LES investigation on cavity shedding of a Clark-Y hydrofoil under different attack angle with an integration method

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Abstract. Numerical simulations of unsteady cavitating turbulent flow around a Clark-Y hydrofoil were performed using the Large Eddy Simulation method under different attack angles. The shedding of the cavity cloud was captured and the numerical results of the total vapour volume accords well with the experimental results. In this paper the concept of entrainment ability (generally adopted in water jet) was firstly introduced to describe the process of the cavity shedding. Based on the integral time averaged Navier-Stokes equations, the integration of $p + \rho v^2$ was adopted to describe the entrainment ability. A horizontal line from upstream to the leading edge was defined to monitor the flow rate and entrainment ability being transferred to the suction side. It is found that the entrainment ability directly determines the level of the re-entrant jet, which sees a sudden rise at the breakdown of the cavity cloud with the re-entrant jet reaching to the leading edge. Moreover, compared with no-cavitating flow, the entrainment ability was greatly hindered by the intensive cavity cloud over the foil. The pressure fluctuation at the trailing edge can be transferred upstream through the pressure side and influences the entrainment ability and flow rate across Line B ($Q_B$, Line B is defined as a horizontal line from upstream to the leading edge). In addition, as the attack angle increases, the time averaged $Q_B$ greatly increases, while the time averaged entrainment ability experiences a marginal rise. Thus, the variation of attack angle only impacts the entrainment ability from Line A ($m_A$), since $m_A$ varies linearly with $Q_B$.

1. Introduction
Cavitation, which presents serious devastations to devices and destroys the stability of system, is the most common phenomenon in hydraulic machine, such as pumps, nozzles, injectors, marine propellers, hydrofoils and valves [1]. When the local pressure drops below the vapour pressure, cavitation emerges with the negative pressures relieved by means of forming gas filled or gas and vapour filled cavities [2]. Generally, due to bubbles collapse and instability of cavitation, a hydraulic machine under cavity flow may experiences alteration of the performance, the appearance of additional forces on the
solid structures, production of noise and vibrations, and wall erosion [1]. Especially for hydrofoils, the unsteady behaviour of cavity flows and cavity shedding seriously impacts the performance of pumps and propellers, and consequently attracts great engineering attentions.

In the past decades, a series of researches based on experimental and numerical methods have been conducted to study the mechanisms of unsteady cavity shedding [3]-[6]. De Lange and De Bruin [7] tested transparent hydrofoils in a cavitation tunnel to show that the re-entrant jet velocity component normal to the cavity closure line was reflected into the cavity in the three-dimensional case, though the jet for the two-dimensional hydrofoil was directed upstream. O. Coutier-Delgosha et al. [4] applied a single-fluid model associated with a barotropic state law that governs the mixture density evolution to investigate the sheet cavitation and the simulated and experimental results see a reliable agreement on the internal structure of the cavity for incidence varying between 3° to 6°. Laberteaux and Ceccio [8] studied a series test of swept wedges to confirm that the cavity instability is greatly influenced by the span-wise pressure gradients and the re-entrant jet might be directed away from the cavity interface, allowing sheet cavitation to become cloud cavitation far downstream. Dular et al. [9] also numerically and experimentally investigated the re-entrant jet reflection at an inclined cavity closure line around a hydrofoil with an asymmetric leading edge, In addition, the Partial-Averaged Navier-Stokes (PANS) approach based on a $k-\varepsilon$ turbulence model, initially developed for aerodynamic flows, was tested for a cavitating flow around a marine propeller and the twisted hydrofoil by Ji et al. [10][11].

Recently more and more researchers resort to LES method to investigate the instability of partial cavitation which is more suitable to capture the flow details of the cavity shedding. G. Wang et al. [12] first presented a single-phase modeling concept of cavitation called a fifth-order polynomial weakly-compressible-fluid cavity model and studied the sheet/cloud cavitation on a NACA0015 hydrofoil with LES method. Luo et al. [13] simulated the cavity shedding around a three-dimensional twisted hydrofoil by LES method coupling with a mass transfer cavitation model based on the Rayleigh-Plesset equation and indicated that the re-entrant flow consisting of a re-entrant jet and a pair of side-entrant jets leaded to the cavity shedding. Using volume of fluid (VOF) technique to track the interface of liquid and vapour phase, Roohi et al. [14] also simulated the cavitating flow over a Clark-Y hydrofoil by LES method.

