A first asteroseismological analysis on WD J1916+3938, the ZZ Ceti star discovered in the *Kepler* mission field

A. H. Córresco$^{1,2}$, A. D. Romero$^{1,2}$, L. G. Althaus$^{1,2}$, and M. M. Miller Bertolami$^{1,2}$

*Facultad de Ciencias Astronómicas y Geofísicas, Universidad Nacional de La Plata, Argentina*

**ABSTRACT**

DAV stars, also called ZZ Ceti variables, are pulsating white dwarfs with atmospheres rich in H. Asteroseismology of DAV stars can provide valuable clues about the origin, structure and evolution of DA white dwarfs. Recently, a new DAV star, WD J191643.83+393849.7, has been discovered in the field of the *Kepler* spacecraft. It is expected that further monitoring of this star in the next years will enable astronomers to obtain the best lightcurve of a pulsating DA white dwarf ever recorded, and thus to know with unprecedented precision the hidden details of the internal structure of this star.

In this paper, we perform a first asteroseismological analysis of WD J191643.83+393849.7 on the basis of fully evolutionary DA white-dwarf models. Specifically, we employ a complete set of evolutionary DA white-dwarf structures covering a wide range of effective temperatures, stellar masses, and H envelope thicknesses. These models have been obtained on the basis of a complete treatment of the evolutionary history of progenitors stars. We compute $g$-mode adiabatic pulsation periods for this set of models and compare them with the pulsation periods exhibited by WD J191643.83+393849.7. Based on a tentative estimation of the mean period spacing of the star, we find that the stellar mass should be substantially large ($\gtrsim 0.80 M_\odot$), in agreement with the spectroscopically derived stellar mass. Also, from period-to-period fits we find an asteroseismological model characterised by a low effective temperature, rather high stellar mass and a thin H envelope. The possibility that this rather massive pulsating white dwarf can be further monitored with *Kepler* with a high degree of detail turns the star WD J191643.83+393849.7 into a promising and unique object to study the physics of crystallization and carbon/oxygen phase diagrams at high densities.

**Key words:** asteroseismology – stars: evolution – stars: interiors – stars: oscillations – stars: individual: ZZ Ceti stars – stars: white dwarfs

1 INTRODUCTION

Pulsating DA (H-rich atmospheres) white dwarfs, also known as ZZ Ceti or DAV variable stars, comprise the most numerous and studied class of degenerate pulsators. Their instability strip, probably a pure one, is centered at an effective temperature of around 12 000 K (Winget & Kepler 2008; Fontaine & Brassard 2008; Althaus et al. 2010a). The luminosity variations (of up to 0.30 mag) are caused by non-radial $g$-mode pulsations of low degree ($\ell \lesssim 2$) and periods between $\approx 70$ and $\approx 1500$ s, excited through a combination of the $\kappa-\gamma$ mechanism acting in the hydrogen partial ionization zone (Dolez & Vauclair 1981; Winget et al. 1982) and the “convective driving” mechanism — once a sufficiently thick outer convective zone has been developed (Brickhill 1991; Goldreich & Wu 1999). ZZ Ceti asteroseismology — the comparison between the observed periods and the periods computed for a suite of representative stellar models — has the potential of disentangle the internal structure of DA white dwarfs, allowing to place constraints on the stellar mass, the thickness of the outer envelopes, the core chemical composition, weak magnetic fields and slow rotation rates from the observed period patterns (Winget & Kepler 2008; Fontaine & Brassard 2008; Althaus et al. 2010a).

A total of 148 ZZ Ceti stars are currently known (Castanheira et al. 2010). To this list we must add the recently discovered DAV star WD J191643.83+393849.7 (*Kepler* ID 4552982 hereinafter WD J1916+3938), a ZZ Ceti
located in the *Kepler* mission field and identified through ground-based times series photometry (Hermes et al. 2011; hereinafter HEA11). This star ($T_{\text{eff}} = 11,129 \pm 115$ K; log $g = 8.34 \pm 0.06$) exhibits low-amplitude luminosity variations with periods between $\approx 800$ and $\approx 1450$ s (see column 2 of Table 1). It is expected that an extended monitoring of this object with *Kepler* could bring the best lightcurve of a pulsating white dwarf ever recorded (HEA11), even with a better quality than the lightcurves of the brighter stars observed through uninterrupted ground-based campaigns of the Whole Earth Telescope (WET; Nather et al. 1990).

In order for white dwarf asteroseismology to provide realistic constraints on the internal structure of white dwarfs, it is crucial the use of stellar models characterised by consistent and detailed chemical profiles to accurately assess the adiabatic pulsation periods. This requirement has been tackled successfully in the case of the hot DOVs or GW Vir stars (see Córsico et al. 2007a, 2007b, 2008, 2009). Very recently, Romero et al. (2011) have performed the first asteroseismological analysis of a set of 44 ZZ Ceti stars based on the new generation of fully evolutionary DA white dwarf models presented in Althaus et al. (2010b). These models are characterised by realistic chemical profiles throughout the star and cover a wide range of stellar parameters, masses, thicknesses of the H envelope and effective temperatures. The analysis of a large number of ZZ Ceti stars has the potential to characterise the global properties of the class, in particular the thicknesses of the H envelope and the stellar masses. The study of Romero et al. (2011) revealed that DA white dwarfs in the solar neighbourhood could harbor a broad range of H-layer thickness. On the other hand, this analysis was successful in finding, for the first time, an univocal asteroseismological model for the archetypal ZZ Ceti star G117–B15A, one of the targets of that analysis.

