Development and Performance Evaluation of a Two-Stage Cascade Refrigeration System for Ice Block Production

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ABSTRACT

A development and performance evaluation of a two-stage cascade refrigeration system for ice block production was carried out in this work. Two single stage vapour compression refrigeration systems were thermally coupled. The cascade refrigeration system thus formed enhances cooling effect and fast track ice production. This machine was designed for a refrigeration capacity of 3kW to achieve the conversion of 128.25m³ of water at 30°C to ice block at -15°C using R407A/R410A as working fluids. Experimental test of the machine was conducted under fixed and variable load conditions with the temperature and pressure both at the inlet and exit of each of evaporator, compressor and condenser taken. From the data obtained the refrigerating effect, COP and overall efficiency were determined. The result of the performance evaluation shows that as the evaporator temperature increases from -15°C to -3°C, the refrigerating effect decreases from 189.17kJ/kg to 184.37kJ/kg, the COP decreases from 4.13 to 3.80 and the overall efficiency of the system decreases from 61.03% to 64.27%. As the condenser temperature increases from 40°C to 49°C keeping the evaporator temperature and temperature difference in the cascade condenser constant, the refrigerating effect decreases from 189.17kJ/kg to 184.37kJ/kg, the COP decreases from 4.13 to 3.80 and the overall efficiency of the system decreases from 61.03% to 50.92%. However, as the temperature difference in the cascade condenser decreases from 6°C to 2°C keeping the evaporator and condenser temperature constant, the refrigerating effect increases from 190.76kJ/kg to 197.06kJ/kg, the COP increases from 4.18 to 4.62 and the overall efficiency of the system increases from 60.64% to 63.25%. The machine achieved the designed condition in six (6) hours and the ice blocks so produced retained its solid state for 48 hours with the cover remained closed which denote a very impressive transformation capacity and reliability of the device compare with other homomcopes.

Keywords: Two-stage, Cascade refrigeration, Cooling effect, Refrigerating capacity, Working fluids.

Nomenclature

COP - Coefficient of performance
GWP - Global Warming Potential
HTC - High Temperature Cycle
LTC - Low Temperature Cycle
\(m_{HT}\) - HTC refrigerant mass flow rate (kg/s)
\(m_{LT}\) - LTC refrigerant mass flow rate (kg/s)
ODP - Ozone Depletion Potential
\(P_{CH}\) - Compressor power for HTC (kW)
\(P_{CL}\) - Compressor power for LTC (kW)
\(R_{e}\) - Refrigerating effect (kJ/kg)
\(t_{i}\) - Temperature of ice (°C)
\(t_{a}\) - Ambient temperature (°C)
\(T_{CTL}\) - Condenser temperature for LTC (°C)
\(T_{CTH}\) - Condenser temperature for HTC (°C)
\(T_{ETH}\) - Evaporator temperature for HTC (°C)
\(T_{EL}\) - Evaporator temperature for LTC (°C)
\(\Delta T_{CT}\) - Change in temperature difference in cascade condenser (°C)

\(W_{CH}\) - Compressor work for HTC (kJ/kg)
\(W_{CL}\) - Compressor work for LTC (kJ/kg)
\(\eta\) - Efficiency of the system (%)
water into portable containers such as polythene bags and then placed them in a freezer or ice block machine over a period of time (depending on sizes) to solidify. It is made by a deliberate conditioning of water so as to change phase from liquid to solid (refrigeration). Refrigeration is the achievement of a temperature below that of the immediate surroundings. It is a process of removing heat from a substance or space under controlled conditions [5, 6]. Vapour compression refrigeration system based applications make use of refrigerants such as chlorofluorocarbons (CFCs) and hydro-chlorofluorocarbons (HCFCs) which are responsible for greenhouse gases, global warming and ozone layer depletion effects (such refrigerants have high Global Warming Potential, GWP and or Ozone Depletion Potential, ODP). Conventional single-stage system does not give satisfactory result at very low evaporator temperature due to the generation of very high compression ratio that further leads to high discharge problem and low volumetric efficiencies [7]. Therefore, some form of multi-stage compression must be employed in order to avoid excessive discharge temperature and to maintain reasonable operating efficiencies. In larger installations, multi-stage compressor should be considered for any evaporator temperature below -18°C [8].

