Anomaly-free deformations of spherical general relativity coupled to matter

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Abstract
A systematic approach is developed in order to obtain spherically symmetric midisuperspace models that accept holonomy modifications in the presence of matter fields with local degrees of freedom. In particular, starting from the most general Hamiltonian quadratic in radial derivatives of the variables, we obtain a family of effective modified constraints that satisfy Dirac’s deformation algebra, which encodes the covariance of general relativity, and show that (scale-dependent) holonomy corrections can be consistently implemented. In vacuum, the deformed anomaly-free Hamiltonian is explicitly written in terms of three free functions and we obtain a weak observable that can be interpreted as the mass of the model. Finally, as a particular example, we present a specific covariant polymeric model that remains regular for any value of the connection components. Some of its physical implications and the relation with previous studies in the literature are commented.

1 Introduction
In the context of effective theories, holonomy-modified cosmological models provide an accurate description when compared to the full dynamics predicted by the equations of loop quantum cosmology. The usual approach to obtain such effective descriptions is to modify the Hamiltonian by hand so that the expected effects from loop quantum gravity are included. In loop quantum cosmology, holonomy corrections are directly related to the spacetime discreteness and they are shown to solve the initial singularity. Hence, in this paper, we will focus on this kind of modifications in our study of effective spherically symmetric midisuperspaces.

Homogeneous configurations present an off-shell vanishing diffeomorphism constraint and one can include by hand a wide variety of modifications in the Hamiltonian, as the closure of the Poisson algebra is trivially guaranteed. The initial cosmological studies yielding a quantum bounce at early times for isotropic models were rapidly extended to anisotropic Kantowski-Sachs spaces. These models describe the homogeneous interior region of black holes [1–10] and showed interesting predictions. For instance, the implementation of holonomy corrections points towards a transition from black to white holes.

However, obtaining a consistent theory becomes rather challenging when non-homogeneous scenarios (particularly those with local degrees of freedom) are considered. In these cases, the diffeomorphism constraint is no longer vanishing and one finds that the quantized notion of spacetime collides with the continuous diffeomorphism symmetry of general relativity. To circumvent these difficulties, several approaches have been presented in the literature. For instance, the hybrid quantization program [11–15] considers a homogeneous background spacetime quantized via loop-quantum-gravity techniques, and incorporates inhomogeneities by means of a Fock quantization. Similar assumptions are made in the dressed-metric approach [16–18], providing a completely quantized theory to study cosmological perturbations. However, it remains unclear whether these frameworks respect general covariance [19].
Spherically symmetric configurations are of particular relevance as they would provide a first step towards the description of black holes and gravitational collapse. It was found that the Abelianization of constraints could lead to a consistent quantization even under the presence of a scalar matter field \([20,21]\). These studies have been extended to include a possible polymerization of the matter component \([22,24]\), so that holonomy effects can be explored even in the radial gauge with the polar foliation condition. However, recent discussions cast doubts on the covariance of these models \([25,26]\). In addition, one should also mention current works where, as in the improved dynamics scheme of loop quantum cosmology, scale-dependent holonomies are implemented in vacuum spherical symmetry \([27,29]\). Nevertheless, these models break covariance when they are weakly coupled to a matter field without further modifications.

In this context, we find the consistent constraint deformation formalism, which searches for effective theories that explicitly satisfy covariance through Dirac’s deformation algebra. This methodology has been successfully implemented in the study of cosmological perturbations \([30,36]\) and spherically symmetric models \([37,40]\). Particularly, it is argued that the effective line element corresponding to the holonomy-corrected constraints might present a signature change when approaching the classical singularity \([39]\). From this perspective, some interior region of the black hole would be Euclidean, and the usual notions of causality and time evolution would not apply. Unfortunately, strong no-go results point out that these models can not be extended to include matter fields with local degrees of freedom \([11]\), although it may be possible to evade such no-go results by using self-dual variables \([12,43]\). This could mean that the possibility of polymerizing the curvature components is a consequence of an excessive symmetry assumption. Indeed, homogeneous models in effective loop quantum cosmology accept any kind of polymerization for the connection components but even the simplest non-homogeneous cases (spherically symmetric vacuum) present limitations regarding polymerization due to the first-class nature of Dirac’s deformation algebra. These restrictions are stronger when one adds matter with local degrees of freedom to the system.

In this paper, we study a simple midisuperspace model and show that one can in fact consider holonomy corrections (even scale-dependent ones) in the presence of local degrees of freedom. In Sec. 2 we briefly review the classical canonical formulation of spherically symmetric spacetimes and introduce the dynamics through a scalar matter field. In Sec. 3 we will show that the requirement of anomaly freedom gives rise to severe consistency conditions, which are necessary for our theory to remain covariant. More precisely, we will compute Dirac’s algebra for a generic Hamiltonian quadratic in radial derivatives of the configuration variables along with the classical diffeomorphism constraint. Although straightforward, these computations are extremely lengthy and, in certain cases, we will omit some intermediate steps in order to show only the final result. In Sec. 4, we completely solve the vacuum anomaly equations and provide the most general anomaly-free Hamiltonian that is quadratic in radial derivatives of the triad components. Moreover, we find an observable of this system, which we identify with the mass of the model. In Sec. 5 we reconsider the case coupled to matter and obtain a specific polymeric constraint that shows a regular behavior for any value of the angular component of the connection. The previous notion of mass will be extended to this model and the deformation of the algebra will be given in terms of scalar functions. Finally, Sec. 6 summarizes the results and presents our conclusions.

2 Classical midisuperspace formulation

In the canonical formalism of general relativity the total Hamiltonian is a combination of first-class constraints. These constraints govern the dynamics and, at the same time, encode the covariance of the theory, which is not explicit in this approach. Rather the commutation relations between constraints, the so-called Dirac’s hypersurface algebra, ensure that the theory remains covariant.

