Predicate Specialization for Definitional Higher-order Logic Programs

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Abstract. Higher-order logic programming is an interesting extension of traditional logic programming that allows predicates to appear as arguments and variables to be used where predicates typically occur. Higher-order characteristics are indeed desirable but on the other hand they are also usually more expensive to support. In this paper we propose a program specialization technique based on partial evaluation that can be applied to a modest but useful class of higher-order logic programs and can transform them into first-order programs without introducing additional data structures. The resulting first-order programs can be executed by conventional logic programming interpreters and benefit from other optimizations that might be available. We provide an implementation and experimental results that suggest the efficiency of the transformation.

1 Introduction

Higher-order logic programming has been long studied as an interesting extension of traditional first-order logic programming and various approaches exist with different features and semantics [2, 4, 12]. Typically, higher-order logic programs are allowed to define predicates that accept other predicates as arguments and variables can appear in places where predicate constants typically occur. Higher-order logic programs enjoy similar merits as their functional counterparts. The support of higher-order features however, usually comes with a price, and the efficient implementation in either logic or functional programming is a non-straightforward task.

The use of higher-order constructs is a standard feature in every functional language in contrast to the logic programming languages. As a result, there exists a plethora of optimizations that target specifically the efficient implementation of such features. A popular direction is to remove higher-order structures altogether by transforming higher-order programs into equivalent first-order ones, with the hope that the execution of the latter will be much more efficient. Reynolds, in his seminal paper [17], proposed a defunctionalization algorithm that is complete,
i.e. it succeeds to remove all higher-order parameters from an arbitrary functional program. There is however a tradeoff; his algorithm requires the introduction of data structures in order to compensate for the inherent loss of expressivity [7]. Other approaches [6, 13, 14] have been proposed that do not use data structures, but share the limitation that are not complete.

In the logic programming context there exist many transformation algorithms with the purpose of creating more efficient programs. Partial evaluation algorithms [6, 11, 9], for example, can be used to obtain a more efficient program by iteratively unfolding logic clauses. Most of the proposals, however, focus on first-order logic programs. Proposals that can be applied to higher-order programs are limited. The prominent technique that targets higher-order logic programs proposed in [21, 4] and adopted from Hilog. It employs the Reynolds’ defunctionalization adapted for logic programs. As a consequence it naturally suffers from the same shortcomings of the original technique: the resulting programs are not natural and the conventional logic programming interpreters fail to identify potential optimizations without specialized tuning [18].

In this paper, we propose a partial evaluation technique that can be applied to higher-order logic programs. The technique propagates only higher-order arguments and avoids to change the structure of the original program. Moreover, it differs from Reynolds’ style defunctionalization approaches as it does not rely on any type of data structures. As a result, the technique will only guarantee to remove the higher-order arguments in a well-defined subset of higher-order logic programs. The main contributions of the present paper are the following:

1. We propose a technique based on the abstract framework of partial evaluation that targets higher-order arguments. We have identified a well-defined fragment of higher-order logic programming that the technique terminates and produces a logic program without higher-order arguments.
2. We provide an implementation of the proposed technique and we experimentally assess its performance. We also compare it with the Reynolds’ defunctionalization implemented in Hilog. Moreover, we experiment with the ability of combining this technique with the well-known tabling optimization.

The rest of the paper is organized as follows. In Section 2 we give an intuitive overview of our method using a simple example. In Section 3 we formally define the fragment of the higher-order logic programs we will use. Section 4 describes the abstract framework of partial evaluation and Section 5 introduces the details of our method. Section 6 discusses some implementation issues and Section 7 discusses the performance of our transformation on various experiments. Lastly, we compare our method with related approaches in Section 8 and we conclude the paper with possible future work.

2 A Simple Example

We start with an introductory example so as to give an informal description of our technique. We borrow an example from the area of knowledge representation which deals with the expression of user preferences [3].
The following program selects the most preferred tuples $T$ out of a given unary relation $R$, based on a binary preference predicate $P$. The preference predicate given two tuples it succeeds if the first tuple is more preferred than the second.

\[
\begin{align*}
\text{winnow}(P,R,T) & :\neg R(T), \neg \text{bypassed}(P,R,T). \\
\text{bypassed}(P,R,T) & :\neg R(Z), P(Z,T).
\end{align*}
\]

The program contains *predicate variables* (for example, $P$ and $R$), that is variables that can occur in places where predicates typically occur.

Assume that we have a unary predicate `movie` which corresponds to a relation of movies and a binary predicate `pref` which given two movies succeeds if the first argument has a higher ranking than the second one. Now, suppose that we issue the query:

?- `winnow(pref,movie,T)`.

We expect as answers the most “preferred” movies, that is all movies with the highest ranking.

