Motion of an Impurity in a Bose-Einstein Condensate with Weyl Spin-Orbit Coupling: Non-collinear Drag Force and Anisotropic Critical Velocity

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We consider the motion of a point-like impurity through a three-dimensional two-component Bose-Einstein condensate subject to Weyl spin-orbit coupling. Using linear-response theory, we calculate the drag force felt by the impurity and the associated anisotropic critical velocity from the spectrum of elementary excitations. The drag force is shown to be generally not collinear with the velocity of the impurity. This unusual behavior is a consequence of condensation into a finite-momentum state due to the spin-orbit coupling.

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Degenerate quantum gases of neutral atoms1,2, polaritons3 as well as the recently discovered condensation of light3 have provided new opportunities for studying superfluidity. One of the most remarkable manifestations of superfluidity is that impurities immersed into such systems propagate without dissipation if their velocities do not exceed the Landau critical velocity4.

\[ v_c = \min_q \left[ \frac{\omega(q)}{q} \right], \]

where \( \omega(q) \) is the spectrum of elementary excitations. As long as the impurity moves slower than the critical velocity, the superfluid cannot absorb any of its energy and therefore the impurity motion is frictionless. Experiments with atomic Bose-Einstein condensates (BEC) have provided evidence for a critical velocity associated with emission of elementary excitations5–9 as well as more complex excitations like vortices and solitons10–14. Intense theoretical efforts15–28 have been undertaken to study the stability of superfluidity and explain the mechanisms of dissipation in BECs.

Recently, the experimental realization of various synthetic gauge fields including one-dimensional and two-dimensional spin-orbit coupling (SOC) in quantum gases29–32 enabled the prediction of a number of novel interesting properties in these new types of condensates31,33. Among them is condensation of Bose atoms at some finite momentum, thus breaking simultaneously the conventional U(1) gauge symmetry associated with condensation as well as rotational symmetry. Moreover, SOC breaks the Galilean invariance of the system33–35, making the applicability of the Landau criteria of superfluidity in the new BECs questionable. This calls for a better understanding of the critical velocity and dissipation mechanism of this new superfluid.

In this Letter, we examine superfluidity in a two-component Bose gas with three-dimensional Weyl SOC by studying the drag force felt by a moving point-like impurity36. The Weyl SOC can be realized using powerful quantum technology31. Here we calculate the drag force using linear response theory from the elementary excitation spectrum through the dynamical structure factor17. The drag force demonstrates the presence of an anisotropic critical velocity. We also find that the drag force is not generally collinear with the velocity of the impurity, in stark contrast to a conventional superfluid. This fact can be used to probe SOC by the scattering of heavy molecules by the condensate.

The Weyl-type SOC takes its name from a seminal paper by Hermann Weyl37 predicting fermions with a high degree of symmetry. Although there is currently no evidence for Weyl fermions to exist as fundamental particles in our universe, Weyl-like quasiparticles have been detected recently in condensed-matter systems38,39. In light of these discoveries, the study of Weyl SOC in ultra-cold atom systems becomes particularly relevant, since the ability to manipulate the Weyl-SOC strength creates interesting opportunities for the discovery of effects not predicted in the realm of particle physics.

The second-quantized Hamiltonian for the BEC with a point-like impurity moving with velocity \( v \) is

\[
H = \int d^3r \Psi^\dagger \left[ \left(-\frac{\hbar^2 \nabla^2}{2m} - \mu \right) I + \lambda \sigma \cdot P \right] \Psi + \int d^3r \left[ \frac{g}{2} n^2 + (g_{1\uparrow} - g)n_{\uparrow}n_{\downarrow} + g_{n}\delta(\mathbf{r} - \mathbf{vt}) \right].
\]
Here $\Psi(\mathbf{r}) = (\psi^\dagger, \psi)^T$ is the two-component condensate quantum field, $I$ is the $2 \times 2$ identity matrix, $n(\mathbf{r}) = n_\uparrow(\mathbf{r}) + n_\downarrow(\mathbf{r}) = \psi^\dagger_\uparrow \psi_\uparrow + \psi^\dagger_\downarrow \psi_\downarrow$ is the density operator, $\mu$ is the chemical potential, $\lambda$ is the strength of the spin-orbit coupling, $g_\lambda$ is the particle-impurity coupling constant, and the strengths of the intra-species interaction and inter-species interaction are $g$ and $g_{\lambda \uparrow \downarrow}$, respectively. For brevity, we set $\hbar = 2m = 1$ from now on.

