Multi-diagnostic approach to geodesic acoustic mode study

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ABSTRACT: Multi-diagnostic approach developed for the GAM research in the spherical tokamak Globus M is described. Doppler backscattering (DBS) method as the tool for the GAM study, together with the diagnostics of plasma density and magnetic field GAM oscillations, were simultaneously used in experiments. The version of the DBS diagnostics with two cut-offs positioned at different poloidal angles of the minor cross-section was employed in Globus-M. For the GAM plasma density oscillation study, the $D_{\alpha}$ emission was observed at different angles to restore the spatial mode structure of the GAM plasma density oscillations. At the same time, the array of Mirnov coils was used for the GAM-like magnetic oscillation study, and that made it possible to restore the magnetic field perturbation spatial structure. The coherent and cross-bicoherence analyzes were employed to identify the interaction between the GAM velocity oscillation and plasma turbulent fluctuations.

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1 Introduction

Geodesic acoustic modes (GAMs) as a class of high-frequency zonal flows manifest themselves in tokamak plasma as localized flows directed on ExB drift in radial electric field [1]. The use of multi-diagnostic approach is mainly imposed by the fact that a lot of plasma parameters such as the level and spectra of plasma turbulence, zonal flows with different spectral characteristics, mean flows determined by the ion pressure gradient, magnetic field oscillations [2] and other plasma parameters involved in the process of GAM occurrence [3]. These parameters are of different spatial scales and mode numbers and can’t be studied simultaneously even by using only the universal DBS method. That is why the GAM investigations involve applying various diagnostics of plasma parameters.

Multi-diagnostic approach to the GAM study has been first employed on the T-10 tokamak [4] and then recently on the DIII-D [5] and TCV [6] tokamaks. In this paper we present a similar multi-diagnostic approach which was first applied for the GAM investigations in spherical tokamak. We have tried to emphasize, where it is necessary, the specificity of the diagnostic use in a tokamak with a small aspect ratio. A set of different GAM diagnostics is described, and some experimental data are given to illustrate the diagnostic capabilities. The main tool for the GAM flow study was Doppler backscattering (DBS) (the other name is Doppler reflectometry (DR) diagnostics [7]). The method of the rotational velocity deriving from the Doppler shift of the backscattered radiation at oblique incidence was first proposed and applied in the TUMAN-3M tokamak [8]. Then, a similar experiment was carried out on the Tore Supra tokamak [9]. The method was proposed to be called the Doppler reflectometry in the presentation of stellarator W7AS experiments [10]. The method
allowed to measure the ExB velocity oscillations and thus to reveal the GAMs. The peculiar ray tracing analysis of the incident beam was previously performed to satisfy the DBS specific requirements in a spherical tokamak characterized by a significant pitch angle at the low magnetic field side. The version of the DBS diagnostics with two cut-offs positioned at different poloidal angles of the minor cross-section was employed in Globus-M to study the spatial GAM velocity structure. The observations of the $D_\alpha$ emission along six lines of sight were successively employed to restore the poloidal mode structure of plasma density. The spatial structure of magnetic field oscillations has been investigated in the tokamak Globus-M with the usage of Mirnov coil array. In conclusion, the methodological aspects of the DBS diagnostic data usage for bispectral analysis have been discussed. The conditions under which the bispectral analyses actually reveal the nonlinear mutual influence of GAM and drift turbulence have been identified.

2 Set of GAM diagnostics on the Globus-M tokamak

Multi-diagnostic approach has been applied for GAM investigation in the spherical tokamak Globus-M ($R = 0.36\,\text{m}$, $a = 0.24\,\text{m}$). The GAM itself has been discovered in Ohmic heating discharge ($\langle n \rangle \approx (2–3) \cdot 10^{19} \,\text{m}^{-3}$, $I_p = 150\,\text{kA}$, $B_T = 0.4\,\text{T}$). GAM oscillations do not develop during the H-mode. When the toroidal ion drift is directed to the X-point the H-mode is mostly observed during the whole ohmic heating discharge except the initial phase of the current and plasma density rise. At this phase, Doppler reflectometry is not applicable due to deeper cut-off layer locations. Therefore, the opposite direction of the toroidal ion drift was chosen when L-H transition occurs during a quasi-stationary stage [11]. The minor cross-section of the Globus-M and the separatrix are shown in figure 1.

