Hyperon production from neutrino-nucleon reaction

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Abstract

The neutrino induced hyperon production processes $\bar{\nu}_e/\mu + p \rightarrow e^+/\mu^+ + \pi + \Lambda/\Sigma$ may provide a unique clean place for studying low energy $\pi\Lambda/\Sigma$ interaction and hyperon resonances below $K\bar{N}$ threshold. The production rates for some neutrino induced hyperon production processes are estimated with theoretical models. Suggestions are made for the study of hyperon production from neutrino-nucleon reaction at present and future neutrino facilities.

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I. INTRODUCTION

The nature of the lowest hyperon resonances is one of the key puzzling issues in hadron spectroscopy [1]. The lowest iso-scalar excited hyperon $\Lambda^*(1405)$ has been ascribed as a $\bar{K}N-\Sigma\pi$ dynamically generated state [2–6], or a pure $|qqq\rangle$ state [7, 8], or a $|qqq\rangle + |qqqq\rangle\bar{q}$ state [1, 9]. Where is its iso-vector partner is still not settled. Evidence is accumulating for a low-lying $\Sigma^*$ around 1390 MeV [10, 11]. Up to now, most experimental data on these hyperon resonances came from $\bar{K}+N$ reaction. Since these lowest hyperon resonances have masses below $\bar{K}N$ threshold, they can only be observed in multi-hadron final states, suffering complicated strong final state interactions as well as initial state interaction. To avoid such complicated strong initial and final state interactions, the $\bar{\nu}_l+p\rightarrow l+\Lambda^*/\Sigma^*$ reactions would be an ideal clean place to study the low energy $\pi\Lambda/\Sigma$ interaction and hyperon resonances below $\bar{K}N$ threshold.

On the other hand, in recent years, because of the hot topic study of neutrino oscillation, the neutrino experiments develop rather fast [12]. With increasing intensity of neutrino fluxes, there are many new measurements of neutrino nuclear scattering from various groups, such as MINOS [13], NuTeV [16], SciBooNE [14], ArgoNeuT [15], MiniBooNE [17] at Fermi Lab, and NOMAD [18] at CERN, and K2K [19] in Japan. Although the main purpose of these experiments is to get neutrino-nuclear interaction information needed for the study of neutrino oscillation, these experiments may also provide us a good opportunity to study baryon spectroscopy and baryon structure by the neutrino probes. For the $\bar{\nu}_l+p\rightarrow l+\Lambda^*/\Sigma^*$ reactions, the interaction vertices are neutrino-lepton-W boson coupling and s quark-u quark-W boson coupling, which are both well defined in the Standard Model (SM) [12]. If the quark wave functions of baryons are known, the reaction would be well described. In other words, the $\bar{\nu}_l+p\rightarrow l+\Lambda^*/\Sigma^*$ reactions may provide us a nice place to explore the quark distribution in the hyperon resonances.

With these potential advantages for the study of hyperon spectroscopy and hyperon structure in our mind, here we estimate the $\Sigma^*_0(1193)$, $\Sigma^*_0(1385)$, $\Lambda(1115)$ and $\Lambda(1405)$ production rates in the $\bar{\nu}_l+p\rightarrow l+\Lambda^*/\Sigma^*$ reactions with some theoretical models to get some ideas about the possibility to study hyperon resonances from these reactions.

In this paper, three theoretical models are used, namely, the non-relativistic 3-quark model (NR3QM), the vector and axial vector currents (V-A) with form factors from either some experiment data (EXP) or some relativistic three quark model (R3QM).

For the non-relativistic 3-quark model (NR3QM), the baryon wave functions from some non-relativistic 3-quark model and the standard vertices of $\nu_l$-W and $u$-$s$-W are used to compute the total cross sections of $\bar{\nu}_l+p\rightarrow l^++\Sigma^0(1193)/\Sigma^*(1385)/\Lambda(1115)/\Lambda(1405)$. The SU(3) symmetry breaking effect is studied in this model.

For the V-A approach, the effective form factors are assumed for the p-$\Lambda/\Sigma$-W vertices [20] to calculate the $\bar{\nu}_l+p\rightarrow l^++\Sigma^*/\Lambda$ reactions. The effective form factors can be either extracted from the data on weak decays of $\Lambda$ and $\Sigma$ (EXP) [21, 22] or from some relativistic three quark model (R3QM) [23]. However, due to lack of data of hyperon excited resonances’ weak decays, the EXP method can be used only for the calculation of $\Lambda(1115)/\Sigma^0(1193)$ production cross sections from the reactions $\bar{\nu}_l+p\rightarrow l^++\Lambda(1115)/\Sigma^0(1193)$. These results can help us to cross check whether the predictions from the non-relativistic 3-quark model are reasonable or not. Actually, our calculation shows that even two sets of form factor parameters from the same $\Lambda(1115)$ weak decay experiment data [21] give very different $\Lambda(1115)$ neutrino production rate with a factor of 5 difference. Obviously, the detection of...
\( \bar{v}_l + p \rightarrow l^+ + \Lambda (1115) \) reaction will help us to understand the detail of the form factors of p-\( \Lambda \)-W vertex. Furthermore, the precise knowledge of these form factors are extremely important to precisely measure \( |v_{us}| \) in the Cabibbo-Kobayashi-Maskawa (CKM) matrix.

In short, the neutrino induced hyperon production processes \( \bar{\nu}_{e/\mu} + p \rightarrow e^+/\mu^+ + \pi + \Lambda/\Sigma \) may provide a unique clean place for studying low energy \( \pi \Lambda/\Sigma \) interaction and hyperon resonances below \( K\Lambda \) threshold, while the \( \bar{\nu}_{e/\mu} + p \rightarrow e^+/\mu^+ + \Lambda/\Sigma \) may be used to explore the internal structure of \( \Lambda/\Sigma \) as well as for the precise measurement of \( |v_{us}| \).

In the next section, we present the formulae for calculating the cross sections of neutrino nucleon reactions and branch ratios of \( \Lambda (1115) \) and \( \Sigma (1193) \) weak decays by three different models. Our results are presented in section III. In section V, we give a summary and discuss the necessary future work.

II. FORMALISM

The Feynman diagrams for some typical neutrino nucleon reactions and \( \Lambda (1115)/\Sigma (1193) \) weak decays by the W boson exchange are shown in Fig.1

There are two vertices: \( T^\mu_1 \) for the neutrino-lepton-W boson interaction and \( T^\nu_2 \) for the nucleon-baryon-W boson interaction.

The standard vertex for the neutrino-lepton-W boson in SM is:

\[
T^\mu_1 = \sqrt{\frac{G_F m_W}{2}} (\bar{l} \gamma^\mu (1 - \gamma^5) l + h.c.),
\]

where \( G_F \) is the Fermi constant; \( m_W \) is the mass of W boson.

For the nucleon-baryon-W boson interaction, \( T^\nu_2 \), we use NR3QM, and V-A theory with the form factors from EXP [21] and R3QM [23] to perform our calculation. The detailed formulae for \( T^\nu_2 \) are given in the next two subsections.

