Geometric thinking in learning distance and angle

Anton Jaelani
Universitas Muhammadiyah Purwokerto
E-mail: antonjaelani@ump.ac.id

Abstract. Solid geometry is an essential science that students should master because students encounter it in the real world. Students' geometric thinking ability is still low because they have difficulty understanding the concepts. The purpose of this study was to determine the comparison of the achievement and increase in geometric thinking skills of students who received generative learning assisted by SketchUp and Direct Learning. This research is quasi-experimental, and the research instrument is a test of geometric thinking skills. The results showed that there was no difference in achievement and improvement. The results of this study suggest that researchers and teachers look for other alternatives to improve students' geometric thinking skills.

1. Introduction
To see the students' ability to master geometry, van Hiele provides a thinking level model called geometric thinking. Van Hiele's levels of geometric thinking require previous mastering levels of thinking in order to reach a higher level. Van Hiele's geometric thinking model focuses on students' mastery of understanding the concepts of geometry related to the properties of an object. The mastery of geometric concepts, according to van Hiele's geometric thinking model, can be used as a basis for students to solve geometric problems. For example, knowing that the distance is the shortest path, students will be able to determine the distance between the point and the plane. Through van Hiele's geometric thinking model, the mental development of students can be seen when they begin to break away from dependence on using real objects as a means of learning. With high geometric thinking skills, students will practice logic and abstract thinking. Increasing the level of geometric thinking skills is one of the main goals of mathematics education because geometric thinking is critical in the fields of science, engineering, and the world of work [1]. Furthermore, one-third of the math questions on university entrance tests in Turkey consisted of geometry [1].

In studying three-dimensional geometry, students often have difficulty understanding and relating postulates, definitions, and properties of an object. Students also cannot look flexibly at the parts of the object that are projected on the image plane and from specific points of view so that students usually find it difficult to visualize these objects in their minds. Difficulty learning three-dimensional objects can be overcome by facilitating students to help visualize these objects using specific media.

In their daily life, humans are surrounded by geometric objects located in a room of various shapes and sizes. In understanding and solving problems related to these objects, humans should have the necessary capital by mastering geometry. Without geometry, creative ideas cannot be realized in reality. Geometry will lead students to other abilities related to numbers, measurement, and relationship. By learning geometry, students will practice creative thought patterns and think in ways that other people do not usually do or have never thought of before. The relationships between objects,
their elements, and their concepts will be carried and/or transformed by students into thinking patterns in their daily lives. Learning geometry is an excellent way to practice doing activities and using concrete or abstract materials or objects.

Researchers have made various ways to contribute to improving students' geometric skills. Abu and Abidin stated from the results of their research that the level of geometric thinking of students who experienced learning using van Hiele theory-based geometry learning videos was significantly different compared to before using it [2]. Further information describes that the average increased from 2.71 to 3.00 for level 1, from 2.20 to 10.03 for level 2, and 4.87 to 11.73 for level 3. Siew, Chong, & Abdullah (2013) have observed that learning using tangram can significantly improve students' geometric thinking skills. In the study, it was explained that there was a decrease in the average difference in the geometric thinking ability of students with high, medium, and low initial mathematical abilities, namely between high, medium, high and low, and medium and low respectively, namely 8.27; 17.61; and 9.37. Liu and Cummings have tried to stimulate students' geometric thinking by using PCLogo and Geometer's Sketchpad in learning [3].

The ability to think geometrically is taken from a phrase in English, namely geometric or geometrical thinking ability. Ability means the ability, skill, or physical or mental strength needed to do something. Thinking is a noun which means the process of considering something mentally and geometrically is an adjective of geometry [4]. From this explanation, the ability to think geometrically can be interpreted as a skill in processing to mentally consider something related to the nature of geometry.

Researchers put forward various understandings about the ability to think geometrically. The ability to think geometrically is defined as the ability to conceptualize and manipulate forms, concepts, properties, and relationships in geometric content [5]. Geometric thinking is the ability to think and reason in the context of geometry [6]. Geometric thinking is a form of mathematical thinking with a particular content domain, namely geometry [7].

