1. Introduction

In this talk, I will give a brief description of the \( \phi \) factory DA\( \Phi \)NE at Frascati, explaining why a \( \phi \) factory is an interesting place to do new physics, and then discuss the physics that can be done at DA\( \Phi \)NE. Since Ulf Meissner has already told you about \( K \) decays at DA\( \Phi \)NE and their relevance to CHPT (Chiral Perturbation Theory), and Pierluigi Campana will cover some of the other DA\( \Phi \)NE physics topics such as \( \gamma \gamma \) physics, meson spectroscopy and \( \Delta I = \frac{1}{2} \) violation in hypernuclei, I will concentrate on CP violation as it can be studied at DA\( \Phi \)NE. This is, after all, the \textit{raison d'\'etre} of DA\( \Phi \)NE. I start with a brief general introduction to CP violation in the \( K \bar{K} \) system, and the distinction between mass-mixing CP violation (\( \epsilon \)) and \textit{intrinsic} CP violation (\( \epsilon'/\epsilon \)). After presenting a summary of \( \epsilon'/\epsilon \) measurements up to now, and briefly discussing the theory of \( \epsilon'/\epsilon \) (the so-called ‘penguins’), I will cover the particularities of measuring \( \epsilon'/\epsilon \) at a \( \phi \) factory, such as \textit{tagging} and \textit{interferometry}. Finally, I will say a few words about searching for CP violation in modes where it has never before been seen. I will end my talk with a list of other physics topics at DA\( \Phi \)NE, and rare decay branching ratio limits that can be achieved there, just to give a flavor of what else can be done. For a starting point for more information about physics at DA\( \Phi \)NE, see Ref. 1; for \( \epsilon'/\epsilon \) theory and history see Ref. 2.

2. Why a \( \phi \) factory? What is DA\( \Phi \)NE?

First of all, why do we want a \( \phi \) factory? Because, essentially, a \( \phi \) factory is a \( K \) factory. And these kaons are not just any kaons, but kaons in a well-defined quantum-mechanical and kinematic state. This, as we shall see, is very important. The \( \phi(1019) \) is the lowest lying \( J^{PC} = 1^{--} \) bound state of a strange quark and a strange anti-quark. DA\( \Phi \)NE is an \( e^+e^- \) collider optimized to run at the center-of-mass (c.m.) energy \( M_\phi \), due to deliver a luminosity \( \mathcal{L} = 10^{32}\text{cm}^{-2}\text{s}^{-1} \) in 1996. The cross section for \( e^+e^- \to \phi \) at the \( \phi \) resonance peak is about 5 \( \mu \)b, meaning that at the eventual target luminosity \( \mathcal{L} = 10^{33}\text{cm}^{-2}\text{s}^{-1} \), 5000 \( \phi \)'s are produced per second. Using the canonical high energy physics definition of one ‘machine year’=10\(^7\) seconds, giving a leeway of about a factor of \( \pi \) to account for the various integrated luminosity degradation factors (e.g. machine study, maintenance and down periods; detector down periods; and peak versus average luminosity), this means \( 5 \times 10^{10} \phi \)'s per year!
The $\phi(1019)$, with a mass $M_{\phi} = 1019.412 \pm 0.008$ MeV, total width $\Gamma = 4.41$ MeV, and leptonic width $\Gamma_{ee} = 1.37$ keV, decays into the modes given in Table 1. Here BR is the branching ratio, in percent; $\beta_K$ is the $\beta$ (= velocity/c) of the kaon; $\gamma_{\beta c\tau}K$ its mean path length in centimeters; and $p_{\text{max}}$ is the momentum of the resultant particles in MeV (maximum momentum if there are three particles). The last column gives the resultant number of such decays in the canonical year, demonstrating that DAΦNE is indeed a factory of neutral kaons in a well prepared quantum state, and of charged $K$ pairs, as well as of $\rho$’s, $\eta$’s (and rarer $\eta’$’s). These numbers of kaons will have to be reduced by the tagging efficiencies of about 30–80%, depending on kaon species, in order to get the number of useful, well-identified kaons (see Sec. 6). The high luminosity of DAΦNE will also allow measurements of rare $\phi$ radiative decays (see Sec. 9).

| Mode     | BR % | $\beta_K$ | $\gamma_{\beta c\tau}K$ cm | $p_{\text{max}}$ MeV/c | #         |
|----------|------|-----------|-----------------------------|------------------------|-----------|
| $K^+K^-$ | 49   | 0.249     | 95.4                        | 127                    | $2.5 \times 10^{10}$ pairs |
| $K_S$, $K_L$ | 34   | 0.216     | 343.8                       | 110                    | $1.5 \times 10^{10}$ |
| $\rho\pi$ | 13   | -         | -                           | 182                    | $6 \times 10^9$ |
| $\pi^+\pi^-\pi^0$ | 2   | -         | -                           | 462                    | $1 \times 10^9$ |
| $\eta\gamma$ | 1.3  | -         | -                           | 362                    | $6 \times 10^8$ |
| other    | $\sim$1 | -         | -                           | -                      | $5 \times 10^8$ |

Table 1. $\phi$ decays.

