Event-Triggered Fixed-Time Control for Steer-by-Wire Systems With Prespecified Tracking Performance

Bingxin Ma · Gang Luo · Yongfu Wang

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Abstract This paper addresses the event-triggered output feedback control problem for (steer-by-wire) SbW systems with uncertain nonlinearity and time-varying disturbance. First, a new framework of event-triggered control systems is proposed to eliminate the jumping phenomenon of event-based control input, and the tradeoff between saving communication resources and attenuating jumping phenomenon can be removed. Then, the adaptive disturbance observer and fuzzy-based state observer are developed to estimate the external disturbance and unavailable state of augmented SbW systems, respectively. Third, an event-triggered fixed-time control is developed for SbW systems to achieve prespecified tracking accuracy while saving communication resources of the controller area network (CAN). Furthermore, theoretical analysis based on Lyapunov stability theory is provided to verify the tracking error of SbW systems can converge to the prespecified neighborhood of the origin in fixed time regardless of the initial tracking error. Finally, simulations and experiments are given to evaluate the effectiveness and superiority of the proposed methods.

Keywords Steer-by-wire (SbW) system · Event-triggered communication · Fuzzy-based state observer · Fixed-time control · Prespecified tracking performance

1 Introduction

The steer-by-wire (SbW) system is one of the main subsystems of autonomous vehicles, realizing the steering control of front-wheels. Compared with the conventional steering system, SbW systems have two distinct characteristics: 1) the mechanical linkage between the steering wheel and front-wheels is no longer required; 2) the additional sensors, steering motor actuator, control unit, and controller area network (CAN) are necessary for SbW systems. Although the steering control of autonomous vehicles can be achieved through the SbW system, it is still a challenge to achieve accurate modeling and control of the SbW system with measurement and communication limitations [1–5].

Significant contributions have been made to the literature of SbW control systems; see, e.g., [6–15] and the references therein. Representatively, the model-based control method, such as proportional-integral-derivative control [6], proportional-derivative control with feedforward compensation [7], and model predictive control [8] are investigated for SbW systems. Considering the SbW system’s parametric uncertainties and disturbances, the fairly accurate state model and robust sliding mode control are first reported in [9]. Then, the adaptive sliding mode control method of SbW systems is investigated in [10–13] without a priori bounds of uncertainties and disturbances. References [14, 15] address the active fault-tolerant control problem of SbW systems subject to modeling uncertainties and disturbances with a priori bounds. A detailed literature review of the SbW control system can be found in Tab.1. Although the methods mentioned above have obtained fruitful achievements in SbW control systems, there are still the following limitations: 1) multi-sensor measurement technology is required in the control application,
which increases the hardware complexity and cost of SbW systems, and 2) the uninterrupted transmission of the control input to the steering motor is required, which will occupy unnecessary CAN communication resources. In practical applications, the vehicle-mounted CAN with limited communication channel bandwidth is usually shared by different nodes, so addressing the communication constraints is of great significance.

To reduce the use of sensors in control systems, significant contributions have been made to the literature of the state estimator. Representatively, the sliding mode differentiator is proposed in [16] for the signal with the k-th derivative having a known positive Lipschitz constant. Reference [17] develops a robust kth-order differentiation for signals with a given functional bound of the (k + 1)-th derivative. Reference [18] proposes the sliding mode differentiator for nonlinear mechanical systems with bounded-input-bounded-state (BIBO) property. Reference [19, 20] proposes a global sliding mode observer for nonlinear mechanical systems subject to the Coriolis term and uncertainty with a priori bound. The first-order low-pass is considered the differentiator for signals with bounded derivatives [21–23], and the high-gain observer is developed for nonlinear systems with accurate model [24, 25]. Although the above state estimators are useful in some particular applications, they are challenging to achieve satisfactory estimation performance for SbW systems without BIBO property, bounded derivatives of state, accurate model, and a priori bounds of uncertainty/disturbance.

### Table 1 A summary of SbW control systems

| Reference | Technique | Main limitations |
|-----------|-----------|-----------------|
| References [6–8] | Model-based control method | 1) Multi-sensor measurement technology is required in the control application, which increases the hardware complexity and cost of SbW systems, and 2) the uninterrupted transmission of the control input to the steering motor is required, which occupies unnecessary CAN communication resources. |
| References [9–13] | Sliding mode control for uncertain SbW systems with disturbance | |
| References [14, 15] | Model predictive control for SbW systems with actuator fault | |

### Table 2 A summary of state estimators

| Reference | Technique | Assumptions or limitations |
|-----------|-----------|---------------------------|
| References [16, 17] | Sliding mode differentiator | k-th derivative of signal with the known Lipschitz constant or bounded function. |
| Reference [18] | Sliding mode differentiator for nonlinear mechanical systems | The input and output of controlled systems should be bounded. |
| References [19, 20] | Sliding-mode observer of uncertain Lagrangian systems | The bounds of uncertainties must be a priori. |
| References [21–23] | First-order low-pass | The derivative of signals must be bounded. |
| References [24, 25] | High-gain observer | The accurate model of nonlinear system are known. |
| References [26–31] | Approximator-based observer | The convergence of observation errors is asymptotic rather than fixed-time. |

### Table 3 A summary of event-triggered control systems

| Reference | Technique | Main limitations |
|-----------|-----------|-----------------|
| References [32–34] | ISS-based event-triggered control for nonlinear systems | 1) The jumping phenomenon of the control input caused by event-triggered communication has not been considered in [27, 28, 31–38]; 2) it is difficult to solve the trade-off between saving communication resources and attenuating jumping phenomenon, and the jumping phenomenon still exists in [39, 40]; and 3) the transient and steady performance of control systems cannot be guaranteed at the same time. |
| References [35, 36] | Model-based event-triggered control for nonlinear systems | |
| References [27, 28, 31, 37, 38] | Adaptive Event-triggered control for uncertain nonlinear systems | |
| References [39, 40] | Jumping-attenuation event-triggered control | |
To this end, the approximator-based observer is proposed in [26–31], but the convergence of observation errors is asymptotic rather than lemma fixed-time time. For this reason, this paper proposes an adaptive state observer for an uncertain SbW system without the above assumptions and limitations shown in Tab. 2.

In network control systems, the communication networks are usually shared by different system nodes, while the network resources including communication channel bandwidth and computation abilities are limited. To reduce the unnecessary waste of communication resources, great efforts have been made to develop event-triggered control, in which information transmission occurs only when necessary, rather than continuously; see, e.g., [27, 28, 31–40] and the references therein. Specifically, with the assumption of the input-to-state stability (ISS), the event-triggered stabilization and tracking problem of nonlinear systems are addressed in [32–34], respectively. In [35, 36], the model-based event-triggered control method is proposed for network-based plants. Considering the model uncertainty of nonlinear systems, the event-triggered state and output feedback control are developed in [37, 38] and [27, 28, 31], respectively. It is worth mentioning that the control input is updated and transmitted only at event-triggered instants, so the jumping phenomenon of control input caused by communication is inevitable. Consequently, the large impulse will be applied to the system, especially in event-triggered control systems with relative threshold, which certainly affects the smoothness of actuator output and degrade the system performance [39, 40].

To this end, the switching triggered strategy, including fixed threshold strategy and relative threshold strategy, is proposed in [39, 40] to attenuate the jumping phenomenon of event-based control input. As shown in Fig. 1(a), the core idea of the switching triggered strategy is that the fixed threshold strategy will be applied when the amplitude of control input is large, and the relative scheme will be applied when the amplitude of control input is small. Thereby, the event-triggered error has been constrained within the bounds of a constant, and the jumping phenomenon can be attenuated while saving communication resources as much as possible in this context. Unfortunately, it is difficult to solve the trade-off between saving communication resources and attenuating the jumping phenomenon, and the jumping phenomenon still exists in [39, 40]. This makes it difficult for the existing event-triggered control system to be applied to electromechanical systems with higher requirements for the smoothness of their actuators. Much importantly, the practical stability can be achieved asymptotically or within finite-time rather than fixed-time, which cannot guarantee the transient performance of closed-loop systems.

It is essential to save communication resources and reduce hardware costs from applying and developing network-controlled SbW systems. Simultaneously, to ensure vehicles’ safety and comfort, it is necessary to achieve fixed-time prespecified tracking performance while guaranteeing the smoothness of the SbW system steering motor’s output. As mentioned in Tab. 1 and Tab. 3, the existing SbW control systems and event-triggered control systems cannot meet these requirements and should be improved. For this reason, this paper proposes an even-triggered output feedback control method for SbW systems. The contributions of this paper are summarized in the following aspects:

1. **From the perspective of even-triggered control system design:** A new framework of the event-triggered control system is proposed in this paper. 1) Combined with the event-triggered control systems in [27, 28, 31–38], as shown in Fig. 1(b), the jumping phenomenon of control input can be eliminated, and 2) compared with the event-triggered control systems in [39, 40], the trade-off between saving communication resources and attenuating jumping phenomenon can be removed.

2. **From the perspective of state observer design:** An adaptive state observer for an uncertain SbW system without the above assumptions and limitations shown in Tab. 2 is proposed in this paper. 1) Compared with recent researches on state observer [16–19, 41, 42], there is no need for additional assumptions, such that bounded-state property, bounded higher-order derivatives of output, and a priori known nonlinearity, and 2) compared with recent researches on FLS/NN-based state observer [27, 28, 30, 43, 44], the convergence speed of the observation error can be improved in this paper, i.e., the observation error can converge to the adjustable neighborhood of the origin in finite-time instead of asymptotically converging to the small neighborhood of the origin.
(3) From the perspective of control method design: An observer-based event-triggered fixed-time control is proposed for uncertain SbW systems regardless of the initial tracking error. 1) Compared with SbW control systems [9–15], the unnecessary sensor can be removed and CAN communication resources can be saved, 2) compared with event-triggered output feedback control [28, 31, 40], the prescribed tracking performance can be achieved within fixed time, so both the transient and steady performance of closed-loop systems can be guaranteed, and 3) compared with [27], the bounds of the initial state of the controlled system is no longer required.

