Optical Coherence Tomography with an SU(1,1) interferometer in the high parametric gain regime

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We demonstrate optical coherence tomography based on an SU(1,1) nonlinear interferometer with high-gain parametric down-conversion. For imaging and sensing applications, this scheme promises to outperform previous experiments working at low parametric gain, since higher photon fluxes provide better sensitivity and lower integration times for obtaining high-quality images. Moreover, there is no need to use single-photon detectors and standard spectrometers can be used instead.

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I. INTRODUCTION

Optical Coherence Tomography (OCT) is a 3D optical imaging technique that permits high-resolution tomographic imaging. It is applied in many areas of science and technology, from medicine to art conservation studies. To obtain good transverse resolution (perpendicular to the beam propagation axis), OCT focuses light into a small spot that is scanned over the sample. To obtain good resolution in the axial direction (along the beam propagation), OCT uses light with a large bandwidth to do optical sectioning of the sample.

Standard OCT schemes make use of a Michelson interferometer, where light in one arm illuminates the sample and light in the other arm serves as a reference. In the last few years, there has been a growing interest in a new type of OCT scheme that uses so-called nonlinear interferometers based on optical parametric amplifiers. The latter can be realized with parametric down-conversion (PDC) in nonlinear crystals or four-wave mixing (FWM) in fibers or atomic systems.

Some of these OCT schemes are based on the idea of induced coherence, a particular class of nonlinear interferometer originally introduced the very same year as OCT. A parametric down-converter generates pairs of signal and idler photons. The idler beam is reflected from a sample with reflectivity \( r_i \) before being injected into a second parametric down-converter. The signal beams coming from the two coherently-pumped parametric sources show a degree of rst-order coherence that depends linearly on the reflectivity. If not only the idler but also the signal from the first down-converter are injected into the second crystal, the scheme turns into an SU(1,1) interferometer. Some OCT schemes use this configuration.

Nonlinear interferometers are key elements in numerous applications beyond OCT, namely in imaging, sensing, spectroscopy, and microscopy. From a practical point of view, an advantage of these systems is that one can choose a wavelength for the idler beam that interacts with the sample and is never detected, and another wavelength for the signal beam, at which the photo-detection efficiency is high. This is why the general term ‘measurements with undetected photons’ is used for such systems.

Broadly speaking, optical parametric amplifiers can work in two regimes. In the low parametric gain regime, the number of photons per mode generated is much smaller than one. Many applications have been demonstrated in this regime. In the high parametric gain regime, the number of photons per mode is higher than one. Applications for imaging have been considered in both regimes and an OCT scheme based on induced coherence and large parametric gain has been demonstrated.

Despite the availability of strongly pumped SU(1,1) interferometers, recent experiments in OCT using the SU(1,1) scheme work at low parametric gain. In this scenario, the generated photon pair flux is low, which is detrimental for OCT applications. Here we demonstrate an OCT scheme that makes use of an SU(1,1) interferometer at high parametric gain, generating thus high photon fluxes.

The regime of high parametric gain has several advantages for sensing and imaging. Higher photon fluxes allow using conventional charge-coupled device (CCD) cameras or spectrometers, instead of single-photon detectors, and images with high signal-to-noise ratio can be obtained with shorter acquisition times. The nonlinear dependence of the interference visibility on the idler

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loss makes this regime more sensitive to small reflectivities. Moreover, it has been observed\textsuperscript{25} that larger down-conversion bandwidths can be generated in this regime, which would yield higher resolution in OCT schemes.

II. OPTICAL COHERENCE TOMOGRAPHY SCHEME

Figure 1 depicts the experimental setup. A Nd:YAG laser generates pulses of light at wavelength $\lambda_p = 532$ nm. The pulse duration is 18 ps and the repetition rate is 1 kHz. The pump illuminates an $L = 1$ mm periodically poled lithium niobate (PPLN) crystal and generates signal and idler beams at different central wavelengths (810 nm and 1550 nm, respectively) with the same vertical polarization as the pump beam. The bandwidth of the signal spectrum is $8 \pm 1$ nm full-width at half maximum (FWHM) and the bandwidth of the idler is $30 \pm 3$ nm.

