A generalized solution of a modified Cauchy problem of class $R_2$ for a hyperbolic equation of the second kind

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Abstract. One of the main problems in the theory of partial differential equations is the study of equations of mixed type. The modified Cauchy problem for some values of $\alpha$ is stated and investigated. The modified Cauchy problem for some values of $\alpha$ is stated and investigated. A convenient representation of the generalized solution of the modified Cauchy problem is obtained.

1. Introduction

One of the main problems of the theory of partial differential equations is the study of equations of mixed type, which is of both theoretical and practical interest. The first fundamental research in this area was carried out by F. Tricomi [1] in the early twenties of the last century.

The mixed-type equations began to be studied systematically, after F.I. Frankl [2] pointed out their applications to the problems of transonic and supersonic gas dynamics. In this regard, the purpose of this work was to find out whether it is possible to find a more convenient form of representation of the solution of the Cauchy problem for a differential equation, with the help of which it would be possible to solve boundary value problems for a mixed type equation of both parabolic-hyperbolic and elliptic-hyperbolic types. The modified Cauchy problem for some values of $\alpha$ is stated and investigated. A convenient representation of the generalized solution of the modified Cauchy problem is obtained.

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2. Statement of the modified Cauchy problem and illustration of obtaining its solution

We study the equation

\[
0 = \begin{cases} 
\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial y} + a(x,y)u, & y \geq 0, \\
\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y}, & y < 0, 
\end{cases}
\]  

where \( a(x,y) \) - given function, and

\[ a(x,y) < 0, \quad \forall (x,y) \in D_1, \quad a(x,y) \in C^{(0,h)}(D_1), \quad 0 < h < 1. \]

in the region \( D=D_1 \cup D_2 \cup AB \), and the region \( D_1 \) is bounded for \( y>0 \) by the segments \( AB, BB_0, A_0B_0, AA_0 \) of straight lines \( y=0, x=1, y=1, x=0 \), respectively, and the region \( D_2 \) is bounded for \( y<0 \) characteristics of equation (2):

\[ AC: x - 2\sqrt{-y} = 0, \quad BC: x + 2\sqrt{-y} = 1; \quad AB: y=0, \quad 0 \leq x \leq 1. \]

Consider the equation

\[
0 = \begin{cases} 
\frac{\partial^2 u}{\partial x^2} - \frac{\partial u}{\partial y} + a(x,y)u, & y \geq 0, \\
\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial u}{\partial y}, & y < 0, 
\end{cases}
\]

in the region \( D_2 \), i.e.

\[ L_\alpha u = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \alpha \frac{\partial u}{\partial y} = 0, \quad y < 0. \]

Continuous solution of the modified Cauchy problem for the equation \( L_\alpha u = 0 \) in the domain \( D_2 \) for values \( \alpha \in (-3;2.5) \) with initial data

\[ u(x,0) = \tau(x) \]

\[ \lim_{y \to -0} (y)^{-\alpha} [u - A_\alpha(\tau)] = \nu(x) \]

in characteristic variables \( (\xi, \eta) \) has the form [28], [29]

\[ u(\xi, \eta) = \gamma_1 (\eta - \xi)^{-\beta-\gamma} \left\{ \int_\xi^\eta (t)(\eta - t)^{\beta+1} (t - \xi)^{-\gamma} \, dt + \frac{2}{(\beta + 3)(2\beta + 1)} \int_\xi^\eta \int_\xi^t (t)(\eta - t)^{\beta+3} (t - \xi)^{-\gamma} \, dt \, dt 
+ \frac{3}{4(\beta + 3)(\beta + 4)(2\beta + 1)(2\beta + 3)} \int_\xi^\eta \int_\xi^t \int_\xi^t (t)(\eta - t)^{\beta+4} (t - \xi)^{-\gamma} \, dt \, dt \, dt 
+ \frac{1}{8(\beta + 3)(\beta + 4)(2\beta + 1)(2\beta + 3)(2\beta + 5)} \int_\xi^\eta \int_\xi^t \int_\xi^t \int_\xi^t (t)(\eta - t)^{\beta+5} (t - \xi)^{-\gamma} \, dt \, dt \, dt \, dt \right\} + \right. \]

