Photon drag of superconducting fluctuations in 2D systems

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The theory of photon drag of superconducting fluctuations in the two-dimensional electron gas is developed. It is shown that the frequency dependence of the induced current is qualitatively similar to the case of photon drag of conventional two-dimensional degenerate electron gas. With the decreasing temperature the magnitude of the effect increases dramatically and the current of superconducting fluctuations carries an additional power of reduced temperature in comparison with the Aslamazov-Larkin contribution. The scope of the developed effect is expected sufficient to be visible against the conventional photocurrent background.

I. INTRODUCTION

The investigation of superconductivity phenomenon in two-dimensional systems takes a great part in condensed matter physics. Starting from thin metallic films, the samples fabrication technologies and experimental tools become suitable for the study of highly crystalline superconductors possessing extremely small thicknesses down to a monolayer. Among atomically thin superconductors, the systems based on transition metal dichalcogenides (TMD), e.g. MoS$_2$, have aroused interest in recent years. The remarkable feature of such systems is the use of ionic liquid gate for creating a large density of electrons, reaching the values up to $3 \cdot 10^{14}$ cm$^{-2}$.

To date, the transition of TMD and main-group metal dichalcogenide flakes from the normal (resistive) to superconductive phase have been studied in experiments. However, in the range of temperature close to the phase transition, $T \gtrsim T_c$, the behaviour of TMD flakes in the electromagnetic (EM) field has not been completely studied experimentally, as well as theoretically. In this direction, it was observed that the superconductive fluctuations in the normal phase make the effect of magnetochiral anisotropy be noticeably more distinct.

In present work, we suggest the additional approach for the investigations of transport features of 2D superconductors, namely, it is the photon drag effect. This effect is routinely used in the research of various 2D systems. We explore the classical limit of this effect. It means that no transition between the subband happens. In other words, it is supposed that the incident EM-wave frequency is much less than any energy gap in the systems. In this case, the physical mechanism of photon drag just consists in the momentum transfer from the EM-wave to fluctuations.

To develop the theory, the Ginsburg-Landau (GL) approach is used. Although the microscopic treatment is more exact, it is simultaneously more difficult and cumbersome than the GL one. Since our aim is to achieve a qualitative picture, the GL theory seems to be appropriate as a good approximation.

FIG. 1. System sketch. The EM-wave falls onto the 2D superconductor with some angle $\Theta$ and produces the electrical current in $x$ and $y$ directions.

II. MODEL

Let us consider the 2D-superconductor being in the normal phase and irradiated by an electromagnetic wave with an electric field amplitude $E_0$, wave vector $K$ and frequency $\Omega$ (Fig.1). In present work we consider a purely 2D system. Thus, the electron motion in the $z$-direction is neglected and the superconducting fluctuations respond to the projection of EM-wave amplitude $E$ on a superconductor surface only. For later computations, it is convenient to define $E$ in the complex form:

$$E(R,t) = E e^{i(kR - \Omega t)} + E^* e^{-i(kR - \Omega t)},$$ (1)

where $E$ and $E^*$ are complex amplitudes of electromagnetic wave and $k$ is a projection of $K$ to the superconductor plane. We focus on the stationary and homogenous part of the current, which does not vanish after the averaging in space and time. Thus, in the lowest order of the wave amplitude, the photon drag current corresponds to...
the second-order response:

\[ j_\alpha = \sigma_{\alpha\beta\gamma}(k, \Omega)E_\beta E_\gamma^* \]

(2)

because any odd term of expansion will give zero contribution after averaging. In Eq. (2) \( \sigma \) is the second-order conductivity and the subscripts denote components in the Cartesian axes. For convenience, let \( k \) to be oriented along the x-axis (Fig. 1). Then the system is symmetric under reflection \( y \to -y \) and, therefore, only \( \sigma_{xx\gamma}, \sigma_{xy\gamma}, \sigma_{yy\gamma} \) are nonzero. After separating \( \sigma \) to symmetric and antisymmetric parts in accordance with \( \sigma_{\alpha\beta\gamma} = \sigma_{\alpha\gamma\beta}^s - \sigma_{\alpha\gamma\beta}^a \), the current reads

\[
\begin{pmatrix}
\sigma_{xx\gamma}^s|E_x|^2 + \sigma_{xy\gamma}^s|E_y|^2 \\
\sigma_{yx\gamma}^s(E_x E_y^* + E_y E_x^*) - i\sigma_{xy\gamma}^a(E_x E_y^* - E_y E_x^*)
\end{pmatrix},
\]

(3)

where the imaginary unit in the second line is introduced to make \( \sigma_{xy\gamma}^a \) real.

