Case study: coordinating among multiple semiotic resources to solve complex physics problems

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Abstract
This work examines student meaning-making in undergraduate physics problem solving. We use a social semiotic perspective to sketch a theoretical framework. The social semiotic approach focuses on all types of meaning-making practices that are accomplished through different semiotic modes including visual, verbal (or aural), written and gestural modes as well as language, text, algebra, diagrams, sketches, graphs, body movements, signs, and gestures. We use the theoretical framework to investigate how semiotic resources might be combined to solve physics problems. Data for this study are drawn from an upper-division Electromagnetism I course and a student (‘Larry’) who is engaged in an individual oral exam. We identify the semiotic and conceptual resources that Larry uses. We use a resource graph representation to show Larry’s coordination of resources in his problem-solving activity. Larry’s case exemplifies coordination between multiple semiotic resources with different disciplinary affordances to build up compound representations. Our analysis of this case illustrates a novel way of thinking about what it means to solve physics problems using semiotic resources.

Supplementary material for this article is available online

Keywords: disciplinary affordances, semiotic resources, mathematics, undergraduate physics, problem solving

(Some figures may appear in colour only in the online journal)
1. Introduction

During problem solving, particularly at the upper-division, students must coordinate multiple representations: algebraic, gestural, graphical, and verbal. While much literature in problem solving in physics focuses on the introductory level, we argue that the harder problems of the upper-division allow for more nuanced views of how students can connect these multiple representations to build meaning and construct compound representations which bridge multiple modes. The ability to construct representations plays an important role in helping students to make physics knowledge and communicate [1]. The process of constructing a representation of a problem makes it easier for the problem solver to make appropriate decisions about the solution process.

In this paper, we investigate how one exemplary student solves a problem involving Ampere’s law. In the course of his problem solving, he constructs a compound representation which includes algebraic, gestural, graphical, and verbal components. To explain his problem solving, we turn to semiotic resources (section 2). After analyzing his work on this problem (section 5), we discuss limitations to this research approach (section 6) and implications for instruction (section 7).

2. Theory

During problem solving, students coordinate multiple representations—algebraic, graphical, verbal, etc—to construct arguments, abstract physical phenomena, and ultimately solve problems. Expertise in problem solving includes being able to represent physical phenomena using several representations and the ability to coordinate among these representations. Research across the STEM courses about students’ use of multiple representations [2, 3] shows that the use of multiple representations plays a critical role in the effectiveness of the interactive engagement between students and instructors in the teaching-learning environment [2–5]. In addition, the ability to translate between different representations tends to enhance students’ sense-making abilities [5].

To understand how students make sense of and solve physics problems, we turn to social semiotic theory. Social semiotics is an approach to communication that seeks to understand how people communicate by a variety of means in particular social settings. In a broad sense the research on multiple representations concerns how multiple representations of scientific concepts in classroom practices affect students’ understanding, while the research on social semiotic resources deals with students’ understanding of scientific concepts through the simultaneous use of various modes of semiotic resources within and across representations.

The social semiotic approach focuses on all types of meaning-making practices that are accomplished through different semiotic modes that include visual, verbal (or aural), written and gestural modes. We couple this theory to Hammer’s conceptual resources [6] to identify small, reusable, nameable [7] chunks of student reasoning which present in a specific semiotic mode. For example, the graphical semiotic resource arrow as vector [8] says that students draw arrows to represent vectors, with the length of the arrow proportional to the magnitude of the vector. It combines a conceptual idea (vectors) with a particular representation (drawn arrows).

Within the context of physics problem solving, student meaning-making can be modeled as a process of using multiple semiotic resources to realize and communicate physics knowledge [9]. These multiple semiotic resources may come from varied semiotic modes, so the process of problem solving involves coordinating ideas across multiple semiotic
modalities. The **disciplinary affordance** of a given semiotic resource is ‘the inherent potential of that [semiotic resource] to provide access to disciplinary knowledge’ [10], allowing researchers to investigate how semiotic resources’ affordances connect to disciplinary ideas. We are especially interested in how different affordances and constraints of different semiotic resources promote students’ meaning-making as they solve physics problems.

