Thermodynamic properties of a magnetically modulated graphene monolayer

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Abstract
The effect of magnetic modulation on thermodynamic properties of a graphene monolayer in the presence of a constant perpendicular magnetic field is reported here. One-dimensional spatial electric or magnetic modulation lifts the degeneracy of the Landau levels and converts into bands and their bandwidth oscillates with magnetic field, leading to Weiss-type oscillations in the thermodynamic properties. The effect of magnetic modulation on the thermodynamic properties of a graphene sheet is studied and then compared with electrically modulated graphene and magnetically modulated conventional two-dimensional electron gas (2DEG). We observe Weiss-type and de Haas–van Alphen oscillations at low and high magnetic fields, respectively. There is a definite phase difference in Weiss-type oscillations in thermodynamic quantities of magnetically modulated graphene compared to electrically modulated graphene. On the other hand, the phase remains the same and the amplitude of the oscillation is large when compared with the magnetically modulated two-dimensional electron gas (2DEG). Explicit asymptotic expressions of the density of states and the Helmholtz free energy are provided to understand the phase and amplitude of the Weiss-type oscillations qualitatively. We also study thermodynamic properties when both electric and magnetic modulations are present. The Weiss-type oscillations still exist when the modulations are out-of-phase.

1. Introduction
Graphene is a two-dimensional sheet of a carbon allotrope with a honeycomb lattice structure. It can be considered as a basic building block of all other dimensional carbon allotropes [1–4]. The relativistic-like, massless and linear energy spectrum of graphene’s quasi-particles in the low-energy range close to the Dirac points in its band structure reflect its different properties like transport properties, optical properties, magnetic properties, etc, in different ways than for the conventional two-dimensional electron gas (2DEG) formed in the semiconductor heterostructures. The massless linear energy dispersion and charge conjugation symmetry cause some unusual phenomena like the Klein paradox, anomalous quantum Hall effect and non-zero Berry phase [5–9].

The effect of electric and/or magnetic modulations on transport and thermodynamic properties of quantum 2DEG systems is continuing to be an active research field. The magnetotransport properties of a conventional 2DEG in the presence of a one-dimensional (1D) weak electric modulation have been studied in great detail experimentally and theoretically by various groups [10–15]. Later, the magnetotransport properties of a magnetically modulated 2DEG in the presence of a constant perpendicular magnetic field have been studied theoretically [16–19] and also experimentally [20–22]. The presence of weak electric/magnetic modulation broadens the Landau energy levels into bands. The bandwidths oscillate with the magnetic field and its oscillations are also reflected in magnetotransport properties. It has been observed that the magnetoresistivity tensor oscillates with the inverse of the magnetic field at very low magnetic field. This oscillation is known as the Weiss oscillation, which is completely different from the Shubnikov–de Haas (SdH) oscillations observed at higher magnetic field. The period of the Weiss oscillation varies
with electron density $n_e$ as $\sqrt{n_e}$, whereas for SdH it depends linearly on $n_e$. The Weiss oscillation is due to the effect of the commensurability between two length scales in the system: the cyclotron diameter at the Fermi energy and the period of the modulated electric/magnetic potential. Magnetotransport properties of electrically and magnetically modulated graphene have been studied recently.

The effect of weak electric and magnetic modulation, on thermodynamic properties for 2DEG in the presence of a perpendicular constant magnetic field have been studied theoretically [12, 26, 27]. It is observed that the Weiss-type oscillation in various thermodynamic properties is present. The Weiss-type oscillations are completely different in origin from the usual de Haas–von Alphen (dHvA) oscillations which occur at high magnetic field. The dHvA oscillation is the effect of the formation of discrete Landau energy levels due to the quantization of magnetic field. Recently, thermodynamic properties of monolayer graphene in the presence of weak electric modulation have been studied and the Weiss-type oscillations are seen [28]. These results inspired us to study the thermodynamic properties of a magnetically modulated graphene sheet in the presence of a constant magnetic field.

The source of the magnetic modulation (magnetic stripes or superconducting materials) acts like electrical gates and produces an electric modulation. The transport properties of a 2DEG [16] and a graphene sheet [29] in the presence of both modulations were studied.

In this paper we study the effect of magnetic modulation on thermodynamic properties of a graphene sheet in the presence of a constant magnetic field. We compare our results with electrically modulated graphene and with magnetically modulated conventional 2DEG. We also calculate an asymptotic expression of the density of states (DOS) and the Helmholtz free energy of a magnetically modulated graphene in the presence of a constant magnetic field at low temperature. In addition, we also study thermodynamic properties of graphene and 2DEG when both electric and magnetic modulations are present.

