Distributed information encoding and decoding using self-organized spatial patterns

Highlights

- Self-organized patterns can be used for secure information encoding
- A machine learning-mediated decoding method is proposed
- Encoding capability and security are tunable through modulating system properties

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In brief

This work demonstrates the feasibility of using self-organized patterns for secure information encoding. It advances the use of these systems by using machine learning-mediated decoding. It provides an empirical analysis that would allow for fast prototyping and implementation of dynamical system-based communication platforms, and a means to measure the convergence of biological system outputs.
Dynamical systems often generate distinct outputs according to different initial conditions, and one can infer the corresponding input configuration given an output. This property captures the essence of information encoding and decoding. Here, we demonstrate the use of self-organized patterns that generate high-dimensional outputs, combined with machine learning, to achieve distributed information encoding and decoding. Our approach exploits a critical property of many natural pattern-formation systems: in repeated realizations, each initial configuration generates similar but not identical output patterns due to randomness in the patterning process. However, for sufficiently small randomness, different groups of patterns that arise from different initial configurations can be distinguished from one another. Modulating the pattern-generation and machine learning model training can tune the tradeoff between encoding capacity and security. We further show that this strategy is scalable by implementing the encoding and decoding of all characters of the standard English keyboard.

SUMMARY

Dynamical systems often generate distinct outputs according to different initial conditions, and one can infer the corresponding input configuration given an output. This property captures the essence of information encoding and decoding. Here, we demonstrate the use of self-organized patterns that generate high-dimensional outputs, combined with machine learning, to achieve distributed information encoding and decoding. Our approach exploits a critical property of many natural pattern-formation systems: in repeated realizations, each initial configuration generates similar but not identical output patterns due to randomness in the patterning process. However, for sufficiently small randomness, different groups of patterns that arise from different initial configurations can be distinguished from one another. Modulating the pattern-generation and machine learning model training can tune the tradeoff between encoding capacity and security. We further show that this strategy is scalable by implementing the encoding and decoding of all characters of the standard English keyboard.

INTRODUCTION

Information encoding is a process of converting information, such as text and images, from its original representation to an output format following defined rules. Dynamical systems have this information encoding capability as they can generate specific outputs according to given inputs. Conversely, decoding can be achieved if one can infer the input corresponding to an output. Depending on the system, decoding could be obvious, challenging, or impossible. As self-organization systems can...
generate high-dimensional outputs, they are particularly useful for encoding rich information.

One example is to use cellular automaton (CA) that converts a grid of cells from a simple initial configuration into a self-organized sequence or spatial pattern according to a set of update rules. Wolfram proposed to use a chaotic rule to generate random sequences to encode information. Here, the encoding is deterministic—each initial configuration corresponds to a unique output pattern. Because of the chaotic nature of the rule, however, decoding the input from a given output pattern is computationally prohibitive without prior knowledge of the update rules. As such, the system in theory can serve as the foundation for digital cryptography. Unless the encoding and transmission are noise-free, the decoding is prone to errors even if the rules are known.

In contrast to these chaotic systems, many natural systems are convergent. That is, for the same or similar input configurations and environmental conditions, the final patterns share global similarity despite local variances. This property is sometimes referred to as “edge of chaos.” Examples are chemical reaction and cortical networks. Many biological patterning systems also fall into this category. Despite minute variances, coat patterns are largely determined by animal genomes and systems also fall into this category. Despite minute variances, colony morphology can serve as a crude signature to distinguish sizes under different growth conditions. Consequently, colony morphology can serve as a crude signature to distinguish environmental conditions and chemical cues, as well as the stage of infectious diseases. Despite these empirical examples, the potential and limitations of information encoding and decoding using biological self-organization remain unexplored. Here, we use these systems to establish distributed information encoding. Coupled with machine learning (ML)-mediated decoding, our system illustrates a scalable strategy for information encoding and decoding with quantifiable reliability and security (Figure 1A).