The main goal of DAΦNE is to measure direct CP violation ($\epsilon'/\epsilon$) to accuracies of about $1 \times 10^{-4}$ by observing $K_{L,S} \rightarrow \pi^0\pi^0$, $\pi^+\pi^-$. In general, also, DAΦNE is exciting not only because it is the first $\phi$ factory, but because it is the next new particle physics accelerator (which means new results!). This statement may be somewhat contestable, depending on what one calls particle physics and what one calls new, so a more unambiguous claim is that it will be the first machine of the factory era, the first of a new generation of super-high luminosity (in the $10^{33}$cm$^{-2}$s$^{-1}$ range, to be contrasted with $10^{31}$cm$^{-2}$s$^{-1}$ for existing machines) $e^+e^-$ colliders, designed to stay within a narrow range of energy and produce a large number of a given particle (or family of particles).

What makes a factory? The luminosity of a collider is given by the following formula,

$$L = fn \frac{N_1N_2}{A},$$

(2.1)

where $f$ is the revolution frequency, $n$ is the number of bunches, $N_i$ is the number of particles per bunch for each species of particle, and $A$ is the area of the beams (for this formula to be valid, the beams must be 100% overlapping). Thus, many bunches
of many particles, going around at a high frequency, focussed tightly into beams of a very small cross section, produce a high luminosity. Nonetheless, whichever of these parameters are modified to produce a larger luminosity, without radically new technology, the luminosity in a single ring machine will be limited by beam-beam interactions to be about that of current machines \((10^{31} \text{cm}^{-2}\text{s}^{-1})\). Synchrotron oscillations ‘shake’ up the beams and destroy the small bunch size needed for high luminosity; bunches containing more particles lead to stronger oscillations. Multiple bunches in a single ring do not help; each bunch ‘sees’ \(n\) of its counterparts and gets successively more and more perturbed.

The solution generally found in ‘factories’ is to have two separate rings (hence the DA in DAΦNE, for Double Annular), which cross each other at a small but non-zero crossing angle (20 to 30 mrad for DAΦNE). This crossing angle is needed, even though head-on collisions are less disruptive to the beam, because if the two beams were parallel even for a few meters, each bunch would then pass several of its counterparts in a small machine like DAΦNE, where there will be more than one bunch per meter. The DAΦNE main rings are 98 meters in perimeter, in a roughly rectangular shape, of 32 by 23 meters. The machine will commence operation with 30 bunches of \(9 \times 10^{10} e^\pm\) per bunch, yielding a luminosity of about \(1 \times 10^{32}\text{cm}^{-2}\text{s}^{-1}\). At this level already, that means about the same number of particles in a 98 m ring as LEP currently has in its 27 km ring. DAΦNE will then go to 120 bunches; this is the maximum possible number of bunches at the planned RF frequency of 368.25 MHz. Fine-tuning of the machine parameters is then expected to bring the luminosity to its final level of \(1 \times 10^{33}\text{cm}^{-2}\text{s}^{-1}\). The bunches are long and flat: 3 cm long in the beam direction, 2-3 mm in the horizontal direction perpendicular to the beam, and only 20 \(\mu\)m thick in the vertical direction. The beams cross at a small angle in the horizontal plane. Historically, at DORIS, synchro-betatron oscillations were a problem with vertical crossings; with horizontal crossings, the particles of one flat bunch will be well embedded in those of the other flat bunch, and thus not disturb each other as they do when they are in different planes.

The machine complex of DAΦNE is shown in Fig. 1. There will be two interaction areas, for the two experiments KLOE (particle physics) and FINUDA (nuclear physics), which will be discussed by Pierluigi Campana. There is as well the possibility for medical research and so on with the ultraviolet and x-ray beams of the DAΦNE-L(ight) facility. The DAΦNE beam energy will be 0.51 GeV \(\pm\) 0.4 MeV, the same as the injection energy, which means there will be no acceleration in the ring in the normal mode of running, on the \(\phi\) resonance. The \(e^\pm\) are accelerated in the linac, a linear accelerator consisting of two pieces, one that imparts a maximum of 250 MeV, the other 550 MeV. The electrons thus could in principle be accelerated to a maximum of 800 MeV. The positrons are created in the first segment and accelerated in the second. The beams are then stored and ‘cooled’ (meaning that the momentum spread – and as a result, the position spread – is decreased) in the accumulator, a small ring, before injection into the main rings.
Figure 1. The DAΦNE machine complex.
In Fig. 2, the luminosity versus center of mass energy \((E_{\text{c.m.}})\) for existing single ring \(e^+e^-\) colliders (the maximums that have been achieved) are contrasted with the expected range for factories-to-be. Beauty factories have been approved for construction at SLAC and at KEK, while the \(\tau\)-charm factory is so far only intermittently under consideration.