This paper is organized as follows. In section 2, we summarize the standard results of the energy spectra and the corresponding eigenstates for electrically and magnetically modulated graphene layers and 2DEG in the presence of a constant magnetic field. In section 3, we numerically calculate the thermodynamic quantities like Helmholtz free energy, internal energy, entropy, heat capacity and magnetization. We analyze our numerical results and compare them with electrically modulated graphene and magnetically modulated 2DEG in section 4. We discuss the behavior of the asymptotic expression of the DOS and the Helmholtz free energy for magnetically modulated graphene in section 5. In section 6, we study the thermodynamic properties of graphene and 2DEG in the presence of both electric and magnetic modulations. We present a summary of our work in section 7. The detailed calculation of the DOS by using the Green’s function method is presented in the appendix.

### 2. Energy spectrum

We consider a monolayer graphene sheet subjected to a perpendicular constant magnetic field $\mathbf{B}_0 = B_0\mathbf{e}_z$. The Hamiltonian of an electron with charge $-e$ is given by

$$H^G_0 = v_F \sigma \cdot (p + e\mathbf{A}_0),$$

(1)

where $\sigma = (\sigma_x, \sigma_y)$ are the Pauli matrices, $v_F \approx 10^6$ m s$^{-1}$ is the Fermi velocity and $\mathbf{A}_0$ is the magnetic vector potential. Here, we have chosen the Landau gauge $\mathbf{A}_0 = B_0 y \mathbf{e}_x$. The energy eigenvalues are $E^m_n = h_0 a_e \sqrt{2n}$, where $n = 0, 1, 2, 3, \ldots$ is the Landau level index and $a_e = v_F \sqrt{e B_0}/\hbar$ is the cyclotron frequency. The corresponding normalized eigenstates are

$$\Psi_{n,k_y}(x,y) = \frac{e^{i k_y y}}{\sqrt{2\pi \sqrt{1/2}}} \left[ -i \phi_{n-1}(x+x_0)/l_0 \right] \phi_n(x+x_0)/l_0,$$

(2)

where $\phi_{n}(x)$ is the known harmonic oscillator wavefunction, $l_0 = \sqrt{\hbar/(e B_0)}$ is the magnetic length, $x_0 = -k_y l_0^2/\hbar$ is the center of the cyclotron orbit and $L_y$ is the width of the graphene sheet in the y direction.

We consider the perpendicular magnetic field is modulated very weakly by $B_{\perp} = B_1 \cos(q x) \mathbf{e}_z$, where $q = 2\pi/a$ and $a$ is the modulation period. The total Hamiltonian can be written as $H = H^G_0 + H_1$, where $H_1 = V_{\perp}^e B_1/q$ is the strength of the effective magnetic potential determined by the amplitude $B_1$ and the period $a$ of the magnetic modulation. Using first-order perturbation theory, the energy correction to the unperturbed Hamiltonian $H^G_0$ is given as [25]

$$\Delta E^e_{n,k_y} = V_{\perp}^e \sqrt{\pi/2 u} \left[ L_{n-1}(u) - L_n(u) \right] \cos(q x_0),$$

(3)

where $L_n(u)$ is the Laguerre polynomial and $u = q^2 l_0^2/2$. So the total energy up to first order in $V_{\perp}^e$ is given by $E_{n,k_y}^e = E_n^0 + \Delta E^e_{n,k_y}$. The bandwidth in the presence of the magnetic modulation is $\Delta_{\perp} \sim \sin(2\sqrt{m u} - \pi/4)$. Using the flat-band condition, we get $2R_c = a(j + 1/4)$, with $j = 0, 1, 2, 3, \ldots$ and $R_c = k_y l_0^2/\hbar$.

The energy correction due to the weak electric modulation $U(x) = V_{\perp}^e \cos(q x)$ can be obtained by the same method and it is given as [23]

$$\Delta E^e_{n,k_y} = \frac{V_{\perp}^e}{2} e^{-u/2} \left[ L_n(u) + L_{n-1}(u) \right] \cos(q x_0),$$

(4)

So the total energy up to the order of $V_{\perp}^e$ is $E_{n,k_y}^{e,e} = E_n^0 + \Delta E^e_{n,k_y}$. The bandwidth in the presence of the electric modulation is $\Delta_e \sim \cos(2\sqrt{m u} - \pi/4)$. The bandwidths due to magnetic modulation and due to electric modulation are out of phase. The band will be flat when $2R_c = a(j + 3/4)$, which is different from what we get in the magnetic modulation case.

