In this paper we investigate an $SU(3)$ extension of the chiral quark-meson model. The spectra of baryons with strangeness, charm and bottom are considered within a "rigid oscillator" version of this model. The similarity between the quark part of the Lagrangian in the model and the Wess-Zumino term in the Skyrme model is noted. The binding energies of baryonic systems with baryon number $B = 2$ and $3$ possessing strangeness or heavy flavor are also estimated. The results obtained are in good qualitative agreement with those obtained previously in the chiral soliton (Skyrme) model.

1 Introduction

The chiral soliton models, Skyrme model first of all [1], are attractive because they are simple, elegant and allow to describe the properties of lowest baryons with a rather good accuracy. At the same time, since the quark degrees of freedom are excluded from the beginning, the Skyrme model is not completely realistic: it is generally believed that quarks should be explicitly present in the baryons. Consideration of more realistic models with explicit quark degrees of freedom included into Lagrangian seems to be necessary.

For the case of nonstrange baryons it was done in papers [2]-[6] within the chiral quark meson model (CQM), where the mean field approximation for the quark wave functions was an important ingredient of the model. From a theoretical point of view, the CQM models have an advantage that there is no question about the choice of the terms in the Lagrangian responsible for the stabilization of the soliton: the stabilization takes place due to the quark-meson interaction. Such models are minimal in the sense that only the second order terms in chiral fields derivatives are present in the effective Lagrangian [2]-[5].

Here we extend such models for the consideration of baryons with strangeness, charm and bottom, for the sector with $B = 1$, first of all. These degrees of freedom are treated in the same manner as in the bound state approach to heavy flavors proposed in [7, 8] and a "rigid oscillator" version of which was developed in [9]-[11]. Within this model the deviations of quark fields and solitons into "strange" ("charm" or "bottom") directions are considered as small ones, and a corresponding expansion of the Lagrangian is made. The results obtained confirm the assumption concerning the smallness of these deviations.

The sectors with $B = 2$ and $3$ are also briefly discussed. Previously, the question of existence of baryonic systems (BS) with strangeness different from zero was a subject of intensive studies beginning with papers [12]-[15]. Some review of theoretical predictions, mainly for the sector with $B = 2$, can be found in [16]. The question of existence of BS with flavor different from $u$ and $d$ is quite general. Charm, bottom or top quantum numbers are also of interest, and their consideration can be performed in the framework of chiral soliton models, in particular, the bound
state approach to heavy flavors [7]-[10]. As it was shown recently, within the rigid oscillator model the BS with charm or bottom have even more chances to be bound relative to the strong interactions than strange baryonic systems [11]. Here we present some estimates for the binding energies of lightest BS with nontrivial flavor in the chiral quark-meson model and show that these estimates are in qualitative agreement with those obtained in [10], [11].

In section 2 we consider the SU(3) extension of the chiral quark-meson Lagrangian. In the next section, we give an explicit expression for the Hamiltonian of the BS in the leading order in $N_c$ in terms of the flavor (antiflavor) excitation frequencies. In section 4 the $B = 1$ sector is considered and hyperon -nucleon mass differences are estimated, including the zero modes corrections of the order of $1/N_c$. In section 5 sectors with $B \geq 2$ are discussed and binding energies of some few-baryon systems are estimated.

2 SU(3) extension of the chiral quark-meson Lagrangian

The SU(3) extension of the chiral quark-meson Lagrangian density can be written in the following way [2, 3]:

$$
\mathcal{L} = i\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi - gF_\pi(\bar{\Psi}L\Psi R + \bar{\Psi}RU^\dagger\Psi L) - \frac{\alpha_F}{3}gF_\pi[\bar{\Psi}L(1-\sqrt{3}\lambda_8)U\Psi R + \bar{\Psi}R U^\dagger(1-\sqrt{3}\lambda_8)\Psi L] +
$$

$$
+ \frac{F_\pi^2}{16}Trl_\mu l^{\mu} + \frac{F_\pi^2m_\pi^2}{16}Tr(U + U^\dagger - 2) + \frac{F_Dm_D^2 - F_\pi^2m_\pi^2}{24}Tr(1-\sqrt{3}\lambda_8)(U + U^\dagger - 2) +
$$

$$
+ \frac{F_\pi^2 - F_D^2}{48}Tr(1-\sqrt{3}\lambda_8)(ULl_\mu l^{\mu} + l_\mu l^{\mu}U^\dagger). \tag{1}
$$

Here $\Psi$ is a triplet of quark fields, $u, d, s$, or $u, d, c$, or $u, d, b$. $U \in SU(3)$ is a unitary matrix incorporating chiral (meson) fields, and $l_\mu = U^\dagger \partial_\mu U$. In this model $F_\pi$ is fixed at the physical value, $F_\pi = 186$ Mev, and $gF_\pi = 500$ Mev characterizes the effective $u, d$ quarks mass. The interaction between quarks is not considered explicitly, but it is present in this mean-field description of the quarks due to the quark-meson coupling. Here, we have included a term in the Lagrangian which describes flavor symmetry breaking (FSB) in the constituent quark masses and in the quark-meson coupling and is proportional to a parameter $\alpha_F$. The FSB in the meson part of the Lagrangian is of usual form, and was sufficient to describe the mass splittings of the octet and decuplet of baryons [17]. Here we consider first the case of flavor symmetry in decay constants, i.e. $F_D = F_\pi$. Even for realistic values of $F_D$ the last term in (1) is small and can be omitted.

The important property of the quark part of the Lagrangian is that it reproduces the properties of the Wess-Zumino term written in the simple form by Witten [18]. First, the baryon number is given by this term and second, when the field $\Psi$ is rotated into "strange", or other direction, the quark Lagrangian gives the contribution coinciding with that coming from the WZ term in the Skyrme model.

