An Alternative Approach to Jaynes-Cummings Model with Dissipation at Finite Temperature

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Abstract

An alternative approach to the Jaynes-Cummings model (JCM) with dissipation at a finite environmental temperature is presented in terms of a new master equation under Born-Markovian approximations. An analytic solution of the dissipation JCM is obtained. A variety of physical quantities of interest are calculated analytically. Dynamical properties of the atom and the field are investigated in some detail. It is shown that both cavity damping and environmental temperature strongly affect nonclassical effects in the JCM, such as collapse and revivals of the atomic inversion, oscillations of the photon-number distribution, quadrature squeezing of the field and sub-Poissonian photon statistics.
1 Introduction

In the past few years there has been considerable interest in studying the Jaynes-Cummings (JCM) [1] with dissipation since the experimental verification of the collapse and revivals of Rabi oscillations [2-4]. A number of authors have treated the JCM with dissipation by means of analytic approximation [5,6] as well as numerical calculations [7-11]. Most of these work dealt with the JCM with dissipation only at the zero environmental temperature due to some technical difficulties. As is well known, in the JCM one must solve a master equation with cavity damping. Generally speaking, it is more difficult to solve the master equation analytically. To our knowledge, no analytic solution for this model has been given for general initial conditions and environmental temperatures. At zero temperature, Agaral and Puri [12] presented an analytic solution for the initial state of the field being a vacuum state; And Daeubler et al. [13] found an analytic expression for the atomic inversion and the intensity of the cavity field for the initial state of the field being a coherent state by using the quasiprobability-distribution technique [14]. At a finite temperature, only some numerical solutions [8,9,11] have been reported. The purpose of the this paper is intend to present an alternative approach to the dissipation JCM at a finite temperature by introducing a new master equation which comes from a simple coupling between the system and its environment. We study dynamical properties of the field in the JCM analytically. In particular, we show that both cavity damping and environmental temperature strongly affect nonclassical effects in the JCM, such as collapses and revivals of the atomic inversion, oscillations of the photon-number distribution, quadrature squeezing of the field, and sub-Poissonian photon statistics.

This paper is organized as follows: In Sec.2, a new master equation is presented and an explicit expression of the master equation for the JCM is obtained. In Sec.3, dynamical properties of the atom is investigated. The atomic inversion, the dipole moment of the atom and the atomic entropy are calculated analytically. Sec.4 is devoted to dynamics of the field. Oscillations of the photon-number distribution, quadrature squeezing of the field and sub-Poissonian photon statistics are discussed in detail. Finally, in Sec.5 we will summarize our results and give some concluding remarks.
2 Master Equation and its Analytic Solution for the JCM

Let $\hat{H}$ and $\hat{\rho}$ to be the Hamiltonian and the density operator of the JCM, respectively. We use a bath of harmonic oscillators to model the cavity damping which describes the irreversible motion of $\hat{\rho}$ caused by the environment consisting of an infinite set of harmonic oscillators. We assume that the atom-field system of the JCM the system interacting with the environment (or a reservoir) can be described by the total Hamiltonian

$$\hat{H}_T = \hat{H} + \sum_i \hbar \omega_i \hat{b}_i\hat{b}_i^+ + \hbar \hat{H} \sum_i C(\omega_i)(\hat{b}_i + \hat{b}_i^+) + \hbar^2 \hat{H}^2 \sum_i \frac{|C(\omega_i)|^2}{\omega_i} \quad (2.1)$$

where $\hat{b}_i$ and $\hat{b}_i^+$ are the boson annihilation and creation operators for the environment, in Eq.(2.1) the second term is the Hamiltonian of the reservoir, the third one represents the interaction between the system and the reservoir, and the last one is the renormalization term which compensates for the coupling-induced renormalization of the potential [15,16].

In the total Hamiltonian we have adopted a simple coupling between the system and the reservoir such that it satisfies the condition $[V, H] = 0$ ($V$ denotes the interaction term in (2.1)), which is required in the back-action evading and quantum-nondemolition schemes [17] and applied to decoherence of quantum system [18-20].

Following the standard master-equation approach [21], making use of the Hamiltonian (2.1) we can derive the following master equation for the reduced density operator in the Schrödinger picture under Born-Markovian approximations:

$$\frac{d\hat{\rho}_s(t)}{dt} = \frac{1}{i\hbar}[\hat{H}, \hat{\rho}(t)] - \gamma [\hat{H}, [\hat{H}, \hat{\rho}(t)]] - \Delta \omega [\hat{H}, \hat{\rho}(t)] \quad (2.2)$$

where

$$\gamma = \Delta \omega' + \frac{kT}{\hbar} \lim_{\omega \to 0} \frac{J(\omega)|C(\omega)|^2}{\omega}, \quad (2.3)$$

$$\Delta \omega = \Delta \omega' + 2i\mathcal{P} \int_0^\infty d\omega \frac{J(\omega)|C(\omega)|^2}{\omega} \quad (2.4)$$

with

$$\Delta \omega' = i\hbar \int_0^\infty d\omega \frac{J(\omega)|C(\omega)|^2}{\omega} \quad (2.5)$$

In the above equations, $J(\omega)$ is the spectral density of the reservoir, $\mathcal{P}$ is the Cauchy principal part of the integration [21], $T$ is the temperature of the environment and $k$ is the
Boltzmann constant. In the derivation of the master equation, we have assumed that the temperature $T$ is high enough so that the Markovian approximation is valid.

If we neglect the the Lamb shift term, i.e., the last term on rhs of Eq.(2.1), the master equation becomes

$$\frac{d\hat{\rho}(t)}{dt} = \frac{1}{i\hbar} [\hat{H}, \hat{\rho}(t)] - \gamma [\hat{H}, [\hat{H}, \hat{\rho}(t)]] \quad (2.6)$$

Notice that this equation has the same form as the Milburn’s equation [22] under diffusion approximation. However, they come from completely different physical mechanism. The former originates from the dissipation while the latter is from the uncontinuous and stochastic unitary evolution. In what follows we will use the master equation (2.6) to study the JCM with dissipation.

The resonant Jaynes-Cummings Hamiltonian describing an interaction of a two-level atom with a single-mode cavity field in the rotating-wave approximation is given by

$$\hat{H} = \hbar \omega (\hat{a}^+ \hat{a} + \frac{m}{2} \hat{\sigma}_3) + \hbar \lambda (\hat{\sigma}_- \hat{a}^+ + \hat{\sigma}_+ \hat{a}), \quad (2.7)$$

where $\omega$ is the resonant frequency of the cavity field and the atomic transition, $\lambda$ is the atom-field coupling constant; $\hat{a}$ and $\hat{a}^+$ are the field annihilation and creation operators, respectively; $\hat{\sigma}_3$ is the atomic-inversion operator and $\hat{\sigma}_\pm$ are the atomic “spin flip” operators which satisfy the relations $[\hat{\sigma}_+, \hat{\sigma}_-] = 2\hat{\sigma}_3$ and $[\hat{\sigma}_3, \hat{\sigma}_\pm] = \pm 2\hat{\sigma}_\pm$. For simplicity, we take $\hbar = 1$ throughout the paper.

