A CLASS OF SEQUENCE SPACES DEFINED BY \( l \)-FRACTIONAL DIFFERENCE OPERATOR

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Abstract. In this paper, we generalize the fractional order difference operator using \( l \)-Pochhammer symbol and define \( l \)-fractional difference operator. The \( l \)-fractional difference operator is further used to introduce a class of difference sequence spaces. Some topological properties and duals of the newly defined spaces are studied.

1. Introduction

Fractional order difference operators defined on the set \( w \) of all sequences of complex numbers have recently got an attention to many researchers because of its applicability in the Numerical analysis [3], statistical convergence [16], approximation theory [14] etc. Fractional order difference operators are basically generalizations of \( m \)-th order difference operators, where \( m \) is a nonnegative integer. For a proper fraction \( a \), Baliarsingh in [3] and Dutta and Baliarsingh in [8] introduced some fractional order difference operators \( \Delta^a, \Delta^{(a)}, \Delta^{-a} \) and \( \Delta^{(-a)} \). The fractional order difference operator \( \Delta^{(a)} : w \rightarrow w \) is defined by

\[
\Delta^{(a)}x = \left\{ \sum_{i=0}^{\infty} (-1)^i \frac{\Gamma(a+1)}{i!\Gamma(a-i+1)} x_{k-i} \right\}_k
\]

for all \( x = \{x_k\} \in w \). Assuming \( x_{-s} = 0 \) for all positive integers \( s \), the fractional order difference operator (1.1) can be restated in terms of Pochhammer symbol as

\[
\Delta^{(a)}x = \left\{ \sum_{i=0}^{k} \frac{(-a)_i}{i!} x_{k-i} \right\}_k,
\]

where the Pochhammer symbol \( (a)_k \) is defined by

\[
(a)_k = \begin{cases} 1, & \text{when } k = 0 \\ a(a+1)(a+2)(a+3)\ldots(a+k-1), & \text{when } k \in \mathbb{N} \setminus \{0\} \end{cases}
\]

A sequence space is a linear subspace of the set \( w \) of all sequences of complex numbers. Difference operators are used to construct sequence spaces which are called difference sequence spaces. Kizmaz [17] was the first to develop the idea of difference sequence space. For the spaces \( \mu = l_\infty, c \) and \( c_0 \), he defined and studied some Banach spaces

\[
\mu(\Delta) = \{ x = \{x_k\} \in w : \Delta x \in \mu \},
\]

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where $\Delta x = \{x_k - x_{k+1}\}$. Et and Colak [10] replaced the first order difference operator by an $m$-th order difference operator in $\mu(\Delta)$ and defined $BK$-spaces (1.5) $$\mu(\Delta^m) = \{x = \{x_k\} \in w : \Delta^m x \in \mu\}.$$ Later on, many authors such as Mursaleen [19], Et et al. [9], Aydin and Başar [4], Isik [13], Srivastava and Kumar [20], Maji and Srivastava [18], Bhardwaj and Bala [3], Djolović and Malkowsky [7], Chandra and Tripathy [5], Hazarika and Savas [12], Hazarika [11] etc. have generalized (1.4) and (1.5) and studied other difference sequence spaces. Recently, fractional difference operators have been used to define fractional difference sequence spaces such as in [2], [8], [15] and many more.

One way to enhance the fractional order difference operator (1.2) is to apply $l$-Pochhammer symbol [6] instead of applying simply Pochhammer symbol. The $l$-Pochhammer symbol $(a)_{k,l}$, for a complex number $a$ and a real number $l$, is introduced by Diaz and Pariguan in [6] as follows:

$$\begin{cases} 1, & \text{when } k = 0 \\ a(a + l)(a + 2l)(a + 3l) \ldots (a + (k - 1)l), & \text{when } k \in \mathbb{N} \setminus \{0\} \end{cases} 
(1.6)$$

In this paper, we introduce $l$-fractional difference operator $\Delta^{(a;l)}$ employing $l$-Pochhammer symbol. We further establish a class of sequence spaces $\mu(\Delta^{(a;l)}, v)$, where $\mu \in \{l_\infty, c_0, c\}$ which is defined by the use of $l$-fractional difference operator $\Delta^{(a;l)}$. We study some topological properties and determine $\alpha$, $\beta$ and $\gamma$ duals of the spaces $\mu(\Delta^{(a;l)}, v)$.

