Emergence of time in quantum gravity: 
is time necessarily flowing?

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Abstract

We discuss the emergence of time in quantum gravity, and ask whether time is always “something that flows”. We first recall that this is indeed the case in both relativity and quantum mechanics, although in very different manners: time flows geometrically in relativity (i.e. as a flow of proper time in the four dimensional space-time), time flows abstractly in quantum mechanics (i.e. as a flow in the space of observables of the system). We then ask the same question in quantum gravity, in the light of the thermal time hypothesis of Connes and Rovelli. The latter proposes to answer the question of time in quantum gravity (or at least one of its many aspects), by postulating that time is a state dependent notion. This means that one is able to make a notion of time-as-an-abstract-flow - that we call the thermal time - emerge from the knowledge of both:

- the algebra of observables of the physical system under investigation,
- a state of thermal equilibrium of this system.

Formally, this thermal time is similar to the abstract flow of time in quantum mechanics, but we show in various examples that it may have a concrete implementation either as a geometrical flow, or as a geometrical flow combined with a non-geometric action. This indicates that in quantum gravity, time may well still be “something that flows” at some abstract algebraic level, but this does not necessarily imply that time is always and only “something that flows” at the geometric level.

I Introduction

We question the notion of time-flow, and the way it emerges is some approaches to quantum gravity. In particular, we ask whether time is necessarily “something that flows”.

In a first part, we recall how, in relativity as well as in quantum mechanics, time is indeed “something that flows”, although the flows do not take place at the same levels: in relativity time is a geometrical flow in space-time whereas, in quantum mechanics, it is a flow in an abstract space, describing the algebraic structure of the theory. In quantum gravity, as shown in the second part, these two figures of temporality need to be reconciled. A way of reconciliation is offered by thermodynamics. This is the thermal time hypothesis of Connes and Rovelli [4], who propose to extract a time-as-geometrical-flow from a time-as-abstract-flow. Mathematics and theoretical physics provide a natural framework to test this idea. In the last part of this contribution, we thus investigate various physical situations in which the pertinence of the thermal time hypothesis is
verified. In the first two examples, the so-called Unruh effect and a variation on the Unruh effect for non-eternal observers, we find that the concrete physical implementation of the thermal time is a physically meaningful geometrical flow. We then exhibit a third example where the concrete realisation of the thermal time combines the previous geometric flow on space-time with a purely non-geometric action on the physical observables. In other terms, time is only partially flowing.

As a conclusion, we show that this multiplicity of modes of emergence of time suggests non-trivial answers to classical philosophical questions such as “is there something rather than nothing ?”, or to seemingly naive questions such as “is there more light at day or night ?”.

II Time-as-geometrical-flow vs. time-as-abstract-flow

II.1 The geometric time of relativity

In relativity, time-evolution has a clear geometrical interpretation: the movement of an observer, that is the evolution of its position as time passes, is described by a line of universe, namely a one-dimensional trajectory in a mathematical space-time of dimension four. The distinction between space and time, however, is not univocal. Each observer makes the separation of the four-dimensional space-time into a three-dimensional space and a one-dimensional time in his own way, depending on his state of movement. For instance in Minkowski space, that is the flat space-time of special relativity in the absence of gravity, the temporal evolution of a static observer is described by a line parallel to the T axis, and the surfaces of simultaneity (in a two dimensional space-time, see figure 1) are parallel to the X axis. Meanwhile, for an observer whose speed is constant with respect to the static observer, the temporal evolution as well as the surfaces of simultaneity are no longer parallel to any of the axes. For an observer with constant acceleration, the line of universe is an hyperboloid, and simultaneity surfaces are lines through the origin (see figure 2).

In other words, and unlike Newton mechanics, in relativity there is no absolute time, that is to say no global object which flows “everywhere in the same way”. Nevertheless, although the relativistic time is not everywhere the same, it is locally unique. Each observer along his line of universe does experiment a single time, his proper time. The proper time of a first observer may not be identical to the proper time of a second observer, but one knows (in principle at least, the explicit computations can be involved) how to go from one to the other.