The time-dependent mean-field Gross-Pitaevskii (GP) equation found from Eq. (2) reads

$$
\left[ (-i\partial_t - \nabla^2 - \mu) I - i\lambda \sigma \cdot \nabla \right] \Psi_0 + g_\lambda \delta(r - vt) \Psi_0 = 0. \quad (3)
$$

To proceed, we split the field $\Psi_0(\mathbf{r}, t) = \Phi_0(\mathbf{r}) + \Phi(\mathbf{r}, t)$, where $\Phi_0(\mathbf{r}) = \sqrt{\frac{n_\lambda}{2}} (1, 1)^T e^{iK \cdot r}$ is the mean-field solution without the impurity, and $\Phi(\mathbf{r}, t)$ is the perturbation caused by the impurity. Without loss of generality, we choose the condensation momentum to be $K = (-\lambda/2, 0, 0)$, and the chemical potential becomes $\mu = n_\lambda (g + g_{\lambda \uparrow \downarrow})/2 - K^2$. Linearizing GP in $\Phi$, we obtain

$$
\left[ (-i\partial_t - \nabla^2 + K^2 + \frac{g_\lambda n_\lambda}{2}) I - i\lambda \sigma \cdot \nabla + \frac{g_\lambda n_\lambda}{2} \sigma_x \right] \Phi + \left( \frac{g_\lambda n_\lambda}{2} \sigma_x + \frac{g_\lambda n_\lambda}{2} I \right) e^{2iK \cdot r} \Phi^* + g_\delta \delta(r - vt) \Phi_0 = 0. \quad (4)
$$

The ansatz $\Phi(\mathbf{r}, t) = e^{iK \cdot r} \sum q \varphi_q e^{iq \cdot (r - vt)}$ yields

$$
[(-i \mathbf{v} \cdot \nabla + q^2 + 2K \cdot q) I + i\lambda \sigma \cdot (K + q)] \varphi_q + \left( \frac{g_\lambda n_\lambda}{2} I + \frac{g_\lambda n_\lambda}{2} \sigma_x \right) (\varphi_q + \varphi_q^*) = -g_\delta i \sqrt{\frac{n_\lambda}{2}} \left( \begin{array}{c} 1 \\ 1 \\ 1 \\ 1 \end{array} \right). \quad (5)
$$

Combining Eq. (4) with its complex conjugate, we obtain

$$
[H_\lambda - \mathbf{v} \cdot \mathbf{I}] \left( \begin{array}{c} \varphi_q \\ \varphi_q^* \\ \varphi_q^\dagger \\ \varphi_q^\dagger \end{array} \right) = -g_\delta i \sqrt{\frac{n_\lambda}{2}} \left( \begin{array}{c} 1 \\ 1 \\ 1 \\ 1 \end{array} \right). \quad (6)
$$

with the matrices

$$
H_\lambda = \left( \begin{array}{cc} M_q & B \\ B & M_{\lambda \uparrow \downarrow} \end{array} \right), \quad \mathbf{I} = \left( \begin{array}{cc} I & 0 \\ 0 & -I \end{array} \right). \quad (7)
$$

Here $B = \frac{g_\lambda n_\lambda}{2} I + \frac{g_\lambda n_\lambda}{2} \sigma_x$ and

$$
M_q = (q^2 + 2K^2 + 2K \cdot q) I + i\lambda \sigma \cdot (K + q) + B. \quad (8)
$$

Solving Eq. (4) for $\varphi_q \uparrow$ and $\varphi_q \downarrow$, the force acting on the impurity is found as

$$
F = -\int d^2\mathbf{r} \Psi^\dagger \nabla [g_\delta \delta(r - vt)] \Psi_0 \equiv g_\delta \nabla \left[ \Psi_0(\mathbf{r}, t) \right]_{r = vt}, \quad (9)
$$

$$
\approx g_\delta i \sqrt{\frac{n_\lambda}{2}} \sum q \int d^2\mathbf{r} \left( \varphi_q^\dagger + \varphi_q + \varphi_q^\dagger + \varphi_{q \dagger} \right), \quad (10)
$$

The infinitesimal imaginary part was added following the usual causality rule [17, 20, 40].

