Non-Minimal Higgs Sectors: The Decoupling Limit and its Phenomenological Implications

Howard E. Haber

SCIPP

Abstract

In models with a non-minimal Higgs sector, a decoupling limit can be defined. In this limit, the masses of all the physical Higgs states are large (compared to the scale of electroweak symmetry breaking) except for one neutral CP-even Higgs scalar, whose properties are indistinguishable from the Higgs boson of the minimal Standard Model. The decoupling limit of the most general CP-conserving two-Higgs doublet model is formulated. Detection of evidence for a non-minimal Higgs sector at future colliders in the decoupling limit may present a formidable challenge for future Higgs searches.

Invited Talk presented at the Workshop on Electroweak Symmetry Breaking, Eötvös University Budapest, Hungary, 11–13 July 1994, and at the Workshop on Physics from the Planck Scale to the Electroweak Scale, University of Warsaw, Poland, 21–24 September, 1994.

* Work supported in part by the U.S. Department of Energy.
1. Introduction

With the recent “discovery” of the top quark, the only missing piece of the Standard Model is the Higgs boson. The Standard Model posits the existence of one complex weak doublet (with hypercharge $Y = 1$) of elementary scalar (Higgs) fields. The dynamics of these fields is assumed to trigger electroweak symmetry breaking. Three Goldstone bosons are absorbed by the $W^\pm$ and $Z$ leaving one CP-even neutral Higgs scalar to be discovered.

Despite the enormous success of the Standard Model, the existing data sheds little light on the spectrum and dynamics of the electroweak symmetry breaking sector. The scalar spectrum could be minimal (as in the Standard Model) or non-minimal, with a rich spectrum of scalar states (including perhaps bound states of higher spin). The dynamics could involve either weakly interacting or strongly interacting forces. Many examples displaying each of the above features have been studied in the literature; comprehensive reviews can be found in refs. 1 and 2. In this paper, I will assume that electroweak symmetry breaking is a result of the dynamics of a weakly-coupled Higgs sector. Such a theory may be technically natural if embedded in a theory of low-energy supersymmetry. However, the results obtained in this work do not necessarily require the existence of low-energy supersymmetry.

In this paper, I shall pose the following questions. Assume that a scalar state (i.e., a candidate for the Higgs boson) is discovered in a future collider experiment. What are the expectations for its properties? Will it resemble the Higgs boson of the minimal Standard Model or will it possess some distinguishing trait? If the properties of this scalar state are difficult to distinguish from the Standard Model Higgs boson, what are the requirements of future collider experiments for detecting the existence (or non-existence) of a non-minimal Higgs sector? Some of these questions have also been addressed by other authors; see e.g., refs. 3 and 4.

These questions would be moot if the experiment that first discovers the lightest Higgs scalar also discovers a second scalar state. For example, in a theory with a light CP-even scalar ($h^0$) and a light CP-odd scalar ($A^0$), both states would be produced in $e^+e^-$ collisions via $s$-channel $Z$-exchange ($e^+e^- \rightarrow Z \rightarrow h^0A^0$) if kinematically allowed. The simultaneous discovery of $h^0$ and $A^0$ would clearly indicate the existence of a non-minimal Higgs sector. In this paper, I shall not consider such a scenario. As we shall see, the case where only one light scalar state is initially discovered may present a formidable challenge to unraveling the underlying structure of the electroweak symmetry breaking sector.

For simplicity, I focus in this paper on the (CP-conserving) two-Higgs doublet model. In this model, the physical scalar states consist of a charged Higgs pair ($H^{\pm}$), two CP-even scalars ($h^0$ and $H^0$), with $m_{h^0} \leq m_{H^0}$ and one CP-odd scalar ($A^0$). The ultimate conclusions of this paper will survive in models with more complicated scalar sectors. Following the discussion above, the working hypothesis of this paper
is that $h^0$, assumed to be the lightest scalar state, will be the first Higgs boson to be discovered. Moreover, the mass gap between $h^0$ and the heavier scalars is assumed to be sufficiently large so that the initial experiments which can detect $h^0$ will not have sufficient energy and luminosity to initially discover any of the heavier scalar states.

How will $h^0$ be discovered and where? Present LEP bounds$^5$ imply that $m_{h^0} \gtrsim 60$ GeV. This bound will be improved by LEP-II,$^6$ which will be sensitive to Higgs masses up to roughly $\sqrt{s} - m_Z - 10$ GeV. The LEP search is based on $e^+e^- \rightarrow Z \rightarrow Zh^0$ where one of the two $Z$’s is on-shell and the other is off-shell. At hadron colliders, an upgraded Tevatron with an integrated luminosity of 10 fb$^{-1}$ can begin to explore the intermediate-mass Higgs regime$^7$ ($80 \lesssim m_{h^0} \lesssim 130$ GeV). The Higgs search at the LHC will significantly extend the Higgs search to higher masses$^8$ (although the intermediate mass regime still presents some significant difficulties for the LHC detector collaborations). The dominant mechanism for Higgs production at hadron colliders is via $gg$-fusion through a top-quark loop. If $m_{h^0} > 2m_Z$, the “gold-plated” detection mode is $h^0 \rightarrow ZZ$; each $Z$ subsequently decays leptonically, $Z \rightarrow \ell^+\ell^-$ (for $m_{h^0} \gtrsim 130$ GeV, $h^0 \rightarrow ZZ^*$, where $Z^*$ is off-shell, provides a viable signature). Other decay modes are required in the case of the intermediate mass Higgs (for recent reviews, see ref. 9). At a future $e^+e^-$ linear collider (NLC), the Higgs mass reach of LEP-II will be extended.$^{10}$ In addition, with increasing $\sqrt{s}$, Higgs boson production via $W^+W^-$ fusion begins to be the dominant production process. Finally, one novel possibility is to run the NLC in a $\gamma\gamma$-collider mode. In this mode, Higgs production via $\gamma\gamma$-fusion (which is typically dominated by a $\wp W^-$ and/or a $t\bar{t}$ loop) may be detectable, depending on the particular Higgs final state decay.$^{11,12}$ Note that almost all of the Higgs search techniques outlined above involve the $h^0 ZZ$ (and in some cases the $h^0 \wp W^-$) vertex. In a few cases, it is the $h^0 t\bar{t}$ vertex (and possibly the $h^0 b\bar{b}$ vertex) that plays the key role. These observations are relevant for the phenomenological considerations of this paper.

