Dynamics in spinor condensates controlled by a microwave dressing field

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(Dated: October 28, 2013)

We experimentally study spin dynamics in a sodium antiferromagnetic spinor condensate with off-resonant microwave pulses. In contrast to a magnetic field, a microwave dressing field enables us to explore rich spin dynamics under the influence of a negative net quadratic Zeeman shift $q_{\text{net}}$. We find an experimental signature to determine the sign of $q_{\text{net}}$, and observe harmonic spin population oscillations at every $q_{\text{net}}$ except near each separatrix in phase space where spin oscillation period diverges. In the negative and positive $q_{\text{net}}$ regions, we also observe a remarkably different relationship between each separatrix and the magnetization. Our data confirms an important prediction derived from the mean-field theory: spin-mixing dynamics in spin-1 condensates substantially depends on the sign of the ratio of $q_{\text{net}}$ and the spin-dependent interaction energy. This work may thus be the first to use only one atomic species to reveal mean-field spin dynamics, especially the separatrix, which are predicted to appear differently in spin-1 antiferromagnetic and ferromagnetic spinor condensates.

PACS numbers: 32.60.+i, 67.85.Hj, 03.75.Kk, 03.75.Mn

An atomic Bose-Einstein condensate (BEC) is a state where all atoms have a single collective wavefunction for their spatial degrees of freedom. The key benefit of spinor BECs is the additional spin degree of freedom. Together with Feshbach resonances and optical lattices which tune the interatomic interactions, spinor BECs constitute a fascinating collective quantum system offering an unprecedented degree of control over such parameters as spin, temperature, and the dimensionality of the system [1, 2]. Spinor BECs have become one of the fastest moving research frontiers in the past fifteen years. A number of atomic species have proven to be perfect candidates in the study of spinor BECs, such as $F=1$ and $F=2$ hyperfine spin states of $^{87}\text{Rb}$ atoms [1–7], and $F=1$ hyperfine spin manifolds of $^{23}\text{Na}$ atoms [8–12]. Magnetic fields can induce the quadratic Zeeman energy shift $q_{B}$. Many interesting phenomena driven by an interplay between $q_{B}$ and the spin-dependent interaction energy $c$ have been experimentally demonstrated in spinor BECs, such as spin population dynamics [1, 9], quantum number fluctuation [10, 13], various quantum phase transitions [1, 9, 11, 12], and quantum spin-nematic squeezing [14]. Such systems have been successfully described with a classical two-dimensional phase space [11, 14, 17], a rotor model [15], or a quantum model [13, 17].

In this paper, we experimentally study spin-mixing dynamics in a $F=1$ sodium spinor condensate starting from a nonequilibrium initial state, as a result of antiferromagnetic spin-dependent interactions and the quadratic Zeeman energy $q_{M}$ induced by an off-resonant microwave pulse. In contrast to a magnetic field, a microwave dressing field enables us to explore rich spin dynamics under the influence of a negative net quadratic Zeeman energy shift $q_{\text{net}}$. A method to characterize the microwave dressing field is also explained. In both negative and positive $q_{\text{net}}$ regions, we observe spin population oscillations resulting from coherent collisional interconversion among two $|F=1, m_{F}=0\rangle$ atoms, one $|F=1, m_{F}=+1\rangle$ atom, and one $|F=1, m_{F}=-1\rangle$ atom. In every spin oscillation studied in this paper, our data shows that the population of the $m_{F}=0$ state averaged over time is always larger (or smaller) than its initial value as long as $q_{\text{net}}<0$ (or $q_{\text{net}}>0$). This observation provides an experimental signature to determine the sign of $q_{\text{net}}$. We also find a remarkably different relationship between the total magnetization $m$ and a separatrix in phase space where spin oscillation period diverges: the position of the separatrix moves slightly with $m$ in the positive $q_{\text{net}}$ region, while the separatrix quickly disappears when $m$ is away from zero in the negative $q_{\text{net}}$ region. Our data confirms an important prediction derived by Ref. [17]: the spin-mixing dynamics in $F=1$ spinor condensates substantially depends on the sign of $R=q_{\text{net}}/c$. This work may thus be the first to use only one atomic species to reveal mean-field spin dynamics, especially the separatrix, which are predicted to appear differently in $F=1$ antiferromagnetic and ferromagnetic spinor condensates.