RESULTS

Criteria for choosing an encoding system
Any dynamical systems, including those generating self-organized patterns, can serve as the foundation for information encoding and decoding. However, to ensure secure encoding and reliable decoding, we reason that the system dynamics need to meet a set of heuristic criteria. First, the output patterns are sufficiently complex and diverse such that different initial configurations would generate distinguishable output patterns. Second, the pattern generation is subject to stochasticity but remains convergent. That is, in repeated pattern-generation processes, the same initial configuration with small noise or perturbations should generate output patterns that are approximately the same but differ in minor details. Importantly, the differences between patterns generated from replicated simulations should be smaller than those between patterns generated from different inputs. Third, while different groups of patterns arising from different initial conditions can be decoded by a properly constructed decoder, their differences are difficult to discern by the naked eye. We note that the degree by which different groups of patterns can be distinguished often has to be established empirically (if a reliable decoder can indeed be constructed).

As a proof of principle, we focus on a coarse-grained model of self-organized pattern formation (Figure 1, also see “mathematical modeling” in methods). The model was developed to simulate qualitative aspects of branching dynamics of Pseudomonas aeruginosa colony growth. In it, each simulation initiates from a predefined cell seeding configuration and the cells develop into a branching colony (Figure S1). The patterning process is influenced by two sources of random noise. One comes from the variability in the initial distribution of seeding cells; the other comes from the underlying growth kinetics. With appropriate choice of parameters (including noise levels), the patterning dynamics satisfy all criteria listed above.

In addition, another rationale for choosing this model is its simplicity and versatility. It can generate diverse patterns by adjusting model parameters and be solved in a computationally efficient manner (one simulation takes several minutes on a cluster compute node to solve). These features allow us to probe this platform’s security, reliability, and scalability (see “tradeoff among encoding capacity, security, and decoding reliability”).

Distributed encoding and decoding by spatial patterns
To demonstrate encoding, we represent a dictionary of 15 characters—letters A–E and numbers 0–9—using binary numbers 0001–1111 (Table S1). Each binary number then corresponds to a seeding configuration of cells in a braille-like array at time 0 (Figure 1B): a digit “1” corresponds to a spot seeding indicating the presence of cells, whereas a digit “0” indicates no cells. In each simulation, the colony grows from its initial configuration into a final pattern. As mentioned above, the simulation is subject to two noise sources: the variability in seeding and during growth. The former could originate from the marginal but unavoidable uneven cell seeding, and the latter could originate from the inherent heterogeneity of cell gene expression, motility, or small external perturbation. Therefore, repeated simulations from the same initial seeding configuration generate similar final patterns with minor differences, which collectively encode the identity of the input configuration (Figure 1C). We chose to encode in seeding configuration because of its simplicity, one may also choose to encode in other parameters influencing pattern formation.

We configure our simulations such that neither the mapping between the initial configurations and the colony patterns nor the difference between patterns corresponding to different inputs is obvious to the naked eye. To allow reliable decoding, we need a robust method to navigate through this visual complexity. A direct method is brute-force search, whereby all the possible patterns for each initial configuration are simulated to establish an empirical mapping between the input and the output. While apparently straightforward, this approach is computationally prohibitive and impractical because the training
patterns are 8-bit, 80 × 80 pixels grayscale images, resulting in up to $2^{8 \times 80 \times 80} \sim 10^{15412}$ possible patterns.

Alternatively, image classification using convolutional neural networks (CNNs) has been successful for numerous applications. Through observing sufficient examples, a CNN learns to cluster images by their categories. Here, we built a CNN to decode the colony patterns via multiclass classification (Figure S2, see “CNN training” in methods). During training, our CNN decoder takes pattern images (generated by repeated simulations) as input and updates its trainable parameters to classify patterns based on initial seeding configurations. With sufficient replicates in each class, our trained CNN was able to distinguish patterns corresponding to the 15 characters with high accuracy (Figure 1D). For instance, greater than 93% of decoding accuracy can be achieved by having 800 replicate patterns in the training set.

In an actual application of this encoding/decoding strategy, we assume the channel is public while the pattern generator, model parameters, training set, and the trained CNNs are private to the end users (Figure 1A). The recipient chooses the correct, trained CNN to decode a pattern according to the model parameters transmitted through another private channel (not shown in the figure) as prior knowledge.

