Surface crack reconstruction from eddy current images using a direct semi-analytic model

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Abstract. A method relying on a direct semi-analytic model is proposed for reconstructing cracks from eddy current images. We consider systems featuring a uniform excitation flow which can be modelled by means of fictitious current sources distributed in the crack volume. Thanks to the relative simplicity of the model a reconstruction method based the comparison of EC images is considered. The sensitivity of different images comparison criteria is studied for 2D surface cracks, leading to a reconstruction method based on a genetic algorithm. Numerical experiments are carried out to examine the performances of the algorithm in terms of convergence and accuracy.

1. Introduction
Non destructive evaluation (NDE) contributes to the security of industrial facilities as a means of preventing accidents by early detecting and characterizing defects. With ultrasounds [1], eddy current (EC) methods [2] are among the most widespread NDE techniques as they are particularly useful for evaluating corrosion and cracks in electrically conductive structures. However, the accurate reconstruction of the shape of cracks and defects by means of EC data is still an open scientific stake because the inversion of EC data is an ill-posed problem [3-5].
To cope with this issue several approaches are therefore investigated, among which total variation regularisation methods [6] or the use a fast interpolation models (such as meta-models obtained from databases) and the estimation of a restricted amount of parameters characterising the shape of the defects [7]. Since a main difficulty of the inversion relies on the establishment of a confidence criterion on the results of the algorithm, statistical approaches (such as Monte Carlo [8]) also are considered which require a high number of resolutions of the direct problem. Non-iterative algorithms also are envisaged based on punctual tests of the inspected structure with near real-time NDE applications in view [9].
An NDE technique usually involves both a sensing device (composed of the excitation and sensing elements) and computations. With a view to optimize the coupling of the excitation signal in the inspected structure and the characteristics of the sensed one, the EC device shall be designed taking the geometry and characteristics of the inspected structure into account. It is also desirable to adapt the inversion method and the experimental setup to each other; which understates the adaptation of the
modelling of the EC problem when the inversion method draws on a model. Such an approach is followed in this paper.

We consider EC sensing systems relying on harmonic excitation and designed to induce a uniform EC flow within the inspected structure (as opposed to circular excitation configurations) [10]. Such systems are sensitive to the presence of cracks and generate a significant magnetic field perpendicular to the surface of the inspected structure. Furthermore, it has been demonstrated that the interactions between a uniform EC flow and a crack could suitably and rather simply be modelled by means of fictitious current sources placed within the volume of the crack [11]. Moreover, the implementation of this model by means of the distributed point source method (DPSM), which is a generic semi-analytic approach, can provide with fast computation [12].

Armed with this direct model, we propose a crack reconstruction method based the comparison of EC images (i.e. cartographies of the magnetic field component perpendicular to the surface of the structure), focusing on the reconstruction of 2D surface cracks.

The paper is organized as follows. Section 2 reports on the description of the direct model and its implementation using DPSM. In section 3 the sensitivity of different image comparison criteria to the dimensions of surface cracks is studied, leading to a reconstruction method based on a genetic algorithm (GA) [14]. In section 4, a GA is applied to the reconstruction of 2D surface cracks.

2. The direct model

The modelling of the interactions between a uniform EC flow and a crack allows for certain simplifications: assuming a superposition hypothesis, the EC surrounding the crack induce a magnetic field distribution similar to the one virtual EC flowing within the crack volume would induce provided they are equal in amplitude and in phase opposition to those which would be present in the same volume in a crack-free material [11]. This model implies that the crack is of small dimensions and represents a small perturbation of the EC flow [13], which means that the amount of EC deviated by the crack is negligible. However approached, this model can prove to be sufficient [11]. Moreover, with the inversion of EC data in view, even more so for iterative inversion methods, its use may be worth considering as it should require a limited computation time.

The model lends itself to an implementation by means of the DPSM, a semi-analytic approach for which the active sources present in the workspace are modelled by a discrete ensemble of elementary radiating sources (according to Green functions). In addition to the active sources, the DPSM implements virtual interface sources calculated according to the boundary conditions. Only the location of the active and virtual interface sources and those where the physical quantities are wished to be computed (as the superposition of the contributions of the radiating sources) require to be meshed, which makes the DPSM computationally efficient.

We will not go into the details of the DPSM here, a comprehensive description being available in [12]. Our purpose will be limited to the main elements of the implementation of the simplified EC modelling proposed above (Figure 1). In that case, the workspace is divided into two media: air and metal, and the physical quantity of interest (i.e. the distribution of the vertical magnetic field $b_z$ in the air above the surface of the metallic structure) calculated as the sum of the contributions of the virtual interface sources radiating in the air. In practice, the problem is formulated in such a way that $b_z$ is computed as the product of a matrix by the current sources placed in the crack volume.

