LYMAN LIMIT SYSTEMS IN COSMOLOGICAL SIMULATIONS

KATHARINA KOHLER\textsuperscript{1,2} AND NICKOLAY Y. GNEDIN\textsuperscript{2,3}

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ABSTRACT

We used a cosmological simulation with self-consistent radiative transfer to investigate the physical nature of Lyman limit systems at $z = 4$. In agreement with previous studies, we find that most of the Lyman limit systems are ionized by the cosmological background, while higher column density systems seem to be illuminated by the local sources of radiation. In addition, we find that most of the Lyman limit systems in our simulations are located within the virial radii of galaxies with a wide range of masses and are physically associated with them ("bits and pieces" of galaxy formation). While the finite resolution of our simulations cannot exclude an existence of a second population of self-shielded, neutral gas clouds located in low-mass dark matter halos ("minihalos"), our simulations are not consistent with minihalos dominating the total abundance of Lyman limit systems.

Subject headings: cosmology: theory — large-scale structure of universe — quasars: absorption lines

1. INTRODUCTION

The hydrogen absorption systems observed in spectra of distant quasars are traditionally subdivided into Ly$\alpha$ forest systems ($N_{H} \leq 1.6 \times 10^{17}$ cm$^{-2}$), Lyman limit systems ($1.6 \times 10^{17}$ cm$^{-2} \leq N_{H} \leq 10^{20}$ cm$^{-2}$), and damped Ly$\alpha$ systems ($N_{H} \geq 10^{20}$ cm$^{-2}$). Amazingly, among these three classes, the Lyman limit systems are least understood, both observationally and theoretically. This is quite unfortunate, given that the Lyman limit systems dominate the absorption of ionizing photons in the universe (Miralda-Escude 2003) and are therefore crucial for understanding the transfer of ionizing radiation in the intergalactic medium (IGM) and the interactions between the early galaxies and quasars and their environments.

A limited number of observational studies conducted so far have steadily identified Lyman limit systems with slowly evolving (if at all) absorbers located within gaseous halos of normal galaxies (Tytler 1982; Lanzetta et al. 1989; Steidel 1990; Mo & Miralda-Escudé 2003). A few kpc in size, predominantly ionized (Prochaska 1999). In many respects, the high-redshift Lyman limit systems appeared to resemble the high-velocity clouds observed around the Milky Way (Oort 1966; Verschuur 1969; Blitz et al. 1999; Maloney & Putman 2003; Putman et al. 2003).

Surprisingly (or, perhaps, not), theoretical studies of Lyman limit systems tended to ignore the observational work, attempting to identify Lyman limit systems with either low-mass dark matter halos, filled with neutral, self-shielded gas (Katz et al. 1996; Gardner et al. 1997; Abel & Mo 1998; Gardner et al. 2001), or with gaseous disks of small galaxies (Maller et al. 2003). In fact, theoretical attempts to identify the Lyman limit systems with large galaxies were not entirely successful (Mo & Miralda-Escudé 1996).

In this paper we reconsider the theoretical view of the Lyman limit systems as they appear in cosmological simulations of the standard ΛCDM model. Our simulations include a self-consistent, albeit approximate, treatment of the transfer of ionizing radiation in the IGM and are designed to achieve a reasonable agreement with the Sloan Digital Sky Survey (SDSS) observational data on the Ly$\alpha$ forest at $z > 5$ (Fan et al. 2006; Gnedin & Fan 2006).

2. SIMULATIONS

Simulations used in this paper have been run with the "softened Lagrangian hydrodynamics" (SLH) code (Gnedin 2000, 2004). Simulations include dark matter, gas, star formation, chemistry, and ionization balance in the primordial plasma, and three-dimensional radiative transfer. We adopt a flat ΛCDM cosmology with values of cosmological parameters as determined by the first year of Wilkinson Microwave Anisotropy Probe (WMAP) data (Spergel et al. 2003).

