NEUTRINOLESS DOUBLE $\beta$-DECAY: STATUS AND FUTURE

S. M. Bilenky

Joint Institute for Nuclear Research, Dubna, R-141980, Russia, and SISSA, I-34014 Trieste, Italy.

Abstract

A brief summary of the status of neutrino masses, mixing and oscillations is presented. Neutrinoless double $\beta$-decay is considered. Predictions for the effective Majorana mass are reviewed. A possible test of the calculations of nuclear matrix elements of the $0^{\nu}\beta\beta$-decay is proposed.

1 Introduction

Observation of neutrino oscillations in the Super-Kamiokande \cite{1}, SNO \cite{2}, KamLAND \cite{3}, K2K \cite{4} and other neutrino experiments \cite{5, 6, 7, 8, 9, 10} is one of the most important recent discovery in particle physics. There are no natural explanations of the smallness of neutrino masses in the framework of the Standard Model. A new, beyond the SM mechanism of neutrino mass generation is required. Several such mechanisms were proposed (see \cite{11}). In order to ensure a progress in the understanding of the origin of small neutrino masses and peculiar neutrino mixing new experimental data are definitely necessary.

One of the most important problem which must be addressed by the future experiments is the problem of the nature of massive neutrinos: are they Dirac or Majorana particles? Investigation of the neutrinoless double $\beta$-decay ($0^{\nu}\beta\beta$-decay) is the most effective method which could allow to resolve this dilemma.

We will review here the status of the $0^{\nu}\beta\beta$-decay. Calculation of the nuclear matrix elements of the process is a complicated theoretical problem. We will discuss here a possible method which could allow to check the calculations.

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We will present first a brief summary of the neutrino oscillations (theory and experimental data).

The theory of the neutrino oscillations (see [12, 13]) is based on the assumption that field $\nu_{lL}(x)$ in the standard charged and neutral currents

$$j^\text{CC}_\alpha(x) = 2 \sum_{l=e,\mu,\tau} \bar{\nu}_{lL}(x) \gamma_\alpha l_L(x); \quad j^\text{NC}_\alpha(x) = \sum_{l=e,\mu,\tau} \bar{\nu}_{lL}(x) \gamma_\alpha \nu_{lL}(x)$$

are “mixed” fields ²

$$\nu_{lL}(x) = \sum_{i=1}^{3} U_{li} \nu_{iL}(x); \quad l = e, \mu, \tau.$$  

Here $U$ is PMNS [14, 15] mixing matrix, $\nu_i$ is the field of neutrino with mass $m_i$.

From Eqs. (1) and (2) follows that flavor lepton numbers $L_e, L_\mu$ and $L_\tau$ are not conserved. If total lepton number $L = L_e + L_\mu + L_\tau$ is conserved, $\nu_i(x)$ is Dirac field of neutrinos $\nu_i (L = 1)$ and antineutinos $\bar{\nu}_i (L = -1)$.

If total lepton number $L$ is not conserved, $\nu_i(x)$ is the field of Majorana neutrinos. The Majorana field $\nu_i(x)$ satisfies the condition

$$\nu_i^c(x) = C \bar{\nu}_i^T(x) = \nu_i(x),$$

where $C$ is the matrix of the charge conjugation.

The state of the flavor neutrino $\nu_l$ ($l = e, \mu, \tau$), produced in a CC weak process together with the lepton $l^+$, is described by ”mixed” vector of state

$$|\nu_l\rangle = \sum_{i=1}^{3} U_{li}^* |\nu_i\rangle.$$  

Here $|\nu_i\rangle$ is the state of neutrino with mass $m_i$, momentum $\vec{p}$ and energy

$$E_i \simeq p + \frac{m_i^2}{2p}$$

²We are assuming here that the number of massive neutrinos is equal to the number of flavor neutrinos (three). All existing data (with the exception of the data of the LSND experiment [17]) are in a perfect agreement with this assumption. The data of the LSND experiment will be checked by the running MiniBooNE experiment [18].
Let us assume that at $t = 0$ flavor neutrinos $\nu_l$ with momentum $\vec{p}$ are produced. At the time $t$ for the neutrino state we have

$$|\nu_t\rangle = \sum_i e^{-iE_i t} U^*_{li} |\nu_i\rangle = \sum_{\nu'} U_{\nu'i} e^{-iE_i t} U^*_{li},$$

(5)

Probability of the transition $\nu_l \rightarrow \nu_{l'}$ is given by

$$P(\nu_l \rightarrow \nu_{l'}) = |\delta_{l'l} + \sum_{i \geq 2} U_{l'i} (e^{-i \Delta m^2_{li} L/E} - 1) U^*_{li}|^2.$$  

(6)

Here $\Delta m^2_{ik} = m_i^2 - m_k^2$ and $L \simeq t$ is the distance between neutrino production and detection points. In the general case the probability $P(\nu_l \rightarrow \nu_{l'})$ depends on six parameters: two mass-squared differences $\Delta m^2_{12}$ and $\Delta m^2_{23}$, three mixing angles $\theta_{12}, \theta_{23}, \theta_{13}$ and $CP$ phase $\delta$.

