A DIFFERENT VIEW OF DEEP INELASTIC ELECTRON-PROTON SCATTERING

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Deep inelastic electron-proton scattering is analyzed in the target rest frame using a theoretical approach suitable to describe many-body systems of bound constituents subject to interactions. At large three-momentum transfer $|q|$, this approach predicts the onset of scaling in the variable $\tilde{y} = \nu - |q|$, where $\nu$ denotes the energy transfer. The present analysis shows that the data, plotted at constant $|q|$, exhibit a remarkable scaling behavior in $\tilde{y}$ and manifestly display the presence of sizable interaction effects.

1 Introduction

The inclusive cross section for deep inelastic scattering (DIS) of unpolarized electrons by unpolarized protons can be written in terms of two structure functions, $W_1$ and $W_2$, according to

$$\frac{d^2\sigma}{d\Omega d\nu} = \sigma_M \left[ W_2(|q|, \nu) + 2W_1(|q|, \nu) \tan^2 \frac{\theta}{2} \right].$$

(1)

In the above equation $\theta$ is the electron scattering angle, $q$ and $\nu$ denote the three-momentum and energy transfer, and the Mott cross section is defined as

$$\sigma_M = \alpha^2 \cos^2(\theta/2)/[4E_0^2 \sin^4(\theta/2)],$$

$\alpha$ and $E_0$ being the fine structure constant and the beam energy, respectively.

In general, $W_1$ and $W_2$ depend upon both $q$ and $\nu$. However, at large $Q^2$ ($Q^2 = |q|^2 - \nu^2$) $F_1 = mW_1$ and $F_2 = \nu W_2$ have been observed to scale, i.e. to depend primarily on the Bjorken variable $x = Q^2/2(mp)$, where $p$ and $q$ denote the four momentum carried by the target proton and the four momentum transfer, respectively. In the target rest frame $x = Q^2/2m\nu$, $m$ being the proton mass. The physical interpretation of the Lorentz scalar variable $x$ becomes apparent in the infinite momentum frame, defined by the condition $p \to \infty$, where it can be identified with the momentum fraction carried by the proton constituent involved in the scattering process.

The scaling behavior of the observed proton $F_1$ is illustrated in fig. The weak dependence upon $Q^2$, mostly due to gluonic radiative corrections, is well described by QCD evolution equations.

In this note I discuss an approach to DIS based on many body theory, in which proton constituents are treated as pointlike bound particles. Within
Figure 1. Proton $F_1$ obtained from the MRS(A) parton distributions, plotted as a function of the Bjorken scaling variable $x$ for a range of values of $Q^2$.

In this framework, the proton response is studied at constant $|q|$ as a function of the energy transfer $\nu$. This analysis naturally leads to predict the onset of scaling in the new variable $\tilde{y} = \nu - |q|$ at large $|q|$.

Section 2 describes the many body treatment of the proton response at large momentum transfer, the basic assumptions underlying the Plane Wave Impulse Approximation (PWIA), and the emergence of the new variable $\tilde{y}$, closely related to the standard scaling variable of many body theory, $y$. In Section 3 the proton $W_1$ obtained from the MRS(A) parametrization of parton distributions is shown to scale in $\tilde{y}$ and exhibit large effects driven by the binding energy of proton constituents. Finally, section 4 is devoted to a summary and a discussion of the differences between the present approach and the standard parton model description of DIS. The implications of the results presented in this paper for the interpretation of different DIS observables are also briefly analyzed.

## 2 Proton response within many-body theory and $\tilde{y}$-scaling

The proton response to a scalar probe delivering a large momentum $q$ can be written

$$S(q, \nu) = \sum_F |\langle F | \sum_i e^{iq \cdot r_i} |0\rangle|^2 \delta(\nu + m - E_F) ,$$

(2)
where \( r_i \) specifies the position of the i-th proton constituent. The above equation shows that \( S(q, \nu) \) is given by the distribution of the strength of the state \( \sum_i i e^{i q \cdot r_i} |0 \rangle \), created by the probe, among the eigenstates \( |F \rangle \) of the system belonging to momentum \( q \).

In general, the response (2) is nonzero at both \( \nu \leq |q| \) and \( \nu > |q| \). As a consequence, in many-body theory \( S(q, \nu) \) is usually studied at fixed \( |q| \) as a function of the energy transfer \( \nu \). The difference between this type of analysis and the one carried out at fixed \( Q^2 \) as a function of \( x \) is illustrated in fig. 2. The dashed lines show the parabolae \( Q^2 = 5 \) and \( 10 \) GeV\(^2 \). They intersect the curve \( \nu = \sqrt{|q|^2 + m^2} - m \), corresponding to elastic scattering kinematics, at \( x = 1 \) and approach the thick solid line \( \nu = |q| \), separating the spacelike (\( \nu < |q| \)) and timelike (\( \nu > |q| \)) regions as \( \nu \to \infty \) (i.e. \( x \to 0 \)). The standard analysis of the structure functions is performed along these parabolae, that do not enter the timelike region. In the present paper I will discuss the behavior of the proton response along the horizontal dash-dot lines of constant \( |q| \), which extend into the \( \nu > |q| \) region.