According to the fluctuation-dissipation theorem, the drag force should be related to the fluctuation properties (i.e., the spectrum of elementary excitations) of the unperturbed system. This fact will assist us to analyze the drag force in more detail. We consider the system without the impurity by setting $g_\delta = 0$ in Eq. (2). The partition function of the system can be conveniently casted as imaginary time field integral [41]

$$
Z = \int d[\psi^\dagger, \psi] e^{-S[\psi^\dagger, \psi]} \text{ with the action given by } S = \int_0^\beta d\tau \left[ \int d\sigma \sum_{\sigma} \psi^\dagger_{\sigma} \partial_\tau \psi_{\sigma} + H(\psi^\dagger_{\sigma}, \psi_{\sigma}) \right], \quad (11)
$$

where $\tau = i\tau$ is the imaginary time. We replace the Bose field with a static part and a fluctuating part as $\psi_{\sigma} = \phi_{\sigma 0} + \phi_{\sigma 1}$. Within the Bogoliubov approximation, we expand the action up to quadratic orders in the fluctuating fields, and approximate the action as $S \approx S_{\text{eff}} = S_0 + S_g$. Here $S_0$ is the saddle point action containing only the static fields $\phi_{\sigma 0}$, and $S_g$ is the Gaussian action containing fluctuating fields $\phi_{\sigma 1}$ of quadratic orders. By defining column vectors $\Xi_q = (\phi_{K + q \uparrow}, \phi_{K + q \downarrow}, \phi_{K - q \uparrow}, \phi_{K - q \downarrow})^T$, we may write the Gaussian action in a compact form as

$$
S_g = \frac{1}{2} \sum_{q, i\omega_n} \Xi_q^\dagger \tilde{G}^{-1} \Xi_q, \quad (12)
$$

where $\gamma = (q, i\omega_n)$ with $\omega_n = 2n\pi/\beta$ being bosonic Matsubara frequencies. The inverse Green’s function is given by $\tilde{G}^{-1}(q, i\omega_n) = \mathcal{H}_q - i\omega_n \mathbf{I}$. Comparison with Eq. (10) yields the drag force in terms of the Green’s function of the unperturbed system:

$$
F = -\frac{g_\lambda^2 n_\lambda}{2} \sum_{q} \int d\mathbf{r} \left[ \mathcal{H}_q - \mathbf{v} \cdot \mathbf{I} \right]_{r = vt}, \quad (13)
$$

Thus the drag force can be also expressed in terms of the dynamic structure factor:

$$
F = -\frac{g_\lambda^2 n_\lambda}{2} \sum_{q} \int d\mathbf{r} \mathcal{S}(q, i\omega_n) \rightarrow \mathbf{v} + i0^+. \quad (14)
$$

The spectrum $\omega_\lambda(q)$ of elementary excitations is found by solving $\text{Det}[\tilde{G}^{-1}(q, i\omega_n)] = 0$ with subsequent analytic continuation $i\omega_n \rightarrow \omega_\lambda(q)$. Using this fact, we arrive at the following expression for the drag force:

$$
F = 2\pi g_\lambda^2 n_\lambda \sum_{q} \frac{4}{4} \int_{j \neq i} d\mathbf{r} \left[ \mathcal{J}(q, \omega_\lambda(q)) \delta(q \cdot \mathbf{v} - \omega_\lambda(q)) \right] \quad (15)
$$

Here we used the abbreviation

$$
\mathcal{J}(q, i\omega_n) = q^2 (i\omega_n + 2\lambda q^2) - (q^2 + \lambda^2) (q^2 + \lambda^2 q^2) - (g^2 - g_{\lambda \uparrow \downarrow} n_\lambda q^2) (q^2 + \lambda^2 q^2 - \lambda^2 q^2). \quad (15)
$$

