Sampling-based 3-D Line-of-Sight PWA Model Predictive Control for Autonomous Rendezvous and Docking with a Tumbling Target

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ABSTRACT

In this paper, a model predictive control (MPC) framework is employed to realize autonomous rendezvous and docking (AR&D) with a tumbling target, using the piecewise affine (PWA) model of the 3-D line-of-sight (LOS) dynamics and Euler attitude dynamics. Consider the error between the predictions obtained by the approximate linear model and the actual states of nonlinear dynamics, a sampling-based PWA MPC is proposed to sample the predictions in the closer neighborhood of the actual states. Besides, novel constructions of constraints are presented to reduce the on-board computation cost and time-delay. Furthermore, a singularity-free strategy is provided to realize crossing the singularity of angle states smoothly. Then, the mission is achieved by continuous 6-DOF pose (position and attitude) tracking of the target’s docking port, with the coupling between the position and attitude of the target’s docking port is taken into account. Finally, numerical results are presented to demonstrate the above theories.

1. Introduction

With rapid development of aerospace technology, autonomous rendezvous and docking (AR&D) has attracted growing interest of researchers. In recent years, impressive achievements have been acquired in the studies of AR&D with a cooperative target (e.g., [1][2][3]), which requires the target spacecraft is attitude stable. However, due to the increase of malfunctioning satellites and space debris, AR&D with a tumbling target is required, which is more complicated but closer to the practical applications.

Since MPC [4][5] can achieve optimal performance and respond to various constraints, several excellent works (e.g., [6][7][8][9], etc.) have been proposed by using MPC in the AR&D missions. Firstly, coordinates in the orbital frame are adopted to describe the relative position, by employing the C-W or T-H equations. Consequently, the navigation information has to be transformed into the orbital frame, which can cause the increase of computation efforts and time-delay. To address this drawback, the in-plane LOS dynamics was used to control a spacecraft to rendezvous with an attitude stable target in [10], by controlling the states which can be measured directly.

Besides, the modeling of constraints are complex and need linear approximation in existing papers. The distance between two spacecrafts was calculated by the 2-norm of the coordinates in [6], which caused the collision avoidance constraint was quadratic and cannot be taken into the optimization index directly. The azimuth angle in [2] was calculated by the trigonometric function’s operation. The entry cone in [8][9] was linearized by an inscribed pyramid, the constraint was achieved by keeping the inner product of the position vector and the normal external vectors of the pyramid’s side is negative.

Furthermore, since the states in LOS dynamics and Euler dynamics can be measured directly, to the best knowledge of the authors, there is no paper adopt the 3-D LOS dynamics and Euler attitude dynamics together to realize 6-DOF pose tracking, by using MPC. However, since the nonlinear dynamics is coupling, the real time performance can not be guaranteed if solving a coupling nonlinear optimization problem in a standard nonlinear MPC. The existing approach to deal with this kind of model is to utilize approximate linear models, e.g., linearize the nonlinear model with Taylor expansion at each sampling instant [11]; adopting the multi-model predictive control [12]; using the neural networks to approximate the nonlinear characteristics [13][14]. Consider the linearization needs to calculate the covariance matrix of the states in real time which will increase the computation cost and time-delay, the PWA model is adopted in [15][10]. However, there exist error between the predictions obtained by the approximate linear model and the actual states of nonlinear dynamics, which can influence the control performance of MPC.

Moreover, there exists singularity of angle states, which means two different values represent a same position physically. For instance, an angle state is set within [−π, π], −π and π represent the same position physically but differ by 2π. Consider a tracking problem, the desired states will jump from π to −π once it reaches the singular point π. However, it is impossible for an input-constraint system to track the mutation signal in an extraordinarily short time. The phenomenon extremely limits the application of the LOS dynamics and Euler attitude dynamics in AR&D.

In this paper, the piecewise affine (PWA) systems based on the 3-D LOS dynamics and the Euler attitude dynamics are firstly adopted together to realize AR&D with a tumbling target. To reduce the error between the predictions made by the PWA model and the actual nonlinear states, this paper provides a sampling-based PWA MPC strategy to realize sampling the predictions which describe the ac-
ual states more accurately. Since the time-varying position of the docking port is coupled with the target’s rotation, this problem is also considered and solved in this study. In addition, based on the adopted united models, novel construction strategy of constraints is proposed, which is more suitable for practical space missions for the advantages of more concise and without any approximation. Furthermore, a singularity-free strategy is proposed, which can simultaneously realize continuous pose tracking by crossing the singular points smoothly.

The rest of this paper is arranged as follows. Section 2 provides the control model and the novel modeling of constraints. In section 3, the sampling-based PWA MPC is proposed. Section 4 illustrates the AR&D and the singularity-free strategies. In section 5, numerical simulations are presented to demonstrate the effectiveness of proposed strategies. Finally, the conclusions are given in section 6.

2. Problem formulation

In this paper, the control objective is to design a position and an attitude MPC controller separately to track the pose of the tumbling target’s docking port. Before introducing the LOS dynamics and the Euler dynamics, the following coordinate frames are defined and illustrated in Fig. 1.

- $F_t = \{O_t, \hat{x}_t, \hat{y}_t, \hat{z}_t\}$: the local-vertical-local-horizontal (LVLH) frame, its origin is fixed at the center of the target, $\hat{z}_t$-axis points towards the Earth's center, $\hat{y}_t$-axis is along the direction of the orbital angular rate, $\hat{x}_t$-axis completes the triad.
- $F_{bt} = \{O_t, \hat{x}_{bt}, \hat{y}_{bt}, \hat{z}_{bt}\}$ and $F_{bc} = \{O_c, \hat{x}_{bc}, \hat{y}_{bc}, \hat{z}_{bc}\}$: the body-fixed coordinate frames of the target and the chaser.
- $F_s = \{O_s, \hat{x}_s, \hat{y}_s, \hat{z}_s\}$: the LOS frame, its origin is fixed at the center of the target, $\hat{x}_s$-axis points towards the chaser, $\hat{y}_s$-axis is along the direction of angular momentum of the rotation of $\hat{x}_s$, $\hat{z}_s$ completes the triad. Denote $\epsilon$ as the elevation angle between $x_t$ and its projection on $O_s\hat{x}_s\hat{z}_s$ plane, $\beta$ as the azimuth angle between the projection and $x_t$.

