On the role of Collins effect in the Single Spin Asymmetry $A_N$ in $p^\uparrow p \rightarrow h X$ processes

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The much debated issue of the transverse single spin asymmetry $A_N$ observed in the inclusive large $P_T$ production of a single hadron in $p p$ interactions, $p^\uparrow p \rightarrow \pi X$, is considered in a TMD factorization scheme. A previous result [1, 2] stating that the maximum contribution of the Collins effect is strongly suppressed, is revisited, correcting a numerical error. New estimates are given, adopting the Collins functions recently extracted from SIDIS and $e^+ e^-$ data, and phenomenological consequences are discussed.

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I. INTRODUCTION AND FORMALISM

The understanding of spin effects, and in particular, for parity conserving processes, of transverse Single Spin Asymmetries (SSAs) has always been one of the major challenges for QCD and any fundamental quantum field theory. Such effects, abundantly observed in experiments, are not generated by the perturbative dynamical properties of the Standard Model elementary interactions and originate from more profound phenomena related to the intrinsic structure of the nucleons. Thus, the study of these often neglected effects has recently opened a new phase in our exploration of the partonic composition of hadrons.

Most progress has occurred in the study of the rich azimuthal dependences measured in Semi-Inclusive Deep Inelastic Scattering (SIDIS) of leptons off polarized nucleons by dedicated experiments, HERMES (DESY), COMPASS (CERN) and JLab. These SIDIS azimuthal asymmetries are interpreted and discussed in terms of new, unintegrated, Transverse Momentum Dependent distribution and fragmentation functions (shortly, TMDs); these offer new information on the properties of quarks and gluons, which go well beyond the usual one-dimensional description of Partonic Distribution Functions (PDFs) in terms of longitudinal momentum fraction only.

In particular the Sivers distributions [3–5] and the Collins fragmentation functions [6] have been extracted [7–12] from SIDIS data, and, thanks to complementary information from Belle on the Collins function [13, 14], a first extraction of the transversity distribution has been possible [15, 16].

All these analyses have been performed in the $\gamma^* p \rightarrow p$ center of mass (c.m.) frame, within a QCD factorization scheme, according to which the SIDIS cross section is written as a convolution of TMDs and elementary interactions:

$$d\sigma^{p^\uparrow p \rightarrow h X} = \sum_q \hat{f}_{q/p}(x, k_\perp; Q^2) \otimes d\hat{\sigma}^{q\rightarrow q\ell}\otimes \hat{D}_{h/q}(z, p_\perp; Q^2),$$  \hspace{1cm} (1)

where $k_\perp$ and $p_\perp$ are, respectively, the transverse momentum of the quark in the proton and of the final hadron with respect to the fragmenting quark. At order $k_\perp/Q$ the observed $P_T$ of the hadron is given by

$$P_T = z k_\perp + p_\perp. \hspace{1cm} (2)$$

There is a general consensus [17–19] that such a scheme holds in the kinematical region defined by

$$P_T \simeq k_\perp \simeq \Lambda_{QCD} \ll Q. \hspace{1cm} (3)$$

The presence of the two scales, small $P_T$ and large $Q$, allows to identify the contribution from the unintegrated partonic distribution ($P_T \approx k_\perp$), while remaining in the region of validity of the QCD parton model. At larger values of $P_T$ other mechanisms, like quark-gluon correlations and higher order perturbative QCD contributions become important [19, 20]. A similar situation [18, 22–27] holds for Drell-Yan processes, $AB \rightarrow \ell^+ \ell^- X$, where the two scales are the small transverse momentum, $q_T$, and the large invariant mass, $M$, of the dilepton pair.
The situation is not so clear for processes in which only one large scale is detected, like the inclusive production, at large \( P_T \), of a single particle in hadronic interactions, \( AB \to CX \). However, the most striking and large SSAs have been \([28, 31]\) and keep being measured \([32, 33]\) in these reactions. The TMD factorization for these processes was first suggested in Refs. \([8, 10]\) and adopted in Refs. \([38, 40]\) to explain the large single spin asymmetries observed by the E704 Collaboration \([29]\). The same approach led to successful predictions \([41, 42]\) for the values of \( A_N \) measured at RHIC \([43]\).