The rest of the paper is organized as follows. In Section II, problem formulation and preliminaries are given. The fuzzy-based state-observer and event-triggered fixed-time control is presented in Section III. The simulation and experiment are given in Section IV. Section V is the concluding remarks.

Notations: For a matrix $A$, $\|A\|$ denotes the Euclidean norm of $A$, and $\lambda_{\text{max(min)}}(A)$ denotes its maximum (minimum) eigenvalue. $|x|^\rho = |x|^{\rho}\text{sign}(x)$ with $\rho \geq 0$.

2 Problem Formulation and Preliminaries

Fig. 2 shows a schematic diagram of the SbW system for automatic vehicles. To describe the dynamics model clearly, Tab. 4 is given to clarify each of the components in Fig. 2.

2.1 Problem formulation

According to the researches [7, 9, 10, 45], the dynamics model of the steering motor can be established as

$$J_m \ddot{\theta}_m + B_m \dot{\theta}_m + \tau_{12} = \tau_m + \tau_d.$$  (1)

The rotation of the front-wheels around their vertical axes can be modeled as

$$J_f \ddot{\theta}_f + H_f(\theta_f, \dot{\theta}_f) = \tau_s,$$  (2)

where $H_f(\theta_f, \dot{\theta}_f) = \tau_e + \tau_f$ denotes the uncertain non-linearity. The transmission ratio between the steering motor and front-wheels is

$$\frac{\dot{\theta}_f}{\dot{\theta}_m} = \frac{\dot{\theta}_f}{\dot{\theta}_m} = \frac{\tau_{12}}{\tau_e} = \frac{1}{\mu},$$  (3)

which together with (1)-(2) gets

$$J_e \ddot{\theta}_f + \mu^2 B_m \dot{\theta}_f + H_f(\theta_f, \dot{\theta}_f) = \mu(\tau_m + \tau_d)$$  (4)

with $J_e = J_f + \mu^2 J_m$. For brevity, the dynamics model (4) can be rewritten as

$$\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= f_o(x_o) + g u + d_o(t) \\
y &= x_1
\end{align*}$$  (5)

where $x_o = [x_1, x_2]^T = [\theta_f, \dot{\theta}_f]^T \in \mathbb{R}^2$, $u$ is the real control input which is represented as $\tau_m$ in (4), $f_o(x_o) = -(\mu^2 B_m x_2 + H_f(x))/J_e$ denotes the lumped uncertain nonlinearity of the SbW system, $g = \mu/J_e$, and $d_o(t) = \mu \tau_d/J_e$ denotes the lumped motor pulsation disturbance [9, 10].

Control Objective: This paper addresses the event-triggered output feedback control problem for SbW systems, such that

(1) the tracking error between the front-wheels steering angle and its reference signal can converge to the prespecified neighborhood of the origin in fixed time, and
(2) the jumping phenomenon of the control input caused by event-triggered communication can be eliminated.

The following assumptions are made on the reference signal and external disturbance, respectively.

**Assumption 1** The reference signal $y_d(t)$ is known, and there exist unknown positive constants $\bar{y}_d$ and $\bar{y}_d$ such that $|y_d(t)| \leq \bar{y}_d$ and $|\dot{y}_d(t)| \leq \bar{\bar{y}}_d$. 

![Fig. 2 Schematic diagram of the SbW system. (a) Schematic diagram of the SbW system, (b) Physical diagram of the SbW system.](image-url)
table Variables and parameters of SbW systems

| Symbol | Model variable |
|--------|----------------|
| $\bar{B}_m$ | Viscous friction coefficient of steering motor assembly |
| $J_m$ | Rotational inertia of steering motor assembly |
| $J_f$ | Rotational inertia of front-wheels |
| $J_e$ | Rotational inertia of equivalent system |
| $\theta_m$ | Steering motor assembly steering angle |
| $\theta_f$ | Front-wheels steering angle |
| $\tau_m$ | Output torque of steering motor assembly |
| $\mu$ | Steer torque of front-wheels |
| $\tau_1$ | Load torque of steering motor assembly |
| $\tau_s$ | Input torque on the steering arm |
| $\tau_c$ | Self-aligning torque of the front-wheels |
| $\tau_f$ | Friction torque |
| $\tau_d$ | Motor torque pulsation disturbance [9, 10] |
| $\beta$ | Slip angle |
| $\gamma$ | Yaw rate |
| $C_f$ | Front wheel cornering stiffness |
| $C_r$ | Rear wheel cornering stiffness |
| $v$ | Longitudinal velocity |
| $m$ | Vehicle mass |
| $I_z$ | Polar moment of inertia |
| $l_f$ | Distance from mass center to front axle |
| $l_r$ | Distance from mass center to rear axle |
| $l_m$ | Mechanical trails |
| $l_p$ | Pneumatic trails |
| $\rho_r$ | Coefficient for various road conditions |

Assumption 2: There exists an unknown positive constant $d$ such that $|\dot{d}(t)| \leq \bar{d}$.

Assumption 3: Considering the SbW system (5), assume that $f_0(x_o)$ is first-order differentiable.

Remark 1: The above assumptions are relatively general, even compared with the existing researches on time-triggered tracking control of SbW systems [9–13]. Specifically, 1) for the assumption of the external disturbance $d(t)$, the same assumption as Assumption 2 can be found in [9–12]. Moreover, the assumptions that both the external disturbance and its time derivative are bounded is made in [15]. 2) for the assumption of the reference signal $y_d(t)$, both the time derivative of the reference signal $\dot{y}_d(t)$ and its second-order time derivative $\ddot{y}_d(t)$ are required in the control design [9–13]. It is worth noting that to ensure driving safety and vehicle comfort, the reference path is usually smooth and its change rate is usually bounded in the practice application. Thereby, we can find that the time derivative of the reference signal, i.e., $\dot{y}_d(t)$, may not be known but can be considered bounded in this context. So compared with the related works, e.g., [9–13], Assumptions 1-2 are rather mild.

In [7, 10, 46], the self-aligning torque $\tau_c$ can be obtained

$$\tau_c = -C_f(t_m + t_p) \left( \beta + \frac{\gamma l_f}{v} - \delta f_w \right)$$

where $\beta$ and $\gamma$ can be obtained from the following the two-degree-of-freedom model of the vehicle

$$\begin{bmatrix} \dot{\beta} \\ \dot{\gamma} \end{bmatrix} = \begin{bmatrix} -\frac{C_r-C_f}{m v} & -1 + \frac{C_r v}{m v} \\ \frac{C_r v}{m v} & -\frac{C_r}{I_z} \end{bmatrix} \begin{bmatrix} \beta \\ \gamma \end{bmatrix} + \begin{bmatrix} C_f \\ \frac{C_r v}{I_z} \end{bmatrix} \theta_f$$

where the parameters of the model are defined in Table 4.

Besides, the following simplified model of self-aligning torque can be obtained [9, 11, 47, 48]

$$\tau_c = \rho_r \tan \left( x_1 \right)$$

where $\rho_r$ denotes a time-varying coefficient for various road conditions, and $\theta_f$ means the steering angle of the front-wheels. From (6)-(7), one can find that $\tau_c$ is first-order differentiable.

Furthermore, the SbW system is a typical electromechanical system that can be expressed by the general Euler-Lagrange formulation. According to the existing researches [49, 50], the friction torque $\tau_f$ can be presented as the following parameterized form

$$\tau_f = \alpha_1 (\tanh(\beta_1 x_2) - \tanh(\beta_2 x_2)) + \alpha_2 \tanh(\beta_3 x_2)$$

with $\alpha_i$ and $\beta_i$, $i = 1, 2, 3$ being the positive constants to be defined, which means that $\tau_f$ is differentiable.

From the above analysis, one can find that the self-aligning torque $\tau_c$ and friction torque $\tau_f$ of SbW systems are first-order differentiable, i.e., the nonlinearity $f_0(x_o)$ of (5) is differentiable. From a practical point of view, the self-aligning torque $\tau_c$ and friction torque $\tau_f$ of SbW systems have physical meaning. So their rate of change exists and is continuous. Therefore, the assumption that the nonlinearity of the SbW system is first-order differentiable is true in practice.