Our analysis will be restricted to spherically symmetric models. In this case, there is only one non-homogeneous space direction and thus two components of the diffeomorphism constraint will vanish by choosing coordinates adapted to the symmetry. Instead of the four constraints from the full theory, we retain only two. These are the Hamiltonian \(\mathcal{H}^{\text{class}}\) and the radial component of the diffeomorphism constraints \(\mathcal{D}\) which, in a smeared form, will be written as,

\[
\mathcal{H}^{\text{class}}[N] := \int dr \, N \mathcal{H}^{\text{class}}, \quad \text{and} \quad \mathcal{D}[N^r] := \int dr \, N^r \mathcal{D}.
\]  

(1)
Here, $r$ denotes the non-homogeneous coordinate, $N$ is the lapse and $N^r$ is the only non-vanishing component of the shift. In addition, the dependence on the homogeneous coordinates has been integrated out and all global constants have been set to one. The geometric degrees of freedom will be described by the $U(1)$ invariant components of the spherically symmetric triad, $E^r$ and $E^\varphi$, combined with their conjugate momenta, $K_r$ and $K_\varphi$. The symplectic structure is given by,
\[
\{E^r(r_a), K_r(r_b)\} = \{E^\varphi(r_a), K_\varphi(r_b)\} = -\delta(r_a - r_b).
\] (2)

As we want to study dynamical scenarios, we need to couple some matter field to the model. Particularly, we will consider a scalar field $\phi$, with canonical Poisson brackets with its conjugate momentum $P_\phi,$
\[
\{\phi(r_a), P_\phi(r_b)\} = \delta(r_a - r_b).
\] (3)

In terms of the Ashtekar variables, the classical diffeomorphism constraint takes the form,
\[
\mathcal{D} = -E^r K_r + E^\varphi K_\varphi + \phi P_\phi.
\] (4)

whereas the classical Hamiltonian constraint reads,
\[
\mathcal{H}^{(\text{class})} = -\frac{E^\varphi}{2\sqrt{E^r}} - \frac{E^r K_r^2}{2\sqrt{E^\varphi}} - 2\sqrt{E^r}K_r K_\varphi + \frac{(E^r)^2}{8\sqrt{E^r E^\varphi}} - \frac{\sqrt{E^r}}{2E^\varphi}E^r E^\varphi + \frac{\sqrt{E^r}}{2E^\varphi}E^\varphi'' + \mathcal{H}_m,
\] (5)

where we have considered the positive orientation of the triad. The prime denotes the derivative with respect to $r$ and $\mathcal{H}_m = \mathcal{H}_m(E^r, E^\varphi, \phi, P_\phi)$ is the matter contribution which, for the minimally coupled scalar field under consideration, is given by:
\[
\mathcal{H}_m = \frac{P_\phi^2}{2\sqrt{E^r E^\varphi}} + \frac{(E^r)^{3/2}}{2E^\varphi}(\phi')^2 + \sqrt{E^r}E^\varphi V(\phi).
\] (6)

In terms of these variables, the spacetime line element can be written as follows,
\[
ds^2 = -N^2 dt^2 + \frac{(E^\varphi)^2}{E^r}(dr + N^r dt)^2 + E^r d\Omega^2,
\] (7)

and Dirac’s deformation algebra reads,
\[
\{D[N_1], D[N_2]\} = D[N_1 N_2' - N_1' N_2], \quad (8a)
\]
\[
\{D[N], H^{(\text{class})}[N]\} = H^{(\text{class})}[N^r N^r], \quad (8b)
\]
\[
\{H^{(\text{class})}[N_1], H^{(\text{class})}[N_2]\} = D \left[ E^r (E^\varphi)^{-2} (N_1 N_2' - N_1' N_2) \right]. \quad (8c)
\]

Note that the structure function in the last bracket is an element of the inverse metric.

### 3 Modified constraint algebra

The main goal of this paper is to construct the most general Hamiltonian constraint $\mathcal{H}$ that is quadratic in radial derivatives of the variables of our model $(E^r, E^\varphi, \phi)$ and that respects the covariance of the system. For such a purpose, we begin with an ansatz of the form,
\[
\mathcal{H} = a_0 + (E^r)^2 a_{rr} + (E^\varphi)^2 a_{\varphi\varphi} + (\phi')^2 a_{\phi\phi} + E^r E^\varphi a_{r\varphi} + E^r' \phi a_r + E^\varphi' \phi a_\varphi + E^{\varphi''} a_2,
\] (9)

where all quadratic combinations of the radial derivatives of $E^r, E^\varphi$ and $\phi$ are included multiplied by a free function $a_{ij} = a_{ij}(E^r, E^\varphi, \phi, K_r, K_\varphi, P_\phi)$, with $i, j = r, \varphi, \phi$, that depends on all the variables and momenta of the model. Following the form of the classical Hamiltonian $\mathcal{H}$, only one second-order derivative term has been included, that is, $E^{\varphi''}$, with a free coefficient $a_2 = a_2(E^r, E^\varphi, \phi, K_r, K_\varphi, P_\phi)$. Other second-order derivatives $(E^{\varphi''}, \phi'')$ are not expected to affect possible holonomy corrections and would further complicate the algebraic computations. On the other hand, the free function $a_0 = a_0(E^r, E^\varphi, \phi, K_r, K_\varphi, P_\phi)$ includes all
the terms that do not depend on radial derivatives. Finally, note that no radial derivatives of the curvature components are considered and, hence, we only expect to be able to describe pointwise holonomy corrections. Further modifications involving derivatives of \( K_r \) and \( K_\phi \) are beyond our assumptions.

In order to fix these free functions, two conditions will be imposed. First, the above generic expression \( [9] \) will be required to form an anomaly-free algebra along with the classical diffeomorphism constraint \( \mathcal{D} [5] \). Second, the classical limit for the constraint \( \mathcal{H} \) should be given by the classical Hamiltonian \( \mathcal{H}^{(\text{class})} [5] \). As we will see, these two requirements severely restrict the form of the free functions in \( \mathcal{H} \).

As a side note, since the density weight of the different objects will be of key relevance in the following discussion, let us recall that the Hamiltonian \( [9] \) must be a weight-one density. Moreover, from each conjugate couple, one of them is a scalar whereas the other one is a weight-one density. In fact, the diffeomorphism constraint \( [4] \) shows explicitly this weight distribution: those variables that appear primed in \( [4] \), that is \((E^r, K_\phi, \phi)\), are scalars while their conjugates \((K_r, E^\phi, P_\phi)\) are densities. To reach that conclusion, one only needs to identify the gauge transformation generated by the constraint \( D[N^r] \) on each canonical variable with its Lie derivative along the shift vector.