**Tradeoff among encoding capacity, security, and decoding reliability**

In this platform, we aim to maximize the capability of the patterns to encode information, termed encoding capacity, and our platform’s robustness against data leakage to unauthorized parties, termed encoding security. We consider that a system has higher encoding capacity if it can encode more characters correctly with adequate data, while we consider our encoding scheme
The required data size increases exponentially as the desired accuracy increases. (D) Required training replicates per class as a function of dictionary size. The green, orange, and blue lines represent accuracy of 0.9, 0.5, and 0.1, respectively, to reach the same accuracy. Data are represented as mean ± standard deviation.

Decoding is trivial. Notably, when the patterns become more complicated (e.g., larger growth noise, smaller spacing, or larger dictionary), more data are required to reach the same accuracy. Data are represented as mean ± standard deviation. (D) Required training replicates per class as a function of dictionary size. The green, orange, and blue lines represent accuracy of 0.9, 0.5, and 0.1, respectively. The required data size increases exponentially as the desired accuracy increases. Being more secure when the attacker cannot build a successful decoder from the leaked data. For example, the accuracy of a separate decoder built on only 10 replicates per class drops to less than 20% (Figure 1D), which is only slightly better than random guessing (1/15). Note that the efficacy of our platform depends on the complexity of the generated patterns, our desired accuracy, and the amount of available training data.

We can tune our scheme’s performance by modulating parameters in the pattern-generation model. We constructed 16 simulated training datasets of diverse patterns by tuning these 2 parameters (see “mathematical modeling” in methods, Figure S3A). Based on their final appearance, we categorized our results into three subgroups: disk-like (a large disk occupying the entire growth domain), trivial (final pattern is identical to initial configuration), and branching. Disk-like colonies cannot be distinguished regardless of the training data size—thus, the input information was obscured and “lost” after growth (Figures S3B–S3D). Conversely, trivial patterns allow perfect but insecure decoding since the reverse mapping is obvious. Ultimately, the intricate branching patterns allow secure encoding and reliable decoding as demonstrated previously.

We can also modulate encoding capacity and security by tuning the noise during the patterning process. Without noise, one pattern per input is sufficient for perfect decoding as long as output patterns are distinguishable (Figure 2A). Too much noise would introduce too many variations in the replicate patterns generated from each input. If these intra-category variations (between replicate patterns) approach or exceed the inter-category differences (between sets of patterns corresponding to different inputs), the decoding accuracy would deteriorate significantly (Figure 2A). Depending on the magnitude of the noise, this loss in accuracy can be alleviated by increasing the number of replicate patterns per class. A similar tradeoff exists for other parameters as well, such as the spacing between spots in the initial configuration (Figure 2B). When spacing decreases, patterns grown from different configurations appear more alike and indistinguishable. Moreover, a larger dictionary with all else being equal would also reduce the decoding accuracy (Figure 2C). Again, expanding the number of replicate patterns per class can compensate for losses in accuracy, thus increasing the encoding capacity (Figure 2D). Similar tradeoff was also observed in patterns arrested from growth at different time points (see “temporal information encoding and decoding” in supplemental information).

In principle, the encoding–decoding scheme is applicable to any dynamical systems where the input-output mapping satisfies the criteria listed above. To illustrate this point, we chose an elementary CA model with weakly chaotic dynamics (see “encoding and decoding using elementary cellular automaton” in supplemental information). Given the set of rules, we chose the model parameters (including noise levels) such that the resulting dynamics can allow secure encoding and reliable decoding. Again, we encoded characters in binary numbers, which is then converted into 1D initial configuration in a similar manner as in 2D. Noise was imposed on the initial sequence, and the latter develops into a final sequence following the evolution rules (Figure S4). A feedforward neural network was trained to code the final sequence. As expected, higher complexity leads to worse decoding accuracy, and it can be remedied by increasing training data size (Figure S5).

**Enhancing encoding security and integrity**

To enhance security, we evaluated utilizing encryption to prevent unauthorized access during communication. A secret key is
implemented during encoding and successful decoding requires the correct key (Figure 3A). For pattern formation systems, the geometry of the patterning domain is a feasible choice of secret key as it can influence the patterning process and is easily tunable.\textsuperscript{24–26} In our system, the boundary suppresses bacteria colonization, and the strength of the impact decreases exponentially as the distance from every location in the colony to the boundary increases (see “encryption dataset generation” in methods). As such, the boundary exhibits a time-invariant, long-range, and weak inhibitive force on colony expansion. As this force is anisotropic due to asymmetric boundary geometry, the patterns are encrypted by the domain shape.

