Impact Time Control Guidance Law for Large Initial Lead Angles Based on Sliding Mode Control

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Abstract: In view of the interception scenario where a missile intercepts a stationary target in the longitudinal plane, an impact time control guidance (ITCG) law is proposed by utilizing the sliding mode control (SMC). An amendatory approach of \( \text{tgo} \) (time-to-go) is adopted where there is no need to take the small-angle assumption into consideration. Under the basis of SMC, the error of impact time is selected as a sliding mode variable, which can converge to be zero in finite time by using the fast power reaching law (FPRL). The guidance law performance has been conducted in one-to-one engagement with different desired impact time. Simulation results demonstrate that the proposed guidance scheme can operate effectively even with a large initial lead angle of the missile, which broadens the initial launch conditions of the missile as well as improving the application capability of the guidance law.

1. Introduction

With the increasing threat coming from modern anti-missile systems, it has become more and more difficult to break through the defense of anti-missile systems. Since it is difficult for the anti-missile systems to intercept all missiles at the same time, the saturation attacks can greatly increase the damage effect on the target. As a result, it is necessary to pay attention to the problem of controlling the impact time of missiles.

Impact time control guidance (ITCG), which is firstly put forward by Kim[1], has been broadly applied in cooperative attack. ITCG is derived on the basis of proportional navigation guidance (PNG). In[2], an ITCG scheme applied to salvo attack of anti-ship missiles law was investigated. The ITCG problem with missile’s field-of-view (FOV) constraint was taken into consideration by adding a new part to the guidance instruction based on PNG[3][4]. Although the ITCG schemes based on PNG are simple in form and convenient to be applied in practice, the guidance law performance is easily affected by the estimation accuracy of \( \text{tgo} \). Therefore, the core of ITCG is to estimate the \( \text{tgo} \) (time-to-go) of the missile accurately, which has enormous influence on the attack accuracy of the missiles.

The research in regard to the estimation of \( \text{tgo} \) can be divided into two categories. The first one is based on the concept of trajectory tracking so that the problem of estimating \( \text{tgo} \) can be converted into the tracking of missile’s trajectory indirectly. In[5], a recursive algorithm was applied to compensate for the \( \text{tgo} \) estimation error caused by the flight lead angle of the missile, so as to obtain a more estimation of accurate \( \text{tgo} \). By obtaining the ballistic trajectory between missile and attack point through numerical integration, Ref[6] obtained \( \text{tgo} \) indirectly. The second one is to calculate \( \text{tgo} \) through the specific method.
A new derivation of an optimal missile guidance law was presented in[7], in which it was then incorporated into a hybrid guidance scheme that allows the specification and satisfaction of a desired time-of-arrival constraint. To examine the effects of system lag on performance of a generalized impact-angle-control guidance law, analytic solutions of the guidance law for a first-order lag system were investigated in[8].

In fact, the estimation of $t_{go}$ were considered in the mentioned guidance schemes. However, for these studies, there are some disadvantages in the estimation of $t_{go}$ at a large initial lead angle or in a specific ITCG design. On the one hand, the method based on the concept of trajectory tracking is difficult to achieve the ideal tracking of the trajectory. On the other hand, the method based on the assumption of small angle approximation is insufficient under the large lead angles, which will consume a lot of computing resources and time.

Sliding mode control (SMC) has been widely used in guidance design due to its high precision, rapidity and strong robustness[9]. A terminal guidance law based on stochastic fast smooth second-order SMC was proposed in order to handle the track imprecision caused by the model uncertainties and atmospheric environment disturbances existing in the guidance system[10]. In[11], based on the SMC theory, a nonsingular and finite time fast convergent guidance law was designed to attack the targets with zero miss-distance and terminal impact angle constraint. A three-dimensional guidance was proposed with the consideration of maneuvering acceleration and input saturation based on SMC and adaptive control[12]. SMC shows excellent performance in the numerical simulation of the above-mentioned guidance law, and can meet the requirements of guidance goal.

Therefore, in view of intercepting the target with a large initial lead angle, motived by Ref[13], an ITCG scheme is proposed based on SMC method, in which an improved method of estimating $t_{go}$ is adopted so that there is no need to apply the assumption of small angle approximation.

2. Problem Statement

The interception scenario where a missile intercepts a stationary target in the longitudinal plane, which is shown in Figure 1.

