Methods for measuring the auxiliary modulation step in interferometric fiber optic sensor

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Abstract. In the current paper we consider the case of the sawtooth-type of phase modulation of the interference signal argument and demonstrate a method for calculating the phase modulation step of interferometric signal in a fiber optic sensor. Two methods of calculation the modulation step are proposed: for 4-point and 5-point digital demodulation algorithm based on ordinary least squares approach. The effect of target phase on the results of calculating modulation step has been demonstrated. The dependence of the error in the calculation of the auxiliary modulation step on the amplitude of the target signal is also shown. The correspondence between analytical expressions and real modulation step of interferometric signal has been verified by means of numeric simulation.

1. Introduction

Fiber optic sensors are intensively developing over the past decades. Fiber optic interferometric sensors require a demodulation system for extracting the target phase. One of the most widely used and efficient method is the use of auxiliary phase modulation of the interference signal argument (phase-shifting interferometry, PSI). The choice of the modulation type depends on the features of the optical scheme, as well as by the signal processing unit. Harmonic or sawtooth type of phase modulation is most often used as auxiliary modulation signals. However, the use of a digital sawtooth modulation with amplitude of 2\(\pi\) is advantageous since in this case the interferometric signal is a cosine signal, which may be convenient from the point of view in the diagnostic system of interferometric signal prior to phase detection in the data processing unit [1, 2].

The interferometric sensor interrogation system may include a laser, phase modulator, other optical and optoelectronic modules, and data processing system [1]. During operation, some parameters of the nodes of the sensor interrogation system may change slightly, primarily due to changes in ambient temperature. Specifically, the effect of temperature on the change in the parameters of integrated optical or piezoceramic phase modulators is well known. As a result, over time, the auxiliary phase modulation parameters are become miscalibrated. It means that the auxiliary modulation signal in the interferometer signal does not correspond to the auxiliary modulation value in the data processing unit. The phase modulator can be calibrated before the start of work. For this purpose, the half-wave voltage of the phase modulator is measured when the temperature is in the required range. Then the measurement result for a phase modulator is entered into the program to account for the coefficients in the data processing unit. This method does not take into account the residual contribution of temperature changes.
It seems appropriate, more efficient to use the received interferometric signal, and in addition to phase demodulation to add to the algorithm the calculation of the auxiliary modulation amplitude (below we will use the term “auxiliary modulation step”, which is more suitable in case of digital representation of the auxiliary phase modulation). This approach implies using well-known [1-3] algorithms of phase demodulation based on the least squares methods (for 4 and 5 points in the auxiliary modulation interval) to phase demodulation of the interferometric signal. The refined value of the auxiliary modulation step should be taken into account in the phase modulation adjustment system for further phase demodulation.

2. The algorithms for calculating the amplitude of the phase modulation
In case of sampled interference signal with sampling frequency $f_d$, one has to deal with stream of discrete samples with the following form

$$u_i = U_0 + U_m \cos(\phi + \psi_i),$$

where $i$ – stream sample number, $\phi$ – target phase difference carrying information about the actual the measured physical quantity; $U_0$ – constant part, and $U_m$ – the amplitude of the interference signal; $\psi_i$ – known periodical signal of phase modulation with frequency $f_M$. The sawtooth shape of the auxiliary modulation results in linear increase of the values $\psi_i$ during the modulation period. In addition, it is convenient to choose the sample rate $f_d$ multiple of $f_M$, so that the number of samples during one modulation period is an integer $N = f_d/f_M$.

Interference signal (1) has at least three parameters: $\phi$, $U_0$ and $U_m$, which are assumed to be unknown. Therefore, to find these parameters at least $Q$ readings $u_i$ are required, thus forming as a solution of a system of linear equations. So, the shortest possible demodulation interval corresponds to $Q=N=3$ and each single value of target phase $\phi$ is calculated based on three readings $u^{(q)}$, where $q = 0, 1, 2 ..., Q-1$ [4]. If the number of $Q=N=4$ or more, it becomes possible to find the current value of the modulation amplitude.

