On simple and subtle properties of neutrinos

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Abstract

Neutrino flavors, light and heavy, are discussed in their interdependence within the minimal unifying gauge group of SO10. The general situation which excludes the existence of an exact symmetry from which exactly vanishing light neutrino masses would follow is discussed. Subtle and simple consequences for 'low-low' oscillation phenomena are presented in a general framework.

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1 SO10 extension of the standard model:

\[
(\nu, N)^{\dot{\gamma}}_{(J)}
\]

In the title above the label \(\dot{\gamma}\) denotes, for \(\dot{\gamma} = 1, 2\), the two left-chiral components of the spin \(1/2\) fields \(\nu\) and \(N\) respectively.

The label \((J)\), for \(J = 1, 2, 3\), denotes family number. So we define the (minimal) fermionic extension of the three standard model fermion families to three families of 16-representations of SO10:

\[
\{ f \}^{\dot{\gamma}}_{(J)} = \left\{ \begin{array}{c}
\nu_e \\
d^{1, 2, 3}_e
\end{array} \bigg| N_e, \hat{u}^{1, 2, 3}_1 \right\}^{\dot{\gamma}}_{(J)}
\]

(1)

Under the standard model gauge group \(SU_3^c \times SU_2^L \times U_1^Y\) the fermion flavors in eq. (1) transform as

\[
\left\{ \begin{array}{c}
u_e \\
e^-
\end{array} \right\} = (1, 2, -\frac{1}{2})
\]

(2)

We ask as a guiding question whether the associated B - L (baryon number - lepton number) charge with quantum numbers

\[
\left\{ \begin{array}{c}
\frac{1}{3}, \frac{1}{3}, -1 \\
\frac{1}{3}, \frac{1}{3}, -1
\end{array} \bigg| +1, -\frac{1}{3}, -\frac{1}{3}, \frac{1}{3}
\right\}
\]

(3)

can be conserved - if not gauged - in the limit \(m^{N_{(J)}} \rightarrow \infty\).

The limit of infinite mass for the (standard model gauge group) singlet neutrino flavors \(N_e^{(j)}\) implies a Majorana mass matrix of the form

\[
\mathcal{H}_m = \frac{1}{2} m^{N_{(J)}} N^{\dot{\gamma}}_{(J)} \hat{N}^{\dot{\gamma}}_{(J)} + h.c.
\]

\[
m^{N_{(J)}}_{(J)} = m^{N_{(J)}} \rightarrow M_{(J)} ; \quad M \rightarrow \infty
\]
Apart from being symmetric the mass matrix $m_N$ in eq. (4) is complex general and the physical masses in the infinite mass limit involve the eigenvalues of the associated hermitian matrices $(m^\dagger m)^{1/2}$ and (equivalently) $(m^\dagger m)^{1/2}$.

Including the $N_J$ flavors (before taking the infinite mass limit) the B - L current is of vectorial form. In a chiral basis this amounts to a matching number of positive and negative eigenvalues of the B - L charge as exhibited in eq. (3):

$$j_{\mu}^{B-L} (16) = \sum J \sum_{r}^{16} (B - L) r f_r^* \beta f_r^\gamma \gamma (J)$$

For finite masses $m_N$ the B - L current is broken by these masses, but in the infinite mass limit the remaining (45) flavors are constrained to form a chiral current:

$$j_{\mu}^{B-L} (15) = \sum J \sum_{r}^{15} (B - L) r f_r^* \beta f_r^\gamma \gamma (J)$$

This reduced chiral current ($j_{\mu}^{B-L} (15)$ in eq. (6)) develops a gravitational anomaly and thus fails to be conserved in a gravitational environment:

$$D_\mu j_{\mu}^{B-L} (15) = 3 c_1 (\text{spin}) p_1 (R) = 3 \mathcal{A}_1 (R)$$

$$c_1 (\text{spin}) = -\frac{1}{11}$$

In eq. (7) $D_\mu$ denotes the covariant derivative with respect to the vierbein. Since the current is equivalent to an antisymmetric 3-form (in four dimensions) the divergence does not involve the vierbein or the metric and thus the right hand side of eq. (7) necessarily defines a topological 4-form.