All the methods of accomplishing multi-stage compression can be grouped into direct staging and cascade staging. The direct staging method employs two or more compressors connected in series to compress a single refrigerant in successive stages. The pressure of the refrigerant vapour is raised from the evaporator pressure to the condenser pressure in stages on incremental basis, the discharge vapour from the lower stage compressors is piped to the next suction of the higher stage compressor [8]. Cascade staging involves the use of two or more separate refrigerant circuits which employ refrigerants having progressively lower boiling points. The compressed refrigerant vapour from the lower stage is condensed in a heat exchanger, usually called a cascade condenser, which is also the evaporator of the next higher stage refrigerant [9]. However, both methods of multistage have relative advantages and disadvantages. The particular method which will produce the best results in a given installation depends for the most part on the size of the installation and on the degree of low temperature which must be attained. In some instances, a combination of the cascade and direct staging methods can be used to an advantage. In these cases, the compound compression (direct staging) is usually applied to the lower stage of the cascade [10]. Cascade system was first used by Pietet in 1877 for liquefaction of oxygen, employing sulphur dioxide (SO₂) and carbon dioxide (CO₂) as intermediate refrigerants. The additional values given by cascade refrigeration system include energy conservation, stable ultra-low-temperature operation, easy repair and low running cost [11-13].

Meanwhile, a lot of designs on cascade refrigeration systems for ice block production had been made available but are yet to achieve the desired considerations. Most part of scientific work dealt with theoretical performance evaluation of cascade systems with establishing their optimum Low Temperature condensing conditions. Only few experimental works have been reported up till now and most of them dealt with the use of refrigerants that are facing out of market or worked with NH₃/CO₂ systems for industrial applications, which are not recommended for commercial systems because of environmental reasons. Hence, the present devices used in making ice block are of low productivity due to their prolong period of time taken for the freezing process, higher energy consumption coupled with inhabitable heat release. Therefore this decision for development of two-stage cascade refrigeration system with environmental friendly refrigerants for rapid production of ice block was made to solve these problems.
2. METHODOLOGY

In this study, the work done is focusing on the utilization of two-stage cascade refrigeration system for quick ice block production. This aspect explicitly describes the design, working operations, fabrication procedures, test-run and experimentation carried out in actualizing the desired purpose. Towards achieving this, the operating conditions were determined and heat load to be removed were estimated using thermodynamics principles. System refrigerating capacity was calculated and the materials were carefully selected.

The refrigerator was constructed by cutting the selected plate and insulator to suitable sizes which were joined together to form a rigid box while the evaporating coils were lined inside. Every other components such as compressors, condenser, expansion valves and driers are predesigned and were bought (base on the estimated heat load) and fixed. The system is charged with the respective working fluid and connected to electric supply for test-run. Pressure and temperature sensors (gauge and digital thermometer) were fixed at the inlet and exit of each of the compressors, condenser and evaporator to measure the pressure and temperature respectively during experimentation on the system. Constant supply of electric power was ensured by using a generator. The system was made to run under no load, constant load and varying load conditions while corresponding readings at each sensor location were taken at regular interval of time.

2.1 System Description

The schematic diagram of basic components of the cascade refrigeration system is shown in Fig.1. It consists of two stages “A” and “B” which are connected by cascade condenser. Stage A, cycle 5-6-7-8 is called High Temperature Cycle (HTC) and stage B, cycle 1-2-3-4 is called Low Temperature Cycle (LTC). The main components of the system are: a compressor each for HTC and LTC, a condenser, an evaporator (cooling box), a cascade condenser and two expansion valves (throttling devices) one for the HTC and the other one for the LTC. The two stages (units A and B) of vapour compression refrigeration system are thermally connected together through the cascade condenser.

![Fig.1 - The schematic diagram of the two-stage cascade refrigeration system](image)

![Fig. 2a – P-h diagram of the two-stage cascade refrigeration system](image)
2.2 System Operation

With reference to Fig. 1, the vapourized refrigerant flows from the evaporator of unit B, cycle 1-2-3-4 and gets compressed by the LTC compressor (process 1-2) to a high pressure and saturation temperature. Then the high pressure, high temperature refrigerant flows through a concentric counter-flow heat exchanger known as cascade condenser (process 2-3). This brings about condensation of the refrigerant from vapour into liquid by heat rejection to unit A, cycle 5-6-7-8. The cascade condenser serves dual purpose in the system. It serves as condenser (process 2-3) for the unit B and at the same time serves as evaporator (process 8-5) for unit A. Hence, unit A absorbs the heat from the refrigerant of unit B and then discharges the resultant heat to the environment through the HTC condenser. The high pressure, low temperature liquid refrigerant of unit B moves from the cascade condenser and flows through an expansion valve (process 3-4) which reduces its pressure and further lower the temperature by throttling. At low temperature and pressure, the refrigerant leaves the expansion device and flows to the evaporator (process 4-1) where it converts to vapour by taking away heat from the cold region. These processes repeats continuously till the desired refrigeration is attained. The $p-h$ and $T-s$ diagrams for the system are shown in Fig. 2a and 2b respectively. However, unit A is another refrigerating cycle called the HTC. Unit B (LTC) removes heat from the cooling box (evaporator) while the HTC removes heat from the refrigerant of LTC and discharged it into the environment. Both refrigerating cycles (LTC and HTC) are thermally connected through the cascade condenser. Therefore, the desired cooling effect is obtained in the evaporator of unit B that is cycle 1-2-3-4. The whole system operates in closed cycle which required mechanical work as input to the compressors so as to drive the refrigerants through the systems.

