Adversarial Models and Resilient Schemes for Network Coding

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Abstract

In a recent paper, Jaggi et al. [12], presented a distributed polynomial-time rate-optimal network-coding scheme that works in the presence of Byzantine faults. We revisit their adversarial models and augment them with three, arguably realistic, models. In each of the models, we present a distributed scheme that demonstrates the usefulness of the model. In particular, all of the schemes obtain optimal rate \( C - z \), where \( C \) is the network capacity and \( z \) is a bound on the number of links controlled by the adversary.

1 Introduction

Network coding is a powerful paradigm for network communication. In “traditional” networks, internal nodes simply transmit packets that arrive to them (without any substantial change of their content). In contrast, when performing network coding, internal nodes of the network are allowed to mix the information from different packets they receive before transmitting on outgoing edges. This mixing may substantially improve the throughput of a network, it can be done in a distributed manner with low complexity, and is robust to packet losses and network failures, e.g., [1, 17, 15, 13, 8].

The focus of this paper is network coding for multicast networks (where a single sender wants to transmit the same information to several receivers), at the presence of Byzantine network faults. A Byzantine adversary that may maliciously introduce erroneous messages into a network may be especially disruptive when network coding is applied. The simple reason is that any message (including the faulty ones) affect all messages on its path to the recipient. Therefore, a single faulty message may contaminate many more messages down the line.

Motivated by the above difficulty, there has been some work on detecting and correcting Byzantine faults. We distinguish between computationally unbounded and computationally bounded adversaries. For computationally unbounded Byzantine adversaries, error detection was first addressed in [9]. This was followed by the work of Cai and Yeung [18, 2], who generalize standard bounds on error-correcting codes to networks, without providing any explicit algorithms for achieving these bounds. Jaggi et al. [11], consider an information-theoretically rate-optimal solution to Byzantine attacks, which however requires a centralized design. Finally, a distributed polynomial-time rate-optimal network-coding scheme was recently obtained (independently) by Jaggi et al. [12] and Koetter and Kschischang [14]. Error detection for multicast network coding in the presence of computationally bounded Byzantine adversaries was also considered in the past [16, 4]. In these works various authentication schemes are performed at internal nodes of the network.

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In [16] a centralized trusted authority is assumed to provide hashes of the original packets to each node in the network, [3] obviates the need for a trusted entity under the assumption that the majority of packets received at terminal nodes is uncorrupted.

This paper builds on the scheme of [12] to obtain distributed polynomial-time rate-optimal network-coding schemes in three realistic adversarial models. Our schemes, as well as those of [12] assume no knowledge of the topology of the network and follow the distributed network coding protocol of [8]. Namely, their implementation involves only a slight modification of the source and destination while the internal nodes can continue to use the standard protocol of [8]. Before we mention our contribution in detail, we present a brief description of the adversarial models studied in [12]. In the following informal summary, a sender named ‘Alice’ is interested in the transmission of information to a group of receivers named ‘Bob’ over a given network. The Byzantine adversary, ‘Calvin’, controls some of the links of the network and injects erroneous messages into the network in aim to corrupt the communication between Alice and Bob.

**Omniscient adversary model:** In this model Calvin is all-powerful and all-knowing, and is limited only by the number of links $z$ under his control. [12] obtained a network coding scheme with optimal rate for this model of $C - 2z$, where $C$ is the network capacity.

**Secret channel model:** This model allows Alice to send to Bob a short (low rate) secret, which is completely hidden from Calvin (who is again all-powerful and all-knowing (excluding the secret), and is limited by the number of links $z$ under his control). [12] obtained a network coding scheme with optimal rate of $C - z$ for this model. Notice that the rate achievable in this model is strictly higher than that in the Omniscient model. This secret channel model was originally referred to in [12] as the ‘shared secret model’. We rename it here to secret channel model as the secret shared in this model between Alice and Bob may depend on Alice’s message. We elaborate on this point in detail shortly.

**Limited eavesdropping model:** The last model, which is the least relevant to our work, limits the number of links on which Calvin can eavesdrop (it was originally named “limited adversary model” and we rename it here for concreteness). In this model [12] obtained a network coding scheme with rate $C - z$, as long as Calvin can eavesdrop on at most $C - z$ links (in addition to the $z$ links under his complete control).

**Our Contribution**

In this work we introduce three additional adversarial models, and give optimal rate efficient distributed network-coding schemes in each of the models. As mentioned above, our schemes (as well as those of [12]) assume no knowledge of the topology of the network and follow the distributed network coding protocol of [8]. Roughly speaking, we obtain an optimal rate of $C - z$ on all the adversarial models described below (the optimality of our schemes follow, e.g., from [11]).

**Random-Secret Model:** The first model we present is the random secret model in which Alice and Bob share a short (uniformly distributed) random secret which is completely hidden from Calvin. Calvin is all-powerful and all-knowing (excluding the secret), and is limited by the number of links $z$ under his control. This model differs from the ‘secret channel’ model discussed in [12] in the sense that the secret that is shared by Alice and Bob is not constructed as a function of Alice’s message, but rather is uniformly distributed and independent of Alice’s message. The independence of the secret shared by Alice and Bob from the actual message $M$ being transmitted by Alice has several advantages. This allows Alice and Bob to share their secret prior to the act of communicating $M$. For example, one may consider the scenario where Alice and Bob are able to meet (or communicate) in advance and share a large source of completely random bits (such as a CD of uniformly generated bits). As long as these bits are unknown to Calvin, they can be used overtime.
to communicate at high rate over the network (without the need of an additional low rate channel connecting Alice and Bob). Moreover, as we will see shortly, communication in this model sets the foundations for communicating at high rate in the setting in which Calvin is computationally bounded (more specifically, in the symmetric key cryptographic setting). We would like to note that in the scheme of [12] for their ‘secret channel’ model, the secret information that Alice and Bob share indeed strongly depends on the message \( M \) Alice transmits to Bob and hence cannot extend naturally to the examples mentioned above.

For the random secret model we obtain a network coding scheme with optimal rate of \( C - z \). Our scheme is obtained by a transformation of the scheme of [12] for the secret channel model. In our proof we do not need to get into the finer details of the original scheme and instead observe and exploit a useful property of the original secret composition.

**Causal-Omniscient Model:** As in the Omniscient model of [12], in this model we assume Calvin is all-powerful and all-knowing, and is again limited by the number of links \( z \) under his control. However, to obtain rate greater than \( C - 2z \), we slightly restrict Calvin. Namely, we assume that Calvin is causal. Specifically, when Calvin injects messages into the network at time step \( t \), he only has access to messages sent by Alice at time steps at most \( t + \Delta \). Here, \( \Delta \) is some parameter of the network which is considered small compared with the length of the communication stream.