The Eqs. (3) are just a general form of the second-order response. However, the explicit expressions for conductivity are the goal. To succeed in it, we start from the definition of an electric current in the form of the variational derivative (further we will omit variables \( R, t \) for brevity):

\[ j = -\frac{\delta F[\Psi]}{\delta A}. \]

(4)

where the GL free energy has the form:22

\[ F[\Psi] = \alpha T_c \int dV \left\{ \epsilon |\Psi|^2 + \xi^2 (|\hat{\mathbf{p}}|^2 - 2eA)|\Psi|^2 \right\}. \]

(5)

\( \Psi \) is the order parameter, \( A \) is the EM-wave vector potential, \( \alpha = (4mT_c\xi^2)^{-1} \) is the GL expansion coefficient, \( m \) is the electron mass, \( T_c \) is the temperature of transition to the superconducting state, \( \xi \) is the coherent length, \( \hat{\mathbf{p}} = -i\nabla \), \( \epsilon = \ln(T/T_c) \approx (T - T_c)/T_c \) is the reduced temperature. In writing (4) it is supposed that the EM-field does not change the coefficients in the GL free energy expansion and is just included via the minimal coupling \(-i\nabla \to -i\nabla - 2eA \). Combining (4) and (5) we can see that, as usual, the current proves to be a sum of dia- and paramagnetic terms:

\[ j^D = -8e^2 \alpha T_c \xi^2 \mathbf{A} |\Psi|^2, \quad (6a) \]

\[ j^P = 4e\alpha T_c \xi^2 \epsilon \mathbf{p} \Re \{ \hat{\Psi}^* \hat{\mathbf{p}} \hat{\Psi} \}. \]

(6b)

In (6) the order parameter is still undefined. To proceed, let us note that including the vector potential to the GL free energy makes the order parameter dependent on it, \( \Psi = \Psi(A) \). To obtain the explicit expression of this dependence, we explore the Time-Dependent Ginzburg-Landau (TDGL) equation:23

\[ \{ \frac{\partial}{\partial t} + \alpha T_c \left[ \epsilon + \xi^2 (|\hat{\mathbf{p}} - 2eA|^2) \right] \} \Psi(r, t) = f(r, t), \]

(7)

where parameter \( \gamma \) has both real and imaginary parts, \( \gamma = \gamma' + i\gamma'' \). The explicit expression for \( \gamma \) is derived from the microscopic theory and, further, it is assumed that \( \gamma''/\gamma' \ll 1 \). In eq. (7), \( f \) is a Langevin random force, which defines the white noise in the system and is completely uncorrelated:

\[ \langle f^*(r, t)f(r', t') \rangle = 2T\gamma' \delta(r - r') \delta(t - t'). \]

(8)

Here the angle brackets designation \( \langle \ldots \rangle \) means fluctuations averaging. In writing the TDGL equation, we choose the gauge of EM-wave with zero scalar potential that means the connection \( \mathbf{E} = -\partial_t \mathbf{A} \). Assuming the vector potential to be a perturbation, let us utilize the method of progressive approximation, i.e. we will find the solution of (7) in the form of expansion in the powers of \( A \):

\[ \Psi = \Psi_0 + \Psi_1 + \Psi_2 + \ldots \]

(9)

where \( \Psi_i \sim A^i \). Since the second order response is needed, we should keep the terms \( \sim A^2 \) after the substitution of expansion (9) to (6) yielding:

\[ \langle j^D \rangle \approx -8e^2 \alpha T_c \xi^2 \mathbf{A} \{ \langle \hat{\Psi}_0^* \hat{\Psi}_0 \rangle + \langle \hat{\Psi}_1^* \hat{\Psi}_0 \rangle \}, \]

(10a)

\[ \langle j^P \rangle \approx 4e\alpha T_c \xi^2 \epsilon \Re \{ \langle \hat{\Psi}_0^* \hat{\mathbf{p}} \hat{\Psi}_2 \rangle + \langle \hat{\Psi}_1^* \hat{\mathbf{p}} \hat{\Psi}_1 \rangle + \langle \hat{\Psi}_2^* \hat{\mathbf{p}} \hat{\Psi}_0 \rangle \}. \]

