DETECTION OF AIR GAPS IN COPPER-MINE CEILING
BY ELECTRICAL IMPEDANCE TOMOGRAPHY

Tomasz Rymarczyk¹, Paweł Tchórzewski¹, Jan Sikora²,³
¹Netrix S.A., Research and Development Center, ul. Związkowa 26, 20-148 Lublin, ²Lublin University of Technology, Institute of Electronics and Information Technology, ³Electrotechnical Institute

Abstract In this paper, we investigate the inverse problem for the electric field so-called copper mine problem. In general, this task assumes detection of all air gaps. Gaps are localised above ceiling in a copper mine. Such task can be considered as application of the electrical impedance tomography. In order to solve forward problem there was used the boundary element method or the finite element method. The inverse problem is based on the level set method. There was considered extension of boundary element method (BEM). For simplicity zero order approximation has been chosen. The BEM has been connected with the infinite boundary elements. Hence, open domain problems with infinite boundary curves can be analysed. For such domain, we have solved the Dirichlet problem for two-dimensional Laplace’s equation. The proposed numerical model has been verified.

Keywords: inverse problem, boundary element method, electrical impedance tomography

Introduction

In this paper, we propose algorithm based on the combination of the boundary element method (or the finite element method) and the level set method to solve the inverse problem arising from electrical impedance tomography (EIT) [11–14]. The representation of the boundary shape and its evolution during an iterative reconstruction process is achieved by the level set method [2, 5–8, 15]. In our numerical algorithm we have used the gradient technique in order to calculate the velocity. This idea has been applied successfully in the context of inverse problem [3, 12, 16].

We focus our attention on so-called copper mine problem. Generally, this problem involves the detection of all air gaps are located above the roof of the copper mine (see Fig. 1). This task is very important for safety reasons. It could be done EIT.

The electrical impedance tomography is very important field of research nowadays. It possesses many applications, for example this technique may be used in medical imaging, geophysics and other scientific areas. However, EIT is not easy to use due to necessity of solving the inverse problem during calculations. In three dimensional cases this requires a lot of computational effort. Using the level set method and the finite element method coupled together is proper way to solve the inverse problem. In particular all gaps in copper-mine ceiling can be localized.

1. Boundary element method

Physical phenomena are described usually by sets of differential equations. Numerical techniques give us opportunity to find approximate solutions of differential equations which cannot be solved by means of analytical ones. Among various numerical tools let us concentrate our attention on the boundary element method. BEM can be effectively employed on condition that partial differential equation can be transformed to integral form. Additionally, the Green’s function has to be calculated. The explicit form of this one is desired. Often BEM can be easy coupled with other numerical methods or even analytical ones [1, 4]. Application of infinite boundary elements (IBEs) in BEM is rather less common because of difficulties with accurate numerical calculations of integrals with infinite limits of integration. In this paper we propose the generalisation of classical BEM with constant element interpolation for field function and its normal derivative by coupling with IBEs. For simplicity we consider Laplace’s equation in two spatial dimensions.

Exterior domain with open boundary curve cannot be completely discretised by standard boundary elements with finite length. Hence, in the numerical model the IBEs should be introduced. So far several kinds of such boundary elements have been researched [9, 10, 17]. In case of IBEs one has to select interpolation functions which carry out appropriate conditions. In this paper we propose utilisation of the interpolation functions with exponential decay. This approach assumes that along IBEs the solution of differential equation and its normal derivative tend exponentially to zero. Speed of decay is described by one positive parameter.

It is clear that in realistic technical problems domains are usually finite. However, some engineering phenomena can be considered as unbounded domain problems and they are effectively solved using infinite elements.

Section 3 is the heart of this work and contains explanation of the theoretical model. The third section is devoted to numerical results and conclusions.

Let us consider the Laplace’s equation in two-dimensional Cartesian coordinate system:

$$\frac{\partial^2 u(x,y)}{\partial x^2} + \frac{\partial^2 u(x,y)}{\partial y^2} = 0$$  \hspace{1cm} (1)
where \((x, y) \in \Omega\). We assume that \(\Omega\) is homogeneous open set in general. Additionally electrical potential \((u)\) or its normal derivative \((q)\) is known for all boundary points. The differential problem defined in described manner may be regarded as forward problem for the electric field. Starting point for our research is typical for BEM integral equation, where the boundary curve is divided into \(N\) elements:

\[
c(\vec{r}_i)u(\vec{r}_i) + \sum_{j=1}^{N} \int_{\Gamma_j} u(\vec{r}_i)q(\vec{r}_j)d\gamma_j = \int_{\Gamma_i} \phi(\vec{r}_i)d\gamma_i
\]

(2)