We use two sets of simulations in this paper. The two sets differ by the size of the computational box: in the first set the computational box has a size of 4$h^{-1}$ comoving Mpc on a side, while in the second set the box is 8$h^{-1}$ comoving Mpc on a side.

The primary simulations in each of the two sets are runs L4N128 and L8N128 from Gnedin & Fan (2006). Both of them include 128$^3$ dark matter particles and the same number of quasi-Lagrangian mesh cells for the gas evolution, as well as a smaller number of stellar particles that formed continuously during the simulation. The dynamic range of the simulations is fixed to 2000, so the effective spatial resolution (twice the Plummer softening length) of the 4$h^{-1}$ and 8$h^{-1}$ Mpc simulations is 2$h^{-1}$ and 4$h^{-1}$ comoving kpc, respectively.

As Gnedin & Fan (2006) emphasized, these simulations provide an adequate fit to the evolution of the mean transmitted flux in the Ly$\alpha$ forest in the range $5 < z < 6.2$, but they overproduce the opacity in the forest at $z < 5$. This is caused mainly by the limited size of the computational volume, which leads to a substantial (even serious for the case of the 4$h^{-1}$ Mpc box) underestimate of the star formation rate at $z < 5$. As we show below, the simulations also overproduce Lyman limit systems.

The simulations, however, contain adjustable parameters; the one of importance here is the ionizing efficiency parameter, $\epsilon_{\text{UV}}$, that controls the amount of ionizing radiation emitted per unit mass of stars formed in the simulation. This parameter depends on the numerical resolution of the simulation and on the details of stellar feedback and radiative transfer implementations, and

\footnote{These values are $\Omega_M = 0.27$, $\Omega_{\Lambda} = 0.73$, $h = 0.71$, $\Omega_{b} = 0.04$, $n_s = 1$, and $\sigma_8 = 0.9$.}

\textsuperscript{1} JILA, University of Colorado, Boulder, CO.
\textsuperscript{2} Particle Astrophysics Center, Fermi National Accelerator Laboratory, Batavia, IL; kkohler@fnal.gov, gnedin@fnal.gov.
\textsuperscript{3} Department of Astronomy and Astrophysics, University of Chicago, Chicago, IL.
it has no clear physical interpretation. It can be thought of as a fraction of ionizing radiation escaping from the spatial scales unresolved in the simulation (it has no direct relation to the widely used “escape fraction” parameter).

Since we treat the ionizing efficiency as a free parameter, we can adjust it to obtain a better agreement with the observational data. For that purpose, we run additional simulations in which we increased the ionizing efficiency by an additional factor of $q$. Because in this paper we only concentrate on comparing the simulations with the data at $z = 4$, in order to save computational resources, we only rerun the simulations with different values of the ionizing intensity from $z = 4.3$ to $z = 4$. The redshift interval of $\Delta z = 0.3$ corresponds to a time interval of 130 Myr, which is some 4000 times larger than the mean photoionization time at this redshift (McDonald et al. 2002) and thus should be more than sufficient for establishing a new photoionization equilibrium even in the lowest density regions. To be on the safe side, we also rerun one of the simulations from $z = 4.6$ to $z = 4$, and the results for the $L_{\gamma \alpha}$ mean transmitted flux and the column density distribution of the Lyman limit systems from that simulation are indistinguishable from the analogous run started at $z = 4.3$.

In order to make reference to specific simulations transparent, we label each simulation with a letter “L” followed by the value of the linear size of the computational volume (measured in $h^{-1}$ Mpc), followed by the letter “q” and the value of the ionization enhancement parameter $q$. For example, the primary simulations in each set are thus labeled $L_{4q1}$ and $L_{8q1}$, while a $4 h^{-1}$ Mpc simulation with 6 times higher ionizing efficiency is labeled $L_{4q6}$. In the $4 h^{-1}$ Mpc set, we run simulations $L_{4q1}$, $L_{4q2}$, $L_{4q6}$, and $L_{4q10}$, while in the $8 h^{-1}$ Mpc set, we run $L_{8q1}$ and $L_{8q6}$.

