Nonlocality – The party may be over!

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Abstract. We demonstrate that the phenomenon known as Spontaneous Parametric Down Conversion is really an amplification, in a nonlinear crystal pumped by a laser, of certain pairs of modes of the electromagnetic zeropoint field. The demonstration is achieved by showing the existence of a related phenomenon, Spontaneous Parametric Up Conversion. This phenomenon, once observed, will cast doubt on the quantum-optical theory, which treats photons as the elementary objects of the light field. It will also lend greater credibility to the zeropoint-field description of optical entanglement phenomena. That description is based on the unquantized light field and is consistently local, in contrast with the nonlocal description of Quantum Optics.

1 Introduction

In 1948 Albert Einstein[1] wrote a letter to Max Born in which he enunciated the Principle of Local Action$^1$ (PLA), which states that any physical action is transmitted from one point to another by a field which propagates outwards in space and forwards in time at a speed no greater than $c$. The usual form of Quantum Optics$^2$ is based on the notion of photons as the elementary objects of the light field, whose corpuscular nature is revealed, through the collapse mechanism, whenever a detection event occurs. The PLA is violated by Quantum Optics$^3$, because a light signal, in that description, may undergo collapse as a result of making an observation on another signal with which it is correlated. The nonlocal correlation between photons has been given the title of entanglement, in order to emphasize that, following an analysis first made by Bell$^4$, there is no way, based on normal probability theory, in which the correlation of the two photons may be explained in terms

$^1$Born translated Einstein’s Nahewirkung as ‘contiguity’. I think my translation of ‘local action’ is better; the opposite of Nahewirkung is Fernwirkung, which is usually translated as ‘action at a distance’.
of their common origin. The latter may be in the mutual annihilation of a positron-electron pair\cite{5}, or in an atomic cascade\cite{6} or in the nonlinear optical process known as Spontaneous Parametric Down Conversion\cite{7} (SPDC). Observations of all these give correlations which confirm the entanglement phenomenon.

Does this mean that Einstein was wrong, and that the PLA has to be abandoned? Before attempting to answer that we should note that, in his letter to Born, Einstein expressed the opinion that the PLA is the basis of all experimental science. So mere corroboration of the entanglement predictions is not sufficient. If anyone seriously wishes to take on the PLA, then they must be able to point to an experimental situation where that principle is tested \emph{without reference to any other assumptions, however plausible those assumptions may appear to be}.

Such a necessity was recognized by Clauser and Horne\cite{8}, who acknowledged that a local description of an entanglement phenomenon would not necessarily be in terms of photons. They proposed a plausible assumption which made it possible to transform the untestable Bell inequality into a testable Freedman inequality, namely the \emph{no-enhancement hypothesis} that a given light signal has a certain probability of activating a detector, and that, if a polarizer is interposed between the source and the detector, that detection probability cannot increase. The experiment of Freedman and Clauser showed a violation of that inequality, and most people, including the experimenters, came to the breathtakingly naive conclusion that the PLA had been violated. The simple alternative of holding on to the PLA (that is, according to Einstein, to the standard procedure of experimental science) and concluding that a hypothesis which had seemed very plausible was nevertheless wrong, seems not to have occurred to anybody! Those few of us who did hold on were occasionally acknowledged in a footnote. According to one overworked phrase we were said to be “exploiting the detection loophole”\cite{9} in the proof of nonlocality.

We have shown\cite{10, 11, 12, 13, 14, 15, 16, 17}, that entanglement in SPDC may be interpreted as a correlation between modes of the electromagnetic field, in which both the parts above and below the “vacuum”, or zeropoint field (ZPF), level participate. In this interpretation the observed enhancement phenomena are all consistently local. We have previously reviewed\cite{18, 19} the earlier generation of Bell tests using atomic cascades, and shown that any theory of light detectors which recognizes that they subtract the ZPF violates the no-enhancement hypothesis. We have
now developed a full theory, at least in the context of nonlinear crystals, of the production, propagation and detection of weak signals. That theory now also leads to the prediction of the new phenomenon which we call Spontaneous Parametric Up Conversion (SPUC).

So what is “exploiting the detection loophole”? It is simply following the normal method of experimental science. We use Einstein’s PLA to identify the weak point of the prevailing theory, in this case the standard quantum-optical theory of detection. That theory is based on the normal-ordering algorithm, which in turn comes from the collapse mechanism, already identified, by Einstein at the Solvay Conference of 1927, as a weak point. Following that indication we have been able to show that the normal-ordering algorithm may be shown to be the linear limit of a realist detection theory, that is one in which the activation of a detector depends only on the local values of the electric field within a certain region of space and within a certain time window. And that enables us to make further experimental predictions, in addition to that of SPUC.

