In Search of the Quark Spins in the Nucleon: 
A Next–to–Next–to–Leading Order QCD Analysis 
of the Ellis–Jaffe Sum Rule†

Paolo M. Gensini
Dip. di Fisica dell’Università di Perugia, Perugia, Italy, and
Sezione di Perugia dell’I.N.F.N., Perugia, Italy

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ABSTRACT

The data from the last seven experiments performed on polarized deep-inelastic scattering on proton and neutron (or deuteron) targets have been analyzed in search of a precise determination of the spin fraction carried by the quarks in the nucleon. We find that this fraction can be of the size expected from naïve quark model arguments, provided the gluon axial anomaly is explicitly included and the isosinglet axial charge normalization is fixed at a suitably low momentum scale, such that a) the running, strong coupling constant is about unit, and b) the orbital angular momentum inside the nucleon vanishes.

We also find that, despite the appeal of this solution of the “nucleon spin crisis”, a solution where the axial anomaly is absent and its effects are traded for an appreciable strange quark polarization can not however be excluded — because of the limited accuracy of the data — unless this latter and/or the gluon polarization in the nucleon are explicitly measured.

1. Introduction.

Just after the 1988 publication by the European Muon Collaboration (EMC) at CERN of their preliminary\(^1\) results on the asymmetry in muon polarized deep-inelastic scattering (PDIS) on a polarized hydrogen (actually spin–frozen ammonia) target, the particle physics community was somewhat shocked to learn that the total sum of the quark spins in the proton seemed to be very close to zero, rather than somewhere between 1/2 and 3/4, as generally expected from naïve quark–model arguments.

As for any unexpected discovery in our field, to begin perhaps with that of the muon, the following years have seen both the planning and running of new experiments, and the deepening of the theoretical studies (not completely free of much heated controversies) trying to clarify this mystery, often know under the name of “nucleon spin crisis”.

The aim of this paper, relying heavily on the results thus accumulated, is
to use i) all PDIS data taken on both proton and neutron (or deuteron) targets, ii) current phenomenological ideas on the behaviour of the parton distributions at both large and small values of the Bjorken variable $x$, and iii) the results from perturbative QCD (PQCD) at next–to–next–to–leading order (NNLO), to produce an internally consistent estimate of the quark spin content of the nucleon.

It will turn out that one can not, at the present level of experimental accuracy, unambiguously separate the different flavour components from PDIS data alone, nor directly verify the PQCD predictions for the isoscalar sum rule: namely, one can not decide on a purely experimental basis on the nature of its unitary–singlet component, given the (basically two) different possibilities offered in principle by the stage at which one decides to send the quark masses (used as regulators in the calculations of the splitting functions) to zero.

Despite this persistent lack of experimental proof of all PQCD arguments, it can however be shown that, for the “most natural” (in the parton model framework) of these two possibilities, one can interpret the data as being consistent with a negligible intrinsic strange component of the quarks’ spins at a suitably low mass scale, such that $\alpha_s \simeq 1$, consistent with the naive quark model expectation.

There are two primary reasons why this result contradicts the initial findings from the EMC data\(^1\): the first, of experimental nature, is that it is now clear that the $g_1^p$ values given by the EMC were too low because of their choice of the $F_2^p$ data employed to normalize them; the second, of theoretical nature, is that PQCD corrections were included only at leading order, while it is only at next–to–leading order that peculiar features of the polarized deep–inelastic scattering (and particularly for its unitary–singlet piece) become to emerge, as it will be clear in next section.

The rest of this paper will be divided in three parts: a presentation of the PQCD NNLO corrections to the first–moment isovector and isoscalar sum rules, a careful enumeration of the constraints that can be imposed on the various polarized distribution functions of the quarks, in terms of which one can describe the polarized structure functions $g_1^{p,n}$, and then a short discussion on the results obtained fitting to the data simple parametrizations satisfying these constraints. This discussion will be centred on the spin composition of the nucleon, and in particular on the need (or absence of any need) for an intrinsic strange component $\Delta s$ in it.

2. The First Moments of $g_1^{p,n}$ and PQCD at NNLO.

The sum rules for the first moments $I_0^{p,n}(Q^2) = \int_0^1 dx g_1^{p,n}(x, Q^2)$ are often discussed separating the non–singlet, $(p - n)$ combination from the isoscalar one, $(p + n)$, which contains both a non–singlet and a singlet part. Actually, despite the technical difficulties involved in dealing with the latter, both sum rules have equal footing in PQCD and should not be considered as separate entities, apart
from historical, or practical, considerations.

Their only difference lies indeed in the fact that the axial coupling involved in the first is extremely well measured from neutron $\beta$–decay, while the couplings involved in the second are outside direct measurement, and can be arrived at only through more or less founded theoretical arguments. It is therefore expedient to separate them, since the first can either be considered a good test of higher–order PQCD, or, alternatively (and this will be the attitude taken in the present paper), a useful normalization for the non–singlet part of the moments $I_0^{p,n}(Q^2)$.

The first–moment sum rule for the difference between proton and neutron polarized structure functions, known as the Bjorken sum rule (BjSR), reads

$$I_0^{p-n}(Q^2) = \int_0^1 dx [g_1^p(x, Q^2) - g_1^n(x, Q^2)] = \frac{1}{6} \cdot C_8(\alpha_s) \cdot g_A + (h.t.)_{I_t=1}, \quad (1)$$

where the coefficient $C_8(\alpha_s)$ includes the PQCD corrections to the parton–model result, and the symbols $(h.t.)_{I_t=0,1}$ stand for the higher–twist contributions (HTC), behaving as integer inverse powers of $Q^2$. Since experiments extend from $< Q^2 > = 2.0 \text{ GeV}^2$ (SLAC E142 Collaboration) to the 10.7 GeV$^2$ of the EMC, inclusion of these effects turns out to be crucial to a test of BjSR, Eq. (1). This is indeed also the attitude taken by Ellis and Karliner in their analysis of PDIS data. HTC are also essential to connect the PDIS sum rules for proton and neutron targets to the Gerasimov–Drell–Hearn sum rules at $Q^2 = 0$.

An estimate of the HTC was originally given — and revised — by Balitskii, Braun and Kolesnichenko, and then repeated by several groups of authors: in this paper preference will be given to the estimate by Ross and Roberts, mostly because of its widespread use in other analyses of the same data.

Let us now focus our attention on some aspects of the recently improved evaluation of $C_8(\alpha_s)$ and on their implications. This coefficient is now known to better than $O(\alpha_s^3)$, and to NNLO accuracy it can be written as

$$C_8(\alpha_s) = 1 - \frac{\alpha_s}{\pi} \cdot [1 + c_1^{(8)}(N_f) \cdot \frac{\alpha_s}{\pi} + c_2^{(8)}(N_f) \cdot \left(\frac{\alpha_s}{\pi}\right)^2 + ...], \quad (2)$$

where the numerical values of $c_{1,2}^{(8)}$ are listed, for $N_f$ from 3 to 5, in Table I, and their complete expressions can be found in the original paper by Larin and Vermaseren. Due to the large values of the constants $c_{1,2}^{(8)}$ for $N_f = 3, 4$, $C_8(\alpha_s)$ is evidently decreasing with $\alpha_s$ much more steeply than expected on the basis of the leading order estimate $C_8 = 1 - \alpha_s^3/\pi$ at low values of $Q^2$, such as those of SLAC experiments E142 (< $Q^2$ > = 2.0 GeV$^2$) and E143 (< $Q^2$ > = 3.0 GeV$^2$), and of the preliminary deuteron data from the Spin Muon Collaboration (SMC) at CERN (< $Q^2$ > = 4.6 GeV$^2$).
An initial comment is in order here: actually, the expansion in $\alpha_s$ for $C_8(\alpha_s)$ is known to one order beyond $15$, what has been written in Eq. (2), but its unitary–singlet partner, $C_1(\alpha_s)$, has been estimated only to $O(\alpha_s^5)$, as most of the quantities computed in PQCD to extract the coupling $\alpha_s$: for its consistency, any PQCD analysis must be performed to the same, fixed order in the coupling, using the $\beta$–function at that order to express the running of the coupling $\alpha_s(Q^2)$.

Accordingly, here only NNLO expansions will be used, together with the expansion for $\alpha_s(Q^2)$ (and the estimate of its scale $\Lambda_{\overline{MS}}(N_f)$) at the same order: a recent PQCD analysis by Bethke and Catani of all available data (both space– and time–like, but with no inclusion of PDIS ones) led to a conservative estimate of $\Lambda_{\overline{MS}}(5) = 200 \pm 50$ MeV, and therefore to the value $\Lambda_{\overline{MS}}(3) = 410 \pm 100$ MeV that will be employed throughout. There is a reason to overlook more recent compilations: since these tend to include also $\alpha_s$ determinations from PDIS data to enrich the available points at low momenta, one should exclude these latter determinations from the analysis to be free of possible internal biases, and to treat PQCD corrections as external inputs rather than as parameters to be fit to the data. This is simply and easily accomplished, rather than by re–doing all fits, by using the $\Lambda_{\overline{MS}}(3)$ from the slightly older compilation of Ref. 17.

| $N_f$ | $c_1(N_f)$ | $c_2(N_f)$ |
|------|-----------|-----------|
| 3    | 3.5833    | 20.2153   |
| 4    | 3.2500    | 13.8503   |
| 5    | 2.9167    | 7.8402    |

Table I
Coefficients of higher QCD corrections to BjSR

A question better addressed at this point is the actual value of $N_f$, the number of active flavours, to be used at each $Q^2$ in the PQCD expansions of the coefficients $C_{1,8}(\alpha_s)$ and of the anomalous dimension of the unitary–singlet axial charge. For time–like $Q^2$, there is no ambiguity, since for heavy quarks the flavour thresholds at $Q^2 = 4m_i^2$ can be fixed by setting the mass $m_i$ of the $i$–th flavour quark approximately equal to that of its lowest–mass pseudoscalar meson. In deep–inelastic scattering, however, a flavour is active only when appreciably contributing to the moment sum rules, i.e. when produced a) in a really inclusive manner (read: not only in low–multiplicity events), and b) over an appreciable range in Bjorken’s variable $x$ (say up to $x \simeq 1/3$). If one sets the beginning of the scaling region at $Q^2 \simeq 2$ GeV$^2$ (as indicated by the “classic” SLAC–MIT experiments), the previous requirements ask for a $Q^2 \gtrsim 17$ GeV$^2$ for charm to be an active flavour in DIS, and so mandates $N_f = 3$ for the PQCD analysis of all available PDIS data.

