Self-Completeness of Einstein Gravity.

Gia Dvali$^{a,b,d,c}$ and Cesar Gomez$^e$

$^a$Arnold Sommerfeld Center for Theoretical Physics
Department für Physik, Ludwig-Maximilians-Universität München
Theresienstr. 37, 80333 München, Germany

$^b$Max-Planck-Institut für Physik
Föhringer Ring 6, 80805 München, Germany

$^c$CERN, Theory Division
1211 Geneva 23, Switzerland

$^d$CCPP, Department of Physics, New York University
4 Washington Place, New York, NY 10003, USA

$^e$Instituto de Física Teórica UAM-CSIC, C-XVI
Universidad Autónoma de Madrid, Cantoblanco, 28049 Madrid, Spain

Abstract

We argue, that in Einsteinian gravity the Planck length is the shortest length of nature, and any attempt of resolving trans-Planckian physics bounces back to macroscopic distances due to black hole formation. In Einstein gravity trans-Planckian propagating quantum degrees of freedom cannot exist, instead they are equivalent to the classical black holes that are fully described by lighter infra-red degrees of freedom and give exponentially-soft contribution into the virtual processes. Based on this property we argue that pure-Einstein (super)gravity and its high-dimensional generalizations are self-complete in deep-UV, but not in standard Wilsonian sense. We suggest that certain strong-coupling limit of string theory is built-in in pure Einstein gravity, whereas the role of weakly-coupled string theory limit is to consistently couple gravity to other particle species, with their number being set by the inverse string coupling. We also discuss some speculative ideas generalizing the notion of non-Wilsonian self-completeness to other theories, such as the standard model without the Higgs.
1 The Shortest Scale of Nature

In gravity the Planck length is the shortest length-scale of nature, and any attempt of probing the shorter distances will instead probe the larger length-scales.

We start with the pure Einsteinian theory of gravity in four-dimensions, in which below the Planck scale the only propagating degree of freedom is a massless spin-2 graviton, $h_{\mu\nu}$. No other propagating species are assumed at this point. All the sources, other than the ones composed out of gravitons, will be considered as external sources that are not associated with any new degrees of freedom. This requirement uniquely fixes the action in form of the Einstein-Hilbert action,

$$S_{EH} = \int d^4x \, M_P^2 \sqrt{-g} \, R.$$  \hspace{1cm} (1)

For definiteness, we have set the cosmological constant to zero. In this way, we shall always consider observers on asymptotically-flat spaces.

We shall first clarify the field-theoretic meaning of the Planck mass $M_P \sim 10^{19}$GeV and of the corresponding Planck length, $L_p \equiv M_P^{-1} \sim 10^{-33}$ cm. The overall numerical factors of order one will be ignored throughout the paper.

In pure Einstein gravity, the Planck scale plays the central role. It defines the coupling of graviton to energy momentum sources universally,

$$h_{\mu\nu} \frac{T^{\mu\nu}}{M_P},$$  \hspace{1cm} (2)

where $T_{\mu\nu}$ is an arbitrary energy-momentum tensor. The important fact is, that the above universality property is true to all orders in non-linear interactions of graviton. That is, one can either think of the above coupling as the coupling to an external source, or as a self-coupling of graviton to its own energy-momentum tensor in a given non-linear order. Due to the above crucial property, Einstein’s gravity viewed as a quantum field theory possesses an universal strong coupling scale, $M_P$. This fact shall play the key role in what follows.

The existing common knowledge about Einstein gravity is, that it becomes unapplicable in deep-UV, at distances $L \ll L_p$ and must be completed by a more powerful theory that will restore consistency at sub-Planckian distances.

We wish to question the above statement and suggest, that the pure Einstein’s gravity is self-complete in deep-UV. In other words, we argue that for restoring consistency no new
propagating degrees of freedom are necessary at energies $\gg M_P$, and moreover, even if one tries to introduce such new states, they will not have any physical meaning, since the corresponding distances can never be probed. All the information that such new states can in principle carry, will be identical to the information carried by the semi-classical macroscopic black holes of the same mass, whose properties are completely determined by the IR gravity.

The reason behind our claim is, that in Einstein’s gravity, $L_P$ represents the absolute lower bound on any distance that can ever be resolved. Distances $L \ll L_P$, cannot be probed, *in principle*.

A version of the above statement sometimes goes under the name of Generalized Uncertainty Principle [1, 2].

More precisely, any attempt of resolving physics at the distance scales $L \ll L_p$, will inevitably bounce us back to much larger distances $L_P^2/L \gg L_P$, which are completely insensitive to any short-distance physics and are entirely governed by the massless graviton, which is the only king at any scale longer than the Planck length.

Namely, physics that one can decode at sub-Planckian distances is identical to the physics at macroscopic distances,

$$L \leftrightarrow \frac{L_P^2}{L}.$$  

(3)

The fundamental reason for such an obstacle is the existence of black holes (BH). BH-formation interferes with any attempt of extracting information from beyond the Planck length and produces an insuperable barrier. In fact, harder we try to go beyond $L_P$, with a larger and more classical BH we shall end up. Physics of such a BH has nothing to do with short distances and is entirely governed by the infra-red (IR) gravity.

In other words, the key reason for our claim can be formulated in the following way,

$$\text{Deep − UV Gravity } = \text{ Deep − IR Gravity}.$$  

(4)

Some crucial aspects of the above connection has been stressed by the previous authors [4, 5]. However, we shall suggest that it is absolute in field-theoretical sense. Namely, in Einstein gravity trans-Planckian propagating quantum degrees of freedom cannot exist, instead they are mapped on (non-propagating) classical states, fully described by the dynamics of lighter propagating IR degrees of freedom, such as the massless graviton. Any attempt of integrating-in trans-Planckian quantum fields that avoid such correspondence is bound to fail. Since, dynamics of quantum field theories is formulated in terms of
propagating degrees of freedom, our conclusion is, that Einstein’s gravity is self-UV-complete, but in the sense that is different from the notion of standard Wilsonian UV-completeness.

In order to see how the concept of the minimal length arises, consider a generic thought experiment that attempts to resolve the physics at distance $L$. An elementary act of such a measurement is a scattering process in which one has to localize the minimum amount of energy $E > 1/L$ within the space-time box of size $L$. The corresponding Schwarzschild radius of such a localized energy portion is,

$$R(L) = L^2_P/L.$$

Notice, that for $L \ll L_P$ the above Schwarzschild radius exceeds both $L$ and the Planck length. Thus, any attempt of probing length scales $L \ll L_P$ will require localization of energy within the radius much smaller than the corresponding Schwarzschild, $R(L) \gg L_P$. The corresponding act of measurement thus will lead to a formation of a macroscopic classical BH, way before it has any chance of probing distance $L$.

The above conclusion is completely insensitive to what formally happens to the gravitational dynamics in the trans-Planckian region $L \ll L_P$. The BH shield is turned on way before this dynamics has any chance to get excited.

To put it in different terms, the maximal information that can be extracted from the measurements of a sub-Planckian distance $L$ is equal to the information that can be encoded at the horizon of a classical BH of size $L^2_P/L$.

The above reasoning is in full accordance with the ideas of holography \cite{3}. It can only be violated if the theory possesses an agent that could violate energy positivity condition. Then, using such an agent, one could encode (and extract) information at arbitrarily short distances, without paying the energy price.

Let us suppose we attempt to change the laws of gravity dramatically at distances $L \ll L_P$. For this we have to introduce new gravitational degrees of freedom with masses $m = L^{-1} \gg M_P$. In the other words, we try to introduce new poles in the graviton propagator at some $p^2 = L^{-2} \gg M^2_P$. Naively, such poles will change gravitational dynamics at distances $L$, but this is a complete illusion, since corresponding change can never be probed. Any observer that will attempt to probe the physics of the trans-Planckian pole, will not learn anything new other than what he/she can learn in Einsteinian gravity at distance $L^2_P/L$. 

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We shall now attempt to give a field theoretic perspective of the above phenomenon. A related work with applications to particular UV-completions of gravity shall appear in [6].

We shall discuss this on a concrete example. Let us say, we modify the graviton propagator by adding a new trans-Planckian pole at $1/L$,

$$\frac{1}{M_P^2} T^{\mu\nu} \langle h_{\mu\nu} h_{\alpha\beta} \rangle \tilde{t}^{\alpha\beta} = \frac{1}{M_P^2} \left( \frac{T^{\mu\nu} t^{\mu\nu} - \frac{1}{2} T_{\mu}^{\mu} t^{\nu\nu}}{p^2} + \frac{a T^{\mu\nu} t^{\mu\nu} - b \frac{1}{3} T_{\mu}^{\mu} t^{\nu\nu}}{p^2 + (1/L)^2} \right),$$

where we have convoluted the propagator with the two sources, $T_{\mu\nu}$ and $t_{\mu\nu}$. The pole at $p^2 = 0$ in the first term corresponds to the Einsteinian massless graviton. The parameters $a$ and $b$ are fixed according to the spin of the new pole ($a = b$ for spin-2, and $a = 0, b < 0$ for spin-0).

In order to probe a pole at $p^2 = L^{-2} \gg M_P^2$, we need to consider an experiment with the momentum-transfer $\sim L^{-1}$. In any such process we need to localize energy $1/L$ within the distance $L \ll L_P$. But, this is impossible without first forming a classical BH of size $R(L) \gg L_P$. For example, we can scatter gravitons with an impact parameter $L$ and center of mass energy $1/L$. But since the Schwarzschild radius is much larger than the impact parameter,

$$R(L) = L_P^2 / L \gg L_{\text{impact}} = L,$$

the classical BH will form way before the scattering gravitons have any chance of approaching the distance $L$. Thus, a trans-Planckian pole in the graviton propagator remains completely shielded by the BH barrier and is unaccessible, in principle. Or to be more precise, accessing such a pole is the same as accessing the classical BH of the same mass, and thus former cannot carry any other information that the latter.

Thus, our attempt to integrate in a propagating quantum degree of freedom of trans-Planckian mass failed and we ended up with a classical BH instead. This means, that representation of the graviton propagator in the form [6] for trans-Planckian poles is inconsistent. Contribution from such poles must be exponentially suppressed. We can estimate the required suppression factor (up to a numerical coefficient in the exponent) as,

$$e^{-(L_P/L)^2}.$$

The necessity of this factor can be understood at least in two ways. First, it can be interpreted as the entropy suppression, $e^{-S}$, where $S = (L_P/L)^2$ is the Bekenstein-
Hawking entropy. Secondly, it can be interpreted as the Boltzmann suppression in the evaporation of a classical BH.