(4)
Continuous solution of the modified Cauchy problem for the equation \( L_2 u = 0 \) in the domain \( D \) for values \( \alpha \in (-3,5;3) \) with initial data
\[
\lim_{y \to -0} \left[ u(x,0) - A_4 \tau(x) \right] = v(x)
\]
in characteristic variables \((\xi, \eta)\) has the form \([28],[29]\)
\[
u(\xi,\eta) = \gamma_1 (\eta - \xi)^{-2\beta} \int_{\xi}^{\eta} \tau(t)(\eta - t)^{\beta+3}(t - \xi)^{\beta+3} dt + \\
+ \frac{2}{(\beta + 4)(2\beta + 1)} \int_{\xi}^{\eta} \tau(2)(t)(\eta - t)^{\beta+4}(t - \xi)^{\beta+4} dt + \\
+ \frac{3}{2(\beta + 4)(\beta + 5)(2\beta + 1)(2\beta + 3)} \int_{\xi}^{\eta} \tau(4)(t)(\eta - t)^{\beta+5}(t - \xi)^{\beta+5} dt + \\
+ \frac{1}{16(\beta + 4)(\beta + 5)(\beta + 6)(\beta + 2)(2\beta + 1)(2\beta + 3)(2\beta + 5)} \int_{\xi}^{\eta} \tau(6)(t)(\eta - t)^{\beta+6}(t - \xi)^{\beta+6} dt + \\
+ 1 \int_{\xi}^{\eta} \tau(8)(t)(\eta - t)^{-\beta}(t - \xi)^{-\beta} dt \equiv \gamma_2 \int_{\xi}^{\eta} \nu(t)(\eta - t)^{-\beta}(t - \xi)^{-\beta} dt = \\
= A_4 \tau(x) - 2^{2(\beta - 1)} \gamma_2 \int_{\xi}^{\eta} \nu(t)(\eta - t)^{-\beta}(t - \xi)^{-\beta} dt,
\]
\[
\tau(x) \in C^{(10)}[0,1]; \quad v(x) \in C^{(2)}[0,1], \quad \xi = x - 2\sqrt{-y}, \quad \eta = x + 2\sqrt{-y};
\]
\[
\gamma_1 = \frac{\Gamma(2\beta + 8)}{\Gamma(\beta + 4)}, \quad \gamma_2 = \left(1 - \frac{1}{2}\right)^{-1} \frac{\Gamma(2\beta - 2\beta)}{\Gamma(1 - \beta)}, \quad \beta = \alpha - \frac{1}{2}.
\]

**Definition.** The function \( u(\xi, \eta) \) defined by formula \((4) \) or \((5) \) is called a generalized solution of the equation \( L_2 u = 0 \) of class\([33]\) \( R \) in the domain \( D \) if the function \( \tau(x) \) can be represented in the form
\[
\tau(x) = \int_{0}^{x} (x - t)^{-2\beta} T(t) dt,
\]
where \( u(x) \) and \( T(t) \) are some continuous integrable functions on \((0; 1)\), also \( 6 < -2\beta < 7 \) and \( 7 < -2\beta < 8 \), respectively.

The generalized solution to the \( u \in R_2 \) is continuous in \( D_2 \), and the derivatives \( u_x \) and \( u_y \) are continuous in \( D_2 \), and the function \( u - A(\tau) \), \( i=3,4 \), is continuous up to the line of type change.

Based on (6), we find

\[
\begin{align*}
\tau^{(2)}(x) &= 2\beta(2\beta+1) \int_0^1 (x-t)^{-2\beta-2} T(t) dt, \\
\tau^{(4)}(x) &= 2\beta(2\beta+1)(2\beta+2)(2\beta+3) \int_0^1 (x-t)^{-2\beta-4} T(t) dt, \\
\tau^{(6)}(x) &= 2\beta(2\beta+1)(2\beta+2)(2\beta+3)(2\beta+4)(2\beta+5) \int_0^1 (x-t)^{-2\beta-6} T(t) dt, \\
\tau^{(8)}(x) &= 2\beta(2\beta+1)(2\beta+2)(2\beta+3)(2\beta+4)(2\beta+5)(2\beta+6)(2\beta+7) \int_0^1 (x-t)^{-2\beta-8} T(t) dt.
\end{align*}
\]

The proof of the representation of the generalized solution of the class \( R_2 \) for \( \alpha \in (-3, -2.5) \) is similarly investigated as a change in the parameter \( \alpha \in (-2.5, -2) \) [29].

