Multi-element coupling model of rising bubble in water

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Abstract. Multi-factor coupling modeling is carried out in the process of floating bubbles in water in this paper, and the model is employed to analyze the full force of bubbles in water, and to consider the change of pressure inside bubbles and the mass exchange of gas and liquid during bubble floatation. By numerical simulation of the coupling model, it is confirmed that the Non-equilibrium mass transfer condition is more consistent with the actual situation, and the relationship between the elements in the process of bubble floatation is analyzed, which provides a theoretical basis for the subsequent study of the acoustic characteristics of the bubble wake of the ship.

1. Introduction

When the ship sails on the surface of the sea, it will form a long wake behind the ship’s crotch due to the entanglement of the ship’s waves and the entrapment of air bubbles into the sea and the high speed rotation of the ship’s propeller[1]. Since the nature of the seawater in the wake of the ship is different from the physical properties of the surrounding seawater medium, different physical properties of these properties can be used to detect and track the ship[2]. Therefore, the study of various physical properties of ship wakes has important military significance and has long been valued by various maritime powers. Researchers have begun to study the bubble wake of ships and have received continuous attention. The study of the motion characteristics and distribution characteristics of bubbles in the wake of a ship's bubble is the basis for the study of the bubble wake of the ship, because this directly determines whether the ship can be detected in complex sea conditions and the longest time that the ship can be detected[3].

The basis of the motion characteristics of the bubble in the wake of the ship's bubble is the floating motion model of the bubble. Although the existing model has a complete force analysis on the bubble in the water[4,5], it also consider the change of the bubble radius during the floating process and the gas-liquid exchange between the gas in the bubble and the surrounding liquid[6].

However there are also some shortcomings. During the process of accelerating the bubble from the stationary state, bubble’s velocity, bubble’s depth, pressure and gas density inside the bubble all change. These changes will change the force of the bubble in the water, and then continue to affect the speed, depth, pressure inside the bubble, gas density in the bubble, while the bubble is in the gas and liquid exchange with the surrounding liquid. Existing models do not fully consider the coupling relationship of these elements. In view of the existing problems, this paper fully considers the change of the force in the floating process of bubbles in the water, and considers the changes in the depth of the bubble after floating, the pressure inside the bubble and the gas density in the bubble, as well as the gas-liquid exchange between the gas in the bubble and the surrounding liquid. The multi-element
coupling model of the bubble floating up. Through the established coupling model, the bubble floating process with different initial depths of different radii is simulated. The effects of Basset and two mass transfer conditions on the bubble floating process are analyzed. The changes of various parameters in the bubble floating process are also analyzed. The situation laid the foundation for subsequent research.

2. Multi-element coupling model of bubble rising

2.1 Basic equation of bubble rising

The movement of the bubble in the liquid is mainly determined by the force of the water in the water. Under the assumption that the liquid is stationary and the interaction force between the multiple bubbles is not considered, the bubble of the radius accelerates from the static direction in the vertical direction to the water. The balance speed is mainly affected by the following forces.

Buoyancy: The vertical upward force of a bubble in liquid, $F_b = \rho_g \rho V_s^4 / 3 \pi R \rho g$, where $\rho_g$ is the density of seawater, $g$ is the acceleration of gravity, and $V_s$ is the volume of bubbles.

Gravity: The force that a bubble is attracted to the earth in liquid, $F_g = m_b g = V_s \rho g = 4 / 3 \pi R \rho g$, where $m_b$ is the mass of the bubble and $\rho_s$ is the density of the gas in the bubble.

Viscous resistance: The force of bubbles in a liquid and liquid due to mutual movement, $F_v = 0.5 C_D \rho_v v^2$, where $C_D$ is the drag coefficient. In the Rayleigh coefficient $Re = \rho_v R v / \mu < 300$, it can be approximated as $C_D = 12 / Re \left(1 + 0.168Re^{0.72}\right)$ [7]. Where $v$ is the bubble floating speed, which $\mu$ is the viscosity coefficient of seawater.

Additional mass force: When the bubble accelerates in the liquid, it will drive the surrounding liquid to accelerate. The mass of the surrounding accelerated liquid is converted into the additional mass. The force that accelerates the acceleration of the surrounding liquid is the additional mass, $F_a = 1/2 \rho_v V_b dV/dt = 2/3 \pi R \rho \rho_a dv/dt$.