2.1 Doppler backscattering

The main tool for GAM studies in the Globus-M tokamak was Doppler microwave backscattering. The method is based on the backscattering of microwave radiation at oblique incidence in the presence of a cut-off layer for the probing beam. Backscattering is mostly localized near the cut-off layer. The scattering occurs only when the Bragg condition is fulfilled that is when the scattering wave vector $k$ is perpendicular to the magnetic field lines in the vicinity of the reflection region. If so, it is possible to determine the component of the fluctuation velocity $V_\perp$ in the direction of the diamagnetic or $E \times B$ drift in the radial electric field by measuring the Doppler frequency shift $\Delta \omega_D = k \cdot V = k_\perp V_\perp$ of backscattered radiation. There are some specific requirements for a DBS application in the spherical tokamak Globus-M which is characterized by significant pitch angle (over 20 degrees) at the low magnetic field side. Therefore, to fulfil the Bragg condition the antenna had to be tilted in both poloidal and toroidal directions.

If the Bragg condition is satisfied one can try to detect the GAM oscillation as an oscillation of the perpendicular velocity $V_\perp(t)$ or radial electric field (see figure 2 — $E_{DR}^r$). Moreover, the spectral power $|n(k_\perp, \omega)|^2$ of plasma density fluctuations for selected $k_\perp$-values can be estimated by measuring the backscattered power.
Figure 1. Main diagnostics for GAM detection and ray tracing of DBS measurements in a minor cross-section of Globus-M.  a-f are lines of sight for $D\alpha$ emission detection; numbered squares are Mirnov coils.

2.1.1 Adjustment of DBS to the specific geometry of the GLOBUS-M spherical torus

To evaluate the needed antenna tilt angles, the incident beam propagation had to be determined. It was performed in quasi-optical approximation. The ray tracing of incident beam was obtained for the actual 3D geometry of Globus-M tokamak using the full dispersion expression for anisotropic plasma with allowance for spatial dispersion.

$$H \left( N_\perp^2, N_\parallel^2, \nu, u \right) = 0;$$
$$\nu = \frac{\omega_{pe}}{\omega^2}, \quad u = \frac{\omega_{ce}}{\omega^2} \tag{2.1}$$

Here $N_\perp$ and $N_\parallel$ are the perpendicular and parallel projections of normalized wave vector. $\omega_{pe}$ and $\omega_{ce}$ are the electron plasma and cyclotron frequencies respectively. In our description, we are forced to apply a non-orthogonal system of coordinates for the case of pronounced elongation and triangularity of flux surfaces in the spherical torus.

$$x = R(\rho, \theta) \cos(\phi), \quad y = R(\rho, \theta) \sin(\phi), \quad z = Z(\rho, \theta) \tag{2.2}$$

where $\theta$ and $\phi$ are the poloidal and toroidal angles. The frame of reference was selected in the way that the poloidal flux $\psi_p$ and plasma density $n$ were determined by only one variable $\rho$. The functions $R(\rho, \vartheta)$ and $Z(\rho, \vartheta)$ were selected so that the coordinate lines at the constant $\rho$-values coincided with the flux surface cross-sections obtained from the EFIT code application. The functions $R(\rho \vartheta)$ and $Z(\rho \vartheta)$ are selected so that surfaces with $\rho = \text{const}$ coincide with the flux surfaces.
Figure 2. Normalized auto-power spectra of radial electric field extracted by DR $E^{\text{DR}}_r$ and Langmuir probes $E^{\text{LP}}_r$, GAM magnetic poloidal component $B^{\text{MP}}$, $D_\alpha$ emission signal and probe ion saturation current $I_{\text{sat}}$. The time interval for calculating all spectra is 1 ms.

Figure 3. Flux surface cross-sections. Red curves are closed flux surfaces, black ones are open surfaces. Blue curves are $\rho$-constant surfaces. Location of Doppler reflectometer antenna. Magenta curve is ray tracing.

