Research Article

A New Flexible Statistical Model: Simulating and Modeling the Survival Times of COVID-19 Patients in China

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The spread of the COVID-19 epidemic, since December 2019, has caused much damage around the world, disturbed every aspect of daily life, and has become a serious health threat. The COVID-19 epidemic impacted nearly 150 countries around the globe between December 2019 and March 2020. Since December 2019, researchers have been trying to develop new suitable statistical models to adequately describe the behavior of this deadly pandemic. In this paper, a flexible statistical model has been proposed that can be used to model the lifetime events associated with this deadly pandemic. The new distribution is derived from the combination of an extended Weibull distribution and a trigonometric strategy referred to as the arcsine-X approach. Hence, the new model may be referred to as the arcsine new flexible extended Weibull model. The proposed model is capable of capturing five different behaviors of the hazard rate function. The model parameters are estimated via the maximum likelihood approach. Furthermore, a Monte Carlo study is conducted to assess the behavior of the estimators. Finally, the applicability of the new model is demonstrated using the data of fifty-three patients taken from a hospital in China.

1. Introduction

COVID-19 (coronavirus disease 2019) was first spotted in China in December 2019 and then spread worldwide exponentially. The main reasons for the exponential growth of COVID-19 include unawareness and ignorance of this deadly virus, inadequate tools, low efficiency in detection, and delays in formulating appropriate and effective policies during the initial stage of the epidemic [1]. As of April 04, 2021, 08:47 GMT, 131,437,582 cases have been confirmed, 2,860,569 deaths have occurred, and 105,852,256 patients have been luckily recovered.

The top ten countries with the higher number of total confirmed COVID-19 cases include the USA with 32,669,121 cases, India with 16,263,695 cases, Brazil with 14,172,1399 cases, France with 5,408,606 cases, Russia with 4,744,961 cases, Turkey with 4,501,382 cases, the UK with 4,398,431 cases, Italy with 3,920,945 cases, Spain with 3,456,886 cases, and Germany with 3,238,054 cases.

The comparison of the COVID-19 epidemic between different countries is worth studying and is of great concern. In this regard, researchers are devoting serious efforts to make comparisons between different countries. The problems related to the COVID-19 epidemic in Italy have been discussed in [2]. Comparison of COVID-19 in some European countries, South Korea, and the USA has been done in [3]. The COVID-19 pandemic in Australia has been discussed in [4]. The phenomena of the spread of COVID-19
in Lebanon are provided in [5]. A case study from Spain has been discussed in [6]. Mathematical analysis of COVID-19 in Mexico is provided in [7]. A case study from Brazil is discussed in [8]. The issues related to the COVID-19 epidemic in Pakistan are provided in [9]. A mathematical model developed for assessing the transmission of the COVID-19 epidemic in India is introduced in [10]. Among the Asian countries, the phenomena of the spread of the COVID-19 epidemic are discussed in [11]. The comparison between Iran and mainland China has appeared in [12]. The comparison between two neighbor countries Iran and Pakistan has appeared in [13]. The problems related to the COVID-19 pandemic in Indonesia have been discussed in [14]. From the literature cited above, we see that there is a great interest to learn and know more about the COVID-19 pandemic. In the field of big data science and other related sectors, providing the best description of the real phenomena is a prominent research topic (refer [15–20] for details). Recent studies have pointed out the potentiality of the statistical models in different sectors of applied sciences. The goal of this article is to carry out this research area of distribution theory and propose a new statistical model to provide a better fit to the survival times data.

Recently, Liao et al. [21] introduced a modification of the Weibull distribution called a NFEW (new flexible extended Weibull) and suggested its application in modeling lifetime events. Let X have the NFEW distribution with two shape parameters \((\eta_1, \eta_2 > 0)\) and two scale parameters \((\kappa_1, \kappa_2)\); then, its DF (distribution function) \(F(x; \Xi)\) is as follows:

\[
F(x; \Xi) = 1 - \exp\left\{-e^{\left(k_1 x^{\eta_1} - k_2 x^{\eta_2}\right)}\right\}, \quad x \geq 0, \tag{1}
\]

where \(\Xi = (\eta_1, \eta_2, \kappa_1, \kappa_2)\). The corresponding PDF (probability density function) \(f(x; \Xi)\) is as follows:

\[
f(x; \Xi) = \frac{k_1 \eta_1 x^{\eta_1 - 1} + k_2 \eta_2}{x^{\eta_1 + 1}} e^{\left(k_1 x^{\eta_1} - k_2 x^{\eta_2}\right)}\cdot \exp\left\{-e^{\left(k_1 x^{\eta_1} - k_2 x^{\eta_2}\right)}\right\}, \quad x > 0. \tag{2}
\]

In this work, we propose a new modification of the NFEW model called ASNFEW (arcsine new flexible extended Weibull) distribution using the arcsine-X strategy [22], which can be obtained as a subcase of [23]. The DF and PDF of the arcsine-X distributions are given by

\[
G(x) = \frac{2}{\pi} \arcsin e(F(x; \Xi)), \quad x \in \mathbb{R}, \tag{3}
\]

\[
g(x) = \frac{2}{\pi} \frac{f(x; \Xi)}{\sqrt{1 - F^2(x; \Xi)}}, \quad x \in \mathbb{R}, \tag{4}
\]

respectively.

