A Joint Positioning Algorithm of TDOA and TOF Based on Ultra-wideband

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Abstract. Ultra-wideband (UWB) positioning is a mainstream positioning technology in recent years, and the main positioning algorithms used in UWB based positioning systems are TDOA and TOA. Aiming at the problem of unstable positioning accuracy of TDOA algorithm in traditional UWB positioning system, this paper proposes a joint positioning algorithm of TDOA and TOF based on UWB. The algorithm needs only one time of TOF ranging and combined with the TDOA algorithm, the distances between all base stations and a single tag in the positioning area can be obtained. Compared with the single TDOA algorithm, the algorithm has stable measurement accuracy and can achieve the same positioning accuracy whether the tag is inside or outside the triangle formed by the base station. The simulation results show that the average positioning error of the joint positioning algorithm of TDOA and TOF is less than 10 cm. The algorithm is effective and reliable, and meets the positioning requirements.

1. Introduction

With the development of positioning technology, the demand for location-based services is growing. The mainstream wireless positioning technologies include ultra-wideband (UWB), ZigBee, Wi-Fi, Bluetooth and satellite-based positioning, such as GPS[1]. The UWB technology, as a carrier-free communication technology, uses a short pulse of nanosecond level for signal transmission, and completes the accurate distance measurement by high clock resolution at the sub-nanosecond level[2], which gives UWB technology decimetre-level positioning accuracy. In addition, UWB technology has the advantages of high transmission rate, strong anti-interference ability and long transmission distance. Therefore, UWB technology is widely used in positioning tracking and navigation[3].

Wireless communication ranging is the basis of UWB location. The TOA (time of arrival) algorithm based on TOF (time of flight) has a high positioning accuracy. The symmetric double-sided two-way ranging (SDS-TWR) algorithm is used to complete the TOF ranging process, which can realize the high precision positioning of decimetre-level[4]. However, the large amount of UWB communication required for TOF ranging limits the positioning capacity and refresh rate of the system. The TDOA algorithm[5] requires less UWB traffic makes it have higher positioning capacity. But the single TDOA algorithm will lead to unstable positioning accuracy due to the error amplification caused by its hyperbolic geometric properties[6].
In order to improve the stability and accuracy of TDOA algorithm in UWB positioning system and maintain a high positioning capacity, the paper proposes a joint positioning algorithm of TDOA and TOF based on UWB. The distance between a specific base station and a tag can be obtained by only one time of TOF ranging, and then the distance between all the base stations and a single tag in the location area can be obtained by TDOA algorithm. The hyperbolic problem of TDOA algorithm can be transformed into the circle problem of TOA algorithm. The reliability of the algorithm is verified by MATLAB.

2. Basic Location Algorithm

The traditional UWB positioning system usually consists of several observation base stations and tags[7]. A tag sends the ranging packet to the observation base station and records the sending time. The base station receives the packet from the tag in real time and records the time of arrival. The information is transmitted by the UWB signal, and then the time information is converted into distance to calculate the position coordinates of the tag.

2.1. TDOA Algorithm

TDOA algorithm is a method of location using time difference of arrival. The location is obtained by determining the hyperbolic intersection with a priori information[8]. At least three base stations are required to complete the positioning at the two-dimensional level, and the positioning principle in the ideal case of the two-dimensional plane is shown in figure 1. A1, A2, A3 are positioning base stations and T is tag.

2.2. TOA Algorithm

TOA is a classical positioning algorithm[9], in the case of two-dimensional plane, at least 3 base stations are required for positioning. And the positioning principle in the ideal case of the two-dimensional plane is shown in figure 2.

