Gravitational Particle Production in Gravity Theories with Non-minimal Derivative Couplings

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Abstract

We study the gravitational production of heavy X-particles of mass of the order of the inflaton mass, produced after the end of inflation. We find that, in the presence of a derivative coupling of the inflaton field or of the X-field to the Einstein tensor, the number of gravitationally produced particles is suppressed as the strength of the coupling is increased.

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1 Introduction

Gravitational particle production (for a review see [1, 2]) is a generic mechanism for quantum fields in a curved spacetime background and are analogs of particle creation in strong electric fields. This mechanism has been used to explain the presence of dark matter (DM), which is believed to constitute most of the mass of the Universe, by generating superheavy long-lived particles after inflation in the preheating process [3, 4].

The inflationary paradigm is well known and extensively studied (for a review see [5]). During inflation, driven by a scalar field $\phi$, the Universe expands exponentially solving in this way the horizon and flatness problems of the standard cosmology. The large scale structure of the Universe is generated through fluctuations during the inflationary period. This fixes the mass of the inflaton field to a value of $m_\phi \sim 10^{13}$ GeV. During inflation, the inflaton field slowly rolls down towards the minimum of its potential. Inflation ends when the potential energy associated with the inflaton field becomes comparable to the kinetic energy. When this happens all the energy of the Universe is contained entirely in the form of coherent oscillations of the inflaton field around the minimum of its potential. At that time, through a purely classical mechanism of parametric resonance [6], studied in detail in [7, 8], stable very heavy particles will be produced in excess. These entities are known as X-particles, with masses $m_X \gtrsim m_\phi$ and will in general overclose the Universe. Parametric resonance for X-particles is ineffective if X is either a fermion field or its coupling to inflaton is small [7].

Another mechanism of generating heavy DM has been proposed in [4]. The DM is produced in the transition between an inflationary and a matter-dominated universe due to the “nonadiabatic” expansion of the background spacetime during the transition acting on the vacuum quantum fluctuations. It was shown that for a particular range of the masses of the X-particles the DM needed for the closure the Universe can be produced gravitationally, independently of the details of the transition between the inflationary phase and the matter dominated phase.

In an attempt to explain the spectrum of the highest cosmic rays the authors in [3] also proposed the generation of DM through vacuum fluctuations during inflation. They showed that to have the right number of gravitationally produced X-particles their masses should be comparable to the inflaton mass and their couplings to the inflaton field should be weak.

In this work we study the gravitational particle production in gravity theories where the inflaton field couples to the Einstein tensor and the X-field in addition to its coupling to the inflaton field it also couples to the Einstein tensor. We show that these couplings give a generic suppression mechanism for the production of the heavy X-particles. As the strength of the coupling to the Einstein tensor of the inflaton field or of the X-field is increased, less particles are produced.

The paper is organized as follows. In Section 2 we review the formalism of gravitational particle production. In Section 3 we discuss the inflationary phase allowing the inflaton to couple to the Einstein tensor. In Section 4 we make a systematic study of gravitational particle production for a wide range of parameters. Section 5 contains our conclusions.
2 Gravitational Particle Production

In this section we will review the basic formalism of gravitational particle production in a curved spacetime and we will derive the Bogolyubov coefficients. We will apply this formalism to a FRW background and we will extend the formalism to the case of a scalar field coupled to Einstein tensor.

2.1 Basic Formalism

We consider a scalar field $\phi$ with the Lagrange density

$$L = \sqrt{-g} \left\{ \frac{1}{2} g^{\alpha\lambda} \partial_\alpha \phi \partial_\lambda \phi - V(\phi) \right\},$$

from which we get the equation of motion for the scalar field

$$\Box \phi + V_\phi = 0,$$

where

$$\Box \phi = (g)^{-1/2} \partial_\mu \left( (g)^{1/2} g^\mu_\nu \partial_\nu \phi \right).$$

For the spacelike hypersurfaces defined by a constant value for $t$ we define the inner product of the solutions of (2.2) as

$$(\phi_1, \phi_2) \equiv i \int (-g)^{1/2} g^0_\nu (\phi_1^*(x) \partial_\nu \phi_2(x) - \phi_2(x) \partial_\nu \phi_1^*(x)) d^3x .$$

We expand the field $\phi$ in terms of the complete set of the modes $\chi_\vec{k}$

$$\hat{\phi}(x) = \sum_\vec{k} (\hat{a}_\vec{k} \chi_\vec{k}(x) + \hat{a}_\vec{k}^\dagger \chi_\vec{k}^*(x)) ,$$

with $\chi_\vec{k}$ satisfying

$$(\chi_\vec{k}, \chi_\vec{k}^\prime) = \delta_{\vec{k}\vec{k}^\prime} , \quad (\chi_\vec{k}^*, \chi_\vec{k}^\prime) = -\delta_{\vec{k}\vec{k}^\prime} , \quad (\chi_\vec{k}, \chi_\vec{k}^*) = 0 .$$

The operators $\hat{a}_\vec{k}$ and $\hat{a}_\vec{k}^\dagger$ satisfy the commutation relations

$$[\hat{a}_\vec{k}, \hat{a}_\vec{k}^\dagger] = 0 , \quad [\hat{a}_\vec{k}^\dagger, \hat{a}_\vec{k}^\dagger] = 0 , \quad [\hat{a}_\vec{k}, \hat{a}_\vec{k}^\dagger] = \delta_{\vec{k}\vec{k}^\prime} .$$

The $n_\vec{k}$ excitations are given by

$$|n_\vec{k}\rangle = \frac{1}{\sqrt{n_\vec{k}!}} (\hat{a}_\vec{k}^\dagger)^n_\vec{k} |0\rangle .$$

Consider now another complete set of modes $\psi_\vec{k}$. The transformation connecting the two sets of modes $\chi_\vec{k}$ and $\psi_\vec{k}$

$$\chi_\vec{k}(x) = \sum_\vec{k}' (\alpha_{\vec{k}\vec{k}'} \psi_\vec{k}'(x) + \beta_{\vec{k}\vec{k}'} \psi_\vec{k}'^*(x))$$

1Throughout the paper we use the “mostly minus” convention $(+ - -)$. 
is the Bogolyubov transformation and the $\alpha_{\vec{k}\vec{k}'}$, $\beta_{\vec{k}\vec{k}'}$ are the Bogolyubov coefficients.