Disciplinary affordances allow us to connect ideas in physics with the kinds of representations that best express them. As an example, spoken language is better for certain tasks, and diagrams are better for other tasks; different semiotic resources access and fabricate different aspects of physics knowledge. Taking up disciplinary affordances allows us to focus on knowledge production and communication within the discipline (physics) more than focusing on the view or the experience of an individual student.

Fredlund’s [10] research on the disciplinary affordances of different semiotic resources uses two versions of basic RC-circuit diagrams to show the importance of unpacking the affordances for effective learning in student laboratories. First, the students were given a circuit diagram that can be connected in eight possible ways, but only one way is correct. Students have difficulty connecting the circuit to get an appropriate output. To help students, researchers introduced a modified circuit diagram that shows the positions for connecting the signal input and the ground cable with color coded dots. The modified circuit diagram was a semiotic resource with different disciplinary affordances than the original circuit diagram; though an expert could look at both diagrams and glean the same information, the revised diagram’s different disciplinary affordances made this aspect more clear to students. These new disciplinary affordances foregrounded disciplinary relevant aspects, helping students make better connections among different semiotic resources and allowing the students to make meaning of the circuit.

In another attempt to visualize the effect of disciplinary affordances of semiotic resources, Fredlund [3] investigated a group of third year physics undergraduates who selected among semiotic resources as they described the refraction of light. During the task, students produced two semiotic resources: a ray diagram and a wavefront diagram. The diagrams have different potentials to provide access to different aspects of disciplinary knowledge. The ray diagram could help students to reason about the refraction angles at the boundary and also about the direction of propagation, but it could not help students reason about speed changes in the two media. In contrast, the wavefront diagram promoted reasoning about speed changes but obscured reasoning about angles and directionality.

In both of these studies [3, 10], Fredlund *et al* provided students with two different semiotic resources in the same semiotic mode, showing that different semiotic resources have different disciplinary affordances. In addition to the use of individual semiotic resources or selecting among two semiotic resources in student meaning-making practices, research in science classrooms [4, 10–13] highlights the importance of using several semiotic resources to mediate classroom interactions [14, 15]. Additionally, getting students to use multiple semiotic resources helps shift their focus away from algebraically manipulating symbols and towards understanding the scientific processes and concepts [16, 17].

### 3. The study

We argue that, to better represent an idea or a concept, students should be able to strategically combine multiple semiotic resources. Expanding on this idea, we adopt a social semiotic perspective to sketch a theoretical framework (figure 1) that accounts for how semiotic resources should be combined to solve problems. In our work, we take up the idea of different
semiotic resources having different disciplinary affordances. Under this framing, the process of making meaning is about coordinating different semiotic resources with different disciplinary affordances across multiple semiotic modes. By coordination we mean how the disciplinary affordances of each semiotic resource reinforce each other or hinder the use of other semiotic resources towards building representations. We introduce the idea of compound representations, composed of two or more parts or semiotic modes and linking at least two semiotic resources in the same representation. Then these compound representations can be used to solve problems.

In this study, we are interested in how students coordinate among multiple semiotic resources. In particular, we consider how students construct compound representations using semiotic resources, not how they select between researcher-provided representations. In this sense, our work extends Fredlund et al.’s prior work to be closer to authentic classroom practice and student problem solving.

We explore the research question: How do students coordinate among different semiotic resources to build up compound representations in the process of solving complex physics problems?

4. Context and method

4.1. Context

This research was carried out at Kansas State University; the data for this study were collected from an upper-division Electromagnetism I course, which had about 20 students enrolled. As at many similar institutions, our course is a textbook-centric 4 credit-hour course with a solid foundation in the basics of theoretical physics. The class was taught by a white female instructor who had previously taught the same course at a different, smaller institution. This course covers the first seven chapters of Introduction to Electrodynamics (3rd edition) by
David J. Griffiths [18]. The class meets four hours per week and students work on tutorials and small group problem solving. In this course, students are highly encouraged to work in groups and think aloud while solving problems; class time is divided about equally among small group problem solving, interactive lecture, and problem-solving worksheets [19]. This class has a strong focus on problem solving and sense-making, and we observe that the instructor engages and gives hints in a way that aligns with the sense-making goals.

