Features of the electron avalanche formation process in a strongly inhomogeneous electric field under high overvoltages

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Abstract. The simulation of the electron avalanche formation process in subnanosecond discharges of high pressure was carried out by means of the Monte-Carlo approach. The discharge gap under consideration was of the configuration “the finger-shaped cathode – the hemispherical anode”. The presence of a conic-shaped microprotrusion on a cathode surface was assumed. Such the electrode configuration provided the strongly inhomogeneous distribution of an electric field. A gas simulated was nitrogen at a pressure of 6 atm. An average electric field strength across the discharge gap was varied from 200 kV/cm up to 400 kV/cm. The critical size and formation time of an electron avalanche were determined under various conditions simulated. The threshold electric field strength for electrons to transit into the continuous accelerating regime was calculated for various heights of the microprotrusion. The applicability of the non-self-consistent Monte-Carlo technique for the investigation of the runaway electron kinetics and the correct simulation of the runaway electron beam transport across the discharge gap was shown.

1. Introduction
In gas discharge physics, this is an electron avalanche development process that determines largely the dynamics of the discharge formation. Under given conditions (a gas type and pressure, a voltage applied to a discharge gap, etc.), the rate of the electron number exponential growth and the critical number of electrons comprising the avalanche have a strong effect on the distribution of an electric field across a discharge gap, as well as on plasma processes taking place, and, eventually, on a gas discharge type. So, the Townsend and streamer mechanisms of the discharge development are usually distinguished. The first one is usually valid for relatively low gas pressures of about 1 Torr and less (that is typical for a glow discharge), while the second one applies to the investigation of high-pressure discharges (e.g. a corona discharge). However, in subnanosecond discharges of high pressure, the explanation of the discharge dynamics using these mechanisms may be inapplicable. So, in [1], it was shown that the subnanosecond discharge in nitrogen at a pressure of 6 atm developed in a way that differed substantially from the streamer discharge theory commonly used for the description of similar discharges. In particular, it was shown [1] that at the initial stage before the spark channel formation the discharge was of a volumetric form. Wherein, the discharge contraction stared from a cathode and an anode almost simultaneously that was found to be unusual for such a type of discharges. It was suggested that the phenomena observed may be caused by the presence of runaway electrons (RE) at the prebreakdown stage of the discharge. In laboratory experiments with subnanosecond gas
discharges, REs are usually generated if the reduced electric field strength $E/p$ (where $E$ is electric field strength and $p$ is a gas pressure) is high enough. In this case, energy acquired by free electrons in gas from an electric field can exceed inelastic energy losses caused by electron collisions with gas particles [2-4]. Hence, such electrons are continuously accelerated by the field. REs can preionize a gas medium, and the formation of secondary electron avalanches produced by REs may lead to a decrease in the discharge gap commutation time and the formation of the discharge in a volume form before the spark channel formation [5-7]. However, in [1], the maximal magnitude of the electric field reached at the prebreakdown stage was believed to be not sufficient for the generation of REs under the experimental conditions described. In [1], the acceleration of free electrons field-emitted from irregularities taking place on the cathode surface was proposed to be one of the possible mechanisms leading to the RE appearance in relatively weak electric fields. For example, a microprotrusion can act as such an irregularity.

The present paper is devoted to a numerical investigation of the electron avalanche critical parameters in subnanosecond discharges of high pressure at the prebreakdown stage in the vicinity of a microprotrusion located on the surface of a cathode. As a numerical model, the 3D Monte-Carlo code was used [8]. The discharge gap under consideration in the configuration “a finger-shaped cathode – a hemispherical anode” generating the inhomogeneous distribution of an electric field was the same as the one described in [1]. As a result, the critical size and formation time of an electron avalanche were determined under the conditions using in [1]. The threshold electric field strength $E_{th}$ for electrons to transit into the continuous accelerating regime was calculated for various heights of the considered microprotrusion. Special attention was paid to the problem of the electron and ion space charge consideration in the investigation of the RE kinetics and the RE beam propagation process. The paper is a logical continuation of our previous works [1,8-11].