### 3.1 \( \{\mathcal{D}, \mathcal{H}\} \) bracket

The Poisson bracket between the classical diffeomorphism \( [4] \) and the modified Hamiltonian constraint \( [9] \) is quite lengthy and contains several anomalous terms. In order to classify and solve these anomalies systematically, we write the result in a unique way by removing all the derivatives from the shift vector through integration by parts. In this way, the result can be schematically written as,

\[
\{D[N^r], H[N]\} = \int dr \ N^r (N F_0 + N' F_1 + N'' F_2),
\]

where \( H[N] := \int dr N \mathcal{H} \) denotes the smeared form of \( \mathcal{H} \); and \( F_0, F_1 \) and \( F_2 \) are complicated expressions of the free functions \( a_{ij}, a_0 \) and \( a_2 \), the basic variables of our model and their radial derivatives. For instance, the term \( F_2 \) is given by

\[
F_2 = (a_2 + E^\phi a_{r\phi}) E'' + 2 E^\phi a_{\phi\phi} E'\phi' + E^\phi a_{\phi\phi} \phi'.
\]

Since the free functions \( a_0, a_{ij} \) and \( a_2 \) do not depend on radial derivatives, \( F_2 \) can not be written as a linear combination of the constraints \( (\mathcal{D} \text{ and } \mathcal{H}) \) and it thus must be regarded as anomalous. The requirement \( F_2 = 0 \) is then translated to three independent equations,

\[
a_2 + E^\phi a_{r\phi} = 0, \quad a_{\phi\phi} = 0, \quad a_{r\phi} = 0,
\]

which can readily be solved for \( a_2, a_{\phi\phi} \) and \( a_{r\phi} \). Once these relations are enforced, one can continue solving the remaining anomalies. For example, the coefficient of the term \( N^r N^r E^r \phi' \) on the right-hand side of \( [10] \) reads,

\[
E^\phi \frac{(a_{r\phi})^2}{a_2} \left( K_r \frac{\partial}{\partial K_r} + E^\phi \frac{\partial}{\partial E^\phi} + P_\phi \frac{\partial}{\partial P_\phi} - 1 \right) \frac{a_{r\phi}}{a_{r\phi}}.
\]

This anomaly will vanish if the function \( a_{r\phi} \) takes the form,

\[
a_{r\phi} = E^\phi a_{r\phi} b_{r\phi} (E^r, \phi, K_r, K_\phi, P_\phi, E^\phi),
\]

for a generic function \( b_{r\phi} \). Note that the weight-one densities \((E^\phi, K_r, P_\phi)\) appear as ratios in the arguments of \( b_{r\phi} \), ensuring that it behaves as a scalar function. We must remark that, in order to obtain this solution, \( a_2 \) has been assumed to be non-vanishing. Otherwise, there would not be any second-order derivative in the Hamiltonian \( [9] \) and the basis of our study would change. With this assumption, \( a_{r\phi} \) can neither be vanishing as can be read from relation \( [12] \).

In fact, it is possible to analytically solve all the anomalies that appear in \( [10] \), and the commented dependence on scalar quantities is dragged along for the rest of the free functions. More explicitly, the
modified Hamiltonian constraint that provides a weakly vanishing bracket with the classical diffeomorphism constraint takes the form,

$$\mathcal{H} = -\sqrt{E^r} \frac{g}{2} \left( E^\varphi f_0 + \frac{K_r}{E^\varphi} f_1 + \frac{P_\phi}{E^\varphi} h_0 + \frac{P^2_\phi}{E^\varphi^2} f_3 \right) + \frac{(E^\varphi')^2}{E^\varphi} \left( f_2 + \frac{K_r}{E^\varphi} h + \frac{P_\phi}{E^\varphi} h_1 + \frac{P^2_\phi}{E^\varphi^2} h_2 \right) \right)$$

where $b_0 = b_0(E^r, \phi, K_r/E^\varphi, K_\varphi, P_\varphi/E^\varphi)$ and $b_{ij} = b_{ij}(E^r, \phi, K_r/E^\varphi, K_\varphi, P_\varphi/E^\varphi)$, with $i, j = r, \phi$, are free functions of all possible scalar combinations of the model. In addition, we have defined $g(E^r, E^\varphi, \phi, K_r, K_\varphi, P_\varphi)$ as a generic global factor. The anomaly-free bracket between this constraint and the diffeomorphism constraint reads,

$$\{ D|N^r, H[N] \} = H[N^r N'] + H[g],$$

where the smearing $G$ is a function of $N$, $N^r$, $g$ and their derivatives. However, for the Hamiltonian to be a density of weight +1, the global factor $g$ must be a scalar function, and therefore all its arguments must behave as scalar quantities: $g = g(E^r, \phi, K_r/E^\varphi, K_\varphi, P_\varphi/E^\varphi)$. Regarding the Poisson structure, this condition imposes the vanishing of $G$. Therefore, the constraint (17), with any scalar global factor $g$, provides the classical result (15) for the bracket with the diffeomorphism constraint. Note again that the term inside brackets in (17) is a weight-one density and that the arguments of the different free functions are all the scalar objects that can be constructed within the model.

### 3.2 $\{H, H\}$ bracket

We continue our study by computing the Poisson bracket of the constraint (17) with itself. In order to analyze the result, we define the combination,

$$N := N_1 N'_2 - N'_1 N_2.$$  

It can be shown that, by performing a number of integration by parts, the commented Poisson bracket can be written as,

$$\{H[N_1], H[N_2]\} = \int dr NF,$$  

where $F$ is again a long expression but it does not depend on the smearing functions $N_1$ and $N_2$. Just as in the previous section, each coefficient in front of a given radial derivative must vanish on-shell by itself. In particular, several of these coefficients involve partial derivatives of the free functions $b_0$ and $b_{ij}$ with respect to $K_r$ and $P_\varphi$, and the requirement of anomaly freedom immediately fixes the dependence of the Hamiltonian on all the densities $(K_r, E^\varphi, P_\varphi)$, except in the global factor $g$. Hence, let us define the new modified constraint:

$$\mathcal{H} = -\sqrt{E^r} \frac{g}{2} \left( E^\varphi f_0 + \frac{K_r}{E^\varphi} f_1 + \frac{P_\phi}{E^\varphi} h_0 + \frac{P^2_\phi}{E^\varphi^2} f_3 \right) + \frac{(E^\varphi')^2}{E^\varphi} \left( f_2 + \frac{K_r}{E^\varphi} h + \frac{P_\phi}{E^\varphi} h_1 + \frac{P^2_\phi}{E^\varphi^2} h_2 \right) \right)$$

