Finite Temperature Effective Action in Monopole Background

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We compute the CP-odd part of the finite temperature effective action for massive Dirac fermions in the presence of a Dirac monopole. We confirm that the induced charge is temperature dependent, and in the effective action we find an infinite series of CP-violating terms that generalize the familiar zero temperature $F \tilde{F}$ term. These results are analogous to recent results concerning finite temperature induced Chern-Simons terms.

Recent finite temperature studies\cite{1-8} of 2+1 dimensional Chern-Simons systems\cite{9} have revealed several interesting new features of induced parity violating terms at finite temperature. It is well known\cite{10,11} that at $T=0$, for fermions in the presence of a static magnetic flux $\Phi = \int d^2 x \frac{eB}{2\pi}$ there is an induced charge

$$Q = -\frac{e}{2} \Phi$$ \hfill (1)

and this corresponds to an induced Chern-Simons term in the zero temperature Euclidean effective action:

$$S_{P-\text{odd}} = -i\frac{e^2}{8\pi} \int d^3 x \epsilon^{\mu\nu\rho} A_\mu \partial_\nu A_\rho = -i \frac{\Phi}{2} \int e A_0 \, d\tau$$ \hfill (2)

But at finite temperature, in such a static background, the induced charge is\cite{12}

$$Q = -\frac{e}{2} \Phi \tanh\left(\frac{\beta m}{2}\right)$$ \hfill (3)

where $\beta = \frac{1}{T}$ is the inverse temperature. The corresponding parity-odd part of the (Euclidean) finite temperature induced effective action is\cite{1-8}

$$S_{P-\text{odd}} = -i \Phi \arctan\left[ \tanh\left(\frac{\beta m}{2}\right) \tanh\left(\frac{\beta}{2} \int_0^\beta e A_0 d\tau\right) \right]$$

$$= -i \Phi \left[ \tanh\left(\frac{\beta m}{2}\right) \left( \frac{1}{2} \int_0^\beta e A_0 d\tau \right) + \frac{1}{3} \tanh\left(\frac{\beta m}{2}\right) - \frac{1}{3} \tanh^2\left(\frac{\beta m}{2}\right) \left( \frac{1}{2} \int_0^\beta e A_0 d\tau \right)^2 + \ldots \right]$$ \hfill (4)

The perturbative expansion in the second line of (4) shows that at nonzero temperature there is an infinite series of parity violating terms, of which the Chern-Simons term, with temperature dependent charge (3), is only the first. The entire series is required at finite $T$ to show that $S_{P-\text{odd}}$ is invariant under “large” gauge transformations $A_0 \to A_0 + \frac{2\pi}{e} N, N \in \mathbb{Z}$\cite{1-8}. The higher terms are non-extensive (i.e. they are not integrals of a Lagrangian density), but they all vanish at zero temperature, leaving just the Chern-Simons term (2).

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In this Letter we investigate similar phenomena for the finite temperature induced charge and induced effective action in 3 + 1 dimensional field theory. The analogue discrete symmetry we consider is CP, and we compute the finite $T$ induced charge, and the corresponding finite $T$ induced effective action, for massive Dirac fermions in the presence of a Dirac monopole. This should be distinguished from earlier work \cite{13}, in a similar spirit, concerning CP-violating terms at high temperature and finite fermion density in the effective action for an even number of (massless) chiral fermions.

We recall that the quantum mechanics of a particle of electric charge $e$ and a magnetic pole of magnetic charge $g = \frac{1}{16\pi} \int d^3x \vec{\nabla} \cdot \vec{B}$, is consistent only if these charges satisfy \cite{14–15}

$$2eg \equiv 2q = \text{integer} \quad (5)$$

But the Dirac Hamiltonian

$$H_D = -\vec{\alpha} \cdot (i\vec{\nabla} + e\vec{A}_D) + m\gamma_0 \quad (6)$$

for a fermion in the presence of a Dirac monopole $\vec{A}_D$ is not self-adjoint, and so requires the introduction of a self-adjoint extension parameter $\theta$ that characterizes the boundary conditions (in the s-wave sector) at the location of the monopole \cite{17–20}. The spectrum of the fermions becomes $\theta$ dependent, and there is an induced charge of the fermion vacuum given by \cite{21–23}

$$Q = -\frac{e\theta}{2\pi} 2q \quad (7)$$

This induced charge corresponds to an induced CP-violating term in the zero temperature Euclidean effective action \cite{21,22}:

$$S_{CP-\text{odd}} = \frac{e^2}{16\pi^2} \int d^4x F_{\mu\nu} \tilde{F}_{\mu\nu} = -\frac{e^2\theta}{4\pi^2} \int d^4x A_0 \vec{\nabla} \cdot \vec{B} = -i2q \frac{\theta}{2\pi} \int eA_0 d\tau \quad (8)$$

