Towards accurate modelling of the integrated Sachs–Wolfe effect: the non-linear contribution

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ABSTRACT

In a universe with a cosmological constant, the large-scale gravitational potential varies in time and this is, in principle, observable. Using an N-body simulation of a Λ cold dark matter universe, we show that linear theory is not sufficiently accurate to predict the power spectrum of the time derivative, Φ, needed to compute the imprint of large-scale structure on the cosmic microwave background (CMB). The linear part of the Φ power spectrum [the integrated Sachs–Wolfe effect (ISW)] drops quickly as the relative importance of ΩΛ diminishes at high redshift, while the non-linear part [the Rees–Sciama effect (RS)] evolves more slowly with redshift. Therefore, the deviation of the total power spectrum from linear theory occurs at larger scales at higher redshifts. The deviation occurs at k ∼ 0.1 h Mpc⁻¹ at z = 0. The cross-correlation power spectrum of the density δ with Φ behaves differently from the power spectrum of Φ. First, the deviation from linear theory occurs at smaller scales (k ∼ 1 h Mpc⁻¹ at z = 0). Secondly, the correlation becomes negative when the non-linear effect dominates. For the cross-correlation power spectrum of galaxy samples with the CMB, the non-linear effect becomes significant at l ∼ 500 and rapidly makes the cross-power spectrum negative. For high-redshift samples, the cross-correlation is expected to be suppressed by 5–10 per cent on arcminute scales. The RS effect makes a negligible contribution to the large-scale ISW cross-correlation measurement. However, on arcminute scales it will contaminate the expected cross-correlation signal induced by the Sunyaev–Zel’dovich effect.

Key words: cosmology: theory – large-scale structure of Universe.

1 INTRODUCTION

The most intriguing topic in contemporary cosmology is the nature of the dark energy, which appears to dominate the energy density of the Universe at late times. Strong evidence for the existence of dark energy comes from both the combined analysis of the cosmic microwave background (CMB) radiation and the galaxy large-scale structure (LSS; e.g. Efstathiou et al. 2002; Spergel et al. 2003), and from high-redshift Type Ia supernovae (e.g. Riess et al. 1998; Perlmutter et al. 1999). Both of these techniques infer the presence of dark energy from geometrical measures. A complementary probe of dark energy is provided by techniques that measure the dynamical effect of dark energy through its influence on the rate of growth of structure. Large deep galaxy redshift surveys [like the EUCLID, the ESA Mission to Map the Dark Universe and the Joint Dark Energy Mission (JDEM)] are being planned that will exploit the redshift space anisotropy of galaxy clustering, caused by coherent flows into overdense regions and outflows from underdense regions, to measure directly the growth rate as a function of redshift.

The integrated Sachs–Wolfe (ISW) effect (Sachs & Wolfe 1967), in which the decay of the large-scale potential fluctuations induces CMB temperature perturbations, provides another measure of the dynamical effect of dark energy. In principle, the ISW effect could be detected directly in the CMB power spectrum at very low multipole. In the Λ cold dark matter (ΛCDM) cosmology, it would boost the plateau in the power spectrum at l ∼ 10. However, as the increase of the power is not large in comparison to the cosmic variance, it cannot be unambiguously detected even in the Wilkinson Microwave Anisotropy Probe (WMAP) data (Hinshaw et al. 2009). A more sensitive technique is to search for the ISW signal in the cross-correlation of the LSS with the CMB. As the expected signal is weak and occurs on large scales, a very large galaxy survey is needed to trace the LSS. Currently, individual detections based on surveys such as the Automatic Plate Measuring machine (APM), the Two-Micron All-Sky Survey (2MASS), the NRAO VLA Sky Survey (NVSS) and the Sloan Digital Sky Survey (SDSS) (e.g. Fosalba, Gaztaña & Castander 2003; Afshordi, Loh & Strauss 2004; Fosalba & Gaztaña 2004; Padmanabhan et al. 2005; Cabré et al. 2006; McEwen et al. 2007; Rassat et al. 2007; Racanelli et al. 2008) are not of very high statistical significance (but see Granett, Neyrinck & Szapudi 2008). There are also analyses of the ISW