We now find the analytic solution of the master equation (2.6) applied to the Hamiltonian (2.7). Moya-Cessa et al. [23] have obtained an formal solution of (2.6) for a Hamiltonian with a little difference from (2.7). In what follows we will present an explicit solution of the master equation for the Hamiltonian under our consideration. Following the approach in refs.[23,24], we introduce three superoperators $\hat{R}$, $\hat{S}$ and $\hat{T}$ which are defined through their actions on the density operator, respectively,

$$\exp(\hat{R}t)\hat{\rho}(0) = \sum_{k=0}^{\infty} \frac{(2\gamma t)^k}{k!} \hat{H}^k \hat{\rho}(0) \hat{H}^k \quad (2.8)$$

$$\exp(\hat{S}t)\hat{\rho}(0) = \exp(-i\hat{H}t)\hat{\rho}(0) \exp(i\hat{H}t) \quad (2.9)$$

$$\exp(\hat{T}t)\hat{\rho}(0) = \exp(-\gamma t\hat{H}^2)\hat{\rho}(0) \exp(-\gamma t\hat{H}^2) \quad (2.10)$$

where $\hat{\rho}(0)$ is the initial density operator of the atom-field system, and the Hamiltonian $\hat{H}$ is given by Eq.(2.7).
It can be checked that the master equation (2.6) has the following formal solution:

$$\dot{\rho}(t) = \exp(\hat{R}t) \exp(\hat{S}t) \exp(\hat{T}t)\rho(0)$$  \hspace{1cm} (2.11)

We assume that initially the field is prepared in the coherent state \( |z\rangle \) defined by

$$|z\rangle = \sum_{n=0}^{\infty} Q_n |n\rangle, \quad Q_n = \exp(-\frac{1}{2}|z|^2) \frac{z^n}{\sqrt{n!}}$$  \hspace{1cm} (2.12)

and the atom was prepared in its excited state \( |e\rangle \), so that the initial density operator of the atom-field system takes this form:

$$\hat{\rho}(0) = \begin{pmatrix} |z\rangle\langle z| & 0 \\ 0 & 0 \end{pmatrix}$$  \hspace{1cm} (2.13)

We divide the Hamiltonian (2.7) into a sum of two terms which commute with each other, i.e.,

$$\hat{H} = \hat{H}_o + \hat{H}_I, \quad [\hat{H}_o, \hat{H}_I] = 0$$  \hspace{1cm} (2.14)

with

$$\hat{H}_o = \omega \left( \begin{array}{cc} \hat{n} + \frac{1}{2} & 0 \\ 0 & \hat{n} - \frac{1}{2} \end{array} \right), \quad \hat{H}_I = \left( \begin{array}{cc} 0 & \lambda \hat{a} \\ \lambda \hat{a}^+ & 0 \end{array} \right)$$  \hspace{1cm} (2.15)

where \( \hat{n} = \hat{a}^\dagger \hat{a} \).

Similarly, we can make the following decomposition:

$$\hat{H}^2 = \hat{A} + \hat{B}, \quad [\hat{A}, \hat{B}] = 0$$  \hspace{1cm} (2.16)

where the representations of the operators \( \hat{A} \) and \( \hat{B} \) in the two-dimensional atomic basis take the following forms:

$$\hat{A} = \begin{pmatrix} \hat{S}_{n+1} & 0 \\ 0 & \hat{S}_n \end{pmatrix}, \quad \hat{B} = 2\lambda \omega \begin{pmatrix} 0 & \hat{a}(\hat{n} - \frac{1}{2}) \hat{a}^+ \\ (\hat{n} - \frac{1}{2}) \hat{a}^+ & 0 \end{pmatrix}$$  \hspace{1cm} (2.17)

where

$$\hat{S}_n = \omega^2 (\hat{n} - \frac{1}{2})^2 + \lambda^2 \hat{n}$$  \hspace{1cm} (2.18)

For convenience, we introduce the following auxiliary operator:

$$\hat{\rho}_2(t) = \exp(\hat{S}t) \exp(\hat{T}t)\hat{\rho}(0)$$  \hspace{1cm} (2.19)

From the definition of the superoperators and the initial condition (2.13), we find that

$$\hat{\rho}_2(t) = \exp(-i\hat{H}_It) \exp(-\gamma t\hat{B})\hat{\rho}_1(t) \exp(-\gamma t\hat{B}) \exp(i\hat{H}_It)$$  \hspace{1cm} (2.20)
Here the operator $\hat{\rho}_1(t)$ is defined by
\begin{equation}
\hat{\rho}(0) = \left( \begin{array}{cc} |\Psi(t)\rangle\langle\Psi(t)| & 0 \\ 0 & 0 \end{array} \right)
\end{equation}

where
\begin{equation}
|\Psi(t)\rangle = \exp\{-\gamma t[\omega^2(\hat{n} + \frac{1}{2})^2 + \lambda^2\hat{a}\hat{a}^+] \} |ze^{-i\omega t}\rangle
\end{equation}

For the exponential operators on the rhs of Eq.(2.20) we can find that
\begin{align}
\exp(-\gamma t\hat{B}) &= \left( \begin{array}{cc} \hat{X}_{n+1} & -\hat{Y}_n(t) \\ -\frac{\hat{Y}_{n+1}(t)}{\sqrt{n+1}} & \hat{X}_n(t) \end{array} \right) \\
\exp(-i\hat{H}_I t) &= \left( \begin{array}{cc} \hat{C}_{n+1} & -i\hat{S}_n(t) \\ -i\frac{\hat{S}_{n+1}(t)}{\sqrt{n+1}} & \hat{C}_n(t) \end{array} \right)
\end{align}

where
\begin{align}
\hat{C}_n(t) &= \cos(\lambda t \sqrt{n}), \quad \hat{S}_n(t) = \sin(\lambda t \sqrt{n}) \\
\hat{X}_n(t) &= \cosh[2\lambda\gamma\omega t(\hat{n} - \frac{1}{2})\sqrt{n}], \quad \hat{Y}_n(t) = \sinh[2\lambda\gamma\omega t(\hat{n} - \frac{1}{2})\sqrt{n}]
\end{align}