In order to explain the latter suppression, let us denote the (would-be) degree of freedom corresponding to the massive pole, \( p^2 = L^{-2} \), by \( \phi_{\mu\nu} \). The spin (2 versus 0) is unimportant for the present discussion and we shall leave it unspecified. Let the source \( T_{\mu\nu} \), to which \( \phi \) is coupled, be an energy momentum tensor of some light (not necessarily massless) particle \( q \). Then, the interaction vertex \( \phi_{\mu\nu} T^{\mu\nu} \) sets the decay rate of \( \phi \) into a particle anti-particle pair,

\[
\phi \rightarrow q + \bar{q} .
\]

As long as \( L^{-1} \ll M_P \), this is an ordinary quantum decay of a heavy particle into the lighter ones. However, for \( L^{-1} \gg M_P \), the same vertex describes evaporation of a classical BH of mass \( 1/L \) into a single particle-anti-particle pair. This process is obviously suppressed by the Boltzmann factor \( e^{-(1/LT)} \), since the particle pair has energy \( 1/L \), which exceeds the Hawking temperature \( T = L/L_P^2 \). This Boltzmann suppression matches \( \text{[N]} \).

Thus, the two conclusions follow from the above analysis. First, the effects of the trans-Planckian poles are exponentially softened, and secondly, such poles no longer describe quantum propagating degrees of freedom, but rather the classical states.

The above reincarnation of trans-Planckian degrees of freedom into the classical states is one of the key points of our analysis. What we are observing is, that in gravity there are no propagating quantum degrees of freedom above the Planck mass, instead they become classical states that are fully described by other lighter propagating degrees of freedom.

Generic quantum field theories are defined by quantum propagating degrees of freedom and by their interactions. Field theories also describe classical states, such as solitons or other classical solutions. The defining property of the classical states is, that they are not independent entities, and at least in principle can be fully described by quantum degrees of freedom. For example, solitons can be understood as coherent superpositions of quantum degrees of freedom with large occupation numbers.

The intrinsic property of classical states is that they cannot probe distances shorter than the characteristic wavelength of their constituent quantum particles, which is typically given by the size of the classical configuration in question. For example, the size of a 't Hooft-Polyakov magnetic monopole is given by the Compton wave-length of the massive gauge fields. Because of this, despite the fact that monopole can be much heavier
than the gauge field, the former is no better probe of the short-distance physics than the latter.

The above statements are true both for gravity and for other field theories. However, the crucial peculiarity of Einstein gravity is the following. In ordinary quantum field theories, with suitable arrangements, one can include arbitrarily heavy quantum degrees of freedom. By making their mass higher, heavy degrees of freedom probe shorter and shorter distances. Of course, they do gradually decouple from the low energy processes, but their effects can in principle be detected in precise measurements at arbitrarily large distances.

The story in Einstein gravity is dramatically different. By becoming heavier than $M_P$ particles simply stop existence as independent quantum degrees of freedom and become classical states. These classical states are no longer independent entities, but instead are fully described by other already-existing light fundamental degrees of freedom, such as the massless graviton.

This transition of the heavy would-be degrees of freedom into non-fundamental classical states is intrinsic property of gravity, and the key to its self-completeness. From this behavior it also follows that at the boundary of the two regimes, some quantum degrees of freedom of mass $M_P$ must be present in Einstein gravity.

Thus, the built-in spectrum of quantum degrees of freedom of Einstein gravity includes massless graviton plus new quantum degrees of freedom in a narrow mass interval around $M_P$. As we shall show, existence of the latter degrees of freedom is not an additional assumption, but follows from the smooth transition between the quantum particles and classical states. However, their presence plays essentially no role neither in deep-UV nor in deep-IR. The rest of the states in Einstein gravity are not fundamental and are described by the dynamics of the massless graviton.

This findings lead us to the conclusion that Einstein gravity is self-complete in a sense that is very different from the standard notion of the Wilsonian completeness.

In Wilsonian sense a quantum field theory is defined as a relevant perturbation of an UV fixed point CFT. The UV CFT sets the real degrees of freedom of the theory...
as well as those unitarizing the high energy scattering S matrix. In this frame, given a particular low energy physics, the corresponding UV completion is obtained by embedding that IR dynamics into a quantum field theory flowing in the UV to a CFT fixed point. As already pointed out the essential aspect of gravity is the existence of a BH barrier for resolving scales smaller than Planck length. A natural consequence of this barrier is to unitarize high energy scattering amplitudes using black hole production. This led to the hypothesis known as asymptotic darkness [4] and to postulate BHs as the real UV states of the theory. However this potential UV description of quantum gravity in terms of BHs does not easily fit with any UV Wilsonian CFT. Indeed the essential property of BHs is to carry entropy and therefore any UV CFT Wilsonian description of quantum gravity - consistent with the BH barrier to short scales resolution - should be able to account for the BH Bekenstein entropy. The BH entropy for asymptotically flat BHs in generic dimension \(d\) scales with energy as \(E^{d-2\over d-3}\), while the entropy for a CFT scales as \(E^{d-1\over d-2}\). This mismatch for the entropy formula for asymptotically flat BHs indicates that quantum gravity is not a Wilsonian quantum field theory. The situation changes drastically in the case of negative cosmological constant. For asymptotically AdS black holes the Bekenstein entropy goes like \(E^{d-2\over d-1}\) and therefore we can account for that entropy using a CFT in one less dimension. This leads to the famous AdS/CFT [7] definition of quantum gravity in five dimensions in terms of the \(N=4\) SYM CFT.

Independently of what could the microscopic theory that accounts for the BH entropy be, the crucial consequence of describing the UV quantum gravity in terms of BHs of masses bigger than \(M_P\) is that those UV degrees of freedom are - in contrast to what happens in Wilsonian-complete quantum field theories - perfectly well defined low energy states of the theory. Therefore we can map the UV degrees of freedom into the space of states describing gravity in the IR. It is in this sense that Einstein gravity is self-complete, although not Wilsonian.

Although the UV/IR transformation \(L \rightarrow L^{2\over L^2}\) bounces deep UV probes into classical macroscopic BHs, we may wonder how this correspondence works near the Planck length itself. Fortunately, thanks to the fact that in pure Einsteinian gravity BHs evaporate, we can define the relevant quantum degrees of freedom at the Planck length by parametrically reducing the BH mass until reaching a quantum mechanical regime with the corresponding Compton length bigger than the Schwarzschild radius. This happens when the BH becomes of mass \(M_P\). As we shall explain in more details later, this crossover
between the two regimes implies existence of propagating quantum degrees of freedom with mass around $M_P$. Thus, we can conclude that in addition to macroscopic BHs the UV description of quantum gravity requires inclusion of quantum species of mass $M_P$. We shall discuss later the possible role of these quantum species in connection to string theory and to microscopic description of BH entropy.

In summary although the concepts of $L_P$ being a minimal length [2] as well as UV-IR connection through the BHs [4][5] have been around for some time, the goal of the present paper is to push these concepts to certain extreme and to show that they take us to self-completeness of Einstein gravity. We do this by deriving the above concepts from the quantum field theoretic perspective, basing our reasoning on fundamental notions such as local propagating degrees of freedom and the scattering amplitudes. This language allows us to circumvent secondary (but otherwise very important) issues, e.g., such as BH information loss, and to unambiguously identify the true physical meaning of trans-Planckian degrees of freedom as of classical IR states, which is the key for understanding the self-UV completeness of gravity. We see that any propagating quantum degree of freedom when being pushed into the trans-Planckian region becomes a non-propagating classical state belonging to the deep IR sector of the theory. In this way, there are no poles on the complex plane that are able to probe short distances. In what follows we shall discuss this from various angles.

2 Being Patient: Can Trans-Planckian Physics be Probed by Waiting Longer?

As we have seen from the previous section, BHs make it impossible to probe distances smaller than the Planck length in any measurement process. The question we would like to ask now is, whether it is possible to circumvent the BH barrier by waiting a long enough time. To formulate the question more precisely, can we probe a new heavy pole $p^2 \gg M^2_P$, by waiting for the final stages of the BH evaporation?

In general, the physics of the heavy particles, of mass $M_X$, can be probed in the following two ways:

1) Observe the processes among the light fields mediated by the high-dimensional operators generated after integrating-out the heavy quanta;
2) Detect a direct production of heavy states in the high-energy processes.

As we shall now see, none of the above is possible for $M_X \gg M_P^2$. We consider the two options separately.

### 2.1 Can Operators Induced by Trans-Plankian Quanta Serve as Probes of Trans-Planckian Physics?

In many cases the processes mediated by high-dimensional operators induced by heavy states can be probes of high-energy physics. The well known example is the proton decay mediated by the baryon and lepton number violating operators, such as,

\[
\frac{qql}{M_X^2},
\]

where $q$ and $l$ stand for quark and lepton fields respectively. For example, in standard grand unified theories (GUTs), such operators are generated by the exchange of $X$ and $Y$ gauge bosons and colored Higgs states of mass $M_X \sim 10^{16}\text{GeV}$. Despite of the huge suppression, the operators of the above sort represent direct low-energy probes of the GUT-scale physics, since the tiny decay rate can be overcompensated by the huge number of baryons in the sample and by the possibility of performing observations over long time-scales.

Can a similar reasoning be applied to the trans-Planckian physics, at least in principle? The answer to this question is negative. As we have explained, any quantum of mass $M_X \gg M_P$ is no longer a perturbative state, but rather is a macroscopic object, a classical BH. This fact immediately implies the following.

First, in accordance with (9), the operators obtained by integrating out such a state must be exponentially suppressed at least by the entropy (or Boltzmann) factor $e^{-S}$, where $S = (M_X/M_P)^2$ is the Bekenstein-Hawking entropy. For example, the operator (10) can be generated as a result of collapsing two quarks into a virtual classical BH with the subsequent evaporation of the latter into a quark and a lepton. As we have discussed earlier in the paper, the effective form-factor describing evaporation of a classical BH into a two-particle final state, must be suppressed by the Boltzmann factor, $e^{-M_X/T}$, which gives (9).