The choice of $N_f = 3$ (rather than 4) has no great effect on $C_8(\alpha_s)$, as one can read from Table I, but for the $Q^2$–evolution of the unitary–singlet piece the
coefficient of the first–order term in the expansion in powers of $\alpha_s$ of its integrated anomalous dimension almost cancels, for $N_f = 4$, the first–order one in $C_1(\alpha_s)$. It is therefore important, for the extraction of the quarks’ spin fraction,

$$\Sigma = \sum_i \Delta q_i = \Delta u + \Delta d + \Delta s \text{ for } N_f = 3,$$

(3)

to use the value of $N_f$ appropriate to the range of values of $Q^2$ where the actual data have been taken. This, unfortunately, has not always been done consistently by some authors\(^1\).

In 1974, Ellis and Jaffe\(^2\), faced with the problem of how to use a sum rule akin to Eq. (1) with only hydrogen data, used parton–model ideas, flavour SU(3) symmetry and the Okubo–Zweig (OZI) rule to derive a sum rule for the first moment of $g_1^p$ alone; as stated above, the PQCD–corrected version of such a sum rule (when freed of OZI–rule restrictions) is a part of PQCD as fundamental as the BjSR. For the isoscalar combination of PDIS structure functions this sum rule becomes the (PQCD corrected) Ellis–Jaffe sum rule (EJSR)

$$I_0^{p+n}(Q^2) = \int_0^1 dx [g_1^p(x, Q^2) + g_1^n(x, Q^2)] =$$

$$= \frac{1}{18} \cdot C_8(\alpha_s) \cdot g_8 + \frac{2}{9} \cdot C_1(\alpha_s) \cdot g_0(Q^2) + (h.t.)_{t=0},$$

(4)

not to be confused with the original EJSR\(^2\), which was derived for $I_0^p(Q^2)$ only, and without any PQCD corrections to the parton–model plus OZI–rule predictions.

Additional complications with respect to the BjSR, Eq. (1), arise from the facts a) that the isoscalar axial charges of the nucleon are not directly measurable, and $g_8$ is indeed derived via flavour–symmetry arguments (apart from symmetry–breaking effects), and b) that the unitary–singlet one $g_0(Q^2)$ couples not only to the quarks, but also to the gluons via the axial anomaly and possesses therefore anomalous dimensions\(^2\), so that its evolution with $Q^2$ is not exhausted by the PQCD coefficient $C_1(\alpha_s)$ and must be explicitly computed.

Since this coupling is scheme dependent, this point has been the focus of a very heated theoretical debate\(^2\). To cut a long history short, one can summarise it by saying that, in conventional parton language where the masses of the partons $m_i$ are neglected with respect to the momentum scale $Q$, and wishing to identify $\Sigma$ with the spin fraction carried by the quarks actually present in the target proton, one has to put

$$g_0(Q^2) = \Sigma - N_f \frac{\alpha_s}{2\pi} \Delta G(Q^2),$$

(5)

where $\Delta G$ is the first moment of the gluon polarized distribution function $\delta G(x) = G_+(x) - G_-(x)$, and determine its evolution via the equation (where $t = \log Q^2/\mu^2$)

$$\frac{d}{dt}g_0(t) = -N_f \frac{\alpha_s}{2\pi} \gamma qq(\alpha_s)g_0(t),$$

(6)
which relates to the anomalous dimension of the axial anomaly via

\[-N_f \frac{\alpha_s}{2\pi} \gamma_{gg}(\alpha_s) = \gamma_{gg}(\alpha_s) - \beta(\alpha_s) \frac{2\pi}{\alpha_s}\]  

(6')

where the various \(\gamma_{ij}\) \(\{i, j\} = \{g, q\}\) represent the coefficients giving \(d\Sigma/dt\) and \(d\Delta G/dt\) in terms of \(\Sigma\) and \(\Delta G\), whose matrix is diagonalized (in any scheme where quark mass regulators are sent to zero) building the combination in Eq. (5) on one side, which evolves anomalously as in Eq. (6), and leaving \(\Sigma\) on the other, free of anomalous dimensions since the constraint in Eq. (6') makes the determinant of the 2 \(\times\) 2 matrix of the (final) coefficients (whose eigenvalues are the anomalous dimensions of the two operators mixed by the evolution) vanish identically. This has been verified step by step at next–to–leading order\(^{22}\): that it should hold also at higher orders is inferred from the fact that conserved (and partially conserved) charges should be free of anomalous dimensions on one side, and on the other there is nothing, apart from the mixing with the unitary singlet of the pure gluonic world, to distinguish an SU(3)– from a U(3)–symmetric fermionic world, so that it is “natural” to expect all purely fermionic operators to be free of anomalous dimensions.

After integrating in \(\alpha_s\) from a normalization scale \(\mu^2\) to \(Q^2\), one obtains the integrated anomalous dimension at NNLO

\[
\log \frac{g_0(Q^2)}{g_0(\mu^2)} = \tilde{\gamma}(\alpha_s(Q^2)) - \tilde{\gamma}(\alpha_s(\mu^2)) = \frac{6N_f}{33 - 2N_f} \alpha_s(Q^2) - \alpha_s(\mu^2) \frac{33 - 2N_f}{\pi}
\cdot \left[ 1 + \left( \frac{83}{24} + \frac{N_f}{36} - \frac{33 - 2N_f}{8(153 - 19N_f)} \right) \frac{\alpha_s(Q^2) + \alpha_s(\mu^2)}{\pi} + \ldots \right],
\]

(7)

with the NNLO calculation by Larin\(^{23}\) of the anomalous dimension.

As one can read from the above expression, \(g_0(Q^2)\) can be drastically reduced (still at \(N_f = 3\)) from its value at the normalization scale \(\mu^2\) for \(Q^2 \gg \mu^2\); on the other hand, one can not set \(\mu^2 \to \infty\) and thus drop \(\alpha_s(\mu^2)\) from Eq. (7) for two reasons: first, that the anomaly contribution to Eq. (5) is not definable in this limit\(^{21,22}\), due to \(\Delta G(Q^2)\) diverging asymptotically as \(\alpha_s(Q^2)^{-1}\) from Eqs. (5) and (7), and, second, that we can not keep \(N_f = 3\) while letting \(Q^2 \to \infty\).

There is another point which deserves consideration: the expression in Eq. (7) is of second order in \(\alpha_s\), while it was said from the beginning that the analysis of the PDIS data will be done consistently at NNLO, or \(O(\alpha_s^3)\). Indeed the anomalous dimension \(\gamma_{gg}(\alpha_s)\) has been computed to \(O(\alpha_s^3)\) by Larin\(^{23}\): it is however a logarithmic derivative in the variable \(t\), which to be integrated has to be divided by the \(\beta\)–function to make the variable change from \(t\) to \(\alpha_s\), losing in this step a power in the coupling \(\alpha_s\). This gives no problem if the anomalous dimension is left where it belongs, i.e. as the argument of an exponential function,
\[ g_0(Q^2) = g_0(\mu^2) \cdot \exp[\tilde{\gamma}(\alpha_s(Q^2)) - \tilde{\gamma}(\alpha_s(\mu^2))], \]

where \( \tilde{\gamma}(\alpha_s) \) is the integrated anomalous dimension, but has caused unnecessary problems whenever \( \exp \tilde{\gamma}(\alpha_s) \) has been expanded in powers of the coupling, since then one had either to go one step further in the expansion of the anomalous dimension to find the “missing” power\(^{15,16} \), or to truncate “prematurely” the series in the coefficient \( C_1(\alpha_s) \). Both procedures are inconsistent from a PQCD point of view, since they tend to mix one order with the next, while the running coupling \( \alpha_s \) must be defined order by order: it is clear that in this way one will end up using \( \alpha_s \) at a given order in at least one perturbative expansion calculated at a different order (not to be confused with the power of \( \alpha_s \) appearing in the final expression, which could depend, as is the case here, on additional mathematical manipulations).

Another, equally unnecessary, but luckily only semantic problem has been created by the persons who first misnamed the scale \( \mu^2 \) in the same equation a renormalization scale, while it is clear enough that one has to do with just a normalization scale, which has absolutely nothing to share with the actual renormalization procedure\(^{16} \). The scale \( \mu^2 \) is thus free both to appear explicitly in the physical expressions, and to be chosen to follow the author’s (or authors’) theoretical prejudices; certain precautions must however be followed: for instance, working at a fixed number of flavours \( N_f = 3 \), it would be rather unwise to set it to infinity, where \( N_f \) will be six at the best of our knowledge, so that the \( g_0(\infty)^{N_f=3} \) so obtained would hardly be connected to the physical limit of \( g_0(Q^2) \) for \( Q^2 \to \infty \). Note also that this \( g_0(\infty)^{N_f=3} \) (whatever its meaning) can not be identified with \( \Sigma \) as defined by Eq. (3): if one believed the singlet axial coupling \( g_0(Q^2) \) in the EJSR, Eq. (4), to be given by \( \Sigma \) alone, from the diagonalization of the unitary–singlet operators one had also to put \( \tilde{\gamma} = 0 \) accordingly. In line of principle, these two definitions for \( g_0(Q^2) \) could be distinguished on the basis of the different evolutions with \( Q^2 \) they predict for the EJSR integrals \( I_0^{p+n}(Q^2) \): making the two to coincide at a scale \( Q_0^2 \), the difference for the EJSR between the case of a running \( g_0(Q^2) \) and that of \( g_0 = \Sigma \) at a scale \( Q^2 \) would correspond to

\[ \Delta I_0^{p+n}(Q^2) = \frac{2}{9} \cdot C_1(\alpha_s(Q^2)) \cdot \{ \exp[\tilde{\gamma}(\alpha_s(Q^2)) - \tilde{\gamma}(\alpha_s(Q_0^2))] - 1 \} \cdot g_0(Q_0^2) . \] (8)

We shall return later on this point, when analysing our numerical results in Section 4.