In section 2, I review the Higgs sector of the minimal supersymmetric extension of the Standard Model (MSSM). In this context, I discuss why one might expect that $h^0$ is the lightest scalar whose properties are nearly identical to that of the Standard Model Higgs boson. In section 3, I place the results of section 2 in a more general context. I define the “decoupling limit” of the general two-Higgs doublet model; in this limit, $h^0$ is indistinguishable from the Standard Model Higgs boson. In section 4, I discuss the phenomenological challenges of the decoupling limit for the Higgs search at future colliders. After briefly mentioning the prospects for non-minimal Higgs detection at the LHC, I consider in more detail the prospects for the discovery of the non-minimal Higgs sector at the NLC. Conclusions are presented in section 5.
2. The Higgs Sector of the MSSM—A Brief Review

In this section, I provide a very brief review of the Higgs sector of the minimal supersymmetric extension of the Standard Model (MSSM)\textsuperscript{13} let us consider the MSSM Higgs potential at tree-level which depends on two complex doublet scalar fields $H_1$ and $H_2$ of hypercharge $-1$ and $+1$, respectively:

$$V = m_{11}^2|H_1|^2 + m_{22}^2|H_2|^2 - m_{12}^2(\epsilon_{ij} H_1^i H_2^j + \text{h.c.}) + \frac{1}{8}(g^2 + g'^2)(|H_1|^2 - |H_2|^2)^2 + \frac{1}{2} g^2 |H_1^* H_2|^2,$$

where $m_{ii}^2 \equiv |\mu|^2 + m_i^2$ ($i = 1, 2$). The parameters $m_{11}^2$, $m_{22}^2$ and $m_{12}^2$ are soft-supersymmetry-breaking parameters, $\mu$ is the Higgs superfield mass parameter, and $g$ and $g'$ are the $\text{SU}(2) \times \text{U}(1)$ gauge couplings.

The three parameters $m_{11}^2$, $m_{22}^2$ and $m_{12}^2$ of the Higgs potential can be expressed in terms of the two Higgs vacuum expectation values, $\langle H^0_i \rangle \equiv v_i/\sqrt{2}$ and one physical Higgs mass. One is free to choose the phases of the Higgs fields such that $v_1$ and $v_2$ are positive. Then, $m_{12}^2$ must be positive, in which case it follows from eq. (2.1) that

$$m_{A^0}^2 = \frac{m_{12}^2}{\sin \beta \cos \beta}. \quad (2.2)$$

Note that $m_W^2 = \frac{1}{4} g^2 (v_1^2 + v_2^2)$, which fixes the magnitude $v_1^2 + v_2^2 = (246 \text{ GeV})^2$. This leaves two parameters which determine all the Higgs sector masses and couplings: $m_{A^0}$ and $\tan \beta \equiv v_2/v_1$. The charged Higgs mass is given by

$$m_{H^\pm}^2 = m_W^2 + m_{A^0}^2. \quad (2.3)$$

The neutral CP-even Higgs bosons, $H^0$ and $h^0$, are obtained by diagonalizing a $2 \times 2$ mass matrix. The eigenstates are

$$H^0 = (\sqrt{2} \text{Re} H_1^0 - v_1) \cos \alpha + (\sqrt{2} \text{Re} H_2^0 - v_2) \sin \alpha$$

$$h^0 = -(\sqrt{2} \text{Re} H_1^0 - v_1) \sin \alpha + (\sqrt{2} \text{Re} H_2^0 - v_2) \cos \alpha$$

which defines the CP-even Higgs mixing angle $\alpha$. The corresponding CP-even Higgs mass eigenvalues are

$$m_{H^0, h^0}^2 = \frac{1}{2} \left( m_{A^0}^2 + m_Z^2 \pm \sqrt{(m_{A^0}^2 + m_Z^2)^2 - 4 m_Z^2 m_{A^0}^2 \cos^2 2\beta} \right), \quad (2.5)$$

where by definition, $m_{h^0} \leq m_{H^0}$. Explicit formulae for $\alpha$ can also be derived. Here,
I shall note one particularly useful relation

$$\cos^2(\beta - \alpha) = \frac{m_{h^0}^2 (m_Z^2 - m_{h^0}^2)}{m_{A^0}^2 (m_{H^0}^2 - m_{h^0}^2)}.$$  \hfill (2.6)

Consider the limit where \( m_{A^0} \gg m_Z \). Then, from the above formulae,

$$m_{h^0}^2 \approx m_Z^2 \cos^2 2\beta,$$

$$m_{H^0}^2 \approx m_{A^0}^2 + m_Z^2 \sin^2 2\beta,$$

$$m_{H^±}^2 = m_{A^0}^2 + m_W^2,$$

$$\cos^2(\beta - \alpha) \approx \frac{m_Z^2 \sin^2 4\beta}{4m_{A^0}^4}.$$  \hfill (2.7)

Two consequences are immediately apparent. First, \( m_{A^0} \approx m_{H^0} \approx m_{H^±} \), up to corrections of \( \mathcal{O}(m_Z^2/m_{A^0}) \). Second, \( \cos(\beta - \alpha) = 0 \) up to corrections of \( \mathcal{O}(m_Z^2/m_{A^0}) \).

