Expectations on $B \to (K_0^*(1430), K_2^*(1430)) \phi$ decays

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Abstract

As the annihilation contributions play important roles in solving the puzzle of the small longitudinal polarizations in $B \to K^*\phi$ decays, we examine the similar effects in the decays of $B \to K_0^*(1430)\phi$. For the calculations on the annihilated contributions, we adopt that the form factors in $B \to K^*(\phi)$ decays are parameters determined by the observed branching ratios (BRs), polarization fractions (PFs) and relative angles in experiments and we connect the parameters between $B \to K_0^*(1430)\phi$ and $B \to K^*(\phi)$ by the ansatz of correlating $\langle K_0^*(1430)\phi|(V-A)\mu|0\rangle$ to $\langle K^*(\phi)|(V-A)\mu|0\rangle$. We find that the BR of $B_d \to K_0^*(1430)\phi$ is $(3.69 \pm 0.47) \times 10^{-6}$. By using the transition form factors of $B \to K_2^*(1430)$ in the light-front quark model (LFQM) and the 2nd version of Isgur-Scora-Grinstein-Wise (ISGW2), we show that BR of $B_d \to K_2^*(1430)\phi$ is a broad allowed value and $(1.70 \pm 0.80) \times 10^{-6}$, respectively. In terms of the recent BABAR’s observations on BRs and PFs in $B_d \to K_2^*(1430)\phi$, the results in the LFQM are found to be more favorable. In addition, due to the annihilation contributions to $B \to K_2^*\phi$ and $B \to K^*\phi$ being opposite in sign, we demonstrate that the longitudinal polarization of $B_d \to K_2^*(1430)\phi$ is always $O(1)$ with or without including the annihilation contributions.

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I. INTRODUCTION

Since the transverse decay amplitudes of vector meson productions in $B$ decays are associated with their masses, by naive estimations, the longitudinal polarization (LP) of $B$ decaying into two light vector mesons is close to unity. The expectation is confirmed by BELLE \cite{1} and BABAR \cite{3,4} in $B \to \rho(\omega)\rho$ decays, in which the longitudinal parts occupy over 88%. Furthermore, the LPs could be small if the final states include heavy vector mesons. The conjecture is verified in $B \to J/\Psi K^*$ decays \cite{5,6}, in which the longitudinal contribution is only about 60%. However, the rule for the small LPs seems to be violated in $B \to K^*\phi$ decays. From the measurements of BELLE \cite{7} and BABAR \cite{3,8}, it is quite clear that the LPs in $B \to K^*\phi$ are only around 50%. To solve the unexpected observations, many mechanisms have been proposed, such as those with new QCD effects \cite{9,10,11,12} as well as extensions of the standard model (SM) \cite{13,14}.

Recently, the BABAR Collaboration has observed the branching ratios (BRs) and polarization fractions (PFs) for the decays of $B_d \to K_{0,2}^*(1430)\phi$ \cite{15}, given by

$$
\begin{align*}
BR(B_d \to K_{2}^*(1430)\phi) & = (7.8 \pm 1.1 \pm 0.6) \times 10^{-6}, \\
R_L(B_d \to K_{2}^*(1430)) & = 0.853^{+0.061}_{-0.069} \pm 0.036, \\
R_\perp(B_d \to K_{2}^*(1430)) & = 0.045^{+0.049}_{-0.040} \pm 0.013, \\
BR(B_d \to K_{0}^*(1430)\phi) & = (4.6 \pm 0.7 \pm 0.6) \times 10^{-6}.
\end{align*}
$$

By the observations, it seems that the LP of the p-wave tensor-meson production is much larger than those of the s-wave vector mesons in $B$ decays. To find out whether the data are just the statistical fluctuation or the correct tendency for the behavior of the p-wave productions in $B$ decays, it is important to study the phenomena from theoretical viewpoint.

It is known that the annihilation contributions play important roles in the PFs of $B \to K^*\phi$ decays \cite{9,11,12}. As the corresponding time-like form factors are more uncertain than those of the transition form factors, we first adopt that the form factors of the annihilation contributions on $B \to K^*\phi$ are parameters fixed by the data in $B \to K^*\phi$ and then connect them to those in $B_d \to K_{2}^*(1430)\phi$. To have more illustrating examples, we also examine the decays of $B \to K_{0}^*(1430)\phi$ simultaneously.

The paper is organized as follows. In Sec. II, we carry out a general study on the decay amplitudes and hadronic matrix elements. We present our numerical analysis in Sec. III.
Our conclusions are presented in Sec. IV.

II. DECAY AMPLITUDES AND HADRONIC MATRIX ELEMENTS

It is known that the effective interactions for the decays of \( B \to K^{*}(1430)\phi \) (\( n = 0, 2 \)) are described by \( b \to sq\bar{q} \), which are the same as \( B \to K^{(*)}\phi \), given by \[16\]

\[
H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{q=u,c} V_q \left[ C_1(\mu)O_1^{(q)}(\mu) + C_2(\mu)O_2^{(q)}(\mu) + \sum_{i=3}^{10} C_i(\mu)O_i(\mu) \right],
\]

where \( V_q = V_{qs}V_{q\phi} \) are the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements \[17\] and the operators \( O_1-O_{10} \) are defined as

\[
\begin{align*}
O_1^{(q)} &= (\bar{s}_a q_\beta)_{V-A}(\bar{q}_\beta b_\alpha)_{V-A}, & O_2^{(q)} &= (\bar{s}_a q_\alpha)_{V-A}(\bar{q}_\beta b_\beta)_{V-A}, \\
O_3 &= (\bar{s}_a b_\alpha)_{V-A} \sum_q (\bar{q}_\beta q_\beta)_{V-A}, & O_4 &= (\bar{s}_a b_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V-A}, \\
O_5 &= (\bar{s}_a b_\alpha)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V+A}, & O_6 &= (\bar{s}_a b_\beta)_{V-A} \sum_q (\bar{q}_\beta q_\alpha)_{V+A}, \\
O_7 &= \frac{3}{2}(\bar{s}_a b_\alpha)_{V-A} \sum_q e_q(\bar{q}_\beta q_\beta)_{V+A}, & O_8 &= \frac{3}{2}(\bar{s}_a b_\beta)_{V-A} \sum_q e_q(\bar{q}_\beta q_\alpha)_{V+A}, \\
O_9 &= \frac{3}{2}(\bar{s}_a b_\alpha)_{V-A} \sum_q e_q(\bar{q}_\beta q_\alpha)_{V-A}, & O_{10} &= \frac{3}{2}(\bar{s}_a b_\beta)_{V-A} \sum_q e_q(\bar{q}_\beta q_\alpha)_{V-A},
\end{align*}
\]

