Anisotropic in-plane resistivity in the nematic phase of the iron pnictides

Rafael M. Fernandes, Elihu Abrahams, and Jörg Schmalian

Ames Laboratory and Department of Physics and Astronomy, Iowa State Univ., Ames, IA 50011, USA
Department of Physics and Astronomy, University of California Los Angeles, Los Angeles, CA 90095, USA

(Dated: January 21, 2013)

We show that the interference between scattering by impurities and by critical spin fluctuations gives rise to anisotropic transport in the Ising-nematic state of the iron pnictides. The effect is closely related to the non-Fermi liquid behavior of the resistivity near an antiferromagnetic quantum critical point. Our theory not only explains the observed sign of the resistivity anisotropy \( \Delta \rho \) in electron doped systems, but also predicts a sign change of \( \Delta \rho \) upon sufficient hole doping. Furthermore, our model naturally addresses the changes in \( \Delta \rho \) upon sample annealing and alkaline-earth substitution.

In many materials, anisotropic properties are related to the underlying crystalline structure. However, when correlations are present, the electronic states can themselves become anisotropic \([1]\). Recent transport measurements in detwinned crystals of the iron pnictide compounds \( AEFe_2As_2 \) (\( AE = Ba, Ca, Sr \)) found an in-plane anisotropy that cannot be attributed only to lattice distortions, unveiling an anisotropic electronic state \([2,3]\). Its existence is also supported by the observations of an orbital polarization \( O = n_{xz} - n_{yz} \) of \( d_{xz} \) and \( d_{yz} \) Fe states in angle-resolved photoemission spectroscopy (ARPES) \([4,5]\), local anisotropies in scanning tunneling microscopy \([6]\), and anisotropies in the optical spectrum \([6,8]\).

One candidate for such unconventional electronic state is the Ising-nematic order, which emerges from the combination of magnetic fluctuations and frustration \([6,13]\). In a strong-coupling approach, frustration is promoted by the competing \( J_1 - J_2 \) exchange interactions, while in a weak-coupling approach, it follows from the degeneracy of the magnetic ground state due to the nesting properties of the Fermi surface (FS). Indeed, the iron pnictides support two magnetic instabilities with order parameters \( \Delta_1, \Delta_2 \) corresponding to the in-plane ordering vectors \( Q_1 = (\pi, 0), Q_2 = (0, \pi) \), respectively. The electronic structure \([14,13]\), as well as the coupling to the lattice \([13]\), give rise to the coupling \( g\Delta_1^2\Delta_2^2 \) in the free energy, with \( g > 0 \). As a result, a discrete Ising-nematic degree of freedom \( \varphi \propto \langle \Delta_1^2 \rangle - \langle \Delta_2^2 \rangle \) emerges, which labels the two degenerate states \( \Delta_1 \neq 0, \Delta_2 = 0 \) and \( \Delta_1 = 0, \Delta_2 \neq 0 \). Most importantly, \( \varphi \) is able to order at a temperature \( T_\text{N} \) above the onset of antiferromagnetic (AFM) at \( T_\text{AFM} \), breaking the tetragonal symmetry already in the paramagnetic (PM) phase - hence the term nematic.

By symmetry, Ising-nematic order induces the orbital polarization \( O \) and a shear distortion \( \varepsilon_s \), driving a structural transition from the tetragonal (Tet) to the orthorhombic (Ort) phase at \( T_s \). It explains why \( T_s \) and \( T_N \) track each other closely across every pnictide phase diagram, even inside the superconducting (SC) dome \([17,18]\). Recently, it has been shown that nematic fluctuations can explain the dramatic softening of the lattice observed experimentally in the Tet phase \([13]\). It remains an important issue whether this scenario is also able to address the observed in-plane resistivity anisotropy \( \Delta \rho \equiv \rho_{xy} - \rho_a \), where \( b \) refers to the direction with shorter (longer) lattice constant of the Ort state. The induced orbital polarization \( O \) has been alternatively proposed to explain such anisotropy, but different authors disagree on the suitability of this proposal \([19,20]\).

In this paper we demonstrate that the observed anisotropic transport above \( T_N \) can be understood within the Ising-nematic picture. Our model explains in a unified way several experimental observations, such as the sign of the anisotropy \([2,3]\) and the changes in \( \Delta \rho \) upon sample annealing \([8]\) or alkaline-earth substitution \([21]\). We also predict a sign change of \( \Delta \rho \) for hole-doped systems, thus addressing the peculiar observations in K-doped compounds \([22,23]\). Our results support the view that nematic order is the driving force for the structural transition and the orbital polarization in the pnictides. Note that our theory does not address the anisotropy observed below \( T_N \), in the AFM state. However, in all observations, the sign of \( \Delta \rho \) is already set above \( T_N \), where the slope \( d(\Delta \rho)/dT \) is the largest \([2,3]\). Concentrating on the regime above \( T_N \) allows for a more transparent and less model-dependent understanding of the resistiv-

Figure 1: (a) Schematic temperature-doping \((T, x)\) phase diagram in the absence of SC. The yellow region corresponds to enhanced nematic fluctuations, as seen by elastic modulus measurements \([13]\). (b) Temperature dependence of the correlation lengths \( \xi \) associated with the \((\pi, 0)\) and \((0, \pi)\) magnetic instabilities above the magnetic QCP.
ity anisotropy.