2. The numerical model description
The discharge system under consideration is given in figure 1. The system comprised of two hemispherical electrodes: a cathode and an anode. The electrode curvature radii were 10 mm. In addition, a cylinder of 2 mm diameter and 3 mm height was mounted at the top of the cathode along the discharge system symmetry axis. The cylinder was rounded with a hemisphere of a 1 mm radius. So, the electrode configuration “a finger-shaped cathode – a hemispherical anode” was considered. This was the same electrode configuration used in [1]. Besides, a cone-shaped microprotrusion was assumed to be located on the cathode surface on the axis of symmetry of the discharge system. The microprotrusion was of high $h$, its base diameter was set 0.5$h$. To avoid signatures, the apex of the cone was rounded with a hemisphere of 0.01$h$ radius. So, it was a typical microprotrusion for the cathode field emission analysis [12]. In the present paper, $h$ was varied from 0 $\mu$m up to 30 $\mu$m. The electrode configuration described gave the strongly inhomogeneous distribution of an electric field. To describe the distribution, the electric field enhancement factor term was used. Since the enhancement was provided by both the electrode geometry and the microprotrusion, the macroscopic $\beta_{macr}$ and microscopic $\beta_{micr}$ enhancement factors were distinguished. The first one concerned the electrode geometry, and the second one – the microprotrusion parameters. The factors $\beta_{macr}$ and $\beta_{micr}$ can be determined in the following way. If $E_{av}$ is an average electric field across the discharge gap determined as $E_{av} = U/d$ where $U$ is a voltage applied to the discharge gap and $d$ is a cathode-anode distance, the factor $\beta_{macr}$ is as follows: $\beta_{macr} = E/E_{av}$, where $E$ is a local value of the electric field strength. $\beta_{micr}$ was determined in an analogous way. The discharge gap length $d$ was 5 mm. In the simulation, to calculate a real value of $E$ in the vicinity of the cathode microprotrusion, the $\beta_{micr}$ and $\beta_{macr}$ factors were multiplied:

$$E = \beta_{macr} \beta_{micr} E_{av}.$$  

Since $h$ and $d$ differed approximately 100 times, $\beta_{micr}$ and $\beta_{macr}$ acted at quite different spatial scales. Also, $\beta_{macr}$ changed little over the area near the microprotrusion. Hence, such an approach of $E$ determination was valid. To calculate the spatial distributions of $\beta_{macr}$ and $\beta_{micr}$, Laplace’s equation was
employed. The equation was solved numerically using the ANSYS software package. The $\beta_{\text{macr}}$ and $\beta_{\text{micr}}$ distributions obtained are given in figure 2 (for $\beta_{\text{micr}}$, the distribution corresponding to the microprotrusion with $h = 10 \mu m$ is shown).

**Figure 1.** A sketch of the simulated discharge system. A shaded area indicates a space domain considered in a simulation. On the inset, a microprotrusion is shown. The units (if any) are millimetres.

**Figure 2.** Macroscopic electric field enhancement factor $\beta_{\text{macr}}$ distribution along the $z$-axis of the discharge gap considered. On the inset, microscopic electric field enhancement factor $\beta_{\text{micr}}$ distribution is shown for $h = 10 \mu m$.

How it has been mentioned above, in this paper, the electron avalanche formation process was investigated by means of the Monte-Carlo approach. It allowed one to consider the motion of individual charged particles through a gas under the impact of an electric field of a given spatial configuration. The 3D Monte-Carlo code developed by us was used. During the simulation, two main problems were solved at each simulation time step. The first one was integrating of electron motion equation in a 3D approximation for each particular particle (an electron):

$$m_e \frac{dV(r,t)}{dt} = -eE(r),$$  \hspace{1cm} (2)

where $m_e$ is electron mass, $V$ is an electron velocity vector, $r$ is an electron coordinate vector, $E$ is an electric field strength vector, $e$ is an electron charge, $t$ is time. The value of $E$ at each spatial point was determined using (1) through decomposition of the factors $\beta_{\text{micr}}$ and $\beta_{\text{macr}}$ in two components: the one longitudinal to the discharge system symmetry axis (the $z$-axis) and the one orthogonal to this axis (belonging to the $xy$ plane). It should be noted that a space charge of secondary particles was not taken into account in the calculation of $E$ here. The time step $\Delta t$ for the numerical integration of (2) was set in such a way to ensure a meeting with the energy conservation law (ECL). So, $\Delta t$ was $5 \cdot 10^{-17}$ s and meeting with ECL was not worse than 0.1%. The computational domain of equation (2) was limited by the cubic domain with the dimensions $x:y:z = d:d:d$ (see figure 1). The start point $(0, 0, 0)$ for the simulation of the electron avalanche formation process (the coordinate system initial point) was assumed to be at the apex of the microprotrusion located on the cathode surface. So, the actual dimensions of the computational domain $x:y:z$ were $d:d:(d - h)$ cm. In addition, since Laplace’s equation was solved independently for electrodes of the discharge system and the microprotrusion, the multiplication of $\beta_{\text{micr}}$ and $\beta_{\text{macr}}$ factors gave uncertainty in the voltage drop determination across the discharge gap. However, taking into account $d = 0.5$ cm and $h \sim 10 \mu m$, relative uncertainty in the
interelectrode distance determination and the calculation of the voltage drop did not exceed several tenths percent, and it was neglected.

