Equivalent Performance Analysis of Thermal-Mechanical of 3D Lattice Structure

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Abstract. In order to achieve the efficient and accurate thermal stress analysis of 3D lattice structure, basing on the principles of statics and thermodynamics, the elastic modulus and thermal conduction equivalent calculation formula of the lattice structure are deduced. The theoretical relationship between the relative elastic modulus, relative thermal conductivity and relative density of equivalent cells is obtained. Finally, the numerical simulation method is used to compare the simulation results and experimental results of the equivalent model and the meticulous model, and the efficiency and accuracy of the equivalent model mechanical analysis and heat conduction analysis are proved. On this premise, designers can quickly determine the size and thermodynamic properties of lattice cells when using a multi-layer pyramid structure. And this research lays the foundation for the rapid analysis of structural thermo-mechanical coupling.

1. Introduction
3D Lattice Structure is composed of lattice cells arranged periodically in three directions. It has attracted attention in the fields of aerospace, medical devices and robots, because of its excellent mechanical properties such as lightweight, high porosity, superior specific strength and specific stiffness [1], as well as special properties such as sound absorption, heat dissipation, shock absorption and electromagnetic shielding.

Multilayer pyramid structure is a kind of 3D lattice structure, which is composed of body-centered cubic (BCC) cells arranged orderly. The BCC cell structure is shown in figure 1. This structure has been widely studied and applied because of its simple configuration, large internal space, symmetrical cell and bending dominated strut [2] which can meet the requirements of lightweight, functional, load-bearing and thermal protection of spacecraft structure [3].

Figure 1. Schematic diagram of BCC type lattice cell.
However, lattice cells are very small, and the mechanical structure usually contains thousands of cells. The traditional finite element method is used to build a precise thermodynamic model of lattice structure, which needs to be divided into many elements, resulting in a serious increase of calculation time and workload of designers, and unable to provide timely design guidance. In view of the above problems, scholars use representative volume element (RVE) to predict the equivalent performance of the structure according to the periodic arrangement characteristics of lattice cells and the influence of cell equivalent density on the thermal-mechanical performance. At present, it is only limited to the separate research of heat transfer performance or structural performance. In the real application scenario, the separate performance research can no longer meet the increasing demand of rapid design of structural thermal-mechanical integration.

Therefore, in this paper, the theoretical derivation and finite element method are used to find the relationship between the thermal-mechanical performance parameters and the relative density of lattice cells, and through the representative volume element method, the lattice cells are simplified as a solid element to carry out the finite element simulation calculation, and finally the relationship between the relative density, the relative elastic modulus and the relative thermal conductivity of lattice cells is obtained.

2. The Relationship between Relative Density, Elastic Modulus and Thermal Conductivity

2.1. The Relationship between the Equivalent Elastic Modulus and the Relative Density of BCC Cell

Figure 1 shows the spatial structure of BCC cell. The cell is composed of eight struts with length of \( l \), diameter of \( d \), angle between the struts and the horizontal plane is \( \theta \). The space occupied by the cell is cube, and side length is \( h \). Therefore, the relative density of cell can be calculated according to the ratio of the volume of struts \( V_s \) to the volume of solid space occupied by cell \( V_c \).

So the relative density of cell can be expressed as

\[
\bar{\rho} = \frac{V_s}{V_c} = \frac{3\sqrt{3}\pi}{4} \left( \frac{d}{l} \right)^2
\]  

According to the analysis of the elastic constitutive relation of lattice structure by Fang [4], the elastic modulus of BCC cell is deduced as follows.

Figure 2 shows the deformation of structure when the cell under loading of an axial stress \( \sigma_z \) in Z direction. We can take a strut EC for analysis because the structure is symmetrical in the x, y, z direction, and the stress of each strut is the same. The strut can be regarded as a cantilever beam. The C-end is subject to tensile, shear and bending forces, in which the axial pressure is \( N \), the shear force perpendicular to the member direction is \( F \), and the bending moment is \( M \).

