Exploring Fairness in District-based Multi-party Elections under different Voting Rules using Stochastic Simulations

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Abstract
Many democratic societies use district-based elections, where the region under consideration is geographically divided into districts and a representative is chosen for each district based on the preferences of the electors who reside there. These representatives belong to political parties, and the executive powers are acquired by that party which has a majority of the elected district representatives. In most systems, each elector can express preference for one candidate, though they may have a complete or partial ranking of the candidates/parties. We show that this can lead to situations where many electors are dissatisfied with the election results, which is not desirable in a democracy. The results may be biased towards the supporters of a particular party, and against others. Inspired by current literature on fairness of Machine Learning algorithms, we define measures of fairness to quantify the satisfaction of electors, irrespective of their political choices. We also consider alternative election policies using concepts of voting rules and rank aggregation, to enable voters to express their detailed preferences without making the electoral process cumbersome or opaque. We then evaluate these policies using the aforementioned fairness measures with the help of Monte Carlo simulations. Such simulations are obtained using a proposed stochastic model for election simulation, that takes into account community identities of electors and its role in influencing their residence and political preferences. We show that this model can simulate actual multi-party elections in India. Through extensive simulations, we find that allowing voters to provide 2 preferences reduces the disparity between supporters of different parties in terms of the election result.

1 Introduction
In a democracy, it is impossible to satisfy the preferences of all the people, all the time. Especially in multi-layered, multi-ethnic societies, the preferences of different people are often contradictory, and the democratic system often relies on numbers to decide which preferences to accept. However, this approach has a number of problems: various minority groups may remain permanently unrepresented or left out of the power structure, while political parties or other powerful players may game the system to maximize their benefits, even if it means that the preferences of a large number of electors are ignored. Such gaming may be through explicit steps like re-drawing the district boundaries (Gerrymandering) or through implicit steps like forging or breaking coalitions between different parties, setting up dummy candidates to wean away votes from opponent parties, and so on. To make the democratic process truly robust and egalitarian where everyone’s preferences are respected as far as possible, it is necessary to explore alternate ways of eliciting people’s preferences and aggregating them to choose representatives, without making the electoral process too cumbersome and opaque. It should be the aim of any democracy to give every person a direct or at least indirect access to the power structure and find a way for their voice to reach the decision makers, regardless of whether their opinions are finally accepted. This requires us to examine various electoral systems over a wide variety of social settings.

The aim of this work is to lay a framework to explore and evaluate various electoral settings, using concepts of Rank Aggregation and fairness in Machine Learning. We consider a number of voting rules and seat assignment policies, and define criteria to evaluate the fairness of these rules and policies to the electors. We also propose a stochastic model to simulate the results of elections under the different rules and policies discussed above, and evaluate them using the aforementioned fairness criteria. We show that the proposed model is sophisticated enough to capture various features of actual elections in multi-party electoral democracies like India. Finally, we use this model to generate a wide number of election scenarios (electors’ preferences) and evaluate electoral policies over them. We point out the relative strengths and weaknesses of different policies.

2 Related Work
There is a huge body of work related to elections and voting rules in the context of Computational Social Choice Theory. Some works have pointed out how social cleavages influence the choice of residential areas [Dawkins, 2004; Dawkins, 2007; Bharathi et al., 2018] in many multi-cultural
District-based electoral systems often produced results that are not compatible with the popular support for different parties. In certain situations, a party with a higher popular support may win fewer seats than a less popular party. This is known as Referendum Paradox, which occurs due to differences in geographical distributions of the supporters of the parties. This may happen naturally (as discussed in [Chen et al., 2013; Borodin et al., 2018]) or due to deliberate tampering of district boundaries, popularly called Gerrymandering [Lewenberg et al., 2017; Erdélyi et al., 2015]. Various studies have tried to quantify the representation of electoral biases of elections [Deford et al., 2020; Bachrach et al., 2016], and many mathematical models and algorithms have been proposed to alter the electoral system to make the results more competitive between parties and more representative of their popular support [Deford et al., 2020; Borodin et al., 2018; Erdélyi et al., 2015], usually by re-defining districts [Stoica et al., 2019; Lasisi, 2018] or by considering mobility of electors [Lev and Lewenberg, 2019]. Recently, there was a study [Mitra, 2020] to develop stochastic models to simulate outcomes of district-based elections, taking into account the aforementioned factors like social cleavage and geographical distributions.

