Power-split transmission performance research with the method of lever analogy

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Abstract. Power-split transmission system was very complex in structure; the continuously variable transmitting performance of this system is difficult to analysis, which impeded the further research of this kind of transmission. In this research, universal series mathematical models of power-split transmission system were set up firstly. Then, the lever analogy analysis method was introduced into the power-split transmission performance analysis process, and the multi-range continuously variable transmitting performance of power-split transmission was described comprehensive. Finally, the method was used to analysis one real power-split transmission systems, ZF Friedrichshafen power, and the accuracy, validity and convenience of this method were verified. The result show that the method of mathematical model combine with lever analogy could analysis the power-split transmission system continuously variable transmitting performance more correctly, easily and rapidly, it could provide a more intuitive and efficiently analysis method for the further research of power-split transmission system.

Keywords: Wheel loader; power-split transmission; continuously variable transmission; lever analogy; compatible coupling.

1. Introduction

Power-split transmission was coupled hydrostatic transmitting branch and mechanical transmitting branch with the planetary. The hydrostatic branch could vary ratio continuously, the mechanical branch could transmit power with higher torque and more efficiency, which made the power-split transmission was more efficiency, ratio rang was wider, transmit more power and the ratio varied continuously, these performances could satisfy the power transmit requirements of heavy-duty vehicles such as wheel loader and tractor etc.

Kumar R studied all kinds of power-split structural scheme, and evaluated the energy saving potential of each scheme [1]. Nilsson T researched the fuel economy of wheel loader with power-split transmission by simulation and test, the result show that it could save more than 20% fuel [2]. Alarico Macor&Antonio Rossetti found the proportion of power distribution of hydrostatic branch and mechanical branch was the key factor of the transmission efficiency and the ratio variable performance. When the hydrostatic branch transmitted more power, the transmission system became lower efficiency, but the ratio continuously variable rang became wider, and vice versa [3]. Horst Schulte propose a
control algorithm base on S-T fuzzy control, which could fit wheel loader with power-split transmission work condition, the simulation and test research show that this method could track the target ratio very well[4].

Until now, most researches about power-split transmission focused on structural scheme design, transmission efficiency improvement and target ratio tracking, there less research on continuously variable transmitting performance and ratio vary mechanism. This research would study the continuously variable transmitting performances and ratio variation mechanism of power-split transmission by the means of mathematical model combine with lever analogy.

2. Continuously variable transmitting mechanism of Power-split transmission

2.1. Structure of power-split transmission

The structural scheme of output coupling power-split transmission was shown in Fig 1. Hydrostatic branch ratio was controlled by displacement ratio of pump and motor. The two ports of hydrostatic branch were connected with two parts of planetary, so one freedom of planetary was restricted by the speed ratio of hydrostatic branch, and then the planetary could transmit power with one freedom.

2.2. Ratio variable mechanism of hydrostatic branch

In the power-split transmission system, hydrostatic branch most commonly adopt the closed loop, the scheme shown in fig 1, suppose the closed loop without leakage, and based on the flow balance of closed loop relation, the following equation could be obtained:

\[ n_p \cdot q_p = n_m \cdot q_m \]  

\[ n_p, n_m \] -Speed of variable pump and motor  
\[ q_p, q_m \] -Displacement of variable pump and motor

Speed ratio and displacement ratio of variable pump and motor could be expressed as:

\[ i_e = \frac{n_p}{n_m} = \frac{q_m}{q_p} = \frac{1}{\varepsilon} \]  

\[ \varepsilon \] -Displacement ratio of hydrostatic transmitting system  
\[ i_e \] -Speed ratio of hydrostatic transmitting system

Equation (2) indicated that speed ratio of hydrostatic branch was reciprocal of displacement ratio, and by adjusting displacement ratio could control speed ratio.

Planetary compatible coupling mechanism of power-split transmission
Different parts of planetary motion relation were shown in following equation:

\[ n_s - (1 + \rho) \cdot n_H + \rho \cdot n_s = 0 \] (3)

\( n_R, n_H, n_S \) - Rotation speed of ring gear, planet carrier and sun gear

\( \rho \) - Structural parameters of planetary, which could be expressed as teeth number ratio of sun gear and ring gear.