![Figure 1. The positioning principle of TDOA](image1)

![Figure 2. The positioning principle of TOA](image2)

The coordinates of A1, A2, A3 are \((x_1, y_1), (x_2, y_2), (x_3, y_3)\), T is the tag and its coordinate is \((x, y)\). The distance from T to A1, A2, A3 are measured by time information, which is expressed as \(d_1, d_2, d_3\). Then the equation is constructed according to the circular geometry, such as equation (1).

\[
\begin{align*}
(x_1 - x)^2 + (y_1 - y)^2 &= d_1^2 \\
(x_2 - x)^2 + (y_2 - y)^2 &= d_2^2 \\
(x_3 - x)^2 + (y_3 - y)^2 &= d_3^2
\end{align*}
\]  

(1)
3. Joint Positioning Algorithm of TDOA and TOF

Joint positioning algorithm of TDOA and TOF requires at least 3 base stations to locate in a two-dimensional plane. One of the base stations is selected as the specific base station for TOF ranging and one is selected as the main base station for time synchronization.

3.1. UWB Signal Communication Process

The positioning algorithm of TDOA and TOF communication flow is shown in figure 3, the specific communication steps are as follows:

- The tag T broadcasts the Init Packet in the positioning area, and records the sending time $t_1$.
- Main base station A and slave base stations B and C record the time $a_1$, $b_1$, $c_1$ of receiving Init Packet respectively.
- The specific base station C records $c_1$ and sends the Response Packet to the tag at moment $c_2$.
- The tag receives the Response Packet to record the receive time $t_2$ and sends the End Packet at moment $t_3$.
- The specific base station C receives the End Packet and records the receive time $c_3$.

![Figure 3. UWB signal communication process](image)

Then the joint positioning algorithm of TDOA and TOF converts the time information into the distance information to obtain the location coordinates of the tag T.

3.2. Joint Positioning Algorithm of TDOA and TOF

As the communication flow, an SDS-TWR ranging is completed between the specific base station C and the tag T[10]. The distance between base station C and tag T can be calculated by equation (2).

$$d_{CT} = c \times \left[ \left( t_2 - t_1 \right) - \left( c_2 - c_1 \right) \right] + \left[ \left( c_3 - c_2 \right) - \left( t_3 - t_2 \right) \right] / 4 \quad (2)$$

Based on the time difference idea of TDOA algorithm, the distance difference between tag T to main base station A and tag T to specific base station C, and the distance difference between tag T to base station B and tag T to specific base station C can be obtained from $a_1 b_1 c_1$, as shown in equation (3) and equation (4).

$$\Delta d_{AC} = c \times (a_1 - c_1) \quad (3)$$

$$\Delta d_{BC} = c \times (b_1 - c_1) \quad (4)$$

The distance $d_{CT}$ between the specific base station C and the tag T obtained by the TOF algorithm is combined to calculate the distance $d_{AT}$ between the main base station A and the tag and the distance $d_{BT}$ between the base station and the tag, as shown in equation (5) and equation (6).

$$d_{AT} = d_{CT} + \Delta d_{AC} \quad (5)$$

$$d_{BT} = d_{CT} + \Delta d_{BC} \quad (6)$$
Then the hyperbolic problem of TDOA algorithm is transformed into the circle problem of TOA algorithm. Figure 2 shows that the three circles intersect at exactly one point in the ideal case. But in practice, there are often errors in the measured values, resulting in the three circles not intersecting at one point exactly. In this case, it is necessary to linearize equation (1) to derive an approximate coordinate. As shown in equation (7).

\[
X_r = A^{-1}B
\]  

\(X_r\) is the coordinate vector of the tag, where:

\[
A = \begin{pmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \end{pmatrix}
\]

\[
B = \frac{1}{2} \begin{pmatrix} (x_2^2 + y_2^2) - (x_1^2 + y_1^2) + d_{AT}^2 - d_{BT}^2 \\ (x_3^2 + y_3^2) - (x_1^2 + y_1^2) + d_{AT}^2 - d_{CT}^2 \end{pmatrix}
\]

4. Simulation and Result Analysis

4.1. Positioning Results Analysis

The simulation uses 3 base stations to complete the 2-D localization. In centimeters, the coordinates of the main base station are set to (0, 0), the specific base station to (300, 200), the slave base station to (100, 500), and the real coordinates of the tag to (150, 250). The random ranging error is added to the simulation, and the results obtained from the simulation of TDOA and TOF algorithm are expressed in circles. The distribution of measurement points after 500 times of simulation is shown in figure 4.