In the following, we will show how we can create a first-order version of the original program specialized for this specific query. Notice that the atom `winnow(pref,movie,T)`, that makes up our given query, does not contain any free predicate variables, but on the contrary, all of its predicate variables are substituted with predicate names. Therefore, we can specialize every program clause that defines `winnow` by substituting its predicate variables with the corresponding predicate names. By doing so, we get a program where our query yields the same results as to those in the original program:

\[
\begin{align*}
\text{winnow}(pref,movie,T) & :\neg \text{movie}(T), \neg \text{bypassed}(pref,movie,T). \\
\text{bypassed}(P,R,T) & :\neg R(Z), P(Z,T).
\end{align*}
\]

We can continue this specialization process by observing that in the body of this newly constructed clause there exists the atom `bypassed(pref,movie,T)`, in which all predicate variables are again substituted with predicate names. Therefore, we can specialize the second clause of the program accordingly:

\[
\begin{align*}
\text{winnow}(pref,movie,T) & :\neg \text{movie}(T), \neg \text{bypassed}(pref,movie,T). \\
\text{bypassed}(pref,movie,T) & :\neg \text{movie}(Z), \text{pref}(Z,T).
\end{align*}
\]

There are no more predicate specializations to be performed and the transformation stops. Notice that the resulting program does not contain any predicate variables, but it is not a valid first-order one. Therefore, we have to perform a simple rewriting in order to remove all unnecessary predicate names that appear as arguments.

\[
\begin{align*}
\text{winnow1}(T) & :\neg \text{movie}(T), \neg \text{bypassed2}(T). \\
\text{bypassed2}(T) & :\neg \text{movie}(Z), \text{pref}(Z,T).
\end{align*}
\]

Due to this renaming process, instead of the initial query, the user now has to issue the query `?- `winnow1(T)`. Comparing the final first-order program with the original one it is easy to observe that no additional data structures were
introduced during the first-order transformation, a characteristic that leads to performance improvement (ref. Section 7).

This technique, however, cannot be applied in every higher-order logic program. Notice that the resulting program of the previous example does not contain any predicate variables. This holds due to the fact that in the original program, every predicate variable that appears in the body of a clause it also appears in the head of this clause. By restricting ourselves to programs that have this property we ensure that the transformation outputs a first-order program. Moreover, the transformation in this example terminates because the set of the specialization atoms (ie. winnow(pref,movie,T) and bypassed(pref,movie,T)) is finite, which is not the case in every higher-order logic program. To solve this, we need to keep the set specialization atoms finite. This is achieved in two ways. Firstly, we ignore all first-order arguments in every specialization atom, meaning that in a query of the form ?- winnow(pref,movie,m_001), we will specialize the program with respect to the atom winnow(pref,movie,T). Secondly, we impose one more program restriction; we focus in programs where the higher-order arguments are either variables or predicate names. Since the set of all predicate names is finite and since we ignore all first-order values, the set of specialization atoms is also finite and as a result the algorithm is ensured to terminate.

3 Higher-order Logic Programs

In this section we define the higher-order language of our interest. We begin with the syntax of the language $\mathcal{H}$ we use throughout the paper. $\mathcal{H}$ is based on a simple type system with two base types: $o$, the boolean domain, and $\iota$, the domain of data objects. The composite types are partitioned into three classes: functional (assigned to function symbols), predicate (assigned to predicate symbols) and argument (assigned to parameters of predicates).

Definition 1. A type can either be functional, argument, or predicate, denoted by $\sigma, \rho$ and $\pi$ respectively and defined as:

\[
\begin{align*}
\sigma & := \iota \mathrel{|} (\iota \rightarrow \sigma) \\
\pi & := o \mathrel{|} (\rho \rightarrow \pi) \\
\rho & := \iota \mathrel{|} \pi
\end{align*}
\]

Definition 2. The alphabet of the language $\mathcal{H}$ consists of the following:

1. Predicate variables of every predicate type $\pi$ (denoted by capital letters such as $P, Q, R, \ldots$).
2. Individual variables of type $\iota$ (denoted by capital letters such as $X, Y, Z, \ldots$).
3. Predicate constants of every predicate type $\pi$ (denoted by lowercase letters such as $p, q, r, \ldots$).
4. Individual constants of type $\iota$ (denoted by lowercase letters such as $a, b, c, \ldots$).
5. Function symbols of every functional type $\sigma \neq \iota$ (denoted by lowercase letters such as $f, g, h, \ldots$).
6. The inverse implication constant $\leftarrow$, the negation constant $\sim$, the comma, the left and right parentheses, and the equality constant $\approx$ for comparing terms of type $\iota$.

The set consisting of the predicate variables and the individual variables of $\mathcal{H}$ will be called the set of argument variables of $\mathcal{H}$. Argument variables will be usually denoted by $V$ and its subscripted versions.

**Definition 3.** The set of expressions of $\mathcal{H}$ is defined as follows:

- Every predicate variable (resp. predicate constant) of type $\pi$ is an expression of type $\pi$; every individual variable (resp. individual constant) of type $\iota$ is an expression of type $\iota$;
- if $f$ is an $n$-ary function symbol and $E_1, \ldots, E_n$ are expressions of type $\iota$ then $(f \ E_1 \cdots E_n)$ is an expression of type $\iota$;
- if $E$ is an expression of type $\rho_1 \rightarrow \cdots \rightarrow \rho_n \rightarrow o$ and $E_i$ an expression of type $\rho_i$ for $i \in \{1, \ldots, n\}$ then $(E \ E_1 \cdots E_n)$ is an expression of type $o$.
- if $E_1, E_2$ are expressions of type $\iota$, then $(E_1 \approx E_2)$ is an expression of type $o$.