where $h = h(E^r, \phi, K_\varphi)$, $f_i = f_i(E^r, \phi, K_\varphi)$ and $h_i = h_i(E^r, \phi, K_\varphi)$, with $i = 0, 1, 2, 3, 4$, are free functions of the scalar objects $(E^r, \phi, K_\varphi)$. In addition, all the functions $h_i$ and $h$ must disappear in the classical limit, whereas the classical expression (15) for the Hamiltonian is obtained for the following values,

$$f_0 \to \frac{1}{E^r}(1 + K_\varphi^2), \quad f_1 \to 4K_\varphi, \quad f_2 \to -\frac{1}{4E^r}, \quad f_3 \to -\frac{1}{E^r}, \quad f_4 \to -E^r, \quad g \to 1.$$  

At this point, the fixed dependence of the above Hamiltonian on the densitized variables demands that each coefficient that multiplies a given power of the form $(K_\varphi)^m(E^\varphi)^n(P_\varphi)^i$ on the right-hand side of (20) must vanish by itself. In this way, it is possible to rewrite the requirement of anomaly freedom of the bracket (20) as the following system of differential equations:
\[ \frac{\partial f_0}{\partial K_\varphi} = 2 f_0 h - 2 f_1 f_2 - h_0 h_3 + \frac{\partial f_1}{\partial \varphi}, \quad (23a) \]
\[ \frac{\partial h_0}{\partial K_\varphi} = -2 f_1 h_1 + h_0 (2 h - 4) - 2 f_3 h_3, \quad (23g) \]
\[ \frac{\partial f_1}{\partial K_\varphi} = f_1 h_4 + 4 f_4 f_2, \quad (23b) \]
\[ \frac{\partial h_1}{\partial K_\varphi} = -h_1 h_4 - 2 h_2 h_3, \quad (23h) \]
\[ \frac{\partial f_2}{\partial K_\varphi} = -h_1 h_3 + \frac{\partial h}{\partial \varphi}, \quad (23c) \]
\[ \frac{\partial h_2}{\partial K_\varphi} = -2 h_2 h_4, \quad (23i) \]
\[ \frac{\partial f_4}{\partial K_\varphi} = 2 f_4 (h - h_4), \quad (23d) \]
\[ \frac{\partial h_3}{\partial K_\varphi} = h_3 (h - h_4) - 2 h_1 f_4 + \frac{\partial h}{\partial \varphi}, \quad (23j) \]
\[ \frac{\partial f_3}{\partial K_\varphi} = -2 f_1 h_2 + 2 f_3 (h - h_4), \quad (23e) \]
\[ \frac{\partial h_4}{\partial K_\varphi} = h_4 (h - h_4) - 4 h_2 f_4 + \frac{\partial h}{\partial \varphi}. \quad (23k) \]
\[ \frac{\partial f_1}{\partial \varphi} = f_1 h_3 + 2 h_0 f_4, \quad (23f) \]

This system is composed by eleven first-order partial differential equations for the eleven free functions \( f_1, h_i, \) and \( h \). Note in particular that the global factor \( g \) does not enter the anomaly resolution. Even if this system is linear in derivatives, the quadratic combinations of functions make it very difficult to provide a general solution. Nonetheless, since all the equations, except (23i), contain derivatives with respect to \( K_\varphi \), it is clear that the dependence on this variable will be severely restricted. In particular, in Sec. 5, we will solve this system for possible polymeric deformations of the Hamiltonian and, under some specific choices, we will be able to keep just one free function of \( K_\varphi \) in the Hamiltonian.

Whenever the above differential relations are satisfied, the constraint (21) forms a first-class algebra with the classical diffeomorphism constraint and it weakly commutes with itself: \( \{ H[N_1], H[N_2] \} \approx 0 \). The result of this last bracket is rather complicated. Nonetheless, it can be simplified by choosing a global factor that only depends on the scalar quantities, i.e. \( g = g(E^\varphi, \phi, K_\varphi) \). In this case, the off-shell bracket reads,

\[ \{ H[N_1], H[N_2] \} = D \left[ \mathcal{N} \frac{E^\varphi}{E^{\varphi^2}} \frac{g^2}{4} \left( \frac{\partial f_1}{\partial K_\varphi} + \left( \frac{E^\varphi}{E^\varphi} \right)^2 \frac{\partial h}{\partial K_\varphi} \right) \right] - H \left[ \mathcal{N} \frac{E^\varphi}{E^{\varphi^2}} \sqrt{E^{\varphi^2}} \left( h g + \frac{1}{2} \frac{\partial g}{\partial K_\varphi} \right) \right], \quad (24) \]

where it is immediate to check that both smearing functions are of the appropriate weight. This result differs from the classical case \( \mathcal{H} \) by an extra \( H \) term on the right-hand side, which means that the constraint \( \mathcal{H} \) generates a combination of normal and tangential deformations on the hypersurfaces. In order to recover the canonical form of this bracket, so that the Hamiltonian constraint becomes the infinitesimal generator of normal transformations, one only needs to demand the vanishing of the expression inside the last round brackets in (21). This leads to choose a specific global factor \( g \) in terms of the function \( h \), that is, \( g = \exp(-2 \int h \, dK_\varphi) \) \( ^1 \) and further restricts the allowed dependence on \( K_\varphi \) of the Hamiltonian. Note that, in particular, this choice of global factor is consistent with the classical limit (22).

