Design of Optical Fibers and Calculate their Guided Modes Properties at 1550 nm

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Abstract. There is no doubt that optical fiber technology is one of the most important stages of the communications revolution at all and it is of utmost importance in our daily life. In this work, five fibers with core radii 2.5, 4.5 and 6.5–8.5 µm were designed. The properties of all guided modes have been calculated at a wavelength of 1550 nm by using RP Fiber Calculator. A single-mode fiber is obtained when the core radius approaches the wavelength. As the core radius is increased, the fiber becomes a multimode. The percentage power in the core increases with increasing core radius. The modes profiles were illustrated and compared with the modern references.

Keywords: Single-mode fiber, Multimode fibers, Step-index fibers, Guided modes, Third window (1550 nm), RP Fiber Calculator.

1. Introduction

A thin strand of glass with a higher refractive index core surrounded by a lower refractive index cladding forms an optical fiber [1]. A step-index fiber (SIF) has an abrupt index change at the core-cladding boundary [2]. Figure 1a shows a multimode (MM) SIF which allows the propagation of multiple modes within the fiber core. Figure 1b shows a single-mode (SM) SIF which allows the propagation of only one mode. The refractive index profile in both cases may be defined as [3]:

\[ n(r) = \begin{cases} 
  n_1 & \text{for } r < a \text{ (core)} \\
  n_2 & \text{for } r \geq a \text{ (cladding)} 
\end{cases} \]  

(1)

where \( r \) is the radial position, \( n_1 \) and \( n_2 \) are the refractive indices of core and cladding, respectively and \( a \) is the radius of the fiber core. MMFs and SMFs only differ in the radius of the light-guiding core [4]. Fabrication of MMFs is easy. Further the launching of light into MMFs is also easy. These fibers are generally used for short distances, like local area networks (LANs). Launching of light into SMFs and fabrication of SMFs are difficult and so the fiber is expensive. Generally in the SMFs, the transmission loss and dispersion are very small. So the SMFs are very useful in long distance communication [5].
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In 2017, Mohammed and Al-Hindawi [6] studied and calculated the design parameters such as operating wavelength, core radius, numerical aperture, acceptance angle, attenuation, dispersion and information capacity for SIFs and graded-index fibers for different sources.

In 2018, Gulistan et al. [7] demonstrated an innovative approach to enhance mode stability by reducing mode coupling between the propagation modes of a few-mode fiber by using commercially available finite element method (FEM)-based COMSOL software.

In 2020, Salih [8] designed a SMF at 1310 nm and 1550 nm wavelengths by using RP Fiber Calculator. Also, Ibrahim and Salih [9, 10] used this program to study properties of propagated transverse modes through SIFs at 850 nm and 1300 nm.

In this work, SM and MM optical fibers have been designed and their properties were calculated at 1550 nm.

2. Theoretical Part

When the incidence angle is greater than the critical angle, the light is totally internally reflected at the core-cladding interface \((r = a)\). The critical angle is given by [11]:

\[
\theta_c = \sin^{-1} \left( \frac{n_2}{n_1} \right)
\]

The acceptance angle of the fiber can be calculated by [12]:

\[
\theta_a = \sin^{-1} \left( \frac{1}{n_o} \sqrt{n_1^2 - n_2^2} \right)
\]

where \(n_o\) is the refractive index of the incident medium \((n_o = 1\) for air\).

The numerical aperture (NA) measures the amount of light that collects by or emerges from the fiber core. The NA is defined as [13]:

\[
\text{NA} = \sin \theta_a
\]

The normalized frequency is a dimensionless parameter which determines the number of modes. It is calculated by [3]:

\[
V = \frac{2\pi}{\lambda} \text{aNA}
\]

where \(\lambda\) is the free space wavelength. A fiber has the lowest loss at 1550 nm [14].

The propagation constant is given by [12]:

\[
\beta = \frac{2\pi}{\lambda} n_{\text{eff}}
\]

where \(n_{\text{eff}}\) is the effective refractive index which lies within the range of cladding and core indices.

The effective area is a quantitative measure of the area which a fiber mode effectively covers in the transverse dimensions. It is defined as:
The effective area is:

\[ A_{\text{eff}} = \frac{\left[ \int |E|^2 \, dA \right]^2}{\int |I|^2 \, dA} = \frac{\left[ \int I \, dA \right]^2}{\int I^2 \, dA} \]  

(7)

where \( E \) is the amplitude of the electric field, \( I \) is the corresponding intensity and \( dA \) signifies integration over the whole plane.