2.3 Design Analysis

2.3.1 Determination of heat load

Heat enters and leaves the cooling box through many ways. In this work, the heat load involved is categorized into three different parts, namely:

i. Heat conducted through the walls of cooling space; ii. Product heat load; iii. Service load.

Heat conducted through the walls of evaporator

![Fig. 2b – T-s diagram of the two-stage cascade refrigeration system](image-url)
The quantity of heat conducted through the walls of evaporator depends on the overall coefficient of heat transfer, surface area and temperature difference between the inside and outside of the box. Thus, in this design, the followings are maintained:

Outer dimension = 0.90m x 0.70m x 0.56m (length x breadth x depth)

Inner dimension = 0.80m x 0.60m x 0.46m (length x breadth x depth)

Ambient temperature, \( t_o = 30^\circ C \);

Temperature of ice, \( t_i = -15^\circ C \); \( \Delta T = t_o - t_i = 45^\circ C \)

Plate = galvanized metal sheet (0.001m thick); Insulator = polyisocyanurate (0.05m thick)

Let: the surface area covered by insulator = \( A_{\text{ins}} \)

\[
A_{\text{ins}} = [(0.90 \times 0.70) + (0.90 \times 0.56) + (0.70 \times 0.56)] \times 2 \text{ m}^2
\]

\[
A_{\text{ins}} = 3.052 \text{ m}^2
\]

The rate of heat conduction through the walls of the refrigerated space, \( q_{\text{cond}} \) is calculated as follows:

\[
q_{\text{cond}} = UA\Delta T
\]

Where: \( U = \) Overall heat transfer coefficient;

\( A = \) Surface area of the insulator = \( A_{\text{ins}} = 3.052 \text{ m}^2 \)

\( \Delta T = \) Change in temperature = 45\(^\circ\)C;

\[
q_{\text{cond}} = \text{rate of heat transfer by conduction}
\]

\[
\frac{1}{U} = \frac{1}{h_i} + \frac{X_1}{K_1} + \frac{X_2}{K_2} + \ldots + \frac{X_n}{K_n} + \frac{1}{h_o}
\]

Thus: \[
\frac{1}{U} = \frac{1}{h_i} + \frac{X}{K} + \frac{1}{h_o}
\]

Where: \( x = \) thickness of insulator = 0.05m;

\( k = \) thermal conductivity of insulator

\( k = 0.022 \text{ W/mK} \)

\[
\frac{1}{h_i} = \text{inside convection coefficient};
\]

\[
\frac{1}{h_o} = \text{outside convection coefficient}
\]

\( h_i = 9.3 \text{ W/m}^2\text{K} \) and \( h_o = 22.7 \text{ W/m}^2\text{K} \)

Then: \[
\frac{1}{U} = \frac{1}{9.37} + \frac{0.05}{0.022} + \frac{1}{227}
\]
\[ U = 0.4126 \]
\[ q_{\text{cond}} = UA \Delta T = (0.4126 \times 3.052 \times 45) \text{ W} \]
\[ = 56.67 \text{ W} = 0.0567 \text{ kJ/s} \]

The quantity of heat conducted, \( Q_{\text{cond}} \), through the walls of the evaporator in six hours of steady operation is estimated as follows:

\[ Q = q \times t \]

Thus:
\[ Q_{\text{cond}} = q_{\text{cond}} \times t \]

Where: \( Q_{\text{cond}} \) = Heat transferred by conduction
\( t = \) time in second = \((6 \times 60 \times 60)\) seconds

Therefore:
\[ Q_{\text{cond}} = 0.0567 \text{ kJ/s} \times (6 \times 60 \times 60) \text{ s} \]
\[ = 1224.72 \text{ kJ} \]

**Product heat load**

In order to estimate the product heat load, the capacity of the evaporator (cooling box) is first determined so as to know the quantity of ice block that could be produced per batch.