Below $T_s$, the onset of a finite Ising-nematic order parameter $\varphi \neq 0$ gives rise to two sources of anisotropy: in the electronic structure itself and in the spectrum of magnetic fluctuations. The former is rather weak, due to the smallness of the shear distortion $\varepsilon_s = (a - b) / (a + b)$ - for instance, in optimally doped Ba(Fe$_{1-x}$Co$_x$)$_2$As$_2$, $\varepsilon_s \approx 5 \times 10^{-4}$ [4, 17]. On the other hand, the latter is strong, since only fluctuations associated with one of the magnetic instabilities diverge. The impact of this anisotropy on the transport depends on the contribution of the scattering of electrons by spin fluctuations around the hot spots of the Fermi surface, defined via $\varepsilon_{\lambda,k} = \varepsilon_{\lambda',k + Q_i}$, with $\varepsilon_{\lambda,k}$ denoting the dispersion of band $\lambda$. In ultra-clean systems, this scattering channel is short-circuited by the contribution from the other regions of the FS [24]. However, in the presence of significant impurity scattering, the scattering by spin fluctuations becomes important, leading to non-Fermi liquid behavior [25], and, in addition, to anisotropic transport. To see this, we follow Rosch [26] and solve the Boltzmann equation (BE) for scattering by impurities and spin fluctuations, which yields features not captured by the relaxation time approximation [26]. In the regime where impurity scattering dominates, we obtain the resistivity along the $\alpha$ ($\alpha = x, y, z$) direction:

$$\rho_{\alpha\alpha} - \rho_{\text{imp}} = \sum_{k,k',\lambda,\lambda'} \left( \Phi^{\alpha}_{k,k',\lambda} - \Phi^{\alpha}_{k,k',\lambda'} \right)^2 D^{\lambda\lambda'}_{kk'}$$

with $D^{\lambda\lambda'}_{kk'} = f^0_{k',\lambda'} \left( 1 - f^0_{k,\lambda} \right) \eta^{\lambda\lambda'}_{\text{sf,k,k'}}$. Here, $k$ is the electron momentum, $\lambda$ is the band index, $f^0_{k,\lambda}$ is the Fermi-Dirac distribution function, and $\rho_{\text{imp}}$ is the (isotropic) residual resistivity. $\eta^{\lambda\lambda'}_{\text{sf,k,k'}} = \sum_{n=1}^{\infty} g_{\lambda,\lambda'}^2 \int d\omega \imath \chi_i (q, \omega)$ is the spin-fluctuations collision term, where $\omega = \varepsilon_{\lambda,k} - \varepsilon_{\lambda',k'}$ and $q = k - k'$ are, respectively, the transferred energy and momentum, and $\chi_i (q, \omega)$ is the dynamic susceptibility associated with the ordering vector $Q_i$. $g_{\lambda,\lambda'}$ is the scattering amplitude and $\Phi^{\alpha}_{k,k',\lambda}$ is the deviation of the electron distribution function from equilibrium due to scattering by impurities only, as a consequence of an electric field applied along the $\alpha$ direction. It only depends on the impurity scattering potential $V^{\lambda\lambda'}_{\text{imp}}$ and on the shape of the FS (see supplementary material for more details).

The resistivity anisotropy data in Ba(Fe$_{1-x}$Co$_x$)$_2$As$_2$ show that the temperature-dependent contribution is smaller than the residual resistivity, particularly close to optimal doping [2]. Therefore, we can calculate $\rho_{\alpha\alpha}$ using Eq. (11), whose inputs are the band dispersions $\varepsilon_{\lambda,k}$, which determine the impurity-only solution $\Phi^{\alpha}_{k,k',\lambda}$, and the dynamic susceptibilities $\chi_i (q, \omega)$, which determine the spin-fluctuation scattering amplitude $t^{\lambda\lambda'}_{\text{sf,k,k'}}$. In the Ising-nematic phase, the latter are:

$$\chi_i (q, \omega) = \frac{C_0}{r_i (q + Q_i)} \frac{1}{\varphi - i\omega / \gamma},$$