In case when $N=4$ on the demodulation interval, the values of the auxiliary modulation are specified by a set $\psi^{(q)}=[0, \beta, 2\beta, 3\beta]$. It is known that in this case minimum of phase noise is obtained, when we choose $\beta = \pi/2$ [5], and the solution for estimating $\phi_r$ of the target phase $\phi$ based on the ordinary least squares method (OLS) can be written as

$$\phi_r = \text{atan2} \left( \frac{u^{(1)} - u^{(0)}}{u^{(3)} - u^{(0)}} \right).$$

The function atan2 is commonly written as a function of two arguments atan(y, x). The atan2 function takes into account signs of x and y and represents nominator and denominator of atan function and therefore results in $\phi_r$ within $[-\pi, \pi]$ range. The eq. (2) will cause errors if the $\beta$ will deviate from the required value $\pi/2$. However, in case $Q = 4$ it becomes possible to find the parameter $\beta$ from what has been said above and correct the signal of auxiliary modulation for provide the needed value of $\beta$. According to the equations described in [4, 6] for case $N = Q = 4$ and $\psi^{(q)}=[0, \beta, 2\beta, 3\beta]$ of the following formulas

$$\phi_r = \text{atan2} \left( \frac{u^{(1)} - u^{(0)}}{u^{(3)} - u^{(0)}} \right).$$
a) \[ \cos(\beta) = \frac{u^{(0)} - u^{(1)} + u^{(2)} - u^{(3)}}{2u^{(1)} - u^{(2)}}. \]

b) \[ \tan(\varphi_r) = \left[ \frac{1 - \cos(\beta)}{\sqrt{1 - \cos^2(\beta)}} \right] \frac{u^{(0)} - u^{(1)} - 2(u^{(0)} - u^{(1)}(1 + \cos(\beta)))}{u^{(0)} - u^{(2)} - 2(u^{(0)} - u^{(1)})\cos(\beta)}. \]

Detailed analysis of eq. (3) shows that there take place singularities in points \( \varphi = -3\beta/2 + \pi k \), where \( k \) is integer (for \( \beta = \pi/2 \varphi = \pi/4 \) and \(-3\pi/4\)). In this case calculation of \( \beta \) is not correct. Therefore, calculate \( \varphi_r \), of target phase \( \varphi \) by eq. (2) helps to calculate \( \beta \) correctly. It should be noted, that calculation of target phase \( \varphi \) given in eq. (3) is not optimal in terms of noise impact given that \( \beta = \pi/2 \).

In case \( N = Q = 5 \) the solution for estimating \( \varphi_r \), of the target phase \( \varphi \) based on the OLS method for \( \beta = 2\pi/5 \) can be written as

\[ \varphi_r = \arctan\left[ \frac{-\sum_{q=1}^{5} u_q \cdot \sin\left( \frac{2\pi}{5} \cdot q \right)}{\sum_{q=1}^{5} u_q \cdot \cos\left( \frac{2\pi}{5} \cdot q \right)} \right]. \]

Since demodulation interval includes 5 samples, we have five values of the modulating signal and can find the value \( \beta \) in several formulas [7-10]. To calculate this value the following three formulas are most suitable

a) \[ \beta_r = a \cos\left( \frac{u^{(0)} - u^{(1)} + u^{(2)} - u^{(3)}}{2u^{(1)} - u^{(2)}} \right), \]

b) \[ \beta_r = a \cos\left( \frac{u^{(1)} - u^{(2)} + u^{(3)} - u^{(4)}}{2u^{(2)} - 2u^{(3)}} \right), \]

c) \[ \beta_r = a \cos\left( \frac{u^{(0)} - u^{(4)}}{2u^{(1)} - 2u^{(3)}} \right). \]

In the same way as the eqs. (3a) and (3b) were derived, one can determinate singularity points, where the denominator is zero. In this case, for each equation can be a singularity point can be written as

a) \[ U_m \cdot 4 \cdot \sin\left( \frac{\varphi + 5\beta}{2} \right) \cdot \sin\left( \frac{\beta}{2} \right) = 0 \rightarrow \varphi = -\frac{5}{2} \beta + \pi k; \]

b) \[ U_m \cdot 4 \cdot \sin\left( \frac{\varphi + 7\beta}{2} \right) \cdot \sin\left( \frac{\beta}{2} \right) = 0 \rightarrow \varphi = -\frac{7}{2} \beta + \pi k; \]

c) \[ U_m \cdot 4 \cdot \sin(\varphi + 3\beta) \cdot \sin(\beta) = 0 \rightarrow \varphi = -3\beta + \pi k. \]