Let us first calculate the drag force in the absence of SOC by setting $\lambda = 0$. In this case, the four branches
of excitations are the Bogoliubov-type modes $\omega_{1,2}^0 = \pm q\sqrt{q^2 + (g + g_{1z})n_0}$ and $\omega_{3,4}^0 = \pm q\sqrt{q^2 + (g - g_{1z})n_0}$. The former is the spectrum of density waves propagating with the speed of sound $c = \sqrt{(g + g_{1z})n_0}$, while the latter is the spectrum of spin waves. Using these analytical expressions, we evaluate the sum in Eq. (13) and find the drag force $F_0 = vuy^2n_0(1 - c^2/v^2)^2/(16\pi)\Theta(v - c)$, in agreement with Ref. [17]. The drag force in this case is collinear with the impurity’s direction of motion. Note also that the lower-lying spin-wave mode is not excited because the impurity couples only to density waves.

The presence of SOC modifies the above result. Due to condensation into a finite-momentum state, the ground state breaks rotational symmetry, and the spectrum of elementary excitations becomes anisotropic. For our choice of condensate momentum, the spectrum is invariant under flipping the direction of $q_y$ and/or $q_z$, namely $\omega_i(q_xq_yq_z) = \omega_i(q_xq_yq_z)$. As a result, the $x$-direction is distinguished from the $y$ and $z$ axes. To be specific, let’s assume that the impurity moves along the $z$ axis. We can write $\delta(q - v - \omega) = \delta(q - q_{0z})/|v_z - \partial\omega_0/\partial q_z|$, where $q_{0z} = h(q_xq_yq_z)$ is some function reflecting symmetry properties, and the detailed form of $h_i$ is unimportant for our further analysis. Carrying out the integration in Eq. (14), one immediately finds that $F_y$ vanishes and both $F_x$ and $F_z$ survive, by repeating the integration of odd or even functions within a symmetrical interval. This argument can be repeated for different directions of the velocity yielding an additional contribution to the drag force along the $x$ axis. Therefore, in addition to the conventional force component along the velocity vector, a force component along $x$ axis is generated, due to the asymmetrical excitation spectrum with regard to $x$ axis. For small spin-orbit coupling, corrections in the drag force brought about by SOC may be estimated by performing an expansion in the parameter $\lambda$. Up to the first non-vanishing order in the SOC strength, after lengthly but straightforward calculations, we obtain $F = F_0 + F_\parallel + F_x$ with $F_\parallel \approx i\nu\mathcal{O}(\lambda^2)$ and $F_x \mathcal{O}(\lambda^3)$. Evidently, SOC produces an additional drag along the direction of condensation.

To substantiate the above argument we now calculate the drag force numerically. For our numerics we choose $gn_0$ as the energy scale and $\sqrt{gn_0}$ as the momentum scale. Let us first consider simple situations where the velocity of the moving impurity is along the $x$, $y$ and $z$ axes, respectively. Results obtained for these cases are shown in Fig. 1. When the velocity is along the $x$ axis, the drag force is also along $x$ axis, and is asymmetrical between positive and negative direction of velocity, reflecting the spontaneously broken symmetry of the ground state with finite condensate momentum in the negative $x$ direction. It requires more force to drag the impurity against the direction of condensation than along with it. When the velocity is along the $y$ axis or the $z$ axis, the magnitude of force does not change upon reversing the direction of the velocity. However, it is remarkable that in both cases a non-vanishing force component along the $x$ axis emerges. Fig. 1 also shows that there exists a critical velocity $v_c$ below which there is no drag force. Its magnitude decreases when the strength of SOC is increased. We calculated the critical velocity in three orthogonal planes by examining the lower bound of the velocity where the drag force becomes nonzero. These results are presented in Fig. 2. As shown in panel (a), the critical velocity along the negative $x$ axis remains