If we expand the field $\phi$ also in terms of the modes $\psi_{\vec{k}'}$,

$$\hat{\phi}(x) = \sum_{\vec{k}'} (\hat{b}_{\vec{k}'} \psi_{\vec{k}'}(x) + \hat{b}^*_{\vec{k}'} \psi^*_{\vec{k}'}(x)) \ ,$$

then one can prove that:

$$\hat{a}_{\vec{k}} = \sum_{\vec{k}'} (\alpha_{\vec{k}\vec{k}'}^* \hat{b}_{\vec{k}'} - \beta_{\vec{k}\vec{k}'}^* \hat{b}^*_{\vec{k}'} ) \ ,$$

$$\hat{b}_{\vec{k}'} = \sum_{\vec{k}} (\alpha_{\vec{k}\vec{k}'} \hat{a}_{\vec{k}'} + \beta_{\vec{k}\vec{k}'}^* \hat{a}^*_{\vec{k}} ) .$$

Imagine that an observer is in the $|0_{\psi}\rangle$ vacuum where there are no $\psi$ particles. Using (2.11) we can calculate the average number of $\chi$ particles

$$\langle 0_{\psi}|\hat{n}_{\chi_{\vec{k}}}|0_{\psi}\rangle = \sum_{\vec{k'}} |\beta_{\vec{k}\vec{k}'}|^2 .$$

Therefore knowledge of the Bogolyubov coefficients $\beta_{\vec{k}\vec{k}'}$ is essential in order to calculate the $\chi$ particles produced.

### 2.2 Particle Production in a FRW Universe

Consider a 4-dimensional Friedmann-Robertson-Walker Universe with metric

$$ds^2 = dt^2 - a^2(t)dx^2 \ ,$$

where $a(t)$ is the scale factor. Then equation (2.2) for the scalar field becomes

$$\ddot{\phi}(x) + 3 \frac{\dot{a}(t)}{a(t)} \dot{\phi}(x) - a^{-2}(t) \sum_{i=1}^{3} \partial_i^2 \phi(x) + V_\phi = 0 \ .$$

We consider the potential function $V(\phi) = \frac{1}{2}[m_\phi^2 \phi^2 + \zeta R(t) \phi^2]$, where we have included a coupling term of the scalar field $\phi$ to the curvature $R$. Then (2.14) becomes

$$\ddot{\phi}(x) + 3 \frac{\dot{a}(t)}{a(t)} \dot{\phi}(x) - a^{-2}(t) \sum_{i=1}^{3} \partial_i^2 \phi(x) + [m_\phi^2 + \zeta R(t)]\phi = 0 \ .$$

We expand $\phi$ in modes as in (2.5) with $\chi_{\vec{k}} \sim e^{i\vec{k}\vec{x}} \chi_{\vec{k}}(t)$. Then (2.15) becomes

$$\ddot{\chi}_{\vec{k}}(t) + 3 \frac{\dot{a}(t)}{a(t)} \dot{\chi}_{\vec{k}}(t) + \left[ \frac{\vec{k}^2}{a^2(t)} + m_\phi^2 + \zeta R(t) \right] \chi_{\vec{k}}(t) = 0 \ .$$

With the transformation $\chi_{\vec{k}}(t) = f(t) h_{\vec{k}}(t)$, where $f(t) = 1/a^{2/3}(t)$, we can eliminate the first derivative in (2.16) and we obtain

$$\ddot{h}_{\vec{k}}(t) + \left[ m_\phi^2 + \frac{\vec{k}^2}{a^2(t)} + \zeta R(t) - \frac{3}{2} \frac{\dot{a}(t)}{a(t)} - \frac{3}{4} \frac{\dot{a}^2(t)}{a^2(t)} \right] h_{\vec{k}}(t) = 0 \ .$$
We assume that asymptotically the spacetime is Minkowski and define two complete sets of modes for the scalar field $\phi$,

$$
\chi^\text{in}_k(x) = \frac{e^{i\vec{k} \cdot \vec{x}} h^\text{in}_k(t)}{V^{1/2} a(t)^{3/2}}, \quad \chi^\text{out}_k(x) = \frac{e^{i\vec{k} \cdot \vec{x}} h^\text{out}_k(t)}{V^{1/2} a(t)^{3/2}},
$$

(2.18)

with $h^\text{in}_k(t)$ and $h^\text{out}_k(t)$ solving (2.17) with the relevant boundary conditions, and the volume $V$ appears in (2.18) to get the correct commutation relations of the $h_k$ modes in the case of $\vec{k} = \vec{k}'$, so that the modes are orthonormal.

