The essential role of string–derived symmetries in ensuring proton–stability and light neutrino masses

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Abstract

The paper addresses the problem of suppressing naturally the unsafe $d = 4$ as well as the color-triplet mediated and/or gravity-linked $d = 5$ proton-decay operators, which generically arise in SUSY-unification. It also attempts to give light masses to the neutrinos, of the type suggested by current experiments. It is noted that neither the symmetries in $SO(10)$, nor those in $E_6$, suffice for the purpose – especially in the matter of suppressing naturally the $d = 5$ proton-decay operators. By contrast, it is shown that a certain string-derived symmetry, which cannot arise within conventional grand unification, but which does arise within a class of three-generation string-solutions, suffices, in conjuction with $B - L$, to safeguard proton-stability from all potential dangers, including those which may arise through higher dimensional operators and the color-triplets in the infinite tower of states. At the same time, the symmetry in question permits neutrinos to acquire appropriate masses. This shows that string theory plays an essential role in ensuring natural consistency of SUSY-unification with two low-energy observations – proton-stability and light masses for the neutrinos. The correlation between the masses of the extra $Z'$-boson (or bosons), which arise in these models, and proton-decay rate is noted.

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1 Introduction

While supersymmetry is an essential ingredient for higher unification, it is known that it poses the generic problem of rapid proton decay [1]. This is because, in accord with the standard model gauge symmetry \(SU(2)_L \times U(1)_Y \times SU(3)^C\), a supersymmetric theory in general permits, in contrast to non-supersymmetric ones, dimension 4 and dimension 5 operators which violate baryon and lepton numbers. Using standard notations, the operators in question which may arise in the superpotential are as follows:

\[
W = [\eta_1 \overline{UD}D + \eta_2 \overline{QL}D + \eta_3 \overline{LE}] + [\lambda_1 \overline{QQQL} + \lambda_2 \overline{U} \overline{D} E + \lambda_3 \overline{L} \overline{H}_2 \overline{H}_2] / M. \tag{1}
\]

Here, generation, \(SU(2)_L\) and \(SU(3)^C\) indices are suppressed. \(M\) denotes a characteristic mass scale. The first two terms of \(d = 4\), jointly, as well as the \(d = 5\) terms of strengths \(\lambda_1\) and \(\lambda_2\), individually, induce \(\Delta(B - L) = 0\) proton decay with amplitudes \(\sim \eta_1 \eta_2 / m_q^2\) and \((\lambda_{1,2}/M)(\delta)\) respectively, where \(\delta\) represents a loop-factor. Experimental limits on proton lifetime turns out to impose the constraints: \(\eta_1 \eta_2 \leq 10^{-24}\) and \((\lambda_{1,2}/M) \leq 10^{-25}\) GeV\(^{-1}\) [2]. Thus, even if \(M \sim M_{\text{string}} \approx 10^{18}\) GeV, we must have \(\lambda_{1,2} \leq 10^{-7}\), so that proton lifetime will be in accord with experimental limits.

Renormalizable, supersymmetric standard-like and \(SU(5)\) [3] models can be constructed so as to avoid, by choice, the \(d = 4\) operators (i.e. the \(\eta_{1,2,3}\)-terms) by imposing a discrete or a multiplicative \(R\)-parity symmetry: \(R \equiv (-1)^{3(B-L)}\), or more naturally, by gauging \(B - L\), as in \(G_{224} \equiv SU(2)_L \times SU(2)_R \times SU(4)^C\) [4] or \(SO(10)\) [5]. Such resolutions, however, do not in general suffice if we permit higher dimensional operators and intermediate scale VEVs of fields which violate \((B - L)\) and \(R\)-parity (see below). Besides, \(B - L\) can not provide any protection against the \(d = 5\) operators given by the \(\lambda_1\) and \(\lambda_2\) - terms, which conserve \(B - L\). These operators are, however, expected to be present in any theory linked with gravity, e.g. a superstring theory, unless they are forbidden by some new symmetry.

For SUSY grand unification models, there is the additional problem that the exchange of color-triplet Higgsinos which occur as partners of electroweak doublets (as in \(5 + \overline{5}\) of \(SU(5)\)) induce \(d = 5\) proton-decay operators [1]. Thus, allowing for suppression of \(\lambda_1\) and \(\lambda_2\) (by about \(10^{-8}\)) due to the smallness of the Yukawa couplings, the color-triplets still need to be superheavy (\(\geq 10^{17}\) GeV) to ensure proton-stability [2], while their doublet
partners must be light \((\leq 1 \text{ TeV})\). This is the \textit{generic problem of doublet-triplet splitting} that faces all SUSY GUTS. Solutions to this problem needing either unnatural fine-tuning as in SUSY \(SU(5)\) \cite{3}, or suitable \textit{choice} of large number and/or large size Higgs multiplets and discrete symmetries as in SUSY \(SO(10)\) \cite{6} and missing-partner models \cite{7}, are technically feasible. They, however, do not seem to be compelling because they have been invented for the sole purpose of suppressing proton-decay, without a deeper reason. Furthermore, such solutions are not easy to realize, and to date have not been realized, in \textit{string-derived} grand unified theories \cite{8}.

These considerations show that, in the context of supersymmetry, the extraordinary stability of the proton is in fact surprising. As such it deserves a natural explanation. Rather than being merely accommodated, it ought to emerge as a \textit{compelling feature}, owing to symmetries of the underlying theory, which should forbid, or adequately suppress, the unsafe operators in Eq. (1). As discussed below, the task of finding such symmetries becomes even harder, if one wishes to assign non-vanishing light masses \((\leq \text{ few eV})\) to neutrinos. The purpose of this letter is to propose a class of solutions, within supersymmetric theories, which \(a\) naturally ensure proton-stability, to the extent desired, and \(b\) simultaneously permit neutrinos to acquire light masses, of a nature that is relevant to current experiments \cite{9}. These solutions need \textit{either} \(I_{3R}\) and \(B - L\) as \textit{separate} gauge symmetries, as well as \textit{one extra abelian symmetry} that lies beyond even \(E_6\) \cite{5}; \textit{or} the weak hypercharge \(Y = I_{3R} + (B - L)/2\) accompanied by \textit{two extra symmetries} beyond those of \(E_6\). The interesting point is that while the extra symmetries in question can not arise within conventional grand unification models, including \(E_6\), they do arise within a class of string-derived three generation solutions. This in turn provides \textit{a strong motivation} for symmetries of string-origin. The extra symmetries lead to extra \(Z'\)-bosons, whose currents would bear the hallmark of string theories. It turns out that there is an interesting correlation between the masses of the \(Z'\)-bosons and observability of proton decay.

2 \textbf{The need for symmetries beyond \(SO(10)\) and \(E_6\)}

In what follows, we assume that operators (with \(d \geq 4\)), scaled by Planck or string scale-mass, that respect all symmetries, exist in the effective superpotential of any theory which is linked to gravity, like a superstring theory \cite{10,11}. For reasons discussed
before, the class of theories – string-derived or not – which contains $B-L$ as in $G_{2311} \equiv SU(2)_L \times SU(3)_C \times U(1)_{I_{3R}} \times U(1)_{B-L}$ as a symmetry, the $d=4$ operators in Eq.(1) are naturally forbidden. They can in general appear however through non-renormalizable operators if there exist VEVs of fields which violate $B-L$. This is where neutrino-masses become relevant. The familiar see-saw mechanism [12] that provides the simplest reason for known neutrinos $\nu^i_L$’s to be so light assigns heavy Majorana masses $M^i_R$ to the right-handed neutrinos $\nu^i_R$, and thereby light masses $m^i_L \sim (m^i_D)^2/M^i_R$ to the left-handed ones, where $m^i_D$ denotes a typical Dirac mass for the $i$th neutrino. These masses would have just the right pattern to be relevant to the neutrino-oscillation experiments [9, 13] and to $\nu_\tau$ being hot dark matter, with $m^i_L \sim (10^{-8}, 3 \times 10^{-3}$ and 1-10) eV for $i = e, \mu, \tau$, if $M^i_R \sim 10^{12}$ GeV (within a factor of 10). Generating heavy Majorana masses for $\nu_R$’s, however, needs spontaneous violation of $B-L$ at a heavy intermediate scale.