Take the structural scheme of output coupling power-split transmission as an example, the scheme was shown in fig 1. The engine power was input planetary form the sun gear, which means engine speed equal sun gear speed:

\[ n_s = n_E \] (4)

\( n_E \) - Engine speed;

The two ports of hydrostatic branch were connected to ring gear and planet carrier of planetary, ratio of hydrostatic branch could be controlled by the displacement ratio of pump and motor, so the ring gear and sun gear speed relation is:

\[ n_R = i_1 \cdot i_e \cdot i_2 \cdot n_H \] (5)

\( i_1 \) - The ratio of ring gear to pump

\( i_2 \) - The ratio of combination hydrostatic and mechanical branch

\( i_e \) - The ratio of pump and motor

Take the equation (4) and (5) into equation (3), equation (6) was obtained:

\[ \frac{n_E}{n_H} = \frac{1 + \rho \cdot i_1 \cdot i_e \cdot i_2}{\rho} \] (6)

In order to reduce speed and increase torque of output power, and widen continuously variable rang of speed ratio, a gear box was equipped after the planet carrier. The total ratio of the whole power-split transmission system was:

\[ i_\Sigma = \frac{1 + \rho \cdot i_1 \cdot i_e \cdot i_2}{\rho} \cdot i_g \cdot i_0 = f(i_e, i_0) \] (7)

\( i_g \) - Step ratio of gear box

\( i_0 \) - The fix ratio of drive axle

2.3. Continuously variable transmitting preference analysis with lever analogy

The planetary compatible coupling relation of output coupling power-split transmission system could be expressed by the lever analogy, shown as fig 2. 3 vertical coordinates marked speed of 3 parts of planetary, and the scale of coordinate was uniform. The planet carrier coordinate was in the middle, the relative distance of 3 coordinates could be expressed by \( \rho \), if the distance of planet carrier speed coordinate and ring gear speed coordinate was \( \rho \), then the distance of planet carrier speed coordinate and sun gear speed coordinate was 1. With this arrangement, the speed of 3 planetary parts in different coordinates would always keep in one line, just like a lever with 3 speed point as pivots [5].

Lever analogy could be used to analysis the continuously variable transmitting performance of power-split transmission system, while the hydrostatic branch ratio varied continuously. Take the structural scheme of fig 1 as an example, the speed relation of different parts of planetary was shown in fig 2, if engine (sun gear) speed was constant, the pump P (ring gear) speed could continuously increase from negative maximum to positive maximum, then the output axis (planet carrier) speed continuously
increase from zero (point ①) to certain speed (point ②), by this way the power-split transmission system could vary speed ratio continuously.

![Diagram](image)

Fig 2. Continuously variable transmitting characteristic of power-split transmission system

2.4. Multi-range continuously variable transmitting performance of power-split transmission

It could be learned from equation (7), the whole power-split transmission system ratio $i_\Sigma$ was changed with $i_e$ & $i_g$. Because the hydrostatic branch could vary the displacement ratio continuously, $i_e$ made the total ratio $i_\Sigma$ could vary continuously too, but the range was narrower. $i_g$ was ratio of gearbox, it was a series ratio in steps, with the shifting of $i_g$ the range of power-split transmission total ratio $i_\Sigma$ could be widened. If $i_\Sigma$ skip distance of different gear was smaller than the vary range of $i_e$ in $i_\Sigma$, the total ratio $i_\Sigma$ of power-split transmission could vary continuously in a wider range, and the total ratio $i_\Sigma$ could be expressed as by uniform equation:

$$i_\Sigma_k = f(i_e, i_g), \ k = \text{I, II, \ldots, N} \tag{8}$$

Where: $i_e = \{ x | i_e_{\text{min}} \leq x \leq i_e_{\text{max}}, x \in R \}$

- $i_e_{\text{min}}$, $i_e_{\text{max}}$ - Minimum and maximum of hydrostatic branch ratio
- $i_g = \{ x | i_g_1, i_g_2, \ldots, i_g_N \geq i_g_{\text{min}}, x \in R \}$
- $i_g_1$, $i_g_2$, \ldots, $i_g_N$ - Gearbox ratio in series