We will omit parentheses when no confusion arises. Expressions of type $o$ will often be referred to as atoms. We write $\text{vars}(E)$ to denote the set of all variables in $E$. We say that $E_i$ is the $i$-th argument of an atom $E \ E_1 \cdots E_n$. A ground expression $E$ is an expression where $\text{vars}(E)$ is the empty set.

**Definition 4.** A clause is a formula

$$p \ V_1 \cdots V_n \leftarrow L_1, \ldots, L_m, \sim L_{m+1}, \ldots, \sim L_{m+k}$$

where $p$ is a predicate constant of type $\rho_1 \rightarrow \cdots \rightarrow \rho_n \rightarrow o$, $V_1, \ldots, V_n$ are distinct variables of types $\rho_1, \ldots, \rho_n$ respectively, and $L_1, \ldots, L_{m+k}$ are expressions of type $o$, such that every predicate argument of $L_i$ is either variable or ground.

A program $P$ of the higher-order language $\mathcal{H}$ is a finite set of program clauses.

The syntax of programs given in Definition 4 differs slightly from the usual Prolog-like syntax that we have used in Section 2. However, one can easily verify that we can rewrite every program from the former syntax to the latter. For instance, we could use the constant $\approx$ in order to eliminate individual constants that appear in the head of a clause that uses the Prolog-like syntax.

**Example 1.** Consider the following program in Prolog-like syntax, in which we have three predicate definitions, namely $p : \iota \rightarrow o$, $q : \iota \rightarrow \iota \rightarrow o$, and $r : (\iota \rightarrow o) \rightarrow (\iota \rightarrow o) \rightarrow (\iota \rightarrow \iota) \rightarrow o$.

$$p(a).$$
$$q(X, X).$$
$$r(P, Q, f(X)) :- P(X), Q(Y).$$

In our more formal notation, these clauses can be rewritten as:

$$p \ X \leftarrow (X \approx a).$$
$$q \ X \ Y \leftarrow (X \approx Y).$$
$$r \ P \ Q \ Z \leftarrow (Z \approx f(X)), (P \ X), (Q \ Y).$$

Notice that all clauses are now valid $\mathcal{H}$ clauses.
Notice that in a $\mathcal{H}$ program, all arguments of predicate type are either variables or predicate names, which as discussed in Section 2 leads to the termination of our technique. However, in a $\mathcal{H}$ program all head predicate variables to be distinct. That implies that checking for equality between predicates (higher-order unification) is forbidden. In other words, the higher-order parameters can be used in ways similar to functional programming, namely either invoked or passed as arguments. We decided to impose this restriction because equality between predicates is treated differently in various higher-order languages \cite{2, 3, 12}. Moreover, in Section 2, we briefly discussed that the reason why our technique can produce a first-order program is due to the following property:

**Definition 5.** A clause will be called definitional iff every predicate variable that appears in the body appears also as a formal parameter of the clause. A definitional program is a finite set of definitional clauses.

**Example 2.** Consider the following program in Prolog-like syntax:

\begin{verbatim}
p(Q,Q) :- Q(a).
qu(X) :- R(a,X).
\end{verbatim}

This program does not belong to our fragment, because the first clause is a non-$\mathcal{H}$ clause and the second clause is a non-definitional clause. Regarding the first clause, the predicate variable $Q$ appears twice in the head, therefore the formal parameters are not distinct. Regarding the second clause, the predicate variable $R$ that appears in the body, does not appear in the head of the clause.

We extend the well-known notion of substitution to apply to $\mathcal{H}$ programs.

**Definition 6.** A substitution $\theta$ is a finite set \{\(V_1/E_1, \ldots, V_n/E_n\}\} where the $V_i$’s are different argument variables and each $E_i$ is a term having the same type as $V_i$. We write $\text{dom}(\theta) = \{V_1, \ldots, V_n\}$ to denote the domain of $\theta$.

**Definition 7.** Let $\theta$ be a substitution and $E$ be an expression. Then, $E\theta$ is an expression obtained from $E$ as follows:

- $E\theta = E$ if $E$ is a predicate constant or individual constant;
- $V\theta = \theta(V)$ if $V \in \text{dom}(\theta)$; otherwise, $V\theta = V$;
- $(f \ E_1 \cdots \ E_n)\theta = (f \ E_1\theta \cdots \ E_n\theta)$;
- $(E \ E_1 \cdots \ E_n)\theta = (E\theta \ E_1\theta \cdots \ E_n\theta)$;
- $(E_1 \approx E_2)\theta = (E_1\theta \approx E_2\theta)$;
- $(L_1, \ldots, L_m, \sim L_{m+1}, \ldots, \sim L_n)\theta = (L_1\theta, \ldots, L_m\theta, \sim (L_{m+1}\theta), \ldots, \sim (L_n\theta))$.

Let $\theta$ be a substitution and $E$ an expression. Then, $E\theta$ is called an instance of $E$.

