Two-surface wave decay

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Abstract

The parametric excitation of pairs of electron surface waves (ESW) in the interaction of an ultrashort, intense laser pulse with an overdense plasma is discussed using an analytical model. The plasma has a simple step-like density profile. The ESWs can be excited either by the electric or by the magnetic part of the Lorentz force exerted by the laser and, correspondingly, have frequencies around $\omega/2$ or $\omega$, where $\omega$ is the laser frequency.
I. INTRODUCTION

Surface wave (SW) excitation has been widely studied in the past as a mechanism for electromagnetic (EM) wave absorption in highly inhomogeneous plasmas in several regimes and for different target geometries [1,2] and, more recently, in the specific case of solid targets irradiated by intense, ultrashort laser pulses [3,4]. The key problem in the linear mode conversion of the EM wave (the laser pulse) into an SW of the same frequency $\omega$ is that the phase matching between the two waves is not possible at a simple plasma-vacuum interface. The reason for this is that for an SW wave the wavevector along the plasma surface $k_s$ is larger than $\omega/c$, and thus cannot be equal to the component of the EM wavevector in the same direction $k_t = (\omega/c) \sin \theta$, where $\theta$ is the incidence angle of the laser pulse. Linear mode conversion of the laser pulse into an SW is possible in specifically tailored density profiles, e.g. for a “double-step” density profile (i.e. at the interface between an underdense plasma region where $\omega > \omega_p$ and an overdense region where $\omega < \omega_p$, with $\omega_p$ the plasma frequency), or if a periodic surface modulation with wavevector $k_p = k_t - k_s$ exists, such that the matching condition for the wavevectors is

$$k_t = k_s + k_p.$$  

In practical applications, corrugated targets are used to optimize SW generation at a given incidence angle. Other possible schemes of SW excitation are discussed, e.g., in Refs. [2,3].

In this paper, we discuss a different mechanism of electron surface wave (ESW) excitation by ultrashort laser pulses, based on the generation of two counterpropagating ESWs. The basic idea is as follows. An intense laser pulse impinging on an overdense, step-boundary plasma drives electron oscillations at the frequencies $\omega$ and $2\omega$ by the electric and the magnetic part of the Lorentz force, respectively. The electron response may be viewed as a superposition of “forced” surface modes with frequency $\omega_0 = \omega$ or $2\omega$, respectively, and wavevector $k_0 = (\omega_0/c) \sin \theta$. Each of these forced modes may “decay” parametrically into two ESWs (labeled “+” and “−”, respectively) provided that the matching conditions for a three-wave process hold:
$$k_0 = k_+ + k_-,$$

$$\omega_0 = \omega_+ + \omega_-.$$  \hspace{1cm} (2) \hspace{1cm} (3)

No pre-imposed target modulation is required to satisfy these relations, as depicted in Fig.1. The ESW frequencies may be written as \(\omega_\pm = \omega_0/2 \pm \delta\omega\). Thus, the “decay” of the mode excited by the electric (magnetic) force leads to the generations of ESWs with frequencies around \(\omega/2\) (\(\omega\)). We call this process “two-surface wave decay” (TSWD) in analogy to the well-known “two-plasmon decay” parametric instability in laser-plasma interactions.

Earlier investigations of parametric and three-wave nonlinear processes involving ESWs have been previously carried out in different regimes \([5,6]\). In particular, the parametric excitation of pairs of subharmonic ESWs in a magnetized plasma by an external EM wave was studied in Refs. \([7]\). These studies were restricted to the electrostatic limit in which the ESW frequency is \(\omega_s \simeq \omega_p/\sqrt{2}\) (see \([8]\) and eq.(11) below) and does not depend on the wavevector \(k_s\) in a semi-infinite, step boundary plasma (e.g. with a density profile \(n_i = n_i \Theta(x)\), where \(\Theta(x)\) is the Heaviside step function). In the electrostatic limit the phase matching with an impinging EM wave is possible only for particular target geometries where the frequency of electrostatic SWs depends on \(k_s\) (e.g. a plasma slab of thickness \(d\) where \(\omega_s\) depends on the product \(k_s d\) \([4]\)). In our model, we assume a semi-infinite geometry but we do not use the electrostatic approximation, and therefore in general the frequency \(\omega_s\) of ESWs is significantly lower than \(\omega_p/\sqrt{2}\) and depends on \(k_s\), approaching the EM limit \(\omega_s \simeq k_s c\) when \(\omega_s \rightarrow 0\). This allows the matching conditions \([3]\) to be satisfied in a semi-infinite plasma. It is interesting to notice that in Ref. \([4]\) it has been shown that two counterpropagating ESWs generate nonlinear radiation at a frequency that is the sum of the ESW frequencies; this process may be regarded, in some sense, as the inverse process of the TSWD.

One of the main results of this paper is that the intense \(v \times B\) force at \(2\omega\) of the laser pulse may drive a “\(2\omega \rightarrow \omega + \omega\)” decay with a growth time of a few laser cycles. In this paper, we focus on this case because of its direct relevance to the interpretation of recent numerical simulations in two dimensions (2D) for normal laser incidence, which
have provided evidence for this process \[10\]. Note that the surface oscillations observed in Ref. \[10\] and theoretically studied in the present paper are different from the grating-like surface inhomogeneities induced by the magnetic force that were studied in Ref. \[11\]; these latter “non-parametric” surface structures oscillate at the same frequency \(2\omega\) of the driving force and are generated at oblique incidence only. In the case of a \(p\)-polarized laser pulse at oblique incidence, we show below that the parametric process may be driven by the electric force leading to the generation of subharmonic ESWs around the frequency \(\omega/2\).

The primary aim of this paper is to describe the “basic principle” of the TSWD as a possible route to the generation of surface waves and to outline an analytical model for the TSWD. Therefore, for the sake of simplicity, we use a cold fluid, non-relativistic plasma model that can be tackled analytically. However, we believe that the TSWD is a general process that may occur for laser and plasma parameters beyond the limits of the approximations used in the present paper. Relativistic and kinetic effects, the inclusion of damping mechanisms and the fully nonlinear evolution of the TSWD are left for future investigations.

II. THE MODEL

In our model, we consider a cold plasma with immobile ions and a step-like density profile \(n_i(x) = n_i\Theta(x)\). The laser pulse has frequency \(\omega\), linear polarization and wavevector \(\mathbf{k} = (\omega/c)(\mathbf{x}\cos\theta + \mathbf{y}\sin\theta)\). Translational invariance along \(z\) is assumed. For both \(s\)- and \(p\)-polarizations, all fields oscillating at the frequency \(\omega\) can be determined inside and outside the plasma from general Fresnel formulae for refraction and transmission at a boundary \[12\] between vacuum and a medium with an (imaginary) refractive index \(n\) given by \(n^2 = 1 - \omega_p^2/\omega^2 < 0\), \(\omega_p = (4\pi n_i e^2/m_e)^{1/2}\) being the plasma frequency.

In what follows we study the excitation of “\(H\)” ESWs with the magnetic field in the \(z\) direction. Thus, we deal with an effectively 2D geometry, in which \(\mathbf{B} = B\mathbf{\hat{z}}\) and the Maxwell-Euler equations read
\[ \nabla \cdot \mathbf{E} = 4\pi e(n_0 - n_e), \quad (4) \]
\[ \nabla \times \mathbf{E} = -\frac{1}{c} \partial_t B_z, \quad (5) \]
\[ \nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{j} + \frac{1}{c} \partial_t \mathbf{E}, \quad (6) \]
\[ m_e \partial_t \mathbf{v} = -m_e \mathbf{v} \cdot \nabla \mathbf{v} - e \left( \mathbf{E} + \frac{\mathbf{v}}{e} \times \mathbf{B} \right). \quad (7) \]

and the current density is given by \( \mathbf{j} = -en_e \mathbf{v} \).

We adopt the following expansion for all fields

\[ F(x, y, t) = F_i(x) + \epsilon F_0(x, t - y \sin \theta/c) + \epsilon^2 [f_+(x, y, t) + f_-(x, y, t)], \quad (8) \]

where \( \epsilon \) is a small expansion parameter and \( f \) stands either for the electron density or the velocity or for the EM fields in the \((x, y)\) plane. In this expansion, \( F_i \) represents unperturbed fields of zero order (e.g., \( n_i \)). The term \( F_0 \) represents the “pump” field at the frequency \( \omega_0 \), that is taken to be of order \( \epsilon \) and is written as

\[ F_0 = \tilde{F}(\omega_0)(x) e^{i\omega_0(t - y \sin \theta/c)} + c.c. \quad (9) \]

In what follows the cases with \( \omega_0 = \omega \) and \( \omega_0 = 2\omega \) are studied separately. The last term in \((8)\) is the sum of two counterpropagating surface modes, which are assumed to be of order \( \epsilon^2 \) and are written as

\[ f_\pm = \frac{1}{2} \left[ \tilde{f}(\omega_\pm)(x) e^{i k y - i \omega \pm t} \right] + c.c. \quad (10) \]

Using expansion \((8)\), the coupling between the pump and the surface modes is of order \( \epsilon^3 \). Thus, from eqs.\((4)-(5)-(6)-(7)\) we obtain to order \( \epsilon \) the pump fields. To order \( \epsilon^2 \), we obtain a set of linearized, homogeneous 2D Maxwell-Euler equations which have ESWs as normal modes. Note that terms such as \( \mathbf{v}_0 \cdot \nabla \mathbf{v}_0 \) in the Euler equation are also of order \( \epsilon^2 \), but are non resonant with the ESWs at the frequencies \( \omega_\pm \). Such terms represent a source term for harmonic oscillations at \( 2\omega_0 \) and can be neglected in our model.

ESWs are “H” waves which can propagate in a dielectric medium along a layer of discontinuity of the dielectric function, with the latter changing sign across the boundary [13].
For a cold, step-boundary plasma there is no volume charge density perturbation associated with SWs, i.e.
\[ \nabla \cdot \mathbf{E} = -4\pi e \delta n_e = 0. \]
At a vacuum-plasma interface, electrons do not enter the vacuum side, but are “stopped” at the interface \((x = 0)\) forming a surface charge layer.

The dispersion relation for an SW of frequency \(\omega_s\) and wavevector \(k_s\) is
\[
k_s^2 = \frac{\omega_s^2 \omega_p^2 - \omega_s^2}{c^2 \omega_p^2 - 2\omega_s^2} = \frac{\omega_s^2 \alpha_s - 1}{c^2 \alpha_s - 2}. \tag{11}
\]
We have set for convenience \(\alpha_s = \omega_p^2/\omega_s^2\). The dispersion relation (11) is shown in Fig.1.

The EM field envelopes (see eq.(10)) are given by
\[
\tilde{E}_y^s(x) = \tilde{E}_y^s(0) \left[ \Theta(-x)e^{q<x} + \Theta(x)e^{-q>x} \right],
\]
\[
\tilde{B}_z^s(x) = \frac{i\omega_s}{q<} \tilde{E}_y^s(0) \left[ \Theta(-x)e^{q<x} + \Theta(x)e^{-q>x} \right],
\]
\[
\tilde{E}_x^s = ik \tilde{E}_y^s(0) \left[ \Theta(-x) \frac{e^{q<x}}{q<} - \Theta(x) \frac{e^{-q>x}}{q>} \right], \tag{12}
\]
where \(q_> = (\omega_s/c)(\alpha_s - 1/\sqrt{\alpha_s - 2}), q_< = (\omega_s/c)(1/\sqrt{\alpha_s - 2})\). The velocity fields are found from \(\tilde{v}^s = (ie/m_e\omega_s)\mathbf{E}^s\).

