Polarization of the final nucleon in quasi-elastic neutrino scattering and the axial form factor of the nucleon

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Abstract

Measurements of the polarization of the final proton in elastic $e-p$ scattering drastically changed our knowledge about the electromagnetic form factors of the proton. Here we present our results of the calculation of the polarization of the final nucleon in charged current quasi-elastic neutrino nucleon scattering. Relations which connect the axial form factor with the polarization, the cross section and the electromagnetic form factors of the nucleon are derived. Measurements of the polarization of the nucleon in the high-statistics short baseline neutrino experiments (or in near detectors of long baseline experiments) could provide important information on the axial form factor of the nucleon.

1 Introduction

Weak and electromagnetic nucleon form factors are an important source of information about the structure of the nucleon. Their study is one of the central issues in high energy physics.

The electromagnetic form factors are determined via investigation of elastic scattering of electrons or muons on proton, deuterium and other nuclei (see, for example, reviews [1, 2]).

Starting from the famous Hofstadter experiments in the 50’s and up to the middle of the 90’s, information about the electromagnetic form factors of the proton and the neutron was obtained from measurements of the differential cross section of unpolarized electrons on unpolarized nucleons. The electric
$G_E(Q^2)$ and magnetic $G_M(Q^2)$ form factors of the nucleon were extracted from these data by the Rosenbluth procedure based on one-photon exchange approximation. Until the recoil polarization measurements, from compilation of the data it was found:

1. The proton form factors satisfy the approximate scaling relation:

$$R(Q^2) = \frac{\mu_p G_E^p(Q^2)}{G_M^p(Q^2)} \approx 1,$$

where $\mu_p$ is the total magnetic moment of the proton (in nuclear Bohr magnetons), $Q^2$ is the squared four-momentum transfer.

2. At relatively small $Q^2$ ($Q^2 \leq 6 \text{ GeV}^2$) the $Q^2$-dependence of the proton form factors and the magnetic form factor of the neutron are described by the dipole formula

$$G_E^p(Q^2) \approx \mu_p G_D(Q^2), \quad G_M^p(Q^2) \approx \mu_n G_D(Q^2)$$

(2)

Here $\mu_n$ is the magnetic moment of the neutron and

$$G_D(Q^2) = \frac{1}{(1 + \frac{Q^2}{M_D^2})^2},$$

where $M_D^2 = 0.71 \text{ GeV}^2$.

In the late 90’s series of experiments on measurement of the polarization of the recoil protons in elastic scattering of longitudinally polarized electrons on unpolarized protons started.

In ref. [3] it was shown that measurement of polarization effects in elastic $e-p$ scattering provides a sensitive way for determination of the electric form factor of the proton. For the ratio of the transverse $P_\perp$ and longitudinal $P_\parallel$ polarizations of the proton it was found [3, 4]:

$$\frac{P_\perp}{P_\parallel} = - \frac{G_E^p}{G_M^p} \sqrt{\frac{2\varepsilon}{\tau(1 + \varepsilon)}},$$

(3)

where $\tau = Q^2/4M^2$ ($M$ is the nucleon mass) and $\varepsilon = [1 + 2(1 + \tau)\tan^2\theta/2]^{-1}$ ($\theta$ is the scattering angle).
Thus, measurement of the ratio $P_\perp/P_\parallel$ allows to determine the ratio of the electric and magnetic form factors in a direct model independent way.

Such measurements were done in experiments performed in the Jefferson Lab: in experiments [5] in the $Q^2$ range from 0.5 to 5.6 $GeV^2$, in the experiment [6] the $Q^2$ range was extended up to $Q^2 \simeq 8.5 GeV^2$. It was established that eq.(1) does not hold and the ratio $R$ of the electric and magnetic form factors of the proton is not a constant but decreases linearly with $Q^2$ starting from $R \simeq 1$ at $Q^2 \simeq 1 GeV^2$ and falling down to $R = 0.28 \pm 0.09$ at $Q^2 = 5.6 GeV^2$.