In this study, the 3-D LOS dynamics is described in frame $F_s$, which is obtained by the Euler rotation of $F_t$ with 3 $\epsilon$ -2 $\beta$ sequence. It should be noted that $\epsilon$ is set within $(-\pi/2, \pi/2)$ to avoid the gimbal lock phenomenon. The attitude of target (chaser) is defined by the Euler rotation from frame $F_s$ to frame $F_{bt}$ ($F_{bc}$) with 3-2-1 sequence. Therefore, it is easier to solve the coupling between the position and attitude of the tumbling target, by defining the same rotation sequence from $F_t$ to $F_s$ and from $F_t$ to $F_{bt}$.

2.1. Relative motion dynamics

2.1.1. 3-D Line-of-sight dynamics

The relative translation between the chaser and the target formulated in the LOS frame is described as follows [16] [17],

$$
\dot{x}_p = A_p^c(x_p)\dot{x}_p(t) + B_p^c(x_p)u_p(t),
$$

where $A_p^c(x_p)$ and $B_p^c(x_p)$ are time-varying state-dependent matrices with

$$
A_p^c = \begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & \frac{1}{s_{p1}} & 0 & 0 \\
0 & 0 & 0 & 0 & \frac{1}{s_{p1}} & 0 \\
a_{d41} & 0 & 0 & \frac{s_{p1}}{s_{p1}} & a_{d46} & 0 \\
a_{d51} & 0 & 0 & 0 & \frac{-s_{p1}}{s_{p1}} & a_{d56} \\
a_{d61} & 0 & 0 & 2\omega - \frac{s_{p1}}{s_{p1}} & 2\tan x_{p2}(\omega + \frac{x_{p6}}{x_{p1}}) & 0
\end{bmatrix},
$$

where

$$
a_{d41} = \omega^2 \cos^2 x_{p2} - \frac{\mu}{R^3} (1 - 3\cos^2 x_{p2} \sin^2 x_{p3}),
$$
\[ a_{46} = \left(-2\omega + \frac{x_{26}}{s_{51}}\right) \cos^2 x_{p2}, \]
\[ a_{51} = \left(-\omega^2 - 3 \frac{s_{16}}{R^3} \sin x_{p1}\right) \cos x_{p2} \sin x_{p2}, \]
\[ a_{62} = \left(2\omega - \frac{x_{26}}{s_{51}}\right) \cos x_{p2} \sin x_{p2}, \]
\[ a_{61} = \dot{\omega} + \frac{3y}{R^3} \sin x_{p3} \cos x_{p3}, \]
and
\[ B_c^e = \begin{pmatrix}
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
\end{pmatrix}. \]

### 2.1.2. Euler attitude dynamics

The attitude kinematics of the chaser is described by

\[
\begin{pmatrix}
\dot{\phi}_c \\
\dot{\theta}_c \\
\dot{\psi}_c 
\end{pmatrix} = \frac{1}{c(\theta)} \begin{pmatrix}
c(\theta) & s(\phi)s(\theta) & c(\phi)s(\theta) \\
0 & c(\phi)c(\theta) & -s(\phi)c(\theta) \\
s(\phi) & 0 & c(\phi) 
\end{pmatrix} \begin{pmatrix}
o_{c,1} \\
o_{c,2} \\
o_{c,3} 
\end{pmatrix},
\]

where \(s(\cdot) \triangleq \sin(\cdot), c(\cdot) \triangleq \cos(\cdot); \phi_c(t), \theta_c(t), \) and \(\psi_c(t)\) (rad) denote the chaser’s roll, pitch, and yaw angles, respectively; \(o_{c,1}(t), o_{c,2}(t),\) and \(o_{c,3}(t)\) (rad/s) denote the chaser’s angular velocities. To avoid the gimbal lock phenomenon, the roll angle of the target is set within \([-\pi/2, \pi/2]\), and pitch angles are set within \([0, \pi]\).

The attitude dynamics is formulated as follows,

\[
\begin{align*}
J_1 \dot{o}_{c,1} &= (J_2 - J_3) o_{c,2} o_{c,3} + M_1, \\
J_2 \dot{o}_{c,2} &= (J_3 - J_1) o_{c,1} o_{c,3} + M_2, \\
J_3 \dot{o}_{c,3} &= (J_1 - J_2) o_{c,1} o_{c,2} + M_3,
\end{align*}
\]

where \(J_1, J_2,\) and \(J_3\) denote the principal moments of inertia of the chaser, \(M_1, M_2,\) and \(M_3\) are the input moments.

Consider the reaction wheels equipped along each principal body axis as the actuators, the relation between the wheels’ dynamics and the moments of the chaser is defined as

\[
\begin{align*}
M_1 &= -J_1(\dot{o}_{c,1} + \ddot{\alpha}_1 + \dot{\alpha}_1 o_{c,2} o_{c,3} - \dot{\alpha}_3 o_{c,2}), \\
M_2 &= -J_2(\dot{o}_{c,2} + \ddot{\alpha}_2 + \dot{\alpha}_2 o_{c,1} o_{c,3} - \dot{\alpha}_3 o_{c,1}), \\
M_3 &= -J_3(\dot{o}_{c,3} + \ddot{\alpha}_3 + \dot{\alpha}_3 o_{c,1} o_{c,2} - \dot{\alpha}_1 o_{c,2}),
\end{align*}
\]

where \(\ddot{\alpha}_1, \ddot{\alpha}_2, \) and \(\ddot{\alpha}_3\) denote the speed of wheels. Consider the following linearized relationship between the chaser’s angular velocities and the wheels’ acceleration,

\[
\dot{\omega}_{c,i} = -\frac{\ddot{\alpha}_i}{J_i}, \quad i = 1, 2, 3.
\]

Denote \(x_{a} = [\phi, \theta, \omega_{c,1}, \omega_{c,2}, \omega_{c,3}]^T\), then (4)-(7) can be reduced to the following first-order differential equation,

\[ x_{a}(t) = A_a^e(x_{a})x_{a}(t) + B_a^e u_a(t), \]

where \(u_a\) denotes the control input acting on the chaser, described by

\[ u_a = [u_{a,\phi}, u_{a,\theta}, u_{a,p}]^T. \]

### 2.2. Novel construction strategy of constraints

By employing the 3-D LOS dynamics and Euler dynamics in this study, an advantage is the constraints can be put on the states directly without any transformation, which have more practical application significance.