All feedback effects of the nonlinear coupling on the pump fields are neglected. The coupling terms that lead to the excitation of the ESW with frequency \(\omega_\pm\) may be represented by the nonlinear force
\[
f_\pm^{(NL)}(x, ɛt) \rightarrow f_\pm^{(NL)}(x, ɛt) = - \left[ m_e (v_\mp \cdot \nabla v_0 + v_0 \cdot \nabla v_\mp) + \frac{e}{c} (v_0 \times B_\mp + v \times B_0) \right]_{res}. \tag{13}
\]
In the term in square brackets on the r.h.s of (13) only “resonant” terms with the same phase of \(f_\pm^{(NL)}\) are included. Obviously, these terms exist if the matching conditions (3) hold.

The nonlinear coupling leads to the parametric excitation and growth of ESWs. We thus let the ESW envelopes vary slowly in time, i.e.
\[
\tilde{f}_\pm(x) \rightarrow \tilde{f}_\pm(x, ɛt) \tag{14}
\]
To evaluate the growth rate, we use an energy approach. The surface energy density of the ESW that will be needed in the calculations is given by
\[ U_\pm = \frac{k}{2\pi} \int_{-\pi/k}^{+\pi/k} dy \int_{-\infty}^{+\infty} dx \left\langle u_\pm^{(K)} + u_\pm^{(F)} \right\rangle, \]  

(15)

where the brackets denote average over one laser cycle and \( u_\pm^{(K)} \) and \( u_\pm^{(F)} \) are the volume densities of the kinetic and EM fields energies, respectively:

\[ u_\pm^{(K)} = \frac{m_e n_0}{2} |v_\pm|^2, \]  

(16)

\[ u_\pm^{(F)} = \frac{1}{8\pi} \left( |E_\pm|^2 + |B_{\pm,z}|^2 \right). \]  

(17)

The integral over \( y \) yields a factor 1/2. The kinetic energy contribution vanishes for \( x < 0 \).

Using eqs. (12) and \( \tilde{v}_{\pm, x}(0^+) = \tilde{v}_{\pm, y}(0)/\sqrt{\alpha - 1} \), and finally summing the two contributions we obtain for the total surface energy \( U_\pm = M_\pm |\tilde{v}_{\pm, y}|^2/2 \) where

\[ M_\pm = \frac{m_e n_i c \alpha_\pm (\alpha_\pm - 2)^{1/2}(\alpha_\pm^2 - 2\alpha_\pm + 2)}{(\alpha_\pm - 1)^2}. \]  

(18)

To obtain an expression for the temporal variation of \( U_\pm \) due to the nonlinear coupling forces, we write the Euler and the Ampere equation for the ESW fields keeping terms up to order \( \epsilon^3 \) and disregarding terms that are non-resonant with the oscillation:

\[ m_e \partial_t \mathbf{v}_\pm = -eE_\pm + \epsilon \mathbf{f}_\pm^{(NL)}, \]  

(19)

\[ \nabla \times \mathbf{B}_\pm = \frac{4\pi}{c} \left( -en_o \mathbf{v}_\pm + \epsilon \mathbf{J}_\pm^{(NL)} \right) + \frac{1}{c} \partial_t \mathbf{E}_\pm, \]  

(20)

where \( \mathbf{f}_\pm^{(NL)} \) is given by eq. (13), and \( \mathbf{J}_\pm^{(NL)} = -e\delta n_\pm^{(\omega_0)} \mathbf{v}_\pm \). Using eqs. (19-20) together with Maxwell’s equations we obtain for the temporal variation of the energy densities

\[ \partial_t u_\pm^{(K)} = -en_o \mathbf{v}_\pm \cdot \mathbf{E}_\pm + \epsilon \mathbf{f}_\pm^{(NL)} \cdot \mathbf{v}_\pm, \]  

(21)

\[ \partial_t u_\pm^{(F)} = -\nabla \cdot \mathbf{S}_\pm - \epsilon \mathbf{J}_\pm^{(NL)} \cdot \mathbf{E}_\pm, \]  

(22)

where \( \mathbf{S}_\pm = (c/4\pi)\mathbf{E}_\pm \times \mathbf{B}_\pm \) is the Poynting vector of the ESW. We integrate the sum of the energy densities over space and average over time. Noting that the total flux of \( \mathbf{S}_\pm \) vanishes because of the evanescence of the ESW fields, we finally obtain the equation for the evolution of the total energy \( U \) of the ESW:
\[
\partial_t U_\pm = \frac{2\pi}{k} \int_{-\pi/k}^{+\pi/k} dy \int_0^{+\infty} dx \partial_t \left\{ \frac{m_e n_0}{2} |v_\pm|^2 + \frac{1}{8\pi} \left( |E_\pm|^2 + |B_\pm|^2 \right) \right\}
\]
\[
\quad = \frac{2\pi}{k} \int_0^{+\infty} dx \int_{-\pi/k}^{+\pi/k} dy \left\{ v_\pm \cdot \left( e \delta n_c^{(\omega_0)} E_\pm + f_{NL}^{(\pm)} \right) \right\}.
\]

The integral extends for \( x > 0 \) only since there are no electrons for \( x < 0 \).

From eq. (23), performing the integral and neglecting high order terms one obtains two coupled equations in the general form

\[
\frac{M_\pm}{2} \partial_t |\tilde{v}_\pm|^2 = A_0 \omega |\tilde{v}_+||\tilde{v}_+| S_\pm \sin \varphi,
\]

where \( A_0 \) is the amplitude of the pump mode in dimensionless units, \( S_\pm \) are positive factors depending on \( \omega_p/\omega \) and \( \theta \), and \( \varphi \) is obtained from the phase factors of the pump and ESW modes as \( \varphi = \phi_+ + \phi_- - \phi_0 \). From (24) one easily obtains

\[
\partial_t^2 |\tilde{v}_\pm| = (A_0 \omega)^2 \left( \frac{S_+ S_-}{M_+ M_-} \right) \sin^2 \varphi |\tilde{v}_\pm|.
\]

Setting \( \sin \varphi = 1 \) simply corresponds to finding the relative phase of the growing modes with respect to the phase of the pump mode. The growth rate of ESWs is then

\[
\Gamma = A_0 \omega \sqrt{\frac{S_+ S_-}{M_+ M_-}}.
\]

In the next section we use the analytical method outlined above to study TSWD in two cases of particular relevance: TSWD driven by the \( \mathbf{v} \times \mathbf{B} \) force for normal laser incidence (which we name “\( 2\omega \to \omega + \omega \)” decay), and TSWD driven by the electric force for oblique laser incidence and \( p \)-polarization (“\( \omega \to \omega/2 + \omega/2 \)” decay).

**III. “\( 2\omega \to \omega + \omega \)” DECAY**

To discuss the “\( 2\omega \to \omega + \omega \)” decay, we restrict ourselves for simplicity to the case of normal incidence, as was done in the simulations reported in Ref. [10]. Thus, the ESW frequency \( \omega_\pm = \omega \) and \( k_+ = -k_- \). The case of oblique incidence is indeed very similar, the main difference being that the matching condition \( k_+ + k_- = (2\omega/c) \sin \theta \) holds, causing a
shift $\delta \omega \neq 0$ between the two ESWs. It is found that the growth rate decreases monotonically with increasing incidence angle, since the pump force has a maximum at normal incidence [14].

The laser wave can be represented by a single component of the vector potential, $A_z = A_z(x, t)$, that for $x > 0$ is given by

$$A_z(x, t) = A_z(x) \cos \omega t = A_z(0)e^{-x/l_s} \cos \omega t,$$

where $l_s = c/(\omega_p^2 - \omega^2)^{1/2}$ is the screening length. Imposing boundary conditions for the incident and reflected waves one finds $A_z(0) = 2A_i(\omega l_s/c)(1 + \omega^2 l_s^2/c^2)^{-1/2}$, where $A_i$ is the amplitude of the incident field.

Electrons perform their quiver motion in the $z$ direction. Thus, the magnetic force term is in the $x$ direction and has a secular term ($0\omega$), named the ponderomotive force, and an oscillating term ($2\omega$) that, in what follows, we simply name the $v \times B$ force. The secular term corresponds to radiation pressure and creates a surface polarization of the plasma. The $v \times B$ force drives a longitudinal, electrostatic oscillation that acts as a pump mode for TSWD. In the expansion (8), the pump amplitude is supposed to be of order $\epsilon$. As will be shown below, coupling between 1D and 2D fields occurs only in the overdense plasma ($x > 0$). Thus, since the $v \times B$ force is quadratic in the laser field, the expansion (8) also implies $a(0) \sim \epsilon^{1/2}$, where $a(0)$ is the (dimensionless) laser amplitude at the surface of the plasma, e.g $a(0) = (eA_z(0)/mc^2)$. For overdense plasmas $a(0) \sim (\omega/\omega_p)a_i < a_i$, where $a_i = (eA_i/mc^2)$. This yields $\epsilon \sim (\omega/\omega_p)^2 a_i^2 = (n_c/n_e)a_i^2$, where $n_c = m_e\omega^2/4\pi e$ is the “critical” density. We therefore expect our expansion procedure to be valid even at relativistic fields amplitudes $a_i \geq 1$ for high enough plasma densities.

We now derive the solutions for the 1D pump fields at $2\omega$. The electron oscillation velocity is obtained from the conservation of canonical momentum as $m_e v_z = eA_z/c$. For normal laser incidence, the longitudinal $v \times B$ force is given by

$$-\frac{e}{c}v_zB_y = -\frac{e^2}{m_e c^2}A_z \partial_x A_z \equiv F^0(x)(1 + \cos 2\omega t),$$

(28)
where we have set $F^0(x) = (m_e c^2 / 2 l_s) a_s^2 e^{-2x / l_s} \equiv F^0 e^{-2x / l_s}$. To order $\epsilon$, one obtains the following equations for the longitudinal, electrostatic motion:

$$m_e \partial_t V_x^{(2\omega)} = -e(E_x^{(0)} + E_x^{(2\omega)}) + F^0(x)(1 + \cos 2\omega t),$$

$$\partial_x (E_x^{(0)} + E_x^{(2\omega)}) = -4\pi e(\delta n_e^{(0)} + \delta n_e^{(2\omega)}),$$

$$\partial_t \delta n_e^{(2\omega)} = -n_0 \partial_x V_x^{(2\omega)}.$$  

All fields in the equations above decay inside the plasma as $\exp(-2x / l_s)$. The secular part simply gives $eE_x^{(0)}(x) = F^0(x)$ and $\delta n_e^{(0)} = -\partial_x F^0(x) / 4\pi e^2 = F^0(x) / 2\pi e^2 l_s$. For the motion at $2\omega$ one obtains

$$\bar{V}_x^{(2\omega)} = -\frac{i F^0}{\omega m_e D} \left( \frac{\omega^2}{\omega_p^2} \right) e^{-2x / l_s},$$

$$\delta \bar{n}_e^{(2\omega)} = n_i \frac{F^0}{\omega_p^2 l_s m_e D} e^{-2x / l_s},$$

$$e \bar{E}_x^{(2\omega)} = \frac{F^0}{2 D} e^{-2x / l_s}.$$  

The denominator $D$ is given by

$$D = 1 - \frac{4\omega^2}{\omega_p^2} = 1 - \frac{4n_e}{n_i},$$

which shows the well-known resonance at $n_e = 4n_c$ due to excitation of plasmons with frequency $\omega_p = 2\omega$ by the $\mathbf{v} \times \mathbf{B}$ force.

