Magnetorheological Fluid Yield Stress Prediction Using Particle Swarm Optimization at Low and High Shear Rate Region

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Abstract. Yield stress is an important parameter to measure the performance of a magnetorheological (MR) fluid. The parameter can be obtained by fitting a flow curve consisting of a shear rate-shear stress dataset to a Bingham plastic equation. However, the dataset selection is usually determined by trial and error by selecting the data either at a low or high shear rate region due to there is no standardized selection method. Therefore, this paper attempts to develop a platform to predict the yield stress automatically using particle swarm optimization (PSO). The PSO objective function is inspired by the Biplastic Bingham model. The results have shown that the prediction has shown a good agreement when fitting to the experimental data. Furthermore, the obtained yield stress values at high and low shear rate regions also were discussed from the point of view of the difference and possible effect if the wrong variables are chosen. The evaluations have shown that the gap between the yield stress at low and high regions can be relatively high, which is about more than 10 kPa. The wrong selection of the yield stress at an MR device possibly bring inaccuracy performance prediction/design, especially at high magnetic field value.

1. Introduction
The magnetorheological fluid is a class of smart materials that the yield stress can be changed by applying external magnetic fields. The reason for the phenomena is the stronger alignment and bonding created by the dispersed magnetic particles as the results of the stronger magnetic field exposure. The physical properties representation of the bond strength is yield stress that can be defined as the minimum value of force to break the particle alignment; thus, the fluid can flow. In practice, the yield stress values are employed to design MR devices, such as in brake [1] and damper [2]. Yield stress values increase as the rising of the magnetic fields. The changing time can be very fast within millisecond fraction. Because the unique characteristic of the fluids, the researches about the fluid yield stress have been discussed in various literature including how to improve the yield stress [3], characterization of yield stress [4], and the yield stress modeling [5].
The values of yield stress can be derived from the graph describing the relation between shear stress and shear rate or also known as flow curve obtained from a rotational rheological test [6,7]. The derivation process involved rheological equations, such as Bingham Plastic, Herschel Bulkley, Biplastic Bingham, Papanastasiou, and others [8]. Then, the equations are fitted to a flow curve to obtain rheological parameters. Each equation has its own advantages and disadvantages, such as the high accuracy and the capability to predict the shear-thinning or thickening behavior of a fluid of the Herschel Bulkley model. Bingham plastic equation is the most popular due to its simplicity and easiness to use, especially the yield stress and plastic viscosity. Bingham plastic has some disadvantages, such as the limitation of the shear rate region coverage. Because of the limitation, it is difficult to determine which data is suitable for prediction.

In general, there are two methods to determine the yield stress from the point of view data selections, by selecting a certain number of data in the near-zero value of shear rate [9] or by determining a certain shear rate range after observing the flow curve [10]. Those trial and error selection methods may bring the inaccuracy interpretation on the obtained yield stress and cannot be compared with other works. A metaheuristics method is possibly capable of replacing the trial and error selection. Some metaheuristic methods are available in the literature, such as genetic algorithm [11], particle swarm optimization [12], and ant colony [13]. In rheology, genetic algorithm and particle swarm optimization have been employed for MR fluid [14] and other materials [15]. Although various works have tried to predict the yield stress using optimization algorithms, the prediction by considering shear rate regions can be considered rare.

Therefore, this paper aims to further develop an optimization platform to predict the yield stress at both low and high shear rate regions automatically using particle swarm optimization (PSO). The objective function is developed based on the previous work by employing an inspired Biplastic Bingham objective function of the optimization platform [16]. PSO algorithm is first described. Then, the derivation process of the objective function and the characterized materials are then provided. Finally, the discussion in terms of convergent diagram, obtained parameters, and the application in a meandering MR valve is then described.