#### 2.2.1. Control input constraints

In practical space missions, the torque generated by the actuators is limited. The constraints of the thrusters and the reaction wheels are described by

\[ |u_p| \leq u_{p,\max}, \]

and

\[ |u_a| \leq u_{a,\max}, \]

where \(u_p, u_a\) are described in (3) and (9). \(u_{p,\max}, u_{a,\max}\) denote the maximum control force of the thrusters and reaction wheels, respectively.

#### 2.2.2. Collision avoidance constraint

For safe docking, collision avoidance is realized by maintaining the chaser outside a keep-out zone around the tumbling target. The keep-out zone is described by a sphere with a fixed radius,

\[ \rho \geq r_{\text{safe}}, \]
where $\rho$ is the LOS range, $r_{\text{safe}}$ denotes the minimum radius of the keep-out sphere.

2.2.3. Entry cone constraints

As shown in Fig. 2(a), the chaser should be kept in an approaching cone around the target’s docking port during the final phase of docking. In this paper, we give the following assumptions: (1) the target’s docking port is fixed at the $\hat{s}_{\text{bt}}$ axis in $F_{\text{bt}}$; (2) the attitude of the target is described by Euler rotation from $F_1$ to $F_{\text{bt}}$ with 3-2-1 sequence; (3) half of the entry cone angle is $\gamma_c$. By defining the same rotation sequence from $F_1$ to $F_f$ and from $F_1$ to $F_{\text{bt}}$, it can be drawn that the desired elevation angle is equal to the target’s pitch angle, and the desired azimuth angle is equal to the target’s yaw angle. Then the entry cone constraint is described by

$$
\begin{align*}
\theta_e(t) - \gamma_e &\leq \varepsilon \leq \theta_e(t) + \gamma_e, \\
-\pi/2 < \varepsilon < \pi/2, \\
-\psi_f(t) - \gamma_e &\leq \beta \leq -\psi_f(t) + \gamma_e, \\
-\pi \leq \beta \leq \pi,
\end{align*}
$$

(13)

where $\varepsilon$ and $\beta$ have been shown in Fig. 1. The above relations can be rewritten as

$$
\begin{align*}
\varepsilon_{\text{min}}(t) < \varepsilon < \varepsilon_{\text{max}}(t), \\
\beta_{\text{min}}(t) < \beta < \beta_{\text{max}}(t),
\end{align*}
$$

(14)

where $\varepsilon_{\text{min}}(t) = \max\{-\pi/2, \theta_e(t) - \gamma_e\}$, $\varepsilon_{\text{max}}(t) = \min\{\pi/2, \theta_e(t) + \gamma_e\}$, $\beta_{\text{min}}(t) = \max\{-\pi, -\psi_f(t) - \gamma_e\}$, and $\beta_{\text{max}}(t) = \min\{\pi, -\psi_f(t) + \gamma_e\}$ are time-varying functions.

Figure 2: Entry cone and the field of view

2.2.4. Field of view constraint

As shown in Fig. 2(b) for an active chaser spacecraft equipped with vision sensors, it should be guaranteed that the vision sensors can obtain the information of the target. The field of view constraint is realized by limiting the attitude angles of the chaser on the condition of accurate attitude tracking. Consider the coupling, the field of view constraint is expressed as follows,

$$
\begin{align*}
-\pi \leq \phi_c \leq \pi, \\
\theta_{c_{\text{min}}}(t) < \theta_c < \theta_{c_{\text{max}}}(t), \\
\psi_{c_{\text{min}}}(t) < \psi_c < \psi_{c_{\text{max}}}(t),
\end{align*}
$$

(15)

where $\phi_c$, $\theta_c$, $\psi_c$ are the attitude angles of the chaser, $\theta_{c_{\text{min}}}(t) = \max\{-\pi/2, \varepsilon(t) - \gamma_f\}$, $\theta_{c_{\text{max}}}(t) = \min\{\pi/2, \varepsilon(t) + \gamma_f\}$,

$$
\psi_{c_{\text{min}}}(t) = \max\{-\pi, \beta(t) - \gamma_f\}, \psi_{c_{\text{max}}}(t) = \min\{\pi, \beta(t) + \gamma_f\}
$$

are time-varying functions, and $\gamma_f$ denotes half of the angle of the field of view. For understanding convenience, the constraint of the pitch angle $\theta_c$ is taken as an example to illustrate detailed. As shown in Fig. 3, the following relation should be guaranteed, i.e.,

$$
\theta_c - \gamma_f \leq \varepsilon(t) \leq \theta_c + \gamma_f.
$$

The above relation can be rewritten as

$$
\varepsilon(t) - \gamma_f \leq \theta_c \leq \varepsilon(t) + \gamma_f.
$$

The construction of constraint on $\psi_c$ is similar to the above process.

Figure 3: Field of view constraint on the pitch angle

From the above process of modeling, the novel construction in this paper is more concise and without any linear approximation. Since the constraints are imposed on the states directly, the on-board computational efforts and time-delay are reduced without losing any control precision.

3. Sampling-based PWA model predictive control

Model predictive control (MPC) is widely employed for achieving optimal performance and treating various constraints. At each sampling instant, the prediction is made by the plant model, the basic idea of MPC is to take the prediction into a constrained optimization index, converting the index to a convex quadratic programming (QP) problem and minimizing it online to obtain a sequence of control input. Subsequently, only the control input related to the sampling instant is applied, according to the receding horizon strategy.