Geometrically, the unicity of the proper time means that along each line of universe, at any point one has one and only one unit tangent vector (here “unit” means of length 1). In the absence of gravity, the switching of proper time between two observers of constant speed is done thanks to some geometrical transformations called Poincaré transformations. Consider for instance the lines of universe of two static observers (the two lines on the right of figure 1). A vector tangent to the first is mapped to a vector tangent to the second by the simplest Poincaré transformation, that is a translation. The same is true for two observers with the same constant speed (parallel lines on the left of figure 1). For observers with different - but constant - speeds (the two lines that intersect at the origin in figure 1), one uses another Poincaré transformation to map a vector tangent to the first curve to a vector tangent to the second one, namely a Lorentz boost (in essence: a rotation in the T,X plane).

To get an intuitive idea of what is going on, it could be useful to draw a simple analogy with the usual time measured by our clocks and watches on Earth. There is no absolute time on Earth: noon -bells do not ring simultaneously in Kathmandu or Haiti. But there is a universal time, divided in time-zones, and each observer knows how to regulate his clock according to it: noon-bells should ring when the Sun is at the highest in the sky of Kathmandu or Haiti. A traveller knows very well how to pass from one time-zone to another, by simply put one’s watch forward or back a few

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a The Unruh effect is defined later on the text. In short, it is the prediction that an accelerating observer holding a “quantum thermometer” that measures the temperature of the quantum vacuum would measure a non-zero temperature where an inertial observer would see a zero temperature.

b To avoid confusion, let us insist that the notion of time flow should not be understood as a property of a point, but as a property of a trajectory in space-time.: in figure 1, at any given point one can attach as many arrows as lines of universe that can go through this point: that is infinitely many!
Figure 1: *Separation of space and time in the 2-dimensional Minkowski space-time:* the $T$ axis is the line of universe of an observer who stays at the same place $X = 0$ at any time. In this space-time, being immobile at a point $K$ or $H$ in space corresponds to lines of universe parallel to this $T$ axis (on the right). On the left, the lines of universe of two observers with the same non-zero constant speed (parallel lines), and a third one with a lower speed. The dashed lines are the surfaces of simultaneity. The latter are parallel to the $X$ axis only for static observers. The arrows are the vectors tangent to the lines of universe. Each of them has length 1: the apparent difference between the length of the arrows is an effect of hyperbolic geometry (Minkowski space-time is not an Euclidean geometry).

hours. Putting forward or back are precisely translations. In the geometrical picture above, this means that the origin of time on the line of universe $X=Kathmandu$ is not on the same surface of simultaneity as the origin of time on the line of universe $X=Haiti$.

To illustrate why the translations do not cover all the possible cases and why other Poincaré transformations are required, let us imagine that on Earth one takes into account a second system of time-zones, for instance based on the Moon rather than the Sun. Inside this lunar time system, the lunar noon is defined as the moment of time where the Moon is highest in the sky. Each observer knows which lunar time-zone he is belonging to, and he knows the lunar-time difference between the lunar noon of Kathmandu and the lunar noon of Haiti. The change of lunar time-zone is not more difficult than the usual change of solar time-zone (in the same way as the mapping of tangent vectors between two same-speed lines of universe is a translation, whatever the speed). The difficulty arises when one intends to switch from a lunar time-zone to a solar time-zone (in the same way: a translation does not allow to map tangent vectors between two constant-speed lines of universe with different speed). An observer in the solar time who aims at regulating his clock on the lunar time can not simply put back or forward his clock. He needs to modify all the inside mechanism of his clock, for a lunar day cannot be divided into twenty four solar hours. Nevertheless this change of time system is possible, and well described by the formalism of relativity.

When gravity enters the game and curves the space-time, the multiplicity of time systems according to which one could regulate one’s clock might become even more involved (technically speaking, this means there may exist many *time-like Killing vector fields*). On Earth, this is as if one were considering not only the solar or lunar times, but imagine more systems, for instance based on Mars, Jupiter, Halley’s comet etc. Each system can equally be promoted as "the" time system, and the effective choice will be made on practical reasons: it is easy to indicate solar time
Figure 2: The Rindler wedge: the hyperboloids are the lines of universe of observers with constant acceleration, and the dashed lines are the surfaces of simultaneity. The two asymptotes to these trajectories are lines of universe of light. The wedge W encompassed by these two light-like trajectories is the region of space-time causally connected to the accelerated observers.

thanks to a sundial, it not easy to build a Jupiter-dial! The same is true in relativity: on a given spacetime, one will choose the coordinate system most appropriate to the system under study. But in any case, besides the diversity of the clocks used by different observers to measure their own proper time, all observers will agree: there is “something flowing”. Only the ways of flowing are different.