If $B-L$ is violated by the VEV of a field by two units, an effective $R$-parity would still survive [14], which would forbid the $d=4$ operators. That is precisely the case for the multiplet 126 of $SO(10)$ or $(1,3,10)$ of $G_{224}$, which have commonly been used [12] to give Majorana masses to $\nu_R$’s. Recent works show, however, that 126 and very likely $(1,3,10)$, as well, are hard – perhaps impossible – to obtain in string theories [15]. We, therefore, assume that this constraint holds. It will become clear, however, that as long as we demand safety from both $d=4$ and $d=5$ operators, our conclusion as regards the need for symmetries beyond $E_6$, would hold even if we give up this assumption.

Without 126 of Higgs, $\nu_R$’s can still acquire heavy Majorana masses utilizing product of VEVs of sneutrino-like fields $\tilde{N}^i_R$ and $\tilde{N}^i_L$, which belong to $16_H$ and $\overline{16}_H$ respectively. (as in Ref. [16], see also [17].) In this case, an effective operator of the form $16 \cdot 16 \cdot \overline{16}_H \cdot \overline{16}_H / M$ in $W$, that is allowed by $SO(10)$, would induce a Majorana mass $(\nu^i R C^{-1} \nu^T_R) (\langle \tilde{N}^i_L \rangle \langle \tilde{N}^i_L \rangle / M) + hc$ of magnitude $M_R \sim 10^{12.5}$ GeV, as desired, for $\langle \tilde{N}^i_L \rangle \sim 10^{15.5}$ GeV and $M \sim 10^{18}$ GeV. However, consistent with $SO(10)$ symmetry and therefore its subgroups, one can have an effective $d=5$ operator in the superpotential $16^a \cdot 16^b \cdot 16^c \cdot 16_H / M$. This would induce the terms $URDRDR\langle \tilde{N}^i_R \rangle / M$ and $QLDL\langle \tilde{N}^i_R \rangle / M$ in $W$ (see Eq.(1)) with strengths $\sim \langle \tilde{N}^i_R \rangle / M \sim 10^{15.5}/10^{18} \sim 10^{-2.5}$, which would lead to unacceptably short proton lifetime $\sim 10^{-6}$ yrs. [18]. We thus see that, without having the 126 or $(1,3,10)$ of Higgs, $B-L$ and therefore $SO(10)$ does not suffice to suppress even the $d=4$ - operators adequately while giving appropriate masses to neutrinos. As
Table 1:

| Operators            | \(I_{3R}\) | \(B - L\) | \(Y\) | \(Q_{\psi}\) | \(Q_{T}\) |
|----------------------|-----------|----------|-------|-------------|---------|
| \(\bar{U}D\bar{D}, Q\bar{L}\bar{D}\) | \(1/2\)  | -1       | 0     | 3           | 4       |
| \(LLE\)             | \(1/2\)  | -1       | 0     | 3           | 4       |
| \(QQQL/M\)          | 0         | 0        | 0     | 4           | 4       |
| \(UUDE/M\)          | 0         | 0        | 0     | 4           | 4       |
| \(LLH_2H_2/M\)      | 1         | -2       | 0     | -2          | 0       |
| \(\bar{N}_R\)       | -1/2      | 1        | 0     | 1           | 0       |
| \((H_1, H_2)\)      | (-1/2, 1/2)| -1/2, 1/2|(-1/2, 1/2)| 0 | 4 | 4 |
| \(\chi\)            | 0         | 0        | 0     | 4           | 4       |

mentioned before, \(B - L\) does not of course prevent the \(d = 5\), \(\lambda_1\) and \(\lambda_2\) - terms, regardless of the Higgs spectrum, because these terms conserve \(B - L\).

To cure the situation mentioned above, we need to utilize symmetries beyond those of \(SO(10)\). Consider first the presence of at least one extra \(U(1)\) beyond \(SO(10)\) of the type available in \(E_6\), i.e. \(E_6 \to SO(10) \times U(1)_{\psi}\), under which 27 of \(E_6\) branches into \((16_1 + 10_{-2} + 1_4)\), where 16 contains \((Q, L | U_R, D_R, E_R, \nu_R)\), with \(Q_{\psi} = +1\); while 10 contains the two Higgs doublets \((H_1, H_2)^{(0,-2)}\) and a color-triplet and an anti-triplet \((H_3^{(-2/3,-2)} + H_3^{(2/3,-2)})\), where the superscripts denote \((B - L, Q_{\psi})\). Assume that the symmetry in the observable sector just below the Planck scale is of the form:

\[
G_{st} = [G_{fc} \subseteq SO(10)] \times \hat{U}(1)_{\psi} \times [U(1)'s].
\]

It is instructive to first assume that \(\hat{U}(1)_{\psi} = U(1)_{\psi}\) of \(E_6\) \([13]\) and ignore all the other \(U(1)\)'s. Ignoring the doublet-triplet splitting problem for a moment, we allow the flavor-color symmetry \(G_{fc}\) to be as big as \(SO(10)\). The properties of the operators in \(W\) given in Eq.(1), and of the fields \(\bar{N}_R, (H_1, H_2)\) and the singlet \(\chi \subset 27\), under the charges \(Y, I_{3R}, B - L, Q_{\psi}\) and \(Q_T \equiv Q_{\psi} - (B - L)\), are shown in Table 1. We see that the \(d = 4\) operators (\(\eta_i\) -terms) are forbidden by \(B - L\), as well as by \(Q_{\psi}\) and \(Q_T\). Furthermore, note that when \(\bar{N}_R \subset 16\) and \(\bar{N}_L' \subset \bar{16}\) acquire VEV, and give Majorana masses to the \(\nu_R\)'s, the charges \(I_{3R}, B - L\) as well as \(Q_{\psi}\) are broken, but \(Y\) and \(Q_T\) are preserved. Now \(Q_T\) would be violated by the VEVs of \((H_1, H_2) \sim 200\) GeV and of the singlets \(\chi^{(27)}\) and \(\chi^{(27)}\). Assume that \(\chi\) and \(\chi\) acquire VEVs \(\sim 1\) TeV.
through a radiative mechanism, utilizing Yukawa interactions, analogous to \((H_1, H_2)\). The \(d = 4\) operators can be induced through nonrenormalizable terms of the type \(16 \cdot 16 \cdot 16 \cdot [\langle \tilde{N}_R \subset 16 \rangle / M], [\langle 10 \rangle \langle 10 \rangle / M^2 \) or \([\langle \chi \subset 27 \rangle / M]\), where the effective couplings respect \(SO(10)\) and \(U(1)_\psi\). Thus we get \(\eta_i \leq (10^{15.5 \cdot 10^{18}}) (1 \text{ TeV} / 10^{18} \text{ GeV}) \sim 10^{-18}\), which is below the limit of \(\eta_1 \eta_2 \leq 10^{-24}\). Thus, \(B - L\) and \(Q_\psi\), arising within \(E_6\), suffice to control the \(d = 4\) operators adequately, while permitting neutrinos to have desired masses.