Besides the vertices, the propagator of W boson is also needed:

\[
G^\mu_\nu = \frac{-g^\mu_\nu + p^\mu p^\nu_W/m_W^2}{p^2_W - m_W^2},
\]

where \( p_W \) is the momentum of W boson.
where $p_W$ is the four momentum of the W boson. Because of the heavy mass of W boson, we neglect $p_W^2/m_W^2$ in the following calculation. With the $T_1^\mu$, $T_2^\nu$ and $G^\mu\nu_W$, the amplitudes of the reactions can be easily obtained:

$$\mathcal{M} = T_1^\mu T_2^\nu G_{W \mu\nu}.$$  

(3)

The cross section of $\nu_i(\bar{\nu}_i) + N_1 \rightarrow l^- (l^+) + N_2$ can be calculated from the amplitude $\mathcal{M}$:

$$d\sigma = \frac{(2\pi)^4}{2E_\nu} \frac{1}{4} \sum_{s_i'} \sum_{s_iN} |\mathcal{M}|^2 \delta^{(4)}_{(p_\nu + p_{N_1} - p_l - p_{N_2})} \frac{d^3\bar{p}_N m_{N_1}}{(2\pi)^3 E_{N_1}} \frac{d^3\bar{p}_m m_i}{(2\pi)^3 E_i},$$  

(4)

where $p_i$ and $\bar{p}_i$ are the four and three momentum of particle $i$, respectively; $E_i$ is the energy of particle $i$; $s'_z$ is the $z$ component of spin of particle $i$. All of these momenta are in the initial baryon rest system.

The formula for calculating the width of $\Lambda \rightarrow \bar{\nu}_l + l^- + p$ is:

$$d\Gamma = (2\pi)^{-1} \sum_{s'_z} \sum_{s_z} \sum_{s_z} |\mathcal{M}|^2 \delta^{(4)}_{(p_\nu + p_l + p_{N_1})} \frac{d^3\bar{p}_m m_p}{(2\pi)^3 E_p} \frac{d^3\bar{p}_m m_i}{(2\pi)^3 E_i}.$$  

(5)

### A. The $T_2^\nu$ in the NR3QM

In our NR3QM calculation, the wave functions of baryons are taken from Refs.\[7, 9, 24\] as the following:

$$|N(939)\rangle = 0.90|N_8^2S_s\rangle + 0.34|N_8^2S'_s\rangle - 0.27|N_8^2S_M\rangle,$$  

(6)

$$|\Lambda(1115)\rangle = 0.93|\Lambda_8^2S_s\rangle + 0.30|\Lambda_8^2S'_s\rangle - 0.20|\Lambda_8^2S_M\rangle,$$  

(7)

$$|\Sigma(1193)\rangle = 0.97|\Sigma_8^2S_s\rangle + 0.18|\Sigma_8^2S'_s\rangle - 0.16|\Sigma_8^2S_M\rangle,$$  

(8)

$$|\Lambda^*(1405)\rangle = 0.90|\Lambda_8^2P_A\rangle - 0.43|\Lambda_8^2P_M\rangle + 0.06|\Lambda_8^4P_M\rangle,$$  

(9)

$$|\Delta^{++}\rangle = |\Delta^{++}S\rangle,$$  

(10)

$$|\Sigma^{*0}(1385)\rangle = |\Sigma^{*0}(1385)S\rangle.$$  

(11)

where the wave functions of $N(939)$, $\Lambda(1115)$, $\Sigma(1193)$, and $\Lambda^*(1405)$ have taken into account the SU(3) breaking effect, while the wave functions of $\Delta$ and $\Sigma^*(1385)$ follow the SU(3) symmetry. The detailed wave functions are given in Appendix A. In the following calculations, we consider two cases: including the SU(3) breaking effect (named NR3QM-Full) or not (named NR3QM-Single). For the latter, only the first term in Eq.\[6\] is kept, i.e., $|N(939)\rangle = |N_8^2S_s\rangle$, $|\Lambda(1116)\rangle = |\Lambda_8^2S_s\rangle$, $|\Sigma(1193)\rangle = |\Sigma_8^2S_s\rangle$ and $|\Lambda^*(1405)\rangle = |\Lambda_8^2P_A\rangle$.

Unfortunately, we do not find the wave functions of $\Delta^{++}$ and $\Sigma^{*0}(1385)$ with the SU(3) breaking effect, so we only use NR3QM-Single to calculate. The parameters in the NR3QM are the masses of u, d, and s quark, as well as $\omega_3$, the oscillator parameter for the orbital wave function.

The nucleon-baryon-W boson interaction can be described at quark level as shown in Fig.\[2\] where $Q_1$ and $Q_2$ represent the baryon in the initial state and final state, respectively. The Jacobin momenta used for the initial and final state spatial wave functions are defined as:

$$\vec{p}_1 \rho = \frac{\vec{p}_a - \vec{p}_b}{\sqrt{2}},$$  

(12)
\[ \vec{p}_1 \lambda = \frac{\vec{p}_a + \vec{p}_b - 2\vec{p}_{q1}}{\sqrt{6}} = -\frac{3\vec{p}_{q1}}{\sqrt{6}}, \] (13)

\[ \vec{p}_2 \rho = \frac{\vec{p}_a - \vec{p}_b}{\sqrt{2}}, \] (14)

\[ \vec{p}_2 \lambda = \frac{\vec{p}_a + \vec{p}_b - 2\vec{p}_{q2}}{\sqrt{6}} = -\frac{3\vec{p}_{q2} + 2\vec{p}_w}{\sqrt{6}}. \] (15)

\[ \begin{array}{cccc}
Q_1 & \vec{p}_1 & q_1 & \vec{p}_{q_1} \\
 & \vec{p}_a & & \\
d & \vec{p}_b & & \\
Q_2 & \vec{p}_2 & q_2 & \\
 & & & \end{array} \]

FIG. 2: Diagram of the nucleon-baryon-W interaction at quark level.

In the NR3QM, the \( T_2^\nu \) can be calculated in the initial nucleon at rest system as the following:

\[
T_2^\nu(\vec{p}_w, s_z^{Q1}, s_z^{Q2}) = \int d\vec{p}_a d\vec{p}_b d\vec{p}_{q1} d\vec{p}_{q2} \delta(\vec{p}_a + \vec{p}_b + \vec{p}_{q1}) \delta(\vec{p}_{q2} - \vec{p}_{q1} - \vec{p}_w) \times \sqrt{\frac{G_{Fm_W}}{\sqrt{2}}} |v_{us}| (V^\mu + A^\mu)
\]

\[
\times \sum s_z^{q1} s_z^{q2} <X^{Q2}, s_z^{Q2}, \Phi^{Q2}|\chi^+_{q2}, s_z^{q2}, X^{Q1}, s_z^{Q1}, \Phi^{Q1}|X^{Q1}, s_z^{Q1}, \Phi^{Q1}> \times \vec{u}_q(\vec{p}_{q2}, s_z^{q2}) \gamma^\nu (1 - \gamma^5) u_q(\vec{p}_{q1}, s_z^{q1}), \] (16)

where \( s_z^i \) is the z component of spin of particle \( i \); \( |X^{Q_i}, s_z^{Q_i}, \Phi^{Q_i}> \) is the flavor, spin and spatial wave function of baryon \( Q_i \); \( v_{us} \) is the CKM element of quark \( q_1 \) and \( q_2 \); \( \chi^+_{q2}, s_z^{q2} \) and \( \chi_{q1}, s_z^{q1} \) is the creation and annihilation operator of quark \( q_i \) with spin \( s_z^{q_i} \), respectively; \( u_q(\vec{p}_q) \) is the spinor wave function of quark \( q_i \). We also give an example of \( T_2^\nu \) in Appendix B.

