Role of electron-phonon interaction in the strongly correlated cuprates superconductors

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Using high resolution angle-resolved photoemission data in conjunction with that from neutron and other probes, we show that electron-phonon (el-ph) coupling is strong in cuprates superconductors and it plays an important role in pairing. In addition to the strong electron correlation, the inclusion of phonons provides a theoretical framework explaining many important phenomena that cannot be understood by a strongly correlated electronic model alone. Especially it is indispensable to explain the difference among materials. The phonons with the wave number around the \((0, q_x)\) and \((q_y, 0)\) axes create the d-wave pairing while that near \((\pi, \pi)\) are pair breaking. Therefore the half-breathing mode of the oxygen motions helps d-wave superconductivity.

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It has been a long-standing question whether a strongly correlated electronic model of the CuO\(_2\) plane, such as the t-J or Hubbard model alone, can explain the essential experimental observation of superconductivity in cuprates oxides. On the one hand, such a model has been remarkably successful in explaining many important physical properties, most notably the property of the undoped insulator and the renormalization of charge dynamics in it, by spin dynamics from t to J scale and in predicting a spin gap. On the other hand, important questions have been raised. The first is the observation that, while the CuO\(_2\) plane conductivity is essentially the same for various families of cuprates, their \(T_c\) vary by at least an order of magnitude. The second is the observation that the phonons and lattice effects are more important among materials. The phonons with the wave number around the \((0, q_x)\) and \((q_y, 0)\) axes create the d-wave pairing while that near \((\pi, \pi)\) are pair breaking. Therefore the half-breathing mode of the oxygen motions helps d-wave superconductivity.

Fig 1a also clearly shows a discrepancy between p- and n-type materials, as superconductivity in a n-type material appears to be very fragile with a much weaker pairing strength. The electron-hole asymmetry is particularly perplexing in the context of spin pairing only, as magnetism is very strong and persists to a much wider range in the n-type superconductor. It is natural to ask whether one can gain more insight into this issue by considering the lattice degree of freedom.

Considering cuprates superconductors in a larger context, there are other reasons to investigate the role of phonon as exemplified by the following questions. Why the lattice effect is so important to the properties of manganite and nickelate, but completely unimportant to cuprates? Why all the high-temperature superconductors with \(T_c\) larger than 30K, MgB\(_2\), doped C\(_{60}\), BKCO and cuprates, contains light elements of boron, carbon and oxygen and have strong bonds? The standard theory of electron-phonon interaction does not give high-\(T_c\) and d-wave pairing, but the issue of electron-phonon coupling was not scrutinized to the same extent as the strongly correlated theoretical models.

This paper addresses these problems by providing experimental evidence pointing toward phonon to be also an essential player: (i) the phonon strongly influences the electronic dynamics, (ii) the strength of the el-ph coupling correlates with that of pairing. These findings suggest that both electron-electron (el-el) and el-ph interactions are essential ingredients for superconductivity in cuprates. Especially phonon effect is indispensable to explain the difference among materials showing different superconducting \(T_c\). There are two types of the el-ph interaction, i.e., the diagonal and off-diagonal one. The former is the coupling of the atomic (oxygen) displacement to the charge at the copper site, while the other is the...
FIG. 1. In panel (a) we plot the superconducting gap determined by photoemission spectroscopy (full symbols) and tunneling spectroscopy (empty symbols) as reported in the literature as well as determined from our data. In panel (b) the doping dependence of the coupling constant is reported for LSCO, Bi2201 and Bi2212 (right side) and NCCO (left side) as determined by angle resolved photoemission, along the nodal direction ($\Gamma - Y$). In panel (c) (right side), we report the doping dependence of the critical temperature for four different p-type systems: Hg1223, Bi2212, Bi2201 and LSCO as reported in the literature. On the left side of the same panel, we report the critical temperature for the NCCO system. The shaded area are a guide to the eyes.

modulation of the transfer integral due to the phonons. Deriving the effective t-J model from the three-band d-p model according to Zhang-Rice, we obtain both diagonal and off-diagonal interactions whose ratio are different between p- and n-type cuprates. The off-diagonal el-ph interaction creates the d-wave pairing while the diagonal one tends to suppress it. Therefore the weaker d-wave superconductivity in n-type cuprates, where the diagonal el-ph interaction is dominant, is consistent with the less phonon contribution to the pairing. We also discuss the issue of vertex correction for the electron-phonon coupling.