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cross-correlation, which combine multiple galaxy survey samples and achieve \( \sim 4\sigma \) detection of the ISW effect (Gianantonio et al. 2008; Ho et al. 2008). These measurements may be the best that can be obtained before the next generation of surveys (BOSS,\(^1\) Pan-STARRS1\(^2\)) come to fruition and make redshift tomography possible. If such surveys are to place robust, meaningful constraints on the properties of the dark energy, it is important to take full account of other processes beyond the (linear) ISW effect that may contribute to the cross-correlation signal. Here, we focus on deviations caused by non-linear gravitational evolution, the Rees–Sciama (RS) effect (Rees & Sciama 1968).

Other processes are known to contribute to the cross-correlation signal. First, the thermal Sunyaev–Zel’dovich (SZ) effect (Sunyaev & Zeldovich 1972) caused by hot ionized gas in galaxy clusters induces an anticross-correlation signal which can cancel the ISW effect on small scales. Its statistical contribution can be modelled and subtracted given the value of \( \sigma_8 \) (the rms linear mass fluctuations within a sphere of 8 h\(^{-1}\) Mpc) which determines the abundance of galaxy clusters (e.g. White, Efstathiou & Frenk 1993; Fan & Chiueh 2001; Mei & Bartlett 2004). Also, since the thermal SZ effect is frequency-dependent, it can be subtracted in frequency space given sufficient spectral coverage. Secondly, the redshift dependence of galaxy bias, if not properly taken into account, can introduce systematic effects in the determination of dark energy parameters. Other effects such as lensing magnification and the Doppler redshift effect can also boost the cross-correlation signal, but are only important at high redshift (Gianantonio & Crittenden 2007; Loverde, Hui & Gaztañaga 2007). These effects are well documented and can be calibrated and removed.

In this paper, we will solely explore the contribution of the non-linear terms, or the RS effect, on the cross-correlation signal. The RS effect arises from the non-linear evolution of the potential (Rees & Sciama 1968). It is believed to be much smaller than the CMB signal at all scales (Seljak 1996; Puchades et al. 2006). Indeed, compared with the CMB power spectrum, the RS effect is orders of magnitude lower. Also, compared with the complete integrated ISW power spectrum, the RS effect has been shown, using the halo model approach (e.g. Cooray 2002,a,b), to be unimportant if \( t < 100 \). However, the RS effect has not been taken into account in cross-correlation analyses, and it is important to assess its importance ahead of the completion of the next generation of large deep galaxy surveys.

We use a large N-body simulation to investigate the effect of the non-linear contribution on the interpretation of the ISW cross-correlation signal. We use the 488\(^3\)-particle L-BASICC simulation described by Angulo et al. (2008) which, with a box size of 1340 h\(^{-1}\) Mpc, is ideal for this purpose because not only does it enable us to extrapolate our analysis to non-linear scales at different redshifts, but it also includes the very large-scale power necessary to check the agreement with linear theory. The cosmology adopted in the L-BASICC simulation is \( \Lambda \)CDM, with \( \Omega_m = 0.75, \Omega_b = 0.25, \Omega_{\Lambda} = 0.024, \sigma_8 = 0.9 \) and \( H_0 = 73 \text{ km s}^{-1} \text{ Mpc}^{-1} \).

This paper is organized as follows. In Section 2, we compute the power spectrum of the ISW plus RS effects from our simulation and compare them with linear theory. In Section 3, we analyse these two effects in terms of the cross-correlation of the LSS with the CMB. Finally, in Section 4, we discuss our results and present our conclusions.

