The $S_3$ Symmetric Model with a Dark Scalar

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Abstract

We study the $S_3$ symmetric extension of the Standard Model in which all the irreducible representations of the permutation group are occupied by $SU(2)$ scalar doublets, one of which is taken as inert and can lead to dark matter candidates. We perform a scan over parameter space probing points against physical constraints ranging from unitarity tests to experimental Higgs searches limits. We find that the latter constraints severely restrict the parameter space of the model. For acceptable points we compute the value of the relic density of the dark scalar candidates and find that it has a region for low dark matter masses which complies with the Higgs searches bounds and lies within the experimental Planck limit. For masses $\gtrsim 80$ GeV the value of the relic density is below the Planck bound, and it reaches values close to it for very heavy masses $\sim 5$ TeV. In this heavy mass region, this opens the interesting possibility of extending the dark sector of the model with additional particles.

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1. Introduction

One of the main challenges in particle physics at present is to find the nature of dark matter (DM). It is generally accepted that it should be neutral, cold and weakly interacting (although other possibilities exist), and there are proposals for scalar, fermion and vector particles that satisfy the criteria [1]. Among the scalar candidates a very interesting proposal is to introduce inert Higgs scalars, i.e. that do not couple to matter, which is usually achieved by introducing an extra $Z_2$ discrete symmetry, and that do not acquire a vacuum expectation value (vev), thus guaranteeing the stability of the DM candidate; for a single inert scalar this model is referred to in the literature as Inert Doublet Model (IDM or i2HDM) [2–11].

Among the numerous proposals to extend the scalar sector of the standard model, the 3-Higgs Doublet Model (3HDM) with an $S_3$-family symmetry (S3-3H) presents interesting phenomenology, such as the prediction of a non zero reactor neutrino mixing angle $\theta_{13}$ and of a CKM matrix in accordance with the experimental results [12]. The 3HDM with $S_3$ symmetry has been extensively studied in different contexts (see for instance, [12–19], and references therein). The aim of this project is to extend this model to a 4HDM in order to have dark matter candidates, without spoiling the good features of the S3-3H model in the quark and lepton sectors. To do this we occupied all irreducible representations of the $S_3$ symmetry: one symmetric singlet, one antisymmetric singlet and one doublet. The S3-3H model is constituted by the singlet symmetric and doublet representations, with all these Higgs scalars acquiring vacuum expectation values. The fourth Higgs doublet is assigned to the antisymmetric singlet representation and assumed to be inert, it does not acquire a vev and we impose a $Z_2$ symmetry to ensure the stability of the potential dark matter candidates.

In this letter we present an analysis of the parameter space of the model focusing in the determination of the relic density of the dark scalar for physically acceptable points by requiring them to satisfy numerous physical constraints. In the study presented here we limit the calculations to include only tree level quantities, for example the values of the quartic couplings are approximated by on shell values. While higher order corrections can be of sizeable importance for non-supersymmetric models (see e.g. [20, 21]), an analysis including full loop corrections is outside the scope of this work, and we leave it for future research.
2. The model

In the $S^3$ symmetric model the scalar sector of the SM is extended with additional $SU(2)$ scalar doublets $\Phi_k$ with definite transformation properties with respect to the permutation symmetry. Whilst the matter sector content of the model remains the same as the SM, the Yukawa lagrangian is required to be invariant also with respect to $S^3$. This, together with the mixing of the scalars after electroweak symmetry breaking, leads to Yukawa terms that can be very different from the SM, for example the proportionality of the fermion masses to single Yukawa couplings is in general lost in the extended model. Since our primary focus in this letter will be centred in the properties of the dark scalar, in particular a probe for the values of the relic density of this particle in the model parameter space, we’ll make simplifying assumptions over the Yukawa lagrangian’s explicit form which we shall argue not to have a strong impact in the conclusions driven from our results.