The DF of the proposed ASNFEW distribution is obtained by inserting equation (1) in (3). In the next section, we introduce the proposed model and sketch some possible behaviors of the PDF and HRF (hazard rate function) of the ASNFEW distribution.

### 2. The Arcsine New Flexible Extended Weibull Model

A random variable \(X\) has the ASNFEW model if its DF \(G(x)\) is given by

\[
G(x) = \frac{2}{\pi} \arcsin e\left(1 - \exp\left\{-e^{(k_1 x^{\eta_1} - k_2 x^{\eta_2})}\right\}\right), \quad x \geq 0, \tag{5}
\]

with SF (survival function) denoted by \(S(x)\), and it can be expressed as follows:

\[
S(x) = 1 - \frac{2}{\pi} \arcsin e\left(1 - \exp\left\{-e^{(k_1 x^{\eta_1} - k_2 x^{\eta_2})}\right\}\right), \quad x \geq 0. \tag{6}
\]

For \(\eta_1 = 0.5, \kappa_1 = 1.2, \eta_2 = 0.5, \kappa_2 = 0.9\), the plots for the DF and SF of the ASNFEW model are sketched in Figure 1.

The PDF corresponding to equation (5) is given by

\[
g(x) = \frac{2}{\pi} \frac{k_1 \eta_1 x^{\eta_1 - 1} + \left(k_2 \eta_2 x^{\eta_2 + 1}\right)}{1 - \exp\left\{-e^{(k_1 x^{\eta_1} - k_2 x^{\eta_2})}\right\}} e^{(k_1 x^{\eta_1} - k_2 x^{\eta_2})} \exp\left\{-e^{(k_1 x^{\eta_1} - k_2 x^{\eta_2})}\right\}, \quad x > 0. \tag{7}
\]

Some possible behaviors of the PDF of the ASNFEW model are presented in Figure 2. The plots in Figure 2 are sketched for \(\eta_1 = 2.1, \eta_2 = 0.5, \kappa_1 = 0.2, \text{ and } \kappa_2 = 0.9\) (red line), \(\eta_1 = 2.5, \eta_2 = 2.2, \kappa_1 = 0.05, \text{ and } \kappa_2 = 0.3\) (green line), \(\eta_1 = 0.5, \eta_2 = 0.1, \kappa_1 = 0.3, \text{ and } \kappa_2 = 0.5\) (black line), and \(\eta_1 = 0.6, \eta_2 = 1.4, \kappa_1 = 1.3, \text{ and } \kappa_2 = 1.4\) (blue line). From the plots sketched in Figure 2, we can see that the ASNFEW distribution is capable of capturing four different patterns of the PDF including bimodal (red line), modified unimodal (green line), reverse J-shaped (black line), and unimodal (blue line).

The HRF of the ASNFEW distribution is given by
Figure 1: The (a) DF and (b) SF plots of the ASNFEW distribution.

Figure 2: Different PDF plots of the ASNFEW distribution.
Some possible behaviors of the HRF of the ASNFEW distribution are shown in Figure 3. The plots in Figure 3 are sketched for $\eta_1 = 0.6, \eta_2 = 1.2, \kappa_1 = 1.3, \text{and } \kappa_2 = 1.2$ (red line), $\eta_1 = 0.5, \eta_2 = 0.5, \kappa_1 = 0.1, \text{and } \kappa_2 = 0.3$ (green line), $\eta_1 = 0.5, \eta_2 = 1.5, \kappa_1 = 0.1, \text{and } \kappa_2 = 0.5$ (blue line), $\eta_1 = 1.9, \eta_2 = 0.5, \kappa_1 = 0.5, \text{and } \kappa_2 = 0.9$ (magenta line), and $\eta_1 = 2.1, \eta_2 = 0.3, \kappa_1 = 0.4, \text{and } \kappa_2 = 0.9$ (black line). From the plots sketched in Figure 3, we can see that the ASNFEW distribution is capable of capturing five different patterns of the HRF including increasing (red line), decreasing (green line), unimodal (blue line), modified unimodal (magenta line), and bathtub (black line).

3. Basic Mathematical Properties

In this section, we will establish some statistical properties of the ASNFEW distribution.

Using expression (12) in (10), we have

$$
\mu_\prime = \frac{2}{\pi} \int_0^\infty \frac{x^\prime}{\sqrt{1 - \left(1 - \exp\left\{-e^{(\kappa_1 x^{\eta_1} - \kappa_2 x^{\eta_2})}\right\}\right)^2}} \exp\left\{-e^{(\kappa_1 x^{\eta_1} - \kappa_2 x^{\eta_2})}\right\} \, dx.
$$

3.1. Quantile Function. Let $X$ denote the ASNFEW random variable with DF given by equation (5); then, the QF (quantile function) of $X$, denoted by $Q(u)$, is given by

$$
Q(u) = e^{\eta_1 x^{\eta_1} + \eta_2 x^{\eta_2} - \log\left(1 - \sin\left(\frac{\pi}{2} u\right)\right)}.
$$

The QF (also called inverse DF) can be used to generate random numbers. Later, in Section 5, we will use the inverse DF method to carry out the simulation study.

