Cosmological Constraints on a Massive Neutrino

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The paper by Sato and Kobayashi in 1977 studied the cosmological effects of a massive neutrino and obtained constraints on its properties. This paper initiated many studies to use cosmology as a laboratory of particle physics or to use particle physics to explore the very early universe.

§1. Introduction

In 1977 Kobayashi and one of the present author (K.S.) studied cosmological effects of a massive neutrino and published the paper entitled "Cosmological Constraints on the Mass and the Number of Heavy Lepton Neutrinos".\textsuperscript{1)} Hereafter we call it Sato-Kobayashi 1977. This study was motivated by the discovery of anomalous lepton production in $e^+e^-$ annihilation ($e^+ + e^- \rightarrow e^{\pm} + \mu^{\mp} + \text{missing energy}$) in 1975,\textsuperscript{2)} which strongly suggested the existence of a heavy lepton having a mass in the range 1.6 to 2.0 GeV. [Later, the existence of new lepton was confirmed and named tau ($\tau$).]

Assuming the heavy lepton in the third family, the Sato-Kobayashi 1977 addressed two issues; one was to constrain the mass of the associated neutrino and the other was limiting the number of massive neutrinos. The latter issue had been partly investigated by Steigman et al.\textsuperscript{3)} for massless neutrinos and Kobayashi-Sato 1977 gave a more generic limit which applies to massive neutrinos. As for the first issue the authors were first to study the cosmological effects of a massive neutrino and obtained constraints on its mass and lifetime. In deriving the cosmological constraints, the effects on the cosmic matter density, cosmic background radiation and big bang nucleosynthesis were considered. Today this cosmological approach is the standard way to reveal properties of particles predicted in new models of particle physics or to explore the very early universe by using new particle theories. The research field is now called particle cosmology. Therefore, Sato-Kobayashi 1977 is regarded as a historic paper that took the initiative in development of particle cosmology.

In this paper we review Sato-Kobayashi 1977 and the subsequent progress. The paper is organized as follows. In section \textsuperscript{2} we review Sato-Kobayashi 1977. In section \textsuperscript{3} the subsequent progress is shown, and we conclude in section \textsuperscript{4}. 

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2. Review of Sato-Kobayashi 1977

2.1. Lifetime of the massive neutrino

In the 1977 paper,\(^1\) a neutrino, which is associated with the heavy lepton (now called ‘tau’) suggested in the \(e^+e^-\) collider experiment, was considered. If it has a finite mass and quark-like mixings with other neutrinos (\(\nu_e, \nu_\mu\)), the heavy neutrino is unstable and can decay into a lighter neutrino via

\[ \nu_h \rightarrow \nu_{e,\mu} + \gamma. \]  

(2.1)

The neutrino with mass \(m_{\nu_h} > 2m_e\) can also decay into an electron-positron pair as

\[ \nu_h \rightarrow \nu_e + e^+ + e^- . \]  

(2.2)

With use of the mass eigenstates \(\nu_i \ (i = 1, 2, 3)\) the weak interaction eigenstate \(\nu_h\) of the heavy neutrino is written as

\[ \nu_h = \sum_i U_{hi} \nu_i, \]  

(2.3)

where \(U_{hi}\) is the neutrino mixing matrix. In Ref.\(^1\), the lifetimes for the decay processes (2.1) and (2.2) were estimated in the framework of Weinberg-Salam theory. For decay mode (2.1)

\[ \tau(\nu_h \rightarrow \nu_{e(\mu)} + \gamma) = \frac{128\pi^3}{G_F m_{\nu_h}^5} \frac{16}{9} \left( \sum_i U_{ih} U_{ie(\mu)^*} m_i^2 \right) m_{\nu_h}^5 \]  

\[ \simeq 2 \times 10^6 \text{yr} \left( \frac{m_{\nu_h}}{\text{MeV}} \right)^{-5} |U_{he(\mu)}|^{-2}, \]  

(2.4)

where \(m_{\nu_h}\) is the mass of the heavy neutrino, \(m_i\) is the charged lepton mass, \(\alpha\) is the fine structure constant, \(G_F\) is the Fermi constant and \(M_W\) is the W boson mass. In the second line of Eq.\(^2\), we have used the measured \(\tau\) lepton mass, but in 1977, the heavy lepton (= \(\tau\)) was not confirmed, so the upper bound on the lifetime was obtained from unitarity of the mixing matrix, which leads to the following inequality:

\[ \sum_{i' < i} \left( \sum_j U_{ji} U_{ji'}^* m_j^2 \right)^2 \leq \sum_j U_{ij} U_{ij}^* m_j^4 - \left( \sum_j U_{ji} U_{ji}^* m_j^2 \right)^2 \]  

\[ \leq \frac{1}{4} \max_{j,j'} [(m_j^2 - m_{j'}^2)^2]. \]  

