ON THE GRAVITATIONAL COLLAPSE
OF A MASSIVE STAR

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ABSTRACT. The celebrated treatment of continued gravitational collapse by Oppenheimer and Snyder is revisited and emended from some inherent flaws. The star contracts itself into a material point, not into a black hole.

1.– The gravitational spherically-symmetric collapse of a massive (i.e., of mass greater than a few solar masses) star, simply described as “dust” cluster of a spherical shape (with negligible pressure), has been firstly investigated in 1939 by Oppenheimer and Snyder [1], whose approach is now a standard reference [2]. However, treatments [1] and [2] are not exempt from conceptual flaws, and therefore the problem deserves a re-examination.

2.– The above mentioned collapse is described with the same mathematical formalism of one of Friedmann’s cosmological models, and precisely the oscillating model in its contraction phase. Friedmann’s universe – as it is well known – consists of a spherical “dust” cluster of “point” galaxies interacting only gravitationally, and it is based on the principle of homogeneity and isotropy of the three-dimensional space, according to which at any instant of time it is seen similarly by all galactic observers [3].

First of all, we remember that for a dust cluster of particles, which interact only gravitationally, the world lines of the particles are geodesic lines.

Quite generally, in a Gaussian-normal (or synchronous) frame of reference – for which

\[ g_{00} = 1 \ ; \ g_{0\alpha} = 0 \ ; \ (\alpha = 1, 2, 3) \quad , \]

the time lines are geodesic lines. Quite generally, in a co-moving frame of reference the world lines coincide with the time lines.

Accordingly, if we choose for our dust cluster (our star) a Gaussian-normal frame \( S \), the time lines of the particles are both geodesic and world lines – and the frame \( S \) is also a co-moving one. We have:

\[ \frac{dx^0}{ds} = 1 \ ; \ \frac{dx^\alpha}{ds} = 0 \quad . \]

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The matter tensor of the dust is:

\[ T^{jk} = \rho \frac{\partial x^j}{\partial s} \frac{\partial x^k}{\partial s} , \quad (j, k = 0, 1, 2, 3) , \quad (c = 1) , \]

where \( \rho \) is the invariant mass density. In the above frame \( S \) it is simply:

\[ T_{00} = T^{00} = \rho ; \quad T^{0\alpha} = T^{\alpha\beta} = 0 . \]

The \( ds^2 \) of our problem is given by Friedmann-Robertson-Walker metric:

\[ ds^2 = (dx^0)^2 - A^2(r^2)F^2(x^0) \left[ (dx^1)^2 + (dx^2)^2 + (dx^3)^2 \right] , \]

where:

\[ A(r^2) \equiv \left[ 1 + \frac{r^2}{4} \right]^{-1} ; \quad r^2 = (x^1)^2 + (x^2)^2 + (x^3)^2 , \]

and \( F(x^0) \) must be determined by Einstein field equations:

\[ R_{jk} - \frac{1}{2}g_{jk}R = -\kappa T_{jk} , \quad (\kappa \equiv 8\pi G) , \]

from which:

\[ \frac{1}{F} + \frac{\dot{F}^2}{F} + 2\ddot{F} = 0 , \]

\[ \frac{1}{F^2} + \frac{\dot{F}^2}{F^2} - \frac{1}{3}\kappa \rho = 0 . \]

Their solution describes a periodic oscillation between \( F = 0 \) and a given maximum value \( F_{\text{max}} \). For \( F = 0 \) the density \( \rho \) is infinite and the gravitational field is singular. Starting from \( F_{\text{max}} \) our star contracts itself, in a finite time, into a material point corresponding to \( F = 0 \).

3. In the Newtonian model analogous to the relativistic model of sect.2., a particle on the spherical surface of radius \( \Re(t) \) of the star is attracted by the mass \( M \) within the sphere according to Newton’s law:

\[ \ddot{\Re} = -\frac{\kappa M}{8\pi \Re^2} , \quad \text{with} \quad M = \frac{4}{3}\pi \Re^3(t)\rho(t) = \text{constant} ; \]

thus

\[ \ddot{\Re} + \frac{\kappa}{6}\Re \rho = 0 . \]
from which

(12) \[ \frac{1}{\mathcal{R}^2} + \frac{\dot{\mathcal{R}}^2}{\mathcal{R}^2} - \frac{1}{3} \kappa \rho = 0. \]

Therefore we can write the following equations, which are formally identical to eqs. (5) and (6):

(13) \[ \frac{1}{\mathcal{R}} + \frac{\dot{\mathcal{R}}^2}{\mathcal{R}} + 2 \ddot{\mathcal{R}} = 0 \]

(14) \[ \frac{1}{\mathcal{R}^2} + \frac{\dot{\mathcal{R}}^2}{\mathcal{R}^2} - \frac{1}{3} \kappa \rho = 0. \]

They give a periodic oscillation between \( \mathcal{R} = 0 \) and a given \( \mathcal{R}_{\text{max}} \). Starting from \( \mathcal{R}_{\text{max}} \) the star contracts itself into a material point (for which \( \rho = \infty \)) corresponding to \( \mathcal{R} = 0 \).