Let: \( C_{\text{eva}} \) = the capacity of the evaporator

\[ C_{\text{eva}} = L_i \times W_i \times D_i \]

Where: \( L_i \), \( W_i \) and \( D_i \) are inner length, width and depth of evaporator for product loading and are 0.75m, 0.45m and 0.38m respectively.

\[ C_{\text{eva}} = 0.75 \text{ m} \times 0.45 \text{ m} \times 0.38 \text{ m} = 0.12825 \text{ m}^3 \]

\[ 1 \text{ m}^3 \text{ of water} = 1000 \text{ L of water} = 1000 \text{ kg of water} = 1000 \text{ kg of ice block (approximation)} \]

Thus: 0.12825 m\(^3\) of water =128.25kg of ice block

Hence: \( C_{\text{eva}} = 128.25 \text{ kg of ice block} \)

The product heat load, \( Q_{\text{prod}} \) is the addition of the heat gained, \( H_a \) in cooling water from 30°C to 0°C (above the freezing point of water); the heat gained, \( H_{ph} \) in changing water phase at 0°C to ice at 0°C (latent heat of freezing of water); and the heat gained, \( H_b \) in further cooling of ice from 0°C to -15°C (below the freezing point of water). This could be represented as follows:

\[ Q_{\text{prod}} = H_a + H_{ph} + H_b \]
\[ H_a = m_w c_w \Delta T_1 \]
\[ H_{ph} = m_w L_w \]
\[ H_b = m_w c_i \Delta T_2 \]
Where: \( m_w = \) mass of water = 128.25kg; 
\( c_w = \) specific heat capacity of water 
\[ = 4.187 \text{kJ/kgK} \]
\( \Delta T_1 = \) temperature change in cooling water from \( 30^0\text{C} \) to \( 0^0\text{C} = 30^0\text{C} \); 
\( L_w = \) latent heat of water changing to ice at \( 0^0\text{C} \) 
\[ = 334 \text{kJ/kg} \]
\( c_i = \) specific heat of ice = 2.108kJ/kgK
\( \Delta T_2 = \) temperature change in cooling ice below freezing point at \( 0^0\text{C} \) to \( -15^0\text{C} = 15^0\text{C} \)

Thus: 
\[
H_a = m_w c_w \Delta T_1 = (128.25 \times 4.187 \times 30) = 16109.48 \text{kJ}
\]
\[
H_{ph} = m_w L_w = (128.25 \times 334) = 42835.50 \text{kJ}
\]
\[
H_b = m_w c_i \Delta T_2 = (128.25 \times 2.108 \times 15) \text{kJ}
\]
\[ = 4055.27 \text{kJ} \]

Thus: 
\[
Q_{prod} = (16109.48 + 42835.50 + 4055.27) = 63000.25 \text{kJ}
\]

**Service load**

The amount of heat added by such operations as lighting, opening and the like are called service load. Heat gained by this source is usually not easy to determine accurately. Meanwhile, it is dealt with collectively and assumed to be equal to one percent of heat load from the other two sources.

Let: \( Q_{serv} = \) the service load 
\[
Q_{serv} = (Q_{cond} + Q_{prod}) \times 1% 
\]
\[ = [(1224.72 + 63000.25) \times 1%] = 642.25 \text{kJ} \]

**The total heat load**

This is the addition of all the heat loads and is denoted in this work as \( Q_{Total} \)

\[
Q_{Total} = Q_{cond} + Q_{prod} + Q_{serv}
\]
\[ = (1224.72 + 63000.25 + 642.25) \text{kJ} \]
\[ = 64867.22 \text{kJ} \]

However, refrigeration system is rated by amount of heat it will remove within a time frame. Taking the time to absorb above heat load to be 6hours, the refrigeration capacity, \( Q_{Ref} \) for the system is estimated as:
2.3.2 System operating conditions

For the LTC (processes 1-2-3-4):
Ref: R410A; T_{EL} = -15^0C; T_{CL} = 16^0C

For the HTC (process 5-6-7-8):
Ref: R407A; T_{EH} = 10^0C; T_{CH} = 40^0C

For the system, Q_{Ref} = 3 kW; t_o = 30^0C;
Temperature overlap = T_{CL} – T_{EH} = 6^0C

With the aid of p-h and T-s diagram of the system (fig. 2a and fig. 2b respectively), the value of each property at various points are calculated by applying thermodynamic property of fluids transport table with respect to the operating conditions.

