A nuclear-style mean field model for the Sivers effect in nucleon-hadron single spin asymmetries and Drell-Yan.

A. Bianconi

Dipartimento di Chimica e Fisica per l’Ingegneria e per i Materiali, Università di Brescia, I-25123 Brescia, Italy, and
Istituto Nazionale di Fisica Nucleare, Sezione di Pavia, I-27100 Pavia, Italy

Abstract

I study the effect of scalar and spin-orbit absorption potentials, in the production of a nonzero Sivers-like asymmetry in hadron-hadron high energy collisions (Drell-Yan and single spin asymmetries). A basic model is built for the intrinsic state of a quark in the projectile hadron. S-wave and P-wave 2-component states are considered. Before the hard event, this quark is subject to absorbing mean fields simulating interactions with a composite target. The relevant interaction terms are found to be the imaginary diagonal spin-orbit ones. Spin rotating terms, and scalar absorption, seem not to be decisive. For $x = 0$ the found Sivers asymmetry vanishes, while at larger $x$ its qualitative dependence on $x$, $K_T$ follows the usual trends met in available models and parameterizations. Given the present-day knowledge of the considered phenomenological interactions, it is not possible to establish whether the related Sivers-like asymmetry is a leading twist-one.

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Email address: andrea.bianconi@bs.infn.it (A. Bianconi).
1 Introduction

1.1 General background

The problem of the study and measurement of T-odd distributions in hadron-hadron scattering has recently acquired a certain relevance and quite a few related experiments have been thought or scheduled for the next ten years [1, 2, 3, 4, 5].

In particular, several studies and models have been proposed for the Sivers distribution function [6]. Its possible existence as a leading-twist distribution was demonstrated [7, 8, 9, 10] recently, and related [11] to previously studied T-odd mechanisms [12, 13]. Some phenomenological forms for its dependence on $x$ and $K_T$ have been extracted [14, 15, 16, 17] from available data [18, 19, 20, 21].

While studies of general properties [22, 23, 24, 25, 26, 27, 28] of T-odd functions relate these functions with a wide spectrum of phenomena, quantitative models mostly follow the general scheme suggested in [7]. A known quark-diquark spectator model [29] is extended by including single particle (meson or gluon) exchange [30, 31, 32, 33]. In the case of [34] the unperturbed starting model was a Bag model.

1.2 This work

The class of processes I want to consider here is the one of single spin asymmetries in collisions between an unpolarized hadron and a transversely polarized proton. In particular, azimuthal asymmetries in Drell-Yan dilepton production and hadron semi-inclusive production, where in both cases one of the colliding hadrons is normal-polarized. I consider phenomena that may be present in the beam energy range 10–300 GeV (so, not necessarily leading twist effects).

The present work uses phenomenological schemes that are not typical of perturbative QCD, but rather of high-energy nuclear physics. It is inspired by previous works on T-odd structure functions in high-energy nuclear physics [35, 36], by a previous work on nuclear-target induced polarization in Drell-Yan [37], and by the results from the theory and phenomenology of spin-orbit interactions in high-energy hadron-hadron exclusive processes (see e.g. [38, 39, 40] and references therein).

The goal is not to reproduce precisely some phenomenology. Rather, it is to establish whether scalar and spin-orbit interactions dominated by their absorption part are able to build a nonzero Sivers-like asymmetry with a reasonable shape, possibly of higher-twist nature.
For this reason, the model for both the initial “intrinsic” state of a quark in the proton, and for initial state interactions is built in such a way to be as simple as possible. All the necessary functions have been chosen in Gaussian form and the parameter number has been reduced to the minimum necessary to explore the interesting independent physical situations.

1.3 The general scheme

Contrary to the ordinary treatment of the problem, where one works on a two-point correlation operator deriving from a set of squared one-point amplitudes, I develop most of the work at the level of one-point amplitudes, square them and then sum over the relevant states. I imagine, for a hadron with a given spin projection \( S_y = +1/2 \), a two-component quark spinor \( (f_+, f_-) \), with \( f_\pm \) associated with the transverse quark spin. In this scheme, the \( \gamma_+ \)-trace normally calculated on the correlation operator simply corresponds to the sum \( |f_+|^2 + |f_-|^2 \). This quantity is the final goal of the calculation, and the distribution functions associated to an unpolarized quark, including Sivers’ one, are extracted from it.

Concerning the initial, “intrinsic” state of a quark in the hadron, I assume that the relevant quantity defining this state is the quark total angular momentum \( \vec{J} \) in the hadron rest frame. In this state a nonzero correlation \( \langle \vec{S}_{\text{hadron}} \cdot \vec{J}_{\text{quark}} \rangle \sim +1/4 \) is present. In other words, the quark \( \vec{J} \) coincides with the parent hadron spin. This may be realized both in S-wave and in P-wave, with spin-spin correlation \( \langle \vec{S}_{\text{hadron}} \cdot \vec{S}_{\text{quark}} \rangle \sim \pm 1/4 \). A nonzero correlation between the hadron and the quark angular momentum is necessary in any model, since a spin-related effect is impossible if a quark transports no information on the parent hadron spin. Clearly, we have different effects in the S and P-wave cases.

This correlation alone would not produce a single-spin asymmetry of naive Time-odd origin because of global invariance rules. Initial state interactions between the two hadrons, or between the quarks of one hadron and the quarks of the other one, must be introduced\[7,8\].

