On the progenitor quark mass matrix

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We determined the quark mass matrix in terms of a small expansion parameter \( \sqrt{\epsilon} \), which gives correctly all the quark masses and the CKM matrix elements at the electroweak (EW) scale, and obtain a progenitor form at the GUT scale by running the EW scale mass matrix. Finally, a possible texture form for the progenitor quark mass matrix is suggested.

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I. INTRODUCTION

The huge hierarchy on the quark masses is one of the most important flavor problems in the Standard Model (SM) of particle physics. The charged current (CC) weak interactions in the SM are well established, which are summarized by the Cabibbo–Kobayashi–Maskawa (CKM) and Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrices [1–5]. The CKM matrix being close to the identity is due to this huge hierarchy [6]. But neutrino masses do not reveal a hierarchy even in the real world with three families, the expansion in terms of \( \sqrt{\epsilon} \) has been the main reason [8]. To obtain this relation, the texture scenarios of the quark mass matrix have boomed in the last four decades [11–13]. The texture zero entries are assumed to arise from some symmetry. In this paper, we do not follow this line of argument, but take next guide to some symmetry based on this phenomenologically determined progenitor mass matrix. If some elements of the CKM matrix are smaller than those of our ansatz, we regard them as possible texture zeros.

In general, the CKM matrix, \( U^{(u)}U^{(d)\dagger} \), is given by the product of two left-hand (L-hand) unitary matrices \( U^{(u)} \) and \( U^{(d)} \) diagonalizing respectively the up-type (i.e. \( \frac{Q_{em}}{m_0} = +\frac{2}{3} \) quarks) and the down-type mass matrices. The matrix \( U^{(u)}U^{(d)\dagger} \) is a unitary matrix, and hence can be written as a unitary matrix \( V_{CKM} \). Therefore, defining \( V_{CKM} \) with general up-type and down-type weak eigenstates is equivalent to defining it with a general up-type weak eigenstate and diagonal (or mass) down-type eigenstates. These two cases have the same number of physical parameters. Let us choose this simple case. We can choose the diagonal bases as those of up- or down-type quarks. But, since \( m_c/m_t \) is smaller than \( m_s/m_b \), we use the bases where the down-type quark mass matrix is already diagonalized [6]. In this case, the CKM matrix \( V_{CKM} = U^{(u)} \) which can be expanded in terms of the small expansion parameter, \( \sqrt{\epsilon} \equiv \sqrt{m_c/m_t} \approx 0.0584 \). Out of 9 parameters in \( U^{(u)} \), two phases can be absorbed to two up-type quark phases. One overall phase cannot be used for this purpose because of the baryon number conservation in the SM. The remaining 7 parameters encode three up-type quark masses, three real angles, and a phase \( e^{i\theta} \). Instead of texture zeros, we assume the following mass matrix, for its determinant being of \( O(\epsilon^3) \) [6],

\[
M = \begin{pmatrix}
O(\epsilon^2), & O(\epsilon^{3/2}), & O(\epsilon) \\
O(\epsilon^{3/2}), & O(\epsilon), & O(\sqrt{\epsilon}) \\
O(\epsilon), & O(\sqrt{\epsilon}), & 1
\end{pmatrix}.
\]

The electroweak (EW) scale is defined usually as the \( Z^0 \) boson pole, but here we define the EW scale as the top mass pole which is the highest scale among SM poles. Then, above the EW scale, we need not consider threshold effects. At the EW scale or at the top mass pole \( m_t = 172.5 \) [14], \( m_c/m_t |_{m_t} = 0.00341 \pm 0.00046 \) where \( m_c = 1.25 \) GeV of
the charm mass is runned to the EW scale \[^{[15]}\]. \(m_c \approx 2.5\) MeV \[^{[16]}\] is run to the electroweak scale to 0.392 MeV.\(^1\) Then, \(m_u = m_c = 2.5\) MeV \[^{[10]}\] is run to the electroweak scale to 0.392 MeV.\(^\dagger\) \[^{[10]}\] is run to the electroweak scale to 0.392 MeV.\(^\dagger\) Then, \(m_u = m_c = 2.5\) MeV \[^{[10]}\] is run to the electroweak scale to 0.392 MeV.\(^\dagger\)

To place zero entries at some places \[^{[12]}\], some kind of symmetry is needed, for example by a kind of the Froggatt–Nielson mechanism \[^{[18]}\]. Theoretically, it is important to know the progenitor mass matrix determined in this kind of way. So far, there has not appeared any reliable progenitor mass matrix. The essence of this paper is to determine the progenitor form starting from Eq. \[^{[1]}\], giving correctly all the quark masses by fitting to the experimentally determined CKM matrix elements as accurately as possible. This progenitor mass matrix can be very useful for future texture studies of the quark mass matrix.