3.2. Moments. This section deals with the derivation of the $r^{th}$ moment of the ASNFEW distribution that can be further used to obtain important characteristics. The $r^{th}$ moment of the ASNFEW distribution can be obtained as follows:

$$
\frac{1}{\sqrt{1 - t^2}} = \sum_{n=0}^{\infty} \frac{1 \times 3 \times 5 \times \cdots \times (2n-1)}{n!2^n} t^{2n}.
$$

Using equation (11), we have

Using expression (12) in (10), we have

$$
\mu_\prime = \frac{2}{\pi} \sum_{n=0}^{\infty} \frac{2n}{2n} \left(\begin{array}{c} 2n \\ i \end{array}\right) (-1)^i \frac{1 \times 3 \times 5 \times \cdots \times (2n-1)}{n!2^n} \times \exp\left\{-i e^{(\kappa_1 x^{\eta_1} - \kappa_2 x^{\eta_2})}\right\}.
$$

$$
\frac{2}{\pi} \sum_{n=0}^{\infty} \frac{2n}{2n} \left(\begin{array}{c} 2n \\ i \end{array}\right) (-1)^i \frac{1 \times 3 \times 5 \times \cdots \times (2n-1)}{n!2^n} \times \exp\left\{-i e^{(\kappa_1 x^{\eta_1} - \kappa_2 x^{\eta_2})}\right\}.
$$

$$
\mu_\prime = \frac{2}{\pi} \sum_{n=0}^{\infty} \frac{2n}{2n} \left(\begin{array}{c} 2n \\ i \end{array}\right) (-1)^i \frac{1 \times 3 \times 5 \times \cdots \times (2n-1)}{n!2^n} \times \exp\left\{-i e^{(\kappa_1 x^{\eta_1} - \kappa_2 x^{\eta_2})}\right\} \int_0^\infty x^\prime \left(\kappa_1 x^{\eta_1} + \kappa_2 x^{\eta_2}\right) e^{(\kappa_1 x^{\eta_1} - \kappa_2 x^{\eta_2})} \exp\left\{-e^{(\kappa_1 x^{\eta_1} - \kappa_2 x^{\eta_2})}\right\} \, dx.
$$
Using the series $e^{-t}$, we have
\[ e^{-t} = \sum_{j=0}^{\infty} \frac{(-1)^j}{j!} t^j. \]  
(14)

Let $t = (i+1)e^{(\kappa_1 x^{\eta_1} - \kappa_2 x^{\eta_2})}$; then, using expression (14), we get

\[ \exp \left\{ -(i+1)e^{(\kappa_1 x^{\eta_1} - \kappa_2 x^{\eta_2})} \right\} = \sum_{j=0}^{\infty} \frac{(-1)^j (i+1)^j}{j!} e^{(\kappa_1 x^{\eta_1} - \kappa_2 x^{\eta_2})}. \]  
(15)

Using expression (15) in (13), we get

\[
\mu'_r = \frac{2}{\pi} \sum_{n,j=0}^{\infty} \sum_{i=0}^{2n} \binom{2n}{i} (-1)^i (i+1)^{i+1} \frac{k_1 \times 3 \times 5 \times \cdots \times (2n-1)}{j!n!2^n} \times \int_0^{\infty} x^{r+\kappa_1 x^{\eta_1} \eta_1 - 1 + \frac{\kappa_2 \eta_2}{x^{\eta_2 + 1}}} \left( x^{(j+1)(\kappa_1 x^{\eta_1} - \kappa_2 x^{\eta_2})} \right) dx,
\]

\[
\mu'_r = \frac{2}{\pi} \sum_{n,j=0}^{\infty} \sum_{i=0}^{2n} \binom{2n}{i} (-1)^i (i+1)^{i+1} \frac{k_1 \times 3 \times 5 \times \cdots \times (2n-1)}{j!n!2^n} \times \int_0^{\infty} x^{r+\kappa_1 x^{\eta_1} \eta_1 - 1 + \frac{\kappa_2 \eta_2}{x^{\eta_2 + 1}}} e^{-((j+1)\kappa_2 x^{\eta_2})} dx,
\]

\[
\mu'_r = \frac{2}{\pi} \sum_{n,j=0}^{\infty} \sum_{i=0}^{2n} \binom{2n}{i} (-1)^i (i+1)^{i+1} \frac{k_1 \times 3 \times 5 \times \cdots \times (2n-1)}{j!n!2^n} \times \int_0^{\infty} x^{r+\kappa_1 x^{\eta_1} \eta_1 - 1 + \frac{\kappa_2 \eta_2}{x^{\eta_2 + 1}}} e^{-(j+1)\kappa_2 x^{\eta_2})} dx,
\]
3.3. Moment-Generating Function. This section offers the MGF (moment-generating function) of the ASNFEW model. Let $X$ have the ASNFEW model; then, the MGF of $X$ is derived as

$$M_X(t) = \sum_{m=0}^{\infty} \frac{t^m}{m!} \mu_r.$$  \hfill (17)