I reproduce these interactions in eikonal approximation \( \exp(\oint \vec{T} d\xi) \), where \( \vec{T} \) is a 2x2 space-time dependent matrix reproducing a mean field acting on the projectile quark, and \( \xi \) is a light-cone coordinate. Then the full matrix element, affected by this operator, is calculated in space-time representation.

Although it may seem more natural to adopt a mean field treatment for problems where a projectile is subject to multiple soft scattering\[42\], there is also...
a tradition\textsuperscript{1} for such recipes in hadronic problems dominated by a single or at most double hard scattering event, when the scatterer belongs to a composite structure.

The way it is used here, the above eikonal approximation is only a short-wave approximation, that alone does not support the persistence of initial state interaction effects at very large energies. This derives from the fact that we do not know the asymptotic properties of the operator $\hat{T}$ (see below). In addition, the above eikonal approximation does not support automatically factorization, since it is applied at single point amplitude level.

### 1.4 Anti-hermitean initial state interactions

The added initial state interactions (the $\hat{T}$ matrix) consist of two terms: (i) anti-hermitean scalar mean field, (ii) anti-hermitean spin-orbit mean field\textsuperscript{2}

Hermitean terms have been tested and they affect the results. Alone, these terms do not produce T-odd distributions (by definition, since they are intrinsically T-even) and do not change the main qualitative features of the presented results. Not to overload this work with a many-parameter phenomenology, I have limited myself to terms that are not intrinsically T-even.

A remark is important: strong and electromagnetic interactions are T-even and hermitean. As well known in nuclear physics\textsuperscript{12}, relevant anti-hermitean terms originate in the projection of hermitean interactions on a subspace, including only a part of all those degrees of freedom that are able to exchange energy/momentum within a characteristic interaction time relevant for the problem. If one were able to include all the relevant degrees of freedom in the formalism, there would be no room for anti-hermitean interactions (see \textsuperscript{28} for a long discussion about these points).

### 1.5 Spin-orbit terms

The results of this calculation show that also most of the considered anti-hermitean terms are not effective, for the purpose of a Sivers asymmetry. The key interaction term is one of the three components of the spin-orbit scalar

\textsuperscript{1} the most obvious reference is \textsuperscript{43}, but see also ref.\textsuperscript{44} for an exposition of the application of these techniques to quark scattering on composite hadronic structures in light-cone formalism.

\textsuperscript{2} In the following the words “real” and “imaginary” are sometimes used instead of “hermitean” and “anti-hermitean” when speaking of operators.
product. A chain of qualitative arguments presented in section III relates imaginary spin-orbit terms to the high-energy hadron-nucleon analyzing power and recoil polarization. These observables are nonzero at as large beam energies as 300 GeV (for the case of quasi-forward scattering\cite{38}) or 25-30 GeV (for the case of large transferred momenta up to 7 GeV/c, see e.g.\cite{40}). Their behavior at larger energies is not known, and there is no commonly accepted model that allows for an extrapolation\cite{38,39,40}.

This has two consequences. On the one side, at energies $\sim 100$ GeV we face the possibility of relevant Sivers-like asymmetries with this origin. On the other side, it is impossible to decide whether this is a leading twist effect. For this reason, as above anticipated, it is impossible to decide whether the $\hat{T}$ operator appearing in the rescattering factor $\exp(\int \hat{T} d\xi)$ is nonzero at very large energies. Because of this, in the following the terms “Sivers asymmetry” and “Sivers effect” are preferred to “Sivers function”. The latter is appropriate in the case of a leading twist contribution. Experimentally, it may be impossible to distinguish between the two at the presently available energies.

2 The general formalism

Where not differently specified, all variables will refer to the center of mass of the colliding hadrons. Let $\vec{b} = (b_x, b_y)$, be the quark impact parameter and $\vec{K}_T = (k_x, k_y)$ the transverse momentum conjugated with it. Let $P_+$ be the large light-cone component of the hadron momentum, so that $xP_+$ is the quark (+) momentum conjugated with $z_-$. I substitute $z_-$ with the rescaled coordinate

$$\xi \equiv P_+ z, \quad \rightarrow \quad \xi = P_+ \int dz_- \exp(-ixP_+ z_-) = \int d\xi \exp(-ix\xi) \quad (1)$$

not to work with a singularity of the Fourier transform in the infinite momentum limit $P_+ \to \infty$.

Since the inclusive process is described here in terms of squared amplitudes, and these amplitudes are calculated before being squared, $\xi$ is not bound to be positive, as it happens in the ordinary treatment based on a two-point correlator with intermediate real states. In that case $\xi$ has the meaning of the difference between the light-cone positions of two points. Here it describes the light-cone position of one of the two only.
2.1 Basic structure of the quark unperturbed state and insertion of initial state interactions

I represent the initial “unperturbed” quark state in the form

\[ \tilde{\psi}(\xi, \vec{b}) \equiv \int d^2K_T \tilde{f}(x, \vec{K}_T)e^{ix\xi}e^{i\vec{K}_T \cdot \vec{b}}, \quad \tilde{f}(x, \vec{K}_T) \equiv \begin{pmatrix} f_+(x, \vec{K}_T) \\ f_-(x, \vec{K}_T) \end{pmatrix} \]  

So our hadron consists in a coherent superposition of plane wave states with given \( x, \vec{K}_T \) and transverse spin, each with amplitude \( f_+(x, \vec{K}_T) \) or \( f_-(x, \vec{K}_T) \).

