Understanding BBN: the physics and its history

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Abstract

Big-bang nucleosynthesis (BBN), today a pillar of modern cosmology, began with the trailblazing 1948 paper of Alpher, Bethe and Gamow [1]. In it, they proposed non-equilibrium nuclear processes in the early Universe ($t \sim 1000 \text{ sec}$) and an early radiation-dominated phase to explain the abundances of all the chemical elements. Their model was fundamentally flawed, but initiated a complex and interesting path to the modern theory of BBN, which explains only the abundances of the lightest chemical elements (mostly $^4\text{He}$) and the discovery of the cosmic microwave background (CMB). The purpose of this paper is to clarify the basic physics of BBN, adding some new insights, and to describe how the modern theory developed. I finish with a discussion of two misunderstandings about BBN that still persist and the tale of the pre-discovery predictions of the temperature of the CMB and the missed opportunity it turned out to be.

1 Introduction

The landmark 1948 paper by Alpher, Bethe and Gamow [1] that started it al was motivated by the failure of equilibrium processes in stars to account the abundances of the chemical elements. Th Their paper – hereafter $\alpha\beta\gamma$ – broke new ground with its addition of a radiation component to the Universe and attention to particle interactions (here, nuclear reactions) during the earliest moments. Both are hallmarks of our understanding of the early Universe today.

1For a through review of this point, see Ref. [3].
Because the energy density in photons scales as $R^{-4}$ while that in matter scales as $R^{-3}$, a hot, radiation-dominated beginning is a natural consequence of considering a photon component in the Universe, even the tiniest. (Today, photons contribute about 0.01% of the total energy density, yet earlier than 60,000 yrs radiation dominated the energy density of the Universe.) As the Universe cools and becomes matter-dominated, the embers of the hot beginning remain with us today in the form of the CMB (here $R$ is the cosmic scale factor).

Gamow’s theory as it was often referred to, attracted a lot of attention, in no small part because of his stature and big personality. However, the basic nuclear physics underlying their paper – *non-equilibrium* neutron capture – was wrong and it took almost 30 years to straighten out, culminating in the seminal 1967 paper by Wagoner, Fowler and Hoyle. Moreover, when all the dust settled, there is no significant nucleosynthesis beyond $^4$He. As I describe, the modern understanding of big-bang nucleosynthesis is mostly *equilibrium* nuclear physics. I suspect that the failure of equilibrium nuclear physics in stars to account for the chemical abundances strongly influenced the starting point, leading to the long delay in arriving at the correct, mostly equilibrium picture.

In the next section, with the benefit of more than 70 years hindsight, I elucidate the basic physics of BBN, with some new insights. I go on to discuss my reconstruction of the physics history, paying close attention to the thread that led to the modern theory.

Because there is still some confusion about the physics of BBN, in the next section I clear up some persistent misunderstandings about BBN, the so-called deuterium bottleneck and the supposed importance of the absence of stable nuclei of mass 5 and 8, both of which are relics of the original neutron-capture model. After that, I discuss the interesting tale of the pre-discovery predictions of the CMB temperature that could have been made and were made by various people including Gamow, Alpher and Herman and Zeldovich. The missed opportunity of predicting the CMB temperature rivals Einstein’s missed opportunity for predicting the expansion of the Universe.

Lastly, I write this paper as a practicing scientist who has written close to sixty papers on big bang nucleosynthesis, and not as a science historian. And I do so at the risk of eliciting the ire of both communities, too much history for the scientist and too much science for the historian. However, I find the history of the physics underpinning this pillar of modern cosmology fascinating, with its big ideas, twists and turns and missed opportunities. I hope that the reader comes away with a better understanding of the physics, how that understanding developed and some appreciation of how foggy it often is at the frontiers of discovery.

\footnote{Further, the germ of the idea traces back to an earlier sole-authored paper he wrote and talks he gave.}
2 Physics of BBN

Like much of the history of the early Universe, the starting point for the discussion is thermal equilibrium. This is because the high-densities and particle energies in the early Universe almost always win out over the time scale set by the expansion rate $H \equiv \dot{R}/R \sim 1/t$, which determines the rate at which the temperature decreases since $T \propto 1/R$.

Of course, if thermal equilibrium were the entire story of the Universe, the state of affairs today would be very boring: a Universe comprised of the iron-group elements with no nuclear free energy to power stars and life here on Earth! The deviations from thermal equilibrium, even though relatively rare, are crucial.

Today, BBN is a mature subject, and my focus here is to elucidate at a high level the basic physics that underpins it and not the details. The top-level story is simple: At the earliest times ($T \gg 1$ MeV and $t \ll 1$ sec), the energy density of the Universe was dominated by that of radiation. Thermal equilibrium existed because particle interactions occurred rapidly compared to the rate at which the temperature decreased. Free nucleons were thermodynamically favored and weak interactions keep the ratio of neutrons to protons at its equilibrium value, $n/p = \exp[-\Delta m/T]$, close to unity ($\Delta m \approx 1.3$ MeV).

Very late ($T \ll 0.01$ MeV and $t \gg 10^4$ sec), nuclei are thermodynamically favored, any free neutrons have long-since decayed (neutron mean life $\tau_n \sim 900$ sec), and most importantly, nuclear reactions have ceased ("frozen out") owing to Coulomb barriers between the charged nuclei. Nuclear physics does not become relevant again until the first stars form a billion years or so later.

What happened in between very early and very late involves two departures from thermal equilibrium which determine the outcome of BBN. And this outcome only depends upon the baryon-to-photon ratio $\eta$, whose value remains constant as the Universe evolved and today is known to be $6.03 \pm 0.04 \times 10^{-10}$.

The inverse of $\eta$ tells us there are about 2 billion photons per baryon.