Figure 2. Luminosity versus center of mass energy for existing single ring \(e^+e^-\) colliders (dots) contrasted with the expected ranges for factories-to-be. At the top of the figure are indicated the energies of the various resonances to be studied at these factories.

3. CP violation in the \(K\bar{K}\) system

Kaons contain a strange quark \((s)\) or antiquark \((\bar{s})\), and are composed of two isospin doublets: \(K^0 = d\bar{s}, K^+ = u\bar{s}\), the \(S = +1\) doublet, and \(\bar{K}^0 = \bar{d}s, K^- = \bar{u}s\), \(S = -1\). Kaons are produced by strangeness conserving strong interactions. Weak interactions do not conserve \(S\), and mix \(K^0\) and \(\bar{K}^0\) via the transition \(K^0 \leftrightarrow 2\pi \leftrightarrow \bar{K}^0\), represented at the quark level by the box diagrams in Fig. 3. C violation allows these transitions to
occur; CP need not be violated. Under CP, the $K^0$ and $\bar{K}^0$ transform into each other:

$$CP |K^0\rangle = |\bar{K}^0\rangle \quad CP |\bar{K}^0\rangle = |K^0\rangle.$$  \hfill (3.1)

We can then write down the linear combinations of these states that are CP eigenstates; if CP is conserved, these will be the physical eigenstates as well:

$$|K_1\rangle = \frac{|K^0\rangle + |\bar{K}^0\rangle}{\sqrt{2}} \quad |K_2\rangle = \frac{|K^0\rangle - |\bar{K}^0\rangle}{\sqrt{2}}.$$  \hfill (3.2)

$|K_1\rangle$ is thus CP even ($CP|K_1\rangle = +|K_1\rangle$) and must decay to two pions, a CP even state. $|K_2\rangle$ is CP odd and must decay to three pions.

![Figure 3. The box diagrams mediating the $K^0 - \bar{K}^0$ transition.](image)

Thus, if one has a beam of kaons, one will see several two pion decays, close together, near the origin of the beam; then several meters down the beamline, a spread-out sprinkling of three pion decays. However, in 1964, Cronin and Fitch [3] observed long lived kaons decaying to two pions! This was the first evidence for CP violation. Physical states are thus no longer CP eigenstates. We instead have physical states called $K_S$, the short-lived state, and $K_L$, the long-lived state. $K_S$ will have a small admixture of CP-odd $K_2$, while $K_L$ has a small admixture of CP-even $K_1$, in other words:

$$|K_S\rangle = \frac{|K_1\rangle + \epsilon |K_2\rangle}{\sqrt{1 + |\epsilon|^2}} = \frac{1 + \epsilon}{\sqrt{1 + |\epsilon|^2}} |K^0\rangle + \frac{1 - \epsilon}{\sqrt{1 + |\epsilon|^2}} |\bar{K}^0\rangle$$ \hfill (3.3)

$$|K_L\rangle = \frac{|K_2\rangle + \epsilon |K_1\rangle}{\sqrt{1 + |\epsilon|^2}} = \frac{1 + \epsilon}{\sqrt{1 + |\epsilon|^2}} |K^0\rangle - \frac{1 - \epsilon}{\sqrt{1 + |\epsilon|^2}} |\bar{K}^0\rangle.$$ \hfill (3.4)

In this picture, $K_L$ can decay to two pions simply because it has a small component of $K_1$. If $K_2$ has no intrinsic CP violation, i.e.,

$$\langle K_2 | H_W | \pi\pi \rangle = 0,$$ \hfill (3.5)

then

$$\frac{\langle K_L | H_W | \pi\pi \rangle}{\langle K_S | H_W | \pi\pi \rangle} = \frac{\epsilon}{\langle K_1 | H_W | \pi\pi \rangle} = \frac{\epsilon}{\langle K_1 | H_W | \pi\pi \rangle} = \frac{\epsilon}{\langle K_1 | H_W | \pi\pi \rangle} = \epsilon.$$ \hfill (3.6)

CP violation was discovered in 1964, implying a non-vanishing value for $\epsilon$, which has
been quite well-measured for quite some time, with value

$$|\epsilon| = 2.259 \pm 0.018 \times 10^{-3}. \quad (3.7)$$

The question now is, is there *intrinsic* CP violation? Thirty years after the discovery of CP violation, this question still remains unanswered.

