Modeling of X-ray images of Tycho’s supernova remnant

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Abstract. High-resolution Chandra X-ray observations of Tycho’s supernova remnant (SNR 1572) have revealed several series of bright, nearly parallel stripes in some regions of the SNR clearly seen in 4-6 keV band images. The observed radiation of Tycho’s SNR is most likely the synchrotron radiation of electrons with energies well above TeV. In this paper we present the modeling of synchrotron X-ray images of Tycho’s SNR in order to reveal the physical mechanism of the observed coherent structures formation. It is shown that the mirror instability, which evolves in plasma near SNR shock as a result of anisotropic distribution function of accelerated particles, can be the reason of the bright stripes.

1. Introduction

Tycho’s supernova remnant is one of the most famous objects in space, which has been observed with different instruments in different energy ranges. Eriksen et al.[1] observed Tycho in 2009 with the Chandra X-ray Observatory in 4-6 keV energy range. The observations have revealed several groups of nearly-regularly spaced stripes. The exact location, size and brightness of stripes are given in [1]. Tycho’s image with the sets of stripes is shown in Figure 1.

The problem is to understand what is the origin of the observed coherent stripes and what can we learn from the observations. Since the stripes are prominent mostly in 4-6 keV X-ray range, they are likely produced by synchrotron radiation of ultrarelativistic electrons. Earlier, Bykov et al.[2] performed modeling of the stripes, caused by synchrotron radiation of shock-accelerated electrons in the magnetic field, amplified by Bell instability. Laming et al.[3] suggested that the stripes are connected with the Alfvén wave propagating in the oblique SNR shock.

Local maxima and minima in the synchrotron emission can be provided with the modulation of the magnetic field or electron distribution function, or both. We are taking into account a specific mirror-type instability, which arises when there is an anisotropy of parallel and perpendicular pressure in plasma. The mirror instability doesn’t provide strong enough amplification of the magnetic field to make visible stripes, but it affects also the distribution function of electrons, background plasma density and the magnetic field, amplified by Bell instability, as will be shown later. We suggest that the observed stripes are the result of all these effects combined. Furthermore, unlike Bell instability, the mirror instability is long-wavelength, so it can provide a good correspondence with the observational data.
2. Mirror instability
The mirror instability is observed in the Earth’s magnetosphere. The mirror mode propagates perpendicular to the magnetic field and grows, when the perpendicular pressure is larger than the parallel pressure. The physical mechanism of the mirror instability in thermal plasma is described in [4]. The instability region was recorded by spacecraft as oscillations of the magnetic field and plasma density, as illustrated in Figure 2. The mirror instability can also occur in SNRs.

![Figure 1. Chandra image of Tycho’s SNR in 4.0-6.0 keV. a) The main ensemble of stripes b) The fainter series of stripes.](image)

![Figure 2. The sketch of the mirror-unstable region. Grey arrows point at particle motions in the magnetic mirrors. The insert at the bottom illustrates the magnetic field and plasma density in the mirror-unstable region. [4]](image)

In our model we consider the mirror instability in SNR driven by particles accelerated at the shock wave[5]. The mirror instability occurs, when the pressure of the energetic particles
is anisotropic, or, in other words, when the quadrupole anisotropy of the energetic particles distribution function is non-zero.

The distribution function of accelerated particles with the quadrupole anisotropy has the form:

\[ f_0^{ap} (p, \mu) = \frac{n_{ap} N(p)}{4\pi} \left[ 1 + \frac{\chi}{2} (3\mu^2 - 1) \right], \]

where \( \chi \) is the anisotropy parameter, \( n_{ap} \) is the concentration of accelerated particles, \( p \) is their momentum, \( f_0^{\infty} N(p) p^2 dp = 1 \) and \( \mu = \cos \theta \), where \( \theta \) is an angle between particle velocity \( v \) and \( z \)-axis (which is parallel to the background magnetic field: \( e_z = B_0/B_0 \)). Calculations (see [5]) show that near the shock wave \( \chi \simeq -0.05 \).

The dispersion relation for the mirror instability can be derived from linearized fluid mechanics equations (for background plasma) and kinetic equation (for energetic particles), where anisotropic distribution function (1) should be used. All the parameters of plasma are defined as \( \xi = \xi_0 + \delta \xi, \delta \xi \propto \exp(ikr - i\omega t) \). The wavevector is \( k = e_\perp k_\perp + e_\parallel k_\parallel \), where the parallel direction is \( e_\parallel = e_\parallel \), and \( e_\perp \) is the transverse direction. We examine the long-wavelength regime, i.e. \( kr_g \ll 1 \), where \( r_g = cp/eB_0 \) is the gyroradius of accelerated particles, \( B_0 = 3\mu G \) is the background (interstellar) magnetic field, \( c \) is the speed of light, \( e \) is the particle charge. The dispersion relation has the form

\[ \omega_{mir}^2 = \left( \frac{v_a^2 + a_0^2}{\rho_0} + 2 \frac{\delta P}{\rho_0} \right) k^2 \]

Here, \( v_a \) is the Alfvén velocity, \( a_0 = \sqrt{\frac{\gamma_g p_0}{\rho_0}} \), where \( \gamma_g \) is the adiabatic index, \( p_0 \) and \( \rho_0 \) are unperturbed pressure and density of background plasma. \( \delta P \) is the difference between parallel and perpendicular pressure of energetic particles.

\[ \delta P = P_\parallel - P_\perp = \frac{3\chi}{5} P_0, \text{ where } P_0 = \frac{n_{ap}}{3} \int_0^{\infty} vp^3 N(p) dp. \]

Equation (2) shows that the magnetic field modes (\( \delta B \propto \exp(ikr - i\omega t) \)) are growing, when \( \delta P \propto \chi < 0 \). The complete derivation of the formula (2) can be found in [5].

