A quantum router for high-dimensional entanglement

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Abstract

In addition to being a workhorse for modern quantum technologies, entanglement plays a key role in fundamental tests of quantum mechanics. The entanglement of photons in multiple levels, or dimensions, explores the limits of how large an entangled state can be, while also greatly expanding its applications in quantum information. Here we show how a high-dimensional quantum state of two photons entangled in their orbital angular momentum can be split into two entangled states with a smaller dimensionality structure. Our work demonstrates that entanglement is a quantum property that can be subdivided into spatially separated parts. In addition, our technique has vast potential applications in quantum as well as classical communication systems.

Considered one of the most seminal experiments in quantum physics, the Stern–Gerlach experiment provided the first direct evidence of quantization in quantum mechanics [1]. It is now well known that in addition to their spin, photons can also carry quantised values of orbital angular momentum (OAM) [2]. Unlike spin, the OAM of a photon can take on a large range of discrete values. In direct analogy to the Stern–Gerlach experiment with atoms, recent demonstrations have shown that single photons carrying OAM can be spatially separated into directions corresponding to discrete linear momenta [3–6]. The OAM of light also enabled the first laboratory studies of high-dimensional entanglement [7, 8], with recent experiments demonstrating entangled states of ever-increasing dimensionality [9–11]. In this letter, we show how a quantum state composed of many entangled modes can be split into two smaller entangled states with exactly opposite parity. The original Stern–Gerlach experiment demonstrated that a single particle has discrete quantum properties that can be physically separated. Similarly, we show that entanglement, which is a property unique to quantum systems of two or more particles, can also be separated into two distinct parts.

The OAM of light is a physical property that appears due to a spatially varying field distribution [2]. Photons carrying OAM have a helical wavefront where the phase, given by \( \exp ( i \ell a ) \), winds azimuthally around the optical axis. The number of twists \( \ell \) in a \( 2\pi \) period indicate the amount of OAM carried by one photon in units of \( \hbar \) [12]. The OAM state space is discrete and unbounded, which makes it particularly suitable as an information carrier in both quantum and classical communication [13]. Photons carrying OAM have been used to push the capacity of a quantum key distribution system beyond one bit per photon [14] and for reaching dazzling data rates in classical communication systems [15]. Photons entangled in their OAM have enabled superdense quantum communication [16] and were recently used to teleport a photon in a hybrid polarization-OAM space [17].

Here we demonstrate how a high-dimensional OAM-entangled state \( | \psi \rangle \) can be split into two smaller entangled states \( | \pi \rangle \) and \( | \alpha \rangle \) of different parity through the use of a high-dimensional entanglement router (figure 1). This device uses an interferometric sorting method to distribute a high-dimensional entangled state originally shared between two spatially separated parties to three such parties. We quantify the entanglement dimensionality of these states and demonstrate that the high-dimensional non-classical correlations do indeed survive the process of subdivision. Furthermore, we demonstrate precise and stable control over the routing process for an extended period of time and show how the two entangled states can be exchanged between their respective parties on-demand and extremely fast. Besides being of use for fundamental tests, the entanglement
router has immense potential applications in high-dimensional quantum communication systems [14, 18], as a quantum state sorter for example. Furthermore, this device can be readily used as an OAM switch in classical communication systems that exploit the spatial modes of light [15, 19, 20].

The design of the high-dimensional entanglement router is based on an interferometer that was initially developed to sort single photons carrying OAM [3]. This device consists of a modified Mach–Zehnder interferometer with a dove prism in each arm, rotated at 90° with respect to each other. The effect of a dove prism oriented at angle α is to rotate an incoming photon by an angle 2α. This introduces an OAM-dependent phase on the photon such that

$$\exp(2i\alpha)$$

(note that a single reflection through the prism flips the OAM sign). Thus, a dove prism oriented at 90° adds a phase of $\pi$ (modulo 2π) to photons carrying odd OAM and a phase of 0 to ones with even OAM. Sending an OAM-carrying photon through such an interferometer results in destructive or constructive interference (out of one port), depending on whether the photon was carrying odd or even values of $\ell$. In this manner, the interferometer sorts input photons based on the parity of their OAM quantum number. By changing the relative path length of the interferometer by $\pi$, the sorting direction can be changed.