The Hamiltonian of a conventional 2DEG in the presence of a perpendicular constant magnetic field $\mathbf{B}_0$ is

$$H_{0d}^d = \frac{p_x^2}{2m^*} + \frac{1}{2m^*} \left( p_y + eA_y \right)^2.$$

(5)
The energy spectrum is $E_{2d}^n = h\omega_c(n + 1/2)$, where $n = 0, 1, 2, 3, \ldots$ and $\omega_0 = eB_0/m^*$ is the cyclotron frequency. The corresponding eigenstates are

$$\Psi_{n,k_y}(x, y) = \frac{e^{ik_y y}}{\sqrt{L_y}} \phi_n(x + x_0)/l_0. \quad (6)$$

In the presence of a weak magnetic modulation $B_1$, the total Hamiltonian is $H = H_0^{2d} + H_1^{2d}$, where $H_0^{2d} = [V_m^{2d}/(\hbar q)](p_y + eB_0 x)\sin(qx)$ with $V_m^{2d} = h(eB_1/m^*)$ the strength of the effective magnetic potential determined by the amplitude of the magnetic modulation. The first-order energy correction due to the weak magnetic modulation is given by [16, 17]

$$\Delta E_{n,k_y}^{2d,m} = V_m^{2d} e^{i\frac{\pi}{2} - \frac{\pi}{2}u} \left[ \frac{u - 2n}{2\mu} L_n(u) + \frac{n}{u} L_{n-1}(u) \right] \cos(qx_0). \quad (7)$$

The total energy of a 2DEG in the presence of the magnetic modulation is then $E_{n,k_y}^{2d,m} = E_{n,k_y}^{2d} + \Delta E_{n,k_y}^{2d,m}$. The energy correction due to electric modulation is given by

$$\Delta E_{n,k_y}^{2d,e} = V_e^{2d} e^{-u/2} L_n(u) \cos(qx_0). \quad (8)$$

All these standard results will be used to calculate the thermodynamic properties numerically in section 3.

3. Thermodynamic quantities

In this section we discuss all standard thermodynamic equations to be used for calculating the chemical potential, Helmholtz free energy, internal energy, entropy, magnetization and specific heat.

The DOS of a magnetically modulated graphene sheet in the presence of a constant magnetic field can be written as

$$D(E) = \frac{A}{\pi l_0^2} \sum_{n,k_y} \delta(E - E_{n,k_y}), \quad (9)$$

where $A = L_x L_y$ is the area of the graphene sheet and $E_{n,k_y}$ is an energy dispersion of a given system like $E_{n,k_y}^{2d,m}, E_{n,k_y}^{2d,e}, E_{n,k_y}^{2d,m}$. The dependence of chemical potential $\mu(B, T)$ on temperature ($T$) and magnetic field ($B$) can be obtained numerically by using the following normalization condition:

$$N = \int_0^\infty D(E) f(E) \, dE, \quad (10)$$

where $N$ is the total number of electrons, $f(E) = [\exp(E/\mu_B T) + 1]^{-1}$ is the Fermi–Dirac distribution function and $k_B$ is the Boltzmann constant. Using the expression of $D(E)$ given in the above equation we get

$$n_e \pi l_0^2 = \frac{1}{\pi} \sum_{n=0}^\infty \int_0^\pi f(E_{n,t}) \, dt, \quad (11)$$

where $n_e$ is the electron density and $t = qx_0$.

The total internal energy can be written as

$$U = \int_0^\infty E D(E) f(E) \, dE. \quad (12)$$

The internal energy per unit area is

$$U/\mathcal{A} = \frac{1}{\pi^2 l_0^2} \sum_{n=0}^\infty \int_0^\pi E_{n,t} f(E_{n,t}) \, dt. \quad (13)$$

Now for a system of non-interacting electrons, the Helmholtz free energy density [30] is given by

$$F/\mathcal{A} = \mu n_e - k_B T \sum_{n=0}^\infty \int_0^\pi \ln \left[ 1 + \exp \left( \frac{\mu - E_{n,t}}{k_B T} \right) \right] \, dt. \quad (14)$$

From the above equations it is clear that the DOS plays an important role in the behavior of the thermodynamic properties. In the presence of a perpendicular magnetic field the DOS shows a series of delta functions because of the quantized energy spectrum. The fact that graphene and conventional 2DEG have different energy spectra is reflected in their differing thermodynamic properties. By using the above results we compute entropy via $S = (U - F)/T$, orbital magnetization $M = -(\partial F/\partial B_0)_{A,N}$ and heat capacity $C = T (\partial S/\partial T)_{A,N} = -T (\partial^2 F/\partial^2 B_0)_{A,N}$. For better visualization of the effect of the magnetic modulation we plot the fluctuation $\Delta \Pi = \Pi(B_1) - \Pi(B_1 = 0)$, where $B_1$ is the strength of the magnetic modulation and $\Pi$ is a thermodynamic quantity like $\mu, F, U, M, S$ or $C$.