We shall consider the collective coordinates rotation of the quark field $\Psi$ and the meson fields incorporated into the matrix $U$, in the spirit of the bound state approach to the description of strangeness proposed in [7]-[9] and used in [10], [11]:

$$
\Psi(r, t) = R(t)\Psi_0(O(t)\vec{r}), \quad U(r, t) = R(t)U_0(O(t)\vec{r})R^\dagger(t), \quad R(t) = A(t)S(t), \tag{2}
$$

where $\Psi_0$ is originally a two-component spinor in $(u, d)$ SU(2) subgroup, $U_0$ is the SU(2) soliton embedded into SU(3) in the usual way (into left upper corner), $A(t) \in SU(2)$ describes SU(2) rotations, $S(t) \in SU(3)$ describes rotations in the "strange", "charm" or "bottom" direction, and $O(t)$ describes rigid rotations in real space.
For definiteness we shall consider the extension of the \( (u, d) \) \( SU(2) \) Skyrme model in strange direction, when \( D \) is the field of \( K \)-mesons. But it is clear that quite similar extension can be made in the charm or bottom directions, also.

\[
S(t) = \exp(iD(t)), \quad D(t) = \sum_{a=4\ldots7} D_{a}(t) \lambda_{a},
\]

\( \lambda_{a} \) are Gell-Mann matrices of \( (u, d, s) \), \( (u, d, c) \) or \( (u, d, b) \) \( SU(3) \) subgroups. The \( (u, d, c) \) and \( (u, d, b) \) \( SU(3) \) subgroups are quite analogous to the \( (u, d, s) \) one. For the \( (u, d, c) \) subgroup, a simple redefinition of hypercharge should be made. For the \( (u, d, s) \) subgroup, \( D_{4} = (K^{+} + K^{-})/\sqrt{2}, \) \( D_{5} = i(K^{+} - K^{-})/\sqrt{2}, \) etc. For the \( (u, d, c) \) subgroup \( D_{4} = (D^{0} + D^{0})/\sqrt{2}, \) etc.

Consider first the contribution due to the time dependence of the collective rotations in the quark part of the Lagrangian:

\[
\mathcal{L}_{q} = \sum_{q} i\bar{\Psi} \dot{\Psi} \text{collective} = \sum_{q} \Psi^{\dagger} \left[ iS^{\dagger} \dot{S} + \frac{1}{2} S^{\dagger} \vec{\omega} \vec{\tau} S + i(\vec{r} \vec{\tau} \vec{0}) \right] \Psi
\]

\( \vec{\omega} \) and \( \vec{\eta} \) are the angular velocities of the isospin and usual space rotations defined in the standard way:

\[
A^{\dagger} A = -i\vec{\omega} \vec{\tau}/2, \quad \dot{O}_{kn} = \epsilon_{ikm} \Omega_{m}
\]

The field \( D \) is small in magnitude, of order \( 1/\sqrt{N_{c}} \), where \( N_{c} \) is the number of colors in \( QCD \). Therefore, an expansion of the matrix \( S \) in \( D \) can be made. Collecting all the terms up to \( O(1/N_{c}) \), \( \mathcal{L}_{q} \) can be presented in the following form:

\[
\mathcal{L}_{q} \approx \sum_{q} \Psi^{\dagger} \left[ i\left( \frac{1}{2} (\dot{D}^{\dagger} D - D^{\dagger} \dot{D}) - \frac{2}{3} D^{\dagger} D - \frac{1}{2} \vec{\tau} D^{\dagger} \vec{\tau} D \right) + \frac{1}{2} (\vec{\omega} - \vec{\beta}) \vec{\tau} - \frac{1}{2} (\vec{\omega} \vec{\tau} D^{\dagger} D + \vec{\tau} D^{\dagger} \vec{\tau} D) \right] \Psi_{0}
\]

\[
+ \frac{1}{12} D^{\dagger} D \vec{\tau} \vec{\beta} + i(\vec{r} \vec{\tau} \vec{0}) \right] \Psi_{0}
\]

Here

\( \vec{\beta} = i(D^{\dagger} \vec{\tau} \dot{D} - \dot{D}^{\dagger} \vec{\tau} D) \)

is the angular velocity of rotation in the "flavor" direction, \( D \) is the doublet of heavy meson fields, kaons, \( D \)- or \( B \)-mesons. Expression (5a) does not depend on the color of the quark directly, but some of the terms in (5a) depend on the orientation of the quark in isospace and on its radial wave function which need not be the same for all the quarks. However, for an arbitrary \( B \), we always find the following term containing the factor \( N_{c} \) after summation over quarks and integration over space:

\[
L_{q} = \frac{N_{c} B s_{d}^{2}}{2} d^{2} \left[ i(\dot{D}^{\dagger} D - D^{\dagger} \dot{D}) - \vec{\omega} D^{\dagger} \vec{\tau} D \right],
\]

which is valid in any order in \( d^{2} = 2D^{\dagger} D \). It is assumed in (6) that the quark wave functions \( \Psi \) are properly normalized. This contribution coincides with that obtained from the Wess-Zumino term in the action in the collective coordinates quantization procedure \([\text{19}, \text{13}]\).

The general parametrization of \( U_{0} \) for an \( SU(2) \) soliton we use here is given by

\[
U_{0} = c_{f} + s_{f} \vec{r} \vec{n}
\]

with \( n_{z} = c_{n}, \) \( n_{x} = s_{n} c_{\beta}, \) \( n_{y} = s_{n} s_{\beta} \), \( s_{f} = \sin f, \) \( c_{f} = \cos f \), etc. The mass term of the Lagrangian (1) can be calculated exactly, without expansion in the field \( D \) because the matrix

\[
S = 1 - iD \sin d - D^{2}(1 - \cos d)/d^{2}
\]

\[
\Delta \mathcal{L}_{M} = \frac{F_{D}^{2} m_{D}^{2} - F_{\pi}^{2} m_{\pi}^{2}}{4}(1 - c_{f}) s_{d}^{2}
\]
The expansion of this term can be done easily up to any order in \( d \). The comparison of this expression with \( \Delta L_M \) within the collective coordinates approach allows to establish the relation \( \sin^2 d = \sin^2 \nu \), where \( \nu \) is the angle of the \( \lambda_4 \) rotation, or rotation into ”strange” direction. The so called strangeness (or flavor) content of the quark fields can be calculated easily, \( C_s \approx D^c D \). It should be remembered that in the collective coordinates method strangeness content of the soliton \( C_s = (\sin^2 \nu)/2 \).