where the upper (lower) signs refers to $Q_1$ ($Q_2$), $\gamma$ is the Landau damping parameter, and $r_i (q + Q_i) = a_0^2 q^2 + a_0^2 q_y^2 (1 + \eta) + a_0^2 q_y^2 (1 + \eta) + \eta_i^2 \cos^2 (\frac{2\pi q}{a}),$ which yields features not captured by the relaxation time approximation [26]. In the paraelectric phase, the correlation length $\xi$, the in-plane anisotropy $\eta$, and the out-of-plane anisotropy $\eta_z$. The dependence of $\xi$ and $\varphi$ is calculated within a self-consistent mean-field approach [11, 12], yielding the equations:

$$1 = \frac{T}{2} \sum_{\omega_n} \left[ \chi_1 (q, \omega_n) + \chi_2 (q, \omega_n) \right]$$

$$\varphi = \frac{T}{2} \sum_{\omega_n} \left[ \chi_1 (q, \omega_n) - \chi_2 (q, \omega_n) \right] + h_{\text{strain}},$$

where $\omega_n$ are bosonic Matsubara frequencies and $h_{\text{strain}} \propto P$, with $P$ denoting the external strain applied to detwin the samples. For $h_{\text{strain}} = 0$, Eqs. (4) have a non-trivial solution with $\varphi \neq 0$ below a temperature $T_s$, describing the PM-Ort (Ising-nematic) phase. This model is similar to the one used in Ref. [13], which successfully describes the lattice softening above $T_s$. Since $\xi^{-2} = \xi^{-2} \pm \varphi$, when $\varphi > 0$ (i.e. $a \parallel x$ and $b \parallel y$) the correlation length $\xi (\pi, 0)$ associated with the $Q_1$ ordering vector lengths while $\xi (\pi, 0)$, associated with $Q_2$, shortens (see Fig. 1). In the figure, we present results for an AFM quantum critical point (QCP), $\xi^{-1} (\pi, 0) (T = 0) = 0$, where fluctuations are strongest.
Substituting the susceptibilities given by (2) in the BE solution (1), we find that the low-T transport is dominated by the subset of hot spots connected by the soft ordering vector \((\mathbf{Q}_1)\) for \(\varphi > 0\), \(\varepsilon_{\lambda,\mathbf{k}_{\parallel}+\mathbf{Q}_1} = \varepsilon_{\lambda',\mathbf{k}_{\parallel}+\mathbf{Q}_1}\), while contributions due to \(\mathbf{Q}_2 = (0, \pi)\) are negligible. It follows that

\[
\rho_{\alpha\alpha} - \rho_{\text{imp}} = \kappa T \langle \Phi_{\mathbf{k}_{\parallel}+\mathbf{Q}_1,\lambda}^a - \Phi_{\mathbf{k}_{\parallel}+\mathbf{Q}_1,\lambda}^b \rangle^2 \tag{4}
\]

where \(\kappa > 0\) is a constant. The exponent \(\zeta\) determines the T-dependence of the resistivity and is given by \(\zeta = 3/2\) (\(\zeta = 1\)) for \(T \lesssim \Delta^2\eta_2^2\) (\(T \gtrsim \Delta^2\eta_2^2\)), depending on whether the spin fluctuation spectrum is two- or three-dimensional (\(\Delta\) is the energy cutoff). While the experiments of Ref. [29] on BaFe\(_2\)(As\(_{1-x}\)P\(_x\))\(_2\) materials support \(\zeta = 1\), Eq. (4) is always dominant when compared to the Fermi-liquid contribution \(\rho - \rho_{\text{imp}} \propto T^2\). The distinction into active and passive hot spots is not restricted to the AFM QCP and persists everywhere along the finite temperature phase boundary where soft and hard magnetic fluctuations exist. Thus, the Ising-nematic order causes an anisotropy in the spectrum of magnetic fluctuations, which also induces an anisotropy in the resistivity via scattering by spin fluctuations (see Fig. 2). To determine the amplitude and sign of \(\Delta\rho\), all that is left is to calculate \(\Phi_{\mathbf{k}_{\parallel}+\mathbf{Q}_1,\lambda}^a\), i.e. the solution of the BE with impurities only, which depends solely on the band dispersions \(\varepsilon_{\lambda,\mathbf{k}}\).