Then, from Eqs.(2.23) and (2.24) it follows that
\begin{equation}
\exp(-i\hat{H}_I t) \exp(-\gamma t\hat{B}) = \left( \begin{array}{cc} \hat{R}_{n+1}(t) & -\hat{a}\frac{\hat{Y}_n(t)}{\sqrt{n}} \\ -\hat{a}^+\frac{\hat{Y}_{n+1}(t)}{\sqrt{n+1}} & \hat{R}_n(t) \end{array} \right)
\end{equation}

where
\begin{align}
\hat{R}_n &= \hat{C}_n(t)\hat{X}_n(t) + i\hat{S}_n(t)\hat{Y}_n(t) \\
\hat{Y}_n &= \hat{C}_n(t)\hat{Y}_n(t) + i\hat{S}_n(t)\hat{X}_n(t)
\end{align}

Substituting Eq.(2.27) into Eq.(2.20), we can obtain an explicit expression for the operator $\hat{\rho}_2(t)$ as follows:
\begin{equation}
\hat{\rho}_2(t) = \left( \begin{array}{cc} \hat{\Psi}_{11}(t) & \hat{\Psi}_{12}(t) \\ \hat{\Psi}_{21}(t) & \hat{\Psi}_{22}(t) \end{array} \right)
\end{equation}

where we have used the following symbol:
\begin{equation}
\hat{\Psi}_{ij}(t) = |\Psi_i(t)\rangle\langle\Psi_j(t)|, \quad (i,j = 1,2)
\end{equation}

with
\begin{align}
|\Psi_1(t)\rangle &= \hat{R}_{n+1}(t)|\Psi(t)\rangle, \quad |\Psi_2(t)\rangle = -\frac{\hat{V}_n(t)}{\sqrt{n}}\hat{a}^+|\Psi(t)\rangle
\end{align}
where $|\Psi(t)\rangle$ is given by equation (2.22).

Taking into account the definition of the superoperator $\hat{R}$, through the action of the operator $\exp(\hat{R}t)$ on the operator $\hat{\rho}_2(t)$ one can obtain the following formal solution:

$$\hat{\rho}(t) = \sum_{k=0}^{\infty} \frac{(2\gamma t)^k}{k!} \hat{H}^k \hat{\rho}_2(t) \hat{H}^k$$  \hspace{1cm} (2.33)

where the operator $\hat{H}$ and $\hat{\rho}_2(t)$ are given by Eqs.(2.7) and (2.30), respectively.

Indeed, Eq.(2.33) is the exact solution of the master equation (2.6) for the resonant Jaynes-Cummings Hamiltonian (2.7) in the operator form. Although the form of the solution (2.33) is pleasant, it is unconvenient in use. In most cases of practical interest, one needs to know the matrix elements of the density operator $\hat{\rho}(t)$ in the two-dimensional atomic basis to calculate expectation values of observables.

In order to find the explicit form of the solution, we need that

$$\hat{H}^k = \left( \begin{array}{cc} \hat{f}_{n+1}(k) & \hat{a}^{\dagger} \sqrt{n} \hat{g}_n(k) \\ \hat{a}^{\dagger} \sqrt{n+1} \hat{g}_{n+1}(k) & \hat{f}_n(k) \end{array} \right)$$  \hspace{1cm} (2.34)

where

$$\hat{f}_n(k) = \frac{1}{2}(\varphi_n^k + \phi_n^k), \hspace{1cm} \hat{g}_n(k) = \frac{1}{2}(\varphi_n^k - \phi_n^k)$$  \hspace{1cm} (2.35)

with

$$\varphi_n = \omega(\hat{n} - \frac{1}{2}) + \lambda \sqrt{n}, \hspace{1cm} \phi_n = \omega(\hat{n} - \frac{1}{2}) - \lambda \sqrt{n}$$  \hspace{1cm} (2.36)

For convenience, we define the following matrix:

$$\hat{M}^{(k)}(t) = \hat{H}^k \hat{\rho}_2(t) \hat{H}^k$$  \hspace{1cm} (2.37)

From Eqs.(30) and (2.34) we can obtain its matrix elements as follows:

$$\hat{M}^{(k)}_{11}(t) = \hat{f}_{n+1}(k) \hat{\Psi}_{11}(t) \hat{f}_{n+1}(k) + \hat{a}^{\dagger} \hat{g}_n(k) \hat{\Psi}_{21}(t) \hat{f}_{n+1}(k) + \hat{f}_{n+1}(k) \hat{\Psi}_{12}(t) \hat{g}_n(k) \hat{a}^{\dagger} + \hat{a}^{\dagger} \hat{g}_n(k) \hat{\Psi}_{22}(t) \hat{g}_n(k) \hat{a}^{\dagger}$$  \hspace{1cm} (2.38)

$$\hat{M}^{(k)}_{22}(t) = \hat{g}_n(k) \hat{a}^{\dagger} \hat{\Psi}_{11}(t) \hat{a}^{\dagger} \hat{g}_n(k) \hat{\Psi}_{21}(t) \hat{g}_n(k) \hat{a}^{\dagger} + \hat{g}_n(k) \hat{\Psi}_{12}(t) \hat{f}_n(k) + \hat{f}_n(k) \hat{\Psi}_{22}(t) \hat{f}_n(k)$$  \hspace{1cm} (2.39)

$$\hat{M}^{(k)}_{21}(t) = (\hat{M}^{(k)}_{12}(t))^{\dagger}$$

$$= \hat{g}_n(k) \hat{a}^{\dagger} \hat{\Psi}_{11}(t) \hat{a}^{\dagger} \hat{f}_n(k) + \hat{f}_n(k) \hat{\Psi}_{21}(t) \hat{f}_n(k) + \hat{g}_n(k) \hat{\Psi}_{12}(t) \hat{g}_n(k) \hat{a}^{\dagger} + \hat{g}_n(k) \hat{\Psi}_{22}(t) \hat{g}_n(k) \hat{a}^{\dagger}$$  \hspace{1cm} (2.40)
where
\[
\hat{g}'_n(k) = \frac{1}{\sqrt{n}} \hat{g}_n(k)
\]  (2.41)

Therefore, the analytic solution of the master equation (2.6) for the Jaynes-Cummings Hamiltonian (2.7) can be expressed explicitly as follows:
\[
\hat{\rho}(t) = \left( \sum_{k=0}^{\infty} \frac{(2\gamma t)^k}{k!} \hat{M}_{11}^{(k)}(t) \sum_{k=0}^{\infty} \frac{(2\gamma t)^k}{k!} \hat{M}_{12}^{(k)}(t) \right)
\]  (2.42)

Making use of this solution, one can evaluate mean values of operators of interest. In the next two sections we will use it to investigate various dynamical properties of the dissipation JCM.