2.2 Fuzzy logic system and Useful Lemmas

A typical FLS consists of four parts: knowledge base, fuzzifier, fuzzy inference engine and defuzzifier. The
knowledge base consists of a series of fuzzy IF-THEN inference rules:

\[ R_j : \text{IF } \chi_1 \text{ is } \mathcal{F}_1^j \ldots \text{ and } \chi_n \text{ is } \mathcal{F}_n^j, \text{THEN } Y \text{ is } \mathcal{G}_j^j, j = 1, \ldots, m \]

where \( \chi_l, l = 1, \ldots, n, \) and \( Y \) denote the inputs and output of FLS, respectively, \( \mathcal{F}_j^j \) and \( \mathcal{G}_j^j \) are fuzzy sets and their membership functions are \( \mu_{\mathcal{F}_j^j}(\chi_l) \) and \( \mu_{\mathcal{G}_j^j}(Y) \), respectively, \( m \) is the number of fuzzy rules. Then, through the singleton fuzzifier, center average defuzzification, and product inference, the output of FLS can be expressed as

\[ Y = \Theta^T \xi(\chi) \quad (9) \]

where \( \Theta = [\Theta_1, \ldots, \Theta_m]^T = [\Phi_1, \ldots, \Phi_m]^T \) with \( \Phi_j = \max_{Y \in \mathcal{G}_j^j} \mu_{\mathcal{G}_j^j}(Y) \) is the parameter vector, and the fuzzy basis function vector is \( \xi(\chi) = [\xi^1(\chi), \ldots, \xi^n(\chi)]^T \) with

\[ \xi^j(\chi) = \frac{n}{\sum_{l=1}^n \mu_{\mathcal{F}_j^j}(\chi_l)} \sum_{l=1}^n \mu_{\mathcal{F}_j^j}(\chi_l) \quad (10) \]

**Lemma 1** (see[51]) Suppose that the input universe of discourse \( \Omega \) is a compact set in \( \mathbb{R}^n \). Then, for the continuous function \( f(\chi) \) on \( \Omega \) and arbitrary \( \omega > 0 \), there exists an FLS (9) with an optimal parameter vector \( \Theta^* \) such that

\[ |\Theta^{*T} \xi(\chi) - f(\chi)| \leq \omega. \quad (11) \]

**Lemma 2** (see[52]) Consider a class of systems \( \dot{x} = f(x) \), there exist a smooth positive-definite function \( V(x) \) and some positive scalars \( \alpha > 0, \beta > 0, p > 0, \) and \( k > 0 \) such that

\[ \dot{V}(x) \leq -[\alpha V^p(x) + \beta V^q(x)]^k. \quad (12) \]

Then the fixed-time stability can be guaranteed with the setting time \( T_r = \frac{1}{\alpha \Gamma(1-p)} + \frac{1}{\beta \Gamma(qk-1)} \).

**Lemma 3** Consider a class of systems \( \dot{x} = f(x) \), there exist a smooth positive-definite function \( V(x) \) and some positive scalars \( \alpha_1 > 0, \alpha_2 > 0, \alpha_3 > 0, 1 > \beta_1 > 0, \) and \( 1 > \beta_2 \geq 0 \) such that

\[ \dot{V}(x) \leq -\alpha_1 V(x) - \alpha_2 V^{\beta_1}(x) + \alpha_3 V^{\beta_2}(x) \quad (13) \]

then system \( \dot{x} = f(x) \) is semi-global practical finite-time stable, i.e., the following inequality can be obtained if \( t \geq T_r \)

\[ V(x) \leq \begin{cases} \left[ \frac{\alpha_3}{(1 - \xi_1) \alpha_1} \right]^\frac{1}{1-\xi_2}, & \text{if } \beta_1 \leq \beta_2 \\ \min \left\{ \left[ \frac{\alpha_3}{(1 - \xi_1) \alpha_1} \right], \left[ \frac{\alpha_3}{(1 - \xi_2) \alpha_2} \right] \right\}, & \text{else} \end{cases} \quad (14) \]

where \( 1 > \xi > 0, \) \( T_r \) is bounded as

\[ T_r \leq \begin{cases} T_0 + \frac{1}{\alpha_1(1 - \beta_1)} \ln \left( \frac{\alpha_1 V^{\beta_1}(T_0) + \alpha_2}{\alpha_2} \right), & \text{if } \beta_1 \leq \beta_2 \\ T_0 + \max \left\{ \frac{1}{\alpha_1(1 - \beta_1)} \ln \left( \frac{\alpha_1 V^{\beta_1}(T_0) + \alpha_2}{\alpha_2} \right), \frac{1}{\alpha_1(1 - \beta_1)} \ln \left( \frac{\alpha_1 V^{\beta_2}(T_0) + \alpha_2}{\alpha_2} \right) \right\}, & \text{else} \end{cases} \]

with \( T_0 \) being the initial time.

The proof of Lemma 3 is given in Appendix A.

**Remark 2** To improve the closed-loop system’s transient performance, finite-time control methods have been developed for various nonlinear systems during the past few years [53–55]. Generally, the practical finite-time stability of closed-loop systems can be guaranteed for uncertain nonlinear systems. As shown in Tab.5, the conclusion of practical finite-time stability theory can be obtained from the following research. From Tab. 5, one can find that [53] provides a more general Lyapunov condition of the practical finite-time stability. However, when \( \beta_2 \neq 0 \), the trajectory of system \( \dot{x} = f(x) \) is difficult to judge based on the conclusions of [53] and [54]. This Lyapunov condition (i.e., \( \beta_2 \neq 0 \)) is corresponds to the stability analysis of the state observer in this paper. Thus, the following Lemma is given in this paper.

### Table 5 Sufficient condition of finite-time stability and its residual set and settling time

| Reference | Sufficient condition | Residual set | Setting time |
|-----------|----------------------|--------------|--------------|
| Zhu et al. [53] | \( \dot{V}(x) \leq -cV^\beta(x) + \varepsilon \) with \( c, \varepsilon > 0, 1 > \beta > 0 \). | \( V(x) \leq \left[ \frac{\varepsilon}{(1 - \xi)c} \right]^\frac{1}{\beta} \), with \( 1 > \xi > 0 \). | \( T \leq \frac{\varepsilon^{1-\beta}c}{\xi^2} \) with \( 1 > \xi > 0 \). |
| Yu et al. [54] | \( \dot{V}(x) \leq -\lambda_1 V(x) - \lambda_2 V^\gamma(x) + \eta \), with \( \lambda_1, \eta > 0, 1 > \gamma > 0 \). | \( V(x) \leq \min \left\{ \frac{\eta}{(1 - \xi)\lambda_1}, \left( \frac{\eta}{(1 - \xi)\lambda_2} \right)^\frac{1}{\gamma} \right\} \), with \( 1 > \xi > 0 \). | \( T \leq \max \left\{ \frac{1}{\lambda_1(1 - \gamma)}, \frac{1}{\lambda_2(1 - \gamma)} \right\} \times \ln \left( 1 + \frac{\lambda_1 V^{1-\gamma}(0)}{\lambda_2} \right) + \ln \left( 1 + \frac{\lambda_2 V^{1-\gamma}(0)}{\lambda_1} \right) \) |
3 State Observer and Event-Triggered Control Design

3.1 Fuzzy-based state observer design

To eliminate the jumping phenomenon of control input, a new framework of the event-triggered control system, as shown in Fig. 3, is proposed. In this context, based on [16, 56], the following augmented system (15) can be established from the system (5) by introducing the auxiliary variable ζ = u

\[
\begin{align*}
\dot{x}_1 &= x_{i+1}, \; i = 1, 2 \\
\dot{x}_3 &= f(x) + g\dot{u} + d(t) \\
y &= x_1
\end{align*}
\]

(15)

where \( x = [x_1, x_2, x_3]^T \), \( f(x) = df(x)/dt \), \( d(t) = d_{ob}(t) \), \( \dot{u} \) is considered as the "new" control signal to be designed.

As shown in Fig. 4, the angle position \( x_1 \) of the augmented system (15) can be measured by a linear sensor. In practical applications, the angular velocity of the front wheel’s steering angle can be obtained through a sensor such as an encoder or gyroscope. However, the introduction of non-essential sensors will increase the hardware complexity and cost, thereby reducing the reliability of SbW systems. For this reason, the state observer is proposed for the augmented system (15), i.e.,

\[
\begin{align*}
\dot{x}_1 &= -k_1\phi_1(x_1) + \dot{x}_{i+1}, \; i = 1, 2 \\
\dot{x}_3 &= -k_3\phi_2(x_1) + \gamma + g\dot{u} + \ddot{d}(t)
\end{align*}
\]

(16)

where \( \chi = [x_1, \dot{x}_2, \dot{x}_3]^T \) and \( \gamma = \Theta^T \xi(\chi) \) denotes the input and output of FLS, \( \dot{x}_1 = \dot{x}_1 - x_1, \ddot{d}(t) \) is the estimation of the disturbance \( d(t) \), the parameters \( k_1, k_2 \) and \( k_3 \) satisfy that \( A = [-k_1, 1, 0; -k_2, 0, 1; -k_3, 0, 0] \) is Hurwitz, \( \phi_1(x_1) = (\mu_1 + \mu_2|x_1|)^{3/4} |\dot{x}_1|^0 + \dot{x}_1, \phi_2(x_1) = \phi_1(x_1)\phi_1'(x_1), \) with \( \mu_1, \mu_2 > 0 \), and

\[
\phi_1'(x_1) = \frac{3\mu_2}{4} (\mu_1 + \mu_2|x_1|)^{-\frac{1}{4}} + 1
\]

(17)

Besides, the adaptive laws of \( \Theta \) and \( \dot{d}(t) \) are designed as

\[
\begin{align*}
\dot{\Theta} &= \gamma_1\psi(S)\xi(\chi) - \sigma_1\Theta \\
\dot{d}(t) &= \gamma_2\psi(S) - \sigma_2\dot{d}(t)
\end{align*}
\]

(18)

(19)

where \( \psi(\chi) = -b(t)\phi_1(\dot{x}_1)/(|\phi_1(\dot{x}_1)| + \varepsilon_{ob}), \sigma_1, \sigma_2, \gamma_1, \gamma_2 \) and \( \varepsilon_{ob} \) are positive constants, and \( b(t) \) with \( \varepsilon_{ob} > 0 \) is

\[
b(t) = \begin{cases} 
\sin \left( \frac{\pi t}{2\varepsilon} \right), & \text{if } |t| < \varepsilon_{ob} \\
1, & \text{else}
\end{cases}
\]

(20)

For the adaptive state observer (16), the following main results can be obtained in this section.

