Highlights

Spin-resolved electron transport
in nanoscale heterojunctions. Theory and applications.

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- The extension of the conductance model in nanoscale point-like contacts for the spin-resolved transport and magnetic contacts.

- Unified description of the contact resistance from Maxwell diffusive through the quasi-ballistic to ballistic and purely quantum transport regimes without residual terms.
Spin-resolved electron transport in nanoscale heterojunctions. Theory and applications.

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Abstract

The work represents the extended theoretical model of the electrical conductance in nanoscale magnetic point-like contacts. The developed approach describes diffusive, quasi-ballistic, ballistic and quantum regimes of the spin-resolved conductance that is important for further development of the contact Andreev reflection spectroscopy, heterojunction models, scanning tunnel microscopy techniques. As a benefit, the model provides a unified description of the contact resistance from Maxwell diffusive through the ballistic to purely quantum transport regimes without residual terms. The model of the point contact assumes that the contact area can be replaced by a complicated object (i.e. the tunnel barrier or complicated one with nanoparticles, narrow domain wall, etc.), where the potential energy profile determines its electrical properties. The model can be easily adapted to particular contact materials, its physical properties and species of the contact area.

Keywords: Spintronics, interconnects, heterojunctions, point contact, ballistic magnetoresistance, spin-resolved conductance, spin-resolved contact resistance, domain wall resistance, tunnel magnetoresistance.

1. Introduction

Quantitative theory of conductance G in various electronic systems with restricted geometry has numerous important applications, e.g., in a case of point contacts it solves the problem of determining the size of the contact [1, 2, 3]. The conductivity of point contacts (PCs) has been studied during many years in the past [4]. At present time, great efforts have been made to create reliable PCs or nanocontacts (NCs) with predictable properties, considering the interface matching of the nanowire connections between normal, semiconductor, ferromagnetic (FM) and superconducting materials in nanoscale spintronics devices [5, 6, 7, 8, 9, 10, 11].

A simplest, but relevant in most cases, solvable model for the PC is a circular constriction of the radius a, which connects two large electron reservoirs. It is convenient to quantify the conducting properties of the NC via the dimensionless ratio of the geometrical size a to the bulk electron mean free path l. The a/l or its inverse, the Knudsen ratio K = l/a, becomes an output of a fitting of the theory to experimental data on the resistance of the PCs [12]. Once l is known from resistivity measurements of the material, the effective diameter of the contact can be estimated from the fitted K.

The model diameter d = 2a can be identified as the size of the contact, if information about the contact shape is unavailable. Two limiting regimes of the conductance through NCs are commonly discussed. The first one is the Maxwell, or diffusive conductance GM, when the contact size much larger than l (K ≪ 1.0) [13, 14, 15],

\[ G_M = 2\alpha /\rho_N, \]

where \( \rho_N \) is the bulk resistivity, which can be expressed in terms of bulk conductivity \( \sigma_N \) of the isotropic metal as follows:

\[ \rho_N^{-1} = \sigma_N = \frac{e^2 n l}{h k_F} = \frac{e^2 p_F^2 l}{3\pi^2 h^3}, \]

where e, k_F = p_F/h and n = \( k_F^3/3\pi^2 \) are the electron charge, Fermi wave-number and free electron density in
G-way, the resistance measurements give a tool to estimate the point of view: once the contact size is known in some or combining the resistivity with specific heat measurements, which show the connection between the resistance model, the bulk mean free path \( l \) and electron-phonon scattering via the size effects in thin films and nanowires (NWs), respectively. Within the model, the bulk mean free path \( l = \hbar k_F^2/\pi m_e \) (\( m_e \) is the electron mass) depends on impurities concentration, defects, electron-electron and electron-phonon scattering via the averaged time \( \tau \) between collisions.

The second regime refers to the ballistic conductance through the contact area when no any collisions occur during the electron transmission [16], \( K \gg 1.0 \). In this case, there is no place or information about the product \( \rho \) \( l \) - product, i.e. establish the contact material parameter from the single kind of measurements. Indeed, it has been done for Au-Au nancontacts [18], where it was pointed out that the procedure to extract \( l \) from \( \rho \gamma l = \text{constant} \) has yielded \( l \approx 3.8 \text{ nm} \), which is an order of magnitude below the bulk \( l \approx 38 \text{ nm} \) for 99.99% pure gold [22] at room temperature. Moreover, it was noticed that the range of applicability of the ballistic Sharvin approach, (3) or (4), is restricted to a smallest radius of the contacts close to 1 nm, otherwise, the accordance of the theory with the relevant experiment is poor. Thus, both diffusive Maxwell and ballistic Sharvin limits of the NC conductance cover extreme limits keeping unexplored a wide gap of most accessible and relevant sizes from 1 nm to 100 nm.

