ON THE CANONICAL FORMULA OF C LÉVI-STRAUSS, II

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§1 INTRODUCTION AND ORGANIZATION

We venture a leap: we grant *ab initio* that there is ‘something there’ to be translated . . .

George Steiner, quoted in [3] (p 19)

1.1 This is a sequel to, and elaboration of, earlier work [21, 22] on a possible formal model for the canonical formula

\[ CF : F_x(a) : F_y(b) \simeq F_x(b) : F_{a^{-1}}(y) \]

of C Lévi-Strauss, which he proposed [11, 14, 15, 16], *cf.* [Appendix] as a tool for the structural analysis of mythological systems; see [12] (p 562) for a discussion of the hair-raising example

marriage\textsubscript{solidarity} : rape\textsubscript{hostility} \simeq \text{marriage}\textsubscript{hostility} : dissociation\textsubscript{rape}.

The model proposed here is based on the study of small finite groups, which have proved useful in the classification of kinship systems, crystallography, and other fields [8, 19, 25, 27]; a short account of some of the mathematics involved is postponed till §3. For clarity, a sketch of the model, with technical details backgrounded, is displayed immediately below. However, because translation across the conceptual and cognitive gulfs separating anthropology and mathematics raises significant questions, a short discussion (§2) of models and metaphors precedes the (not actually very complicated) verification of the details of the model, in §4.

1.2 A MODEL

The first formulation of the real situation would be to state that the speaking subject perceives neither idea \textit{a} nor form A, but only the relation a/A . . . What he perceives is the relation between the two relations a/AHZ and abc/A, or b/ARS and blr/B, etc. This is what we term the LAST QUATERNION . . .

F de Saussure, *Writings.* . . . [26] (p 22), [16]

(Date: Imbolc 2020.)
**Proposition:** To elements \( \{x, a, y, b\} \), e.g. \( \{1, i, j, k\} \) of the group \( Q \) of Lipschitz unit quaternions [3.2.1], the function

\[
x, a \mapsto \Phi_x(a) = 2^{-1/2}(x - a) \in 2 \cdot O
\]

assigns elements of the binary octahedral group [3.4.2] such that the anti-automorphism \( \lambda = \ast I \sigma^2 \) [3.3iii] of that group transforms the (noncommutative [3.2.2]) ratio \( \{\Phi_x(a) : \Phi_y(b)\} \) into

\[
\{\Phi_{\lambda(x)}(\lambda(a)) : \Phi_{\lambda(y)}(\lambda(b))\} = \{\Phi_x(b) : \Phi_{a^{-1}}(y)\}.
\]

Terms such as \( x, a, y, b \) or \( 1, i, j, k \) will be referred to here as 'values', while functions such as \( F \) and \( \Phi \in 2 \cdot O \) of ordered pairs of values will be called 'valences'. [Citations such as [1.2] refer to sections of this paper.]

§2 Wider Questions

It takes a while to learn that things like tenses and articles

...are not 'understood' in Burmese [as] something not

uttered but implied; they just aren't there...

AL Becker, *Beyond Translation* [3] (p 8)

2.1 For the philologist Becker it is fundamental that languages (and cultures)

have 'exuberances and deficiencies' that can make translation problematic;

in the present case these issues are acute.

It seems generally agreed [21] that the CF is underdetermined. One of the

first things a mathematician notices (cf. the original statement, included be-

low as an appendix) is the absence of quantifiers, which usually convey information about the circumstances under which a proposition holds; moreover, the reversal of function and term [condition 2] is mysterious. One issue - the

assumption that the element \( a \) has a natural or canonical dual \( a^{-1} \) - can perhaps be resolved by reading the CF as asserting, in the context \( \{x, a, y, b\} \), the

existence of \( a^{-1} \); for example, as in [12]. On the other hand, an exuberance of the paraphrase above is a precise specification of the binary octahedral group as a repository for the values of our analog \( \Phi \) of Lévi-Strauss’s \( F \).