It is better to take bases where both the CKM and the PMNS matrices are defined by the charge raising CCs or by the charge lowering CCs. The currently used convention is that the CKM matrix is defined by the charge raising CCs but the PMNS matrix by the charge lowering CCs \[^{[17]}\]. It has its own reason that the potential \(V\) is important in mixing in the quark sector while the kinetic energy is more important in the oscillation of neutrinos. Here, however, we try to describe both the CKM and PMNS matrices in a unified form by the charge raising CCs. Therefore, our PMNS matrix is the hermitian conjugate of that defined in the PDG book \[^{[17]}\]. In this paper, however, we pay attention to the CKM matrix only.

In Sec. \[^{[II]}\] we obtain the up-type quark mass matrix at the EW scale in the bases where the down-type quark mass matrix is already diagonalised. In Sec. \[^{[III]}\] we run this EW mass matrix to the GUT scale \((\sim 2.5 \times 10^{16}\) GeV\)), obtaining the progenitor quark mass matrix. In Sec. \[^{[IV]}\] we discuss possible zeros in the mass matrix obtained in Secs. \[^{[II]}\] and \[^{[III]}\]. Finally, a brief conclusion is given in Sec. \[^{[V]}\].

### II. DETERMINATION OF THE R-HAND UNITARY MATRIX

We choose the bases where the down-type quarks are mass eigenstates. To unify with leptons, we generalize this to choose the \(T_3 = -\frac{1}{2}\) members in the L-hand SM doublets are mass eigenstates, \(i.e.\) \(Q_{em} = 1\) leptons and down-type quarks are mass eigenstates. This choice is simple because the renormalizable couplings are enough for generating the mass terms of these \(T_3 = -\frac{1}{2}\) members in both the quark and lepton sectors. Since neutrino masses are not so hierarchical as the up-type quarks, we do not apply the same criteria for the PMNS and neutrino mass matrices. Even though the CKM and PMNS matrices are of very different forms, they can be successfully unified in GUTs from string compactification \[^{[19]}\]. An explicit example with the tetrahedral symmetry is already given in Ref. \[^{[20]}\].

Let us consider a realistic \(3 \times 3\) matrix,

\[
M = \begin{pmatrix}
q \varepsilon^2, & f \varepsilon^{3/2}, & c \varepsilon \\
\varepsilon^{3/2}, & p \varepsilon, & a \sqrt{\varepsilon} \\
d \varepsilon, & b \sqrt{\varepsilon}, & 1
\end{pmatrix},
\]

(2)

where \(a, b, c, d, f, g, p, q\) are O(1) numbers. Note the possibility that higher order corrections by loops and gravitational interactions can affect the numbers \(q, f\) and \(g\). \(M\) is diagonalized by the following L-hand matrix \(U\),

\[
U = \begin{pmatrix}
c_1, & s_1 c_3, & s_1 s_3 \\
-c_2 s_1, & c_1 c_2 c_3 + s_2 s_3 e^{-i\delta}, & c_1 c_2 s_3 - s_2 c_3 e^{-i\delta} \\
-s_1 s_2 e^{i\delta}, & -c_2 s_3 + c_1 s_2 c_3 e^{i\delta}, & c_2 c_3 + s_1 s_2 s_3 e^{i\delta}
\end{pmatrix},
\]

(3)

where \(c_i\) and \(s_i\) are cosines and sines of three real angles \(\theta_i (i = 1, 2, 3)\), and the right-hand (R-hand) matrix \(W\), by \(W M U^\dagger = M^{(diag)}\), \(W\) is determined if the matrix \(M\) of the form (2) is given. The unitarity condition introduces 9 parameters in \(W\) out of which two phases\(^2\) can be absorbed to the R-hand up-type quarks. Thus, we can consider

\(^1\) From the QCD scale to the scale \(m_c\), we multiplied a factor 1/3.