This is an extraordinarily high entropy; for comparison, the value at the center of the sun is only around $10^{-3}$ photons per baryon and at the center of a newly-formed neutron star about 1 photon per baryon. Further, $\eta$ is related to the fraction of critical density contributed by baryons:

$$\eta_0 \equiv \frac{\eta}{10^{-10}} = \frac{274 \Omega_B h^2}{T_{2.73}^3},$$

where the Hubble constant today $H_0 \equiv 100h \text{ km/s/Mpc}$, $T_{2.73} \equiv T_0/2.73$ K.

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The baryon-to-photon ratio remains constant so long as there is no entropy creation or transfer because the number density of baryons decreases as $R^{-3}$ as does the number density of photons, which is proportional to $T^3$ and $T \propto R^{-1}$. Around a temperature of 0.5 MeV, the entropy in $e^\pm$ pairs is transferred to the photons, increasing the photon number density by a factor of 11/4. While the outcome of nucleosynthesis depends upon the temperature $T$ and the nucleon density $n_B$; because of the constancy of $\eta$, the nucleon number density is simply $n_N \propto \eta T^4$; see Refs. [12, 13]. The baryon-to-entropy ratio is exactly constant in the absence of significant entropy production, and today it is simply related to $\eta$: $\eta = 7.04 n_B/s$ [13].
Figure 1: Key temperatures during BBN vs. the baryon-to-photon ratio \( \eta \): 
- \( T_F \), the freeze in of the n/p ratio;
- \( T_{\text{nuclei}} \), the temperature below which nuclei are thermodynamically favored over free nucleons;
- \( T_{\text{Coulomb}} \), the temperature below which charged-particle nuclear reactions cease occurring; and
- \( T_n \), the temperature at which the age of the Universe is the lifetime of a free neutron.

Any significant nucleosynthesis beyond deuterium requires \( T_{\text{nuclei}} \geq T_{\text{Coulomb}} \), or \( \eta \geq 10^{-11} \). The vertical line marked “time” shows the timeline of successful BBN: freeze in of the n/p ratio at \( T = T_F \rightarrow \) period of waiting until nuclei are favored \( T_F > T > T_{\text{nuclei}} \rightarrow \) nucleosynthesis \( T_{\text{nuclei}} > T > T_{\text{Coulomb}} \rightarrow \) frozen out nuclear reactions \( T_{\text{Coulomb}} > T \rightarrow \) any free neutrons remaining decay \( T_n > T \).

\[ \Omega_B \equiv \frac{\rho_B}{\rho_{\text{crit}}} \] is the fraction of critical density contributed by baryons and the critical density \( \rho_{\text{crit}} = 1.88h^210^{-29} \text{g/cc} \).

Three events are crucial in the all-important “in-between phase,” where there is neither thermal equilibrium nor frozen-out nuclear reactions. First, the weak interactions freeze out around \( T \sim 1 \text{MeV} \) and the ratio of neutrons-to-protons freezes in at a value of about \( 1/7 \), rather than exponentially decreasing to zero. This means the first step of any BBN is an EM interaction, \( n + p \rightarrow d + \gamma \). 

This fact is crucial: unlike the center of a star which remains hot for billions of years, there is not enough time in the early Universe for weak interactions to take place.

Next, a simple calculation of Nuclear Statistical Equilibrium (NSE) shows that nuclei become favored over free nucleons at a temperature \( 13 \):

\[ T_{\text{nuclei}} \simeq \frac{B_A/(A-1)}{\ln \eta^{-1} + 1.5\ln(m/T)} \simeq 0.25 \text{MeV}[1 + \ln \eta_{10}/35] , \quad (1) \]

\footnote{It began slightly larger and slowly decreased to this value due to neutron decays.}
where $B_A$ is the binding energy of nucleus $A$ and $m \simeq 1$ GeV is the nucleon mass. Because of the very large number of photons per baryon ("high entropy"), nuclei are not thermodynamically favored until a temperature much, much less than a typical nuclear binding energy (here taken to be that of $^4$He, or about 7 MeV per nucleon). Note too, more photons per baryon (smaller \( \eta \)) lowers the temperature at which nucleons become favored and delays BBN.

Finally, any nucleosynthesis beyond deuterium must involve charged-particle on charged-particle reactions, e.g., $d + d \rightarrow ^4$He + $\gamma$ or $t + D \rightarrow n + ^4$He, and so Coulomb barriers – the electrostatic repulsion between charged nuclei – are important. They suppress nuclear reaction rates by an exponential factor; for $d(d,\gamma)^4$He, that factor is approximately $\exp[-2(T/{\text{MeV}})^{-1/3}]$. Because of this, the nuclear reactions that are needed to produce anything beyond deuterium will cease occurring ("freeze out") at a temperature that can be estimated by comparing the expansion rate $H \simeq T^2/m_{\text{pl}}$ to the nuclear reaction rate $\Gamma$ (see appendix for more details):

$$T_{\text{Coulomb}} \sim \frac{0.03\text{ MeV}}{[1 + \ln \eta_{10}/7]^{3/8}}.$$  \hspace{1cm} (2)

In this case, "higher entropy" – that is, smaller \( \eta \) – means nuclear reactions freeze out at a higher temperature.

Bringing it all together, for there to be any nucleosynthesis beyond deuterium, nuclei must become thermodynamically favored before Coulomb barriers freeze out nuclear reactions, or $T_{\text{nuclei}} \geq T_{\text{Coulomb}}$. This corresponds to $\eta > 10^{-11}$ (see Fig. 1). In this case, essentially all the neutrons end up in the thermodynamically-favored $^4$He nuclei, around 25% by mass. By the time other nuclei become more favored than $^4$He, Coulomb barriers have shut off any additional nuclear reactions.