(10b)

For the next step the explicit form of approximate solution is required. To derive it, we rewrite (7) in terms of operators:

\[ \left\{ \hat{L}^{-1} - \hat{M}_1 - \hat{M}_2 \right\} \hat{\Psi}(R, t) = f(R, t), \]

(11)

where

\[ \hat{L}^{-1} = \gamma \frac{\partial}{\partial t} + \alpha T_c [\epsilon + \xi^2 |\hat{\mathbf{p}}|^2], \]

(12a)

\[ \hat{M}_1 = \alpha T_c \xi^2 \epsilon (\hat{\mathbf{p}} \hat{\mathbf{p}} + \hat{\mathbf{A}} \hat{\mathbf{A}}), \]

(12b)

\[ \hat{M}_2 = -\alpha T_c \xi^2 (2e \hat{\mathbf{A}})^2. \]

(12c)

Thus, the formal solution of (11) can be obtained with multiplying (11) by \( \hat{L} \) from the left. So, we find the following expressions for the terms in expansion (10):

\[ \Psi_0(R, t) = \hat{L} f(R, t), \]

(13a)

\[ \Psi_1(R, t) = \hat{L} \hat{M}_1 \hat{L} f(R, t), \]

(13b)

\[ \Psi_2(R, t) = (\hat{L} \hat{M}_1 \hat{L} \hat{M}_2) f(R, t). \]

(13c)

Returning to Eq. (12) we can see that operator (12a) is diagonal in the plane wave basis and has the eigenvalue:

\[ L_{q} = \frac{1}{\epsilon_q - i\gamma}, \]

(14)

where

\[ \epsilon_q = \alpha T_c [\epsilon + \xi^2 q^2]. \]

(15)

So, it is convenient to deal with Fourier transformed functions, \( \hat{\Psi}(R, t) = \sum_{q} \hat{\Psi}_q e^{iq(R - wt)} \) and \( f(R, t) = \sum_{q} f_q e^{iq(R - wt)} \). Substituting (13) to (11), performing Fourier transformation and assuming \( \gamma'' \ll \gamma' \), after some computations, we arrive at expressions:
\[
\langle j^D \rangle = -8e^3 T (\alpha T \xi \xi^2)^2 \sum_p \frac{1}{\xi} \left\{ \frac{\text{Re}[A(p\alpha^*)]^{\gamma''\Omega}}{(\varepsilon_- + \varepsilon_+)^2 + \gamma^2 \Omega^2} \left[ 1 + \frac{2(\varepsilon_- - \varepsilon_+)(\varepsilon_- + \varepsilon_+)}{(\varepsilon_- + \varepsilon_+)^2 + \gamma^2 \Omega^2} \right] + \frac{2\text{Im}[A(p\alpha^*)]^{\gamma'\gamma''\Omega^2(\varepsilon_- - \varepsilon_+)}}{(\varepsilon_- + \varepsilon_+)^2 + \gamma^2 \Omega^2} \right\}
\]

\[
\langle j^P \rangle = 8e^3 T (\alpha T \xi \xi^2)^3 \sum_p \frac{(p - k)|pA|^2}{\xi} \left\{ \frac{\gamma''\Omega}{(\varepsilon_- + \varepsilon_+)^2 + \gamma^2 \Omega^2} \left[ 1 + \frac{(\varepsilon_- - \varepsilon_+)(\varepsilon_- + \varepsilon_+)}{(\varepsilon_- + \varepsilon_+)^2 + \gamma^2 \Omega^2} \right] + \frac{(p + k)|pA|^2 \gamma''\Omega(\varepsilon_- - \varepsilon_+)(\varepsilon_- + \varepsilon_+)}{(\varepsilon_- + \varepsilon_+)^2 + \gamma^2 \Omega^2} \right\}
\]

where \( \varepsilon_\pm = \varepsilon(p\pm k)/2 \) and \( A \) is a complex amplitude of vector potential \( \mathbf{A}(\mathbf{R}, t) = A e^{i(k\mathbf{R} - \Omega t)} + c.c. \). The full integration of expressions (16) is quite difficult but the polar angle integration can be performed. To make the text be not overloaded, we set the cumbersome integrals to the appendix section and produce the second-order conductivity in the following form:

\[
\sigma_{\alpha\beta\gamma}^{\alpha'} = \frac{\gamma'' e^3 T \xi^2 I_{\alpha\beta\gamma}(\bar{T}, \Omega, \Theta)}{\hbar^2 c T^4 \cos^3(\Theta)}, \quad (17a)
\]

\[
\sigma_{xy}^{n} = -\frac{\pi \gamma'' e^3 T \xi^2 \varepsilon_{xy}^n (\bar{T}, \Omega, \Theta)}{2\gamma' \hbar^2 c T^2 \cos^3(\Theta)}, \quad (17b)
\]

where dimensionless factors \( I_{\alpha\beta\gamma}^{\alpha'} \) and \( \varepsilon_{xy}^{n} \) are given in the appendix, \( \bar{T} = \Omega \xi / c \) and \( T(c) = k_B T(c) \xi / \hbar c \).

### III. RESULTS AND DISCUSSION

The qualitative dependence of (17a) on dimensionless frequency \( \Omega \) is shown in Fig. 2. It is proved that the absolute value of each component of the symmetrical part of the second-order conductivity monotonically increases, while the frequency decreases and, furthermore, it converges to the constant value at \( \Omega = 0 \). It is important to note that, in fact, no all components are independent and the following equality is obeyed:

\[
\sigma_{xxx} - \sigma_{xyy} = 2\sigma_{xxy}'. \quad (18)
\]

This relation is a result of the system symmetry with regard to the rotation around the z-axis. Indeed, if we suppose the smallness of a wave vector, then the second-order conductivity can be represented in the form of the convolution of vector \( k \) with a forth-rank tensor, \( \sigma_{\alpha\beta\gamma}(k) \rightarrow D_{\alpha\beta\gamma\delta}\xi_3 \), where we take into account that \( \sigma_{\alpha\beta\gamma}(k = 0) = 0 \) as a result of inversion symmetry. The conventional symmetry analysis of tensor \( D_{\alpha\beta\gamma} \) then gives the formula (15). In practice, the light polarization is often defined by Stokes parameters. So, it is convenient to rewrite the first line of (16) in the corresponding form:

\[
J_x = \frac{\sigma_{xxx} + \sigma_{xyy}(|E_x|^2 + |E_y|^2) + \sigma_{xxy}(|E_x|^2 - |E_y|^2)}{2}, \quad (19)
\]

Further, the dependence of drag current magnitude on temperature arouses great interest. But, to begin with, it is necessary to confine the temperature range of applicability of the theory. First, the inequality \( \epsilon \approx (T - T_c) / T_c \ll 1 \) should be obeyed because the GL free energy \( F \) is derived under this condition. Second, the presented theory does not include the effects of interaction between superconductive fluctuations because we omit the term \( \sim |\Psi|^4 \) in the GL free energy. At an essentially small \( \epsilon \) the fluctuations become strong and this contribution cannot be neglected. The analysis (23) produces the so-called Ginsburg-Levanyuk parameter \( g_i \approx T_c / E_F \) which characterizes the temperature range of strong fluctuations. Our theory is correct for the case of weak fluctuations only, i.e. under the condition \( \epsilon \gg g_i \). For MoS\textsubscript{2} systems the estimation gives \( g_i \approx 10^{-4} / 10^{-3} \) at sufficiently high density of electron gas, thus, we suppose that \( \epsilon \in [10^{-3}, 10^{-1}] \).
FIG. 3. The frequency dependence of $\sigma_{yxy}(\Omega \rightarrow 0) = \frac{\gamma'' e^{3} T \xi^{2} \cos(\Theta)}{48 \hbar^{2} c T_{c} e^{2}}$. We can see that the reduced temperature dependence at zero frequency, $\sigma^{\ast} \sim 1/e^{2}$, is rather dramatic and includes an additional power of $\epsilon$ in comparison with the Aslamazov-Larkin conductivity.

The frequency dependence of $\sigma_{yxy}$ is non-monotonic and possesses its extremum at a small value of $\Omega$ (Fig. 3). With the decreasing temperature the extremum depth increases and moves towards zero frequency. We want to remind here that the current, defined by the asymmetric component of conductivity, is nonzero in response to the circular-polarized EM-wave only, which is characterized by the direction of vector $E$ rotation. Thus, the switching from the clockwise polarization to the reverse one changes the sign of $E_{x}E_{y}^{*} - E_{y}E_{x}^{*}$ as well as the direction of current $y$-projection.