\(^2\) We cannot use the overall phase of the R-hand matrix for removing a phase.
the 7 parameter $W$. Since a 7 parameter unitary matrix is not known, we use the following form for $W$ (i.e. the same form as Eq. (3))

$$W = \begin{pmatrix} c_1, & s_1 c_3, & s_1 s_3 \\ -c_2 s_1, & c_1 c_2 c_3 + s_2 s_3 e^{-i\alpha}, & c_1 c_2 s_3 - s_2 c_3 e^{-i\alpha} \\ -s_1 s_2 e^{i\alpha}, & -c_2 s_3 + c_1 s_2 c_3 e^{i\alpha}, & c_2 c_3 + c_1 s_2 s_3 e^{i\alpha} \end{pmatrix},$$  

(4)

where $c_i$ and $s_i$ are cosines and sines of three real angles $\varphi_i$ ($i = 1, 2, 3$). The angles $\varphi_i$ ($i = 1, 2, 3$) will introduce more parameters in them, as they are expanded in powers of $\sqrt{\varepsilon}$,

$$\varphi_i = \phi_i[0] + \phi_i[1/2] \sqrt{\varepsilon} + \phi_i[1] \varepsilon + \cdots$$

(5)

Let us use the central points of data evaluated by the Kim–Seo (KS) form [21] for the CKM matrix [22],

$$V_{\text{CKM}}^{\text{KS}} = \begin{pmatrix} 0.975188, & 0.221345, & 0.003888 \\ -0.221226, & 0.974365 + 0.00065 e^{-i\delta}, & 0.01712 - 0.03712 e^{-i\delta} \\ -0.00822 e^{i\delta}, & -0.017551 + 0.03620 e^{i\delta}, & 0.999156 + 0.00064 e^{i\delta} \end{pmatrix},$$

$$V_{\text{CKM}}^{\text{KS}} = \begin{pmatrix} 0.975188, & 0.221345, & 0.555429 \varepsilon \\ -0.221226, & 0.974365 + 1.10986 e^{-i\delta} \varepsilon^{3/2}, & (0.204623 - 0.443669 e^{-i\delta}) \varepsilon^{3/2} \\ -1.17429 e^{i\delta} \varepsilon, & (-0.209775 + 0.432623 e^{i\delta}) \varepsilon^{3/2}, & 0.999156 + 1.09278 e^{i\delta} \varepsilon^{3/2} \end{pmatrix},$$

(6)

which gives $J = (3.114 \pm 0.325) \times 10^{-5} |\sin \delta|$ [23]. We scanned $W$ to make $WMU^\dagger$ diagonal to $M^{(\text{diag})} \simeq \text{diag} \left( |0.195 e^2|, |\varepsilon|, 1 \right)$. Since $M^{(\text{diag})}U$ is $(\varepsilon = 0.00341, \sqrt{\varepsilon} = 0.0584)$

$$M^{(\text{diag})}U = \begin{pmatrix} 0.28768 e^2, & 0.0652968 e^2, & 0.163516 e^3 \\ -0.221156 \varepsilon, & 0.974365 \varepsilon + 1.10986 e^{-i\delta} \varepsilon^{5/2}, & (0.204623 - 0.443669 e^{-i\delta}) \varepsilon^{3/2} \\ -1.17429 e^{i\delta} \varepsilon, & (-0.209775 + 0.432623 e^{i\delta}) \varepsilon^{3/2}, & 0.999156 + 1.09278 e^{i\delta} \varepsilon^{3/2} \end{pmatrix},$$

which is not close to Eq. (1) or Eq. (2), we need an appropriate $W$ to make $M$ match with the form Eq. (1). We find that the real angles of $W$, giving the form (2) approximately, are

$$\phi_1 = p_1[1] \varepsilon + \cdots, \quad \phi_2 = p_2[0] + p_2[1/2] \sqrt{\varepsilon} + p_2[1] \varepsilon + \cdots, \quad \phi_3 = p_3[0] + p_3[1] \varepsilon + \cdots$$