From the data of the neutrino oscillation experiments it was found that

1. $\Delta m^2_{12} \ll \Delta m^2_{23}$.
2. The angle $\theta_{13}$ is small.

From analysis of the data of the reactor CHOOZ experiment [16] it was obtained that

$$\sin^2 \theta_{13} \lesssim 5 \cdot 10^{-2}$$

(7)

If we neglect in transition probabilities small terms proportional to $\frac{\Delta m^2_{23}}{\Delta m^2_{12}}$ and $\sin^2 \theta_{13}$, then in the region of $\frac{L}{E}$ sensitive to $\Delta m^2_{23}$ (atmospheric and accelerator long baseline experiments) dominant transitions are (see [13])

$$\nu_\mu \leftrightarrow \nu_\tau \quad (\bar{\nu}_\mu \leftrightarrow \bar{\nu}_\tau).$$

Probability of $\nu_\mu$ ($\bar{\nu}_\mu$) to survive is given by

$$P(\nu_\mu \rightarrow \nu_\mu) = P(\bar{\nu}_\mu \rightarrow \bar{\nu}_\mu) = 1 - \frac{1}{2} \sin^2 2\theta_{23} (1 - \cos \frac{\Delta m^2_{23} L}{2E}).$$

(8)

The data of the atmospheric Super-Kamiokande experiment are perfectly described by Eq. (8). For the parameters $\Delta m^2_{23}$ and $\sin^2 2\theta_{23}$ the following 90 % CL ranges were obtained [11]:

$$1.5 \cdot 10^{-3} \leq \Delta m^2_{23} \leq 3.4 \cdot 10^{-3} \text{ eV}^2; \quad \sin^2 2\theta_{23} > 0.92.$$  

(9)
In the regions sensitive to $\Delta m_{12}^2$ (solar, KamLAND experiments) effects of "large" $\Delta m_{23}^2$ is averaged. In the leading approximation vacuum probability of $\bar{\nu}_e$ to survive is given by

$$P(\bar{\nu}_e \to \bar{\nu}_e) = 1 - \frac{1}{2} \sin^2 2\theta_{12} \left(1 - \cos \Delta m_{12}^2 \frac{L}{2E}\right).$$

(10)

The probability of $\nu_e$ to survive in matter is given by

$$P(\nu_e \to \nu_e) = P_{\nu_e \to \nu_e}^{\text{mat}}(\Delta m_{12}^2, \sin^2 \theta_{12}, \rho_e),$$

(11)

where $P_{\nu_e \to \nu_e}^{\text{mat}}$ is two-neutrino probability of $\nu_e$ to survive in matter ($\rho_e$ is the electron density). From global analysis of solar and KamLAND data for the parameters $\Delta m_{12}^2$ and $\sin^2 \theta_{12}$ it was found \cite{2}

$$\Delta m_{12}^2 = (8.0^{+0.6}_{-0.4}) \times 10^{-5} \text{ eV}^2; \quad \tan^2 \theta_{12} = 0.45^{+0.09}_{-0.07}$$

(12)

Information on the lightest neutrino mass $m_0$ can be inferred from the measurement of the high-energy part of the $\beta$- spectrum of tritium. From the data of Mainz \cite{19} and Troitsk \cite{20} experiments the following upper bound was obtained

$$m_0 \leq 2.3 \text{ eV}$$

(13)

From several analysis of cosmological data, in which results of different measurements were used, for the sum of the neutrino masses upper bounds in the range

$$\sum_i m_i \leq (0.4 - 1.7) \text{ eV}$$

(14)

were deduced \cite{21}. Investigation of neutrino oscillations can not reveal the nature of neutrinos with definite masses $\nu_i$ \cite{22}. In fact, Majorana and Dirac mixing matrices are connected by the relation

$$U^M = U^D S(\beta),$$

(15)

where $S_{ik}(\beta) = e^{i\delta_{ik}} \delta_{ik}$ is phase matrix ($\beta_3 = 0$). From (6) and (15) it is obvious that phase matrix $S(\beta)$ drops out from the expression for the transition probability. We have

$$P^M(\nu_l \to \nu_r) = P^D(\nu_l \to \nu_r).$$

(16)
2 Neutrinoless double $\beta$-decay

The search for neutrinoless double $\beta$-decay

$$ (A, Z) \rightarrow (A, Z + 2) + e^- + e^- $$ \hspace{1cm} (17)

of $^{76}$Ge, $^{130}$Te, $^{136}$Xe, $^{100}$Mo and other even-even nuclei is the most sensitive and direct way of the investigation of the nature of neutrinos with definite masses $\nu_i$ (see reviews [23, 24]). The total lepton number in $0\nu\beta\beta$-decay is violated and the process is allowed only if $\nu_i$ are Majorana particles [25].