I suppose that the parent hadron has \( y \)-polarization \( +1/2 \), and that one initial state only contributes to the final distribution function. The expected distribution has the form\(^3\)

\[ q(x, \vec{K}_T) = |f_+(x, \vec{K}_T)|^2 + |f_-(x, \vec{K}_T)|^2 \equiv q_U(x, K_T) + \frac{K_x}{M} q_S(x, K_T). \]  

The Sivers asymmetry can of course be isolated by subtracting two terms like the previous one, corresponding to opposite hadron polarizations. Here I limit myself to searching for \( k_x \)-asymmetric terms in the above unpolarized quark distribution corresponding to one assigned hadron transverse polarization.

To introduce initial state interactions, I identically write \( \tilde{f}(x, \vec{K}_T) \) as a twice iterated Fourier transform, and in the intermediate stage I substitute each plane wave spinor by a spinor that contains the distortion due to the initial state interactions. Writing only the \( x, \xi \) dependence for simplicity, it means that in the undistorted plane wave

\[ \tilde{f}_{PW}(x) \equiv \int d\xi e^{-ix\xi} \int dx' e^{ix'\xi} \tilde{f}_{PW}(x') \equiv \int d\xi e^{-ix\xi} \int dx' [e^{ix'\xi} \hat{I}] \tilde{f}_{PW}(x') \]  

(where \( \hat{I} \) is the identity matrix and in the last passage I have only highlighted the piece to be modified) the free field operator \( \exp(ix'\xi)\hat{I} \) is substituted by \[3\] This definition is the one given by the so-called “Trento convention”\[^{45}\] for polarization oriented as written. It assumes that the second term is scale-independent and in this case \( q_S \) is the Sivers function. Since this work refers to energies \( \sim 10 \div 300 \) GeV, I will speak of “Sivers asymmetry” referring to the full second term \( q_S K_x/M \).
the more general matrix operator $\hat{\Psi}(x', \xi)$ reproducing a field subject to the action of initial state interactions:

$$
\tilde{f}_{PW}(x) \rightarrow \tilde{f}_{DW}(x) \equiv \int d\xi e^{-ix\xi} \int dx' \hat{\Psi}(x', \xi) \tilde{f}_{PW}(x')
$$

(5)

More precisely, initial state interactions in eikonal approximation affect the quark light-cone path starting from $\xi = \infty$ and reaching the hard interaction point $\xi$, along fixed impact parameter lines (see the discussion in refs. [8] and [24], and compare the figures describing final state interactions for Deep Inelastic Scattering in ref. [7] with those for initial state interactions in Drell-Yan in ref. [30]).

These initial state interactions are here averaged by an effective mean field containing absorbing and spin orbit terms. So, each plane wave is substituted by a wave with eikonal phase distorted by this local field:

$$
e^{ix\xi} e^{i\vec{K}_T \cdot \vec{b}} \begin{pmatrix} f_+ \\ f_- \end{pmatrix} \rightarrow e^{ix\xi} e^{i\vec{K}_T \cdot \vec{b}} \exp \left( \int_{-\infty}^{\xi} \hat{T}(\xi', \vec{b}) d\xi' \right) \cdot \begin{pmatrix} f_+ \\ f_- \end{pmatrix}
$$

(6)

where $\hat{T}$ is a 2x2 matrix operator.

### 2.2 The undistorted quark state

In this subsection I refer the quark spin, orbital and total angular momentum to the parent hadron rest frame.

In absence of initial state interactions, we may assume that we are able to calculate the Fourier transform eq.(2) and write it directly in impact parameter representation as ($PW$ means “plane wave”)

$$
\tilde{\psi}_{PW}(\xi, \vec{b}) \equiv \phi(\xi) \phi'(|b|) \cdot |J_y = +1/2 >
$$

(7)

where my main interest is for the very simple S-wave state

$$
|J_y = +1/2 >_S \equiv \begin{pmatrix} 1 \\ 0 \end{pmatrix}
$$

(8)

and as a second choice for the P-wave state
\[ J_y = +1/2 >_p \equiv ib_x \begin{pmatrix} 0 \\ 1 \end{pmatrix}. \] (9)

Eq. (7) reproduces a space-time fluctuation of the hadron ground state into a quark+spectator state. Eqs. (8, 9) are the impact parameter space projections of the states \( Y_{00}1/2 \rangle_y, Y_{11}(\theta_y, \phi_y)\rangle - 1/2 \rangle_y \). Of course, other terms may be included. I limit to these two possibilities.

The above states eqs. (8) and (9) imply a positive/negative transversity (since these states transport information on the polarization of the parent hadron) but do not allow for any \( \vec{K}_T \)–odd asymmetry, including a Sivers asymmetry, in absence of initial state interactions.

