D-Brane solutions in a light-like linear dilaton background

Rashmi R. Nayak\textsuperscript{a,}\textsuperscript{*}, and Kamal L. Panigrahi\textsuperscript{a,b}\textsuperscript{†}

\textsuperscript{a} Dipartimento di Fisica, Universita' & INFN Sezione di Genova, “Genova”
Via Dodecaneso 33, 16146, Genova, Italy

\textsuperscript{b} Physics Department, Indian Institute of Technology, Guwahati, 781 039 India

Abstract

The light-like linear dilaton background presents a simple time dependent solution of type II supergravity equations of motion that preserves 1/2 supersymmetry in ten dimensions. We construct supergravity D-brane solutions in a linear dilaton background starting from the known intersecting brane solutions in string theory. By applying a Penrose limit on the intersecting (NS1 − NS5 − NS5')-brane solution, we find out a D5-brane in a linear dilaton background. We solve the Killing spinor equations for the brane solutions explicitly, and show that they preserve 1/4 supersymmetry. We also find a M5-brane solution in eleven dimensional supergravity.

\textsuperscript{*}e-mail: Rashmi.Nayak@ge.infn.it
\textsuperscript{†}e-mail: Kamal.Panigrahi@ge.infn.it,panigrahi@iitg.ernet.in
Study of time dependent physics is interesting and challenging. Till now our knowledge of time dependent phenomena in string theory is quite limited, and hence it is important to learn more about them. Often they contain space time singularities, and the challenge is that in order to make definite statements about the perturbative physics we should learn how to resolve the singularities. One of the key problems of quantum gravity is to solve the cosmic singularity, which is widely believed to be resolved in the framework of string theory. Unlike the orbifold or the conifold singularity, the big bang singularity is space-like, and its resolution requires knowledge of string theory in time dependent or cosmological backgrounds. For example, the time dependent orbifold models have been constructed to address the issues of resolution of and physics near the cosmological singularities [1, 2, 3]. The time dependent orbifold models are promising as they are supersymmetric and there is no null killing vectors. So the problem of particle production was absent and it was fairly simple to solve these models. However, it turns out that these models are unstable due to the large blue shift effect and so on [4, 5, 6]. There are several other attempts for studying string theory in time dependent background with cosmological singularity and some related behavior in (see for example: [7]-[19]). Another class of time dependent model has been proposed in the form of the ‘rolling’ of the open string tachyon [20] on an unstable brane (brane-anti brane pair). Recently, there are attempts to replace the cosmological singularity by a closed string tachyon condensation phase and study the perturbative string amplitudes [21]. More recently, the “matrix big bang” proposal has been put forth in [22]. It has been argued that in a light-like linear dilaton background the matrix degrees of freedom, rather than the point particles or the perturbative string states, explain the correct physics near the big-bang singularity. In this context, the dual matrix string description has been given in terms of a 2-dimensional supersymmetric Yang-Mills theory on a time dependent worldsheet that is the Milne orbifold of a 2-dimensional flat Minkowski space. This background, in particular, presents a simple time dependent solution of type II supergravity equations of motion which preserves 1/2 supersymmetry in ten dimensions. In order to

\[\text{1 see [23]-[43] for related work along this line.}\]
have a complete picture of what is going on, one should try to learn more about the dual gauge theory of some class of time dependent solutions with a linear dilaton. It might help us to understand the physics near the singularity from the dual gauge theory viewpoint. Supergravity backgrounds often help in understanding the nature of time dependent sources in gauge theory, and hence it is important to investigate such solutions in detail. In view of finding out time dependent solutions in string theory, and to learn about the physics near the space-like singularity, in this paper we construct D-brane solutions in a linear dilaton background. Starting from the intersecting brane solutions of type $1_{NS} + 5_{NS} + 5'_{NS}$ in supergravity, we obtain D5-brane solutions by applying a particular Penrose limit \[22\]. We further analyze the supersymmetric properties of the solution that we found.