2. Methodology

2.1. Particle Swarm Optimization

PSO is an algorithm inspired by the behavior of the birds while look for the foods. They coordinate each other to pinpoint the food location. PSO has some variants, such as classical, dynamic weight, and constrictions. For yield stress prediction applications, constriction PSO has the best performance compared to the other type of PSO [16]. The general equations of the algorithm can be described in the following equations of Eq. (1), (2), and (3). Eq. (1) describe the update process of the particle velocity $\mathbf{V}_p$ with constant of weight factor of particle cognitive $c_p$ and of group cognitive $c_g$. In the equation, random parameters are also applied to each particle and group components denoted by $r_p$ and $r_g$, respectively. The update is based on the best particle position $\mathbf{P}(t)$, the current particle position $\mathbf{X}_p(t)$, group best position $\mathbf{G}(t)$. The constriction factor $\chi$ is determined using Eq. (2), where $k$ and $\Phi$ are determined using Eq. (3). According to literature [16–18], the value of 2.05 is applied in $c_p$ and $c_g$ to fulfill the condition of Eq. (16).

$$
\mathbf{V}_p(t + 1) = \chi \left( \mathbf{V}_p(t) + c_p \ast r_p \ast (\mathbf{P}(t) - \mathbf{X}_p(t)) + c_g \ast r_g \ast (\mathbf{G}(t) - \mathbf{X}_p(t)) \right),
$$

$$
\chi = \frac{2 \ast k}{2 - \sqrt{\Phi^2 - 4 \ast \Phi}}
$$

$$
\Phi = c_p + c_g, \Phi > 4, k \in [0:1]
$$
The general workflow of the algorithm is as follows:

a. Determine search space
   The search space needs to be determined first to make sure the iteration process can be carried out effectively and efficiently.

b. The initialization process
   The initialization process of the swarm is then carried out. The initialization process for particle positions is determined based on the available search space data or decided as in Eq. (4) where \( x \) is the particle position, subscript \( p, 0 \) means initial position, \( p, L \) is the minimum search space, \( p, H \) is the maximum search space.
   \[
   X_{p,0} = (X_{p,H} - X_{p,L}) \times \text{rand} + X_{p,L} \tag{4}
   \]
   The initial velocity \( v_{p,0}(t) \) of all the particles is zero (0). Each \( X \) is a matrix containing the predicted rheological parameters. The swarm size is also determined in the initialization process. All of the particle’s initialization values are inputted in the objective or cost function then memorized as the best value for particle and group best particle positions.

c. Iteration
   In every iteration, the best objective function is updated by selecting the best value compared to the saved best values of group and particle. Based on the obtained group and particle best, the particle positions and velocities are then updated according to the PSO equation in Eq. (1), (2), and (3).

d. Iteration termination
   The iteration is terminated based on pre-determined conditions, which are the maximum iteration is reached, or the expected value is reached. In this paper, the maximum iteration is 50 iteration.

2.2. Objective Functions
The objective function is an essential part of an optimization process. As discussed in the introduction, the predicted parameters are the yield stress values in the high and low shear rate regions. There are several possible candidates of the existing rheological equations that can be applied for predicting the yield stress values. In the work of [16], some rheological equations have been evaluated in terms of the capability to predict the yield stress values at both shear rate regions simultaneously. The possible models are Papanastasiou and biplastic Bingham. Based on the comparison, Biplastic Bingham (BB) is selected because of the straightforwardness to predict both variables, as shown in Eq. (5). \( \tau_{y1} \) and \( \tau_{y2} \) are the yield stress at low and high region, respectively. Another variables are also defined which are the plastic viscosity at low \( \eta_1 \) and high shear rates and \( \eta_2 \). To differentiate both regions, a critical value of the shear rate \( \dot{\gamma}_c \) is defined.
   \[
   \tau = \begin{cases} 
   \tau_{y1} + \eta_1 \dot{\gamma} & \text{if } \dot{\gamma} \leq \dot{\gamma}_c \\
   \tau_{y2} + \eta_2 \dot{\gamma} & \text{if } \dot{\gamma} > \dot{\gamma}_c
   \end{cases} \tag{5}
   \]
   A cost function is defined based on this model by calculating the error between data reference and the output of the model, as shown in Eq. (6). The difference between shear stress of the reference data \( \tau_{ref,h} \) and prediction data \( \tau_{p,h} \) as a function of shear rate of the reference data \( \dot{\gamma}_{ref,h} \) are sum squared and averaged in the form of mean square error (mse) where \( h \) is the data sequence from one (1) to the total number of data \( K \).