We will discuss now the process (17). The effective Hamiltonian of the process is given by

$$ H_{CC}^{\nu} = \frac{G_F}{\sqrt{2}} 2\bar{e}_L\gamma_\alpha\nu_{\alpha L} j^\alpha + \text{h.c.}. \hspace{1cm} (18) $$

Here $j^\alpha$ is the hadronic charged current, $G_F$ is the Fermi constant and

$$ \nu_{\alpha L} = \sum_i U_{ei}\nu_{iL}, \hspace{1cm} (19) $$

where $\nu_i$ are Majorana fields.

The neutrinoless double $\beta$-decay is the second order in $G_F$ process with virtual neutrinos. The neutrino propagator is given by the expression

$$ <0|T(\nu_{eL}(x_1) \nu_{eL}^T(x_2))|0> = m_{ee} \left(\frac{i}{2\pi}\right)^4 \int e^{-ip(x_1-x_2)} \frac{1}{p^2 - m_i^2} \frac{1 - \gamma_5}{2} d^4p C. \hspace{1cm} (20) $$

Here

$$ m_{ee} = \sum_i U_{ei}^2 m_i, \hspace{1cm} (21) $$

is effective Majorana mass. For small neutrino masses $m_i^2 \ll p^2$ and we can safely neglect $m_i^2$ in the denominator of the neutrino propagator. Therefore, the matrix element of the $0\nu\beta\beta$ -decay is factorized in the form of a product of the effective Majorana mass, which is determined by neutrino masses $m_i$ and $U_{ei}$, and nuclear matrix element, which is determined only by nuclear properties and does not depend on neutrino masses and mixing.

The half-life of $0\nu\beta\beta$ decay $T_{1/2}^{0\nu}(A, Z)$ is given by the expression

$$ \frac{1}{T_{1/2}^{0\nu}(A, Z)} = |m_{ee}|^2 |M^{0\nu}(A, Z)|^2 G^{0\nu}(E_0, Z), \hspace{1cm} (22) $$
where $M^{0\nu}(A, Z)$ is nuclear matrix element (NME) and $G^{0\nu}(E_0, Z)$ is known phase-space factor.

The results of several experiments on the search for $0\nu\beta\beta$ -decay are available at present. No commonly accepted evidence in favor of the decay was obtained so far.\footnote{The recent claim \cite{26} of evidence of the $0\nu\beta\beta$ -decay of $^{76}$Ge will be checked by the GERDA experiment \cite{27}.} The most stringent lower bounds for the half-time of the $0\nu\beta\beta$- decay were obtained in the Heidelberg-Moscow \cite{28} and CUORICINO \cite{29} experiments.

In the Heidelberg-Moscow experiment the following lower bound for the half-life of $^{76}$Ge was found

$$T_{1/2}^{0\nu} \geq 1.9 \cdot 10^{25}\text{ years}$$  \hspace{1cm} (23)

In the CUORICINO experiment for half-life of $^{130}$Te the following bound was obtained

$$T_{1/2}^{0\nu} \geq 1.8 \cdot 10^{23}\text{ years}$$  \hspace{1cm} (24)

Taking into account different calculations of the nuclear matrix elements, for the effective Majorana mass from these results the following upper bounds were deduced

$$|m_{ee}| \leq (0.3-1.2) \text{ eV (H – M)};$$

$$|m_{ee}| \leq (0.2-1.1) \text{ eV (CUORICINO)}. \hspace{1cm} (25)$$

New experiments on the search for neutrinoless double $\beta$-decay of different nuclei (CUORE ($^{130}$Te), EXO ($^{136}$Xe), MAJORANA ($^{76}$Ge), SuperNEMO ($^{82}$Se), MOON ($^{100}$Mo) and others) are in preparation \cite{30}. In these experiments large detectors (about one ton) will be used. In future experiments on the search for $0\nu\beta\beta$-decay the sensitivity

$$|m_{ee}| \simeq \text{ a few } 10^{-2}\text{ eV}. \hspace{1cm} (26)$$

are planned to be achieved.

\section{Effective Majorana mass and neutrino oscillation data}

The expected values of the effective Majorana neutrino mass, which can be inferred from current neutrino oscillation data, were considered in numerous papers (see \cite{31} and references therein). We will discuss here some issues.