The rest of the paper is organized as follows. In section-2 we make a quick review of the linear dilaton background that can be obtained by taking a particular Penrose limit of a stack of NS5-branes in the near horizon region. Section-3, is devoted to the construction of D-brane supergravity solutions by the application of a Penrose limit on a configuration of intersecting branes in supergravity. We also investigate the geodesic equations, construct a M5-brane solution in eleven dimensional supergravity. In section-4, we investigate the space-time supersymmetry of the D5-brane solution. We present our conclusions in section-5.

2 Light-like linear dilaton background as a Penrose limit

It was pointed out in \[22\] that a light-like linear dilaton can be obtained by taking a particular ‘Penrose limit’ on a stack of NS5-branes in type II string theory. The supergravity solution of a stack of NS5-branes can be written in string frame as,

\[
\begin{align*}
\text{ds}^2 &= -dt^2 + dy_5^2 + H(r) \left( dr^2 + r^2 (d\theta^2 + \cos^2 \theta d\psi^2 + \sin^2 \theta d\phi^2) \right), \\
H^{(3)} &= N\epsilon_3, \\
e^{2\Phi} &= g_s^2 H(r), \\
H(r) &= 1 + \frac{Nl_5^2}{r^2},
\end{align*}
\]  

(2.1)

where $H^{(3)}$ is the NS-NS field strength, $\epsilon_3$ is the volume form on the transverse $S_3$ and $N$ is the NS5-brane charge. $H(r)$ is the harmonic function in the transverse space. The near-horizon geometry corresponds to the limit $r \to 0$ which removes the 1 in $H(r)$ and, on rescaling the time ($t = \sqrt{NL_5} \tilde{t}$), leads to the following,

\[
\begin{align*}
\text{ds}^2 &= Nl_5^2 \left( -d\tilde{t}^2 + \frac{dr^2}{r^2} + \cos^2 \theta d\psi^2 + d\theta^2 + \sin^2 \theta d\phi^2 \right) + dy_5^2, \\
e^{2\Phi} &= \frac{Nl_5^2 g_s^2}{r^2},
\end{align*}
\]

(2.2)

there is also the three form NS-NS field strength whose explicit form we are not mentioning here. To take the Penrose limit, one is interested in boosting along a
radial rather than an angular null geodesic \[44\]. The starting point is to make the following replacements

\[(\tilde{t}, x) \rightarrow (x^+, x^-), \quad \tilde{t} = x^+ - x^-, \quad r = \sqrt{Nl_s e^{x^+}}.\] (2.3)

This gives the metric and the dilaton in the following form

\[ds^2 = Nl_s^2 \left[ 2dx^+ dx^- - (dx^-)^2 + d\theta^2 + \cos^2 \theta d\psi^2 + \sin^2 \theta d\phi^2 \right] + dy_a^2 \]
\[e^{2\Phi} = g_s^2 e^{-2x^+}\] (2.4)

Next step is to rescale

\[x^- \rightarrow \frac{x^-}{N}, \quad \theta \rightarrow \frac{\theta}{\sqrt{N}}, \quad \psi \rightarrow \frac{\psi}{\sqrt{N}}\] (2.5)

and take the limit \(N \rightarrow \infty\). This sends the \(H^{(3)} \sim N \sin \theta \ d\theta \ d\psi \ d\phi \rightarrow N^{-1/2} \rightarrow 0\), and finally one is left with a metric and a dilaton that is linear along \(x^+\),

\[ds^2 = l_s^2 \left( 2dx^+ dx^- + \sum_{i=1}^{3} (dx^i)^2 \right) + \sum_{a=1}^{5} (dy_a)^2, \quad \Phi = \Phi_0 - x^+,\] (2.6)

which describes the light-like linear dilaton in a flat space-time. It is fairly straightforward to check the space-time supersymmetry of this solution, that is given by the condition

\[\Gamma^+ \epsilon = 0\] (2.7)

where \(\epsilon\) represents the space-time Killing spinor in ten dimensions and the hat represents the corresponding tangent space index. Hence the background preserves half of the supersymmetry. In the subsequent analysis, we would like to see how the supersymmetry changes in the presence of D-branes.

