Consequence for Wavefunction Collapse Model of
the Sudbury Neutrino Observatory Experiment

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It is shown that data on the dissociation rate of deuterium obtained in an
experiment at the Sudbury Neutrino Observatory provides evidence that the
Continuous Spontaneous Localization wavefunction collapse model should have
mass–proportional coupling to be viable.
Key words: wavefunction collapse, CSL model, dissociation of deuterium, SNO
Neutrino Observatory, mass-proportional coupling.

1 Introduction

The “measurement problem” can be thought of as indicating a serious flaw in
the foundations of quantum theory [1], and therefore a potential clue to new
physics. When a measurement is described by Schrödinger’s equation, the state-
vector evolves to a superposition of apparatus states, each recording a different
outcome. This conflicts with the interpretation of the statevector as describing
“reality” since only one of these apparatus states exists in reality.

One resolution of this problem is to modify Schrödinger’s equation, adding a
term so that the “collapse” of the statevector to a macroscopically unique state
occurs dynamically. The added term depends upon a randomly fluctuating
quantity. The particular realization of that quantity determines the particu-
lar macroscopic collapse outcome, and the statistics of the quantity gives the
outcomes with the Born probabilities [2, 3].

Arguably, the Continuous Spontaneous Localization (CSL) model [4, 5] is at
present the most successful dynamical collapse model [6]. In CSL, the fluctuating
quantity is a scalar field. In each added term it is coupled to the operator
representing the number density of particles of a particular type (e.g., electrons,
protons, neutrons...) by a coupling constant \( g_\alpha \) (e.g., \( g_e, g_p, g_n \)...).

The construction of CSL crucially depended upon the advent of a model of
Ghirardi, Rimini and Weber (GRW) [7]. In the GRW model, the overall rate of
collapse for any isolated particle in a superposition of “widely separated” states
is chosen to be \( \lambda = 10^{-16} \text{ sec}^{-1} \). “Widely separated” refers to greater than the
mesoscopic distance $a = 10^{-5}$ cm: for separation less than $a$, the collapse rate is less than $\lambda$. These features of the GRW model are carried over into CSL, except that the rate of collapse for particles of type $\alpha$ becomes $\sim \lambda g^2_{\alpha}$. The GRW collapse rate $\lambda$ is assigned to the proton, so $g_p \equiv 1$: experimental limits on the parameters $g_e/g_p \equiv g_e$, $g_n/g_p \equiv g_n$ are discussed in this paper.

Since CSL is different than standard quantum theory, in some instances it makes different predictions which are experimentally testable. The most direct test is to put an object into a superposition of spatially separated states for a while, and then bring the states together and measure their interference\cite{8} (see also the scheme of reference\cite{9}). A sufficiently sensitive direct test has not yet been proposed. However, there are "side effects" predicted by CSL which are testable. One is that a small object will undergo random walk\cite{10}. Another is that a charged particle will be shaken and therefore radiate\cite{11}. Another, the one which concerns us here, is that a bound state will be "spontaneously excited" by the collapse mechanism\cite{12}.

2 Collapse–Induced Bound State Excitation

Collapse narrows wavefunctions and so, by the uncertainty principle, the particles described by the narrowed wavefunction possess greater momentum and, therefore, energy. (The energy gain may be attributed to a loss of energy of the fluctuating field that causes the collapse\cite{13}.) When particles are in a bound state, their wavefunction continually undergoes a slight modification. This modified wavefunction may be written as the sum of a complete set of states, bound (with the coefficient of the original bound state of magnitude slightly less than one) and excited. Those particles which are excited move apart, due to the ever-present usual Schrödinger evolution, and they can collide with and give their energy to other particles placed nearby for the purpose of detection. The probability/sec of excitation of a bound state $|\psi\rangle$, to an excited state $|\phi\rangle$ (normalized to 1), when expanded in a power series in (size of bound state/$a$), is\cite{14}

$$\frac{dP}{dt} = \frac{\lambda}{2a^2} |\langle \phi | \sum_{\alpha,i} g_{\alpha} r_{\alpha i} |\psi\rangle|^2 + O[\lambda(\text{bound state size}/a)^4],$$

(1)

where $r_{\alpha i}$ is the position operator (or center of mass position operator for an extended particle like a nucleon) of the $i$th particle of type $\alpha$.