Both the above states imply \( J_y = +1/2 \) for the quark and the parent hadron in the hadron rest frame. This is the limit possibility. More in general we may imagine that a state where the parent hadron is fully polarized with transverse spin \( S_y = +1/2 \), corresponds to a quark mixed configuration of the kind

\[ a \left| J_y = 1/2 > \right|^2 + b \left| J_y = -1/2 > \right|^2. \] (10)

For \( a \approx b \) evidently the quark transports little or no information on the parent hadron polarization state, so it is logically impossible to get a nonzero hadron-polarization-related function, unless some very indirect mechanism is imagined. So, in the following I exclude this possibility. For nonzero \( b \neq a \), the results I get must be diluted by the factor \( |a-b|/(a+b) \). Indeed, substituting the quark \( \left| J_y = 1/2 > \right. \) initial state with the quark \( \left| J_y = -1/2 > \right. \) initial state, the asymmetries that are calculated in the following simply reverse their sign.

For \( \phi(\xi) \) and \( \phi'(b) \), I simply take Gaussian shapes \( \exp(-y^2/y_o^2) \). The same is done for the relevant functions \( \rho(\xi) \) and \( \rho'(b) \) later introduced to describe initial state interactions. In practice, the underlying hadron-quark-spectator vertex is a space-time version of the vertex adopted in [31] (a Gaussian quark-diquark vertex).

The parameters for these gaussian functions are chosen not to get too unrealistic distributions. Since for \( x = 0 \) the distribution is large, the above must be considered an implementation of a sea+valence state. This is not a divisive detail, because the used initial state interactions lead to a zero Sivers effect at \( x = 0 \). For \( x = 1 \) the parameters are tuned so to have a small distribution value, that cannot be zero however. With a logarithmic mapping it could be possible to produce a distribution that is zero at \( x = 1 \). This increase of com-
plication would not be worthwhile since this work does not focus on the \( x \approx 1 \) region where a completely different physics should be included.

### 2.3 Initial state interactions

I assume that the distorting factor \( \hat{D} \equiv \exp\left[ \int d\xi \hat{T}(\xi, \vec{b}) \right] \) of eq.\((6)\) does not depend on \( x \) or \( \vec{K}_T \). This simplifies much the calculations since it allows for transporting \( \hat{D} \) out of the Fourier transform eq.\((2)\) and applying directly it to the function \( \tilde{\psi}(\xi, \vec{b}) \) of eq.\((7)\)\(^4\).

So, this equation is modified to (DW means “Distorted Wave”):

\[
(2)_{DW} = \phi(\xi)\phi'(|b|) \cdot \exp\left( \int_{-\infty}^{\xi} \hat{T}(\xi', \vec{b})d\xi' \right) \cdot |J_y| = +1/2 >_{S,P}. \tag{11}
\]

In the calculations, the exponential operator is approximated by a quasi-continuous product:

\[
\int_{-\infty}^{\xi} \hat{T}(\xi', \vec{b})d\xi' \approx \prod (1 + \hat{T}d\xi)
\tag{12}
\]

where the product starts from a negative and large enough \( \xi' \) value where interactions may be neglected, and stops at \( \xi \). The \( \hat{T} \) matrix is

\[
\hat{T} \equiv \begin{pmatrix}
-(\delta + \alpha_b x) & -i\alpha_b y \\
i\alpha_b y & -(\delta - \alpha_b x)
\end{pmatrix} \rho(\xi)\rho'(b).
\tag{13}
\]

All the coefficients are supposed to introduce reasonably small corrections, at least for \( K_T \lessapprox 3 \text{ GeV/c} \) where we know that any asymmetry due to initial state interactions is at most 30 %.

The \( O(\delta) \) term is a scalar absorption term, associated with spreading of the quark momentum and so to damping of the quark initial state. It assumes underlying chaotic interactions, that because of this lack of coherence deplete any given \( |x, \vec{K}_T > \) state without a direct coherent enhancement of another one, as it would happen in the case of a hermitean interaction. A part of the

\(^4\) Else, one could directly define the action of an \( x \)-dependent initial state interaction starting from eq.\((7)\), sacrificing something at interpretation level.
lost flux is recovered because of diffraction, but in the average some flux is lost from the elastic channel.

The $O(\alpha)$ terms are spin-orbit terms, since in a basis where $S_y$ is diagonal, as the one I am using here, we may write

$$\hat{T}d\xi = \rho \rho' \left( -\delta \hat{I} - \alpha b_x \hat{\sigma}_y + \alpha b_y \hat{\sigma}_x \right) P_z dz_-$$  \hspace{1cm} (14)$$

Defining

$$\delta \equiv \delta' x/\sqrt{2}, \quad \alpha \equiv \alpha' x/\sqrt{2},$$  \hspace{1cm} (15)$$

and remembering that, at large $P_+$, in the hadron collision c.m. frame

$$k_z \approx x P_+ / \sqrt{2}, \quad L_y \approx -k_z b_x, \quad L_x \approx k_x b_y, \quad L_z \ll L_x, L_y,$$  \hspace{1cm} (16)$$

(here $\vec{L}$ is referred to the hadron collision c.m. frame) the above may be rewritten as

$$\hat{T}d\xi = \rho \rho' \left( -\delta' k_z \hat{I} - \alpha' \vec{L} \cdot \vec{\sigma} \right) dz_-$$  \hspace{1cm} (17)$$

and we see that $\hat{T}$ contains a scalar absorption term plus a spin-orbit term.

Since it appears in a real exponential (without an explicit factor "i" in the argument), for real $\alpha$ the spin-orbit term is a anti-hermitean one. For imaginary $\alpha$, it is hermitean. Since by definition the latter cannot produce $T$–odd effects, I have focused my attention on the case of real $\alpha$. This corresponds to the nuclear physics case of an imaginary spin-orbit potential. More in general, we will have a complex potential, able to introduce $T$–odd effects if its imaginary part is nonzero. Aiming at studying the simplest possible case, I limit to a pure anti-hermitean term.