On the ground of the previous investigations, we conducted a new integral method to analysis the dynamics of the cavity shedding, especially the re-entrant jet, and LES method was adopted to simulate the cavitating flow around a Clark-Y hydrofoil. The integral method can be referred to the derivation of Hill number in confined turbulent jet [15]. The re-entrant jet attaching to the foil resembles the recirculation in the confined turbulent jet under a low flow rate ratio and the common ground of the two kinds of flow is the adverse pressure gradient. Hence the concept of entrainment ability, which is generally utilized to describe the status of the recirculation in confined turbulent jet, was introduced to describe the movement of the re-entrain jet during the cavity shedding. In addition, the relationship between the attack angle and the entrainment ability of the main flow was also concerned in this paper.

2. Numerical method

2.1. Cavitation model

As for the vapour/liquid two-phase mixture model, the fluid was assumed to be homogeneous, so the multiphase fluid components share the same velocity and pressure. When cavitation emerges, the control equations for vapour phase is

$$\frac{\partial}{\partial t}(\alpha, \rho_v) + \nabla(\alpha, \rho_v \vec{V}) = \dot{m}_v + \dot{m}_c$$

(1)

where $\alpha_v$ is the volume fraction of vapour, $\rho_v$ is the density of vapour, and the source terms $\dot{m}_v$ and $\dot{m}_c$ represent the mass transfer rate of evaporation and condensation.
The correlation of $\alpha$ and the number of bubbles per volume of liquid $n_b$ is determined by the direction of phase change. For the process of incipient vaporization or bubble growth, the main content of bubbles may be non-condensable gas and $n_b$ is calculated from:

$$n_b = (1 - \alpha_nuc) \frac{3\alpha_{nuc}}{4\pi R^3_a}$$

where $\alpha_{nuc}$ is the fraction of non-condensable gas in liquid. However during the condensation process, vapour fills the bubbles and the non-condensable gas can be neglected. Thus, $n_b$ is expressed as:

$$n_b = \frac{3\alpha_v}{4\pi R^3_a}$$

On the ground of Rayleigh-Plesset equations, neglecting the second-order terms and the surface tension force, the following equation is derived:

$$\frac{dR^3_b}{dt} = \frac{2}{3} \frac{|P_v - P_l|}{\rho_l}$$

where $P_v$ is the vapour pressure and $\rho_l$ is the density of liquid. Combining (3), (4) and (5), the algebraic expression of the source term $\dot{m}$ for the interphase mass transfer rate can be written as:

$$\dot{m} = n_b \frac{d\left(\rho_v 4\pi R^3_b / 3\right)}{dt} = 4\pi n_b \rho_v R^3_b \frac{dR^3_b}{dt}$$

where $P_v$ is the vapour pressure and $\rho_l$ is the density of liquid. Combining (3), (4) and (5), the algebraic expression of the source term $\dot{m}$ for the interphase mass transfer rate can be written as:

when $P \leq P_v$

$$\dot{m}_+ = F_{vap} \frac{3(1 - \alpha_nuc) \alpha_{nuc} \rho_v}{R^3_l} \left(\frac{2}{3} \frac{P_v - P_l}{\rho_l}\right)$$

when $P > P_v$

$$\dot{m}_- = F_{cond} \frac{3\alpha_v \rho_v}{R^3_l} \left(\frac{2}{3} \frac{P_v - P_l}{\rho_l}\right)$$

where $\alpha_{nuc}=5 \times 10^{-4}$ and $R^3_l=\times 10^{-6}$ in practical. According to Zwart’s recommendation [16], $F_{vap}$ and $F_{cond}$, the coefficient of evaporation and condensation, are set as 50 and 0.01.