From this basic picture, we can say that BBN makes four robust predictions:

- A significant amount of $^4$He (10% to 30%) is produced provided $\eta > 10^{-11}$
- A large amount of hydrogen remains (70% to 90% by mass) because nucleosynthesis is limited by the available neutrons ($n/p \sim 1/7$)
- Little nucleosynthesis beyond $^4$He occurs because the binding energy of $^4$He is so large and other nuclei are not thermodynamically favored until Coulomb barriers have already precluded further nucleosynthesis
- Some deuterium remains unburnt, with an abundance that decreases rapidly with $\eta$ since more deuterium can be used up when the nucleon density is larger

For $\eta < 10^{-11}$, only deuterium is synthesized (as much $D/H \sim 10^{-2}$), and for the very smallest values, $\eta \ll 10^{-11}$, even deuterium is not produced because all of the neutrons have decayed by the time deuterium is thermodynamically favored. The detailed calculations of Wagoner, Fowler and Hoyle [5] (and all other calculations since) bear out this picture; see Fig. 2. The physics of BBN is explained in more detail elsewhere; see e.g., [13, 19].
Figure 2: Abundances calculated by Wagoner, Fowler and Hoyle [5]; cross sections and physics has since been updated, but the results are largely correct, especially when viewed on a log-log plot. Note: their “little $h$” is not the same as mine; their baryon mass density and $\eta$ are related by, $\eta \approx 1.1 \times 10^{-12}/(\rho/10^{-33} \text{g/cc})$, so that their calculations span nine orders-of-magnitude, $\eta \approx 10^{-12}$ to $10^{-3}$.

3 History of BBN physics

The fog at the frontiers of discovery is always thick, and the path to a clear understanding of a new idea with all the details in place usually takes awhile. BBN was no different. It was the early days of nuclear physics and nuclear data was sparse; particle physics was not yet a field; scientific computing was in its infancy, and astronomy was done with photographic plates and visible light.

This section describes my reading of the original papers. It begins with the landmark $\alpha\beta\gamma$ paper in 1948, and ends almost 30 years later with the equally important paper by Wagoner, Fowler and Hoyle [5] in 1967. Because of the strong initial focus on non-equilibrium nucleosynthesis, the story began far from where it would end, a mostly equilibrium story with a couple of significant deviations from thermal equilibrium.
In the neutron-capture model, the Universe begins entirely with neutrons. The needed protons are produced by neutron decays, setting the timescale for nucleosynthesis, \( t \sim \tau_n \sim 1000 \text{ sec} \). In \( \alpha\beta\gamma \) the limited neutron-capture data were used to identify the nucleon density needed to move up the periodic table. Shortly thereafter, additional papers were published with numerical calculations of the production of deuterium and other light elements \([14, 15, 16]\) and made predictions for the temperature of the CMB. The key assumptions were an initial state of pure neutrons, with only neutron decays and captures taken into account.

Two criticisms of Gamow’s theory were immediately raised. In an unpublished correspondence with the authors, Fermi and Turkevich pointed out that the absence of long-lived isotopes with mass 5 and 8 prevents significant nucleosynthesis beyond \( ^4\text{He} \) in their neutron capture model (all the nuclides of mass 5 have lifetimes in the yoctosecond range; and for mass 8 at most in the msec range). While a real concern for the neutron-capture model, this point is not relevant for what actually occurs and leads to one of the BBN myths as I discuss in the next section.

Hayashi \([17]\) made an even more important point: the weak interactions – through the so-called beta reactions – involving neutrons and protons \((n + e^+ \leftrightarrow p + \bar{\nu}_e)\) should occur rapidly and regulate the ratio of neutrons-to-protons precluding the initial state of all neutrons assumed by \( \alpha\beta\gamma \). This led to a series of papers, including one by Alpher, Follin and Herman \([8]\), that would focus on computing the neutron-to-proton ratio by numerical integration. (While the discovery of the neutrino did not happen until 1956, Fermi’s theory of the weak interactions was well developed and well known.)

The other key element on the path to the modern theory was to incorporate a nuclear-reaction network that included charged-particle reactions as well as neutron captures. That was done around the same time – circa 1949 – in an unpublished paper by Fermi and Turkevich, entitled, *Thermonuclear reactions in the first hour of an expanding Universe*. While unpublished, it is described in detail in the 1950 RMP article by Alpher and Herman \([3]\). Their network contained 28 reactions, and they tracked the abundances of H, He, Li, and Be.

However, they too assumed an all neutron initial state; they did not include the beta reactions that set the neutron-to-proton ratio nor the inverse reactions that are so important because of the high entropy of the Universe. Their predicted \(^4\text{He} \) mass fraction for \( \eta \approx 2 \times 10^{-11} \) of 0.28 is about a factor of 2 high compared to the correct calculation, though a comparison senses a bit non-sensical since their calculation is conceptually wrong. Nucleosynthesis beyond \(^4\text{He} \) does not occur because of the absence of stable mass 5 and 8 nuclei.

