Profiles of dark matter haloes at high redshift

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ABSTRACT
I study the evolution of halo density profiles as a function of time in the SCDM and ΛCDM cosmologies. Following Del Popolo, I calculate the concentration parameter $c = r_c/a$ and study its time evolution. For a given halo mass, I find that $c(z) \propto 1/(1 + z)$ in both the ΛCDM and SCDM cosmology, in agreement with the analytic model of Bullock et al. and N-body simulations. In both models, $a(z)$ is roughly constant. The present model predicts a stronger evolution of $c(z)$ with respect to the Navarro, Frenk & White model. Finally I show some consequences of the results on galaxy modelling.

Key words: galaxies: formation – cosmology: theory – large-scale structure of Universe.

1 INTRODUCTION

The structure of dark matter haloes is of fundamental importance in the study of the formation and evolution of galaxies and clusters of galaxies. From the theoretical point of view, the structure of dark matter haloes can be studied both analytically and numerically. A great part of the analytical work done so far is based on the secondary infall model (SIM) introduced by Gunn & Gott (1972). Calculations based on this model predict that the density profile of the virialized halo should scale as $r \propto r^{-0.5}$. Self-similar solutions were found by Fillmore & Goldreich (1984) and Bertschinger (1985), who obtained a profile of $r \propto r^{-2.25}$. Hoffman & Shaham (1985) (hereafter HS) considered a scale-free initial perturbation spectra, $P(k) \propto k^n$. They showed that $r \propto r^{-\alpha}$ with $\alpha = 3(3 + n)/(4 + n)$, thus recovering Bertschinger’s (1985) profile for $n = 0$ and $\Omega = 1$. They also showed that, in an open Universe, the slopes of the density profiles steepen with increasing values of $n$ and with decreasing $\Omega$, reaching a profile $r \propto r^{-3}$ for $\Omega \rightarrow 0$.

N-body simulations, such as those of Quinn, Salmon & Zurek (1986), West, Dekel & Oemler (1987) and Efstathiou et al. (1988) arrived at conflicting results implying that better numerical resolution were needed to settle the issue. Recent results from higher resolution simulations (Navarro, Frenk & White 1995, 1996, 1997, hereafter NFW95, NFW96, NFW97); Lemson 1995; Cole & Lacey 1996; Tormen, Diaferio & Syer 1998) obtained, using different codes and different set-ups for the initial conditions, agreement in the conclusion that halo density profiles do not follow a power law but develop a universal profile, a one-parameter functional form that provides a good fit to haloes over a large range of masses and for any scenario in which structures form owing to hierarchical clustering, characterized by a slope $\beta = d \ln \rho / d \ln r = -1$ near the halo centre and $\beta = -3$ at large radii. In that approach, density profiles can be fitted with the functional form:

\[
\frac{\rho(r)}{\rho_0} = \left(\frac{r}{a(1 + \frac{r}{a})}\right)^{\gamma},
\]

where $\rho_0$ is the background density and $\delta_b$ is the central overdensity [below I shall refer to equation (1) (NFW97) as the NFW profile]. The scale radius $a$, which defines the scale where the profile shape changes slope from $\beta < -2$ to $\beta > -2$, and the characteristic overdensity, $\delta_b$, are related because the mean overdensity enclosed within the virial radius $r_v$ is $\approx 180$. I recall that according to NFW96 and NFW97, $a$ is linked to a ‘concentration’ parameter, $c$, by the relation $a = r_v/c$ and the parameter $c$ is linked to the characteristic density, $\delta_b$, by the relation:

\[
\delta_b = \frac{200}{3} \frac{c^3}{\ln(1 + c) - c/(1 + c)}.
\]

The scale radius and the central overdensity are directly related to the formation time of a given halo (NFW97). The power spectrum and the cosmological parameters only enter to determine the typical formation epoch of a halo of a given mass, and thereby the dependence of the characteristic radius, $a$, or the overdensity $\delta_b$ on the total mass of the halo. $\delta_b$ increases for decreasing virial mass, $M_v$. A natural reason for the fact that low-density haloes tend to show higher densities is that they typically collapse earlier, when the Universe was denser. To model this trend, NFW97 proposed a step-by-step calculation of the density profile assuming that the characteristic density, $\delta_b$, is proportional to the density of the universe at the corresponding collapse redshift, $z_c$. This model successfully predicts the $\delta_b - M_v$ relation for different cosmological models at $z = 0$. The model has been also extended in NFW97 to predict the redshift dependence of the halo profile...
parameters but, as shown by Bullock et al. (1999, hereafter B99) for the ΛCDM cosmology, the evolution of \( c \) (and consequently of \( \delta_n \)) is much stronger than in the NFW97 model.

