On the Capacity of Bi-static Modulated Re-scatter Systems

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Abstract

In ambient re-scatter communications, devices convey information by modulating and re-scattering the radio frequency signals impinging on their antennas. In this correspondence, we consider a system consisting of a modulated continuous carrier multiple-input multiple-output (MIMO) link (primary) and a multi-antenna modulated re-scatter (MRS) node (secondary), where the MRS node uses the signal generated by the primary transmitter. The receiver tries to decode both the original transmitted message and the information added by the MRS antennas. We show that the sum capacity of this system exceeds that, which the MIMO system could achieve alone. We also consider the impact of channel estimation errors under least squares channel estimation. The results suggest that the estimation error has negative impact on the capacity, however, its severity can be minimised by increasing the number of receiver antennas.

Index Terms

Backscatter, re-scatter, bi-static channel, MIMO, keyhole channel, modulation coding, polyphase coding.

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1Widely used term back-scatter refers to reflecting the received signals back in the direction of arrival. In this work, the scattering direction is not constraint so that the term re-scatter is used instead.
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I. INTRODUCTION

One of the limiting factors of connecting things to the Internet using wireless technologies is the availability of energy. One solution is provided by the modulated back-scattering (MBS) systems, such as RFID, where the nodes (tags) modulate their information onto a carrier generated by a reader, and the reader decodes the modulated information. In the advanced forms of MBS systems, multiple-antenna techniques are applied to increase the capacity \(^1\) and improve the reliability \(^2\). The communication range can be increased by allowing a carrier transmitter and a reader (receiver) to be physically separated, referred to as a bi-static MBS system \(^3\). Recently, a new communication technology is emerging based on the fact that the tags can re-scatter even modulated signals, often referred to as ambient MBS system \(^4\). These systems enhance the use-case possibilities of MBS systems since the information is exchanged through modulating and back-scattering the RF signals without power hungry transceivers.

In this correspondence, we propose a system that extends a bi-static MBS system to an ambient bi-static modulated re-scatter (MRS) system, which allows information transmission between a multi-antenna transmitter and a multi-antenna receiver (primary MIMO system). The multi-antenna MRS node (secondary MRS system) bear additional information on the signal emitted by the transmitter, and the receiver decodes the information of both sources as shown in Fig. \(^1\).

In the following sections, we provide a detailed analysis of the impact of different components on the capacity of the overall system, and a joint pilots design using Hadamard matrix for channel estimation. For the system model defined in Sec. \(\text{\textsection}\) II, we assume that the primary system uses a Gaussian codebook and the secondary system uses a polyphase coding scheme. Under these assumptions, in Sec. \(\text{\textsection}\) III we consider the case that the receiver has perfect channel state information (CSI). First, the joint capacity available for the two systems is derived. Then, it is shown that the sum capacity is larger than that, which the primary system could achieve alone. The excess capacity of the system can be utilised by the primary system alone or it can be shared with the secondary system. We show that the limiting capacity of the overall system coincides
with the sum of the limiting capacity of a rich scattering MIMO channel [5] and a multiple-keyhole channel [6]. The capacity that the secondary could obtain in the case it has a single antenna is also derived. In Sec. [VI] we first propose a pilot structure that allows joint estimation of the primary and secondary channels, and then consider the impact of channel estimation error by deriving the sum capacity lower bound as a function of estimation errors. We present the simulation results in Section [V] Section [VI] concludes this correspondence.

II. SYSTEM MODEL

We consider an ambient MRS MIMO system with an $n_t$-antenna transmitter, an $n_r$-antenna receiver, and a $K$-antenna MRS node shown in Fig. 1. The MRS antennas are located in an array without coupling between the antenna elements. Otherwise, coupling would contribute to channel uncertainty. The MRS is synchronised to the primary system. Information transmitted from the transmitter to the receiver goes through the direct links and links passing through the MRS. The signals passing through the MRS are modulated accordingly. We conduct the analysis by considering fixed channels.

