Applications of graph theory in district heat network analysis at national scale

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Abstract. District heat networks can make a significant contribution to decarbonising domestic heat supply and have been successfully applied in a number of countries. Traditionally, district heat networks have been analysed at the local or city district scale. Performing this analysis at national scale presents new methodological challenges. Efficient techniques are required to work with large datasets and enable rapid iteration of results. As district heating fundamentally involves networks, graph theory is highly relevant for providing well established analysis methods that can scale to very large datasets. Two applications of graph theory are presented for this work. The first presents a method for estimating minimum pipe lengths needed to connect buildings, from which demand densities may be derived. The second presents a graph-based clustering method for connecting distributed supplies and demand regions. These methods have been applied as Swiss national scale.

1. Introduction

District heat networks (DHN) can make a significant contribution to decarbonising domestic heat supply and have been successfully applied in a number of countries [1,2]. District heat networks also make it simpler to integrate decarbonised heat sources into the energy mix, notably waste heat streams. Currently however, they only contribute a less than 5% of the Swiss heating demand [3]. Ongoing work [4,5] aims to map district heat network potential and perform spatio-temporal analysis in order to link district heat networks with suitable renewable heat supplies as well as assess the potential of seasonal storage.

Traditionally, district heat networks have been analysed at the local or city district scale. Performing this analysis at national scale presents new methodological challenges. Efficient techniques are required to work with large datasets and enable rapid iteration of results. As district heating fundamentally involves networks, graph theory is highly relevant for providing well established analysis methods. Two applications of graph theory are presented for this work.

The first application described is the use of minimum spanning tree analysis for the rapid estimation of heat network linear demand densities. The linear demand density of a district heat network is an important parameter for determining its viability in a given region. Enabling its rapid estimation is therefore key to performing large scale mapping exercise. A map of the linear demand density per hectare for Switzerland is presented, as well as a general model for linear demand density that can be applied in a large variety of situations.
The second application of graph theory presented is the use of connected component analysis to discover geographically coherent clusters of heat source and heat demand clusters. Specifically, we overlay mapping of industrial waste heat source regions with potential district heat networks with the aim of determining the contribution of ‘free’ waste heat to heating demand. By forming a graph of connectable heat sources and sinks we perform connected component analysis to discover independent clusters within which heat balances can be calculated. Furthermore, load curve analysis is performed to estimate the potential for seasonal storage.

These two examples present methodological innovations in the field of district heat network analysis, which can be applied generally for any geographical region. They allow the processing of geographical information at big-data scales, enabling forms of analysis that have been traditionally applied only regionally to districts and cities but could instead be applied across entire countries. These new big-data oriented methods furthermore blur the lines between regional and national analysis by removing limitations that have traditionally prevented national-scale analysis from taking into account specific local conditions.

2. Method for estimating DHN pipe network lengths

To-date, thermal-atlas approaches to DHN analysis have generally not attempted to calculate pipe lengths. Optimal pipe layouts are more commonly estimated for specific DHNs with a small set of buildings, which tend further to focus on balancing the thermal and hydraulic flows within the network [4–6]. These can use information on local road layouts to estimate an optimal route [7], however this detailed approach is not suitable at national scale. However, pipe lengths are important for estimating linear thermal demand densities, which have a significant impact on the network system costs.

This work takes advantage of the existence of the comprehensive Swiss building registry – the Registre des Batiments et Logements (RegBL), maintained by the Swiss Federal Office for Statistics (SFOS), which includes for each building a unique identifier (EGID) and location.

This work proposes an approach for rapidly estimating the minimum pipe network needed to connect a set of buildings, which can be later used to estimate a maximum linear demand density at a hectare level for the whole country. Planning a real pipe network is a complex process that requires accounting for detailed local conditions such as road layout, site access, and existing underground installations. In contrast, for calculating maximum potential linear demand density, only the length of the shortest possible pipe network is required.