3. Generalized solution results

Here we give a representation of the generalized solution [32-34] of class \( R_2 \) for \( \alpha \in (-3, -2.5, -3) \). Substituting the above equalities and (6) into (5), we have

\[
\begin{align*}
u(x, t) &= \int_0^t T(t) \left[ (\eta-t)^{\beta+3}(t-\xi)^{\beta+3}(t-\zeta)^{-2\beta} + \frac{4\beta}{\beta+4} (\eta-t)^{\beta+4}(t-\xi)^{\beta+4}(t-\zeta)^{-2\beta-2} + \frac{6\beta(\beta+1)}{(\beta+4)(\beta+5)} (\eta-t)^{\beta+5}(t-\xi)^{\beta+5}(t-\zeta)^{-2\beta-4} + \frac{4\beta(\beta+1)(\beta+2)}{(\beta+4)(\beta+5)(\beta+6)} (\eta-t)^{\beta+6}(t-\xi)^{\beta+6}(t-\zeta)^{-2\beta-6} + \frac{\beta(\beta+1)(\beta+2)(\beta+3)}{(\beta+4)(\beta+5)(\beta+6)(\beta+7)} (\eta-t)^{\beta+7}(t-\xi)^{\beta+7}(t-\zeta)^{-2\beta-8} \right] d\zeta dt - 2^{2(2\beta-1)} \gamma_1 \int_0^\eta \nu(t)(\eta-t)^{-\beta}(t-\xi)^{-\beta} dt \equiv \gamma_1(\eta-\xi)^{-2\beta-7} J_1 - J_2.
\end{align*}
\]

In the expression for \( J_1 \), we divide the interval of integration over \( \zeta \) into two, \((0, \xi)\) and \((\xi, t)\). Then, changing the order of integration, we get

\[
J_1 = \int_0^\xi I_1(\xi, \eta; \xi) T(\xi) d\xi + \int_\xi^\eta I_2(\xi, \eta; \xi) T(\xi) d\xi, \quad (7)
\]

where
To calculate these expressions, we use the integral representation of hypergeometric functions [31]:

\[
\int_\zeta^\eta (\eta - t)^{\alpha_1} (\eta - \zeta)^{\beta_1} dt = \frac{\Gamma(k+1)\Gamma(l+1)}{\Gamma(k+l+2)} (\eta - \zeta)^{k+l+1}(\eta - \zeta)^m \cdot (k+1,-m,k+l+2;\frac{\eta - \zeta}{\eta - \zeta})
\]

We have

\[
I_1 = \frac{\Gamma^2(\beta + 4)}{\Gamma(2\beta + 8)} (\eta - \zeta)^{2\beta} F(\beta + 4,2\beta,2\beta + 8;z) + \frac{2\beta}{2\beta + 9} z^2 F(\beta + 5,2\beta + 2,2\beta + 10;z) + \frac{3(\beta + 1)}{2(2\beta + 9)(2\beta + 11)} z^4 F(\beta + 6,2\beta + 12;z) + \frac{\beta(\beta + 1)(\beta + 2)}{2(2\beta + 9)(2\beta + 11)(2\beta + 13)} z^6 F(\beta + 7,2\beta + 14;z) + \frac{\beta(\beta + 1)(\beta + 2)(\beta + 3)}{16(2\beta + 9)(2\beta + 11)(2\beta + 13)(2\beta + 15)} z^8 F(\beta + 8,2\beta + 16;z),
\]

\[
z = \frac{\eta - \zeta}{\eta - \zeta};
\]

\[
I_2 = \frac{\beta(\beta + 1)(\beta + 2)(\beta + 3)\Gamma(\beta + 4)\Gamma(-2\beta - 7)}{\Gamma(1 - \beta)} (\eta - \zeta)^{\beta} (\eta - \zeta)^{\beta} \times F(\beta + 8,-7,1 - \beta;z_i) + \frac{8(2\beta + 7)}{1 - \beta} z_i F(\beta + 7,-6,2 - \beta;z_i) + \frac{2(2\beta + 7)(2\beta + 5)}{1 - \beta}(\beta + 4,2\beta + 11) z_i^2 F(\beta + 6,\beta - 5,3 - \beta;z_i) + \frac{3(2\beta + 7)(2\beta + 5)(2\beta + 3)}{(1 - \beta)(2 - \beta)(3 - \beta)} z_i^3 F(\beta + 5,\beta - 4,4 - \beta;z_i) + \frac{16(2\beta + 7)(2\beta + 5)(2\beta + 3)(2\beta + 1)}{(1 - \beta)(2 - \beta)(3 - \beta)(4 - \beta)} z_i^4 F(\beta + 4,\beta - 3,5 - \beta;z_i),
\]

\[
z_1 = \frac{\eta - \zeta}{\eta - \zeta}.
\]