With the parameter values here assumed (see below), the combined action of nonzero $\delta$ and $\alpha$ is such as to produce absorption through all the region affected by serious initial state interactions. This absorption is spin-orbit-selective.

The functions $\rho(\xi)$, $\rho'(b)$ have been chosen with gaussian form, and their widths satisfy the conditions: $\rho(\xi) \approx |\phi(\xi)|^2$, $\rho'(b) \approx |\phi'(b)|^2$. This is motivated by the following facts: (i) initial state interactions cannot take place too far from the hard quark-antiquark vertex; (ii) the projectile and the target are supposed to have similar shapes; (iii) in terms of the longitudinal rescaled quantity $\xi = P_+ z_-$, leading twist effects (if any) must take place over a finite $\xi$ range in the scaling limit $P_+ \to \infty$; if they are next-to-leading, at any finite $P_+$ for which they assume a non-negligible value they take place over a finite (scale-dependent) $\xi$ range; (iv) since I assume that initial state interactions
have incoherent character, the $\phi(\ldots)$ functions are wavefunctions, while the $\rho(\ldots)$ functions are densities $\sim |\phi(\ldots)|^2$.

The choice of using all gaussian functions, with correlated widths, is aimed to simplicity and to reducing the number of independent parameters.

3 Some general comments on the use of a non-hermitean spin-orbit mean field in hadron-hadron semi-inclusive processes

Presently, nonzero effects of spin-orbit terms are measured in exclusive hadron-nucleon interactions at rather large energies. Their origin in terms of fundamental interactions is not fully explained. So they must be considered phenomenological interactions. What is argued in the present work is that these interactions are present at quark-quark level, where they preserve the same generic structure they have at hadron-hadron level. As a consequence, their effect should be visible not only in a few exclusive channels, but in a wider class of hadron-hadron induced processes, including semi-inclusive scattering and Drell-Yan. Although elastic channel measurements are the most precise available, up to now spin-orbit effects have been found in any exclusive channel where they have been searched for via dedicated experiments.

In the following I give a qualitative account (i) of the present-day knowledge on spin-orbit interactions in hadron-hadron interactions at high energies, (ii) of the assumptions that are necessary to pass from hadron-hadron spin-orbit interactions to the quark-mean field interactions introduced in the previous section.

3.1 Present status of spin-orbit interactions in exclusive hadronic processes

The arguments summarized in this subsection may be reconstructed from refs.\[38,39,40\], that also contain a long list of references concerning theoretical models and previous measurements. Several papers on this subject, or touching this subject, have been written especially in the years before 1985.

Writing the amplitude for the elastic scattering between a normally polarized beam particle and an unpolarized target particle as $A(\vec{K}_T) \equiv A_{\text{even}}(\vec{K}_T) + A_{\text{odd}}(\vec{K}_T)$, the spin-orbit potential is the core part of the impact parameter space representation of $A_{\text{odd}}(\vec{K}_T)$. In other words, a nonzero normal spin analyzing power in hadron elastic scattering on hadron targets is equivalent to a spin-orbit coupling like the one appearing in eq.\(14\) of the previous section. As a specific example, from eqs.\(49\) and \(53\) of ref.\[39\] one may directly de-
duce the spin-orbit terms in eq. (14) of the present work, taking into account that in the case of a Gaussian density one has $\partial \rho(b^2)/\partial b_x \propto -b_x \rho(b^2)$ (in [39], the same 2-dimensional formalism used here is employed).

Experimentally, an unexpectedly large normal analyzing power (or, equivalently, a large recoil polarization) is found in nucleon-hadron exclusive processes like elastic scattering. In the quasi-forward diffraction-dominated region, this is nonzero up to beam energies 300 GeV. At large transferred momenta (up to 7 GeV/c) it is rather large up to beam energy 30 GeV. What happens at larger energies is not known, and it has been guessed but not demonstrated that such terms can survive asymptotically. The separation between the two regimes (small and large transferred momenta) is not obvious in the data. Comparing small and large angle data (that normally refer to different energy regimes) taking as a starting point the peak at very small angles due to electromagnetic-strong interference, and then increasing the angle, the transferred momentum dependence of the analyzing power or recoil polarization shows a series of diffraction or interference peaks and changes of sign. The peaks do not decrease in magnitude, and at the largest transferred momenta the size of the effect seems to increase in an uncontrollable way, together with error bars. I remark that the region that theoretically should imply the transition between the two regimes (transferred momenta between 0.5 and 3 GeV/c) is also the most important for a nonzero and measurable Sivers asymmetry.

At intermediate energies the origin of the measured spin-orbit terms may be reasonably interpreted for pion-proton diffractive elastic scattering. In this case two dominating Regge trajectories (pomeron and rho) mix, leading to interference between two terms with phase difference $90^\circ$ in the helicity-flip and helicity-non-flip amplitudes. In the case of quasi-forward elastic proton-proton scattering, several Regge poles and cuts potentially contribute and the situation is less clear. Because the presence of the discussed observables requires a phase difference $90^\circ$ between the interfering helicity-flip and helicity-non-flip amplitudes, contributions from two poles/cuts are needed to explain a nonzero analyzing power or recoil polarization. So, an unexpected survival of these observables at large energies would contradict the standard idea that only pomeron contributes asymptotically to small-angle elastic scattering.