Basset force: When a bubble moves in a linear motion in a viscous liquid, the instantaneous resistance acting on the bubble in addition to the additional mass force is also called Basset force because the bubble is accelerated in the viscous liquid. Expressed by the following formula,

$$F_b = 6R^2 \sqrt{\pi \rho_v \mu} \int_0^t \frac{dv/d\tau}{\sqrt{t - \tau}} d\tau.$$

According to the above several forces, the force analysis of the bubble in the liquid is shown in Figure 1, and the vertical direction is taken as the positive direction.

![Figure 1. Force condition of bubble.](image)

According to the force analysis

$$\frac{dv}{dt} = F_b - F_g - F_v - F_a - F_b$$

Substituting the expressions of the individual forces into the above formula

$$\frac{dv}{dt} = \frac{2(\rho_s - \rho_a) g}{2 \rho_s + \rho_a} - \frac{3 C_D \rho_v v^2}{4 (2 \rho_s + \rho_a) R} - \frac{9 (\rho_s - \rho_a) \mu}{\pi} \int_0^t \frac{dv/d\tau}{\sqrt{t - \tau}} d\tau$$

(1)
The above formula is the expression of the acceleration of the bubble in the liquid. When the parameters other than the velocity are considered to be constant, the acceleration of the bubble is only related to the bubble radius and the bubble motion process. However, as the bubble moves, the depth of the bubble becomes smaller, and the pressure of the bubble from the liquid decreases, so that the bubble increases, so the floating process of the bubble needs to take into account the change in depth.

2.2 Change in radial radius of the bubble caused by depth variation and gas-liquid exchange
Since the bubble moves vertically upwards in the water, the depth changes with time, the pressure of the liquid at the position gradually decreases, and the gas pressure in the bubble decreases, so that the volume of the bubble increases, that is, the radius of the bubble increases, and the radius of the bubble increases. It affects the floating speed of the bubble, so it is necessary to consider the change of the bubble radius when modeling the floating motion of the bubble.

The pressure inside the bubble at depth \( h \) is

\[
P = P_{\text{atm}} + \rho_g h + 2\sigma / R
\]

(2)

Where \( P_{\text{atm}} \) is the standard atmospheric pressure, \( h \) is the depth at which the bubble is located, and \( \sigma \) is the surface pressure of the bubble.

When the bubble is in the process of floating, the quality at any time can be expressed as

\[m_b = \frac{3}{4} \cdot \pi R^3 \rho_a\]

(3)

and both sides are simultaneously, derivative both sides by \( t \), there is

\[
\frac{dm_b}{dt} = \frac{4}{3} \pi R^3 \left( \frac{dR}{dt} \cdot \frac{d\rho_a}{dt} \right)
\]

The ideal bubble state equation \( PV = nRT \) is expressed as a form \( PM = \rho RT \) with respect to density. In the case where the liquid temperature is constant throughout the floating process, there is

\[P / P_{\text{atm}} = \rho_a / \rho_{\text{atm}}\]

(4)

\[\rho_{\text{atm}} = \rho_{\text{atm}} \left( \frac{P_{\text{atm}} + \rho_g h + 2\sigma / R}{P_{\text{atm}}} \right)
\]

(5)

derivative both sides by \( t \), there is

\[
\frac{d\rho_a}{dt} = \frac{\rho_{\text{atm}} \cdot d\rho_a}{\rho_{\text{atm}} \left( \frac{d\rho_a}{dt} \right) - \left( \frac{d\rho_a}{dt} \right) \cdot \left( \frac{dR}{dt} \right) / R^2 / dt}
\]

(6)

It can be known from the literature [8] that the gas-liquid exchange rate of the bubbles in the liquid is satisfied.

\[
\frac{d\rho_a}{dt} / \rho = 8 \left( C_a - C_j \right) D_{ab}^{2/3} v^{1/3} R^{1/3}
\]

(7)

Where \( C_a \) is the mass concentration of the gas in the liquid, \( C_j \) is the mass concentration of the gas at the junction of the bubble and the liquid, and \( D_{ab} \) is the diffusion coefficient of the gas in the liquid.