![Figure 2. Cell deformation analysis.](image)

Assuming that the axial force in Z direction of each strut is \( P \), we have

\[
P = \frac{\sigma_z h^2}{4}
\]
The component forces $N$ and $F$ of load $P$ can be obtained. If point E is the origin of x-axis, the direction EC is the positive direction of x-axis, and the moment at each node of cell is the same, then the moment equation of the strut is

$$M(x) = \frac{1}{2} Fl - Fx$$

(3)

According to the energy method, the deformation of a single cell $\Delta h$ is calculated, and the torsion force is ignored in the deformation of the member. Therefore, the deformation energy $U$ is composed of the tensile deformation energy $U_F$ and the bending deformation energy $U_M$, as following

$$U = 8(U_F + U_M) = 8\frac{N^2l}{2EsA} + \int_0^l \frac{M(x)^2I}{2EsI} \, dx$$

(4)

where $A$ is the cross-sectional area of the strut, $A = \pi \left(\frac{d}{2}\right)^2$, $Es$ is the material Elastic modulus, $I$ is the section moment of inertia of the strut, $I = \pi \frac{d^4}{64}$.

External work can be expressed as

$$W = \frac{1}{2} 4P\Delta h$$

(5)

According to the unit load method, $U = W$, we have

$$\Delta h = \frac{2\sigma_x h^2l(4\cos^2\theta l^2 + 3\sin^2\theta d^2)}{3E\pi d^4}$$

(6)

The elastic modulus of equivalent element in Z direction is

$$E_z^* = \frac{\sigma_z}{\varepsilon_z} = \frac{\sigma_x h}{\Delta h} = E_s\frac{d^2}{2\pi (2 + 3\sin^2\theta)\rho^2}$$

(7)

The cell lattice is square, the single cell structure equivalent compressive modulus of elasticity in the three directions is the same, relative elastic modulus as following

$$\bar{E} = E^*/E_s = \frac{1}{2\sqrt{3\pi + \rho}}$$

(8)

2.2. The Relationship between Equivalent Thermal Conductivity and Relative Density of BCC Cell

Lattice cells are usually in a closed space, and usually maintain a certain temperature in the mechanical structure. Therefore, the convection heat transfer and heat radiation in the cells are ignored, and only heat conduction is considered. According to Fourier’s law, we have

$$\Phi = -\lambda_x \times A \frac{dT}{dx}$$

(9)

where $\Phi$ is the thermal conductivity of a given section with an area of $A$ in unit time, $\lambda_x$ is the thermal conductivity of the material, reflecting the thermal conductivity of the material, $dT$ is the temperature difference perpendicular to the section.

The temperature of the upper and lower interface of the cell is $T_1$ and $T_2$, respectively. As shown in figure 3, there are four heat conduction paths along the FAE, FBE, FCE and FDE. The size of the heat transfer area has a great influence on the thermal conductivity. In this paper, the calculation of the area is not simplified, so the equivalent analysis model of heat conduction established by Deng [5] is improved.
Therefore, under the current temperature difference, the thermal conductivity $\Phi_{\text{BCC}}$ of lattice cell is

$$\Phi_{\text{BCC}} = \lambda_s \sqrt{3} \pi d^2 \frac{T_1 - T_2}{2l}$$

(10)

where $\lambda_s$ is the thermal conductivity of the material used in the struts. The heat conduction $\Phi_{\text{RVE}}$ of the solid space occupied by the cell is

$$\Phi_{\text{RVE}} = \lambda^* h^2 \frac{T_1 - T_2}{h}$$

(11)

where $\lambda^*$ is the thermal conductivity of the equivalent cell.

When $\Phi_{\text{BCC}} = \Phi_{\text{RVE}}$, we have an expression for $\lambda^*$, it can be seen that the thermal conductivity of equivalent structure is related to the relative density of cell and the thermal conductivity of material. Then the relative thermal conductivity $\tilde{\lambda}$ can be expressed as

$$\tilde{\lambda} = \frac{\lambda^*}{\lambda_s} = \frac{1}{3\sqrt{3} \bar{\rho}}$$

(12)

Combining equations (8) and (12), the relationship among $\bar{E}$, $\tilde{\lambda}$ and $\bar{\rho}$ are shown as figure 4.