**Lemma 4** Consider the adaptive laws (18)-(19), there exist unknown positive constants \( \Theta \) and \( \dot{d}(t) \) such that \( ||\Theta|| \leq \tilde{\Theta} \) and \( |\dot{d}| \leq \tilde{d}, \forall t > 0 \) with \( \tilde{\Theta} = \Theta - \Theta^\star \) and \( \tilde{d} = \dot{d} - d \). Moreover, there also exist \( \tilde{\Theta} \geq ||\Theta|| \) for \( t > 0 \).

The proof of Lemma 4 is given in Appendix B.

**Theorem 1** Consider the observer (16), if parameters meet \( \lambda_{\min}(Q) - 3\mu_2\|B_1P\|(2\mu_1^{-1})^4 \geq \kappa_{ob} \) with \( B_1 = [0, 1, 1]^T \), the estimation error of state can converge to the small neighborhood of the origin in finite time. i.e.,

\[
||\tilde{x} - x|| \leq C_{ob}, \forall t \geq T_{ob}
\]

(21)

with \( C_{ob} = \max \left( \frac{2\kappa}{\lambda_{\min}(Q)}, \frac{1}{\lambda_{\min}(P)} \right)^{1/2}, \) and

\[
T_{ob} \leq \max \left\{ \frac{8\lambda_{\max}(P)}{\kappa_{ob}} \ln \left( 1 + \frac{4(\lambda_{\min}(Q) + \mu_2)^2}{3\mu_1 \lambda_{\min}(Q) \kappa_{ob} V_{ob}(0)} \right) \right\}
\]

\[
+ \max \left\{ \frac{8\lambda_{\max}(P)}{\kappa_{ob}} \ln \left( 1 + \frac{4(\lambda_{\min}(Q) + \mu_2)^2}{3\mu_1 \lambda_{\min}(Q) \kappa_{ob} V_{ob}(0)} \right) \right\}
\]

\[
C_{ob} = \min \left\{ \frac{2\phi_1^{-1}_{\max}(P)}{(1 - \gamma_{ob})}, \frac{8\lambda_{\max}(Q) + \mu_2}{3\mu_1 (1 - \gamma_{ob}) \lambda_{\min}(Q)} \right\}
\]

where \( P = P^T > 0 \) is the solution of \( A^T P + PA = -Q \) with \( Q \) being the designed positive definite matrix, \( \Theta^\star \) has been defined in Lemma 4, \( \Phi = \Theta + ||\Theta|| + \dot{d} + \dot{\omega} \), \( \dot{\omega} \) has been defined in Lemma 1, and \( \tilde{\Theta} \) and \( \tilde{d} \) have been defined in Lemma 4.
Proof: Combined with (5) and (16), the following dynamics of observation errors can be obtained

\[
\begin{align*}
\dot{x}_1 &= - k_1 \phi_1(x_1) + \tilde{x}_{i+1}, \quad i = 1, 2 \\
\dot{x}_3 &= - k_3 \phi_2(x_1) + \Phi(t)
\end{align*}
\]  
(22)

where \( x = [\tilde{x}, \tilde{x}_3] \) and \( P = P^T > 0 \). Note that \( \tilde{x}_1 = 0 \) means \( \tilde{x} = 0 \) is also hold, and from (22), one can find that \( \tilde{V}_0 = 0 \) will be always hold in this context. Thus the following analysis is given for the condition that \( \tilde{x}_1 \neq 0 \).

From (22), the following equation can be obtained

\[
\dot{V}_0 = -\phi_1(x_1)Q_0 + 2B_1^T \Phi(t) \Phi(t)^T + 3P_0
\]

with \( B_1 = [0, 0, 1]^T \) and \( B_2 = [0, 0, 1]^T \). Combined with (23), the derivative of \( V_0 \) can be obtained

\[
\dot{V}_0 \leq -\kappa_0 \| \xi \|^2 - \frac{3\mu_1 \lambda_{\min}(Q)}{4(\lambda_{\max}(Q) + \mu_2)} \| \xi \|^2 + 2\lambda_{\max}(P) \| \Phi(t) \| \| \xi \|
\]

(23)

This ends the proof of Theorem 1.

Remark 3: As can be seen from Lemma 1, the approximation property of FLS can only be established in a convex region of interest, which implies that the initial states are within the bounded set. The same is true for all FLS/NN-based methods.

3.2 Event-triggered fixed-time control design

The following change of coordinates is given

\[
\begin{align*}
z_1 &= x_1 - y_d - \dot{y}_1 \\
z_2 &= \dot{x}_2 + \eta_1 z_1 - \dot{y}_2 \\
z_3 &= \dot{x}_3 + \eta_2 z_2 + \eta_1 (\dot{x}_2 - \dot{y}_1) - \dot{y}_3
\end{align*}
\]

(26)

where \( \eta_1 > 0 \) and \( \eta_2, \eta_1 > \frac{1}{2} \) are positive constants to be designed, \( i = 1, 2, 3 \) can be obtained from following dynamics

\[
\begin{align*}
\dot{\psi}_1 &= \psi_1 [-\ell_1 \psi_1 + \text{sign}(z_1)], \quad \psi_1(0) = 0, \quad i = 1, 2, 3 \\
\dot{\psi}_2 &= \psi_2 [-\ell_2 \psi_2 + \text{sign}(z_2)], \quad \psi_2(0) = 0, \quad i = 1, 2, 3
\end{align*}
\]

(27)

with \( \ell_1, \ell_2 \) and \( \tau \) being positive constants to be designed, and \( \psi_1 \) and \( \psi_2 \) being designed as

\[
\begin{align*}
\psi_1 &= \eta_0 ([z_1^2 + |z_1| z_2^2] + \eta_2 \dot{\psi}_1 (t_k^1) + m_1 \\
\psi_2 &= \eta_0 ([z_2^2 + |z_2| z_3^2] + \eta_2 \dot{\psi}_2 (t_k^1) + m_2 \\
\psi_3 &= g_m + 2\eta_0 \lambda_1 \psi_3 (z_1^2 + z_2^2) + \eta_1 z_2 [\eta_1 \lambda_{\max}(P) + \mu_2 \phi_2(\lambda)] + \eta_2 [\eta_1 \lambda_{\max}(P) + \mu_2 \phi_2(\lambda)] + \eta_3 [\eta_1 \lambda_{\max}(P) + \mu_2 \phi_2(\lambda)] + \eta_4 [\eta_1 \lambda_{\max}(P) + \mu_2 \phi_2(\lambda)] + \eta_5 [\eta_1 \lambda_{\max}(P) + \mu_2 \phi_2(\lambda)] + \eta_6 [\eta_1 \lambda_{\max}(P) + \mu_2 \phi_2(\lambda)] + \eta_7 [\eta_1 \lambda_{\max}(P) + \mu_2 \phi_2(\lambda)] + \eta_8 [\eta_1 \lambda_{\max}(P) + \mu_2 \phi_2(\lambda)]
\end{align*}
\]

(28)

with \( \lambda_1 \geq \lambda_{\max}(P) \). Moreover, the dynamic gain \( \kappa \) can be obtained from the following adaptive scheme

\[
\dot{k} = \kappa_0 \text{sgn} \left( \sum_{i=1}^{3} \psi_i^2 \right), \quad \kappa(0) = 0
\]

(29)

This ends the proof of Theorem 1.
by combining with $|\varphi| \leq \ell_2^1$ and $\Psi_2 \leq \Psi_2(t_k^r)+m_2$. From (16), (28) and (29), one gets

$$z_3 \dot{z}_3 = z_3 [-k_3 \phi_2(\tilde{x}_1) + \Theta T \chi + g \hat{u} + \hat{d}(t)] + (\eta_{11} + \eta_{21})$$

by combining with $\Psi_3 \leq \Psi_3(t_k^r)+m_3$. As shown in Fig. 5, the event-triggered control and the event-triggering mechanism of the augmented system (15) are designed as

$$\dot{u}(t) = v(t_k), \quad \forall t \in [t_k, t_{k+1}), \quad t_{k+1} = \{ t > t_k : |e(t)| \geq \delta \mid v(t_k) \mid + m \}, \quad t_1 = 0$$

where $e = v(t) - v(t_k)$ denotes the event-triggered error and $\dot{u}(t) = v(t_k)$ denotes the “new” control signal with $v(t)$ being designed as

$$v(t) = -\frac{1 + \delta}{g} (\eta_{31} z_3 + \Gamma)$$

with $\eta_{31} > \frac{3}{2}$ being positive constant to be designed.

Theorem 2 Consider the augmented system (15) under the developed state observer (16) and event-triggered control (39), the tracking error can converge to the prespecified neighborhood of the origin in fixed-time, i.e.,

$$|y - y_d| \leq \ell_3^1, \quad \forall t \geq T_{tr}$$

where $\ell_1$ has been defined in (29), and $T_{tr}$ is defined as

$$T_{tr} = \frac{\max(\bar{C} + \|\bar{x}(0)\|, C_{ob}) + \frac{2}{c_2} + \frac{1}{\eta_0}}{\kappa_d}$$

with $c_2 = \min(\eta_{12}, \eta_{22}, \eta_{13})$ and $\bar{C} \geq \|x\|$ for $x \in \Omega$.