The difference between the total number of electrons and ions for $x > 0$ is, to order $\epsilon$,

$$\Delta N_e^{(x>0)} = \int_0^{+\infty} dx \left( \delta n_e^{(2\omega)} + \delta n_e^{(0)} \right) = \frac{F^0}{2\pi e^2} \left( 1 + \frac{\cos 2\omega t}{D} \right).$$

The fact that $\Delta N_e^{(x>0)} > 0$ during most of the oscillation implies that, due to compression from the ponderomotive and $\mathbf{v} \times \mathbf{B}$ forces, electrons leave behind a charge depletion layer of thickness $\zeta = \Delta N_e^{(x>0)} / n_i$. We note that $\zeta$ is of order $\epsilon$, thus to lowest order it is correct to treat the charge depletion layer as a surface layer. Heuristically, the oscillating behavior of $\zeta$ describes the plasma “moving mirror” effect [15]: due to charge depletion the laser is reflected at $x = \zeta$ rather than exactly at $x = 0$. This leads to the appearance of high harmonics in the reflected light.
Since $|D| < 1$, there is always a time interval in which electrons are pulled into vacuum forming a cloud of negative charge. This interval is very short for $n_e/n_c \gg 4$, i.e. $D \approx 1$, but for lower densities it is relevant in the interaction process. The motion in vacuum is strongly anharmonic and more difficult to solve analytically than the motion inside the plasma. However, it is found in section [III] that the surface modes gain energy during the phase of electron motion inside the plasma only, so that the expressions of fields for $x < 0$ are not needed. Nevertheless, we have to assume that the density of the electron cloud for $x < 0$ is low enough for the laser propagation not to be affected.

Using eq. (32), the NL force (13) may be written in terms of the oscillation velocity $V_x^{(2\omega)}$ as follows:

$$f_{NL}^\pm = -m_e \left( V_x^{(2\omega)}(x) \partial_x v_\mp + v_{x,\pm} \partial_x V_x^{(2\omega)}(x) \hat{x} \right) - \frac{e}{c} V_x^{(2\omega)}(x) B_{z,\mp} \hat{y}. \quad (37)$$

Because of the symmetry with respect to inversion of the $y$ axis, the two ESWs may differ only by a phase factor and have the same amplitude and energy. Inserting the force (37) in the energy equation (23) and performing the integral in $y$ we obtain for the ESW energy $U = U_\pm$

$$\partial_t U = \frac{1}{8} \sum_{l=+k,-k} \int_0^{+\infty} dx \mathbf{V}_x^* \cdot \left( e \delta n_e^{(2\omega)}(x) \mathbf{E}_l^* - m_e n_0 \mathbf{v}_x^* \partial_x V_x^{(2\omega)}(x) + m_e n_0 \mathbf{v}_x^* \partial_x V_x^{(2\omega)}(x) \right)$$

$$- n_0 \mathbf{v}_x^{(2\omega)}(x) \partial_x \mathbf{v}_l^* + n_0 \frac{e}{c} \mathbf{V}_x^{(2\omega)}(x) \mathbf{B}_{z,\mp} \hat{y} + \text{c.c.} \quad (38)$$

We rewrite all fields as functions of $\mathbf{V}_x^{(2\omega)}$ and $\mathbf{v}_\pm$. We obtain, after some algebra

$$\partial_t U = \frac{m_e n_1}{2(1 + q_\perp l_s)} \Re \left\{ \mathbf{V}_x^{(2\omega)} \left[ (1 + q_\perp l_s) \mathbf{v}_+^* \mathbf{v}_-^* + 2 \mathbf{v}_x^* \mathbf{v}_x^* \right] - \frac{\mathbf{v}_y^* + \mathbf{v}_y^*}{q_\perp l_s} \right\}. \quad (39)$$

All the terms of eq. (39) are proportional to

$$\Re \left( \mathbf{V}_x^{(2\omega)} \mathbf{v}_+^* \mathbf{v}_-^* \right) = |\mathbf{V}_x^{(2\omega)} \mathbf{v}_\parallel|^2 \cos(\phi + \pi/2), \quad (40)$$

where $\phi = \phi_+ + \phi_-$, $\phi_\pm$ being the phase angles of $v_\pm$, and we used eq. (32) and the fact that $F^0$ is real and positive. Thus, the phase of the growing ESWs with respect to the $\mathbf{v} \times \mathbf{B}$ force

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is such that $\phi = -\pi/2$, the value for which the growth rate is positive and has a maximum. The overlap of the two ESWs gives
\[
\frac{\tilde{v}_+}{2} e^{i k_y - i \omega t} + \frac{\tilde{v}_-}{2} e^{-i k_y - i \omega t} = \frac{|\tilde{v}_\pm|}{2} e^{-i \omega t} \left( e^{i k_y + i \phi_+} + e^{-i k_y + i \phi_-} \right) = |\tilde{v}_\pm| e^{-i \omega t - i \pi/4} \cos \left( k y + \Delta \phi \right),
\]
where $\Delta \phi = (\phi_+ - \phi_-)/2$. This is a standing wave which has a temporal phase shift $-\pi/4$ with respect to the $v \times B$ force of eq. (28). The phase is such that, at a given position in $y$, the temporal maxima of $V_x^{(2\omega)}$ and $v^{(\omega)}$ overlap once for laser cycle. The angle $\Delta \phi$ gives the location of the maxima of the standing wave on the $y$ axis, which depends on the arbitrary choice of the initial phase.

Using eq. (40), we can rewrite the energy variation as
\[
\partial_t U = m_e n_i |\tilde{V}_x^{(2\omega)}|^2 \left[ |\tilde{v}_+|^2 + \frac{2|\tilde{v}_{x,k}|^2}{1 + q_> l_s} - \frac{2|\tilde{v}_{y,k}|^2}{q_> l_s (1 + q_> l_s)} \right].
\]
Note that $\partial_t U > 0$ since $q_> l_s = \sqrt{(\alpha - 1)(\alpha - 2)^{-1}} > 1$, where $\alpha = \omega_p^2/\omega^2 = n_e/n_c$. Eliminating $\tilde{V}_x^{(2\omega)}$ as a function of $a_i$ and $\alpha$, after some algebra the growth rate $\Gamma$ is obtained:
\[
\Gamma = 4 \omega a_i^2 \frac{(\alpha - 1)^{3/2}}{\alpha |\alpha - 4| |(\alpha - 1)^2 + 1| \alpha^{-1/2}} \left[ 1 + \frac{2}{\alpha (1 + q_> l_s)} - \frac{2(\alpha - 1)}{\alpha q_> l_s (1 + q_> l_s)} \right] \approx 4 \omega a_i^2 \frac{(\alpha - 1)^{3/2}}{\alpha |\alpha - 4| |(\alpha - 1)^2 + 1| (\alpha - 2)^{1/2}}.
\]
The leading contribution was highlighted in the last equality.

A plot of the growth rate as a function of $\alpha$ is given in Fig. 2. For moderately overdense plasmas, $\Gamma$ is a considerable fraction of the laser frequency. The resonance at $n_e = 4 n_c$ is due to the resonant behavior of the pump field. The growth rate diverges also in the electrostatic limit when $n_e \to 2 n_c$. However, in this limit the value of $k$ tends to infinity, i.e. the wavelength becomes very small and one expects that this second resonance might be damped by thermal effects, which are neglected in our model and are left for future investigations.

The comparison of spatial and temporal scales predicted from this model with the simulation results shows reasonable agreement for laser and plasma parameters such that our
expansion procedure is valid [10]. We note that simulation results for high intensities in the relativistic regime suggest that a $2\omega \to \omega + \omega$ TSWD-like process still occurs and produces strong rippling of the plasma surface; the spatial scales are different from those predicted by our analytical, cold fluid model, but still of the same order of magnitude.

We conclude this section by noting that, although we took the laser polarization along $z$, the whole derivation is valid at normal incidence for any polarization direction. The only difference in 2D geometry is that, for the laser polarization along $y$, the quiver oscillations at $\omega$ and the ESW oscillations at the same frequency overlap. This is observed in numerical simulations [14].

IV. “$\omega \to \omega/2 + \omega/2$” DECAY

A case of particular interest is that of oblique incidence and $p$-polarization of the laser pulse, since these conditions lead in general to a stronger plasma coupling. In the preceding section we have found that the $2\omega \to \omega + \omega$ decay is driven by the longitudinal component of the velocity field, as shown by eq.(37). For $p$-polarization and $\theta \neq 0$ the electric field drives a strong longitudinal oscillation at the frequency $\omega$; thus, one expects that the TSWD process leads to the generation of subharmonic ESWs, having frequencies around $\omega/2$, with peak efficiency at some angle of incidence. As shown below, this picture is substantially correct. However, there is a small but non-vanishing growth rate for the $\omega \to \omega/2 + \omega/2$ process even at normal incidence. In fact, the pump force (13) is now given by

$$f_{\pm}^{(NL)} = -m_e(v_{\pm,x}\partial_x V^{(\omega)} + V_x^{(\omega)}\partial_x v_{\pm}) - m_e(v_{\pm,y}\partial_y V^{(\omega)} + V_y^{(\omega)}\partial_y v_{\pm})$$

$$-e/c(V^{(\omega)} \times B_{\pm} + v_{\pm} \times B^{(\omega)}),$$

and does not vanish for $\theta = 0$.

The case of oblique incidence and $p$-polarization of the laser pulse may be tackled with an approach analogous to that of section [11]. The main physical difference with the previous case is that now the “pump” is a divergence-free velocity field at the laser frequency $\omega$, and
its amplitude is now proportional to the laser field rather than to the laser intensity. In applying the expansion procedure (8) again, now \( \epsilon \sim (\omega/\omega_p) a_i \) (appendix A). Thus, the limits of validity of this expansion are more restrictive in the present case: the expansion tends to be valid only for rather high densities or rather low fields. One must also note that temperature effects, neglected in the cold plasma approximation, are important for low fields.

The calculation of the growth rate for subharmonic ESWs can be performed by the same method as in the preceding sections in a straightforward way. The details are reported in appendix A. As a final result, the growth rate \( \Gamma = \Gamma(\alpha, \theta) \) may be written as

\[
\Gamma = a_i \omega |F_B| \left( \frac{n_e}{n_c} \right)^{5/2} K(\alpha, \theta),
\]

where \( F_B \) is the Fresnel factor for the magnetic field and the factor \( K(\alpha, \theta) \) scales weakly with density. A plot of \( \Gamma \) as a function of \( \theta \) for different values of \( \alpha \) is reported in Fig.3.

The frequency shift \( \delta \omega \) of the two ESWs can be calculated as a function of \( \theta \) and \( \alpha \) from the matching condition \( k_+ + k_- = (\omega/c) \sin \theta \). The result is shown in Fig.4. Comparing with Fig.3, one finds that \( \delta \omega \approx 0.1 \omega \) in conditions favorable to the \( \omega \rightarrow \omega/2 + \omega/2 \) TSWD.

V. DISCUSSION

The TSWD concept has been investigated analytically in two cases that clarify its basic features in the context of the cold fluid plasma approximation. This process may lead to the excitation of ESWs by an ultrashort laser pulse in a solid target without any special (e.g., grating-like) structure, and even for normal incidence and \( s \)-polarization.

In both the cases discussed in this paper, we found a strong decrease of the TSWD growth rate for increasing densities. This does not necessarily imply that TSWD is not relevant to short pulse interaction with solid targets which have \( n_e/n_c \gg 1 \). In fact, important processes such as high harmonic generation or fast electron production are more efficient when the laser pulse interacts with a moderate density plasma. This is the case for most of the experiments.
of short pulse interaction with solid targets, since a moderate density “shelf” is usually
produced at the time of peak pulse intensity by target ablation and plasma hydrodynamic
expansion during the leading edge of the short pulse or during the long prepulse. For
instance, the “$2\omega \rightarrow \omega + \omega$” decay appears to be very efficient exactly in conditions favorable
for high harmonics production via the “moving mirror” effect \cite{15,16}. Simulations \cite{10}
suggest that TSWD in strongly nonlinear and relativistic regimes (beyond the limits of the
analytical approach of this paper) may produce surface perturbations acting as a ”seed” for,
e. g. , electron instabilities leading to current filamentation \cite{17,18} or detrimental distortions
of the moving mirrors, and for Rayleigh-Taylor-like (RT) hydrodynamic instabilities occurring
on the time scale of ion motion. In Ref. \cite{10} it is estimated that the growth rate of the RT
instability is much slower than that of TSWD. A different hydrodynamic instability with a
surprisingly high growth rate has been reported in Ref. \cite{19}).