The evaluation of the singlet coefficient \( C_1(\alpha_s) \) runs along the same lines as that of the non–singlet one \( C_8(\alpha_s) \) and it can be similarly expanded in powers of \( \alpha_s \):

\[ C_1(\alpha_s) = 1 - \frac{\alpha_s}{\pi} \cdot \left[ 1 + c_1^{(1)}(N_f) \cdot \frac{\alpha_s}{\pi} + c_2^{(1)}(N_f) \cdot \left( \frac{\alpha_s}{\pi} \right)^2 + ... \right] . \] (9)

There are however additional complications, for this coefficient receives contributions from graphs, not included in the other, whose number grows with the order of the calculation. Thus Kataev\(^ {16} \) has produced only an estimate of the constant \( c_2^{(1)} \)
for \( N_f = 3 \) (note that one of the conditions under which this was possible breaks down for \( N_f = 4 \)).

To close this section, the numerical values of the constants for the perturbative expansions in orders of \((\alpha_s/\pi)\) of \(C_8(\alpha_s)\), \(C_1(\alpha_s)\), and \(\tilde{\gamma}(\alpha_s) = \sum_k \gamma_k \cdot (\alpha_s/\pi)^k\), are listed in Table II. Two points must be noted before going to the next section: the constants in the singlet part are systematically lower than in the corresponding, non–singlet one, giving a slower evolution with \(Q^2\), and the trends of the integrated anomalous dimension \(\exp \tilde{\gamma}(\alpha_s)\) and of the coefficient \(C_1(\alpha_s)\) run in opposite directions, tending to some extent to compensate each other.

Also, the first line in Table II makes clear enough the essential difference between leading– and higher–order PQCD treatments: besides introducing an anomalous dimension for \(g_0(Q^2)\), the latter break strongly the accidental, lowest–order degeneracy of the coefficient functions \(C_{1,8}(\alpha_s)\).

| \(k\) | \(c_k^{(8)}\) | \(c_k^{(1)}\) | \(\gamma_k\) |
|-------|-------------|-------------|-----------|
| 0     | 1           | 1           | 0         |
| 1     | 3.5833      | 1.0959      | 0.6667    |
| 2     | 20.2153     | \(\sim 3.7\) | 0.9907    |

3. Measurements and parametrizations for \(g_1(x)\).

What is actually measured by experiments in PDIS are not the structure functions \(g_1\) themselves, but rather the polarization asymmetries \(A_1\), related to the polarized cross sections \(\sigma^{\uparrow\uparrow}, \sigma^{\uparrow\downarrow}\) by

\[
D \cdot A_1 = (\sigma^{\uparrow\uparrow} - \sigma^{\uparrow\downarrow})/(\sigma^{\uparrow\uparrow} + \sigma^{\uparrow\downarrow}) ,
\]

where \(D\) is the target polarization fraction, and \(g_2\) is neglected: \(g_1\) is then related to \(A_1\) by

\[
g_1(x, Q^2) = \frac{A_1 \cdot F_2(x, Q^2)}{2x \cdot [1 + R(x, Q^2)]} .
\]

It is obvious that the factor \(2x\) in the denominator makes a direct determination of \(g_1\) at \(x \to 0\) impossible for finite–accuracy data. The evaluation of the EJSR, Eq. (4), depends thus on the parametrization assumed to extrapolate \(g_1\) to \(x = 0\): the usual treatment of this point has till now assumed it to extrapolate smoothly to a constant, as \(g_1 \sim \alpha + \beta \cdot x\), in accord with the pion–pole trajectory
intercept being close to zero. However, since one does not expect the sea distributions to couple dominantly to an isovector, pseudoscalar trajectory, but rather to an isoscalar one such as the eta, one should rather have for these a behaviour $x^{-\alpha_{\eta}(0)}$, with $\alpha_{\eta}(0) \simeq -1/4$, which, together with the expected negative sign for the sea contribution, produces a spike in the isoscalar part of $g_1$ as $x \to 0$, of the type $g_1 \sim \alpha - \beta \cdot x^{1/4}$ (with $\alpha, \beta > 0$), perhaps just appearing\textsuperscript{24} at very low values of the Bjorken variable $x$ in the proton data taken by the Spin Muon Collaboration at CERN at $<Q^2> = 10.0$ GeV$^2$. Of course, such a spiky behaviour does not show in the integrand of the BjSR, which can therefore be extrapolated smoothly to $x = 0$ according to the conventional practice in this matter.

This point could have a non-negligible influence on the evaluation of the EJSR, Eq. (4), raising the low-$x$ contribution to its left-hand side well above the conventional estimates. A special comment is in order here on the EMC published data\textsuperscript{5} for $g_1^p$: rather than using them, one should use instead only their values for $A_1^p$ with an adequate set of values for $F_2^p$ and $R^p$. Indeed, the EMC calculated $g_1^p$ using a) the ratio $R^p$ predicted by PQCD, systematically smaller than experiment since finite-mass corrections are dominant at low values of $Q^2$ and $x$ (though the effect of this choice is not too important at $<Q^2> = 10.7$ GeV$^2$), and b) their values for the unpolarized structure function $F_2^p$, systematically lower than those by the BCDMS collaboration\textsuperscript{25} (and than the recent New Muon Collaboration (NMC) data\textsuperscript{26} as well) by as much as 13% at the lowest values of $x$. Even using their measured values of $A_1$ together with a phenomenological parametrization for $F_2^p(x,Q^2)$ (and $R^p(x,Q^2)$) to produce $g_1^p(x,Q^2)$ at a reference, fixed value of $Q^2$ (and assuming $A_1$ to vary little\textsuperscript{6,15,27} with $Q^2$, an assumption which a recent analysis from the E143 Collaboration at SLAC seem to corroborate\textsuperscript{28}) is not completely free of the above, last source of error: indeed only the latest, post-NMC parametrizations\textsuperscript{29} have dropped the unpolarized EMC data altogether, while all previous analyses ended up averaging over the two, conflicting sets of data for small values of $x$. We have therefore re-normalized the published EMC data\textsuperscript{5} for $g_1^p$ with the known ratio of BCDMS to EMC data, and this will be the data set referred to as EMC$^p_{rev}$ in the rest of the paper.

As just mentioned two paragraphs above, to build the first moments the actual measurements of $g_1^{p,n}$ have to be extrapolated in $x$ to $x = 0$ and to $x = 1$, to cover parts of the integration range not coverable by the experiments on the asymmetry $A_1^{p,n}$. The second extrapolation does not pose any problem, since $A_1$ tends to unit (and $R$ to zero) in the limit $x \to 1$ (this is a well-known feature\textsuperscript{30} of the nucleon wavefunction: at high values of $x$ the proton — neutron — structure is dominated by the $u_+ - d_+$ quark distribution: some parametrizations\textsuperscript{31} violate this constraint gaining thus additional but unphysical freedom in dealing with the small sea components), and Eq. (11) is thus reducing simply to $g_1 \sim F_2/(2x)$, whose behaviour as $x \to 1$ is largely determined by well known “counting rules” of conventional parton models.
In this paper, instead of extrapolating separately in the two extreme ranges of \(x\), the choice has been made to fit the data to simple functional forms incorporating at least three elements: a) the counting rules, b) reasonable Regge (or other, QCD–motivated) behaviours for \(x \to 0\) (separately for valence and sea contributions), and c) constraints on the integrals of the parton distribution functions coming from a connection between the constituent–quark picture and the quark–parton model, originally introduced by Altarelli, Cabibbo, Maiani and Petronzio\cite{32}, and recently recovered in this context by Fritzsch\cite{33}.

In this picture, the structure functions \(g_{1}^{p,n}\) are decomposed in terms of the helicity distribution functions \(\delta q_{i}'\) for each active quark flavour \((q_{i} = u, d, s)\), where the prime indicates that the contribution from the axial anomaly, formally of order \(\alpha_{s}\), has been included in each flavour’s sea distribution, \(\delta q_{i}' = \delta q_{i} + k_{qg} \otimes \delta G\) (the symbol \(\otimes\) will stand from here on for a convolution integral in Bjorken variables), with \(k_{qg}\) the appropriate gluon–to–quark splitting function. It has been sometimes said in the literature that the anomaly contributes to \(g_{1}\) only at very small \(x\) values, so that it should be hardly seen in the \(x\)–regions covered by experiments: this is true (in the scheme where quark masses \(m_{i}\) are sent to zero) only if the polarized gluon distribution is assumed to peak at \(x = 0\), as e.g. in the intrinsic gluon distribution proposed by Brodsky and Schmidt\cite{34}. Unfortunately, their distribution for \(\delta G(x)\) has the wrong Regge behaviour for \(x \sim 0\), contrary to their statement, since it requires dominance of \(\delta G\) by the pion trajectory, with intercept \(\alpha_{\pi}(0) \simeq 0\), while it should be dominated instead by a pseudoscalar–glue ball trajectory, with an expected intercept \(\alpha_{G}(0) \lesssim \alpha_{\eta'}(0) \simeq -1\). When this constraint is imposed on the Brodsky–Schmidt formulae, \textit{ceteris paribus}, most of the anomaly contribution falls in the \(x\) interval covered by the experiments (e.g. 80% of it in the case of the EMC \(x\)–range), making it virtually indistinguishable from the other, intrinsic sea distributions, while the integral \(\Delta G\) remains of the same magnitude as in Ref. 34, being solidly tied to the momentum fraction \(< x_{G} > \simeq 1/2\) carried by the gluons at the momentum scale \(< Q^{2} >_{SLAC} \simeq 1 \sim 2\) GeV\(^{2}\) at which these intrinsic components are defined.

