Study on a combined prediction method based on BP neural network and improved Verhulst model

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ABSTRACT
The cost prediction is an important part of macroeconomic prediction. The fitting degree of the prediction model not only directly influences the prediction accuracy but also determines the effectiveness of the information provided to the decision maker, especially in the defense economy area. This paper uses the model improving method of reverse prediction and makes the best of the advantage of the Verhulst model of reverse prediction which can solve the problem of ‘small sample, uncertainty’ for the prediction of the defense expenditure. Based on the establishment of a reverse prediction Verhulst model which corrects the initial value, the BP neural network and the combined prediction model based on the BP neural network and improved Verhulst model is further established from the residual sequence dimension with the introduction of the BP neural network, and the analysis validation is conducted through the comparison to the Verhulst-BP model established based on the residual sequence of the traditional Verhulst model. China defense expenditure data collected is tested by practice, showing that this combined prediction model can improve the prediction accuracy greatly.

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Gray Verhulst model; neural network; reverse prediction; defense expenditure prediction

1. Introduction
The grey model is a kind of prediction model established on the grey theory and has been widely used in the study of uncertainties such as small samples, less data, and poor information (Dai, Gao, & Zhou, 2016; Xie & Liu, 2008). As one kind of the grey nonlinear models, the Verhulst model is mainly used to describe the process in saturation state, namely, the ‘S’ type process, and is widely applied to the cost prediction, population prediction, species reproduction prediction and product economic life prediction, solving many practical problems (Chen & Zhang, 2010; Dang, Yao, Cui, & Xiong, 2012; Wang & Wang, 2011; Yan, Liu, Liu, & Zhao, 2014; Zhang & Peng, 2016). Now many scholars at home and abroad have made lots of improvement on the traditional Verhulst model and the reverse prediction method is the correction of the traditional Verhulst model with the use of the reverse prediction method. After macroeconomic data verification, the fitting accuracy is significantly improved. The model correction method of the reverse prediction is to obtain the \( n+1 \) (or \( n+j, j \leq n \)) prediction values first by using the traditional Verhulst model for the prediction and at this time, the initial value selected by this model is the first value of the original data. Then it uses the obtained \( n+1 \) (or \( n+j, j \leq n \)) predicted values and \( n-1 \) actual values for reverse prediction to get the initial values and the corrected initial value can be obtained through the weighted array of the predicted initial values and the initial values of the original data.

However, for a certain actual prediction problem, due to the different modelling mechanisms and information presentation angles of various models, the economic prediction conclusions can only reflect the data information collection of a certain aspect, but cannot reflect the complete information of the prediction problem (Liu, Huang, & Zhang, 2017), showing certain limitations. While the combined model can make full use of the known information from multiple dimensions to solve this problem better. Therefore, the combined prediction model has been widely used in many fields such as transportation (Cheng, Liu, Lu, & Song, 2017), stock analysis (Wu & Feng, 2016), economic prediction (Liu, Huang, & Zhang, 2018), gas load (Chen, He, & Chen, 2019) and so on.

In this paper, the residual sequence is established with the use of Verhulst model of inverse prediction and the actual data, and the BP neural network is introduced from the residual sequence dimension. The combined prediction model based on BP neural network and improved Verhulst model can be established through the fitting prediction of the residual sequence then the advantage
that the grey model can solve the problems of small samples and uncertainties and the advantage that the neural network uses lots of neurons to establish the relationship for the solution of the non-linearity can be combined fully. Through the verification analysis of actual defense expenditure data, it can be found that this model can further improve the fitting accuracy to provide effective support for prediction.

2. Verhulst model of reverse prediction

The Verhulst model is a special case that the parameter \( \alpha = 2 \) in the power model (Deng & Deng, 2002; Wang, Wu, & Zhang, 1998). This model is a nonlinear model proposed by German mathematician and biologist Verhulst according to the change law of biological reproduction over time and is mainly applied to the data with saturation state process, that is 'S' type data. We can get highly reliable prediction results by this model with less data.