2.1. The scalar sector

We accommodate four $SU(2)$ doublets into the irreducible representations of the permutation group $S^3$, denoting the symmetric and antisymmetric scalars by $\Phi_s$ and $\Phi_a$ respectively, while the remaining two doublet scalars $\Phi_1$ and $\Phi_2$ are arranged in a column vector conforming the $S^3$ doublet. The (invariant and renormalizable) scalar potential is a mixture of the potentials known from the studies of the three Higgs model with the permutation symmetry (see for instance [17–19, 22, 23]), and can be written as:

$$V = V_2 + V_4 + V_{4s} + V_{4a} + V_{4sa},$$

where $V_2$ comprises the quadratic terms:

$$V_2 = \mu_0^2 \Phi_s^\dagger \Phi_s + \mu_1^2 (\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2) + \mu_2^2 \Phi_a^\dagger \Phi_a,$$

$V_4$ contains quartic terms involving $\Phi_1$ and $\Phi_2$ only:

$$V_4 = \lambda_1 (\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2)^2 + \lambda_2 (\Phi_1^\dagger \Phi_2 - \Phi_2^\dagger \Phi_1)^2 + \lambda_3 [(\Phi_1^\dagger \Phi_2)^2 + (\Phi_2^\dagger \Phi_2)^2],$$

while $V_{4s}$ and $V_{4a}$ represent the quartic terms involving $\Phi_s$ and $\Phi_a$ respectively:
\[ V_{4a} = \lambda_4 ([\Phi_s^\dagger \Phi_1](\Phi_1^\dagger \Phi_2 + \Phi_2^\dagger \Phi_1) + (\Phi_s^\dagger \Phi_2)(\Phi_1^\dagger \Phi_1 - \Phi_2^\dagger \Phi_2) + \text{h.c.}] \\
+ \lambda_5 (\Phi_s^\dagger \Phi_a)(\Phi_1^\dagger \Phi_1 + \Phi_2^\dagger \Phi_2) \\
+ \lambda_6 ([\Phi_s^\dagger \Phi_1](\Phi_1^\dagger \Phi_s) + (\Phi_s^\dagger \Phi_2)(\Phi_2^\dagger \Phi_s)] \\
+ \lambda_7 ([\Phi_s^\dagger \Phi_1](\Phi_s^\dagger \Phi_1) + (\Phi_s^\dagger \Phi_2)(\Phi_s^\dagger \Phi_2) + \text{h.c.}] \\
+ \lambda_8 (\Phi_s^\dagger \Phi_s)^2 \\
\] (4)

The expression for \( V_{4a} \) is very similar to eq. (4) with \( \Phi_a \) replacing \( \Phi_s \) and quartic couplings \( \lambda_9, \ldots, \lambda_{13} \), except that the \( \lambda_9 \) term analogous to the \( \lambda_4 \) term has \( \Phi_1 \) and \( \Phi_2 \) interchanged. Finally the mixed \( \Phi_s, \Phi_a \) term is given by:

\[ V_{4sa} = \lambda_{14} (\Phi_s^\dagger \Phi_a \Phi_a^\dagger \Phi_s) + \lambda_{15} (\Phi_s^\dagger \Phi_s \Phi_a^\dagger \Phi_a + \text{h.c.}) \] (5)

In the following we will assume no additional sources of CP violation other than those present in the SM and hence we shall take the quartic couplings \( \lambda_i, i = 1 \ldots 14 \) to be real and also restrict their absolute values with the usual perturbativity condition \( |\lambda_i| < 4\pi \). In order to force the scalar \( \Phi_a \) to be inert we introduce an additional discrete \( Z_2 \) symmetry with respect to which all fields are even except those with subindex \( a \), taken as \( Z_2 \)-odd. This gets rid of the \( \lambda_9 \) and \( \lambda_{15} \) term in the potential leaving only terms with even powers of \( \Phi_a \). Incidentally this also leads to the appearance of an additional symmetry of the potential under the interchange \( \Phi_1 \to -\Phi_1 \), this fact explains the absence of vertices with odd number of fields with subindex 1 in the Feynman rules.