Next consider the \(LLH_2 H_2\)-term. While it violates \(I_3^R\), \(B - L\) and \(Q_\psi\), it is the only term that is allowed by \(Q_T\). Such a term can arise through an effective interaction of the form \(16 \cdot 16 \cdot (H_2 \subset 10)^2 \cdot [\langle \tilde{N}_R \subset 16 \rangle / M]^2\), and thus with a strength \(\sim 10^{-5} \cdot (10^{18} \text{GeV})^{-1}\), which is far below the limits obtained from \(\nu\)-less double \(\beta\)-decay.

Although the two \(d = 5\) operators \(QQQL / M\) and \(U \bar{U} D \bar{E} / M\) are forbidden by \(Q_\psi\) and \(Q_T\), the problem of these two operators still arises as follows. Even for a broken \(E_6\)-theory, possessing \(U(1)\)-symmetry, the color-triplets \(H_3\) and \(H_3^*\) of \(27\) still exist in the spectrum. They are in fact needed to cancel the anomalies in \(U(1)^3\) and \(SU(3)^2 \times U(1)_\psi\) etc. They acquire masses of the form \(M_3 H_3 H_3^* + h c\) through the VEV of singlet \(\langle \chi \rangle\) which breaks \(Q_\psi\) and \(Q_T\) by four units. With such a mass term, the exchange of these triplets would induce \(d = 5\) proton-decay operators, just as it does for SUSY \(SU(5)\) and \(SO(10)\). We are then back to facing either the problem of doublet-triplet splitting (i.e. why \(M_3 \geq 10^{17} \text{ GeV}\) or that of rapid proton-decay (for \(M_3 \sim 1 \text{ TeV}\)). In this sense, while the \(E_6\)-framework, with \(U(1)_\psi\), can adequately control the \(d = 4\) operators and give appropriate masses to the neutrinos (which \(SO(10)\) cannot), it does not suffice to control the \(d = 5\) operators, owing to the presence of color-triplets. As we discuss below, this is where string-derived solutions help in preserving the benefits of a \(Q_\psi\)-like charge, while naturally eliminating the dangerous color-triplets.

**Doublet-Triplet Splitting In String Theories: A Preference For Standard-like Symmetries over GUTS:** While the problem of doublet-triplet splitting does not have a compelling solution within SUSY GUTS and has not been resolved within string-derived GUTS \([8]\), it can be solved quite simply within string-derived standard-like \([20, 21]\) or the \(G_{224}\)-models \([16]\), because in these models, the electroweak doublets are naturally decoupled from the color-triplets after string-compactification. As a result, invariably, the same set of boundary conditions (analogous to “Wilson lines”) which
break SO(10) into a standard-like gauge symmetry such as $G_{2311}$, either project out, by GSO projections, all color-triplets $H_3$ and $H'_3$, from the “massless” spectrum $\{21\}$, or yield some color-triplets with extra $U(1)$ - charges which make them harmless $\{20\}$, because they can not have Yukawa couplings with quarks and leptons. In these models, the doublet triplet splitting problem is thus solved from the start, because the dangerous color - triplets simply do not appear in the massless spectrum $\{22\}$.

At the same time, owing to constraints of string theories, the coupling unification relations hold $\{23\}$ for the standard-like or $G_{224}$-models, just as in GUT. Furthermore, close to realistic models have been derived from string theories only in the context of such standard-like $\{20, 21\}$, flipped $SU(5) \times U(1)$ $\{22\}$ and $G_{224}$ models $\{16\}$, but not yet for GUTS. For these reasons, we will consider string-derived non-GUT models, as opposed to GUT-models, as the prototype of a future realistic string model, and use them as a guide to ensure (a) proton - stability and (b) light neutrino masses.

Now, if we wish to preserve the benefits of the charge $Q_\psi$ (noted before), and still eliminate the color-triplets as mentioned above, there would appear to be a problem, because, without the color-triplets, the incomplete subset consisting of $\{16_1 + (2, 2, 1)_{-2} + 1_4\} \subset 27$ of $E_6$ would lead to anomalies in $U(1)_\psi^3$, $SU(3)^2 \times U(1)_\psi$ etc. This is where symmetries of string-origin come to the rescue.

### 3 The crucial role of string-derived symmetries

The problem of anomalies (noted above) is cured within string theories in a variety of ways. For instance, new states beyond those in the $E_6$-spectrum invariably appear in the string-massless sector which contribute toward the cancellation of anomalies, and only certain combinations of generators become anomaly-free. We must then examine whether such anomaly-free combinations can help achieve our goals. To proceed further, we need to focus on some specific solutions. For this purpose, we choose to explore here the class of string-derived three generation models, obtained in Refs. $\{20\}$ and $\{21\}$, which is as close to being realistic as any other such model that exists in the literature (see e.g. Refs. $\{10\}$ and $\{22\}$). In particular, they seem capable of generating qualitatively the right texture for fermion mass-matrices and CKM mixings. We stress, however, that the essential feature of our solution, relying primarily on the existence of extra symmetries analogous to $U(1)_\psi$, is likely to emerge in a much larger class of string-derived solutions.
We refer the reader to Refs. [20] and [21] and references therein for the procedure of choosing string-boundary conditions, applying GSO projections, and deriving the effective low-energy theory. After the application of all GSO projections, the gauge symmetry of the models developed in these references, at the string scale, is given by:

\[ G_{st} = [SU(2)_L \times SU(3)_C \times U(1)_{I_{3R}} \times U(1)_{B-L}] \times [G_M = \prod_{i=1}^{6} U(1)_{i}] \times G_H. \] (3)

Here, \( U(1)_{i} \) denote six horizontal-symmetry charges which act non-trivially on the three families and distinguish between them. In the models of Refs. [20], [21], \( G_H = SU(5)_H \times SU(3)_H \times U(1)_{2H} \). There exists “hidden” matter which couples to \( G_H \) and also \( U(1)_{i} \).

Thus the gauge interactions of the sector \( G_M = [U(1)]^6 \) serve as the messenger between the hidden and the observable matter. The form of \( G_M \) varies from model to model, but its occurrence seems to be a generic feature (see e.g. Refs. [22], [16], [17]).

A partial list of the massless states for the solution derived in Ref. [20], together with the associated \( U(1)_i \)-charges, is given in Table 2. We have not exhibited a host of other states including (a) 10 pairs of \( SO(10) \)-singlets, with \( U(1)_i \)-charges, (b) three universal singlets \( \xi_{1,2,3} \), and (c) states with fractional charges which either get superheavy or get confined [24]. The table reveals the following features:

(i) There are three families of quarks and leptons (1, 2 and 3), each with 16 components, including \( \nu_R \). Their quantum numbers under the symmetries belonging to \( SO(10) \) are standard and are thus not shown. Note that the \( U(1)_i \) charges differ from one family to the other. There are also three families of hidden sector multiplets \( V_i, \overline{V}_i, T_i \) and \( \mathcal{T}_i \) which possess \( U(1)_i \)-charges.