2.2. Governing equations and LES method

Based on the incompressible Navier-Stokes equations, the governing equations consisting of the mass and momentum conservation equation can be written as follows:

$$\frac{\partial \rho_m}{\partial t} + \frac{\partial (\rho_m u_j)}{\partial x_j} = 0$$

$$\frac{\partial (\rho_m u_j)}{\partial t} + \frac{\partial (\rho_m u_j u_i)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j} \left(\mu \frac{\partial u_i}{\partial x_j}\right)$$

where $\mu$ is the dynamic viscosity and $\rho_m$ the mixture density, which are defined as:

$$\mu = \sum \alpha_i \mu_i = \alpha_v \mu_v + (1 - \alpha_v) \mu_l$$

$$\rho_m = \sum \alpha_i \rho_i = \alpha_v \rho_v + (1 - \alpha_v) \rho_l$$

In this work, a filtered variable shown as follow is adopted:
\[
\overline{f}(x) = \int_D f(x') G(x, x') dx'
\]  
(12)

Here, \(D\) is the fluid domain, and \(G\) is the function that determines the scale of the resolved eddies

\[
\overline{f}(x) = \frac{1}{V} \int_D f(x') dx', \quad x' \in D
\]
(13)

where \(V\) is the volume of a computational cell. The filter function, \(G(x, x')\), implied here is then

\[
G(x, x') = \begin{cases} 
\frac{1}{V},& x' \in V \\
0, & \text{otherwise}
\end{cases}
\]
(14)

Filtering the mass and momentum conservation equations by equation (14), LES equations are obtained:

\[
\frac{\partial \rho u_i}{\partial t} + \frac{\partial (\rho u_i u_j)}{\partial x_j} = 0
\]
(15)

\[
\frac{\partial (\rho u_i u_j)}{\partial t} + \frac{\partial (\rho u_i u_j u_j)}{\partial x_j} = \nu \frac{\partial^2 u_i}{\partial x_j \partial x_j} - \frac{\partial p}{\partial x_j} - \frac{\partial \tau_{ij}}{\partial x_j}
\]
(16)

where \(\tau_{ij}\) is the sub-grid scaled stress defined by

\[
\tau_{ij} = \rho_m \left( \frac{\partial \bar{u}_i}{\partial x_j} - \frac{\partial \bar{u}_j}{\partial x_i} \right)
\]
(17)

A popular sub-grid turbulence models (SGS model), on the ground of Boussinesq hypothesis, is referred to resolve the sub-grid-scales stresses, which is defined as

\[
\tau_{ij} = \frac{1}{3} \tau_{ii} \delta_{ij} = -2 \mu \bar{S}_{ij}
\]
(18)

where \(\bar{S}_{ij}\) is the rate-of-strain tensor for the resolved scale calculated by

\[
\bar{S}_{ij} = \frac{1}{2} \left( \frac{\partial \bar{u}_i}{\partial x_j} + \frac{\partial \bar{u}_j}{\partial x_i} \right)
\]
(19)

the eddy-viscosity \(\mu_t\) is modeled by a simple method proposed by Smagorinsky[17]

\[
\mu_t = \rho L_s^2 \left| \bar{S} \right|
\]
(20)

\[
\left| \bar{S} \right| = \sqrt{2 \bar{S}_{ii} \bar{S}_{ij}}
\]
(21)

where \(L_s\) is the mixing length for sub-grid scales computed using

\[
L_s = \min(\kappa d, C_s \Delta)
\]
(22)

where \(\kappa\) is the von Kármán constant, \(d\) is the distance to the closest wall, \(C_s\) is the Smagorinsky constant, and \(\Delta\) is the local grid scale (\(\Delta = \sqrt[3]{V}\), \(V\) is the volume of the computational cell). In addition, \(C_s\) in this work is set as 0.1 according to the reference [18].

### 2.3. Numerical strategy and mesh

The Clark-Y hydrofoil was chosen as the simulating prototype and Figure 1 shows the calculating domain. The chord length of the foil \((L_c)\) is 70 mm. The whole size of the calculating domain is \(14.4L_c \times 21.4L_c\). The distance between the domain inlet and the leading edge of the hydrofoil is \(5.7L_c\). The original point is set on the leading edge. As for the boundary condition, the inlet is set as a
uniform velocity inlet and the outlet a constant static pressure. The remained boundaries are walls. By
regulating the outlet pressure, we can obtain the certain inlet pressure corresponding to the cavitation
number demanded.

The pressure-velocity direct coupling method is used to solve the flow in the present simulation. The
pressure term was discretized by PRESTO scheme and the momentum terms by second order
upwind scheme. QUICK scheme is used for the vapour mass fraction transport equation. In addition,
the time-dependent second order implicit algorithm is used for transient formulation.