Next, let us review some of the geometric features of the background with a linear dilaton. The geometry specified by (2.6), from a string frame viewpoint, contains a time-dependence in the coupling constant but the metric is flat. However, to study the geodesic equations for such a background, we pass on to the Einstein frame. By using the relation

\[ds_E^2 = e^{-\Phi/2} ds_{\text{string}}^2\] (2.8)

we get the metric in the Einstein frame

\[ds_E^2 = e^{x^+/2} \left( l_s^2 \left( 2dx^+ dx^- + \sum_{i=1}^{3} (dx^i)^2 \right) + \sum_{a=1}^{5} (dy_a)^2 \right)\] (2.9)

Viewed from an Einstein frame, the space-time originates at a big-bang singularity as \(x^+ \rightarrow -\infty\), as the scale factor goes to zero. The geodesic equation for this background in Einstein frame, along \(x^+\) (for \((x^-, x^i, y^a) = \text{constant}\)) is written as

\[\frac{d^2 x^\mu}{d\sigma^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{d\sigma} \frac{dx^\beta}{d\sigma} = 0\] (2.10)
which for $\Gamma_{++}^+ = 1/2$, can be given by

$$ \frac{d^2 x^+}{d\sigma^2} + \frac{1}{2} \left( \frac{dx^+}{d\sigma} \right)^2 = 0. \quad (2.11) $$

Hence the affine parameter is given by

$$ \sigma = e^{x^+/2}, \quad (2.12) $$

upto an affine transformation. So the singularity at $x^+ \to -\infty$ correspond to $\sigma \to 0$.

Written in $\sigma$ variable, we get the following metric

$$ ds^2 = l_s^2 \left( 4d\sigma dx^- + \sigma \sum_{i=1}^3 (dx^i)^2 + \sigma \sum_{a=1}^5 (dy^a)^2 \right) \quad (2.13) $$

In this frame, the Riemann tensors diverge at $\sigma = 0$, thereby showing a curvature singularity. In other words this gives a divergent tidal force felt by an inertial observer. In the next section, we would like to see if this behavior is affected by the introduction of non-perturbative objects like D-branes.

### 3 Classical D-brane solutions in a light-like linear dilaton background

To obtain our solution, we now begin by writing the $1_{NS} + 5_{NS} + 5'_{NS}$ solution given in [45] with the metric:

$$ ds^2 = g_1^{-1}(x,y)(-dt^2 + dz^2) + H_5(x)dx_1^2 + H'_5(y)dy_m^2, \quad (3.1) $$

supported by the NS-NS field strengths

$$ dB = dg_1^{-1} \wedge dt \wedge dz + *dH_5 + *dH'_5, \quad (3.2) $$

and the dilaton

$$ e^{2\Phi} = \frac{H_5(x)H'_5(y)}{g_1(x,y)}, \quad (3.3) $$

where

$$ [H'_5(y)\partial_x^2 + H_5(x)\partial_y^2]g_1(x,y) = 0. \quad (3.4) $$

A particular solution for $g_1$ is given as:

$$ g_1(x,y) = H_1(x)H'_1(y), \quad (3.5) $$

with the harmonic functions given by the following expression

$$ H_1 = 1 + \frac{N_1^2 l_s^2}{x^2}, \quad H_5 = 1 + \frac{N_5^2 l_s^2}{x^2}, \quad H'_1 = 1 + \frac{N'_1^2 l_s^2}{y^2}, \quad H'_5 = 1 + \frac{N'_5^2 l_s^2}{y^2}. \quad (3.6) $$
We would like to note that the D5-branes wrapping \( \text{AdS}_3 \times S^3 \) has been obtained in [15] by taking a particular near horizon limit. We also note that by taking a Penrose limit along a null geodesic, D-brane solutions in the pp-wave background with R-R three form flux has been obtained in [46]. However, in the present paper we are interested in another Penrose limit proposed in [22] in order to get D-brane solutions in a light-like linear dilaton background.