The second problem solved at each time step was a calculation of electron collisions with gas particles. The electron avalanche formation in nitrogen at a pressure of 6 atm was considered. A number of kinetic processes were taken into account (ionization, excitation, elastic scattering, etc.). Respective total cross sections for electron-molecular collision processes were taken from [13]. To estimate the probability of a collision in each time step, the null-collision technique was applied [14]. For electrons with energy \( e < 500 \) eV, scattering angles \( \theta \) were calculated using differential cross-section data [15,16]. For \( e \geq 500 \) eV, the differential cross section \( d\sigma \) was determined by means of the equation [17]

\[
d\sigma/d\theta = [1 - (1 - 2\mu(e))\cos \theta]^2,
\]

where \( \mu(e) \) is a screening parameter determined as

\[
\mu(e) = 0.6 \cdot [1 + (e / 50)^{0.5} + (e / 20)^{1.01}]^{-0.99}.
\]

For ionization collisions, the energy of secondary electrons \( e_s \) was calculated as follows [18, 19]:

\[
e_s = I \tan \left[ N \arctan \left( (e - e_i)/2I \right) \right],
\]

where \( N \) is a uniform random number in a range from 0 up to 1, \( I \) is an adjustable parameter, \( e_i \) is an ionization potential. For molecular nitrogen, \( I = 13.0 \) eV and \( e_i = 15.6 \) eV. Also, within an ionization collision, the appearance of a positively charged ion was considered, but, for the sake of computational simplicity, its motion and scattering were not taken into account. Such a simplification was valid since the typical simulation time did not exceed 100 ps, and during this period, the ion motion and scattering did not have any significant effect on the simulation results.

3. Estimation of the runaway threshold electric field strength for the electrons accelerated in the vicinity of the microprotrusion

The simulation process was as follows. First, the values of the average electric field across the discharge gap \( E_{av} \) and the high of the microprotrusion \( h \) were set. Next, an initial electron that was assumed to be of the field-emissive nature was placed manually at the zero point of the computational domain considered (the apex of the microprotrusion). Then, the simulation was started. A motion and scattering of the primary electron and secondary electrons were simulated. Also, the generation of positively charged ions was taken into account. In the simulation, a full number of electrons \( N_e \) comprising the avalanche being simulated as well as the stochastic appearance of REs were monitored. As an example of the simulation results, the dependence \( N_e(t) \) for \( E_{av} = 280 \) kV/cm and \( h = 15 \) \( \mu \)m is given in figure 3.

One of the main tasks of the paper was to estimate the dependence of the runaway threshold electric field strength \( E_{th} \) for electrons that appeared in the immediate vicinity of the microprotrusion on the height of the microprotrusion \( h \) under the experimental conditions described in [1] (the finger-shaped cathode and nitrogen at the pressure of 6 atm). In numerical simulations, the procedure of the \( E_{th} \) determination is always complicated by the stochastic nature of the runaway phenomenon. Within the present paper, the dependence \( E_{th}(h) \) was determined as follows. First, for a given \( h \) and varied \( E_{av} \), the series of simulations was carried out to determine an approximate range of \( E_{av} \) within which the probability of the generation of at least a single RE changed from 0% up to 100%. Then, such a value of \( E_{av} \) was chosen to provide a small probability (explicitly less than 50%) for an electron to transit into the runaway regime, and this value of \( E_{av} \) was considered as \( E_{th} \). It should be noted that only the electrons having reached the anode of the discharge gap in time comparable to the time of their flight in vacuum through the discharge gap under the impact of the same \( E_{av} \) value were considered runaway electrons. As simulations showed, near the microprotrusion, the majority of electrons having initially acquired the energy that exceeded the typical thermal electron energy lost it in inelastic collisions and
became slow plasma electrons. This was the direct demonstration of the criterion on the electron transition into the continuous accelerating regime proposed in [8,20,21]: despite the acceleration of electrons near the microprotrusion up to the energies of about several keV, it was necessary to apply some additional field to prevent the deceleration of REs at the periphery of the discharge gap. Correspondingly, it was found that if $E_{av}$ was decreased by 10 kV/cm from the $E_{th}$ value determined, the electrons having reached the anode were not registered in the simulation at all. On the contrary, if $E_{av}$ was increased by 10 kV/cm from the $E_{th}$ value, the probability of the generation of at least a single RE was close to 100%. So, the error in the $E_{th}$ determination was estimated to be about 10 kV/cm. The results of the $E_{th}$ estimation for various $h$ are given in figure 4.