(2.5)

Using this relation and \(m_j/M_W < 1\), the lower bound on the lifetime was obtained as

\[ \tau(\nu_h \rightarrow \nu_{e(\mu)} + \gamma) > \frac{128\pi^3}{G_F m_{\nu_h}^5} \frac{64}{9} \left( \frac{\max_j [m_j^4]}{m_{\nu_h}^4} \right)^{-2} > 2 \text{ yr} \left( \frac{m_{\nu_h}}{\text{MeV}} \right)^{-5}. \]  

(2.6)
On the other hand, the lifetime for decay into $e^+ e^-$ is given by

$$
\tau(\nu_h \to \nu_e + e^+ + e^-) = \frac{192\pi^3}{G_F^2 m_{\nu_h}^5} |U_{1h} U_{1e}|^{-2} \simeq 3 \times 10^4 \text{ sec} \left(\frac{m_{\nu_h}}{\text{MeV}}\right)^{-5} |U_{h1}|^{-2},
$$

(2.7)

for $m_{\nu_h} > 2m_e$. The mixing $U_{1h}$ is restricted with use of $\mu$-$e$ universality. The pion decay process $\pi \to e + \nu_h$, if exists, is enhanced because the amplitude is proportional to $m_{\nu_h}$. So in Ref. 1) requiring that the contribution of $\pi \to e + \nu$ to the total decay width of $\pi \to e + \nu$ should be less than 5%, the limit on the $U_{1h}$ was estimated as

$$
|U_{1h}|^2 (m_{\nu_h}/m_e)^2 < 0.05,
$$

(2.8)

which led to the lower limit to the lifetime,

$$
\tau(\nu_h \to \nu_e + e^+ + e^-) \gtrsim 2 \times 10^8 \text{ sec} \left(\frac{m_{\nu_h}}{\text{MeV}}\right)^{-3}.
$$

(2.9)

Eqs. (2.6) and (2.9) show that the massive neutrino decays with cosmological time scale and the decay into an electron-positron pair, if kinematically possible, may be faster than that into a photon.

2.2. Cosmological constraints

Since the massive neutrino is long-lived ($\tau > 1$ sec), it can affect the thermal history of the universe. In Sato-Kobayashi 1977 [1]) the constraints on mass and lifetime were obtained by considering the following cosmological effects:

1. When the massive neutrino is stable or has a longer lifetime than the present age of the universe, its present mass density may exceed the observed upper limit.
2. Photons and electrons produced in the decay may cause a spectral distortion of the cosmic microwave background (CMB) radiation, or if the decay takes place after recombination, the emitted photons may be directly observed as the background radiation.
3. The massive neutrino can affect the big bang nucleosynthesis (BBN) by speed-up of the cosmic expansion and/or entropy production.

First let us consider the cosmic density of the heavy neutrino. The neutrinos are in thermal equilibrium via weak interaction ($\nu_h + \bar{\nu}_h \leftrightarrow e^- + e^-$) in the very early universe. The annihilation cross section is

$$
\langle \sigma v \rangle \sim \begin{cases} 
G_F^2 T^2 & m_{\nu_h} \lesssim T \\
G_F^2 m_{\nu_h}^2 & m_{\nu_h} \gtrsim T
\end{cases},
$$

(2.10)

where $v$ is the relative velocity and $\langle \cdots \rangle$ means thermal average. When the interaction rate $\Gamma = \langle \sigma v \rangle n_\nu$ becomes less than the cosmic expansion rate $H \sim T^2/M_p (M_p$:...
Planck mass $\simeq 2.4 \times 10^{18}$ GeV), the neutrino decouples from the thermal bath. After decoupling, the neutrino does not interact with other particles and its comoving density freezes out. The freezing temperature is determined from $\Gamma \simeq H$, leading to $T_f \simeq \max[1, (m_{\nu_h}/10\text{MeV})]$ MeV. Then the number ratio of the massive neutrino to the photon is estimated as

$$\left(\frac{n_{\nu_h}}{n_\gamma}\right) \simeq \frac{3}{11} \min\left\{1, \left(\frac{10 \text{ MeV}}{m_{\nu_h}}\right)^3\right\}. \quad (2.11)$$

Notice that the neutrino number density decreases for $m_{\nu_h} \gtrsim 10$ MeV because such heavy neutrino becomes non-relativistic at decoupling and its number density suffers from the Boltzmann suppression ($\sim \exp(-m/T)$).