### 4. Measuring direct CP violation in the processes $K_{L,S} \to \pi\pi$

A state consisting of a pair of pions must have an isospin of zero or two. The Clebsch-Gordan table for the combination of two isospin one states is the following.

| 1 × 1  | 2 | 1 | 0 |
|-------|---|---|---|
| 1     | 1/√6 | 1/√2 | 1/√3 |
| 0     | √2/3 | 0 | −1/√3 |
| −1    | 1/√6 | −1/√2 | 1/√3 |

Thus we have, for appropriately symmetrized dipion states,

$$\frac{1}{\sqrt{2}}(|\pi^+\pi^-\rangle + |\pi^-\pi^+\rangle) = \sqrt{\frac{2}{3}}|I = 0\rangle + \sqrt{\frac{1}{3}}|I = 2\rangle, \quad (4.1)$$

$$|\pi^0\pi^0\rangle = -\sqrt{\frac{1}{3}}|I = 0\rangle + \sqrt{\frac{2}{3}}|I = 2\rangle. \quad (4.2)$$

Let us define the amplitudes

$$\langle \pi\pi I = 0| H |K^0\rangle \equiv A_0 e^{i\delta_0}, \quad \langle \pi\pi I = 0| H |\bar{K}^0\rangle = -A_0^* e^{i\delta_0} \quad (4.3)$$

$$\langle \pi\pi I = 2| H |K^0\rangle \equiv A_2 e^{i\delta_2}, \quad \langle \pi\pi I = 2| H |\bar{K}^0\rangle = -A_2^* e^{i\delta_2}. \quad (4.4)$$

Recall eq. (3.1). In the CP conservation limit, $A_{0,2} = A_{0,2}^*$, i.e., the amplitudes for $K^0$ and $\bar{K}^0$ differ only by a sign, and have no phase difference. The phases $\delta_{0,2}$ are the phases that $K^0$ and $\bar{K}^0$ have in common, while an imaginary part to $A_{0,2}$ indicates a CP violating phase difference between $K^0$ and $\bar{K}^0$. Actually, there is one unphysical phase (since we can only measure intensities, not amplitudes), so it is the difference in phase between $A_0$ and $A_2$ that indicates direct CP violation.

One common phase convention is to take $A_0$ real. One can then derive (an exercise left for the reader) from eqs. (3.3), (3.4), and (4.1)–(4.4), dropping higher order terms...
(and bearing in mind that $\epsilon \ll 1$, $\epsilon' \ll \epsilon$ and $A_2 \ll A_0$), the relations

$$
\eta^\pm \equiv \frac{\langle \pi^+\pi^- | H | K_L \rangle}{\langle \pi^+\pi^- | H | K_S \rangle} \approx \epsilon + \epsilon' \quad \eta^{00} \equiv \frac{\langle \pi^0\pi^0 | H | K_L \rangle}{\langle \pi^0\pi^0 | H | K_S \rangle} \approx \epsilon - 2\epsilon'
$$

(4.5)

where $\epsilon'$ is given by the expression

$$
\epsilon' = \frac{i \, \text{Im} A_2 \, e^{i(\delta_2 - \delta_0)}}{\sqrt{2} A_0}.
$$

(4.6)

Note that here $\langle \pi^+\pi^- |$ is taken to imply a symmetrized dipion state.

Thus, the following three statements are equivalent:

1. $\epsilon' \neq 0$;
2. $A_2$ is complex in the basis in which $A_0$ is real;
3. there is direct CP violation, i.e., there is CP violation in the decay $K_2 \to \pi\pi$.

Moreover, from the expressions for $\eta^\pm$ and $\eta^{00}$, we see that $\epsilon'/\epsilon$ can be measured via the so-called double ratio

$$
\frac{N(K_L \to \pi^+\pi^-)}{N(K_S \to \pi^+\pi^-)} \left/ \frac{N(K_L \to \pi^0\pi^0)}{N(K_S \to \pi^0\pi^0)} \right. \approx \frac{|\epsilon + \epsilon'|^2}{|\epsilon - 2\epsilon'|^2} \approx 1 + 6\text{Re} \frac{\epsilon'}{\epsilon}.
$$

(4.7)

Other similar ways of measuring $\epsilon'/\epsilon$ are ratios such as the following:

$$
\frac{N(\pi^+\pi^-\pi^+\pi^-)}{N(\pi^0\pi^0\pi^0\pi^0)} \times \left( \frac{BR(K_S \to \pi^0\pi^0)}{BR(K_L \to \pi^+\pi^-)} \right)^2 \approx 1 + 6\text{Re} \frac{\epsilon'}{\epsilon}
$$

(4.8)

and

$$
\frac{N(\pi^+\pi^-\pi^+\pi^-)}{N(\pi^+\pi^-\pi^0\pi^0)} \times \left( \frac{BR(K_S \to \pi^0\pi^0)}{BR(K_S \to \pi^+\pi^-)} \right) \approx 1 + 3\text{Re} \frac{\epsilon'}{\epsilon}.
$$

(4.9)

The statistical uncertainties on these three and similar ratios will all be the same, but the systematic errors may differ. Thus different ratios may be optimal for different experiments, or different ratios may serve as valuable counterchecks in a given experiment.