4. Numerical results and discussions

Thermodynamic properties of magnetically modulated graphene sheets in the presence of a constant magnetic field are studied. The aim is to study the effect of magnetic modulation on graphene in comparison with the electrically modulated graphene and magnetically modulated conventional 2DEG. We plot the fluctuation due to weak modulation in the chemical potential, Helmholtz free energy, magnetization, internal energy, entropy and specific heat with the magnetic field $B_0$. We have used the following parameters for numerical calculation—electron density $n_e = 3.16 \times 10^{15} \text{ m}^{-2}$, effective mass of an electron $m^* = 0.067 m_e$ with $m_e$ the bare electron mass, temperature $T = 2 \text{ K}$ and modulation period $a = 382 \text{ nm}$—as done in [26, 28]. For these parameters, $E_F^{2d} = 0.1 \text{ eV}$ and $E_F^{2d} = 14.25 \text{ meV}$.

In figures 1–6, we have plotted the fluctuations in various thermodynamic properties, $\Delta \Pi$, due to both magnetic and electric modulations. We have scaled all the thermodynamic quantities per electron. Each figure contains two panels: the upper panel shows the effect of the magnetic modulation on graphene (solid line) and conventional 2DEG (dashed line) and the lower panel shows this fluctuation for electrically modulated (dashed line) and magnetically modulated (solid line) graphene. We have enlarged the oscillations at low magnetic field and shown them in the insets of figures 1–5. It clearly shows that a weak 1D periodic potential, either electric or magnetic in nature, induces new oscillations at low magnetic field. These modulation-induced oscillations are due to the commensurability of the two length scales present in the system. These oscillations are similar to the Weiss oscillations observed in the magnetoresistance at low magnetic fields.
Figure 1. Plots of the fluctuation in chemical potential versus magnetic field at $T = 2$ K.

Figure 2. Plots of the change in free energy versus magnetic field. To make the fluctuation dimensionless we use $F_0 = E^{2d}_F/2$ in the lower panel but, as $E_F$ is different for the two systems, we show fluctuations in units of $k_B T$ in the upper panel.

Figure 3. Plots of the fluctuation in magnetization versus magnetic field. In the lower panel $\Delta M$ is scaled by $M_0 = \mu_B$, where $\mu_B = e\hbar/2m_g$ is the effective Bohr magneton with $m_g = E_g F/v_F^2$ the cyclotron mass.

Figure 4. Plots of the fluctuation in the internal energy versus magnetic field. In the lower panel $\Delta U$ is scaled by $U_0 = E^2_F/2$.

It is clear from the upper panels of figures 1–6 that the fluctuation in the thermodynamic properties, $\Delta \Pi$, of the graphene sheet is quite large compared to the 2DEG system. This can be understood qualitatively from the following arguments. The energy corrections for the magnetically modulated graphene and 2DEG systems are $V_m^g = 1$ meV and $V_m^{2d} = 0.046$ meV for the same strength of magnetic modulation $B_1 = 0.02$ T. Clearly, the energy correction due to the magnetic modulation in graphene is quite large compared to that of the 2DEG. This is the origin of the higher amplitude fluctuation of the Weiss-type and dHvA oscillations in graphene compared to the 2DEG. It is interesting to note that the amplitude of diffusive conductivity in magnetically modulated graphene is small compared to magnetically modulated 2DEG [25].

The lower panels of figures 1–6 show that the Weiss-type oscillation has a definite phase difference in the thermodynamic properties between magnetic and electric modulation cases, which is due to the following reasons. The phase difference in the fluctuations between electrically and magnetically modulated graphene comes from the nature of
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Figure 5. Plots of the fluctuation in entropy versus magnetic field. In the lower panel $\Delta S$ is scaled by $S_0 = k_B$.

Figure 6. Plots of the fluctuation in specific heat versus magnetic field.

The energy correction. In the electric modulation case the energy correction contains the addition of two successive Laguerre polynomials and in the magnetic modulation case it is the subtraction of two successive Laguerre polynomials, giving rise to the cosine and sine term, respectively. We also observe that the dHvA-type oscillations for the two different kinds of modulations remain in the same phase as each other. The phase difference between the two different kinds of modulated systems is shown explicitly by using the analytical expressions of the DOS in the next section.

Figures 1 and 3 show the fluctuation in chemical potential and magnetization with magnetic field, respectively. The fluctuation in magnetically modulated graphene is several times higher than the magnetically modulated 2DEG in the Weiss-type oscillation, but their phases remain the same. On the other hand, magnetically modulated graphene shows a $\pi$ phase difference in the Weiss-type oscillation compared to the electrically modulated graphene with the same amplitude.