4 Partial Evaluation of Logic Programs

Partial evaluation \cite{8} is a program optimization that specializes a given program according to a specific set of input data, such that the new program is more efficient than the original and both programs behave in the same way according to
1: **Input:** a program \( P \) and a goal \( G \)
2: **Output:** a specialized program \( P' \)
3: \( S := \{ A : A \text{ is an atom of } G \} \)
4: repeat
5: \( S' := S \)
6: \( P' := \text{Unfold}(P, S) \)
7: \( S := S \cup \{ A : A \text{ is an atom that appears in a body of a clause in } P' \} \)
8: \( S := \text{Abstract}(S) \)
9: until \( S' = S \) (modulo variable renaming)
10: return \( P' \)

**Fig. 1.** Basic Algorithm for Partial Evaluation.

the given data. In the context of logic programming [6, 11, 9], a partial evaluation algorithm takes a program \( P \) and a goal \( G \) and produces a new program \( P' \) such that \( P \cup \{G\} \) and \( P' \cup \{G\} \) are semantically equivalent. In Figure 1 we illustrate a basic scheme that aims to describe every partial evaluation algorithm in logic programming, which is based in similar ones in the literature [6, 9]. Notice that this general algorithm depends on two operations, namely \( \text{Unfold} \) and \( \text{Abstract} \), which can be implemented differently in several partial evaluation systems.

Firstly, the algorithm uses an unfolding rule [19] in order to construct a finite and possibly incomplete proof tree for every atom in the set \( S \) and then creates a program \( P' \) such that every clause of it is constructed from all root-to-leaf derivations of these proof trees. This part of the process is referred as the local control of partial evaluation. There are many possible unfolding rules, some of which being more useful for a particular application than others. Examples include determinate, leftmost non-determinate, loop-preventing or depth-bound unfolding strategies [6, 9]. In some cases though, taking a simple approach which performs no unfolding at all, or in other words by using **one-step unfolding strategy**, may result in useful program optimizations. In such a case, \( \text{Unfold} \) exports a program that is constructed by finding the clauses that unify with each atom in \( S \) and then by specializing these clauses accordingly, using simple variable substitutions.

Secondly, the algorithm uses an \( \text{Abstract} \) operation, which calculates a finite abstraction of the set \( S \). We say that \( S' \) is an abstraction of \( S \) if every atom of \( S \) is an instance of some atom in \( S' \), and there does not exist two atoms in \( S' \) that have a common instance in \( S' \). This operation is used to keep the size of the set of atoms \( S \) finite, which will ensure the termination of the algorithm. This part of the process is referred as the global control of partial evaluation. Examples of abstraction operators include the use of a most specific generalizer and a finite bound in the size of \( S \) [9], or by exploiting a distinction between static and dynamic arguments for every atom in \( S \) [10].

A partial evaluation algorithm should ensure termination in both levels of control. Firstly, we have the _local termination problem_, which is the problem of the non-termination of the unfolding rule, and the _global termination problem_.
which is the problem of the non-termination of the iteration process (ie. the repeat loop in the algorithm). As we stated earlier, the global termination problem is solved by keeping the set $S$ finite through a finite abstraction operation. Regarding the local termination problem, one possible solution is ensuring that all constructed proof trees are finite. The one-step unfolding rule is by definition a strategy that can ensure local termination.

5 Predicate Specialization

In the following, we define our technique using the standard framework of partial evaluation (ref. Section 4), by specifying its local and global control strategies (namely UNFOLD and ABSTRACT operations). In particular, we will use a one-step unfolding rule and an abstraction operation which generalizes all individual (ie. non-predicate) arguments from all atoms of the partial evaluation.

**Definition 8.** Let $P$ be a program and $S$ be a set of atoms. Then,

$$\text{UNFOLD}(P, S) = \left\{ p \; E_1 \cdots E_n \leftarrow B \theta : \begin{array}{l}
(p \; E_1 \cdots E_n) \in S, \\
(\theta \; V_1 \cdots V_n \leftarrow B) \in P,
\end{array} \theta = \{V_1/E_1, \ldots, V_n/E_n\}\right\}$$

**Definition 9.** Let $S$ be a set of atoms. Then,

$$\text{ABSTRACT}(S) = \{ p \; E'_1 \cdots E'_n : (p \; E_1 \cdots E_n) \in S \}$$

where $E'_i = E_i$ if $E_i$ is of predicate type, otherwise $E'_i = V_i$, where $V_i$ is a fresh variable of the same type as of $E_i$.

In the following, we will show some properties of our transformation. Firstly we will need the following lemma:

**Lemma 1.** Let $P$ be a program, $S$ be a (possibly infinite) set of atoms. Then:

1. If $S$ is finite, then UNFOLD($P, S$) is finite.
2. ABSTRACT($S$) is a finite abstraction of $S$.
3. If every element of $S$ does not contain any free predicate variables, then every atom of UNFOLD($P, S$) does not contain any free predicate variables.

**Proof.**

1. Obvious from the construction of UNFOLD($P, S$).
2. Every predicate argument of every atom that appears in $P$ is either a variable or a predicate name, therefore ABSTRACT($S$) is finite.
3. Suppose that UNFOLD($P, S$) contains an atom $A$ that contains a free predicate variable $V$. If $A$ appears in the head of a clause, then from the construction of UNFOLD($P, S$), $S$ must contain $A$. If $A$ appears in the body of a clause, then since $P$ is definitional, $V$ also appears in the head of this clause. In any case, $S$ must contain an atom that contains the free predicate variable $V$. 