The time-dependent part of the second order term in the Lagrangian density (1) due to rotations in the configuration space leads to the following contribution:

\[
\mathcal{L}_2 = 2Tr[\dot{S} S + S^\dagger \dot{U}_0 S^\dagger \dot{U}_0 + \dot{A} A^\dagger + 2A^\dagger AS] + S^\dagger A^\dagger ASU_0 + S^\dagger \dot{S} U_0 S^\dagger + S^\dagger A^\dagger ASU_0 + S^\dagger A^\dagger ASU_0 S^\dagger \dot{S} U_0]
\]

Making an expansion of the matrix \( S \) and adding also contributions from the usual space rotations we obtain:

\[
\mathcal{L}_2 \approx \frac{F^2}{8} \left\{ 4(1 - c_f) [D^\dagger \dot{D}(1 - \frac{2}{3}D^\dagger D) - \frac{2}{3}(D^\dagger \dot{D}D - (D^\dagger \dot{D})^2) + \omega \dot{\beta}/2] + 
+ s_1^2 [\dot{\omega} - \dot{\beta} - (\dot{\omega}n - \dot{\beta}n)^2] + (\dot{\alpha}_{f} \dot{r} \hat{\Omega})^2 + s_1^2 (\dot{\alpha}_{f} \dot{r} \hat{\Omega})^2 + 2s_1^2 (\dot{\omega}n \dot{\beta}n \epsilon_{ijkl} \epsilon_{ikl} r_k \Omega_l) \right\}
\]

The moments of inertia of the configuration - coefficients in the quadratic form in angular velocities of rotation - can be extracted easily from (9).

The interaction of quarks and mesons gives the contribution proportional to the new parameter \( \alpha_F \), after integration over space:

\[
L_{\text{int}} = -\alpha_F E_{qm} D^\dagger D(1 - \frac{2}{3}D^\dagger D)
\]

where, according to [2, 3], \( E_{qm} < 0 \) is the quark-meson interaction energy.

Equation (9) simplifies considerably for spherically symmetrical configurations (hedgehogs) for \( B = 1 \), as well as for \( B \geq 2 \) solitons described by axially symmetrical configurations (see Sections 4 and 5 for details).

After some calculation, the Lagrangian of the chiral quark-meson model in the lowest order in field \( D \) can be written in the form below which is similar to that of the bound state approximation to the topological soliton model [7-10]:

\[
L = -M_{d,B} + 4\Theta_{F,B} D^\dagger \dot{D} - [\Gamma_B (m_0^2 - m_\gamma^2) + \alpha_F E_{qm}] D^\dagger D - i \frac{N_c B}{2} (D^\dagger \dot{D} - \dot{D}^\dagger D).
\]

We have ignored the effect of the difference between \( F_K \) and \( F_\pi \) through the last term in (1), in the above expression. We have maintained our former notation for the moment of inertia for the rotation into ”strange”, ”charm” or ”bottom” direction \( \Theta_c = \Theta_b = \Theta_s = \Theta_F \) (the index \( \epsilon \) means the charm quantum number, except in \( N_c \)). In the present model, this moment of inertia has a simple analytical form for arbitrary starting \( SU(2) \) skyrmion, regardless of its symmetry properties:

\[
\Theta_{F,B} = \frac{F^2}{8} \int (1 - c_f) d^3r.
\]

Note that since the Skyrme term is absent in the \( CQM \) model, this formula is especially simple.

The quantity \( \Gamma_B \) defines the contribution of the mass term in the Lagrangian:

\[
\Gamma_B = \frac{F^2}{2} \int (1 - c_f) d^3r,
\]
so, the following relation is valid in \( CQM \):

\[
\Gamma_B = 4\Theta_{F,B} \tag{14}
\]

The term in (11) proportional to \( N_c B \) which comes from the quark part here, is responsible for the splitting between excitation energies of strangeness and anti-strangeness (flavor and antiflavor in general case) \([8]-[10]\).

3 Flavor excitation frequencies

After the canonical quantization procedure the Hamiltonian of the system including the terms of the order of \( N^0_c \), takes the form which is similar to that in the topological soliton models \([9, 10]\):

\[
H_B = M_{cl,B} + \frac{1}{4\Theta_{F,B}} \Pi^\dagger \Pi + [\Gamma_B m_D^2 + \alpha_F E_{qm} + \frac{N_c^2 B^2}{16\Theta_{F,B}}] D^\dagger D + \frac{N_c B}{8\Theta_{F,B}} (D^\dagger \Pi - \Pi^\dagger D). \tag{15}
\]

\( m_D^2 = m_D^2 - m_\pi^2 \). The momentum \( \Pi \) is canonically conjugate to variable \( D \) (see Eq. (23) below). Eq. (15) describes the oscillator-type motion of the field \( D \) in the background formed by the \((u,d) \) \( SU(2) \) soliton. After the diagonalization which can be done explicitly according to \([9, 10]\) the normal-ordered Hamiltonian can be written as

\[
H_B = M_{cl,B} + \omega_{F,B} a^\dagger a + \bar{\omega}_{F,B} b^\dagger b + O(1/N_c) \tag{16}
\]

with \( a^\dagger, b^\dagger \) being the operators of creation of strangeness (i.e., antikaons) and antistrangeness (flavor and antiflavor) quantum number, \( \omega_{F,B} \) and \( \bar{\omega}_{F,B} \) being the frequencies of flavor (antiflavor) excitation. \( D \) and \( \Pi \) are connected with \( a \) and \( b \) in the following way \([9, 10]\):

\[
D^i = \frac{1}{\sqrt{N_c B \mu_{F,B}}} (b^i + a^\dagger i), \quad \Pi^i = \frac{\sqrt{N_c B \mu_{F,B}}}{2i} (b^i - a^\dagger i) \tag{17}
\]

with

\[
\mu_{F,B} = [1 + 16(m_D^2 \Gamma_B + \alpha_F E_{qm})\Theta_{F,B}/(N_c B)^2]^{1/2}.
\]

For the lowest states the values of \( D \) are small:

\[
D \sim [16\Gamma_B \Theta_{F,B} m_D^2 + N_c^2 B^2]^{-1/4},
\]

and increase with increasing \(|F| \) like \((2|F| + 1)^{1/2}\) As it was noted in \([10]\), deviations of the field \( D \) from the vacuum decrease with increasing mass \( m_D \), as well as with increasing number of colors \( N_c \) and the method works for any \( m_D \) - for charm and bottom quantum number also.