### 2.2.1. Stable Neutrino

If the massive neutrino is stable or has a longer lifetime than the age of the universe, the neutrino density should be less than the upper limit of the present density of the universe. The present mass density of the heavy neutrino is given by

$$\rho_{\nu_h} = \frac{3}{11} g_s m_{\nu_h} n_{\gamma,0} \min\left\{1, \left(\frac{10 \text{ MeV}}{m_{\nu_h}}\right)^3\right\}, \quad (2.12)$$

where $g_s$ is the statistical weight of the massive neutrino. In term of the density parameter $\Omega \equiv \rho/\rho_c$, where $\rho_c$ is the critical density ($= 1.054 \times 10^4 h^2$ eV/cm$^3$) and $h$ is the Hubble constant in units of 100 km/s/Mpc, the above equation is rewritten as

$$\Omega_{\nu_h} = 1.0 \ h^{-2} \ g_s \left(\frac{m_{\nu_h}}{100\text{eV}}\right) \min\left\{1, \left(\frac{10 \text{ MeV}}{m_{\nu_h}}\right)^3\right\}. \quad (2.13)$$

In Ref.1, from observations of the deceleration parameter [$q_0 = (\ddot{a}/a^2)$] and the age of the universe at that time, $\Omega h^2 < 1.45$ was adopted as the upper limit of the density parameter, which resulted in

$$m_{\nu_h} < 70 \text{ eV} \quad \text{or} \quad m_{\nu_h} > 3.7 \text{ GeV}. \quad (2.14)$$

As mentioned before, since the Boltzmann suppression reduces the number density of the heavy neutrino, the neutrino with large mass $m_{\nu_h} \gtrsim 3$ GeV is allowed as well as the light neutrino with mass less than $O(10)$ eV. Unfortunately, the lower bound on the heavy neutrino mass is often called “Lee-Weinberg limit” because Lee and Weinberg obtained the limit at the same time.

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* The statistical weight $g_s$ is always 1 for a Majorana neutrino. As for a Dirac neutrino $g_s$ is $1 - 2$ depending on its mass. Because of chirality of the weak interaction, the rate of the interaction of a right-handed neutrino is suppressed by a factor $(m_{\nu}/E)^2$ where $E$ is the relevant energy. Thus, neutrinos with mass less than 2 keV are never in thermal equilibrium in the early universe and $g_s = 1$ for that case. However, in Ref.1) $g_s = 2$ was assumed for any massive neutrinos.

** The both papers were submitted in May 1977. We should also note that the lower bound on the heavy neutrino mass was also obtained by P.Hut, Dicus, Kolb and Teplitz and Vysotskii, Zel’dovich and Dolgov. 

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2.2.2. Unstable Neutrino

Next, we consider the constraint on the unstable massive neutrino which decays into a photon or an electron-positron pair. If the neutrino decays sufficiently early \( t \lesssim 10^{10} \) sec, the emitted photons are quickly thermalized through Compton scattering and bremsstrahlung. However, when decay takes place at later epochs, the emitted photons may partially thermalized via Compton scattering but cannot form the Planck distribution because photon number changing processes such as bremsstrahlung are inefficient. Thus, the CMB spectral shape is distorted, which leads to a constraint on the lifetime \( \tau_{\nu} \) of the massive neutrino.

When photons are emitted in the neutrino decay, the ratio of the extra photon density \( \Delta \rho_\gamma \) to the CMB density \( \rho_\gamma \) at the decay time is estimated as

\[
\frac{\Delta \rho_\gamma (\tau_{\nu})}{\rho_\gamma (\tau_{\nu})} = \frac{(3/11)g_s(m_{\nu h}/2)n_\gamma F}{2.7T_d^2n_\gamma} \simeq 0.05g_s \frac{m_{\nu h}F}{T_d},
\]

where \( T_d \) is the cosmic temperature at \( t = \tau_{\nu} \) and \( F = \min[1, (10 \text{ MeV}/m_{\nu h})^3] \).

With relation between the cosmic temperature and time, \( t = 10^{12} \text{ sec} (T/eV)^{-3/2} \), and \( \Delta \rho_\gamma/\rho_\gamma \lesssim 0.1 \), the constraint was obtained as

\[
\tau_{\nu} \lesssim \begin{cases} 
10^{12} \text{ sec} & m_{\nu h} \lesssim 20 \text{ eV} \\
10^{10} \text{ sec} & 20 \text{ eV} \lesssim m_{\nu h} \lesssim 2 \text{ GeV} \\
10^9 \text{ sec} & m_{\nu h} \gtrsim 2 \text{ GeV} 
\end{cases}.
\]

The above constraint obtained for decay into photons also applies to the case where electron-positron pairs are produced in the decay, i.e., for \( m_{\nu h} > 2m_e \) because the energetic electrons (positrons) scatter off the CMB photons to produce energetic photons or the positrons annihilate into photons.