To gain analytical insight, we introduce a simplified model with a circular hole pocket at the center of the Brillouin zone and elliptical electron pockets displaced from the center by \(\mathbf{Q}_1\) and \(\mathbf{Q}_2\) [14, 15], \(\varepsilon_{1,\mathbf{k}_{\parallel}} = \varepsilon_0 - k_x^2/2m - \mu\), \(\varepsilon_{2,\mathbf{k}+\mathbf{Q}_1} = -\varepsilon_0 + k_x^2/2m_x + k_y^2/2m_y - \mu\), and \(\varepsilon_{3,\mathbf{k}+\mathbf{Q}_2} = -\varepsilon_0 + k_x^2/2m_y + k_y^2/2m_x - \mu\). Here, \(\varepsilon_0\) is the shift in the band energies, \(m_i\) are the band masses, and \(\mu\), the chemical potential. The calculation of \(\Phi_{\mathbf{k}_{\parallel}+\mathbf{Q}_1,\lambda}^a\) for this case is straightforward and is presented in the supplementary material. In particular, we find that the sign and amplitude of the anisotropy depends on the projections along \(k_x\) (parallel to \(a\)) and \(k_y\) (parallel to \(b\)) of the Fermi velocity at the hot spots \(\varepsilon_{1,\mathbf{k}_{\parallel}} = \varepsilon_{2,\mathbf{k}_{\parallel}+\mathbf{Q}_1}\) (see Fig. 2). When the hot spots are close to a certain axis, there is stronger scattering by spin fluctuations when the electrons move parallel to this axis, and the resistivity is larger along this direction. Interestingly, ARPES measurements in the optimally doped Ba(Fe\(_{1-x}\)Co\(_x\))\(_2\)AS\(_2\), which is close to a possible AFM QCP covered by the SC dome [33], reveal the presence of soft hot spots close to the \(k_y\) (\(b\)) axis [31]. According to our model, this implies \(\rho_b > \rho_a\), in agreement with transport measurements [2].

To go beyond the low-T limit in Eq. (1), we numerically calculated \(\Delta\rho\) from Eq. (1) for fixed band structure parameters, \(m_x = 1.25m\), \(m_y = 0.83m\), and \(\mu = 0.05\varepsilon_0\). For the dynamic susceptibility in Eq. (2), based on previous works [13] and neutron scattering data [28], we used \(g/C_0 \approx 2 \times 10^{-2}\), \(\eta_s \approx 0.1\), \(\eta_f \approx 0.5\), \(\gamma \approx 350\) meV, and \(C_0 \approx 1\) meV, yielding \(T_a = 40\) K and \(T_N = 0\). We also considered \(g_{\lambda'}^{\lambda} \approx 2\lambda\lambda'\) and expressed the results in terms of the residual resistivity \(\rho_{\text{imp}}\).

The behavior of \(\rho_a\) and \(\rho_b\) as function of temperature is shown in Fig. 3, where we considered \(\varphi > 0\), i.e. \(x\) and \(y\) parallel to \(a\) and \(b\), respectively. In accordance to our discussion of the low-T limit, we find \(\rho_b > \rho_a\) for any temperature below \(T_s\), a behavior that lingers even when one moves away from the QCP. As expected, we also find that \(\Delta\rho \propto \varphi\) close to \(T_s\) [22]. Note that the sign of \(\Delta\rho\) does not depend on the sign of the in-plane momentum anisotropy \(\eta\) of \(\chi(\mathbf{q},\omega)\) in Eq. (2). As \(T \rightarrow 0\), spin fluctuations are suppressed and \(\Delta\rho \rightarrow 0\). In the actual data of Ref. [2], these curves are cut off by the onset of SC, which is not considered in our model.

To make the comparison with experiments more realistic, we also included the effects of a small strain \(\delta\) of \(0.1\) MPa, which is applied to detwin the sample. This strain breaks the tetragonal symmetry at any temperature, and there is not a well-defined structural transition, since \(\varphi \sim \delta^{1/\beta}\) is finite at \(T_s^{\text{win}}\), where \(\beta\) is the corresponding Ising critical exponent. Our calculations show that the AFM transition is practically unchanged for small strain, in agreement with the observations in Ref. [2]. If the structural transition in the twin sample is strongly first order such that \(\delta^{1/\beta}\) is small compared to the jump of \(\varphi\) at \(T_s^{\text{win}}\), the tail in \(\Delta\rho\) above \(T_s^{\text{win}}\) becomes negligible. This is the case in the parent compounds with Ba replaced by Ca or Sr, as observed by Ref. [21].

Our model also explains recent experiments that found a suppression of \(\Delta\rho\) after annealing the sample [8]. As we mentioned earlier, in the ultra-clean limit the contribution of the hot spots to the resistivity is short-circuited by the other regions of the FS [24]. Since the anisotropy is governed by scattering processes near the hot spots, \(\Delta\rho\) becomes smaller for cleaner samples. Our results can also be rationalized in terms of a \(T\)-dependent anisotropic dressing of the impurity scattering by spin fluctuations.
Via this many-body mechanism, an s-wave scattering center acquires effectively an anisotropic cross-section, breaking the $C_4$ symmetry.