The meshes conducted by commercial code ANSYS ICEM are all structural quadrilateral
elements. The mesh size was initially set as 120,000 (coarse) and then increased to 560,000 (medium)
and 1,100,000 (fine) by doubling the cell number in x and y direction. The mesh independence for
cavitating flow was confirmed as shown in Table 1 and the medium mesh with 560,000 elements was
chosen in the following simulation. Figure 9 indicates the details of the final mesh along the foil. The
meshes near the wall was refined and the wall-normal distance of the first layer grid is \( \Delta y/L_c = 1.4 \times 10^{-5} \)
and the Reynolds number is around \( 7 \times 10^5 \). Y-plus \( (y^+) \) defined as \( y^+ = \Delta y u_\tau/\nu \) [19] is adopted to exam
the mesh resolution for LES method and the maximum \( y^+ \) on the wall of the foil is less than 1 during
the simulating work.

3. Integration method

Referring to the theory of confined jet, we conducted an integral method. Since the inception and
development of cavitation in shear flows are mainly controlled by their non-cavitating structure, the
integration is conducted under the steady and no-cavitating flow and the flow upstream is assumed to
be uniform. Figure 10 presents the schematic domain for integral. Three lines (Line A, Line B and Line
C as shown in Figure 10) were drawn to represent the integral boundary. Line A is a vertical line
passing through point \((-\infty, 0)\) and Line B a horizontal line cross point \((0, 0)\). Line C is an assumed
streamline to close the integral domain with Line A and Line B and the function expression is denoted
as \( y = C(x) \) or \( x = C^{-1}(y) \).

3.1. Conservation of mass flow rate

According to the integral continuous equation, the total flow rate across the closed region bounded by
Line A, B and C is zero and the following equation is derived,
\[ Q_A \cdot Q_B + Q_C = 0 \]  

Since Line C is defined as a streamline running upstream to original point which is across the separation point as drawn in Figure 10, the flow rate of Line C is 0. Then,

\[ Q_A = Q_B \]  

(24)

In addition, \( Q_A \) and \( Q_B \) can be calculated by the following equations:

\[ Q_A = u_{\infty} L \Delta z \]  

(25)

\[ Q_B = \int_0^\infty \rho v dx \Delta z \]  

(26)

where \( \Delta z \) is the unit length on spanwise, \( u_{\infty} \) is the velocity at the inlet and \( L \) is the length of Line A. Combine (24), (25) and (26), then \( L \) is calculated

\[ L_A = \frac{Q_B}{u_{\infty} \Delta z} = \int_0^\infty \rho v dx \Delta z \]  

(27)

3.2. Integration of the momentum equations

At first, the momentum equations in the Cartesian system can be written as follows:

\[
\begin{align*}
\rho \frac{\partial u}{\partial x} + \rho \frac{\partial v}{\partial y} &= -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \\
\rho \frac{\partial u}{\partial x} + \rho \frac{\partial v}{\partial y} &= -\frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)
\end{align*}
\]  

(28)

(29)

As for any horizontal line across the integral domain in Figure 10, the integral momentum equation on \( x \) direction can be written as:

\[ \int_{-\infty}^{C(y)} \rho \frac{\partial v}{\partial x} dx + \int_{-\infty}^{C(y)} \rho \frac{\partial \nu}{\partial y} dx = -\int_{-\infty}^{C(y)} \frac{\partial p}{\partial x} dx + \int_{-\infty}^{C(y)} \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 \nu}{\partial y^2} \right) dx \]  

(30)

Each term in equation (30) can be transformed as follows:

\[
\begin{align*}
\int_{-\infty}^{C(y)} \rho \frac{\partial v}{\partial x} dx &= (\rho \nu) \int_{-\infty}^{C(y)} - \int_{-\infty}^{C(y)} \rho \nu \frac{\partial u}{\partial x} dx = (\rho \nu) \int_{-\infty}^{C(y)} + \int_{-\infty}^{C(y)} \rho \nu \frac{\partial v}{\partial y} dx \\
&= (\rho \nu) \int_{-\infty}^{C(y)} + \frac{d}{dy} \int_{-\infty}^{C(y)} \frac{1}{2} \rho v^2 dx - \frac{1}{2} (v(C^{-1}(y), y))^2 \frac{dC^{-1}(y)}{dy} \\
\int_{-\infty}^{C(y)} \rho \frac{\partial \nu}{\partial y} dx &= \frac{d}{dy} \int_{-\infty}^{C(y)} \frac{1}{2} \rho v^2 dx - \frac{1}{2} (v(C^{-1}(y), y))^2 \frac{dC^{-1}(y)}{dy} \\
\int_{-\infty}^{C(y)} \frac{\partial p}{\partial x} dx &= -\frac{d}{dy} \int_{-\infty}^{C(y)} p(C^{-1}(y), y) \frac{dC^{-1}(y)}{dy}
\end{align*}
\]  

(31)

(32)

(33)

Substitute equations (31), (32) and (33) into (30) and the following expression is obtained:

\[
\frac{d}{dy} \int_{-\infty}^{C(y)} (p + \rho v^2) dx = \left[ p(C^{-1}(y), y) \right]_{-\infty}^{C(y)} \frac{dC^{-1}(y)}{dy} - (\rho \nu) \left[ \frac{d}{dy} \int_{-\infty}^{C(y)} + \int_{-\infty}^{C(y)} \mu \left( \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 \nu}{\partial y^2} \right) dx \right] (34)
\]

In the equation (34), the left term represents the variation of the entrainment ability of the flow across the integrated line on \( y \) direction. Term (a) and (b) are determined by flow condition on Line C.
(c) denotes the shearing force on Line C. Hence this equations means that the entrainment ability of the flow cross Line B is varied by the effect conducted from Line C. Then the entrainment ability of the flow crossing Line B can be expressed as the following dimensionless form,

\[ m_B = \frac{\int_0^\infty (p + \rho u^2) dx}{\rho u^2 L_z \Delta z} \]  

(35)

Note that this part of flow and the attached entrainment ability is transferred from the pressure side to the suction side.

Using the same integral method on Line A, we can also obtain the same definition of dimensionless entrainment ability, which is calculated as follows:

\[ m_A = \frac{\int_{C(-\infty)}^0 (p + \rho u^2) dy}{\rho u^2 L_z \Delta z} = \frac{\left( p_{\infty} + \rho u^2 \right)}{\rho u^2 L_z \Delta z} Q_B \]  

(36)

\[ m_A \propto Q_B \]  

(37)

To sum up, the flow crossing Line A is transferred to Line B and \( m_A \) sees a linear relationship with \( Q_B \). Moreover as we vary the attack angle \( \alpha \), \( m_A \) and \( m_B \) varies simultaneously. Hence there must be a correlation between \( \alpha \), \( m_A \) and \( Q_B \). More details will be analyzed in the following section on the ground of the simulated results.

4. Results and discussion

4.1. Experimental validation

The simulated results of cavity shedding is firstly compared with the experimental results \([5]\) with \( \alpha=8^\circ \) and \( \sigma=0.8 \) as shown in Figure 11. When \( t=0 \), the cavity cloud begins to grow at the leading edge and the shedding of previous circle occurs at the trailing edge. Then the attached cavity cloud grows more and the shed vapour moves downstream (Figure 11 (b) and (c)). In Figure 11 (d) the cavity cloud occupies most of the hydrofoil and experiences its maximum extent. Subsequently, breakdown of cavity starts \( (t=4T/9) \) and the re-entrant jet moves upstream which promotes the cavity shedding \( (t=5T/9 \text{ and } 6T/9) \). As the re-entrant jet reaches to the leading edge, the cavity cloud breaks thoroughly and the detached cavity cloud accumulated at the trailing edge as shown in Figure 11 (h) and (i). After that, a new circle happens with the cavity cloud starts again at the leading edge. Compare with the experimental results, the simulated cavity shedding process sees a good accordance. The breakdown of the cavity cloud is also captured by numerical method.
4.2. Influence of entrainment ability

