

K\bar{K} photoproduction and S – P wave interference

Łukasz Bibrzycki†, Leonard Leśniak†, Adam P. Szczepaniak‡

† Department of Theoretical Physics,
The Henryk Niewodniczański Institute of Nuclear Physics,Polish Academy of Sciences, PL 31-342 Kraków, Poland
‡ Physics Department and Nuclear Theory Center,Indiana University, Bloomington, IN 47405, USA

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Abstract

Results of a new analysis of the $K^+K^-$ photoproduction at two photon energies $E_\gamma = 4$ GeV and $5.65$ GeV with a particular emphasis on the $S$-wave production are presented. We show that the proper treatment of all the helicity components of the $S$- and $P$-waves enables one to eliminate the reported discrepancies in the extraction of the $S$-wave photoproduction cross section from experimental data.

1 Introduction

From the early days of QCD, light meson spectroscopy played an important role in development of the theory and in understanding of its low energy structure. The flavour symmetry of QCD originates in part from the observed $SU(3)$ multiplet structure of the light pseudoscalar and vector mesons. More recently, scalar and tensor spectra have provided evidence for a possible over-population of the $Q\bar{Q}$ spectrum and for the existence of gluon-rich states [1-4]. The possible existence of gluonic excitations is one
of the most intriguing features of the meson spectroscopy. There is tantalizing evidence of exotic, hybrid mesons in the spectrum around \(1.6\) GeV \([5-7]\), and future experiments proposed for JLab and GSI in the light and charm sector respectively will map out the exotic spectrum. The glueball signature comes primarily from the analysis of the Crystal Barrel \(p\bar{p}\) and WA102 central production data \([1, 3]\). Detailed mapping of various scalar meson decay channels led to the identification of \(f_0(1370)\), \(f_0(1500)\) and \(f_0(1710)\) states, all expected to contain significant gluon components. While the genuine QCD resonances in the scalar channel are not expected to occur below \(1\) GeV, the low energy region can be studied using standard, low energy expansion techniques. These include the effective range expansion, \(N/D\) and other methods based on analyticity coupled with truncation of the unitarity condition. The parameters of the soft meson-meson interactions \(e.g.\) subtraction constants, form factors, coupling strengths, \textit{etc.} effectively correspond to local potentials smeared over distance scales of the orders of \(\sim 1\) fm, \textit{i.e.} close to the pion root mean square radii. The strength of the interaction can be constrained from chiral symmetry or simply fitted to the data \([8-11]\). As a result one obtains a very good description of the spectrum including the resonance region. Furthermore, the behavior of the scattering amplitude in the complex energy plane enables one to establish the existence of dynamical resonances such as the isoscalar \(\sigma(600)\) and \(f_0(980)\) and the isovector \(a_0(980)\) mesons. The last two are particularly interesting as they are very sensitive to the details of the meson-meson interactions. This is due to the proximity of the \(K\bar{K}\) threshold. In particular the detailed structure of the \(f_0(980)\) \(e.g.\) whether it is a genuine bound state or a virtual bound state, strongly depends on the threshold \(K\bar{K}\) interaction parameters. Since the \(K\bar{K}\) channel also influences the higher mass region a proper description of its dynamics is crucial for a partial wave analysis and the identification of exotic, hybrid or glueball signals. Furthermore the \(K\bar{K}\) system is relevant for testing the origins of CP violation and possible signals of CPT violation via hadronic interferometry \([12]\).

The data on the near threshold \(K\bar{K}\) production are very scarce and come mainly from high-energy peripheral production \([13, 14]\). In the medium-energy region, \(E_{\gamma\text{lab}}\) \(\sim\) a few GeV, it is advantageous to study the \(K\bar{K}\) system in photoproduction. A real photon couples strongly to vector mesons and near the \(K\bar{K}\) threshold \(\phi\) photoproduction dominates the \(K\bar{K}\) spectrum. In this energy range the \(\phi\) photoproduction cross section is large,
\( \sigma(\gamma P \rightarrow \phi p) \sim 0.5 \mu b \), and has an energy dependence characteristic for diffraction. The \( s \)-channel baryon resonance production cross section is highly suppressed at these energies, and the \( t \)-channel meson exchange (\( \pi \)) is marginally relevant only at \( E_\gamma \) very close to threshold [15]. Finally, the \( S \)-wave \( K\bar{K} \) state, dominated in this mass range by the \( f_0(980) \) in the isoscalar and by the \( a_0(980) \) in the isovector channel, can be accessed via interference with the \( \phi \) meson in the \( P \)-wave. This \( S - P \) interference was first explored in experiments at DESY [16, 17] and Daresbury [18], and could be further studied with high statistics at an energy upgraded JLab.

In the analysis of the data from the DESY and Daresbury experiments, however, the rich spectrum of the \( S \)-wave \( K\bar{K} \) has not been fully explored. In both cases the \( S \)-wave was parameterized as either a simple Breit-Wigner resonance or a nonresonant background. The two analyses yielded results for the \( S \)-wave production cross section different from each other by more than one order of magnitude. Finally, apart of the \( K^+K^- \) mass spectrum, only one moment \( \langle Y_{10}^l \rangle \), describing the \( K^+ \) angular distribution, has been analyzed and very simplified assumptions about the nucleon spin dependence have been done.

In this paper we use the results of a recent calculation of the coupled channel scalar-isoscalar and scalar-isovector spectra together with a diffractive model for \( P \)-wave production to interpret the existing \( S - P \) interference data and estimate the \( S \)-wave production cross section with a better accuracy than in the phenomenological analyses already published in [16-18]. Here we shall discuss a complete set of six moments \( \langle Y_{LM}^l \rangle \) of spherical harmonics including \( L = 0 \) and \( L = 1 \). In the following section we shall present the theoretical foundation of the \( S \)- and \( P \)-wave photoproduction and discuss the main features of the existing data. In Sect. 3 we present results of the numerical analysis and fits to the data. Conclusions and outlook are given in Sect. 4.