B. The \( T_2^\nu \) in the EXP and R3QM approaches

At the hadronic level, the baryon-baryon-W boson interaction can be expressed as effective vector and axial vector current as usual. In the V-A theory, the \( B-N-W \) (\( B=\Lambda(1115), \Sigma(1193) \) and \( N=p, n \)) boson vertex can be written as:

\[
T_{2 BNW}^\nu = \sqrt{\frac{G_{Fm_W}}{\sqrt{2}}} |v_{us}| (V^\mu + A^\mu)
\]

with \( V^\mu = B \left( f_1(q^2) \gamma^\mu - i f_2(q^2) \sigma^\mu_5 q^\mu \frac{m_B}{m_B} + f_3(q^2) \frac{q^\mu}{m_B} \right) N + h.c. \).
\[ A^\mu = B \left( g_1(q^2) \gamma^\mu - i \frac{g_2(q^2) \sigma^{\mu\nu} q_\nu}{m_B} + g_3(q^2) \frac{q_\mu}{m_B} \right) \gamma^5 N + h.c. \] (19)

where \( q = p_B - p_N \). According to Refs. [20, 21, 23], the contribution of \( f_3, g_2 \) and \( g_3 \) are negligibly small. Therefore in the following calculations we only keep the vector, axial-vector and weak magnetism form factors, \( f_1, g_1 \) and \( f_2 \). They are parameterized slightly different in the literalism. As usual, the dipole form for \( f_1 \) and \( g_1 \), and constant form for \( f_2 \) are used:

\[
f_1(q^2) = \frac{f_1(0)}{(1 - q^2/M_1^2)^2},
\]

\[
g_1(q^2) = \frac{g_1(0)}{(1 - q^2/M_1^2)^2},
\]

\[
f_2(q^2) = f_2(0).
\]

These form factor parameters are obtained from fits to experiment data of hyperon weak decays [21, 22] and are shown in Table I.

On the other hand, in Ref. [23], these form factors are obtained from a relativistic quark model calculation with the parametrization form as:

\[
f_1(q^2) = \frac{f_1(0)}{1 - q^2/\Lambda_1^2 + q^4/\Lambda_2^4},
\]

\[
g_1(q^2) = \frac{g_1(0)}{1 - q^2/\Lambda_1^2 + q^4/\Lambda_2^4},
\]

\[
f_2(q^2) = f_2(0).
\]

The parameters are also listed in Table I.

TABLE I: Parameters of form factors from fits of hyperon weak decays and relativistic quark model.

| Model         | Ref.   | \( f_1(0) \) | \( \Lambda_1(\text{GeV}) \) | \( \Lambda_2(\text{GeV}) \) | \( g_1(0) \) | \( \Lambda_1(\text{GeV}) \) | \( \Lambda_2(\text{GeV}) \) | \( f_2(0) \) |
|---------------|--------|---------------|-----------------------------|-----------------------------|-------------|-----------------------------|-----------------------------|-----------|
| \( \Lambda(1115)-p-W \) | R3QM [23] | -1.19         | 0.71                        | 0.98                        | 0.99        | 0.81                        | 1.12                        | -0.95     |
|               | EXP-A  [21, 25] | -\sqrt{3/2}   | \( M_v \) = 0.97            | 0.88                        | \( M_a \) = 1.25 | -1.19                      |
|               | EXP-B  [21, 25] | -\sqrt{3/2}   | \( M_v \) = 0.97            | 0.90                        | \( M_a \) = 1.25 | -0.18                      |
| \( \Sigma(1193)-p-W \) | R3QM [23] | -0.69         | 0.64                        | 0.84                        | -0.19       | 0.83                        | 1.16                        | 0.53      |
|               | EXP [22] | -\sqrt{1/2}   | \( M_v \) = 0.97            | -0.231                      | \( M_a \) = 1.25 | 0.68                       |
| \( \Sigma^- (1193)-n-W \) | R3QM [23] | -0.97         | 0.64                        | 0.84                        | -0.27       | 0.83                        | 1.16                        | 0.74      |
|               | EXP [22] | -1            | \( M_v \) = 0.97            | -0.327                      | \( M_a \) = 1.25 | 0.96                       |

III. RESULTS AND DISCUSSION

With the formalism and ingredients given in last section, we compute the total cross sections of \( \bar{\nu}_\mu + p \rightarrow \mu^+ + n, \nu_\mu + n \rightarrow \mu^- + p, \nu_\mu + p \rightarrow \mu^- + \Delta^{++}, \bar{\nu}_e/\mu + p \rightarrow e^+/\mu^+ + \Lambda(1115), \)
\[ \bar{\nu}_{e/\mu} + p \rightarrow e^+ / \mu^+ + \Lambda^+(1405), \bar{\nu}_{e/\mu} + p \rightarrow e^+ / \mu^+ + \Sigma^0(1193), \bar{\nu}_{e/\mu} + p \rightarrow e^+ / \mu^+ + \Sigma^{0*}(1385) \]

for the neutrino beam energies \( E_\nu \) up to 2 GeV, and the decay widths of \( \Lambda(1115) \rightarrow \bar{\nu}_{e/\mu} + e^- / \mu^- + p \) and \( \Sigma(1193) \rightarrow \bar{\nu}_{e/\mu} + e^- / \mu^- + N \). For the \( \nu N \) reactions, when \( E_\nu = 2 \) GeV, the momentum-transfer \( Q^2 = (p_B - p_N)^2 \) is about \( 0 \sim -5.5 \) GeV\(^2\), and the three momentum of outgoing baryon is about \( 0.2 \sim 3.2 \) GeV.

A. Results from the NR3QM

In this model, two types of wave function of baryons, with and without including SU(3) breaking effect, are used in our calculations. As mentioned in Sec. II A, the parameters in the NR3QM are the masses of \( u, d, s \) quark, and \( \omega_3 \) in the orbital wave function. By fitting the spectrum of baryons, the values of these parameters are all fixed, \( m_u = m_d = 340 \) MeV, \( m_s = 430 \) MeV, and \( \omega_3 = 340 \) MeV \[9\]. Our strategy for this model approach is: 1) to examine whether the results from this model is good enough to explain the existing experiment data of \( \bar{\nu}_\mu + p \rightarrow \mu^+ + n, \nu_\mu + n \rightarrow \mu^- + p, \bar{\nu}_\mu + p \rightarrow \mu^- + \Delta^{++} \) and \( \Lambda(1115) \rightarrow \bar{\nu}_{e/\mu} + e^- / \mu^- + p, \Sigma^-(1193) \rightarrow \bar{\nu}_{e/\mu} + e^- / \mu^- + n \); 2) we will use this model with proper parameters to predict the cross section of \( \bar{\nu}_{e/\mu} + p \rightarrow e^+ / \mu^+ + \Lambda(1115), \bar{\nu}_{e/\mu} + p \rightarrow e^+ / \mu^+ + \Sigma^0(1193), \bar{\nu}_{e/\mu} + p \rightarrow e^+ / \mu^+ + \Sigma^{0*}(1385) \).

Before showing our results, we want to discuss the status of existing experiment data. The experiment data from the MiniBooNE \[17\] (solid circle) are larger than those from others \[26, 27\] as shown in Fig. 3. A possible reason may be that the MiniBooNE used carbon as the target while others used light target (\( H_2 \) or \( D_2 \)). The heavier nuclear targets may bring complicated nuclear effects, such as correlations between target nucleons. However, the light target (\( H_2 \) or \( D_2 \)) would avoid such problem by detecting both muon and nucleon in the final state. Thus, the data from Argonne National laboratory (ANL) \[26, 28\] are more suitable for making comparison with our calculation. We only include the ANL’s data for \( \Delta^{++} \) production \[28\]. The cross section of single \( \pi^+ \) production in MiniBooNE \[29\] by \( CH_2 \) target is much larger than that of ANL \[28\] by \( H_2 \) target with a factor about 10.

