Computer analysis of three mode filters through optical couplers for single longitudinal mode fiber laser

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Abstract. In this paper, three basic mode filters based on optical couplers (OCs) for single longitudinal mode (SLM) fiber lasers are theoretically analyzed and compared by numerical simulation, providing a reference for the selection of filters according to the corresponding requirement. The three basic mode filters are single-coupler fiber ring (SCFR), double-coupler fiber ring (DCFR), and Mach-Zehnder filter (MZF). The three mode filters are acclaimed for the effective selection of SLM in fiber lasers. The transfer functions of the SCFR, DCFR, and MZF are derived and numerically simulated, respectively. Consequently, the bandwidths and amplitude characteristics of the three mode filters are analyzed and compared. The simulation results show that the bandwidths of DCFR and SCFR are mainly affected by the intensity coupling coefficient of the OC and the length of the ring. However, the bandwidth of MZF is hardly influenced by the intensity coupling coefficient but the difference of length between its upper and lower arm. The intensity coupling coefficient primarily impacts the amplitude of the three mode filters.

Keywords: single-coupler fiber ring (SCFR), double-coupler fiber ring (DCFR), Mach-Zehnder filter (MZF), single longitudinal mode (SLM).

1. Introduction
Single longitudinal mode (SLM) fiber lasers are used extensively in applications such as coherent laser radar, optical time and frequency transfer, optical sensors and high-resolution spectroscopy [1-2] due to their advantages of monochromaticity, high coherence and narrow linewidth. In order to achieve the SLM operation of a fiber laser, there are several methods utilized in the laser cavity, such as unpumped EDF based saturable absorbers (SAs) [3], ultra-narrow pass-band optical filter [4], Mach-Zehnder filter (MZF) [3], optical injection technique [5] and the fiber ring filter [6,7]. Among these methods, the MZF and fiber ring filter based on OCs are more popular for their low cost and simplicity.

In fiber lasers, the use of fiber ring filter and MZF can effectively extend the longitudinal mode spacing or the free spectral range (FSR) due to the Vernier effect [8], making it easier to select SLM. There are two commonly used basic fiber ring filters. One is built with a single OC, called single-coupler fiber ring (SCFR) [9]; the other is composed of two OCs with identical coupling ratios, called double-coupler fiber ring (DCFR) [10]. Lee C C et al. proposed a SLM fiber laser by cascading three SCFRs [9]. Yang X X et al. demonstrated a SLM fiber laser by utilizing a DCFR [11]. Feng T et al. nested a SCFR into a DCFR in the SLM fiber laser [6]. Li L et al. employed a MZF with one SCFR nested in its...
upper and lower arms, respectively [3]. Wang Z et al. reported a stable SLM fiber laser with ultra-narrow linewidth by exploiting two DCFRs and one SCFR [12]. Thus, it can be seen that the three mode filters play an essential role in the achievement of SLM operation for fiber lasers. However, the three mode filters behave differently on the filtering performances. Therefore, it is necessary and beneficial to analyze the features of three mode filters to provide a reference for subsequent usage.

In this paper, based on the coupling equation of the OC, the transfer functions of the SCFR, DCFR and MZF are derived, respectively. The bandwidths and amplitude characteristics of the three mode filters are analyzed and compared by the numerical simulation of the transfer functions. From the simulation results, we could see that the bandwidths of DCFR and SCFR are mainly affected by the intensity coupling coefficient of the OC and the length of the ring. However, the bandwidth of MZF is scarcely influenced by the intensity coupling coefficient but the difference of length between its upper and lower arm. The amplitude of the three mode filters is primarily impacted by the intensity coupling coefficient.

2. Setup and Principles

2.1. SCFR

Fig. 1 shows the schematic diagram of the SCFR. Referring to Fig. 1, the input electric field of OC at port 1 and 2, the output electric field of OC at port 3 and 4, are $E_1$, $E_2$, $E_3$ and $E_4$, respectively. The SCFR is formed by fusing port 2 and port 4 and the electric field propagates anti-clockwise around the ring. According to the coupled equation of OC [13], we could obtain:

$$E_3 = \sqrt{1 - \gamma_0} \left( \sqrt{1 - k} E_1 + j\sqrt{k} E_2 \right)$$

$$E_4 = \sqrt{1 - \gamma_0} \left( j\sqrt{k} E_1 + \sqrt{1 - k} E_2 \right)$$

$E_2$ and $E_4$ are further related by:

$$E_2 = \sqrt{1 - \gamma_1} e^{-\alpha L} e^{j\beta L} E_4$$

Substitute equation (3) into equation (1):

$$E_3 = \sqrt{1 - \gamma_0} \left( \sqrt{1 - k} E_1 + j\sqrt{k} \sqrt{1 - \gamma_1} e^{-\alpha L} e^{j\beta L} E_4 \right)$$