In summary, we conclude that the generator of infinitesimal normal transformations is given by the expression (21), with the global factor \( g = \exp(-2 \int h \, dK_\varphi) \). As long as the free functions \( f_1, h_i, \) and \( h \) obey the above consistency equations (24), the Poisson bracket of this constraint with itself will be given by,

\[ \{ H[N_1], H[N_2] \} = D \left[ \mathcal{N} \frac{E^\varphi}{E^{\varphi^2}} \frac{g^2}{4} \left( \frac{\partial f_1}{\partial K_\varphi} + \left( \frac{E^\varphi}{E^\varphi} \right)^2 \frac{\partial h}{\partial K_\varphi} \right) \right]. \quad (25) \]

## 4 Reduction to vacuum

In the absence of matter, the generic Hamiltonian (21) derived in the previous section reads,

\[ \mathcal{H}^{(\text{vacuum})} = -\sqrt{E^{\varphi^2}} \frac{g}{2} \left[ E^{\varphi^2} f_0 + K_r f_1 + \left( \frac{E^{\varphi^2}}{E^\varphi} \right)^2 \left( f_2 + \frac{K_r}{E^\varphi} h \right) + \frac{E^{\varphi^2} E^\varphi}{E^{\varphi^2}} - \frac{E^{\varphi^2}}{E^\varphi} \right], \quad (26) \]

\(^1\)In principle one could take \( g = \tilde{g}(E^\varphi, \phi) \exp(-2 \int h \, dK_\varphi) \) and leave a free global factor \( \tilde{g} \) that depends on \( E^\varphi \) and \( \phi \). Nonetheless, for simplicity we will fix \( \tilde{g}(E^\varphi, \phi) = 1 \).
where now the free functions \( f_1 \) and \( h \) depend only on \( E^r \) and \( K_\varphi \), and the global factor is still given by the relation \( g = \exp \left[ -2 \int \! h dK_\varphi \right] \). These functions are not completely independent since they must satisfy the following two conditions:

\[
\frac{\partial f_0}{\partial K_\varphi} = 2f_0 h - 2f_1 f_2 + \frac{\partial f_1}{\partial E^r}, \quad (27a) \quad \frac{\partial f_2}{\partial K_\varphi} = \frac{\partial h}{\partial E^r}. \quad (27b)
\]

In contrast to the case coupled to matter presented in the previous section, in vacuum it is possible to explicitly solve these equations. The general solution can be written in terms of the global factor \( g(E^r, K_\varphi) \) and two additional integration functions \( f = f(E^r, K_\varphi) \) and \( v = v(E^r) \), as follows,

\[
f_0 = \frac{2}{g} \left( \frac{\partial f^2}{\partial E^r} + \left(1 + f^2\right) \frac{\partial \ln v}{\partial E^r} \right), \quad (28a) \quad f_2 = -\frac{1}{2} \frac{\partial \ln (gv)}{\partial E^r}, \quad (28c) \quad f_1 = \frac{2}{g} \frac{\partial f^2}{\partial K_\varphi}, \quad (28b) \quad h = -\frac{1}{2} \frac{\partial \ln g}{\partial E^r}, \quad (28d)
\]

where (28d) comes directly from the definition of the global factor. In this way, the most general anomaly-free vacuum Hamiltonian with quadratic dependence on first-order radial derivatives takes the form,

\[
\mathcal{H} = -\sqrt{E^r} \left[ E^r \left(1 + f^2 \frac{\partial v}{\partial E^r} + \frac{\partial f^2}{\partial E^r} \right) + K_\varphi \frac{\partial f^2}{\partial K_\varphi} - \frac{g}{2E^r} \frac{E^{\prime r}}{E^r} + \frac{g}{2E^r} \frac{E^{\prime r}E^r}{E^r} - \left( \frac{E^{\prime r}}{2E^r} \right)^2 \frac{E^r}{v} \frac{\partial (gv)}{\partial E^r} + K_\varphi \frac{\partial q}{\partial K_\varphi} \right], \quad (29)
\]

for any \( f(E^r, K_\varphi) \), \( g(E^r, K_\varphi) \) and \( v(E^r) \) arbitrary functions. The classical constraint is directly obtained for the values \( g = 1 \), \( f = K_\varphi \) and \( v = \sqrt{E^r} \). This Hamiltonian constraint generalizes previous models in the literature. For instance, the usual scale-independent polymerization \[37\] can be recovered for \( g = 1 \), \( v = \sqrt{E^r} \) and \( f = \lambda^{-1} \sin(\lambda K_\varphi) \); while the scale-dependent holonomy corrections in \[28\] correspond to the choice \( g = 1 \), \( v = \sqrt{E^r} \) and \( f = \sqrt{E^r/\Delta} \sin(\sqrt{\Delta/E^r} K_\varphi) \) along with the partial gauge fixing \( K_\varphi = E^r K_\varphi/(E^r)^{\prime} \) and \( E^r = r^2 \).

In addition, it is a straightforward computation to check that the following expression,

\[
m = \frac{v}{2} \left(1 + f^2 - g \left( \frac{E^{\prime r}}{2E^r} \right)^2 \right), \quad (30)
\]

commutes on-shell with the total Hamiltonian. That is, \( \hat{m} = \{m, H[N] + D[N^r]\} \approx 0 \), and thus \( m \) is a weak Dirac observable of the system. Moreover, its classical limit takes the form,

\[
m \rightarrow \frac{\sqrt{E^r}}{2} \left(1 + K_\varphi^2 - \left( \frac{E^{\prime r}}{2E^r} \right)^2 \right), \quad (31)
\]

which is the expression of the Schwarzchild mass. In order to check this statement, one can compute the classical Hawking mass for a spherically symmetric configuration:

\[
M_H = \frac{\sqrt{E^r}}{2} \left(1 - \frac{1}{4E^r} \partial^\mu E^r \partial_\mu E^r \right) = \frac{\sqrt{E^r}}{2} \left(1 + K_\varphi^2 - \left( \frac{E^{\prime r}}{2E^r} \right)^2 \right), \quad (32)
\]

where in the last step the classical evolution equation for \( E^r \) has been used. Therefore, we have obtained an observable \[30\] that can be interpreted as the mass of the deformed vacuum system described by the Hamiltonian \[29\].

In particular, the expression \[30\] can be used to remove the radial derivative that appears on the right-hand side of the bracket between two Hamiltonian constraints and write it in terms of the mass and the three free functions \( f, g \) and \( v \) as follows:

\[
\{H[N_1], H[N_2]\} = D \left[ N \frac{E^r}{E^r} \frac{2}{g} \left( \frac{\partial f^2}{\partial K_\varphi} \left[ \frac{1}{g} \frac{\partial f^2}{\partial K_\varphi} \right] - \frac{1}{g} \frac{\partial^2 \ln g}{\partial K_\varphi^2} \left(1 + f^2 - \frac{2m}{v} \right) \right) \right]. \quad (33)
\]
In principle, this expression has been derived only for the vacuum case. But note that, whenever the model coupled to matter admits a vacuum limit, the specific form of the functions $f$, $g$ and $v$ could be read off from the corresponding vacuum Hamiltonian constraint and the mass would be the scalar quantity defined by the expression [33]. In this way, the deformed algebra could also be expressed in this last form, even under the presence of matter. Particularly, we will make use of this fact for the polymeric model that will be presented in the next section.