Remark, in conclusion, that: i) in both cases – Einsteinian Friedmann’s model and Newtonian model – we have employed only one co-ordinate frame; ii) the gravitational field outside the star has caused no problem, and it has been ignored.

4. – The approach by Oppenheimer and Snyder ([1], [2]) can be criticized for the following reasons: i) the consideration of the Einsteinian field outside the star is a useless superfetation, because the Gaussian-normal frame of FRW-metric characterizes exhaustively the phenomenon of collapse; ii) in [1], [2] the external gravitational field is described by the standard form of solution of the Schwarzschild problem – erroneously called “Schwarzschild solution” [3] –, which is properly valid only for the values of radial co-ordinate \( r > \frac{\kappa M}{4\pi} \).

At any rate, if one wishes to take into consideration also the outside field, it is suitable to proceed as follows.

As it is well known, the necessary and sufficient condition that a Riemann-Einstein manifold admit the group of spatial rotations is that its \( ds^2 \) be reducible to the following form, which holds both internally and externally to the matter distribution:

(15) \[ ds^2 = B_1(r, t)dt^2 - B_2(r, t)dr^2 - r^2d\omega^2, \quad (r > 0; c = 1), \]

where

(16) \[ d\omega^2 \equiv d\theta^2 + \sin^2 \theta d\phi^2, \quad (0 \leq \theta < \pi; \quad 0 \leq \phi < 2\pi). \]
By virtue of a famous Birkhoff’s theorem, the outside field is time-independent, and consequently the general form of external metric is given by the following formula, as it was emphasized by Eddington [5]:

\[ ds^2 = \left(1 - \frac{2m}{f(r)}\right) dt^2 - \left(1 - \frac{2m}{f(r)}\right)^{-1} \left[ df(r)^2 - [f(r)]^2 d\omega^2 \right] \]

here: \( m \equiv \kappa M/(8\pi) \), and \( f(r) \) is any regular function of \( r \). No physical result depends on the choice of \( f(r) \).

If we put

\[ f(r) \equiv r + 2m \]

we obtain the form of solution first investigated by M. Brillouin [6], which holds for \( r > 0 \). Putting

\[ f(r) \equiv \left[r^3 + (2m)^3\right]^{\frac{1}{3}} \]

we obtain the original form of solution given by Schwarzschild [7], which is valid for \( r > 0 \); thus, Schwarzschild’s and Brillouin’s forms are maximally extended. If we put simply

\[ f(r) \equiv r \]

we obtain the standard, or Hilbert-Droste-Weyl, form of solution, which holds only for \( r > 2m \). Remark that Brillouin’s form and Schwarzschild’s original form are diffeomorphic to the exterior part \( r > 2m \) of HDW-form. Within the singular locus \( r = 2m \) of this last form the time co-ordinate \( t \) takes the role of the radial co-ordinate \( r \), and vice-versa. The solution loses its physical Eigentlichkeit (“appropriateness”, according to Hilbertian terminology), and becomes further non-static. Consequently, the notion of black hole – the “globe” \( r = 2m \) – is destitute of any meaning. As all the Fathers of Relativity perfectly knew!

**Conclusion.** Take ideally an instantaneous photograph of the contracting sphere at any time \( \bar{t} \), and call \( \bar{r} \) its co-ordinate radius at this time. If we choose the regular function \( f(r) \) of Eddington’s formula [17] so that the corresponding form of solution be valid for \( r > 0 \), we can say: since \( \bar{t} \) is just any time and \( \bar{r} \) tends to zero, at the final stage of its contraction our star will become a point mass. Thus, also with this schematic, model-independent, consideration we see that no celestial body can convert itself into a black hole.

**APPENDIX A**

From the experimental standpoint we can affirm that no black hole (BH) has ever been detected. Indeed, an accurate scrutiny of the papers in which observational discoveries of stellar-mass, or of supermassive, BH’s are cried up, shows that in reality the authors have discovered only celestial bodies of large, or enormously large, masses concentrated in very small volumes [8].
It is interesting to read the comments of Wolfgang Kundt, a renowned astrophysicist, concerning the BH’s [9]. He does not criticize the current theoretical viewpoint on the question, but limits himself to significant remarks of the following kind.

a) Page 37 of [9]: “The critical mass [...] which determines whether a star eventually turns into a white dwarf or something more compact – if such a mass is well defined – is [...] controversial. It should be consistent with (i) the birthrate of white dwarfs, (ii) the birthrate of neutron stars, (iii) the PN [planetary nebula] rate, (iv) the SN [supernova] rate, (v) the supernova remnant (SNR) rate, and (vi) the initial mass function (IMF) which counts the number of stars as a function of their mass at birth. In view of the many neutron stars in the Galaxy – detected as pulsars, binary X-ray sources, or even invisible (when screened, without accreting) – I favour a critical mass of some 5 solar masses (over larger values, like 8 solar masses). The bias would become even more severe if a large number of massive stars would end up as black holes (BHs); in my own judgment, none of the BH candidates (BHCs) do involve BHs, rather they are neutron stars surrounded by massive disks [...]: The proposed BHCs have too much spectral and variability structure, reminiscent of a rotating inclined magnet at their center [...].”

