Analysis on Wave-Velocity Inverse Imaging for the Supporting Layer in Ballastless Track

Yong YANG††, Junwei LU†, Nonmembers, Baoxian WANG††, Member, and Weigang ZHAO†††, Nonmember

SUMMARY The concrete quality of supporting layer in ballastless track is important for the safe operation of a high-speed railway (HSR). However, the supporting layer is covered by the upper track slab and the functional layer, and it is difficult to detect concealed defects inside the supporting layer. To solve this problem, a method of elastic wave velocity imaging is proposed to analyze the concrete quality. First, the propagation path of the elastic wave in the supporting layer is analyzed, and a head-wave arrival-time (HWAT) extraction method based on the wavelet spectrum correlation analysis (WSCA) is proposed. Then, a grid model is established to analyze the relationships among the grid wave velocity, travel route, and travel time. The loss function based on the total variation is constructed, and an inverse method is applied to evaluate the elastic wave velocity in the supporting layer. Finally, simulation and field experiments are conducted to verify the suppression of noise signals and the accuracy of an inverse imaging for the elastic wave velocity estimation. The results show that the WSCA analysis could extract the HWAT efficiently, and the inverse imaging method could accurately estimate wave velocity in the supporting layer.

key words: supporting layer, WSCA, wave velocity estimation, inverse imaging, total variation

1. Introduction

Due to construction quality and long-term effect of external loads, the concrete of the supporting layer in ballastless track of high-speed railway (HSR) may experience surface peeling and the flowing out of internal mortar, as shown in Fig. 1. This will reduce the strength of the support layer concrete, and even threaten the operational safety. The supporting layer is located at the bottom of the ballastless track, and the upper part is covered by a functional layer and track slab, which is only partially exposed. It is impossible to detect internal defects in the supporting layer by direct observation.

The elastic wave velocity is an important characterization of concrete performance [1]. However, the existing detection method for wave velocity uses a single-point measurement to obtain the mean wave velocity in a local area of the concrete. It cannot be directly employed to detect the velocity of the supporting layer covered by the track slab. Therefore, it is of great significance to propose a new method to estimate the wave velocity.

The accurate extraction of the head-wave arrival-time (HWAT) is the crux to improve the precise estimation of wave velocity. This problem was first proposed in seismic signal processing. Prior to the 1980s, HWAT extraction primarily depended on manual methods. In 1987, Ramananantoandro [2] stated the importance of automatic extraction of the HWAT by large-scale data processing, and a convolution method based on a half-period width of the refracted wave was proposed to suppress random noise. Wang [3] studied the similarity between two transmission path signals and extracted HWAT using the dispersion spectrum between multi-channel signals. Cong [4] used the measured time difference and the calculated time difference of the HWAT as constraints to evaluate the mine echo velocity. Those methods have primarily focused on the seismic signal. The signal propagation distance is several hundred meters to several kilometers, so the accuracy of HWAT has not been particularly precise. However, the width of the supporting layer is centimeters. For example, the width of the supporting layer of a CRTSII ballastless track is 2.9 m. When the wave velocity is 4000 m/s, the HWAT error of 10 μs will produce a 50 m/s wave velocity error. In addition, based on HWAT, the relationship between the arrival time difference and the travel distance can be directly used to obtain the velocity. However, this velocity is an average wave velocity based on the assumption of isotropy of the medium [5], [6]. Therefore, it is necessary to exploit a high-precision algorithm to extract HWAT.

In recent decades, a time-frequency analysis, represented by the wavelet transform [7], turned a 1D time signal into a 2D time and scale (frequency) signal. This provided a finer description of the signal component. This method made it possible to improve the accuracy of the HWAT extraction.

This study presents a model of supporting layer wave
velocity detection and proposes a calculation method for the HWAT based on a wavelet spectrum correlation analysis (WSCA). Then the loss function of the wave velocity inversion is constructed, and wave velocity imaging is achieved using the total variation (TV).