The simulations include both galaxies and quasars as sources of ionizing radiation. However, while galaxies are treated as individual distributed sources of ionizing radiation, the contribution of quasars is added as incident background radiation. Because the relative contribution of quasars and galaxies to the ionizing radiation is not very well established but is certainly comparable, we assume for simplicity that galaxies and quasars contribute equally to the ionizing background at $z < 5$. Changing the $q$-parameters then changes the respective contribution of galaxies and quasars, but does not affect their relative (equal) contributions. This is broadly consistent with the observational constraints on the relative contributions of galaxies and quasars to the intergalactic ionizing background (Shull et al. 2004; Bolton et al. 2005, 2006; Shapley et al. 2006), although the observational determinations remain somewhat uncertain.

The Lyman limit systems are found by casting 10,000 lines of sight through the simulation box, allowing us to sample the simulation box finely enough to find even rare absorbers. The lines of sight are cast in random directions and cover a redshift range of $\delta z = 0.033$ for the $8 h^{-1}$ Mpc boxes. Since finding maxima numerically is much easier than finding steep gradients, we search for peaks in $J^2 d\tau_{LL}/d\lambda$ instead of $\tau_{LL}$. The two approaches are completely equivalent, but searching for peaks is similar to searching for an absorption line, which allows us to reuse the tools developed for creating synthetic $L_{\gamma \alpha}$ spectra.

Indeed, a $L_{\gamma \alpha}$ absorption-line profile can be described with (we ignore natural width of the line here)

$$
\tau_\alpha = \frac{\tau_{\alpha 0}}{b\sqrt{\pi}} \int d\lambda' e^{-[(\lambda' - \lambda_0)^2/b^2]} \delta(\lambda' - \lambda).
$$

When considering $d\tau_{LL}/d\lambda$ instead of $\tau_{LL}$, the Lyman limit edge becomes the delta function in frequency, in full analogy with the absorption-line profile. Thus, a Lyman limit system appears as a peak in the quantity

$$
\tilde{\tau}_{LL} = \frac{1}{b\sqrt{\pi}} \int d\lambda' e^{-[(\lambda' - \lambda_0)^2/b^2]} \lambda' \frac{d\tau_{LL}}{d\lambda'}.
$$

In particular, the value of $\tilde{\tau}_{LL}$ at the “line” center is directly proportional to the neutral hydrogen column density, $\tilde{\tau}_{LL} = \sigma_{LL} N_{H I}$, where $\sigma_{LL} = \epsilon_{H I} 10^{-16} \text{ cm}^{-2}$ is the hydrogen photoionization cross section. All regions in the spectra where $\tilde{\tau}_{LL} \geq 1$, corresponding to column densities of $N_{H I} > 1.6 \times 10^{17} \text{ cm}^{-2}$, are selected as Lyman limit systems.

After casting the lines of sight, the positions of these regions in real space (by inverting the real space–redshift space mapping), as well as their opacities, are determined. We also compute other characteristics such as the local photoionization rate, the column density, and the neutral fraction of the gas. This information allows us to investigate whether the systems are ionized by local sources or by the cosmological background radiation.

Finding the positions of the Lyman limit systems in the simulation box and determining the position of the galaxies in the simulation allows us to determine whether each Lyman limit system can be associated with a galaxy. Galaxies in the simulation are identified with gravitationally bound objects, which are found with the DENMAX algorithm (Bertschinger & Gelb 1991). The DENMAX algorithm works by constructing the finite-resolution smooth total (dark matter + gas + stars) density field from the simulation data. It then identifies density maxima and associates them with bound objects. This approach has a major advantage for our purposes here, since it allows us to assign a specific meaning to the word “associated.” Since each galaxy is represented by DENMAX as a density hill, we define all points in space that lie on that hill as being “associated” with that galaxy. In other words, a point of space is associated with a given galaxy if, by moving against the density gradient from that point, one would eventually end up at the center of the galaxy. We use 1/5 of the mean interparticle separation as the DENMAX resolution, as this value gives results most consistent with other halo finders.