### 2 TIME DERIVATIVE OF THE POTENTIAL

The ISW effect results from the late time decay of gravitational potential fluctuations. The net blueshift or redshift of the CMB photons caused by the change in the potential during the passage of the photons induces net temperature fluctuations of the blackbody spectrum,

\[
\frac{\Delta T(t)}{T_0} = -\frac{2}{c^2} \int_0^t \Phi(t', \hat{n}) \, dt',
\]

where \( \Phi \) is the time derivative of the gravitational potential, \( t \) is the look-back time, with \( t = 0 \) at the present and \( t = t_l \) at the last scattering surface. The angular power spectrum of these temperature fluctuations (see the Appendix) is given by

\[
C_l = \frac{4}{c^4} \frac{2}{\pi^2} \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} P_{\Phi\Phi}(k) \, d\theta \, d\varphi,
\]

where \( k \) is the comoving distance to look-back time, \( t, j_j \) is the spherical Bessel function and \( P_{\Phi\Phi}(k, r) \) is the 3D power spectrum of \( \Phi \) fluctuations. To derive the final expression, we have used Limber’s approximation by assuming \( k \approx 1/r \) (Limber 1954; Kaiser 1992; Hu 2000; Verde, Heavens & Matarrese 2000, also see the Appendix).

The ISW effect consists of the temperature fluctuations described by these equations when linear theory is used to compute \( \Phi \) and its fluctuation power spectrum \( P_{\Phi\Phi} \). Using a simulation to determine the non-linear contributions, we can quantify the full ISW plus RS effect. In Fourier space, the time derivative of the gravitational potential can be expressed as

\[
\Phi_t(k, t) = \frac{3}{2} \left( \frac{H_0}{k} \right)^2 \Omega_m^2 \left[ \frac{\dot{\delta}(k, t)}{a} - \frac{\dot{a}(k, t)}{a^2} \delta(k, t) \right],
\]

where \( a \) is the expansion factor, \( H_0 \) is the Hubble constant, \( \Omega_m \) is the present mass density parameter and \( \dot{\delta} \) is the time derivative of the density fluctuation. Combining this with the Fourier space form of the continuity equation, \( \delta_t(k, t + i \mathbf{k} \cdot \mathbf{p}(k, t) = 0 \) gives

\[
\Phi_t(k, t) = \frac{3}{2} \left( \frac{H_0}{k} \right)^2 \Omega_m^2 \left[ \frac{\dot{a}(k, t) + i \mathbf{k} \cdot \mathbf{p}(k, t)}{a} \right],
\]

where \( \mathbf{p}(k, t) = \{1 + \delta(k, t)\} v(k, t) \) is the momentum density field in Fourier space divided by the mean mass density. This enables us to estimate the Fourier transform of the field of the simulation from the Fourier transforms of the density and momentum fields. Using equation (3), the resulting power spectrum, \( P_{\Phi\Phi}(k, t) = (2\pi)^{-3} \Phi_t(k, t) \Phi_t(k', t) \), can be written as

\[
P_{\Phi\Phi}(k, t) = \frac{9}{4} \left( \frac{H_0}{k} \right)^4 \Omega_m^2 \left[ \frac{\dot{a}(k, t) - \dot{a}_s^2 P_{\delta\delta}(k)}{a^2} + \frac{1}{a^2} P_{\delta\delta}(k) \right].
\]

In linear theory, \( P_{\delta\delta}(k, t) = k^2 P_{\delta\delta}(k) = D(t)^2 P_{\delta\delta}^{\text{lin}}(k) \) and \( P_{\Phi\Phi}(k, t) = k P_{\Phi\Phi}(k, t) = k P_{\Phi\Phi}(k) = D(t) D(t') P_{\delta\delta}^{\text{lin}}(k) \), where \( P_{\delta\delta}^{\text{lin}}(k) \) is the linear density power spectrum at the present time and \( D(t) \) is the growth factor normalized to be unity at present. Therefore, the power spectrum of the linear ISW effect is

\[
P_{\delta\delta}^{\text{lin}}(k) = \frac{9}{4} \left( \frac{H_0}{k} \right)^4 \Omega_m^2 \left[ \frac{H(t) D(t)(1 - \beta)^2}{a} \right] \frac{P_{\delta\delta}^{\text{lin}}(k)}{D(t)}.\]
This result \( \equiv B \) power spectrum are shown in ISW angular power spectrum coming from different redshift/\( \Omega_1 \). The overall result is shown in Fig. 1. The results from linear theory are also plotted. We find the total scaled \( \Phi \) power spectrum can be well fitted by a broken power law plus the linear-scaled \( \Phi \) power spectrum, namely their deviation from linear theory occurs at smaller scales for the density field than for the other fields.