The finite temperature induced charge for fermions in the presence of a Dirac monopole has been computed recently \cite{24,25}, and the zero temperature result (7) becomes temperature dependent:

$$Q = -\frac{e}{\pi} (2q) \sin \theta \sum_{n=0}^{\infty} \frac{1}{(2n+1)^2 + x^2 + x \cos \theta \sqrt{(2n+1)^2 + x^2}} \quad (9)$$

where $x \equiv m_0 \frac{\theta}{\pi}$. In \cite{24}, it was checked that the zero temperature limit (i.e. $x \to \infty$) of the expression (8) agrees with the zero temperature result (7) \cite{[see also (19) below].

These results should be compared with the related work of Le Guillou and Schaposnik \cite{26}, who considered the problem of the electric charge of a non-Abelian dyon at finite T with CP violation introduced by an explicit $F\tilde{F}$ term, but no fermions. There, Witten’s relation (7) was found to be unchanged at finite T. However, as discussed in \cite{25}, the analysis of \cite{26} was effectively at tree level, and the inclusion of light fermions was shown \cite{23} (in the limit of an infinitely massive ’t Hooft-Polyakov monopole) to make the charge temperature dependent in a manner analogous to (9).

In this Letter we confirm the result (8) using a slightly different approach, and we then compute the corresponding CP-odd part of the finite temperature induced effective action for fermions in the background of a static Dirac monopole. For this purpose it is convenient to express everything in terms of the spectral function (the density of states) of the Dirac equation for fermions in the background of a static Dirac monopole. This spectral function is computed in the appendix using an elementary calculation of the resolvent of the (s-wave) Dirac Hamiltonian. This leads to a simple integral representation of the summation formula for the charge in (9). We then obtain a spectral representation of the CP-odd part of the induced effective action at finite temperature, and study its dependence on temperature and on $\theta$.

The finite temperature induced charge is expressed in terms of the spectral function as \cite{25}
\[ Q = -\frac{e}{2} \int_{-\infty}^{\infty} d\omega \rho_D(\omega) \tanh\left(\frac{\beta \omega}{2}\right) \]  

(10)

The spectral function \( \rho_D(\omega) \) is defined in the usual manner as

\[ \rho_D(\omega) = \frac{1}{\pi} \text{Im} \text{Tr} \left( \frac{1}{H_D - \omega - i\epsilon} \right) \equiv \frac{1}{\pi} \text{Im} \Gamma_D(\omega + i\epsilon) \]  

(11)

where \( H_D \) is the Dirac Hamiltonian in (6), and (11) defines \( \Gamma_D(\omega) \), the trace of the resolvent of \( H_D \).

In all but the lowest partial wave, the spectrum of the Dirac Hamiltonian \( H_D \) has a symmetry between positive and negative energy states \([17,19,20]\), so that only the lowest partial wave contributes to the induced charge in (10). Thus we only need to consider this lowest partial wave sector, which is effectively a one dimensional radial problem (i.e. one dimensional, but on the half-line \( r \geq 0 \)). The effective one dimensional Hamiltonian is \([17–20]\) (we choose \( q > 0 \))

\[ H = -i\gamma_5 \frac{d}{dr} + m\gamma_0 \]  

(12)

acting on two-component spinors \( \chi(r) \) with the “bag” boundary condition

\[ i\gamma_1 \chi(0) = \exp(-i\gamma_5 \theta) \chi(0) \]  

(13)

Here, the \( 2 \times 2 \) Dirac matrices are:

\[ \gamma_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \gamma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \]  

(14)

The boundary condition (13) ensures self-adjointness of the reduced Hamiltonian \( H \) in (12), with the spinor inner product being \( \langle \chi|\psi \rangle = \int_0^\infty dr \chi^\dagger(r)\psi(r) \). We also note here that we can restrict our attention to \( 0 < \theta < \pi \), since a CP-inversion takes \( \theta \to -\theta \), and CP-odd observables are correspondingly odd in \( \theta \).