These observations significantly changed the theoretical models for the structure of the nucleon.

Direct information about the axial form factor of the nucleon, which characterizes the one-nucleon matrix element of the charged weak current, can be obtained from measurement of the cross sections of the charged current quasi-elastic (CCQE) neutrino processes:

$$\nu_\mu + n \rightarrow \mu^- + p$$

and

$$\bar{\nu}_\mu + p \rightarrow \mu^+ + n,$$

which are the dominant neutrino processes at relatively small neutrino energies ($E \leq 1 GeV$).

Starting with the earlier bubble chamber experiments many experiments on the measurements of the cross sections of these processes in a wide range of $Q^2$ have been done. However, the results of these experiments are not compatible with each other: from analysis of recent lower $Q^2$-data significantly larger values of the parameter $M_A$, which characterizes the $Q^2$-dependence of the axial form factor, have been obtained. Among different reasons for such a disagreement nuclear effects are actively discussed.

The axial form factor of the nucleon is of fundamental importance for the theory. A knowledge of the cross sections of the CCQE processes (4) and (5) in a wide range of energies is extremely important for a correct interpretation of the high-precision neutrino oscillation experiments. At present, several new experiments (MINER$\nu$A[7], T2K[8], ArgoNeuT [9]) on a detailed study of CCQE neutrino scattering are going on. In the next Section we will briefly summarize the present day status of the axial form factor of the nucleon.
Measurement of the polarization of the recoil nucleons in the CCQE processes could be a source of an important information on the axial form factor of the nucleon. Such measurement, like in the electromagnetic case, could change our ideas about the $Q^2$-dependence of the axial form factor, about nuclear effects etc. It is worthwhile and timely to consider the possibility for measurement of the recoil polarization in modern short baseline neutrino experiments in which thousands of neutrino events are detected.

In this paper we shall present the results of the calculations of the recoil polarization of the nucleon in the CCQE neutrino processes (4) and (5) in the case of the monochromatic neutrino beam on a free nucleon.

However, in order to obtain measurable quantities in neutrino (antineutrino) experiments one has to average over the neutrino (antineutrino) spectrum. This implies that to obtain the measurable polarization one needs to average the expressions presented below over this spectrum. Note that the numerator and the denominator in the expressions (18) and (19) must be averaged separately. Also, in modern neutrino experiments nuclear targets are used. Here we do not consider nuclear effects.

2 The axial form factor of the nucleon

The determination of the axial form factor of the nucleon is a very challenging experimental problem due to the fact that in neutrino experiments nuclear targets (C, Fe, etc.) are used, the neutrino beams are not monochromatic, they are normalized in different ways etc.

In analogy with the electromagnetic form factors the axial form factor is usually parameterized by the dipole formula:

$$G_A(Q^2) = \frac{g_A}{(1 + \frac{Q^2}{M_A^2})^2}. \quad (6)$$

Here $g_A = 1.2701 \pm 0.0025$ [10] is the axial constant, determined from the neutron $\beta$-decay data and $M_A$ is a parameter (the ”axial mass”).

The values of the parameter $M_A$ determined from the data of different experiments, under the assumption that neutrinos interact with a quasi-free nucleon in a nuclei and other nucleons are spectators (impulse approximation), are quite different.

From analysis of the data on measurements of the cross section of the process $\nu_\mu + n \rightarrow \mu^- + p$ on deuterium target and of the process $\bar{\nu}_\mu + p \rightarrow \mu^+ + n$
on proton target it was found [11]:

\[ M_A = 1.016 \pm 0.026 \text{ GeV}. \]  

(7)

The value of the parameter \( M_A \) obtained from the data of the NOMAD experiment (carbon target) [12] is in agreement with (7):

\[ M_A = 1.05 \pm 0.02 \pm 0.06 \text{ GeV}. \]  

(8)

Let us note that the value (8) was found from the total cross section averaged over the neutrino spectrum. In the same experiment, but from the \( Q^2 \)-distribution a value of the parameter \( M_A \) [12]: \( M_A = 1.07 \pm 0.06 \pm 0.07 \text{ GeV} \) was extracted, which is compatible with (8), but has larger statistical and systematic errors.