Using equations (16) in (17), we get the MGF of the ASNFEW model:

$$\mu_r = \frac{2}{\pi} \sum_{n=0}^{\infty} \left( \frac{2n}{i} \right) (-1)^{i+1} (i+1)^{i+1} (j+1)^{j+1} \Gamma \left( \frac{r+i+1}{2} \right) \Gamma \left( \frac{r+j+1}{2} \right) \eta_1^{r+i+1} \eta_2^{r+j+1} e^{-x^2/\eta_1 \eta_2},$$  \hfill (16)

For $\eta_1 = \eta_2 = 1$ and different values of $\eta_1$ and $\eta_2$, plots of the important characteristics of the ASNFEW distribution are sketched in Figure 4.

4. Maximum Likelihood Estimation

This section deals with the computation of the MLEs (maximum likelihood estimators) of the ASNFEW model parameters. Let $x_1, x_2, \ldots, x_k$ be the observed values of a sample randomly selected from the ASNFEW distribution. These moments can be used to derive the following important characteristics of the ASNFEW distribution:

(i) The CV (coefficient of variation):

$$CV = \sqrt{\frac{\mu'_3}{(\mu')^2}} - 1.$$  \hfill (18)

(ii) The CS (coefficient of skewness):

$$CS = \frac{\mu'_4 - 3 \mu'_2 \mu'_2 + 2(\mu')^3}{(\mu'_2 - (\mu')^2)^{3/2}}.$$  \hfill (19)

(iii) The CK (coefficient of kurtosis):

$$CK = \frac{4 \mu'_4 - 6 \mu'_2 (\mu'_2)^2 - 3 (\mu')^4}{(\mu'_2 - (\mu')^2)^3}.$$  \hfill (20)

For $\eta_1 = \eta_2 = 1$ and different values of $\eta_1$ and $\eta_2$, the log-likelihood function of the respective sample denoted by $\ell(\eta_1, \eta_2, \kappa_1, \kappa_2)$ is

$$\ell(\eta_1, \eta_2, \kappa_1, \kappa_2) = n \log \left( \frac{2}{\pi} \right) + \sum_{i=1}^{k} \log \left( \eta_i \kappa_1 x_i^{\eta_1-1} + \eta_2 \kappa_2 x_i^{\eta_2-1} \right) - \frac{k}{2} \sum_{i=1}^{k} \log \left( 1 - A_i \right)^2,$$

where $A_i = \exp \left\{ -e^{x_i^{\eta_1-1} - k_i^{\eta_2-1}} \right\}$. By computing the partial derivatives of $\ell(\eta_1, \eta_2, \kappa_1, \kappa_2)$ on behalf of the model parameters $(\eta_1, \eta_2, \kappa_1, \kappa_2)$, we have

$$\frac{\partial}{\partial \eta_1} \ell(\eta_1, \eta_2, \kappa_1, \kappa_2) = \kappa_1 \sum_{i=1}^{k} \left( \eta_i x_i^{\eta_1-1} \log(x_i) + x_i^{\eta_1-1} \right) + \kappa_1 \sum_{i=1}^{k} x_i^{\eta_1} \log(x_i) + \kappa_1 \sum_{i=1}^{k} \frac{x_i^{\eta_1} \log(x_i)}{(1 - (1 - A_i)^2)} - \kappa_1 \sum_{i=1}^{k} \frac{x_i^{\eta_1} e^{x_i^{\eta_1-1} - k_i^{\eta_2-1}} A_i (1 - A_i)}{(1 - (1 - A_i)^2)}$$

$$= 0,$$

$$\frac{\partial}{\partial \kappa_1} \ell(\eta_1, \eta_2, \kappa_1, \kappa_2) = \sum_{i=1}^{k} \left( \eta_i x_i^{\eta_1-1} \right) + \kappa_1 \sum_{i=1}^{k} x_i^{\eta_1} e^{x_i^{\eta_1-1} - k_i^{\eta_2-1}} A_i (1 - A_i) + \kappa_1 \sum_{i=1}^{k} \frac{x_i^{\eta_1} e^{x_i^{\eta_1-1} - k_i^{\eta_2-1}} A_i (1 - A_i)}{(1 - (1 - A_i)^2)}$$

$$= 0,$$
\[
\frac{\partial}{\partial \kappa_2} \ell (\eta_1, \eta_2, \kappa_1, \kappa_2) = \sum_{i=1}^{k} \frac{\eta_1 / \kappa_i^{\eta_1 + 1}}{(\eta_1 \kappa_i^{\eta_1} + (\eta_2 \kappa_2^{\eta_2 + 1}))} - \sum_{i=1}^{k} \frac{1}{\kappa_i^{\eta_1}} + \sum_{i=1}^{k} \frac{e^{(\eta_1 x_i^{\eta_1} - (x_i / \kappa_i^{\eta_1}))}}{\kappa_i^{\eta_1 + 1} x_i^{\eta_1}} - \kappa_1 \sum_{i=1}^{k} \frac{e^{\kappa_2 x_i^{\eta_2} - x_i / \kappa_i^{\eta_1}} A_i (1 - A_i)}{\kappa_i^{\eta_1} x_i^{\eta_1} (1 - (1 - A_i)^2)}
\]