Finally, we discuss a general prediction of our model. When doping is introduced and the chemical potential shifts, the positions of the hot spots change accordingly. Without relying on details of $\varepsilon_{\lambda,k}$, one expects that by hole-doping the samples (i.e. decreasing $\mu$ towards negative values), the area of the hole pocket decreases, whereas the area of the electron pocket decreases. Consequently, active hot spots, which initially are close to the $k_y$ ($b$) axis, will move towards the $k_x$ ($a$) axis, and $\Delta \rho$ will eventually change its sign. This is illustrated in Fig. 4, where we present a phase diagram for the sign of $\Delta \rho$ for different values of the chemical potential $\mu$ and the ellipticity of the electron pockets $\propto m_x - m_y$, keeping the other parameters unchanged. It is interesting to note that measurements of $\Delta \rho$ in hole-doped samples ($\text{Ba}_1-x\text{K}_x\text{Fe}_2\text{As}_2$) found a vanishingly small resistivity anisotropy \cite{22}, as expected near the region of the phase diagram of Fig. 4 where $\Delta \rho$ changes sign. Most interestingly, very recent data on samples with higher doping revealed a negative $\Delta \rho$, in accordance to our prediction \cite{22}.

In our approach, the resistivity anisotropy above $T_N$ is a consequence of anisotropic scattering rates along $a$ and $b$ directions. Thus, optical conductivity at low frequencies is the ideal tool to verify our results. Unfortunately, as stated in Ref. \cite{20}, the experimental conditions in this temperature regime need to be refined before definite conclusions can be drawn. Yet, it is encouraging that the available data above $T_N$ on electron-doped samples indicate a larger scattering rate along the $b$ direction \cite{20,21}, in agreement with our results. We stress that our model is suitable for the paramagnetic phase, where the sign of the resistivity anisotropy is determined, according to the experimental results on various compounds \cite{2,3,21,33}. Below $T_N$, the spin fluctuations associated with the magnetically ordered phase are naturally anisotropic, and also give rise to anisotropic scattering. Both this anisotropic scattering and the details of the anisotropic reconstruction of the FS \cite{20,33,31} will govern the temperature evolution of $\Delta \rho$ below $T_N$.

In summary, we showed that the onset of Ising-nematic order leads to an anisotropy in the magnetic fluctuations associated with the two magnetic ground states of the iron pnictides. As a result, the same physics responsible for non-Fermi liquid behavior - the interference of scattering by spin fluctuations and impurities \cite{22} - also gives rise to anisotropic transport properties. The latter are in agreement with several different experimental observations, such as the opposite signs of $\Delta \rho$ in electron-doped and hole-doped materials, as well as the suppression of $\Delta \rho$ upon sample annealing.

The authors thank J.-H. Chu, I. R. Fisher, R. Prozorov, and M. Tanatar for useful discussions. Research at Ames Lab was supported by the U.S. DOE, Office of BES, Materials Sciences and Engineering Division.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure4.png}
\caption{Sign of the low-temperature resistivity anisotropy $\Delta \rho = \rho_b - \rho_a$ as function of the ellipticity $m_x - m_y$ of the electron pockets and the chemical potential $\mu$. Note the asymmetry between electron and hole doping.}
\end{figure}
Supplemental Material

The linearized Boltzmann equation determines the electronic distribution function $f_{k,\lambda}$:

$$-e v_{k,\lambda} \cdot n \left( \frac{\partial f_{k,\lambda}^0}{\partial \epsilon_{k,\lambda}} \right) = \frac{1}{T} \sum_{k',\lambda'} (\Phi_{k,\lambda} - \Phi_{k',\lambda'}) f_{k',\lambda'}^0 (1 - f_{k,\lambda}^0) t_{k,k'}^{\lambda,\lambda'}$$  \hspace{1cm} \text{(S1)}$$

Here, $\Phi_{k,\lambda}$ is the non-equilibrium distribution function, defined as $f_{k,\lambda} \equiv f_{k,\lambda}^0 - \left( \frac{\partial f_{k,\lambda}^0}{\partial \epsilon_{k,\lambda}} \right) \Phi_{k,\lambda}$, with $f_{k,\lambda}^0$ denoting the Fermi-Dirac distribution at energy $\epsilon_{k,\lambda}$. $k$ is the momentum, $\lambda$ is the band index, $v_{k,\lambda} = \partial \epsilon_{k,\lambda}/\partial k$, and $n$ is the unit vector parallel to the external electric field. The collision term $t_{k,k'}^{\lambda,\lambda'}$ comprises scattering by impurities and spin fluctuations \cite{24,25}:

$$t_{\text{imp},k,k'}^{\lambda,\lambda'} = \left( g_{\text{imp}}^2 \delta_{\lambda,\lambda'} + \bar{g}_{\text{imp}}^2 (1 - \delta_{\lambda,\lambda'}) \right) \delta (\epsilon_{k,\lambda} - \epsilon_{k',\lambda'})$$

