Radiation pressure in finite Fabry-Pérot cavities

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Abstract

In optomechanical systems radiation pressure couples electromagnetic radiation and a mechanical mode. This leads to a displacement of the mechanical system which in turn yields a well-known phase-sensitive response of the intracavity field. We analyze this phase sensitivity in a standard setup of a driven cavity with one moveable mirror. For high reflectivities of the mirrors, a very sharp resonance in the the effective intensity impinging on the moveable mirror evolves, implying an extreme sensitivity to the phase covered by the field in a round trip. This phase-shift is directly linked to the actual displacement of the mirror. By lowering the reflectivities, this high sensitivity is traded with a larger range of phases (displacements) over which the change of the intensity is detectable. While the phase shift itself is hard to measure, the intracavity response also appears prominently in the output intensity in front of the fixed mirror. More importantly, while the trade-off between sensitivity and phase-range persists, the appearing intensity changes are significantly less effected by the actual reflectivities of the mirrors.

We also analyze all the above effects for a finite cavity height with a limited number of times the light can hit the cavity walls. Similar to lowering the reflectivities we find finite size leads to a lowering of the amplitude and broadening of the phase range covered by the main resonance. Also in the output field, this limitation persists. Yet for some scenarios, the phase range may even be broader for a limited cavity as for the infinite case and the same reflectivities. Due to the one-to-one correspondence between phase-shift and mirror displacement, the latter can in some ranges be extremely well measured from the output field. To motivate this idea, we calculate the effect of thermal mechanical noise. For realistic microscopic mirrors, there exist easy to reach regimes with negligible thermal noise. Thus, using the output field to determine the movement of this mirror may be an alternative method to analyze that movement based on external sources.

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I. INTRODUCTION

Optomechanics (OM) studies the interaction of an electromagnetic radiation field with a mechanical oscillator. The dynamics of each subsystem, well-described within the classical and quantum frameworks, become non-linearly coupled, revealing a plethora of new fundamental effects as well as applications [1-3]. OM systems range from macroscopic mirrors used to detect gravitational waves [4, 5] to microscopic cantilevers [6-8] and membranes [9] used in the search for macroscopic quantum state engineering. Applying a well-defined driving field, the state of the mechanical oscillator can be prepared, and then coherently controlled [10-12]. This allows, for example, to either cool the mechanical motion or amplify the forces acting on it solely by manipulating the electromagnetic field [13]. This is the basis for all kinds of high-precision measurements or quantum information processes.

Radiation pressure is the origin behind optomechanical coupling [14, 15]. The first experimental proof of such an effect dates back to 1967 [16], while the interaction between visible light and a macroscopic mechanical oscillator was demonstrated in 1983 [17]. In short, the momentum of the cavity light field is transferred to the mechanical system. Thus, in a resonator with one movable mirror, the latter is pushed outward by the radiation pressure of the cavity field. In turn, as the distance between the mirrors increases, the resonance frequency of the cavity field decreases. Law provided the first consistent second quantization of the model in 1995 [18], showing that the cavity frequency change leads, in general, to a complex coupling between creation and annihilation operators of the optical and mechanical subsystems. This coupling is usually restricted to the linear regime in quantum OM due to the limited amplitude of the mechanical oscillation [19]. However, recently technical developments has put the nonlinear range of interaction within reach and jump-started research into corrections to the standard OM description [20].

Here, we focus on these nonlinear effects, including the finite size of the resonator, from a classical point of view and following early studies where the optical response function defines the radiation pressure in the cavity [17, 21, 22]. In the following, we will discuss a driven high-finesse Fabry-Perot (FP) resonator made from two plane, highly-reflecting mirrors [23]. In this setup radiation pressure leads to a well-known high sensitivity of the intracavity field with respect to the phase accumulated within one circulation time. We call this effect FP radiation pressure (FPRP). In general there is a trade-off between that sensitivity and the broadness of the FPRP resonance when decreasing the reflectivities of the mirrors. As a measurable quantity we demonstrate that this
phase-sensitive response due to FPRP also appears in the output field in front of the fixed mirror. In this case, however, the scaling of the intensity is less affected by the reflectivities, allowing a very stable measurement. Then, we will explore the effect of finite size on the FPRP. The response for a limited number of impinges is scaled with a fast oscillating factor. Like for lowering the reflectivities, this causes a broadening and lowering of the FPRP resonance around its maximum value. Nevertheless, the phase sensitivity and the easier-to-measure output field behaviour can be preserved. As there is a direct relation between phase and mirror displacement, we analyze if a determination of that displacement is possible with the inclusion of thermal mechanical noise. Finally, we provide some conclusions and an outlook.