In order to varying continuously from minimum to maximum, the total ratio $i_\Sigma$ of adjacent ranges should not be discontinuity, that means the adjacent ranges of total ratio $i_\Sigma$ should be satisfied following equation:

$$i_\Sigma_{k\text{min}} \leq i_\Sigma_{k+1\text{max}}, \ k = \text{I, II, \ldots, N} \tag{9}$$

- $i_\Sigma_{k\text{min}}$ - minimum total ratio when the gearbox ratio was $i_g_k$;
- $i_\Sigma_{k+1\text{max}}$ - maximum ratio when the gearbox ratio was $i_g_{k+1}$;

Because of the hydrostatic branch structure restricting, its continuously variable transmitting range was narrow, and the planetary compatible coupling relation made the total ratio $i_\Sigma$ of power-split transmission system even narrower. In order to keep the total ratio $i_\Sigma$ continuously varying, hydrostatic branch need to change the regulation of displacement variation, and mechanical branch need to shift gearbox ratio, by which made the power-split transmission system transmitting variable continuously in multi-range.

There are two regulations of displacement variation for the hydrostatic branch: one was named coherent displacement variation regulation, which required the displacement of hydrostatic branch change from one extreme position to another extreme position orderly and continuously, when gearbox
was changing ratio. The total ratio $i_{\Sigma}$ trend was kept by the means of changing the monotonicity of hydrostatic branch ratio $i_e$ to the total ratio $i_{\Sigma}$, the total ratio $i_{\Sigma}$ of adjacent ranges partial derivative on hydrostatic branch ratio $i_e$ satisfied following equation:

$$\frac{\partial i_{\Sigma_k}}{\partial i_e} \cdot \frac{\partial i_{\Sigma_{k+1}}}{\partial i_e} < 0, \ k=I, \ II, \ \cdots, \ N$$

(10)

The coherent displacement variation regulation required hydrostatic branch ratio $i_e$ and gearbox ratio $i_g$ combine total ratio $i_{\Sigma}$ in different ranges as the rule show in fig 3(a) [6].

![Fig 3](image)

**Fig 3.** Ratio composition relationship of power-split transmission

(a) Coherent displacement variation regulation; (b) Reset displacement variation regulation

Another was named reset displacement variation regulation, which required the displacement of hydrostatic branch reset from one extreme position to another extreme position, and repeated the displacement variation again and again, when gearbox shifted ratio. The total ratio $i_{\Sigma}$ monotonicity about hydrostatic branch ratio $i_e$ should be remained, the total ratio $i_{\Sigma}$ of adjacent ranges partial derivative on hydrostatic branch ratio $i_e$ satisfied following equation:

$$\frac{\partial i_{\Sigma_k}}{\partial i_e} \cdot \frac{\partial i_{\Sigma_{k+1}}}{\partial i_e} > 0, \ k=I, \ II, \ \cdots, \ N$$

(11)

The reset displacement variation regulation required hydrostatic branch ratio $i_e$ and gearbox ratio $i_g$ combine total ratio $i_{\Sigma}$ in different ranges as the rule show in fig 3(b).

3. **cPower scheme analyses**

cPower was created by ZF Friedrichshafen, its all range was power-split transmitting, the structural scheme was shown in fig 4[7].

Power transmitting rout and control logic

When the clutch CF and C1 were engaged, cPower worked at I range, engine power allocated to power train was input from clutch CF, gear Z4 and planet carrier to planetary, the power was split as hydrostatic and mechanical branch on planetary, the hydrostatic branch power was input from the first sun gear, through pump-motor, output to gear Z8, the mechanical branch power input from ring gear, through Z9 to gear Z8, the power of hydrostatic and mechanical branch was combined at gear Z8, through gear Z10 and clutch C1 to drive axle.
When the clutch CF and C2 were engaged, cPower worked at I range, the power was split into hydrostatic and mechanical branch on planetary too, hydrostatic branch power was input from ring gear, through gear Z9, motor (used as pump), pump (used as motor), first sun gear to duplex planet gears, mechanical branch power input from planet carrier to duplex planet gears, hydrostatic and mechanical branch power was combined on duplex planet gears, the combined power through second sun gear, gear Z5, Z7 and clutch C2, output to drive axle.