For three neutrino masses two neutrino mass spectra are possible:

\begin{itemize}
  \item \textbf{Type I} (normal hierarchy: $m_1 < m_2 < m_3$)
  \item \textbf{Type II} (inverted hierarchy: $m_1 > m_2 > m_3$)
\end{itemize}
1. Normal spectrum

\[ m_1 < m_2 < m_3; \quad \Delta m^2_{12} \ll \Delta m^2_{23} \] (27)

2. Inverted spectrum

\[ m_3 < m_1 < m_2; \quad \Delta m^2_{12} \ll |\Delta m^2_{13}| \] (28)

In the framework of the leading approximation it is not possible to distinguish normal and inverted spectra. In order to reveal the type of the neutrino mass spectrum necessary to study small effects beyond the leading approximation. The size of such effects and possibilities to measure them depend on the value of the parameter \( \sin^2 \theta_{13} \). We will demonstrate here that the effective Majorana mass \( m_{ee} \) strongly depends on the type of the neutrino mass spectrum.

In the standard parametrization for the elements \( U_{ei} \) we have

\[ U_{e1} = \cos \theta_{13} \cos \theta_{12} e^{i \beta_1}; \quad U_{e2} = \cos \theta_{13} \sin \theta_{12} e^{i \beta_2}; \quad U_{e3} = \sin \theta_{13} e^{i \beta_3} \] (29)

The value of the angle \( \theta_{12} \) is known from analysis of the data of the solar and KamLAND experiments (see (12)). The angle \( \theta_{13} \) is small and limited by the CHOOZ bound (7)). Majorana phases \( \beta_i \) are unknown.

From the analysis of the neutrino oscillation data two neutrino mass-squared differences \( \Delta m^2_{12} \) and \( |\Delta m^2_{23}| \) were determined. For neutrino masses in the case of the normal spectrum we have

\[ m_2 = \sqrt{m_1^2 + \Delta m^2_{12}}, \quad m_3 = \sqrt{m_1^2 + \Delta m^2_{12} + \Delta m^2_{23}}. \] (30)

In the case of the inverted spectrum neutrino masses are given by

\[ m_1 = \sqrt{m_3^2 - \Delta m^2_{13}}, \quad m_2 = \sqrt{m_3^2 - \Delta m^2_{13} + \Delta m^2_{12}}. \] (31)

The lightest neutrino mass \( m_0 = m_1(m_3) \) is unknown at present. Upper bound of \( m_0 \), obtained from the data of the latest tritium experiments, is given in (14).

We will consider three characteristic neutrino mass spectra

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4In order to keep for solar-KamLAND neutrino mass-squared difference notation \( \Delta m^2_{12} \) different labeling of neutrino masses are used in the case of normal and inverted spectra. For neutrino mixing angles in both cases the same notations can be used.
1. Neutrino mass hierarchy

\[ m_1 \ll m_2 \ll m_3 \]  

This type of the neutrino mass spectra is suggested by \( SO(10) \) and other GUT models which unify quarks and leptons (see [11]).

In the case of the hierarchy \((32)\) neutrino masses are determined by the neutrino mass squared differences and are known from the oscillation data:

\[ m_2 \simeq \sqrt{\Delta m_{12}^2} \simeq 8.9 \cdot 10^{-3} \text{eV}; \quad m_3 \simeq \sqrt{\Delta m_{23}^2} \simeq 4.9 \cdot 10^{-2} \text{eV} \]  

(33)

Neglecting the contribution of the lightest neutrino mass \( m_1 \) for the effective Majorana mass we obtain the following expression

\[ |m_{ee}| \simeq \left| \sin^2 \theta_{12} \sqrt{\Delta m_{12}^2} + e^{i \beta_{32}} \sin^2 \theta_{13} \sqrt{\Delta m_{23}^2} \right|, \]  

(34)

where \( \beta_{32} = \beta_3 - \beta_2 \) is the Majorana phase difference.

The first term of \((34)\) is small because of the smallness of \( \sqrt{\Delta m_{12}^2} \). Contribution of “large” \( \sqrt{\Delta m_{23}^2} \) is suppressed by the small factor \( \sin^2 \theta_{13} \).

If we will use the CHOOZ bound \([17]\) the modulus of both terms in \((34)\) are approximately equal. Hence the terms in \((34)\) could cancel each other and \( |m_{ee}| \) could be very small. From \([7, 9, 12]\) for the upper bound of \( |m_{ee}| \) we find

\[ |m_{ee}| \leq 6.4 \cdot 10^{-3} \text{eV} \]  

(35)

Thus, in the case of the hierarchy of neutrino masses, even upper bound of the effective Majorana mass is about two times smaller that the expected sensitivity of future experiments on the search for \( 0\nu\beta\beta \)-decay.