To increase the parametric gain $G$, the pump is focused onto the nonlinear crystal by means of lens $L_1$ with the focal length $f_1 = 200$ mm. The beam size at the crystal has FWHM $40 \pm 10 \mu m$. The signal and idler beams are separated by long-pass dichroic mirror $DM_2$ with the transmission edge at 950 nm: the pump and the signal are transmitted, while the idler is reflected. The pump and signal beams form the reference arm of the interferometer and are collimated using lens $L_2$ with the focal length $f_2 = 200$ mm. The idler beam constitutes the probing arm of the interferometer and is collimated using lens $L_3$ with the focal length $f_3 = 150$ mm.

The pump and signal beams are reflected by a mirror and both are focused back onto the nonlinear crystal by means of lens $L_2$. We consider a reflectivity $r_s$ for the signal beam that takes into account losses in optical elements along the signal path. The distance traveled by the signal beam $s_1$ before reaching the nonlinear crystal is $z_s$. The idler beam $i_1$ interacts with an object with reflectivity $r_i(\Omega)$ and is focused back on the crystal by lens $L_3$. It propagates a total distance $z_i$. Finally, after parametric amplification in the second pass of the pump by the nonlinear crystal, the signal ($s_2$) and idler ($i_2$) beams are transmitted by the dichroic mirror $DM_1$. The idler radiation is filtered out by the short-pass filter $F$.

A flip mirror allows us to switch the detection between a CCD camera and a spectrometer. In the first case, we reflect signal $s_2$ to the CCD camera placed in the Fourier plane of lens $L_4$ (focal length $f_4 = 100$ mm). In this scenario, if the arms of the interferometer are balanced up to the coherence length of PDC radiation, interference fringes appear when the phase of the pump beam is scanned by means of piezoelectric actuator PA.

To measure the spectrum, the signal beam $s_2$ is spatially filtered in the Fourier plane of lens $L_4$ and fiber-coupled to a visible spectrometer. In this scenario, spectral interference is observed without scanning the phase, regardless of the optical path difference\textsuperscript{25}. However, it will only be discernible if the interference fringes are resolved by the spectrometer.

OCT requires the use of a large bandwidth ($\Delta \omega_{DC}$) of the PDC spectrum for obtaining high axial resolution of samples. Under the approximation of continuous-wave pump\textsuperscript{25,26}, valid also for picosecond pump pulses, the signal beam spectrum $S(\Omega)$ is (see Section I of Supplementary Material)

$$S(\Omega) = \left[1 - |r_i(-\Omega)|^2\right]|V_s(\Omega)|^2$$

$$\quad + |r_s U_s(\Omega)V_s(\Omega)\exp[i\varphi_s(\Omega)]$$

$$\quad + r^*_i(-\Omega)U^*_i(-\Omega)V_s(\Omega)\exp[-i\varphi_i(-\Omega)]|^2,$$  \hspace{1cm} (1)

with

$$U_{s,i}(\Omega) = \left\{ \cosh(\Gamma L) - i \frac{\Delta s,i}{2\Gamma} \sinh(\Gamma L) \right\}$$

$$\times \exp\left\{ i \left[k_{s,i}(\Omega) - k_{i,s}(-\Omega)\right]\frac{L}{2}\right\},$$ \hspace{1cm} (2)

$$V_{s,i}(\Omega) = -i\frac{\sigma}{\Gamma} \sinh(\Gamma L) \exp\left\{ i \left[k_{s,i}(\Omega) - k_{i,s}(-\Omega)\right]\frac{L}{2}\right\}.$$  \hspace{1cm}

Here $\Omega$ designates the frequency deviation from the central frequencies $\omega_{s,i}$ and phases are $\varphi_{s,i}(\Omega) = (\omega_{s,i} + \Omega)z_{s,i}/c$. The subscripts $s,i$ refer to the signal and idler beams, $\Gamma = (\sigma^2 - \Delta_{s,i}^2)/4$ and the phase-matching functions are $\Delta_s = -\Delta_i = (D_i - D_s)\Omega$. $D_{s,i}$ are inverse group velocities at the signal and idler central frequencies, respectively. The nonlinear coefficient $\sigma$ is

$$\sigma = \left( \frac{\hbar c}{8\epsilon_0}\right)^{1/2} \frac{R_p}{R_p(\hbar \omega_p) / T_0} \frac{1}{N_{p,s,i}}$$  \hspace{1cm} (3)

where $\omega_{p,s,i}$ are angular frequencies and $n_{p,s,i}$ are refractive index at the corresponding frequencies. $A$ is the effective area of interaction and $R_p$ is the flux rate of pump photons. We estimate $R_p$ as $R_p = E_p/(\hbar \omega_p) / T_0$ where $T_0$ is the pulse duration, and $E_p$ is the energy per pump pulse.