Solved by (3)(4)(5), and \( dh / dt = -v \), rearranged

\[
\frac{dR}{dt} = \frac{R \rho_g g v + 6/3 \cdot P_{\text{atm}} / \rho_{\text{atm}} \cdot (C_a - C_j) D_{ab}^{2/3} v^{1/3} R^{1/3}}{3P_{\text{atm}} + 3 \rho_g gh + 4\sigma / R}
\]

(8)

The first term on the left side of the upper equal sign is the change in radius of the bubble due to floating, and the second term is the change in radius caused by the gas-liquid exchange of the gas in the bubble with the surrounding liquid.

2.3 Multi-element coupling model for bubble floating
Combining equation (1) and (7) and \( dh / dt = -v \), we can get a multi-element coupling model of single bubble floating in liquid.
According to the formula (7), by setting different initial conditions, it is possible to obtain various data in which the bubble floats up to disappear.

3. Model calculation and result analysis

Set the basic parameters in the model as follows:

\[ C_s = 0 \]
\[ \rho_s = 1025 \text{ kg/m}^3 \]
\[ \rho_w = 1.205 \text{ kg/m}^3 \]
\[ \sigma = 0.072 \text{ N/m} \]
\[ \mu = 1.01 \times 10^{-1} \text{ Pa·s} \]
\[ P_{atm} = 101325 \text{ Pa} \]
\[ H = 2.0 \times 10^{-7} \text{ kg/(m·s·Pa)} \]
\[ D_{ab} = 2.0 \times 10^{-3} \text{ m}^2/\text{s} \]
\[ g = 9.8 \text{ m/s}^2 \]

3.1 Influence of different mass transfer conditions

The numerical simulation of the floating process of the bubble with the initial radius \( R_0 = 100, 200, 300 \mu m \) is carried out, the initial depth \( h_0 = 10 \text{ m} \) and the initial velocity \( v_0 = 0 \), case 1 unbalanced mass transfer condition and case 2 equilibrium mass transfer condition, respectively, where \( H \) is Henry coefficient, according to the complete coupling model represented by equation (7), solves the entire floating process of the bubble, and the bubble radius changes with the depth as shown in Figure 2.

![Figure 2. Comparison of different mass transfer conditions.](image)

It can be seen from Figure 2 that the calculation of the equilibrium mass transfer condition results in a decrease in the bubble radius with depth, while the bubble floating process using non-equilibrium mass transfer conditions. The smaller bubble radius decreases as the depth decreases, and the larger bubble radius increases as the depth decreases. The reason is that the bubble dissolution rate using equilibrium mass transfer conditions is faster than that of non-equilibrium mass transfer conditions, and it cannot compensate for the pressure reduction radius of the bubble floating. The effect of the increase makes the calculation result survival time in water much shorter than the non-equilibrium mass transfer condition. Compared with previous experiments, the bubble survival time calculated by non-equilibrium mass transfer conditions is more reasonable. Therefore, the non-equilibrium mass transfer conditions are used in this paper.

3.2 The effect of Basset force on bubble floating

The numerical simulation of the floating process of the bubble with the initial radius \( R_0 \) of 100, 200, 300 \( \mu m \) is carried out. The initial depth \( h_0 = 10 \text{ m} \) and the initial velocity \( v_0 = 0 \), case 1 without considering the upward acceleration of the bubble acceleration process; case 2 the bubble floating without considering the Basset force; case 3 complete coupled model is represented by three cases of
floating up by equation (7), respectively, where cases 1 and 2 are without considering the variation of bubble radius with depth and mass transfer conditions. The graphs of the speed changes over time in 0.3 seconds and 20 seconds are shown in Figure 3 and Figure 4, respectively.

Figure 3. The relationship between the first 0.3s floating speed and the time.

Figure 4. The relationship between the first 20s floating speed and the time.

