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Phys. Rev. Lett. 109, 036406 — Published 20 July 2012
DOI: 10.1103/PhysRevLett.109.036406
First-principles calculations of quasiparticle excitations of open-shell condensed matter systems

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We develop a Green's function approach to quasiparticle excitations of open-shell systems within the GW approximation. It is shown that accurate calculations of the characteristic multiplet structure require a precise knowledge of the self energy and, in particular, its poles. We achieve this by constructing the self energy from appropriately chosen mean-field theories on a fine frequency grid. We apply our method to a two-site Hubbard model, several molecules and the negatively charged nitrogen-vacancy defect in diamond, and obtain good agreement with experiment and other high-level theories.

Introduction.—In nature, there exists a wide range of electronic systems with open shells, including most atoms and many molecules, but also defects in crystalline solids. These systems play important roles in many areas of condensed matter physics, chemistry and biology: for example, the negatively charged nitrogen-vacancy (NV−) defects in diamond are used for biological imaging [1, 2] and are also promising candidates for qubits in quantum computers [3–5].

It is therefore important to develop theoretical methods to study open-shell systems and their properties. While for closed-shell systems a well-established set of methods exists, ranging from wave function-based quantum chemistry approaches to density-functional theory (DFT) and Green’s function based many-body perturbation theory, the accuracy of these methods when applied to open-shell systems is less certain: Even the application of wave function-based methods to small open-shell molecules is far from straightforward [6] and standard density functionals are known to break the orbital and spin degeneracy of the ground state [7, 8].

A Green’s function approach to electron excitations in open-shell molecules was first considered by Cederbaum and coworkers [9, 10] in the 1970’s. Later highly accurate Green’s function approaches were developed for quantum chemistry applications including open-shell systems [11–15]. Previous applications of Green’s function theory to open-shell systems using the GW approximation [16, 17] had either carefully selected reference states to avoid complications associated with the open-shell [18] or ignored the degenerate ground-state problem [19, 20].

In this Letter, we extend the GW approach to open-shell systems. Calculations on several prototypical systems are performed: a two-site Hubbard cluster, four molecules (nitrogen dioxide, oxygen, nitrogen difluoride, chlorine dioxide) and the NV− center in diamond. We find our approach is capable of describing these systems with quantitative accuracy. We have identified and implemented two important elements for accurate results in GW calculations of open-shell systems: i) a careful choice of the mean-field starting point providing accurate self-energy pole positions, and ii) a method for evaluating the self energy on a fine frequency grid.

Theory.—In a photoemission experiment with photons of energy \(\omega_{\text{photon}}\) (setting \(\hbar = 1\)), the photocurrent \(J(\epsilon_k)\) due to photoelectrons with momentum \(k\) and energy \(\epsilon_k\) is given by [21]

\[
J(\epsilon_k) = \sum_{ij} \Delta_{ij} \langle \epsilon_k - \omega_{\text{photon}} \rangle,
\]

where \(\Delta_{ij} = \langle k|\Delta_{\text{dipole}}|\psi_i \rangle\) and \(A_{ij}(\omega) = \langle \psi_i|A(r',r,\omega)|\psi_j \rangle\) denote matrix elements of the dipole operator and the spectral function, respectively, with \(\psi_i\) being an appropriate single-particle orbital. Neglecting off-diagonal matrix elements for an appropriately chosen physical set of orbitals, we obtain \(A_{jj}(\omega) = 1/\pi|\text{Im}G_{jj}(\omega)|\) by computing the interacting Green’s function (here we give the electron removal part)

\[
G_{jj}(\omega) = \sum_{\lambda} |\langle N - 1, \lambda|c_j|N, 0 \rangle|^2 \frac{1}{\omega - E_{\lambda} - i\eta}
\]

with \(E_{\lambda} = E_0^{(N)} - E_\lambda^{(N-1)}\). Here, \(|N,0\rangle\) and \(E_0^{(N)}\) denote the \(N\)-particle ground state and its energy, respectively, while \(|N - 1, \lambda\rangle\) denotes an \((N - 1)\)-particle state (with \(\lambda\) being an appropriate set of quantum numbers) with energy \(E_\lambda^{(N-1)}\). Also, \(c_j\) is the destruction operator for an electron in orbital \(j\) and \(\eta = 0^+\).