However, from fit of the data of more recent experiments larger average values of the parameter \( M_A \) (with larger errors) were obtained.

From the data of the MINOS experiment (iron target) it was found [13]:

\[ M_A = 1.26^{+0.12+0.08}_{-0.10-0.12} \text{ GeV}. \]  

(9)

In the K2K experiment (\( H_2O \) target) it was obtained [14]:

\[ M_A = 1.20 \pm 0.12 \text{ GeV}. \]  

(10)

From the data of the MiniBooNE experiment (carbon target) it was found [15]:

\[ M_A = 1.23 \pm 0.20 \text{ GeV}. \]  

(11)

From the analysis of data of the later high-statistic experiment[16] (1.4 \cdot 10^5 events) it was inferred:

\[ M_A = 1.35 \pm 0.17 \text{ GeV}. \]  

(12)

There could be many different reasons for the disagreement of the average values of \( M_A \) obtained from the data of the different experiments. It could be a problem of systematics and normalization (see [17]). Target nuclei in the different experiments are different. The difference of the values of \( M_A \) could be due to such nuclei effects as interaction of neutrinos with correlated pairs of nucleons (see [18, 19]). Experiments on the study of CCQE neutrino processes were done in different ranges of \( Q^2 \). The difference between the different values of \( M_A \) could be a signature that the dipole parametrization
(2) may not be the correct parametrization of the axial form factor in the whole region of $Q^2$ studied (like in the case of the electromagnetic form factors).

A measurement of the polarization of the recoil protons produced in the CCQE neutrino process $\nu_\mu + n \rightarrow \mu^- + p$ could change the situation with axial form factor $G_A(Q^2)$. Taking into account that in short baseline neutrino experiments thousands of neutrino events are observed it is worthwhile to consider a possibility of measuring of the polarization of the protons by the observation of left-right asymmetry in the scattering of the recoil protons in a neutrino detector.

In the next section we will present our results of the calculation of the polarization of final nucleon in the CCQE neutrino processes.

3 Polarization of the final nucleons in CCQE processes

Here we shall present the results of the calculations of the polarization of final nucleons in the CCQE neutrino processes (4) and (5).

Process (4) is a charged current process and its matrix element is characterized by the four weak form factors of the nucleon:

$$\langle f \mid (S - 1) \mid i \rangle = -i \frac{G_F \cos \theta_c}{\sqrt{2}} N_k N_{k'} \bar{u}(k') \gamma^\alpha (1 - \gamma_5) u(k). \quad p \langle p' \mid J^{(1+\frac{1}{2})}_\alpha \mid p \rangle_n$$

$$\times (2\pi)^4 \delta(k + p - k' - p').$$ (13)

Here $G_F$ is the Fermi constant, $\theta_c$ is the Cabibbo mixing angle. The hadronic matrix element $p \langle p' \mid J^{(1+\frac{1}{2})}_\alpha \mid p \rangle_n$ is:

$$p \langle p' \mid J^{(1+\frac{1}{2})}_\alpha \mid p \rangle_n = N_p N_{p'} \bar{u}(p') (V_\alpha - A_\alpha) u(p),$$ (14)

where

$$V_\alpha = \gamma_\alpha F^{CC}_1(Q^2) + \frac{i}{2M} \sigma_{\alpha\beta} q^\beta F^{CC}_2(Q^2), \quad A_\alpha = \gamma_\alpha \gamma_5 G_A(Q^2) + q_\alpha \gamma_5 G_P(Q^2).$$ (15)

$F^{CC}_{1,2}$, $G_A$ and $G_P$ are the CC weak vector, axial and pseudoscalar form factors, respectively, $k$ and $p$ ($k'$ and $p'$) are the initial neutrino and neutron (final muon and proton) momenta, $N_p = \frac{1}{(2\pi)^{3/2}} \sqrt{2p_0}$ is the standard normalization factor, $q = p' - p = k - k'$ is momentum transfer, $Q^2 = -q^2$. 

