Origin of Nonlinear Damping due to Mode Coupling in Auto-Oscillatory Modes Strongly Driven by Spin-Orbit Torque

Inhee Lee,† Chi Zhang,‡ Simranjeet Singh,† Brendan McCullian,† and P. Chris Hammel†,‡

†Department of Physics, The Ohio State University, Columbus, OH 43210, USA

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We investigate the physical origin of nonlinear damping due to mode coupling between several auto-oscillatory modes driven by spin-orbit torque in constricted Py/Pt heterostructures by examining the dependence of auto-oscillation on temperature and applied field orientation. We observe a transition in the nonlinear damping of the auto-oscillation modes extracted from the total oscillation power as a function of drive current, which coincides with the onset of power redistribution amongst several modes and the crossover from linewidth narrowing to linewidth broadening in all individual modes. This indicates the activation of another relaxation process by nonlinear magnon-magnon scattering within the modes. We also find that both nonlinear damping and threshold current in the mode-interaction damping regime at high drive current after transition are temperature independent, suggesting that the mode coupling occurs dominantly through a non-thermal magnon scattering process via a dipole or exchange interaction rather than thermally excited magnon-mediated scattering. This finding presents a promising pathway to overcome the current limitations of efficiently controlling the interaction between two highly nonlinear magnetic oscillators to prevent mode crosstalk or inter-mode energy transfer and deepens understanding of complex nonlinear spin dynamics in multimode spin wave systems.

Spin-orbit torque driven magnetic nano-oscillators have recently emerged as potential charge current tunable microwave sources for spintronics devices[1–13], as well as fundamental elements for neuromorphic computing[14]. These oscillators use spin Hall effect to convert the charge current into a pure spin current which is injected into the ferromagnet, exerting an anti-damping torque on the magnetization. Above the threshold current, coherent magnetic auto-oscillations (AO) are generated at microwave frequencies.

In principle, these planar spin Hall nano-oscillators (SHNO) need not be limited in size and are expected to provide larger AO power because the Oersted field is relatively small and uniform in the device unlike nanopillar spin-torque nano-oscillators (STNO). However, additional damping channels arise in extended thin film structures through nonlinear magnon scattering that prevents auto-oscillation[3]. These damping channels can be suppressed by restricting the area of the AO by fabricating a nano-constriction[3,11] as well as other methods of spatial mode confinement such as dimensional reduction[4,12] and local dipolar field[16,19].

Nevertheless, in constriction-based SHNOs, mode splittings are often observed[3,10], leading to substantial linewidth broadening which degrades performance in terms of AO coherence and power. So far, little is known about the origins of nonlinear damping as a consequence of multimode excitation in this system, and the mechanism underlying mode coupling is still unknown. In nano-pillar STNOs, similar multimode behaviors such as mode hopping[20,26], mode coexistence[27,29] and 3-magnon process[30,31], have been reported along with theoretical studies[20–26,32,33] of mode coupling, including its description[21,29,33], its effect on mode decoherence[22,24,25] and its temperature dependence[22,24,25,33], however, much remains to be understood.

For this study, we fabricate Pt(5nm)/Py(5nm) bilayer devices in the form of a bow tie with an active center area of 600 nm × 1 μm, as shown in Fig. 1(a). This sample structure allows the generation of multiple AO modes by applying a high current density \( J_c \) that is converted into a spin current \( J_s \) that, in turn, exerts a spin torque on the magnetization of the Py (See Fig. 1(b)). By changing the orientation of the magnetic field \( \mathbf{H}_0 \) applied at angle \( \theta \) with respect to the x-axis of the microstrip line, we tune the eigenmodes of the spin waves defined by the spatial pattern of the internal field in Py, which is significantly modified by its dipolar, or demagnetizing field.