2. Methodology

2.1 Elastic Wave Propagation Model

Using one side of the supporting layer as excitation and the other side as reception, an elastic wave propagation model can be developed, as shown in Fig. 2.

The detection equipment was primarily composed of an exciter and receiver. Ideally, the waveform excited by the exciter was regarded as a unit pulse signal:

$$x(t) = \delta(t - t_0)$$

(1)

With regards to the supporting layer as the transport channel, an elastic wave will undergo a process of attenuation and oscillation. Time consumption will occur to reach the receiver. Therefore, let the signal received by the receiver be \(y(t)\), and \(y(t)\) can be expressed in the time domain as follows:

$$y(t) = f(t) * x(t - \tau)$$

(2)

where \(\tau\) denotes the delay time, determined by the distance, \(l\), between the exciter and the receiver and the wave velocity \(v\); \(f(t)\) denotes the transfer function of supporting layer in time domain. In this case, define \(t_0\) as HWAT of the exciter, and \(t_1 = t_0 + \tau\) as that of receiver.

2.2 HWAT Extraction

Ideally, \(t_0\) and \(t_1\) can be obtained by judging the starting point (jump-off point) of the exciter and receiver. However, due to the influence of white noise in field experiments, it is difficult to accurately judge the starting point of the signal according to a threshold-search method.

(1) Wavelet transform and its frequency resolution

A time-frequency analysis expands the 1D signal to 2D, and this characteristic can describe the data feature from multiple perspectives.

The wavelet transform uses the scaling and translation of the wavelet base (mother wavelet) to obtain a multiscale analysis of the signal. Let the signal be \(y(t)\), and the wavelet transform is defined as follows:

$$WT_y(a, b) = \int_{-\infty}^{\infty} y(t) \frac{1}{\sqrt{a}} \psi \left( \frac{t - b}{a} \right) dt$$

(3)

where \(\psi(t)\) is the mother wavelet; and the parameters \(a\) and \(b\) denote the scale and translation factor, respectively.

The resolution of the wavelet transform is higher in the low-frequency portion and lower in the high-frequency portion. The frequency component of the received wave signal during a field experiment is primarily located in the low-frequency portion, as shown in Fig. 3. Obviously, the primary frequency of the collected signal is in the low-frequency portion with a higher frequency resolution.

(2) Wavelet spectrum correlation Analysis (WSCA)

Let the wavelet spectrum of standard signal be \(WT_y(a, b)\); the signal to be calibrated is \(y_i\); and its wavelet spectrum is \(WT_y(a, b)\). Then define the coefficient of the WSCA, \(\eta_i\), as follows:

$$\eta_i(t) = \frac{\langle WT_y(a, b), WT_y(a, b) \rangle}{|WT_y(a, b)|^2 |WT_y(a, b)|^2}$$

(4)

The HWAT, \(t_i\), is expressed as:

$$t_i = \arg\max_{t} \eta_i(t)$$

(5)

2.3 Wave Velocity Imaging of the Supporting Layer

Wave velocity imaging must be a MIMO system. \(m\) exciters and \(m\) receivers were placed on both sides of the supporting layer, and the supporting layer was divided into grids \((m \times n)\), as shown in Fig. 4; where \(v_{ij}\) denotes the wave velocity in a grid \((i, j)\). Therefore, when the exciter is in position \(p\), and receiver in position \(q\), and the travel route through the grid, \((i, j)\), is denoted as \(l_{pq}\). Then elastic wave travel time, \(t_{pq}\), can be written as follows:

$$WT_y(a, b) = \int_{-\infty}^{\infty} y(t) \frac{1}{\sqrt{a}} \psi \left( \frac{t - b}{a} \right) dt$$

(3)
The travel time, \( t_{pq} \), can be expressed as the matrix \( T \),

\[ T = \{T_1, T_2, \ldots, T_m\} \tag{7} \]