To test this notion, we generated patterns within different boundary shapes. For each shape, the resulting patterns would occupy the entire space. We removed the information of the boundary in the output by cropping out a smaller, circular area at the center of each pattern (Figure 3B). We found that only the decoders trained on the correct datasets can decode at high accuracy (Figure 3C), indicating that knowledge of the domain shape (i.e., the secret key) is critical for selecting the right CNN decoder to accurately decode. Note that since the x and y axes are datasets and models, respectively, we do not expect symmetric accuracies in the off-diagonal cases. Similarly, we evaluated the potential of other secret key choices, such as the seeding spacing (Figure S7) and patterning domain size.

We have also considered the threat to information integrity during communication, in which the attackers could alter the output patterns or replace them with fake ones, thus deceiving the intended information receiver. We demonstrated that the noise in the patterning dynamics could be used to ensure the integrity (see “authenticating patterns using noise signatures” in supplemental information). In brief, the noise leaves a unique signature for each correct pattern, which can be used to authenticate a received pattern.

Improving decoding performance by ensemble learning
All else being equal, the reliability of decoding can be improved by increasing the number of replicates per class when training the decoder. However, the degree of improvement diminishes for an increasing number of replicates (Figure 2C). For instance, for a dictionary of 63 characters, the decoding accuracy increases by \(~30\)-fold by increasing the number of replicates from 10 to 100; it only increases by \(~1.5\)-fold by increasing from 100 to 800. To more effectively use the available data, we adopted ensemble learning—a class of ML techniques.\textsuperscript{27–29} Staked generalization combines the knowledge learned by individual ML models (base model) for better prediction.\textsuperscript{30–33} We first trained multiple-base CNN decoders on a dataset with random initialization using the same protocol in the previous sections, then trained an ensemble decoder to combine their
prediction capabilities. The ensemble model was then used for final decoding (Figure 4A, see “ensemble learning and uncertainty estimation” in methods). For patterns generated with moderate growth noise, the prediction performance of the ensemble decoder excels that of the base models for up to 22% in accuracy (Figure 4B). Receiver operating characteristic (ROC) curves and confusion matrices also show significant improvement with ensemble model (Figures 4C, S8, and S9). As expected, the ensemble model generally outperforms the base models regardless of the training data size. Notable improvement in accuracy occurs when a moderate amount of data was available for training, whereas the improvement is less significant with adequate or scarce data. Data are represented as mean ± standard deviation.

Ensemble learning not only improves the decoding accuracy, but also sheds light on the prediction uncertainty. According to Lakshminarayanan et al., the base models trained with random initialization explore the entirely different modes of function space,\textsuperscript{34} thus their independent predictions can be used to estimate well-calibrated uncertainty.\textsuperscript{35} We adopted this notion and estimated decoding uncertainty through multiple metrics, including log likelihood, mean square error (MSE), top 1 and top 5 errors (see “ensemble learning and uncertainty estimation” in methods). A higher metric value indicates larger uncertainty or lower confidence. As expected, the uncertainty reduces as more training data are available (Figure S12). Having more base models does not necessarily reduce the uncertainty (Table S2).

**Distributed encoding of English in Emorfi**

Our distributed encoding-decoding platform is scalable for practical applications. We constructed 100 sets of patterns
to encode all printable ASCII characters including English letters in upper and lower cases, digits, punctuations, and whitespaces (Figure 5A; Appendix A). A 7-bit seeding array was used to create the training dataset, in which 100 of the unique initial configurations corresponded to the printable characters. Each of the initial configurations was then used to generate 1,000 patterns. We term this collection of patterns Emorfi, which represents a new, digitally generated coding scheme. When encoding text, each character is represented by one or multiple newly generated patterns with the same setup, and the patterns are then arranged to assemble a video (Figure 5B).