When the decay takes place after recombination the emitted photons freely stream and may be observed as the background radiation after suffering redshift. This extra background radiation should not exceed the observed one, from which Sato-Kobayashi 1977 obtained the constraint on the lifetime of the massive neutrino as

\[
\tau_{\nu} \gtrsim 10^{18} \text{ sec}
\]

Finally, let us show how Sato-Kobayashi 1977 derived the constraint on the lifetime and mass of the massive neutrino from consideration of nucleosynthesis in the early universe. Two effects both of which increase the \( ^4\text{He} \) abundance were studied in Ref.1).

The first effect is entropy production by the neutrino decay. It is well known that the abundances of light elements synthesized in the standard BBN only depend on the baryon density of the universe which is parametrized by “photon-baryon ratio”,

\[
\eta_B \equiv \frac{n_B}{n_\gamma} = 2.7 \times 10^{-8} \Omega_B h^2.
\]

In Ref.1) the different parameter \( h_N = \rho_B/T_0^3 \) (\( T_0 = T/10^9 \text{K} \)) was used but here we have adopted the standard parameter used in recent literature. \( h_N \) is related to \( \eta_B \)
\[ h_N = 3.4 \times 10^{-6} (\eta_B/10^{-10}) \text{g/cm}^3. \] (2.19)

When the neutrino decay produces many photons, the baryon-photon ratio at BBN epoch (\(\eta_{\text{BBN}}\)) is larger than the present one and is roughly given by
\[ \eta_{\text{bbn}} \simeq \left( 1 + \frac{\Delta \rho_\gamma(\tau_\nu_h)}{\rho_\gamma(\tau_\nu_h)} \right) \eta_B, \] (2.20)

where \(\Delta \rho_\gamma/\rho_\gamma\) is given by (2.15). Using the lower bound on the present baryon density (\(\Omega_B > 0.05\)), which predicts the lowest \(^{4}\text{He}\) abundance because the theoretical prediction of \(^{4}\text{He}\) abundance is an increasing function of \(\eta_{\text{bbn}}\), \(\eta_{\text{bbn}}\) is estimated as
\[ \eta_{\text{bbn}} \lesssim 3.5 \times 10^{-10} \left( 1 + \frac{\Delta \rho_\gamma(\tau_\nu_h)}{\rho_\gamma(\tau_\nu_h)} \right). \] (2.21)

The other effect is speed-up of the universe due to the massive neutrino. As compared with massless case, the massive neutrino can have a larger energy density, which increases the expansion rate of the universe and leads to earlier freeze-out of neutron-proton exchange interactions (e.g. \(n + \nu_e \leftrightarrow p + e^-\)). Since the neutron-proton ratio at the freeze-out temperature \(T_f\) is given by \(n/p \simeq \exp(-1.29\text{MeV}/T_f)\), the earlier freeze-out (higher \(T_f\)) results in more abundance of \(n\) and hence more production of \(^{4}\text{He}\). The change of the cosmic expansion rate is represented by \(\xi\) defined as
\[ \xi \equiv \left( \frac{\rho(T)}{\rho_{\text{std}}(T)} \right)^{1/2}, \] (2.22)

where \(\rho_{\text{std}}\) is the cosmic density at temperature \(T\) for the standard case without massive neutrinos. The densities \(\rho_{\text{std}}\) and \(\rho\) are given by
\[ \rho_{\text{std}} = \frac{\pi^2}{30} \left( \frac{11}{2} T^4 + \frac{7}{4} N_{\text{std}} T^4 \right), \quad \rho \simeq \rho_{\text{std}} + (m_{\nu_h} + 3.15 T_{\nu}) n_{\nu_h}, \] (2.23)

where \(T_\nu\) and \(N_{\text{std}}\) are the neutrino temperature and the number of massless neutrino species. As mentioned before, in 1977 the existence of the tau lepton was not confirmed, thus \(N_{\text{std}} = 2\) was taken as the standard value. Since the speed-up effect is important when the neutron-proton ratio freezes out (\(T_f \simeq 1\text{ MeV}\)), we can assume \(T > m_\nu\) and hence \(T_\nu = T\). Thus \(\xi\) is written as
\[ \xi^2 = 1 + 0.062 g_s \left( \frac{m_{\nu_h}}{T} + 3.15 \right). \] (2.24)