![Figure 1 Plane interception engagement geometry](image)

$$
\begin{align*}
\dot{R} &= V \cos \eta \\
R \dot{\gamma} &= -V \sin \eta \\
\dot{\eta} &= \theta - \gamma \\
\dot{\theta} &= a / V
\end{align*}
$$

where $R$ is the relative range between missile and target, $\gamma$ represents the LOS angle of missile, $a$ stands for the acceleration instruction normal to LOS of missile, $\eta$ is the lead angle of missile, $\theta$ represents the ballistic inclination angle of missile and $V$ stands for the speed of missile.

Thus, the objective of guidance law is aimed at pursuing an appropriate guidance instruction $a$ to achieve the constraint of impact time to finish the specific interception mission.
where $t_{in}$ represents the total impact time during interception and $t_d$ is the desired impact time.

### 3. Main Results

#### 3.1. Estimation of Impact Time

For the whole interception process, the total impact time $t_{in}$ can be expressed as the following

$$t_{in} = t_{el} + t_{go}$$

where $t_{el}$ is the elapsed interception time and $t_{go}$ stands for the remaining interception time. 

Inspired by Ref.[13], an improved calculation method for estimating $t_{go}$ is adopted in Eq.(7), in which there is no need to satisfy the small-angle assumption.

$$t_{go} = \frac{R}{V} \mu = \frac{R}{V} \left( 1 + \frac{\sin \eta^2}{4N - 2} \right)$$

where $\mu$ is an adjustment factor. Thus, by using Eq.(7) to estimate $t_{go}$, the missile can be ensured to have the ability to intercept target at a specific impact time even if with a large initial lead angle.

#### 3.2. Guidance Law Design

Selecting the regulation error of impact time as a moderating variable and defining a sliding mode surface variable $s$ as follows

$$s = t_{in} - t_d = t_{el} + t_{go} - t_d$$

**Theorem 1** In view of the interception engagement geometry whose dynamics is described as in Eq.(1)–(4), the guidance instruction with impact time constraint for a stationary target is proposed as

$$a = \frac{V^2 \tau (\cos \eta - 1)}{r \sin 2\eta} + \frac{1}{2} \frac{V^2 \tau}{r \sin 2\eta} \dot{s}$$

where $\tau = 4N - 2$ and $\dot{s} = -k_1 |s|^r \text{sgn}(s) - k_2 s$ is the dynamic of the sliding mode surface where both $k_1, k_2$ are positive parameters need to be chosen.

**Proof**

Taking the derivative of sliding mode surface $s$ along the close-sloop system

$$\dot{s} = 1 + t_{go}$$

Substituting the derivative of Eq.(7) into Eq.(10), one can yield

$$\dot{s} = \frac{R}{V} \left( \frac{\sin 2\eta}{\tau} (\dot{\theta} - \dot{\gamma}) \right) + \frac{\dot{R}}{V} \left( 1 + \frac{\sin^2 \eta}{\tau} \right) + 1$$

Then, combining Eq.(4) with Eq.(11), it can be obtained that

$$\dot{s} = \frac{\dot{R} V \left( \tau + \sin^2 \eta \right) + (R \sin 2\eta) \dot{a} - \dot{R} \dot{V} \sin 2\eta + V^2 \tau}{V^2 \tau}$$

As is shown in Eq.(12), guidance instruction $a$ exists in the dynamics of sliding mode surface manifold $s$, which shows that the dynamics of $t_{go}$ has a relative one with respect to $a$. Thus, this motives us to apply the fast power reaching law (FPRL) by choosing

$$\dot{s} = -k_1 |s|^r \text{sgn}(s) - k_2 s$$

As a result, the guidance instruction can be rendered in turn as the following

$$a = \frac{V^2 \tau (\cos \eta - 1)}{r \sin 2\eta} + \frac{1}{2} \frac{V^2 \tau}{r \sin 2\eta} \times \left( k_1 |s|^r \text{sgn}(s) + k_2 s \right)$$

which is the same as Eq.(9). Thus, the proof is completed.

Then, it is essential to analyze the stability analysis of the dynamics of $\dot{s}$ existing in guidance...
instruction. Indeed, \( \hat{s} \) can be regarded as made up of two parts \( \dot{s}_1 = -k_1 |s|^{\alpha} \text{sign}(s) \) and \( \dot{s}_2 = -k_2 s \). The following theorem gives the finite time convergence obtained by virtue of such dynamics.

**Theorem 2** Considering the system (13), if \( k_1, k_2 > 0 \) and \( \alpha \in (0,1) \), then the sliding mode surface \( s \) and its first derivative \( \dot{s} \) will converge to be zero in finite time. And the convergence time \( T \) is a continuous function associated with the initial condition of \( s \).