By doing so, all standard English text can be encoded in Emorfi and decoded back. For instance, we encoded the public speech “I have a dream” by Martin Luther King Jr. containing 8,869 individual characters as a video (Video S1). Accommodating majority voting, each character was represented by five different patterns, and 99.8% of the text can be correctly decoded (Appendix B). The same approach was also used to encode the poem “Auguries of Innocence” by William Blake as a video (Video S2), and 99.6% of the text was correctly decoded (Appendix C). In another example, using a 5-bit seeding array, we encoded the GFP protein sequence (238 amino acids) as a video (Video S3) and 100% was correctly decoded (Appendix D). In these real-world use cases, attackers with limited access to the training data cannot decode successfully. For example, having access to 10 patterns per class would only lead to decoding 1.3% of “Auguries of Innocence,” which is much lower than using a properly trained decoder.

**DISCUSSION**

Our encoding and decoding framework is applicable to diverse dynamics systems, as long as they have three key properties: (1) an approximately convergent mapping between initial input and output, (2) complex output signals, and (3) the output patterns are difficult to distinguish to the naked eye. While past studies have explored the possibility of using chaos to encode information and to provide security, unavoidable noise and error in numerical simulation (e.g., finite precision computing) or transmission (e.g., channel noise) can alter the output despite these systems being deterministic. In contrast, the convergent nature of our system ensures patterns that originate from the same initial configurations share common features (recognizable by a trained NN) despite small variances. Although noise is often considered undesirable in biological studies—such as masking ground truth or disrupting interactions between components—we take advantage of the variance in our system to ensure information security and to authenticate each pattern. These features distinguish our methods from other biology-based information encoding, encryption, or storage methods, such as DNA sequences, DNA origami, and arrays of microbial colony, which mostly rely on one-to-one mapping between the information to encode and the encoded format.

Our proposed criteria together contribute to the sufficient encoding capacity and tunable information security of our platform. Many systems satisfy these criteria. With appropriate parameterization and boundary conditions, many reaction-diffusion models exhibit considerable robustness in output patterns and
sensitivity to initial conditions.\textsuperscript{48,49} In addition to the example we demonstrated (Figures 2, S4, and S5), many CA models with asynchrony update rules also show convergence.\textsuperscript{50,51} Biological systems, such as biofilm morphology, butterfly wing scale pattern, and human fingerprint, have also evolved to exhibit common features but vary in detail. Their convergent nature results from the rich multiscale, multidimensional interactions between different system components, such as chemical reactions and diffusion, gene circuits, and cell-cell interactions.\textsuperscript{52–56} Our work can motivate future studies of utilizing other types of dynamical system outputs or implementing information encoding using controllable experimental patterns. Similar to the computational examples, the methods of selecting a suitable system, and balancing encoding capacity and security are also applicable for experimental systems.

However, our work does bring up a fundamental question: given a dynamical system with stochasticity, how do we know the dynamics are convergent enough while the output signals from different initial conditions are also distinguishable? We suspect that the question has to be addressed empirically for each specific system. In ours, each initial configuration generates an ensemble of output patterns following a distribution (visualized using t-SNE in Figure S13). It is difficult to determine this distribution by solely inspecting the pattern-generation model, even if parameters and noise magnitudes are known. However, whether each distribution corresponding to an input can be distinguished from another distribution arising from another input is established by ML. In essence, the trained CNN provides an empirical estimate on the extent by which the pattern generation is convergent. To this end, our work has implications for quantifying the convergence for a dynamical system by using ML.

As we have demonstrated with Emorfi, the pattern-based encoding-decoding platform is scalable and generalizable for information in various formats. We envision that the platform could be extended to other languages, such as alphabetic languages with different letters or diacritics (e.g., French, Hebrew) and logographic scripts consisting of thousands of characters (e.g., Chinese, Japanese). It could also be applicable for communicating science and protecting intellectual properties by incorporating Greek alphabet, mathematical symbols, nucleic acid bases, etc. In addition, one may increase the information density from one-character-per-pattern to multiple-characters-per-pattern by using more complex initial conditions, or improve the information efficiency by choosing more convergent systems. The encoding speed could be accelerated by using a faster pattern generator.