This 4-form defines, apart from the overall factor 3, the Hirzebruch-Atiah index form. The integral of this form yields the chiral topological invariant of the (full) Dirac operator pertaining to the curvature 2-form of the metric (or vierbein).

It is also referred to as $\mathcal{A}$ genus in the (mathematical) literature and extends to any (Euclidean (compact)) space with dimension divisible by 4 : $d_n = 4 n$. The coefficient $c_n (\text{spin})$ gives the relation between the Hirzebruch-Atiah- and Pontrjagin classes in $d_n = 4 n$ dimensions. The latter are generically (for all n simultaneously) defined through the relations
\[
\overline{R}^a_b = \frac{1}{2\pi} \frac{1}{2} d x^\mu \wedge d x^\nu \left( R^a_b \right)_{\mu\nu}
\]

\[
Det \left( 1 - \lambda \overline{R} \right) = \sum_n \lambda^{2n} p_1 (R) \tag{8}
\]

\[
p_1 = \frac{1}{16 \pi} R^a_b \mu \nu \tilde{R}^b_a \mu \nu \ ; \ \tilde{R}^b_a \mu \nu = \frac{1}{2} \varepsilon^{\mu\nu\sigma\tau} \left( R^a_b \right)_{\sigma\tau}
\]

\[
p_2 = -\frac{1}{4} tr \overline{R}^4 + \frac{1}{8} \left( tr \overline{R}^2 \right)^2 \ ; \ \cdots
\]

with

\[
A(R; \lambda) = Det \left( \frac{\lambda^{1/2} \overline{R}}{\sin \left( \lambda^{1/2} \overline{R} \right)} \right) = \sum_n \lambda^{2n} c_n (R) \tag{9}
\]

\[
c_1 = -\frac{1}{24} p_1 \ ; \ c_2 = \frac{1}{2^2 3^2 5} \left( -4 p_2 + 7 p_1^2 \right) \ ; \ \cdots
\]

- the classes (Hirzebruch-Atiyah and Pontrjagin classes) are simple.
- the relations are obvious but not clearly simple.
- the consequence is subtle: There does not exist any exact symmetry, which is necessary to maintain exactly massless neutrino flavors.

This is so even if the overall (6 chiral flavor) neutrino mass matrix would exactly conserve B - L.

In order to have exact B - L symmetry the six chiral neutrino flavors have to combine into triply doubled (Dirac) pairs with equal overall B - L quantum numbers:

\[
\{ \nu_A \}_{(J)} = \left( \begin{array}{c}
\varepsilon_{\alpha\beta} \left( N^{\beta} \right)^* (J) \\
\nu^{\dot{\gamma}} (J)
\end{array} \right) ; \ A = 1, \cdots, 4 \tag{10}
\]

\[
A = 1, 2 \leftrightarrow \alpha = 1, 2 \ ; \ A = 3, 4 \leftrightarrow \dot{\gamma} = 1, 2
\]

with a B - L conserving mass matrix of the form equivalent to the case of (electrically) charged flavors:
\[ 
\mathcal{H}_{m}^{\nu N} = m_{J J'}^{\nu N} \bar{\nu} J \left( 1 + \gamma^5 L \right) \nu J' + h.c. 
\]

\[ \gamma^5 L = -\gamma^5 R = i \gamma^0 \gamma^1 \gamma^2 \gamma^3 \]

The reduced mass matrix \( m_{J J'}^{\nu N} \) is an unrestricted complex 3 by 3 matrix. The physical masses are the eigenvalues of the hermitian (nonnegative) combination \( \left( m_{J J'}^{\nu N} m_{J J'}^{\nu N} \right)^{1/2} \).

While in the above situation B - L is exactly conserved and therefore not gauged, three arbitrary neutrino masses are allowed.