Unless otherwise indicated, we adopt the following main assumptions: 1) the transmitted symbols of the primary system are Gaussian distributed; 2) the CSI is available at the receiver only, and the distribution of the CSI is known at the transmitter; 3) the channels is flat and quasi-static, i.e., the coherence time of the channel is much longer than the frame duration. When
channel estimation is considered, the training-based channel estimation and data transmission are assumed to be completed in one frame; 4) the channel vectors are independently and identically distributed (i.i.d) standard circularly symmetric complex Gaussian (CSCG) distributed, and are independent across the antennas of the primary and the MRS systems. These assumptions have been considered in literature, for instance, [7], [8] among others.

Notations: small and capital bold letters denote vectors and matrices, respectively; $A^\dagger$ and $A^T$ denote the Hermitian transpose and transpose of matrix $A$, respectively; $\mathcal{CN}(0, B)$ denotes the circular complex Gaussian distribution with zero mean and covariance matrix $B$; $\text{tr}(X)$ and $\text{det}(X)$ denote the trace and determinant of a matrix $X$, respectively; $\otimes$ denotes the Kronecker product operator; $I_n$ is an $n \times n$ identity matrix, and the subscript $n$ may be omitted for simplicity; $|| \cdot ||$ denotes the Frobenius norm for matrices and Euclidean norm for vectors; $\mathbb{E}\{X\}$ is the expectation of $X$.

A. Channel Model

Following the assumptions, the resulting complete channel matrix between the transmitter and the receiver in $\mathbb{C}^{n_r \times (K+1)n_t}$ can be written as

$$ G = \begin{bmatrix} G_0 & G_1 & \cdots & G_K \end{bmatrix}, \quad (1) $$

where $G_0 \in \mathbb{C}^{n_r \times n_t}$ denotes the channel matrix of the direct links, and the channel matrix passing through the $k$th MRS antenna $G_k \in \mathbb{C}^{n_r \times n_t}$ for $k = 1, \cdots, K$ is defined as

$$ G_k = \alpha g_{rk} g_{tk}^\dagger, \quad (2) $$

where $\alpha \in \mathbb{C}$ is the constant scale factor, $g_{tk} \in \mathbb{C}^{n_t}$ denotes the complex channel gains between transmit antennas and the $k$th MRS antenna, and $g_{rk} \in \mathbb{C}^{n_r}$ is the channel vector between the $k$th MRS antenna and the receiver antennas.

In this correspondence, the channels passing through the MRS antenna $k$ resemble a MIMO keyhole channel modulating the primary signals, which is different from the unmodulated multiple-keyhole channels in [8]. If $G_0 = 0$, our general channel model coincides with the MIMO backscatter channel in [9], [10].

$^2\alpha$ could be utilized to model the impact of pathloss and possible absorption at the MBS antenna. The model could easily be extended to allow antenna specific scaling factors.
B. Signal Model

We consider a polyphase coding modulation scheme at the MRS antennas. Polyphase codes have been studied to have some important properties, such as periodic orthogonality, and the constant amplitude zero auto-correlation property [11], [12].

The equivalent input over the considered MIMO channels in (1) is denoted by $\psi$ (within one symbol) with

$$\psi = \left[ \begin{array}{c} 1 \\ x_1 \end{array} \right] \otimes x_0 = \left[ x_0 \ x_0 x_{11} \ \cdots \ x_0 x_{1K} \right]^T,$$

where $x_0 \sim \mathcal{CN}(0, \rho_d I_{n_t})$ is the channel input vector of the transmitter at a given time instant, where $\rho_d$ denotes the data symbol power per antenna. The reflection coefficients of the MRS antennas are denoted by $x_1 \in \mathbb{C}^K$, where the polyphase coding is applied such that $E\{x_1^* x_1\} = I_K$, $|x_{1k}|^2 = 1$, and $E\{x_{1k}\} = 0$ for each $k = 1, \cdots, K$ [12]. Since each element of $x_1$ is unitary, each element of $\psi$ has the same distribution as the channel input, i.e., $\psi \sim \mathcal{CN}(0, \rho_d I_{(K+1)n_t})$.

