PARTICLE BASIS OF FEIGIN-STOYANOVSKY’S TYPE SUBSPACES OF LEVEL 1 $\tilde{\mathfrak{sl}}_{\ell+1}(\mathbb{C})$-MODULES

GORAN TRUPČEVIĆ

Abstract. We construct particle basis for Feigin-Stoyanovsky’s type subspaces of level 1 standard $\tilde{\mathfrak{sl}}_{\ell+1}(\mathbb{C})$-modules. From the description we obtain character formulas.

1. Introduction

A problem of finding a monomial basis of a standard module is part of the Lepowsky-Wilson’s program of studying Rogers-Ramanujan type identities through representation theory of affine Lie algebras ([LW], [LP], [MP]). Description of basis was used to obtain graded dimension of these modules, which gave the sum-side in Rogers-Ramanujan-type identities.

B. Feigin and A. Stoyanovsky initiated another approach to Rogers-Ramanujan type identities by considering what they called a principal subspace of a standard $\mathfrak{sl}_2(\mathbb{C})$-module ([FS]). These subspaces were further studied by G. Georgiev ([G]), C. Calinescu, S. Capparelli, J. Lepowsky and A. Milas ([CLM1,2], [CalLM1,2,3], [C1,2]), C. Sadowski ([S1,2]), E. Ardonne, R. Kedem and M. Stone ([AKS]).

Another type of principal subspace, called Feigin-Stoyanovsky’s type subspace, was introduced and studied by M. Primc who constructed a basis of this subspace and from it he obtained basis of the whole standard module ([P1,2]). For $\tilde{\mathfrak{sl}}_{\ell+1}(\mathbb{C})$, these bases were parameterized by $(k, \ell+1)$-admissible configurations ([FJLMM]), combinatorial objects introduced and further studied by Feigin et al. in [FJLMM] and [FJMNT], where bosonic and fermionic formulas for characters were obtained. Primc and M. Jerković obtained fermionic formulas for characters of standard $\mathfrak{sl}_3(\mathbb{C})$-modules by using intertwining operators and admissible configurations ([J2]), or by using quasi-particle bases [JP]. In our previous work ([T]), we have used $(1, \ell+1)$-admissible configurations to combinatorially obtain character formulas for Feigin-Stoyanovsky’s type subspaces of level 1 standard $\tilde{\mathfrak{sl}}_{\ell+1}(\mathbb{C})$-modules. In this note we use an approach similar to Georgiev and Jerković and Primc to construct particle basis for Feigin-Stoyanovsky’s type subspaces of level 1 standard $\tilde{\mathfrak{sl}}_{\ell+1}(\mathbb{C})$-modules. From this description we immediately obtain character formulas.

2. Affine Lie algebra $\tilde{\mathfrak{sl}}_{\ell+1}(\mathbb{C})$

Let $\mathfrak{g} = \tilde{\mathfrak{sl}}_{\ell+1}(\mathbb{C})$ be a simple finite-dimensional Lie algebra of type $A_{\ell}$. Fix a Cartan subalgebra $\mathfrak{h} \subset \mathfrak{g}$ and denote by $R$ the corresponding root system; $\mathfrak{g}$ has a root decomposition $\mathfrak{g} = \mathfrak{h} \oplus \bigoplus_{\alpha \in R} \mathfrak{g}_{\alpha}$. Let $\Pi = \{\alpha_1, \ldots, \alpha_\ell\}$ be a basis of the root system $R$, and let $\{\omega_1, \ldots, \omega_\ell\}$ be the corresponding set of fundamental weights, $\langle \omega_i, \alpha_j \rangle = \delta_{ij}$. Set $\omega_0 = 0$ for convenience. Let $\langle \cdot, \cdot \rangle$ be a a normalized invariant
bilinear form on \( g \); we identify \( \mathfrak{h} \) with \( \mathfrak{h}^* \) via \( \langle \cdot, \cdot \rangle \). Denote by \( Q \) the root lattice, and by \( P \) the weight lattice of \( g \). Also for each root \( \alpha \in R \) fix a root vector \( x_\alpha \in g_\alpha \).

Let \( \tilde{g} \) be the associated untwisted affine Lie algebra (\cite{K}),

\[
\tilde{g} = g \otimes \mathbb{C}[t, t^{-1}] \oplus \mathbb{C}c \oplus \mathbb{C}d.
\]

Denote by \( x(m) = x \otimes t^m \) for \( x \in g \), \( m \in \mathbb{Z} \), and define formal Laurent series \( x(z) = \sum_{m \in \mathbb{Z}} x(m)z^{-m-1} \). Denote by \( \Lambda_0, \ldots, \Lambda_\ell \) fundamental weights for \( \tilde{g} \).