Following the formalism of Sec. 2.1 the average number of $\chi^\text{in}$ particles can be calculated from

$$
\langle 0^\text{out} \mid \hat{n}_{\chi^\text{in}_k} \mid 0^\text{out} \rangle = \sum_{\vec{k}'} |\beta_{\vec{k}'}|^2 = \sum_{\vec{k}'} |\beta_{\vec{k}'-\vec{k}}|^2 = |\beta_{\vec{k}}|^2,
$$

(2.19)

where $\beta_{\vec{k}}$ are the Bogolyubov coefficients given by

$$
\beta_{\vec{k}} = -(\chi^\text{out*}_{-\vec{k}}, \chi^\text{in}_{\vec{k}}) = i(h^\text{in}_k(t) h^\text{out}_k(t) - h^\text{out}_k(t) h^\text{in}_k(t)) \, .
$$

(2.20)

### 2.3 Particle Production from a Scalar Field Coupled to Einstein Tensor

So far we have considered a quantized scalar fields minimally coupled to gravity. We will introduce a coupling of a scalar field to the Einstein tensor and we will study how the formalism of gravitational particle production is modified in the presence of this coupling.

The non-minimal couplings between derivatives of a scalar field and curvature are types of scalar-tensor theories [9] in which the field equations can be reduced to second-order differential equations [10]. This kind of interaction belongs to a wider class of scalar-tensor theories having galilean symmetry [11]. These properties of the derivative coupling of the scalar field to the curvature have triggered the interest of the study of the cosmological implications of this new type of scalar-tensor theory [12, 13, 14, 15, 16]. Also local black hole solutions were discussed in [17] and applications to holographic superconductivity were presented in [18, 19].

To introduce the coupling of the scalar field to the Einstein tensor the Lagrangian density (2.1) is modified to

$$
\mathcal{L} = \sqrt{-g} \left\{ \frac{1}{2} g^{\mu\nu} + \chi G^{\mu\nu} \right\} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right\} ,
$$

(2.21)

where the coupling $\lambda$ has the units $[\lambda] = M_{pl}^{-2}$. Then, the equation of motion for the scalar field (2.2) in the cosmological background (2.13) becomes

$$
\left[ 1 - 3\lambda \frac{\dot{a}^2(t)}{a^2(t)} \right] \ddot{\phi}(x) + 3 \left[ \frac{\dot{a}(t)}{a(t)} - \lambda \left( \frac{\ddot{a}(t)}{a^3(t)} + \frac{2\dot{a}(t)\ddot{a}(t)}{a^2(t)} \right) \right] \dot{\phi}(x) +
$$

$$
- \left. \frac{a^2(t)}{3} \sum_{i=1}^{3} \partial_i^2 \phi(x) + \lambda \frac{2\dot{a}(t)\dot{a}(t)}{a^4(t)} \right| \frac{\ddot{a}(t) + \dot{a}^2(t)}{a^2(t)} \sum_{i=1}^{3} \partial_i^2 \phi(x) \right] + V_\phi = 0 \right. .
$$

(2.22)
The definition of the inner product for scalar fields which are coupled non-minimally with the Einstein tensor is a tricky matter. For our case, which involves a diagonal metric and Einstein tensor, a hint may come from the conserved current resulting from the application of the Noether theorem. The relevant symmetry is the phase transformation \( \phi(x) \to e^{iq\theta} \phi(x) \), \( \phi^\dagger(x) \to e^{-iq\theta} \phi^\dagger(x) \). The resulting current is proportional to the expression:

\[
i \int (-g)^{1/2} \left[ g^{0\nu} + \lambda G^{0\nu} \right] (\phi^*(x) \partial_\nu \phi(x) - \phi(x) \partial_\nu \phi^*(x)) d^3 x.
\]

We define the inner product of two different fields (in constant \( t \) hypersurfaces, for instance) to read:

\[
(\phi_1, \phi_2) \equiv i \int (-g)^{1/2} \left[ g^{0\nu} + \lambda G^{0\nu} \right] (\phi_1^*(x) \partial_\nu \phi_2(x) - \phi_2(x) \partial_\nu \phi_1^*(x)) d^3 x.
\]

One may check that in the specific model we are dealing with, this form of the inner product does not depend on the hypersurface chosen.

Using the same potential as before and expanding the scalar field in modes of \( \chi_\vec{k} \), we get

\[
\left[ 1 - 3\lambda \frac{a^2(t)}{a^2(t)} \right] \ddot{\chi}_\vec{k}(t) + 3 \left[ \frac{\dot{a}(t)}{a(t)} - \lambda \left( \frac{\dot{a}^3(t)}{a^3(t)} + \frac{2\dot{a}(t)\ddot{a}(t)}{a^2(t)} \right) \right] \dot{\chi}_\vec{k}(t) + \left[ \frac{k^2}{a^2(t)} - \lambda k^2 \frac{2\dot{a}(t)a(t) + \dot{a}^2(t)}{a^4(t)} + m_\phi^2 + \zeta R(t) \right] \chi_\vec{k}(t) = 0.
\]

Following the discussion in Sec. 2.2, after eliminating the first derivatives in (2.24), we get

\[
\dot{h}_\vec{k}(t) + \Omega_\vec{k}^2(t) h_\vec{k}(t) = 0,
\]

where

\[
\Omega_\vec{k}^2(t) = B(t) - \frac{A(t)}{2} - \frac{A^2(t)}{4}
\]

and

\[
A(t) = \frac{3 \left[ \frac{\dot{a}(t)}{a(t)} - \lambda \left( \frac{\dot{a}^3(t)}{a^3(t)} + \frac{2\dot{a}(t)\ddot{a}(t)}{a^2(t)} \right) \right]}{1 - 3\lambda \frac{a^2(t)}{a^2(t)}},
\]

\[
B(t) = \frac{k^2}{a^2(t)} - \lambda k^2 \frac{2\dot{a}(t)a(t) + \dot{a}^2(t)}{a^4(t)} + m_\phi^2 + \zeta R(t)}{1 - 3\lambda \frac{a^2(t)}{a^2(t)}}.
\]