![FIG. 1. (color online) Drag force (measured in units of $g_{1z}n_0$) experienced by an impurity moving in a Weyl-spin-orbit-coupled BEC with condensate wave vector parallel to the negative $x$ direction and $g_{1z}/g = 1/2$. Results for different spin-orbit coupling strength $\lambda$ are given. For the case of the impurity velocity being parallel to the $x$ axis, only the $x$ component $F_x$ of the drag force is finite even for $\lambda \neq 0$ [panel (a)]. When the velocity is along the $y$ axis, the drag force has components along the $y$ axis [panel (b)] and, for finite $\lambda$, also along the $x$ axis [panel (c)]. When the velocity is along the $z$ axis, the drag force has finite components in the $z$ and $x$ directions when $\lambda \neq 0$ [panels (d) and (e)].](image1)

![FIG. 2. (color online) Directional dependence of the critical velocity for an impurity moving in a Weyl-spin-orbit-coupled BEC with condensate wave vector parallel to the negative $x$ direction and $g_{1z}/g = 1/2$.](image2)
unchanged as SOC strength is increased, however, it decreases significantly along the positive $x$ axis. As can be seen in panel (b), the critical velocity in the $yz$ plane is slightly deformed from a circle, signalling the inequivalence between $y$ and $z$ directions.

Let us now examine the drag force in more detail. Fixing the velocity of the impurity to lie in the $xy$ plane we show the behavior of the corresponding drag force in Fig. 3. Here we define the azimuth of the drag force to be $\phi_F = \arg(F_x + iF_y)$, i.e. the angle in the $xy$ plane, and the azimuth of the velocity to be $\phi_v = \arg(v_x + iv_y)$. In panel (a), the $x$ component for the scaled drag force $F_x$ has the symmetry of $F_x(\pi - \phi_v) = F_x(\pi + \phi_v)$, namely it has reflection symmetry with respect to the $x$ axis. The $y$ component $F_y$ entails the symmetry of $F_y(\pi - \phi_v) = -F_y(\pi + \phi_v)$, as indicated in panel (b). The $z$ component of the drag force vanishes. In panel (c), we show the difference between the azimuth of the drag force and the azimuth of the velocity. It is quite remarkable that the direction of the drag force is not aligned with the velocity, as is the case in a conventional superfluid. For a better visualization, we show the force vector in panel (d), where the arrow sitting on constant circle of speed indicates the force vector.

Now we fix the velocity vector to lie in the $yz$ plane. The drag force is shown in Fig. 4. Panel (a) illustrates that the drag force has an $x$ component that is independent of the direction of the velocity within the $yz$ plane for a fixed small spin-orbit coupling strength $\lambda$. For large $\lambda$, $F_x$ oscillates slightly with varying directions of the velocity in $yz$ plane. This inequivalence between $y$ axis and $z$ axis is due to the breaking of spin-rotational invariance by the non-linear interaction potential $g - g_{z\perp} \neq 0$. In panel (b), $F_y$ is shown to be symmetric with respect to $y$ axis while antisymmetric with respect to $z$ axis. In panel (c), $F_z$ is antisymmetric with respect to $y$ axis and symmetric with respect to the $z$ axis. Interestingly, as can been seen in panel (d), the direction of the force is not lying in the $yz$ plane, but is tilted towards the $z$ axis.

Results for the case when the velocity vector lies in the $zx$ plane are shown in Fig. 5. The behavior of the drag force looks quite similar to the situation when the velocity is in the $xy$ plane. Panel (a) illustrates that $F_x$ is symmetric with respect to $x$ axis, while panel (b) shows that $F_z$ is antisymmetric with respect to the $z$ axis. As seen in panel (c), the difference of azimuthal angles for the force and velocity is antisymmetric with respect to the $z$ axis. In panel (d), the force vector is visualized.

In summary, we have studied the motion of a point-like impurity in a three-dimensional two-component BEC.
with Weyl SOC. We calculated the drag force and the associated critical velocity. At small SOC strength, we showed that the drag force can be decomposed into two parts. One is along the direction of the moving velocity, and the other one is along the direction of the condensation momentum. Hence, unlike in non-spin-orbit-coupled superfluids, the drag force is not generally collinear with the velocity of the impurity. This unusual feature can be utilized to probe SOC in bosonic superfluids.

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