Defined through EFIT. Therefore, in the cast the value $\rho = \text{const}$ determines the flux surface. An example of comparison with the flux surface cross-section is shown in figure 3. The needed $n(\rho)$ and $\psi_p(\rho)$ dependencies were extracted from the EFIT data. As a result, the equations for the ray tracing and the covariant normalized wave vector projections $N_1, N_2, N_3$ take the forms:

$$
\frac{d\rho}{ds} = -\frac{\delta H}{\delta N_1}, \quad \frac{d\theta}{ds} = -\frac{\delta H}{\delta N_2}, \quad \frac{d\phi}{ds} = -\frac{\delta H}{\delta N_3}
$$

(2.3)

Previously projections $N_\perp$ and $N_\parallel$ in the expression $H \left( N_\perp^2, N_\parallel^2, \nu, u \right)$ were rewritten through $N_1, N_2, N_3$ and projections of the contravariant vector of magnetic field $B^1, B^2, B^3$. The result of a solution of the set of equations (4) is the ray tracing along which the dispersion equation $H \left( N_\perp^2, N_\parallel^2, \nu, u \right) = 0$ is satisfied. The initial $N_\perp$ and $N_\parallel$ values were determined for low plasma density near the boundary of a discharge where different modes of microwave propagation can be separated. The computation allows us to estimate $k$ near the cut-off layer and the cut-off layer...
position. The number of calculations has been performed with the initial $k$-parallel closed to zero near the antenna mouth. In experiment, it can be achieved with the both poloidal and toroidal antenna inclinations. An example of the relevant $k_{\|}$ and $k_{\perp}$ dependencies on major radius are shown in figure 4. The calculations show that the ratio of $k$-parallel/$k$-perpendicular could change near the cutoff in the course of experiments within 25%. The dependence of the DBS signals on the antenna tilt in the toroidal direction was specifically verified by the method which was also used in the MAST tokamak [12]. It was found that the ratio change within 25% did not affect the level of backscattered radiation. This weak dependence is apparently associated with the final $k$-resolution of our reflectometer, which was estimated as 30%. Moreover, it was found that the estimated $k$-perpendicular in the cut-off region is very close to $k_{\perp}$, which was calculated with the usage of simple expiration for slab geometry — $k_{\perp} = 2k_0 \sin(\alpha)$ (figure 5). It is possible due to the small curvature of the flux surface near mid-plane at large elongation of the D-shaped cross-section of the Globus-M and as well due to small distance from antenna mouth and the cut-off when the magnetic shear still does not play any role. It is clear that differences in the $k$-values will increase with decreasing of the cut-off radius.

### 2.1.2 Poloidal correlation DBS

The version of the DBS layout with two microwave schemes with the cut-offs positioned at different poloidal angles ($-30^\circ$ and $45^\circ$) of the same minor cross-section was employed in Globus-M for the first time. Each reflectometer is based on a monostatic antenna scheme, which allows to probe the plasma by O-mode microwaves in the frequency band 20–36 GHz. Photograph of one of the steerable antenna horn is shown in figure 6. There is the possibility to change the incident frequency from shot to shot. The toroidal tilt angles for the both reflectometers were in the range of $2^\circ$–$5^\circ$.
and poloidal angles were in $6^\circ$–$9^\circ$ respectively. The relevant wavelength of the scattering plasma fluctuations was in the range 1.1–2.4 cm. The cut-off positions were in the vicinity of the separatrix at high $q$-values (in the interval of major radii $R$ from 53 cm to 58 cm). The radial resolution of the method was estimated to be about 0.5 cm. Dual homodyne detection [13] was employed to receive the backscattered radiation (figure 7). It was previously defined a set of the probing frequencies at which the needed phase difference in the IQ detector was $\pi/2$. Namely these frequencies can be specified from shot to shot. The sine and cosine signals of the quadrature (IQ) detector were used for calculation of complex signal spectrum and subsequent determination of the Doppler frequency shift as a center of gravity of the spectrum or as a phase derivative of the complex signal:

$$
S(\omega) = FT[I\cos(t) + iI\sin(t)],
$$

$$
\Delta f_{Dopp}(t) = (1/2\pi) \int \omega S(\omega)d\omega / \int S(\omega)d\omega I\cos(t) + iI\sin(t) = A(t) \exp[i\Phi(t)],
$$

$$
\Delta \omega_{Dopp}(t) = d\Phi(t)/dt
$$

(2.4)