The definition of the inner product (2.23) adopted above allows us to normalize the \( \chi_\vec{k} \) modes, so that they are orthonormal. The normalized solutions of (2.24) will have the form

\[
\chi_\vec{k}(x) = \frac{e^{ikx}}{V^{1/2} \sqrt{a(t)} \sqrt{3\lambda \dot{a}^2(t) - a^2(t)}}.
\]

Then, the average number of particle produced in the presence of the derivative couplings is given by (2.19) and (2.20).
We will calculate the Bogolyubov coefficients (2.20) solving numerically the scalar equation (2.24). We rewrite (2.24) introducing the dimensionless time $\tau = M_{\text{pl}} t$

$$
\left[ 1 - 3\lambda M_{\text{pl}}^2 \frac{\dot{a}^2(\tau)}{a^2(\tau)} \right] \ddot{\chi}_k(\tau) + 3 \left[ \dot{\chi}_k(\tau) - \lambda M_{\text{pl}}^2 \frac{\dot{a}^3(\tau)}{a^3(\tau)} \right] \dot{\chi}_k(\tau) + \left[ \frac{k^2}{M_{\text{pl}}^2 a^2(\tau)} - \lambda k^2 \frac{2\dot{a}(\tau)a(\tau) + \dot{a}^2(\tau)}{a^2(\tau)} \right] \chi_k(\tau) = \frac{m_\phi^2}{M_{\text{pl}}} + \frac{\zeta R(\tau)}{M_{\text{pl}}} \chi_k(\tau) = 0 . \tag{2.29}
$$

Then (2.25) becomes

$$
\ddot{h}_k^i(\tau) + \Omega_k^2 h_k^i(\tau) = 0 , \tag{2.30}
$$

with

$$
A(\tau) = \frac{3 \left[ \frac{\dot{a}(\tau)}{a(\tau)} - \lambda M_{\text{pl}}^2 \frac{\dot{a}^2(\tau)}{a^2(\tau)} \right]}{1 - 3\lambda M_{\text{pl}}^2 \frac{\dot{a}^2(\tau)}{a^2(\tau)}} ,
$$

$$
B(\tau) = \frac{\frac{k^2}{M_{\text{pl}}^2 a^2(\tau)} - \lambda k^2 \frac{2\dot{a}(\tau)a(\tau) + \dot{a}^2(\tau)}{a^2(\tau)} + \frac{m_\phi^2}{M_{\text{pl}}} + \frac{\zeta R(\tau)}{M_{\text{pl}}}}{1 - 3\lambda M_{\text{pl}}^2 \frac{\dot{a}^2(\tau)}{a^2(\tau)}} . \tag{2.31}
$$

If we choose a scale factor of the form $a^2(\tau) = C + D \tanh \left( \frac{\rho \tau}{M_{\text{pl}}} \right)$ the spacetime is asymptotically Minkowski. The first and second derivatives of the scale factor and the curvature are asymptotically zero. The initial and final values of $\Omega$ are

$$
\omega_{\text{in}} = \sqrt{\frac{k^2}{M_{\text{pl}}^2(C - D)} + \frac{m_\phi^2}{M_{\text{pl}}^2}} ,
$$

$$
\omega_{\text{out}} = \sqrt{\frac{k^2}{M_{\text{pl}}^2(C + D)} + \frac{m_\phi^2}{M_{\text{pl}}^2}} . \tag{2.32}
$$

We solve numerically the differential equation (2.30) first with the initial condition $h_{k}^{i}(\tau_{-}) = \frac{e^{-i\omega_{\text{in}}\tau_{-}}}{\sqrt{2\omega_{\text{in}}}}$ and then with the initial condition $h_{k}^{i}(\tau_{+}) = \frac{e^{-i\omega_{\text{out}}\tau_{+}}}{\sqrt{2\omega_{\text{out}}}}$. The resulting solutions will give the Bogolyubov coefficients from the relation (2.20).

In Fig. 11 we show the average number of particles produced, $|\beta_{k}|^2$, in the quantum state with momentum $\vec{k}$. In principle the sign of $\lambda$ can be positive or negative. However, in [13] stability arguments were put forward and the avoidance of ghosts restricts the sign of $\lambda$ to be negative. Also, an instability was found in [17] looking for local black hole solutions of a gravity theory in the presence of an electromagnetic and a scalar field coupled to Einstein tensor with positive coupling strength $\lambda$. Therefore, for all our numerics we take $\lambda$ to be negative.

We observe that for a fixed value of the mass of the scalar field, the number of particles produced is suppressed as the coupling $\lambda$ is increased.

In the next sections we will apply this formalism to calculate the average particle production after the end of inflation in the preheating epoch. After discussing the inflationary phase of an expanding Universe with an inflaton field coupled to the Einstein tensor, in Sec. 4 we will discuss the particle production of heavy X-particles.
3 Inflationary Phase of an Expanding Universe

In this section after reviewing the basic formalism of the slow roll inflationary phase we will discuss the inflationary phase in the presence of the coupling of the inflaton field to the Einstein tensor.