After electroweak symmetry breaking all the scalar doublets acquire a vacuum expectation value (vev) denoted by \( v_s, v_1, v_2 \) and \( v_a \) respectively. However, in order to avoid the explicit breaking of the \( Z_2 \) symmetry we fix \( v_a = 0 \) and henceforth from the minimization conditions for the scalar potential (tadpole equations) the fourth equation \( \partial V/\partial v_a = 0 \) is automatically satisfied \(^1\). The three minimization equations \( \partial V/\partial v_i = 0, i = s, 1, 2 \) reduce to those of the three Higgs doublet model with \( S_3 \) symmetry (e.g. \cite{17, 18}),

\(^1\)Many of the expressions presented here can be obtained using SARAH \cite{24–27}; the model files can be downloaded from the SARAH model database \cite{28}.
\[ 
\begin{align*}
\mu_0^2 &= -\frac{1}{2}(\lambda_5 + \lambda_6 + 2\lambda_7)(v_1^2 + v_2^2) - \lambda_8 v_s^2 + \frac{\lambda_4(v_2^2 - 3v_1^2)v_2}{2v_s} \\
\mu_1^2 &= -\frac{1}{2}(\lambda_5 + \lambda_6 + 2\lambda_7)v_s^2 - (\lambda_1 + \lambda_3)(v_1^2 + v_2^2) - 3\lambda_4v_2v_s \\
\mu_1^2 &= -\frac{1}{2}(\lambda_5 + \lambda_6 + 2\lambda_7)v_s^2 - (\lambda_1 + \lambda_3)(v_1^2 + v_2^2) + \frac{3\lambda_4v_s(v_2^2 - v_1^2)}{2v_2}
\end{align*}
\]

whose more general CP preserving solution is \( v_1 = \sqrt{3}v_2 \), with the usual SM vev given by \( v = \sqrt{v_s^2 + 4v_2^2} = 246 \text{ GeV} \). It is convenient to parametrize in spherical coordinates,

\[ v_s = v \cos \theta, \quad v_1 = v \sin \theta \cos \phi, \quad v_2 = v \sin \theta \sin \phi, \quad (7) \]

where \( \theta \in (0, \pi) \) and \( \phi \in (0, 2\pi) \). With this parametrization we get \( \tan^2 \phi = \frac{1}{3} \) and \( v_s = v \cos \theta, \quad v_2 = \frac{1}{2} v \sin \theta \). We choose \( \tan \theta = 2v_2/v_s \) as one of the independent parameters in the numerical calculations.

Parametrizing the Higgs doublets as

\[ \Phi_s = \left( \begin{array}{c} \frac{h_s^+}{\sqrt{2}} \\ (v_s + h_s^m + ih_s^p)/\sqrt{2} \end{array} \right) \quad (8) \]

and similarly for \( \Phi_1, \Phi_2 \) and \( \Phi_a \) (here the indices \( n \) and \( p \) refer to neutral scalar and pseudoscalar respectively, and we use primes to distinguish from the mass eigenstates which we will denote with the same letters except when explicitly stated otherwise) it is straightforward to obtain the mass and mixing matrices and mass eigenstates for the scalar fields. The \( Z_2 \) odd fields do not mix (e.g. \( h_n^m = h_n^p \)) and therefore the mass and mixing matrices have block diagonal form. In the case of the neutral scalar fields the submatrix mixing the \( Z_2 \) even fields \( h_s^m, h_1^m \) and \( h_2^m \) into the mass eigenstates \( H, H_3 \) and \( h \) takes the form:

\[ Z = \left( \begin{array}{ccc} \cos(\alpha) & 0 & \sin(\alpha) \\ 0 & 1 & 0 \\ -\sin(\alpha) & 0 & \cos(\alpha) \end{array} \right) \left( \begin{array}{ccc} 1 & 0 & 0 \\ 0 & \frac{1}{2} & -\frac{\sqrt{3}}{2} \\ 0 & \frac{\sqrt{3}}{2} & \frac{1}{2} \end{array} \right) \quad (9) \]