\footnote{All told, more than a dozen papers on BBN were published by Alpher and various co-authors, mostly with Robert Herman and only the first with George Gamow and/or Hans Bethe as co-authors. The ones cited in this paper are the most relevant for the history of BBN physics.}

\footnote{Unlike weak interactions amongst nuclei, the beta interactions are important because they involve nucleons interacting with the thermal photons, electron/positron pairs and neutrinos which are \( 10^{10} \) times more abundance than nucleons and thus occur at a much fast rate.}
Taken together, the Hayashi insight and the Fermi/Turkevich network if combined would take one almost all the way to WFH; however, the paper of Fermi and Turkevich was never published.\(^7\)

In 1964, just before the discovery of the CMB, Hoyle and Tayler put the two together in a paper entitled, *The mystery of the the cosmic Helium abundance* \(^6\). Here they combined approximate tracking of the \(n/p\) ratio and a small nuclear network (not described in detail) to predict a \(^4\)He abundance of around 36\%, which they concluded was consistent with estimates of the observed He abundance, whose value was not well determined.\(^8\)

Hoyle and Tayler get the \(^4\)He abundance high, stating in the penultimate paragraph on page 1109, “we can now say that if the Universe originated in a singular way the He/H cannot be less than about 0.14,” which corresponds to a \(^4\)He mass fraction of about 36\%.\(^9\) Their statement is based upon the two paragraphs above, in which they argue that any variations in the cosmological conditions and parameters would only increase \(^4\)He production. Of course, we know that for \(\eta < 10^{-11}\) there is essentially no nucleosynthesis and only at very high values of \(\eta\) does \(^4\)He production reach 36\%. There is not enough detail in the paper to see where they went wrong.

Just after the discovery of the CMB, Peebles carried out a similar calculation, with a limited reaction network and an approximate treatment of the \(n/p\) ratio, arriving at a \(^4\)He abundance of between 27\% and 30\% (for \(\eta = 4 \times 10^{-10}\)). Both

\[^7\]One can speculate why; among other things, Turkevich was deeply involved in work involving nuclear weapons test monitoring at Los Alamos during this period [18]. Nonetheless, the calculations were state of the art for the time and used some of the most powerful computers at the time, e.g., the ENIAC and the MANIAC at Los Alamos.

\[^8\]They also noted that the cosmic Helium problem could be solved if similar conditions were achieved in a supermassive star. This is a theme that would continue in the WFH paper and presumably had to do with Hoyle’s disdain for the hot big-bang model.

\[^9\]Various authors track different quantities; under the (good) assumption that all the neutrons wind up in \(^4\)He, \(Y = 2(n/p)/(1 + (n/p)) = 4(n_{He}/n_H)/(1 + 4(n_{He}/n_H))\), where the Helium-to-Hydrogen ratio \(n_{He}/n_H = (n_n/2)/(n_p - n_n)\).
Peebles and Hoyle & Tayler halted their numerical integrations at a temperature of around $5 \times 10^9$ K, without a clear argument for doing so. Unlike, the Hoyle-Tayler paper, Peebles' paper does show the correct qualitative variation of $^4$He production with baryon density (or $\eta$) in Fig. 1.

A few months after Peebles' paper, the 47-page paper of Wagoner, Fowler and Hoyle put it all together. The abundances of nuclides $A \leq 23$ were tracked individually and for $A \geq 24$ collectively; the thermal bath included photons, neutrinos, and electron/positron pairs; no approximations were made in tracking the n/p ratio and the nuclear-reaction network contained 79 reactions; and the numerical integration began at $T \sim 10^{11}$ K and continued down to $T \sim 5 \times 10^7$ K, until well after Coulomb barriers had quenched all nuclear reactions. The results are shown in Fig. 2.

Since then, BBN calculations have been updated for changes in nuclear-reaction rates (most importantly the neutron lifetime) and small physics effects at the percent level have been accounted for to obtain greater precision (e.g., finite nucleon mass, radiative corrections, finite temperature quantum corrections, and the slight heating of the neutrinos by $e^\pm$ annihilations). Nevertheless, it is the WFH paper that finally put all the physics together and is the forerunner of all modern calculations.

Summing up, the theory of BBN started with the non-equilibrium, neutron-capture model, with the all-important neutrons being supplied as an initial condition. The weak interactions that regulate the n/p ratio and the crucial inverse reactions that would reflect the high entropy of the Universe were absent. The timescale in the $\alpha\beta\gamma$ model was set by the neutron lifetime – then determined to be around 2000 sec – because neutron decays were needed to produce the protons necessary for BBN.

The modern picture of BBN is a largely a story of thermal equilibrium, with the high entropy preventing significant nucleosynthesis until a temperature of about 0.2 MeV. The neutrons needed for BBN arise because the weak reactions that regular the n/p ratio cease to be effective at around 1 sec (temperature of 1 MeV) and the ratio freezes in at a value of 1/7. Without the freeze out of the weak interactions there would be no neutrons around by the time nuclei became thermodynamically favored and hence no BBN. The falling temperature and the exponentially rising effect of Coulomb barriers ends BBN at around 200 sec (temperature of around 0.05 MeV), shortly after $^4$He is statistically favored (provided that $\eta > 10^{-11}$). That is why Steven Weinberg’s famous book is entitled, The First Three Minutes, and not, as the Fermi/Turkevich paper is entitled, the first hour of the Universe!

If there is one number that screams out the difference between the neutron-capture model and the modern theory of near-equilibrium physics it is when nucleosynthesis took place: around 2000 sec in the former and around 200 sec in the latter. The reason is very simple: in the neutron-capture model the needed protons are produced by neutron decays which take almost an hour, and in the modern theory nucleosynthesis must occur before the neutrons decay and before
Coulomb barriers become insurmountable.\footnote{A theorist can’t resist asking if there is any scenario in which the neutron-capture model would be relevant. It might be a good approximation for a Universe that began with pure neutrons at an age of around 500 sec and temperature of about 0.05 MeV. In this case, the beta-reactions would be ineffective, and inverse reactions with photons that break up nuclei would be as well. Thus both could be neglected, and the calculations of Fermi and Turkevich might be a good approximation. Unfortunately, the computed $^4$He mass fraction is only 20\% (too low) and the deuterium abundance is 1\% (too high); see Fig. 20 of Ref. \cite{5}.}

Figure 4: Left: Time evolution in the neutron-capture model \cite{14} (left), and in the modern theory of BBN \cite{5} (right). The factor of 10 extra time in the neutron-capture model is for neutrons to decay and produce the protons needed for nucleosynthesis. In the modern theory, the neutrons are relic of the freeze out of the beta reactions (at around 1 sec); nucleosynthesis is over by 200 sec because Coulomb barriers preclude any further nuclear reactions.