Currently, the most precise determinations of $\epsilon'/\epsilon$ are those of the experiments NA31\,[4] at CERN, and E731\,[5] at Fermilab. They find

$$
\text{Re}(\epsilon'/\epsilon) = 2.3 \pm 0.65 \times 10^{-3} \quad \text{(NA31)}
$$

$$
0.74 \pm 0.60 \times 10^{-3} \quad \text{(E731)}
$$

(4.10)

the one consistent with $\epsilon'/\epsilon \neq 0$, the other consistent with $\epsilon'/\epsilon = 0$, and yet both consistent with each other. These experiments create high-energy $K$ beams (of the order of 100 GeV) by collisions of high-energy protons (450 GeV for NA31, 800 GeV for E731) with a fixed target. NA31 alternates operation with a $K_L$ beam and with a $K_S$ beam, while E731 uses simultaneously two $K_L$ beams, one of which, by virtue of regenerators,*
produces the required $K_S$’s. In a few years, these two experiments also hope to come out with measurements of $\epsilon'/\epsilon$ at the same accuracies that DAΦNE is aiming at, namely at the $10^{-4}$ level, with clearly very different, and thus very complementary, systematic effects.

Fig. 4 shows the evolution of the measured value of $\epsilon'/\epsilon$ over the last two decades.

Two decades is a convenient cutoff, since many of the results before 1970 tend to be based on measurements of $\eta^\pm$ and $\eta^{00}$ by separate experiments. The results here are, in chronological order: 1972, Holder et al.,[6] CERN; 1972, Banner et al.,[7] Brookhaven/Princeton; 1979, Christenson et al.,[8] Brookhaven/NYU; 1985, Black et al.,[9] Brookhaven/Yale; 1985, E731,[10] 1988, E731,[11] 1988, NA31,[12] 1990, E731[13] and finally the two most recent measurements, quoted above, whose errors are so small that they are denoted by

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An initially pure $K^0$ beam quickly becomes essentially a $K_L$ beam as the $K_S$ part decays. However, a $K_L$ beam passing through matter will lose more $K^0$ than $\bar{K}^0$, as $\bar{K}^0$ reacts more in nuclear collisions (since $\bar{K}^0$ contains $\bar{d}$, a light antiquark). This passage through matter thus regenerates the $K_S$ component.

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Figure 4. Evolution of experimental measurements and theoretical estimates of $\mathrm{Re}\; \epsilon'/\epsilon$ (in units of $10^{-3}$) over the last two decades. See text for detailed description.
the size of the points alone.† The errors of the next generation of experiments, nearly ten times smaller than these, will require a new graph with a different scale! In this figure is also shown the evolution of theoretical predictions for $\epsilon'/\epsilon$, which will be described in the next section. These predictions are all within the framework of the Standard Model.

5. Theory of $\epsilon'/\epsilon$ in a nutshell

We have seen in the previous sections that if CP violation is due only to the mixing of $K_1$ and $K_2$, $\epsilon' = 0$. If there is CP violation in the decay of $K_2$ as well, $\epsilon' \neq 0$. Mixing is described by the box diagrams in Fig. 3. The decay is described by analogous box diagrams, and the so-called penguin diagram, shown in Fig. 5 (which I have taken pains to draw as much like a penguin as possible — nonetheless...).

In the Standard Model (SM) both penguin diagrams and box diagrams are proportional to $\sin \delta$, where $\delta$ is the one non-trivial (and unknown) Cabibbo-Kobayashi-Maskawa (CKM) phase. In 1973, Kobayashi and Maskawa[14] showed explicitly that a three generation model can account for CP violation. They proved that with a two generation model, one has no non-trivial phase, and thus no CP violation (in the SM). Thus, in the SM, barring accidental cancellations that we will see later, both $\epsilon$ and $\epsilon'/\epsilon$ should be non-zero.

![Figure 5. The gluonic penguin](image)

In contrast, in the ‘superweak’ model of Wolfenstein[15] (1964), some small new interaction is postulated to contribute at lowest order to the $\Delta S = 2$ mass matrix, while the CKM phase $\delta = 0$, yielding a scenario where $\epsilon \neq 0$ but $\epsilon' = 0$ to extremely good accuracy.

Within the standard model, theoretical predictions for $\epsilon'/\epsilon$ have varied a lot, and generally shrunk, for two main reasons. The first is that our expectation for $m_t$, the top quark mass, has been steadily growing and as $m_t$ grows, the amplitude of the gluonic penguin in Fig. 5 decreases. In the 1970’s, 20 to 30 GeV was considered reasonable; towards the end of the 80’s estimates rose to around 100 GeV, with indirect evidence from sources such as $B\bar{B}$ mixing measurements; now, from LEP and Fermilab data we believe $m_t$ to be in the vicinity of 170 GeV! The second is that not only the contribution of

† That there are less points shown for NA31 than E731 does not mean that NA31 did not publish any results between 1988 and 1993, but as their results did not change dramatically I have not shown their intermediate results.
the gluonic penguin decreases as $m_t$ grows, but as $m_t$ enters the 100 to 200 GeV range, the 
*electroweak* penguins (like Fig. 5, only with a photon or a $Z$ replacing the gluon) can no longer be neglected as they were originally. In particular the $Z$ penguin has significant and *cancelling* contributions. A novel feature was thus that $\epsilon'/\epsilon$ could actually pass through zero, for large enough $m_t$ (around 200 GeV). In Fig. 6 I have shown some sample recent predictions (from Buras and Harlander\[16\] in 1992) for $\epsilon'/\epsilon$ versus $m_t$, in the two allowed quadrants for the phase $\delta$, for the parameter ranges $0.09 \leq |V_{ub}|/|V_{cb}| \leq 0.17$, $0.036 \leq |V_{cb}| \leq 0.046$ (CKM matrix elements), $0.1$ GeV \(\leq \Lambda_{QCD} \leq 0.3\) GeV (QCD scale), $0.5 \leq B_K \leq 0.8$ (bag factor), and $125$ MeV \(\leq m_s \leq 200\) MeV (strange quark mass).