For the case of large transferred momentum, there are a lot of competing models (e.g. references 5-22 in ref. [40]). In all of them some non-trivial mechanism is added to a standard PQCD set of processes. Indeed, PQCD may be properly applied in this region, leading to zero normal analyzing power because of helicity conservation. So, PQCD cannot be ignored, and at the same time it

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5 The case of the electromagnetic-strong interference at small angle must be excluded from this discussion. It may be asymptotically relevant without implying conclusions about spin-orbit terms of strong origin.
cannot be applied in the most straightforward way.

3.2 Assumptions about spin-orbit interactions at quark level

To pass from the measurements of normal spin asymmetries in elastic scattering to the spin-orbit potential contained in the previous section, some assumptions are needed.

Assumption (A): At least a part of the high-energy spin-orbit interactions between hadrons can be reduced to an incoherent sum of spin-orbit interactions between quarks.

Assumption (B): The absorption part of these quark-quark spin-orbit interactions is relevant.

These two assumptions are plausible and questionable at the same time, because of our lack of knowledge about the fundamental mechanisms determining the spin-orbit coupling at hadron level.

In the case of large angle scattering, assumption (A) could be naively justified because of the short wavelength perturbative regime. On the other side, in this regime PQCD may be applied, and it says that spin-orbit interactions should not exist at all (both at quark and at hadron level), while they are there.

In the case of quasi-forward scattering, we may imagine that the spin-orbit term in hadron-hadron scattering is associated with an interference term between two t-channel Regge pole exchanges. Although it is possible to imagine the same process with the same exchanged poles taking place between two individual quarks \( q_1 \) and \( q_2 \), small transverse momenta imply a large degree of coherence between two processes like \( q_1 - q_2 \) and \( q_1 - q_3 \) scattering. So the possibility of extracting the incoherent part of these interactions and estimate its relevance would require at least to know what exactly is exchanged, and this is not completely clear presently.

Concerning assumption (B), it is easy to imagine a large absorption part in quark-hadron scattering, for the same reasons why hadron-hadron scattering is absorption-dominated. However, this argument cannot be transferred directly from the hadron level to the quark level. Indeed, the absorption part of the hadron-hadron scattering potential derives from the inelasticity of the process. In other words, from the direct loss of flux of the initial channel. But what is a hard inelastic process at hadron level may be a perfectly elastic process on the quark level.

\footnote{The word “direct” reminds that a part of this lost flux does anyway contribute to the elastic channel because of diffraction.}
at quark level, in the normally employed approximation where quarks are free particles.

For the specific case of the production of T-odd effects, I have long discussed this point in my previous work ref. [28]. Summarizing very briefly, since a quark is never asymptotically free, inelasticity is not determined by a modification of the quark internal state or by extra particle production, but also by the behavior of the surrounding environment. This has little relevance in the analysis of T-even effects in hard processes, because of the separation between hard and soft scales. In T-odd effects however, soft scales acquire indirect relevance. In particular, elastic quark-quark in-medium scattering leads to a finite imaginary part in the quark propagator, i.e. the quark behaves like a free but unstable particle.  

As a consequence of this inelasticity, if spin-orbit interactions may be transferred from hadron to quark level an imaginary part is likely to be present, giving an argument for the plausibility of assumption (B).  

3.3 Mean field potential, exponentiation and asymptotic behavior

At finite beam energies $\sim 10\text{-}100 \text{ GeV}$, from assumption (A) and (B) the possibility follows of writing the cumulative effect of spin-orbit quark-quark interactions in the exponentiated form eq.(11) where a quark interacts with a mean-field potential along a straight path. Exploiting the fact that for finite $P_+\xi$ may be one-to-one related in a non-singular way to the longitudinal space variable $z$ in the rest frame of the unpolarized particle, we may just treat the interaction in Glauber-like style[43] and proceed as in high-energy hadron-nucleus collisions. With things done this way, exponentiation is just a useful short-wavelength approximation, but does not prove that one is considering leading twist terms. In other words, I cannot presently support the following assumption:  

Assumption (C): The operator $\hat{T}$ of eq.(6) is finite in the limit $P_+ \to \infty$.  

Formal considerations apart, it is obvious that such an assumption could make sense if we knew the $P_+ \to \infty$ behavior of spin-orbit effects in nucleon-hadron elastic scattering.

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7 Of course the quark itself is stable from the point of view of its internal structure, what is unstable is the quark “free” state we are probing.
8 I cannot argue that this imaginary part is also dominant, as it happens e.g. for the unpolarized part of the forward hadron-hadron scattering process.
4 Results

All calculations refer to $k_y = 0$, so in the following only the $k_x$–dependence appears explicitly.

Figures 1 to 4 have been calculated with the S-wave state, figures 5 and 6 with the P-wave one.

The free parameters have the same values in all figures 1 to 6, and their list follows:

For the gaussian function/density widths:

$\phi(\xi): \Delta \xi = 3.5$. $\rho(\xi): \Delta \xi = 2$. $\phi'(b): \Delta b = 0.9$. $\rho'(b): \Delta b = 0.6$.

I have used $\rho(\xi) \approx |\phi(\xi)|^2$ and $\rho'(b) \approx |\phi'(b)|^2$, so to have two independent parameters only.

