The adaptive algorithm of change of structural redundancy network information systems on the basis of the spectral theories of graphs

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Abstract. Operation is devoted obtaining, on the basis of the spectral theory of graphs, the adaptive algorithm of conversion of basic structural models of network information systems for synthesis of topology with the minimum redundancy meeting requirements of structural stability by the given criteria. On the basis of switchings of Zajdel the method providing on each step of operation of cycles of algorithm a finding cospectral structures of high stability is developed. During researches concepts cospectral structures in narrow and wide sense are formulated. Modeling of operation of algorithm and an estimation of information efficiency cospectral network information systems on the basis of the generalized index – performance coefficient of information transfer taking into account influence of noises is led. Results of modeling confirmed a realizability and serviceability of the received algorithm, and also allowed to generate practical recommendations about its application in a structural circuit of complex multiple loop adaptation.

1. Introduction

The analysis of approaches to implementation of structure analysis and synthesis of network information systems (NIS) with demanded topological characteristics shows that in the standard setting the decision of the given tasks, presented the generalized type in operation [1], is based on usage of methods of the theory of graphs and topology. However application of known methods does not allow to receive the strict analytical description of all main structural parameters NIS, and also to define on their basis criteria of topological synthesis of structures with demanded characteristics. Thereupon in the given operation the nonconventional approach to the effective decision of tasks of structure analysis and synthesis NIS with usage of the spectral theory of graphs [2, 3] is offered. The spectral method of the analysis graphs models NIS provides the strict analytical description of the main structural parameters, allows to generate physically explainable criteria of structural synthesis on their basis, and also to receive cospectral topology NIS (with an identical spectrum and structural characteristics) high structural stability and minimum structural redundancy with usage of serial conversions of basic structure of system [4]. The given process can be presented in the form of iterated algorithm which allows to receive topology NIS simultaneously meeting requirements on structural redundancy and stability for finite number of steps.

The purpose of the given operation consists in determination of the main structural parameters and criteria of synthesis structurally steady NIS, and also in development on their basis of the adaptive
algorithm of conversion of the elementary basic topology in cospectral structures of minimum redundancy and high structural stability.

2. Materials and methods
The initial data for structure analysis and synthesis NIS are: an amount of switching nodes (SN) \( N = N_{SN} \) and communication channels (CC) \( M \); basic structural model NIS in a type of the graph \( G(M,N) \) and its matrix of a contiguity \( A \). Using the theorem of Hamilton-Kelli the characteristic equation for a contiguity matrix \( A \) registers expression

\[
P_G(\lambda) = |I - A| = 0,
\]

which decision defines an ordinary spectrum graphs model NIS

\[
Sp(G) = [\lambda_1, \lambda_2, ..., \lambda_N].
\]

where \( \lambda \) – coefficient, \( I \) – identity matrix, \( \lambda_1, \lambda_2, ..., \lambda_N \) – own values of a spectrum.

For each own value of a spectrum there is own vector \( x_1, x_2, ..., x_N \) [3, 4], and the system of own vectors \( X \) and an ordinary spectrum unambiguously describe any graphs model NIS since define a contiguity matrix

\[
A = XX^{-1}.
\]

Using an ordinary spectrum graphs models NIS, it is possible to find following major structural parameters:

- diameter of structure NIS

\[
d \leq m - 1;
\]

- the generalized structural index

\[
OCII = \frac{d}{M} = 0.75d / M;
\]

- an index (a regular level) structures

\[
r = \bar{r} = \frac{1}{N} \sum_{i=1}^{N} \lambda_i^2;
\]

- an inequality of Chiger, defining presence of “a narrow throat” in structure NIS

\[
\frac{1}{2}(r - \lambda_2) \leq h(G) \leq \sqrt{2r(r - \lambda_2)};
\]
- number of internal stability

\[ \alpha(G) \leq p_0 + \min(p_-, p_+); \tag{8} \]

- number of routes of length \( k \)

\[ N_k = \sum_{i=1}^{N} X_i \lambda_i^k, \tag{9} \]

where \( m \) – an amount of various own values of a spectrum, \( \frac{d}{2} \leq \bar{d} \leq d \) – average diameter NIS, \( h(G) \) – a constant of Chiger, \( p_0, p_-, p_+ \) – amounts of own values of the spectrum equal to zero, smaller or big zero, accordingly, \( X_v = \sum_{i=1}^{N} x_{iv} \) – own vectors.

