Simulation of the temperature state of the spacecraft shell in the shadow part of the orbit

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Abstract. The thermal model describing the steady-state temperature state of the aluminized polymer spherical shell for the calibration spacecraft located in the shadow part of the Earth orbit is constructed. This model takes into account transfer of the thermal energy in the shell cavity by means of radiation and allows us to formulate the dependence of the temperature distribution over the shell surface on the spacecraft height above the Earth and on the radiation coefficients of the outer and inner surfaces of the shell. The quantitative analysis of the obtained dependence showed that when the mentioned radiation coefficients were increased, the temperature distribution would equalized over the shell surface. At the same time, the isothermal section of the shell with the lowest temperature located on the opposite from the Earth side of the shell was remained. When the spacecraft height above the Earth increases the area of isothermal section will increase, too, and a significant reduction of the average temperature will have place. This fact must be considered when estimating the availability of the used polymer material of the shell.

1. Introduction
Calibration spacecrafts (CS) are used to estimate an energy potential of a radar channel of a ground complex for a control of space objects motion [1, 2]. One of the types of a geometric shape for such spacecrafts is a high-precision spherical shell. A similar shape is typical for passive signal repeaters and for some types of standard reflectors [1, 3]. In addition to orbits close to the polar ones, these CS can also be situated either in circular or in elliptical near-earth orbits [2, 3], which have areas shaded from the solar irradiation by the Earth.

It can be assumed that in the shadow part of the Earth orbit the CS shell is only exposed to the Earth own radiation, which intensity is significantly less than the one of the Sun in the near-earth space. It means that the shell temperature in the shadow part of the orbit will be lower than one in the part of the orbit under the Sun [4, 5]. So, it will cause a significant change of the CS shell temperature within one period of its revolution around the Earth [6]. Long-term cyclic changing of the CS temperature state can decrease the service life of the shell material.

Usually the CS shell is made of a polymer film with the thickness of several tens of micrometers, covered with a thin layer (the thickness of several nanometers) of sprayed aluminum [7], which is needed because of the operational requirements of the CS. When the CS is placed into the near-earth orbit, the shell will take a spherical shape due to its filling with gas at a relatively low pressure. So it is possible to get a spherical shell with a sufficiently large diameter, which is usual for modern trends of the deployment of large transformable structures in orbit [8].
To predict the service life of the shell material, it is necessary to have information about the temperature distribution over its surface not only in the illuminated, but also in the shaded portion of the orbit. Quantitative analysis of the temperature state of the CS shell can be carried out by mathematical modeling methods using its thermal model. In this work, the formation of such a model for a spherical shell in relation to the conditions on the shaded portion of the orbit is carried out in three stages.

2. Earth radiation distribution over the shell surface
When observing from the Earth, then, starting from the height of the CS above the Earth \( H = 200 \text{ km} \), the angular size of the CS does not exceed \( 10^{-4} \text{ rad} \), even if the radius \( r_0 \) of its spherical shell is about 10 m. Therefore, for a given value \( H \) of the CS current position above the Earth (the distance between the points \( O \) and \( O'' \) in figure 1) we can assume that the Earth own radiation falls on the CS shell from the part of the surface which area depends on the central angle \( \gamma_m = \arccos \left[ \frac{R_0}{r_0} \right] \). On this surface area, let us select the annular layer of the infinitely small width \( R_0 d\gamma \) (\( \gamma \in [0, \gamma_m] \)) which area is equal to \( dS' = 2\pi R_0^2 \sin \gamma d\gamma \) (in figure 1 this layer is shaded).

Due to the negligible angular size of the CS when observed from the Earth, the angle \( \varphi \) between the normal to the selected annular layer and directions to different parts of the CS surface can be taken equal to \( \beta + \gamma \), where \( \beta \) is the angle between the rays starting from the center of the spherical CS shell to the center of the Earth and to this layer (figure 1). According to Lambert’s law, this layer sends to the CS surface the flux of radiation \( dQ = q_0 (r_0/l)^2 \cos \varphi dS' \), where \( l^2 = R_0^2 + (R_0 + H)^2 - 2R_0(R_0 + H) \cos \gamma, q_0 \approx 215 \text{ W/m}^2 \) is the flux density of the own Earth radiation. Finally, the total flux of radiation falling on the surface of the spherical CS shell is equal to

\[
Q = 2\pi R_0^2 q_0 \int_0^{\gamma_m} \frac{\cos(\beta + \gamma)}{R_0^2 + (R_0 + H)^2 - 2R_0(R_0 + H) \cos \gamma} \sin \gamma d\gamma. \tag{1}
\]

It can be noted that the largest value of \( \beta_m \) is equal to \( \pi/2 - \gamma_m \), which corresponds to the angle \( \varphi = \pi/2 \).

Let us replace the function \( \cos(\beta + \gamma) \) from the integrand in the right side of equality (1) by the expression \( \cos \beta \cos \gamma - \sin \beta \sin \gamma \) and assume in accordance with figure 1

\[
\sin \beta = \frac{R_0}{l} \sin \gamma, \quad \cos \beta = \frac{R_0(1 - \cos \gamma) + H}{l}. \tag{2}
\]
Then we can obtain instead of equality (1)

\[ Q = 2\pi R_0^2 q_0 r_0^2 \int_0^{\gamma_m} \frac{[R_0(1 - \cos \gamma) + H] \cos \gamma - R_0(1 - \cos^2 \gamma)}{[R_0^2 + (R_0 + H)^2 - 2R_0(R_0 + H) \cos \gamma]^{3/2}} \sin \gamma \, d\gamma. \]

From the last expression, substituting \( x = \cos \gamma \) and taken into account that \( dx = -\sin \gamma \, d\gamma \) and \( \cos \gamma_m = R_0/(R_0 + H) \), we can write

\[ Q = 2\pi r_0^2 q_0 \int_{\gamma_m}^{1} \frac{(1 + \eta)x - 1}{1 + (1 + \eta)^2 - 2(1 + \eta)x} \, dx = 2\pi r_0^2 q_0 \left[ 1 - \frac{\sqrt{\eta(2 + \eta)}}{1 + \eta} \right], \tag{3} \]

where \( \eta = H/R_0 \).