![Figure 3. Heat transfer model of BCC cell.](image)

3. Finite Element Simulation Analysis

3.1. Finite Element Model

The detailed and equivalent BCC lattice cell analytical models were established using ABAQUS, and the cell size is $5 \times 5 \times 5\text{mm}$. The element size of the detailed cell finite element model is 0.1mm, and 17280 8-node hexahedral elements are divided, including 19718 nodes. The equivalent finite element model replaces the corresponding detailed model with one 8-node hexahedral solid element.

When simulating quasi-static compression experiment, two rigid structures with two reference points (RP1 and RP2) were built to simulate the compression head and the bottom support of the testing machine. Friction constraints are established between the compression head and the bottom support and the cell respectively, and the friction coefficient is 0.4 [6]. As shown in figure 5, symmetric boundary conditions were established. RP2 is set as encastré. In order to ensure that the structure is in
the linear elastic stage, a vertical downward displacement of 0.25 mm is applied to RP$_1$ according to the experimental results [6].

![Meticulous model](image1) ![Relative model](image2)

**Figure 5.** Schematic diagram and mechanical constraints of the two models.

![Meticulous model](image3) ![Relative model](image4)

**Figure 6.** Temperature boundary of models.

In the heat conduction analysis, both models are divided by 8-node hexahedral heat transfer unit DC3D8. According to the data in Table 2, the equivalent model and the original model of lattice cell heat conduction are established. Heat is the product of heat flow density and action area. In the equivalent calculation, if the input heat load is the surface heat flux, in order to ensure that the heat input of the equivalent model and the non-equivalent model are the same, the input value of heat flux should be inversely proportional to the action area. Then the relationship between the two models is as follows:

$$\frac{q_R}{q_o} = \frac{S_o}{S_R} = \frac{1}{\bar{\rho}}$$

(13)

where $S_o$ and $S_R$ are the upper surface area of the original lattice model and the equivalent model respectively, $q_o$ and $q_R$ are the heat flux input by the lattice model and the equivalent model respectively, and the heat flux ratio is related to the equivalent density $\bar{\rho}$. As shown in figure 6, the heating flux of the upper surface of the lattice model is 50 mW/mm$^2$, the temperature boundary of the lower surface of the model is 20 °C, the overall initial temperature is 20 °C, and other surfaces have no load constraints.

The material of lattice model is Ti64, and the performance parameters are obtained from the experimental data [6, 7], as shown in table 1.

| Density $\rho$ (T/mm$^3$) | Elastic modulus $E$ (MPa) | Poisson’s ratio $\nu$ | Conductivity $\lambda_s$ (mW/mmK) | Specific heat $C$ (mJ/t°C) |
|--------------------------|--------------------------|-----------------------|----------------------------------|---------------------------|
| 4.43e-9                  | 118000                   | 0.3                   | 6.8                              | 6.11e8                    |
From section 2.2, it is known that the elastic modulus and thermal conductivity of BCC cell are related to the relative density, which can be expressed by the aspect ratio \( l/d \). The theoretical values of elastic modulus, thermal conductivity and equivalent density are shown in table 2 when \( l \) is constant and only the diameter \( d \) is changed.

