Flavor-Changing Top Quark Rare Decays in the Littlest Higgs Model with T-Parity

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Abstract

We analyze the rare and flavor changing decay of the top quark into a charm quark and
a gauge boson in the littlest Higgs model with T-parity (LHT). We calculate the one-loop
level contributions from the T-parity odd mirror quarks and gauge bosons. We find that
the decay $t \to cV,$ ($V = g, \gamma, Z$) in the LHT model can be significantly enhanced relative
to those in the Standard Model. Our numerical results show that the top quark FCNC
decay can be as large as $\text{Br}(t \to cg) \sim 10^{-2}, \text{Br}(t \to cZ) \sim 10^{-5}$ and $\text{Br}(t \to c\gamma) \sim 10^{-7}$
in the favorite parameter space in the LHT model.

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I. Introduction

The detailed study of the dynamics of top quark production and decay is an important objective of experiments at the Tevatron, CERN Large Hadron Collider (LHC) and a possible International Linear $e^+e^-$ Collider (ILC). The enormous production rates for the top quark at the future colliders, reaching $10^7 \bar{t}t$ pairs per year at the LHC with luminosity of $10fb^{-1}/y$ and $10^5$ per year at a 500 GeV $e^+e^-$ Collider with luminosity of $100fb^{-1}/y$, will allow to perform precision studies of top quark properties. Due to the large mass of the top quark, it may be more sensitive to new physics than other fermions and it may serve as a window to probe new physics beyond the Standard Model (SM).

In the SM, the flavor changing neutral current (FCNC) decays of the top quark, $t \to cV$ with $V = \gamma, Z, g$, are absent at tree level and are extremely suppressed at one-loop level due to the GIM mechanism. Their branching ratios predicted in the SM are of $O(10^{-10})$ or smaller [1, 2], far below the detectable level at the Colliders. The top quark FCNC decays have been extensively studied in various extensions of the SM, e.g. two-Higgs Doublet model (2HDM), left-right models, SUSY, top-color assisted technicolor models (TC2), 331 models and models with extra singlet quarks[3]. It has been realized that the top quark FCNC decay modes can be enhanced by several orders of magnitude in some scenarios beyond the SM and might fall in the reach of the future Colliders.

The Little Higgs mechanism [4, 5] offers an alternative approach to SUSY for weakly coupled electroweak symmetry breaking (EWSB) without fine-tuning. The most compact implementation of the Little Higgs mechanism is known as the Littlest Higgs model [6, 7]. In this model, there are new vector bosons, a heavy top quark and a triplet of heavy scalars in addition to SM particles. The original Little Higgs models suffer strong constraints from electroweak precision data[8]. To solve this problem, a $Z_2$ discrete symmetry named "T-parity" (analogous to R-parity in SUSY) is introduced in Refs. [9, 10]. In the Littlest Higgs model with T parity (LHT), a large part of the model parameter space is consistent with data, and values of the symmetry breaking scale $f$ can be as low as 500 GeV [11].
The LHT model requires the introduction of "mirror fermions" for each SM fermion doublet. The mirror fermions are odd under T-parity and can be given large masses. We can introduce new flavor interactions in the mirror quark sector that could have a very different pattern from the ones present in the SM. In Refs. [13, 14, 16], the impact of the mirror fermions on FCNC processes such as neutral meson mixing and rare $K, B$ meson decays in the LHT model are studied in detail. In this paper, we will concentrate on the top quark FCNC decays $t \to cV$ in the LHT model.

The structure of this paper is as follows: In Sec. II, we briefly review the ingredients of the LHT model that are relevant to our calculation. In Sec. III, numerical analysis of $t \to cV$ in the LHT is presented. Finally, we give a short conclusion in Sec. IV.

II. The Model

A detailed description of the LHT model can be found in the literature [12, 13], here we only review the ingredients which are relevant to the analysis in this paper.