Proof: From the designed event-triggering mechanism (40), one can find that there exist $|z_i(t)| \leq 1$ ($i = 1, 2$) such that $\dot{z} = [v(t) + m_2 c_2(t)]/[1 + c_1(t) \delta]$. This together with (31), (38) and (41) yields

$$z_3 \dot{z}_3 \leq - (\eta_{31} - \frac{1}{2}) z_3^2 - \eta_{21} z_3| + k_3(\|\phi_2(\tilde{x}_1)| - \phi_2(\kappa))|z_3|$$

$$+ k_2(\eta_{11} + \eta_{21})(|\phi_2(\tilde{x}_1)| - \phi_2(\kappa))|z_3| + \eta_{11} \eta_{21}$$

$$\times (\|\tilde{x}_2| + \|\tilde{u}_d - \kappa\|)|z_3| - \eta_0 (z_3^4 + 2 z_3^2 z_3^2).$$

Consider the following Lyapunov function

$$V_{Tr} = \frac{1}{2} \sum_{i=1}^{3} \dot{z}_i^2.$$
with \( c_1 = \min(\eta_{11}, \eta_{21}, \eta_{31}) - 1, \) \( c_2 = \min(\eta_{12}, \eta_{22}, \eta_{32}) \) and 
\( \gamma_1^2 + \gamma_2^2 + \gamma_3^2 + 2\gamma_1^2\gamma_2^2 + 2\gamma_2^2\gamma_3^2 + 2\gamma_1^2\gamma_3^2 = (\gamma_1^2 + \gamma_2^2 + \gamma_3^2)^2. \)
Consider that \( x \in \Omega \), there existing the positive constant \( \mathcal{C} \) such that \( \| x \| \leq \mathcal{C} \). This together with the conclusion of Theorem 1 yields that \( \| \hat{x} \| \leq \max(\mathcal{C} + \| \tilde{x}(0) \|, C_{ob}) \) is always hold. Therefore, there exists a time instants \( T_0 \) that \( \kappa(T_0) \geq \max(\mathcal{C} + \| \tilde{x}(0) \|, C_{ob}) + \tilde{y}_d. \)
Thus, the following dynamics can be obtained
\[
\dot{V}_r \leq -c_1 V_r - c_2 \dot{V}^2_r - \theta_0 V_r^2,
\]
This together with Lemma 2 yields that \( V_r \) can converge to the origin within fixed time with the setting time \( T_r \leq \max(\mathcal{C} + \| \tilde{x}(0) \|, C_{ob}) + \frac{c_2}{\theta_0} + \frac{1}{m}. \) This together with (26) yields
\[
|y - y_d| \leq \ell_1^{-1}, \quad \forall t \geq T_r
\]
This ends the proof of Theorem 2.

**Remark 4:** In the practical application of the digital control system, the following dead-zone technique can be used to prevent the parameter drift problem of the adaptive law (35)
\[
\dot{\kappa} = \kappa_d \max[0, \min(\text{sign}(V), \text{sign}([z_1 - \varepsilon_c])]]
\]
where \( \kappa(0) = 0, \varepsilon_c \) is the small positive constant to be designed. Combined with (48) and the analysis of Theorem 2, it is not difficult to find that \( V \leq \varepsilon_c \) can be achieved within fixed time. This together with (26) and (45) yields that
\[
|y - y_d| \leq \ell_1^{-1} + \varepsilon_c
\]
can be achieved in fixed time.

**Remark 5:** In this paper, the contradiction method to prove that Zeno-behavior [57] is avoided. Suppose that \( \Delta t_k = t_{k+1} - t_k = 0 \). Due to the function \( v(t) \) is always continuous, so one has
\[
\lim_{\Delta t_k \to 0} |e(t_k + \Delta t_k)| = \lim_{\Delta t_k \to 0} |v(t_k + \Delta t_k) - v(t_k)| = 0.
\]
However, it can be seen from (40) that
\[
|e(t_k)| = \lim_{\Delta t_k \to 0} |e(t_k + \Delta t_k)| > m > 0.
\]
The above analysis indicates that (50) contradicts the event-triggering condition (40), which means that the Zeno-behavior can be strictly avoided under the triggering mechanism (40).

### 4 Simulation and Experiment

#### 4.1 Numerical simulation

(1) **Simulation model of the ShW system**

According to existing researches [10, 11], the friction torque \( \tau_f \) in (4) are regarded as \( \tau_f = 0.2(\tanh(100x_2) - \tanh(x_2)) + 30\tanh(100x_2) + 10x_2 \), and self-aligning torque \( \tau_e \) and the parameters of the system (4) can be easily obtained in [10]. The different stiffness coefficients and vehicle velocity with respect to \( \tau_e \) are respectively used in the two simulations

\[
\begin{align*}
C_f &= C_r = 80000, \quad v = 5m/s, \quad \text{Simulation I} \\
C_f &= C_r = 60000, \quad v = 10m/s, \quad \text{Simulation II}
\end{align*}
\]

The initial condition of state is chosen as \( [x_1(0), x_2(0)] = [0.1, 0]^T \), the disturbance is assumed as \( d_{ob}(t) = 10\sin(2\pi), \) and the reference angle is considered as \( \gamma(t) = 5(\gamma_{ob} - y_d)t/(\text{rad}) \), with \( y_m = 0.4\sin(0.8t), \) \( \forall t \in [0, 10] ; \) \( y_m = 0.35\sin(0.6(t - 10)), \) \( \forall t \in [10, 20] ; \) \( y_m = 0.5\sin(0.4(t - 20)), \) \( \forall t \in [20, 30] ; y_m = 0.3\sin(0.8(t - 30)), \) \( \forall t \in [30, 40] ; y_m = 0.4\sin(0.4(t - 40)), \) \( \forall t \in [40, 50] ; y_m = 0.5\sin(0.2(t - 50)), \) \( \forall t \in [50, 60] ; y_m = 0.3\sin(0.4(t - 60)), \) \( \forall t \in [60, 70] . \)

(2) **Parameter selection of the designed model**

The parameters for the state observer (16)-(19) are selected as shown in Tab.6, where

| Table 6 The rule base of the fuzzy logic system |
| --- |
| \( \chi_3 \) | \( \chi_2 \) | \( \chi_1 \) | \( F_1 \) | \( F_2 \) | \( F_3 \) |
| \( F_1^1 \) | \( F_1^2 \) | \( F_1^3 \) | \( F_2^1 \) | \( F_2^2 \) | \( F_2^3 \) | \( F_3^1 \) | \( F_3^2 \) | \( F_3^3 \) |
| \( G_1 \) | \( G_2 \) | \( G_3 \) | \( G_4 \) | \( G_5 \) | \( G_6 \) | \( G_7 \) | \( G_8 \) | \( G_9 \) | \( G_{10} \) | \( G_{11} \) | \( G_{12} \) | \( G_{13} \) | \( G_{14} \) | \( G_{15} \) | \( G_{16} \) | \( G_{17} \) | \( G_{18} \) | \( G_{19} \) | \( G_{20} \) | \( G_{21} \) | \( G_{22} \) | \( G_{23} \) | \( G_{24} \) | \( G_{25} \) | \( G_{26} \) | \( G_{27} \) |

\( G_j, j = 1 \cdots 27 \) are the fuzzy sets of the fuzzy output, and the center points of the sets \( G_j \) are defined as \( \Theta_j \) with \( \Theta_j \) being the \( j \)-th element of the vector \( \Theta \). The parameters of (26)-(41) and (48) are \( \ell_1 = 2200, \ell_2 = 40, \ell_3 = 5, \) \( m_1 = m_2 = m_3 = 0.01, \) \( \kappa_d = 0.01, \) \( \eta_{11} = 80, \)
where $A_h = [0, 1, 0, 0, 0, 1; 0, 0, 0, 1]$, $B_h = [0, 0, 1]^T$, $C = [1, 0, 0]$, and $L_h$ is the observer gain vector. According to Theorem 3.2 and the equation (19) of [25], $L_h = [0.9918, 35297]^T$ is chosen in simulation by solving matrix inequality (24) of [25].

The adaptive event-triggered control [39] and observer-based fuzzy event-triggered control [27] are chosen for comparison in this paper. For the SbW system (5), the adaptive state observer designed in [27] can be expressed as

$$
\begin{align*}
\dot{x}_1 &= \dot{x}_2 + k_1 \phi_1(x_1 - \hat{x}_1) \\
\dot{x}_2 &= f(\dot{x}_2, u_f(\dot{\theta}) + u + k_2 \phi_1(x_1 - \hat{x}_1)
\end{align*}
$$

(54)

where $b_1$ and $b_2$ satisfy that $A = [-k_1, 1; -k_2, 0]$ is Hurwitz, and $u_f = H_L(s)u$ with $H_L(s)$ being the Butterworth low-pass filters (as described in [27]). The event-triggered control designed in [27] can be expressed as

$$
u(t) = r(t_k), \quad \forall t \in [t_k, t_{k+1})$$

(55)

$$t_{k+1} = \{t > t_k : |r(t) - u(t)| \geq \varepsilon_2\}, \quad t_1 = 0$$

(56)
Table 7 Control performance in the simulation I

| Index | Method | 0~10π/8 | 10π/8~20π/8 | 20π/8~30π/8 | 30π/8~40π/8 | 40π/8~50π/8 | 50π/8~60π/8 | 60π/8~70π/8 | 0~70π/8 |
|-------|--------|---------|------------|------------|------------|------------|------------|------------|--------|
| RMSE  | ETC of [39] | 0.0077  | 0.0066     | 0.0094     | 0.0057     | 0.0076     | 0.0094     | 0.0057     | 0.0076  |
|       | ETC of [27] | 0.0104  | 0.0079     | 0.0100     | 0.0054     | 0.0066     | 0.0078     | 0.0044     | 0.0078  |
|       | New ETC   | 0.0097  | 0.0049     | 0.0054     | 0.0048     | 0.0034     | 0.0029     | 0.0031     | 0.0053  |
| IAE   | ETC of [39] | 0.2142  | 0.1882     | 0.2666     | 0.1607     | 0.2152     | 0.2671     | 0.1623     | 1.4744  |
|       | ETC of [27] | 0.2884  | 0.2223     | 0.2834     | 0.1527     | 0.1886     | 0.2200     | 0.1256     | 1.4810  |
|       | New ETC   | 0.3858  | 0.2531     | 0.2666     | 0.2435     | 0.1223     | 0.0955     | 0.1041     | 1.4720  |
| NoE   | ETC of [39] | 5.139   | 3.758      | 4.121      | 3.693      | 3.416      | 2.834      | 2.958      | 2.5919  |
|       | ETC of [27] | 2.700   | 3.229      | 3.882      | 4.600      | 5.061      | 5.671      | 5.877      | 3.1020  |
|       | New ETC   | 3.122   | 3.387      | 4.106      | 4.371      | 3.541      | 2.747      | 4.606      | 2.5880  |