It is known that one-loop radiative corrections have a significant impact on the MSSM Higgs sector.\textsuperscript{14} Nevertheless, the conclusions just stated are not modified. For example, in the limit where \( m_{A^0} \gg m_Z \) and \( m_Z \ll m_t \ll M_t \), the one-loop radiatively corrected Higgs squared masses are\textsuperscript{15}

$$m_{h^0}^2 \approx m_Z^2 \cos^2 2\beta + \frac{3g^2}{8\pi^2 m_W^2} \left[ m_t^4 + \frac{1}{4}m_t^2 m_Z^2 \cos 2\beta \right] \ln \left( \frac{M_t^2}{m_t^2} \right),$$

$$m_{H^0}^2 \approx m_{A^0}^2 + m_Z^2 \sin^2 2\beta + \frac{3g^2 \cos^2 \beta}{8\pi^2 m_W^2} \left[ \frac{m_t^4 \sin^2 \beta}{m_W^2} - m_t^2 m_Z^2 \right] \ln \left( \frac{M_t^2}{m_t^2} \right),$$

$$m_{H^±}^2 \approx m_{A^0}^2 + m_W^2 + \frac{3g^2}{32\pi^2} \left[ \frac{2m_t^2 m_b^2}{m_W^2 \sin^2 \beta \cos^2 \beta} - \frac{m_t^2}{\sin^2 \beta} - \frac{m_b^2}{\cos^2 \beta} \right] \ln \left( \frac{M_t^2}{m_t^2} \right).$$  \hfill (2.8)

The formula for \( m_{h^0}^2 \) exhibits the importance of the one-loop radiative corrections. The tree-level upper bound, \( m_{h^0} \leq m_Z \) is significantly modified by terms that are enhanced for large values of \( m_t \) and \( M_t \). Nevertheless, the numerical value of the radiative corrections to the squared masses is never greater than \( \mathcal{O}(m_Z^2) \). Thus, in the limit of \( m_{A^0} \gg m_Z \), one again finds that \( m_{A^0} \approx m_{H^0} \approx m_{H^±} \), up to corrections of \( \mathcal{O}(m_Z^2/m_{A^0}) \). One can also show that \( \cos(\beta - \alpha) = \mathcal{O}(m_Z^2/m_{A^0}) \) as before.

The phenomenological implications of these results may be discerned by reviewing the coupling strengths of the Higgs bosons to Standard Model particles (gauge bosons, quarks and leptons) in the two-Higgs doublet model. The coupling of \( h^0 \) and \( H^0 \) to vector boson pairs or vector-scalar boson final states is proportional to either \( \sin(\beta - \alpha) \) or \( \cos(\beta - \alpha) \) as indicated below.\textsuperscript{1,13}
Note in particular that all vertices in the theory that contain at least one vector boson and exactly one heavy Higgs boson state \((H^0, A^0 \text{ or } H^\pm)\) is proportional to \(\cos(\beta - \alpha)\). This can be understood as a consequence of unitarity sum rules which must be satisfied by the tree-level amplitudes of the theory.\(^\text{16}\)

In models with non-minimal Higgs sectors, the Higgs couplings to quarks and leptons are model-dependent. Typically, one imposes constraints on the Higgs-fermion interaction in order to avoid Higgs mediated tree-level flavor changing neutral currents. For example, in the MSSM, one Higgs doublet couples exclusively to down-type fermions and the second Higgs doublet couples exclusively to up-type fermions. In this model, the couplings of the neutral CP-even Higgs bosons to \(f\bar{f}\) relative to the Standard Model value are given by (using 3rd family notation)

\[
\begin{align*}
    h^0 b\bar{b} : & \quad -\frac{\sin \alpha}{\cos \beta} = \sin(\beta - \alpha) - \tan \beta \cos(\beta - \alpha), \\
    h^0 t\bar{t} : & \quad \frac{\cos \alpha}{\sin \beta} = \sin(\beta - \alpha) + \cot \beta \cos(\beta - \alpha), \\
    H^0 b\bar{b} : & \quad \frac{\cos \alpha}{\cos \beta} = \cos(\beta - \alpha) + \tan \beta \sin(\beta - \alpha), \\
    H^0 t\bar{t} : & \quad \frac{\sin \alpha}{\sin \beta} = \cos(\beta - \alpha) - \cot \beta \sin(\beta - \alpha).
\end{align*}
\]

In contrast to the Higgs couplings to vector bosons, none of the couplings in eq. (2.9) vanish when \(\cos(\beta - \alpha) = 0\). This is a model-independent feature of the Higgs couplings to fermions. One finds a similar behavior for the Higgs self-couplings. Namely, the various Higgs self-couplings are model-dependent since they depend on the form of the scalar potential. Nevertheless, one can show that the Higgs self-couplings do not vanish when \(\cos(\beta - \alpha) = 0\).

The significance of \(\cos(\beta - \alpha) = 0\) is now evident: in this limit, couplings of \(h^0\) to gauge boson pairs and fermion pairs are identical to the couplings of the Higgs
boson in the minimal Standard Model. More precisely, in the limit of \( m_{A^0} \gg m_Z \), the effects of the heavy Higgs states \((H^\pm, H^0, A^0)\) decouple, and the low-energy effective scalar sector is indistinguishable from that of the minimal Standard Model.

In the MSSM, the decoupling regime \((m_{A^0} \gg m_Z)\) sets in rather quickly once \( m_{A^0} \) is taken above \( m_Z \). Although \( m_{A^0} \) is a free parameter of the MSSM, its origin is intimately connected to the scale of supersymmetry breaking. From eq. (2.2), we see that \( m_{12}^2 \) is proportional to the soft-supersymmetry breaking parameter \( m_{12}^2 \). Generically, one would expect that \( m_{A^0} \) is of the same order as a typical soft-supersymmetry-breaking mass parameter. In supergravity based models, \( m_{12}^2 \equiv B \mu \), where \( B \) is a soft-supersymmetry-breaking parameter. Models of this type with universal soft-supersymmetry-breaking scalar masses at the Planck scale tend to yield values of \( m_{A^0} \) that typically lie above \( m_Z \). In such approaches, one would expect the couplings of \( h^0 \) to be nearly identical to those of the Standard Model Higgs boson.