Dina and Pierre van Hiele discovered geometric thinking in the late 1950s. Liu states that van Hiele's geometric thinking model describes the level of thinking and does not explain the process [3]. Van Hiele's geometric thinking model can be used to guide learning and assess students' abilities [8]. Most of the references that use the term geometric thinking ability refer to the concepts put forward by van Hiele.

Van Hiele's theory provides a level of thinking that spans the ages of five to academic adulthood. Initially, there were five levels, which were later adapted and renamed by many researchers, but now, van Hiele is concentrating its discussion on the three levels taken during the regular school period. The main content focus is on two-dimensional forms [9].

Aspects of geometric thinking can be seen from the level of ability to think. There are different terms and codes used by the authors in ordering the levels. Abdullah and Zakaria mentioned five levels of van Hiele's geometric thinking skills, namely van Hiele's geometric thinking level, namely visualization, analysis, informal deduction, formal deduction, and rigor [10]. Abu and Abidin mentioned these levels differently, namely, recognition as level 0, analysis as level 1, order as level 2, deduction as level 3, rigor as level 4 [2]. In other literature, Liu and Cummings only mentioned three levels of geometric thinking, according to van Hiele, namely visual level, level descriptive/analytical, and the abstract/relation level [3]. According to Mason, in his original work, van Hiele numbered the levels from 0 to 4, and Americans began numbering the levels from 1 to 5 [11]. This numbering scheme from 1 to 5 allowed the precognition levels to be numbered 0. Clements and Battista also propose a level 0, which is called precognition. Students at this level know only a subset of the visual properties of shapes, resulting in an inability to distinguish between shapes [12]. For example, they may be able to distinguish between a triangle and a quadrilateral but not a rhombus and a parallelogram. Pierre van Hiele's new jobs describe only three levels of thinking rather than five levels.

In contrast, to van Hiele and other researchers, Hoffer uses the term expertise in geometry and has his classification of it [13]. However, he linked the classification he presented with the level of geometric thinking conveyed by van Hiele. Moreover, the examples it gives are closer to geometric thinking concepts than spatial abilities. Therefore, the true meaning of the ability in geometry is the
ability to think geometrically as previously described. The difference is that the classification is not a form of level or level but a separate classification from one another.

The five classifications of geometric thinking skills presented by Hoffer are visual skills, verbal skills, drawing skills, logic skills, and application abilities [13]. Each of these abilities can be included in level 1 to level 5 van Hiele's geometric thinking. Hoffer does not explicitly define each of these geometric thinking abilities, but he provides clear examples of each of these abilities [13]. One example of a problem that tests visual abilities at the deduction level mentioned by Hoffer is "A rectangular sheet of paper is rolled to form a vertical tube, what form of paper is needed to form an inclined tube?" [13]. Another example for the ability to draw at the level of analysis is "Make a rectangular drawing that has one length and one of the diagonals known first." An example of a logic capability, including level ordering would be "Is every rectangle a square?"

The purpose of this study was to determine the effect of generative learning assisted by SketchUp on the achievement and improvement of geometric thinking skills. The achievement and improvement of geometric thinking skills involve the students' initial level of mathematical ability. Study shows the achievement and improvement of geometric thinking skills for each of the indicators.

2. Method

This type of research is quantitative research, and the research subjects are 219 grade XII high school students in Banyumas, Central Java, Indonesia. The research was conducted from August to November 2018. The research instrument used a spatial ability test. The instrument is geometric thinking test and the data analysis used statistics and the average test.

3. Result and Discussion

The calculation of the free sample t-test on the data on the achievement of GTA shows that the 2-tailed significance value obtained is 0.848, so the 1-tailed significance value obtained is 0.848 divided by two so that it becomes 0.424. This value is more than 0.05, so H0 is accepted. The acceptance of H0 can also be seen through the results of the t value, which is 0.192. This value is smaller than the critical value, which is 1.652. Therefore, the conclusion of this free sample t-test is that with a confidence level of 95%, the average GTA achievement of students who get SAGL is the same as the average GTA achievement of students who get DL.