β) Page 81 of [9]: “Another disk peculiarity is expected at the centers of galaxies [...] The galaxy feeds an active, nuclear-burning nucleus, a burning disk [Kundt, 2000]. – Instead, most of my colleagues prefer to think of a supermassive black hole as the central engine of all the active galactic nuclei (AGN). They have not convinced me, after more than 20 years. AGN activity requires a refilling engine, with nuclear burning, magnetic reconnections, and explosive ejections of the ashes [...]. Black-hole formation would require distinctly higher mass concentrations than are even reached in galactic nuclei, by a factor of $10^2$. The quasar phenomenon is a simple consequence of a permanent inward galactic mass flow, at an average of $< 1$ (or $\sim 1$) solar mass per year, which piles up at the center.”

γ) Page 101 and 102 of [9]: “How about stellar-mass holes? [...] Over 45 black-hole candidates have been proposed during the past 30 years from the class of binary X-ray sources, both high-mass and low-mass – among them Cyg X-1 and A0620-00 – on account of their large mass function, absence of strict periodicities, and absence of type-I bursts (understood as nuclear detonations at neutron-star surfaces). To me, all of them look like neutron stars surrounded by massive ($\approx 5$ solar masses) accretion disks, because of their often hard spectra (up into the $\gamma$-ray range), highly structured, [...] and because of their indistinguishable further properties, as a class, from all the established neutron-star binaries [...]. They just fill the gap between the high-mass and low-mass compact binary systems. – And the postulated supermassive black holes at the centers of (all the active) galaxies? They were once believed to be required for energy reasons. The nuclear burning (of H to Fe) is almost as efficient a lamp as black-hole accretion, yielding a guaranteed $< 1\%$ (or $\sim 1\%$) of the rest energy [...]. Besides[...], the universality of the jet phenomenon suggests a universal engine which we know is a fast rotating magnet in the cases of newly forming stars, binary neutron stars, and forming binary white dwarfs [Kundt, 1996, 2000]. – [...] I share the
doubts of a few other people, among them (the late) Viktor Ambartsumyan and Hoyle et al. [2000], in the widely accepted black-hole paradigm. [...] Active galactic nuclei may owe their extreme properties to those of their central disks.”

δ) Pages 131 and 132 of [9]: “As already mentioned [...], [see supra in γ]), the list of stellar-mass black-hole candidates contains presently over 45 entries, five of them with high-mass companions (> 6 (or ∼ 6) solar masses), the rest with low-mass ones (< 2 (or ∼ 2) solar masses). Their defining property is a mass in excess of 3 solar masses of their compact component. All the high-mass BHCs are persistent sources whereas most of the low-mass ones are transient, with recurrence times of decades. Every year, two or more X-ray novae joint the list. [...] – My suspicion of the BH interpretation comes from (i) a number of spectral and lightcurve properties which require a hard surface, an oblique magnetic dipole, and two dense, interacting windzones; (ii) the indistinguishability, as a class, of the BHCs from the neutron-star binaries in all properties other than their inferred mass; and (iii) the missing intermediate-mass systems which should naturally evolve into neutron-star binaries with massive disks. [...] – Among the long list of remarkable properties in which the BHCs are indistinguishable, as a class, from neutron-star binaries are (j) the presence of a third (precessional) period of several months, 294 d in the case of Cyg X-1; (jj) a hard-soft state spectral bimodality, pivoting around 6 keV, and extending up to MeV; (jjj) their flickering, expressed by their X-ray power spectra which range from mHz to > kHz (or ∼ kHz) and show various quasi-periods, in particular of several 10^2 Hz, up to 1.2 kHz, reminiscent of innermost Kepler periods, of a spin period, and/or of beat frequencies thereof; (jv) their jet-formation capability [...].”

APPENDIX B

Perhaps the belief of most of the theoretical astrophysicists in the physical existence of BH’s begins now to get cracked. We report below the abstract of a recent paper by Frønsdal [10], an author who anticipated in 1959 [11] the celebrated result by Kruskal [12] and Szekeres [13], which contributed so much to the conviction of the real existence of the BH’s.

Here is the mentioned abstract [10]: “This paper studies the interpretation of physics near a Schwarzschild black hole. A scenario for creation and growth is proposed that avoids the conundrum of information loss. In this picture the horizon recedes as it is approached and has no physical reality. Radiation is likely to occur, but it cannot be predicted.”

(Of course, with the phrase “Schwarzschild black hole” the author denotes the fictive object derived from the current unphysical interpretation of the part r ≤ 2m of the standard HDW-form of solution of Schwarzschild problem – as we have previously emphasized; and “horizon” denotes in this interpretation the singular locus r = 2m.)

Accordingly, the observational astrophysicists are warned: the existence of the black holes cannot be detected – exactly as the existence of cosmic ether.
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