The analysis of the electron transport through a circular constriction at arbitrary temperature between the orifice radius \( a \) and the mean free path \( l \) has been made by Wexler [12]. It is based on the variation solution for the Green function (GF) of the Boltzmann kinetic equations. The obtained solution for the resistance was represented as follows [12]:

\[
R_W = \frac{1}{G_W} = \frac{1}{G_M} \gamma(K) + \frac{1}{G_S},
\]

where \( G_W \) is defined as the Wexler conductance, \( \gamma(K) \) is a slowly varying function with the asymptotic values \( \gamma(K \to 0) = 1.0 \) and \( \gamma(K \to \infty) = 9\pi^2/128 = 0.694 \). Expression (5) has the form of an interpolation formula combining additively the diffusive Maxwell and ballistic Sharvin resistances, the relevant terms are vanishingly small when one of them approaches the related limit. The gamma factor gives a smooth transition from one regime to the other by inclining the asymptotics to the correct values.

In 1999 Nikolic and Allen [23] reconsidered the Wexler solution for the orifice conduction for the nonmagnetic junctions. The stationary Boltzmann and Poisson equations for the electric potential were solved taking into account the Bloch-wave propagation and Fermi-Dirac statistics in presence of an electric field. It is worthy to note, that this solution is referred to in literature as the most accurate solution [24] (see, however, strong assumptions after Eq. (59) in Ref. [23]). It was formulated in the Wexler solution with a proper \( \gamma(K) \) re-definition, Fig. 2 in Ref. [23]). At the same time, Mikrajuddin et al. [25] proposed the approach of the resistance model, which is based on the solution of the electrostatic Laplace problem, summing up the resistances of the infinitesimal shells between equipotential surfaces in the orifice.
constriction. The result is represented in the form of Eq. (5) with the re-defined $\gamma(K)$. The comparison of the Nikolic-Allen and Mikrajuddin et al. solutions shows the significant difference between them, which again refreshes the interest to the problem. To summarize, the theoretical approach of the orifice constriction, which is determined via the classical electrodynamics, results in the sum of the diffusive and ballistic terms with a complex transition between them.

We propose an alternative approach, which is based on the quasiclassical transport formalism [26, 27, 28]. The outcome and advantage of our solution is a simple integral expression, which provides smooth functional transition between the Sharvin and Maxwell limits without residual terms or counterparts. Moreover, this result is derived as a limiting case from a general quantum model of the NC, where NC can be built from different magnetic metals or metal alloys. As example of verification, the theoretical model is applied to explain experimental data for the golden NCs (symmetric, non-magnetic limit of the general theory) as well as to explain the resistance impact of the single domain wall (DW) in magnetic NWs.

2. Theoretical model of the spin-resolved electron transport in heterojunction

In this section, the model of the NC is considered in terms of the extended quasiclassical approach, which is based on solution of the transport differential equations for the quasiclassical GFs. The model is formulated as a boundary problem in which two large electron reservoirs (leads) are linked via the general NC’s interface, [dataset] Supplementary Material. The NC itself can be a simple constriction or a complicated structure containing e.g. a tunnel barrier. It is important only that the internal NC’s structure could be solved quantum-mechanically, and then the electric current through the NC is expressed in terms of the boundary solution, solving the problem of the conduction. The application of this method is suitable for the heterostructure dimensions larger than the Fermi wavelength of a free electron, $\lambda_F = 2\pi/k_F \approx 0.5 \text{ nm}$.