In this paper, I use the improved SIM introduced by Del Popolo et al. (2000, hereafter DP2000) to determine \( c(z) \) for both SCDM and ΛCDM models and to compare it with prediction of NFW97 and B99 models.

The plan of the paper is the following. In Section 2, I introduce the model. In Section 3, I show the results of the model and Section 4 is devoted to the conclusions.

2 TIME-EVOLUTION OF THE CONCENTRATION PARAMETER

The simplest version of SIM considers an initial point mass, which acts as a non-linear seed, surrounded by a homogeneous uniformly expanding universe. Matter around the seed slows down because of its gravitational attraction, and eventually falls back in concentric spherical shells with pure radial motions. The assumptions of SIM that are most often questioned are the spherical symmetry and the absence of peculiar velocities (non-radial motions): in the 'real' collapse, accretion does not happen in spherical shells but by aggregation of subclumps of matter which have already collapsed; a large fraction of observed clusters of galaxies exhibit significant substructure (Kriessler et al. 1995). Motions are not purely radial, especially when the perturbation detaches from the general expansion. Nevertheless, the SIM gives good results in describing the formation of dark matter haloes, because in energy space the collapse is ordered and gentle, differently from what is seen in N-body simulations (Zaroubi, Naim & Hoffman 1996). As I showed in a recent paper (DP2000), the discrepancies between the SIM and some high-resolution N-body simulations are not caused by the spherical symmetry assumption of the SIM but arise because of some non-accurate assumptions used in its implementation. As I showed in DP2000, the predictive power of the SIM is greatly improved when some of the problems of the previous implementations are removed.

To begin with, the conclusion \( \rho \propto r^{-2} \) for \( n < -1 \), claimed by HS, is not a direct consequence of the HS model, but it is an assumption made by the quoted authors, following the study of self-similar gravitational collapse by Fillmore & Goldreich (1984). In fact, as reported by the same authors, in deriving the relation between the density at maximum expansion and the final one, HS assumed that each new shell that collapses can be considered as a small perturbation in the gravitational field of the collapsed halo. This assumption breaks down for \( n < -1 \).

Secondly, the assumption made by Hoffman & Shaham (1985) that \( \delta(r) \propto \xi(r) \propto r^{-(3+n)} \) is not good for regions internal to the virial radius, \( r_v \) (see Peebles 1974; Peebles & Groth 1976; Davis & Peebles 1977; Bonometto & Lucchin 1978; Peebles 1980; Fry 1984). In the inner regions of the halo, scaling arguments plus the stability assumption tell us that \( \xi(r) \propto r^{-(3+n)} \) and we expect a slope different from that of HS. In other words, HS's (1985) solution applies only to the outer regions of collapsed haloes, and consequently the conclusion, obtained from that model, that dark matter haloes density profiles can be approximated by power-laws on their overall radius range is not correct. It is then necessary to introduce a model that can also make predictions on the inner parts of haloes.

Thirdly, according to Bardeen et al. (1986, hereafter BBKS), the mean peak profile depends on a sum involving the initial correlation function, \( \xi(r) \propto r^{-(3+n)} \), and its Laplacian, \( \nabla^2 \xi(r) \propto r^{-(5+n)} \) (BBKS; Ryden & Gunn 1987):

\[
\delta(r) = \frac{\nu(\xi(r))}{\xi(0)} - \frac{\xi(0)}{\xi(1)} \left[ \left( 1 - g^2 \right) r^2 + \frac{R^2}{3} \nabla^2 \xi(r) \xi(0)^{-1/2}. \right.
\]

where \( \nu \) is the height of a density peak:

\[
\nu = \frac{\delta(0)}{\sigma[R, z]).}
\]