(7)

where $p_i[x]$ are the O(1) coefficient of $e^x$ for the real angle $\varphi_i$ in Eq. (4). There are more than six real numbers $(p_1[1], p_2[0], p_2[1/2], p_2[1], p_3[1/2], p_3[1])$ and a phase $e^{i\alpha}$ in our expansion of $W$. Using $V_{\text{CKM}}^{\text{KS}}$ as $U$, $M = W^\dagger M^{(\text{diag})}U$ becomes for $\alpha = 0$,

$$M = \begin{pmatrix} 0, & \left(0.300555 p_1[1] S[p_2[0]] - 0.61914 e^{i\delta} p_1[1] S[p_2[0]] \right) e^{3/2}, & -0.999156 p_1[1] S[p_2[0]] \varepsilon \\ e^{3/2} \left(0.221226 p_2[1/2] S[p_2[0]] - 2.41056 e^{i\delta} p_2[0] C[p_2[0]] \right) + \varepsilon \left(-0.221226 C[p_2[0]] \right) + C[p_2[0]] \left(0.974365 + 0.61914 e^{i\delta} \right) + 0.999156 p_2[0] \varepsilon \\ \left(0.221226 S[p_2[0]] - 2.41056 e^{i\delta} C[p_2[0]] \right) + 0.61914 e^{i\delta} C[p_2[0]] \varepsilon, & 0.999156 C[p_2[0]] \end{pmatrix}$$

(8)

where $p_{23}^{-} = p_2[1/2] - p_3[1/2]$, and we kept only the leading terms. $S[p_2[0]]$ cannot be zero or the determinant is zero. We can obtain an upper bound on the $S[p_2[0]]$ such that its contribution to the determinant is less than
O(ε^3) from every element of M. For a small S[p_2[0]], M_{12}M_{23}M_{31}, M_{13}M_{32}M_{21}, M_{31}M_{22}M_{13} and M_{13}M_{12}M_{21} are O(ε^3), O(ε^{5/2}), O(ε^{5/2}) and O(ε^{5/2}), respectively. Therefore, from M_{13}M_{32}M_{21}, M_{31}M_{22}M_{13} and M_{33}M_{12}M_{21}, we have bounds S[p_2[0]] < √ε, 2.4S[p_2[0]]^2 < √ε, {0.22S[p_2[0]], 2.4S[p_2[0]]^2} < √ε, respectively. These give S[p_2[0]] < 0.0584, 0.156, 0.265, and 0.156, respectively. Thus, we have the common region

0 < |S[p_2[0]]| < 0.0584. \hspace{1cm} (9)

In a somewhat large region of Eq. \([9]\) for an illustration, we take \(S[p_2[0]] = 0.05\) (with \(C[p_2[0]] = 0.99875\)) for which \(M\) is

\[
M \left( p_2[0] = 2.866^\circ \right) = \begin{pmatrix}
0, & (0.0150231 - 0.030986\cos \epsilon)p_1[1] \epsilon^{3/2}, & -0.0499422p_1[1] \epsilon \\
(0.220949 - 0.12049e^{i\epsilon}) \epsilon & (0.0150231 + 0.030986\cos \epsilon) \sqrt{\epsilon} & 0.0499422 \\
-2.40754e^{i\epsilon} p_2[1] & -0.30018p_2[1] \epsilon & 0.997907 \\
(0.0110578 - 2.40754e^{i\epsilon}) \epsilon & -0.30018 + 0.619139e^{i\epsilon} \sqrt{\epsilon} & 0.997907 \\
\end{pmatrix}
\]

where we used \(\sqrt{\epsilon} \approx 0.05840\) at the top mass pole for the (22) element. The matrix \([10]\) is almost the one given in Eq. \([1]\). Indeed, using Eqs. \([6]\) and \([7]\), the product \(WMU^\dagger\) is shown to be diagonal with the error of \(O(10^{-5})\) for the off-diagonal elements,

\[
WMU^\dagger = \begin{pmatrix}
2.267 \times 10^{-6}, & -1.499 \times 10^{-10}, & 0 \\
-2.255 \times 10^{-7}, & 3.41 \times 10^{-3} - 4.334 \times 10^{-6} \cos \delta, & 1.525 \times 10^{-8} + 6.21 \times 10^{-9} e^{-i\delta} \\
-1.076 \times 10^{-7}, & 4.471 \times 10^{-6} + 1.369 \times 10^{-6} e^{i\delta}, & 0.9999 + 0.8227 \times 10^{-5} \\
-3.566 \times 10^{-6} e^{i\delta}, & -2.268 \times 10^{-7} e^{2i\delta}, & 0.9999 + 0.8227 \times 10^{-5} \\
\end{pmatrix}
\]