Considering the general case of FM hetero-contact, which is composed of different FM metals, we assume that the spin-dependent Fermi wave-numbers in both sides of the contact $k_{F,a}$ as well as $l_{m,\sigma}$ ($\sigma = \uparrow, \downarrow$) are accounted as arbitrary parameters. The NC is modeled by a conductive circular orifice of the radius $a$ obtained in an impenetrable membrane. This membrane divides the space into the left ($L$) and right ($R$) half-spaces and each half-space is assigned to a single magnetic domain,

\[
F'_{\alpha} = \frac{e^2(k_{\min})^2 a^2 V}{2\pi \hbar} \int_0^\infty dk \frac{J_2(ka)}{k} F'_{\alpha}(k),
\]

where $k$ is the wave-number conjugated to the radial variable in the contact plane; $k_{\min}$ is minimum one of the two wave-numbers: $k_{F,R}$ and $k_{F,a}$. $J_2(ka)$ is the Bessel function, appearing after the integration over the contact plane, the detailed derivation is given in the [dataset] Supplementary Material. Despite the external similarity of expression (6) with those given earlier in our works [28, 29, 30], the integrand function $F'_{\alpha}(k)$ is completely reconsidered:

\[
F'_{\alpha}(k) = \langle \chi_{\alpha} \Delta_{\alpha} \rangle_{\alpha} \left\{ N_{1} \langle \chi_{\alpha} \chi_{L} \rangle_{\alpha} + N_{2} \langle \chi_{\alpha} \chi_{R} \rangle_{\alpha} \right\},
\]

where $\Delta_{\alpha}$ is the quantum-mechanical transmission coefficient; $\chi_{L} = \cos(\theta_{L})$. The angle between the $z$-axis and direction of the electron trajectory is $\theta_{\alpha}, \alpha$, which is related to the contact side $c = L(R)$, see Fig. 1. The averaging over solid angle is given in spherical coordinate system $[k, \theta, \varphi]$, and $\langle \ldots \rangle_{\alpha}$ is equivalent to $\frac{4\pi}{2\pi} \int_0^{2\pi} d\varphi \int_0^\pi \sin(\theta) \ldots d\theta \sim \frac{1}{2\pi} \int_0^{2\pi} \int_0^\pi \sin(\theta) \ldots d\theta d\chi_{L}$, where the limit $\tilde{\chi} = \cos(\theta_{\alpha})$ appears as a result of the electron momentum conservation along the direction of the contact’s plane. The index $\alpha$ is hidden, but refers to all variables throughout. Further quantities are displayed
as follows:

\[ N_1 = \left\langle (D_\alpha)_{10} \right\rangle_{\theta_x} [2(1 - \lambda_R) + \lambda_3] - \left\langle (D_\alpha)_{01} \right\rangle_{\theta_x} \lambda_4 \Delta^{-1}, \]

\[ N_2 = \left\langle (D_\alpha)_{00} \right\rangle_{\theta_x} [2(1 - \lambda_L) + \lambda_3] - \left\langle (D_\alpha)_{01} \right\rangle_{\theta_x} \lambda_4 \Delta^{-1}, \]

\[ \Delta = 4(1 - \lambda_L)(1 - \lambda_R) + 2[\lambda_1(1 - \lambda_R) + \lambda_2(1 - \lambda_L)] - \lambda_3, \lambda_4 + \lambda_1 \lambda_2, \]

where

\[ \lambda_{LR} = \frac{1}{1 + \frac{k_F^2}{r_{20}^2}}, \]

\[ \lambda_1 = \frac{\left\langle \frac{\delta x_{\alpha} \theta_x}{n r_{20}^2} \right\rangle_{\theta_x} \left\langle \frac{\beta x_{\alpha} \theta_x}{n r_{20}^2} \right\rangle_{\theta_x}}{\left\langle \frac{\delta x_{\alpha} \theta_x}{n r_{20}^2} \right\rangle_{\theta_x}}, \]

\[ \lambda_2 = \frac{\left\langle \frac{\delta x_{\alpha} \theta_x}{n r_{20}^2} \right\rangle_{\theta_x}}{\left\langle \frac{\beta x_{\alpha} \theta_x}{n r_{20}^2} \right\rangle_{\theta_x}}, \]

\[ \lambda_3 = \frac{\left\langle \frac{\delta x_{\alpha} \theta_x}{n r_{20}^2} \right\rangle_{\theta_x}}{\left\langle \frac{\beta x_{\alpha} \theta_x}{n r_{20}^2} \right\rangle_{\theta_x}}, \]

\[ \lambda_4 = \frac{\left\langle \frac{\delta x_{\alpha} \theta_x}{n r_{20}^2} \right\rangle_{\theta_x}}{\left\langle \frac{\beta x_{\alpha} \theta_x}{n r_{20}^2} \right\rangle_{\theta_x}}, \]