2. Inverted hierarchy of neutrino masses

\[ m_3 \ll m_1 < m_2 \]  

(36)

Such neutrino mass spectrum requires a special flavor symmetry of the neutrino mass matrix. (for example, conservation of \( L_e - L_\mu - L_\tau \)). For neutrino masses in the case of the inverted hierarchy we have

\[ m_2 \simeq m_1 \simeq \sqrt{|\Delta m_{13}^2|}; \quad m_3 \ll \sqrt{|\Delta m_{13}^2|} \]  

(37)
Neglecting small contribution of the lightest neutrino mass, for the effective Majorana mass we have the following expression

\[ |m_{ee}| \simeq \sqrt{\Delta m_{13}^2} \left( 1 - \sin^2 2 \theta_{12} \sin^2 \beta_{21} \right)^{\frac{1}{2}} \]  

(38)

The only unknown parameter in (38) is \( \sin^2 \alpha_{21} \). This parameter characterize CP violation in the case of the Majorana neutrino mixing.

From the condition of the CP invariance in the lepton sector we have

\[ U_{ei} = \eta_i U_{ei}^* \]

(39)

where \( \eta_i = \pm i \) is the CP parity of the Majorana neutrino with the mass \( m_i \). From (39) for the Majorana CP phase \( \beta_i \) we find

\[ 2 \beta_i = \frac{\pi}{2} \rho_i + 2\pi n_i. \]

(40)

Here \( n_i \) is an integer number and \( \rho_i = \pm 1 \) is determined by the relation \( \eta_i = e^{i \frac{\pi}{2} \rho_i} \). Thus, in the case of the CP invariance in the lepton sector from (38) and (40) we find

- \( |m_{ee}|_{CP1} = \sqrt{\Delta m_{13}^2} \) (the same CP parities of \( \nu_2 \) and \( \nu_1 \))
- \( |m_{ee}|_{CP2} = \sqrt{\Delta m_{13}^2} \cos \theta_{12} \) (opposite CP parities of \( \nu_2 \) and \( \nu_1 \))

From (38) for the effective Majorana mass we have the range

\[ \cos 2 \theta_{12} \sqrt{\Delta m_{13}^2} \leq |m_{ee}| \leq \sqrt{\Delta m_{13}^2} \]

(41)

Upper and lower bounds in (41) correspond to the case of the CP conservation. Other values of the effective Majorana mass corresponds to the case of the CP non conservation. Let us notice that the parameter \( \sin^2 \beta_{21} \) is determined by the measurable quantities

\[ \sin^2 \beta_{21} = \frac{1}{\sin^2 2 \theta_{12}} \left( 1 - \frac{|m_{ee}|^2}{|\Delta m_{13}^2|} \right) \]

(42)

From analysis of the solar oscillation data it was found that \( \theta_{12} < \pi/4 \) (see (12)). Thus, the lower bound of the effective Majorana mass in
is different from zero. From (9), (12) and (41) we find the following 90 % CL range

\[ 1.0 \cdot 10^{-2} \leq |m_{ee}| \leq 5.5 \cdot 10^{-2} \text{ eV} \] (43)

The sensitivities to \( |m_{ee}| \) of the most ambitious future experiments on the search for 0\( \nu \beta \beta \) are in the range (43). Thus, next generation of the 0\( \nu \beta \beta \)-experiments could probe the inverted hierarchy of the neutrino masses.

3. Quasi degenerate neutrino mass spectrum

If the lightest neutrino mass satisfies inequality

\[ m_1 \gg \sqrt{\Delta m_{23}^2} \text{ (} m_3 \gg \sqrt{|\Delta m_{13}^2}| \text{)} \] (44)

neutrino mass spectrum is practically degenerate

\[ m_1 \simeq m_2 \simeq m_3 \simeq m_0 \] (45)

This spectrum requires symmetry of the neutrino mass matrix and only marginally is compatible with neutrino oscillation data (see [33]).

For the effective Majorana mass in the case of the quasi degenerate spectrum we have

\[ |m_{ee}| \simeq m_0 \left(1 - \sin^2 2 \theta_{12} \sin^2 \alpha_{21}\right)^{1/2} \] (46)

There are two unknown parameters in Eq. (46): \( m_0 \) and \( \sin^2 \alpha_{21} \). From the measurement of \( |m_{ee}| \) for the lightest neutrino mass the following range can be obtained

\[ |m_{ee}| \leq m_0 \leq 2.4 |m_{ee}| \] (47)

The future tritium experiment KATRIN [34] will be sensitive to \( m_0 \simeq 0.2 \text{ eV} \).