so that the neutral scalar mass matrix \( m_{\Phi_s}^2 \) is diagonalized by

\[ \text{diag}(m_H^2, m_{H_3}^2, m_h^2) = Zm_{\Phi_s}^2Z^T \quad (10) \]
here we have resorted to a notation for the neutral scalar mass eigenstates that facilitates comparison with analogue expressions found in the literature of the Two Higgs Doublet Model (THDM) (see e.g. [29]), thus \( H \) and \( h \) are the physical fields encountered also in the THDM, with the latter being the lightest of the two. On the other hand \( H_3 \) is an additional field not present in THDMs and cannot be related in any form with the SM Higgs, since it does not have couplings to the vector bosons [17]. From the first rotation in (10) the submatrix \( M_{ij} \) with \( i, j = 1, 2 \), analogue to the mixing matrix of the THDM is found to be:

\[
\begin{align*}
M_{11} & = \frac{1}{2} v^2 (4 \lambda_8 \cos^2 \theta - \lambda_4 \sin^2 \theta \tan \theta) \\
M_{12} & = \frac{1}{2} v^2 \sin \theta (2(\lambda_5 + \lambda_6 + 2\lambda_7) \cos \theta + 3\lambda_4 \sin \theta) \\
M_{22} & = \frac{1}{2} v^2 \sin \theta (3\lambda_4 \cos \theta + 4(\lambda_1 + \lambda_3) \sin \theta)
\end{align*}
\]

and defines the rotation angle \( \alpha \) through the expressions (e.g. see [30]):

\[
\begin{align*}
\sin (2\alpha) & = \frac{2M_{12}}{\sqrt{(M_{11} - M_{22})^2 + 4(M_{12})^2}} \\
\cos (2\alpha) & = \frac{M_{11} - M_{22}}{\sqrt{(M_{11} - M_{22})^2 + 4(M_{12})^2}}
\end{align*}
\]

The mass eigenvalues corresponding to \( H \) and \( h \) are neatly expressed in terms of the elements of \( M \):

\[
m_{H,h}^2 = \frac{1}{2} \left( M_{11} + M_{22} \pm \sqrt{(M_{11} - M_{22})^2 + 4(M_{12})^2} \right)
\]

where the lower sign corresponds to \( h \). The mass eigenvalue of \( H_3 \) is simply:

\[
m_{H_3}^2 = -\frac{9}{4} \lambda_4 v^2 \sin (2\theta)
\]

The connection of \( H \) and \( h \) with the SM Higgs is done as usual through the decoupling limit defined by the relation \( \cos (\theta - \alpha) \approx 0 \), in this limit \( h \) has SM-like couplings and can be identified with the SM Higgs. In the numerical calculation we impose the mass of \( h \) to be always around 125 GeV, so that
for points satisfying the decoupling limit we recover the measured mass of the scalar discovered at CERN [31, 32] within experimental error bars.

The dark matter candidate can be either the neutral scalar $h_n$ or the pseudoscalar $h_p$, both come from the inert doublet and have odd charges with respect to the $Z_2$ symmetry, and their masses are found to be:

$$M_{h_n}^2 = \mu_2^2 + \frac{1}{2}v^2(\lambda_{14}\cos^2\theta + \lambda_\chi^+ \sin^2\theta)$$

$$M_{h_p}^2 = \mu_2^2 + \frac{1}{2}v^2(\lambda_{14}\cos^2\theta - \lambda_\chi^- \sin^2\theta)$$

where the effective parameter $\lambda_\chi^\pm = \lambda_{10} + \lambda_{11} \pm 2\lambda_{12}$ characterizes the difference between the masses of the dark neutral particles.

