$B_c$ mesons in a Bethe-Salpeter model

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Abstract

We apply our Bethe-Salpeter model for mesons to the $B_c$ family with parameters fixed in our previous investigation. We evaluate the mass of the pseudo-scalar $B_c$ meson as 6.356 GeV/$c^2$ and 6.380 GeV/$c^2$ and the lifetime as 0.47 ps and 0.46 ps respectively in two reductions of the Bethe-Salpeter Equation, in good agreement with the recently reported mass of $6.40 \pm 0.39$ (stat.) $\pm 0.13$ (syst.) GeV/$c^2$ and lifetime of $0.46^{+0.18}_{-0.16}$ (stat.) $\pm 0.03$ (syst.) ps by the CDF Collaboration. We evaluate the decay constant of the $B_c$ meson and compare different contributions to its decay width.
Recently the CDF Collaboration reported the observation of the bottom-charmed mesons $B_c$ in 1.8 TeV $p\bar{p}$ collisions using the CDF detector at the Fermilab Tevatron [1]. This pseudo-scalar state is the lowest energy state of the family of mesons composed of a $\bar{b}$ anti-quark and a $c$ quark. Since this state lies below the $(BD)$ threshold and has non-vanishing flavor quantum numbers, it decays only through weak interactions. This eliminates uncertainties encountered in strong decays and renders the decay width of $B_c$ more theoretically tractable.

Different approaches have been used to evaluate the spectrum of $B_c$ mesons. Non-relativistic potential models have been used by Eichten and Quigg [2] where they discussed four potentials and gave mass values for the $B_c$ meson in the range 6.248–6.266 GeV/$c^2$. Gershtein et al. [3] used two potentials and reported predictions of 6.253 and 6.264. QCD sum rules have been used by Chabab [4] where he predicted a prediction of 6.25 GeV/$c^2$.

In this paper, we extend our model [5, 6] based on the Bethe-Salpeter Equation (BSE) to include the bottom-charmed mesons. BSE provides an appealing starting point to describe hadrons as relativistic bound states of quarks, just as the Dirac Equation provides a relativistic description of a fermion in an external field. The BSE for a bound state may be written in momentum space in the form

$$G^{-1}(P,p)\psi(P,p) = \int \frac{1}{(2\pi)^4} V(P,p-p')\psi(P,p')d^4p'$$

(1)

Where $P$ is the four-momentum of the bound state, $p$ is the relative four-momentum of the constituents. The BSE has three elements, the interaction kernel ($V$) and the propagator ($G$) which we provide as input, and the amplitude ($\psi$) obtained by solving the equation. We also solve for the energy, which is contained in the propagator.

Different approaches have been developed to make the four dimensional problem BSE more tractable and physically appealing. These include the Instantaneous Approximation (IA) and Quasi-Potential Equations (QPE) [7, 8, 9]. In the IA, the interaction kernel is taken to be independent of the relative energy. In QPE, the two particle propagator is modified in a way which keeps covariance and reduces the 4-dimensional BSE to a 3-dimensional equation. Of course, there is considerable freedom in carrying out this reduction.

We have used two reductions of the QPE to study the meson spectrum [3, 5]. These reductions correspond to different choices of the two particle propagator used to reduce the problem into three dimensions. We refer to these reductions as A and B. Reduction A corresponds to a spinor form of the Thompson equation [10] and reduction B corresponds to a new QPE introduced in Ref. [11]. These two reductions are chosen because they are shown to give good fits to the meson spectrum.
We assume the interaction kernel to consist of a one gluon exchange interaction, $V_{OGE}$, in the ladder approximation, and a phenomenological, long range scalar confinement potential, $V_{CON}$ given in the form

$$V_{OGE} + V_{CON} = -\frac{4}{3} \alpha_s \gamma_\mu \otimes \gamma_\mu + \sigma \lim_{\mu \to 0} \frac{\partial^2}{\partial \mu^2} \frac{1 \otimes 1}{-(p-p')^2 + \mu^2}$$

Here, $\alpha_s$ is the strong coupling, which is weighted by the meson color factor of $\frac{4}{3}$, and the string tension $\sigma$ is the strength of the confining part of the interaction. We adopt a scalar Lorentz structure $V_{CON}$ as discussed in [5, 6].

