_{\text{LSS}}$ need to pass through only $\sim 1/4$ of the
thickness of the LSS, $g_{\text{NEAR}} \sim 0.75$. On the other hand, $g_{\text{FAR}}$ is relatively small, $\sim 0.25$, since photons in the redshift interval $z_{\text{LSS}}$ to $z_{\text{MAX}}$ need to pass through $\sim 3/4$ of the LSS.

The illusion of multiple copies of sources arises from the identification of pairs of faces of the FP (Section II). We not only observe $P_1$ in the redshift interval $z_{\text{MIN}}$ to $z_{\text{LSS}}$ in the direction of face A, but also in the redshift interval $z_{\text{LSS}}$ to $z_{\text{MAX}}$ in the direction of face B. Likewise, we observe $P_2$ in the redshift interval $z_{\text{LSS}}$ to $z_{\text{MAX}}$ in the direction of face A as well as in the redshift $z_{\text{MIN}}$ to $z_{\text{LSS}}$ in the direction of face B. Thus, looking at the opposite side of FP in the direction of face B, the radiation from $P_1$ and $P_2$ in terms of an effective temperature $T_{21}$, is now

$$T_{21} = g_{\text{NEAR}} T_2 + g_{\text{FAR}} T_1.$$  \hspace{1cm} (2)

The sum of the temperatures $T_{S1}$ from $T_{12}$ and $T_{21}$ in case I is

$$T_{S1} = g_{\text{NEAR}} T_1 + g_{\text{FAR}} T_2 + g_{\text{NEAR}} T_2 + g_{\text{FAR}} T_1$$  \hspace{1cm} (3)
$$= [g_{\text{NEAR}} + g_{\text{FAR}}] [T_1 + T_2].$$

Both $T_{12}$ and $T_{21}$ involve two plasma regions. Thus, the sum $T_{S1}$ involves four terms. However, only two regions are independent since the other two are repetitions of the first.

When the LSS is within in the FP and doesn’t intersect its faces, the non-trivial topology is not evident. Thus the temperature sum in two different directions involves four different plasma regions of the LSS, instead of only two. The sum of the temperatures $T_{S\text{II}}$ from the two directions in case II is

$$T_{S\text{II}} = g_{\text{NEAR}} T_1 + g_{\text{FAR}} T_2 + g_{\text{NEAR}} T_3 + g_{\text{FAR}} T_4$$  \hspace{1cm} (4)

Because this temperature sum involves four different regions and not just two, as in $T_{S1}$, this temperature sum has smaller fluctuations, as we shall now show.

Let us assume that each different plasma region, from $z_{\text{MIN}}$ to $z_{\text{LSS}}$ and from $z_{\text{LSS}}$ to $z_{\text{MAX}}$, has a random probability of having a temperature $T_0 + \delta T$ or $T_0 - \delta T$, where $T_0$ is the mean CMB temperature. From measurements of the regions near the intersection of the LSS with the faces A and B of the FP in case I, we obtain a probability distribution function of obtaining a temperature $T_{S1} = 2T_0 + \Delta T_1, 2T_0 + \Delta T_2, 2T_0 + \Delta T_3, \text{ etc.}$ The values $\Delta T_1, \Delta T_2, \Delta T_3, \text{ etc.}$ describe a probability distribution function whose width at half height is

$$\Delta T_I = [g_{\text{NEAR}} + g_{\text{FAR}}] 2\delta T \simeq 2\delta T.$$ \hspace{1cm} (5)

In the same way, the value of the width at half height of the probability distribution function in case II is

$$\Delta T_{\text{II}} = 0.84 [g_{\text{NEAR}} + g_{\text{FAR}}] 2\delta T_{\text{II}}$$
$$\simeq 0.84 \times (2\delta T_{\text{II}}).$$ \hspace{1cm} (6)

Thus, $\Delta T_I$ has a value which is 16% greater than $\Delta T_{\text{II}}$.

III. CONCLUSIONS AND DISCUSSION

We suggest here the comparison of temperature fluctuations near the intersections of the LSS and a pair of faces of the FP with those taken where the LSS does not intersect the
faces of the FP. The temperature fluctuations are predicted to be appreciably different. In the worked-out example given, a 16% difference was found.

It is interesting to note from Eqs. (1) and (2) that, in general $T_{12} \neq T_{21}$. This indicates a weakness of the CIS method.

We strongly suggest that the new method for observing the topology of the universe, discussed here, be explored with present and future CMB data. A telescope with a resolution on the order of the thickness of the LSS should be used. The recent WMAP data indicates that $\Delta z_{\text{LSS}} \simeq 195$ and $\Omega \simeq 10^2$\footnote{16}. This corresponds to a proper thickness of the LSS $\sim 37$ Mpc and an angle $\sim 20$ arcmin. The resolution of the WMAP data is $\sim 42$ arcmin.

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