6
Under isotopic $SU(2)$ transformations the weak charged current $J_{\alpha}^{1+i2}$
is transformed as the "plus component" of the conserved isovector current. Taking into account that the third component of this isovector is the isovector part of the electromagnetic current – the hypothesis for conservation of the vector current (CVC), from isotopic $SU(2)$ invariance for the weak vector form factors we obtain:

$$F_{CC}^{1,2}(Q^2) = F_{p}^{1,2}(Q^2) - F_{n}^{1,2}(Q^2),$$

(16)

where $F_{p}^{1,2}(Q^2)$ and $F_{n}^{1,2}(Q^2)$ are the Dirac and Pauli electromagnetic form factors of the proton and the neutron. These form factors are known at present in a wide region of $Q^2$ (see, for example, the review [1, 2]).

From the hypothesis for partial conservation of the axial current (PCAC) it follows that the contribution of the pseudoscalar form factor $G_{P}(Q^2)$ to the matrix element (15) can be neglected. Thus, from study of the CCQE process (4) an information about the axial form factor $G_{A}(Q^2)$ can be obtained.

The matrix elements of the processes (4) and (5) are characterized by the same form factors. In fact from charge symmetry we have:

$$p \langle p' | J_{\alpha}^{1+i2} | p \rangle_n =_n p \langle p' | J_{\alpha}^{1-i2} | p \rangle_p.$$

(17)

The polarization 4-vector of the final proton in process (4) is given by the expression:

$$\xi^\rho = \frac{\text{Tr} [\gamma^\rho \gamma_5 \rho_f]}{\text{Tr} [\rho_f]},$$

(18)

where $\rho_f$ is the final spin density matrix. Using the relation

$$\Lambda(p') \gamma^\rho \gamma_5 \Lambda(p') = 2M \left( g^{\rho\sigma} - \frac{p'_{\rho} p'_{\sigma}}{M^2} \right) \Lambda(p') \gamma_\sigma \gamma_5$$

(19)

and performing integration over the momenta of the final lepton and nucleon, for the polarization 4-vector of the final nucleon we have:

$$\xi_\alpha = \left( g_{\alpha\beta} - \frac{p'_{\alpha} p'_{\beta}}{M^2} \right) \frac{\text{Tr} \left[ N \Lambda(p) \gamma^\beta \gamma_5 \Lambda(p') \right]}{\text{Tr} \left[ N \Lambda(p) N \Lambda(p') \right]}. $$

(20)

Here

$$N = \bar{u}(k') \gamma_\alpha (1 - \gamma_5) u(k) \left( V^\alpha - A^\alpha \right),$$

(21)
\( \Lambda(p) = p' + M \). In eq. (20) the projection operator \( (g_{\alpha\beta} - p'_\alpha p'_\beta/M^2) \) guarantees the condition \( (\xi \cdot p') = 0 \).

The vector \( \xi^\alpha \) can be decomposed along the following three independent 4-vectors \( Q^\alpha_i \) orthogonal to \( p'^\alpha \):

\[
Q^\alpha_+ = k^\alpha_+ = \frac{(p'k_+)}{M^2} p'^\alpha, \quad Q^\alpha_- = k^\alpha_- = \frac{(p'k_-)}{M^2} p'^\alpha, \quad Q^\alpha_p = p^\alpha - \frac{(p'p)}{M^2} p'^\alpha
\]

where

\[
k_+ = (k + k'), \quad k_- = (k - k') = q.
\]