Separately, the question of ranking and rank aggregation, along with notions of fairness and bias have also been studied thoroughly in Machine Learning over the past decade. This problem usually deals with the situation where each user has a ranked preference list over a set of items, and an overall ranking has to be produced which is unacceptable to the least number of users. A survey of initial rank aggregation methods is found in [Lin, 2010]. A major aspect of this problem is how to compare different sets of complete and incomplete rankings [Jagathula and Shah, 2008; Rajkumar et al., 2015; Negahban et al., 2012] or to evaluate any rank aggregation method by using criteria such as the Condorcet Criteria, or their statistical properties [Rajkumar and Agarwal, 2014]. Several rank aggregation algorithms are based on probabilistic models of permutations, such as the Plackett-Luce and Mallows models and their extensions [Lu and Boutilier, 2011], [Guiver and Snelson, 2009], [Qin et al., 2010], [Volkovs and Zemel, 2012]. These ideas of rank aggregation have also been used in the context of computational social choice for comparing different voting and scoring rules for problems like predicting election winners from small samples [Dey and Bhattacharyya, 2015], candidate unavailability [Boutilier et al., 2014], victory margin estimation [Xia, 2012] and choosing multiple winners by proportional representation [Lu and Boutilier, 2013].

A recent trend in Machine Learning is to quantify the fairness of different algorithms [Feldman et al., 2015], in terms of biases in their predictions that may be explicit or implicit. The question of fairness in ranking and rank aggregation algorithms has been investigated recently [Kuhlman and Rundensteiner, 2021], where the basic aim is to come up with an optimal ranking that will be sufficiently representative of different groups of candidates as identified by a protected attribute (eg. gender, race, nationality) [Zehlike et al., 2017; Geyik et al., 2019]. Statistical parity is considered as a desirable property to be maximized [Yang and Stoyanovich, 2017; Asudeh et al., 2019]. The idea of fairness has also been used to promote representation and diversity in electoral outcomes [Relia, 2021].

### 3 Motivating Examples

First of all, let us illustrate a few situations, when district-based elections can throw up unfair results. Consider a population of 25, divided into 5 districts. There are two parties (A and B). Each person casts their vote for any one of the two parties, and the district representative is chosen by the plurality voting rule, i.e. the party which received the higher number of votes wins the corresponding seat. The number of votes obtained by the parties in the 5 districts in three scenarios is shown in Table 1.

| District | Party A | Party B |
|----------|---------|---------|
| D1       | 10      | 15      |
| D2       | 15      | 10      |
| D3       | 20      | 5       |
| D4       | 25      | 0       |
| D5       | 0       | 25      |

Table 1: 2-party election over 5 districts D1,...,D5. In all cases, party A has 15 votes and party B has 10. In the upper case, Party A wins all 5 seats, in the middle case they win 3 seats, while in the lower case Party B wins 3 of the 5 seats. The voters whose choices are represented are shown in bold, those whose choices are indirectly represented through their favoured party are shown in italics.
where the governing body is dominated by the representatives of the party that has less supporters overall.

Based on the above analysis, we find that the second scenario is most fair, while the first one is most unfair. In the second and third cases, no one is left out of the system. In the second case 6 people are indirectly represented, and they are from both parties in equal number, while in the third case only 5 people are indirectly represented, but all of them are from party A. So we can say that the first scenario is strongly unfair to supporters of party B, and the third is weakly unfair to supporters of party A. However all of these scenarios can arise with occurrence probabilities depending on the dynamics of the political process in this hypothetical society. If scenarios 1 or 3 do arise, the setup (plurality voting rule) is unable to prevent them.