Figure 2 shows the fluctuation in the Helmholtz free energy where the lower panel shows $\pi/2$ phase differences between the electric and magnetic modulation cases in graphene with the same amplitude. The amplitude of the fluctuation of graphene is several times higher than in the 2DEG but the phases remain the same. From the flat-band condition, the minima of the bandwidth occur at $B(T) = 0.092, 0.113, 0.148, 0.214$ and 0.385. On the other hand, the fluctuation in the free energy vanishes at $B(T) = 0.092, 0.115, 0.150, 0.214$ and 0.385. It shows that the minima of the free energy fluctuation occur at those values of the magnetic field where the bandwidth minima occur.

We have plotted internal energy fluctuations in figure 4. The lower panel of figure 4 shows that the Weiss-type oscillation appears with the same amplitude but a $\pi/2$ phase difference when compared with the electrical modulation case. When we compare it with conventional 2DEG, it is similar to the case of the Helmholtz free energy, i.e. the amplitude is higher in magnitude.

Figures 5 and 6 show the entropy and specific heat fluctuation, respectively. The fluctuation in graphene is higher by several times than that of the 2DEG. The phase relationship is not discernible in these figures.

In all the above cases, the dHvA-type oscillation remains in the same phase and does not depend on the modulation type as it is the manifestation of the quantized Landau levels rather than the periodic perturbation. Figure 7 shows the damping of the fluctuation in chemical potential and free energy with increasing temperature. The temperature dependence in Weiss-type and dHvA oscillations is independent of
Table 1. The phase shifts in the Weiss-type oscillations appear in the fluctuation of thermodynamic quantities between electrically and magnetically modulated graphene.

| Case   | $\Delta \mu$ | $\Delta F$ | $\Delta M$ | $\Delta U$ | $\Delta S$ | $\Delta C$ |
|--------|-------------|-------------|-------------|-------------|-------------|-------------|
| Phase shift | $\pi$ | $\pi/2$ | $\pi$ | $\pi/2$ | Indiscernible | Indiscernible |

the type of modulation and is already discussed in the electrical modulation case. The phase relationships for all the thermodynamic quantities are given in table 1.

Even at higher magnetic field, the effect of modulation on the fluctuation of the thermodynamic quantities still exists which is the manifestation of the modulated DOS. The dHvA-type oscillation corresponds to the crossing of each Landau level one by one through the Fermi level. The density of available states per Landau level is $1/2\pi l_0^2$ and it increases linearly with magnetic field, which results in an increasing amplitude of the dHvA oscillation with magnetic field.

5. Asymptotic results

Here we derive an asymptotic expression for the DOS and the Helmholtz free energy. For weak magnetic modulation and under a quasi-classical limit, we calculate the DOS by using the Green’s function technique (see the appendix) and written as a sum of the modulated and unmodulated parts as $D(\epsilon) = D_u(\epsilon) + D_m(\epsilon)$, where

$$D_u(\epsilon) = \frac{1}{\pi l_0^2 \hbar \omega_g} \left[ 1 + 2 \cos(\pi \epsilon^2) \right],$$

$$D_m(\epsilon) = -\frac{2\Omega_m}{\pi l_0^2 \hbar \omega_g} \left[ \epsilon e^{3/2} \cos(\pi \epsilon^2) \sin^2 \left( q_0 l_0 - \frac{\pi}{4} \right) \right],$$

and

$$\Omega_m = \frac{(V_m^g)^2}{\pi^2} \left( \frac{a}{l_0} \right)^3 \left( \frac{1}{\hbar \omega_g} \right)^3 \sin^2 \left( q_0 \frac{g_F}{2 \hbar^2} \right).$$

Here, $\epsilon = E/(\hbar \omega_g)$ and $e_F^g = E_F^g/(\hbar \omega_g)$.

The fluctuation in the DOS for electromagnetically modulated graphene [28] is proportional to $\cos^2(q_0 \hbar \omega_g^2 - \pi/4)$. The appearance of the square of the sine term in equation (16) instead of the square of the cosine is the reason for definite phase differences in the fluctuation of all the thermodynamic quantities.

Using the two equations (15) and (16) separately, we get an approximate analytical expression of the free energy and its fluctuation. Our aim is to study the magnetic modulation effect in comparison to the electrical modulation in graphene [28]. The change in the free energy due to the magnetic modulation can be expressed as

$$F_m = \frac{-k_B T \hbar \omega_g}{\pi l_0^2} \int_0^\infty D_m(\epsilon) \ln \left[ 1 + \exp \left( \frac{\mu - \hbar \omega_g \epsilon}{k_B T} \right) \right] d\epsilon$$

$$= -\frac{\Omega_m}{\pi l_0^2} \left( e_F^g \right)^4 \left( k_B T \hbar \omega_g \sin^2 \left( q_0 \frac{g_F}{2 \hbar^2} - \frac{\pi}{4} \right) \right) \int_0^\infty \cos(\pi \epsilon^2) \ln \left[ 1 + \exp \left( \frac{\mu - \hbar \omega_g \epsilon}{k_B T} \right) \right] d\epsilon.$$