Similar to Ref. [1, 11], we take into account the conservation of $m$ and the total atom number. Because no spin domains and spatial modes are observed in our system, the single spatial mode approximation (SMA), in which all spin states have the same spatial wavefunction, appears to be a proper theoretical model to understand our data. Spin-mixing dynamics in a $F=1$ spinor BEC can thus be described with a two-dimensional ($\rho_{0}$ vs $\theta$) phase space, where the fractional population $\rho_{\text{mp}}$ and the phase $\theta_{\text{mp}}$ of each $m_{F}$ state are independent of position. The BEC energy $E$ and the time evolution of $\rho_{0}$ and $\theta$ may be expressed as [11, 16]

\begin{equation}
E = q_{\text{net}}(1-\rho_{0}) + c\rho_{0}[(1-\rho_{0}) + \sqrt{(1-\rho_{0})^{2} - m^{2}} \cos \theta],
\end{equation}

\begin{equation}
\dot{\rho}_{0} = -(2/h)\partial E/\partial \rho_{0}, \quad \dot{\theta} = (2/h)\partial E/\partial \rho_{0}.
\end{equation}

Here $q_{\text{net}} = q_{B} + q_{M}$, $\theta = \theta_{+1} + \theta_{-1} - 2\theta_{0}$ is the relative phase among the three $m_{F}$ spin states, and $h$ is the reduced Planck constant. The induced linear Zeeman shift

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remains the same during the collisional spin interconversion and is thus ignored. The spin-dependent interaction energy is $c = c_2 \langle n \rangle$, where $\langle n \rangle$ is the mean BEC density and $c_2$ is the spin dependent interaction coefficient. The total magnetization is $m = \rho_{+1} - \rho_{-1}$. It is well known that $q_{\text{net}} \propto B^2 > 0$, and $c_2 > 0$ (or $c_2 < 0$) in $F=1$ $^{23}$Na (or $^{87}$Rb) spinor BECs. Spin-dynamics in $F=1$ antiferromagnetic and ferromagnetic spinor BECs have been studied in magnetic fields where $q_{\text{net}} > 0$ with $^{23}$Na and $^{87}$Rb atoms, respectively [1]. A few methods have been explored for generating a negative quadratic Zeeman shift, such as via a microwave dressing field [1, 11, 19–21] or through a linearly polarized off-resonant laser beam [22]. In this paper, we choose the first method.

The experimental setup is similar to that illustrated in our previous work [23]. Hot $^{23}$Na atoms are slowed by a spin-flip Zeeman slower, captured in a standard magneto-optical trap, cooled through a polarization gradient cooling process to 40 µK, and loaded into a crossed optical dipole trap originating from a linearly-polarized high power IR laser at 1064 nm. After an optimized 6 s forced evaporative cooling process, a pure $F=1$ BEC of $1 \times 10^5$ sodium atoms is created. We can polarize atoms in the $F=1$ BEC fully to the $|F = 1, m_F = -1 \rangle$ state by applying a weak magnetic field gradient during the first half of the forced evaporation (or fully to the $|F = 1, m_F = 0 \rangle$ state by adding a very strong magnetic bias field during the entire 6 s forced evaporation). We then ramp up a small magnetic bias field with its strength $B$ being 270 mG, while turning off the field gradient. An rf-pulse resonant with the linear Zeeman splitting is applied to prepare an initial state with any desired combination of the three $m_F$ states. To generate sufficiently large $q_{\text{net}}$ with off-resonant microwave pulses, a microwave antenna designed for a frequency near the $|F = 1 \rangle \leftrightarrow |F = 2 \rangle$ transition is placed a few inches above the center of the magnetooptical trap, and connected to a function generator outputting a maximum radiation power of 10 W. After various hold time $t$ in the optical dipole trap, populations of the three spin states are then measured via the standard absorption imaging preceded by a 3 ms Stern-Gerlach separation and a 7 ms time of flight.

We observe spin oscillations at every given value of $q_{\text{net}}$ within a wide range, i.e., $-240 \text{ Hz} \leq q_{\text{net}}/h \leq 240 \text{ Hz}$. Here $h$ is the Planck constant. Typical time evolutions of $\rho_0$ starting with the same nonequilibrium initial state at a negative and a positive $q_{\text{net}}$ are shown in Fig. 1(a). We find that these evolutions can be well fit by sinusoidal functions of the similar oscillation period $T$ and amplitude $A$. On the other hand, our data in Fig. 1(a) shows that the value of $\langle \rho_0 \rangle$ is drastically different in the two spin oscillations: $\langle \rho_0 \rangle > \rho_0 |z=0 \rangle$ as long as $q_{\text{net}} < 0$, while $\langle \rho_0 \rangle < \rho_0 |z=0 \rangle$ if $q_{\text{net}} > 0$. Here $\langle \rho_0 \rangle$ is the average