In our model the strong coupling is assumed to run as in the leading log expression for $\alpha_s$,

$$\alpha_s(Q^2) = \frac{4 \pi \alpha_s(\mu^2)}{4 \pi + \beta_1 \alpha_s(\mu^2) \ln(Q^2/\mu^2)}$$

where $\beta_1 = 11 - 2n_f/3$ and $n_f= 4$ is the number of quark flavors taken to be fixed. At the scale of the $Z$-boson, $\alpha_s(\mu^2 = M_Z^2) \simeq 0.12$ and $Q^2$ is related to the meson mass scale through,

$$Q^2 = \gamma^2 M_{meson}^2 + \beta^2,$$

where $\gamma$ and $\beta$ are parameters determined by a fit to the meson spectrum. In our formulation of BSE there are therefore seven parameters: four masses, $m_u=m_d$, $m_s$, $m_c$, $m_b$; the string tension $\sigma$, and the parameters $\gamma$ and $\beta$ used to govern the running of the coupling constant.

| Table 1: Values of the parameters used in reductions A, B |
|----------------------------------------------------------|
| Parameter      | Reduction A | Reduction B |
|----------------|-------------|-------------|
| $m_b$ (GeV)    | 4.65        | 4.68        |
| $m_c$ (GeV)    | 1.37        | 1.39        |
| $m_s$ (GeV)    | 0.397       | 0.405       |
| $m_u$ (GeV)    | 0.339       | 0.346       |
| $\sigma$ (GeV$^2$) | 0.233     | 0.211       |
| $\gamma$      | 0.616       | 0.444       |
| $\beta$ (GeV) | 0.198       | 0.187       |

In Ref. [5, 6] we fitted the meson spectrum using these seven parameters. However we did not include the $B_c$ mesons in our fit. Table 1 shows the values of the parameters obtained by the fits in reductions A and B.
In this paper we extend our model to evaluate the properties of the $B_c$ mesons using these same values of the parameters. In Table 2 we compare the spectrum of $B_c$ mesons obtained in reduction A and B with the work of Eichten and Quigg [2], Gershtein at al [3] using Martin potential and Gershtein et al. [3] using Buchmuller-Tye (BT) potential. The first row compares with the experimental results [1] of $6.40 \pm 0.39$ (stat.) $\pm 0.13$ (syst.) GeV/$c^2$. Both reduction A and B compare reasonably well with the experimental results though the experimental uncertainties are large.

Table 2: spectrum of $B_c$ mesons in different channels (GeV/$c^2$).

| State | This work Reduction A | This work Reduction B | Eichten and Quigg Ref. [2] | Gershtein et al. Martin potential | Gershtein et al. BT potential |
|-------|-----------------------|-----------------------|----------------------------|----------------------------------|-----------------------------|
| $1^1S_0$ | 6.356 | 6.380 | 6.264 | 6.253 | 6.246 |
| $1^3S_1$ | 6.397 | 6.415 | 6.337 | 6.317 | 6.337 |
| $1^3P_0$ | 6.673 | 6.692 | 6.700 | 6.683 | 6.700 |
| $1^3P_2$ | 6.751 | 6.773 | 6.747 | 6.743 | 6.747 |
| $1^1P_1$ | 6.752 | 6.777 | 6.729 | 6.729 | 6.736 |
| $2^1S_0$ | 6.888 | 6.874 | 6.856 | 6.867 | 6.856 |
| $2^3S_1$ | 6.910 | 6.891 | 6.899 | 6.902 | 6.899 |
| $1^3D_1$ | 6.984 | 6.955 | 7.012 | 7.008 | 7.012 |