\[ \left\langle \chi_L W_L \right\rangle_{\theta_x} = \frac{\left\langle \frac{\delta x_{\alpha} \theta_x}{n r_{20}^2} \right\rangle_{\theta_x}}{\left\langle \frac{\beta x_{\alpha} \theta_x}{n r_{20}^2} \right\rangle_{\theta_x}}, \]

\[ \left\langle \chi_R W_R \right\rangle_{\theta_x} = \frac{\left\langle \frac{\delta x_{\alpha} \theta_x}{n r_{20}^2} \right\rangle_{\theta_x}}{\left\langle \frac{\beta x_{\alpha} \theta_x}{n r_{20}^2} \right\rangle_{\theta_x}}, \]

The expressions above include \( (D_\alpha)_{10} = \left\langle \frac{\delta x_{\alpha} \theta_x}{n r_{20}^2} \right\rangle_{\theta_x} \) and \( \delta = k_F^L / k_F^R(V) \), where \( \alpha \) is conserved for \( k_F^R(V) = \sqrt{\left( k_F^L \right)^2 + \left( 2m_\text{eff} / \hbar^2 \right) V} \) according the assumption that the spin diffusion length is larger than the contact dimension. The lower integral limit becomes \( \tilde{x} = 0 \), \( \theta_{\text{c,r}} = \pi / 2 \) at the condition \( \delta \leq 1 \), otherwise \( \tilde{x} = \sqrt{\delta^2 - 1} / \delta^2 \), or both conditions can be joined as following: \( \tilde{x} = \Re \sqrt{\delta^2 - 1} / \delta^2 \). The solution for the reversed bias \( V \) with negative terminal on the right-hand side can be retrieved using the symmetry of the system: \( k_F^L \rightarrow k_F^R(V) \), \( k_F^L(V) \rightarrow k_F^L \), that gives again the positive terminal on the right side. It is assumed that the left side is grounded and the conduction band edge does not move with \( V \), the Fermi level is fixed, otherwise: \( k_F^R(V) = \sqrt{\left( k_F^L \right)^2 + \left( 2m_\text{eff} / \hbar^2 \right) V/2} \) and \( k_F^L(V) = \sqrt{\left( k_F^L \right)^2 - \left( 2m_\text{eff} / \hbar^2 \right) V/2} \), and \( \delta = k_F^L(V) / k_F^R(V) \).

The transmission coefficient \( T_\alpha \) is a function of the applied voltage \( V \) and parameters of the potential energy profile within NC area. It should be noticed that transmission \( T_\alpha \) for 2D and 3D electron transport, which is characterized originally by 1D potential energy profile \( U(z) \), becomes a function of \( \theta_{\text{c,r}} \) and \( V \), consisting the projections of the Fermi wave-vectors on the \( z \)-axis: \( k_F^L = k_F^L \cos(\theta_{\text{c,r}}) \) and \( k_F^R = k_F^R \cos(\theta_{\text{c,r}}) \). It should be noticed, that the derived set of the variables, such as \( \alpha, \lambda_{1,2,4, \chi_LW_L} \) as a functions of \( k \) in (6) and (7), is significantly different from the set that in our previous works [28, 29, 30], while the ballistic and tunnel-responsible term \( \chi_L D_\alpha \) is conserved. The origin of this difference is the accurate solution of the integro-differential equation which takes into account the second-order derivatives of the GFs by \( \lambda \). The mathematical derivation of the Eq. (6) is collected in [dataset] Supplementary Material.

The general solution (6) can be verified applying it to the case of symmetric non-magnetic contact: \( \theta_{\text{c,r}} = 1 \), \( I_{\text{c,r}} = 1 \), \( k_F^L = k_F^R \), \( k_F^L \), and thus \( F_1(k) = F_1(k) \), \( F_2 = F_2 \). The replacement of the variable \( y = ka \) in Eq. (6) results to \( \int_0^\infty dy F_2(y)/y = 1/2 \). For the infinitesimal applied voltage \( V \) the conductance \( G = \frac{dI}{dV} = \left( F_1 + F_2 \right) / V \)

\[ G = 4G_S \left( \frac{1}{4} - \int_0^\infty dy \frac{F_2(y)}{y + 1 + y^2 K^2 + \sqrt{1 + y^2 K^2}} \right), \]

which satisfies the exact Maxwell and Sharvin limit automatically. It is an advantage of the revisited derivation of the present work against the previous ones [28, 29, 30]. One might expect that it gives also more precise \( I - V \) curves for the non-magnetic NCs. The analytical solution Eqs. (6)-(8) is applied in the next section for the comparison with alternative theoretical approaches and fitting the experimental data available in literature.