## 2 S- and P-wave photoproduction

In this paper we consider unpolarized \( K\bar{K} \) photoproduction reaction

\[
\gamma p \rightarrow K^+K^-p, \tag{1}
\]

for incident photon laboratory energies \( E_\gamma \) of the order of a few GeV. This is an optimum energy range for the \( S \)-wave \( K\bar{K} \) production. In this en-
nergy range the process is dominated by the pomeron exchange, leading to \( \phi \)-meson production which becomes even more important as the photon energy increases. However, \( t \)-channel processes expected to be responsible for the \( S \)-wave production decrease rapidly with energy. The experimental evidence for the \( S - P \) interference in the \( E_\gamma \) range between 2.8 and 6.7 GeV was presented in [16-18]. The values of the \( S \)-wave photoproduction cross sections found in these experimental analyses varied between 2.7 and 96 nb. In both cases, in the effective mass range \( 1.00 \text{ GeV} < M_{KK} < 1.04 \text{ GeV} \), in the rest frame of the \( K \bar{K} \) system an asymmetry in the kaon polar angle distribution was observed. This can only be the case if there are odd powers of \( \cos \theta \) in the angular distribution. For low partial waves this implies presence of both \( S \)- and \( P \)-waves. This feature of the angular distribution is independent of the magnetic-quantum number \( \text{i.e. choice of the quantization axis.} \)

Due to the empirical \( s \)-channel helicity conservation, diffractive production is most naturally analyzed in the \( s \)-channel helicity frame, which in our case corresponds to the rest frame of the \( K \bar{K} \) pair with the \( z \)-axis anti-parallel to the direction of the recoiling nucleon. In another reference frame, called the \( t \)-channel frame or the Gottfried-Jackson frame, the \( z \)-axis is chosen along the direction of the photon beam with the \( K \bar{K} \) pair at rest [19]. In both cases the \( y \)-axis is perpendicular to the production plane.

As discussed in [20], in this energy range, the \( S \)-wave production is expected to be dominated by vector \( \rho \), \( \omega \) and pseudoscalar \( \pi \), \( K \) \( t \)-channel exchanges.

As a function of the momentum transfer squared \( t \), the kaon pair invariant mass \( M_{KK} \) and the \( K^+ \) decay angles \( \Omega = (\theta, \phi) \) in the \( s \)-channel frame the photoproduction amplitude restricted to \( S \)- and \( P \)-waves can be written as

\[
T_{\lambda,\lambda'}(t, M_{KK}, \Omega) = \sum_{L=S,P} T_{\lambda,\lambda M}^L(t, M_{KK}) Y_M^L(\Omega). \tag{2}
\]

Here \( \lambda, \lambda' \) denote the photon, target proton and recoil proton helicities, respectively, and \( Y_M^L(\Omega) \) are the spherical harmonics. The corresponding four-momenta will be denoted by \( q, p, p' \), and \( k_1 \) and \( k_2 \) will be used for the \( K^+ \) and the \( K^- \) momenta, respectively. The unpolarized differential cross section is given by

\[
\frac{d\sigma}{dt \ dM_{KK} \ d\Omega} = \frac{\kappa_f}{32 (2\pi)^3 m_p^2 E_\gamma^2} \frac{1}{4} \sum_{\lambda, \lambda'} |T_{\lambda,\lambda'}(t, M_{KK}, \Omega)|^2, \tag{3}
\]
where \( m_p \) is the proton mass, \( m_K \) is the kaon mass and \( \kappa_f = \sqrt{\frac{M^2_{KK}}{4} - m^2_K} \) is the kaon momentum in the rest system of the \( KK \) pair. The \( S \)-wave amplitude \( T^S \) and the \( P \)-wave amplitude \( T^P \) have been described in \([20, 21]\), respectively. Here we will briefly summarize the basic properties of these amplitudes. The \( S \)-wave \( KK \) production is parameterized as a double \( t \)-channel exchange. In the upper meson vertex we use a simple meson exchange and allow for an interaction of the two produced mesons in the final state. The dominant exchanges for the \( S \)-wave \( KK \) production are shown in Fig. 1. At the nucleon vertex we use either normal or Regge propagators of the exchanged vector mesons. The normal propagator of the vector meson of the mass \( m_e \) is equal to \( (t - m^2_e)^{-1} \) and the Regge type propagator reads

\[
-\left[1 - e^{-i\pi\alpha(t)}\right] \Gamma(1 - \alpha(t)) \frac{(\alpha's)^{\alpha(t)}/(2s^\alpha)}{\alpha'(t - m^2_e)}/(2s^\alpha),
\]

where we have the vector meson trajectory \( \alpha(t) = \alpha_0 + \alpha'(t - m^2_e) \), \( \alpha_0 = 1 \) and \( \alpha' = 0.9 \) GeV\(^{-2} \). The final state interactions include the \( \pi\pi \) and \( KK \) channels. Both interactions can have a resonant character. This is important since the \( S \)-wave in the mass region of interest, \( M_{KK} \sim 1 \) GeV, is dominated by the \( f_0(980) \) and \( a_0(980) \) resonances. We should notice that the \( K^+K^- \) system is an equal mixture of the isospin 0 and isospin 1 states. The isoscalar

![Fig. 1: The amplitude for the S-wave KK production. The a in the Born amplitude \( T_j^B \) stands for \( \pi, K, \rho \) and \( \omega \) mesons, the b stands for \( \rho \) or \( \omega \) mesons and \( m\bar{m} \) denotes either a \( \pi\pi \) or \( KK \) pair. The oval represents the final state rescattering amplitude.](image-url)
The $f_0(980)$ resonance has a main branching to $\pi\pi$ and in addition an important coupling to $K\bar{K}$. The isovector $a_0(980)$ resonance lies also very close to the $K\bar{K}$ threshold so one should take it into account in the calculations of the final state interactions. The $S$-wave $K^+K^-$ photoproduction amplitude $T^S_f$ can therefore be written as

$$T^S_{\lambda,\lambda'} = \bar{u}(p',\lambda')J^S_\mu(p',p,q,M_{\bar{K}K})\epsilon^\mu(q,\lambda)u(p,\lambda)$$

and decomposed as

$$T^S_f = \frac{1}{2}[T^S_f(I = 0) + T^S_f(I = 1)].$$

Here $\epsilon$ is the photon helicity four-vector and $J^S_\mu$ is the $S$-wave projection of the appropriate current operator. As illustrated in Fig. 1 each $S$-wave amplitude is a sum of products of the Born amplitudes $T^B_j(I)$ describing the $t$-channel meson or Regge exchanges and the final state interaction factors $F_{jjf}(I)$:

$$T^S_f(I) = \sum_{j=\pi\pi,K\bar{K}} T^B_j(I)F_{jjf}(I).$$

If we restrict ourselves to the on-shell part of the final state coupled channel interactions, represented by the $2\times2$ $S$-matrix elements $S^I_{jjf}$, then the factors $F_{jjf}(I)$ can be written as

$$F_{jjf}(I) = \frac{1}{2}(\delta_{jjf} + S^I_{jjf})\sqrt{\frac{\kappa_j}{\kappa_f}},$$