We present an optimal rate distributed network coding scheme for this model. Just as in the omniscient adversary model, our scheme requires \( C > 2z \). However, in such a case we obtain a rate of \( C - z \) (compared with \( C - 2z \) in the omniscient adversary model). Our scheme is obtained by a fully modular composition of two network coding schemes from [12]: one for the omniscient adversary model and the other for the secret channel model. The study of the causal-omniscient model will set the foundations for communicating at high rate in yet an additional setting in which Calvin is computationally bounded, the public key cryptographic setting.

We would like to note that causal adversaries were also implicitly studied in [11] in the centralized setting, while in this work we focus on the distributed setting. Nevertheless, the upper bounds proven in [11] imply that the requirement of \( C > 2z \) is necessary (otherwise no information can be transmitted).

**Computationally-Bounded Adversary Model:** While our previous models did not make any computational assumptions on the parties involved, we now turn to study the case in which Calvin is computationally bounded (as before Calvin is all-knowing, excluding any secret keys, and limited by the number of links \( z \) under his control). In this setting, we present two results. The first result uses the notion of symmetric key cryptography and is based on our random secret model. Roughly speaking, in the case Calvin is computationally bounded, one may replace the random secret in the random secret model, by a series of pseudo-random bits: bits that would still look completely random (i.e., uniform) to Calvin. Now, to generate an (effectively) unlimited amount of shared pseudo-random bits, to be used in several executions of the random secret protocol, all Alice and Bob need to do is exchange a single short secret key prior to the communication process. This single key, and the bits it generates may be used over essentially unlimited time to communicate at high rate.

The second result addresses the public key cryptographic setting. In this setting, each of the parties, Alice and Bob, hold a pair of keys: a private key (known only to itself) and a public key (known to all — including Calvin). Encrypted point to point communication between Alice and Bob can be done using these public and private keys; without Alice and Bob ever meeting in advance to exchange a shared secret key. However, in the model we study, no point to point channel is available - and Alice would like to communicate at high rate to Bob over a given network. We present a network coding scheme for the model.
at hand. Our scheme is based on the scheme we present for the causal-omniscience model, with the sole difference that public-key encryption is used to hide some of Alice’s information from Calvin.

As common in the study of cryptographic primitives, both our results are conditional — in the sense that they hold assuming that certain cryptographic primitives exist (such as the assumption that factoring is hard). Under such assumptions, we prove in the symmetric key setting that our scheme obtains an optimal rate of $C - z$, and in the (weaker) public key setting we obtain the same optimal rate under the condition that $C > 2z$. In this model Calvin is no longer causal, however, as in the causal-omniscient model, it can be seen that the upper bounds of [11] imply that the latter requirement of $C > 2z$ is necessary.

We note that in the public key scenario, we assume that Alice knows Bob’s public key. For this reason, the public-key model seems particularly suitable in settings where cryptography is already involved (e.g., to ensure privacy and integrity of the communication). In such a scenario, a public-key infrastructure may already be available and computational limitations on the adversary are usually already assumed.

The remainder of the paper is organized as follows. Section 2 contains the model definitions and notation. Sections 3, 4 and 5 discuss the three new models and schemes presented above.

## 2 Preliminaries

In this section we give the definitions and notation that are required to model network coding in various adversarial models. Our definitions and notation mostly follow [12].

### Network Model
The network will be modelled as a graph. We assume our graphs are acyclic, and the communication over them is done in a synchronous manner. Namely, in each time step a single packet of information can traverse an edge of the network.

### Network-coding schemes
We will consider the task of routing information over the network from a single sender Alice to multiple receivers Bob (the setting of multicast). In fact, in our analysis, it will usually be sufficient to consider a single receiver Bob. The reason is that in the schemes we suggest, neither Alice nor the network need to be aware of the location of Bob in the network. Therefore, it will be possible to extend each one of our schemes from the case of a single receiver to the case of multiple receivers (this state of affairs is common in the study of multicast network coding, e.g., [12]). We will therefore continue the formalization, assuming a single receiver (and will address the setting of multicast separately for each one of our schemes).

We will not assume that Alice, Bob or any other internal node is aware of the network topology or of the location of Alice and Bob in the network. The network topology will only influence the maximal achievable rate. A network-coding scheme is defined by Alice’s encoder, Bob’s decoder, and the coding performed in internal nodes. We will now discuss those three components.

Let $M$ be the message Alice wishes to transmit to Bob. The encoding algorithm of Alice adds some redundancy into the message, thus obtaining an encoded message $X$. This information is routed through the network, where it is further encoded (as a result of the network coding). Bob receives encoded information that may also encompass network faults. Bob’s decoding algorithm, applied to the encoded information, is supposed to factor out the network faults and retrieve the original message $M$.

It is convenient to assume that Alice’s encoded message $X$ is represented by a $b \times n$ matrix, where every entry of the matrix is an element from a finite field $F_q$. We refer to a column as a slice, to a row as a packet, and to each entry as a symbol. It is also useful to note that in all of the schemes of [12], as well as ours, $X$
is composed of the original message $M$, and in addition some $\delta n$ slices of redundancy. In other words, the size of $M$ is $(1-\delta)$ that of $X$.

The specific coding performed by internal nodes is less relevant to our work, as it is inherited without change from \cite{12}. For concreteness, let us mention that internal nodes, as well as Alice herself, perform random linear network coding a la \cite{8}. Namely, for each of its outgoing links, a node selects random coefficients of a linear transformation over $\mathbb{F}_q$ (the number of coefficients is $b$ for Alice and equals the indegree for any internal node). The network coding of each of the slices of $X$ goes as follows: First Alice sends on each of her out-going edges the corresponding linear transformation of the symbols in the slice. Whenever an internal node receives a symbol on each of its incoming edges (which is in itself a linear transformation of the slice’s symbols), it sends on its outgoing edges the corresponding transformation of those symbols. As common in the literature of network coding, as our graphs are acyclic, we assume that information from different slices is not mixed throughout the communication process. This can be established by sufficient memory at internal nodes of the network.

**Adversarial Model** Each one of the adversarial models we consider in this paper is specified by the exact power of the adversary Calvin. We mention here the common properties of Calvin.

Calvin has under his control $z$ network’s links of his choice\footnote{The parameter $z$ represents Calvin’s power. It is possible to define $z$ as the min cut between Calvin’s links and Bob. This is at most, but may be strictly smaller than the number of links under Calvin’s control.}. On these links Calvin may inject his own packets, disguised as part of the information flow from Alice to Bob. Calvin succeeds if Bob decodes a message different than Alice’s original $M$. The goal of the network-coding scheme is to ensure this only happens with very small probability while maximizing the rate in which information flows from Alice to Bob.

We do not assume that Alice, Bob or any internal node are aware of the links under Calvin’s control. On the other hand, Calvin has full knowledge of the network topology as well as the identity of Alice and Bob. In all of our models we assume that Calvin has full eavesdropping capabilities (i.e., Calvin can monitor the entire communication on each one of the links). Calvin knows the encoding and decoding schemes of Alice and Bob, and the network code implemented by the internal nodes (including the random linear coefficients). Furthermore, in our proofs, we assume that Calvin selects the message $M$ that Alice transmits. This ensures that our schemes work for every message $M$ Alice sends to Bob.

