Brane-Bulk Interaction in Topological Theory.

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Abstract

In this letter we address the problem of inducing boundary degrees of freedom from a bulk theory whose action contains higher-derivative corrections. As a model example we consider a topological theory with an action that has only a “higher-derivative” term. By choosing specific coupling of the brane to the bulk we show that the boundary action contains gravity action along with some higher-derivative corrections. The co-dimension of the brane is more than one. In this sense the boundary is singular.
1 Introduction

The subject of inducing boundary degrees of freedom from the bulk has a very rich history. There exists a rather general mechanism for gauge degrees of freedom in a topological theory to become dynamical after introduction of a boundary (see e.g. [1], [2], [3]).

In this paper we consider a similar mechanism but with one unusual ingredient: the boundary of the manifold will be singular. It is “singular” in the sense that its co-dimension is more than one (e.g. marked four dimensional sub-manifold embedded in six dimensions). To treat the sub-manifold as a regular boundary of co-dimension one we “regularize it”. We blow it up to a cylinder and then work with the boundary of this cylinder. At the end we take the limit in which the cylinder shrinks back to the original singular sub-manifold.

The motivation to study singular boundaries comes from the problem of studying the dynamics of solitonic (brane) backgrounds. One often uses the approximation in which the theory of localized zero-modes is separated from the rest of the bulk modes. Instead of original theory one considers the theory on trivial (e.g. flat) background in the bulk plus the lower-dimensional theory in the world-volume of the brane as a theory of localized zero modes of the brane. The total action of the system is a sum of two actions corresponding to each theory. In such an approximation there is a question of how an interaction between the brane and the bulk should be taken into account. One possible regime is when both theories decouple from each other. However, sometimes it is impossible to neglect the interaction. E.g. if the dimension of the sub-manifold (brane’s world-volume) is even and some of zero modes are chiral, the world volume theory could suffer from gauge and gravitational anomalies. In this case the world-volume theory is inconsistent by itself (which actually means that it is the decoupling approximation which is inconsistent) and it is necessary to take into account an interaction between bulk and world-volume which makes the whole theory anomaly free. This is called “inflow mechanism” [4] of anomaly cancellation. There are several important examples of such cancellation in field and M theory [5], [6], [7], [8]. In general the bulk theory has non-zero gauge variation which is non-zero only on the sub-manifold and cancels the anomalous gauge variation of the world-volume theory.

In this paper we study some other example of such an interaction though we use the same setup. The bulk action is purely topological. Topological terms are not unusual for string theory. Some of M-theory corrections to 11-d SUGRA action have structure of lower dimensional topological terms embedded in eleven dimensions [9]. In general such terms can appear as a higher-derivative correction to some more complicated system. These corrections can have very different origin depending on the bulk theory. For simplicity we consider only topological term by itself to study a new interaction it can be responsible for.

The interaction with the brane is specified by choosing boundary conditions for the
bulk fields. Our goal will be to show that introduction of the boundary in this topological theory under some rather general boundary conditions generates the boundary term that contains lower dimensional gravity action.

This work generalizes the result of [10]. That paper gave the realization to the idea suggested by ’t Hooft of canceling the four dimensional cosmological constant by inducing the gravity from topological six dimensional theory. Here, without any relation to cosmological constant problem, we give more accurate mathematical formulation of the mechanism of inducing gravity on the brane from topological term in the bulk. We generalize the construction to the case of higher co-dimensions which can arise in other applications. This generalization is not quite trivial since in the case of co-dimension higher than two the angular form has more complicated dependence on the normal bundle gauge connection.

Besides, we make one more improvement of the construction used in [10]. In this work the boundary conditions were used which violated the covariance under rotations of normal bundle. In this work we resolve that difficulty by finding suitable boundary conditions that are covariant under normal bundle rotations. These boundary conditions have a natural physical interpretation.

One of the interesting implications of our result is in the context of brane-worlds. It can offer mechanism of localizing gravity. We will give a short discussion of that in the conclusion.