For the interaction matrix: $\delta = 0.2$, $\alpha = 0.1$.

These mean an overall 20 % reduction of the quark distribution for $K_T = 0$, entirely due to the parameter $\delta$. At large $K_T$ on the contrary the distribution is enhanced. The spin-orbit parameter $\alpha$ produces local flux modifications with zero $\vec{b}$–average. The Fourier transform for $k_x = 0$ is not sensible to these. For nonzero $k_x$, $\alpha$ produces asymmetry.

In fig.1 I show the distribution function $q(x, k_x)$ for $k_x = 0$ in the $x$–range (0,1).

In fig.2 I show the corresponding $k_x$–distribution $q(x, k_x)$ for $x = 0.6$. A logarithmic plot has been chosen to give more evidence to the large $-k_x$ asymmetry, caused by a nonzero $\alpha$.

In fig.3 I show the asymmetry $A \equiv (q_+ - q_-)/(q_+ + q_-)$, for $x = 0.6$, as a function of $k_x$ in the $k_x$ range (0,5) GeV/c. With $q_{\pm}$ I mean $q(x, \pm |k_x|)$.

In fig.4 I show the same asymmetry for fixed $k_x = 4$ GeV/c (i.e. at its peak value) as a function of $x$.

Many more (not shown) distributions and asymmetries have been produced changing the values of all the above parameters. The result is that figs.1 to 4 are general enough and contain all the relevant qualitative features of the found asymmetries. By tuning parameter values, the asymmetry may be made larger/smaller, and its peak may be shifted towards larger/smaller $x$ or $k_x$.\footnote{I refer to reasonable parameter values only. E.g., there is no hint in the literature}
Fig. 1. S-wave: The \( q(x, k_x) \) distribution function for \( k_x = 0 \), as a function of \( x \). \( k_x \) is the component of \( \vec{K}_T \) orthogonal to the initial hadron polarization \( \vec{S} \propto \hat{y} \). For the parameter values, see the beginning of the “Results” section in the text.

Fig. 2. S-wave: \( k_x \)–dependence of the quark distribution \( q(x, k_x) \) for \( x = 0.6 \). The asymmetry is made more evident by the logarithmic plot.

Summarizing the general properties deduced by systematic parameter tuning of unpolarized \( k_x \)–integrated distribution functions being strongly affected by initial state interactions, so this is considered a boundary condition to be respected.
Fig. 3. S-wave: $k_x$–distribution of the asymmetry $(q_+ - q_-)/(q_+ + q_-)$ for $x = 0.6$. It is the left-right asymmetry of the distribution reported in the previous figure.

Fig. 4. S-wave: Asymmetry $(q_+ - q_-)/(q_+ + q_-)$ as a function of $x$ for $k_x = 4$. At this $k_x$ we have the peak value of the asymmetry, for each $x$.

activity I find:

1) For small values of the parameters $\delta$, $\alpha$ (both $\leq 0.2$) one already obtains peak values of the asymmetry $|A(x, K_{max})| \sim 1$ (see fig.4). This is however reached at $k_x \gtrsim 4$ GeV/c, where experimental data would easily present large
Fig. 5. P-wave: $k_x$–dependence of the quark distribution $q(x, k_x)$ for $x = 0.6$.

Fig. 6. P-wave: The asymmetry $(q_+ - q_-)/(q_+ + q_-)$ of the quark distribution reported in the previous figure.

error bars making it difficult to distinguish between e.g. 60 % and 30 % asymmetries. \[^{10}\] For $k_x$ up to 3 GeV/c the asymmetry is smaller than 30 %, i.e. much

\[^{10}\] For asymmetries over 30 % the error is given by the error on the less populated of the two subsets in comparison. E.g. with total population 300 event, 33 % asym-
smaller than the peak value. A consequence of the use of gaussian shapes is the presence of this rather pronounced peak at large $k_x$, that however would have scarce influence on an event-weighted asymmetry (dominated by $k_x \approx 1/2$ GeV/c).

2) Changing the gaussian distribution parameters it is possible to change the shape of the asymmetries, so to have the peak asymmetry e.g. at 3 GeV/c. This would be far from anything observed up to now (compare with the fits by refs. [14, 15, 16, 17]).

3) For $k_x > 5$ GeV/c, we find oscillations near zero, that could be of numeric origin. The fast decrease immediately following the peak at $k_x = 4$ GeV/c is however stable with respect to changes of the numerical parameters (number of integration points, integration range cutoffs).

4) Asymmetries obtained via anti-hermitean spin-orbit terms are zero at $x = 0$. $|A(x, K_{max})|$ increases with $x$ up to a maximum and then decreases at larger $x$, seemingly to reach zero at the unphysical value $x = \infty$. The fact that the asymmetry is zero for $x = 0$ is a consequence of the symmetries of the model, and of the fact that the $x = 0$ component of the Fourier transform is just a plain $\xi$—average. In a more realistic model this property should be anyway present for another reason. Here, we have assumed $x$—independent interactions, but accordingly to eqs. (14-17), the spin-orbit potential is $O(x)$ at small $x$ (in a semi-classical approximation, assuming continuous $\vec{L}$). At a semi-classical level, it is reasonable to imagine that a wee parton has comparatively small $L_x$ and $L_y$ in the hadron collision c.m. frame, and so negligible spin-orbit interactions.

