Robust Stability of DC Microgrid Under Distributed Control

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ABSTRACT Compared with AC microgrid, DC microgrid has attracted more and more attention due to their high reliability and simple control. In this paper, we analyze the existence and stability of equilibrium of DC microgrid under distributed control. Firstly, the power-flow equation of the DC microgrid with n DGs and m CPLs is built. On this basis, under the limitation of currents of DGs, the sufficient solvability conditions are obtained based on Brouwer’s fixed-point theorem. Secondly, we build the small-signal model to forecast the system qualitative behavior around equilibrium. The eigenvalue analysis based on inertial theorem provides the analysis basis for system stability, then we obtain the robust stability condition based on the properties of a special matrix. The obtained stability conditions provide a reference for establishing a stable DC microgrid. Lastly, simulation results verify the correctness of the proposed theorems.

INDEX TERMS Distributed control, solvability condition, inertia theorem, robust stability.

I. INTRODUCTION
Microgrid includes alternating-current (AC) microgrid and direct-current (DC) microgrid [1]. With the rapid development of power electronic technology, the DC power system with power electronic equipment as the core device returns to people’s vision again. Compared with AC microgrid, DC microgrid has great advantages: There are fewer conversion links and has no problems of frequency and phase synchronization [2], [3].

In DC microgrid, most loads connected to the DC bus by a point-of-load converter which makes the load behave as a constant power load (CPL) [4]. CPL usually leads to instability and the loss of equilibrium (voltage collapse) [5]. Therefore, the control objectives of DC microgrid can be briefly summarized as the following three objectives: current sharing, voltage recovery and maintaining system stability. For these three objectives, scholars at home and abroad have done a lot of research and made great progress. In order to realize current sharing and voltage recovery of DC microgrid, decentralized control and distributed control are usually adopted. The traditional droop control is a typical decentralized control, which can realize current sharing roughly. However, this control method has two drawbacks: voltage drop and biased current sharing [6], [7].

To solve this problem, scholars have proposed a distributed control strategy based on consensus algorithm. The core idea of this method is to achieve global current sharing and voltage recovery by keeping consistent with adjacent DGs [8], [9]. However, in the above study, only the stability under conventional resistive load is considered. DC microgrid usually contains a large number of CPLs which easily lead to the instability of the system. Therefore, the design of DC microgrid should overcome the instability of CPLs.

Aiming at the stability problem of DC microgrid under current sharing control, the research contents can be divided into four categories according to the differences of topology and control method: Star DC microgrid (Multiple DGs with single CPL) under droop control; Star DC microgrid under distributed control; Meshed DC microgrid (Multiple DGs with multiple CPLs) under droop control; Meshed DC microgrid under distributed control. At present, the stability analysis of the first three types of systems has been reported, the fourth type is more complex and rarely reported.

Small-signal analysis is a typical method to analyze system stability when subject to small disturbances. Existing approaches for the small-signal stability of power electronic systems are mainly based on impedance-based and
eigenvalue-based method [10]. Many papers have carried out the stability analysis by impedance-based method. For instance, [11] use the nodal admittance matrix of the system and the output admittance of the converters to formulate the return-ratio matrix, and then the stability of the system can be judged using generalized Nyquist criterion. However, if there is a large number of DGs and loads exist in the power network, the orders of the nodal admittance and the output admittance matrix will be very high. For this reason, [12] propose the bus node impedance criterion. It shows that the system is stable if and only if the bus impedance at any node does not have any right-half-plane poles which avoid a large number of calculations. A weakness of the impedance-based method is that it cannot predict the stability of the entire system from a particular interfacing point, this paper focuses on the stability analysis through eigenvalue-based method in order to overcome this problem.

For eigenvalue-based method, the state-space model of the whole system needs to be established. A high-dimensional model has been proposed for analyzing the transient processes of the converter in [5]. By using the quadratic eigenvalue problem theories, the stability of the linearized system is analyzed, and the stability conditions are obtained. Some studies about the stability of the system under distributed control have been carried. The impedance matrix of microgrid with one CPL has only a negative eigenvalue has been proved in [5], and the stability condition of the system is derived through quadratic eigenvalue theory. On this basis, [13] studies the stability of star DC microgrid under distributed control and reveals that negative impedance and time delay are the causes of system instability. Then, stability conditions are obtained. When the equilibrium meet the proposed conditions, system is stable. Unluckily, the equilibrium of the system may change due to the change of CPLs. This paper doesn’t tell us the robust stability condition when loads change over time. Moreover, the stability analysis will be more difficult for the DC microgrid with multiple CPLs.

In star DC microgrid, there is only one CPL. The existence of equilibrium is determined by a quadratic equation with one unknow which is easily to obtain the sufficient condition [14]. However, there are multiple CPLs in the meshed DC microgrid. As a result, the system equilibrium is determined by a multi-dimensional quadratic equation (MDQE). Much of current studies about MDQE consider the voltage constraints of DGs, but seldom considers the constraints of the current of DGs. Thus, it is significant to derive the solvability conditions for the power-flow equation of the system considering current constraints.

We hope to determine the stability of the system regardless of the location of the source of instability, thus this paper is addressed through eigenvalue-based method. This paper is based on the previous research of [15]. In [15], under the current limit, we investigate the existence of the feasible power-flow solution of DC microgrid considering current constraints of DGs under distributed control. However, the stability analysis of DC microgrid ignored which is essential to establish a reliable microgrid. CPLs will lead to system unstable due to its negative impedance characteristic, and the access of distributed control will bring difficulties to the analysis of system stability. In this study, we supplement the existing research about the solvability conditions of power flow equation and introduce a distributed control aiming at current sharing and voltage regulation. Compared with the stability analysis through impedance-based method in [10, 12], and [11], the stability analysis in this paper can determine the stability of the entire system. What’s more, high precision current sharing and voltage regulation are considered. Compared with decentralized control in [5], the topology of DC microgrid in this paper is more general, and the distributed control method in this paper can achieve more accurate current sharing.