5 Polymeric deformations

In this section, we will focus on constructing a deformed Hamiltonian with corrections inspired by loop quantum gravity, in particular with the so-called holonomy modifications. In effective models for cosmological scenarios, these corrections are responsible for the quantum bounce that replaces the initial singularity. Holonomy corrections are usually implemented by a polymerization procedure, which lies in replacing connection components by periodic and bounded functions. Note that, in our setup, the polymerization of $K_r$ and $P_\phi$ is completely ruled out as the dependence of the modified Hamiltonian (21) on these momenta is completely fixed. Therefore, our goal will be to explore the possibility of considering free functions of $K_\phi$ in (21) that satisfy the consistency equations (23).

5.1 An effective Hamiltonian with holonomy corrections

We will try to construct the effective Hamiltonian in such a way that it remains as close as possible to its classical form. More precisely, we will choose the classical form for the correction functions related to the matter Hamiltonian, that is, $f_3 = -1/E r$ and $f_4 = -E r$ (though the global factor $g$ might still couple holonomy corrections to the matter variables). In addition, the dependence on the variable $\phi$ will be assumed to appear only in a classical potential term $V(\phi)$; that is, the function $f_0$ in (21) will be decomposed as $f_0(E r, K_\phi, \phi) := f_0(\phi) - 2V(\phi)/g(E r, K_\phi)$.

Under these assumptions, it is possible to obtain the general solution to the system (23):

$$f_0 = \frac{2}{g} \left( \frac{\partial f^2}{\partial E^r} + (1 + f^2) \frac{\partial \ln v}{\partial E^r} - V(\phi) \right),$$  \hspace{1cm} (34a)

$$f_1 = \frac{2}{g} \frac{\partial f^2}{\partial K_\phi},$$  \hspace{1cm} (34b)

$$f_2 = -\frac{1}{2} \frac{\partial \ln (gv)}{\partial E^r},$$  \hspace{1cm} (34c)

$$h = h_4 = -\frac{1}{2} \frac{\partial \ln g}{\partial K_\phi},$$  \hspace{1cm} (34d)

$$h_0 = h_1 = h_2 = h_3 = 0,$$  \hspace{1cm} (34e)

$$g = \left( \frac{\partial f}{\partial K_\phi} \right)^2 \left( 1 + \frac{w}{f^2} \right)^{-1},$$  \hspace{1cm} (34f)

in terms of the integration functions $f = f(E r, K_\phi)$, $v = v(E r)$ and $w = w(E r)$. Note that we have written this solution in a very similar form to the one found in Sec. 4 for vacuum. In particular, up to the potential term $V(\phi)$, the functions $f_0$, $f_1$, $f_2$ and $h$ take exactly the same form as in the vacuum case. The main difference is that, whereas in the vacuum model the global factor $g$ was completely fixed, in this case $g$ is given in terms of $f$ and $w$ by relation (34f). In essence, the freedom in the deformation functions has been reduced: a free function of two variables $g = g(E r, K_\phi)$ in the vacuum case has been replaced by a free function of just one variable $w = w(E r)$ in the model coupled to matter.

Classically, the free function $f$ is given by $f(E r, K_\phi) = K_\phi$. Therefore, in order to recover the classical Hamiltonian, this imposes the classical limits $v \to \sqrt{E r}$ and $w \to 0$ for the other two free functions. As we are looking for the polymerized Hamiltonian that is closest to its classical form, we fix these last two functions to their classical values.

In this way, gathering together all the above results, we obtain the deformed
Hamiltonian constraint,
\[
\mathcal{H} = -\frac{E^\varphi}{2\sqrt{E^\varphi}} \left( 1 + f^2 + 4E^\varphi f \frac{\partial f}{\partial E^\varphi} \right) - 2\sqrt{E^\varphi} K_r f \frac{\partial f}{\partial K_r} + \left( \frac{(E^\varphi')^2}{8\sqrt{E^\varphi} E^\varphi} - \frac{\sqrt{E^\varphi}}{2E^\varphi} E^\varphi E^\varphi' + \frac{\sqrt{E^\varphi}}{2E^\varphi} E^\varphi'' \right) \left( \frac{\partial f}{\partial K_r} \right)^2 \\
+ \left( \frac{P_\varphi^2}{2\sqrt{E^\varphi} E^\varphi} + \frac{E^{3/2}(\phi')^2}{2E^\varphi} \right) \left( \frac{\partial f}{\partial K_r} \right)^2 + \sqrt{E^\varphi} E^\varphi V(\phi) + \frac{\sqrt{E^\varphi}}{2E^\varphi} E^\varphi E^\varphi'(E^\varphi' K_r + \phi' P_\varphi) \frac{\partial f}{\partial K_r} \frac{\partial^2 f}{\partial K_r^2} \\
+ \frac{\sqrt{E^\varphi}}{2E^\varphi} (E^\varphi')^2 \frac{\partial f}{\partial K_r} \frac{\partial^2 f}{\partial K_r \partial K_r},
\]
which, for any function \( f = f(E^\varphi, K_\varphi) \), forms a first-class algebra along with the classical diffeomorphism constraint. The corresponding Poisson brackets will be given by (34), where one should replace the form (33) for the functions \( g, f_1 \) and \( h \). As already commented above, the particular choice \( f(E^\varphi, K_\varphi) = K_\varphi \) reproduces the classical Hamiltonian. Moreover, the free dependence of \( f \) on \( E^\varphi \) allows us to introduce a scale-dependence in the holonomy corrections. Nonetheless, as noted for similar models in previous studies [37], this freedom spoils the periodicity in \( K_\varphi \), at least for simple sinusoidal choices of the free function \( f \). For instance, if one considers \( f = \sin(\mu(E^\varphi) K_\varphi)/\mu(E^\varphi) \) the derivatives of this function with respect to \( E^\varphi \) that appear in the last expression would introduce a non-periodic dependence on \( K_\varphi \) in the Hamiltonian.