Fix a minuscule weight \( \omega \) and set

\[
\Gamma = \{ \alpha \in R \mid \langle \alpha, \omega \rangle = 1 \} = \{ \alpha_i + \cdots + \alpha_\ell \mid i = 1, \ldots, \ell \}.
\]

Denote by \( \gamma_i = \alpha_i + \cdots + \alpha_\ell \). Then

\[
g = g_{-1} \oplus g_0 \oplus g_1, \quad g_0 = \mathfrak{h} \oplus \sum_{\langle \alpha, \omega \rangle = 0} g_\alpha, \quad g_{\pm 1} = \sum_{\alpha \in \pm \Gamma} g_\alpha,
\]

is a \( \mathbb{Z} \)-gradation of \( g \). Subalgebras \( g_1 \) and \( g_{-1} \) are commutative.

The \( \mathbb{Z} \)-gradation of \( g \) gives the \( \mathbb{Z} \)-gradation of the affine Lie algebra \( \tilde{g} \):

\[
\tilde{g} = \tilde{g}_{-1} = g_{-1} \oplus g_0 \oplus g_1, \quad \tilde{g}_0 = g_0 \otimes \mathbb{C}[t, t^{-1}] \oplus \mathbb{C}c \oplus \mathbb{C}d, \quad \tilde{g}_{\pm 1} = g_{\pm 1} \otimes \mathbb{C}[t, t^{-1}].
\]

Again, \( \tilde{g}_{-1} \) and \( \tilde{g}_1 \) are commutative subalgebras. We will call elements \( \gamma \in \Gamma \) colors and we will say that \( x_\gamma(-m) \) is an element of color \( \gamma \) and degree \( m \).

Let \( L(\Lambda_r) \) be a standard (i.e. integrable highest weight) \( \tilde{g} \)-module of level 1. Denote by \( v_r \) the highest weight vector of \( L(\Lambda_r) \). Define a Feigin-Stoyanovsky’s type subspace

\[
W(\Lambda_r) = U(\tilde{g}_1) \cdot v_r \subset L(\Lambda_r).
\]

By Poincaré-Birkhoff-Witt theorem, we have a spanning set of \( W(\Lambda_r) \) consisting of monomial vectors

\[
\{ bv_r \mid b = x_1(-m_{1,n_1}) \cdots x_1(-m_{1,n_i}) \cdots x_1(-m_{1,1}), m_{i,j+1} \geq m_{i,j} > 0, n_i \geq 0 \},
\]

where we write \( x_i \) instead of \( x_{\gamma_i} \), for short.

3. VOA construction

We briefly recall the vertex operator algebra construction of standard \( \tilde{g} \)-modules \( L(\Lambda_r) \) from \cite{FK}, \cite{S}. For details and notation we turn the reader to \cite{FLM}, \cite{DL} and \cite{LL}.

Consider tensor products \( V_P = M(1) \otimes \mathbb{C}[P] \) and \( V_Q = M(1) \otimes \mathbb{C}[Q] \), where \( M(1) \) is the Fock space for the Heisenberg subalgebra \( h_0 = \sum_{m \in \mathbb{Z} \setminus \{0\}} h \otimes t^m \oplus \mathbb{C}c \), and \( \mathbb{C}[P] \) and \( \mathbb{C}[Q] \) are group algebras of the weight and root lattice with bases consisting of \( \{ e^{\lambda} \mid \lambda \in P \} \), and \( \{ e^{\alpha} \mid \alpha \in Q \} \), respectively. We identify \( \mathbb{C}[P] \) with \( 1 \otimes \mathbb{C}[P] \subset V_P \).

Space \( V_Q \) has a structure of vertex operator algebra and \( V_P \) is a module for this algebra:

\[
Y(e^{\lambda}, z) = E^-(\lambda, z)E^+(\lambda, z) \otimes e^{\lambda}z^{-\lambda}\epsilon(\lambda, \cdot),
\]

where \( E^\pm(\lambda, z) = \text{exp} \left( \sum_{m \geq 1} \lambda(\pm m)z^{\mp m}/\pm m \right) \), \( e^{\lambda} \) is a multiplication operator, \( z^\lambda \cdot e^{\mu} = e^{\mu}z^{(\lambda, \mu)} \) and \( \epsilon(\cdot, \cdot) \) is a 2-cocycle (cf. \cite{FLM}).