On the basis of the received parameters criteria of synthesis structurally steady NIS can be formulated: \( d \to \min \) and \( OCII \to \min \), providing maximization of transmission capacity taking into account a considerable quantity of alternative ways with the minimum length; \( r(\bar{f}) \to \max \) and \( h(G) \to \max \) – the maximum connectivity and absence of “a narrow throat”; \( \alpha(G) \to \min \) and \( N_k \to \max \) at \( k \to \min \) – minimization SN with a small amount of communications and maximization of routes of the minimum length [4, 5].

For finding of the remained structural parameters characterizing stability NIS, the matrix of Kirhgo \( C = D - A \) ( \( D \) – a matrix of valences SN) which on the basis of the equation decision \( C_{\varepsilon}(\lambda) = |I - C| = 0 \), allows to define the C-spectrum of Fidler

\[ \mathcal{S}_C(G) = [\lambda_1, \lambda_2, \ldots, \lambda_N]_C. \tag{10} \]

The C-spectrum allows to find:

- number of spanning trees of model NIS, through coefficients of a characteristic C-multinomial or own values of the C-spectrum, accordingly

\[ t(G) = \frac{1}{N} (-1)^{N-1} c_{N-1}, \quad r(G) = \frac{1}{N} \prod_{i=1}^{N-1} \lambda_i; \tag{11} \]

- k-connectivity graphs models NIS

\[ a(G) = [\lambda_{N-1}]_C, \tag{12} \]

Thus criteria of synthesis structurally steady NIS are: \( a(G) \to \max \) the maximum connectivity SN at maximization \( r(G) \to \max \) – numbers it is linear independent ways, that is not intersected routes of information transfer on the column [4, 5].
Generally, at the decision of the task of synthesis structurally steady NIS, as the initial data demanded values (intervals) presented above the main structural parameters (4) – (12), taking into account requirements to an information exchange in a type are set: a range of change of the input traffic $\Delta T_{in}$, an admissible temporal time delay of packets $T_{adm}$, transmission capacities $CC_{chan}$. Besides, on the basis of the given requirements to structural stability and efficiency of an information exchange basic structures NIS are defined. In operation [5] three main variants of the elementary basic topology NIS (as an example for $N_{SN} = 8$): complete connection, possessing the maximum structural redundancy – for the high input traffic (figure 1a); "star" (without structural redundancy) – for the feeble traffic (figure 1b); cellular, hierarchical structures or their combination (with an average level of structural redundancy) – at the moderate input traffic (figure 1c).

Figure 1. Basic elementary structures NIS.

With usage of mathematical apparatus of the spectral theory of graphs in operation the adaptive algorithm of conversion of basic structures NIS is offered. It includes all possible variants of change of structural redundancy and has iterative character, providing finding of quasioptimal values of structural elements NIS $M$ (or $N_{SN}$ if it is possible, proceeding from operating conditions NIS) for performance of the given requirements of structural stability (figure 2).

In the unit 1 formation of basic model NIS and input of requirements to structural parameters – criteria of structural synthesis is carried out. Then operation is entered at first by a cycle of iterations (the unit 2) lowerings of structural redundancy of model NIS on number $CC$ in a limit to value $M_{min}$ which is defined by a condition of support of minimum connectivity (absence isolated SN). Calculations of an ordinary spectrum of the graph $Sp(G)$ and the C-spectrum $Sp_C(G)$ (the unit 3) on which basis in the unit 4 the main structural indexes NIS (4) – (12) are defined are produced ($Sp(G)$, $Sp_C(G)$). Further in the unit 5 check of satisfaction of requirements of structural stability, by the criteria defined above is carried out. If conditions are fulfilled, algorithm operation proceeds, for received model NIS are cospectral topology (the unit 6) and the following iteration on lowering of structural redundancy of basic model NIS is implemented.