The $2\omega \rightarrow \omega + \omega$ and the $\omega \rightarrow \omega/2 + \omega/2$ processes have been considered independently.
This is appropriate since these are resonant processes which do not interfere with each other.
In principle, both decays may occur during the interaction of an intense laser pulse with
an overdense plasma. According to our analysis the $2\omega \rightarrow \omega + \omega$ process appears to have
a stronger growth rate. However, in the case of the $\omega \rightarrow \omega/2 + \omega/2$ process our analytical
approach is valid for a very narrow range of parameters only. Either an extension of the
present analytical approach or numerical simulations are needed to investigate the TSWD
for a very intense, $p$-polarized laser pulse at oblique incidence.

VI. CONCLUSIONS

We have discussed the Two-Surface Wave Decay as a novel mechanism for the excitation
of electron surface waves in the interaction of ultrashort, intense laser pulses with overdense
plasmas. TSWD is based on the parametric excitation of a pair of ESWs. The “pump” force
may either be the magnetic or the electric force of the laser pulse, and leads respectively
to the generation of two ESWs with frequencies close to the laser frequency of half of it.
An analytical model for TSWD in the cold fluid plasma, non-relativistic approximation has been developed. The model supports the interpretation of recent simulation results [10] and suggests that TSWD may be of relevance in certain regimes of laser interaction with solid targets.

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APPENDIX A: GROWTH RATE OF THE $\omega \rightarrow \omega/2 + \omega/2$ DECAY

We now give the detailed derivation of the growth rate for the $\omega \rightarrow \omega/2 + \omega/2$ TSWD of section IV. The pump fields are easily found with the help of Fresnel formulae and Maxwell equations. For instance, the magnetic field is given by

$$B_2^{(\omega)}(x, t) = B_2^{(\omega)}(0^+)e^{iky\sin \theta - x/l_p - i\omega t} + \text{c.c.},$$

where the screening length is given by

$$l_p = \frac{c}{\omega_p} \left( 1 - \frac{\omega^2}{\omega_p^2} \cos^2 \theta \right)^{-1/2} = \frac{c}{\omega} \frac{1}{\sqrt{\alpha - \cos^2 \theta}},$$

and the magnetic field at the surface is given by the Fresnel formula

$$\frac{B_2(0^+)}{B_{z,i}} = \frac{2n^2 \cos \theta}{\sqrt{n^2 - \sin^2 \theta + n^2 \cos \theta}} = \frac{2(\alpha - 1) \cos \theta}{(\alpha - 1) \cos \theta - i\sqrt{\alpha - \cos^2 \theta}} \equiv F_B(\theta),$$

where $B_{z,i}$ is the incident field amplitude in vacuum. The electric field components are found by using $\nabla \cdot \mathbf{E}^{(\omega)} = 0$ and $\nabla \times \mathbf{E}^{(\omega)} = i(\omega/c)B_2^{(\omega)} + \text{c.c.}$.

The nonlinear force is given by eq. (37). Evaluating the spatial derivatives as

$$\partial_x \mathbf{V}^{(\omega)} = -\mathbf{V}^{(\omega)}/l_p, \quad \partial_y \mathbf{V}^{(\omega)} = i k_l \mathbf{V}, \quad \partial_y \mathbf{v}_\pm = i k_{l_\pm} \mathbf{v}_\pm, \quad \text{and} \quad \partial_x \mathbf{v}_\pm = -q_\pm \mathbf{v}_\pm$$

where $q_\pm \equiv q_{>}(\omega_\pm)$, and keeping resonant terms only we find
It is worth rewriting these equations in a more compact form. Introducing

\[
\tilde{f}_{NL}(\omega) = \frac{m_e}{4} \left( \frac{1}{l_p} + q_\mp \right) \tilde{V}_y^{(\omega)} \tilde{v}^{*}_{\mp,x} - \frac{e}{m_e c} \left( \tilde{V}_y^{(\omega)} \tilde{B}^{*}_{\mp,z} + \tilde{B}_z^{(\omega)} \tilde{v}^{*}_{\mp,y} \right)
\]

\[
-ik_t \tilde{V}_x^{(\omega)} \tilde{v}^{*}_{\mp,y} + ik_t \tilde{V}_y^{(\omega)} \tilde{v}^{*}_{\pm,x} \right) e^{-(q_\mp + 1/l_p)x} + c.c.,
\]

we thus find

\[
\tilde{f}_{NL}(\omega) = \frac{m_e}{4} \frac{1}{l_p} \tilde{V}_y^{(\omega)} \tilde{v}^{*}_{\mp,y} + q_\mp \tilde{V}_x^{(\omega)} \tilde{v}^{*}_{\pm,y} + \frac{e}{m_e c} \left( \tilde{V}_x^{(\omega)} \tilde{B}^{*}_{\mp,z} + \tilde{B}_z^{(\omega)} \tilde{v}^{*}_{\pm,x} \right)
\]

\[
-ik_t (k_t - k_\mp) \tilde{V}_y^{(\omega)} \tilde{v}^{*}_{\mp,y} \right) e^{-(q_\mp + 1/l_p)x} + c.c.
\]

(A4)

We thus obtain

\[
\tilde{f}_{NL}(\omega) = \frac{m_e}{4} \tilde{V}_y^{(\omega)} \tilde{v}^{*}_{\mp,y} \left[ k_\mp \left( k_t + k_t q_\mp l_p + k_\mp \right) + \frac{\omega^2}{q_\mp c^2} (\alpha_\mp - 1) \right] e^{-(q_\mp + 1/l_p)x}
\]

\[
= \frac{m_e}{4} \tilde{V}_y^{(\omega)} \tilde{v}^{*}_{\mp,y} Q_{\pm,x} e^{-(q_\mp + 1/l_p)x}
\]

(A6)

\[
\tilde{f}_{NL}(\omega) = \frac{m_e}{4} \tilde{V}_y^{(\omega)} \tilde{v}^{*}_{\mp,y} i \left[ k_t l_p \left( q_\mp - \frac{1}{l_p} - \frac{k_t k_\mp}{q_\mp} - \frac{\omega^2}{q_\mp c^2} (\alpha_\mp - 1) \right) + k_\mp \right] e^{-(q_\mp + 1/l_p)x}.
\]

(A7)

\[
\equiv \frac{m_e}{4} \tilde{V}_y^{(\omega)} \tilde{v}^{*}_{\mp,y} i Q_{\pm,y} e^{-(q_\mp + 1/l_p)x},
\]

(A8)

The relations \(\omega^2_\pm (\alpha_\pm - 1)/q_\pm = \omega_c \sqrt{\alpha_\pm - 2}, k_\mp/q_\pm = 1/\sqrt{\alpha_\pm - 1}, k_t l_p = \sin \theta (\alpha - \cos^2 \theta)^{-1/2}\)

may be used to simplify the expressions for \(Q_{\pm,x}\) and \(Q_{\pm,y}\). We thus obtain

\[
Q_{\pm,x} = \frac{1}{\sqrt{\alpha_\pm - 1}} \left[ k_t (1 + q_\mp l_p) + k_\mp \right] + \frac{\omega^2_\pm}{c} \sqrt{\alpha_\pm - 2} + \frac{1}{l_p},
\]

(A10)

\[
Q_{\pm,y} = k_t l_p \left( q_\mp - \frac{1}{l_p} - \frac{k_t}{\sqrt{\alpha_\pm - 1}} - \frac{\omega^2_\pm}{c} \sqrt{\alpha_\pm - 2} \right) + k_\mp.
\]

(A11)

In the energy equation (23), since \(\delta n^{(\omega)} = 0\) the integrand, eliminating \(\tilde{v}_{\pm,x}\), is given by

\[
\left\langle v_{\pm}, f_{NL}^{V_{NL}} \right\rangle = \frac{m_e}{16} i \tilde{V}_y^{(\omega)} \tilde{v}^{*}_{\mp,y} \tilde{v}^{*}_{\mp,y} \left( \frac{k_\mp}{q_\pm} Q_{\pm,x} + Q_{\pm,y} \right) + c.c.
\]

\[
= \frac{m_e}{8} \Re \left[ i \tilde{V}_y^{(\omega)} \tilde{v}^{*}_{\mp,y} \tilde{v}^{*}_{\mp,y} \right] \left( \frac{k_\mp}{q_\pm} Q_{\pm,x} + Q_{\pm,y} \right).
\]

(A12)

We thus find

\[
\partial_t U_{\pm} = \frac{m_e}{8} \frac{l_p}{1 + (q_+ + q_-)l_p} \left( \frac{k_\mp}{q_\pm} Q_{\pm,x} + Q_{\pm,y} \right) \left| \tilde{V}_y^{(\omega)} \tilde{v}_{\mp,y} \right| \sin \varphi.
\]

(A14)

Here, \(\varphi = (\phi_+ + \phi_- - \Phi)\), where \(\Phi\) and \(\phi_\pm\) are the phase factors of \(\tilde{V}_y^{(\omega)}\) and \(\tilde{v}_{\pm,y}\), respectively.

It is worth rewriting these equations in a more compact form. Introducing \(a_i = eB_i/m_\omega c\)

and using Fresnel’s formulas to eliminate \(\tilde{V}_y\) we obtain
\[
\frac{\partial_t U_\pm}{m_e} = n_e a_i c |F_B(\theta)| |\bar{v}_{+,y}| |\bar{v}_{-,y}| \sin \varphi G_\pm(\alpha, \theta),
\]  
(A15)

where we have posed \(G_\pm(\alpha, \theta) = G_0(\alpha, \theta) g_\pm(\alpha, \theta)\) and

\[
G_0 = \frac{\sqrt{\alpha - \cos^2 \theta}}{(\alpha - 1)[1 + (q_+ + q_-)l_p]},
\]  
(A16)

\[
g_\pm = l_p \left(\frac{k_\pm}{q_\pm} Q_{\pm,x} + Q_{\pm,y}\right) - \frac{1}{\sqrt{(\alpha_+ - 1)(\alpha_+ - 1)}} [k_1 l_p (1 + q_+ l_p) + k_2 l_p] + \frac{\omega_\pm l_p \sqrt{\alpha_+ - 2}}{c \sqrt{\alpha_+ - 1}} + \frac{1}{\sqrt{\alpha_+ - 1}}
\]

\[
+ k_1 l_p \left(\frac{q_+ l_p - 1 - \frac{k_1 l_p}{\sqrt{\alpha_+ - 1}} - \frac{\omega_\pm l_p}{c \sqrt{\alpha_+ - 2}}}{\sqrt{\alpha_+ - 1}} + k_2 l_p\right).
\]  
(A17)

Using the equations above we obtain the following coupled equations in the form (24) for the field amplitudes

\[
\mu_\pm \partial_t |\bar{v}_{+,y}|^2 = a_i \omega |\bar{v}_{+,y}| |\bar{v}_{-,y}| |F_B| G_\pm \sin \varphi,
\]  
(A18)

where \(\mu_\pm \equiv 4M_\pm/(m_e n_i c)\). Thus, the growing modes have relative phases such that \(\sin \varphi = 1\). The growth rate is given by

\[
\Gamma = a_i \omega |F_B| G_0 \sqrt{\frac{|g_+ g_-|}{\mu_+ \mu_-}} \equiv a_i \omega |F_B| \left(\frac{n_e}{n_c}\right)^{5/2} K(\alpha, \theta)
\]  
(A19)

which is the formula (45). A plot of \(\Gamma\) is depicted in Fig.3.
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FIGURES

FIG. 1. Dispersion relation of electron surface waves (thick lines) eq.(11), and the matching conditions for TSWD, eq.(3).

FIG. 2. The TSWD growth rate $\Gamma$ (normalized to $a_i^2 \omega$) for the $2\omega \rightarrow \omega + \omega$ process at normal incidence, eq.(43), as a function of $\alpha = n_e/n_c$.

FIG. 3. The TSWD growth rate $\Gamma$ (normalized to $a_i \omega$) for the $\omega \rightarrow \omega/2 + \omega/2$ process at oblique incidence, eq.(45), as a function of $\theta$ and $\alpha = n_e/n_c$.