The parametric gain $G = \sigma L$ is measured from the dependence of the intensity $I$ of the output radiation (signal or idler) on the input average pump power $P_{in}$.

FIG. 1. Experimental setup for OCT. $DM_{1,2}$ stands for dichroic mirrors, $L_{1,2,3,4}$ are lenses, PPLN is a periodically poled lithium niobate crystal, $M$ is a mirror and $F$ is a short-pass filter. $V$ designates vertical polarization with sub-indexes indicating the wavelengths of the corresponding beams. PA is a piezoelectric actuator. With a flip mirror we can choose to measure the flux rate of signal photons or its spectrum.
$$I \propto \sinh^2(G) \text{ and } G \propto \sqrt{P}.$$ In our setup we measure the gain for the first pass by the nonlinear crystal to be $G = 1.7 \pm 0.2$. Thus, the total number of idler photons per pulse is estimated to be $\sim 13000$ ($\sim 7$ paired photons per mode), with an idler energy per pulse of 1.6 fJ and a mean power of 1.6 pW.

### III. OCT AT HIGH PARAMETRIC GAIN: RESULTS

#### A. Interference visibility at high parametric gain

To study the dependence of the interference visibility on the reflectivity of the sample, we mimic the latter by a neutral density filter together with a highly reflecting mirror (not shown in Fig. 1 for simplicity), the total reflectivity coefficient being $R_i$. We approximately reduce to zero the path length difference $\Delta z = z_s - z_i$ between signal and idler beams with mirror $M$ on a translation stage. Scanning the phase is done with the PA shown in Fig. 1. We measure the flux rate of signal photons $\bar{N}_s$ and idler photons $\bar{N}_i$ at the maximum and minimum of the flux rate, respectively. We repeat this procedure for several values of the reflectivity.

The visibility of interference fringes for multimode radiation in frequency is (Section II of Supplementary Material considers the single-mode approximation)

$$V = \frac{2|\bar{N}_s||\bar{N}_i|}{(1 - |\bar{N}_i|^2)\alpha + |\bar{N}_i|^2\beta + |\bar{N}_s|^2\gamma},$$

where

$$\nu = \int d\Omega U_s(\Omega)U_i(-\Omega)|V_s(\Omega)|^2 \exp \left\{i \frac{\Omega}{c} \Delta z \right\},$$

$$\alpha = \int d\Omega |V_s(\Omega)|^2,$$

$$\beta = \int d\Omega |U_i(-\Omega)|^2 |V_s(\Omega)|^2,$$

$$\gamma = \int d\Omega |U_s(\Omega)|^2 |V_s(\Omega)|^2.$$  

In order to observe fringes, $|\Delta z|$ should be smaller than the coherence length of PDC $l_c \sim \lambda_s^2/\Delta\lambda_i$, where $\Delta\lambda_i$ is the bandwidth of the idler beam.

Figure 2 shows the visibility measured (blue points) and calculated (solid and dashed lines) for gains $G = 0.4$ (red), 1.7 (green) and 4.8 (blue), with $r_s = 0.6$. The insets show some examples of the interference pattern measured by scanning the phase. The red dashed line corresponds to the low parametric gain regime. In this regime, $|V_{s,i}(\Omega)| < 1$ and $|U_{s,i}(\Omega)| = 1$, which gives a linear dependence of the visibility on the idler reflectivity:

$$V = \frac{2|r_s|}{1 + |r_s|^2 |r_i|}.$$ 

The green line is the case studied in our experiment, and it yields a nonlinear dependence. This is similar to the case of induced coherence schemes where interference visibility also depends in a nonlinear fashion on the sample reflectivities.27,28 The experimental results are in good agreement with the theory. The maximum visibility value measured is $V = 90\%$. High visibility is important to reach high sensitivity in OCT.

The blue dashed line corresponds to $G = 4.8$. In this case the visibility is

$$V = \frac{2|r_s|}{|r_i|^2 + |r_s|^2 |r_i|}.$$ 

If signal and idler losses are equal, $r_i = r_s$, the visibility is equal to 1. Notice that nonlinear relationships have also been observed for configurations where the first parametric down-converter is seeded with an intense signal beam.25,29

The value $r_s = 0.6$ is the estimated reflectivity in the signal path in our setup. Losses in the signal path $s_1$ are not only due to the optics (double pass through an uncoated lens and the dichroic mirror), but also most likely due to spatial mode mismatch.