\section{Misunderstandings and missed predictions}

\subsection{Two persistent misunderstandings}

The path from a neutron-capture model that aspired to synthesize the entire periodic table of elements to the present nearly equilibrium model of the synthesis of the light elements has left its imprint on the discussion of big-bang nucleosynthesis today. The so-called deuterium bottleneck that supposedly delays the synthesis of $^4$He and the supposed importance of the absence of long-lived nuclei with mass 5 and 8 preventing nucleosynthesis beyond Helium, neither of which is true, can be found in review articles \cite{21}, Wikipedia’s discussion of
BBN and in the many on-line lectures about BBN that can be found through Google searches. In reality, the real culprits are the high entropy and Coulomb barriers. In one way or another, both are relics of the pioneering work on BBN.

Early on, Fermi and Turkevich pointed out that the absence of stable nuclei of mass 5 and mass 8 is a roadblock to the neutron capture model going beyond He synthesis. It is the correct argument, for the neutron-capture model. However, in the modern theory, charged-particle reactions bridge the gap; e.g., $^3\text{He} + ^4\text{He} \rightarrow ^7\text{Be} + \gamma$ and so on; see Fig. 3.

Nonetheless, one still sees the absence of significant nucleosynthesis beyond $^4\text{He}$ blamed on the absence of stable nuclei with mass 5 and 8. The real reason is simply thermodynamics: by the time heavier nuclei are thermodynamically favored, Coulomb barriers have shut off any further nucleosynthesis. It is a simple matter to add a stable mass 5 and/or 8 nuclide to the BBN code and find that nothing changes!

Another persistent misunderstanding is that of blaming the late onset of BBN on “the deuterium bottleneck,” namely that the small binding energy of deuterium (about 2.22 MeV) is responsible for BBN beginning at such a low temperature compared to typical nuclear binding energies. As discussed earlier, the cause is really the high-entropy of the Universe, not deuterium.

The term itself seems to have been coined by Weinberg in his discussion of the role of Nuclear Statistical Equilibrium during BBN (see p. 553 of Ref. [19]), where he says, “The low binding energy of deuterium thus serves as a “bottle-neck,” which delays the formation of complex nuclei ...”. His words are correct, but misleading. Considerations of NSE alone would have the $^4\text{He}$ abundance soar at a temperature of about 0.3 MeV, far below the binding energy of 7 MeV/nucleon; in actuality, it occurs closer to 0.1 MeV. The delay is a “supply chain” issue, with many parties to blame.

All of the reactions that produce $^4\text{He}$ have rates that are proportional to $\eta \times X_D \times T^3 \langle \sigma v \rangle$. Each one of the three factors contributes to the brief delay in converting all of the available neutrons into $^4\text{He}$ when $^4\text{He}$ is thermodynamically favored: the small baryon-to-photon ratio $\eta$, the fraction of nucleons which are deuterons, $X_D \sim 10(T/m_N)^{3/2} \exp(2.22 \text{MeV}/T)$, and the thermally-averaged reaction rates which are suppressed by Coulomb barriers.

To blame it on deuterium alone is to neglect the other factors at play. In fact, deuterium is the hero: to move things along it becomes massively overabundant relative to NSE (by a factor of a billion!). Further, it is Coulomb barriers becoming insurmountable before $^4\text{He}$ is thermodynamically favored that leads to no nucleosynthesis for $\eta < 10^{-11}$. Moreover, for $\eta < 10^{-11}$, $^4\text{He}$ production is proportional to $\eta$ (see Fig. 2), just as expected since $^4\text{He}$ production is rate-limited in this regime.

A more spirited defense of deuterium and longer discussion of both misunderstandings is given in Chapter 4 of Ref. [13].
4.2 Missed predictions

The story of the discovery of the CMB and the mis-steps along the way is fascinating and well documented [11, 20, 22]. On the theory side, there was the opportunity of a lifetime – predicting the existence of the CMB and its temperature on the basis of BBN. Here I focus on the predictions that could have been made and the predictions that were actually made, remembering that hindsight is always of great advantage.

A couple of formulae are useful to ground the discussion. Once the baryon and radiation densities are fixed at any epoch, they are specified at all epochs, and the temperature at a given baryon density can be predicted. There are numerous ways to express that crucial ratio, which can make comparisons between different works arithmetically challenging. Since the baryon-to-photon ratio is the single parameter that determines the outcome of BBN, I will use it to specify the ratio of the matter and radiation densities:

\[ \rho_B = 6.86 \times 10^{-32} \eta_{10} T_{2.73}^3 \text{g/cc} \] (1)

\[ T_{\text{CMB}} = \left( \frac{\rho_B / (6.86 \times 10^{-32} \text{g/cc})}{\eta_{10}} \right)^{1/3} 2.73 \text{K} \] (2)

This formula can also be written in terms of the fraction of critical density contributed by baryons,

\[ T_{\text{CMB}} = \left[ \frac{274 \Omega_B h^2}{\eta_{10}} \right]^{1/3} 2.73 \text{K} \] (3)

Without detailed calculations, based solely upon the estimates from §2, we can say: if \( \eta > 10^{-11} \), there is significant nucleosynthesis, i.e., \(^4\text{He} \) mass fraction of up to 30% or so and some leftover D; for \( \eta \) smaller than \( 10^{-11} \), essentially no nucleosynthesis takes place because when nuclei are thermodynamically favored, Coulomb barriers prevent the needed charged particle reactions from taking place.