To proceed further, first we would like to set all the string charges to zero, by putting \( H_1 = H_1' = 1 \). Then the above solution becomes a direct product of \( \text{NS}5 + \text{NS}5' \) configuration. Next, we apply the Penrose limit as [22, 44]. By rescaling the conventional time \( t = \sqrt{N_5}l_s \tilde{t} \), the metric above can be written as

\[
\text{ds}^2 = N_5 l_s^2 \left[ -d\tilde{t}^2 + \frac{dx^2}{x^2} + d\Omega_3^2 \right] + dz^2 + H'_5dy_m^2
\]

As in [22] we will also be interested in boosting along a radial null geodesic in this solution. To do this let’s make the following replacement

\[
(\tilde{t}, x) \rightarrow (x^+, x^-), \quad \tilde{t} = x^+ - x^-, \quad x = \sqrt{N_5}l_s e^{x^+}.
\]

and apply the rescaling

\[
x^- \rightarrow \frac{x^-}{N_5}, \quad \theta \rightarrow \frac{\theta}{\sqrt{N_5}}, \quad \psi \rightarrow \frac{\psi}{\sqrt{N_5}}
\]

and finally take the limit \( N_5 \rightarrow \infty \). In this limit the three form field strength correspond to the first NS5-brane \(*dH_5 \rightarrow 0\). Now the metric for the \( \text{NS}5' \)-brane looks like

\[
\text{ds}^2 = \left[ 2dx^+ dx^- + \sum_{a=1}^{4} dx_a^2 \right] + H'_5 \sum_{m=5}^{8} dy_m^2, \quad H_{mnp} = \epsilon_{mnpq} \partial_q H'_5
\]

The dilaton after this rescaling looks like

\[
e^{2\Phi} = e^{-2x^+} H'_5.
\]

Applying a S-duality\(^2\) we get the following metric, dilaton and the Ramond-Ramond field strength \( (F_{mnp}) \) for the system of D5-branes in a linear dilaton background:

\[
\text{ds}^2 = e^{x^+} H_5^{-\frac{1}{2}} \left[ 2dx^+ dx^- + \sum_{a=1}^{4} dx_a^2 \right] + e^{x^+} H_5^{\frac{1}{2}} \sum_{m=5}^{8} dy_m^2,
\]

\[
e^{2\Phi} = e^{2x^+} H_5^{x^+ - 1}, \quad F_{mnp} = \epsilon_{mnpq} \partial_q H'_5
\]

We have further checked that the above solution solve the type IIB field equations. We interpret the solution as D-brane in a linear dilaton background. We note that

\(^2\)Under S-duality, \( \Phi \rightarrow -\Phi \), \( ds^2_{\text{string}} \rightarrow \exp(-\Phi)ds^2_{\text{string}} \), and the NS-NS fields change to RR fields.
by putting $H'_5 = 1$, we get both the string frame metric and the coupling constant that are time dependent.

Other D-brane solutions can be found out by applying T-dualities along the transverse and worldvolume directions of the above D5-brane solutions. For example, the D4-brane solution can be generated in the following way. First one delocalizes the above D5-brane solution (3.12) along one of the worldvolume directions (say $x^4$), and then apply a T-duality along that. The D4-brane solution can be written by

$$ds^2 = e^{x^+} H_4^{-\frac{q}{4}} \left[ 2dx^+ dx^- + \sum_{a=1}^{3} \sum_{m=4}^{8} dy_m^2 \right] + e^{x^+} H_4^{\frac{q}{4}} \sum_{m=4}^{8} dy_m^2$$

$$e^{2\Phi} = e^{x^+} H_4^{-\frac{q}{4}}, \quad F_{mnpq} = \epsilon_{mnpqr} \partial_r H_4$$

(3.13)

where $H_4 = 1 + \frac{Q_4}{e^4}$ is the Harmonic function in the transverse space.

Next we would like to get a M5-brane solution starting with the above D4-brane solution in the linear dilaton background. Using the well known relation between the 10d and 11d metric:

$$ds_{11}^2 = e^{\frac{2\Phi}{3}} ds_{10}^2 + e^{\frac{4\Phi}{3}} (dx_{11} + A_\mu dx^\mu)^2,$$

(3.14)

where $ds_{11}^2$ and $ds_{10}^2$ represent the metric in eleven and ten dimensions respectively, and $A_\mu$ is the one-form field (which is zero in the present case). One can easily see that the M5-brane solution is given by

$$ds^2 = e^{\frac{2\Phi}{3}} f^{-\frac{1}{3}} \left[ 2dx^+ dx^- + \sum_{a=1}^{3} dx_a^2 + (dx_{11})^2 \right] + e^{\frac{2\Phi}{3}} f^{\frac{2}{3}} \sum_{m=4}^{8} dy_m^2,$$

$$C^{(4)} = F^{(4)}$$

(3.15)

where $F^{(4)}$ is the four form field strength given in eqn. (3.13) and $f = 1 + \frac{Nl_p^3}{e^4}$, with $l_p$ being the eleven dimensional Plank length.