Therefore, in order to obtain a \( K_\varphi \)-periodic Hamiltonian constraint, let us restrict ourselves to the case with a scale-invariant function \( f = f(K_\varphi) \). Furthermore, as we look for bounded functions, we can set \( f = \sin(\lambda K_\varphi)/\lambda \), with a real constant \( \lambda \), which provides the classical limit for \( \lambda \to 0 \) and yields the polymeric constraint,
\[
\mathcal{H}^{(pol)} = -\frac{E^\varphi}{2\sqrt{E^\varphi}} \left( 1 + \sin^2(\lambda K_\varphi) / \lambda^2 \right) - \sqrt{E^\varphi} K_r \sin(2\lambda K_\varphi) / \lambda \left( 1 + \left( \frac{\lambda E^\varphi'}{2E^r} \right)^2 \right) \\
+ \left( \frac{(E^\varphi')^2}{8\sqrt{E^\varphi} E^\varphi} - \frac{\sqrt{E^\varphi}}{2E^\varphi} E^\varphi E^\varphi' + \frac{\sqrt{E^\varphi}}{2E^\varphi} E^\varphi'' + \frac{P_\varphi^2}{2\sqrt{E^\varphi} E^\varphi} + \frac{E^{3/2}(\phi')^2}{2E^\varphi} \right) \cos^2(\lambda K_\varphi) \\
+ \sqrt{E^\varphi} E^\varphi V(\phi) - \frac{\sqrt{E^\varphi}}{4E^\varphi} E^\varphi E^\varphi' \phi' P_\varphi \lambda \sin(2\lambda K_\varphi),
\]
which obeys the deformed Poisson bracket:
\[
\{H^{(pol)}[N_1], H^{(pol)}[N_2]\} = D \left[ \beta E^\varphi E^\varphi^{-2} (N_1 N_2' - N_1' N_2) \right],
\]
with \( H^{(pol)}[N] := \int \text{d}r \mathcal{H}^{(pol)} \) and the deformation function,
\[
\beta := \cos^2(\lambda K_\varphi) \left( 1 + \left( \frac{\lambda E^\varphi'}{2E^r} \right)^2 \right).
\]

The constraint (36) is regular for all possible values of \( K_\varphi \). Concerning dynamical properties, this Hamiltonian is periodic in \( \lambda K_\varphi \) with period \( \pi \). Hence, further dynamical studies can be restricted to the range \( \lambda K_\varphi \in [0, \pi) \). In this range, the value \( \lambda K_\varphi = \pi/2 \), where the deformation function \( \beta \) vanishes, corresponds to a symmetry point. The Hamiltonian is invariant under reflection of \( \lambda K_\varphi \) around that point \( \lambda K_\varphi \to \pi - \lambda K_\varphi \), and the change of sign of the other momenta \( K_r \to -K_r \) and \( P_\varphi \to -P_\varphi \), which reflects a time-symmetry of the model. Regarding the equations of motion, they are also invariant under the commented transformation as long as one also considers the inversion of time \( t \to -t \). As will be explained in the next section, the present model is closely related to the one presented in [23], and the same physical conclusions have been reached in [26]; the surface where the deformation function \( \beta \) vanishes can be understood as a time-reversal surface. However, in [26] it is argued that this model does not respect covariance since, \( K_\varphi \) not being a spacetime scalar, the surface \( \beta = 0 \) is not covariantly defined. Nonetheless, we will show that in the present model the deformation function (and, particularly, its zeros) can indeed be covariantly represented by making use of the mass function (30) defined above.
More precisely, following the derivation performed in Sec. 4, we can define the mass for this polymerized Hamiltonian as follows,

\[ m_{\text{pol}} := \sqrt{E^r} \left( 1 + \frac{\sin^2 (\lambda K_\varphi)}{\lambda^2} - \left( \frac{E^r}{2E^\varphi} \right)^2 \cos^2 (\lambda K_\varphi) \right). \] (39)

In vacuum, this will be a weak observable and thus it will be conserved through evolution, in contrast to the model coupled to matter where it is a dynamical quantity. In either case, the deformation of the structure function can be written in terms of this “polymerized mass”:

\[ \beta = 1 + \frac{2m_{\text{pol}}}{\sqrt{E^r}}. \] (40)

In this way, \( \beta \) is completely determined by scalar functions. Since, by the definition (38), the deformation function is non-negative, this last relation provides a lower bound for the radial component of the triad:

\[ \sqrt{E^r} \geq \frac{2\lambda^2 m_{\text{pol}}}{1 + \lambda^2}, \] (41)

where the saturation of this bound corresponds to the point where \( \beta = 0 \). In vacuum \( m_{\text{pol}} \) represents a generalized Schwarzschild mass, which should be positive, and thus this result ensures that the system can not reach the classical singularity at \( E^r = 0 \). When considering a matter field, \( m_{\text{pol}} \) will not be an observable and it will have a non-trivial evolution. Therefore, in order to check whether the system can reach the classical singularity, one should numerically solve the equations of motion and, in particular, study how \( m_{\text{pol}} \) scales as \( E^r \to 0 \).

5.2 Back to general relativity through a canonical transformation

The first terms in the Hamiltonian (36), that go with a sine function of the curvature component \( K_\varphi \), are the usual correction terms considered in different polymeric models in vacuum. However, these terms by themselves do not remain covariant under the presence of matter with local degrees of freedom. In order to ensure the covariance of the model, our derivation has led us to include two additional corrections: the cosine function of the curvature component that multiplies several terms in (36), as well as the coupling between radial derivatives of matter and geometric variables that appears in the last term of (36). In order to understand better these new contributions, we will relate our derivation to the recent proposal in [23], where a polymerized constraint is obtained by means of a canonical transformation of the classical phase-space variables.