In a strong coupling conventional superconductor the superconducting transition temperature is

$$T_c = \frac{\omega_0}{1.45} \exp \left[ -1.04(1 + \lambda) \frac{\omega_0}{\lambda - \mu^*(1 + 0.62\lambda)} \right]$$  \hspace{1cm} (1)$$

where the notations are standard. For $T_c$ to increase, one in principle should increase the phonon frequency $\omega_0$. This, unfortunately, leads to two problems. The first is the Coulomb repulsion $\mu^* = \mu/(1 + \mu \ln(E_F/\omega_0))$, which increases rapidly with increasing $\omega_0$, so the retardation effect is less effective. The second is that higher frequency means smaller $\lambda$, because $\lambda = C/M\omega_0^2$, where $C$ is a constant for a given class of materials. In order to have a large $\lambda$, as well as high $\omega_0$, one has to increase the constant $C$, which often leads to structural phase transitions. These two factors conspire to limit the value of $T_c$ in conventional superconductors. There are several band calculations including the el-ph interactions. The values shown in Fig. 1 for cuprates make it clear that the conventional wisdom with phonons will not be able to generate the observed gap size, and thus $T_c$.

It is evident that eq.(1) can not be applied directly to cuprates, where the strong el-el interaction plays essential role. However we will use this equation below for a guide of the discussion, which is not unreasonable because the qualitative dependence of the transition temperature on the coupling constant $\lambda$, phonon frequency $\omega_0$, and the Coulomb interaction $\mu^*$ remains to be correct.

Although the conventional el-ph interaction alone can not explain the high-$T_c$, recent high-resolution angle-resolved photoemission (ARPES) data suggest that phonons may nevertheless be an essential player. By analyzing photoemission data from Bi$_2$Sr$_2$CaCuO$_8$ (Bi2212), Bi$_2$Sr$_2$CuO$_6$ (Bi2201) and La$_2-x$Sr$_x$CuO$_4$ (LSCO), in addition to earlier data from Bi2212, we concluded that the quasiparticles in p-type cuprates are strongly coupled to phonons in the frequency range 50-80meV, in contrast to that of the n-type material. We see a dramatic change in the "quasiparticle" velocity in the form of a break in the dispersion near this energy scale, as shown in Fig. 2 (a-c) where the dispersion along the nodal direction for three families of p-type cuprates are plotted. This result is very robust, and the details of the fit can be found elsewhere. In related data, a drop in the quasiparticle scattering rate below this energy scale is observed, which is consistent with optics data reporting a rapid drop of the scattering rate, $1/\tau$, at similar energy. The observed behavior, a sudden change in the dispersion and a drop in the scattering rate, is very reminiscent of quasiparticle coupled to a sharp collective mode. The famous 41 meV neutron mode and phonons are the only two sharp collective modes we are aware of. Since this effect is seen in all compounds and is not associated with the superconducting phase, as the phenomena are seen well above $T_c$, we rule out the neutron mode possibility, as neutron modes are only seen in YBa$_2$Cu$_3$O$_{7-x}$ (YBCO) and Bi2212 and are only associated with superconductivity. We note here that the overdoped Bi2201 data was collected at 30K, six times higher than $T_c$. Alternatively one may try to explain the data by the opening of a gap elsewhere on the Fermi surface. However, the superconducting and pseudogap in LSCO, Bi2201 and Bi2212 are very different, but the "kink" energy is very similar, so this explanation does...
not work. Thus we are left with phonons as the only surviving candidate to explain our data.

The phonon interpretation receives additional support from comparison between photoemission and neutron data. Neutron scattering data from LSCO and YBCO suggest that the zone boundary phonon couples most strongly to doped charge and softens significantly with doping. In the case of LSCO, where a direct comparison between neutron and photoemission is possible, the phonon mode appears at 70meV. As shown by the thick arrow in Fig. 2c, the phonon energy coincides with the “kink” energy in the dispersion, providing a very strong piece of direct evidence for the phonon being the mode responsible for the effect seen in quasiparticle dynamics. In a separate analysis we show the striking similarity between photoemission data from Bi2212 with that of the Be(0001) surface state where the el-ph coupling is known to be strong. In both cases, a peak-dip-hump structure is clearly seen in the spectra, as expected in theories for electrons coupled to a collective mode. This analogy again provides strong support for the phonon interpretation. In summary, we believe that this ensemble of data makes a compelling case for a strong coupling between quasiparticles and phonons.