2.2 I make no claim about the uniqueness of the model proposed here: I

hope there are better ones. Perhaps this is the place to say that my concern

with the CF is not its validity, or 'truth-value'; it is rather whether or not it can be usefully be interpreted as a formal mathematical assertion. Its interest as an empirical hypothesis, like Bohr’s model for the atom, seems well-established. Po moemu the key question is what ‘interpretation’, in this

context, could even mean.

2.3 An answer may lie in the ancient opposition, older than Zeno, between

continuous and discrete. An anthropologist, like William James’ infant [17]
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(Ch 13) is ‘assailed by eyes, ears, nose, skin and entrails at once, feels it all as one great blooming, buzzing confusion’; but ‘nowadays, fundamental psychological changes occur [to the mathematician]. Instead of sets, clouds of discrete elements, we envisage some sorts of vague spaces . . . mapped one to another. If you want a discrete set, then you pass to the set of connected components of spaces defined only up to homotopy’ [20] (p 1274), [17].

I believe these remarks of Manin point to an answer to this question, in terms of cognitive condensation of chaotic clouds of experience into discrete, classifiable conceptual entities, cf. [4].

§3 A SHORT MATHEMATICAL GLOSSARY

This section summarizes more than enough background from the theory of groups for the purposes of this paper:

3.1.1 The objects of the category of groups (for example the integers \( \mathbb{Z} = \{ \ldots, -2, -1, 0, 1, 2, \ldots \} \)) consist of sets \( G \) with an associated multiplication operation

\[ \mu_G : G \times G \ni (g, h) \mapsto g \cdot h \in G \]

and an identity element \( 1 = 1_G \in G \), subject to familiar rules of associativity which I will omit. A homomorphism (or map, or morphism) \( \phi : G \to G' \) between groups respects multiplication (i.e. \( \phi(g \cdot g') = \phi(g) \cdot \phi(g') \) etc.); a composition of homomorphisms is again a homomorphism, thus defining a category. The set of one-to-one self-maps of a set with \( n \) elements, for example, defines the symmetric group \( \Sigma_n \).

A group \( A \) is commutative if \( a \cdot a' = a' \cdot a \) for any \( a, a' \in A \); in such cases the multiplication operation is often written additively, i.e. with \( a + a' \) instead of \( a \cdot a' \), e.g. as in the additive group \( \mathbb{Z} \) of integers. The order of a group is the (not necessarily finite) number of its elements.

Example The set of isomorphisms \( \alpha : G \to G \) with itself is similarly a group \( \text{Aut}(G) \) (of automorphisms of \( G \)). Any element \( h \in G \) defines an inner automorphism

\[ \alpha_h : G \ni g \mapsto hgh^{-1} \in G \]

of \( G \); it is a group homomorphism since

\[ \alpha_h(g \cdot g') = h(g \cdot g')h^{-1} = hgh^{-1} \cdot hg'h^{-1} = \alpha_h(g) \cdot \alpha_h(g') , \]

and \( h \mapsto \alpha_h : G \to \text{Aut}(G) \) is a homomorphism since

\[ (\alpha_h \circ \alpha_k)(g) = \alpha_h(kgk^{-1}) = hkgk^{-1}h^{-1} = hk \cdot g \cdot (hk)^{-1} = \alpha_hk(g) ; \]

much mathematics consists of shuffling parentheses. The subgroup \( \text{In}(G) = \{ \alpha_g \in \text{Aut}(G) \} \) is normal in that

\[ (\forall \beta \in \text{Aut}(G)) \alpha \in \text{In}(G) \Rightarrow \beta \alpha \beta^{-1} \in \text{In}(G) . \]

In particular, if \( G = A \) is commutative, then \( \text{In}(A) = \{ 1 \} \) is the trivial group.
If a subgroup $H$ of $G$ is normal, then the set $G/H = \{gH \mid g \in G\}$ (of ‘orbits’ of elements of $G$ under right multiplication by $H$) is again a group.

