Derivation of Gell-Mann-Nishijima formula from the electromagnetic field modes of a hadron

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Abstract
When an electron probes another elementary particle Q, the wave function of the electron can be separated into two independent parts, the first part represents the electronic motion, the second part represents the electromagnetic field mode around the particle Q. In analogy with optical modes $TEM_{nlm}$ for a laser resonator, when the electromagnetic field around the particle Q forms into a mode, the quantum numbers of the mode satisfy the Gell-Mann-Nishijima formula, these quantum numbers are recognized as the charge number, baryon number and strangeness number. The modes are used as a visual model to understand the abstract baryon number and strangeness number of hadrons.

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1 Introduction
The Gell-Mann-Nishijima formula relates the baryon number $B$, the strangeness number $S$, the isospin $I_3$ of hadrons to the charge number $Q$. It was originally given by Tadao Nakano and Kazuhiko Nishijima in 1953[1, 2]. Murray Gell-Mann proposed the formula independently in 1956[3]. The modern version of the formula relates all flavor quantum numbers with the baryon number and the electric charge. We can understand the charge number $Q$ through the Coulomb’s interaction, but failed to understand the baryon number $B$ and the strangeness number $S$ with reality, so $B$, $S$ and $I_3$ are abstract. Since $B$ and $S$ appear in the formula with the charge number $Q$, they must relate to the electromagnetic interaction in some way, this clue guide us to study the possible electromagnetic field modes around a hadron, in analogy with optical modes $TEM_{nlm}$ for a laser resonator. As a result, it is found that the Gell-Mann-Nishijima formula is the critical condition of the electromagnetic field modes around a hadron. It is important to realize that the electromagnetic field modes around a hadron provide us a visual model to understand the abstract baryon number $B$ and strangeness number $S$ in reality.
2 Electromagnetic field around a particle

Consider an electron probing an elementary particle $Q$, the wave function of the electron is given by

\[(p_\mu - eA_\mu)\psi = -i\hbar \partial_\mu \psi\] (1)

\[\psi(x) = \exp\left[\frac{i}{\hbar} \int (p_\mu - eA_\mu)dx_\mu\right]\] (2)

Where $A_\mu$ is the 4-vector potential around the particle $Q$. Frequently, $p_\mu$ in Eq.(1) is understood as momentum operator of the electron, but here we regard $p_\mu$ as momentum itself as shown in Eq.(2) for calculating the wave function. Because of path independence, $l$ is an arbitrary path from the initial point $x_0$ to the observed point $x$, we can choose two path $l$ and $l + L$ to calculate the wave function, as shown in Figure 1, $L$ is a closed loop linking the observed point $x$ around the particle $Q$, then

\[\psi(x)_l = \psi(x)_{l+L}\] (3)

\[\exp\left[\frac{i}{\hbar} \int_l (p_\mu - eA_\mu)dx_\mu\right] = \exp\left[\frac{i}{\hbar} \int_{l+L} (p_\mu - eA_\mu)dx_\mu\right]\] (4)

We obtain a quantization equation:

\[\frac{1}{\hbar} \int_L (p_\mu - eA_\mu)dx_\mu = 2\pi n\quad n = 0, \pm 1, \pm 2, ...\] (5)

\[\text{Figure 1: The calculation of the wave function around a particle by path integral.}\]

The second part of the loop integral, $\frac{1}{\hbar} \int_L (-eA_\mu)dx_\mu$, is independent from the first part $\frac{1}{\hbar} \int_L (p_\mu)dx_\mu$, no matter how much the electron energy $\varepsilon$ is ($\varepsilon = 1MeV$ or $\varepsilon = 500MeV$), it always hold the quantization equation, so we have

\[\frac{1}{\hbar} \int_L (-eA_\mu)dx_\mu = 2\pi n'\quad n' = 0, \pm 1, \pm 2, ...\] (6)
This part represents the properties of the particle $Q$, and defining

$$\xi(x) = \exp\left[\frac{i}{\hbar} \int (-eA_\mu) dx_\mu\right]$$  \hspace{1cm} (7)

We call $\xi(x)$ as the potential-wave-function of the particle $Q$, Eq. (6) is the quantization equation of the potential-wave-function.