In Fig. 4, the total cross sections of \( \nu_\mu + n \rightarrow \mu^- + p \) and \( \bar{\nu}_\mu + p \rightarrow \mu^+ + n \) as the function of the neutrino beam energy in the lab system are shown. For the \( \nu_\mu + n \rightarrow \mu^- + p \) reaction, if we include the SU(3) breaking effect, the predicted cross sections fit the experiment data well as shown by the black solid line, while the calculated cross sections without including the SU(3) breaking effect are much smaller than experiment data as shown by the red dashed line. We find the results are sensitive to the parameter \( \omega_3 \). A good fit can be obtained by using \( \omega_3 = 430 \) MeV as shown by the red dashed-dotted line. For the \( \bar{\nu}_\mu + p \rightarrow \mu^+ + n \) reaction, the calculation including the SU(3) breaking effect overestimates the cross sections a little bit while the results without including the SU(3) breaking effect fit the data well no matter whether using \( \omega_3 = 340 \) MeV or 430 MeV.

In Fig. 5, the total cross section of \( \nu_\mu + p \rightarrow \mu^- + \Delta^{++} \) vs the beam energy of anti-neutrino is shown. The experiment data is from the ANL \[28\]. Other groups used multi-nucleons target and their results may not be suitable for comparing with our results directly. Because we do not have the \( \Delta^{++} \) wave function with SU(3) breaking effect, all of the results are calculated by the SU(3) symmetry assumption. The results with \( \omega_3 = 340 \) MeV or 430 MeV are both much smaller than the experiment data by a factor of \( 2 \sim 3 \). Only by using \( \omega_3 = 600 \) MeV, the theoretical results are comparable to the experiment data. However, note that the experiment data is \( \nu_\mu + p \rightarrow \pi^+ + p + \mu^- \), and the non-resonance effect is important here as discussed in Refs. \[32, 33\]. Especially, in Ref. \[32\], the non-\( \Delta \) contribution reaches 33% after...
FIG. 3: (Color online) Total cross sections of $\nu_\mu + n \rightarrow \mu^- + p$ (left) and $\bar{\nu}_\mu + p \rightarrow \mu^+ + n$ (right) vs beam energy of $\nu_\mu$ and $\bar{\nu}_\mu$ in the lab system. The black solid lines are calculated by the nucleon wave function of SU(3) break effect with $\omega_3=340$ MeV. The red dashed and dashed-dotted lines are calculated by the nucleon wave function of SU(3) symmetry with $\omega_3=340$ MeV and $\omega_3=430$ MeV, respectively. The experiment data are from Refs. [17, 26, 27, 30, 31].

FIG. 4: (Color online) Total cross section of $\nu_\mu + p \rightarrow \Delta^{++} + \mu^-$ as function of energy of $\nu_\mu$ in the lab system by using the proton and $\Delta$ wave function from SU(3) symmetry. The red dashed, red dashed-dotted and green dotted lines are calculated by using $\omega_3 = 340$ MeV, 430 MeV and 600 MeV, respectively. The experiment data are from Ref. [28] for $\nu_\mu + p \rightarrow \pi^+ + p + \mu^-$. Including the couple channel effect. From these considerations, our results with $\omega_3 = 430$ MeV seems also acceptable here.

In Table III we list the branch ratio of $\Lambda(1115) \rightarrow \bar{\nu}_e/\mu + e^-/\mu^- + p$ and $\Sigma^-(1193) \rightarrow \bar{\nu}_e/\mu + e^-/\mu^- + n$ in the NR3QM (NR3QM-Full, NR3QM-Single$^a$ and NR3QM-Single$^b$) approaches, compared with results from other approaches given in the next subsection and data. And the results in the three cases are slightly larger than the experiment data. And for the $\Sigma^0(1193) \rightarrow \bar{\nu}_e/\mu + e^-/\mu^- + p$, because of the large decay width of $\Sigma^0(1193) \rightarrow \Lambda(1115) + \gamma$, including the couple channel effect.
the branch ratio of its weak decay is very small and not be able to be measured yet.

TABLE II: The branch ratios of Λ(1115) and Σ(1193) weak decays from various models (a: \( \omega_3 = 340\, \text{MeV} \); b: \( \omega_3 = 430\, \text{MeV} \)), compared with data.

| Model            | \( \bar{\nu}_e + e^- + p \) | \( \bar{\nu}_\mu + \mu^- + p \) |
|------------------|-------------------------------|-------------------------------|
| EXP-A            | 8.18                          | 1.36                          |
| EXP-B            | 8.25                          | 1.38                          |
| R3QM             | 9.30                          | 1.55                          |
| NR3QM-Full       | 10.7                          | 1.82                          |
| NR3QM-Single\(^a\) | 11.8                          | 1.95                          |
| NR3QM-Single\(^b\) | 12.1                          | 2.00                          |
| Experiment Data [12] | 8.32 ± 0.14                  | 1.57 ± 0.35                  |

| Model            | \( \bar{\nu}_e + e^- + p \) | \( \bar{\nu}_\mu + \mu^- + p \) |
|------------------|-------------------------------|-------------------------------|
| EXP              | 22.43                         | 10.32                         |
| R3QM             | 20.10                         | 9.35                          |
| NR3QM-Full       | 18.82                         | 8.82                          |
| NR3QM-Single\(^a\) | 23.83                        | 10.80                         |
| NR3QM-Single\(^b\) | 28.19                        | 12.68                         |
| Experiment Data [12] | -                            | -                            |

| Model            | \( \bar{\nu}_e + e^- + p \) | \( \bar{\nu}_\mu + \mu^- + p \) |
|------------------|-------------------------------|-------------------------------|
| EXP              | 9.58                          | 4.51                          |
| R3QM             | 9.30                          | 4.06                          |
| NR3QM-Full       | 8.16                          | 3.95                          |
| NR3QM-Single\(^a\) | 10.35                        | 4.85                          |
| NR3QM-Single\(^b\) | 12.25                        | 5.69                          |
| Experiment Data [12] | 10.17 ± 0.34                 | 4.5 ± 0.4                     |