The rest of the field content of the model include additional pseudo-scalar fields $A, h_p^2$, and additional charged fields $H^\pm, h_2^\pm$ and $h_3^\pm$; $A$ and $H^\pm$ being the analogue of the fields appearing in the THDM. Explicit expressions for the masses of these fields are given below:

$$M_{h_n}^2 = \mu_2^2 + \frac{1}{2}v^2\lambda_{10}\sin^2\theta$$

$$M_{h_p}^2 = \mu_2^2 + \frac{1}{2}v^2(\lambda_6 + 2\lambda_7 + 4\lambda_3 + (\lambda_6 + 2\lambda_7 - 4\lambda_3)\cos 2\theta + 5\lambda_4 \sin 2\theta)$$

$$M_{h_2^\pm} = \frac{1}{2}v^2(\lambda_6 + 2\lambda_7 + \lambda_4 \tan \theta)$$

$$M_{h_3^\pm} = \frac{1}{2}v^2(4\lambda_7 + \lambda_4 \tan \theta)$$

$$M_{h_2^\pm} = \frac{1}{2}v^2 \sin^2\theta(4(\lambda_2 + \lambda_3) + 5\lambda_4 \cot\theta + 4\lambda_7 \cot^2\theta)$$

Following the procedure outlined in [19], the stability constraints for this model can be obtained (see [33]). The (tree level) unitarity conditions are calculated using the LQT prescription [34]; we consider all possible combination of two-particle scattering processes (including charged ones) at high energies involving longitudinal gauge bosons and scalars. As is well known from the Goldstone boson equivalence theorem, these amplitudes can be calculated for high energies from the underlying Higgs-Goldstone system. The resulting $S$ matrix is diagonalized numerically and the imposed condition is for the largest eigenvalue to be less than $8\pi$ in absolute value. To help restrict further the parameter space we impose also unitarity conditions at finite energy $\sqrt{s}$, in which the full tree level (scalar) two particle scattering matrix is constrained in an analogous fashion as in the LQT case but for an entire range of scattering energies above the weak scale. This procedure not
only strengthens the constraints over the quartic couplings but also introduces new ones over the trilinear scalar couplings, which can be important (as demonstrated recently in [35, 36]).

It is convenient to have as many physical observables as possible serving as free parameters to increase the efficiency of the scanning algorithm, for this purpose it is straightforward to invert equations (12) through (17) to work with the set of free parameters given by all the physical masses plus the set \( \mu^2, \lambda_{13}, \lambda_{14}, \tan \theta \) and \( \alpha \), the only subtlety arises when manipulating equations (12) and (13) where there are two solutions for \( (\lambda_1, \lambda_5, \lambda_8) \) in terms of \( (m_H^2, m_h^2, \alpha) \) and care must be taken to choose the one consistent with \( (m_H^2 - m_h^2) \cos 2\alpha = M_{11} - M_{22} \) which follows from the same equations\(^2\).

2.2. The matter sector

In the matter sector, invariance under the \( S_3 \) symmetry implies a lagrangian of the form:

\[
-\mathcal{L}_Y = Y^{D_1}_{\alpha \beta} \bar{Q}_{\alpha L} \Phi_1 q_{\beta R} + Y^{D_1}_{\alpha \beta} \bar{Q}_{\alpha L} \Phi_2 q_{\beta R} + Y^{D_2}_{\alpha \beta} \bar{Q}_{\alpha L} \Phi_2 q_{\beta R} + \ldots \quad (18)
\]

where the transformation properties of the matter fields under \( S_3 \) are taken as in reference [12].

Here \( D \) denotes \( d \)-type quarks and the dots refer to similar expressions for \( u \)-type quarks and leptons plus hermitian conjugate. Flavor indices are denoted by \( \alpha \) and \( \beta \), quark left doublets and right singlets are denoted by \( Q_L \) and \( q_R \) respectively, while unprimed quantities will refer to the mass eigenstate basis. The Yukawa matrices are given by:

\[
Y^{D_1} = \begin{pmatrix}
0 & \frac{y_{d3}}{2} & \frac{y_{d5}}{\sqrt{2}} \\
\frac{y_{d3}}{2} & 0 & 0 \\
\frac{y_{d5}}{\sqrt{2}} & 0 & 0
\end{pmatrix}, \quad Y^{D_2} = \begin{pmatrix}
\frac{y_{d3}}{2} & 0 & 0 \\
0 & \frac{y_{d4}}{2} & \frac{y_{d5}}{\sqrt{2}} \\
0 & \frac{y_{d4}}{\sqrt{2}} & 0
\end{pmatrix} \quad (19)
\]

and

\[
Y^{Ds} = \text{diag}(\frac{y_{d2}}{\sqrt{2}}, \frac{y_{d2}}{\sqrt{2}}, y_{d1}) \quad (20)
\]