3 Dynamical Properties of the Atom

In this section, we shall study dynamical properties of the atom and discuss the influence of the dissipation on them. We will derive analytic expressions of the atomic inversion and the dipole momentum of the atom, and calculate the atomic entropy.

3.1 Collapse and revivals of the atomic inversion

The atomic inversion is defined as the probability of the atom being the excited state minus the probability of being the ground state, that is
\[
W(t) = Tr[\hat{\rho}_A(t)\hat{\sigma}_3]
\]  (3.1)

where \(\hat{\rho}_A(t)\) is the reduced density operator of the atom, it can be obtained through tracing over the field part in (2.42). Making use of the solution (2.42), one can rewrite the inversion (3.1) as
\[
W(t) = \sum_{k,n=0}^{\infty} \frac{(2\gamma t)^k}{k!} [\langle n | \hat{M}_{11}^{(k)} | n \rangle - \langle n | \hat{M}_{22}^{(k)} | n \rangle]
\]  (3.2)

We now calculate the two expectation values on the rhs of the above equation. Making use of Eqs.(2.38), (2.39) and (2.42) we find that
\[
\langle n | \hat{M}_{11}^{(k)}(t) | n \rangle = (f_{n+1}(k))^2 | \psi_1(n,t) |^2 + (g_{n+1}(k))^2 | \psi_2(n+1,t) |^2 \\
+ 2Re\{f_{n+1}(k)g_{n+1}(k)\psi_1^*(n,t)\psi_2(n+1,t)\}
\]  (3.3)

\[
\langle n | \hat{M}_{22}^{(k)}(t) | n \rangle = (g_n(k))^2 | \psi_1(n-1,t) |^2 + (f_n(k))^2 | \psi_2(n,t) |^2 \\
+ 2Re\{f_n(k)g_n(k)\psi_1(n,t)\psi_2^*(n-1,t)\}
\]  (3.4)
where functions \( f_n(k) \) and \( g_n(k) \) are given through replacing the number operator \( \hat{n} \) by the number \( n \) in Eq.(2.35), and we have introduced the following symbols:

\[
\psi_1(n, t) = \langle n | \Psi_1(t) \rangle, \quad \psi_2(n, t) = \langle n | \Psi_2(t) \rangle \tag{3.5}
\]

which can be explicitly written as

\[
\psi_1(n, t) = \frac{1}{2} Q_n \left\{ C_{n+1} \left[ 1 + \exp(4\gamma \lambda \omega t(n + \frac{1}{2})\sqrt{n + 1}) \right] - iS_{n+1} \left[ 1 - \exp(4\gamma \lambda \omega t(n + \frac{1}{2})\sqrt{n + 1}) \right] \right\} \exp(-\gamma t \phi_{n+1}^2) e^{-in\omega t} \tag{3.6}
\]

\[
\psi_2(n, t) = \frac{1}{2} Q_n \left\{ -C_n \left[ 1 - \exp(4\gamma \lambda \omega t(n - \frac{1}{2})\sqrt{n}) \right] + iS_n \left[ 1 + \exp(4\gamma \lambda \omega t(n - \frac{1}{2})\sqrt{n}) \right] \right\} \exp(-\gamma t \phi_n^2) e^{-i(n-1)\omega t} \tag{3.7}
\]

where \( C_n(t) \), \( S_n(t) \), and \( \phi_n \) are obtained from their corresponding operator form through the replacement: \( \hat{n} \rightarrow n \).

From Eqs.(3.3), (3.4), (3.6), and (3.7) it follows that

\[
\langle n | \hat{M}_{11}^{(k)}(t) | n \rangle = \frac{1}{4} |Q_n|^2 \left\{ \varphi_{n+1}^{2k} \exp[-2\gamma t \phi_{n+1}^2] + \phi_{n+1}^{2k} \exp[-2\gamma t \phi_{n+1}^2] \right. \\
+ 2 \varphi_{n+1}^k \phi_n^k \cos[2\lambda t \sqrt{n + 1}] \exp[-2\gamma t \phi_{n+1} \phi_{n+1}] \\
\left. \cdot \exp[-4\lambda^2 \gamma t(n + 1)] \right\} \tag{3.8}
\]

\[
\langle n | \hat{M}_{22}^{(k)}(t) | n \rangle = \frac{1}{4} |Q_{n-1}|^2 \left\{ \varphi_n^{2k} \exp[-2\gamma t \phi_n^2] + \phi_n^{2k} \exp[-2\gamma t \phi_n^2] \\
- 2 \varphi_{n+1}^k \phi_n^k \cos[2\lambda t \sqrt{n}] \exp[-2\gamma t \phi_n \phi_n] \\
\right. \\
\left. \cdot \exp[-4\lambda^2 \gamma t n] \right\} \tag{3.9}
\]

Substituting Eqs.(3.8) and (3.9) into Eq.(3.2), after summing over \( k \), we arrive at the result:

\[
W(t) = \sum_{n=0}^{\infty} |Q_n|^2 \exp[-4\lambda^2 \gamma t(n + 1)] \cos(2\lambda t \sqrt{n + 1}) \tag{3.10}
\]

which indicates that the revivals of the atomic inversion are destroyed in the time evolution due to the appearance of the decay factor \( \exp[-4\gamma \lambda^2 t(n + 1)] \) which comes from the contribution of the damping term in the master equation (2.6). Obviously, when \( \gamma = 0 \), the usual result can be recovered.
3.2 The dipole moment of the atom

As is known, the dynamical properties of the atom involve not only the atomic inversion but also the knowledge of the dynamics of coherence between the two atomic levels, which can be described by the dipole moment defined by

\[ D(t) = Tr[\hat{\rho}_A(t)\hat{\sigma}_-] \]  

(3.11)

which can be expressed explicitly as follows:

\[ D(t) = \sum_{k,n=0}^{\infty} \frac{(2\gamma t)^k}{k!} \langle n | \hat{\mathcal{M}}_{12}^{(k)} | n \rangle \]  

(3.12)

It is not trivial to evaluate the mean value on the rhs of the above equation. From Eq.(2.40) it follows that

\[ \langle n | \hat{\mathcal{M}}_{12}^{(k)}(t) | n \rangle = f_{n+1}(k)f_n(k)\psi_n^*(n,t)\psi_1(n,t) + f_{n+1}(k)g_n(k)\psi_1^*(n-1,t)\psi_1(n,t) + g_n(k)g_{n+1}(k)\psi_1^*(n-1,t)\psi_2(n+1,t) + f_n(k)g_{n+1}(k)\psi_2^*(n,t)\psi_2(n+1,t) \]  

(3.13)