Alternative approaches to explain the origin of SSAs, linking collinear partonic dynamics to higher-twist quark-gluon correlations, were originally proposed in Refs. \([14, 48]\) and phenomenologically used in Refs. \([49, 52]\). These two approaches, the TMD factorization and the higher-twist correlations, have been shown to be related \([55, 54]\) and consistent with each other \([23, 24, 56]\).

However, a definite proof of the validity of the TMD factorization for hadronic inclusive processes with one large scale only is still lacking. Due to this, the study of dijet production at large \( P_T \) in hadronic processes was proposed \([50, 53]\), where the second small scale is the total \( q_T \) of the two jets, which is of the order of the intrinsic partonic momentum \( k_T \). This approach leads to a modified TMD factorization approach, with the inclusion in the elementary processes of gauge link color factors \([60, 62]\). Despite the identification of two separate scales, some problems with the TMD factorization for hadronic processes with the final inclusive production of two jets, or two hadrons, have been recently pointed out \([63, 64]\). TMD factorization is expected to work for the observation, inside a jet with large transverse momentum \( P_T \), of a final hadron with a transverse momentum with respect to the jet direction, like \( p^+ p \to jet + \pi + X \) \([65, 67]\).

In this paper we consider SSAs in \( p^+ p \to \pi X \) processes, with only one large \( P_T \) final pion detected, for which data are available. We adopt the TMD factorization scheme \([1, 2, 38, 42]\),

\[
 d\sigma^{p^{+}p^{\rightarrow}\pi X} = \sum_{a,b,c,d} \hat{f}_{a/p}(x_a,k_{T \perp};Q^2) \otimes \hat{f}_{b/p}(x_b,k_{T \perp};Q^2) \otimes d\sigma^{ab\rightarrow cd} \otimes \hat{D}_{\pi/e}(z,p_{L \perp};Q^2),
\]

as a natural phenomenological extension of the corresponding collinear factorization, based on the convolution of integrated parton distributions (PDFs) and fragmentation functions (FFs) with QCD elementary dynamics; this collinear factorization works well in computing unpolarized cross sections, but fails in explaining SSAs as there is no single spin effect in the collinear PDFs and FFs, and in lowest order QCD dynamics. In the polarized case, for a generic process \((A,S_A) + (B,S_B) \to C + X\), Eq. \([3]\) explicitly reads \([2]\):

\[
 \frac{E_C d\sigma^{(A,S_A)+(B,S_B)\rightarrow C+X}}{d^3p_C} = \sum_{a,b,c,d,(\lambda)} \int \frac{dx_a dx_b dz}{16\pi^2 x_a x_b z^2 S} \frac{d^2k_{T \perp a} d^2k_{T \perp b} d^3p_{L \perp} \delta(p_{L \perp} \cdot \hat{p}_C)}{J(p_{L \perp})} \times \notag
\]

\[
 \times \rho_{\lambda_a,\lambda_b}^{a/A,S_A} \hat{f}_{a/A,S_A}(x_a,k_{T \perp a}) \rho_{\lambda_b,\lambda_c}^{b/B,S_B} \hat{f}_{b/B,S_B}(x_b,k_{T \perp b}) \times \notag
\]

\[
 \times \delta_\pi \cdot \hat{p}_C \notag
\]

\[
 \Delta \notag
\]

\[
 M_{\lambda_c,\lambda_d,\lambda_a,\lambda_b} \delta(s + \hat{t} + \hat{u}) \hat{D}_{\pi/e}(z,p_{L \perp}).
\]

Further details and a full explanation of the notations can be found in Refs. \([1, 2]\) (where \( p_{L \perp} \) is denoted as \( k_{T \perp (C)} \)).

There are many contributions, from different TMDs, to the SSAs according to the above expression. In Ref. \([1]\) it was argued that only the Sivers effect contributes significantly; a further small contribution from the Collins effect is possible (largely suppressed by phase integrations), while all other TMD contributions are utterly negligible. However that conclusion was affected by a wrong sign \([68, 69]\) in one of the elementary interactions and the Collins effect contribution was underestimated. It remains true that all other contributions to the SSAs are negligible.