Due to the assumptions made at the beginning of this section, the resulting discrete time signals for the considered channels Eq. (1) within one frame of $N$ discrete samples can be written as

$$Y = \sqrt{\beta_K} G \Psi + Z,$$

where the $n_r \times N$ matrix $Y$ denotes the received signals, the $n_t \times N$ matrix $\Psi$ represents the equivalent input whose columns $\psi_i$, $\forall i = 1, \cdots, N$ are given in Eq. (5), the entries of the $n_r \times N$ noise matrix $Z$ are i.i.d. $\mathcal{CN}(0,1)$, and the scaling factor is $\beta_K = 1/(n_r(K|\alpha|^2 + 1))$.

For any physical channel, the total received signal energy cannot exceed the total transmit energy. Correspondingly, since the average impact of the channel on the energy is $E\{\|G\|^2\}$, the complete channel matrix $G$ is normalised by $E\{\|G\|^2\}/n_t = n_r(K|\alpha|^2 + 1)$ [7]. With the above notations and the signal model in Eq. (4), the total average received SNR from all transmit antennas reads

$$\gamma = \frac{(P_t/n_t)E\{\|G\|^2\}}{n_r(K|\alpha|^2 + 1)\sigma^2} = \frac{P_t}{\sigma^2},$$

where $P_t = n_t \rho_d$ denotes the total transmit power. Thus, $\gamma/n_r$ and $\gamma/n_t$ represent the SNR per receive antenna and per transmit antenna, respectively, for $K$ antenna MRS node.
III. Capacity with Perfect CSI at the Receiver

In this section, we investigate the capacity of the considered system given in Eq. (4), where the perfect CSI is only available at the receiver, and the transmitter only knows the channel distribution. The Gaussian vector \( \psi \) defined in Eq. (3) can be equivalently written as \( \psi = \sqrt{\rho_d} \mathbf{u} \) with \( \mathbf{u} \sim \mathcal{CN}(0, \mathbf{I}_{n_t(K+1)}) \), such that the received symbol vector reads

\[
y = \sqrt{\rho_d \beta K} \mathbf{G} \mathbf{u} + \mathbf{z}.
\]  

(6)

**Proposition 1:** The sum rate of the system in Eq. (6) is

\[
r = \log_2 \det \left( \mathbf{I} + \frac{\gamma \beta K \mathbf{G} \mathbf{G}^\dagger}{n_t} \right).
\]

**Proof:** The channel model in Eq. (6) corresponds to the standard MIMO system with zero mean circularly symmetric normal input \( \mathbf{u} \) and channel matrix \( \sqrt{\rho_d \beta K} \mathbf{G} \). Hence, the assertion follows from the results of the work [13].

**Proposition 2:** Let \( \mathbf{F} = \gamma_0 \mathbf{G}_0 \mathbf{G}_0^\dagger \) and \( \Delta = \gamma_0 \sum_{k=1}^{K} \mathbf{G}_k \mathbf{G}_k^\dagger \), where \( \gamma_0 = \gamma / (n_t n_r (K|\alpha|^2 + 1)) \). Hence, \( \mathbf{F} \) is a positive-semidefinite Hermitian matrix with eigenvalues \( \lambda_i(\mathbf{F}) \), \( \forall i = 1, 2, \ldots, n_t \), and \( \Delta \) is a positive-semidefinite Hermitian matrix. It follows that

\[
\log_2 \det \left( \mathbf{I} + \mathbf{F} \right) \leq \log_2 \det \left( \mathbf{I} + \mathbf{F} + \Delta \right).
\]

With equality if and only if eigenspace of \( \mathbf{F} \) is a subspace of the null space of \( \Delta \).