The main contributions of this paper can be summarized as follows:

- A distributed control method aims at current sharing and voltage regulation is proposed, which can overcome the instability of CPLs. On this basis, the sufficient conditions of the existence of equilibrium considering current constraint is obtained based on Brouwer’s fixed-point theorem.
- The small signal model of the system with n DGs and m CPLs under distributed control is obtained. Through the analysis of system Jacobian matrix based on inertia theorem, the robust stability condition independent of the loads real-time information is obtained.
- We give a stability analysis method for a class of matrix \( AB + F \) (A is semi-positive definite, B has negative eigenvalues, and F is a rank-one matrix), which usually appears in the stability analysis of DC microgrid under distributed control.

The rest of this paper is organized as follows: Section II introduces the distributed control framework. The sufficient condition for the existence of equilibrium of the system is obtained in section III. The stability analysis and the robust stability condition are introduced in section IV. In section V, simulation results verify the correctness of the proposed theorems. Conclusions are made in section VI. Section VI introduces the preliminaries and notations used in this paper.

II. SYSTEM MODEL OF THE DC MICROGRID

A. GRAPH THEOREM

The graph theorem has been introduced in [15], and this omitted here. It should be noted that for a connected graph, its Laplacian matrix \( \mathbf{L} \) has at least one spanning tree, and \( \ker(\mathbf{L}) = \text{span}(1_n) \), i.e., \( \mathbf{L} 1_n = 0_n \).

B. BASIC MODEL AND ASSUMPTIONS

A general DC microgrid with n DGs and m CPLs is illustrated in FIGURE 1. In communication network, the nodes represent converters and edges represent the communication link for data exchange. In distributed control, each DG only exchanges information with its neighbors. Physical network, the sub-network induced by load nodes and communication network are all strongly connected.
According to Kirchhoff’s and Ohm’s laws, we have
\[
\begin{bmatrix}
\tilde{i}_S \\
i_L
\end{bmatrix} =
\begin{bmatrix}
B_{SS} & B_{SL} \\
B_{LS} & B_{LL}
\end{bmatrix}
\begin{bmatrix}
u_S \\
u_L
\end{bmatrix} = B
\begin{bmatrix}
u_S \\
u_L
\end{bmatrix}
\tag{1}
\]
where \(i_S\), \(u_S\) are the vector of currents and voltages of DGs, respectively. \(i_L\), \(u_L\) are the vector of currents and voltages of loads, respectively. \(B\) is the symmetric admittance matrix of the transmission network.

Since the point of load converters respond quickly, loads are regarded as CPLs, as shown in FIGURE 1. (d). The characteristic of CPLs is obtained
\[
\|u_L\| i_L = -p
\tag{2}
\]
where \(p\) represents the power of loads. As the current reference direction is opposite to the voltage reference direction, the right side of (2) is negative.

C. STABILIZING DISTRIBUTED CONTROL
To realize proportional current sharing and voltage regulation, a distributed control method is proposed. The control diagram is shown as FIGURE 2. The output voltage for each converter can be expressed as
\[
u_{S_i} = \delta i_i + \delta u_i
\tag{3}
\]
where \(u_{S_i}\), \(\delta i_i\) and \(\delta u_i\) represent the output voltage, current-correction term and the voltage-correction term, respectively, for the \(i\)th DG.

The current-correction term is designed as follows:
\[
\delta i_i = \frac{1}{q_i} \int \sum a_{ij} \left( i_j - \frac{i_i}{q_i} \right) dt
\tag{4}
\]
where \(q_i\) is the current sharing proportionality coefficient, \(i_i\) and \(i_j\) are the output current of \(i\)th and \(j\)th DG, respectively. \(a_{ij}\) represents the communication weight. If there is a communication link between nodes \(i\) and \(j\), \(a_{ij} > 0\); otherwise, \(a_{ij} = 0\).

To realize voltage regulation, the voltage-correction term is expressed as follows
\[
\delta u_i = \int v_{ref} - \tilde{u}_S dt
\tag{5}
\]
where \(v_{ref}\) and \(\tilde{u}_S = \frac{1}{n} \sum u_S\) represent the reference voltage and average voltage of DGs, respectively.

Rewrite (3) in matrix form, the distributed control method is proposed as follows:
\[
u_S = \int -Q \xi Q i_S + (v - \tilde{u}_S) dt
\tag{6}
\]
where \(v = v_{ref}\), and \(Q = \text{diag}(1/q_i)(q_i > 0)\) are the rated voltage vector of the DC bus and current sharing proportionality coefficient matrix, respectively. \(\xi = [a_{ij}]\) is the Laplacian matrix of the communication graph.

D. CONTROL OBJECTIVES
Current sharing and voltage regulation are two objectives of the DC microgrid, which are considered in this paper.

Objective 1: In the DC microgrid, current sharing means the output current of DGs is divided equally in proportion in the steady state, that is,
\[
i_1 : i_2 : \cdots : i_n = q_1 : q_2 : \cdots : q_n
\tag{7}
\]

Objective 2: In the DC microgrid, average voltage regulation means in the steady state, following equation holds
\[
\tilde{u}_S = v
\tag{8}
\]

Next, we will prove that in the steady state, two control objectives are realized.