with intraband impurity scattering amplitude $g_{\text{imp}}^2 = \left| V_{\text{imp}}^{\lambda = \lambda'} \right|^2$, interband impurity scattering amplitude $\bar{g}_{\text{imp}}^2 = \left| V_{\text{imp}}^{\lambda \neq \lambda'} \right|^2$, and spin fluctuation scattering amplitude $g_{\text{sf}}^2$. Here, $n(\omega)$ is the Bose-Einstein distribution and $\chi_i(q,\omega)$ is the dynamic susceptibility, given by Eq. 2 of the paper. Since the electric field defines a direction $n$, it is convenient to write the non-equilibrium distribution function as $\Phi_{k,\lambda} \equiv \Phi_{k,\lambda}^\mu n^\mu$, with $\mu = 1, 2, 3$ denoting Cartesian coordinates. Given $\Phi_{k,\lambda}^\mu$, it is straightforward to obtain the conductivity tensor $\sigma_{\mu\nu} = -e \sum_{k,\lambda} \left( \frac{\partial f_{k,\lambda}^0}{\partial \epsilon_{k,\lambda}} \right) t_{k,k'}^{\lambda,\lambda'} \Phi_{k',\lambda'}^\nu$, with $\epsilon_{k,\lambda}$ and $\Phi_{k,\lambda}^\mu$ are vectors ($\mu = 1, 2, 3$), but also matrices in the $k\lambda$ space. $\hat{C}_i$ is defined by the matrix elements:

$$\left( \hat{C}_i \hat{\Phi}^\mu \right)_{k,\lambda} = \frac{1}{T} \sum_{k',\lambda'} (\Phi_{k,\lambda}^\mu - \Phi_{k',\lambda'}^\mu) f_{k',\lambda'}^0 (1 - f_{k,\lambda}^0) t_{k,k'}^{\lambda,\lambda'}$$  \hspace{1cm} \text{(S3)}$$

In this language, it is convenient to introduce the scalar product $\langle \hat{X}^\mu | \hat{Y}^\nu \rangle = \sum_{k,\lambda} X_{k,\lambda}^\mu Y_{k,\lambda}^\nu$, from which $\sigma_{\mu\nu} = \langle \hat{X}^\mu | \hat{\Phi}^\nu \rangle$. We consider that impurities are the dominant scatterers, while scattering by spin fluctuations perturbatively suppresses the conductivity. This is always true at low enough temperatures, where $g_{\text{sf}}^2 (T/\gamma)$ is small compared to $g_{\text{imp}}^2$. To lowest order, the solution of the Boltzmann equation is $\hat{\Phi}^\mu = \hat{\Phi}^\mu_0 + \delta \hat{\Phi}^\mu$, where $\delta \hat{\Phi}^\mu$ is the solution in the presence of impurities only and $\hat{\Phi}^\mu_0$ is the solution of $\hat{C}_{\text{imp}} \delta \hat{\Phi}^\mu = -\hat{C}_{\text{sf}} \delta \hat{\Phi}^\mu$. Therefore, the conductivity becomes:

---

[24] R. Hlubina and T. M. Rice, Phys. Rev. B 51, 9253 (1995).
[25] A. Rosch, Phys. Rev. Lett. 82, 4280 (1999).
[26] A. F. Kemper et al., Phys. Rev. B 83, 184516 (2011).
[27] J. Zhang, R. Sknepnek, and J. Schmalian, Phys. Rev. B 82, 134527 (2010).
[28] S. O. Diallo et al., Phys. Rev. B 81, 214407 (2010); D. S. Inosov et al., Nat. Phys. 6, 178 (2010).
[29] S. Kasahara et al., Phys. Rev. B 81, 184519 (2010).
[30] J. Dai, Q. Si, J.-X. Zhu, and E. Abrahams, PNAS 106, 4118 (2009).
[31] C. Liu et al., Nat. Phys. 6, 419 (2010).
[32] R. M. Fernandes, J. Schmalian, and H. Westfahl Jr, Phys. Rev. B 78, 184201 (2008).
[33] H. H. Kuo et al., arXiv:1103.4535.
[34] Z. P. Yin, K. Haule, and G. Kotliar, Nat. Phys. 7, 294 (2011).
\[
\sigma_{\text{imp}} = \left\langle \Phi^{0,\mu} \left| \hat{C}_{\text{imp}} \right| \Phi^{0,\mu} \right\rangle - \left\langle \hat{\Phi}^{0,\mu} \left| \hat{C}_{\text{st}} \right| \Phi^{0,\mu} \right\rangle \tag{S4}
\]