#### B. Fourier-domain OCT at high parametric gain

We show that we can do Fourier- or spectral-domain OCT (FD-OCT) based on a SU(1,1) interferometer. FD-OCT allows faster data acquisition, is more robust since
it has no movable elements, and shows better sensitivity than time-domain OCT.\cite{chow1996optical}

FD-OCT analyzes the modulation of the spectrum of the output signal beam and requires a non-zero value of the path length difference $\Delta z$, contrary to the case of time-domain OCT that requires a value of $\Delta z$ close to zero (Section III of Supplementary Material shows a way to evaluate $\Delta z$). When considering a single reflection, the average fringe separation in angular frequency, $2\pi c/|\Delta \lambda|$, needs to be smaller than the bandwidth of parametric down-conversion $\Delta \delta \lambda$ and larger than the resolution of the spectrometer, $\delta \omega$, so that the modulation is accurately resolved. Making use of $\delta \omega = (2\pi c/\lambda_2^2)\delta \lambda$ and $\Delta \delta \lambda_c = (2\pi c/\lambda_c^2)\Delta \lambda_c$, $\Delta z$ is constrained by

$$\frac{\lambda_s^2}{\Delta \lambda_c^2} \ll |\Delta z| \ll \frac{\lambda_s^2}{\delta \lambda}. \quad (8)$$

In our setup we have $\Delta \lambda_c = 8$ nm and $\delta \lambda = 1.2$ nm, so $82 \mu m \ll |\Delta z| \ll 546 \mu m$.

The flip mirror at the output of the interferometer is removed to send and fiber-coupled into a spectrometer the signal photons. Mirror $M$ is mounted on a nanometric step translation stage to actively control the reference arm length, so that $\Delta z$ is adjusted to an optimum value.

We probe a $d = 100 \mu m$ thick microscope glass slide with group index $n_g \sim 1.5$, equivalent to a two reflecting layers object with optical path length of $2n_g d \sim 300 \mu m$. The spectrum $S(\lambda)$ is expressed in terms of k-values and Fourier transformed (see Section IV in Supplementary Material for details). $\Delta z_1$ and $\Delta z_2$ are path length differences corresponding to the locations of the two layers and should fulfill Eq. (8). The values of $\Delta \lambda$ and $\delta \lambda$ in our setup do not allow to resolve the sample with $|\Delta z_1|, |\Delta z_2| \gg 2n_g d$. Instead of this, Figure 3 shows the result obtained for the case $\Delta z_1 = -\Delta z_2 = n_g d$. It shows two peaks separated 325 $\mu m$, close to the theoretical expectation of $2n_g d$. The small difference might be due to a slight tilt of the sample or to the large axial resolution ($60 \mu m$), which corresponds to the width of the peaks shown in Fig. 3. The axial resolution can be improved by engineering the phase-matching conditions of nonlinear crystals\cite{icsicr15,icsicr16,icsicr17} and by spectral shaping\cite{icsicr18}.

IV. CONCLUSIONS & OUTLOOK

We have demonstrated a robust and compact OCT scheme based on an SU(1,1) nonlinear interferometer that works in the high gain regime of parametric down-conversion. The setup is versatile since it allows to do both time-domain and Fourier-domain OCT, and can be also easily converted into an induced coherence scheme (see Section V of Supplementary Material for details).

The high parametric gain allows to generate higher flux rates and use standard CCD cameras and spectrometers, avoiding the use of single-photon detectors. We can easily reach powers of interest for many applications. For instance, in ophthalmology light entering the cornea should have a maximum power of 750 $\mu W$. In art restoration studies typical power is a few milliwatts. Moreover the nonlinear dependence of the interference visibility on the reflectivity makes the high-gain OCT especially sensitive to weakly reflecting samples.

The method still benefits from its well-known salutary features: the sample is probed by photons centered at near infrared (NIR), while photodetection takes place in the visible range. This may yield larger penetration depths in samples while measuring at the optimum wavelength with silicon-based photodetectors. In our case, the infrared beam is centered at 1550 nm and the visible at 810 nm. One can generate even bright THz radiation\cite{icsicr15,icsicr16,icsicr17} by clever engineering of nonlinear crystals. Enhanced axial resolution can be achieved by using appropriate parametric down-converters with a broader bandwidth.