The variance \( \sigma[R, z] \) is given by

\[
\sigma^2(R, z) = D^2(\xi, \Omega) \int_0^{1/2} dk k^2 P(k) W^2(2kR),
\]

where the function \( D(\xi, \Omega) \) describes the growth of density fluctuations (section 11 of Peebles 1980) and \( W(kR) \) is a top-hat smoothing function

\[
W(kR) = \frac{3}{(kR)^2} \sin(kR) - kR \cos(kR).
\]

\( \gamma \) and \( R_s \) are two spectral parameters given respectively by

\[
\gamma = \frac{1}{k^2 P(k) \xi} \int \left[ k^2 P(k) \xi \right]^{1/2}
\]

and

\[
R_s = \left[ \frac{3}{k^2 P(k) \xi} \right]^{1/2},
\]

while \( \delta(\nu, \gamma) \) is

\[
\delta(\nu, \gamma) = \frac{3(1 - \gamma^2) + (1.216 - 0.9\gamma^4) \exp \left[ -\left( \frac{\gamma}{2} \right)^2 \right]}{3(1 - \gamma^2) + 0.45 + (\frac{\gamma}{2})^2 \exp \left[ -\left( \frac{\gamma}{2} \right)^2 \right]}
\]

I recall that the \( z \)-dependence of \( \delta \) is

\[
\delta(z) = \delta_0 D(z, \Omega),
\]

where \( \delta_0 \) is the overdensity as measured at current epoch \( t_0 \). As can be seen for example in the case of a scale-free density perturbation spectrum (DP2000, equation 20), the initial mean density obtained using the model of this paper is extremely different from that obtained and used in HS.

The first step to obtain \( c(z) \) is to calculate \( \delta(r) \) for a given cosmology starting from the related spectrum. In order to calculate \( \delta(r) \) in the ΛCDM cosmology (\( \Omega = 1, h = 0.5, n = 1 \)), I use the spectrum given by BBKS.

\[
P(k) = Ak^{-1} \left[ 1 + (\frac{1.464k}{k_0})^2 \right] \times \left[ 192.9 + 1340k + 1.599 \right. \times 10^7 k^2 + 1.78 \times 10^5 k^3 + 3.995 \times 10^3 k^4 \left. \right]^{1/2},
\]

normalized by imposing that the mass variance at \( 8h^{-1} \) Mpc is \( \sigma_8 = 0.63 \). For the ΛCDM model (\( \Omega_m = 0.3, \Omega_L = 0.7, h = 0.7 \)), I also use the BBKS spectrum normalized as \( \sigma_8 = 1 \). Supposing that energy is conserved, the shape of the density profile at maximum of expansion is conserved after the virialization, and is
given by (section 25 of Peebles 1980; HS; White & Zaritsky 1992)

$$\rho(r) = \rho_s \left( \frac{r}{r_s} \right)^3 \frac{\delta(r)}{\delta_s},$$

where $r_s$ and $\rho_s$ are the initial radius and the density respectively, while $\Omega(z)$ is the density parameter at epoch $z$. The final radius, $r$, and the initial one, $r_s$ are connected by

$$r = F r_m = F r_i \frac{1 + \tilde{\delta}}{\tilde{\delta} - (\Omega_i^{-1} - 1)},$$

where $r_m$ is the shell radius at maximum expansion, $F$ is given in DP2000 (equation 26) and the mean fractional density excess inside a given radius, $\delta$, is

$$\delta = 3 \int_0^r \frac{r}{\Omega(z)} dy.$$  

In order to calculate the evolution of $c = r_v/a$, I must calculate the inner radius $a$ and the virial radius, $r_v$. The inner radius, $a$, is characterized by the condition

$$\frac{d \log \rho(r)}{d \log(r)}|_a = -2,$$

while the virial radius, $r_v$, is the radius within which the mean overdensity is $\delta_v$ times the critical density, $\rho_c$, at that redshift.

$$M_v = \delta_v(z) \rho_c(z) \frac{4\pi}{3} r_v(z)^3,$$

where $M_v$ is the virial mass of the halo, and the critical density $\rho_c$ is

$$\rho_c(z) = \rho_c(1 + z)^3 + (1 - \Omega_0).$$

The subscript 0 indicates that the parameter is to be calculated at epoch $t_0$, $H_0$ is the Hubble constant at $t_0$ and $\rho_0 = 8 \pi G \rho_c/3 H_0^2$.