As a part of the course assessments, students are required to complete two 20–30 min individual oral exams with the instructor. These oral exams are used to assess students’ conceptual understanding, problem solving, and scientific communication skills [20]. In this paper, we analyze the case of ‘Larry’ (a pseudonym), who works on an oral exam problem that takes place in the later part of the course. Larry is a strong student whose marks are near the top of his class, and his group discussions are robust and far-ranging. Larry’s approach and reasoning in the oral exam are typical to a student at this level. We select Larry as an exemplary case because he is unusually verbal in oral exams compared to his peers; we get a lot of information on his problem-solving activity as a result. This then gives us lots of insight into how students at this level might solve the problem more generally.

4.2. Physics problem

We selected a problem which is canonical at the upper-division level: Suppose you had an infinite sheet which carries current \( k \) equal to some constant \( (k = \alpha \hat{x}) \). What’s the magnetic field look like? First, Larry determines the direction of the magnetic field created by the current sheet, then the instructor asks Larry to find the magnitude of the magnetic field, how big is it? To solve this problem, one can use the right-hand rule to find the direction of the magnetic field created by the sheet, and use Ampere’s law \( \oint B \cdot dl = \mu_0 I_{	ext{enc}} \) to find the magnitude.

4.3. Methods

This study has three stages of analysis. The first stage involves transcribing Larry’s oral exam and dividing the problem-solving activity into segments of events. From the transcript and the video of the oral exam, each event is described without explicitly mentioning the resources (semiotic or conceptual) to generate a narrative description. Within each event, we then identify key elements using semiotic modes: different types of inscriptions (diagrams, mathematical formulas); extra-linguistic modes of expression (gestures); words (oral); and objects (used by Larry and the instructor).

For the second stage of analysis, we classify semiotic and conceptual resources within each semiotic mode. First, we identify semiotic resources within inscription, verbal, and gestural modes, naming each resource descriptively. Conceptual resources are identified using published guidelines [7], as augmented by work in semiotic resources [10] and procedural resources [21]. Resources are named using descriptive names for the things resources represent [7]. For this study, we used this idea to name resources and link them with their semiotic modes. For example, in the process of solving the problem, when the student uses a right-hand rule gesture (connecting the directions of the inputs of the cross product with the fingers on their right-hand, and making a gripping gesture to perform the cross product), we coded it as the semiotic resource (gestural) right-hand grip rule. Similarly, we identify mathematical resources by naming them—e.g. integral form for Ampere’s law—for the formulas or mathematical actions they describe. Notationally, we denote resource names in italics.
Finally, we make a list of conceptual and semiotic resources. This third stage of analysis involves comparing the resources’ frequency and connections across episodes and connecting the semiotic resources to their disciplinary affordances (tables 1–3). Within Larry’s problem-solving activity, we also look at the extent of his confidence using the loudness of his voice, choice of words, and speed of hand gestures.

A case study methodology enables us to focus on a limited amount of data in a pilot study. The study, in turn, allows us to show that with this type of analysis we can gain an understanding of the subject (Larry), and of the events and processes that are involved in a relatively sophisticated problem-solving situation. Three researchers worked together to establish reliability of coding and to list the disciplinary affordances of each semiotic resource that Larry uses.

5. Analysis

We divided Larry’s oral exam into three episodes. We chose each episode with our research focus to show how the coordination between resources can be used to build different stages of the compound representation. A full analysis of the first episode, and important aspects of all three episodes are presented here. More extensive analyses of the second and third episodes are presented in the supplementary material, available online at stacks.iop.org/EJP/40/025701/mmedia. In our descriptions, we use the resource graph representation to show Larry’s coordination of resources to build different stages of the compound representations. During the first half of his oral exam, Larry works to determine the direction of the magnetic field created by the current sheet and then continues to find the magnitude.