### 3 Magnetism of particle

If the particle $Q$ has magnetic field around it without electric field, $E = 0$, then

$$A_4(x) = \frac{i}{c} \nabla \cdot \mathbf{V}(x) = \frac{i}{c} \int_x^\infty \mathbf{E} \cdot d\mathbf{l} = 0$$  \hspace{1cm} (8)

The quantized Eq. (6) reduces to

$$-\frac{e}{\hbar} \oint_L \mathbf{A} \cdot d\mathbf{l} = 2\pi n \hspace{1cm} n = 0, \pm 1, \pm 2, ...$$  \hspace{1cm} (9)

Because of magnetic field $B = \nabla \times \mathbf{A}$, we find that the magnetic flux $\Phi_m$ through the closed loop $L$ is quantized by

$$\Phi_m = \oint_S \mathbf{B} \cdot ds = \oint_S (\nabla \times \mathbf{A}) \cdot ds = \oint_L \mathbf{A} \cdot d\mathbf{l} = -2\pi n \frac{\hbar}{e}$$  \hspace{1cm} (10)

It is important to note that the loop shape $L$ or surface area $S$ are arbitrary because of path independence for the potential-wave-function, the magnetic flux must concentrate into the particle interior as a tiny solenoid, the particle position is a singularity in mathematics. Experiments have made numerous measurements on the magnetic moments of baryons, but lacking attentions to magnetic fluxes of baryons.

### 4 Gell-Mann-Nishijima formula for hadrons

In order to describe the 4D electromagnetic field around the particle $Q$, we choose a big closed loop $L$ as shown in Figure 2, saying A-B-C-D-A, where A contains the particle at the origin and C at the infinity, from Eq. (9) we have

$$-\frac{e}{\hbar} \oint_L A_r \cdot dr + \frac{e}{\hbar} \oint_L A_\theta \cdot r d\theta + \frac{e}{\hbar} \oint_L A_\phi \cdot r \sin \theta d\phi + \frac{e}{\hbar} \oint_L A_4 \cdot d(ict) = 2\pi n$$  \hspace{1cm} (11)

$$n = 0, \pm 1, \pm 2, ...$$

The big closed loop $L$ makes that the angle $\theta$ varies from 0 to $2\pi$ and the angle $\phi$ varies from 0 to $2\pi$, the radical distance $r$ varies from 0 to $R = \infty$ then returns to 0, time $t$ varies from 0 to $T = \infty$ then returns to 0. Note that $L$ is not the path of the probing electron, is a calculation path for wave function.

We can imagine a conformal transformation for the electromagnetic field around the nucleus, as shown in Figure 3, from 4D $A$ in the circular region to 4D $A$ in the conformal box. (conformal theory in mathematics is a classical branch). We use this conformal box to emphasize that we must fairly treat $A_r$, $A_\theta$, $A_\phi$.
$A_\theta$, $A_\phi$, and $A_4$. Regarding this 4D conformal box, the boundary conditions for the electromagnetic field are given by

$$\frac{-e}{\hbar} \int_0^R A_r \cdot dr = 2\pi Q_m \quad Q_m = 0, \pm 1, \pm 2, \ldots \quad (12)$$

$$\frac{-e}{\hbar} \int_0^{2\pi} A_\theta \cdot r d\theta = 2\pi B_m \quad B_m = 0, \pm 1, \pm 2, \ldots \quad (13)$$

$$\frac{-e}{\hbar} \int_0^{2\pi} A_\phi \cdot r \sin \theta d\phi = 2\pi S_m \quad S_m = 0, \pm 1, \pm 2, \ldots \quad (14)$$

$$\frac{-e}{\hbar} \int_0^T A_4 \cdot d(ict) = 2\pi Q \quad Q = 0, \pm 1, \pm 2, \ldots \quad (15)$$