FIG. 4. The frequency shift $\delta \omega$ for the $\omega \rightarrow \omega/2 + \omega/2$ process at oblique incidence, as a function of $\theta$ and $\alpha = n_e/n_c$. The dashed line gives the shift $\delta \omega^{max} = (1/2) \sin \theta$ that is obtained for $\alpha \rightarrow \infty$. 

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\[ \frac{\Gamma}{d^2 \omega} \]

\[ \alpha = \frac{n_e}{n_c} \]
Two-surface wave decay

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Abstract

The parametric excitation of pairs of electron surface waves (ESW) in the interaction of an ultrashort, intense laser pulse with an overdense plasma is discussed using an analytical model. The plasma has a simple step-like density profile. The ESWs can be excited either by the electric or by the magnetic part of the Lorentz force exerted by the laser and, correspondingly, have frequencies around $\omega/2$ or $\omega$, where $\omega$ is the laser frequency.

\[\delta \omega \approx \frac{1}{2} \alpha \omega \sin \theta\]
Surface wave (SW) excitation has been widely studied in the past as a mechanism for electromagnetic (EM) wave absorption in highly inhomogeneous plasmas in several regimes and for different target geometries \[\text{[1,2]}\] and, more recently, in the specific case of solid targets irradiated by intense, ultrashort laser pulses \[\text{[3,4]}\]. The key problem in the linear mode conversion of the EM wave (the laser pulse) into an SW of the same frequency $\omega$ is that the phase matching between the two waves is not possible at a simple plasma-vacuum interface. The reason for this is that for an SW wave the wavevector along the plasma surface $k_s$ is larger than $\omega/c$, and thus cannot be equal to the component of the EM wavevector in the same direction $k_t = (\omega/c) \sin \theta$, where $\theta$ is the incidence angle of the laser pulse. Linear mode conversion of the laser pulse into an SW is possible in specifically tailored density profiles, e.g. for a “double-step” density profile (i.e. at the interface between an underdense plasma region where $\omega > \omega_p$ and an overdense region where $\omega < \omega_p$, with $\omega_p$ the plasma frequency), or if a periodic surface modulation with wavevector $k_p = k_t - k_s$ exists, such that the matching condition for the wavevectors is

$$k_t = k_s + k_p. \quad (1)$$

In practical applications, corrugated targets are used to optimize SW generation at a given incidence angle. Other possible schemes of SW excitation are discussed, e.g., in Refs. \[\text{[2,3]}\].

In this paper, we discuss a different mechanism of electron surface wave (ESW) excitation by ultrashort laser pulses, based on the generation of two counterpropagating ESWs. The basic idea is as follows. An intense laser pulse impinging on an overdense, step-boundary plasma drives electron oscillations at the frequencies $\omega$ and $2\omega$ by the electric and the magnetic part of the Lorentz force, respectively. The electron response may be viewed as a superposition of “forced” surface modes with frequency $\omega_0 = \omega$ or $2\omega$, respectively, and wavevector $k_0 = (\omega_0/c) \sin \theta$. Each of these forced modes may “decay” parametrically into two ESWs (labeled “+” and “−”, respectively) provided that the matching conditions for a three-wave process hold:
No pre-imposed target modulation is required to satisfy these relations, as depicted in Fig.1. The ESW frequencies may be written as $\omega_\pm = \omega_0/2 \pm \delta \omega$. Thus, the “decay” of the mode excited by the electric (magnetic) force leads to the generations of ESWs with frequencies around $\omega/2$ ($\omega$). We call this process “two-surface wave decay” (TSWD) in analogy to the well-known “two-plasmon decay” parametric instability in laser-plasma interactions.

Earlier investigations of parametric and three-wave nonlinear processes involving ESWs have been previously carried out in different regimes [5,6]. In particular, the parametric excitation of pairs of subharmonic ESWs in a magnetized plasma by an external EM wave was studied in Refs. [7]. These studies were restricted to the electrostatic limit in which the ESW frequency is $\omega_s \simeq \omega_p/\sqrt{2}$ (see [8] and eq. (11) below) and does not depend on the wavevector $k_s$ in a semi-infinite, step boundary plasma (e.g. with a density profile $n_i = n_i \Theta(x)$, where $\Theta(x)$ is the Heaviside step function). In the electrostatic limit the phase matching with an impinging EM wave is possible only for particular target geometries where the frequency of electrostatic SWs depends on $k_s$ (e.g. a plasma slab of thickness $d$ where $\omega_s$ depends on the product $k_s d$ [7]). In our model, we assume a semi-infinite geometry but we do not use the electrostatic approximation, and therefore in general the frequency $\omega_s$ of ESWs is significantly lower than $\omega_p/\sqrt{2}$ and depends on $k_s$, approaching the EM limit $\omega_s \simeq k_s c$ when $\omega_s \rightarrow 0$. This allows the matching conditions (3) to be satisfied in a semi-infinite plasma. It is interesting to notice that in Ref. [9] it has been shown that two counterpropagating ESWs generate nonlinear radiation at a frequency that is the sum of the ESW frequencies; this process may be regarded, in some sense, as the inverse process of the TSWD.

One of the main results of this paper is that the intense $v \times B$ force at $2\omega$ of the laser pulse may drive a “$2\omega \rightarrow \omega + \omega$” decay with a growth time of a few laser cycles. In this paper, we focus on this case because of its direct relevance to the interpretation of recent numerical simulations in two dimensions (2D) for normal laser incidence, which
have provided evidence for this process [10]. Note that the surface oscillations observed in Ref. [10] and theoretically studied in the present paper are different from the grating-like surface inhomogeneities induced by the magnetic force that were studied in Ref. [11]; these latter “non-parametric” surface structures oscillate at the same frequency 2ω of the driving force and are generated at oblique incidence only. In the case of a p-polarized laser pulse at oblique incidence, we show below that the parametric process may be driven by the electric force leading to the generation of subharmonic ESWs around the frequency ω/2.

The primary aim of this paper is to describe the “basic principle” of the TSWD as a possible route to the generation of surface waves and to outline an analytical model for the TSWD. Therefore, for the sake of simplicity, we use a cold fluid, non-relativistic plasma model that can be tackled analytically. However, we believe that the TSWD is a general process that may occur for laser and plasma parameters beyond the limits of the approximations used in the present paper. Relativistic and kinetic effects, the inclusion of damping mechanisms and the fully nonlinear evolution of the TSWD are left for future investigations.

II. THE MODEL

In our model, we consider a cold plasma with immobile ions and a step-like density profile \( n_i(x) = n_i \Theta(x) \). The laser pulse has frequency \( \omega \), linear polarization and wavevector \( \mathbf{k} = (\omega/c)(\hat{x}\cos \theta + \hat{y}\sin \theta) \). Translational invariance along \( z \) is assumed. For both s- and p-polarizations, all fields oscillating at the frequency \( \omega \) can be determined inside and outside the plasma from general Fresnel formulae for refraction and transmission at a boundary [12] between vacuum and a medium with an (imaginary) refractive index \( n \) given by \( n^2 = 1 - \omega_p^2/\omega^2 < 0 \), \( \omega_p = (4\pi n_i e^2/m_e)^{1/2} \) being the plasma frequency.

In what follows we study the excitation of “H” ESWs with the magnetic field in the \( z \) direction. Thus, we deal with an effectively 2D geometry, in which \( \mathbf{B} = B\hat{z} \) and the Maxwell-Euler equations read
\[ \nabla \cdot \mathbf{E} = 4\pi\varepsilon (n_0 - n_e), \quad (4) \]
\[ \nabla \times \mathbf{E} = -\frac{1}{c} \partial_t \mathbf{B}_z, \quad (5) \]
\[ \nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{j} + \frac{1}{c} \partial_t \mathbf{E}, \quad (6) \]
\[ m_e \partial_t \mathbf{v} = -m_e \mathbf{v} \cdot \nabla \mathbf{v} - \varepsilon \left( \mathbf{E} + \frac{\mathbf{v}}{c} \times \mathbf{B} \right), \quad (7) \]

and the current density is given by \( \mathbf{j} = -en_e \mathbf{v} \).

We adopt the following expansion for all fields

\[ F(x, y, t) = F_i(x) + \epsilon F_0(x, t - y \sin \theta/c) + \epsilon^2 [f_+(x, y, t) + f_-(x, y, t)], \quad (8) \]

where \( \epsilon \) is a small expansion parameter and \( f \) stands either for the electron density or the velocity or for the EM fields in the \((x, y)\) plane. In this expansion, \( F_i \) represents unperturbed fields of zero order (e.g., \( n_i \)). The term \( F_0 \) represents the “pump” field at the frequency \( \omega_0 \), that is taken to be of order \( \epsilon \) and is written as

\[ F_0 = \tilde{F}^{(\omega_0)}(x)e^{i\omega_0(t-y\sin\theta/c)} + \text{c.c.} \quad (9) \]

In what follows the cases with \( \omega_0 = \omega \) and \( \omega_0 = 2\omega \) are studied separately. The last term in \((8)\) is the sum of two counterpropagating surface modes, which are assumed to be of order \( \epsilon^2 \) and are written as

\[ f_{\pm} = \frac{1}{2} \left[ \tilde{f}^{(\omega_{\pm})}(x)e^{ik_{\pm}(y-y\sin\theta/c)} \right] + \text{c.c..} \quad (10) \]

Using expansion \((8)\), the coupling between the pump and the surface modes is of order \( \epsilon^3 \). Thus, from eqs.\((4\,5\,6\,7)\) we obtain to order \( \epsilon \) the pump fields. To order \( \epsilon^2 \), we obtain a set of linearized, homogeneous 2D Maxwell-Euler equations which have ESWs as normal modes. Note that terms such as \( \mathbf{v}_0 \cdot \nabla \mathbf{v}_0 \) in the Euler equation are also of order \( \epsilon^2 \), but are non resonant with the ESWs at the frequencies \( \omega_{\pm} \). Such terms represent a source term for harmonic oscillations at \( 2\omega_0 \) and can be neglected in our model.

ESWs are “H” waves which can propagate in a dielectric medium along a layer of discontinuity of the dielectric function, with the latter changing sign across the boundary \[13\].
For a cold, step-boundary plasma there is no volume charge density perturbation associated with SWs, i.e. $\nabla \cdot \mathbf{E} = -4\pi e\delta n_e = 0$. At a vacuum-plasma interface, electrons do not enter the vacuum side, but are “stopped” at the interface ($x = 0$) forming a surface charge layer.

The dispersion relation for an SW of frequency $\omega_s$ and wavevector $k_s$ is

$$k_s^2 = \frac{\omega_s^2}{c^2} \frac{\omega_p^2 - \omega_s^2}{\omega_p^2 - 2\omega_s^2} = \frac{\omega_s^2 \alpha_s - 1}{c^2 \alpha_s - 2}. \quad (11)$$

We have set for convenience $\alpha_s = \omega_p^2/\omega_s^2$. The dispersion relation (11) is shown in Fig.1.

The EM field envelopes (see eq. (10)) are given by

$$\tilde{E}_y(x) = \tilde{E}_y(0) \left[ \Theta(-x)e^{q_x < x} + \Theta(x)e^{-q_x > x} \right],$$
$$\tilde{B}_z(x) = \frac{i\omega_s}{c} \tilde{E}_y(0) \left[ \Theta(-x)e^{q_x < x} + \Theta(x)e^{-q_x > x} \right],$$
$$\tilde{E}_x = ik \tilde{E}_y(0) \left[ \Theta(-x) \frac{e^{q_x < x}}{q_<} - \Theta(x) \frac{e^{-q_x > x}}{q_>} \right], \quad (12)$$

where $q_> = (\omega_s/c)(\alpha_s - 1/\sqrt{\alpha_s - 2}), q_< = (\omega_s/c)(1/\sqrt{\alpha_s - 2})$. The velocity fields are found from $\tilde{v}^s = (ie/m_e\omega_s)\mathbf{E}^s$.