2 Embedding

Consider 4-d sub-manifold embedded into six dimension, $W^4 \subset M^6$. This embedding is “singular” in a sense that it can’t be treated as a boundary because the boundary of 6-d manifold is 5 dimensional. To “regularize “ such embedding it is convenient to introduce tabular neighborhood $W_\epsilon$ of 4-d sub-manifold [11]. Locally $W_\epsilon \times D^2_\epsilon$, where $D^2_\epsilon$ is a 2-d disk of radius $\epsilon$. That is, tabular neighborhood is a cylinder surrounding the brane. The theory in the bulk is defined on the six dimensional manifold with the tabular neighborhood cut out. Its boundary is the boundary of the $\partial W_\epsilon$ of the tabular neighborhood. Introduction of the boundary requires to impose some boundary conditions for the bulk theory. This will specify an interaction between bulk and brane theories.

3 The action

For simplicity the action we want to consider is purely topological. That is, it can be locally expressed as a total derivative. In terms of forms it is written as

$$E_6 = \int_{M^6} \varepsilon_{ABCDEF} \tilde{R}^{AB} \wedge \tilde{R}^{CD} \wedge \tilde{R}^{EF}$$  \hspace{1cm} (1)
Where $\tilde{R}$ is a curvature of Lorenz spin-connection. Thus $E_6$ represents an Euler class. Since the manifold $M^6$ has a boundary $\partial W$ such an action can be written as a surface term only. We will proceed with determining it.

On the brane the original $SO(1,5)$ Lorenz group is broken to $SO(1,3) \times SO(2)$. Let’s split the 6-d Lorenz connection $\tilde{\omega}^{AB}$ into corresponding parts:

$$\tilde{\omega}^{ab} = A^{ab} \quad \tilde{\omega}^{a\alpha} = \pi^{a\alpha} \quad \tilde{\omega}^{\alpha\beta} = \omega^{\alpha\beta}$$ (2)

where $a,b$ are the indexes in the $SO(2)$ part of the bundle and $\alpha, \beta$ in the $SO(1,3)$. In these terms the 6-dim curvature $\tilde{R}^{AB} = d\tilde{\omega}^{AB} + \tilde{\omega}^{AC} \wedge \tilde{\omega}^{CB}$ is :

$$\tilde{R}^{ab} = F^{ab}(A) - \pi^{a\alpha} \wedge \pi^{b\alpha}$$ (3)

$$\tilde{R}^{a\alpha} = D(A,\omega) \pi^{a\alpha}$$ (4)

$$\tilde{R}^{\alpha\beta} = R^{\alpha\beta}(\omega) - \pi^{\alpha a} \wedge \pi^{\beta a}$$ (5)

Where $F^{ab}(A)$ and $R^{\alpha\beta}(\omega)$ are curvatures that correspond to connections $A$ and $\omega$, $D(A,\omega) \pi^{a\alpha}$ is a covariant derivative with respect to both bundles $SO(2)$ and $SO(1,3)$

$$D(A,\omega) \pi^{a\alpha} = d\pi^{a\alpha} + \omega^{\alpha\beta} \wedge \pi^{a\beta} + A^a_b \wedge \pi^{ba}$$ (6)

Now we can express the Euler form as

$$E_6 = \int_{M^6} \varepsilon_{ab\alpha\beta\gamma\delta} \left[3R^{\alpha\beta} \wedge R^{\gamma\delta} \wedge A^{ab} - 6\pi^{a\alpha} \wedge D\pi^{b\beta} \wedge (2R^{\gamma\delta} - \pi^{\gamma c} \wedge \pi^{\delta c})\right] =$$

$$\oint_{\partial W} \varepsilon_{ab\alpha\beta\gamma\delta} \left[3R^{\alpha\beta} \wedge R^{\gamma\delta} \wedge A^{ab} - 6\pi^{a\alpha} \wedge D\pi^{b\beta} \wedge (2R^{\gamma\delta} - \pi^{\gamma c} \wedge \pi^{\delta c})\right]$$ (7)

$$\int_{\partial W} \varepsilon_{ab\alpha\beta\gamma\delta} \left[3R^{\alpha\beta} \wedge R^{\gamma\delta} \wedge A^{ab} - 6\pi^{a\alpha} \wedge D\pi^{b\beta} \wedge (2R^{\gamma\delta} - \pi^{\gamma c} \wedge \pi^{\delta c})\right]$$ (8)