Table 1. Results of the Free Sample t-test on the achievement of GTA

| Instruction | N   | Significance | t Value | Critical t | Decision |
|-------------|-----|--------------|---------|------------|----------|
| SAGL        | 113 | 0.424        | 0.192   | 1.652      | Accept H0 |
| DL          | 106 |              |         |            |          |

This free sample t-test calculation yields a significant value for the 2-tailed of 0.832 so that the significance value for the 1-tailed is 0.832 divided by 2, which is 0.432. The significance value for this 1-tailed is greater than 0.05. Besides, the resulting t value is 0.212. The t value is smaller than the t-critical value, which is 1.652. Therefore, H0 is accepted. Thus, the conclusion of the free sample t-test on the increase in GTA of students who received SAGL and students who received DL was that the average increase in GTA of students who received SAGL was the same as the average increase in GTA of students who received DL.

Table 2. Results of the Free Sample t-test on the achievement of GTA

| Instruction | N   | Significance | t Value | Critical t | Decision |
|-------------|-----|--------------|---------|------------|----------|
| SAGL        | 113 | 0.432        | 0.212   | 1.652      | Accept H0 |
| DL          | 106 |              |         |            |          |

Regarding the GTA, the results of this study state that the achievement of the GTA of students who get SAGL is the same as the achievement of the GTA of students who get the DL. This result contradicts the results of Abdullah and Zakaria's research, which states that the use of geometric tools in mathematics learning has increased the level of geometric thinking skills [10]. The details of the comparison of the average GTA achievement per indicator are presented in Table 3 which shows that...
the average GTA achievement of students who get SAGL is less than the achievement of the GTA of students who get the DL for each indicator except indicator 2 (calculating the distance between points and fields based on definitions and characteristics—it is nature). The difference between the GTA achievement is for indicator 1 (determining the distance between points and lines according to the concept), indicator 3 (sketching the angle between the line and the plane based on its definition), indicator 4 (providing reasons related to the distance between two parallel lines), and indicators 5 (providing logical evidence regarding the angle between two areas of definition and their properties) are 80.66 - 73.19 = 7.47, respectively; 20.57 - 19.47 = 1.10; 28.21 - 24.82 = 3.39, and 33.58 - 28.76 = 4.82. For indicator 2, the difference in GTA achievement between students who get SAGL and students who get DL is 33.54 - 14.81 = 18.73. Even though the achievement of GTA of students who get SAGL is the same as the achievement of GTA of students who get DL, there is quite a big difference in the achievement of GTA per indicator, especially for indicator 1 and indicator 2.

Table 3. Comparison of the Average GTA Achievement per Indicator

| Instruction | Indicator 1 | Indicator 2 | Indicator 3 | Indicator 4 | Indicator 5 |
|-------------|-------------|-------------|-------------|-------------|-------------|
| SAGL        | 79.19       | 33.54       | 19.47       | 24.82       | 28.76       |
| DL          | 80.66       | 14.81       | 20.57       | 28.21       | 22.58       |

Information:
1: Determine the distance between points and lines according to the concept
2: Calculates the distance between a point and a plane based on its definition and properties
3: Gives a reason related to the distance between two parallel lines
4: Gives the reason related to the distance between two parallel lines
5: Provides logical evidence regarding the angle between the two areas of definition and their properties

These results provide information that the achievement of the GTA of students who get SAGL is only better at indicator 2 than the achievement of the GTA of students who get the DL. This can happen because the answers given by students are related to previous knowledge that is strengthened in learning, namely about the similarity of the area of a triangle. The linkage of previous learning and experiences to be used in solving problems in generative learning is very strong so that some students get high scores. This high score contributes greatly to the achievement of GTA for indicator 2.