In conventional theory the phonon correction to the electronic velocity is given by \( v(k) = v_0/(1+\lambda) \), where \( v_0 \) is the bare electron velocity (below the phonon frequency \( \omega_0 \)), \( v(k) \) is the dressed velocity (above the phonon frequency) and \( \lambda \) is the el-ph coupling constant. In a real experiment, one cannot obtain \( \lambda \) without knowing the bare velocity that can be different from band structure value due to el-el interaction. We have extracted a similar quantity which we call \( \lambda' \) using the velocity obtained from the experimental dispersion above the phonon frequency to approximate the bare velocity. In this case, we define the velocity ratio below and above the phonon energy as \( 1/(1+\lambda') \). Because the high energy velocity is an overestimate of the bare velocity, \( \lambda' \) is an overestimate of \( \lambda \). Within a specific model such as Debye model, one can see that this overestimate can be 20-30%. We use the experimental quantity \( \lambda' \) for this discussion as the systematics are the same. We find that the velocity ratio of p-type materials is close to two near optimal doping and increases with underdoping. While the quantitative value is model and procedure dependent, the data suggest that el-ph coupling is strong in these materials. The el-ph coupling constant (\( \lambda' \)) as a function of doping is plotted in Fig. 1b. We note that the frequency scale of the phonon here is an order of magnitude higher than that of conventional superconductors. In principle, given the high \( \lambda' \) value, it can deliver a pairing strength that is an order of magnitude higher with respect to the conventional superconductors, if one can figure out a way to deal with the \( \mu^{+} \) problem. As we will elaborate later, the d-wave pairing helps this issue.

In contrast to p-type materials, the kink effect is much weaker and basically not discernible along the nodal direction for n-type materials, Fig. 2d, indicating a much weaker el-ph coupling. This significantly weakened effect is consistent with optical data, where the drop in \( 1/\tau \) is found to be much smaller in n-type material. Intuitively this is not surprising, as these high energy phonons must involve in-plane oxygen atoms, as suggested by different experiments. For p-type materials the doped carriers predominately go to the oxygen site, while they go to the copper site (upper Hubbard band) in n-type material. This qualitative difference is expected to give a large difference of the el-ph interaction between n- and p-type cuprates as discussed below.

A glance to Fig.1a and Fig.1b makes it clear that there is a correlation, as a function of doping, between the pairing gap size and the el-ph coupling constant. To first order, the most striking feature in both Fig.1a and Fig.1b is that the electron doped and hole doped materials are very different. The much weaker el-ph coupling strength explains smaller pairing gap in electron-doped material. Within a single band t-J or Hubbard model with only nearest neighbor hopping integral, one would expect particle-hole symmetry. The other clear correlation is that the pairing strength of p-type material decreases with doping in Fig.1a, as does the el-ph coupling constant in Fig.1b. We believe that the correlation seen in these figures makes a strong case for the el-ph coupling being a key to pairing.

As for the systematic on the gap size in different p-type materials, the situation is more complex with the following factors being potentially important. It is known that the material dependent copper and apical oxygen distance correlates with \( T_c \) at optimal doping, \( T_c^{\text{max}} \). It has been discussed that this is related to the local stability of the Zhang-Rice singlet and its effective transfer integral. Another electronic structure analysis reduces this down to a correlation between \( T_c^{\text{max}} \) and the range of the intralayer hopping. Hg1201 is found to have not only a larger intralayer hopping than LSCO/Bi2201, while...
but also a larger interlayer hopping. The latter is due to
the on-top stacking of the CuO$_2$ layers in Hg1201.
In LSCO/Bi2201 the interlayer hopping is found to be
substantially smaller due to the body-centered tetra-
agonal stacking. Difference in hopping may be a reason
for $\Delta$ in Bi2212/Hg1201 ($T_{c}^{\text{max}}=90$K) and LSCO/Bi2201
($T_{c}^{\text{max}}=30$-40K) to segregate into two groups. For the
much higher $\Delta_{\text{max}}$ in Hg1223 there is the additional pos-
sibility of a resonance between the phonon and the su-
perconducting gap, being the two energies close to each
other. Optics experiment saw in fact a drop in $1/$
 possibility of a resonance between the phonon and the su-
perconducting gap in Hg1223 is maybe the reduced strain on the CuO
sheets by other layers in this material, a factor related to
the on-top stacking of the CuO
layers in Hg1201.

Now we turn to the more detailed description of the
theoretical consideration. Fig.3a shows the displace-
ment pattern of the zone boundary half-breathing mode
($q=(\pi,0)$), which corresponds to the important phonon
with frequencies 50-80meV. There are several ways to
consider its interaction with the electronic system. The
usual approach is to consider the diagonal el-ph interac-
tion in the particle number, namely the oxygen displace-
ment is coupled to the electron density $n_{i}$ in the
Cu orbitals. When one considers the dispersionless
phonons with the momentum-independent coupling
and, this in-plane oxygen displacement is pair-breaking
for d-wave because it gives rise to the repulsive interac-
tion between the electrons on the nearest neighbor Cu
orbitals. This is more appropriate when the carriers are
doped into the Cu orbital as in n-type cuprates, but not
when they are doped into the oxygen orbital as in p-type
cuprates. With the strong el-el interaction, the vertex
correction to the el-ph interaction becomes crucial.
Especially this diagonal el-ph coupling is irrelevant in
the undoped case because the charge fluctuation is
completely suppressed there. In this case the only relevant
el-ph interaction is in the spin sector, i.e., the, the modula-
tion of the exchange interaction $J$ due to the phonon
which is related to the modulation of $t_{ij}$ described below. With
the small doping concentration $x$, the diagonal el-ph in-
teraction is still not so effective because the vertex cor-
geration gives the reduction factor $x$ to the effective cou-
pling constant. Further, the absolute value of the
dielectric constant $\varepsilon(\omega)$ at the energy of the gap
$\omega=2\Delta$
is much larger than one for the superconducting materi-
als, and the diagonal el-ph interaction is expected to be
screened and reduced considerably.