The mirror instability affects also the background plasma density and the distribution function of accelerated electrons which transforms into \( f = f_0 + \delta f \), where

\[ \delta f \simeq \frac{\delta B_z}{B_0} \frac{n_{cr} N(p)}{4\pi} \frac{3\chi}{2} \sin^2 \theta \]

The background plasma density modulation can be found as \( \delta \rho/\rho_0 \simeq \delta B_z/B_0 \).

Aside from the mode of the magnetic field, amplified by the mirror instability, there is the short-wavelength random magnetic field in the remnant, amplified mostly by Bell instability. Magnetic field is frozen into plasma, which leads to the magnetic field fluctuations as a result of plasma density modulation: \( B_{rand}^2 > 1/2 \propto \rho = \rho_0 + \delta \rho \). This effect provides the stripes contrast.

3. Modeling

We model electron distribution function near the front of a collisionless shock wave using Monte Carlo (MC) simulations (the method is described by Bykov et.al. in [5]). Simulations show that the best amplified mode as a result of the mirror instability is the mode with \( k_0 r_g = 2 \cdot 10^{-7} \), where \( r_g = mcu_{sh}/eB_0, u_{sh} = 5 \cdot 10^8 cm/s \) is a shock velocity, \( m \) is the particle mass. The growth rate of this mode gives \( \delta B_z/B_0 \simeq 3 \) and fluctuations of other parameters depend on this
rate. The total magnetic field in our code is the sum of the background field, mode, amplified by the mirror instability and random magnetic field, also modulated in the region where there are fluctuations of the background plasma density.

\[ B^2 = B_{0}^2 + B_{\text{mirr}}^2 + <B_{\text{rand}}^2> \tag{5} \]

The rate of the total magnetic field is defined by MC simulations.

To get the intensity of synchrotron radiation of electrons we use the well-known formula from [6]. We compute the intensity of synchrotron radiation in a small part of the remnant, where the main ensemble of stripes was discovered. We have a simulation box with the size of 100x300x1200 pixels. To imitate the observed picture we integrate the intensity along the line of sight.

4. Results

The result of the modeling is shown in Figure 3.

**Figure 3.** Simulated image of the intensity of synchrotron emission of Tycho’s SNR in 4.0-6.0 keV in relative units. 1 pixel corresponds to 0.01 pc.
The wavevector of the mirror mode $k = e_\perp k_\perp + e_\parallel k_\parallel$, taking into account that $k_\perp \gg k_\parallel$, is directed upwards in the picture plane (see Figure 3), i.e. is perpendicular to the line of sight. Hence, the magnetic field and the distribution function of electrons change in that direction. This geometry gives us maxima and minima of synchrotron radiation.

We can compare our result with the observations. The two most important parameters of stripes are distance between stripes and brightness contrast of stripes (i.e. the ratio between intensities of stripe and non-stripe regions). The comparison is given in Table 1. As one can see, the parameters that we got with our model correspond well with the observations.

| Point of comparison       | Observations       | Modeling      |
|---------------------------|--------------------|---------------|
| Distance between stripes  | $4.6 \cdot 10^{17}$ cm | $5.2 \cdot 10^{17}$ cm |
| Brightness contrast       | 4–10               | 5–15          |

5. Conclusions

The results, presented in this paper, are in a reasonably good correspondence with the observations by the Chandra X-ray Observatory. Hence, our model gives a satisfactory explanation for appearing of a highly-ordered structure in the radiation of Tycho’s supernova remnant. That means that the mirror instability can evolve in supernova remnants’ plasma and should be considered as a possible mechanism of amplification and modulation of the magnetic field and distribution function of accelerated particles in supernova remnants.

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References

[1] Eriksen K A, Hughes J P, Badenes C, Fesen R, Ghavamian P, Moffett D, Plucinsky P P, Rakowski C E, Reynoso E M and Slane P 2011 ApJ 728 L28
[2] Bykov A M, Ellison D C, Osipov S M, Pavlov G G and Uvarov Y A 2011 ApJ 735 L40
[3] Laming J M 2015 ApJ 805 102
[4] Treumann R A and Baumjohann W 1997 Advanced Space Plasma Physics (London:Imperial College Press)
[5] Bykov A M, Ellison D C and Osipov S M 2017 Phys. Rev. E 95 033207
[6] Ginzburg V L and Syrovatskii S I 1965 ARA&A 3 297