4 Angular form

The boundary term we just obtained contains the integration over the whole boundary $W^4 \times S^1$. We would like to reduce it to integration over $W^4$ only by performing integration over $S^1$ separately. In doing so we define first the form integration of which over the transverse directions is equal to one. This is the volume form. Let us introduce the coordinates on a unit sphere $S^1$, $\hat{y}^a = y^a / y$. Then the volume form can be expressed as

$$\Psi_1 = \frac{1}{2\pi} \varepsilon_{ab} \hat{y}^a d\hat{y}^b$$ (9)

The boundary of the tabular neighborhood is isomorphic to the total space of the $SO(2)$ bundle, normal bundle. The base of the normal bundle is sub-manifold $W^4$ and $S^1$ are fibers. Since we want to perform an integration along the fiber we need to introduce covariant generalization of the volume form which will be globally defined. It requires an introduction of the connection on the normal bundle. The resulting form $e_1$ is called an angular form and has the following properties. Its restriction on the fibers is a volume form and

$$de_1 = \chi(F)$$ (10)
Where $\chi(F)$ is an Euler class of the normal bundle. Such angular forms can be constructed for the case of any even co-dimension. In the case of odd co-dimension the corresponding angular form is closed. We consider the cases of $e_1$. The explicit expression is

$$e_1 = \frac{1}{2\pi} \varepsilon_{ab} \hat{y}^a D\hat{y}^b$$

(11)

Where $D\hat{y}^a = d\hat{y}^a + \Theta^a_{\ b}\hat{y}^b$, $\Theta$ is a connection on the normal bundle.

5 Boundary conditions

Next we impose some boundary conditions on the connection $\tilde{\omega}_{ab}$, that is, specify the coupling of the bulk theory to the brane. We want to do it in such a way that the boundary action Eq. (8) splits into the product of two parts, the angular form and the rest that depends only on the brane coordinates. Then we can perform the integration and get the action defined on the brane only. First, it is required that:

$$\pi_{\alpha\alpha}^{\ a} |_{\partial W} = \hat{y}^\alpha e^\alpha$$

(12)

Next, we require $e^\alpha$ to depend on brane coordinates only and to satisfy no-torsion constrain with respect to connection $\omega^\alpha_{\ \beta}$, that is $D(\omega)e = 0$. Later we’ll see that $e^\alpha$ plays a role of induced veilbein on the brane. Under such conditions

$$\pi^{\alpha\alpha} D(\omega, A) \pi^{\beta\beta} = -\hat{y}^\alpha D(A)\hat{y}^b \wedge e^\alpha \wedge e^\beta$$

(13)

Second, part of the connection $A$ is taken as the connection on the normal bundle $\Theta$, the rest will yield the angular form $e_1$.

$$A^{ab} |_{\partial W} = a\hat{y}^a D(\Theta)\hat{y}^b + \Theta^{ab}$$

(14)

Where brackets stand for anti-symmetrization. With such a choice of the connection $\Theta$, $D(A)\hat{y}^a$ is

$$D(A)\hat{y}^a = D(\Theta)\hat{y}^a + a\hat{y}^a D(\Theta)\hat{y}^b \hat{y}^b = (1 - a) D(\Theta)\hat{y}^a$$

(15)

Since $\hat{y}^a \hat{y}^a = 1$ and $\hat{y}^a D\hat{y}^a = 0$.

Here we should make a short remark. It may look that chosen boundary conditions are very artificial. Nevertheless, one can show though that they are a consequence a very simple requirement of spherical symmetry. All modes which are spherically symmetric in the plane normal to the brane satisfy them.
6 Boundary action

Now we can calculate the boundary action Eq. (8) under chosen above boundary conditions.

\[ E_6 = \oint_{\partial W} \varepsilon_{ab\varepsilon_{\alpha\beta\gamma\delta}} 6\hat{y}^a D\hat{y}^b \]

\[ \wedge \left[ aR^{\alpha\beta} \wedge R^{\gamma\delta} + 2(1-a)R^{\alpha\beta} \wedge e^\gamma \wedge e^\delta - (1-a)e^\alpha \wedge e^\beta \wedge e^\gamma \wedge e^\delta \right] \]

\[ + \oint_{\partial W} \varepsilon_{ab\varepsilon_{\alpha\beta\gamma\delta}} 3\Theta^{ab} \wedge R^{\alpha\beta} \wedge R^{\gamma\delta} \tag{16} \]