In another example, consider 6 electors and 3 candidates. In this case, each candidate has a ranked list of preferences over the candidates. These preferences are shown in Table 2. We see that candidate A is a divisive candidate (either top or bottom choice), but in plurality voting system, candidate A will be elected, which will satisfy voters V4, V5 and V6. In case of Borda count, B will be elected with score 7, satisfying only V3. So in this scenario, Borda count is fairer than plurality voting, despite its logistical disadvantage as each voter must specify a full ranking.

### 4 Problem Formulation

Consider a geographical region that is divided into S districts, and corresponding to each district there is a seat in the governing body like parliament. There are also R extra seats, which may be filled up as necessary. N number of electors are present in the region, partitioned into the districts as $n_1, n_2, \ldots, n_S$, where $\sum_{i=1}^{S} n_i = N$. Also, there are K political parties, and in each district there are K candidates sponsored by them (we ignore independent candidates). In the rest of this paper, we will use the terms party and candidate interchangeably.

In any district, suppose C represents the set of candidates. Each elector may have a ranked list of preference on these candidates. Denote by $R(C)$, the set of possible rankings (total order) over the candidates. Then a aggregation/scoring rule is a function $\bigcup_{n,m \in \mathbb{Z}^+, m \in C} R(C)^n \mapsto C^m$, i.e. it collects the rankings over the candidates from n electors and computes a set of m candidates, who are called the elected. Usually, $m = 1$. The party sponsoring the elected candidate of district s is denoted by $w(s)$, and the number of district seats won by candidates from party k is denoted as $V(k)$. Clearly, $\sum_{k=1}^{K} V(k) = S$.

### 4.1 Fairness Criteria

Based on the previously mentioned motivating examples, we define fairness criteria for elections. For this purpose, we first define the following:

1. **Represented**: An elector $i$ from district $s$ is said to be represented, if the candidate of their top choice $j = R_i(C_s)(1)$ is among the elected candidates from $s$, i.e. $j \in w(s)$

2. **Indirectly represented**: An elector $i$ from district $s$ is said to be indirectly represented if the candidate of their top choice $j$ is not among the elected candidates from $s$, but belongs to the same party as at least one elected candidate from another district.

3. **Unrepresented**: An elector $i$ is unrepresented if no elected candidate from any district belongs to the same party as the top choice of $i$.

4. **K-Dissatisfied**: An elector $i$ from district $s$ is dissatisfied if they are unrepresented, and the elected candidates from $s$ are not within their top K preferences.

Using these definitions, we define fairness criteria as follows:

1. **Net K-Dissatisfied ($ND(k)$)**: The total number of k-dissatisfied electors, across all districts. This number should be minimized.

2. **Net Unrepresented ($NUR$)**: The total number of unrepresented electors, across all districts. This number should be minimized, so that the opinions of as many people as possible have some access to the governing body.

3. **Partywise Indirect Representation Parity ($PIRP$)**: Let $IR(p)$ be the number of indirectly represented electors, whose candidates of first preference belong to party $p$. We define $PIRP$ score as the variance of $IR$ over all the parties, and this score should be minimized, so that the election results are not biased towards or against the supporters of any party.

4. **Net Represented ($NR$)**: The total number of represented electors. This number should be maximized, as it indicates the net satisfaction and empowerment.

5. **Partywise Representation Parity ($PRP$)**: Let $R(p)$ be the number of represented electors whose first preference is party $p$. We define $PRP$ score as the variance of $R$ over all the parties, and this score should be minimized, so that the election results are not biased towards or against the supporters of any party.

