Electroweak Baryogenesis, CDM and Anomaly-free Supersymmetric $U(1)'$ Models

Junhai Kang, Paul Langacker$^a$, Tianjun Li$^{b,c}$ and Tao Liu$^d$

$^a$ School of Natural Science, Institute for Advanced Study, Einstein Drive, Princeton, NJ 08540, USA
$^b$ Key Laboratory of Frontiers in Theoretical Physics, Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100190, P. R. China
$^c$ George P. and Cynthia W. Mitchell Institute for Fundamental Physics, Texas A&M University, College Station, TX 77843, USA
$^d$ Enrico Fermi Institute, University of Chicago, Chicago, IL 60637, USA

Abstract

We construct two anomaly-free supersymmetric $U(1)'$ models with a secluded $U(1)'$-breaking sector. For the one with $E_6$ embedding we show that there exists a strong enough first order electroweak phase transition for electroweak baryogenesis (EWBG) because of the large soft trilinear terms in the Higgs potential. Unlike the Minimal Supersymmetric Standard Model (MSSM), the stop masses can be very heavy. We then discuss possible CP violation in the Higgs sector, which can be both spontaneous and explicit, even at tree level. The spontaneous violation provides a direct source for baryogenesis, while its magnitude is mediated by an explicit phase from the secluded sector. These new CP sources do not introduce significant new contributions to electric dipole moments. EWBG in the thin wall ($\tau$ leptons) and thick wall regimes (top squarks, charginos and top quarks) are systematically discussed. We find that the $CP$-violating stop and chargino currents are very different from those obtained in the MSSM. Due to the space-dependence of the relevant $CP$ phases, they do not require a variation of $\tan \beta$ in the bubble wall to have a non-trivial structure at the lowest order of Higgs insertion. In addition to $\tau$ leptons, top squarks and charginos, we find that top quarks can also play a significant role. Numerical results show that the baryon asymmetry is large enough to explain the cosmological observation today. We illustrate that EWBG and neutralino cold dark matter can be accommodated in the same framework, i.e., there exists parameter space where a strong enough first order EWPT, large $CP$ phase variations across the bubble wall, a reasonable baryon asymmetry as well as an acceptable neutralino dark matter relic density can be achieved simultaneously.
1 Introduction

The baryon asymmetry of the Universe (BAU) has been precisely measured by WMAP. Combining their five year results with other CMB and large scale structure results, they obtain \[\Omega_b h^2 = 0.02265 \pm 0.00059\] ,

in excellent agreement with the 95\% C.L. range 0.017 – 0.024 obtained from big bang nucleosynthesis [2]. This implies

\[\frac{n_B}{n_\gamma} = (6.21 \pm 0.16) \times 10^{-10},\]  

\[\frac{n_B}{s} = (8.82 \pm 0.23) \times 10^{-11},\]

for the ratios of baryon density \((n_B)\) to photon density \((n_\gamma)\) and entropy \((s)\).

To generate the baryon asymmetry, the Sakharov criteria [3] must be satisfied: (1) Baryon number \((B)\) violation; (2) \(C\) and \(CP\) violation; (3) a departure from thermal equilibrium (or \(CPT\) violation). The first criterion is obvious. The second is required because if \(C\) and \(CP\) are exact symmetries, no net baryon number can be produced. Moreover, the equilibrium average of \(B\) is zero in thermal equilibrium by \(CPT\), so the third criterion is necessary.

There are several baryogenesis scenarios: Grand Unified Theory (GUT) baryogenesis [4, 5, 6], Affleck-Dine baryogenesis [7], leptogenesis [8, 9], electroweak baryogenesis (EWBG) [10, 11, 12], etc. (For reviews, see [13, 14, 15, 16] ) We will concentrate on EWBG, where the Sakharov criteria can be satisfied. Baryon number violation is induced by the anomalous sphaleron process, which violates \(B + L\) but preserves \(B - L\) at temperatures higher than the critical temperature \(T_c\) of the electroweak (EW) phase transition (EWPT) [17, 18, 19]. In addition, \(C\) is violated because the left- and right-handed fermions transform differently under \(SU(2)_L \times U(1)_Y\). \(CP\) can be violated in the Standard Model (SM) and Minimal Supersymmetric Standard Model (MSSM) and their extensions by the Yukawa interactions, \(\mu\) term, and soft supersymmetry breaking terms. Non-equilibrium processes may be obtained through the expansion of bubbles containing the true vacuum after the EWPT.

However, to preserve the generated baryon asymmetry after the EWPT, one needs a strong first order EWPT so that the sphaleron effects in the bubbles of true vacuum are shut down quickly. This requirement can be expressed as\[\frac{v(T_c)}{T_c} > 1,\]  

where in the SM \(v(T)\) is the vacuum expectation value of the Higgs field \(H^0\) at temperature \(T\) and \(T_c\) is the critical temperature of the EWPT. Similarly, in the

\[^1\text{Some authors impose the weaker constraint } \sqrt{2} v(T_c/T_c) > 1.\]
where $v_1(T)$ and $v_2(T)$ are respectively the VEVs of the two neutral Higgs fields $H_d^0$ and $H_u^0$. Our normalization is such that $v(0) \sim 174$ GeV.

In the SM, the EWPT can be strongly first order only if the Higgs mass is lighter than 40 GeV, which is inconsistent with the lower bound of 114.4 GeV from LEP $^{[20]}$ (recently, it was noticed that this tension can be relaxed in the SM, with an extra singlet introduced $^{[21]}$). Furthermore, the $CP$ violation from the CKM matrix is too small to generate a sufficient baryon asymmetry during the EWPT. In the MSSM there may be a small parameter space for EWBG $^{[22]-[41]}$. There are additional sources of $CP$ violation associated with the phases of the $\mu$ and supersymmetry breaking parameters. However, a strong enough first order EWPT requires that the lightest stop quark mass be smaller than the top quark mass $\sim 173$ GeV, called the light stop scenario $^{[28, 29, 30, 31, 32]}$ (for recent work, also see $^{[33, 34, 36, 37, 38, 39]}$). Also, the mass of the lightest $CP$-even Higgs must be smaller than 125 GeV, which leaves a small window above the current limit. In the Next-to-Minimal Supersymmetric Standard Model (NMSSM) $^{[42]}$ there is a trilinear term $A_h h S H_u H_d$ in the Higgs potential, where $S$ is a singlet under the SM gauge group. This allows a strong enough first order EWPT $^{[43, 44, 45, 46]}$. In the best-motivated versions of the NMSSM (motivated by the $\mu$ problem $^{[47]}$), the effective $\mu$ parameter is given by $h \langle S \rangle$ (for recent studies, see $^{[48]}$). However, the original versions involve a discrete $Z_3$ symmetry and serious cosmological domain wall problems $^{[49]}$. These can be avoided by introducing an elementary bilinear $\mu$ term in the superpotential (or linear or quadratic terms in $S$), but at the cost of reintroducing the $\mu$ (or analogous) problems $^{[44, 45]}$. The domain wall problem can also be avoided in an alternative version of the NMSSM $^{[50]}$, and in the nearly Minimal Supersymmetric Standard Model (nMSSM) $^{[51, 52, 53]}$, using a different discrete symmetry broken by a loop-induced tadpole. (For reviews, see $^{[54]}$.)

On the other hand, the possibility of an extra $U(1)'$ gauge symmetry $^{[55]}$ is well-motivated in superstring constructions $^{[56]}$, in grand unified theories $^{[57]}$, in models of dynamical symmetry breaking $^{[58]}$, and in little Higgs models $^{[59]}$. In supersymmetric models, an extra $U(1)'$ can provide an elegant solution to the $\mu$ problem $^{[60, 61]}$, with an effective $\mu$ parameter generated by the VEV of the Standard Model singlet field $S$ which breaks the $U(1)'$ symmetry. This is somewhat similar to the effective $\mu$ parameter in the NMSSM $^{[42]}$. However, with a $U(1)'$ there are no extra discrete symmetries or domain wall problems. A closely related feature is that the MSSM upper bound of $M_Z$ on the tree-level mass of the lightest Higgs scalar is relaxed, both in models with a $U(1)'$ and in the NMSSM, because of the $h S H_u H_d$ term in the superpotential and the $U(1)' D$-term $^{[63]}$. Light Higgs masses lower than those allowed in the MSSM are also allowed experimentally because of possible mixings among the Higgs doublets and singlets $^{[64, 65]}$. For specific $U(1)'$ charge assignments for the ordinary and exotic fields in some theories one can simultaneously ensure
the absence of anomalies; that all fields of the TeV-scale effective theory are chiral, avoiding a generalized $\mu$ problem; consistency with gauge coupling unification; and the absence of dimension-4 proton decay operators [66].

There are stringent limits on an extra $Z'$ from direct searches from the Tevatron [67] and from indirect precision tests at the $Z$-pole, at LEP 2, and from weak neutral current experiments [68]. The constraints depend on the particular $Z'$ couplings, but in typical models one requires $M_{Z'} > (800 - 1000)$ GeV and the $Z - Z'$ mixing angle $\alpha_{Z-Z'}$ to be smaller than a few $\times 10^{-3}$. (The specific parameters considered here yield a $Z'$ mass of around 1 TeV.) Thus, explaining the $Z - Z'$ mass hierarchy without fine-tuning is non-trivial. Two of us with J. Erler proposed a supersymmetric $U(1)'$ model with a secluded $U(1)'$-breaking sector (sMSSM) in which the squark and slepton spectra can mimic those of the MSSM, the EW symmetry breaking is driven by relatively large $A$ terms, and a large $Z'$ mass can be generated by the VEVs of additional SM singlet fields that are charged under the $U(1)'$ but do not directly contribute to the effective $\mu$ parameter [69]. If these fields are only weakly coupled to the SM fields, i.e., by $U(1)'$ interactions and possibly soft SUSY-breaking terms, then the scale of VEVs in this sector is only weakly linked to the EW scale.

In particular, we consider the situation in which there is an almost $F$ and $D$ flat direction involving these secluded fields, with the flatness lifted by a small Yukawa coupling $\lambda$. For a sufficiently small value for $\lambda$, the $Z'$ mass can be arbitrarily large.

In this paper, we systematically study EWBG and its correlation with neutralino CDM in the sMSSM (for a discussion on EWBG in the thin wall regime in the sMSSM, also see [70]). We construct two explicit anomaly-free models. In Model I, we embed the $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)'$ gauge symmetry into a subgroup of $E_6$. We consider three families of the SM fermions as arising from three fundamental 27 representations of $E_6$. The 27’s also contain candidates for the Higgs doublets, SM singlets (needed to break the $U(1)'$), and additional exotic fields. Additional vector-like pairs from $27 + 27^*$ representations can also exist at low energy without introducing anomalies. Consistency with gauge coupling unification is achieved by introducing one pair of vector-like Higgs-type doublets from $27 + 27^*$ [71], while the necessary $U(1)'$ breaking and the generation of masses for the exotic fermions is achieved by introducing three pairs of vector-like SM singlets from two pairs of $27 + 27^*$. The spectrum of exotics is rather complicated in Model I. In contrast, in Model II we add the minimal number of exotic particles with rational $U(1)'$ charges to cancel the $U(1)'$ anomalies. This model has the minimal exotic particle content but does not respect the simplest form of gauge coupling unification. We present the general superpotential and the supersymmetry breaking soft terms for both models.

For concreteness, we will work in Model I (the $E_6$ embedding). First, we discuss the one-loop effective potential at finite temperature in the ’t Hooft-Landau gauge in the $\overline{MS}$-scheme. We calculate the gauge boson, Higgs, neutralino, chargino, squark, and slepton mass matrices, and those for the scalar components of the exotic chiral

\footnote{These are in addition to those in the three 27’s. It is not important for our purposes which ones correspond to the two MSSM Higgs doublets.}
superfields. We also calculate the temperature dependent masses for the Higgs fields, the longitudinal components of the gauge bosons, squarks, sleptons, and the scalar components of the exotic chiral superfields. There are two relevant phase transitions as the temperature decreases: the $U(1)'$ symmetry is broken at about 1 TeV, and the EW symmetry at the weak scale. We show that there exists strong enough first order EWPT because of the large trilinear term $A_{h}hSH_{u}H_{d}$ in the tree-level Higgs potential. Thus, unlike the MSSM, the stop masses can be very heavy compared to the top quark mass. The EWPT features in model II or the other possible anomaly-free embeddings are similar, because the exotic particles’ contributions to the one-loop effective potential at finite temperature are suppressed by their heavy masses.

Subsequently, we discuss possible $CP$ violation introduced by the extended Higgs sector of the sMSSM. As first pointed out in [70], unlike the MSSM in which there is no $CP$-violation in the Higgs sector at tree level, both explicit and spontaneous violation (denote them as ECPV and SCPV) can occur. There are five complex parameters in the supersymmetry breaking soft terms and only four gauge-independent Higgs phase degrees of freedom, which implies that one of these complex phases cannot be removed by field redefinition. The $CP$-symmetry of this tree-level potential can therefore be explicitly broken. In addition, as a result of balancing different terms in the minimization of the tree-level neutral Higgs potential, the phases of the Higgs fields can obtain non-trivial VEVs, which implies that $CP$ symmetry can also be spontaneously broken. With loop corrections included, there may coexist vacua at finite temperature with broken and unbroken EW symmetry. The values of the spontaneous $CP$ phases are usually different in these vacua.

In this work SCPV provides a direct source for baryogenesis, while its magnitude is mediated by an explicit phase from the secluded sector. These new $CP$ sources do not introduce significant new contributions to the electric dipole moments (EDMs) of the electron and neutron. After proper field redefinitions, the $CP$-violation phases will only appear in the Higgs mass matrix through soft masses associated with singlet components. Numerical estimates show that, for typical parameter values, their contributions to EDMs will be about six or seven orders of magnitude smaller than the experimental upper limits. These contributions disappear completely in the limit with a trivial explicit $CP$ phase, where the spontaneous $CP$ phases are absent in the true vacuum but not where EWBG occurs.

In the early Universe the first order EWPT is realized by nucleating bubbles of the broken phase. The dynamical properties of these bubbles, such as the wall profile and expansion velocity, can have important influences on the production of the baryon asymmetry. We also study their physics in detail. The VEVs of the Higgs fields (including both their magnitudes and phases) are space-dependent, as one crosses the bubble wall. The bubble wall thickness is estimated by minimizing

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3We turn off all $CP$-violating sources beyond the Higgs sector, e.g., the ones from soft gaugino and sfermion masses, which are generally used for EWBG in supersymmetric models. Though they may provide non-trivial contributions to EWBG, these $CP$ phases usually suffer strong constraints from electric dipole moments.

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the action under the kink ansatz \[72, 73, 74\]. Numerical results show that it varies from \(3 T_c^{-1}\) to \(30 T_c^{-1}\) as an approximately monotonically increasing function of the phase changes of the Higgs fields. In addition, we argue that the wall velocity in the sMSSM cannot be larger than that in the MSSM under the same phase transition condition and thus should be non-relativistic. This fact implies that EW sphaleron processes have more time to occur and hence will enhance the final baryon asymmetry.

We then systematically study non-local EWBG in both the thin wall and thick wall regimes. Non-local baryogenesis has a great advantage over the local one \[75\]: the baryon number violation processes take place in an effective volume extending from the wall surface to a region in the symmetric phase, and thus we may expect an enhancement in the generation of the baryon asymmetry. According to the bubble wall profile, we calculate the contribution from \(\tau\) leptons in the thin wall regime, and the ones from stop, charginos and top quarks in the thick wall regime. We find that the \(CP\)-violating currents induced by stops and charginos are very different from those obtained in the MSSM: (1) the current of the \(v_1 \partial v_2 - v_2 \partial v_1\) type (in our formalism, the concrete expression is a little different) never requires \(\partial \tan \beta \neq 0\) in the bubble wall due to the variance of the relevant \(CP\) phases crossing the wall\[4\]; (2) there is a new type of \(CP\)-violating current at the leading order which is proportional to \(v_i v_i\) with \(i = 1, 2\) and is absent in the MSSM. The \(CP\)-violating current of the new type has important influence on the EWBG. First, the stop contribution can be quadratically enhanced by a large soft \(A_h\) parameter. Second, in addition to \(\tau\) leptons, top squarks and charginos, top quarks can also play a significant role in the EWBG. All of these features are results of the SCPV-driven EWBG, so they are not sensitive to the concrete embeddings of the sMSSM. Numerical calculations show that the produced baryon asymmetry is large enough to explain the cosmological observation today. Though the EWBG is directly driven by SCPV, there is no dilution problem between matter and anti-matter bubbles. Such a dilution is caused by a \(Z_2\) discrete symmetry which usually exists in the SCPV scenarios. In our work, however, this symmetry has been explicitly broken at tree level by the explicit \(CP\)-violating phase.

We also study the correlation between EWBG and cold dark matter (CDM) in the sMSSM. Large trilinear soft parameters (used for strong enough first order EWPT) more often arise from gravity-mediated SUSY breaking, where the lightest supersymmetric particle (LSP) is usually the lightest neutralino. We study the possibility of describing the EWBG and neutralino CDM in the same framework. Though we do not scan the whole parameter space, we find that there indeed exist some regions where strong enough first order EWPT, large \(CP\) phase variations across the bubble wall, reasonable baryon asymmetry, as well as acceptable neutralino LSP relic density can be achieved simultaneously. The relevant neutralino mass \(m_{\chi_1}\) is close to

\[4\] In the MSSM, there also exists a \(CP\)-violating current of the \(v_1 \partial v_2 + v_2 \partial v_1\) type for charginos which arises from a resummation of corrections associated with higher order Higgs insertions or multiple scattering effects in the bubble wall. Though not suppressed by \(\partial \tan \beta\), the current of this type is usually subdominant unless \(\partial \tan \beta\) is suppressed (in our model it is always subdominant). For relevant or more general discussions, see \[76, 77, 26, 27, 29, 30, 31, 32\]. We will therefore not discuss the currents of this type.
Finally, we comment briefly on possible cosmological signals: superconducting cosmic strings and gravitational waves (GWs). We expect that particle emission from the decays of cosmic strings and GWs from EWPT may be observed within the foreseeable future.

This paper is organized as follows. We briefly review the tree-level Higgs potential in the sMSSM and construct two anomaly-free embeddings in Section 2. The one-loop effective potential at finite temperature is considered in Section 3. In Section 4, we show that the EWPT is strongly first order. We study \( CP \) violation in Section 5 and the bubble wall physics in Section 6. In Section 7, we systematically study non-local EWBG in both the thin wall and thick wall regimes, and discuss the contributions from leptons, squarks, charginos and quarks in detail. In Section 8, we illustrate that there exists common parameter space where the baryon asymmetry and the CDM can both be explained by EWBG and neutralino LSP, respectively. Some simple comments on the cosmological signals of cosmic string decays and GWs are given in Section 9. The last section is our discussion and conclusions.

2 Two Anomaly-free \( U(1)' \) Models

Let us briefly review the sMSSM [69]. There are one pair of Higgs doublets, \( H_u \) and \( H_d \), and four SM singlets, \( S, S_1, S_2, \) and \( S_3. \) The \( U(1)' \) charges for the Higgs fields satisfy

\[
Q'_{H_d} + Q'_{H_u} + Q'_S = 0, \quad Q'_S = -Q'_{S_1} = -Q'_{S_2} = \frac{1}{2} Q'_{S_3}. \tag{6}
\]

The superpotential for the Higgs is

\[
W_H = h S H_u H_d + \lambda S_1 S_2 S_3, \tag{7}
\]

where the Yukawa couplings \( h \) and \( \lambda \) are respectively associated with the effective \( \mu \) term and with an (almost) \( F \) and \( D \)-flat direction. For simplicity, we assume that terms such as \( S_2 S_3, S_2 S_3 \) and their associated soft terms are absent, and that there are no bilinears \( SS \) in \( W_H. \) The existence of a number of SM singlets and the non-diagonal nature of the superpotential is in part motivated by explicit superstring constructions [78]. The corresponding \( F \)-term scalar potential is

\[
V_F = h^2 (|H_u^0|^2 |H_d^0|^2 + |S|^2 |H_u^0|^2 + |S|^2 |H_d^0|^2) \\
+ \lambda^2 (|S_1|^2 |S_2|^2 + |S_2|^2 |S_3|^2 + |S_3|^2 |S_1|^2). \tag{8}
\]

The \( D \)-term scalar potential for the neutral fields is

\[
V_D = \frac{G^2}{8} (|H_u^0|^2 - |H_d^0|^2)^2 \\
+ \frac{1}{2} g_z^2 \left( Q'_S |S|^2 + Q'_{H_d} |H_d^0|^2 + Q'_{H_u} |H_u^0|^2 + \sum_{i=1}^{3} Q'_S_i |S_i|^2 \right)^2, \tag{9}
\]
where $G^2 = g_1^2 + g_2^2$; $g_1$, $g_2$, and $g_Z$ are the coupling constants for $U(1)$, $SU(2)_L$ and $U(1)'$; and $Q'_\phi$ is the $U(1)'$ charge of the field $\phi$.

In addition, one introduces the supersymmetry breaking soft terms

$$V^H_{soft} = m_{H_d}^2 |H_d^0|^2 + m_{H_u}^2 |H_u^0|^2 + m_S^2 |S|^2 + \sum_{i=1}^3 m_{S_i}^2 |S_i|^2 - \left( A_h h S H_d^0 H_u^0 + A_\lambda \lambda S S_1 S_2 S_3 + m_{S_1 S_2 S_3}^2 S_1 S_2 S_3 + m_{S_1 S_2 S_3}^2 S_1 S_2 S_3^* + H.C. \right).$$

(10)

$m_{S_i}^2, i = 1, 2$ are needed to break two unwanted global $U(1)$ symmetries (for a recent discussion of such global symmetries, see [79]). There is an almost $F$ and $D$ flat direction involving $S_i$, with the flatness lifted by a small Yukawa coupling $\lambda$. For a sufficiently small value of $\lambda$, the $Z'$ mass can be arbitrarily large. For example, if $h \sim 10\lambda$, one can generate a $Z$ – $Z'$ mass hierarchy in which the $Z'$ mass is of order 1 TeV. The dimensional parameters in $V^H_{soft}$ are specified in arbitrary units, and then rescaled after the (effective) potential is minimized so that the EW scale is

$$v = \sqrt{v_1^2 + v_2^2} \sim 174 \text{ GeV},$$

with $v_1 = \langle H_u^0 \rangle$ and $v_2 = \langle H_d^0 \rangle$.