The same situation occurred in the increase in GTA of students who received SAGL and students who received DL as a whole. The results of the study stated that overall, the increase in the GTA of students who received SAGL was the same as the increase in students who received DL. For indicator 1, indicator 3, indicator 4, and indicator 5, the increase in the GTA of students who get SAGL is smaller than the increase in GTA of students who get DL. The differences were 0.732 - 0.626 = 0.106, respectively; 0.077 - 0.061 = 0.016; 0.627 - 0.204 = 0.423, and 0.323 - 0.278 = 0.045. Conversely, for indicator 2, the increase in GTA of students who received SAGL was also greater than the increase in GTA of students who received DL by a difference of 0.307 - 0.138 = 0.169.

The significant difference in the increase in GTAs can occur because of the existence of one of the strategies in generative learning, namely recalling the knowledge that has been learned and mastered by students. This knowledge is about the Pythagorean theorem and the similarity of the area of the triangle that students get when they are in junior high school. Examples of student answers to indicator 2 are shown in Figure 1. The answer shows that the student calculates the area of a triangle using different bases and heights to get the x value. The base and height of the first are 10√2 and 10 and the base and height of the second are 5√6 and x.
The comparison chart of the average achievement and increase in GTA per indicator also shows that the lowest indicator value is indicator 3. It happens because indicator 3 includes the level of deduction in the ability to think geometrically. According to van Hiele, the higher the level of geometric thinking, the higher the level of difficulty [14]. This result is also in line with the research of Wijayanti, Nikmah, & Pujiastuti which proves that the higher the level of geometric thinking, the more indicators of problem-solving that cannot be achieved by students [15]. Another evidence that is in line with this is the research of Salifu whose results state that only five people can reach the deduction level out of the 131 people he studied [16].

Table 4. Comparison of the Average GTA Enhancement per Indicator

| Instruction | Indicator 1 | Indicator 2 | Indicator 3 | Indicator 4 | Indicator 5 |
|-------------|-------------|-------------|-------------|-------------|-------------|
| SAGL        | 0.626       | 0.307       | 0.061       | 0.204       | 0.278       |
| DL          | 0.732       | 0.138       | 0.077       | 0.267       | 0.323       |

Information:
1: Determine the distance between points and lines according to the concept
2: Calculates the distance between a point and a plane based on its definition and properties
3: Gives a reason related to the distance between two parallel lines
4: Gives the reason related to the distance between two parallel lines
5: Provides logical evidence regarding the angle between the two areas of definition and their properties

At this level of deduction, most students did not get the value of what they answered because they did not show how to get to the right answer, both from students who got SAGL and DL. Some even did not answer at all. Besides, the low achievement and increase in GTA in indicator 3 are also due to its relation to angles. If the material about distances and angles is compared, then the material about angles is more complicated than the material about distances.
Proof:

because GT intersects the LJ line which is the altitude, therefore GT is perpendicular to LJ

Overall, the achievement and improvement of KS students who received SAGL were estimated to be better than the achievement and improvement of GTA who received DL, but the results of this study indicate that the results of achievement and improvement of GTA are not as predicted. Based on observations when learning, one of the reasons for this is that students are not used to regularly and continuously to follow and experience learning that has a constructivist approach. Students are also not used to solving problems based on their thinking skills. Even though sometimes students get an active learning process but the questions in the evaluation test that they have to solve are questions of the same type as what they get when studying so that they do not rely too much on logic thinking and less mathematical thinking skills and this is what they are used to experiencing.

The implementation of new learning methods that are rarely experienced by students can be an obstacle for students to get maximum evaluation test results. Tharayil et. al stated that the new active learning strategy implemented in a classroom looks spontaneous and temporal so that students need to be accustomed to doing activities that support active learning [17]. It is as conveyed by Aksit, Niemi, & Nevgi, namely that some of the obstacles in learning that require student activity are passive learning culture, lack of motivation and initiative, and lack self-confidence [18]. Student passivity and lack of student motivation have been resolved by facilitating worksheets containing questions that can elicit their thinking. However, students did not try harder and did not have the initiative to answer with answers that they did not typically answer. In addition, students are not confident in answering the questions presented in the evaluation test because similar questions that they have answered in learning are not in the evaluation test. Students are used to being in a comfort zone facing evaluation test questions that are similar to the questions they have encountered in learning. When answering questions that demand to reason, students only answer in short sentences without exploring the connection between the knowledge they have acquired. Figure 2 shows the short answer of the students even though this question demands an ample reason.
4. Conclusion
The results of this study indicate that there are no differences in the achievement and increase in geometric thinking skills of students who get generative learning assisted by SketchUp and direct learning. The highest indicator of achievement and improvement is the level 1 indicator. However, generative learning assisted by SketchUp is effective in improving the geometric thinking skills of students whose initial mathematical abilities are moderate.