All feedback effects of the nonlinear coupling on the pump fields are neglected. The coupling terms that lead to the excitation of the ESW with frequency $\omega_\pm$ may be represented by the nonlinear force

$$f_{\pm}^{(NL)}(x) = f_{\pm}^{(NL)}(x)e^{ik_{\pm}y-i\omega_{\pm}t} = - \left[ m_e(\mathbf{v}_\mp \cdot \nabla \mathbf{v}_0 + \mathbf{v}_0 \cdot \nabla \mathbf{v}_\mp) + \frac{e}{c}(\mathbf{v}_0 \times \mathbf{B}_\mp + \mathbf{v} \times \mathbf{B}_0) \right]_{res}. \quad (13)$$

In the term in square brackets on the r.h.s of (13) only “resonant” terms with the same phase of $f_{\pm}^{(NL)}$ are included. Obviously, these terms exist if the matching conditions (3) hold.

The nonlinear coupling leads to the parametric excitation and growth of ESWs. We thus let the ESW envelopes vary slowly in time, i.e.

$$\tilde{f}_{\pm}(x) \rightarrow \tilde{f}_{\pm}(x, \epsilon t) \quad (14)$$

To evaluate the growth rate, we use an energy approach. The surface energy density of the ESW that will be needed in the calculations is given by
\[
U_\pm = \frac{k}{2\pi} \int_{-\pi/k}^{+\pi/k} dy \int_{-\infty}^{+\infty} dx \left< u^{(K)}_\pm + u^{(F)}_\pm \right>,
\]

where the brackets denote average over one laser cycle and \( u^{(K)}_\pm \) and \( u^{(F)}_\pm \) are the volume densities of the kinetic and EM fields energies, respectively:

\[
u^{(K)}_\pm = \frac{m_e n_0}{2} |\mathbf{v}_\pm|^2, \tag{16}
\]

\[
u^{(F)}_\pm = \frac{1}{8\pi} \left( |\mathbf{E}_\pm|^2 + |B_{\pm,z}|^2 \right). \tag{17}
\]

The kinetic energy contribution vanishes for \( x < 0 \). Using eqs.\((12)\) and \( \tilde{v}_{\pm,x}(0^+) = \tilde{v}_{\pm,y}(0)/\sqrt{\alpha - 1} \), and finally summing the two contributions we obtain for the total surface energy \( U_\pm = M_\pm |\tilde{v}_{\pm,y}|^2/2 \) where

\[
M_\pm = \frac{m_e n_i c \alpha_\pm (\alpha_\pm - 2)^{1/2}(\alpha_\pm^2 - 2\alpha_\pm + 2)}{(\alpha_\pm - 1)^2}. \tag{18}
\]

To obtain an expression for the temporal variation of \( U_\pm \) due to the nonlinear coupling forces, we write the Euler and the Ampere equation for the ESW fields keeping terms up to order \( \epsilon^3 \) and disregarding terms that are non-resonant with the oscillation:

\[
m_e \partial_t \mathbf{v}_\pm = -\epsilon \mathbf{E}_\pm + \epsilon \mathbf{f}_{\pm}^{(NL)}, \tag{19}
\]

\[
\nabla \times \mathbf{B}_\pm = \frac{4\pi}{c} \left( -en_o \mathbf{v}_\pm + \epsilon \mathbf{J}_{\pm}^{(NL)} \right) + \frac{1}{c} \partial_t \mathbf{E}_\pm, \tag{20}
\]

where \( \mathbf{f}_{\pm}^{(NL)} \) is given by eq.\((13)\), and \( \mathbf{J}_{\pm}^{(NL)} = -\epsilon \delta n_e^{(\omega_0)} \mathbf{v}_\pm \). Using eqs.\((19)\)\,(20) together with Maxwell’s equations we obtain for the temporal variation of the energy densities

\[
\partial_t \nu^{(K)}_\pm = -en_o \mathbf{v}_\pm \cdot \mathbf{E}_\pm + \epsilon \mathbf{f}_{\pm}^{(NL)} \cdot \mathbf{v}_\pm, \tag{21}
\]

\[
\partial_t \nu^{(F)}_\pm = -\nabla \cdot \mathbf{S}_\pm - \epsilon \mathbf{J}_{\pm}^{(NL)} \cdot \mathbf{E}_\pm, \tag{22}
\]

where \( \mathbf{S}_\pm = (c/4\pi)\mathbf{E}_\pm \times \mathbf{B}_\pm \) is the Poynting vector of the ESW. We integrate the sum of the energy densities over space and average over time. Noting that the total flux of \( \mathbf{S}_\pm \) vanishes because of the evanescence of the ESW fields, we finally obtain the equation for the evolution of the total energy \( U \) of the ESW:
\[
\partial_t U_\pm = \frac{k}{2\pi} \int_{-\pi/k}^{+\pi/k} dy \int_0^{+\infty} dx \partial_x \left\{ \frac{m_e n_0}{2} |v_\pm|^2 + \frac{1}{8\pi} \left( |E_\pm|^2 + |B_\pm|^2 \right) \right\}
\]
\[
= \epsilon \frac{k}{2\pi} \int_0^{+\infty} dx \int_{-\pi/k}^{+\pi/k} dy \left\langle v_\pm \cdot \left( e^{\delta_n (\omega_0)} E_\pm + f_{NL}^\pm \right) \right\rangle.
\]

(23)

The integral extends for \( x > 0 \) only since there are no electrons for \( x < 0 \).

From eq. (23), performing the integral and neglecting high order terms one obtains two coupled equations in the general form

\[
\frac{M_\pm}{2} \partial_t |\tilde{v}_\pm|^2 = A_0 \omega |\tilde{v}_+| |\tilde{v}_+| S_\pm \sin \varphi,
\]

(24)

where \( A_0 \) is the amplitude of the pump mode in dimensionless units, \( S_\pm \) are positive factors depending on \( \omega_p/\omega \) and \( \theta \), and \( \varphi \) is obtained from the phase factors of the pump and ESW modes as \( \varphi = \phi_+ + \phi_- - \phi_0 \). From (24) one easily obtains

\[
\partial_t^2 |\tilde{v}_\pm| = (A_0 \omega)^2 \left( \frac{S_+ S_-}{M_+ M_-} \right) \sin^2 \varphi |\tilde{v}_\pm|.
\]

(25)

Setting \( \sin \varphi = 1 \) simply corresponds to finding the relative phase of the growing modes with respect to the phase of the pump mode. The growth rate of ESWs is then

\[
\Gamma = A_0 \omega \sqrt{\frac{S_+ S_-}{M_+ M_-}}.
\]

(26)

In the next section we use the analytical method outlined above to study TSWD in two cases of particular relevance: TSWD driven by the \( \mathbf{v} \times \mathbf{B} \) force for normal laser incidence (which we name “\( 2\omega \rightarrow \omega + \omega \)” decay), and TSWD driven by the electric force for oblique laser incidence and \( p \)-polarization (“\( \omega \rightarrow \omega/2 + \omega/2 \)” decay).

III. “\( 2\omega \rightarrow \omega + \omega \)” DECAY

To discuss the “\( 2\omega \rightarrow \omega + \omega \)” decay, we restrict ourselves for simplicity to the case of normal incidence, as was done in the simulations reported in Ref. [10]. Thus, the ESW frequency \( \omega_\pm = \omega \) and \( k_+ = -k_- \) The case of oblique incidence is indeed very similar, the main difference being that the matching condition \( k_+ + k_- = (2\omega/c) \sin \theta \) holds, causing a
shift $\delta \omega \neq 0$ between the two ESWs. It is found that the growth rate decreases monotonically with increasing incidence angle, since the pump force has a maximum at normal incidence [14].

The laser wave can be represented by a single component of the vector potential, $A_z = A_z(x,t)$, that for $x > 0$ is given by

$$A_z(x,t) = A_z(x) \cos \omega t = A_z(0)e^{-x/l_s} \cos \omega t,$$

where $l_s = c/(\omega_p^2 - \omega^2)^{1/2}$ is the screening length. Imposing boundary conditions for the incident and reflected waves one finds $A_z(0) = 2A_i(\omega l_s/c)(1 + \omega^2 l_s^2/c^2)^{-1/2}$, where $A_i$ is the amplitude of the incident field.

Electrons perform their quiver motion in the $z$ direction. Thus, the magnetic force term is in the $x$ direction and has a secular term $(0\omega)$, named the ponderomotive force, and an oscillating term $(2\omega)$ that, in what follows, we simply name the $v \times B$ force. The secular term corresponds to radiation pressure and creates a surface polarization of the plasma. The $v \times B$ force drives a longitudinal, electrostatic oscillation that acts as a pump mode for TSWD. In the expansion (8), the pump amplitude is supposed to be of order $\epsilon$. As will be shown below, coupling between 1D and 2D fields occurs only in the overdense plasma ($x > 0$). Thus, since the $v \times B$ force is quadratic in the laser field, the expansion (8) also implies $a(0) \sim \epsilon^{1/2}$, where $a(0)$ is the (dimensionless) laser amplitude at the surface of the plasma, e.g $a(0) = (eA_z(0)/mc^2)$. For overdense plasmas $a(0) \sim (\omega/\omega_p)a_i < a_i$, where $a_i = (eA_i/mc^2)$. This yields $\epsilon \sim (\omega/\omega_p)^2 a_i^2 = (n_c/n_e)a_i^2$, where $n_c = m_e\omega^2/4\pi e$ is the “critical” density. We therefore expect our expansion procedure to be valid even at relativistic fields amplitudes $a_i \geq 1$ for high enough plasma densities.

We now derive the solutions for the 1D pump fields at $2\omega$. The electron oscillation velocity is obtained from the conservation of canonical momentum as $m_e v_z = eA_z/c$. For normal laser incidence, the longitudinal $v \times B$ force is given by

$$-\frac{e}{c}v_zB_y = -\frac{e^2}{m_e c^2}A_z \partial_x A_z \equiv F^0(x)(1 + \cos 2\omega t),$$

(28)
where we have set $F^0(x) = \left( m_e c^2 / 2 l_s \right) a_s^2 e^{-2x/l_s} \equiv F^0 e^{-2x/l_s}$. To order $\epsilon$, one obtains the following equations for the longitudinal, electrostatic motion:

$$m_e \partial_t V^{(2\omega)}_x = -e(E^{(0)}_x + E^{(2\omega)}_x) + F^0(x)(1 + \cos 2\omega t), \quad (29)$$

$$\partial_x (E^{(0)}_x + E^{(2\omega)}_x) = -4\pi e (\delta n^{(0)}_e + \delta n^{(2\omega)}_e), \quad (30)$$

$$\partial_t \delta n^{(2\omega)}_e = -n_0 \partial_x V^{(2\omega)}_x. \quad (31)$$

All fields in the equations above decay inside the plasma as $\exp(-2x/l_s)$. The secular part simply gives $eE^{(0)}_x(x) = F^0_0(x)$ and $\delta n^{(0)}_e = -\partial_x F^0(x)/4\pi e^2 = F^0(x)/2\pi e^2 l_s$. For the motion at $2\omega$ one obtains

$$\tilde{V}^{(2\omega)}_x = -i F^0 \frac{\omega^2}{\omega_p D} \left( \frac{\omega^2}{\omega_p^2} e^{-2x/l_s} \right), \quad (32)$$

$$\delta \tilde{n}^{(2\omega)}_e = n_0 \frac{F^0}{\omega_p^2 l_s m_e D} e^{-2x/l_s}, \quad (33)$$

$$e \tilde{E}^{(2\omega)}_x = \frac{F^0}{2D} e^{-2x/l_s}. \quad (34)$$

The denominator $D$ is given by

$$D = 1 - \frac{4\omega^2}{\omega_p^2} = 1 - \frac{4n_e}{n_i}, \quad (35)$$

which shows the well-known resonance at $n_e = 4n_i$ due to excitation of plasmons with frequency $\omega_p = 2\omega$ by the $v \times B$ force.