By using vertex operators, one can define the structure of \( \tilde{g} \)-module on \( V_P \) by setting \( x_\alpha(z) = Y(e^{\alpha}, z) \) for \( \alpha \in R \). This gives \( V_Q \cong L(\Lambda_0) \) and \( V_Q e^{v_r} \cong L(\Lambda_r) \), with highest weight vectors \( v_0 = 1 \) and \( v_r = e^{v_r} \), and \( V_P \cong L(\Lambda_0) \oplus \cdots \oplus L(\Lambda_\ell) \).
From vertex operator formula (2) one easily obtains the following relations on $L(\Lambda_r)$

\begin{align}
(3) \quad x_i^2(z) &= 0, \quad 1 \leq i \leq \ell, \\
(4) \quad x_i(z)x_j(z) &= 0, \quad 1 \leq i < j \leq \ell, \\
(5) \quad x_i(m)v_r &= 0, \quad m \geq -\delta_{i\leq r}, \\
(6) \quad x_r(-1)v_{r-1} &= Ce^{\omega_r+\omega_r} = Ce^{\omega_r}v_r,
\end{align}

for some $C \in \mathbb{C}^\times$. Here, $\delta_{i\leq j}$ is 1 if $i \leq j$, 0 otherwise.

For the proof of linear independence we will be using certain coefficients of intertwining operators

\[ \mathcal{Y}(e^\lambda, z) = Y(e^\lambda, z)e^{i\pi\lambda}c(\cdot, \lambda), \]

for $\lambda \in P$, where $c(\cdot, \lambda)$ is a commutator map (cf. [DL]). Let $\lambda_i = \omega_i - \omega_{i-1}$ for $i = 1, \ldots, \ell$. From Jacobi identity ([DL]) we see that operators $\mathcal{Y}(e^\lambda, z)$ commute with the action of $\tilde{g}_1$. Define the following coefficients of intertwining operators (cf. [P3])

\[ [i] = \text{Res} \ z^{-i - (\lambda_i, \omega_{i-1})} \mathcal{Y}(e^{\lambda_i}, z), \]

for $i = 1, \ldots, \ell$. From (2), it follows (cf. [P3])

\[ [i]v_{i-1} = Cv_i, \]

for some $C \in \mathbb{C}^\times$.

We will also be using simple current operators $e^{\omega_i}$, $i = 1, \ldots, \ell$. For $\alpha \in R$ and $\lambda \in P$ from (2) we get the following commutation relation

\[ x_\alpha(z)e^\lambda = \epsilon(\alpha, \lambda)z^{(\alpha, \lambda)}e^\lambda x_\alpha(z). \]

By comparing coefficients, we get

\[ x_\alpha(m)e^\lambda = \epsilon(\alpha, \lambda)e^\lambda x_\alpha(m + (\alpha, \lambda)). \]

In particular, for $\alpha = \gamma_i$ and $\lambda = \omega_j$, we get

\[ x_i(m)e^{\omega_j} = \epsilon(\gamma_i, \omega_j)e^{\omega_j}x_i(m + \delta_{i\leq j}). \]

4. Basis of $W(\Lambda_r)$

To reduce the spanning set (1) and to prove linear independence, we need a linear order on monomials. Define a linear order $x_i(n) < x_j(m)$ if either $i > j$ or $i = j$ and $n < m$. We assume that in all monomials factors are sorted descendingly from right to left, like in (1). We compare two monomials $b_1$ and $b_2$ by comparing their factors from right to left (reverse lexicographic order): $b_1 < b_2$ if either $b_2 = bb_1$ or $b_1 = b_1'x_i(n)b$, $b_2 = b_2'x_j(m)b$ and $x_i(n) < x_j(m)$, for some monomials $b, b_1, b_2$.

This linear order is compatible with multiplication: if $b > c$, then $ab > ac$.

For a monomial $b = x_i(-m_{\ell,n_\ell}) \cdots x_\ell(-m_{\ell,1}) \cdots x_1(-m_{1,n_1}) \cdots x_1(-m_{1,1})$ define its degree, weight and length by $d(b) = m_{\ell,n_\ell} + \cdots + m_{\ell,1} + \cdots + m_{1,n_1} + \cdots + m_{1,1}$, $w(b) = n_1g_1 + \cdots + n_{\ell}g_{\ell}$ and $l(b) = n_1 + \cdots + n_{\ell}$.