Substitute equation (3) into equation (2):

$$E_4 = \frac{j\sqrt{1 - \gamma_0} \sqrt{k}}{1 - \sqrt{1 - \gamma_0} \sqrt{1 - \gamma_1} \sqrt{1 - k} e^{-\alpha L} e^{j\beta L}} E_1$$

Substitute equation (5) into equation (4) and simplify:

$$\frac{E_3}{E_1} = \sqrt{1 - \gamma_0} \frac{\sqrt{1 - k} - \sqrt{1 - \gamma_1} e^{-\alpha L} e^{j\beta L}}{1 - \sqrt{1 - \gamma_0} \sqrt{1 - \gamma_1} \sqrt{1 - k} e^{-\alpha L} e^{j\beta L}}.$$  

Thus, the transmission function of the SCFR can be obtained:

$$T_{SCFR} = \frac{l_3}{l_1} = \left| \frac{E_3}{E_1} \right|^2 = (1 - \gamma_0) \cdot \frac{(1 - k) + (1 - \gamma_0)(1 - \gamma_1)e^{-2\alpha L} - 2\sqrt{1 - \gamma_0} \sqrt{1 - \gamma_1} \sqrt{1 - k} e^{-\alpha L} \cos \beta L + \gamma_0 (1 - \gamma_1)(1 - k)e^{-2\alpha L} - 2\sqrt{1 - \gamma_0} \sqrt{1 - \gamma_1} \sqrt{1 - k} e^{-\alpha L} \cos \beta L}{1 + (1 - \gamma_0)(1 - \gamma_1)(1 - k)e^{-2\alpha L} - 2\sqrt{1 - \gamma_0} \sqrt{1 - \gamma_1} \sqrt{1 - k} e^{-\alpha L} \cos \beta L}.$$  

In these equations: $j=\sqrt{-1}$; $\gamma_0$ and $k$ are the intensity coupling loss and intensity coupling coefficient of the OC, respectively; $\gamma_1$ is the fusion loss coefficient of the ring; $\alpha$ and $\beta$ are attenuation and propagation coefficient of the fiber, respectively; $L$ is the length of SCFR.

We know that:

$$\beta = \frac{2\pi nf}{c}$$  \hspace{1cm} (8)

Where $n$ is the fiber refractive index (about 1.5), $f$ is the propagating frequency and $c$ is the velocity of light in vacuum. Substituting equation (8) into equation (7), we could obtain the relationship between transfer function TSCFR and frequency $f$.

![Fig. 1 Schematic diagram of the SCFR. OC, optical coupler.](image)

2.2. DCFR

The schematic diagram of the DCFR is given as Fig. 2. Referring to Fig. 2, the input electric field of OC1 at port 1 and 2, the output electric field of OC1 at port 3 and 4, are $E_1, E_2, E_3$ and $E_4$, respectively. The input electric field of OC2 at port 5 and 6, the output electric field of OC2 at port 7 and 8, are $E_5, E_6, E_7$ and $E_8$, respectively. The DCFR is constructed by splicing the port 2 and 8, the port 3 and 5 respectively. The electric fields propagate anti-clockwise around the ring. According to the coupled equation of OC, we could obtain:

$$E_3 = \sqrt{1-\gamma_0^2}(\sqrt{1-k_1}E_1 + j\sqrt{k_1}E_2)$$  \hspace{1cm} (9)

$$E_7 = \sqrt{1-\gamma_1^2}(\sqrt{1-k_2}E_5 + j\sqrt{k_2}E_6)$$  \hspace{1cm} (10)

$$E_8 = \sqrt{1-\gamma_1^2}(j\sqrt{k_2}E_5 + \sqrt{1-k_2}E_6)$$  \hspace{1cm} (11)