DENMAX separates all points of space into four distinct categories:

1. Points gravitationally bound to a given galaxy.
2. Points associated with a given galaxy but not gravitationally bound to it (i.e., points lying on the galaxy density hill, but outside the virial radius).
3. Unassociated points (i.e., points not associated with any galaxy, such as points in the middle of the void).
4. Points associated with unresolved galaxies (i.e., points belonging to a density hill too small to be qualified as a resolved galaxy).

We use this classification below to describe locations of Lyman limit systems in space.

3. RESULTS

The main goal of this work is not only to investigate the physical properties of the Lyman limit systems themselves, such as the neutral fraction and the photoionization rate, but also to investigate what kinds of galaxies they are associated with. Since, as we have mentioned above, the primary simulations $L_{4q1}$ and $L_{8q1}$ underproduce the intergalactic ionizing background and overproduce the abundance of Lyman limit systems at $z = 4$, we adjust the $q$-parameter to achieve a rough agreement with observations. In this paper we are mainly interested
in the general properties of Lyman limit systems, so we do not attempt to provide a statistically acceptable fit to the observational data; rather, we are satisfied with being in the ballpark for the abundance of the Lyman limit systems at $z = 4$.

Figure 1 shows column density distributions of the Lyman limit systems for three different runs with varying values of ionization efficiency $q$. The upper run, with its fitted line overlaid, corresponds to L4q1, which is the run with the default value of the ionization efficiency. The middle run is L4q6, where the ionization efficiency is increased by a factor of 6, and the lower run is run L4q10, with an ionization efficiency of $q = 10$. The overlaid fits are best-fit power laws for the region $17.2 < \log N_{\text{HI}} < 18$, with the slope kept fixed at the observational value of $-1.5$ (Petitjean et al. 1993; Miralda-Escude 2003). This specific choice of the column density range is motivated by the canonical definition of a Lyman limit system of $\tau_{\text{LL}} > 1$ ($\log N_{\text{HI}} > 17.2$) on the lower end and by the highest column density for systems fully sampled in our small-box simulations (for $\log N_{\text{HI}} > 18.0$, our best-fit L4q6 sample starts being incomplete).

For the L4q6 run the fitted line closely corresponds to the observed abundance of Lyman limit systems from Storrie-Lombardi et al. (1994), $dN/dz \approx 3.5$. The L4q1 run has a number density of Lyman limit systems too high to fit the observations. This illustrates that the original simulations, while fitting the mean transmitted flux in the Lyman limit systems for three different runs with varying values of the ionization efficiency $q$, the increased ionizing intensity ionizes more of the initially neutral gas. The dashed lines show the amplitude of the power law that is needed to fit the number density of Lyman limit systems as described above in Figures 1 and 2. Here, for lower ionization efficiencies the offset is lower, and for $q = 6$ the offset is unity, since the number density distribution is best fitted by the observational value. The horizontal line shows the observational values for both parameters, since we normalize by the observed value. The gray lines correspond to a 20% error, as is expected from the observations. This plot illustrates that we can fit two different observables with just one parameter, the ionization efficiency, and that increasing this parameter by a factor of 6 from the values used in Gnedin & Fan (2006) fits the observations at $z = 4$.

Figures 4 and 5 show the physical properties of the Lyman limit systems from our simulations: neutral hydrogen fraction, density, characteristic size (defined as $L_{\text{eff}} = N_{\text{HI}}/n_{\text{HI}}$), and the local photoionization rate in units of the mean photoionization

For higher values of $q$ there is more flux transmitted, as the increased ionizing intensity ionizes more of the initially neutral gas. The dashed lines show the amplitude of the power law that is needed to fit the number density of Lyman limit systems as described above in Figures 1 and 2. Here, for lower ionization efficiencies the offset is lower, and for $q = 6$ the offset is unity, since the number density distribution is best fitted by the observational value. The horizontal line shows the observational values for both parameters, since we normalize by the observed value. The gray lines correspond to a 20% error, as is expected from the observations. This plot illustrates that we can fit two different observables with just one parameter, the ionization efficiency, and that increasing this parameter by a factor of 6 from the values used in Gnedin & Fan (2006) fits the observations at $z = 4$.