\(^2\text{We also take } \cos 2\alpha > 0 \text{ to ensure the correctness of the diagonalization (10) which is only valid for } |M_{11} - M_{22}| - (M_{11} - M_{22}) = 0.\)
and similar expressions for $u$-quarks and leptons. It is well known from the studies of multi-Higgs models that the coupling of the fermions to extra scalar doublets can induce flavor changing currents even at tree level. Nevertheless, it has been shown in previous research concerning the $S_3$ model (e.g. \cite{12, 15}) that values of the parameters $y_{d_1}$, \ldots calculated from fits to the CKM and PMNS mixing matrices are naturally small, so that experimental bounds on flavor changing processes are not violated. For our purposes, we will assume in the following that the off-diagonal Yukawa couplings, given its small size, do not contribute noticeably to the physical processes calculated in the next section, and thus we simply take this couplings identically zero. Writing explicitly the quark fields for each family, with these assumptions after EWSB the parts in the Yukawa lagrangian involving neutral scalar fields become:

$$-\mathcal{L}_Y = y_{d_1} (\bar{b}'_L h^n_s b'_R) + \frac{y_{d_2}}{\sqrt{2}} (\bar{d}'_L h^n_s d'_R + \bar{s}'_L h^n_s s'_R) + \ldots$$  \hspace{1cm} (21)

After rotating the scalars to the mass eigenstate basis using (9) we find

$$h^n_s = H \cos \alpha - h \sin \alpha$$  \hspace{1cm} (22)

and thus $H_3$ does not couple to the fermions in this limit. In the numerical analysis we make the approximation of massless fermions for the first two families since the contributions to the relevant observables are dominated by the masses of the third family of fermions.

3. Numerical analysis and results

The implementation of the model is made through \texttt{SARAH} \cite{24–27} taking advantage of this package’s functionality to generate model files for other tools. We perform a random scan of the parameter space filtering points that do not satisfy all the conditions mentioned before; the generation of the scattering matrix to calculate unitarity constraints in the large $s$ limit is done using \texttt{FeynArts} \cite{37} and \texttt{FormCalc} \cite{38}, whereas the calculation of the finite energy unitarity constraints is done with the latest \texttt{SARAH} update \cite{39} only that we chose to port this specific part of the \texttt{SARAH}-generated \texttt{SPheno} code to our scanning module for purposes of optimization\footnote{A slight optimization per point is gained by only executing \texttt{SPheno} for points that comply with all unitarity constraints, this augments the efficiency of the code particularly where necessary.}; for the handling of $t$...
Figure 1: Mass of the DM candidate as a function of $\tan \theta$ (left panel), and value of the DM relic density as a function of the DM mass. The dark blue points (set A) are the ones that comply with stability and unitarity constraints, the light blue points (set B) are also compatible with the experimental bounds for extra scalar searches (see text), the red points also satisfy the decoupling limit and the green points in the right panel lie within the experimental Planck bound.

and $u$ poles in the calculation of the scattering amplitudes, with hindsight we chose the weakest limits described in [39] since already for this choice finding physical points is computationally very expensive; the energy interval defined for these computations is taken to be 500 to 5000 GeV. The generation of SLHA [40, 41] input files for HiggsBounds [42–46] and MicrOMEGAS [47] is done using the SARAH–SPheno [48–50] framework. We use HiggsBounds to further filter points that do not comply with current experimental limits from Higgs searches, and finally MicrOMEGAS is utilized to compute the value of the relic density and annihilation cross section of the dark particle (the lightest of the $Z_2$-odd neutral scalars) for points that satisfy all the constraints. We only show results for the case where the dark scalar $h_n^a$ is the dark matter candidate and we take its mass in the range 10 to 5000 GeV; similar conclusions are obtained when the candidate is the pseudo-scalar $h_p^a$. All other dark particle masses are taken randomly in the range $\gtrsim M_{h_n^a}$ to $\sim 5000$ GeV, while the heavy scalar masses take values in the range $\gtrsim M_h$ to $\sim 5000$ GeV. For the parameter $\mu_2^2$ due to the first equation in (17) we generate random values for it in the interval $(\sim (-M_{h_n^a}^2), \sim M_{h_n^a}^2)$, this should be a large interval to probe and in any case the value of $\lambda_{10}$ will be limited by the unitarity bounds and we don’t expect large differences if this interval is enlarged. Fur-