One can see that under the conditions described in [1] running away of electrons without the contribution of the microprotrusion ($h = 0 \mu m$) is possible only for $E_{th} = 400$ kV/cm. Also, it was found that the microprotrusions with a height of less than 8 $\mu m$ had almost the same magnitude of $E_{th}$ – about 400 kV/cm. A sufficient decrease in $E_{th}$ was found to be only for $h = 10 \mu m$. In this case, $E_{av} = 350$ kV/cm. For $h = 30 \mu m$, $E_{th} = 200$ kV/cm. Hence, under the conditions given in [1], the largest microprotrusions only may give a significant contribution to the RE generation. However, it was found that the presence of even small microprotrusions located on the cathode surface led to the generation of electrons with the energy $\sim 1$ keV near the cathode, that provided a sharp increase in the ionization frequency and fast growth of the electron avalanche (see Section 4).

4. Estimation of critical parameters of an electron avalanche

It is commonly accepted that the electron avalanche critical size corresponds to such a number of electrons $N_{e,c}$ comprising the avalanche that electrons and ions generated produce an electric field comparable in magnitude to an electric field in the discharge gap. So, to monitor the electron avalanche formation process, the electric field distribution across the discharge gap simulated was estimated for a number of the $N_{e}$ values during the simulation. It was done with a help of the numerical solution of one-dimensional Poisson’s equation along the $z$-axis. Since the electric field strength $E$ and the concentration of charged particles were expected to be maximal near the discharge gap symmetry axis (the $z$-axis), this solution may serve as the upper limit estimation of $E$ across the discharge gap. During the simulation of the electron avalanche formation, the electric field distribution calculation procedure was repeated each time $N_{e}$ increased by $10^5$ electrons to register the $N_{e}$ value corresponding

\[ \text{Absence of REs} \]
\[ \text{Generation of a single RE} \]

Figure 3. The dependence $N_{e}$ from time for $E_{av} = 280$ kV/cm and $h = 10 \mu m$ for two situations: absence of REs and the stochastic generation of a single RE.

\[ E_{av}, \text{kV/cm} \]
\[ 0, 10, 20, 30, 40, 50, 60, 70, 80, 90, 100 \]

Figure 4. The dependence of the runaway threshold electric field strength $E_{th}$ on the microprotrusion height $h$. 

\[ t, \text{ps} \]
\[ 0, 1, 2, 3, 4, 5 \]

Figure 3.
to significant distortion of an electric field under simulation conditions given. The criterion on the
distortion was a decrease in the absolute value of \( E \) to the level at least \( E_{av}/2 \) or less at some point over
the discharge gap. If the criterion was met, the simulation of thermal electrons comprising the
avalanche was stopped and it was assumed that the avalanche reached the critical size. A typical view
of the electric field distribution evolution during the simulation is given in figure 5. The distributions
shown were obtained for \( E_{av} = 280 \text{ kV/cm} \) and \( h = 15 \mu \text{m} \).