\[
= 0,
\]

\[
\frac{\partial}{\partial \eta_2} \ell (\eta_1, \eta_2, \kappa_1, \kappa_2) = \sum_{i=1}^{k} \frac{\kappa_2}{(\eta_1 \kappa_i^{\eta_1} + (\eta_2 \kappa_2^{\eta_2 + 1}))} \left( \frac{X_i^{\eta_2 + 1} - \eta_2 X_i^{\eta_2 + 1} \log(x_i)}{X_i^{\eta_2 + 1}} \right) x_i^{\eta_1}
\]

\[
+ \kappa_2 \sum_{i=1}^{k} \log(x_i) \frac{e^{\kappa_2 x_i^{\eta_2} - x_i / \kappa_i^{\eta_1}} A_i (1 - A_i)}{\kappa_i^{\eta_1} x_i^{\eta_1} (1 - (1 - A_i)^2)}
\]

\[
- \kappa_2 \sum_{i=1}^{k} \frac{e^{(\eta_1 x_i^{\eta_1} - (x_i / \kappa_i^{\eta_1})) \log(x_i)}}{\kappa_i^{\eta_1} x_i^{\eta_1}}
\]

\[
= 0.
\]
The expressions \( \frac{\partial}{\partial \eta_1} \ell (\eta_1, \eta_2, \kappa_1, \kappa_2), \frac{\partial}{\partial \kappa_1} \ell (\eta_1, \eta_2, \kappa_1, \kappa_2), \frac{\partial}{\partial \eta_2} \ell (\eta_1, \eta_2, \kappa_1, \kappa_2), \frac{\partial}{\partial \kappa_2} \ell (\eta_1, \eta_2, \kappa_1, \kappa_2) \) do not have closed forms. Henceforth, the numerical estimates of the parameters can be obtained via an iterating process using computer software (see Supplementary Materials (available here)). In order to show the uniqueness of the MLEs of the ASNF EW distribution, the profiles of the log-likelihood functions of the parameters are sketched in Figure 5.

5. Simulation Study

This section deals with the Monte Carlo simulation study to evaluate the MLEs of the ASNF EW distribution parameters. The ASNF EW distribution is easily simulated by inverting equation (5) as follows:

\[
e^{\kappa_1 x^{\eta_1} - \kappa_2 / x^{\kappa_2}} - \log \left( 1 - \sin \left( \frac{\pi x}{2} \right) \right) = 0.
\]  

(23)

The simulation results are obtained for two different sets of parameters: (i) \( \eta_1 = 1.5, \kappa_1 = 0.5, \eta_2 = 0.5, \) and \( \kappa_2 = 1.2 \) and (ii) \( \eta_1 = 0.7, \kappa_1 = 0.5, \eta_2 = 0.9, \) and \( \kappa_2 = 1. \)

The random number generation is done using the inverse DF approach. Note that the inverse process and simulation results are obtained using a well-known statistical software R with a (rootSolve) library through the command \texttt{optim()}. For \( i = 1, 2, \ldots, 1000, \) the MLEs \((\hat{\eta}_1, \hat{\kappa}_1, \hat{\eta}_2, \hat{\kappa}_2)\) of \((\eta_1, \kappa_1, \eta_2, \kappa_2)\) are obtained for each set of simulated data. The bias and mean square error (MSE) were then computed for accessing the model adequacy. These quantities are calculated as follows:

\[
\text{bias}(\hat{\tau}) = \frac{1}{1000} \sum_{i=1}^{1000} (\hat{\tau} - \tau),
\]

\[
\text{MSE}(\hat{\tau}) = \frac{1}{1000} \sum_{i=1}^{1000} (\hat{\tau} - \tau)^2,
\]

(24)

where \( \tau = (\eta_1, \kappa_1, \eta_2, \kappa_2) \).

Corresponding to the first set of parameter values \((\eta_1 = 1.5, \eta_2 = 0.5, \kappa_1 = 0.5, \) and \( \kappa_2 = 1.2)\), the simulation results are provided in Table 1, whereas for the second set of parameter values \((\eta_1 = 0.7, \eta_2 = 0.9, \kappa_1 = 0.5, \) and \( \kappa_2 = 1. \)), the simulation results are provided in Table 2.

6. An Application to the Survival Times of COVID-19 Patients in China

A clear motivation behind proposing new statistical distributions is to increase the level of flexibility and applicability of the existing distributions. The main objective of proposing the ASNF EW distribution is its utilization in describing different types of time-to-event data. In this section, the respective fact is demonstrated using the data of fifty-three patients taken from a hospital in China. The data set is given as follows: 0.054, 0.064, 0.704, 0.816, 0.235, 0.976, 0.865, 0.364, 0.479, 0.568, 0.352, 0.978, 0.787, 0.976, 0.087, 0.548, 0.796, 0.458, 0.087, 0.437, 0.421, 1.978, 1.756, 2.089, 2.643, 2.869, 3.867, 3.890, 3.543, 3.079, 3.646, 3.348, 4.093, 4.092, 4.190, 4.237, 5.028, 5.083, 6.174, 6.743, 7.274, 7.058, 8.273, 9.324, 10.827, 11.282, 13.324, 14.278, 15.287, 16.971, 17.209, 19.092, and 20.083. The basic summary measures of the data are given in Table 3.