More importantly, since it is a classical BH, by BH no-hair theorem (9), it cannot be distinguished from any other BH of the same mass and the spin, obtained by collapse
of the low energy particles. Therefore, an operator obtained by integrating out such a trans-Planckian state cannot be distinguished from the analogous operator obtained by integrating out any other classical BH of the same characteristics. Since the latter object obviously cannot probe any trans-Planckian physics, the same applies to the former one. In conclusion, operators mediated by trans-Planckian quanta are unable to give any information about the deep-UV physics, in principle.

### 2.2 Direct Production of trans-Planckian Quanta in BH-Evaporation?

Another question is, can one probe trans-Planckian degrees of freedom by their direct production in BH evaporation? Again, the answer to this question is negative. The reason is simple. First, by conservation of energy, BH can produce a particle of mass $M_X$ only until its mass drops below $M_X$. But, because $M_X \gg M_P$, the BH of corresponding mass is a classical BH of temperature $T_H = M_P^2 / M_X \ll M_X$. Thus, the trans-Planckian state of the mass $M_X$ itself is a classical BH of the same mass and vice-versa. This closes the issue. Since, first the production of such a heavy state will be suppressed at least by a Boltzmann factor $e^{-(M_X/M_P)^2}$. And secondly, since the state itself is a classical BH of the same mass, it will not carry any message about the deep-UV physics, but rather only about the IR physics corresponding to distances $M_X / M_P^2$.

### 2.3 Jumping into a Black Hole?

Finally, we briefly note, that jumping into the BH and trying to probe physics of singularity will not give any new information about the trans-Planckian physics. An observer falling towards the singularity is not in any respect in a better position to perform the measurement experiment than a flat space observer. If he/she wants to probe trans-Planckian physics, he/she cannot avoid localizing the energy $L$ within the interval $1/L$, with all the above-considered consequences.

### 3 Influence of a Possible Black Hole Information Loss

The question we would like to address is, whether our reasoning about deep-UV-completeness of Einstein gravity is sensitive to the possible information loss by a BH [9]. The answer to this question is negative, as we shall now explain.
As discussed above, the deep-UV-completeness of Einstein gravity follows from the impossibility of probing distances \( L \ll L_P \). The reason for this barrier, as we have explained, is in the fact that in order to probe a distance \( L \), one has to pump energy \( E = 1/L \) within that distance. But, for \( L \ll L_P \) the Schwarzschild radius of this localized portion of energy is much larger than the Planck length. As a result, any such attempt will end up by a formation of a BH with horizon \( R = L_P^2/L \), way before one can approach the sub-Planckian distances. Thus, the entire information extracted from such a measurement will be restricted by the information encoded at the horizon of a resulting classical BH.

We now wish to make the following two comments.

First, regardless whether the subsequent evaporation of a classical BH violates information or not, its formation does represent the insuperable barrier for the short distance measurements.

Secondly, since the dynamics of a large semi-classical BH is governed by IR Einsteinian gravity, any inconsistency (e.g., such as information loss, or violation of unitarity) would signal the incompleteness of Einstein gravity in IR, rather than in UV.

Our assumptions exclude such an inconsistency. We rely on the fact that pure Einstein (super)gravity is a consistent theory in IR. Existence of any inconsistency in IR would mean, that new light degrees of freedom must be integrated in, which would contradict to our starting point that the only propagating IR degree of freedom is the massless graviton.

Finally, since we are working in pure Einstein gravity without any non-gravitational species, the only information encoded in the BH can be in the form of gravitons. Discussions about the BH information loss, typically involve other probe states (e.g., such as fermions with baryon number). As we shall see, this seemingly innocent deformation of the theory dramatically affects its properties. In particular, in such a case existence of extra propagating gravitational degrees of freedom is a must, and analysis has to be changed accordingly.

## 4 Large Distance Effects of Trans-Planckian States

We wish to discuss, why trans-Planckian physics cannot have any observable long-distance effects that could show up in very precise measurement. For example, why their influence
could not modify the metric of gravitating sources, and for instance, affect the dynamics of
BH formation. The reason why long-distance measurements cannot establish any contact
with trans-Planckian physics, is again rooted in the fact that trans-Planckian degrees
of freedom cannot be perturbative quantum states. The only physical meaning they can
carry is of the macroscopic classical object that belong to the deep IR region of the theory.
In this respect, any trans-Planckian state is not any better probe of UV physics than any
other macroscopic BH of the same mass.

In order to make this discussion more concrete, we shall first consider a simplified toy
model with two scalar “gravitons”, which crudely captures the essence of the phenomenon.
Let us consider a theory with the graviton that propagates two spin-0 degrees of freedom.
One, call it $\chi$, will be assumed to be massless and will be the analog of Einstein graviton.
The other one, $\phi$, will be a heavy state with the mass $m$, which at the beginning we
shall take below $M_P$ and later push into the trans-Planckian region, $m \gg M_P$. The two
degrees of freedom couple to the energy momentum sources through an effective metric,

$$g_{\mu \nu} = \eta_{\mu \nu} + \eta_{\mu \nu}(\chi + \phi)/M_P. \quad (11)$$

We wish to study the long distance corrections to the metric produced by a heavy source
$T_{\mu \nu}$. For our purposes it will be enough to work up to the second order in $G_N$. Therefore,
we restrict ourselves by considering up to trilinear couplings of the gravitons. As said
above, we shall first keep the mass of the heavy graviton below $M_P$, and later take trans-
Planckian limit. The Lagrangian is:

$$\left( \partial_\mu \chi \right)^2 + \left( \partial_\mu \phi \right)^2 - m^2 \phi^2 + \frac{1}{M_P}(\chi + \phi)(\partial_\mu \chi)^2 + \frac{1}{M_P}(\chi + \phi)T, \quad (12)$$

with the corresponding equations of motion,

$$\partial^\mu((1 + \frac{1}{M_P}(\chi + \phi))\partial_\mu \chi) - \frac{1}{2M_P}(\partial_\mu \chi)^2 = \frac{T}{2M_P}, \quad (13)$$

and

$$\Box \phi + m^2 \phi - \frac{1}{2M_P}(\partial_\mu \chi)^2 = \frac{T}{2M_P}. \quad (14)$$

Let us evaluate the above system for a static localized source of mass $M$. Since we
are interested in the metric outside the source, the latter can be approximated by $T =
8\pi\delta(r)M$. In the linear order the two gravitons contribute into the metric as,

$$\frac{\chi^{(1)}}{M_P} = \frac{R}{r} \quad \text{and} \quad \frac{\phi^{(1)}}{M_P} = \frac{R}{r}e^{-mr}. \quad (15)$$
where \( R \equiv M/M_P^2 \) is the gravitational radius of the source. Thus, to the linear order, the correction to the metric from the heavy scalar relative to the massless one is exponentially small. The stronger relative correction occurs in the next order in \( R/r \). In the diagrammatic language this corresponds to a Feynman diagram with a cubic vertex from which the two graviton lines are ending on the source. As it is easily checked from the equations, the second order correction to the metric are,

\[
\frac{\chi^{(2)}}{M_P} \propto \frac{R^2}{r^2} \quad \text{and} \quad \frac{\phi^{(2)}}{M_P} \propto \frac{R^2}{r^2} \frac{1}{(rm)^2}.
\]

(16)

We see that unlike linear order, in the second order the relative correction from the heavy state is suppressed only by the power \((mr)^{-2}\). This fact can be understood as the correction to the non-linear coupling of the massless graviton \( \chi \) to the source, due to the exchange of the heavy state \( \phi \). The reason, why this effective interaction is not exponentially suppressed is because the virtual heavy state does not have to propagate distances larger than its inverse mass. Indeed, if we explicitly integrate out the heavy scalar, we will induce the following effective coupling between the massless graviton and the source

\[
\frac{(\partial_{\mu} \chi)^2}{M_P^2 m^2} T.
\]

(17)

The following two points emerge from the above consideration.

First, as long as \( m < M_P \), the corrections to the metric coming from the heavy state is suppressed at least by the powers of \((mr)^{-1}\). This implies that the heavy state cannot interfere with gravitational processes at distances larger than \( m^{-1} \). For example, formation of a BH of the gravitational radius \( R \gg m^{-1} \) will not be affected.

Notice that the gravitational radius in Einstein theory can precisely be deduced from equating the leading and subleading contributions. That is, we approach the “horizon” when \( \chi^{(1)} \sim \chi^{(2)} \). At the horizon all higher order corrections become equally important, and the series have to be re-summed.

Secondly, for \( m < M_P \), one can argue, that although the fact of large BH formation is unaffected, the deviation from the Einsteinnian dynamics should be observable by precise measurements of the corrections to the metric. In other words, by measuring corrections of order \( \sim 1/(mr)^2 \) to the metric, we can deduce the information about the heavy physics. This is certainly true as long as the mass \( m \lesssim M_P \), but for trans-Planckian states the situation changes dramatically.
Again, the reason is, that as soon as we push the mass of the $\phi$ into the trans-Planckian region, $m \gg M_P$, $\phi$ stops to be a quantum particle and becomes a classical object, and this must be taken into the account. For instance, the operator (17) in reality will be exponentially suppressed at least by the entropy factor $e^{-s} = e^{-m^2/M_P^2}$, but more importantly, it should be indistinguishable from the operator obtained by integrating out any other classical BH of the same mass $m$. Thus, operators generated by integrating out $\phi$, stop revealing information about the length scale $m^{-1}$ as soon as the latter becomes shorter than $L_P$. Starting from this point, $\phi$ becomes less and less efficient probe of the short-distance physics and only carries information about the scale $R_\phi = (L_P^2 m)$ rather than $m^{-1}$.

We can now repeat the same analysis replacing the massless scalar graviton by the real spin-2 Einstein graviton, $h_{\mu\nu}$. The equation (14) is now replaced by the Einstein equation,

$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

which to the linear order in graviton can be written as (in harmonic gauge $\partial^\mu h_{\mu\nu} = \frac{1}{2} \partial_\nu h$):

$$\Box h_{\mu\nu} = -16\pi G_N (T_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} T^\alpha_\alpha)$$

where, $h \equiv h^\mu_\mu$. To the linear order in $G_N$ this gives a familiar result,

$$\frac{h^{(1)}_{\mu\nu}}{M_P} = \frac{1}{M_P} \frac{T_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} T}{\Box},$$

which for a static point-like source $T_{\mu\nu} = \delta^0_\mu \delta^0_\nu M \delta(r)$ becomes,

$$\frac{h^{(1)}_{\mu\nu}}{M_P} = \delta_\mu^\nu \frac{R}{r}.$$ 

The horizon corresponds to a distance $r$ for which the above contribution becomes order one. In the same time, all the higher order (in $G_N$) contributions, by consistency, become equally important, and the series have to be re-summed.