These quantization equations are obvious when thinking about that the four components of the vector potential are independent from each other. The path from the corner ($r = 0, \theta = 0, \phi = 0, t = 0$) in the 4D conformal box to the corner ($R, \pi, \pi, T$) corresponds to the half loop of the big closed loop $L$ as shown in Figure 2 saying A-B-C. Considering the half loop A-B-C in Figure 2, then Eq. (11) becomes

$$-\frac{e}{\hbar} \int_0^R A_r \cdot dr - \frac{e}{2\hbar} \int_0^{2\pi} A_\theta \cdot r d\theta - \frac{e}{2\hbar} \int_0^{2\pi} A_\phi \cdot r \sin \theta d\phi - \frac{e}{\hbar} \int_0^T A_4 \cdot d(ict) = \frac{2\pi n}{2} \quad (16)$$

Substituting Eq. (12)-(15) into Eq. (16), we obtain

$$Q_m + \frac{B_m + S_m}{2} + Q = \frac{n}{2} \quad n = 0, \pm 1, \pm 2, \ldots \quad (17)$$

We immediately recognize that this equation is the Gell-Mann-Nishijima relation for hadrons: According to hadron proprieties, we have to define $Q$ as
Figure 3: A conformal transformation from the circular region to the conformal box.

the charge number, to define $-B_m$ as the baryon number $B$, to define $-B_m$ as strangeness number $S$, to define $\frac{2}{3}$ as the third component of isospin $I_3$, typically, $A_r = 0$, $Q_m = 0$, Eq.(17) becomes the Gell-Mann-Nishijima formula

$$Q - \frac{B + S}{2} = I_3 \quad I_3 = 0, \pm \frac{1}{2}, \pm 1, ...$$  \hspace{1cm} (18)

To note that the charge number $Q$ is conserved, therefore we deduce that the baryon number $B$ is conserved and the strangeness number $S$ is conserved too for all hadron reactions. It is important to realize that the electromagnetic field of a hadron can be regarded as the higher order mode in terms of its potential-wave-function, marked by three quantum numbers: charge number $Q$, baryon number $B$ and strangeness number $S$. This result has three significations: (1) the formalism we developed about the potential-wave-function is success and useful for elementary particle classification. (2) the potential-wave-function provides us a visual model for abstract baryon number and strangeness number, in analogy with the optical mode $TEM_{qlm}$ in a laser resonator, it provides us an important work platform for extending our knowledge. (3) the Gell-Mann-Nishijima formula is the critical condition of the electromagnetic field modes around a hadron.

5 Charge quantization

Consider the fourth component quantization Eq.(15) and the path independence, we have

$$\frac{-e}{\hbar} \int_0^T A_4(r) \cdot d(ict) = \frac{-eA_4(r)}{\hbar} \int_0^T d(ict) = \frac{-eA_4(r)ict}{\hbar}$$ \hspace{1cm} (19)

$$A_4(r) = i\frac{e}{c}V(r) = i\int_r^\infty \mathbf{E} \cdot d\mathbf{l} = \frac{ikr}{cr}$$ \hspace{1cm} (20)
Where we have used the Coulomb’s law $E = \frac{k_e q}{r^2}$, we have

$$-\frac{e}{\hbar} \int_0^T A_4(r) \cdot d(ict) = \frac{k_e q}{r} \cdot T \cdot \frac{1}{\hbar} = 2\pi Q \quad Q = 0, \pm 1, \pm 2, \ldots \quad (21)$$

From the Heisenberg’s uncertainty principle, we immediately recognize that the potential energy is the uncertain energy $\triangle E = \frac{k_e e}{r}$ for the observed point $x$, and the time limit $T$ for the observed point $x$ is the uncertain time $\triangle t = T$, both satisfies the charge quantization and the uncertainty principle, because from Eq (21):

$$q = Q e \quad Q = 0, \pm 1, \pm 2, \ldots \quad (22)$$

$$\triangle E \cdot \triangle t = \left( \frac{k_e e}{r} \right) \cdot T = 2\pi \hbar \quad (23)$$

The charge $q$ of the particle $Q$ has been quantized by Eq. (22). Here for the first time, the charge quantization can be derived from the quantum mechanics. In fact, the uncertainty principle Eq. (23) uses the energy $k_e e / r$ to determine time limit $T$ without regarding the elementary particle charge number $Q$, it is in minimum case for the Heisenberg’s uncertainty principle, or:

$$\triangle E \cdot \triangle t = \left| \frac{k_e e Q e}{r} \right| \cdot T \geq 2\pi \hbar \quad Q = \pm 1, \pm 2, \ldots \quad (24)$$

Where we discard $Q = 0$.