\[
\beta = \frac{4h^2 D}{ab \pi} \simeq \Omega_0^2 (t).
\]

For easy comparison at different redshifts, we defined a scaled \( \Phi \) power spectrum, \( \mathcal{P}_{\Phi}(k, z) = P_{\Phi}(l)/(\Omega_0^2 z)^2 \), which from equation (5) is simply

\[
\mathcal{P}_{\Phi}(k, z) = P_{\Phi}(k, z) - 2 \frac{P_{\delta\delta}(k, z)}{H(z)} + \frac{P_{\delta\delta}(k, z)}{H(z)}.
\]

Equation (7) is shown in Fig. 1. The results from linear theory are also plotted. We find the total scaled \( \Phi \) power spectrum can be well fitted by a broken power law plus the linear-scaled \( \Phi \) power spectrum, namely their deviation from linear theory occurs at smaller scales for the density field than for the other fields.

\[
\mathcal{P}_{\Phi}(k, z) = \frac{A}{(10k)^{4/75} + (10k)^{4/91}}.
\]

where \( A \) and \( B \) are two free parameters that we use to fit the model to the simulation results at each redshift up to \( z = 6 \). To interpolate the model to intermediate redshifts, we linearly interpolate the values of \( A \) and \( B \) from the nearest two simulation outputs. Our model is compared to the simulation results in Fig. 1.

We see in Fig. 1 that the linear theory reproduces the ISW+RS \( P_{\Phi}(k, z) = 0 \) only at \( k < 0.1 h \, \text{Mpc}^{-1} \). It fails at progressively larger scales as the redshift increases. By \( z = 2 \), linear theory agrees with the simulation results only at \( k < 0.02 h \, \text{Mpc}^{-1} \). The reason for this surprising behaviour is that the linear part of the \( P_{\Phi} \) drops quickly to zero as the relative importance of \( \Omega_\Lambda \) diminishes at high redshift, while the non-linear part evolves more slowly with redshift. Therefore, the deviation of the total power spectrum from linear theory happens at larger scales at high redshifts. We find that the momentum power spectrum, \( P_{m^2} \), and the correlation power spectrum of the density and momentum, \( P_{\delta\delta} \), behave similarly to the \( P_{\Phi} \) power spectrum, namely their deviation from linear theory occurs at larger scales at higher redshift. This is in contrast with the power spectrum of the density field, which deviates from linear theory on progressively larger scales at lower and lower redshift. In another words, at the same redshift, the deviation from linear theory occurs at smaller scales for the density field than for the other fields.

The sharp increase of \( P_{\Phi} \) measured from the simulation at small scales (\( k > 1 h \, \text{Mpc}^{-1} \)) is due to discreteness in the 448$^3$ particle L-BASICC simulation. We used the much higher resolution 2160$^3$ particle Millennium simulation (Springel et al. 2005) to verify that our model remains accurate at smaller scales and is robust to shot noise corrections.

We can now compute the induced angular power spectrum of CMB temperature fluctuations by performing the integral in equation (2) over the redshift range \( 0 < z < 6 \) using our model for the 3D power spectrum, \( P_{\Phi}(k, z) \). The overall result is shown in Fig. 2 along with the contributions coming from different redshift intervals. For the overall angular power spectrum, the deviation of the model from the linear theory happens at \( l \sim 100 \). This result confirms the prediction of Cooray (2002b) based on the halo model. However, we also see that the failure of linear theory, as judged by our simulation results, occurs at smaller and smaller \( l \) as redshift increases. For example, above \( z = 5 \), the deviation occurs at \( l < 20 \) and, for larger values of \( l \) than this, linear theory becomes extremely inaccurate.

