Entanglement, CPT and neutral kaons

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Direct tests of T, CP, CPT symmetries in transitions using entangled neutral kaons produced at a φ-factory are briefly reviewed.

1. Introduction

Neutral kaons produced at a φ-factory combine their peculiar flavour oscillations, charge-parity (CP) and time-reversal (T) violation, into an Einstein-Podolsky-Rosen (EPR) entangled system, revealing surprising features. Here the possibility of exploiting their entanglement to make direct tests of T, CP, CPT symmetries is briefly reviewed.

2. Direct test of discrete symmetries with neutral kaons

In order to implement direct tests of T, CP, CPT symmetries in transitions, the preparation of the initial kaon state is performed exploiting the entanglement of $K^0\bar{K}^0$ pairs produced at a φ-factory. In this way, taken a kaon transition process as a reference, the exchange of $m$ and $out$ states required for a genuine test involving an anti-unitary transformation implied by time-reversal, can be easily implemented. In fact, the initial kaon pair produced in $\phi \rightarrow K^0\bar{K}^0$ decays can be rewritten in terms of any pair of orthogonal states:

$|i\rangle = \frac{1}{\sqrt{2}} \{ |K^0\rangle |\bar{K}^0\rangle - |\bar{K}^0\rangle |K^0\rangle \} = \frac{1}{\sqrt{2}} \{ |K^+\rangle |K^-\rangle - |K^-\rangle |K^+\rangle \}$. (1)

Here the states $|K^-\rangle$, $|K^+\rangle$ are defined as the states which cannot decay into pure CP $= \pm 1$ final states, $\pi\pi$ or $3\pi^0$, respectively. The condition of orthogonality $\langle K_- | K_+ \rangle = 0$, corresponds to assume negligible direct CP (or CPT) violation contributions, while the $\Delta S = \Delta Q$ rule is also assumed, so that the two flavor orthogonal eigenstates $|K^0\rangle$ and $|\bar{K}^0\rangle$ are identified by the charge of the lepton in semileptonic decays.
Thus, exploiting the perfect anticorrelation of the states implied by Eq. (1), it is possible to have a “flavor-tag” or a “CP-tag”, i.e. to infer the flavor ($K^0$ or $\bar{K}^0$) or the CP ($K_+ \rightarrow K_-$ or $K_- \rightarrow K_0$) state of the still alive kaon by observing a specific flavor decay ($\pi^+ \ell^- \nu$ or $\pi^- \ell^+ \bar{\nu}$) or CP decay ($\pi\pi$ or $3\pi^0$) of the other (and first decaying) kaon in the pair. Then the decay of the surviving kaon into a semileptonic ($\ell^+$ or $\ell^-$), $\pi\pi$ or $3\pi^0$ final state, filter the kaon final state as a flavor or CP state.

In this way one can identify a reference transition (e.g. $K^0 \rightarrow K_-$) and its symmetry conjugate (e.g. the CPT-conjugated $K_- \rightarrow K^0$), and directly compare them through the corresponding ratios of probabilities. The observable ratios for the various symmetry tests can be defined as follows:

\[ R_{2,T}^{\text{exp}}(\Delta t) \equiv \frac{I(\pi, \ell^+; \Delta t)}{I(\pi, \ell^+; \Delta t)} \cdot \frac{1}{D_{\text{CPT}}} \]

\[ = \frac{1}{1 - 4\Re \epsilon + 4\Re x_+ + 4\Re y} \]

\[ \times \left| 1 + \left( 2\epsilon + \epsilon'_{3\pi^0} + \epsilon'_{3\pi^0} \right) e^{-i(\lambda_S - \lambda_L)\Delta t} \right|^2 , \]  

\[ R_{4,T}^{\text{exp}}(\Delta t) \equiv \frac{I(\pi, \ell^-; \Delta t)}{I(\pi, \ell^-; \Delta t)} \cdot \frac{1}{D_{\text{CPT}}} \]

\[ = \frac{1}{1 + 4\Re \epsilon + 4\Re x_+ - 4\Re y} \]

\[ \times \left| 1 - \left( 2\epsilon + \epsilon'_{3\pi^0} + \epsilon'_{3\pi^0} \right) e^{-i(\lambda_S - \lambda_L)\Delta t} \right|^2 , \]  

\[ R_{2,\text{CP}}^{\text{exp}}(\Delta t) \equiv \frac{I(\pi, \ell^+; \Delta t)}{I(\pi, \ell^+; \Delta t)} \cdot \frac{1}{D_{\text{CPT}}} \]

\[ = \frac{1}{1 - 4\Re \epsilon - 4\Re x_- + 4\Re y} \]

\[ \times \left| 1 + \left( 2\epsilon + \epsilon'_{3\pi^0} - \epsilon'_{3\pi^0} \right) e^{-i(\lambda_S - \lambda_L)\Delta t} \right|^2 , \]  

\[ R_{4,\text{CP}}^{\text{exp}}(\Delta t) \equiv \frac{I(\pi, \ell^-; \Delta t)}{I(\pi, \ell^-; \Delta t)} \cdot \frac{1}{D_{\text{CPT}}} \]

\[ = \frac{1}{1 + 4\Re \epsilon - 4\Re x_- - 4\Re y} \]

\[ \times \left| 1 - \left( 2\epsilon + \epsilon'_{3\pi^0} + \epsilon'_{3\pi^0} \right) e^{-i(\lambda_S - \lambda_L)\Delta t} \right|^2 , \]  

\[ R_{2,\text{CPT}}^{\text{exp}}(\Delta t) \equiv \frac{I(\pi, \ell^+; \Delta t)}{I(\pi, \ell^+; \Delta t)} \cdot \frac{1}{D_{\text{CPT}}} \]

\[ = \frac{1}{1 - 4\Re \delta + 4\Re x_+ + 4\Re x_-} \]

\[ \times \left| 1 + \left( 2\delta + \epsilon'_{3\pi^0} - \epsilon'_{3\pi^0} \right) e^{-i(\lambda_S - \lambda_L)\Delta t} \right|^2 , \]  