**Proof:** Above all, following equation is satisfied when system is stable
\[
-Q \xi Q i_S + v - \tilde{u}_S = 0
\tag{9}
\]
Clearly, \(Q\) is invertible, then there is a matrix \(Q^{-1}\) such that
\[
-L Q i_S + Q^{-1}(v - \tilde{u}_S) = 0
\tag{10}
\]
Because $£$ is the Laplacian matrix of a strongly connected graph, then $£$ has a zero eigenvalue and its corresponding eigenvector is $1_n$, that is, $1_n^T £ = 0_n^T$.

Left multiplied by $1_n^T$, (10) becomes

$$1_n^T Q^{-1} (v - \tilde{u}_S) = 0$$  \hspace{1cm} (11)

Since $1_n^T Q^{-1} > 0_n$, then (11) is true if and only if $v - \tilde{u}_S = 0$, thus (8) is obtained.

By substituting (8) into (9), we obtain $Q E \tilde{q}_S = 0_n$, i.e. $Q \tilde{q}_S \in \text{span}(1_n)$, then (7) is obtained.

Remark 1: In the steady state, current sharing and average voltage regulation are realized. In the next section, we will obtain the sufficient existence conditions of equilibrium considering the current constraints.

III. SOLVABILITY CONDITION FOR POWER-FLOW EQUATION

A. PROPERTIES OF NETWORK

In this section, we analyze the properties of the sub-network, which is necessary for the analysis of power-flow equation. The main results are as follows.

Theorem 1: For a microgrid with strongly physical network and sub-network, the following statements hold:

1. $B_{LL}$ is irreducible and $B_{LL}^{-1} > O$.
2. $-B_{LL}^{-1} B_{LS} v_{ref} 1_n = v_{ref} 1_m$.

Proof: For the statement (1), suppose $B_{LL}$ is a reducible matrix, according to the definition of reducible matrix, there exist a permutation matrix $E$ such that

$$E^T B_{LL} E = \begin{bmatrix} \bar{B}_{11} & \bar{B}_{12} \\ \bar{O} & \bar{B}_{22} \end{bmatrix}$$

Because $B_{LL}$ is a symmetric matrix, then we have $E^T B_{LL} E$ is also a symmetric matrix, i.e., $E^T B_{LL} E$ can be written as

$$E^T B_{LL} E = \begin{bmatrix} \bar{B}_{11} & \bar{O} \\ \bar{O} & \bar{B}_{22} \end{bmatrix}$$

Thus, the sub-network induced by load nodes is divided into two independent networks which is contrary to the assumption that the sub-network is strongly connected. Then $B_{LL}$ is an irreducible $M$-matrix, then $B_{LL}^{-1} > O$.

The statement (1) is proved.

For the statement (2), since the physical network is strongly connected, then $B$ has only one zero eigenvalue. Moreover, $\ker(B) = 1_n + 1_m$.

Then, considering $\ker(B) = 1_n + 1_m$, we obtain $B_{LS} 1_n + B_{LL} 1_m = 0_n$, then we have $-B_{LL}^{-1} B_{LS} 1_n = 1_m$. Thus, the statement (2) is obtained.

The proof is accomplished.

B. POWER-FLOW EQUATION OF THE DC MICROGRID

By Combining (1), (2) and (8), the power-flow equation is obtained as

$$[u_L] B_{LL} u_L + [u_L] B_{LS} v + p = 0_m$$  \hspace{1cm} (12)

Left multiplied by $B_{LL}^{-1} [u_L]^{-1}$, (12) becomes

$$u_L = v_{ref} 1_m - M u_L^{-1}$$  \hspace{1cm} (13)

where $M = B_{LL}^{-1} [p]^{-1}$.

Define $G(u_L) = v_{ref} 1_m - M u_L^{-1}$. Since $B_{LL}^{-1} > 0_m$, i.e., $M > 0$, then for any $u_L > 0_m$, $G(u_L) < 1_m$.

C. RECENTLY RESULTS

In steady state, $u_S$ is a known positive constant vector. Let $u_S^*$ denote the steady-state value of $u_S$. Define $u_S^* = -(B_{LL} + R_L^{-1}) B_{LS} u_S$, $u_L = [u_S^*]^{-1} u_L$ and $A = [u_S^*]^{-1} (B_{LL} + B_{LS}^{-1} p) [u_S^*]^{-1}$. Clearly, (12) is equivalent to the following form

$$u = f(u) = 1_m - A u^{-1}$$  \hspace{1cm} (14)

To obtain the solvability condition of the power flow equation, two mainly methods are proposed, namely “weighted sum of sub-equations” [11], and “fixed point theorem” [12]. The main ideas of these methods are as follows. For the first method, if there exists a diagonal matrix $H$ such that (15) is not solvable, then (14) has no solution.

$$u^T H B_{LL} u + u^T H B_{LS} u_S + 1_m^T H p = 0_m$$  \hspace{1cm} (15)

In fact, (15) is the sum of sub-equations of (14), consequently, (15) must be solvable if (14) is solvable. By completing the quadratic form, a necessary solvability condition for (14) is obtained as follows.

Proposition 1: If (14) is solvable, then there is no diagonal matrix $H$ such that

$$\begin{bmatrix} H B_{LL} & H B_{LS} u_S \\ u_S^* B_{SL} H & 2 p^T H 1_m \end{bmatrix} > 0$$  \hspace{1cm} (16)

For the second method, if there is a compact set $D$ such that $f(u)$ is a continuous self-mapping, contraction mapping and concave increasing self-mapping on $D$, according to Brouwer’s, Banach’s and Taski’s fixed-point theorem, there is a $u^* \in D$ such that $f(u^*) = u^*$.