3. Decoupling Properties of the Two-Higgs Doublet Model

The decoupling properties of the MSSM Higgs sector are not special to supersymmetry. Rather, they are a generic feature of non-minimal Higgs sectors. In this section, I demonstrate this assertion in the case of the most general CP-conserving two-Higgs doublet model. Let \( \Phi_1 \) and \( \Phi_2 \) denote two complex \( Y = 1 \), SU(2) \( _L \) doublet scalar fields. The most general gauge invariant scalar potential is given by:

\[
\mathcal{V} = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - [m_{12}^2 \Phi_1^\dagger \Phi_2 + \text{h.c.}] \\
+ \frac{1}{2} \lambda_1 (\Phi_1^\dagger \Phi_1)^2 + \frac{1}{2} \lambda_2 (\Phi_2^\dagger \Phi_2)^2 + \lambda_3 (\Phi_1^\dagger \Phi_1)(\Phi_2^\dagger \Phi_2) + \lambda_4 (\Phi_1^\dagger \Phi_2)(\Phi_2^\dagger \Phi_1) \\
+ \left\{ \frac{1}{2} \lambda_5 (\Phi_1^\dagger \Phi_2)^2 + \left[ \lambda_6 (\Phi_1^\dagger \Phi_1) + \lambda_7 (\Phi_2^\dagger \Phi_2) \right] \Phi_1^\dagger \Phi_2 + \text{h.c.} \right\}.
\]

In principle, \( m_{12}^2, \lambda_5, \lambda_6 \) and \( \lambda_7 \) can be complex. However, I shall ignore the possibility of CP-violating effects in the Higgs sector by choosing all coefficients in

* Likewise, the \( h^0 h^0 h^0 \) and \( h^0 h^0 h^0 \) couplings also reduce to their Standard Model values when \( \cos(\beta - \alpha) = 0 \).

† In terms of the \( Y = \pm 1 \) fields of the previous section, \( H_1^i = \epsilon_{ij} \Phi_1^j \) and \( H_2 = \Phi_2 \).

‡ In most discussions of two-Higgs-doublet models, the terms proportional to \( \lambda_6 \) and \( \lambda_7 \) are absent at tree-level. This can be achieved by imposing a discrete symmetry \( \Phi_1 \rightarrow -\Phi_1 \) on the model. Such a symmetry would also require \( m_{12} = 0 \) unless we allow a soft violation of this discrete symmetry by dimension-two terms. This latter requirement is sufficient to guarantee the absence of Higgs-mediated tree-level flavor changing neutral currents.
eq. (3.1) to be real. The scalar fields will develop non-zero vacuum expectation values if the mass matrix $m^2_{ij}$ has at least one negative eigenvalue. Imposing CP invariance and U(1)$_{EM}$ gauge symmetry, the minimum of the potential is

$$
\langle \Phi_1 \rangle = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 0 \\ v_1 \end{array} \right), \quad \langle \Phi_2 \rangle = \frac{1}{\sqrt{2}} \left( \begin{array}{c} 0 \\ v_2 \end{array} \right),
$$

where the $v_i$ can be chosen to be real and positive. It is convenient to introduce the following notation:

$$
v^2 \equiv v_1^2 + v_2^2 = \frac{4m_W^2}{g^2}, \quad t_\beta \equiv \tan \beta \equiv \frac{v_2}{v_1}.
$$

The mass parameters $m^2_{11}$ and $m^2_{22}$ can be eliminated by minimizing the scalar potential. The resulting squared masses for the CP-odd and charged Higgs states are

$$
m^2_{A^0} = \frac{m^2_{12}}{s_\beta c_\beta} - \frac{1}{2} v^2 (2\lambda_5 + \lambda_6 t_\beta^{-1} + \lambda_7 t_\beta),
$$

$$
m^2_{H^\pm} = m^2_{A^0} + \frac{1}{2} v^2 (\lambda_5 - \lambda_4),
$$

where $s_\beta \equiv \sin \beta$ and $c_\beta \equiv \cos \beta$. The two CP-even Higgs states mix according to the following squared mass matrix:

$$
\mathcal{M}^2 = m^2_{A^0} \left( \begin{array}{cc} s_\beta^2 & -s_\beta c_\beta \\ -s_\beta c_\beta & c_\beta^2 \end{array} \right) + v^2 \left( \begin{array}{cc} \mathcal{M}^2_{11} & \mathcal{M}^2_{12} \\ \mathcal{M}^2_{12} & \mathcal{M}^2_{22} \end{array} \right),
$$

where

$$
\begin{pmatrix} \mathcal{M}^2_{11} & \mathcal{M}^2_{12} \\ \mathcal{M}^2_{12} & \mathcal{M}^2_{22} \end{pmatrix} \equiv \begin{pmatrix} \lambda_1 c_\beta^2 + 2\lambda_6 s_\beta c_\beta + \lambda_5 s_\beta^2 & (\lambda_3 + \lambda_4)s_\beta c_\beta + \lambda_6 c_\beta^2 + \lambda_7 s_\beta^2 \\ (\lambda_3 + \lambda_4)s_\beta c_\beta + \lambda_6 c_\beta^2 + \lambda_7 s_\beta^2 & \lambda_2 s_\beta^2 + 2\lambda_7 s_\beta c_\beta + \lambda_5 c_\beta^2 \end{pmatrix}.
$$

It is convenient to define four squared mass combinations:

$$
m^2_L \equiv \mathcal{M}^2_{11} \cos^2 \beta + \mathcal{M}^2_{22} \sin^2 \beta + \mathcal{M}^2_{12} \sin 2\beta,
$$

$$
m^2_D \equiv (\mathcal{M}^2_{11} \mathcal{M}^2_{22} - \mathcal{M}^2_{12})^{1/2},
$$

$$
m^2_T \equiv \mathcal{M}^2_{11} + \mathcal{M}^2_{22},
$$

$$
m^2_S \equiv m^2_{A^0} + m^2_T.
$$
In terms of these quantities,
\[ m_{H^0,H^0}^2 = \frac{1}{2} \left[ m_S^2 \pm \sqrt{m_S^4 - 4m_A^2m_L^2 - 4m_D^4} \right], \tag{3.9} \]
and
\[ \cos^2(\beta - \alpha) = \frac{m_L^2 - m_{h^0}^2}{m_{H^0}^2 - m_{h^0}^2}. \tag{3.10} \]

Suppose that all the Higgs self-coupling constants \( \lambda_i \) are held fixed such that \( \lambda_i \lesssim 1 \), while taking \( m_{A^0}^2 \gg \lambda_i v^2 \). Then the \( M_{ij}^2 \sim \mathcal{O}(v^2) \), and it follows that:
\[ m_{h^0} \simeq m_L, \quad m_{H^0} \simeq m_{A^0} \simeq m_{H^\pm}, \tag{3.11} \]
and
\[ \cos^2(\beta - \alpha) \simeq \frac{m_L^2(m_T^2 - m_L^2) - m_D^4}{m_{A^0}^4}. \tag{3.12} \]

Comparing these results with those of eq. (2.7), one sees that the MSSM results are simply a special case of the more general two-Higgs doublet model results just obtained.