### 6 Confinement of quarks

Quarks are abstract particles; here we focus on how to form the quarks by using our theory. The charge number $Q$, baryon number $B$ and strangeness number $S$ are three freedoms for classifying elementary particles, we can build a frame in which $Q$, $B$ and $S$ are the three independent axes as shown in Figure 4. But mathematicians have rights to rotate the frame $(Q, B, S)$ to a new orientation and let it becomes a new frame $(u, d, s)$ as shown in Figure 4, the transformation equations between the old coordinates $(Q, B, S)$ and new coordinates $(u, d, s)$ are given by

$$\begin{bmatrix} Q \\ B \\ C \end{bmatrix} = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix} \cdot \begin{bmatrix} u \\ d \\ s \end{bmatrix} \quad (25)$$

It is an easy thing to get the transformation matrix $R$ after reading the basic data of baryons and three quarks, finally it is

$$\begin{bmatrix} Q \\ B \\ C \end{bmatrix} = \begin{bmatrix} 2/3 & -1/3 & -1/3 \\ 1/3 & 1/3 & 1/3 \\ 0 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} u \\ d \\ s \end{bmatrix} \quad (26)$$

Where the axis $u$ represents the up-quark number, the axis $d$ represents the down-quark number; the axis $s$ represents the strange-quark number. In our viewpoint, the three quark types are another mathematical representation for the higher order modes of the electromagnetic field around a particle.
The motivation of this section is to make readers to understand with reality: (1) The electromagnetic filed modes around a hadron have three quantum numbers: $Q$, $B$ and $S$, therefore the quark type number through the mathematical transformation in Eq. (26) are three. (2) The three quark types have been confined in Eq. (26), we have no way to let the three quark types being free particles mathematically or physically, experiments have no chance to discover free quark in reality, like virtual phonons in metals.

7 Six components of electromagnetic field

Generally speaking, the electromagnetic field around the particle $Q$ has six component: $(B_r, B_\theta, B_\phi, E_r, E_\theta, E_\phi)$, we can not naively think that $E_\theta = E_\phi = 0$, specially the lift time for some hadrons has only $10^{-10}$ seconds or even shorter.

The electrical potential $V$ in the fourth component quantization Eq.(15) is written as

$$\frac{-e}{\hbar} \int_0^T A_4 \cdot d(ict) = \frac{eT}{\hbar} \int_{r}^{\infty} \mathbf{E} \cdot d\mathbf{l}$$

It is also path independent about the electric field, so we have

$$\frac{eT}{\hbar} \oint_L \mathbf{E} \cdot d\mathbf{l} = 2\pi n \quad n = 0, \pm 1, \pm 2, \ldots$$

Where $L$ is the big close loop linking to the observed point $x$, as shown in Figure 2 saying A-B-C-D-A. We are now in a position to realize: (1) the electric field around the particle has been quantized by Eq. (28) mathematically. (2) if the quantum number $n = 0$, then $E_\theta = E_\phi = 0$, then Eq. (28) is the Ampere’s law for the electrostatic field. (3) if $n \neq 0$, then, $E_\theta \neq 0$ or $E_\phi \neq 0$, the electric field $\mathbf{E}$ is recognized as a higher order mode ($n$ denotes the mode rank) with
a mathematical singularity at the particle. (3) regarding the potential-wave-
function, the boundary conditions for the electric field are

\[
\frac{eT}{\hbar} \int_R^\infty E_r \cdot dr = 2\pi Q \quad Q = 0, \pm 1, \pm 2, \ldots \\
\frac{eT}{\hbar} \int_0^{2\pi} E_\theta \cdot r \sin \theta d\phi = 2\pi S_e \quad S_e = 0, \pm 1, \pm 2, \ldots
\]