Indeed, when the orientation of the applied field changes relative to the device edges causing mode constrictions, spectral and spatial distributions of the resulting AO modes vary significantly. Fig. 2(a) shows the angle \( \theta \) dependence of the AO spectrum measured with bias current \( I_{DC} = 7 \) mA at temperature \( T = 77 \) K in an applied field \( H_0 = 570 \) Oe. As \( \theta \) increases, more excitation modes appear over a wider frequency range as a result of stronger mode constrictions. We perform micromagnetic simulations using MuMax3[33](see Appendix B) to understand the complex spectra at various \( \theta \) and to identify the relevant AO modes. Fig. 2(b) shows the angle dependence of the AO spectra obtained from simulations conducted with the current density distribution and Oersted field in Fig. 10 of Appendix. It well describes the overall evolution of the experimental AO spectrum with increasing angle in Fig. 2(a), although more accurate spatial information reflecting the high current den-
by thermally excited magnons relative to the x-axis. (c) Four magnon scattering mediated spin polarization required for anti-damping torque. Which is injected into the ferromagnet Py with the appropriate thermal reservoir. The angle made by the direction of in-plane applied field $H$ and $\theta_i$ is the angle made by the direction of in-plane applied field $H_0$ relative to the x-axis. (c) Four magnon scattering mediated by thermally excited magnons $\omega_{th,i}$ and $\omega_{th,f}$ in equilibrium with thermal reservoir. $\omega_i + \omega_{th,i} = \omega_f + \omega_{th,f}$. (d) Four magnon scattering by intermode interactions. $d\omega$ is the frequency shift within the mode caused by dipole or exchange interactions between $\omega_i$ and $\omega_f$.

FIG. 1. (a) Optical image of the auto-oscillation device. (b) Schematic diagram of the 2D constricted Pt(5 nm)/Py(5 nm) bilayer structure of the bow tie shape in the dashed box of (a). Due to the strong spin-orbit coupling in Pt, the charge current density $J_s$ flowing in the x direction along the axis of microstrip line is converted to pure spin current density $J_c$ which is injected into the ferromagnet Py with the appropriate spin polarization required for anti-damping torque. $\theta$ is the angle made by the direction of in-plane applied field $H_0$ relative to the x-axis. (c) Four magnon scattering mediated by thermally excited magnons $\omega_{th,i}$ and $\omega_{th,f}$ in equilibrium with thermal reservoir. $\omega_i + \omega_{th,i} = \omega_f + \omega_{th,f}$. (d) Four magnon scattering by intermode interactions. $d\omega$ is the frequency shift within the mode caused by dipole or exchange interactions between $\omega_i$ and $\omega_f$.

There are other possible causes for the nonlinear damping transition at $I_{MI}$. The transition in the dependence of power on current shown in Fig. 3(d) may be described by the selective excitation and selective saturation of each mode. However, C2 and C3 in multimode excitation cannot be explained this way because their linewidth broadening (Fig. 3(b)), ii) a power increase in the C2 and C3 modes despite their linewidth broadening (Fig. 3(c) and 3(d)), and iii) an increase in the total power (Fig. 3(d)). All of these imply that the magnon population is growing faster than excited magnons can decay to other thermal reservoirs. Therefore, we conclude that nonlinear damping in the high current regime occurs through magnon redistribution from low-frequency modes to high-frequency modes via nonlinear magnon-magnon scattering as shown schematically in Fig. 1(c) or 1(d).

The spectral and spatial mode profiles for AO in the active region seem necessary to fully account for the spectral shape details. Fig. 2(c)-(g) show the spatial profiles of the spin wave eigenmodes corresponding to the spectral peaks in Fig. 2(b) at various angles $\theta$. We find that for $\theta \leq 65^\circ$, the edge and bulk modes are combined, whereas for $\theta \geq 70^\circ$, the edge and bulk modes are spatially separated due to the significant difference in the demagnetizing field between the edge and center regions. This separation also occurs spectrally (see Fig. 2(a) and 2(b)) and the largest AO peak appears in one of the bulk modes at frequencies higher than edge modes, which is in good agreement with previous AO results obtained on nanowires with $\theta = 85^\circ$.