Both approaches exhibited approximately the same temporal behavior of the Doppler frequency shift (figure 8). Here the phase derivative is smoothed over the time interval of 8 mks. The same time interval (8 mks) was used for complex signal spectral analysis in the “center of gravity” method. Some Doppler frequency shifts were estimated by fitting a Gaussian function to

![Figure 6. Photograph of the steerable antenna horn.](image)

![Figure 7. Microwave layout of dual homodyne Doppler reflectometer.](image)

![Figure 8. Doppler frequency shift calculated as center of gravity and phase derivative.](image)
the backscattered spectra. The detailed statistical analysis shows that the difference between the estimations obtained by these three methods in the GAM frequency band GAM was no more than 25%. This difference may be regarded as an error in the absolute estimation of the GAM amplitude.

The antennas and relevant ray tracing beams are schematically represented in figure 1. The two microwave schemes were designed to determine the phase relation between the GAM velocity oscillations at different poloidal angles. The location of the cut-offs on the same flux surface was controlled by the mean rotation velocity measurements. An example of the similar temporal behaviour of the mean velocities is demonstrated in figure 9 derived from the average rotation velocity measurements. In fact, the $E \times B$ velocity can be different on the same surface because of the poloidal dependence of the radial electric field due to the flux surface compression as well as an increase in the toroidal magnetic field while reducing of the major radius. Estimates of the impact of these effects were performed on the assumption that the electric potential is a flux surface value. The results of estimations are shown in figure 10 as a poloidal angle dependence of the distance $\Delta r$ between neighboring flux surfaces near the cut-off and the toroidal magnetic field. It is seen that the corresponding velocities for the poloidal angle $-30^\circ$ and $45^\circ$ differ by less than 2%.

The cross-spectral analysis has been performed to study a poloidal mode structure of the GAM velocity perturbation. The coherent spectrum $C(f)$ shown in figure 11 exhibits a maximum at the GAM frequency. The coherence spectrum in figure 11 was computed with averaging over 15

**Figure 9.** Temporal behavior of rotational velocities recovered from the different DB scheme data.

**Figure 10.** Magnetic field $B$ and effective distance between flux surfaces $\Delta r$ as functions of poloidal angle $\theta$.

**Figure 11.** Coherence spectrum $C(f)$ and cross-phase spectrum $\phi(f)$ of two Doppler shift oscillations recorded by reflectometers positioned at poloidal angles $-30^\circ$ and $45^\circ$ (#34504, time = 170 ms).
samples of 128 µs duration. The relevant cross-phase is equal to zero just at this frequency. The observation consistent with $m = 0$ mode structure of the GAM $E \times B$ flow predicted by theory.

2.2 Langmuir probe

Plasma density and electric field oscillations at the GAM frequency were detected (see figure 2 — $I_{\text{sat}}$ and $E_{LP}^{r}$) by a row of four Langmuir probes (figure 12) placed in the mid-plane of the tokamak at the low magnetic field side. Each probe head is equipped with cylindrical tip 2.5 mm in diameter and 2 mm in length. The high resistant to thermal loads nitride-bore isolators were used in construction of probes for providing measurements near the separatrix. There is a possibility to move the probes along the major radius in the range from $R = 63$ cm (wall) to $R = 57$ cm (1 cm inside the separatrix) from shot to shot. The probe row makes it possible to detect simultaneously both the plasma density and the radial electric field oscillations by measuring of ion saturation current and the floating potentials of radially separated probes. There are some probe application limitations inherent only in the Globus-M experiment. Firstly, even the Langmuir probes even with high resistance to thermal loads could not be used in the inner region at a distance of more than 1 cm from the separatrix. Secondly, the Langmuir probes do not allow us to define mode structure of the GAM which would be required to use the probe array. So, Langmuir probes were mostly employed in determining the phase relationship between the plasma density and $D_{\alpha}$ emission oscillations.