In addition, let \( L = \{L_{pq}\} \), \( L_{pq} = \{t_{pq}^l\} \) be the travel route matrix; \( V = \{v_{ij}\} \) the velocity matrix; and \( S = \{s_{ij}\} \) the slowness matrix \( s_{ij} = 1/v_{ij} \). By combining Eqs. (6) and (7), the relationship among \( T, S, \) and \( L \) can be expressed as follows:

\[ SL = T \tag{8} \]

When \( n \geq m \), the matrix \( L \) is singular, and the slowness matrix, \( S \), cannot be obtained directly. Therefore a loss function based on the TV [8], [9] is constructed as follows:

\[ \min_{L} \sum_{i,j=1}^{m} (SL - T)^2 + \lambda \|L_1S\|_1 \tag{9} \]

where \( L_1 \) is the first derivative matrix of \( S \), and \( \lambda \) is a constant.

3. Result Analysis and Discussion

3.1 Suppression of the HWAT Noise Signal

White noise is a kind of noise signal often mixed in the process of data acquisition. Let the true signal be \( y(t) \), and the white noise be \( n(t) \). The noise is often presented as additive noise. Therefore, the collected signal, \( y'(t) \):

\[ y'(t) = y(t) + n(t) \tag{10} \]

According to the linear feature of the wavelet transform:

\[ WT_y(a, t) = WT_y(a, t) + WT_n(a, t) \tag{11} \]

Figure 5 shows the random noise, \( n(t) \), with a uniform distribution and its continuous wavelet transform. Compared with the signal in Fig. 3, the wavelet transform of \( n(t) \) is more in the high-frequency portion, while the true signal, \( y(t) \), is more in the low-frequency portion.

To further compare and analyze the suppression of the HWAT noise between the WSCA and the time domain, the echo signal in Fig. 3 was extracted as reference data. Then the noise signals with different signal-to-noise ratios (SNRs) were added for a comparative analysis. In the time domain extraction method, 10 percent of the maximum value extracted using the envelope theorem was regarded as HWAT, \( t_1 \).

Table 1 shows a comparison between the two methods to extract the HWATs. (a) Random signal with a uniform distribution and (b) wavelet transform of the random signal.

### Table 1: Comparison between the two methods to extract the HWATs.

| SNR(dB) | time | HWAT \( t_1 \)(\( \mu \)s) | mean square error(\( \mu \)s) |
|---------|------|----------------|------------------|
| 1       | 10   | 47.20          | 0.43             |
| 2       | 7.5  | 47.19          | 0.51             |
| 3       | 5    | 47.09          | 0.70             |
| 4       | 3    | 46.28          | 4.29             |
| 5       | 1    | 36.57          | 14.76            |

3.2 Simulation Analysis of Wave Velocity Imaging

A simulation analysis was employed to verify the preciseness of the TV inverse method.

(1) Travel route matrix \( L \)

In this study, an image-processing-based method was used to determine the travel route matrix, \( L = \{t_{pq}^l\} \). Suppose an image with a resolution of \( M \times N \) is divided into grids \((m \times n)\), and each grid contains the number of pixels \( M/m \times N/n \); the actual physical size represented by each pixel is \( x_{res} \times y_{res} \). According to Fig. 4, a line is drawn to connect the exciter, \( p \), and the receiver, \( q \), and set all points on the line to 1 and other points to 0. Travel route matrix, \( L \), with grids \( 16 \times 16 \) and a resolution \( 200 \times 100 \) is shown in Fig. 7.

(2) Wave velocity imaging

Similar to the grids in (1), \( v_{ij} \) denotes the wave velocity in grid \((i, j)\) and obeys the orthodox distribution \( v_{ij} \sim \) (3500, 120).

Figure 8 represents the true wave velocity imaging and inverse imaging. The red portion represents the wave velocity at \( v < 3500 \) m/s, and the green portion \( v > 3500 \) m/s.
3.3 Field Experiment

A field experiment was conducted at the Shijiazhuang Railway University Training Center to detect the wave velocity in the supporting layer of ballastless track. From the top to the bottom, the ballastless track included a track slab, a CA mortar layer, a plain concrete supporting layer, and a subgrade surface layer. The Bruel & Kjaer accelerometer 4533-B-002 used as the sensor, and the DongHua test 8302 was used as the acquisition equipment. The acquisition parameters and sensor layouts are shown in Table 2.