II. INFINITE FABRY-PÉROT CAVITY RADIATION PRESSURE

![Schematic of a cavity with one moveable (left) and one fixed (right) mirror. Laser driving light impinges on the cavity at a given angle, akin to a FP resonator, creating multiple but finite reflections on the mirrors.]

Let us first recap the main, well-known results for the radiation-pressure-induced response of the optomechanical system depicted in Fig. 1 see e.g. [20]. A cavity made of two highly-reflective mirrors, one allowed to undergo harmonic motion and the other fixed, separated by a distance $L$ can be excited to a certain resonance, $\omega_0 = n \pi c \cos \theta / L$ with $n \geq 1$ the number of the excited harmonic and $\theta$ the angle between the normal of the cavity walls and the direction of the incident driving monochromatic laser of frequency $\omega_L$ [24]. Radiation pressure arises from momentum transfer of the light field to the moving mirror and is proportional to the Poynting vector $|S| \propto |E|^2$, with $E$ the total electric field impinging on the moveable mirror. We assume
the transverse area of the laser constant as the laser travels through the cavity. The total field amplitude $E$ can be given in terms of the response function $J(\varphi) = |E/E_L|^2$ that relates it with respect to the driving laser intensity $E_L$. For an infinitely long cavity, we can calculate the transfer function,

$$J(\varphi) = J_{\text{max}} A(\varphi),$$

(1)

in terms of the ideal maximum response,

$$J_{\text{max}} = \frac{1 - R_f}{(1 - \sqrt{R_f R_m})^2},$$

(2)

obtained at the half-round trip phase $\varphi = n\pi$, the Airy function, and the coefficient of finesse $[23]$,

$$A(\varphi) = \frac{1}{1 + F \sin^2(\varphi)}, \quad F = \frac{4\sqrt{R_f R_m}}{(1 - \sqrt{R_f R_m})^2},$$

(3)

in that order. Here the reflectivities of the moveable and fixed mirrors are given by the parameters $R_m$ and $R_f$, respectively. The half-round trip phase gained by the field inside the cavity,

$$\varphi = \frac{\omega L}{c} \cdot \frac{L + x}{\cos \theta} = n\pi \frac{\omega L}{\omega_0} \left(1 + \frac{x}{L}\right) = \varphi_0 \left(1 + \frac{x}{L}\right),$$

(4)

is proportional to the fixed resonator half-round trip phase $\varphi_0$ and the position of the moveable mirror $x$.

We find it useful to consider two particular cases of mirror configurations: (I) one where the mirrors are identical, $R_f = R_m = R$, and (II) another where the moveable mirror is ideal, $R_f = R < R_m = 1$. In the first case, we find that the response function changes from $(1 - R)^{-1} \gg 1$ to approximately $(1 - R)/4$. In the second, it changes from $(1 + \sqrt{R})/(1 - \sqrt{R})$ to $(1 - \sqrt{R})/(1 + \sqrt{R})$. For a high reflectivity of $R = 0.999$, this means a change of $4 \times 10^6$ for case (I) and $1.6 \times 10^7$ for case (II). Such an enormous change in the response function leads to the question of phase sensitivity. The phase that provides us with $J(\varphi) = 1$ will be denoted $\varphi_{eq}$ and is given by

$$\sin(\varphi_{eq}) = \sqrt{\frac{1}{2} \left[1 - \sqrt{R_f \left(\frac{\sqrt{R_m} + \sqrt{R_m}^{-1}}{2}\right)}\right]} .$$

(5)

At this point the response of the resonator becomes as small as if the laser light would directly impinge just once on the moveable mirror. For our two cases, this phase reduces to

$$\sin(\varphi_{eq,\text{I}}) = \sqrt{\frac{1 - R}{4}}, \quad \sin(\varphi_{eq,\text{II}}) = \sqrt{\frac{1 - \sqrt{R}}{2}},$$

(6)
and we find $\varphi_{\text{eq}, I} \approx \varphi_{\text{eq}, II} \approx 0.906^\circ$ for a reflectivity value of $R = 0.999$. We see that even the tiniest deviation from resonance drastically diminishes the effect of FPRP. For both cases, if the following condition holds,

$$R_t > \frac{4R_m}{(1 + R_m)^2} \geq R_m,$$

there is no real solution for $\varphi_{\text{eq}}$, as $J(\varphi) < 1$ for all values of $\varphi$. It is thus paramount to have the reflectivity of the moveable mirror at least as large as, if not significantly larger than, the reflectivity of the fixed mirror.