II.2 The algebraic time of quantum mechanics

In quantum mechanics, time is not a geometrical flow. Time-evolution is characterized as a transformation that preserves the algebraic relations between physical observables. If at a time $t = 0$ an observable - say the angular momentum $L(0)$ - is defined as a certain combination (product and sum) of some other observables - for instance positions $X(0)$, $Y(0)$ and momenta $P_X(0)$, $P_Y(0)$, that is to say

$$L(0) = X(0)P_Y(0) − Y(0)P_X(0), \tag{2.1}$$

then one asks that the same relation be satisfied at any other instant $t$ (preceding or following $t = 0$),

$$L(t) = X(t)P_Y(t) − Y(t)P_X(t). \tag{2.2}$$

The quantum time-evolution is thus a map from an observable at time 0 to an observable at time $t$ that preserves the algebraic form of the relation between observables. Technically speaking, one talks of an automorphism of the algebra of observables.

At first sight, this time-evolution has nothing to do with a flow. However there is still “something flowing”, although in an abstract mathematical space. Indeed, to any value of $t$ (here time is an absolute parameter, as in Newton mechanics) is associated an automorphism $\alpha_t$ that allows to
deduce the observables at time $t$ from the knowledge of the observables at time 0. Mathematically, one writes

$$L(t) = \alpha_t(L(0)), X(t) = \alpha_t(X(0))$$

and so on for the other observables.

The term “group” is important for it precisely explains why it still makes sense to talk about a flow. Group refers to the property of additivity of the evolution: going from $t$ to $t'$ is equivalent to going from $t$ to $t_1$, then from $t_1$ to $t'$. Considering small variations of time $\frac{t_n}{n}$, where $n$ is an integer, in the limit of large $n$ one finds that going from $t$ to $t'$ consists in flowing through $n$ small variations, exactly as the geometric flow consists in going from a point $x$ to a point $y$ through a great number of infinitesimal variations $\frac{x-y}{n}$. That is why the time-evolution in quantum mechanics can be seen as a “flow” in the (abstract) space of automorphisms of the algebra of observables.

To summarize, in quantum mechanics time is still “something that flows”, although in a less intuitive manner than in relativity. The idea of “flow of time” makes sense, as a flow in an abstract space rather than a geometrical flow.

### III The question of time in quantum gravity

As we have seen in the previous section, the time in relativity does not flow in the same manner as the time in quantum mechanics: the former is a geometrical flow in space-time, the latter is a 1-parameter group in the space of automorphisms of the quantum observables. Combining quantum physics with gravity thus inevitably yields to a conflict between these two “ways of flowing”.

The question of time in quantum gravity takes many aspects (see [6], [5]). Here, we will consider the problem of the unicity of the classical limit of the flow of time, when one passes from quantum gravity to general relativity. By classical, we intend the relativistic, non-quantum, limit of quantum gravity. In a rather unexpected way, the abstract nature of the flow of time in quantum mechanics - rather than being a problem - brings a solution.

#### III.1 Uniqueness of the classical limit

A founding principle of quantum mechanics may be summarized as “everything is possible, but with a certain probability”. The evolution of this probability is perfectly deterministic. Quantum uncertainties appear when one effectively measures a physical observable: the theory predicts the probability that such or such measure gives such or such result, but it does not predict the outcome of one single measure. Before the measurement process, a quantum system is thus described by a state which is the sum of all the possibles, each of it with a given probability. We speak of a statistical state.

The most famous example is the Schrödinger cat which, as long as no observer verifies his health, is described as a superposition of a dead cat and a living cat (this is called the principle of superposition of the wave packet).

In the same way, in quantum gravity one expects the gravitational field to be described by a superposition of all possibles. Since in general relativity the gravitational field determines the metric of space-time, and the latter indicates how space can be separated from time, on expects that at the quantum level any direction can be picked out as the direction of time. But in general relativity, time is locally unique (as explained in section II.1, an observer experiences one and only one proper time). How can one reconcile the freedom given by quantum physics in the choice of the time direction, with the unicity of the proper time, locally imposed by general relativity?