Using the (Dirac doubled) field variables \( \{ \nu J \} \) as defined in eqs. (10) and (11) the conserved B - L current is given by

\[ 
\bar{j}_{B-L}^{\mu} (16) = \sum_{J} \left( \begin{array}{c} 
\frac{1}{3} \sum_{q=u,d} \bar{q}^{c} (J) \gamma_{\mu} q^{c} (J) \\
- \bar{\nu} (J) \gamma_{\mu} \nu (J) \\
- \bar{e} (J) \gamma_{\mu} e (J) 
\end{array} \right) 
\]

2 The diploma thesis cited contains common work with P.M.
- the mixing matrix of any three out of the six neutrino flavors is \textit{not} unitary.
- the number of real CP violating parameters, associated \textit{just} with the mass matrix of the minimally six neutrino flavors is 15. These are all observable, albeit with variable sensitivity (at relatively) low energies.

\textbf{A simple phenomenological excursion}

Let us 'fix ideas' with respect to the three light neutrino flavors, and restricting the \textit{assumed} heavy ones to three, \textit{assuming} hierarchical masses \cite{footnote1}:

\[
\{ m^{\nu N} \} = \{ m_{1,2,3}, M_{1,2,3} \} \\
M_{1,2,3} \gg 1 \text{ GeV} ; m_{1,2,3} \ll 1 \text{ GeV} \\
m_3 \gg m_2 > \text{ or } \gg m_1 ; \Delta_{ij}^2 = m_i^2 - m_j^2 \\
\Delta_{32}^2 = 3 \times 10^{-3} \text{ eV}^2 ; \text{ to be modified eventually} \quad (13) \\
\Delta_{32} = 0.055 \text{ eV} = 632 \circ K \\
\Delta_{21}^2 = 3.5 \times 10^{-5} \text{ eV}^2 ; \text{ to be modified eventually} \\
\Delta_{21} = 0.0059 \text{ eV} = 68.5 \circ K
\]

This yields, reemphasizing the elimination \textit{by assumption} of a common mass for the three light neutrino flavors much larger than the mass differences

\[
m_3 \simeq 0.055 \text{ eV} = 632 \circ K \\
m_2 \simeq 0.0059 \text{ eV} = 68.5 \circ K \quad (14) \\
m_1 \gg 2 \circ K
\]

Then the mass eigenstates 3 and 2 constitute hot dark matter at the time of nucleosynthesis but cold today, whereas the lightest neutrino flavor 1 may be still in relativistic (mean) motion today.

The general (\(\nu, N\)) mass term is of the form

\footnote{The cited paper gives an extensive phenomenological review.}
\[ H^{\nu N} = \frac{1}{2} \nu_i^\delta \varepsilon_{\gamma \delta} M_{ik} \nu_k^\gamma + h.c. \; ; \; M_{ik} = M_{ki} \]

\[ i, k = 1, \ldots, 3 + N_h ; \; r = k - 3 = 1, \ldots, N_h ; \; \varepsilon_{\gamma \delta} N_{r}^\delta = N_{\gamma r} \]

\[ M = \begin{pmatrix} 0 & \mu^T \\ \mu & M \end{pmatrix} \]

(15)

In eq. (15) we generalize to \( N_h \geq 3 \) heavy neutrino flavors, while the minimal SO10 scenario has \( N_h = 3 \).

The mass matrix \( M \) is compatible with the so extended standard model renormalizability conditions, provided the \( N_h \times 3 \) matrix \( \mu \) and its transpose are generated through Yukawa couplings of one or several electroweak scalar doublets.

\[ -L_Y = g^{(\alpha)}_{ki} N_{\gamma k-3} \left( \phi^0_{(\alpha)}, \phi^-_{(\alpha)} \right)^* \nu_i^\gamma e^{-i \gamma} + h.c. \]

\[ i = 1, 2, 3 ; \; k = 4, \ldots, 3 + N_h ; \; \alpha = 1, \ldots, n_{sc} \]

\[ \mu_{ki} = g^{(\alpha)}_{ki} v_{c(\alpha)} ; \; v_{c(\alpha)} = \langle \Omega | \phi^0_{(\alpha)} | \Omega \rangle^* \]

(16)