The spectral and spatial mode profiles for AO in the active mode shown in Fig. 2 differ significantly from those of the linear spin wave mode (see Fig. 2 in Appendix). This demonstrates that the dynamics of the AO mode are different from those of the linear spin wave mode, arising from nonlinear effects such as complex bullet mode dynamics involved with mode size reduction [4, 9, 35] or mode size oscillation [5, 36]. In particular, this bullet mode effect appears to be more pronounced in bulk mode 3a and 3b in Fig. 2(c) and bulk mode 3 in Fig. 2(d), away from the edge effect. The frequency jump as the signature nonlinearity and structural defects in the AO active region seems necessary to fully account for the spectral shape details. Fig. 2(c)-(g) show the spatial profiles of the spin wave eigenmodes corresponding to the spectral peaks in Fig. 2(b) at various angles $\theta$. We find that for $\theta \leq 65^\circ$, the edge and bulk modes are combined, whereas for $\theta \geq 70^\circ$, the edge and bulk modes are spatially separated due to the significant difference in the demagnetizing field between the edge and center regions. This separation also occurs spectrally (see Fig. 2(a) and 2(b)) and the largest AO peak appears in one of the bulk modes at frequencies higher than edge modes, which is in good agreement with previous AO results obtained on nanowires with $\theta = 85^\circ$.

In the evolution of the auto-oscillation modes with increasing bias current, we observe a transition in nonlinear damping due to mode-mode coupling. As can be seen in Fig. 3(a) obtained by measuring at $\theta = 65^\circ$, as the current $I_{DC}$ and hence the anti-damping torque increases, the three main modes C1, C2, and C3 (labelled in Fig. 3(c)) appear in the spectrum above each threshold current. We characterize these modes evolving at various $I_{DC}$ with the resonance frequency $f_0$, linewidth $\Delta f$, and power $P$ obtained from Lorentzian fits, which are shown in Fig. 3(b)-(d). Associated with the transition of nonlinear damping, we observe noticeable abrupt changes at the current $I_{MI} = 7.1$ mA, such as the turnover from the linewidth narrowing to linewidth broadening (sign change of $\Delta f/I_{DC}$) for all excitation modes (Fig. 3(c)) and power saturation in C1 mode (Fig. 3(d)). This transition marks the onset of another relaxation process with additional damping. However, as $I_{DC}$ increases in the high current regime ($I_{DC} > I_{MI}$), we also observe i) a monotonic redshift of the frequency for all three modes reflecting a monotonic reduction in saturation magnetization due to magnon excitation [37] (Fig. 3(b)), ii) a power increase in the C2 and C3 modes despite their linewidth broadening (Fig. 3(c) and 3(d)), and iii) an increase in the total power (Fig. 3(d)). All of these imply that the magnon population is growing faster than excited magnons can decay to other thermal reservoirs. Therefore, we conclude that nonlinear damping in the high current regime occurs through magnon redistribution from low-frequency modes to high-frequency modes via nonlinear magnon-magnon scattering as shown schematically in Fig. 1(c) or 1(d).

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FIG. 2. (a) Angle dependence of the auto-oscillation spectrum measured at $I_{\text{DC}} = 7$ mA, $T = 77$ K and $H_0 = 570$ Oe. We vary the angle $\theta$ between $H_0$ and $I_{\text{DC}}$ in our experiments as shown in the diagram in the lower right corner of the figure. In the spectra for 65° and 70°, ‘C’, ‘E’ and ‘B’ represent a combined edge-bulk mode, edge mode, and bulk mode, respectively. (b) Spectra for various $\theta$ obtained from micromagnetic simulations using Mumax3 performed with the current density distribution and Oersted field in Fig. 10 of Appendix. The corresponding $I_{\text{DC}}$ are 7.92 mA for $\theta = 55$°, 7.26 mA for $\theta = 60$°, 6.92 mA for $\theta = 65$°, 6.52 mA for $\theta = 70$°, and 6.36 mA for $\theta = 75$°, about 0.2 - 0.3 mA larger than the respective threshold current at each $\theta$. In (a) and (b), each spectrum is offset. (c)-(g) Spatial eigenmode profiles corresponding to the spectral peaks indicated by the numbers in (b) at various $\theta$. The dynamic magnetization amplitude $m$ is normalized on the color scale of each image.