After standard calculations we obtain:

\[
\xi^\alpha = \frac{M}{(kp)} J_0 \left[ Q^\alpha_+ P_+ + Q^\alpha_- P_- + Q^\alpha_p P_p \right]
\]

\[
P_+ = \left[ y G_{CC}^M + (2 - y) G_A \right] G_{EC}^C
\]

\[
P_- = - G_A \left[ y G_{CC}^M + (2 - y) G_A \right] + F_2^{CC} \left[ (2 - y) \tau G_{MM}^C + y (1 + \tau) G_A \right]
\]

\[
P_p = \frac{F_2^{CC}}{y} \left[ 2y(2 - y) \tau G_{MM}^C + [2\tau[1 + (1 - y)^2] + y^2] G_A \right].
\]

Here

\[
G_{EC}^C = F_1^{CC} - \tau F_2^{CC}, \quad G_{MM}^C = F_1^{CC} + F_2^{CC}
\]

\[
J_0 = \frac{Tr [\Lambda(p)\Lambda(p')]}{82^2 (kp)^2}, \quad y = \frac{(pq)}{(pk)}, \quad \tau = \frac{Q^2}{4M^2}.
\]

From (16) it follows:

\[
G_{MM}^{CC} = G_{MM}^p - G_{MM}^n, \quad G_{EC}^{CC} = G_{EC}^p - G_{EC}^n,
\]

where \( G_{MM}^{p,n} \) and \( G_{EC}^{p,n} \) are the magnetic and charge form factors of proton and neutron.

From (24) one can easily find the polarization vector of the proton in the laboratory frame. We have:

\[
\vec{\xi} = \frac{1}{J_0 E} \left\{ (\vec{k} + \vec{k}') P_+ + \vec{q} \left[ -\frac{E + E'}{M} P_+ + (1 + \frac{E - E'}{M}) (P_- - P_p) \right] \right\}.
\]
Here \( E \) and \( E' \) are the energies of the neutrino and the final muon:

\[
E' = \frac{E}{1 + (2E/M) \sin^2(\theta/2)}, \quad y = \frac{E - E'}{E},
\]

(31)

\( \theta \) is the angle between the vectors \( \vec{k} \) and \( \vec{k}' \).

The polarization vector lays in the scattering plane.\(^1\) For the longitudinal \( \xi_\parallel \) and transverse \( \xi_\perp \) components of the polarization we have:

\[
\vec{\xi} = \xi_\perp \vec{e}_\perp + \xi_\parallel \vec{e}_\parallel,
\]

(32)

where \( \vec{e}_\perp \) and \( \vec{e}_\parallel \) are two orthogonal unit vectors in the scattering plane:

\[
\vec{e}_\parallel = \frac{\vec{p}'}{|\vec{p}'|} = \frac{\vec{q}}{|\vec{q}|}, \quad \vec{e}_\perp = \vec{e}_\parallel \times \vec{n}, \quad \vec{n} = \frac{\vec{k} \times \vec{k}'}{|\vec{k} \times \vec{k}'|}.
\]

(33)

From (30), (32) and (33) we obtain:

\[
s_\perp = \xi_\perp = \left( \frac{1}{J_0} \right) \frac{-2E' \sin \theta}{|\vec{q}|} \left[ G_A(2 - y) + G_{\text{CC}}^{\text{MC}} y \right] G_{\text{CE}}^{\text{CC}}.
\]

(34)

\[
s_\parallel = \frac{M}{p'_0} \xi_\parallel =
\]

\[
= - \frac{1}{J_0} \frac{q_0}{|\vec{q}|} \left[ G_A(2 - y) + G_{\text{CC}}^{\text{MC}} y \right] \left[ G_{\text{MC}}^{\text{CC}}(2 - y) + G_A(y + \frac{2M}{E}) \right].
\]

(35)

Here \( s_\parallel \) and \( s_\perp \) are the longitudinal and transverse components of the polarization vector in the rest frame of the recoil nucleon:

\[
s^\alpha = (0; s_\parallel, s_\perp)
\]

(36)

\( M/p'_0 \) is the Lorentz boost along \( \vec{p}' \).