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DATA AVAILABILITY STATEMENT

The data that support the findings of this study are available from the corresponding author upon reasonable request.
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Supplementary Material

I. DERIVATION OF THE SPECTRUM OF SIGNAL PHOTONS GIVEN BY EQ. (1) IN THE MAIN TEXT.

The input signal and idler beams are in the vacuum state. The relationship between the input annihilation operators $b_s$ and $b_i$ and the output operators $a_s$ and $a_i$ of signal and idler beams generated after the first pass of the pump pulse by the nonlinear crystal is described by the Bogoliuov transformation:

\begin{align*}
  a_s(\Omega) &= U_s(\Omega) b_s(\Omega) + V_s(\Omega) b_i^\dagger(-\Omega), \\
  a_i(\Omega) &= U_i(\Omega) b_i(\Omega) + V_i(\Omega) b_s^\dagger(-\Omega), \\
\end{align*}  \tag{S1}

We assume that $r_s(\Omega)$ is frequency independent. If we substitute Eq. (S1) into $S(\Omega) = \langle a_s^\dagger(\Omega) a_s(\Omega) \rangle$ we obtain the expression of the spectrum given in Eq. (1) of the main text.

II. VISIBILITY OF INTERFERENCE IN THE SINGLE MODE APPROXIMATION

Some of the results obtained in the experiments described in the main text can be also be well described qualitatively using the single-mode approximation. In this case, the expression for the visibility equivalent to Eq. (4) in the main text is

\begin{equation}
  V = \frac{2|r_s||r_i||U_1||U_2||V_1||V_2|}{(1 - |r_i|^2)|V_1|^2 + |r_i|^2|U_1|^2|V_2|^2 + |r_s|^2|U_2|^2|V_1|^2},
  \tag{S5}
\end{equation}

where $U_1$ and $V_1$ refer to the first pass by the nonlinear crystal and $U_2$ and $V_2$ to the second pass. If we write $|U_j| = \sinh(G_j)$, $|V_j| = \cosh(G_j)$ and make use of $|U_j|^2 - |V_j|^2 = 1 \ (j = 1, 2)$, we have

\begin{equation}
  V = \frac{\sinh^2(G_1 + G_2) - \sinh^2(G_1 - G_2)}{2 \sinh^2(G_2) + |r_s|^2 \sinh^2(G_1) + (|r_s|^2 + |r_i|^2) \sinh^2(G_1) \sinh^2(G_2)}. \tag{S6}
\end{equation}

For $r_s = r_i$ we recover Eq. (S1) in Supplementary Material.

We consider two important limits. In the low parametric gain regime, the values of $G_{1,2}$ are very small, so $\sinh(G_j) \sim G_j$ and $G_1^2 G_2^2 \ll G_1^2, G_2^2$. We thus have

\begin{equation}
  V = \frac{2|r_s|G_1 G_2}{G_2^2 + |r_s|^2 G_1^2} |r_i| \tag{S7}
\end{equation}

that it is the expected linear relationship on $|r_i|$. For $G_1 = G_2$ this is Eq. (6) in the main text.

In the very high parametric gain regime, when the difference of gains is small, $\sinh^2(G_1 + G_2) \sim \exp[2(G_1 + G_2)]/4 \gg \sinh^2(G_1 - G_2)$ and $\sinh^2(G_1) \sinh^2(G_2) \sim \exp[2(G_1 + G_2)]/16 \gg \sinh^2(G_1) \sinh^2(G_2)$. The visibility is

\begin{equation}
  V = \frac{2|r_s||r_i|}{|r_i|^2 + |r_i|^2}. \tag{S8}
\end{equation}

This is Eq. (7) in the main text. Notice two important
III. MEASUREMENT OF THE PATH LENGTH DIFFERENCE $\Delta z$.

We determine the value of the path length difference $\Delta z$ moving the position of a mirror located in the signal path, and looking at the modulation of the signal beam spectrum as a function of the path length difference. Figures S1(a) and (b) show the spectrum obtained experimentally for two values of the path length difference: $\Delta z_1 = 220 \mu m$ and $\Delta z_2 = 300 \mu m$. The visibility of the spectral modulation is affected by the losses in the signal path and by the resolution of the spectrometer ($\delta \lambda \sim 1.2$ nm). In spite of this, we still can get the information of interest. Figure S1(c) shows the Fourier transform of the signals shown in Figs. S1(a) and (b). The separation of the two peaks is 80 $\mu m$, in perfect agreement with the difference between the two values of $\Delta z$ measured. Figure S1(d) shows the position of the peaks of the Fourier transform as a function of the path length difference $\Delta z$. There is good agreement between theoretical and experimental values.