The excitation frequencies \( \omega \) and \( \bar{\omega} \) are:

\[
\omega_{F,B} = \frac{N_c B}{8\Theta_{F,B}} (\mu_{F,B} - 1), \quad \bar{\omega}_{F,B} = \frac{N_c B}{8\Theta_{F,B}} (\mu_{F,B} + 1) \tag{18}
\]

As it was observed in \([11]\), the difference \( \bar{\omega}_{F,B} - \omega_{F,B} = N_c B/(4\Theta_{F,B}) \) coincides in the leading order in \( N_c \) with that obtained in the collective coordinates approach \([15]\).

To get an idea about the value of the parameter \( \alpha_F \), we can write a relation between \( \alpha_F g_{F\pi} \) and the effective quark mass:

\[
(1 + \alpha_F) g_{F\pi} \simeq m_{F\pi}^{eff} \tag{20}
\]
Since the quark-meson interaction energy is negative - it leads to the stabilization of the whole configuration - the term $\alpha FE_{qm}$ makes the flavor excitation frequencies smaller. The relative role of this effect decreases with increasing mass of the flavor, and is most important for strange baryons. For the $B = 1$ configuration the quark-meson interaction energy, $E_{qm} = -1.127$ Gev [1]. For strange baryons, to have constituent strange quark mass greater than that of nonstrange quarks mass by about $\sim 0.2$ Gev we should have $\alpha_s \simeq 0.4$. Similarly, we can obtain the crude estimates, $\alpha_c \simeq 2.7$ and $\alpha_\theta \simeq 9.4$.

The $FSB$ in the flavor decay constants, i.e. the fact that $F_K/F_\pi \simeq 1.23$ and $F_D/F_\pi \simeq 1.7 \pm 0.2$, should be taken into account as well. In the Skyrme model it leads to the increase of the flavor excitation frequencies which changes the spectra of flavored baryons in better agreement with data [21, 22], and leads also to some changes of the binding energies of $BS$ [1]. It was mainly due to the large contribution of the Skyrme term in the Lagrangian to the inertia $\Theta_F$. Since the Skyrme term in the $CQM$ model under consideration is absent - we obtain the relation $\Gamma_B = 4\Theta_{F,B}$ as a result - the influence of $FSB$ in decay constants is much less important in the chiral quark-meson model.

The terms of the order of $N_\pi^2$ in the Hamiltonian depending on the angular velocities of rotations in the isospin and the usual space and describing the zero-modes contributions are not crucial but also important for numerical estimates of baryons spectra. They will be considered in the next Sections.

4 $B = 1$ hedgehog and estimates of baryon spectra

The $B = 1$ hedgehog configuration in the chiral quark-meson model can be treated in same manner as the corresponding one in the topological (Skyrme) model. The unit vector $\vec{n}$ characterizing the chiral meson field configuration is $\vec{n} = \vec{r}/r$, and the spinor $\Psi_0$ has the structure

$$\Psi_0 = \begin{pmatrix} G(r)\chi_h \\ i\vec{\sigma}\bar{F}(r)\chi_h \end{pmatrix}$$

where $\chi_h$ is the hedgehog spinor

$$\chi_h = \frac{1}{\sqrt{2}}(u \downarrow - d \uparrow).$$

It can be checked that for hedgehogs the terms in (5) which depend on the orientation of the quarks in iso- and spin space, i.e. those proportional to $\vec{r}$ give zero contribution into Lagrangian. Rotations in the iso- and usual spaces are equivalent for hedgehogs, and the contribution to the energy depends on one common moment of inertia, $\Theta_{T,B}$.

From equations (1), (6),(9) and (10) in section 2, we obtain the following expression for the Lagrangian including all the terms upto $O(1/N_c)$:

$$L \simeq -M_{\omega} + 4\Theta_F[D\bar{D}(1 - \frac{2}{3}D^\dagger D) - \frac{2}{3}(D^\dagger D\bar{D} - (D^\dagger D)^2) + 2\Theta_F(\vec{\omega}\bar{\beta}) + \frac{\Theta_T}{2}(\vec{\omega} - \bar{\beta})^2$$

$$-(\Gamma_B m_\pi^2 + \alpha_F E_{qm})D^\dagger D(1 - \frac{2}{3}D^\dagger D) + i\frac{N_c B}{2}(1 - \frac{2}{3}D^\dagger D)(D^\dagger D - D^\dagger D) - \frac{N_c B}{2}\vec{\omega}D^\dagger D. \quad (22)$$

From this expression we can find the canonical variables,

$$\Pi = \frac{\partial L}{\partial D^\dagger} = 4\Theta_F[D(1 - \frac{2}{3}D^\dagger D) - \frac{2}{3}D^\dagger \bar{D} D + \frac{4}{3}D^\dagger D D] + i(\Theta_F - 2\Theta_F\vec{\omega}D - i\Theta_T\vec{\beta}D + i\frac{N_c B}{2}(1 - \frac{2}{3}D^\dagger D) D, \quad (23)$$
\[
\tilde{I}_f = \frac{\partial L}{\partial \tilde{\omega}} = \Theta_T \tilde{\omega} + (2\Theta_F - \Theta_T)\tilde{\beta} - \frac{N_c B}{2} D^1 \tilde{\tau} D.
\]  
(24a)

or

\[
\tilde{I}_f = \Theta_T \tilde{\omega} + (1 - \frac{\Theta_T}{2\Theta_F})\tilde{I}_F - \frac{N_c B\Theta_F}{4\Theta_F} D^1 \tilde{\tau} D
\]  
(24b)

with \(\tilde{I}_F = (b^1 \tilde{\tau} b - a \tilde{\tau} a^1)/2\).