![FIG. 1. (color online) (a). Time evolutions of $\rho_0$ at $q_{\text{net}}/h = +93 \text{ Hz} > 0$ (solid blue triangles) and $q_{\text{net}}/h = -83 \text{ Hz} < 0$ (solid red circles) with $m = 0$ and $c/h = 52 \text{ Hz}$. It is important to note that the two curves start from the same initial state which is marked by the solid black circles. Dashed black lines represent the energy of the above two experimental conditions shown in Fig. 1(a), i.e., $q_{\text{net}} > 0$ (left) and $q_{\text{net}} < 0$ (right). The heavy solid blue and red lines represent the energy of the separatrix between the running and oscillatory phase solutions. Darker colors correspond to lower energies.](image1)

![FIG. 2. (color online) $q_{\text{net}}$ as a function of $\Delta$. The residual magnetic field is $B = 270 \text{ mG}$. Dashed blue lines and solid red lines represent the predictions derived from Eq. 2 when the microwave pulse is purely $\pi$-polarized and when the pulse has a specially-chosen polarization, respectively (see text). In this paper, $\Delta$ is tuned within the range of $-190 \text{ kHz}$ to 190 kHz.](image2)
and its on-resonance Rabi frequency is $\Omega$.

For a given polarization $k$, we define $k$ used in this paper has a specially-chosen polarization in applications \[1, 11, 19–21\]. However, the microwave pulse is detuned by $\Delta$ from the microwave pulses. Every microwave pulse used in this paper is detuned by $\Delta$ from the microwave function generator, we obtain a desired value of $\Omega_{\text{net}}$ by monitoring the number of atoms excited by a resonant microwave pulse as a function of the pulse duration. The solid line is a sinusoidal fit to extract the on-resonance Rabi frequency $\Omega_{1,2}$. Inset (b): amplitudes $A$ of spin oscillations shown in the main figure as a function of $q_{\text{net}}$ at $m = 0$. The dashed black line is a fit based on Eq.\(1\) with the same set of fit parameters as that applied in the main figure.

On the other hand, the exact value of $q_{\text{net}}$ is carefully calibrated based on Eq.\(2\) with a few experimental parameters, such as the polarization and frequency of a microwave pulse. Every microwave pulse used in this paper is detuned by $\Delta$ from the $|F = 1, m_F = 0 \rangle \leftrightarrow |F = 2, m_F = 0 \rangle$ transition. A purely $\pi$-polarized microwave pulse has been a popular choice in some publications \[1, 11, 13, 21\]. However, the microwave pulse used in this paper has a specially-chosen polarization in order to easily access every positive and negative value of $q_{\text{net}}$. Jointly by continuously tuning $\Delta$, as shown in Fig.\(2\). We define $k$ as 0 or ±1 for a $\pi$ or a $\sigma_\pm$ polarized microwave pulse, respectively. For a given polarization $k$, the allowed transition is $|F = 1, m_F \rangle \leftrightarrow |F = 2, m_F + k \rangle$ and its on-resonance Rabi frequency is $\Omega_{m_F, m_F + k} \propto \sqrt{\Delta C_{m_F, m_F + k}}$, where $C_{m_F, m_F + k}$ is the Clebsch–Gordan coefficient of the transition and $I_k$ is the intensity of this purely polarized microwave pulse. We also define $\Delta_{m_F, m_F + k} = \Delta - [(m_F + k)/2 - (-m_F/2)]\mu_B B$ as the frequency detuning of the microwave pulse with respect to the $|F = 1, m_F \rangle \rightarrow |F = 2, m_F \rangle$ transition, where $\mu_B$ is the Bohr magneton. Similar to Refs. \[19, 21\], we express the value of $q_{\text{net}}$ as

$$q_{\text{net}} = q_B + q_M = a B^2 h + (\delta E)_{m_F = 1} + \delta E_{m_F = -1} - 2\delta E_{m_F = 0}/2,$$

where $a \approx 277 \text{ Hz}/\text{G}^2$ \(\text{or } a \approx 71 \text{ Hz}/\text{G}^2\) for $F = 1$ $^{23}$Na \(\text{or } ^{87}\text{Rb}) atoms. Due to the limited power of our microwave function generator, we obtain a desired value of $q_{\text{net}}$ by choosing a proper $\Delta$ within the range of $-190 \text{ kHz}$ to $190 \text{ kHz}$ at a fixed intensity, as shown in Fig.\(2\). The on-resonance Rabi frequencies of our microwave pulses are $\Omega_{-1,2} = 3.6 \text{ kHz}$, $\Omega_{0,-1} = 2.1 \text{ kHz}$, $\Omega_{1,0} = 1.5 \text{ kHz}$, $\Omega_{1,-1} = \Omega_{0,0} = \Omega_{1,1} = 0$, $\Omega_{-1,0} = 1.6 \text{ kHz}$, $\Omega_{0,1} = 2.8 \text{ kHz}$, and $\Omega_{1,2} = 4.0 \text{ kHz}$. Another advantage of choosing such microwave pulses is to conveniently place the microwave antenna on our apparatus without blocking optical components. In order to ensure an accurate calibration of $q_{\text{net}}$ based on Eq.\(2\), we measure $\Omega_{m_F, m_F + k}$ everyday by monitoring the number of atoms excited by a resonant microwave pulse to the $F = 2$ state as a function of the pulse duration. A typical example of the Rabi frequency measurement is shown in the inset (a) in Fig.\(3\).