The virial overdensity, $\delta_v$, is provided by the spherical top-hat collapse model, which, for the family of flat cosmologies ($\Omega_0 + \Omega_\Lambda = 1$), gives

$$\delta_v(z) = (18 \pi^2 + 82 y - 39 y^2),$$

(Bryan & Norman 1998), where $y = \Omega(z) - 1$ and $\Omega(z)$ is

$$\Omega(z) = \frac{\Omega_0 (1 + z)^3}{\Omega_0 (1 + z)^3 + (1 - \Omega_0)}.$$  

In the limit $z \rightarrow \infty$, equations (16)–(19) give the corresponding quantities for the SCDM model.

### 3 RESULTS

In NFW96 and NFW97, the $N$-body simulations were interpreted by means of a model, a step-by-step calculation of the density profile that is also useful to calculate the mass and redshift dependence of the concentration parameter, $c$. This model assigns to each halo of mass $M_h$ identified at $z = z_0$ a collapse redshift, $z_c$, defined as the time at which half of the mass of the halo was first contained in progenitors more massive than some fraction $f$ of the final mass, $M_c$. Lacey & Cole (1993) showed that a randomly chosen mass element from a halo having mass $M_h$, identified at redshift $z_0$, was part of a progenitor with mass exceeding $f M_h$ at the earlier redshift $z$ with probability

$$P(M_h, z; M_c, z_0) = \text{erfc} \left( \frac{\delta_m(z_0) - \delta_m(z)}{\sqrt{2} \sigma_m^2(M_h) - \sigma_m^2(M_c)} \right),$$

where $\sigma_m^2$ is the linear variance of the power spectrum and $\delta_m(z)$ is the density threshold for spherical collapse at redshift $z$. The collapse redshift $z_c$ is determined setting $P = 1/2$. Assuming that the characteristic density of a halo is proportional to the density of the universe at the corresponding $z_c$ then we have (NFW97)

$$\delta_v(z_0) = C \rho_m(z_0) \left( \frac{1 + z_c}{1 + z_0} \right)^3.$$  

Given $M_c$ and $z_0$, it is possible to obtain $z_c$ from equation (20) and $\delta_v$ from equation (21), thus completely specifying the density profile.

The NFW97 model is in agreement with $N$-body simulations at $z = 0$, for several different cosmological models (NFW96; NFW97; DP2000; B99), but as shown by B99, it does not reproduce properly the redshift dependence of the halo profiles as seen in their simulation: it over-predicts the concentration, $c$, at early times, $z \approx 1$.

In DP2000, I showed that the improved SIM model, introduced in that paper, gives good results in predicting the shape of the dark halo profiles and the mass dependence of the concentration parameter, $c$, both in a SCDM model and in a scale-free universe. In that paper, I did not study the redshift dependence of the concentration parameter. Here, in order to answer this question, I calculated the evolution of $c$ for two different cosmologies, namely SCDM ($\Omega = 1, \ h = 0.5, \ n = 1$; $\sigma_8 = 0.63$) and $\Lambda$CDM ($\Omega_m = 0.3, \ \Omega_\Lambda = 0.7, \ h = 0.7, \ \sigma_8 = 1$). The results are plotted in Figs 1–3.

The solid line of Fig. 1 represents the expected behaviour for
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8 \times 10^{14} \, h^{-1} \, M_{\odot} haloes as predicted from the NFW97 model in a \Lambda CDM model. The dotted line with error bars represents the \( c(z) \) median as obtained in N-body simulations by B99, the Poisson error bars were obtained by the quoted authors from the profile fitting procedure: after identifying a centre for the halo, they count error bars were obtained by the quoted authors from the profile fitting procedure: after identifying a centre for the halo, they count for the values of the parameters, constant in time, \( f = 0.01 \) and \( C = 3 \times 10^5 \), defined in NFW96 and NFW97, while the dashed lines represent the predictions of B99 model for the values of the two parameters, constant as a function of both \( z \) and mass, \( F = 0.01 \), \( K = 3.8 \), defined in their paper. The filled hexagons represent the concentration \( c \) at \( z = 0 \) obtained by NFW96 and NFW97 N-body simulations.

\( \hat{c} \) was described by B99: in the present model the scale radius, \( a \), is roughly constant, then the \( z \) dependence of \( c = r_c/a \) comes from the virial radius \( r_v \), aside from the \( z \) dependence of \( \hat{\delta} \), both in the SCDM and \Lambda CDM model \( r_c \propto 1/(1 + z) \).