To apply the above idea in a nonlinear system, one existing practice is to convert the continuous nonlinear model to a pseudo linear form with the state matrix and control input matrix are state-dependent, discretize it at each sampling instant to get a linear time-invariant (LTI) model corresponding to each interval of states, the set of discrete models is the piecewise affine (PWA) model. By this way, we can
utilize linear models corresponding to different interval of states to approximate the nonlinear model, then the control is completed by a standard MPC. However, the error between the predictions made by the approximate linear model and the actual states cannot be neglected, especially with the increase of prediction horizon. In this study, a method of reducing this error is provided by an additional sampling control input. By determining the sampling direction from the actual nonlinear model and adjusting the value of sampling input, the proposed method can sample the predictions in the closer neighborhood of the actual states.

3.1. Basic idea

The general process of nonlinear MPC based on the linear approximate model is given in Algorithm 1. Obviously, the control performance depends on the accuracy of the predictions describing the actual states. Denote the n step ahead prediction obtained by the linear approximate model at tk+n as \(x^l(k+n)\) (n = 1, 2, \cdots), and the actual state as \(x(k+1)\), there exist error between \(x(k+n)\) and \(x^l(k+n)\). Consider the relation between the linear approximate model and the actual nonlinear model at each sampling state, even if the specific value of the error is unknown, the symbol of the error can be obtained from the information of the nonlinear model. Therefore, an additional sampling control input is introduced as follows,

\[
u_s = w_s \text{sign}(\tilde{x})[u(k)] \text{rand}(1), \tag{16}
\]

where \(w_s \in [0, 1]\) is the sampling factor to control the value of sampling input, \(\text{sign}(\tilde{x}) = \pm 1\) is an auxiliary item of the actual nonlinear model to control the sampling direction. \(u(k)\) denotes the predictive control input at \(x_k\), \(\text{rand}()\) is a random value. Since the optimal predictions made by the linear approximate model that describe the actual states are unknown, the random value’s effect is to reduce the mean error of all predictions. If the model is concave at \(x(k)\), the predictive state \(x^l(k+1)\) is lower than \(x(k+1)\), the positive \(u_s\) can help to decrease the error. On the contrary, if the model is convex at \(x(k)\), the predictive state \(x^l(k+1)\) is larger than \(x(k+1)\), the negative \(u_s\) can help to decrease the error. If adopting the sampling control input at each sampling instant, the proposed method can realize sampling the predictions in the closer neighborhood of the actual nonlinear states.

Remark 1. Both the PWA model and linearized model (e.g., Taylor expansion) are belong to the way of approximating nonlinear characteristics with linear model, it should be noted that the approximating accuracy of Taylor expansion is higher than the employed PWA model in this study. However, consider the on-board computation efforts and real time performance, it is enough for the PWA model to complete the AR&D well. It should also be noted that the proposed strategy is also applicable to cases of employing the model of Taylor expansion, if the computation cost are not considered firstly.

3.2. Controller design

Consider the continuous LOS dynamics \(\dot{x}(k) = Ax(k) + Bu(k)\), and discrete \(\Delta x(k) = Ax(k) + Bu(k)\) at each sampling instant, the approximate PWA model corresponding to the state at \(t_k\) is obtained by holding the system matrix and the control input matrix are constant during each sampling interval.

Denote \(\dot{x}_p^l(k+1)\) as the one step ahead prediction of position state made by the PWA model,

\[
\dot{x}_p^l(k+1) = A_p(k)x_p^l(k) + B_p(k)u_p^l(k), \tag{17}
\]

where \(A_p(k) = e^{AT_s(k)}\), \(B_p(k) = (\int_0^{T_s} e^{AT_s(t)}d\tau)B_o\), and \(T_s\) is the sampling interval. In view of the sampling-based idea in Section 3.1, the sampling prediction \(x_p^l(k+1)\) is obtained by applying the addition sampling input \(u_{s,p}\),

\[
x_p^l(k+1) = A_p(k)x_p^l(k) + B_p(k)u_p^l(k) + B_p(k)u_{s,p}(k), \tag{18}
\]

where

\[
u_{s,p}(k) = [u_{s,p}(k), u_{s,e}(k), u_{s,b}(k)]^T, \tag{19}
\]

with

\[
\begin{cases}
  u_{s,p}(k) = w_{s,p}\text{sign}[\beta(k)] \text{rand}(1), \\
  u_{s,e}(k) = w_{s,e}\text{sign}[\varepsilon(k)] \text{rand}(1), \\
  u_{s,b}(k) = w_{s,b}\text{sign}[\beta(k)] \text{rand}(1).
\end{cases}
\]

Combining (18) and (19) yields

\[
x_p^l(k+1) = A_p(k)x_p^l(k) + B_p(k)u_p^l(k) + B_p(k)W_p(k)u_{s,p}(k), \tag{20}
\]

where

\[
W_p(k) = \text{rand}(1)
\]

\[
\begin{pmatrix}
  w_{s,p} & 0 & 0 \\
  0 & w_{s,e} & 0 \\
  0 & 0 & w_{s,b}
\end{pmatrix}
\]
where \( \text{sign}[u_{p,b}(k)] \), \( \text{sign}[u_{p,c}(k)] \), \( \text{sign}[u_{p,d}(k)] \) are obtained from the relation between the current states and the next desired states. If the next desired state is larger than the current state, then the value of the symbolic function is 1, conversely, the symbol is -1.

By introducing an augmented vector \( \dot{x}_p(k) \in \mathbb{R}^{6N_p} \), the \( N_p \) ahead predictions of \( \dot{x}_p(k) \) can be described as

\[
x_p(k) = [x_p^T(k+1|k), x_p^T(k+2|k), \ldots, x_p^T(k+N_p|k)]^T.
\]

By iteration, rewriting the above predictions in a compact form, i.e.,

\[
x_p^*(k) = A_p^*x_p(k) + B_p^*u_p + \bar{B}_p^*\bar{W}_p^*(k)\bar{u}_p^*.
\]

Define the optimization index as

\[
\min J(k) = \sum_{i=1}^{N_p} \|x_p(k+i)-x^*_d,p(k+i)\|^2 + \sum_{i=0}^{N_p-1} \|\Delta u_p(k+i)\|^2,
\]

and substitute (30) into (29), yields

\[
\frac{1}{2} \Delta u_p^T \bar{H}_p \Delta u_p + f_p^T \bar{u}_p + E_p^T \bar{Q} E_p.
\]

where \( \bar{H}_p = 2[\Gamma \bar{B}_p + B_p^*W_p^*] + \bar{P} \) and \( f_p = -2[\Gamma \bar{B}_p^* + B_p^*W_p^*]^T \bar{Q} E_p \).