For a Gaussian beam, the effective area is:

\[ A_{\text{eff}} = \pi \omega^2 \]  

(8)

where the radius \( \omega \) can be determined from [2]:

\[ \omega \approx (0.65 + 1.619V^{-3/2} + 2.879V^{-6})a \]  

(9)

This is accurate within 1% for 1.2 < V < 2.4. For V > 2.405, it applies to the first mode.

There is also a modified approximate formula [12]:

\[ \omega \approx (0.65 + 1.619V^{-3/2} + 2.879V^{-6})a - (0.016 + 1.561V^{-7})a \]  

(10)

The percentage power propagating in the core is [2]:

\[ P_{\text{in core}} = \left(1 - e^{-2a^2/\omega^2}\right) \times 100\% \]  

(11)

The cut-off wavelength is given by [3]:

\[ \lambda_c = \frac{2\pi}{V_c} aN_A \]  

(12)

where \( V_c \) is the cut-off normalized frequency for the linearly-polarized (LP) mode. For example, the LP\(_{01}\) mode has \( V_c = 0 \) and the LP\(_{11}\) mode has \( V_c = 2.405 \).

3. Results and Discussion

RP Fiber Calculator can be used to make calculations on optical fibers with radially symmetric index profiles. Inputs in this program as shown in Figure 2 are core radius, core refractive index (1.45), cladding refractive index (1.44) and wavelength (1550 nm). Outputs are: numerical aperture (NA = 0.17), normalized frequency (V), number of modes (M), properties and profiles of guided modes.

![Figure 2. RP Fiber Calculator.](image)

Table 1 shows normalized frequencies and number of modes. The normalized frequency and number of modes increase with increasing core radius. The fiber is SM when V < 2.405; while at V > 2.405, the fiber is MM.

| a (µm) | V    | M |
|-------|------|---|
| 2.5   | 1.723| 1 |
| 4.5   | 3.101| 2 |
| 6.5   | 4.479| 4 |
| 7.5   | 5.168| 5 |
| 8.5   | 5.858| 6 |
3.1. Single-Mode Fiber

A single-mode fiber can be obtained when the radius of the core is a small multiple of the operating wavelength. Table 2 shows LP$_{01}$ mode properties of this fiber calculated from RP Fiber Calculator. These properties include propagation constant, effective refractive index, effective area and percentage power in the core.

Table 2. Properties of LP$_{01}$ mode ($a=2.5$ µm).

| $\beta$ ($/\mu$m) | $n_{\text{eff}}$ | $A_{\text{eff}}$ ($\mu$m$^2$) | P in core (%) |
|-----------------|-----------------|-------------------|-------------|
| 5.84977         | 1.443081        | 38.4              | 64.5        |

Effective area and power in core for this mode calculated from Equations (8) and (11) are listed in Table 3. There is a small difference when compared with those in Table 2.

Table 3. Effective area and power in core for LP$_{01}$ mode ($a=2.5$ µm).

| From Equations (8) and (10) | From Equations (10) and (11) |
|-----------------------------|-----------------------------|
| $A_{\text{eff}}$ ($\mu$m$^2$) | $P$ in core (%) |
| 39.9                        | 62.7                        |

Figure 3 shows LP$_{01}$ mode profile (single spot in the 2D plot and Gaussian in the radial plot).

3.2. Multimode Fibers (First Design)

Multimode fibers can be obtained when the core radius is large compared to the operating wavelength. Four core radii were compared 4.5, 6.5–8.5 µm. The properties of guided modes of these fibers calculated from RP Fiber Calculator are listed in Tables 4 to 7. From these tables, the following can be observed:

1. The mode LP$_{01}$ has the highest $\beta$, $n_{\text{eff}}$, P in core and has no cut-off.
2. Increasing core radius results in increasing $\beta$, $n_{\text{eff}}$, $A_{\text{eff}}$, P in core and cut-off wavelength of the mode.