\(E_{\lambda}\) solves the quasiparticle equation

\[
E_{\lambda} = \epsilon_j + \Sigma_{jj}(E_{\lambda}) - V_{jj}^{xc},
\]

where \(\epsilon_j\) and \(V_{jj}^{xc}\) denote the orbital energy and a diagonal matrix element of the exchange-correlation potential from a mean-field calculation, respectively, while \(\Sigma_{jj}(\omega)\) is a diagonal matrix element of the self-energy operator.

The quasiparticle equation [Eq. (3)] follows from Dyson’s equation [22]

\[
G_{ij}^{-1}(\omega) = G_{0,ij}^{-1}(\omega) - \Sigma_{ij}(\omega) + V_{ij}^{xc},
\]

where \(G_{ij}^{-1}(\omega)\) and \(G_{0,ij}^{-1}(\omega)\) denote the exact and non-interacting Green’s functions, respectively.
which relates the interacting Green’s function to the mean-field Green’s function $G_{0,jj}(\omega)$ via the self energy. The standard derivation of Dyson’s equation [22] assumes the existence a nondegenerate interacting ground state which evolves into a nondegenerate single Slater determinant state as the interactions are adiabatically turned off. The hallmark of open-shell systems, however, is the existence of multiple degenerate ground states which do not generally evolve into noninteracting single Slater determinants [23]. If — for a particular ground state — the resulting noninteracting state is a sum of Slater determinants, one has to employ the methods of quantum field theory with initial correlations and replace Dyson’s equation with a more complicated expression [23, 24]. In our calculations, we avoid this difficulty by carefully choosing a ground state (among the multiple ground states) which evolves into a single Slater determinant such that Dyson’s equation is valid. As shown in Ref. [10], the Green’s function computed from such a specific ground state provides information on all the quasiparticle multiplet levels of the N-1 and N+1 system in an ensemble averaged measurement. In particular, we work with the ground state with the highest magnetic quantum number because there exists a corresponding single Slater determinant with the same properties (i.e., it is also an eigenstate of the total spin and/or orbital angular momentum operator with the same eigenvalue) [10]. An approximation to this particular ground state is provided by standard spin-polarized mean-field calculations. We note that it is not always possible to find a single determinant ground state. However, such a state must exist whenever Hund’s rules apply indicating a broad range of validity of our approach including many systems containing d- and f-electrons.

In closed-shell systems, Eq. (3) typically has a single solution leading to a pronounced quasiparticle peak in $A_{jj}(\omega)$ which corresponds to the removal of an electron from orbital $j$ [25]. In open-shell systems, the orbital and spin angular momenta of the electrons in the unoccupied orbitals can couple in various ways resulting in multiple low-energy eigenstates of the $(N-1)$-particle system. The coupling of angular momenta generally produces eigenstates which are sums of multiple Slater determinants [26]. As a consequence, multiple eigenstates of the $(N-1)$-particle system can make significant contributions to $G_{jj}(\omega)$ if their matrix element in the numerator of Eq. (2) is large. $G_{jj}(\omega)$ then has multiple poles and we expect to find multiple solutions to Eq. (3) [9, 10].

If $G_{jj}(\omega)$ has multiple poles, Eq. (4) shows that the self energy $\Sigma_{jj}(\omega)$ must also have poles occurring between the poles of $G_{jj}(\omega)$. The occurrence of poles in $\Sigma_{jj}(\omega)$ near $E_\lambda$ is a particular feature of open-shell systems and a direct consequence of the electronic multiplet structure. We note that multiple solutions of Dyson’s equation can also occur in closed-shell systems where the extra solution results from the coupling of electrons to plasmons[17].