The process of applying the sampling-based PWA MPC to the attitude dynamics is similar to the above process. Consider the attitude dynamics (4), introduce an augmented state vector \( \dot{x}_a^*(k) \in \mathbb{R}^{6N_p} \), which represent \( N_p \) ahead prediction states of \( \dot{x}_a(k) \). According to the iterative relation, rewrite the prediction states \( \dot{x}_a(k) \) in a compact form such that,

\[
x_a^*(k) = A_a^*x_a(k) + B_a^*W_a^*u_a + B_a^*u_a^*.
\]
where the form of $A^*_p$, $B^*_p$, and $W^*_p$ are similar to $A^*_a$, $B^*_a$, and $W^*_a$ in (26). Consider the following optimization index,

$$
\min J_a(k) = \left[ x^+_a(k) - x^-_a(k) \right]^T \bar{Q} \left[ x^+_a(k) - x^-_a(k) \right] + \Delta \bar{u}^T \bar{P} \Delta \bar{u}_a,
$$

where $x^+_a(k) = \sum_{i=1}^{N_p} x_{d,a}(k+i|i) \Delta$ denotes the augmented desired states. Define

$$
E_a = x^+_a(k) - A^*_a x_a(k) - (B^*_a + B^*_a W^*_a) \Delta u_a(k-1),
$$

and substitute (34) into (33), yields

$$
\min J_a(k) = \frac{1}{2} \Delta \bar{u}^T H_a \Delta \bar{u}_a + \bar{f}^T_a \Delta \bar{u}_a + E_a^T \bar{Q} E_a.
$$

(35)

where $H_a = 2[\Gamma^T (B^*_a + B^*_a W^*_a)^T (\bar{Q} (B^*_a + B^*_a W^*_a)^T + \bar{P})]$, $f_a = -2 \Gamma (B^*_a + B^*_a W^*_a) \bar{Q} E_a$.

### 3.3. Constraints recreation

Based on Section 2.3, the constraints related with the augmented vectors $x^+_a(k)$ and $x^-_a(k)$ which are employed in the convex QP problem are reconfigured in this subsection.

#### 3.3.1. Control input constraints

The control input constraint of thrusters is described as,

$$
-\bar{u}^\text{max}_p \leq \Delta u_p(k-1) + \Gamma \Delta \bar{u}_p \leq \bar{u}^\text{max}_p,
$$

where $\bar{u}^\text{max}_p = \sum_{i=1}^{N_p} u^\text{max}_p(i) \in \mathbb{R}^{3 N_p}$ is the augmented vector of $u^\text{max}_p$ defined in (10).

Converting (36) to the following form,

$$
G_c \Delta \bar{u}_p \leq g_{c,p}
$$

where

$$
G_c = \left( \begin{array}{c} \Gamma \\ -\Gamma \end{array} \right),
\quad g_{c,p} = \left( \begin{array}{c} \bar{u}^\text{max}_p - \Delta u_p(k-1) \\ \bar{u}^\text{max}_p + \Delta u_p(k-1) \end{array} \right).
$$

Similarly, the control input constraint on the reaction wheels is given by

$$
G_c \Delta \bar{u}_a \leq g_{c,a}
$$

where

$$
G_c = \left( \begin{array}{c} \Gamma \\ -\Gamma \end{array} \right),
\quad g_{c,a} = \left( \begin{array}{c} \bar{u}^\text{max}_a - \Delta u_a(k-1) \\ \bar{u}^\text{max}_a + \Delta u_a(k-1) \end{array} \right).
$$

where $\bar{u}^\text{max}_a = \sum_{i=1}^{N_p} u^\text{max}_a(i) \in \mathbb{R}^{3 N_p}$ is the augmented vector of $u^\text{max}_a$ described in (11).

#### 3.3.2. Collision avoidance constraint

The collision avoidance constraint is reconfigured as follows,

$$
\tilde{f}_1 x^+_a(k) \geq \tilde{r}_\text{safe},
$$

where $\tilde{f}_1 = E_{N_p} \otimes f_1 \in \mathbb{R}^{N_p \times 6 N_p}$ with $\otimes$ being the Kronecker product of two matrices, $E_{N_p} \in \mathbb{R}^{N_p \times N_p}$ is an identity matrix with $N_p$ dimention, $f_1 = [1, 0, 0, 0, 0, 0]$, and $\tilde{r}_\text{safe} = \sum_{i=1}^{N_p} \tilde{r}_\text{safe}(i) \in \mathbb{R}^{N_p}$ is the augmented vector of $r\text{safe}$ described in (12).

Substituting (27) into (39), yields

$$
G_a \Delta \bar{u}_p \leq g_a.
$$

(40)

where

$$
g_a = -\tilde{f}_1 (B^*_p + B^*_p W^*_p)^T \Gamma, 
\quad g_a = -\tilde{f}_1 (B^*_p + B^*_p W^*_p)^T \Delta u_p(k-1).
$$

#### 3.3.3. Entry Cone Constraint

The entry cone constraint is described by

$$
\bar{e}^\text{max}(k) \leq \tilde{f}_2 x^+_a(k) \leq \bar{e}^\text{max}(k),
$$

and

$$
\bar{\rho}^\text{max}(k) \leq \tilde{f}_2 x^+_a(k) \leq \bar{\rho}^\text{max}(k),
$$

where $\tilde{f}_2 = E_{N_p} \otimes f_2$ with $f_2 = [0, 1, 0, 0, 0, 0]$, $\tilde{f}_3 = E_{N_p} \otimes f_3$ with $f_3 = [0, 0, 1, 0, 0, 0]$. $\bar{e}^\text{min}(k) = \sum_{i=1}^{N_p} e^\text{min}(k+i|i)$ is the augmented vector of $e^\text{min}(k)$ shown in (15). Similarly, $\bar{e}^\text{max}(k) = \sum_{i=1}^{N_p} e^\text{max}(k+i|i)$, $\bar{\rho}^\text{min}(k) = \sum_{i=1}^{N_p} \rho^\text{min}(k+i|i)$, $\bar{\rho}^\text{max}(k) = \sum_{i=1}^{N_p} \rho^\text{max}(k+i|i)$.