Diagrammatically, these corrections correspond to the processes when multiple gravitons emitted by the source interact non-linearly. This is equivalent to solving the Einstein equation in the given order in $G_N$, which effectively takes into the account self-sourcing of graviton by its energy-momentum tensor. For example, to second order in $h_{\mu\nu}$ we have,

$$8\pi G_N T_{\mu\nu}(h) = -\frac{1}{2} h^{\alpha\beta} \partial_\mu \partial_\nu h_{\alpha\beta} + ....$$
Evaluating this for the linearized graviton contribution $h_{\mu\nu}^{(1)}$, we get the corrections to the metric in the second order in $G_N$. For example,

$$\frac{h_{00}^{(2)}}{M_P} = \frac{1}{2} \frac{R^2}{r^2}, \quad \frac{h^{(2)}}{M_P} = -\frac{1}{2} \frac{R^2}{r^2}. \quad (22)$$

Again, these corrections confirm, that the horizon is at $r = R$. Beyond this point, the expansion in $G_N$ is no longer valid and the series have to be re-summed.

The effect of the massive scalar graviton $\phi$ is, that $h_{\mu\nu}^{(2)}$ gets corrections also from the coupling to the energy momentum of $\phi$,

$$T_{\mu\nu}(\phi) = \partial_{\mu}\phi\partial_{\nu}\phi - \frac{1}{2} \eta_{\mu\nu}(\partial_{\alpha}\phi\partial^{\alpha}\phi + m^2 \phi^2) + \ldots. \quad (23)$$

This has to be evaluated on the first order solution $\phi^{(1)} = e^{-mr}(R/r)$, and obviously gives an exponentially-suppressed contributions to $h_{\mu\nu}^{(2)}$.

A more important, power-law suppressed, corrections can also appear if there are couplings between $\phi$ and $h$ of the form

$$\frac{\phi\partial^n h^k}{M_P^{n+k-3}}, \quad (24)$$

(gauge invariant contraction of indexes is assumed). Just as in the scalar example case, after integrating out $\phi$, we will induce corrections to the effective metric,

$$g_{\mu\nu} = \eta_{\mu\nu} + \frac{h_{\mu\nu}}{M_P} + \eta_{\mu\nu} \frac{(\partial^n h^k)}{M_P^{n+k-3}} + \ldots, \quad (25)$$

which after being evaluating on the solution for $h$ will give power low corrections to the long-distance metric. Although not playing a significant role in long distance gravity, these corrections can certainly be measured and serve as a probe of short distance physics, as long as $m < M_P$.

However, for $m \gg M_P$ the new degree of freedom is no longer a perturbative state, but a macroscopic BH, and belongs entirely to large-distance sector of the theory. It becomes a classical BH of horizon $R_\phi = m/M_P^2 \gg L_P$.

Again, $\phi$ now has to be considered as a sequence of sources that emit arbitrary number of gravitons that merge in non-linear vertexes. These corrections contribute powers of $(R_\phi/r)$ to the metric, which have to be re-summed at $r \sim R_\phi$.

This is the diagrammatic indication of the non-perturbative fact that $\phi$ is a BH and develops horizon. Thus, at this point $\phi$ can no longer be regarded as the propagating quantum degree of freedom, and integration over $\phi$ has to be performed as the integration
over a classical object. Again, as in the examples considered earlier, the interaction vertex between $\phi$ and other quantum propagating degrees of freedom now has to be understood as an effective vertex controlling the quantum decay (i.e., Hawking evaporation) of a classical BH into the quantum particles in question. For example, any vertex of the form \[ 24 \] now describes the evaporation of $\phi$-BH into $k$-number of massless gravitons, \[ \phi \rightarrow k - \text{number of gravitons}. \] (26)

Since this process describes a quantum decay of a semi-classical thermal object of temperature \( T = M_p^2/m \) in $k$-number of quantum particles of energy \( m \gg T \), the rate of this decay must be exponentially-suppressed by the Boltzmann factor $e^{-(m/T)}$. This suppression factor has to be included in the effective strength of the vertex.

Thus, in any process in which $\phi$ appears as an internal virtual state, the contribution is exponentially suppressed at least by $e^{-\left(R_\phi m\right)}$. At this point, running $\phi$ in a virtual line is no any different than running any other classical BH in the same line.

To summarize, operators that we can obtain by integrating out $\phi$ cannot be any different from what we would obtain by integrating out an ordinary classical BH of the same mass. In other words, by becoming trans-Planckian, $\phi$ stopped to be a quantum degree of freedom and became classical, with the minimal size given by $R_\phi$.

5 Difference of Gravity from Other Non-Renormalizable Interactions

From the above reasoning it is clear that because of BH barrier the trans-Planckian region of Einstein gravity is equivalent to the deep-IR region. The maximal information that can be extracted from any sub-Planckian distance $L$ cannot exceed the information carried by a classical BH of size $L_p^2/L$. In this way, Einstein gravity is self-UV-complete.

In order to stress the profound uniqueness of gravity, let us compare it to any other non-renormalizable interaction. Consider, for instance, the interaction of the Nambu-Goldstone bosons in $O(n)$ sigma model, with the following action,

\[ V^2 \partial_\mu O(x)^T \partial^\mu O(x), \] (27)

where $O(x)_a \equiv O(x)_{ab}n_b$ denotes an arbitrary local $O(n)$-transformation acting on a constant fundamental $n$-vector $n_a \equiv (0, 0, ....1)$ ( with $a, b = 1, 2,...n$) and $V$ is the
scale. The angular degrees of freedom represent Nambu-Goldstone bosons, which after canonical normalization acquire derivative interactions suppressed by the scale $V$. These derivative interactions are seemingly similar to gravity. First, they both decouple at low energies, and become strong at the scale $V$ above which the perturbative unitarity is violated. So naively, the scale $V$ for Nambu-Goldstone bosons plays the role which is similar to the one that $M_P$ plays for gravity. But, the difference between the two cases is fundamental. In contrast with gravity, in the above theory nothing prevents us from probing distances shorter than the scale $1/V$.

In order to restore the consistency above the scale $V$, we need to integrate in a new radial degree of freedom, by allowing the absolute length of the unit vector to fluctuate. In the other words we promote the scale $V$ into a vacuum expectation value (VEV) of a fundamental scalar $\Phi(x)$ in the following way,

$$\Phi_a(x) = \left(1 + \frac{\rho(x)}{V}\right)\mathcal{O}(x)_a,$$

(28)

where $\rho(x)$ is the radial mode. In this way, the $O(N)$ sigma model is promoted into a Nambu-Goldstone model with the spontaneously-broken $O(N)$-symmetry at the scale $V$,

$$\partial_\mu \Phi(x)_a \partial^\mu \Phi_a - \lambda(\Phi_a \Phi_a - V^2)^2.$$

(29)

The field $\Phi_a$ differs from the sigma model field only through the existence of the radial mode $\rho(x)$.

In sharp difference with gravity, the existence of the radial mode is crucial for restoring unitarity at all the energies above $V$. This is because in the case of the sigma-model there is no BH barrier, and physics can be probed down to arbitrarily short lengths.

In case of pure Einstein gravity, even if we introduce some new degrees of freedom right at $M_P$, these will not play any role in restoring consistency of the theory in deep-UV. This role is taken up by the massless graviton. Moreover, we want to stress, that even if we do not introduce any new degrees of freedom, some quantum particles will nevertheless appear around $M_P$. The existence of such states follows from the fact, that at the very last stage of evaporation BHs are essentially indistinguishable from quantum particles. But, again, these states play no role in deep-UV.
6 Quantum Particles of Planck Scale Mass in Spectrum of Einstein Gravity

We wish to point out that Hilbert space of Einstein gravity contains quantum particle states with mass $\sim M_P$. Existence of such states is not an additional assumption, but is built-in in Einstein gravity. Their presence follows from the existence of classical BHs.

Let us consider Einstein gravity at large distances. This sector of theory contains classical BHs of mass $M \gg M_P$. The half evaporation time of these objects is given by

$$\tau_{BH} = c L_P (M L_P)^3,$$

where $c$ is a numerical constant. For us, the important fact is that $c$ is sufficiently larger than one, which implies that semi-classical black holes live much longer than their inverse mass. The black holes with the Schwarzschild radius $R \gg L_P$ are classical objects, since their Schwarzschild radius exceeds the Compton wave-length $M^{-1}$. We shall only be interested in BHs that do not carry excessive charges (such as an electric charge) that could stabilize them in the classical region.

Let us parametrically decrease the mass $M$. Once $R \sim L_P$, the BH crosses into the quantum region, since its Compton wavelength exceeds the Schwarzschild radius. At this point, the semi-classical description of the BH breaks down, and it has to be treated as a quantum state. Of course, in this regime the eq(30) is no longer applicable, but by continuity, the balance between the mass and the decay width should be maintained. In other words, because in the semi-classical domain there is a strong hierarchy between the decay width and the mass,

$$\Gamma_{BH} = \tau_{BH}^{-1} \ll M,$$

it is parametrically impossible to cross over from the semi-classical long-lived state directly into a quantum broad resonance, without passing an intermediate stage of a sharp quantum resonance. This intermediate state corresponds to a quantum particle of mass $\sim M_P$. To make a more precise estimate is hard with the current knowledge of properties of the micro BHs. However, for us the important thing is the very fact of existence of such quantum states.

One may wonder, how robust is the existence of quantum states around $M_P$. For example, what if evaporating BHs either never reach the Planck mass, or cross over and
continue existence with masses $\ll M_P$? In fact, none of the above is possible in Einstein gravity in which only propagating degree of freedom below $M_P$ is a massless graviton.

First, in Einstein gravity there always will be BHs that will reach the $M_P$ mass in their evaporation process. The BHs with size $R \gg L_P$ are in the classical regime and their properties are well understood. Such a BH can only stop evaporation if it becomes an extremal state. That is, its charge $Q$ (under some gauge symmetry that must be the part of the IR sector of the theory) has to become equal to its mass measured in $M_P$-units. Thus, only the BHs with excessive charge ($Q \gg 1$) can be stabilized in trans-Planckian mass region, $M = Q M_P \gg M_P$. Notice, that this charge must be pre-existing, and cannot be acquired in the evaporation process. This is because the neutral semi-classical BHs evaporate democratically in particles and anti-particles and thus cannot accumulate any net charge in the evaporation process. Thus, any BH of sufficiently small charge is bound to shrink down to the Planck size.