**Lemma.** The identities are valid:

\[
F(\beta + 4,2\beta,2\beta + 8;z) + \frac{2\beta}{2\beta + 9} z^2 F(\beta + 5,2\beta + 2,2\beta + 10;z) + \frac{3(\beta + 1)}{2(2\beta + 9)(2\beta + 11)} z^4 F(\beta + 7,2\beta + 14;z) + \frac{\beta(\beta + 1)(\beta + 2)}{2(2\beta + 9)(2\beta + 11)(2\beta + 13)} z^6 F(\beta + 8,2\beta + 16;z) = (1 - z)^p,
\]

\[
\frac{\beta(\beta + 1)(\beta + 2)(\beta + 3)}{16(2\beta + 9)(2\beta + 11)(2\beta + 13)(2\beta + 15)} z^8 F(\beta + 8,2\beta + 16;z) = (1 - z)^p.
\]
\begin{align*}
F(\beta + 8, -\beta - 7, 1 - \beta; z_i) + \frac{8(2\beta + 7)}{1 - \beta} \zeta F(\beta + 7, -\beta - 6, 2 - \beta; z_i) + \\
+ \frac{24(2\beta + 7)(2\beta + 5)}{(1 - \beta)(2 - \beta)} z^2 F(\beta + 6, -\beta - 5, 3 - \beta; z_i) + \\
+ \frac{32(2\beta + 7)(2\beta + 5)(2\beta + 3)}{(1 - \beta)(2 - \beta)(3 - \beta)} z^3 F(\beta + 5, -\beta - 4, 4 - \beta; z_i) + \\
+ \frac{16(2\beta + 7)(2\beta + 5)(2\beta + 3)(2\beta + 1)}{(1 - \beta)(2 - \beta)(3 - \beta)(4 - \beta)} z^4 F(\beta + 4, -\beta - 3, 5 - \beta; z_i) = (1 - z_i)^{\beta} \\
\end{align*}

Evidence:

Proof. We use the expression for the hypergeometric function in the form of a series

\[ F(a,b,c; z) = \sum_{i=0}^{\infty} \frac{(a)_i (b)_i}{(c)_i i!} z^i \]

where \((a)_i = a(a+1)(a+2)...(a+i-1)\), \((a)_0 = 1\).

\[(1 - z)^{\beta} = 1 + \beta z + \frac{\beta(\beta + 1)}{2!} z^2 + \frac{\beta(\beta + 1)(\beta + 2)}{3!} z^3 + ... + \frac{\beta(\beta + 1)\ldots(\beta + n - 1)}{n!} z^n + ... \]

Let us turn to the proof of identity (8). For this, we expand the hypergeometric functions on the left-hand side and the polynomial on the right-hand side of identity (8) in a series:

\[
\sum_{i=0}^{\infty} \frac{(\beta + 4)_i (\beta + 1)_i}{(2\beta + 8)_i i!} z^i + \frac{2\beta}{2\beta + 9} \sum_{i=0}^{\infty} \frac{(\beta + 5)_i (2\beta + 2)_i}{(2\beta + 10)_i i!} z^i + \frac{3\beta(\beta + 1)}{2(2\beta + 9)(2\beta + 11)} z^4 .
\]

\[
\sum_{i=0}^{\infty} \frac{(\beta + 6)_i (2\beta + 4)_i}{(2\beta + 12)_i i!} z^i + \frac{\beta(\beta + 1)(\beta + 2)}{2(2\beta + 9)(2\beta + 11)(2\beta + 13)} z^6 \sum_{i=0}^{\infty} \frac{(\beta + 7)_i (2\beta + 6)_i}{(2\beta + 14)_i i!} z^i + \\
\frac{\beta(\beta + 1)(\beta + 2)(\beta + 3)}{2(2\beta + 9)(2\beta + 11)(2\beta + 13)(2\beta + 15)} z^8 \sum_{i=0}^{\infty} \frac{(\beta + 8)_i (2\beta + 8)_i}{(2\beta + 16)_i i!} z^i = \\
= 1 + \beta z + \frac{\beta(\beta + 1)}{2!} z^2 + \frac{\beta(\beta + 1)(\beta + 2)}{3!} z^3 + ... + \frac{\beta(\beta + 1)\ldots(\beta + n - 1)}{n!} z^n + ... \\
\]