Eq. (1) holds irrespective of the excitation of the center of mass of the bound state. If $Q$ is the center of mass position operator, the states $|\psi\rangle$ and $|\phi\rangle$ are defined so that

$$Q |\psi\rangle = Q |\phi\rangle = 0,$$

(2)

where $Q \equiv \sum_{\alpha,i} m_{\alpha,i} r_{\alpha i}/\sum_{\alpha,i} m_{\alpha,i}$. An interesting consequence of Eqs. (1),

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is that, for mass–proportional coupling \((g_\alpha \sim m_\alpha)\), the first term in (1) vanishes identically [14]. Mass–proportional coupling is suggestive of a connection between collapse and gravity [15]. There is an interpretation of CSL [16] which requires mass–proportional coupling, although other interpretations [17] are possible. There also is an argument [18] applicable to a nonrelativistic collapse theory (i.e., where mass and energy are distinctly different entities), that the sum of couplings of the individual constituents of a compound object ought to be the coupling of the object, which may be implemented by mass-proportional coupling.

The term in (1) \(\sim (\text{size of bound state}/a)^4\) is too small to be observed in present experiments. However, the first term in (1) is experimentally testable. A very low noise experiment exists which observes radiation appearing in a slug of Ge (order of a kg), over a period of time (order of a year) [19]. What is looked for here is collapse–induced ionization of a 1s electron in Ge, which should give a photon pulse of 11.1 keV (the binding energy of the 1s electron, emitted by the remaining electrons in the atom as they readjust) plus kinetic energy deposited in the Ge (which is also the detector) by the ejected electron. Analysis of the experimental data gives [20]

\[
|g_e - M_e/M_p| < 12M_e/M_p \left(\frac{\lambda/a^2}{\lambda/a^2}\right)^{1/2}.
\]

So, if \(\lambda/a^2\) has the GRW value, it follows from (3) that \(g_e\) is within 1200% of mass proportionality:

\[
0 \leq g_e < 13\frac{M_e}{M_p}.
\]

### 3 Collapse–Induced Deuteron Excitation

In this paper we consider observation of deuteron excitation in order to obtain a limit on \(|g_n - M_n/M_p|\). (In what follows we neglect the electron contribution.) For deuterons, Eq. (2) implies that \(r_p = -r_n(M_n/M_p)\) (more precisely, this operator equation holds when acting on \(|\psi\rangle\) or \(|\phi\rangle\), so the deuteron relative coordinate \(r \equiv r_p - r_n = -r_n[(M_n/M_p) + 1].\) Then (1) becomes

\[
\frac{d\delta P}{dt} = \frac{\lambda}{2a^2} |\langle \phi | r_p + g_n r_n | \psi \rangle|^2 \]

\[
= \frac{\lambda}{2a^2} \left[ \frac{g_n - M_n/M_p}{1 + M_n/M_p} \right]^2 d\|k| r \rangle |^2.
\]

In Eq. (3) we have replaced \(|\phi\rangle\langle \phi |\) by \(d\|k|) |k\rangle\) (\(|k\rangle\) is the relative momentum
eigenstate of the dissociated neutron and proton) and we have replaced $P$ in (1) by $\delta P$ since there is an infinitesimal probability of excitation to any $|k\rangle$ state.