Previously, I showed that the model proposed in this paper, in agreement with claims by B99, predicts a different redshift dependence for the concentration \( c(z) \) with respect to NFW97. It is time to discuss the reasons behind the quoted discrepancy and, at the same time, the agreement of the results of the present paper with the B99 paper.

The different result of these two models with respect to NFW97 is fundamentally because of the different way of defining the collapse redshift, \( z_{\text{cr}} \): as previously reported in this section, NFW97 defines \( z_{\text{cr}}(M_c, f, z_0) \) as the time at which half the mass of the halo was first contained in progenitors more massive than some fraction \( f \) of the final mass, \( M_c \). Then the collapse redshift is obtained through the Press–Schechter formalism, equation (20) and finally the characteristic overdensity of the halo, \( \delta_c \) (which according to equation 2 is connected to the concentration, \( c \)), is assumed to be proportional to the density of the universe at \( z_c \). As a consequence of this way of defining \( z_{\text{cr}} \), the NFW97 prediction for \( c \) eventually goes to a constant value at high redshift, because \( z_{\text{cr}} \) becomes closer and closer to \( z_0 \), as can easily be found from equation (20). In other words, at sufficiently high redshift the halo collapse redshift, \( z_{\text{cr}} \), becomes essentially indistinguishable from the redshift, \( z_0 \), at which they are analysed and their concentrations tend to a constant.

Things are different with respect to the previous discussion, both in the present paper and in that of B99. In the present paper, I use the SIM, which, as it is well known, allows one to establish, for a given power spectrum, a relationship between the mass \( M \) and its formation (collapse) epoch, \( z_c \) (see Peebles 1980, section 19; Gunn & Gott 1972; Avila-Reese, Firmani & Hernandez 1998). In fact, given a density perturbation, a shell with initial comoving radius \( r_i \) and mass \( M = (4\pi/3)r_i^3(1 + \hat{\delta}) \), will expand to a maximum radius \( r_m \), given by equation (13) or by \( r_m / r_i = 1/\hat{\delta} \) in an Einstein–de Sitter universe, and then collapse will occur. The time of
maximum expansion in an Einstein–de Sitter universe is given by
\[ t_m = \frac{t_c}{2} = \frac{\pi}{2H_0\delta_c}, \]  
(22)

where \( t_c \) is the collapse time and \( \delta_c \) is the mean fractional density excess measured at time \( t_c \). In other words, equation (13) and equation (22) tell us that the collapse redshift \( z_c \) is a function only of the virial radius (and then of the virial mass, \( M_v \)), and is independent of \( z_0 \), in agreement with the assumption made by B99 in their equation (8). As a consequence, the condition \( z_c = z_0 \), encountered in the NFW97 model at high redshift, implying that \( c(z) \) tends to a constant value, is no longer present in the model of this paper. This implies that the slope of the relation \( c-z \) remains constant. In conclusion, \( c(z) \propto 1/(1+z) \) and \( c \) never tends to a constant. The same result is found in the B99 model. I also would like to remark that in the present model and in that of B99, it is possible, at high redshift, that \( z_c < z_0 \) if a halo is much more massive than the characteristic mass, \( M_v \) (we recall that, as shown in DP2000, for the adopted normalization of the CDM spectrum \( M_v = 3 \times 10^{13} \text{M}_\odot \)). This is a function only of mass and not on \( z_0 \). Hence haloes at high redshift are just beginning to form and are therefore more diffuse.

