Deeply Virtual Compton Scattering on the Deuteron

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We study deeply virtual Compton scattering (DVCS) on a deuteron target. We model the Generalized Quark Distributions in the deuteron by using the impulse approximation for the lowest Fock-space state. Numerical predictions are given for the unpolarized cross sections for the kinematical regimes relevant for JLab and HERMES at HERA. Differential cross sections show the same pattern as for the proton case and at low values of $-t$ they are of comparable size.

1. INTRODUCTION

In recent years it has become clear that hard exclusive processes, such as DVCS and deeply exclusive meson production (DEMP), play a unique role in offering a rather complete picture of the hadronic structure (for a recent review see [1] and references therein). The information which can be accessed through these experiments is encoded by the Generalized Parton Distributions (GPDs), whose physical interpretation has been elucidated by some authors [2]. Recent measurements of the azimuthal dependence of the beam spin asymmetry in DVCS [3,4] have provided experimental evidence to support the validity of the formalism of GPDs and the underlying QCD factorization theorems.

The theoretical arguments used in deriving factorization theorems in QCD for the nucleon can be applied for the deuteron as well, and therefore one can develop the formalism of GPDs for the deuteron. From the theoretical viewpoint, it is the simplest and best known nuclear system and represents the most appropriate starting point to investigate hard exclusive processes off nuclei. On the other hand, these processes could offer a new source of information about the partonic degrees of freedom in nuclei, complementary to the existing one from deep inelastic scattering. Experimentally, deuteron targets are quite common and as a matter of fact DVCS experiments are planned or being carried out at facilities like JLab and HERA.

A parameterization of the non-perturbative matrix elements which determine the amplitudes in DVCS and DEMP on a spin-one target were given in terms of nine GPDs for the quark sector [3]:

$$V_{\chi\lambda} = \int \frac{d\kappa}{2\pi} e^{i\kappa x 2 P.n} \langle P', \lambda' | \bar{\psi}(-\kappa n) \gamma.n \psi(\kappa n) | P, \chi \rangle = \sum_{i=1,5} \epsilon^{\alpha\beta} V_{\beta\alpha}^{(i)} H_i(x, \xi, t)$$

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Figure 1. Deeply Virtual Compton Scattering off Deuteron in the impulse approximation. The interaction of the two photons with the active nucleon is determined by the nucleon GPDs.

\[
A_{\lambda'\lambda} = \int \frac{d\kappa}{2\pi} e^{ix\kappa P.n} \langle P', \lambda' | \bar{\psi}(-\kappa n) \gamma_5 \psi(\kappa n) | P, \lambda \rangle = \sum_{i=1,4} \epsilon^{\alpha\beta} A^{(i)}_{\beta\alpha} \epsilon^\alpha_H(x, \xi, \Omega)
\]

where \(|P, \lambda\rangle\) represents a deuteron state of momentum \(P\) and polarization \(\lambda\), \(P = (P + P')/2\), and \(n^\mu\) is a light-like vector with \(n^+ = 0, n_\perp = 0\). Due to the spin-one character of the target, there are more GPDs than in the nucleon case, but at the same time the set of polarization observables which in principle could be measured is also richer.

Our first priority is to obtain some quantitative estimates of the expected counting rates for some basic observables like the unpolarized cross section in DVCS, in order to assess the feasibility of experiments and check which regions are optimal for measurements.

2. THE IMPULSE APPROXIMATION

Most of the models for the nucleon GPDs take advantage of the known properties of GPDs in some limits, such as their relationship with form factors and parton distributions, supplemented with some assumptions about the \(\xi\) dependence. For the deuteron this is a more difficult task, since nothing is known experimentally about axial form factors or one of the parton distributions \((b_1)\). Moreover, four GPDs have vanishing first moments and then no information can be inferred about their \(t\) dependence.

The strategy that we will follow is to use the impulse approximation to describe DVCS off the deuteron, i.e. to assume that only one nucleon is active and participates in the absorption and emission of photon, the other being spectator, Fig. 1. We will also retain only the lowest Fock-space state and neglect other components in the deuteron wave function. Within this approximation it is straightforward to link the light-cone wave function of the deuteron to the wave function obtained in non-relativistic quantum mechanics \([3]\).