According to the previous derivation, \( Q_B \) and \( m_B \) represent the entrainment ability integrated on Line A and Line B respectively. Figure 12 (a) the time evolution of \( Q_B \) and \( m_B \) with \( \alpha=8^\circ \) and \( \sigma=0.8 \) and the variation of \( Q_B \) under no-cavitating flow (\( \sigma=8.0 \)) is attached for comparing. Simultaneously the time evolution of the total vapour volume (\( V_{cav} \)) under the same period is also presented in Figure 12 (b). Obviously, \( m_B \) experiences a sudden rise at the breakdown of the cavity cloud with \( V_{cav} \) drops to the valley. As the breakdown of cavity cloud happens, the re-entrant jet moves to the leading edge and the entrainment ability shifting across Line B to the suction side enhanced greatly. Moreover, the flow rate across Line B (\( Q_B \)) under cavitating flow shares the same tends with \( m_B \), while it sees an apparent drop when \( m_B \) reaches the peak value as shown in Figure 12 (a) by red line. At this time, the re-entrant jet exerts a great effect on impeding the flow across Line B and subsequently increases the static pressure on Line B. Hence the entrainment ability directly correlated to the level of the re-entrant jet. The stronger the entrainment ability is, the stronger the re-entrant jet is.

In addition, \( Q_B \) under no-cavitating flow with \( \sigma=8.0 \) (shown by blue line in Figure 12 (a)), even though fluctuating slightly, varies little when compared with that under cavitating flow. Hence the fluctuation of \( Q_B \) induced by the shedding of the vortex at the trailing edge is negligible as compared to the influence of cavity cloud breakdown. Besides, the average value of \( Q_B \) under no-cavitating flow is far higher than that subjecting to cavitation. Thus, the cavity cloud tends to hinder the entrainment ability being transferred to the suction side.

![Figure 12](image_url)

**Figure 12** (a) Time evolution of entrainment ability, flow rate across Line B under cavitating and no-cavitating flow; (b)Time evolution of total vapour volume \( V_{cav} \) (\( \alpha=8^\circ \), \( \sigma=0.8 \))
4.3. Pressure distribution on suction side

Figure 13 presents the schematic diagram of these six points. There are six points set uniformly along the chord with point 1 at leading edge and point 6 at trailing edge, which is used for monitoring the variation of pressure along the foil.

The time evolutions of pressure coefficient at these six points are indicated in Figure 15. Except Point 1, the static pressure of the other five points drops to the vapour pressure when cavitation emerges and the cavity cloud covers the foil. The breakdown of cavity cloud can conducts a sudden shock to the pressure distribution at each suction point. As for Point 5 and Point 6, the pressure fluctuation tends to be more intensive than that at the other points due to the vortex shedding and cavity closure at the trailing edge. As for the other monitored points at the fore part, the pressure pulsation at the cavity breakdown only dominated by the re-entrant jet and the slight fluctuation generated at the trailing edge cannot be conveyed upstream to influence the pressure distribution at fore part. The cavity cloud on the suction side can counteract this kind of pressure pulsation.

However, $Q_B$ and $m_B$ as shown in Figure 12 (a) sees the same fluctuation being similar to the pressure distribution at Point 6. According to the previous equation (34), the flow condition on Line C plays a main role on influencing the entrainment ability. The fluctuation at the trailing edge can be transferred upstream through the pressure side, varies the flow condition on Line C, and consequently impacts on the entrainment ability conveyed to the suction side.

4.4. Influence of attack angle

The influences of the attack angle $\alpha$ on $m_B$ and $Q_B$ is also studied in this work. Figure 15 and Figure 16 present the time evolution of $Q_B$ and $m_B$ under each $\alpha$. When $\alpha=0^\circ$, there is only a few bubbles detached from the suction side and collapse immediately. The flow structure resembles that under no-cavitation condition. Hence, $Q_B$ keeps constant (Figure 8 in the blue line) and $m_B$ sees a slight fluctuation as bubbles generate and separate from the foil as shown in Figure 9 (a). However as $\alpha$ ranges from $4^\circ$ to $16^\circ$ and partial cavitation happens, $Q_B$ and $m_B$ experience a fierce fluctuation which is synchronized with the cavity shedding. The main fluctuating frequency of $Q_B$ and $m_B$ decreases with the decreasing shedding frequency. Since the thickness and length of cavity cloud tends to be larger when $\alpha$ increased, the re-entrant jet may cost more time on reaching to the leading edge.