2 Down conversion

Parametric Down Conversion (PDC) is the optical analogue for a well established classical phenomenon of wave propagation in a nonlinear dispersive medium. It was first demonstrated in the 1960s, when high-intensity coherent sources (lasers) became available. When two lasers, one ultraviolet (frequency $\omega_1$) and the other infrared (frequency $\omega_2$), are incident on one of a wide class of nonlinear crystals (NLC), then, for a certain combination of incidence angles, a signal at the difference frequency ($\omega_3 = \omega_1 - \omega_2$) is emitted (see Fig.1). For example, if the ultraviolet wavelength is 351nm and the infrared wavelength is 845nm, then the signal wavelength is 600nm, which is in the visible spectrum.
The angle between the two lasers, and the exit angle of the signal, are determined through the *phase matching relations* between the frequencies and wave vectors \((k_1, k_2, k_3)\) of the three modes, that is

\[
\begin{align*}
\omega_3 &= \omega_1 - \omega_2, \\
k_3 &= k_1 - k_2,
\end{align*}
\]

which have to be combined with the Sellmeier relations for the refractive index of the crystal, as a function of frequency and of the angle between the wave vector and the polar axis. Account also has to be taken of the polarization of the three modes; in a typical arrangement, called Type-I PDC, \(\omega_1\) is extraordinary, and \(\omega_2\) and \(\omega_3\) are ordinary.

Spontaneous Parametric Down Conversion (SPDC) is the name given to the phenomenon which occurs when we remove the laser \(\omega_2\); a weak signal, which is nevertheless visible to the unaided eye, remains in the \(\omega_3\) channel. This is because, in the vacuum, there is a *zeropoint field (ZPF)* intensity in all modes, corresponding to half a “photon”. The radiation pressure of this ZPF field is what gives rise to the Casimir effect\(^{[21, 22]}\). Through its nonlinear interaction with the hydrogen atom, the ZPF also gives rise to the greater (that is nonrelativistic) part of the Lamb shift in the hydrogen spectrum\(^{[22]}\). In SPDC, as opposed to PDC, the laser mode \(k_2\) is replaced by the same, but very much less intense, ZPF mode, so \(k_3\) is still emitted. The SPDC phenomenon actually manifests itself as a rainbow (see Fig.\(\PageIndex{3}\)). Since all frequencies and directions of modes are present in the ZPF, all the corresponding down converted signals also appear at those angles satisfying the phase matching relations.
Figure 2: Position of the SPDC rainbow produced when a 351nm laser is normally incident on a BBO crystal cut with its axis at 37 degrees to the incident wave vector. The 600, 700 and 800nm components are shown; the 600 being the inner one and the 800 the outer one.

The above phenomenon is nowadays often called simply PDC, because it has become popular to view the process as one in which laser photons $\omega_1$ down convert into $\omega_2$ and $\omega_3$. In this description the phase matching relations linking the wave vectors of the three coupled modes in SPDC are considered to express the conservation of four-momentum between the three participating photons, that is

$$\hbar \omega_1 = \hbar \omega_2 + \hbar \omega_3 ,$$
$$\hbar k_1 = \hbar k_2 + \hbar k_3 .$$

These equations are, of course, trivially equivalent to (1), but their theoretical underpinning is entirely different. We may write eq.(1) in the descriptive form

laser ultraviolet wave + vacuum infrared wave $\rightarrow$ visible signal ,
whereas the corresponding description for eq. (2) is

\[
\text{laser ultraviolet photon} \rightarrow \text{visible signal} + \text{infrared idler}.
\]

It is an unfortunate and highly misleading consequence of this latter description that certain correlations in the intensities of the outgoing SPDC signals are now widely interpreted as showing rather bizarre connections between the corresponding photons. All of these allegedly nonlocal phenomena \cite{1, 23, 24, 25, 26, 27}, including those which purport to test the Bell inequalities \cite{7, 24}, have the entanglement feature in common, and our analysis, based on the wave description of light, gives a consistently local description for all of them. The difference is, perhaps, emphasized by the absence of \( \hbar \) in eq. (1), as opposed to its presence in (2). The optical theory we are proposing is a semiclassical one, that is it combines a wave description of the light field with a quantum description of atoms. However, the interaction of light with a nonlinear crystal does not require any recognition of the atomic constitution of the crystal. Consequently our description of such phenomena, in contrast with our description of atomic cascades, is purely classical. Planck’s constant enters only when we want to calculate the intensity of the ZPF, which we shall do in the next section. That is because \( \hbar \) is the constant determining the intensity of the ZPF. I think Max Planck would have approved, because he originated the idea of the ZPF in 1911 \cite{21, 22, 28}.