In Fig. 4 we show the temperature dependence of the nonlinear damping of the AO system along with the threshold current $I_{\text{th}}$ and transition current $I_{\text{MI}}$. Fig. 4(a) shows the total power $P_{\text{total}}$ summed for all excitation modes as a function of $I_{\text{DC}}$ obtained at various temperatures for $\theta = 65^\circ$. For each temperature, $P_{\text{total}}$ has two current regimes, ‘A’ for $I_{\text{DC}} < I_{\text{MI}}$ in which a single mode is dominant, and ‘B’ for $I_{\text{DC}} \geq I_{\text{MI}}$ in which mode-mode interaction is active amongst multiple excitation modes, separated by the kink in $P_{\text{total}}$ at which $P$ in the lowest frequency mode saturates. One of our remarkable findings is that total power $P_{\text{total},B}$ for all temperatures in regime B falls onto a single common curve (black line in Fig. 4(a)), whereas total power $P_{\text{total},A}$ in regime A is temperature dependent. This suggests that the mode-mode coupling occurs through a non-thermal process (Fig. 4(d)) originating from exchange or dipole interactions instead of the thermally excited magnon-mediated scattering (Fig. 4(c)). The rapid growth of $P_{\text{total},A}$ (color lines in Fig. 4(a)) from each threshold current $I_{\text{th}}$ is eventually limited by $P_{\text{total},B}$ (black line in Fig. 4(a)) at each corresponding transition current $I_{\text{MI}}$ for all temperatures. This indicates that the much faster relaxation process arising from the temperature-independent mode-mode interactions in regime B predominates over the temperature dependent single mode relaxation processes occurring in regime A. Note that magnon thermalization through redistribution within the dynamic magnetic system is possible only if the relaxation via nonlinear magnon scattering arising from mode coupling is much faster than the relaxation to the external non-magnetic systems in individual modes by intrinsic Gilbert damping or spin pumping.

In order to quantify the nonlinear damping of the AO system, we discuss the parameter $Q$ that represents the change in positive nonlinear damping $\Gamma_+ \approx \Gamma_G (1 + Q_{\text{p}})$ with increasing AO power, where $\Gamma_G$ is the Gilbert damping. $Q_{\text{p}}$ as well as $I_{\text{th}}$ can be obtained from the relationship between the normalized AO power $p$ and $I_{\text{DC}}$ (see Appendix C). Fig. 4(b) and 4(c) show the temperature dependence of the AO parameters in the regimes...
FIG. 3. (a) Evolution of power spectral density (PSD) with increasing bias current $I_{DC}$, (b) resonance frequency $f_0$ vs. $I_{DC}$, (c) linewidth $\Delta f$ vs. $I_{DC}$, (d) power $P$ vs. $I_{DC}$ for the main modes C1, C2 and C3 for $\theta = 65^\circ$. The FMR parameters in (b)-(d) are extracted by fitting the data in (a) to Lorentzian functions. (e) Representative spectra at high $I_{DC}$ for $\theta = 65^\circ$ showing the redistribution of power among modes at high drive current. The FMR parameters in (g)-(i) are extracted by fitting the data in (f) to Lorentzian functions. (j) Evolution of PSD with increasing $I_{DC}$, (k) $f_0$ vs. $I_{DC}$, (l) $\Delta f$ vs. $I_{DC}$, (m) $P$ vs. $I_{DC}$ of the main edge (E1, E2) and bulk (B1, B2, B3) modes for $\theta = 70^\circ$. The FMR parameters in (g)-(i) are extracted by fitting the data in (f) to Lorentzian functions. (l) Representative spectra at high $I_{DC}$ for $\theta = 70^\circ$ showing the redistribution of power among modes at high drive current. In (d) and (i), the black diamond markers are the total power $P_{\text{total}}$ summed over all excited modes, and $I_{MI}$ represents the current at which the nonlinear damping transition occurs. At very high $I_{DC}$, the main modes shown here are not clearly identified due to the large linewidth broadening and the emergence of other excited modes even though $P_{\text{total}}$ can be obtained. Here $H_0 = 570$ Oe and $T = 77$ K for all measurements.