**Theorem 1.** A spanning set of $W(\Lambda_r)$ is given by the set of monomial vectors (1) satisfying initial conditions

\[ m_{i,n} \geq 1 + \sum_{i < j} n_i + \delta_{j \leq r} \]

and difference conditions

\[ m_{i,n+1} \geq m_{i,n} + 2, \quad 1 \leq n \leq n_i - 1. \]
Proof: Difference conditions follow from (3). Assume that \( b \) doesn’t satisfy (10). Then \( b = b' x_j(-m) x_j(-m') \), for some monomial \( b' \) and \( m' \leq m \leq m' + 1 \). By (1) and (3), on \( W(\Lambda_r) \) we have

\[
x_j(-m) x_j(-m') = C_1 x_j(-m-1) x_j(-m'+1) + \cdots + C_{m'-1} x_j(-m-m'+1) x_j(-1),
\]

for some \( C_i \in \mathbb{C}^\times \). Multiply this by \( b' \) to obtain \( b \) expressed as a linear combination of greater monomials of the same degree and weight.

Now assume that \( b \) doesn’t satisfy (10); let \( b = b_2 x_j(-m) b_1 \) where \( b_1 \) contains all factors of colors \( \gamma_1, \ldots, \gamma_{j-1} \) and

\[
m < 1 + \sum_{i<j} n_i + \delta_{j \leq r}.
\]

We will prove that \( b \) can be expressed in terms of greater monomials of the same degree and weight. The proof is done by induction on the length \( l(b_1) = \sum_{i<j} n_i \).

If \( l(b_1) = 0 \), then (5) gives \( x_j(-m) v_r = 0 \). Now, assume that all monomials \( a \) with \( l(a) < l(b_1) \) can be expressed with greater monomials of the same degree and weight. We can also assume that \( m = \sum_{i<j} n_i + \delta_{j \leq r} \). Let \( x_k(-n) \) be the smallest factor in \( b_1 \); \( b_1 = x_k(-n) b'_1 \). By (1) and (3) we have

\[
x_j(-m) x_k(-n) = C_1 x_j(-1 - \delta_{j \leq r}) x_k(-n-m+1+\delta_{j \leq r}) + \cdots + C_{m-1} x_j(-m-1) x_k(-n+1) + \cdots
\]

for some \( C_i \in \mathbb{C} \). Multiply this with \( b_2 b'_1 \) and obtain

\[
b = b_2 x_j(-1 - \delta_{j \leq r}) x_k(-n-m+1+\delta_{j \leq r}) b'_1 + \cdots + b_2 x_j(-m+1) x_k(-n-1) b'_1 + b_2 x_j(-m-1) x_k(-n+1) b'_1 + \cdots
\]

On the right-hand side we have monomials

\[
b_2 x_j(-m-1) x_k(-n+1) b'_1, b_2 x_j(-m-2) x_k(-n+2) b'_1, \ldots
\]

which are greater than \( b \). But we also have the first few monomials

\[
b_2 x_j(-1 - \delta_{j \leq r}) x_k(-n-m+1+\delta_{j \leq r}) b'_1, \ldots, b_2 x_j(-m+1) x_k(-n-1) b'_1.
\]

Consider their factors \( x_j(-1 - \delta_{j \leq r}) b'_1, \ldots, x_j(-m+1) b'_1 \). By the inductive assumption, they can be expressed as linear combinations of greater monomials of the same degree and weight. Then it is obvious that by multiplying these linear expressions by \( b_2 x_j(-n-m+1+\delta_{j \leq r}), \ldots, b_2 x_k(-n-1) \) we obtain linear expressions for monomials in (13) in terms of greater monomials. Moreover, these monomials will also be greater than \( b \). Hence, we have expressed \( b \) in terms of greater monomial. □

**Theorem 2.** A spanning set

\[
B = \{ b v_r | b \text{ satisfies (10} \text{ and (3)} \}
\]

is a basis of \( W(\Lambda_r) \).

Proof: Let \( b \in B \). We first prove a particular case: if

\[
C b v_r = 0
\]

then \( C = 0 \). We prove this by induction on degree of \( b \). Let \( x_i(-n) \) be the greatest factor in \( b \); \( b = b' x_i(-n) \).