Let us note that at \( G_A = 0 \), using the kinematic relations

\[
|\vec{q}| = Ey\sqrt{\frac{1 + \tau}{\tau}}, \quad \frac{q_0}{|\vec{q}|} = \sqrt{\frac{\tau}{1 + \tau}},
\]

(37)

\(^{1}\)It is obvious that the component orthogonal to the scattering plane disappears due to \( T \)-invariance.
one can show that eqs. (34) and (35) coincide with the well known expressions for the transverse and longitudinal polarizations of the recoil protons in elastic scattering of longitudinally polarized leptons on unpolarized protons (see, for example, [1]).

Taking into account that the hadronic part of the processes $\nu_{\mu} + n \rightarrow \mu^- + p$ and $\bar{\nu}_{\mu} + p \rightarrow \mu^+ + n$ are the same, we easily obtain the polarizations of the final nucleons for both processes:

$$\left(J_0 s_\perp\right)_{\nu, \bar{\nu}} = \frac{-2E' \sin \theta}{|q|} \left[\pm y G^{CC}_M + (2 - y)G_A\right] G^{CC}_E$$  \hspace{0.5cm} (38)$$

and

$$\left(J_0 s_\parallel\right)_{\nu, \bar{\nu}} = -\frac{q_0}{|q|} \left[\pm y G^{CC}_M + (2 - y)G_A\right] \left[(2 - y) G^{CC}_M \pm \left(y + \frac{2M}{E}\right) G_A\right].$$  \hspace{0.5cm} (39)$$

Here and further the upper (lower) sign corresponds to neutrino (antineutrino) scattering.

The quantity $J^{\nu, \bar{\nu}}_0$ is determined from the differential cross section:

$$J^{\nu, \bar{\nu}}_0 = \frac{d\sigma^{\nu, \bar{\nu}}}{dQ^2} \cdot \frac{4\pi}{G_F^2 \cos^2 \theta_c}.$$  \hspace{0.5cm} (40)$$

In terms of the form factors it is given by the expression:

$$J^{\nu, \bar{\nu}}_0 = 2(1 - y) \left(G_A^2 + \frac{\tau(G^{CC}_M)^2 + (G^{CC}_E)^2}{1 + \tau}\right) + \frac{My}{E} \left[G_A^2 - \frac{\tau(G^{CC}_M)^2 + (G^{CC}_E)^2}{1 + \tau}\right] + y^2 \left(G^{CC}_M \mp G_A\right)^2 \pm 4y G^{CC}_M G_A.$$  \hspace{0.5cm} (41)$$

4 Comments

- From eq.(38) we obtain a rather simple expression for $G_A$:

$$G_A = \frac{-1}{2 - y} \left\{ \frac{M \sqrt{\tau(1 + \tau)}}{E' \sin \theta} \frac{(J_0 s_\perp)_{\nu, \bar{\nu}}^{\nu, \bar{\nu}}}{G^{CC}_E} \pm y G^{CC}_M \right\}.$$  \hspace{0.5cm} (42)$$

- Note, that the electric form factor does not enter eq.(39). Thus the axial form factor $G_A$ is determined only by the cross section, the longitudinal
polarization $\xi_\parallel$ and the best known magnetic form factors of the proton and neutron.