Given that only this lowest partial wave contributes to the induced charge, we obtain from (10) and (11) the following contour integral representation of the induced charge

\[ Q = -\frac{e}{2} \left( 2q \right) \oint_C dz \frac{dz}{2\pi i} \tanh\left(\frac{\beta z}{2}\right) \Gamma(z) \]  

(15)

where \( C \) is the contour in Figure 1, and \( \Gamma(z) \) is the trace of the resolvent of the reduced Hamiltonian \( H \). The overall factor of \( 2q \) arises as the degeneracy of the lowest partial wave (viz, \( 2j + 1 \), with \( j = q - \frac{1}{2} \)).

In the appendix, we present an elementary derivation of \( \Gamma(z) \) in terms of the diagonal resolvent (i.e. diagonal Green’s function) \( \Gamma(r; z) \) of \( H \) [see Eqs. (39,40)]:

\[ \Gamma(z) \equiv \int_0^\infty dr \Gamma(r; z) = -\frac{m}{2k^2} \left( \frac{z \sin \theta - m - i k \cos \theta}{z - m \sin \theta} \right) \]  

(16)

Here \( z^2 = k^2 + m^2 \), and \( \text{Im} k > 0 \) on the physical sheet, and we have dropped a trivial \( \theta \)-independent term from \( \Gamma(z) \) (see the appendix). Notice that for (real) infinitesimal \( \lambda \) and \( \epsilon \),

\[ \Gamma(m \sin \theta + \lambda + i\epsilon) = -\frac{\Theta(-\cos \theta)}{\lambda + i\epsilon} \]  

(17)

and thus, in addition to scattering states, there is a bound state at \( z = m \sin \theta \), provided \( \cos \theta < 0 \). Using the result (16) and deforming the contours around the cuts and around the bound state pole (which is only present if \( \cos \theta < 0 \)) we obtain the following expression for the induced charge as a real integral:

\[ Q = -\frac{q \epsilon}{\pi} \left[ \pi \tanh\left(\frac{m \beta}{2} \sin \theta\right) \Theta(-\cos \theta) + \sin \theta \cos \theta \int_1^\infty \frac{du}{\sqrt{u^2 - 1}} \left( \frac{\tanh\left(\frac{m \beta}{2} u\right)}{u^2 - \sin^2 \theta} \right) \right] \]  

(18)
We plot this induced charge in Figure 2 as a function of $\theta$ for various values of $m\beta$, and in Figure 3 as a function of $m\beta$ for various values of $\theta$. It is clear from these Figures that in the zero temperature limit ($m\beta \to \infty$), $-\pi Q_{eq}$ saturates to $\theta$, in agreement with the zero temperature result (7). This is easy to verify analytically from the integral representation (18), since

$$Q(T = 0) = -\frac{qe}{\pi} \left[ \pi \Theta(-\cos \theta) + \sin \theta \cos \theta \int_1^\infty \frac{du}{\sqrt{u^2 - 1}} \left( \frac{1}{u^2 - \sin^2 \theta} \right) \right]$$

(19)

In the high temperature limit ($m\beta \to 0$), the induced charge vanishes for any $\theta$, as is clear from Figures 2 and 3. Analytically,

$$Q(T \to \infty) \sim -\frac{qe}{\pi} \left[ \frac{m\beta}{2} \sin \theta \Theta(-\cos \theta) + \frac{m\beta}{2} \sin \theta \cos \theta \int_1^\infty \frac{du}{\sqrt{u^2 - 1}} \left( \frac{u}{u^2 - \sin^2 \theta} \right) \right]$$

(20)

When $\theta = \frac{\pi}{2}$, the finite temperature induced charge (18) reduces to the simple form:

$$Q(\theta = \frac{\pi}{2}) = -\frac{qe}{\pi} \frac{m\beta}{2}$$

(21)

As mentioned in [24], this is consistent with the $2 + 1$ dimensional result [2], since it is known [25] that for $\theta = \frac{\pi}{2}$, the monopole problem with $2q = 1$ reduces to that of an Aharonov-Bohm flux string with flux $\Phi = \frac{1}{2}$.