Only three values of function \(c\) are possible for constant boundary elements. If a given point belongs to boundary of domain \(\Omega\), then the value equals 0.5. The value of function \(c\) equals 1, when a given point lies inside of \(\Omega\) and equals 0 in other cases. The Green’s function \(u^*\) may be obtained by solving the fundamental equation and is given by the following formula:

\[
u^*(\vec{r}, \vec{r}_i) = \frac{1}{2\pi} \ln \left( \frac{|\vec{r} - \vec{r}_i|}{|\vec{r} - \vec{r}_j|} \right)
\]

(3)

where \(A\) is a positive constant. Function \(q^*\) represents derivative of the Green’s function in normal direction appointed by unit vector \(\vec{n}(\vec{r})\). After elementary calculations we get:

\[
q^*(\vec{r}, \vec{r}_i) = -\frac{1}{2\pi} \frac{\vec{n}(\vec{r}) \cdot (\vec{r} - \vec{r}_i)}{|\vec{r} - \vec{r}_i|^2}
\]

(4)

The vector formula for \(j\)-th constant boundary element is given by:

\[
q^*(\vec{r}, \vec{r}_i) = -\frac{1}{2\pi} \frac{\vec{n}(\vec{r}) \cdot (\vec{r} - \vec{r}_i)}{|\vec{r} - \vec{r}_i|^2}
\]

(5)

In order to extend our theoretical formalism into cases of open domain problems with infinite boundary curves, we have to introduce some modifications. Those modifications are attributable to infinite elements, which must be added to the set containing all finite boundary elements. If both quantities \(u\) and \(q\) are constant along given BIE, then some integrals appearing in fundamental equation and is given by the following formula:

\[
\int_{\Gamma_1} \phi(\vec{r}_i)d\gamma_i = \sum_{j=1}^{N} \int_{\Gamma_j} \phi(\vec{r}_i)d\gamma_j
\]

(6)

where \(Q(\xi)\) are so-called interpolation functions:

\[
Q_\xi(\xi) = \sum_{a=f,m,l} \sum_{s_0=\pm1} N_a(\xi) \eta_{s_0}
\]

(7)

One should notice that the function \(Q_\xi(\xi)\) takes value 1 for his own node and it takes value 0 for all other nodes. First of all, in order to expand the theory on IBEs we should modify equations (7) by appropriate exponential factors. Every factor not have to modify the value of the interpolation function for her “own” node and it have to tends to zero for asymptotical values of the parameter \(\xi\). For subsequent considerations let us assume \(\xi_\varepsilon(-1, \infty)\). These conditions are satisfy when:

\[
M_\xi(\xi) = 0.5(\xi - 1)e^{-\frac{(1 + \xi)}{\varepsilon}}
\]

\[
M_\varepsilon(\xi) = 1 - (1 - \xi)e^{-\frac{(1 - \xi)}{\varepsilon}}
\]

(8)

Positive and dimensionless parameter \(\varepsilon\) is responsible for speed of decay of electrical potential and its normal derivative along IBEs. Since for constant elements we have \(u_f = u_m = u_l\) and \(q_f = q_l = q_m\), so after changing \(N_a(\xi)\) to \(M_a(\xi)\) formulas (6) becomes:

\[
u(\xi) = u_m S_\varepsilon(\xi)
\]

\[
q(\xi) = q_m S_\varepsilon(\xi)
\]

(9)

where:

\[
S_\varepsilon(\xi) = \sum_{a=f,m,l} M_a(\xi)
\]

(10)

is the sum of the interpolation functions providing exponential decay. Generally, the sum (10) can be expressed as follows:

\[
S_\varepsilon(\xi) = \left[ 2(\xi_\varepsilon)^2 \sinh^2 \left( \frac{1}{\varepsilon} \right) + \xi_\varepsilon \sinh \left( \frac{1}{\varepsilon} \right) + 1 \right] e^{-\frac{1}{\varepsilon}(11)}
\]

In above expression we assume that \(s_0 = +1\) when the set of admissible values for parameter representing the IBE takes form \(\Gamma = (-1, +\infty)\), and \(s_0 = -1\) when \(\Gamma = (-\infty, +1)\). Let us notice from formula (11) that in case of discussed elements the sum of interpolation functions is not equal one. The graph of the function \(S_\varepsilon(\xi)\) for several selected parameters \(\varepsilon\) is shown in Figure 2. This figure shows unequivocal oscillations for small values of \(\varepsilon\). Therefore, during numerical calculations the condition \(\xi_\varepsilon \geq 2\) should be satisfied. One can notice that the amplitude of unphysical oscillations decreases when \(\varepsilon\) is larger and larger.