The illustration of the shear rate region division for the shear-thinning case is shown in Figure 1. The region of low shear rate region (region I) has a maximum value of \( \dot{\gamma}_c \). After the value of the critical shear rate, the region is called as region II/high shear rate region.
However, this formulation can produce a high accuracy in one region while abandoning another region. Therefore a formulation based on [16] is employed as in Eq. (7) for mse in low shear rate region mse_I, Eq. (8) for mse in high shear rate region mse_II, and Eq. (9) for total root mse rmse_t. In the total rmse equation, the weighting values are defined for low shear rate region w_1 and high shear rate region w_2. The weighting value can be defined based on various factors, such as the importance, the data distribution, or others. In this paper, both values are designated to be one (1). In other words, the weighting is set to be the same, considering the similar importance of both regions.

\[
\text{mse}_I = \frac{k_1}{K} \sum_{h=1}^{k_1} \left( \tau_{\text{ref},h} - \tau_{p,h}(\dot{\gamma}_{\text{ref},h}) \right)^2 \quad \text{for } \dot{\gamma}_{\text{ref},h} \leq \dot{\gamma}_c, \tag{7}
\]

\[
\text{mse}_{II} = \frac{k}{K-k_1} \sum_{h=k_1+1}^{K} \left( \tau_{\text{ref},h} - \tau_{p,h}(\dot{\gamma}_{\text{ref},h}) \right)^2 \quad \text{for } \dot{\gamma}_{\text{ref},h} > \dot{\gamma}_c, \tag{8}
\]

\[
\text{rmse}_t = \sqrt{w_1 \text{mse}_I + w_2 \text{mse}_{II}} / w_1 + w_2 \tag{9}
\]

Based on Eq. (5), there are four variables that need to be predicted. Besides shear stress at low \(\tau_{y1}\) and high shear rate region \(\tau_{y2}\), the corresponding plastic viscosities (\(\eta_1\) and \(\eta_2\)) are other variables that needed to be predicted. Furthermore, \(\dot{\gamma}_c\) also needs to be predicted. In sum, five variables need to be tuned using PSO. To reduce the number of variables, plastic viscosity variables for low shear rate region is defined as the following in Eq. (10).

\[
\eta_1 = \frac{\tau_{y2} + \eta_2 \dot{\gamma}_c - \tau_{y1}}{\dot{\gamma}_c} \tag{10}
\]
The same concept can be further developed for not only material with two regions, but also for more regions, especially for fluid with thickening fluids that usually have at least three regions, which are shear thinning, shear thickening, and the shear thinning again.

2.3. Materials and models
The deployed materials are based on a steady-state experiment test of MRF 132 DG from Lord Corporation with properties as provided in Table 1 and an extreme learning machine model as described in [16]. The data is a result of steady-state experiment using rotational mode rheometer produced by Anton Paar, Physical, MCR 302, GmbH, Austria. The data contain flow curves under exposure of various magnetic fields at 0, 100, 200, 300, 400, and 500 mT and shear rate between 0.01 and 2000 s\(^{-1}\). The data is then modeled using extreme learning machine method. The dataset is then simply addressed as reference data. Since the extreme learning machine method itself has been widely described in other literature [10,19,20], the description is not detailed in this paper.

| Table 1. Properties of MRF 132 DG |
|------------------|----------------|
| Variable         | Value          |
| Solid weight percentage | 80.98 wt% |
| Density          | 2.95–3.15 g/cm\(^3\) |
| Operating temperature | -40 to +130 °C |
| Viscosity        | 0.112 ± 0.02 Pa s at 40 °C |

2.4. Simulation setup
The PSO platform has the detailed parameters, as shown in Table 2. The simulations were conducted at least 20 times to ensure consistency, especially due to the existence of a random generator constant in the updating function.

| Table 2. PSO parameters |
|------------------------|
| Parameter       | CPSO          |
| \(w\)           | 0.7           |
| \(c_p\)         | 2.05          |
| \(c_g\)         | 2.05          |
| Minimum search space for \(\eta_2\) | 0          |
| Maximum search space for \(\eta_2\) | 1.2        |
| Minimum search space for \(\gamma_c\) | 10         |
| Maximum search space for \(\gamma_c\) | 1000      |
| Swarm size      | 20, 30, 50, 100, 200 |