These results (18,19,20,21) are in complete agreement with the summation expression (9) for the finite temperature induced charge found by Coriano and Parwani in [24]. Indeed, the integral expression (18) is in fact the Sommerfeld-Watson representation of the sum in (9); and equivalently, the sum in (9) can be obtained from (15) by deforming the contour around the poles of the $\tanh(\beta z)$ function on the imaginary $z$ axis. The main utility of the integral representation (18), as compared to the summation representation (9), becomes apparent when we compute the CP-odd part of the finite temperature induced effective action.

To compute the finite temperature effective action, we work in Euclidean space and consider fermions in the presence of the background gauge field $A_\mu = (A_0, \vec{A}_D)$, where $A_0$ is a constant (this can always be achieved by a small gauge transformation), and $\vec{A}_D$ is the static vector potential for a Dirac monopole. The finite temperature induced effective action is

$$S = \int_{-\infty}^\infty d\omega \rho_D(\omega) \log \cosh \left( \frac{\beta}{2}(\omega - i e A_0) \right)$$

(22)

As we are only interested in the CP-odd part of the induced effective action, two simplifications occur. First, since only the lowest partial wave sector of the spectrum has a CP-odd piece, we can (just as we did for the charge) use the reduced spectral function $\rho(\omega)$ of the s-wave Hamiltonian (14), rather than the full spectral function $\rho_D(\omega)$ of the Dirac Hamiltonian $H_D$ in (8). Second, since the CP-odd part must be odd in $A_0$, we can compute the difference as:

$$S_{CP-odd} = \frac{1}{2} (2q) \int_{-\infty}^\infty d\omega \rho(\omega) \log \left[ \frac{\cosh \left( \frac{\beta}{2}(\omega - i e A_0) \right)}{\cosh \left( \frac{\beta}{2}(\omega + i e A_0) \right)} \right]$$

$$= -i q \int_{-\infty}^\infty d\omega \rho(\omega) \left[ \tanh \left( \frac{\beta \omega}{2} \right)(e\beta A_0) + \frac{1}{12} \tanh \left( \frac{\beta \omega}{2} \right) \sech^2 \left( \frac{\beta \omega}{2} \right)(e\beta A_0)^3 + \ldots \right]$$

(23)

In this expansion, the first term, which is linear in $A_0$, corresponds to the familiar zero temperature induced effective action in (8), but with a temperature dependent induced charge multiplying $A_0$. The higher
order terms in the perturbative expansion \((23)\) of the CP-odd part of the induced effective action are not of this \(\int F \tilde{F}\) form. They are non-extensive in Euclidean time, as they involve powers of \(e^\beta A_0 = (e^\beta \int_0^\beta A_0 d\tau)\). Nevertheless, all these non-extensive terms vanish at zero temperature due to factors of \(\text{sech}^2(\frac{m\beta}{2})\), which vanishes exponentially fast as \(T \to 0\). Thus, at zero temperature only the first term survives, and the CP-odd part of the finite temperature induced effective action \((23)\) reduces to the familiar zero temperature expression \((8)\).

Given that we know the relevant spectral function \(\rho(\omega)\), it is straightforward to write down an integral representation of any term in the perturbative expansion in \((23)\). The coefficient of \((i \int_0^\beta A_0 d\tau)\) is just the induced charge \((18)\), while the coefficient of \((i \int_0^\beta A_0 d\tau)^3\) is

\[
J \equiv -\frac{e^3q}{12\pi} \int \frac{\pi \text{tanh}(\frac{m\beta \sin \theta}{2}) \text{sech}^2(\frac{m\beta \sin \theta}{2}) \Theta(-\cos \theta) + \sin \theta \cos \theta}{\int_1^\infty \frac{du}{\sqrt{u^2 - 1} \text{sech}(\frac{m\beta u}{2})}}
\]

In Figure 4 we plot this first correction term \(J\) as a function of \(m\beta\), for various values of \(\theta\). It is clear that \(J\) vanishes exponentially fast as \(m\beta \to \infty\) (i.e. as \(T \to 0\)). Using the integral representation \((24)\), we find that (for \(\theta \neq \frac{\pi}{2}\))

\[
J(T \to 0, \theta \neq \frac{\pi}{2}) \sim -\frac{e^3q}{12\pi} \left[ 4\pi e^{-m\beta \sin \theta} \Theta(-\cos \theta) + 2\sqrt{2} \pi \tan \theta \frac{e^{-m\beta}}{\sqrt{m\beta}} \right], \quad m\beta \to \infty
\]