Here, we can take a stop and make two assumptions: (i) the driving laser is dominant, thus the field quadratures follow only this laser oscillation in the long-time limit, and (ii) the position of the moveable mirror can be approximated by a constant in the long-time limit, $\lim_{t \to \infty} x \approx x_s$, due to the absence of direct driving and the presence of damping. This steady state can either be numerically evaluated from the balance of forces, or measured indirectly. These assumptions yield for the long-time limit of the half-round trip phase

$$\varphi_s = \lim_{t \to \infty} \varphi = \varphi_0 \left(1 + \frac{x_s}{L}\right).$$

(8)

The driving laser and cavity field pressure contributions provide only positive moveable mirror displacements, $x_s > 0$, for all cases. Thus, we are required to set the laser below the bare resonance, $\omega_L < \omega_0$, in order to reach the maximum of the response function, $J_{\text{max}}$, in the long-time limit. For three different reflectivities in case (II) the response functions are depicted in Fig. 2(a). It is clear that there is a tradeoff between a large intensity jump in the response function, and its phase broadness around $\varphi = n\pi$. That means that for high reflectivities, small amounts of shift of the moveable mirror may lead to big variations in the response function. Let us reconsider Eq. (6) for case (II) and express the phase shift $\delta \varphi$ from FPRP resonance in terms of the cavity wavelength $\lambda$ corresponding to $\omega_0$. We find

$$\delta \varphi = \varphi - \varphi_0 = \frac{n\pi}{\omega_0} \frac{\omega_L}{L} x_s = \frac{2\pi \omega_L}{\omega_0 \cos \theta} \frac{x_s}{\lambda}.$$  

(9)

For the laser being on bare cavity resonance and almost in normal incidence ($\theta \approx 0$), the mirror shift compared to the wavelength is just $\delta \varphi / (2\pi)$. For a high reflectivity $R = 0.999$ a phase-shift to $\varphi_{\text{eq}}$ thus corresponds to a movement of $x_s \approx 0.25\% \lambda$. For a He-Ne-laser driven cavity this equals 1.6 nm. Within a movement of the mirror of 1.6 nm, the response function changes by a factor of 4000. That is, if we can experimentally determine this quantity we would be extremely
sensitive in a very small region. On the other hand, for \( R = 0.9 \) and case (II), the response changes only by a factor of roughly 38, but this change occurs over a range of \( x_s \approx 2.56\% \lambda \) or 16 nm.

The actual displacement of the moveable mirror \( x_s \) is generally hard to measure. Thus, we analyze the output intensity \( I_o \propto |E_o|^2 \), transmitted back through the fixed mirror, instead. Making a similar calculation as for the field hitting the moveable mirror we obtain

\[
\frac{I_o}{I_L} = \left| \frac{E_o}{E_L} \right|^2 = R_f + [R_m(1 - 3R_f) + 2\sqrt{R_f R_m} \cos(2\varphi)]J(\varphi). \tag{10}
\]

The first term on the right-hand side of Eq. (10), \( R_f \), stems from the initial reflection when entering the cavity. The left-hand side would just be this \( R_f \) without a second mirror, \( R_m \to 0 \), implying that a deviation from that value is fully based on the constructive interference displayed by the response function. Both cases (I) and (II) for different reflectivities are depicted in Fig. 2(b).

The maximum value, given for FPRP resonance in both cases, can be shown to be

\[
\frac{I_o}{I_L} = 4R \lesssim 4. \tag{11}
\]

for case (I) and

\[
\frac{I_o}{I_L} = (1 + 2\sqrt{R})^2 \lesssim 9 \tag{12}
\]

for case (II). In contrast to the response function \( J(\varphi) \) this maximum is little effected by variations of \( R \lesssim 1 \), making this maximum resistant to the actual mirror parameters. For large reflectivities the phase dependence of the output field mimics the behaviour of the reflected field, just for a smaller parameter region. Thus, we only see a sharp peak around \( \delta \varphi = 0 \), whereas for all other phases the intensity is almost constant. If the reflectivities go down on the other hand, we again see this tradeoff between lower intensity change and broader range of phase deviations for which this change occurs. That means higher reflectivities should allow an extremely sensitive measurement of the mirror movement, but only for a very small range of movements. On the other hand lower reflectivities are less sensitive, but on a broader range.