Power model GM(1, 1, \( \alpha \)) expression is:

\[
x^{(0)}(k) + ax^{(1)}(k) = b(z^{(1)}(k))^{\alpha}
\]  

(1)

According to the least-squares method, through the parameter sequence \( \hat{a} = [a, b]^T \) of the power model GM(1, 1, \( \alpha \)), we can get:

\[
\hat{a} = (B^T B)^{-1} B^T y
\]  

(2)

\[
B = \begin{bmatrix}
-z^{(1)}(2) & (z^{(1)}(2))^\alpha \\
-z^{(1)}(3) & (z^{(1)}(3))^\alpha \\
\vdots & \vdots \\
-z^{(1)}(n) & (z^{(1)}(n))^\alpha
\end{bmatrix}, \quad Y = \begin{bmatrix}
x^{(0)}(2) \\
x^{(0)}(3) \\
\vdots \\
x^{(0)}(n)
\end{bmatrix}
\]  

(3)

The whitening equation of the power model GM(1, 1, \( \alpha \)) is:

\[
dx^{(1)} + a x^{(1)} = b(x^{(1)})^{\alpha}
\]  

(4)

When \( \alpha = 2 \), GM(1, 1, 2) is the Verhulst model. Through the solution to the Siluet equation of the Verhulst model, we can get:

\[
x^{(1)}(t) = \frac{1}{e^{at}[1/x^{(1)}(0) - (b/a)(1 - e^{-at})]}
\]  

(5)

Then the time response equation of the Verhulst model is:

\[
\hat{x}^{(1)}(k + 1) = \frac{a x^{(0)}(1)}{b x^{(0)}(1) + [a - bx^{(0)}(1)] e^{ak}}
\]  

(6)

\[
\hat{x}^{(0)}(k + 1) = \hat{x}^{(1)}(k + 1) - \hat{x}^{(1)}(k)
\]  

(7)

The reverse prediction makes most of the model prediction value, and corrects the traditional Verhulst model by being substituted into the original model inversely together with other original data. This paper uses the idea to correct the initial value with the reverse prediction to establish the Verhulst-BP model and the followings describe the reverse prediction mechanism briefly:

Definition Set \( \hat{x}^{(0)}_1 \) as a reverse prediction original sequence

\[
\hat{x}^{(0)}_1 = (x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n))
\]  

(8)

Step 1. Using the traditional Verhulst model to fit the original data with \( n \) terms to predict that

\[
\hat{x}^{(0)}(n + 1)
\]  

Step 2. Establishing reverse prediction (9) (10)

\[
\text{Set } x^{(0)}_1(k) = x^{(0)}(n + 2 - k), k = 2, 3, \ldots n. \quad \text{(9)}
\]

\[
\text{Set } x^{(0)}_1(1) = \hat{x}^{(0)}(n + 1) \quad \text{(10)}
\]

Step 3. Establishing a Verhulst grey model for the newly established first reverse predicted original sequence \( x^{(0)}_1 \) to get \( \hat{x}^{(0)}_1(n + 1) \):

Step 4. The first reverse prediction corrected initial value \( C^1 \) can be obtained through the weighted array of \( x^{(0)}_1(1) \) and \( \hat{x}^{(0)}_1(1) \) as:

\[
C^1 = \eta x^{(0)}_1(1) + (1 - \eta)\hat{x}^{(0)}_1(1),
\]

\[
\eta \text{ is the adjustment coefficient} \quad \text{(11)}
\]

Step 5. Precision test

Using the corrected initial value to establish an improved grey Verhulst model, we can get the fitted numerical sequence as

\[
\hat{x}^{(0)}_{C^1}(1), \hat{x}^{(0)}_{C^1}(2), \ldots, \hat{x}^{(0)}_{C^1}(n).
\]

(12)

The fitting accuracy of the new model can be judged according to different accuracy-test indicators. If the requirements are met, use this model for the prediction. Or carry out the next reverse prediction until get the initial value \( C^0 \) that meets the accuracy requirement after \( n \) iterations before using this model for the corresponding predictions.