Taking into account Eq.(2.35), one can express (3.13) as a more useful form:

\[ \langle n | \hat{\mathcal{M}}_{12}^{(k)}(t) | n \rangle = \frac{1}{4} \varphi_{n+1}^k \varphi_n^k [\psi_1^*(n-1,t)\psi_1(n,t) + \psi_2^*(n,t)\psi_1(n,t) + \psi_1^*(n-1,t)\psi_2(n+1,t) + \psi_2^*(n,t)\psi_2(n+1,t)] \]  

(3.14)

Substituting Eq.(3.14) into (3.12) and making use of Eqs.(3.6) and (3.7), after summing over \( k \) and through a lengthy calculation we obtain the result:

\[ D(t) = \frac{1}{4} \sum_{n=0}^{\infty} Q_n Q_n^* \exp \left\{ -\gamma t [\omega - \lambda a_-(n)]^2 \right\} \]  

\[ \cdot \left\{ \exp [4\gamma \omega t (na_+(n) - 1) \frac{1}{2} a_-(n)] \cdot e^{-i[\lambda a_-(n) + \omega]t} \right\} \]
\[ -\exp\left[-4\gamma \lambda \omega t \left( (n + \frac{1}{2})a_- (n) + \lambda \sqrt{n(n+1)} \right) \right] \cdot e^{i [\lambda a_+ (n) - \omega]t} \\
+ \exp\left[4\gamma \lambda \omega t (n - \frac{2}{2})a_- (n) - \lambda \sqrt{n(n+1)} \right] \cdot e^{-i [\lambda a_+ (n) + \omega]t} \\
- \exp\left[-4\gamma \lambda \omega t (na_+ (n) + \frac{1}{2})a_- (n)) \right] \cdot e^{-i [\lambda a_+ (n) - \omega]t} \] 

(3.15)

where

\[ a_{\pm} (n) = \sqrt{n} \pm \sqrt{n+1} \]  

(3.16)

### 3.3 The entropy of the atom

It is well known that if we assume that initially the atom and the field in the JCM are in a pure state, then at \( t > 0 \), the atom-field system evolves into an entangled state. In this entangled state the field and the atom separately are mixed states. Phoenix and Knight [25] have shown that entropy is the most appropriate quantity of measuring the purity of the quantum state in the JCM. The time evolution of the atomic (field) entropy reflects the time evolution of the degree of entanglement between the atom and the field. The higher the entropy is, the greater the entanglement between the atom and the field becomes.

The atomic entropy is defined in terms of the reduced density operator of the atom in this form:

\[ S_A(t) = -Tr[\hat{\rho}_A(t) \ln \hat{\rho}_A(t)] \]  

(3.17)

where the reduced density operator of the atom can be written as

\[ \hat{\rho}_A(t) = \begin{pmatrix} \lambda_{11} & \lambda_{12} \\ \lambda_{21} & \lambda_{22} \end{pmatrix} \]  

(3.18)

where

\[ \lambda_{ij} = \sum_{k,n=0}^{\infty} \frac{(2\gamma t)^k}{k!} \langle n | \hat{M}_{ij}^{(k)} | n \rangle, \quad (i, j = 1, 2) \]  

(3.19)

It is easy to see that the off-diagonal element \( \lambda_{12} \) equals the dipole moment of the atom, i.e., \( \lambda_{12} = D(t) \). Making use of Eqs. (3.8) and (3.9), from Eq. (3.19) one can find that

\[ \lambda_{11} = \frac{1}{2} \sum_{n=0}^{\infty} |Q_n|^2 \left\{ 1 + \exp[-4\lambda^2 \gamma t (n + 1)] \cos(2\lambda t \sqrt{n+1}) \right\} \]  

(3.20)

\[ \lambda_{22} = \frac{1}{2} \sum_{n=0}^{\infty} |Q_{n-1}|^2 \left\{ 1 - \exp[-4\lambda^2 \gamma t n] \cos(2\lambda t \sqrt{n}) \right\} \]  

(3.21)
Since the trace is invariant under a similarity transformation, one can go to a basis in which the reduced density operator of the atom is diagonal. Then, the atomic entropy (3.17) can be expressed as follows:

\[ S_A(t) = -\alpha_+ \ln \alpha_+ - \alpha_- \ln \alpha_- \]  

(3.22)

where

\[ \alpha_\pm = \frac{1}{2} \{ 1 \pm \sqrt{1 - 4(\lambda_{11}\lambda_{22} - |\lambda_{12}|^2)} \} \]  

(3.23)

4 Dynamical Properties of the Field

As is known, the field in the JCM can exhibit a lot of nonclassical effects in the time evolution, such as oscillations of the photon-number distribution, quadrature squeezing and sub-Poissonian photon statistics. In this section, we shall investigate dynamical properties of the cavity field in the presence of dissipation, and discuss the influence of the dissipation on these nonclassical effects.

4.1 Oscillations of the photon-number distribution

It is easy to show that the probability \( p(n, t) \) of finding \( n \) photons in the cavity field is given by

\[ p(n, t) = \sum_{k=0}^{\infty} \frac{(2\gamma t)^k}{k!} \left[ \langle n | \hat{M}_{11}^{(k)} | n \rangle + \langle n | \hat{M}_{22}^{(k)} | n \rangle \right] \]  

(4.1)

where two expectation values on the rhs of the above equation have been given explicitly in the previous section.

Substituting Eqs.(3.8) and (3.9) into (4.1), after summing over \( k \) we obtain

\[ p(n, t) = \frac{1}{2} \frac{|Q_n|^2}{|Q_n|^2} \left( 1 + \exp[-4\lambda^2\gamma t(n + 1)] \cos(2\lambda t \sqrt{n + 1}) \right) \]  

(4.2)

\[ + \frac{1}{2} \frac{|Q_{n-1}|^2}{|Q_{n-1}|^2} \left( 1 - \exp[-4\lambda^2\gamma tn] \cos(2\lambda t \sqrt{n}) \right) \]

With the help of this probability distribution, it is straightforward to obtain the mean number of photons in the cavity field with the result:

\[ \langle \hat{n}(t) \rangle = \bar{n} + \frac{1}{2} \frac{e^{-\bar{n}}}{\bar{n}!} \sum_{n=0}^{\infty} \frac{\bar{n}^n}{n!} \exp[-4\lambda^2\gamma t(n + 1)] \cos(2\lambda t \sqrt{n + 1}) \]  

(4.3)

where \( \bar{n} = |z|^2 \) is the initial mean photon number in the field. As expected, from expressions (4.2) and (4.3) we can see that there is two decay factors \( \exp[-4\lambda^2\gamma t(n + 1)] \)
and \(\exp[-4\lambda^2\gamma tn]\) which come from the damping term in the master equation (2.6). The oscillatory behaviors of \(p(n, t)\) and \(\langle \hat{n}(t) \rangle\) are weakened with increasing of the parameter \(\gamma\). Since the damping is proportional to the temperature of the environment, these oscillatory behaviors deteriorate with the increasing of the environmental temperature. When the damping vanishes, (4.2) and (4.3) reduce to

\[
p(n, t) = |Q_n|^2 \cos^2(\lambda t \sqrt{n + 1}) + |Q_{n-1}|^2 \sin^2(\lambda t \sqrt{n})
\]

\[
\langle \hat{n}(t) \rangle = \bar{n} + \frac{1}{2} - \frac{m}{2}e^{-\bar{n}} \sum_{n=0}^{\infty} \frac{\bar{n}^n}{n!} \cos(2\lambda t \sqrt{n + 1})
\]

which are just the usual expressions without dissipation.