It is easy to see that the solutions presented above contain singularity at $x^+ \to -\infty$, as the metric components goes to zero. It is worthy however to know the geometric properties of the background. Let us focus on the geodesic at constant $x^-, x^a$, and at $x^m = 0$, that is the trajectory along $x^+$:

$$\frac{d^2 x^+}{d\sigma^2} + \left( \frac{dx^+}{d\sigma} \right)^2 = 0$$

(3.16)

which gives

$$e^{x^+} \frac{dx^+}{d\sigma} = \text{constant}.$$ 

(3.17)

Hence the affine parameter is given by

$$\sigma = e^{x^+}$$

(3.18)

up to an affine transformation. Therefore the singularity at $x^+ \to -\infty$ correspond to $\sigma = 0$ and it has finite affine distance to all points inside.
4 Spacetime Supersymmetry

Next, we would like to check the supersymmetric properties of the D5-brane presented in (3.12). The supersymmetry variation of dilatino and gravitino fields of type IIB supergravity in ten dimension, in string frame, is given by [47, 48]:

\[
\delta \lambda_\pm = \frac{1}{2}(\Gamma^\lambda \partial_\lambda \Phi \mp \frac{1}{12} \Gamma^{\mu \nu \rho} H_{\mu \nu \rho}) \epsilon_\pm + \frac{1}{2} e^\Phi (\pm \Gamma^M F_M^{(1)}) + \frac{1}{12} \Gamma^{\mu \nu \rho} F_{\mu \nu \rho}^{(3)} \epsilon_\mp, \tag{4.1}
\]

\[
\delta \Psi^\pm_\mu = \left[ \partial_\mu + \frac{1}{4}(w_{\mu \hat{a} \hat{b}} \mp \frac{1}{2} H_{\mu \hat{a} \hat{b}}) \Gamma^{\hat{a} \hat{b}} \right] \epsilon_\pm + \frac{1}{8} e^\Phi \left[ \mp \Gamma_\lambda F_\lambda^{(1)} - \frac{1}{3!} \Gamma^{\mu \nu \rho} F_{\mu \nu \rho}^{(3)} + \frac{1}{2.5!} \Gamma^{\mu \nu \rho \sigma \beta} F_{\mu \nu \rho \sigma \beta}^{(5)} \right] \Gamma_\mu \epsilon_\mp, \tag{4.2}
\]

where we have used \((\mu, \nu, \rho)\) to describe the ten dimensional space-time indices, and the hat’s to represent the corresponding tangent space indices. Solving the dilatino variation for the D5-brane solution presented above, we get the following two conditions to be obeyed by the Killing spinors

\[
\Gamma^{\hat{+}} \epsilon_\pm = 0, \tag{4.3}
\]

and

\[
\Gamma^{\hat{m}} \epsilon_\pm + \frac{1}{3!} \epsilon^{\hat{m} \hat{n} \hat{p} \hat{r}} \Gamma^{\hat{n} \hat{p} \hat{r}} \epsilon_\mp = 0. \tag{4.4}
\]

It is easy to see that the first condition comes purely from the background, whereas the second condition is the usual D5-brane supersymmetry condition even in the flat space time. We note that for the dilatino variation to be satisfied we indeed need both the conditions. Now we would like to analyze the gravitino variations for the solution (3.12). The gravitino variations give the following conditions on the spinors

\[
\delta \Psi_\mp^+ \equiv \partial_+ \epsilon_\pm = 0, \quad \delta \Psi_\mp^- \equiv \partial_- \epsilon_\pm = 0, \quad \delta \Psi_a^\pm \equiv \partial_a \epsilon_\pm = 0,
\]

\[
\delta \Psi_m^\pm \equiv \partial_m \epsilon_\pm + \frac{1}{8} \frac{H_5'}{H_5} \epsilon_\pm = 0. \tag{4.5}
\]

