Research on Denoising Method for Improving the Identification Accuracy of Structural Damage

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Abstract. Under the influence of equipment and environment, gaussian white noise will be accompanied in the process of obtaining the measured vibration mode of the structure, thus masking the effective features of signal and disturbing the effect of damage identification. To avoid this problem, this paper presented a new denoising method. Firstly, the multi-index evaluation function constructed by analytic hierarchy process is used to select the optimal wavelet base and the decomposition layers’s number. Secondly, we use the newly constructed wavelet threshold function to research the wavelet coefficients in the high frequency part of the signal to remove the noise coefficients. Simulation results show: the new denoising method is contributing in improving the SNR of vibration signals and reducing the RMS error. The experimental results testified that the new method can effectively increase the accuracy of structural damage identification.

Keywords. Vibration signal, damage identification, wavelet denoising, analytic hierarchy process, intelligence algorithm.

1. Introduction
During the service period, the structure is subjected to various factors such as environmental erosion and load cycle, which leads to the continuous accumulation of structural damage. The damage identification method [1-3] based on signal processing technology can effectively detect the structural damage, thus avoiding the occurrence of engineering accidents. However, the acquisition of structural vibration signals is often disturbed by environmental noise, which makes it difficult to identify the effective features. Therefore, the denoising effect is particularly important to improve the accuracy of structural damage identification.

Wavelet analysis has been extensively used in signal-denosing domain because of its good time-frequency local characteristics, multi-resolution and low entropy. In recent years, the research on signal denoising mainly focuses on the improvement of threshold function and the combination of empirical decomposition and threshold denoising. Lu Jingyi et al. [4] putted forward a new threshold function based on the soft threshold function, used the regulator factors and applied it to the denoising of various signals. Zhang Xun et al. [5] constructed the objective function using the generalized crossover criterion, searched the optimal threshold through the chaotic swarm algorithm, and remove the noise part of the vibration signal of high-rise structures. Zhou Jianhe xiang Peiping [6] took Shannon entropy as the coefficient of the regulating parameter in the threshold function and obtained the adaptive threshold function. Li Mingzhu et al. [7] used neural network to find the best threshold and presented a denoising method based on threshold neural network (TNN). Liu Yong [8] combined savitzky-Golay smooth filter with wavelet threshold denoising to remove the white noise in the approximate coefficient and detail coefficient of the signal respectively. Liu Hu [9] proposed the use...
of two-state Gaussian mixture model to classify high frequency’s coefficients on the minimum scale and settled the situation that the accuracy of traditional noise variance estimation is enormously disturbed by noise value’s fluctuate.

The essence of wavelet threshold denoising is to distinguish useful signal and noise signal by mathematical operation according to their different characteristics in the wavelet domain. The selection of threshold function, threshold and decomposition layer number has a direct impact on the effect of denoising [10-13], while the traditional hard threshold function has a breakpoint at the position of threshold, which will easy to lead to false Gibbs phenomenon in the process of signal reconstruction. The soft threshold function has a great overall continuity at the threshold, but has a constant deviation, which may lead to distortion of reconstructed signal.

The analytic 2 hierarchy process is used to construct the evaluation function in this paper, and a new threshold function suitable for vibration signal is proposed. The optimal decomposition layer and wavelet basis are selected by the new evaluation function. The denoising effect of new threshold function and traditional threshold function is simulated and analyzed. The results show that the denoising result of the new threshold function is better than the traditional method. Finally, a new threshold function is used to denoise the measured signals with damaged structures, which can effectively improve the identification accuracy of structural damage.

2. Improve the Threshold Function and Evaluation Index

2.1. Principle of Wavelet Threshold Denoising

Assume that the model of one-dimensional measured signal is

$$ S(n) = X(n) + N(n) \quad (n=1,2,\ldots,N) $$

where, $S(n)$ is the measured noise signal, $X(n)$ is the original signal, $N(n)$ is the noise signal.

The wavelet threshold denoising of $S(n)$ is the threshold quantization processing of each layer coefficient obtained after wavelet decomposition. The detailed steps are as follows

Wavelet decomposition. Choose the suitable wavelet base and decomposition layer number to perform wavelet decomposition for the measured noise signal, and the decomposition equation is

$$ \begin{align*}
  c_{L,K} &= \sum c_{L-1,n} h_{n-2K} \\
  d_{L,K} &= \sum c_{L-1,n} g_{n-2K}
\end{align*} \quad (K=0,1,\ldots,N) $$

where, $c_{L,K}$ is the approximate coefficient, $d_{L,K}$ is the detail coefficient; $L$ is the decomposition layer, $h$ and $g$ are low and high pass filters respectively, $N$ is signal’s length.