Therefore we will focus below on the off-diagonal el-
ph interaction. For the holes, doped predominantly into
the oxygen orbitals, the direct el-ph coupling originat-
ing from the shift of the d-level due to the oxygen dis-
placement is missing. In this case, the el-ph interaction
should be derived in the framework of the effective t-
J model where the strong correlation and the exchange
interaction are taken into account. A realistic model of
electronic structure is the d-p model, which considers the
d- and p- orbitals as well as the strong Coulomb interac-
tions. When we reduce the d-p model into the effective
single-band t-J model, the transfer integral $t_{ij}$ between
the Zhang-Rice (ZR) singlet states, at $i$ and $j$ sites, is
given by the second order process in the hybridization
$t_{dp} \sim 0.8$eV between the d- and p-orbitals as

$$t_{ij} \approx \frac{t_{dp}^{2}}{\Delta_{dp}} \tag{2}$$

where $\Delta_{dp}$ is the energy difference between the p- and
d-levels. When the negatively charged oxygen ions
approach (go away from ) the copper, the d-level energy
increases (decreases). This reflects the fact that the oxy-
gen displacement modulates the energy level of d-orbital
as

$$\delta \varepsilon_{d} = gu(i) \tag{3}$$

where $u(i)$ is the linear combination of the oxygen dis-
placements surrounding the copper $i$ given by

$$u(i) = u_{x}(i-x/2) - u_{y}(i+x/2) + u_{y}(i+y/2) - u_{y}(i+y/2), \tag{4}$$

where $u_{\mu}(i \pm \mu/2) (\mu=x,y)$ is the displacement along
the $\mu$-axis of the oxygen between the two coppers at $i$
and $i \pm \mu$. On the other hand the energy level of the
oxygen p-orbitals does not change in linear order in $u$’s.
Therefore the charge transfer energy $\Delta_{dp} = \varepsilon_{p} - \varepsilon_{p}$ in
the hole picture is given by $\Delta_{dp} = \Delta_{0} - gu$, which leads to
the off-diagonal el-ph interaction through eq.(2). $\Delta_{0}$ is
about 2eV, while $g$ and $u$ are the el-ph coupling constant
and the displacement, respectively. Then the modula-
tion of the transfer integrals $t_{ij}$ is given by

$$\delta t_{ij} \approx -g_{dp}^{2} \frac{u(i) + u(j)}{\Delta_{0}} \tag{5}$$

This is the off-diagonal coupling. It is noted here that
$\Delta_{dp}(i)$ and $\Delta_{dp}(j)$ ( $u(i)$ and $u(j)$ ) appear symmetrically
in $t_{ij}$. On the other hand, the modulations of $t_{ij}$ due to
those of $t_{dp}$’s cancel in linear order in $u$’s. In Fig. 3a, we
take the other view focusing on the displacement of one
oxygen, and see the modulation of the transfer integrals
$t_{ij}$. Taking other sites into account the modulation in
$t_{ij}$, parallel to the displacement (green colored), cancel out
and only the $t_{ij}$ perpendicular to the displacements
are modulated.

It is noted here that the transfer integrals $t_{pp}$’s be-
tween the oxygen p-orbitals are also relevant to $t_{ij}$ in
the t-J model, and the modulation of $t_{pp}$’s due to the
oxygen displacements gives another channel for the off-
diagonal el-ph interaction. We believe this contribution
is the main reason why the off-diagonal el-ph interac-
tion is different between the p- and n-type materials.