3.1.2 A composition

$$H \xrightarrow{\phi} G \xrightarrow{\psi} K$$

of homomorphisms is exact if the image

$$\text{im} \phi = \{g \in G \mid \exists h \in H, \phi(h) = g\}$$

of $\phi$ equals the kernel

$$\ker \psi = \{g \in G \mid \psi(g) = 1_K\}$$

of $\psi$; this implies in particular that the composition $\psi \circ \phi$ is trivial (i.e. maps every element of $H$ to the identity element of $K$), but is more restrictive. A sequence

$$1 \longrightarrow H \xrightarrow{\phi} G \xrightarrow{\psi} K \longrightarrow 1$$

of groups and homomorphisms is exact if its consecutive two-term compositions are exact; this implies that

- $\phi$ is one-to-one (or is a monomorphism, or has trivial kernel),
- $\psi$ is ‘onto’ (or surjective, or $K = \text{im} \psi$), and
- $\text{im} H$ is normal in $G$, and $\psi$ factors through an isomorphism $G/H \cong K$.

3.1.3 Such an exact sequence is said to split, if there is a homomorphism $\rho : K \rightarrow G$ inverse to $\psi$ in the sense that the composition $\psi \circ \rho$

$$K \xrightarrow{\rho} G \xrightarrow{\psi} K$$

is the identity map $K$. In that case there is a unique homomorphism

$$\varepsilon : K \rightarrow \text{Aut}(H)$$

such that (the ‘semi-direct product) $G \cong H \rtimes K$ is isomorphic to the group defined on the set product $H \times K$, with twisted multiplication

$$(h_0, k_0) \cdot (h_1, k_1) = (h_0 \cdot \varepsilon(k_0) h_1, k_0 k_1);$$

such a split sequence will usually be displayed below as

$$1 \longrightarrow H \longrightarrow G \cong H \rtimes K \longrightarrow K \longrightarrow 1.$$  

3.2 Some topological groups and algebras

3.2.1 The quaternion group

$$Q = \{\pm 1, \pm i, \pm j, \pm k\}$$

of order eight is defined by three elements $i, j, k$ with multiplication $i^2 = j^2 = k^2 = -1$ and

$$ij = k = -ji, \quad jk = i = -kh, \quad ki = j = -ik.$$
The (noncommutative) division algebra
\[ H = \{ q = q_0 + q_1 i + q_2 j + q_3 k \mid q_i \in \mathbb{R} \} \cong \mathbb{R}^4 \]
of Hamiltonian quaternions is the four-dimensional real vector space with multiplication extended from \( Q \); alternately, it is the two-dimensional complex vector space
\[ H = \{ z_0 + z_1 j \mid z_i \in \mathbb{C} \} , \]
where \( z_0 = q_0 + iq_1, \ z_1 = q_2 + iq_3. \) The quaternions thus extend the field \( \mathbb{C} \) of complex numbers much as \( \mathbb{C} \) extends the field \( \mathbb{R} \) of real numbers.

The quaternion conjugate \( q^* = q_0 - q_1 i - q_2 j - q_3 k \) to \( q \) has positive product
\[ q^* \cdot q = q \cdot q^* = |q|^2 = \sum q_i^2 > 0 \]
with \( q \) if \( q \neq 0 \), implying the existence of a multiplicative inverse \( q^{-1} = |q|^{-2} q^* \). This defines an isomorphism
\[ H^* \ni q \mapsto (|q|^{-1} q, |q|) \in S^3 \times \mathbb{R}_+^x \]
making the three-dimensional sphere \( S^3 \) a group under multiplication. This notation is nonstandard but convenient; note that \( q^{**} = q \) and that quaternion conjugation \( * : q \mapsto q^* \) is an anti-homomorphism, i.e.
\[ (u \cdot v)^* = v^* \cdot u^* . \]

Similarly,
\[ \mathbb{R}^x \cong S^0 \times \mathbb{R}_+^x, \ S^0 = \{ \pm 1 \} , \]
while
\[ \mathbb{C}^x \cong S^1 \times \mathbb{R}_+^x \]
(where \( S^n = \{ x \in \mathbb{R}^{n+1} \mid |x|^2 = 1 \} \) is the \( n \)-dimensional sphere of radius one, e.g. the circle when \( n = 1 \)).