Otherwise, passage (it is designated by a letter "And" on figure 2) on a branch of algorithm of magnification of structural redundancy (units 7 – 11) is produced. Thus operation of a cycle of iterations (the unit 7) provides serial increase of structural redundancy of model NIS on number $CC$ to the value $M_{max}$ corresponding complete connection topology. To similarly unit 5, on each iteration of a cycle 7 in the unit 10 performance of requirements of structural stability by the given criteria is checked. If conditions in the unit 10 are fulfilled, in the unit 11 (it is similar to the unit 6) are defined cospectral topology NIS. Then algorithm operation is completed, and results (from units of cycles of
iterations 2 or 7) in the form of variants cospectral structures NIS with the minimum redundancy, providing requirements of structural stability, are produced on level of an estimation of information efficiency NIS. For topology on figure 1a and 1b, accordingly, the presented algorithm is degenerated by possessing maximum and minimum structural redundancy in the simplified variant, that is implemented only on right (for complete connection NIS) or the left branch (for structure “star”), therefore is general-purpose.

![Diagram](image)

**Figure 2.** Adaptive algorithm of transformation of base structure NIS.

Separately we consider operation of the unit 6 which implements a finding method cospectral structures of high stability with usage of matrix switchings of Zajdel [1, 2, 6, 7]. For this purpose with usage of a matrix of a contiguity of Zajdel \( S = J - 2A - I \) (where \( J \) – the square matrix of dimensionality \( N_{SN} \times N_{SN} \), value of which all elements are equal 1) the decision of the characteristic equation \( S_G(\lambda) = |J - S| = 0 \) allows to define a S-spectrum of the graph
With application of a matrix of switchings \( U \) (the diagonal matrix containing values \( \pm 1 \)), the switching of Zajdel consisting in a finding of a “new” S-matrix cospectral the column under the formula is carried out

\[
S_1 = U \cdot S \cdot U .
\]

For the received matrix \( S_1 \) by the technique presented above the S-spectrum of the new graph \( G_1 \) which coincides with a spectrum of the initial graph \( G \)

\[
Sp_{S_1}(G_1) = \{ S_{G_1}(\lambda) \} = \{ \lambda_1, ..., \lambda_{N_{xx}} \} .
\]

Applying the similar approach, Zajdel was restricted to reviewing of strictly regular graphs and proved them cospectral on S-spectrums. However, as show the conducted researches, all columns without dependence from properties of a regular as a result of such conversion appear S-cospectral, that is cospectral in narrow sense [2, 5, 6]. Thus, with usage of expression for a S-matrix “the normal” matrix of a contiguity \( A_i \) of the new graph has been received \( G_1 \)

\[
A_i = \frac{J - I - S_1}{2} ,
\]

and with its help the ordinary spectrum of the graph is defined also \( Sp(G_1) \).

As a result of researches it has been installed that graphs models NIS having deviations of a level (a regular index \( r \)) peaks more than two, have cospectral no graphs with an ordinary spectrum and the S-spectrum, accordingly. On the contrary, graphs models NIS with the regular structure, with a deviation of an index of a regular \( r \pm 1 \) and \( r \pm 2 \), have cospectral columns with ordinary \( Sp(G_1) \) and the S-spectrum \( Sp_{G_1}(G_1) \), and, hence, possess identical structural parameters (4) – (12), and also characteristics in sense of stability and support of requirements of an effective information exchange in system. There fore given graphs models NIS have been defined, how cospectral in a broad sense. In the course of correction of basic structures it is necessary to be guided by the topological conversions leading to obtaining close to regular models NIS which possess a row of essential advantages [2 – 7]. It is proved that amounts of components \( +1 \) and \( -1 \) on a principal diagonal of a matrix of switchings \( U \), should correspond to an amount of coefficients of a characteristic polynom \( S_{G}(\lambda) \) with appropriate signs. Thus signs on components can change on opposite, but their ratio should correspond strictly to a ratio between the negative and positive coefficients of a characteristic polynom \( S_{G}(\lambda) \), and the amount cospectral in a broad sense graphs models NIS is restricted to number of possible combinations of layout of components \( +1 \) and \( -1 \) on a diagonal of a matrix of switchings \( U \). Thus, the presented method to the full allows to implement determination procedure cospectral topology (units 6 and 11) with current structural characteristics in the course of operation of cycles of iterations of the adaptive algorithm of conversion of basic structural model NIS (figure 2).
3. Results and discussion