- If the neutrino detector is in a magnetic field, then both the transverse and longitudinal polarizations could be measured (like in the case of elastic $e - p$ scattering). For their ratio we have:

$$
\left( \frac{s_\parallel}{s_\perp} \right)^{\nu,\bar{\nu}} = \frac{q_0}{2E' \sin \theta} \left[ \frac{(2 - y) \, G^{CC}_M \pm G_A (y + 2M/E)}{G^{CC}_E} \right].
$$

(43)

Then for the axial form factor we obtain:

$$
G_A = \pm \frac{E + E'}{E - E' + 2m} \left[ \frac{2EE' \sin \theta}{E^2 - E'^2} \, G^{CC}_E \left( \frac{s_\parallel}{s_\perp} \right)^{\nu,\bar{\nu}} - G^{CC}_M \right].
$$

(44)

- Finally, let us notice the relations:

$$
(J_0 s_\perp)^{\nu,\bar{\nu}} + (J_0 s_\perp)^{\nu,\bar{\nu}} = -\frac{4 E' \sin \theta}{|\vec{q}|} (2 - y) G_A G^{CC}_E
$$

(45)

$$
(J_0 s_\parallel)^{\nu,\bar{\nu}} + (J_0 s_\parallel)^{\nu,\bar{\nu}} = -\frac{4q_0}{|\vec{q}|} G_A G^{CC}_M \left\{ 1 + (1 - y)^2 \right\} + \frac{My}{E} \}
$$

(46)

5 Numerical results

Here we present a numerical study of the sensitivity of the discussed recoil nucleon polarization to the different choices of the axial mass $M_A$.

We use the following commonly used parameterizations for the form factors, summarized in [1]:

$$
G_D = \frac{1}{\left(1 + \frac{Q^2}{M_V^2}\right)^2}, \quad M_V^2 = 0.71
$$

$$
G_{M,p} = \mu_p \, G_D, \quad G_{M,n} = \mu_n \, G_D
$$

$$
G_{E,p} = (1.06 - 0.14 \, Q^2) \, G_D
$$

$$
G_{E,n} = -a \, \frac{\mu_n \tau}{1 + b\tau} \, G_D, \quad a = 1.25, \quad b = 18.3
$$

(47)

where $\mu_p = 2.79$ and $\mu_n = -1.91$ are the magnetic moments of the proton and neutron. We calculate the effect of the different axial form factors on the
longitudinal and transverse polarizations, considering the following values of $M_A$:

1) $M_A = 1.016$ — full line
2) $M_A = 1.20$ — dashed line
3) $M_A = 1.35$ — dotted line

(48)

We examined the polarizations at fixed neutrino energies as functions of $Q^2$ in the energy range $Q^2_{\text{min}} \leq Q^2 \leq Q^2_{\text{max}}$. Here $Q^2_{\text{min}}$ and $Q^2_{\text{max}}$ are fixed by the condition $0 \geq \cos \theta \leq 1$ and the scattering angle $\theta$ is determined via (37). We have:

$$
\begin{align*}
\cos \theta &= 1 - \frac{MQ^2}{E(2ME - Q^2)} \\
Q^2_{\text{min}} &= 0, \quad Q^2_{\text{max}} = \frac{2ME^2}{M + E} \\
\sin \theta &= \frac{MQ^2}{E(2ME - Q^2)} \sqrt{\frac{4E^2}{Q^2} - \frac{2E}{M} - 1}.
\end{align*}
$$

(49)

On Figs.(1) and (2) we show the dependence of $s_\parallel/s_\perp$ and $s_\perp$ on the choice of $M_A$ for the two considered processes: $\bar{\nu} + p \rightarrow \mu^+ + n$ (left) and $\nu + n \rightarrow \mu^- + p$ (right).

We found that the polarization of the final proton in $\nu_\mu + n \rightarrow \mu^- + p$ practically does not depend on the value of $M_A$. However, polarization of the final neutron in $\bar{\nu}_\mu + p \rightarrow \mu^+ + n$ is rather sensitive to the value of the axial mass. It is most clearly pronounced for the longitudinal polarization $s_\parallel$ and, respectively, for the ratio $s_\parallel/s_\perp$, shown on Fig.(1). Note that an advantage of $s_\parallel/s_\perp$ is that many of the systematic uncertainties and radiative corrections cancel, however a magnetic field should be applied to the detector in order to measure $s_\parallel$. This sensitivity is exhibited in the whole $Q^2$-range. As higher $Q^2$ are accessed through higher neutrino energies (and also the measured quantities are averaged over the neutrino spectra), we have presented the polarizations for three values of the neutrino energies: E=1, 3.5 and 5 GeV.