Separating further valence \((u_{v}, d_{v})\) from sea \((u_{s}', d_{s}', s')\) components one can write, with self–explanatory notations,

\[
g_{1}^{p}(x, Q^{2}) = \frac{2}{9} \cdot [\delta u_{v}(x, Q^{2}) + \delta u_{s}'(x, Q^{2})] + \\
\quad + \frac{1}{18} \cdot [\delta d_{v}(x, Q^{2}) + \delta d_{s}'(x, Q^{2}) + \delta s'(x, Q^{2})], \quad (12)
\]

and

\[
g_{1}^{n}(x, Q^{2}) = \frac{2}{9} \cdot [\delta d_{v}(x, Q^{2}) + \delta d_{s}'(x, Q^{2})] + \\
\quad + \frac{1}{18} \cdot [\delta u_{v}(x, Q^{2}) + \delta u_{s}'(x, Q^{2}) + \delta s'(x, Q^{2})]. \quad (13)
\]
The valence and sea distributions (with the latter primed to distinguish them from the intrinsic ones, free of the gluonic contribution, which relate to the quarks spin contents \( \Delta q_i \), entering e.g. in the angular momentum sum rule), will in general be expressed by the general functional form

\[
\delta q'_{v,s}(x, Q^2) = \sum_j x^{-\alpha_j(0)} \cdot (1 - x)^{n_{v,s}} \cdot P_j(x, Q^2),
\]

where the sum runs over the Regge singularities assumed to dominate the distribution at \( x \sim 0 \), and the \( P_j(x, Q^2) \) will be polynomials in \( x \) with \( Q^2 \)-dependent coefficients.

For the powers \( n_{v,s} \) the parton–model counting rules\(^{36}\) give \( n = 2N_s - 1 \) (where \( N_s \) is the number of “spectator” quarks), or \( n_v = 3 \) and \( n_s = 7 \) (for gluons the rule gives \( n_g = 5 \)): their further suppression in the distributions \( \delta q'_s \) at high \( x \)-values comes from the splitting function): it is however known\(^{35,36}\) that the valence \( u \)-quark dominates at high \( x \) values over the \( d \)-quark in their unpolarized distributions, and this fact is commonly “explained” as a consequence of the Pauli principle, barring two quarks of the same quantum numbers from being close to each other in phase space, and often expressed as a “penalty factor” \((1 - x)\) in the odd flavour distribution function. The same mechanism should operate in the polarized valence distributions as well, as in the sea distributions (both polarized and unpolarized), suppressing here \( u \)-quarks with respect to \( d \)-quarks, the “penalty” being now paid when an \( u \)-antiquark is produced at \( x \sim 1 \); while there is undisputable evidence of this effect for unpolarized valence distributions in the ratio \( F_2^u/F_2^D \), tending almost linearly to the value \( 1/4 \) as \( x \to 1 \), the same effect in the unpolarized sea ones could be responsible\(^{37}\) for the defect of the so–called Gottfried sum rule (one can simply check that the figures are indeed of the right order of magnitude, though a detailed model would require a complete re–fitting of all unpolarized parton distributions).

For the PDIS data, this Pauli–principle “penalty factor” will be included only in the polarized valence distributions, for the effects of its presence in the sea ones would be so small vis–à–vis the experimental errors that its inclusion would only lead to unnecessary mathematical complications in the fitting procedures.

With this additional factor omitted, and reducing the polynomials \( P_j(x, Q^2) \) to \( Q^2 \)-dependent factors, independent of \( x \), the sea contributions to the PDIS structure functions will reduce to an isoscalar term, simply expressed as

\[
g_1^{p,n}(x, Q^2)_{sea} = P(Q^2) \cdot x^\frac{2}{7} \cdot (1 - x)^\frac{7}{14},
\]

where we have assumed all sea components to couple dominantly to the \( \eta \)-meson trajectory with intercept \( \alpha_\eta(0) \simeq -1/4 \). Note that under this hypothesis the sea would contribute to the BjSR only through the difference between \( u \)-quark and \( d \)-quark seas eventually brought about by the Pauli–principle “penalty factor”: once this latter is dropped, the BjSR contains only the valence distributions and is therefore offering a mean to define their normalizations in terms of \( g_A \).

This is accomplished by using the connection between constituent–quark and parton model languages of Refs. 32–33, i.e. by writing \( \delta u = \delta U \otimes \delta u_U + \delta D \otimes \delta u_D \) (and so on for all active flavours and for gluons as well), where the notation \( \delta q_Q \) represent the \( q \)-flavoured parton polarized density inside the \( Q \)-flavoured constituent.
quark. Assuming SU(2) symmetry to hold for the partons inside the constituent quarks \((Q = U, D\) only), and integrating over all Bjorken variables, one can put \(\Delta u_U = \Delta d_D = \Delta (qQ)_v + \Delta (qQ)_s, \Delta d_U = \Delta u_D = \Delta (qQ')_s\) and \(\Delta s_U = \Delta s_D = \Delta s\), and one finds the relation for the spin content of the valence partons

\[
\frac{\Delta u_v}{\Delta d_v} = \frac{\Delta U}{\Delta D} = -4
\]  

(15)

from the constituent quark model results \(\Delta U = 4/3, \Delta D = -1/3\): note that, since these constituent quarks are structured objects, these values do not imply \(g_A = 5/3\) (modulo small recoil corrections), as in naïve treatments of the constituent quark model. The same hypothesis leads only to the generous bounds \(-1/4 < \Delta u'_v/\Delta d'_s < 1\) on the non–strange sea distributions, from the expected sea spin–component ratio \(0 < \Delta (qQ)_s/\Delta (qQ')_s < 1\) for sea partons inside a constituent quark, where the upper and lower bounds are justified respectively by the Pauli–principle “penalty factor” mentioned above and by the similarity in their production mechanisms.

With the constraint of Eq. (15) imposed on the polarized valence distributions joined with the neglect of the small isovector part in the polarized sea ones, the distributions \(\delta u_v\) and \(\delta d_v\) can be normalized to the BjSR, independent of their functional forms, since Eq. (1) can be reduced to

\[
I_{P-n}^0(Q^2) = \frac{1}{6} \int_0^1 dx (\delta u_v - \delta d_v) = \frac{1}{6} \cdot C_8(\alpha_s) \cdot g_A + (h.t.)_{I_t=1} .
\]  

(16)

For these forms, three parametrizations will be adopted, to check the systematic effects on the EJSR integrals of the behaviour assumed in the isoscalar valence distributions as \(x \to 0\).

The first parametrization (which will be labeled FRP, for fully Reggeized parametrization) breaks down each of the polarized distributions for the valence quarks into an isoscalar and isovector part, the first dominated at \(x \sim 0\) by the \(\eta\)–meson trajectory, and the second by the pion one, so that, after introducing the Pauli–principle “penalty factor” \((1 - x)\) in \(\delta d_v\) and imposing the conditions in Eqs. (13) and (14), with the \(x\)–polynomials \(P_j(x, Q^2)\) again reduced to \(Q^2\)–dependent coefficients, one gets

\[
\delta u^{(1)}_v(x, Q^2) = C(Q^2) \cdot \frac{231}{820} \cdot B\left(\frac{5}{4}, 4\right)^{-1} \cdot x^{\frac{1}{4}} + \frac{85}{41} \cdot (1 - x)^3 ,
\]  

(17)

\[
\delta d^{(1)}_v(x, Q^2) = C(Q^2) \cdot \frac{231}{820} \cdot B\left(\frac{5}{4}, 4\right)^{-1} \cdot x^{\frac{1}{4}} - \frac{85}{41} \cdot (1 - x)^4 ,
\]  

(17')

where the normalization \(C(Q^2) = C_8(\alpha_s) \cdot g_A + 6 \cdot (h.t.)_{I_t=1}\) is fixed from Eq. (16) to automatically satisfy the BjSR, and the function \(B(\alpha, \beta)\) is the well known Euler’s beta function.
For the second set of model distribution functions (to be labeled SRP, or simplified Regge–pole parametrization) one takes the limit \( \alpha_\eta(0) \to \alpha_\pi(0) \to 0 \), still keeping the same constraints, so that Eqs. (17), (17') become

\[
\delta u_v^{(2)}(x, Q^2) = \frac{16}{5} \cdot C(Q^2) \cdot (1 - x)^3, \tag{18}
\]

\[
\delta d_v^{(2)}(x, Q^2) = -C(Q^2) \cdot (1 - x)^4, \tag{18'}
\]

where the normalization is again the same. This parametrization has the advantage over the previous one of producing an isoscalar structure function \( g_1^p + g_1^n \) non–vanishing in the limit \( x \to 0 \), but at the cost of violating the isotopic spin properties of the Reggeon exchanges.

A third set of model distribution functions has been built following the suggestion by Bass and Landshoff\(^{38}\) that a non–perturbative, two–gluon exchange term might develop at small \( x \) a behaviour \( g_1 \sim \log(1/x) \), obviously only in the isoscalar part, for the PDIS structure function \( g_1 \). Thus one can write, again with the same number of parameters and the same constraints, the functions (labeled, from the names of the above–mentioned authors, as BLP)

\[
\delta u_v^{(3)}(x, Q^2) = C(Q^2) \cdot \left( \frac{66}{131} \log \frac{1}{x} + \frac{2817}{1310} \right) \cdot (1 - x)^3, \tag{19}
\]

\[
\delta d_v^{(3)}(x, Q^2) = C(Q^2) \cdot \left( \frac{66}{131} \log \frac{1}{x} - \frac{2817}{1310} \right) \cdot (1 - x)^4; \tag{19'}
\]

this time the \( \eta \)–meson Regge–pole term has been left out to avoid including one more parameter and so keep all three parametrizations on equal statistical footing. Taking the PQCD coefficients listed above (with the value for \( \Lambda_{\overline{MS}}(5) \) of Ref. 17) and the HTC of Ref. 9 as inputs, there is only one free parameter left for each set of data, i.e. the sea normalization \( P(Q^2) \). Note that, to account for possible systematics either in the data or in the theoretical inputs (particularly the possible inadequacy both of the PQCD approximation and of the phenomenological parametrization used, as well as of the evaluation of the HTC), different parameters will be used for different data, even when these latter have been normalized at the same value of \( < Q^2 > \).