Other relations are:

$$E_6 = 0$$  \hspace{1cm} (12)

$$E_2 = \sqrt{1-\gamma_2^2}E_8 e^{-\alpha L_1} e^{j\beta L_1}$$  \hspace{1cm} (13)

$$E_5 = \sqrt{1-\gamma_3^2}E_3 e^{-\alpha L_2} e^{j\beta L_2}$$  \hspace{1cm} (14)
Substitute equation (12) and (14) into equation (11):

\[ E_8 = j\sqrt{1 - \gamma'_1'}\sqrt{1 - \gamma'_3'}\sqrt{k_2 e^{-\alpha L_2}} e^{j\beta L_2} E_3 \]  

(15)

Substitute equation (15) and (13) into equation (9) and simplify:

\[ \frac{E_3}{E_1} = \frac{\sqrt{1 - \gamma'_0'}}{1 + \sqrt{1 - \gamma'_0'}\sqrt{1 - \gamma'_1'}\sqrt{1 - \gamma'_3'}\sqrt{1 - \gamma'_2'}\sqrt{k_1 k_2}} \]  

(16)

Substitute equation (12), (14) and (16) into equation (10) and simplify:

\[ \frac{E_7}{E_1} = \frac{(1 - \gamma'_0')(1 - \gamma'_1')(1 - \gamma'_2')(1 - k_1)(1 - k_2)e^{-2\alpha L_2}}{1 + 2\sqrt{1 - \gamma'_0'}\sqrt{1 - \gamma'_1'}\sqrt{1 - \gamma'_2'}(1 - \gamma'_3')\sqrt{k_1 k_2} e^{-\alpha L_1 + L_2} \cos(\beta (L_1 + L_2)) + (1 - \gamma'_0')(1 - \gamma'_1')(1 - \gamma'_2')(1 - \gamma'_3') k_1 k_2 e^{-2\alpha (L_1 + L_2)}} \]  

(17)

Thus, the transmission function of the DCFR can be obtained:

\[ T_{DCF R} = \frac{E_7}{E_1} = \frac{I_7}{I_1} = \left| \frac{E_7}{E_1} \right|^2 \]

(18)

In these equations: \( j = \sqrt{-1} \); \( \gamma'_0' \) and \( \gamma'_1' \) are the intensity coupling loss coefficients of OC\(_1\) and OC\(_2\), respectively; \( k_1 \) and \( k_2 \) are intensity coupling coefficients of OC\(_1\) and OC\(_2\), respectively; \( \gamma'_2' \) and \( \gamma'_3' \) are the fusion loss coefficients of DCFR’s upper arm and lower arm, respectively; \( \alpha \) and \( \beta \) are attenuation and propagation coefficient of the fiber, respectively; \( L_1 \) and \( L_2 \) are the lengths of DCFR’s upper arm and lower arm, respectively. Substituting equation (8) into equation (18), we could obtain the relationship between transfer function \( T_{DCF R} \) and frequency \( f \).

![Fig.2 Schematic diagram of the DCFR. OC, optical coupler.](image-url)
2.3. MZF

Fig. 3 depicts the schematic diagram of the MZF. Referring to Fig. 3, the input electric field of OC1 at port 1 and 2, the output electric field of OC1 at port 3 and 4, are \( E_1 \), \( E_2 \), \( E_3 \), and \( E_4 \), respectively. The input electric field of OC2 at port 5 and 6, the output electric field of OC2 at port 7 and 8, are \( E_5 \), \( E_6 \), \( E_7 \), and \( E_8 \), respectively. The MZF is composed by fusing port 4 and 6, the port 3 and 5, respectively. According to the coupled equation of OC, we could obtain:

\[
E_3 = \sqrt{1 - \gamma_0''(1 - k_1 E_1 + j\sqrt{k_1} E_2)} \tag{19}
\]

\[
E_4 = \sqrt{1 - \gamma_1''(j\sqrt{k_1} E_1 + \sqrt{1 - k_1} E_2)} \tag{20}
\]

\[
E_7 = \sqrt{1 - \gamma_3''(1 - k_2 E_5 + j\sqrt{k_2} E_6)} \tag{21}
\]

Other relations are:

\[
E_2 = 0 \tag{22}
\]

\[
E_6 = \sqrt{1 - \gamma_2'' E_4 e^{-\alpha L_1} e^{j\beta L_1}} \tag{23}
\]

\[
E_5 = \sqrt{1 - \gamma_3'' E_3 e^{-\alpha L_2} e^{j\beta L_2}} \tag{24}
\]

