Comment on “Can there be a quark-matter core in a magnetar?”

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Abstract

We comment on the paper by Tanusri Ghosh and Somenath Chakrabarty, Phys. Rev. D63, 043006 (2001). In that paper it was argued that a first-order transition to quark matter in the core of a magnetar is absolutely forbidden if its magnetic field strength exceeds $10^{15}$ G. However, we show in this comment that if the quark anomalous magnetic moment and the population of higher Landau levels is taken into account, it may still be possible for a first-order phase transition from nuclear matter to quark matter to occur in the core of a magnetar. These effects may also obscure the question of whether beta equilibrium favors strange quark matter in the core of a magnetar.

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Ghosh and Chakrabarty [1,2] have recently investigated the effects of a strong quantizing magnetic field on the nucleation of quark-matter droplets in the core of a magnetar. They argued that since the surface energy of the quark phase diverges logarithmically for the zeroth Landau level, a first-order phase transition from hadronic to quark matter would be absolutely forbidden at the core of a magnetar. This is true as long as the magnetic field strength exceeds $10^{15}$ G and only the zeroth Landau levels of light $(u, d)$ quarks are populated. Here, however, we argue that if the quark anomalous magnetic moment and higher Landau levels are taken into account, it is possible to avoid the divergence in the zeroth Landau level so that a first-order phase transition from nuclear matter to quark matter may still be possible in the core of a magnetar.

They also computed the chemical equilibrium and concluded that beta equilibrium does not favor the existence of strange quark matter in the core of a magnetar. Here, we argue that, this conclusion may also be obscured by effects of the quark anomalous magnetic moment and higher Landau levels, though their conclusion probably remains valid.

The argument of Ghosh and Chakrabarty regarding the phase transition is as follows. The rate of nucleation of stable quark-matter droplets due to fluctuations in a metastable hadronic medium is given by [1]

$$
\Gamma = \Gamma_0 e^{-\sigma^3/C},
$$

where $\Gamma_0$ and $C$ are finite constants, and $\sigma$ is the surface tension of the quark phase in metastable hadronic matter.

Then, using the MIT bag model of color confinement, the expression for the surface tension of the quark phase in the presence of a strongly quantizing magnetic field of strength $B_m$ is given by [1,2]

$$
\sigma_i = \frac{2TB_m}{8\pi}Z_i^q \sum_{\nu=0}^{\infty} \int_{\nu=0}^{\infty} \frac{dk_z}{(k_z^2 + k_{\perp}^2)^{1/2}} \ln \left( 1 + \exp \left[ -\frac{E_{\nu(i)} - \mu_i}{T} \right] \right) G_i,
$$

where $i$ indicates the quark flavor $(u, d, \text{or } s)$, $Z_i^q$ is the flavor charge, $\nu$ is the Landau
quantum number, $k_z$ and $k_\perp$ are the components of momentum along and perpendicular to the $z$-axis, and $\mu_i$ is the chemical potential. The quark-energy eigenvalues are given by

$$E_{\nu(i)} = \left[m_i^2 + k_z^2 + k_{\perp}^2\right]^{1/2},$$ (3)

with

$$k_{\perp(i)} = \sqrt{2\nu Z_i^q B_m},$$ (4)

and the quantity $G$ in Eq. (2) is defined as $G = 1 - 2\tan^{-1}(k/m_i)/\pi$.

Now, through simple integration by parts, we can easily derive the surface tension of the quark phase and show that it diverges logarithmically for the lowest Landau level, $\nu = 0$. That is, the surface tension goes as $\sim -\ln(\nu)$ as $\nu \rightarrow 0$ in the presence of a quantizing magnetic field. There is also a factor $1/k = 1/(k_z^2 + k_{\perp}^2)^{1/2}$ in Eq. (2) which will diverge for $k_z = 0$ and $\nu = 0$. Therefore, the rate of nucleation of quark-matter droplets as given by Eq. (1) becomes identically zero. It thus follows that if the magnetic field at the core of a neutron star is strong enough to populate the Landau levels of quarks, the nucleation of quark droplets becomes impossible, which means that under such conditions a first-order phase transition from hadronic matter into quark matter is absolutely forbidden. Ghosh and Chakrabarty also have insisted that if higher order as well as the zeroth order Landau levels are populated simultaneously, the conclusions do not change.

However, as mentioned in Ref. [5], if ultra-strong magnetic fields can exist in the interior of neutron stars, such fields will affect the behavior of the residual charged particles. Moreover, contributions from the anomalous magnetic moment (AMM) of the particles in a strong magnetic field should also be significant. Therefore, we should include the AMM to derive a modified energy spectrum of the particles.