(ii) The charge \( Q_1 \) has the same value (\( \frac{1}{2} \)) for all sixteen members of family 1, similarly \( Q_2 \) and \( Q_3 \) for families 2 and 3 respectively. In fact, barring a normalization difference of a factor of 2, the sum \( Q_+ \equiv Q_1 + Q_2 + Q_3 \) acts on the three families and on the three Higgs doublets \( \overline{h}_1, \overline{h}_2, \overline{h}_3 \) in the same way as the \( Q_\psi \) of \( E_6 \) introduced before. The analogy, however, stops there, because the solution has additional Higgs doublets (see table) and also because there is only one pair of color triplets \( (D_{45}, \overline{D}_{45}) \) instead of three. Furthermore, the pair \( (D_{45}, \overline{D}_{45}) \) is vector-like with opposite \( Q_\psi \)-charges, while \( (H_3, H^\prime_3) \), belonging to \( 27 \) of \( E_6 \), have the same \( Q_\psi \)-charge. In fact the pair \( (D_{45}, \overline{D}_{45}) \) can have an invariant mass conserving all \( Q_\psi \)-charges, but \( (H_3, H^\prime_3) \) can not.

(iii) It is easy to see that owing to different \( U(1)_i \)-charges, the color-triplets \( D_{45} \) and \( \overline{D}_{45} \) (in contrast to \( H_3 \) and \( H^\prime_3 \)) can not have allowed Yukawa couplings to \( (qq) \) and \( (ql) \).
- pairs. Thus, as mentioned before, they can not mediate proton decay.

(iv) Note that the solution yields altogether four pairs of electroweak Higgs doublets: \((h_1, h_2, h_3, h_{45})\) and \((\bar{h}_1, \bar{h}_2, \bar{h}_3, \bar{h}_{45})\). It has been shown \([20]\) that only one pair – i.e. \(h_1\) or \(h_{45}\) – remains light, while the others acquire superheavy or intermediate scale masses. The cubic level superpotential has the form \([20]\):

\[
W = \left\{ \sum_{i=1}^{3} U_i Q_i \bar{h}_i + \sum_{i=1}^{3} N_i^C L_i \bar{h}_i + \sum_{i<j} h_i h_j \phi_{ij} + \sum_{i<j} \bar{h}_i h_j \phi_{ij} \right\} + O(\phi^3) + \left\{ \frac{\xi_3}{2}(D_{45}\bar{D}_{45} + h_{45}\bar{h}_{45} + O(\phi^2)) \right\} + \cdots ,
\]

where \(\phi\)’s denote \(SO(10)\) - singlets, possessing \(U(1)_i\)-charges. Note that, owing to differing \(U(1)_i\)-charges, the three families have Yukawa couplings with three distinct Higgs doublets. Since only one pair \((h_1\) and \(h_{45}\)\) remains light and acquires VEV, it turns out that families 1,2 and 3 get identified with the \(\tau, \mu\) and \(e\)-families respectively \([20]\).

The mass-heirarchy and CKM mixings arise through higher dimensional operators, by utilizing VEVs of appropriate fields and hidden-sector condensates.

Including contributions from the entire massless spectrum, one obtains: \(TrU_1 = TrU_2 = TrU_3 = 24\) and \(TrU_4 = TrU_5 = TrU_6 = -12\). Thus, all six \(U(1)_i\)’s are anomalous. They give rise to five anomaly-free combinations and one anomalous one:

\[
\begin{align*}
U_1' &= U_1 - U_2, \quad U_2' = U_4 - U_5, \quad U_3' = U_4 + U_5 - 2U_6, \\
\hat{U}_\psi &= U_1 + U_2 - 2U_3, \\
\hat{U}_\chi &= (U_1 + U_2 + U_3) + 2(U_4 + U_5 + U_6), \\
U_A &= 2(U_1 + U_2 + U_3) - (U_4 + U_5 + U_6).
\end{align*}
\]

One obtains \(TrQ_A = 180\) \([20]\). The anomalous \(U_A\) is broken by the Dine-Seiberg-Witten (DSW) mechanism \([23]\), in which the anomalous D-term generated by the VEV of the dilaton field is cancelled by the VEVs of some massless fields which break \(U_A\), so that supersymmetry is preserved. The solutions (i.e. the choice of fields with non-vanishing VEVs) to the corresponding F and D - flat conditions are, however, not unique. A few alternative possibilities have been considered in Ref. \([20]\) (see also Refs. \([16]\) and \([22]\) for analogous considerations). Following our discussions in Sec. 2 as regards non-availability of 126 of \(SO(10)\) or \((1, 3, \overline{10})\) of \(G_{224}\), we assume, for the sake of simplicity in estimating strengths of relevant operators, that \(B - L\) is violated spontaneously at a scale \(\sim 10^{15}-10^{16}\) GeV by one unit (rather than two) through the VEVs of elementary sneutrino-like
fields $\widetilde{N}_R \subset 16_H$ and $\widetilde{N}_L \subset 16_H$ (as in Ref. [16]). Replacing VEVs of these elementary fields by those of products of fields including condensates, as in Ref. [20], would only lead to further suppression of the relevant unsafe higher dimensional operators and go towards strengthening our argument as regards certain symmetries being sufficient in preventing rapid proton-decay [26].

Proton-Decay Revisited: We now reexamine the problem of proton-decay and neutrino-masses by assuming that in addition to $I_{3R}$ and $B - L$, or just $Y$, either $\hat{Q}_\psi \equiv Q_1 + Q_2 - 2Q_3$, or $\hat{Q}_\chi \equiv Q_1 + Q_2 + Q_3 + 2(Q_4 + Q_5 + Q_6)$ (see Eq. (5)), or both emerge as good symmetries near the string scale and that suitable combinations of these symmetries (analogous to $Q_T = Q_\psi - (B - L)$ of Sec. 2) survive up to some scale $M_t \ll M_{st}$, even when sneutrino-like fields acquire VEVs $\sim 10^{15} - 10^{16}$ GeV. $M_t$ is determined in part by the VEVs of electroweak doublets and singlets (denoted by $\phi$’s). Generated radiatively, these are expected to be of order 1 TeV. $M_t$ can also receive contributions from the hidden-sector condensates which can be much larger than 1 TeV. As explained below, to ensure proton-stability, we need to assume that the condensate-scale is $\leq 10^{-2.5} M_{st}$. With the gauge coupling $\alpha_X$, at the unification-scale $M_X$, having nearly the MSSM value of .04 – .06, or even an intermediate value $\approx .16 - .2$ (say), as suggested in Ref. [27], this seems to be a safe assumption for most string models (see discussions later). The roles of the symmetries $Y$, $B - L$, $\hat{Q}_\psi$, $\hat{Q}_\chi$ and $(\hat{Q}_\chi + \hat{Q}_\psi)$ in allowing or forbidding the relevant $(B, L)$ - violating operators, including the higher dimensional ones, which allow violations of these symmetries through appropriate VEVs, are shown in Table 3. Based on the entries in this table, the following points are worth noting:

(i) Individual Roles of $\hat{Q}_\psi$ and $\hat{Q}_\chi$: Once $Q_\psi$ of Sec. 2 is replaced by the anomaly-free but family-dependent charge $\hat{Q}_\psi = Q_1 + Q_2 - 2Q_3$, it no longer forbids the $d = 4$ operators $\overline{U}D\overline{D}$, $QLD$ and $LLE$, when the three fields belong to three different families. $\hat{Q}_\psi$ still forbids the $d = 5$ operators – i.e. $QQQL/M$ and $\overline{U}\overline{U}D\overline{E}/M$ etc. – for all family - combinations. The charge $\hat{Q}_\chi$, which is family - universal, forbids all three $d = 4$ operators, but it effectively allows them utilizing VEVs of sneutrino-like fields through operators like $\overline{U}D\overline{D}/M$ and $QLD/\widetilde{N}_R/M$, which are unsafe. Furthermore, it allows the $d = 5$ operator $QQQL/M$, which is also unsafe. We see from these discussions and Table 3 that no single charge provides the desired protection against all the unsafe operators. Let us next consider pairs of charges.
(ii) Inadequacy of the Pairs \((Y, B - L); (Y, \hat{Q}_\psi); (Y, \hat{Q}_\chi)\) and \((B - L, \hat{Q}_\chi)\): Table 3 shows that the pair \((Y, B - L)\) does not give adequate protection against some of the unsafe operators. Neither do the pairs \((Y, \hat{Q}_\psi), (Y, \hat{Q}_\chi)\) and \((B - L, \hat{Q}_\chi)\). Amusingly enough, the charge \((\hat{Q}_\chi + \hat{Q}_\psi)\) by itself forbids all unsafe operators except when all four fields of the \(d = 5\) operator \(\mathcal{U} \mathcal{U} \mathcal{D} \mathcal{E} / M\) belong to family 3. This is unsafe if the identification of family 3 with the electron - family \([20]\) is rigid. We see that just one additional charge beyond \(Y\) is not adequate. Let us next consider other pairs of charges.