The limit \( m_{A^0}^2 \gg \lambda_i v^2 \) (subject to \( \lambda_i \lesssim 1 \)) will be called the decoupling limit of the model. From eq. (3.4), it follows that \( m_{12}^2 \gg \lambda_i v^2 \) (assuming that neither \( t_\beta \) nor \( t_\beta^{-1} \) is close to 0). This condition clearly depends on the original choice of scalar field basis \( \Phi_1 \) and \( \Phi_2 \). For example, I can diagonalize the squared mass terms of the scalar potential [eq. (3.1)] thereby setting \( m_{12} = 0 \). In the decoupling limit in the new basis, eq. (3.4) would then imply that \( \beta \) must be near 0 or near \( \pi/2 \). A basis independent characterization of the decoupling limit is simple to formulate. Starting from the scalar potential in an arbitrary basis, form the matrix \( m_{ij}^2 \). Denote the eigenvalues of this matrix by \( m_a^2 \) and \( m_b^2 \) respectively; note that the eigenvalues are real but can be of either sign. By convention, I shall take \( |m_a^2| \leq |m_b^2| \). Then, the decoupling limit corresponds to \( m_a^2 < 0, m_b^2 > 0 \) such that \( m_b^2 \gg |m_a^2|, v \) (with \( \lambda_i \lesssim 1 \)).

We are now ready to consider the questions posed in section 1. Let us assume that \( m_{h^0} \ll m_{H^0}, m_{A^0}, m_{H^\pm} \). From eq. (3.9), it follows that
\[ 0 < m_{A^0}^2m_L^2 + m_D^4 \ll m_S^4. \tag{3.13} \]
Eq. (3.12) implies that in the decoupling limit, \( \cos(\beta - \alpha) = \mathcal{O}(m_Z^2/m_{A^0}^2) \), which means that the \( h^0 \) couplings to Standard Model particles match precisely those of the Standard Model Higgs boson. However, in the following, I shall not impose the decoupling limit. Rather, I shall only require eq. (3.13), which reflects my basic assumption that one scalar state, \( h^0 \), is lighter than the other Higgs bosons. Eq. (3.13) is satisfied in one of two cases:
(i) $m_Z^2, m_L^2, m_D^2/m_{A^0}^2 \ll m_{A^0}^2, m_S^2$. That is, each term on the left-hand side of eq. (3.13) is separately smaller in magnitude than $m_S^4$, or

(ii) $m_{A^0}^2 m_L^2$ and $m_D^4$ are both of $O(m_{A^0}^4)$, but due to cancelation of the leading behavior of each term, the sum satisfies eq. (3.13).

In case (i), one finds that

$$m_{h^0}^2 \simeq \frac{m_{A^0}^2 m_L^2}{m_S^2} + \frac{m_D^4}{m_S^2} + \frac{m_{A^0}^2 m_L^2}{m_S^4} + O\left(\frac{m_L^4}{m_S^4}\right),$$

(3.14)

and $m_{h^0}^2 \sim O(m_S^2)$. In the same approximation,

$$\cos^2(\beta - \alpha) \simeq \frac{m_S^2}{m_L^2} \left(1 - \frac{m_{A^0}^2}{m_S^2}\right) + \frac{1}{m_S^4} \left[ m_L^2 \left(\frac{2m_{A^0}^2}{m_S^2} - \frac{3m_{A^0}^4}{m_S^4}\right) - m_D^4 \right].$$

(3.15)

The behavior of $\cos(\beta - \alpha)$ depends crucially on how close $m_{A^0}^2/m_S^2$ is to 1. If $m_T^2 \ll m_S^2$ [see eq. (3.8)], we recover the results of the decoupling limit [eqs. (3.11) and (3.12)]. On the other hand, if $m_T^2 \sim O(m_S^2)$, then eq. (3.15) implies that $\cos(\beta - \alpha) \sim O(m_Z^2/m_{A^0})$. This is a particular region of the parameter space where some of the $\lambda_i$ are substantially larger than 1, and yet the $h^0$ couplings do not significantly deviate from those of the Standard Model. Nevertheless, the onset of decoupling is slower than the $\cos(\beta - \alpha) \sim O(m_Z^2/m_{A^0})$ behavior found in the decoupling regime. In order to find a parameter regime which exhibits complete non-decoupling, one must consider case (ii) above. In this case, despite the fact that $m_{h^0} \ll m_{H^0}, m_{A^0}, m_{H^+}$, one nevertheless finds that $\cos(\beta - \alpha) \sim O(1)$, which implies that the couplings of $h^0$ deviate significantly from those of the Standard Model Higgs boson. Although it might appear that case (ii) requires an unnatural cancelation, it is easy to construct a simple model of non-decoupling. Consider a model where: $m_{t_2}^2 = \lambda_6 = \lambda_7 = 0, \lambda_3 + \lambda_4 + \lambda_5 = 0, \lambda_2 \approx O(1)$, and $\lambda_1, \lambda_3, -\lambda_5 \gg 1$. This model yields: $m_{h^0}^2 = \lambda_2 v^2 s_\beta^2, m_{H^0}^2 = \lambda_1 v^2 c_\beta^2, m_{A^0}^2 = -\lambda_5 v^2, m_{H^+}^2 = \frac{1}{2} \lambda_3 v^2$, and $\cos^2(\beta - \alpha) = c_\beta^2$. Note that in this model, the heavy Higgs states are not approximately degenerate (as required in the decoupling limit).

### 4. Phenomenological Challenges of the Decoupling Limit

We have seen that in the decoupling limit, the couplings of $h^0$ to Standard Model gauge bosons and fermions approach those of the Standard Model Higgs boson. Suppose that a future experiment has already discovered and studied the properties of $h^0$. What are the requirements of experiments at future colliders
for proving the existence or non-existence of a non-minimal Higgs sector? To be specific, let us assume in this section that $h^0$ has been discovered with couplings approximating those of the Standard Model Higgs boson and $m_{A^0} > 250$ GeV.