**Reconsideration of the Collins contribution**

We reconsider here the Collins contribution to the SSA

\[
 A_N = \frac{d\sigma^+ - d\sigma^-}{d\sigma^+ + d\sigma^-} \quad \text{where} \quad d\sigma^\pm = \frac{E_\pi d\sigma^{p^{+}p^{\rightarrow}\pi X}}{d^3p_\pi}.
\]

Such a contribution can be computed according to the TMD factorized expression \([1, 2]\):

\[
 [d\sigma^+ - d\sigma^-]_{\text{Collins}} = \sum_{q_a,b,q_c,d} \int \frac{dx_a dx_b dz}{16\pi^2 x_a x_b z^2 S} \frac{d^2k_{T \perp a} d^2k_{T \perp b} d^3p_{L \perp} \delta(p_{L \perp} \cdot \hat{p}_C)}{J(p_{L \perp})} \delta(s + \hat{t} + \hat{u}) \times \notag
\]

\[
 \times \Delta \pi q_a(x_a,k_{T \perp}) \cos(\phi_1 - \phi_2 + \phi_a^H) \times \notag
\]

\[
 f_{b/p}(x_b,k_{T \perp}) \left[ M^0_1 M^0_2 \right]_{q_a,b \rightarrow q_c,d} \Delta^N D_{\pi/e}(z,p_{L \perp}),
\]
In addition, we correct the numerical error in one of the elementary interactions mentioned above, which affected the process. A simple factorized form of the TMD functions was adopted, using a Gaussian shape for their dependence on the corresponding set of parameterizations as the “SIDIS-2” fit. In this case, the set of pion and kaon FFs by de Florian, Sassot and Stratmann was adopted. An updated extraction of the transversity and Collins functions was presented in Ref. [16]. We will refer to it as the “SIDIS-1” fit. In this extraction, the Kretzer set of unpolarized FFs [72] was used for the unpolarized parton distribution and fragmentation functions coming from SIDIS and annihilation data.

Thus, in this section, we investigate in much detail the phenomenology of the Collins effect for pion SSAs at STAR, BRAHMS and E704 kinematics, according to Eqs. (55)-(58), exploiting all the available and updated information on the transversity distribution and the Collins fragmentation functions coming from SIDIS and $e^+e^-$ annihilation data. In addition, we correct the numerical error in one of the elementary interactions mentioned above, which affected the conclusions of Ref. [1].

The first combined extraction of the quark transversity distribution and the Collins function was presented in Ref. [15]. We will refer to it as the “SIDIS-1” fit. In this extraction, the Kretzer set of unpolarized FFs [72] was adopted. An updated extraction of the transversity and Collins functions was presented in Ref. [16]. We will refer to the corresponding set of parameterizations as the “SIDIS-2” fit. In this case, the set of pion and kaon FFs by de Florian, Sassot and Stratmann [12] became available at that time, was considered.

Let us recall the main features of the parameterizations adopted in Refs. [15, 16]. The analysis of SIDIS and $e^+e^-$ data is performed at leading order, $O(k_\perp/Q)$, in the TMD factorization approach, where $Q$ is the large scale in the process. A simple factorized form of the TMD functions was adopted, using a Gaussian shape for their $k_\perp$ dependent component. For the unpolarized parton distribution and fragmentation functions we have:

$$f_{q/p}(x, k_\perp) = f_{q/p}(x) \frac{e^{-k_\perp^2/(k_\perp^2)}}{\pi(k_\perp^2)}, \quad D_{h/q}(z, p_\perp) = D_{h/q}(z) \frac{e^{-p_\perp^2/(p_\perp^2)}}{\pi(p_\perp^2)},$$

where $\langle k_\perp^2 \rangle$ and $\langle p_\perp^2 \rangle$ have been fixed by analyzing the Cahn effect in unpolarized SIDIS processes, see Ref. [7]:

$$\langle k_\perp^2 \rangle = 0.25 \text{ GeV}^2, \quad \langle p_\perp^2 \rangle = 0.20 \text{ GeV}^2.$$

For the usual integrated PDFs $f_{q/p}(x)$ we adopted the GRV98 set [74] and, as said above, for the integrated FFs $D_{h/q}(z)$ we used the Kretzer set [72] for the SIDIS-1 fit and the DSS one [73] for the SIDIS-2 fit. We have taken into account their DGLAP QCD evolution.
The quark transversity distribution, $\Delta_T q(x, k_\perp)$, and the Collins fragmentation function, $\Delta^N D_{h/q^+}(z, p_\perp)$, have been parametrized as follows:

$$\Delta_T q(x, k_\perp) = \frac{1}{2} \mathcal{N}_q^T(x) \left[ f_{q/p}(x) + \Delta q(x) \right] e^{-k_\perp^2/(k_\perp^2)^T} {\pi(k_\perp^2)^T},$$  