### 4.2 Squeezing properties of the cavity field

We now study the quadrature squeezing of the field in the cavity field. We introduce the two slowly varying Hermitian quadrature components of the field \(\hat{X}_1\) and \(\hat{X}_2\) defined by, respectively,

\[
\hat{X}_1 = \frac{1}{2}(\hat{a}e^{i\omega t} + \hat{a}^* e^{-i\omega t}), \quad \hat{X}_2 = \frac{1}{2i}(\hat{a}e^{i\omega t} - \hat{a}^* e^{-i\omega t})
\]

The commutation of \(\hat{X}_1\) and \(\hat{X}_2\) is \([\hat{X}_1, \hat{X}_2] = \frac{i}{2}\). The variances \(\langle (\Delta X_i)^2 \rangle \equiv \langle \hat{X}_i^2 \rangle - \langle \hat{X}_i \rangle^2\) \((i = 1, 2)\) satisfy the Heisenberg uncertainty relation \(\langle (\Delta X_1)^2 \rangle \langle (\Delta X_2)^2 \rangle \geq \frac{1}{16}\). A state of the field is said to be squeezed when one of the quadrature components \(\hat{X}_1\) and \(\hat{X}_2\) satisfies the uncertainty relation \(\langle (\Delta X_i)^2 \rangle < \frac{1}{4}\). The degree of squeezing can be measured by the squeezing parameters [26] \(S_i\) \((i = 1, 2)\) defined by

\[
S_i = \frac{\langle (\Delta X_i)^2 \rangle - \frac{1}{2}|\langle [\hat{X}_1, \hat{X}_2] \rangle|}{\frac{1}{2}|\langle [\hat{X}_1, \hat{X}_2] \rangle|}
\]

which can be expressed in terms of the annihilation and creation operators of the field as follows:

\[
S_1 = 2\langle \hat{a}^+ \hat{a} \rangle + 2\text{Re}\langle \hat{a}^2 e^{i2\omega t} \rangle - 4(\text{Re}\langle \hat{a} e^{i\omega t} \rangle)^2
\]

\[
S_2 = 2\langle \hat{a}^+ \hat{a} \rangle - 2\text{Re}\langle \hat{a}^2 e^{i2\omega t} \rangle - 4(\text{Im}\langle \hat{a} e^{i\omega t} \rangle)^2
\]

Then, the condition for squeezing in the quadrature component can simply be written as \(S_i < 0\).
In principle, expectation values for any function $F(\hat{a}^+, \hat{a})$ are calculated by the following formula:

$$\langle F(\hat{a}^+, \hat{a}) \rangle = \sum_{n=0}^{\infty} \sum_{k=0}^{\infty} \frac{2^{\gamma}}{\sqrt{t}} \kappa! \left[ \langle n| \hat{M}_{11}^{(k)}(t) F(\hat{a}^+, \hat{a}) |n \rangle + \langle n| \hat{M}_{22}^{(k)}(t) F(\hat{a}^+, \hat{a}) |n \rangle \right]$$  (4.10)

However, it is generally not an easy matter to calculate expectation value of an arbitrary function $F(\hat{a}^+, \hat{a})$ for the field in the JCM. For the expectation value $\langle \hat{a} e^{i\omega t} \rangle$, through a tedious calculation we find that

$$\langle \hat{a} e^{i\omega t} \rangle = \frac{1}{4} \sum_{n=0}^{\infty} Q_n Q_n^* \exp[\gamma t (S_{n+1} + S_n)]$$

$$\cdot \left\{ a_+(n) T_1 \exp(2\gamma t \varphi_{n+1} \varphi_n) + a_-(n) T_2 \exp(2\gamma t \varphi_{n+1} \phi_n) + a_+(n) T_3 \exp(2\gamma t \varphi_{n+1} \phi_n) + a_+(n) T_4 \exp(2\gamma t \phi_{n+1} \phi_n) \right\}$$  (4.11)

where $S_n$, $\varphi_n$, and $\phi_n$ are given by the replacement: $\hat{n} \to n$ in Eqs.(2.18) and (2.36), and

$$T_1 = [C_{n+1}(t) - iS_{n+1}(t)][C_n(t) + iS_n(t)]$$

$$\cdot [X_{n+1}(t) - iY_{n+1}(t)][X_n(t) - iY_n(t)]$$  (4.12)

$$T_2 = [C_{n+1}(t) - iS_{n+1}(t)][C_n(t) - iS_n(t)]$$

$$\cdot [X_{n+1}(t) - iY_{n+1}(t)][X_n(t) + iY_n(t)]$$  (4.13)

$$T_3 = [C_{n+1}(t) + iS_{n+1}(t)][C_n(t) + iS_n(t)]$$

$$\cdot [X_{n+1}(t) + iY_{n+1}(t)][X_n(t) + iY_n(t)]$$  (4.14)

$$T_4 = [C_{n+1}(t) - iS_{n+1}(t)][C_n(t) - iS_n(t)]$$

$$\cdot [X_{n+1}(t) + iY_{n+1}(t)][X_n(t) + iY_n(t)]$$  (4.15)

where the functions $C_n(t)$, $S_n(t)$, $X_n(t)$, and $Y_n(t)$ are given by the replacement: $\hat{n} \to n$ in Eqs.(2.25) and (2.26).