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The first part of the lemma ensures local termination and the second part of the lemma ensures global termination. The third part identifies that the transformation to first-order succeeds, provided that the program belongs to our fragment and the initial goal does not contain free higher-order variables. In the following corollaries, by $\Phi$ we denote the algorithm of Figure 1 combined with the operations in Definitions 8 and 9.

**Corollary 1.** Let $P$ be a $\mathcal{H}$ program and $G$ an goal. Then, the computation of $\Phi(P, G)$ terminates in a finite number of steps.

**Corollary 2.** Let $P$ be a definitional program and $G$ an goal that does not contain any free predicate variables. Then, the output of $\Phi(P, G)$ does not contain any free predicate variables.

The result of $\Phi$ is neither a valid $\mathcal{H}$ program since it contains predicate names as arguments in the heads, nor a valid first-order program since some symbols appear both as arguments and as predicate symbols. Therefore, we must apply a simple renaming [6, Section 3] in order to construct a valid first-order output. In our case, at the end of the partial evaluation algorithm, every atom $p \ E_1 \cdots E_n$ of $S$ is renamed into $p' \ V_1 \cdots V_m$, where $p'$ is a fresh predicate symbol and $\{V_1 \cdots V_m\} = \text{vars}(p \ E_1 \cdots E_n)$. Moreover, all instances of every atom of $S$ in the resulting program are renamed accordingly.

### 6 Implementation

We have developed a prototype implementation of our predicate specialization technique. Instead of developing a tailor-made higher-order language only for the purpose of demonstrating the benefits of the transformation, we build upon an existing higher-order logic programming language. The source programs in have to be written in the higher-order language Hilog [4], a mature and well-known language with a stable implementation within the XSB system [20].

A feature that we need and is not supported in Hilog though, is the use of types. Our algorithm needs types not only for deciding whether the input program belongs to our fragment, but also for the abstraction operation in Definition 9. Since the process of extending Hilog with types is outside of the scope of this paper, we assume that the input programs are well-typed and accompanied with type annotations for all predicates that contain predicate arguments.

The fragment that we discussed in Section 3 consists of programs that the only elements that can appear as predicate arguments are variables and predicate constants. However, most higher-order languages (and Hilog among them) allow more complex expressions to appear as predicate arguments. One such example is the use of *partial applications*, i.e., the ability to apply a predicate to only some of its arguments. Consider the following simple program.

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3 The implementation of the transformation is open source and can be accessed at [http://bitbucket.org/antru/firstify](http://bitbucket.org/antru/firstify)
conj2(P, Q, X) :- P(X), Q(X).
conj3(P, Q, R, X) :- conj2(P, conj2(Q, R), X).

In the second clause the expression \texttt{conj2(Q, R)} is a partial application where only the first two arguments are defined. A partial application effectively produces a new relation and therefore typically occur in higher-order arguments.

In the implementation we are able to handle programs that make a limited use of complex predicate expressions, as a syntactic sugar for our initial fragment. In particular, we allow non-variable and non-constant predicate arguments in an expression of the form \( p \ E_1 \cdots E_n \) that appears in the body of a clause \( q \ V_1 \cdots V_m \leftarrow B \) only if \( p \) and \( q \) do not belong in the same cycle in the \textit{predicate dependency graph}. The transformation in this case is also ensured to terminate (because due to the form of the program all predicate variables of a predicate that depends on itself have to be specialized only with predicate names and therefore the set of all possible specialization atoms will remain finite). As we mentioned earlier, this class of programs has the same expressive power as our initial fragment. For example, the aforementioned logic program is equivalent to the following program that does not use any partial applications.

\begin{align*}
\text{conj2}(P, Q, X) & : - P(X), Q(X). \\
\text{conj31}(P, Q, R, X) & : - \text{conj22}(P, Q, R, X). \\
\text{conj22}(P, Q, R, X) & : - P(X), \text{conj2}(Q, R, X).
\end{align*}

Interestingly, we can use our algorithm to convert a program of the extended fragment into its equivalent \( \mathcal{H} \) program. This can be done by initializing the transformation process with the top predicate (here \( \text{conj3}(P, Q, R, X) \)).

7 Experiments

In this section we present some experiments to illustrate that our technique can lead to the improvement of the execution runtime of higher-order logic programs.

We have tested our method with a set of benchmarks that include the computation of the transitive closure of a chain of elements, a \( k \)-ary disjunction and \( k \)-ary conjunction of \( k \) relations (for \( k = 5, 10 \)), the computation of the shortest path programs of a directed acyclic graph and a set of programs that deal with preference representation \cite{3}. The higher-order program is expressed in Hilog and executed using the Hilog module of XSB. XSB essentially transforms Hilog programs into first-order programs using the techniques and optimizations described in \cite{16}, and it also uses an optimized WAM instruction set to efficiently execute Hilog. The measurements obtained include these optimizations. We compare them with the execution of the Prolog programs produced by the our predicate specialization technique. Apart from XSB \cite{3}, we also consider for the execution

\begin{itemize}
\item An edge from the predicate \( p \) to predicate \( q \) in the predicate dependency graph means that there exists a clause that \( p \) appears in the head and \( p \) appears in the body of the same clause.
\item version 3.7, cf. \url{http://xsb.sourceforge.net/}
\end{itemize}
of the specialized program in other Prolog engines. The Prolog engines that we use are SWI-Prolog\textsuperscript{6} and YAP\textsuperscript{7}. Every program is executed several times, each time with a predefined set of facts. All data has been artificially generated.