5.1. Episode 1

The episode starts with the instructor presenting the problem verbally, writing the current on the board as $k = \alpha \hat{x}$. The problem statement activates the conceptual resources current sheet and magnetic field direction. This leads Larry to represent the current sheet on the board (figure 2(a)) using the semiotic resource parallelogram as current sheet. After visually representing the current sheet, Larry looks at the mathematical equation ($k = \alpha \hat{x}$) on the board. The directional information ($\hat{x}$) embodied in the equation prompts Larry to think of a way to represent this detail. He uses the visual semiotic resource coordinate system to add the directional information to his diagram and draws three arrows with their tails together, labeling them $\hat{x}$, $\hat{y}$, and $\hat{z}$ (figure 2(a)).

Larry: So the coordinate ...$x$ hat, $y$ hat, $z$ hat, $xyz$ I mean.

Figure 2(a) shows the initial compound representation which has a coordinate system and a parallelogram representing the current sheet. Figure 2(b) shows the resource graph, the resources (coordinate system, current sheet, and parallelogram as current sheet) that are connected to produce the initial compound representation. In our representation of resource graphs, each circle represents either a semiotic or a conceptual resource. After generating the initial compound representation, Larry moves to describe how he imagines the current sheet is built up from a collection of wires. Larry then focuses on a single wire to find the magnetic field that can represent the effect of the whole current sheet (part of a whole).

Larry: um...m okay, so think of this, a sheet is kind of having a bunch of infinite wires (up and down movement of hand) one next to each other and the current from a single wire curls around this (applies right-hand grip rule).
Larry starts using the gestural semiotic resource *hand for wires* in free space to show adjacent current wires. Then he focuses on a single wire and applies the gestural semiotic resource *right-hand grip rule* to figure out and then to demonstrate the *magnetic field direction* from a single current wire. While applying the right-hand grip rule, Larry does not indicate a specific current direction (whether or not he uses the given current direction). Soon after, Larry decides to use the diagram on the board (figure 2(a)) to continue with this argument and concentrates on a point above the current sheet by using the semiotic resource *pinpointing gesture*.

**Larry:** So, I think that like if you look at a point above it (pinpoints to a location), then uh... from a single wire ... will pointing ...

After pinpointing to a location above the sheet, Larry considers an imaginary wire and applies the *right-hand grip rule*. He applies the *right-hand grip rule* for the second time without specifying the current direction and on this occasion, we observe Larry get stuck. Instead of making a clear conclusion, he ends up repeatedly changing the orientation of his right-hand gesture.

In order to apply the right-hand grip rule, Larry has to have a certain current direction; but when he applies the right-hand grip rule in free space, he should not have to specifically mention the current direction. Because at that point the semiotic resource *hand for wires* allowed Larry to show the existence of a current wire in space, the right-hand grip rule helps him to get the *magnetic field direction* using an arbitrary current direction. When Larry moves to build his argument using the diagram on the board, the diagram itself contains a coordinate system that affords to define the direction in space. So, unlike using free space, Larry has to specify the current direction before applying the right-hand rule when using the diagram on the board. The missing detail of specific current direction leads Larry to decide between the orientations of his gesture. Here the semiotic resource *coordinate system* hinders the use of the gestural semiotic resource *right-hand grip rule* without a specific current direction.

Finally, Larry decides to add the current direction information to his diagram and uses the semiotic resource *arrow as vector* to represent the current direction.

**Larry:** So, if the current is in x hat (records an arrow on diagram).

The coordinate system on the compound representation (figure 2(a)) allows Larry to show the direction in space (table 1). That also permits representing the given current direction along the x-axis (figure 3(a)) using the semiotic resource *arrow as vector* along with the mathematical symbol ($k$).

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**Figure 2.** (a) Initial compound representation. (b) Resource graph for the initial compound representation.
Larry: Then above the sheet it would be pointing out of the board (records on diagram), from one wire, so above one wire. So, I think it would be true for the rest of the sheet as well.

After recording the current direction, the rest is straightforward for Larry. He repeats the same procedure and focuses on a single wire above the sheet to apply right-hand grip rule once more. Then he uses the semiotic resource arrow as vector to record the resulting magnetic field direction on the diagram (figure 3(a)). Finally, Larry refers back to his initial assumption (part of a whole) and concludes, ‘I think it would be true for the rest of the sheet as well.’