In [10], the compact set $D$ is constructed as $D = \{ x \mid \frac{1}{2} 1_m \leq x \leq 1_m \}$. Then, $f(u)$ is a contraction mapping on $D$ if

$$4 \| A \| \infty < 1$$  \hspace{1cm} (17)

By invoking Banach’s fixed-point theorem, there exists a unique vector $u^* \in D$ that is a solution to (5). Considering

$$f(tx + (1 - t)y - (tf(x) + (1 - t)f(y)) = tA x^{-1} + (1 - t)A y^{-1} - A (tx + (1 - t)y)^{-1} = (1 - t) A \| x \| \| y \| (x + (1 - t)y)^{-1}$$  \hspace{1cm} (18)

Clearly, $f(tx + (1 - t)y) > (tf(x) + (1 - t)f(y))$ for any $0 < t < 1$, $x > 0_m$ and $y > 0_m$, i.e., $f(u)$ is a concave increasing function. Then, $f(u)$ is a concave self-mapping on $\{ x \mid y \leq x \leq 1 \}$ if there exists a positive vector $y$ such that $y < f(y)$. 

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According to Taski’s fixed-point theorem, there exists a unique vector that is a solution of (14). Moreover, a stronger explicit solvability condition than (17) is obtained as follows

$$\min\{4\|A\|_\infty, \frac{\eta_{\text{max}} + \eta_{\text{min}}}{\sqrt{\eta_{\text{max}} \eta_{\text{min}}}} \sqrt{\rho(A)}\} < 1$$ (19)

where $\eta_{\text{max}}$ and $\eta_{\text{min}}$ are the maximum and minimum Perron eigenvalue of $A$, respectively.

The results in (19) are also obtained in [21] based on nested interval theorem. In summary, if there is a compact set $D$ such that $f(u)$ is a self-mapping on $D$, then (14) is solvable. In the next section, we obtain the sufficient solvability conditions under the limits of currents of DGs.

**D. SUFFICIENT SOLVABILITY CONDITIONS CONSIDERING CURRENT CONSTRAINTS**

In the steady state, current sharing and average voltage regulation are realized. Current sharing control enables the current of DGs are divided equally in proportion. However, when the loads are small, the current of DGs will be large. Next, we will obtain the condition to ensure the current of DGs doesn’t exceed maximum current. From (1), considering the current constraints of DGs, we obtain the following equation

$$i_s = B_{SS}u_S + B_{SL}u_L \leq i$$ (20)

where $i$ represents the maximum of current of DGs.

Left multiplied by $B_{SS}^{-1}$, (20) becomes

$$u_S I_n + B_{SS}^{-1} B_{SL} u_L \leq B_{SS}^{-1} i$$ (21)

Since $B$ is a Laplacian matrix, then $B_{SS} I_n + B_{SL} I_m = 0_n$, i.e., $B_{SS}^{-1} B_{SL} I_m = 1_n$. For simplifying calculation, let $u_L = u_{1m}, u_S = v_{\text{ref}}$, thus we obtain

$$v_{\text{ref}} I_n - u_{1n} \leq B_{SS}^{-1} i$$ (22)

According to property of matrix norm, following statement holds

$$B_{SS}^{-1} \leq \|B_{SS}\|_\infty I_n$$ (23)

Substituting (23) into (22), we obtain

$$v_{\text{ref}} I_n - u_{1n} \leq \|B_{SS}\|_\infty i$$ (24)

From inequality (24), we obtain

$$u > v_{\text{ref}} - \|B_{SS}\|_\infty \min\{i\}$$ (25)

where $\min\{i\}$ represents the minimum value of $i$.

Combine $u_L$ and (25), the domain of $u_L$ is obtained as follows

$$S = \{u_L | (v_{\text{ref}} - \|B_{SS}\|_\infty \min\{i\}) I_m \leq u_L \leq v_{\text{ref}} I_m\}$$

If (13) has a solution in $S$, the currents of DGs will not exceed $i$. Then, the following question naturally arises:

Q1: How to guarantee that power-flow equation of DC microgrid has a solution in $S$?

According to Brouwer’s fixed-point theorem, if there exists a nonempty compact set $S$ such that $G(u_S)$ is a contraction-mapping, then there exists a unique $u_S^* \in S$ such that $G(u_S^*) = u_S^*$. Base on this idea, a sufficient solvability condition for (13) is obtained as follows.

**Theorem 2:** If the following holds, equation (13) admits a unique solution in $S$

$$v_{\text{ref}} > \frac{\|M\|_\infty}{\|B_{SS}\|_\infty \min\{i\}} + \|B_{SS}\|_\infty \min\{i\}$$ (26)

**Proof:** Clearly, $S$ is a compact convex set. If $G(u_L)$ is a self-mapping in $S$, according to Brouwer’s fixed-point theorem, (13) admits a unique solution in $S$.

Firstly, we prove $G(u_L) \in S$ for any $u_L \in S$.

Since $M > 0$, then for any $u_L \in S$, $G(u_L) < v_{\text{ref}}$, and when condition (26) holds we obtain

$$G(u_L) > v_{\text{ref}} - \|B_{SS}\|_\infty \min\{i\}$$

When (26) holds, we have

$$\frac{\|M\|_\infty}{\|B_{SS}\|_\infty \min\{i\}} - \|B_{SS}\|_\infty \min\{i\} I_m > \|B_{SS}\|_\infty \min\{i\} I_m$$

Then we obtain

$$G(u_L) > (v_{\text{ref}} - \|B_{SS}\|_\infty \min\{i\}) I_m$$

Then we have for any $u_L \in S$, $G(u_L) \in S$, i.e., $G(u_L)$ is self-mapping.