On this stage of consideration it is possible to write down the new version of integral equation (2). Collecting previous modifications and making discretization we obtain:

\[
c(\vec{r}_i)u(\vec{r}_i) + \sum_{j=1}^{N} \int_{\Gamma_j} q^*(\vec{r}_i, \vec{r}_j)d\gamma_j
\]

+ \(u_1\int_{\xi_\varepsilon(-1, \infty)} S_\varepsilon(\xi)q^*(\vec{r}_i, \vec{r}_i)d\gamma_1 + u_N\int_{\xi_\varepsilon(-\infty, +1)} S_\varepsilon(\xi)q^*(\vec{r}_i, \vec{r}_i)d\gamma_N = \sum_{j=1}^{N} \int_{\Gamma_j} u^*(\vec{r}_i, \vec{r}_j)d\gamma_j + q_1\int_{\xi_\varepsilon(-1, \infty)} S_\varepsilon(\xi)q^*(\vec{r}_i, \vec{r}_i)d\gamma_1 + q_N\int_{\xi_\varepsilon(-\infty, +1)} S_\varepsilon(\xi)q^*(\vec{r}_i, \vec{r}_i)d\gamma_N
\]

(12)

It is natural that the first and the last element of the boundary are infinite. The other \(N - 2\) boundary elements have finite length. Equation (12) represents model in which area \(\Omega\) is described by one open curve only. However, generalisation of the formula (12) is easy and may be done by adding appropriate terms.

![Fig. 2. The sum of the interpolation functions for IBEs defined by equation (11) (where \(s_0 = +1\)) versus the curve parameter \(\xi\) for selected coefficients \(\varepsilon\)](image-url)
3. Numerical results

The definite integrals present in equation (12) have been calculated by means of Gauss-Legendre quadrature, Gauss-Laguerre quadrature and Gauss quadrature with logarithmic weight function. Some diagonal integrals have been calculated analytically. The appropriate method has been chosen according to existence of singularity and type of domain of integration (finiteness or infiniteness). In numerical experiments we set $A = 10^{15} \text{m}$ and $\xi_e = 50$. Let us introduce open domain, where unit of length is the meter. The sketch of geometrical structure is given in Figure 4. One can notice that our geometry contains four IBEs. Whole boundary of the domain $\Omega$ is divided into forty elements ($N = 40$). Exact boundary conditions are expressed as:

$$\left\{\begin{align*}
u(x,y) &= -5V \quad \text{for} \quad x = 0 \\
&= +5V \quad \text{for} \quad x = m
\end{align*}\right. \quad (13)$$

In case of Laplace’s equation (1), Dirichlet problem (13) has analytical solution. Normal derivative $q$ is trivial to obtain and it takes following form on the boundary of the domain $\Omega$:

$$q(x,y) = \left\{\begin{align*}
&= -10V/m^2 \quad \text{for} \quad x = 0 \\
&= +10V/m^2 \quad \text{for} \quad x = m
\end{align*}\right. \quad (14)$$

However, one should remember that electrical potential tends exponentially to zero along IBEs. Therefore, parameter $\xi_e$ should be large enough in order to create good approximation of conditions (13). Fig. 5a shows electrical potential for all nodes.

The solution of considered problem is given in Fig. 5b. From Fig. 6 we can see that percent errors are less than 0.7%. Good agreement of the numerical result with exact solution (14) is demonstrated. As we expected, reflectional symmetry exists in Fig. 6. Additionally, percent errors are the same for each straight line $(x = 0; x = m)$. The numerical results show that our theoretical formalism gives appropriate approach to forward problem described by open boundary curves.

Fig. 3. Block diagram of the optimisation algorithm

Fig. 4. The boundary problem: unit normal vectors are always outward pointing, positions of infinite elements are indicated

Fig. 5. The left graph a) presents the boundary conditions imposed on Laplace’s equation, whereas the right graph b) shows the numerical solution

Fig. 6. The percent error obtained for the numerical solution of the Dirichlet problem

Fig. 7. Boundary problem: normal vectors and nodes on boundary elements are indicated

The definition of the boundary problem is shown in Figure 7. Normal vectors and nodes on boundary elements are indicated here. Figure 8 presents the image reconstruction of the copper-mine ceiling using BEM. The picture shows different object and the reconstructed image. The original object is noted by the blue line and the final figure is red. Figure 9 presents the image reconstruction in EIT obtained through the finite element method. The final contour represents the zero value of the level set function. The process of reconstruction is good, because the region borders are located nearly the object edges.
In this paper, there was presented the method to solve the inverse problem for the electric field so-called copper mine problem. The level set idea is the good tool to the topological changes of the interface and gives the good quality reconstruction of unknown areas with one or many objects. All gaps in copper-mine ceiling were properly localized. This problem was motivated by electrical impedance tomography. Gaps were localised above ceiling in a copper mine. The applications were depended on a specially built model. There were used the boundary element method or the finite element method with the level set method to solve this problem. The level set function techniques were shown to be useful in this system. The boundary (finite) element method with the level set method gave the successfully results to identify the unknown properties of the object.