Motivated by the exciting discovery of the first pulsating DA white dwarf in the field of the *Kepler* mission, and encouraged by the availability of the most complete set of detailed evolutionary and pulsation models for DAV stars up to date (Romero et al 2011), we present in this paper a first asteroseismological analysis of WD J1916+3938. We are well aware that the asteroseismological potential of this pulsating star will probably be fully exploited with further observations by *Kepler*, which will allow to have available a period spectrum substantially richer than that we have at hand at present just from ground-based photometry. However, we think that a first asteroseismological analysis of this star based on the seven periods — a representative number of periods for ZZ Ceti standards — currently available is worth being attempted. By considering the mean period spacing exhibited by the star, we found that WD J1916+3938 should be substantially massive, in excellent agreement with the spectroscopic inference. We also found that period-to-period fits favour an asteroseismological model characterised by a high stellar mass, a low effective temperature, and a rather thin H envelope.

The paper is organized as follows. In the next Section we briefly describe the computer codes and stellar models employed. The short Section 3 is devoted to the derivation of the spectroscopic mass of WD J1916+3938. We make inferences about the stellar mass from the period spacing data in Section 4 taking into account its dependence not only with the effective temperature and mass, but instead also with the thickness of the H envelope. In Section 5 we extract information of the star through asteroseismological period fits. Section 6 is devoted to infer a rotation period of the star. Finally, we enumerate our results in the Conclusions (Section 7).

### 2 NUMERICAL TOOLS AND MODELS

The present asteroseismological study is based on the fully evolutionary DA white dwarf models described in Althaus...
et al. (2010b) and Romero et al. (2011), generated with the LPCODE evolutionary code. To the best of our knowledge, these models are the first complete set of DA white dwarf models characterised by consistent chemical profiles for both the core and envelope simultaneously. This feature renders these models particularly suitable for asteroseismological studies of DA white dwarfs. Details about the input physics and numerical methods included in LPCODE, and relevant issues related to the evolutionary computations are given in Althaus et al. (2010b) and Romero et al. (2011), and we refer the interested reader to those papers (and references therein). Here, we briefly note that the DA white dwarf models employed in this study are the result of complete evolutionary calculations of progenitor stars for solar-like metallicity ($Z = 0.01$). It is worth mentioning that these models are in agreement with the semi-empirical white dwarf initial-final mass relationships (Salaris et al. 2009; Althaus et al. 2010b). The evolution of eleven evolutionary sequences with initial stellar mass in the range $1−5 M_\odot$ has been computed from the ZAMS through the thermally-pulsing and mass-loss phases on the AGB and finally to the domain of planetary nebulae and white dwarfs. The values of the stellar mass and the H envelope thickness of our set of white dwarf models are shown in Table 1 of Romero et al. (2011). Convection is modeled by using the standard mixing length theory with $\alpha = 1.61$. For our asteroseismological analysis, we have considered a large bank of models with stellar mass and mass of the H envelope varying in the ranges $0.525 < M_\star < 0.877 M_\odot$, and $−9.4 \lesssim \log(M_H/M_\star) \lesssim −3.6$, respectively, being the range of the values of $M_\star$ dependent on $M_\bullet$ (see Table 1 of Romero et al. 2011). For simplicity, the mass of He has been kept fixed at the value predicted by the evolutionary computations for each sequence. A detailed justification of this assumption is given in Romero et al. (2011). The thickest H envelopes (or “canonical” H envelopes) considered are characterised by the maximum allowed values of $M_H$ as predicted by standard stellar evolution calculations, which are dependent on the stellar mass of the white dwarf. Note that the largest value of $M_H/M_\star$ is usually arbitrarily fixed around $10^{-4}$ in previous DA asteroseismological studies, regardless the value of the stellar mass. H envelopes thinner than the canonical ones have been generated in order to extend the exploration of the parameter space of the models for asteroseismology. The properties of our DA white dwarf models have been exhaustively described in Romero et al. (2011). In particular, we emphasise again that these models are characterised by consistent and detailed chemical profiles from the centre to the surface, which are needed to correctly assess the adiabatic pulsation periods of DAVs, a crucial aspect of white dwarf asteroseismology. The pulsation computations have been performed with the adiabatic nonradial pulsation code described in Córredo & Althaus (2006) that uses the so-called “Ledoux modified” treatment (Tassoul et al. 1990) to assess the Brunt-Väisälä frequency.