Substitute equation (22) into equation (19) and (20):

\[
E_3 = \sqrt{1 - \gamma_0'' \sqrt{1 - k_1} E_1} \tag{25}
\]

\[
E_4 = j\sqrt{1 - \gamma_1'' \sqrt{1 - k_1} E_1} \tag{26}
\]

Substitute equation (23), (24), (25) and (26) into equation (21):

\[
\frac{E_2}{E_1} = \frac{\sqrt{1 - \gamma_0'' \sqrt{1 - \gamma_1'' \sqrt{1 - \gamma_3'' \sqrt{1 - k_1 \sqrt{1 - k_2 e^{-\alpha L_2} e^{j\beta L_2} - \sqrt{1 - \gamma_2'' \sqrt{1 - \gamma_1'' \sqrt{1 - \gamma_3'' \sqrt{1 - k_1 \sqrt{1 - k_2 e^{-\alpha L_1} e^{j\beta L_1}}}}}}}}}}}{E_1} \tag{27}
\]

Thus, the transmission function of the MZF can be obtained:

\[
T_{MZF} = \frac{T_2}{T_1} = \left[ \frac{E_2}{E_1} \right]^2 = (1 - \gamma_0'')(1 - \gamma_1'')(1 - \gamma_3'')(1 - k_1)(1 - k_2)e^{-2\alpha L_2} + (1 - \gamma_0')(1 - \gamma_1')k_1 k_2 e^{-2\alpha L_1} - 2(1 - \gamma_0')(1 - \gamma_1')(1 - \gamma_3'') \sqrt{1 - \gamma_2'' \sqrt{1 - \gamma_1'' \sqrt{1 - \gamma_3'' \sqrt{1 - k_1 \sqrt{1 - k_2 e^{-\alpha L_1} e^{j\beta L_1}}}}}} \tag{28}
\]

In these equations: \( j = \sqrt{-1} \); \( \gamma_0'' \) and \( \gamma_1'' \) are the intensity coupling loss coefficients of OC1 and OC2, respectively; \( k_1 \) and \( k_2 \) are intensity coupling coefficients of OC1 and OC2, respectively; \( \gamma_2'' \) and \( \gamma_3'' \) are the fusion loss coefficients of MZF’s upper arm and lower arm, respectively; \( \alpha \) and \( \beta \) are attenuation and propagation coefficient of the fiber, respectively; \( L_1 \) and \( L_2 \) are the lengths of MZF’s upper arm and lower arm, respectively. Substituting equation (8) into equation (28), we could obtain the relationship between transfer function \( T_{MZF} \) and frequency \( f \).
In SLM fiber lasers, the use of fiber ring filters and MZFs act as mode filters, suppressing a large number of mode oscillations due to the providing of a large free spectral range (FSR). The FSR is given as follows:

\[
\text{FSR} = \frac{c}{nL}.
\]  

Where \( c \) is the speed of light in vacuum, \( n \) is the effective refractive index of the fiber; \( L \) is the filter ring length in fiber ring filter and the difference of length between the upper and lower arm in MZF. According to the Vernier effect, the effective FSR is the least common multiple of all FSRs: 

\[
FSR_e = m_1FSR_1 = m_2FSR_2 \ldots = m_nFSR_n. 
\]

### 3. Results and Discussions

Considering the actual experimental conditions that the fusion loss of the filter construction and the intensity coupling loss of the OC are small, the OC coupling coefficient \( k \) and filter length \( L \), which is the ring length in SCFR and DCFR and is the difference of length between the upper and lower arm in MZF, are taken into account to the influence on the three mode filters. Ignoring the fusion loss of construction, the other fixed settings of common parameters in the numerical simulation are shown in Table 1.

| Intensity coupling loss coefficient \((\gamma_0, \gamma_0', \gamma_0'', \gamma_0''')\) | Fiber attenuation coefficient \((\alpha)\) | Fiber refractive index \((n)\) | Light velocity in vacuum \((c)\) | Frequency \((f)\) |
|---------------------------------|-----------------|-----------------|-----------------|-----------------|
| 0.1087                        | 0.3 dB/km       | 1.5             | \(3 \times 10^8\) m/s | 1:0.1:500 MHz |

#### 3.1. SCFR

Set the filter length is 2 m. Fig.4 (a) and (b) illuminate the changes of the transmission curve, the bandwidth, and the amplitude extremum of SCFR, with the coupling coefficient varying from 0.1 to 0.9 in step of 0.2, respectively. When the coupling coefficient increases, the amplitude minimum increases greatly as well. However, the amplitude maximum slightly decreases. The bandwidth of SCFR reduces by 29% with the coupling coefficient variation of 0.1 to 0.9.
Fig 4 Changes of the transmission curve, the bandwidth and the amplitude extremum of SCFR with variation of coupling coefficient.