III. A PROGENITOR FORM

The pole mass is related to the running mass, up to three gluon loops, as

\[
M_q = m_q(\mu) \left[ 1 + \frac{\alpha_s(\mu)}{3} + K_q^{(2)} \left( \frac{\alpha_s(\mu)}{\pi} \right)^2 + K_q^{(3)} \left( \frac{\alpha_s(\mu)}{\pi} \right)^3 + K_{tt} \delta_{3q} \right] = m_q(\mu) \left( R_q(\mu) + K_{tt} \delta_{3q} \right) \hspace{1cm} (12)
\]

where \(K_q^{(2)} = 1.121, K_q^{(2)} = 9.13, K_q^{(3)} = 123.8, \) and \(K_{tt}^{(3)} = 80.4\) are taken from [15]. The running masses at \(M_t\) are \(m_t(M_t) = 162.75\) GeV and \(m_c(M_t) = 554.5\) MeV are found at the top mass pole, using \(\alpha_s = 0.108 \pm 0.002\) at \(\mu = M_t\).

Above the EW scale or the top mass scale, we can consider the running of \(M_{ij}\), considering the gluon loops of Fig. 1 [21]. The vector couplings of gluons to quarks allow the same couplings to L-hand and R-hand quarks, \(u_R^{(i)}\) and \(u_L^{(i)}\).

The element \(M_{ij}\) have the same factor after the running. The reason is the following. The diagonalized mass matrix at \(m_t\) is \(M^{\text{diag}}(m_t)\). The R-hand and L-hand diagonalizing matrix at \(m_t\) relate it to the weak eigenstate mass matrix

\[
M(m_t)_{ij} = \left( W^\dagger(m_t)M^{\text{diag}}(m_t)U(m_t) \right)_{ij} \hspace{1cm} (13)
\]

To obtain a progenitor mass matrix at the GUT scale, each factor in Eq. \([13]\) is runned to the GUT scale. For the diagonal masses, the knowledge on the anomalous dimension suffices. For the R- and L-hand unitary matrices, strong interaction corrections do not distinguish them because the perturbative QCD conserves parity. The \(\times\) in Fig. 1 corresponds to \(M(\mu)_{ij}\) and every gluon vertex on the fermion line has the same 1-loop factor \((1 + a_1\alpha_s)\) because it is flavor blind. Therefore, we can take out \((1 + a_1\alpha_s)\) at 1-loop level

\[
(1 + a_1\alpha_s(\mu))^2 \left( W^\dagger(m_t)M^{\text{diag}}(m_t)U(m_t) \right)_{ij} = W^\dagger(\mu)M(\mu)_{ij}U(\mu) = R(\mu)M_{ij}(m_t). \hspace{1cm} (14)
\]
Taking a ratio $M(\mu)_{ij}/M(\mu)_{33}$, we obtain

$$M(\mu)_{ij} = \left( \frac{W^\dagger(m_t)M_{\text{diag}}(\mu)U(m_t)}{W^\dagger(m_t)M_{\text{diag}}(\mu)U(m_t)} \right)_{ij},$$

where the flavor independence of gluon couplings is used. Therefore, the progenitor mass matrix is proportional to Eq. (10), except the (33) element. Let $M_{ij}(\mu) = R(\mu)M_{ij}(m_t)$ where $R(m_t) = 1$, but the (33) element may be modified significantly due to a large top quark Yukawa coupling constant. The Higgs loop for is considered as

$$K_{tt} = -\frac{3}{16\pi^2}\left(\frac{m_t^2}{v_u^2}\right)$$

where the Higgs doublet $H_u$ couples to the top quark as

$$m_t\sqrt{2}v_u(H_u^\dagger \bar{t}R_{q^3L} + h.c., H_u = (H_u^+\sqrt{2}v_u + \frac{h_u}{\sqrt{2}})).$$

If $H_u$ is the only electroweak Higgs doublet, the (33) element runs to the scale $M_{\text{GUT}} \simeq 2.5 \times 10^{16}$ GeV to $R_t(M_{\text{GUT}}) = 1.01123$ for $\alpha_s(M_{\text{GUT}}) \simeq \frac{1}{30}$, and $K_{tt}(M_{\text{GUT}}) = -0.0094$. So, the (33) element changes factor 1 to factor 0.9703. Even if we added the top quark Yukawa coupling, the progenitor form obtained from Eq. (10) is almost intact. Now, we study the form (8) from the point of view of introducing texture zeros.