3 SPECTROSCOPIC MASS DETERMINATION

The spectroscopic mass of WD J1916+3938 can be derived simply interpolating from the tracks in the log $g − T_{\text{eff}}$ diagram of Fig. 1 given the value of log $g$ and $T_{\text{eff}}$ inferred from the spectroscopic analysis of HEA11. If we consider for the moment only the tracks corresponding to DA white dwarf models with thick (canonical) H envelopes, we found a rather high stellar mass, $M_\star = (0.805 \pm 0.040) M_\odot$. If, instead, we consider the evolutionary tracks of models with thin H envelopes, the stellar mass of WD J1916+3938 turns be slightly lower, of $M_\star = (0.797 \pm 0.040) M_\odot$. These values are somewhat smaller ($\sim 2.4\%$) than the value derived by HEA11 using the older tracks of Wood (1990), $M_\star = (0.82 \pm 0.04) M_\odot$.

4 INFERENCES FROM THE PERIOD SPACING

WD J1916+3938 exhibits seven periods in the range $824$−$1437$ s with amplitudes up to 0.44%. The large aliasing present in the ground-based data set of HEA11 prevents from a secure identification of true periodicities. Here, in absence of any additional observational constraint, we shall assume that all the periods detected correspond to genuine eigenmodes of the star. An inspection of the list of periods shown in the second column of Table 1 suggests that, due to their proximity, the periods at 823.9 and 834.1 s could correspond to different values of the harmonic degree $\ell$. If we look for a nearly constant period spacing among the periods, we found that a period separation of about 38.5 s could exist if we discard for the moment the period at 834.1 s. Thus, somewhat arbitrarily, we can assume that this period is associated to a $\ell = 2$ mode and that the remainder ones constitute a series of $\ell = 1$ modes. If we take this assumption as a valid one, we can compute an observed mean period spacing for WD J1916+3938 by means of a nonlinear least-squares fit by considering these six remainder periods. We obtain $\overline{\Delta \Pi}_{\text{obs}} = 38.54 \pm 0.29$ s. In column 1 of Table 1 we list the observed periods and compare six of them (those supposed to be $\ell = 1$) with the periods associated to a phenomenological model characterised by a fixed period spacing of 38.54 s (column 5). The periods of this model are obtained by means of the expression $\Pi_\text{mod} = 38.54 \times \Delta k + 742.92$ [s], where $\Delta k$ (the relative radial order of the modes) is shown in column 4. The residuals between $\Pi_\text{obs}$ and $\Pi_\text{mod}$ are shown in the sixth column, being the average of these residuals 2.44 s.

For $g$-modes with high radial order $k$ (long periods), the separation of consecutive periods ($|\Delta k| = 1$) becomes nearly constant at a value given by the asymptotic theory of nonradial stellar pulsations. Specifically, the asymptotic period spacing (Tassoul et al. 1990) is given by:

$$
\Delta \Pi_k = \sqrt{\frac{\alpha}{\ell(\ell + 1)}}
$$

where

$$
\Pi_0 = 2\pi^2 \left[ \int_{r_1}^{r_2} \frac{N}{r} dr \right]^{-1}.
$$

Other possibilities are that these periods correspond to components of a $\ell = 1$ triplet induced by rotation, or that both periods are associated to modes with the same $\ell$ and consecutive radial order $k$, being their very short separation due to strong mode trapping.
For DAV stars, the asymptotic period spacing is sensitive to the stellar mass, the effective temperature, the mass of the H envelope, and the convective efficiency (CE) (see Tassoul et al. 1990 for an excellent discussion of this issue). For simplicity, in our analysis we shall neglect the dependence of $\Delta \Pi_2$ on the convective efficiency. In principle, one can compare the asymptotic period spacing computed from a grid of models with different masses, effective temperatures, and H envelope thicknesses with the mean period spacing exhibited by the star, and then infer the value of the stellar mass by adopting the effective temperature from the spectroscopic analysis. This method has been applied in numerous studies of pulsating PG 1159 stars (see, for instance, Córscico et al. 2007a,b, 2008, 2009 and references therein). For the approach to be valid, the periods exhibited by the pulsating star must be associated with high order $g$-modes ($k \gg 1$), that is, the star must be within the asymptotic regime of pulsations. Fortunately, this condition is fulfilled by WD J1916+3938, because it pulsate in quite long periods with (presumably) $k \gtrsim 16$ for $\ell = 1$ and $k \gtrsim 30$ for $\ell = 2$ if the stellar mass and effective temperature of this star are assumed to be $M_* \approx 0.8 \ M_\odot$ and $T_{\text{eff}} \approx 11 \ 000$ K, as suggested by spectroscopy.

In Fig. 2 we show the run of the dipole ($\ell = 1$) asymptotic period spacing in terms of the effective temperature for our complete set of DA white dwarf sequences in the case of thick (canonical) H envelopes. Also displayed is the location of WD J1916+3938 with its associated uncertainties. By adopting the spectroscopic effective temperature of WD J1916+3938 ($T_{\text{eff}} = 11 \ 129 \pm 15 \ K$; HEA11) we found that the mass of the star according to our computed mean period spacing is $M_* = 0.851 \pm 0.031 \ M_\odot$, which is $\approx 5.5\%$ larger than — but still compatible with — the spectroscopic estimate ($M_* \approx 0.8 \ M_\odot$). This comparison makes sense because both estimates of the stellar mass of WD J1916+3938 are based ultimately on the same DA white dwarf evolutionary tracks. The above method to estimate the stellar mass from the asymptotic period spacing strongly depends on the precise value of $T_{\text{eff}}$ as derived from the spectroscopic analysis. If we ignore the constraint imposed by the effective temperature, and look for the possible range of stellar mass just for the star to be within the ZZ Ceti instability strip, we found that $M_* \gtrsim 0.77 \ M_\odot$. This lower limit is free of the uncertainties inherent to the spectroscopic effective temperature determination, and is still indicating a rather high stellar mass for WD J1916+3938.