In writing down the above variations, we have made use of the conditions (4.3) and (4.4). Now the conditions (4.3) and (4.4) breaks 1/4 of the spacetime supersymmetry, as we can integrate out the last equation of (4.5) with the solution given by

\[
\epsilon_\pm = \exp \left( -\frac{1}{8} \ln H_5' \right) \epsilon_\mp^0 \tag{4.6}
\]

with a constant spinor \(\epsilon_\mp^0\). So the D5-brane solution presented in (3.12) preserves 1/4 supersymmetry.

5 Conclusion

In this paper, we have constructed classical solutions for D-branes in a light-like linear dilaton background. These brane solutions have been obtained by taking a particular
Penrose limit on an intersecting brane solution in type II supergravity. We have found out that starting with the intersecting $NS1 - NS5 - NS5'$ brane solution, one can apply the near horizon limit followed by a Penrose limit along a radial null geodesic to obtain branes in a light-like linear dilaton background. We further have obtained a M5-brane solution in eleven dimensional supergravity. The supersymmetry variations reveal that these branes preserve 1/4 of the full type IIB space-time supersymmetry. We have also pointed out the geodesic equations and the nature of singularity at $x^+ \to -\infty$. One can possibly try to extend the present analysis to all the D-branes including the intersecting ones in type II string theory. In particular, one can try to find out an intersecting $D1 - D5$ brane solution in a light-like or null linear dilaton background. The near horizon limit of such brane configuration can be thought of as a deformation of AdS$_3 \times S^3$ space time. It might also help us in understanding the physics near the singularity from the view point of the underlying gauge theory. The construction of the corresponding matrix model for these class of branes would be very interesting. We hope to come back to some of these issues in near future.

Acknowledgements: We would like to thank A. Kumar for reading the manuscript. We thank S. Siwach for useful discussions. The work of RRN was supported by INFN. The work KLP was supported partially by PRIN 2004 - ”Studi perturbativi e non perturbativi in teorie quantistiche dei campi per le interazioni fondamentali”.

References

[1] H. Liu, G. W. Moore and N. Seiberg, JHEP 0206 (2002) 045 [arXiv:hep-th/0204168].
[2] H. Liu, G. W. Moore and N. Seiberg, JHEP 0210, 031 (2002) [arXiv:hep-th/0206182].
[3] L. Cornalba and M. S. Costa, Phys. Rev. D 66, 066001 (2002) [arXiv:hep-th/0203031].
[4] G. T. Horowitz and J. Polchinski, Phys. Rev. D 66, 103512 (2002) [arXiv:hep-th/0206228].
[5] A. Lawrence, JHEP 0211, 019 (2002) [arXiv:hep-th/0205288].
[6] M. Berkooz, B. Craps, D. Kutasov and G. Rajesh, JHEP 0303, 031 (2003) [arXiv:hep-th/0212215].
[7] G. T. Horowitz and A. R. Steif, Phys. Rev. Lett. 64, 260 (1990).
[8] V. Balasubramanian, S. F. Hassan, E. Keski-Vakkuri and A. Naqvi, Phys. Rev. D 67, 026003 (2003) [arXiv:hep-th/0202187].
[9] N. A. Nekrasov, Surveys High Energ. Phys. 17, 115 (2002) [arXiv:hep-th/0203112].
[10] S. Elitzur, A. Giveon, D. Kutasov and E. Rabinovici, JHEP 0206, 017 (2002) arXiv:hep-th/0204189.

[11] A. Hashimoto and S. Sethi, Phys. Rev. Lett. 89, 261601 (2002) arXiv:hep-th/0208126.

[12] J. Simon, JHEP 0210, 036 (2002) arXiv:hep-th/0208165.

[13] R. G. Cai, J. X. Lu and N. Ohta, Phys. Lett. B 551, 178 (2003) arXiv:hep-th/0210206.