Consider an OAM-entangled 11-dimensional two-photon state, initially shared between two parties, Alice and Bob:

$$|\psi\rangle_{AB} = \sum_{\ell=-5}^{5} c(\ell) |\ell\rangle_A |-\ell\rangle_B.$$  \hspace{1cm} (1)

Here, $c(\ell)$ are complex probability amplitudes describing the state. The effect of sending one photon from this entangled state through the high-dimensional router is as follows:

$$|\psi\rangle_{AB}^{\text{HDR}} \rightarrow \sum_{\ell=-5}^{5} c(\ell) |\ell\rangle_A [-\ell\rangle_B' + |-\ell\rangle_C']$$

$$= \sum_{\ell=-2}^{2} c(2\ell) |2\ell\rangle_A |-2\ell\rangle_B$$

$$+ \sum_{\ell=-3}^{3} c(2\ell + 1) |2\ell + 1\rangle_A |2\ell - 1\rangle_C$$

$$= |\alpha\rangle_{AB} + |\pi\rangle_{AC}.$$  \hspace{1cm} (2)

The superscripts e and o represent even and odd modes respectively. Thus, the 11-dimensional state $|\psi\rangle_{AB}$ is split into a five-dimensional entangled state $|\alpha\rangle_{AB}$ comprised of even OAM quanta $\ell \in \{-4, -2, 0, 2, 4\}$, and a six-dimensional entangled state $|\pi\rangle_{AC}$ comprised of odd OAM quanta $\ell \in \{-5, -3, -1, 1, 3, 5\}$.

First, we generate the high-dimensionally entangled state $|\psi\rangle_{AB}$ using the experimental setup depicted in figure 2(a). This consists of a femtosecond pulsed laser operating at 404 nm that is used to pump a 1 mm thick, periodically poled potassium titanyl phosphate (ppKTP) crystal. The laser is focused into the crystal with a beam waist of 240 μm. This ensures that the distribution of OAM values is very broad and that our state contains sufficient higher-order modes. Entangled photon pairs at 808 nm are generated via a type-II collinear spontaneous parametric downconversion process, and are separated from the pump by a dichroic mirror. A polarizing beam splitter separates the two photons, directing each to a detection system consisting of a spatial
light modulator (SLM), a single mode fiber (SMF) and a single photon detector with \( \approx 65\% \) detection efficiency. These devices are used in combination to perform projective measurements of OAM modes and their superpositions. The SLM flattens the phase of an incoming mode, allowing it to couple efficiently into the SMF. A home-built electronic circuit records coincidence counts between the two detectors. This allows us to make suitable measurements on the generated two-photon state and verify its entanglement dimensionality. With this experimental configuration we measure a total rate of state detection of approximately 30,000 per second. This in combination with the total efficiency of \( \eta_{\text{total}} \approx 33\% \) leads to an estimated total state production of 273,000 per second, see appendix for details.

The high-dimensional entanglement router (shown in the green square in figure 2(b)) is inserted into arm B, effectively splitting it into two paths B and C. Instead of the original Mach–Zehnder design [3], the HD router is implemented in a folded, double-path Sagnac configuration. The input beam is split into two counter-propagating Sagnac loops that meet back up the beam splitter. Since these two loops use the same mirrors and follow a similar path through the interferometer, path length fluctuations due to mechanical stresses or vibrations are effectively cancelled out. Each loop contains a dove prism (DP1 and 2). One of the mirrors of the interferometer is connected to a computer-controlled piezo actuator (P) that allows fine control over the alignment. This feature is also used for rapid automatic realignment of the interferometer in order to compensate for temperature drifts. The piezo also allows us to rapidly switch the routing direction (even/odd). The entire HD router is enclosed in a plastic box which reduces air flow and provides temperature stability to within \( \pm 0.02 \) °C. This configuration provides a very high alignment stability and allows the router to operate over several days, as is shown later.