In the above derivation of the stellar mass, we have neglected the dependence of the asymptotic period spacing with the thickness of the H envelope. As is well known, for a given stellar mass the asymptotic period spacing is larger for models with thinner H envelopes (Tassoul et al. 1990). This is shown in Fig. 3 where we depict the run of $\Delta \Pi_2$ for the sequences with $M_* = 0.837 \ M_\odot$ and $M_* = 0.877 \ M_\odot$ and several values of the H envelope thickness for each sequence. Note that the curves for different H envelope thicknesses corresponding to the two different values of the stellar mass overlap, somewhat that can complicate the analysis. For instance, regarding the value of $\Delta \Pi_2$, and for a fixed $T_{\text{eff}}$, a DA white dwarf model with $M_* = 0.837 \ M_\odot$ and log($M_H/M_\odot$) = −7.36 can readily mimic a model with

---

*For DAV stars, the asymptotic period spacing is sensitive to the stellar mass, the effective temperature, the mass of the H envelope, and the convective efficiency (CE) (see Tassoul et al. 1990 for an excellent discussion of this issue). For simplicity, in our analysis we shall neglect the dependence of $\Delta \Pi_2$ on the convective efficiency. In principle, one can compare the asymptotic period spacing computed from a grid of models with different masses, effective temperatures, and H envelope thicknesses with the mean period spacing exhibited by the star, and then infer the value of the stellar mass by adopting the effective temperature from the spectroscopic analysis. This method has been applied in numerous studies of pulsating PG 1159 stars (see, for instance, Córscico et al. 2007a,b, 2008, 2009 and references therein). For the approach to be valid, the periods exhibited by the pulsating star must be associated with high order $g$-modes ($k \gg 1$), that is, the star must be within the asymptotic regime of pulsations. Fortunately, this condition is fulfilled by WD J1916+3938, because it pulsate in quite long periods with (presumably) $k \gtrsim 16$ for $\ell = 1$ and $k \gtrsim 30$ for $\ell = 2$ if the stellar mass and effective temperature of this star are assumed to be $M_* \approx 0.8 \ M_\odot$ and $T_{\text{eff}} \approx 11 \ 000$ K, as suggested by spectroscopy.

In Fig. 2 we show the run of the dipole ($\ell = 1$) asymptotic period spacing in terms of the effective temperature for our complete set of DA white dwarf sequences in the case of thick (canonical) H envelopes. Also displayed is the location of WD J1916+3938 with its associated uncertainties. By adopting the spectroscopic effective temperature of WD J1916+3938 ($T_{\text{eff}} = 11 \ 129 \pm 15 \ K$; HEA11) we found that the mass of the star according to our computed mean period spacing is $M_* = 0.851 \pm 0.031 \ M_\odot$, which is $\approx 5.5\%$ larger than — but still compatible with — the spectroscopic estimate ($M_* \approx 0.8 \ M_\odot$). This comparison makes sense because both estimates of the stellar mass of WD J1916+3938 are based ultimately on the same DA white dwarf evolutionary tracks. The above method to estimate the stellar mass from the asymptotic period spacing strongly depends on the precise value of $T_{\text{eff}}$ as derived from the spectroscopic analysis. If we ignore the constraint imposed by the effective temperature, and look for the possible range of stellar mass just for the star to be within the ZZ Ceti instability strip, we found that $M_* \gtrsim 0.77 \ M_\odot$. This lower limit is free of the uncertainties inherent to the spectroscopic effective temperature determination, and is still indicating a rather high stellar mass for WD J1916+3938.

In the above derivation of the stellar mass, we have neglected the dependence of the asymptotic period spacing with the thickness of the H envelope. As is well known, for a given stellar mass the asymptotic period spacing is larger for models with thinner H envelopes (Tassoul et al. 1990). This is shown in Fig. 3 where we depict the run of $\Delta \Pi_2$ for the sequences with $M_* = 0.837 \ M_\odot$ and $M_* = 0.877 \ M_\odot$ and several values of the H envelope thickness for each sequence. Note that the curves for different H envelope thicknesses corresponding to the two different values of the stellar mass overlap, somewhat that can complicate the analysis. For instance, regarding the value of $\Delta \Pi_2$, and for a fixed $T_{\text{eff}}$, a DA white dwarf model with $M_* = 0.837 \ M_\odot$ and log($M_H/M_\odot$) = −7.36 can readily mimic a model with