4. The isosinglet sum rule and the quark spins in the nucleon.

Before turning to fitting the parametrizations to the data available (as of November 1995) on the PDIS structure functions \( g_1 \), it is better to consider the information we possess of the axial couplings appearing in the right–hand sides of the first–moment sum rules, Eqs. (1) and (4). All pieces of information available have been summarized in Table III, which deserves some comments. Usually most
analyses\textsuperscript{39} of these couplings stop at the information coming from asymmetries in
the decay products momenta and/or polarizations: while this is enough in most
of the cases, in some no such measurements are either available or possible, and
disregard of the information coming from the rates can have drastic effects, even
on the correctness of the conclusions inferred. Enough to say that dropping the
information on the $\Delta S = 0$, $\Sigma \rightarrow \Lambda$ couplings leads to the conclusion that flavour
SU(3) symmetry breaking effects are negligible\textsuperscript{39}, relying in fact on a single point,
being the coupling of the $\Xi$ to both $\Lambda$ and $\Sigma$ not as well known as those of the latter
two to the nucleon. Accordingly, Table III includes evidence from both asymmetries
and rates, while the detailed analysis of these latter can be found elsewhere\textsuperscript{40}. One
further thing evident from Table III is that the two sources of information yield fully
compatible values for the axial couplings, contrary to the statement of Jaffe and
Manohar\textsuperscript{41}, which originated from a completely outdated treatment of the decay
rate data.

| Table III |
| --- |
| Data on octet baryon axial couplings |

| Decays | from rates\textsuperscript{40} | from asymmetries\textsuperscript{42} | SU(3) formulæ\textsuperscript{43} |
| --- | --- | --- | --- |
| $n \rightarrow p e^- \bar{\nu}_e$ | $1.2553 \pm 0.0018$ | $1.2573 \pm 0.0028$ | $F + D$ |
| $\Sigma^- \rightarrow \Lambda e^- \bar{\nu}_e$ | $0.723 \pm 0.017$ | | $D - \sqrt{3} \tan \phi F$ |
| $\Sigma^+ \rightarrow \Lambda e^+ \nu_e$ | $0.750 \pm 0.094$ | | $D + \sqrt{3} \tan \phi F$ |
| $\Sigma^- \rightarrow n e^- \bar{\nu}_e$ | $-0.330 \pm 0.023$ | $-0.340 \pm 0.017$ | $F - D$ |
| $\Sigma^- \rightarrow n \mu^- \bar{\nu}_\mu$ | $-0.237 \pm 0.078$ | | $F - D$ |
| $\Lambda \rightarrow p e^- \bar{\nu}_e$ | $0.729 \pm 0.011$ | $0.718 \pm 0.015$ | $F + \frac{1}{3}(1 + \frac{4}{\sqrt{3}} \tan \phi) D$ |
| $\Lambda \rightarrow p \mu^- \bar{\nu}_\mu$ | $0.756 \pm 0.139$ | | $F + \frac{1}{3}(1 + \frac{4}{\sqrt{3}} \tan \phi) D$ |
| $\Xi^- \rightarrow \Lambda e^- \bar{\nu}_e$ | $0.265 \pm 0.044$ | $0.250 \pm 0.050$ | $F - \frac{1}{3}(1 + \frac{4}{\sqrt{3}} \tan \phi) D$ |
| $\Xi^- \rightarrow \Lambda \mu^- \bar{\nu}_\mu$ | $0.76 \pm 0.47$ $0.76$ | | $F - \frac{1}{3}(1 + \frac{4}{\sqrt{3}} \tan \phi) D$ |
| $\Xi^- \rightarrow \Sigma^0 e^- \bar{\nu}_e$ | $1.216 \pm 0.147$ | | $F + (1 - \frac{4}{\sqrt{3}} \tan \phi) D$ |

Table III does not contain, for reasons of internal clarity, an additional piece
of information, coming from the study of the asymmetries in the $\Sigma^- \rightarrow \Lambda \beta$–decay,
i.e. the ratio\textsuperscript{42} $g_V/g_A = -0.01 \pm 0.10$, which can be considered a bound on the
size of the $\Sigma^0 - \Lambda$ mixing effects, which have been included following the analysis
by Karl\textsuperscript{43}, and fixing $\tan \phi$ to the value given by him.

The SU(3)–symmetric fit to all data listed in Table III, plus the ratio $g_V/g_A$
for the decay $\Sigma^- \rightarrow \Lambda e^- \bar{\nu}_e$, yields the parameters $F = 0.4678$ and $D = 0.7876$, or
$g_8 = 3F - D = 0.616 \pm 0.022$ and $g_A = F + D = 1.2554 \pm 0.0020$ (note that this
latter acts almost as a constraint due to the very high precision of neutron data
on both asymmetries and rates); the $\chi^2$ of the fit is not very high with respect to
the number of data points, being 20.17 (about 2 units better than without $\Sigma^0$–$\Lambda$ mixing) versus 15, but is concentrated almost exclusively in the $\Sigma^- \rightarrow \Lambda$ rate, whose datum on the axial coupling lies $3\sigma$ below the SU(3)–symmetric fit value. Coming this datum from a good–quality experiment, this fact has to be taken as initial evidence for some flavour SU(3) breaking in these couplings, being the mixing required to explain this discrepancy more than five times the one calculated by Karl, and thus giving a ratio $g_V/g_A$ more than twice its experimental $1\sigma$ limit. This must be remembered when using flavour symmetry to extract the nucleon spin composition. A further point raised in this context is also the possible effect (particularly on the $\Delta S = 1$ transitions) of the tensor components in the weak axial current: the good agreement between the two columns of data in Table III seems to indicate that the effect is not as large as indicated by some fits, since it would affect the axial couplings extracted from the asymmetries differently from those extracted from the rates.

Since it is the aim of the present paper to concentrate on the analysis of the spin composition of the nucleon, the PDIS structure functions will not be fit treating both $C(Q^2)$ and $P(Q^2)$ as free parameters, but rather taking the first from the right–hand side of the BjSR, Eq. (1), and fitting only the second to the data, by experiment. Table IV will present the expectations for the right–hand side of the BjSR in NNLO PQCD, including the HTC of Ross and Roberts, together with the integrals evaluated by two experimental groups, the E143 Collaboration at SLAC and the Spin Muon Collaboration at CERN. In the same table, we shall also display the two expectations for the right–hand side of the EJSR obtained assuming validity of the OZI rule, i.e. $\Sigma = g_8$, and either a) the naïve parton model identification $g_0 = \Sigma$ (i.e. decoupling the contribution from the axial anomaly, as is the case for massive quarks), or b) the definition in Eq. (5) with the normalizations scale $\mu^2$ fixed so that $\alpha_s(\mu^2) = 1$ (and $< L_z >= 0$): here $g_8$ is assumed equal to the above flavour SU(3) symmetry prediction. The choice a) amounts to using an isoscalar, PQCD corrected version of the original EJSR, while b) serves to give an idea of the effect induced by a very reasonably sized axial anomaly contribution.

From this table one can read two facts, already mentioned above: the running with $Q^2$ of the EJSR is slower than that of the BjSR, and therefore harder to see in the data unless higher precisions are reached, and the difference between the “anomalous” and “non–anomalous” versions of the EJSR, described by Eq. (8) (apart from the lower asymptotic value of the first, which can always be traded for a breaking of the OZI rule, i.e. a $\Delta s \neq 0$, in the second), is even smaller, and thus much harder to see.
Table IV
Sum rule expectations for BjSR and naïve EJSR

| $Q^2$/GeV$^2$ | EJSR (a) | EJSR (b) | BjSR | BjSR integrals | Ref.   |
|--------------|----------|----------|------|----------------|--------|
| 2.0          | 0.1278   | 0.0917   | 0.1492 | 0.163 ± 0.019  | 13     |
| 3.0          | 0.1378   | 0.0993   | 0.1636 |                |        |
| 4.6          | 0.1448   | 0.1045   | 0.1733 |                |        |
| 6.0          | 0.1479   | 0.1068   | 0.1776 |                |        |
| 8.0          | 0.1506   | 0.1088   | 0.1813 |                |        |
| 10.0         | 0.1523   | 0.1099   | 0.1836 | 0.194 ± 0.038  | 46     |
| 10.7         | 0.1528   | 0.1103   | 0.1843 |                |        |
| 12.0         | 0.1535   | 0.1108   | 0.1853 |                |        |

In Table V there will be displayed the parameters $P(Q^2)$ obtained from the fits to the seven sets of PDIS data analyzed here: the re–normalized EMC set of data on proton$^{1,5}$ at $<Q^2>=10.7$ GeV$^2$, the E142 neutron data$^4$ at $<Q^2>=2.0$ GeV$^2$, the SMC preliminary deuteron data$^{14}$ at $<Q^2>=4.6$ GeV$^2$, the SMC proton data$^{24}$ at $<Q^2>=10.0$ GeV$^2$, the E143 data on proton$^{12}$ and deuteron$^{13}$ at $<Q^2>=3.0$ GeV$^2$, and the SMC new deuteron data$^{40}$ at $<Q^2>=10.0$ GeV$^2$. The very preliminary data of the SLAC–Yale E80 and E130 Collaborations$^{47}$ have not been included in the fits, since they cover either very low values of $Q^2$ or a very limited range in $x$ values, and would have been, besides, of very little statistical significance; also, the recently appeared E143 data$^{28}$ are not included, since they have become available only during the final redaction of the present paper. The factor $\frac{1}{2} \cdot (1 - \frac{3}{2} \omega_D)$, with $\omega_D = 0.05$, has been explicitly included for deuteron data, so that all $P(Q^2)$ obtained from the fits have the same normalization. All are relatively large and negative, implying a strong reduction from the pure valence contribution to the EJSR, which under our constraint Eq. (15) would be $I_0^{p+n}(Q^2)_{val} = I_0^{p-n}(Q^2)_{val} = \frac{1}{6} \cdot C(Q^2)$, i.e. equal to the BjSR integral: from this and the previous table one can see that some negative sea is already expected even at the level of a naïve formulation of the (PQCD corrected) EJSR. The table displays also systematic variations in the parameter both with the model used for the valence distributions and with the nature of the target, variations which in the opinion of the author cast some doubt on the possibility of really testing PQCD in the isoscalar combination: since the two kind of variations are of the same order of magnitude, and comparable to the statistical errors from the fits, their origin is difficult to trace.
Table V
Values of $P(Q^2)$ from fits to the data
(Shown in parenthesis are the $\chi^2$–values per data point)

| Expt. | $Q^2$/GeV$^2$ | FRP fit | SRP fit | BLP fit |
|-------|---------------|---------|---------|---------|
| E142$^n$ | 2.0           | $-0.38 \pm 0.07$ (0.7) | $-0.35 \pm 0.07$ (0.5) | $-0.23 \pm 0.13$ (3.1) |
| E143$p$  | 3.0           | $-0.69 \pm 0.04$ (1.9) | $-0.72 \pm 0.05$ (2.2) | $-0.73 \pm 0.08$ (6.2) |
| E143$d$  | 3.0           | $-0.53 \pm 0.04$ (1.3) | $-0.54 \pm 0.04$ (1.2) | $-0.51 \pm 0.08$ (4.7) |
| SMC$d$   | 4.6           | $-0.53 \pm 0.24$ (0.6) | $-0.59 \pm 0.24$ (0.7) | $-0.66 \pm 0.27$ (1.1) |
| SMC$p$   | 10.0          | $-0.70 \pm 0.18$ (1.7) | $-0.75 \pm 0.14$ (1.0) | $-0.80 \pm 0.13$ (0.7) |
| SMC$d$   | 10.0          | $-0.44 \pm 0.18$ (2.7) | $-0.49 \pm 0.18$ (2.5) | $-0.53 \pm 0.22$ (3.8) |
| EMC$^p_{rev}$ | 10.7 | $-0.74 \pm 0.18$ (0.4) | $-0.79 \pm 0.18$ (0.8) | $-0.82 \pm 0.20$ (1.1) |
| $\chi^2/N_{pts}$ | 149.7 / 102 | 152.3 / 102 | 373.7 / 102 |