3. The BP neural network model based on the traditional Verhulst residual sequence

The artificial neural network is a kind of artificial network composed by a large amount of simple information-processing elements called neurons or nodes. The multi-node model and its error back propagation (Back Propagation, BP) algorithm is one of the mature and widely used artificial neural networks and algorithms (Qiu & Li, 2017; Wang, Dong, & Wang, 2017). The BP neural network
model consists of three layers, namely, the input layer, the hidden layer, and the output layer (see Figure 1). Among them, the hidden layer can consist of one or more layers and the specific number of layers shall be determined according to data debugging. The model can summarize the law from mass data and use input data to establish the corresponding relationship with the output data. Meanwhile, this method uses the information highly related to variables themselves to reach the expected value of the data. The advantage is that it can establish the corresponding relationship with the use of the attribute or connotation of the provided data variables themselves to reach the expected value of the data. Meanwhile, this method uses the information highly related to variables themselves to reach the expected value of the data. The advantage is that it can establish the corresponding relationship with the use of the attribute or connotation of the provided data variables themselves to reach the expected value of the data. The advantage is that it can establish the corresponding relationship with the use of the attribute or connotation of the provided data.

Definition: The residual sequence of the traditional Verhulst model is $E^{(0)}$

$$E^{(0)} = (e^{(0)}(1), e^{(0)}(2), \ldots, e^{(0)}(n))$$

(13)

Step 1. Using the traditional Verhulst model to calculate the analog value $\hat{x}^{(0)}(i)$ of time $i$

Using the whitening equation of Verhulst model:

$$\frac{dx^{(1)}(1)}{dt} + ax^{(1)} = b(x^{(1)})^2$$

(14)

Restore the solutions $\hat{x}^{(1)}(i)$, $\hat{x}^{(1)}(i - 1)$ and obtain the analog value

$$\hat{x}^{(0)}(i) = \hat{x}^{(1)}(i) - \hat{x}^{(1)}(i - 1)$$

(15)

Step 2. Establishing a residual sequence

The difference between the original data $x^{(0)}(i)$ of the time $i$ and the analog value $\hat{x}^{(0)}(i)$ of the traditional Verhulst model is called the residual of the time $i$

$$e^{(0)}(i) = x^{(0)}(i) - \hat{x}^{(0)}(i)$$

(16)

Step 3. Establishing BP artificial neural network model for the residual sequence

Set $E^{(0)}$ as the residual sequence. If the predicted order is $Q$ then use $e^{(0)}(i - 1), e^{(0)}(i - 2), \ldots, e^{(0)}(i - Q)$ to predict the value of the time $i$, that is, to use the values of $e^{(0)}(i - 1), e^{(0)}(i - 2), \ldots, e^{(0)}(i - Q)$ as the input values of the BP neural network and $e^{(0)}(i)$ as the expected value of the BP neural network. The adaptive learning training is performed with a series of samples to achieve the expected error requirements that people allow and the trained BP neural network can have the residual prediction.

Step 4. Predicting residual sequence

Outputting the predicted residual values $\hat{e}^{(0)}(i)$, $i = 1, 2, \ldots, n$ of the trained BP neural network, and establishing the prediction residual sequence

$$\hat{E}^{(0)} = (\hat{e}^{(0)}(1), \hat{e}^{(0)}(2), \ldots, \hat{e}^{(0)}(n))$$

(17)

Step 5. Constructing the traditional Verhulst-BP neural network predicted value sequence $\hat{x}^{(0)}_{BP}$

$$\hat{x}^{(0)}_{BP} = (\hat{x}^{(0)}_{BP}(1), \hat{x}^{(0)}_{BP}(2), \ldots, \hat{x}^{(0)}_{BP}(n))$$

(18)

where,

$$\hat{x}^{(0)}(i) = \hat{x}^{(0)}_{BP}(i) + e^{(0)}(i)$$

(19)

$\hat{x}^{(0)}_{BP}(i)$ is the predicted value of the Verhulst-BP neural network model.