In comparison of our NR3QM results with the existing data on the total cross sections of \( \bar{\nu}_\mu + p \rightarrow \mu^- + n \), \( \nu_\mu + n \rightarrow \mu^- + p \), \( \nu_\mu + p \rightarrow \mu^- + \Delta^{++} \) and the branch ratio of \( \Lambda(1115) \rightarrow \bar{\nu}_e/\mu + \mu^-/e^- + p \) and \( \Sigma^-(1193) \rightarrow \bar{\nu}_e/\mu + e^-/\mu^- + n \), we find two cases fitting all data reasonably well: \( \omega_3 = 340 \, \text{MeV} \) with the SU(3) breaking effect and \( \omega_3 = 430 \, \text{MeV} \) with the SU(3) symmetry. Then with these model parameters, we can make predictions for the corresponding \( \Lambda(1115), \Lambda^*(1405), \Sigma(1193) \) and \( \Sigma^*(1385) \) production rates as shown in Figs. 5 and 6. At \( E_\nu = 2.0 \, \text{GeV} \) in the lab system, the predicted total cross section for \( \bar{\nu}_e/\mu + p \rightarrow e^-/\mu^- + \Lambda(1115) \) is \( (0.07 \sim 0.2) \times 10^{-39} \, \text{cm}^2 \), for \( \bar{\nu}_e/\mu + p \rightarrow e^-/\mu^- + \Lambda^*(1405) \) is \( (2 \sim 7) \times 10^{-42} \, \text{cm}^2 \), for \( \bar{\nu}_e/\mu + p \rightarrow e^-/\mu^- + \Sigma^0(1193) \) is \( (0.009 \sim 0.035) \times 10^{-39} \, \text{cm}^2 \), and for \( \bar{\nu}_e/\mu + p \rightarrow e^-/\mu^- + \Sigma^+(1385) \) is \( (15 \sim 27) \times 10^{-42} \, \text{cm}^2 \). The predicted total cross section for \( \Lambda(1115) \) is much larger than that for \( \Sigma^-(1193) \), while their \( J^p \) are both \( \frac{1}{2}^- \). The reason is that the flavor wave functions of \( \Lambda(1115) \) and \( \Sigma^-(1193) \) are different. For the \( \Lambda(1115) \) and \( \Sigma(1193) \), only \( \rho(\Lambda|\chi_s^+\chi_s|p)_{\rho} \neq 0 \) and \( \lambda(\Sigma|\chi_s^+\chi_s|p)_{\lambda} \neq 0 \), respectively, in the NR3QM. As a result, the main contribution of \( \Lambda\text{-p-W} \) and \( \Sigma\text{-p-W} \) is \( \rho(\frac{1}{2}, s^\Lambda_{\rho} |\hat{O}_\nu|^\frac{1}{2}, s^0_{\rho}) \).
and $\chi(\frac{1}{2}, s_2^A, \hat{O}^{\nu}\frac{1}{2}, s_2^B)_\lambda$, respectively. As shown in Appendix B, these two terms are totally different. On the other hand, the predicted total cross section for $\Sigma^{*0}(1385)$ is much larger than that for $\Lambda^*(1405)$. This may be because the $J^P$ of $\Sigma^{*0}(1385)$ is $\frac{3}{2}^+$ while $\Lambda^*(1405)$ is $J^P = \frac{1}{2}^-$. Note that the photo-production rate of $\Delta$ with $J^P = \frac{3}{2}^+$ is much larger than that of $N^*(1535)$ with $J^P = \frac{1}{2}^-$. However, as we know, $\Lambda^*(1405)$ may include large five quark components ($uds[\bar{u}\bar{u}] + uds[\bar{d}\bar{d}]$) with all quarks in the S-wave. The transition of 3-quark configuration to 5-quark configuration may play an important role for the $\Lambda^*(1405)$ production, as in the case for the photo-production of $N^*(1535)$ [34]. This may make the actual total cross section for the $\Lambda^*(1405)$ production larger than our prediction assuming

FIG. 5: (Color online) NR3QM prediction of total cross sections for $\bar{\nu}_e + p \to \Lambda + e^+$ (left) and $\bar{\nu}_\mu + p \to \Lambda + \mu^+$ (right) vs neutrino beam energy in the lab system. The thin and thick lines are for $\Lambda(1115)$ and $\Lambda^*(1405)$, respectively. The line types are defined as the same as in Fig. 3.

FIG. 6: (Color online) NR3QM prediction of total cross sections for $\bar{\nu}_e + p \to \Sigma + e^+$ (left) and $\bar{\nu}_\mu + p \to \Sigma + \mu^+$ (right) vs neutrino beam energy in the lab system. The thin and thick lines are for $\Sigma^0(1193)$ and $\Sigma^{*0}(1385)$, respectively. The line types are defined as the same as in Fig. 3.
only 3-quark configuration.

B. Comparison with V-A models for \( \Lambda/\Sigma \) weak decays and neutrino production

To further check the reliability of our predictions from the simple non-relativistic 3-quark model, we also use the V-A models to calculate the \( \Lambda(1115)/\Sigma(1193) \) weak decays and neutrino production rates. The V-A models with effective form factors either from fitting hyperon weak decay data or from relativistic 3-quark model are used for our calculation.

In Table II the branch ratios of \( \Lambda(1115) \) and \( \Sigma(1193) \) weak decays calculated from various models are listed. The results from the non-relativistic model are larger than those from V-A models, and also 30% larger than experiment data. However, all of the results are consistent in the order of magnitude.

Comparison of predictions from various models for the total cross sections of \( \bar{\nu}_{e/\mu} + p \rightarrow e^+/\mu^+ + \Lambda(1115) \) and \( \bar{\nu}_{e/\mu} + p \rightarrow e^+/\mu^+ + \Sigma^0(1193) \) are shown in Fig.7 and Fig.8, respectively.

In Fig.7 we see that the different parameters sets (EXP-A and EXP-B), which are from the same experiment [21], give quite different predictions for the total cross sections of \( \bar{\nu}_{e/\mu} + p \rightarrow e^+/\mu^+ + \Lambda(1115) \), 0.8 and 0.15 \( \times 10^{-39} \text{ cm}^2 \) at \( E_\nu = 2.0 \text{ GeV} \) in the lab system. This means that we can distinguish two sets of parameters in this reaction. Comparing the results from different models, we find that the predictions of NR3QM-Single with \( \omega_3 = 430 \text{ MeV} \) are well consistent with those from the EXP and R3QM models, while the predictions of NR3QM-Full are smaller than those from other models. In short, the range of total cross section of \( \bar{\nu}_{e/\mu} + p \rightarrow e^+/\mu^+ + \Lambda(1115) \) from various models is about \( (0.07 \sim 0.8) \times 10^{-39} \text{ cm}^2 \); the NR3QM-Full gives the lower bound of the range.

In Fig.8 the estimations of \( \Sigma(1193) \) production from the NR3QM are all much smaller than those from the EXP and R3QM by a factor about 5 – 20. At the \( E_\nu = 2.0 \text{ GeV} \) in the lab system, the predicted total cross section is 0.2 and 0.12 \( \times 10^{-39} \text{ cm}^2 \) in the EXP and R3QM model, while it is \( (0.009 \sim 0.035) \times 10^{-39} \text{ cm}^2 \) in the NR3QM. Although the predictions for the \( \Sigma(1193) \) production suffer larger model dependence than the case for the \( \Lambda(1115) \) production, predictions in all models indicate that the total cross sections for the \( \Sigma(1193) \) production is much smaller than for the \( \Lambda(1115) \) production by a factor about 7\( \pm 3 \). In the previous subsection, we have explained why the \( \Sigma(1193) \) production is different from the \( \Lambda(1115) \) production, and by the comparison in this section, it seems that the wave functions of \( \Lambda(1115) \) and \( \Sigma^0(1193) \) reasonably reflect the nature of these hyperons.

From these model calculations, we believe, roughly speaking, the neutrino production rates of \( \Lambda(1115) \), \( \Lambda^*(1405) \), \( \Sigma(1193) \) and \( \Sigma^0(1385) \) hyperons are about 1 to 2 orders of magnitude smaller than the production of \( \Delta \). Recently, the MiniBooNE Collaboration reported a collection of about 50000 candidate \( \pi^+ \) events with \( CH_2 \) target at \( E_\nu \sim 1 \text{ GeV} \) with an overall signal efficiency of 12.7% [29]. This means about a few ten thousands hyperons were also produced in their target. We would suggest the MiniBooNE Collaboration to consider to study hyperon production from their experiment. As the first step, the \( \Lambda(1115) \) production should be studied with its \( p\pi^- \) decay mode to set up a calibration for possible study of other hyperon resonances. This may open a new window for the study of low energy hyperon resonances in the future accelerator-based neutrino experiments with higher neutrino flux.
In summary, we use non-relativistic 3-quark model to calculate the total cross sections of \( \bar{\nu}_e + p \rightarrow \mu^+ + n, \nu_\mu + n \rightarrow \mu^- + p, \) and \( \nu_\mu + p \rightarrow \mu^- + \Delta^{++} \), and the branch ratios of \( \Lambda(1115) \rightarrow \bar{\nu}_{e/\mu} + e^-/\mu^- + p \) and \( \Sigma^-(1193) \rightarrow \bar{\nu}_{e/\mu} + e^-/\mu^- + n \). All the data can be reproduced reasonably well in two cases: \( \omega_3=340 \text{ MeV} \) with SU(3) breaking effect and

IV. SUMMARY AND OUTLOOK

FIG. 7: (Color online) Comparison of predictions from various models for the total cross section of \( \bar{\nu}_e + p \rightarrow \Lambda(1115) + e^+ \) (left) and \( \bar{\nu}_\mu + p \rightarrow \Lambda(1115) + \mu^+ \) (right) vs neutrino beam energy in the lab system. The black solid, red dashed and red dash-dotted lines are the same as the thin lines in Fig. 5. The blue short-dashed, blue dash-dot-dotted, and green dotted lines are from “Exp-A”, “Exp-B”, “R3QM” models, respectively.