In order to evaluate how the breakdown of linear theory depends on redshift, we plot the evolution of the \( \Phi \) power at a given scale as a function of redshift in Fig. 3. Generally, the deviations of linear theory from the simulation results decrease with scale and increase with redshift. At \( k = 0.01 h \, \text{Mpc}^{-1} \), deviations start to be seen at \( z \sim 3 \) and, at \( k = 0.1 h \, \text{Mpc}^{-1} \), linear theory has become inaccurate at all redshifts. In the right-hand panel, which shows results in \( l \) space, we find no deviations up to \( l \sim 10 \), but for \( l > 50 \), linear theory has clearly broken down at all redshifts. Interestingly, at high redshift, the \( \Phi \) power in the simulation appears to be independent of \( z \) while, in linear theory, this quantity drops monotonically with \( z \).
Figure 3. Evolution of the $\Phi$ power spectrum $\Delta^2(k) \equiv k^3 P_{\Phi\Phi}(k)/2\pi^2$ for specific spatial (left-hand panel) and angular (right-hand panel) modes. The solid lines show our model of the simulation results. The dashed lines show the results of linear theory. The power $\Delta^2(k)$ decreases monotonically as a function of $z$ at all scales in linear theory. However, the total power seems to be independent of $z$ at high redshift. The deviation from linear theory increases with $z$ and $k$. The angular power spectrum $\Delta^2(l)$ shows no deviation from linear theory up to $l = 10$. For $l > 50$, deviations appear at all redshifts.

3 THE LSS–CMB CROSS-CORRELATION

To illustrate the contribution of the RS effect to the cross-correlation of the density field $\delta$ with $\dot{\Phi}$, we can compute the 3D cross-correlation power spectrum $P_{\dot{\Phi}\delta}(k, z) = \langle \dot{\Phi}(k, z)\delta^*(k, z) \rangle$ from our simulations

$$P_{\dot{\Phi}\delta}(k, z) = P_{\delta\delta}(k, z) - P_{\dot{\Phi}\dot{\Phi}}(k, z)/H(z).$$

In linear theory, $P_{\dot{\Phi}\delta}(k, z) = D^2(1 - \beta)P_{\Phi\delta}^\text{lin}(k)$, where $P_{\Phi\delta}^\text{lin}(k)$ is the linear density power spectrum at the present time. Results from our simulations are shown in Fig. 4. Comparing with linear theory, we find that the non-linear contribution appears at somewhat smaller scales than that of the autocorrelation power spectrum of $\dot{\Phi}$. At $z = 0$, the deviation occurs at $k \sim 1 \ h \ Mpc^{-1}$. However, it then dominates rapidly, making the cross-correlation power spectrum negative. This indicates that once the non-linear effect dominates,
the potential of overdense regions evolves faster than the expansion of the universe, becoming deeper and thus imparting a net redshift to CMB photons passing through them. The sense of the effect from underdense regions is reversed, but the effect generated by the overdense regions is dominant and induces a negative cross-correlation between the CMB and the LSS.3

To quantify the effect on the large-scale ISW cross-correlation measurements, we model the 3D cross-power spectrum from the simulations at each redshift output. We use linear theory to model the linear regime but once the cross-power spectrum starts to deviate from linear theory, we fit it with a function of the form

\[ k^3 P_{\delta\delta}(k) = A_1 + A_2 k + A_3 k^2, \]

where \( A_1, A_2, \) and \( A_3 \) are free parameters. Interpolating to intermediate redshifts, we are then able to calculate the projected 2D power spectrum.