**Proof**

It can be seen that \( \dot{s} \) is continuous in the right side. And it is local Lipschitz except \( s(0) = 0 \). Thus, there exists the only solution of forward time under any condition \( s(0) \in \mathbb{R} \setminus \{0\} \). Inspired by Ref[14], Eq.(15) can be obtained by multiplying both sides of Eq.(13) by \( e^{k_1 t} \) at the same time.

\[
\frac{d(e^{k_1 t}s)}{dt} = -k_1 |s|^{\alpha} e^{((1-\alpha)k_1)|s|} \text{sign}(s)
\]

(15)

For the convenience of further analysis, it can be written as

\[
\frac{d(e^{k_1 t}s)}{e^{k_1 t}s} = -k_1 e^{((1-\alpha)k_1)|s|} dt
\]

(16)

Then, by integrating both side of Eq.(16), the solution of Eq.(13) can be expressed as follows

\[
s(t) = \begin{cases} 
\text{sign}(s(0)) e^{k_1 t} \left[ |s(0)|^{\alpha} + \frac{k_1}{k_2} \frac{k_1}{k_2} e^{((1-\alpha)k_1)|s|} \right]^{\frac{1}{\alpha}} \\
\ln \left( 1 + \frac{k_1}{k_2} |s(0)|^{\alpha} \right) \\
\ln \left( 1 + \frac{k_1}{k_2} |s(0)|^{\alpha} \right) \\
0, \ t \geq \frac{-k_1(1-\alpha)}{k_2}, \ s(0) \neq 0 \\
0, \ t \geq \frac{k_2}{k_1(1-\alpha)}, \ s(0) = 0 \\
0, \ t \geq 0, \ s(0) = 0
\end{cases}
\]

(17)

where \( s(0) \) is the initial condition of \( s(t) \). It can be proved by Eq.(17) that \( s(t) \) will converge to be zero in finite time, and the convergence time \( T \) can be described as the following

\[
T = \frac{\ln \left( 1 + \frac{k_1}{k_2} |s(0)|^{\alpha} \right)}{k_2(1-\alpha)}
\]

(18)

Moreover, it can be obviously proved that \( \dot{s} = 0 \) holds when \( s = 0 \) according to Eq.(13). Therefore, the prove is completed.

4. **Simulation Results**

In this section, the effectiveness of the proposed guidance law is verified. The position of missile and target are selected as (2km,4km) and (8km,8km). The missile moves at the speed of 300m/s. The guidance parameters of acceleration instruction proposed in Eq.(9) are selected with

\( k_1 = 0.2, k_2 = 0.01, \alpha = 1/3 \).

The limit of acceleration instruction is \( a_{\text{max}} = 300m/s^2 \) separately. The proposed guidance law is switched to PNG when \( s \) is less than a small threshold which is chosen as 0.01 to guarantee the numerical stability.
Simulation is performed with different specific impact time 26s, 30s and 33s to intercept a stationary target with the initial lead angle 30° separately. The simulation results are shown in Table 1 and Figure 2. It can be seen from Figure 2 (a) that the missile can intercept target successfully at different desired impact time with the miss distance 0.2967m, 0.2982m, 0.1069m. As is shown in Figure 2(b), $t_{go}$ varies gradually at the beginning of terminal guidance process to satisfy the constraint of impact time. It can be demonstrated form Figure 2(c) that the value of acceleration instruction is positively related to the impact time so that it can provide enough power for the missile to ensure that the missile can adjust its posture to satisfy the constraint of impact time. As is shown in Figure 2(d), the sliding mode surface $s$ can converge to be zero under three initial conditions with impact time errors 0.011s, 0.010s and 0.001s separately, which proves that the proposed guidance law is effective to guarantee the missile intercept target with impact time constraint.

Table 1 Analysis of simulation results

| Impact time $t_u$ (s) | Miss distance (m) | Impact time error (s) |
|-----------------------|-------------------|-----------------------|
| 26                    | 0.2967            | 0.011                 |
| 30                    | 0.2982            | 0.010                 |
| 33                    | 0.1069            | 0.001                 |

5. Conclusion

In view of the problem of intercepting a stationary with impact time constraint, an ITCG scheme is proposed based on the SMC theory. An estimation of $t_{go}$ which is suitable for a large initial lead angle is adopted to allow more applicability in achievement of ITCG. On the basis of the FPRL, the guidance law is designed by choosing the impact time error as the sliding mode variable, which performs with
greater practical application value in actual interception process. The extension of the proposed guidance law to the inclusion of impact angle constraint is an important area for future extension.

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