In actual calculations for open-shell systems, a precise knowledge of the frequency dependence of the self energy is necessary to locate its poles and obtain accurate multiplet splittings. In contrast, for closed-shell systems it is usually sufficient to employ a simple linear expression for the frequency dependence of the self energy in the vicinity of the quasiparticle energy [25].

In this work, we employ the GW approximation to the self energy following the first-principles method of Hybertsen and Louie [25]. To obtain $\Sigma_{jj}(\omega)$ at many frequencies, we make use of a specific form of the evaluation of the frequency dependence of the dielectric response and self energy as proposed in Refs. [27] and [18]. In this approach, $\Sigma_{jj}(\omega)$ is separated into a frequency-independent bare exchange part $\Sigma_{jj}^{(x)}$ and a frequency-dependent correlation part $\Sigma_{jj}^{(c)}(\omega)$ given by

$$\Sigma_{jj}^{(c)}(\omega) = \sum_{nI} \frac{|V_{jnI}|^2}{\omega - \epsilon_n - \Omega_I \text{sgn}(\epsilon_n - \mu)},$$

where $\mu$ denotes the chemical potential and $\Omega_I$ is a neutral excitation energy of the $N$-particle system obtained by solving Casida’s equation in the random-phase approximation [27]. Also, $V_{jnI}$ denotes a Coulomb matrix element between the product $\psi_j^\dagger \psi_n$ and the fluctuation charge density $\rho_I$ [27] (see Supplementary Material for details on the approach).

Equation (5) shows that the poles of $\Sigma_{jj}(\omega)$ are determined by the mean-field electron removal (or addition) energies $\epsilon_n$, which are the poles of $G_0$, and by the neutral excitation energies $\Omega_I$, which are the poles of the screened interaction $W_0$ in the random-phase approximation. Both $\epsilon_n$ and $\Omega_I$ depend on the mean-field theory used to compute $G_0$ and $W_0$, implying an analogous dependence on the choice of the mean-field starting point for the poles of $\Sigma_{jj}(\omega)$.

In principle, the self energy should be computed from the interacting Green’s function $G$, whose poles are at $E_\lambda$, and the exact screened interaction $W$ [25]. For closed-shell systems, it is possible to carry out self-consistent GW$_0$ calculations where the self energy is recomputed using the iterated Green’s functions such that $\Sigma$ becomes independent of the mean-field starting point [25]. For open-shell systems, self-consistent calculations are more difficult because of the more complicated structure of $G$ and additional problems to be discussed below. To obtain accurate self-energy pole positions we instead carefully choose mean-field theories that yield $\epsilon_n$ and $\Omega_I$ which are good approximations to $E_\lambda$ and the poles of the exact $W$, respectively. In general, one finds that the poles of $W_0$ obtained from standard density-functional calculations are good approximations to neutral excitation energies. In contrast, the poles of $G_0$ obtained from density-functional theory often differ from the exact removal or addition energies (i.e. the quasiparticle energies) by several electron volts. Such an error in the poles of $G_0$ leads
to a similar-sized error in the self-energy pole locations and to a large error in the multiplet splittings. To obtain the best \( G_0 \), we construct it from mean-field calculations using the static COHSEX approximation [16, 25].

In addition, if the result of a calculation depends on a particular self-energy pole we carry out partially self-consistent calculations where we only update the particular \( \varepsilon_n \) in Eq. (5) which determines the position of the self-energy pole under consideration.

**Molecules.**—First, we study the quasiparticle multiplet structure of four small molecules for which accurate experimental data is available.

Nitrogen dioxide (NO\(_2\)) has a doublet ground state. We first carry out DFT calculations [28, 29] at the experimental geometry [30] using the spin-polarized LDA exchange-correlation functional, norm-conserving pseudopotentials, a plane-wave basis (50 Ry cutoff) and a cubic supercell with linear dimension of 10.6 \( \AA \).