Table 8 Control performance in the simulation II

| Index | Method | 0~10π/8 | 10π/8~20π/8 | 20π/8~30π/8 | 30π/8~40π/8 | 40π/8~50π/8 | 50π/8~60π/8 | 60π/8~70π/8 | 0~70π/8 |
|-------|--------|---------|------------|------------|------------|------------|------------|------------|--------|
| RMSE  | ETC of [39] | 0.0125  | 0.0110     | 0.0156     | 0.0093     | 0.0126     | 0.0157     | 0.0095     | 0.0126  |
|       | ETC of [27] | 0.0214  | 0.0166     | 0.0213     | 0.0114     | 0.0142     | 0.0167     | 0.0095     | 0.0164  |
|       | New ETC   | 0.0107  | 0.0050     | 0.0061     | 0.0050     | 0.0037     | 0.0028     | 0.0028     | 0.0057  |
| IAE   | ETC of [39] | 0.3518  | 0.3107     | 0.4433     | 0.2639     | 0.3569     | 0.4451     | 0.2684     | 2.4400  |
|       | ETC of [27] | 0.6040  | 0.4698     | 0.6025     | 0.3233     | 0.4031     | 0.4724     | 0.2691     | 3.1442  |
|       | New ETC   | 0.4510  | 0.2735     | 0.3116     | 0.2638     | 0.1413     | 0.0814     | 0.0889     | 1.6510  |
| NoE   | ETC of [39] | 10.659  | 8.580      | 8.966      | 8.808      | 8.676      | 4.834      | 5.180      | 5.3794  |
|       | ETC of [27] | 2.838   | 3.188      | 3.756      | 4.398      | 4.957      | 5.544      | 6.051      | 3.0732  |
|       | New ETC   | 3.443   | 3.165      | 3.279      | 3.383      | 2.884      | 2.820      | 2.919      | 2.1893  |

Fig. 8 Observation result in the simulation I. (a) State variable $x_2$ and its observation $\hat{x}_2$, (b) State variable $x_3$ and its observation $\hat{x}_3$.

Fig. 9 Observation result in the simulation II.

with $g_2 > 0$, and $r(t)$ being designed as $r(t) = \alpha_2(t) - g_1 \tanh(\frac{c_2 g_1}{\epsilon})$, where $g_1 > g_2 > 0$, $\epsilon > 0$, $\xi_2 = \hat{x}_2 - x_1$, and $\alpha_1$ and $\alpha_2$ can be found in [27]. For the event-triggered control [27], the parameters of equations (7), (27)-(29), (31), (45), (46), (60), and (61) of [27] are chose as $c_1 = 5$, $c_2 = 10$, $k_1 = 5$, $k_2 = 2000$, $\omega = 1.5$, $\omega_0 = 0.8$, $\eta_{\max} = 0.5$, $\eta_{\min} = 0.4$, $\sigma = 0.01$, $\epsilon = 0.1$, $\gamma_2 = 0.001$, $g_1 = 1$, and $g_2 = 2$. Besides, the event-triggered control with switching threshold strategy [39] can be expressed as

$$u(t) = \omega(t_k), \quad \forall t \in [t_k, t_{k+1})$$

$$t_{k+1} = \begin{cases} \{t > t_k : |e(t)| \geq \delta |u(t)| + m_1\}, & |u(t)| < D \\ \{t > t_k : |e(t)| \geq m_1\}, & |u(t)| \geq D \end{cases}$$
where \( e(t) = u(t) - \omega(t) \), \( \delta \), \( m \), \( m_1 \) and \( D \) are positive constants to be designed, and \( \omega(t) \) being designed as
\[
\omega(t) = 1 + \frac{\varphi}{g} \left( (\alpha_2 + \beta_3) \tanh \left( \frac{\beta_2 (\alpha_2 + \beta_3)}{\varepsilon} \right) + m_1 \tanh \left( \frac{m_1 \alpha_2 - \beta_3}{\varepsilon} \right) \right)
\]
where \( \alpha_2 \) can be obtained from [39], and the related parameters in [39] are chosen as \( c_1 = 180, \varphi_1 = 0 \), \( c_2 = 160 \), \( \varphi_2 = \pi_1 \), \( \Gamma = 1 \), \( \sigma = 0.1 \), \( \varepsilon = 1 \), \( \delta = 0.4 \), \( m = 0.2 \), and \( D = 30 \).

(4). Simulation results and analysis

Fig. 6 and Tab. 7 give the control performance of the different ETCs in the simulation I. From Fig. 6 and Tab. 7, one can find that in the designed event-triggered control system, the tracking error can converge to the smaller residual set of the origin, and the jumping phenomenon of control input can be avoided while saving more communication resources. Fig. 7 and Tab. 8 give the control performance of the different ETCs in simulation II. One can find that the ETC designed in this paper also has better adaptability to model uncertainty. Fig. (8) and Fig. 9 give the observation results in simulation I and simulation II. From Fig. 8, one can find that both the designed adaptive state observer and model-based HGO can achieve satisfactory observation performance without model uncertainty. As shown in Fig. 9, when the model uncertainty is considered in simulation II, i.e., when the model parameters are changed, the observation performance of the adaptive observer designed in this paper is better than that of the model-based HGO.

4.2 Experiment

The experiment platform of SbW systems is shown in Fig. 10. In this platform, the single board computer (dSPACE-ds1202) is used as the control unit of SbW systems, and the servo motor driver (XinJIE DS2-20P7) is used for driving the steering motor (XinJIE MS80ST-M02430B-20P7) equipped with a reducer. The linear sensor (KTR11-10) fixed on the steering arm measures the steering angle of the front-wheels. A computer is applied to display the experimental results of the experiment on-line and store the experimental data. The sampling period is chosen as 0.001s.

(1). Parameter selection of the proposed method

The parameters for the state observer (16)-(19) are selected as \( m_1 = 0.5, \mu_2 = 0.1, k_1 = 5, k_2 = 15, k_3 = 10, b_{ab} = 0.2, \gamma_1 = 360, \sigma_1 = 0.01, \gamma_2 = 120, \sigma_2 = 0.01, \varepsilon = 0.1 \), and \( Q = diag(10^5, 0^5, 0^5) \). The initial values \( \Theta(0) = \text{zeros}(27, 1) \) and \( \tilde{d}(0) = 0 \) are used. For the fuzzy logic system, the membership function and fuzzy rule base are the same as those in simulation. The parameters of (26)-(41) and (48) are \( \ell_1 = 120, \ell_2 = 10, \ell_3 = 2, m_1 = m_2 = m_3 = 0.01, \kappa_4 = 0.01, \eta_1 = 40, \eta_2 = 15, \eta_3 = 30, \eta_4 = \eta_5 = \eta_6 = 0.1, m = 15, \delta = 0.1 \) and \( \varepsilon = 0.06 \). To verify the robustness of the designed controller to time-varying disturbance, \( u_{real} = u + d_{e}(t) \) is considered as the real control input of the SbW system with \( u \) being the designed control input. Besides, the following cases are considered

\[
\begin{cases}
\frac{d_{e}(t)}{5 \cos(2t)} = 2 m/s, & \text{Experiment I} \\
\frac{d_{e}(t)}{2 \sin(4t)} = 5 m/s, & \text{Experiment II}
\end{cases}
\]

(2). Parameter selection of the compared methods

To verify the adaptability and superiority of the designed observer, HGO [25] is used to estimate the system (15) for comparison in experiment. According to Theorem 3.2 and the equation (19) of [25], \( L_h = [14.16, 339.4, 7965.86]^T \) is chosen in experiment by solving matrix inequality (24) of [25]. Besides, the following low-pass filter [21] is used for comparison.

\[
\tau \ddot{x} = x(t) - \hat{x}
\]

where \( \hat{x} \) is the estimation of \( x \) and \( \tau \) is the small positive constant to be designed. \( \tau = 0.005 \) is chosen in experiment.

To verify the jumping phenomenon of control input caused by the event-triggered communication can...
be eliminated under the designed event-triggered control system, the adaptive event-triggered control [39] and observer-based fuzzy event-triggered control [27] are chosen for comparison. For the event-triggered control [39], the parameters and functions of the equations (5), (7), (21), (22) and (28) of [39] are chosen as $c_1 = 20$, $\varphi_1 = 0$, $c_2 = 35$, $\varphi_2 = x_1$, $I' = 1$, $\sigma = 0.1$, $\varepsilon = 1$, $\delta = 0.4$, $m = 0.2$, and $D = 20$. For the event-triggered control [27], the parameters of equations (7), (27)-(29), (31), (45), (46), (60), and (61) of [27] are chose as $c_1 = 10$, $c_2 = 25$, $k_1 = 1$, $k_2 = 200$, $\omega_0 = 2$, $\omega_\infty = 1$, $\eta_{\max} = 0.8$, $\eta_{\min} = 0.6$, $\sigma = 0.01$, $\varepsilon = 1$, $\gamma_2 = 0.001$, $\varrho_1 = 5$, and $\varrho_2 = 2$.