The time evolution of $\rho_0$ is fit by a sinusoid to extract the spin oscillation period $T$ and amplitude $A$ at a given $q_{\text{net}}$, as shown in Fig.\(3a\). The value of $T$ as a function of $q_{\text{net}}$ is then plotted in Fig.\(3\) for $m = 0$ and $m = 0.2$, which demonstrates two interesting results. First, when $m = 0$, the spin oscillation is harmonic except near the critical values \(\text{i.e., } q_{\text{net}}/h = \pm 52 \text{ Hz}\) where the period diverges. This agrees with the predictions derived from

![FIG. 3.](color online) The spin oscillation period as a function of $q_{\text{net}}$ for $m = 0$ (open red circles) and $m = 0.2$ (open blue triangles). The lines are fits based on Eq.\(1\) which yield the fit parameters: $\rho_0|_{t=0} = 0.48$, $\theta|_{t=0} = 0$, and $c/h = 52 \text{ Hz}$ for $m = 0$; and $\rho_0|_{t=0} = 0.48$, $\theta|_{t=0} = 0$, and $c/h = 47 \text{ Hz}$ for $m = 0.2$. The fit parameters are within the 5% uncertainty of our measurements. Note the different scales of the left and right vertical axes. Inset (a): the number of $F = 2$ atoms excited by a resonant microwave pulse as a function of the pulse duration. The solid line is a sinusoidal fit to extract the on-resonance Rabi frequency $\Omega_{1,2}$. Inset (b): amplitudes $A$ of spin oscillations shown in the main figure as a function of $q_{\text{net}}$ at $m = 0$. The dashed black line is a fit based on Eq.\(1\) with the same set of fit parameters as that applied in the main figure.
Eq. [1] as shown by the dotted red line in Fig. 3. The energy contour $E_{\text{sep}}$ where the oscillation becomes anharmonic is defined as a separatrix in phase space. A point on the separatrix satisfies the equation $\rho_0 = \dot{\theta} = 0$ according to the mean-field SMA theory. In fact for our sodium system with $c > 0$, $E_{\text{sep}} = q_{\text{net}}$ for $q_{\text{net}} > 0$, while $E_{\text{sep}} = 0$ at $m = 0$ for $q_{\text{net}} < 0$. Figure 3 shows that the $T$ vs $q_{\text{net}}$ curve is symmetric with respect to the $q_{\text{net}} = 0$ axis at $m = 0$. The period $T$ decreases rapidly with increasing $|q_{\text{net}}|$ when $|q_{\text{net}}|$ is large, which corresponds to the “Zeeman regime” with running phase solutions. In the opposite limit, the period only weakly depends on $|q_{\text{net}}|$, which represents the “interaction regime” with oscillatory phase solutions. Here $|q_{\text{net}}|$ is the absolute value of $q_{\text{net}}$. The value of $\theta$ is (or is not) restricted in the regions with oscillatory (or running) phase solutions.

In conclusion, we have experimentally studied spin dynamics in a sodium spinor condensate controlled by a microwave dressing field. In both negative and positive $q_{\text{net}}$ regions, we have observed harmonic spin oscillations and found that the sign of $q_{\text{net}}$ can be determined by comparing $\langle \rho_0 \rangle$ to $\rho_0|_{t=0}$. Our data also demonstrates that the position of the separatrix in phase space moves slightly with $m$ in the positive $q_{\text{net}}$ region, while the separatrix quickly disappears when $m$ is away from zero in the negative $q_{\text{net}}$ region. Our data confirms that the spin-mixing dynamics in $F=1$ spinor condensates substantially depends on the sign of $R$. As a matter of fact, our results in the negative $q_{\text{net}}$ region are similar to those reported with a $F=1$ ferromagnetic $^{87}$Rb spinor condensate in magnetic fields where $q_{\text{net}} > 0$ [1, 3]. Although the relationship between the separatrix and $m$ in the ferromagnetic Rb system has not been experimentally explored yet, our data in Fig. 3 can be extrapolated to understand this relationship. This paper may thus be the first to use only one atomic species to reveal mean-field spin dynamics, especially the separatrix, which are predicted to appear differently in $F=1$ antiferromagnetic and ferromagnetic spinor condensates.

We thank the Army Research Office, Oklahoma Center for the Advancement of Science and Technology, and Oak Ridge Associated Universities for financial support. M.W. thanks the Niblack Research Scholar program.

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