6. **Net Borda Count ($NBC$)**: The Borda count of an elector $i$ for an elected candidate $j$ indicates the number of opponent candidates who ranks below $j$ in the ranked preference list of $i$. A high Borda count indicates the satisfaction of the elector. The Net Local Borda Count is the sum of the Borda counts of all electors for the candidates elected from their respective districts. This score should be maximized to indicate satisfaction of the electors over the elected candidates.

|   | V1 | V2 | V3 | V4 | V5 | V6 |
|---|----|----|----|----|----|----|
| P1 | A  | A  | A  | B  | B  | C  |
| P2 | B  | B  | C  | C  | C  | B  |
| P3 | C  | C  | B  | A  | A  | A  |

Table 2: The ranked list of 5 voters V1-V6 over 3 candidates A,B,C.
7. **Partywise Borda Score (PBS):** Let $BC(p)$ indicate the Net Borda count of those electors whose first preference is party $p$. We define $PBS$ as the variance of $BC$ over all the parties. This score too be minimized, like the other partywise scores.

### 4.2 Voting and Aggregation Rules

As already discussed, a number of voting rules have been proposed in social choice theory. However, these require the voters to specify their preferences over the given set of candidates as a ranked list. This is often difficult for electors, especially if there is a long list of candidates, which happens in multi-party democracies. So in such systems, electors are usually given the option of specifying only one choice. Here we consider a set of voting rules that may be used in actual elections, without making the process too cumbersome for electors.

- **$k$-approval**: Each elector is allowed to specify $k$ choices. The standard rule is 1-approval. For modeling purposes, it may be assumed that each voter has total or partial ranking over the candidates, of which they choose the top $k$.

- **Weighted $k$-approval**: Each elector is allowed to specify $k$ choices, but each choice has a weight, specified by a vector $\{\alpha_1, \alpha_2, \ldots, \alpha_k\}$.

- **approval**: Each elector can either approve or disapprove each candidate. For modeling purposes, it may be assumed that the elector assigns a score to each candidate, and approves only those candidates whose score exceeds a cutoff.

- **negative vote**: Each elector can cast a positive vote for, and/or a negative vote against one candidate. This is a restricted version of the approval rule.

- **transferable vote**: Each elector can cast one vote and also mark another candidate as second choice, to whom the vote will be transferred, in case the preferred candidate is unable to be elected.

Once the votes are collected from all the electors, it is necessary to aggregate them using **scoring rules**. This may be done using various rank aggregation techniques, as already discussed. However, most of these techniques require each elector to provide a complete ranking over the candidates, which is not possible according to the voting rules mentioned above. So we adapt the scoring rules to suit the voting rules, as follows:

- **plurality**: The positive or approval votes in favour of each candidate from all the electors are added up and sorted in descending order. This is possible for the $k$-approval and approval voting rules. The winners can be elected in the following ways:
  - The candidate with maximum number of votes is elected
  - The top-$k$ candidates from the sorted list are elected, for a fixed $k$
  - All candidates having a minimum number of votes are elected

- **Weighted plurality**: In case of the weighted $k$-approval, all votes in favour of each candidate are added up according to their weights. The candidates are then sorted in the descending order of this weighted sum, and elected according to any of the 3 policies mentioned above.

- **Net plurality**: This is applicable in case approval and negative vote rules. Each positive vote or approval is considered to have a weight of +1, while each negative vote has a weight of -1. Weighted plurality scoring rule (mentioned above) is applied using these weights.

- **plurality with transfer**: This scoring rule is applied for transferable vote. The votes for each candidate are first counted, and the candidates ranked in descending order of votes polled. Then the candidate with least number of votes is eliminated, and the votes in their favor are transferred as specified by the corresponding electors. The remaining candidates are re-ordered according to the direct and transferred votes. The process continues till candidates can be elected according to any of the aforementioned ways.

Each (voting rule, scoring rule) combination will be referred to as **electoral policy**. Having defined these voting and scoring rules, we wish to evaluate them according to the fairness criteria defined above. It is likely that different scenarios regarding the choices of the electors, the most fair rule will be different. However, not all scenarios are equally likely. To make an average-case analysis, we use a Monte Carlo approach where scenarios will be sampled from an election simulation model, the results computed according to various voting and scoring rules, and their fairness scores measured accordingly.