(iii) Adequate Protection Through the Pair \((B - L \ and \ \hat{Q}_\psi)\) or the Pair \((\hat{Q}_\chi \ and \ \hat{Q}_\psi)\): Using Table 3, we observe that the pair \((B - L \ and \ \hat{Q}_\psi)\), as well as the pair \((\hat{Q}_\chi \ and \ \hat{Q}_\psi)\), forbid all unsafe operators, including those which may arise from higher dimensional ones, with or without hidden-sector condensates. In fact, members of the pairs mentioned above complement each other in the sense that when one member of a pair allows an unsafe operator, the other member of the same pair forbids it, and vice versa – a remarkable team effort. Note that the strengths of the \(d=4\) and \(d=5\) operators are controlled by the VEVs \(<\mathcal{T}_1/M>^2, <\Phi/M>^n\) and \(<T_i \mathcal{T}_j/M^2>^2\), which give more than necessary suppression (see estimates below).

(iv) \(\hat{Q}_\psi\) removes Potential Danger From Triplets in The Heavy Tower As Well: Color triplets in the heavy infinite tower of states with masses \(M \sim M_{st} \sim 10^{18}\) GeV in general pose a potential danger for all string theories, including those for which they are projected out from the massless sector \([20]\). The exchange of these heavy triplets, if allowed, would induce \(d = 5\) proton-decay operators with strengths \(\sim \kappa/M\), where \(\kappa\) is given by the product of two Yukawa couplings. Unless the Yukawa couplings are appropriately suppressed \([28]\) so as to yield \(\kappa \leq 10^{-7}\) \([4]\), these operators would be unsafe. Note, however, that string-derived solutions possessing symmetries like \(\hat{Q}_\psi\) are free from this type of danger. This is because, if \(\hat{Q}_\psi\) emerges as a good symmetry near the string-scale, then the spectrum, the masses and the interactions of the color-triplets in the heavy tower would respect \(\hat{Q}_\psi\). As a result, the exchange of such states can not induce \(d = 5\) proton-decay operators, which violate \(\hat{Q}_\psi\) (see Table 3).

In fact, for such solutions, the color-triplets in the heavy tower can appear only as vector-like pairs, with opposite \(\hat{Q}_\psi\)-charges (like those in 10 and \(\mathbf{10}\) of \(SO(10)\), belonging to \(2\mathbf{7}\) and \(\overline{27}\) of \(E_6\) respectively), so that they can acquire invariant masses of the type \(M\{(H_3 \overline{H}_3 + H_3' \overline{H}'_3) + hc\}\), which conserve \(\hat{Q}_\psi\). Such mass-terms cannot induce proton
decay. By contrast, if only $I_{3R}$ and $B - L$, but not $\hat{Q}_\psi$ (or something equivalent), emerged as good symmetries, the mass-term of the type $M(H_3H'_3 + hc)$ for the triplets in the heavy tower would be permitted, which violates $\hat{Q}_\psi$ and can in general induce proton-decay at an unacceptable rate.

Thus we see that a symmetry like $\hat{Q}_\psi$ plays an essential role in safeguarding proton-stability from all angles. Since $\hat{Q}_\psi$ distinguishes between the three families [21], it cannot, however, arise within single-family grand unification symmetries, including $E_6$. But it does arise within string-derived three-generation solutions (as in Ref. [20]), which at once know the existence of all three families. In this sense, string theory plays a vital role in explaining naturally why the proton is so extraordinarily stable, in spite of supersymmetry, and why the neutrinos are so light.

4 $Z'$-mass and proton decay rate

If symmetries like $\hat{Q}_\psi$ and possibly $\hat{Q}_\chi$, in addition to $I_{3R}$ and $B - L$, emerge as good symmetries near the string scale, and break spontaneously so that only electric charge is conserved, there must exist at least one extra $Z'$-boson (possibly more), in addition to a superheavy $Z_H$ (that acquires mass when sneutrino acquires a VEV) and the (almost) standard $Z$ [30]. The extra $Z'$ boson(s) will be associated with symmetries like $\hat{Q}_T \equiv 2\hat{Q}_\psi - (B - L)$ and $\hat{Q}_\chi + \hat{Q}_\psi$, in addition to $Y$, that survive after sneutrinos acquire VEVs. The $Z'$ bosons can acquire masses through the VEVs of electroweak doublets and singlets ($\phi$’s), as well as through the hidden-sector condensates like $\langle \mathbf{T}_i \mathbf{T}_j \rangle$, all of which break $\hat{Q}_T$ and $\hat{Q}_\chi + \hat{Q}_\psi$ (see Table 2). As mentioned before, we expect the singlet $\phi$’s to acquire VEVs, at least radiatively (like the electroweak doublets), by utilizing their Yukawa couplings with the doublets, which at the string-scale is comparable to the top-Yukawa coupling (see Eq. (4)). Since the $\phi$’s do not have electroweak gauge couplings, however, we would expect that their radiatively-generated VEV, collectively denoted by $v_0$, to be somewhat higher than those of the doublets ($v_{EW} \sim 200$ GeV) - i.e., quite plausibly, $v_0 \sim 1$ TeV. Ignoring possible contribution from the hidden sector, we would thus expect the extra $Z'$ to be light $\sim 1$ TeV.

To be specific, consider the case when the string-scale symmetry (suppressing $SU(3)^C$ and $G_H$) is given by $G_{st} = G_1 = SU(2)_L \times I_{3R} \times (B - L) \times \hat{Q}_\psi$, which breaks at a superheavy scale into $SU(2)_L \times Y \times [\hat{Q}_T = \hat{Q}_T + Y = 2\hat{Q}_\psi + (I_{3R} - (B - L)/2)]$ due to
VEVs of sneutrino-like fields $\langle \tilde{N}_R^i \rangle \neq 0$ (choose $i = 1$ or 2, to be concrete). Now the VEVs of electroweak doublets and singlets ($\leq \mathcal{O}(1 \text{ TeV})$) would break $SU(2)_L \times Y \times \tilde{Q}_T$ to just $U(1)_{em}$. This second stage of SSB would produce two relatively light $Z$-bosons, to be called $Z_1$ and $Z_2$. $Z_1$ is the almost standard $Z$ with a mass $m_Z[1 - \mathcal{O}(v_{EW}/v_0)^2]$; $Z_2$ is the non-standard $Z$ with a mass $M_{Z_2} = (g v_0)[1 + \mathcal{O}(v_{EW}/v_0)]$; the $Z$-$Z'$ mixing angle is $\theta \sim (v_{EW}/v_0)^2$. Such a light $Z_2$ is compatible with known data \textsuperscript{[31]}, if $v_0 \geq 10v_{EW} \approx 2$ TeV (say). Alternative cases of $\mathcal{G}_{st}$ — e.g. $\mathcal{G}_{st} = \mathcal{G}_2 = SU(2)_L \times Y \times \tilde{Q}_{\psi} \times \tilde{Q}_{\chi}$ and $\mathcal{G}_{st} = \mathcal{G}_3 = SU(2)_L \times I_{3R} \times (B - L) \times \tilde{Q}_{\psi} \times \tilde{Q}_{\chi}$ — can be treated similarly. These will break in the first step of SSB respectively into $SU(2)_L \times Y \times (\tilde{Q}_{\psi} + \tilde{Q}_{\chi})$ and $SU(2)_L \times [3 \text{ orthogonal combinations of } Y, \tilde{Q}_T \text{ and } (\tilde{Q}_{\psi} + \tilde{Q}_{\chi})]$, which in the second step will produce one and two extra $Z'$-bosons, in addition to the almost standard $Z$. Details of this analysis will be presented in a separate note.