when large amounts of points are being probed.
Figure 2: Histograms of the frequency of predominant annihilation channels (for a full description see text).

Furthermore, this parameter also must satisfy the conditions (analogous to the IDM) $\mu_1^2/\sqrt{\lambda_{13}} > \text{Max}\{\mu_0^2/\sqrt{\lambda_8}, \mu_1^2/\sqrt{\lambda_1}\}$ in order to prevent the possibility of tunnelling to a $v_a \neq 0$ vacuum [3], to achieve this we tune the value of $\lambda_{13}$ until the inequalities are satisfied, note that unphysical values of $\lambda_{13}$ will always be casted out by the subsequent filters. Finally the values of the rest of the free parameters are taken in the ranges $\lambda_{14} \in [-4\pi, 4\pi]$, $\tan \theta \in (0, 100]$ and $\alpha \in [-\pi/4, \pi/4]$. We present our results in figures (1) through (3).

The first observation that we want to make is that from the entire sample of points probed, those that satisfy the first line of constraints (close to $10^5$)\textsuperscript{4} i.e. stability and unitarity conditions, only a small proportion (around 10%) passed the HiggsBounds tests. While the size of this sample\textsuperscript{5} is rather small compared to the size of a parameter space of such dimensionality (15 total free parameters), we believe that the main conclusions drawn from our findings show important properties of the model.

In the left panel of figure (1) we present a scatter plot of the points in our scan projected in the plane DM mass vs $\tan \theta$. We refer to the points that pass the first line of constraints as set $A$ (dark blue points), while set $B$ (light blue points) are points that additionally satisfy the HiggsBounds limits; the subset of $B$ that satisfy the decoupling limit are shown in red color. From this figure we observe that stability and unitarity constraints severely restrict

\textsuperscript{4}Note that the total number of scanned points is far greater than this number since many of the randomly generated points are already discarded at the first line of constraints.

\textsuperscript{5}Larger sample sizes can easily become too expensive in computational time terms.
the values of $\tan \theta$ to $\lesssim 10$, with a handful of points reaching up to 19, while the vast majority of the points in set $B$ lie below the 700 GeV mass mark. We also show in the right panel of this figure the value of the relic density for each point as a function of the dark matter candidate’s mass, with the color code for the points identical to the previous case except that points in set $B$ within the measured experimental value of Planck [51] are highlighted in green. From this figure we draw attention to the fact that for a large amount of the potential physical points found the dark scalar doublet of the model contributes only a fraction of the experimentally measured value of the relic density, particularly in the region of masses above 80 GeV where we found no points in or above the Planck value. On the other hand for masses below 80 GeV plentiful of the points shown have values of the relic well above the Planck limit and thus are unphysical; but there are also points in and below this bound, the latter are not necessarily unphysical since there could be other sources of dark matter not taken into account by the model.

In figure (2) we show histograms of the frequency of the dominant annihilation channels that contribute to the value of the relic density for each point of the scan in set $B$, where we distinguish the mass region where all points are below the Planck bound from the region of small masses where points are found within the experimental interval. The color keys defined in the histograms display the percentage of points in the corresponding sets with given predominant annihilation channel (PAC), defined as the channel with the highest branching ratio for each particular point of parameter space. For the small mass range the dominant channel is by far annihilation into $b \bar{b}$ pair since almost 80% of the points prefer this channel; interestingly we also see a few points where co-annihilation with $h_2^\pm$ is important. For the high DM mass range (right panel histogram) around 50% of the points annihilate into gauge bosons pairs and we find that around a third of the points annihilate predominantly into pairs $h_2^\pm h_2^\mp$, $t \bar{t}$ and $A A$ equally; and we don’t find instances where co-annihilation with other particles is important.