The first term is the constant impurity contribution to the conductivity, \(\sigma_{\text{imp}}\), whereas the second term is the temperature dependent contribution from spin fluctuations (see Eq. 1 of the main text). The calculation of \(\hat{\Phi}^{0,\mu}\), which satisfy \(\hat{X}^\mu = \hat{C}_{\text{imp}} \hat{\Phi}^{0,\mu}\), is tedious but straightforward. We use the variational principle, as outlined in \cite{29}, and the three-band model discussed in the paper, and obtain, for the electric field along \(\hat{x}\):

\[
\Phi_{1,\theta}^{0,x} = - \frac{e v_F}{\nu \left( g_{\text{imp}}^2 + 2 g_{\text{imp}}^2 \right)} \cos \theta
\]

\[
\Phi_{(2,3),\theta}^{0,x} = \frac{e v_F}{\nu \left( g_{\text{imp}}^2 + 2 g_{\text{imp}}^2 \right)} \left( \frac{1 + \delta_0 \mp \delta_2 \pm 2 \delta_0 \cos 2\theta}{\sqrt{1 + \delta_0 \pm \delta_2 \cos 2\theta}} \right) \cos \theta \tag{S5}
\]

Here, \(\theta\) is the polar angle and \(v_F, \varepsilon_F\), and \(\nu\) are, respectively, the Fermi velocity, Fermi energy, and density of states of the hole pocket. \(\delta_i = \delta_i / \varepsilon_F\) depends only on the band structure parameters and is given by:

\[
\begin{align*}
\frac{\delta_0}{\varepsilon_F} &= 2 \tilde{\mu} + \left( 1 - \frac{\bar{m}_x + \bar{m}_y}{2 \bar{m}_x \bar{m}_y} \right) \\
\frac{\delta_2}{\varepsilon_F} &= \frac{\bar{m}_x - \bar{m}_y}{2 \bar{m}_x \bar{m}_y}
\end{align*} \tag{S6}
\]

with \(\bar{m}_i = m_i / m\) and \(\tilde{\mu} = \mu / \varepsilon_F\). The residual conductivity due to impurities is:

\[
\sigma_{\text{imp}} = \frac{e^2 v_F^2}{2 \left( g_{\text{imp}}^2 + 2 g_{\text{imp}}^2 \right)} \left[ 1 + 4 \left( 1 + \delta_0 \right) - 2 \sqrt{(1 + \delta_0)^2 - \delta_2^2} \right] \tag{S7}
\]

For an electric field parallel to the \(\hat{y}\) direction, we can use the transformation properties of the non-equilibrium distribution functions under a \(\pi/2\) rotation: \(\Phi_{1,\theta}^{0,y} = \Phi_{1,\theta}^{0,x}\), \(\Phi_{2,\theta}^{0,y} = \Phi_{3,\theta}^{0,x}\), and \(\Phi_{3,\theta}^{0,y} = \Phi_{2,\theta}^{0,x}\), with \(\tilde{\theta} = \theta - \pi/2\).

We can now evaluate the matrix elements \(\left\langle \Phi^{0,\mu} \left| \hat{C}_{\text{st}} \right| \Phi^{0,\mu} \right\rangle\) using the solutions in Eqs. S5. After defining:

\[
\omega_{\theta,\theta'}^{(i)} = \left( \xi_i / a_0 \right)^{-2} + \left( 2 + \delta_0 \mp \delta_2 \cos 2\theta' \right) - 2 \cos (\theta - \theta') \sqrt{1 + \delta_0 \pm \delta_2 \cos 2\theta'} \pm \eta \left[ \cos 2\theta - 2 \cos (\theta + \theta') \sqrt{1 + \delta_0 \pm \delta_2 \cos 2\theta'} \cos 2\theta' \left( 1 + \delta_0 \pm \delta_2 \cos 2\theta' \right) \right] \tag{S8}
\]

we obtain (hereafter we omit the superscript 0 in \(\Phi_{1,\theta}^{0,\mu}\) for convenience):

\[
\rho^{\mu\nu} - \rho_{\text{imp}} = \zeta t \sum_{i=2,3} \int d\theta d\theta' \left( \Phi_{1,\theta}^{0,\mu} - \Phi_{i,\theta'}^{0,\mu} \right)^2 \sqrt{\left( \omega_{\theta,\theta'}^{(i)} + \frac{2\pi t}{3} \right) \left( \omega_{\theta,\theta'}^{(i)} + \frac{2\pi t + \eta_2}{3} \right) - \sqrt{\left( \omega_{\theta,\theta'}^{(i)} + \frac{2\pi t}{3} \right) \left( \omega_{\theta,\theta'}^{(i)} + \frac{2\pi t + \eta_2}{3} \right)}} \tag{S9}
\]

with \(\rho_{\text{imp}} = \sigma_{\text{imp}}^{-1}\), \(t = T / \gamma (k_F a_0)^2\) and the pre-factor:

\[
\zeta = \frac{2 \eta_2^2}{g_{\text{imp}}^2 + 2 g_{\text{imp}}^2} \left[ 1 + 4 \left( 1 + \delta_0 \right) - 2 \sqrt{(1 + \delta_0)^2 - \delta_2^2} \right]^{-1} \tag{S10}
\]