If the hidden sector condensates like $\langle T_i T_j \rangle$ form, they would also contribute to the $Z'$-boson masses. If the strength of $\langle T_i T_j \rangle$ is denoted by $\Lambda_c^2$, and if $\Lambda_c \sim \Lambda_H$, where $\Lambda_H$ is the confinement-scale of the hidden sector, their contribution to $Z'$-mass, would typically far supercede that of the singlets, because $\Lambda_H$ is expected to be superheavy $\sim 10^{15}$-$10^{16}$ GeV, or at least medium-heavy $\sim 10^8$-$10^{13}$ GeV (see below). Nevertheless, with our present ignorance of the hidden sector, it seems prudent to keep open the possibility that its contribution to $Z'$-mass is even zero \textsuperscript{[32]}, and that $Z'$ is light $\sim 1$ TeV.

The mass of the $Z'$-boson is correlated with the proton decay-rate. The heavier the $Z'$, the faster is the proton-decay. Looking at Table 3, and allowing for the hidden sector - condensates of strength $\Lambda_c^2$, we see that the strength of the effective $d = 4$ operators $(\tilde{U} \tilde{D} \tilde{D} \text{ etc.})$ is given by $\left( \langle \tilde{N}_R^i / M \rangle \right)^2 / M^2 \sim 10^{-2.5}(\Lambda_c^2 / M)^4$, and that of the $d = 5$ operator $(QQQL / M)$ is given by $\left( \langle T_i T_j \rangle / M^2 \right)^2 \sim (\Lambda_c^2 / M)^4$. The observed bound on the former ($\eta_{1,2} \leq 10^{-12}$) implies a rough upper limit of $(\Lambda_c / M)^4 \leq 10^{-9.5}$ and thus $\Lambda_c \leq 10^{15.5}$ GeV, while that on the latter (i.e. $\lambda_{1,2} \leq 10^{-7}$) implies that $\Lambda_c \leq 10^{16.2}$ GeV, where, for concreteness, we have set $M = 10^{18}$ GeV.

Thus, if $\Lambda_c \leq 1$ TeV \textsuperscript{[32]}, $Z'$ would be light $\sim 1$ TeV, and accessible to LHC and perhaps NLC. But, for this case, and even for $\Lambda_c \leq 10^{15}$ GeV (say), proton-decay would be too slow ($\tau_p \geq 10^{42}$ yrs.) to be observed. On the other hand, if $\Lambda_c \sim 10^{15.4} - 10^{15.6}$ GeV, the $Z'$-bosons would be inaccessible; but proton decay would be observable with
a lifetime $\sim 10^{32}-10^{35}$ years [33]. To see if such a superheavy $\Lambda_c$ is feasible, we note the following. It has recently been suggested [27], that an intermediate unified coupling $\alpha_X \approx 0.2-0.25$ at $M_X \sim 10^{17}$ GeV (as opposed to the MSSM-value of $\alpha_X \approx 1/26$) is desirable to stabilize the dilaton and that such a value of $\alpha_X$ would be realized if there exists a vector-like pair of families having the quantum numbers of $16 + \overline{16}$ of $SO(10)$, in the TeV-region. With $\alpha_X \approx 0.16-0.2$ (say), and a hidden sector gauge symmetry like $SU(4)_H$ or $SU(5)_H$ [21], a confinement scale $\Lambda_H \sim \Lambda_c \sim 10^{15} - 10^{16}$ GeV would in fact be expected. Thus, while rapid proton decay is prevented by string-derived symmetries of the type discussed here, *observable rate for proton decay* ($\tau_p \sim 10^{32}-10^{34}$ yrs.), *which would be accessible to Superkamiokande and ICARUS*, seems perfectly natural and perhaps called for [33, 34].

Before concluding, the following points are worth noting:

(i) The Messenger Sector: The existence of a messenger sector $G_M$, which is $[U(1)]^6$ for the case considered here, is a generic property of a large class of string-solutions (see e.g. [24, 16, 17, 22]). We have utilized this sector to find symmetries like $\hat{Q}_\psi$ and $\hat{Q}_\chi$ to prevent rapid proton decay. It is tempting to ask if the gauge interactions of this sector, as opposed to standard model gauge interactions [35], can help transmit SUSY-breaking efficiently from the hidden to the observable sector and thereby ensure squark-degeneracy of at least the electron and the muon families. This question will be considered separately.

(ii) $\hat{Q}_\psi$ – The Prototype of A Desirable Symmetry: $\hat{Q}_\psi$ is a good example of the type of symmetry that can safeguard, in conjunction with $B - L$ or $\hat{Q}_\chi$, proton-stability from *all angles*, while permitting neutrinos to have desired masses. It even helps eliminate the potential danger from contribution of the color-triplets in the heavy tower of states. In this sense, $\hat{Q}_\psi$ plays a very desirable role. We do not, however, expect it to be the only choice. Rather, we expect other string-solutions to exist, which would yield symmetries like $\hat{Q}_\psi$, serving the same purpose [36]. At the same time, we feel that *emergence of symmetries like $\hat{Q}_\psi$ is a very desirable constraint that should be built into the searches for realistic string-solutions*.

To conclude, the following remark is in order. For the sake of argument, one might have considered an $SO(10)$-type SUSY grand unification by including 126 of Higgs to break $B - L$ and ignoring string-theory constraints [15]. One would thereby be able
to forbid the $d = 4$ operators and give desired masses to the neutrinos [14]. But, as mentioned before, the problems of finding a compelling solution to the doublet-triplet splitting as well as to the gravity-linked $d = 5$ operators would still remain. This is true not just for SUSY $SO(10)$, but also for SUSY $E_6$, as well as for the recently proposed SUSY $SU(5) \times SU(5)$ - models [37]. By contrast, a string-derived non-GUT model, possessing a symmetry like $\hat{Q}_\psi$, in conjunction with $B - L$ or $\hat{Q}_\chi$, meets naturally all the constraints discussed in this letter. This shows that string theory is not only needed for unity of all forces, but also for ensuring natural consistency of SUSY-unification with two low-energy observations – proton stability and light masses for the neutrinos.