In the experiment under consideration at the Sudbury Neutrino Observatory (SNO)\[21\], deuterium (contained in heavy water inside a 12m diameter spherical shell) is dissociated by incident neutrinos via the neutral current reaction $\nu + d \rightarrow p + n + \nu$ (any flavor neutrino). The putative CSL deuteron dissociation occurs in addition to this. The free neutrons created by either mechanism are thermalized by travelling through the deuterium bath until they are either captured by a deuteron and form tritium (with the resulting emission of a 6.25 MeV gamma) or captured by $^{35}$Cl (since NaCl is dissolved in the heavy water) with typical emission of multiple gammas. The gammas Compton scatter electrons, whose Cerenkov radiation is detected by a surrounding array of photomultiplier tubes. Thus the experiment is not sensitive to the energy spectrum of the ejected neutrons, but only to their number. Accordingly, we integrate Eq. (3.1) over all $k$ to obtain

$$\frac{dP}{dt} = \frac{\lambda}{2a^2} \left[ \frac{g_n - M_n/M_p}{1 + M_n/M_p} \right]^2 \langle \psi | r^2 | \psi \rangle. \quad (6)$$

Various models\[22\] give $\langle \psi | r^2 | \psi \rangle \approx (3 \cdot 10^{-13})^2 \text{cm}^2$ to about 10% accuracy. If the volume of heavy water under observation is $V$ in units of thousand cubic meters and it is observed for $T$ years, we obtain from (6) (taking the number of deuterons in heavy water to be $(2/3) \cdot 10^{23}/\text{cc}$) the expected number of neutrons ejected from deuterium due to the CSL collapse mechanism:

$$N \approx 2.4 \cdot 10^7 \frac{\lambda/a^2}{(\lambda/a^2)_{GRW}} [g_n - M_n/M_p]^2 TV. \quad (7)$$

### 4 Comparison With Experiment

In the cited experiment, $T=254.2/365=.70\text{yrs}$ and, since only events were accepted whose origin lay within a radius of 5.5m, then $V = (4\pi/3)(5.5)^3/10^3 = .70\text{km}^3$, so Eq. (7) becomes

$$N \approx 1.2 \cdot 10^7 \frac{\lambda/a^2}{(\lambda/a^2)_{GRW}} [g_n - M_n/M_p]^2. \quad (8)$$

The number of detected neutron events was $1344.2 \pm 69.8/-69.0$ (statistical error) +98.1/-96.8 (systematic uncertainty) = $1344.2 \pm 120/-119$. Here and below we are adding errors in quadrature. The reported detection efficiency is .40, which gives $N_{\text{expt}} = 3361 \pm 300/-298$.

Since the Standard Solar Model (SSM) is well established, it is reasonable to consider the measured excess neutrons beyond those predicted by the SSM as due to CSL. The SSM prediction of the $^8B$ flux \[23\], applied to the SNO
experiment\[24\], is 13 +2.6/-2.08 neutrons/day or, for 254.2 days, \(N_{SSM}=3305+661/-529\) neutrons. Therefore, the excess number of neutrons which may be attributed to CSL is

\[N_{CSL} \equiv N_{\text{expt}} - N_{SSM} = 56 + 608/-725. \quad (9)\]

That is, according to the experiment (to one sigma accuracy), the number of excess neutrons produced due to the CSL excitation mechanism is not larger than 664, although it could be as small as 0, in which case the theory is incorrect. The negative value of the error in Eq. (9) simply quantifies the possibility that, in repeated identical experiments, the mean value obtained by the experimenters could be less than was obtained in this run. Of course, whatever mean value is measured, the putative CSL excitation would be a positive addition to the neutrino excitation. Thus, it is the positive error estimate that is relevant for our considerations.

If we take from Eq. (9) the experimental result \(N_{CSL} < 664\), we obtain from (8) the inequality

\[|g_n - M_n/M_p| < 8 \cdot 10^{-3} \left[\frac{(\lambda/a^2)_{GRW}}{\lambda/a^2}\right]^{1/2}. \quad (10)\]

(The result is actually .0074 which we have rounded up to .008). So, if \(\lambda/a^2\) has the GRW value, it then follows from (10) that \(g_n\) is within 1% of mass–proportionality:

\[g_n = \frac{M_n}{M_p} \pm .008. \quad (11)\]

We note that a limiting factor in improving this result is the 20% uncertainty in the SSM calculation which is the major source of the uncertainty in (11). Reduction of the combined theoretical and experimental uncertainty by a factor of about 30 is needed to reduce the uncertainty in \(g_n\) shown in Eq. (11) to the order of \((M_n - M_p)/M_p \approx 1.4 \cdot 10^{-3}\) [25].