For the construction of \( W_0 \) we use wave functions and energies from the DFT calculation. We use 300 empty states and a 15 Ry momentum space cutoff for the dielectric response. For \( G_0 \) we use wave functions and energies from a static COHSEX calculation. Table I shows that the COHSEX single-particle energies are much closer to the experimental ionization potentials than the DFT energies, but the multiplet structure is still missing in this calculation. For the calculation of the self-energy matrix element we use 300 empty states and a modified static remainder correction [31, 32] which extends the sum over \( n \) in Eq. (5) to all empty states and greatly improves convergence. This choice of parameters results in multiplet splittings converged to within \( \sim 0.1 \) eV.

Figure 1(a) shows the self energy and spectral function for the removal of a down-spin electron from the 4\( b_2 \) orbital [see insert in Fig. 1(a)]. We do not expect any multiplet structure for this process because the up-spin hole can only couple to the up-spin electron in the 6\( a_1 \) orbital to give a triplet state. Indeed, the spectral function exhibits a single peak corresponding to the triplet \((3B_2)\) state.

Figure 1(b) shows results for the removal of an up-spin electron from the 4\( b_2 \) orbital. The down-spin hole can now couple to the up-spin electron in the 6\( a_1 \) orbital to yield either a singlet \((1B_2)\) or a triplet \((3B_2)\) state. Indeed, we find two solutions of Eq. (3) resulting in two poles of the Green’s function and two peaks in the spectral function with a singlet-triplet splitting of 1.8 eV which compares favorably with the experimental splitting of 1.5 eV (Table I). In contrast, the singlet-triplet splitting from \( G_{LDA}W_{LDA} \) is 2.9 eV highlighting the importance of an accurate mean-field starting point. To make sure that the two solutions are indeed multiplet states we traced back the low lying self-energy pole to open-shell features in \( G_0 \) and \( W_0 \): namely, to the pole in \( G_0 \) due to the unpaired up-spin 6\( a_1 \) state and the pole in \( W_0 \) due to the 4\( b_{2\downarrow} \) \( \rightarrow \) 6\( a_{1\downarrow} \) transition between the two open shells. Hund’s rule suggests that the lower energy solution is the triplet state: an exact diagonalization analysis in the subspace consisting of the 6\( a_1 \) and the 4\( b_2 \) orbitals shows that the singlet is higher in energy than the triplet solution by \( 2J \) with \( J \) being the positive exchange integral.

Inspection of Table I shows that we obtain two values for the energy of the triplet state \( 3B_2 \), one from the re-

### Table I: Comparison of calculated ionization potentials for NO\(_2\) with experiment [30].

| orbital    | LDA      | COHSEX   | \( G_{LDA}W_{LDA} \) | GW exp. |
|------------|----------|----------|----------------------|---------|
| \( 6a_1(\uparrow) \) | -6.6     | -12.0    | -10.7                | -11.2   |
| \( 4b_2(\downarrow) \) | -8.7     | -14.1    | -12.5                | -12.8   |
| \( 4b_2(\uparrow) \) | -9.3     | -14.9    | -10.5                | -13.6   |
| \( 4b_2(\downarrow) \) | -9.3     | -14.9    | -13.4                | -15.4   |

**FIG. 1:** Self energy \( \Sigma_{jj}(\omega) \) and spectral function \( A_{jj}(\omega) \) for (a) the removal of a down-spin electron from the \( j = 4b_2 \) orbital in NO\(_2\) and (b) the removal of an up-spin electron from the \( j = 4b_2 \) orbital. A Lorentzian broadening of 20 meV is used for each curve.
moval of an up-spin electron from the $4b_2$ orbital, one from the removal of a down-spin electron from the same orbital. These values differ by 0.8 eV and bracket the experimental result. There are two factors which contribute to this discrepancy: i) remaining errors in the positions of the self-energy poles which contaminate only solutions of the up-spin quasiparticle equation and ii) missing vertex corrections which contaminate solutions of the up- and down-spin quasiparticle equations in different amounts [33]. We expect that the inclusion of vertex corrections will reduce the difference. Nevertheless, as shown above, accurate multiplet splittings can be extracted from our calculations if the energy differences are calculated from solutions of the quasiparticle equation for a particular spin direction because errors due to missing vertex correction cancel.