Thus, according to Brouwer’s fixed-point theorem, when condition (26) satisfied, equation (13) admits a unique solution in $S$.

**Remark 2:** The power-flow equation has feasible solution if (26) holds, thus Q1 is answered.

**IV. ROBUST STABILITY CONDITIONS OF THE SYSTEM**

**A. SMALL-SIGNAL MODEL NEAR THE EQUILIBRIUM**

Linearizing (2) near the equilibrium, we obtain

$$\Delta u_L = R_L \Delta i_L$$ (27)

where $R_L = \frac{u^2_L}{p}$ is the equivalent impedance matrix of loads.

Linearizing (1) and (6), the following is obtained

$$\begin{bmatrix} \frac{d \Delta u_S}{dt} \\ \Delta i_S \\ \Delta u_L \end{bmatrix} = \begin{bmatrix} B_{SS} & B_{SL} & 0 \\ B_{LS} & B_{LL} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \Delta u_S \\ \Delta u_L \end{bmatrix} = B \begin{bmatrix} \Delta u_S \\ \Delta u_L \end{bmatrix}$$ (28)

Then, we obtain $\Delta i_S = B_{eq} \Delta u_S$, where $B_{eq} = B_{SS} - B_{SL}(B_{LL} - R^{-1}_L)^{-1} B_{LS}$.

Combine (27) and (28), the dynamic of the system is described as follows

$$\frac{d \Delta u_S}{dt} = -(Q^T B_{eq} + 1_n n^T) \Delta u_S$$
B. ROBUST STABILITY CONDITIONS

Jacobian matrix of the system is given as follows

\[ J_j = -Q\xi QB_{eq} - 1_n 1_n^T \]

Clearly, according to Lyapunov stability theory, if every eigenvalue of \( J_j \) has negative real part, the system is stable.

Define \( J_1 = Q\xi QB_{eq}, J_2 = 1_n 1_n^T \), and \( J = J_1 + J_2 \). The system is stable if the real part of every eigenvalue of \( J \) is positive. Reference [16] points out that matrix \( B_{eq} \) must has at least one negative eigenvalue, and we know that matrix \( J \) is not symmetric, which will make it difficult to obtain the sufficient condition of the real part of every eigenvalue of \( J \) is positive. What’s worse, CPLs usually change over time and different CPLs will cause different \( B_{eq} \). We assume all the CPLs are bounded in \( \psi = \{ p | 0 < p < p_{max}\} \). Thus, following question arises:

Q2: Under what conditions, the system is robust stable when CPLs change in \( \psi \)?

This paper will analyze it and the main conclusions are as follows.

**Theorem 3**: \( J_1 \) is semi-positive stable if the following holds

\[ p_{max} < (v_{ref} - |B_{SS}^{-1}\|\infty \min(i)|)^2 (\rho(A) - \rho_{n-1}(A)) \quad (29) \]

where \( A = tl - B, t = \max(B_{eq}) \), clearly, we can obtain \( A \geq 0 \).

**Proof**: Since \( \xi \) is the Laplacian matrix which has at least one spanning tree, we obtain that \( \xi \) has \( n \) - 1 positive eigenvalues and one zero eigenvalue. As \( Q \) is symmetric and nonsymmetric matrix, we have \( i(i\xi Q) = i(\xi) \). Since \( Q\xi Q \) is the semi-positive definite real symmetric matrix with one zero eigenvalue, according to Lemma 3, if \( i_i(B_{eq}) = 1 \) and \( \eta^T B_{eq}^{-1}\eta < 0 \) (\( \eta \) is the zero eigenvector of \( Q\xi Q \)), we obtain \( i_i(J_1) = i_i(\xi) = 0 \), that is, \( J_1 \) is semi-positive stable. Next, we will obtain the sufficient condition to make \( i_i(B_{eq}) = 1 \) and \( \eta^T B_{eq}^{-1}\eta < 0 \).

According to statement (1) in Lemma 4, \( i_i(B_{eq}) = 1 \) if \( B_{eq} \) is \( N_0^{-} \) matrix. Moreover, we obtain \( B_{eq}^{-1} < O \) according to the statement (2) in Lemma 4. Since \( \eta \) is the zero eigenvector of \( Q\xi Q \), we can express \( \eta \) as \( \eta = cQ^{-1}1_n \), where \( c \in \mathbb{R} \). And because the entries of \( Q^{-1}1_n \) are all positive, we obtain \( \eta^T B_{eq}^{-1}\eta = c 1_n^T Q^{-1}B_{eq}Q^{-1}1_n < 0 \). Then, the focus of the problem is proving that when (29) holds, \( B_{eq} \) is \( N_0^{-} \) matrix.

Define \( B_1 = B - R \), where \( R = \begin{bmatrix} O \\ R^{-1}_1 \end{bmatrix} \).

Since \( B_{eq} \) is a Schur complement of \( B_1 \), then \( B_{eq} \) is \( N_0^{-} \) matrix if \( B_1 \) is \( N_0^{-} \) matrix according to the statement (3) in Lemma 4. Then, we will prove is matrix if (29) holds.