Quantitatively we can use the results for the response function again, because, one can easily show that

\[
\frac{I_o(\varphi_{eq})}{I_L} = R(3 - 2R) \approx 1 \tag{13}
\]

for case (I) and

\[
\frac{I_o(\varphi_{eq})}{I_L} = 1 \tag{14}
\]
FIG. 2. (Color online) (a) ideal response function, $J(\varphi_s)$, in terms of the fixed half-round trip phase, $\varphi_0$, for $n = 1$ in case II with $\varphi_s = \varphi_0(1 + x_s/L)$ and $x_s/L = 0.1$. The different colors represent $R = 0.999$ (solid black), 0.99 (dashed blue), 0.9 (wider dashed red). (b): Output intensity $I_o/I_L$ as a function of the deviation $\delta\varphi$ from the FPRP resonance $n\pi$ for case (I) (solid lines) and (II) (dashed lines) with $R = 0.999$ (black), $R = 0.99$ (red), $R = 0.9$ (blue).

for case (II). In other words, in case (I) and for large reflectivity $R = 0.999$, the output intensity changes from $4I_L$ to $I_L$ when the moveable mirror position $x_s$ changes by 0.25 % of the cavity wavelength.

III. FINITE SIZE EFFECTS

The finite length of planar mirror walls implies that light at non-normal incidence, $\theta \neq 0$, will only stay a limited amount of time inside the cavity, independent of the reflectivities. In this case, the geometric series at the heart of the response function becomes limited, yielding just a scaled version of the ideal response function,

$$J_N(\varphi) = (1 - \sqrt{R_f R_m})^2 + 4 \sqrt{R_f R_m} \sin^2(N\varphi),$$

where $N$ is the number of hits on the back mirror. This number can be calculated from the system geometry or recovered from the maximum of the finite response function at $\varphi = n\pi$ for non-ideal reflectivities $R_{f,m} < 1$,

$$N = \frac{\ln \left[ 1 - \sqrt{J_N(n\pi)/J(n\pi)} \right]}{\ln \sqrt{R_f R_m}} \approx \sqrt{\frac{J_N(n\pi)}{1 - R_f}}.$$

In this approximation, we used the restrictions $1 - \sqrt{R_f R_m} \ll 1$ and $\sqrt{J_N(n\pi)/J(n\pi)} \ll 1$. For example, for $R_f = 0.999$, we need 32 bounces to obtain $J_N(n\pi) = 1$, which yields a mirror size.
of at least $63L \tan \theta$ for the fundamental mode $n = 1$. This means mirrors must be at least 348 nm in size for He-Ne laser light, $\lambda = 633$ nm, at 1.00° incident angle without considering the obvious diffraction issues.

![Diagram](image)

**FIG. 3.** (Color online) (a): finite response function, $J_N(\varphi_s)$, for $N = 2$ (solid black), 32 (dashed blue), and 64 (dotted red) for case (II) and $R = 0.999$. (b): Output field as in Fig. 2 for $R = 0.9$ with the case $N = \infty$ depicted by the thin lines for comparison. For the thick lines $N = 8$.

The maxima of $J_N(\varphi)$ are broader and have lower values than the ideal case of $J(\varphi)$, and slowly approach the ideal response with increasing $N$. Figure 3 shows the finite response function for the cases of non-normal incidence allowing for $N = 2$ bounces in solid black, where the FPRP on resonance is still smaller than if the laser light would impinge directly on the moveable mirror just once. The case delivering a maximum unit finite response of one, $N = 32$ bounces on the moveable mirror, is shown in dashed blue and the response for double that number, $N = 64$, is shown in dotted red lines yielding a maximum response slightly below to a value of four. The influence of the FPRP is quite fragile towards the geometry of the resonator, in particular mirror size related to incident angle and the ratio between the driving laser and the long-time cavity frequencies.

A similar but lengthy formula as Eq. (10) can be derived for the output field of a finite mirror and $J_N(\varphi)$. The dependence of the output field in this case is shown in Fig. 3(b). In this case we consider the low reflectivity of $R = 0.9$ and only $N = 8$ hits on the back mirror. The maximum amplitude at $\delta \varphi = 0$ does not yet approach its ideal value, thus limiting the sensitivity in that range. However, in a medium range of phases, the intensity is even higher for low $N$ than for the infinite case. That means, while a precise determination of the phase and thus the mirror displacement is hard to practically impossible, over a broader range of phase a rough estimation,
between resonance and far off-resonance is possible.

