2D Rutherford-Like Scattering in Ballistic Nanodevices

S. Toussaint, B. Brun-Barrière, S. Faniel, L. Desplanque, X. Wallart, V. Bayot, and B. Hackens

1 Université catholique de Louvain, Institute of Condensed Matter and Nanosciences (IMCN/NAPS), B-1348 Louvain-la-Neuve, Belgium
2 Université Lille, CNRS, Centrale Lille, ISEN, Univ. Valenciennes, UMR 8520 - IEMN, F-59000 Lille, France

Ballistic injection in a nanodevice is a complex process where electrons can either be transmitted or reflected, thereby introducing deviations from the otherwise quantized conductance. In this context, quantum rings (QRs) appear as model geometries: in a semiclassical view, most electrons bounce against the central QR antidot, which strongly reduces injection efficiency. Thanks to an analogy with Rutherford scattering, we show that a local partial depletion of the QR close to the edge of the antidot can counter-intuitively ease ballistic electron injection. On the contrary, local charge accumulation can focus the semi-classical trajectories on the hard-wall potential and strongly enhance reflection back to the lead. Scanning gate experiments on a ballistic QR, and simulations of the conductance of the same device are consistent, and agree to show that the effect is directly proportional to the ratio between the strength of the perturbation and the Fermi energy. Our observation surprisingly fits the simple Rutherford formalism in two-dimensions in the classical limit.

I. INTRODUCTION

Controlling collisions and scattering has always played an essential role in physics. Thanks to model experiments ranging from collisions of alpha particles with gold foils, conducted more than a century ago [1, 2], to high energy collisions between hadrons at the LHC [3], a wealth of intimate information were revealed about the nature of atoms and elementary particles as well as their interactions. In this framework, the most fundamental description of the interaction of a beam of particles and a scatterer is the famous Rutherford formula, describing the differential cross section dependence on the scattering angle, energy of incident beam, and potential shape of the scatterer [4]. Collisions are also ubiquitous in solid state physics, in particular when considering charge transport. Charge carriers indeed scatter on a large variety of "defects": lattice vacancies, phonons, potential of remote ionized impurities, etc. Due to this complexity, it is almost impossible to reach the same degree of control in charge transport scattering experiments as in the case of collisions involving beams of elementary charged particles propagating in vacuum.

However, in the ballistic regime of charge transport, the bulk carrier mean free path becomes larger than the device size, and transport properties can be tailored by tuning the device geometry [5]. This is of course achieved most favorably in nanodevices, which are probably the most adequate system to attempt to perform "ideal" scattering experiments with electrons in solids and their associated quasiparticles. Nevertheless, even in the ballistic regime, a full treatment of scattering in solid-state devices requires to take into account complex many-body interactions with the Fermi sea [6, 7].

The archetypal ballistic device is the so-called quantum point contact (QPC). Thanks to a metallic split gate deposited on top of a semiconductor heterostructure hosting a high mobility two-dimensional electron gas (2DEG), one can create a constriction whose width can be varied at will with gate voltage. The smooth resulting potential ensures adiabaticity, which leads to a quantized conductance of the QPC [8, 9]. This canonical realization of ballistic transport allowed to go one step further, in particular when combining transport measurements with a local electrostatic perturbation by a scanning probe. This method lead to explore deviations from this perfect picture of QPCs, such as the observation of branched electron flow in the leads or rich many-body physics [10-13]. In other studies, geometric scatterers with an asymmetric shape were designed to act as mirrors redirecting electrons towards a particular lead through specular reflection [14], leading to a rectifying behavior similar to diode bridges. Such devices could yield applications at high frequency, given the short electron transit time in the ballistic regime [13, 16]. In addition, the magnetic field is a particularly useful knob to focus electrons at desired locations through the so-called "magnetic focusing" effect [17, 18]. In a surprising way, up to our knowledge, there are much less examples where fine tuning of the electrostatic potential is used for similar lensing purposes [19].