Thermodynamic properties with respect to the operating conditions

Determination of the properties at the specified operating temperatures using tables:

i. With R410A as refrigerant for the low temperature cycle;

At -15^0C:

P_1 = 0.480 MPa; h_1 = h_{g1} = 415.63 kJ/kg;
S_1 = 1.8382 kJ/kgK; T_1 = 258 K

At 16^0C:

P_2 = 1.287 MPa; T_2^f = 289 K; S_2 = S_1 = 1.8382 kJ/kgK; T_2 = 307.7 K; h_2 = 446.00 kJ/kg

h_3 = h_{f3} = 224.87 kJ/kg; h_4 = h_{f4} = 224.87 kJ/kg

Refrigerating effect, Re = h_1 – h_4

Re = (415.63 – 224.87) kJ/kg = 190.76 kJ/kg

\[ Q_{Ref} = \dot{m}_L Re \]

Thus: \[ \dot{m}_L = \frac{Q_{Ref}}{Re} = \frac{3\ kW}{190.76\ kJ/kg} = 0.01573\ kg/s \]

Work done by LTC compressor, W_{CL} = h_2 – h_1

= (446.00 – 415.63) kJ/kg = 30.37 kJ/kg

Compressor power, P_{CL} = \dot{m}_L (h_2 – h_1) = \dot{m}_L (W_{CL}) = [0.01573(30.37)] kW = 0.478 kW

1hP = 0.746 kW
Hence: \[ P_{CL} = \frac{0.478}{0.746} \times hP = 0.641 \times hP \]

Meanwhile, compressors size is best chosen in such a way that about half of its relative output power should be able to carry the required heat load. Therefore, a 1.5\(hP\) compressor is selected for the LTC.

Heat rejected by the low temperature cycle’s condenser, \(Q_{CL}\) is given by:

\[ Q_{CL} = \dot{m}_L(h_2 - h_3) = 0.01573(446.00 - 224.87) \text{ kW} \]

\[ = 3.478 \text{ kW} \]

LTC coefficient of performance, \(\text{COP}_{L}\)

\[ \text{COP}_{L} = \frac{\text{Heat absorbed from Evaporator}}{\text{Compressor Work}} = \frac{\text{Re}}{W_{CL}} \]

\[ \text{COP}_{L} = \frac{190.76 \text{ kJ/kg}}{30.37 \text{ kJ/kg}} = 6.28 \]

Assuming Carnot cycle, the LTC coefficient of performance, \(\text{COP}_{\text{ideal(L)}}\) is given by;

\[ \text{COP}_{\text{ideal(L)}} = \frac{T_{EL}}{T_{CL} - T_{EL}} = \frac{258K}{289K - 258K} = 8.32 \]

The efficiency of the system, \(\eta_L = \frac{\text{COP}_{L}}{\text{COP}_{\text{ideal(L)}}} \)

\[ \eta_L = \frac{6.28}{8.32} \times 100\% = 75.48\% \]

ii With R407A as refrigerant for the high temperature cycle;

At 10\(^{\circ}\)C:

\[ P_5 = 0.694 \text{ MPa}; h_5 = h_{g5} = 400.69 \text{ kJ/kg}; \]
\[ S_5 = 1.7161 \text{ kJ/kgK}; T_5 = 283 K \]

At 40\(^{\circ}\)C:

\[ P_6 = 1.645 \text{ MPa}; T_6^{\prime} = 313K; S_6 = S_5 = 1.7161 \text{ kJ/kgK}; T_6 = 320 K; h_6 = 416.46 \text{ kJ/kg} \]
\[ h_7 = h_{f7} = 259.68 \text{ kJ/kg}; h_8 = h_7 = 259.68 \text{ kJ/kg} \]

The heat rejected by the low temperature cycle’s condenser is equal to the heat gained by the evaporator of the high temperature cycle (activities at the cascade condenser).

Thus; \(\dot{m}_L(h_2 - h_3) = \dot{m}_H(h_5 - h_8)\)

\[ \dot{m}_H = \frac{\dot{m}_L(h_2 - h_3)}{(h_5 - h_8)} = \frac{3.478 \text{ kW}}{(400.69 - 259.68) \text{ kJ/kg}} \]

\[ \dot{m}_H = 0.02467 \text{ kg/s} \]

Work done by compressor, \(W_{CH} = h_6 - h_5\)

\[ W_{CH} = (416.46 - 400.69) \text{ kJ/kg} = 15.77 \text{ kJ/kg} \]
Compressor power, \( P_{CH} = \dot{m}_L(h_6 - h_5) = \dot{m}_L(W_{CL}) P_{CL} = (0.02467 \text{ kg/s}) \times (15.77 \text{ kJ/kg}) = 0.389 \text{ kW} \)

Recalled: \( h_P = 0.746 \text{ kW} \)

\[ P_{CH} = \frac{0.389}{0.746} \times h_P = 0.521h_P \]

Likewise, \( h_P \) compressor capacity is selected for the high temperature system for effective and smooth operation.