The **network capacity**, denoted by $C$, is the maximum number of symbols that can be delivered on average, per time step, from Alice to Bob, assuming no adversarial interference (i.e., the max flow of information from Alice to Bob). The network capacity is known to equal the min-cut from Alice to Bob. (For the corresponding multicast case, $C$ equals the minimum of the min-cuts over all destinations.) For a message $M$, the **error probability** $e(M)$ is the probability that Bob reconstructs a message different than Alice’s original $M$. The (maximum) error probability of the encoding scheme is defined to be $e = \max_M \{e(M)\}$ (Here the maximization is taken over the message $M$ of Alice). The **rate** is the number of information symbols that can be delivered on average, per time step, from Alice to Bob. In the parameters above, the rate equals $(1-\delta) \cdot b$ (recall that $\delta$ is the fraction of redundant slices). Rate $R$ is said to be achievable if for any $\alpha > 0$ and $\epsilon > 0$ there exists a coding scheme of block length $n$ with rate $\geq R - \alpha$ and error probability $e \leq \epsilon$.\footnote{The parameter $z$ represents Calvin’s power. It is possible to define $z$ as the min cut between Calvin’s links and Bob. This is at most, but may be strictly smaller than the number of links under Calvin’s control.}
2.1 Building Blocks of our Schemes

Our network-coding schemes rely on the schemes of [12], given in two adversarial models: the omniscient adversary model and the secret channel model. We discuss those schemes here.

2.1.1 A scheme in the omniscient-adversary model

In the omniscient adversary model, we put no restrictions on the knowledge and ability of Calvin (see discussion in Section 2). In this model, [12] gave a distributed polynomial-time scheme $A_{omn}$, and proved for it the following theorem:

**Theorem 2.1 ([12])** $A_{omn}$ achieves a rate of $C - 2z$, in the omniscient-adversary model, with code-complexity $O((nC)^3)$.

2.1.2 A scheme in the secret-channel model

In the secret channel model, we assume that Alice can secretly send Bob a (short) message that is completely hidden from Calvin. We put no additional restrictions on the knowledge and ability of Calvin (see discussion in Section 2). In this model, [12] gave a distributed polynomial-time scheme $A_{sc}$, and proved for it the following theorem:

**Theorem 2.2 ([12])** $A_{sc}$ achieves a rate of $C - z$, in the secret-channel model, with code-complexity $O(nC^2)$. The communication on the secret channel consists of at most $C^2 + C$ symbols.

In Section 3 we give some more details on the way the secret message is defined in $A_{sc}$.

2.2 Proof techniques: reduction and worst case analysis

A scheme is said to be (information theoretically) secure against an adversarial entity Calvin, if for any behavior of Calvin, Alice is able to communicate her information to Bob (with high probability). Loosely speaking, we think of Calvin as an algorithmic procedure, which given certain inputs (such as the network topology, Alice’s information and the network code applied by the network), computes which edges in the network to corrupt and which error message to transmit.

There are several proof paradigms that can be used in an attempt to establish the correctness of a given coding scheme. In this work, the correctness of our coding schemes will be proven by means of reduction. Namely, we build upon the results of [12], and prove that any adversarial entity Calvin that breaks our schemes will imply an additional adversary (usually referred to as Calvin’) that will not allow communication in one of the schemes presented in [12].

More specifically, our proofs can be outlined as follows. We first define our coding schemes. We will then assume for sake of contradiction that they are not secure. As we would like our schemes to be secure for any message $M$ of Alice, this will imply the existence of an adversary Calvin that first chooses which message $M$ Alice should send to Bob, and then is able to corrupt the communication of $M$ between Alice and Bob. Thinking of Calvin as an algorithmic procedure, we show how to define the additional adversary Calvin’ — which is a procedure based on Calvin. Finally we show that Calvin’ is able to break one of the (provably secure) schemes presented in [12] — this suffices to conclude our proof.
3 Random-Secret Model

The random-secret model is similar to the secret-channel model of [12] with the difference that the secret information sent from Alice to Bob should be random and independent of Alice’s input message $M$. Formally, we allow Alice to share with Bob a short secret (which is uniformly distributed). This secret will stay hidden from Calvin.

Alice’s secret and message encoding in $\mathcal{A}_{SC}$ Recall that $\mathcal{A}_{SC}$ is the scheme presented in [12] for communication in the secret-channel model. We will show how to transform $\mathcal{A}_{SC}$ to a comparable scheme which works in the random secret model. The only ingredients of $\mathcal{A}_{SC}$ we need to recall is the structure of Alice’s secret and of Alice’s encoder. The encoding of $M$ into $X$ is very simple: We assume that Alice’s message $M$ is a $b \times (n - b)$ matrix over $\mathbb{F}_q$. The matrix $X$ is $M$ concatenated with the $b \times b$ identity matrix, $I$. Namely, $X = [M I]$.

Alice’s secret message is computed in two steps. She first chooses $C$ parity symbols uniformly at random from the field $\mathbb{F}_q$. The parity symbols are labelled $r_d$, for $d \in \{1, \ldots, C\}$. We denote by $R$ the vector of parity symbols. Corresponding to the parity symbols, Alice’s parity-check matrix $P$ is defined as the $n \times C$ matrix whose $(i, j)^{th}$ entry equals $(r_j)^i$, i.e., $r_j$ taken to the $i^{th}$ power. The second part of Alice’s secret message is the $b \times C$ hash matrix $H$, computed as the matrix product $X \cdot P$. The secret message sent by Alice to Bob on the secret channel is composed of both $R$ and $H$. As $n \leq C$, we indeed have a secret of at most $C^2 + C$ symbols.

A useful property of $\mathcal{A}_{SC}$ Note that the vector $R$ of Alice’s secret is already uniform and independent of the message $M$. On the other hand, the hash $H$ is a deterministic function of $R$ and $M$ (given by the equation $H = X \cdot P$). Our main observation (which we will prove below) is the following: for almost every value of $R$, when $M$ is uniform then $H$ is uniform as well. Furthermore, it is enough that a small chunk of $M$ will be uniform to guarantee the uniformity of $H$. This suggests the following idea: instead of selecting $H$ as a function of Alice’s message, we can select both $R$ and $H$ uniformly at random. Later, Alice can tweak the message a bit such that we indeed get $H = X \cdot P$. We continue to formalizing this idea.

3.1 Defining the new scheme $\mathcal{A}_{RS}$

We now show how to transform $\mathcal{A}_{SC}$ into a scheme $\mathcal{A}_{RS}$ with comparable performance in the random secret model. To define the scheme we now define the random secret, Alice’s encoder, Bob’s decoding, and the coding in internal nodes.

The random secret The secret shared between Alice and Bob is composed of a length-$C$ vector $R$ over $\mathbb{F}_q$ and a $b \times C$ matrix $H$ over $\mathbb{F}_q$. Both are selected uniformly at random (and independently of each other).