Table 4. Properties of LP modes ($a=4.5$ µm).

| Number of the mode | Mode | $\beta$ ($/\mu$m) | $n_{\text{eff}}$ | $A_{\text{eff}}$ ($\mu$m$^2$) | P in core (%) | cut-off (nm) |
|-------------------|------|------------------|-----------------|-----------------|--------------|-------------|
| 1                 | LP$_{01}$ | 5.86421 | 1.446643 | 58.4 | 90.7 |     |
| 2                 | LP$_{11}$ | 5.84550 | 1.442028 | 73.2 | 68.8 | 1965.75 |
Table 5. Properties of LP modes (a=6.5 µm).

| Number of the mode | Mode   | $\beta$ (µm) | $n_{\text{eff}}$ | $A_{\text{eff}}$ (µm$^2$) | P in core (%) | cut-off (nm) |
|-------------------|--------|--------------|------------------|--------------------------|----------------|--------------|
| 1                 | LP$_{01}$ | 5.87003      | 1.448080         | 97.2                     | 96.4           |              |
| 2                 | LP$_{11}$ | 5.85846      | 1.445225         | 95.3                     | 89.4           | 2839.42      |
| 3                 | LP$_{21}$ | 5.84419      | 1.441703         | 117.6                    | 75.9           | 1797.05      |
| 4                 | LP$_{02}$ | 5.84085      | 1.440880         | 143.5                    | 60.7           | 1796.87      |

Table 6. Properties of LP modes (a=7.5 µm).

| Number of the mode | Mode   | $\beta$ (µm) | $n_{\text{eff}}$ | $A_{\text{eff}}$ (µm$^2$) | P in core (%) | cut-off (nm) |
|-------------------|--------|--------------|------------------|--------------------------|----------------|--------------|
| 1                 | LP$_{01}$ | 5.87169      | 1.448488         | 122.5                    | 97.5           |              |
| 2                 | LP$_{11}$ | 5.86247      | 1.446213         | 116.4                    | 92.8           | 3276.25      |
| 3                 | LP$_{21}$ | 5.85076      | 1.443325         | 130.0                    | 85.0           | 2073.52      |
| 4                 | LP$_{31}$ | 5.83740      | 1.440030         | 169.1                    | 67.7           | 1553.47      |
| 5                 | LP$_{02}$ | 5.84732      | 1.442476         | 122.5                    | 79.1           | 2073.31      |

Table 7. Properties of LP modes (a=8.5 µm).

| Number of the mode | Mode   | $\beta$ (µm) | $n_{\text{eff}}$ | $A_{\text{eff}}$ (µm$^2$) | P in core (%) | cut-off (nm) |
|-------------------|--------|--------------|------------------|--------------------------|----------------|--------------|
| 1                 | LP$_{01}$ | 5.87285      | 1.448774         | 150.2                    | 98.2           |              |
| 2                 | LP$_{11}$ | 5.86531      | 1.446915         | 140.3                    | 95.0           | 3713.09      |
| 3                 | LP$_{21}$ | 5.85563      | 1.444526         | 150.9                    | 89.9           | 2349.99      |
| 4                 | LP$_{31}$ | 5.84417      | 1.441699         | 163.9                    | 81.4           | 1760.60      |
| 5                 | LP$_{02}$ | 5.85256      | 1.443769         | 133.9                    | 86.7           | 2349.75      |
| 6                 | LP$_{12}$ | 5.83930      | 1.440498         | 208.4                    | 61.1           | 1637.92      |

Table 8 shows a comparison between the effective refractive index differences of the last two core radii. It can be observed that the difference between LP$_{21}$ and LP$_{02}$ modes is the lowest. A low $\Delta n_{\text{eff}}$ between the modes may result in energy loss because of the interference between the adjacent optical modes or energy transfer due to inter-mode mixing.

Table 8. Effective index differences between LP modes.

| a (µm) | LP$_{01}$–LP$_{11}$ | LP$_{11}$–LP$_{21}$ | LP$_{21}$–LP$_{02}$ | LP$_{02}$–LP$_{31}$ |
|--------|---------------------|---------------------|---------------------|---------------------|
| 7.5    | 0.002275            | 0.002888            | 0.000849            | 0.002446            |
| 8.5    | 0.001859            | 0.002389            | 0.000757            | 0.00207             |