Here, we study the geometry presented in Fig. 1 where specular electron reflection on the hard-wall facing the entrance of a quantum ring is either enhanced or reduced by tailoring the local electrostatic potential in the vicinity of the wall. The idea is that a Rutherford-like scattering effect - induced by an attractive/repulsive potential - should deflect electron trajectories and hence ease or un eas electron injection in the QR arms. Using simulations we indeed show that even small changes in the electrostatic potential at a specific location in the device have strong impacts on ballistic charge transmission, and hence on the device conductance. Experiments fully re-
yield a strong signature in the device conductance $G$ which may look counter-intuitive at first sight: simulations indeed predict that a repulsive perturbation should increase $G$ while an attractive potential should degrade it. Furthermore, one can wonder how sensitive is this peculiar focusing/defocusing behavior with respect to the amplitude, spatial extension and location of the introduced perturbation. The effects of all these parameters will be simulated in detail later in the paper where transmission through the device - converted in conductance - will be computed. In addition, it is tempting to test these predictions by measuring the conductance of a real-world device.

We thus carved out a ring-like structure from an InGaAs/InAlAs heterostructure hosting a 2DEG. The device geometry shown in Fig. 2a is lithographically very comparable to the one simulated above (the layer structure is similar to the one described in Ref. [21], except for the doped substrate). The 2DEG density and mobility can be tuned thanks to an applied electrostatic back-gate potential ($V_{BG}$). The following data were measured at the maximal accessible charge carrier density ($\sim 10^{16} \text{ m}^{-2}$) and mobility ($\sim 10 \text{ m}^2/\text{Vs}$) corresponding to $V_{BG} = 4 \text{ V}$. The Fermi energy is thus $E_F = 55 \text{ meV}$ and the Fermi wavelength is $\lambda_F = 25 \text{ nm}$. The 4-contacts conductance measurements were performed at a temperature $T = 40 \text{ mK}$ using a standard lock-in technique with a polarization that remained comparable to $k_B T / e$. It is important to note one difference with the simulation results: since the conductance is measured using an alternative current, it is averaged over two different current signs contrary to simulations where current flows only from one side to the other. The physical characteristics of the host heterostructure allowed the modeling of a fixed disorder potential represented in Fig. 2b that will be used in the forthcoming simulations.

Experimentally, a convenient way to generate the kind of perturbation potential used in the simulations presented above is by approaching an electrically biased nanoscale tip ($V_{tip}$) at a distance $d_{tip}$ above the patterned quantum ring (as illustrated in Fig. 3a). The tip can then be scanned along the transport direction, i.e. along the dashed line in Fig. 2b. In order to achieve a large effect, we brought the tip to a distance $d_{tip} = 60 \text{ nm}$ above the sample surface, and polarized the tip with large positive and negative voltages up to $|V_{tip}| = 14 \text{ V}$.

The presence of the polarized conductive AFM tip is numerically modeled using a Lorentzian-shape perturbation potential $\varphi_p(x, y)$ - illustrated in Fig. 1a - parametrized by the position of its center $(x_{tip}, y_{tip})$, height $\varphi_{p, max}$, and width $R_p$ which is half the potential FWHM. The superposition of $\varphi_p(x, y)$ on the modeled disordered potential $\varphi_d$, together with the hard-wall boundaries that mark the edges of the nanodevice, define the potential landscape used in the simulations.

In Fig. 3b, the conductance is computed as $\varphi_p$ moves along the axis joining the entrance and the exit con-
tacts (dashed line in Figs. 2a and 3a). Remarkably, when the tip position stands nearby the location of the hard-wall (vertical dashed lines), the conductance significantly deviates from that in the absence of perturbation ($\sim 11 \times \frac{2e^2}{h}$), e.g., when the tip stands at the center of the device. Beyond fluctuations originating from the presence of the random disorder, the effect is symmetric as positioning the tip near both T-junctions gives the same result. In other words, the Lorentzian potential has a similar effect on conductance when it modifies either the entry or the exit conditions. More importantly, this behavior is somewhat counter-intuitive: while a repulsive potential close to both T-junctions actually helps electrons crossing the overall structure (enhanced conductance), an attractive perturbation reduces their ability to pass through the device.