---

* $\Delta \Pi_2$ increases when the convective efficiency is increased; see Fig. 36 of Tassoul et al. (1990).
$M_\ast = 0.877M_\odot$ and $\log(M_H/M_\ast) = -8.37$. Obviously, much more ambiguities can still be found if we consider the curves of $\Delta \Pi^\ell_i$ corresponding to the remainder sequences with lower stellar masses, not shown in the plot in the interest of clarity. Now, going to the case of WD J1916+3938, we could naively conclude that that this star should have a stellar mass $M_\ast \sim 0.877M_\odot$ and a rather thick H envelope of $\log(M_H/M_\ast) \sim -5.40$. But we also could easily accommodate a sequence with a stellar mass $M_\ast > 0.877M_\odot$ and a thinner H envelope, if we had available a sequence with these characteristics. Thus, given the intrinsic degeneracy of $\Delta \Pi^\ell_i$ with $M_\ast$ and $M_H/M_\ast$, all we can do is just to place a lower limit for the stellar mass of WD J1916+3938. That is, for a $\Delta \Pi_{\text{obs}} = 38.54$ and a $T_{\text{eff}} = 11 129$ K, the stellar mass of the star must be $M_\ast \gtrsim 0.837M_\odot$ and we cannot say anything about the thickness of the H envelope. Again, if we relax the constraint imposed by the spectroscopic value of $T_{\text{eff}}$, and we look for the allowed values of $M_\ast$ for the star to be residing within the ZZ Ceti instability strip, we again obtain the lower limit $M_\ast \gtrsim 0.77M_\odot$, which is completely independent of the estimation of the effective temperature of WD J1916+3938.

We close this Section by noting that, if the convective efficiency characterising our set of DA white dwarf models were larger than assumed, then the asymptotic period spacing should be larger for each sequence. As a result, the lower limit for the stellar mass of WD J1916+3938 derived from the period spacing should be larger, thus pointing again to the conclusion that this DA white dwarf must be rather massive.

5 INFORMATION FROM THE INDIVIDUAL PULSATION PERIODS

Besides using the information implicit in the period spacing, we also can place constraints on the stellar mass, effective temperature, and details of the internal structure of WD J1916+3938 through its individual pulsation periods. In this approach we seek a pulsation DA white dwarf model that best matches the pulsation periods of WD J1916+3938. We assume that all of the observed periods correspond to normal modes, that is, they are associated with physical displacements and carry out information of the internal structure of the star. The goodness of the match between the theoretical pulsation periods ($\Pi^\ell_i$) and the observed individual periods ($\Pi_{\text{obs}}^\ell$) is measured by means of a quality function defined as:

$$\Phi = \Phi(M_\ast, M_H, T_{\text{eff}}) = \frac{1}{N} \sum_{i=1}^{N} |\Pi^\ell_{\text{th}} - \Pi^\ell_{\text{obs}}|$$  \hspace{1cm} (3)

where $N (= 7)$ is the number of observed periods. The DA white dwarf model that shows the lowest value of $\chi^2$ is adopted as the “best-fit model”. We evaluate the function $\Phi = \Phi(M_\ast, M_H, T_{\text{eff}})$ for stellar masses in the range $0.525 \gtrsim M_\ast/M_\odot \gtrsim 0.877$. For the effective temperature we adopt the range $14 000 \gtrsim T_{\text{eff}} \gtrsim 9 000$ K and employ a more much finer grid ($\Delta T_{\text{eff}} = 10 - 30$ K). Finally, we vary the thickness of the H envelope in the range $-9.4 \leq \log(M_H/M_\ast) \leq -3.6$, respectively, being the ranges of the values of $M_H$ dependent on $M_\ast$ (see Table 1 of Romero et al. 2011). In our analysis, we employ a set of about 15 500 evolutionary DA white dwarf models. Each of our models have about 2000 mesh points, which guarantee us that eigenfunctions of $g$-modes with radial orders reasonably high (for $\ell = 1, 2$) can be spatially resolved.

We first assumed that the harmonic degree of the seven observed periods of WD J1916+3938 is $\ell = 1$ from the outset. We found a best-fit solution for a model with $M_\ast = 0.837M_\odot$, $T_{\text{eff}} = 11 391$ K and $\log(M_H/M_\ast) = -7.36$. In Fig. 4 we show the quality function $\Phi$ in terms the effective temperature for the case of $M_\ast = 0.837M_\odot$ and for all the values of the thickness of the H envelope considered for this stellar mass. The best-fit solution corresponds to the minimum of $\Phi$. Our asteroseismological solution is a bit ($\approx 2.4\%$) hotter than WD J1916+3938. The stellar mass of the best-fit model is in good agreement with the spectroscopic estimate ($M_\ast \approx 0.8M_\odot$) and with the derivation from the period spacing ($M_\ast \gtrsim 0.837M_\odot$). This is quite encouraging, because the three estimates have been derived from very different approaches and assumptions. In Table 2 we show a comparison between the observed periods and the periods of the best-fit model (model 1). The value of the quality function is $\Phi = 3.18$ s. Note that, not surprisingly, the periods at 823.9 s and 834.1 s are associated to a single theoretical period at 825.25 s. This because these two periods are very close, and probably correspond to different $\ell$ values.

Next, we tried to find a best-fit model by discarding the period at 834.1 s (so, $\ell = 6$), to be consistent with the assumption made to derive a mean period spacing of the star in Sect. 4. We found the same best-fit model than before (see Table 2 model 2), but the quality of the fit is, as expected, substantially improved ($\Phi = 2.24$ s).