with \( \alpha \) and \( \beta \) being the color indices and \( C_1-C_{10} \) the corresponding Wilson coefficients (WCs). In Eq. \[2\], \( O_1-O_2 \) are from the tree level of weak interactions, \( O_3-O_6 \) are the so-called gluon penguin operators and \( O_7-O_{10} \) are the electroweak penguin operators. Using the unitarity condition, the CKM matrix elements for the penguin operators \( O_3-O_{10} \) can also be expressed by \( V_u + V_c = -V_t \). Besides the weak effective interactions, to study exclusive two-body decays, we should know how to calculate the transition matrix elements such as \( \langle M_1M_2|H_{\text{eff}}|B \rangle \), where nonperturbative effects dominate the uncertainties. By taking the heavy quark limit, we consider that the productions of light mesons satisfy the assumption of color transparency \[18\], i.e., the final state interactions are subleading effects and negligible. Hence, the decays of \( B \to K^{*}_{a}(1430)\phi \) could be treated as short-distance dominant processes. As the wave functions of p-wave states are quite uncertain, unlike those of s-wave states which are known at least in the leading twist-2 and twist-3 \[19\], in our calculations we will employ the generalized factorization assumption \[20, 21\], in which the factorized parts are
regarded as the leading effects and the nonfactorized effects are lumped and characterized by the effective number of colors, denoted by $N_c^{\text{eff}}$.  

Based on the effective interactions of Eq. (2), the matrix elements $\langle K^*_n(1430)\phi|H_{\text{eff}}|B\rangle$ could be classified by various flavor diagrams displayed in Fig. 1, where (a) and (b) denote the penguin emission topologies, while (c) and (d) are the annihilation topologies for penguin and tree contributions, respectively.  

Furthermore, in terms of the flavor diagrams, we can group the effects of Eq. (2) for the transition matrix elements to be

FIG. 1: Flavor diagrams for $B \to (K^*_0(1430), K^*_2(1430))\phi$ decays, where (a) and (b) denote the penguin emission topologies, while (c) and (d) are the annihilation topologies for penguin and tree contributions, respectively.

$$X^{(BK^*_n\phi)} = \langle \phi|(\bar{s}s)_{V \pm A}|0\rangle \langle K^*_n|(\bar{b}s)_{V-A}|B\rangle,$$

$$Z_1^{(B,K^*_n\phi)} = \langle K^*_n\phi|(\bar{q}s)_{V-A}|0\rangle \langle 0|(\bar{b}q)_{V-A}|B\rangle,$$

$$Z_2^{(B,K^*_n\phi)} = \langle K^*_n\phi|(\bar{q}s)_{S-P}|0\rangle \langle 0|(\bar{b}q)_{S+P}|B\rangle,$$

(4)

where $X^{(BK^*_n\phi)}$ represent the factorized parts of the emission topology and $Z_{1,2}^{(B,K^*_n\phi)}$ stand for the factorized parts of the annihilation topology. Note that the currents associated with $(S+P) \otimes (S-P)$ in Eq. (4) are from the Fierz transformations of $(V - A) \otimes (V + A)$. From
Eqs. (2)-(4), the decay amplitudes for $B \to K^*_n(1430)\phi$ can be written as

$$A(B_d \to K^*_n(1430)\phi) = \frac{G_F}{\sqrt{2}} \left\{ -V_{tb}V_{ts}^* \left[ \tilde{a}^{(s)} X^{(BK^*_n,\phi)} + a_4^{(s)} Z_1^{(B.K^*_n,\phi)} - 2a_6^{(s)} Z_2^{(B.K^*_n,\phi)} \right] \right\},$$

$$A(B_u \to K^*_n(1430)\phi) = \frac{G_F}{\sqrt{2}} \left\{ V_{us} V_{ub}^{*} a_1 Z_1^{(B.K^*_n,\phi)} - V_{tb} V_{ts}^{*} \left[ \tilde{a}^{(s)} X^{(BK^*_n,\phi)} + a_4^{(s)} Z_1^{(B.K^*_n,\phi)} - 2a_6^{(s)} Z_2^{(B.K^*_n,\phi)} \right] \right\},$$

with $\tilde{a}^{(s)} = a_3^{(s)} + a_4^{(s)} + a_5^{(s)}$. To be more convenient for our analysis, we have redefined the useful WCs by combining the gluon and electroweak penguin contributions to be

$$a_1 = C_2^{\text{eff}} + \frac{C_1^{\text{eff}}}{N_c^{\text{eff}}},$$

$$a_2 = C_3^{\text{eff}} + \frac{C_2^{\text{eff}}}{N_c^{\text{eff}}},$$

$$a_3^{(q)} = C_3^{\text{eff}} + \frac{C_4^{\text{eff}}}{N_c^{\text{eff}}} + \frac{3}{2} e_q \left( C_9^{\text{eff}} + \frac{C_{10}^{\text{eff}}}{N_c^{\text{eff}}} \right),$$

$$a_4^{(q)} = C_4^{\text{eff}} + \frac{C_3^{\text{eff}}}{N_c^{\text{eff}}} + \frac{3}{2} e_q \left( C_9^{\text{eff}} + \frac{C_{10}^{\text{eff}}}{N_c^{\text{eff}}} \right),$$

$$a_5^{(q)} = C_5^{\text{eff}} + \frac{C_6^{\text{eff}}}{N_c^{\text{eff}}} + \frac{3}{2} e_q \left( C_7^{\text{eff}} + \frac{C_{8}^{\text{eff}}}{N_c^{\text{eff}}} \right),$$

where the effective WCs of $C_i^{\text{eff}}$ contain vertex corrections for smearing the $\mu$-scale dependences in the transition matrix elements and the effective color number of $N_c^{\text{eff}}$ is a variable, which may not be equal to 3.

The hadronic matrix elements defined in Eq. (1) are the essential quantities that we have to deal with in the two-body exclusive B decays. In the following discussions, we analyze the quantities $X^{BK^*_n,\phi}$, $Z_1^{BK^*_n,\phi}$ and $Z_2^{BK^*_n,\phi}$ individually. Since the degrees of freedom of $K^*_0$ are less than those of $K^*_2$, we start with $B \to K^*_0(1430)\phi$. As usual, we define the various normal hadronic matrix elements as follows:

$$\langle 0|\bar{b}\gamma^\mu\gamma_5 q|B(p_B)\rangle = i f_B p_B^\mu, \quad \langle \phi(q, h)|\bar{s}\gamma^\mu s|0\rangle = m_\phi f_\phi \varepsilon_{\phi}^{\mu*}(h),$$

$$\langle K^*_0(p)|V_\mu - A_\mu|B(p_B)\rangle = i \left[ \left( P_\mu - \frac{m_B^2 - m_{K^*_0}^2}{q^2} q_\mu \right) F_1^{BK^*_0}(q^2) + \frac{m_B^2 - m_{K^*_0}^2}{q^2} q_\mu F_0^{BK^*_0}(q^2) \right],$$