In [69], $m_{S_1 S_2}^2$ was set to 0 for simplicity. In that case, the fields can be defined so that $A_h$, $A_\lambda$, $m_{S_1 S_2}^2$, and $m_{S_1 S_2}^2$ are all real and positive. The minimum then occurs at a point at which all of the fields are real and positive, i.e., there is no $CP$ violation in the tree-level Higgs potential. For $m_{S_1 S_2}^2 \neq 0$, however, there will in general be $CP$ violation.

The $U(1)'$ charges for the fermions and the anomaly cancellations were not discussed in Ref. [69], so in the following two subsections we construct two anomaly-free models, one of which is consistent with minimal gauge coupling unification.

### 2.1 Model I: sMSSM with $E_6$ Embedding

$U(1)'$ models necessarily imply that new fermions are needed for anomaly cancellations [55, 66]. Since all complete representations of $E_6$ are anomaly-free, it is convenient and conventional to simply consider the $U(1)'$ charge assignment and exotic particle content of $E_6$ as an example of an anomaly-free construction. We are not considering a full $E_6$ grand unified theory, because a light $Z'$ would prevent a large doublet-triplet splitting, leading to rapid proton decay if the $E_6$ Yukawa relations were enforced. (Detailed studies of $E_6$ theories with broken Yukawa relations may be found in [80].)

The $E_6$ gauge group can be broken as [81, 82]

$$E_6 \rightarrow SO(10) \times U(1)_\psi \rightarrow SU(5) \times U(1)_\chi \times U(1)_\psi .$$

(11)

The $U(1)_\psi$ and $U(1)_\chi$ charges for the $E_6$ fundamental 27 representation are given in Table 1. The $U(1)'$ in Model I is one linear combination of the $U(1)_\chi$ and $U(1)_\psi$

$$Q' = \cos \theta \ Q_\chi + \sin \theta \ Q_\psi ,$$

(12)
Table 1: Decomposition of the $E_6$ fundamental $27$ representation under $SO(10)$, $SU(5)$, and the $U(1)_X$, $U(1)_Y$ and $U(1)_R$ charges.

| $SO(10)$ | $SU(5)$ | $2\sqrt{10}Q_X$ | $2\sqrt{6}Q_R$ | $2\sqrt{15}Q_L$ |
|----------|---------|-----------------|----------------|-----------------|
| 16       | 10 $(u, d, \bar{u}, \bar{d})$ | -1 | 1 | -1/2 |
|          | 5 $(\nu, e, \bar{e})$ | 3 | 1 | 4 |
|          | $1\bar{N}$ | -5 | 1 | -5 |
| 10       | 5 $(D, H'_u)$ | 2 | -2 | 1 |
|          | 5 $(\bar{D}, H'_d)$ | -2 | -2 | -7/2 |
| 1        | 1 $S_L$ | 0 | 4 | 5/2 |

where the angle $\theta$ will be chosen to ensure the needed $U(1)'$ charges for the SM singlets. For simplicity, we assume that the other $U(1)$ gauge symmetry from the orthogonal linear combination of the $U(1)_X$ and $U(1)_Y$ is absent or broken at a high scale.

We assume three $27$s, which include three families of the SM fermions, one pair of Higgs doublets ($H_u$ and $H_d$), and a number of SM singlets, extra Higgs-like doublets, and other exotics. The embeddings of the SM fermions are obvious. For definiteness, we assume that the Higgs doublets ($H_u$ and $H_d$) are the doublets in 10 (or 5 and $\bar{5}$) in the third 27. (Of course, the exact identification of the $H_u$, $H_d$ and $S$ with specific 27s is irrelevant.) In addition, we assume that the four SM singlets $S$, $S_1$, $S_2$, $S_3$ are the $S_L$, $S_L^*$, $S_R^*$ and $\bar{N}^*$, respectively, in two (partial) pairs of 27 and $27^*$. To cancel the $U(1)'$ anomalies, we introduce $X$ and $X_3$, which are $S_L$ and $\bar{N}$, respectively, in the two pairs. Thus, $(S, S_1)$, $(X, S_2)$ and $(X_3, S_3)$ are three pairs of vector-like SM singlets and will not introduce any anomalies. To be consistent with minimal gauge coupling unification, we introduce one pair of vector-like doublets $H'_u$ and $\bar{H}'_d$ from a pair of $27 + 27^*$ [74]. For simplicity, we assume that the other particles in the $27 + 27^*$ pairs are absent or are very heavy and decouple at low energy. From the requirement $Q_S = \frac{1}{2}Q_{S3}$, i.e., $Q_{SL} = \frac{1}{2}Q_{N*}$, we obtain

$$\tan \theta = \frac{\sqrt{15}}{9}. \quad (13)$$

The exotic particles in Model I are $D_i$, $\bar{D}_i$, $S_L^i$, $\bar{N}_i$, where $i = 1, 2, 3$; $H_u^k$ and $\bar{H}_d^k$, where $k = 1, 2$; $X$, $X_3$; and $H'_u$ and $\bar{H}'_d$. The $U(1)'$ charges for the Standard Model fermions and exotic particles are also given in Table I.

The general superpotential is $W_H + W_Y$, where

$$W_Y = \alpha SXX + \alpha^N SS_L^i \bar{N}_j + \mu' \bar{H}'_u H'_u + h_{ij}^u Q_i H_u \bar{u}_j + h_{ij}^d H_d Q_i \bar{d}_j + h_{ij}^e H_d \bar{e}_j$$
$$+ h_{ij}^u H_u L_i \bar{N}_j + \alpha^D SD_i \bar{D}_j + \alpha^H SH_u^i H_d^j + \lambda_{ij}^u Q_i H'_u \bar{u}_j$$
$$+ \lambda_{ij}^H H_u^i L_i \bar{N}_j + \lambda_{ijm}^D D_i \bar{u}_j \bar{e}_m + \lambda_{ijm}^2 D_i Q_j L_m + \lambda_{ijm}^3 D_i \bar{d}_j \bar{N}_m$$
$$+ \lambda_{ijm}^4 D_i Q_j Q_m + \lambda_{ijm}^5 D_i \bar{u}_j \bar{e}_m + \lambda_{ijm}^6 \phi S_k S_L^i + M_\phi \phi^2, \quad (14)$$
with \( Q_i = (u_i, d_i) \), \( L_i = (\nu_i, e_i) \), and \( i, j, m = 1, 2, 3 \) while \( k, l = 1, 2 \). For simplicity, we will assume that the Higgs couplings \( h^u_{ij} \), \( h^d_{ij} \), \( h^\nu_{ij} \) and \( h^N_{ij} \) and those associated with the exotics are diagonal in the numerical calculations. In addition, to explain the neutrino masses and mixings, we introduce a SM singlet \( \phi \) which is neutral under \( U(1)' \). Integrating out \( \phi \), we obtain the non-renormalizable terms \( S_k S_i S'_j S'_L / M_\phi \), and then Majorana mass terms for \( S'_L S'_L \) will appear after \( U(1)' \) breaking. Thus, the light neutrino masses and mixings in Model I can be generated via a double-see-saw mechanism naturally [83, 84]. We see from Eq. (14) that all of the exotics except \( H'_u \) and \( H'_u \) can have masses generated by the VEV of \( S \). The latter can be given a supersymmetric mass \( \mu' \). This reintroduces a form of the \( \mu \) problem, though not for the Higgs fields associated with EW symmetry breaking, and is clearly a flaw of the specific construction. However, this segment of the model is only loosely connected with the issue of concern in this paper, i.e., EWBG, so we will tolerate it rather than going to a more complicated model.

The new scalar supersymmetry breaking soft terms (in addition to Eq. (10)) are

\[
V_{soft}^A = -A_{\alpha} S \gamma X_3 - A_{\alpha} \gamma S S_i \bar{N}_i - B_{\mu'} \mu' \bar{H}_u' H'_u - A_{\mu} \mu Q_i H_u \bar{u}_j \\
- A_{\mu} h^d_{ij} H_u Q_d \bar{d}_j - A_{\mu} h^e_{ij} H_u L_i \bar{e}_j - A_{\mu} h^N_{ij} H_u L_i \bar{\nu}_j - A_{\mu} h^N_{ij} S D_i \bar{D}_j \\
- A_{\lambda} \lambda' \gamma S H_u H'_d - A_{\lambda} \lambda' \gamma \lambda U_i Q_i \bar{u}_j - A_{\lambda} \lambda' \gamma \lambda H'_u H'_u \bar{N}_j \\
- A_{\lambda} \lambda' \gamma D_i \bar{u}_j \bar{e}_m - A_{\lambda} \lambda' \gamma \lambda' \bar{Q}_i \bar{Q}_m - A_{\lambda} \lambda' \gamma \lambda' \bar{D}_i \bar{D}_m - A_{\lambda} \lambda' \gamma \lambda' \bar{S}_i \bar{S}_m \\
-B \phi M_\phi \phi^2 + H.C.,
\]

(15)

\[
V_{soft}^m = \sum_{i=1}^{3} \left( m^2_{Q_i} |\tilde{Q}_i|^2 + m^2_{u_R} |\tilde{u}_i|^2 + m^2_{d_R} |\tilde{d}_i|^2 + m^2_{L_i} |\tilde{L}_i|^2 + m^2_{e_R} |\tilde{e}_i|^2 \\
+ m^2_{\bar{N}_i} |\tilde{\bar{N}}_i|^2 + m^2_{\bar{S}'_L} |\tilde{\bar{S}}'_L|^2 + m^2_{\bar{D}_i} |\tilde{\bar{D}}_i|^2 + m^2_{\bar{D}'_i} |\tilde{\bar{D}}'_i|^2 \right) \\
+ \sum_{k=1}^{2} \left( m^2_{H_u^k} |H_u^k|^2 + m^2_{H_d^k} |H_d^k|^2 \right) + m^2_{\phi} |\phi|^2 \\
+ m^2_{X} |\tilde{X}|^2 + m^2_{\tilde{X}} |\tilde{\tilde{X}}|^2 + m^2_{H_u'} |H_u'|^2 + m^2_{H_d'} |H_d'|^2,
\]

(16)

where we have assumed that the mass-squared terms are diagonal. We assume that only \( S, S_1, S_2, S_3, H_u^0 \) and \( H_d^0 \) acquire VEVs.

The vector-like particles \( H_u' \) and \( H_d' \) can decay via the Yukawa \( \lambda_u^i \) and \( \lambda_d^i \) terms in Eq. (14). In addition, the charged exotic particles \( D_i \) and \( \bar{D}_i \) can decay through the Yukawa \( \lambda_{ijm}^n \) terms with \( n = 1, 2, \ldots, 5 \) in Eq. (14) [82, 83]. However, to avoid the proton decays via dimension-5 operators, we must forbid some of the Yukawa \( \lambda_{ijm}^n \) terms. For example, we can choose \( \lambda_{ijm}^n = 0 \) either for \( n = 1, 2, 3 \) or for \( n = 4, 5 \).
2.2 Model II: Minimal Anomaly-free Model

In this subsection, we consider the minimal sMSSM in which all the particles have rational $U(1)'$ charges. In order to cancel the gauge and gauge-gravity mixed anomaly due to the $U(1)'$ gauge symmetry, we introduce the exotic particles $D_1$, $\bar{D}_1$, $D_2$, $\bar{D}_2$, $V$, $\bar{V}$, $X_1$, $\bar{X}_1$, $X_2$, $\bar{X}_2$, $X_3$, $\bar{X}_3$, $X_4$, and $\bar{X}_4$. The quantum numbers under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)'$ for the SM fermions, Higgs fields and extra exotic particles are given in Table 2. If one allowed irrational $U(1)'$ charges, only $\bar{D}_1$, $D_2$, $\bar{D}_2$, $V$, and $\bar{V}$ would be needed.

With the $U(1)'$ charge assignments for the particles in Table 2, the conditions for $[SU(3)_C]^2U(1)'$ and [Gravity]$^2U(1)'$ are automatically satisfied, and the condition for $[SU(2)_L]^2U(1)'$ is the same as that for $[U(1)_Y]^2U(1)'$. The anomaly-free conditions for $[SU(2)_L]^2U(1)'$, $U(1)_Y[U(1)]^2$ and $[U(1)']^3$, respectively, are

$$c_1 + c_2 + 3(a + 3b) = 0 ,$$

$$-3a^2 + 3b^2 + 3(a + c_1)^2 - 6(b + c_2)^2 + 3(b + c_1)^2 - c_1^2 + c_2^2$$

$$-d_1^2 + (s + d_1)^2 - d_2^2 + (2s + d_2)^2 - d_3^2 + (2s + d_3)^2 = 0 ,$$

$$6a^3 - 3(a + c_2)^3 - 3(a + c_1)^3 + 18b^3 - 9(b + c_2)^3 - 9(b + c_1)^3$$

$$+3d_1^3 - 3(s + d_1)^3 + 3d_2^3 - 3(2s + d_2)^3 + d_3^3 - (2s + d_3)^3$$

$$+2d_4^3 - (d_4 - s)^3 - (d_4 + s)^3 + 7s^3 + 2c_1^3 + 2c_2^3 = 0 .$$

The simplest solution is $c_1 = 9/10$, $c_2 = 9/10$, $s = -9/5$, $a = -18/5$, $b = 1$, $d_1 = -2$, $d_2 = -1/5$, $d_3 = 9/5$, $d_4 = 27/10$, $d_5 = 9/10$, and $d_6 = -3$.

The general superpotential in Model II is

$$W = \alpha_1 S D_1 \bar{D}_1 + \alpha_2 S D_2 \bar{D}_2 + \alpha_3 S_3 V \bar{V} + \alpha_4 S_1 X_1 \bar{X}_1 + \alpha_5 S_2 X_1 \bar{X}_1$$

$$+\alpha_6 S X_2 \bar{X}_2 + \alpha_7 S_1 X_3 \bar{X}_3 + \alpha_8 S_2 X_3 \bar{X}_3 + \alpha_9 S X_4 \bar{X}_4$$

$$+\alpha_1' S_1 X_1 \bar{X}_1 + \alpha_2' S_1 X_1 \bar{X}_1 + \alpha_3' S_2 X_1 \bar{X}_1 + \alpha_4' S_2 X_1 \bar{X}_1$$

$$+h_{ij}^2 Q_i H_u \bar{u}_j + h_{ij}^2 H_u Q_i \bar{d}_j + h_{ij}^2 H_d L_i \bar{e}_j + h_{ij}^2 H_u L_i \bar{N}_j$$

$$+\lambda_{ij}^2 D_1 Q_i + \lambda_{ij}^2 \bar{D}_1 \bar{u}_i \bar{d}_j + \lambda_3^2 V \bar{e}_i X_1 + \lambda_3^2 V \bar{e}_i \bar{X}_1 .$$

All of the exotics can be made massive by $U(1)'$ breaking.

The charged exotic particles ($D_i$, $\bar{D}_i$), and ($V$, $\bar{V}$) can decay through the Yukawa $\lambda_{ij}^1$, $\lambda_{ij}^2$, $\lambda_{ij}^3$ and $\lambda_3^i$ terms in Eq. (20) [82,83], where we have assumed that $m_{\nu} > m_{X_1}$. After the $U(1)'$ breaking, the vector-like particles ($X_i$, $\bar{X}_i$) are neutral. There is no discrete symmetry after $U(1)'$ breaking, so the vector-like particles ($X_i$, $\bar{X}_i$) can decay via higher dimensional operators.
Table 2: Minimal Anomaly-free Model. Quantum numbers under $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)'$ for the left-chiral Standard Model fermions ($Q_i, \bar{u}_i, \bar{d}_i, L_i, \bar{N}_i, \bar{e}_i$), Higgs fields ($H_u, H_d, S, S_1, S_2, S_3$), and extra exotic particles ($D_1, \bar{D}_1, D_2, \bar{D}_2, V, \bar{V}, X_1, \bar{X}_1, X_2$ and $\bar{X}_2$).

| Particles | Quantum Numbers | Particles | Quantum Numbers |
|-----------|-----------------|-----------|----------------|
| $L_i$     | (1; 2; $-1/2; a$) | $Q_i$     | (3; 2; 1/6; $b$) |
| $\bar{N}_i$ | (1; 1; 0; $-(a + c_2)$) | $\bar{u}_i$ | (3; 1; $-2/3; -(b + c_2)$) |
| $\bar{e}_i$ | (1; 1; 0; $-(a + c_1)$) | $d_i$     | (3; 1; $1/3; -(b + c_1)$) |
| $H_d$     | (1; 2; $-1/2; c_1$) | $H_u$     | (1; 2; 1/2; $c_2$) |
| $S$       | (1; 1; 0; $s$)   | $S_3$     | (1; 1; 0; $2s$)   |
| $S_1$     | (1; 1; 0; $-s$)  | $S_2$     | (1; 1; 0; $-s$)  |
| $D_1$     | (3; 1; $-1/3; d_1$) | $D_1$     | (3; 1; 1/3; $-(s + d_1)$) |
| $D_2$     | (3; 1; $-1/3; d_2$) | $D_2$     | (3; 1; 1/3; $-(2s + d_2)$) |
| $V$       | (1; 1; $-1; d_3$) | $V$       | (1; 1; $- (d_3 + 2s)$) |
| $X_1$     | (1; 1; 0; $d_4$) | $X_1$     | (1; 1; 0; $-(d_4 - s)$) |
| $X_2$     | (1; 1; 0; $d_5$) | $X_2$     | (1; 1; 0; $-(d_5 + s)$) |
| $X_3$     | (1; 1; 0; $d_6$) | $X_3$     | (1; 1; 0; $-(d_6 - s)$) |
| $X_4$     | (1; 1; 0; $d_6$) | $X_4$     | (1; 1; 0; $-(d_6 + s)$) |

3 One-Loop Effective Potential at Finite Temperature

In the 't Hooft-Landau gauge and in the $\overline{MS}$-scheme, the one-loop effective potential at finite temperature is [86]

$$V_{\text{eff}}(\phi, T) = V_0(\phi) + V_1(\phi, 0) + \Delta V_1(\phi, T) + \Delta V_{\text{daisy}}(\phi, T),$$  \hspace{1cm} (21)

where $V_0(\phi)$ is the tree-level potential, $V_1(\phi, 0)$ is the one-loop zero temperature correction, $\Delta V_1(\phi, T)$ is the temperature dependent one-loop correction, and $\Delta V_{\text{daisy}}(\phi, T)$ is the multi-loop daisy correction. The explicit expressions for $V_0(\phi), V_1(\phi, 0), \Delta V_1(\phi, T)$ and $\Delta V_{\text{daisy}}(\phi, T)$ are

$$V_0(\phi) = V_F + V_D + V_{\text{soft}}^H, \hspace{1cm} (22)$$

$$V_1(\phi, 0) = \sum_i \frac{n_i}{64\pi^2} m_i^4(\phi) \left[ \log \frac{m_i^2(\phi)}{Q^2} - C_i \right], \hspace{1cm} (23)$$

$$\Delta V_1(\phi, T) = \frac{T^4}{2\pi^2} \left\{ \sum_i n_i J_i \left[ \frac{m_i^2(\phi)}{T^2} \right] \right\}, \hspace{1cm} (24)$$
\[ \Delta V_{\text{daisy}}(\phi, T) = -\frac{T}{12\pi} \sum_{i=\text{bosons}} n_i \left[ M_i^4(\phi, T) - m_i^2(\phi) \right], \tag{25} \]

where \( m_i(\phi) \) are field-dependent masses, \( n_i \) are the number of degrees of freedom, and \( C_i \) are constants dependent on the regularization scheme: in the \( \overline{\text{MS}} \)-scheme that we are assuming, \( C_i = 5/6 \) for the gauge bosons and \( 3/2 \) for scalars and fermions.\(^5\) The function \( J_i \) comes from the one-loop corrections to the effective potential at finite temperature \([86]\). For bosons

\[ J_B(m^2(\phi)/T^2) = \int_0^\infty dx \ x^2 \log \left[ 1 - e^{-x^2 \frac{m^2(\phi)}{T^2}} \right]. \tag{26} \]

At relatively high temperature, \( \frac{m(\phi)}{T} < 2.2 \), one can expand \([86]\)

\[ J_B(m^2(\phi)/T^2)^{\text{HT}} = -\frac{\pi^4}{45} + \frac{\pi^2}{12} \frac{m^2(\phi)}{T^2} - \frac{1}{6} \left( \frac{m^2(\phi)}{T^2} \right)^{3/2} \]

\[ - \frac{1}{32} \frac{m^4(\phi)}{T^4} \log \frac{m^2(\phi)}{a_b T^2} - 2\pi^{7/2} \sum_{\ell=1}^\infty (-1)^\ell \frac{\zeta(2\ell + 1)}{(\ell + 1)!} \Gamma \left( \ell + 1, \frac{m^2(\phi)}{4\pi^2 T^2} \right)^{\ell + 2}, \tag{27} \]

where \( a_b = 16\pi^2 \exp(3/2 - 2\gamma_E) \) (log \( a_b = 5.4076 \)) and \( \zeta \) is the Riemann \( \zeta \)-function.