One can see that at the time moment \( t = 2.6 \text{ ps} \) (see figure 5 (a)) that corresponded to \( N_e = 10^5 \)
there was no any significant distortion of the electric field, although concentrations of charged particles
were quite large. For \( t = 2.6 \text{ ps} \), the electron \( n_e \) and ion \( n_i \) concentrations were about \( 2 \times 10^{14} \text{ cm}^{-3} \). On
the contrary, at the time moment \( t = 4.2 \text{ ps} \) (see figure 5 (b)), severe distortion of the electric field was
observed. The minimal value of \( E \) was about 200 \text{kV/cm}, hence, it was less than \( E_{av} \) and much less
than a magnitude of the undistorted field at the minimum point – about \( 10^3 \text{kV/cm} \). So, one may
conclude that at the time moment \( t = 4.2 \text{ ps} \) the avalanche approached its critical size. Also, it was
found that for all the values of \( h \) and \( E_{av} \) investigated typical values of \( n_e \) and \( n_i \) for the avalanches
having reached the critical size were in the range from \( 10^{15} \) up to \( 10^{16} \text{ cm}^{-3} \).

The reasoning given above was applied to the estimation of the avalanche critical size \( N_{e\text{cr}} \) and its
growth time \( t_{e\text{cr}} \) for various values of \( h \) under the runaway near-threshold conditions (that is, for the
corresponding \( E_{th} \) values). For \( E_{av} = E_{th} \) in the simulations, two cases were observed: the first one
corresponding to the successful RE generation (a RE reached the anode) and the second one
corresponding to the thermalization of electrons initially accelerated near the microprotrusion (the
absence of REs). Therefore, \( N_{e\text{cr}} \) and \( t_{e\text{cr}} \) were determined for these two typical cases. It should be noted
that the approach described gave unavoidable errors in the \( N_{e\text{cr}} \) and \( t_{e\text{cr}} \) values. So, the obtained values
of \( E_{th} \) had the error \( \sim 10 \text{kV/cm} \). Performing the electric field distribution calculation procedure at
some particular time moments (corresponding to different \( N_e \)) did not allow one to register the exact
moment when the avalanche had reached the critical size. In addition, because of the random nature of
electron-neutral collisions and the unpredictability of electron trajectories, the estimated values of \( N_{e\text{cr}} \)
and \( t_{e\text{cr}} \) had a significant stochastic spread. Taking into account the methodological and statistical
uncertainties, the summary error in the \( N_{e\text{cr}} \) determination was estimated to be \( \sim 0.25 \times 10^6 \) electrons,
and in \( t_{e\text{cr}} \sim 0.5 \text{ ps} \). The results obtained are given in figure 6.
It was found that for $h = 0 \mu m$ (the absence of the microprotrusion) $t_{cr}$ reached 7 ps both in the case when a RE was generated and when it was not. But if $h \neq 0$ a sharp decrease in $t_{cr}$ was observed down to $\sim 3.5$ ps (see figure 6 (a)). $h$ being increased, a significant change in $t_{cr}$ was not found. A sharp peak at $h = 15 \mu m$ had to be stochastic. In addition, some decrease in $t_{cr}$ was observed in the simulations in which the generation of a single RE was registered. So, for $h = 20 \mu m$, for the cases when a RE was generated and when it was not the difference between $t_{cr}$ was about 2 ps, but for $h = 10 \mu m$ this difference was much less than 1 ps. This discrepancy was believed to be of stochastic origin. Wherein, the apparent correlation between the microprotrusion height $h$ and the avalanche critical size $N_{e,cr}$, as well as between $N_{e,cr}$ and the participation of a single RE, was not found. So, one may conclude that for the conditions described in [1], a typical size of an electron avalanche was $\sim 2.5-3.0 \times 10^6$ electrons, and a typical formation time was varied in the range between 2.8-7.0 ps. This time range was in agreement with the estimation of the characteristic ionization time given in [22] (about 2.4 ps). The ionization time was associated with the formation of dense plasma near the cathode that governed the generation and termination of the RE beam in the discharge gap. However, since the emission task and photoionization of gas molecules were out of the scope of the paper, the propagation of the avalanche (the ionization wave) toward the cathode was not taken into account, and the simulation approach described may give slightly higher $t_{cr}$ values.