Further, substituting the expressions for the hypergeometric functions using a series into the last equation and calculating the coefficients at the same powers of \(z\), we have:

\[
z^0 : \frac{(\beta + 4)_0 (2\beta)_0}{(2\beta + 8)_0 0!} = 1, \quad z^1 : \frac{(\beta + 4)_1 (2\beta)_1}{(2\beta + 8)_1 1!} = \beta \\
z^2 : \frac{(\beta + 4)_2 (2\beta)_2}{(2\beta + 8)_2 2!} + \frac{2\beta}{2\beta + 9} = \frac{(\beta + 4)(\beta + 5)(2\beta)(2\beta + 1)}{(2\beta + 8)(2\beta + 9)2!} + \frac{2\beta}{2\beta + 9} = \frac{\beta(\beta + 1)}{2!} \\
z^3 : \frac{(\beta + 4)_3 (2\beta)_3}{(2\beta + 8)_3 3!} + \frac{2\beta}{2\beta + 9} = \frac{(\beta + 4)(\beta + 5)(\beta + 6)(2\beta)(2\beta + 1)(2\beta + 2)}{(2\beta + 8)(2\beta + 9)(2\beta + 10)3!} + \\
+ \frac{2\beta(\beta + 5)(2\beta + 2)}{(2\beta + 9)(2\beta + 10)} = \frac{\beta(\beta + 1)(\beta + 2)}{3!} \\
\]

Similarly, continuing the calculation of the coefficient at \(z^n\), we obtain the following:
\[ z^n = \frac{(\beta + 4)n(2\beta_n)}{(2\beta + 8)_n \cdot n!} + \frac{2\beta}{2\beta + 9} \frac{(\beta + 5)_n \cdot (2\beta + 2)_n \cdot n!}{(2\beta + 10)_{n-2} \cdot (n-2)!} + \frac{3\beta(\beta + 1)}{2(2\beta + 9)(2\beta + 11)} \frac{(\beta + 6)_n \cdot (2\beta + 4)_n \cdot n!}{(2\beta + 12)_{n-4} \cdot (n-4)!} + \frac{\beta(\beta + 1)(\beta + 2)}{2(2\beta + 9)(2\beta + 11)} \frac{(\beta + 7)_n \cdot (2\beta + 6)_n \cdot n!}{(2\beta + 13)(2\beta + 15)} \frac{(\beta + 2)(\beta + 3)}{2(2\beta + 9)}(2\beta + 11)(2\beta + 13)_{n-6} \cdot (n-6)! + \frac{\beta(\beta + 1)(\beta + 2)(\beta + 3)(\beta + 8)_{n-8} \cdot (2\beta + 8)_n \cdot n!}{(2\beta + 16)_{n-8} \cdot (n-8)!} .
\]

Using the expansion \( a_r \) and grouping the corresponding terms in the last equality, we obtain the following common factor:
\[ \frac{\beta(\beta + 1)(\beta + 2)(\beta + 3)(\beta + 8)_{n-8} \cdot (2\beta + 8)_n \cdot n!}{(2\beta + 9)(2\beta + 11)(2\beta + 13)(2\beta + 15)(2\beta + 16)_{n-8} \cdot (n-8)!} .
\]

and taking this into account, from the last expression we have the following coefficient for \( z^n \):
\[ \frac{\beta(\beta + 1)(\beta + 2)(\beta + 3)(\beta + 8)_{n-8} \cdot (2\beta + 8)_n \cdot n!}{(2\beta + 9)(2\beta + 11)(2\beta + 13)(2\beta + 15)(2\beta + 16)_{n-8} \cdot (n-8)!} \cdot \frac{16(\beta + n)(\beta + n + 1)\beta + n + 2) \cdot (\beta + n + 3)(\beta + 1)(\beta + 2)(\beta + 3)(\beta + 2)(\beta + 5) + 12(\beta + n)(\beta + n + 1)(\beta + n + 2)\beta + 3(\beta + 5) \cdot (\beta + 2)(n(n - 1) + 24(\beta + n)(\beta + n + 1)(\beta + 5)(\beta + 7) \cdot n(n - 1)(n - 2)(n - 3)(n - 4)(n - 5) + n(n - 1)(n - 2)(n - 3)(n - 4)(n - 5)(n - 6)(n - 7) .
\]