The cross-correlation between LSS and CMB maps has been shown to be a powerful tool for verifying the existence of dark energy and constraining its properties. Current measurements of the cross-correlation have low statistical significance because the volumes probed by LSS surveys are relatively small, but this situation will improve greatly with upcoming surveys. For example, Pan-STARRS1 will survey three quarters of the sky, obtaining photometry for galaxies up to \( \sim 24.6 \) mag in the \( g \) band. The mean galaxy redshift in this ‘3rd survey’ will be \( \bar{z} \sim 0.5 \). Pan-STARRS1 will also carry out a deeper but smaller ‘Medium Deep Survey’ (‘MDS’) covering 84 sq deg of the sky to \( \sim 27.3 \) mag in \( g \) for which \( \bar{z} \sim 0.8 \) (Cai et al. 2008). Cross-correlating such photometric redshift galaxy samples with a CMB map (from WMAP or Planck) will make it possible for the first time to perform ISW tomography. Galaxy samples would be divided into different redshift slices, and each one cross-correlated with the CMB map. Values of the dark energy equation of state parameter, \( w \), could then be measured using the results from the different redshift slices, effectively constraining the evolution of \( w \).

To illustrate how ISW tomography may work, we follow Baugh & Efstathiou (1993) and model the redshift distribution of galaxies tracing the LSS as

\[ N(z) \propto \begin{cases} (z - z_c)^2 \exp \left(-\frac{(z - z_c)^2}{\sigma^2}\right) & \text{if } z < z_c, \\ 0 & \text{if } z > z_c, \end{cases} \]

but then choose the parameters \( z_0 \) and \( z_c \) to emulate plausible photometric redshift slices. [The same functional form was also taken by Cabrè et al. (2006) to model the SDSS LRG sample.] We assume \( z_0 = 0.2 \) so that the width of \( N(z) \) is much greater than the expected photometric redshift errors. We shift the function into different redshift intervals by using \( z_c = 0, 1, 2 \) and 3. The median redshift of these samples is \( \bar{z} \approx 1.4 z_0 + z_c = 0.28, 1.28, 2.28 \) and 3.28, respectively. The cross-correlation power spectrum (derived in an analogous way to the autocorrelation function detailed in the Appendix) is given as

\[ C_{\Phi\Phi}^{\Phi\tilde{g}} = \frac{2}{c^2} \int_0^{\bar{z}} \int_0^{\bar{z}} P_{\delta\delta}(k = l/r_c, z) b(z) N(z) H(z) r^2 dz, \]

where \( P_{\delta\delta}(k, z) \) is the cross-power spectrum of the potential field and the galaxy density field, \( b(z) \) is the galaxy bias parameter at redshift \( z \) and \( N(z) \) is the normalized galaxy selection function, where \( \int N(z) dz = 1 \). We adopt the small angle approximation in which \( k = l/r(z) \), where \( r(z) \) is the comoving distance. For simplicity, in this illustration, we assume the galaxy bias parameter to be unity. In angular space, the cross-correlation becomes

\[ w^{\Phi\tilde{g}}(\theta) = \sum_l \frac{2l + 1}{4\pi} P_l(\cos \theta) C_{l}^{\Phi\tilde{g}}, \]

where \( P_l \) are Legendre polynomials. In actual measurements of CMB fluctuations, the monopole and dipole are subtracted. Therefore, we set the power at \( l = 0 \) and 1 to zero before converting the signal into real space. To ensure that the results at smaller angles (\( \theta < 1^\circ \)) converge accurately, we sum the power up to \( l = 10,000 \).

The cross-correlation results are shown in Fig. 5. The contribution from the non-linear RS effect can be seen to become increasingly important as the redshift of the sample increases. The cross-correlation power spectrum decreases and deviates from linear theory rapidly at \( l \sim 500 \) due to the non-linear effect. It turns negative at \( l \sim 1000 \). This result is consistent with that obtained by Nishizawa et al. (2008) who used a similar method to illustrate the impact of the ISW and RS effects on the CMB-weak lensing cross-correlation. In angular coordinates, shown in the right-hand panel, the RS effect is negligible at \( \theta > 1^\circ \) at all redshifts. For the high-redshift samples, it becomes important at subdegree scales for the high-redshift samples where it suppresses the cross-correlation power spectrum by about 5–10 per cent at arcminute scales.