where $\kappa_j = \sqrt{M_{m\pi}^2 - m_j^2}$ and $M_{m\pi}$ is the effective mass of the $m\pi$ pion or kaon pair and $m_j$ is the pion or kaon mass. The explicit forms of the $S$-wave Born amplitudes $T^B_j$ are given in [20]. The magnitude of the off-shell part of the final state interaction amplitude is much less certain than the on-shell part as has been shown in [20]. Thus we prefer here to use only the on-shell part of the amplitude and correct it later by a constant modification factor. In comparison with [20] we have added in these amplitudes a $t$-dependent factor $exp(B \epsilon t)$, where the parameter $B_S = 1.07$ GeV$^{-2}$ is responsible for a spatial dimension of the meson (kaon or pion) coupled to the photon. In [20] the meson in the upper vertex in Fig. 1 was treated as a pointlike particle. The isoscalar $S^I_{jjf}$ matrix elements can be expressed by the scalar- isoscalar phase shifts $\delta$ and inelasticity $\eta$ in two channels $\pi\pi$ and $K\bar{K}$:

$$S^I_{jjf} = \begin{pmatrix} \eta e^{2i\delta_{\pi\pi}^l} & i \sqrt{1 - \eta^2} e^{i(\delta_{\pi\pi}^l - \delta_{\bar{K}K}^l)} \\ i \sqrt{1 - \eta^2} e^{i(\delta_{\bar{K}K}^l + \delta_{\pi\pi}^l)} & \eta e^{2i\delta_{\bar{K}K}^l} \end{pmatrix}. \quad (8)$$
These elements have been computed in [22] and here we use the solution A corresponding to the so-called "down-flat" data of the phase shift analysis [23].

The $I=1$ $KK$ interaction near 1 GeV is very strongly influenced by the $a_0(980)$ resonance which decays dominantly into the $\pi\eta$. A coupled channel model including the $\pi\eta$ and $KK$ states has been formulated in [24] and recently compared to the existing data [25]. As a result one obtains the $KK$ scalar-isovector $S^{I=1}$ which differ from the scalar-isoscalar $S$-matrix elements written in (8). This new information has been incorporated into calculations of the final state $K^+K^-$ interactions.

The $P$-wave amplitudes corresponding to a given projection $M$ of the $K^+K^-$ angular momentum on the quantization axis can be handled in a similar way as the $S$-wave, i.e. we write

$$T^P_{\lambda,\lambda'M} = \bar{u}(p',\lambda') J^P_{\mu M}(p,p',q, M_{KK}) e^{\mu}(q,\lambda) u(p,\lambda),$$

with $J^P_{\mu M}$ being the $P$-wave projection of the current. The $P$-wave current is described in detail in [21]. In particular, it is saturated by the diffractive $\phi$-meson production,

$$J^P_{\mu M} = \frac{i F(t)}{M^2 - M^2_{KK} - i M\Gamma_{\phi}} \left[ \gamma^{\nu} q_{\nu} (k_1 - k_2)^{\mu} - q^{\nu} (k_1 - k_2)_\nu \gamma^{\mu} \right],$$

where $\gamma^\mu$ are the Dirac matrices, and $M_{\phi}$ and $\Gamma_{\phi}$ are $\phi$ mass and width, respectively. The Lorentz-Dirac structure of the current is motivated by the Donnachie-Landshoff model for the pomeron exchange which assumes vector coupling of the Pomeron to hadrons [26]. The two terms in (10) are needed to preserve electromagnetic current conservation, $q^\mu J^P_{\mu M} = 0$. One can show that $q^\mu J^P_{\mu M} = 0$ is satisfied independently by each projection of the $KK$ spin, $M = \pm 1, 0$. Then in the phenomenological analysis of the data one can separately modify different $J^P_{\mu M}$ components by multiplying them by constant factors. The function $F(t)$ is a phenomenological function which will be suitably parameterized to reproduce, at fixed energy, the $t$-dependence of the observed $\phi$ photoproduction. Its analytical form will be specified in the next section. We note, however, that this model for the $P$-wave does not lead to significant suppression of the single helicity flip amplitude. In particular in the high energy limit of $s \simeq 2E\gamma m_\phi \gg |t|$ and $s \gg M_{KK}$ the helicity non-flip, single-flip and double-flip amplitudes with $\lambda_\gamma = 1$ behave as
$$T_{1\lambda'\lambda} \propto 2N_{\lambda'\lambda} M_{K\bar{K}}, \quad T_{1\lambda'\lambda_0} \propto N_{\lambda'\lambda}\sqrt{-2t}, \quad T_{1\lambda'\lambda_1} \to O(1/s), \quad (11)$$

respectively. The proportionality constant $N_{\lambda'\lambda}$ contains the Breit-Wigner propagator of (10) and is finite as $t' = t - t_{min} \to 0$. The diagonal elements of the matrix $N$ are finite and the off-diagonal ones, corresponding to nucleon helicity flip, are $O(1/s)$. Thus the only suppression of the photon-meson coupling comes from the angular momentum conservation factor $t^{|\lambda' - M|/2}$, but it is otherwise finite at high energy. The existing data on $\phi$ photoproduction suggest that the helicity flip amplitude is of the order of 10% of the dominant, helicity non-flip one at photon energies below 10 GeV [18]. Thus, qualitatively the model has the correct features but the quantitative agreement may require adjusting the normalization and the phase of the helicity flip amplitudes. This can be done by multiplication of these amplitudes by a constant complex factor $C_{10} \exp(i\phi_{10})$, where $C_{10}$ is a real positive number and $\phi_{10}$ is an additional phase of the $P$-wave amplitude with $M = 0$. We will not modify phases of the dominant helicity non-flip amplitudes, however we do change slightly their moduli as well as the moduli of small double helicity flip amplitudes to keep the total $P$-wave cross-section untouched. The phases of the small double helicity flip amplitudes are the same as the phases of the corresponding non-flip amplitudes. Similarly, for the $S$-wave amplitudes we introduce a constant modification factor equal to $C_{00} \exp(i\phi_{00})$. It is assumed that the parameters $C_{10}, \phi_{10}, C_{00}$ and $\phi_{00}$ do not depend on proton helicities $\lambda$ nor $\lambda'$. The experimental data on the angular distribution of the $K\bar{K}$ pair are given in terms of the moments $\langle Y^L_M \rangle$ of the angular distribution evaluated in the $s$-channel helicity frame,

$$\langle Y^L_M \rangle \equiv \int_{t_1}^{t_2} dt \int d\Omega \ Y^L_M(\Omega) \frac{d\sigma}{dt \ dM_{K\bar{K}}} d\Omega = \mathcal{N} \int dt \int d\Omega \sum_{\lambda',\lambda\lambda'} |T_{\lambda\gamma,\lambda'\lambda_0}|^2 Y^L_M(\Omega), \quad (12)$$

where $\mathcal{N}$ takes into account the photon flux and the 1/4 factor standing before the sum in (3). The lowest moment is normalized to the $K^+K^-$ mass distribution integrated over the momentum transfer squared range limited by $t_1$ and $t_2$:

$$\langle Y^0_0 \rangle = \frac{1}{\sqrt{4\pi}} \frac{d\sigma}{dM_{K\bar{K}}}, \quad (13)$$