In our formalism the mesons are taken as bound states of a quark and an anti-quark. The wavefunctions for the mesons are calculated by solving reductions of Bethe-Salpeter equation [4, 5]. We construct the meson states as [12]

$$|M(P_M, J, m_J)\rangle = \sqrt{2M} \int d^3p \langle Lm_LSm_S|Jm_J\rangle \langle sm_s\bar{sm}_\bar{s}\rangle \langle \bar{sm}_\bar{s}Sm_S\rangle \Phi_{Lm_L}(p)|\bar{q}(\frac{m_q-P_M}{M_{q\bar{q}}}-p, m_s)|q(\frac{m_q-P_M + p, m_s})\rangle \tag{5}$$

where the quark states are given by

$$|q(p, m_s)\rangle = \sqrt{\frac{(E_q + m_q)}{2m_q}} \left( \frac{\chi^m_s}{\langle \sigma \cdot p \rangle} \chi^m_s \right)$$

$$M_{q\bar{q}} = m_q + m_{\bar{q}}$$

$$E_q = \sqrt{m^2_q + p^2}$$

$$\langle \sigma \cdot p \rangle = \frac{E_q + m_q}{2m_q} \chi^m_s$$

$$\chi^m_s = \frac{(E_q + m_q)^m_q}{2m_q}$$
In Eq. 5 $M$ is the meson mass. The meson and the constituent quark states satisfy the normalization condition.

\[ \langle M(P'_M, J', m'_J)|M(P_M, J, m_J) \rangle = 2E\delta^3(P'_M - P_M)\delta_{J', J}\delta_{m'_J, m_J} \] (7)

\[ \langle q(p', m'_s)|q(p, m_s) \rangle = \frac{E_q}{m_q}\delta^3(p' - p)\delta_{m'_s, m_s} \] (8)

In previous works \cite{13,14,15}, we have used the wavefunctions of our model to evaluate the semi-leptonic form factors for $B$ to $D$ and $D^*$ mesons, and the leptonic decay constants. Here, we are interested in the leptonic decay constant. The weak decay constants for the pseudo-scalar and vector mesons are defined by

\[ <0|J_\mu|P(p)> = if_{P}\epsilon_{\mu} \]
\[ <0|J_\mu|V(p)> = M_Vf_V\epsilon_{\mu} \]

\[ J_\mu = V_\mu - A_\mu \] (9)

where $P$ and $V$ are pseudo-scalar and vector states and $V_\mu$ and $A_\mu$ are the vector and axial vector currents.

Taking into account the relativistic effects, the expressions of the decay constants in terms of the wavefunctions are given by \cite{16}

\[ f_i = \sqrt{\frac{12}{M}}\int_0^\infty dp \frac{E_q}{2\pi^3 \epsilon_{\mu}}F_i(p) \] (10)

\[ F_i(p) = \left[ 1 - \frac{p^2}{(m_q + E_q)(m_\bar{q} + E_\bar{q})} \right] \psi_i(p) \] (11)

where the subscript $i$ represents $P$ or $V$ and

\[ F_P(p) = \left[ 1 - \frac{p^2}{(m_q + E_q)(m_\bar{q} + E_\bar{q})} \right] \psi_P(p) \] (12)

\[ F_V(p) = \left[ 1 - \frac{p^2}{3(m_q + E_q)(m_\bar{q} + E_\bar{q})} \right] \psi_V(p) \] (13)

where $\psi_P$, $\psi_V$ are the wavefunctions of the pseudo-scalar and vector states respectively. The non-relativistic limit of these expressions yields a relation between $f_i$ and the wavefunction at the origin in coordinate space, $R(0)$,

\[ f_i = \sqrt{\frac{3}{\pi M}}R(0). \] (14)
Table 3: Leptonic decay constant of $B_c$ ($f_{B_c}$) in MeV.

| This work A | This work B | Eichten and Quigg Ref. [2] | Gershtein et al. Martin potential | Gershtein et al. BT potential |
|-------------|-------------|-----------------------------|----------------------------------|------------------------------|
| 578         | 490         | 500                         | 512                              | 500                          |

The leptonic decay constant ($f_{B_c}$) is relevant for the annihilation channel of the $B_c$ pseudo-scalar meson. In Table 3 we compare different predictions for this quantity.