The wave function of ZR-singlet has more weight on the
oxygen p-orbitals in the p-type case and is expected to have larger el-ph interaction due to this mechanism. It should be noted here that the el-ph interaction corresponding to the modulation of the exchange interaction is also present, and is expected to be dominant in the underdoped region \[2\]. The exchange interaction can be represented as \( J \hat{S}_i \cdot \hat{S}_j = -J x_{ij}^\dagger \chi_{ij} + \text{constant} \) in terms of the bond variable \( \chi_{ij} = \sum \sigma C_{i\sigma} C_{j\sigma} \). By replacing \(-J x_{ij}^\dagger \chi_{ij} \to -J (\langle \chi_{ij}^\dagger > \chi_{ij} + \chi_{ij}^\dagger < \chi_{ij} >)\) in the mean field approximation, this interaction can be reduced to the same interaction as eq.(5). An important feature of this exchange modulation el-ph interaction is that it is free from the reduction due to the vertex correction by the factor of \( x \) as shown below in eq.(6). It represents the fact that it survives in the half-filled case. In Table 1 are summarized the el-ph interactions in p-type and n-type materials.

| diagonal el-ph | off-diagonal el-ph |
|----------------|--------------------|
| p-type weak and reduced by \( x \) | strong |
| n-type strongest but is reduce by \( x \) | weak |

Table 1. Classification of electron-phonon interactions.

Below we focus on the off-diagonal el-ph interaction, which is most relevant in the underdoped region due to the vertex corrections and can be represented by the standard el-ph Hamiltonian as

\[
H_{\text{el-ph}} = \frac{1}{\sqrt{N}} \sum_{k,q,\sigma} g(k,q) u_q C_{k+q\sigma}^\dagger C_{k\sigma}. \tag{6}
\]

where \( g(k,q) \) has the characteristic \( k,q \)-dependence for off-diagonal interactions as \( g(k,q) \propto (\sin(q_x/2) + \sin(q_y/2)) \times (\cos(k_x + q_x) + \cos(k_x) + \cos(k_y + q_y) + \cos(k_y)) \). These momentum dependence is essential for the pair-creating of d-wave superconductivity as shown below.

The above off-diagonal el-ph interaction, which is tied to the change of bond (rather than site) by lattice vibration, is significantly enhanced by the strong Coulomb interaction on the Cu site \[4\]. This is expressed by the vertex correction for the el-ph interaction, which is represented by the following formula for the effective el-ph coupling constant \[3\]

\[
g_{\text{eff}}(k,q) = \gamma(q) g(k,q) = \frac{g(k,q)}{1 - J \Pi_\chi(q)} \tag{7}
\]

where \( \Pi_\chi(q) \) is the generalized susceptibility for bond variable \( \chi_{ij} = \sum \sigma C_{i\sigma}^\dagger C_{j\sigma} \), and the factor \( \gamma(q) = 1 \frac{1}{1 - J \Pi_\chi(q)} \) is the usual Stoner enhancement factor in the RPA approximation. As described below, this interaction creates d-wave pairing. According to the BCS theory, we integrate over the phonons to create the effective el-el interaction.

\[
H_{\text{eff}} = -\frac{1}{N} \sum_{k,k',q,\sigma,\sigma'} g_{\text{eff}}(k,q) g_{\text{eff}}(k',q',-q) < u_q u_{-q} > \\
\times C_{k+q\sigma}^\dagger C_{k\sigma} C_{k'-q\sigma'} C_{k'\sigma'}. \tag{8}
\]

Here we have assumed for simplicity the limit of the high phonon frequency, which is justified marginally as follows. In the present case, \( \omega_0 \cong 50 - 80 \text{meV} \) is comparable to \( J \cong 140 \text{meV} \) which determines the effective band-width. We have found that the qualitative conclusion does not change when we consider the case of more realistic bandwidth determined experimentally.

In the case of s-wave pairing, the Debye cut-off is critical and only below it the effective attractive force is effective. Therefore the smaller frequency reduce the pairing force and enhances the pair-breaking. For the d-wave pairing as will be assumed below, on the other hand, the momentum-dependence is more important, and the sign of the interaction is not relevant. It is because the constant part of the effective interaction does not contribute due to \( \sum_k < C_{k\uparrow} C_{-k\downarrow} >= 0 \). We have also checked that the retardation effect does not change the conclusion qualitatively. Our argument is based on the fact that the strong correlation and exchange interaction leads to d-wave superconductivity. We calculated the energy in the BCS approximation for d-wave superconductivity, with the order parameter \( \Delta \chi = < C_{k\uparrow} C_{-k\downarrow} >= (\cos k_x - \cos k_y) / \sqrt{\cos^2 k_x + \cos^2 k_y} \) appropriate for the half-filled case. Then we obtain