Calculating the shortest set of connections between a set of nodes, known as the Minimum Spanning Tree (MST), is a known problem in graph theory. The Kruskal algorithm [6] can be used to calculate the MST given a set of node and edge weights (connections between nodes having a given ‘length’ or ‘cost’). Concretely, the nodes are building locations and edge weights are the linear (Euclidean) distances between buildings. As the Swiss building registry provides exact building locations, the minimum pipe length for a set of N buildings can be calculated as follows:

1. Calculate the pairwise distances between every building (resulting in N² distances).
2. Create the graph connection representation as an NxN matrix where the value [i, j] is the linear distance between building i and j.
3. Calculate the MST using the Kruskal algorithm. The implementation in the Scipy Python package was used [7].
4. Sum the length of the resulting graph edges to calculate the minimum total pipe length.
Figure 1. Illustration of the generation of a Minimum Spanning Tree (MST) network for a randomly selected hectare containing 20 buildings. a) shows the initial fully connected network which consists of every connection between every building, b) shows the MST network produced by the Kruskal algorithm. Background image by Google Maps.

The pipe length can be calculated for a given hectare by selecting all buildings within the hectare and applying the algorithm above. However, calculating pipe lengths for every hectare is unnecessary and somewhat impractical, particularly as the number of buildings per hectare may change depending on possible pre-filtering of buildings e.g. in the case of comparing different scenarios which excluded certain buildings from being connected to a DHN such as those already equipped with high efficiency heating systems.

Figure 2. Boxplot of minimum pipe lengths calculated for samples of hectares as a function of number of buildings per hectare. The orange line represents logarithmic model fit to the distribution medians.

Instead, we use the observation that the number of buildings in a hectare is the main determinant of total pipe length and proceed with a sampling approach. For each N buildings per hectare from N=2 to N=50, 500 hectares were selected randomly with replacement. The minimum pipe length was calculated for each sample and the median length as a function of N buildings per hectare was also determined. The results are shown in Figure 2, clearly demonstrating the non-linear growth of the pipe length as a function of the number of buildings. A larger variability was found for large N (N>35) as there were relatively few hectares for that number. The trend for pipe length $l$ can be accurately described by a
logarithmic model fit, which is performed using a linear regression of pipe length against $\ln(N_{\text{buildings}})$ (regression $R^2=0.984$):

$$l = 130.6 \times \ln(N_{\text{buildings}}) - 84.5 \quad (\text{eq. 1})$$

This enables rapid calculation of minimum pipe length on a hectare grid using only the hectare grid of count of buildings per hectare.

It is important to highlight that the derived relation between building density and minimum pipe length should hold for other European countries, given the broad similarity in spatial distribution of buildings. Therefore, for any location the minimum pipe length can be estimated given a value for the building density per hectare.

In [4] a validation is performed of the DHN areas calculated with the aid of this method against regional case studies and comparing national aggregate DHN potential against previous national estimates, wherein a good agreement was found with both local and national analyses.

3. Method for spatial clustering of supply and demand for district heat networks

A key benefit of DHN is to make use of local energy resources. Frequently, such resources are geographically distributed and there are constraints placed upon how they may be connected to demand regions. In general, heat transmission cannot occur over very long distances, therefore analysis of supply potentials requires a spatial clustering of supply and demand. This section presents a network-theory based method for spatial clustering of source and demand regions, which allows rapid and scalable analysis. It makes it possible to take into account local constraints while still performing nation-wide calculations.

In this section, an example will be given using Industrial Excess Heat (IEH) sources which are geographically distributed as they flow from buildings associated with individual businesses. The question is how to analyze the balance of IEH heat sources and DHN heat demand taking into account spatial constraints. Furthermore, it is not feasible to perform a detailed local optimization of the supply/demand matching. A full treatment of this question would require detailed allocation of resources between the different demands according to a chosen allocation strategy, e.g. completely fair distribution, prioritization of a single demand, etc. This can be considerably simplified by aggregating inter-connected regions using graph theory and calculating the net heat balance for each region.

The collection of possible connections between heat sources and demands can be considered as a directed graph where nodes are the sources or demands and possible heat supply connections are the directed edges of the graph. We may form a graph for the entire country by generating possible connections between source and demands according to certain threshold criteria (described below). Within this graph there will exist clusters of supply/demand nodes which are connected to each other but not to the rest of the graph. These are known in graph theory as weakly connected, which consist of subgraphs wherein there exists an undirected path between every source/demand pair.