3.1 Slow-roll Inflationary Phase

We consider a FRW Universe with a metric (2.13) and a scalar field with the action

\[ S_\phi = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right] . \tag{3.1} \]

Varying the above action we get the Friedmann equation

\[ H^2(t) = \frac{8\pi}{3M_{pl}^2} \left[ \frac{\dot{\phi}^2}{2} + V(\phi) \right] , \tag{3.2} \]

with \( H(t) \equiv \frac{\dot{a}(t)}{a(t)} \) and the acceleration equation

\[ 2 \frac{\ddot{a}(t)}{a(t)} + \frac{\dot{a}^2(t)}{a^2(t)} = -\frac{8\pi}{M_{pl}^2} \left[ \frac{\dot{\phi}^2}{2} - V(\phi) \right] \tag{3.3} \]

while the Klein-Gordon equation for the scalar field \( \phi \) is

\[ \ddot{\phi}(t) + 3H(t) \dot{\phi}(t) + V_\phi = 0 . \tag{3.4} \]

Inflation takes place provided that

\[ [\ddot{a}(t) > 0] \Leftrightarrow \left[ \frac{d}{dt} \left( \frac{H^{-1}(t)}{a(t)} \right) < 0 \right] \tag{3.5} \]
and using (3.3) we get the first slow-roll condition $\frac{\ddot{\phi}^2}{2} \ll V(\phi)$, while the second slow-roll condition is given by $\dot{\phi} \ll 3H\dot{\phi}$. Using the slow-roll conditions, equations (3.2) and (3.4) become

$$H^2 \simeq \frac{8\pi}{3M_{pl}^2}V(\phi), \quad 3H\dot{\phi} \simeq -V_\phi,$$

from which we can find the evolution of the scale factor and the inflaton field during the inflationary phase

$$a(t) = a_0 \exp\left[-\frac{8\pi}{M_{pl}^2} \int_{\phi_0}^{\phi(t)} \frac{V}{V_\phi} d\phi\right],$$

$$\phi(t) = \phi_0 - \frac{M_{pl}}{\sqrt{24\pi}} \int_{t_0}^{t} \frac{V_\phi}{\sqrt{V}} dt',$$

where $a_0 = a(t_0)$, $\phi_0 = \phi(t_0)$ and $t_0$ is the time when the inflation starts. The inflation ends time $t_f$, when the kinetic energy becomes comparable to the potential energy. We may define $t_f$ through the relation

$$\frac{V(\phi(t_f))}{V_\phi(\phi(t_f))} = \frac{M_{pl}}{\sqrt{48\pi}}.$$

To study the inflationary phase numerically, we fix the potential to $V(\phi) = \frac{1}{2}M_\phi^2\phi^2$. Then inflation ends at

$$t_f = -\frac{M_{pl} + 2\phi_0\sqrt{3\pi}}{M_\phi M_{pl}}.$$

We define a dimensionless time $\tau \equiv M_\phi t$ and a dimensionless field $\psi(\tau) \equiv \frac{\phi(t)}{M_{pl}}$ so previous relations can be rewritten as

$$a(\tau) = a_0 \exp\left[\sqrt{\frac{4\pi}{3}} \left(\psi_0 \tau - \frac{\tau^2}{\sqrt{48\pi}}\right)\right], \quad \psi(\tau) = \psi_0 - \frac{\tau}{\sqrt{12\pi}},$$

$$\tau_f = 2\sqrt{3\pi}\psi_0 - 1, \quad \psi(\tau_f) = \frac{1}{\sqrt{12\pi}}, \quad \dot{\psi}(\tau_0) = -\frac{1}{\sqrt{12\pi}}.$$

We use the following values for the numerics

$$M_{pl} = 10^{19} GeV, \quad \psi_0 = 3.5, \quad t_0 = 0, \quad a_0 = 1, \quad M_\phi = 10^{-6} M_{pl}.$$

The value of $\psi_0 = 3.5$ is fixed from the requirement to have the right number of e-folds given by the relation

$$N \equiv \ln \frac{a(\tau_f)}{a(\tau_0)}.$$

In Fig. 2 we show the evolution of $\psi(\tau)$ as it results from the numerical solution of the initial equations (3.2), (3.4) and the slow-roll approximation. Until the end of inflation the numerical and the approximate solution coincide, while at the end of the inflation the $\psi$ field oscillates at the bottom of the potential producing particles until it discharges.
Figure 2: The evolution of $\psi$ as function of $\tau$. The continuous line shows the numerical solution of $\psi$, the dashed line the solution from the slow-roll approximation, while the vertical line indicates the end of slow-roll phase.

3.2 Inflationary Phase with the Inflaton Field Coupled to the Einstein Tensor

In the presence of the derivative coupling of the inflaton field to the Einstein tensor the action is modified to

$$S_\phi = \int d^4x \sqrt{-g} \left\{ \frac{R}{16\pi G} + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{2} \lambda_1 G^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \right\}.$$  \hspace{1cm} (3.14)

We have denoted by $\lambda_1$ the coupling of the inflaton field to the Einstein tensor; later on a similar coupling $\lambda_2$ will appear in connection with the X-field. Varying the above action we get the Einstein equations [10, 20]

$$G_{\mu\nu} = -8\pi G [T_{\mu\nu} + \lambda_1 \Theta_{\mu\nu}],$$  \hspace{1cm} (3.15)

with

$$T_{\mu\nu} = \partial_\mu \phi \partial_\nu \phi - \frac{1}{2} g_{\mu\nu} g^{ab} \partial_a \phi \partial_b \phi + g_{\mu\nu} V(\phi),$$  \hspace{1cm} (3.16)

and

$$\Theta_{\mu\nu} = - \frac{1}{2} \nabla_\mu \phi \nabla_\nu \phi R + 2 \nabla_a \phi \nabla_{(\mu} \phi R_{\nu)}^a - \frac{1}{2} G_{\mu\nu} (\nabla \phi)^2 + \nabla^a \phi \nabla_b \phi R_{\mu\nu \alpha\beta} + \nabla_\mu \nabla^a \phi \nabla_\nu \nabla_a \phi$$

$$- \nabla_\mu \nabla_\nu \phi \Box \phi + g_{\mu\nu} \left[- \frac{1}{2} \nabla^a \phi \nabla_a \nabla_b \phi + \frac{1}{2} (\Box \phi)^2 - \nabla_b \phi \nabla_b \phi R_{ab} \right].$$  \hspace{1cm} (3.17)