The ratio of the areas under the singlet and the triplet peaks in Fig. 1(b) should be the experimentally observed ratio of ensemble-averaged photoemission intensities, the so-called multiplet ratio [10, 34]. We find in our calculations that the multiplet ratios are much more sensitive to the positions of the self-energy poles than the multiplet splittings. We do not expect that these ratios can be computed reliably with our current GW approach because of the remaining uncertainties in the self-energy pole locations. However, Schirmer and coworkers found a relatively simple analytical procedure for calculating these ratios based on the addition of angular momenta [34]. We expect that the combination of their approach for the multiplet ratios and the GW approach for the multiplet splittings offers a reliable and complete description of the multiplet structure of open-shell systems.

We also investigated the multiplet structure of the oxygen (O$_2$), nitrogen difluoride (NF$_2$) and the chlorine dioxide (ClO$_2$) molecules. The GW multiplet splittings are 2.4 eV for O$_2$, 2.3 eV for NF$_2$ and 2.5 eV for ClO$_2$. They compare favorably with experimental splittings: 2.3 eV for O$_2$, 1.8 eV for NF$_2$ and 2.4 eV for ClO$_2$ [35–37]. However, splittings obtained from G$_{LDA}W_{LDA}$ can deviate from experimental findings by several electron volts (see Supplementary Materials).

We also applied our method to a two-site Hubbard model with three electrons finding good agreement for multiplet splitting between the exact diagonalization result and the GW theory over a large range of the interaction parameter (see Supplementary Materials).

**NV$^-$ center.**—Next, we apply our approach to the NV$^-$ center in diamond which has a triplet ground state. This defect currently attracts much attention because of its extraordinary properties, such as long coherence times and potential application to quantum computing [3–5].

Figure 2 shows results for the removal of an up-spin electron from the $j = \nu$ defect level in the band gap of diamond (see Supplementary Materials for computational details). As in the NO$_2$ calculation, the self energy exhibits a low-lying pole leading to two solutions of the quasiparticle equation. To understand which many-body states these solutions correspond to, we compare our findings to the results of the exact diagonalization of the extended Hubbard model of Choi, Jain and Louie[38] (see Supplementary Materials). These authors fit the parameters of an extended Hubbard model for the defect levels to ab initio static COHSEX results and show that this model describes accurately neutral excitations. The model predicts four many-body states $\Psi_j$ for the $\nu^1e^2$ configuration. However, only two of the four states, namely $^4A_2$ and $^2A_2$, are observed in our calculations because by symmetry only these states have a non-vanishing matrix element $\langle \Psi_x|c_{j\uparrow}|\nu\nu\nu e_{j\uparrow}e_{j\uparrow}\rangle$ with the ground state. Table II shows that our GW multiplet splittings agree well with the results of the exact diagonalization of the extended Hubbard model. Note that the $^4A_2$-$^2A_2$ splitting in Fig. 2 corresponds to the last row in Table II. The first row in Table II shows the splitting between the $^2E$ state (obtained by removing an up-spin electron from the $e$ defect orbital) and the $^2A_2$ state. Again, our findings agree well with the result from the exact diagonalization of the extended Hubbard model.

Future work is needed to investigate the performance of the open-shell GW method for systems with more complicated multiplet structures, such as systems containing d- and f-electrons, where additional low-energy solutions of Casida’s equation can give rise to multiple poles of the multiplet structure.
self energy.

J.L. acknowledges useful discussions with SangKook Choi and Dr. Eric Shirley. J.D. acknowledges support under National Science Foundation Grant No. DMR10-1006184. J.L. and M.J. acknowledge support from the Director, Office of Science, Office of Basic Energy Sciences, Materials Sciences and Engineering Division, U.S. Department of Energy under Contract No. DE-AC02-05CH11231. Computational resources have been provided by DOE at Lawrence Berkeley National Laboratory's NERSC facility.

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