Figure 3 shows an overview of the analysis workflow to obtain clusters:

1. Each source polygon and DHN zone was converted to GIS polygons with unique identifiers.
2. For each excess heat source polygon, district heat regions within radius $R_{\text{max}}=5\text{km}$ were found (calculated as closest distance between the polygons boundaries). The linear connection demand density was calculated as the yearly demand of the DHN region divided by the distance to the source region.
3. Connections with a density of more than $1.8\text{MWh}/\text{m}$ were retained, resulting in a set of source/demand identifier pairs. The threshold is selected based on the work of [8], but may be adjusted according to selected connection criteria and will be affected by uncertainty in the spatial definition of the polygons.
4. The complete set of connected source/demand ID pairs were converted into a directed graph where each node was a source or a demand and the connection defined above formed the graph edges. This graph was split into a set of weakly connected components using the NetworkX package [9,10].
5. We define the resulting weakly connected components as clusters of sources and demands.

![Diagram](image)

**Figure 3.** Overview of method to combine polygons defining district heating regions and polygons defining IEH supply regions.

Figure 4 illustrates one weakly connected component cluster, showing the interconnection between heat source zones (in this case zones containing industrial activity which produces waste heat) and potential DHN zones (in this case for low temperature district heating technology). The method can readily generate clusters for the whole country and therefore enables national-scale analysis which takes into account specific local conditions. Energy balance analysis can be performed within each cluster to determine suitable local energy distribution, which can then be used as part of local energy planning or aggregated to national scale.

![Map](image)

**Figure 4.** Example of weakly connected component generated by graph clustering method.
Application and evaluation of this method is performed in [5], wherein the potential for IEH utilization in Switzerland is calculated. It should be noted that the method is suitable for a range of spatially distributed energy sources.

4. Conclusion
Application of graph analytics methods is important for analysis of district heat networks and enables regionally-specific analysis to be applied nationally in an automated fashion. Application of mature graph-theoretical results and software implementation allow the expansion of DHN analysis to national scale, blurring the line between local case studies and national analysis through the usage of big data processing methods and appropriate algorithms.

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References
[1] EU. Commission Staff Working Document on an EU Strategy for heating and cooling 2016:1689–99. doi:10.1017/CBO9781107415324.004.
[2] International Renewable Energy Agency, IRENA. Renewable Energy in District Heating and Cooling: A sector roadmap for REmap 2017.
[3] Narula K, Chambers J, Streicher KN, Patel MK. Strategies for Decarbonising the Swiss Heating System. Energy 2018;169:1119–31. doi:10.1016/J.ENERGY.2018.12.082.
[4] Chambers J, Narula K, Sulzer M, Patel MK. Mapping district heating potential under evolving thermal demand scenarios and technologies: A case study for Switzerland. Energy 2019;176:682–92. doi:10.1016/J.ENERGY.2019.04.044.
[5] Chambers J, Zuberi MJS, Narula K, Patel MK. Spatiotemporal analysis of industrial excess heat supply for district heat networks in Switzerland. Under Submiss 2019.
[6] Kruskal JB. On the shortest spanning subtree of a graph and the traveling salesman problem. Proc Am Math Soc 1956;7:48–48. doi:10.1090/S0002-9939-1956-0078686-7.
[7] Jones E, Oliphant T, Peterson P, others. SciPy: Open source scientific tools for Python 2015.
[8] Sres A, Nussbaumer T. Weissbuch Fernwärme Schweiz – VFS Strategie Langfristperspektiven für erneuerbare und energieeffiziente Nah- und Fernwärme in der Schweiz 2014.
[9] Weisstein EW. Weakly Connected Component. MathWorld–A Wolfram Web Resour n.d. http://mathworld.wolfram.com/WeaklyConnectedComponent.html (accessed December 7, 2018).
[10] Hagberg AA, Schult DA, Swart PJ. Exploring Network Structure, Dynamics, and Function using NetworkX. In: Varoquaux G, Vaught T, Millman J, editors. Proc. 7th Python Sci. Conf., Pasadena, CA USA: 2008, p. 11–5.