Effective area and power in core for LP$_{01}$ mode calculated from Equations (8) and (11) are listed in Table 9. These values are smaller than those in Tables 4 to 7 with a small difference in power.
From Equations (8) and (9)

| a (µm) | $A_{\text{eff}}$ ($\mu$m²) | P in core (%) |
|--------|-----------------|---------------|
| 4.5    | 57.4            | 89.9          |
| 6.5    | 89.5            | 95.4          |
| 7.5    | 109.7           | 96.5          |
| 8.5    | 132.6           | 97.2          |

Figures 4 to 7 show LP mode profiles: LP$_{01}$ (single spot, Gaussian), LP$_{11}$ (two spots), LP$_{21}$ (four spots), LP$_{31}$ (six spots). Each of these modes have one maximum in the radial plot. Both LP$_{02}$ and LP$_{12}$ modes have two maxima. The intensity is proportional to the square of the amplitude.

| Mode | 2D profile | Radial |
|------|------------|--------|
|      | Amplitude | Intensity | Amplitude | Intensity |
| LP$_{01}$ |          |          |          |          |
|   | ![LP$_{01}$](image) | ![LP$_{01}$](image) | ![Amplitude](image) | ![Intensity](image) |
| LP$_{11}$ |          |          |          |          |
|   | ![LP$_{11}$](image) | ![LP$_{11}$](image) | ![Amplitude](image) | ![Intensity](image) |

Figure 4. Profiles of LP modes (a=4.5 µm).
| Mode    | 2D profile | Radial  |
|---------|------------|---------|
|         | Amplitude  | Intensity |
|         | Amplitude  | Intensity |
| LP\(_{01}\) | ![LP\(_{01}\) image] | ![LP\(_{01}\) intensity graph] |
|         | ![LP\(_{01}\) image] | ![LP\(_{01}\) intensity graph] |
| LP\(_{11}\) | ![LP\(_{11}\) image] | ![LP\(_{11}\) intensity graph] |
|         | ![LP\(_{11}\) image] | ![LP\(_{11}\) intensity graph] |
| LP\(_{21}\) | ![LP\(_{21}\) image] | ![LP\(_{21}\) intensity graph] |
|         | ![LP\(_{21}\) image] | ![LP\(_{21}\) intensity graph] |
| LP\(_{02}\) | ![LP\(_{02}\) image] | ![LP\(_{02}\) intensity graph] |
|         | ![LP\(_{02}\) image] | ![LP\(_{02}\) intensity graph] |

Figure 5. Profiles of LP modes (a=6.5 µm).
| Mode | 2D profile | Radial |
|------|-------------|--------|
| LP₀₁ | ![LP₀₁ 2D profile](Image) | ![LP₀₁ Radial profile](Image) |
| LP₁₁ | ![LP₁₁ 2D profile](Image) | ![LP₁₁ Radial profile](Image) |
| LP₂₁ | ![LP₂₁ 2D profile](Image) | ![LP₂₁ Radial profile](Image) |
| LP₃₁ | ![LP₃₁ 2D profile](Image) | ![LP₃₁ Radial profile](Image) |
| LP₀₂ | ![LP₀₂ 2D profile](Image) | ![LP₀₂ Radial profile](Image) |

Figure 6. Profiles of LP modes (a=7.5 µm).
| Mode    | 2D profile | Radial |
|---------|------------|--------|
| **LP\(_0^1\)** | ![LP\(_0^1\)](image) | ![LP\(_0^1\)](image) |
| **LP\(_1^1\)** | ![LP\(_1^1\)](image) | ![LP\(_1^1\)](image) |
| **LP\(_2^1\)** | ![LP\(_2^1\)](image) | ![LP\(_2^1\)](image) |
| **LP\(_3^1\)** | ![LP\(_3^1\)](image) | ![LP\(_3^1\)](image) |
| **LP\(_0^2\)** | ![LP\(_0^2\)](image) | ![LP\(_0^2\)](image) |
| **LP\(_1^2\)** | ![LP\(_1^2\)](image) | ![LP\(_1^2\)](image) |

Figure 7. Profiles of LP modes (\(a=8.5 \mu m\)).
In order to compare the results, a second design of MMFs is considered. The core and cladding refractive indices are taken as 1.45 and 1.4403, respectively. Table 10 shows a comparison between this work and Reference [7].

### Table 10. Comparison between this work and Reference [7].

| Software                  | This work (second design) | Reference [7] |
|---------------------------|---------------------------|----------------|
| Free RP Fiber Calculator  | 1.45                      | 1.45           |
| Commercial COMSOL         | 1.4403                    | 1.4403         |
| A (nm)                    | 1550                      | 1550           |
| a (µm)                    | 7.5                       | 8.5            |
| V                         | 5.091                     | 5.769          |

Tables 11 and 12 show modes properties for the second design of this work.

