Axial Anomaly, Mismatched Fermi Surfaces and Vector Interaction in Dense Neutral Quark Matter

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We report on effects of the q-\overline{q} vector interaction and/or the U(1)\(_A\)-anomaly-induced chiral-diquark coupling on the charge-neutral quark matter in \(\beta\)-equilibrium. We show that when the vector coupling is absent, there can appear a cross-over region sandwiched by two critical points in the intermediate temperature (\(T\)) region, while the phase transition in the low-\(T\) region including zero temperature keeps being first order until the strength of the anomaly term is increased to have a critical value. On the other hand, when the vector coupling is also present, there appears a crossover region in the low-\(T\) area including zero temperature with a new critical point, as was first demonstrated by Kitazawa et al and the present authors without and with the charge-neutral condition, respectively. We remark that the possible chromomagnetic instability is suppressed and can be even completely absent owing to the enhanced diquark coupling due to the anomaly term and/or by the vector interaction.

\section{1. Introduction}

In this report, we discuss the phase diagram of the charge-neutral quark matter under beta-equilibrium constraint, taking into account the q-\overline{q} vector interaction and/or U(1)\(_A\)-anomaly-induced chiral-diquark coupling\textsuperscript{1}.

\subsection{1.1. Effect of the vector-type interaction in chiral transition}

The significance of the vector-vector interaction \(\sim G_V(\overline{q}\gamma^\mu q)^2\) on the chiral phase transition in hot and dense quark matter is known for a long time, and clearly demonstrated by Kitazawa et al\textsuperscript{2} who showed that the increase of the vector coupling \(G_V\) enlarges the crossover region, and the celebrated QCD critical point\textsuperscript{3} eventually disappears completely in the phase diagram with a sufficiently large coupling: See Fig. 13 of [2].

When the possible color-superconducting(CSC) phase transition is taken into consideration, the phase boundary for the chiral-to-CSC phase transition becomes of crossover in the low temperature (\(T\)) region including vanishing temperature, as is shown in Fig.8 of [2] by Kitazawa et al.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1}
\caption{The schematic figures to show that the diquark gap plays a similar role as temperature for the chiral transition: (a) the distribution function at finite \(T\), \(n = 1/(e^{(p-\mu)/T} + 1)\) and (b) that for CSC phase with a diquark gap \(\Delta\). Taken from [4].}
\end{figure}
The reason why the phase transition involving the chiral restoration becomes so weak and turns to crossover is a smearing of the Fermi surface due to the diquark gap, which is analogous to that due to finite $T$ where the quark distribution function is smeared and the chiral transition is known to become crossover at zero chemical potential: See Fig. 1. Notice that a positive energy state of the fermion has a positive matrix element $\langle \bar{q}q \rangle$ while the vacuum chiral condensate has a negative value, and hence the presence of the positive-energy fermions act to decrease the chiral condensate in the absolute value.

Such a competition between the chiral and CSC phase transition is enhanced by the vector interaction because the repulsive term $G_V \rho^2$ postpones the emergence of the high density, although the large Fermi sphere is preferable for the formation of the CSC phase.

1.2. **Effect of electric charge neutrality**

In the asymmetric homogeneous quark matter under the electric charge neutrality, u, d and s quarks have different densities, namely $\rho_d > \rho_u > \rho_s$, or the mismatched Fermi surfaces, which disfavors the pairing. However, when the system is heated, the resulting smeared Fermi surfaces at finite $T$ make the diquark pairing with mismatched Fermi surfaces possible, and hence the diquark gap shows an abnormal temperature dependence that it has a maximum value at a finite temperature.

Then, the competition between the chiral and the diquark correlations becomes largest at an intermediate $T$, which can lead to a nontrivial impact on the chiral-to-CSC transition. Indeed, Zhang, Fukushima and Kunihiro showed that there can appear a crossover region in the phase boundary sandwiched by novel two critical end points, although the resulting phase diagram is strongly dependent on the choice of the parameters in the model Lagrangian.

1.3. **Combined effect of vector interaction and electric charge neutrality**

The combined effect of the vector interaction and the charge-neutrality condition was examined by Zhang and Kunihiro. Although the structure of the phase diagram depends on the choice of the strengths of the interaction, there can appear three cross over region or two first-order critical lines, the ends of which are attached by critical points: the appearance of a crossover region in the low $T$ region including zero temperature is due to the vector coupling, whereas that in the intermediate $T$ region is understood to be due to the abnormal temperature dependence of the diquark gap inherent for the asymmetric quark matter with mismatched Fermi surfaces.