The coefficient of performance, \( COP_H \)

\[ COP_H = \frac{\text{Heat absorbed from Evaporator}}{\text{Compressor Work}} = \frac{h_2 - h_8}{h_6 - h_5} \]

\[ COP_H = \frac{(400.69 - 259.68) \text{ kJ/kg}}{(416.46 - 400.69) \text{ kJ/kg}} = 8.94 \]

Also: Assuming Carnot cycle, the HTC coefficient of performance, \( COP_{ideal(H)} \) is given by;

\[ COP_{ideal(H)} = \frac{T_{EH}}{T_{CH} - T_{EH}} = \frac{283K}{313K - 283K} = 9.43 \]

The efficiency of the HTC, \( \eta_H = \frac{\text{COP}_{actual(H)}}{\text{COP}_{ideal(H)}} \)

\[ \eta_H = \frac{8.94}{9.43} \times 100\% = 94.8\% \]

However, for the whole system, the coefficient of performance, \( COP_{whole} \) is estimated as follows:

\[ COP_{whole} = \frac{\text{Heat Absorbed from Evaporator}}{\text{Total Input Work to the System}} \]

\[ COP_{whole} = \frac{h_1 - h_4}{W_{CL} + W_{CH}} = \frac{415.63 - 224.87}{30.37 + 15.77} = 4.13 \]

The efficiency of the entire system, \( \eta_{whole} \)

\[ \eta_{whole} = \frac{\text{Reversible work input}}{\text{Actual work input}} \times 100\% = \frac{W_{rev}}{W_{actual}} \times 100\% \]

Reversible work input, \( W_{rev} = \dot{m}_L(h_1 - h_4)\frac{T_2}{T_1} - 1 \)

\[ W_{rev} = 0.01573(190.76) \left(\frac{303}{258} - 1\right) = 0.5234 \text{ kW} \]

Actual work input, \( W_{actual} = P_{CL} + P_{CH} \)

\[ W_{actual} = (0.478 + 0.389) \text{ kW} = 0.867 \text{ kW} \]

By substitution: \( \eta_{whole} = \frac{0.5234}{0.867} \times 100\% = 60.37\% \)

### 2.3.3 Design of evaporator

Refrigeration capacity, \( Q_{Ref} = UA\Delta T = 3\text{kW} \)

Where: \( U = \) Overall heat transfer coefficient;

\( A = \) Surface area through which heat is being transferred;

\( \Delta T = \) Change in temperature between outside and inside of evaporator = 303K – 258K = 45K
\[
\frac{1}{U} = \frac{1}{h_i} + \frac{x}{K_1} + \frac{x}{K_2} + \ldots + \frac{x}{K_n} + \frac{1}{h_o}
\]

Thus:
\[
\frac{1}{U} = \frac{1}{h_i} + \frac{x}{k} + \frac{1}{h_o}
\]

Where: \(x\) = thickness of copper tube = 0.001m; 
k = thermal conductivity of insulator

\(= 0.4 \text{ kW/mK}\)

\(h_i = \) inside convection coefficient; \(h_o = \) outside convection coefficient

The value of \(h_i\) and \(h_o\) is given from table to be 0.081 kW/m\(^2\)K and 0.094 kW/m\(^2\)K respectively

Thus:
\[
\frac{1}{U} = \frac{1}{0.081} + \frac{0.001}{0.4} + \frac{1}{0.094}
\]

Therefore:
\[
U = 0.04348 \text{ kW/m}^2\text{K}
\]

\(Q_{\text{Ref}} = 3 \text{ kW} = AU \Delta T\)

\[
A = \frac{3 \text{ kW}}{0.04348 \text{ kW/K (45 K)}} = 1.5333 \text{ m}^2
\]

Also: \(A = \pi DL\)

Where: \(\pi = 3.142;\)

\(D = \) diameter of copper pipe = 0.01m;

\(L = \) length of coil

\[
L = \frac{A}{\pi D} = \frac{1.5333}{3.142 \times 0.01} = 48.8 \text{ m}
\]

Let: \(L_{\text{te}} = \) Length of coil in one turn,

\(L_{\text{te}} = (0.7 \times 7) + (0.6 \times 2) = 6.1\text{ m}\)

Number of turns of coil round the inner part of evaporator, \(N_{te}\) is given by:

\[
N_{te} = \frac{\text{Length of coil}}{\text{Length of coil in one turn}} = \frac{L_{\text{te}}}{L_{\text{te}}} = \frac{48.8 \text{ m}}{6.1 \text{ m}} = 8
\]