The distribution of the radiation flux falling on the spherical shell surface is uneven. It is reasonable to describe this distribution as the dependence \( q'(\psi) \) of the falling radiation flux density on the angle \( \psi \) counted from the point on the shell surface closest to the Earth’s surface (figure 1) along any arc of a large circle of the radius \( r_0 \) passing through that point. The greatest density of the falling radiation flux \( q'(0) \) will be exactly in this point at \( \psi = 0 \).

To find the value \( q'(0) \) in the vicinity of the indicated above point, let us select the area perpendicular to the line \( O'O \) connecting the center of the Earth and the center of the spherical shell (figure 1). Then, taking into account the fact that the radiation from the annular layer on the Earth surface falls on this area at the angle \( \beta \) and applying the second equality (2), we can obtain

\[ q'(0) = 2q_0 \int_{\psi_m}^{1} \frac{(1 + \eta)x - 1}{1 + (1 + \eta)^2 - 2(1 + \eta)x} \, dx = \frac{q_0}{1 + \eta^2}. \tag{4} \]

If the angle \( \psi \) increases, the density of the radiation flux falling on the spherical shell will decrease and in accordance with the above assumption about the possibility to neglect the angular size of the shell will take a zero value at \( \psi_m = \pi - \gamma_m \), i.e. \( q'(\psi_m) = 0 \) and, in addition, \( dq'(\psi)/d\psi|_{\psi = \psi_m} = 0 \). From the symmetry of the distribution \( q'(\psi_m) \) relatively the line \( O'O \) follows that \( dq'(\psi)/d\psi|_{\psi = \psi_m} = 0 \). These conditions are satisfied by the approximating ratio

\[ q'(\psi) = q'(0) \cos^2 \frac{\pi \psi}{2\psi_m} + q'_1 \sin^2 \frac{\pi \psi}{\psi_m}. \tag{5} \]

in which the coefficient \( q'_1 \) can be found from the balance of the falling radiation fluxes

\[ Q = 2\pi r_0^2 \int_{0}^{\psi_m} q'(0) \cos^2 \frac{\pi \psi}{2\psi_m} + q'_1 \sin^2 \frac{\pi \psi}{\psi_m} \sin \psi \, d\psi. \tag{6} \]

The integral on the right side of this equality can be represented by the sum of two integrals

\[ I_0 = \frac{q'(0)}{2} \int_{0}^{\psi_m} \left( 1 + \cos \frac{\pi \psi}{\psi_m} \right) \sin \psi \, d\psi, \quad I_1 = \frac{q'_1}{2} \int_{0}^{\psi_m} \left( 1 - \cos \frac{2\pi \psi}{\psi_m} \right) \sin \psi \, d\psi, \]

which can be reduced to the tabular ones [9] and after calculation can be written as

\[ I_0 = q'(0) \frac{1 - \cos \psi_m - 2(\psi_m/\pi)^2}{2 - 2(\psi_m/\pi)^2}, \quad I_1 = 2q'_1 \frac{1 - \cos \psi_m}{4 - (\psi_m/\pi)^2}. \]

Hence, taking into account the equalities (3), (4), and (6), we can obtain

\[ q'_1 = q_0 \frac{4 - (\psi_m/\pi)^2}{2(1 - \cos \psi_m)} \left\{ 1 - \frac{\sqrt{\eta(2 + \eta)}}{1 + \eta} - \frac{1 - \cos \psi_m - 2(\psi_m/\pi)^2}{2(1 + \eta)^2[1 - (\psi_m/\pi)^2]} \right\}. \tag{7} \]
Since $\psi_m = \pi - \gamma_m = \pi - \arctan \sqrt{(2 + \eta)}$ and $\cos \psi_m = -\cos \gamma_m = -\frac{1}{1+\eta}$, the equations $\bar{q}(0) = q'(0)q_0$, $\bar{q}_1 = q'_1/q_0$ and $\bar{Q} = Q/(\pi r_0^2 q_0)$ as well as the angle $\psi_m$ can be represented as functions of the only argument $\eta$ (figure 2). In figure 3, dependency graphs of $\bar{q}(\psi) = q'(\psi)/q_0$, $\bar{\psi} = \psi/\psi_m$ are constructed using relation (4). It can be noted, that the density distribution of the radiation falling on the shell surface has an axial symmetry with respect to a straight line $O'O$ (figure 1).

Conclusions
The thermal model describing the steady temperature state of the aluminized polymer spherical shell of the CS located in the shadow section of the Earth orbit allows determination of the dependence of temperature distribution over this shell surface on the height of the spacecraft above the Earth and on the radiation coefficients of the external and internal shell surfaces.

This dependence was used to estimate the deviation of the shell shape from the spherical shape caused by the uneven temperature distribution. The results obtained can be used to predict the operability of the CS in the shadow section of the Earth orbit. The quantitative analysis of the obtained dependence showed that when the mentioned radiation coefficients were increased, the temperature distribution would equalized over the shell surface. At the same time, the isothermal section of the shell with the lowest temperature located on the opposite from the Earth side of the shell was remained. When the spacecraft height above the Earth increases the area of isothermal section will increase, too, and a significant reduction of the shell average temperature will have place. This fact must be considered when estimating the availability of the used polymer material of the shell.

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