Now let us ask if a BH could cross over and continue existence with mass $M \ll M_P$. This can only happen, if in the IR region of the theory there is a quantum state to which the BH can evolve. For example, an electron can easily be an end result of evaporation a BH of unit electric charge and $1/2$ spin, but this is a triviality, since the corresponding quantum state, the electron, is already part of the IR spectrum of the theory. In other words, the end results of the BH evaporation cannot add any new state to the IR region of the theory. Thus, in pure Einstein gravity the new quantum states appear only around $M_P$.

The reason why we were able to deduce the existence of the above quantum states in pure Einstein gravity is the UV-IR connection. This connection follows from the fact that any perturbative degree of freedom whenever pushed into the trans-Planckian region bounces back to an IR sector of the theory in form of a classical BH.

In other words, in gravity, although heavy states decouple from the quantum processes, they inevitably become part of the classical IR sector of the theory.

In contrast, in ordinary field theories the heavy perturbative states simply decouple without having any IR counterparts.

In order to understand this profound difference, consider the example of an $O(3)$ sigma model described by the action (27). In this theory there are no classical object that indicate existence of new degrees of freedom at the scale $V$. Of course, there are classical solutions, but they do not correspond to a massive limit of perturbative states. Here
we are not talking about correspondence between the solitons and perturbative states in terms of electric/magnetic type duality. We are interested in perturbative and classical states that cross to each other within the same weakly-coupled description.

For instance, the $O(3)$ sigma model admits a monopole solution,

$$n_a = \frac{x_a}{r}, \quad (32)$$

where $x_a$ are cartesian coordinates, and $r$ is the radial one. But, monopole carries no information about existence of any massive quantum degree of freedom around the scale $V$.

The same is true in the gauged version of the theory. Gauging of non-linearly realized $O(3)$ symmetry introduces a massive $W_\mu$-boson and a massless photon. The configuration now acquires an $U(1)$-magnetic charge, and a mass $M_{\text{mon}} = \frac{M_W}{g^2}$, but again, this classical solution carries no information about the existence of extra degrees of freedom. For instance, from the existence of the magnetic monopole of the finite mass, we can certainly deduce the existence of the $W$-bosons, but this is only because both the monopole and the $W$ boson probe exactly the same length-scale, the Compton wave-length of the $W$-boson. The monopole is just a coherent state of $W$-bosons. However, from the existence of monopole, we cannot deduce the existence of a heavy Higgs particle (28).

The reason is that Higgs particle decouples without leaving any classical trace in IR. If we make it arbitrarily heavy, Higgs will probe arbitrarily short distances, and will decouple from classical IR.

Such a decoupling is impossible in gravity, since by making a quantum state heavy, we will eventually make it classical and vice versa, by making a classical BH lighter we will sooner or later cross in the quantum regime. Because of this fundamental property, at the crossover of the two domains the presence of heavy quantum species is inevitable. These are the states with masses $\sim M_P$, existence of which is built-in in Einstein gravity.

7 Beyond Einstein: Generalizing the Notion of the Planck Length

We now wish to extend our analysis to the theories that include new gravitational degrees of freedom on top of the Einsteinian massless graviton, and derive a criterion of
deep-UV completion of such theories by the BH barrier.

### 7.1 What is Gravity?

Quantum field theories are fully characterized by the propagating degrees of freedom and by their interactions. Here we shall be interested in theories that include only gravitational degrees of freedom. We shall define the latter as the propagating degrees of freedom that are sourced by the conserved energy momentum tensor $T_{\mu\nu}$ and which in the classical limit and for localized sources amount to the local deformations of an asymptotically-flat metric $g_{\mu\nu} = \eta_{\mu\nu} + \bar{h}_{\mu\nu}$. By conservation of the source, to the linear order these include only spin-0 and spin-2 fields. We shall therefore restrict ourselves by considering theories with maximal spin equal to 2. Thus, a small metric perturbation around the flat background $(\bar{h}_{\mu\nu})$ in such theories will contain not only the massless spin-2 state, $h_{\mu\nu}$, but also an arbitrary number of massive spin-2 ($h_{\mu\nu}^i$) and spin-0 ($\phi_{\mu\nu}^j$) fields:

$$\bar{h}_{\mu\nu} = \frac{1}{M_P} \left( h_{\mu\nu} + \sum_i c_2(i) h_{\mu\nu}^i + \sum_j c_0(j) \phi_{\mu\nu}^j \right). \tag{33}$$

Possible spin-1 fields cannot couple to the conserved sources to the linear order, and therefore do not appear in the above decomposition. However, they can participate in non-linear interactions.

The one-graviton exchange amplitude among the two sources $T_{\mu\nu}$ and $t_{\mu\nu}$ is highly restrictive and takes the following form,

$$T^{\mu\nu} \langle \bar{h}_{\mu\nu} T_{\alpha\beta} \rangle t^{\alpha\beta} = \frac{1}{M_P^2} \left( \frac{T_{\mu\nu} t_{\mu\nu}}{p^2} - \frac{1}{2} T_{\mu\nu}^{\alpha\beta} T^{\alpha\beta}_{\mu\nu} - \frac{1}{3} T^{\mu\nu}_{\mu\nu} \left( \frac{T^{\alpha\beta}_{\mu\nu}}{p^2} + \frac{m_i^2}{p^2 + m_i^2} \right) \right), \tag{34}$$

where we have separated the contributions from massless spin-2, massive spin-2 and spin-0 poles. We have normalized the relative strengths to the one of a zero mode graviton. In case of continuum, the discrete sum has to be replaced by the integral. The crucial point is, that all the spectral densities $\rho_2(i) \equiv |c_2(i)|^2$ and $\rho_0(j) \equiv |c_0(j)|^2$ must be semi-positive,

$$\rho_2(i) > 0, \quad \rho_0(j) > 0, \tag{35}$$

in order for the theory to be ghost-free. This structure uniquely defines the linearized metric produced by an arbitrary source. For instance, the gravitational potential produced
by a localized non-relativistic mass $T_{\mu\nu} = \delta^0_{\mu} \delta^0_{\nu} M \delta(r)$ is give by,

$$\bar{h}_{00}(r) = \frac{M}{M_P r} \left( \frac{1}{2} + \frac{2}{3} \sum_i \rho_i(m_i) e^{-m_i r} + \sum_j \rho_0(m_j) e^{-m_j r} \right).$$

(36)

What would be an analog of $L_P$ as of the shortest observable distance in such a theory? Without the knowledge of non-linear interactions it is impossible to answer this question. However, we shall formulate a sufficient condition for the existence of such a length scale in terms of the strong coupling scale of gravitational degrees of freedom. Let $\Lambda_{str}$ be a lowest energy scale at which some of the gravitational degrees of freedom become strongly-coupled. Notice, that due to the very constrained tensorial structure and the positive-definiteness of the spectral functions, the strength of linearized gravity can only grow at short scales. Due to this, $\Lambda_{str} = M_P$ is the upper bound, since even if all the massive degrees of freedom remain weakly-coupled, the Einsteinian massless graviton becomes strongly coupled at the Planck energies. So let us consider the case $\Lambda_{str} < M_P$. This means that (some) interactions become strong at the scale $\Lambda_{str}$, so that in naive perturbative approach, theory requires an UV completion at the distances $L \ll 1/\Lambda_{str}$. So let us ask the question, under what circumstances the BH barrier shields such distances and UV-completes the theory?

The criterion can be formulated in terms of the BH properties in the following way. First, since we are interested in theories that in deep-IR flow to Einsteinian gravity with the only propagating degree of freedom being a massless spin-2, we shall assume the existence of a mass gap $M_c \equiv 1/R_c$ corresponding to a first massive excitation in expansions and . Later, in some examples, we shall consistently take the continuum limit, $R_c \to \infty$, but for a moment we shall keep the gap finite.

We thus have a hierarchy of scales,

$$L_P \lesssim 1/\Lambda_{str} \lesssim R_c.$$  

(37)

In such a theory, at distances $r \gg R_c$, the only propagating degree of freedom is a massless graviton and gravity is pure Einsteinian, with all the usual properties. In particular, in such a theory there must exist Schwarzschild BHs of radius $R \gg R_c$. The mass-to-radius dependence for such a BH is given by the usual Einsteinian relation, $R(M) = M L_P^2$. Now, since $R \gg L_P$, we have $R \gg M^{-1}$. In other words, by taking $M$ large, the Schwarzschild radius of such a BH can be made arbitrarily larger than its Compton wavelength.
Now, let us start decreasing $M$ parametrically. Of course, for $R(M) > R_c$ we are in Einstein’s gravity and $R(M)$ decreases linearly with $M$. Once $R(M)$ drops below $R_c$, we are no longer in Einsteinian regime and dependence of $R(M)$ on $M$ can change, in general.

If $R(M)$ and $M^{-1}$ meet for some $M = M_*$, then the (first) meeting point,

$$R(M_*) = M_*^{-1}, \quad (38)$$

marks the start of the BH barrier. The corresponding length scale $L_* \equiv M_*^{-1}$ is the shortest observable length of nature. This scale plays the same role as the Planck length plays in Einstein gravity.

The measurement attempted at any shorter scale $L \ll L_*$, will result into the formation of a classical BH of size $R(1/L) > L_*$. For example, in Einstein gravity, the relation is $R(M) = M L_P^2$ and the meeting point is $M = M_P$.

Thus, the whole issue, when is gravity able to UV-complete a strongly-coupled interaction, is reduced to the question, whether $R(M)$ and $M^{-1}$ meet before $R(M)$ crosses with $1/\Lambda_{\text{str}}$. That is, whether $M_* < \Lambda_{\text{str}}$. Obviously, the physically-observable strong coupling scale $\Lambda_{\text{str}}$ can only be at or below the meeting point $M_*$, but never above. The meeting point marks the beginning of the BH barrier, and any coupling that formally gets strong above this energy is shielded by the BH physics.

On the other hand, if $M_* \gg \Lambda_{\text{str}}$, interactions become strongly-coupled way before the BH barrier can interfere and restore consistency. In such a case, theory requires an independent UV-completion above the scale $\Lambda_{\text{str}}$.