Looking further in the simulation results, we observe that reversing the sign of $\varphi_p$ essentially reverses the change in conductance. Surprisingly, the back-scattering to the leads, due to current focusing on the hard-wall potential of the antidot (described in Figs. 1a and c), is similar in amplitude to the enhanced transmission due to defocusing (Figs. 1a and b). On the other hand, we observe that the symmetry naturally breaks when the tip locates above the leads. In that case, while depleting the lead strongly reduces the conductance, accumulating electrons has naturally a much weaker effect. Finally, when moving the perturbation from the T-junction area towards the center of the device, the effect on $G$ naturally vanishes over a distance corresponding roughly to $R_p$ (Fig. 3b).

Beside moving the tip along the device axis, one can also wonder how sensitive $G$-variations are to the perturbation position in the $(x, y)$ plane. This aspect is examined in the $G$ maps plotted in Figs. 3c and d obtained for locally raised or lowered moving potentials, respectively. The main contrast is observed over the T-junctions as well as over the device leads for a depleting potential. In both $x$ and $y$ directions, this contrast fades away over distances comparable to $R_p$. When positioning the perturbation potential over the device arms and their vicinities, the $G$ map is decorated with short characteristic length scale fluctuations which are similar to those reported in previous works [22, 24]. This weaker amplitude contrast was attributed to the perturbation of resonant states in the local density of states (LDOS) by the moving potential [22, 23], as well as to the electrostatic Aharonov-Bohm effect [24]. Note that here the mapping conditions are not suitable for imaging the LDOS because the moving potential is in the strong perturbation and not in the linear regime discussed in Ref. [22]. In this framework we are not using the scanning gate with microscopy purposes in mind.

It is now time to compare these predictions with experimental results on the sample described above. Figure 4 summarizes the data in a way to ease the compari-

---

**FIG. 2.** a) Scanning electron micrograph of the fabricated sample in an InGaAs/InAlAs heterostructure. b) Computed real-space disorder potential ($\varphi_d$) at the level of the 2DEG that will be used in the forthcoming simulations. The disorder standard deviation ($S_d$) is 4.78 meV. Calculated taking into account a distribution of Si ionized dopants located 20 nm above the 2DEG (i.e., thickness of the InAlAs spacer). The inset to b) shows a map of the autocorrelation as the correlation lag becomes a vector in the $x - y$ plane.

**FIG. 3.** a) Illustration of the potential used for the simulations. It is composed of a disorder potential $\varphi_d$ and a Lorentzian-shaped perturbation potential $\varphi_p$ caused by a polarized conductive AFM tip located above the 2DEG. b) Simulated conductance profiles as $\varphi_p$ is swept along the dashed line in a). Simulation parameters are as follows (same conditions as in Fig. 1): the red profile corresponds to $\varphi_p^{\text{max}} = 0.9 E_F$ (depletion) and $R_p = 150$ nm; the blue profile corresponds to a reversed perturbation potential (accumulation; $\varphi_p^{\text{max}} = -0.9 E_F$). These profiles are extracted from the conductance mapping obtained when $\varphi_p$ is swept in the $(x, y)$ plane. They are presented in c) ($\varphi_p > 0$) and d) ($\varphi_p < 0$). The vertical dashed lines correspond to the locations of the hard-walls along the scanned line in a).
son with the simulations. We first scanned the biased tip along a line linking the device leads for two opposite sign polarities. Figure 4a shows, like simulations in Fig. 3b, that a depleting (red) potential, near the border of the inner quantum dot, eases electron injection, while an accumulating potential (blue) located at the same place tends to reduce electron transmission through the device.

FIG. 4. a) Experimental conductance profile as a voltage biased tip is scanned along the dashed line presented in Fig. 2a. The tip is scanned at a distance of 60 nm from the sample surface with $V_{\text{tip}}=-14$ V (red curve) or $+8$ V (blue curve). A qualitative scenario is also illustrated for the peculiar electron forth-scattering (red) and back-scattering (blue). b) Conductance map as the polarised tip ($V_{\text{tip}}=-14$ V - depleting) is scanned in a plane at the same constant distance from the sample surface. c) Same map as the one presented in b) but with $V_{\text{tip}}=+8$ V (accumulation). Note that Fig. S4 presents the same data as c), with an enhanced contrast.