(11)

$$\Delta^N D_{h/q^+}(z, p_\perp) = 2 \mathcal{N}_q^C(z) D_{h/q}(z) h(p_\perp) e^{-p_\perp^2/(p_\perp^2)^T},$$  

(12)

where $\Delta q(x)$ is the usual collinear quark helicity distribution,

$$\mathcal{N}_q^T(x) = N_q^T x^{q_0} (1-x)^{\beta_q} \left( \frac{\alpha_q + \beta_q}{\alpha_q \beta_q} \right) \frac{\alpha_q}{\beta_q},$$  

(13)

and

$$\mathcal{N}_q^C(z) = N_q^C z^{q_0} (1-z)^{\delta_q} \left( \frac{\gamma_q + \delta_q}{\gamma_q \delta_q} \right),$$  

(14)

with $|N_q^{T(C)}| \leq 1$. Moreover,

$$h(p_\perp) = \sqrt{2e} \frac{p_\perp}{M_h} e^{-p_\perp^2/M_h^2}.$$  

(15)

With these choices, the transversity and Collins functions automatically fulfill their proper Soffer and positivity bounds respectively, for any values of the $(x, k_\perp)$ and $(z, p_\perp)$ variables. The quark helicity distributions $\Delta q(x)$, required for the Soffer bound, are taken from Ref. [23]. The term $\left[ f_{q/p}(x) + \Delta q(x) \right]$ in Eq. (11) is evaluated at the initial scale and evolved at the appropriate $Q^2$ values using the transversity evolution kernel. Similarly, for the $Q^2$ evolution of the Collins function, which remains so far unknown, we considered the unpolarized DGLAP evolution of its collinear factor $D_{h/q}(z)$.

Despite the simplicity of these functional forms, they still involve, in the most general case, a huge number of free parameters. In Refs. [13, 16] we therefore adopted some additional, physically motivated assumptions, in order to keep the number of free parameters reasonably low. First of all, for the transversity distribution we used only valence quark contributions. In addition, for the fragmentation functions we considered two different expressions for $N_q^C$, corresponding to the so-called “favoured” and “unfavoured” FFs, $N_q^{C,\text{fav}}(z)$ and $N_q^{C,\text{unf}}(z)$; for example, for pions, we had:

$$N_{\pi^+}^{C,\text{fav}}(z) = N_{\pi^+}^{C,\text{unf}}(z) = N_{\pi^-}^{C,\text{fav}}(z) = N_{\pi^-}^{C,\text{unf}}(z),$$  

(16)

$$N_{\pi^+}^{C,\text{unf}}(z) = N_{\pi^+}^{C,\text{unf}}(z) = N_{\pi^-}^{C,\text{unf}}(z) = N_{\pi^+}^{C,\text{unf}}(z) = N_{\pi^-}^{C,\text{unf}}(z).$$  

(17)

Notice, however, that our complete parameterization of the Collins FFs, Eqs. (12), allows for further differences among parton flavours, possibly contained in the usual unpolarized FFs.

In addition, we kept a flavour dependence in the coefficients $N_{u,d}^T$ and $N_{u,d}^C$, while the parameters $\alpha_q$, $\beta_q$, $\gamma_q$, $\delta_q$, and $M_h$ were taken to be flavour independent. For simplicity we also assumed that $\langle k_\perp^2 \rangle_T = \langle k_\perp^2 \rangle_T$. With these choices, we were left with a total of 9 free parameters for the SIDIS-1 and SIDIS-2 fit parameterizations:

$$N_{u}^T, N_{d}^T, N_{u}^{C,\text{fav}}, N_{d}^{C,\text{unf}}, \alpha, \beta, \gamma, \delta, M_h.$$  

(18)

Both fits gave good results. However, a study of the uncertainties of the best fit parameters, and a comparison of the two sets of parameterizations, SIDIS-1 and SIDIS-2, clearly shows that SIDIS data are not presently able to constrain the large $x$ behaviour of the quark $(u, d)$ transversity distributions, leaving a large uncertainty in the possible values of the parameter $\beta$. In fact, the range of Bjorken $x$ values currently explored by HERMES and COMPASS experiments is limited to $x_B \lesssim 0.3$.