**Proof:** It is enough to show that the eigenvalues of an Hermitian positive-semidefinite matrix \( \mathbf{F} \) are strictly less than the eigenvalues of \( \mathbf{F} + \Delta \). The Corollary of Weyl’s theorem [14, Corollary 4.3.3] states that the eigenvalues fulfil

\[
\lambda_i(\mathbf{F} + \Delta) \geq \lambda_i(\mathbf{F})
\]

(7)

with equality for some \( i \) if and only if \( \Delta \) is singular and \( \exists \mathbf{x} \neq 0 \) such that \( \mathbf{F} \mathbf{x} = \lambda_i(\mathbf{F}) \mathbf{x}, \Delta \mathbf{x} = 0, \text{ and } (\mathbf{F} + \Delta) \mathbf{x} = \lambda_i(\mathbf{F} + \Delta) \mathbf{x} \). Since \( \log_2 \det(\cdot) \) is equal to matrix trace, the capacity is increased in case only one of the eigenvalues satisfy Eq. (7) with strict inequality. Conversely, if all eigenvectors of \( \mathbf{F} \) are in the null space of \( \Delta \) then \( \mathbf{F} \) and \( \mathbf{F} + \Delta \) have the same trace.

Proposition 2 states that the consider system has larger capacity than the system without the MRS if and only if the effect of reflectors (\( \Delta \)) does not cancel out with respect to \( \mathbf{F} \).

\(^3\)Remark: The primary system can achieve this capacity alone if polyphase-coded \( \mathbf{x}_1 \) is a known pseudo-random sequence at the receiver.
A. Asymptotic Analysis

In this subsection, we conduct asymptotic analysis of the considered system. First, note that

\[
\log_2 \det(I_n + AA^\dagger) = \log_2 \det(I_m + A^\dagger A),
\]

where \(A\) is an \(n \times m\) matrix. Second, considering the channel gain vectors of the MRS system \(g_{rk}\) defined earlier, we have

\[
\lim_{n_r \to \infty} \frac{1}{n_r} \mathbb{E} \{\|g_{rk}\|^2\} = 1.
\]

**Corollary 1:** As \(n_r \to \infty\), the capacity of the considered system is the sum of limiting capacities of a multiple-keyhole MIMO channel in [6] and a rich scattering MIMO channel in [5], i.e.

\[
r = n_t \log_2 \left(1 + \frac{\gamma}{n_t(K|\alpha|^2 + 1)}\right) + \sum_{k=1}^{K} \log_2 \left(1 + |\alpha|^2 \|g_{tk}\|^2 \frac{\gamma}{n_t(K|\alpha|^2 + 1)}\right).
\]

**Proof:** Using Proposition [1] and Eq. (8) and Eq. (9), as \(n_r \to \infty\) we have

\[
\frac{1}{n_r} G^\dagger G \to \begin{bmatrix}
I_{n_t} & 0 & \cdots & 0 \\
0 & |\alpha|^2 g_{t1} g_{t1}^\dagger & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & |\alpha|^2 g_{tk} g_{tk}^\dagger
\end{bmatrix}
\]

with eigenvalues

\[
\underbrace{1, \cdots, 1}_{n_t}, |\alpha|^2 \|g_{t1}\|^2, \cdots, |\alpha|^2 \|g_{tk}\|^2, 0, \cdots, 0.
\]

We obtain the result from Proposition [1].

**Remark:** Corollary 1 indicates that the additional MRS antennas increase the overall system capacity due to the increased rank of the overall channel matrix.

