Cellular Automata under the influence of noise

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Abstract Noise in the local transition function is compared to fluctuations in the updating times of the cells. Obtained results are shown to be quite different in both cases. In this extended abstract we briefly explain the problem and present results obtained and comment them.

1 Introduction

In their original definition, Cellular Automata (CA) are synchronous \cite{1}. This means that all cells are updated in parallel simultaneously. However it has been shown that this updating policy induces artificial structure in the CA states \cite{2,3,4,5}. Moreover, natural collective systems are not synchronous, in the sense that their components do not react to a signal exactly at the same time \cite{6}. Therefore, asynchronous CA have been studied and defended as more realistic models of natural systems \cite{7,8}.

By comparison to synchronous models, asynchronous ones may be regarded as having noise \cite{9,5}. In fact, we may consider that besides signal amplitude fluctuations, timing fluctuations are also a form of noise \cite{10}. In the latter case it takes the form of delays or speedups in the updating time instead of changes in the output of the local function.

It has been hypothesised that these two forms of noise may qualitatively produce the same or a similar effect \cite{6,11}. In this paper we show that this is not the case. Qualitatively different results are produced by noise in the local function and by noise in the updating moments. We used 1D binary CA with neighbourhood of radius 1.

The next section presents the models considered, the following one shows results obtained and the paper ends with an analysis to the results and ideas to develop further.

2 Models

In asynchronous CA, several updating policies are possible, producing different results \cite{9,5} and possessing more or less realism. According to a statistical anal-
ysis in [5] the most realistic asynchronous updating model is time driven, with independent timings for each cell. The waiting times of each timer are exponentially distributed, with mean 1.

The step-driven method with uniform choice is shown to produce approximate results for large CA grids. It just randomly picks the next cell to be updated from all the cells in the grid. Therefore, for a grid with \( N \) cells, after \( N \) updates, some cells may have been updated more than once while others may not have been updated at all. This method is simpler to compute and may also be called unfair. Nevertheless, over time all cells will tend to equal number of updates. It has been shown [5] that for updates in which the local function is independent of time, this model is equivalent to the time driven above described.

Fairness in each set of \( N \) updates may be forced originating the asynchronous step-driven random new sweep method [5]. In this model the \( N \) cells are randomly sorted and this gives the update order for the next individual \( N \) updates. Therefore, in a cycle of \( N \) updates, all the cells are updated exactly once, although in a different random order in each cycle. For this reason we may designate this updating by fair.

The fair model, in spite of the artificiality of guaranteeing exactly one update per each cycle of \( N \), may be quite realistic for models of natural systems. We may consider that the result of \( N \) updates is just a snapshot that we may observe cyclically. If each component of a natural system has a latency time (minimum time between two successive state changes) that is long compared to the state changing time and the state changes roughly self-synchronise among cells, then the fair model may be quite realistic. It should be noticed that self-synchronisation in natural systems is quite a frequent process (see for instance [6]).

The synchronous model is the commonly used. Each cell update is computed for all cells but the new states are only loaded into the cells after all updates performed.

For any of this updating models we may consider noise in the local function. It the most general form, for these CA, it may introduce an error in the output with two given probabilities, \( p_{10} \) for an output error of 1 to become a 0 and \( p_{01} \) for the other way round.

3 Results

We tested the three models with and without errors in the local function. Results are not qualitatively different between the two asynchronous updates (which confirms results obtained in [5] without error in the local function). Therefore, we only present a comparison between the asynchronous unfair and the synchronous updatings.

In introducing an error in the local function we opted for considering only errors from an active to a quiescent state. Otherwise, the error would introduce a kind of seed from which active structures could be built in the CA. In modeling
of natural systems this is an important issue. Simulated systems have a kind of background (the quiescent state) over which the action takes place.

The synchronous updating is much sensitive to noise in the local function, which is specially visible in complex and in periodic rules. For chaotic and fixed point rules its behaviour is more stable.

In Fig. 1 we show results for rule 110, complex. It can be clearly noticed that the complex character of the rule in synchronous update changed into a chaotic behaviour with only an error of 1% (1→0). The behaviour of the asynchronous unfair update is approximately the same in the two cases.

The periodic rule 38 is depicted in Fig. 2. The small amount of noise (1%) in the local function is enough to destroy the periodic behaviour of the synchronous update. In fact, after some more iterations it converges to an all zero state. Again, the behaviour of the asynchronous unfair update does not sensibly change with the addition of errors in the local function output.

Notice, in both cases, that the behaviour of the two updatings is quite different already in the case with no errors. Asynchronous updates, in general, present much less structure in the patterns.
4 Discussion

It was already known that synchronous updates artificially introduce structure in the patterns displayed by CA. In this work we show that the introduction of errors in the local function is also critical for the structure of synchronous updates.

Errors in the local function, however, result in qualitatively different behaviour from asynchronous updates. A synchronous update with local function errors still maintains some amount of structure correlated with the errorless synchronous update. This effect is quite resilient. To eliminate it we need to increase error percentage to very high values, near random noise.

A negative answer can thus be given to the hypothesis raised in [5] of stochasticity acting the same way unregarding whether it is introduced via asynchronous update or via stochastic local functions. The two processes are qualitatively different, in spite of both being able to modify the behaviour of the artificial synchronous updating model. Synchronous CA are very sensitive to noise, whether in amplitude (local function) or in time (asynchrony), but in different ways.

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