Then, the energy dispersion relation for quarks $E_{q(i)}$ for an arbitrary Landau level in a magnetic field becomes [6]:

$$E_{q(i)} = k_z^2 c^2 + \left\{m_i c^2 \left[1 + 2\gamma_q n_{q(i)}\right]^{1/2} - s_q Z_i^q B_m \right\}^2,$$ (5)
where \( n_f^{q(i)} = n + \frac{1}{2} - s_{z}^{q} \), \( n \) is the principal quantum number of the Landau level, \( s_{z}^{q} = \pm \frac{1}{2} \) is the \( z \) component of the quark spin, and \( \mu_{f}^{i} \) is the quark magnetic moment discussed below. The summation over \( \nu \) in Eq. (2) is now replaced with two summations over \( n \) and \( s_{z}^{q} \), i.e.,

\[
\sum_{\nu=0}^{\infty} \Rightarrow \sum_{n=0}^{\infty} \sum_{s_{z}^{q}}.
\]

(6)

An estimate of the quark anomalous magnetic moment (QAMM) can be obtained as follow. In the Pauli notation for fermions with charge \( e_{f} \), the electronmagnetic current up to first order in the photon momentum \( q_{\nu} \) is given by

\[
J^{\mu} = e_{f} \bar{u} \Gamma^{\mu} u = e_{f} \bar{u} \left[ F_{1}(q^{2}) \gamma^{\mu} + F_{2}(q^{2}) \frac{i\sigma^{\mu\nu}}{2mc} q_{\nu} \right] u,
\]

(7)

where the form factors \( F_{1}(q^{2}) \) and \( F_{2}(q^{2}) \) describe the electromagnetic structure of the charged particle. For free photons, \( q^{2} = 0 \), and thus, the charge form factor \( F_{1}(0) = 1 \). The magnetic form factor \( F_{2}(0) \equiv \kappa_{f} \) stands for the anomalous part of the magnetic moment \( \mu_{f} \). It is given by

\[
\mu_{f} = \mu_{0} \kappa_{f}, \quad \text{with} \quad \mu_{0} = \frac{e_{f} c}{2mc},
\]

(8)

and \( m \) stands for the particle mass. For example, the magnetic moment of ground-state nucleons has the experimentally measured values of \( \kappa_{p} \simeq 2.79 \) for the proton and \( \kappa_{n} \simeq -1.91 \) for the neutron. Therefore, in the constituent quark model for light hadrons, we have

\[
\mu_{p} = \frac{1}{3}(4\mu_{u} - \mu_{d}), \quad \mu_{n} = \frac{1}{3}(4\mu_{d} - \mu_{u}),
\]

(9)

which leads to a nonzero quark anomalous magnetic moment of

\[
\mu_{u} = 1.852\mu_{N}, \quad \mu_{d} = -0.972\mu_{N},
\]

(10)

with \( \mu_{N} \), the nuclear magneton.

There are several theoretical and experimental studies which indicate that quarks have a QAMM \([7-11]\). Recently Bicudo et al. \([7]\) have shown that in the case of massless-current quarks, chiral symmetry breaking usually triggers the generation of an anomalous magnetic
moment for the quarks. [Notice that the hadron ↔ quark phase transition is deeply related to chiral symmetry breaking.] They also computed the QAMM in several quark models. Similarly, Singh [8] has also proven that, in theories in which chiral symmetry breaks dynamically, quarks can have a large anomalous magnetic moment. In particular, Köpp et. al. [9] have provided a stringent bound on the QAMM from high-precision measurements at LEP, SLC, and HERA. To fit the measured magnetic moment of the baryon octet, it is found that the quarks must have a QAMM [10]. Therefore, on the chiral symmetry breaking scale, if one considers the phase transition from hadronic matter to quark matter in a strong magnetic field, one should include the QAMM in the quark-energy spectrum.

Given that a QAMM likely exists, then our main point is that if instead of Eq. (3), we use Eq. (5) which includes the QAMM, we can at least avoid a divergence from the lowest Landau level, $\nu = 0$, in calculations of the surface tension [Eq. (2)] as well as in the curvature energy [2] which are included in the bubble nucleation rate. Therefore, a finite value of the bubble nucleation rate can be obtained. This means that it is still possible for a first-order phase transition to quark matter to occur in a magnetar.

Even if a first-order phase transition to quark matter is possible, however, one must still consider whether $uds$-quark matter is the energetically preferred state in beta equilibrium. In [1,3], the chemical evolution of quark matter was studied. They concluded that in the presence of a strong magnetic field $\geq 4.4 \times 10^{13} \text{G}$, the matter in beta equilibrium is energetically unstable with respect to normal neutron matter at the core. Hence, they concluded that the existence of quark matter is impossible if the magnetic field exceeds the quantum limit for electrons. An alternative way to understand this is that, since the quantum mechanical effect of a strong magnetic field on strange quarks is negligible unless the magnetic field strength is $\geq 10^{20} \text{G}$, it is energetically favorable for the system to produce $d$-quarks whose Landau levels are populated, or if the magnetic field is low enough so that only the electrons are affected, then it is energetically much more favorable to produce electrons and $u$-quarks (to balance the charge). As a result, the system in equilibrium is not $uds$-quark matter.
Here we point out that their conclusions were based upon the presumption that only the lowest Landau level was populated for all charged particles (except the s-quark). As shown in [5], the Landau levels at high density depend upon both the strength of the magnetic field and the Fermi energy. At high density, beta equilibrium (i.e., charged particle fraction) is absolutely different between the case in which only the lowest Landau level is considered and the case in which higher Landau levels are included. In particular, when the anomalous magnetic moment is included, one must consider that the maximum landau level has a strong correction in the spin-polarization term (third term in Eq. (5)). There is a good chance that their conclusion may remain even if the ground states are recalculated using the higher Landau levels. Nevertheless, it is at least conceivable that a revised calculation could shift the equilibrium back to strange quark matter. This calculation should be redone before we can categorically conclude that strange quark matter core is absolutely impossible for a magnetar.

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