At the LHC, the rate for $gg \rightarrow A^0$, $H^0$ and $gb \rightarrow H^- t$ provides a substantial number of produced Higgs bosons per year (assuming that the heavy Higgs masses are not too large). Unfortunately, there may not be a viable final state signature. For example, since $\cos^2(\beta - \alpha) \ll 1$, the branching ratio of $H^0 \rightarrow ZZ$ is significantly suppressed, so that the gold-plated Standard Model Higgs signature is simply absent. At present, there is no known proven technique for detecting $A^0$, $H^0$ and $H^\pm$ signals at the LHC in the decoupling regime of parameter space. An attempt to isolate a Higgs signal in $t\bar{t}$ final states has been discussed in ref. 20. Another method consists of a search for Higgs signals in $t\bar{t}t\bar{t}$, $t\bar{t}b\bar{b}$ and $b\bar{b}b\bar{b}$ final states. These can arise from $gg \rightarrow QQ'(H^0, A^0 \text{ or } H^\pm)$, where $Q$ is a heavy quark ($b$ or $t$), and the Higgs boson subsequently decays into a heavy quark pair. As noted below eq. (2.9), even in the decoupling limit, the couplings of $H^0$, $A^0$ and $H^\pm$ to heavy quark pairs are not suppressed. Whether such signals can be extracted from the substantial QCD backgrounds (very efficient $b$-tagging is one of the many requirements for such a search) remains to be seen.

Let us now turn to $e^+ e^-$ colliders. First, consider the process $e^+ e^- \rightarrow h^0 A^0$ (via $s$-channel $Z$-exchange). Since the $Zh^0 A^0$ coupling is proportional to $\cos(\beta - \alpha)$, the production rate is suppressed in the decoupling regime. For example, in the MSSM, if $m_{A^0} > m_{h^0}$, then LEP-II will not possess sufficient energy and/or luminosity to directly produce the $A^0$. Of course, with sufficient energy, one can directly pair-produce the heavy Higgs states via $e^+ e^- \rightarrow H^+ H^-$ and $e^+ e^- \rightarrow H^0 A^0$ without a rate suppression. At the NLC, with $\sqrt{s} = 500$ GeV and 10 fb$^{-1}$ of data, it has been shown that no direct signals for $A^0$, $H^0$, and $H^\pm$ can be seen if $m_{A^0} \geq 230$ GeV. Although this result was obtained in the MSSM, it also applies to the decoupling regime of more general Higgs sectors. These results seem to imply the following rather general conclusion: evidence for the non-minimal Higgs sector at the NLC requires a machine with energy $\sqrt{s} > 2m_{A^0}$, sufficient to pair-produce heavy Higgs states.

Can this conclusion be avoided? There are two possible methods.

1. Produce one heavy Higgs state in association with light states.

2. Make precision measurements of $h^0$ couplings to Standard Model particles in order to detect a deviation from the Standard Model expectations.

First, consider production mechanisms which result in a singly produced heavy Higgs state. It was noted in section 2 that whenever a single heavy Higgs state

* In this context, light states refer to all Standard Model fermions and bosons, with masses of order $m_Z$ or less. Thus, the gauge bosons, $h^0$, and even the top-quark will be considered among the light states!
couples directly to a gauge boson plus other particles, the coupling is suppressed by \(\cos(\beta - \alpha)\). To avoid this suppression, one must couple the heavy Higgs state to either fermions or scalars. For example, consider \(e^+e^- \rightarrow Q\bar{Q}(H^0, A^0, \text{or } H^\pm)\), where \(Q = b \text{ or } t\). The production rates have been computed by Djouadi et al.\(^{25}\)

Unfortunately, the three-body phase space greatly suppresses the rate once the heavy Higgs state is more massive than the \(Z\). In particular, for \(m_{A^0} > \sqrt{s}/2\), these processes do not provide viable signatures for the heavy Higgs states. A similar conclusion is obtained when the heavy Higgs state couples to light scalars. Scott Thomas and I have computed the rate for \(e^+e^- \rightarrow h^0H^0\). We assumed that the dominant contribution arises in the \(s\)-channel \(Z\)-exchange, where the virtual \(Z^*\) decays via \(Z^* \rightarrow Zh^0 \rightarrow Zh^0H^0\). In the limit where \(m_{H^0} \gg m_{h^0}, m_Z\), we obtained\(^{18}\)

\[
\frac{\sigma(e^+e^- \rightarrow h^0H^0Z)}{\sigma(e^+e^- \rightarrow h^0Z)} \sim \frac{g_{H^0h^0h^0}^2}{32\pi^2s^3m_{H^0}^2} \left\{ (s - m_{H^0}^2) \left[ (s - m_{H^0}^2)^2 + 12sm_{H^0}^2 \right] - 6m_{H^0}^2s(s + m_{H^0}^2) \ln \left( \frac{s}{m_{H^0}^2} \right) \right\}, \tag{4.1}
\]

where the \(H^0h^0h^0\) coupling in the decoupling limit [\(i.e., \text{when } \cos(\beta - \alpha) = 0\)] is given by

\[
g_{H^0h^0h^0} = \frac{-3igm_Z}{4\cos\theta_W} \sin 4\beta. \tag{4.2}
\]

This rate [eq. (4.1)] is also too small for a viable Higgs signal.