This uncertainty in the knowledge of the transversity distribution at large $x$ values has relevant consequences when one uses the parameterizations extracted from SIDIS and $e^+e^-$ data for the study of single spin asymmetries in hadronic collisions. In this case the largest pion asymmetries have been measured at large Feynman $x$ values, $x_F \gtrsim 0.3$; then, kinematical cuts imply that the transversity distribution is probed at even larger $x$ values.

In order to assess the possible relevance of the Collins effect in explaining a large value of $A_N$ in $pp$ collisions we should explore in greater detail the large $x$ contribution of the transversity distribution. We have then devised a simple analysis, to which we will refer to as the “scan procedure” and which is based on the following considerations.
• In our parameterizations, the large $x$ behaviour of the quark transversity distributions is driven by the parameters $\beta_d$, the exponents of the $(1-x)$ factor in $N_q^T(x)$, see Eqs. (11) and (13). Not surprisingly, for the dominant $u$ and $d$ contributions, the values of the single $\beta_u = \beta_d = \beta$ parameter are indeed very different in the SIDIS-1 and SIDIS-2 sets, despite the fact that they offer comparably good fits of SIDIS and $e^+e^-$ data. As a consequence, the two sets give strongly different estimates of the pion SSAs at large $x_F$ in hadronic collisions. It is then natural to conclude that the choice of a flavour-independent $\beta$ parameter, good for SIDIS data, is a much too strong assumption for the hadronic collisions, and must be released in this analysis. We also notice that both in SIDIS-1 and SIDIS-2 fits the best-fit values of $\delta_{uu} = \delta_{ud} = \delta$ are very close to zero.

• We therefore start the scan procedure by performing a preliminary 9-parameter “reference fit” to SIDIS and $e^+e^-$ data taking, w.r.t. Eq. (13), $\beta_u \neq \beta_d$ and $\delta = 0$. We then let the two parameters $\beta_u$ and $\beta_d$ vary independently over the range 0.0–4.0 by discrete steps of 0.5. Larger values of $\beta$ would give negligible contribution to $A_N$. For each of the 81 points in this two-dimensional grid in $(\beta_u, \beta_d)$ space we perform a new 7-parameter fit to SIDIS and $e^+e^-$ data, keeping $\beta_u,d$ fixed and $\delta = 0$, but leaving all remaining 7 parameters in Eq. (15) free.

• As a next step we select only those sets of parameters from the scan procedure over the $(\beta_u, \beta_d)$ grid leading to an increment of the total $\chi^2$ of the fit, as compared to the corresponding 9-parameter reference fit, smaller than a given chosen value, $\Delta \chi^2$. Notice that, since the reference fit and the scan fits have a different number of free parameters, the selection criterium is applied to the total $\chi^2$ rather than to the $\chi^2$ per degree of freedom, $\chi^2_{\text{dof}}$. The chosen value of $\Delta \chi^2$ is the same as that used in Refs. [13, 10] to generate the error band, following the procedure described in Appendix A of Ref. [8]. It is worth noticing that all 81 points of our grid in $(\beta_u, \beta_d)$ lead to acceptable fits; this further confirms the observation that the SIDIS data cannot constrain the large $x$ behaviour of the transversity distribution.

• For each of the selected sets, we calculate the Collins pion SSA for polarized $pp$ collisions, Eqs. (10–13), in the kinematical regions of the available data from the E704 Collaboration at Fermilab and the STAR (for $\pi^0$) and BRAHMS (for charged pions) Collaborations at RHIC.

Finally we generate a “scan band”, by taking the envelope of ALL curves for $A_N(\pi)$ obtained by using the sets selected in the scan procedure, and compare this band with the experimental data available. This band shows the potentiality of the Collins effect alone to account for $A_N(p^+p \rightarrow \pi X)$ data while preserving a combined fair description (quantified by $\Delta \chi^2$) of the SIDIS and $e^+e^-$ data on Collins azimuthal asymmetries.

• Notice that in the data sets used for the fits of the scan procedure we have also included the recent preliminary data by the COMPASS Collaboration on SIDIS off a transversely polarized proton target [74], which were not available at the time when Refs. [13, 10] were published. Strictly speaking, therefore, the parameterizations used as starting point of the scan procedure are not the same as those published in Refs. [13, 10]. However, we have verified that the new parameterizations are only slightly different from the previous ones and are qualitatively consistent with them. Therefore, here we will not present and discuss them further, referring for more details to a future complete upgrade of our parameterizations. Apart from the insertion of this new set of COMPASS data, all technical aspects of the fitting procedure followed here are the same as in Refs. [13, 10], with the difference that, for the QCD evolution of the Collins function, limited to its collinear $z$-dependent $D_{h/q}(z)$ factor, we have attempted, in addition to an unpolarized-like evolution, also a transversity-like one, driven by the transversity evolution kernel.