From the above results we deduce that the dark matter relic density dependence with the mass of our candidate follows a similar pattern to the i2HDM [2]. Since we have more scalars than in the i2HDM there will be more decay channels available, but similar features remain. The left part (low mass region) will be dominated by Higgs mediated diagrams in the s-channel decaying into light fermions, then, there is a sharp dip where the candidate annihilates through the SM Higgs boson, for the most part this is a consequence of the resonance around the value of the Higgs mass due to the s-
pole in the annihilation amplitude (for example two DM particles annihilating at rest will hit the resonance at around a mass of 62 GeV and thus the location of the dip in the figure); note that resonances due to the additional diagrams with heavy scalars do not appear as sharp dips because their masses are not fixed like the SM Higgs mass. Higher order corrections will shift the locations of the poles but these corrections are not taken into account by MicrOMEGAS. In the large mass region the quartic interaction and the s-channel decays into gauge bosons are dominant, followed by the quartic channel dominated decay into charged scalars, s-channel dominated decay into top-anti-top pair, and quartic channel dominated decay into pseudoscalars. With rising DM mass the interplay between these decays, together with the values of the effective coupling $\lambda^\pm_X$ and the heavy scalar masses, will lead to a slow increase of the relic density with the dark matter candidate mass. In our case this increase is less steep than in the i2HDM, and it reaches values close to the Planck bound for masses $\sim 5$ TeV.

Finally in figure (3) we present the annihilation cross section (relevant for Indirect Detection Experiments) as a function of the DM mass for points with masses below 100 GeV; to highlight the points that lie within the Planck
bound we define a likelihood function $\mathcal{L}$ with respect to the value of the relic density, namely a gaussian centered in the Planck value with width equal to the 68% experimental interval [51], the points are colored according to their $\mathcal{L}/\mathcal{L}_{\text{max}}$ value, where $\mathcal{L}_{\text{max}}$ corresponds to the maximum value attained within the set of points. Thus, the darkest points in this figure lie above or below the Planck bound whilst the light pink points have relic density values within the experimental limits. In the figure we also show the FermiLAT combined limits from dwarf spheroidal galaxies [52] for the $b\bar{b}$ channel (for this range of masses other channel limits are indistinguishable). We see that the FermiLAT exclusion curve rules out many of the points compliant with the value of the experimental relic density, but still some of these points lie safely below this bound.

4. Conclusions

We have performed an analysis of the $S3$ symmetric model where the number of scalar doublets fill in all the irreducible representations of the permutation group; by taking one of the scalars as inert we are retaining convenient features of the 3-Higgs Doublet Model with $S3$ symmetry whilst additionally extending the model with a dark sector. We have probed random samples of parameter space points and find that the combination of physical constraints severely limits the parameter space, in particular the most stringent constraints come from current experimental limits on Higgs searches. For a set of potentially physical points found in the scan we calculated the value of the relic density of the dark scalar. The results obtained are similar to the i2HDM. There are points that comply with the Higgs searches bounds all along the dark matter mass spectrum probed. There is a low dark matter mass region $10 \sim 100$ GeV with points that have an overproduction of dark matter, but others that comply with the Planck bound and some that are below it. In this small mass region there are also points that satisfy the scalar searches bounds and relic density abundance Planck bound, as well as the FermiLAT combined limits for the annihilation cross section for the $b\bar{b}$ channel. For the rest of the mass values there is an underproduction of dark matter due to an interplay between the annihilation channels, the values of the effective parameter $\lambda^X_\chi$, and the heavy scalar masses. In this case, for very heavy masses $\sim 5$ TeV there are some points that comply with the Higgs searches and which have a DM relic density close to the Planck measurement.
Our results suggest the possibility of extending the dark sector of the model with additional particles, potentially enriching the phenomenology of this type of models. The inclusion of loop corrections to our results might change the outcome of the analysis, especially in the Higgs sector.

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