\[
< H_{\text{eff}} >_{\text{BCS}} = -\frac{1}{N} \sum_{k,q} |g(k,q)|^2 |\gamma(q)|^2 < u_q u_{-q} > \Delta_{k+q} \Delta_{k+q} \\
= \sum_q < u_q u_{-q} > e(q) |\gamma(q)|^2 \tag{9}
\]
where we have done only the \( k \)-integral in the last line of the above equation. It is noted here that \(< u_{q}u_{-q} > \propto \omega_{\pi}^{-2} \) and hence the experimental information can be put through this factor. Therefore \( e(q) \) is weighted more in the momentum region where the strong el-ph coupling is observed and the dispersion shows softening, i.e., along the axis \((q_{x}, 0)\) and \((0, q_{y})\). In Fig. 3(b) we show the contour plot of \( e(q) \). It is noted here that the oxygen displacement contributes strongest to the d-wave pairing in exactly the region of q-space, namely along the line \( q = (q_{x}, 0) \) and \( q = (0, q_{y}) \), where the strong coupling to the electrons is observed experimentally. It should be noted here that \( u(i) \) is given by the linear combination of the various phonon modes other than the half-breathing mode in general, and also even the displacements of the apical oxygen play similar roles to give off-diagonal el-ph interaction. On the other hand the phonons with the momentum around \( q = (\pi, \pi) \) are decoupled from the electrons due to the momentum dependence of the off-diagonal el-ph coupling constant \( g(k, q) \) given below eq.(5). In this momentum region, the pair-breaking effect originates in the case of diagonal el-ph coupling. In experiments, the anomalous dispersion of this half-breathing mode has been observed and the zone boundary region modes show the softening and couple to the electrons more strongly than the zone center region modes. In another paper, it is found that el-ph coupling is strong from \((0.5\pi, 0)\) to \((\pi, 0)\). In any case the finite wave number phonons play important role. Integration of this range gives pair creating results. Especially it is found that \( |\gamma(q)|^{2} \) strongly depends on \( q \) and enhances the coupling around \((0, \pi)\) and \((\pi, 0)\) points. This corresponds to the tendency towards the spin-Peierls (dimer) state or stripe formation, and favors the d-wave pairing because the pair-creating momentum region is enhanced by this vertex correction.

Now we comment on the n-type cuprates. Although our theory can not conclude definitely that el-ph interaction is weaker in n-type, the smaller kink in Fig. 2 and the smaller scattering rate \( 1/\tau(\omega) \) in optical experiment strongly suggests it. In particular, the el-ph coupling in n-type cuprates is much weaker along the \((\pi, \pi)\)-direction (although el-ph coupling, or the kink effect is seen along other directions), this is consistent with the weaker off-diagonal el-ph interaction in the n-type material. Also the tunneling data from NCCO has been interpreted by the phonons with lower frequencies, which gives lower \( T_{c} \). Therefore we can not expect much from the el-ph interaction for the pairing force. It is not clear if this reduced pairing force due to el-ph interaction is enough to explain the \( T_{c} \) in n-type material. It is possible that the antiferromagnetic fluctuations contribute to the d-wave pairing as discussed extensively. Therefore certain phonon channels can be compatible with d-wave pairing. Here we note that our identification of the phonon structure in ARPES by the half-breathing \( q = (\pi, 0) \) phonon is mostly motivated by the neutron experiment showing it to soft significantly with doping. The energy consideration of the photoemission data is also consistent with this mode. However, the experimental uncertainty in the photoemission data analysis also allows the possibility of the lower energy phonons in the range of 40-50 meV. Part of this is related to the uncertainty of the superconducting gap size which modifies the kink position. Therefore the half-breathing mode in our discussion should be regarded as a linear combination of various modes. This is particularly true for the in-plane buckling mode (40-50 meV). According to a band calculation, both the half-breathing mode and this buckling mode show d-wave pair creating tendency near \( q \sim (0.5\pi - \pi, 0) \) and pair breaking near \( q \sim (\pi, \pi) \). Raman experiments have shown very strong softening of this phonon through \( T_{c} \). We hope that our discussion stimulates more theoretical investigations on the el-ph interaction in strongly correlated electron systems, as the data in Fig. 1 and Fig. 2 strongly suggest phonon to be a key player for superconductivity, independent of specific theory. It is probably true that many phonons need to be considered.

Fig. 4 proposes a phase diagram that contains essential ingredients for the key physics. At very high energy and temperature, there is an energy scale (or cross-over line) \( T_{0} \), which is related with J scale physics caused by strong Coulomb interactions. At lower energy scale (or temperature) phonons helps pairing which is allowed with compatible magnetism to deliver the pairing, yielding the intermediate pairing energy scale \( T_{\text{phonon}} \). This two scales energy scheme naturally explains the observation that there are two kinds of pseudogaps in underdoped cuprates, as strongly indicated by ARPES data near \((\pi, 0)\). The high energy one (lump) connects smoothly to that of the insulator. The low energy one (leading edge) is related to the superconducting gap.

