Infrared Small Target Detection based on Principal Component Tracing

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Abstract. According to the characteristics of infrared small target images, an infrared small target detection method based on principal component tracing is proposed. The method by minimizing the combination of the nuclear matrix norm and L1 norm implementation principal component tracing, using principal component tracing to the infrared small target image is decomposed into sparse matrix and low rank matrix, sparse matrix for segmented small target detection. The proposed method is tested and compared with the existing method. The experimental results show that the proposed infrared small target detection method has better small target detection performance.

1. Introduction
The infrared small target detection includes two aspects: single frame image detection and sequence image frame detection. This paper studies small infrared target detection method based on single frame image (hereinafter referred to as infrared small target detection). At present, there are two kinds of infrared small target detection methods: image filtering based detection method and machine learning detection method. According to the correlation and continuity of the gray distribution of the natural background space in the infrared image, some scholars have proposed an infrared dim target detection method based on image filtering. Max-Median[1], Top-Hat[2] and TDLMS[3] are typical representatives. The basic ideas of this method are: first, the background of the infrared small target image is estimated (also called the background estimation). Then the original image is subtracted from the background image to get the image containing the target and the noise, and then the target position is obtained by the threshold processing or other methods. The image filtering method has a good effect on the specific noise suppression, and the noise in the actual image is often mixed with many kinds of noise, which makes the image filtering method have some limitations. In recent years, some scholars have proposed a small infrared target detection method based on machine learning, which transforms small target detection problem into classification problem. Among them, the representative methods are Principle Component Analysis (PCA)[4,5], probability principal component analysis (Probability Principle Component Analysis, PPCA)[6] and sparse representation(SR)[7]. In this method, the target model and background model are trained by the machine learning algorithm according to the training sample. Then the target model and the background model are used to classify the detection image, that is, the sub image is extracted in turn and the target is determined by the rule. Although the machine learning method has good detection performance, it is greatly influenced by the training sample, and the target and background in the actual image often have diversity, which restricts its application in the actual small target detection system.

In 2009, Wright[8] proposed robust principal component analysis (Robust Principal Component Analysis, RPCA), also known as the principal component tracing (Principal Component Tracing,
PCP\cite{9}. PCP has been applied to computer vision problems such as sequence image background estimation, sequence face image registration\cite{10} and low rank texture restoration\cite{11}. Based on the low rank characteristics of the infrared small target background image and the sparsity of the small target image, this paper proposes an infrared small target detection method based on the principal component tracing literature \cite{8-11}. In this method, the infrared small target image is decomposed into sparse matrix and low rank matrix by principal component tracing, and the sparse matrix is segmented to achieve small target detection.

2. Principal Component Tracing

When the observation noise obeys the Gauss distribution, the classical PCA gives the optimal low rank representation of the observation matrix, but the validity of the classical PCA is destroyed when the observation matrix is damaged. In order to overcome this shortcoming of classic PCA, robust principal component analysis (Robust Principal Component Analysis, RPCA) is proposed in Literature\cite{8}, also known as the principal component pursuit (Principal Component Pursuit, PCP)\cite{9}. The purpose of PCP is to solve the low rank and sparse matrix of the known observation matrix $M \in \mathbb{R}^{m \times n}$. The model is as follows:

$$M = L + S$$

Among them, $L$ and $S$ respectively represent low rank matrices and sparse matrices. Literature\cite{8} proves that under very broad conditions, the problem (1) can be solved accurately by minimizing the combination of matrix singular values (sum of singular values of matrices) and $l_1$ norms. The solution of the model (1) is limited to the observation matrix containing only the low rank and the sparse component, but the actual observation data will be polluted by the noise, and the observation model is as follows:

$$M = L + S + Z$$

Among them, $Z$ is noise item, it may exist in every item of data matrix. In order to solve the low rank matrix $L$ and sparse matrix $S$ in (2), PCP method is proposed by literature \cite{9}, and it has been proved that under the same conditions given in the literature, the solution formula (2) low rank matrix $L$ and sparse matrix $S$ can be solved by convex optimization.