[14] A. Giveon, E. Rabinovici and A. Sever, Fortsch. Phys. 51, 805 (2003) arXiv:hep-th/0305137.

[15] B. Pioline and M. Berkooz, JCAP 0311, 007 (2003) arXiv:hep-th/0307280.

[16] J. L. Hovdebo and R. C. Myers, JCAP 0311, 012 (2003) arXiv:hep-th/0308088.

[17] M. Berkooz, B. Durin, B. Pioline and D. Reichmann, JCAP 0410, 002 (2004) arXiv:hep-th/0407216.

[18] Y. Hikida, R. R. Nayak and K. L. Panigrahi, JHEP 0505, 018 (2005) arXiv:hep-th/0503148.

[19] Y. Hikida, R. R. Nayak and K. L. Panigrahi, JHEP 0509, 023 (2005) arXiv:hep-th/0508003.

[20] A. Sen, JHEP 0204, 048 (2002) arXiv:hep-th/0203211.

[21] J. McGreevy and E. Silverstein, JHEP 0508, 090 (2005) arXiv:hep-th/0506130.

[22] B. Craps, S. Sethi and E. P. Verlinde, JHEP 0510, 005 (2005) arXiv:hep-th/0506180.

[23] M. Li, Phys. Lett. B 626, 202 (2005) arXiv:hep-th/0506260.

[24] S. Kawai, E. Keski-Vakkuri, R. G. Leigh and S. Nowling, Phys. Rev. Lett. 96, 031301 (2006) arXiv:hep-th/0507163.

[25] M. Li and W. Song, JHEP 0510, 073 (2005) arXiv:hep-th/0507185.

[26] S. R. Das and J. Michelson, Phys. Rev. D 72, 086005 (2005) arXiv:hep-th/0508068.

[27] B. Chen, Phys. Lett. B 632, 393 (2006) arXiv:hep-th/0508191.

[28] J. H. She, JHEP 0601, 002 (2006) arXiv:hep-th/0509067.

[29] B. Chen, Y. l. He and P. Zhang, Nucl. Phys. B 741, 269 (2006) arXiv:hep-th/0509113.
[30] T. Ishino, H. Kodama and N. Ohta, Phys. Lett. B 631, 68 (2005) arXiv:hep-th/0509173.

[31] D. Robbins and S. Sethi, JHEP 0602, 052 (2006) arXiv:hep-th/0509204.

[32] S. Kalyana Rama, arXiv:hep-th/0510008.

[33] Y. Hikida and T. S. Tai, JHEP 0601, 054 (2006) arXiv:hep-th/0510129.

[34] J. H. She, arXiv:hep-th/0512299.

[35] M. Li and W. Song, arXiv:hep-th/0512335.

[36] T. S. Tai, arXiv:hep-th/0601039.

[37] B. Craps, A. Rajaraman and S. Sethi, arXiv:hep-th/0601062.

[38] C. S. Chu and P. M. Ho, arXiv:hep-th/0602054.

[39] S. R. Das and J. Michelson, arXiv:hep-th/0602099.

[40] S. R. Das, J. Michelson, K. Narayan and S. P. Trivedi, arXiv:hep-th/0602107.

[41] E. J. Martinec, D. Robbins and S. Sethi, arXiv:hep-th/0603104.

[42] H. Z. Chen and B. Chen, arXiv:hep-th/0603147.

[43] T. Ishino and N. Ohta, arXiv:hep-th/0603215.

[44] V. E. Hubeny, M. Rangamani and E. P. Verlinde, JHEP 0210, 020 (2002) arXiv:hep-th/0205258.

[45] G. Papadopoulos, J. G. Russo and A. A. Tseytlin, Class. Quant. Grav. 17, 1713 (2000) arXiv:hep-th/9911253.

[46] A. Kumar, R. R. Nayak and S. Siwach, Phys. Lett. B 541, 183 (2002) arXiv:hep-th/0204025.

[47] J. H. Schwarz, Nucl. Phys. B 226, 269 (1983).

[48] S. F. Hassan, Nucl. Phys. B 568, 145 (2000) arXiv:hep-th/9907152.