In terms of the $S$- and $P$-partial waves, the non-vanishing moments are
given by
\[
\begin{align*}
\langle Y_0^0 \rangle &= \frac{N}{\sqrt{4\pi}} \left( |S|^2 + |P_+^2| + |P_-^2| + |P_0^2| \right), \\
\langle Y_1^1 \rangle &= \frac{N}{\sqrt{4\pi}} (SP_0^* + S^*P_0), \quad \langle Y_1^0 \rangle = \frac{N}{\sqrt{4\pi}} (P^+S^* - SP^*), \\
\langle Y_2^2 \rangle &= \frac{N}{\sqrt{4\pi}} \sqrt{\frac{1}{5}} \left( 2P_0P_0^* - P_+P_0^* - P_-P_0^* \right), \\
\langle Y_2^1 \rangle &= \frac{N}{\sqrt{4\pi}} \sqrt{\frac{3}{5}} \left( P_+P_0^* - P_0P_0^* \right), \quad \langle Y_2^0 \rangle = \frac{N}{\sqrt{4\pi}} \sqrt{\frac{6}{5}} \left( -P_+P^*_0 \right).
\end{align*}
\]

(14)

Here \( S, P \) stand for \( T^S \) and \( T^P \) amplitudes, respectively, and summation over photon and nucleon spin indices is implicit, \( e.g. \),
\[
P_+P_0^* = \sum_{\lambda, \lambda'} T^P_{\lambda, \lambda' \lambda_1} T^{*P}_{\lambda, \lambda' \lambda_0}.
\]

The dominant \( P_+ \) and \( P_- \) waves originating from \( M = \lambda_\gamma = +1 \) and \( M = \lambda_\gamma = -1 \) helicity non-flip production will manifest themselves in a large, positive \( \langle Y_0^0 \rangle \) and a large negative \( \langle Y_2^0 \rangle \) near \( M_{K\bar{K}} = 1.02 \) GeV – the mass of the \( \phi \) resonance. This is indeed the dominant feature of the data as shown in Figs. 3 and 4. Since \( |S|^2 \ll |P\pm|^2 \) the \( S \)-wave is not expected to be significant in the mass spectrum \( i.e. \), in the \( \langle Y_0^0 \rangle \) moment. It will primarily contribute to the cos \( \theta \) asymmetry measured by the \( \langle Y_1^1 \rangle \) and the \( \langle Y_1^0 \rangle \) moments. For diffractive \( P \)-wave production with equal phases of all \( P \)-amplitudes it is expected that \( |\langle Y_1^1 \rangle| > |\langle Y_0^0 \rangle| \) since the latter describes the interference between the \( S \)- and the helicity flip \( P_0 \)-wave which has a smaller magnitude than \( P_+ \) one. The data suggests, however, that the two moments are comparable with more structure actually seen in the \( \langle Y_1^1 \rangle \) moment. This can only be possible if we allow a well defined pattern of phases in the three waves. This justifies our choice of additional parameters as already described above. In the following section we discuss the results of fitting this model to the experimental data.

### 3 Numerical results

In the analysis of the \( S \)- and \( P \)-wave production we compare the model described above to the \( t \), \( M_{K\bar{K}} \) and angular distribution of the \( K\bar{K} \) system.
at two photon energies, \(E_\gamma = 4 \text{ GeV} \ [18]\) and \(E_\gamma = 5.65 \text{ GeV} \ [16, 17]\). The two photon energies represent averages of the photon beam energies used in the experiments performed at Daresbury and at DESY, respectively. At first we discuss the momentum transfer dependence of the cross section integrated over the \(K\bar{K}\) effective mass and the kaon emission angles.

### 3.1 Momentum transfer distributions and integrated cross sections at \(E_\gamma = 4 \text{ GeV}\)

At 4 GeV photon energy the comparison between the experimentally measured differential cross section \(d\sigma/dt\) and the model cross section integrated over the \(K\bar{K}\) mass \(0.997 \text{ GeV} < M_{K\bar{K}} < 1.042 \text{ GeV}\) is shown in Fig. 2. For this comparison, the \(t\)-dependent normalization \(F(t)\) in the \(P\)-wave was chosen as

\[
F(t) = \frac{D_1 e^{bt}}{(1 - t/a)^2},
\]

(16)

where the normalization constant \(D_1\) has been adjusted to reproduce the very forward value of \(d\sigma/dt\left(t_0\right) = 1.852 \mu b/GeV^2\) at small argument, \(t_0 = -0.0225 \text{ GeV}^2\), and the remaining parameters have been chosen as \(a = 0.7 \text{ GeV}^2\) and \(b = 0.05 \text{ GeV}^{-2}\). The value of \(d\sigma/dt\left(t_0\right)\) has been obtained from the experimental fits of Barber et al. at low momentum transfers and in the energy range between 3.4 and 4.8 GeV (see Fig. 7 of [18]). We took \(d\sigma/dt = (2.13 \pm 0.38)\mu b/GeV^2 \ exp\left[ (6.2 \pm 1.3)\text{GeV}^{-2} t\right]\) and calculated its value at the minimum \(|t_0|\) argument corresponding to the \(K\bar{K}\) effective mass 1.042 GeV, which was the upper limit studied in [18]. Then the constant \(D_1\) equals to

\[
D_1 = \left(\frac{d\sigma/dt\left(t_0\right) BR}{Int}\right)^{\frac{1}{2}},
\]

(17)

where BR=0.486 is the branching ratio for the \(\phi\) decay into \(K^+K^-\) used in [18] and

\[
\text{Int} = \int_{M_1}^{M_2} \frac{d\sigma^P}{dt\ dM_{K\bar{K}}}(D_1 = 1, t_0) \ dM_{K\bar{K}},
\]

(18)