**Corollary 2:** As \(n_t \to \infty\), and \(K \leq n_r < n_t\), the capacity of the considered system is given by

\[
r = (n_r - K) \log_2 (1 + \gamma \beta_K) + \sum_{k=1}^{K} \log_2 \left(1 + |\alpha|^2 \|g_{tk}\|^2 \gamma \beta_K\right).
\]

**Proof:** Following the same line of reasoning as the previous corollary, as \(n_t \to \infty\), we have

\[
\frac{1}{n_t} G G^\dagger \to I_{n_r} + \sum_{k=1}^{K} |\alpha|^2 g_{rk} g_{rk}^\dagger.
\]
When $K \leq n_r$, the vectors $g_{rK}$ for each $k = 1, \cdots, K$ become asymptotically orthogonal as $n_r$ grows. Hence the eigenvalues of $\frac{1}{n_t}GG^\dagger$ become

$$\frac{1 + |\alpha|^2||g_{t1}||^2}{K}, \cdots, \frac{1 + |\alpha|^2||g_{tK}||^2}{n_r-K}.$$ 

The result follows.

**B. Capacity of MRS when $K = 1$**

The exact capacity for a constant envelope MIMO system is difficult to obtain. In order to gain an insight into the capacity available for the MRS, we assume that the receiver first decodes $x_0$, and then employs the zero-forcing technique to decode each of the streams separately (assuming that $n_r > K$). The resulting capacity of the MRS system in the presence of a modulated carrier can be obtained by treating $x_0$ to be part of the known channel. The received signal reads

$$y = \sqrt{\beta_1}G_0x_0 + x_1\sqrt{\beta_1}G_1x_0 + z.$$  

(10)

where $G_0$ and $G_1$ are defined in Eq. (1), and $\beta_1$ is given by $\beta_K$ when $K = 1$. The achievable rate for the primary system $r_0$ by treating the transmission of the MRS as noise can be obtained as

$$r_0 \leq \log_2 \det \left( I_{n_r} + (I_{n_r} + W)^{-1} \gamma G_0G_0^\dagger\beta_1/n_t \right),$$  

(11)

where $W = \gamma G_1G_1^\dagger\beta_1/n_t$.

Note that since $G_0x_0$ is known, it can be canceled out. In this case, the SNR for the MRS flow is given by

$$\gamma_1(x_0) = \gamma \beta_1 G_1^*G_1/n_t.$$  

(12)

Consequently, for a Gaussian codebook the channel capacity is given as

$$C_G(\gamma_1) = \log (1 + \gamma_1),$$  

(13)

where the dependence of $\gamma_1$ on $x_0$ is dropped from the notation for simplicity. For Wyner polyphase coding [12], the channel capacity is

$$C_{WP}(\gamma_1) = -\int_0^\infty f(u, \gamma_1) \ln \left( \frac{f(u, \gamma_1)}{u} \right) du + \ln \left( \frac{2\gamma_1}{e} \right),$$  

(14)

$^4$The proof is straightforward, and it is omitted due to limited space.
where
\[
f(u, \gamma_1) = 2u\gamma_1 e^{-\gamma_1(1+u^2)} J_0(2u\gamma_1),
\]
and \(J_\nu(x)\) is the modified Bessel function of \(\nu\)th order. If \(\gamma_1\) is close to zero, \(C_{WP}(\gamma_1)\) can be approximated to \(C_{WP}(\gamma_1) \approx \gamma_1 [12]\). In Fig. 2, the achievable rate region of a scenario of \(4 \times 4\) primary system and a single-antenna MRS system is studied when the polyphase coding scheme is applied at the MRS system. The solid curves show the region that the MRS system employs the polyphase coding scheme, and the dashed ones represent that using the Gaussian codebook for both systems.³

³For instance, we select \(\gamma = 20\) dB. Points A and E show the maximum achievable rate for the MRS system using the polyphase codes and the Gaussian codebook, respectively. Point C shows the achievable sum rate, and is achievable by the primary system alone when \(z_1\) is known and a joint detection scheme is used at the receiver. Segments BC and EF can be achieved by the primary and the MRS systems through, for instance, a time-duplexing scheme. Point D shows the achievable rate by the primary system when the MRS signal is unknown by the primary system and treated as noise.
IV. CAPACITY WITH CHANNEL ESTIMATION ERROR

A. Joint channel estimation

Suppose that the data transmission consists of two phases: a training phase and a data transmission phase, as it has been considered in [15]. In the training phase, a pilot sequence is transmitted to the receiver to estimate the channel matrix $G$ in Eq. (1) using acquired $N_p$ digital samples. In this section, our aim is to investigate the impact of channel estimation on the system capacity. For this purpose, we first describe the pilot sequence design. The least estimation error and its impact on the capacity are studied for the designed pilots.