To summarize, the BH barrier restores consistency in a strongly-coupled theory as long as,

$$M_* < \Lambda_{\text{str}} \quad (39)$$

This criterion can be re-formulated in terms of holography. For this, in any theory that satisfies standard energy-positivity conditions we can define a shortest length scale $L_*$ on which one can store (and retrieve) information. Since the storage of a single information bit on a scale $L$ costs energy $1/L$, the scale $L_*$ is the minimal wavelength that exceeds its Schwarzschild radius, which is the same as $L_P$. In Einstein $L_* = L_P$. Now whenever in a given theory $\Lambda_{\text{str}}^{-1} \gg L_*$, such theory requires UV completion at the scale $\Lambda_{\text{str}}$ and cannot be saved by BHs.

Let us now consider some examples.
7.2 Kaluza-Klein Theory

An example of UV complete gravity that satisfies relation (39) is provided by Kaluza-Klein theories. Consider for example a 5-dimensional theory compactified on a circle of radius $R_c$. As it is well-known, from the 4-dimensional point of view this is a theory of the tower of massive spin-2 states. The spectral decomposition (34) in this case takes the form,

$$T^{\mu\nu} \langle \tilde{h}_{\mu\alpha} \tilde{h}_{\alpha\beta} \rangle t^{\alpha\beta} = \frac{1}{M_P^2} \sum_n \frac{T^{\mu\nu} t^{\mu\nu}}{p^2 + n^2/R_c^2}.$$

(40)

Although each KK graviton couples by $1/M_P$ suppressed interaction, because of multiplicity the strong coupling universally happens at the scale $\Lambda_{\text{str}}^3 = M_P^2/R_c$, which is simply a 5-dimensional Planck mass. The same scale sets the crossing point for the BH Schwarzschild radius $R(M)^2 = M R_c/M_P^2$ and its inverse mass. Theory contains no other strong coupling scale, and because of this, distances shorter than $L_5 \equiv (R_c/M_P^2)^{-1/3}$ cannot be probed, in principle. Theory is deep-UV-complete. This result is not surprising since in deep-UV the KK theory is just a five dimensional pure-Einstein gravity.

Similar properties must be shared by pure supergravity theories in $D$-dimensions since by supersymmetry the only strong coupling scale for all degrees of freedom is a $D$-dimensional Planck scale.

7.3 Examples Not UV-Completed by Black Hole Barrier

We shall now consider examples that cannot be UV-completed by Einsteinian BHs, and require some additional physics in order to restore consistency at short distances. Essentially any physics for which the probe of strong coupling is not accompanied by BH formation serves as such an example.

For instance, consider a non-linear sigma model with the scale $\Lambda_{\text{str}} \ll M_P$ coupled to Einsteinian gravity. The ordinary pions coupled to gravity would serve as simplest realistic example of this sort. Obviously, pion interactions are getting strong at energies above the pion decay constant $f_\pi \sim \text{GeV}$, and theory requires UV-completion at distances $\ll f_\pi^{-1}$. However, Einsteinian gravity cannot provide such an UV-completion, since distances $\ll L_\pi$ can be probed without encountering any BH barrier. Thus, such a theory requires an independent physics for UV-consistency, and as we know, QCD provides one.

Another example of the same sort is given by the electroweak non-abelian gauge field with a hard mass, $M_W$. As it is well known, scattering of longitudinal $W$-bosons
becomes strong at energies above $M_W/g_W$, where $g_W$ is the gauge coupling. Again, for $M_W/g_W \ll M_P$, such a theory cannot be UV-completed by Einsteinian gravity, and requires additional physics, such as the Higgs field (however, see the discussion in the outlook section, about possibility of completing by KK gravity).

In both above examples, the UV-incomplete physics comes from non-gravitational dynamics. We shall come back to this issue in more details later. Now we wish to provide an example in which UV-incomplete physics comes from gravitational degrees of freedom. That is, the degrees of freedom sourced by the energy momentum tensor.

An example of the above sort is a theory which on a Minkowski background together with Einstein’s graviton $h_{\mu\nu}$ propagates an additional scalar, $\phi$. The non-linear Interactions of graviton are fixed by the general covariance and are controlled by $M_P$. However, let us assume that the scalar possesses a self-coupling of the following sort,

$$\frac{1}{\Lambda_{str}^3} \Box \phi (\partial \phi)^2,$$  \tag{41}

where $\Lambda_{str}$ is some scale that can be taken arbitrarily smaller than $M_P$. The scalar of the above sort appears in the model of [15]. However, the model we are considering now is not this theory, which would be much harder to analyze in the present context. Rather, the above theory is a simplified prototype, which is substantially different. The difference is, that in the original model of [15] $\phi$ is not an independent scalar, but rather a helicity-zero polarization of a massive spin-2. The different helicities decouple only in a very special limit [18, 20], in which $M_P$ is taken to infinity. Since we don’t want to take such a limit, and moreover we wish to keep the graviton massless, we therefore introduce $\phi$ as an independent field, coupled to Einstein gravity. The only similarity we borrow from the model of [15] is the self-coupling [11]. The reason why we choose such a form of the interaction is, that on one hand it is becoming strong at the scale $\Lambda_{str}$, and on the other hand theory is ghost-free on the Minkowski background. Thus, such a theory in IR seems to be perfectly consistent. In order to see what is happening in UV, we need to recall few properties of the classical solutions.

Notice, that in such a theory a static localized source of mass $M$, produces two types of gravitational radii. First is the usual Schwarzschild radius $R(M) = ML_P^2$, which in case of a BH marks the horizon. But in addition [17, 19] there is a second scale, the so-called Vainshtein radius [22],

$$R_V(M) = \Lambda_{str}^{-1} \left( \frac{R(M)}{L_P} \right)^{1/3}.$$  \tag{42}
The physical meaning of the above radius can be read from the spherically-symmetric solution for $\phi$, which in the limit of the decoupled Einstein gravity can be found exactly and takes the form \[20\],

$$\partial_r \phi(r) = \frac{\Lambda_{\text{str}}^3}{4r} \left( \sqrt{9r^4 + \frac{1}{2\pi} R_V^3 r^3} - 3r^2 \right).$$

From this expression it is clear that Vainshtein radius marks the place where non-linearities in $\phi$ become important. For $r \gg R_V$, we have $\phi(r) \propto R/r$, whereas for $r \ll R_V$ the non-linear interaction takes over and we have $\phi(r) \propto \Lambda_{\text{str}}^3 R_V^{3/2} \sqrt{r}$. Crudely speaking, $R_V$ plays the role for $\phi$ somewhat analogous to the Schwarzschild radius for Einstein graviton.

However, there is a crucial difference. The Vainshtein’s radius is not a horizon. So information can be readily retrieved from beyond the $R_V$-sphere, without encountering any obstacle. This is the source of the problem, since the strongly coupled region can be experimentally probed and is not protected by the BH barrier.

Indeed, we can perform a scattering experiment that probes distances $\ll 1/\Lambda_{\text{str}}$. Again, we have to localize energy $\sim \Lambda_{\text{str}}$, within the distance $1/\Lambda_{\text{str}}$. The Schwarzschild and Vainshtein radii of this localized energy are $R(\Lambda_{\text{str}}) = \Lambda_{\text{str}} L_P^2$ and $R_V(\Lambda_{\text{str}}) = \Lambda_{\text{str}}^{-1} (\Lambda_{\text{str}} L_P)^{1/3}$ respectively. So we see, that both radii are much smaller than the size of the region in which the probe energy is spread,

$$R(\Lambda_{\text{str}}) \ll \Lambda_{\text{str}}^{-1}. \tag{44}$$

Thus, Einsteinian BHs are powerless in preventing the access to the distances $\ll \Lambda_{\text{str}}$. As a result, the theory as it stands is strongly-coupled at such energies and requires UV completion by some non-Einsteinian physics.

Before abandoning the above example, we wish to stress the following subtlety. In concluding that the above theory requires UV-completion, we were assuming that distances $\ll \Lambda_{\text{str}}^{-1}$ can be probed by sources external to $\phi$. What, happens if such probes are forbidden, that is if we decouple $\phi$ from all the other degrees of freedom, is a separate question and requires an independent investigation.

8 Gravity and Species

We have argued that Einsteinian (super)gravity and its KK extensions are self-complete in deep-UV. Why is this not an end of the story? In fact, the reason why we cannot
declare the success in producing a realistic model of UV-complete theory of gravity is the existence of non-gravitational particle species with the mass much below $M_P$. Such are the species of the Standard Model, and the problem is their consistent coupling to gravity. It turns out, that interactions of species with gravity dramatically affects gravitational dynamics \cite{10,12}. We shall now discuss this phenomenon.

Consider Einstein gravity in $D$ space-time dimensions, in which gravitational interaction is mediated by a $D$-dimensional massless particle of spin-2, the graviton $h_{\mu\nu}$. The corresponding $D$-dimensional Planck mass we shall denote by $M_D$. As we have discussed, the shortest observable distance in such a theory is given by $D$-dimensional Planck length $L_D \equiv M_D^{-1}$, and this fact makes the theory self-complete in deep-UV. Let us now try to couple this theory to $N$ particle species. For simplicity we shall take the species to be light. As we shall see, this seemingly-innocent deformation of the theory dramatically affects the gravitational dynamics. In fact, in the presence of light species, it is no longer consistent to assume that gravity is mediated by a single massless graviton, but rather the new gravitational degrees of freedom must be introduced necessarily \cite{10,12}. These degrees of freedom are necessary in order to UV-complete theory at the new fundamental scale, $L_N$, which is larger than the $D$-dimensional Planck length $L_D$,

$$L_N \equiv N^{1/D} L_D.$$ \hfill (45)

We shall refer to $L_N$ as the species scale.

Effect of species on gravity was also considered in some perturbative \cite{13} and cosmological \cite{14} contexts.

For the detailed proofs we refer the reader to the original papers \cite{10,12}. Here we shall reproduce the argument of \cite{11}.

This argument is based on impossibility of resolving species identities at the length scales shorter than $L_N$. This is a fundamental obstacle created by gravity. For a physicists, existence of $N$ distinct particle species means that he/she can label them, at least in principle, and further distinguish these labels by physical measurements. Let $\Phi_j$ be the particle species and $j = 1, 2, ..., N$ be their labels. Let us now show, that resolving these labels at distances beyond $L_N$ is fundamentally impossible. The reasoning is similar to why one cannot resolve distances beyond $L_P$ in theory without species, with the difference that the existence of species forces this minimal length to grow.