The statistical significance of current measurements of the CMB-LSS cross-correlation is not yet high enough to detect the effects we are discussing. Most current measurements can only determine the cross-correlation at degree scales or above (e.g. Cabrè et al. 2006). Future CMB or SZ surveys with high resolution might be able to resolve this contribution.

### 4 CONCLUSIONS

We have used an \( N \)-body simulation to calculate the non-linear (RS) contribution to the ISW effect. The comparison of the 3D and 2D power spectra measured from the simulation with those given by linear theory reveals a strong non-linear contribution whose physical scale increases with redshift. We investigated the strength of this effect on the cross-correlation of the CMB with galaxy samples in terms of angular power spectra and in angular coordinates at different redshifts. We find that there is a non-linear contribution to the cross-correlation signal at subdegree scales. The non-linear effect alters not only the amplitude, but also the shape of the cross-correlation power spectrum. With current galaxy samples, which cover relatively small volumes, it is not yet possible to disentangle the contribution of the RS effect from that of the ISW effect within the noise. However, in future surveys like Pan-STARRS and LSST, for which the number of galaxies and the sky coverage will increase dramatically, the error bars on the cross-correlation will be much smaller. In this case, the importance of the RS effect may become significant for high-redshift samples. The effect of the non-linear cross-correlation at scales of arcminutes would contaminate the SZ signal in the CMB, and this could confuse its interpretation (e.g. Diego et al. 2003; Fosalba, Gaztañaga & Castander 2003; Myers et al. 2004; Cao, Chu & Fang 2006; Lieu, Mittaz & Zhang 2006; Blinby & Shanks 2007).

Our analysis is based on a simulation that assumes a \( \Lambda \)CDM cosmology. The non-linear contribution depends on the values of
the cosmological parameters. In a flat universe with a cosmological constant, the RS effect will become increasingly dominant relative to the ISW effect as the value of $\Omega_\Lambda_0$ decreases. In the most extreme case, if $\Omega_\Lambda_0 = 0$, the ISW effect will vanish, leaving only the RS effect (Seljak 1996). The analysis of this paper could be generalized using either renormalized perturbation theory (e.g. Crocce & Scoccimarro 2006) or simulations with different dark energy models. In any case, more general modelling of the non-linear effect will be required for an accurate interpretation of future measurements of the LSS–CMB cross-correlation.

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APPENDIX: ANGULAR POWER SPECTRA

Here, we derive the relationship between the 3D power spectrum of gravitational potential fluctuations, \( P_{\phi\phi}(k, t) = (2\pi)^{-3} \langle \Phi(k, t) \Phi^*(k, t) \rangle \), and the resulting angular power spectrum of the induced CMB temperature fluctuations. Expanding the pattern of temperature fluctuations, \( \Delta T(\hat{r})/T_0 \), in terms of spherical harmonics, we have

\[
\alpha_{lm} = \int \frac{\Delta T(\hat{r})}{T_0} Y_{lm}^*(\hat{r}) d\hat{r}
\]

which using equation (1) becomes

\[
\alpha_{lm} = -\frac{2}{c^2} \int Y_{lm}^*(\hat{r}) \int_0^{4\pi} \Phi(\hat{r}, t) r d\theta d\phi.
\]

Writing \( \Phi(\hat{r}, t) \) in terms of a Fourier expansion and using the spherical harmonic expansion of a plane wave, \( \exp(i \mathbf{k} \cdot \mathbf{r}) = 4\pi \sum_{lm} i^l j_l(k r) Y_{lm}^*(\hat{k}) Y_{lm}(\hat{r}) \) this becomes

\[
\alpha_{lm} = -\frac{2}{(2\pi)^2c^2} \int Y_{lm}^*(\hat{r}) \int_0^{4\pi} \Phi(\hat{r}, t) \exp(i \mathbf{k} \cdot \mathbf{r}) dk \, dr \, d\hat{r}.
\]

\[
= -\frac{2 \times 4\pi}{(2\pi)^2c^2} \int Y_{lm}^*(\hat{r}) \int_0^{4\pi} \Phi(\hat{r}, t) \times \sum_{l'm'} i^l j_l(k r) Y_{l'm'}^*(\hat{k}) Y_{l'm}(\hat{r}) \, dk \, dr \, d\hat{r}
\]

\[
= -\frac{1}{\pi^2c^2} \int_0^{2\pi} \int_0^\pi \Phi(\hat{r}, t) i^l j_l(k r) Y_{lm}^*(\hat{k}) \, dk \, dr.
\]