\(M_1\) being the lower effective mass limit 0.997 GeV and \(M_2\) being the upper mass limit 1.042 GeV. In the above equation the unnormalized double differential cross section \(d\sigma^P/(dt\ dM_{K\bar{K}})\) corresponds to the \(P\)-wave amplitudes
Fig. 2: Differential cross section at $E_\gamma = 4$ GeV. The solid line shows the model $t$-distribution for the $\phi$ photoproduction, the dotted line is the $P$-wave contribution with $M=0$ multiplied by the branching ratio of the $\phi$ decay into the $K^+K^-$ pair. The dashed line is the $S$-wave part of the $K^+K^-$ cross section calculated for normal $\rho, \omega$ propagators, while the double dotted-dashed line corresponds to the Regge propagators. Model parameters are given in Table 1. Data are from Fig. 6b of [18].
calculated at fixed $t_0$. A variation of the $t_0$ with the effective mass $K^+K^-$ has been neglected in this narrow band of $M_{K\pi}$.

As expected, the $t$-distribution is dominated by the helicity non-flip $P$-wave. The $P_0$-wave and the $S$-wave are kinematically suppressed at low $t$ and are two orders of magnitude smaller than the dominant wave. After integration over $t$ in the range up to $-t = 1.5\text{ GeV}^2$ the total $\phi$ photoproduction cross section equals $0.449\,\mu b$ and is in very good agreement with the measured value of $(0.450 \pm 0.019)\,\mu b$ presented in Table 1 of [18] for the photon energy range between 2.8 and 4.8 GeV. The $K^+K^-$ part of the integrated $P$-wave cross section equals $(0.218 \pm 0.039)\,\mu b$. In the model fits to data, the decomposition of the total $P$-wave cross section in its parts corresponding to $M = 1$, $M = 0$ and $M = -1$ components depends to some extent on the contribution of the $S$-wave cross section and in particular on the choice of the type of propagators included in the $S$-wave amplitude. For normal $\rho, \omega$ propagators the integrated cross sections for the $P_0$-wave and the $S$-wave are equal to $(6.4^{+5.5}_{-4.8})\,\text{nb}$ and $(4.9^{+5.8}_{-3.6})\,\text{nb}$, respectively. The corresponding numbers for the Regge propagators are $(4.7^{+5.7}_{-4.5})\,\text{nb}$ and $(4.3^{+6.6}_{-3.6})\,\text{nb}$. The errors of the cross sections have been evaluated using the limits of the model parameters obtained from the fitting program MINUIT [27]. These numbers are smaller than our early estimates of the $S$-wave integrated cross sections written in [20]. One reason of this change is related to a presence of the $S$-wave form factor $\exp(B_st)$ introduced in the previous section and the second one comes from the diminution of the $S$-wave modulus obtained in the fitting procedure which will be explained later.

3.2 Momentum transfer distributions and integrated cross sections at $E_\gamma = 5.65$ GeV

The DESY data [17] have been taken at much lower momentum transfers than the Daresbury data [18]. The most precise data of Behrend et al. lie within the range $|t - t_{\text{min}}| < 0.2 \text{ GeV}^2$. The average energy $E_\gamma = 5.65$ GeV corresponds to the photon energy range between 4.6 and 6.7 GeV. The $\phi$ production differential cross section $d\sigma/dt$ can be fitted as $d\sigma/dt = n\,\exp(bt)$, where $n = (2.40 \pm 0.15)\,\mu b/\text{GeV}^2$ and $b = (6.11 \pm 0.53)\,\text{GeV}^{-2}$ represents the average slope of $d\sigma/dt$ for the four upper energy bins given in Table 3 of [17]. Taking the appropriate $t_0$ value equal to $-0.0113\,\text{GeV}^2$ we calculate $d\sigma/dt (t_0) = 2.24\,\mu b/\text{GeV}^2$. Then we define a simple form of the $t$-dependent
normalization factor at $E_\gamma = 5.65$ GeV as 

$$F(t) = D_2 \ e^{\frac{1}{2}bt}, \quad (19)$$

and use once again (17) and (18) to find a new constant $D_2$ taking into account that the $\phi$ branching ratio used in [17] was $BR=1/2.14$, $M_1 = 1.01$ GeV and $M_2 = 1.03$ GeV. The $K^+K^-$ photoproduction cross section at $E_\gamma = 5.65$ GeV integrated over $|t|$ up to 0.2 GeV$^2$ is 120.5 nb. Its $P_0$- and $S$-wave parts are $(13.8^{+5.3}_{-4.7})$ nb and $(7.0^{+6.8}_{-4.4})$ nb for normal propagators and $(14.0^{+5.3}_{-4.8})$ nb and $(6.8^{+6.6}_{-4.3})$ nb for Regge propagators, respectively. Let us notice that the $P_0$- and $S$-wave cross sections are comparable and do not vary too much with energy bearing in mind rather large errors. The $S$-wave cross section at 5.65 GeV is comparable in its magnitude with the estimate of the upper limit $(2.7 \pm 1.5)$ nb quoted in [16] and [17]. We finally note that the total $\phi$ photoproduction cross section of $(0.25 \pm 0.2) \ \mu b$ given in [16] corresponds to $M_{KK}$ integrated over the range 1.0 GeV to 1.024 GeV. When integrated in this mass range our model gives $0.23 \ \mu b$ in good agreement with the measurement.

3.3 $K^+K^-$ mass distributions and moments at $E_\gamma = 4$ GeV

Next let us describe calculations of the $K^+K^-$ differential cross section integrated over a certain range of the momentum transfer at fixed $M_{KK}$ mass. We first discuss the results corresponding to $E_\gamma = 4$ GeV where the upper limit of $-t$ was 1.5 GeV$^2$. In Fig. 3 we show the results of the simultaneous fit to the $K^+K^-$ effective mass distribution $d\sigma/dM_{KK}$ in the mass range $0.992 < M_{KK} < 1.037$ GeV and to the moments $\langle Y_L^L \rangle$ at $0.997$ GeV $< M_{KK} < 1.042$ GeV. To account for a possibly large $\pi\pi$ experimental background in [18] we have introduced an additional linear term

$$\frac{d\sigma_b}{dM_{KK}} = A + B(M_{KK} - M_{av}), \quad (20)$$

where $A$ and $B$ are free parameters and $M_{av} = (M_1 + M_2)/2$ is the average $K\bar{K}$ effective mass in the range chosen above. The parameters $A$ and $B$ will be fitted to the experimental data. The background cross section integrated over the mass range between $M_1$ and $M_2$ is equal to $A(M_2 - M_1)$. It is
Fig. 3: $K^+K^-$ mass spectrum and moments of angular distribution in the helicity frame for an incident photon energy 4 GeV. Solid lines are results of model calculations for the normal $\rho, \omega$ propagators, while the dashed lines correspond to the Regge propagators. The dotted line shows the background contribution to the mass spectrum, while the short dashed line represents the $S$-wave part. The phenomenological parameters are given in Table 1. The data points are from [18].
Table 1: Fitted values of model parameters for $E_\gamma = 4$ GeV. Units of $A$ and $B$ are $\mu b$/GeV and $\mu b$/GeV$^2$, respectively.