Under the assumption of very low temperature the above integration results in

$$F_m = \frac{-\Omega_m}{\pi l_0^2} \left( e_F^g \right)^4 \left( \sin(\pi (e_F^g)^2) \right)$$

$$= \left( 1 - \frac{T/T_{dHvA}}{\sinh(T/T_{dHvA})} \right) \frac{1}{2} \cos(\pi (e_F^g)^2) \right] \right]$$

$$\times \sin^2 \left( q_0 \frac{g_F}{2 \hbar^2} - \frac{\pi}{4} \right),$$

where $T_{dHvA} = (\hbar \omega_g)/(2\pi^2 \hbar \omega_g^2)$ is the critical temperature for the dHvA-type oscillations in graphene. The ratio of the amplitude of the free energy fluctuations of magnetically and electrically modulated graphene is

$$\frac{\lambda_m^g}{\lambda_e^g} \approx \left( \frac{V_m^g}{V_e^g} \right)^2.$$
6. Electric and magnetic modulations

In this section, we study how the thermodynamic properties discussed in the previous sections are changed in the presence of an additional electric modulation with the same period. We consider two different cases: when both modulations are in phase and when they are \( \pi/2 \) out of phase. The fluctuation in thermodynamic quantities like the chemical potential and the Helmholtz free energy are calculated numerically. Other thermodynamic quantities can easily be obtained by taking a suitable numerical derivative of the Helmholtz free energy fluctuation.

6.1. In-phase modulations

We consider a weak electric modulation described by the periodic potential \( U(x) = V_e \cos(qx) \), which is in phase with the magnetic modulation \( B_1(x) = B_1 \cos(qx) \hat{z} \). When both modulations are in the same phase, the total energy correction for graphene in a weak magnetic field can be written as

\[
\Delta E^g = \sqrt{f_m^2 + f_e^2} \sin \left( 2\sqrt{n_{\mu}} - \frac{\pi}{4} + \delta_f^g \right) \cos(qx_0),
\]

where \( f_m = \left( 2V_{m}^{2} / \sqrt{\pi} \right) \left( n / \sqrt{4n} \right) \) \( 1/4 \sin \left( \sqrt{4n} / \sqrt{4n} \right) \), \( f_e = (V_e^2 / \sqrt{\pi}) \left( 1 / \sqrt{n \mu} \right) \cos \left( \sqrt{4n} / \sqrt{4n} \right) \) and \( \delta_f^g = \tan^{-1} \left( f_m / f_e \right) \). The flat-band condition at the Fermi energy gives the positions of the minima in the free energy fluctuation as \( B_j = 2 \pi B_{m} / (j + 1/4 - \delta_f^g) \). Here, \( j \) is an integer, \( p_F = ak_F \) is a dimensionless momentum and \( B_{m} = h / (e \alpha^2) \) is the characteristic magnetic field. In this case, \( \delta_f^g = \pi / 4 \) and then \( B_j = 2 \pi B_{m} / j \).

Similarly for conventional 2DEG, the total energy correction in the low magnetic field can be written as

\[
\Delta E^{2d} \simeq \sqrt{w_m^2 + w_e^2} \sin \left( 2\sqrt{\mu_{\nu}} - \frac{\pi}{4} + \delta_f^{2d} \right) \cos(qx_0),
\]

where \( w_m \simeq V_m^{2d} \sqrt{n_{\epsilon}(\pi u/\sqrt{\mu_{\nu}})} \), \( w_e \simeq V_e^{2d} / (\sqrt{\pi} \sqrt{\mu_{\nu}}) \) and \( \delta_f^{2d} = \tan^{-1} (w_m / w_e) \). The flat-band condition at the Fermi energy is now \( B_j = 2 \pi B_{m} / (j + 1/4 - \tan^{-1} \{ V_m^{2d} / (2 \pi V_e^{2d}) \}) \).

In figures 8 and 9, we plot the chemical potential and free energy fluctuations in the presence of both electric and magnetic modulations, respectively. The upper panel shows the thermodynamic fluctuations of graphene (solid) and 2DEG (dashed) when the modulations are in phase and the lower panel shows the thermodynamic fluctuations of graphene (solid) and 2DEG (dashed) when the modulations are out of phase.