**EXPERIMENTAL PROCEDURES**

**Resource availability**

**Lead contact**

Further information and requests for resources should be directed to and will be fulfilled by the lead contact, Lingchong You (you@duke.edu).

**Materials availability**

This study did not utilize any materials aside from the code noted below and did not generate new unique reagents.

**Data and code availability**

The mathematical simulation and machine learning codes used in this study are available on Github: https://github.com/youlab/Information_encoding.

The platform for encoding text in the format of video is available at https://www.patternencoder.com/.

**Mathematical modeling**

The simple colony pattern-generation model accounts for several driving forces. In particular, it uses a kernel-based method to capture the high-level positive (expansion) and negative (inhibition) effects on patterning, regardless of the specific mechanism. The model is formulated as the following equations:

\[
N (x, y) = \int K (d) N (x, y) dx dy
\]

\[
K (d) = b 2 \left( \frac{2}{d^2} \right) - 2 \left( \frac{1}{d} \right)^{r + 3}.
\]

Here, \( N \) is the colonization of the bacteria over the growing medium, \( K \) is the growth kernel that is the addition of the expansion and (negative) repulsion kernels, \( b \) is the relative magnitude of expansion to repulsion, \( d_1 \) and \( d_2 \) are the distances that characterize half of the maximum effect of expansion and repulsion respectively, \( d_{xy} \) is the distance of a position \((x, y)\) to \((x', y')\). We used \( d_1 = 0.4, d_2 = 0.4, h_1 = 1,000, h_2 = 2,000,000, and b = 6.5 \) as the default parameter values unless otherwise mentioned. This parameter set generates complex branching patterns.

To adapt the published model for our study, we made several modifications. First, we implemented various seeding configuration, such as the spot seeding arrangement for encoding binary representations of characters (Figure 1B). The size and spacing of the spots were subjected to modulation. As the default setting, we used spacing = 15 and spot radius = 5. Second, we implemented white Gaussian noise with varying signal-to-noise (SNR) ratios to the growth kernel at each time step. The noise (\( \eta \)) mimics the heterogeneity and small perturbations in growth. Thus, the kernel equation becomes:

\[
K (d_{xy}) = b 2 \left( \frac{2}{d^2} \right) - 2 \left( \frac{1}{d} \right)^{r + 3} + \eta.
\]

We also implemented uneven cell seeding by assigning random intensities drawn from a truncated Gaussian distribution (mean = 0.5, deviation varies) to the pixels within the spot configurations. Both noise sources contribute to the variation in patterns given the same model parameters and initial configurations. As default, we used random seeding without growth noise.

The model was implemented in MATLAB 2017b and solved numerically. The simulation terminates once the colony stops growing. The simulation outputs an 8-bit, 451 x 451 pixel grayscale image. Except for the encryption experiments, the patterns were formed on a circular growth domain of a diameter of 451 pixels.

To generate different patterns, we modulated the relative acting distance (\( d_1', d_2' \)) and magnitude of colony expansion versus repulsion processes (\( b \)). Large relative distance and magnitude (i.e., higher colony expansion) result in thick branches, whereas small relative distance and magnitude (i.e., higher repulsion) result in thin, sparse branches. In extreme cases, these conditions can result in large disks or small circular colonies, respectively. When these two forces are intermediate and comparable, the system generates branching colonies.

**CNN training**

For CNN training, we numerically simulated datasets with equal numbers of replicates for each encoding character. For evaluation, test datasets made of 100 replicates per class were used. The pattern images were rescaled to 80 x 80 pixels before training or testing.

The CNN (Figure S3) and the ensemble model (Figure 4A) were implemented in Python 3, TensorFlow 1.15.2, and Keras 2.4.0. The CNN uses pattern images as inputs and outputs N features, where N is the dictionary size (i.e., number of characters in a dictionary). It consists of two convolutions, each followed by max pooling and rectified linear unit (ReLU). Then their output is passed onto two fully connected layers, followed by ReLU and softmax, respectively. Here, the softmax function turns it into categorical probabilities. For training, we used Glorot normal initializer, categorical cross entropy loss, and Adam optimization algorithm with learning rate subject to tuning. Keras early...
stopping function was also implemented to stop the training once the loss metric stopped improving. We carried out hyperparameter tuning (including learning rate, batch size, early stopping patience, and delta) to obtain the best performing models for analysis. The data generation and training were conducted on Duke Compute Cluster and Google Cloud Platform.