(29)

(30)

(31)

Considering the half loop A-B-C in Figure 2, then Eq.(15) becomes

\[
\begin{align*}
-\frac{e}{\hbar} \int_0^R A_r \cdot dr &+ \frac{e}{2\hbar} \int_0^{2\pi} A_\theta \cdot r d\theta + \frac{e}{2\hbar} \int_0^{2\pi} A_\phi \cdot r \sin \theta d\phi \\
+ \frac{eT}{\hbar} \int_R^\infty E_r \cdot dr &+ \frac{eT}{2\hbar} \int_0^{2\pi} E_\theta \cdot r d\theta + \frac{eT}{2\hbar} \int_0^{2\pi} E_\phi \cdot r \sin \theta d\phi \\
&= 2\pi n \\
&\quad n = 0, \pm 1, \pm 2, \ldots
\end{align*}
\]

(32)

Substituting Eq.(12-15) and Eq.(29) into Eq.(32), we obtain

\[
Q_m + \frac{B_m + S_m}{2} + Q + \frac{B_e + S_e}{2} = \frac{n}{2} \quad n = 0, \pm 1, \pm 2, \ldots
\]

(33)

We immediately recognize that this equation is the Gell-Mann-Nishijima relation for hadrons. Therefore, the electromagnetic field mode around a particle has six quantum numbers: \((Q, B_e, S_e, Q_m, B_m, S_m)\), thus the independent quark type number relating to the six quantum numbers are actually six. Recalling up to now we have known that the quarks have six types \((u, d, s, c, b, t)\) and the leptons have six type too, and the electromagnetic field has six components, we realize that 6 is a magic number for physics: all physical quantities are sharing the 6 freedoms originating from the 6 components of electromagnetic field in explicit or implicit ways[4].

Because of the charge conservation, we deduce that these six quantum numbers are conserved, in agreement with the experiments of hadrons in conservation law number: six. Of cause, the details of the relationships concerning quarks seem to be complex and abstract.

8 Gell-Mann-Nishijima formula in hydrogen atom

Hydrogen atom consists of a single electron bound to its central nucleus, a single proton, by the attractive Coulomb force that acts between them. In the theory of relativistic quantum mechanics, the 4-vector momentum of an electron \(p\) is given by

\[
(p_\mu + qA_\mu)\psi = -i\hbar\partial_\mu \psi \quad \mu = 1, 2, 3, 4
\]

(34)

Where \(A\) is the 4-vector potential of the electromagnetic field, \(\psi\) is the wave-function of the electron. We have adopted momentum \(p\) rather than momentum
operator \( \hat{p} \) in Eq.(34) [4]. In our viewpoint, the wave-function \( \psi \) is the potential-wave-function of the atom, because

\[
\psi = \exp\left(\frac{i}{\hbar} \int_L (p_\mu + qA_\mu) \cdot dx_\mu \right)
\]

(35)

Where \( l \) is an arbitrary path from the observed point \( x \) to the infinity. In analogy with the electric field modes discussed in the preceding sections, for any closed loop \( L \) containing the nucleus in Figure 1, the wave function \( \psi \) for the electron in a hydrogen atom is quantized by

\[
\frac{1}{\hbar} \oint_L (p_\mu + qA_\mu) \cdot dx_\mu = 2\pi n \quad n = 0, \pm 1, \pm 2, ...
\]

(36)

The nucleus of the hydrogen atom provides the 4-vector potential for the electron:

\[
A_r = A_\theta = A_\phi = 0; \quad A_4 = \frac{ie}{cr}
\]

(37)

Where we take \( k_e = 1 \) in the Coulomb’s law: Gaussian units. Because the electron keeps the angular momentum conservation, the angular momentum magnitude \( J = r\sqrt{\hat{p}_r^2 + \hat{p}_\theta^2} \) is a constant and its z-axis component magnitude \( J_z = r \sin \theta \hat{p}_\phi \) is a constant too. The energy of the electron \( E \) also is a constant for its stationary state: \( \psi = \psi(r)e^{-iEt/\hbar} \), thus

\[
\exp(-iEt/\hbar) = \exp\left(\frac{i}{\hbar} \int_L (p_4 + qA_4) \cdot dx_4 \right)
\]