The difference between the total number of electrons and ions for $x > 0$ is, to order $\epsilon$,

$$\Delta N^{(x>0)}_e = \int_{0}^{+\infty} dx \left( \delta n^{(2\omega)}_e + \delta n^{(0)}_e \right) = \frac{F^0}{2\pi e^2} \left( 1 + \frac{\cos 2\omega t}{D} \right). \quad (36)$$

The fact that $\Delta N^{(x>0)}_e > 0$ during most of the oscillation implies that, due to compression from the ponderomotive and $v \times B$ forces, electrons leave behind a charge depletion layer of thickness $\zeta = \Delta N^{(x>0)}_e/n_i$. We note that $\zeta$ is of order $\epsilon$, thus to lowest order it is correct to treat the charge depletion layer as a surface layer. Heuristically, the oscillating behavior of $\zeta$ describes the plasma “moving mirror” effect [15]: due to charge depletion the laser is reflected at $x = \zeta$ rather than exactly at $x = 0$. This leads to the appearance of high harmonics in the reflected light.
Since $|D| < 1$, there is always a time interval in which electrons are pulled into vacuum forming a cloud of negative charge. This interval is very short for $n_e/n_c \gg 4$, i.e. $D \simeq 1$, but for lower densities it is relevant in the interaction process. The motion in vacuum is strongly anharmonic and more difficult to solve analytically than the motion inside the plasma. However, it is found in section III that the surface modes gain energy during the phase of electron motion inside the plasma only, so that the expressions of fields for $x < 0$ are not needed. Nevertheless, we have to assume that the density of the electron cloud for $x < 0$ is low enough for the laser propagation not to be affected.

Using eq.(32), the NL force (13) may be written in terms of the oscillation velocity $V_x^{(2\omega)}$ as follows:

$$f^{\text{NL}}_{\pm} = - \left[ m_e \left( V_x^{(2\omega)}(x) \partial_x v_{\mp} + v_{x,\mp} \partial_x V_x^{(2\omega)}(x) \right)\hat{x} - \frac{e}{c} V_x^{(2\omega)}(x) B_{z,\mp} \hat{y} \right].$$

(37)

Because of the symmetry with respect to inversion of the $y$ axis, the two ESWs may differ only by a phase factor and have the same amplitude and energy. Inserting the force (37) in the energy equation (23) and performing the integral in $y$ we obtain for the ESW energy $U = U_{\pm}$

$$\partial_t U = \frac{1}{8} \sum_{l=+k,-k} \int_0^{+\infty} dx \tilde{V}_x^{(2\omega)} \cdot \left( e \delta n_e^{(2\omega)}(x) \tilde{E}_x^{*} - m_e n_0 \tilde{v}_x^{*} \partial_x V_x^{(2\omega)}(x) 
-m_e n_0 \tilde{V}_x^{(2\omega)}(x) \partial_x \tilde{v}_x^{*} + n_0 \frac{e}{c} \tilde{V}_x^{(2\omega)}(x) \tilde{B}_{z,\mp} \hat{y} \right) + \text{c.c.}$$

(38)

We rewrite all fields as functions of $\tilde{V}_x^{(2\omega)}$ and $\tilde{v}_\pm$. We obtain, after some algebra

$$\partial_t U = \frac{m_e n_i}{2(1 + q_s l_s)} \Re \left\{ \tilde{V}_x^{(2\omega)} \left[ (1 + q_s l_s) \tilde{\nabla}_+^{*} \tilde{\nabla}_-^{*} + 2 \tilde{v}_x^{*} \right] + \frac{\tilde{v}_x^{*} \tilde{y}_x^{*}}{q_s l_s} \right\}. $$

(39)

All the terms of eq.(39) are proportional to

$$\Re \left( \tilde{V}_x^{(2\omega)} \tilde{v}_+^{*} \tilde{v}_-^{*} \right) = \left| \tilde{V}_x^{(2\omega)} \tilde{v}_+^{*} \right| \cos(\phi + \pi/2),$$

(40)

where $\phi = \phi_+ + \phi_- + \phi_\pm$ being the phase angles of $v_\pm$, and we used eq.(32) and the fact that $F_0$ is real and positive. Thus, the phase of the growing ESWs with respect to the $\mathbf{v} \times \mathbf{B}$ force
is such that \( \phi = -\pi/2 \), the value for which the growth rate is positive and has a maximum.

The overlap of the two ESWs gives

\[
\frac{\tilde{v}_+}{2} e^{iky-i\omega t} + \frac{\tilde{v}_-}{2} e^{-iky-i\omega t} = \frac{|\tilde{v}_\pm|}{2} e^{-i\omega t} \left( e^{iky+i\phi_+} + e^{-iky+i\phi_-} \right)
\]

\[
= |\tilde{v}_\pm| e^{-i\omega t-i\pi/4} \cos (ky + \Delta \phi),
\]

where \( \Delta \phi = (\phi_+ - \phi_-)/2 \). This is a standing wave which has a temporal phase shift \(-\pi/4\) with respect to the \( \mathbf{v} \times \mathbf{B} \) force of eq.(28). The phase is such that, at a given position in \( y \), the temporal maxima of \( V_x^{(2\omega)} \) and \( V^{(\omega)} \) overlap once for laser cycle. The angle \( \Delta \phi \) gives the location of the maxima of the standing wave on the \( y \) axis, which depends on the arbitrary choice of the initial phase.

Using eq.(40), we can rewrite the energy variation as

\[
\partial_t U = m_e n_d |\tilde{V}_x^{(2\omega)}| \left[ |\tilde{v}_+|^2 + \frac{2|\tilde{v}_{x,k}|^2}{1 + q_s l_s} - \frac{2|\tilde{v}_{y,k}|^2}{q_s l_s (1 + q_s l_s)} \right].
\]

(42)

Note that \( \partial_t U > 0 \) since \( q_s l_s = \sqrt{(\alpha - 1)(\alpha - 2)^{-1}} > 1 \), where \( \alpha = \omega_p^2/\omega^2 = n_e/n_c \).

Eliminating \( \tilde{V}_x^{(2\omega)} \) as a function of \( a_i \) and \( \alpha \), after some algebra the growth rate \( \Gamma \) is obtained:

\[
\Gamma = 4\omega a_i^2 \frac{(\alpha - 1)^3/2}{\alpha|\alpha - 4|[(\alpha - 1)^2 + 1](\alpha - 2)^{1/2}} \left[ 1 + \frac{2}{\alpha(1 + q_s l_s)} - \frac{2(\alpha - 1)}{\alpha q_s l_s (1 + q_s l_s)} \right] 
\]

\[
\simeq 4\omega a_i^2 \frac{(\alpha - 1)^{3/2}}{\alpha|\alpha - 4|[(\alpha - 1)^2 + 1](\alpha - 2)^{1/2}}.
\]

(43)

The leading contribution was highlighted in the last equality.

A plot of the growth rate as a function of \( \alpha \) is given in Fig.2. For moderately overdense plasmas, \( \Gamma \) is a considerable fraction of the laser frequency. The resonance at \( n_e = 4n_c \) is due to the resonant behavior of the pump field. The growth rate diverges also in the electrostatic limit when \( n_e \to 2n_c \). However, in this limit the value of \( k \) tends to infinity, i.e. the wavelength becomes very small and one expects that this second resonance might be damped by thermal effects, which are neglected in our model and are left for future investigations.

The comparison of spatial and temporal scales predicted from this model with the simulation results shows reasonable agreement for laser and plasma parameters such that our
expansion procedure is valid [10]. We note that simulation results for high intensities in the relativistic regime suggest that a $2\omega \rightarrow \omega + \omega$ TSWD-like process still occurs and produces strong rippling of the plasma surface; the spatial scales are different from those predicted by our analytical, cold fluid model, but still of the same order of magnitude.

We conclude this section by noting that, although we took the laser polarization along $z$, the whole derivation is valid at normal incidence for any polarization direction. The only difference in 2D geometry is that, for the laser polarization along $y$, the quiver oscillations at $\omega$ and the ESW oscillations at the same frequency overlap. This is observed in numerical simulations [14].

IV. “$\omega \rightarrow \omega/2 + \omega/2$” DECAY

A case of particular interest is that of oblique incidence and $p$-polarization of the laser pulse, since these conditions lead in general to a stronger plasma coupling. In the preceding section we have found that the $2\omega \rightarrow \omega + \omega$ decay is driven by the longitudinal component of the velocity field, as shown by eq.(37). For $p$-polarization and $\theta \neq 0$ the electric field drives a strong longitudinal oscillation at the frequency $\omega$; thus, one expects that the TSWD process leads to the generation of subharmonic ESWs, having frequencies around $\omega/2$, with peak efficiency at some angle of incidence. As shown below, this picture is substantially correct.

However, there is a small but non-vanishing growth rate for the $\omega \rightarrow \omega/2 + \omega/2$ process even at normal incidence. In fact, the pump force (13) is now given by

$$
\mathbf{f}_{\pm}^{(NL)} = -m_e (v_{\mp,x} \partial_x \mathbf{V}^{(\omega)} + V_{x}^{(\omega)} \partial_x \mathbf{v}_{\mp}) - m_e (v_{\mp,y} \partial_y \mathbf{V}^{(\omega)} + V_{y}^{(\omega)} \partial_y \mathbf{v}_{\mp}) - e \left( \mathbf{V}^{(\omega)} \times \mathbf{B}_{\mp} + \mathbf{v}_{\mp} \times \mathbf{B}^{(\omega)} \right),
$$

and does not vanish for $\theta = 0$.

The case of oblique incidence and $p$-polarization of the laser pulse may be tackled with an approach analogous to that of section [II]. The main physical difference with the previous case is that now the “pump” is a divergence-free velocity field at the laser frequency $\omega$, and
its amplitude is now proportional to the laser field rather than to the laser intensity. In applying the expansion procedure (8) again, now \( \epsilon \sim \left(\omega / \omega_p\right) a_i \) (appendix A). Thus, the limits of validity of this expansion are more restrictive in the present case: the expansion tends to be valid only for rather high densities or rather low fields. One must also note that temperature effects, neglected in the cold plasma approximation, are important for low fields.

The calculation of the growth rate for subharmonic ESWs can be performed by the same method as in the preceding sections in a straightforward way. The details are reported in appendix A. As a final result, the growth rate \( \Gamma = \Gamma(\alpha, \theta) \) may be written as

\[
\Gamma = a_i \omega |F_B| \left(\frac{n_e}{n_c}\right)^{5/2} K(\alpha, \theta),
\]

where \( F_B \) is the Fresnel factor for the magnetic field and the factor \( K(\alpha, \theta) \) scales weakly with density. A plot of \( \Gamma \) as a function of \( \theta \) for different values of \( \alpha \) is reported in Fig. 3.

The frequency shift \( \delta \omega \) of the two ESWs can be calculated as a function of \( \theta \) and \( \alpha \) from the matching condition \( k_+ + k_- = \left(\omega / c\right) \sin \theta \). The result is shown in Fig. 3. Comparing with Fig. 3, one finds that \( \delta \omega \approx 0.1 \omega \) in conditions favorable to the \( \omega \rightarrow \omega/2 + \omega/2 \) TSWD.

V. DISCUSSION

The TSWD concept has been investigated analytically in two cases that clarify its basic features in the context of the cold fluid plasma approximation. This process may lead to the excitation of ESWs by an ultrashort laser pulse in a solid target without any special (e.g., grating-like) structure, and even for normal incidence and s-polarization.