Threshold quantization. By selecting the appropriate threshold and threshold function, the mathematical operations such as selection and contraction of the decomposed detail coefficient are carried out. Traditional hard threshold function is as follows

$$ \hat{c}_{L,K} = \begin{cases} 
  c_{L,K}, & (|c_{L,K}| \geq T) \\
  0, & (|c_{L,K}| < T)
\end{cases} \quad (3) $$

The above equation shows that the hard threshold function directly reserved the wavelet coefficient greater than the threshold, and zeros the other part. There is a discontinuous point at the threshold point, which easily leads to the vibration of reconstructed signal. Traditional soft threshold function shrinks the part which is larger than the threshold. This function is as follows

$$ \hat{c}_{L,K} = \begin{cases} 
  \text{sign}(c_{L,K})(|c_{L,K}| - T), & (|c_{L,K}| \geq T) \\
  0, & (|c_{L,K}| < T)
\end{cases} \quad (4) $$
where, $\text{Sign}(x)$ is the sign function, $T$ is the threshold

Coefficient reconstruction. The detail coefficient quantified by threshold function and the approximate coefficient at the highest level are reconstructed by wavelet, and the reconstruction equation is as follows

$$c_{L-1,n} = \sum c_{L,n} h_{K-2n} + \sum d_{L,n} g_{K-2n}$$ (5)

2.2. Improved Threshold Function

Aiming at the deficiency of traditional threshold function, the new function constructed in this paper is

$$\hat{c}_{L,K} = \begin{cases} 
(1 - \mu)c_{L,K} + \mu \text{sign}(c_{L,K}) \left[ |c_{L,K}| - T e^{-\left(\frac{|c_{L,K}| - T}{t}\right)^2} \right], & (|c_{L,K}| \geq T) \\
0, & (|c_{L,K}| < T)
\end{cases}$$ (6)

where, $\mu = e^{-\alpha(|c_{L,K}| - T)^2}, t, \alpha$ is positive number, $T$ is the threshold

According to equation 6, this new threshold function is strictly monotone and continuous in the definition domain where the wavelet coefficient is larger than threshold. And with the increase of the wavelet coefficient, this function will approach the hard threshold function, so as to give full play to the advantages of the hard threshold function, the large part of the wavelet coefficient can be retained directly, and the noise content will gradually decrease with the increase of the wavelet coefficient. In this way, the function can not only retain useful signals to a greater extent, avoid overkill phenomenon, but also compensate for the constant deviation of soft threshold function.

When the wavelet coefficient tends to the threshold, this function shows an exponential decay, which is in line with the law of exponential decay of the modulus of noise wavelet transform. Moreover, the function can control the decay rate through coefficient adjustment, and there is no crossover and overlap in the curve family [14], so the denoising method is closer to the actual situation.

The evaluation function is as follows:

$$F = 0.637 \text{SNR} + 0.258 \frac{1}{\text{MSE}} + 0.105 \frac{1}{\text{S}}$$ (7)

From the above equation, it can be seen that the greater the value of judging function $F$ is, the better the denoising effect will be.

3. Simulation Verification

To verify the veracity of this method, doppler signals in Matlab were selected to verify the validity of the new method. Add Gaussian white noise to the original-signal, as shown in figure 1.

![Original signal and noise-containing signal](image)

**Figure 1.** Original signal and noise-containing signal.
3.1. Determination of Wavelet Basis and Decomposition Layer Number
The wavelet bases of symN and dbN suitable for vibration signal de-noising with good orthogonality, high order vanishing moment and appropriate tight width were selected, and the de-noising effects of different decomposition layers were analyzed and compared. The final analysis result is given by the judging function F. Based on the analysis in figures 2 (a-b), it shows that when choose Sym6 as the wavelet basis and the decomposition layer is 3, the value of the multi-indicator function F is the largest, 0.991. That is, for the vibration signal, the best de-noising effect is selected when the wavelet basis of Sym6 and the decomposition layer of 3 layers are used.

![Wavelet basis of symN.](image1)
(a) Wavelet basis of symN.

![Wavelet basis of dbN.](image2)
(b) Wavelet basis of dbN.

**Figure 2.** The denoising effects of different decomposition layers and wavelet base.

3.2. Comparison of Different Threshold Functions
To further verify the availability of this new threshold function and threshold, both the traditional threshold function (soft and hard threshold function) and the new threshold function are selected, and the signal is de-noising by combining the fixed threshold and the new threshold. The new threshold is shown in equation (8). According to the above analysis, this signal was decomposed into 3 layers by using Sym6. This final de-noising results were shown in figure 3.

\[ T = \left( \frac{\sigma^2}{2} \right)^{\frac{1}{2}} \sigma \sqrt{2 \ln N} \]  

(8)

where, J is the decomposition level, N is this signal’s length, \( \sigma = \frac{\text{median}(C_{\text{Lk}})}{0.6745} \), median is the median function.

![Denoising effect diagram of each threshold function combined with the new threshold.](image3)

**Figure 3.** Denoising effect diagram of each threshold function combined with the new threshold.

To show the de-noising validity, SNR, MSE, smoothness (S) and function F were used to compare this de-noising validity of each function and threshold. As shown in table 1.

To sum up, the best denoising effect is achieved when the vibration signal is decomposed by the wavelet base of Sym6. Moreover, compared with the traditional threshold function, the new threshold function improves the denoising effect by about 7%-15% when combined with fixed threshold threshold. Combined with the new threshold threshold, the denoising effect is improved by about
12%-21%, and the value of the evaluation function (F) is much larger than that of the soft and hard threshold functions, indicating that the new de-noising method is more superior.