### Table 2. Relationship between cell relative density \( \bar{\rho} \), elastic modulus \( E^* \), thermal conductivity \( \lambda^* \) and equivalent density \( \rho^* \) and the member’s \( l/d \).

| \( l/d \) | 4     | 5     | 6     | 7     | 8     | 9     | 10    | 11    | 12    |
|---------|-------|-------|-------|-------|-------|-------|-------|-------|-------|
| \( \bar{\rho} \)      | 0.255 | 0.163 | 0.113 | 0.083 | 0.064 | 0.050 | 0.041 | 0.034 | 0.028 |
| \( E^* \) (MPa)      | 689.26| 283.83| 138.00| 74.67 | 43.88 | 26.98 | 18.16 | 12.30 | 8.69  |
| \( \lambda^* \) (mW/mmK) | 0.578 | 0.370 | 0.256 | 0.188 | 0.145 | 0.113 | 0.093 | 0.077 | 0.063 |
| \( \rho^* \) \((10^{-10} \text{g/mm}^3)\) | 11.3  | 7.72  | 5.01  | 3.68  | 2.84  | 2.22  | 1.82  | 1.51  | 1.24  |

### 3.2. The Relationship between Relative Elastic Modulus and Relative Density of Cell

The nominal stress-strain curve of the cell can be calculated. In figure 7, when the cell relative density \( \bar{\rho} \) is 0.163, 0.064 and 0.028, the theoretical value of elastic modulus is compared with the simulation value calculated by the detailed model, and it’s found that the two values are very close. According to the curves of simulation results, eight kinds of lattice cell elastic modulus with different equivalent densities are fitted, and the fitting factors are all bigger than 0.999. Sum up the theoretical value \( E^* \) of elastic modulus, simulation value \( E_F \) and draw the curve in figure 8.

**Figure 7.** Comparison of \( E^* \) with \( E_F \).

**Figure 8.** Relationship between \( E^* \), \( E_F \) and \( \bar{\rho} \).

It can be seen from figure 8 that the simulation values of lattice cell elastic modulus are smaller than the theoretical values, and the error is 3%-9% of the theoretical values. The elastic modulus of lattice cell is directly proportional to \( \bar{\rho} \), and the smaller the \( \bar{\rho} \) is, the smaller the error between theoretical value and simulation value is.

The equivalent model is used to replace the lattice cell, and the material parameters are set according to table 2. Using the simulation results \( E^k \) to fit the elastic modulus of the equivalent model, four groups of data with \( \bar{\rho} \) of 0.225, 0.113, 0.064 and 0.041 are selected, which are compared with the theoretical value \( E^* \) of the elastic modulus and the experimental data [6] \( S_1 \) and \( S_2 \), and the results are summarized in table 3.
Table 3. Comparison of numerical analysis of equivalent elastic modulus of test pieces and experimental values.

| $\bar{\rho}$ | $E^*$ | $S_1$ [6] | $S_2$ [6] | $E^R$ |
|-------------|-------|-----------|-----------|-------|
| 0.255       | 689.26| 840.15    | 758.48    | 691.896 |
| 0.113       | 138.00| 146.77    | 143.42    | 138.818 |
| 0.064       | 43.88 | 60.65     | 51.13     | 44.141  |
| 0.041       | 18.16 | 24.20     | 30.82     | 18.266  |

According to the results, the theoretical value $E^*$ of lattice structure is almost the same as the simulation value $E^R$ of equivalent model. The increase of $E^R$ to $E^*$ is a fixed value of 0.6%, and the error comes from the problem of simulation parameters. The experimental values of the two groups are greater than the theoretical values, which are caused by the processing technology, forming defects and experimental errors of lattice structure.

4. Conclusions

According to the characteristics of load-carrying and heat transfer of lattice structure, a simple equivalent analysis method of lattice structure is proposed in this paper. The theoretical formula is obtained and verified by numerical simulation, which provides guidance for efficient analysis of lattice structure.

(1) The relationship between the relative density, the relative modulus of elasticity and the relative thermal conductivity of BCC pyramidal lattice cell is studied, and the theoretical analysis is carried out, which provides a rapid guidance for designers;

(2) By using the numerical simulation method, the relationship between the relative density, elastic modulus and the relative thermal conductivity of BCC pyramidal lattice cell is studied comparatively. The results show that the conclusions of numerical simulation and theoretical derivation are consistent;

(3) Using a single solid element to establish the equivalent model, instead of the complex mesh of the detailed model, achieved similar accuracy, which provides help for reasonably simplifying the design model and speeding up the design speed.

References

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