where $(V_\mu, A_\mu) = \bar{b}(\gamma_\mu, \gamma_\mu\gamma_5)s$, $m_{B,\phi,K^*_0}$ are the meson masses of $B$, $\phi$ and $K^*_0$, $P = p_B + p$, $q = p_B - p$ and $P \cdot q = m_B^2 - m_{K^*_0}^2$. Similarly, the time-like form factors for $\langle K^*_0|V_\mu - A_\mu|0\rangle$...
could be defined by

$$\langle K_0^*(p)\phi(q, \varepsilon_\phi) | V_\mu - A_\mu | 0 \rangle = -i \frac{V_{K_0^*}}{m_\phi - m_{K_0^*}} \varepsilon^{\mu\nu\rho\sigma} \varepsilon_{\phi}^{\nu} Q^\rho s^\sigma,$$

$$+ \left\{ 2m_\phi A_0^{K_0^*}(q^2) \frac{\varepsilon_\phi^* \cdot Q}{m_\phi - m_{K_0^*}} + (m_\phi - m_{K_0^*}) A_1^{K_0^*}(q^2) \left( \frac{\varepsilon_{\phi\mu}^* - \frac{\varepsilon_\phi^* \cdot Q}{Q^2 Q_\mu}}{m_\phi - m_{K_0^*}} \right) - A_2^{K_0^*}(q^2) \left( s_\mu - \frac{\varepsilon_\phi^* \cdot Q}{Q^2 Q_\mu} \right) \right\}$$

with $Q_\mu = (p + q)_\mu = p_{B\mu}$ and $s_\mu = (p - q)_\mu$. In terms of form factors in Eqs. (7) and (8), Eq. (4) could be rewritten as

$$X(BK_0^*\phi) = i 2m_\phi f_\phi F_1^{BK_0^*}(m_\phi^2) \varepsilon_\phi^* \cdot p_B,$$

$$Z_1^{(B,K_0^*)} = -i 2m_\phi f_B A_0^{K_0^*}(m_B^2) \varepsilon_\phi^* \cdot p_B,$$

$$Z_2^{(B,K_0^*)} = i 2m_\phi f_B \frac{m_B^2}{(m_b + m_q)(m_s + m_q)} A_0^{K_0^*}(m_B^2) \varepsilon_\phi^* \cdot p_B.$$

To compare with the s-wave states, we also give the hadronic matrix elements in $B \to K\phi$ as

$$X^{(B,K}\phi) = 2m_\phi f_\phi F_1^{BK}(m_\phi^2) \varepsilon_\phi^* \cdot p_B,$$

$$Z_1^{(B,K\phi)} = -2m_\phi f_B A_0^{K\phi}(m_B^2) \varepsilon_\phi^* \cdot p_B,$$

$$Z_2^{(B,K\phi)} = -2m_\phi f_B \frac{m_B^2}{(m_b + m_q)(m_s + m_q)} A_0^{K\phi}(m_B^2) \varepsilon_\phi^* \cdot p_B.$$

We note that except the factor of $-i$ associated with the p-wave states, the definitions of the form factors for $\langle K|V_\mu - A_\mu | B \rangle$ and $\langle K\phi|V_\mu - A_\mu | 0 \rangle$ are similar to those for $\langle K_0^*|V_\mu - A_\mu | B \rangle$ and $\langle K_0^*\phi|V_\mu - A_\mu | 0 \rangle$, respectively. We will discuss the behaviors of $A_0^{K_0^*\phi}$ and $A_0^{K\phi}$ later on.

We now investigate the decays of $B \to K_2^*(1430)\phi$, which are similar to $B \to K^*\phi$. The
analogy of Eq. (4) for $B \to K^*\phi$ can be presented by

$$X^{(BK^*,\phi)} = -im_\phi f_\phi \left\{ (m_B + m_{K^*})A_{1}^{BK^*}(q^2)\varepsilon_\phi^* \cdot \varepsilon_{K^*}^* - \frac{2A_{2}^{BK^*}(q^2)}{m_B + m_{K^*}}\varepsilon_\phi^* \cdot p_B \varepsilon_{K^*}^* \cdot p_B \\
+ i\frac{2V^{BK^*}(q^2)}{m_B + m_{K^*}}\varepsilon_{\mu\nu\rho\sigma}^\mu \varepsilon_{\phi}^\nu \varepsilon_{K^*}^\rho \varepsilon_{\phi}^\sigma \right\},$$

$$Z_1^{(BK^*,\phi)} = -if_B \left\{ m_B^2 V_{1}^{K^*\phi}(Q^2)\varepsilon_\phi^* \cdot \varepsilon_{T}^* - V_{2}^{K^*\phi}(Q^2)\varepsilon_{\phi}^* \cdot Q \varepsilon_{K^*}^* \cdot Q \\
+ i2A_{1}^{K^*\phi}(Q^2)\varepsilon_{\mu\nu\rho\sigma}^\mu \varepsilon_{\phi}^\nu \varepsilon_{K^*}^\rho \varepsilon_{\phi}^\sigma \right\},$$

$$Z_2^{(BK^*,\phi)} = -i \frac{m_B^2 f_B}{m_b + m_q} \left\{ \frac{m_B^2 V_{1}^{K^*\phi}(Q^2)}{m_s - m_q} \varepsilon_\phi^* \cdot \varepsilon_{T}^* - \frac{V_{2}^{K^*\phi}(Q^2)}{m_s - m_q} \varepsilon_{\phi}^* \cdot Q \varepsilon_{K^*}^* \cdot Q \\
+ i2A_{2}^{K^*\phi}(Q^2)\varepsilon_{\mu\nu\rho\sigma}^\mu \varepsilon_{\phi}^\nu \varepsilon_{K^*}^\rho \varepsilon_{\phi}^\sigma \right\},$$

where we have used the form factors in the transition of $B \to K^*$ and $\langle K^*\phi|V_\mu(A_\mu)|0\rangle$, defined by [23]

$$\langle K^*(p, \varepsilon_{K^*})|V_\mu|B(p_B)\rangle = -\frac{V^{BK^*}(q^2)}{m_B + m_{K^*}}\mu_{\varepsilon_{K^*}} P^\mu P^\rho q^\rho,$$

$$\langle K^*(p, \varepsilon_{K^*})|A_\mu|B(p_B)\rangle = i \left\{ 2m_B V_{0}^{BK^*}(q^2)\varepsilon_{K^*}^* \cdot q \varepsilon_{\phi}^* \\
+ (m_B + m_V) A_{1}^{BK^*}(q^2)\left( \varepsilon_{\phi}^* - \frac{\varepsilon_{K^*}^* \cdot q}{q^2} q_\mu \right) \\
- A_{1}^{BK^*}(q^2)\frac{\varepsilon_{K^*}^* \cdot q}{m_B + m_{K^*}} \left( P_\mu - \frac{P \cdot q}{q^2} q_\mu \right) \right\},$$

and [24]