Even though $R$ and $H$ are selected uniformly, their function in $\mathcal{A}_{RS}$ is identical to the function of $R$ and $H$ in $\mathcal{A}_{SC}$. We therefore use the same notation as given above. In particular, we refer to $H$ as the hash matrix. The elements of $R$ are referred to as the parity symbols and denoted $r_d$, for $d \in \{1, \ldots, C\}$. Furthermore, we define the corresponding parity-check matrix $P$ as before.

Alice’s encoder We allow Alice to encode a slightly shorter input message $M$ assumed to be $b \times (n - b - C)$ matrix over $\mathbb{F}_q$. Alice encodes $M$ into a $b \times n$ matrix $X = [L M I]$, where $L$ is a $b \times C$ matrix and $I$ is the $b \times b$ identity matrix. The matrix $L$ is defined (arbitrarily) such that $H = X \cdot P$. We show shortly that this
system of linear equations (on the elements of $L$) will have a unique solution with high probability over $H$ and $P$. If this system has no solution or more than a single solution we define $L$ arbitrarily (say, to be the all-zero matrix).

**Network coding and Bob’s decoder** Both the network coding and Bob’s decoder are defined in the same way as in $A_{SC}$. Once Bob decodes a matrix $[L \ M]$, Bob discards of the $b \times C$ prefix $\bar{L}$ and outputs $\bar{M}$.

### 3.2 Properties of $A_{RS}$

We now state and prove the properties of $A_{RS}$ that are almost identical to those of $A_{SC}$:

**Theorem 3.1** $A_{RS}$ is a distributed polynomial-time scheme. $A_{RS}$ achieves a rate of $c - z$, in the random-secret model, with code-complexity $O(nC^2)$. The random secret consists of at most $C^2 + C$ symbols.

**Proof:** We will prove that the probability that Bob decodes correctly in $A_{RS}$ is almost identical to the probability that Bob decodes correctly in $A_{SC}$. The theorem will then follow immediately from the definition of $A_{RS}$ and from Theorem 2.2. We note that even though Alice is able to send to Bob a little bit less information in $A_{RS}$ than in $A_{SC}$ (specifically, Alice sends $b \cdot C$ fewer elements of $\mathbb{F}_q$), the rate in both schemes is identical (as we consider the rate as $n$ goes to infinity).

Let us consider an adversary Calvin that makes $A_{RS}$ fail with probability $\epsilon$. In particular, Calvin may choose a message $\bar{M}$ for Alice to send s.t. with probability $\epsilon$, Bob reconstructs $\bar{M}$ which is different than $M$. We will define an adversary Calvin’ that makes $A_{SC}$ fail with probability $\epsilon' \geq \epsilon - C^2/q$. This will conclude our proof.

Calvin’ is defined as follows. First Calvin’ imitates the message selection of Calvin (namely, Calvin’ uses the message $M'$ Calvin would have chosen given the topology and the code of the network). If Calvin sets Alice’s input to the message $M$ then Calvin’ sets Alice’s input to $M' = [L \ M]$, where $L$ is a uniformly chosen $b \times C$ matrix. Then Calvin’ continues to mimic Calvin, and behaves identically (in particular Calvin’ sends the same messages as Calvin would on the same corrupted links).

As we see, Calvin’ tries to fail $A_{SC}$ by mimicking an attack of Calvin on the execution of $A_{RS}$. The success of Calvin’s attack on the execution of $A_{RS}$ depends both on the message $X = [L \ M \ I]$ transmitted by Alice and the secret information $R, H$ shared by Alice and Bob. Let $D$ be the distribution over triplets $(R, H, L)$ obtained when $R$ and $H$ are selected uniformly at random (and independently of each other) and the matrix $L$ is defined to satisfy $H = X \cdot P$ if a single such $L$ exists, and is defined to be the all-zero matrix otherwise. Let $A$ be the set of triplets $(R, H, L)$ on which Calvin’s attack succeeds (here we are assuming Calvin to be a deterministic adversary, however our analysis extends naturally to the case in which Calvin may act based on random decisions also). Namely, the success probability of Calvin can be formalized as $\Pr[A]$, where the probability is over the distribution $D$.

Now consider the success probability of Calvin’ on $A_{SC}$ averaged over messages of the form $M' = [L \ M]$ (where $L$ is chosen at random). As before this probability depends on the message $X = [L \ M \ I]$ sent by Alice and by the information $R, H$ shared by Alice and Bob. Let $D'$ be the distribution over triplets $(R, H, L)$ obtained when $R$ and $L$ are selected uniformly at random (and independently of each other) and $H$ is defined to be $X \cdot P$. Recall that Calvin’ mimics the behavior of Calvin, thus Calvin’ succeeds on the triplet $(R, H, L)$ iff Calvin succeeds on $(R, H, L)$. Hence, the average success probability of Calvin over messages of the form $M' = [L \ M]$ can be formalized as $\Pr[A]$, where the probability is now over $D'$. Notice that the subset $A$ of triplets $(R, H, L)$ is the set used above in the discussion on $A_{RS}$.

In what follows we show that $D$ and $D'$ are almost identical. This will suffice to prove our assertion.
Definition 3.2  The event $\mathcal{E}_{\text{bad}}$ on $R$ happens either if one of the parity symbols is selected to be zero or if any two of the parity symbols are identical. In other words, $\mathcal{E}_{\text{bad}}$ happens if there exists $d \in \{1, \ldots, C\}$ such that $r_d = 0$, or if for two distinct $d, d' \in \{1, \ldots, C\}$, we have that $r_d = r_d'$.

Note that $\mathcal{E}_{\text{bad}}$ is defined both for $D$ and for $D'$. In both cases, $R$ is uniformly distributed. Therefore $\Pr[D]\{\mathcal{E}_{\text{bad}}\} = \Pr[D']\{\mathcal{E}_{\text{bad}}\}$. Furthermore, it is easy to argue that this probability is at most $C^2/q$ (simply, each of the $C$ parity symbols is zero or identical to a previously selected parity symbol with probability at most $C/q$). We are now able to formalize our main observation:

Lemma 3.3 Conditioned on $\mathcal{E}_{\text{bad}}$ not happening, the two distributions $D$ and $D'$ are identical.

Proof: (of lemma) Let us fix any value of $M$. Let us also fix any value of $R$ such that $\mathcal{E}_{\text{bad}}$ does not happen. We will show that conditioned on every such fixings, the distributions $D$ and $D'$ are identical.

Let us decompose the $n \times C$ parity-check matrix $P$ into a $C \times C$ matrix $V$ and an $(n-C) \times C$ matrix $P'$, such that $P = \begin{bmatrix} V \\ P' \end{bmatrix}$. By the definition of $P$, the matrix $V$ is the Van der Monde matrix that corresponds to the parity symbols in $R$. Since we assumed that $\mathcal{E}_{\text{bad}}$ does not happen, we have that the parity symbols are all distinct and non zero. Therefore $V$ is invertible.