Figure 4 shows that the positioning results obtained by the TDOA and TOF joint algorithm are basically distributed around the real coordinate position of the tag, which indicates that the joint algorithm is effective.

Calculate the root mean square error (RMSE) of the distance from the measured coordinate position to the real coordinate position of the tag. The RMSE curve is shown in figure 5.

It can be seen from figure 5 that when the joint algorithm of TDOA and TOF is used for localization, the error of the location result is less than 10 cm. The accuracy is in accordance with the expected result, indicating that this algorithm is effective.

4.2. Comparative Analysis

In order to reflect the advantages of joint positioning algorithm of TDOA and TOF, the single TDOA algorithm and joint positioning algorithm of TDOA and TOF are compared in the same simulation environment. Figure 6 shows the schematic diagram of base station and tag arrangement. In centimeters, the coordinates of main base station are set to (0, 0), the specific base station to (100, 400),
the slave base station to (200, 200), and the real coordinates of the tag to (150, 250). 15 tag points to be measured are selected.

![Figure 6. Simulation arrangement](image)

Tags are measured by a single TDOA algorithm and a joint positioning algorithm of TDOA and TOF. And calculate the RMSE of the distance from the coordinate position to the real coordinate position of the tag, so as to compare the measurement accuracy in various cases. The simulation results of TDOA algorithm are shown in figure 7, and the simulation results of the joint positioning algorithm of TDOA and TOF are shown in figure 8.

![Figure 7. The RMSE of single TDOA](image)  
![Figure 8. The RMSE of joint algorithm](image)

Analysis of simulation results. When the tag is inside the triangle formed by the base station, the RMSE can maintain good accuracy and the TDOA positioning algorithm is effective. However, when the tag is located outside the triangle formed by the base station, its RMSE increases rapidly, and the farther the range from the triangle, the larger the accuracy deviation, the maximum error reaches 3 meters in this simulation.

This rapid decline in localization accuracy is not due to the data, but to the algorithm[11]. The single TDOA algorithm is unstable in its localization accuracy due to the amplification of errors brought about by its hyperbolic geometric nature[12]. From the shape characteristics of hyperbolic curve, the curvature of the position near the focus changes largely, the farther away from the focus, the smaller the curvature changes, and the hyperbolic is closer to the straight line[13]. Measurement error has a large effect on the slope of the straight line, then the farther away from the hyperbolic focus, the error in the intersection of the two approximate straight lines will be larger.
As shown in figure 8, the RMSE of the joint positioning algorithm of TDOA and TOF are kept within 10 cm, and the data are very stable. The simulation results show that the joint algorithm of TDOA and TOF can effectively solve the problem that the localization accuracy of TDOA algorithm is unstable and the accuracy tends to drop rapidly when the tag is outside the range of the base station. Moreover, compared with a single TOF algorithm, joint positioning algorithm of TDOA and TOF can effectively improve the system capacity. With the single TOF algorithm, the tag needs to be ranged with each base station. The TOF positioning algorithm can solve the problem of accuracy degradation when the tag is outside the base station, but it will lead to excessive UWB traffic, which severely limits the capacity of the system.

5. Conclusion
This paper introduces the TOA algorithm and the TDOA algorithm, analyses in detail the error caused by the TDOA algorithm when the tag is outside the base station range, and proposes an improved joint positioning algorithm of TDOA and TOF. The simulation results show that the error of joint positioning algorithm of TDOA and TOF is within 10 cm, which effectively solves the problem of unstable positioning accuracy caused by hyperbolic properties of TDOA algorithm. Only one time of TOF ranging process solves the problem that the system capacity of TOA algorithm decreases with the increase of base station number. In this paper, only the static target positioning is simulated. The next research will be carried out on the high-speed moving target positioning algorithm.

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