3. Discussion: applicability of the model

The general approach, that we developed in the present work, covers a variety of the NC realizations, which might be involved further in the development of the quantum integrated circuits for the next generation of electronics below 10 nm. Potentially, the model can deal with spin-resolved conducting properties of the nanoscale elements such as interconnects, complicated magnetic tunnel junctions (MTJs), quantum dots, spin field-effect transistors (FETs), etc. As a first-order approximation, the contact area can be replaced by a simple or composite quantum object, where the electric properties are determined by the internal structure of the energy levels and/or the potential energy profile across the junction. For instance, the spin-resolved quantum term \( F_\alpha = \chi_L D_\alpha \theta_x \) of the model was successfully applied for the simple MTJs [31, 32, 33] as well as for MTJs with embedded nanoparticles [34, 35] explaining
the voltage dependence of the tunnel magnetoresistance TMR(V), the quantized conductance behavior [34] and \( R(V) \) curves. The improved model Eqs. (6)-(8) extends further the range of applicability, making it more adequate to the real systems.

3.1. The orifice conductance: comparison with alternative theoretical models

One of the goals of this work is to compute the classical conductance for the non-magnetic junction and compare it with that obtained in the earlier theories. The proposed approach allows to reproduce the Maxwell and Sharvin analytical limits in such terms that they smoothly transform from one to the other exactly without some additional factors like \( \gamma \) in Refs. [12, 23, 25]. Indeed, at \( K \to \infty (a/l \to 0) \) the integral in Eq. (8) vanishes, hence, the conductance transforms into the ballistic Sharvin one, \( G = G_S \). The integral in Eq. (8), at small \( K \ll 1 (a/l \gg 1) \), reads \( \left( \frac{1}{4} - \frac{2K_0}{\pi} \right) \) that gives accurate solution for the diffusive limit, \( G = (8/3\pi) KG_S = G_M \). In contrast to the Wexler and the followers’ solutions, in which the ballistic term always exists in (5) for any \( K \), our solution (8) exactly transforms from the ballistic to the diffusive limit of the conductance.

The normalized conductance by the Sharvin limit is given in Fig. 2a for the four solutions of the problem, the result of the present work is shown as \( R_1 = G/G_S \). The ratios \( R_2 = G/G_S \), \( R_3 = G'/G_S \) and \( R_4 = G'/G_S \) correspond to the solutions by Mikrajuddin et al. with \( \gamma \approx \frac{1}{2} \int_0^{\infty} e^{-K_x} \text{sinc}(x) \, dx \), by Nikolic and Allen with \( \gamma_{\text{fit}} \approx (1+0.83K)/(1+1.33K) \) and, finally, by the Wexler approach with flexible \( \gamma \), respectively. The comparison of the relative differences of the conductance to \( R_1 \), which is displayed in Fig. 2b, shows that the Mikrajuddin solution is the closest one to our result. The Nikolic-Allen solution \( G' \) with relevant \( \gamma_{\text{fit}} \) shows the maximal difference of 15.8% with ours at \( a/l \approx 1 \). It should be noticed, the presented \( G'_W \) is the lowest order solution with a maximal deviation of 1.0% against the most exact summed-up series solution in [23]. The Wexler solution shows the intermediate deviation of 12.9% at \( a/l \approx 0.75 \). We believe that the strong assumption created by Wexler [Ref. [12], the paragraph after Eq. (42)], where the numerical coefficient 9/8 is replaced by 1 at the Knudsen-Sharvin limit, and the one which is made by Nikolic and Allen [Ref. [23], the paragraph after Eq. (59)], where the numerical coefficient 3/4 is also replaced by 1 at the same limit, could be a cause of the deviations in the vicinity of the Sharvin limit reported above.