In eq. (16) \( \alpha \) numbers scalar doublets. For one doublet in the SM or two doublets in the MSSM the vacuum expected values are

\[ SM : n_{sc} = 1 ; \; v^c_{(1)} \rightarrow \frac{1}{\sqrt{2}} v ; \; v = \left( G_F \sqrt{2} \right)^{-1/2} \]

\[ MSSM : n_{sc} = 2 ; \left( v^c_{(1)} \rightarrow \frac{1}{\sqrt{2}} v^u \right) ; \; v^u = \sin \beta v \]

(17)

**Generic estimate of light versus heavy flavors**

A simple consequence of the structure of \( M \) as defined in eq. (15) concerns the absolute value of the determinant, given the assumption that each (complex) eigenvalue of the \( N_h \times N_h \) matrix \( M \) is much larger in absolute value than (the absolute value of) any given element of the \( N_h \times 3 \) matrix \( \mu \).
It then follows for the (complex) determinant of $M$:

$$Det \ M = Det \ M' ; \ M' = \left( \begin{array}{cc}
0 & \mu' T \\
\mu' & M'
\end{array} \right)$$

$$\mu' = \left( \begin{array}{cccc}
k = 4 & \hat{\mu}_{11} & \hat{\mu}_{12} & \hat{\mu}_{13} \\
k = 5 & \hat{\mu}_{21} & \hat{\mu}_{22} & \hat{\mu}_{23} \\
k = 6 & \hat{\mu}_{31} & \hat{\mu}_{32} & \hat{\mu}_{33} \\
k = 7 & 0 & & \\
\vdots & 0 & & \\
k = N_h & 0 & & \\
\end{array} \right)$$

$$M' = \left( \begin{array}{cc}
0_{3 \times 3} & 0_{3 \times 3} \\
0_{3 \times 3} & \hat{M}
\end{array} \right) ; \ \hat{M} = \hat{M}_{\kappa \lambda}$$

$$k, l = \kappa + 3, \lambda + 3 = 7, \cdots, N_h$$

In eq. (18) $\hat{\mu}$ is formed from any subset of three out of the $N_h$ (three component -) row vectors of $\mu$ with maximal rank, which we generically assume to be maximal, i.e. three.

For the case of minimal heavy neutrino flavors $N_h = 3$, we have simply $\hat{\mu} = \mu$ and $\hat{M} \rightarrow 1$.

Modulo simple permutations according to the subset of three row vectors in $\mu$ chosen to form $\hat{\mu}$, the $N_h - 3 \times N_h - 3$ matrix $\hat{M}$ is of the form

$$\hat{M}_{\kappa \lambda} = \left[ M_{\kappa \lambda} + \left( L^{(i)}_\kappa M_{(i)\lambda} + \kappa \leftrightarrow \lambda \right) \right] + L^{(i)}_\kappa M_{(i)(j)} L^{(j)}_\lambda$$

$$| L^{(i)}_\kappa | = O(1) ; \ (i), (j) = 1, 2, 3$$

Except for non-generic matrices $\mu$ and $M$ the eigenvalues of $\hat{M}$ ($N_h - 3$ in number) are of the same order of magnitude than those of $M$ ($N_h$ in number).
From the structure of $M'$ in eq. (18) it follows

$$| \text{Det} \ M | = \left( \prod_{i}^{3} m_{i} \right) \left( \prod_{J}^{N \ h} M_{J} \right) = | \text{Det} \ 3 \times 3 \ \hat{\mu} |^{2} | \text{Det} \ N_{h-3 \times N_{h-3}} \ \hat{M} |$$

(20)

In eq. (20) (extending eq. (13)) $M_{1}, \cdots, M_{N_{h}}$ denote the masses of the heavy flavors.

The above simple relations are known as 'sea-saw' mechanism [3].