With this notation, we can rewrite the equation $H = X \cdot P$ as follows:

$$H = [L \ M \ I] \cdot \begin{bmatrix} V \\ P' \end{bmatrix} = L \cdot V + [M \ I] \cdot P'.$$

Since we already fixed $M$ and $R$, we have that $[M \ I] \cdot P'$ is a fixed matrix, which we will denote as $H'$. We also have that $V$ is a fixed invertible matrix. We denote by $V^{-1}$ its inverse. Now we have that $H = L \cdot V + H'$, or alternatively that $L = (H - H') \cdot V^{-1}$. We can conclude that for every value of $H$ there is exactly one value of $L$ for which $H = X \cdot P$. We therefore have that the equation $H = X \cdot P$ forces a one-to-one correspondence between the values of $L$ and the values of $H$. Therefore, the uniform distribution over $L$ induces the uniform distribution over $H$ and vice versa. The lemma follows.

Recall that we defined $A$ be the set of triplets $(R, H, L)$ on which Calvin’s attack succeeds. It follows from the lemma that conditioned on $\mathcal{E}_{\text{bad}}$ not happening, $\Pr[A]$ is identical under $D$ and $D'$. Since we already argued that $\Pr[D]\{\mathcal{E}_{\text{bad}}\} = \Pr[D']\{\mathcal{E}_{\text{bad}}\} \leq C^2/q$, we can conclude that the probability that $\mathcal{E}_{\text{bad}}$ does not happen and $A$ does happen is at least $\epsilon - C^2/q$ (regardless of whether the probability is taken over $D$ or $D'$). We can finally conclude that Calvin' succeeds in failing $\mathcal{A}_{\text{SC}}$ with probability at least $\epsilon - C^2/q$.

The case of multicast  In the above description of $\mathcal{A}_{\text{RS}}$, we considered for simplicity the case of a single Bob. In the setting of multicast, there are two possible scenarios. First, it may be the case that Alice and each of the Bobs share the same random secret. $\mathcal{A}_{\text{RS}}$ extends to this scenario with no change (simply because $\mathcal{A}_{\text{RS}}$ completely ignores the location of Bob in the network). We now address the more general scenario, where Alice may share a different secret with each one of the Bobs.

Our main observation is that almost all of the information Alice transmits (the matrix $X$) is independent of the random secret. The only part of $X$ that does depend on the secret is the matrix $L$. This matrix is rather small and its size is independent of the block-length $n$. Therefore to extend $\mathcal{A}_{\text{RS}}$ to the setting of multicast, all we need to do is to have Alice send a different matrix $L_i$ for each of the secrets she shares. Since the number of Bobs is bounded by the size of the graph, this only results in negligible rate loss. To decode, each
one of the Bobs ignores the communication which relates to other secrets and only keeps the communication related to his $L_i$. Bob then decodes exactly as in $A_{RS}$.

It remains to argue that with high probability each one of the Bobs will decode Alice’s message $M$ correctly. Let us consider Calvin’s attempt to fail the receiver Bob whose secret corresponds to the matrix $L_i$. Our previous analysis implies that each one of the matrices $L_j$ for $j \neq i$ are with high probability uniform and independent of both $L_i$ and $M$. Therefore, these additional matrices cannot assist Calvin in the attempt to fail this particular Bob. We conclude that each of the receivers will decode correctly with high probability, and therefore all of them are likely to decode correctly.

**Remark 3.4** In the above we assumed that the secret shared between Alice and each receiver Bob includes the index $i$, such that $L_i$ corresponds to their shared secret. It is possible to avoid this assumption as follows: (1) Let the random secret between Alice and Bob also contain a random (almost pair-wise independent) hash function $g_i$. Alice augments the message $M$ with $g_i(M)$ for all of those hash functions $g_i$. (2) Continue as before and have Bob decode according to each of the $L_j$’s (as now we assume that Bob does not know $i$ such that $L_i$ corresponds to his secret). Some of these decodings may result in $\bar{M} \neq M$. But with very high probability none of the erroneous decodings will be authenticated by a correct hash $g_i(M)$ (as for every $M$ and $\bar{M}$ we have that $g_i(\bar{M})$ is almost uniform and independent of $g_i(M)$).

### 4 Causal-Omniscient Model

Recall that in our model of communication, the columns of the matrix $X$ (namely, each slice of information from $X$) is encoded independently over time. Given the network’s latencies (the number of steps it takes for a message to traverse the network), we have that while an internal node $v$ sends messages that correspond to the $t^{th}$ column of $X$, Alice may already be sending messages that correspond to column $t' > t$. Therefore, in the model in which Calvin can eavesdrop on all links, it inherently has a “pick into the future”. Namely, when sending messages which correspond to the $t^{th}$ column of $X$, we assume that Calvin knows all the columns of $X$ up to column $t + \Delta$, where $\Delta$ is some fixed parameter of the network. It is not hard to verify that $\Delta$ is at most the size of the edge set $E$. However, it may be the case that Calvin does not necessarily know any later columns of $X$. This motivates the definition of the Causal-Omniscient model.

In the Causal-Omniscient model, Calvin has unlimited computational power. He has under his control $z$ network links of his choice. On these links Calvin may inject his own packets, disguised as part of the information flow from Alice to Bob. We do not assume that Alice, Bob or any internal nodes are aware of the links under Calvin’s control. On the other hand, Calvin has full knowledge of the network topology as well as the identity of Alice and Bob. Calvin has full eavesdropping capabilities (i.e., Calvin can monitor the entire communication on each one of the links). Calvin knows the encoding and decoding schemes of Alice and Bob, and the network code implemented by the internal nodes (including the random linear coefficients). Furthermore, we assume that Calvin knows which message $M$ Alice is sending to Bob.

The only limitation on Calvin is the following: while Calvin is allowed access to the internal state and randomness of all parties, he does not get such access to Alice’s state and randomness. Note that such a limitation is implicit in all other limited adversarial models considered here and in \[12\]. The desired implication of this limitation for the Causal-Omniscient model is the following: let $\Delta$ be a fixed parameter of the network that specifies a bound on the latency of the network. By the discussion above, if Calvin’s

\[\text{For example, in the random-secret model, one has to hide the secret-key which is expressed in various computations of both Alice and Bob.}\]
messages correspond to columns of $X$ up to its $t^{th}$ column then we assume that all columns beyond column number $t + \Delta$ are hidden from Calvin.