Throughout this paper, we use the following symbols: $w$, the set of all complex sequences; $\mathbb{C}$, the set of complex numbers; $\mathbb{R}$, the set of real numbers; $\mathbb{N}$, the set of natural numbers; $\mathbb{N}_0 = \mathbb{N} \cup \{0\}$; $l_\infty$, the space of bounded sequences; $c$, the space of convergent sequences; and $c_0$, the space of null sequences.

2. $l$-FRACTIONAL DIFFERENCE OPERATOR

In this section, we define $l$-fractional difference operator and study some properties of this operator. Results in this section are equivalent to the results in [2].

**Definition 2.1.** Let $a$ be a real number. Then $l$-fractional difference operator $\Delta^{(a;l)} : w \to w$ is defined by

$$\Delta^{(a;l)} x = \left\{ \sum_{i=0}^{k} \frac{(-a)^{i,l}}{i!} x_{k-i} \right\}_k$$

(2.1)

for all $x = (x_k) \in w$. For convenience we write the transformation of the $k^{th}$ term of a sequence $x = (x_k)$ by $\Delta^{(a;l)}$ as

$$\Delta^{(a;l)} x_k = \sum_{i=0}^{k} \frac{(-a)^{i,l}}{i!} x_{k-i}$$

(2.2)

For some particular values of $a$ and $l$, we have the following observations:

- For $l = 1$, the fractional difference operator $\Delta^{(a;l)}$ is reduced to the operator $\Delta^{(a)}$ (Equation (1.4)).
- If $l = 1$ and $a = m$, a positive integer, then $\Delta^{(a;l)}$ is basically the $m$-th order backward difference operator $\Delta^{(m)}$, where $\Delta^{(1)} x_k = x_k - x_{k-1}$ and $\Delta^{(m)} x_k = \Delta^{(m-1)} (\Delta^{(1)} x_k)$.
- $\Delta^{(a;a)} x_k = x_k - a x_{k-1}$. 


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- For \( a = 2l \), we have \( \Delta^{(a;l)} x_k = x_k - 2lx_{k-1} + l^2x_{k-2} \).
- \( \Delta^{(\frac{1}{2};l)} x_k = x_k - \frac{1}{2}x_{k-1} + \frac{1}{10}x_{k-2} \).
- \( \Delta^{(-\frac{1}{2};l)} = x_k + \frac{1}{2}x_{k-1} + \frac{3}{40}x_{k-2} + \frac{3}{850}x_{k-3} + \frac{5}{2560}x_{k-4} + \ldots + \frac{1}{(2l)^3} \ldots \frac{1}{(2l)^{k-1}} x_0 \).

**Theorem 2.2.** The l-fractional difference operator \( \Delta^{(a;l)} \) is a linear operator.

**Proof.** As the proof of this theorem is straightforward, we omit the proof. \( \square \)

**Theorem 2.3.** For real numbers \( a \) and \( b \), the following results hold:

1. \( \Delta^{(a;l)}(\Delta^{(b;l)} x_k) = \Delta^{(a+b;l)} x_k = \Delta^{(b;l)}(\Delta^{(a;l)} x_k) \)
2. \( \Delta^{(a;l)}(\Delta^{(-a;l)} x_k) = x_k \)

**Proof.** We give the proof of the first result only. The second result can be deduced from the first result. We consider

\[
\Delta^{(a;l)}(\Delta^{(b;l)} x_k) = \Delta^{(a;l)} \sum_{i=0}^{k} \frac{(-b)_i l}{i!} x_{k-i} \]

As the l-fractional difference operator is linear,

\[
\Delta^{(a;l)}(\Delta^{(b;l)} x_k) = \sum_{i=0}^{k} \frac{(-b)_i l}{i!} \Delta^{(a;l)} x_{k-i} = \sum_{i=0}^{k} \frac{(-b)_i l}{i!} \sum_{j=0}^{k-i} \frac{(-a)_j l}{j!} x_{k-i-j}. \tag{2.4}
\]

Taking the transformation \( i + j = m \) or arranging the coefficients of \( x_{k-m} \) for \( m = 0, 1, \ldots, k \), the Equation \( 2.4 \) can be rewritten as

\[
\Delta^{(a;l)}(\Delta^{(b;l)} x_k) = \sum_{m=0}^{k} \left( \sum_{i=0}^{m} \frac{(-b)_i l}{i!} \frac{(-a)_{(m-i)j}}{(m-i)!} \right) x_{k-m}. \tag{2.5}
\]