Thus we succeeded in separating angular form and fields on the brane. The \( \Theta \)-dependent term doesn’t contribute since the integrand form \( \Theta RR \) doesn’t have any transverse components. We can perform integration of the rest to get

\[ E_6 = \oint_{\partial W} 12\pi e_1 \]

\[ \wedge \varepsilon_{ab\varepsilon_{\alpha\beta\gamma\delta}} \left[ aR^{\alpha\beta} \wedge R^{\gamma\delta} + 2(1-a)R^{\alpha\beta} \wedge e^\gamma \wedge e^\delta - (1-a)e^\alpha \wedge e^\beta \wedge e^\gamma \wedge e^\delta \right] \tag{17} \]

\[ = 12\varepsilon_{ab\varepsilon_{\alpha\beta\gamma\delta}} \int_{W^4} a \cdot R^{\alpha\beta} \wedge R^{\gamma\delta} + 2(1-a)R^{\alpha\beta} \wedge e^\gamma \wedge e^\delta - (1-a)e^\alpha \wedge e^\beta \wedge e^\gamma \wedge e^\delta \tag{18} \]

Thus the \( E_6 \) term is equivalent to the following action on the brane: a topological term, Hilbert-Einstein action and a cosmological term.

7 Generalization to co-dimension 4

The whole framework can be easily generalized to the cases of higher co-dimension. We consider the case of co-dimension four. Thus we have four dimensional sub-manifold \( W^4 \) embedded into eight dimensions. The action is taken to be eight dimensional Euler class

\[ E_8 = \int_{M^8} \varepsilon_{ABCDEFGH} \hat{R}^{AB} \wedge \hat{R}^{CD} \wedge \hat{R}^{EF} \wedge \hat{R}^{GH} \tag{19} \]

On the brane the original \( SO(1,7) \) Lorenz group is broken to \( SO(1,3) \times SO(4) \). Splitting of the 8-dimensional spin-connection \( \tilde{\omega} \) and the curvature tensor \( \hat{R} \) stays the same as in Eq. (2) and Eq. (3) correspondingly except that index \( a \) in \( A = (\alpha, a) \) is in \( SO(4) \) group now. Since \( E_8 \) is a closed form it can be written locally as a total derivative. In terms of the decomposition of \( SO(1,7) \) fields into \( SO(1,3) \times SO(4) \) fields it reads

\[ E_8 = \int_{M^8} \varepsilon_{abcd} \varepsilon_{\alpha\beta\gamma\delta} d \left[ 6R^{\alpha\beta} \wedge R^{\gamma\delta} \wedge CS(A)^{abcd} \right. \]

\[ + \pi^{\alpha\alpha} \wedge D\pi^{b\beta} \wedge \left( 16D\pi^{\gamma\gamma} \wedge D\pi^{d\delta} + 24(R^{\gamma\delta} \wedge \phi^{cd} + F^{cd} \psi^{\gamma\delta}) \right) \]

\[ - 48R^{\gamma\delta} \wedge F^{cd} - 16\phi^{cd} \wedge \psi^{\gamma\delta} \right] = \tag{20} \]
\[\begin{align*}
&= \int_{\partial W} \varepsilon_{abcd} \varepsilon_{\alpha\beta\gamma\delta} \left[ 6R^{\alpha\beta} \wedge R^{\gamma\delta} \wedge CS(A)^{abcd} \\
&+ \pi^{\alpha\alpha} \wedge D\pi^{\beta\beta} \wedge \left( 16D\pi^{\gamma\gamma} \wedge D\pi^{\delta\delta} + 24(R^{\gamma\delta} \wedge \phi^{cd} + F^{cd}\psi^{\gamma\delta}) \\
&- 48R^{\gamma\delta} \wedge F^{cd} - 16\phi^{cd} \wedge \psi^{\gamma\delta} \right) \right] 
\end{align*}\]  
(21)