Acknowledgement
Researchers would like to thank Prof. Yaya Sukjaya Kusumah, M.Sc., Ph.D. and Prof. Turmudi, M.Ed., M.Sc., Ph.D. for guidance in carrying out this research.

References
[1] Olkun S, Sinoplu N B, and Deryakulu D 2005 Geometric explorations with dynamic geometry applications based on van Hiele levels International Journal for Mathematics Teaching and Learning Online p 1 – 8
[2] Abu M S and Abidin Z Z 2013 Improving the levels of geometric thinking of secondary school students using geometry learning video based on van Hiele theory International Journal of Evaluation and Research in Education (IJERE) 2 p 16 – 22
[3] Liu L and Cummings R 2001 A Learning model that stimulates geometric thinking through use of PCLogo and geomter’s sketchpad Computers in the Schools 17 p 85 – 104
[4] Hornby A S 1995 Oxford advanced learner’s dictionary of current english (Oxford: Oxford University Press) p 2 – 493
[5] Evans, M A and Wilkins J L M 2011 Social interactions and instructional artifacts: emergent socio-technical affordances and constraints for children’s geometric Thinking Journal of Educational Computing Research 44 p 141 – 71.
[6] Van de Walle J A 2001 Elementary and middle school mathematics: teaching developmentally (Boston: Pearson Education) p 306 – 312
[7] Dindyal J 2007 The need for an inclusive framework for students’ thinking in school geometry The Montana Mathematics Enthusiast 4 p 73 – 83
[8] Crowley M L 1987 Learning and teaching geometry K – 12 (Virginia: National Council of Teachers of Mathematics) p 1 – 16
[9] Way J 2012 The development of spatial and geometric thinking: the importance and instruction (Cambridge: University of Cambridge) p 1 – 5
[10] Abdullah A H and Zakaria E 2013 Enhancing students’ level of geometric thinking through van Hiele’s phase-based learning Indian Journal of Science and Technology 6 4432 – 46.
[11] Mason M 2002 Professional handbook for teachers, geometry: exploration and applications (Georgia: McDougal Litterll Inc) p 1 – 8
[12] Clements D H and Battista M T 1992 Handbook of research on mathematics teaching and learning (New York: Macmillan) p 420-464
[13] Hoffer A 1981 Geometry is more than proof The Mathematics Teacher 74 p 11 – 8
[14] Van Hiele P M 1999 Developing geometric thinking through activities that begin with play Teaching Children Mathematics 6 p 310 – 316.
[15] Wijayanti K, Nikmah A, and Pujiastuti E 2018 Problem-solving ability of seventh-grade students viewed from geometric thinking levels in search-solve-create-share learning model UNNES Journal of Mathematics Education 7 p 8 – 16.
[16] Salifu A S 2018 Gender geometric reasoning stages and gender differences in achievements of preservice teachers of e. p. college of education, Bimbilla, Ghana International Journal of Research and Development 7 p 13 – 25.
[17] Tharayil S, Borrego M, Prince M, Nguyen K A, Shekhar p, Fineli C I, and Waters C 2018 Strategies to mitigate students resistance to active learning. International Journal of STEM Education 5 p 1 – 16
[18] Aksit F, Niemi H, and Nevgi A 2016 Why is active learning so difficult do implement: the Turkish case Australian Journal of Teacher Education 41 p 94 – 109