6.2. Out-of-phase modulation

We consider the same electric modulation \( U(x) = V_e \cos(qx) \) and assume the magnetic modulation is given by \( B_1(x) = B_1 \sin(qx) \hat{z} \) so that the two modulations are \( \pi/2 \) out of phase. To first order in \( V_e \) and \( B_1 \), the total energy correction for graphene can be written as

\[
\Delta E^g = f_m \left[ 1 + \left( \frac{f_e}{f_m} \right)^2 - 1 \right] \cos^2 (2 \sqrt{\mu_{\nu}} - 1) \times \sin (qx_0 + \delta_f^g),
\]

where \( \tan(\delta_f^g) = (f_m / f_e) \tan(2 \sqrt{\mu_{\nu}} - \pi / 4) \). Similarly, the total
energy correction for 2DEG is written as
\[
\Delta E^{2d} = w_m \left[ 1 + \left( \left( \frac{w_e}{w_m} \right)^2 - 1 \right) \cos^2(2\sqrt{nu} - 1) \right] \times \sin(qx_0 + \delta_0^{2d}),
\]
where \( \tan(qx_0 + \delta_0^{2d}) = (w_m/w_e) \tan(2\sqrt{nu} - \pi/4) \). In the case of graphene, though the bandwidth is almost constant at the Fermi energy, the magnetic-field-dependent phase factor \( \delta_0 \) plays an important role in the fluctuation of the thermodynamic quantities. This phase factor produces the Weiss-type oscillations in the thermodynamic properties at low magnetic field. In conventional 2DEG, the bandwidth oscillates with the magnetic field when \( w_e \neq w_m \) in our case but the Weiss-type oscillation is due to the magnetic-field-dependent phase factor \( \delta_0^{2d} \) in the total energy correction. When \( w_e = w_m \), the bandwidth becomes non-oscillatory, but the Weiss-type oscillation still exists due to the magnetic-field-dependent phase factor. It is interesting to contrast our result with the results of the Weiss oscillations in the conductivity in the presence of both modulations \[16, 29\]. The Weiss oscillations in the conductivity are suppressed when the modulations are out of phase. On the other hand, the Weiss-type oscillations in the thermodynamic properties are enhanced when the modulations are out of phase. In the out-of-phase case, the Weiss-type oscillation in the thermodynamic quantities is due to the magnetic-field-dependent phase factor in the energy correction.

7. Summary

We have studied the effect of magnetic modulation on the thermodynamic properties of a graphene sheet. The results of magnetically modulated graphene are compared with electrically modulated graphene and magnetically modulated conventional 2DEG. It is observed that, in the case of magnetically modulated graphene, a definite phase difference appeared in the Weiss-type oscillation in comparison to electrically modulated graphene for all thermodynamic quantities. But when we compare our results with magnetically modulated conventional 2DEG, the amplitude of the fluctuations is found to be higher in graphene than in 2DEG, but the phases remain the same. It is interesting to note that the amplitude of diffusive conductivity in magnetically modulated graphene is small compared to that of magnetically modulated 2DEG. It should be noted here that for 2DEG the magnetic modulation strength achieved in the experiments is much smaller than the electric modulation. Therefore, the amplitude of the Weiss-type oscillations is not as large as predicted from the electric and magnetic modulations of equal strength. We calculate the DOS and the Helmholtz free energy analytically, and explain the origin of this phase difference in the Weiss-type oscillation. The enhancement of the fluctuation in magnetically modulated graphene in comparison to the 2DEG and the definite phase difference between magnetically and electrically modulated graphene are explained by using the asymptotic results of the DOS in general and the Helmholtz free energy in particular.

We have also studied the thermodynamic properties such as the chemical potential and the Helmholtz free energy of graphene and conventional 2DEG in the presence of both magnetic and electric modulations with the same period. The combined effect of both modulations does not modify the nature of the Weiss-type oscillation when they are in the same phase except for a finite phase shift in the fluctuation. The effect of the out-of-phase modulations on thermodynamic fluctuations is different from that of in-phase modulation. We found a large-amplitude Weiss-type oscillation at very low magnetic field even when the bandwidth becomes non-oscillatory. For conventional 2DEG, the effect of the out-of-phase modulation remains the same as in graphene though the bandwidth of 2DEG is oscillatory. This high-amplitude Weiss-type oscillation is due to the magnetic-field-dependent phase factor which plays an important role here, unlike the Weiss oscillation in electrical transport properties where the oscillation is washed out for the same amplitude of energy correction due to the two modulations.