![Figure 12. Photograph of the Langmuir probes.](image)

2.3 $D_{\alpha}$ emission detection

The $D_{\alpha}$ emission oscillations at the GAM frequency were found on the tokamak Globus-M (see figure 2 — $D_{\alpha}$). The oscillations were treated as oscillations caused by the GAM density oscillations. Indeed, $D_{\alpha}$ emission could depend on plasma density, electron temperature and neutral density. However, there is no reason for the density of neutrality to oscillate at relatively high frequency of GAM (about 20–30 kHz). As for the electron temperature, there is no theoretical prediction of the electron temperature oscillations associated with the GAM development. Even if such oscillations occur, their impact on the $D_{\alpha}$ emission is expected to be very weak due to very weak dependence of the $D_{\alpha}$ emission on the electron temperature in the range of 50 to 100 eV. Therefore it is possible to assume that the $D_{\alpha}$ emission oscillations at the GAM frequency are mainly dependent on the plasma density oscillations with the same frequency. The oscillations of ion saturation current of
Langmuir probe at the GAM frequency were found to be in phase with the $D_\alpha$ emission oscillations. This fact is in line with the assumption made above. The relevant phase relation is seen in figure 13. The cross-phase spectrum and coherence spectrum were calculated with averaging over 15 samples of 256 $\mu$s duration.

The lines of sight of $D_\alpha$ emission detectors are schematically shown in the figure 1. Three vertical observation chords in the poloidal cross-section at toroidal angle $\phi = 0^\circ$ (a, b and c in figure 1) and three horizontal chords at toroidal angle $\phi = 22^\circ$ (d, e and f) and also one horizontal line of sight in the poloidal cross-section at toroidal angle $\phi = 180^\circ$ were used. Each $D_\alpha$ detector and its aperture are at a distance 60 mm from one another. The detector’s radius is 2.5 mm, the aperture’s radius is 5 mm. The distance from each detector to the vacuum chamber is 0.7 m. The $D_\alpha$ emission emerges mainly from the vicinity of the separatrix for the Globus-M conditions. The poloidal mode structure of plasma density oscillations with the GAM frequency was restored by the cross-spectral analysis of all mentioned above $D_\alpha$ signals. It was found that all signals oscillate in phase (figure 14). Thus we forced to conclude that the plasma density GAM perturbations have an $m/n = 0/0$ topology that is inconsistent with present theoretical predictions of an $m/n = 1/0$.

Possible nature of the occurrence of GAM plasma density fluctuations with $m = 0$, which were recorded as fluctuations in the radiation $D_\alpha$ emission, is discussed in [11].

### 2.4 Mirnov coils

To study the magnetic field oscillation associated with the GAM development poloidal and toroidal Mirnov coil arrays were used. The poloidal array of 28 Mirnov coils was placed in poloidal cross-section as shown in figure 1. Each probe is a pick-up coil 16 mm in height and 8.5 mm in diameter. The probe resistance is 9 $\Omega$, and the product of the effective probe area by the number of coil turns is $sw \approx 63 \text{ cm}^2$. The coils were tuned to measure the magnetic field fluctuations within a frequency band of up to 100 kHz. This poloidal Mirnov coil array was used in order to determine the poloidal number of the GAM perturbation through a reduced analysis of the recorded signals [14]. The frequency of the perturbation is determined from the Fourier spectrum of the probe signal (figure 2 — $B^{\text{MP}}$). The digital band filtration is applied to improve the signal-to-noise ratio. Since the probe signals are close to sinusoids over the time interval under study, the phase delay of the probe signal can...
be determined from the approximation of the recorded signal by a harmonic function. Figure 15a shows the reconstructed poloidal structure of the GAM perturbation for three moments of time separated by a quarter of the GAM oscillation cycle. The signals were normalized by their amplitudes, and therefore the presented values are varied in a ±1 interval. The magnetic field GAM perturbations have an \( m = 2 \) topology whereat the toroidal effect seems to lead to a drastic deformation.

**Figure 14.** Coherence spectra \( C(f) \) and cross-phase \( \phi(f) \) of the \( D_\alpha \) signals for lines of sight: (a) “a” and “b”, (b) “c” and “f”, (c) “c” and “d” (\#36124) and (\( n = 0 \)) for horizontal lines of sight at toroidal angles \( \phi = 0^\circ \) and \( \phi = 180^\circ \) (\#34504). Averaging over 15 samples of 128 \( \mu \)s duration.