IV. FOURIER TRANSFORM ANALYSIS

Here we explain in detail how we obtain Figure 3 in the main text. Figure S2 depicts the step-by-step procedure.

Figure S2(a) shows the spectrum $S(\lambda)$ measured with a spectrometer sensitive in the visible range. The spectrum is rewritten as function of the wavenumber $k = 2\pi/\lambda$. Considering the Jacobian of the transformation, the spectrum is

$$ S(k) = \frac{2\pi}{k^2} S(\lambda) \quad (S9) $$

The spectrum is re-sampled to obtain a function $S(k)$ with equally-spaced $k$-values [Fig. S2(b)]. The Fourier transform ($\sim \int dk S(k) \exp(ikz)$) of the re-sampled spectrum is shown as function of the axial position $z$ in Fig. S2(c). Finally, we show a zoom of the FT showing only the peak of the FT at a positive value of the $z$-coordinate [Fig. S2(d)].

Let us assume that the resolution is good enough to unveil all peaks of the FT. The number of peaks, and its positions, depends on values of the path length differences $\Delta z_1$ and $\Delta z_2$. If $\Delta z_1 \times \Delta z_2 > 0$, there are five peaks. The separation between the two peaks at positive (or negative) values of the $z$-coordinate is the optical thickness $2n_g d$. If $\Delta z_1 \times \Delta z_2 = 0$ or $\Delta z_1 = -\Delta z_2$, three peaks can be observed and the separation between the non-DC peaks, one located at a positive value of the $z$-coordinate and the other at a negative value, is $2n_g d$. Figure 3 of the main text shows the case when $\Delta z_1 = -\Delta z_2$. The separation between the non-DC peaks is $2n_g d$. 

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**FIG. S1.** (a,b) Spectra measured for two different optical path differences $\Delta z_{1,2}$ (solid line). The dotted lines stand for the spectrum when the idler arm is blocked. (a) $\Delta z_1 = 300 \mu m$; (b) $\Delta z_2 = 220 \mu m$. (c) Zoom of the Fourier transforms of the (a,b) spectra after resampling to wavenumbers. The positions of the peaks reveal directly the unbalancing between the arms of the interferometer. The peak separation is 80 $\mu m$, corresponding to the path difference between $\Delta z_1 - \Delta z_2$. (d) Position of the Fourier transform peak versus the path difference. Stars are the experimental data, and the dashed line is the theoretical dependence assuming exact equality.

**FIG. S2.** Step-by-step procedure to obtain the depth profile of the OCT sample. (a) Spectrum measured with a visible spectrometer, $S(\lambda)$. (b) Spectrum re-sampled to wavenumbers, $S(k)$. (c) Fourier transform of the re-sampled spectrum. (d) Zoom of the Fourier transform showing the peak at a positive value of the $z$-coordinate.
V. HOW TO TRANSFORM THE SU(1,1) INTERFEROMETER INTO AN INTERFEROMETER BASED ON INDUCED COHERENCE

The experimental setup depicted in Figure 1 of the main text corresponds to an OCT system based on an SU(1,1) interferometer. We show here how we can easily transform this experimental scheme into an OCT system that makes use of the concept of induced coherence.

To change the experimental setup from an SU(1,1) interferometer (Fig. S3(a)) to a scheme based on the induced coherence effect, one should prevent the signal wave from being amplified on the second pass through the nonlinear crystal. This can be done (see Fig. S3(b)) by changing the polarization of signal beam $s_1$ to an orthogonal one with the help of a quarter-wave plate (QWP). Then, only the idler beam $i_1$ would seed the parametric amplification process in the second pass through the nonlinear crystal. In this case, one would distinguish three beams at the output of the nonlinear crystal: idler $i_2$ and two signal beams with orthogonal polarizations: $s_1$ and $s_2$. The detection stage would measure coherence induced between signal beams $s_1$ and $s_2$. Importantly, the QWP should not affect the pump polarization (be a full-wave plate for the pump).

**FIG. S3.** (a) Sketch of the experimental setup corresponding to an SU(1,1) interferometer described in the main text. (b) Sketch of a new experimental setup based on the idea of induced coherence. The pump beams are represented by green lines, the signals are represented by blue lines and the idlers by red lines. A quarter-wave plate QWP placed into the signal beam prevents it from being amplified on the second pass through the nonlinear crystal.