One of the assumptions in Sec. II is that the MRS is fully synchronised to the primary transmitter. This assumption casts the current pilot design problem to the design problem for the multi-relay systems, studied earlier in [16]. In these systems, one of the proposed techniques is to generate the pilot sequences using Hadamard matrices, which has recursive generation property. Let $H_n$ denote a $2^n \times 2^n$ dimensional Hadamard matrix. Then, starting from

$$H_1 = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix},$$

a $2^n$-dimensional matrix can be generated from the recursion

$$H_n = H_1 \otimes H_{n-1}.$$

In this work, we assume that the primary system uses orthogonal pilots, and the MRS system uses rows of a Hadamard matrix such that the resulting joint pilot becomes orthogonal.

Let $X_{0p} \in \mathbb{C}^{n_t \times m_0}$ and $X_{1p} \in \mathbb{C}^{(K+1) \times m_1}$ be the orthogonal sequences of the primary system and MRS with length of $m_0$ and $m_1$, respectively, yielding $N_p = m_0 m_1$. The two sequences have the properties $\mathbb{E}\{X_{0p}X_{0p}^\dagger\} = m_0 \rho_p I_{n_t}$ and $\mathbb{E}\{X_{1p}X_{1p}^\dagger\} = m_1 I_{K+1}$, where $\rho_p$ denotes the power of a pilot and could be different from $\rho_d$ the data symbol power$^6$. All elements of the first row of $X_{1p}$ are equal to 1, which correspond to the paths $G_0$ with constant non-controllable channel gains. Hence, $X_{1p}$ could be selected to correspond the first $K+1$ rows of the Hadamard matrix so that the composite pilot, with dimension $n_t(K+1) \times (m_0 m_1)$,

$$\Psi_p = X_{1p} \otimes X_{0p}$$

is orthogonal, and $\Psi_p \Psi_p^\dagger = m_0 m_1 \rho_p I_{n_t(K+1)}$.

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$^6$see [15] for the impact of $\rho_p$ and $\rho_d$ on the capacity.
The composite pilots in Eq. (15) require the MRS pilot symbol length to be \( m_0 \) primary symbols. In this case, the training phase can be realised by transmitting the same pilot sequence of the primary systems \( K + 1 \) times so that during the transmission of one primary pilot sequence, a MRS pilot is constant. Following this strategy and using the signal model in Eq. (4), the received \( n_r \times m_0 m_1 \) pilots matrix \( Y_p \) is

\[
Y_p = \sqrt{\beta K} G \Psi_p + Z_p, \tag{16}
\]

where the entries of the noise matrix \( Z_p \) are i.i.d. standard complex Gaussian random variables. Knowing the transmitted pilots \( \Psi_p \) and the received pilots matrix \( Y_p \), the least squares (LS) estimate of the (normalized) channel matrix is given by

\[
\sqrt{\beta K} \hat{G} = Y_p \Psi_p^\dagger (\Psi_p \Psi_p^\dagger)^{-1} = \frac{1}{m_0 m_1 \rho_p} Y_p \Psi_p^\dagger. \tag{17}
\]

The (normalized) channel estimation error is

\[
\sqrt{\beta K} \tilde{G} = Z_p \Psi_p^\dagger (\Psi_p \Psi_p^\dagger)^{-1} = \frac{1}{m_0 m_1 \rho_p} Z_p \Psi_p^\dagger. \tag{18}
\]

\[\text{B. Capacity with Channel Estimation Error}\]