$$\langle K^*\phi|q_\gamma\mu\gamma_5 s Q^\mu|0\rangle = -i A_{2}^{K^*\phi}(q^2)\varepsilon_{\mu\nu\rho\sigma}^\mu \varepsilon_{\phi}^\nu \varepsilon_{K^*}^\rho Q^\sigma s,$$

$$\langle K^*\phi|q_\gamma\gamma_5 s Q^\mu|0\rangle = m_B^2 V_{1}^{K^*\phi}(q^2)\varepsilon_{\phi}^* \cdot \varepsilon_{K^*}^* - V_{2}^{K^*\phi}(q^2)\varepsilon_{\phi}^* \cdot Q \varepsilon_{K^*}^* \cdot Q,$$

respectively. Here, $\varepsilon_{K^*}$ denotes the polarization vector of the $K^*$ meson. To study the production of a tensor meson such as $K_2^*(1430)$ in $B$ decays, we need to introduce the properties of polarization vectors for the tensor meson. It is known that the polarization tensor $\varepsilon^{\mu\nu}$ of a tensor meson satisfies

$$\varepsilon^{\mu\nu}(p, h) = \varepsilon^{\mu\nu}(p, h), \quad \varepsilon^{\mu\nu}(p, h)p_\nu = \varepsilon^{\mu\nu}(p, h)p_\mu = 0, \quad g_{\mu\nu} \varepsilon^{\mu\nu} = 0,$$

where $h$ is the meson helicity. The states of a massive spin-2 particle can be constructed by using two spin-1 states. To analyze PFs in the production of the tensor mesonic $B$ decays,
we explicitly express $\tilde{e}^{\mu\nu}(p, h)$ to be

$$
\tilde{e}^{\mu\nu}(\pm 2) = e^\mu(\pm)e^\nu(\pm),
$$

$$
\tilde{e}^{\mu\nu}(\pm 1) = \frac{1}{\sqrt{2}} [e^\mu(\pm)e^\nu(0) + e^\nu(0)e^\mu(\pm)],
$$

$$
\tilde{e}^{\mu\nu}(0) = \frac{1}{\sqrt{6}} [e^\mu(+e^\nu(-) + e^\mu(-)e^\nu(+)] + \sqrt{\frac{2}{3}} e^\mu(0)e^\nu(0),
$$

where $e^\mu(0, \pm)$ denote the polarization vectors of a massive vector state and their representations are chosen as

$$
e^\mu(0) = \frac{1}{m_T}(|p|, 0, 0, E_T), \quad e^\mu(\pm) = \frac{1}{\sqrt{2}}(0, \mp 1, -i, 0)
$$

with $m_T(|p|)$ being the mass (momentum) of the particle. Since the $B$ meson is a spinless particle, the helicities carried by decaying particles in the two-body $B$ decay should be the same. Moreover, although the tensor meson contains 2 degrees of freedom, only $h = 0$ and $\pm 1$ give the contributions. Hence, it will be useful to redefine the new polarization vector of a tensor meson to be $\varepsilon_{T\mu}(h) \equiv \varepsilon_{\mu\nu}(p, h)p_2^\nu$, where

$$
\varepsilon_{T\mu}(\pm 2) = 0, \quad \varepsilon_{T\mu}(\pm 1) = \frac{1}{\sqrt{2}}e(0) \cdot p_B e^\mu(\pm), \quad \varepsilon_{T\mu}(0) = \sqrt{\frac{2}{3}} e(0) \cdot p_B e^\mu(0),
$$

with $e(0) \cdot p_B = m_B|p|/m_T$. Based on the new polarization vector $\varepsilon_T$, the transition form factors for $B \to K_2^*$ could be defined by

$$
\langle K_2^*(p, \varepsilon_T)|V_\mu|B(p_B)\rangle = h(q^2)\varepsilon_{\mu\rho\sigma}\varepsilon^\mu_{T\rho}P^\sigma q^\rho,
$$

$$
\langle K_2^*(p, \varepsilon_T)|A_\mu|B(p_B)\rangle = -i [k(q^2)\varepsilon^\mu_{T\mu} - \varepsilon^\mu_{T}\gamma^\nu P_{\mu\nu} + h(q^2)\varepsilon_{\mu\rho\sigma}e^\mu_{\phi} q^\rho q^\sigma]
$$

and the time-like form factors for $\langle K_2^*\phi|V_\mu - A_\mu|0\rangle$ could be parametrized as

$$
\langle K_2^*(p, \varepsilon_T)\phi(q, \varepsilon_\phi)|(V_\mu - A_\mu)Q^\mu|0\rangle = -i A_K^2\phi(Q^2)\varepsilon_{\mu\rho\sigma}e^\mu_{\phi} \varepsilon^\rho_{T\phi} Q^\sigma
$$

$$
- V_1 K_2^2\phi(Q^2)\varepsilon^\mu_{T\phi} \varepsilon^\nu_{T\phi} Q^\nu \cdot Q^\nu \cdot Q^\nu \cdot Q.
$$

Consequently, the analogy of Eq. (14) for $B \to K_2^*(1430)\phi$ could be explicitly expressed by

$$
X^{(BK_2^*\phi)} = im_{B}\alpha \{k(q^2)e_0^\phi e^\nu_T - 2b_\mu(q^2)e^\mu_\phi p_B e^\nu_T - p_B + i 2h(q^2)\varepsilon_{\mu\rho\sigma}e^\mu_{T\phi} q^\rho q^\sigma\},
$$

$$
Z_1^{(B, K_2^*\phi)} = if_B \left\{V_1 K_2^2\phi(\varepsilon_0^\phi) \varepsilon^\mu_T - V_2 K_2^2\phi(\varepsilon_0^\phi) \varepsilon^\mu_T \cdot Q^\nu \cdot Q^\nu \cdot Q \right. \varepsilon^\mu_T
$$

$$
+ i 2 A_K^2 \phi (Q^2) \varepsilon_{\mu\rho\sigma}e^\mu_{T\phi} q^\rho q^\sigma\right\},
$$

$$
Z_2^{(B, K_2^*\phi)} = -\frac{m_B}{m_B + m_q} \left\{V_1 K_2^2\phi(\varepsilon_0^\phi) \varepsilon^\mu_T \cdot Q - V_2 K_2^2\phi(\varepsilon_0^\phi) \varepsilon^\mu_T \cdot Q^\nu \cdot Q^\nu \cdot Q
$$

$$
+ i 2 A_K^2 \phi (Q^2) \varepsilon_{\mu\rho\sigma}e^\mu_{T\phi} q^\rho q^\sigma\right\}.
$$
Since the final states in the decays $B \to VV$ and $B \to TV$ carry spin degrees of freedom, the decay amplitudes in terms of helicities can be generally described by

$$\mathcal{M}^{(h)} = \varepsilon^*_{1\mu}(h)\varepsilon^*_{2\nu}(h) \left[ a g^{\mu\nu} + \frac{b}{m_1 m_2} p_B^{\mu} p_B^{\nu} + i \frac{c}{m_1 m_2} \epsilon^{\mu\nu\alpha\beta} p_1 p_2 p_3 \right],$$

which can be decomposed in terms of

$$H_{00} = - \left[ ax + b \left( x^2 - 1 \right) \right],$$

$$H_{\pm\pm} = a \pm c\sqrt{x^2 - 1},$$

(21)

and

$$H_{00} = - \sqrt{\frac{2}{3}} e(0) \cdot p_B \left[ ax + b \left( x^2 - 1 \right) \right],$$

$$H_{\pm\pm} = \frac{1}{\sqrt{2}} e(0) \cdot p_B \left( a \pm c\sqrt{x^2 - 1} \right)$$