In a further step, we relaxed the constraint of $\ell = 1$ for all the observed periods of WD J1916+3938, and we redone our period fit by allowing the values $\ell = 1$ or $\ell = 6$.
Table 2. Possible asteroseismological solutions for WD J1916+3938. Model 1 is the best-fit model when we assume that the seven periods exhibited by the star are $\ell = 1$; model 2 corresponds to the case in which we discard the period at 834.1 and again assume that all the six remainder periods are $\ell = 1$; model 3 is the best-fit model when we consider the seven periods and allow the values $\ell = 1$ and $\ell = 2$ for them; and finally, model 4 correspond to the case in which we consider a period at 829 s instead of the periods at 823.9 s and 834.1 s, and again suppose that all the periods are $\ell = 1$.

| Model | $T_{\text{eff}}$ [K] | $M_*/M_\odot$ | log($M_{\text{HI}}/M_*$) | log($M_{\text{H}}/M_*$) | $\Pi^{\text{obs}}$ [s] | $\Pi^{\ell=1}$ [s] | $\ell$ | $k$ | $|\Delta|$ [s] | $\Phi$ |
|-------|----------------|-------------|-----------------|-----------------|----------------|----------------|-----|-----|----------------|------|
| 1     | 11391          | 0.837       | $-2.50$         | $-7.36$         | 823.9          | 825.25         | 1   | 18  | -1.35          | 3.18 |
|       |                |             |                 |                 | 834.1          | 825.25         | 1   | 18  | 8.85           |      |
|       |                |             |                 |                 | 934.5          | 930.08         | 1   | 20  | 4.42           |      |
|       |                |             |                 |                 | 968.9          | 971.58         | 1   | 21  | -2.68          |      |
|       |                |             |                 |                 | 1089.0         | 1089.11        | 1   | 24  | -0.11          |      |
|       |                |             |                 |                 | 1169.9         | 1169.59        | 1   | 26  | 0.31           |      |
|       |                |             |                 |                 | 1436.7         | 1432.14        | 1   | 32  | 4.56           |      |
| 2     | 11391          | 0.837       | $-2.50$         | $-7.36$         | 823.9          | 825.25         | 1   | 18  | -1.35          | 2.24 |
|       |                |             |                 |                 | 834.1          | 832.76         | 2   | 32  | 1.34           |      |
|       |                |             |                 |                 | 934.5          | 932.69         | 2   | 36  | 1.81           |      |
|       |                |             |                 |                 | 968.9          | 971.58         | 1   | 21  | -2.68          |      |
|       |                |             |                 |                 | 1089.0         | 1089.11        | 1   | 24  | -0.11          |      |
|       |                |             |                 |                 | 1169.9         | 1169.59        | 1   | 26  | 0.31           |      |
|       |                |             |                 |                 | 1436.7         | 1437.09        | 2   | 56  | -0.39          |      |
| 3     | 11391          | 0.837       | $-2.50$         | $-7.36$         | 823.9          | 825.25         | 1   | 18  | -1.35          | 1.14 |
|       |                |             |                 |                 | 834.1          | 832.76         | 2   | 32  | 1.34           |      |
|       |                |             |                 |                 | 934.5          | 932.69         | 2   | 36  | 1.81           |      |
|       |                |             |                 |                 | 968.9          | 971.58         | 1   | 21  | -2.68          |      |
|       |                |             |                 |                 | 1089.0         | 1089.11        | 1   | 24  | -0.11          |      |
|       |                |             |                 |                 | 1169.9         | 1169.59        | 1   | 26  | 0.31           |      |
|       |                |             |                 |                 | 1436.7         | 1433.51        | 1   | 32  | 3.19           |      |
| 4     | 11376          | 0.837       | $-2.50$         | $-7.36$         | 829.0          | 825.88         | 1   | 18  | 3.12           | 2.56 |
|       |                |             |                 |                 | 834.1          | 832.76         | 2   | 32  | 3.63           |      |
|       |                |             |                 |                 | 934.5          | 932.69         | 2   | 36  | 3.96           |      |
|       |                |             |                 |                 | 968.9          | 972.42         | 2   | 21  | -3.52          |      |
|       |                |             |                 |                 | 1089.0         | 1090.11        | 1   | 24  | -1.11          |      |
|       |                |             |                 |                 | 1169.9         | 1170.68        | 1   | 26  | -0.78          |      |
|       |                |             |                 |                 | 1436.7         | 1433.51        | 1   | 32  | 3.19           |      |

We obtain exactly the same best-fit model as before. In Table 2 we show the theoretical periods and the $k$- and $\ell$-identifications (model 3). For this period fit, we obtain $\Phi = 1.14$. This is the best agreement with the observed periods we obtain, although in this case the $\ell$-identification is not the same as that assumed in Sect. 4 to derive the period spacing value of the star.