4. Conclusion

References

[1] Brebbia C.A., Dominguez J.: Boundary Elements – An Introductory Course, WIT Press, UK, 1992
[2] Burger M.: Level-Set-Marching: Level set methods for inverse obstacle problems, Inverse Problem 20, 2004, 259–282.
[3] Ito K., Kunish K., Li Z.: The Level-Set Function Approach to an Inverse Interface Problem, 2001, Vol. 17, 11.
[4] Kythe P.K.: An introduction to Boundary Element Methods, CRC Press, USA, 1995.
[5] Li C., Xu C., Gui C., Fox M. D.: Level set evolution without re-initialization: A new variational formulation, IEEE Conference on Computer Vision and Pattern Recognition (CVPR), volume 1, 2005, 430–436.
[6] Osher S., Fedkiw R.: Level Set Methods and Dynamic Implicit Surfaces, Springer, New York 2003.
[7] Osher, S., Fedkiw, R.: Level Set Methods: An Overview and Some Recent Results. J. Comput. Phys. 169, 2001, 463–502.
[8] Osher S., Sethian J.A.: Fronts Propagating with Curvature Dependent Speed: Algorithms Based on Hamilton-Jacobi Formulations, Journal of Computational Physics 79, 1988.
[9] Pańczyk M.: Elementy nieskończone w metodzie elementów brzegowych, PhD thesis, Lublin University of Technology, 2009.
[10] Pańczyk M., Sikora J.: Geometry and physical quantity transformations in 2D boundary element method with infinite elements. Proceedings of Electrotechnical Institute 3, 2007, 233.
[11] Rymarzyk T.: Using electrical impedance tomography to monitoring flood banks, International Journal of Applied Electromagnetics and Mechanics 45, 2014, 489–494.
[12] Rymarzyk T.: Characterization of the shape of unknown objects by inverse numerical methods, Przegląd Elektrotechniczny, R. 88 NR 7b/2012, 138–140.
[13] Rymarzyk T., Sikora J., Waleda B.: Coupled Boundary Element Method and Level Set Function for Solving Inverse Problem in EIT, 7th World Congress on Industrial Process Tomography, WCPT7, 2-5 September 2013, Krakow, Poland
[14] Rymarzyk T., Adamkiewicz P., Duda K., Szumowski J., Sikora J.: New Electrical Tomographic Method to Determine Dampness in Historical Buildings, Achieve of Electrical Engineering 65, 2016, 273–283.
[15] Sethian J.A.: Level Set Methods and Fast Marching Methods. Cambridge University Press, 1999.
[16] Tai C., Chung E., Chan T.: Electrical impedance tomography using level set representation and total variational regularization. Journal of Computational Physics, vol. 205, no. 1, 2005, 357–372.
[17] Xia K., Zhang Z.: Three-dimensional finite/infinite elements analysis of fluid flow in porous media, Applied Mathematical Modelling, 30, 6, 2005.

Ph.D. Eng. Tomasz Rymarzyk
e-mail: tomasz.rymarzyk@netrix.com.pl
Director in Research and Development Center Netrix S.A. His research area focuses on the application of non-invasive imaging techniques, electrical tomography, image reconstruction, numerical modelling, image processing and analysis, process tomography, software engineering, knowledge engineering, artificial intelligence and computer measurement systems.

M.A. Paweł Tchorzewski
e-mail: pawel.tchorzewski@netrix.com.pl
Researcher in R&D Department – Netrix S.A. A graduate of the Maria Curie-Skłodowska University Lublin on the physics (specializing in theoretical physics). Currently, the work carries out tasks profile research and development in the field of numerical methods for solving partial differential equations, electrical tomography in image reconstruction, forward and inverse problem. He is a Ph.D. student of the Institute of Electrical Engineering in Warsaw.

Prof. Jan Sikora

e-mail: sak59@wp.pl

Prof. Jan Sikora (Ph.D. D.Sc. Eng.) graduated from Warsaw University of Technology Faculty of Electrical Engineering. During 40 years of professional work he has proceeded all grades, including the position of full professor at his alma mater. Since 1998 he has worked for the Institute of Electrical Engineering in Warsaw. In 2008 he has joined Electrical Engineering and Computer Science Faculty In Lublin University of Technology. During 2001-2004 he has worked as a Senior Research Fellow at University College London in the prof. S. Arridge’s Group of Optical Tomography. His research interests are focused on numerical methods for electromagnetic field.