If \( i \leq r \) then, since \( v_r = e^{i r} = e^{i r} v_0 \), we have

\[
C b v_r = C b e^{i r} v_0 = e^{i r} C' b' v_0 = 0,
\]

where \( C' \in \mathbb{C}^\times \) and \( b' \) is obtained from \( b \) by decreasing degrees of factors of color \( \gamma_1, \ldots, \gamma_r \) by 1 (see (3)). Since \( e^{i r} \) is injective, \( C' b' v_0 = 0 \). Monomial \( b' \) satisfies difference and initial conditions for \( W(\Lambda_0) \) and it is of a smaller degree then \( b \). By the inductive assumption, we conclude that \( C = 0 \).
If \( i > r \) and \( n > 1 \) then use operators \([i][i-1] \cdots [r+1]\) to obtain
\[
C bv_i = 0.
\]
Then, by (8),
\[
C bv_i = C b e^{\omega_i} v_0 = e^{\omega_i} C' C b v_i v_0 = 0,
\]
where \( C' \in \mathbb{C}^\times \) and \( b'' \) is obtained from \( b \) by decreasing degrees of factors of color \( \gamma_i \) by 1 (see (17) and (8)). Again, \( b'' \) satisfies difference and initial conditions for \( W(\Lambda_i) \) and it is of a smaller degree then \( b \). By induction, we conclude that \( C = 0 \).

If \( i > r \) and \( n = 1 \) then use operators \([i-1] \cdots [r+1]\) to obtain
\[
C bv_{i-1} = 0.
\]
Let \( b_{\text{min}} \) be the smallest monomial in (15). We use the same operators as in the previous case to peel down \( C_{b_{\text{min}}} b_{\text{min}} v_i \) to \( C_{b_{\text{min}}} v_{j} \), for some \( j \). Note that operators that we used above at some point annihilate other monomial vectors in (15). We will get
\[
C_{b_{\text{min}}} v_j = 0
\]
and we conclude \( C_{b_{\text{min}}} = 0 \). We proceed inductively to conclude that all coefficients \( C_b \) in (15) are 0.

From this combinatorial description of basis of \( W(\Lambda_i) \) we immediately obtain character formulas (cf. [J1], [T]): for \( n_1, \ldots, n_\ell \geq 0 \), set \( \alpha = n_1 \gamma_1 + \cdots + n_\ell \gamma_\ell \) and
\[
\chi_{W(\Lambda_i)}^\alpha(q) = \sum_q q^{\text{card} \{b \mid w(b) = \alpha, d(b) = i\}}.
\]

**Corollary 3.**
\[
\chi_{W(\Lambda_i)}^\alpha(q) = \frac{q^{\sum_{i=1}^\ell n_i^2 + \sum_{1 \leq i < j \leq \ell} n_i n_j + \sum_{i=1}^\ell n_i}}{(q)_{n_1} (q)_{n_2} \cdots (q)_{n_\ell}},
\]
where \((q)_n = (1 - q) \cdots (1 - q^n)\).

**References**

[AKS] E. Ardonne, R. Kedem, M. Stone, Fermionic characters and arbitrary highest-weight integrable \( sl_{r+1} \)-modules, Comm. Math. Phys. 264 (2006), 427–464.

[C1] C. Calinescu, Intertwining vertex operators and certain representations of \( sl(n) \), Commun. Contemp. Math. 10 (2008), 47–79.

[C2] C. Calinescu, Principal subspaces of higher-level standard \( sl(3) \)-modules, J. Pure Appl. Algebra 210 (2007), 559–575.

[CalLM1] C. Calinescu, J. Lepowsky, A. Milas, Vertex-algebraic structure of the principal subspaces of certain \( A_1^{(1)} \)-modules, I: level one case, Int. J. Math. 19 (2008), 71–92.

[CalLM2] C. Calinescu, J. Lepowsky, A. Milas, Vertex-algebraic structure of the principal subspaces of certain \( A_1^{(1)} \)-modules, II: higher-level case, J. Pure Appl. Algebra, 212 (2008), 1928–1950.

[CalLM3] C. Calinescu, J. Lepowsky, A. Milas, Vertex-algebraic structure of the principal subspaces of level one modules for the untwisted affine Lie algebras of types \( A,D,E \), to appear in Journal of Algebra, math.QA/0908.4054

[CLM1] S. Capparelli, J. Lepowsky and A. Milas, The Rogers-Ramanujan recursion and intertwining operators, Comm. Contemporary Math. 5 (2003), 947–966.