By mathematical induction, it is easy to see that

\[
\frac{-(a + b)_{m,l}}{m!} = \left( \sum_{i=0}^{m} \frac{(-b)_i l}{i!} \frac{(-a)_{(m-i)l}}{(m-i)!} \right). \tag{2.6}
\]

Then Equations \( 2.5 \) and \( 2.6 \) imply that

\[
\Delta^{(a;l)}(\Delta^{(b;l)} x_k) = \sum_{m=0}^{k} \frac{-(a + b)_{m,l}}{m!} x_{k-m} = \Delta^{(a+b;l)} x_k.
\]

Similarly, we can prove that

\[
\Delta^{(b;l)}(\Delta^{(a;l)} x_k) = \Delta^{(a+b;l)} x_k.
\]

This proves the theorem. \( \square \)
3. Sequence spaces $\mu(\Delta^{(a:l)}, v)$ for $\mu = l_\infty, c_0,$ and $c.$

In this section, we introduce a class of sequence spaces $\mu(\Delta^{(a:l)}, v),$ where $\mu \in \{l_\infty, c_0, c\},$ by the use of $l$-fractional difference operator $\Delta^{(a:l)}.$ Let $V$ be the set of all real sequences $v = \{v_n\}$ such that $v_n \neq 0$ for all $n \in \mathbb{N}_0.$ Then the class of sequence spaces $\mu(\Delta^{(a:l)}, v)$ is defined by

$$\mu(\Delta^{(a:l)}, v) = \left\{ x = \{x_n\} \in w : \left\{ \sum_{j=0}^{n} v_j \Delta^{(a:l)} x_j \right\}_n \in \mu \right\}.$$  

Let $y_n$ denotes the $n^{th}$ term of the sequence $\left\{ \sum_{j=0}^{n} v_j \Delta^{(a:l)} x_j \right\},$ then

$$y_n = \sum_{j=0}^{n} v_j \Delta^{(a:l)} x_j$$

$$= \sum_{j=0}^{n} v_j \left( \sum_{i=0}^{j} \frac{(-a)_i}{i!} x_{j-i} \right)$$

Taking the transformation $j - i = m$ or arranging the coefficients of $x_m$ for $m = 0, 1, \ldots, n,$ the Equation (3.1) can be rewritten as

$$y_n = \sum_{m=0}^{n} \left( \sum_{i=0}^{n-m} \frac{(-a)_i}{i!} v_{i+m} \right) x_m.$$  

Now, we consider a matrix $C = (c_{nm})$ such that

$$c_{nm} = \begin{cases} \sum_{i=0}^{n-m} \frac{(-a)_i}{i!} v_{i+m}, & \text{when } n \geq m \\ 0, & \text{when } n < m \end{cases}. $$

Then the sequence $\{y_n\}$ is $C$-transform of the sequence $\{x_n\}$.

**Theorem 3.1.** The space $\mu(\Delta^{(a:l)}, v)$ for $\mu = l_\infty, c_0,$ or $c$ is a normed linear space with respect to the norm $\|x\|_{\mu(\Delta^{(a:l)}, v)}$ defined by

$$\|x\|_{\mu(\Delta^{(a:l)}, v)} = \sup_n \left|\sum_{j=0}^{n} v_j \Delta^{(a:l)} x_j\right|.$$  

**Proof.** We observe that $\|\alpha x\|_{\mu(\Delta^{(a:l)}, v)} = |\alpha| \|x\|_{\mu(\Delta^{(a:l)}, v)}$ for all $\alpha \in \mathbb{C}$ and $\|x + y\|_{\mu(\Delta^{(a:l)}, v)} \leq \|x\|_{\mu(\Delta^{(a:l)}, v)} + \|y\|_{\mu(\Delta^{(a:l)}, v)}$ for all sequences $x = \{x_n\}$ and $y = \{y_n\}$ of $\mu(\Delta^{(a:l)}, v).$ Now suppose that $x = \{x_k\} \in \mu(\Delta^{(a:l)}, v)$ is such that $\|x\|_{\mu(\Delta^{(a:l)}, v)} = 0.$ Then