In addition to the standard execution for Hilog and Prolog code, we also perform a \textit{tabled execution} of both the higher-order and the first-order programs in XSB. Tabling is a standard optimization technique that is widely used in Prolog systems. In this optimization, a re-evaluation of a \textit{tabled predicate} is avoided by memoizing (ie. remembering) its answers. The XSB system is known for its elaborate and efficient implementation of tabling for first-order logic programs. For higher-order Hilog programs however, XSB’s tabling mechanism may not be as effective as it is for first-order ones. The reason is that in order to table any Hilog predicate one has to table all Hilog code. This may lead to high memory consumption, and can be problematic for large-scale program development. We decided to table all predicates of the first-order programs as well, despite the fact that it might have been possible to make a more efficient use of tabling in this case. The idea behind this decision is to enable us to draw a fair comparison between tabled Hilog and tabled Prolog.

Table 1 summarizes the experimental results. The average execution time is depicted in seconds for each program and for each engine. The execution time is measured using the standard \texttt{time/1} predicate. Apart from the execution time, the table also contains the number of the (non-fact) clauses of the original higher-order program, the number of the (non-fact) clauses of the resulting first-order program after the transformation, and the ranges of the number of the corresponding facts. We do not show the runtime of each transformation from the higher-order to first-order since the execution of process was negligible (e.g. less than 0.01 seconds in all cases).

Firstly, we observe that the first-order programs are in general much faster than the higher-order ones. Even in the context of XSB which offers a native support of Hilog, the Prolog code is in almost all cases faster than the Hilog code. Especially in the transitive closure and the \textit{k}-ary conjunction, we have an improvement by one or more of orders of magnitude. In most programs in our experiment, we noticed that the ratio between the execution time of Prolog code and the execution time of Hilog code does not change much if we increase the number of facts, with the exception of the transitive closure benchmark, in which the more we increase the number of facts, the more this ratio decreases. The most important advantage of executing standard Prolog though, is that it allows us to choose from a wide range of available Prolog engines. From the three Prolog engines that we used, YAP is the most performant one. Therefore, we can get a further decrease in execution times by simply choosing a different Prolog engine, a fact that is not possible if we want to execute Hilog code directly.

As we stated earlier, tabling is another standard optimization technique that is widely used in Prolog systems. Tabling was very effective in many cases in the experiment, especially in the preference operations (\textit{winnow}, \textit{w} and \textit{wt}) and

\textsuperscript{6} version 7.2.3, cf. \url{http://www.swi-prolog.org/}
\textsuperscript{7} version 6.2.2, cf. \url{http://www.dcc.fc.up.pt/~vsc/Yap/}
Table 1. Experiment results. All execution times are in seconds.

| program      | hilog | prolog | hilog | prolog | program size |
|--------------|-------|--------|-------|--------|--------------|
|              | xsb   | swi    | xsb   | swi    |              |
| closure      | 1744.829 | 17.426 | 15.813 | 8.782  | 3 3 1000-8000 |
| closure_1000 | 12.132 | 0.801  | 0.609  | 0.372  | 3 3 1000     |
| closure_2000 | 91.284 | 2.884  | 2.644  | 1.332  | 3 3 2000     |
| closure_4000 | 709.356 | 11.336 | 10.918 | 5.464  | 3 3 4000     |
| closure_6000 | 2365.728 | 25.536 | 23.459 | 13.532 | 3 3 6000     |
| closure_8000 | 5545.644 | 46.576 | 41.433 | 23.208 | 3 3 8000     |
| conj5        | 9.887 | 1.090  | 0.026  | 0.010  | 3 6 1000-8000 |
| genconj(5)   | 9.921 | 1.101  | 0.028  | 0.011  | 4 4 1000-8000 |
| conj10       | 21.676 | 2.414  | 0.023  | 0.015  | 3 11 1000-8000 |
| genconj(10)  | 21.580 | 2.415  | 0.039  | 0.013  | 4 4 1000-8000 |
| union5       | 0.035 | 0.028  | 0.030  | 0.023  | 9.618  |
| genunion(5)  | 0.034 | 0.030  | 0.025  | 0.021  | 0.037  |
| union10      | 0.063 | 0.062  | 0.046  | 0.036  | 0.075  |
| genunion(10) | 0.062 | 0.079  | 0.054  | 0.035  | 0.091  |
| path_dag     | 971.326 | 679.557 | 975.027 | 54.156 | 6 6 10-80    |
| path_naive   | 5.725 | 4.248  | 6.661  | 0.407  | 0.021  |
| winnow       | 0.147 | 0.130  | 0.117  | 0.039  | 1.107  |
| w(2)         | 3.920 | 3.257  | 3.844  | 0.527  | 0.168  |
| w(3)         | 129.457 | 107.183 | 122.556 | 21.103 | 0.119  |
| wt(2)        | 4.146 | 3.288  | 3.857  | 0.530  | 0.144  |
| wt(3)        | 130.540 | 108.048 | 126.876 | 21.360 | 0.100  |

in the path programs (notice the dramatic decrease in the execution times for the path_dag benchmark). It seems that the performance of this optimization offers the same performance gain for both Hilog and Prolog code, since the execution times are in most cases similar. A notable exception is that of the \( k \)-ary conjunction benchmark, in which the tabled Prolog code is 5 to 10 times faster than that of the tabled Hilog code. Also, the fact that we table all Hilog and Prolog code did not have much negative effect in our experiment after all, because (with the sole exception of the winnow benchmark) the tabled executions are not slower than their non-tabled counterparts.