3. Detection of Small Infrared Target by Principal Component Tracing

Infrared small target image $f(x,y)$ can usually be described as follows:

$$f(x,y) = f_B(x,y) + f_T(x,y) + n(x,y)$$

Among them, $f_B(x,y)$ is background image, $f_T(x,y)$ is small target image, $n(x,y)$ is measured noise image, they have the following characteristics. First, because the rows of the background image $f_B(x,y)$ or the vectors formed by each column have strong gray spatial correlation, the space formed by the column vector or row vector is a limited low dimensional linear subspace, so if the background image $f_B(x,y)$ is used as a matrix $B$, the matrix $B$ has a low rank structure, that is, the rank of matrix $B$ is rather small. A background image of 200 small infrared targets with a size of 320 * 240 is selected, and the singular values of each background image are decomposed and the mean value of each singular value of these infrared background images is calculated. Figure 1 shows some samples of infrared background images, and the curves of the singular values of these background images varying with the singular value index. From Figure 1, it can be seen that the main energy of the infrared background image is concentrated on a few singular values ahead. This phenomenon indicates that the infrared background image has a low rank structure. Secondly, because the area of the infrared small target is small, if the small target image $f_T(x,y)$ is used as a matrix $E$, then the matrix $E$ has the sparsity, that is, the non-zero term of the matrix $E$ is less. Finally, Due to the constraints of the infrared imaging mechanism itself, the observed noise $n(x,y)$ is covered with all pixels, and its energy is limited\cite{12], that
is, if the noise image \( n(x,y) \) is used as a matrix, the noise will affect every item of the matrix and its energy is limited.

![Figure 1. Infrared background images and its curve of mean singular value](image)

The assumption matrix \( D \) represents the infrared small target image \( f(x,y) \). According to the above analysis, we can see that the problem of infrared small target detection and background estimation can be modeled as the following optimization problem:

\[
\min_{E, B} \text{rank}(B) \quad \text{s.t.} \quad \|D - B - E\|_F \leq \delta, \|E\|_0 \leq k
\]  

Among them, \( \text{rank}(\cdot) \) represents the rank of the matrix, \( \|\cdot\|_F \) is the Frobenius norm of the matrix, \( \delta \) is a constant greater than 0, and \( \|\cdot\|_0 \) is the \( \ell_0 \) norm of the matrix (the number of non-zero terms in the matrix), and \( k \) is a constant, which represents the maximum number of non-zero pixels in the sparse matrix \( E \). The optimization problem shown in (4) is easy to write in the following form of Lagrange:

\[
\min_{B, E} \text{rank}(B) + \lambda \|E\|_0 \quad \text{s.t.} \quad \|D - B - E\|_F \leq \delta
\]  

Among them, \( \lambda > 0 \) is a weighted parameter used to balance the sparsity of the rank sum matrix \( E \) of matrix \( B \).

Although the rank of the matrix and the \( \ell_0 \) norm have non-convexity, this makes the optimization problem shown in formula(5) become a NP problem, and the rank of the matrix and the \( \ell_0 \) norm are discrete value functions, and the solution of the formula(5) may be unstable, but literature [9] has proved that the optimization questions shown in formula(5) can be used to minimize the kernel norm of the matrix and \( \ell_1 \). The combination of norms is solved accurately and accurately. The optimization models are as follows:

\[
\min_{B, E} \|B\|_* + \lambda \|E\|_1 \quad \text{s.t.} \quad \|D - B - E\|_F \leq \delta
\]  

Among them, \( \|\cdot\|_* \) and \( \|\cdot\|_1 \) are the norm and \( \ell_1 \) norm of the matrix respectively, and the parameter \( \lambda \) is \( 1/\sqrt{m} \) (\( m \) is the row of matrix \( D \)). In this paper, we use the augmented Lagrange multiplier (Augmented Lagrange Multiplier, ALM) algorithm to solve the solution (6).
4. Experimental Results

In order to verify the effectiveness of the proposed method (for short, PCP method), the proposed method is implemented on the Matlab 2009 as the development tool, Pentium (R) Dual-Core E5000, the main frequency 2.8GHz, and the memory of 2G, and is compared with the present excellent methods: SPCA\[5\] and SR\[7\]. In the experiment, the parameter setting and training sample generation of SPCA and SR two methods are the same as those of literature \[5\] and \[7\] respectively.