4.1 The scheme $A_{\text{CO}}$

We now define the scheme $A_{\text{CO}}$ for the Causal-Omniscient model. The scheme is obtained by a completely modular composition of two schemes: A scheme $A_{\text{SC}}$ in the secret-channel model, and a scheme $A_{\text{OMN}}$ in the omniscient-adversary model. See more details on the schemes in Section 2. The idea of the composition is simple: first Alice, Bob (and the network) execute $A_{\text{SC}}$ with Alice’s input $M$, but without Alice sending the message on the secret channel (simply because a secret channel is not available in this model). Unfortunately, without the secret message, Bob cannot decode $M$ correctly yet. Therefore, to transmit this secret information, we suggest that Alice and Bob execute $A_{\text{OMN}}$ with the secret message as Alice’s new input. Unfortunately, $A_{\text{OMN}}$ may reveal the secret message to Calvin as well. Our simple observation is that as long as the secret message is revealed after the execution of $A_{\text{SC}}$ ends, it is too late for Calvin to cause any harm. Therefore, all that we need (so that $A_{\text{CO}}$ works) is for Alice to send $\Delta$ “garbage” columns between the executions of $A_{\text{SC}}$ and of $A_{\text{OMN}}$. We turn to a formal definition of $A_{\text{CO}}$:

Alice’s encoder

Alice invokes the encoding and secret generating algorithms of $A_{\text{SC}}$ on her input $M$. Denote by $X_M$ the output of the encoding and $S$ the message to be sent on the secret channel. Now Alice invokes an independent execution of the encoding algorithm $A_{\text{OMN}}$ on $S$ as input. Denote by $X_S$ the output of the encoding. For reasons that will be made clear shortly, Alice encodes a secret $S$ such that $X_S$ will be of block length $n_S = (n/C)^{1/3}$ (here $n$ is the block length of our scheme and $C$ is the capacity). Recall, that the size of $S$ (and thus the block length of $X_S$) in the secret channel scheme is independent of $n$ and significantly smaller than $n_S$. Hence, such a blowup in the size of $X_S$ can be obtained for example by an arbitrary padding of $S$ with irrelevant information. As we will see, this blowup will enable our scheme to have a low probability of error (without significantly increasing the code-complexity). As $n_S$ is much smaller than $n$, our rate remains optimal. Alice’s encoder now outputs $X = [X_M \ 0 \ X_S]$, where $0$ denotes the zero matrix with $\Delta$ columns.

Network coding

As in $A_{\text{SC}}$ and $A_{\text{OMN}}$, the network coding is the standard random-linear coding of $[8]$.

Bob’s decoding

Bob first uses the decoder of $A_{\text{OMN}}$ on the suffix of the communication (which corresponds to the columns of $X_S$). Denote by $\bar{S}$ the decoded message. Bob now applies the decoder of $A_{\text{SC}}$ on the prefix of the communication (which corresponds to the columns of $X_M$), with the (relevant parts of the) secret message set to $\bar{S}$. Bob outputs the decoded message, which we denote by $\bar{M}$.

4.2 Properties of $A_{\text{CO}}$

We state the parameters obtained by $A_{\text{CO}}$ in the following theorem.

Theorem 4.1 $A_{\text{CO}}$ is a distributed polynomial-time scheme. $A_{\text{CO}}$ achieves a rate of $C - z$, as long as $C > 2z$, in the Causal-Omniscient model, with code-complexity $\mathcal{O}(nC^2)$.

Proof: Most of the properties of $A_{\text{CO}}$ follow from the related properties of $A_{\text{SC}}$ and $A_{\text{OMN}}$, as given by Theorems 2.2 and 2.1. The restriction that $C > 2z$ guarantees positive rate for $A_{\text{OMN}}$ (as the rate of $A_{\text{OMN}}$ is $C - 2z$). Other than that, $A_{\text{CO}}$ inherits its rate from $A_{\text{SC}}$. There is some loss of rate in $A_{\text{CO}}$ (compared with
A_{SC}) due to the communication related to the zero columns and to $X_S$. Nevertheless, this loss is negligible as $n$ tends to infinity. The choice of $n_S$ (the block length of $X_S$), guarantees that the code complexity due to both building blocks ($A_{SC}$ and $A_{omn}$) will equal $O(nC^2)$.

It remains to bound the error probability $\epsilon$ of $A_{co}$. Obviously, $\epsilon \leq \epsilon_1 + \epsilon_2$, where $\epsilon_1$ is the probability that $\bar{S} \neq S$ while $\epsilon_2$ is the probability that $\bar{S} = S$ but $\bar{M} \neq M$. It follows that $\epsilon_1$ is bounded by the error probability of $A_{omn}$ when applied to messages of block length $n_S$ that corresponds to $X_S$. In [12] this error is shown to be vanishing as the block length tends to infinity (note that when $n$ tends to infinity so does $n_S = (n/C)^{1/3}$). To bound $\epsilon_2$ notice that Calvin does not get access to $X_M$ until he is done corrupting $X_S$. Thus $\epsilon_2$ is bounded by the error probability of $A_{SC}$ when applied to messages of block length that correspond to $X_M$. As the block length of $X_M$ is proportional to $n$, we conclude our assertion.

**Remark 4.2** While we described $A_{co}$ for the case of a single Bob, it also applies with no change to the setting of multicast (simply because $A_{co}$ completely ignores the location of Bob in the network, and there is nothing that distinguishes one Bob from the other).

### 5 Computationally-Bounded Adversary Model

In this section we consider a limitation of a different flavor on the strength of the adversary Calvin. Namely, we assume that Calvin is computationally bounded. Assuming so allows us to employ powerful cryptographic tools. The two results in this section correspond to cryptographic tools that are applicable in two different settings: (1) Symmetric-key cryptography (discussed in Section 5.1), and (2) Public-key cryptography (discussed in Section 5.2). As common in the study of cryptographic primitives, both our results are conditional — in the sense that they hold assuming that certain cryptographic primitives exist (such as the assumption that factoring is hard).

Note that apart from the computational limitations on Calvin his powers are intact. In particular, Calvin has full eavesdropping capabilities and has full knowledge of the network topology as well as the identity of Alice and Bob. Calvin knows the encoding and decoding schemes of Alice and Bob, and the network code implemented by the internal nodes (including the random linear coefficients).

#### 5.1 Symmetric-key cryptography

Recall our scheme $A_{RS}$ in the random secret model. Assuming that Alice and Bob share a short random secret, this scheme allows them to communicate a significant amount of information which is specified by the block length $n$. Unfortunately, Bob will only be able to decode Alice’s message after receiving the entire block of communication. Therefore, it is natural to assume that every time slot (e.g., every hour or every day, depending on the rate of communication), Alice and Bob would like to terminate the previous execution of $A_{RS}$ and to start a new execution. Let $S_1, S_2, \ldots, S_l$ be the sequence of secrets used in these executions. In general, the secrets should be independent of each other, which implies that Alice and Bob may need to share a long secret if they communicate over a long period of time.