It is to be noted that for the sake of accuracy of the modelling it is suitable that the interface be meshed respecting a distance between the sources lower than a third of the characteristic distance of the electromagnetic problem, which is typically in the order of the skin depth $\delta$:

$$\delta = 1/\sqrt{\pi f \sigma \mu_0 \mu_r} \quad (1)$$

where $f$ is the frequency of the EC, $\mu_0$ is the magnetic permeability in a vacuum, $\mu_r$ is the relative magnetic permeability and $\sigma$ is the electric conductivity of the medium.
3. EC images parametric study

The computational efficiency of the simplified EC direct model proposed above makes it reasonable to envisage iterative crack reconstruction methods relying on the comparison of EC images obtained from the direct model. We focus here on the development of such an approach applied to the reconstruction of surface cracks. Previous to the design of a 2D reconstruction algorithm, we analyze the relevance of different criteria of image comparison.

The EC model is implemented via DPSM with the discretization configuration described in Table 1. The metal is assumed to be an aluminum alloy featuring $\sigma = 16 \text{ MS.m}^{-1}$ and $\mu_r = 1$. $f$ is fixed at 400 Hz and the meshing of the interface fulfills the rule stated above, the interface sources being spaced out of $\delta 3 \approx 6 \text{ mm}$. The EC flow along the $x$ axis like in Figure 1.

| z position (mm) | x dimension (mm) | y dimension (mm) |
|----------------|------------------|------------------|
| Active sources plane | $-2$ | $20$ | $n = 40$ | $20$ | $m = 40$ |
| $b_z$ cartography | $0$ | $54$ | $n = 40$ | $54$ | $m = 40$ |

Let us denote $b_z \in \mathbb{C}^M$ the vertical magnetic field at the surface of the metallic structure, where $M = m \times m$ is the image (i.e. cartography) dimension. Let us consider 3 criteria of comparison of images: a correlation error $\varepsilon_{\rho}$, a normalized mean square error $\varepsilon_{\text{mse}}$ and a normalized weighted mean square error $\varepsilon_{\text{wmse}}$ defined as relative errors between a magnetic field $b_z$ and a reference $b_{z0}$:

$$
\varepsilon_{\rho} = \frac{|b_z - E(b_z)|}{|b_z - E(b_{z0})|}
$$

$$
\varepsilon_{\text{mse}} = \frac{|b_z - b_{z0}|}{b_{z0} \cdot b_{z0}}
$$

$$
\varepsilon_{\text{wmse}} = \frac{|b_z - b_{z0}|}{|b_{z0} - E(b_{z0})|}
$$

Figure 1. DPSM modelling of the interactions between a uniform EC flow and a 3D crack featuring a parallelepiped shape; cartography of the modulus of the vertical component of the magnetic field above the surface of the metallic part.
where \( W = \text{diag} \{ |b_{z0}| \} \) is the diagonal matrix supporting \( |b_{z0}| \) on the diagonal. In (3) and (4), the normalization is performed with respect to the total energy (or weighted energy) of the reference crack \( b_{z0} \).

\[
\epsilon_{wmse} = \frac{|b_z - b_{z0}|/W|b_z - b_{z0}|}{b_{z0}}
\]

(4)

Figure 2. \( \epsilon_{\rho} \), \( \epsilon_{mse} \), and \( \epsilon_{wmse} \) for 2D rectangular cracks compared to a 0.5 mm by 4 mm reference crack (1 pixel \( \Leftrightarrow \) 0.5 mm).

Figure 2 illustrates the sensitivity of \( \epsilon_{\rho} \), \( \epsilon_{mse} \), and \( \epsilon_{wmse} \) to the width \( W_x \) and length \( L_y \) of a rectangular surface crack compared to a 0.5 mm wide and 4 mm long reference. Hence, every error is null at \( \{ W_x = 0.5 \text{ mm}, L_y = 4 \text{ mm} \} \). It appears that \( \epsilon_{\rho} \) is rather sensitive to \( L_y \) and much less to \( W_x \). Indeed, the correlation is reputed sensitive to the variations of the shape of the images, which are sharper along the longitudinal direction of the crack (as shown in Figure 1). Compared to \( \epsilon_{\rho} \), the variations of \( \epsilon_{mse} \) happen to be 5 times higher. Moreover this criterion is sensitive both to \( W_x \) and \( L_y \). Regarding \( \epsilon_{wmse} \), a much higher dynamic of variations is observed and the criterion is sensitive both to \( W_x \) and \( L_y \). Such a result is consistent with the fact that the higher the signal corresponding to a pixel, the higher its weight (4). This suggests that \( \epsilon_{wmse} \) could be the most relevant criterion with implementation in a 2D reconstruction algorithm in view. Furthermore, it is to be noted that the three errors feature multiple minima. It follows that a reconstruction based on a GA seems more suitable than a gradient algorithm since the room left to random evolution of the individuals in a GA enables minimizing the risk of converging towards a local minimum.