At relatively low temperature, \( \frac{m(\phi)}{T} > 2.2 \), we can expand \([87]\)

\[ J_B(m^2(\phi)/T^2)^{\text{LT}} = -\frac{\pi^4}{45} + \left( \frac{\pi}{2} \right)^{1/2} \left( \frac{m(\phi)}{T} \right)^{3/2} e^{-m(\phi)/T} \]

\[ \left[ 1 + \frac{15}{8} \frac{T}{m(\phi)} + ... \right]. \tag{28} \]

The function \( J_F \) for fermion \( \psi \) with mass \( m \) is \([86]\)

\[ J_F(m^2(\psi)/T^2) = \int_0^\infty dx \ x^2 \log \left[ 1 + e^{-x^2 \frac{m^2(\psi)}{T^2}} \right], \tag{29} \]

For \( \frac{m(\psi)}{T} < 1.8 \), one finds \([86]\)

\[ J_F(m^2(\psi)/T^2)^{\text{HT}} = \frac{7\pi^4}{360} - \frac{\pi^2}{24} \frac{m^2(\psi)}{T^2} - \frac{1}{32} \frac{m^4(\psi)}{T^4} \log \frac{m^2(\psi)}{a_f T^2} \]

\[ - \frac{\pi^{7/2}}{4} \sum_{\ell=1}^\infty (-1)^\ell \frac{\zeta(2\ell + 1)}{(\ell + 1)!} \left( 1 - 2^{-2\ell-1} \right) \Gamma \left( \ell + 1, \frac{m^2(\psi)}{\pi^2 T^2} \right)^{\ell + 2}, \tag{30} \]

\(^5\)For the \( \overline{\text{DR}} \)-scheme, \( C_i \) in Eq. \([23]\) is \( 3/2 \) for gauge bosons, scalars, and fermions. Our discussions and conclusions do not depend on the scheme.
where \( a_f = \pi^2 \exp(3/2 - 2\gamma_E) \) (log \( a_f = 2.6351 \)). At low temperature, \( \frac{m(\psi)}{T} > 1.8 \) \[87\],

\[
J_F(m^2(\psi)/T^2)^{LT} = \frac{7\pi^4}{360} + \left( \frac{\pi}{2} \right)^{1/2} \left( \frac{m(\psi)}{T} \right)^{3/2} e^{-m(\psi)/T} \left[ 1 + \frac{15}{8} \frac{T}{m(\psi)} + \ldots \right].
\] (31)

The \( m(\phi) \) and \( m(\psi) \) are the Higgs field dependent masses from the eigenvalues of the mass matrices. Thus, we need the mass matrices for all the particles in the models. The tree-level mass matrices\[6\] for the particles in Model I are presented in Appendix A. (Those for the ordinary and exotic fermions are trivial.)

There are small discontinuities between the \( J_B(m^2(\phi)/T^2)^{HT} \) and \( J_B(m^2(\phi)/T^2)^{LT} \) at \( \frac{m(\phi)}{T} \sim 2.2 \), and between the \( J_F(m^2(\phi)/T^2)^{HT} \) and \( J_F(m^2(\phi)/T^2)^{LT} \) at \( \frac{m(\phi)}{T} \sim 1.8 \). When we numerically calculate the minimum of the potential, we need smooth interpolations for \( J_B(m^2(\phi)/T^2)^{HT} \) and \( J_F(m^2(\phi)/T^2)^{HT} \). Thus, in our numerical calculation we use the following approximation

\[
J_B(m^2(\phi)/T^2) = \frac{\tanh(-m_i^2(\phi)/T^2 + 2.2^2) + 1}{2} \times J_B(m^2(\phi)/T^2)^{HT} + \frac{\tanh(m_i^2(\phi)/T^2 - 2.2^2) + 1}{2} \times J_B(m^2(\phi)/T^2)^{LT},
\] (32)

\[
J_F(m^2(\phi)/T^2) = \frac{\tanh(-m_i^2(\phi)/T^2 + 1.8^2) + 1}{2} \times J_F(m^2(\phi)/T^2)^{HT} + \frac{\tanh(m_i^2(\phi)/T^2 - 1.8^2) + 1}{2} \times J_F(m^2(\phi)/T^2)^{LT}.
\] (33)

The temperature dependent scalar mass-squared \( M_i^2(\phi,T) \) is obtained from the \( m_i^2(\phi) \) by adding the leading temperature dependent self-energy contributions \( \Pi_\phi(T) \),

\[
M^2(\phi,T) = m^2(\phi) + \Pi_\phi(T),
\] (34)

where \( \Pi_\phi(T) \) is proportional to \( T^2 \) \[88\]. For the gauge bosons, only the longitudinal components receive such contributions. The \( \Pi_\phi(T) \)'s for the particles in Model I are presented in Appendix B. The daisy corrections are of higher order, but important at high temperature.

\[6\]The use of tree-level mass matrices leads to a slight inconsistency when they are evaluated at the minimum of the full one-loop effective potential. In particular, the tree-level approximations to the Goldstone Boson masses are sometimes shifted slightly from zero. We avoid singularities by setting slightly negative mass-squares to zero. We have checked that these small shifts have negligible effects on the phase structure, as should be apparent from the formula \[27\] relevant for small \( \frac{m(\phi)}{T} \).
Table 3: Three sets of typical parameter values. The energy units are 69 GeV, 92 GeV and 120 GeV, corresponding to $m_{H_d}^2 = 5.0, 9.0$ and 13.0, respectively. In this paper, we will denote them as cases a, b and c. Note that the soft mass squares in the superpotential are relatively small, i.e., the symmetry breaking is $A$-term dominant. $M'_1$, $M_1$ and $M_2$ denote the three neutral gaugino soft masses.

| $h$ | $A_h$ | $\lambda$ | $A_\lambda$ | $m_{SS_1}^2$ | $m_{SS_2}^2$ | $m_{S_1S_2}^2$ | $m_{H_d}^2$ |
|-----|-------|-----------|-------------|--------------|--------------|--------------|---------------|
| 0.8 | 4.2   | 3.3       | 0.02        | 0.1          |              | $4.8 \times 10^{-4}$ | $5.0 / 9.0 / 13.0$ |
| $m_{H_d}^2$ | $m_{S_1}^2$ | $m_{S_2}^2$ | $m_{S_3}^2$ | $M_1^2$ | $M_1$ | $M_2$ |
| -0.1 | 0.5   | 0.03      | 0.03        | 1.5          | 1.5          | 3.0          |
| $m_{Q_1}^2$ | $m_{\tilde u_R}^2$ | $m_{\tilde u_R}^2$ | $m_{\tilde Q_1,2}^2$ | $m_{\tilde d_{1,2}}^2$ | $m_{\tilde d_{1,2}}^2$ | $m_{\tilde l_{1,2,3}}^2$ | $m_{\tilde l_{1,2,3}}^2$ |
| 8   | 8     | 25        | 25          | 25           | 25           | 25           | 25             |

4 The Electroweak Phase Transition

Generally, the EWPT in the sMSSM is more complicated than those in the SM, MSSM, and NMSSM due to the secluded $U(1)'$-breaking sector. The temperature dependent corrections to the effective potential are a function of the masses $M_i(\phi, T)$. At low temperature, the temperature dependent corrections are negligible, while at very high temperature they are dominant. At the critical temperature, the temperature dependent corrections are comparable to the effective potential at zero temperature. Our one-loop effective potential is a function of 10 physical variables (the complex VEVs of the six neutral Higgs fields minus two gauge degrees of freedom). There might exist local minima near the critical temperature. In our analysis we always choose the global minimum (at one-loop level plus daisy correction).

Because the $Z'$ mass is assumed to be of order 1 TeV, the VEVs of the SM singlets $S_1$, $S_2$, and $S_3$ are about an order of magnitude larger than those of the Higgs doublets and $S$\(^6\). There are therefore two phase transitions: the $U(1)'$ symmetry breaking at TeV scale and the EW symmetry breaking at the weak scale.

In Figure 1 we plot the Higgs VEVs versus temperature for the given parameters values in case b (see Table 3). From the figure, one can see that the $U(1)'$ phase transition occurs just below 1200 GeV, and the EWPT at about 120 GeV. We are interested in whether the EWPT is strongly first order. From the figure, we also see that the VEVs of $H_d$, $H_u$ and $S$ all experience a first order transition due to their discontinuity at the weak scale. The magnitudes of the VEVs of the Higgs fields are plotted at the global minimum of the potential for the given temperature. There is an abrupt change in the $S_i$ VEVs at temperatures around 400–450 GeV because another

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6We ignore the possibility of charge or color breaking minima.
Figure 1: The Higgs VEVs vs. temperature $T$. The VEVs of $H^0_{u,d}$ and $S$ are multiplied by 10. The VEVs of $S_1$, $S_2$ and $S_3$ are indistinguishable.

local minimum becomes the global one.

After the $U(1)'$ symmetry is broken, $S_1$, $S_2$, and $S_3$ have non-zero VEVs of order 2 TeV, while the VEVs for $H^0_d$ and $H^0_u$ are still zero. The supersymmetry breaking soft terms $m^2_{S_1}, m^2_{S_2}$ generate a linear term for $S$ in the potential after the $U(1)'$ symmetry breaking, which induces a non-zero VEV for $S$. At high temperature, $\langle S \rangle$ is tiny because $m^2_{S_1}$ and $m^2_{S_2}$ are small and the temperature dependent corrections to the $S$ mass are large. At lower temperature, $\langle S \rangle$ becomes larger, as can be seen from Figure 1. At the EWPT the term $A_h hSH_uH_d$ induces a first order phase transition in the VEV of $S$ as well.

The first order EWPT can be strong because of the trilinear term $A_h hSH_uH_d$ in the tree-level potential. Unlike in the MSSM, in which the phase transition is associated with loop effects, there is no upper bound on the lightest stop mass. This feature is independent of the concrete embeddings of the sMSSM, though we are using an effective potential of finite temperature in Model I as an illustration. This feature is also shared by the NMSSM, the nMSSM, the $U(1)'$ models with no secluded sector, and the singlet extensions of the SM.

In Table 4, we calculate $v(T_c)/T_c$ versus the stop soft mass square $m^2_{\tilde{Q}_3}$, $m^2_{\tilde{u}_R}$ and $A_h$ while the other parameters are fixed to the values given by Table 3 with the

8In a real transition the fields would most likely remain in the same minimum, but that would have little effect on our conclusions. Detailed investigations of issues such as these are very model dependent and beyond the scope of this investigation.
Table 4: $v(T_c)/T_c$ versus the stop soft mass squares $m^2_{\tilde{Q}_3}$, $m^2_{\tilde{u}_R}$, and $A_h$, where the other parameters are fixed to the values in case b (see Table 3). For simplicity, we assume $m^2_{\tilde{Q}_3} = m^2_{\tilde{u}_R}$. The scales are different for different points, but typically they are about 100 GeV. The slashes in the table mean that there is no first order EWPT.

| $m^2_{\tilde{Q}_3}$ = $m^2_{\tilde{u}_R}$ | $A_h$ | 2.0 | 3.0 | 4.0 | 5.0 |
|---|---|---|---|---|---|
| 4.0 | / | 1.29 | 1.59 | 2.06 |
| 8.0 | 0.91 | 1.02 | 1.21 | 1.50 |
| 12.0 | / | 0.56 | 1.03 | 1.22 |
| 16.0 | / | / | / | 1.37 |

choice $m^2_{H_d^0} = 9.0$. The scales are different for different points, but typically they are about 100 GeV. As expected, when $A_h$ decreases (with fixed $m^2_{\tilde{Q}_3}$ and $m^2_{\tilde{u}_R}$), $v(T_c)/T_c$ decreases. The first order EWPT is not strong enough when $A_h \leq 2 - 3$. This table also displays the dependence of $v(T_c)/T_c$ on $m^2_{\tilde{Q}_3}$ and $m^2_{\tilde{u}_R}$. For fixed $A_h$, the strength of the first order EWPT usually becomes weaker as $m^2_{\tilde{Q}_3}$ increases. Unlike the MSSM, even for heavy stop soft masses (or physical masses) one can still have a strong enough first order EWPT.

For simplicity, we did not turn on any CP phases for the discussion in this section. We will see later that the results obtained in this section are insensitive to these phases.

5 Generation of CP Violation

5.1 CP Violation at Tree Level

One nice feature of the sMSSM is that both ECPV and SCPV can happen in the Higgs sector at tree level [70]. This model involves six complex neutral fields. To study the CP-violation, we define

$$H^0_d \equiv |H^0_d|e^{i\theta_1}, \quad H^0_u \equiv |H^0_u|e^{i\theta_2}, \quad S \equiv |S|e^{i\alpha},$$

$$S_1 \equiv |S_1|e^{i\alpha_1}, \quad S_2 \equiv |S_2|e^{i\alpha_2}, \quad S_3 \equiv |S_3|e^{i\alpha_3}. \quad (35)$$

Among these six phase degrees of freedom, only four of them, which are given by

$$\beta_1 = \alpha + \alpha_1, \quad \beta_2 = \alpha + \alpha_2,$$

$$\beta_3 = \alpha + \theta_1 + \theta_2, \quad \beta_4 = \alpha_1 + \alpha_2 + \alpha_3. \quad (36)$$
are invariant under the $U(1)_Y$ and $U(1)'_Y$ gauge transformations. The two remaining
degrees of freedom

$$\Sigma_{i}Y_{\phi_i}\theta_{\phi_i}, \Sigma_{i}Q'_{\phi_i}\theta_{\phi_i},$$

(37)
correspond to Goldstone bosons associated with the two spontaneously broken gauge
symmetries, and decouple from the Higgs sector. Here, $Y_{\phi_i}$ and $Q'_{\phi_i}$ are the $U(1)_Y$ and
$U(1)'_Y$ charges for $\phi_i$, $\theta_{\phi_i}$ is the phase of $\phi_i$, and the index “$\phi_i$” runs over all six complex
Higgs fields. The complex phases of the Higgs fields $\theta_{\phi_i}$ are linear combinations of the
gauge-invariant and gauge-dependent components. Of course, physical consequences
only depend on the gauge-invariant components.

There are five supersymmetry breaking complex parameters in the tree-level
Higgs potential Eq (8-10), $A_{h}, A_{\lambda}, m_{2SS_1}, m_{2SS_2}^2, m_{S_1S_2}^2$. Four of the five complex
phases can be resolved by $\beta_{1,2,3,4}$. Without loss of generality, we assume that

$$m_{S_1S_2}^2 = |m_{S_1S_2}^2|e^{i\gamma}.$$ (38)
is complex and the other parameters are real and positive. If $\gamma$ is not equal to zero
or $\pi$, there will be ECPV at the tree level. Unlike the MSSM, SCPV can also occur
at tree level in this model. Let us rewrite the supersymmetry breaking soft terms for
the neutral Higgs bosons as

$$V_{soft}^H = m_{H_d}^2|H_d^0|^2 + m_{H_u}^2|H_u^0|^2 + m_{S}^2|S|^2 + \sum_{i=1}^{3} m_{S_i}^2|S_i|^2$$

- $2A_{h}|S||H_d^0||H_u^0|\cos \beta_3 - 2A_{\lambda}|S_i||S_2||S_3|\cos \beta_4$
- $2m_{2SS_1}^2|S||S_1|\cos \beta_1 - 2m_{2SS_2}^2|S||S_2|\cos \beta_2$
- $2|m_{S_1S_2}^2||S_1||S_2|\cos(-\beta_1 + \beta_2 + \gamma).$ (39)

It is easy to see that a mildly dominant $m_{S_1S_2}^2$ soft term will make $\beta_1$ or $\beta_2 \neq 0, \pi$ for
$\gamma \neq 0, \pi$ and hence lead to SCPV. Meanwhile, the strength of SCPV can be controlled
by the explicit $CP$ phase $\gamma$.

5.2 $CP$ Violation at Finite Temperature

Explicit $CP$ phases from soft parameters beyond the Higgs sector could also con-
tribute to EWBG. However, it is advantageous to turn off these phases and use the
spontaneous $CP$ phases as the source for baryogenesis. To implement the EWPT, we
need to include radiative corrections to the Higgs potential, as discussed in Section 3.
At finite temperature there may coexist multiple vacua, in which spontaneous $CP$
phases usually have different values. If the phase transition between two vacua is
realized through bubble nucleation, as in a first order EWPT, these $CP$ phases are
space-dependent as one crosses the bubble wall. In the next sections we will see that
the generated baryon asymmetry depends on these variations.
To make this more explicit, it is convenient to write the particle interactions during EWBG using the individual Higgs phases. We define the two gauge-dependent phases as $A$ and $B$, given by

\[
Y_{H_d} \theta_1 + Y_{H_u} \theta_2 + Y_S \alpha + \Sigma_i Y_S \alpha_i = A,
\]
\[
Q'_{H_d} \theta_1 + Q'_{H_u} \theta_2 + Q'_S \alpha + \Sigma_i Q'_S \alpha_i = B.
\]

(40)

Inverting Eq. (36) and Eq. (40), we obtain

\[
\theta_1 = -\frac{1}{5} \beta_1 - \frac{1}{5} \beta_2 + \frac{7}{15} \beta_3 + \frac{2}{15} \beta_4 - \frac{11}{25} A - \frac{4}{15} B,
\]
\[
\theta_2 = -\frac{1}{5} \beta_1 - \frac{1}{5} \beta_2 + \frac{7}{15} \beta_3 + \frac{2}{15} \beta_4 + \frac{14}{25} A - \frac{4}{15} B,
\]
\[
\alpha = \frac{2}{5} \beta_1 + \frac{2}{5} \beta_2 + \frac{1}{15} \beta_3 - \frac{4}{15} \beta_4 - \frac{3}{25} A + \frac{8}{15} B,
\]
\[
\alpha_1 = \frac{3}{5} \beta_1 - \frac{2}{5} \beta_2 - \frac{1}{15} \beta_3 + \frac{4}{15} \beta_4 + \frac{3}{25} A - \frac{8}{15} B,
\]
\[
\alpha_2 = \frac{2}{5} \beta_1 + \frac{3}{5} \beta_2 - \frac{1}{15} \beta_3 + \frac{4}{15} \beta_4 + \frac{3}{25} A - \frac{8}{15} B,
\]
\[
\alpha_3 = \frac{1}{5} \beta_1 - \frac{1}{5} \beta_2 - \frac{1}{15} \beta_3 + \frac{7}{15} \beta_4 - \frac{6}{25} A + \frac{16}{15} B,
\]

(41)

where we have used the rescaled $U(1)_Y$ and $U(1)'$ charges for the Higgs particles in Model I

\[
Y_{H_d} = -Y_{H_u} = -1, \quad Y_S = Y_{S_1} = Y_{S_2} = Y_{S_3} = 0;
\]
\[
Q'_{H_d} = -\frac{7}{20}, \quad Q'_{H_u} = \frac{1}{10}, \quad Q'_{S_1} = -Q'_{S_2} = -\frac{1}{2} Q'_{S_3} = -\frac{1}{4}.
\]

(42)

(43)

Because $A$ and $B$ correspond to the Goldstone bosons of the $U(1)_Y$ and $U(1)'$ breaking, they can be rotated away by gauge transformations.

The variations of the Higgs phases crossing the bubble wall are given by

\[
\Delta \langle \theta_{\phi_i} \rangle = \langle \theta_{\phi_i} \rangle_t - \langle \theta_{\phi_i} \rangle_f,
\]

(44)

where $\langle \theta_{\phi_i} \rangle_{f,t}$ are the values of the Higgs phases in the false and true vacua, respectively. More explicitly,

\[
\Delta \theta_1 = \Delta \theta_2 = -\frac{1}{5} \Delta \beta_1 - \frac{1}{5} \Delta \beta_2 + \frac{7}{15} \Delta \beta_3 + \frac{2}{15} \Delta \beta_4
\]
\[
\Delta \alpha = \frac{2}{5} \Delta \beta_1 + \frac{2}{5} \Delta \beta_2 + \frac{1}{15} \Delta \beta_3 - \frac{4}{15} \Delta \beta_4,
\]
\[
\Delta \alpha_1 = \frac{3}{5} \Delta \beta_1 - \frac{2}{5} \Delta \beta_2 - \frac{1}{15} \Delta \beta_3 + \frac{4}{15} \Delta \beta_4,
\]
\[
\Delta \alpha_2 = \frac{2}{5} \Delta \beta_1 + \frac{3}{5} \Delta \beta_2 - \frac{1}{15} \Delta \beta_3 + \frac{4}{15} \Delta \beta_4,
\]
\[
\Delta \alpha_3 = \frac{1}{5} \Delta \beta_1 - \frac{1}{5} \Delta \beta_2 + \frac{2}{15} \Delta \beta_3 + \frac{7}{15} \Delta \beta_4.
\]

(45)
These quantities are not uniquely defined, due to the uncertainties in the winding numbers of the $\langle \beta_i \rangle_{f,t}$. If there is a large difference between the winding numbers of $\langle \beta_i \rangle_f$ and $\langle \beta_i \rangle_t$, then $\Delta \theta_{\phi_i}$ can be large, and hence the variation of $\theta_{\phi_i}$ in the bubble wall is also large. However, the nucleation rate for a phase transition between two such vacua is generically small [40], so we will require $|\beta_i| < \pi$.

Let us consider how $\Delta \theta_{\phi_i}$ is affected by the behavior of the related field variables in the wall. For a qualitative discussion, we neglect all loop as well as finite temperature corrections, even though they are important for the complete physical picture. Due to $h \gg \lambda$, the $|S_i|$ are fixed to almost the same large VEVs in both the symmetric and broken phases, which leads to $\langle \beta_i \rangle_{f,t} = 0$ and $\Delta \beta_4 = 0$. The change in $\beta_3$, however, is subtle. For the unbroken phase, $v_1 = v_2 = 0$ and hence $\beta_3$ is not fixed. But for non-local EWBG the relevant scattering processes mainly occur in the bubble wall where $v_{1,2}$ do not vanish, so we can understand $\beta_3$ in the unbroken phase as its asymptotic value as $v_1$ and $v_2$ approach zero. From the tree-level potential Eq. (39), it is easy to see that the non-vanishing Higgs VEVs lead to $\beta_3 = 0$ inside the bubble wall except the boundary region between the wall and the false vacuum. We simply assume $\Delta \beta_3 = 0$. The phase changes of the Higgs fields therefore are mainly due to $\Delta \beta_1$ and $\Delta \beta_2$. To illustrate how to obtain large $\Delta \beta_1$ and $\Delta \beta_2$, let us consider their first order differential equations from the tree-level potential Eq. (39),

$$
\begin{align*}
\left( m_{SS_1}^2 |S||S_1| \cos \beta_1 + |m_{S_1S_2}|^2 |S_1||S_2| \cos(-\beta_1 + \beta_2 + \gamma) \right) \Delta \beta_1 \\
-|m_{S_1S_2}|^2 |S_1||S_2| \cos(-\beta_1 + \beta_2 + \gamma) \Delta \beta_2 = -m_{S_1S_1}^2 |S_1| \sin \beta_1 \Delta |S| \\
\left( m_{SS_2}^2 |S||S_2| \cos \beta_2 + |m_{S_1S_2}|^2 |S_1||S_2| \cos(-\beta_1 + \beta_2 + \gamma) \right) \Delta \beta_2 \\
-|m_{S_1S_2}|^2 |S_1||S_2| \cos(-\beta_1 + \beta_2 + \gamma) \Delta \beta_1 = -m_{S_2S_2}^2 |S_2| \sin \beta_2 \Delta |S|.
\end{align*}
$$

(46)

Obviously, large $\Delta \theta_{\phi_i}$ or large $\Delta \beta_1$ and $\Delta \beta_2$ requires large $\Delta |S|$, and also depends significantly on $\gamma$.