Substituting (27) into (41) and (42), i.e.,

$$
G_{e1} \Delta \bar{u}_p \leq g_{e1},
$$

where

$$
G_{e1} = \left( \begin{array}{c} \tilde{f}_2 (B^*_p + B^*_p W^*_p)^T \Gamma \\ -\tilde{f}_2 (B^*_p + B^*_p W^*_p)^T \Gamma \end{array} \right),
\quad g_{e1} = \left( \begin{array}{c} e^\text{max}(k) - \tilde{f}_2 [A^*_p x^+_a(k) + (B^*_p + B^*_p W^*_p)^T \Delta u_p(k-1)] \\ -e^\text{min}(k) + \tilde{f}_2 [A^*_p x^+_a(k) + (B^*_p + B^*_p W^*_p)^T \Delta u_p(k-1)] \end{array} \right).
$$

and

$$
G_{e2} \Delta \bar{u}_p \leq g_{e2},
$$

where

$$
G_{e2} = \left( \begin{array}{c} \tilde{f}_3 (B^*_p + B^*_p W^*_p)^T \Gamma \\ -\tilde{f}_3 (B^*_p + B^*_p W^*_p)^T \Gamma \end{array} \right),
\quad g_{e2} = \left( \begin{array}{c} \bar{\rho}^\text{max}(k) - \tilde{f}_3 [A^*_p x^+_a(k) + (B^*_p + B^*_p W^*_p)^T \Delta u_p(k-1)] \\ -\bar{\rho}^\text{min}(k) + \tilde{f}_3 [A^*_p x^+_a(k) + (B^*_p + B^*_p W^*_p)^T \Delta u_p(k-1)] \end{array} \right).
$$

#### 3.3.4. Field of view constraints

The constraint on the $N_p$ predictions of the roll angle is reconfigured as

$$
-\bar{\alpha} \leq \tilde{f}_1 x^+_a(k) \leq \bar{\alpha},
$$

where $\bar{\alpha} = \sum_{i=1}^{N_p} \alpha(i)$. Substituting (32) into (45), yields

$$
G_{f1} \Delta \bar{u}_p \leq g_{f1},
$$
where
\[
G_{f1} = \left( \begin{array}{c}
\tilde{f}_1(B_+^* + B_+ W_+^*) T \\
\tilde{f}_1(B_a^* + B_a W_a^*) T 
\end{array} \right),
\]
\[
g_{f1} = \left( \begin{array}{c}
\tilde{g}_1(A_+^* x_a(k) + (B_+^* + B_+ W_+^*) \Delta u_a(k-1)) - \tilde{g}_1(A_a^* x_a(k) + (B_a^* + B_a W_a^*) \Delta u_a(k-1))
\end{array} \right).
\]

The constraint on the \(N_p\) predictions of the pitch angle is reconfigured as
\[
\hat{\theta}_c^{\min}(k) \leq \tilde{f}_c x_a^p(k) \leq \hat{\theta}_c^{\max}(k),
\]
where \(\hat{\theta}_c^{\min}(k) = \sum_{i=1}^{N_p} \theta_c^{\min}(k+i)(i)\) is the augmented vector of \(\theta_c^{\min}(k)\) described in (15). Similarly, \(\hat{\theta}_c^{\max}(k) = \sum_{i=1}^{N_p} \theta_c^{\max}(k+i)(i)\). Substituting (32) into (47), yields
\[
G_{f2} \Delta \hat{u}_p \leq g_{f2},
\]
where
\[
G_{f2} = \left( \begin{array}{c}
\tilde{f}_2(B_+^* + B_+ W_+^*) T \\
\tilde{f}_2(B_a^* + B_a W_a^*) T 
\end{array} \right),
\]
\[
g_{f2} = \left( \begin{array}{c}
\tilde{g}_2(A_+^* x_a(k) + (B_+^* + B_+ W_+^*) \Delta u_a(k-1)) - \tilde{g}_2(A_a^* x_a(k) + (B_a^* + B_a W_a^*) \Delta u_a(k-1))
\end{array} \right).
\]

The constraint on the \(N_p\) predictions of the yaw angle is reconfigured as follows.
\[
\hat{\psi}_c^{\min}(k) \leq \tilde{f}_c x_a^y(k) \leq \hat{\psi}_c^{\max}(k),
\]
where \(\hat{\psi}_c^{\min}(k) = \sum_{i=1}^{N_p} \psi_c^{\min}(k+i)(i)\), and \(\hat{\psi}_c^{\max}(k) = \sum_{i=1}^{N_p} \psi_c^{\max}(k+i)(i)\). Substituting (32) into (49), gives,
\[
G_{f3} \Delta \hat{u}_p \leq g_{f3},
\]
where
\[
G_{f3} = \left( \begin{array}{c}
\tilde{f}_3(B_+^* + B_+ W_+^*) T \\
\tilde{f}_3(B_a^* + B_a W_a^*) T 
\end{array} \right),
\]
\[
g_{f3} = \left( \begin{array}{c}
\tilde{g}_3(A_+^* x_a(k) + (B_+^* + B_+ W_+^*) \Delta u_a(k-1)) - \tilde{g}_3(A_a^* x_a(k) + (B_a^* + B_a W_a^*) \Delta u_a(k-1))
\end{array} \right).
\]

3.4. Implement of sampling-based PWA MPC

Consider the optimization index (31) and (35) derived by the sampling-based PWA MPC strategy, and the reconfiguration of constraints on the relative position states and the attitude states, the optimal control problem over the predictive horizon can be converted to the following standard QP problems.
\[
\Delta \hat{u}_p = \arg \min \Delta \hat{u}_p(k),
\]
s.t.
\[
\begin{align*}
\Delta \hat{u}_p & \leq g_{p}, \\
\hat{x}_p(k) & = x_p(k), \\
\hat{x}_p(k) & = A_p^* x_p(k) + B_p^* W_p^* u^* + B_p^* u^*, \\
G_p^* \Delta \hat{u}_p & \leq g_p^*,
\end{align*}
\]
where \(G_p^* = [G_{c,p}^*, G_{e1}^*, G_{e2}^*]^T, g_p^* = [g_{c,p}^*, g_{e1}^*, g_{e2}^*]^T, g_p^T, g_{c,p}^T, g_{e1}^T, g_{e2}^T\).