Results and comments

We have computed $A_N(p^+p \rightarrow \pi X)$ adopting, as explained above, a single set of collinear parton distributions [74, 75], two different sets for the pion collinear FFs [72, 73] and two different (partial) evolution schemes for the Collins function. In Figs. 1–3 we show some of our results, avoiding the explicit presentation of other cases with very similar outcomes (some further comments are given below). For all results presented the Kretzer set for the unpolarized FFs and the unpolarized-like Collins evolution have been used.

In Fig. 1 the scan band for $A_N$, as a function of $x_F$ at fixed scattering angles, is shown for charged pions and BRAHMS kinematics, while in Fig. 2 the same result is given, at fixed rapidity values, for neutral pions and STAR kinematics; analogous results, as a function of $P_T$ at several fixed $x_F$ values, are shown for STAR kinematics in Fig. 3. These results allow to draw some first qualitative conclusions:

• The Collins contribution to $A_N$ is not as tiny as claimed in Ref. [1];

• The Collins effect alone might in principle be able to explain the BRAHMS charged pion results on $A_N$ in the full kinematical range so far explored;
FIG. 1: Scan band (i.e., the envelope of possible values) for the Collins contribution to the charged pion single spin asymmetries $A_N$, as a function of $x_F$ at two different scattering angles, compared with the corresponding BRAHMS experimental data [33]. The shaded band is generated, adopting the GRV98 and GRSV2000 sets of collinear PDFs, the Kretzer FF set and an “unpolarized-like” evolution for the Collins function, following the procedure explained in the text.

FIG. 2: Scan band (i.e., the envelope of possible values) for the Collins contribution to the neutral pion single spin asymmetry $A_N$, as a function of $x_F$ at two different rapidity values, compared with the corresponding STAR experimental data [34]. The shaded band is generated, adopting the GRV98 and GRSV2000 sets of collinear PDFs, the Kretzer FF set and an “unpolarized-like” evolution for the Collins function, following the procedure explained in the text.

- The full amount of the $\pi^0$ STAR data on $A_N$ cannot be explained by the Collins contribution alone. The Collins effect might be sufficient for the small $x_F$ portion of the data; however, it is not sufficient for the medium-large $x_F$ range of STAR data, $x_F \gtrsim 0.3$.

The results obtained with a different choice of the fragmentation functions (the DSS set) are qualitatively very similar in the large $x_F$ regions. They are instead smaller in size at smaller $x_F$, due to the large gluon contribution in the leading order (LO) DSS fragmentation functions. The use of a transversity-like Collins evolution, rather than the unpolarized one, does not lead to any significant difference, in all cases.

At this point, in order to fully assess the role of the Collins effect in understanding the large SSAs for neutral pions
measured at large $x_F$ by the STAR Collaboration at RHIC, we have performed several further tests.

First of all, we should make it clear that the scan bands presented in our plots have nothing to do with the statistical error bands presented in Refs. [15, 16]. There, the error bands are generated by estimating the uncertainty in the best fit values of the parameters, according to the procedure described in detail in Appendix A of Ref. [8]. Instead, the scan bands in this paper are obtained by simply taking the envelope of all curves generated by the selected best fit sets within the full grid in $\beta_{u,d}$.

It is not clear how to combine the statistical error band, associated with the full 7 or 9 free-parameter best fits of SIDIS and $e^+e^-$ data, with the scan bands. Therefore, in order to understand to what extent the statistical errors on the best fit parameters may affect the capability of the Collins effect to reproduce the large $x_F$ STAR data, we have adopted the following strategy: besides considering the envelope of the full set of curves produced by the scan procedure, we have considered explicitly each of these curves, isolating the set leading to the largest asymmetries in the large $x_F$ region; we have then evaluated, as in Appendix A of Ref. [8], the corresponding statistical error band, which covers larger values of the asymmetry. Our result is presented in Fig. 4. Again, it appears that the Collins effect alone cannot account for the large $x_F$ data. Notice also that trying to fit the large $x_F$ data on $A_N$ might lead to an overestimation of the same data at smaller $x_F$, which have tinier error bars.
There is still another issue that deserves some attention. Although simplified, our parameterization of the TMDs and of their functional shape involves in principle a huge number of fit parameters. Since most of these parameters are highly correlated, adopting larger set of parameters would lead to larger uncertainties in their value. Therefore, reasonable fits require a reduction in the number of fit parameters and involve a careful choice of the most significant ones. This choice may have consequences on the allowed values of the asymmetries, particularly for kinematical regions not covered by the data sets used for the fitting procedure. In the present scan procedure we use a 7-parameter fit and a grid of values for the two additional parameters $\beta_u, \beta_d$. In order to investigate if a larger set of free parameters for the scan procedure could modify our conclusions about the Collins effect for the STAR data in the large $x_F$ range, we have repeated our scan procedure by starting from a preliminary reference fit with 13 free parameters, 