The depth of loading space inside the evaporator is 0.38m. Gap between each turn of the coil, \(G_{\text{coil}}\) is given as:

\[
G_{\text{coil}} = \frac{0.38\text{m}}{8} = 0.0475 \text{ m}
\]

2.3.4 Design of the cascade condenser

Heat rejected by LTC condenser, \(Q_{\text{CL}} = 3.478 \text{ kW}\)

Volumetric flow rate of liquid refrigerant, \(V = \frac{m_l}{\rho_l}\)
Where: \( \dot{m}_L = 0.01573 \text{ kg/s} \)

\[ \rho_l = \text{Density of refrigerant} = 1228 \text{ kg/m}^3 \]

Therefore: \( V = \frac{0.01573}{1228} = 1.281 \times 10^{-5} \text{ m}^3/\text{s} \)

Cross section area, \( A \) of the condenser tube is given by:

\[ A = \frac{\text{volumetric flow rate}}{\text{Velocity of liquid refrigerant}} \]

Where: Volumetric flow rate \( =1.281 \times 10^{-5} \text{ m}^3/\text{s} \); Velocity of the liquid refrigerant \( = 0.508 \text{ m/s} \)

Therefore: \( A = \frac{1.281 \times 10^{-5}}{0.508} = 2.522 \times 10^{-5} \text{ m}^2 \)

Inner diameter of the condenser tube is calculated thus from:

\[ d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4 \times 2.522 \times 10^{-5}}{3.142}} = 0.006 \text{ m} = 6 \text{ mm} \]

Therefore, a tube with inner diameter of 6 mm is selected for the design.

### 2.3.5 Design of the HTC condenser

Heat rejected by HTC condenser, \( Q_{CH} \) is given as:

\[ Q_{CH} = \dot{m}_H (h_6 - h_7) \]

Where: \( \dot{m}_H = 0.02467 \text{ kg/s} \)

\( (h_6 - h_7) = (416.46 - 259.68) \text{ kJ/kg} \)

Therefore: \( Q_{CH} = 0.02467 \times (416.46 - 259.68) \text{ kW} \quad Q_{CH} = 3.868 \text{ kW} \)

Volumetric flow rate of liquid refrigerant, \( V = \frac{\dot{m}_H}{\rho_l} \)

Where: \( \dot{m}_H = 0.02467 \text{ kg}; \)

\( \rho_l = \text{density of refrigerant} = 1070.70 \text{ kg/m}^3 \)

Therefore: \( V = \frac{0.02467}{1070.70} = 2.3041 \times 10^{-5} \text{ m}^3/\text{s} \)

Let: \( A = \text{Cross section area of the condenser tube} \)

\[ A = \frac{\text{volumetric flow rate}}{\text{Velocity of liquid refrigerant}} \]

Where: Volumetric flow rate \( =2.3041 \times 10^{-5} \text{ m}^3/\text{s} \)

Velocity of the liquid refrigerant \( = 0.508 \text{ m/s} \)

Therefore: \( A = \frac{2.3041 \times 10^{-5}}{0.508} = 4.54 \times 10^{-5} \text{ m}^2 \)
Inner diameter of the condenser tube is calculated from: \[ A = \frac{\pi d^2}{4} \]

Thus: \[ d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4 \times 4.54 \times 10^{-5}}{3.142}} = 0.0076 \text{ m} \]

Therefore, a tube with inner diameter of 8mm is selected for the design.

### 2.4 Fabrication and Assembly

The machine has an evaporator (cooling box) with outer dimensions of 900mm x 700mm x 560mm and internal dimensions of 800mm x 600mm x 460mm (representing length, breadth and depth respectively). It is built of materials comprising of galvanized steel sheet metal (plate) of 1mm thick and polyisocyanurate (insulator) of 50mm thickness. The plate and the insulating materials are cut to size and riveted together to form a rigid box. The box is divided into six compartments to enhance effective flow of heat between evaporating coils and water. Within the surrounding walls of each compartment are fitted the evaporating coils (Fig. 3). The cover for the evaporator is also made of the same materials in form of slab of 900mm x 700mm x 50mm with insulation and covered all round with galvanized steel sheet metal. In order to maintain good thermal efficiency a seal made of flat and straight edge gasket material is laid at the tip of the slab positioned to form an airtight between it and the box when the evaporator cover is closed. Every other component such as compressors, condensers, expansion valves and drier filters are predesigned and are readily available in the market. They come in different sizes and capacities thus their selection are based on the cooling load expected to carry. Hence, they are bought and fixed. The piping networks are made of copper tube and are brazed together with the aid of sivrons. Circuit breakers are used as a safety device from over load. Digital temperature meter is mounted on the evaporator to display the evaporator temperature as the device works. The compressors are installed inside the same casing with the respective condenser using bolts and nuts. Other components such as the expansion valves and filters are fixed by welding. All the components of the refrigeration system are arranged in compact form into three units. One unit is the evaporator mounted on a separate frame support, the second unit houses the other components of LTC system while the third unit houses the other components of HTC system. The last two units are mounted on a single support frame to minimize space. The supports are constructed of 5cm x 5cm angled bar which were cut to sizes and welded together to form rigid frame.