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[19] Our discussion in this section is similar in spirit, though not in content, to that of Weinberg in Ref. [1]. Weinberg assumed a symmetry of the form $SU(3)^C \times SU(2)_L \times U(1)_Y \times \tilde{U}(1)$. He, however, rejected its possible origin in $E_6$ and proposed instead an alternative that had no $\nu_R$, and contained a color octet (instead of triplets $3 + 3^*$) and an $SU(2)_L$-triplet. One reason for his rejection is tied to the need for a heavy $\nu_R$-mass in $E_6$. This does not, however, apply if one gauges $I_{3R}$ and $B - L$ separately, in addition to $Q_\psi$, as we do, rather than just $Y$ and $Q_\psi$. His other two reasons, tied to the problem of color-triplets mediating proton-decay and the need for a separate Higgs-doublet for each generation to cancel anomalies, do indeed apply to $E_6$, as we also discuss in the text. But string-derived solutions of the type we explore neatly remove both concerns.

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[22] A different solution to the problem of doublet-triplet splitting arises for the flipped $SU(5) \times U(1)$ - model, owing to a natural missing-partner mechanism: I. Antoniadis, J. Ellis, J. Hagelin and D.V. Nanopoulos, Phys. Lett. B 231, 65 (1989). For a review, see J. L. Lopez and D. V. Nanopoulos, hep-ph/9511266. One still needs to examine if the problems of $d = 4$ and gravity-linked $d = 5$ operators, which would seem to exist in this model, can be resolved by utilizing extra symmetries of the type discussed in the text.
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[26] We remark that, while the vector-like $(16_H + \overline{10}_H)$ - pair or equivalently the $(B = (1, 2, \overline{3}) + \overline{B'} = (1, 2, 4))$ - pair, does not appear in the specific solutions of Ref. [20], existence of such pairs is fairly generic in string theories (see e.g. Refs. [16] and [17]). It is important to note, however, that if sneutrino-like fields belonging to $16_H$ and $\overline{10}_H$ acquire large VEVs (as in Ref. [16]), one must ensure that strengths of operators like $16_i \cdot 16_H \cdot 10_H$ and $16_i \cdot \overline{10}_H \cdot 1_H$ in the superpotential are strongly suppressed, so that $\nu_L$-Higgsino mixing mass (which these operators would induce) remains below about 1 MeV. While such a suppression could happen through constraints of symmetries and the allowed pattern of VEVs, it turns out that this specific problem does not arise in models like those of Ref. [20]. This is because, as mentioned in the text, $<\overline{\tilde{N}}_R>$ is effectively replaced in these models by VEVs of product of fields and a condensate, which naturally lead to sufficiently strong suppression of $\nu_L$ - $\tilde{H}$ mixing (especially if the condensate-scale is $\leq 10^{14}$ GeV) through relevant higher dimensional operators. I thank K.S. Babu for drawing my attention to this potential problem.

[27] K.S. Babu and J.C. Pati, hep-ph/9606215, Phys. Lett. (to appear).

[28] As far as we can see, Yukawa couplings of the color-triplets in the heavy tower need not all be suppressed to the same extent as those of the electroweak doublets.

[29] Note $\hat{Q}_\psi$ assigns the same charge to all sixteen members of a given family. In this sense, $\hat{Q}_\psi$ (though not $\hat{Q}_\chi$) commutes with $SO(10)$.

[30] The possibility of extra $Z'$ bosons, with or without a string-origin, has been considered by several authors. See e.g. M. Cvetic and P. Langacker, hep-ph/9511378 for a recent string-motivated analysis. The role of a string-derived symmetry (associated with an extra $Z'$) in preventing rapid proton-decay, has not however been noted in these previous works.
[31] See e.g. G. Altarelli et al. (hep-ph/9601324) and P. Chiappetta et al. (hep-ph/9601306) for recent analyses of $Z-Z'$ - system based on LEP data.

[32] This would be the case if, for a certain string-solution, the hidden-sector condensates, even if they form, do not break the corresponding proton-stabilizing symmetry. In this case, they will not contribute to either the $Z'$-mass or to the strength of the proton-decay operator.

[33] If $<\tilde{N}_R> \neq 0$, the d=4 operators (see table 3) lead to proton decay rate $\propto (\eta_1\eta_2)^2 \propto (\Lambda_c/M)^{16}$, which varies extremely rapidly with $\Lambda_c$. It is interesting that, despite this rapid variation, proton decay is observable for a plausible though narrow range of values of $\Lambda_c \approx 10^{15.5}$ GeV. If $<\tilde{N}_R> = 0$, as in Ref. [20] (see also remarks in Ref. [22]), only the d=5 operators with strengths $\sim (T_i T_j > /M^2)^2$ are relevant (see table 3). These will lead to proton decay rate $\propto (\Lambda_c/M)^8$, which would be observable if $\Lambda_c \sim 10^{16}$ GeV (see text).

[34] In all these considerations, contribution of d=6 operators are neglected, because they are strongly suppressed if the relevant scale ($M_X$ or $M$) exceeds $10^{16}$ GeV.

[35] See e.g. M. Dine, A. Nelson and Y. Shirman, Phys. Rev. D 51, 1362 (1995) and M. Dine, A. Nelson, Y. Nir and Y. Shirman, hep-ph/9507378.

[36] For example, other pairs of string-derived symmetries such as $(\tilde{F}, B - L)$, where $\tilde{F} \equiv F_1 + F_2 - 2F_3$ and $F_i$ acts as the fermion number on the $i$th family (i.e. $F_i \equiv (3B + L)_i = (1, 1 | -1, -1)$ for $(q, l | q, l)_i$), may also play a role analogous to the pair $(\tilde{Q}_\psi, B - L)$. As an additional possibility, it would be interesting to explore whether the recently proposed variants of the solutions in Refs. [20] and [22], which yield a leptophobic $Z'$ (see e.g. A. Faraggi, hep-ph/9604302 and J. L. Lopez and D. V. Nanopoulos, hep-ph/9605359), can be developed to provide a consistent Higgs-mechanism and also prevent rapid proton decay, utilizing extra symmetries, as discussed here. These issues will be considered elsewhere.

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| Family | States | $Q_1$ | $Q_2$ | $Q_3$ | $Q_4$ | $Q_5$ | $Q_6$ | $Q_\psi$ | $Q_\chi$ |
|--------|--------|-------|-------|-------|-------|-------|-------|--------|--------|
| 1      | $q_1$  | 0     | 0     | -1/2  | 0     | 0     | 1/2   | -1/2   |         |
|        | $L_1$  | 0     | 0     | 1/2   | 0     | 0     | 1/2   | 3/2    |         |
|        | $(\bar{U}, \bar{E})_1$ | 0 | 0 | 1/2 | 0 | 0 | 1/2 | 3/2 |         |
|        | $(D, \nu\bar{\pi})_1$ | 0 | 0 | -1/2 | 0 | 0 | 1/2 | -1/2 |         |
| 2      | $q_2$  | 0     | 1/2   | 0     | 0     | -1/2  | 0     | 1/2   | -1/2   |
|        | $L_2$  | 0     | 1/2   | 0     | 1/2   | 0     | 1/2   | 3/2    |         |
|        | $(\bar{U}, \bar{E})_2$ | 0 | 0 | 1/2 | 0 | 0 | 1/2 | 3/2 |         |
|        | $(D, \nu\bar{\pi})_2$ | 0 | 0 | -1/2 | 0 | 0 | 1/2 | -1/2 |         |
| 3      | $q_3$  | 0     | 0     | 1/2   | 0     | 0     | -1/2  | -1    | -1/2   |
|        | $L_3$  | 0     | 0     | 1/2   | 0     | 0     | 1/2   | -1    | 3/2    |
|        | $(\bar{U}, \bar{E})_3$ | 0 | 0 | 1/2 | 0 | 0 | 1/2 | -1/2 |         |
|        | $(D, \nu\bar{\pi})_3$ | 0 | 0 | -1/2 | 0 | 0 | 1/2 | -1/2 |         |
|        | $D_{45} = (3, -2/3, 1_L, 0)$ | -1/2 | -1/2 | 0 | 0 | 0 | 0 | -1 | -1 |
|        | $\overline{D_{45}} = (3^*, +2/3, 1_L, 0)$ | 1/2 | 1/2 | 0 | 0 | 0 | 0 | +1 | +1 |
|        | $\bar{h}_1 = (1, 0, 2_L, 1/2)$ | -1 | 0 | 0 | 0 | 0 | 0 | -1 | -1 |
|        | $\bar{h}_2 = (1, 0, 2_L, 1/2)$ | 0 | -1 | 0 | 0 | 0 | 0 | -1 | -1 |
|        | $\bar{h}_3 = (1, 0, 2_L, 1/2)$ | 0 | 0 | -1 | 0 | 0 | 0 | +2 | -1 |
|        | $\bar{h}_{45} = (1, 0, 2_L, 1/2)$ | 1/2 | 1/2 | 0 | 0 | 0 | 0 | 1 | 1 |
|        | $V_1, \overline{V}_1$ | 0 | 1/2 | 1/2 | 1/2 | 0 | 0 | -1/2 | 2 |
|        | $T_1, \overline{T}_1$ | 0 | 1/2 | 1/2 | -1/2 | 0 | 0 | -1/2 | 0 |
|        | $V_2, \overline{V}_2$ | 1/2 | 0 | 1/2 | 0 | 1/2 | 0 | -1/2 | 2 |
|        | $T_2, \overline{T}_2$ | 1/2 | 0 | 1/2 | 0 | -1/2 | 0 | -1/2 | 0 |
|        | $V_3, \overline{V}_3$ | 1/2 | 1/2 | 0 | 0 | 0 | 1/2 | 1 | 2 |
|        | $T_3, \overline{T}_3$ | 1/2 | 1/2 | 0 | 0 | 0 | -1/2 | 1 | 0 |