Figure2 (the rectangle in the figure) gives three typical infrared small target images and the detection-result image obtained by this method. These three frames are infrared small target images in the sky, sea sky and the land background respectively. From the three-dimensional display of the original image, it can be seen that the energy of the small target is weak, almost inundated by the background clutter and noise, especially the first image, the background cloud is more, the target and the background are very similar. Figure2(c) and Figure2(d) are the detection results of this method and their three dimensional display. From Figure2 (c) and Figure2(d), we can see that this method can effectively suppress background and convex objects.

In order to further verify the performance of this method, the method is compared with the SPCA and SR methods. The experimental comparison results are shown in Figure3. From the test results in Figure 3, we can see that compared with the other two methods, this method can suppress background clutter better. For further compare the performance of the three detection methods, the local signal to noise ratio (Local Signal-to-Noise Ratio, LSNR) and the local signal to noise ratio gain (Local Signal-to-Noise Ratio Gain, LSNRG) are selected to evaluate these three detection methods. LSNR is defined as: LSNR = Sarea /Narea, in which Sarea represents the local signal value, and the maximum value of the pixel gray in the target area is taken in the experiment; the Narea represents the local background value, and the maximum of the background area is taken in the experiment. The larger LSNR shows that the target is more significant than the background in the local area, and the detection effect is better. LSNRG is defined as LSNRG=LSNRout/LSNRin, where LSNRin and LSNRout are LSNR respectively in the target area before and after target detection. The larger LSNRG shows that the detection method is more effective in enhancing LSNR and better in detecting performance. Table1 gives the LSNR and LSNRG values of the three detection methods after testing the image in Figure3(a) (the best value of the index is shown in BOLD). From table 1, we can see that in the three detection
methods, the results obtained by this method are both LSNR and LSNRG, which shows that the performance of the proposed method is better.

![Test image and SPCA detection results](image1.png)

**Figure3.** Comparisons of detection results

![SR detection result and PCB detection result](image2.png)

**Table1.** Quantitative evaluation index

| Target | LSNR SPCA | LSNR SR | LSNR PCP | LSNRG SPCA | LSNRG SR | LSNRG PCP |
|--------|-----------|---------|----------|------------|----------|-----------|
| 1      | 1.78      | 1.90    | 2.83     | 1.95       | 2.49     | 2.76      |
| 2      | 1.13      | 2.32    | 3.52     | 1.24       | 2.37     | 3.56      |
| 3      | 1.50      | 1.85    | 3.73     | 1.47       | 1.51     | 2.61      |
| 4      | 1.06      | 2.53    | 3.25     | 1.09       | 2.24     | 3.56      |
| 5      | 1.85      | 3.14    | 3.26     | 1.62       | 2.81     | 2.95      |
| 6      | 1.43      | 2.74    | 2.97     | 1.27       | 3.16     | 3.87      |
| 7      | 1.17      | 3.88    | 3.59     | 1.24       | 3.50     | 4.02      |

5. Conclusions

According to the low rank structure of the infrared small target background image, the sparsity of the small target image and the energy finiteness of the noisy image, a new detection method of small red target is proposed based on the principle of principal component tracing. In this method, the infrared small target image is decomposed into a low rank matrix and a sparse matrix by principal component tracking, and the small target detection is realized by segmentation of the sparse image. The experimental verification of the proposed infrared small target method is carried out and compared
with the existing methods. The experimental results show that compared with the existing methods, the method has better detection performance.

6. References

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