Indeed, any process of decoding the label of an unknown particle localized within the
space-time box of size $L$, involves comparing the unknown particle with all $N$ different sample species. Thus, any such measurement must involve localization of $N$ various species (or of the equivalent information) within the same box.

This fact automatically limits the size of the box from below. Indeed, localization of each sample particle within the size $L$ costs energy $E = 1/L$, which implies that the total energy localized within the region $L$ is at least $E_{Total} = N/L$. The corresponding Schwarzschild radius then is,

$$R(E_{Total})^{D-3} = \frac{N}{L} L_D^{D-2}.$$  \hspace{1cm} (46)

The key point is, that species measurement scale $L$ cannot be decreased arbitrarily, since eventually its Schwarzschild radius will exceed its own size, and localized species can no longer be resolved. Any further attempt to decrease $L$ will result into the creation of an even bigger BH. The critical size, beyond which the resolution of species is no longer possible is readily derived by equating the Schwarzschild radius to the localization size $L$,

$$L^{D-3} = R^{D-3} = \frac{N}{L} L_P^{D-2},$$  \hspace{1cm} (47)

which gives the bound on $L$ set by $L_N$. This constraint is very powerful. It tells us that gravity makes it impossible to resolve the species beyond the scale $L_N$, which means that the new gravitational degrees of freedom must enter into the game.

So far, we did not specify the nature of species. Now it is time to discuss this issue.

8.1 KK Species

The simplest case is when species have just right quantum numbers to fill the KK tower of new $d$ compact flat dimensions of radius $R_c$. In such a case everything falls nicely into the places and relation between $L_N$ and $L_D$ exactly reproduces the usual geometric relation between the $D + d$-dimensional ($L_{D+d}$) and $D$-dimensional Planck lengths,

$$L_N = L_{D+d} = L_D (R_c/L_{D+d})^d.$$  \hspace{1cm} (48)

The reason why $L_N$ came out equal to the $L_{D+d}$, becomes immediately clear if we notice that $N = (R_c/L_{D+d})^d$ is simply the number of KK species. It is obvious, that the fundamental length scale of the theory is $L_{D+d}$, and this is precisely what species counting tells us. As discussed previously, this theory has a single strong coupling scale, and is self-complete in deep-UV.
8.2 Non-Gravitational Species.

We shall now discuss the effect of non-gravitational species. Under this name, we shall refer to the light particles, with the quantum numbers and interactions that do not fill gravitational (super)multiplets. For instance, such are the Standard Model particles that cannot be identified with the fragments of pure high-dimensional supergravity compactified on a smooth manifold. For simplicity, let us consider theory with $N$ massless real scalar fields $\phi_j$, coupled to Einsteinian gravity in 4-dimensions, with the action,

$$\int d^4x \sqrt{-g} g^{\mu\nu} \partial_\mu \phi_j \partial_\nu \phi_j .$$

We do not put any specific requirement on the mass and interaction terms of the scalars, as long as they are light (with masses $\ll L_N^{-1} = M_p/\sqrt{N}$) and weakly coupled. For instance, scalars that inter-couple only through gravity would be sufficient. The above action describes a theory in which below Planck energy the only propagating degrees of freedom are the massless Einsteinian graviton $h_{\mu\nu}$ plus $N$ light weakly-interacting scalars. Seemingly, there is nothing wrong in considering the above action as an effective low energy theory below $M_p$ energies. However, as we know from the previous analysis, this theory as it stands is inconsistent. Namely, it is impossible to avoid the existence of the additional propagating gravitational degrees of freedom with the Compton wavelengths larger than $L_N$. Absence of such new degrees of freedom, would be in contradiction both with breakdown of BH semi-classicality, as well as with impossibility of resolving species identities at distances shorter than $L_N$. Thus, what we are learning is, that introduction of $N$ non-gravitational species alone is impossible. By consistency, such species must be accompanied by new gravitational species that make sure that gravity goes out of semi-classical regime at the scale $L_N$. Thus, the theory requires an UV-completion at the scale $L_N$. Can such a theory maintain the self-completeness properties of the pure-gravity?

Naively, from our previous experience such an UV completion looks pretty straightforward. We know, that by integrating in new KK gravitons, we can make $L_N$ to be equal to a fundamental Planck length of a higher dimensional theory. Then, we can expect that the high-dimensional gravity self-completeness itself, just in the same way as this happens in a pure-gravity theory. The problem, however, is the existence of $N$ zero modes, which do not come from the high-dimensional graviton multiplet. These degrees of freedom require pre-existence of the “parent” non-gravitational species in a high-dimensional theory, and the issue of UV-completion is lifted to the next level.
Let us explain the latter concern by considering an attempt of UV-completing the above scalar example by the BH barrier. As a first step, we can restore the consistency of the theory with the BH requirements, by adding $N$ KK gravitons and completing the theory to a $4 + d$-dimensional theory with the fundamental Planck length being equal to $L_N$. This step takes care of the consistency with the fact that fundamental length is $L_N$ rather than $L_P$. But the remaining issue is to fit $N$ zero mode scalars to the completed theory without jeopardizing the property that the strong couplings must be shielded by the BH formation. This is how the new challenge arises. The most straightforward possibility would be to promote our $N$ massless scalars into the zero modes of $N$ 4 + $d$-dimensional fields. But this will not work, since now the fundamental scale of 4 + $d$ dimensional theory has to be lower than the 4 + $d$-dimensional Planck length as

$$L_N^{(4+d)} = N^{\frac{1}{4+d}} L_{4+d}.$$  

(50)

So the issue of UV completion procreates.

We may attempt to introduce species not in form of the high-dimensional fields, but of zero modes that are localized on some branes. However, this again does not avoid the problem, since localized species count as much as the bulk ones [11]. Thus, it is a challenge to consistently introduce non-gravitational species, without creating a strong coupling scale below the Planck length.

9 The Role of String Theory

UV-IR connection exhibited by string theory is strikingly similar to UV-IR connection of pure Einstein gravity. This similarity between the deep UV-IR properties of the two theories suggests some intrinsic connection at the most fundamental level.

However, string theory introduces a new scale, the string tension scale, $M_s$. Veneziano-type softening of scattering amplitudes starts precisely at the string scale, which in a weakly coupled string theory can be arbitrarily lower than $M_P$. So, what is the role of string theory in UV-completion of gravity?

In this note, we shall limit ourselves by suggesting a possible line of thought in this direction. One idea is, that in fact string theory and Einstein gravity are non-separable. In other words, a string theory with order one string coupling $g_s \sim 1$ is built-in in Einstein gravity. To put it differently, by writing down Einstein’s action, we
are committing ourselves to a string theory. But, where are the string excitations coming from in pure Einstein gravity? In this approach, the first string massive excitations have mass $\sim M_P$, and these are precisely quantum states that are suggested by particle-BH transition discussed above. Heavier Regge resonances are simply classical states, and are indistinguishable from classical solutions of Einstein gravity such as heavy BHs or other classical states of IR gravity (e.g., loops of long strings).

Another idea is, that string theory is necessary for consistent coupling of Einstein gravity to particle species. This idea is supported by several findings. First, as we have seen, in the presence of species, a new scale, $L_N$, parametrically larger than $L_P$ inevitably appears. The role of this scale can be naturally played by the string length $l_s$. This matches the previous findings [11], that string theory is a theory of species with their effective number being $N = 1/g_s^2$.

The latter picture also suggests, that for $g_s \sim 1$ the difference between string theory and Einstein gravity is essentially erased. Since, the both descriptions predict existence of quantum degrees of freedom around $M_P$. These may be thought as the species arising as quantum limit of lightest BHs, or equivalently as first string resonances. Only if one needs to introduce many light species, one inevitably has to open up a finite energy window below $M_P$, and string theory becomes weakly coupled in order to accommodate this window. Next, we sketch a possible strategy to figure out how this could happen.

The BH induced UV/IR ”bounce”

$$L \rightarrow L' = \frac{L_P^2}{L}, \quad (51)$$

lead us to assume that the relevant quantum degrees of freedom controlling the self dual Planckian region are the quantum resonances of mass $M_P$ we get when the BH enters into a quantum regime. In order to make contact with string theory we shall make use of existence of these particle species. At a subplanckian scale $L'$ we will assume that the effective number of species is,

$$N(L') = \frac{L^2}{L_P^2}, \quad (52)$$

and therefore on the basis of the results of [11] the effective ”string coupling” will be given by $g(L') = \frac{1}{\sqrt{N(L')}}$ and the species scale by $L_s = \sqrt{N(L')L_P}$. Note, that, as it should be, in this deep UV region we get weak string coupling. If now we consider a state with mass $M = \sqrt{N(L')}M_P$ that is the mass of the effective BH we will create whenever we try to probe scale $L'$ and we write the Planck length in terms of the species scale, we
get a string mode with string entropy being the BH entropy. It is amusing to notice, that the transformation from $L_P$ into the species scale $L_s = \sqrt{N(L')}L_P$ is precisely the transformation on the Planck length induced by a string T-duality transformation \[51\]. Indeed if we interpret the UV/IR bounce \[51\] as a T duality, we will need to change the Planck length as,

$$L_P \rightarrow L'_P = L_P\left(\frac{L}{L_P}\right),$$

which gives precisely the species relation we have used.

In summary, once we assume that Einstein gravity by itself sets the bound on information storage, a string theory can be naturally built-in by identifying species identities as the information bits. These bits count elementary quantum states of mass $M_P$. In this frame of string theory as of theory of species, the string coupling flows in the UV to a weak coupling regime, reflecting the standard way used by string theory for completing gravity.

A final question that naturally appears is, if any theory satisfying holography - in the sense of possessing an absolute bound on information storage - should necessarily contain extra quantum states of mass $\frac{1}{L_H}$ with $L_H$ being the holographic scale (the shortest scale that can store a minimal information bit) as well as to enjoy some form of duality invariance under the UV/IR bounce $L \rightarrow \frac{L_H}{L}$.

10 On Self-UV-Completeness of 11-Dimensional Supergravity

An interesting evidence in favor of UV-completeness of pure-Einsteinian supergravity can be obtained if we combine our notion of the BH barrier with the fact that in a certain consistent decoupling limit string theory is reduced to a pure gravitational theory, with the Planck length much longer than the string length ($L_s$). We wish to suggest that the existence of such a decoupling limit, is an indication that the supergravity theory on its own is UV-complete.