3 Up conversion

The SPUC phenomenon, which was mentioned in the Introduction, is a simple modification of SPDC. It has not yet been observed, but any well equipped optical laboratory with a suitably cut specimen of the nonlinear crystal beta barium borate (BBO) may see it by following the design details below. Essentially all that needs to be done is to return to Fig. 1 and, instead of removing the infrared laser (that is \( \omega_2 \)), we remove the ultraviolet one (that is \( \omega_1 \)). Then the signal \( \omega_3 \) persists. We may describe this process as

\[
\text{laser infrared wave} + \text{vacuum ultraviolet wave} \rightarrow \text{visible signal},
\]

but, in contrast with the corresponding SPDC process, there is no photonic equivalent. However, the phase matching relations for the SPDC and SPUC processes are identical. Therefore we can be assured that, if one of them is
a real process, then so is the other. It only remains to calculate the position and the intensity of the SPUC rainbow.

For definiteness we assume that the crystal is cut with its optic axis at 37 degrees to the incident face, and that the infrared laser has a wavelength of 845nm normally incident. Because the ultraviolet mode is extraordinarily polarized, its refractive index depends on its orientation, which results in the SPUC rainbow being off centred with respect to the laser beam. We depict the position of the rainbow in Fig.3. We note that phase match-

![Figure 3: Position of the SPUC rainbow produced when a 845nm laser is normally incident on a BBO crystal cut with its axis at 37 degrees to the incident wave vector. The arcs of the 600, 700 and 800nm components are shown. Note that, in contrast with SPDC (Fig.2), the rainbow is not a complete circle, neither is it centred around the direction of the pumping laser.

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ing is not possible for all azimuthal angles, the polar axis having azimuth \( \phi = 0 \). The angles at which the visible SPUC modes exit from the crystal
are substantially larger than those of the SPDC rainbow, depicted in Fig.2, especially since, as we shall see shortly, the most intense part of the rainbow is near $\phi = 180$ degrees, for which $\theta$ is around 50 degrees. It is therefore not surprising that the SPUC rainbow has never been accidentally observed.

The intensities of the SPDC and SPUC rainbows are calculated from the mode coupling equations\[16\]. We make the linearization approximation, which consists of treating the amplitude of the laser, that is $A_1$ in SPDC and $A_2$ in SPUC, as a constant. Then, for SPUC, taking the normal to the incident face of the crystal as the $z$-direction, $A_1$ and $A_3$ are slowly varying functions of $z$ satisfying the coupling equations

$$
\frac{dA_1}{dz} = i\beta_1 A_2 e^{i\Delta z} A_3 ,
$$

$$
\frac{dA_3}{dz} = i\beta_3 A_2^* e^{-i\Delta z} A_1 ,
$$

where our model assumes perfect phase matching in the $x$ and $y$ directions, and $\Delta$ is the mismatch in the $z$ components of $k_1, k_2, k_3$, that is

$$
\Delta = k_{1z} - k_{2z} - k_{3z} ,
$$

and $\beta_1, \beta_3$ are constants related to the Pockels coefficients of the crystal. Putting the single-mode intensities $I_r = |A_r|^2$ ($r = 1, 2, 3$), then, for a crystal of depth $l$, the relation between incoming and outgoing intensities is

$$
I_1(l) - I_1(0) = \beta_3 I_2 l^2 [\beta_3 I_1(0) - \beta_1 I_3(0)] \text{sinc}^2(\frac{1}{2}\Delta l') ,
$$

$$
I_3(l) - I_3(0) = \beta_1 I_2 l^2 [\beta_1 I_3(0) - \beta_3 I_1(0)] \text{sinc}^2(\frac{1}{2}\Delta l') ,
$$

where

$$
sinc(x) = \frac{\sin x}{x} , \quad \Delta' = \sqrt{\Delta^2 + \beta_1 I_2 \beta_3} .
$$

The total intensity in the $\omega_3$ channel is obtained by summing $I_3(l) - I_3(0)$ over all the relevant pairs of modes. For details refer to Ref.\[20\]. The results are depicted in Fig.4. Over a large part of the visible frequency range, the SPUC intensity is about half the SPDC intensity, based on a pumping laser for the latter at 442nm, having the same intensity as the SPUC pump at 845nm. So, with a suitable camera, the SPUC rainbow will be seen in glorious technicolor!
Figure 4: The SPUC and SPDC cross sections plotted against the azimuthal angle in degrees (upper figure, outgoing wavelength $0.6 \mu m$) and wavelength in $\mu m$ (lower figure, outgoing azimuth 180 degrees) for a BBO crystal cut with its optic axis at 37 degrees to the normal of the incident face. The normally incident lasers have wavelengths $0.442 \mu m$ (SPDC) and $0.845 \mu m$ (SPUC)

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