Clearly, we have \( B_1 = tl - (A + R) \). As \( A \geq 0 \) and \( R > O \), then we obtain \( t = \rho(A) < \rho(A + R) \) according to Lemma 5. Since \( v_{ref} - |B_{SS}^{-1}\|\infty \min(i)| < u < v_{ref} \), the following is obtained

\[ \rho(R) < \max(R_{11}^{-1}) < p_{max}/(v_{ref} - |B_{SS}^{-1}\|\infty \min(i)|)^2 \quad (30) \]

When (29) holds, the following is obtained

\[ \rho_{n-1}(A + R) < \rho_{n-1}(A) + \rho_{n-1}(R) \]
\[ < \rho_{n}(A) + \rho(R) \]
\[ < \rho_{n-1}(A) + p_{max} / (v_{ref} - |B_{SS}^{-1}\|\infty \min(i)|)^2 < \rho(A) \]

As \( \rho_{n-1}(A + R) < t < \rho(A + R) \) and \( A + R > O \), we have \( B_1 \) is \( N_0^{-} \) matrix according to Lemma 4. Thus, when (29) holds, \( J_1 \) is semi-positive stable.

The proof is finished.

If (29) holds, the eigenvalues of \( J_1 \) are all nonnegative, however, how is it going when a rank-one matrix \( J_2 \) is added? In the following, we answer this question through two Theorems proposed in the following.

**Theorem 4**: Matrix \( J_1 \) is diagonalizable if (29) is satisfied.

**Proof**: To begin with, matrix \( Q\xi Q \) is symmetric, so it is diagonalizable. Define \( P_1 \) as the orthogonal matrix such that \( P_1^T Q\xi Q P_1 = \Lambda_1 \oplus 0 \), where \( \Lambda_1 = \text{diag}(\lambda_2(Q\xi Q), \ldots, \lambda_n(Q\xi Q)) \). Define \( J_3 = P_1^T J_1 P_1 \), and it can be written as \( J_3 = (\Lambda_1 \oplus 0) P_1^T B_{eq} P_1 \). Note \( M_1 = P_1^T B_{eq} P_1 \), write \( M_1 \) as a block matrix, and \( J_3 \) can be expressed as

\[ J_3 = \begin{bmatrix} \Lambda_1_o \\ \Lambda_1_o \end{bmatrix} M_1 = \begin{bmatrix} \Lambda_1_o \\ \Lambda_1_o \end{bmatrix} \begin{bmatrix} M_2 \beta_1 \\ \beta_1^T a \end{bmatrix} = \begin{bmatrix} \Lambda_1 M_2 \Lambda_1 \beta_1 \\ 0 \end{bmatrix} \]

Since \( \Lambda_1 \) is positive definite and \( M_2 \) is symmetric, there is a nonsymmetric matrix \( P_2 \) such that \( P_2^T \Lambda_1 M_2 P_2 = \Lambda_2 \), where \( \Lambda_2 = \text{diag}(\delta_1, \ldots, \delta_{n-1}) \), \( \delta_i \) is the eigenvalue of \( J_1 \). Define \( J_4 = P_2^T J_3 P_2 = P_2^T \Lambda_1 \beta_1 \), where \( P_3 = P_2 \oplus 1 \). Then \( J_4 \) can be written as \( J_4 = \begin{bmatrix} \Lambda_2 \beta_2; 0^T \end{bmatrix} \).

Define \( P_4 = \begin{bmatrix} I - \Lambda_2^{-1} \beta_2; 0^T \end{bmatrix} \). We obtain the following equation

\[ P_4^{-1} J_4 P_4 = P_4^{-1} P_3^{-1} P_1^T J_1 P_1 P_3 P_4 = \begin{bmatrix} \Lambda_2 \beta_2; 0^T \end{bmatrix} \]

According to (31), matrix \( J_1 \) is diagonalizable. The proof is accomplished.

For convenience, note \( P_5 = P_1 P_3 P_4 \) and \( \Lambda_3 = \Lambda_2 \oplus 0 \).

Define \( J_5 = P_5^{-1} J_4 P_5 \), then \( J_5 \) is cospectral with \( J \). Note \( \eta = P_5^{-1}_1 \eta \) and \( \xi = P_5^T 1_n \), then \( J_5 = \Lambda_3 + \eta \xi^T \).

If \( J_5 \) is positive definite, we can make the empirical conclusion that \( J_5 \) will be positive stable if the absolute value of \( b_2 \) is small enough.

Unfortunately, \( \Lambda_3 \) has a zero root. Consequently, it will need more math tricks to obtain the stability conditions.

**Theorem 5**: If (29) is satisfied, system is robust stable when loads change in \( \psi \).

**Proof**: According to the theory of the field of values, we have \( i(J_5) = i(J_5 + J_2^T) \). Note \( J_6 = J_5 + J_5^T \) and \( J_7 = \eta \xi^T + \xi \eta^T \), then \( J_6 = 2\Lambda_3 + J_7 \). According to the corollary in [17], the spectral of matrix \( J_7 \) can be expressed as

\[ \lambda_1 = \eta T \xi + \|\eta\|_2 \|\xi\|_2 \]
\[ \lambda_2 = \eta \xi^T \|\eta\|_2 \|\xi\|_2 \]
\[ \lambda_3 = \cdots = \lambda_n = 0 \]

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According to Cauchy-Schwartz inequality, matrix $J_7$ has at most one negative root. Since $i(A_5) = [n - 1 0 1]$, $J_n$ has at most one negative root according to Lemma 5. Thus, $J_5$ is positive definite if and only if the determinant of $J_5$ is positive.

According to Lemma 6, the determinant of $J_5$ can be obtained

$$
\det(J_5) = \eta_n \xi_n \prod_{i=1}^{n-1} \sigma_i
$$

Since $\sigma_i(i = 1 \cdots n - 1)$ is a positive scalar, so only $\eta_n \xi_n$ is positive can make that the inequality $\det(J_5) > 0$ hold.