Finally, consider the programs that deal with the \( k \)-ary conjunction and disjunction, i.e. the pairs conj5 – genconj(5), conj10 – genconj(10), union5 – genunion(5) and union10 – genunion(10). Both programs of each of these pairs are making the same computation, with the former expressed in a non-recursive way and the latter in a recursive way. These programs differ also in the size of their first order counterparts. The first-order form of the non-recursive version has more clauses than the first-order form of the recursive version. We observe that both the higher-order and the first-order versions of the same computation have similar execution times, even though the first-order versions have different num-
bers of clauses. As a result, an increase on the size of the first-order program did not produce any overhead in the overall program execution time.

8 Related Work

The proposed predicate specialization is closely connected with related work on partial evaluation of logic programs [11, 6, 9]. More specifically, the proposed technique is a special form of partial evaluation which targets higher-order arguments and uses a simple one-step unfolding rule to propagate the constant higher-order arguments without changing the structure of the original program. Consequently, first-order programs remain unchanged. To the extend of our knowledge, partial evaluation techniques have not been previously applied directly to higher-order logic programming with the purpose to produce a simpler first-order program.

Other techniques, however, have been proposed that focus on the removal of higher-order parameters in logic programs. Warren, in one of the early papers that tackle similar issues [21], proposed that simple higher-order structures are non-essential and can be easily encoded as first-order logic programs. The key idea is that every higher-order argument in the program can be encoded as a symbol utilizing its name and a special apply predicate should be introduced to distinguish between different higher-order calls. A very similar approach has been employed in Higlog [4]; a language that offers a higher-order syntax with first-order semantics. A Hilog program is transformed into an equivalent first-order one using a transformation similar to Warren’s technique [21]. Actually, these techniques are closely related to Reynolds’ defunctionalization [17] that has been originally proposed to remove higher-order arguments in functional programs. These techniques are designed to be applied in arbitrary programs in comparison to our approach. In order to achieve this they require data structures in the resulting program. However, on a theoretical view this imposes the requirement that the target language should support data structures even if the source language does not support that. This is apparent when considering Datalog; transforming a higher-order Datalog program will result into a first-order Prolog program. On a more practical point, the generic data structures introduced during the defunctionalization render the efficient implementation of these programs challenging. The wrapping of the higher-order calls with the generic apply predicate makes it cumbersome to utilize the optimizations in first-order programs such as indexing and tabling. In comparison, our technique produce more natural programs that do not suffer for this phenomenon. Moreover, it does not introduce any data structures and as a result a higher-order Datalog program will be transformed into a first-order one amenable to more efficient implementation.

In order to remedy the shortcomings of defunctionalization there have been proposed some techniques to improve the performance of the transformed programs. Sagonas and Warren [18] proposed a compile-time optimization of the classical Hilog encoding that eliminates some partial applications using a family
of apply predicates thus increasing the number of the predicates in the encoded program, which leads to a more efficient execution. The original first-order encoding of Hilog as well as this optimization are included in the XSB system [20]. In the context of functional-logic programming, there exist some mixed approaches that consider defunctionalization together with partial evaluation for functional-logic programs [1, 16], where a partial evaluation process is applied in a defunctionalized functional-logic program. Even though these approaches can usually offer a substantial performance improvement, the resulting programs still use a Reynolds’ style encoding; for instance, the performance gain of the optimizations offered by XSB is not sufficient when compared to the technique presented in this paper, as presented in Section 7.

The process of eliminating higher-order functions is being studied extensively in the functional programming domain. Apart from defunctionalization, there exist some approaches that do not introduce additional data structures while removing higher-order functions. These techniques include the higher-order removal method of Chin and Darlington [5], the firstification technique of Nelan [14] and the firstify algorithm of Mitchell and Runciman [13]. The removal of higher-order values here is achieved without introducing additional data structures, so the practical outcome is that the resulting programs can be executed in a more efficient way than the original ones. The basic operation of these transformation methods is function specialization, which involves generating a new function in which the function-type arguments of the original definition are eliminated. A predicate specialization operation is also the core operation in our approach, so in this point these approaches are similar to ours. The remaining operations that can be found in those approaches (e.g. simplification rules, inlining, eta-abstractions etc.), are either inapplicable to our domain or not needed for our program transformation. Contrary to Reynolds’ defunctionalization, these higher-order removal techniques [5, 13, 14] are not complete, meaning that they do not remove all higher-order values from a functional program, and therefore the resulting programs are not always first order. This phenomenon would happen in our case as well if we considered the full power of higher-order programming. However, because of the fact that we focus on a smaller but still useful class of higher-order logic programs, we are sure that the output of our transformation technique will produce a valid first-order program for every program that belongs to our fragment.