Let us consider a parameter subspace: $100|m_{S_1S_2}|^2 > m_{S_1S_1}^2 \sim 0.1m_{SS_2}^2 > 10|m_{S_1S_2}|^2$. Assume that $\langle S \rangle/\langle S_{1,2,3} \rangle$ is small ($\sim 0.01$) in the false vacuum and large ($\sim 0.1$) in the true vacuum. In the broken phase, the $m_{S_1S_1}$ and $m_{SS_2}$ terms will dominate over the $m_{S_1S_2}$ term in the tree-level potential and thus favor small $\beta_1$ and $\beta_2$. In the symmetric phase, the dominant terms change to $m_{SS_2}$ and $m_{S_1S_2}$ due to the sharp decrease of the VEV of $S$. For $\gamma \sim \pi$, $\beta_1$ is pushed to a large value in this region. A large $\Delta \beta_1$ is then generated. We will follow this line to generate large $\Delta \theta_{\phi_i}$ in our analyses. In Figure 2 we illustrate the $\gamma$-dependence of $\beta_1$ and $\beta_2$, in the false and true vacua respectively. The relevant $\gamma$-dependence of $\Delta \theta_{\phi_i}$ is illustrated in Figure 2. For both of them, the parameter values are from case b (see Table 3).

Besides the baryon asymmetry, it would be desirable to also account for the CDM. In the scenario of the lightest neutralino $\chi_1^0$ as a CDM candidate, the CDM generation has a strong dependence on the mass $(m_{\chi_1})$ and composition of $\chi_1^0$. Since $m_{\chi_1}$ is sensitive to the soft mass terms of $H_d$, $H_u$ and $S$ due to their roles in determining tan $\beta$, we will consider EWBG with respect to $\gamma$ in three cases first (cases a, b and c) which are specified by different $m_{H_d}^2$ values (see Table 3), and then discuss
Figure 2: $\langle \beta_{1,2} \rangle_{f,t}$ vs. $\gamma(\pi)$, in case b.

Figure 3: $\Delta \theta_{\phi_i}$ vs. $\gamma(\pi)$, in case b.
the continuous dependence of EWBG and the $\chi_1^0$ relic density on $m_{H_d}^2$ for a given $\gamma$ value. In Figure 4 we show the $\gamma$ dependence of $\Delta \theta_1 = \Delta \theta_2$ and $\Delta \alpha$ in cases a, b and c (since the baryon asymmetry generation mainly couples to these three phases). From this figure we see that for a given $\gamma$ value a larger $m_{H_d}^2$ value gives a smaller phase variation crossing the wall. This is because a large $m_{H_d}^2$ value make $|S|$ small in both vacua, which further leads to a small $\Delta|S|$ (or small $\Delta\beta_i$ due to Eq. (46)). As expected from the previous discussion, the large $|\Delta \theta_{\phi_i}|$ appears in the large $\gamma$ region. For $\gamma = \pi$ there exists a $Z_2$ discrete symmetry in the neutral Higgs potential which might cause a dilution between matter and anti-matter bubbles for SCPV-induced EWBG (the explicit CP breaking from the fermion sector avoids problems with cosmological domain walls [106]). The vacua degeneracy is lifted as long as $\gamma$ deviates from $\pi$. Because the nucleation rates of these two vacua are exponentially biased by the difference of their energy densities, a slight shift from $\pi$ for $\gamma$ can greatly diminish the dilution. In this article therefore we will choose $\pi/2 \leq \gamma < \pi$.

5.3 New Contributions to Electric Dipole Moments

In this subsection, let us discuss possible new contributions to the EDMs in the SCPV-induced baryogenesis (also see [70]). According to the discussions in the previous subsection, no CP violation occurs at zero temperature for $\gamma = 0, \pi$, so there is no EDM problem.

For $\gamma \neq 0, \pi$, the Higgs fields may obtain nonzero phases at zero temperature.
However, their contributions to the EDMs are highly suppressed. Among the four gauge independent phases, \( \beta_3 \) and \( \beta_4 \) are zero due to the \( A_h \) and \( A_\lambda \) terms, so there will be no phases entering the \( h \) and \( A_h \) terms after we do the field redefinitions. The only phases are associated with the \( m_{SS_1}^2 \), \( m_{SS_2}^2 \) and \( m_{S_1S_2}^2 \) terms. (The phases will enter the fermion Yukawa couplings also, but they only lead to overall phases in the mass matrices which do not affect the CKM matrix.) Therefore, the \( CP \) violation will only affect the Higgs sector.

The Feynman diagrams contributing to the electron and quark EDMs which are mediated by the Higgs fields involve two vertices containing the Yukawa couplings, and include a mass insertion on the fermion line to flip the chirality. The EDMs will therefore be proportional to the cube of Yukawa coupling, making it very small. The EDMs are also proportional to the \( CP \) violation term which comes from the secluded sector and couples very weakly to the ordinary fermions. In particular, if any of the three soft mass terms \( m_{SS_1}^2 \), \( m_{SS_2}^2 \) and \( m_{S_1S_2}^2 \) vanish, there will be no tree-level \( CP \) violation. Also, for the parameters that we consider, the imaginary part of \( m_{S_1S_2}^2 \) is very small. Combining all these effects, new EDM contributions from the Higgs sector are very small. Our numerical estimate shows that they are 6 to 7 orders smaller than the experimental upper limit.

### 6 Wall Thickness and Velocity

In the early Universe the first order EWPT is realized by nucleating bubbles of the broken phase. To calculate the baryon asymmetry, understanding the dynamical behavior of these bubbles, such as the nucleation rate, expansion velocity, and bubble wall profile, is important but analytically difficult. In this section, we will focus on the wall profile and wall velocity.

#### 6.1 Wall Thickness

The kinetic Lagrangian for the complex scalar fields can be written as

\[
\mathcal{L} \sim \sum_i \left( \frac{1}{2} \partial_\mu \phi_i \partial^\mu \phi_i + \frac{\phi_i^2}{2} \partial_\mu \theta_{\phi_i} \partial^\mu \theta_{\phi_i} \right),
\]

where \( \phi_i \) and \( \theta_{\phi_i} \) collectively denote the magnitudes and phases of the Higgs fields, respectively. For the unitary gauge, we are left with 10 dynamical equations (with respect to 6 magnitudes and 4 gauge-invariant phase variables). Once the stationary solutions to these 10 equations are found, we can obtain the relevant physical information on the bubble wall, such as the wall thickness and velocity. However, these classical field equations are extremely complicated, so we have to solve them numerically.

A useful numerical method was devised in [72, 73, 74]. The basic idea [72] is that to solve the classical field equations one has to find the field configuration for
which
\[
S_A = \int_{-\infty}^{+\infty} dz \left[ \sum_i E_{\phi_i}(z)^2 + \frac{1}{T_c^2} \sum_j E_{\beta_j}(z)^2 \right] = 0, \tag{48}
\]
where \(E_{\phi_i}(z)\) and \(E_{\beta_j}(z)\) are the relevant field equations under the planar wall approximation. \((E_{\beta_j}(z)\) is rescaled by a factor \(\frac{1}{T_c}\), so that \(E_{\text{magnitude}}(z)\) and \(E_{\text{phase}}(z)\) have the same dimensions.) This can be achieved by searching for the absolute minimum of the action \(S_A\). We need to find an appropriate ansatz satisfying the boundary conditions at infinity as a first step. The kink ansatz is especially suitable since it satisfies the boundary conditions very well and has a smooth, gradually changing behavior as one moves from negative infinity to positive infinity. Therefore, we will take the kink ansatz for the independent field variables
\[
\phi_i(z) = \frac{\langle \phi_i \rangle_t + \langle \phi_i \rangle_f}{2} + \frac{\langle \phi_i \rangle_t - \langle \phi_i \rangle_f}{2} \tanh \left( \frac{z}{\delta} \right), \tag{49}
\]
\[
\beta_j(z) = \frac{\langle \beta_j \rangle_t + \langle \beta_j \rangle_f}{2} + \frac{\langle \beta_j \rangle_t - \langle \beta_j \rangle_f}{2} \tanh \left( \frac{z}{\delta} \right), \quad j = 1, 2, 4
\]
where the index "\(i\)" runs over the six complex neutral Higgs fields: \(H^0_d, H^0_u, S, S_1, S_2, S_3\), and "\(t\)" and "\(f\)" denote the VEVs in the true and false vacua, respectively. In our convention the true vacuum is located at \(z \to \infty\) and the false one is located at \(z \to -\infty\). For simplicity, we assume that there are no offsets which shift the fields against each other.

However, this assumption does not hold for \(\beta_3\). From the \(A_h\) term in Eq. (39), it is easy to see that, as \(\langle H^0_d \rangle\) and \(\langle H^0_u \rangle\) become non-trivial crossing the bubble wall, \(\langle \beta_3 \rangle\) will be suppressed to zero quickly. So a good ansatz for \(\beta_3\) should be
\[
\beta_3(z) = \frac{\langle \beta_3 \rangle_f}{2} \left[1 - \tanh \left( \frac{z + a}{\delta} \right) \right]. \tag{50}
\]
This is an anti-kink function and \(a \geq \delta\). For simplicity, we set \(a = \delta\), so all kink ansatzes are mediated by only one parameter – the wall thickness \(\delta\). This discussion does not contradict the assumption \(\Delta \beta_3 = \beta(\infty) - \beta(-\infty) = 0\) in Subsection 5.2 since there we are analyzing the phase change crossing the wall while \(\beta_3 \sim 0\) inside the wall.

Figure 5 describes the dependence of the wall thickness on \(\gamma\) \(\left(\frac{\pi}{2} \leq \gamma < \pi\right)\) by minimizing the action \(S_A\) over \(\delta\) for the parameter sets in Table 3. Again, the three curves correspond to the three different \(m^2_{H^0_d}\) values: 2, 4 and 6. These plots tell us that the wall thickness becomes larger as \(\gamma\) increases and reaches its peak value around \(\gamma = \pi\). In the action \(S_A\) the kinetic terms and potential slope terms are proportional to \(\delta^{-3}\) and \(\delta\), respectively. The wall thickness is a result of the balance between them. According to the discussions in Section 5, large \(\gamma\) induces large magnitudes for \(\Delta \beta_j\) or \(\Delta \theta_{\phi_i}\), which can increase the weight of the kinetic terms in \(S_A\). The wall thickness therefore is an approximately monotonic increasing function with respect to \(\gamma\). Numerical simulation shows that the wall thickness varies in the range \((3 - 30)T_c^{-1}\).
6.2 Wall Velocity

The velocity of the bubble wall also plays a crucial role in EWBG. As a technical requirement, the wall velocity cannot be smaller than 0.01. Otherwise, the effect of EW sphalerons and Higgs-mediated (in the thin wall regime) decay processes cannot be neglected while the $CP$-violating current is propagating in the false vacuum, and the new relaxation effect of EW sphalerons will appear before the wall recaptures the left-chiral particles composing the flux.

The wall velocity of a new-born bubble is mainly mediated by three kinds of forces: 1) the driving force ($F_d$) responsible for the bubbles’ expansion; 2) the shrinking force ($F_s$) due to the surface tension of the bubbles ($T_s$); and 3) the friction coming from the plasma ($F_f$). Though some progress has been reported in the MSSM recently [91], a systematic study on the dynamics of the bubble wall is still absent in the sMSSM. In the following, we will argue that the wall velocity in these models should be comparable to or smaller than that in MSSM or NMSSM under the thin wall approximation [92].

Consider two bubbles from the sMSSM and MSSM, respectively, and assume that the energy density differences across the wall, $\varepsilon = V_{\text{out}} - V_{\text{in}}$, are the same for both bubbles. In this section, a prime will be used to denote the quantities from the sMSSM. The driving force per area acting on the walls is $F_d = \varepsilon$ and the shrinking force is $F_s = 2T_s/r$, so we can approximately estimate the critical radii for both kinds

9This “thin wall” is different from the one defined in next section in that the comparison scale is the bubble radius rather than the mean free path of the scattered particles.
of bubbles by employing the equation $F_d = F_s$, which gives

$$r_c = \frac{2T_s}{\varepsilon},$$

$$r'_c = \frac{2T'_s}{\varepsilon}.$$  \hfill (51)

(52)

For $T'_s = aT_s$, we have $r'_c = ar_c$. Here $a \gg 1$ since the large VEVs of the Higgs fields in the secluded sector will contribute significantly to $T'_s$ according to the second term of Eq. (47).

Now suppose that both bubbles sweep a distance $\Delta r$, i.e., the bubble radii respectively increase to $r_c + \Delta r$ and $r'_c + \Delta r$, and that the wall tensions $T_s$ and $T'_s$ are left invariant during this process; then the shrinking forces due to the surface tension will become smaller. The net force for both bubbles are changed from zero to

$$\Delta F = F_d - F_s = \varepsilon - \frac{2T_s}{r_c + \Delta r},$$

$$\Delta F' = F'_d - F'_s = \varepsilon - \frac{2T'_s}{r'_c + \Delta r},$$

and

$$\Delta F - \Delta F' = 2T_s \left( \frac{-1}{r_c + \Delta r} + \frac{1}{r'_c + \Delta r/a} \right) > 0.$$  \hfill (55)

That is, in the sMSSM the shrinking force decreases more slowly if there is no other physics involved (note that the driving forces stay unchanged during this process).

On the other hand, due to thermal scattering from particles in the hot plasma, the bubble wall will suffer friction or thermal pressure $F_f$ during its expansion, where $F_f$ is determined by the thermal environment and the wall velocity relative to the plasma. Now suppose that bubbles in both models expand at the same velocities, microscopically this thermal pressure is mainly induced by momentum transfer of the scattered particles, particularly those with

$$m(\infty) \gg m(-\infty), \frac{k_i k_f}{|k_i||k_f|} < 0.$$  \hfill (56)

Besides the ones in the MSSM, the sMSSM contains extra light degrees of freedom, e.g., the singlets in the Higgs sector and the additional exotic particle species. We may well expect that in the sMSSM the thermal pressure is not smaller than that in the MSSM given $T_c = T'_c$ and $v_w = v'_w$. Therefore, the wall velocity in the sMSSM should be non-relativistic. In this article, we will set $v_w = 0.05$.

7 Non-local Electroweak Baryogenesis

In EWBG scenarios, the non-local baryogenesis or charge transport mechanism \cite{75} refers to the case where particles have $CP$-violating interactions with a bubble wall.
These $CP$-violating interactions cause an asymmetry in some $CP$-odd quantum numbers or transport charges other than $B$ carried by particle currents into the false vacuum. In front of the expanding bubble wall, where the thermal equilibrium is broken to some extent, the asymmetry of transport charges induces a non-vanishing number density for left-chiral particles carrying EW charges. This asymmetry is subsequently converted by the $B+L$-violating EW sphaleron processes, such as $t^R t^L \rightarrow 0$ and $b^R b^L \rightarrow 0$, into an asymmetry in baryon number. Some instant later the wall sweeps over the region. The generated net $n_B$ is then frozen in the bubble due to the exponentially suppressed EW sphaleron effects. There are two typical regimes relevant to non-local baryogenesis, depending on the particles under consideration: the thin wall regime for $\delta/\Delta < 1$ and the thick wall region for $\delta/\Delta \geq 1$, where $\delta$ is the wall thickness and $\Delta$ is the particle’s mean free path or propagation length.

For the thin wall regime [93, 94], the physical picture is relatively simple since we may neglect the damping effect on the $CP$-violating source caused by the self-energy thermal correction. The reflectivity and transmittivity of particle scattering can be calculated in quantum mechanics, where the fermions interacting with the bubble wall are treated as free particles with space-dependent masses, and the $CP$-violating source can be modeled as a delta function current.

For the thick wall regime, the effects of additional decay processes in the wall cannot be neglected, and hence modeling the $CP$-violating source will be very complicated. Generally, two parallel approaches are available: the semiclassical force method [95, 96] and spontaneous baryogenesis [97]. The first one is based on the fact that, under the WKB approximation ($p \gg \delta^{-1}$), a $CP$-violating mass term in the Lagrangian can induce a $CP$-violating shift in gradients of the dispersion relation at first order. This $CP$-violating source then appears in the diffusion equations, interpreted as one kind of “semi-classical” source. This approach has been applied to the MSSM [90] and the non-minimal NMSSM [45]. However, this approach neglects the contributions from the non-WKB region, which may greatly enhance the produced baryon asymmetry. The second approach is a little like the one used in the thin wall regime: the $CP$-violating source is the particle current reflected by the bubble wall, but the damping and multiple-scattering effects are included.

With $CP$-violating sources produced, the particles carrying the $CP$-odd transport charges will undergo a diffusion process outside the bubble. Dynamical baryogenesis requires a slight departure from thermal equilibrium in front of the wall, implying that not all particle “decay” processes (i.e., those which affect the charge number density) are fast or adiabatic compared to the expanding velocity of the bubble wall. If

$$\frac{D \Gamma}{v_w^2} > 1$$

(57)

with $\Gamma$ being the decay rate and $D$ the associated diffusion constant, the processes will be considered as fast, and vice versa. In our calculations, almost all processes included in the diffusion equations are fast, otherwise they will not have enough time to significantly affect the charge number density. The only exception is the EW
sphaleron process. We assume that they are slow during the diffusion and can be neglected in mediating the charge number density. We can therefore deal with the diffusion of the particles carrying transport charges and the generation of the net baryon number separately. We must then: (1) solve the diffusion equations of the particles carrying the transport charges, and find the number density of all left-chiral particles $n_L$ which carry EW gauge charges; (2) insert $n_L$ into the baryon diffusion equation, to find the generated baryon asymmetry. The description of the particle transport in front of the bubble wall by the diffusion equations is well-justified because $v_w \ll v_s$. Here $v_s = 1/\sqrt{3}$ is the sound speed in the plasma.

In the following, we will study the roles in EWBG played by different particles in the sMSSM. We will calculate the contributions from leptons in the thin wall regime, and the contributions from squarks, charginos and quarks in the thick wall regime by assuming the spontaneous baryogenesis mechanism.

### 7.1 The Lepton Contribution – Thin Wall Regime

In the supersymmetric extension of the SM, leptons generally have a mean free path in the thermal plasma much larger than the bubble wall thickness. To consider their contribution to EWBG, we can work in the relatively simple thin wall regime [93]. In the thin wall regime, the scattering of particles by the bubble wall is described in terms of freely propagating particles with space-dependent masses, with the Lagrangian

$$\mathcal{L} = \overline{\psi}_L i\gamma^\mu \partial_\mu \psi_L + \overline{\psi}_R i\gamma^\mu \partial_\mu \psi_R - m(z)\overline{\psi}_L \psi_R - m^*(z)\overline{\psi}_R \psi_L ,$$  

with the mass term determined by the Yukawa coupling. For charged leptons it is

$$m_l(z) = y_l v_1(z) e^{i\theta_1(z)} .$$  

(59)

Since the Yukawa coupling $y_\tau \gg y_{e,\mu}$, we will only study the contributions from $\tau$ leptons.

The $CP$-violating current in this case is produced by $\tau_R \rightarrow \tau_L$, where the $CP$-odd transport charge is axial $\tau$ number

$$Q^\tau_{\tau_L,R} = \text{Diag}(1,-1) .$$  

(60)

Explicitly solving the Dirac equation and only considering the contribution from the non-WKB momentum subspace ($k_i^z < \delta^{-1}$) as a conservative approximation, one obtains the $CP$-odd reflection asymmetry [93, 94]

$$\Delta R_{\tau_R \rightarrow \tau_L}(k_i, k_f) = 4t_\phi(1 - t_\phi^2) \int_{-\infty}^{\infty} dz \cos(2k_i^z z)\text{Im} \left[ m_\tau(z) m_\tau(\infty)^* \right]$$  

$$= -\frac{\sqrt{\pi}}{2} m_\tau(\infty)^2 \delta \Delta \theta_1 e^{-\frac{4}{k_i^2} \delta k_i^2} ,$$  

(61)

where $t_\phi = \tanh \phi$ and $\tanh(2\phi) = |m_\tau(\infty)|/|k_i^z|$. In the second line, the kink ansatz defined by Eq. (49) and the assumption of $\Delta \theta_1/2$ as perturbative (it turns out that $\frac{1}{2} \Delta \theta_1 \lesssim 0.3$ in the parameter region we are interested in) have been used.