Now, opening all the inner brackets and performing calculations, we get:
\[ \frac{\beta(\beta + 1)(\beta + 2)(\beta + 3)(\beta + 8)_{n-8} \cdot (2\beta + 8)_n \cdot n!}{(2\beta + 9)(2\beta + 11)(2\beta + 13)(2\beta + 15)(2\beta + 16)_{n-8} \cdot (n-8)!} \cdot \frac{500 n + 13068 n^2 + 13132 n^3 + 88}{500 n + 13068 n^2 + 13132 n^3 + 88} .
\]

It is easy to make sure that the coefficients for \( z^n \) in the left and right sides have the appropriate form:
\[ 500 n + 13068 n^2 + 13132 n^3 + 6769 n^4 + 1960 n^5 + 322 n^6 + 28 n^7 + n^8 + 10080 \beta + 52272 n\beta + 78792 n^2\beta + 54152 n^3\beta + 19600 n^4\beta + 392 n^5\beta + 16 n^6\beta + 52272 n^2\beta + 157584 n^3\beta + 162456 n^4\beta + +112 n^5\beta + 105056 \beta^2 + 216608 n\beta^3 + +156800 n^2\beta^3 + 78400 n^3\beta^3 + 448 n^4\beta^3 + 108304 n^5\beta^3 + 156800 n^6\beta^3 + +77280 n^7\beta^3 + 15680 n^8\beta^3 + 1120 n^9\beta^3 + 62720 n^10\beta^3 + 61824 n^11\beta^3 + 18816 n^{12}\beta^3 + +192 n^{13}\beta^3 + 12544 n^14\beta^3 + 256 \beta^8} .
\]

\[ = (n + 2\beta)(n + 2\beta + 1)(n + 2\beta + 2)(n + 2\beta + 3)(n + 2\beta + 4) \cdot (n + 2\beta + 5)(n + 2\beta + 6)(n + 2\beta + 7) .
\]
Therefore,
\[
\frac{(\beta + 8)_{n}}{n!} \beta + 1(\beta + 2)(\beta + 3)(\beta + 5)(\beta + 6)(\beta + 7) = \frac{\beta}{n!}
\]
Thus, the first identity in the lemma is proved. The second identity (9) is proved in a similar way. Based on the lemma, the expressions \( I_1(\xi, \eta; \zeta) \) and \( I_2(\xi, \eta; \zeta) \) take the form
\[
I_1(\xi, \eta; \zeta) = \Gamma(\beta + 4) (\eta - \xi)^{2\beta - 7} (\eta - \zeta)^{\beta} (\xi - \zeta)^{-\beta}
\]
\[
I_2(\xi, \eta; \zeta) = \frac{\beta(\beta + 1)(\beta + 2)(\beta + 3)\Gamma(\beta + 4)\Gamma(-2\beta - 7)}{\Gamma(1 - \beta)} \times (\eta - \xi)^{2\beta - 7} (\eta - \zeta)^{\beta} (\xi - \zeta)^{-\beta}.
\]
Substituting (10), (11) into (7), we obtain a representation of the generalized solution of the class \( R_2 \)
\[
u(\xi, \eta) = \int_0^\xi (\eta - \zeta)^{-\beta} (\xi - \zeta)^{-\beta} T(\zeta) d\zeta + \int_\xi^\eta (\eta - \zeta)^{-\beta} (\zeta - \xi)^{-\beta} N(\zeta) d\zeta,
\]
where
\[
N(\zeta) = \frac{1}{2\cos \pi \beta} T(\zeta) - 2^{4\beta - 2} \gamma_1 v(\zeta).
\]
For all values of \( \alpha \) under consideration, a representation of the generalized solution of the \( R_2 \) class is obtained.

4. Conclusions
Got the result of the modified Cauchy problem for some values of \( \alpha \) is stated and investigated and a convenient representation of the generalized solution of the modified Cauchy problem is obtained.

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