Figure 3(a) shows the compound representation at the end of first episode. Figure 3(b) shows the resources that are connected to produce the compound representation in figure 3(a). We can see Larry brings in and combines more resources as he progresses with the problem at hand. At the beginning of this episode, Larry does not have a diagram to start with; but after a few steps, Larry builds up a compound representation that contains a coordinate system and a parallelogram for the current sheet. Then he adds an arrow to represent the current direction with the help of the coordinate system. Larry builds up his compound representation to include more details in it. By the end of this episode, Larry adds the magnetic field direction onto his diagram by thinking of a single wire and applying the right-hand grip rule to figure out the direction.

5.2. Episode 2

After finding the magnetic field direction above the current sheet, Larry could have continued with the same argument using the diagram on the board to figure out the direction below the sheet. The instructor introduces a sheet of paper to represent the current sheet, however. Then Larry switches to the sheet of paper and successfully reconstructs the magnetic field direction above the sheet from a single current wire. Table 1 summarizes the semiotic resources and disciplinary affordances involved in this process.

**Table 1.** Disciplinary affordances of the semiotic resources coordinated to build the compound representation of the current sheet.

| Semiotic resource       | Disciplinary affordance                                                                 |
|-------------------------|----------------------------------------------------------------------------------------|
| Paper as current sheet  | The paper allows Larry to generate a visual representation of the current sheet in free space. Larry refers to this while considering multiple current wires in free space and also while gesturing for the loop in episode 3 |
| Hands for wire          | Larry repeats the up and down movement of his hand to show the current wires in free space. This gesture allows Larry to simplify the sheet to a bunch of wires and then focus on a single current wire, to which he applies the right-hand rule to get the magnetic field for that wire |
| Line as wire            | This helps Larry to visually represent the current wire in the space of the sheet in the diagram (on the board) and allows him to locate the wire on the sheet. It also helps Larry to keep track of his reference current wires (with embodied current direction) while applying the right-hand grip rule, which leads Larry’s effort to a successful conclusion |

*Larry:* Then above the sheet it would be pointing out of the board (records on diagram), from one wire, so above one wire. So, I think it would be true for the rest of the sheet as well.
Table 2. Disciplinary affordances of the semiotic resources coordinated to build the compound representation of the right-hand grip rule.

| Semiotic resource         | Disciplinary affordance                                                                                                                                 |
|---------------------------|--------------------------------------------------------------------------------------------------------------------------------------------------------|
| Coordinate system         | A coordinate system enables Larry to define the locations of points, distances between points, and directions in space (on a planar surface). This allows Larry to represent the given current direction (using an arrow) that he uses to find the magnetic field direction. |
| Arrow as vector           | Arrow as vector permits Larry to visually represent the vector direction. As the coordinate system permits showing direction in space, using an arrow to represent a vector allows Larry to represent the given current direction along the x-axis. In this situation, visually representing the current direction helps Larry to apply the right-hand rule towards determining the magnetic field direction. Later, the same semiotic resource allows Larry to visualize the resulting magnetic field directions. |
| Finger pointing in direction | Larry uses the index finger to the side while gesturing for direction. This allows him to show the direction of the current on the surface of the paper and also in the space of the sheet in the diagram (on the board), helping him to apply the right-hand grip rule |

Larry: So, if this is the sheet (refers to the sheet of paper), the current is going this way (across the surface of sheet-right to left), and looking at a point above it, then from one wire, magnetic field will be pointing in that way (away from him).

After some effort, which is described in the supplementary material, the compound representation of the right-hand grip rule allows Larry to figure out the resulting direction of the magnetic field lines from a current-carrying wire. Table 2 summarizes this process.