At large momentum transfer, scattering off a many-body system can be described as the incoherent sum of elementary processes in which the momentum transfer \( q \) is delivered to the i-th constituent, carrying initial momentum \( k_i \), while the residual system, acting as a spectator, is left in the state \( |R \rangle \) with total momentum \( -k_i \). In PWIA, i.e. neglecting final state interactions (FSI) between the struck constituent and the residual system, the energy of the state

\[
|F \rangle = |i(k + q); R(-k)\rangle
\]

(3)
can be written \( (k_z = k \cdot q/|q|) \)

\[
E_F = |q| + k_z + E_R + O\left(\frac{1}{|q|}\right).
\]

(4)

Combining the requirement of energy conservation, \( \nu + m = E_F \), and the above equation we obtain, in the \( |q| \to \infty \) limit:

\[
\tilde{y} = \nu - |q| = k_z + E_R - m ,
\]

(5)

implying in turn that as \( |q| \to \infty \) \( S(q, \nu) \to S(\tilde{y}) \), i.e. that \( S(q, \nu) \) scales in the new variable \( \tilde{y} \).

The above discussion can be easily generalized to the case of electron scattering. It should kept in mind, however, that \( W_1 \) and \( W_2 \) extracted from the measured electron-proton scattering cross section are not trivially related to the proton response. For example, while the proton response is in general nonvanishing in the \( \nu \geq |q| \) region, at \( \nu = |q| \) \( W_2 \) vanishes due to gauge
Figure 2. The thick solid line separates the spacetime and timelike regions. The thin solid line corresponds to elastic electron-proton scattering, while the dashed lines correspond to constant $Q^2$.

Invariance and $W_1$ does not contribute to the cross section (see eq.(1)). In this note I will focus on $W_1$ only.

The expression of $W_1$ within PWIA can be easily obtained from eq.(2) replacing

$$\sum_F \langle F | F \rangle \to \sum_R \int d^3k \ |i(k+q); R(-k) \rangle \langle R(-k); i(k+q)| ,$$

and including the transverse cross section for electron scattering off a bound pointlike constituent carrying spin 1/2, defined as $\sigma_T = \sigma_M \sigma_1$. The results is

$$W_1(|q|, \nu) = \sum_i \int d^3k \ dE \ \sigma_1(k, E, q, \nu) \ P_i(k, E) \ \delta(\bar{y} - k_z - E) ,$$

where

$$P_i(k, E) = \sum_R \ |\langle R(-k); i(k)|0 \rangle|^2 \ \delta(m - E_R - E) .$$

In the limit of vanishing constituent mass $\sigma_1 \to e_i^2$, $e_i$ being the electric charge of the i-th constituent, and we get

$$W_1(|q|, \nu) = \sum_i e_i^2 \int d^3k \ dE \ P_i(k, E) \ \delta(\bar{y} - k_z - E) = \sum_i e_i^2 \ \bar{f}(\bar{y}) .$$
showing that i) the structure function $W_1$ scales in $\tilde{y}$ and ii) it provides a direct measurement of the response. It is apparent that $\tilde{y}$ closely resembles the scaling variable $y$ used in quasi-elastic electron-nucleus scattering, where it is associated with the component of the momentum of the struck nucleon parallel to the momentum transfer. However, it should be pointed out that since $\tilde{y} = -m\xi$, where $\xi$ is the Nachtmann variable, $\tilde{y}$-scaling is related to Bjorken scaling as well.

The simple picture of the proton response described so far will be obviously modified by color confining interactions. While the mass of the nucleon contains the contribution of confinement interactions, this contribution is omitted in the energy of the struck quark, $|q| + k_z$. Therefore, it must be included in the energy $E(R)$ of the residual system. We expect that the confinement energy does not change significantly in the time duration of DIS, and its main influence is via the wave functions $|0\rangle$ and $|R\rangle$. However, it could also contribute to FSI.

3 Data analysis

Fig. 3 shows the proton $W_1(q, \nu)$, obtained from the MRS(A) fit of parton distributions, plotted at several values of $|q|$ as a function of $\tilde{y}$. The MRS(A) parton distributions provide a smooth description of the data on the structure function $F_2(Q^2, x)$ all the way from $Q^2 = 10^{-1}$ GeV to $Q^2 = 10^3$ GeV. It clearly appears that at large values of $|q|$ $W_1(q, \nu)$ depends primarily on $\tilde{y}$. As pointed out in the previous section, this scaling has a simple interpretation within many-body theory, related to the well known $y$-scaling observed in electron-nucleus scattering.