![Figure 6. The dependencies of the electron avalanche formation time $t_{cr}$ (a) and the critical size $N_{e,cr}$ of the avalanche on the microprotrusion height $h$. All the points were obtained for corresponding $E_{th}(h)$ points (see figure 4).](image)

It should be noted, for the simulation results shown in figure 5 and figure 6, the electron avalanche formation was accompanied by the generation of a single RE. In figure 5, a blue vertical line indicates the RE position at the time moments corresponding to the electric field distribution calculation. One can see that during the simulation the RE was ahead of the main part of the avalanche, that is, the RE moved in the space domain of the undistorted electric field. This observation was also confirmed by the results of other research groups on the RE beam transport (e.g. see [22, 23]). So, one may conclude that the Monte-Carlo approach that did not take into account the electric field provided by space charges was valid for the investigation of the kinetics of REs and the correct simulation of the RE beam transport through the discharge gap due to a low concentration of the electrons comprising the beam, although it can not give full information about the plasma processes taking place near the cathode of the discharge gap that provide the conditions for the generation of REs and the termination of such conditions.
5. Conclusion
In the present paper, the electron avalanche formation process in nitrogen at a pressure of 6 atm in the vicinity of a cone-shaped microprotrusion located on a cathode surface was investigated employing the Monte-Carlo technique [8]. The simulation conditions were the same as the ones described in [1]. As a result, the dependence of the runaway threshold electric field strength on the microprotrusion height was obtained. The critical parameters of the electron avalanche simulated were estimated. It was found that the typical size of the electron avalanche was \( \sim (2.5-3.0) \times 10^6 \) electrons and it did not depend on the microprotrusion height. The typical avalanche formation time was in the range between 2.8 and 7.0 ps, and it was found to be in agreement with the estimations of the near-cathode dense plasma formation time performed in [22]. Also, the avalanche formation time was found to be somewhat less if the runaway electron generation took place during the avalanche formation. In addition, it was concluded that the non-self-consistent Monte-Carlo technique was applicable to the investigation of the runaway electron kinetics and the runaway electron transport through a discharge gap since during the simulation REs moved ahead of the avalanche in the space domain of the undistorted electric field.

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References
[1] Ivanov S N, Lisenkov V V and Mamontov Y I 2021 Plasma Sources Sci. T. 30 075021
[2] Gurevich A V 1961 J. Exp. Theor. Phys. 12 904–12
[3] Babich L P 2003 High-Energy Phenomena in Electric Discharges in Dense Gases: Theory, Experiment and Natural Phenomena (Arlington: Futurepast) p 372
[4] Korolev Y D and Mesyats G A 1998 Physics of Pulsed Breakdown in Gases (Yekaterinburg: URO) p 274
[5] Mesyats G A and Yalandin M I 2009 IEEE T. Plasma Sci. 37 785–9
[6] Mesyats G A and Yalandin M I 2019 Phys.-Usp. 62 699–703
[7] Babaeva N Yu, Zhang C, Qiu J, Hou X, Tarasenko V F and Shao T 2017 Plasma Sources Sci. T. 26 085008
[8] Mamontov Y I, Uimanov I V, Kozyreva A V, Zubarev N M and Semeniuk N S 2020 Prog. 7th Int. Congress on Energy Flux. and Rad. Effects (EFRE) (Tomsk: Publishing House of IAO SB RAS) 140–6
[9] Lisenkov V V, Ivanov S N, Mamontov Y I and Tikhonov I N 2018 Tech. Phys. 63 1872–5
[10] Lisenkov V V and Mamontov Y I 2018 J. Phys. Conf. Ser. 1141 012051
[11] Lisenkov V V 2020 Tech. Phys. 65 710–4
[12] Korolev Y D and Mesyats G A 1982 Field-Emission and Explosive Processes in Gas Discharges (Novosibirsk: Nauka) p 253
[13] Itikawa Y 2006 J. Phys. Chem. Ref. Data 35 31–53
[14] Lin S L and Bardsley J N 1978 Comput. Phys. Commun. 15 161–3
[15] Shyn T W, Stolarски R S and Carignan G R 1972 Phys. Rev. A 6 1002–12
[16] DuBois R D and Rudd M E 1976 J. Phys. B.–At. Mol. Phys. 9 2657–67
[17] Phelps A V and Pitchford L C 1985 Phys. Rev. A 31 2932–49
[18] Opal C B, Peterson W K and Beaty E C 1971 J. Chem. Phys. 55 4100–6
[19] Moss G D, Pasko V P, Liu N and Veronis G 2006 J. Geophys. Res. 111 A02307
[20] Zubarev N M, Mesyats G A and Yalandin M I 2017 JETP Lett. 105 537–41
[21] Zubarev N M et al 2018 J. Phys. D: Appl. Phys. 51 284003
[22] Mesyats G A et al 2020 Appl. Phys. Lett. 116 063501
[23] Shklyaev V A, Belomyttsev S Ya and Ryzhov V V 2012 J. Appl. Phys. 112 113303