At the beginning of episode 2, the compound representation includes a coordinate system, a parallelogram to represent the current sheet, arrows representing the magnetic field direction above the sheet, and the current direction. By the end of episode 2, Larry has added a line to represent a current-carrying wire and an arrow to represent net magnetic field direction below the current sheet. Larry has connected the resources to produce this compound representation and we see Larry continue with the same combination of resources as he is still looking for the magnetic field direction. As the only modification, he adds the line as wire to represent the current wire and further build up his compound representation to include the magnetic field direction below the current sheet.

5.3. Episode 3

Larry completes the first task of finding the magnetic field direction, and now he has to figure out the magnitude of the magnetic field. Finding the magnitude of the magnetic field is a new task for Larry, and we see Larry uses gestures and words to communicate with the instructor in attempting to accomplish this task. While progressing with the mathematical manipulations, Larry receives some help from the instructor similar to what he gets in the first two episodes. To start the process, Larry records the semiotic resource of mathematical formula for the integral form of the Ampere’s law ($\oint B \cdot dl = \mu_0 I_{enc}$). In order to continue with the mathematical manipulations, Larry has to pick an Amperian loop. First he gestures for the
loop using the semiotic resources *hand for loop* and then he uses the semiotic resource *square as loop* to record an Amperian loop on his diagram. Larry then simplifies the right-hand side of the integral equation by figuring out the current passing through the loop. Later, Larry adds the loop dimensions and direction on the loop using the *Arrow as vector*. The inclusion of the direction on the loop helps him to figure out the relative orientation with the magnetic field direction and to manipulate the left-hand side of the integral equation. Figure 4 and table 3 further illustrate this process.

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**Table 3.** Disciplinary affordances of the semiotic resources coordinated to build the compound representation of the Amperian loop.

| Semiotic resource   | Disciplinary affordance                                                                                                                                 |
|---------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------|
| Hand for loop       | Larry uses his hand to represent the location and orientation of the loop with respect to the orientation of the paper sheet. Even after recording the loop on the diagram, Larry reuses hand for loop to best communicate his idea to the instructor. |
| Square for loop     | This helps to generate a visual representation of the loop in the space of the board (diagram on the board) and allows Larry to show the orientation of the loop with respect to the orientation of the current sheet (parallelogram for sheet). This recording step helps Larry to continue with the mathematical manipulations as he labels the loop dimensions and to figure out the current enclosed in the Amperian loop. |
| Arrow as vector     | Arrow as vector permits Larry to visually represent the given current direction and the resulting magnetic field directions. While Larry is trying to simplify the left-hand side of the integral form of the Ampere’s law equation during the latter part of episode 3, either gesturing for the loop or just recording the loop does not help Larry to figure out the relative orientation between the magnetic field direction and the unit length on the Amperian loop. Then the use of arrow as vector to represent the direction on the loop helps Larry to build a complete argument and advance with the mathematical manipulation. |
5.4. Connections among the episodes

Our study shows the disciplinary affordances of some semiotic resources hindering the use of other semiotic resources. In episode 1, Larry first applies the right-hand grip rule in free space, without specifying the current direction to figure out the magnetic field direction. Then he continues to apply the right-hand grip rule considering the diagram on the board again without specifying the current direction. The directional information embodied in the diagram on the board by coordinate system obstructs the use of right-hand grip rule, and Larry ends up changing his gestural orientation. Later the inclusion of the current direction information helps Larry to figure out the magnetic field direction.

During the first two episodes, Larry works to get the magnetic field direction, and we observe him using the semiotic resource right-hand grip rule. Once he gets the magnetic field direction and builds onto his compound representation, he no longer uses the right-hand grip rule. As Larry moves to use the mathematical formulas in episode 3 to get the magnitude, the disciplinary affordance of mathematical formulas integral form of the Ampere’s law, current enclosed, current density and magnitude of the magnetic field prevents the use of the semiotic resource right-hand grip rule. The right-hand grip rule had been important for getting the direction of the magnetic field in episodes 1 and 2 but was no longer useful for getting the magnitude of the magnetic field in episode 3.