Note $P^{-1} = \begin{bmatrix} \vartheta_1 & \vartheta_2 & \cdots & \vartheta_n \end{bmatrix}$, and $P \psi = \begin{bmatrix} \gamma_1 & \gamma_2 & \cdots & \gamma_n \end{bmatrix}$, where $\vartheta_i, \gamma_i$ represents the left and right eigenvector of $J_1$, respectively. $\vartheta_n, \gamma_n$ is the left and right zero eigenvector of $J_1$, respectively. Since $1^T_n Q^{-1} J_1 = 0_n^T$ and $J_1 B_{eq}^{-1} Q^{-1} 1_n = 0_n$, we obtain that $Q^{-1} 1_n$ and $B_{eq}^{-1} Q^{-1} 1_n$ is the left and right zero eigenvector, respectively. Obviously, $\eta_n = \vartheta_n^T 1_n$ and $\xi_n = 1_n^T \gamma_n$. Thus we can know that $\vartheta_n = c_1 Q^{-1} 1_n$ and $\gamma_n = c_2 B_{eq}^{-1} Q^{-1} 1_n$, where $c_1, c_2 \in \mathbb{R}$. Then, $\eta_n$ and $\xi_n$ can be obtained as

$$
\eta_n = c_1 1_n^T Q^{-1} 1_n \xi_n = c_2 1_n B_{eq}^{-1} Q^{-1} 1_n
$$

(32)

Since $P_5^{-1} P_5 = I$, we obtain

$$
\vartheta_n^T \gamma_n = c_1 c_2 1_n^T Q^{-1} B_{eq}^{-1} Q^{-1} 1_n = 1
$$

(33)

Combining with (32) and (33), we have

$$
\eta_n \xi_n = (1_n^T Q^{-1} B_{eq}^{-1} Q^{-1} 1_n)^{-1} (1_n^T Q^{-1} 1_n) (1_n^T B_{eq}^{-1} Q^{-1} 1_n)
$$

(34)

According to Theorem 3, when (29) holds, $B_{eq}$ is $n_0$-matrix. Then we obtain

$$
\begin{bmatrix}
1_n^T Q^{-1} 1_n < 0 \\
1_n^T B_{eq}^{-1} Q^{-1} 1_n < 0
\end{bmatrix}
$$

(35)

Because $1_n^T Q^{-1} 1_n > 0$, we have $\eta_n \xi_n > 0$. As a result, $\det(J_5) > 0$ is satisfied, that is, $J$ is positive stable which shows that the system is robust stable when loads change in $\psi$. Thus, $Q$ is answered.

V. SIMULATION

In this section, we simulate a meshed DC microgrid with 20 DGs and 50 CPLs which is shown in FIGURE 3. The square and circle represent DGs and CPLs, respectively. The DGs are controlled as (6). The black line represents the transmission line resistance. The green numbers are the resistances of transmission line, and the black numbers are the identifiers of nodes.

According to Theorem 1, the power-flow equation of the DC microgrid considering current constraints has the feasible solution. And current sharing and voltage regulation are achieved. Let $\min(i_0) = 300A$. Denote $x$ as $x = 1 + \frac{1}{\sum_{i=1}^{10} \max(i)}$.

We assume the proportion of current among DGs is $3 : \cdots : 3 : 2 : \cdots : 2$, then take $Q = diag(2 \times 1_{10}^T, 3 \times 1_{10}^T)$.

The Laplacian matrix of the communication graph is designed as

$$
\mathbf{L} = \begin{bmatrix}
20 & -10 & \cdots & 0 & -10 \\
-10 & 20 & \cdots & 0 & \vdots \\
\vdots & \ddots & \ddots & \vdots & \vdots \\
0 & \cdots & -10 & 20 & -10 \\
-10 & 0 & \cdots & -10 & 20
\end{bmatrix}
$$

To verify the existence condition for the power-flow equation, we design two cases.

Case 1: The CPL vector is as $p = [3 \times 1_{20}^T, 4 \times 1_{20}^T, 5 \times 1_{10}^T]kW$. Then $x$ is obtained as $x = 1005.7$. Let $\nu = 1006$.

Case 2: The power of CPLs is the same with Case 1. Let $\nu = 807$.

According to Theorem 1 and 3 in [18], if (13) is solvable, Newton-Raphson Method in Feasible Power-Flow is convergent as long as the power-flow equation is solvable. Thus, we will verify whether (13) has a solution according to Newton-Raphson Method.

The simulation results are in FIGURE 4 (a) and (b). FIGURE 4 (a) shows that Newton-Raphson Method converges to the solution if conditions (26) is satisfied. FIGURE 4 (b) shows that the proposed algorithm may not be convergent if conditions (26) is not satisfied. The maximum
value of $i_S$ calculated by Case 1 is $265 < 300$, which shows that $i_S$ does not exceed the current constraint value $i$. The result in FIGURE 4 (c) shows that Case 1 is stable, which verifies the correctness of the proposed solvability conditions considering current constraint.

Thus, Theorem 2 is verified.

Let $\tau = (v_{\text{ref}} - \|R_S\|_{\infty} \min(|i|))^2(\rho(A) - \rho_{n-1}(A))$. To verify the robust stability condition, we design two cases.

Case 3. Let $v = 400$, then $\tau = 1.72 \text{kW}$. The CPL vector is $p = [0.8 \times 1_{25}, 0.7 \times 1_{25}]$ for $t < 0.2s$, $p = [1.3 \times 1_{25}, 1.2 \times 1_{25}]$ for $0.2 \leq t < 0.4s$, $p = [1.7 \times 1_{25}, 1.5 \times 1_{25}]$ for $t > 0.4s$.