A key issue for the classical phonon theory is the problem with \( \mu^{*} \). Since J has higher energy (Fig. 4), the anti-ferromagnetic interaction dictates that the pairing state can only be of d-wave symmetry. This suppresses the s-wave pairing instabilities of some phonons while cooperates with the d-wave pairing processes of other phonons. Since the d-wave state has a node at the origin, this significantly reduces the \( \mu^{*} \) problem or even changes sign, because there is no amplitude on the same site, and the on-site repulsion is not an issue. We note that while \( \mu^{*} \) in eq. (1) is introduced in the context of s-wave superconductor as a pair-breaker, the \( \mu^{*} \) in the discussion here should be either positive or negative depending on the model without the el-ph interaction. Another critical role of the strong Coulomb interaction is that it significantly enhances the off-diagonal el-ph coupling near the quasi-resonance of the p- and d- levels of cuprates. This is understood in the vertex correction in eq.(7) due to the near instability of the system towards the spin-Peierls type ordering and/or the bond-centered stripe formation. As we have indicated earlier, the high phonon frequency of 50-80meV and the strong coupling constant can deliver a pairing strength that is an order of magnitude higher (and thus high-\( T_{c} \)) if the \( \mu^{*} \) problem can be significantly
FIG. 4. A schematic phase diagram for high temperature materials is presented. The black line represent the $T_0$ line, which is particle-hole symmetric in the t-J model. The purple shaded area represents the range where the phonons become important. The superconducting regime (SC) is indicated by yellow shaded area and the antiferromagnetic regime (AF) by blue shaded area.

Another factor to differentiate $\Delta_{\text{max}}$ among families of compounds is the presence/absence of the static distortion. Aside from theory, that is based on the average structural data, this tendency is re-enforced by local structural data. As shown by local structural probes, the 70meV zone boundary phonon couples with dynamic local structural distortions. The onset temperatures of the distortion ($T^*$) are significantly lower in LSCO, compared to Bi2212 and YBCO, likely related to stripes.

This coupling with more static structural distortion in LSCO is harmful to the pairing and superconductivity explaining the lower value of $\Delta_{\text{max}}$, although $\omega_0$ is comparable. For a similar reason we do not observe any clear change of $\omega_0$ in the La$_{1.28}$Nd$_{0.6}$Sr$_{0.12}$CuO$_4$ (NdLSCO) system with respect to optimally doped LSCO. The stripes and distortion in NdLSCO are static, further suppressing pairing. This may also be a factor for $\Delta_{\text{max}}$ to be the largest in Hg1223 as the CuO$_2$ planes are the flattest there.

The above interpretation receives support from the large change in the onset temperature $T^*$ of the distortion (order of 80K) with isotope substitution. The opposite change in the $T^*$ respect to the one in the $T_c$ ($T^*$ increases with increasing the mass of the isotope), is consistent with the idea that more static lattice distortions suppress pairing. The local structural distortion is likely to be similar in Bi2201 and this may explain why the maximum $T_c$ (30K) for the single layer is much lower than that of the isostuctural single layer of Tl and Hg compounds. The difference in the dynamics of local structural distortion between different families and different dopings is probably related to the mismatch between the CuO$_2$ plane and the block layers.

As one introduces more CuO$_2$ planes in a unit cell, their structures are less vulnerable to strains from other layers, and this may explain why $T_c$ increases with number of layers in Bi, Tl and Hg based families of cuprates.

We now briefly discuss the connection between the picture which emerges here and those in the literature. First there are several authors who considered the interplay between the electron correlation and el-ph interaction. One important conclusion there is that even starting from the momentum independent el-ph coupling, the vertex function due to el-el interaction suppresses the large momentum transfer process, and hence the transport $\lambda_{tr} \ll \lambda_c$. This is expected to resolve the apparent contradiction between the ARPES and transport data. Namely the el-ph interaction of the order of $\lambda_{tr} \sim 1$ does not appear in the resistivity while $\lambda \sim 1$ is needed to explain the ARPES data. It has been pointed out before that the resistivity data is not incompatible with the presence of el-ph coupling, contrary to general belief. However these works does not take into account the modulation of the exchange interaction, which we believe is the most important for underdoped region. One clue is the $x$-dependence of the coupling constant. The usual treatment in terms of the $1/N$ expansion and/or the slave boson method leads to the conclusion that the effective coupling constant is proportional to $x$ for small $x$, which is not consistent with the observed $x$-dependence shown in Fig. On the other hand, when one consider the off-diagonal coupling described in eq. (7), the doping will reduce the Stoner enhancement factor of the bond-order, and hence the coupling constant is expected to be decreased. We have concentrated on the zero-temperature superconducting state, and the transport properties are beyond the scope of this paper. At finite temperature above $T_c$, the spin-charge separation may play an important role, and the charge carriers are not the simple electrons. If the momenta of the carriers are smaller than the Fermi momentum, the large momentum phonon scattering does not contribute so much to the resistivity. However this issue is a subtle one, and we do not go further in detail on it. As for the coupling to the spin fluctuation as the origin of the kink in the dispersion and the peak-hump-dip structure, we believe the small spectral weight of the 41meV-resonant peak invalidates that scenario.