We can conclude a few things. First, the relation between input and output field at the fixed mirror is connected to the same response function $J(\varphi)$ as the reflection field at the moveable mirror. Second, the output field is limited to values below 9 and this maximum is less sensitive to the actual reflectivities than the response function. Third, for high reflectivities a very small range of phase changes, i.e. movements of the mirror, can be sensitively detected by comparing the output intensity with the laser intensity. In contrast, for lower reflectivity, a broader range of phases yields a less varying output intensity. There is a general tradeoff between the range of mirror movements that yield a variation of the output intensity and the amplitude of this variation, which we called the sensitivity. If the variation is detectable, it allows a one-to-one relation between the movement of the mirror and $I_0$. Fourth, when only a finite amount of mirror reflections is considered, the tradeoff above appears to favor a broader range while the sensitivity decreases. Additionally, for case (II), we find oscillations appearing at larger $N$, leading to unwanted ambiguities in the determination of $x_s$.

IV. OUTLOOK OF APPLICATIONS

In order to better motivate the idea of using the output field of the cavity as a measure of the mirror position, let us consider the mechanical thermal noise of the moveable mirror. Following the standard description of a classical mechanical damped harmonic oscillator with $m, \Omega, \Gamma_c$ being mass, eigenfrequency and damping constant, respectively, and temperature $T$, one can calculate a thermal noise spectrum, see e.g. [25], of

$$ S_{xx}(\omega) = \frac{4k_B T}{m} \frac{\Gamma_c}{(\omega^2 - \Omega^2)^2 + (\Gamma_c \omega)^2}. $$

The average thermal motion from this spectrum follows as

$$ \Delta x = \sqrt{\frac{k_B T}{m \Omega^2}}. $$

(18)

The range of optomechanical devices and thus parameters for a moveable mirror is vast, so let us consider the statement more generally. Within the output field the phase is changed by the mirror movement as $\varphi = \varphi_0(1 + \Delta x/L)$. The latter value is fixed for a given setup and temperature. In Fig. 4 we depicted the noise for $\Delta x/L = 0.5$ and 0.05.

For the larger noise any measurability of the phase gets easily lost. However, for the lower value the mechanical noise is almost invisible. Roughly speaking, one generally requires $\Delta x/L < 0.1$
for low noise, leading for room temperature to the condition
\[ m\Omega^2L^2 > 2.5 \text{ eV}. \] (19)

This should be easily obtainable with microscopic mirrors \((m \sim 10^{-12} \text{ kg})\) and He-Ne lasers \((L \approx 316 \text{ nm})\) leading to a frequency restriction of \(\Omega \geq 2\pi \cdot 320 \text{ rad/s}\). We thus conclude that it should be possible in metrological setups to measure the displacement of the moveable mirror via the FPRP effect by determining the output field.

Now, let us think about a setup where the fixed mirror is movable via a linear actuator or step motor. These are used to automate tiny movements in larger structures. Modern step motors allow a controlled periodic increment of motion by around 5 nm, not accounting for friction limitations, e.g. \([26]\). Staying with our example of a He-Ne laser, we stated above that for mirror reflectivities around 90%, we can be sensitive in a range of 16 nm mirror displacement. Thus, we might include linear actuators to switch between finder, low-precision/high-range, and measuring, high-precision/low-range, set-ups with the same device. Conversely, including the low mechanical noise, we would also be in the range to actually control the step motor, or be significantly more precise on below microstep level.

V. CONCLUSIONS

We have studied the influence of radiation pressure in a cavity setup with one moveable mirror for both an infinite and a finite cavity size. Radiation pressure pushes the cavity walls apart and creates a well-known phase-sensitive response of the intracavity field. For the infinite cavity and highly-reflecting mirrors the evolving extreme sensitivity in a very narrow region around the
resonance of the maximal response function implies that tiniest movements of the mirror yield strong variations of the field amplitude. For lower reflectivities, this sensitivity decreases, but in exchange the phase range of the resonance increases. As a measurable quantity we calculated the output field in front of the fixed mirror. While displaying the same sensitivity-range trade-off for measuring the phase-shift, the range of output intensities is much smaller and little effected by the actual reflectivities. Hence, similar to the response function, we can have extreme sensitivity in a very small range of mirror displacements (sub-percent of the cavity wavelength) or reasonable sensitivity in a broader range.

For non-normal incidence of laser light, it can only impinge on the mirrors a limited number of times before leaving the cavity, independent of the reflectivities. In this case, as for lower reflectivities, the resonance becomes broader but also smaller. Additionally, highly oscillating terms appear. However, for limited resonances and a medium number of impinges, the range of phases covered by the resonance may be even broader as for the case of infinite impinges. We calculated the expectable mechanical noise in such a setup to motivate using the output field to measure mirror displacements. For reasonable setups of microscopic mirrors, this appears easy to realize. Thus, this method could be an alternative option to analyse movements of the mirror induced by external sources.

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