The small scaling violations seen in fig. 3 can be ascribed to gluonic radiative corrections, as in the standard $x$-scaling analysis. Note however that in $\tilde{y}$-scaling $Q^2$ has a large variation, ranging from zero at $|q| = \nu$ ($\tilde{y} = 0$) to $\sim 2m|q|$ at $\nu = |q| - m$ ($\tilde{y} = m$), which does not appear to spoil the quality of scaling.

An interesting feature of fig. 3 concerns the width of the response, which turns out to be few hundred MeV only, independent of the value of $|q|$. This implies that DIS has an intrinsic energy scale of few hundred MeV. The main part of the energy transfer $\nu$, of order $|q|$, goes into the kinetic energy of the struck constituent, and does not play any interesting role in the dynamics of the target system. It follows that changes in the energy $E_R$ of the residual system of order 100 MeV significantly affect $W_1(\tilde{y})$.

The effects of $E_R$ on the response can be clearly seen in the following example, first studied by Close and Thomas. Let us consider the difference
between the responses due to valence $u$ and $d$ quarks in the proton. When the electron strikes the valence $d$ quark, the remaining two valence $u$ quarks are left in the residual state $\mathcal{R}_1$ with spin 1. On the other hand, when a valence $u$ quark is struck, the residual $ud$ pair can be found in the spin 0 state, $\mathcal{R}_0$, with probability 0.75, or in the spin 1 state, $\mathcal{R}_1$, with probability 0.25. Denoting by $V_u(\bar{y})$ and $V_d(\bar{y})$ the contributions of valence $u$ and $d$ quarks to $W_1(\bar{y})$, we can write the response due to the final state $\mathcal{R}_1$ (normalized to unit constituent charge)

$$
\chi_1(\bar{y}) = 9V_d(\bar{y}) ,
$$

while that associated with $\mathcal{R}_0$ is

$$
\chi_0(\bar{y}) = \frac{3}{2}[V_u(\bar{y}) - 2V_d(\bar{y})] .
$$

In perturbation theory

$$
E_{\mathcal{R}_1} - E_{\mathcal{R}_0} \sim \frac{2}{3}(m_\Delta - m) \sim 0.2 \text{ GeV} ,
$$

and $\chi_1(\bar{y})$ is expected to be shifted to larger $\bar{y}$ by $\sim 0.2 \text{ GeV}$, with respect to $\chi_0(\bar{y})$.

Fig. 4 shows that $\chi_0$ and $\chi_1$ obtained from the MRS(A) parton distributions at $|q| = 10 \text{ GeV}$ are indeed shifted by $\sim 0.1 \text{ GeV}$ from each other at $\bar{y} < -0.2 \text{ GeV}$. According to PWIA this shift should be independent of $\bar{y}$, provided the color magnetic interaction can be treated perturbatively. The
fact that the shift is only $\sim 0.1$ GeV indicates that it has nonperturbative contributions. Differences in FSI can also have an influence.

4 Summary and conclusions

The results presented in this paper show that the structure function $W_1$, which is proportional to the proton response in the limit of vanishingly small constituent mass, scales in the impulse approximation variable $\tilde{y}$ at large $|q|$. The occurrence of $\tilde{y}$ scaling emerges in a most natural way from the many body treatment of DIS in the target rest frame.

Unlike the standard parton model of DIS, many body theory treats proton constituents as bound particles. This difference has a number of relevant implications. As pointed out in Section 2, within the present approach the response is in general nonzero in the $\nu > |q|$ region, not accessible by electron scattering. A timelike response can occur either due to initial state interactions, which can make $E_R$ large enough to give a positive right hand side of eq. (5), or because of FSI. The initial energy of the struck constituent is identified with $e = m - E_R$ and not with the on-shell energy $\sqrt{m_i^2 + |k|^2}$, where $m_i$ is the mass of the i-th constituent. On the other hand, in the parton model the struck particle is assumed to be on mass-shell before and after the
interaction with the electron. In this case
\[ \nu = \sqrt{m_i^2 + (k + q)^2} - \sqrt{m_i^2 + k^2} \leq |q|, \quad (13) \]
and all of the response is at negative \( \tilde{y} \), in the spacelike region. The same conclusion applies to the leading twist-two order of the operator product expansion. The fact that putting the quarks on mass-shell, as commonly done in perturbative QCD, is not legal has been recently pointed out by Bjorken.\(^{10}\)

In conclusion, new insights in DIS off the proton can be obtained using standard many-body theory and relating the scaling function to the distribution of proton constituents in the proton rest frame. While \( \tilde{y} \) scaling is derived assuming bound constituents subject to interactions, the occurrence of \( \xi \)- or \( x \)-scaling is obtained under the assumption of free, on-shell, constituents. Thus, the occurrence of scaling cannot automatically be taken as evidence of scattering off free constituents.

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