These results for the sea parameter $P(Q^2)$ can now be turned into values for the EJSR integrals, since these can be written as

$$I_0^{p+n}(Q^2) = \frac{1}{6} \cdot C(Q^2) + 2 \cdot B\left(\frac{5}{4}, 8\right) \cdot P(Q^2) .$$  (20)

Note however that the right–hand side of this equation corresponds to what is actually interpolated by the fits only in the case of a deuterium target, when $I_0^d(Q^2) = \frac{1}{7} \cdot (1 - \frac{3}{7} \omega_D) \cdot I_0^{p+n}(Q^2)$: for a proton or neutron target the fits measure, respectively,

$$I_0^p(Q^2) = \frac{1}{6} \cdot C(Q^2) + B\left(\frac{5}{4}, 8\right) \cdot P(Q^2) .$$  (20$'$)

in the proton case, and

$$I_0^n(Q^2) = B\left(\frac{5}{4}, 8\right) \cdot P(Q^2) .$$  (20$''$)

in the neutron one, to which one must, respectively, either subtract or add the BjSR right–hand side:

$$I_0^{p+n}(Q^2) = 2 \cdot I_0^p(Q^2) - \frac{1}{6} \cdot C(Q^2) .$$  (21)

in the first case, and

$$I_0^{p+n}(Q^2) = 2 \cdot I_0^n(Q^2) + \frac{1}{6} \cdot C(Q^2) .$$  (21$'$)

in the second. In these cases the last terms are no longer part of the valence distribution normalization, and therefore they carry a theoretical error, difficult to
quantify\textsuperscript{48}, but approximable with that of the most precise experimental determination of the BjJSR integral (i.e. with that of Ref. 46).

Taking these aspects into account, the fits yield for the EJSR the integrals listed in Table VI for the three valence parametrizations, which show a marked reduction with respect the naïve expectations for this quantity, tabulated in the second column of Table IV. As already said, this is evidence for either presence of a sizeable reduction of $g_8$ and/or $g_0$ (due to a polarized strange–quark density or the breaking of flavour SU(3) symmetry\textsuperscript{49}) or an appreciable contribution from the axial anomaly (or both).

From the previous table one can also see that the three parametrizations, though behaving quite differently at $x \rightarrow 0$ in the combination $g_1^{p+n}$, do not produce neither appreciably different $\chi^2$’s (the BLP fit turns worse than the other two, with only by a factor $\sim 2.5$), nor large variations (contrary to naïve expectations) for $I_0^{p+n}(Q^2)$: this is due to the normalization of the valence components to the BjJSR imposed by Eq. (15)–(16), valid for all parametrizations regardless of their behaviour as $x \rightarrow 0$ and quite robust, since the only isovector contribution from the sea could come, under the hypotheses adopted here, from the Pauli principle “penalty factor”. The different behaviours as $x \rightarrow 0$ of the valence parametrizations reflect thus only on the quality of the fits, tending to prefer distributions which stay finite in this limit, since diverging ones become harder to accommodate to the behaviours of $g_1$ for neutron and deuteron targets at moderate and high values of the Bjorken variable $x$.

| Expt. | $Q^2$/GeV$^2$ | FRP fit $\pm$ | SRP fit $\pm$ | BLP fit $\pm$ |
|-------|---------------|----------------|----------------|----------------|
| E142$^n$ | 2.0 | 0.098 ± 0.020 | 0.103 ± 0.020 | 0.118 ± 0.025 |
| E143$p$ | 3.0 | 0.073 ± 0.020 | 0.068 ± 0.020 | 0.068 ± 0.022 |
| E143$d$ | 3.0 | 0.094 ± 0.005 | 0.092 ± 0.005 | 0.096 ± 0.010 |
| SMC$d$ | 4.6 | 0.103 ± 0.032 | 0.095 ± 0.032 | 0.086 ± 0.036 |
| SMC$p$ | 10.0 | 0.090 ± 0.032 | 0.084 ± 0.028 | 0.077 ± 0.028 |
| SMC$d$ | 10.0 | 0.124 ± 0.024 | 0.119 ± 0.023 | 0.114 ± 0.029 |
| EMC$^p_{rev}$ | 10.7 | 0.086 ± 0.032 | 0.080 ± 0.032 | 0.075 ± 0.034 |

Any interpretation of these data (beyond the existence of a sizeable, negative polarization in the sea) has to make clear the nature of the unitary–singlet piece in the EJSR. If one follows the naïve parton model picture to the end, and identifies $g_0$ with $\Sigma$, one must also have $\tilde{\gamma} = 0$ from the vanishing of its anomalous dimension, so that the EJSR becomes now, since $g_8 = \Sigma - 3\Delta s$,

$$I_0^{p+n}(Q^2) = \frac{2}{9} \cdot C_1(\alpha_s) + \frac{1}{18} \cdot C_8(\alpha_s) \cdot \Sigma - \frac{1}{6} \cdot C_8(\alpha_s) \cdot \Delta s + (h.t.)_{I_t=0}, \quad (22)$$
which, putting $\delta I = -(h.t.)|_{t=0}$, $F(Q^2) = \frac{2}{3}C_1(\alpha_s) + \frac{1}{18}C_8(\alpha_s)$ and $R(Q^2) = C_8(\alpha_s)/[6 \cdot F(Q^2)]$, can be turned into a linear relation between $\Sigma$ and $\Delta s$,

$$\Sigma = \frac{I_0^{p+n}(Q^2) + \delta I}{F(Q^2)} + R(Q^2) \cdot \Delta s = \Sigma_0(Q^2) + R(Q^2) \cdot \Delta s , \quad (23)$$

to be eventually combined with the flavour SU(3) prediction for $g_8 = \Sigma - 3\Delta s$ (and with $g_A = \Delta u - \Delta d$) to disentangle the quark spin components in the proton (modulo flavour SU(3) symmetry violation effects).

Though simple and attractive, Eq. (22) runs however against the PQCD prediction on the evolution of light parton densities: when the mass–parameter of these latter is neglected, the gluon–to–quark splitting function dictates for $g_0$ an anomalous evolution and at the same time identifies it with the combination in Eq. (5). One is now faced with the additional problem of choosing a normalization scale $\mu^2$ to relate it to the spin composition of the nucleon. For this, let us consider the angular momentum sum rule

$$\Sigma + 2 \cdot [\Delta G(Q^2) + <L_z>] = 1 , \quad (24)$$

which has to be satisfied at all momentum scales. Here both $\Delta G$ and $<L_z>$ must run with the momentum scale, and from the evolution equation for $g_0$ one can see that the first will run like

$$\Delta G(Q^2) = \frac{\alpha_s(\mu^2)}{\alpha_s(Q^2)} \cdot \exp[\tilde{\gamma}(\alpha_s(Q^2)) - \tilde{\gamma}(\alpha_s(\mu^2))] \cdot \{ \Delta G(\mu^2) +$$

$$+ \frac{2\pi}{3\alpha_s(\mu^2)} \cdot \Sigma \cdot \{ \exp[\tilde{\gamma}(\alpha_s(\mu^2)) - \tilde{\gamma}(\alpha_s(Q^2))] - 1 \} \} , \quad (25)$$

while $<L_z> = (1 - \Sigma)/2 - \Delta G(Q^2)$, where the first term $(1 - \Sigma)/2$ is positive in all reasonably conceivable models of the nucleon spin structure.

It is therefore “natural” to assume that $<L_z>$ will vanish at some low mass scale, where $\Sigma < 1$ will be balanced by some polarized glue distributed between the constituent quarks. If we take $\mu^2$ as such a scale, then $\Delta G(\mu^2) = (1 - \Sigma)/2$ and the EJSR becomes

$$I_0^{p+n}(Q^2) = \left\{ \frac{2}{9} [1 + 3\alpha_s(\mu^2)/4\pi] \cdot \exp[\tilde{\gamma}(\alpha_s) - \tilde{\gamma}(\alpha_s(\mu^2))] \cdot C_1(\alpha_s) + \frac{1}{18} \cdot C_8(\alpha_s) \right\} \cdot \Sigma -$$

$$- \frac{1}{6} \cdot C_8(\alpha_s) \cdot \Delta s + (h.t.)|_{t=0} - \frac{\alpha_s(\mu^2)}{6\pi} \cdot \exp[\tilde{\gamma}(\alpha_s) - \tilde{\gamma}(\alpha_s(\mu^2))] \cdot C_1(\alpha_s) \quad (26)$$

The linear relation between $\Sigma$ and $\Delta s$, Eq. (23) is still holding, but now $F(Q^2)$ is changed including the integrated anomalous dimension factor $[1 + 3\alpha_s(\mu^2)/4\pi]$. 

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exp[\(\tilde{\gamma}(\alpha_s) - \tilde{\gamma}(\alpha_s(\mu^2))\)] to multiply the PQCD coefficient \(C_1(\alpha_s)\), and to the correction \(\delta I\) one has to add a positive contribution — equal to minus the last term in Eq. (26) — coming again from the anomaly contribution at the normalization scale \(\mu^2\). This last term becomes thus comparable to the EJSR integral \(I_0^{p+n}(Q^2)\), so that it is not too difficult in this approach to recover a quark content of the nucleon spin very close to the flavour SU(3) prediction for \(g_8\), as people were naïvely expecting before the EMC results became public.