3.2.2 The subalgebra (i.e. closed under addition and multiplication, but not division) of Lipschitz quaternions in \( H \) is the set of \( q \) with integral coordinates \((q_i \in \mathbb{Z})\), while the subalgebra of Hurwitz quaternions consists of elements \( q \) with all coordinates either integral or half-integral (i.e. such that each \( q_i \) is half of an odd integer). Finally, the subalgebra of Lipschitz (integral) quaternions has an additional (commutative but nonassociative) Jordan algebra product
\[ u, v \mapsto \{ u, v \} = \frac{1}{2}(u \cdot v + v \cdot u) = \{ v, u \} , \]
e.g.
\[ \{ 1, 1 \} = 1, \ {i, i} = \{ j, j \} = \{ k, k \} = -1, \ {i, j} = \{ j, k \} = \{ k, i \} = 0 . \]
This allows us to define a (non-commutative) Jordan ratio
\[ \{ u : v \} = \{ u, v^* \} = |v|^{-2}\{ u, v^{-1} \} \]
for \( u, v \in H \), distributive
\[ \{ u + u' : v \} = \{ u : v \} + \{ u : v' \}, \ \{ u : v + v' \} = \{ u : v \} + \{ u : v' \} \]
in both variables, satisfying
\[ \{u : v\}^* = \{u^* : v^*\} . \]

**Remark** $\mathbb{H}$ can be regarded as a subalgebra of the $2 \times 2$ complex matrices $M_2(\mathbb{C})$, in such a way that the quaternion norm $|q|^2$ equals the determinant of $q$, regarded as a matrix. This identifies the $3$-sphere $S^3$ with a subgroup of the Lie group $\text{SL}_2(\mathbb{C})$ of complex $2 \times 2$ matrices with determinant one; as such, it is the maximal compact (special unitary) subgroup $\text{SU}(2)$ of $\mathbb{H}^\times$. The special orthogonal group $\text{SO}(3) \cong \text{SU}(2)/\{\pm 1\}$ (of rotations in three dimensions) is a quotient of this group, and the various ‘binary’ (tetrahedral, octahedral etc.) groups lift the symmetry groups of the classical Platonic solids [12] to subgroups of the three-sphere. See [2], and its comments, for some very pretty animations of a certain ‘24-cell’ associated [10] (cf. note ar) to the octahedral and tetrahedral groups. [The noncompact group $\text{SL}_2(\mathbb{C})$ is similarly a double cover of (the identity component of) the physicists’ Lorentz group.]

**3.3** The subset $Q \subset \mathbb{H}^\times$ (of Lipschitz units) is a finite subgroup of $\text{SU}(2)$. Similarly, the subset
\[ Q \subset A_{24} \subset \text{SU}(2) \]
(of Hurwitz units) is the union of $Q$ with the set of sixteen elements of the form
\[ \frac{1}{2}[\pm 1 \pm i \pm j \pm k] . \]
It is known as well [5] as the binary tetrahedral group $2 : T$.

Klein’s (commutative) ‘Vierergruppe’
\[ V = \{1, I, J, K\} \]
with multiplication $I^2 = J^2 = K^2 = 1$ and $IJ = JI = K$, $JK = KJ = I$, $KI = IK = J$ can be regarded as the subgroup
\[ V = \text{In}(Q) \subset \text{Aut}(Q) \]
defined by $\alpha_i = I$, $\alpha_j = J$, $\alpha_k = K$.