Hence, the angular power spectrum, \( C_l \), is given by

\[
C_l \delta_{l'm'} \equiv \langle \alpha_{lm} \alpha_{l'm'}^* \rangle
\]

\[
= \left[ \frac{1}{\pi^2c^2} \right]^2 \int_0^{4\pi} \int_0^{4\pi} \Phi(\hat{r}, t) i^l j_l(k r) Y_{lm}^*(\hat{k}) \, dk \, dr \times \int_0^{4\pi} \Phi^*(\hat{k}', t') i^l j_l(k' r') Y_{l'm'}(\hat{k}') \, dk' \, dr'.
\]

Using the identity \( \langle \Phi(\mathbf{k}) \Phi^*(\mathbf{k}') \rangle \equiv (2\pi)^3 \delta(\mathbf{k} - \mathbf{k}') P_{\phi\phi}(k) \) and the orthogonality relationship of spherical harmonics

\[
\int_{4\pi} Y_{lm}^*(\hat{r}) Y_{l'm'}(\hat{r}) \, d\hat{r} = \delta_{l'l} \delta_{m'm'}
\]

this becomes

\[
C_l \delta_{l'm'} \delta_{m'm} = \frac{8}{\pi c^2} \int_0^{2\pi} \int_0^{4\pi} P_{\phi\phi}(k, r, r') i^l j_l(k r) Y_{lm}^*(\hat{k}) \times i^{l'} j_{l'}(k'r) Y_{l'm'}(\hat{k}) \, dr' \, dk
\]

\[
= \frac{8}{\pi c^2} \int_0^{2\pi} \int_0^{4\pi} k^2 P_{\phi\phi}(k, r, r') j_l(k r) j_{l'}(k'r) \, dr' \, dk \, dk' \, \delta_{l'l} \delta_{m'm}.
\]

\[
C_l = \frac{8}{\pi c^2} \int_0^{2\pi} \int_0^{4\pi} k^2 P_{\phi\phi}(k, r, r') j_l(k r) j_{l'}(k'r) \, dr' \, dk.
\]

This exact relationship can be simplified by using Limber’s approximation. For small angular separations, \( \theta \), at comoving distance, \( r \), the wavenumber, \( k \), can be expressed in terms of its components parallel and perpendicular to the line of sight and approximated by

\[
k = \sqrt{k_x^2 + k_y^2} \approx k_z, \text{ where } k_z = 2\pi/r \theta \approx l/r \gg k_1 \sim 1/\Delta r
\]

namely, the power is dominated by that perpendicular to the line of sight and there is no correlation between different shells of \( \Delta r \) along the line of sight. Combining this with the orthogonality relation for spherical Bessel functions,

\[
\frac{2}{\pi} \int k^2 j_l(k r) j_{l'}(k'r) \, dk = \delta(r - r')/r^2,
\]

we arrive at

\[
C_l \approx \frac{4}{c^2} \int_0^{2\pi} P_{\phi\phi}(k) \left( \frac{1}{r} \right) r^2 \, dr.
\]

(see also Limber 1954; Kaiser 1992; Hu 2000; Verde et al. 2000). Verde et al. (2000) find that the difference between this approximation and the full calculation is less than 3 per cent at \( l > 20 \). In this paper, we are mainly concerned with even smaller scales (\( l > 500 \)), and so we are justified in using Limber’s approximation. We also tested the difference between using \( k = l/r \) and the more accurate \( k = (l + 1/2)/r \) (Loverde & Afshordi 2008) and find very little difference to our results.

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