Introducing the geometric mean (nonnegative) masses

$$\prod_{i}^{3} m_{i} = m_{\text{light}}^{3} ; \quad \prod_{J}^{N \ h} M_{J} = M_{\text{heavy}}^{N \ h}$$

(21)

the relation in eq.(20) takes the form

$$m_{\text{light}} \ M_{\text{heavy}} = \bar{\mu}^{2} \left( \frac{M}{M_{\text{heavy}}} \right)^{N \ h/3 - 1}$$

(22)

The estimate of the Yukawa induced (doublet-singlet) mass $\bar{\mu}$ is derived from the simple $16 \times 16 \times 10$ SO10 mass relation evolved for quark flavors to a unification mass $M_{\text{unif}} \sim 10^{16} \text{GeV}$:

$$\bar{\mu} = C_{\nu} \frac{1}{3} (m_{u} m_{c} m_{t})^{1/3} \sim C_{\nu} \ 0.35 \text{GeV} \ ; \ C_{\nu} = O (1)$$

(23)

Let us measure the geometric mean light neutrino mass $m_{\text{light}}$ in units of $10^{-2} \text{eV}$. Then the estimate in eq. (22) takes the form

$$m_{\text{light}} = K_{\text{light}} \ 10^{-11} \text{GeV} \ \rightarrow$$

$$\ M_{\text{heavy}} \sim 1.2 \ 10^{10} \text{GeV} \ (C_{\nu}/K_{\text{light}}) \left( \frac{M}{M_{\text{heavy}}} \right)^{N \ h/3 - 1}$$

(24)
For given light neutrino mass (in geometric mean) and given $\mu$, $M_{\text{heavy}}$ can be reduced through the ratio $M/M_{\text{heavy}}$ even considerably if there are many heavy neutrino flavors beyond the minimal three. However this emerges as the simple consequence of the light neutrino flavors mixing predominantly to the heaviest three, while only little to the lighter $N_h - 3$ ones.

I wish to emphasize here, that any mass relations inside and outside of a unifying gauge group out of the increasing sequence $SO10 < E6 < ... < E8$ cannot explain the pattern of light and heavy neutrino flavors. E.g. the mass matrix $M$, remains totally unconstrained within the SM, yet within SO10 it has the quantum numbers of the 126 irreducible representation. If there exist elementary scalars transforming as this 126 (complex) representation the induced structure of $M$ is intrinsically tied to rest symmetries remaining after the breakdown of SO10 gauge invariance, beyond the symmetries of the SM. If these symmetries are associated with an N=1 or 2 susy structure again no clear mass relations among the known fermion families arise naturally.

On the other hand the mass matrix $M$ can be induced through finite loop effects involving e.g. the square of scalar 16 representations, but again no direct structure reflecting this situation on the known fermion families can be derived.

On the other hand the generic ratio

$$m_{\text{light}} / M_{\text{heavy}} \sim 10^{-21} \leftrightarrow M_{\text{heavy}} \sim 10^{7} \, \text{TeV} \quad (25)$$

is a subtle measure of lepton number violation at electroweak and lower energies, as well as of associated CP violation. The generic mass scale $M_{\text{heavy}}$ is well above the assumed susy scale - by seven orders of magnitude - if the latter is assumed to be 1 TeV. From this a dangerous enhancement of both CP-violation beyond the CKM matrix and lepton number violation by these seven orders of magnitude indirectly affect susy induced contributions to all electric dipole moments (transition dipole moments for neutrinos) including the neutron, and directly the charged leptons, as well as to lepton number violating processe like $\mu \rightarrow e \gamma$ and $\mu \rightarrow 3 e$.

This in my opinion augments the necessity to look for a theoretically viable explanation of mass and mixing structures but it must be sought along hitherto unexplored paths. Further new phenomena beyond the SM hopefully will give some clues.
3 The Majorana equations for $\nu$, $N$ flavors

In the following we restrict the discussion to three heavy neutrino flavors for simplicity. Then the field equations for the associated chiral fields $\nu^j, \nu^\ast_{\alpha j}, j = 1,...,6$ take the form

\begin{align*}
\nu^j_{j+3} &= N^j_j; \quad j = 1,2,3 \\
\nu^\ast_{\alpha j} &= \varepsilon_{\alpha\beta} \left( \nu^\beta_j \right)^\ast \\
i D^{\gamma\alpha} \nu^\ast_{\alpha} &= M \nu^\gamma \\
i D_{\alpha\gamma} \nu^j &= M^\dag \nu^\ast_{\alpha} \\
M^\dag &= \overline{M}; \quad M = M^T
\end{align*} (26)

In eq. (26) the covariant derivatives are restricted to gravity.