In the Computationally-Bounded Adversary model, Alice and Bob can execute $A_{RS}$ many times while still only exchanging a short random string $s$. For that purpose, Alice and Bob may use a pseudorandom generator. For a definition and thorough discussion of pseudorandom generators see [5]. Essentially, an efficiently computable function $G$ is a pseudorandom generator, if (1) $G$ is length increasing (i.e., for every input its output is longer than its input) and (2) $G(x)$ is computationally indistinguishable from a uniform string, as long as $x$ is uniformly distributed. In other words $G(x)$ is effectively random. It is known that the existence of pseudorandom generators is essentially the minimal cryptographic assumption (as it is
equivalent to the assumption that one-way functions exist). A pseudorandom generator that expands the length of its input implies a pseudorandom generator with arbitrary polynomial expansion (again, the reader is referred to [5] for a detailed discussion and references therein).

The relation to our context is now simple: Alice and Bob can exchange a single short secret key \( s \) prior to the communication process. Applying a pseudorandom generator to this single key, they can obtain many pseudorandom keys \( G(s) = S_1, S_2, \ldots, S_\ell \) to be used in repeated executions of \( A_{\text{RS}} \) (in fact, \( \ell \) need not be known in advance as it is possible to keep on expanding \( G \)'s output on the fly). The proof that such repeated executions of \( A_{\text{RS}} \) are still secure is rather immediate from the definition of a pseudorandom generator and we therefore only sketch it here. For any \( i \), if \( S_i \) is truly random Calvin will fail the execution of \( A_{\text{RS}} \) with very small probability. Assume for the sake of contradiction that this is not the case when \( S_i \) is taken from the output of \( G \). This gives a way to distinguish the output of \( G \) from random. The distinguisher simply simulates the repeated execution of \( A_{\text{RS}} \) (playing the roles of Alice, Bob, internal nodes, and Calvin). Now if Calvin succeeds then the distinguisher can deduce with non-negligible probability that \( S_i \) is not truly random.

**Remark 5.1** As discussed in Section 3, the scheme \( A_{\text{RS}} \) can be extended to the case of multicast. The idea described here, of replacing the random keys in multiple executions of \( A_{\text{RS}} \) with pseudorandom keys, applies in the setting of multicast as well (in that setting, Alice and each one of the Bobs will share a short key that will be expanded to many pseudorandom keys using the pseudorandom generator).

### 5.2 Public-key cryptography

A disadvantage of the scheme \( A_{\text{RS}} \) is that Alice and Bob need to share a common key. In this section we relax this setup requirement and only ask that Bob holds a pair of keys: a private key (known only to itself) and a public key (known to all — including Calvin). In such a setup, without Alice and Bob ever meeting or exchanging private information, we are able to give a network-coding scheme, \( A_{\text{PK}} \), against a computationally-bounded Calvin with very similar parameters to those of \( A_{\text{CO}} \) (which was given in the Causal-Omniscient model).

### 5.3 The scheme \( A_{\text{PK}} \)

We present a network coding scheme, \( A_{\text{PK}} \), for the public-key model, that is very similar to our scheme for the Causal-Omniscient model. Again, we compose two schemes: A scheme \( A_{\text{SC}} \) in the secret-channel model, and a scheme \( A_{\text{Omn}} \) in the omniscient-adversary model. (See more details on the schemes in Section 2) Alice, Bob (and the network) execute \( A_{\text{SC}} \) with Alice’s input \( M \), but without Alice sending the secret message \( S \) on the secret channel (simply because a secret channel is not available in this model). We would like to execute \( A_{\text{Omn}} \) with \( S \) as Alice’s new input. Unfortunately, as in this model all of the information sent by Alice (i.e., the matrix \( X \) in its entirety) is known to Calvin from the start, \( S \) will be available to Calvin during the execution of \( A_{\text{SC}} \) and the scheme may fail. The solution is simple: instead of sending \( S \), Alice will first encrypt \( S \) using Bob’s public key, and send the encryption to Bob using \( A_{\text{Omn}} \). We now describe our scheme and proof in more detail.

**Public-key encryption** A central ingredient in building \( A_{\text{PK}} \) is a public-key encryption (PKE) scheme. For a thorough discussion of public-key encryptions see [6]. We describe here the relevant definitions for completeness.
A PKE scheme gives a way for two parties to communicate securely even though they did not previously meet and exchange secrets. The scheme is composed of three probabilistic polynomial time algorithms – the key generating algorithm $Gen$ and the encryption and decryption algorithms $Enc$ and $Dec$: (1) The input of $Gen$ is the security parameter $k$ (we will shortly discuss the role of $k$), and its output is a pair of keys – the secret key $sk$ and the public key $pk$ (both are of length polynomial in $k$). (2) The public key $pk$ (which is known to everyone) is used for encryption. The encryption of a message $m$ is a ciphertext $y = Enc(pk, m)$. The plaintext $m$ may be of arbitrary length and the length of $y$ is polynomial in the length of $m$ and in $k$. (3) The secret key $sk$ allows decryption. For every $y$ as above we have that $Dec(sk, y) = m$.

The security requirement from a PKE scheme is that a ciphertext $y$ gives no information on the plaintext $m$ to a computationally bounded adversary. More formally, we will use a PKE scheme which is semantically-secure against chosen-plaintext attack (CPA) \[7\]. There are various equivalent formalizations of this security requirement and the one that seems most convenient for us is based on the notion of indistinguishability. Loosely, this means that no efficient adversary $Adv$ can distinguish an encryption of a message $m_0$ from an encryption of a message $m_1$, (where $Adv$ is also allowed to select $m_0$ and $m_1$). In other words, given an encryption $y = Enc(pk, m_0)$, where $\sigma$ is a uniformly selected bit, an adversary cannot guess $\sigma$ with probability significantly better than half. This is exactly where the security parameter $k$ comes into play: the advantage over half of the adversary in guessing $\sigma$ is smaller than $1/poly(k)$ for every polynomial $poly$ (under stronger assumptions we may require the advantage to be exponentially small in $k$).

We are now ready to define $A_{pk}$ formally.

**Security parameter** In this model, the network coding scheme is defined per security parameter $k$. This parameter should be chosen as to make the encryption scheme $(Gen, Enc, Dec)$ secure enough. As the errors we seek are of the order of $1/n$, it is enough to take $k < n^\alpha$ for some small constant $\alpha > 0$ (under stronger assumptions, $k$ may even be logarithmic in $n$). This will imply that all the ciphertexts used in our scheme are of negligible length compared with $n$ (e.g. smaller than $n^{\alpha'}$ for any $\alpha' > 0$ of our choosing).

**Bob’s keys** Bob runs $Gen$ and gets as output the pair $(sk, pk)$. Bob publishes $pk$ as his public key and saves his secret key $sk$.

**Alice’s encoder** Alice invokes the encoding and secret generating algorithms of $A_{SC}$ on her input $M$. Denote by $X_M$ the output of the encoding and $S$ the message to be sent on the secret channel. Alice invokes $Enc(pk, S)$ and gets as output $Y$. Now Alice invokes (using fresh randomness) the encoding algorithm of $A_{omn}$ on $Y$ as input. Denote by $X_S$ the output of the encoding. Alice now outputs $X = [X_M X_S]$. As in Section $4$, we will pad $X_S$ if necessary to ensure that it has block length $n_S = (n/C)^{1/3}$.