Coming back to the analogy with time on Earth, this is a bit as if, assuming the gravitational field in the Solar System is completely determined by the mass of the Sun, one considers a quantum state of the Solar system as a superposition of all the “would be” Solar Systems, corresponding to Suns with different masses. Then, a quantum-gravity clock should be described as a superposition of as many clocks as possible values of the Sun mass. Each of these clocks measures the “something flowing” discussed at the end of section II.1 (whose concrete realization as a proper time will depend

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*Let us warn the reader that this analogy, for what it be worth, only aims at giving a reader with no particular background in physics a “taste” of what is going on. By no mean it should be taken as a serious physical explanation. For instance we are mixing without any caution macroscopic and quantum effects, and from an astrophysical point of view it does not make much sense to imagine a quantum superposition of Suns with various masses!"
on the observer), corresponding to a value of the Sun mass. Locally, for an observer who has chosen once for all his time system - say the solar time - the quantum-gravity clock will take the form of a quantum sundial, that is a superposition of many sundials, each of them measuring the solar time corresponding to one of the possible values of the Sun mass. At the non-quantum limit, how does this quantum sundial collapse to the usual sundial?

Before examining a possible solution to this problem, let us emphasize that the problematic is, in a sense, opposite to the one that might have been expected at first sight: time in quantum mechanics is an absolute parameter, unique, whereas proper-times in general relativity are as many as observers. The principle of superposition of the wave packet, applied to the gravitational field, reverses the problem: quantum mechanics offers the multiplicity, while relativity, restricted to one single observer, asks for unicity.

III.2 The thermal time hypothesis

A solution to the uniqueness problem of the classical limit of the time flow has been proposed by Connes and Rovelli under the name of thermal time hypothesis [4] (see also [14] for a recent analysis). The idea is to extract from an equilibrium state the time-as-abstract-flow of quantum mechanics (i.e. a flow of automorphisms), then to turn it into the locally unique time-as-geometrical-flow of relativity. The almost miraculous aspect of this proposition is that it has an immediate translation in a branch of mathematics which has a priori nothing to do with the question, namely the modular group in the theory of von Neumann algebra. Quoting R. Haag [7], this is an example of the remarkable "pre-harmony between mathematics and physics".

To understand well the solution proposed by the thermal time hypothesis, it could be useful to make a short digression regarding the notion of state and equilibrium. Let us begin by emphasizing the difference between a physical system (e.g. the cat), and its states (e.g. a living cat, or a dead cat, or a superposition of both). In a similar way as, for Heidegger [5] it not so much the greek temple that is made of stone than the stone that comes to existence through the temple, a physical system comes to existence - in our case: become accessible to the experiment - through its various possible states. The Schrödinger cat by itself has no existence, what appears in the equations of quantum mechanics are the mathematical objects encoding its various possible states of existence (alive, dead, half alive/half dead). Describing a system by a state is typical of thermodynamics. Consider a gas in a box: one has no access to the movement of each atom of gas, however one is able to obtain relevant physical information by studying the statistical behaviour of these atoms. Said differently, one does not know the microscopic behavior of each particle (when will this precise particle hit the edge of the box ?), however it makes sense to define, at the macroscopic scale, the pressure of the gas on the edge of the box. Thus, there is a distinction between a macroscopic state - intended as a set of values of the thermodynamical quantities describing the system, like energy, temperature, pressure - and the microscopic states, intended as the values of the position and speed of all the atoms of the gas. An equilibrium state is a macroscopic state in which the value of the thermodynamical quantities are constant in time. This does not mean that the system is frozen: the atoms of gas do not stop moving, so that the microscopic state is changing. There is no contradiction, because to one macroscopic state correspond many microscopic states. For instance by reversing the speed of each of the atoms of gas, the values of the thermodynamical quantities is not not modified: one changes the microscopic state but not the macroscopic one.