The lifetime of $B_c$ is a very important quantity which may help us understand the basic properties of the weak interaction at a fundamental level especially since the strong interaction effects can be estimated reliably. The total width can be approximated by the sum of the widths of $\bar{b}$-quark decay with the spectator $c$-quark, the $c$-quark decay with the spectator $\bar{b}$-quark, and the annihilation channel $B_c^+ \to l^+\nu_l(c\bar{s}, u\bar{s}), l = e, \mu, \tau$. Since all these decays lead to different final states, we have no interference between different amplitudes. The total width is then given by

$$\Gamma(B_c \to X) = \Gamma(b \to X) + \Gamma(c \to X) + \Gamma(\text{ann}) .$$

(15)

Neglecting the quark binding effects, we obtain for the $b$ and $c$ inclusive widths in the spectator approximation,

$$\Gamma(b \to X) = \frac{G_F^2 |V_{cb}|^2 m_b^5}{192 \pi^3} \cdot 9 ,$$

$$\Gamma(c \to X) = \frac{G_F^2 |V_{cs}|^2 m_c^5}{192 \pi^3} \cdot 5 ,$$

(16)

The width of the annihilation channel is given by

$$\Gamma(\text{ann}) = \sum_i \frac{G_F^2}{8 \pi} |V_{bc}|^2 f_{B_c}^2 M_{bc} m_i^2 \left(1 - \frac{m_i^2}{m_{B_c}^2}\right)^2 \cdot C_i ,$$

(17)

where $C_i = 1$ for the $\tau\nu_\tau$ channel and $C_i = 3|V_{cs}|^2$ for the $\bar{c}s$ channel, and $m_i$ is the mass of the heaviest fermion ($\tau$ or $c$). Table 4 shows various contributions to the width of $B_c$ in our model.

We have used $V_{cb} = 0.041$, and $V_{cs} = 0.96$. From Table 4 we see that both reductions predict that the $b$ decay dominates $c$ decay in $B_c$ meson.

In Table 5, we compare the lifetime of $B_c$ in different models with the CDF experimental result. The experimental result indicates that the binding effects may not be very important as suggested by Quigg [17].
Table 4: Various contributions to the decay width of $B_c$ in $10^{-12}$ GeV.

|          | $\Gamma(b \to X)$ | $\Gamma(c \to X)$ | $\Gamma$ (ann) |
|----------|-------------------|-------------------|---------------|
| Reduction A | 0.75              | 0.51              | 0.14          |
| Reduction B | 0.78              | 0.55              | 0.11          |

Table 5: Comparison of the lifetime of $B_c$ meson (in ps) in different models.

|                  | Experiment [1]  | Reduction A | Reduction B | Quigg [17] | Gershtein et al. [3] |
|------------------|-----------------|-------------|-------------|-------------|-----------------------|
|                  | $0.46^{+0.18}_{-0.16}$ (stat.) $\pm 0.03$ (syst.) | 0.47        | 0.46        | 1.1 − 1.4    | 0.55 ± 0.15            |

In conclusion, we have evaluated the meson spectrum of the $B_c$ mesons in two reductions of BSE. We used parameters fixed from our previous fits and our results for properties of $B_c$ agree with the recent measurement of the CDF Collaboration of $B_c$ mass. We also predicted the leptonic decay constant and evaluated various contributions to the decay width of $B_c$. The partial width of $B_c$ due to b-quark decay dominates that due to the c-quark decay. Our result for the $B_c$ lifetime is in good agreement with the CDF measurement.

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