The result (11) for \(g_n\) is 1600 times stronger than the comparable result (4) for \(g_e\). It strongly suggests that, if CSL with the GRW parameter values is to be physically viable, it should have mass–proportional coupling.

We conclude by considering other possible parameter values, since the GRW parameter values are ad hoc, although intelligently chosen. In a recent paper, experimental and theoretical constraints on these parameters were discussed (see the last paper in reference [10]). What is of interest here are the constraints on \(\lambda/a^2\), where \((\lambda/a^2)_{GRW} = 10^{-6}\) sec\(^{-1}\) cm\(^{-2}\).

The upper limit \(\lambda/a^2 < 2.5\) sec\(^{-1}\) cm\(^{-2}\) was obtained by Fu [11] from comparison of the CSL prediction of radiation of electrons in the conduction band of Ge with experimental data [11].

The lower limit is a “theoretical constraint.” The theory is supposed to describe the world as we see it, e.g., it should not leave uncollapsed for any “appreciable amount of time” a superposed state of a ball in two “discernibly
different” places. This constraint can be roughly quantified by requiring that
a just visible object, a sphere of diameter $d \approx 4 \cdot 10^{-5}$ cm, in a superposition
of two just touching states (centers separated by $d$), collapse in less than the
human perception time $\approx 0.1$ sec. The constraint formulas below are given in the
first paper in reference [20].

For $a >> d/2$, the constraint is $[\lambda N^2 d^2 / 4a^2]^{-1} < .1$. $N$ is the number of
nucleons in the sphere: for a sphere of density 1 gm/cc, $N \approx 2 \cdot 10^{10}$. This
constraint becomes $\lambda / a^2 > .6 \cdot 10^{-10}$ sec$^{-1}$ cm$^{-2}$.

For $a << d/2$, the constraint is $[\lambda N^2 a^3 (4\pi)^{3/2} / V]^{-1} < .1$, where
$V$ is the volume of the sphere. This constraint becomes $\lambda / a^2 > 2 \cdot 10^{-35} / a^5$ sec$^{-1}$ cm$^{-2}$. For $a \approx 10^{-5}$ cm (a value somewhat large for the applicability of this equation),
this becomes $\lambda / a^2 > 2 \cdot 10^{-10}$ sec$^{-1}$ cm$^{-2}$.

Therefore, for $(\lambda / a^2)_{GRW} = 10^{-6}$ sec$^{-1}$ cm$^{-2} < \lambda / a^2 < 2.5$ sec$^{-1}$ cm$^{-2}$, the
error is even smaller than that in Eq. (11), making mass-proportional coupling
even more likely. On the other hand, at the “theoretical constraint” limit $\lambda / a^2 \approx 10^{-10}$ sec$^{-1}$ cm$^{-2}$, the error in Eq. (11) becomes $\approx \pm .8$. One would need an even
better experimental constraint on the “spontaneous excitation” of bound states
than given here to conclude in this case that mass-proportional coupling is
inescapable for a viable collapse model.

References

[1] J. S. Bell, in Sixty-Two Years of Uncertainty, edited by A. Miller, (Plenum,
New York 1990), p. 17.

[2] P. Pearle, Phys. Rev. D 13, 857 (1976).

[3] P. Pearle, Int’l. Journ. Theor. Phys. 48, 489 (1979); Found. Phys. 12, 249
(1982).

[4] P. Pearle, Phys. Rev. A 39, 2277 (1989).