Figure 9 shows the detection process and collected data. The lateral dimension of the supporting layer (perpendicular to the rail direction) was 2.9 m, and the survey length was 1.5 m. Hence, the survey range was 2.9 m × 1.5 m. Similar to (1), the grid was 16 × 16, and inverse calculation was employed based on Sects. 3.2–3.3. The result is shown in Fig. 10.

The mean velocity of the inverse result was 3144 m/s. The red portion \( v < 3144 \) m/s, and the green portion \( v > 3144 \) m/s. This result shows that the velocity of the inner portion of the supporting layer was slightly lower, and velocity of the left portion was obviously smaller than that of other places.

To verify the accuracy of wave velocity estimation, the elastic wave velocity of the supporting layer exposed to air on the left side was surveyed using a MIRA A1040 ultrasonic imager. Figure 11 shows the surveying velocity and inverse result. Table 3 represents the features data of the surveying velocity and inverse result. The parameter, \( \alpha \), denotes the coefficient between the two methods. The results show that the inverse result was in good agreement with the surveying velocity found using Mira in terms of the mean and correlation coefficient.

4. Conclusions

To solve the difficulty in detecting concealed defects in the supporting layer of ballastless track, a wave velocity estimation method was developed. The following conclusions are drawn:

1. The HWAT extraction method based on WSCA effectively suppressed the errors caused by random noise, and
the accuracy was significantly higher than that of the time domain analysis method.

(2) The inverse imaging method based on TV could accurately estimate wave velocity in the supporting layer. The correlation coefficient between the true wave velocity and the inverse wave velocity in simulation is 92.04%, and the mean error is 34.54 m/s.

Acknowledgments

This research was financially supported by (1) National Natural Science Foundation of China (No. 51978423), (2) Natural Science Foundation in Hebei Province (No. E2019210214, 19210804D)

References

[1] C.V.D. Haar and S. Marx, “Development of stiffness and ultrasonic pulse velocity of fatigue loaded concrete,” Structural Concrete, vol.17, no.4, pp.630–636, 2016.

[2] R. Ramananantoandro and N. Bernitsas, “A computer algorithm for automatic picking of refraction first-arrival time,” Geoexploration, vol.24, no.2, pp.147–151, 1987.

[3] B.-L. Wang, “Automatic pickup of arrival time of channel wave based on multi-channel constraints,” Applied Geophysics, vol.15, no.1, pp.118–124, 2018.

[4] S. Cong, Y.-H. Wang, and J.-Y. Cheng, “Coal mine microseismic velocity model inversion based on first arrival time difference,” Arabian Journal of Geosciences, vol.12, no.1, pp.1–9, 2019.

[5] A. Wiedmann, F. Weise, E. Kotan, H.S. Müller, and B. Meng, “Effects of fatigue loading and alkali–silica reaction on the mechanical behavior of pavement concrete,” Structural Concrete, vol.18, no.4, pp.1–11, 2017.

[6] X.-Y. Li and S. Crampin, “Approximations to shear-wave velocity and moveout equation in anisotropic media,” Geophysical Prospecting, vol.41, no.7, 833–857, 1993.

[7] I. Daubechies, “The wavelet transform, time-frequency localization and signal analysis,” Journal of Renewable & Sustainable Energy, vol.36, no.5, pp.961–1005, 1990.

[8] T.F. Chan and S. Esedoglu, “Aspects of Total Variation Regularized L1Function Approximation,” Siam Journal on Applied Mathematics, vol.65, no.5, pp.1817–1837, 2005.

[9] A. Beck and M. Teboulle, “Fast gradient-based algorithms for constrained total variation image denoising and deblurring problems,” IEEE Trans. Image Process., vol.18, no.11, pp.2419–2434, 2009.