This is a well-known example of the strong coupling limit of type IIA string theory \[25, 26\]. This example is analyzed in \[24\] from the point of view of the behavior of the species scale in strong coupling limit, and the resulting view is, that string theory becomes theory of gravity when other species decouple.
We shall not repeat all the details of this construction, but only the aspects that are crucial for our argument. The key feature is the existence of the consistent limit in which string theory decouples and the low energy theory is given by the theory of pure Einstein supergravity in 11-dimensions. As explained in [24], from the point of view of the remaining supergravity theory, the species scale in this limit becomes the 11-dimensional Planck length.

The decoupling of strings is achieved by taking the limit $g_s \to \infty$ and $L_s \to 0$, but keeping the 11-dimensional Planck scale, $M_{11}^9 \equiv M_s^9/g_s^3$, fixed. In this limit strings decouple, but the $D_0$-branes give rise to the perturbative states. The interpretation of these new perturbative states is, that they form the KK species of 11-th dimension that opens up in this limit. The radius of this 11-th dimension is,

$$R = g_s L_s .$$ (54)

The 10 and 11 dimensional Planck lengths satisfy the usual geometric relation,

$$L_{10}^8 = L_{11}^9/R .$$ (55)

The crucial fact is, that $L_{11}$ is much larger than the string length,

$$L_{11} = g_s^{1/3} L_s .$$ (56)

Since at distances larger than $L_{11}$ the theory is a pure-Einstein supergravity, in the light of our BH arguments, $L_{11}$ automatically becomes a shortest length scale of nature. In the other words, any measurement that attempts to retrieve the information at a length scale $L \ll L_{11}$ bounces us back to the deep IR physics, corresponding to the length $L_{11}^2/L$,

$$L \to \frac{L_{11}^2}{L} .$$ (57)

Now, since by design, there are no non-gravitational degrees of freedom until the scale $M_s$, which is infinitely above $M_{11}$, the 11-dimensional supergravity must be UV-complete on its own. In the other words, by becoming trans-Planckian the stringy physics automatically became unreachable, and the remaining gravitational physics has no choice other than completing itself by the available gravitational tools, the BHs.

Starting from a consistent UV-complete theory, the ten-dimensional string theory, we have obtained a pure-gravity theory, with the shortest scale $L_{11}$. As explained above, because $M_{11}$ is a boundary between the quantum and classical objects, certain quantum
degrees of freedom are necessarily "stuck" there. These quantum degrees of freedom, together with 11-dimensional graviton multiplet, must play the crucial role in self-completing the theory.

11 On Entropy Count

An independent issue although intimately connected is how to build -with the tools of pure gravity- a microscopic theory that accounts for the BH entropy. The standard answer to this question is that in order to reach this microscopic understanding of BH entropy we need to move into string theory and it is this claim/hope what normally leads to think of string theory as a way to UV complete gravity.

It is well known that the string length sets an absolute bound on physical space resolution [2]. This bound as well as the Hagedorn bound on temperatures is intimately related with the extended nature of strings. Moreover this property of strings becomes manifest with the discovery of T-duality. In string theory what prevent us to resolve small scales (or to exceed Hagedorn temperature) is that whenever we pump more energy we end up with a longer string probe. As we are pointing out the ancestor of this phenomena is already present in pure Einsteinian gravity, where we find that whenever we pump more energy to localize a probe below Planck scale we end up with a BH larger than the Planck length. Thus, independently of any other consideration, quantum gravity should possesses the Planck length as the minimal length.

In Einstein gravity, an immediate consequence of this is the existence of a gap between the massless graviton and the first excited state of the theory that should have mass equal to $M_p$. Moreover if we assume holography then the expression of the BH mass spectrum written in "information" terms, namely $M^2 = \frac{N}{L_p^2}$ seems strikingly similar to the standard Regge spectrum of string theory for $L_s = L_p$.

In spite of this strong similarity we do not need to jump, at least not too quickly, into a string UV completion of gravity where the first excited quantum state appears as a string vibration mode. As already pointed out, we can follow a different path to identify this state using just standard quantum field theory and assuming that Einsteinian gravity is complete in the IR and therefore able to fully describe large semiclassical black holes. Indeed we can start with a large BH and identify the first quantum excited state as the remnant of the semiclassical black hole evaporation.
String theory is avoiding this complicated path involving a classical/quantum transition and it is replacing it by a sort of RG flow in the string coupling. Indeed we can just take an arbitrary BH and reach the string state by simply continuously moving the string coupling $g$ appropriately (keeping $L_s$ fixed). By this flow in $g$, we smoothly “degravitate” the theory until reaching the point where the string shows up as the Wilsonian UV completion of the theory. In other words, by flowing in $g$ we effectively can move from an effective GR-IR regime (strong coupling in $g_s$) with $L_s << L_P$ into a stringy-UV regime (weak coupling in $g_s$) with $L_s >> L_P$. In this sense string theory defines an UV completion in the Wilsonian sense identifying as the UV degrees of freedom the string vibration modes.

How this string picture can fit with the Einsteinian requirement of having $L_P$ as the minimal observable physical length in nature? Part of the answer is contained in the string/BH correspondence [21] or in other words in the microscopic meaning of the BH entropy. In fact if we consider the BH mass written in ”information” variables as,

$$M_{BH} = \frac{\sqrt{N}}{L_P},$$

(58)

with the corresponding entropy $S = N$, the understanding of this entropy as string entropy simply requires to define $L_s$ as,

$$L_s = \sqrt{NL_P}.$$  

(59)

By doing so the BH mass becomes in ”string” variables $M_{BH} = \frac{N}{L_s}$, with string entropy $S = N$. It is in the transformation from ”information” variables $L_P$ into ”string” variables $L_s$ where we implicitly introduce a string coupling $g = \frac{1}{\sqrt{N}}$. However as we have argued in reference [24], the previous transformation is simply the definition, within pure Einsteinian gravity, of the species scale for a number $N$ of quantum species. In other words, pure Einsteinian gravity by setting $L_P$ as the minimal physical length is also setting the bound on information storage and fixing the ”information” variables in terms of which the spectrum of BH masses organizes itself in a sort of Regge trajectory. This Regge trajectory becomes stringy once we move into the ”species frame”, with the species scale defined relative to a number of species equal to the amount of information. The string flow in $g_s$ that was crucial to the stringy Wilsonian UV completion of gravity becomes a physical flow in information. In pure Einstein gravity this flow in information in encoded in the quantum BH evaporation process.

We can extract some general lessons from the previous picture. In particular, we can
imagine as a general feature of any theory that sets by itself the bound on information storage, let us say $L_H$, that mass spectrum could be organized in ”information” variables as Regge trajectories. However, the actual decoding of a given amount of information sets the corresponding species scale leading to a string frame with the flow in $g_s$ the flow in information. It seems that Einstenian gravity contains the essential ingredients of this general picture.

12 Outlook

To summarize, we have argued that the Planck length is the absolute shortest length-scale of nature, and any attempt of probing the physics beyond it automatically bounces us back to the physics at large distances. The maximal physical information $I_{\text{max}}(L)$ that can be extracted in any measurement at distance $L \ll L_P$, is bounded by the information contained at the horizon of a classical BH of radius $L_P^2/L$,

$$I(L)_{\text{max}} = I(L_P^2/L),$$

and thus, is intrinsically IR in nature. This property indicates that Einstein gravity is self-UV-complete. Of course, the information stored in a classical BH obeys the holographic bound. But this is an intrinsic property of Einstein BHs rather than an extra assumption.

Regarding the role of string theory, the emerging conclusion is, that certain strong coupling limit of string theory is built-in in pure Einstein gravity, and that the role of weakly-coupled string theory is to consistently couple gravity to the other particle species, with their number being set by $N = 1/g_s^2$, and the species scale $L_N$ being set by $L_s$. String theory decouples together with species, and the resulting limit is a pure gravity theory. In other words, string theory is the UV-completion of theory of species coupled to gravity above the scale $L_N$ \[24\].

An interesting question is what is the connection (if any) of our results with the recent hints of possible perturbative finiteness of $N = 8$ supergravity theory \[27\]. A priory, since our argument is fully non-perturbative, no perturbative finiteness is necessary. From our perspective even if loop diagrams are badly divergent, some resummation that will guarantee the finiteness of the theory must take place. At the moment we do not see anything in our arguments that would demand cancellations at loop by loop level. However, it may be that the loop-by-loop finiteness is the way the theory perturbatively
reconciles itself with the existence of the BH barrier.

12.1 Non-Wilsonian “Higgless” UV-Completion by Gravity?

Finally, an interesting phenomenological question is, whether one can use gravity for the UV completion of the Standard Model without any need of a Higgs particle? For this, first one needs to add to Einstein gravity the extra gravitational degrees of freedom in order to stretch the fundamental Planck length \(L_\ast\) all the way to the electroweak distances. In such a case the strong coupling in scattering of longitudinal \(W\)-bosons will be UV-completed by classical BHs. This can certainly be achieved by introducing large extra dimensions, as it was already done for solving the hierarchy problem \[28\]. However, the question is not just lowering the scale \(M_\ast\), but whether one can get away without introducing the Higgs. This idea would be in spirit somewhat similar to the “Higgless” models \[29\], except the known Higgless models rely on Wilsonian completion of the theory, in which the Higgs particle is substituted by other quantum degrees of freedom that restore unitarity up to sufficiently high scales.

In contrast, our idea is to look for non-Wilsonian UV-completion in which in deep-UV the quantum degrees of freedom become classical states. To fix terminology, completion that we are trying to suggest will not be truly Higgless, in the sense that the Higgs-like degree of freedom will inevitably appear as the state at the boundary that separates classical BHs from quantum degrees of freedom. To be more precise, once we couple the standard model without the Higgs to gravity, the Higgs particle will appear automatically at the scale \(M_\ast\), as the latest stage of the evaporation of a BH with the quantum numbers of the Higgs doublet. Indeed, such a BH can always be formed by scattering quarks and leptons with appropriate gauge quantum numbers. The existence of a particle-like state in the spectrum will result from the latest stages of the evaporation, at which the BH is indistinguishable from a heavy particle. So in this sense, gravity automatically provides a composite Higgs in form of a quantum BH, even if initially there was no Higgs scalar in the spectrum of the SM species. In this respect this idea also shares some similarity with the idea of the top quark condensate \[30\].

Of course, there are many phenomenological questions that make it unclear if such a scenario can ever work, the electroweak precision parameters being an immediate concern. Another question is, why is the BH-type Higgs condensing? We shall postpone answering
these question for future.

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