Using the relations

\[-i \tilde{\beta} \tilde{\tau} D = 2D^1 D \dot{D} - (\dot{D}^1 D + D^1 \dot{D})D\]

and

\[\tilde{\beta}^2 = 4D^1 D \dot{D} \dot{D} - (\dot{D}^1 D + D^1 \dot{D})^2,\]

one can see that \(L, I, I_f\) have essentially the same structures as the corresponding expressions in [10]. This is true for the Hamiltonian also, and we find that the \(\sim 1/N_c\) zero modes quantum correction to the energies of hedgehogs in the CQM has a structure which is very similar to the correction term in the Skyrme model and can be estimated according to the expression [3, 10]:

\[
\Delta E_{1/N_c} = \frac{1}{2\Theta_F} \left[c_{F,B} T_r (T_r + 1) + (1 - c_{F,B}) I (I + 1) + (\tilde{c}_{F,B} - c_{F,B}) I_F (I_F + 1)\right],
\]  
(25)

where \(I = I_f\) is the isospin of the baryon or BS, \(T_r\) is the quantity analogous to the "right" isospin \(T_r\) in the collective coordinates approach [20, 19], and \(\tilde{T}_r = \tilde{I}_f - \tilde{I}_F, \tilde{I}_F = (b^1 \tilde{\tau} b - a \tilde{\tau} a^1)/2\).

\[c_{F,B} = 1 - \frac{\Theta_T B}{2\Theta_F B \mu_{F,B}} (\mu_{F,B} - 1), \quad \tilde{c}_{F,B} = 1 - \frac{\tilde{\Theta}_T B}{\tilde{\Theta}_F B (\mu_{F,B} - 1)} (\mu_{F,B} - 1).
\]  
(26)

In the case of antiflavor excitations, we have the same formula (25), with the substitution \(\mu \rightarrow -\mu\) in (26). For example,

\[\tilde{c}_{F,B} = 1 + \frac{\tilde{\Theta}_T B}{\tilde{\Theta}_F B \mu_{F,B}} (\mu_{F,B} + 1).
\]  
(27)

According to (9) the isotopic inertia

\[\Theta_T = \frac{F^2}{6} \int s^2 d^3 r,
\]  
(28)

but it receives some contribution, about 30%, also from the quark part of the Lagrangian due to the cranking procedure described in [1]. For numerical estimates here we take the value of \(\Theta_T\) obtained in [3] in the linear \(\sigma\) model since the differences of all calculated quantities in the linear and the nonlinear versions of the \(\sigma\) model are negligible.

In the rigid oscillator model the states predicted are not identified with definite \(SU(3)\) or \(SU(4)\) representations. However, it can be done, as shown in [10]. The quantization condition \((p + 2q)/3 = B\) for arbitrary \(N_c\) is changed to \((p + 2q) = N_c B + 3n_{q\bar{q}}\), where \(n_{q\bar{q}}\) is the number of additional quark-antiquark pairs present in the quantized states. For example, the state with \(B = 1, |F| = 1, I = 0\) and \(n_{q\bar{q}} = 0\) should belong to the octet of \((u, d, s)\), or \((u, d, c)\), etc. \(SU(3)\) group, if \(N_c = 3\), see also [14]. If \(\Theta_T \rightarrow \infty\), Eq. (25) goes over into the expression obtained for axially symmetrical BS in the collective coordinate approach [15]. In realistic case with \(\Theta_T = 2.9\), the structure of (25) is more complicated.
We will first summarise the results for B=1 in the 'rigid oscillator' approach to heavy flavours in CQM, without including the effect of flavour symmetry breaking in the quark-meson couplings (that is, $\alpha_F = 0$). We find that the excitation frequencies $\omega_F$ are in general higher than in the Skyrme model. This can be attributed to the fact that the value of $\Gamma$ in the present model is higher than the value in the Skyrme model. The mass difference $m_\Lambda - m_N$ comes out to be 284 MeV compared to the experimental value of 176 MeV. However it is to be noted that the value of $\omega_S$ in the model in the rigid oscillator approach used here is close to the value of 315 MeV obtained in a random phase approximation to CQM with broken $SU(3)$.

| $F$ | $\omega_F$ | $\bar{\omega}_F$ | $\omega_{Sk}$ | $\bar{\omega}_{Sk}$ | $<D^1D>$ | $\Delta M_{\Lambda F}-N$ | $\Delta M_{\Sigma F}-N$ | $\Delta M_{Z F}-N$ | $\tilde{e}_F$ |
|-----|-------------|-------------------|---------------|---------------------|--------|-------------------|-------------------|-------------------|--------|
| $s$  | 0.326       | 0.69              | 0.196         | 0.12                | 0.28   | 0.176             | 0.44             | 0.78             | 0.34   |
| $c$  | 1.687       | 2.05              | 1.18          | 0.032               | 1.67   | 1.346             | 1.89             | 2.07             | 0.75   |
| $b$  | 5.098       | 5.461             | 3.66          | 0.011               | 5.09   | 4.702             | 5.32             | 5.47             | 0.90   |

Table: The excitation frequencies for flavor $F$, $\omega_F$, antiflavor, $\bar{\omega}_F$ and the energy differences of baryons with different flavors and the nucleon, in GeV. $\omega_{Sk}$ are the flavor excitation frequencies in the Skyrme model shown here for comparison. For $B = 1$ soliton we use the values of mass $M_\Lambda = 1149$ MeV, flavor inertia $\Theta_F = 2.06$ Gev$^{-1}$, and isotopic inertia $\Theta_T = \Gamma = 5.93$ Gev$^{-1}$. The estimate used here, $<D^1D> = (N_cB\mu)^{-1}$, is valid for the lowest state of oscillator with $|F| = 0$, i.e. for the nucleon.

It should be noted that the values of inertia obtained within the chiral quark-meson model are close to those obtained in the Skyrme model. E.g. the flavor inertia $\Theta_F = 1.86$ Gev$^{-1}$ in the Skyrme model with $F_\pi = 108$ MeV and $e = 4.84$ (nucleon an $\Delta$-isobar masses are fitted), and $\Theta_F = 2.03$ Gev$^{-1}$, $\Theta_T = 5.55$ Gev$^{-1}$ in the Skyrme model variant with $F_\pi = 186$ MeV, $e = 4.12$.