9 Conclusions and Future Work

In this paper we presented a program transformation technique that reduces higher-order programs into first-order ones through argument specialization. The transformation does not introduce additional data structures and therefore the resulting programs can be executed efficiently in any standard Prolog system. We do not consider the full power of higher-order logic programming, but we focus on a modest but useful class of programs; in these programs we do not allow partial applications or existential predicate variables in the body of a clause.
In our actual implementation we considered a slightly broader class than the fragment discussed before; we allowed a limited use of partial applications in the case of predicates that do not belong to the same cycle in the predicate dependency graph. This extension however does not increase the expressive power of the language. An interesting open question that arises is whether this technique can be used as a first-order reduction method only for programs that belong to our fragment (or a fragment that have the same expressive power as ours) or if it can be used for a wider class of programs that are more expressive than our fragment. Moreover, any expansion of the supported class would be desirable, even if it has the same expressivity as our current fragment.

Until now, we have used and evaluated our transformation technique only as an optimization method for performance improvement. However, in the functional programming domain, such techniques have been used in additional applications, such as program analysis \[13\] and implementation of debuggers \[15\]. Therefore, an interesting aspect for future investigation would be the search of similar or completely new applications of our higher-order removal technique in the logic programming domain.

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**References**

[1] Albert, E., Hanus, M., Vidal, G.: A practical partial evaluation scheme for multi-paradigm declarative languages. Journal of Functional and Logic Programming 2002 (2002)

[2] Charalambidis, A., Handjopoulos, K., Rondogiannis, P., Wadge, W.W.: Extentional higher-order logic programming. ACM Transactions on Computational Logic 14(3), 21 (2013)

[3] Charalambidis, A., Rondogiannis, P., Troumpoukis, A.: Higher-order logic programming: An expressive language for representing qualitative preferences. Science of Computer Programming 155, 173 – 197 (2018)

[4] Chen, W., Kifer, M., Warren, D.S.: Hilog: A foundation for higher-order logic programming. Journal of Logic Programming 15(3), 187–230 (1993)

[5] Chin, W., Darlington, J.: A higher-order removal method. Lisp and Symbolic Computation 9(4), 287–322 (1996)

[6] Gallagher, J.P.: Tutorial on specialisation of logic programs. In: Proceedings of the ACM SIGPLAN Symposium on Partial Evaluation and Semantics-Based Program Manipulation, PEPM’93, Copenhagen, Denmark, June 14-16, 1993. pp. 88–98 (1993)

[7] Jones, N.D.: The expressive power of higher-order types or, life without CONS. J. Funct. Program. 11(1), 5–94 (2001)

[8] Jones, N.D., Gomard, C.K., Sestoft, P.: Partial evaluation and automatic program generation. Prentice Hall (1993)
[9] Leuschel, M.: Logic program specialisation. In: Partial Evaluation - Practice and Theory, DIKU 1998 International Summer School, Copenhagen, Denmark, June 29 - July 10, 1998. pp. 155–188 (1998)
[10] Leuschel, M., Vidal, G.: Fast offline partial evaluation of logic programs. Inf. Comput. 235, 70–97 (2014)
[11] Lloyd, J.W., Shepherdson, J.C.: Partial evaluation in logic programming. J. Log. Program. 11(3&4), 217–242 (1991)
[12] Miller, D., Nadathur, G.: Programming with Higher-Order Logic. Cambridge University Press, New York, NY, USA, 1st edn. (2012)
[13] Mitchell, N., Runciman, C.: Losing functions without gaining data: another look at defunctionalisation. In: Proceedings of the 2nd ACM SIGPLAN Symposium on Haskell, Haskell 2009, Edinburgh, Scotland, UK, 3 September 2009. pp. 13–24 (2009)
[14] Nelan, G.: Firstification. Ph.D. thesis, Arizona State University (1991)
[15] Pope, B.J., Naish, L.: Specialisation of higher-order functions for debugging. Electr. Notes Theor. Comput. Sci. 64, 277–291 (2002)
[16] Ramos, J.G., Silva, J., Vidal, G.: Fast narrowing-driven partial evaluation for inductively sequential programs. In: Proceedings of the 10th ACM SIGPLAN International Conference on Functional Programming, ICFP 2005, Tallinn, Estonia, September 26-28, 2005. pp. 228–239 (2005)
[17] Reynolds, J.C.: Definitional interpreters for higher-order programming languages. In: Proc. of the 25th ACM Nat. Conf. pp. 717–740. ACM (1972)
[18] Sagonas, K., Warren, D.S.: Efficient execution of hilog in wam-based prolog implementations. In: Proceedings of the 12th International Conference on Logic Programming, Tokyo, Japan, June 13-16, 1995. pp. 349–363 (1995)
[19] Shepherdson, J.C.: Unfold/fold transformations of logic programs. Mathematical Structures in Computer Science 2(2), 143–157 (1992)
[20] Swift, T., Warren, D.S.: XSB: extending prolog with tabled logic programming. TPLP 12(1-2), 157–187 (2012)
[21] Warren, D.H.: Higher-order extensions to prolog-are they needed. Machine Intelligence 10, 441–454 (1982)