4. Combined prediction based on BP neural network and improved Verhulst model

Definition: The Verhulst model residual sequence of the reverse prediction is $E^{(0)}_{C}$

$$E^{(0)}_{C} = (e^{(0)}_{C}(1), e^{(0)}_{C}(2), \ldots, e^{(0)}_{C}(n))$$

(20)

Step 1. Establishing the Verhulst model for the corrected initial value of the reverse prediction according to Equations (12)–(16) above.

Step 2. Using Verhulst model of inverse prediction $n$ to calculate the analog value $\hat{x}^{(1)}_{C}(i)$ of time $i$

Using Equation (18) to find the solutions of $\hat{x}^{(1)}_{C}(i)$, $\hat{x}^{(1)}_{C}(i - 1)$, and restore them to obtain the analog value

$$\hat{x}^{(0)}_{C}(i) = \hat{x}^{(1)}_{C}(i) - \hat{x}^{(1)}_{C}(i - 1)$$

(21)

Step 3. Establishing a residual sequence $E^{(0)}_{C}$

The difference between the original data $x^{(0)}(i)$ of the
Figure 2. Modelling solutions.

Table 1. Actual military expenditure of China from 2009 to 2016.

| Number | Year | Military expenditure (One hundred million yuan) |
|--------|------|-------------------------------------------------|
| 1      | 2009 | 4951.1                                          |
| 2      | 2010 | 5333.37                                         |
| 3      | 2011 | 6027.91                                         |
| 4      | 2012 | 6691.92                                         |
| 5      | 2013 | 7410.62                                         |
| 6      | 2014 | 8289.5                                          |
| 7      | 2015 | 9087.84                                         |
| 8      | 2016 | 9765.84                                         |

Similarly, using the residual sequence to establish the neural network and after the completion of the training, this BP neural network can have the residual prediction.

Step 5. Predicting residual sequence

Outputting the predicted residual values \( \hat{e}^{(0)}(i) \), \( i = 1, 2, \ldots, n \) of the trained BP neural network, and establishing the prediction residual sequence

\[
\hat{e}_C^{(0)} = (\hat{e}_C^{(0)}(1), \hat{e}_C^{(0)}(2), \ldots, \hat{e}_C^{(0)}(n))
\]  

Step 6. Constructing the Verhulst-BP neural network predicted value sequence \( \hat{X}_{BPC}^{(0)} \) of the reverse prediction

\[
\hat{X}_{BPC}^{(0)} = (\hat{X}_{BPC}^{(0)}(1), \hat{X}_{BPC}^{(0)}(2), \ldots, \hat{X}_{BPC}^{(0)}(n))
\]  

where,

\[
\hat{X}_{BPC}^{(0)}(i) = \hat{x}_{BPC}^{(0)}(i) + \hat{e}_{BPC}^{(0)}(i)
\]  

\( \hat{X}_{BPC}^{(0)}(i) \) is the predicted value of the Verhulst-BP neural network mode of the reverse prediction (Figure 2).
5. Example verification

The study on defense expenditure has been the classical issue in the defense economy area. As we can find through the development of human history that the national defense construction and economical construction are very important for a country and the two are indispensable (Jia, Zeng, Zeng, & Yang, 2018). With the national economic development, the overall trend of defense expenditure has been gradually improved and is basically meets the data requirements of the Verhulst model. Therefore, in order to further test the accuracy of macroeconomic cost prediction from the combined prediction model based on the BP neural network and improved Verhulst model and better analyze the China’s economy or budget development trend, in this paper, the military expenditures (see Table 1) from 2009 to 2016 in China have been taken as the samples (all data are from the National Bureau of Statistics) to analyze and verify the prediction accuracy of the combined model through the comparison of the traditional Verhulst model, the traditional Verhulst-BP model and the combined prediction model based on BP neural network and improved Verhulst model.