For the attitude controller, the optimal control problem is converted to the following standard QP problem,
\[
\Delta \hat{u}_c = \arg \min \Delta \hat{u}_c(k),
\]
s.t.
\[
\begin{align*}
\hat{x}_c(k) & = x_c(k), \\
\hat{x}_c(k) & = A_c^* x_c(k) + B_c^* W_c^* u^* + B_c^* u^*, \\
G_c^* \Delta \hat{u}_c & \leq g_c^*,
\end{align*}
\]
where \(G_c^* = [G_{c,a}^*, G_{f1}^*, G_{f2}^*, G_{f3}^*]^T, g_c^* = [g_{c,a}^*, g_{f1}^*, g_{f2}^*, g_{f3}^*]^T\).

Then the optimal control input in the optimization problem can be solved by a QP solver.

4. Rendezvous and docking strategy

4.1. AR&D strategy

In this paper, the control objective is to track the time-varying position and attitude of the target’s docking port. Consider the coupling between the position and attitude, the desired state of relative position is described by \([\rho_2(t), \theta_2(t), \psi_2(t), \theta_p(t), \psi_p(t), \theta_\psi(t)]^T\), and the desired state of relative position is described by \([\phi_2(t), \phi_\theta(t), \phi_\psi(t)]^T\) with the field of view constraint being related with the azimuth and angle and the elevation angle. In practical space missions, the AR&D mission is divided into a rendezvous phase and a docking phase. In the rendezvous phase, the chaser is driven to a safe position around the target, approaching rapidity is mainly concerned. Subsequently, the chaser is controlled to track the target’s docking port. More focus is put on the safe-docking, which is realized through constraining the approach velocity, entry cone, and the keep-out zone. Besides, since there exist sensor uncertainty in practice, existing paper [18] has stated that the uncertainty can be described by Gaussian distributions with certain standard deviation, the Kalman filter can be combined with the proposed sampling-based PWA MPC framework. To focus on the effect of proposed sampling-based PWA MPC and constrains construction, this study directly control the chaser to track the target’s docking port without a two-phase AR&D process.

4.2. Singularity-free strategy

As shown in Fig. 2, the singularity of the angle states means two values represent a same position physically but differ widely in mathematics. Once the desired tracking state reaches the singularity, the actual state can not track the next jump desired state immediately, which greatly limit the application of the LOS dynamics and Euler dynamics in the 6-DOF pose tracking. In this subsection, a singularity-free strategy is provided. Denote a control period as the period before the state reaches the singular point, the basic idea is to design a mechanism which can automatically reset the controller to a new control period. The specific practice is to...
choose the correct value from different singular values according to the continuity of motion, and reset the mathematical expression of the state (doesn’t change physically) once it reaches the singular point. The previous tracking error in the final instant of the previous period is regarded as the initial error of the new period.

![Figure 4: Singularity of angle state](image)

There exist two situations in a tracking problem as follows. Assume that the desired state is monotonic and won’t repeatedly switch in the neighbourhood of the singular point. One situation is the desired state reaches the singular point but the actual state doesn’t. The other is the actual state reaches the singular point but the desired state doesn’t. For the first situation, we reset the mathematical expression of the desired state temporarily to ensure the actual state moves in the original direction until the actual state reaches the singularity. For the second situation, we reset the actual state firstly, and change the mathematical expression of the desired state temporarily to ensure the actual state moves in the original direction until desired state reaches the singularity. The detailed strategy of singularity-free is presented in Algorithm 2.

It should be noted that the discrete state can’t always reach the singular point precisely. Our method is to set a neighbourhood field near the singular point, once the state reaches the neighborhood, we consider the state reaches the singular point approximately. In this way, there exists inevitable error, our approach is to calculate the previous error in the final instant of the previous period and take it as the initial error of the new period.

### 5. Numerical simulations

To illustrate the advantages of the proposed strategies, numerical simulations of different AR&D scenarios are presented. Consider different conditions of the target’s motion, we investigate two cases. In case 1, the rotation of the tumbling target is assumed to be uncontrolled. In case 2, we consider a more general situation that the angular velocity of the tumbling target changes regularly under control. The absolute position of the target is considered not change, since the whole process won’t take long time.

Assuming the target is running in an elliptical orbit, with the eccentricity $e$ is 0.3, semi-major axis $a$ is 10000 km, initial true anomaly $f$ is 0 deg, and the initial position of the target is assumed at the perigee. The dimension of the target is $6 \times 6 \times 6$ m. The coordinate of the target’s docking port is assumed $[0.5, 0, 0]^T$ m in the body frame $F_{bt}$.

$F_{bt}$. The chaser’s inertia matrix in the body frame $F_{bt}$ is $\text{diag}(50, 35, 40)$ kg · m². The wheels’ inertia matrix in the body frame $F_{bt}$ is $\text{diag}(5, 5, 5)$ kg · m². The dimension of the chaser is $6 \times 6 \times 6$ m. The coordinate of the chaser’s docking port is assumed $[-0.5, 0, 0]^T$ m in the body frame $F_{ct}$.

For the parameters in the sampling-based PWA MPC, the prediction horizon $N_p$ is 30, the control horizon $N_c$ is 15, the sampling factor is $0.4, 0.25, 0.25$. The simulation duration is set as 500 s and the sampling interval $T_s$ is 0.1 s. The weight matrices $P$ and $Q$ in the relative position controller is set as $\text{diag}(100, 100, 100)$ and $\text{diag}(1000, 30000, 30000, 10000/\rho, 3000/\rho, 3000/\rho)$. The weight matrices $P$ and $Q$ in the relative attitude controller are set $\text{diag}(100, 100, 100)$ and $\text{diag}(30000, 30000, 30000, 30000, 30000)$. In this study, the neighbourhood of the singularity is set as $[-90i, -90i + 0.5] \deg$ and $[90i - 0.5, 90i] \deg$ for $i = 1, 2$. The state is considered reaches the singularity once it is within the neighbourhood. The AR&D conditions are shown in Table 1.