4. Surface crack reconstruction

Considering the reconstruction of rectangular surface cracks (and assuming such a feature as an a priori for the reconstruction), we have implemented a GA [14] involving a population of 4 individuals, i.e. rectangular. It is to be noted that the population size was not optimized for minimizing the number of iterations, but simply chosen as the “magic number” 4. At each iteration of the GA, the fitness of the individuals is evaluated thanks to an objective function based on a comparison between the EC images \( b_z \) of the individuals (obtained from the direct model described above) and the image \( b_{z0} \) of the
crack to be reconstructed. A selection of the individuals is performed according to a fortune wheel and followed by a mutation of the population (by randomly varying according to a given Gaussian law the dimensions of the selected individuals). The initialization of the population consists in randomly choosing the individuals while ensuring a minimum fitness of each individual.

Table 2. GA convergence statistics.

| Objective function | Average number of iterations | Standard deviation |
|--------------------|-------------------------------|--------------------|
| $\varepsilon_p$    | 23.13                         | 20.10              |
| $\varepsilon_{mse}$| 22.69                         | 17.68              |
| $\varepsilon_{wmse}$| 16.83                         | 13.65              |

Statistics based on 100 drawings of the GA for each objective function. Researched crack $\{W_x = 0.5 \text{ mm}, L_y = 4 \text{ mm}\}$ with $b_0$ featuring $30 \text{ dB \ SNR}$. Convergence statistics of the GA towards the right rectangular defect were performed for objective functions based on each of the 3 image comparison criteria considered in section 3 (Table 2). Whatever the objective function the GA converges towards the right $\{W_x, L_y\}$. Nonetheless, the objective function based on $\varepsilon_{wmse}$ requires less iterations than the others to converge, which is consistent with the results of the previous sensitivity study.

To further examine the estimation performances, numerical experiments have been carried out with only 15 iterations per drawing, which is lower than the average number required for the GA to converge. 250 drawings of the GA were made with each objective function and for 5 signal to noise ratios (SNR) (Figure 3). Whatever the objective function the resulting average errors are very small whether regarding $L_y$ or $W_x$, even though the SNR considered is low. However, the results tend to confirm that the objective function based on $\varepsilon_p$ provides with more accurate estimation of $L_y$ than $W_x$, while using $\varepsilon_{wmse}$ leads to the overall most accurate estimation of both $L_y$ and $W_x$.

Figure 3. Errors $\varepsilon_{W_x}$ and $\varepsilon_{L_y}$ (expectancy and standard deviation of $W_x$ and $L_y$ for 250 drawings of the GA limited to 15 iterations). $\{W_x = 0.5 \text{ mm}, L_y = 4 \text{ mm}\} \Leftrightarrow \{W_x = 1 \text{ pixel}, L_y = 8 \text{ pixels}\}$. 

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Another aspect of numerical experiments is the computational efficiency of the reconstruction method. Still in the conditions described in section 3, the computation time of the DPSM transfer matrix (calculated once for all prior to iterating the GA) was in the order of 12 seconds, while an iteration of the GA did last about 3.5 seconds. The algorithm was implemented by means of Matlab™ on a standard PC.

5. Conclusion
In this work, an iterative approach based on the use of a semi-analytic direct model of EC images has been proposed for reconstructing surface cracks. The method which applies to systems featuring homogeneous EC induction has been tested via a GA with simulated data in the presence of noise. Convergence of the GA towards the exact solution was obtained for several fitness functions based on different criteria of comparison of EC images. The relevance of the considered criteria to the estimation of the length and width of cracks also was studied by implementing the GA with a small number of iterations. Based on these results the reconstruction of 3D cracks (either surface or buried cracks) can be envisaged. Therefore, further works will focus on a multi-frequency approach taking the skin effect into account in order to reconstruct the depth of structures layer by layer. The application of the method to experimental data will also be considered.

6. References

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