Table VII lists the numerical values for the terms \(\delta I(Q^2)\), \(F(Q^2)\) and \(R(Q^2)\) in Eq. (23) under the two hypotheses: for the second one, \(\mu^2\) is selected here by putting \(\alpha_s(\mu^2) = 1\), a not unreasonable choice in view of the fact that we expect it to lie at the bottom end of the PQCD validity range, due to the smallness of the \(\Delta G\) involved, whose size at a scale \(Q^2 \simeq 1 \sim 2\) GeV\(^2\) is expected to be comparable with \(<x_G> \simeq 1/2\). One can note that the PQCD evolution has its largest effect in the factor \(F(Q^2)\), rescaling \(I_0^{p+n}(Q^2)\) to \(\Sigma_0\), which in the parton model limit should take the value 5/18, while \(R(Q^2)\) in both cases considered deviates from its parton model value 3/5 only at the lowest values of \(Q^2\).

Table VII

| \(Q^2/\text{GeV}^2\) | \(F(Q^2)\) | \(\delta I\) | \(R(Q^2)\) |
|-----------------|-------------|-------------|-------------|
|                 | Eq. (22)    | Eq. (26)    | Eq. (22)    | Eq. (26)    | Eq. (22)    | Eq. (26)    |
| 2.0             | 0.2310      | 0.2315      | 0.0145      | 0.0509      | 0.5566      | 0.5553      |
| 3.0             | 0.2394      | 0.2368      | 0.0097      | 0.0465      | 0.5712      | 0.5775      |
| 4.6             | 0.2453      | 0.2403      | 0.0063      | 0.0435      | 0.5795      | 0.5915      |
| 10.0            | 0.2520      | 0.2442      | 0.0029      | 0.0404      | 0.5874      | 0.6062      |
| 10.7            | 0.2525      | 0.2444      | 0.0027      | 0.0403      | 0.5878      | 0.6072      |

Using these values one can extract the terms \(\Sigma_0(Q^2)\) in the linear relationship of Eq. (23), \(\Sigma = \Sigma_0(Q^2) + R(Q^2) \cdot \Delta s\), listed in Table VIII.

To sum the results into single values, the different experiments can be averaged à la PDG, so as to weight their different (both experimental and theoretical) accuracies: the outcomes are

\[
\Sigma = (0.418 \pm 0.021) + 0.5582 \cdot \Delta s
\]

for the “non–anomalous” treatment of the EJSR, while the correct inclusion of the axial anomaly (with the choices \(\alpha_s = 1\), \(< L_z >= 0\) at its normalization scale \(Q^2 = \mu^2\) leads to an average

\[
\Sigma = (0.579 \pm 0.022) + 0.5796 \cdot \Delta s
\]
One can note that the treatments proposed here lead in both cases to higher values for $\Sigma$ than those to be found in the recent literature$^{6,15,27,50}$, though of course in the second case much higher than in the first. It also noteworthy (in the author’s opinion) that the second average for $\Sigma_0$ is remarkably close to the flavour SU(3) value derived above for $g_8$, since estimates on its flavour–symmetry breaking part led to expect a value slightly lower than the symmetry prediction$^{49}$.

Note also that this averaging does not produce results much different from those obtainable selecting the deuterium data alone, which should be better from the point of view of theoretical systematics, since in the handling of the data one does not have to correct them with the BjSR integrals. The reduction in the scale of the error is of course a product of the use of the full statistics of the (almost) complete world data set.

Intersecting the above error bands on $\Sigma$ and $\Delta s$ with that from the flavour SU(3) result

$$g_8 = \Sigma - 3 \cdot \Delta s = 0.616 \pm 0.022 ,$$  \hspace{0.5cm} (29)

one can now obtain the results (modulo of course SU(3) violations)

$$\Sigma = 0.373 \pm 0.026 \hspace{0.5cm} \text{and} \hspace{0.5cm} \Delta s = -0.081 \pm 0.012$$  \hspace{0.5cm} (30)

for the “non–anomalous” treatment of the EJSR, and

$$\Sigma = 0.570 \pm 0.028 \hspace{0.5cm} \text{and} \hspace{0.5cm} \Delta s = -0.015 \pm 0.013 ,$$  \hspace{0.5cm} (31)

for the description of $g_0(Q^2)$ including the axial anomaly, not very far from the naïve OZI expectations $\Sigma \simeq g_8$ and $\Delta s \simeq 0$, and practically consistent with them, if the theoretical uncertainties$^{49}$ entailed by the use of flavour SU(3) symmetry are duly considered.
Comparing the present analysis with the recently published ones\textsuperscript{15,27,50}, the values of Eq. (31) reproduce within 1σ those for the asymptotic limit of the unitary–singlet axial charge $g_0(\infty)^{N_f=3}$, misnamed $\Sigma$ in much of the current literature,

$$g_0(\infty)^{N_f=3} = \exp -\tilde{\gamma}(\alpha_s(\mu^2)) \cdot [\Sigma - \frac{3\alpha_s(\mu^2)}{4\pi}(1 - \Sigma)] = 0.342 \pm 0.025,$$

but note also that neglecting the anomaly (and trading it for a sizeable $\Delta s$) we find a value for the spin sum $\Sigma$ still 20% higher than the currently quoted (and, in the author’s opinion, erroneously derived) values. Note, as well, that, unless one assumes SU(3) to be a good symmetry for the axial charges, one can not conclude that the low $\Sigma_0$ obtained with the identification $g_0 = \Sigma$ is evidence for a large and negative $\Delta s$, since the value $0.418 \pm 0.021$ lies inside the allowed range of flavour–SU(3)–violating effects expected\textsuperscript{49} for $g_8$.

A test of these two possible ways out of the conundrum on the smallness of $g_0$, in the range of $Q^2$ explored by the PDIS experiments, is offered by the possibility of measuring the size and sign of $\Delta G(Q^2)$, on which the “anomalous” interpretation gives sharp predictions: indeed with the above normalizations one can rewrite Eq. (25) as

$$\Delta G(Q^2) = \frac{\alpha_s(\mu^2)}{\alpha_s(Q^2)} \cdot \exp[\tilde{\gamma}(\alpha_s(Q^2)) - \tilde{\gamma}(\alpha_s(\mu^2))] \cdot \left\{ \frac{1}{2} + \Sigma \cdot \left[ \frac{2\pi}{3\alpha_s(\mu^2)} \cdot (\exp[\tilde{\gamma}(\alpha_s(\mu^2)) - \tilde{\gamma}(\alpha_s(Q^2))] - 1) - \frac{1}{2} \right] \right\}$$

(33)

which, due to the smallness of the coefficient of $\Sigma$ for $Q^2 \geq 2 \text{ GeV}^2$, gives quite accurate predictions on the gluonic polarization $\Delta G$ in the nucleon, tabulated below for the value of $\Sigma$ given by Eq. (31): these predictions could be tested, for instance, measuring the polarization asymmetries for the semi–inclusive process $\ell^\pm N \rightarrow \ell^\pm J/\psi + X$, probably a much clearer signature for $\Delta G$ than that proposed via semi–inclusive kaon deep–inelastic production\textsuperscript{30} to test the size and sign of $\Delta s$.

The third column in this last table displays clearly the effect of the higher orders of PQCD in lowering the values of $\Delta G$ at low $Q^2$ with respect to the leading–order prediction, which would lead to a constant value, even higher than the asymptotic limit listed in the last line, due to the large higher–order terms in the coefficients $C_{1,8}(\alpha_s)$ (see Table II).
4. Summary and conclusions.

The several experiments now conducted on the PDIS asymmetries (SLAC experiments E142\textsuperscript{4} and E143\textsuperscript{12,13}, and the collaborations EMC\textsuperscript{1,5} and SMC\textsuperscript{14,24,46} at CERN) do not contradict conventional expectations on the spin structure of the nucleon, namely that one should have a strange spin component $\Delta s$ much smaller than the non–strange–quark sea ones on one side, but on the other no pure valence–quark spin components either, as known since more than twenty years (and a complete list of references would be almost as long as this paper: it is enough to refer to the recent, illuminating papers by Lipkin\textsuperscript{51}) from the reduction in $g_A$ with respect to the na"ive constituent quark model prediction $g_A = 5/3$.

One finds that it is possible to reduce drastically $g_0(Q^2)$ from the parton model plus OZI rule expectation $g_0 \simeq g_8$, due both to the presence of the QCD axial anomaly\textsuperscript{22} and to its anomalous dimension\textsuperscript{23} (and more to the second than to the first reason): a correct use of QCD with high orders included (which make these effects even larger than the next–to–leading–order alone) is necessary to describe the EJSR without conflict with both the experiments and our expectations. The “spin crisis” of 1988 was the result as much of an inadequate theoretical description as of the low normalization in the $F_2^p$ values used by the EMC.

However, the presence of the anomaly is not strictly required by the data\textsuperscript{52}: from a purely statistical point of view, these could be described also by trading it
for a sizeable (not necessarily strange) sea component in the nucleon spin: but an adequate parton model description excluding the contribution of the anomaly has also to exclude the anomalous dimension it carries (for the diagonalization of the evolution matrix for the pair $\Sigma$ and $\Delta G$ requires one of the eigenvalues to vanish), thus increasing by as much as 20% the total spin sum $\Sigma$ over currently quoted values, and making it possible to explain the data through a sizeable, though not impossible, reduction in $g_8 \simeq g_0 = \Sigma$ due to flavour–SU(3)–breaking effects $^{49}$.

Direct measurements of either a large $\Delta G$ or a large $\Delta s$ — or even of the absence of both, in the case of a large, negative SU(3)–breaking effect — are clearly called for to make a definitive choice between these possible ways out of the “nucleon spin crisis”.