4 Problem of nuclear matrix elements

The observation of the 0\( \nu \beta \beta \)-decay would be of a profound importance for our understanding of the origin of small neutrino masses. The establishment
of the Majorana nature of neutrinos with definite masses would be a strong support of the most plausible see-saw mechanism of neutrino mass generation, which connect the smallness of neutrino masses with the violation of the total lepton number at a large scale.

If $0\nu\beta\beta$-decay would be observed, it will be very important to obtain precise value of the effective Majorana mass $|m_{ee}|$. As we have seen, the determination of $|m_{ee}|$ would allow to obtain an important information about neutrino mass spectrum, mass of the lightest neutrino and, possibly, Majorana CP phase difference. From experimental data, however, only the product of the effective Majorana mass and nuclear matrix element can be determined. Nuclear matrix elements must be calculated.

The calculation of NME is a complicated nuclear problem (see reviews[24]). NME is the matrix element of an integral of the T-product of two hadronic charged weak currents and neutrino propagator. Many intermediate nuclear states must be taken into account in calculations.

Two approaches, which are based on different physical assumptions, are usually used for the calculation of NME: Nuclear Shell Model (NSM) and Quasiparticle Random Phase Approximation (QRPA). In literature exist many QRPA-based models. As a result different calculations of the same NME differ by factor 2-3 or even more.

We will discuss here possible method which could allow to test NME calculations in a model independent way[38]. We will use factorization property of matrix elements of $0\nu\beta\beta$-decay which is based on the assumption that Majorana neutrino mass mechanism is the dominant mechanism of the $0\nu\beta\beta$-decay.

Several future experiments on the search for $0\nu\beta\beta$-decay of different nuclei will have comparable sensitivity to $|m_{ee}|$. Thus, if $0\nu\beta\beta$-decay of one nuclei will be discovered in a future experiment it is probable that the process will be observed also in other experiments with different nuclei. The effective Majorana mass, which can be determined from the measurement of half-lives of the $0\nu\beta\beta$-decay of different nuclei, must be the same. From this requirement we obtain the following relations between half-lives of nuclei $A_i, Z_i$ and $A_k, Z_k$

$$T_{1/2}(A_i, Z_i) = X_M(i; k) T_{1/2}(A_k, Z_k),$$

where coefficients $X_M(i; k)$ are given by the expression

$$X_M(i; k) = \left(\frac{|M^{0\nu}(A_k, Z_k)|^2}{|M^{0\nu}(A_i, Z_i)|^2}\right)_M \frac{G^{0\nu}(E_{0k}^i, Z_i)}{G^{0\nu}(E_{0i}^k, Z_k)}.$$
The values of these coefficients depend on the model of the calculation of NME. A model $M$ is compatible with data if relations (48) are satisfied. Let us stress, however, that this does not mean that the model $M$ allows to obtain the correct value of the effective Majorana mass.

For illustration we calculated coefficients $X_M(i; k)$ for three latest models of NME calculations:

1. $(M_1)$ Shell Model (E. Courier et al.\cite{35})

2. $(M_2)$ QRPA (V. Rodin et al.\cite{36}; important QRPA parameter $g_{pp}$ is fixed by the data of the experiments on the measurement of half-lives of the $2\nu\beta\beta$-decay.)

3. $(M_3)$ QRPA (O. Civitarese, J. Suhonen\cite{37}; parameters of the QRPA model were fixed by the $\beta$-decay data of nearby nuclei)

The results of the calculation are presented in the Table I.

|                        | $M_1$ | $M_2$ | $M_3$ |
|------------------------|-------|-------|-------|
| $X(^{100}\text{Mo};^{76}\text{Ge})$ | 0.59  | 0.17  |       |
| $X(^{130}\text{Te};^{76}\text{Ge})$ | 0.25  | 0.49  | 0.13  |
| $X(^{136}\text{Xe};^{76}\text{Ge})$ | 0.55  | 0.80  | 0.07  |

From the Table I we see that the measurement of $0\nu\beta\beta$-decay of $^{76}\text{Ge}$ and $^{130}\text{Te}$ (or $^{76}\text{Ge}$ and $^{136}\text{Xe}$ or $^{76}\text{Ge}$ and $^{100}\text{Mo}$) can tell us which model ($M_1$, $M_2$ or $M_3$) is compatible with data (if any). This conclusion, however, depends on the choice of nuclei. Let us consider, for example, the pair $^{100}\text{Mo}$ and $^{130}\text{Te}$. We have

$$X(^{100}\text{Mo};^{130}\text{Te}) = 1.2 \ (M_2); \ 1.3 \ (M_3) \quad (50)$$
Thus, if the relation (48) for $^{100}$Mo and $^{130}$Te is satisfied, say, for the model $M_2$, it will be difficult to exclude also the model $M_3$. However, the values of the effective Majorana mass which can be obtained with the help of these two models are quite different:

$$|m_{ee}|_{M_2} = 2.6 \ |m_{ee}|_{M_3}$$

We come to the conclusion that the observation of $0\nu\beta\beta$-decay of three (or more) nuclei would be an important tool for the test of the models of NME calculation and for the determination of the value of the effective Majorana mass $|m_{ee}|$.