Likewise, Larry uses the semiotic resource hand for loop to gesture how he picks the Amperian loop in the third episode. Using hand for loop helped Larry to show the orientation of the Amperian loop relative to the orientation of the current sheet. As Larry switches to work on mathematical formulas to get the magnitude of the magnetic field, he ends up associating mathematical formulas: integral form of the Ampere’s law, current enclosed and current density. The inclusion of mathematical formulas constrains the use of the semiotic resource hand for loop. We observe Larry switching to the diagram on the board to represent the loop using a square as loop. The hand gesture alone is not helpful in determining the current piercing the loop or in determining the left-hand side of the Ampere’s law integral.

Figure 4. The compound representation of the Amperian loop is built and represented through the gesture and a drawing of a square on the board with an arrow to represent the direction on the loop.
6. Discussion

The process of student meaning-making is not a purely cognitive one; in physics classes, both students and teachers use a number of semiotic resources in addition to speech and writing. The approach [9] considering the involvement of all artifacts, objects, and actions to describe student meaning-making broadens the boundaries in physics as a discipline. The idea of disciplinary affordance allows us to connect ideas in physics with the kinds of representations which best express them.

The problem Larry solves is a canonical problem at the upper-division level. Larry solving this problem gives us lots of insight into how students at this level might solve this problem. But, we are not looking for prevalence, and we are not trying to make a normative argument that all the students should solve this problem in the same way. This particular problem required Larry to start from a diagram and move to the mathematics at the later part of his solution; this process required Larry to coordinate between multiple semiotic resources. We observe Larry combine a series of semiotic resources with other available conceptual resources, some of which he keeps coming back to and some of which he discards after using.

There are three compound representations which give the features (direction and magnitude) of the magnetic field to solve the given problem (figure 5). First, Larry builds up the compound representation of the current sheet (episodes 1 and 2) that he uses to build up the direction and the magnitude of the magnetic field. The iconic parts of the current sheet are represented using a parallelogram with a line to represent the current wire, a sheet of paper and some hand gestures. In this compound representation, the parallelogram and the sheet of paper (table 4) allow Larry to visually represent the current sheet. The inclusion of hand gestures and a line to represent the wire helps to simplify the structure of the current sheet.

Then, he does some different things with the right-hand grip rule to build up magnetic field direction. First, the compound representation of the right-hand grip rule (episodes 1 and 2) is developed using a gesture and a drawing of a line to represent the current-carrying wire with an arrow to represent the given current direction. Then, the curled right-hand gesture visualizes the application of the right-hand grip rule. Later, Larry builds up the compound representation of the Amperian loop (episode 3) using gesture and a drawing of a square on the board with an arrow to represent the direction on the loop. In this compound representation, gesturing and drawing (square) on the board (table 4) allow Larry to visually
represent the existence of the loop with respect to the orientation of the current sheet. Then, the inclusion of the direction on the loop helps him figure out the relative orientation with the magnetic field direction.

The compound representation of the right-hand grip rule allows Larry to figure out the resulting direction of the magnetic field lines from a current-carrying wire. In order to apply the right-hand grip rule, Larry has to have a specified current-carrying wire with a certain current direction. Larry’s approach to first simplify the current sheet into individual wires and then to focus on a single wire to apply the right-hand grip rule helps to build a valid argument towards finding the net magnetic field direction above and below the sheet. The use of the parallelogram (episode 1) to visually represent the loop in the space of the diagram (on the board) helps Larry to include the arrow to represent the direction on the Amperian loop. This allows Larry to figure out the relative orientation between the magnetic field direction and the unit length on the Amperian loop. Finally, Larry builds a complete argument and advances

Table 4. Disciplinary affordances of three representations incorporated to find direction and magnitude of the magnetic field.