Encryption dataset generation
The geometry of the growth domain impacts the growth and pattern formation through exerting a negative effect on the colony in the vicinity of the boundary, such that the colony does not reach the edge. The plate influence is formulated as:

\[ I = -k z^{-4}. \]

The model is:

\[ N_{t+1}(x, y) = \int \int (K(d_{xy}) + I(d_{xy})) N_t(x, y) \, dx \, dy. \]

Here, \( d \) is the Euclidean distance of a position \((x, y)\) in the space to the boundary, and \( k = 1,000 \). \( R \) is the plate radius. For irregular domains, contour lines are drawn to determine \( \frac{d}{R} \), where \( R \) is the value of the highest contour line. \( \epsilon \) regulates the shape of the impact function. We deducted the influence from the colony after each discrete time step. For the purpose of encryption, we maximized the influence of the geometry by modulating \( \epsilon \), such that the negative plate impact reached as far as the center of the patterns. We used \( \epsilon = 1 \) for generating the encryption datasets, and 2,000 for any other dataset. When using the shape of the growing medium as the secret key, we simulated the colony patterns on circular-, diamond-, square-, and equilateral triangular-shaped domains. The area of each geometry was kept the same to compare the effect of the geometry. We removed the information of growth domain shape by cropping out a smaller, circular area at the center of each pattern, and only the processed pattern images were used for CNN training.

ENSEMBLE LEARNING AND UNCERTAINTY ESTIMATION
The training of ensemble model was carried out in two steps. First, we trained several base CNN models using the same protocol described in “CNN training.” Their probabilistic predictions on the training set were then linearly combined to constitute a new dataset. Next, we used the new dataset to train an ensemble model from scratch. We tested several ensemble model architectures, including logistic regression and feedforward neural networks with different numbers of hidden layers and nodes. In the ensemble model, we used ReLU activation function for the input and hidden layers and passed the model output into softmax function to turn it into categorical probabilities. For its training, we used Glorot uniform initializer, categorical cross entropy loss, and Adam optimization algorithm with learning rate = 0.0001. Keras early stopping was used to stop the training once the loss metric stopped improving. The patience was 5 and the minimum change was 0.0001. We evaluated the model performance on a balanced dataset of 100 datapoints per class through metrics such as precision, recall, ROC, AUC ROC using scikit-learn (0.22.2).

We evaluated the prediction uncertainty based on the output of base models. We used common metrics, such as log likelihood, MSE, and top 1 and top 5 errors, for estimating the uncertainty. Specifically, the log likelihood is

\[ \log \sum_{j=1}^N \frac{1}{M} \sum_{i=1}^M \log(p_{ij}) \]

and the MSE is

\[ \frac{1}{M} \sum_{i=1}^M \sum_{j=1}^N \left( y_{ij} - p_{ij} \right)^2. \]

For the \( i \)th data point, \( y_{ij} \) is the true label for class \( j \) (1 if the data point belongs to class \( j \), otherwise 0), \( p_{ij} \) is the predicted probabilities for class \( j \). \( M \) indicates the total number of data points, \( N \) indicates the dictionary size, and top 1 and top 5 indicate the fraction of data points whose correct label is not among their top 1 or 5 probable predictions, respectively.

SUPPLEMENTAL INFORMATION
Supplemental information can be found online at https://doi.org/10.1016/j.patter.2022.100590.

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AUTHOR CONTRIBUTIONS
J.L. and L.Y. conceived the research and wrote the manuscript. J.L. designed and performed mathematical modeling and machine learning training and carried out data analysis. N.L. assisted with modeling. L.Y., R.T., N.L., and Y.H. assisted with results interpretation. J.L., M.K., Y.B., N.M., S.T., and A.Z. developed the website. R.T., N.L., Y.H., S.W., and N.G. contributed to manuscript revisions.

DECLARATION OF INTERESTS
The authors declare no competing interests.

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