5 Conclusion

The establishment of the nature of neutrinos with definite masses $\nu_i$ (Majorana or Dirac?) will have a profound importance for the understanding of the origin of small neutrino masses and neutrino mixing. Investigation of the neutrinoless double $\beta$-decay is the most sensitive probe of the Majorana nature of neutrinos. Today’s limit on the effective Majorana mass is $|m_{ee}| \leq (0.2-1.2) \text{ eV}$. The sensitivity $|m_{ee}| \simeq$ a few $10^{-2}\text{eV}$ is a challenging goal of future experiments.

If $|m_{ee}|$ is determined the pattern of the neutrino mass spectrum, lightest neutrino mass and, possibly, Majorana CP phase can be inferred. Calculation of nuclear matrix elements is a very important, challenging nuclear problem. Observation of $0\nu\beta\beta$-decay of several nuclei could allow to test NME calculations.

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References

[1] Super-Kamiokande Collaboration, S. Fukuda et al., Phys. Rev. Lett. **81**, 1562 (1998); Y. Ashie et al., Phys. Rev. Lett. **93**, 101801 (2004); Phys. Rev. Lett. **93**, 101801 (2004); Y. Ashie et al., hep-ex/0501064.

[2] SNO collaboration, Q.R. Ahmed et al., Phys. Rev. Lett. **87**, 071301 (2001); Phys. Rev. Lett. **89**, 011301 (2002); Phys. Rev. Lett. **89**, 011302 (2002); B. Aharmin et al., nucl-ex/0502021.
[3] KamLAND collaboration, T. Araki et al., Phys. Rev. Lett. 94, 081801 (2005).

[4] K2K Collaboration, M.H. Alm et al., Phys. Rev. Lett. 90, 041801 (2003); E. Aliu et al., Phys. Rev. Lett. 94, 081802 (2005).

[5] B. T. Cleveland et al., Astrophys. J. 496, 505 (1998).

[6] GALLEX Collaboration, W. Hampel et al., Phys. Lett. B 447, 127 (1999); GNO Collaboration, M. Altmann et al., Phys. Lett. B 490, 16 (2000); Nucl. Phys. Proc. Suppl. 91, 44 (2001).

[7] SAGE Collaboration, J. N. Abdurashitov et al., Phys. Rev. C 60 (1999) 055801; Nucl. Phys. Proc. Suppl. 110 (2002) 315.

[8] Super-Kamiokande Collaboration, S. Fukuda et al., Phys. Rev. Lett. 86, 5651 (2001) 5651; M. Smy, Nucl. Phys. Proc. Suppl. 118, 25 (2003).

[9] Soudan 2 Collaboration, W.W.M. Allison et al., Physics Letters B 449 (1999) 137.

[10] MACRO Collaboration, M. Ambrosio et al. hep-ex/0106049; Phys. Lett. B 517 (2001) 59 M. Ambrosio et al. NATO Advanced Research Workshop on Cosmic Radiations, Oujda (Morocco), 21-23 March, 2001.

[11] G. Altarelli and F. Feruglio, New J. Phys. 6 (2004) 106, hep-ph/0405048; R.N. Mohapatra et al., hep-ph/0412099; S. F. King, Rept. Prog. Phys. 67 (2004) 107, hep-ph/0310204.

[12] S. M. Bilenky and S. T. Petcov, Rev. Mod. Phys. 59 (1987) 671.

[13] S. M. Bilenky, C. Giunti and W. Grimus, Prog. Part. Nucl. Phys. 43 (1999) 1, hep-ph/9812360; M.C. Gonzalez-Garcia, Y. Nir, Rev. Mod. Phys. 75 (2003) 345, hep-ph/0202058.

[14] B. Pontecorvo, J. Exptl. Theoret. Phys. 34, 247 (1958) [Sov. Phys. JETP 7, 172 (1958)]; B. Pontecorvo, Zh. Eksp. Teor. Fiz. 53, 1717 (1967) [Sov. Phys. JETP 26, 984 (1968)]; V. Gribov and B. Pontecorvo, Phys. Lett. B 28, (1969) 493.