![Fig 4. Structural scheme of cPower](image)

When the revers clutch CR was engaged, and engaged the range clutch at same time, cPower worked at revers range corresponding. The power transmitting rout of cPower was controlled by the clutch CF, CR, C1 and C2, the control logic was shown as table 1, cPower transmitting rout need to control two clutches at same time.

| Range | CF | CR | C1 | C2 |
|-------|----|----|----|----|
| F1    | ●  | ○  | ●  | ○  |
| F2    | ●  | ○  | ○  | ●  |
| R1    | ○  | ●  | ●  | ○  |
| R2    | ○  | ●  | ○  | ●  |

○ disengage; ● engage

3.1. Continuously variable transmitting performance analysis
There are 4 key components in the duplex planetary of cPower. 4 vertical coordinate in fig 5 indicated 4 key components of cPower, the lever analogy pivots indicated speed relation of different components, where \( \beta \) was the distance of second sun gear and planet carrier, it indicated the speed correlation coefficient of front planetary and real planetary[8].

When cPower worked at I range, hydrostatic and mechanical branch power were combined with the output coupling rule. Sun gear connected with pump(P), ring gear connected with motor(M), motor(M) was connected with drive axle too, planet carrier connected with engine. The pump(P) conjugated to motor with 45° phase difference, so the displacement of pump(P) and motor(M) were always varying inverse phase, that means when the pump(P) displacement was increasing, the motor(M) displacement was reducing, vice versa. If engine (planet carrier) speed was constant, the pump(P) speed varied from
maximum to minimum and motor(M) speed varied from minimum to maximum by the means of adjusting pump(P) and motor(M) displacement, which made the cPower output speed of ring gear increasing from ① to ②, shown as fig 5(a).

When the cPower worked at II range, hydrostatic and mechanical branch power were combined with composite coupling. Sun gear connected with pump(used as motor M), ring gear connected with motor(used as pump P), planet carrier connected with engine, the drive axle of cPower was connected with the second sun gear. When the cPower was shifted II range, the motor (used as pump) displacement was minimum and speed was maximum(maintained the work state in I range), pump (used as motor) displacement was maximum and speed was minimum(maintained the work state in I range). The engine (planet carrier) speed maintained constant, the pump (used as motor) speed varied from minimum to maximum and the motor (used as pump) speed varied from maximum to minimum by the means of adjusting pump and motor displacement, which made the cPower output speed of ring gear increasing from ③ to ④, shown as fig 5(b).

![Fig 5. Continuously variable transmitting performance of cPower](image)

(a) I range (b) II range

With optimizing the teeth of gear Z5, Z7, Z8 and Z10, speed of ② could joint with speed of ③ exactly, then cPower total transmitting ratio could vary continuously in two ranges, which could satisfy equation (9). Because I and II range used different combining means, different power transmitting rout, and the function of pump and motor was exchanged when cPower was shifting, so the hydrostatic branch of cPower followed coherent displacement variation regulation, and the total ratio \( \frac{\Sigma i}{i} \) expressions of I and II range partial derivative on hydrostatic branch ratio \( \frac{\Sigma e}{\Sigma} \) satisfied the equation (10), and the impact was soft when cPower shifted range.

4. Conclusion
A series of mathematical models of power-split transmission system was set up, multi-range continuously variable transmitting performance was described comprehensively, a convenient tool was provided to analysis performance of power-split transmission system. Lever analogy was introduced into the performance analysis of power-split transmission system, the performance analysis process of power-split transmission was more intuitive and more convenient by this method. The multi-range continuously variable transmitting performance of cPower was analyzed with the method proposed by this research, the validity and convenience was verified.
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