When channel estimation error is taken into account, the capacity of a MIMO system is not known \cite{15}, \cite{17}. However, the channel uncertainty can be treated as noise when the channel estimate \( \hat{G} \) is available at the receiver. As the considered estimator is unbiased and the estimation error in Eq. (18) is Gaussian, the received data signal sequence during data transmission reads

\[
Y_d = \sqrt{\beta K} \hat{G} \Psi_d + \sqrt{\beta K} \tilde{G} \Psi_d + Z_d, \tag{19}
\]

where \( \Psi_d \) denotes the transmit data sequence whose columns are of the form in Eq. (3), and the entries of the noise matrix \( Z_d \) are i.i.d. and follow the standard complex Gaussian distribution. The received signal vector at a moment reads

\[
y = \sqrt{\beta K} \hat{G} \psi + \sqrt{\beta K} \tilde{G} \psi + z. \tag{20}
\]

The covariance matrix of \( \sqrt{\beta K} \tilde{G} \psi \) is given by

\[
R_{\tilde{G} \psi} = (m_0 m_1 \rho_p)^{-2} \mathbb{E} \left\{ Z \Psi_p^\dagger \psi \psi^\dagger \Psi_p Z^\dagger \right\}. \tag{21}
\]

As the pilots are known at the receiver, we obtain

\[
R_{\tilde{G} \psi} = \frac{\rho_d \sigma^2 I_{n_r}}{(m_0 m_1 \rho_p)^2} \operatorname{tr} (\Psi_p \Psi_p^\dagger) = \frac{n_t (K + 1) \rho_d \sigma^2 I_{n_r}}{\rho_p m_0 m_1}. \tag{22}
\]
Consequently, a lower bound of the sum capacity is obtained as [17]

\[ r \geq \frac{N - N_p}{N} \log_2 \det \left( I + W_K^{-1} \hat{G} \hat{G}^\dagger \gamma \beta_K \right), \] (23)

where \( W_K = (I_{n_r} + R_{G \psi}/\sigma^2) \).

V. SIMULATION RESULTS

In this section, we show the simulation results of the sum capacity of the considered system with perfect and imperfect CSI. In particular, we investigate the impacts of the MRS system on the capacity of the overall system. We also depict the achievable rate of the MRS system using a polyphase coding scheme for \( K = 1 \). The results of this section are averaged over \( 10^5 \) realisations, where for each realisation the pilot and data symbols power are the same, \( \rho_d = \rho_p \), and the amplitude of the attenuation shift factor of the MRS is \( |\alpha| = -3 \text{ dB} \).
Fig. 3 shows the achievable rate by the whole system as a function of $\gamma$ for $K = 0, 1, 10$ and $n_t = n_r = 4$. The achievable rate by the MRS system alone ($K = 1$) is plotted when the MRS employs polyphase coding and Gaussian coding schemes, respectively. The solid and dashed curves depict the sum capacity with perfect and imperfect CSI at the receiver, respectively. The asymptotic sum capacity as $n_r \to \infty$ is plotted using dotted curves. First, results show that the sum capacity of the considered system that provided perfect CSI is always larger than that of the primary system alone as stated by Proposition 2. However, this statement is not always valid when CSI is not perfect as can be seen from the figure.

VI. Conclusion

In this correspondence, we propose a system that extends bi-static MBS system to an ambient bi-static MRS system which allows information transmission between the multi-antenna transmitter and the multi-antenna receiver (primary MIMO system) as well as between the MRS antennas and the receiver (secondary MRS system). We showed that the capacity of such a system exceeds that, which the traditional MIMO system could achieve alone. The excess capacity can be achieved by the primary system alone or it can be shared by the two systems. In a rich scattering environment and perfect CSI the limiting sum capacity of the ambient bi-static MRS channel approaches the sum of the limiting capacities of a MIMO channel and a multiple-keyhole MIMO channel. In practice, the capacity is limited by the channel estimation errors. We discussed how a joint pilot signal could be designed to estimate the channel and provided a lower bound for the capacity with channel estimation errors.

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