The important feature of the obtained frequency dependence of the second-order conductivity consists in that its qualitative behaviour is the same as for the case of photon drag effect in conventional systems, for example, based on graphene\textsuperscript{24,25}. Apparently, the reason of such similarity lies in a certain affinity of the TDGL-equation and the Boltzmann one, which is widely used for analyzing the nonlinear response of 2D electron gas.

At the end of this subsection, we discuss the scope of the examined effect. For this purpose, let us compare the contribution of superconducting fluctuations with the one of normal electron gas. For the estimation, it is enough to use the simplest classical expression for the photon drag current in 2D systems, which has the following form\textsuperscript{26}:

$$J_{n} = \frac{2 e^{3} n}{1 m^{2} \rho} \frac{\tau^{2}}{1 + (\Omega \tau)^{2}} |E|^2 \mathbf{k},$$  \hspace{1cm} (21)

where $n$ is the electron gas density and $\tau$ is the momentum relaxation time. With utilizing Eqs. (20) and (21), the ratio of two contributions in the zero-frequency limit can be easily composed:

$$\frac{\gamma''}{\gamma'} = \frac{8 \gamma'' T_{n} \xi^{2}}{3 \gamma'} \frac{T_{n}^{2}}{T_{c}} \left( \frac{\sigma_{AL}}{\sigma_{n}} \right)^{2},$$  \hspace{1cm} (22)

where $\sigma_{n}$ is the Drude conductivity and $\sigma_{AL} = e^{2}/16 \hbar c$ is the Aslamazov-Larkin conductivity for the 2D system. Eq. (22) is convenient to be considered piecemeal. First, the ratio $T/T_{c} \approx 1$ and it does not play any role. Second, the ratio of conductivities should be less than unity, $\sigma_{AL} < \sigma_{n}$. However, the AL-contribution may be commensurate with the normal one at sufficient proximity of $T$ to $T_{c}$. Further, the quantity $(\gamma''/\gamma')$ is still undefined and we only say that it is much less than unity. The rigorous physical sense of this quantity is given from the microscopic theory. It is found that the non-zero $\gamma''$ is the result of asymmetry of electron and hole spectra in the vicinity to Fermi energy, while $\gamma' = \pi \nu/8 T_{c}$, where $\nu$ is the density of states per single spin at Fermi energy. Besides, the ratio $(\gamma''/\gamma')$ is often $\sim T_{c}/E_{F}$ and it
is small indeed. Nevertheless, we should note that, in fact, this quantity is still not computed for a real TMD system. The above arguments give an idea that the ratio (22) takes a small magnitude. However, the rest dimensionless factor $n_\xi^2$ is expected to be very large and, thus, the contribution of fluctuations should be visible in the experiment.

IV. CONCLUSION

In the presented work we developed the theory of photon drag of the superconducting fluctuations based on using the TDGL-equation. It was proved that the qualitative dependence of drag current on the EM-frequency is similar to the one in conventional systems. The estimation showed that the drag current magnitude is expected to reach a considerable value and to be sufficient for experimental observation.

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Appendix: Explicit expressions for integrals $I_{\alpha\beta\gamma}$

The resulting expressions (17) contain the dimensionless factors which have the following form:

$$I_{yxy} = \int_{1 + \frac{4\epsilon}{\Omega^2 \cos^2(\Theta)}}^{\infty} dy \left\{ y - \sqrt{y - 2} \left[ y - \frac{16e}{\Omega^2 \cos^2(\Theta)} \right] \right\}^2,$$

$$I_{xxx} = \int_{1 + \frac{4\epsilon}{\Omega^2 \cos^2(\Theta)}}^{\infty} dy \left[ 2F_1 + F_2 + F_3 \right],$$

$$I_{xxy} = \int_{1 + \frac{4\epsilon}{\Omega^2 \cos^2(\Theta)}}^{\infty} dy \left[ F_4 + F_5 \right],$$

$$I_{yxy} = \int_{1 + \frac{4\epsilon}{\Omega^2 \cos^2(\Theta)}}^{\infty} dy \left[ F_1 + F_5 + F_6 \right],$$