$$\epsilon'/\epsilon \left(10^{-3}\right)$$

![Figure 6](image)

**Figure 6.** The upper and lower limits of $\epsilon'/\epsilon$ in the first (I) and second (II) quadrant of the CKM phase $\delta$ (from Ref. 16).

Some of the historical progress of theoretical predictions, which bears some resemblance to the evolution of the measurements, is summarized in Fig. 4. The first prediction, by Ellis, Gaillard and Nanopoulos,\[17\] was quite small. The second prediction shown, of Gilman and Wise,\[18\] covered a range only starting at 0.01 and going all the way to 0.08 for $\epsilon'/\epsilon$. The next prediction is a more detailed calculation by Gilman and Wise.\[19\] The next two predictions are *lower limits* only, both by Gilman and Hagelin.\[20\] Neither bound is still valid, with current knowledge and a large top mass. The next rectangle shows the range of $\epsilon'/\epsilon$ that was considered reasonable\[21\] in 1987, with a reasonably large $m_t$ (in the range of 100 GeV) and $B\bar{B}$ mixing constraints taken into account.
Finally the last rectangle is representative of the predictions in the last couple of years. This is a collection: individual preferred ranges may resemble the NA31 measurement or the E731 measurement. See for example Ref. 22 for a recent look at $\epsilon'/\epsilon$ and references to other work. While $\epsilon'/\epsilon$ at or below zero is in principle allowed, it is not very favored, as it would require a very heavy top quark (over 200 GeV).

6. $\epsilon'/\epsilon$ at a $\phi$-factory

At a $\phi$-factory, $K$'s are produced strongly in pairs in the reaction $e^+e^- \rightarrow \gamma^* \rightarrow \phi \rightarrow K^0\bar{K}^0$ so that

$$C(K^0\bar{K}^0) = C(\phi) = C(\gamma) = -1.$$  
(6.1)

The initial state is thus

$$\frac{1}{\sqrt{2}} (|K^0, \bar{p} \rangle |\bar{K}^0, -\bar{p} \rangle - |\bar{K}^0, \bar{p} \rangle |K^0, -p \rangle)$$  
(6.2)

which can be shown to be equal to

$$\frac{1}{\sqrt{2}} \frac{1 + |\epsilon|^2}{1 - |\epsilon|^2} (|K_S, -\bar{p} \rangle |\bar{K}_L, \bar{p} \rangle - |K_S, \bar{p} \rangle |\bar{K}_L, -\bar{p} \rangle)$$  
(6.3)

Thus the $\phi$ decays to a pure $K_SK_L$ state, with no admixture of $K_SK_S$ or $K_LK_L$. Since $K_S$ and $K_L$ are weak eigenstates, in vacuum the pair of kaons remain in a state of pure $K_S$, $K_L$. As a result DAΦNE benefits from ‘tagging’: by observing a clear signal for a $K_L$ ($K_S$) one can be sure that one has a $K_S$ ($K_L$) as the other particle in the event, independent of what this other particle decays to. More specifically, a ‘V’ 20 to 180 cm from the interaction point signals a $K_L \rightarrow \pi^+\pi^-\pi^0$, $\pi\mu\nu$, or $e\mu\nu$ with no background ($< 10^{-6}$) and is thus a perfect $K_S$ tag. The efficiency for this tagging is 28%, since $K_L$'s are very long lived (recall the mean path of 3.43 m) and thus do not all decay inside the detector). In other words, for a well-defined sample of one-third of the all K’s, one can be very sure one has a $K_S$. Similarly, a $\pi^+\pi^-$ reconstructing to within 2 cm of the interaction point tags the $K_L$. The background is two pion decays that come from $K_L$ instead of $K_S$, and thus is of the order of $8 \times 10^{-6}$ (the branching ratio for $K_L$ going to two pions, times the probability that it does so, so close to the interaction point). Here the efficiency is essentially the percentage of $K_S$’s decaying to charged rather than neutral pions, 2/3. These efficiencies drop out identically from the double ratio defined in Eq. (4.7), and thus produce no systematic errors, only a loss in statistics. Another feature of $K$’s at a $\phi$ factory is that they have a very precise — and small ($\beta = 0.2$) — momentum.