For a strongly depleting potential ($V_{\text{tip}}=-14$ V - red curve in Fig. 4a), corresponding roughly to $\varphi_p^{\text{max}} \approx 0.4 \cdot E_F$ (see Fig. S2), $G$ exhibits local maxima when the tip is located above the limit of the etched area in front of the entrance and exit leads (dashed lines in Fig. 4a). As expected, the conductance is reduced all the more as the tip increases the 2DEG density over the leads. But, counter-intuitively, a strongly accumulating potential ($V_{\text{tip}}=8$ V) brings $G$ to a minimum. Moreover, the effect is essentially symmetric when the tip moves from one T-branch to the other. The qualitative match with the curves presented in Fig. 3b (obtained for $\varphi_p^{\text{max}} = \pm 0.9 \cdot E_F$) is striking, and the experimental conductance maps presented in Figs. 4c and d compare well with the simulations presented in Figs. 3c and d. We observe a remarkable coincidence of simulated and experimental positions and lateral extensions of the peaks and dips located around the hard-walls in the T-junctions.

Resonant features along the ring circumference are also observed in all cases, but the smallest ones that are visible in simulated $G$ maps in Figs. 3c and d, in particular those with concentric shape observed mostly outside the device area, are absent in the experimental data. This is most probably related to thermal averaging, which is not taken into account in the simulations.

At this stage, we can conclude that the experiments confirm, at least qualitatively, that a focusing/defocusing can be induced by a Lorentzian perturbation combined to a hard-wall potential in a ballistic device. While defocusing (Fig. 4a red) is clearly reminiscent of the Rutherford scattering - here in 2D -, focusing on the hard-wall induces a peculiar back-scattering mechanism as the lensing is combined with the specular reflection illustrated in Fig. 4a (blue).

At first sight, the weaker absolute value of the voltage applied on the tip in accumulation (blue in Fig. 4a) could explain why the effect on the conductance is weaker than in depletion (red in Fig. 4a). However, we need to dig deeper in the simulations to test the quantitative correspondence between experiments and predictions.

Figure 5 shows the evolution of the conductance when $\varphi_p$ travels along the axis of the quantum ring, and when either $\varphi_p^{\text{max}}$ or $R_p$ is varied, the other parameters remaining constant (a similar map with a varying disorder amplitude $S_d$ is shown in supplementary materials - Fig. S1). We obviously focus our attention on the two regions near the edge of the inner QR, i.e. $x \sim \pm 800$ nm (dashed lines in Figs. 5a and b). We first observe no obvious threshold when $|\varphi_p^{\text{max}}|$ increases (Fig. 5a). $G$ undergoes a smooth evolution at least up to $2 \cdot E_F$. However, on the depletion side ($\varphi_p > 0$), the positions of the local $G$ maxima are gradually shifting towards the center of the device as $\varphi_p^{\text{max}}$ is made more positive. This reflects the fact that roughly identical potential perturbation conditions are found in the T-junctions both for a weakly perturbing potential ($\varphi_p^{\text{max}} < E_F$) centered close to the hard-wall, and a strongly perturbing potential ($\varphi_p^{\text{max}} > E_F$) centered further away from the hard-wall. On the accumulation side ($\varphi_p < 0$), the position of the dips’ centers remains essentially unaffected: charge accumulation in the T-junctions does not modify its geometry.

Varying $R_p$ has an interesting effect on the conductance peaks and dips. Beyond a few tens of nm, and up to 200 nm where the arms themselves start to be narrowed, varying $R_p$ has essentially no effect on the amplitude of conductance extrema, either for negative (Fig. 5b) or positive (Fig. 5c) perturbation potentials. Indeed, the amplitude of conductance peaks and dips saturates for $R_p \geq \lambda_F = 25$ nm, i.e. in the classical regime (Fig. S3).

On the other hand, the evolution of the width of conductance extrema (Figs. 5b and c) is smoother and gives us the possibility to determine the value of $R_p^{\text{exp}}$ that characterizes our experimental configuration. Based on the FWHM of the strongest (red) conductance peaks in Fig. 4a, we obtain that $R_p^{\text{exp}} \sim 135$ nm. This value is
well in the range investigated in the simulations and indeed consistent with data discussed in the supplementary informations.