When \(\theta = \frac{\pi}{2}\) we can evaluate \(J\) exactly:

\[
J(\theta = \frac{\pi}{2}) = -\frac{e^3q}{24} \int \frac{\pi \text{tanh}(\frac{m\beta \sin \theta}{2}) \text{sech}(\frac{m\beta \sin \theta}{2})}{\Theta(-\cos \theta)} \tan(\frac{e^\beta \int_0^\beta \Theta(-\cos \theta) + \sin \theta \cos \theta}{\sqrt{u^2 - 1} \text{sech}(\frac{m\beta u}{2})})
\]

These asymptotic formulae \((25, 26)\) match the large \(m\beta\) behaviour in Figure 4, but note that when \(\theta \neq \frac{\pi}{2}\), the sign of \(\cos \theta\) determines which exponential factor dominates in \((24)\).

At high temperatures \((m\beta \to 0)\),

\[
J(T \to \infty) \sim -\frac{e^3q}{12\pi} \int \frac{\pi m\beta \sin \theta}{2} \Theta(-\cos \theta) + \frac{m\beta}{2} \sin \theta \cos \theta}{\int_1^\infty \frac{du}{\sqrt{u^2 - 1} \text{sech}(\frac{m\beta u}{2})}}
\]

This involves the same \(\frac{\sin \theta}{T}\) factor as the high temperature limit of the induced charge \(Q\) in \((21)\). In fact, this is true of the high \(T\) behaviour of every term in the expansion in \((23)\), so we find that as \(T \to \infty\), the CP-odd induced effective action behaves as

\[
S_{CP-odd} \sim -i \left(\frac{q m}{2T}\right) \sin \theta \tan(\frac{e}{2} \int_0^\beta A_0 d\tau)
\]

Finally, we comment on some special values of \(\theta\). When \(\theta = 0\) or \(\pi\), the CP-odd finite temperature effective action \((23)\) vanishes, as is easy to verify term by term in the perturbative expansion. This is of course consistent with the fact that CP is not violated for these values of \(\theta\) \([17, 20]\). When \(\theta = \frac{\pi}{2}\), we can evaluate every term in the perturbative expansion in \((23)\). Each term contributes one half of the bound state contribution, so we find the simple closed form expression:

\[
S_{CP-odd}(\theta = \frac{\pi}{2}) = -iq \arctan \left[ \text{tanh}(\frac{\beta m}{2}) \tan(\frac{1}{2} \int_0^\beta A_0 d\tau) \right]
\]
Note that this is exactly the same form as the finite temperature induced parity-odd effective action (4) found in $2+1$ dimensions, with the identification $q = \Phi$. This is consistent with the fact \[28,24\] that the s-wave spectral properties of the $\theta = \frac{\pi}{2}$ fermion plus monopole system coincide with those of the planar fermion plus Aharonov-Bohm string system with flux $\Phi = q$.

To conclude, we note that it would be of interest to extend this analysis to fermions in the presence of a 't Hooft-Polyakov monopole. The finite temperature induced charge has been computed \[25\] in the limit of an infinitely massive monopole coupled to light fermions, but it would be interesting to try to go beyond this point monopole limit. For the effective action, another interesting question is to go beyond the static background ansatz, to find other structures that generalize the zero temperature $\tilde{F}F$ term for nonzero temperature. Even in the simpler Chern-Simons case, the calculation of induced terms at finite temperature is much more complicated for a non-static background \[29,30\].

**APPENDIX**

In this appendix we present an elementary derivation of the spectral function for the reduced s-wave hamiltonian $H$ in (12). The resolvent (or Green’s function) $G(r, r'; z)$ for $H$ is defined by:

$$
(H - z)G(r, r'; z) = \delta(r - r')1
$$

This is a $2 \times 2$ matrix equation, so we write

$$
G(r, r') = \begin{pmatrix} a(r, r') & b(r, r') \\ c(r, r') & d(r, r') \end{pmatrix}
$$

where we have suppressed the $z$ dependence. Using the Hamiltonian in (12), we find that $a$ and $c$ satisfy:

$$
-\frac{\partial^2 a(r, r')}{z + m} - (z - m) a(r, r') = \delta(r - r')
$$

$$
c(r, r') = -i \frac{\partial_r a(r, r')}{z + m}
$$

with $d$ and $b$ satisfying similar equations as $a$ and $c$ (respectively) with $m \to -m$. It is trivial to solve these equations in terms of the fundamental solutions

$$
\psi_{\pm}(r) = e^{\pm ikr}
$$

where $z^2 = m^2 + k^2$, and $\Im(m(k)) > 0$ on the physical sheet. We find

$$
a(r, r') = -\left(\frac{z + m}{2ik}\right) \left[ \Theta(r - r')e^{ik(r-r')} + \Theta(r' - r)e^{-ik(r-r')} + \alpha e^{ik(r+r')} \right]
$$

where the last term is a solution of the homogeneous equation. (Note that the homogeneous solution $e^{-ik(r+r')}$ is excluded by its large $r$ behaviour for $r = r'$ and $\Im(m(k)) > 0$). The coefficient $\alpha$ in (34) is fixed by the boundary condition (13) which, when applied to the resolvent $G(r, r')$, requires that

$$
a(0, r') = i \left(\frac{1 + \sin \theta}{\cos \theta}\right) c(0, r') , \quad r' > 0
$$

This determines the constant $\alpha$ to be:

$$
\alpha = \frac{z \sin \theta - m - ik \cos \theta}{z - m \sin \theta} = \frac{k \sin \theta - i z \cos \theta}{k + i m \cos \theta}
$$

Thus, the diagonal part of $a(r, r')$ is...
\[ a(r, r) = -\left( \frac{z + m}{2ik} \right) \left( 1 + \alpha e^{2ikr} \right) \]  

(37)

A similar calculation for \( d(r, r') \) leads to

\[ d(r, r) = -\left( \frac{z - m}{2ik} \right) \left( 1 - \alpha e^{2ikr} \right) \]  

(38)

Thus, the diagonal resolvent is

\[ \Gamma(r; z) \equiv \text{Tr}[G(r, r; z)] = -\frac{1}{ik} \left[ z + \alpha m e^{2ikr} \right] \]  

(39)

The first term in (39) corresponds to the free case and may be dropped (also it is independent of \( \theta \) and so does not contribute to induced CP-violating terms). Thus, the trace of the diagonal resolvent is

\[ \Gamma(z) \equiv \int_0^\infty dr \Gamma(r; z) = -\frac{\alpha m}{2k^2} \]  

(40)

with \( \alpha \) given in (36). Note that \( \alpha \) has a pole at \( k = -im \cos \theta \) in the \( k \) upper half plane provided \( \cos \theta < 0 \). Thus, when \( \cos \theta < 0 \) there is a bound state of energy \( z_b = m \sin \theta \), in addition to scattering states.

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FIG. 1. Contour of integration $C$, in the complex energy plane, for the induced charge in (15). Note the branch cuts beginning at the continuum thresholds $z = \pm m$, and the bound state pole at $z = m \sin \theta$, which is present only if $\cos \theta < 0$. 

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FIG. 2. Plot of $-\pi Q/qe$ in (18) as a function of $\theta$ for various values of $m\beta = \frac{m}{T}$. The short-dash curve is for $m\beta = 0.5$, the long-dash curve is for $m\beta = 2$, and the solid curve is for $m\beta = 30$. As $T \to 0$, the plot tends to $\theta$, while at high $T$ it is proportional to $\sin \theta$. This type of plot, based on the summation formula (9), appeared in the work of Coriano and Parwani; here for completeness we have re-plotted it using the integral representation (18) of the finite $T$ induced charge.

FIG. 3. Plot of $-\pi Q/qe$ in (18) as a function of $m\beta = \frac{m}{T}$ for various values of $\theta$. The solid curve is for $\theta = \frac{\pi}{4}$, the long-dash curve is for $\theta = \frac{\pi}{2}$, and the short-dash curve is for $\theta = \frac{3\pi}{4}$. As $T \to 0$, each curve saturates rapidly to $\theta$. 
FIG. 4. Plot of $-\frac{2\pi J}{q^3}$ in (24) as a function of $m\beta = \frac{m}{T}$ for various values of $\theta$. The solid curve is for $\theta = \frac{\pi}{4}$, the long-dash curve is for $\theta = \frac{\pi}{3}$, and the short-dash curve is for $\theta = \frac{3\pi}{4}$. Note the exponential decay at low temperatures.