Finally, it should be noticed that some experimental works manipulate with \( \gamma \) in Eq. (5), in order to achieve the desired fitting, considering \( \gamma \approx 0.7 - 0.75 \) as a reduced constant, e.g. [2, 18, 19]. We assume that their reduced value of \( \gamma \) originates from the striving to compensate the inaccuracy of the Wexler model for the quasi-ballistic region \( a/l \approx 0.25 - 4.0 \), Fig. 2b, nevertheless that it can give a valuable deviation from a real value for \( l \), being estimated at the larger scales, e.g. for \( a/l \approx 10 \).
3.2. Conductance of the golden nanocontacts

The experimental data by Erts et al. [18] is considered for a quantitative comparison with our theory. The conductance for the golden NCs was measured with different dimensions and fitted by Wexler’s model finally resulting in $l \approx 3.8$ nm [18]. The drastic reduction of $l$ in the NCs was attributed to a high density of scattering centers, which are created during the point contact formation process.

Considering the golden contacts in the ballistic conductance regime, we found that the experimental points from Ref. [18] lie predominantly between the straight lines of the Sharvin conductance at $k_F^{Au} = 0.8 \text{Å}^{-1}$, $k_F^{Au} = 0.9 \text{Å}^{-1}$, $k_F^{Au} = 1.0 \text{Å}^{-1}$, $k_F^{Au} = 1.1 \text{Å}^{-1}$ and $k_F^{Au} = 1.2 \text{Å}^{-1}$, respectively. Fig. 3a. The Fermi wave-number in the bulk for the gold can be estimated using the electron density $n = 5.9 \times 10^{22} \text{cm}^{-3}$, and thus $k_F^{Au} = (3\pi^2n)^{1/3} = 1.205 \text{Å}^{-1}$, Ref. [36] (Chapter 1, Table 1.1) and compared with that corresponding to the lines 1 – 5 of the Fig. 3a.

It makes sense to go beyond a ballistic conductance regime in analysis of the experimental data of Erts et al. [18], because most probably, they refer to the quasi-ballistic regime of the conductance (see Fig. 2). Figure 3b shows theoretical curves of the contact conductance derived from (8), where the $k_F^{Au}$ and $l$ values were considered as independent parameters. The fitted curves 1 – 4 refer to $k_F^{Au} = 1.1 \text{Å}^{-1}$, $k_F^{Au} = 1.0 \text{Å}^{-1}$, $k_F^{Au} = 0.9 \text{Å}^{-1}$ and $k_F^{Au} = 0.85 \text{Å}^{-1}$ with $l = 4.0 \text{nm}$, $l = 6.0 \text{nm}$, $l = 10.0 \text{nm}$ and $l = 38.0 \text{nm}$, respectively. Utilizing (2) in the form

$$n = k_F^{Au}/(\pi l G_0 \rho_N^{Au})$$

where $\rho_N^{Au} = 22.14 \Omega \text{nm}$, the related parameters correspond to $n = 5.1 \times 10^{23} \text{cm}^{-3}$, $n = 3.09 \times 10^{23} \text{cm}^{-3}$, $n = 1.67 \times 10^{23} \text{cm}^{-3}$ and $n = 4.15 \times 10^{22} \text{cm}^{-3}$ for the curves 1 – 4, respectively. It seems that the experimental points lie predominantly on the curve 4. Moreover, the curve 4 has the closest value by $n$, which is estimated in the Ref. [36].
The conductance of the golden NCs with the various contact radius. The red circles are the data adapted from Fig. 4 in Ref. [37]. The ballistic limit is estimated at $k_{\text{Au}}^F = 0.9 \text{ Å}^{-1}$, while the curve 1 is obtained from eq. (8) with $k_{\text{Au}}^F = 0.9 \text{ Å}^{-1}$ and $l = 38 \text{ nm}$.

The experimental data, which cover not only quasi-ballistic but also the diffusive regimes of the conductance as well, might be determinative to verify our theoretical model. Fortunately, experiments of Jensen et al. [37] with golden NCs extend further the measurement range, which was partly covered by Erts et al., towards the diffusive regime of the conductance. Figure 4 shows the best fit of our model Eq. (8) to the Jensen data. The theory matches the experimental data almost ideally with the fitting parameters $k_{\text{Au}}^F = 0.9 \text{ Å}^{-1}$ and $l = 38 \text{ nm}$. Both datasets by Erts and Jensen are collected in the inset of Fig. 4 together, keeping the linear scale for $G$.

The remaining discrepancies in $k_F^{\text{Au}}$ values with respect to the textbook references, as well as the large scatter in the estimated $l$, which may satisfactorily describe the existing experimental data, should not raise doubts about the correctness of the approach: the lateral shape deviation from the ideal circular orifice and the opening angle of the constriction might also influence the conductance quantitatively, giving a correction up to $\sim 50\%$ [38].