29
The axial \( \tau \) current \( j_{\tau L} \) induced by the scattering of leptons is \[93, 94\]

\[
j_{\tau L} = \int_{k^2_i > m^2} \frac{d^3k_i}{(2\pi)^3} \frac{f^\tau_f(k^2_i)}{E} \left( f^\tau_f(k^2_i) - f^\tau_f(-k^2_i) \right) \Delta R_{\tau R \to \tau L}(k_i, k_f) Q^\tau_{\tau L} .
\] (62)

Here \( f_t \) and \( f_f \) in Eq. (62) are the Fermi-Dirac distributions in the wall frame for the particles localized in and outside the bubble, respectively, given by

\[
f^\tau(k^2_i) = \frac{\gamma^2_w E^{\gamma_w}(1 - v_w k^z_i E)}{e^{\gamma_w(E - v_w k^z_i E)}/T_c + 1} .
\] (63)

The chemical potential in the exponential factor is suppressed due to the large value of the critical temperature \( T_c \). Similarly, due to the exponential suppression for \( E > T_c \), we may only consider the contribution from the small energy region \( E < T_c \) and make a perturbative expansion with respect to \( E/T_c \), approximately yielding the formula for the injected \( CP \)-violating current

\[
j_{\tau L} = -\frac{m_{\tau}(\infty)^2 v_w \delta \Delta \theta_1}{(2\pi)^2} h(\delta, T_c) ,
\] (64)

where

\[
h(\delta, T_c) = \int_0^{T_c} dk_{i\perp} \int_0^{1/\delta} dk^z_i \frac{k_{i\perp}}{\sqrt{k^2_{i\perp} + (k^2_i)^2}} \left( \frac{\gamma_w k^z_i}{T_c} - \frac{k^z_i}{\sqrt{k^2_{i\perp} + (k^2_i)^2}} \right) + O[(E/T_c)^3] \) (65)

is a function of the wall thickness and critical temperature. The first term in Eq. (65) exactly leads to Eq. (25) in Ref. [94] up to an ansatz-dependent factor after \( h(\delta, T_c) \) is integrated out. The second term, which will suppress the injected chiral flux, is neglected in Ref. [94]. Note that the \( CP \)-violating current \( j_{\tau L} \) is proportional to \( m_{\tau}(\infty)^2 \). Given that the associated calculations can be applied to the other two families of leptons as well, we are justified to only consider the \( \tau \).

While propagating in the false vacuum, for a wall velocity that is not too large the behavior of the scattered leptons can be approximately described by diffusion processes. The injected current will lead to a “diffusion tail” of the particles in front of the moving wall. If this diffusion tail or persistence length \( \xi \) is much larger than the width of the wall, it will be a good approximation to describe the injected current as a delta-function-like source. While the current is propagating in the false vacuum, leptons will undergo some Higgs-mediated or chiral mixing decay processes which may have a negative effect on the \( CP \)-violating current amplitude. A reasonable assumption is that the characteristic decay time for leptons is much longer than the time it takes for the wall to recapture them. We can then neglect lepton decays or chiral mixing processes and only consider the left-chiral leptons propagating in the false vacuum. Furthermore, we also need to make an assumption that the EW sphaleron processes are too slow to alter the density in front of the wall significantly and thus can be neglected except for their role in generating baryons. These two
assumptions require $\frac{D_{\tau}}{v_{w}} < 1$ for the associated processes. For $\tau$ leptons, they work well if $v_{w} > 0.01$ [93]. Neglecting these decay processes, the diffusion equation in the bubble wall frame can be rewritten as [93]

$$\tau_{\tau L} n''_{\tau L}(z) - v_{w} n'_{\tau L}(z) + J_{\tau L} = 0,$$  \hspace{1cm} (66)

with

$$n'_{\tau L} = \frac{\partial n_{\tau L}(z)}{\partial z},$$

$$J_{\tau L} = -\xi_{\tau L} j_{\tau L} \delta'(z),$$  \hspace{1cm} (67)

where the persistence length $\xi_{\tau L}$ is given by $\xi_{\tau L} \sim 6 D_{\tau L} \langle v_{\tau L} \rangle$ [93], with $D_{\tau L} \sim 100 T_{c}^{-1}$ being the related diffusion constant [93] and $\langle v_{\tau L} \rangle$ being the average velocity of the injected current relative to the wall; $\delta'(z) = \partial \delta(z)/\partial z$ and $z$ is related to the coordinates in the plasma frame by $z = r + v_{w} t$. Solving Eq. (67) by combining with the boundary conditions

$$\lim_{|z| \to \infty} n_{\tau L}(z) = 0,$$  \hspace{1cm} (68)

one obtains

$$n_{\tau L}(z) = \begin{cases} C_{\tau L} e^{\frac{v_{w}}{D_{\tau L}}(z)} & \text{for } z < 0 \\ 0 & \text{for } z > 0 \end{cases},$$  \hspace{1cm} (69)

with

$$C_{\tau L} = 6 \langle v_{\tau L} \rangle j_{\tau L}.$$  \hspace{1cm} (70)

The baryon generation is described by the diffusion equation

$$D_{L} n''_{B} - v_{w} n'_{B} - n_{f} \Gamma_{\text{ws}} n_{L} = 0.$$  \hspace{1cm} (71)

Here $n_{L}$ is the net number density of left-chiral particles carrying EW charges. Only such particles are involved in the EW sphaleron processes and hence able to source the baryon generation. $D_{L}$ is effective diffusion constant. In the thin wall region, the main contribution to $n_{L}$ is from the scattering of $\tau$ leptons, so we have $n_{L} = n_{\tau L}$ and $D_{L} = D_{\tau L}$. Under the approximation ($\Theta(-z)$ is step function)

$$\Gamma_{\text{ws}}(z) = \Theta(-z) \Gamma_{\text{ws}}(-\infty) = 6 \kappa \alpha_{5} T_{c} \Theta(-z),$$  \hspace{1cm} (72)

with $\kappa \sim 21$ for $T_{c} > T_{EW} \sim 100 \text{ GeV}$ [98, 99], we find for $z \geq 0$

$$n_{B} = \frac{-n_{f} D_{\tau L} \Gamma_{\text{ws}} C_{\tau L}}{v_{w}^{2}}.$$  \hspace{1cm} (73)
Here the boundary condition \( n_B(-\infty) = 0 \) has been used. Then with \( s = \frac{2\pi^2 g_* T_c^3}{45} \) the baryon asymmetry is solved to be

\[
\frac{n_B}{s} = \frac{540\gamma_w^3(v_{\tau L}) D_{\tau L} m_{\tau}(\infty) \delta \Delta \theta_1 h(\delta, T_c) \Gamma_{ws}}{(2\pi)^4 v_w g_* T_c^3}.
\]

Eq. (74) tells us that in the thin wall regime of the non-local EWBG scenario, three model-dependent physical quantities play critical roles: the variation of the \( CP \) phase crossing the wall \( \Delta \theta_1 \), the wall thickness \( \delta \), and its velocity \( v_w \). \( \Delta \theta_1 \) will bias the symmetry between the dynamical behaviors of particles and anti-particles. For the perturbative approximation in the non-WKB momentum region, \( i.e., \) both \( \Delta \theta_1 \) and momentum are not large (\( \Delta \theta_1/2 < 1 \) and \( k_i^2 < \delta^{-1} \)), the \( CP \)-violating current and baryon asymmetry are proportional to \( \Delta \theta_1 \). It turns out that the contribution from the thin wall regime is non-trivial compared to the ones from the thick wall regime. The wall configuration is also essential. It measures the size of the region in which the \( CP \)-violating current is generated, and it provides the \( p_z \) upper limit or definition standard of non-WKB particles. From Eqs. (64) and (74), the \( CP \)-violating current and baryon asymmetry are proportional to the wall thickness, and are also affected by the wall thickness through the integration upper limit on \( p_z \). In addition, \( 1/v_w \) measures the time for which the EW sphaleron processes continue before the particles in the \( CP \)-violating current are recaptured by the bubble wall, so it is natural that the baryon produced asymmetry is inversely proportional to \( v_w \).

### 7.2 The Squark, Chargino and Quark Contributions – Thick Wall Regime

In this subsection we will calculate the baryon asymmetry generated by squarks, charginos and quarks by employing the spontaneous baryogenesis mechanism. Compared to the thin wall regime, there are two new effects which are important for our understanding of the interactions between particles and the bubble wall:

1. The effect of thermal scattering. This can lead to a self-energy correction to the particle propagators and hence suppress the \( CP \)-violating source (due to the shortened life-time of the particles), and will be taken into account by including the imaginary part of the fermion self-energy in the dispersion relation, \( i.e., \)

\[
[\omega(k) + i\tilde{\gamma}]^2 = k^2 + m^2,
\]

where \( \tilde{\gamma} \) is the associated damping rate. This immediately gives

\[
|\Im(k)| = \frac{1}{\Delta},
\]

with

\[
\Delta = \tau \left| \frac{\Re(k)}{\omega} \right|, \quad \tau = \frac{1}{\tilde{\gamma}}.
\]
Since $\tau$ is the lifetime of the scattered particles, and $\Re(k)/\omega$ is their velocity, $\Delta$ defines the propagation length of the scattered particles in the wall. In addition, from Eq. (75), we have

$$\omega^2 = \Re(k)^2 + m^2 + \tilde{\gamma}^2(1 - \frac{\omega^2}{\Re(k)^2}).$$

(78)

We will work in the limit $\tilde{\gamma}^2 \ll \Re(k)^2 + m^2$, which gives the relation

$$\omega^2 = \Re(k)^2 + m^2.$$  

(79)

(2) The effect of multiple scattering by the wall. This effect can be interpreted as multiple insertions in a perturbation regime. Currently, there are two main insertion methods: mass insertion [76] and Higgs insertion [31, 77]:

Mass insertion assumes that the tree-level physics is described by kinetic energy terms and considers the space-dependent mass terms as a perturbation, which can be denoted by

$$\mathcal{L} = \mathcal{L}_{\text{tree}} + \mathcal{L}_{\text{pert}} = K - M,$$

(80)

where $K$ is kinetic energy terms and $M$ is mass terms. Since tree-level physics in the mass insertion regime is a Lorentz-invariant free theory of massless particles, this method is effective only if $K \gg M$. However, due to the Boltzmann factor suppression for the injected $CP$-asymmetric current, the main contribution to the $CP$-violating source comes from the non-WKB momentum region, which usually makes multiple mass insertions unstable.

Compared with mass insertions, Higgs insertion is more extensively assumed in the literature. Its starting point is to separate the mass terms into a free part and perturbative part, i.e.,

$$\mathcal{L} = \mathcal{L}_{\text{tree}} + \mathcal{L}_{\text{pert}} = (K - M^0(z)) - \delta M(z, z_i),$$

(81)

where $z$ denotes the point where the $CP$-violating current is calculated, and also the mass eigenstates are defined, and $z_i$ denotes the point where the scattering occurs. Using these local mass eigenstates as approximate asymptotic states, one can calculate the associated $S$ matrix and the $CP$-violating source. If these local mass eigenstates are not the gauge eigenstates in the false vacuum, after the $CP$-violating source is found one needs to go back to the gauge eigenstates by taking an inverse unitary transformation to evaluate the diffusion equations in the false vacuum. This method is justified for any scattered particle species, but it requires that the Higgs field variations be small within a propagation length. This implies $\Delta \ll \delta$. This relation adds an upper bound on the particle momentum in the $z$ direction. Numerically, its effect is weak, since the contributions from the small momentum region to
the $CP$-violating current are more important, due to Boltzmann factor suppression in the high momentum region.

We will use Higgs insertion method. For given momentum and energy, the $CP$-violating current at point “$z$” is mainly generated in the layer between $z$ and $z + \Delta$. Since $\Delta \ll \delta$, at the lowest order we can use a formalism similar to that in the thin wall regime to approximately calculate the current. Then the generated baryon asymmetry can be solved by embedding this $CP$-violating source into the diffusion equations in the thick wall regime. This approximation is adequate for our purpose, i.e., for presenting typical features of EWBG in the sMSSM.

(I) The squark sector

The mass square matrix of the up-type squark is

$$M_{\tilde{u}} = \begin{pmatrix} M_{\tilde{u}_L}^2 & M_{\tilde{u}_{LR}}^2 \\ M_{\tilde{u}_{RL}}^2 & M_{\tilde{u}_R}^2 \end{pmatrix},$$

(82)

where the diagonal entries are defined by

$$M_{\tilde{u}_L}^2 = m_{\tilde{Q}_i}^2 + m_{\tilde{u}_i}^2 + \Delta_{\tilde{u}_L}^2 + \Delta_{\tilde{u}_L}'^2,$$

$$M_{\tilde{u}_R}^2 = m_{\tilde{u}_i}^2 + m_{\tilde{u}_i}^2 + \Delta_{\tilde{u}_R}^2 + \Delta_{\tilde{u}_R}'^2,$$

$$M_{\tilde{u}_{LR}}^2 = y_u (h s v_1 e^{i(\alpha + \theta_1)} - A_{h u} v_2 e^{-i\theta_2}),$$

$$M_{\tilde{u}_{RL}}^2 = y_u (h s v_1 e^{-i(\alpha + \theta_1)} - A_{h u} v_2 e^{i\theta_2}),$$

(83)

with

$$m_{\tilde{u}_i}^2 = g_{u_i}^2 v_1^2,$$

$$\Delta_{\tilde{u}_L}^2 = \left( \frac{1}{2} - \frac{2}{3} \sin^2 \theta_W \right) \frac{G_F^2}{2} (v_1^2 + v_2^2),$$

$$\Delta_{\tilde{u}_R}^2 = \frac{2}{3} \sin^2 \theta_W \cos(2\beta) \frac{G_F^2}{2} (v_1^2 + v_2^2),$$

$$\Delta_{\tilde{u}_L}' = \frac{1}{8} \Delta_{\tilde{u}_R}' = \frac{-1}{4\sqrt{15}} g_Z^2 (v_1^2 - v_2^2).$$

(84)

$M_{\tilde{u}}$ can be perturbatively expanded at “$z$” as

$$M_{\tilde{u}}^0 = M_{\tilde{u}}(z) = \begin{pmatrix} M_{\tilde{u}_L}^2(z) & M_{\tilde{u}_{LR}}^2(z) \\ M_{\tilde{u}_{RL}}^2(z) & M_{\tilde{u}_R}^2(z) \end{pmatrix} \approx \begin{pmatrix} m_{\tilde{Q}_i}^2 & 0 \\ 0 & m_{\tilde{u}_i}^2 \end{pmatrix},$$

$$\delta M_{\tilde{u}}^0 = \begin{pmatrix} M_{\tilde{u}_L}^2(z_i) - M_{\tilde{u}_L}^2(z) \\ M_{\tilde{u}_{RL}}^2(z_i) - M_{\tilde{u}_{RL}}^2(z) \end{pmatrix} \begin{pmatrix} M_{\tilde{u}_{LR}}^2(z_i) - M_{\tilde{u}_{LR}}^2(z) \\ M_{\tilde{u}_R}^2(z_i) - M_{\tilde{u}_R}^2(z) \end{pmatrix}. $$

(85)
In the first line, we have made a first order approximation for $M^0_u$. In the second line, “$z_i$” denotes the space point where the particle scattering occurs.

According to the bilinear interactions, a $CP$-violating left-chiral squark current $j_{\tilde{u}_L}$ is produced through $\tilde{u}_R \rightarrow \tilde{u}_L$, where the $CP$-odd transport charge is axial top number

$$Q^{t}_{L,R} = \text{Diag}(1, -1).$$ (86)

Considering that this is mediated by the Yukawa couplings of the up-type quarks and that the Yukawa coupling of the top quark $y_t = y_{u3}$ is much larger than the other two, $i.e.$, $y_{u1,u2}$, we neglect the contributions from the first two family squarks. After a field theory calculation (for details, see Appendix C), we find

$$\Delta R_{i_R \rightarrow i_L}(k_i, k_f, z) = \frac{-1}{4E_iE_f} \frac{1}{v^z_i v^z_f} \int_J dz_1 dz_2 \sin((k^z_i - k^z_f)(z_1 - z_2))$$

$$\{w_1(z_1)w_1(z_2)\sin(\theta'_1(z_1) - \theta'_1(z_2))$$

$$+ w_2(z_1)w_2(z_2)\sin(-\theta_2(z_1) + \theta_2(z_2))$$

$$+ w_1(z_1)w_2(z_2)\sin(\theta'_1(z_1) + \theta_2(z_2))$$

$$+ w_2(z_1)w_2(z_2)\sin(-\theta_2(z_1) - \theta'_1(z_2))$$

$$(z_1 \rightarrow z) - (z_2 \rightarrow z)\},$$ (87)

with

$$E_i^2 = E_f^2 = (k_i)^2 + m_{iR}^2 = (k_f)^2 + m_{iL}^2,$$

$$v^z_i = \frac{k^z_i}{E_i}, \quad v^z_f = \frac{k^z_f}{E_f},$$

$$w_1(z) = y_h s(z) v_1(z),$$

$$w_2(z) = -y_t A_{h_t} v_2(z),$$

$$\theta'_1(z) = \alpha(z) + \theta_1(z),$$

$$\tilde{\gamma}_i = \tilde{\gamma}_{i_R} + \tilde{\gamma}_{i_L}.$$ (88)

Here the indices “$i$” and “$f$” denote the initial and final states of the scattering, respectively; and $(z_i \rightarrow z)$ denotes terms similar to the first four ones in $\{\}$, but with $z_i$ replaced by $z$. In this paper, we set the damping rates $\tilde{\gamma}_{i_R} = \tilde{\gamma}_{i_L} = 0.1T_c \sim \alpha_s T_c$ as an approximation, since an analytical calculation for them is still absent. $\Delta R_{i_R \rightarrow i_L}(k_i, k_f, z)$ is sensitive to the $A_{h_t}$ soft parameter due to its dependence on $w_2$. This makes its contribution to EWBG very different for small and large $A_{h_t}$ (EWPT is not sensitive to $A_{h_t}$, since it enters the neutral Higgs effective potential only at loop level). Particularly, this contribution can be quadratically enhanced by a large $A_{h_t}$ through the $w_2 w_2$ term. We will discuss more on these issues in the next subsection. Using these results, the space-dependent $CP$-violating current is solved to be

$$j_{\tilde{u}_L}(z) = \int \frac{d^3k_i}{(2\pi)^3} \frac{k^z_i}{E_f} \left( f^z_{z+\Delta(\tilde{\gamma}_i)}(-k^z_i) - f^z_{z}(k^z_i) \right) \Delta R_{i_R \rightarrow i_L}(k_i, k_f, z) Q^t_{i_L}.$$ (89)
Here $f_{z_i}^R(k_i^z)$ and $f_{z_i+\Delta}^R(-k_i^z)$ are the thermal distributions of $\tilde{t}_R$ in the wall frame at 
"z" and 
"z + \Delta", respectively. Compared to the one defined in Eq. (63), they have a 
factor 3 difference due to QCD color as well as a sign-flip for the second term in the 
denominator.

(II) The chargino sector

The chargino mass matrix is

$$M_{\tilde{\chi}_\pm} = \begin{pmatrix} M_2 & u_2 e^{-i\theta_2} \\ u_1 e^{-i\theta_1} & \mu e^{i\alpha} \end{pmatrix}, \quad (90)$$

with $u_i = g_2 v_i/\sqrt{2}$ and $\mu \equiv h s$. It can be perturbatively expanded around $z$ as

$$M_{\tilde{\chi}_\pm}^0 = M_{\tilde{\chi}_\pm}(z),$$

$$\delta M_{\tilde{\chi}_\pm} = \begin{pmatrix} 0 & u_2(z_1)e^{-i\theta_2(z_1)} - u_2(z)e^{-i\theta_2(z)} \\ u_1(z_1)e^{-i\theta_1(z_1)} - u_1(z)e^{-i\theta_1(z)} & \mu(z_1)e^{i\alpha(z_1)} - \mu(z)e^{i\alpha(z)} \end{pmatrix}. \quad (91)$$

For the parameter values that we are using, $|u_{1,2}(z)| \ll M_2, |\mu(z)|$, so we simply 
neglect the off-diagonal elements in $M_{\tilde{\chi}_\pm}^0$. In this sector, the 
$CP$-violating Higgsino current is produced through $\tilde{W}^c \rightarrow \tilde{H}^c$, where the 
$CP$-odd transport charge is vector Higgs number

$$Q^H_{\tilde{W}^+,\tilde{H}^+,\tilde{W}^+\tilde{H}^-} = \text{Diag}(0, 1, 0, 1). \quad (92)$$

After a field theory calculation (for details, see Appendix [C]), we find

$$\Delta R_{\tilde{W}^c \rightarrow \tilde{H}^c}(k_i, k_f, z) = \frac{1}{E_i E_f v_i^z v_f^z} \int_{z_1, z_2 < 0}^{z_1 + \Delta(\tilde{\gamma}_c)} dz_1 dz_2 \sin((k_i^z - k_f^z)(z_1 - z_2))$$

$$\{k_i^z k_f^z[u_1(z_1)u_1(z_2) \sin(\theta_1(z_1) - \theta_1(z_2))$$

$$+ u_2(z_1)u_2(z_2) \sin(-\theta_2(z_1) + \theta_2(z_2)) +$$

$$M_2\mu(z)[u_1(z_1)u_2(z_2) \sin(\theta_1(z_1) + \theta_2(z_2) + \alpha(z))$$

$$+ u_2(z_1)u_1(z_2) \sin(-\theta_2(z_1) - \theta_1(z_2) - \alpha(z))]$$

$$-(z_1 \rightarrow z) - (z_2 \rightarrow z)\} = \frac{1}{E_i E_f v_i^z v_f^z} \int_{z_1, z_2 < 0}^{z_1 + \Delta(\tilde{\gamma}_c)} dz_1 dz_2 \sin((k_i^z - k_f^z)(z_1 - z_2))$$

$$\{k_i^z k_f^z[u_1(z_1)u_1(z_2) \sin(\theta_1(z_1) - \theta_1(z_2))$$

$$+ u_2(z_1)u_2(z_2) \sin(-\theta_2(z_1) + \theta_2(z_2)) +$$

$$M_2\mu(z)[u_1(z_1)u_2(z_2) \sin(\theta_1(z_1) + \theta_2(z_2) + \alpha(z))$$

$$+ u_2(z_1)u_1(z_2) \sin(-\theta_2(z_1) - \theta_1(z_2) - \alpha(z))]$$

$$-(z_1 \rightarrow z) - (z_2 \rightarrow z)\}, \quad (93)$$

where

$$E_i^2 = E_f^2 = (k_i)^2 + M_2^2 = (k_f)^2 + \mu(z)^2,$$

$$v_i^z = \frac{k_i^z}{E_i}, \quad v_f^z = \frac{k_f^z}{E_f},$$

$$\tilde{\gamma}_c = \tilde{\gamma}_{\tilde{W}^c} + \tilde{\gamma}_{\tilde{H}^c}. \quad (94)$$

\[10\]In the chargino sector, the $CP$-odd transport charge can also be axial Higgs number. But this 
source usually is suppressed by a fast Higgsino violating process $\Gamma_{\mu}$ corresponding to the 
h($S\tilde{h}_1\tilde{h}_2$) term in the Lagrangian [31]. We will therefore not consider it here.
For the damping rates, we will use the results in the MSSM, which are given by
\[ \tilde{\gamma}_{H^c} = 0.025T_c \] and \[ \tilde{\gamma}_{W^c} = 0.065T_c \] [100]. The CP-violating current is solved to be
\[ j_{\tilde{H}^c}(z) = \int_{k_i>0} \frac{d^3k_i}{(2\pi)^3} \frac{k_i^z}{E_f} \left( f_{z}^{\tilde{W}^c}(k_i^z) - f_{z+\Delta(\tilde{\gamma}_{W^c})}^{\tilde{W}^c}(-k_i^z) \right) \Delta R_{\tilde{W}^c \rightarrow \tilde{H}^c}(k_i, k_f, z) Q^H_{\tilde{H}^c, \tilde{H}^c}, \] (95)
where \( f_{z}^{\tilde{W}^c}(k_i^z) \) and \( f_{z+\Delta}^{\tilde{W}^c}(-k_i^z) \) are the thermal distributions of the charged gaugino \( \tilde{W}^c \) in the wall frame, which have a factor 2 difference from that defined in Eq. (63) due to \( \tilde{W}^c = \tilde{W}_L^c + \tilde{W}_R^c \).