Second, consider precision measurements of \(h^0\) branching ratios at the NLC. In a Monte Carlo analysis, Hildreth et al.\(^{26}\) evaluated the anticipated accuracy of \(h^0\) branching ratio measurements at the NLC, assuming \(\sqrt{s} = 500 \text{ GeV}\) and a data set of 50 fb\(^{-1}\). For example, taking \(m_{h^0} = 120 \text{ GeV}\), they computed an extrapolated error of \(\pm 7\%\) for the 1-\(\sigma\) uncertainty in \(BR(h^0 \rightarrow b\bar{b})\) and \(\pm 14\%\) for \(BR(h^0 \rightarrow \tau^+\tau^-)\). Other channels yielded substantially larger uncertainties. To see whether these are significant measurements, one can compare these results with the theoretical expectations of the MSSM as a function of \(m_{A^0}\) and \(\tan\beta\). Hildreth and I have found\(^{27}\) that deviations in \(BR(h^0 \rightarrow b\bar{b})\) and \(BR(h^0 \rightarrow \tau^+\tau^-)\) from the Standard Model can be about 7\% for values of \(m_{A^0}\) as large as 450 GeV and about 15\% for values of \(m_{A^0}\) as large as 250 GeV. (See ref. 23 for a related analysis.) These results imply that a precision measurement of \(h^0 \rightarrow b\bar{b}\) has the potential for detecting the existence of the non-minimal Higgs sector even if the heavier Higgs states cannot be directly detected at the NLC. Of course, one will have to push the precision of this measurement beyond its present expectations, since a 2-\(\sigma\) deviation is not compelling evidence for new physics. Other Higgs decay channels are not competitive.
There is one novel approach which could extend the discovery limits for heavy Higgs bosons at the NLC. A high energy, high luminosity photon beam can be produced by the Compton backscatter of an intense laser beam off a beam of electrons. This provides a mechanism for turning the NLC into a high energy, high luminosity γγ collider. All neutral Higgs states couple to γγ at one-loop via loops of charged matter. Since the couplings of the heavy Higgs states to fermions and scalars are not suppressed in the decoupling limit, the γγ couplings of the heavy Higgs states are also not suppressed relative to the $h^0\gamma\gamma$ coupling. Thus, one can search for the non-minimal Higgs sector at the γγ collider by either measuring the $h^0\gamma\gamma$ coupling with sufficient precision or by directly producing $A^0$ and/or $H^0$ in γγ fusion. In the decoupling regime, the $h^0\gamma\gamma$ coupling approaches the corresponding coupling of the Standard Model Higgs boson. As a result, this is not a viable method for detecting deviations from the Standard Model. Thus, one must focus on $\gamma\gamma \rightarrow (A^0, H^0)$. In ref. 11, Gunion and I showed that parameter regimes exist where one could extend the heavy Higgs mass reach above $\sqrt{s}/2$. For example, at a 500 GeV γγ collider, a statistically significant $A^0$ signal in $b\bar{b}$ and $Zh^0$ final states could be seen above backgrounds if $m_{A^0} < 2m_t$.

5. Conclusions

In the most general CP-conserving two Higgs doublet model, a decoupling limit can be defined in which the lightest Higgs state is a CP-even neutral Higgs scalar, whose properties approach those of the Standard Model Higgs boson. This result is more general, and applies to non-minimal Higgs sectors that contain the Standard Model Higgs doublet and respect standard phenomenological constraints (such as $m_W = m_Z \cos \theta_W$ at tree-level). In the MSSM, the decoupling limit corresponds to $m_{A^0} \gg m_Z$ (independent of $\tan \beta$). Moreover, the approach to decoupling is rapid once $m_{A^0}$ is larger than $m_Z$. Thus, over a very large range of MSSM parameter space, the couplings of $h^0$ to gauge bosons, quarks and leptons are nearly identical to the couplings of the Standard Model Higgs boson.

If the $h^0$ is discovered with properties approximating those of the Standard Model Higgs boson, then the discovery of the non-minimal Higgs sector will be difficult. At the LHC, $A^0$, $H^0$ and $H^\pm$ production rates via gluon-gluon fusion are not suppressed. However, isolating signals of the heavy Higgs states above background presents a formidable challenge. At the NLC, if $\sqrt{s} > 2m_{A^0}$, then pair production of $H^+H^-$ and $H^0A^0$ is easily detected. However, below pair-production

\[†\] The contributions of supersymmetric loops to the $h^0\gamma\gamma$ amplitude vanish in the limit of large supersymmetric particle masses.
threshold, detection of the non-minimal Higgs sector is problematical. For example, the cross sections for single heavy Higgs boson production (in association with light particles) are too small to be observed. However, experiments at the $\gamma\gamma$ collider may extend the NLC discovery limits of the heavy Higgs states (via $\gamma\gamma$ fusion to $H^0$ or $A^0$). Precision measurements of $h^0 \rightarrow b\bar{b}$ could provide additional evidence for a non-minimal Higgs sector. However, current experimental expectations at the NLC predict only a 2-$\sigma$ deviation from Standard Model expectations if $m_{A^0}$ is just beyond the kinematic limit of direct NLC detection.

If $h^0$ is discovered with distinguishable properties relative to Standard Model predictions, then there are three possibilities. The simplest possibility is that other scalar states of the non-minimal Higgs sector lie close in mass to $h^0$ and will be discovered via direct production soon after the discovery of $h^0$. If the other Higgs states are not discovered (and if appropriately strong mass limits are obtained), then two alternative scenarios emerge. If the Higgs parameters lie in the non-decoupling regime (i.e., there exists at least one or more large Higgs self-couplings), then the Higgs sector is probably strongly coupled, which suggests the existence of a rich electroweak symmetry breaking sector at an energy scale near 1 TeV. On the other hand, if the Higgs sector is weakly coupled, then there must exist new non-Standard Model decay channels accessible in $h^0$ decay. For example, in Majoron models, $h^0 \rightarrow JJ$ (where $J$ is the Majoron) provides an invisible decay mode for $h^0$,\textsuperscript{30} which would cause deviations from Standard Model expectations for $h^0$ branching ratios to gauge bosons, quarks and leptons. A similar possibility exists in some supersymmetric models where the decay $h^0 \rightarrow \tilde{\chi}_1^0\tilde{\chi}_1^0$ (where $\tilde{\chi}_1^0$ is the lightest neutralino) can be significant,\textsuperscript{31} leading again to deviations in the $h^0$ branching ratios to Standard Model final states.

Once the first evidence for the Higgs boson is established, it will be crucial to ascertain the underlying dynamics of the electroweak symmetry breaking sector. If the data reveals that the Higgs sector is non-minimal, then we will have an important clue to the structure of the electroweak symmetry breaking sector. However, one must be prepared for the more pessimistic scenario—the discovery of a Higgs boson whose properties are not experimentally distinguishable from the Higgs boson of the minimal Standard Model. This scenario presents a formidable challenge for future collider experiments in their attempt to probe the physics of the electroweak symmetry breaking sector and the nature of TeV-scale physics beyond the Standard Model.