FIG. 8: (Color online) Comparison of predictions from various models for the total cross section of \( \bar{\nu}_e + p \rightarrow \Sigma^0(1193) + e^+ \) (left) and \( \bar{\nu}_\mu + p \rightarrow \Sigma^0(1193) + \mu^+ \) (right) vs neutrino beam energy in the lab system. The black solid, red dashed and red dash-dotted lines are the same as the thin lines in Fig. 6. The blue short-dashed and green dotted lines are from “EXP” and “R3QM” models, respectively.
\[ \omega_3 = 430 \text{ MeV} \] with SU(3) symmetry. Then we use this model to predict the cross sections of \( \bar{\nu}_{e/\mu} + p \rightarrow e^+/\mu^+ + \Sigma^0(1193) \), \( \bar{\nu}_{e/\mu} + p \rightarrow e^+/\mu^+ + \Sigma^0(1385) \), \( \bar{\nu}_{e/\mu} + p \rightarrow e^+/\mu^+ + \Lambda(1115) \) and \( \bar{\nu}_{e/\mu} + p \rightarrow e^+/\mu^+ + \Lambda^*(1405) \) to be about 9–35, 15–27, 70–200 and 2–7 \( \times 10^{-42} \text{ cm}^2 \), respectively, at \( E_\nu \sim 2 \text{ GeV} \). Furthermore, we use the vector and axial vector current approach with effective form factors determined either by fitting the \( \Lambda(1115) \) and \( \Sigma(1193) \) weak decay data or from relativistic quark model to predict the \( \Lambda(1115) \) and \( \Sigma(1193) \) neutrino production. The results are larger but of the same order of magnitude with the results from the non-relativistic 3-quark model. Based on our predictions in the simple non-relativistic 3-quark model, we estimate that corresponding to about 50000 candidate \( \pi^+ \) events collected by the MiniBooNE Collaboration with \( CH_2 \) target at \( E_\nu \sim 1 \text{ GeV} \), there should also be about a few ten thousands hyperons produced in their target. Including more elaborate 5-quark configurations for hyperon resonances may predict higher production rates for them, but not by order of magnitude.

We suggest the MiniBooNE Collaboration and future accelerator-based neutrino experiments with higher neutrino flux to study hyperon production processes. The neutrino induced hyperon production processes \( \bar{\nu}_{e/\mu} + p \rightarrow e^+/\mu^+ + \pi + \Lambda/\Sigma \) may provide a unique clean place for studying low energy \( \pi \Lambda/\Sigma \) interaction and hyperon resonances below \( K\bar{N} \) threshold. It would be very helpful for clarifying the puzzling nature of the lowest \( J^P = 1/2^- \) hyperon states: \( \Lambda^*(1405) \) and possible \( \Sigma^*(1390) \).

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Appendix A: The baryon’s wave function

The baryon’s wave functions are taken as the same as those in Ref. [9]. For the configurations of the octet baryons, the flavor-spin-orbital wave functions are:

\[ |B^2 S_s, \frac{1}{2}^+ \rangle = \frac{1}{\sqrt{2}} \left( |B\rangle_{\lambda} |\frac{1}{2}, s_z\rangle_{\lambda} + |B\rangle_{\rho} |\frac{1}{2}, s_z\rangle_{\rho} \right) \Phi_{000}(\vec{q}_\lambda, \vec{q}_\rho), \]  
(A1)

\[ |B^2 S_s, \frac{1}{2}^+ \rangle = \frac{1}{\sqrt{2}} \left( |B\rangle_{\lambda} |\frac{1}{2}, s_z\rangle_{\lambda} + |B\rangle_{\rho} |\frac{1}{2}, s_z\rangle_{\rho} \right) \Phi_{200}^e(\vec{q}_\lambda, \vec{q}_\rho), \]  
(A2)

\[ |B^2 S_M, \frac{1}{2}^+ \rangle = \frac{1}{2} \left[ \left( |B\rangle_{\lambda} |\frac{1}{2}, s_z\rangle_{\rho} + |B\rangle_{\rho} |\frac{1}{2}, s_z\rangle_{\lambda} \right) \Phi_{200}^e(\vec{q}_\lambda, \vec{q}_\rho) \right. 
- \left. \left( |B\rangle_{\lambda} |\frac{1}{2}, s_z\rangle_{\rho} + |B\rangle_{\rho} |\frac{1}{2}, s_z\rangle_{\lambda} \right) \Phi_{200}^e(\vec{q}_\lambda, \vec{q}_\rho) \right] . \]  
(A3)

For the baryon-decuplet \( \Delta^{++} \) and \( \Sigma^0(1385) \), their flavor-spin-orbital wave functions are:

\[ |\Delta^{++} S, \frac{3}{2}^+ \rangle = |\Delta^{++}\rangle |\frac{3}{2}, s_z\rangle \Phi_{000}(\vec{q}_\lambda, \vec{q}_\rho), \]  
(A4)
\[ |\Sigma^0(1385)S, \frac{3}{2}^+ \rangle = |\Sigma^0(1385)| \frac{3}{2}, s_z \rangle \Phi_{000}(q_\lambda, q_\rho). \] (A5)

For the configurations of \( \Lambda^*(1405) \), the flavor-spin-orbital wave functions are:

\[ |\Lambda^2 P_A, \frac{1}{2}^- \rangle = \frac{1}{\sqrt{6}} |\Lambda\rangle \ X_\lambda \Phi_\Lambda(q_\lambda, q_\rho), \] (A6)

\[ |\Lambda^2 P_M, \frac{1}{2}^- \rangle = -\frac{1}{2\sqrt{3}} (|\Lambda\rangle \ X_\lambda + |\Lambda\rangle \ X_\rho) \Phi_\Lambda(q_\lambda, q_\rho), \] (A7)

\[ |\Lambda^4 P_M, \frac{1}{2}^- \rangle = \frac{1}{2\sqrt{3}} (|\Lambda\rangle \ X_\lambda + |\Lambda\rangle \ X_\rho) \Phi_\Lambda(q_\lambda, q_\rho). \] (A8)

In these flavor-spin-orbital wave functions, \( |B\rangle_{\lambda/\rho}, |\Lambda\rangle_{\alpha/\beta} \), and \( |\Delta^{++}\rangle \) are the flavor wave functions; \( X_\alpha, X_{\lambda/\rho} \), and \( X_{\lambda/\rho}' \) are the spin-orbital coupled wave functions; \( |\frac{3}{2}, s_z\rangle_{\lambda/\rho} \) and \( |\frac{1}{2}, s_z\rangle_{\lambda/\rho} \) are the spin wave functions. All of these are used to construct \( |X^Q, s^Q_z\rangle \) in Eq. (10). \( \Phi_{000}(q_\lambda, q_\rho), \Phi_{200}(q_\lambda, q_\rho), \Phi_{000}(q_\lambda, q_\rho), \Phi_{000}(q_\lambda, q_\rho), \Phi_{000}(q_\lambda, q_\rho), \Phi_{000}(q_\lambda, q_\rho), \Phi_{000}(q_\lambda, q_\rho), \Phi_{000}(q_\lambda, q_\rho) \) denote the orbital wave functions; and the Jacobin momenta are related to those of the quarks by

\[ \tilde{q}_\rho = \frac{q_1 - q_2}{\sqrt{2}}, \] (A9)

\[ \tilde{q}_\lambda = \frac{q_1 + q_2 - 2q_3}{\sqrt{6}}. \] (A10)

The involved flavor wave functions, spin wave functions and orbital wave functions are given explicitly as the following.