Finally, our results show that increasing the disorder dampens the effect but no qualitative change is observed even when multiplying the initial disorder (Fig. 2b) by a factor of four (see supplementary materials, Fig. S1). This robustness is a clear signature that distinguishes the present effect from universal conductance fluctuations [25], even sensitive to a change of potential amplitude on a single tight-binding site.

To go beyond the good qualitative correspondence between Figs. 3 and 4, we now need to question the experimental data more quantitatively. This is the purpose of Fig. 6 that addresses the effect of the density, or Fermi energy, and finally provides a quantitative comparison between experiments and simulations.

The variation of the conductance with $E_F$, while keeping the absolute value of the ratio $\varphi_p^{max}/E_F = 0.9$, is presented in Figs. 6a and b. Since $G$ increases with $E_F$, it makes sense to examine the relative change of conductance $\Delta G/G_0 = (G - G_0)/G_0$, where $G_0$ is the conductance of the device when the tip is above the device center ($x = 0$). It is immediately apparent that $\Delta G/G_0$ is insensitive to $E_F$. In other words, the efficiencies of both focusing and defocusing are not sensitive to $E_F$ alone, but, as Fig. 6c reveals clearly, to the ratio $\varphi_p^{max}/E_F$. More precisely, Fig. 6c shows a linear dependence of $\Delta G/G_0$ as a function of $\varphi_p^{max}/E_F$, up to $\varphi_p^{max}/E_F \approx 1$ in the depletion regime. For $\varphi_p^{max}/E_F \geq 1$, the symmetry of defocusing with respect to entry and exit breaks down and defocusing becomes less efficient as the arms themselves start to shrink. No such deviation from either linearity or symmetry is observed in the case of depletion (blue lines in Fig. 6c). The counter-intuitive entry/exit symmetry persists in all the range investigated and the linearity with respect to $\varphi_p^{max}/E_F$, is preserved.

How can we understand this linear dependence, at least in the depletion regime? The defocusing of ballistic electrons facing a Lorentzian-shape repulsive potential is clearly reminiscent of the Rutherford scattering. The original Rutherford formalism provides an expression for the differential cross-section in three dimensions (3D) for a scattering potential $\frac{E}{r}$ - where $C$ is the amplitude and $r$ the distance from the scattering center - as a function of the energy of incident particles $E$ and of the scattering angle $\theta$. Since the arms of the quantum ring capture electrons in a finite angle range from the leads, one can consider the differential cross-section at a given angle as related to the conductance of our ballistic device. Coincidentally, in the 3D case, the Rutherford formula is independent of wether you treat particles classically or quantumly [4]. In 2D, this elegant result is no longer valid in general. In the 2D quantum regime, one has to find an analytical expression of the differential cross-section $\frac{d\lambda}{d\theta}$ by solving the 2D version of the Lippmann-Schwinger equation [26] with a Lorentzian-shaped potential distribution, which is far beyond the scope of the present work. In the 2D classical regime however, an equivalent formula was derived [27]. For the same $\frac{E}{r}$ potential:

$$\frac{d\lambda}{d\theta} = \frac{|C|}{4E\sin^2(\theta/2)}$$  \hspace{1cm} (1)

One readily finds from Equ. (1) that, for a given angle $\theta$, the scattering amplitude is fully determined by the ratio between the amplitude $C$ of the perturbative potential and the energy of the particles. In the case of our device, this ratio would correspond to $\frac{\varphi_p^{max}}{E_F}$. The linear response of $\Delta G/G_0$ with respect to changes in $\frac{\varphi_p^{max}}{E_F}$ revealed in Fig. 6c is thus reminiscent of the 2D Rutherford scattering in the classical regime (note that Equ. 1
The phenomena is similar to the 2D Rutherford scattering assuming classical electron dynamics. The applicability of this relatively simple classical formalism in the case of a 2DEG-based device was not expected. Indeed, the scattering amplitude for the interaction between charged particles and a sharp electrostatic potential should in principle be governed by complex interactions related to the presence of the many-particle background of the Fermi sea [7]. Other unexpected results of this work resides in two symmetries. The first symmetry concerns the effect of the scattering potential with respect to incoming and outgoing electrons in the T-junctions (i.e. the left-right symmetry in the simulated results). While it is quite straightforward to understand the focusing or defocusing effect of a locally accumulating or depleting potential for incoming electrons, one could not anticipate that a similar effect would be visible for outgoing electrons (i.e. not impinging the hard wall close to normal incidence), in particular in the case of an accumulating potential. A second unexpected symmetry was revealed between the amplitude of the Rutherford defocusing effect (when a depleting potential is applied) and reflective focusing, as experienced by electrons scattered by an accumulating potential in front of a hard wall. All these puzzling fundamental questions will require additional scrutiny and will probably foster further experimental and theoretical work.