II.1 Scalar sector

The Littlest Higgs model embeds the electroweak sector of the standard model in an $SU(5)/SO(5)$ non-linear sigma model. It begins with an $SU(5)$ global symmetry with a locally gauged subgroup $[SU(2) \times U(1)]^2$. The $SU(5)$ symmetry is spontaneously broken down to $SO(5)$ via a vacuum expectation value (VEV) of order $f$. At the same time, the $[SU(2) \times U(1)]^2$ gauge symmetry breaks to its diagonal subgroup $SU(2)_L \times U(1)_Y$ which is identified as the SM electroweak gauge group.

From the $SU(5)/SO(5)$ breaking, there arise 14 Goldstone bosons which are described by the "pion" matrix $\Pi$, given explicitly by

$$
\Pi = \begin{pmatrix}
-\omega \left( \frac{\eta}{\sqrt{20}} \right) & -i \pi^+ \left( \frac{\eta}{\sqrt{20}} \right) & -i \phi^+ \left( \frac{\eta}{\sqrt{20}} \right) & -i \phi^+ \left( \frac{\eta}{\sqrt{20}} \right) \\
-i \omega \left( \frac{\eta}{\sqrt{20}} \right) & -\omega \left( \frac{\eta}{\sqrt{20}} \right) & -i \omega \left( \frac{\eta}{\sqrt{20}} \right) & -i \omega \left( \frac{\eta}{\sqrt{20}} \right) \\
i \phi^- \left( \frac{\eta}{\sqrt{20}} \right) & -i \phi^- \left( \frac{\eta}{\sqrt{20}} \right) & -i \phi^- \left( \frac{\eta}{\sqrt{20}} \right) & -i \phi^- \left( \frac{\eta}{\sqrt{20}} \right) \\
i \phi^- \left( \frac{\eta}{\sqrt{20}} \right) & -i \phi^- \left( \frac{\eta}{\sqrt{20}} \right) & -i \phi^- \left( \frac{\eta}{\sqrt{20}} \right) & -i \phi^- \left( \frac{\eta}{\sqrt{20}} \right)
\end{pmatrix}.
$$

(2.1)
where it consists of a doublet $H$ and a triplet $\Phi$ under the unbroken $SU(2)_L \times U(1)_Y$ group which are given by

$$H = \left( \begin{array}{c} -i \frac{\pi}{\sqrt{2}} + \sqrt{2} v + i \pi^0 \frac{v}{2} \\ -i \frac{\phi^+}{\sqrt{2}} + i \phi^0 + \phi^0 + i \phi^0 + \phi^0 \end{array} \right), \quad \Phi = \left( \begin{array}{c} -i \frac{\phi^+}{\sqrt{2}} + i \phi^0 + \phi^0 \\ -i \frac{\phi^+}{\sqrt{2}} - i \phi^0 + \phi^0 \end{array} \right). \quad (2.2)$$

Here, $H$ plays the role of the SM Higgs doublet, $h$ is the physical Higgs field and $v \simeq 246$ GeV. The fields $\eta$ and $\omega$ are eaten by heavy gauge bosons when the $[SU(2) \times U(1)]^2$ gauge group is broken down to $SU(2)_L \times U(1)_Y$, whereas the $\pi$ fields are absorbed by the standard model $W/Z$ bosons after EWSB. The field $h$ and $\Phi$ remain in the spectrum.

In the LHT model, a T-parity discrete symmetry is introduced to make the model consistent with the data. Under the T-parity, the fields $\Phi, \omega$ and $\eta$ are odd, and the SM Higgs doublet $H$ is even. The Goldstones $\omega$ and $\eta$ are present in our analysis.

**II..2 Gauge boson sector**

In the LHT model, the T-even gauge boson sector consists only of the SM gauge bosons

$$W_L^\pm, \quad Z_L, \quad A_L \quad (2.3)$$

with masses given to lowest order in $v/f$ by

$$M_{W_L} = \frac{g v}{2}, \quad M_{Z_L} = \frac{M_{W_L}}{\cos \theta_W}, \quad M_{A_L} = 0, \quad (2.4)$$

where $\theta_W$ is the weak mixing angle, and $g$ is the SM $SU(2)$ gauge couplings.