The results of measurements performed in the OAM basis, both before and after routing, are shown in figure 3. As can be seen in figure 3(a), the two-photon state is strongly anti-correlated in OAM. After passing through the HD router, the state is split into two smaller states containing only even and odd OAM terms (figure 3) that are still strongly anti-correlated in OAM. Note that these matrices only show all the diagonal elements of each state, which are insufficient to show entanglement. In order to prove that these states are indeed high-dimensionally entangled, we use an entanglement witness that puts a bound on the entanglement dimensionality of the state [21]. First, we calculate the fidelity \( F_{\text{exp}} = \text{Tr}(\rho |\chi\rangle \langle \chi|) \) between the experimentally measured state \( \rho \) and the theoretically expected state \( |\chi\rangle = \sum_{\ell=-L}^{L} c_{\ell}|\ell\rangle \) which is \( d \)-dimensionally entangled \( (d = 2L + 1) \). Then, we calculate the maximal possible fidelity between a \( (d - 1) \)-dimensionally entangled state \( |\phi_{d-1}\rangle \) with the assumed \( d \)-dimensionally entangled state \( |\chi\rangle \) to obtain an entanglement bound.
which are verified to have dimensionalities of 5 and 6 each. While we only show elements of the HD router into two smaller states such that the state \( \psi_{AB} \) that measures the distance between the experimental fidelity \( F_{\exp} \) and the entanglement bound \( B_{d-1} \) always leaves out the smallest Schmidt coefficient of \( \chi \), which is directly connected to its smallest probability amplitude. Thus, as the state gets more asymmetric, the entanglement bound gets closer to one. In order to optimise this, we define a parameter \( \xi \left( \{ c_i \} \right) = F_{\exp} \left( \{ c_i \} \right) - B_{d-1} \left( \{ c_i \} \right) \) that measures the distance between the experimental fidelity \( F_{\exp} \) and the entanglement bound \( B_{d-1} \). Note that both the experimental and the maximal fidelity depend on the choice of the coefficients \( \{ c_i \} \). Thus, by maximizing \( \xi \left( \{ c_i \} \right) \) over all \( \{ c_i \} \), we find the minimal entanglement dimensionality \( d \) of the experimentally measured state \( \rho \). Table 1 shows the measured fidelity and entanglement dimensionality of the states \( \psi_{AB}, \alpha_{AB}, \) and \( \pi_{AC} \). In order to calculate the fidelity of each state, it is sufficient to measure its density matrix elements that are expected to be non-zero. In the case of \( \psi_{AB} \), there are 121 such elements. Details on how to reconstruct these elements can be found in [111]. These elements are plotted in figure 4. In order to illustrate the sorting process, the density matrix plots of states \( \alpha_{AB} \) and \( \pi_{AC} \) also show the odd and even parity elements that are expected to be zero in dark blue. From these measurements, we are able to show that the state \( \psi_{AB} \) before the router has an entanglement dimensionality of 10. The state is then split by the HD router into two smaller states \( \alpha_{AB} \) and \( \pi_{AC} \), which are verified to have dimensionalities of 5 and 6 each. While we only show elements of \( \psi_{AB} \) up to \( \ell' = \pm 5 \), we needed to take the next higher order modes of \( \ell' = \pm 6 \) into account in order to achieve a dimensionality of 10. These results give rise to an interesting question: how can a 10 dimensionally entangled state be split up into a five and six-dimensionally entangled state? The answer is that the state \( \psi_{AB} \) is very close to the bound of being 11 dimensionally entangled. The HD router acts like a filter for even/odd modes, which results in a lower crosstalk between these modes after the routing process. This increases the visibility of the density matrix measurements and results in a higher quality state at both \( AB \) and \( AC \).

Finally, we perform long term measurements of the fidelity of states \( \alpha_{AB} \) and \( \pi_{AC} \) over a continuous period of 39 h (figure 5(a)). During this time, both states have a fidelity that is well above the entanglement

\[ B_{d-1} = \max_{\{ c_i \}} |\langle \phi_{d-1} | \chi \rangle|^2 = \sum_{i=1}^{d-1} c_i^2, \] for a detailed proof see [21]. This is the sum of the squares of all but the smallest Schmidt coefficient \( c_i \) of the target state \( |\chi\rangle \). Note that here the Schmidt coefficients are sorted such that \( c_1 \geq c_2 \geq \cdots \geq c_i \). If the calculated experimental fidelity exceeds this bound \( F_{\exp} > B_{d-1} \) for a \((d-1)\)-dimensional entangled state, then the measured correlations can only be explained with a \( d \)-dimensionally entangled state.