|       | Normal propagators         | Regge propagators         |
|-------|----------------------------|---------------------------|
| $\phi_{00}$ | $122.3^0 _{-21.5^0}$ | $74.5^0 _{-27.0^0}$ |
| $\phi_{10}$ | $87.6^0 _{-11.1^0}$ | $86.8^0 _{-23.1^0}$ |
| $C_{00}$ | $0.33 \pm 0.16$ | $0.72 _{-0.43} ^{+0.44}$ |
| $C_{10}$ | $0.44 ^{+0.16} _{-0.22}$ | $0.37 _{-0.31} ^{+0.18}$ |
| $A$ | $6.65 ^{+0.22} _{-0.23}$ | $6.67 _{-0.24} ^{+0.22}$ |
| $B$ | $133.0 \pm 11.9$ | $133.1 \pm 11.9$ |
| $v_{10}$ | $(-12.2 ^{+6.0} _{-6.7}) \times 10^{-3}$ | $(-11.3 \pm 6.5) \times 10^{-3}$ |
| $v_{11}$ | $(-2.0 \pm 5.5) \times 10^{-3}$ | $(-1.2 ^{+8.9} _{-5.6}) \times 10^{-3}$ |
| $v_{20}$ | $(-7.8 ^{+8.9} _{-9.0}) \times 10^{-3}$ | $(-6.5 ^{+8.9} _{-9.1}) \times 10^{-3}$ |

rather large, attaining a value of about 300 nb. Similarly we have added the background terms to three moments:

$$
\langle Y^1_{0} \rangle_b = v_{10} \frac{d\sigma_b}{dM_{K\bar{K}}}, \quad \langle Y^1_{1} \rangle_b = v_{11} \frac{d\sigma_b}{dM_{K\bar{K}}}, \quad \langle Y^2_{0} \rangle_b = v_{20} \frac{d\sigma_b}{dM_{K\bar{K}}},
$$

where $v_{10}$, $v_{11}$ and $v_{20}$ are constants. The remaining two moments $\langle Y^2_{1} \rangle$ and $\langle Y^2_{2} \rangle$ have not been corrected for background since their experimental values fluctuate around zero [18]. In the fits to data a finite experimental resolution of the effective $K^+K^-$ mass distribution was taken into account by choosing the effective $\phi$ width equal to 5.6 MeV according to the $K^+K^-$ spectrum in Fig. 5 of [18]. The theoretical effective mass distribution and the moments have been smeared over the mass interval $\Delta M_{K\bar{K}} = 5$ MeV equal to the mass bin size. Both effects lead to broadening of the $\phi$ resonance spectrum and to a widening of the structure of moments near the $\phi$ mass. The overall number of free parameters in our fit is 9 (or 7 if $v_{11}$ and $v_{20}$ are set to zero, which they are within errors). Table 1 presents the values of fitted model parameters in cases of normal and Regge propagators used in the $S$-wave amplitudes. The total number of experimental data is 60. We see in Fig. 3 that all the moments are well fitted including the moment $\langle Y^1_{0} \rangle_b$. One has the impression that the theoretical curve corresponding to this moment has a slightly too small amplitude near 1.015 GeV, but the $\chi^2$ value for 10 data points is good:
equal to 8. The $S$-wave cross section is small, almost invisible in comparison with the large peak corresponding to the $\phi$ resonance and the very important background which we attribute to an experimental misidentification of the $\pi\pi$ events as $K\bar{K}$ events. In our fits of the moments we have not assumed like in [18] that the nucleon non-flip $S$-wave amplitudes vanish. In fact, they do not vanish and become more important at higher values of $t$. Thus we have included the moment $\langle Y^1_1 \rangle$ in our analysis without making the ad hoc assumption that it is zero from the beginning. We see in Fig. 3 that $\langle Y^1_1 \rangle$ is in general non zero and has some structure near 1.02 GeV related to the position of the maximum of the dominant $P$-wave. The experimental values of the moment $\langle Y^2_1 \rangle$ are particularly small. In our model this moment is also small due to a large phase difference $\phi_{10}$ between the $P$-wave amplitudes with $M = 1$ and $M = 0$. This phase is close to 90° as seen in Table 1. The $S$-wave phase has also a large correction $\phi_{00}$, which depends on the type of propagators used in the model. This phase is smaller for the Regge propagators since they are complex and vary with the momentum transfer. On the average the Regge propagators add about 50° to the phase of the $S$-wave. This increase is compensated by a decrease of $\phi_{00}$.

Among the $S$-wave amplitudes, the proton spin non-flip components are the most important ones, although their dominance is not so strong as in the case of the $P$-wave. This feature of the $S$-wave amplitudes is related to an important contribution of the $\rho$ exchange. The phase difference between the $S$-wave proton spin non-flip and the $P_0$-wave proton spin non-flip amplitudes is larger than 90° for the $M_{K\bar{K}}$ masses smaller than the $\phi$ mass and it becomes smaller than 90° on the right hand side of the $\phi$ resonance. This happens due to the rapid phase increase of the resonant $P_0$-amplitude. As a consequence of this phase variation the moment $\langle Y^1_0 \rangle$ has a minimum to the left of $\phi$ resonance position and the maximum to its right. Let us stress here that only this moment has been analyzed in [18] as a source of the $S - P$ wave interference. We should remark, however that the moment $\langle Y^1_1 \rangle$ also depends sensitively on the $S$-wave amplitudes.