The T-odd gauge boson sector consists of three heavy "partners" of the SM gauge bosons

$$W_H^\pm, \quad Z_H, \quad A_H \quad (2.5)$$

with masses given to lowest order in $v/f$ by

$$M_{W_H} = g f, \quad M_{Z_H} = g f, \quad M_{A_H} = \frac{g' f}{\sqrt{5}}, \quad (2.6)$$

where $g'$ is the SM $U(1)$ gauge couplings. All these heavy gauge bosons will be involved in our analysis.
II..3 Fermion sector

The T-even fermion sector consists of the SM quarks, leptons and an additional heavy quark $T_+$. The T-odd fermion sector consists of three generations of mirror quarks and leptons and an additional heavy quark $T_-$. The $T_-$ will not be present in our analysis for the reason discussed in appendix A of Ref. [14]. Only the mirror quarks are involved in this paper. We denote them by

$$
\begin{pmatrix}
u^1_H \\
d^1_H
\end{pmatrix}, \quad
\begin{pmatrix}
u^2_H \\
d^2_H
\end{pmatrix}, \quad
\begin{pmatrix}
u^3_H \\
d^3_H
\end{pmatrix}.
$$

(2.7)

To the first order of $v/f$, their masses satisfy

$$m_{u^1_H} = m_{d^1_H} \equiv m_{H1}, \quad m_{u^2_H} = m_{d^2_H} \equiv m_{H2}, \quad m_{u^3_H} = m_{d^3_H} \equiv m_{H3}.$$  

(2.8)

II..4 T-odd flavor mixing

In the LHT, the mirror fermions open up a new flavor structure in the model. As discussed in Ref. [13, 14, 16], there are four CKM-like unitary mixing matrices in the mirror fermion sector:

$$V_{H_u}, \quad V_{H_d}, \quad V_{H_l}, \quad V_{H_\nu}.$$  

(2.9)

$V_{H_u}$ and $V_{H_d}$ are for the mirror quarks which are present in our analysis. These mirror mixing matrices are involved in the flavor changing interactions between SM fermions and T-odd mirror fermions which are mediated by the T-odd heavy gauge and Goldstone bosons ($W_H, Z_H, A_H$ and $\omega^\pm, \omega^0, \eta$). $V_{H_u}$ and $V_{H_d}$ satisfy the relation

$$V_{H_u}^\dagger V_{H_d} = V_{\text{CKM}}.$$  

(2.10)

Following the method in [15, 16], we parameterize the $V_{H_d}$ with three angles $\theta^d_{12}, \theta^d_{23}, \theta^d_{13}$ and three phases $\delta^d_{12}, \delta^d_{23}, \delta^d_{13}$

$$V_{H_d} = \begin{pmatrix}
cd_{12}^{d}e^{i\delta_{13}} & -sd_{12}^{d}e^{i\delta_{13}} & cd_{12}^{d}\sqrt{2}e^{-i\delta_{12}}
cd_{23}^{d}e^{i\delta_{12}+\delta_{23}} & -sd_{23}^{d}e^{i(\delta_{12}+\delta_{13})} & cd_{23}^{d}\sqrt{2}e^{-i\delta_{12}}
cd_{13}^{d}e^{i\delta_{12}} & -sd_{13}^{d}e^{i\delta_{12}} & cd_{13}^{d}\sqrt{2}e^{-i\delta_{12}}
\end{pmatrix}.$$  

(2.11)
The matrix $V_{Hu}$ is then determined through $V_{Hu} = V_{Hd} V^{\dagger}_{CKM}$. The Feynman rules for the flavor violating interactions which are involved in our analysis can be found in Appendix B of Ref. [16].