The histogram and box plot of COVID-19 data along with the total time test (TTT) plot are sketched in Figure 6.

We illustrate the best fitting power of the ASNF EW model and compare its fitting results with the other (i) three-parameter distributions such as Marshall–Olkin–Weibull (MOW) [24], (ii) four-parameter distributions such as Kumaraswamy–Weibull (Ku-W) [25], odd log-logistic modified Weibull (OLL-MW) [26], transmuted modified Weibull (T-MW) [27], and Frechet–Weibull (FrW) [28], and (iii) five-parameter model called beta modified Weibull (B-MW) [29].

The PDFs of the competitive models are as follows:

(i) MOW distribution:

\[
f(x) = \frac{\alpha \kappa_1 x^{\alpha - 1} e^{-\kappa_1 x^\alpha}}{1 - (1 - \sigma) e^{\eta_1 x} z^{\kappa_1}}, \quad x > 0.
\]

(25)

(ii) Ku-W distribution:

\[
f(x) = abx_1x^{a - 1}e^{-\kappa_1 x^a}(1 - e^{-\kappa_1 x^a})^{a - 1}
\]

\[
\left[ 1 - (1 - e^{-\kappa_2 x^\kappa_2})^{b - 1}, \quad x > 0.
\]

(26)

(iii) OLL-MW distribution:

\[
f(x) = \frac{\alpha (\kappa_1 x^a + \kappa_2 e^{-\kappa_1 x^a - \kappa_2 x})^{a - 1}}{e^{ak_1 x^a} - e^{ak_2 x}}, \quad x > 0.
\]

(27)

(iv) T-MW distribution:

\[
f(x) = \frac{\alpha \kappa_1 x^{a - 1} + \kappa_2 e^{-\kappa_1 x^{-1} x - \kappa_2 x}}{(1 - \lambda + 2\lambda e^{-\kappa_1 x^{-1} x - \kappa_2 x})^{a - 1}}, \quad x > 0.
\]

(28)

(v) FrW distribution:

\[
f(x) = \frac{abx_1 x^{a - 1} + \kappa_2 e^{-\kappa_1 x^a - \kappa_2 x}}{(1 - \lambda + 2\lambda e^{-\kappa_1 x^a - \kappa_2 x})^a}, \quad x > 0.
\]

(29)

(vi) B-MW distribution:

\[
f(x) = \frac{\kappa_1 x^{a - 1} (\alpha + \kappa_2 x)}{B(a, b)} \left( 1 - e^{-\kappa_1 x_1 x^{a - 1}} \right)^{a - 1}, \quad x > 0.
\]

(30)

The decision about the best fitting of the competing distributions is made by considering certain criteria selected for comparison. These criteria consist of some discrimination and goodness-of-fit measures. The expressions of the discrimination measures (DM) are given as follows:

(i) The AIC (Akaike information criterion):

\[
\text{AIC} = 2k - 2\ell(\Phi).
\]

(31)
(ii) The BIC (Bayesian information criterion):  \[ \text{BIC} = k \log(n) - 2\ell(\Phi). \]  

(32)

(iii) The HQIC (Hannan–Quinn information criterion):  \[ \text{HQIC} = 2k \log(\log(n)) - 2\ell(\Phi). \]  

(33)

(iv) The CAIC (corrected Akaike information):  \[ \text{CAIC} = \frac{2nk}{n-k-1} - 2\ell(\Phi). \]  

(34)

The expressions of the goodness-of-fit measures are given as follows:

(i) The AD (Anderson–Darling) test statistic:  \[ \text{AD} = -n - \frac{1}{n} \sum_{i=1}^{n} (2i - 1) \left[ \log G(x_i) + \log \left[ 1 - G(x_{n-i+1}) \right] \right]. \]  

(35)

(ii) The CM (Cramer–von Mises) test statistic:  \[ \text{CM} = \frac{1}{12n} + \sum_{i=1}^{n} \left[ \frac{2i-1}{2n} - G(x_i) \right]^2. \]  

(36)

(iii) The KS (Kolmogorov–Smirnov) test statistic:  \[ \text{KS} = \sup_x |G_n(x) - G(x)|. \]  

(37)
### Table 1: Simulation results of the ASNFEW distribution for the first set of parameter values.