(III) The quark sector

The top quark mass is
\[ m_t = h_t v^2 e^{i\theta_2} \] (96)
which can be perturbatively expanded around \( z \) as
\[ m_t^0 = m_t(z), \]
\[ \delta m_t = h_t v_2 (z_i e^{i\theta_2(z_i)} - h_t v_2 (z) e^{i\theta_2(z)}). \] (97)
In this sector, the CP-violating current is produced by \( t_R \rightarrow t_L \), where the CP-odd transport charge is axial top number
\[ Q_{t_{L,R}} = \text{Diag}(1, -1). \] (98)
Then (for details, see Appendix C)
\[ \Delta R_{t_R \rightarrow t_L}(k_i, k_f, z) = \frac{1}{E_i E_f v_i^z v_f^z} \int_z^{\Delta(\tilde{\gamma}_t)} dz_2 dz_1 \sin((k_i^z - k_f^z)(z_1 - z_2)) \]
\[ \{h_i^2 k_i^z k_f^z v_2(z_1) v_2(z_2) \sin(\theta_2(z_1) - \theta_2(z_2)) \]
\[ -(z_1 \rightarrow z) - (z_2 \rightarrow z) \}, \] (99)
where
\[ E_i^2 = E_f^2 = (k_i)^2 + |m_t^0(z)|^2 = (k_f)^2 + |m_t^0(z)|^2, \]
\[ v_i^z = \frac{k_i^z}{E_i}, \quad v_f^z = \frac{k_f^z}{E_f}, \]
\[ \tilde{\gamma}_t = \tilde{\gamma}_{t_R} + \tilde{\gamma}_{t_L}. \] (100)
Again, we approximate the damping rates by \( \tilde{\gamma}_{t_L} = \tilde{\gamma}_{t_R} = 0.1 T_c \sim \alpha_s T_c \). The trace in the \( S \) matrices of the chargino and top quark scattering gives an extra factor “4”,
so the factor "4" in the denominator of Eq. (C.11) disappears in both Eq. (93) and Eq. (99). The $CP$-violating current is solved to be

$$\mathbf{f_{t_L}(z) = \int_{k_i^+ > \Lambda_{t_L}} \frac{d^3k_i}{(2\pi)^3 E_f} \left( f_{z}^{TR}(k_i^2) - \frac{k_i^2}{\sqrt{k_i^2 - \Delta m_i^2}} f_{z+\Delta(\bar{\gamma}_i)}^{TR}(-k_i^2) \right) \Delta R_{tR} \rightarrow t_L(k_i, k_f, z) Q_{t_L}^i} \tag{101}$$

where $f_{z}^{TR}(k_i^2)$ and $f_{z+\Delta}(k_i^2)$ are the thermal distributions of the right chiral top quark in the wall frame, which have a factor 3 difference from that defined in Eq. (63) due to color,

$$\Delta m_i^2 = m_i^2(z + \Delta(\bar{\gamma}_i)) - m_i^2(z) \tag{102}$$

is the difference of the quark mass square at $z$ and $z + \Delta(\bar{\gamma}_i)$, and

$$\Lambda_{k_i}^L = \sqrt{m_i^2(\infty) - m_i^2(z)} \tag{103}$$

is the momentum cutoff. The factor in front of $f_{z}^{TR}(-k_i^2)$ in Eq. (101) is due to the large mass difference of the scattered particles inside and outside the bubble. For $\tau$ leptons, stops and charginos, this difference is small compared to $T_c$, so this factor is approximately equal to 1.

Before moving to the calculation of the produced baryon asymmetry, we have some comments on the $CP$-violating currents obtained from the lowest Higgs insertion. First, according to the discussions in Subsection 5.2, the single Higgs phases $\theta_{\phi_i}$ are not physical since their values depend on two decoupled degrees of freedom: $A$ and $B$. Indeed, though working in the $\theta_{\phi_i}$ basis, from Eq. (87), Eq. (93) and Eq. (99) we see that all of these currents depend on some special linear combinations of $\theta_{\phi_i}$, where their dependences on $A$ and $B$ are cancelled and they are only sensitive to $\beta_i(z)$. Second, for the $CP$-violating currents from the stop and the chargino sectors, the terms $\propto \omega_1 \omega_2$ and $\propto u_1 u_2$ will be reduced to the $v_1 \partial v_2 - v_2 \partial v_1$ type sources in the MSSM if $\theta_{1,2}(z) = 0$ and $\alpha(z)$ = constant. Though in this limit these contributions are trivial for $\partial \tan \beta = 0$, they are not in Eq. (87) and Eq. (93) due to the space-dependence of $\theta_{1,2}$ in the wall. Third, the terms $\propto v_1 v_i$ in Eq. (87) and Eq. (93) are zero in the limit $\theta_{1,2}(z) = 0$ and $\alpha(z)$ = constant. They therefore never appear in the MSSM currents and denote a new-type of $CP$-violating current in the stop and chargino sector. This is also the only type of $CP$-violating current at the leading order in the quark sector. The vector Higgsino current of the $v_1 \partial v_2 + v_2 \partial v_1$ type does not appear in our calculations since it denotes a resummation effect from higher order Higgs insertions [77, 31, 32]. Finally, we emphasize that all of these features are results of SCPV-driven EWBG, so they are not sensitive to the concrete embeddings of the sMSSM. (For different embeddings, the $U(1)_Y$ and $U(1)'$ gauge charges
in Eq. (43) and hence the formulas in Eq. (41) are different. However, we did not use these results when deriving the formulas of the $CP$-violating currents.)

The field theory calculations used in the thick wall regime cannot be directly applied to the thin wall regime. In the thin wall regime, the main contribution is from an interference effect between a $CP$ and a non-$CP$ term (see the Appendix of [94]). The non-$CP$ term is induced by the effect that the fundamental solutions inside and outside the bubble are different due to the mixing between left- and right-chiral fermions. In the quantum mechanics calculations of the reflection and transmission coefficients, this effect is explicitly included. Its strength can be measured by the parameter $t\phi$ which is defined in Subsection 7.1. In the thick wall regime, this effect is neglected in the field theory calculations. Instead we assume the same asymptotic states between the two sides of a layer inside the wall, which greatly simplifies the calculations. This assumption is justified by the fact that $t\phi$ is approximately estimated to be

$$t\phi \lesssim \frac{\Delta m(z)}{\Delta m} \sim \frac{\Delta}{2\delta} \ll 1$$

in the thick wall limit. Here $\Delta m(z) = m(z + \Delta) - m(z)$ is the mass difference of the incident particles between the two sides of the layer, $\Delta m = m(\infty) - m(-\infty)$ is their mass variation crossing the wall, and $\Delta$ is the thickness of the layer (see Eq. (77)).

The main contribution in the thick wall regime therefore is from an interference effect of the $CP$-violating terms. Actually, if we apply the field theory calculation to leptons (similar to what we did for quarks), we will get their subleading contribution, which is given in the Appendix of [94].

While these injected $CP$-violating Higgsino or squark currents propagate in the false vacuum, the original thermal equilibrium among different particle species is broken and these species may obtain nonvanishing net number densities due to the associated decay processes. In the thick wall regime, the main decay processes include [76]: (1) the Yukawa interaction $\Gamma_{yT}$ corresponding to all supersymmetric and soft trilinear interactions arising from the $y_T H_2 Q_L T_R$ term in the superpotential; (2) the Higgsino violating process $\Gamma_{\mu}$ corresponding to the $h(S)\tilde{h}_1 \tilde{h}_2$ term in the Lagrangian and supergauge interaction $\Gamma_{y}$; (3) the axial top number violating process $\Gamma_{m}$ and the Higgs-violating process $\Gamma_{h}$ corresponding to top quark mass effects and Higgs self interactions, respectively; (4) the strong sphaleron process $\Gamma_{ss}$ [101] and weak sphaleron process $\Gamma_{ws}$. $\Gamma_{m}$ and $\Gamma_{h}$ are suppressed in the false vacuum and $\Gamma_{ws}$ is suppressed in the true vacuum (for a strong enough first order EWPT); all of the others can occur both inside and outside of the bubble. For simplicity, we neglect all processes with a singlet component involved in the current propagation. In the concrete calculations, further simplifications are also made [76].

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As a first order approximation, we neglect the tau and bottom Yukawa interactions. For relevant discussion, see [102].
(a) We assume fast enough $\Gamma_g$ which thermally equilibrates $n_q$ and $n_{\bar{q}}$; thus we can describe the system by the densities $n_{QL} = n_{qL} + n_{\bar{q}L}$, $n_{TR} = n_{\tau R} + n_{\bar{\tau}_R}$, $n_{H_1} = n_{h_1} + n_{\tilde{H}_1}$ and $n_{H_2} = n_{h_2} + n_{\tilde{H}_2}$. (b) We assume fast enough $\Gamma_\mu$; then we have $n_{H_1} = n_{H_2}$ and hence are able to simplify the two quantities $n_{H_1}$ and $n_{H_2}$ into one $n_H = n_{H_1} + n_{H_2}$. (c) We assume slow $\Gamma_{ws}$, which allows us to ignore leptons for the particle diffusion and the $n_L$ generation. Under these approximations, the diffusion equations in the wall frame are\(^{12}\) [76]

\[
\begin{align*}
  v_\omega n'_{QL} &= D_{QL} n''_{QL} - \Gamma_{yt} \left[ \frac{n_{QL}}{k_{QL}} - \frac{n_{TR}}{k_{TR}} - \frac{n_H}{k_H} \right] - \Gamma_m \left[ \frac{n_{QL}}{k_{QL}} - \frac{n_{TR}}{k_{TR}} \right] \\
  &= 6\Gamma_{ss} \left[ 2 \frac{n_{QL}}{k_{QL}} - n_{TR} + 9 \frac{n_{QL} + n_{TR}}{k_{BR}} \right] + J_{iL} + J_{tL}, \quad (105) \\
  v_\omega n'_{TR} &= D_{QL} n''_{TR} + \Gamma_{yt} \left[ \frac{n_{QL}}{k_{QL}} - \frac{n_{TR}}{k_{TR}} - \frac{n_H}{k_H} \right] + \Gamma_m \left[ \frac{n_{QL}}{k_{QL}} - \frac{n_{TR}}{k_{TR}} \right] \\
  &= 3\Gamma_{ss} \left[ 2 \frac{n_{QL}}{k_{QL}} - n_{TR} + 9 \frac{n_{QL} + n_{TR}}{k_{BR}} \right] - J_{iL} - J_{tL}, \quad (106) \\
  v_\omega n'_{H} &= D_{H} n''_{H} + \Gamma_{yt} \left[ \frac{n_{QL}}{k_{QL}} - \frac{n_{TR}}{k_{TR}} - \frac{n_H}{k_H} \right] - \Gamma_h \frac{n_H}{k_H} + J_{\tilde{H}^c}, \quad (107)
\end{align*}
\]
with

\[
\begin{align*}
  J_{iL}(z) &= -\partial_z j_{iL}(z) \sim \frac{\tilde{\gamma}_i}{\langle v_{iL} \rangle} j_{iL}(z), \\
  J_{tL}(z) &= -\partial_z j_{tL}(z) \sim \frac{\tilde{\gamma}_t}{\langle v_{tL} \rangle} j_{tL}(z), \\
  J_{\tilde{H}^c}(z) &= -\partial_z j_{\tilde{H}^c}(z) \sim \frac{\tilde{\gamma}_{\tilde{H}^c}}{\langle v_{\tilde{H}^c} \rangle} j_{\tilde{H}^c}(z), \quad (108)
\end{align*}
\]

where $\langle v_{iL,tL,\tilde{H}^c} \rangle$ is average velocity of the scattered particles and $\{k_{QL}, k_{TR}, k_{BR}, k_H\}$ are statistical factors.

For light particles compared to $T_c$, their statistical factor is 2 for bosons and 1 for fermions. If these particles are very heavy, then their statistical factors are suppressed exponentially. Explicitly, this suppression factor is (e.g., see [103])

\[
12 \left( \frac{m}{2\pi T_c} \right)^{3/2} \exp \left( -\frac{m}{T_c} \right), \quad (109)
\]

with $m$ being the particle mass. In this paper, we assume that all charginos and neutralinos are light for simplicity. As for squark masses, we take $m_{\widetilde{Q}_3}^2 = m_{\widetilde{u}_R}^2 = 8$ while keeping the other squark soft mass squares to be 25 (see Table 3). This soft

\(^{12}\)These equations are only valid in the limit of non-relativistic wall velocity. This is also true for Eq. [103].
mass pattern is phenomenologically favored due to the large beta functions for $m_{Q_b}^2$ and $m_{u_R}^2$. According to Eq. (109), we immediately find\(^{13}\)

\[ k_{Q_L} \approx 10, \quad k_{T_R} \approx 5, \quad k_{B_R} \approx 3, \quad k_H \approx 12, \quad (110) \]

and

\[ \bar{D} = \frac{5}{28} D_{Q_L} + \frac{23}{28} D_H, \]
\[ \bar{\Gamma} = \frac{23}{336} (\Gamma_m + \Gamma_h), \]
\[ \bar{J} = \frac{23}{28} (J_{t_L} + J_{t_R} + J_{H_{t}}) \quad (111) \]

Here $\bar{D}$, $\bar{\Gamma}$, and $\bar{J}$ are the effective diffusion constant, decay rate and $CP$-violating current (for their definition, see \(^{76}\)). It has been implicitly assumed that the other Higgs singlets are not thermally equilibrated with the EW charged ones. Using $D_H \sim 110/T_c$ and $D_{Q_L} \sim 6/T_c$ \(^{93}\), one finds $\bar{D} \sim 91/T_c$. This result further constrains the wall velocity: slow weak sphaleron processes require

\[ \bar{D} \Gamma_{ws}/v_w^2 < 1, \quad (112) \]

or

\[ v_w > 0.02. \quad (113) \]

As for $\bar{\Gamma}$, using the estimation for $\Gamma_m(z) + \Gamma_h(z)$ with $\tan \beta > 1$ in \(^{76}\), we have

\[ \bar{\Gamma}(\infty) \approx 10^{-2} T_c. \quad (114) \]

To solve Eq. (105-107), we further assume that $\Gamma_{ss}$ and $\Gamma_{yT}$ are large, and approximate

\[ \Theta(z) \bar{J}(z) \rightarrow \bar{J}(z), \]
\[ \Theta(z) \bar{\Gamma}(\infty) \rightarrow \bar{\Gamma}(z) \quad (115) \]

following \(^{31, 76}\). Then using the boundary conditions $n_L(\pm \infty) = 0$, we find

\[ n_L = \sum_i (n_{q_i^L} + n_{\tilde{q}_i^L}) = C_{Q_L} e^{z v_w / \bar{D}} \quad (116) \]

for $z < 0$, where

\[ C_{Q_L} = \frac{5}{46} \frac{1}{D\lambda^+} \int_0^\infty d\zeta \bar{J}(\zeta) e^{-\zeta \lambda^+}, \quad (117) \]

\(^{13}\)As an approximation, we use their soft masses as the squarks’ physical masses.
with
\[ \lambda_+ = \frac{1}{2D} \left\{ v_\omega + \sqrt{v_\omega^2 + 4\Gamma(\infty)D} \right\} \approx 10^{-2}T_c. \] (118)

This solution is the same as the MSSM one obtained in [31, 76] except the coefficient, which is caused by the difference of the \( k_{Q_L}, k_{T_R}, k_{B_R}, k_H \) values. Solving the diffusion equation for \( n_B \), which has exactly the same form as Eq. (71) with \( D_L \to \bar{D} \) and \( C_L \to C_{Q_L} \), we obtain the baryon number density for the \( z \geq 0 \) region
\[ n_B = -\frac{n_f\bar{D}\Gamma wsC_{Q_L}}{v_\omega^2}, \] (119)

and finally
\[ n_B s = -\frac{45}{2\pi^2 g_s T_c^3} \frac{15\Gamma ws}{46v_\omega^2\lambda_+} \int_0^\infty d\zeta \bar{J}(\zeta) e^{-\zeta\lambda_+}. \] (120)

### 7.3 Numerical Results of Non-local EWBG

According to the previous sections, the explicit \( CP \) phase, \( \gamma \), plays a crucial role in mediating the EWBG. Both the spontaneous \( CP \) phases and the bubble wall physics show a strong dependence on it. There should be a similar \( \gamma \) dependence, therefore, for the generated baryon asymmetry.

Let us first consider the \( \gamma \) dependence of \( v(T_c)/T_c \), the measure of the EWPT strength. Some related issues on EWBG have been discussed in Section 4 where no
Figure 7: $\frac{n_B}{s}$ vs. $\gamma(\pi)$ for $\nu_w = 0.05$, in case a. There is a sign flip for $n_B/s$ in the region denoted by black-dashed line for the top quark curve.

Figure 8: $\frac{n_B}{s}$ vs. $\gamma(\pi)$ for $\nu_w = 0.05$, in case b. There is a sign flip for $n_B/s$ in the region denoted by black-dashed line for the top quark curve. The stop contribution is smaller than the scale of this figure.
Figure 9: $n_B/s$ vs. $\gamma(\pi)$ for $v_w = 0.05$, in case c. There is a sign flip for $n_B/s$ in the region denoted by black-dashed line for the top quark curve. The stop contribution is smaller than the scale of this figure.

ECPV is implicitly assumed. As expected, $v(T_c)/T_c$ shows a strong dependence on the soft SUSY breaking parameters $m_{\tilde{Q}_3}^2$, $m_{\tilde{u}_R}^2$ and $A_h$. In Figure 6, we illustrate the $\gamma$ dependence of $v(T_c)/T_c$. The weak $\gamma$ dependence is because $\gamma$ is from the secluded sector of the neutral Higgs potential, which has no direct coupling to $H_d^0$ and $H_u^0$. For all of the three cases a, b and c, the first order EWPT is strong enough for implementing the EWBG.

In Figure 7-9, we show the $\gamma$ dependence of the baryon asymmetry $|n_B/s|$ produced by $\tau$ leptons, top squarks, charginos and top quarks. We set $v_w = 0.05$. The produced baryon asymmetry is sensitive to $\gamma$. This is easy to understand since both the magnitude of SCPV and the bubble physics are sensitive to $\gamma$. Within theoretical uncertainties, the observed value $n_B/s = (8.82 \pm 0.23) \times 10^{-11}$ can be explained for a large range of $\gamma$ values. Top quarks play a significant role in all three cases because of the top Yukawa coupling enhancement. In addition, due to the large change of the top quark mass crossing the bubble wall, the cancellation between $f_{zR}^{\tau}(k^z)$ and $f_{z+\Delta}^{\tau}(k^z)$ in Eq. (101) is relatively weak. The stop contribution depends on the magnitude of the soft parameter $A_{h_t}$. For a small $A_{h_t}$, it is dominated by the $w_1 w_1 \sim s^2 v_1^2$ term in Eq. (87). As $m_{H_d^0}^2$ increases, $v_1$ and $s$ usually becomes smaller.

We did not consider the contribution from neutralinos. Since the relevant $CP$-violating sources and the interactions with the bubble wall are similar for neutralinos and charginos, their contributions are expected to be comparable with those from charginos. For relevant discussions in the MSSM, e.g., see [31].
and smaller. The stop contribution is then quartically suppressed. This explains
why the stop contributions are smaller in case b and c than in case a, and also
explains why the stop contribution is small for a small $m_{\chi_1^0}$ in Figure 15 of the next
section (for all figures in this article, we chose $A_{h_t} = 0.1$). For a large $A_{h_t}$, the stop
contribution is dominated by the $w_3 w_2 \sim A_{h_t}^2 v_2^2$ term in Eq. (87). It is not sensitive to
the soft parameter $m_{H_d}^2$ and can be comparable with or even larger than the chargino
contribution. Numerical results show that the stop contribution becomes comparable
with the chargino one as $A_{h_t}$ increases to $\sim T_c$ and with the top quark one as $A_{h_t}$
increases to $\sim 4T_c$. As pointed out in the previous section, the $v_2 v_2$-type $CP$-violating
current is absent at the leading order in the MSSM. Both the large contribution by top
quarks and the enhancement effect of the stop contribution by a large $A_{h_t}$ therefore
are absent in the MSSM.