On the other hand the theoretical prediction for the \(^{4}\text{He}\) abundance was calculated numerically by Wagoner\(^9\) and he gave the empirical formula,
\[ Y = 0.264 + 0.0195 \log(\eta_{\text{bbn}}/10^{-10}) + 0.380 \log \xi, \] (2.25)

where \(Y\) is the mass fraction of \(^{4}\text{He}\), \(Y = \rho_{^{4}\text{He}}/\rho_B\). Using Eqs. (2.21), (2.24) and (2.25) together with observational upperbound \(Y < 0.29\), the following constraint
Cosmological constraint on massive neutrino

on the mass and lifetime was obtained.\(^1\)

\[
\tau_{\nu_h} \lesssim \begin{cases} 
1 \text{ sec} \left( \frac{m_{\nu_h}}{\text{MeV}} \right)^{-2} & (m_{\nu_h} \lesssim 1 \text{ MeV}) \\
1 \text{ sec} \left( \frac{m_{\nu_h}}{\text{MeV}} \right)^4 & (m_{\nu_h} \gtrsim 1 \text{ MeV})
\end{cases} \quad (2.26)
\]

The cosmological constraints obtained in Ref.\(^1\) were so stringent that a large region in the parameter space \((\tau_{\nu_h}, m_{\nu_h})\) was excluded as shown in Fig.4 of Ref.\(^1\). Furthermore, together with the lower limits on the lifetime of the massive neutrino estimated in the Weinberg-Salam theory \((2.6)\) and \((2.9)\), the neutrino in the mass range,

\[
70 \text{ eV} < m_{\nu_h} < 23 \text{ MeV} \quad (2.27)
\]

was found to be excluded.

2.3. Number of Massive Neutrinos

Furthermore, Sato-Kobayashi 1977 discussed the limit on the number of massive neutrinos. As seen in the previous subsection, the masses of neutrinos should be less than 70 eV or more than 23 MeV. So we can assume that \(N_1\) types of neutrinos have masses less than 70 eV and \(N_2\) types of neutrinos have masses larger than 23 MeV.

The constraint on \(N_1\) is obtained by considering the speed-up effect in BBN. In this case, assuming massless \(\nu_e\) and \(\nu_\mu\), Eq.(2.24) is rewritten as

\[
\xi^2 = 1 + 0.196 \ g_s N_1. \quad (2.28)
\]

Using Eq.(225) and \(Y < 0.29\) the constraint is obtained as

\[
N_1 \leq 3 \ g_s^{-1}. \quad (2.29)
\]

As for \(N_2\), suppose that \(\nu_m\) has the longest lifetime among neutrinos with mass larger than 23 MeV. Then the lifetime is bounded as

\[
\tau_{\nu_m} \gtrsim \frac{192\pi^3}{G_F^2} \left( \frac{1}{N_2} \sum_i m_{\nu_i} \left| U_{1i} \right|^2 \right)^{-1} \gtrsim \frac{192\pi^3}{G_F^2 m_{\nu_N}^2 m_e^2} \left( \frac{1}{N_2} \sum_i \left| U_{1i} \right|^2 (m_{\nu_i}/m_e)^2 \right)^{-1}, \quad (2.30)
\]

where \(\nu_{N_2}\) is the neutrino with the largest mass. Using \(\mu-e\) universality Sato-Kobayashi 1977 obtained

\[
\tau_{\nu_m} \gtrsim 2.2 \times 10^6 N_2 (m_{\nu_{N_2}}/\text{MeV})^{-3} \text{ sec}. \quad (2.31)
\]

In Ref.\(^1\), applying the BBN constraint, \(N_2\) was limited as

\[
N_2 \lesssim \left( \frac{m_{\nu_{N_2}}}{23 \text{ MeV}} \right)^7. \quad (2.32)
\]

§3. Progress after 1977

Since Sato and Kobayashi first obtained the constraint on the massive neutrino in 1977, the constraints have been improved according to development of cosmology, laboratory experiments and observations.
3.1. Stable Neutrino

The neutrino, if stable, cannot have a mass larger than about 70 eV from the cosmic density constraint. In other words, the neutrino can be dark matter of the universe if its mass is $O(10)$ eV. In fact, the neutrino with $O(10)$ eV attracted much interest in cosmology in 1980's. In addition, at that time it was reported that a Russian group discovered the electric neutrino mass in the tritium beta decay experiment \((^3\text{H} \rightarrow ^3\text{He} + e^- + \bar{\nu}_e)\) experiment [ which was later denied by other experiments]. However, it was soon found that a large velocity dispersion of the neutrino erases the small scale density fluctuations by free-streaming. So if the neutrino is dark matter, very large structure such as clusters of galaxies are formed first and smaller structure like galaxies are formed later. This “top-down” scenario was studied extensively by N-body simulations and it was concluded that the neutrino cannot be a main component of the dark matter.\(^{11}\)