A possible question that may arise at this point is why the NFW97 model works well at \( z = 0 \) and not at higher redshift. In fact at \( z = 0 \), the NFW97 model correctly predicts the mass–density relation obtained from \( N \)-body simulations and the result is also in agreement with B99 and DP2000. The answer to the previous question is that the problem of NFW97, seen at \( z > 0 \), is no longer present at \( z = 0 \) because at this redshift the extended Press–Schechter formula that NFW97 used to determine \( z_c \) never gives \( z_0 = z_c \), so the problem of the wrong prediction of \( c(z) \), seen at high redshift, is no longer present at \( z = 0 \). In order to clarify further the previous discussion about the differences between the model of this paper and NFW97, I calculated the concentration \( c \) as a function of mass, \( M \), in a CDM model for different values of redshift in the case of the NFW97 model, in that of B99 and in the model of this paper. The result is shown in Fig. 2. The solid lines represent the prediction of the model of this paper at three different redshifts, \( z = 0, 2 \) and 4. The dotted lines represent the prediction of NFW97 for the quoted redshifts, while the dashed lines are the predictions of B99 model. The filled hexagons represent the concentration \( c \) at \( z = 0 \) obtained by NFW96 and NFW97 in their \( N \)-body simulations. Fig. 2 shows that for a fixed mass and at \( z = 0 \), the evolution of \( c \) as a function of mass \( M \) is, as is known, successfully predicted by the NFW97 model for the values of the parameters, constant in time, \( f = 0.01 \) and \( C = 3 \times 10^2 \), defined in NFW96 and NFW97. The B99 model (for the two parameters, constant as function of mass and \( f = 0.01 \), \( K = 3.8 \), see their paper for a definition) and the model of this paper reproduce the \( z = 0 \) results of NFW97 quite well over the range \( M/M_v = 0.01-100 \) (given as previously reported \( M_v = 3 \times 10^{13} \text{M}_\odot \)).

\(^1\) I recall that, as shown in DP2000, for the adopted normalization of the CDM spectrum \( M_v = 3 \times 10^{13} \text{M}_\odot \).

direct comparison of the three models, however, shows that the NFW97 model prediction is shallower than the other two. The same situation was present in the \( \Lambda \)CDM model, as shown in fig. 2 of B99. Also in this case, the NFW97-predicted slope for \( c \) is shallower than the B99 model which reproduces the simulation data better. At \( z = 2 \), the discrepancy between the NFW97 and the other two models is evident. Fig. 2 evidently shows that, going from \( z = 0 \) to \( z = 2 \), the slope of the relation \( c-M \), predicted by B99 and the model of this paper, remains roughly constant and moreover gives lower values of \( c \) with respect to the NFW97 model. The figure also shows that the NFW97 model for \( z = 2 \) has a smaller slope with respect to the \( z = 0 \) case. The situation now described is even more evident in the \( z = 4 \) case. This situation was expected because, as previously quoted in the precedent discussion, with increasing \( z \), \( z_c \to z_0 \), and one expects that \( c \) goes to a constant; in fact we see that the slope of the \( c-M \) relation predicted by the NFW97 models decreases going from \( z = 0 \) to \( z = 4 \). In other terms, the B99 model and the model of this paper predict that \( c(z) \propto 1/(1+z) \) and that the relation never tends to a constant for a particular high redshift value, which is the case for the NFW97 model. As previously quoted, the reason why the NFW97 model gives a successful description of the \( c-M \) relation in the case of \( z = 0 \) is because of the fact that in this case it never happens that \( z_c = z_0 \).

Finally, I want to remark that although the B99 model gives predictions for the \( c(M,z) \) relation in good agreement with \( N \)-body simulations and with the model of this paper, there is a fundamental difference between these two models. The B99 model makes the ‘ad-hoc’ assumption that \( z_c \) depends only on mass and not on \( z_0 \), in order to explain the results of the \( N \)-body simulations performed in the same paper. The model of the present paper is a direct consequence of SIM, without additive assumptions.

Now I want to discuss some consequences of the results obtained for \( c(z) \) on galaxy modelling at high redshift. The discussion is intended to gain only a qualitative understanding of how \( c(z) \) may affect some features of galactic formation (a more complete discussion is given in B99).