$$\sup_n \left|\sum_{j=0}^{n} v_j \Delta^{(a:l)} x_j\right| = 0.$$  

This implies that

$$\sum_{j=0}^{n} v_j \Delta^{(a:l)} x_j = 0 \text{ for all } n \in \mathbb{N}_0.$$
This gives,
\[ v_0 \Delta^{(a,j)} x_0 = 0 \]
\[ v_1 \Delta^{(a,j)} x_1 = 0 \]
\[ \vdots \]
This shows that \( x_n = 0 \) for all \( n \in \mathbb{N}_0 \). That is, \( x = (0, 0, \ldots) \). Thus \( \| \cdot \|_{\mu(\Delta^{(a,j)}, v)} \) is a norm and \( \mu(\Delta^{(a,j)}, v) \) is a normed linear space for \( \mu = l_\infty, c_0 \) or \( c \). \( \square \)

**Theorem 3.2.** The space \( \mu(\Delta^{(a,j)}, v) \) for \( \mu = l_\infty, c \) or \( c_0 \) is a complete normed linear space.

**Proof.** As proof of the theorem for the spaces \( \mu(\Delta^{(a,j)}, v) \) for \( \mu = l_\infty, c \), and \( c_0 \) run along similar lines, we just prove that the space \( l_\infty(\Delta^{(a,j)}, v) \) is a complete normed linear space. For this, let \( \{x^i\}_i = \{x^1 = \{x^i_{k=0}^1, x^2 = \{x^i_{k=0}^2, \ldots\} \) be a Cauchy sequence in \( l_\infty(\Delta^{(a,j)}, v) \). Then, from definition of Cauchy sequence, there exists a natural number \( n_0(\epsilon) \) corresponding to each real number \( \epsilon > 0 \) such that

\[
\| x^i - x^m \|_{\mu(\Delta^{(a,j)}, v)} < \epsilon
\]

for all \( i, m \geq n_0(\epsilon) \). By definition of the norm \( \| \cdot \|_{\mu(\Delta^{(a,j)}, v)} \), Equation (3.3) gives

\[
\sup_n \left| \sum_{j=0}^n v_j \Delta^{(a,j)} x^i_j - \sum_{j=0}^n v_j \Delta^{(a,j)} x^m_j \right| < \epsilon
\]

for all \( i, m \geq n_0(\epsilon) \). Inequality (3.4) can be rewritten with respect to \( C \)-transformation (see (3.2)) as

\[
\sup_n \left| (Cx^i)_n - (Cx^m)_n \right| < \epsilon
\]

for all \( i, m \geq n_0(\epsilon) \). Inequality (3.5) shows that for a fixed nonnegative integer \( n \) the sequence \( \{(Cx^i)_n\}_i = \{(Cx^1)_n, (Cx^2)_n, \ldots\} \) is a Cauchy sequence in the set of real numbers \( \mathbb{R} \). As \( \mathbb{R} \) is complete, the sequence \( \{(Cx^i)_n\}_i \) converges to \( (Cx)_n \) (say). Letting \( m \) tends to infinity in Inequality (3.5), we get

\[
\| (Cx^i)_n - (Cx)_n \| < \epsilon
\]

for all \( i \geq n_0(\epsilon) \). Since Inequality (3.6) holds for each nonnegative integer \( n \), we have

\[
\sup_n \left| (Cx^i)_n - (Cx)_n \right| < \epsilon
\]

for all \( i \geq n_0(\epsilon) \), that is

\[
\| x^i - x \|_{\mu(\Delta^{(a,j)}, v)} < \epsilon
\]

for all \( i \geq n_0(\epsilon) \). Now, it remains to show that the sequence \( x = \{x_n\} \) belongs to \( \mu(\Delta^{(a,j)}, v) \). Since the sequence \( \{x^i\} \) belongs to \( \mu(\Delta^{(a,j)}, v) \), there exists a real number \( M \) such that

\[
\sup_n |(Cx^i)_n| \leq M.
\]
With the help of Inequalities (3.7) and (3.9), we conclude that
\[
\sup_n |(Cx)_n| = \sup_n |(Cx)_n - (Cx')_n| + \sup_n |(Cx')_n| \leq M + \epsilon
\]
This shows that the sequence \( x = \{x_n\} \in l_\infty(\Delta^{(a;i)}, v) \). Thus, the space \( l_\infty(\Delta^{(a;i)}, v) \) is a complete normed linear space.