In a broader context, our observations help in the understanding of charge carrier injection in ballistic devices, as it shows that fine tuning of the potential in the vicinity of the entrance and exit leads can have huge effects on transmission through the whole device. In turn, this work provides useful tools in the perspective of building ‘electron optics’ devices, where a local modulation of the electrostatic potential inside a device redirects the electron flow in a similar way as a optical lens curves light rays [29]. In this framework, scanning gate microscopy can play an important role, as pointed out in various theoretical proposals where scattering is investigated by tuning the electrostatic potential at the local scale using a charged metallic tip [7, 30, 31]. Although the description of scattering in two spatial dimensions was considered as a curiosity up to the early eighties [27], nowadays high mobility two-dimensional charge systems gives this fundamental question a complete relevance and the possibility of testing this description, even with relativistic Dirac particles [32–35], also opens new directions of research.

ACKNOWLEDGEMENTS

This work was funded by the Fonds de la Recherche Scientifique FRS-FNRS (Grants No. J.0067.13, T.0172.13, 326 U.025.14, J.0009.16, and 2450312F) and by the Communauté Française de Belgique (ARC Grant No. 11/16-037, Stresstronics Project and ARC Grant No. 16/21-077, NATURIST Project). S.T. is funded by a Fonds pour la Formation à la Recherche dans l’Industrie et dans l’Agriculture FRIA fellowship. B.H. is FRS-FNRS research associate. Computational resources have been provided by the supercomputing facilities of the Université catholique de Louvain (CISM/UCL) and the Consortium des Equipements de Calcul Intensif en Fédération Wallonie Bruxelles (CECI) funded by the...
Fonds de la Recherche Scientifique de Belgique (F.R.S.-FNRS). S. T. addresses a special thank to D. François for his valuable help concerning parallel computing.