This lower limit to \( \eta \) and a baryon density today of \( 5 \times 10^{-31} \text{g/cc} \) (the mean of what Peebles and Zel’dovich assumed; see below), predicts a temperature \( T_{\text{CMB}} < 11 \text{K} \). Or approaching it another way, taking \( \Omega_B \sim 0.1 \) and \( h \sim 1 \), values reasonable at the time, one would get: \( T_{\text{CMB}} < 20 \text{K} \). In either case, the prediction is an upper limit, between \( 10 \text{K} \) and \( 20 \text{K} \), for the temperature of the relic radiation. Going beyond this, to an actual prediction, requires both detailed calculations and knowledge of the primordial \(^4\text{He} \) abundance.

In discussing the history of BBN, I did not mention Zel’dovich and the Soviet school of cosmology. In part, because much of the work in the Soviet Union found its way into western science slowly if at all. He wrote a comprehensive review of cosmology just before the discovery of the CMB [23]. In it, there is a remarkable paragraph on page 491, which discusses BBN. While it lacks

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\(^{11}\)Knowledge of the baryon density is less important since the predicted temperature varies as the cube root of the baryon density.
equations or any detail, it is fully consistent with the picture I have discussed here and makes it clear that he had zeroed in on the correct physics. In that paragraph, Zel’dovich describes two possibilities, a low-entropy universe and a high-entropy universe, and dismisses them both.\footnote{That paragraph is expanded upon in a paper published a year later by Smirnov \cite{24}, where Smirnov describes calculations that he carried out at the behest of Zel’dovich and which underpin the conclusions in the paragraph. Smirnov discusses the context of his work in Ref. \cite{22}.}

For the low-entropy universe, $\eta \sim 10^{-8}$, the $^4$He production is 30%, which Zel’dovich concluded exceeds that seen in old stars. For the baryon density he uses, $\rho_B = 3 \times 10^{-31} \text{ g/cc}$, this corresponds to a CMB temperature of 1 K. For the high-entropy universe, $\eta \sim 10^{-12}$, he states that $^4$He production is less than 10% (also true) and the CMB temperature today would be 20 K. He goes on to say that the Universe would still be radiation-dominated (preventing structure formation), and argues against the high-entropy case as well.

Instead of considering an intermediate case – say $T_{\text{CMB}} \simeq 3$ K – where the $^4$He production would take on the Goldilocks value, Zel’dovich dismisses BBN and goes on to describe his cold big-bang theory. Two years later, after the discovery of the CMB, he writes another review of cosmology \cite{25}, enthusiastically discussing all of the consequences of a hot big bang, including BBN.

Also prior to the discovery of the CMB, Hoyle and Tayler \cite{6} carried out an almost-modern calculation of BBN, but made no prediction for the temperature of the CMB. Two other things worked against them making a prediction: first, their calculations overestimated the amount of $^4$He production (because of the lack of detail, it is hard to figure out the source of this error); and second, they did not explore the dependence of the $^4$He production on the present temperature.

This brings me to the most controversial, well-publicized and confusing prediction of the CMB temperature, that made by Alpher and Herman in 1948 \cite{15}. Their paper is a short note to Nature, correcting some numerical errors in the paper published by Gamow two weeks earlier \cite{14}. Gamow’s paper itself is based calculations from Alpher’s PhD thesis \cite{16}; and, he adds a new element to the discussion: the formation of galaxies. He notes that for his favored value ($\eta = 6 \times 10^{-11}$), when the Universe became matter-dominated and structure could begin to grow, the Jeans mass is comparable to that of small galaxies.\footnote{Not surprisingly, Gamow was unaware of Lifshitz’s pioneering and more extensive work on the evolution of small density perturbations and the growth of structure \cite{27}.}

While Gamow’s paper all but predicts a CMB temperature, he stops short of doing so. However, after correcting his numerical errors and near the end of their paper, Alpher and Herman predict a present temperature of 5 K for the relic radiation, remarkably close to the actual value. While Alpher often touts this prediction (see e.g., \cite{26}), as I now explain, it is fundamentally wrong. Read carefully, it is a little confusing.

Based upon Alpher’s numerical calculations of the neutron-capture model, Alpher and Herman chose a matter-to-radiation ratio that corresponds to $\eta = 4 \times 10^{-12}$. This is a very high-entropy Universe, where essentially no nucle-
osynthesis would take place. Further, such a universe would still be radiation-dominated today. This is essentially the model Zel’dovich correctly dismissed. In any case using a matter density of \(5 \times 10^{-31} \text{g/cc}\), this would lead to a prediction of \(T_{\text{CMB}} = 15 \text{K}\). However, recognizing a radiation-dominated Universe today would be problematic, Alpher and Herman go on to invoke large negative-curvature to solve the problem and find a present temperature of \(T_{\text{CMB}} = 5 \text{K}\).

So what is wrong with their close prediction? First, and foremost, it is based upon a model of nucleosynthesis – neutron-capture – that is incorrect! Close to the right answer, but for the wrong reason. Next, they have invoked another parameter, the curvature radius, not considered in any of the other calculations. While it was not known at the time that the Universe was flat, introducing curvature adds more problems than it solves.

Using the value they did, the Universe enters a negative-curvature dominated phase, where the scale factor increases linearly with time, at an age of about \(10^8\) yrs. At the present age of the Universe (they assume about 3 Gyr), the baryon density would have fallen to a very low value, about \(2 \times 10^{-32} \text{g/cc}\) corresponding to \(\Omega_B = 2 \times 10^{-4}\), much, much lower than any estimates of the matter density at the time. Further, this model is never matter-dominated, transitioning directly from radiation-dominated to curvature-dominated at \(10^8\) yrs. While not generally appreciated at the time, Lifshitz had shown that small density inhomogeneities only grow in a matter-dominated Universe, and so the structure we see today would never have formed \[27\].