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The effective size serves as an estimate to a typical scale over which the given column density is accumulated, and, for a Gaussian distribution, is very close to the full width at half-maximum.
rate in the L8q6 simulation (the properties of Lyman limit systems from the L4q6 run are statistically identical, but the sample size is 8 times smaller). There exist general trends of a higher neutral fraction, higher density, higher photoionization rate, and smaller size with higher optical depth, but the scatter in all cases is about an order of magnitude, so these trends cannot be interpreted as physical relations.

We note that our simulated Lyman limit systems are substantially (but not highly) ionized and are not fully neutral. This implies that the Lyman limit systems detected in our simulations are not small self-shielded clumps of neutral gas, but rather diffuse, partially shielded clouds of gas. Their orientation toward a nearby source plays an important role in determining their ionization state; otherwise a cloud with $n_{\text{H}}/C_{28}^{\text{LL}}/C_{24}^{\text{LL}} \approx 100$ would be completely neutral.

For lower $n_{\text{H}}$ systems, the normalized photoionization rate $\Gamma/\langle \Gamma \rangle$, where $\Gamma$ is the average $\Gamma$, is about unity. This means that these systems are ionized by the cosmological background and not by local sources. Systems with higher $n_{\text{H}}$ appear to be ionized by local sources, because their normalized photoionization rate is higher. This result is in agreement with previous findings by Miralda-Escudé (2005) and Schaye (2006).

One of the questions we investigate is whether the Lyman limit systems are associated with high- or low-mass galaxies. At first, we tried to use a method similar to one used by observational studies of the environment of the Lyman limit systems at low redshifts (Penton et al. 2002). In observations, the Lyman limit systems are often associated with a galaxy by finding the nearest object, either along a line of sight or near it. A similar method was also used to analyze the simulations in Katz et al. (1996). There, a relation between the distance to the nearest galaxy and the $N_{\text{H}}$ of the Lyman limit systems was shown, and it was concluded that the column density correlates inversely with projected distance and that Lyman limit systems either lie in the outer parts of massive protogalaxies or closer to the center of less massive galaxies.

In our case, we found that associating the Lyman limit systems with the nearest object is not necessarily the best option, because such an association is strongly dependent on the galaxy sample. When we decrease the mass limit of the sample, thus including less and less massive galaxies in the sample, we find progressively lower mass galaxies that are closer and closer to a typical Lyman limit system. This, of course, does not imply that the Lyman limit systems are physically associated with these low-mass galaxies, because low-mass galaxies are more numerous, and the distances between even a random subset of points in space to galaxies of progressively smaller masses will be progressively smaller.

Thus, associating the Lyman limit systems with the nearest galaxies could lead to the conclusion that most of the Lyman limit systems are located in small galaxies. However, by using the physical association as permitted by the DENMAX algorithm (§ 2)—that is, associating a Lyman limit system with the galaxy whose “density hill” (i.e., dark matter halo) the Lyman limit system is located in—we find a different result: the Lyman limit systems are associated with galaxies in a wide range of...
masses, and the small galaxies near them are often physically unrelated satellites of a larger galaxy.

Figure 6 shows the masses of the galaxies that the Lyman limit systems are associated with, versus the systems’ optical depths. There is no strong dependence on galaxy mass for the optical depth, meaning that the size of the galaxy does not appear to strongly influence the opacity of the Lyman limit system.

The bottom panel of Figure 7 is a graph of the distance of the Lyman limit system to its associated galaxy and the mass of that galaxy. Overlaid on this correlation is the distance that corresponds to the virial radius for each galaxy mass. The resolution limit of our simulation is about 1 kpc (in physical units), so we resolve most of the distances to the galaxies. The top panel shows the number of galaxies per mass bin for the same mass range as the bottom panel. It is clear from this histogram that there are many more small galaxies that have no Lyman limit systems associated with them.