[CLM2] S. Capparelli, J. Lepowsky, A. Milas, The Rogers-Selberg recursions, the Gordon-Andrews identities and intertwining operators, Ramanujan J. 12 (2006), no. 3, 379–397
C. Dong and J. Lepowsky, Generalized Vertex Algebras and Relative Vertex Operators, Progress in Math. 112, Birkhäuser, Boston, 1993.

B. Feigin, M. Jimbo, S. Loktev, T. Miwa and E. Mukhin, Bosonic formulas for \((k, \ell)\)-admissible partitions, Ramanujan J. 7 (2003), no. 4, 485–517.; Addendum to 'Bosonic formulas for \((k, \ell)\)-admissible partitions', Ramanujan J. 7 (2003), no. 4, 519–530.

B. Feigin, M. Jimbo, T. Miwa, E. Mukhin and Y. Takeyama, Fermionic formulas for \((k, 3)\)-admissible configurations, Publ. RIMS 40 (2004), 125–162.

A. V. Stoyanovsky and B. L. Feigin, Functional models of the representations of current algebras, and semi-infinite Schubert cells, (Russian) Funktsional. Anal. i Prilozhen. 28 (1994), no. 1, 68–90, 96; translation in Funct. Anal. Appl. 28 (1994), no. 1, 55–72; preprint B. Feigin and A. Stoyanovsky, Quasi-particles models for the representations of Lie algebras and geometry of flag manifold, hep-th/9308079, RIMS 942.

I. Frenkel, V. Kac, Basic representations of affine Lie algebras and dual resonance models, Invent. Math. 62 (1980), 23–66.

I. Frenkel, J. Lepowsky and A. Meurman, Vertex Operator Algebras and the Monster, Pure and Appl. Math. Vol. 134, Academic Press, Boston, 1988.

G. Georgiev, Combinatorial constructions of modules for infinite-dimensional Lie algebras, I. Principal subspace, J. Pure Appl. Algebra 112 (1996), 247–286.

M. Jerković, PhD thesis, University of Zagreb, 2007.

M. Jerković, Character formulas for Feigin-Stoyanovsky's type subspaces of standard \(\hat{\mathfrak{sl}}(3, \mathbb{C})\)-modules, Ramanujan journal 27 (2011), 357–376.

M. Jerković, M. Primc, Quasi-particle fermionic formulas for \((k, 3)\)-admissible configurations, Central European journal of mathematics 10 (2011), 703–721.

V.G. Kac, Infinite-dimensional Lie algebras, 3rd ed. Cambridge University Press, Cambridge, 1990.

J. Lepowsky, H.-S. Li, Introduction to Vertex Operator Algebras and Their Representations, Progress in Math. 227, Birkhäuser, Boston, 2004.

J. Lepowsky, M. Primc, Structure of the standard modules for the affine Lie algebra \(A_1^{(1)}\), Contemp. Math. 46 (1985), 1–84.

J. Lepowsky, R. L. Wilson, The structure of standard modules, I: Universal algebras and the Rogers-Ramanujan identities, Invent. Math. 77 (1984), 199–290.

A. Meurman, M. Primc, Annihilating fields of standard modules of \(\hat{\mathfrak{sl}}(2, \mathbb{C})\) and combinatorial identities, Memoirs Amer. Math. Soc. 652 (1999)

M. Primc, Vertex operator construction of standard modules for \(A_1^{(1)}\), Pacific J. Math 162 (1994), 143–187.

M. Primc, Basic Representations of classical affine Lie algebras, J. Algebra 228 (2000), 1–50.

M. Primc, \((k, r)\)-admissible configurations and intertwining operators, Contemp. Math. 442 (2007), 245–434.

C. Sadowski, Presentations of the principal subspaces of the higher level \(\hat{\mathfrak{sl}}(3)\)-modules, Journal of Pure and Applied Algebra, 219 (2015) 2300-2345.

C. Sadowski, Principal subspaces of standard \(\hat{\mathfrak{sl}}(n)\)-modules, International Journal of Mathematics, to appear, [arXiv:1406.0095] [math.QA]

G. Segal, Unitary representations of some infinite-dimensional groups, Commun. Math. Phys. 80 (1981), 301–342.

G. Trupčević, Characters of Feigin-Stoyanovsky’s type subspaces of level one modules for affine Lie algebras of types \(A_1^{(1)}\) and \(D_4^{(1)}\), Glasnik Matematički 46 (2011), 49–70.

Faculty of Teacher Education, University of Zagreb, Trg Matice hrvatske 12, Petrinja, Croatia

E-mail address: goran.trupcevic@ufzg.hr