The $Z_F$ baryons included in the Table have $\bar{F}$ quantum number and are true exotic because they cannot be made of $N_c$ valence quarks: one $\bar{q}q$ pair is necessary for this. These states belong to $\mathbf{10}$ representation of corresponding $SU(3)$ (the upper state with isospin $I = 0$). The mass of the state with $S = +1$ calculated first in [22] within collective coordinates approach to the quantization of zero modes in the Skyrme model was found about $\sim 740$ MeV above the nucleon. Later this anti-strange baryon was considered in more details in [24] where the $M_Z - M_N$ mass difference was found to be $\sim 590$ MeV, also within Skyrme model, but with the additional assumption that the $N^*(1710)$ resonance is the nonstrange component of the $\mathbf{10}$ of baryons. The CQM model prediction for $S = +1$ baryon (see the Table) is in better agreement with predictions of the collective coordinate method [24, 26].

The inclusion of $FSB$ in the quark-meson coupling improves the situation. We take the values of the parameter $\alpha_F$ to be: $\alpha_s = 0.4$, $\alpha_c = 2.7$, $\alpha_b = 9.4$ which allow to obtain the effective quark masses in the Lagrangian close to the known values. Then we obtain, $\omega_s = 0.27$ Gev, $\omega_c = 1.58$ Gev, $\omega_b = 4.97$ Gev. The values of the mass differences now are, in Gev: $\Delta M_{\Lambda-N} = 0.229(0.176)$, $\Delta M_{\Sigma-N} = 0.371(0.254)$, $\Delta M_{\Lambda_c-N} = 1.57(1.346)$, $\Delta M_{\Sigma_c-N} = 1.788(1.516)$, $\Delta M_{\Lambda_b-N} = 4.968(4.702)$, $\Delta M_{\Sigma_b-N} = 5.196$, where the figures in the parantheses correspond to the experimental values. We see that the values are now in better agreement with data. The relative role of the $\alpha_F$ term decreases with increasing mass of the quark, as expected.
5 Binding energy estimates for dibaryons with strangeness, charm and bottom

It was shown in [3, 6] that, in the chiral quark-meson model there are bound states of solitons with $B = 2$ and greater, similar to the topological soliton models [20]. Therefore, one should expect the predictions of the dibaryons, tribaryons, etc. with different values of flavor quantum number, $s, c$ or $b$, stable relative to strong interactions, similar to the Skyrme model.

The structure of the toroidal configurations with $B = 2$ should be described first. For $B=2$, $\Psi_0$ has the structure

$$\Psi_0 = \left( \begin{array}{c} G(\rho, z) \chi_{1,2} \\ i\vec{\sigma}F(\rho, z)\chi_{1,2} \end{array} \right)$$

where

$$\chi_1 = \frac{1}{\sqrt{2(1-\cos \theta \cos \alpha)}} [\sin \alpha u \downarrow - \sin \theta e^{i\phi} d \uparrow - (\cos \alpha - \cos \theta) e^{2i\phi} d \downarrow]$$

and

$$\chi_2 = \frac{1}{\sqrt{2(1-\cos \theta \cos \alpha)}} [\sin \alpha d \uparrow - \sin \theta e^{-i\phi} u \downarrow + (\cos \alpha - \cos \theta) e^{-2i\phi} u \uparrow]. \quad (29)$$

In the $B=2$ soliton, $N_c$ quarks are in the state $\chi_1$ and $N_c$ quarks are in the state $\chi_2$. Similar considerations apply for higher $B$. Then equations (5) and (9) for the Lagrangian simplify, in particular, the terms in (5) proportional $\Psi^\dagger \vec{r} \Psi$ cancel, similar to the hedgehog case.

In [3, 23] the following values of the binding energy of quark-meson solitons have been obtained: $\epsilon_2 = 279$, $\epsilon_3 = 226$ and $\epsilon_4 = 192$ Mev for baryon numbers 2, 3 and 4. These values can be compared with the values of binding energy in the Skyrme model, 74, 72 and 14 Mev, for smaller value of the constant, $F_{\pi} = 108$ Mev [20]. For $F_{\pi} = 186$ Mev and $e = 4.12$ $\epsilon_2 = 142$ Mev. It makes sense to give the binding energies in units, e.g. of the mass of the $B = 1$ soliton: although the symmetry violating mass terms in the Lagrangian violate the scaling, such comparison gives an information which does not depend strongly on the value of $F_{\pi}$. In CQM, $\epsilon_2 = 0.24 M_1$, $\epsilon_3 = 0.20 M_1$ and $\epsilon_4 = 0.17 M_1$ to be compared with 0.086, 0.083 and 0.016 $M_1$ in the Skyrme model.

Let us consider here the state with $B = 2$ and $|F| = 2$ with the lowest value of isospin, $I = 0$ which can belong to the 27-plet of corresponding $SU(3)$ group, $(u, d, s)$ or $(u, d, c)$, etc. For 27-plet of dibaryons $T_r = 1$, for antidecuplet $T_r = 0$. The quantum correction due to usual space rotations, also of the order of $1/N_c$ is exactly of the same form as obtained in [13], see [9, 10]. Since we are interested in the lowest energy states, we discuss here the baryonic systems with the lowest allowed angular momentum, $J = 0$ for $B = 2$, and $J = 3/2$ for $B = 3$. The latter value is due to the constraint because of symmetry properties of the configuration. The value $J = 1/2$ is allowed for the configuration found in [27].