Because the estimation \( \beta_r \) become difficult when \( \varphi \) is close to mentioned singular values, the possible errors of calculated \( \beta \) depends on current value \( \varphi \) and target oscillation of \( \varphi \).
In case when we use only one equation from eqs. (5) to estimate the \( \beta \), we will have to set a boundary of the target phase \( \phi \) for the denominator value above, which may be considered adequate to use. The higher is this border, the lower will be the distortion in the worst case. However, the higher is the probability during a given period of time, the current value of \( \phi \) will allow us to assess the magnitude of parameter \( \beta \). If we use three eqs. (5), we can evaluate the step always.

There are two possible variants of calculations for the estimation of the value \( \beta \). The first variant is to derive approximation equation for the errors of \( \beta \) determination under the influence of various distortions for eqs. (5). As a result of approximation, from eq. (5) we may choose the value at a given minimum error. The problem is that in practice we do not know the current value of the target phase \( \phi \), we only know the estimate \( \phi_r \), which can be quite different from the true value in absolute value.

A simpler second variant is to choose one of the three eqs (5) whose denominator is greater in absolute value. In addition reduction of the errors in the estimation of the value \( \beta \) can be achieved by averaging over several estimates of the value \( \beta_r \), since the value of \( \phi \) varies in time.

3. Numerical simulation
In order to estimate effectiveness of the algorithms given in eqs. (5), the numeric modelling was performed in the following way – interference signal was modelled according to eq. (1), where the amplitude of phase modulation is specified with shift 0.1 rad, so there was \( \beta = 1.237 \) radian (the optimal value is \( 2\pi/5 = 1.257 \) radian). This shift imitates the change of modulator efficiency and allows to demonstrate estimations of \( \beta \) by means of interference signal processing. The samples of interference signals were calculated for a time interval 0.1 sec length. An example of interference signal on time interval 0 – 0.4 ms is shown in figure 1. Points correspond to signal samples \( u^{(q)} \).

![Figure 1. Fragment of interference signal in case \( f_d = 125 \) kHz, \( f_M = 25 \) kHz.](image)

In case a wider range of the target phase (-\( 2\pi \) to \( 2\pi \)), a numerical simulation of the dependence of \( \beta_r \) on the target phase amplitude was performed (figure 2). In the simulation model, the value of the target phase increased linearly over 65 ms. As can be seen from figure 2, when making calculation using a single eq. 5 (dashed line), there is an area where it is impossible to perform an adequate estimate of \( \beta \). In the case of using three equations near the singularity points, also there is a region of values of the target phase, where the error of estimation of parameter \( \beta \) is greater. However, in this case, the values of \( \beta_r \) is close to true.
Figure 2. Estimation of $\beta$ depending on time given in case of linearly increasing of the signal target phase $\varphi$.

Figure 3 shows the result of numerical simulation of the estimate of the value $\beta$ in the presence of the target phase oscillation $\varphi$ with 4 radian amplitude and 1 kHz frequency. The dashed line in figure 3 represents the result of $\beta$ estimation using three eqs. (5) and the solid line while using only one eq. (5). As can be seen from figure 3, the estimate varies depending on the current value of the target phase. In addition, when using one eq. (5), there is an outlier, which indicates approximation of the current value $\varphi$ to the singularity point. In this case, averaging the $\beta$ estimate over several points will minimize the effect of the target phase signal on the error.

Figure 3. Estimation of $\beta$ value depending on time under the signal target phase $\varphi$.

4. Conclusion
In this article we proposed an algorithm for calculating the step of auxiliary sawtooth phase modulation $\beta$ for the interferometric signal of a fiber-optic sensor. For the case of $N = 4$, the results of numerical simulation will be similar to the case of $N = 5$ using the same eq. (5). For the case $N = 4$, there is a range of values of the target phase $\varphi$ for which the estimate of the modulation step $\beta$ will be incorrect. Therefore, to calculate the auxiliary modulation step $\beta$ in fiber-optic interferometric sensors,
it is advisable to use algorithms with $Q = N = 5$. In addition, when correcting the auxiliary modulation signal, it is necessary to perform averaging on a sufficient time interval to eliminate the influence of the target phase oscillations on the result of the estimation of the value $\beta$.

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