Here, before discussing the recent cosmological bound on the neutrino mass, we briefly describe the experimental progress towards measurement of neutrino masses. The experimental evidence for massive neutrinos was first obtained by the Super-Kamiokande(SK) experiment\(^{12}\) which measured atmospheric neutrinos and discovered the neutrino oscillation between $\mu$- and $\tau$-neutrinos. The experiment suggests \[ |m_{\nu_\tau}^2 - m_{\nu_\mu}^2| \approx 3 \times 10^{-3} \text{ eV}^2. \] (3.1)

Moreover, the solar neutrino experiments\(^{13,14}\) and the reactor experiment\(^{15}\) also see the oscillation between $e$- and $\mu$- neutrinos and obtained \[ |m_{\nu_e}^2 - m_{\nu_\mu}^2| \approx 7 \times 10^{-5} \text{ eV}^2. \] (3.2)

Notice that the oscillation experiments can measure only the differences of neutrino mass squares and do not give the absolute values. The most severe mass limit is still obtained by the tritium beta decay experiment. The present best limit is\(^{16}\) \[ m_{\nu_e} < 2 \text{ eV}. \] (3.3)

As for the other neutrinos, the present mass bounds are\(^{16}\) \[ m_{\nu_\mu} < 0.19 \text{ MeV} \] (3.4) \[ m_{\nu_\tau} < 18.2 \text{ MeV} \] (3.5)

Let us return to cosmological consideration. Even if the massive neutrino is a minor component of the dark matter, its existence can give a significant effect on the density fluctuations on small scales. Fukugita, Liu and Sugiyama,\(^{17}\) comparing the predicted density fluctuations with that inferred from the cluster abundance, derived the neutrino mass limit as \[ \sum m_{\nu_i} \lesssim 3 \text{ eV}, \] (3.6) where the sum is taken over the three species of neutrinos. The result of the oscillation experiments strongly suggest that the three neutrino masses are degenerate if their masses are larger than 0.1 eV, i.e. $m_{\nu_e} \simeq m_{\nu_\mu} \simeq m_{\nu_\tau}$. 

\[ 8 \]

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The neutrino mass limit was significantly improved by Wilkinson Microwave Anisotropy Probe (WMAP) observation which obtained the full sky map of the CMB temperature and thereby determined the cosmological parameters precisely.\textsuperscript{18} The neutrinos with mass $\gtrsim \mathcal{O}(0.1)$ eV become non-relativistic before or during recombination and they change the epoch of matter-radiation equality, which alters the amplitude of the temperature fluctuations through change of the gravitational potential from radiation dominated era to matter dominated one. The recent WMAP five year data alone gives the upper bound on the neutrino mass as\textsuperscript{19}

$$\sum m_{\nu_i} < 1.7 \text{ eV } \ (95\% \text{CL}),$$

(3.7)

and a more stringent upper bound is obtained if the other observational data (baryon acoustic oscillation and supernovae) are added,\textsuperscript{19}

$$\sum m_{\nu_i} < 0.61 \text{ eV } \ (95\% \text{CL}).$$

(3.8)

3.2. Unstable Neutrino

In the 1977 paper, the cosmological constraints on the radiatively decaying neutrino were obtained by considering effects of the emitted photons on the background radiation and big bang nucleosynthesis. Even today these effects lead to the most stringent constraint on long-lived radiatively decaying particles including massive neutrino.

3.2.1. CMB spectral distortion

As for the effect on the background radiation, the emitted photons (or electron-positron pairs) cause the spectral distortion of CMB as pointed out in Sato-Kobayashi 1977.\textsuperscript{1} If the neutrino decays early enough the photons injected in the decay are fully thermalized by photon number violating processes. In Ref.1, the bremsstrahlung ($e+p \rightarrow e+p+\gamma$) was considered as the main photon number violating process. Afterwards, double Compton scattering ($e+\gamma \rightarrow e+2\gamma$) was recognized to thermalize the background photons more efficiently by Lightman.\textsuperscript{20} If we include the double Compton scattering, it is found that the injected photons are thermalized and the Planck distribution is recovered quickly for $t \lesssim 10^6$ sec.