**Theorem 3.3.** The space \( \mu(\Delta^{(a;i)}, v) \) (where \( \mu = l_\infty, c_0 \) or \( c \)) is linearly isomorphic to the space \( \mu \).

*Proof.* To prove this theorem, we have to show that there exists a mapping \( T : \mu(\Delta^{(a;i)}, v) \rightarrow \mu \), which is linear and bijective. For our purpose, we suppose that \( T \) is defined by
\[
T x = \left\{ \sum_{j=0}^{n} v_j \Delta^{(a;i)} x_j \right\}_n = \{y_n\}
\]
for all \( x = \{x_n\} \in \mu(\Delta^{(a;i)}, v) \). We observe that the mapping \( T \) is linear and it satisfies the property "\( T x = \theta \) implies \( x = \theta' \), where \( \theta = \{0, 0, \ldots\} \). The property "\( T x = \theta \) implies \( x = \theta' \)" shows that the mapping \( T \) is injective. Now it remains to prove that the mapping \( T \) is surjective. Therefore, we consider an element \( \{y_n\} \in \mu \) and determine a sequence \( \{y_n\} \) such that
\[
x_n = \Delta^{(-a;i)} \left( \frac{y_n - y_{n-1}}{v_n} \right)
\]
for all \( n \in \mathbb{N}_0 \). Then we see that
\[
\sum_{j=0}^{n} v_j \Delta^{(a;i)} x_j = \sum_{j=0}^{n} v_j \Delta^{(a;i)} \Delta^{(-a;i)} \left( \frac{y_j - y_{j-1}}{v_j} \right).
\]
By Theorem 2.3, the Equation (3.13) gives
\[
\sum_{j=0}^{n} v_j \Delta^{(a;i)} x_j = \sum_{j=0}^{n} (y_j - y_{j-1}) = y_n
\]
This shows that \( \left\{ \sum_{j=0}^{n} v_j \Delta^{(a;i)} x_j \right\}_n = \{y_n\} \in \mu \). Using Definition (2.1), we have \( \{x_n\} \in \mu(\Delta^{(a;i)}, v) \). Thus we have shown that for every sequence \( \{y_n\} \in \mu \), there exists a sequence \( \{x_n\} \in \mu(\Delta^{(a;i)}, v) \). That is, the mapping \( T \) is a surjective mapping. Hence the space \( \mu(\Delta^{(a;i)}, v) \) is linearly isomorphic to the space \( \mu \). □

4. \( \alpha-, \beta- \) AND \( \gamma- \) DUALS OF THE SPACES \( \mu(\Delta^{(a;i)}, v) \) FOR \( \mu = l_\infty, c_0 \), AND \( c \).

In this section, we determine \( \alpha-, \beta- \) and \( \gamma- \) duals of the spaces \( \mu(\Delta^{(a;i)}, v) \) for \( \mu = l_\infty, c_0 \) and \( c \). For two sequence spaces \( \mu \) and \( \lambda \), the multiplier space of \( \mu \) and \( \lambda \) is defined by
\[
S(\mu, \lambda) = \{ z = \{z_k\} \in w : zx = \{z_k x_k\} \in \lambda \text{ for all } x = \{x_k\} \in \mu \}.
\]
In particular, \( \alpha-, \beta- \), and \( \gamma- \) duals of a space \( \mu \) are defined by \( \mu^\alpha = S(\mu, l_1) \), \( \mu^\beta = S(\mu, c_0) \) and \( \mu^\gamma = S(\mu, bs) \) respectively, where \( l_1, c_0 \) and \( bs \) are the space of absolutely summable sequences, the space of convergent series and the space of bounded series respectively. To find duals of the spaces \( \mu(\Delta^{(a;i)}, v) \), we have used the following lemmas that are given by Stieglitz and Tietz in [21].
Lemma 4.1. \( B \in (l_\infty : l_1) = (c_0 : l_1) = (c : l_1) \) if and only if

\[
\sup_{K \in F} \sum_n \left| \sum_{k \in K} b_{nk} \right| < \infty.
\]

Lemma 4.2. \( B \in (l_\infty : c) \) if and only if

\[
\lim_{n \to \infty} b_{nk} \text{ exists for all } k \quad \text{and} \quad \lim_{n \to \infty} \sum_k |b_{nk}| = \sum_k \lim_n |b_{nk}|.
\]

Lemma 4.3. \( B \in (c : c) \) if and only if \((4.2)\), \(\sup_n \sum_k |b_{nk}| < \infty\) and \((4.4)\).