**Exercise:**
i) $I$ sends $i$ to itself, and $j, k$ to $-j, -k$; similarly $J$ sends $j$ to itself, while $i, k \mapsto -i, -k$, etc. The cyclic permutation
\[ (abc) = (a \to b \to c \to a) \in \Sigma_3 \]
of three things defines a homomorphism $C_3 = \{1, \sigma, \sigma^2\} \to \text{Aut}(Q)$ sending $\sigma$ to $(ijk)$.

ii) The map $C_2^2 = C_2 \times C_2 \to V$ defined by $(1, 0) \mapsto I$, $(0, 1) \mapsto J$, $(1, 1) \mapsto K$ (and $1 \mapsto (0, 0)$) is a (nonunique) isomorphism.
iii) For example, in $V \rtimes C_3 = A_4$ (i.e. the alternating subgroup of order 12 (see below) of $\Sigma_4 \cong \text{Aut}(Q)$) we have
\[(I\sigma^2) \cdot (I\sigma^2) = I\sigma^2(I) \cdot \sigma^4 = IK\sigma = J\sigma .\]
The anti-automorphism $\lambda = *I\sigma^2$ of $Q$ satisfies
\[\lambda(i) = k, \lambda(j) = -i, \lambda(k) = j,\]
e.g. $\lambda(j) = (I\sigma^2(j))^* = (Ii)^* = i^* = -i$, cf. [21 §5].

3.4.1 Some useful small groups

| order & name | order & name |
|--------------|--------------|
| $n$ | cyclic $C_n = \{0, \ldots, n - 1\}, n \geq 1$
| $1 \rightarrow \mathbb{Z} \xrightarrow{n} \mathbb{Z} \xrightarrow{n \mathbb{Z}} C_n \xrightarrow{1}$ |
| $n!$ | symmetric $\Sigma_n, n \geq 1$ |
| $4$ | Klein Vierergruppe $V$
| $V = \{1, I, J, K\} \cong C_2 \times C_2$ (non-uniquely) |
| $6$ | symmetric $\Sigma_3$
| $1 \rightarrow C_3 \rightarrow \Sigma_3 \cong C_3 \rtimes C_2 \rightarrow C_2 \rightarrow 1$ |
| $8$ | (Lipschitz) quaternion units $Q = \{\pm 1, \pm i, \pm j, \pm k\}$
| $1 \rightarrow C_2 \rightarrow Q \rightarrow V \rightarrow 1$ |
| $12$ | alternating or tetrahedral $(T = A_4)$
| $1 \rightarrow V \rightarrow A_4 \cong V \rtimes C_3 \rightarrow C_3 \rightarrow 1$ |
| $24$ | $\Sigma_4$ as above; binary tetrahedral $2 \cdot T = \text{Hurwitz units } A_{24}$
| $1 \rightarrow C_2 \rightarrow 2 \cdot T = Q \rtimes C_3 \rightarrow A_4 = V \rtimes C_3 \rightarrow 1$ |
| $48$ | binary octahedral $2 \cdot O$
| $1 \rightarrow C_2 \rightarrow 2 \cdot O \rightarrow 2 \cdot T \rightarrow 1$ |
| as well as | $1 \rightarrow Q \rightarrow 2 \cdot O \rightarrow \Sigma_3 \rightarrow 1$ . |

3.4.2 The binary octahedral group $2 \cdot O$, regarded as a subgroup of the unit quaternions $[2, 6]$, is the disjoint union of $A_{24}$ with the set of twenty-four special elements
\[q = 2^{-1/2}[q_0 + q_1i + q_2j + q_3k] .\]
in which exactly two of \(q_0, \ldots, q_3\) are nonzero and equal \(\pm 1\), in which \(\Phi\) takes its values.

3.4.3 We have

\[
\text{Aut}(Q) \cong \Sigma_4 \cong \text{Aut}(2 \cdot T)
\]

and

\[
\text{Aut}(2 \cdot O) \cong C_2 \times (2 \cdot T \cong A_{24})
\]

3.5 Some of the groups above can be presented in terms of matrices over finite Galois fields \(F\): in particular, \(\Sigma_3 \cong \text{Sl}_2(F_2)\) and \(A_{24} \cong \text{Sl}_2(F_3)\). Similarly, the binary icosahedral group (which plays no role in this paper) is isomorphic to \(\text{Sl}_2(F_5)\).