The DNN vertex is described by the wave function \(\chi_\lambda(\alpha, \vec{k}_\perp, \lambda_1, \lambda_2)\) where \(\alpha\) is the fraction of \('+'\) momentum carried by one of the nucleons and \(\vec{k}_\perp\) is its transverse momentum in a frame where the total transverse momentum of the deuteron vanishes. The indexes \(\lambda, \lambda_i\) refers to the polarization of the deuteron and nucleons respectively. Recall that in Light-front dynamics the two nucleons are on-shell but off (light-cone) energy shell.

The matrix elements \([1][2]\), or equivalently, the deuteron GPDs, can be written, in a somewhat symbolic notation, as a convolution between the deuteron wave functions and
the isoscalar flavour combination of the appropriate nucleon GPDs:

\[ V_{\lambda\lambda}(x,\xi,t) = \chi^*_\lambda(\alpha',\vec{k}_\perp',\lambda_1',\lambda_2') \otimes H, E(x_N,\xi_N,t) \otimes \chi_\lambda(\alpha,\vec{k}_\perp,\lambda_1,\lambda_2) \]  
\[ A_{\lambda\lambda}(x,\xi,t) = \chi^*_\lambda(\alpha',\vec{k}_\perp',\lambda_1',\lambda_2') \otimes \tilde{H}, \tilde{E}(x_N,\xi_N,t) \otimes \chi_\lambda(\alpha,\vec{k}_\perp,\lambda_1,\lambda_2) \]  

where \( \alpha = \frac{p_1^+}{p^+} \), \( \alpha' = \frac{p_1'^+}{p^+} \), according to the notation of Fig. 1. We will not consider here the gluon sector of the GPDs and we will limit ourselves to the quark content of the nucleon. The arguments of the nucleon GPDs are given by

\[ \xi_N = \frac{\xi}{\alpha(1+\xi) - \xi} \]  
\[ x_N = \frac{x}{\xi} \xi_N \]  

Notice that the variables \( x \) and \( \xi \) refer to fractions of plus momentum with respect to \( P^+ \) whereas \( x_N \) and \( \xi_N \) are defined with respect to \((p_1^+ + p_1'^+)/2\). From the relations above one can see that when \( x < \xi \) then \( x_N < \xi_N \), i.e. when we enter the kinematic region where we are testing the meson distribution amplitude in the deuteron we are actually looking at this region at the nucleon level. This is just a consequence of retaining only the lowest two-nucleon state in the deuteron.

The skewness parameter \( \xi \) determines the momentum transfer in the longitudinal direction:

\[ \Delta^+ \equiv (P'^+ - P^+) = -2\xi \bar{P}^+ \]  

and in the generalized Bjorken limit this is entirely fixed by the kinematics of the virtual photon \( (\xi \approx x_B/2) \). In the impulse approximation, this momentum transfer has to be provided by the active nucleon, and after that, the final state of this active nucleon still has to fit into the final deuteron. However, the deuteron is a loosely bound system, which means that one cannot have a very asymmetrical sharing of longitudinal momentum between the nucleons and therefore the formation of the coherent final state will be strongly suppressed in the impulse approximation for large skewness.

To be more quantitative let us define the longitudinal momentum distribution of the nucleon in the deuteron as:

\[ n_\lambda(\alpha) = \sum_{\lambda_1,\lambda_2} \int \frac{d\vec{k}_\perp d\beta}{(16\pi)^3} \left| \chi_\lambda(\beta,\vec{k}_\perp,\lambda_1,\lambda_2) \right|^2 \delta(\alpha - \beta) \]  

which is normalized according to

\[ \int d\alpha \ n_\lambda(\alpha) = 1 \]  

In Fig. 2 we show \( n_0(\alpha) \) evaluated with the wave function from the Paris potential [7]. This distribution is strongly peaked at \( \alpha = 0.5 \) and its width is of the order of the ratio of the binding energy divided by the nucleon mass.
In the impulse approximation, the active nucleon after the interaction with the photon carries a fraction of longitudinal momentum which is given by

$$\alpha' = \alpha - \frac{x_{Bj}}{1 - x_{Bj}}(1 - \alpha) .$$

(10)

In Fig. 2 we plot the difference $\alpha - \alpha'$ as a function of $\alpha$ and for several values of the skewness. We see that for $x_{Bj} \gtrsim 0.1$ this difference is larger than the width of the momentum distribution, and therefore, we will inevitably have a too fast or too slow nucleon (in the longitudinal direction). In this case the central region of momentum, where a maximal contribution is expected, is missed and then the cross sections will decrease very fast with $x_{Bj}$. In other words, there is an increasing difficulty in forming a coherent final state as the longitudinal momentum transfer, i.e. $x_{Bj}$ increases. In that case other coherent mechanisms, which could involve higher Fock-space components, will presumably become dominant. Not much is known about these states, but it should be emphasized that the suppression of the diagram of Fig. 1 occurs at $x_{Bj}$ as low as 0.2, so that there is room to check the importance of the contribution of these 'exotic' states.