where

$$F_1 = -\frac{y - \sqrt{y - 2} \left[ y - \frac{16e}{\Omega^2 \cos^2(\Theta)} \right]}{\sqrt{(y - 2)^2 + \frac{16\epsilon}{\Omega^2 \cos^2(\Theta)}}} \left[ y^2 + \left( \frac{\pi}{4\xi_\Omega \cos^2(\Theta)} \right)^2 \right]^2,$$

$$F_2 = \frac{4}{y^2 + \left( \frac{\pi}{4\xi_\Omega \cos^2(\Theta)} \right)^2} \left\{ y - 1 - \frac{y^2(y - 1) - 2 \left( y - 1 - \frac{4\epsilon}{\Omega^2 \cos^2(\Theta)} \right) (3y - 4)}{(y - 2)^2 + \frac{16\epsilon}{\Omega^2 \cos^2(\Theta)}}^3/2 \right\} \left[ y^2 + \left( \frac{\pi}{4\xi_\Omega \cos^2(\Theta)} \right)^2 \right]^2$$

$$\times \left[ y(3y - 4) + 2 \left( y - 1 - \frac{4\epsilon}{\Omega^2 \cos^2(\Theta)} \right) \right] + \frac{y^2 \left[ 8 \left( y - 1 - \frac{4\epsilon}{\Omega^2 \cos^2(\Theta)} \right) (2y - 3) - y^2(3y - 4) \right]}{(y - 2)^2 + \frac{16\epsilon}{\Omega^2 \cos^2(\Theta)}}^{3/2} \right\}.$$
\[ F_3 = \frac{-2y}{y^2 + \left( \frac{x}{2T, \Omega \cos^2(\Theta)} \right)^2} \left\{ -2 \left( y - 1 - \frac{4\epsilon}{\Omega^2 \cos^2(\Theta)} \right) + y^2 \right\} + \frac{y^3}{(y - 2)^2 + \frac{16\epsilon}{\Omega^2 \cos^2(\Theta)}}^{1/2} \] (A.7)

\[ F_4 = \frac{4}{y^2 + \left( \frac{x}{2T, \Omega \cos^2(\Theta)} \right)^2} \left\{ 1 - y + \frac{y(y - 1) - 2 \left( y - 1 - \frac{4\epsilon}{\Omega^2 \cos^2(\Theta)} \right)}{(y - 2)^2 + \frac{16\epsilon}{\Omega^2 \cos^2(\Theta)}}^{1/2} - \frac{y}{2y^2 + \left( \frac{x}{2T, \Omega \cos^2(\Theta)} \right)^2} \right\} \times \left\{ 2 \left( y - 1 - \frac{4\epsilon}{\Omega^2 \cos^2(\Theta)} \right) - y(3y - 4) + \frac{y^2(3y - 4) - 8 \left( y - 1 - \frac{4\epsilon}{\Omega^2 \cos^2(\Theta)} \right) (y - 1)}{(y - 2)^2 + \frac{16\epsilon}{\Omega^2 \cos^2(\Theta)}}^{1/2} \right\} \] (A.8)

\[ F_5 = \frac{-2y}{y^2 + \left( \frac{x}{2T, \Omega \cos^2(\Theta)} \right)^2} \left\{ y^2 - 2 \left( y - 1 - \frac{4\epsilon}{\Omega^2 \cos^2(\Theta)} \right) - y \left( (y - 2)^2 + \frac{16\epsilon}{\Omega^2 \cos^2(\Theta)} \right)^{1/2} \right\} \] (A.9)

\[ F_6 = \frac{4}{y^2 + \left( \frac{x}{2T, \Omega \cos^2(\Theta)} \right)^2} \left\{ -y + \frac{y^2 - 2 \left( y - 1 - \frac{4\epsilon}{\Omega^2 \cos^2(\Theta)} \right)}{(y - 2)^2 + \frac{16\epsilon}{\Omega^2 \cos^2(\Theta)}}^{1/2} - \frac{y}{2y^2 + \left( \frac{x}{2T, \Omega \cos^2(\Theta)} \right)^2} \right\} \times \left\{ 2 \left( y - 1 - \frac{4\epsilon}{\Omega^2 \cos^2(\Theta)} \right) - 3y^2 + \frac{y(3y^2 - 8 \left( y - 1 - \frac{4\epsilon}{\Omega^2 \cos^2(\Theta)} \right) )}{(y - 2)^2 + \frac{16\epsilon}{\Omega^2 \cos^2(\Theta)}}^{1/2} \right\} \] (A.10)
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