Set the coupling coefficient is 0.5. Fig 5 (a) and (b) show the variations of the transmission curve, the bandwidth, and the amplitude extremum of SCFR, with the filter length changing from 2 m to 5 m in step of 1 m, respectively. When the ring length increases, obviously the FSR decreases, and the amplitude extremum hardly changes as well. The bandwidth of SCFR reduces by 60% with the length variation of 2 m to 5 m.
fig 5 Variations of the transmission curve, the bandwidth and the amplitude extremum of SCFR with changes of the ring length.

3.2. DCFR
Assume that the two couplers forming the DCFR filter are exactly the same. Set the ring length is 2 m (upper arm of 1 m and lower arm of 1 m). Fig.6(a) and (b) depict the changes of the transmission curve, the bandwidth, and the amplitude extremum of DCFR, with the coupling coefficient varying from 0.1 to 0.9 in step of 0.2, respectively. When the coupling coefficient increases, the amplitude maximum and minimum decrease as well, but not at the same rate. The bandwidth of DCFR reduces by 84.2% with the coupling coefficient variation of 0.1 to 0.9.
Changes of the transmission curve, the bandwidth and the amplitude extremum of DCFR with variation of coupling coefficient.

Set the coupling coefficient is 0.5 and fix the upper arm length of 1 m. Fig.7(a) and (b) describe the variations of the transmission curve, the bandwidth, and the amplitude extremum of DCFR, with the lower arm length changing from 1 m to 4 m in step of 1 m, that is, the total length of the DCFR changing from 2 m to 5 m, respectively. When the ring length increases, obviously the FSR decreases, and the amplitude extremum hardly changes as well. The bandwidth of DCFR reduces by 59.8% with the length variation of 2 m to 5 m.
3.3. MZF

Assume that the two couplers forming the MZF are exactly the same. Set the length difference between upper arm (3 m) and lower arm (1 m) is 2 m. Fig.8(a) and (b) depict the changes of the transmission curve, the bandwidth, and the amplitude extremum of MZF, with the coupling coefficient varying from 0.1 to 0.9 in step of 0.2, respectively. When the coupling coefficient increases, the amplitude maximum and the bandwidth of MZF hardly vary as well. However, the amplitude minimum decreases first and then increases, and the inflection point happens when the coefficient is 0.5, which the minimum value is nearly 0.

Fig 7 Variations of the transmission curve, the bandwidth and the amplitude extremum of DCFR with changes of the ring length.

(a) transmission curves of DCFR
(b) bandwidths and amplitude extremums of DCFR
Fig 8 Changes of the transmission curve, the bandwidth and the amplitude extremum of MZF with variation of coupling coefficient.

Set the coupling coefficient is 0.5 and fix the lower arm length of 1 m. Fig 9(a) and (b) represent the variations of the transmission curve, the bandwidth, and the amplitude extremum of MZF, with the upper arm length changing from 3 m to 6 m in step of 1 m, that is, the difference of length between the upper arm and lower arm changing from 2 m to 5 m, respectively. When the difference of length increases, obviously the FSR decreases, and the amplitude extremum hardly changes as well. The bandwidth of MZF reduces by 60% with the difference of length variation of 2 m to 5 m.
4. Conclusion
In summary, we have theoretically analyzed and numerically simulated three mode filters based on optical couplers for single longitudinal mode fiber lasers, which benefit the SLM operation. The three mode filters are single-coupler fiber ring (SCFR), double-coupler fiber ring (DCFR) and Mach-Zehnder filter (MZF). The bandwidths and amplitude characteristics of the three mode filters are analyzed and compared through numerical simulation. The results show that, for the SCFR and DCFR, the intensity coupling coefficient of OC and length of the ring are main factors affecting the bandwidths. However, the bandwidth of MZF is hardly influenced by the intensity coupling coefficient but the difference of length between its upper and lower arm. The amplitude extremum of the three mode filters is primarily impacted by the intensity coupling coefficient.

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