As pointed out by B99, one expects that the described behaviour of \( c(z) \) should have a large impact on galaxy modelling at high-redshift and for interpreting high-redshift data [e.g. evolution of the Tully–Fisher relation (Vogt et al. 1997) and the nature of Lyman break galaxies (Steidel et al. 1996)]. According to the standard picture of galaxy formation, structures grow hierarchically from small, initially Gaussian density fluctuations. Collapsed, virialized dark matter haloes condense out of the initial fluctuation field. Gas associated with such dark haloes cools and condenses within them, eventually forming galaxies. In this scenario, the growth of the dark matter haloes is not much affected by the baryonic components, but determines how they are assembled into non-linear units. The halo density profile determines many of the properties of galaxy discs, e.g. their size and surface brightness. In order to show the effect of \( c \) evolution on disc properties, it is possible to use a fitting formula given by Mo, Mao & White (1998) based on several assumptions about the halo and disc make-up and depending on some free parameters, or more easily the fitting formula introduced in B99 (their equation 7):
model, assuming that $M_c = 8 \times 10^{11} h^{-1} M_\odot$, as assumed also in Fig. 1. In both cases the lower concentration of haloes at high redshift produces a disc size larger than that obtained using the NFW97 model for the evolution of $c$. In fact, in the ACDM model, taking account of the $c-z$ dependence found in this paper, I find the short-dashed line, while the long-dashed line is obtained with the NFW97 $c-z$ dependence. A similar situation is found in the SCDM model: using the $c-z$ dependence of this paper, I obtained the dotted line, while using that of NFW97 I obtained the solid line. I want to stress that the last result must be taken with caution, because it strongly depends on the precision of the fitting formula and on its free parameters (if it contains any). In fact, comparing Figs 1 and 3, it is immediately evident that, while the concentration found in the model of the present paper differs from that predicted by NFW97 evolution quite a lot, the differences are reduced when we deal with $R_d$. If I had used the fitting formula of Mo et al. (1998) to predict the variation of $R_d$ with redshift, I should have found that small variations in the parameters of that model, at a level of 10 per cent, would have produced change in $R_d$ larger than that caused by the redshift evolution of $c$. Hence the validity of the last result, and that regarding the surface brightness, $I(z)$, described in the following, is entirely based upon the reliability of the B99 fitting formula.

As the surface brightness, $I(z)$, scales as

$$I(z) \propto R_d^{-2}(z)$$  \hspace{1cm} (24)

both in the SCDM and ACDM model, we may expect that this parameter is also modified by the strong evolution of $c$ with redshift. Using Fig. 3 and equation (24) it is easy to find that, according to the model of the present paper, discs are dimmer with respect to NFW prediction and that the effect increases with increasing $z$. Besides the two examples that I have just given, the strong evolution of the concentration parameter, $c$, affects several other properties of discs and, as previously reported, has some implications for galaxy modelling. For example, the shape of a disc rotation curve depends on, among other factors, the concentration, $c$, of its halo. As more strongly peaked curves are found in more concentrated haloes and evolution produces a reduction of the concentration of haloes, one expects that evolution tends to produce less peaked rotation curves. Further implications of the $c(z)$ dependence shown in this paper are discussed in B99. However, as stressed by the quoted author, much work remains to be done in order to have a deeper understanding of the implications of the results obtained in this paper and their paper on galaxy modelling.

4 CONCLUSIONS

In this paper I studied the evolution of the concentration parameter, $c(z)$, for fixed-mass haloes, in the SCDM and ACDM model by means of the improved SIM introduced in DP2000. The results of the paper can be summarized as follows.

(i) Both in the SCDM and ACDM model the evolution of $c(z)$ is much stronger than that expected from the NFW97 model: in the model of the present paper $c(z) \propto l/(1 + z)$, while the NFW97 model overpredicts the concentration $c$ by 50 per cent at $z = 1$ with respect to the model of the present paper, and by 40 per cent with respect to that of B99. NWF97 predicts that the concentration $c$ tends to a constant value for large $z$, so the quoted disagreement increases with increasing $z$.

(ii) A comparison of the results of the present paper with the high-resolution N-body simulations of B99 shows a good agreement both for the SCDM and ACDM model.

(iii) The different redshift dependence of $c(z)$, obtained in the present paper and B99, with respect to NFW97, is fundamentally because of the different way of defining the collapse redshift, $z_c$: as a consequence of the way of defining $z_c$ in NFW97, at sufficiently high redshift the collapse redshift of the haloes, $z_c$, becomes essentially indistinguishable from the redshift, $z_0$, at which the haloes themselves are analysed and their concentrations tend to a constant.

(iv) The present model has an advantage with respect to B99, namely B99 results are obtained assuming the ‘ad-hoc’ assumption that $z_c$ depends only on mass, and not on $z_0$, while the results of the present paper are direct consequence of SIM, without additive assumptions.

(v) The result has important consequences on galaxy modelling at high redshift and on some disc characteristics. In particular, the disc size obtained in the present model is larger than that obtained using the B99 fitting formula together with NFW prediction for $c(z)$. The reverse is true for the disc brightness.

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