To evaluate the matrix elements (1,2) according to the expressions (3,4) we have used the parameterization of the deuteron wave function provided by the Paris potential [7] which contains a s-wave and a d-wave component. For the nucleon GPDs we have considered only the contribution of $H$ and $\tilde{H}$ since $E$ and $\tilde{E}$ go with suppressing kinematical prefactors. The flavour combination which enters in (3,4) is the isoscalar one so that there is no dangerous pion pole contribution to $\tilde{E}$.

To model $H$ and $\tilde{H}$ we have used a phenomenological parameterization based on double distributions [8], without considering any D-term at this stage. The $t$ dependence has been taken into account in a factorized form by multiplying the double distributions by the corresponding quark form factors extracted from empirical (dipole) parameterizations of nucleon form factors. Notice that even if we take a factorized $t$-dependence for the nucleon GPDs, the resulting $t$-dependence in the deuteron GPDs, obtained through the expressions (3,4), cannot be factorized out and, in fact, the stronger $t$ dependence comes from the $\vec{k}_\perp$ in the deuteron wave function.

To evaluate the differential cross sections for the process $ld \rightarrow ld\gamma$ we also need to calculate the contribution of the concurring Bethe-Heitler (BH) mechanism. The amplitude for this process can be calculated exactly and the resulting squared amplitude is written in terms of the elastic structure functions of the deuteron, which also appear in the elastic reaction $ed \rightarrow ed$ [11]:

$$|T^{BH}|^2 = \frac{(4\pi\alpha_{em})^3}{t^2}[K_aA(t) + K_bB(t)] ,$$

(11)

where $K_a$ and $K_b$ are kinematical coefficients that will be detailed elsewhere.

3. RESULTS

In Fig. 3 we show the unpolarized differential cross section for kinematical regions which can be accessed at JLab and HERMES. We see that the cross sections are of the
order of the nb. in the central region, with a very fast fall-off. The order of magnitude of the cross sections in this region is comparable to the one obtained for the proton.

Concerning the separate contributions of BH and VCS we observe also the same patterns as for the nucleon case [10], with a dominance of the VCS contribution in the region of negative $\theta_{\gamma}^{\text{LAB}}$ (for the in-plane production it corresponds to the case where the scattered lepton and deuteron are detected on the same side of the detector). For the positive $\theta_{\gamma}^{\text{LAB}}$ region, the balance between BH and VCS strongly depends on the energy of the lepton.

This similarities with the nucleon case leads one to think that we will also have for the deuteron sizeable values for other observables like the beam spin asymmetry, which can be measured more easily than the normalization in any case. Recently, an estimate was made in [11] based on some simplifying assumptions, leading to a beam spin asymmetry of $-13 \sin \phi$ for the kinematics of HERMES shown in fig. 3, and comparable to the one obtained for the nucleon under the same kinematical conditions. Our estimates within the impulse approximation confirm asymmetries roughly of this size though with an important $\sin 2\phi$ component.

One important issue in DVCS off deuteron is the dynamical target mass corrections in the VCS sector. At $Q^2 = 4$ GeV$^2$ these corrections could be moderate but at $Q^2 = 1.6$ GeV$^2$ they may be important. This problem was addressed in [12] for a spin-1/2 target but nothing is known for a spin-1 target.

As stated above, at larger $x_{\text{Bj}}$ the cross sections die away very rapidly though observables like the beam spin asymmetry remain still sizeable. For JLab kinematics, it would allow to reach larger $Q^2$ of the order of 2-3 GeV$^2$ and stay on a less dangerous ground with respect to target mass corrections.
Figure 3. Unpolarized Differential Cross section for DVCS for typical kinematics at JLab (left) and HERMES (right). Dashed-dotted line: BH only; dashed line: VCS only; full line: BH + VCS + Interference.

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