Substituting the explicit expressions of the functions $C_n(t)$, $S_n(t)$, $X_n(t)$, and $Y_n(t)$ into Eqs.(4.12) ~ (4.15), from Eq.(4.11) we find that

$$\langle \hat{a} e^{i\omega t} \rangle = \frac{1}{4} \sum_{n=0}^{\infty} Q_n Q_n^* \left\{ a_+(n) \exp[i\lambda a_-(n) t] \exp[-b_-(n) \gamma t] + a_-(n) \exp[-i\lambda a_-(n) t] \exp[-b_+(n) \gamma t] + a_-(n) \exp[i\lambda a_+(n) t] \exp[-c_-(n) \gamma t] + a_+(n) \exp[-i\lambda a_+(n) t] \exp[-c_+(n) \gamma t] \right\}$$  (4.16)
where
\[ b_\pm(n) = [\omega \pm \lambda a_-(n)]^2, \quad c_\pm(n) = [\omega \pm \lambda a_+(n)]^2 \] (4.17)
Similarly, we can find that
\[ \langle \hat{a}^2 e^{i2\omega t} \rangle = \frac{1}{4} \sum_{n=0}^{\infty} \sqrt{n} Q_n Q_{n-2} \exp[-\gamma t(S_{n+1} + S_{n-2})] \]
\[ \cdot \{ a'_\pm(n) T'_1 \exp(2\gamma t \varphi_{n+1} \varphi_{n-1}) + a'_-(n) T'_2 \exp(2\gamma t \varphi_{n+1} \phi_{n-1}) \]
\[ + a'_- T'_3 \exp(2\gamma t \varphi_{n-1} \phi_{n+1}) + a'_+ T'_4 \exp(2\gamma t \phi_{n+1} \phi_{n-1}) \} \] (4.18)
where
\[ a'_\pm = \sqrt{n-1} \pm \sqrt{n+1} \] (4.19)
and
\[ T'_1 = [C_{n+1}(t) - iS_{n+1}(t)][C_{n-1}(t) + iS_{n-1}(t)] \]
\[ \cdot [X_{n+1}(t) - iY_{n+1}(t)][X_{n-1}(t) - iY_{n-1}(t)] \] (4.20)
\[ T'_2 = [C_{n+1}(t) - iS_{n+1}(t)][C_{n-1}(t) - iS_{n-1}(t)] \]
\[ \cdot [X_{n+1}(t) - iY_{n+1}(t)][X_{n-1}(t) + iY_{n-1}(t)] \] (4.21)
\[ T'_3 = [C_{n+1}(t) + iS_{n+1}(t)][C_{n-1}(t) + iS_{n-1}(t)] \]
\[ \cdot [X_{n+1}(t) + iY_{n+1}(t)][X_{n-1}(t) - iY_{n-1}(t)] \] (4.22)
\[ T'_4 = [C_{n+1}(t) + iS_{n+1}(t)][C_{n-1}(t) - iS_{n-1}(t)] \]
\[ \cdot [X_{n+1}(t) + iY_{n+1}(t)][X_{n-1}(t) + iY_{n-1}(t)] \] (4.23)
Substituting the explicit expressions of the functions \( C_n(t), S_n(t), X_n(t), \) and \( Y_n(t) \) into the above equations, from Eq.(4.18) we find that
\[ \langle \hat{a}^2 e^{i2\omega t} \rangle = \frac{1}{4} \sum_{n=0}^{\infty} \sqrt{n} Q_n Q_{n-2} \cdot \{ a'_\pm n \exp[i\lambda a'_-(n)t] \exp[-2b'_-(n)\gamma t] \]
\[ + a'_+ n \exp[-i\lambda a'_-(n)t] \exp[-b'_-(n)\gamma t] \]
\[ + a'_-(n) \exp[i\lambda a'_+(n)t] \exp[-c'_-(n)\gamma t] \]
\[ + a'_-(n) \exp[-i\lambda a'_+(n)t] \exp[-c'_+(n)\gamma t] \} \] (4.24)
where
\[ b_+'(n) = [2\omega + \lambda a_-(n)]^2, \quad c_+'(n) = [2\omega + \lambda a_+(n)]^2 \] (4.25)

So far we have completed calculations of all expectation values needed in the squeezing parameters Eqs.(4.8) and (4.9). It is straightforward to obtain the two squeezing parameters \( S_1 \) and \( S_2 \) through the simple substitution of the expectation values (4.3), (4.11), and (4.24) into Eqs.(4.8) and (4.9).

We now analyze influence of the dissipation on the quadrature squeezing of the cavity field. It is a well-known fact that without dissipation, the field in the JCM governed by the von Neumann equation exhibits quadrature squeezing [27,28]. From Eq.(2.6) we can see that when the damping vanishes, it reduces to the conventional von Neumann equation. This means that in the JCM with dissipation there exists quadrature squeezing of the field when \( \gamma \rightarrow 0 \). Then, taking into account Eqs.(4.3), (4.11) and (4.24), from Eqs.(4.8) and (4.9) we can see that the damping term in equation (2.6) leads to the appearance of decay factors in each term in the expressions of the squeezing parameters. Thus, each term of \( S_1 \) and \( S_2 \) decays with the decrease of the damping parameter \( \gamma \). In particular, at an arbitrary time \( t \), \( \gamma t \gg 1 \), we find that
\[ S_1 = S_2 = 2|z|^2 + 1 > 0 \] (4.26)
which means that the quadrature squeezing of the field vanishes with the increase of the damping. Therefore, we can conclude that the dissipation suppresses the quadrature squeezing of the field in the JCM.

### 4.3 Sub-Poissonian photon statistics

A good measure of the extent to which the photon statistics of a state of the light is sub-Poissonian is the Mandel’s \( Q \) parameter [29] defined by
\[ Q = \frac{\langle \hat{n}^2 \rangle - \langle (\Delta \hat{n})^2 \rangle}{\langle \hat{n} \rangle} \] (4.27)
which characterizes the departure from Poissonian photon statistics. The Mandel parameter vanishes for a Poissonian distribution. When \( Q < 0 \), the photons are sub-Poissonian while for \( Q > 0 \) the photons are super-Poissonian.
The expectation value \( \langle \hat{n} \rangle \) in (4.27) has been evaluated in Eq.(4.3). And the expectation value \( \langle \hat{n}^2 \rangle \) can be expressed as

\[
\langle \hat{n}^2 \rangle = \sum_{n=0}^{\infty} n^2 p(n, t) \tag{4.28}
\]

From Eqs.(4.2) and(4.28), it follows that

\[
\langle \hat{n}^2 \rangle = (1 + \bar{n})^2 - \frac{1}{2} e^{-\bar{n}} \sum_{n=0}^{\infty} \frac{(2n + 1)}{n!} (\bar{n})^n \exp[-4\lambda^2 \gamma t (n + 1)] \cos[2\lambda t \sqrt{n + 1}] \tag{4.29}
\]

where \( \bar{n} = |z|^2 \).

It is straightforward to get the exact expression of the Mandel’s parameter \( Q \) through the substitution of Eqs.(4.28) and (4.29) into Eq.(4.27).