Case 4. Let $v = 300$, then $\tau = 1.1 \text{kW}$. The CPL vector is $p = [0.8 \times 1_{20}, 0.7 \times 1_{20}]$ for $t < 0.2s$, $p = [1 \times 1_{20}, 1 \times 1_{20}]$ for $0.2 \leq t < 0.4s$, $p = [1.3 \times 1_{20}, 1.3 \times 1_{20}]$ for $t > 0.4s$.

FIGURE 4. (d) and (e) shows that if $p_{\text{max}} < \tau$, that is, (29) holds, the system is robust stable when loads change in $\psi$. What’s more, current sharing and average voltage regulation are realized as shown in FIGURE 4. (d) and (f).

FIGURE 4. (g) and (h) show that if (29) doesn’t hold, the system is unstable and voltage collapse occurs.

Thus, Theorem 5 is verified.

VI. CONCLUSION

In this paper we design a distributed control method in DC microgrid. Firstly, a distributed control method aiming at current sharing and average voltage regulation is proposed. Secondly, the properties of the sub-network are obtained, on this basis, the solvability conditions of the nonlinear power-flow equations are obtained based on Brouwer’s fixed-point theorem. Thirdly, the robust stability condition independent of the loads real-time information is obtained based on the property of $N_0$- matrix. Moreover, we give an effective method to analyze the inertia of a typical matrix. Finally, the effectiveness of the proposed theorem is verified by using simulations.

APPENDIX

The definitions and lemmas used in this paper are shown as follows.

**Definition 1:** For a positive vector $x=[x_1\ x_2\ \cdots\ x_n]^T$, we define $x^{-1}=[x_1^{-1}\ x_2^{-1}\ \cdots\ x_n^{-1}]^T$. Let $O$ be a matrix whose elements are all 0. Define $I_n(0_a)$ as the $n$-dimensional vector which all entries are 1(0) and $I$ is the unit matrix with appropriate dimension.

**Definition 2:** Let $\mathbb{R}^m$, $\mathbb{R}^{m \times m}$ denote the set of the real numbers, real $m$-dimensional vector, and real $n \times m$ matrices, respectively. Suppose $M$ is a matrix partitioned into the form $M = \begin{bmatrix} A & B \\ C & D \end{bmatrix}$, the Schur complement of $A$ in $M$ is $S_A = D - CA^{-1}B$ if $A$ is nonsingular.

**Definition 3:** If there is no permutation matrix $T$, such that matrix $T^TA$ has the following form

\[
T^TA = \begin{bmatrix} A_{11} & A_{12} \\ O & A_{22} \end{bmatrix},
\]

then $A$ is irreducible matrix.
Definition 4: For $B \in \mathbb{R}^{m \times m}$, define $i_+(B)$, $i_-(B)$, $i_0(B)$ as the numbers of eigenvalues of $B$, counting multiplicities, with positive, negative and zero real parts, respectively. The row vector $ii(A) = [i_+(A) \ i_-(A) \ i_0(A)]$ is called the inertia of $A$.

Lemma 1: Brouwer’s fixed-point theorem [19]. Let $D \in \mathbb{R}^n$ be a nonempty compact set and $G(x) : \mathbb{R}^n \to \mathbb{R}^n$ satisfies the condition: $G(x)$ is a self-mapping, i.e., $\forall x \in D, G(x) \in D$. Then, there is a unique vector $x^* \in D$ such that $x^* = G(x^*)$.

Lemma 2: Let $A \in \mathbb{R}^{m \times m}$ be a Laplacian matrix of a strongly connected graph, then all the leading principal submatrices are $M$-matrices. If $B$ is an irreducible $M$-matrix, $B^{-1} > O$ [20].

Lemma 3: For a real symmetric semi-positive definite matrix $A \in \mathbb{R}^{m \times m}$ with only one zero eigenvalue and a symmetric matrix $B$ with negative eigenvalue, we have $i_-(AB) = 0$ if $i_-(B) = 1$ and $\eta^T B^{-1} \eta < 0$, where $\eta$ is the zero eigenvalue of $A$ [13].

Lemma 4: Let $M \in \mathbb{R}^{n \times n}$ takes the form $M = \alpha I - N$, $M$ is $N_0$-matrix if $N \geq O$ and $\rho_{n-1}(N) \leq \rho < \rho(N)$, where $\rho()$ and $\rho_{n-1}()$ are the spectral radius of matrix and the maximum of the spectral radius of all principal submatrices of $N$ of order $n-1$, respectively. And the following statements hold:

1. $M$ has exactly one negative eigenvalue.
2. If $A$ is a nonsingular principal submatrix of $M$, then $(M/A)$ is also $N_0$-matrix.
3. $M^{-1}$ exists and $M^{-1} \leq O$ [21].

Lemma 5: Let $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_n$ and $\beta_1 \leq \beta_2 \leq \cdots \leq \beta_n$ be the eigenvalues of the real symmetric matrix $A$ and $B$, respectively, then

$$\lambda_i + \beta_1 \leq \eta_i \leq \lambda_i + \beta_n$$

where $\eta_1 \leq \eta_2 \leq \cdots \leq \eta_n$ are the eigenvalues of matrix $A + B$. If $A > B > O$, then $\rho(A) > \rho(B)$ [13].

Lemma 6: Define $M_1 = \Lambda + ab^T$, where $\Lambda = c_1 \oplus c_2 \oplus \cdots \oplus c_{n-1} \oplus 0$, $a = [a_1 \ a_2 \ \cdots \ a_n]$, $b = [b_1 \ b_2 \ \cdots \ b_n]$, then we obtain

$$\det(M_1) = a_nb_n \prod_{i=1}^{n-1} c_i$$ [20].

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