For the mass of the state with $B = |F| = 2$ one obtains [10]:

$$M(B = 2, |27; Y = 0, I = 0>) = M_{cl} + 2\omega_{F:2} + \frac{\tilde{c}_{F:2}}{\Theta_{T:2}}. \quad (30)$$

The binding energy of this state relative to the two $\Lambda_F$-particles:

$$\epsilon(|27; Y = 0, I = 0>) = \epsilon_2 + 2(\omega_{F:1} - \omega_{F:2}) + \frac{3\tilde{c}_{F:1}}{4\Theta_{T:1}} - \frac{\tilde{c}_{F:2}}{\Theta_{T:2}} \quad (31)$$

As always, we define the binding energies relative to the decay into $B$ baryons, nucleons or flavored hyperons.
If the moments of inertia of $BS$ at small values of $B$ were proportional to the baryon number $B$, then the values of $\mu$, excitation frequencies $\omega_T$ and coefficients $c$ would not depend on $B$ at all. In this case the binding energy consisted only of its classical part, and some contribution from zero modes, the difference of $\omega$’s would not contribute. Within the $CQM$ model the moments of inertia for $B \geq 2$ have not been calculated, still. Therefore, we shall make a natural assumption that the ratios of moments of inertia for different values of $B$ in the $CQM$ model are the same as in the Skyrme model $[26]$. For $B = 2 \Theta_{F;2}/\Theta_{F;1} = 2.038$, $\Theta_{T;2}/\Theta_{T;1} = 2.053$ $[26]$.

With this assumption, we obtain the following numerical values: $\epsilon_{AA}(S=-2) = 0.29\ Gev$, $\epsilon_{AA}(c=2) = 0.31\ Gev$, $\epsilon_{AA}(b=-2) = 0.32\ Gev$, from expression (31). It should be compared with the binding energy of the deuteron $\epsilon_D = 351\ Mev$ and the binding energy of the NN scattering state with $J = 0$ and isospin $I = 1$, $\epsilon_{D'} = 321\ Mev$. After renormalization which is necessary to produce the NN scattering state on the right place, i.e. near threshold, we obtain that the strange dibaryon with $s = -2$ is unbound but close to the threshold, charmed as well as bottomed dibaryon are also unbound but even more near to the $\Lambda_F\Lambda_F$-threshold. This renormalization procedure is justified by the fact that the number of quantum effects like loop corrections and nonzero-modes contributions have not been, but should be taken into account (see also discussion of Casimir energy in Conclusions). The binding energy of the deuteron is $30\ Mev$ instead of measured $2.23\ Mev$, so $\sim 30\ Mev$ is the uncertainty of our approach.

The dibaryons with $|F| = 1$ should be considered also. The lowest states belong to antidecuplet of corresponding $SU(3)$, $(p,q) = (0,3)$ and have isospin $I = 1/2$. They all are bound within the approach developed here, and become close to the threshold, even unbound after the renormalization procedure.

These results are in qualitative agreement with those obtained in the chiral soliton models, but it should be noted that in the Skyrme model the states with charm and bottom remain bound after such renormalization $[11]$.

For 35-plet of tribaryons $T_r = 1/2$ (for arbitrary $(p,q)$ irrep which the $BS$ belongs to $T_r = p/2$ if $n_{q\bar{q}} = 0$). $I$ and $T$ take the lowest possible values, 0 or 1/2 for $|F|=1$, and 1/2, 0 for $|F| = 2$. The binding energies are of the same order of magnitude as for the $B = 2$ case if we make similar assumption concerning the behaviour of moments of inertia. But after renormalization the flavored states become unbound, although very close to thresholds.

### 6 Conclusions

We found that, as far as we are concerned with the spectra of baryons, there is no difference of principle between topological (Skyrme) soliton models and chiral quark meson model $[2,3]$. The $CQM$ model is more realistic, but, as is usual for more realistic models, it contains an additional parameter which defines the flavor symmetry breaking in the part of the Lagrangian describing the quark-meson interaction. When this parameter is omitted, the flavor excitation frequencies are too large in comparison with the data and with the topological Skyrme model also. Reasonable values of this parameter make the excitation frequencies smaller, in better agreement with data.

We have estimated the spectra of baryons with flavor different from $u,d$ in the simplest $SU(3)$ extension of the chiral quark-meson model proposed in $[2,3]$. One can note that the approach developed here - the rigid oscillator version of the $CQM$ - works even better for $c$ and $b$ flavor in comparison with strangeness.

There are predictions of the baryonic systems with $B = 2,3...$ and flavors $s, c, b$ similar to that in topological soliton (Skyrme) models $[15,11,11]$. In the
CQM model, due to the absence of the Skyrme term in the Lagrangian, the attraction of heavy flavors by \((u,d)\) solitons is, after all the renormalization procedures, somewhat weaker than in topological models. Similar predictions can also be made for systems with top-number. However, because of the large width of the \(t\)-quark, the spectroscopy of the baryonic systems as well as hadrons containing the \(t\)-quark will not be available, most probably.

The apparent drawback of the approach exploited in the present paper is that the motion of the system into the "strange", "charm" or "bottom" direction is considered independently from other motions. Consideration of the \(BS\) with "mixed" flavors is possible in principle, but it demands a more complicated treatment, technically.

There is some difference between the rigid oscillator variant of the CQM we considered here and the collective coordinates approach to soliton models widely exploited previously. In the collective coordinates approach to the zero modes of solitons with a rigid or a soft rotator variant of the model, the masses of baryons are usually considerably greater than in the bound state approach, when the Casimir energies are not taken into account \[28, 29\]. One of the sources of this difference is the presence of a term of order \(N_c/\Theta_F\) in the zero-modes contribution to the rotation energy, which is absent in the bound state model. It was shown recently by Walliser for the \(B = 1\) sector within the \(SU(3)\) symmetrical \((m_K = m_{\pi})\) variant of the Skyrme model \[29\] that, this large contribution is cancelled almost completely by the kaonic 1-loop correction to the zero-point Casimir energy which is of the same order of magnitude, \(N_c^0\) \[29\]. This correction has been calculated recently also within the bound state approach to the Skyrme model \[30\]. The consideration of loop corrections to the energies of quantized states is necessary also in the hybrid models similar to CQM.

Recently it was shown within the Skyrme model \[31\] that, one should expect the existence of strange baryonic systems close to the strong decay threshold, for baryon numbers up to 17. They are obtained by means of quantization of bound \(SU(2)\) skyrmions found previously in \[27, 32\]. The charmed baryonic systems with \(B = 3, 4\) were considered in \[33\] within a potential approach. The \(B = 3\) systems were found to be very near the threshold and the \(B = 4\) system was found to be stable relative to the strong decay, with a binding energy of \(\sim 10\ Mev\).