**Figure 15.** Poloidal structure of GAM mode magnetic field oscillations. a — Normalized magnitudes of Mirnov coil signals at GAM frequency as a function of poloidal angle. Black curve — \( t_1 = 173.936 \) ms, blue curve \( t_2 = 173.946 \) ms, red curve — \( t_3 = 173.956 \) ms. \( t_3 - t_1 = 0.5/f_{\text{GAM}} \). Ohmic heating discharge \#33203. b — Relative phase angular \( \Delta \phi() \) distribution of magnetic field fluctuations at GAM frequency.
Since the magnetic field oscillations at GAM frequency show a standing wave feature the statistical method was used as well to reconstruct its poloidal structure. The phase differences at GAM frequency between the signal of Mirnov coil positioned at 180° poloidal angle and the signals of all other Mirnov coils were measured. The phase differences were found as the values of cross-spectra phase at GAM frequency for each pair of signals. Statistically averaged cross-spectra were calculated over 63 samples of 256 µs duration picked out with the step of 128 µs. Figure 15b shows the relative phase angular distribution of magnetic field fluctuations at GAM frequency. Radial coordinate of polar diagram is a phase difference; inner circle corresponds to phase difference $-\pi$ rad, outer dashed circle corresponds to phase difference 0 rad. Similarity of two methods is clearly visible.

3 Bicoherent analysis at Doppler backscattering diagnostics

It is widely believed that GAM, as a high frequency kind of zonal flows, develops due to Reynolds stress provided by turbulent small-scale fluctuations. On the other hand, intensive localized GAM $E \times B$ velocity oscillations can suppress the plasma turbulence through sheared rotation. These two types of mutual influence of GAM and drift turbulence can be interpreted as three-wave interaction which, in turn, can be extracted via bicoherent analysis. The cross-bicoherence as a measure of phase coupling between three spectral components of signals is described by the following expressions:

$$b^2(f_1, f_2) = \frac{\left| \langle Y^*_k(f_3)Y_i(f_1)Y_j(f_2) \rangle \right|^2}{\langle |Y_k(f_3)|^2 \rangle \langle |Y_i(f_1)Y_j(f_2)|^2 \rangle}; \quad f_3 = f_1 \pm f_2$$

(3.1)

where $Y_i(f), Y_j(f), Y_k(f)$ are spectra of different diagnostic signals. The summed cross-bicoherence $b^2_1(f_2)$ used to reveal the interaction of certain harmonic $f_2$ with all other spectral components is as follows:

$$b^2_1(f_2) = \frac{1}{s(f_2)} \sum_{f_1} b^2(f_1, f_2)$$

(3.2)

where $s(f_2)$ is the number of summands.

It was found in the FT-2 tokamak with the using of enhanced microwave backscattering technique [15] that the cross-bicoherence $b^2(f_1, f_2)$ plot has a clearly visible values at the GAM frequency that significantly exceeds the statistical significance level 1/M (M is the number of realizations for averaging) when the cross-bicoherent spectrum was computed for the following combination of the spectra: $Y_i(f) = Y_k(f) = Y_{BS}(f)$ is the spectrum of sine (or cosine) signal of the IQ detector, $Y_j(f) = Y_{E \times B}(f)$ is the spectrum of $E \times B$ velocity recovered from the Doppler frequency shift. Nevertheless, this positive result does not mean that the mutual nonlinear influence of GAM and drift turbulence has been discovered due to strong modulation of the phase of complex signal. It can simply indicate the presence of velocity fluctuation at GAM frequency. Indeed, the sine and cosine signals in this case contain high frequency components which phases are coupled with $E \times B$ velocity signal component at GAM frequency that may cause the significant values of bicoherence for such combination of signals.
3.1 Simple simulation of DBS data