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Appendix

Here we calculate the asymptotic expression of the DOS of graphene in the presence of a modulated magnetic field: \( B = [B_0 + B_1 \cos(qx_0)] \). The total energy up to first order in \( V_m^{\pm} \) is \( E_{n,k}^{\pm} = h\omega_0 \sqrt{2n} + G_n \cos(qx_0) \), where \( G_n = V_m^{\pm} \sqrt{n/\pi} e^{-n^2/2} \{ L_n^{-1}(u) - L_n(u) \} \) and \( u = q^2 \omega_0^2/2 \). Using \( e^{-n^2/2} L_n(u) \sim (\pi \sqrt{nu})^{-1/2} \cos[2\sqrt{nu} - \pi/4] \) for higher values of \( n \), we get the asymptotic expression of \( G_n \) as
\[
G_n = \frac{2 V_m^{\pm}}{\sqrt{\pi}} \left( \frac{\pi}{\nu^3} \right)^{1/4} \sin \left( \frac{u}{4\nu} \right) \sin \left[ 2\sqrt{nu} - \frac{\pi}{4} \right],
\]
\( n \) for higher values of \( n \), we get the asymptotic expression of \( G_n \) as
\[
G_n = \frac{2 V_m^{\pm}}{\sqrt{\pi}} \left[ \frac{2\nu}{q0} \right]^{1/2} \frac{1}{q0} \sin \left( q0 \epsilon \right) \sin \left[ q0 \epsilon - \frac{\pi}{4} \right],
\]
where \( \epsilon = E/(\hbar\omega_0) \). Now we use impurity broadened Landau levels in the limiting case. The self-energy can be written as \[31, 32\]
\[
\Sigma^- (\epsilon) = \frac{1}{1} \sum_n \int_0^a \frac{dx_0}{a} \frac{1}{\epsilon \hbar \omega_0 - E_n^{\pm} - \Sigma^- (\epsilon)},
\]
where \( \Gamma_0 \) is the broadening of the Landau levels due to impurities. By determining the imaginary part of the self-energy we can obtain the DOS through
\[
D(\epsilon) = \text{Im} \left[ \frac{\Sigma^- (\epsilon)}{\pi^2 \epsilon_0^2 \Gamma_0^2} \right].
\]
By using the residue theorem we get

\[ \Sigma^{-}(\epsilon) = 2\pi i^{2} \int_{0}^{\infty} \frac{dx_{0}}{a} \epsilon \hbar \omega_{g} - \Sigma^{-}(\epsilon) - G_{n} \cos(q_{0}a) \left( \sqrt{2} \hbar \omega_{g} \right)^{2} \]

\[ \times \cot \left[ \frac{\pi}{(\sqrt{2} \hbar \omega_{g})^{2}} \{ \epsilon \hbar \omega_{g} - \Sigma^{-}(\epsilon) - G_{n} \cos(q_{0}a) \} \right] \]

\[ \simeq \frac{\pi i^{2} e}{(\hbar \omega_{g})} \int_{0}^{\infty} \frac{dx_{0}}{a} \sin u + i \sin v \cosh v - \cos u \]

where

\[ u = \pi \epsilon \{ \epsilon - 2(\hbar \omega_{g})^{-1}\Delta(\epsilon) + G_{n} \cos(q_{0}a) \} \]

and

\[ v = \frac{\pi \epsilon}{\hbar \omega_{g}} \Gamma(\epsilon) \].

In the limit of large collisional broadening, \( \pi \Gamma \gg \hbar \omega_{g} \), after expanding the hyperbolic term with respect to the small quantity \( \exp(-v) \) up to first order one obtains

\[ \Gamma(\epsilon) = \frac{\pi i^{2} e}{(\hbar \omega_{g})} \left[ 1 + 2 \exp \left\{ -\frac{\pi \epsilon \Gamma}{\hbar \omega_{g}} \right\} \right. \]

\[ \times \left. \int_{0}^{\infty} \frac{dx_{0}}{a} \cos \left( \pi \epsilon - 2(\hbar \omega_{g})^{-1} G_{n} \cos(q_{0}a) \right) \right] \].

(A.5)

After the first iteration, we have \( \Gamma(\epsilon)/2 = \pi i^{2} e/(\hbar \omega_{g}) \), and then putting it back into equation (A.5), we get

\[ \Gamma(\epsilon) = \frac{\pi i^{2} e}{(\hbar \omega_{g})} \left[ 1 + 2 \exp \left\{ -\frac{\pi \epsilon \Gamma}{\hbar \omega_{g}} \right\} \right. \]

\[ \times \left. \left[ 1 - \Omega_{m} (\epsilon \hbar \omega_{g})^{3} \sin^{2} \left( q_{0} \epsilon - \frac{\pi}{4} \right) \right] \right] \].

(A.6)

where

\[ \Omega_{m} = \frac{(l_{0})^{2}}{\pi^{2}} \left( \frac{a}{l_{0}} \right)^{3} \left( \frac{1}{\hbar \omega_{g}} \right) \sin^{2} \left( q_{0} \epsilon - \frac{\pi}{4} \right) \].

(A.7)

Using equations (A.3) and (A.6), we get the DOS \( D(\epsilon) \) as given in equations (15) and (16).

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