**Set 1: \( \eta_1 = 1.5, \eta_2 = 0.5, \kappa_1 = 0.5, \text{ and } \kappa_2 = 1.2 \)**

| \( n \) | Par. | MLE      | MSEs       | Biases  |
|---------|------|----------|------------|---------|
| 10      | \( \eta_1 \) | 2.309541 | 2.5797791 | 1.109540 |
|         | \( \eta_2 \) | 0.948288 | 1.4612937 | 0.848288 |
|         | \( \kappa_1 \) | 0.880968 | 1.611005  | 0.819031 |
|         | \( \kappa_2 \) | 1.781896 | 1.622253  | 0.881895 |
| 50      | \( \eta_1 \) | 1.847816 | 1.9800997 | 0.947816 |
|         | \( \eta_2 \) | 0.817015 | 1.244477 | 0.710701 |
|         | \( \kappa_1 \) | 0.726937 | 1.221523 | 0.737306 |
|         | \( \kappa_2 \) | 1.485075 | 1.149649 | 0.714924 |
| 100     | \( \eta_1 \) | 1.789175 | 1.701112 | 0.889175 |
|         | \( \eta_2 \) | 0.754896 | 0.925639 | 0.654896 |
|         | \( \kappa_1 \) | 0.668901 | 0.952687 | 0.543109 |
|         | \( \kappa_2 \) | 1.483073 | 0.911437 | 0.501692 |
| 200     | \( \eta_1 \) | 1.607212 | 1.394093 | 0.547211 |
|         | \( \eta_2 \) | 0.655961 | 0.788634 | 0.385961 |
|         | \( \kappa_1 \) | 0.564289 | 0.720769 | 0.295710 |
|         | \( \kappa_2 \) | 1.382883 | 0.708039 | 0.217116 |
| 400     | \( \eta_1 \) | 1.577385 | 1.456242 | 0.427348 |
|         | \( \eta_2 \) | 0.625364 | 0.657897 | 0.293646 |
|         | \( \kappa_1 \) | 0.532675 | 0.542984 | 0.144732 |
|         | \( \kappa_2 \) | 1.340878 | 0.564954 | 0.151220 |
| 600     | \( \eta_1 \) | 1.546882 | 0.894580 | 0.288252 |
|         | \( \eta_2 \) | 0.553193 | 0.453964 | 0.153193 |
|         | \( \kappa_1 \) | 0.569309 | 0.231666 | 0.103069 |
|         | \( \kappa_2 \) | 1.319894 | 0.257282 | 0.111053 |
| 800     | \( \eta_1 \) | 1.511490 | 0.318580 | 0.121489 |
|         | \( \eta_2 \) | 0.516691 | 0.149273 | 0.096914 |
|         | \( \kappa_1 \) | 0.517491 | 0.120298 | 0.022250 |
|         | \( \kappa_2 \) | 1.238630 | 0.104759 | 0.011369 |
| 1000    | \( \eta_1 \) | 1.484861 | 1.494595 | 1.198486 |
|         | \( \eta_2 \) | 1.885658 | 1.235921 | 0.981434 |
|         | \( \kappa_1 \) | 0.952314 | 0.932594 | 0.857468 |
|         | \( \kappa_2 \) | 1.530815 | 1.132786 | 0.933081 |

### Table 2: Simulation results of the ASNFEW distribution for the second set of parameter values.

**Set 1: \( \eta_1 = 0.7, \eta_2 = 0.9, \kappa_1 = 0.5, \text{ and } \kappa_2 = 1.2 \)**

| \( n \) | Par. | MLE      | MSEs       | Biases  |
|---------|------|----------|------------|---------|
| 10      | \( \eta_1 \) | 1.484861 | 1.494595 | 1.198486 |
|         | \( \eta_2 \) | 1.885658 | 1.235921 | 0.981434 |
|         | \( \kappa_1 \) | 0.952314 | 0.932594 | 0.857468 |
|         | \( \kappa_2 \) | 1.530815 | 1.132786 | 0.933081 |
| 50      | \( \eta_1 \) | 1.293477 | 1.346326 | 1.043477 |
|         | \( \eta_2 \) | 1.622404 | 1.145529 | 0.912240 |
|         | \( \kappa_1 \) | 0.865659 | 0.886875 | 0.784343 |
|         | \( \kappa_2 \) | 1.316718 | 1.016277 | 0.891671 |
| 100     | \( \eta_1 \) | 1.184970 | 1.224046 | 0.949701 |
|         | \( \eta_2 \) | 1.497052 | 1.017400 | 0.870520 |
|         | \( \kappa_1 \) | 0.800769 | 0.828047 | 0.649230 |
|         | \( \kappa_2 \) | 1.257491 | 0.976931 | 0.757491 |
| 200     | \( \eta_1 \) | 1.108270 | 1.109240 | 0.827004 |
|         | \( \eta_2 \) | 1.448372 | 0.984237 | 0.744837 |
|         | \( \kappa_1 \) | 0.749676 | 0.730843 | 0.560323 |
|         | \( \kappa_2 \) | 1.135901 | 0.876300 | 0.635900 |
| 400     | \( \eta_1 \) | 0.950793 | 0.927471 | 0.730793 |
|         | \( \eta_2 \) | 1.397758 | 0.872203 | 0.677582 |
|         | \( \kappa_1 \) | 0.651045 | 0.653168 | 0.448954 |
|         | \( \kappa_2 \) | 1.102208 | 0.747960 | 0.520848 |
The term “best fitting” is used in the sense of a model having smaller values of the considered criterion. The MLEs and DM of the fitted distributions are provided in Tables 4 and 5, respectively, whereas the goodness-of-fit measures are provided in Table 6.