In addition to $\gamma$, there are two other important factors which can influence
the produced baryon asymmetry. The first is $v_w$. One can see this from Eq. (74)
and Eq. (119). A larger baryon asymmetry can be generated if the bubble wall is
expanding at a more non-relativistic velocity; and vice versa. The second is the stop
masses. In this paper we assume the stop soft mass $|m_{\tilde{Q}_3}| = |m_{\tilde{u}_3^R}|$ to be heavier
than $T_c$, even though they are lighter than those of the other squarks and sleptons.
If they (or more precisely, the stop physical masses) are light compared to $T_c$, then
the associated statistical factors will be modified to [76]

$$ k_{QL} \approx 18, \quad k_{TR} \approx 9, \quad k_{BR} \approx 3, \quad k_H \approx 12, \quad (121) $$

which gives an additional factor 3 in the numerator of Eq. (120).

8 Neutralino CDM and Non-local EWBG

Besides the baryon asymmetry, another mystery is dark matter (DM), which has been
measured precisely by astrophysical and cosmological probes [104]:

$$ \Omega_{DM} h^2 = 0.106 \pm 0.008. \quad (122) $$

Given that supersymmetry with $R$-parity conservation could have a neutral LSP as
a good candidate for the latter, e.g., the lightest neutralino $\chi_1^0$, one naturally wants
to know whether or not these two cosmological problems can be solved in the same
framework.

To answer this question, we will not scan the whole space, since it is large.
Instead, we vary parameters to which the neutralino DM relic density or $m_{\chi_1^0}$ is
sensitive. From the discussions in the previous sections, $m_{H_d}^2$ can be such a parameter.
Explicitly, we use parameter values given in Table 4 except that we vary $m_{H_d}^2$ between
$10^4 \sim 5 \times 10^5 \text{ GeV}^2$ and set $\gamma = 0.95 \pi$. The relation between $m_{\chi_1^0}$ and $m_{H_d}^2$
is illustrated in Figure 10. As $m_{H_d}^2$ increases, $m_{\chi_1^0}$ decreases from more than 90 GeV to
35 GeV. This is because $m_{\chi_1^0} \sim h v_1$ in the limit of large $\langle S_{1,2,3} \rangle$ and $v_1 \ll v_2 \sim \langle S \rangle$ (see
Figure 10: $m_{\chi_0^1}$ vs. $m_{H_d^0}^2$.

Figure 11: $\Omega_{\chi_0^1} h^2$ vs. $m_{\chi_0^1}$. The two black parallel lines give the upper and lower bounds on the DM relic density measured by astrophysical and cosmological probes [104].
Figure 12: $\frac{v(T_c)}{T_c}$ vs. $m_{\chi_1}^0$.

Figure 13: $\Delta \theta_{1,2}$ and $\Delta \alpha$ vs. $m_{\chi_1}^0$. 
Figure 14: $\delta$ vs. $m_{\chi_1^0}$.

Figure 15: $\frac{n_B}{s}$ vs. $m_{\chi_1^0}$. There is a sign flip for $n_B/s$ in the region denoted by black-dashed lines. The two black solid lines specify the $m_{\chi_1^0}$ values for which the observed dark matter relic density can be explained by $\chi_1^0$. 

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Eqs. (A.25-A.27) while a larger \( m_{H_d}^2 \) implies a smaller \( v_1 \). This feature is important for the DM relic density, since the \( Z \) pole region is covered. As is well-known, the relic density of neutralino CDM is usually overproduced. One way to reduce the abundance is from the \( Z \)-pole region, where we have \( m_Z \sim 2m_{\chi_0^0} \) and the annihilation of two \( \chi_0^0 \) through a \( Z \) boson is greatly enhanced. In this region, therefore, we can easily obtain an acceptable value for the \( \chi_0^0 \) relic density. As a first-order approximation, we calculate the \( \chi_0^0 \) relic density today, only counting in the processes mediated by gauge bosons (for general discussions on the relevant calculations, e.g., see [105]). The results are shown in Figure 11.

Let us come back to EWBG for this parameter region. With \( m_{\chi_0^i} \) (or equivalently \( m_{H_d}^2 \)) as a varying parameter, the strength of the EWPT, the magnitude of \( CP \)-violation, the thickness of the bubble wall and the produced baryon asymmetry by different particle species are shown in Figures 12-15. From Figure 12 it is easy to see that the measure of the EWPT strength \( v(T_c)/T_c \) is always larger than 1.1. As \( m_{\chi_0^i} \) increases or equivalently \( m_{H_d}^2 \) decreases, its value increases. This can be understood because a smaller scaled \( m_{H_d}^2 \) (the real \( m_{H_d}^2 \) value is also smaller) usually leads to a larger scaled \( v_1(T_c) \) in the true vacuum, while keeping the scaled \( v_2(T_c) \) and \( T_c \) almost invariant (\( T_c \sim 1.4 \) in our simulation). In Figure 13 we see that \( \Delta \theta_{1,2} \) and \( \Delta \alpha \) are close to zero around \( m_{\chi_0^0} \approx 40 \) GeV, which explains the small wall thickness in Figure 14 (recall that a large phase variation crossing the wall usually leads to a small wall thickness (see discussion in Subsection 6.1)) and a sign flip of \( n_B/s \) for the same \( m_{\chi_0^0} \) value in Figure 14. Another sign flip for the top curve in Figure 14 is related to the ones shown in Figures 7-9 which is a result of balance between the contributions from the particle reflection and transmission. Finally, for most region of \( m_{\chi_0^i} \sim 35 - 90 \) GeV a strong enough first order EWPT and a reasonable baryon asymmetry can be obtained, and in the \( Z \) pole region the right \( \chi_0^0 \) relic density can be produced (see Figure 15). The two cosmological mysteries, baryon asymmetry and DM, can therefore be explained simultaneously.

9 Some Other Cosmological Effects

9.1 Superconducting Cosmological Strings

There may exist phase transitions in the early Universe due to gauge symmetry breaking, and then topological defects, such as monopoles and/or cosmic strings, etc, can be produced [106, 107]. If the gauge group \( G \) is broken down to a subgroup \( H \), we will have monopoles and cosmic strings if the homotopy group \( \pi_2(G/H) \neq I \) and \( \pi_1(G/H) = Z \neq I \), respectively, where \( Z \) is the group of integers. In particular, the cosmic strings may be superconducting because of the presence of a charged field condensate in the core of the string [108]. The gauge symmetry in the simplest cosmic string model is \( U(1) \times U(1)' \) where the \( U(1)' \) gauge symmetry is broken via the Higgs mechanism [108]. Note that \( \pi_1(U(1)') = Z \), so we can have a topologically stable
cosmic string solution. Although the $U(1)$ symmetry is respected by the vacuum, it can be broken in the string core and then give rise to a bosonic condensate. Moreover, there may exist fermionic superconducting strings that have fermionic zero modes in its core \cite{108}. These fermions can have non-zero $U(1)$ charges, and arise from Yukawa couplings to the scalar fields. In the non-supersymmetric models, we only need to introduce two complex scalar fields: one is charged under $U(1)$ while the other is charged under $U(1)'$ \cite{108}. However, in order to have the bosonic superconductivity for cosmic strings in the supersymmetric $U(1) \times U(1)'$ model, one needs to introduce at least five chiral superfields \cite{109}. Interestingly, one can realize the fermionic superconductivity in the simplest supersymmetric Abelian Higgs models \cite{110}.

In the sMSSM, we have the fermionic superconductivity, and supersymmetry is broken in the core of the cosmic strings (supersymmetry is not broken outside of the string core) which is similar to that in Ref. \cite{110}. However, we do not have the bosonic superconductivity since all the Higgs fields are charged under the $U(1)'$ gauge symmetry. Because the $U(1)'$ gauge symmetry is broken around the TeV scale, the string tension $T_S$ should be around the TeV scale as well. The gravitational effect of cosmic strings with $T_S^2/M_{Pl}^2 \sim 10^{-30}$ is extremely weak and is not within foreseeable detection capabilities. Interestingly, we can search for the signatures of particle emissions, such as positrons or other decaying particles produced where the current on the string quenches. Similar to the discussions in Ref. \cite{111}, which also has strings around 1 TeV, the sMSSM might explain the observed 511 keV emission from the electron-positron annihilation in the Galactic bulge \cite{112,113,114}.

9.2 Gravitational Waves

There are a few experiments under consideration that might detect for the first time a stochastic background of GWs, for example, the planned Laser Interferometer Space Antenna (LISA) \cite{115}, and the Big Bang Observer (BBO) \cite{116}, which is the follow-on mission to LISA. One of the well-motivated and convincing sources for GWs is inflation, and the discovery of a relic gravitational background could be a smoking gun signal from inflation. Another source of the stochastic GWs are strong first-order phase transitions. In contrast with the inflationary spectrum, the spectrum from a phase transition is not flat and has a characteristic peak related to the temperature at which the phase transition took place. Especially, LISA and BBO will have fairly good sensitivity at the frequencies that coincide with the redshifted spectrum of GWs produced during an electroweak phase transition at temperatures around 100 GeV. Thus, the GW observations will give us information about EWBG since the strong first-order electroweak phase transition is a necessary requirement.

There are two distinct mechanisms that can generate the GWs during a first-order electroweak phase transition: the bubble collisions \cite{117,118,119,120} and the turbulent motion of the primordial plasma \cite{121,122,123,124,125}. The GWs from the electroweak phase transition have been studied in a model independent way in Refs. \cite{126} and \cite{127}. The results can be presented as a function of two main parameters: the typical size of the colliding bubbles and the available energy. It
was concluded that a sufficiently strong electroweak phase transition could lead to an observable GW signal at LISA. However, for a definite model, the bubble size and the available energy are correlated. Thus, it is still very important to study the concrete models in details.

It was found previously that in the MSSM the produced amount of GWs is orders of magnitude below the LISA sensitivity. The situation in the NMSSM seems much more promising because the trilinear term $S H_d H_u$ can naturally lead to a much stronger electroweak phase transition $^{[128]}$. Recently, it was pointed out that in Ref. $^{[128]}$ a rough approximation was used to calculate the bubble configurations, which overestimated the strength of the phase transition and the GW signal $^{[129]}$. Moreover, in Refs. $^{[129, 130]}$ the nMSSM, which solves the $\mu$ problem and domain wall problem simultaneously, was considered. It was shown that the GW signals can be detected at BBO for part of parameter space, but cannot be observed at LISA $^{[129, 130]}$.

In the sMSSM the strong first-order electroweak phase transitions arises from the trilinear term $S H_d H_u$ in the superpotential, which is similar to the NMSSM and nMSSM. The GW signals might therefore be detectable at BBO for part of the parameter space, but cannot be observed at LISA $^{[129, 130]}$. As a side comment, the phase transition for the $U(1)'$ gauge symmetry breaking may not be strong first order since the coupling $\lambda$ in the trilinear term $\lambda S_1 S_2 S_3$ is small, and one might not be able to observe the GWs from the $U(1)'$ gauge symmetry breaking.

10 Discussion and Conclusions

In this paper we discussed EWBG and its correlation with the neutralino CDM in the sMSSM. We first constructed two anomaly-free models. In Model I, we embedded the $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)'$ gauge symmetry into a larger gauge symmetry, $E_6$, whose representations are anomaly-free. We considered three families of SM fermions and one pair of Higgs doublets arising from three $E_6$ fundamental representation $^{27}s$. To include the SM singlet Higgs particles, achieve gauge coupling unification and cancel the $U(1)'$ anomaly, we assumed that three pairs of vector-like SM singlets and one pair of vector-like Higgs doublets from two pairs of $^{27}$ and $^{27^*}$ are light while the other particles in the two pairs of $^{27}$ and $^{27^*}$ are absent or very heavy. In Model II, we calculated the anomaly-free conditions, and added the minimal number of exotic particles to cancel the $U(1)'$ anomalies. We required that all the particles have rational $U(1)'$ charges. Although we had minimal exotic particles, we lost the gauge coupling unification. We also presented the general superpotential and the supersymmetry breaking soft terms in both models.

We discussed the one-loop effective potential at finite temperature in the ’t Hooft-Landau gauge and in the $MS$-scheme. We showed that there exists a strong enough first order EWPT because of the large trilinear term $A_t h S H_u H_d$ in the tree-level Higgs potential. Unlike the MSSM, the stop masses can be very heavy. The EWPT features in both models are quite similar because the exotic particles’ contri-
butions to the one-loop effective potential at finite temperature are suppressed due to their heavy masses. We therefore restricted the detailed analysis to Model I.

In the early Universe the first order EWPT is realized by nucleating bubbles of the broken phase. Crossing the bubble walls, the VEVs of the Higgs fields (including their magnitudes and phases) are space-dependent. Because the dynamical behaviors of these bubbles, such as wall profile and expansion velocity, can have important influences on the production of the baryon asymmetry, we also studied their physics in detail. Numerical results show that the bubble wall thickness varies from $3 \sim 30 T_c^{-1}$ as a monotonically increasing function of the phase changes of the Higgs fields. We also argued that the wall velocity in the sMSSM cannot be larger than that in the MSSM under the same phase transition condition and thus should be non-relativistic. This fact implies that EW sphaleron processes have more time to occur and hence will enhance the final baryon asymmetry.

We also discussed possible $CP$ violation introduced by the extended Higgs sector. Unlike the MSSM, where there is no $CP$-violation at tree level in the Higgs sector, the $CP$ symmetry can be broken spontaneously as well as explicitly. With loop corrections included, there may coexist vacua at finite temperature with broken and unbroken EW symmetry. The values of these spontaneous $CP$ phases usually are different in these vacua. In our work, the SCPV provides a direct source for baryogenesis while its magnitude is mediated by an explicit one from the secluded sector. These new $CP$ sources do not introduce significant new contributions to EDMs. After proper field redefinitions, the $CP$-violation phases will only appear in the Higgs mass matrix through soft masses associated with singlet components. Numerical results show that, for typical parameter values, their contributions to EDMs will be about six or seven orders smaller than the experimental upper limits. These contributions disappear completely in the limit with a trivial explicit $CP$ phase, where the spontaneous $CP$ phases are trivial in the true vacuum but not where EWBG occurs.

Subsequently, we systematically studied non-local EWBG in both the thin wall and thick wall regimes. We calculated the contribution from leptons in the thin wall regime, and the ones from squarks, charginos and quarks in the thick wall regime in terms of the profile of bubble wall. For leptons and quarks, the induced $CP$-violating currents outside the bubbles are proportional to Yukawa couplings squared, so we only considered the contributions from $\tau$ leptons and top quarks. We found that the $CP$-violating currents for stops and charginos are very different from those obtained in the MSSM. Due to the space-dependence of the relevant $CP$ phases, they do not require a variation of $\tan \beta$ in the bubble wall to have a non-trivial structure at the lowest order of Higgs insertion. In addition, there exists a new type of $CP$-violating current at the leading order which can contribute to the generation of the baryon asymmetry. The $CP$-violating current of the new type has important influence on the EWBG. First, the stop contribution can be quadratically enhanced by a large $A_t$. Second, in addition to $\tau$ leptons, top squarks and charginos, top quarks can also play a significant role in the EWBG. We emphasize that all of these features are results of the SCPV-driven EWBG, so they are not sensitive to the concrete embeddings of the sMSSM. Numerical results show that the produced baryon asymmetry is large.
enough to explain the cosmological observations.

After that, we studied the correlation between EWBG and neutralino CDM in the sMSSM. Though we did not scan the whole parameter space, we found that there exists a region where a strong enough first order EWPT, large $CP$ phase variations crossing the bubble wall, a reasonable baryon asymmetry, as well as an acceptable neutralino LSP relic density can be achieved simultaneously. We also commented on possible cosmological signals of the model: superconducting cosmic strings and GWs. Particle emission from the decays of cosmic strings and GWs from EWPT could be observed within the foreseeable future.

The secluded $U(1)'$-breaking sector plays an essential role. It induces ECPV and SCPV at tree level in the Higgs sector, helping avoid significant new contributions to the EDMs which may hinder a successful EWBG in supersymmetric models, and also influences the bubble wall physics. It provides more degrees of freedom to drive the realization of EWBG, and makes the EWBG physics in the sMSSM very different from those in the MSSM and NMSSM. These differences make it interesting to further study the relevant collider implications for the LHC [64], which we will leave to future exploration.

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A Mass Matrices for the Particles in Model I

For the Higgs mass matrices, here we only present the results in the limit \( m_{S_1S_2}^2 = 0 \), where there is no \( CP \) violation. It is not difficult to extend them to the general case with non-zero \( m_{S_1S_2}^2 \) and \( CP \) violation, as considered in the main text. Let us define

\[
v_1 = \langle H^0_d \rangle , \ v_2 = \langle H^0_u \rangle , \ \tan \beta = \frac{v_2}{v_1}, \quad (A.1)
\]

\[
s = \langle S \rangle , \ s_i = \langle S_i \rangle . \quad (A.2)
\]

Then the mass-square matrix for the \( CP \)-odd neutral Higgs particles in the basis \( \{H^0_d, H^0_u, S^0, S_1^0, S_2^0, S_3^0\} \) is

\[
M_{A^0}^2 = \begin{pmatrix} O_{A^0} & C_{A^0} \\ C_{A^0}^T & S_{A^0} \end{pmatrix}, \quad (A.3)
\]

where

\[
O_{A^0} = \begin{pmatrix} \beta_H^2 & A_h h s & A_h h v_2 \\ A_h h s & \beta_H^2 & A_h h v_1 \\ A_h h v_2 & A_h h v_1 & \beta_S^2 \end{pmatrix}, \quad (A.4)
\]

\[
S_{A^0} = \begin{pmatrix} \beta_{S_1}^2 & A_\lambda \lambda s_3 & A_\lambda \lambda s_2 \\ A_\lambda \lambda s_3 & \beta_{S_2}^2 & A_\lambda \lambda s_1 \\ A_\lambda \lambda s_2 & A_\lambda \lambda s_1 & \beta_{S_3}^2 \end{pmatrix}, \quad (A.5)
\]

\[
C_{A^0} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ -m_{S1}^2 & -m_{S2}^2 & 0 \end{pmatrix}, \quad (A.6)
\]

and

\[
\beta_{H_d}^2 = m_{H_d}^2 + h^2(v_2^2 + s^2) + \frac{G^2}{4}(v_1^2 - v_2^2) + g_Z^2 Q_{H_d} \Delta, \quad (A.7)
\]

\[
\beta_{H_u}^2 = m_{H_u}^2 + h^2(v_1^2 + s^2) + \frac{G^2}{4}(v_2^2 - v_1^2) + g_Z^2 Q_{H_u} \Delta, \quad (A.8)
\]

\[
\beta_S^2 = m_S^2 + h^2(v_1^2 + v_2^2) + g_Z^2 Q_S \Delta, \quad (A.9)
\]

\[
\beta_{S_i}^2 = m_{S_i}^2 + \lambda^2 \sum_{i \neq i} s_j^2 + g_Z^2 Q_{S_i} \Delta, \quad (A.10)
\]
where

\[ \Delta \equiv Q_S s^2 + Q_{H_d} v_1^2 + Q_{H_u} v_2^2 + \sum_{i=1}^{3} Q_{S_i} s_i^2 . \] (A.11)

Similarly, in the basis \( \{H_{d}^{0r} = Re(H_{d}^{0}), H_{u}^{0r}, S_{1}^{0r}, S_{2}^{0r}, S_{3}^{0r}\} \), the mass-square matrix for the \( CP \)-even neutral Higgs particles is

\[ M_{H}^{2} = \begin{pmatrix} O_{H}^{0} & C_{H}^{0} \\ C_{H}^{0T} & S_{H}^{0} \end{pmatrix} , \] (A.12)

where

\[ O_{H}^{0} = \begin{pmatrix} \kappa_{H_d}^2 & \kappa_{H_d,H_u} & \kappa_{H_d,S} \\ \kappa_{H_d,H_u} & \kappa_{H_u}^2 & \kappa_{H_u,S} \\ \kappa_{H_d,S} & \kappa_{H_u,S} & \kappa_S^2 \end{pmatrix} , \] (A.13)

\[ S_{H}^{0} = \begin{pmatrix} \kappa_{S_1}^2 & \kappa_{S_1,S_2} & \kappa_{S_1,S_3} \\ \kappa_{S_1,S_2} & \kappa_{S_2}^2 & \kappa_{S_2,S_3} \\ \kappa_{S_1,S_3} & \kappa_{S_2,S_3} & \kappa_S^2 \end{pmatrix} , \] (A.14)

\[ C_{H}^{0} = \begin{pmatrix} \kappa_{H_d,S_1} & \kappa_{H_d,S_2} & \kappa_{H_d,S_3} \\ \kappa_{H_u,S_1} & \kappa_{H_u,S_2} & \kappa_{H_u,S_3} \\ \kappa_{S_1}^2 + m_{S_{S_1}}^2 & \kappa_{S_2}^2 + m_{S_{S_2}}^2 & \kappa_{S_3}^2 \end{pmatrix} , \] (A.15)

and

\[ \kappa_{H_d}^2 = 2 \left( \frac{G^2}{4} + g_{Z'}^2 Q_{H_d}^2 \right) v_1^2 + m_{H_d}^2 + h^2 (v_2^2 + s^2) + \frac{G^2}{4} (v_1^2 - v_2^2) + g_{Z'}^2 Q_{H_u} \Delta , \] (A.16)

\[ \kappa_{H_u}^2 = 2 \left( \frac{G^2}{4} + g_{Z'}^2 Q_{H_u}^2 \right) v_2^2 + m_{H_u}^2 + h^2 (v_1^2 + s^2) + \frac{G^2}{4} (v_2^2 - v_1^2) + g_{Z'}^2 Q_{H_u} \Delta , \] (A.17)

\[ \kappa_{S}^2 = 2 g_{Z'}^2 Q_{S}^2 s^2 + m_{S}^2 + h^2 (v_1^2 + v_2^2) + g_{Z'}^2 Q_{S} \Delta , \] (A.18)

\[ \kappa_{S_i}^2 = 2 g_{Z'}^2 Q_{S_i}^2 s_i^2 + m_{S_i}^2 + \lambda^2 \sum_{j \neq i} s_j^2 + g_{Z'}^2 Q_{S_i} \Delta , \] (A.19)

\[ \kappa_{H_d,H_u} = 2 \left( h^2 - \frac{G^2}{4} + g_{Z'}^2 Q_{H_d} Q_{H_u} \right) v_1 v_2 - A_h h s , \] (A.20)
\[ \kappa_{H_i,s} = 2 (h^2 + g_Z^2 Q_i Q_S) v_i s - |\epsilon_i| A_h v_j, \]  
(A.21)  
\[ \kappa_{H_i,s_j} = 2 g_Z^2 Q_i Q_S v_i s_j, \quad \kappa_{s_i,s_j} = 2 g_Z^2 Q_S Q_S s_i s_j, \]  
(A.22)  
\[ \kappa_{s_i,s_j} = 2 (\lambda^2 + g_Z^2 Q_S Q_S) s_i s_j - |\epsilon_{ijk}| A_\lambda \lambda s_k. \]  
(A.23)