It is worth mentioning that the group of \(2 \times 2\) matrices with entries from \(\mathbb{Z}\) and determinant one is a quotient

\[
1 \longrightarrow \mathbb{Z} \xrightarrow{t} \mathbb{B}_3 \longrightarrow \text{Sl}_2(\mathbb{Z}) \longrightarrow 1
\]

of Artin’s three-strand braid group \([7]\), which thus maps (by \(\mathbb{Z} \rightarrow \mathbb{Z}/3\mathbb{Z} \cong F_3\)) to \(\text{Aut}(2 \cdot O)\).

[The set of braids on \(n\) strands, imagined for example as displayed on a loom, define a group under ‘concatenation’:

Technically, an \(n\)-strand braid can be defined as a smooth path in the space of configurations defined by \(n\) distinct points in the plane, starting for example at time \(t = 0\) at the integral points \((1, 0), (2, 0), \ldots, (n, 0)\) and ending at time \(t = 1\) at the points \((1, 1), (2, 1), \ldots, (n, 1)\), though not necessarily in that order. Such braids can be composed by concatenation (i.e., gluing and rescaling), and define elements of \(\Sigma_n\) (sending \(k\) to \(l\) if the strand starting at \((k, 0)\) ends at \((l, 1)\)). The braid group \(\mathbb{B}_n\) is the set of such things under the equivalence relation roughly described as straightening: thus for example any braid can be parsed into a composition of elementary moves, in which one strand over- or under-passes one of its nearest neighbors. For example, \(\mathbb{B}_3\) can be presented as the group with two generators \(a, b\) satisfying the ‘braid relation’ \(aba = bab\), thus the map \(t\) above sends 1 to the full twist \((aba)^2 = (ab)^3\).]

§4 A Calculation

4.1 Lévi-Strauss’s formula is expressed in terms of formal analogies, e.g. \(F_x(a) : F_y(b)\), understood roughly as a ratio, in a sense going back to Eudoxus; but noncommutative algebra distinguishes the left fraction \(a^{-1}b = a\backslash b\) from the right fraction \(ba^{-1} = b/a\). The noncommutative ratio [3.3.2] splits this difference. Thus if \(x, a, y, b \in Q\) we have
\( \{ \Phi_x(a) : \Phi_y(b) \} = \frac{1}{2} \{ x - a : y - b \} = \frac{1}{2} \left[ (\{ x : y \} + \{ a : b \}) - (\{ a : y \} + \{ x : b \}) \right], \)

so for example if \( x = 1, \ a = i, \ y = j, \ b = k \) we have
\[
\frac{1}{2} \left[ (\{ x : y \} + \{ a : b \}) - (\{ a : y \} + \{ x : b \}) \right] = \frac{1}{2} \left[ (1 - i) : j - k \right],
\]
while
\[
\frac{1}{2} \left[ (-\{ i : j \} - \{ i : j \}) + (\{ k : i \} + \{ k : j \}) \right] = \frac{1}{2} \left[ -i - j + 0 \right] = -\frac{1}{2} (i + j).
\]

But now, applying the anti-automorphisms \( \lambda \) of \([20] (\S 5) \), we have
\[
\lambda(j - k) = -(i + j)
\]
by 3.3iii above, and the proposition is verified. □

Remarks

4.2.1 The automorphism group \( \Sigma_4 \) of \( Q \) preserves the set of special elements \([3.4.2]\) of \( 2 \cdot O \), i.e. the possible values of \( \Phi \), as well as their Jordan ratios; but it differs from the (inner) automorphism group \( A_{24} \) of \( 2 \cdot O \). However, the term \( x \) appears in the CF in the same place on both sides of the equation, and can be interpreted as playing the role of \( 1 \in Q \), fixed by all automorphisms. The canonical formula appears in variant forms in the literature, but (as far as I know) they all feature a term in the same place on both sides of the equation, allowing the variants to be reconciled by a cyclic permutation in \( \Sigma_3 \subset \Sigma_4 \), which lies in a quotient of \( \text{Aut}(2 \cdot O) \).

4.2.2 The classic work of Thom on singularity theory \([24]\) has turned out (e.g. under the influence of Arnol’d, McKay, and others) to have deep connections with the theory of Platonic symmetry groups. The binary orthogonal group, in particular, seems to be related to a certain ‘symbolic umbilic’ singularity \([9, 11]\), a special case of Thom’s original classification.