Finally, we have carried out an additional period-fit, this time assuming that the periods at 823.9 s and 834.1 are the $m = +1$ and $m = -1$ components of a rotational triplet, in which the central component ($m = 0$) is missing (see next Section). Under this assumption, we estimate the period of the $m = 0$ component as the average of the $m = +1$ and $m = -1$ components. Thus, we consider a period of 829 s in our period-fit procedure, instead the two periods at 823.9 s and 834.1. Here, we adopt $\ell = 1$ for the set of $N = 6$ periods at the outset. In this case, we obtain virtually the same asteroseismological model than before, with $M_* = 0.837 M_\odot$, $T_{\text{eff}} = 11376$ K, log($M_{\text{HI}}/M_*$) = $-7.36$, and $\Phi = 2.557$ s. The theoretical periods and $\ell, k$ values are shown in Table 2 (model 4).

We conclude that, even by adopting several different assumptions for the assignation of the $\ell$ and $k$ values to the observed periods, we found the same asteroseismological model which is characterised by an effective temperature of $T_{\text{eff}} = 11380$ K, a surface gravity of $\log g = 8.39$, a stellar mass of $M_* = 0.837 M_\odot$, a mass of H of log($M_{\text{HI}}/M_*$) = $-7.36$, and a He content of log($M_{\text{H}}/M_*$) = $-2.50$. The location of the best-fit model in the $T_{\text{eff}} - \log g$ plane is depicted in Fig. 1.

### 6 STELLAR ROTATION

Here, we suppose that the periods at 823.9 s and 834.1 are the $m = +1$ and $m = -1$ components, respectively, of a rotational triplet in which the central component ($m = 0$) is absent from the pulsation spectrum of WD J1916+3938. If this assumption is valid, we can infer a frequency spacing of $\Delta \nu = \nu(m = +1) - \nu(m = -1) = 14.8$ $\mu$Hz between the extreme components of the triplet. By assuming rigid and slow rotation, we have $\Delta \nu = \nu/2 = 7.4$ $\mu$Hz and $C_{k,\ell} \sim 0.5$ in the asymptotic limit for $\ell = 1$. Then, we can infer a mean rotation period of $P_{\text{rot}} = 1/\nu = 18.77$ hs for WD J1916+3938. This is in line with the values derived for other ZZ Ceti stars from asteroseismology (see, for instance, Table 4 of Fontaine & Brassard 2008). The fact that a DA white dwarf like this, characterised by an effective temperature of $T_{\text{eff}} \sim 11400 - 11100$ K, and pulsating in high order $g$-modes with long periods, which usually have associated rotational kernels that are mostly confined to the outer layers of the star, prevent us...
to attempt a more profound analysis of the internal rotation of WD J1916+3938, like the one performed very recently by Córocado et al. (2011) for the GW Vir star PG 0122+200.

7 CONCLUSIONS

Tempted by the exciting discovery of the first pulsating DA white dwarf in the Kepler mission field by HEA11, and encouraged by the availability of the most complete set of detailed evolutionary and pulsation models for DAV stars up to date (Romero et al. 2011), in this paper we have presented a first asteroseismological analysis of WD J1916+3938. This, even with the certainty that the full asteroseismological potential of this DAV star will be exploited with further observations by Kepler in the next years.

We summarize our main findings below:

- We derive a spectroscopic mass of $M_\star = 0.797 - 0.805 M_\odot$ for WD J1916+3938 on the basis of the new DA white dwarf evolutionary tracks of Romero et al. (2011). This value is smaller than the estimate of HEA11, of $M_\star = 0.82 M_\odot$, based on the older models of Wood (1990). In our analysis, we have taken into account the dependence of the tracks in the $T_{\text{eff}} - \log g$ diagram with the thickness of the H envelope (see Fig. 1).

- By assuming that all the periods exhibited by WD J1916+3938 are normal modes of the star, and supposing that all the periods except one (at 834.1 s) are associated to $\ell = 1$ modes, we derive an average period spacing of $\Delta \Pi_{\text{obs}} = 38.54 \pm 0.29$ s. On the basis of the asymptotic theory of stellar pulsations, we estimate a stellar mass of $M_\star = 0.851 \pm 0.031 M_\odot$, corresponding to a thick H envelope. If we take into account the strong dependence of the asymptotic period spacing with the thickness of the H envelope, we found an evident degeneracy of $\Delta \Pi_\ell$ with $M_\star$ and $M_H$. In virtue of this, we can conclude that $M_\star \gtrsim 0.837 M_\odot$, but we cannot say anything about the thickness of the H envelope. Finally, if we ignore the constraint imposed by the effective temperature as derived by spectroscopy, we found that WD J1916+3938 should have a stellar mass $M_\star \gtrsim 0.77 M_\odot$ in order for the star to be within the ZZ Ceti instability strip.

- By using the individual pulsation periods exhibited by WD J1916+3938, and taking into account several assumptions for their $\ell$- and $k$-identification, we found an asteroseismological model with $T_{\text{eff}} = 11380$ K, $\log g = 8.39$, $M_\star = 0.837 M_\odot$, $\log(M_H/M_\star) = -7.36$, and $\log(M_H/M_\star) = -2.50$. The stellar mass of the asteroseismological model is in good agreement with the spectroscopic estimate ($M_\star \approx 0.8 M_\odot$) and with the derivation from the period spacing ($M_\star \gtrsim 0.837 M_\odot$). This is quite encouraging, because the three estimates have been derived from very different approaches and assumptions. On the other hand, the effective temperature of the asteroseismological model is somewhat higher than the spectroscopic one ($T_{\text{eff}} = 11129 \pm 115$ K).