\[
N_{u,T}, N_{d,T}, \alpha_u, \alpha_d, \beta_u, \beta_d, N_{u,T}^{\text{fav}}, N_{u,T}^{\text{unf}}, \gamma_{\text{fav}}, \gamma_{\text{unf}}, \delta_{\text{fav}}, \delta_{\text{unf}}, M_h ,
\]

and generating again the scan band on the two dimensional grid for the fixed $\beta_{u,d}$ parameters by fitting for each grid point the remaining 11 parameters. This naturally results in a sizably larger scan band, with its upper edge approaching better $A_N$ at the larger $x_F$ values. However, also in this case, looking at all the 81 fit sets we find that the curve with the best behaviour at large $x_F$ approaches the upper edge of the scan band in the full $x_F$ range. It therefore largely misses (overestimates) the lower $x_F$ values of the asymmetry. In Fig. 5 we present the statistical error band on the Collins contribution to $A_N(p^+p \rightarrow \pi^0X)$ generated, following Appendix A of Ref. [8], from the 11-parameter best-fit set which optimizes the agreement with STAR data at large $x_F$.

Let us finally make some comments on the (charged and neutral) pion SSAs for the E704 kinematics [28, 29]. The situation in this case is complicated by the fact that, contrary to the STAR and BRAHMS kinematics, the unpolarized cross sections are largely underestimated within the TMD LO factorized approach when adopting the values of $\langle k_T^2 \rangle$ and $\langle p_T^2 \rangle$ extracted from SIDIS data. Indeed, much larger effective values are required to reconcile the TMD estimates with data, as it was shown in Ref. [41]. However, this fact should have less influence on the SSAs, defined as ratios of (sums and differences of) single-polarized cross sections.

We have therefore directly applied the scan procedure, as illustrated above, also to the E704 results. Again, it turns out that the scan band could cover the data for the neutral pion SSA, with some problem at the largest $x_F$ values. However, it largely misses the huge charged pion SSAs observed in the same kinematical region.
III. CONCLUSIONS

We have investigated the possible role of the Collins effect in explaining the large SSAs observed in $p^+p \rightarrow \pi X$ reactions; we have done so within a TMD factorized scheme, and have revisited a previous work on the same issue, correcting a numerical error and using new experimental data and new phenomenological information on the transversity distribution and the Collins function.

We can conclude that, to the best of our present knowledge, based on SIDIS and $e^+e^-$ data, the Collins effect alone seems to be able to reproduce the available RHIC data on pion single spin asymmetries in polarized $pp$ collisions, only in the small Feynman $x$ region, $x_F \lesssim 0.3$. Above that, which is the region where the values of $A_N$ increase, the Collins effect alone is not sufficient.

Additional mechanisms are required in order to explain the size of the $A_N$ asymmetry in this region. One can obviously think of the Sivers effect [38, 39, 41, 42]. Since TMD factorization has not been proven and is still under active debate for single inclusive particle production in hadronic collisions and since universality breaking effects are possible, one does not know exactly how to use the parameterizations of the quark Sivers functions extracted from SIDIS data in $pp$ collisions. A recent use of the SIDIS Sivers functions in a collinear higher-twist approach to SSAs in $pp$ collisions – rather than in the TMD factorized approach – has been found to give a sizable contribution to $A_N$, but with the wrong sign [77]. A similar conclusion holds in a modified generalized parton model approach with TMD factorization [78]. Notice that such a problem does not occur if one simply adopts the SIDIS Sivers functions in the TMD factorized scheme [79, 80]. Much further investigation is necessary.

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