(3). Experiment results and analysis

Fig. 11 and Tab.9 give control results in experiment I under the different ETCs. It is easy to find that in the designed event-triggered control system, the tracking error can converge to the smaller neighborhood of the origin, and the jumping phenomenon of control input caused by event-triggered communication can be avoided while saving more communication resources can be saved. Fig.12 and Tab.10 give control results in experiment II. It also can be found that the designed ETC can achieve better tracking accuracy, and the control input is without jumping phenomenon. Fig. 13 and Fig.14 give the observation result of the state $x_2$ and $x_3$ under the different experiments. From Fig. 13 and Fig.14, one can find that the estimation results under
the state observer are more smooth and suitable for the design of the output feedback controller. Besides, one can also find that the trend of observation results under the designed state observer is more consistent with the estimation results of the low-pass filter than the model-based HGO.

5 Conclusion

This paper proposes an event-triggered fixed-time control for uncertain SbW systems by considering the bandwidth limitation of CAN. A new framework is proposed to eliminate the jumping phenomenon of the event-based control input. Then, an adaptive fuzzy-based state observer and disturbance observer are proposed to estimate the unavailable state and disturbance of the SbW system. Furthermore, an adaptive event-triggered control is proposed for SbW systems by considering the effect of observation error and event-triggered error, such that the prespecified tracking performance can be guaranteed within fixed time. Finally, simulations and experiments are presented to evaluate the effectiveness and superiority of the proposed methods. Future research on the SbW control system will include the following aspects:

(1) Input saturation and output constraint of the SbW system should be addressed. In the practical application, the control input saturation of SbW systems, i.e., output torque saturation of the steering motor, often occurs due to hardware limitations, limiting the SbW system performance severely even leads to the SbW system instability. Besides, the output constraint of SbW systems, i.e., the steering angle constraint of the front-wheels, is inevitable. So, the input saturation and output constraint of SbW systems will be considered in the control design.

(2) The time delay phenomenon should be considered. In the practical application of the SbW system, the time delay phenomenon caused by measurement and communication is inevitable, which will be considered in future research on SbW control systems.

Appendix A: Proof of Lemma 3

Case 1 $\beta_1 \leq \beta_2$. In this case, the inequality (13) can be expressed as (61) or (62)

$$V(x) \leq -\gamma \alpha_1 V(x) - \zeta \alpha_1 V(x) - \alpha_3 V^{\beta_2}(x) + \alpha_3 V^{\beta_2}(x)$$

(61)

$$\dot{V}(x) \leq -\alpha_1 V(x) - \zeta \alpha_2 V^{\beta_2}(x) - \alpha_3 V^{\beta_2}(x) + \alpha_3 V^{\beta_2}(x)$$

(62)

where $\zeta > 0$ and $\gamma > 0$ satisfy $\gamma + \zeta = 1$. From (61), the following inequality can be obtained if $\alpha_1 V(x) - \alpha_3 V^{\beta_2}(x) > 0$

$$\dot{V}(x) \leq -\zeta \alpha_1 V(x) - \alpha_2 V^{\beta_1}(x).$$

(63)

| Table 9 | Control performance in the experiment I |
| --- | --- |
| Index | Method | 0~10s | 10s~20s | 20s~30s | 30s~40s | 40s~50s | 50s~60s | 60s~70s | 0~70s |
| RMSE | ETC of [39] | 0.0201 | 0.0175 | 0.0251 | 0.0138 | 0.0199 | 0.0252 | 0.0150 | 0.0290 |
| | ETC of [27] | 0.0202 | 0.0139 | 0.0152 | 0.0085 | 0.0091 | 0.0102 | 0.0057 | 0.0127 |
| | New ETC | 0.0192 | 0.0122 | 0.0126 | 0.0132 | 0.0109 | 0.0089 | 0.0094 | 0.0127 |
| IAE | ETC of [39] | 0.5267 | 0.4744 | 0.6904 | 0.3476 | 0.5314 | 0.7001 | 0.4046 | 3.6752 |
| | ETC of [27] | 0.5535 | 0.3854 | 0.4106 | 0.2326 | 0.2419 | 0.2740 | 0.1482 | 2.2562 |
| | New ETC | 0.4789 | 0.3106 | 0.3234 | 0.3443 | 0.2860 | 0.2268 | 0.2437 | 2.2137 |
| NoE | ETC of [39] | 8296 | 7080 | 10786 | 9494 | 7952 | 10654 | 6449 | 6071 |
| | ETC of [27] | 1649 | 1971 | 2972 | 3635 | 5596 | 6066 | 2656 | 2656 |
| | New ETC | 3509 | 3082 | 3903 | 3712 | 3460 | 3762 | 2604 | 2403 |

| Table 10 | Control performance in the experiment II |
| --- | --- |
| Index | Method | 0~10s | 10s~20s | 20s~30s | 30s~40s | 40s~50s | 50s~60s | 60s~70s | 0~70s |
| RMSE | ETC of [39] | 0.0259 | 0.0223 | 0.0317 | 0.0189 | 0.0255 | 0.0317 | 0.0191 | 0.0255 |
| | ETC of [27] | 0.0334 | 0.0226 | 0.0254 | 0.0136 | 0.0152 | 0.0170 | 0.0095 | 0.0209 |
| | New ETC | 0.0232 | 0.0130 | 0.0129 | 0.0151 | 0.0111 | 0.0089 | 0.0096 | 0.0141 |
| IAE | ETC of [39] | 0.7230 | 0.6213 | 0.8879 | 0.5215 | 0.7107 | 0.8914 | 0.5299 | 4.8857 |
| | ETC of [27] | 0.9297 | 0.6350 | 0.7041 | 0.3823 | 0.4176 | 0.4796 | 0.2548 | 3.8031 |
| | New ETC | 0.5747 | 0.3386 | 0.3407 | 0.3944 | 0.2954 | 0.2272 | 0.2506 | 2.4216 |
| NoE | ETC of [39] | 12412 | 12830 | 15312 | 12452 | 13698 | 14997 | 10631 | 92332 |
| | ETC of [27] | 1682 | 1998 | 3022 | 3736 | 4617 | 5748 | 6212 | 27015 |
| | New ETC | 3616 | 3183 | 3877 | 3881 | 3592 | 3850 | 2686 | 24685 |

RMSE=$\sqrt{\frac{1}{n} \sum_{i=1}^{n} s_i^2}$, SD $= \sqrt{\frac{1}{n} \sum_{i=1}^{n} (s_i - \bar{s})^2}$, and IAE $= \int_0^T |s| \, dt$ with $s = y - y_d$. NoE: Number of the event.
This together with Corollary 1 of [54] yields
\[ V^{1-\beta_2}(x) \leq \frac{\alpha_3}{(1-\varsigma)\alpha_1}, \quad \forall \, t \geq T_{re_1} \] (64)
with \( T_{re_1} \leq T_0 + \frac{1}{\alpha_1(1-\beta_2)} \ln \frac{\alpha_2 + \alpha_1 V^{1-\beta_1}(T_0)}{\alpha_3} \) and \( T_0 \) being the initial time. Similarly, from (62), one can get
\[ V^{\beta_2-\beta_1}(x) \leq \frac{\alpha_3}{(1-\varsigma)\alpha_2}, \quad \forall \, t \geq T_{re_2} \] (65)
with \( T_{re_2} \leq T_0 + \frac{1}{\alpha_2(1-\beta_1)} \ln \frac{\alpha_3 + \alpha_2 V^{1-\beta_1}(T_0)}{\alpha_3} \). This completes the proof of Case 1 in Lemma 3.

Case 2. \( \beta_1 < \beta_2 \). In this case, the inequality (13) can be expressed as (61). Thus, the conclusion as in (64) can be obtained, which completes the proof of Case 2 in Lemma 3. This completes the proof of Lemma 3.

Appendix B: Proof of Lemma 4

From (18) one can get
\[ \Theta^T \dot{\Theta} \leq -\sigma_1 \| \Theta \|^2 + \gamma_1 \| \Theta \| \] (66)
with \( |\psi(S)| \leq 1 \) and \( \| \chi(S) \| \leq 1 \). Thus, it is not difficult to get that \( \| \Theta \| \leq T = \max(\gamma_1/\sigma_1, \| \Theta \|(0)) \). This together with the definition \( \dot{\Theta} = \Theta - \Theta^* \) yields \( \| \dot{\Theta} \| \leq \| \Theta^* \| + \dot{\Theta} \). Similarly, the estimation error \( \ddot{d} \) is always bounded. This ends the proof of Lemma 4.

Conflict of interest statement

The authors have no conflicts of interest to declare that are relevant to the content of this article.