It can be seen from Figure 3 and Figure 4 that the larger the bubble is, the higher the floating speed is. Secondly, it can be seen from Figure 3 that if the Basset force is not considered, the floating speed of the bubble will be quickly accelerated until the bubble acceleration process is not considered. The speed of the floating, the larger the bubble, the longer the acceleration time; then, as can be seen in conjunction with Figure 3 and Figure 4, the increase of the buoyancy speed of the bubble in the complete coupling model considering the Basset force is slower than when the Basset force is not considered, but it can also be within a few seconds. Accelerate to not consider the buoyancy speed of the bubble acceleration process, the larger the bubble, the longer the acceleration time; finally, from Figure 4, it can be seen that the initial radius is that the bubble has exceeded the buoyancy speed of the bubble acceleration process at 20s, as the bubble rises, the depth pressure becomes smaller, and its own radius increases, so the floating speed will exceed the floating speed without considering the bubble acceleration process. Therefore, considering the influence of the Basset force during the bubble floating process will make the coupled model more complete.

3.3 Survival time of bubbles with different initial radii

The numerical simulation calculation of the floating process of bubbles with a radius of 5, 10, ..., 100, 110, ..., 300, 320, ..., 700 μm with an initial radius of \(h_0=2, 5, 6, 8, 10, 12.5\) m, initial velocity \(v_0=0\), using non-equilibrium mass transfer conditions, according to the complete coupling model represented by equation (7), the entire floating process of the bubble is solved, and the survival time of the bubbles with different initial radii is shown in Figure 5.

Figure 5. Survival time of bubbles with different initial radii.

It can be seen from Figure 5 that the survival time of bubbles of different initial radii increases with depth, and the bubble radius with the longest survival time increases with depth.

The initial depth \(h_0=10, 15\) m is taken out, and the depth of each depth bubble changes with time as shown in Figure 6 and Figure 7.
Figure 6. Relationship between the time of bubble survival and initial radius in the initial depth of 10 meters.

Figure 7. Relationship between the time of bubble survival and initial radius in the initial depth of 15 meters.

It can be seen from Figure 6 and Figure 7 that the bubble with the longest retention time in the water has a small life cycle final radius.

4. Conclusion

In this paper, through comprehensive analysis of the single bubble in water, considering the influence of single bubble in the water by Basset force, the change of the pressure inside the bubble during the floating process and the mass exchange between the gas in the bubble and the surrounding liquid, a multi-element coupling model for floating of single bubbles in water is constructed.

According to the established model, it is more reasonable to determine the use of non-equilibrium mass transfer conditions; At the same time, it shows that the influence of Basset force on the bubble floating process is mainly in the initial acceleration phase of the bubble, which will increase the time of the bubble acceleration process; the parameters of the bubble life cycle throughout the life cycle were analyzed and the following results were obtained: the bubble with the initial radius between 140–180μm has the longest survival time, and the gas-liquid mass transfer rate and the floating speed are relatively balanced; the deeper the initial depth, the longer the bubble survival time and the longer the peak radius of the longest survival time; bubbles with the longest survival time at each initial depth have a small final radius at the end of their life. These single bubble characteristics are instructive for subsequent research.

References

[1] Zhang, J. (2015) Underwater detection and acoustic homing technology. Naval University of Engineering Press, Wuhan.

[2] Zhang, Q. (2014) Research on the active acoustic characteristics of ship wake. Northwestern Polytechnical University, Xi’an.

[3] Gao, J. Zhang, J. Yang L. (2008) The present situation of research on ship wake characteristic. Ship science and technology, 30(4): 27-32.

[4] Shi, S. Wang, J. Jiang, X. (2008) Mechanics effect study of a rising micro-bubble in still water. Journal of naval university of engineering, 20(3): 83-87.

[5] Feng, Q. Chi, W. (2010) Characterization Model on the Rising Motion of Bubble in Ship Wake. Chinese Journal of Ship Research, 05(3): 27-29.

[6] Zhang, J. Yang, K. Zong S. et al. (2011) Investigation and Measurement of Bubble Characteristic in Waters. Infrared Technology, 33(4): 219-225.

[7] Thorpe, S A. (1982) On the Clouds of Bubbles Formed by Breaking Wind-Waves in Deep Water, and their Role in Air – Sea Gas Transfer. Philosophical Transactions of the Royal Society A Mathematical Physical & Engineering Sciences, 304(1483): 155-210.

[8] Dai, G. Chen, M. (2005) Fluid Mechanics for chemical Engineering. Chemical Industry Press, Beijing.