Lemma 4.4. \( B \in (c_0 : c) \) if and only if \((4.2)\) and \((4.4)\) hold.

Lemma 4.5. \( B \in (l_\infty : l_\infty) = (c_0 : l_\infty) = (c : l_\infty) \) if and only if \((4.4)\) holds.

Now, we determine the duals of the spaces \( \mu(\Delta^{(a,l)}, v) \) for \( \mu = l_\infty, c_0, \) and \( c \) through following theorem.

Theorem 4.6. Let \( D = (d_{nk}) \) and \( E = (e_{nk}) \) are two matrices such that

\[
d_{nk} = \begin{cases} 0; & (k > n) \\ \frac{z_k}{v_k}; & (k = n) \\ z_k \left( \frac{(a)_n - k}{(n-k)!v_k} - \frac{(a)_{n-k-1}}{(n-k-1)!v_{k+1}} \right); & (k < n) \end{cases}
\]

and

\[
e_{nk} = \begin{cases} 0; & (k > n) \\ \frac{z_k}{v_k}; & (k = n) \\ \frac{1}{n} \sum_{i=0}^{n-k} (a)_{i+k} z_{k+i} - \frac{1}{v_{k+1}} \sum_{i=0}^{n-k-1} (a)_{i+k+1} z_{k+i+1}; & (k < n) \end{cases}
\]

and consider the sets \( A_1, A_2, A_3, A_4 \) and \( A_5 \) which are defined as follows:

\[
A_1 = \left\{ z = (z_k) \in w : \sup_{K \in F} \sum \left| \sum_{k \in K} d_{nk} \right| < \infty \right\},
\]

\[
A_2 = \left\{ z = (z_k) \in w : \lim_{n \to \infty} e_{nk} \text{ exists for all } k \right\},
\]

\[
A_3 = \left\{ z = (z_k) \in w : \lim_n \sum_k |e_{nk}| = \sum_k \lim_n |e_{nk}| \right\},
\]

\[
A_4 = \left\{ z = (z_k) \in w : \sup_n \sum_k |e_{nk}| < \infty \right\},\quad \text{and}
\]

\[
A_5 = \left\{ z = (z_k) \in w : \lim_n \sum_k e_{nk} \text{ exists} \right\}.
\]
Proof. We determine duals of the space $c_0(\Delta^{(\alpha;l)}, v)$. Duals of other spaces $l_\infty(\Delta^{(\alpha;l)}, v)$ and $c(\Delta^{(\alpha;l)}, v)$ can be deduced on the similar lines as that of the $c_0(\Delta^{(\alpha;l)}, v)$. By Equation (3.12), we obtain that

$$\{l_\infty(\Delta^{(\alpha;l)}, v)\}^\alpha = A_1, \quad \{l_\infty(\Delta^{(\alpha;l)}, v)\}^\beta = A_2 \cap A_3, \quad \{l_\infty(\Delta^{(\alpha;l)}, v)\}^\gamma = A_4$$

$$\{c_0(\Delta^{(\alpha;l)}, v)\}^\alpha = A_1, \quad \{c_0(\Delta^{(\alpha;l)}, v)\}^\beta = A_2 \cap A_4, \quad \{c_0(\Delta^{(\alpha;l)}, v)\}^\gamma = A_4$$

$$\{c(\Delta^{(\alpha;l)}, v)\}^\alpha = A_1, \quad \{c(\Delta^{(\alpha;l)}, v)\}^\beta = A_2 \cap A_4 \cap A_5 \text{ and } \{c(\Delta^{(\alpha;l)}, v)\}^\gamma = A_4.$$

By introducing $l$- fractional difference operator, we have given a more general fractional order difference operator. Also, we have defined a class of difference sequence spaces with the help of $l$- fractional difference operator. Further, we have studied some topological properties and determined the duals of the spaces.

5. Conclusions

By introducing $l$- fractional difference operator, we have given a more general fractional order difference operator. Also, we have defined a class of difference sequence spaces with the help of $l$- fractional difference operator. Further, we have studied some topological properties and determined the duals of the spaces.

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