[1] E. Rutherford, The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science 21, 669 (1911).
[2] H. Geiger and E. Marsden, in Proc. R. Soc. Lond. A, Vol. 82 (The Royal Society, 1909) pp. 495–500.
[3] G. Aad, E. Abat, J. Abdallah, A. Abdelalim, A. Abdeselam, O. Abdinov, B. Abi, M. Abolins, H. Abramowicz, E. Acerti, and et al., Journal of instrumentation 3, S08003 (2008).
[4] H. Friedrich, Scattering theory, Vol. 872 (Springer, 2013).
[5] S. Datta, Electronic transport in mesoscopic systems (Cambridge university press, 1997).
[6] D. S. Saraga, B. L. Altshuler, D. Loss, and R. M. Westervelt, Phys. Rev. Lett. 92, 246803 (2004).
[7] D. S. Saraga, B. L. Altshuler, D. Loss, and R. M. Westervelt, Phys. Rev. B 71, 045338 (2005).
[8] B. Van Wees, H. Van Houten, C. Beenakker, J. G. Williamson, L. Kouwenhoven, D. Van der Marel, and C. Foxon, Physical Review Letters 60, 848 (1988).
[9] D. A. Wharam, T. J. Thornton, R. Newbury, M. Pepper, H. Ahmed, J. E. F. Frost, D. G. Hasko, D. C. Peacock, D. A. Ritchie, and G. A. C. Jones, Journal of Physics C: Solid State Physics 21, L209 (1988).
[10] K. J. Thomas, J. T. Nicholls, M. Y. Simmons, M. Pepper, D. R. Mace, and D. A. Ritchie, Phys. Rev. Lett. 77, 135 (1996).
[11] S. Cronenwett, H. Lynch, D. Goldhaber-Gordon, L. Kouwenhoven, C. Marcus, K. Hirose, N. Wingreen, and V. Umansky, Physical review letters 88, 226805 (2002).
[12] B. Brun, F. Martins, S. Faniel, B. Hackens, G. Bachelet, A. Cavanna, C. Ulysse, A. Ouerghi, U. Gennser, D. Mailly, S. Huant, B. Vincent, S. Marc, and H. Sellier, Nature communications 5, 4290 (2014).
[13] B. Brun, F. Martins, S. Faniel, B. Hackens, A. Cavanna, C. Ulysse, A. Ouerghi, U. Gennser, D. Mailly, P. Simon, S. Huant, B. Vincent, S. Marc, and H. Sellier, Physical review letters 116, 136801 (2016).
[14] A. Song, A. Lorke, A. Kriele, J. Kotthaus, W. Wegscheider, and M. Bichler, Physical Review Letters 80, 3831 (1998).
[15] A. Song, P. Omling, L. Samuelson, W. Seifert, I. Shorubalko, and H. Zirath, Applied Physics Letters 79, 1357 (2001).
[16] L. Bednarz, B. Hackens, G. Farhi, V. Bayot, and I. Huyven, Solid State Communications 134, 217 (2005).
[17] K. E. Aidala, R. E. Parrott, T. Kramer, E. Heller, R. Westervelt, M. P. Hanson, and A. C. Gossard, Nature Physics 3, 464 (2007).
[18] S. Bhandari, G.-H. Lee, A. Klages, K. Watanabe, T. Taniguchi, E. Heller, P. Kim, and R. M. Westervelt, Nano letters 16, 1690 (2016).
[19] C. Pöllt, A. Kozikov, K. Ensslin, T. Ihn, R. A. Jalabert, C. Reichl, W. Wegscheider, and D. Weinmann, Physical Review B 94, 195304 (2016).
[20] C. W. Groth, M. Wimmer, A. R. Akhmerov, and X. Waintal, New Journal of Physics 16, 063065 (2014).
[21] P. Liu, F. Martins, B. Hackens, L. Desplanque, X. Wallart, M. Pala, S. Huant, V. Bayot, and H. Sellier, Physical Review B 91, 075313 (2015).
[22] F. Martins, B. Hackens, M. Pala, T. Ouisse, H. Sellier, X. Wallart, S. Bollaert, A. Cappy, J. Chevrier, V. Bayot, and S. Huant, Physical review letters 99, 136807 (2007).
[23] R. Crook, C. Smith, A. Graham, I. Farrer, H. Beere, and D. Ritchie, Physical review letters 91, 246803 (2003).
[24] B. Hackens, F. Martins, T. Ouisse, H. Sellier, S. Bollaert, X. Wallart, A. Cappy, J. Chevrier, V. Bayot, and S. Huant, Nature Physics 2, 826 (2006).
[25] P. A. Lee and A. D. Stone, Physical review letters 55, 1622 (1985).
[26] B. A. Lippmann and J. Schwinger, Physical Review 79, 469 (1950).
[27] G. Barton, American Journal of Physics 51, 420 (1983).
[28] H. Hackens, F. Delfosse, S. Faniel, C. Gustin, H. Boutry, X. Wallart, S. Bollaert, A. Cappy, and V. Bayot, Physical Review B 66, 241305 (2002).
[29] P. Boggild, J. M. Caridad, C. Stampfer, G. Calogero, N. R. Papior, and M. Brandbyge, Nature communications 8, 15783 (2017).
[30] M. Braun, L. Chirolli, and G. Burkard, Phys. Rev. B 77, 115433 (2008).
[31] J. Cserti, A. Pályi, and C. Péterfalvi, Phys. Rev. Lett. 99, 246801 (2007).
[32] J.-S. Wu and M. M. Fogler, Physical Review B 90, 235402 (2014).
[33] S. Russo, J. B. Oostinga, D. Wehenkel, H. B. Heersche, S. S. Sobhani, L. M. Vandersypen, and A. F. Morpurgo, Physical Review B 77, 085413 (2008).
[34] D. Cabosart, S. Faniel, F. Martins, B. Brun, A. Felten, V. Bayot, and B. Hackens, Physical Review B 90, 205433 (2014).
[35] D. Cabosart, A. Felten, N. Reckinger, A. Iordanescu, S. Toussaint, S. Faniel, and B. Hackens, Nano letters 17, 1344 (2017).