In the cosmological background (2.13) and with a quadratic potential $V(\phi) = \frac{1}{2} M_\phi^2 \phi^2$ we get the Friedmann equation

$$\frac{\dot{a}^2(t)}{a^2(t)} = \frac{4\pi}{3M_{pl}^2} \left[ \dot{\phi}^2(t) \left(1 - 9\lambda_1 \frac{\dot{a}^2(t)}{a^2(t)}\right) + M_\phi^2 \phi^2(t) \right],$$  \hspace{1cm} (3.18)
while the field equation for the scalar field (2.22) becomes
\[
\left(1 - 3\lambda_1 \frac{\ddot{a}^2(t)}{a^2(t)}\right)\ddot{\phi}(t) + \left(3 \frac{\dot{a}(t)}{a(t)} - 3\lambda_1 \left(\frac{\dot{a}^3(t)}{a^3(t)} + \frac{2\dot{a}(t)\ddot{a}(t)}{a^2(t)}\right)\right)\dot{\phi}(t) + M_\phi^2 \phi(t) = 0 . \tag{3.19}
\]
Defining as before the dimensionless time \(
\tau \equiv M_\phi t\), we get
\[
\dot{a}^2(\tau) \equiv \frac{4\pi}{3} \left[\psi^2(\tau) \left(1 - 9\lambda_1 M_\phi^2 \frac{\dot{a}^2(\tau)}{a^2(\tau)}\right) + \psi^2(\tau)\right] , \tag{3.20}
\]
\[
\left(1 - 3\lambda_1 M_\phi^2 \frac{\dot{a}^2(\tau)}{a^2(\tau)}\right) \ddot{\psi}(\tau) + \left(3 \frac{\dot{a}(\tau)}{a(\tau)} - 3\lambda_1 M_\phi^2 \left(\frac{\dot{a}^3(\tau)}{a^3(\tau)} + \frac{2\dot{a}(\tau)\ddot{a}(\tau)}{a^2(\tau)}\right)\right)\dot{\psi}(\tau) + \psi(\tau) = 0 , \tag{3.21}
\]
with \(\psi(\tau) \equiv \phi(\tau)/M_\phi\).

We will solve (3.20) and (3.21) both numerically and in the slow-roll approximation. For the slow-roll approximation in addition to the conditions \(\frac{\dot{\phi}^2}{2} \ll V(\phi)\) and \(\ddot{\phi} \ll 3H\dot{\phi}\) we also consider the conditions \(3\lambda_1 H^2 \gg 1\) and \(\frac{\dot{H}}{H} \ll 1\). We define the dimensionless parameter \(\lambda_1 \equiv \lambda_1 M_\phi^2\). To have the right number of e-folds (\(N \sim 70\)), we choose the initial conditions \(\psi(\tau_0) = 0.83, a(\tau_0) = 1\) and \(\lambda_1 = -4\). Then the time evolution of \(\psi(\tau)\) is shown in Fig. 3.

Figure 3: Numerical solution of Friedmann and scalar equations (continuous lines), and the solution of the equations in the slow roll approximation (dashed lines).

An important observation is that the introduction of the coupling of the inflaton field to Einstein tensor allows us to go beyond the Chaotic Inflation Scenario [21] in which the inflaton mass is comparable to the Planck scale. By choosing appropriate values of the coupling \(\lambda_1\) we can have the right number of e-folds for much lower values of the \(\psi\) field mass. Another observation is that the coupling to Einstein tensor acts as a friction term, absorbing energy from the kinetic energy of the inflaton field as it rolls down the potential, as first noticed in [15]. This results in prolonging the duration of the inflation. This effect is generic and does not depend on the slow roll approximation, as it can be seen in Fig. 4.

The inflation ends when the kinetic energy becomes comparable to the potential energy
\[
\dot{\phi}^2(t) \left(1 - 9\lambda_1 \frac{\dot{a}^2(t)}{a^2(t)}\right) \simeq 2V(\phi) . \tag{3.22}
\]
This gives
\[ \dot{\phi}^2(\tau) = \frac{\psi^2(\tau)}{1 - 9\lambda_1 H^2(\tau)}. \] (3.23)

The above relation allows a very small window of positive $\lambda_1$
\[ 0 < \lambda_1 < \frac{1}{9H^2(\tau)}. \] (3.24)

We will discuss its significance in Section 4 where we will present the gravitational particle production after inflation in the presence of the derivative coupling.

4 Particle Production after the End of Inflation

We will study the gravitational particle production after the end of inflation using two coupled scalar fields in an expanding Universe. The scalar fields are the inflaton field $\phi(t)$ which drives the inflation and a quantum field $X(x)$ which produces the X-particles. The action of the theory is
\[
S = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} + \int d^4x \sqrt{-g} \left\{ \frac{1}{2} \left[ (g^{\mu\nu} + \lambda_1 G^{\mu\nu}) \partial_\mu \phi \partial_\nu \phi - M^2_\phi \phi^2 \right] \right. \\
+ \left. \frac{1}{2} \left[ (g^{\mu\nu} + \lambda_2 G^{\mu\nu}) \partial_\mu X \partial_\nu X - (M^2_X + \zeta R + g^2 \phi^2) X^2 \right] \right\}. \] (4.1)

The inflaton field couples to Einstein tensor with field strength $\lambda_1$, the X-field couples to the Einstein tensor with field strength $\lambda_2$ and we have also included a direct coupling of the inflaton to X-field with coupling $g$.