Where \(CS(A)\) is Chern-Simons form of the \(SO(4)\) connection \(A^{ab}\)
\[CS(A)^{abcd} = dA^{ab} \wedge A^{cd} + \frac{2}{3} A^{ax} \wedge A^{xb} \wedge A^{cd}\]  
(22)

and \(\phi^{ab}\) and \(\psi^{\alpha\beta}\) are defined as
\[\phi^{ab} = \pi^a \wedge \pi^b, \quad \psi^{\alpha\beta} = \pi^\alpha \wedge \pi^\beta\]  
(23)

Before introducing the boundary conditions we want to discuss the angular form in this case. Its explicit expression is
\[e_3 = \frac{1}{2\pi^2} \varepsilon_{abcd} \left[ \frac{1}{2} \dot{y}^a D\dot{y}^b \wedge D\dot{y}^c \wedge D\dot{y}^d - \frac{1}{3} \dot{y}^a F(\Theta)^{bc} \wedge \dot{y}^d \right] \]  
(24)

\[de_3/2 = \chi(F) = \frac{1}{32\pi^2} \varepsilon_{abcd} F(\Theta)^{ab} \wedge F(\Theta)^{cd}\]  
(25)

Where the covariant derivative is taken with respect to the connection on the normal bundle \(\Theta\). The first term in \(e_3\) contains the volume form on \(SO(4)\), \(\Psi_4 = \varepsilon_{abcd} \dot{y}^a d\dot{y}^b \wedge d\dot{y}^c \wedge d\dot{y}^d\), the rest is required by the condition of Eq. (25).

The boundary conditions in this case are very similar to the case of lower co-dimension.
\[\pi^{\alpha\alpha} \mid_{\partial W} = \dot{y}^\alpha e^\alpha\]  
(26)

That implies the following for \(\phi^{ab}\) and \(\psi^{\alpha\beta}\)
\[\phi^{ab} \mid_{\partial W} = 0, \quad \psi^{\alpha\beta} \mid_{\partial W} = e^\alpha \wedge e^\beta\]  
(27)

Next, we require \(e^\alpha\) to depend on brane coordinates only and to satisfy no-torsion constrain with respect to connection \(\omega^{\alpha\beta}\), that is \(D(\omega)e = 0\). Under such conditions
\[\pi^{\alpha\alpha} D(\omega, A) \pi^{\beta\beta} = -\dot{y}^a D(A) \dot{y}^b \wedge e^\alpha \wedge e^\beta\]  
(28)

Second, part of the connection \(A\) is taken as the connection on the normal bundle \(\Theta\)
\[A^{ab} \mid_{\partial W} = a \dot{y}^a D(\Theta) \dot{y}^b + \Theta^{ab}\]  
(29)

The term \(\varepsilon_{abcd} \dot{y}^a D(\theta) \dot{y}^b F(A)^{cd}\) will give the angular form \(e_3\)
\[F(A)^{cd} \mid_{\partial W} = F(\Theta)^{cd} + a \dot{y}^d F(\Theta) d\dot{x}^d \dot{y}^x + a(2 - a) D(\Theta) \dot{y}^c \wedge D(\Theta) \dot{y}^d\]  
(30)

\[\varepsilon_{abcd} \dot{y}^a D(\theta) \dot{y}^b F(A)^{cd} = \varepsilon_{abcd} \dot{y}^a D(\theta) \dot{y}^b \wedge \left( F(\Theta)^{cd} + a(2 - a) D(\Theta) \dot{y}^c \wedge D(\Theta) \dot{y}^d \right)\]  
(31)

With such choice of the connection \(\Theta\), \(D(A) \dot{y}^a\) is
\[D(A) \dot{y}^a = (1 - a) D(\Theta) \dot{y}^a\]  
(32)
And thus the term $\pi D\pi D\pi D\pi$ gives

$$\pi^a D(\omega, A)\pi^{b\gamma} \wedge D\pi^{c\delta} \big|_{\partial W_i} = (1 - a)^3\hat{y}^a \wedge D(\Theta)\hat{y}^b \wedge D\hat{y}^c \wedge D\hat{y}^d \wedge e^\alpha \wedge e^\beta \wedge e^\gamma \wedge e^\delta$$