| Compound representation | Disciplinary affordance |
|-------------------------|-------------------------|
| Current sheet           | In this compound representation, the sheet of paper and the drawing of the parallelogram on the board generate visual representations of the current sheet. Larry keeps referring back to the diagram on the board as he progresses on this task, and it allows Larry to add different features: current direction (episode 1), current-carrying wire (episode 2), Amperian loop (episode 3), and findings: magnetic field direction (episodes 1 and 2) on to his compound representation. The inclusion of a line helps Larry to visually represent the current-carrying wire in the space of the diagram (on the board), and this allows him to consider multiple current-carrying wires while figuring out the net magnetic field above and below the sheet (episode 2) |
| Right-hand grip rule    | Right-hand grip rule helps to define the magnetic field. It reveals the connection between the current direction and the magnetic field lines for the magnetic field created by a current. In this situation, the right-hand grip allows Larry to manipulate this phenomenon by hand and to visualize the resulting magnetic field direction. In order to apply the right-hand grip rule, Larry must have a certain current direction. Early in episode 1, we observe the missing detail of current direction prevents Larry from making a conclusion while reasoning using the diagram on the board. But the inclusion of the current direction and the line to specify the current wire in episode 2 helps Larry to figure out the magnetic field direction above and below sheet |
| Amperian loop           | In this compound representation, gesturing and drawing (square) on the board allow Larry to visually represent the existence of the loop with respect to the orientation of the current sheet. We observe these visual representations do not help Larry to simplify the left-hand side of the integral form of the Ampere’s law equation. Then, the inclusion of the direction on the loop helps to figure out the relative orientation of the unit length on loop with the magnetic field direction. This leads Larry to find the magnitude of the magnetic field |
with the mathematical manipulation to simplify the left-hand side of the integral form of the Ampere’s law equation.

The class from which the data is drawn has a strong focus on problem solving and sense-making. The instructor involved in this study conducts the oral exam and provides hints to Larry in a certain way that aligns with other practices used in the classroom, but we think this interaction does not materially change our argument about how semiotic resources can come together to build representations and arguments to solve problems. Having a single student and a single instructor is a limitation of our study, but we note that this is an exemplary case with an existing approach that is worth paying attention to.

Research has shown that the use of multiple representations can greatly enhance students’ understanding of mathematical and physical concepts and in physics problem solving. Students can connect these multiple representations to build meaning and construct representations that help them to make appropriate decisions about the solution process. The case of Larry exemplifies the coordination between multiple semiotic resources with different disciplinary affordances to build up compound representations to solve complex physics problems. Our analysis of this case illustrates a novel way of thinking about what it means to solve physics problems and, we hope, contributes to the application of social semiotics in teaching and learning university physics [9, 22]. We hope this approach will help to stress the importance of the use of multiple semiotic resources in student problem solving, and this will help us to minimize student difficulties associated with working with multiple representations.

7. Implications

The process of constructing an effective representation of a problem makes it easier for the problem solver to make appropriate decisions about the solution process. In addition, this process symbolizes the student’s work on that particular problem. If the student constructs an effective representation, then the student is more likely to progress towards solving the problem [23]. If the student constructs an inappropriate representation, however, then the process is unlikely to make any progress until the student re-represents the problem accurately. This is evident in Larry’s case, he could not continue to consider multiple current-carrying wires while using the sheet of paper representation (episode 2), but his decision to switch to the diagram on the board helps him to consider multiple wires and leads him to draw a conclusion about the net magnetic field above and below the sheet.

Further, during episode 3, either gesturing for the loop or recording the loop does not help Larry to figure out the relative orientation between the magnetic field direction and the unit length on the Amperian loop until he adds the direction on the loop. In some cases, students get stuck and cannot identify the nature of the sticking point. We observe in some occasions, Larry gets stuck and his voice level, choice of words, and the speed of gestures indicate his confusion. But after adding new features or switching to a different semiotic resource to re-represent the idea or concept Larry’s reasoning goes back to normal, and he moves forward to solve the problem. As the case of Larry demonstrates, one of the important problem-solving skills is that of effectively representing and re-representing problems. This skill includes both students and teachers being aware of the nature of the disciplinary affordances of semiotic resources that students bring together to construct representations [3].

The research findings presented above suggest that it is important to highlight the complex use of multiple semiotic recourses in student problem solving. An implication is that instructors need to identify the disciplinary affordances of the different semiotic resources in different modalities so that instructors could demonstrate and help students to become better
problem solvers. Further research could involve interviews or classroom observations with a series of typical problems to further explore how the representations are developed, how those representations are determined to be insufficient and replaced, or how those representations are augmented by new ones brought in by the students.

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