Experimental searches for the baryonic systems with flavor different from \(u\) and \(d\) could shed more light on the dynamics of heavy flavors in few-baryon systems. The threshold for the charm production on a free nucleon is about 12\(Gev\), and for double charm it is \(\sim 25.2\ Gev\). For bottom, the threshold on nucleon is \(\sim 70\ Gev\). However, for nuclei as targets the thresholds are much lower due to two-step processes with mesons in intermediate states and due to normal Fermi-motion of nucleons inside the target nucleus (see, e.g. \[34\]). Therefore, the production of baryons or baryonic systems with charm and bottom will be possible on accelerators with energy of several tens of \(Gev\).

Acknowledgements
The authors are very indebted to J.Segar for help in numerical computations and to H.Walliser for useful discussions.

One of the authors (MSS) thanks the Russian Academy of Sciences and The Indian National Science Academy for financial support which enabled him to work at INR, Moscow.

References
1. T.H.R. Skyrme, Proc.Roy.Soc., London, A260,127 (1961); Nucl.Phys. 31,556 (1962)
2. S. Kahana, G. Ripka, V. Soni, Nucl. Phys. A415, 351 (1984); M.C. Birse, M.K. Banerjee, Phys.Rev. D31, 118 (1984)
3. T. D. Cohen, W. Broniowski, Phys. Rev. D34, 3472 (1986); M.K. Banerjee, W. Broniowski, T.D. Cohen, A chiral quark soliton model, Univ. of Maryland preprint, April 1986
4. J.A. McGovern, M.C. Birse, Nucl. Phys. A506, 392 (1990)
5. J. Segar, M. Sripriya, M.S. Sriman, Int. J. of Mod. Phys. E3, 769 (1994)
6. J. Segar, M. Sripriya, M.S. Sriman, Phys. Lett. B342, 201 (1995)
7. C.G. Callan, I.R. Klebanov, Nucl. Phys. B262, 365 (1985)
8. N. Scoccola, H. Nadeau, M. Nowak, M. Rho, Phys. Lett. B201, 425 (1988); C.G. Callan, K. Hornbostel, I.R. Klebanov, Phys.Lett. B202, 269 (1988); J.P. Blaizot, M. Rho, N. Scoccola, ibid. 209, 27 (1988); N. Scoccola, D.P. Min, H. Nadeau, M. Rho, Nucl. Phys. A505, 497 (1989)
9. D. Kaplan, I.R. Klebanov, Nucl. Phys. B335, 45 (1990)
10. K.M. Westerberg, I.R. Klebanov, Phys. Rev. D50, 5834 (1994); I.R. Klebanov, K.M. Westerberg, Phys.Rev. D53, 2804 (1996)
11. V.B. Kopeliovich, Pis’ma Zh. Eksp. Teor. Fiz. 67, 854 (1998) [JETP Lett. 67, 896 (1998)]
12. R.L. Jaffe, Phys.Rev.Lett. 38, 195 (1977); R.L. Jaffe, C.L. Korpa, Nucl. Phys. B258, 468 (1985)
13. A.P. Balachandran, A. Barducci, F. Lizzi, V.G.J. Rogers, A. Stern, Phys. Rev. Lett. 52, 887 (1984); A.P. Balachandran et al, Nucl. Phys. B256, 525 (1985)
14. J. Kunz, P. Mulders, Phys. Lett. B215, 449 (1988); Nucl. Phys. A497, 339c (1989)
15. V.B. Kopeliovich, Yad.Fiz. 51, 241 (1990) [Sov.J.Nucl.Phys. 51, 151 (1990)]; Phys.Lett. 259B, 234 (1991); V.B. Kopeliovich, B. Schwesinger, B.E. Stern, Nucl.Phys. A549, 485 (1992)
16. V.B. Kopeliovich, Genshikaku Kenkyu 41, 171 (1997); hep-ph/9712453, Nucl. Phys. A639, 75c (1998); JETP 85, 1060 (1997); Phys. Atom. Nucl. 56, 1084 (1993)
17. B. Schwesinger, H. Weigel, Phys. Lett. B267, 438 (1991)
18. E. Witten, Nucl. Phys. B223, 422; 433 (1983)
19. G. Guadagnini, Nucl.Phys. B236, 35 (1984)
20. G.S. Adkins, C.R. Nappi, E. Witten, Nucl.Phys. B228, 552 (1983); G.S. Adkins, C.R. Nappi, Nucl.Phys. B233, 109 (1984)
21. D.O. Riska, N.N. Scoccola, Phys.Lett. B265, 188 (1991)
22. M. Bjornberg, K. Dambom, D.O. Riska, N.N. Scoccola, Nucl.Phys. A539, 662 (1992)
23. J. Segar, Ph.D. thesis, University of Madras (1994)
24. H. Walliser, Nucl.Phys. A548, 649 (1992)
25. D. Diakonov, V. Petrov, M. Polyakov, Z. Phys. A359, 305 (1997)
26. V.B. Kopeliovich, B.E. Stern, NORDITA-preprint 89-34, Copenhagen (1989); JETP Lett. 45, 203 (1987) [Pis’ma v ZhETF, 45, 165 (1987)]
27. E. Braaten, S. Townsend, L. Carson, Phys.Lett. B235, 147 (1990)
28. B. Moussalam, Ann. of Phys. (N.Y.) 225, 264 (1993); F. Meier, H. Walliser, Phys. Rept. 289, 383 (1997)
29. H. Walliser, hep-ph/9710232; Phys. Lett. B432, 15 (1998)
30. N. Scoccola, H. Walliser, hep-ph/9805340
31. M. Schwelling, N.N. Scoccola, Phys. Lett. B430, 32 (1998); hep-ph/9801347
32. R.A. Battye, P.M. Sutcliffe, Phys.Lett. B391, 150 (1997); Phys.Rev.Lett. 79, 363 (1997)
33. B.F. Gibson, C.B. Dover, G. Bhamathi, D.R. Lehman, Phys.Rev. C27, 2085 (1983)
34. V.B. Kopeliovich, Phys.Rept. 139, 51 (1986); Yad.Fiz. 42, 854 (1985)