In addition to phases the fitting program provides us with the values of the moduli $C_{00}$ and $C_{10}$. At 4 GeV both factors are smaller than one so the integrated cross sections for the $S$-wave and the $P_0$-wave are reduced in magnitude and their final numbers stay below 10 nb as already written.
3.4 $K^+K^-$ mass distributions and moments at $E_\gamma = 5.65$ GeV

We pass to a discussion of the angular momentum structure at the average photon energy 5.65 GeV. The authors of [17] have presented in their Fig. 22 the so-called normalized moments of spherical harmonics $\langle Y^{l}_{M}\rangle /\langle Y^{0}_{0}\rangle$ together with the unnormalized $K\bar{K}$ mass distribution representing numbers of events per 10 MeV bin. We have attempted to make our own normalization of the above data in order to obtain the functional dependence of the moment $\langle Y^{0}_{0}\rangle$ which is equal to the differential cross section $d\sigma/dM_{K\bar{K}}$ divided by $(4\pi)^{1/2}$. We have integrated the $\phi$ production differential cross section $d\sigma/dt$ parameterized above as a simple exponential function in the $-t$ range up to 0.2 GeV$^2$ obtaining the value of $(0.258 \pm 0.020) \mu$b. This value corresponds to the sum of $3927 \pm 82 K^+K^-$ events in the $M_{K\bar{K}}$ range between 1.01 and 1.03 GeV. Taking into account the $K^+K^-$ branching ratio reported as 1/2.14 in [17] we get the normalization constant equal to $(3.068 \pm 0.245) \cdot 10^{-5} \mu$b/event. Knowing this constant one can calculate the moment $\langle Y^{0}_{0}\rangle$ and consequently the values of all other moments $\langle Y^{L}_{M}\rangle$ for L and M up to 2. Then we have performed a common fit to $d\sigma/dM_{K\bar{K}}$ and 5 moments $\langle Y^{L}_{M}\rangle$ for $M_{K\bar{K}}$ between 1.005 and 1.045 GeV, including fully the range of the $\phi(1020)$ resonance as well as a part of $M_{K\bar{K}}$ well above it. Here the width of the mass bins was 10 MeV. Unfortunately the value of the mass distribution $d\sigma/dM_{K\bar{K}}$ corresponding to the extreme experimental data point at 0.995 GeV was not given in [17], even though the $L \neq 0$ normalized moments were presented. For this reason we were unable to include this bin in our fit. The effective mass resolution reported in [17] was about 7 MeV and the effective $\phi$ width chosen in the analysis done by [16] was 8 MeV. We included the finite mass resolution by smearing our theoretical mass spectrum and moments over the 8.5 MeV range around each $M_{K\bar{K}}$ value. In the $P$-wave amplitudes the Breit-Wigner form of the $\phi$ spectrum was used with the $\phi$ mass equal to 1.0194 GeV and the width equal to 4.26 MeV. As at 4 GeV energy we have introduced a linear background term in the effective $K^+K^-$ mass distribution and in the two moments:

$$\langle Y^{2}_{0}\rangle_b = \beta_{20}(M_{KK} - M_{th}), \quad \langle Y^{2}_{1}\rangle_b = \beta_{21}(M_{KK} - M_{th}),$$

(22)

where $\beta_{20}$ and $\beta_{21}$ are parameters, and $M_{th}$ is the threshold mass of the $K\bar{K}$ system. Counting the four parameters in the two complex factors $C_{00} \exp(i\phi_{00})$ and $C_{10} \exp(i\phi_{10})$ which modify the $S$-wave and the $P_0$-wave...
Fig. 4: $K^+K^-$ mass distribution and normalized moments of angular distribution in the helicity system for an incident photon energy 5.65 GeV. The solid lines are results of model calculations; the dashed line is the $S$-wave contribution to the mass distribution. The phenomenological parameters are given in Table 3. The data points are from [17].
Table 2: The fitted values of model parameters for $E_\gamma = 5.65$ GeV. Units of $A$ are $\mu b/GeV$ and $B$, $\beta_{20}$ and $\beta_{21}$ are in $\mu b/GeV^2$.

|       | Normal propagators | Regge propagators |
|-------|--------------------|-------------------|
| $\phi_{00}$ | $106.0^0\pm10.2^0$ | $49.3^0\pm10.1^0$ |
| $\phi_{10}$ | $11.4^0\pm17.3^0$ | $13.0^0\pm17.2^0$ |
| $C_{00}$ | $1.06^{+0.44}_{-0.41}$ | $1.53^{+0.64}_{-0.59}$ |
| $C_{10}$ | $1.59^{+0.28}_{-0.30}$ | $1.60^{+0.28}_{-0.30}$ |
| $A$ | $0.25^{+0.21}_{-0.31}$ | $0.24^{+0.21}_{-0.29}$ |
| $B$ | $7.14^{+4.81}_{-4.80}$ | $7.22^{+4.82}_{-4.80}$ |
| $\beta_{20}$ | $1.20\pm0.45$ | $1.19\pm0.45$ |
| $\beta_{21}$ | $-1.98\pm0.45$ | $-1.98\pm0.45$ |

amplitudes, and adding two background parameters $A$ and $B$, we have altogether eight parameters to be fitted to data at 5.65 GeV. The results of the fits are shown in Fig. 4 and the parameters are written in Table 2. Contrary to the previous case of $E_\gamma = 4$ GeV, the background cross section at 5.65 GeV is much smaller, less than 5 nb. The shape of the $K\bar{K}$ mass spectrum and the general behaviour of the moments are well described by the model perhaps except of the two points of $\langle Y_{20}^2 \rangle$ at 1.005 and 1.015 GeV. The values corresponding to these data points are smaller than $-0.45$. Let us notice that the lowest limit of $\langle Y_{20}^2 \rangle/\langle Y_{00}^0 \rangle$ equals to $-1/\sqrt{5} \approx -0.45$ if one assumes that only $S-$ and $P-$ waves participate in the $K^+K^-$ production process. Strictly speaking, this limit corresponds to the case in which the amplitudes $P_+$ dominate near the position of the $\phi(1020)$ resonance. Any admixture of the $P_0-$, $P_-$ or $S-$ waves must increase the value of $\langle Y_{20}^2 \rangle/\langle Y_{00}^0 \rangle$ above $-1/\sqrt{5}$. A uniform background contribution would have the same effect. Thus, one is tempting to explain the low experimental values at 1.005 and 1.015 GeV by the presence of a background coming from higher waves like $D-$ or $F-$ waves. Interestingly, slightly above the $\phi(1020)$ mass, there is a structure in the ratio of $\langle Y_{40}^4 \rangle/\langle Y_{00}^0 \rangle$ measured in [17]. This structure can be attributed to $D-$ wave or to an interference of the $P-$ wave with the $F-$ wave. Both cases are, however, physically rather improbable, because near the $K\bar{K}$ threshold these waves should be strongly suppressed and we do not know any $D$ or $F$ resonances located closely to 1 GeV. Let us also remark
that a general shape of \( \langle Y^2_0 \rangle / \langle Y^0_0 \rangle \) shown by the line in Fig. 4 is correct near the \( K \bar{K} \) threshold since the extreme experimental point at 0.995 GeV lies above and not too far from the curve. In Fig. 4 we do not show a line corresponding to the small background to \( d\sigma/dM_{K\bar{K}} \), since its magnitude is very close to the \( S \)-wave contribution shown in this figure.