To confirm this assumption and to find out what signals of DBS can be used for indication of nonlinear processes in plasma a simple simulation of the Doppler backscattering was carried out. The actual scattering fluctuations were substituted by a random 2D pattern. The example of simulated fluctuation pattern is shown in figure 16. This random pattern changed in time so that its correlation time was $2 \mu s$. This turbulent media moved with poloidal velocity $V(t)$ including the GAM velocity fluctuations. Subsequently this moving plasma density pattern was used to synthesize IQ detector signals [16]:

$$I(t) = \int \delta n(r, \theta, t) W(r, \theta) dr d\phi$$

(3.3)

where $W(r, \theta)$ is a complex spatial weighting function, $\delta n(r, \theta, t)$ is the density fluctuation and $r$ and $\theta$ are the radial and poloidal variables. The real part of simulated weighting function is shown in figure 17. The validity of the model was checked by comparison of simulated and experimental DBS signals spectra shown in figure 18. Model of density fluctuations does not include any three-wave nonlinear interactions, so the appearance of model bicoherence at the GAM frequency would mean that the selected combination can not be used to identify the nonlinear interaction.

3.2 Model and experimental results of bicoherent analysis

Certainly, there were no imitations of nonlinear processes of mutual interaction between the GAM and drift turbulence in the model. And nevertheless, the cross-bicoherent spectra computed for the same combination of synthesized signals $Y_i(f) = Y_k(f) = Y_{BS}(f), Y_j(f) = Y_{E \times B}(f)$ occurs to be similar to the bispectra obtained by experimental data [15] (figure 19a,b). On the other side the
Figure 19. Experimentally measured (a) and simulated (b) cross-bicoherence for spectral combination \( Y_1(f) = Y_2(f) = Y_{BS}(f) \), \( Y_3(f) = Y_{E\times B}(f) \): (c) simulated cross-bicoherence for spectral combination \( Y_1(f) = Y_2(f) = Y_{E\times B}(f) \), \( Y_3(f) = Y_A(f) \); (d) 2-D plot of cross-bicoherence spectrum \( b^2(f_1, f_2) \) measured in Globus-M tokamak for spectral combination \( Y_1(f) = Y_2(f) = Y_{E\times B}(f) \), \( Y_3(f) = Y_A(f) \). #34503

modelling have shown (figure 19c) that there were no emphasized values at GAM frequency for the combination of the spectra: \( Y_1(f) = Y_2(f) = Y_{E\times B}(f) \), \( Y_3(f) = Y_A(f) \) that is the spectrum of amplitude of the complex output signal of the IQ detector. The calculation of bicoherence with the use of experimental data was performed for this combination of spectra. 2-D plot of cross-bicoherence spectrum \( b^2(f_1, f_2) \) and summed cross-bicoherence measured in Globus-M tokamak are shown in figure 19d and figure 20, respectively. Bicoherence is evidently seen for experimental data whereas it is absent in modeling. The small difference in the amplitude of the IQ signals, and the slight difference of \( \pi/2 \) phase shift in the IQ detector could appear in the experiment. Therefore the bicoherence has therefore been computed with given artificially unequal simulated IQ signals and the phase is not \( 90^\circ \). These calculations have shown that the bicoherence appeared for the combination \( Y_1(f) = Y_2(f) = Y_{E\times B}(f) \), \( Y_3(f) = Y_A(f) \) only when the difference in the levels of IQ signals was more than three times. Bicoherence also appeared only when the phase was nearing \( 0^\circ \) or \( 180^\circ \). Therefore, the appearance of cross-bicoherence in the experiment was not due to...
Figure 20. Summed bicoherence ($b_2^2(f_2)$) in Globus-M tokamak. The statistical significance level is $1/M = 0.0053 \ #34503$

the difference in the amplitude of the IQ signals and possible slight phase difference from $\pi/2$. Discovered bicoherence was apparently associated with modulation amplitude of backscattered radiation at a frequency of GAM. In this regard, it is noteworthy that a significant level of auto-bicoherence for the oscillations of the reflected signal of conventional reflectometer was observed at the GAM frequency in a recent experiment on the FT-2 tokamak [17]. This remarkable observation shows that the level of turbulent low-k fluctuations is modulated at the GAM frequency and it may derive from turbulence suppression caused by the GAM sheared flows. Maybe, this modulation manifests itself in our cross-bicoherence analysis.