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REFERENCES AND FOOTNOTES

1. European Muon Collaboration (J. Ashman, et al.): Phys. Lett. B 206 (1988) 364.
2. J.D. Bjorken: Phys. Rev. 148 (1966) 1467; Phys. Rev. D 1 (1970) 1376.
3. Ya.Ya. Balitskiï, V.M. Braun and A.V. Kolesnichenko: JETP Lett. 50 (1989) 61; Phys. Lett. B 242 (1990) 245, erratum B 318 (1993) 648. See also Refs. 9–10 for more recent re–evaluations.
4. SLAC E142 Collaboration (P.L. Anthony, et al.): Phys. Rev. Lett. 71 (1993) 959.
5. European Muon Collaboration (J. Ashman, et al.): Nucl. Phys. B 328 (1989) 1.
6. J. Ellis and M. Karliner: Phys. Lett. B 313 (1993) 131; “PAN XIII. Particles and Nuclei, Proc. of the 13th Int. Conf., Perugia 1993”, ed. by A. Pascolini (World Scientific, Singapore 1994), p. 48.
7. R.L. Workman and R.A. Arndt: Phys. Rev. D 45 (1992) 1789; V.D. Burkert and B.L. Ioffe: Phys. Lett. B 296 (1992) 223; V.D. Burkert and Z.–J. Li: Phys. Rev. D 47 (1993) 46; V. Bernard, N. Kaiser and U.–G. Meißner: Phys. Rev. D 48 (1993) 3062.
8. S.B. Gerasimov: Yad. Fiz. 2 (1965) 598; 5 (1967) 1263; S.D. Drell and A.C. Hearn: Phys. Rev. Lett. 16 (1966) 908.
9. G.G. Ross and R.G. Roberts: Phys. Lett. B 322 (1994) 425.
10. E. Stein, P. Gornicki, L. Mankiewicz and A. Schäfer: Phys. Lett. B 343 (1995) 369; B 353 (1995) 107.
11. S.A. Larin and J.A.M. Vermaseren: Phys. Lett. B 259 (1991) 345.
12. SLAC E143 Collaboration (K. Abe, et al.): Phys. Rev. Lett. 74 (1995) 346.
13. SLAC E143 Collaboration (K. Abe, et al.): Phys. Rev. Lett. 75 (1995) 25.
14. Spin Muon Collaboration (B. Adeva, et al.): Phys. Lett. B 302 (1993) 533.
15. See the BjSR analysis of J. Ellis and M. Karliner: Phys. Lett. B 341 (1995) 397.
16. A.L. Kataev: Phys. Rev. D 50 (1994) R 5469. This are the PQCD corrections also employed by Ref. 15, although in a way slightly different from here.
17. S. Bethke and S. Catani: preprint CERN–TH. 6484/92 (Geneve 1992), summary talk presented at the “27th Rencontre de Moriond”, Les Arcs, March 1992.
18. Besides PDIS data — see the analysis in Ref. 15 — the only real improvements on $\alpha_s$ come from the analyses of the HERA data: these however confirm essentially the results of Ref. 17. See, for instance, the report from the ZEUS Collaboration (M. Derrick, et al.): preprint DESY 95–182 (Hamburg, October 1995), also available as hep–ex/9510001.
19. See, for instance: G. Preparata and P.G. Ratcliffe: EMC, E142, SMC, Bjorken, Ellis–Jaffe … and All That, Univ. di Milano report (1993); more recently, see again: P.G. Ratcliffe: Nuovo Cimento A 107 (1994) 2211, and references therein.
20. J. Ellis and R.L. Jaffe: Phys. Rev. D 9 (1974) 1444; Phys. Rev. D 10 (1974) 1669.
21. J. Kodaira: Nucl. Phys. B 165 (1980) 129.
22. G. Altarelli and G.G. Roos: Phys. Lett. B 212 (1988) 391; R.D. Carlitz, J.C. Collins and A.H. Mueller: Phys. Lett. B 214 (1988) 229; G. Altarelli and B. Lampe: Z. Phys. C 47 (1990) 315. For the opposite view see: G.T. Bodwin and J. Qiu: Phys. Rev. D 41 (1990) 2755. However the controversy seems to have been settled, hopefully in a definitive way, see: U. Ellwanger: Phys. Lett. B 259 (1991) 469; W. Vogelsang: Z. Phys. C 50 (1991) 275; Nucl. Phys. B 362 (1991) 3.
23. S.A. Larin: Phys. Lett. B 303 (1993) 113, 334.
24. Spin Muon Collaboration (D. Adams, et al.): Phys. Lett. B 329 (1994) 399, erratum B 339 (1994) 332.
25. BCDMS Collaboration (A.C. Benvenuti, et al.): Phys. Lett. B 223 (1989) 485; Phys. Lett. B 237 (1989) 592, 599.
26. New Muon Collaboration (P. Amaudruz, et al): Phys. Lett. B 295 (1992) 159; Nucl. Phys. B 371 (1992) 3.
27. G. Altarelli, P. Nason and G. Ridolfi: Phys. Lett. B 320 (1994) 153, erratum B 325 (1994) 538.
28. SLAC E143 Collaboration (K. Abe, *et al.*): report SLAC–PUB–95–6997 (Stanford, November 1995), also available as hep–ex/9511015. These data are a finer rebinning of those in Refs. 12–13, plus some data at higher $Q^2$ but over a restricted $x$–range, not included in the present analysis, completed before they were available.

29. A.D. Martin, W.J. Stirling and R.G. Roberts: *Phys. Rev.* D 47 (1993) 867; *Phys. Lett.* B 306 (1993) 145, *erratum* B 309 (1993) 492; H. Plothow–Besch: *Comput. Phys. Commun.* 75 (1993) 396.

30. F.E. Close: *Perspectives in Nuclear Physics at Intermediate Energies, 6th Workshop*, ed. by S. Boffi, C. Ciofi degli Atti and M.M. Giannini (World Scientific, Singapore 1994), p. 103, and Ref. 36.

31. See, for instance: J. Bartelski and S. Tatur: report CAMK 95–288 (Warsaw, February 1995), also available as hep–ph/9502271.

32. G. Altarelli, N. Cabibbo, L. Maiani and R. Petronzio: *Phys. Lett.* 48 B (1974) 435, and the review by G. Altarelli: *Riv. Nuovo Cimento* 4 (1974) 335.

33. H. Fritzsch: *Phys. Lett.* B 229 (1989) 122; *Mod. Phys. Lett.* A 5 (1990) 625, 1815; *Phys. Lett.* B 256 (1991) 75; report CERN–TH 7079–93 (Geneve, March 1994), also available as hep–ph/9403206.

34. S.J. Brodsky and I. Schmidt: *Phys. Lett.* B 234 (1990) 144; S.J. Brodsky, M. Burkardt and I. Schmidt: *Nucl. Phys.* B 441 (1995) 197.

35. The author recalls such parametrizations, which are scattered throughout the DIS literature, being presented as “natural parametrizations” in seminars on the parton model given by the late R.P. Feynman at SLAC in the mid–seventies. For a paper reference, see: R.D. Field and R.P. Feynman: *Phys. Rev.* B 15 (1977) 2590, where mentions of Pauli–principle effects can also be found.

36. An interesting reading, which the author finds as relevant to early QCD as the *Blegdamsvej Faust* to quantum mechanics, is: A. De Rújula, J. Ellis, R. Petronzio, G. Preparata and W. Scott: report CERN–TH. 2778 (Geneve, November 1979). For an equally informative, but easier to find, reference see: F.E. Close, *An Introduction to Quarks and Partons* (Academic Press, London 1979).

37. The same conclusion have been independently reached and extensively checked, though with different formulations, by: F. Buccella, G. Miele, G. Migliore and V. Tibullo: *Z. Phys.* C 68 (1995) 631.

38. S.D. Bass and P.V. Landshoff: *Phys. Lett.* B 336 (1994) 537.

39. P.G. Ratcliffe: *Phys. Lett* B 242 (1990) 271, and Sez. I.N.F.N. di Milano report (September 1995), also available as hep–ph/9509237; M. Roos: *Phys. Lett.* B 246 (1990) 179; C. Avenarius: *Phys. Lett.* B 272 (1991) 71; B. Ehrnsperger and A. Schäfer: *Phys. Lett.* B 348 (1995) 619.

40. P.M. Gensini: *Nuovo Cimento* A 103 (1990) 303; P.M. Gensini and G. Violini: $\pi N$ Newslett. 9 (1993) 80, and Univ. di Perugia report, to appear soon.
41. R.L. Jaffe and A.V. Manohar: Nucl. Phys. B 337 (1990) 509.
42. Particle Data Group (L. Montanet, et al.): Phys. Rev. D 50 (1994) 1173.
43. G. Karl: Phys. Lett. B 328 (1994) 149, erratum B 322 (1994) 473. See also E.M. Henley and G.A. Miller: Phys. Rev. D 50 (1994) 7077.
44. CERN–SPS Hyperon Beam Collaboration (M. Bourquin, et al.): Z. Phys. C 12 (1982) 307.
45. S. Hsueh, et al.: Phys. Rev. D 38 (1988) 2056.
46. Spin Muon Collaboration (D. Adams, et al.): Phys. Lett. B 357 (1995) 248.
47. SLAC–Yale E80 Collaboration (M.J. Alguard, et al.): Phys. Rev. Lett. 37 (1976) 1261; 41 (1978) 70; E130 Collaboration (G. Baum, et al.): Phys. Rev. Lett. 45 (1980) 2000; 51 (1983) 1135.
48. The error corridor of the $\alpha_s(Q^2)$ determinations has a discontinuous momentum dependence, and it is advisable to use the error on this quantity rather than that on $\Lambda_{\overline{MS}}$ in error–propagation formulæ, which is in practice what the above recipe is doing.
49. P.M. Gensini: Nuovo Cimento A 103 (1990) 1311; J. Dai, R.F. Dashen, E. Jenkins, A.V. Manohar: report UCSD–PTH–94–19 (San Diego, June 1995), also available as hep–ph/9506273.
50. F.E. Close and R.G. Roberts: Phys. Lett. B 316 (1993) 165; B 336 (1994) 257. See also F.E. Close’s summary talk presented at the 1995 Erice Conference, report RAL TR–95–047 (Chilton, September 1995), also available as hep–ph/9509251.
51. H.J. Lipkin: Phys. Lett. B 237 (1990) 130; B 251 (1990) 613; B 256 (1991) 284; B 337 (1994) 157.
52. Similar conclusions, based on different parametrizations and a study of the $x$–dependence, have been independently reached, although at next–to–leading order only, by F. Buccella, O. Pisanti, P. Santorelli and J. Soffer: report DSF–T–95/26 (Napoli, July 1995), also available as hep–ph/9507251.