### Table 11. Properties of LP modes (a=7.5 µm) (second design).

| Number of the mode | Mode | β (°/µm) | n_eff | A_eff (µm²) | P in core (%) | cut-off (nm) |
|--------------------|------|----------|-------|-------------|---------------|--------------|
| 1                  | LP01 | 5.87172  | 1.448496 | 123.2        | 97.3          |              |
| 2                  | LP11 | 5.86256  | 1.446236 | 117.4        | 92.5          | 3226.90      |
| 3                  | LP21 | 5.85095  | 1.443373 | 132.0        | 84.3          | 2042.29      |
| 4                  | LP02 | 5.84758  | 1.442542 | 125.9        | 77.8          | 2042.08      |

### Table 12. Properties of LP modes (a=8.5 µm) (second design).

| Number of the mode | Mode | β (°/µm) | n_eff | A_eff (µm²) | P in core (%) | cut-off (nm) |
|--------------------|------|----------|-------|-------------|---------------|--------------|
| 1                  | LP01 | 5.87287  | 1.448779 | 151.0        | 98.1          |              |
| 2                  | LP11 | 5.86538  | 1.446931 | 141.2        | 94.7          | 3657.16      |
| 3                  | LP21 | 5.85576  | 1.444558 | 152.5        | 89.4          | 2314.59      |
| 4                  | LP31 | 5.84441  | 1.441758 | 167.4        | 80.3          | 1734.08      |
| 5                  | LP02 | 5.85273  | 1.443811 | 135.9        | 86.0          | 2314.35      |
| 6                  | LP12 | 5.83981  | 1.440623 | 240.8        | 55.9          | 1613.25      |

A comparison between two designs in this work shows that as the cladding index increases slightly, the propagation constant, effective index and effective area are increased; while the normalized frequency, power in core and cut-off wavelength are decreased.

Table 13 shows an excellent agreement between the effective index differences of this work and those of Reference [7]. The second design has a lower effective index difference compared to the first design.

### Table 13. Effective index differences between LP modes.

| a (µm) | 7.5   | 8.5   |
|--------|-------|-------|
| Δn_eff | Lp01–Lp11 | Lp11–Lp21 | Lp21–Lp02 | Lp02–Lp11 | Lp11–Lp21 | Lp21–Lp02 | Lp02–Lp11 |
| This work (second design) | 0.00226 | 0.000831 | 0.001848 | 0.000237 | 0.000747 | 0.000205 |
| Reference [7] | 0.002253 | 0.000821 | 0.001842 | 0.000236 | 0.0007396 | 0.000205 |
Table 14 shows a comparison between the effective areas of this work and Reference [7]. Effective areas in this work are smaller than those of Reference [7], with a small difference for LP$_{01}$ and LP$_{02}$ modes and 1.5 times smaller for LP$_{11}$ and LP$_{21}$ modes.

| Mode                  | a (µm) | 7.5 | 8.5 |
|-----------------------|--------|-----|-----|
| This work (second design) |       |     |     |
| LP$_{01}$             |        |     |     |
| LP$_{02}$             |        |     |     |
| LP$_{11}$             |        |     |     |
| LP$_{21}$             |        |     |     |
| LP$_{01}$             |        |     |     |
| LP$_{11}$             |        |     |     |
| LP$_{21}$             |        |     |     |
| LP$_{02}$             |        |     |     |

| Reference [7]         |       |     |     |
| LP$_{01}$             |        |     |     |
| LP$_{02}$             |        |     |     |
| LP$_{11}$             |        |     |     |
| LP$_{21}$             |        |     |     |
| LP$_{02}$             |        |     |     |

There is an excellent agreement between Figure 8 and Figure 9 in terms of the number of modes and the profiles of amplitude and intensity.

![Figure 8](image1.png)

![Figure 9](image2.png)
4. Conclusions

1. If the core radius is close to the wavelength, the fiber supports only one mode. If the core radius becomes much larger than the wavelength, the fiber supports multiple modes.
2. When the core radius is large, most power is confined in the core. When the core radius becomes smaller, the amount of power in the core decreases.

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