### 5 Election Simulation Model

First of all, we need a model to simulate the results of an election in a multi-party, district-based setting. It has already been discussed how social identities and connections influence the districts of residence, political preference and final voting decision of individual electors. As a result, the result of district-based election, which is understood in terms of the number of seats won by the different parties, is sensitive to a number of factors beyond the overall popularity of the different parties or candidates. Below, we discuss a stochastic model for simulating such elections.

#### 5.1 Model Definition

We have $K$ parties competing over $S$ seats corresponding to districts, in which reside $N$ people. Assume that there are $C$ social communities, and $\theta_i$ denotes the proportion of people from community $i$. $\theta$ is sampled from a Stick-breaking prior. To every person $i$, we assign their community as $C(i) \sim \text{Categorical}(\theta)$. The people from the same community tend to stay together in the same district. Each person $i$ is assigned to district $S(i)$ by following a Chinese Restaurant Process [Pitman, 1995] with parameter $\alpha$, where each district is considered to be a table. Person $i$, resides in district $s$ with probability proportional to $\alpha_n(C(i)) = \alpha \sum_{j=1}^{C(i)} \mathbb{I}(C(j) = n)$.
Each community is associated with a prior over the political preferences of its members. For community $c$ and party $k$, we assign $\phi_{ck} \in \{-1, 0, 1\}$, indicating if the relation between them is bad (-1), neutral (0) or good (1). The values $\phi_{ck}$ are sampled uniformly at random, with the constraint that no party can have good relation with more than half of the total electorate. Also, a variance $\sigma_k$ is associated with each party which may be drawn from a Gamma distribution. Finally, for each elector $i$, their valuation of party $k$ is denoted by $\lambda_{ik} \sim \mathcal{N}(\phi_{ck}, \sigma_k)$ where $c = C(i)$. A party with high $\sigma$ is strongly liked by some and strongly disliked by others (indicating its “polarizing” nature). Clearly, this valuation $\lambda_{ik}$ can be either positive or negative.

Next, in an election each elector casts their votes on the basis of these valuations $\lambda_i$ according to the voting rules. In case of $k$-approval and weighted $k$-approval, each voter chooses the parties according to their top $k$ valuations. In case of approval, each elector approves the candidates from the parties with positive valuations, and disapproves the rest. In case of negative vote, the party with the least valuation gets the negative vote.

In actual elections, electors rarely vote according to their individual inclinations. They are also influenced their social network. We consider another version of the model (Local Influence), where the $i$-th elector combines their own valuations $\lambda_{ik}$ with the mean valuations of other electors in the same district, as $\tilde{\lambda}_{ik} = \kappa \lambda_{ik} + (1 - \kappa)(\lambda_{i,j} \mathbb{1}(S(j) = S(i)))$, where $\lambda_{i,j} = \frac{\sum_{j \neq i} \mathbb{1}(S(j) = S(i)) \lambda_{jk}}{\sum_{j \neq i} \mathbb{1}(S(j) = S(i))}$, and $\kappa \sim \text{Beta}(a, b)$. Influences on an elector need not be local only, it is possible to consider overall and community-wise influence, or the social network of each specific elector.

In a nutshell, the election model may be written as:

$$\theta \sim \text{SBP}(c)$$
$$\phi_{ck} \sim \text{Uniform}\{-1, 0, 1\} \forall c, k$$
$$C(i) \sim \text{Categorical}(\theta) \forall i \in \{1, N\}$$
$$S(i) \sim \text{CRP}(C(i), \alpha) \forall i \in \{1, N\}$$
$$\sigma_k \sim \text{Gamma}(\alpha) \forall k$$
$$\lambda_{ik} \sim \mathcal{N}(\phi_{ck}, \sigma_k) \forall c = C(i), k$$
$$\kappa \sim \text{Beta}(a, b)$$

$$\tilde{\lambda}_{ik} = \frac{\sum_{j \neq i} \mathbb{1}(S(j) = S(i)) \lambda_{jk}}{\sum_{j \neq i} \mathbb{1}(S(j) = S(i))}$$

$$\text{(1)}$$

### 5.2 Model Evaluation

It is important to validate the above model to show that it is capable of producing realistic results. For this purpose, we attempt to simulate actual elections in India - a multi-party democracy. The election results in India are available at https://eci.gov.in/statistical-report/statistical-reports/.