However, photons produced between decoupling of double Compton scattering ($10^8$ sec) and recombination ($10^{13}$ sec) may cause the spectral distortion in CMB. When the energy exchange between electrons and photons occurs sufficiently frequently by Compton scatterings ($t < 10^8$ sec), the spectrum of CMB photons becomes Bose-Einstein distribution with chemical potential $\mu$. On the other hand, if the Compton scattering is less frequent ($t > 10^9$ sec) the spectrum distortion is described by $y$ parameter.\textsuperscript{@} The COBE satellite launched in 1989 showed that the CMB spectrum is that of a nearly perfect blackbody, from which the distortion parameters $\mu$ and $y$ are stringently constrained as\textsuperscript{21}

$$|\mu| < 9 \times 10^{-5} \quad y < 1.2 \times 10^{-6}. $$

(3.9)
These distortion parameters are related to the energy injection $\Delta \rho_\gamma$ as

\[
\frac{\Delta \rho_\gamma}{\rho_\gamma} \simeq \begin{cases} 
0.714\mu & (10^6 \text{ sec} \lesssim \tau_{\nu h} \lesssim 10^9 \text{ sec}) \\
4y & (10^9 \text{ sec} \lesssim \tau_{\nu h} \lesssim 10^{13} \text{ sec})
\end{cases},
\tag{3-10}
\]

where $\Delta \rho_\gamma/\rho_\gamma$ is given by Eq. (2.15) (a factor $(4/3)$ should be multiplied for decay into an electron-positron pair). Thus we can obtain the following constraint:

\[
\tau_{\nu h} \lesssim \begin{cases} 
10^3 \text{ sec} \left(\frac{m_{\nu h}}{\text{eV}}\right) & (m_{\nu h} \lesssim 1 \text{ keV}) \\
10^6 \text{ sec} & (1 \text{ keV} \lesssim m_{\nu h} \lesssim 30 \text{ MeV}) \\
10^{12} \text{ sec} \left(\frac{m_{\nu h}}{\text{eV}}\right)^4 & (m_{\nu h} \gtrsim 30 \text{ MeV})
\end{cases},
\tag{3-11}
\]

where we have taken account of “time dilation”, i.e. $\tau_{\text{eff}}$(effective lifetime)$\simeq (T_d/m_{\nu h})\tau_{\nu h}$ in the small neutrino mass range.\(^{22), 23)}

### 3.2.2. Big Bang Nucleosynthesis

The BBN constraint discussed in Sato-Kobayashi 1977 was rather rough one and the more precise calculation was done by Miyama and one of the present author (KS),\(^{22)}\) which was later updated in Ref.\(^{24).} In those studies, the abundances of the light elements (D, $^3$He, $^4$He, $^7$Li) were estimated by calculation of the nuclear reaction network, and the result was compared with those inferred from observations. The resultant constraint is approximately given by

\[
\tau_{\nu h} \lesssim \begin{cases} 
3 \times 10^2 \text{ sec} \left(\frac{m_{\nu h}}{\text{MeV}}\right)^{-2} & (m_{\nu h} \lesssim 2 \text{ MeV}) \\
10^2 \text{ sec} & (2 \text{ MeV} \lesssim m_{\nu h} \lesssim 20 \text{ MeV})
\end{cases},
\tag{3-12}
\]

The above constraint is imposed mainly from entropy production due to the radiative decay of the massive neutrino. However, Lindley\(^{25)}\) pointed out that the radiative decay of the neutrino with mass larger than 10 MeV has another serious effect on BBN. High energy photons with $O(10)$ MeV produced in the decay can destroy the light elements which are synthesized in BBN through the following reactions:

\[
\begin{align*}
^4\text{He} + \gamma &\to T + p \quad (19.8 \text{ MeV}), \\
^4\text{He} + \gamma &\to ^3\text{He} + n \quad (20.6 \text{ MeV}), \\
^3\text{He} + \gamma &\to D + p \quad (5.5 \text{ MeV}), \\
D + \gamma &\to n + p \quad (2.2 \text{ MeV}),
\end{align*}
\tag{3-13}
\]

where the numbers in parentheses denote the threshold energies. High energy photons and/or electrons injected into the cosmic plasma induce electro-magnetic show- ers and quickly thermalized, and how fast photons lose their energy crucially depends on whether the photon energy $E_\gamma$ is high enough to scatter off the background photon to produce electron-positron pairs ($\gamma + \gamma_{BG} \to e^- + e^+$). For pair creation to
occur both energies of the background photon and the photon in the shower should be large, which leads to the condition,

\[ E_\gamma \geq E_{\text{th}} \equiv \frac{m_e^2}{2\gamma T}. \]  

(3.17)