As is seen in figure 3, the probability distribution of OAM modes is far from flat. Thus, it is advantageous to find a target state \( |\chi\rangle \) that is close to the observed distribution, since this maximises the experimental fidelity \( F_{\exp} \). However, the odd and even terms are deterministically separated after the routing process. This increases the visibility of the density matrix measurements and results in a higher quality state at both \( AB \) and \( AC \).

\[ \text{Table 1. Measured fidelity} \ F_{\exp} \text{and estimated entanglement bounds} \ B(d). \text{Errors are calculated using Monte Carlo simulation with Poissonian distribution of counting statistics.} \]

| State           | \( F_{\exp} \)   | Bound \( B(d) \) | Dimensionality \( d \) |
|-----------------|------------------|-----------------|------------------------|
| \( |\psi\rangle_{AB} \) | 0.757 ± 0.002    | 0.745            | 10                     |
| \( |\alpha\rangle_{AB} \) | 0.937 ± 0.004    | 0.843            | 5                      |
| \( |\pi\rangle_{AC} \) | 0.956 ± 0.004    | 0.898            | 6                      |

Figure 3. OAM correlations between the two photons measured before and after the routing process. The normalised coincidence counts between detectors \( A + B \) and \( A + C \) as a function of measured OAM mode are shown. As can be seen, the states are all strongly anti-correlated in OAM. However, the odd and even terms are deterministically separated after the routing process.
bound for their respective entanglement dimensionality. The HD router is calibrated every 26 min with the piezo crystal to ensure a high sorting visibility. The first fidelity measurements were taken to estimate the optimal target state $|\psi_{AB}\rangle$, which was then kept the same for every successive measurement. Even though the HD router is continuously re-calibrated, the experimental fidelities decrease constantly. The total count rates are also seen to decrease steadily, which points to grey tracking of the ppKTP crystal as a possible source of error. Grey tracking would affect the mode quality of higher order OAM modes and would thus also lower the experimental fidelity over time. Figure 5(b) demonstrates the switching capability of the HD router. By changing the relative path length difference of the interferometer with the piezo crystal, it is possible to choose whether Bob or Carol receive even or odd modes and thus the $|\alpha\rangle$ or the $|\pi\rangle$ state. The coincidence counts between $AB$ and $AC$ are

Figure 4. Measured density matrix elements (absolute values) showing how the entanglement router splits the state $|\psi_{AB}\rangle$ into odd and even states $|\alpha_{AB}\rangle$ and $|\pi_{AC}\rangle$. Note that only the expected non-zero density matrix elements are necessary for calculating the fidelity and are measured. The elements of $|\alpha_{AB}\rangle$ and $|\pi_{AC}\rangle$ that are not measured are depicted in dark blue to illustrate the routing process.

Figure 5. (a) Long-term measurement of the experimental fidelity of $|\pi\rangle$ and $|\alpha\rangle$ demonstrating the presence of six and five-dimensional entanglement respectively, over a period of 39 h. The dotted lines represent the dimensionality bound. The error (shown in gray) is calculated via a Monte Carlo simulation of the experiment. (b) Rapid switching of state $|\alpha\rangle$ between parties $B$ and $C$ at a frequency of 5 Hz. Coincidence counts between detectors $A + B$ (red) and $A + C$ (blue) are displayed, with all detectors measuring an OAM superposition $|0\rangle + |2\rangle$.
shown with an OAM superposition of $|0\rangle + |2\rangle$ being measured at all three detectors. The piezo crystal is programmed such that it switches back and forth between two voltages $(V_1 = 31.5 \, V, \, V_2 = 51.4 \, V)$ with a switching frequency of 5 Hz, which is mainly limited by our count rates. The average visibility of the switching process is $0.97 \pm 0.02$. In principle, one could achieve extremely fast switching frequencies by using a Pockels cell instead of a piezo crystal to introduce a phase shift.

In summary, we have reported the development of a high-dimensional entanglement router that uses an interferometric method to split a high-dimensional entangled state into two smaller states, while maintaining the high-dimensional non-classical correlations that were present originally. This serves as an analog of the Stern–Gerlach effect applied to the quantum property of entanglement. We use the entanglement router to divide a two-photon state entangled in ten-dimensions of its OAM into two states entangled in five and six-dimensions of their OAM. Additionally, we demonstrate stable operation of this device over several days and show how it can rapidly switch the routing direction. The entanglement router will find extensive application in quantum networks relying on high-dimensional entanglement, as well as in high-capacity classical communication links that exploit the OAM of light.

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Appendix

The total efficiency of our setup can be estimated as follows. The SLMs have a diffraction efficiency of $\approx 85\%$. The coupling efficiency into the single mode fiber is estimated to be $\approx 60\%$. The efficiency of the single photon detectors (Excelitas) is $\approx 65\%$. The overall efficiency is thus estimated to be $\eta_{\text{total}} \approx 33\%$. This then leads to an estimated total rate of state production of $\eta^2_{\text{total}} \times \text{rate of state detection} \approx 273,000 \, \text{per second}$. 

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