We carry out two experiments under different settings. In the first experiment, we consider Delhi National Capital Region- a small state assembly with 70 seats. Roughly 9 million people participate in the elections that are primarily between 3 major political parties. We consider $C = 5$ arbitrary communities, and the party-community relations are generated randomly as mentioned above. A large number of election scenarios $\{S(i), \{\lambda_{ik}\}_{k=1}^{K}\}_{i=1}^{N}$ are simulated from the model, from which we try to retrieve the results that are closest to each of the past 5 elections in the region. The simulated and actual election results are compared based on proportions of popular votes and number of seats won by the 3 parties. The results are shown below in Table 3. We find that the model can produce simulations that are reasonably close to the actual results. It must be remembered that in a district-based election, the mapping from popular vote distribution to seat distribution is many-to-many.

In another experiment, we consider two elections held in another Indian state of Odisha, which has 147 seats. Roughly

| Year | M1 | M2 | M3 | M1 | M2 | M3 |
|------|----|----|----|----|----|----|
| 2013 | 0.37 | 0.34 | 0.29 | 0.36 | 0.35 | 0.29 |
| 2014 | 0.48 | 0.35 | 0.17 | 0.44 | 0.39 | 0.17 |
| 2015 | 0.56 | 0.34 | 0.10 | 0.59 | 0.26 | 0.15 |
| 2019 | 0.58 | 0.23 | 0.19 | 0.59 | 0.26 | 0.15 |
| 2020 | 0.55 | 0.40 | 0.05 | 0.53 | 0.34 | 0.14 |

| Year | V1 | V2 | V3 | V1 | V2 | V3 |
|------|----|----|----|----|----|----|
| 2013 | 32 | 30 | 8  | 33 | 29 | 8  |
| 2014 | 60 | 10 | 0  | 56 | 14 | 0  |
| 2015 | 67 | 3  | 0  | 62 | 8  | 0  |
| 2019 | 65 | 5  | 0  | 62 | 8  | 0  |
| 2020 | 62 | 8  | 0  | 58 | 12 | 0  |
23 million people participated in another tri-partite contest. In this case, we had an estimate of the preferences for the 3 parties in 5 social communities on the basis of post-poll surveys\(^1\). The \(\theta\) and \(\phi\) matrices are accordingly specified before-hand. It turns out that the popular vote proportions and seat proportions, as simulated by the models, are reasonably close enough to the actual results, as shown in Table 4. This shows that our models can simulate realistic results. The results shown are in the individual-based version of the model, without considering local influence \((\lambda)\). In presence of \(\lambda\), we see that the seat proportion of different parties is closer to the popular vote proportion than the observations.

### 6 Simulation for Policy Evaluation

Having defined the simulation model, we now proceed with the simulation under a variety of settings. For all the following simulations, we consider a population of \(N = 1\text{ million}\), spread over \(S = 100\) districts.

#### 6.1 A Motivating Example

First of all, let us consider a motivating example where there are \(C = 3\) communities and \(K = 3\) parties. The distribution of the community sizes is \(\theta = \{0.5, 0.3, 0.2\}\). Consider party 1 which is majoritarian, i.e. it patronizes the first (largest) community while victimizing the third (smallest) community, i.e. \(\phi_1 = \{1, 0, -1\}\). The third party represents the interests of the smallest community but is hated by the largest community \(\phi_3 = \{-1, 0, 1\}\) while the centrist second party tries to balance everyone’s interest \(\phi_2 = \{0, 0, 0\}\). We also consider that the first and third parties have higher variance \(\sigma_1 = \sigma_3 = 2\) than the second party \(\sigma_2 = 1\).