(22)

for $B \to K^*\phi$ and $B \to K_2^*\phi$, respectively, with $x = (m_B^2 - m_1^2 - m_2^2)/(2m_1 m_2)$. In addition, we can also write the amplitudes in terms of polarizations as

$$A_L = H_{00}, \quad A_{\parallel(\bot)} = \frac{1}{\sqrt{2}}(H_{++} \pm H_{--}).$$

(23)

As a result, the BRs are given by

$$\text{BR}(B \to M\phi) = \frac{|\vec{p}|}{8\pi m_B^2} \left( |A_L|^2 + |A_{\parallel}|^2 + |A_{\bot}|^2 \right)$$

(24)

where $M = (K^*, K_2^*(1430))$ and $|\vec{p}|$ is the magnitude of the outgoing momentum, and the corresponding PFs can be defined to be

$$R_i = \frac{|A_i|^2}{|A_L|^2 + |A_{\parallel}|^2 + |A_{\bot}|^2}, \quad (i = L, \parallel, \bot),$$

(25)

representing longitudinal, transverse parallel and transverse perpendicular components, respectively. Note that $\sum_i R_i = 1$.

### III. NUMERICAL ANALYSIS

In Tables I and II, we display the meson decay constants and the transition form factors, respectively. Since the numerical values of $B \to K_2^*$ in the LFQM are different from those in the 2nd version of the Isgur-Scora-Grinstein-Wise approach (ISGW2) \([27, 28]\), in the Table II...
TABLE I: Meson decay constants (in units of GeV) of meson.

| $f_K$ | $f_B$ | $f_{K^*}$ | $f_{K^{*+}}$ | $f_\phi$ | $f_{T_\phi}$ |
|-------|-------|-----------|-------------|----------|-------------|
| 0.16  | 0.19  | 0.20      | 0.16        | 0.23     | 0.20        |

TABLE II: Transition form factors by the LFQM and ISGW2.

| $F(m_\phi^2)$ | $F^{BK}_{1}$ | $V^{BK^{*}}_{1}$ | $A^{BK^{*}}_{1}$ | $A^{BK^{*}}_{2}$ | $F^{BK^{*}}_{0}$ | $k$ | $b_+$ | $h$ |
|---------------|--------------|------------------|-----------------|-----------------|----------------|-----|-------|-----|
| LFQM          | 0.37         | 0.33             | 0.27            | 0.26            | 0.275          | 0.013| 0.0065| 0.0087|
| ISGW2         |              |                  |                 |                 |                | 0.217| 0.0045| 0.0045|

we list both results. In Table III we show the results without the annihilated topologies with various effective $N_{c\text{eff}}$, where the $\mu$ scale for the effective Wilson’s coefficients is fixed to be $\mu = 2.5$ GeV which is usually adopted in the literature. For an explicit example of the effective WCs, we have that $\tilde{a}^{(0)}(\mu = 2.5\text{ GeV}) = (-584 - 97i, -418 - 73i, -284 - 55i, 84 - 27i) \times 10^{-4}$ for $N_{c\text{eff}} = 2, 3, 5, \infty$, respectively. In our naive estimations, we see that the BRs for $B_d \rightarrow (K^{(*)0}, K_n^{*0}(1430))\phi$ decays are close to the world average values when $N_{c\text{eff}} = 3$. This could indicate that the nonfactorized contributions in the processes are small. However, from Table III the longitudinal and transverse PFs for $B_d \rightarrow K^{*0}\phi$ are inconsistent with the measurements. In addition, we find that $R_{||} = 1 - R_L - R_{\perp}$ of $B_d \rightarrow K_2^{*0}(1430)\phi$ in the LFQM almost vanishes due to $k(m_\phi^2)_{\text{LFQM}} << k(m_\phi^2)_{\text{ISGW2}}$.

It has been concluded that the annihilation contributions could significantly reduce the longitudinal polarizations in $B \rightarrow K^{*}\phi$ decays [9, 11, 12]. It is interesting to ask whether such effects could also play important roles on BRs and PFs in $B \rightarrow K_n^{*}(1430)\phi$. To answer the question, we start with the annihilation contributions in $B \rightarrow PP$, which could provide useful information on the general properties of the time-like form factors. In the decays, the factorized amplitude associated with the $(V - A) \otimes (V - A)$ interaction for the annihilated topology is given by [29]

$$\langle P_1P_2|\bar{q}_1\gamma^\mu(1 - \gamma_5)q_2 \bar{q}_3\gamma^\mu(1 - \gamma_5)b|B\rangle_a = -if_B(m_1^2 - m_2^2)F^{P_1P_2}_0(m_B^2),$$

where $m_{1,2}$ are the masses of outgoing particles and $F^{P_1P_2}_0(m_B^2)$ correspond to the time-like form factor, defined by

$$\langle P_1(p_1)P_2(p_2)|\bar{q}_1\gamma_\mu q_2|0\rangle = \left[q_\mu - \frac{m_1^2 - m_2^2}{Q^2}Q_\mu\right]F^{P_1P_2}_1(Q^2) + \frac{m_1^2 - m_2^2}{Q^2}Q_\mu F^{P_1P_2}_0(Q^2)$$
TABLE III: BRs (in units of $10^{-6}$) and PFs without annihilation contributions in the LFQM [ISGW2].