(38)

\[
p_4 = -\frac{E - e^2/r}{ic} \quad x_4 = ict
\]

(39)

From \( p_\mu p_\mu = -m_e^2c^2 \), where \( m_e \) is the mass of the electron, we obtain

\[
p_r = \sqrt{-m_e^2c^2 - p_\theta^2 - p_\phi^2} = \sqrt{-m_e^2c^2 - \frac{J^2}{r^2} + \frac{1}{c^2}(E + \frac{e^2}{r})^2}
\]

(40)

\[
p_\theta = \sqrt{(p_\theta^2 + p_\phi^2) - p_\phi^2} = \frac{1}{r} \sqrt{J^2 - \frac{J_z^2}{\sin^2 \theta}}
\]

(41)

Therefore, quantization Eq.(36) for the loop in Figure 2 becomes

\[
\frac{1}{k} \oint_L p_r \cdot dr + \frac{1}{k} \oint_L p_\theta \cdot rd\theta + \frac{1}{k} \oint_L p_\phi \cdot r \sin \theta d\phi = 2\pi n
\]

(42)

\[
n = 0, \pm 1, \pm 2, ...
\]

Where the loop integral about the time \( t \) component vanishes for the constant energy \( E \) in Eq.(38), while the loop becomes three quantization equations

\[
\frac{1}{\hbar} \int_0^\infty p_r \cdot dr = \pi f \quad f = 0, \pm 1, \pm 2, ...
\]

(43)

\[
\frac{1}{\hbar} \int_0^{2\pi} p_\theta \cdot rd\theta = 2\pi g \quad g = 0, \pm 1, \pm 2, ...
\]

(44)

\[
\frac{1}{\hbar} \int_0^{2\pi} p_\phi \cdot r \sin \theta d\phi = 2\pi m \quad m = 0, \pm 1, \pm 2, ...
\]

(45)
They also can be written as

\[
\frac{1}{\hbar} \int_0^\infty \sqrt{-m^2c^2 - \frac{J^2}{r^2} + \frac{1}{c^2} (E + \frac{e^2}{r})^2} \cdot dr = \pi f \quad f = 0, \pm 1, \pm 2, \ldots
\]

\[
\frac{1}{\hbar} \int_0^{2\pi} \sqrt{J^2 - \frac{J_z^2}{\sin^2 \theta}} \cdot d\theta = 2\pi g \quad g = 0, \pm 1, \pm 2, \ldots
\]

\[
\frac{1}{\hbar} \int_0^{2\pi} J_z d\phi = 2\pi m \quad m = 0, \pm 1, \pm 2, \ldots
\]

These definite integrals have been evaluated on the ranges: these integrands take real values in the previous paper, as the result, they are

\[
\frac{\pi E\alpha}{\sqrt{m^2c^4 - E^2}} - \pi \sqrt{(g + |m|)^2 - \alpha^2} = \pi f \quad f = 0, \pm 1, \pm 2, \ldots
\]

\[
\frac{2\pi}{\hbar} (J - |J_z|) = 2\pi g \quad g = 0, \pm 1, \pm 2, \ldots
\]

\[
\frac{2\pi}{\hbar} J_z = 2\pi m \quad m = 0, \pm 1, \pm 2, \ldots
\]

Where the \( \alpha = e^2/(\hbar c) \) is known as the fine structure constant. It is very easy to get the energy levels of the electron from the Eq. (49):

\[
E = mc^2 \left[ 1 + \frac{\alpha^2}{(\sqrt{(g + |m|)^2 - \alpha^2} + f)^2} \right]^{-\frac{1}{2}}
\]

This result, Eq. (52), is completely the same as the calculation of Dirac wave equation for hydrogen atom, it is just the fine structure of hydrogen atom energy.