Table 2: Partial List of Massless States from Ref. [20]. (i) The quark and lepton fields have the standard properties under $SU(3)^C \times U(1)_{B-L} \times SU(2)_L \times U(1)_{T_{3R}}$, which are not shown, but those of color triplets and Higgses are shown. (ii) Here $\hat{Q}_\psi \equiv Q_1 + Q_2 - 2Q_3$ and $\hat{Q}_\chi = (Q_1 + Q_2 + Q_3) + 2(Q_4 + Q_5 + Q_6)$ (see Eq. (5)). (iii) The doublets $\bar{h}_{1,2,3,45}$ are accompanied by four doublets $h_{1,2,3,45}$ with quantum numbers of conjugate representations, which are not shown. (iv) The $SO(10)$-singlets $\{\phi\}$ which possess $U(1)_U$-charges, and the fractionally charged states which become superheavy, or get confined [24], are not shown. In Ref. [20], since only $\bar{h}_1$ and $h_{45}$ remain light, families 1, 2 and 3 get identified with the $\tau$, $\mu$ and $e$ - families respectively. Hidden matter $V_i, \overline{V}_i, T_i$ and $\overline{T}_i$ are $SO(10)$-singlets and transform as $(1, 3), (1, \overline{3}), (5, 1)$ and $(\overline{5}, 1)$, respectively, under $SU(5)_H \times SU(3)_H$. 

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Table 3: The roles of $Y$, $B - L$, $\hat{Q}_\psi$, $\hat{Q}_\chi$ and $\hat{Q}_\chi + \hat{Q}_\psi$ in allowing or forbidding the relevant $(B, L)$ violating operators. Check mark $(\checkmark)$ means “allowed” and cross $(\times)$ means “forbidden”. The mark $\dagger$ signifies that the corresponding operator is allowed if either two of the four fields are in family (1 or 2) and two are in family 3, with $i = 1$ and $j = 3$; or all four fields are in family (1 or 2) with $i = 1$ and $j = 2$. The mark $(\ast)$ signifies that $(\hat{Q}_\chi + \hat{Q}_\psi)$ forbids $UUDE/M$ for all family-combinations except when all four fields belong to family 3, and that it forbids $LL\bar{h}_i \bar{h}_i$ in some family-combinations, but not in others. In labelling the operators as safe/unsafe, we have assumed that $\langle N_R \rangle \sim 10^{15.5}$ GeV, $\langle \phi/M \rangle^n \leq 10^{-9}$ and $M \sim M_{st} \sim 10^{18}$ GeV, and that hidden sector condensate-scale $\Lambda_c \leq 10^{15.5}$ GeV (see text). Note that the pairs $(Y, B - L)$, $(Y, \hat{Q}_\psi)$, $(Y, \hat{Q}_\chi)$ and $(B - L, \hat{Q}_\chi)$ do not give adequate protection against the unsafe operators. But $\hat{Q}_\psi$, in conjunction with $B - L$ or $\hat{Q}_\chi$, gives adequate protection against all unsafe operators. This establishes the necessity of string-derived symmetries like $\hat{Q}_\psi$ (which can not emerge from familiar GUTs including $E_6$) in ensuring proton-stability.

| Operators | Family Combinations | $Y$ | $B - L$ | $\hat{Q}_\psi$ | $\hat{Q}_\chi$ | $\hat{Q}_\chi + \hat{Q}_\psi$ | If Allowed |
|-----------|---------------------|-----|---------|----------------|----------------|-----------------------------|------------|
| $UUDD, QLD, LL\bar{E}$ | (a) All except (b) | $\checkmark$ | $\times$ | $\times$ | $\times$ | $\times$ | unsafe |
| | (b) 3 fields from 3 different families | $\checkmark$ | $\times$ | $\checkmark$ | $\times$ | $\times$ | unsafe |
| $(UUDD$ or $QLD)(N_R/M)$ | All | $\checkmark$ | $\checkmark$ | $\times$ | $\checkmark$ | $\times$ | unsafe |
| | $([h/M]^2$ or (“$\phi$”$/M)^n]$ | All | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\times$ | safe |
| $(UUDD$ or $QLD)(N_R/M)$ | Some($\dagger$) | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | safe |
| $(QQQL/M)(N^i_L/M)_{i=1,2}$ | All | $\checkmark$ | $\times$ | $\checkmark$ | $\times$ | $\times$ | unsafe |
| | e.g. $(1, 2, 1, 3)$ | $\checkmark$ | $\times$ | $\checkmark$ | $\times$ | $\times$ | unsafe |
| $(QQQL/M)(N^i_L/M)(\overline{N}_R^j/M)$ | All | $\checkmark$ | $\times$ | $\checkmark$ | $\checkmark$ | $\times$ | safe(?) |
| $(QQQL/M)(T^I_i \overline{T}_j/M^2)^2$ | Some($\dagger$) | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | $\checkmark$ | safe |
| $UUDE/M$ | All | $\checkmark$ | $\times$ | $\checkmark$ | $\ast$ | unsafe |
| $LL\bar{h}_i \bar{h}_i/M$ | All | $\checkmark$ | $\checkmark$ | $\times$ | $\ast$ | unsafe |

Note that the pairs $(Y, B - L)$, $(Y, \hat{Q}_\psi)$, $(Y, \hat{Q}_\chi)$ and $(B - L, \hat{Q}_\chi)$ do not give adequate protection against the unsafe operators. But $\hat{Q}_\psi$, in conjunction with $B - L$ or $\hat{Q}_\chi$, gives adequate protection against all unsafe operators. This establishes the necessity of string-derived symmetries like $\hat{Q}_\psi$ (which can not emerge from familiar GUTs including $E_6$) in ensuring proton-stability.