3. Results and Discussion

3.1. Optimization Platform Performance
Firstly, the platform is evaluated in terms of the swarm size. As a reference, the flow curve at 400 mT is determined as the reference. Table 3 shows the objective function output at various swarm sizes for the flow curve at 400 mTesla. The platform with swarm size 200 has the lowest objective function output.
when iteration reaches ten times. After 40 times, iterations of 30, 100, and 200 have almost reached the same values. As long as the computational burden is acceptable, swarm size 200 can be employed. On the other hand, the fewer swarm size should be selected if the computational cost is too high. For the current case, 200 is selected as the particle number. To check the convergence of the selected swarm size, Figure 2 provides information about the convergence trend for the flow curve at 200 and 400 mTesla.

**Table 3.** Objective function outputs at various swarm sizes 400 mT

| Swarm Size | Objective function output |
|------------|--------------------------|
|            | 10 Iteration | 40 Iteration |
| 20         | 605          | 310          |
| 30         | 510          | 215          |
| 100        | 390          | 215          |
| 200        | 220          | 215          |

![Figure 2](image.png)

**Figure 2.** Convergence of PSO-BB platform at (a) 400 mT and (b) 200 mT

The platform also has been compared with the conventional method, which is least mean square or gradient descent methods using the same objective function. The comparison is shown in Table 4. In off-state condition, LMS is better than PSO, while in on-state condition, PSO is better. The reason could be the flow curve in no magnetic field condition has almost straight line like form, while in the on-state condition, the low and high shear rate region can have very different values.

**Table 4.** Comparison between PSO-BB, LMS-BB

| B (mT) | PSO RMSE (Pa) | LMS RMSE (Pa) |
|--------|---------------|---------------|
| 0      | 135           | 81            |
| 200    | 132           | 255           |
| 400    | 271           | 396           |

The visual representation of the obtained parameters, while comparing with the experimental data, is shown in Figure 2. Although the reference data has a kind of noise (which is likely white noise from measurement process or others), the algorithm is still capable of predicting the parameter successfully. In other words, the platform can be applied to real-world applications and clean the data selection from the subjectivity of humans.
3.2. Obtained Parameters

The obtained yield stress value after optimization process are shown in Figure 2. The graphic also has shown its significance to compare between the low and high shear rate. In the near zero magnetic field, the difference is only about less than 1 kPa. In 500 mTesla, the yield stress difference between low and high shear rate region can be almost 10 kPa, which can affect significantly the accuracy of the design process of a device, especially in high magnetic field values.

For the plastic viscosity representing the gradient of the straight line of Bingham Plastic equation, the plastic viscosity at low shear rate region has shown a very high value showing the steep gradient. Meanwhile, in the low shear rate region, the value remains less than 3 Pa s and tend to be consistent. Although the value is significantly different, the difference will not affect much in the design process due to the parameter is not usually employed in some of the design processes. However, when the plastic viscosity parameters need to be used, such as for parametric modeling of an MR device, the right plastic viscosity needs to be chosen carefully according to the MR devices operating range.

4. Conclusions

An optimization platform has been developed using constriction PSO and an objective function inspired by the Biplastic Bingham equation. The objective function considers the error in both regions. The swarm size is firstly evaluated in terms of the fewest of the objective function output at 10th and 40th iteration. It has shown that 200 swarm size is selected because it needs a fewer iteration process and more optimized cost functions. Furthermore, the platform also has been compared with the traditional
optimization method called gradient descent or least square method. The results show a better prediction at the on-state condition or with the existence of a magnetic field. The optimization results also have been evaluated visually by comparing the predicted parameters with the reference data. A discussion comparing the predicted yield stress at low and high shear rate conditions also has been carried out. In summary, the difference between the two yield stress types will be more apparent at the high value of magnetic fields. The right yield stress selection needs to be considered carefully to obtain the best results.

In the future, the platform will be developed further, especially to improve the accuracy and explore the possibility to be applied in MR devices application.

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