References

1. A. Kirli, Y. Chen, C. Okwudire, G. Ulsoy, Torque-vectoring-based backup steering strategy for steer-by-wire autonomous vehicles with vehicle stability control, IEEE Trans. Veh. Technol 68 (8) (2019) 7319–7328.
2. M. D. Vaio, F. Falcone, R. Hult, A. Petrillo, S. Santini, Design and experimental validation of a distributed interaction protocol for connected autonomous vehicles at a road intersection, IEEE Trans. Veh. Technol 68 (10) (2019) 9451–9465.
3. H. Peng, W. Wang, Q. An, C. Xiang, L. Li, Path tracking and direct yaw moment coordinated control based on robust mpc with the finite time horizon for autonomous independent-drive vehicles, IEEE Trans. Veh. Technol 69 (6) (2020) 6053–6066.
4. Z. Luan, J. Zhang, W. Zhao, C. Wang, Trajectory tracking control of autonomous vehicle with random network delay, IEEE Trans. Veh. Technol 69 (8) (2020) 8140–8150.
5. S. Tuohy, M. Glavin, C. Hughes, E. Jones, M. Trivedi, L. Kilmartin, Intra-vehicle networks: A review, IEEE Trans. Intell. Transp. Syst. 16 (2) (2015) 534–545.
6. R. Marino, S. Scalzi, M. Netto, Nested pid steering control for lane keeping in autonomous vehicles, Control Eng. Pract. 19 (12) (2011) 1459–1467.
7. P. Yih, J. Gerdes, Modification of vehicle handling characteristics via steer-by-wire, IEEE Trans. Control Syst. Technol. 13 (6) (2005) 965–976.
8. P. Falcone, F. Borrelli, J. Asgari, H. E. Tseng, D. Hrovat, Predictive active steering control for autonomous vehicle systems, IEEE Trans. Control Syst. Technol. 15 (3) (2007) 566–580.
9. H. Wang, H. Kong, Z. Man, D. M. Tuan, Z. Cao, W. Shen, Sliding mode control for steer-by-wire systems with ac motors in road vehicles, IEEE Trans. Ind. Electron. 61 (3) (2014) 1596–1611.
10. M. T. Do, Z. Man, C. Zhang, H. Wang, F. S. Tay, Robust sliding mode-based learning control for steer-by-wire systems in modern vehicles, IEEE Trans. Veh. Technol. 63 (2) (2014) 580–590.
11. Z. Sun, J. Zheng, Z. Man, H. Wang, Robust control of a vehicle steer-by-wire system using adaptive sliding mode, IEEE Trans. Ind. Electron. 63 (4) (2016) 2251–2262.
12. H. Wang, Z. Man, H. Kong, Y. Zhao, M. Yu, Z. Cao, J. Zheng, M. Do, Design and implementation of adaptive terminal sliding mode control on a steer-by-wire equipped road vehicle, IEEE Trans. Ind. Electron. 63 (9) (2016) 5774–5785.
13. X. D. Wu, M. M. Zhang, M. Xu, Active tracking control for steer-by-wire system with disturbance observer, IEEE Trans. Veh. Technol. 68 (6) (2019) 5483–5493.
14. C. Huang, F. Naghdy, H. Du, Delta operator-based fault estimation and fault-tolerant model predictive control for steer-by-wire systems, IEEE Trans. Control Syst. Technol. 26 (5) (2018) 1810–1817.
15. C. Huang, F. Naghdy, H. Du, Fault tolerant sliding mode predictive control for uncertain steer-by-wire system, IEEE Trans. Cybern 49 (1) (2019) 1810–1817.
16. A. Levant, Higher-order sliding modes, differentiation and output-feedback control, Int. J. Control 76 (9-10) (2003) 924–941.
17. A. Levant, M. Livne, Exact differentiation of signals with unbounded higher derivatives, IEEE Trans. Autom. Control 57 (4) (2012) 1076–1080.
18. J. Davila, L. Fridman, A. Levant, Second-order sliding-mode observer for mechanical systems, IEEE Trans. Autom. Control 50 (11) (2005) 1785–1789.
19. A. Apaza-Perez, J. A. Moreno, L. Fridman, Dissipative approach to sliding mode observers design for uncertain mechanical systems, Automatica 87 (2018) 330–336.
20. T. R. Oliveira, V. H. Pereira Rodrigues, L. Fridman, Generalized model reference adaptive control by means of global homs differentiators, IEEE Trans. Autom. Control 64 (5) (2019) 2053–2060.
21. Y. Li, S. Tong, L. Liu, G. Feng, Adaptive output-feedback control design with prescribed performance for switched nonlinear systems, Automatica 80 (2017) 225–231.
22. K. D. V. Ellenrieder, Dynamic surface control of trajectory tracking marine vehicles with actuator magnitude and rate limits - sciencedirect, Automatica 105 (2019) 433–442.
23. S. Fatemeh, A. M. Mehdi, K. Alireza, K. H. Reza, Observer-based fuzzy adaptive dynamic surface control of uncertain nonstrict feedback systems with unknown control direction and unknown dead-zone, IEEE Trans. Syst., Man, Cybern., Syst. 49 (11) (2019) 2340–2351.
24. A. Alessandri, A. Rossi, Increasing-gain observers for nonlinear systems: Stability and design, Automatica 57 (2015) 180–188.
25. A. Zemouche, F. Zhang, F. Mazenc, R. Rajamani, High-gain nonlinear observer with lower tuning parameter, IEEE Trans. Autom. Control 64 (8) (2019) 3194–3209.
26. S. Tong, B. Huo, Y. Li, Observer-based adaptive decentralized fuzzy fault-tolerant control of nonlinear large-scale systems with actuator failures, IEEE Trans. Fuzzy Syst. 22 (1) (2014) 1–15.
27. J. Qiu, K. Sun, T. Wang, H. J. Gao, Observer-based fuzzy adaptive event-triggered control for pure-feedback nonlinear systems with prescribed performance, IEEE Trans. Fuzzy Syst. 27 (11) (2019) 2152–2162.
28. L. Y. Xin, Y. G. Hong, Observer-based fuzzy adaptive event-triggered control design for a class of uncertain nonlinear systems, IEEE Trans. Fuzzy Syst. 26 (3) (2018) 1589–1599.
29. L. Zhang, G. Yang, Observer-based fuzzy adaptive sensor fault compensation for uncertain nonlinear strict-feedback systems, IEEE Transactions on Fuzzy Systems 26 (4) (2018) 2301–2310. doi:10.1109/TFUZZ.2017.2772879.
30. L. Cao, H. Li, N. Wang, Q. Zhou, Observer-based event-triggered adaptive decentralized fuzzy control for nonlinear large-scale systems, IEEE Trans. Fuzzy Syst. 27 (6) (2019) 1201–1214.
31. C. H. Zhang, G. H. Yang, Event-triggered adaptive output feedback control for a class of uncertain nonlinear systems with actuator failures, IEEE Trans. Cybern. 50 (1) (2020) 201–210.
32. P. Tabuada, Event-triggered real-time scheduling of stabilizing control tasks, IEEE Trans. Automa. Control 52 (9) (2007) 1680–1685.
33. T. Liu, Z. P. Jiang, A small-gain approach to robust event-triggered control of nonlinear systems, IEEE Trans. Automa. Control 60 (8) (2015) 2072–2085.
34. P. Tallapragada, N. Chopra, On event triggered tracking for nonlinear systems, IEEE Trans. Au-
35. E. Garcia, P. J. Antsaklis, Model-based event-triggered control for systems with quantization and time-varying network delays, IEEE Trans. Autom. Control 58 (2) (2013) 422–434.

36. A. Adaldo, F. Alderisio, D. Liuzza, G. Shi, D. V. Dimarogonas, M. di Bernardo, K. H. Johansson, Event-triggered pinning control of switching networks, IEEE Trans. Control Netw. Syst. 2 (2) (2015) 204–213.

37. A. Q. Wang, L. Liu, J. B. Qiu, G. Feng, Event-triggered robust adaptive fuzzy control for a class of nonlinear systems, IEEE Trans. Fuzzy Syst. 27 (8) (2019) 1648–1657.

38. Y. X. Huang, Y. G. Liu, Practical tracking via adaptive event-triggered feedback for uncertain nonlinear systems, IEEE Trans. Automa. Control 64 (9) (2019) 3920–3927.

39. L. T. Xing, C. Y. Wen, Z. T. Liu, H. Y. Su, J. P. Cai, Event-triggered adaptive control for a class of uncertain nonlinear systems, IEEE Trans. Automa. Control 62 (4) (2017) 2071–2076.

40. L. T. Xing, C. Y. Wen, Z. T. Liu, H. Y. Su, J. P. Cai, Event-triggered output feedback control for a class of uncertain nonlinear systems, IEEE Trans. Automa. Control 64 (1) (2019) 290–297.

41. B. Xian, M. S. D. Queiroz, D. M. Dawson, M. L. McIntyre, A discontinuous output feedback controller and velocity observer for nonlinear mechanical systems, Automatica 40 (4) (2004) 695–700.

42. W. A. Apaza-Perez, J. A. Moreno, L. Fridman, Global sliding mode observers for some uncertain mechanical systems, IEEE Trans. Automa. Control 65 (3) (2020) 1348–1355.

43. H. Ma, H. Li, H. Liang, G. Dong, Adaptive fuzzy event-triggered control for stochastic nonlinear systems with full state constraints and actuator faults, IEEE Trans. Fuzzy Syst. 27 (11) (2019) 2242–2252.

44. C. H. Zhang, G. H. Yang, Event-triggered adaptive output feedback control for a class of uncertain nonlinear systems with actuator failures, IEEE Trans. Cybern. 50 (1) (2020) 201–210.

45. S. Haggag, D. Alstrom, S. Cetinkunt, A. Egelja, Modeling, control, and validation of an electro-hydraulic steer-by-wire system for articulated vehicle applications, IEEE/ASME Trans. Mechatronics 10 (6) (2005) 688–692.

46. Y. Yamaguchi, T. Murakami, Adaptive control for virtual steering characteristics on electric vehicle using steer-by-wire system, IEEE Trans. Ind. Electron. 56 (5) (2009) 1585–1594.

47. A. Baviskar, J. R. Wagner, D. M. Dawson, An adjustable steer-by-wire haptic-interface tracking controller for ground vehicles, IEEE Trans. Veh. Technol. 58 (2) (2009) 546–554.