The flavor wave functions:

\[ |\Lambda\rangle = \frac{1}{\sqrt{6}} (|uds\rangle + |dsu\rangle + |sud\rangle - |usd\rangle - |sd\rangle - |dsu\rangle), \] (A11)

\[ |\Lambda^0\rangle = \frac{1}{2\sqrt{3}} (|uds\rangle - |dsu\rangle - |sud\rangle + |su\rangle + 2|uds\rangle - 2|dsu\rangle), \] (A12)

\[ |\Lambda^0\rangle = \frac{1}{2} (|uds\rangle + |dsu\rangle - |sud\rangle - |sud\rangle - |su\rangle), \] (A13)

\[ |\Sigma^0\rangle = \frac{1}{2} (|uds\rangle + |dsu\rangle - |sud\rangle - |sud\rangle - |su\rangle), \] (A14)

\[ |\Sigma^0\rangle = -\frac{1}{2\sqrt{3}} (|uds\rangle + |dsu\rangle + |sud\rangle + |sud\rangle + 2|uds\rangle - 2|dsu\rangle), \] (A15)

\[ |\Sigma^-\rangle = \frac{1}{\sqrt{2}} (|dsd\rangle - |sdd\rangle), \] (A16)

\[ |\Sigma^-\rangle = \frac{1}{6} (2|dds\rangle - |sdd\rangle - |dsd\rangle), \] (A17)

\[ |p\rangle = \frac{1}{\sqrt{6}} (2|uud\rangle - |udu\rangle - |udu\rangle), \] (A18)

\[ |p\rangle = \frac{1}{\sqrt{2}} (|uud\rangle - |udu\rangle), \] (A19)

\[ |n\rangle = -\frac{1}{\sqrt{6}} (2|ddu\rangle - |udd\rangle - |udu\rangle), \] (A20)
\[ |n\rangle_\rho = -\frac{1}{\sqrt{2}} (|udd\rangle - |udd\rangle), \quad (A21) \]
\[ |\Delta^{++}\rangle = |uuu\rangle, \quad (A22) \]
\[ |\Sigma^{0}(1385)\rangle = \frac{1}{\sqrt{6}} (|uds\rangle + |dus\rangle + |dsu\rangle + |sdu\rangle + |sud\rangle + |usd\rangle). \quad (A23) \]

The spin-orbital coupled wave functions \[ \uparrow \downarrow \uparrow \] (\( \Delta^{\pm} \) for \( s_z = 1/2 \) and \( -1/2 \)):

\[ X^+_{\lambda} = -\frac{1}{2} \frac{1}{\sqrt{2}} \lambda(\rho, 0) + \sqrt{2} \frac{1}{2} \frac{1}{\sqrt{2}} \lambda(\rho, +1) \]
\[ + \frac{1}{2} \frac{1}{\sqrt{2}} \lambda(\rho, 0) - \sqrt{2} \frac{1}{2} \frac{1}{\sqrt{2}} \lambda(\rho, +1), \quad (A24) \]
\[ X^-_{\lambda} = \frac{1}{2} \frac{1}{\sqrt{2}} \lambda(\rho, 0) + \sqrt{2} \frac{1}{2} \frac{1}{\sqrt{2}} \lambda(\rho, -1) \]
\[ - \frac{1}{2} \frac{1}{\sqrt{2}} \lambda(\rho, 0) + \sqrt{2} \frac{1}{2} \frac{1}{\sqrt{2}} \lambda(\rho, +1), \quad (A25) \]
\[ X^+_{\rho} = \frac{1}{2} \frac{1}{\sqrt{2}} \lambda(\rho, 0) + \sqrt{2} \frac{1}{2} \frac{1}{\sqrt{2}} \lambda(\rho, +1) \]
\[ + \frac{1}{2} \frac{1}{\sqrt{2}} \lambda(\rho, 0) - \sqrt{2} \frac{1}{2} \frac{1}{\sqrt{2}} \lambda(\rho, +1), \quad (A26) \]
\[ X^-_{\rho} = \frac{1}{2} \frac{1}{\sqrt{2}} \lambda(\rho, 0) - \sqrt{2} \frac{1}{2} \frac{1}{\sqrt{2}} \lambda(\rho, -1) \]
\[ - \frac{1}{2} \frac{1}{\sqrt{2}} \lambda(\rho, 0) + \sqrt{2} \frac{1}{2} \frac{1}{\sqrt{2}} \lambda(\rho, +1), \quad (A27) \]
\[ X^+_{\lambda} = \sqrt{3} \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} \lambda(\rho, 0) + |\lambda, 0\rangle + \frac{1}{2} \frac{1}{\sqrt{2}} \lambda(\lambda, 1), \quad (A30) \]
\[ X^-_{\lambda} = \sqrt{3} \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} \lambda(\rho, 0) - \sqrt{2} \frac{1}{2} \frac{1}{\sqrt{2}} \lambda(\rho, 0) + \frac{1}{2} \frac{1}{\sqrt{2}} \lambda(\lambda, -1), \quad (A31) \]
\[ X^+_{\rho} = \sqrt{3} \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} \lambda(\rho, 0) - \sqrt{2} \frac{1}{2} \frac{1}{\sqrt{2}} \lambda(\rho, 0) + \frac{1}{2} \frac{1}{\sqrt{2}} \lambda(\rho, 1), \quad (A32) \]
\[ X^-_{\rho} = \sqrt{3} \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}} \lambda(\rho, 0) + \sqrt{2} \frac{1}{2} \frac{1}{\sqrt{2}} \lambda(\rho, 0) + \frac{1}{2} \frac{1}{\sqrt{2}} \lambda(\rho, -1), \quad (A33) \]

with \( \lambda, 0 = q_{\lambda, z}, (\lambda, \pm 1) = \mp \frac{1}{\sqrt{2}} (q_{\lambda, x} \pm i q_{\lambda, y}), (\rho, 0) = q_{\rho, z}, \) and \( (\rho, \pm 1) = \mp \frac{1}{\sqrt{2}} (q_{\rho, x} \pm i q_{\rho, y}) \).

The spin wave functions:

\[ |\frac{1}{2} \rule[.5ex]{.7em}{.75pt} \uparrow \rangle_\rho = \frac{1}{\sqrt{2}} (|\uparrow \uparrow\rangle - |\downarrow \downarrow\rangle), \quad (A34) \]
\[ |\frac{1}{2} \rule[.5ex]{.7em}{.75pt} \downarrow \rangle_\rho = \frac{1}{\sqrt{2}} (|\uparrow \downarrow\rangle - |\downarrow \uparrow\rangle), \quad (A35) \]
\begin{align*}
|\frac{1}{2} + \frac{1}{2}\rangle & = \frac{1}{\sqrt{6}} (2|\uparrow\uparrow\downarrow\rangle - |\uparrow\downarrow\uparrow\rangle - |\downarrow\uparrow\uparrow\rangle), \\
|\frac{1}{2} - \frac{1}{2}\rangle & = -\frac{1}{\sqrt{6}} (2|\downarrow\downarrow\uparrow\rangle - |\downarrow\uparrow\downarrow\rangle - |\uparrow\downarrow\uparrow\rangle), \\
|\frac{3}{2}, \frac{3}{2}\rangle & = |\uparrow\uparrow\uparrow\rangle, \\
|\frac{3}{2}, \frac{1}{2}\rangle & = \frac{1}{\sqrt{3}} (|\uparrow\uparrow\downarrow\rangle + |\downarrow\uparrow\uparrow\rangle + |\uparrow\downarrow\uparrow\rangle), \\
|\frac{3}{2}, -\frac{1}{2}\rangle & = \frac{1}{\sqrt{3}} (|\uparrow\uparrow\downarrow\rangle + |\downarrow\uparrow\uparrow\rangle + |\uparrow\downarrow\uparrow\rangle), \\
|\frac{3}{2}, -\frac{3}{2}\rangle & = |\downarrow\downarrow\downarrow\rangle.
\end{align*}