**Network coding** As in $A_{SC}$ and $A_{omn}$, the network coding is the standard random-linear coding.

**Bob’s decoding** Bob first uses the decoder of $A_{omn}$ on the suffix of the communication (which corresponds to the columns of $X_S$). Denote by $\bar{Y}$ the decoded message. Bob invokes $Dec(sk, \bar{Y})$, and receives $\bar{S}$ as output. Bob now applies the decoder of $A_{SC}$ on the prefix of the communication (which corresponds to the columns of $X_M$), with the secret message set to $\bar{S}$. Bob outputs the decoded message, which we denote by $\bar{M}$.

\[^{3}\text{In Remark5.3 we note that in some cases one may choose to require security against chosen-ciphertext attack (CCA security).}\]
5.4 Properties of $A_{pk}$

We state the parameters obtained by $A_{pk}$ in the following theorem.

**Theorem 5.2** $A_{pk}$ is a distributed polynomial-time scheme. $A_{pk}$ achieves a rate of $C - z$, as long as $C > 2z$, in the public-key model, with code-complexity $O(nC^2)$.

**Proof:** Similar to the proof of Theorem 4.1 most of the properties of $A_{pk}$ follow from the related properties of $A_{sc}$ and $A_{omn}$, as given by Theorems 2.2 and 2.1.

It remains to bound the error probability $\epsilon$ of $A_{pk}$. Obviously, $\epsilon \leq \epsilon_1 + \epsilon_2$, where $\epsilon_1$ is the probability that $\bar{Y} \neq Y$ while $\epsilon_2$ is the probability that $\bar{Y} = Y$ but $\bar{M} \neq M$. As in the proof of Theorem 4.1 it is not hard to argue that $\epsilon_1$ is bounded by the error probability of $A_{omn}$ (when applied to messages of block length $n_S$). We would now like to argue that $\epsilon_2$ is bounded by the error probability of $A_{sc}$ (when applied to messages of block length corresponding to $X_M$). This will turn out to be correct up to a negligible additional error (which relates to the security property of the PKE scheme).

To bound $\epsilon_2$ we must argue that Calvin cannot use the encryption $Y$ of $S$ to increase his probability of corrupting the communication between Alice and Bob. Intuitively, this is clear - as the encryption $S$ is computationally sound. However, to prove our argument formally, we need to present our claim under the sole assumption that our encryption is secure against a chosen-plaintext attack. To this end, we condition on the fact that $\bar{Y} = Y$ (and hence Bob knows $S$) and show that a successful Calvin in our setting will imply a successful Calvin in an imaginary setting in which Bob is given $S$ (e.g., via a side channel) and the value of $X_S$ transmitted over the network is the encoding of an all zero message. This in turn, implies that Calvin can corrupt the original secret-channel protocol $A_{sc}$ of [12], a contradiction. We now sketch the details.

Let Calvin be an adversary that causes $\bar{Y} = Y$ and $M \neq M$ in an execution of $A_{pk}$, with probability $\epsilon_2$. As a mental experiment assume that Bob’s decoder receives $S$ as an additional input. Bob can then ignore $\bar{Y}$ and simply invoke the decoder of $A_{sc}$. In this experiment we have that the probability that Calvin manages to cause $\bar{Y} = Y$ but $\bar{M} \neq M$ is still $\epsilon_2$. This follows immediately from the properties of a PKE scheme (as if $\bar{Y} = Y$ we also have that $\bar{S} = S$).

Further revising this mental experiment, let us now assume that Alice defines $Y$ as the encryption of the zero-message (or any other fixed message), rather than the encryption of $S$. It is not hard to argue that in this case, the probability that Calvin causes $\bar{M} \neq M$ is at least $\epsilon_2 - \text{neg}(k)$ where $\text{neg}(\cdot)$ is some negligible function (that is asymptotically smaller than $1/poly(\cdot)$ for every polynomial $\text{poly}(\cdot)$). If this is not the case then we can easily devise an adversary $Adv$ that breaks the security of the PKE scheme. $Adv$ will simulate all parties of the network-coding scheme (Alice, Bob, Calvin and the internal nodes). When Alice generates $S$ then $Adv$ will set $m_3 = S$ and will set $m_0$ to be the all-zero message. $Adv$ then receives $y$ which is an encryption of one of these messages. $Adv$ can now continue the simulation of the network-coding scheme with $Y = y$. Finally, when the simulation is over, $Adv$ will output one if $\bar{M} \neq M$ and zero otherwise.

Summing up, we have an adversary Calvin that causes $\bar{M} \neq M$ with probability $\epsilon_2' = \epsilon_2 - \text{neg}(k)$, in the revised mental experiment based on $A_{pk}$. Note that in this mental experiment $X_S$ is completely independent of $S$. Therefore, it is possible to define an adversary Calvin’ that fails $A_{sc}$ with probability $\epsilon_2'$ by simulating the attack of Calvin in the setting of the mental experiment. The theorem therefore follows.

**Remark 5.3** In the definition of $A_{pk}$ we used a PKE scheme which is secure against a chosen-plaintext attack. We proved that one invocation of $A_{pk}$ works when the public-key is only used for this invocation. In case the same public-key is used many times, and especially if it used for messages sent from different
senders it may be safer to use a PKE scheme that is secure against a chosen-ciphertext attack (see \[6\] for more information).

**The case of multicast** In the above description of $A_{pk}$, we considered for simplicity the case of a single Bob. The scheme can be extended to the setting of multicast, in a similar manner to the extension of $A_{RS}$ to multicast (see discussion in Section \[3\]). In fact the extension is a bit simpler in the case of $A_{RS}$ as we describe now.

We assume that the $i$’th receiver knows a pair consisting of a secret key $sk_i$ and public key $pk_i$. The public key is known to everyone including Alice. Now, for every $i$, Alice will transmit (using $A_{omn}$) the pair $(pk_i, Y_i)$, where $Y_i = \text{Enc}(pk_i, S)$ (recall that in the basic scheme, Alice transmits a single $Y$). Based on the properties of $A_{omn}$ we can assume that each of the Bobs correctly retrieves all of the pairs. The $i$th receiver can decode $Y_i$ (which corresponds to its public key $pk_i$) and continue the decoding of $M$ as in the basic scheme. To argue that with high probability each one of the Bobs will decode Alice’s message $M$ correctly, we note that the concatenation of all the different encryptions of $S$ still does not reveal any information on $S$.

6 Conclusions

In this paper we have introduced three adversarial models and have argued that (1) The models may be realistic. (2) The models are useful in the sense that they allow non-trivial improvements in the parameters of network coding schemes. We feel that this calls for more attention into the assumptions regarding adversarial limitations and set-up assumptions that apply in “real life” scenarios. Are the models suggested here indeed applicable? Are there any other realistic and useful models to consider?

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