Acknowledgments

This paper describes work performed in a number of separate collaborations with Jack Gunion, Mike Hildreth, Yosef Nir, and Scott Thomas. I am especially grateful for their insights and diligence. In addition, I am pleased to acknowledge
George Pocsik and his colleagues at Eötvös University and Stefan Pokorski and his colleagues at the University of Warsaw for their warm hospitality and their labors in providing such a stimulating environment during their workshops. This work was supported in part by the Department of Energy.

**References**

1. J.F. Gunion, H.E. Haber, G.L. Kane and S. Dawson, *The Higgs Hunter’s Guide* (Addison-Wesley, Redwood City, CA, 1990).
2. V.A. Miransky, *Dynamical Symmetry Breaking in Quantum Field Theories* (World Scientific, Singapore, 1993).
3. P.H. Chankowski, S. Pokorski and J. Rosiek, *Phys. Lett.* B281 (1992) 100.
4. G.L. Kane, in *Perspectives on Higgs Physics*, edited by G.L. Kane (World Scientific, Singapore, 1993) pp. 223–228.
5. L. Montanet *et al.* [Particle Data Group] *Phys. Rev.* D50 (1994) 1173.
6. A. Sopczak, *Int. J. Mod. Phys.* A9 (1994) 1747.
7. A. Stange, W. Marciano and S. Willenbrock, *Phys. Rev.* D49 (1994) 1354; D50 (1994) 4491; S. Mrenna and G.L. Kane, CALT-68-1938 (1994) [hep-ph 9406334]; J.F. Gunion, UCD-94-24 (1994) [hep-ph 9406405].
8. D. Froidevaux, Z. Kunszt and J. Stirling *et al.*, in *Proceedings of the Large Hadron Collider Workshop*, Aachen 1990, CERN Report 90-10 (1990).
9. S. Dawson, in *Perspectives on Higgs Physics*, edited by G.L. Kane (World Scientific, Singapore, 1993) pp. 129–155; Z. Kunszt, *ibid.* pp. 156–178.
10. V. Barger, K. Cheung, A. Djouadi, B.A. Kniehl, and P. Zerwas, *Phys. Rev.* D49 (1994) 79.
11. J.F. Gunion and H.E. Haber, *Phys. Rev.* D48 (1993) 5109.
12. D.L. Borden, D.A. Bauer and D.O. Caldwell, *Phys. Rev.* D48 (1993) 4018.
13. J.F. Gunion and H.E. Haber, *Nucl. Phys.* B272 (1986) 1; B278 (1986) 449 [E: B402 (1993) 567].
14. For a review of the influence of radiative corrections on the MSSM Higgs sector, see H.E. Haber, in *Perspectives on Higgs Physics*, edited by G.L. Kane (World Scientific, Singapore, 1993) pp. 79–128.
15. See, *e.g.*, H.E. Haber and R. Hempfling, *Phys. Rev.* D48 (1993) 4280.
16. J.F. Gunion, H.E. Haber, and J. Wudka, *Phys. Rev.* D43 (1991) 904.
17. See, *e.g.*, M. Carena, M. Olechowski, S. Pokorski, and C.E.M. Wagner, *Nucl. Phys.* B419 (1994) 213; G.L. Kane, C. Kolda, L. Roszkowski and J.D. Wells, *Phys. Rev.* D49 (1994) 6173; D50 (1994) 3498; P. Nath and R. Arnowitt, CERN-TH.7227/94 (1994) [hep-ph 9406403]; W. de Boer, R. Ehret, W. Oberschulte, D.I. Kazakov, IEKP-KA/94-05 [hep-ph 9405342], to be published in *Z. Phys. C*.
18. H.E. Haber and S. Thomas, SCIPP preprint in preparation.
19. H.E. Haber and Y. Nir, *Nucl. Phys.* **B335** (1990) 363.
20. D. Dicus, A. Stange and S. Willenbrock, *Phys. Lett.* **B333** (1994) 126.
21. For a recent review, see J.F. Gunion, in *Perspectives on Higgs Physics*, edited by G.L. Kane (World Scientific, Singapore, 1993) pp. 179–222.
22. J. Dai, J.F. Gunion and R. Vega, *Phys. Rev. Lett.* **71** (1993) 2699; UCD-94-7 (1994) [hep-ph 9403362].
23. P. Janot, in *Physics and Experiments with Linear e+e− Colliders*, Workshop Proceedings, Waikoloa, Hawaii, 26–30 April, 1993, edited by F.A. Harris, S.L. Olsen, S. Pakvasa and X. Tata (World Scientific, Singapore, 1993) pp. 192–217.
24. A. Djouadi, J. Kalinowski and P.M. Zerwas, *Z. Phys.* **C57** (1993) 569.
25. A. Djouadi, J. Kalinowski and P.M. Zerwas, *Mod. Phys. Lett.* **A7** (1992) 1765; *Z. Phys.* **C54** (1992) 255.
26. M.D. Hildreth, T.L. Barklow, and D.L. Burke, *Phys. Rev.* **D49** (1994) 3441.
27. H.E. Haber and M.D. Hildreth, in preparation.
28. H.F. Ginzburg, G.L. Kotkin, V.G. Serbo and V.I. Telnov, *Nucl. Inst. and Methods* **205** (1983) 47; H.F. Ginzburg, G.L. Kotkin, S.L. Panfil, V.G. Serbo and V.I. Telnov, *Nucl. Inst. and Methods* **219** (1984) 5.
29. A.I. Vainshtein, M.B. Voloshin, V.I. Zakharov and M. Shifman, *Yad. Fiz.* **30** (1979) 1368 [Sov. J. Nucl. Phys. **30** (1979) 711].
30. A.S. Joshipura and J.W.F. Valle, *Nucl. Phys.* **B397** (1993) 105.
31. K. Griest and H.E. Haber, *Phys. Rev.* **D37** (1988) 719.