4.2.1 Clearing the fog a bit

So what have we learned? First, understanding the correct physics and requiring some nucleosynthesis allows one to predict an upper limit to \(T_{\text{CMB}}\) of around 10 K, without a detailed calculation. To make an actual prediction one needs to know the primordial \(^4\text{He}\) abundance and the dependence of \(^4\text{He}\) production upon \(\eta\) as well as the baryon mass density today. Since the predicted temperature depends upon the cube root of \(\rho_B/\eta\), this lessens the dependence of the prediction upon these two parameters. However, poor knowledge of the primordial \(^4\text{He}\) abundance and the relative insensitivity of its predicted value to \(\eta\) near the correct value of 25% work in the other direction. Next, while Zel’dovich apparently had a reasonably accurate model for BBN, he passed up the opportunity to pick a value of \(\eta\) in between the two extreme values he correctly ruled out; had he done so, he would have made a prediction very close to 3 K. Finally, while Alpher and Herman’s “prediction” of 5 K is close to the actual CMB temperature of 2.73 K, it is purely coincidental and hard to take it seriously since is based upon wrong physics and leads to a Universe that was never matter-dominated, in which structure could not form, and for which \(\Omega_B = 2 \times 10^{-4}\).

In closing this section, I note that an observational upper limit to \(T_{\text{CMB}}\) was already known before the discovery of the CMB, at least to Hoyle. In 1940, McKellar’s \[28\] measurements of the population levels of interstellar CN and CH radicals constrained any “universal radiation field,” to have a temperature \(T_{\text{CMB}} < 5 \text{K}\). In fact, Hoyle used McKellar’s result as an argument against
Gamow’s hot big bang model in debates the two engaged in. After the discovery by Penzias and Wilson, the population levels of interstellar CN were used to make important determinations of the CMB temperature at shorter wavelengths [22].

5 Concluding remarks

Big-bang nucleosynthesis is a cornerstone of modern cosmology. Its correct prediction of a large amount of primordial $^4\text{He}$ was one of the first successes of the hot big-bang model. More recently, the measurement of the primordial deuterium abundance was used with BBN calculations to pin down the baryon density to sub-percent-level precision, $\Omega_B h^2 = 0.02166 \pm 0.00015$ [29]. This determination is consistent with a determination based upon measurements of the CMB made by the Planck Satellite, $\Omega_B h^2 = 0.02237 \pm 0.00015$ [30].

This is an impressive consistency check of today’s cosmological paradigm (ΛCDM): Two determinations of the baryon density, one based upon nuclear physics that took place seconds after the big bang and the other based upon the physics of the CMB, some 400,000 yrs later, agree to percent level precision.

This measurement of the baryon density together with a similarly precise determination of the total matter density, which is almost 6 times larger (50σ different), $\Omega_M h^2 = 0.143 \pm 0.0011$, today provides the linchpin in the case for non-baryonic dark matter.

Further, because BBN is so well understood and its predictions so precise, since 1977 it has been used as a probe of particle physics, e.g., by constraining the numbers and kinds of light particle species present in the thermal bath at a temperature of around 1 MeV [31].

The legacy of BBN might also be said to include the r-process in nuclear astrophysics, the process of building up the heaviest elements through rapid neutron capture – sound familiar? While the ambitions of $\alpha\beta\gamma$ were great, by the mid-1950s it was clear that synthesizing all the chemical elements in the big bang was not in the cards. Astrophysicists turned to exploding stars for the non-equilibrium conditions needed to make the heaviest elements [32, 33].

The story of big-bang nucleosynthesis, with its false starts and twists and turns, is just as intriguing as that of the CMB itself. I sang its praises in a Physics Today article I wrote in 2008 which celebrated the 60th anniversary of the $\alpha\beta\gamma$ paper. Here, I have tried to elucidate the simple physics underpinning BBN and clarify the history of that physics. Further, it serves as a case study of big ideas. This big idea began with the lofty goal of explaining the origin of the elements in the periodic table. In the end, only the lightest elements were produced in the big bang. However, the paper opened the door to hot, early phase of the Universe when particle interactions shaped the Universe that we see today – and played a role in inventing the modern r-process theory of the 14

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14 While it has been widely said that Hoyle was spurred by Gamow’s claim to be able make all the elements in the big bang to think more broadly about stellar nucleosynthesis, at least one source disputes this [31].

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heavy elements. While $\alpha\beta\gamma$ did not achieve its original goal, it spurred thinking that achieved even more transformational advances in cosmology.

5.1 Coda

In closing, I mention two other discussions of the history of BBN physics, by Peebles in his recent book [11] and by Alpher and Herman in their 1988 Physics Today article [26]. In addition, there is a book entitled Finding the Big Bang [22] that is largely comprised of essays by astronomers around the world who played a key roles in the discovery of the CMB, establishing its big-bang origin and other related topics including BBN. While important from a science history standpoint, it has very little to say about the physics of BBN, my topic here.

In his recent book, Peebles devotes more than 20 pages to discussing much of the same physics history and many of the same papers as I do here (in addition to discussing the discovery of the CMB). However, I find his presentation of the physics of BBN opaque, in large measure because it is built around making the Gamow theory mesh with the modern understanding of BBN. They are simply not compatible because of their very different starting points: the modern theory involves mostly equilibrium physics and Gamow’s model is built around non-equilibrium neutron capture physics and neglects most of the key physics.