It is important to keep in mind that our simulations do not resolve galactic disks, so any Lyman limit absorption coming from the central galaxies themselves is missed in the simulations. However, it is clear from Figure 7 that the chances of hitting the disk of a galaxy by a random line of sight are much smaller than those of crossing a typical $\tau_1 \sim \text{a few}$ Lyman limit system further out.

Combining the information from this graph, we find that most of the Lyman limit systems are within the virial radius of a galaxy, often a relatively massive one. This means that they are either in orbit around or falling into a galaxy, rather than being isolated clumps of neutral gas. Using the DENMAX classification of points in space discussed in § 2, we find no Lyman limit systems that are unassociated or associated with unresolved galaxies.

Figure 8 extends the discussion from the previous figure by showing the number of Lyman limit systems associated with galaxies of different masses. There are many more systems per galaxy in the high galaxy mass range from $M_{gal} \sim 10^{10}$ to $M_{gal} \sim 10^{11}$ than there are for lower masses. Since $dN_{gal}/d \log M \sim M^{-1}$, the number of galaxies sharply increases at low masses. In Figure 8 the overlaid power law ($\propto M^1$) shows the equal number of Lyman limit systems per logarithmic bin in mass of galaxies they are associated with. The figure shows that only a small fraction of all Lyman limit systems are associated with galaxies of $M_{gal} < 10^{10} M_{\odot}$. However, this may also be a resolution effect, implying that the Lyman limit systems are associated with galaxies of all mass ranges equally.

4. CONCLUSIONS

We show that the column density distribution of Lyman limit systems at $z = 4$ in cosmological simulations that agree with the SDSS measurements of the Ly$\alpha$ absorption at $z > 5$ in the spectra of high-redshift quasars has the same shape as the observed distribution. The abundance of the Lyman limit systems is not necessarily in agreement with the data, but we manage to achieve agreement with both the observed column density distribution of the Lyman limit systems and with the mean transmitted flux in the Ly$\alpha$ forest by adjusting a single parameter—ionizing efficiency—at $z < 5$.

We find that, in the simulation that agrees with the observational data, the lowest column density Lyman limit systems are mainly illuminated by the cosmological background, in agreement with previous findings of Miralda-Escudé (2005) and Schaye (2006). However, we also find that all Lyman limit systems that are resolved in our simulations ($N_{H_1} \leq 10^{20} \text{ cm}^{-2}$) are substantially...
ionized and are physically associated with (are in orbit around or falling into) the gravitational well of galaxies within a large range of masses ($M_{\text{tot}} > 10^{10} M_\odot$). The absorbing systems are typically a few kpc in size and have overdensities on the order of $10^3$ (physical densities of about $0.01–0.1 \text{ cm}^{-3}$). Because of the limited resolution of our simulations, we are unable to understand the physical nature of these systems: they may be high-redshift counterparts of high-velocity clouds (Maller & Bullock 2004), or may be remnants of dense gas stripped off accreted satellites, which we routinely find in higher resolution simulations (based on the published work currently in progress). We do find, however, that Lyman limit systems are not associated with the substructure that we are able to resolve: the absorbers are gravitationally bound to central galaxies and not to the lower mass satellites that are closest to them.

Because of the finite spatial and mass resolution of our simulations, we cannot exclude the existence of a second population of Lyman limit systems composed of “minihalos.” However, the fact that we are able to fit simultaneously both the Lyman limit column density distribution and the mean transmitted flux in the Ly$\alpha$ forest at $z = 4$ with one adjustable parameter may indicate that the population of Lyman limit systems arising in minihalos is not the dominant one. Thus, the population of Lyman limit absorbers that we find in our simulations appears to be quite consistent with the properties of these objects determined from purely observational studies (Steidel 1990).

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