[5] Some references in which appear elucidation and extension of the ideas in-
troduced in Ref[4] are: G. C. Ghirardi, P. Pearle and A. Rimini, Phys. Rev.
A 42, 78 (1990); G. C. Ghirardi and P. Pearle in PSA 1990, Vol.2, A. Fine,
M. Forbes and L. Wessels eds. (Philosophy of Science Association, Michigan,
1990); Aicardi, A. Borsellino, G. C. Ghirardi and R. Grassi, Found.
Phys. Lett. 4, 109 (1991); G. C. Ghirardi and A. Rimini in Sixty-Two Years
of Uncertainty, edited by A. Miller (Plenum, New York, 1990), p.167; P.
Pearle, Phys. Rev. A 48, 913 (1993); P. Pearle in Stochastic Evolution of
Quantum States in Open Systems and in Measurement Processes, edited by L.
Diosi and B. Lukacs (World Scientific, Singapore, 1994), p. 79.; P.
Pearle in The Interpretation Of Quantum Theory; Where Do We Stand?,
edited by L. Accardi (Enciclopedia Italiana, Roma, 1994), p. 187; G. C.
Ghirardi and R. Grassi, Stud. Hist. Philos. Sci. 25, 97 (1994).
[6] The mathematical collapse formalism without commitment to a physical model also appeared in N. Gisin, Helvetica Physica Acta 62, 363 (1989); V. P. Belavkin, Physics Letters A 140, 355 (1989); A Barchielli and V. P. Belavkin, J. Phys. A: Math. Gen. 24, 1495 (1991). Collapse models inapplicable to macroscopic objects are discussed in N. Gisin and I. C. Percival, J. Phys. A: Math. Gen. 25, 5677 (1992) and 26, 2245 (1993); I. C. Percival, Proc. Roy. Soc. A 451, 503 (1995) and Quantum State Diffusion, (Cambridge Univ. Press, Cambridge, 1998); D. I. Fivel, Phys. Rev. A 56, 146 (1997). The gravitationally based model of L. Diosi, Phys. Rev. A 40, 1165 (1989) with the necessary modification of G. C. Ghirardi, R. Grassi and A. Rimini, Phys. Rev. A 42, 1057 (1990) is essentially equivalent to CSL with mass-proportional coupling (the property tested in this paper). The gravitationally based collapse criterion of R. Penrose, Gen. Rel. and Grav. 28, 581 (1996) is not a dynamical theory. The energy-based collapse model of L. P. Hughston, Proc. Roy. Soc. Lond. A, 452, 953 (1996) has been elucidated in S. L. Adler and L. P. Horwitz, Journ. Math. Phys. 41, 2485 (2000); S. L. Adler, D. C. Brody, T. A. Brun and L. P. Hughston, J. Phys. A: Math. Gen: 34, 8795 (2001); D. C. Brody and L. P. Hughston, Journ. Math. Phys. 43, 5254 (2002) and Proc. Roy. Soc. Lon. 459, 2297 (2003); S. L. Adler, J. Phys. A: Math. Gen. 35, 841 (2002); Phys. Rev. D 67, 25007 (2003). This model does not give proper collapse for macroscopic superpositions: see P. Pearle, Phys. Rev. A 69, 042106 (2004).

[7] G. C. Ghirardi, A. Rimini and T. Weber, Phys. Rev. D 34, 470 (1986); Phys. Rev. D 36, 3287 (1987); Found. Phys. 18, 1, (1988).

[8] P. Pearle, Phys. Rev. D 29, 235 (1984); A. Zeilinger in Quantum Concepts in Space and Time, edited by R. Penrose and C. J. Isham (Clarendon, Oxford, 1986), p. 16; A. Zeilinger, R. Gaehler, C. G. Shull, W. Treimer and W. Mampe, Revs. Mod. Phys. 60, 1067 (1988); A.J. Leggett, Found. Phys. 29 445 (1999); J. R. Clauser in Experimental Metaphysics, edited by R. S. Cohen, M. Horne and J. Stachel (Kluwer, Dordrecht, 1997), p. 1; W. Marshall, C. Simon, R. Penrose and D. Bouwmeester, Phys. Rev. Lett. 91, 130401 (2003).

[9] C. Henkel, M. Nest, P. Domokos and R. Folman, “Optical discrimination between spatial decoherence and thermalization of a massive object,” arXiv:quant-ph/0310160.