The strategy which we will follow is to find the value of the inflaton field at the end of inflation with or without its coupling to Einstein tensor and then through its coupling to
the quantum X-field to calculate the number of X-particles produced due to the expansion of the Universe.

Following the analysis of Sec. 2.3 we end up to the differential equation

\[ \ddot{h}_k(\tau) + \Omega^2_k(\tau) h_k(\tau) = 0, \]  

with

\[ \Omega^2_k(\tau) = B(\tau) - \frac{\dot{A}(\tau)}{2} - \frac{A^2(\tau)}{2} \]  

and

\[ A(\tau) = 3 \frac{\dot{a}(\tau) - \lambda_2 \left( \frac{\dot{a}^3(\tau)}{a^4(\tau)} + 2 \frac{\dot{a}(\tau)\ddot{a}(\tau)}{a^2(\tau)} \right)}{1 - 3 \lambda_2 \frac{\dot{a}^2(\tau)}{a^2(\tau)}}, \]

\[ B(\tau) = \frac{k^2}{M_\phi^2 a^2(\tau)} - \lambda_2 k^2 \frac{2\dot{a}(\tau)\ddot{a}(\tau) + \dot{a}^2(\tau)}{a^2(\tau)} + \frac{M^2}{M_\phi^2} + \frac{\xi R(\tau)}{M_\phi^2} + \frac{\zeta R(\tau)}{M_\phi^2} - \frac{\phi^2(\tau) M^2_{pl}}{M_\phi^2}. \]  

We have used the notations \( \tau \equiv M_\phi t, \lambda_2 \equiv \lambda_2 M_\phi^2 \). After the end of inflation we assume that the Universe enters a matter domination epoch with the scale factor \( a(\tau) = \tau^{2/3} \). It is more convenient to calculate the Bogolyubov coefficients from the differential equations

\[ \dot{\alpha}_k(\tau) = \frac{\dot{\Omega}_k(\tau)}{2\Omega_k(\tau)} \exp \left[ 2i \int \Omega_k(\tau')d\tau' \right] \beta_k(\tau), \]

\[ \dot{\beta}_k(\tau) = \frac{\dot{\Omega}_k(\tau)}{2\Omega_k(\tau)} \exp \left[ -2i \int \Omega_k(\tau')d\tau' \right] \alpha_k(\tau). \]  

Then a function of the form

\[ h_k(\tau) = \frac{\alpha(\tau)}{\sqrt{2\Omega_k(\tau)}} e^{-i \int \Omega_k(\tau')d\tau} + \frac{\beta(\tau)}{\sqrt{2\Omega_k(\tau)}} e^{i \int \Omega_k(\tau')d\tau}. \]  

solves (4.2). We will first calculate the number of particles produced without the couplings to the Einstein tensor and then we will switch on \( \lambda_1 \) and \( \lambda_2 \) and compare the results.

### 4.1 Particle Production without Derivative Couplings

We consider the usual slow-roll inflationary phase as it was discussed in Sec. 3.1 without the coupling of the inflaton field to the Einstein tensor \( (\lambda_1 = 0) \). Also the quantum field X does not couple to Einstein tensor \( (\lambda_2 = 0) \). We only allow the coupling of the X-field to the inflaton field. The inflation starts at \( \tau_0 = 0 \) and with the values of the parameters

\( (3.12) \) ends at \( \tau_f = 20 \). Then the inflaton oscillates around the minimum of the potential until it discharges. The Universe enters a matter-dominated epoch and, according to our discussion in Sec. 2.2 X-particles will be produced.
We consider the following initial conditions and values of the parameters
\[
\alpha(\tau_f) = 1, \quad \beta(\tau_f) = 0, \quad M_\phi = 10^{-6} M_{pl}, \quad k = 10^{-5} M_{pl}.
\] (4.7)
Then we have \(\Omega_k^2(\tau) > 0\) and we know that the adiabatic approximation \(\left(\frac{\dot{\Omega}_k(\tau)}{\Omega_k(\tau)} \ll 1\right)\) holds \[24, 25\].

In Fig. 5 we show the average number of particles \(|\beta_k|^2\) as a function of the mass of the X-particle \(M_X\) for \(M_X > 10^{-6} M_{pl}\) for various values of the coupling \(g\) between the inflaton and the X-field and in Fig. 6 we show the average number of particles for various values of the coupling of the X-field to curvature.

In both cases we observe that for a fixed mass of the X-particle we have an enhancement of the particle production as the values of the couplings are increased. Note that in order to have a sizeable effect in the case that the X-field couples to the curvature, the coupling \(\zeta\) should be large.

### 4.2 Particle Production with Derivative Couplings

We now turn on the derivative couplings. We start with the inflaton field coupled to Einstein tensor with \(\lambda_1 = -4\) and the X-field coupled to Einstein tensor. We consider the case in which there is no coupling between the X-field and the inflaton field and also there is no coupling to curvature.

In Fig. 7 we have the first indication that as the coupling \(\lambda_2\) is increased (in absolute values) less particles are produced. However, to have a sizable effect the coupling strength of \(\lambda_2\) should be very large, comparable to \(\lambda_1\), in order to compensate the attenuation of the Einstein tensor due to the expansion of the Universe.