As is known, when the damping vanishes, i.e., \( \gamma = 0 \), the cavity field in the JCM can exhibit sub-Poissonian distribution [30] in the time evolution governed by the von Neumann equation. Moreover, from Eqs.(4.3) and (4.29) we can see that for an arbitrary given time \( t \), when \( \lambda^2 \gamma t \gg 1 \), we find that

\[
\langle \hat{n} \rangle = \bar{n} + \frac{1}{2} \quad \langle \hat{n}^2 \rangle = (1 + \bar{n})^2 \tag{4.30}
\]

which leads to

\[
Q = \frac{4\bar{n} + 1}{4\bar{n} + 2} > 0 \tag{4.31}
\]

This indicates that the state of the cavity field is super-Poissonian. Therefore, we can conclude that the state of the field in the JCM changes from sub-Poissonian (or Poissonian) distribution to the super-Poissonian distribution with the increase of the damping.

5 Conclusions

In this paper we have presented an alternative approach to deal with the JCM with the dissipation at finite temperature by introducing a new master equation for the reduced density operator of the system under the Born-Markovian approximations. We have found the analytic solution of the master equation for the JCM when initially the field is in a coherent state and the atom in its excited state. We have studied dynamical properties of the atom and the field in the JCM in detail. We have obtained analytic expressions of the atomic inversion the atomic dipole moment and the atomic entropy. We have investigated influence of the dissipation on nonclassical effects in the JCM, such as revivals of the
atomic inversion, oscillations of the photon-number distribution, the quadrature squeezing of the cavity field, and sub-Poissonian photon statistics. In particular, we have shown that the dissipation suppresses nonclassical effects in the JCM. In our model, the damping $\gamma$ linearly increases with the environmental temperature, therefore these nonclassical effects deteriorate with increasing the environmental temperature.

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References

[1] Jaynes, E.T., and Cummings, F.W., 1963, *Proc. IEEE* **51**, 89; Eberly, J.H., Narozhny, N.B., and Sanchez-Mondragon, J.J., 1980, *Phys. Rev. Lett.* **44**, 1323; For a recent review, see Shore, B.W., and Knight, P.L., 1993, *J. Mod. Opt.*, **40**, 1195.

[2] Rempe, G., Walther, H., and Klein, N., 1987, *Phys. Rev. Lett.*, **58**, 353.

[3] Brune, M., Raimond, J.M., Goy, P., Davidovich, L., and Haroche, S., 1987, *Phys. Rev. Lett.*, **59**, 1899; Brune, M., Raimond, J.M., and Haroche, S., 1987, *Phys. Rev.* **A35**, 154.

[4] Diedrich, F., Kruse, J., Rempe, G., Scully, M.O., and Walther, H. 1988, *IEEE J. Quantum Electron.*, **24**, 1495.

[5] Barnett, S.N., and Knight, P.L., 1986, *Phys. Rev.*, **A33**, 2444.

[6] Puri, R.R., and Agarwal, G.S., 1987, *Phys. Rev.*, **A35**, 3433.

[7] Quang, T., Knight, P.L., and Bužek, V., 1991, *Phys. Rev.*, **A44**, 6069.

[8] Eiselt, J., and Risken, H., 1989, *Opt. Commun.*, **72**, 351; 1991, *Phys. Rev.*, **A43**, 346.

[9] Werner, M.J., and Hisken, H., 1991, *Phys. Rev.*, **A44**, 4623.

[10] Gea-Banacloche, J., 1993, *Phys. Rev.*, **A47**, 2221.

[11] Englert, B.G., Naraschewski, M., and Schenzle, A., 1994, *Phys. Rev.*, **A50**, 2667.

[12] Agarwal, G.S., and Puri, R.R., 1986, *Phys. Rev.*, **A33**, 1757.

[13] Daeubler, B., Risken, H., and Schoendorff, L., 1992 *Phys. Rev.*, **A46**, 1654.

[14] Cahill K.E., and Glauber, R.J. 1969, *Phys. Rev.*, **177**, 1857; *ibid.*, **177**, 1882.

[15] Caldeira, A.O., and Leggett, A.J. 1983, *Ann. Phys.*, **149**, 374; Leggett, A.J., Chakravarty, S., Dorsey, A.T., Fisher, M.P.A., Garg, A., Zweger, W., 1987, *Rev. Mod. Phys.*, **59**, 1.

[16] Grabert, H., Schramm, P., and Gert-Ludwig Ingold, 1988, *Phys. Rep.*, **168**, 115.
[17] Caves, C., Thorne, K., Drever, R., Sandberg, V., and Zimmermann, M., 1980, Rev. Mod Phys., 52, 341.

[18] Zurek, W., 1981, Phys. Rev., D24, 1516.

[19] Tameshtit, A., and Sipe, J.E., 1992, Phys. Rev., A45, 8280; 1993, ibid, A47, 1697.

[20] Shao, J., Ge, M.L. and Cheng, H., 1994, Decoherence of Quantum-nomdemolition System, preprint.

[21] Louisell, W.H., 1973, Quantum Statistical Properties of Radiation (Wiley, New York)

[22] Milburn, G.J., 1991, Phys. Rev., A44, 5401; 1993, ibid, A47, 2415.

[23] Moya-Cessa, H., Bužek, V., Kim, M.S., and Knight, P.L., 1993, Phys. Rev., A48, 3900; Kuang, L.M., Chen, X., and Ge, M.L., 1994, Influence of Decoherence on Non-classical effects in the Jaynes-Cummings Model, preprint.

[24] L. M. Kuang and X. Chen, J. Phys., A27, L633 (1994); X. Chen and L. M. Kuang, Phys. Lett., A191, 18 (1994)

[25] Phoenix, S.J.D., and Knight, P.L., 1988, Ann. Phys., 186, 381.

[26] Bužek, V., Vidiella-Barranco, A., and Knight, P.L., 1992, Phys. Rev., A45, 6570.

[27] Gerry, C.C., and Mayer, P.J., 1988, Phys. Rev., A38, 5665; Shumovsky, A.S., Kien, F.L., and Aliskenderov, E.I. 1987, Phys. Lett., A124, 351; Goldberg, P., and Harrison, L.C., 1991, Phys.Rev., A43,376.

[28] Meystre, P., and Zubairy, M.S., 1982, Phys. Lett., A89, 390; Peng, J.S., and Li, G.X., 1993, Phys. Rev., A47, 3167; Cohen, D., Ben-Aryeh, Y., and Mann, A., 1994, Phys. Rev., A49, 2040.

[29] Mandel, L., 1979, Opt. Lett. 4, 205.

[30] Aliskenderov, E.I., and Dung, H.T., 1993, Phys. Rev., A48, 1604.