[15] Z. Maki, M. Nakagawa, and S. Sakata, Prog. Theor. Phys. 28, 870 (1962).
[16] CHOOZ Collaboration, M. Apollonio et al., Phys. Lett. B 466, 415 (1999); M. Apollonio et al., Eur. Phys. J. C 27, 331 (2003); hep-ex/0301017

[17] LSND Collaboration, A. Aguilar et al., Phys. Rev. D 64, 112007 (2001). G. Drexlin, Nucl. Phys. Proc. Suppl. 118, 146 (2003).

[18] MiniBooNE Collaboration, A.A. Aguilar-Arevalo, hep-ex/0408074 R. Tayloe et al., Nucl. Phys. B (Proc. Suppl.) 118, 157 (2003); Heather L. Ray et al., hep-ex/0411022

[19] C. Weinheimer et al., Nucl.Phys.Proc. Suppl. 118, 279 (2003); Ch. Kraus et al., hep-ex/0412056

[20] V.M. Lobashev et al., Phys. Lett. B 460, 227 (1999); Nucl. Phys. Proc. Suppl. 91, 280 (2001); Prog. Part. Nucl. Phys. 48, 123 (2002).

[21] G.L. Fogli, E. Lisi et al., Phys. Rev. D 70, 113003 (2004); S. Hannestad, hep-ph/0409108 astro-ph/0505551; M. Tegmark, hep-ph/0503257

[22] S.M. Bilenky, J. Hosek and S.T. Petcov, Phys. Lett.B94 (1980) 495.

[23] M. Doi, T. Kotani and E. Takasugi, Progr. Theor. Phys. Suppl. 53 (1985) 1; S. M. Bilenky and S. T. Petcov, Rev.Mod.Phys. 59 (1987) 671; J.D. Vergados, Phys. Rep. 361, 1 (2002).

[24] A.Faessler and F.Šimkovic, J. Phys. G 24 (1998) 2139; J.Suhonen and O. Civitarese, Phys. Rep.300 (1998) 123; S. R. Elliott and P. Vogel, Annu. Rev. Nucl. Part. Sci. 52 (2002), hep-ph/0202264

[25] J. Schechter and J.W.F Valle, Phys. Rev. D 25, 2951 (1982).

[26] H.V. Klapdor-Kleingrothaus, I.V. Krivosheina, A. Dietz, O. Chkvorets, Phys. Lett. B 586, 198 (2004).

[27] GERDA Collaboration, I. Abt et al., hep-ex/0404039

[28] Heidelberg-Moscow collaboration, H. V. Klapdor-Kleingrothaus et al., Eur. Phys. J. A 12 (2001) 147.

[29] CUORE collaboration, C. Arnaboldi et al., Phys. Lett. B 584 260 (2004); hep-ex/0501034 hep-ex/0505045
[30] F.T. Avignone, Nucl.Phys.Proc.Suppl.143 (2005) 233

[31] S.M. Bilenky, C. Giunti, W. Grimus, B. Kayser, and S.T. Petcov, Phys. Lett. B 465, 193 (1999); M. Czakon, J. Gluza and M. Zralek, Phys. Lett. B 465, 211 (1999); S.M. Bilenky, S. Pascoli and S.T. Petcov, Phys. Rev. D 64 053010 (2001); F. Feruglio, A. Strumia and F. Vissani, Nucl.Phys.B637 (2002) 345; Addendum-ibid. B659 (2003) 359; S. Pascoli and S.T. Petcov, Phys. Lett.B 544(2002) 239 ; Phys. Lett.B 580 (2004) 280; A. Strumia, F. Vissani, hep-ph/0503246.

[32] L. Wolfenstein, Phys. Lett. B 107, 77 (1981); S.M. Bilenky, N.P. Nedelcheva, and S.T. Petcov, Nucl. Phys. B 247, 61 (1984); B. Kayser, Phys. Rev. D 30, 1023 (1984).

[33] G. Altarelli and F. Feruglio, New J. Phys. 6 (2004) 106, hep-ph/0405048

[34] KATRIN Collaboration, V.M. Lobashev et al., Nucl. Phys. A 719, 153 (2003); L. Bornschein et al., Nucl. Phys. A 752, 14 (2005).

[35] E. Caurier, F. Nowacki, A. Poves, and J. Retamosa, Phys. Rev. Lett. 77, 1954 (1996).

[36] V.A. Rodin, A. Faessler, F. Šimkovic, and P. Vogel, Phys. Rev.C 68 (2003) 044302, nucl-th/0503063

[37] O. Civitarese and J. Suhonen, Nucl. Phys. A 729 (2003) 867.

[38] S.M. Bilenky and J.A. Grifols, Phys. Lett. B550 (2002) 154; S.M. Bilenky and S.T. Petcov, hep-ph/0405237