Now we are ready to compute boundary action Eq. (21).

$$E_8 = \oint_{\partial W_i} \epsilon_{\alpha\beta\gamma\delta} \left[(12a + 4a^3)e_3 \wedge R^{\alpha\beta} \wedge R^{\gamma\delta} + 16(1-a)^3e_3 \wedge e^\alpha \wedge e^\beta \wedge e^\gamma \wedge e^\delta - 24(1-a)e_3 \wedge e^\alpha \wedge e^\beta \wedge e^\gamma \wedge e^\delta - 48(1-a)e_3 \wedge R^{\alpha\beta} \wedge e^\gamma \wedge e^\delta \right] \Phi(\Theta, R, e, \hat{y})$$

Where $\Phi(\Theta, R, e, \hat{y})$ represents all terms that do not have enough components in transverse directions to contain the volume form. Thus $\oint \Phi = 0$. Now we can perform the integration to get

$$E_8 = \int_{W_i^4} \epsilon_{\alpha\beta\gamma\delta} \left[(12a + 4a^3)R^{\alpha\beta} \wedge R^{\gamma\delta} + (16(1-a)^3 - 24(1-a))e_3 \wedge e^\alpha \wedge e^\beta \wedge e^\gamma \wedge e^\delta + 48(1-a)R^{\alpha\beta} \wedge e^\gamma \wedge e^\delta \right]$$

We once again see that $E_8$ term with the set above boundary conditions yields a topological, a cosmological and Hilbert-Einstein terms on the brane.

8 Conclusions and discussion.

In this letter we addressed the problem of inducing boundary degrees of freedom from a bulk theory whose action contains higher-derivative corrections. As a model example we considered a topological theory with an action that has only a “higher-derivative” term. By choosing specific coupling of the brane to the bulk we showed that the boundary action contains gravity action along with some higher-derivative corrections. The co-dimension of the brane is more than one. In this sense the boundary was singular.

This result is refinement and generalization of the work done in [10]. First of all we considered the case of higher co-dimension. The non-trivial part of it lies in the difference between $e_1$ and $e_3$ forms, Eq. (11), Eq. (24). The normal bundle connection enters $e_1$ in a straightforward way, it just make the volume form covariant. On the other hand $e_3$ is the first non-trivial case when an angular form contains other terms besides a covariantized volume form.

There is another (more important) new result. The boundary conditions in [10] broke the covariance under rotations in the normal bundle. The analog of the condition in Eq. (12) was that only one component $\pi$ contained 4-dimensional veilbein, the other was set to zero. That corresponded to choosing one fixed normal vector out of all normal
vectors. In this work (as Eq. (12) shows) we keep the normal vector arbitrary and integrate over all of them in the action. In this way the covariance with respect to rotations in the normal bundle is preserved by the boundary conditions. The topological theory considered doesn’t have any metric in the bulk. It has only connection. One can check that if the metric were introduced, the boundary conditions set on the connection would simply require the bulk metric to be spherically symmetric.

Viewed as a new example (relative to inflow mechanism) of the brane-bulk interaction this work has other interesting implementations. It shows how Einstein action on the brane can arise dynamically from higher-derivative terms in the bulk (for a similar result see also [12]). The origin of this terms can be $\alpha'$ corrections of string theory. The inclusion of such the Einstein term changes the problem of localization of gravity in brane-world scenarios. The problem is usually addressed in the following framework. The brane is considered as a source to the gravity in the bulk. By solving equations of motion in the bulk one can find the background induced by the source. Then the gravity on the brane is described as a normalizable zero mode of the bulk fluctuations in this background. The other possibility is to consider the theory on the brane that includes the gravity [13]. The attractive feature of this scenario is that localization can be achieved even when the bulk theory is asymptotically flat. Besides, the short distance behavior of the gravitational potential is modified, it becomes lower-dimensional. Depending on the relative strength of bulk and brane gravity terms there is an interesting switching between low and high dimensional regimes (see also [14]). The mechanism we investigated here can provide an explanation to how the gravity on the brane can be induced from bulk $\alpha'$ corrections.

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