§2. **Incorporation of anomaly-induced chiral-diquark coupling**

Recently, Zhang and Kunihiro investigated the phase diagram taking into account the following anomaly term $L_{\chi_d}^{(6)} = K' / 8 \cdot \sum_{i,j,k=1}^{3} \sum_{\pm} [(\psi_i^T t_i^c (1 \pm \gamma_5) \psi_C) (\bar{\psi}_j^T t_j^c (1 \pm \gamma_5) \psi_j)]$, as well as the standard Kobayashi-Maskawa-'t Hooft term (with $K$ being the strength) in addition to the four-Fermi scalar-pseudoscalar ($G_S$) and the $U(3)_L \times U(3)_R$-invariant vector-axial-vector interactions. Then the con-
stituent quark masses $M_i$ and the dynamical Majorona masses $\Delta_i$ are expressed in terms of the chiral and diquark condensates, $\sigma_i = \langle \bar{\psi}_i \psi_i \rangle$ and $s_i = \langle \bar{\psi}_C i \gamma_5 \gamma_i \bar{t}^j \psi \rangle$, as follows ($G_D$ being the diquark coupling): $M_i = m_i - 4G_S \sigma_i + K |\varepsilon_{ijk} \sigma_j \sigma_k + \frac{K'}{4} s_i|^2$, and $\Delta_i = 2(G_D - \frac{K'}{4} \sigma_i) s_i$, respectively. Here $m_u = m_d = 5.5$ MeV and $m_s = 140.7$ MeV denote the respective current quark masses. The notable point is that $\mathcal{L}_{\chi d}^{(6)}$ induces the chiral-diquark coupling as is manifested in the fourth and second term of $M_i$ and $\Delta_i$, respectively. Adapted from [10].

![Fig. 2](image)

(a) $K'/K = 2.0$ (b) $K'/K = 2.25$ (c) $K'/K = 2.4$ (d) $K'/K = 2.8$

Fig. 2. The phase diagrams for various values of $K'$ in the two-plus-one-flavor Nambu-Jona-Lasinio model with the charge-neutrality and $\beta$-equilibrium being kept, but without the vector interaction. The thick-solid, thin-solid and dashed lines denote the first-order, second-order and chiral crossover critical lines, respectively. Adapted from [1].

2.1. Without the vector term

In Fig.2, we first show the phase diagrams of the charge neutral quark matter for various ratio $K'/K$ when the vector term is absent ($G_V = 0$) [1]. When the ratio $K'/K$ is as small as 2.0, we have the standard phase diagram with a single critical point, although the existence of the combined chiral-diquark phase in the low-$T$ region is notable. Then $K'/K$ is increased up to 2.2, the crossover window opens in the intermediate-$T$ region, inherent for the charge-neutral system with mismatched Fermi surfaces; we note that the competition between the chiral and diquark correlations is enhanced by the new anomaly term. When $K'$ is further increased, the diquark correlation becomes so large that the chiral transition in the CSC phase becomes smooth, and eventually the first-order critical line disappears completely in the phase diagram. Notice, however, that the crossover window never opens in the low-$T$ region including zero temperature, as is the case without the charge neutrality constraint [10] which is contrary to the observation in [9].

![Fig. 3](image)

(a) $K'/K = 0.55$ (b) $K'/K = 0.57$ (c) $K'/K = 0.70$ (d) $K'/K = 1.0$

Fig. 3. The phase diagrams of charge-neutral quark matter for several values of $K'/K$ and fixed $G_V/G_S = 0.25$. Adapted from [1].
2.2. Combined effect of vector and anomaly terms

We show in Fig.3 the phase diagrams when the vector term is also included as well as the anomaly terms under charge-neutrality constraint. Again there appears a crossover region in the intermediate $T$ region but for much smaller $K'/K$ this time. Slightly increasing $K'/K$ up to 0.57, a new crossover window opens with a new critical point attached in the low-$T$ region including zero temperature, which is again due to the chiral-diquark competition enhanced by the vector term as well as the anomaly term. As $K'/K$ is increased further, the island of the first-order critical line for the chiral restoration disappears owing to so strong diquark correlation, and the first-order critical line ceases to exist for a realistic value $K'/K = 1.0$. We note that the unstable region characterized by the chromomagnetic instability (bordered by the dash dotted line) tends to shrink and ultimately vanishes in the phase diagram.

§3. Summary

The diquark-chiral coupling induced by $U(1)_A$ anomaly plays a similar role as the vector interaction for the phase diagram of the quark matter under charge-neutrality constraint. As a result, the phase boundary for the chiral-color-superconducting (CSC) phase transition can have alternate multiple windows of the crossover/first-order transition lines attached with critical point(s). The message to be taken from the present mean-field level calculation is that the QCD matter under the charge neutrality constraint is soft for the simultaneous formation of chiral and diquark condensates around the would-be phase boundary, implying possible absence of the first-order transition line and critical points. Since the chiral transition at finite density involves a change of baryon density, the soft mode is actually a combined fluctuations of chiral, diquark and baryon densities.

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