Likewise, I believe his defense of the Alpher-Herman prediction of the CMB temperature is too generous: getting a number that is coincidentally close to right answer, based upon the wrong physics and at the same predicting $\Omega_B \sim 2 \times 10^{-4}$ doesn’t sound like a correct prediction to me. Where I believe Peebles excels is on the “astronomy side,” especially his discussion of the history of the realization that there is a large primordial $^4\text{He}$ abundance and its value, $Y \sim 0.25$, and on the history of the discovery of the CMB.

In their Physics Today article, Alpher and Herman reflect upon their BBN work and reminisce about their collaboration with the very creative and interesting George Gamow. Their look back, and another one by Alpher’s son [35], describe the joys and difficulties of working with and in the shadow of Gamow. The key idea was his [4] and his involvement brought great attention to their work. At the same time, he captured much of the credit and attention, e.g., it was in his sole author Nature paper [14] where the results of Alpher’s thesis were first shown, and it was Gamow who invited Bethe to be a co-author of $\alpha\beta\gamma$. It may have also been Gamow’s influence that kept Alpher and Herman so closely wedded to the non-equilibrium model. And what most clearly comes out in their reflections is disappointment at not getting more credit for their prediction of the CMB temperature. I took a pass on assigning credit, which is always a complicated task, and chose to tell the story of the physics.

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**Appendix: some technical details**

Here I add a few more details underpinning my estimates in §2, with the key formulae and approximations used in my calculations. These are formulae for the expansion rate $H$, the age of the Universe $t$, the energy density in relativistic particles, reaction rates, and the number density of photons and baryons:

$$H^2 \equiv \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G \rho}{3} \simeq \frac{8\pi G \rho_R}{3}$$

$$H \approx 10T^2/m_{pl}$$

$$t = 1/2H \simeq \frac{1 \text{ sec}}{(T/\text{MeV})^2}$$

$$\rho_R = \frac{g_\star \pi^2}{30} T^4$$

$$\Gamma = n < \sigma v >$$

$$n_\gamma = \frac{2\zeta(3)}{\pi^2} T^3$$

$$n_B \simeq \eta m_\gamma$$

Throughout $k_B = h = c = 1$ so that $G = 1/m_{pl}^2$ ($m_{pl} = 1.22 \times 10^{19}$ GeV is the Planck mass); $g_\star$ counts the number of relativistic degrees of freedom.
During BBN the energy density is dominated by that in thermal, relativistic particles (photons, neutrinos and electron-positron pairs) with $g^* = 10^{7.75}$, and so the expansion rate $H \simeq 10T^2/m_{pl}$. The reaction rate (per particle) is proportional to the number density of targets times the thermally-averaged cross section times relative velocity. Neglecting the factor of $11/4$ increase in the number of photons when electron-positron pairs annihilate, the number density of baryons (nucleons) during BBN is just the baryon-to-photon ratio $\eta$ times the number density of photons.

Crucial to understanding BBN is the concept of Nuclear Statistical Equilibrium (NSE). It means that the various nuclides are in full thermal equilibrium, with number densities in the non-relativistic limit given by,

$$n_A = \left(\frac{m_A T}{2\pi}\right)^{3/2} \exp\left(\frac{\mu_A - m_A}{T}\right),$$

where $m_A$ and $\mu_A$ are the mass and chemical potential of nuclide $A$ and $\mu_A = Z\mu_p + (A - Z)\mu_n$. NSE determines when nuclei are favored over free nucleons, through the balance between the lower energy of nuclei and the higher entropy of free nucleons, cf. Eq. (1) in §2.

The key concept underlying my discussion in §2 is the “freeze out” of particle interactions. Namely, when a reaction rate key to adjusting a particle abundance cannot keep up with the rate at which the temperature is falling, $\dot{T}/T = -\dot{R}/R = H$, it ceases to be relevant and the abundance it is controlling “freezes in.” To a good approximation, a reaction rate is frozen out when $\Gamma < H$, and freeze out occurs when $\Gamma \simeq H$.

For example, the rate for the reactions that maintain thermal balance of neutrons and protons, $p + e^- \leftrightarrow n + \nu_e$ etc, is $\Gamma \approx T^3 \times G_F^2 T^2 \simeq G_F^2 T^5$. Comparing it to the expansion rate, it follows that the freeze of the neutron-to-proton ratio occurs

$$T_F \simeq G_F^{-2/3} m_{pl}^{-1/3} \simeq 1 \text{ MeV}.$$

The rate of any strong/EM charged-particle nuclear interactions will have a Coulomb barrier suppression factor of the form,

$$\exp\left[-2\frac{\bar{A}^{1/3}(Z_1 Z_2)^{2/3}}{(T/\text{MeV})^{1/3}}\right],$$

with an overall nuclear cross section size, $m_{\pi}^{-2} \simeq 2 \times 10^{-26} \text{ cm}^2$, and an additional factor of $\alpha_{EM}/\pi$ for an EM interaction. The controlling factor for freeze out is the exponential Coulomb barrier factor. And the Coulomb barrier is lowest for charge +1 on charge +1 and small $\bar{A} \equiv A_1 A_2/(A_1 + A_2)$. $\bar{A}$ is smallest for $A_1 = 1$ and $A_2 = 2$ (= 2/3) and $A_1 = A_2 = 2$ (= 1). To estimate a charged-particle nuclear rate, I have used $d + d \rightarrow ^4\text{He} + \gamma$,

$$\Gamma \sim \eta T^3 \frac{\alpha_{EM}/\pi}{m_{\pi}^2} \exp[-2(T/\text{MeV})^{-1/3}].$$

Comparing this rate to the expansion rate leads to Eq. (2).

For a more detailed and expansive discussion of BBN and particle interactions in the early Universe see Refs. [13, 20].