(2)

(3)

(4)

(5)

(6)
\[ R_{4,\text{CPT}}^{\text{exp}}(\Delta t) \equiv \frac{I(\ell^+,3\pi^0;\Delta t)}{I(\pi\pi,\ell^+;\Delta t)} \frac{1}{D_{\text{CPT}}} \]

\[ = (1 + 4R\delta + 4Rx_+ + 4Rx_-) \times \left| 1 - (2\delta + \epsilon_3\pi^0 - \epsilon_\pi\pi) e^{-i(\lambda_\text{S} - \lambda_\text{L})\Delta t} \right|^2, \tag{7} \]

where \( I(f_1,f_2;\Delta t) \) are the double decay rates into decay products \( f_1 \) and \( f_2 \) as a function of the difference of kaon decay times \( \Delta t \), with \( f_1 \) occurring before \( f_2 \) decay for \( \Delta t > 0 \), and vice versa for \( \Delta t < 0 \). \( D_{\text{CPT}} \) is a constant factor that can be determined from measurable branching fractions and lifetimes of \( K_\text{S,L} \) states. For \( \Delta t = 0 \) one has by construction, within our assumptions, \( R_{4,\text{CPT}}^{\text{exp}}(0) = 1 \) (with \( S = T, \text{CP}, \text{or CPT}, \text{and } i = 2, 4 \)), and the measurement of any deviation from the prediction \( R_{4,\text{CPT}}^{\text{exp}}(\Delta t) = 1 \) imposed by the symmetry invariance is a direct signal of the symmetry violation built in the time evolution of the system. The following double ratios independent of the factor \( D_{\text{CPT}} \) can also be defined:

\[ DR_{T,\text{CP}}(\Delta t) \equiv \frac{R_{4,\text{T}}^{\text{exp}}(\Delta t)}{R_{4,\text{CP}}^{\text{exp}}(\Delta t)} \]

\[ = (1 - 8R\epsilon + 8Ry) \times \left| 1 + 2(2\epsilon + \epsilon_3\pi^0 + \epsilon_\pi\pi) e^{-i(\lambda_\text{S} - \lambda_\text{L})\Delta t} \right|^2, \tag{8} \]

\[ DR_{\text{CPT}}(\Delta t) \equiv \frac{R_{4,\text{CPT}}^{\text{exp}}(\Delta t)}{R_{4,\text{CPT}}^{\text{exp}}(\Delta t)} \]

\[ = (1 - 8R\delta - 8Rx) \times \left| 1 + 2(2\delta + \epsilon_3\pi^0 - \epsilon_\pi\pi) e^{-i(\lambda_\text{S} - \lambda_\text{L})\Delta t} \right|^2. \tag{9} \]

The r.h.s. of Eqs. (2)-(9) is evaluated to first order in small parameters and for not too large negative \( \Delta t \); \( \epsilon \) and \( \delta \) are the usual T and CPT violation parameters in the neutral kaon mixing, respectively, and \( \epsilon_{S,L} = \epsilon \pm \delta \) the CP impurities in the physical states \( K_\text{S} \) and \( K_\text{L} \); the small parameter \( y \) describes a possible CPT violation in the \( \Delta S = \Delta Q \) semileptonic decay amplitudes, while \( x_+ \) and \( x_- \) describe \( \Delta S \neq \Delta Q \) semileptonic decay amplitudes with CPT invariance and CPT violation, respectively. Therefore the r.h.s. of Eqs. (2)-(9) shows the effect of symmetry violations only in the effective Hamiltonian description of the neutral kaon system according to the Weisskopf-Wigner approximation, without the presence of other possible sources of symmetry violations. The small spurious effects due to the release of our assumptions are also shown, including possible \( \Delta S = \Delta Q \).
rule violations \((x_+, x_- ≠ 0)\) and/or direct CP and/or CPT violation effects \((\epsilon'_3, \epsilon'_\pi, y ≠ 0)\). In particular the \(\epsilon'\) effects are fully negligible in the asymptotic region \(\Delta t \gg \tau_S\). The KLOE-2 collaboration has recently analysed a data sample corresponding to an integrated luminosity \(L = 1.7 \text{ fb}^{-1}\) collected at the DAΦNE \(\phi\)-factory, and measured all eight observables defined in Eqs. (2)-(9) in the asymptotic region \(\Delta t \gg \tau_S\) (with positive \(\Delta t\)).\(^7\) These results constitute the first direct tests of \(T\) and CPT symmetries in kaon transitions. The \(T\) and CPT observables have been measured with a precision of few percent, showing no evidence of symmetry violations, while CP violation has been observed with more than 5\(\sigma\) significance with the observable ratio (5).

The double ratio \(DR_{\text{CPT}}\) in Eq. (9) in the asymptotic regime appears one of the best observable for testing CPT, free from approximations and model independent.\(^3\) Thus it seems best suited to extend the CPT tests to the general framework of the Standard Model Extension (SME) for CPT and Lorentz symmetry breaking,\(^8-14\) e.g. by studying the CPT observables as a function of 4-momenta of the kaons.\(^15\)

Finally it is worth noting that the full exploitation of time correlations of the two entangled kaons, not only \textit{from past to future}, as in the above mentioned direct \(T\), CP, and CPT tests, but also \textit{from future to past}, as described in Ref. 5 to post-tag special kaon states, might lead to the identification of new observables suitable to further extend these symmetry tests.

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