The orbital wave functions [9]:

\begin{align*}
\Phi_{000}(\vec{q}_\lambda, \vec{q}_\rho) & = \frac{1}{(\sqrt{\pi} \omega_3)^3} \exp \left\{ -\frac{\vec{q}_\lambda^2 + \vec{q}_\rho^2}{2\omega_3^2} \right\}, \\
\Phi_{000}^s(\vec{q}_\lambda, \vec{q}_\rho) & = \frac{1}{\sqrt{3}(\sqrt{\pi} \omega_3)^3} \left( 3 - \frac{\vec{q}_\lambda^2 + \vec{q}_\rho^2}{\omega_3^2} \right) \exp \left\{ -\frac{\vec{q}_\lambda^2 + \vec{q}_\rho^2}{2\omega_3^2} \right\}, \\
\Phi_{020}(\vec{q}_\lambda, \vec{q}_\rho) & = \frac{2}{\sqrt{3}\omega_3^2(\sqrt{\pi} \omega_3)^3} (\vec{q}_\lambda \cdot \vec{q}_\rho) \exp \left\{ -\frac{\vec{q}_\lambda^2 + \vec{q}_\rho^2}{2\omega_3^2} \right\}, \\
\Phi_{020}^\lambda(\vec{q}_\lambda, \vec{q}_\rho) & = \frac{1}{\sqrt{3}\omega_3^2(\sqrt{\pi} \omega_3)^3} (\vec{q}_\rho^2 - \vec{q}_\lambda^2) \exp \left\{ -\frac{\vec{q}_\lambda^2 + \vec{q}_\rho^2}{2\omega_3^2} \right\}, \\
\Phi_{020}^\rho(\vec{q}_\lambda, \vec{q}_\rho) & = \frac{\sqrt{2}}{\sqrt{\pi} \omega_3^2} \exp \left\{ -\frac{\vec{q}_\lambda^2 + \vec{q}_\rho^2}{2\omega_3^2} \right\}.
\end{align*}

Appendix B: nucleon-baryon-W boson vertices $T_2^\nu$

In this appendix, we explain how to calculate $T_2^\nu$ using baryon’s wave functions by taken $\Lambda(1115)$-proton-W boson coupling as an example.

For the flavor part:

\begin{align*}
\lambda\langle \Lambda | \chi^+_s \chi_u | p \rangle_\lambda & = 0, \\
\lambda\langle \Lambda | \chi^+_s \chi_u | p \rangle_\rho & = 0, \\
\rho\langle \Lambda | \chi^+_s \chi_u | p \rangle_\lambda & = 0, \\
\rho\langle \Lambda | \chi^+_s \chi_u | p \rangle_\rho & = \frac{\sqrt{6}}{3}.
\end{align*}

For the spin part:

\begin{align*}
\rho\langle \frac{1}{2}, \frac{1}{2} | \hat{O}^\mu | \frac{1}{2}, \frac{1}{2} \rangle & = O^\mu(s^\Lambda_z, s^p_z), \\
\rho\langle \frac{1}{2}, \frac{1}{2} | \hat{O}^\mu | \frac{1}{2}, \frac{1}{2} \rangle & = 0, \\
\lambda\langle \frac{1}{2}, \frac{1}{2} | \hat{O}^\mu | \frac{1}{2}, \frac{1}{2} \rangle & = 0.
\end{align*}
\[ \lambda \left( \frac{1}{2}, s_z^A = \frac{1}{2} \left| \hat{O}^\nu \right| \frac{1}{2}, s_z^p = \frac{1}{2} \right) = \frac{1}{3} \left( O^\mu \left( \frac{1}{2}, \frac{1}{2} \right) + 2 O^\mu \left( -\frac{1}{2}, -\frac{1}{2} \right) \right) \]  
\[ \lambda \left( \frac{1}{2}, s_z^A = -\frac{1}{2} \left| \hat{O}^\nu \right| \frac{1}{2}, s_z^p = \frac{1}{2} \right) = \frac{1}{3} O^\mu \left( -\frac{1}{2}, -\frac{1}{2} \right) \]  
\[ \lambda \left( \frac{1}{2}, s_z^A = \frac{1}{2} \left| \hat{O}^\nu \right| \frac{1}{2}, s_z^p = -\frac{1}{2} \right) = \frac{1}{3} O^\mu \left( \frac{1}{2}, -\frac{1}{2} \right) \]  
\[ \lambda \left( \frac{1}{2}, s_z^A = -\frac{1}{2} \left| \hat{O}^\nu \right| \frac{1}{2}, s_z^p = -\frac{1}{2} \right) = \frac{1}{3} \left( 2 O^\mu \left( \frac{1}{2}, \frac{1}{2} \right) + O^\mu \left( -\frac{1}{2}, -\frac{1}{2} \right) \right) \]

where the vertex \( O^\mu(s_z^A, s_z^p) \) is:

\[ O^\mu(s_z^A, s_z^p) = \bar{u}_s(\vec{q}_s, s_z^A) \gamma^\nu (1 - \gamma^5) u_u(\vec{q}_u, s_z^u) \]  

By using Eq. (12) and above equations, we can get \( T^\mu_{\Lambda-p-W} \) as:

\[
T^\mu_{\Lambda-p-W}(s_z^A, s_z^p) = \int \left\{ \frac{9}{2} d\vec{q}_u d\vec{q}_p \sqrt{\frac{G_{Fm_W^2}}{2\pi}} |v_{us}| \right. \\
\left. \times \left\{ \frac{0.90}{\sqrt{2}} \Phi_{000}(\vec{q}_u^p, \vec{q}_p^\rho) + \frac{0.34}{\sqrt{2}} \Phi_{200}(\vec{q}_u^p, \vec{q}_p^\rho) + \frac{0.27}{2} \Phi_{200}(\vec{q}_u^p, \vec{q}_p^\rho) \right. \\
\left. \times \frac{0.93}{\sqrt{2}} \Phi_{000}(\vec{q}_u^p, \vec{q}_p^\rho) + \frac{0.30}{\sqrt{2}} \Phi_{200}(\vec{q}_u^p, \vec{q}_p^\rho) + \frac{0.20}{2} \Phi_{200}(\vec{q}_u^p, \vec{q}_p^\rho) \right. \\
\times \rho \left( \frac{1}{2}, s_z^A | \hat{O}^\nu | \frac{1}{2}, s_z^p \right) \right. \\
\left. + \frac{0.27}{2} \frac{0.20}{2} \Phi_{200}(\vec{q}_u^p, \vec{q}_p^\rho) \Phi_{200}(\vec{q}_u^p, \vec{q}_p^\rho) \left( \frac{1}{2}, s_z^A | \hat{O}^\nu | \frac{1}{2}, s_z^p \right) \right\} \]  

where \( \vec{q}_u^p = -3\vec{q}_u/\sqrt{6}, \vec{q}_u^A = -(3\vec{q}_u + 2\vec{q}_W)/\sqrt{6}. \)

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