Thus, high energy photons with \( E_\gamma > E_{\text{th}} \) can be thermalized by the photon-photon process so quickly that they have little chance to destroy the light elements. On the other hand, photons with lower energy cannot partake in the photon-photon process and are only thermalized by much slower Compton scatterings. Such photons can destroy the light element if their energy is above the thresholds of the photo-dissociation reactions. Using Eqs. (3.13) – (3.17) it is found that when the heavy neutrino decays at \( t \gtrsim 10^4 (10^6) \) sec, the photons in the induced electromagnetic showers can destroy D \(^4\text{He}\). This photo-dissociation effect was studied in details for radiative decay of massive neutrinos in Refs.24), 26), which leads to the following constraint:

\[ \tau_{\nu h} \lesssim 10^4 \text{ sec} \quad (10 \text{ MeV} \lesssim m_{\nu h} \lesssim 1 \text{ GeV}). \]  

(3.18)

3.2.3. Constraint from Supernova 1987A

So far we have reviewed the progress of the cosmological constraints on the radiatively decaying neutrino from the CMB distortion and BBN. Other astrophysical and experimental constraints are also discussed in the literature. Among these constraints, here, we present the one from the supernova 1987A. SN1987A is a historic supernova since the neutrinos emitted its core were observed first in the human history.

As well known, the neutrinos emitted from the supernova carry almost all explosion energy \( \sim 10^{53} \text{ erg} \) which is much larger than is visible \( \sim 10^{47} \text{ erg} \). Thus, if even a small fraction of the neutrinos decay into photons (or electrons) inside the envelop of the progenitor \( \lesssim 3 \times 10^{12} \text{ cm} \), the visible luminosity of the supernova is significantly changed, from which the stringent constraint on the lifetime is obtained as \(^{27}\)

\[
\tau_{\nu h} \gtrsim \begin{cases} 
10^7 \text{ sec} \left(\frac{m_{\nu h}}{\text{MeV}}\right) & \left( m_{\nu h} \lesssim 10 \text{ MeV} \right) \\
4 \times 10^7 \text{ sec} \left(\frac{m_{\nu h}}{\text{MeV}}\right)^{3/2} \exp \left( -\frac{m_{\nu h}}{\text{MeV}} \right) & \left( m_{\nu h} \gtrsim 10 \text{ MeV} \right)
\end{cases}.
\]  

(3.19)

One can see from Eqs. (3.19), (3.5), (3.11), (3.12), (3.18) and (3.19) that the radiative decay of the massive neutrino is almost forbidden.

3.3. Non-radiative Decay

In some models a massive neutrino can decay non-radiatively. For example, in the majoron (familon) model where a global lepton number (family) symmetry is spontaneously broken and associated Nambu-Goldstone boson called a majoron (familon) appears, a neutrino can decay into a lighter neutrino and a majoron (familon). Non-radiative decay was studied in Ref.29), and more recently in Ref.30). From these studies, it is known that non-radiative decay is also stringently constrained by cosmology.
3.4. Number of Neutrinos

The number of neutrino species $N_\nu$ was determined by measuring the decay width of the $Z$ boson, which leads to\(^{16}\)

$$N_\nu = 2.9840 \pm 0.0082. \quad (3.20)$$

Therefore we now know that the only three types of neutrinos exist in nature.

§4. Conclusion

We have seen that the 1977 paper by Sato and Kobayashi\(^1\) is considered as a historic paper which first showed that valuable information on neutrino properties can be obtained from cosmological consideration. The obtained constraints on radiative decay of a massive neutrino were updated by a number of authors and led to the important conclusion that the neutrino radiative decay is almost forbidden. The value of the paper is not limited in neutrino physics. The arguments also apply to more generic particles that decay on cosmological time scales as the subsequent studies showed.

The paper initiated many studies to use cosmology as a laboratory of particle physics or to use particle physics to explore the very early universe. Today this research field is called particle cosmology and has achieved brilliant success. For example, the idea of inflation\(^{31},^{32}\) was born during effort to make monopole in the grand unified theories compatible with cosmology. Another example is interplay between supersymmetry and cosmology. Theories based on supersymmetry (SUSY) are considered as the most promising candidate for models beyond the standard particle physics, and predict many new particle some of which have long lifetimes. One of such long-lived particles is stable and account for the dark matter of the universe,\(^{33}\) while others often cause serious cosmological problems like gravitino problem.\(^{34}\) In any way, it is now the standard procedure to test particle physics model by cosmology.

In concluding this article, we would like to quote the final sentence of Ref.\(^1\),

*It is very interesting that valuable information on particle physics can be derived from cosmological arguments in spite of large uncertainties inherited there.*

*K. Sato and M. Kobayashi (1977)*

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