Systematic errors are beyond my scope, but I can now at least demonstrate, on the statistical side, the figures justifying the claimed accuracy for $\epsilon'/\epsilon$ at DAΦNE.
abbreviated form, the double ratio is

$$\frac{N_L^+/N_S^+}{N_L^0/N_S^0} \approx 1 + 6\text{Re} \frac{e'}{\epsilon},$$

where each $N$ refers to the number of $K_{L,S}$ decaying to two charged or neutral pions. The $N_S$ will evidently be much larger then the $N_L$; thus, the statistical errors coming from them will be negligible compared to those coming from the $N_L$. We thus have

$$\delta \left( \frac{e'}{\epsilon} \right) = \frac{1}{6} \sqrt{(\Delta N_L^+)^2 + (\Delta N_L^0)^2} = \frac{1}{6} \sqrt{\frac{1}{N_L^+} + \frac{1}{N_L^0}} = \frac{1}{6} \sqrt{\frac{3}{2N_L^0}},$$

since by isospin symmetry there are twice as many charged two pion decays as neutral two pion decays. $N_L^0$ is given by the $\phi$ cross-section, times the integrated luminosity, times the efficiency for $K_L$ tags, times the $BR(\phi \to K_L K_S)$, times the $BR(K_L \to \pi^0 \pi^0)$, times the number of $K_L$’s that are within the fiducial volume (i.e., that are detectable):

$$N_L^0 = 5 \mu b \times 10^{10} \mu b^{-1} \times 2/3 \times 0.34 \times 10^{-3} \times (1 - e^{-150/350}) = 4 \times 10^6,$$

which gives as claimed,

$$\delta \left( \frac{e'}{\epsilon} \right) = 1 \times 10^{-4}.$$

7. Interferometry

Defining $\eta_i = \langle f_i | K_L \rangle / \langle f_i | K_S \rangle$, $\Delta t = t_1 - t_2$, $t = t_1 - t_2$, $\Delta \mathcal{M} = \mathcal{M}_L - \mathcal{M}_S$, $\mathcal{M} = \mathcal{M}_L + \mathcal{M}_S$, and $\mathcal{M} = M_{L,S} - i\Gamma_{L,S}$, the amplitude for decay to states $f_1$ at time $t_1$ and $f_2$ at time $t_2$, without identification of $K_S$ or $K_L$, is:

$$\langle f_1 | t_1, \mathbf{p}; f_2, t_2, -\mathbf{p} | i \rangle = \frac{1}{\sqrt{2}} \frac{1 + |\epsilon|^2}{1 - \epsilon^2} \times$$

$$\langle f_1 | K_S \rangle \langle f_2 | K_S \rangle e^{-i\mathcal{M}\Delta t/2} \left( \eta_1 e^{i\Gamma_L\Delta t/2} - \eta_2 e^{-i\Gamma_S\Delta t/2} \right).$$

The decay intensity $I(f_1, f_2, \Delta t = t_1 - t_2)$ to final states $f_1$ and $f_2$ is obtained from eq. (7.1) above by integrating over all $t_1$, $t_2$, with $\Delta t$ constant. For $\Delta t > 0$:

$$I(f_1, f_2; \Delta t) = \frac{1}{2} \int_{\Delta t}^{\infty} |A(f_1, t_1; f_2, t_2)|^2 dt =$$

$$\frac{1}{2T} \left| \langle f_1 | K_S \rangle \langle f_2 | K_S \rangle \right|^2 \left( |\eta_1|^2 e^{-\Gamma_L\Delta t} + |\eta_2|^2 e^{-\Gamma_S\Delta t} - 2|\eta_1||\eta_2| e^{-\Gamma\Delta t/2} \cos(\Delta m\Delta t + \phi_1 - \phi_2) \right),$$

with $\eta_i = A(K_L \to f_i)/A(K_S \to f_i) = |\eta_i| e^{i\phi_i}$, exhibiting interference terms sensitive to phase differences.
With the double ratio, DAΦNE complements future fixed target $\epsilon'/\epsilon$ experiments, but for interferometry, it is unique. It can even test CPT conservation. If one is completely general, and does not assume CPT conservation, eqs. (3.3) and (3.4) become

$$|K_S\rangle \propto \left((1 + \epsilon_K + \delta_K)|K^0\rangle + (1 - \epsilon_K - \delta_K)|\bar{K}^0\rangle\right)/\sqrt{2}$$

$$|K_L\rangle \propto \left((1 + \epsilon_K - \delta_K)|K^0\rangle - (1 - \epsilon_K + \delta_K)|\bar{K}^0\rangle\right)/\sqrt{2}.$$  

In Fig. 7 I show a sample interference pattern, taken from Ref. 1, for the process shown in Fig. 8. The determination of $\text{Re}(\epsilon'/\epsilon)$ comes from the difference in height of the two ‘shoulders.’ The large value of $\text{Re}(\epsilon'/\epsilon)$ is only to make the difference visible in the graph; KLOE will have a similar sensitivity to $\text{Re}(\epsilon'/\epsilon)$ from interference as that discussed in the previous section.