| Mode                  | (BR, PF) | $N_c^{\text{eff}} = 2$ | $N_c^{\text{eff}} = 3$ | $N_c^{\text{eff}} = 5$ | $N_c^{\text{eff}} = \infty$ | Exp.          |
|-----------------------|----------|-------------------------|-------------------------|-------------------------|-------------------------|--------------|
| $B_d \to K^{0}\phi$  | BR       | 15.77                   | 8.10                    | 3.77                    | 0.35                    | 8.3$^{+1.2}_{-1.0}$ |
| $B_d \to K^{*0}\phi$ | BR       | 7.70                    | 3.95                    | 1.84                    | 0.17                    | 4.6 $\pm$ 0.7 $\pm$ 0.6 |
| $B_d \to K^{*0}\phi$ | $R_L$    | 0.90                    | 0.90                    | 0.90                    | 0.90                    | 0.49 $\pm$ 0.04 |
|                      | $R_{\perp}$ | 0.04                    | 0.04                    | 0.04                    | 0.04                    | 0.27$^{+0.04}_{-0.03}$ |
| $B_d \to K^{*0}\phi$ | BR       | 13.64[7.77]             | 7.0 [3.99]              | 3.26 [1.86]             | 0.30[0.17]              | 7.8 $\pm$ 1.1 $\pm$ 0.6 |
|                      | $R_L$    | 0.96 [0.90]             | 0.96 [0.90]             | 0.96 [0.90]             | 0.96 [0.90]             | 0.853$^{+0.061}_{-0.069}$ $\pm$ 0.036 |
|                      | $R_{\perp}$ | 0.04 [0.02]             | 0.04 [0.02]             | 0.04 [0.02]             | 0.04 [0.02]             | 0.045$^{+0.049}_{-0.040}$ $\pm$ 0.013 |

with $q = p_1 - p_2$ and $Q = p_1 + p_2$. From Eq. (26), it is clear that if $m_1 = m_2$, the factorized effects of the annihilation topology vanish. Consequently, it is concluded that the annihilated effects associated with $(V - A) \otimes (V - A)$ are suppressed and negligible. The conclusion could be extended to any process in two-body $B$ decays [29]. Hence, in the following discussions we will neglect the contributions of $Z^{B,M\phi}_1 (M = K^{(*)}, K^{*}_n)$. However, if the associated interactions are $(S + P) \otimes (S - P)$, by equation of motion, the decay amplitude becomes

$$\langle P_1 P_2 | \bar{q}_1 (1 + \gamma_5) q_2 \bar{b}(1 - \gamma_5) q_3 | \bar{B} \rangle_a = -i f_B \frac{(m_1^2 - m_2^2) m_B^2}{(m_{q_1} - m_{q_2})(m_b + m_{q_3})} F_0^{P_1 P_2} (m_B^2).$$  \hspace{1cm} (28)$$

We see that the subtracted factors appear in the numerator and denominator simultaneously. As a result, the annihilation effects by $(S + P) \otimes (S - P)$ interactions can be sizable due to the cancelation smeared by $(m_1^2 - m_2^2)/(m_{q_1} - m_{q_2}) \propto (m_1 + m_2)$. In comparing with the emission topologies, now the annihilations associated with $(S - P) \otimes (S + P)$ are only suppressed by the factor of $m_{1,2}/m_B$.

Due to the tree contributions arising from the annihilation, as analyzed before, their effects could be neglected. Moreover, except the lifetime, there are no differences in the decay amplitudes between charged and neutral B mesons. Thus, in our numerical estimations, we just concentrate on the neutral modes. According to Eqs. (5), (9) and (10), the decay
amplitudes for $B_d \to K_0^{*0}(1430)\phi$ and $B_d \to K^0\phi$ are given by

$$A(B_d \to K_0^{*0}(1430)\phi) = \frac{G_F}{\sqrt{2}} m_\phi f_\phi F^{B K_0^{*0}}(m_\phi^2) \varepsilon_\phi^* \cdot p_B \left[ 1 - 2r_a \frac{A_0^{K_0^{*0}\phi}(m_B^2)}{F_1^{B K_0^{*0}}(m_\phi^2)} \right],$$

$$A(B_d \to K^0\phi) = \frac{G_F}{\sqrt{2}} m_\phi f_\phi F^{B K}(m_\phi^2) \varepsilon_\phi^* \cdot p_B \left[ 1 + 2r_a \frac{A_0^{K^0\phi}(m_B^2)}{F_1^{B K}(m_\phi^2)} \right],$$

(29)

respectively, where $r_a = (a_6^{(s)}/\bar{a}^{(s)})m_B^2 f_B/(m_b m_s f_\phi)$. Similarly, in terms of the helicity basis and Eq. (21), the corresponding amplitudes for $a$, $b$ and $c$ are given by

$$a^{K^*\phi} = \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \bar{a}^{(s)} m_\phi f_\phi (m_B + m_{K^*}) A_{1}^{B K^*}(m_\phi^2) \left[ 1 + 2R_a \frac{m_B V_1^{K^*\phi}(m_B^2)}{(m_B + m_{K^*}) A_{1}^{B K^*}(m_\phi^2)} \right],$$

$$b^{K^*\phi} = - \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \bar{a}^{(s)} m_\phi^2 m_{K^*} f_\phi \left[ 1 + R_a \frac{(m_B + m_{K^*}) V_2^{K^*\phi}(m_B^2)}{m_B A_{1}^{B K^*}(m_\phi^2)} \right],$$

$$c^{K^*\phi} = \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \bar{a}^{(s)} m_\phi^2 m_{K^*} f_\phi \left[ 1 + 2R_a \frac{m_B V_3^{K^*\phi}(m_B^2)}{m_B A_{1}^{B K^*}(m_\phi^2)} \right],$$

(30)

for $B_d \to K^*\phi$ and

$$a^{K_2^*\phi} = \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \bar{a}^{(s)} m_\phi f_\phi k(m_\phi^2) \left[ 1 + 2R_a \frac{V_1^{K_2^*\phi}(m_B^2)}{m_B k(m_\phi^2)} \right],$$

$$b^{K_2^*\phi} = - \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \bar{a}^{(s)} 2m_\phi^2 m_{K_2^*} f_\phi b_+(m_\phi^2) \left[ 1 + R_a \frac{V_2^{K_2^*\phi}(m_B^2)}{m_B b_+(m_\phi^2)} \right],$$

$$c^{K_2^*\phi} = \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \bar{a}^{(s)} 2m_\phi^2 m_{K_2^*} f_\phi h(m_\phi^2) \left[ 1 + 2R_a \frac{V_3^{K_2^*\phi}(m_B^2)}{m_B h(m_\phi^2)} \right],$$

(31)

for $B_d \to K_2^*(1430)\phi$, respectively, where

$$R_a = \frac{a_6^{(s)}/\bar{a}^{(s)} m_B^2 f_B}{m_\phi m_b m_s f_\phi}.$$  

(32)

We note that although the formulas for $K^*\phi$ and $K_2^*(1430)\phi$ are the same, the signs for the time-like form factors could be different.

To calculate exclusive B decays, we face the theoretical uncertainties, such as those from CKM matrix elements, decay constants and transition form factors. However, these uncertainties could be fixed by the experimental data such as those in the semileptonic decays. In our concerned processes, in fact, the challenge one is how to get the proper information on the time-like form factors for the annihilation contributions. Since the values of the time-like form factors are taken at $q^2 = m_B^2$, in principle, we can employ the perturbative QCD
(PQCD) to do the calculations. Unfortunately, it is known that the predictions of the PQCD on $R_L$ of $B \to K^* \phi$ are much larger than the measured values in the data. As there exist no better methods to evaluate the time-like form factors at the moment, in our approach we regard them in $B \to K^{(*)} \phi$ as free parameters and we determine their allowed ranges by utilizing the experimental data, such as BRs, $R_L$ and $R_\perp$ for $B \to K^{(*)} \phi$.