Several authors looked for the mechanism of superconductivity in the interplay between the electron correlation and el-ph interaction. Here it should be noted that the recent tunneling experiments give evidence that the pseudo-gap and superconducting gap may be distinct, while the low energy part of these gaps seems to be identical. If these picture is correct, which is still an open question, this pseudo-gap play an essential roles in the physics of underdoped and optimally doped cuprates, which can not be captured by the perturbative analysis. It is natural to consider that the strongest interaction, which is the Coulomb interaction and/or the spin exchange interaction, determines the global and high
energy physics, while the phonons play important roles in the lower energy physics. This is the picture we take in this paper, assuming for example that the superconducting gap is d-wave symmetry, which can not be derived by el-ph interaction only. Obviously, diagrams similar to Fig. 4 already exist, except that the data in Fig. 1a and Fig. 1b strongly suggest that the intermediate pairing line is influenced by phonons.

There are many scenarios for the high energy dynamics and pseudo-gap formation. One possibility is the antiferromagnetic short range order based on the Hubbard type models. The second one is the spin singlet formation based on the RVB picture in terms of the t-J model. In these cases, the role of the phonons has been neglected in most of the literature although some authors mentioned it implicitly. For strong coupling theories there is no consensus on the lowest energy state in these strongly coupled models, with flux phase, d-wave pairing, and anti-ferromagnetically ordered ground states being possible candidates. In the intermediate energy scales between the pseudo-gap and superconducting gap, it is expected that the state can be regarded as the thermal and/or the quantum mixture of the possible ground states such as the staggered flux and d-wave superconducting state in SU(2) gauge theory. The role of the el-ph interaction here is to determine the superconducting gap size and transition temperature, which differ significantly from material to material. The third scenario is the charge ordering and/or the stripe formation discussed intensively. These models are related to what we propose, since the stripe approach also includes two energy scales and furthermore the half-breathing mode can be regarded as the instantaneous creation of the dynamic stripe (Fig. 3a). In some theories phonons are explicitly considered, while in others only the electronic degrees of freedom are stressed. For the cases involving the Hubbard-Holstein model, the discussion mainly concentrates on the issue of quantum critical point without specifying the role of the particular phonons. For the cases where only the electronic aspects of the stripes are considered, the two energy scales correspond to the scales when the stripes and pairs are formed respectively. In this case the pairing of the carriers is purely electronic, driven by quasi-1D behavior. Independently of the specific, it would be interesting to see the role of the electronic stripes on the el-ph coupling considered in our case. Naturally, charge ordering couples to lattice distortions. The fact that the el-ph coupling is strong for phonons with q value from (0, 0, 0) to (0, 0, 0) could be interpreted as the system tendency to couple with stripes of 4a to 2a periodicities. This issue of coupling to 2a periodicity has been discussed before. As shown in Fig. 3a, the (0, 0, 0) phonon has a clear 1D character with 2a periodicity. This could be regarded as the instantaneous creation of the dynamic stripes because the charge should be accumulated/diluted alternatively along the Cu-O chain perpendicular to the displacement. Alternatively, one can say that the systems tendency to have stripes will promote el-ph coupling with desired q, that is pair creating for d-wave state. Therefore the stripe theory could be regarded as the strong coupling limit of the el-ph interaction.

Before leaving the subject of theory, we also note that there are extensive work on phonons only, without explicit consideration of the strong Coulomb correlation, notably the polaronic effect and bipolaron condensation. Here we think that the d-wave aspects are important because of the µ∗ problem in conventional el-ph coupling theory. Whatever the specific relationship between our finding and the interesting theoretical ideas discussed above may be, the data hint strongly that the el-ph interaction must be considered explicitly. At the same time, we also stress that the el-ph interaction must be considered in the context of strong electronic correlation, as the t-J model has been remarkably successful to describe the electronic structure of cuprates down to J scale. In this sense the strongly correlated t-J model does provide a basis for cuprates physics, with phonons add to it in delivering d-wave superconductivity with very high-Tc.

In summary, we have shown a comparison of photoemission experiments in conjunction with neutron, and other probes, providing direct evidence for strong el-ph coupling being important for pairing. The inclusion of el-ph interaction explains many experimental observations that cannot be understood by strongly interacting models of CuO2 planes alone.

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