III. $t \to cV$ in the LHT model

In the LHT model, the flavor changing rare top decay $t \to cV$ ($V = \gamma, Z, g$) can be induced by the interactions between SM quarks and T-odd mirror quarks mediated by heavy T-odd gauge bosons and Goldstone bosons at one-loop level. The relevant Feynman diagrams are shown in Fig.1.

\[ \mathcal{M}(t \to cV) = \bar{u}(p_2) V^\mu u(p_1) \epsilon_\mu(k, \lambda), \]  

(3.1)

where $p_1, p_2$ and $k$ are the momenta of the incoming top quark, outgoing charm quark, and outgoing gauge boson, respectively. $\epsilon_\mu(k, \lambda)$ is the polarization vector for the outgoing gauge
boson. At one-loop level, the diagrams in Fig.1 give rise to effective vertices $V^\mu$ which can be written as

\[
V^\mu(tcZ) = ie[\gamma^\mu(P_LA_Z^L + P_RA_Z^R) + k_\mu \sigma^{\mu\nu}(P_LB^Z_L + P_RB^Z_R)]
\]
\[
V^\mu(tc\gamma) = ie[\gamma^\mu(P_LA_\gamma^L + P_RA_\gamma^R) + k_\mu \sigma^{\mu\nu}(P_LB_\gamma^L + P_RB_\gamma^R)]
\]
\[
V^\mu(tcg) = ig_sT^a[\gamma^\mu(P_LA_g^L + P_RA_g^R) + k_\mu \sigma^{\mu\nu}(P_LB_g^L + P_RB_g^R)]
\] (3.2)

where $P_{R,L} = \frac{1}{2}(1\pm\gamma^5), \sigma^{\mu\nu} = \frac{i}{2}[\gamma^{\mu}, \gamma^{\nu}]$ and $T^a$ are the generators of $SU(3)_C$. The form factors $A_{L,R}, B_{L,R}$ are the sums of the contributions from the diagrams in Fig.1. They encode the loop functions and depend on the masses and momenta of the external and internal quarks, heavy quarks, gauge bosons and heavy gauge bosons. For simplicity, we omit the explicit expressions for these form factors. The feynman diagrams and the corresponding amplitudes are generated by using *FeynArts3* [17]. We have added the relevant Feynman rules of the LHT model in *FeynArts* package. In the calculations of the one-loop diagrams we adopt the definitions of one-loop integral functions as in Ref.[18]. The loop integral functions are calculated by using the formulas in Ref.[19].

The new parameters in the LHT which are relevant to our analysis are

\[
f, m_{H1}, m_{H2}, m_{H3}, \theta^d_{12}, \theta^d_{23}, \theta^d_{13}, \delta^d_{12}, \delta^d_{23}, \delta^d_{13}
\] (3.3)

It is convenient to consider several representative scenarios for the structure of the $V_{Hd}$. In Ref.[13], the constraints on the mass spectrum of the mirror fermions have been investigated from the analysis of neutral meson mixing in the $K, B$ and $D$ systems. They found that a TeV scale GIM suppression is necessary for a generic choice of $V_{Hd}$. However, there are regions of parameter space where there are only very loose constraints on the mass spectrum of the mirror fermions. We will study the $t \rightarrow cV$ in the regions of parameter space where the mass spectrum is not severely constrained in Ref.[13]. We concentrate our study in the following two scenarios for the structure of the $V_{Hd}$

- **Case I** $V_{Hd} = 1, V_{Hu} = V_{CKM}^\dagger$
- **Case II** $s^d_{23} = 1/\sqrt{2}, s^d_{12} = 0, s^d_{13} = 0, \delta^d_{12} = 0, \delta^d_{23} = 0, \delta^d_{13} = 0$
In both the cases, the constraints on the mass spectrum of the mirror fermions are very relaxed.

Before presenting numerical the results of $t \rightarrow cV$ in the LHT, we would like to comment on the sensitivity of the future experiments to FCNC top quark decays. At the LHC with a 100 $fb^{-1}$ integrated luminosity, the branching ratios can be measured as small as [20] 

$$Br(t \rightarrow cg) \leq 7.4 \times 10^{-3},$$
$$Br(t \rightarrow cZ) \leq 1.1 \times 10^{-4},$$
$$Br(t \rightarrow c\gamma) \leq 1.0 \times 10^{-4}$$

(3.4)

Figure 2: Case I $V_{Hd} = 1$, $V_{Hu} = V_{\text{CKM}}^\dagger$. For the left figure, $f$ is taken to be 500 GeV. For the right one, $f$ is equal to 1000 GeV.