**Diagonalization to mass eigenstates** $\nu \rightarrow \nu^\ast$

It is simple but somewhat involved [4] to unitarily diagonalize the 6 by 6 matrix $M$

\[ M = U M_D U^T; \quad U = \begin{pmatrix} U_{(11)} & U_{(12)} \\ U_{(21)} & U_{(22)} \end{pmatrix} \]

\[ M_D = \text{diag} \left( m_{1,2,3}; M_{1,2,3} \right) \] (27)

In eq. (27) we break up the 6 by 6 matrix $U$ into 3 by 3 blocks, since neutrino’s prepared in a system which does not allow for the production of heavy neutrino flavors involves only the mass eigenfields $\nu_k; \quad k = 1,2,3$, whereas again at electroweak energy scales only (predominantly) the modes of $\nu_j; \quad j = 1,2,3$ are produced through the electroweak interactions and are inducing a reaction downstream the oscillation path. We refer to this as 'low-low' oscillation physics.

Eq. (26) becomes

\[ \mathcal{M} = U M_D U^T; \quad U = \begin{pmatrix} U_{(11)} & U_{(12)} \\ U_{(21)} & U_{(22)} \end{pmatrix} \]

\[ \mathcal{M}_D = \text{diag} \left( m_{1,2,3}; M_{1,2,3} \right) \]
\[ i \dot{D} \gamma^\alpha (U^T \nu)^* = \mathcal{M}_D (U^T \nu) \gamma^\alpha \]
\[ i \dot{D}_{\alpha\gamma} (U^T \nu) \gamma^\gamma = \mathcal{M}_D (U^T \nu)^\alpha \]
\[ \dot{\nu}_n = U_{mn} \nu_m ; \ m, n = 1, \cdots, 6 \]

and for \(k,l = 1,2,3\):
\[ \nu_k = (\overline{U}(11))_{k \ell} \dot{\nu}_l + (\overline{U}(12))_{k \ell} \dot{\mathcal{N}}_l \]
\[ \mathcal{N}_k = (\overline{U}(21))_{k \ell} \nu_l + (\overline{U}(22))_{k \ell} \dot{\mathcal{N}}_l \]
\[ \nu_k^* = (U(11))_{k \ell} \dot{\nu}_l^* + (U(12))_{k \ell} \dot{\mathcal{N}}_l^* \]
\[ \mathcal{N}_k^* = (U(21))_{k \ell} \nu_l^* + (U(22))_{k \ell} \dot{\mathcal{N}}_l^* \]

'low-low' oscillation states and amplitudes

For 'low-low' oscillations only the 3 by 3 matrices \(U(11)\) and \(\overline{U}(11)\) are operative.

First we consider production of a normalized state (without subscript \((0)\)) at \(t = 0\) of \(nu\) type \(k\):

\[ (U(11))_{k \ell} = \mathcal{A}_{k \ell} ; \ \mathcal{A} = \tau U_0 \]
\[ |\vec{p}^*; t = 0 ; k \rangle (0) = \mathcal{A}_{k \ell} |\vec{p}^*; m \ell ; 0 \rangle \]
\[ |\vec{p}^*; t = 0 ; k \rangle (0) = \mathcal{N}_k |\vec{p}^*; t = 0 ; k \rangle \]
\[ \mathcal{N}_k^2 = (\mathcal{A} \mathcal{A}^\dagger)_{k k} = \tau_{kk}^2 \rightarrow \]
\[ |\vec{p}^*; t = 0 ; k \rangle = \varrho_k \mathcal{A}_{k \ell} |\vec{p}^*; m \ell ; 0 \rangle ; \ \varrho_k = 1 / \mathcal{N}_k \]

In eq. \((29)\) we decomposed the matrix \(U_{(11)} = \mathcal{A}\) into the product of a hermitian positive matrix \(\tau\) and a unitary (3 by 3) matrix \(U_0\). The deviation of \(\tau\) with \(0 \leq \tau \leq 1\) from unity is of the generic order \(m_{light}/M_{heavy} \sim 10^{-21}\), yet it is one of the subtle properties of neutrino flavor mixing, that this deviation is at the very origin of the light neutrino masses. This is only so, if
we assume the generation of these masses through mixing with heavy flavors, as we do here.