Let us consider the following canonical transformation on the classical model, as the generalization of the one proposed in [23]:

\[ K_\varphi \to f(E^r, K_\varphi), \quad E^r \to E^\varphi \left( \frac{\partial f}{\partial K_\varphi} \right)^{-1}, \quad E^r \to E^r, \quad K_r \to K_r + E^\varphi \frac{\partial f}{\partial E^r} \left( \frac{\partial f}{\partial K_\varphi} \right)^{-1}. \] (42)

The transformed Hamiltonian \( \tilde{H} \) is straightforwardly obtained just by considering the above transformation in the classical constraint \( H^{(\text{class})} \) [30]. Concerning the Poisson algebra, the bracket between the transformed Hamiltonian \( \tilde{H} \) and diffeomorphism constraint \( D \) is unchanged but, due to the presence of phase-space variables on the right-hand side of (25), the bracket between two transformed Hamiltonian constraints acquires a deformation,

\[ \{ \tilde{H}[N_1], \tilde{H}[N_2] \} = D \left[ N \frac{E^r}{(E^\varphi)^2} \left( \frac{\partial f}{\partial K_\varphi} \right)^2 \right], \] (43)

where \( \tilde{H} \) stands for the smeared form of \( H \). In fact, since the classical Hamiltonian is linear in \( E^\varphi \) and \( K_r \), the new constraint \( \tilde{H} \) would be divergent in the zeros of the deformation function \( \partial f/\partial K_\varphi \). One could try to get rid of those potential infinities through a rescaling \( \tilde{H}' := \tilde{H} \partial f/\partial K_\varphi \), but the constraint \( \tilde{H}' \) would not be
the canonical generator of infinitesimal transformations since the bracket with itself would be schematically given by,

\[ \{ \tilde{H}', \tilde{H}' \} = \tilde{H}' + D, \]

\( \tilde{H}' \) being the smeared form of \( \tilde{H}' \). Nonetheless, it can be shown that the following combination with the diffeomorphism constraint,

\[ \mathcal{H} = \left( \tilde{H} - \frac{E^r}{2E^r} \frac{\partial^2 f}{\partial K^2} D \right) \frac{\partial f}{\partial K^2}, \]

produces the canonical result for the bracket \( \{ H, H \} = D \). In fact, this last Hamiltonian turns out to be exactly the same as the one we have obtained above \( \text{(35)} \). Therefore, the extra couplings \( K_r E^r \) and \( P_\phi \) that appear in that constraint can be understood as coming from the linear combination with the diffeomorphism constraint performed in the last expression.

Hence, we have provided a way to reach a polymerized constraint \( \text{(36)} \) which, outside the surface where the deformation function vanishes, can be matched to the one constructed in \( \text{(23)} \). Nonetheless, our constraint \( \text{(35)} \) is regular for all possible values of \( K_\phi \), and one avoids the divergences arising from the canonical transformation above, as well as potential ill-defined lapse and shift rescalings at the zeros of the deformation function.

Finally, let us point out that, since the Hamiltonian presented in \( \text{(29)} \) contains derivatives of the momenta \( (K_r, K_\phi, P_\phi) \), it is not included in the family of deformed Hamiltonians \( \text{(21)} \), which has been constructed by assuming the ansatz \( \text{(9)} \). That is why, in order to relate both approaches, it is necessary to consider the above linear combination with the diffeomorphism constraint.

6 Discussion

In this article, we have performed a comprehensive study of Dirac’s deformation algebra for a general constraint with quadratic dependence on radial derivatives. Starting from the ansatz \( \text{(9)} \), this approach has allowed us to systematically construct modified spherically symmetric models coupled to a scalar matter field. The main result of our study is encoded in the family of modified Hamiltonian constraints \( \text{(21)} \), along with the consistency equations \( \text{(23)} \). Any Hamiltonian of this form respects covariance inasmuch as it forms a first-class constraint system alongside with the classical diffeomorphism constraint. In particular, concerning the dependence on momenta \( (K_r, K_\phi, P_\phi) \), it can be seen that, demanding the closure of Dirac’s commutation relations, explicitly fixes the dependence of the Hamiltonian \( \text{(5)} \) on the momenta \( (K_r, P_\phi) \), whereas the consistency equations severely restrict the allowed dependence on \( K_\phi \).

Although the full consistency-equation system is highly coupled, we have been able to provide the general solution for the vacuum case. The resulting vacuum Hamiltonian \( \text{(29)} \) has two free functions, which depend on \( E^r \) and \( K_\phi \), and an additional freedom regarding only \( E^r \). Moreover, we have found a weak Dirac observable \( \text{(30)} \) that can be identified with the mass of the modified system, and its classical limit reproduces the usual Schwarzschild mass.

In the last section, under certain assumptions for some of the free functions coupled to the matter sector, we have solved the consistency equations for the case coupled to matter in order to obtain a modified Hamiltonian with some free dependence of the curvature component \( K_\phi \), which could be understood as a covariant polymerization of the system. This solution has led us to the constraint \( \text{(35)} \), which is parametrized by one free function of \( E^r \) and \( K_\phi \). Therefore, this analysis opens the possibility to study (scale-dependent) holonomy corrections for this model coupled to matter with local degrees of freedom. Nonetheless, let us stress that probably the obtained solution is not unique, and that the complexity of the system \( \text{(23)} \) might leave room for other different polymerized constraints.

Finally, by choosing a specific form for the mentioned function, we have obtained the polymeric constraint \( \text{(36)} \), which is regular for all values of \( K_\phi \) and forms a first-class algebra with the brackets \( \text{(37)} \) corrected with the deformation function \( \beta \). This result can be seen as a family of effective midisuperspaces depending on a free real parameter \( \lambda \), with the specific case \( \lambda = 0 \) corresponding to general relativity. In fact, as has

\[ f = \sin(\lambda K_\phi) \]

and there is an additional transformation for the scalar field \( (\phi, P_\phi) \) which we did not consider here.

4In that reference, \( f = \sin(\lambda K_\phi) \) and there is an additional transformation for the scalar field \( (\phi, P_\phi) \) which we did not consider here.
been shown in the last subsection, these models can be mapped to general relativity through a (divergent) canonical transformation, similar to the one considered in [23], and a rescaling of the lapse and shift. In consequence, the effective midisuperspaces are locally equivalent to the classical theory [23,26] except in the surfaces of vanishing deformation function $\beta$, which can be interpreted as time-reversal surfaces. An issue raised in [26] is that the definition of these surfaces breaks the covariance of the system as they are defined in terms of the values of $K_\phi$, which is not a spacetime scalar. Nonetheless, in our model we have been able to write the deformation function in terms of the mass of the system (39) and the radial component of the triad $E^r$. This has shown that, at least in vacuum, the commented surfaces correspond to a minimum value of $E^r$ and thus the classical singularity is unreachable by the modified system.

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