Substituting the original data into the traditional Verhulst model (set as the model I) to obtain the Table 2:

| Number | Year | Original data | Fitting value | Absolute error | Relative error | Posterior checks |
|--------|------|---------------|---------------|---------------|---------------|-----------------|
| 1      | 2009 | 4951.10       | 4951.10       | 0.00          | 0.000%        |                 |
| 2      | 2010 | 5333.37       | 5530.06       | 196.69        | 3.688%        |                 |
| 3      | 2011 | 6027.91       | 6153.00       | 125.09        | 2.075%        |                 |
| 4      | 2012 | 6691.92       | 6817.94       | 126.02        | 1.883%        |                 |
| 5      | 2013 | 7410.62       | 7521.70       | 111.08        | 1.499%        |                 |
| 6      | 2014 | 8289.50       | 8259.87       | 29.63         | 0.357%        |                 |
| 7      | 2015 | 9087.84       | 9026.86       | 60.98         | 0.671%        |                 |
| 8      | 2016 | 9765.84       | 9816.02       | 50.18         | 0.514%        |                 |
| Mean absolute error |                   | 87.46         |               |               |                |
| Mean relative error |                   | 1.336%        |               |               |                |
| Posterior checks |                   |               |               |               | 0.0510         |

This paper sets the Verhulst-BP neural network prediction order $Q = 3$, that is, uses the fitting residual data from 2009 to 2011 as the input value and the residual of 2012 as the expected value. By analogy, we can finally get the fitted values of the defense expenditures from 2012 to 2016. Then we can establish a three-layer BP neural network with three neurons in the input layer and one in the output layer. The fitting result obtained by repeated test is the best when the hidden layer includes 8 layers. So the topology of the network is $3 \times 8 \times 1$. The parameters of the BP neural network are: training function as trainlm, transfer function as tansig, the maximum network trainings as 1000, the training error as 0.0001, and the rest as default values. The BP neural network is used to establish the BP neural network model (set as model II) based on the traditional Verhulst residual sequence and the combined prediction model (set as model III) based on BP neural network and improved Verhulst model and the fitting data is processed to obtain the Table 3 and Figure 3 as follows:

Two models, namely model II and III are respectively established for the defense expenditure data from 2012 to 2016 for the fitting, which can be seen from Table 3 and Figures 3 and 4:

(1) For the data from 2012 to 2016, the fitting accuracy of the models II and III with BP neural network is
obviously better than the traditional Verhulst model I without BP neural network, and the fitting accuracy is improved more than half from the three indicators $\beta_1$, $\beta_2$ and $\beta_3$, especially, decreasing from 0.0510 of model I to 0.0288 and 0.0181 of model II and III.

(2) The $\beta_1$, $\beta_2$ and $\beta_3$ of the combined prediction model III based on BP neural network and improved Verhulst model are smaller than those of the BP neural network model II and traditional Verhulst model I based on the traditional Verhulst residual sequence. Among them, the index $\beta_1$ is 66.13% lower than that of the model I, and is 35.01% lower than that of the model II; the index $\beta_2$ is 71.71% lower than that of the model I, and is 33.33% lower than that of the model II. The index $\beta_3$ is 64.51% lower than that of the model I and is 37.15% lower than that of the model II. It can be concluded that, according to the above models, the combined prediction model III based on BP neural network and improved Verhulst model has the best fitting effect.

(3) According to the above chart, the difference in fitting accuracy between model III and II is not so obvious as that between model III and I, which also explains the double improvement in reverse prediction idea and the BP neural network from another aspect. The effect is better than the improvement from the single BP neural network to basically be in line with expectations.
6. Conclusions

The study on China defense expenditure plays a certain role in the subsequent discussion of its related influencing factors and balancing the relationship between national defense and the economy (Zheng & Zheng, 2017). Starting from the analysis of the time-order characteristic of defense expenditure, in this paper, based on the traditional Verhulst nonlinear prediction model, the reverse prediction model correction idea is used and the BP neural network is introduced to establish a combined prediction model based on BP neural network and improved Verhulst model from the residual dimension. It is tested with the use of average absolute error, the average relative error, and the posterior error ratio index and is substituted into the actual data for verification to get the following conclusions:

Compared with the traditional Verhulst model, the average absolute error of the combined prediction model based on BP neural network and improved Verhulst model decreases to 29.62 from 87.46 and the accuracy increases by 66.13%; The average relative error decreases to 0.378% from 1.336% of the traditional Verhulst model and the accuracy increases by 71.71%. The average posterior error decreases to 0.0181 from 0.0519 and the accuracy increases by 64.51%. All prove that the combined prediction model based on BP neural network and improved Verhulst model can fit the historical data better and the fitting accuracy is greatly improved. Therefore, the application of this model to the prediction of defense expenditure is more convincing.

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References

Chen, L., & Zhang, L. S. (2010). Combined gray Verhulst model based on initial value correction. Mathematics in Practice and Theory, 40(11), 160–164.

Cheng, L. X., Liu, D., Lu, W., & Song, Y. H. (2017). Research on traffic accident prediction based on combined prediction optimization model. China Safety Science Journal, 27(5), 31–35.

Dai, L. Q., Gao, B., & Zhou, J. M. (2016). Improvement of Verhulst model and its application with the combination of the hyperbolic model. Chinese Journal of Applied Mechanics, 33(1), 86–93.

Dang, Y. G., Yao, T. X., Cui, J., & Xiong, P. P. (2012). Study on the modeling method of background value optimization of the gray Verhulst model. Chinese Journal of Management Science, 20(6), 154–159.

Deng, J. L., & Deng, J. L. (2002). Foundation of gray theory. Wuhan: Huazhong University of Science and Technology Press.

Jia, N. P., Zeng, L. N., Zeng, L., & Yang, Y. (2018). Analysis of the relationship between national defense expenditure and economic growth in China in the past 20 years. Science Technology and Industry, 18(10), 71–79.

Liu, B. P., Huang, D., & Zhang, K. (2017). Gray Verhulst measurement combination prediction model of small sample nonlinear residuals based on EGA algorithm. System Engineering Theory and Practice, 37(10), 2630–2639.

Liu, B. P., Huang, D., & Zhang, K. (2018). Application of multi-objective combined prediction of Elitist Genetic improved nonlinear gray neural network operator and military expenditure. Systems Engineering and Electronics, 40(5), 1070–1078.

Qiu, Z. Z., & Li, S. F. (2017). Modeling and simulation of mooring force prediction based on improved GA-BP network. Journal of System Simulation, 29(7), 1457–1463.

Wang, F. L., Dong, Z. G., & Wang, J. Z. (2017). Power load prediction based on improved BP neural network. Mathematics in Practice and Theory, 47(9), 276–284.

Wang, F. X., & Wang, F. X. (2011). Improved GM(1,1) model and its parameter optimization. Pure and Applied Mathematics, 27(6), 711–714.

Wang, Z. C., Wu, J. W., & Zhang, W. (1998). Improved calculation method of GM(1,1) model parameters. System Engineering and Electronics, 20(8), 58–60.

Wu, J. B., & Feng, L. (2016). Research on market risk measurement of stock market portfolio—based on correlation model. Journal of Systems Science and Mathematical Sciences, 36(12), 2307–2324.

Xie, N. M., & Liu, S. F. (2008). Gray system theory and its application (116–118). Beijing: Science Press.

Yan, L. R., Liu, Y., Liu, S. F., & Zhao, H. H. (2014). Gray Verhulst modeling method and its application based on background value and initial value optimization. Systems Engineering, 32(3), 149–153.

Zhang, C., & Peng, Z. B. (2016). Application of Verhulst optimization model in foundation settlement prediction. Journal of Railway Science and Engineering, 13(8), 1535–1542.

Zheng, W., & Zheng, W. (2017). Association study of China’s national defense expenditure and economic growth. China Management Informationization, 20(24), 124–125.