[10] F. Karolyhazy, Nuovo Cimento 42A, 1506 (1966); F. Karolyhazy, A Frenkel and B. Lukacs in Physics as Natural Philosophy, edited by A. Shimony and H. Feshbach (M.I.T. Press, Cambridge, 1982), p. 204; in Quantum Concepts in Space and Time, edited by R. Penrose and C. J. Isham (Clarendon, Oxford, 1986), p. 109; A. Frenkel, Found. Phys. 20, 159 (1990); Ref. 7; L. Diosi, Phys. Lett. A 132, 233 (1988); L. Diosi, Phys. Rev. A 40, 1165 (1989); F. Benatti, G. C. Ghirardi and R. Grassi, Found. Phys. 35, 5 (1995); A. Bassi and G. C. Ghirardi, Brit. Journ. Phil. Sci. 50, 719 (1999); B. Collett and P. Pearle, Found. Phys. 33, 1495 (2003).
[11] L. Diosi and B. Lukacs, *Phys. Lett. A* 181, 366 (1993); Q. Fu, *Phys. Rev. A* 56, 1806 (1997).

[12] Ref[7]; E. J. Squires, *Phys. Lett. A* 158, 431 (1991); L. E. Ballentine, *Phys. Rev. A* 43, 9 (1991).

[13] P. Pearle, *Found. Phys.* 30, 1145 (2000).

[14] P. Pearle and E. Squires, *Phys. Rev. Lett.* 73, 1 (1994).

[15] L. Diosi in Ref[10]; G. C. Ghirardi, R. Grassi and A. Rimini, *Phys. Rev. A* 42, 1057 (1990); R. Penrose in *Quantum Concepts in Space and Time*, edited by R. Penrose and C. J. Isham (Clarendon, Oxford 1986), p. 129; in *The Emperor’s New Mind*, (Oxford University Press, Oxford, 1992) and in *Shadows of the Mind*, (Oxford University Press, Oxford, 1994) and in Ref[8]; P. Pearle and E. Squires, *Found. Phys.* 26, 291 (1996).

[16] G. C. Ghirardi and T. Weber in *Potentialities, Entanglement and Passion-at-a Distance, Quantum Mechanical Studies for Abner Shimony*, edited by R. S. Cohen, M. Horne and J. Stachel (Kluwer, Dordrecht, 1997), p.89.

[17] Philip Pearle in *Experimental Metaphysics*, edited by R. S. Cohen, M. Horne and J. Stachel (Kluwer, Dordrecht, 1997), p. 143.

[18] A. Rimini, *Found. Phys.* 27, 1689 (1997).

[19] H. S. Miley, F. T. Avignone III, R. L. Brodzinski, J. I. Collar and J. H. Reeves, *Phys. Rev. Lett.* 65, 3092 (1990).

[20] B. Collett, P. Pearle, F. Avignone and S. Nussinov, *Found. Phys.* 25, 1399 (1995); P. Pearle, James Ring, J. I. Collar and F. T. Avignone III, *Found. Phys.* 29, 465 (1999).

[21] The most recent data, which we use, is in SNO Collaboration, *Phys. Rev. Lett.* 92, 181301 (2004). A previous paper is *Phys. Rev. Lett.* 89, 011301 (2002).

[22] E. Segré, *Nuclei and Particles* (W. A. Benjamin, New York 1964), pp.380-382; J. M. Blatt and V. F. Weisskopf, *Theoretical Nuclear Physics* (Wiley, New York, 1960), pp. 49-56.

[23] J. Bahcall, *Astrophys. J.* 555, 990 (2001).

[24] SNO collaboration, *Nucl. Instr. and Meth.* A449, 172 (2000).

[25] We note, until the uncertainty is reduced by that amount, the results are also consistent with Baryon number proportionality. This would imply e.g., that a macroscopically distinguishable superposition of leptons or of mesons would never collapse, and one might regard that as an undesirable feature of a collapse model.