In Fig. 8 and Fig. 9 we show the particles produced if we turn on the coupling of the inflaton field to X-field. We see that with the increase of the coupling \(g\) we have an increase of the number of particles produced. However the X-particles produced are less than the X-particles produced with \(\lambda_1 = \lambda_2 = 0\) as can be seen in Fig. 5.

This effect can be seen more clearly if we increase the coupling of the inflaton field to Einstein tensor as can be seen in Fig. 10 and Fig. 11. In Fig. 12 we fix the value of \(g = 5.10^{-7}\) and the value of \(\lambda_1 = -6\) and we vary the value of \(\lambda_2\). We see that less particles are produced as the absolute value of the coupling \(\lambda_2\) is increased.

For completeness let us discuss the particle production in the very small window of positive values for \(\lambda_1\) given in relation (3.24). In Fig. 13 and Fig. 14 we set \(\lambda_2 = 0\) and we show the particle production for the couplings \(\lambda_1 = 0.0001\) and \(\lambda_1 = 0.001\) respectively. This time we notice an enhancement on the particles produced. We observe the same thing if we set \(g = 5 \cdot 10^{-7}\) and draw the graph for different values of \(\lambda_1\) (Fig. 15). However as we already stated the window of positive values for \(\lambda_1\) is small if one wants to have a proper inflation. This can be checked in Fig. 16 were we show that by increasing the value of a

\[2\text{In } [22] \text{ a model with derivative couplings was discussed in which the inflaton field decayed to particles via a phenomenological field.}\]

\[3\text{Particles will also be produced through the resonance effect. However, we will not discuss this process here. We also assumed that there is no particle production during inflation.}\]
positive $\lambda_1$ one ends up with a more steep roll of the inflaton and questions of instability are arising.

In summary, the introduction of the coupling to Einstein tensor of the inflaton field or of the X-field results in the suppression of the number of particles produced after the end of inflation.

5 Conclusions

We have studied the particle production due to the expansion of the Universe of a scalar field coupled to Einstein tensor. After reviewing the mechanism of gravitational particle production we applied it to a FRW expanding Universe. Introducing a coupling of a scalar field to the Einstein tensor we find that the number of gravitationally produced particles is decreasing as the strength of the coupling to the Einstein tensor is increasing.

In a more realistic setup, we studied the particle production due to the expansion of the Universe after the end of inflation in the preheating epoch. Introducing a X-field coupled to the inflaton field we find that as the strength of the coupling is increased the number of X-particles produced is increased, as expected.

After reviewing the inflationary phase driven by an inflaton field coupled to Einstein tensor we introduced a coupling of the X-field to Einstein tensor. We carried out a detailed study of gravitationally produced heavy X-particles in the presence of the derivative couplings. We found that as the strength of the couplings of either the inflaton field or the X-particles to the Einstein tensor is increased, less particles are produced. The dominant effect comes from the coupling of the inflaton to the Einstein tensor as the Einstein tensor after the end of inflation attenuates due to the expansion of the Universe.

As we discussed the presence of the coupling of the scalar field to Einstein tensor acts as a friction term absorbing energy from the kinetic energy of the scalar field. We can attribute the suppression of the particle production to the same mechanism. As the strength of the coupling $\lambda_1$ is increased, less energy is transferred to the X-particles through the coupling of the inflaton field to the X-field, so less particles are produced. This can also be understood as an effect of the geometry. Curvature effects are strong during inflation absorbing energy, while after inflation curvature is small and this is the reason why the coupling of the X-field to Einstein tensor does not give sizeable effects.

The main theoretical problem of particle production after the end of inflation is to control the number of particles produced in such a way as not to overclose the Universe. This is achieved with a fine-tuning of parameters. The gravitational particle production is a dynamical mechanism between the classical gravitational field and a quantum field. For this reason has less fine-tuning of parameters. However, to have the right number of particles produced, certain assumptions should be fulfilled. It would be interesting to apply this suppression mechanism we discussed in this work to produce a realistic cosmological model of particle production after the end of inflation.
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Figure 5: Bogolyubov coefficients of the X-field for various values of its coupling to ψ-field.

Figure 6: Bogolyubov coefficients of the X-field for various values of its coupling to the curvature.
Figure 7: Bogolyubov coefficients of the X-field for various values of its coupling to the Einstein tensor.

Figure 8: Bogolyubov coefficients of the X-field with no couplings to Einstein tensor, for various values of its coupling to the inflaton.
Figure 9: Bogolyubov coefficients of the X-field coupled to Einstein tensor, for various values of its coupling to the inflaton.

Figure 10: Bogolyubov coefficients of the X-field with no couplings to Einstein tensor, for various values of its coupling to the inflaton.
Figure 11: Bogolyubov coefficients of the X-field coupled to Einstein tensor, for various values of its coupling to the inflaton.

Figure 12: Bogolyubov coefficients of the X-field for various values of its coupling to the Einstein tensor.
Figure 13: Bogolyubov coefficients of the X-field with no couplings to Einstein tensor, for various values of its coupling to the inflaton ($\lambda_1 = 0.0001$).

Figure 14: Bogolyubov coefficients of the X-field with no couplings to Einstein tensor, for various values of its coupling to the inflaton ($\lambda_1 = 0.001$).
Figure 15: Bogolyubov coefficients of the X-field for various couplings of the Einstein tensor to the inflaton

Figure 16: Evolution of inflaton $\psi$ as a function of $\tau$ for positive values of the coupling $\lambda_1$