5) Alone, a nonzero $\delta$ is not able to produce a Sivers-like asymmetry. Nonzero $\alpha$ is required. On the other side, $\alpha$ could produce asymmetry for $\delta = 0$ too, but this combination ($\delta = 0$, $\alpha > 0$) would imply an unphysical local increase of particle flux. With the chosen values of the coefficients we have spin-selective absorption, but anyway absorption, with the exception of large—$b$ regions where initial interactions are anyway suppressed by $\rho(b)$.

6) In the P-wave case the $k_x$—event distribution assumes the obvious shape shown in fig.5. At the distribution peaks of fig.5 the asymmetry is $\sim 5\%$ and has the same sign of the S-wave asymmetry of figs 1-4. In the distribution tails $k_x > 3$ GeV/c the asymmetry reverses its sign. To analyze in detail situations where both waves (S and P) are relevant one should however introduce a more realistic different radius for the S—wave and the P—wave distributions.

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metrty means 100 vs 200 events. A fluctuation like 50 vs 250, leading the asymmetry to 66 %, is quite easy. In a restricted $x$— range like (0.2—0.3) in the valence region, to collect much more than 300 events with $P_T > 4$ GeV/c is not trivial.
7) The non-diagonal terms of the interaction matrix eq. (14) have practically no role. I.e., removing the diagonal $\pm \alpha b_x$ terms from the interaction matrix produces immediately zero asymmetry, while removing the $\pm i \alpha b_y$ terms, or changing their sign, only produces negligible changes in the asymmetry shape of figs. 3 and 4.

The role of the $\alpha b_x$ terms may be understood if one approximates the action of the damping potential in a homogeneous form. Then a nonzero $\alpha$ (together with a nonzero $\delta$) means that the two components $a_+$ and $a_-$ of a spinor $(a_+, a_-)$ are substituted by damped exponentials:

$$
\begin{pmatrix}
a_+ \\
a_-
\end{pmatrix}
\rightarrow
\begin{pmatrix}
a_+ \exp(-\delta + \alpha b_x) \xi \\
a_- \exp(-\delta - \alpha b_x) \xi
\end{pmatrix}
$$

Evidently at large negative values of the product $\xi b_x$ we have dominance of the spin state $(1,0)$, and the opposite at large positive $\xi b_x$. This asymmetry is detected by the $\exp(-i k_x b_x)$ Fourier transform.

Introducing further approximations (substituting the gaussian functions with simple cutoffs of the integration ranges, and taking $\delta = 0$) shows that the so simplified problem enjoys a kind of $b \cdot \xi$ invariance. As a consequence, any time a parameter is changed so to decrease the $\Delta x$ range of the asymmetry, it also increases the corresponding $\Delta k_x$ range and vice versa. A systematic parameter tuning work confirms that this property is approximately present in the full model.

Some final observations:

The author of this work has remarked in [28] that damping terms in initial state interactions contribute producing $T$-odd distributions, so he cannot claim surprise for the results of the present calculation. What is however surprising is the non-effectiveness of the non-diagonal terms in the $\hat{T}$ matrix. Since one cannot exclude that this is due to the exaggerated level of symmetry contained in the here used unperturbed state, a deeper study needs to be devoted to this point.

Also, one cannot exclude that a more complicated structure of initial states may lead to $k_x$-asymmetries in presence of spin-independent initial state interactions. This is what happened e.g. in refs. [35,36], where a $T$-odd structure was produced in presence of a spin-independent (final state) interaction.

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11 The rescattering terms used in those two works had a rather different structure and physical meaning each with respect to the other. However both are based on non-hermitean scalar interaction terms.
that case however, the effect of final state interactions was not spin independent, despite their Hamiltonian was. This was due to the elaborate shell-model structure of the initial states. E.g., in [35] it was possible to get an asymmetry from $^{12}\text{C}$, but not from $^{16}\text{O}$ (fully filled $P_{3/2}$ and $P_{1/2}$ shells in the latter case, only $P_{3/2}$ in the former).

5 Conclusions

Summarizing, starting from the assumption that the quark total angular momentum is dominantly oriented as the parent hadron spin, it is possible to build a Sivers-like asymmetry via mean field initial state interactions of imaginary spin-orbit kind. With these interactions, a nonzero Sivers function is present also starting from a simple $S_y = 1/2$ S-wave ground state for the quark.

Phenomenological interactions of this kind are known in high-energy hadron and nuclear physics. For values of the parameters such as to guarantee a small overall effect of initial state interactions ($\approx 20\%$ distribution damping at small $K_T$) the qualitative $x, k_x$-distribution of the calculated asymmetries follows the typical pattern proposed by widespread parameterizations.

The employed quark ground state has the features of a joint valence+sea state. Despite this, the predicted asymmetries are zero at $x = 0$.

In the chosen interaction matrix, the only effective terms are the (spin-orbit selective) diagonal absorption ones. In other words, spin rotating interactions are not decisive. Also, spin-independent absorption alone is not sufficient. It is not possible however to exclude that the no-effectiveness of these terms is related to the excess of simplicity of the initial configuration.

With the present day knowledge of spin-orbit hadronic interactions, it is not possible to establish whether their effects persist at very large energies. Consequently it is not possible to establish whether a Sivers-like asymmetry generated by them is a leading twist one or just an intermediate energy effect.

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