The charged Higgs mass is

\[ M_{H^\pm}^2 = M_W^2 + \frac{2 A_h h s}{\sin 2\beta} - h^2 (v_1^2 + v_2^2), \]  
(A.24)

where \( M_W^2 = g_2 (v_1^2 + v_2^2)/2. \)

In the basis \( \{ \tilde{B}', B, \tilde{W}_3, \tilde{H}_d^0, \tilde{H}_u^0, \tilde{S}, \tilde{S}_1, \tilde{S}_2, \tilde{S}_3 \} \), the neutralino mass matrix is

\[ M_{\tilde{\chi}^0} = \begin{pmatrix} M_{\tilde{\chi}^0}(6, 6) & M_{\tilde{\chi}^0}(6, 3) \\ M_{\tilde{\chi}^0}(6, 3)^T & M_{\tilde{\chi}^0}(3, 3) \end{pmatrix}, \]  
(A.25)

where

\[ M_{\tilde{\chi}^0}(6, 6) = \begin{pmatrix} M'_1 & 0 & 0 & \Gamma_{H_d^0}^* & \Gamma_{H_u^0}^* & \Gamma_{S}^* \\ 0 & M_1 & 0 & -\frac{1}{\sqrt{2}} g_1 H_d^{0*} & \frac{1}{\sqrt{2}} g_1 H_u^{0*} & 0 \\ 0 & 0 & M_2 & \frac{1}{\sqrt{2}} g_2 H_d^{0*} & \frac{1}{\sqrt{2}} g_2 H_u^{0*} & 0 \\ \Gamma_{H_d^0}^* & -\frac{1}{\sqrt{2}} g_1 H_d^{0*} & \frac{1}{\sqrt{2}} g_2 H_u^{0*} & h S & h H_d^0 & 0 \\ \Gamma_{H_u^0}^* & \frac{1}{\sqrt{2}} g_1 H_d^{0*} & -\frac{1}{\sqrt{2}} g_2 H_u^{0*} & h S & h H_u^0 & 0 \\ \Gamma_{S}^* & 0 & 0 & h H_u^0 & h H_u^0 & 0 \end{pmatrix} \]  
(A.26)

and

\[ M_{\tilde{\chi}^0}(3, 3) = \begin{pmatrix} 0 & \lambda S_3 & \lambda S_2 \\ \lambda S_3 & 0 & \lambda S_1 \\ \lambda S_2 & \lambda S_1 & 0 \end{pmatrix}, \]  
(A.27)

where \( \Gamma_\phi \equiv \sqrt{2} g_Z Q_\phi \phi \); and \( M'_1, M_1 \) and \( M_2 \) are the gaugino masses for \( U(1)' \), \( U(1) \) and \( SU(2)_L \), respectively. The first row of \( M_{\tilde{\chi}^0}(6, 3) \) is given by \( (\Gamma_{S_1}^*, \Gamma_{S_2}^*, \Gamma_{S_3}^*) \), while the other entries are zero.

The chargino mass matrix is

\[ M_{\tilde{\chi}^\pm} = \begin{pmatrix} M_2 & \frac{g_2}{\sqrt{2}} H_u^{0*} \\ \frac{g_2}{\sqrt{2}} H_d^{0*} & h S \end{pmatrix}. \]  
(A.28)

In contrast to the Higgs mass matrix, the neutralino and chargino mass matrices as well as the other ones in the following, are written in a general background with \( CP \) violation.
In the basis \( (B_\mu, W^3_\mu, Z'_\mu) \), the mass matrix for the neutral gauge bosons is

\[
M_{B,W^3,Z'} = \begin{pmatrix}
M_{B,W^3,Z'}^{11} & M_{B,W^3,Z'}^{12} & M_{B,W^3,Z'}^{13} \\
M_{B,W^3,Z'}^{21} & M_{B,W^3,Z'}^{22} & M_{B,W^3,Z'}^{23} \\
M_{B,W^3,Z'}^{31} & M_{B,W^3,Z'}^{32} & M_{B,W^3,Z'}^{33}
\end{pmatrix},
\]

where

\[
M_{B,W^3,Z'}^{11} = \frac{1}{2} g_1^2 [|H_u^0|^2 + |H_d^0|^2],
\]

\[
M_{B,W^3,Z'}^{12} = M_{B,W^3,Z'}^{21} = -\frac{1}{2} g_1 g_2 [|H_u^0|^2 + |H_d^0|^2],
\]

\[
M_{B,W^3,Z'}^{13} = M_{B,W^3,Z'}^{31} = g_1 g_2 [Q_{H_u} |H_u^0|^2 - Q_{H_d} |H_d^0|^2],
\]

\[
M_{B,W^3,Z'}^{22} = \frac{1}{2} g_2^2 [|H_u^0|^2 + |H_d^0|^2],
\]

\[
M_{B,W^3,Z'}^{23} = M_{B,W^3,Z'}^{32} = g_2 g_3 [Q_{H_u} |H_u^0|^2 - Q_{H_d} |H_d^0|^2],
\]

\[
M_{B,W^3,Z'}^{33} = 2 g_2^2 (Q^2_3 |S|^2 + Q^2_{H_u} |H_u^0|^2 + Q^2_{H_d} |H_d^0|^2 + \sum_{i=1}^3 Q^2_{S_i} |S_i|^2).
\]

In addition, for a scalar \( \phi \), we define

\[
\Delta_\phi \equiv [T_3^\phi - Q^\phi_{EM} \sin^2 \theta_W] M^2 Z \cos(2\beta),
\]

\[
\Delta'_\phi \equiv Q^\phi_6 g_2^2 [Q_{H_u} |H_u^0|^2 + Q_{H_d} |H_d^0|^2 + Q_S |S|^2 + \sum_{i=1}^3 Q_{S_i} |S_i|^2].
\]

The up-type squark mass matrix is

\[
M_u = \begin{pmatrix}
M^2_{\tilde{u}_L} & h_{u_i}^*(h^* S^* H^0_d - A^*_{H_u} H^0_u) & M^2_{\tilde{u}_R} \\
h_{u_i} (h^* S^* H^0_d - A^*_{H_u} H^0_u) & h_{u_i}^* (h^* S^* H^0_d - A^*_{H_u} H^0_u) & M^2_{\tilde{u}_R} \\
M^2_{\tilde{u}_L} & M^2_{\tilde{u}_R}
\end{pmatrix},
\]

where

\[
M^2_{\tilde{u}_L} = m^2_{\tilde{u}_L} + m^2_{\tilde{u}_i} + \Delta_{\tilde{u}_L} + \Delta'_{\tilde{u}_L},
\]

\[
M^2_{\tilde{u}_R} = m^2_{\tilde{u}_R} + m^2_{\tilde{u}_i} + \Delta_{\tilde{u}_R} + \Delta'_{\tilde{u}_R},
\]

while the down-type matrix is obtained by \( u \leftrightarrow d \). The mass matrices for the sleptons and exotic scalars are similar.
\section*{B \ \Pi_{\Phi}(T) for Scalar Particles in Model I}

The \( \Pi_{\Phi}(T) \)'s for the Higgs doublets are

\begin{equation}
\Pi_{H_d}(T) = \frac{3}{8} g_2^2 T^2 + \frac{1}{8} g_1^2 T^2 + \frac{1}{2} g_{Z'}^2 Q_{H_d}^2 T^2 + \sum_{i=1}^{3} \left( \frac{3}{4} h_{d_i}^2 + \frac{1}{4} h_{e_i}^2 \right) T^2 + \frac{1}{4} h^2 T^2 , \quad (B.1)
\end{equation}

\begin{equation}
\Pi_{H_u}(T) = \frac{3}{8} g_2^2 T^2 + \frac{1}{8} g_1^2 T^2 + \frac{1}{2} g_{Z'}^2 Q_{H_u}^2 T^2 + \sum_{i=1}^{3} \frac{3}{4} h_{u_i}^2 T^2 + \frac{1}{4} h^2 T^2 . \quad (B.2)
\end{equation}

For the SM singlets:

\begin{equation}
\Pi_{S}(T) = \frac{1}{2} g_{Z'}^2 Q_{S}^2 T^2 + \frac{1}{8} \alpha_2^2 T^2 + \frac{1}{4} h^2 T^2 + \sum_{i=1}^{3} \frac{3}{4} (\alpha_i^P)^2 T^2 \\
+ \sum_{k=1}^{2} \frac{1}{2} (\alpha_k^H)^2 T^2 + \sum_{i=1}^{3} \frac{1}{4} (\alpha_i^N)^2 T^2 . \quad (B.3)
\end{equation}

\begin{equation}
\Pi_{S_i}(T) = \frac{1}{2} g_{Z'}^2 Q_{S_i}^2 T^2 + \frac{1}{4} \lambda^2 T^2 . \quad (B.4)
\end{equation}

For the longitudinal components of the \( SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)' \) gauge bosons, \( g, W, B \) and \( Z' \):

\begin{equation}
\Pi_{g_L}(T) = 6 g_3^2 T^2 , \quad \Pi_{W_L}(T) = 6 g_2^2 T^2 , \quad (B.5)
\end{equation}

\begin{equation}
\Pi_{B_L}(T) = 8 g_1^2 T^2 , \quad \Pi_{Z'_L}(T) = \frac{619}{2} g_{Z'}^2 T^2 . \quad (B.6)
\end{equation}

For \( \tilde{N}_i \) and \( S_i^L \):

\begin{equation}
\Pi_{\tilde{N}_i}(T) = \frac{1}{2} g_{Z'}^2 Q_{\tilde{N}_i}^2 T^2 + \frac{1}{4} (\alpha_i^N)^2 T^2 , \quad (B.7)
\end{equation}

\begin{equation}
\Pi_{S_i^L}(T) = \frac{1}{2} g_{Z'}^2 Q_{S_i^L}^2 T^2 + \frac{1}{4} (\alpha_i^N)^2 T^2 . \quad (B.8)
\end{equation}

For \( \tilde{X} \) and \( \tilde{X}_3 \):

\begin{equation}
\Pi_{\tilde{X}}(T) = \frac{1}{2} g_{Z'}^2 Q_{\tilde{X}}^2 T^2 + \frac{1}{4} \alpha^2 T^2 , \quad (B.9)
\end{equation}

\begin{equation}
\Pi_{\tilde{X}_3}(T) = \frac{1}{2} g_{Z'}^2 Q_{\tilde{X}_3}^2 T^2 + \frac{1}{4} \alpha^2 T^2 . \quad (B.10)
\end{equation}
\[ \Pi_{\chi_3}(T) = \frac{1}{2} g_Z^2 Q_{\chi_3}^2 \bar{\chi}_3^2 T^2 + \frac{1}{4} \alpha^2 T^2. \]  

(B.10)

For \( \tilde{D}_i \) and \( \tilde{D}_i \):

\[
\Pi_{\tilde{D}_i}(T) = \frac{2}{3} g_3^2 T^2 + \frac{1}{18} g_1^2 T^2 + \frac{1}{2} g_Z^2 Q_{\tilde{D}_i}^2 T^2 + \frac{1}{4} (\alpha_{\tilde{D}}^2 T^2 ,
\]

(B.11)

\[
\Pi_{\tilde{D}_i}(T) = \frac{2}{3} g_3^2 T^2 + \frac{1}{18} g_1^2 T^2 + \frac{1}{2} g_Z^2 Q_{\tilde{D}_i}^2 T^2 + \frac{1}{4} (\alpha_{\tilde{D}}^2 T^2 .
\]

(B.12)

For \( H^k_u \) and \( H^k_d \):

\[
\Pi_{H^k_u}(T) = \frac{3}{8} g_2^2 T^2 + \frac{1}{8} g_1^2 T^2 + \frac{1}{2} g_Z^2 Q_{H^k_u}^2 T^2 + \frac{1}{4} (\alpha_{H^k_u}^2 T^2 ,
\]

(B.13)

\[
\Pi_{H^k_d}(T) = \frac{3}{8} g_2^2 T^2 + \frac{1}{8} g_1^2 T^2 + \frac{1}{2} g_Z^2 Q_{H^k_d}^2 T^2 + \frac{1}{4} (\alpha_{H^k_d}^2 T^2 .
\]

(B.14)

For \( H^k_u \) and \( H^k_d \):

\[
\Pi_{H^k_u}(T) = \frac{3}{8} g_2^2 T^2 + \frac{1}{8} g_1^2 T^2 + \frac{1}{2} g_Z^2 Q_{H^k_u}^2 T^2 ,
\]

(B.15)

\[
\Pi_{H^k_d}(T) = \frac{3}{8} g_2^2 T^2 + \frac{1}{8} g_1^2 T^2 + \frac{1}{2} g_Z^2 Q_{H^k_d}^2 T^2 .
\]

(B.16)

For the superpartners of the SM fermions:

\[
\Pi_{\tilde{\chi}_i}(T) = \frac{2}{3} g_3^2 T^2 + \frac{3}{8} g_2^2 T^2 + \frac{1}{72} g_1^2 T^2 + \frac{1}{2} g_Z^2 Q_{\tilde{\chi}_i}^2 T^2 + \frac{1}{4} (h^2_{u_i} + h^2_{d_i}) T^2 ,
\]

(B.17)

\[
\Pi_{\tilde{\psi}_{\tilde{d}_i}}(T) = \frac{2}{3} g_3^2 T^2 + \frac{3}{8} g_2^2 T^2 + \frac{1}{18} g_1^2 T^2 + \frac{1}{2} g_Z^2 Q_{\tilde{d}_i}^2 T^2 + \frac{1}{4} h^2_{d_i} T^2 ,
\]

(B.18)

\[
\Pi_{\tilde{d}_{\tilde{d}_i}}(T) = \frac{2}{3} g_3^2 T^2 + \frac{1}{18} g_1^2 T^2 + \frac{1}{2} g_Z^2 Q_{\tilde{d}_i}^2 T^2 + \frac{1}{2} h^2_{d_i} T^2 ,
\]

(B.19)

\[
\Pi_{\tilde{L}_i}(T) = \frac{3}{8} g_2^2 T^2 + \frac{1}{8} g_1^2 T^2 + \frac{1}{2} g_Z^2 Q_{\tilde{L}_i}^2 T^2 + \frac{1}{4} h^2_{e_i} T^2 ,
\]

(B.20)

\[
\Pi_{\tilde{e}_{\tilde{e}_i}}(T) = \frac{1}{2} g_1^2 T^2 + \frac{1}{2} g_Z^2 Q_{\tilde{e}_i}^2 T^2 + \frac{1}{2} h^2_{e_i} T^2 .
\]

(B.21)
C  Derivation of $\Delta R$ in the Thick Wall Regime

$\Delta R$ is the reflectivity asymmetry between the particle and antiparticle scatterings. It is crucial for the calculation of the $CP$-violating current in the EWBG. In the following, we will show how $\Delta R$ is calculated in the thick wall regime. We will explicitly calculate $\Delta R_{\tilde{W}^c \rightarrow f^c}(k_i, k_f, z)$ (see Eq. (93)) as an illustration. The calculations for $\Delta R_{t_R \rightarrow t_L}(k_i, k_f, z)$ and $\Delta R_{t_R \rightarrow t_L}(k_i, k_f, z)$ (see Eq. (87) and Eq. (99)) are similar, so we will not present their details.

The general formula for the probability calculation in quantum field theory is (eg. see [131])

$$P(\psi_i \rightarrow \psi_f) = \int \frac{d^3p_f}{(2\pi)^3} \frac{1}{2E_f} |\langle p_f | \psi_i \rangle|^2$$

$$= \int \frac{d^3p_f}{(2\pi)^3} \frac{1}{2E_f} \int \frac{d^3p_i}{(2\pi)^3} \frac{\psi_i(p_i)}{\sqrt{2E_i}} \int \frac{d^3\bar{p}_i}{(2\pi)^3} \frac{\psi_i^*(\bar{p}_i)}{\sqrt{2E_i}} \langle p_f | p_i \rangle \langle p_f | \bar{p}_i \rangle^*.$$  \hspace{1cm} (C.1)

Here the subscripts “i” and “f” denote initial and final states, respectively. Given that in the model the Langrangian is approximately invariant under the translations in the $x$, $y$ and $t$ directions, it is natural to define

$$\langle p_f | p_i \rangle = \langle p_f | i \int d^4x L(z) | p_i \rangle$$

$$= iM(p_i \rightarrow p_f)(2\pi)^3 \delta(E_i - E_f) \delta(p_i^x - p_f^x) \delta(p_i^y - p_f^y),$$

$$M = \langle p_f^z | \int dz L(z) | p_i^z \rangle.$$  \hspace{1cm} (C.2)

Embedding Eq. (C.2) into Eq. (C.1), we find

$$P(\psi_i \rightarrow \psi_f) = \frac{1}{4E_i E_f v_i^z v_f^z} |M(k_i^z \rightarrow k_f^z)|^2,$$  \hspace{1cm} (C.3)

where $k$, $E$ and $v$ are particle momentum, energy and velocity, respectively. For a given particle with mass $m$, all of the quantities are functions of $k_i^z$.

For the scattering process $\tilde{W}^c \rightarrow H^c$ in the chargino sector, $M(k_i^z, k_f^z, z)$ is

$$M(k_i^z, k_f^z, z) = \int_z^{z+\Delta(\tilde{\gamma}^c)} dz_1 \exp(i(k_i^z - k_f^z)z_1)$$

$$A(k_i^z, k_f^z, z_1) - A(k_i^z, k_f^z, z_1 \rightarrow z),$$

$$A(k_i^z, k_f^z, z_1) = \bar{u}_{\tilde{H}^c}(k_f^z) [(u_1(z_1) \exp(-i\theta_1(z_1)) - u_1(z) \exp(-i\theta_1(z))P_L$$

$$+ (u_2(z_1) \exp(i\theta_1(z_1)) - u_2(z) \exp(i\theta_2(z))P_R] u_{\tilde{W}^c}(k_i^z),$$  \hspace{1cm} (C.5)
where $P_{L,R}$ denote chirality projection operators. Then,

$$\Delta |M(k_i^z, k_f^z, z)|^2 = |M(k_i^z, k_f^z, z)|^2 - (CP \text{ Conjugate})$$

$$= \Delta A(k_i^z, k_f^z, z_1, z_2) + \Delta A(k_i^z, k_f^z, z_1 \rightarrow z, z_2 \rightarrow z)$$

$$- \Delta A(k_i^z, k_f^z, z_1 \rightarrow z, z_2) - \Delta A(k_i^z, k_f^z, z_1, z_2 \rightarrow z), \quad (C.7)$$

with

$$\Delta A(k_i^z, k_f^z, z_1, z_2) = \int_{z_1}^{z+\Delta(z_{\tilde{\chi}_c})} dz_1 dz_2 \exp(i(k_i^z - k_f^z)(z_1 - z_2))$$

$$[A(k_i^z, k_f^z, z_1)A(k_i^z, k_f^z, z_2)^* - (CP \text{ Conjugate})]. \quad (C.8)$$

For the other $\Delta A$s, only “$z_1$” or “$z_2$” or both in the second line of Eq. (C.8) are replaced by “$z$”. It is easy to calculate

$$\Delta A(k_i^z, k_f^z, z_1, z_2) = \int_{z_1}^{z+\Delta(z_{\tilde{\chi}_c})} dz_1 dz_2 \sin((k_i^z - k_f^z)(z_1 - z_2))$$

$$\{k_i^z k_f^z[u_1(z_1)u_1(z_2) \sin(\theta_1(z_1) - \theta_1(z_2))$$

$$+ u_2(z_1)u_2(z_2) \sin(-\theta_2(z_1) + \theta_2(z_2))] +$$

$$M_2\mu(z)[u_1(z_1)u_2(z_2) \sin(\theta_1(z_1) + \theta_2(z_2) + \alpha(z))$$

$$+ u_2(z_1)u_1(z_2) \sin(-\theta_2(z_1) - \theta_1(z_2) - \alpha(z))\}], \quad (C.9)$$

which implies

$$\Delta A(k_i^z, k_f^z, z_1 \rightarrow z, z_2 \rightarrow z) \equiv 0. \quad (C.10)$$

Finally, embedding Eq. (C.9) into Eq. (C.7), we obtain Eq. (C.11) according to the relation

$$\Delta R_{\tilde{W}_c \rightarrow \tilde{H}_c}(k_i, k_f, z) = \frac{1}{4E_i E_f} \frac{1}{v_i^z v_f^z} \Delta |M(k_i^z, k_f^z, z)|^2. \quad (C.11)$$

### D More Results at $T = 0K$ in Model I

Additional $T = 0K$ results in the sMSSM with both ECPV and SCPV turned on are presented in Figure 16 - Figure 20. These results include the $\gamma$ dependences of $\tan \beta$, $m_{\chi_1^1}$ and $m_{\chi_1^0}$ in cases a, b and c, and the correlations between $\tan \beta$, $m_{\chi_1^1}$ and $m_{\chi_1^0}$ due to the variance of $m_{H_d}^2$. For discussions on the phenomenology in a similar framework, also see [132].
Figure 16: $\tan \beta$ vs. $\gamma(\pi)$.

Figure 17: $m_{\chi_1^c}$ vs. $\gamma(\pi)$. 
Figure 18: $m_{\chi_1^0}$ vs. $\gamma(\pi)$.

Figure 19: $\tan \beta$ vs. $m_{\chi_1^0}$. 
Figure 20: $m_{\chi_1^c}$ vs. $m_{\chi_1^0}$.

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