With this definition of equilibrium state in minds, it is important to stress a tautology: equilibrium has sense only because there exists an a priori notion of time, according to which one checks that the measured values of the thermodynamical quantities are indeed constant. Time defines equilibrium. The thermal time hypothesis consists in reverting the proposition: from the notion of equilibrium, one extracts time. More exactly, starting from a physical system in a given state, one builds a time-flow such that the state one has started with is precisely an equilibrium state with respect to the equilibrium notion defined by this flow. There are two obvious difficulties:

1. first one needs to characterize the would-be equilibrium states, that is those states among all the states of a system for which there exists a time-flow making them equilibrium states;
2. second, for the thermal time hypothesis to be of any use, one should be able to extract explicitly the time-flow from the knowledge of a would-be equilibrium state.
The characterization of states that could be equilibrium states exists, and is mathematically formulated in terms of the so-called KMS conditions. These are mathematical relations involving the state under consideration, the algebra of observables of the system and a group of automorphisms $\alpha_t$ (see section II.2 for the definition of group of automorphisms). If the KMS conditions are satisfied, then the state has, with respect to $\alpha_t$ the same properties as an equilibrium state of a physical system whose quantum time-evolution would be given by the time-as-abstract-flow $\alpha_t$.

Mathematics also provide a way to solve the second difficulty: the modular theory of Connes, Tomita, Takesaki makes possible to extract from the knowledge of an algebra of observables and a state $\Omega$ (both satisfying technical requirements) a 1-parameter group of automorphisms $\alpha_t^\Omega$, called modular group, such that the state $\Omega$ is KMS with respect to $\alpha_t^\Omega$. This is not an easy piece of mathematics, so we shall not try to explain it here. Let us simply stress that the modular theory mainly deals with two simple but fundamental properties of the physical observables:

- one is the algebraic structure of observables, namely the fact that observables can be added and multiplied, this multiplication being noncommutative (an algebra of quantum observables is a noncommutative algebra: position $X$ multiplied by momentum $P$ is not the same as $P$ multiplied by $X$);
- the other is the fact that measured quantities are real numbers, opposed to complex numbers. This is not a trivial requirement, since in quantum mechanics complex numbers play an important role, but they do not come as the possible value of the measurement of a physical quantity (technically speaking: a quantum observable is a selfadjoint operator). The mathematical translation of this requirement is that the algebra of observables comes equipped with a tool allowing to separate real quantities from complex ones, called an involution (for instance the involution on the set of complex numbers is the usual complex conjugation: a real number is fully characterized as a complex number which is equal to its conjugate).

To summarize, assuming one has at his disposal a notion of equilibrium state, the modular theory allows to extract from the algebra of observables of a system a group of automorphisms that has all the properties one could expect from a bona fide time-as-abstract-flow. Furthermore, this flow is unique: a state is a state of equilibrium only with respect to one modular flow. We shall call this time-as-abstract-flow the thermodynamical time, or thermal time for short. To fully realize the program announced at the beginning of this section, it remains to turn the abstract flow of thermal time into a geometrical flow. This is the question we examine in the next section.

### IV Figures of thermal time

For clarity let us repeat how the thermal time hypothesis proposes to solve the problem of the uniqueness of the classical limit of time-flow discussed above. If one knows the algebra of observables of quantum gravity, the thermal time hypothesis allows to associate to an equilibrium quantum state of the gravitational field a unique thermodynamical time. Said differently, thanks to thermodynamics, one is able to reduce the multiplicity of choice of time-flows at the quantum level (recall that a quantum state of the gravitational field is a superposition of many possible choices of time-flow) to one single thermal-time flow. The bet then consists in claiming that this thermal-time flow coincides with a geometrical flow of proper time. The claim immediately raises (at least !) three questions:

1. what is the algebra of observables of quantum gravity ?
2. the thermal-time flow is an abstract flow of automorphisms. How can one retrieve from it a geometrical flow of proper time ?
3. in what sense is the thermal time unique ? What happens if one starts with another equilibrium state ?

#### IV.1 From an abstract flow to a geometrical flow

Let us examine the first points. For the moment, none of the tentative theories of quantum gravity (e.g. loop quantum gravity, string theory) clearly proposes a definition of the algebra of observables. Thus one cannot test the pertinence of the thermal time hypothesis in this context.
However one can check that, in physical situations where the modular group (discussed in section III.2) is known, then its interpretation as a thermal-time makes sense.

The most famous example where this actually happens is the so called Unruh effect. The later states that in Minkowski space-time, the quantum vacuum is not seen as void by a uniformly accelerated observer. It is seen has an equilibrium state with temperature $T$ proportional to the acceleration. This means the following: assume Minkowski space-time is filed with a quantum field, for instance an electromagnetic field. For a static observer, the field is in its vacuum state, meaning the observer does not see any photons (we do not take into account the fluctuations of the vacuum). For an accelerated observer on the contrary, the field is an equilibrium state, that is the observer sees photons, whose energy is similar as if the observer was at rest, but immersed in a thermal bath of photons at temperature $T$.