3.3. Domain wall resistance in magnetic nanowires

We apply the developed model to calculate the conductance of a magnetic NW with and without single DW. It demonstrates the full range of the spin-resolved ballistic and diffusive electron transport regimes, that is suited to explain a DW resistance behavior, for example, in Ni$_{80}$Fe$_{20}$ Permalloy (Py) [39], Co/Ni [40, 41] and Co NWs [42]. Since a difference in resistance of a NWs with and without DW is a subject of our interest, only DW contribution $\Delta R = (R_{\text{DW}} + R_{\text{NW}}) - (R_0 + R_{\text{NW}})$ is calculated for a wide range of a diameters, Fig. 5a. Thus, the resistance of the homogeneous wire’s segments, $R_{\text{NW}} = 4\rho V l_{\text{NW}}/\pi d^2$, cancels in the difference, where $l_{\text{NW}}$ is total length of NW. The other terms $R_{\text{DW(0)}} \simeq V I(\hat{I}_{\uparrow}^{\text{DW(0)}} + \hat{I}_{\downarrow}^{\text{DW(0)}})$ are resistances of a DW or an interface between segments of the composite NW. The spin-dependent currents $\hat{I}_{\uparrow,\downarrow}^{\text{DW(0)}}$ are estimated within the general magnetic case of the heterojunction Eq. (6) with low bias approach, when the integral distribution is voltage-independent. A spin-bands transition has been taken into account for the case with DW ($R_{\text{DW}}$), while the case without DW assumes $D_{1,1} = 1.0$ for $R_0$ in the case of homogeneous NW. The assignment of spin sub-
bands with respect to the quantization axis is opposite in the case of the opposite direction of the domain’s magnetizations, Fig. 5b. The vortex states and the area between them in [39] are simplified to 1D DW representation, Fig. 5b. The vortex magnetic states and electron transitions with the DOS differences: without (left) and with DW (right). The vortex magnetic states are shown similar to that in the experimental paper [39] and marked as the color thin arrows, while the theoretical DW representation is simplified to 1D case and magnetization is shown as the large gray arrows.

In general, it is found that $\Delta R$ rapidly reduces with increasing of $d$, however, the curve’s slope decreases when the conductance transforms from quasi-ballistic to a diffusive regime for the spin-up conductance channel at $d \approx 2l_{f}$. The curve for $\Delta R$ is sensitive to the mean free path ratios. Experimental measurements of the spin-split $l_{f}$ are accessed in [43, 44], theoretical estimations are available in Ref. [22]. The considered $k_{F}$-values are also consistent with the literature data: $k_{F}^{Py}$ are similar to Mu-metal (Py-type) compounds [29]. It should be noticed also that, taking into account the spin-flip effect and spin accumulation might further improve the consistency of the material parameters utilized in the data fittings.

4. Conclusions

In the present work, a quasi-classical transport model is developed as an approach for computing of electron transport through the point-like contact. The spin-resolved quantum, ballistic, quasi-ballistic and diffusive regimes of the transport are covered by the theory. The solution includes the boundary conditions in terms of the quantum-mechanical transmission coefficient for the NC interface. The NC interface potentially
can be replaced by any quantum object, where the transmission coefficient can be spin-resolved, depending on the applied voltage, the strength of the magnetic field or any other external parameter affecting the energy profile properties. As a result, the analytical solution is derived for the general spin-resolved case for the system, obeying cylindrical symmetry. It doesn’t require so much computer programming to represent it into the form, which allows to make the comparison and fitting to the experiment. The theory has great generality: it can handle with spin-resolved conduction of the nanoscale objects such as NCs, single and multi-barrier tunnel junctions, MTJs with embedded nanoparticles, observing in some cases the quantized conductance, etc.

Finally, we applied our general expression for the current through the NC to a particular problem of the conductance between two separated metallic (nonmagnetic) leads, which are connected by a short and small orifice and filled with the same metal. The simple expression for the conductance that we have got from our general solution provides the smooth functional transition between the Sharvin ballistic and Maxwell-Holm diffusive limits without residual terms. The theory fits quite well with the existing experimental data for the golden NCs. Another application is also shown, which concerns the DW resistance in ferromagnetic nanowires. The comparison to the existing experimental data shows a reasonable quantitative agreement, confirming the wide range of applicability of our approach.

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