We consider the results of such an election under the different voting and scoring rules as discussed in Section 4.2. As we saw, in most cases there is a one-to-one mapping between voting and scoring rules (eg. \(k\)-approval \(\rightarrow k\)-plurality). We consider \(k = 1\) and \(k = 2\) for \(k\)-approval, with \(\{1_1 = 1, 2_2 = 1\}\) and \(\{1_1 = 1, 2_2 = 0.5\}\) for 2-approval. We finally calculate the mean statistics of the different fairness measures (Section 4.1) over all the settings for each electoral policy, and use them to compare the different policies.

- **Net Represented**: We compare the percentage of the electorate who are directly represented under each electoral policy, for the 4 cases of \((C, K)\). The results for are shown in Table 6 by considering the mean over a hundred simulations. The results show that in most settings, the standard form of voting (1-approval) is the best, though the differences between the different policies are not large. However, the mean values hide some variance as shown in the box-plots of Fig. 2. We also calculate the number of simulation runs in which each policy returned the best results. We find that 1-approval is the best policy for both voting models, though under the local influence model it is closely followed by transferable vote and approval policies.

- **Net 2-Dissatisfied**: We compare the percentage of the
Table 6: Mean Percentage of the electorate who are directly represented under different electoral policies. Above: Individual voting model, Below: Local influence model. (app: approval, wtd: weighted, neg: negative, trns: transferable)

| (C,K) | 1-app | 2-app | wtd | app | neg | trns |
|-------|-------|-------|-----|-----|-----|------|
| 4, 3  | 39.24 | 38.04 | 39.05 | 38.89 | 39.22 | 39.16 |
| 6, 4  | 29.20 | 28.44 | 29.10 | 28.35 | 28.83 | 28.98 |
| 10, 4 | 28.44 | 27.54 | 28.12 | 27.59 | 27.91 | 28.14 |
| 12, 4 | 28.82 | 28.42 | 28.77 | 28.44 | 28.63 | 28.82 |
| 4, 3  | 43.89 | 42.86 | 43.67 | 43.61 | 43.76 | 43.83 |
| 6, 4  | 32.41 | 31.92 | 32.53 | 31.80 | 32.40 | 32.29 |
| 10, 4 | 31.76 | 31.01 | 31.61 | 31.20 | 31.67 | 31.45 |
| 12, 4 | 32.69 | 32.38 | 32.44 | 32.43 | 32.51 | 32.65 |

Table 7: Mean Percentage of the electorate who are 2-dissatisfied under different electoral policies. Above: Individual voting model, Below: Local influence model. (app: approval, wtd: weighted, neg: negative, trns: transferable)

| (C,K) | 1-app | 2-app | wtd | app | neg | trns |
|-------|-------|-------|-----|-----|-----|------|
| 4, 3  | 29.63 | 29.00 | 28.79 | 28.94 | 28.77 | 28.90 |
| 6, 4  | 45.84 | 45.14 | 45.18 | 45.29 | 45.60 | 45.46 |
| 10, 4 | 46.41 | 46.03 | 46.11 | 46.02 | 46.25 | 46.10 |
| 12, 4 | 46.32 | 45.29 | 45.77 | 45.32 | 45.84 | 45.79 |
| 4, 3  | 25.63 | 25.91 | 25.48 | 25.48 | 25.48 | 25.54 |
| 6, 4  | 41.87 | 41.39 | 41.34 | 41.57 | 41.67 | 41.73 |
| 10, 4 | 42.61 | 42.73 | 42.53 | 42.48 | 42.53 | 42.66 |
| 12, 4 | 41.57 | 41.03 | 41.50 | 40.99 | 41.56 | 41.40 |