Let us now interpret the Unruh effect at the light of the thermal time hypothesis: because no signal can go faster than light, a uniformly accelerated observer may exchange information, that is send a signal and receive an answer, only with a “receptor” located into the region enclosed by the two light-lines asymptotic to its trajectory (see figure 2). This region is called a Rindler wedge, and is traditionally denoted $W$. This region is said causally connected to the uniformly accelerated observer. One then considers the set of observables localized into $W$. Their precise definition is given in algebraic quantum field theory. For our purposes, it is enough to know that such localized observables form an algebra, denoted $\mathcal{A}(W)$, and represent the amount of observable information accessible to the uniformly accelerated observer. The modular group defined by $\mathcal{A}(W)$ and the vacuum state $\Omega$ of the quantum field theory has been computed in [1]. By definition, this is a group of automorphisms of $\mathcal{A}(W)$, but it turns out that this group also has a geometrical action. And this action precisely coincides with the flow of proper time of an accelerated observer. Therefore the abstract flow of thermal-time coincides with the geometrical flow of proper time of a uniformly accelerated observer.

A similar analysis can be done for other regions of Minkowski space-time, like double-cones (figure 3). These are regions of particular interest since in the analysis of the Unruh effect above, we kept silent a important point: the Rindler wedge is the region causally connected to a uniformly accelerated observer whose lifetime is infinite. Indeed, the line of universe depicted in figure 2 extends from $-\infty$ (where it is asymptotic to a light ray propagating towards negative $X$) to $+\infty$ (idem, with light propagating towards positive $X$). The region causally connected to a uniformly accelerated observer with finite lifetime is no longer a Rindler wedge but the double-cone obtained as the intersection of the future cone of the birth of the observer, with the past cone of its death (see figure 4). As in the wedge case, one finds that the modular flow of the algebra of observables localized inside the double-cone has a geometrical action, which coincides with the trajectory of a uniformly accelerated observer being born and dying at the tips of the cone.

The different with the wedge case is that the temperature not only depends on the acceleration, but also on the lifetime of the observer. Since this dependence on causality has been investigated since [10, 11]: since the temperature depends on the lifetime, can an observer deduce the instant of its death from a measurement of the temperature ? The answer is no: causality is protected by the fourth Heisenberg uncertainty relation $\Delta E \Delta t \geq \hbar$, where $E$ is the energy, $t$ the time and $\hbar$ the Planck constant. Indeed, the difference in temperature between the eternal and non-eternal cases is smaller than the smallest temperature $\frac{\hbar}{E}$ that an observer with lifetime $\Delta t$ can measure ($k$ is the Boltzmann constant). In other term, a non-eternal observer does not live long enough to realize he is not eternal.

Both the examples of the wedge and the double-cone illustrate how the thermal time hypothesis can reconcile the time-as-abstract flow of quantum mechanics with the time-as-geometrical flow of relativity discussed in section II. The abstract flow of automorphisms and the geometrical flow...
in space-time are two ways of flowing that are not contradictory. They are two figures of the same “time”, like the proper times of different observers are various figures of a same “geometrical time”. However one should not believe that this situation is generic.

**IV.2 Is time necessarily flowing?**

Nothing in the formalism of the modular group guarantees that the flow of thermal time should admit a geometrical representation. Even in this case, nothing guarantees that this flow can be identified with a flow of proper time (for instance a space-like flow would correspond to an observer moving faster than light, hence non-physical).

Recently an example has been worked out in which the flow of thermal time is not purely geometric [9, 2]. One considers again a double-cone region in a two-dimensional Minkowski space-time, but the quantum field theory is assumed to satisfy particular symmetry conditions (conformal invariance and specific boundary conditions). As a consequence, any observable localized into the double-cone is completely determined by its value on two intervals on the axis (see figure 4).

There exists a certain ad-hoc quantum state (not the vacuum however), whose associated thermal time has a representation in terms of a geometrical flow of proper time. Notice that this flow is not the flow of proper time of a uniformly accelerated observer, but of an observer whose acceleration is not constant, and even change sign (see fig. 4).