In the environment of Maple 15 the analytical model implementing the right branch of received algorithm on lowering of structural redundancy basic complete connection NIS and synthesis of structurally steady topology with minimum structural redundancy, the given criteria providing performance is developed. For the synthesized topological models with usage of the developed method on the basis of switchings of Zajdel are found alternative (cospectral) topology with identical structural characteristics. So for researched variants of conversion of basic structure NIS (figure 1a), containing eight SN have been received two pairs cospectral in a broad sense topology with k-connectivity: $a(G) = 3$ – figure 3a, $a(G) = 5$ – figure 3b.

On the basis of system of the generalized indexes of information efficiency NIS [5] the estimation of efficiency of an information exchange is produced for the received variants cospectral structurally steady models NIS, with usage: tensor orthogonal model [8] and the standard approach on models systems of mass service (SMS) [9, 10]. Following the results of estimation from the received variants cospectral NIS – performance coefficient (efficiency) of information transfer taking into account influence of noises the structure providing the greatest values of an index is selected $\eta_{noi}$ from the given range of an input information highway $\gamma_{in}$ taking into account influence of destabilizing factors [11]. On figure 4 dependences of functions of information efficiency $E_{NIS} = \eta_{noi} (\gamma_{in})$ for the first variant of structure NIS $a(G) = 3$ (CC 3, the graph on top on figure 3a) and its alternative (cospectral) analog (CC 3 *, the graph from below on figure 3a), and also – for structure $a(G) = 5$ (CC 5, the graph on top on figure 3b) and cospectral topology NIS (CC 5 *, the graph from below on figure 3b).

![Figure 3](image-url)

**Figure 3.** Cospectral structures NIS.
The analysis of the received schedules shows that originally received as a result of iterations of algorithm of structural conversions (figure 2) topology NIS for researched conditions of an information exchange provides great values \( \eta_{noi} \) and that the most important thing, support it at more high level at implementation of procedures of structural adaptation of system (concerning set as criterion of information efficiency of trigger level of efficiency of information transfer \( \eta_{trig} \)). It is illustrated on figure 4 coordinates of a point of passage \((\gamma_{in\ pas}^*, \eta_{pas}^*)\) which appears above the result received at an alternative structural variant \((\gamma_{in\ pas}^*, \eta_{pas}^*)\), and can be used in operation of a structural circuit of complex multiple loop adaptation to changing conditions of an information exchange and to destabilizing actions [11].

![Graph](image.png)

**Figure 4.** Estimation of information efficiency cospectral models NIS.

### 4. Conclusion

Thus, in operation the mathematical apparatus of the spectral theory of graphs is applied to determination of main structural parameters NIS and criteria of structural synthesis of steady topology. The adaptive algorithm of conversion of basic structures NIS at which implementation the finding method cospectral structures taking into account the given requirements to topological stability is received is developed. During researches concepts cospectral structures in narrow and wide sense are formulated. Modeling of operation of the synthesized adaptive algorithm and an estimation of information efficiency of received structural models NIS on the basis of the generalized index – efficiency of information transfer taking into account influence of noises is produced \( \eta_{noi} \). In whole, in the course of the led researches the finding task cospectral steady structural models NIS with minimum admissible redundancy for the given requirements is solved. The iterated principle of operation of the adaptive algorithm allows to receive in real time some alternative variants cospectral network models NIS of the raised structural stability which estimations after implementation of information efficiency can be used for formation of a database of reserve topology NIS in operation of
a structural circuit of complex multiple loop adaptation to changing conditions of an information exchange and to influence of destabilizing factors.

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