Table 1. The denoising effect of each threshold function and threshold.

| Threshold     | Method     | SNR | MSE | S  | F  |
|---------------|------------|-----|-----|----|----|
| Fixed threshold | Hard threshold | 21.07 | 0.63 | 0.96 | 0.40 |
|               | Soft threshold  | 19.48 | 0.75 | 0.77 | 0.11 |
|               | New threshold   | 22.55 | 0.53 | 0.85 | 0.74 |
|               | Hard threshold  | 21.86 | 0.57 | 1.14 | 0.54 |
| The new threshold | Soft threshold  | 22.18 | 0.55 | 1.44 | 0.58 |
|               | New threshold   | 23.55 | 0.47 | 0.95 | 0.96 |

4. Experimental Analysis

Q235 solid steel beam with equal section was used for experimental verification. Beam length 1.3 m, section width B=0.06 m, height H=0.08 m, elastic modulus E=2.06x10^11 N/m^2, material density =7800 kg/m^3. The experimental steel beam was divided into 50 units and 51 measuring points, with the measuring points spaced at 26mm apart. A total of 16 sensors were selected for signal acquisition in four batches. The test specimen and layout are shown in figure 4.

Figure 4. Experimental specimens and layout.

The stiffness reduction [15] is adopted to simulate the damage degree of the structure. The damage condition is shown in table 2.

Table 2. Damage conditions of beam structure with equal cross section.

| Damage condition | Damage location | Degree of damage ε |
|------------------|----------------|-------------------|
| Working condition | 25             | 20%               |
| Two damage       | 38             | 10%               |

There were many singular points in the wave-coefficient graph through the wavelet transform of the measured modes, and structural damage’s location could not be accurately identified. The wavelet coefficient is shown in figure 5(a). After analysis, it is found that the disturbance of the coefficient graph is further caused by the ambient noise on site which conceals the useful features of the signal. However, the wavelet coefficients denoised by the traditional soft-hard threshold method have no obvious singularities, as shown in figure 5(b), and the damage location cannot be determined. However, through the method selecting sym6 and the rational decomposition level, using the new threshold function to eliminate the noise, the de-noising effect is shown in figure 6, visible, wavelet coefficients figure at 25 and 38 units are produced obvious modulus maxima points, and caused by environmental noise mutation also had better inhibition, can further through the damage location of modulus maxima points.
The damage location is taken as a known quantity, and the improved particle swarm optimization algorithm [16] is adopted to identify the damage degree. The objective function is constructed as shown in equation (9).

\[
J_1 = C_w \sum_{i=1}^{m} \left( \frac{f_i^{\text{test}} - f_i^{\text{calc}}}{f_i^{\text{test}}} \right)^2 + C_f \sum_{j=1}^{n} \sum_{k=1}^{k} (\phi_j^{\text{test}} - \phi_j^{\text{calc}})^2
\]

In the above equation, \( C_w \), \( C_f \) is the weighted factor, \( f_i^{\text{test}}, f_i^{\text{calc}} \) are the measured and calculated natural frequencies of order \( i \) respectively, \( \phi_j^{\text{test}}, \phi_j^{\text{calc}} \) are measured and calculated modes of mode \( i \) respectively, \( m \), \( n \) represent the order of frequency and mode of vibration, \( k \) is the number of node displacements. After debugging, the particle population’s number in this algorithm is 20, and the learning factor is 2. In order to avoid the randomness and contingency of the algorithm, the algorithm was run ten times and arithmetic mean was taken. The recognition results are shown as table 3.

Table 3. The results of damage degree identification.

| Unit number | Pre-denoising data (mean) | De-noised data (mean) | Run time (s) |
|-------------|---------------------------|-----------------------|-------------|
|             | Degree of damage          | Relative error        | Degree of damage | Relative error  |
| 25(20%)     | 22.40%                    | 12.00%                | 20.99%       | 4.95%          | 328.50        |
| 38(10%)     | 10.95%                    | 9.50%                 | 10.51%       | 5.10%          |               |

By analyzing table 3, it obvious that the damage degree error identified by using the measured noisy mode is too large, and the relative error of the damage degree is 9.50%-12.00%. After denoising, the relative error is reduced to 4.95-5.10%. It shows that this new de-noising method can effectively suppress the interference of noise to degree recognition and improve the accuracy of damage recognition.

5. Conclusion
To improve the accuracy of structural damage identification, a new method is proposed to denoise the measured data and a new evaluation function is constructed by analytic hierarchy process (AHP) to appraise the de-noising effect. The simulation verified that the best denoising effect is achieved when the vibration signal is decomposed into three layers with the wavelet basis of Sym6, through
comparing with the traditional threshold function, the new method possess a higher evaluation function value and SNR, lower MSE and a better effect of de-noising. The experimental results testified that the new threshold function denoising algorithm preserves the effective characteristics of vibration signal while suppressing noise signal and improves the accuracy of damage identification.

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