Table 8: Mean Net Borda Count, i.e. mean position of each elected representative in their ranked preference list. Above: Individual voting model, Below: Local influence model. (app: approval, wtd: weighted, neg: negative, trns: transferable)

| (C,K) | 1-app | 2-app | wtd | app | neg | trns |
|-------|-------|-------|-----|-----|-----|------|
| 4, 3  | 1.05 | 1.05 | 1.05 | 1.05 | 1.05 | 1.05 |
| 6, 4  | 1.55 | 1.56 | 1.56 | 1.56 | 1.56 | 1.56 |
| 10, 4 | 1.55 | 1.55 | 1.55 | 1.55 | 1.55 | 1.55 |
| 12, 4 | 1.55 | 1.57 | 1.56 | 1.57 | 1.56 | 1.56 |
| 4, 3  | 1.09 | 1.08 | 1.10 | 1.09 | 1.09 | 1.09 |
| 6, 4  | 1.6 | 1.62 | 1.61 | 1.61 | 1.61 | 1.61 |
| 10, 4 | 1.59 | 1.59 | 1.58 | 1.59 | 1.59 | 1.58 |
| 12, 4 | 1.62 | 1.63 | 1.62 | 1.63 | 1.62 | 1.62 |

Electorate who are dissatisfied with the results, i.e. the winner from their district does not occur within their top 2 preferences. Again, this is studied under each electoral policy, for the 4 cases of \((C, K)\). The results are shown in Table 7, which show that on average (over hundred runs) 1-approval is quite poor, though none of the alternative policies is consistently on top. By calculating the best performer in each run, we find that 2-approval, i.e. double-choice voting with equal weightage is the best policy for both individual and local influence voting models. The variance in the performance of the policies over the runs are shown in the box-plot of Fig. 3. The least values are usually attained by 2-approval.

- **Net Borda Count** Next, we compare the mean Borda score of each elector with respect to the final result, i.e. the mean position of each elected representative in their ranked list of preferences. High Borda score indicates high level of satisfaction with the result. The results are shown in Table 8, which show very little difference between the policies. On considering the best policies in each run, it turns out that 1-approval and 2-approval are at par. The performances of the policies over the runs are shown in the box-plot of Fig.4, which shows that the best results are achieved by 2-approval and negative voting.

- **Partywise Representation Parity** Next, we compare the variance in the number of represented supporters of the different parties. The results are shown in Table 9, which show that in most cases the standard 1-approval creates a high variance among the represented supporters of different parties, i.e. tends to over-represent one party’s supporters at the cost of others. This is best mitigated by 2-approval. These results are corroborated on considering the best performing policy of each run.

- **Partywise Borda Score** Next, we compare the variance in the Net Borda score of the supporters of the different parties. The results are shown in Table 10. Just like Partywise Representation Parity, it turns out that the standard 1-approval creates disparity where the supporters of one party are more satisfied than the rest, while the situation is improved under most other policies, though most frequently under 2-approval. However, 1-approval works best in the presence of many communities.
We find that in general the standard 1-approval approach gives direct representation to the largest number of electors, but also increases disparity between the supporters of different parties, and the largest number of people are also dissatisfied under this policy. The 2-approval policy is generally the perfect complement of 1-approval, while the other policies like approval, negative vote and transferable vote fail to perform consistently well with respect to any of the considered measures. However, it should be noted that the variance between results of the different simulation runs is high (as indicated by Figs 2,3,4) and the difference between mean performance of different policies is generally low.

7 Conclusion

In this paper, we defined a theoretical framework to compare different electoral policies (voting and aggregation rules) over a number of measures that represent the satisfaction of electors from different political parties with the election outcome. These measures are inspired from fairness measures that are being used in recent Machine Learning literature. To explore the performance of the different policies, we proposed a stochastic model to simulate elections, where we bring in effects like community-based residence and political preference, and influence of neighbors on voting decision. We validated this model on actual election results in India. We found that the common 1-approval rule which allows each elector to specify one choice, often creates disparities between the supporters of different parties, and the best way to prevent this seems to be allowing two choices with equal weightage. However, the intra-policy variance across simulation runs are large, and inter-policy differences are often small, suggesting the need to expand the simulation range so that a wider range of settings and scenarios can be explored, along with posterior distributions over the results.
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