### Algorithm 2 Singularity-free strategy

**Ensure:** $x \in [-n_x \pi, n_x \pi]$ with $n_x = \frac{1}{4}$ or 1

1. if $\{ x_d \}$ is increasing
2. if $\{ x_d(k) = n_x \pi \land x(k) < n_x \pi \}$ then
3. $i = 1$
4. repeat
5. $x_d(k + i) \leftarrow x_d(k + i) + 2n_x \pi$
6. $i \leftarrow i + 1$
7. until $x(k + N) = n_x \pi$
8. $x(k + N + 1) \leftarrow -n_x \pi$
9. else $\{ x_d(k) < n_x \pi \land x(k) = n_x \pi \}$
10. $x(k + 1) = -n_x \pi$
11. $i = 1$
12. repeat
13. $x_d(k + i) \leftarrow x_d(k + i) - 2n_x \pi$
14. $i \leftarrow i + 1$
15. until $x_d(k + N) = n_x \pi$
16. end if
17. else $\{ x_d \}$ is decreasing
18. if $\{ x_d(k) = -n_x \pi \land x(k) > n_x \pi \}$ then
19. $i = 1$
20. repeat
21. $x_d(k + i) \leftarrow x_d(k + i) - 2n_x \pi$
22. $i \leftarrow i + 1$
23. until $x(k + N) = -n_x \pi$
24. $x(k + N + 1) \leftarrow n_x \pi$
25. else $\{ x_d(k) > -n_x \pi \land x(k) = -n_x \pi \}$
26. $x(k + 1) = n_x \pi$
27. $i = 1$
28. repeat
29. $x_d(k + i) \leftarrow x_d(k + i) + 2n_x \pi$
30. $i \leftarrow i + 1$
31. until $x_d(k + N) = -n_x \pi$
32. end if
33. end if

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5.1. Case 1: AR&D with an uncontrolled tumbling target

In this case, the chaser is driven to track the pose of an uncontrolled tumbling target, the angular velocity of which is set as $[0.02, 0.015, 0.02] \text{ rad/s}$ in this case. The sampling-based PWA MPC is employed to complete this mission, and the performance of sampling-based PWA MPC is compared with that of the standard NMPC based on the approximate PWA model.

As shown in Fig. 5, the performance of relative position tracking of the target’s docking port is presented. Both the standard NMPC and sampling-based PWA MPC can track the time-varying signals in about 10 s of the 500 s simulation time. However, the sampling-based PWA MPC can realize smaller overshoot and faster convergence rate, on the condition of all constraints are satisfied. Fig. 6 shows the AR&D process of employing sampling-based NMPC in the LVLH frame, Fig. 6a presents that the tracking error of the time-varying docking port is less than $2 \times 10^{-3}$ (m), and Fig. 6b shows the 3-D process of the tracking. It can be seen that the proposed controller can achieve better rendezvous performance.

Fig. 7 shows the attitude tracking performance without
singularity-free strategy, it can be seen that the tracking can not be achieved once the desired state reaces the singularity. Fig. 8 shows the tracking performance by adopting the singularity-free strategy, the result shows that the proposed strategy can cross the singularity smoothly, and the tracking takes less than 5 s of the 500 s simulation time. In addition, Fig. 9 presents the tracking error of the attitude, however, since we reset the controller to a new control period, the initial error is inevitable. It can be seen that the tracking error fluctuates once the state reaches the singular point, the tracking error can realize lower than 0.05 deg in one control period, and less than 0.6 deg when the controller is reset to a
new period. Fig. [10] shows that the input constrains on both the position and the attitude are satisfied.

5.2. Case 2: AR&D with a controlled tumbling target

In this case, a more general scenario is considered, i.e., the tumbling target is under controlled. The target’s angular velocity is set as $[0.04\sin(\frac{\pi}{100}t), 0.04\sin(\frac{3\pi}{100}t), 0.04\sin(\frac{5\pi}{100}t)]^T$ rad/s in this case. As shown in Fig. [11] the performance of relative position tracking of the target’s docking port is presented. Both the standard NMPC and sampling-based PWA MPC can track the time-varying signals in less than 15 s of the 500 s simulation time. However, the sampling-based PWA MPC can realize smaller overshoot and faster convergence rate, on the condition of all constraints are satisfied. Fig. [12] shows the AR&D process of employing sampling-based PWA MPC in the LVLH frame, it can be seen that the proposed strategy can realize better rendezvous. The tracking error of the time-varying docking port is less than $4 \times 10^{-3}$ (m). Fig. [13] shows the tracking performance of
the controlled attitude. Fig. 14 shows the attitude tracking error, the result shows that the controller can track the time-varying signals in less than 5 s of the 500 s simulation time. In addition, the tracking error can realize lower than about 0.05 deg of tracking the controlled target. Fig. 15 shows that the input constrains on both the position and the attitude are satisfied.

6. Conclusion

This paper proposes a sampling-based PWA predictive control strategy to control a chaser spacecraft to dock with a tumbling target. To reduce the on-board computation efforts and directly use the output of the navigation system, the 3-D LOS dynamics and the Euler attitude dynamics are employed together to realize the 6-DOF pose tracking. It has been shown that the novel constructions of the constraints based on the adopted states are more concise and without
any linear approximation. It can be seen from the simulation results that: (1) compared with the standard NMPC framework, the sampling-based PWA MPC strategy can realize faster tracking and less overshoot in the continuous pose tracking, while satisfying the constraints; (2) the proposed AR&D strategy can complete 6-DOF pose tracking well; (3) the controller can realize singularity-free.

Future work based on this paper may include: (1) improvement of the robustness of the sampling-based PWA MPC; (2) consider the position-attitude coupling in the AR&D and take corresponding measures to deal with it.

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