4 Conclusion

In summary, we have shown that the multi-diagnostic application is very efficient for the GAM phenomena investigations. It is the combination of different methods, which makes it possible to determine the phase relationship between the GAM oscillations of various plasma parameters and restore spatial structure of GAM oscillations. It has been found that the plasma velocity, density and magnetic field oscillations at the GAM frequency have $m = 0, m/n = 0/0$ and $m = 2$ topology respectively. The application of the Doppler backscattering as main method of the GAM study required a detailed prior consideration of the incident wave propagation in the complex geometry of spherical tokamak. A method of simple DBS simulation was proposed, which allows to determine the possibility of bicoherent analysis to reveal mutual influence of the GAM and drift turbulence.

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References

[1] P.H. Diamond et al., Zonal flows in plasma — a review, Plasma Phys. Control. F. 47 (2005) R35.
[2] C. Wahlberg, Low-frequency magnetohydrodynamics and geodesic acoustic modes in toroidally rotating tokamak plasmas, Plasma Phys. Control. F. 51 (2009) 085006.
[3] A. Fujiwara, A review of zonal flow experiments, Nucl. Fusion 49 (2009) 013001.
[4] A.V. Melnikov et al., Investigation of geodesic acoustic mode oscillations in the T-10 tokamak, Plasma Phys. Control. F. 48 (2006) S87.
[5] G. Wang et al., Multi-field characteristics and eigenmode spatial structure of geodesic acoustic modes in DIII-D L-mode plasmas, Phys. Plasmas 20 (2013) 092501.
[6] C.A. de Meijere et al., Complete multi-field characterization of the geodesic acoustic mode in the TCV tokamak, Plasma Phys. Control. F. 56 (2014) 072001.
[7] G. D. Conway et al., Direct measurement of zonal flows and geodesic acoustic mode oscillations in ASDEX Upgrade using Doppler reflectometry, Plasma Phys. Control. F. 47 (2005) 1165.
[8] V.V. Bulanin, D.O. Korneev, V.A. Rozhansky and M. Tendler, Microwave backscattering in Ohmic H-mode discharge in Tuman-3 tokamak, in Proceedings of 22nd EPS Conference on Controlled Fusion and Plasma Physics, Bournemouth, 19C, Part II (1995) pp. 89-92.
[9] X.L. Zou, T.F. Seak, M. Paume, J.M. Chareau, C. Bottereau and G. Leclert, Poloidal Rotation Measurement in Tore Supra by Reflectometry, in Proceedings of the 26th EPS Conference on Controlled Fusion and Plasma Physics, Maastricht, 23J (1999) p. 1041.
[10] M. Hirsch, E. Holzhauer, J. Baldzuhn, B. Kurzan and B. Scott, Doppler reflectometry for the investigation of propagating density perturbations, Plasma Phys. Control. F. 43 (2001) 1641.
[11] A.Y. Yashin et al., Geodesic acoustic mode observations in the Globus-M spherical tokamak, Nucl. Fusion 54 (2014) 114015.
[12] J.C. Hillesheim et al., Doppler backscattering for spherical tokamaks and measurement of high-k density fluctuation wavenumber spectrum in MAST, Nucl. Fusion 55 (2015) 073024.
[13] V.V. Bulanin et al., Study of plasma fluctuations in the Tuman-3m tokamak using microwave reflectometry with an obliquely incident probing beam, Plasma Phys. Rep. 26 (2000) 813-819.
[14] M.I. Patrov et al., Diagnostics of MHD instability in the Globus-M spherical tokamak, Plasma Phys. Rep. 33 (2007) 81-90.
[15] A.D. Gurchenko et al., Spatial structure of the geodesic acoustic mode in the FT-2 tokamak by upper hybrid resonance Doppler backscattering, Plasma Phys. Control. F. 55 (2013) 085017.
[16] V.V. Bulanin et al., Spatial and spectral resolution of the plasma Doppler reflectometry, Plasma Phys. Rep. 32 (2006) 47.
[17] A.D. Gurchenko et al., Turbulence and anomalous tokamak transport control by Geodesic Acoustic Mode, Europhys. Lett. 110 (2015) 55001.