However, in general, since the time-like form factors are complex, while there are only four observables can be used so far, we have to reduce the free parameters. It is known that since the weak phase is very small in $b \to sq \bar{q}$ processes, CP asymmetries in the SM should be negligible. On the other hand, as we will not consider CP violation in the processes, the encountering problems could be simplified by setting the time-like form factors to be real. In addition, our simplification is supported by the analysis of Ref. in which at the lowest order in $\alpha_s$ the annihilation amplitudes are real. Once the parameters in $B \to K^* \phi$ are fixed, we can adopt the ansatz for the corresponding quantities in $B \to K_n^*(1430) \phi$ to be

$$
\frac{|A_{K^\phi}(m_B^2)|}{F_{1B}^{BK}(m_\phi^2)} \approx \frac{|A_{K^{*\phi}}(m_B^2)|}{F_1^{BK\phi}(m_\phi^2)},
$$

$$
\frac{|V_1^{K^\phi}(m_B^2)|}{(m_B + m_{K^{*}})A_{1B}^{BK}(m_\phi^2)} \approx \frac{|V_1^{K^{*\phi}}(m_B^2)|}{k(m_\phi^2)},
$$

$$
\frac{|V_2^{K^\phi}(m_B^2)|}{A_{2B}^{BK}(m_\phi^2)} \approx \frac{|V_2^{K^{*\phi}}(m_B^2)|}{b_+(m_\phi^2)},
$$

$$
\frac{|V_3^{K^\phi}(m_B^2)|}{V_{BK}(m_\phi^2)} \approx \frac{|V_3^{K^{*\phi}}(m_B^2)|}{h(m_\phi^2)},
$$

(33)

where we regard that the ratios on both sides have removed the detailed characters of the different decay modes. From Eq. 26, we know that $Z_1^{(B,P_1P_2)} = -i f_B (m_1^2 - m_2^2) F_0^{P_1P_2}(m_B^2)$. The explicit suppression factor $m_1^2 - m_2^2$ reflects the effects similar to the chirality flipping on $\pi^- \to \mu \bar{\nu}_\mu$. Since the dependence should be universal, although the definitions of the time-like form factors for $\langle K^{(*)} \phi | \bar{s}(V - A)_\mu | 0 \rangle$ and $\langle K^{*\phi} \phi | \bar{s}(V - A)_\mu | 0 \rangle$ do not display it explicitly, for our further numerical analysis, we reparametrize the form factors to have such behavior. In addition, to remove the ambiguity in the sign $m_1^2 - m_2^2$, we set $m_1 = m_\phi$ and $m_2 = (m_{K^{(*)}}, m_{K_n^{(*)}})$. Obviously, due to $m_{K_n^{(*)}} > m_\phi > m_{K^{(*)}}$, the time-like form factors for $K_{0(2)}^{*\phi}$ and $K^{(*)}\phi$, are opposite in sign. Based on the above consideration, we could
are in the same power. In addition, by Eq. (37) we can obtain further information on
in our above ansatz, the independent unknown parameters are only \( \tilde{A}^{K*} \phi \) where the quantities with tildes at the top denote the new unknown parameters. However,

\[ \frac{m_B^2 (m^2 - m_K^2)}{(m_B + m_{K*}) A_{BK*} (m^2)} \approx \frac{m_B^2 (m^2 - m_{K*}^2)}{k(m^2)} \],

\[ (m_B + m_{K*}) \frac{m^2 - m_{K*}^2}{A_{BK*} (m^2)} \approx \frac{(m_B^2 - m_{K*}^2)}{b_+(m^2)} \],

\[ (m_B + m_{K*}) \frac{m^2 - m_{K*}^2}{V_{BK*} (m^2)} \approx \frac{(m_B^2 - m_{K*}^2)}{h(m^2)} \],

where the quantities with tildes at the top denote the new unknown parameters. However, in our above ansatz, the independent unknown parameters are only \( \tilde{A}^{K*} \phi \) and \( \tilde{V}_{1,2}^{K*} \phi \).

Before performing the detailed numerical calculations, it is worth to show the behaviors of the polarized amplitudes and their relationships with the annihilation effects in more concise expressions. In terms of the large energy effective theory (LEET), we may simplify the \( B \to K^* \) form factors to be

\[ V(q^2) = (1 + r_{K*}) \xi_\perp, \quad A_1(q^2) = \frac{2E}{m_B + m_{K*}} \xi_\perp, \]

\[ A_2(q^2) = (1 + r_{K*}) \left\{ \xi_\perp - \frac{m_{K*}^2}{E} \xi_\parallel \right\}, \]

\[ A_0(q^2) = (1 - \frac{m_{K*}^2}{m_B E}) \xi_\parallel + r_{K*} \xi_\perp, \]

where \( E \) is the energy of the \( K^* \) meson, \( \xi_\parallel(\perp) \) denotes the parallel (perpendicular) transverse form factor for \( B \to K^* \) and \( r_{K*} = m_{K*}/m_B \). From Eqs. (23) and (30), the polarized amplitudes for \( B_d \to K^{*0} \phi \) are given by

\[ A_L = C_A \left[ \xi_\parallel + \frac{m_B}{2m_{K*}} R_a \left( 2V_1^{K^*} \phi (m_B^2) - V_2^{K^*} \phi (m_B^2) \right) \right], \]

\[ A_\parallel = \sqrt{2} C_A r_\phi \left[ \xi_\parallel + 2 R_a V_1^{K^*} \phi (m_B^2) \right], \]

\[ A_\perp = \sqrt{2} C_A r_\phi \left[ \xi_\perp + 2 R_a A_1^{K^*} \phi (m_B^2) \right] \]

with \( C_A = G_F V_{tb} V_{ts}^{*} \tilde{a}^{(s)} m_B^2 f_\phi / \sqrt{2} \) and \( r_\phi = m_\phi / m_B \). From the above equations, it is clear that \( A_L : A_\parallel : A_\perp \approx 1 : r_\phi : r_\phi \). Hence, the transverse polarizations have a power suppression in \( r_\phi^2 \). According to our previous analysis, we conclude that the annihilated effects and \( A_\parallel(\perp) \) are in the same power. In addition, by Eq. (37) we can obtain further information on
2V_1^{K*\phi}(m_B^2) - V_2^{K*\phi}(m_B^2). That is, besides the properties displayed in Eq. (35), we expect that $2V_1^{K*\phi}(m_B^2) - V_2^{K*\phi}(m_B^2) = C_\chi(m^2_{K^*}/m_B^2)(m^2_\phi - m^2_{K^*})/m_B^2$ where $C_\chi$ is an unknown parameter, which leads to

$$|A_L|^2 \propto |\xi||^2 \left(1 + C_\chi R_a \frac{m_{K^*} m^2_\phi - m^2_{K^*}}{m_B^2 |\xi|}ight).$$

(38)

Clearly, the annihilation effects on $|A_L|^2$ are associated with $m_K/m_B$, while those on $|A_{||}(\perp)|^2$ are $m^2_\phi/m^2_B$. By this analysis, we speculate that if the annihilation topology in $A_L$ is destructive interference with the emission one, the puzzle on the small value of $R_L(B \rightarrow K^{*\phi})$ could be solved. Similar conclusions could be applied to the decays of $B \rightarrow K^*_2(1430)\phi$. On the contrary, if the interference in $B \rightarrow K^*_2\phi$ is constructive, we will get a large value of $R_L(B \rightarrow K^*_2\phi)$.