In both the cases discussed in this paper, we found a strong decrease of the TSWD growth rate for increasing densities. This does not necessarily imply that TSWD is not relevant to short pulse interaction with solid targets which have \( n_e / n_c \gg 1 \). In fact, important processes such as high harmonic generation or fast electron production are more efficient when the laser pulse interacts with a moderate density plasma. This is the case for most of the experiments.
of short pulse interaction with solid targets, since a moderate density “shelf” is usually produced at the time of peak pulse intensity by target ablation and plasma hydrodynamic expansion during the leading edge of the short pulse or during the long prepulse. For instance, the “2ω → ω + ω” decay appears to be very efficient exactly in conditions favorable for high harmonics production via the “moving mirror” effect \[15,16\]. Simulations \[10\] suggest that TSWD in strongly nonlinear and relativistic regimes (beyond the limits of the analytical approach of this paper) may produce surface perturbations acting as a ”seed” for, e. g., electron instabilities leading to current filamentation \[17,18\] or detrimental distortions of the moving mirrors, and for Rayleigh-Taylor-like (RT) hydrodynamic instabilities occurring on the time scale of ion motion. In Ref. \[10\] it is estimated that the growth rate of the RT instability is much slower than that of TSWD. A different hydrodynamic instability with a surprisingly high growth rate has been reported in Ref. \[19\]).

The 2ω → ω + ω and the ω → ω/2 + ω/2 processes have been considered independently. This is appropriate since these are resonant processes which do not interfere with each other. In principle, both decays may occur during the interaction of an intense laser pulse with an overdense plasma. According to our analysis the 2ω → ω + ω process appears to have a stronger growth rate. However, in the case of the ω → ω/2 + ω/2 process our analytical approach is valid for a very narrow range of parameters only. Either an extension of the present analytical approach or numerical simulations are needed to investigate the TSWD for a very intense, p-polarized laser pulse at oblique incidence.

VI. CONCLUSIONS

We have discussed the Two-Surface Wave Decay as a novel mechanism for the excitation of electron surface waves in the interaction of ultrashort, intense laser pulses with overdense plasmas. TSWD is based on the parametric excitation of a pair of ESWs. The “pump” force may either be the magnetic or the electric force of the laser pulse, and leads respectively to the generation of two ESWs with frequencies close to the laser frequency of half of it.
An analytical model for TSWD in the cold fluid plasma, non-relativistic approximation has been developed. The model supports the interpretation of recent simulation results [10] and suggests that TSWD may be of relevance in certain regimes of laser interaction with solid targets.

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APPENDIX A: GROWTH RATE OF THE $\omega \rightarrow \omega/2 + \omega/2$ DECAY

We now give the detailed derivation of the growth rate for the $\omega \rightarrow \omega/2 + \omega/2$ TSWD of section IV. The pump fields are easily found with the help of Fresnel formulae and Maxwell equations. For instance, the magnetic field is given by

\[ B_z^{(\omega)}(x, t) = B_z^{(\omega)}(0^+)e^{iky\sin \theta - x/l_p - i\omega t} + c.c. \]  

(A1)

where the screening length is given by

\[ l_p = \frac{c}{\omega_p} \left( 1 - \frac{\omega^2}{\omega_p^2} \cos^2 \theta \right)^{-1/2} = \frac{c}{\omega \sqrt{\alpha - \cos^2 \theta}}, \]  

(A2)

and the magnetic field at the surface is given by the Fresnel formula

\[ \frac{B_z(0^+)}{B_{z,i}} = \frac{2n^2 \cos \theta}{\sqrt{n^2 - \sin^2 \theta + n^2 \cos \theta}} = \frac{2(\alpha - 1) \cos \theta}{(\alpha - 1) \cos \theta - i\sqrt{\alpha - \cos^2 \theta}} \equiv \mathcal{F}_B(\theta), \]  

(A3)

where $B_{z,i}$ is the incident field amplitude in vacuum. The electric field components are found by using $\nabla \cdot \mathbf{E}^{(\omega)} = 0$ and $\nabla \times \mathbf{E}^{(\omega)} = i(\omega/c)B_z^{(\omega)} + c.c.$.

The nonlinear force is given by eq.(37). Evaluating the spatial derivatives as $\partial_x \mathbf{V}^{(\omega)} = -\mathbf{V}^{(\omega)}/l_p$, $\partial_y \mathbf{V}^{(\omega)} = ik_1 \mathbf{V}$, $\partial_y \mathbf{V}_\pm = ik_\pm \mathbf{v}_\pm$, and $\partial_x \mathbf{v}_\pm = -q_\pm \mathbf{v}_\pm$ where $q_\pm \equiv q_>(\omega_\pm)$, and keeping resonant terms only we find
We rewrite all “pump” terms as a function of $\tilde{V}_y(\omega)$ by using $\tilde{V}_x(\omega) = ik_l p \tilde{V}_y(\omega)$, $	ilde{B}_z(\omega) = (m_e c/\ell p)(k_l^2 p^2 - 1)\tilde{V}_y(\omega)$. In a second step we rewrite all SW fields as a function of $\tilde{v}_{\pm,y}$ by using $\tilde{v}_{\pm,x} = (ik_\pm/q_\pm)\tilde{v}_{\pm,y}$ and $\tilde{b}_{\pm,z} = -m_e \omega_\pm^2 (\alpha_\pm - 1)\tilde{v}_{\pm,y}/(ecq_\pm)$. We thus obtain

$$\tilde{f}_{\pm,x}^{(NL)} = \frac{m_e}{4} \left[ \frac{k_\pm}{q_\pm} \left( k_\pm + k_\pm q_\pm p + k_\mp \right) + \frac{\omega_\pm^2}{q_\pm c^2} \left( \alpha_\pm - 1 \right) + \frac{1}{l_p} \right] e^{-(q_\mp + 1/l_p) x}$$ (A6)

$$\tilde{f}_{\pm,y}^{(NL)} = \frac{m_e}{4} \tilde{V}_y(\omega) \tilde{v}_{\mp,y}^* \left[ k_\pm p \left( q_\pm - \frac{1}{l_p} - \frac{k_\pm k_\pm}{q_\pm} - \frac{\omega_\pm^2}{q_\pm c^2} \left( \alpha_\pm - 1 \right) \right) + k_\pm \right] e^{-(q_\mp + 1/l_p) x}$$ (A8)

The relations $\omega_\pm^2 (\alpha_\pm - 1)/q_\pm = \omega_\pm c\sqrt{\alpha_\pm - 2}$, $k_\pm/q_\pm = 1/\sqrt{\alpha_\pm - 1}$, $k_l p = \sin \theta (\alpha - \cos^2 \theta)^{-1/2}$ may be used to simplify the expressions for $Q_{\pm, x}$ and $Q_{\pm, y}$. We thus obtain

$$Q_{\pm, x} = \frac{1}{\sqrt{\alpha_\pm - 1}} \left[ k_\pm \left( 1 + q_\pm p \right) + k_\mp \right] + \frac{\omega_\pm}{c} \sqrt{\alpha_\pm - 2} + \frac{1}{l_p},$$ (A10)

$$Q_{\pm, y} = k_\pm p \left( q_\pm - \frac{1}{l_p} - \frac{k_\pm}{\sqrt{\alpha_\pm - 1}} - \frac{\omega_\pm}{c} \sqrt{\alpha_\pm - 2} \right) + k_\pm.$$ (A11)

In the energy equation (23), since $\delta n(\omega) = 0$ the integrand, eliminating $\tilde{v}_{\pm, x}$, is given by

$$\langle v_{\pm} \cdot f_{\pm}^{NL} \rangle = \frac{m_e}{16} \tilde{V}_y(\omega) \tilde{v}_{\mp,y}^* \left[ \frac{k_\pm}{q_\pm} Q_{\pm, x} + Q_{\pm, y} \right] + \text{c.c.}$$ (A12)

$$= \frac{m_e}{8} \Re \left[ \tilde{V}_y(\omega) \tilde{v}_{\mp,y}^* \tilde{v}_{\mp,y}^* \right] \left[ \frac{k_\pm}{q_\pm} Q_{\pm, x} + Q_{\pm, y} \right].$$ (A13)

We thus find

$$\partial_t U_\pm = n_i \frac{m_e}{8} \left[ \frac{l_p}{1 + (q_+ + q_-) l_p} \left( \frac{k_\pm}{q_\pm} Q_{\pm, x} + Q_{\pm, y} \right) \right] \left| \tilde{V}_y(\omega) \tilde{v}_{\mp,y}^* \tilde{v}_{\mp,y}^* \right| \sin \varphi.$$ (A14)

Here, $\varphi = (\phi_+ + \phi_- - \Phi)$, where $\Phi$ and $\phi_\pm$ are the phase factors of $\tilde{V}_y(\omega)$ and $\tilde{v}_{\pm,y}$, respectively. It is worth rewriting these equations in a more compact form. Introducing $a_i = eB_i/m_e c$ and using Fresnel’s formulas to eliminate $\tilde{V}_y$ we obtain
\[ \partial_t U_{\pm} = n_i \frac{m_e}{8} a_i c |\mathcal{F}_B(\theta)||\vec{v}_{+ y}||\vec{v}_{- y}| \sin \varphi G_{\pm}(\alpha, \theta), \]  

where we have posed \( G_{\pm}(\alpha, \theta) = G_0(\alpha, \theta) g_{\pm}(\alpha, \theta) \) and

\[ G_0 = \frac{\sqrt{\alpha - \cos^2 \theta}}{(\alpha - 1)[1 + (q_+ + q_-) l_p]}, \]  

\[ g_{\pm} = l_p \left( \frac{k_{\pm} Q_{\pm x} + Q_{\pm y}}{q_{\pm}} \right) \]

\[ = \frac{1}{\sqrt{(\alpha_+ - 1)(\alpha_- - 1)}} \left[ k_l l_p (1 + q_+ l_p) + k_\mp l_p \right] + \frac{\omega_\pm l_p \sqrt{\alpha_\pm - 1}}{c} \left[ \frac{1}{\sqrt{\alpha_\pm - 1}} \right] + k_\mp l_p, \]  

\[ + k_\mp l_p \left( q_\pm l_p - 1 - \frac{k_l l_p}{\sqrt{\alpha_\pm - 1}} - \frac{\omega_\pm l_p \sqrt{\alpha_\pm - 1}}{c} \right) + k_\mp l_p. \]  

Using the equations above we obtain the following coupled equations in the form (24) for the field amplitudes

\[ \mu_{\pm} \partial_t |\vec{v}_{\pm y}|^2 = a_i \omega |\vec{v}_{\pm y}||\mathcal{F}_B| G_{\pm} \sin \varphi, \]  

where \( \mu_{\pm} \equiv 4M_{\pm}/(m_e n_i c) \). Thus, the growing modes have relative phases such that \( \sin \varphi = 1 \). The growth rate is given by

\[ \Gamma = a_i \omega |\mathcal{F}_B| G_0 \sqrt{\left| \frac{g_+ g_-}{\mu_+ \mu_-} \right|} \equiv a_i \omega |\mathcal{F}_B| \left( \frac{n_e}{n_i} \right)^{5/2} K(\alpha, \theta) \]  

which is the formula (45). A plot of \( \Gamma \) is depicted in Fig.3.
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FIGURES

FIG. 1. Dispersion relation of electron surface waves (thick lines) eq.(11), and the matching conditions for TSWD, eq.(3).

FIG. 2. The TSWD growth rate $\Gamma$ (normalized to $a_i^2 \omega$) for the $2\omega \rightarrow \omega + \omega$ process at normal incidence, eq.(43), as a function of $\alpha = n_e/n_c$.

FIG. 3. The TSWD growth rate $\Gamma$ (normalized to $a_i \omega$) for the $\omega \rightarrow \omega/2 + \omega/2$ process at oblique incidence, eq.(45), as a function of $\theta$ and $\alpha = n_e/n_c$.

FIG. 4. The frequency shift $\delta \omega$ for the $\omega \rightarrow \omega/2 + \omega/2$ process at oblique incidence, as a function of $\theta$ and $\alpha = n_e/n_c$. The dashed line gives the shift $\delta \omega_{\text{max}} = (1/2) \sin \theta$ that is obtained for $\alpha \rightarrow \infty$. 