**subtle things:**

- the normalization $\varrho_k \mathcal{A}_{kl}$ can be enforced by properly tagging $\nu_k$ upon production irrespective of decay rates, e.g. for $\pi^+ \rightarrow \mu^+ + \nu_\mu + n\gamma$.

- spin states, probabilities for helicities

\[
\begin{align*}
-1 & : 1 - (1 - v) / 2 \\
+1 & : (1 - v) / 2 \sim m_\nu^2 / (4 E^2)
\end{align*}
\]

The precise probability of the 'wrong' helicity state does depend on the nature of production and decay amplitudes.

- to go from neutrino flavors to antineutrino flavors amounts to exchange the helicities and to substitute $\mathcal{A} \rightarrow \overline{\mathcal{A}}$.

**Oscillation amplitudes**

We denote the amplitude of transition from $t = 0$ to $t$ and from initially produced neutrino flavor $k$ to neutrino flavor $k'$ by $T_{k' \leftarrow k}$, neglecting the 'wrong' helicity states, and likewise by $AT_{k' \leftarrow k}$ the corresponding antineutrino amplitude.

\[
\begin{align*}
T_{k' \leftarrow k} (t) &= \varrho_{k'} \varrho_k \left( \mathcal{A} U_{\text{diag}} (t) \mathcal{A}^\dagger \right)_{k'k} \\
AT_{k' \leftarrow k} (t) &= \varrho_{k'} \varrho_k \left( \overline{\mathcal{A}} U_{\text{diag}} (t) \overline{\mathcal{A}}^\dagger \right)_{k'k} \\
U_{\text{diag}} (t) &= \text{diag} \left( e^{-i E_1 t}, e^{-i E_2 t}, e^{-i E_3 t} \right) \\
E_j &= \sqrt{p^2 + m_j^2} \sim p + \frac{1}{2} m_j^2 / p
\end{align*}
\]

The amplitudes in eq. (30) describe oscillations in vacuo. Matter effects modify $\mathcal{A}$ in an energy dependent way.

CPT invariance is manifest, whereas T invariance requires $\mathcal{A}$ to be real:
\[ CPT : (A T_k \leftarrow k' (-t))^\ast = T k' \leftarrow k (t) \]
\[ T : (T_k \leftarrow k' (-t))^\ast = T k' \leftarrow k (t) \]
and \( T \leftrightarrow AT \)
\[ \rightarrow A = \overline{A} \]

**Counting phases**

We generalize again the counting of imaginary parameters or equivalently of complex phase factors to \( 3 \rightarrow N_h \) heavy neutrino flavors with \( n = 3 + N_h \). The number of independent imaginary parameters in the mass matrix \( M \) of the form given in eq. (15) is

\[ \# \varphi (M) = \frac{1}{2} n (n + 1) - 6 \quad (32) \]

The number of phases is 15 for 6 neutrino flavors.

### 4 Conclusions and outlook

- The major experimentally accessible features: \( m_{1,2,3} \) and \( U_0 \) as defined in eq. (29), i.e. light neutrino mass and mixing hopefully will establish the specific structure of \( \nu - N \) dynamics.

- The subtle and small effects: 1) \( A = \tau U_0 \) with \( \tau \neq 1 \) and 2) lepton flavor violation in conjunction with the observation of 'wrong' neutrino helicities or neutrinoless double \( \beta \) decay are in generic situations expected to be very small.

- \( CP \leftrightarrow T \) violating effects in the \( \nu, \bar{\nu} \) leptonic sector are much richer than in the case of quarks and antiquarks. They are tied to the small masses of the light neutrino flavors. Despite this 'low' energy obstruction it seems to me to be worthwhile to look for these effects especially at maximally feasible energies, where light - heavy \( \nu \rightarrow N \) transitions begin to play a role.

- "weniges ist mehr ...".
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