In Fig. 3 of the paper, we show the numerical solution of Eq. S9 for the parameters discussed in the main text, using \(\zeta\) [appearing in Eq. (S8)] that comes from the solution of the self-consistent equations (3) of the main text.
At low temperatures \( t \ll 1 \), we can obtain an analytical solution for the resistivity anisotropy. We consider that the system is in the orthorhombic paramagnetic phase, where the correlation length associated with each ordering vector is given by \( \xi^{-2} = \xi^{-2} + \varphi \), where \( \varphi > 0 \) is the Ising-nematic order parameter. To simplify our analysis, we consider a magnetic three-dimensional QCP. However, our results are more general and hold even away from the QCP.

For the hard mode (short correlation length), \( \xi^{-2} = \xi^{-2} + \varphi \), the \( t \ll 1 \) expansion of Eq. (S9) yields:

\[
\left( \frac{\rho^{\mu\mu} - \rho_{\text{imp}}}{\rho_{\text{imp}}} \right)_{\text{hard}} = t^2 \left\{ \zeta \int d\theta d\theta' \left( \frac{f^{(1)}_{\theta, \theta'} - f^{(0)}_{\theta, \theta'}}{3} \left( 2\varphi + f^{(3)}_{\theta, \theta'} + \eta^2 \right) \right)^{3/2} \right\}
\]

(S11)

where we defined the auxiliary function \( f^{(i)}_{\theta, \theta'} = \omega^{(i)}_{\theta, \theta'} - (\xi_i / a_0)^{-2} \) [see Eq. (S8)]. Thus, the contribution of the hard spin fluctuations to the resistivity vanishes as \( t^2 \). For the soft mode \( \xi^{-2} = \xi^{-2} - \varphi \) the situation is different, since \( \xi^{-2} \to 0 \) as \( t \to 0 \). Then, one has to divide the Fermi surface into two regions [24]: one around the hot spots \( \theta_{hs} \), with width \( f^{(2)}_{\theta, \theta'} \sim t \), and another with \( f^{(2)}_{\theta, \theta'} \gg t \), called the “cold region”. Repeating the same steps that led to Eq. (S11), we find that the contribution from the cold region also vanishes as \( t^2 \). Near the hot spots \( \theta_{hs} \), an expansion similar to the one outlined in [24] yields:

\[
\left( \frac{\rho^{\mu\mu} - \rho_{\text{imp}}}{\rho_{\text{imp}}} \right)_{\text{soft}} = \left( \frac{4\sqrt{2\pi}\zeta c}{\sqrt{3}\eta z} \right) t^{3/2} \left( \Phi_{1, \theta_{hs}} - \Phi_{2, \theta_{hs}} \right)^2
\]

(S12)

which is the Eq. (4) of the paper. Here, \( c > 0 \) is a constant coming from the Jacobian of appropriate coordinate transformations. Since the contribution from the soft mode to the resistivity vanishes as \( t^{3/2} \), while the other contributions vanish as \( t^2 \), the former dominates the transport at low temperatures. Therefore, the resistivity anisotropy at \( t \ll 1 \) is:

\[
\frac{\rho^{yy} - \rho^{xx}}{\rho_{\text{imp}}} = \kappa T^{3/2} \left[ \left( \Phi_{1, \theta_{hs}} - \Phi_{2, \theta_{hs}} \right)^2 - \left( \Phi_{1, \theta_{hs}} - \Phi_{2, \theta_{hs}} \right)^2 \right]
\]

with \( \kappa > 0 \). Using Eqs. (S5) and the fact that the hot spots are at \( \theta_{hs} = \arccos(\delta_0 / \delta_2) / 2 \), we obtain:

\[
\frac{\rho^{yy} - \rho^{xx}}{\rho_{\text{imp}}} = \kappa T^{3/2} \left[ \frac{\bar{\delta}_0 (2 - \bar{\delta}_0)}{\delta_2} + (4 - \bar{\delta}_0) \bar{\delta}_2 \right]
\]

(S14)

Thus, for a fixed value of the mass anisotropy \( (\delta_2) \), the sign of the resistivity anisotropy changes from positive to negative if the chemical potential \( (\delta_0) \) is negative and large enough. The “phase diagram” in Fig. 4 of the paper is obtained from this expression.