From Tables 5 and 6, we can see that corresponding to the COVID-19 data set, the values of the DM for the ASNF EW distribution are \( AIC = 270.4675, CAIC = 271.3008, BIC = 278.3487, \) and \( HQIC = 273.4982 \) and the values of the goodness-of-fit quantities with the
### Table 5: DM of the competing models corresponding to COVID-19 data.

| Model   | AIC       | CAIC      | BIC       | HQIC      |
|---------|-----------|-----------|-----------|-----------|
| ASNFEW  | 270.4675  | 271.3008  | 278.3487  | 273.4982  |
| MOW     | 273.3886  | 273.8784  | 279.2994  | 275.6616  |
| Ku-W    | 274.1946  | 275.0279  | 282.0757  | 277.2253  |
| T-MW    | 274.7410  | 275.5743  | 282.6222  | 277.7717  |
| OLL-MW  | 275.1094  | 275.9765  | 284.4875  | 278.0921  |
| FW      | 292.8950  | 293.7280  | 300.7760  | 295.9260  |
| B-MW    | 274.0107  | 275.2873  | 283.8622  | 277.7991  |

### Table 6: Goodness-of-fit measures of the competing models corresponding to COVID-19 data.

| Model   | CM       | AD        | KS        | p value  |
|---------|----------|-----------|-----------|----------|
| ASNFEW  | 0.0501   | 0.3433    | 0.0729    | 0.9407   |
| MOW     | 0.0772   | 0.4957    | 0.1290    | 0.3405   |
| Ku-W    | 0.0740   | 0.4640    | 0.1386    | 0.2600   |
| T-MW    | 0.0756   | 0.4750    | 0.1240    | 0.3885   |
| B-MW    | 0.0797   | 0.4976    | 0.1435    | 0.3024   |
| FrW     | 0.2473   | 1.6930    | 0.1435    | 0.2250   |
| OLL-MW  | 0.0814   | 0.5209    | 0.1506    | 0.2815   |

![Figure 7: Continued.](image-url)
Figure 7: Graphical illustration of the results presented in Table 4. (a) AIC, (b) BIC, (c) CAIC, and (d) HQIC of the fitted models.

Figure 8: Continued.
Figure 8: Graphical illustration of the results presented in Table 5. (a) CM, (b) AD, (c) KS, and (d) p value of the fitted models.

Figure 9: The (a) estimated PDF, (b) estimated DF, (d) PP, and (c) Kaplan–Meier survival plots of the ASNFEW distribution for the COVID-19 data.
corresponding $p$ value are CM = 0.0501, AD = 0.3433, and KS = 0.0729 with $p$ value = 0.9407, whereas the values of these measures for the second best model (MOW distribution) are AIC = 273.3886, CAIC = 273.8784, BIC = 279.2994, HQIC = 275.6616, CM = 0.0772, AD = 0.4957, and KS = 0.1290, with $p$ value = 0.3405. From the above reported results, we can see that the ASNFEW distribution has smaller values of the analytical measures and a high $p$ value indicating a closer fit to data. Furthermore, for the best description of the results provided in Tables 5 and 6, a graphical display of these results is provided in Figures 7 and 8.

Based on the numerical results in Tables 5 and 6 as well as the graphical display of these results in Figures 7 and 8, we can conclude that the ASNFEW is a useful and suitable candidate distribution for modeling COVID-19 and other related data sets. For further confirmation of the best fitting capability of the ASNFEW distribution, the plots of the estimated PDF, DF, SF, and PP (probability-probability) are sketched in Figure 9. The plots sketched in Figure 9 show a closer fit of the ASNFEW distribution to the data under consideration.

7. Concluding Remarks

The statistical distributions have proven to be of great importance and attracted the attention of researchers to use them for modeling data, particularly, in the fields related to lifetime events. In this work, a generalization of the new flexible extended Weibull model is proposed. An example of real-life data related to the survival times of COVID-19 patients in China is used to demonstrate the applicability of the ASNFEW distribution. Comparisons of the proposed model with the other competitors, including four-parameter and five-parameter models, are provided. Certain analytical tools, including four discrimination measures and three goodness-of-fit measures, are considered to compare the fitted distributions. The numerical results of these analytical measures showed that the ASNFEW model provides a better fit, supported by the graphical sketching and numerical tools. In summary, the proposed model has a wide range of applications because of its flexibility in modeling different types of hazard functions. A simulation study also reveals that the proposed model can be valuable in appropriately describing different types of time-to-event data. We hope that beyond the scope of this paper, the ASNFEW distribution can be used to analyze other forms of the data related to COVID-19 events.

Data Availability

The data set used to support this study is included within the article. However, additional data (if required) will be provided upon request to the corresponding author.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Supplementary Materials

The R-code used for analysis in this paper is provided as a supplementary file. (Supplementary Materials)

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