IV. TEXTURE

Since $S[p_2[0]]$ is very small, we can consider $M_{12}$ being zero in the first approximation. But we keep $M_{13}$ as a nonzero value to have a nonvanishing determinant. Comparing the experimentally determined $M$ with the ansatz form, Eq. (1), we note that a texture zero element is possible for the (11) and (12) elements,

$$M = \begin{pmatrix} 0, & 0, & O(\varepsilon) \\ O(\varepsilon^{3/2}), & O(\varepsilon), & O(\sqrt{\varepsilon}) \\ O(\varepsilon), & O(\sqrt{\varepsilon}), & 1 \end{pmatrix}$$

The zeros in the mass matrix determined from Eq. (17) signal the long-awaited texture form for the quark mass matrix. It is surprisingly simple that the mass matrix determines all the quark flavor parameters. In Table I, we present U(1) times parity $P$ quantum numbers to have the form Eq. (17). The L- and R-states are defined as

$$\bar{u}_R M_{ij} q_L^{(j)} = \bar{u}_R W^\dagger W M U^\dagger U q_L = \bar{u}_R^{\text{mass}} M_{\text{diag}} q_L^{\text{mass}}$$

TABLE I: U(1) and P quantum numbers of the quark and Higgs fields.

| Fields | $u_R^{(1)}$ | $u_R^{(2)}$ | $u_R^{(3)}$ | $q_L^{(1)}$ | $q_L^{(2)}$ | $q_L^{(3)}$ | $H_u$ | $H_d$ | $b_R$ | $s_R$ | $\sigma'^2$ |
|--------|-------------|-------------|-------------|-------------|-------------|-------------|-------|-------|-------|-------|-------------|
| U(1)   | -3 | -2 | -1 | -1 | 0 | +1 | 0 | +1 | 0 | -1 | 0 | +1 | 0 |
| P      | - | + | + | + | + | + | + | - | + | + | + | - | - |

The L- and R-states are defined as

$$\bar{u}_R M_{ij} q_L^{(j)} = \bar{u}_R W^\dagger W M U^\dagger U q_L = \bar{u}_R^{\text{mass}} M_{\text{diag}} q_L^{\text{mass}}$$
where \( \bar{u}_R \) and \( u_L \) are in the weak bases, and \( u_{R}^{\text{mass}} \) and \( u_{L}^{\text{mass}} \) are in the mass eigenstates, e.g. \( u_{L}^{\text{mass}} = U u_L \), etc. 
\( \bar{d}_R, \bar{s}_R \) and \( \bar{d}_R \) are the mass eigenstates, \( d_{R}^{(j)\text{mass}} \) and 
\[
q_{L}^{(j)} = \begin{pmatrix} u_{L}^{(j)} \\ d_{L}^{(j)\text{mass}} \end{pmatrix}.
\]

Table I allows the following mass matrix

\[
M = m_t \begin{pmatrix} q \sigma^4 S, & f \sigma^3 S, & c \sigma^2 S \\ g \sigma^3, & p \sigma^2, & a \sigma \\ d \sigma^2, & b \sigma, & 1 \end{pmatrix}
\]

where \( a, b, c, d, f, g, p, \) and \( q \) are O(1) numbers, and \( \sigma \) and \( S \) are dimensionless fields defined at the scale \( M_{\text{GUT}} \), \( \sigma = \frac{\sigma'}{M_{\text{GUT}}} \) and \( S = \frac{S'}{M_{\text{GUT}}} \).

V. CONCLUSION

We determined the quark mass matrix in terms of a small expansion parameter \( \sqrt{\varepsilon} \), which gives correctly all the quark masses and the CKM matrix elements at the EW scale, and obtain a progenitor form at the GUT scale by running this EW scale mass matrix. Finally, a possible texture form for the progenitor quark mass matrix is suggested.

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