We now proceed our numerical analysis. Since the characters of $B_d \rightarrow (K^0, K^{*0}(1430))\phi$ and $B_d \rightarrow (K^{*0}, K^{*0}(1430))\phi$ are quite different, we discuss them separately. First, we analyze the decays of $B_d \rightarrow (K^0, K^{*0}(1430))\phi$. For numerical estimations, besides the input values displayed in Tables I and II to fix the unknown parameter $\tilde{A}^{K\phi}$, we take the world average value with $1\sigma$ error for $BR(B_d \rightarrow K^0\phi)$, i.e.

$$7.3 \leq BR(B_d \rightarrow K^0\phi)10^6 \leq 9.5.$$  

(39)

By using Eqs. (29) and (31), the correlation in BRs between $B_d \rightarrow K^{*0}(1430)\phi$ and $B_d \rightarrow K^0\phi$ decays is presented in Fig. 2 where (a) and (b) denote the cases of $N^{\text{eff}}_c = 2$ and 3, respectively. From the figure, we see clearly that the results are similar in both cases and they are consistent with the observations by BABAR. In addition, due to the signs of the annihilation contributions being opposite in $K^0\phi$ and $K^{*0}(1430)\phi$ modes, from the figure we also see that the BRs of $B_d \rightarrow K^0\phi$ and $B_d \rightarrow K^{*0}(1430)\phi$ increase simultaneously.

Next, we consider the decays of $B_d \rightarrow (K^{*0}, K^{*0}(1430))\phi$. Similar to $B_d \rightarrow K^0\phi$, to determine $\tilde{A}^{K*\phi}$ and $\tilde{V}_{1,2}^{K*\phi}$, we use the world average values with $2\sigma$ errors for BR and PFs and $1\sigma$ errors for relative angles as

$$7.7 \leq BR(B_d \rightarrow K^{*0}\phi)10^6 \leq 11.3,$$

$$0.45 \leq R_L(B_d \rightarrow K^{*0}\phi) \leq 0.53,$$

$$0.21 \leq R_{||}(B_d \rightarrow K^{*0}\phi) \leq 0.35,$$

$$2.25 \leq \phi_{||}(\text{rad}) \leq 2.59,$$

$$2.35 \leq \phi_{\perp}(\text{rad}) \leq 2.69,$$

(40)

where the angles are defined by $\phi_{||(\perp)} = Arg(A_{||(\perp)}/A_L)$. In terms of the constraints in Eq. (40) and Eqs. (21), (25), (30), (31) and Eq. (35), we present the results with the form
FIG. 2: Correlations between $BR(B_d \rightarrow K^0\phi)$ and $BR(B_d \rightarrow K^{*0}(1430)\phi)$ at $\mu = 2.5$ GeV with (a) $N_{\text{eff}}^c = 2$ and (b) $N_{\text{eff}}^c = 3$.

factors in the LFQM (ISGW2) and $N_{\text{eff}}^c = 3$ in Fig. 3-4. In both figures, the plots (a) [(b)] denote the correlations between $BR(B_d \rightarrow K^{*0}\phi)$ and $R_L(B_d \rightarrow K^{*0}\phi)[R_\perp(B_d \rightarrow K^{*0}\phi)]$. We note our results, which are consistent with data, indicate that the annihilated effects are important for the PFs. The plots (c)[(d)] display the correlations between $BR(B_d \rightarrow K_2^{*0}(1430)\phi)$ and $BR(B_d \rightarrow K^{*0}\phi)[R_L(B_d \rightarrow K^{*0}\phi)]$. From Figs. 3c and 4c, we see that $BR(B_d \rightarrow K_2^{*0}(1430)\phi)$ by the LFQM is much larger than that in the ISGW2. Moreover, the former is more favorable to the BABAR’s observation. From Figs. 3d and 4d, although $R_L$ in the ISGW2 could be lower, with the BR observed by BABAR, both QCD approaches predict a large longitudinal polarization in $B_d \rightarrow K_2^{*0}(1430)\phi$. According to our analysis, we conclude that $R_L(B \rightarrow K_2^*(1430)\phi)$ is $O(1)$ with or without including the annihilation contributions. Finally, we note that the results of $N_{\text{eff}}^c = 2$ are similar to those of $N_{\text{eff}}^c = 3$.

IV. CONCLUSIONS

We have studied the BRs for $B_d \rightarrow (K^{(*)0}, K_2^{*0}(1430))\phi$ decays and the PFs for $B_d \rightarrow (K^{*0}, K_2^{*0}(1430))\phi$ with and without the annihilation contributions. Since the QCD calculations on the time-like form factors are not as good as the transition form factors, we regard them as parameters fixed by the experimental measurements in $B_d \rightarrow K^{(*)0}\phi$. In terms of the ansatz by correlating the time-like form factors of $\langle K_2^{*0}(1430)\phi\rangle(V - A)_{\mu}|0\rangle$ to
FIG. 3: Correlations between (a) $BR(B_d \to K^*\phi)$ and $R_L(B_d \to K^*\phi)$, (b) $BR(B_d \to K^*\phi)$ and $R_L(B_d \to K^*\phi)$, (c) $BR(B_d \to K^*\phi)$ and $R_L(B_d \to K^*\phi)$, (d) $R_L$ and $BR(B_d \to K^*\phi)$ in the LFQM.

FIG. 4: Legend is the same as Fig. 3 but in the ISGW2.
those of $\langle K^*(\phi)|(V-A)\mu|0\rangle$, we find that $BR(B_d \to K_0^{*0}(1430)\phi)$ is $(3.86 \pm 0.73) \times 10^{-6}$ and because of the annihilation contributions to $B \to K^*_2\phi$ and $B \to K^*\phi$ being opposite in sign, the longitudinal polarization of $B_d \to K_2^{*0}(1430)\phi$ is always $O(1)$ unlike those in $B \to K^*\phi$ decays. Due to the large differences in the transition form factors of $B \to K_2^*$ between the LFQM and ISGW2, we have shown that the former gives a broad allowed $BR(B_d \to K_2^{*0}(1430)\phi)$ and the latter limits it to be $(1.69 \pm 0.81) \times 10^{-6}$. In terms of the recent BABAR’s observations on BRs and PFs of $B_d \to K_2^{*0}(1430)\phi$, the results based on the LFQM are more favorable.

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