The transverse polarization in $\nu_\mu + p \rightarrow \mu^+ + n$ shows sensitivity to $M_A$ for low neutrino energies, but not so dramatically pronounced for higher energies (see Figs.(2)).
On Figs. (3) we show the differential cross sections (multiplied by $4\pi/G_F^2$) for the processes $\bar{\nu}_\mu + p \to \mu^+ + n$ (left) and $\nu_\mu + n \to \mu^- + p$ (right) for the energies $E=1$ and $3.5$ GeV, and the same $M_A$, eq.(48). From these figures it is clear that it’s a very difficult task to distinguish among the different values $M_A$ solely from measurements of the cross sections.

6 Conclusion

Investigation of the CCQE neutrino processes and determination of the axial form factor of the nucleon is of great importance for the theory and for the modern high-precision neutrino oscillation experiments. Many experiments on measurement of the cross sections of the CCQE neutrino processes in a wide range of neutrino energies have been done. From analysis of the data of these experiments the value of the parameter $M_A$, which determines the $Q^2$-behavior of the axial form factor in the dipole approximation, was determined. Usually in such analysis the impulse approximation for the target nuclei is used. The values of $M_A$ determined from the data of the different experiments in such a way are not compatible. There could be different reasons for such a disagreement: nuclei effects, more complicated than dipole $Q^2$-dependence of the axial form factor etc.

In this paper we present the calculation of the polarization of the final nucleon in CCQE scattering. Relations that express the axial form factor through the polarization of the final nucleon and the electromagnetic form factors are obtained. Our numerical analysis showed that there is a clear sensitivity to $M_A$ in the polarizations of the neutron in $\bar{\nu}_\mu + p \to \mu^+ + n$, most sensitive is the ratio of the longitudinal to transverse polarization $s_\parallel/s_\perp$. This sensitivity is pronounced in the whole $Q^2$-energy range.

We have considered the idealized case of monochromatic neutrinos on a free nucleon. In order to obtain the measurable polarization the procedure of averaging of the corresponding expressions over the neutrino spectrum should be performed and nuclear effects taken into account.

Experiments on measurement of the polarization of the final proton in elastic scattering of longitudinally polarized electrons on unpolarized protons drastically changed our understanding about the electromagnetic form factors of the proton. Analogously, we suggest that measurement of the polarization of the final nucleon in CCQE processes will provide additional information about the axial form factor. It is obvious that such measurement
is a challenge. However, taking into account the importance of the problem of the axial form factor and the rapid progress of the neutrino detection technique it is worth to consider such a possibility.

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Figure 1: The dependence of $s_L/s_T$ on the values of $M_A$ [see(48)] for $\bar{\nu}_\mu + p \rightarrow \mu^+ + n$ (left) and for $\nu_\mu + n \rightarrow \mu^- + p$ (right) at $E=1$ (up), 3.5 (middle) and 5 (down) GeV.
Figure 2: The dependence of the transverse polarization $s_T$ on the values of $M_A$ [see(48)] for $\bar{\nu}_\mu + p \rightarrow \mu^+ + n$ (left) and for $\nu_\mu + n \rightarrow \mu^- + p$ (right) at $E=1$ (up), 3,5 (middle) and 5 (down) GeV.
Figure 3: The dependence of the cross section (multiplied by $4\pi/G_F^2$) on the values of $M_A$ [see (48)] for the processes $\bar{\nu}_\mu + p \rightarrow \mu^+ + n$ (left) and $\nu_\mu + n \rightarrow \mu^- + p$ (right) at $E=1$ (up) and 3.5 (down) GeV.