### 3. EXPERIMENTATION

In setting for test running, the temperature meter mounted on the evaporator (cooling box) is ensured for perfect working condition. A thermocouple is fixed at inlet and exit of evaporator and condenser of each of the LTC and HTC system. A pressure gauge is mounted at inlet and outlet of every compressor to measure the suction and discharge pressure respectively. The system is charged with respective refrigerant (R410A for LTC and R407A for HTC) and tested leak free.
Then the machine is tested at no-load and at varying load conditions. Basically, the analysis of the machine performance was centered on three operating parameters - evaporator temperature ($T_{EL}$), condenser temperature ($T_{CH}$) and temperature difference in cascade condenser ($\Delta T_{CC}$). These parameters were varied one after the other within a limited range keeping other parameters constant and the effect on the refrigerating effect, coefficient of performance and the overall efficiency of the system were analyzed.

4. RESULTS AND DISCUSSION

From the data obtained during experimentation, the refrigerating effect, COP and overall efficiency were determined, analyzed and discussed as follows: Shown in Fig. 4(a - c), when evaporator temperature increases from $-15^\circ$C to $-3^\circ$C at the interval of $3^\circ$C keeping other parameters constant, the refrigerating effect increases from 189.17kJ/kg to 193.48kJ/kg to 196.16kJ/kg to 198.78kJ/kg to 201.34kJ/kg; the coefficient of performance ($\text{COP}_{\text{whole}}$) increases from 4.13 to 4.67 to 5.25 to 5.90 to 6.90 and the overall efficiency ($\text{Ƞ}_{\text{whole}}$) increases from 61.03% to 62.42% to 62.63% to 62.79% to 64.27%.

![Fig. 4a](https://ssrn.com/abstract=3535604) - Effect of variation in evaporator temperature on refrigerating effect

![Fig. 4b](https://ssrn.com/abstract=3535604) - Effect of variation in evaporator temperature on coefficient of performance.

![Fig. 4c](https://ssrn.com/abstract=3535604) - Effect of variation in evaporator temperature on overall efficiency.

![Fig. 5a](https://ssrn.com/abstract=3535604) - Effect of variation in condenser temperature on refrigerating effect
Fig. 5b - Effect of variation in condenser temperature on coefficient of performance.

Fig. 5c - Effect of variation in condenser temperature on overall efficiency.

Fig. 5(a - c) show that as condensing temperature varies from 40°C to 49°C at the interval of 3°C, the refrigerating effect decreases from 189.17kJ/kg to 187.58kJ/kg to 185.98kJ/kg to 184.37kJ/kg, the COP of the system decreases from 4.13 to 3.84 to 3.82 to 3.80 while the overall efficiency of the system decreases from 61.03% to 54.36% to 52.78% to 50.92%.

Fig. 6a - Effect of variation in temperature difference in cascade condenser on refrigerating effect

Fig. 6b - Effect of variation in temperature difference in cascade condenser on coefficient of performance

Fig. 6c - Effect of variation in temperature difference in cascade condenser on overall efficiency

Electronic copy available at: https://ssrn.com/abstract=3535604
As shown in Fig. 6(a – c), when the temperature difference in cascade condenser ($\Delta T_{CC}$) decreases from 6°C to 2°C at the interval of 1°C, the refrigerating effect increases from 190.76kJ/kg to 192.34kJ/kg to 194.46kJ/kg to 195.49kJ/kg to 197.06kJ/kg; the COP of system increases from 4.18 to 4.31 to 4.44 to 4.53 to 4.62 and the overall efficiency increases from 60.64% to 61.71% to 62.63% to 62.76% to 63.25%.

5. CONCLUSION

The result obtained from various tests shows a very high transformation efficiency and the design goal was achieved within the set up time frame which depicts high effectiveness and reliability of the ice block making machine. It also shows that the required time to transform water to ice in a fixed system is dependent of the quantity of water placed in the refrigerator. Apart, the result revealed that the designed machine exhibits a very impressive transformation capacity compared with some other homemade ice making machines. The device is simple, thus with a little skill, the system can be dismantled or assembled for services and repairs.

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