- In passing, we have estimated a rotation period of 18.77 hr for WD J1916+3938. The reliability of this result rests on the assumption that the modes with periods at 823.9 s and 834.1 are the $m = +1$ and $m = -1$ components, respectively, of a $\ell = 1$ frequency triplet due to rotation.

In closing, we think that the conclusion that WD J1916+3938 should be a rather massive ZZ Ceti star, with $M_\star \gtrsim 0.8 M_\odot$, appears to be robust. The possibility that this rather massive pulsating white dwarf can be further monitored with Kepler with a high degree of detail turns this star into a promising and unique target to study the physics of crystallization and carbon/oxygen phase diagrams at high densities. In this regard, we note that, for a stellar mass of $M_\star \sim 0.88 M_\odot$, crystallization of matter should occur at the centre of a white dwarf star at $T_{\text{eff}} \approx 10500$ K according to the phase diagram of Segretain & Chabrier (1993), and at $T_{\text{eff}} \approx 9600$ K if the new phase diagram recently proposed by Horowitz et al. (2010) is considered (Althaus et al. 2011). Indeed, based on direct molecular dynamics simulations for dense carbon-oxygen mixtures, Horowitz et al. (2010) find substantially lower crystallization temperatures and that the shape of the carbon-oxygen phase diagram is of the azeotropic form — and not of the spindle type — as previously believed. Our analysis from the period spacing of WD J1916+3938 suggests that this star could well have a stellar mass much higher than $0.8 M_\odot$ and necessarily a rather thin H envelope. If this were the case, this star could have a substantial fraction of its interior crystallized depending on the adopted carbon-oxygen phase diagram, an issue that could be explored with future asteroseismological analysis that exploit the new high-quality photometric data coming from the ongoing monitoring by the Kepler mission.

ACKNOWLEDGMENTS

Part of this work was supported by AGENCIA through the Programa de Modernizaci´on Tecnol´ogica BID 1728/OC-AR, and by the PIP 112-200801-00940 grant from CONICET. This research has made use of NASA’s Astrophysics Data System.

REFERENCES

Althaus, L. G., García-Berro, E., Isern, J., Córocado, A. H., & Miller Bertolami, M. M. 2011, A&A, in press [arXiv:1110.5665]
Althaus, L. G., Córocado, A. H., Isern, J., & García-Berro, E. 2010a, A&AR, 18, 471
Althaus, L. G., Córocado, A. H., Bischoff-Kim, A., Romero, A. D., Renedo, I., García-Berro, E., & Miller Bertolami, M. M. 2010b, ApJ, 717, 897
Brickhill, A. J. 1991, MNRAS, 251, 673
Castanheira, B. G., Kepler, S. O., Kleinman, S. J., Nitta, A., & Fraga, L. 2010, MNRAS, 405, 2561
Córocado, A. H., Althaus, L. G., Kawaler, S. D., Miller Bertolami, M. M., García-Berro, E., Kepler, S. O. 2011, MNRAS, 418, 2519
Córocado, A. H., Althaus, L. G., Miller Bertolami, M. M., & García-Berro, E. 2009, A&A, 499, 257
Córocado, A. H., Althaus, L. G., Kepler, S. O., Costa, J. E. S., & Miller Bertolami, M. M. 2008, A&A, 478, 869
Córocado, A. H., Althaus, L. G., Miller Bertolami, M. M., & Werner, K. 2007a, A&A, 461, 1095
Córocado, A. H., Miller Bertolami, M. M., Althaus, L. G., Vauclair, G., & Werner, K. 2007b, A&A, 475, 619
Dolez, N., & Vauclair, G. 1981, A&A, 102, 375
Fontaine, G., & Brassard, P. 2008, PASP, 120, 1043
Goldreich, P., Wu, Y. 1999, ApJ, 511, 904
Hermes, J. J., Mullally, F., Østensen, R. H., et al. 2011, ApJ, 741, L16 (HEA11)
Horowitz, C. J., Schneider, A. S., & Berry, D. K. 2010, Physical Review Letters, 104, 231101
Nather, R. E., Winget, D. E., Clemens, J. C., Hansen, C. J., & Hine, B. P. 1990, ApJ, 361, 309
Romero, A. D., Córsico, A. H., Althaus, L. G., Kepler, S. O., Castanheira, B. G., Miller Bertolami, M. M. 2011, MNRAS, in press (arXiv:1109.6682)
Salaris, M., Serenelli, A., Weiss, A., & Miller Bertolami, M. 2009, ApJ, 692, 1013
Segretain, L., & Chabrier, G. 1993, A&A, 271, L13
Tassoul, M., Fontaine, G., Winget, D. E. 1990, ApJS, 72, 335
Winget, D. E., & Kepler, S. O. 2008, ARAA, 46, 157
Winget, D. E., van Horn, H. M., Tassoul, M., Fontaine, G., Hansen, C. J., Carroll, B. W. 1982, ApJ, 252, L65
Wood, M. A. 1990, Ph.D. Thesis, Univ. Texas, Austin