Considering the half loop A-B-C in Figure 2 about the nucleus, then Eq. (42) becomes

\[
\frac{1}{k} \int_0^\infty p_r \cdot dr + \frac{1}{2k} \int_0^{2\pi} p_\theta \cdot r d\theta + \frac{1}{2k} \int_0^{2\pi} p_\phi \cdot r \sin \theta d\phi = \frac{2\pi n}{2}
\]

\[n = 0, \pm 1, \pm 2, \ldots\]

Substituting Eq. (43-45) into Eq. (53), we find that the Gell-Mann-Nishijima formula also exists in hydrogen atom:

\[
\frac{f}{2} + \frac{g + m}{2} = \frac{n}{2} \quad n = 0, \pm 1, \pm 2, \ldots
\]

In the stationary state spectrum of hydrogen atom, frequently \( f = 0 \). This Gell-Mann-Nishijima formula is the critical condition of the wave function modes around a hydrogen atom.

Among Eq. (43-45), it is unfair to \( \psi(r) \), \( \psi(\theta) \) and \( \psi(\phi) \), because we let \( \psi(r) \) form a standing wave (half integral multiple) from \( r = 0 \) to \( r = \infty \) while \( \psi(\theta) \) and \( \psi(\phi) \) are simple wave (integral multiple). As we have mentioned, we must fairly treat the three components in their conformal box (Figure 3). Therefore, Fairly, if we let they all are integral multiple, then their quantum numbers satisfy the standard Gell-Mann-Nishijima formula.
9 Visual Model of hadrons

Consider a baryon of charge number \( Q \), Baryon number \( B \) and strangeness number \( S \), its potential-wave-function can be separated into three parts as

\[
\xi(r) = \xi_1(r)\xi_2(\theta)\xi_3(\phi)
\]

Where \( r \) varies in the range \((0, \infty)\) with \( Q + 1 \) nodes, \( \theta \) varies in the range \((0, 2\pi)\) with \( 2B + 1 \) nodes, \( \phi \) varies in the range \((0, 2\pi)\) with \( 2S + 1 \) nodes, as shown in Figure 5, where only depicting each fragments for them. The picture in Figure 5 is the visual model for hadrons.

![Figure 5](image)

Figure 5: The visual model for hadrons is represented by three potential-wave-functions: \( \xi_1(r), \xi_2(\theta), \xi_3(\phi) \) and their nodes, where only depicting their fragments.

According to Eq. (26), the visual model for quarks can established from

\[
\begin{bmatrix}
u \\
d \\
s
\end{bmatrix} = \begin{bmatrix}
2/3 & -1/3 & -1/3 \\
1/3 & 1/3 & 1/3 \\
0 & 0 & -1
\end{bmatrix}^{-1} \begin{bmatrix}
Q \\
B \\
S
\end{bmatrix}
\]

10 Conclusions

When an electron probes another elementary particle \( Q \), the wave function of the electron can be separated into two independent parts, the first part represents the electronic motion, the second part represents the electromagnetic field mode around the particle \( Q \). In analogy with optical modes \( TEM_{nlm} \) for a laser resonator, when the electromagnetic field around the particle \( Q \) forms into a mode, the quantum numbers of the mode satisfy the Gell-Mann-Nishijima formula, these quantum numbers are recognized as the charge number, baryon number and strangeness number. As a result, it is found that the Gell-Mann-Nishijima formula is the critical condition of the electromagnetic field modes around a hadron. It is important to realize that the electromagnetic field modes around a hadron provide us a visual model to understand the abstract baryon number \( B \) and strangeness number \( S \) in reality.
References

[1] Nakano, Tadao; Nishijima, Kazuhiko. "Charge Independence for V-particles". Progress of Theoretical Physics 10 (5): 581-582. (1953).

[2] Nishijima, Kazuhiko. "Charge Independence Theory of V Particles". Progress of Theoretical Physics 13 (3): 285-304. (1955).

[3] Gell-Mann, Murray. "The Interpretation of the New Particles as Displaced Charged Multiplets". Il Nuovo Cimento 4: 848. (1956).

[4] Cui, H. Y. "Discovery of quantum hidden variable". arXiv:0711.1247 (2007).