Now we present the numerical results for $Br(t \rightarrow cV)$. We take $m_t = 171.4$ GeV, $m_Z = 91.187$ GeV, $m_W = 80.4$ GeV, $m_c = 1.25$ GeV, $G_F = 1.16639 \times 10^{-5} (\text{GeV})^{-2}$, $\alpha = 1/128$ and $\alpha_s = 0.107[21]$. Because the total width of the top-quark is dominated by the leading decay mode $t \rightarrow bW^+$, we define the branching ratio as

$$Br(t \rightarrow cV) = \frac{\Gamma(t \rightarrow cV)}{\Gamma(t \rightarrow bW^+)}.$$  

(3.5)

In fig.2 we present the plot of the branching ratio of $t \rightarrow cV$ ($V = g, \gamma, Z$) as a function of the mass of the third generation mirror quark for Case I. In this case we have taken $V_{Hd} = 1$,
\( V_{Hu} = V_{CKM}^\dagger \), the mixing in the down type gauge and Goldstone boson interactions are absent. In this case there are no constraints on the masses of the mirror quarks at one loop level from the \( K \) and \( B \) systems and the constraints come only from the \( D \) system. The constraints on the mass of the third generation mirror quark are very weak according to the analysis in Ref. \[13\]. From fig.2, we can see that the branching ratios of \( t \to cV \) rise very fast with the increasing mass of the third generation mirror quark. For \( f = 500 \text{ GeV} \) and \( m_{H3} = 5000 \text{ GeV} \), the branching ratios can reach

\[
\begin{align*}
\text{Br}(t \to cg) &\approx 1.33 \times 10^{-3} \\
\text{Br}(t \to cZ) &\approx 3.83 \times 10^{-6} \\
\text{Br}(t \to c\gamma) &\approx 2.33 \times 10^{-7}.
\end{align*}
\] (3.6)

Figure 3: Case II \( f = 500 \text{ GeV} \), \( s_{23}^d = 1/\sqrt{2} \), while other angles are set to zero. For the left figure, \( m_{H1} = m_{H2} \) is taken to be 1000 GeV. For the right one, \( m_{H1} = m_{H2} \) is equal to 3000 GeV.

In fig.3 the dependence of branching ratio of \( t \to cV \) on \( m_{H3} \) is presented for Case II. In this case, the constraints from the \( K \) and \( B \) systems are also very weak. Compared to Case I, the mixing between the second and the third generations are enhanced with the choice of a bigger mixing angle \( s_{23}^d \). The branching ratios of \( t \to cV \) are significantly enhanced even with stricter constraints on the masses of the mirror quarks from the \( D \) system. For \( f = 500 \)
GeV, $m_{H1} = m_{H1} = 3000$ and $m_{H3} = 4000$ GeV, the branching ratios can reach

$$Br(t \rightarrow cg) \approx 1.44 \times 10^{-2}$$
$$Br(t \rightarrow cZ) \approx 2.56 \times 10^{-5}$$
$$Br(t \rightarrow c\gamma) \approx 1.33 \times 10^{-7}. \quad (3.7)$$

In this case, the branching ratio of $t \rightarrow cg$ can reach the detectable level of the LHC. For $t \rightarrow cZ$, the branching ratio is slightly below the expected reach of the LHC.

### IV. Conclusion

We have calculated the one-loop contributions from the T-odd quarks and gauge bosons to the rare and flavor changing decay of the top quark in the LHT model. We find the branching ratios of the $t \rightarrow cV$, ($V = g, \gamma, Z$) in the LHT model can be significantly enhanced relative to those in the SM. In our numerical analysis, we have chosen two favorite scenarios for the structure of the mixing matrix $V_{Hd}$ in the mirror fermion sector. For both of these cases, the constraints on the mass spectrum of the mirror quarks from the data of neutral meson mixing are very weak. Our numerical results show that the branching ratios of top quark decay into a charm quark and a gauge boson can reach to $Br(t \rightarrow cg) \sim 10^{-2}$, $Br(t \rightarrow cZ) \sim 10^{-5}$ and $Br(t \rightarrow c\gamma) \sim 10^{-7}$ for optimistic parameter choice in the LHT model.

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