Table 2 presents the time averaged $Q_B$ and $m_B$ (denoted as $Q_B^{\bar{}}$ and $m_B^{\bar{}}$) under each $\alpha$. As $\alpha$ increases from $0^\circ$ to $16^\circ$, $Q_B^{\bar{}}$ sees a significant rise from 33.84 kg/s to 103.29 kg/s, while $m_B^{\bar{}}$ varies little. Increasing the attack angle hinders more water on the pressure side and transfers more water to the suction side. Consequently the entrainment ability induced from Line A increases. However the time averaged entrainment ability on Line B nearly keeps constant regardless of $\alpha$. The integration of $p$ and $\rho v^2$ constitutes $m_B$. Although the term of $\rho v^2$ sees a certain increase due to the increased $Q_B$, while the main part, the integration of $p$, only experiences a fierce fluctuation cause by cavity cloud breakdown with the times averaged value hovering a certain level.
The entrainment ability directly determines the level of the re-entrant jet. The stronger the entrainment ability is, the greater the re-entrant jet is.

As the breakdown of the cavity cloud happens and the re-entrant jet reaches the leading edge, the entrainment ability sees a considerably rise.

Compared with no-cavitating flow, the entrainment ability was greatly hindered by the intensive cavity cloud over the foil.

The pressure fluctuations at the trailing edge is absorbed by the cavity cloud at the suction side, while it is transferred upstream through the pressure side and consequently influence the entrainment ability and flow rate across Line B.

As the attack angle increases, the time averaged flow rate crossing Line B greatly increases, while the time averaged entrainment ability experiences a marginal rise.

5. Conclusion

Large Eddy Simulation method combined with the mixture cavitation model was conducted to simulate the cavity shedding on a Clark-Y foil with different attack angles. The concept of entrainment ability (generally adopted in cater jet) was firstly introduced to describe the dynamics of cavity shedding. The following conclusions are obtained.

- The entrainment ability directly determines the level of the re-entrant jet. The stronger the entrainment ability is, the greater the re-entrant jet is.
- As the breakdown of the cavity cloud happens and the re-entrant jet reaches the leading edge, the entrainment ability sees a considerably rise.
- Compared with no-cavitating flow, the entrainment ability was greatly hindered by the intensive cavity cloud over the foil.
- The pressure fluctuations at the trailing edge is absorbed by the cavity cloud at the suction side, while it is transferred upstream through the pressure side and consequently influence the entrainment ability and flow rate across Line B.
- As the attack angle increases, the time averaged flow rate crossing Line B greatly increases, while the time averaged entrainment ability experiences a marginal rise.

Nomenclature

- \( C_d \): drag coefficient of hydrofoil defined by
  \[
  C_d = \frac{\text{drag}}{0.5 \rho V^2 \text{LW}}
  \]

- \( C_l \): lift coefficient of hydrofoil defined by
  \[
  C_l = \frac{\text{lift}}{0.5 \rho V^2 \text{LW}}
  \]

- \( C_p \): pressure coefficient defined by

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**Table 2** Time averaged \( Q_n \) and \( m_n \) under each \( \alpha \)

| \( \alpha \) \( \theta \) | \( \alpha = 0^\circ \) | \( \alpha = 4^\circ \) | \( \alpha = 8^\circ \) | \( \alpha = 16^\circ \) |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| \( Q_n \) (kg/s) | 33.84 | 73.32 | 90.32 | 103.29 |
| \( m_n \) | 2.37 | 2.41 | 2.45 | 2.49 |
\[ C_p = \frac{P - P_0}{0.5 \rho V_{cav}^2} \]

\( Q_A \) and \( Q_B \): flow rate across Line A and Line B
\( \bar{Q}_A \) and \( \bar{Q}_B \): time averaged \( Q_A \) and \( Q_B \)
\( L_c \): hydrofoil chord length
\( m_A \) and \( m_B \): entrainment ability on Line A and B
\( \bar{m}_A \) and \( \bar{m}_B \): times averaged \( m_A \) and \( m_B \)

\[ V_{cav}: \text{total vapour volume} \]
\[ \Delta z: \text{the width of the hydrofoil (set as 1000mm)} \]
\( \alpha: \text{attack angle} \]
\( \rho_l \) and \( \rho_v \): density of liquid and vapor
\( \sigma: \text{cavitation number} \]
\[ \sigma = \frac{P_{\infty} - P_1}{0.5 \rho V_{cav}^2} \]

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