Finally, in Table 3 we list the contributions of the individual waves to the \( K \bar{K} \) photoproduction cross section at the two energies studied.

| Table 3: Integrated cross sections in nb |
|-----------------------------------------|
| photon energy | 4 GeV | 5.65 GeV |
|----------------|-----------------|-----------------|
| \( S \)-wave propagator | normal | Regge | normal | Regge |
|-----------------|-----|-----|-----|-----|
| sum of all \( P \)-waves | 218.4 \( \pm \) 39.5 | 120.5 \( \pm \) 9.4 |
| \( P_0 \)-wave | 6.4 \( \pm \) 4.8 | 4.7 \( \pm \) 3.5 | 13.8 \( \pm \) 3.4 | 14.0 \( \pm \) 5.3 |
| \( S \)-wave | 4.9 \( \pm \) 3.6 | 4.3 \( \pm \) 3.6 | 7.0 \( \pm \) 6.8 | 6.8 \( \pm \) 6.3 |
| background | 299.4 \( \pm \) 4.9 | 300.0 \( \pm \) 10.0 | 4.5 \( \pm \) 4.3 | 4.7 \( \pm \) 5.8 |
| \( |t|_{max} \) | 1.5 GeV² | 0.2 GeV² |
| \( M_{K\bar{K}} \) range | (0.997,1.042) GeV | (1.01,1.03) GeV |

### 3.5 Model predictions at the energy upgraded Jefferson Laboratory

We have performed calculations of the \( K^+K^- \) mass spectrum and moments at \( E_\gamma=8 \) GeV which will be a typical energy of the planned upgrade of the CEBAF accelerator operating at the Thomas Jefferson Laboratory. The results presented in Fig. 5 can be directly compared with Fig. 3 corresponding to the much lower energy of 4 GeV. The calculations have been performed for an ideal case in which there is no background and no phenomenological adjustment of the moduli and phases of the \( S^- \) and \( P_0^- \) waves i.e. \( \phi_{00} = \phi_{10} = 0 \), \( C_{00} = C_{10} = 1 \). We have assumed that the \( K^+K^- \) mass resolution is equal to 5 MeV. The parameters of the \( \phi(1020) \) meson, like the mass, width and the \( K^+K^- \) decay fraction, have been taken as 1019.456 MeV, 4.26 MeV and 0.492, respectively. The differential cross section at low \( t \) was parameterized as \( d\sigma/dt = n \exp(bt) \) with \( n = 2.53 \mu b/GeV^2 \) and \( b = 6.11 \text{ GeV}^{-2} \). One can notice that the application of Regge propagators...
Fig. 5: $K^+K^-$ mass spectrum and moments of angular distribution in the helicity frame for incident photon energy 8 GeV. Solid lines are results of model calculations for the normal $\rho, \omega$ propagators while the dashed lines correspond to the Regge propagators in the $S-$ wave amplitudes.
in the $S-$ wave leads to smaller values of the $S-$ wave cross section and to smaller amplitudes of the moments $\langle Y^1_0 \rangle$ and $\langle Y^1_1 \rangle$ sensitive to the interference between the $S-$ and $P-$ waves. Qualitatively we do not observe important differences in a behaviour of the unnormalized moments between the photon energy of 5.65 and 8 GeV (let us recall here that Fig.4 shows the ratios of $\langle Y^L_M \rangle/\langle Y^0_0 \rangle$, not $\langle Y^L_M \rangle$).

The $S-$ wave total cross sections integrated over the $M_{K\bar{K}}$ range between 1.01 and 1.03 GeV and in the $|t|-$ range from 0.0054 up to 0.2 GeV$^2$ are equal to 6.9 nb and 3.0 nb for the normal and Regge propagators, respectively. The $P-$ wave cross section integrated in the same ranges equals 141 nb while the corresponding $P_0-$ cross section is equal to 6 nb.

4 Conclusions

In this paper we presented results of a theoretical analysis of data on photoproduction of the $K^+K^-$ pairs in the laboratory photon energy range $E_\gamma$ between 2.8 and 6.7 GeV. In particular we mapped out the interference pattern between the $S$- and the $P$-wave due to the presence of the scalar $f_0(980)$, $a_0(980)$ and the vector $\phi(1020)$ resonances. In the analyses the $S$-wave was described by a model based on the dominance of the $t$-channel exchange process with the two-meson spectrum described in terms of the coupled channel meson-meson scattering $S$- matrix. The $P$-wave was described in terms of diffractive production of the $\phi$ resonance decaying into the $K^+K^-$ system. The $S$-wave and the $P$-wave contain 4 and 12 independent amplitudes respectively and we included them all while previous analyses made a severe truncation to a single amplitude in each wave. We have shown that amplitudes omitted in the analyses of [16, 18] corresponding to the proton helicity non-flip are large and cannot be ignored. In the previous analyses only the $\langle Y_{10} \rangle$ moment was taken into account and this led to a large variation in the estimate of the $S$-wave photoproduction cross section, between $(2.7 \pm 1.5)$ nb at $E_\gamma = 5.65$ GeV [16, 17] and $(96.2 \pm 20)$ nb at $E_\gamma = 4$ GeV [18]. In this paper we have considered all six moments appearing in the model with $S$- and $P$-waves which enabled us to isolate the $S$-wave production cross section from that of an incoherent background. This led to a significant, order of magnitude, reduction in the uncertainty in the $S$-wave production and reduced the value of cross section at 4 GeV from 96 nb to approximately 5 nb, thus eliminating the discrepancy between the two measurements (also
suggested in [18]). Thus we have found that the S-wave photoproduction cross section integrated over the φ resonance region is between 4 and 7 nb for the two photon energies 4 and 5.65 GeV. The cross section of the photon helicity-flip P_0-wave, which interferes with the S-wave is found to be somewhat larger than the S-wave cross section, in the range between 5 and 14 nb. New more precise measurements of the K\bar{K} photoproduction with a simultaneous determination of the ππ cross-section in the ππ effective mass region near 1 GeV could provide new insight into the still controversial nature of the scalar mesons f_0(980) and a_0(980).

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