Let us consider now the thermal time defined by the vacuum state. One finds that the modular flow of the vacuum, as flow of automorphisms, combine the above geometric action with another non-geometric action, namely an automorphism whose action amounts to mix the fields in the two intervals $I_1, I_2$. As explained in the footnote e p.8, an automorphism - being an object living at the level of the algebra of observables of a system - does not need to admit a geometrical representation. The mixing term of the modular group of the vacuum state is precisely one of these “purely algebraic” automorphisms. The thermal time is thus flowing as an abstract flow, but its concrete realization is no longer simply a geometrical-flow: time is simultaneously “something that flows geometrically” and “something that mixes the components” in a non -geometrical way.

This is a good place to address point 3 in the list of questions made at the beginning of this section. To what extent are these two thermal times - the purely geometrical one defined by the

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1In mathematics, a conformal map is a function which preserves angles.
Figure 4: Double-cone in a bi-dimensional boundary conformal field theory: the value of the field inside the double-cone is completely determined by its value on the two intervals $I_1, I_2$. The curve represents the line of universe associated to the thermal time generated by a certain state (Longo ad-hoc state) of the algebra of observables of the double-cone. This thermal-time is the proper time of an observer with non-constant acceleration. This can be seen by noticing that at various points along the curve, the tangent vector is vertical, meaning that the speed changes signs several times (compare to the double-cone in Minkowski space-time of figure 2). Notice that the line of universe is in reality contained within the double-cone: in the figure, the oscillation of the curve around the diagonal of the double-cone (dashed line) has been artificially exaggerated, so that to make the effect visible on the plot.

ad-hoc state, and the partially geometric one defined by the vacuum - two figures of a still unique “abstract time”? In a seminal result of operator algebra [3], Connes has shown that the modular flows defined by different states are equal up to inner automorphism. Without entering the details, this means that the two thermal times are not “so much different”, they only differ by a unitary transformation. In other terms, given an algebra of observables, up to these inner automorphisms, it makes sense to talk about unique thermal time.

V Conclusion: is there more light at noon or midnight?

To conclude, let us illustrate these various figures of time, not always easy to deal with, coming back to the analogy with the usual time on Earth. The Unruh effect described above can be interpreted as follows: to the question

Why is there something rather than nothing?

the accelerated observer (who sees the quantum vacuum as populated with quanta at temperature $T$, e.g. photons) will answer to his inertial partner (who sees the vacuum as void): “there is something for me because I am accelerating, and there is nothing for you because you are inertial”. It may sound strange that the answer to the question
Is there anything rather than nothing?

depends on the acceleration and thus, since an acceleration characterizes a proper time, depends on the temporal framework in which the question is asked. In terms of time-zones and time-systems, this phenomenon has in fact nothing mysterious: to the question

Is there more light at day or night?

an observer whose clock is regulated on the solar-time will answer “noon”. Indeed, in Kathmandu like in Haiti, there is more light at day than at night. On the other hand, an observer regulated on the lunar-time will answer: “it depends”. His lunar-day (that is the moment of time when the Moon is visible in its sky) may correspond to a solar day (when the Moon is visible by day), but the lunar-day may also corresponds to a solar-night (when the Moon is visible at night). In this case, there is more light at lunar-night than at lunar-day.

Quantizing gravity means loosing the possibility to associate to an observer a unique time-system. Each observer may happen to be regulated on whatever time-system (solar, lunar, jupitarian etc). To the question “when is there more light?”, the same observer may answer “at day”, and one understands that his quantum superposition of clocks has just collapsed to a clock regulated on solar-time, or he may answer “I do not know, your question has no meaning”, and one understands that his quantum clock has collapsed to a clock regulated on e.g. the moon-time.

However, if one forgets the quantum aspect of gravity, this observer lives according to his unique proper time, which is regulated on a a precise time-system. The question of the unicity of the classical limit thus consists in reconciling the superposition of all possible regulations - at the quantum level - with the uniqueness of the local time-system.

Connes and Rovelli propose that time-flow is an emergent notion, stemming from the algebraic structure of the observables, together with a state of the physical system under consideration. Rather than determining the state of the physical system using the point of view of one precise observer, attached to an a priori given time-system (You, whose clock is solar, do you see more light at night or day?), one considers that the state of the system induces a particular time-flow: the state “day has more light than night” implies solar time; the state “sometimes days has more light than night, sometimes no” implies lunar time.

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