**Figure 7.** The interference pattern for $f_1=\pi^+\pi^-$, $f_2=\pi^0\pi^0 \Rightarrow \text{Re}(\epsilon'/\epsilon)$, $\text{Im}(\epsilon'/\epsilon)$.  

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With various final states $f_1$ and $f_2$, all $K$ system parameters can be measured independently. Just a few examples are:

1. $f_1 = f_2$: $\Gamma_S$, $\Gamma_L$ and $\Delta m$ can be measured, since all the phases disappear. Rates can be measured to $\times 10$ improvement in accuracy and $\Delta m$ to $\times 2$.

2. $f_1 = \pi^+\pi^-$, $f_2 = \pi^0\pi^0$: $\text{Re}(\epsilon'/\epsilon)$ and $\text{Im}(\epsilon'/\epsilon)$ can be measured, the former by concentrating on large time differences, the latter for $|\Delta t| \leq 5\tau_s$. $\text{Im}(\epsilon'/\epsilon)$ can be measured to accuracies of $10^{-3}$ (comparable to other future experiments of the same epoch, but using a completely different method).

3. $f_1 = \pi^+\ell^−\nu$ and $f_2 = \pi^-\ell^+\nu$: the CPT–violation parameter $\delta_K$ can be measured, the real part by concentrating on large time difference regions; and the imaginary part for $|\Delta t| \leq 10\tau_s$. (See Ref. 1 for a more complete list.)

8. Other Searches for CP violation

So far CP violation has only been seen in $K_L$ decays ($K_L \to \pi\pi$ and semileptonic decays). DAΦNE can look for $K_S \to \pi^0\pi^0\pi^0$, the counterpart to $K_L \to \pi\pi$. The branching ratio (BR) for this process is proportional to $\epsilon + \epsilon'_{000}$ where $\epsilon'_{000}$ is a quantity similar to $\epsilon'$, signalling direct CP violation. It is not as suppressed as the normal $\epsilon'$, perhaps a factor of twenty less. Nonetheless, as the expected BR is $2 \times 10^{-9}$, the whole signal will be at the 30 event level, and therefore there is here only the chance to see CP violation in a new channel, not direct CP violation. The current limit on this BR is $3.7 \times 10^{-5}$. Another possibility is to look at the difference in rates between $K_S \to \pi^+\ell^-\nu$ and $K_S \to \pi^-\ell^+\nu$, which is expected to be $\sim 16 \times 10^{-4}$ in one year’s running at DAΦNE, with an expected accuracy of $\sim 4 \times 10^{-4}$. Again this would be only a measurement of $\epsilon$, not $\epsilon'$, but the observation for the first time of CP violation in two new channels would be nonetheless of considerable interest.

CP violation can also be looked for in the decays of $K^{\pm}$. Here there will be probably no signal, but limits will be greatly improved.
9. Conclusions

Aside from being an excellent, dedicated environment for studying CP violation, where $\varepsilon'/\varepsilon$ will be determined to an accuracy of $10^{-4}$ in one year’s running, and all the $K$ system parameters will be determined via interferometry, DAΦNE will also be a rich source of many other physics results. DAΦNE will for example be a unique source of pure $K_S$, thanks to tagging, providing up to $10^{10}$ kaons per year, and measuring rare $K_S$ decay modes, most of which have not been measured yet, down to the $10^{-8}$ or $10^{-9}$ level. Rare charged $K$ decay modes will also be studied. DAΦNE will provide much input for CHPT, as Ulf Meissner has discussed. It will help us to understand the enigmatic $f_0$; it will be a place to study many rare decays, such as $\phi \to \eta \gamma$, and $\eta$ decays; to measure $\sigma(e^+e^- \to \text{hadrons})$ at energies up to 1.5 GeV, which is necessary for the calculation of the muon anomaly $a_\mu$; to study photon photon interactions; and even to test the non-local character of quantum theory. Finally, one will be able to understand better the strange sea quark content of the nucleon and study hypernuclei by looking at kaon-nucleon scattering. I conclude by listing in Table 2 some of the improvements that may be made in various rare decay BR limits.

| Decay mode | To date | Limits that can be achieved at DAΦNE |
|------------|---------|-----------------------------------|
| $\eta \to 3\gamma$ | $\text{BR} < 5 \times 10^{-4}$ | $1.4 \times 10^{-8}$ |
| $\eta \to \omega \gamma$ | $\text{BR} < 5 \times 10^{-2}$ | $10^{-9}$ |
| $\phi \to \rho \gamma$ | $\text{BR} < 2 \times 10^{-2}$ | $10^{-9}$ |
| $\phi \to \pi^+ \pi^- \gamma$ | $\text{BR} < 7 \times 10^{-3}$ | $10^{-9}$ |
| $\eta \to \pi^0 e^+ e^-$ | $\text{BR} < 4 \times 10^{-5}$ | $1.4 \times 10^{-8}$ |
| $\eta \to \pi^0 \mu^+ \mu^-$ | $\text{BR} < 5 \times 10^{-6}$ | $1.4 \times 10^{-8}$ |

Table 2. Rare decays at DAΦNE.

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