4.2.3 As a closing remark: the model proposed here is in fact not that complicated. To a mathematician, perhaps the most interesting implication is its connection with the theory of braids, which is arguably related to the processing of recursion, and to cognitive evolution.

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Appendix From The structural study of myth [14], Journal of American Folklore 68 (1955):

7.30 Finally, when we have succeeded in organizing a whole series of variants in a kind of permutation group, we are in a position to formulate the law of that group. Although it is not possible at the present stage to come closer than an approximate formulation which will certainly need to be made more accurate in the future, it seems that every myth (considered as the collection of all its variants) corresponds to a formula of the following type:

\[ F_x(a) : F_y(b) \simeq F_x(b) : F_{a^{-1}}(y) \]

where, two terms being given as well as two functions of these terms, it is stated that a relation of equivalence still exists between two situations when terms and relations are inverted, under two conditions: 1. that one term be replaced by its contrary; 2. that an inversion be made between the function and the term value of the two elements.

References

1. Kwame Anthony Appiah, https://www.nybooks.com/articles/2020/02/13/claude-levi-strauss-key-to-all-mythologies/
2. John Carlos Baez, https://johncarlosbaez.wordpress.com/2019/08/29/the-binary-octahedral-group/
3. AL Becker, Beyond Translation, U of Michigan Press (1998)
4. JL Borges, The analytical language of John Wilkins, in Otras Inquisiciones (1952) Buenos Aires: Sur.
5. https://en.wikipedia.org/wiki/Binary_tetrahedral_group
6. https://en.wikipedia.org/wiki/Binary_octahedral_group
7. https://en.wikipedia.org/wiki/Braid_group
8. https://en.wikipedia.org/wiki/Crystallographic_point_group
9. https://en.wikipedia.org/wiki/Du_Val_singularity
10. https://en.wikipedia.org/wiki/24-cell
11. P Dechant, From the trinity (A3, B3, H3) to an ADE correspondence, Proc. R. Soc A 474 (2018), no. 2220, 20180034, https://arxiv.org/abs/1812.02804
12. A Doja, Politics of mass rapes in ethnic conflict ..., Crime, Law and Social Change (2019) 71 : 541 – 580, https://link.springer.com/article/10.1007/s10611-018-9800-0
13. N Epa, N Ganter, Platonic and alternating 2-groups, Higher Structures 11 122 – 146 (2017), https://arxiv.org/abs/1605.09192
14. CLévi-Strauss, The structural study of myth, Journal of American Folklore 68 no 270 (1955) 428–444
15. https://fr.wikipedia.org/wiki/Formule_canonique_du_mythe
16. https://ncatlab.org/nlab/show/canonical+formula+of+myth
17. W James, Principles of Psychology [1890]
18. A Joyal, Notes on quasicategories, http://www.math.uchicago.edu/~may/IMA/Joyal.pdf
19. https://ncatlab.org/nlab/show/kinship
20. Y Manin, We do not choose our profession ..., AMS Notices 56 (2009) 1268 -â€” 1274 (p 1274)
21. P. Maranda, The double twist: from ethnography to morphodynamics, U of Toronto Press (2001)
22. J Morava, On the canonical formula of CLévi-Strauss, https://arxiv.org/abs/math/0306174
23. ——, From Lévi-Strauss to chaos and complexity, in MS Mosko, FH Damon, On the order of chaos p 47 – 63, Berghan Books (2005)
24. J Petitot, A morphodynamical schematization of the canonical formula for myths, in [19] 267 – 311
25. AV Phillips, A non-commutative marriage system in the South Pacific, http://www.math.stonybrook.edu/~tony/whatsnew/oct09/vanuatu2.html
26. F de Saussure, Writings on General Linguistics, OUP 2006
27. A Weil, On the algebraic study of certain types of marriage laws, appendix (221–232) to C Lévi-Strauss, Elementary structures of kinship, Beacon (1969)

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