FIG. 4. (a) Total power $P_{\text{total}}$ of the AO as a function of $I_{DC}$ at various temperature $T$, (b) the nonlinear damping coefficient $Q$ vs. $T$, (c) the threshold current $I_{th}$ vs. $T$ and the transition current $I_{MI}$ vs. $T$ for $\theta = 65^\circ$. Colored solid curves are the linear fits to Eq. (1) with $I_{th,0} = 7.35$ mA and $\kappa_{th} = -0.0117 \text{ mA/K}$ and Eq. (2) with $I_{MI,0} = 8.34$ mA and $\kappa_{MI} = -0.0147 \text{ mA/K}$ for $I_{th}$ and $I_{MI}$, respectively. Similarly, (d) $P_{\text{total}}$ of the AO as a function of $I_{DC}$ at various $T$, (e) $Q$ vs. $T$, (f) $I_{th}$ vs. $T$ and $I_{MI}$ vs. $T$ for $\theta = 70^\circ$. Colored solid curves are the linear fits to Eq. (1) with $I_{th,0} = 7.43$ mA and $\kappa_{th} = -0.0124 \text{ mA/K}$ and Eq. (2) with $I_{MI,0} = 8.12$ mA and $\kappa_{MI} = -0.015 \text{ mA/K}$ for $I_{th}$ and $I_{MI}$, respectively. Colored solid curves in Fig. 4(a) and 4(d) are the $P_{\text{total}}$ calculated using the theoretical equation (see Eq. (C4) in Appendix C) with $Q_A$ and $I_{th,A}$ in 4(b)-(c) and 4(e)-(f), respectively. The black curves in Fig. 4(a) and 4(d) are the $P_{\text{total}}$ calculated using the theoretical equation (see Eq. (C5) in Appendix C) with the common values of $Q_B$ and $I_{th,B}$ in 4(b)-(c), 4(e)-(f), respectively. Here ‘A’ represents individual mode dominant regime ($I_{DC} < I_{MI}$) and ‘B’ represents mode interaction activation regime ($I_{DC} \geq I_{MI}$). And the normalization factor $N_0 = 9.5$ pW and $8.7$ pW, determined from fitting, for $\theta = 65^\circ$ and $\theta = 70^\circ$, respectively, (see Appendix C) and $H_0 = 490$ Oe. Temperature dependence of (g) minimum linewidth and (h) maximum power of C1 mode for $\theta = 65^\circ$. 

$H_0 = 570$ Oe and $T = 77$ K for all measurements.
demonstrating that thermal fluctuation noise facilitates also, as the existence of another temperature-independent relaxation mechanism below 125 K for an individual AO mode. Also, as T increases, both $I_{th,A}$ and $I_{th,B}$ decrease linearly, demonstrating that thermal fluctuation noise facilitates the generation and stabilization of the AO mode and of mode-mode coupling with the smaller $I_{th}$ such as

$$ I_{th}(T) = I_{th,0} + \kappa_{th} T $$

(1)

$$ I_{MI}(T) = I_{MI,0} + \kappa_{MI} T $$

(2)

where $I_{th,0}$ and $I_{MI,0}$ are the intrinsic threshold and transition currents with thermal fluctuation excitation effects removed, and $\kappa_{th}$ and $\kappa_{MI}$ are coefficients with respect to temperature change. The temperature dependence of the intrinsic threshold, $I_{th,0}(T)$, can be obtained from a separately measured spin torque FMR, where $I_{th,0}$ is estimated to be almost constant in the temperature range of 5 – 300 K (See Appendix E).

There are significant changes in the current evolution of the AO modes when $\theta$ changes from 65° to 70° as shown in Fig. 3a and 3f. Edge-bulk combined modes (C1, C2, C3) at $\theta = 65^\circ$ (see Fig. 2c and Fig. 3a)-(c)) are spectrally and spatially separated into edge (E1, E2) and bulk (B1, B2, B3) modes at $\theta = 70^\circ$ (see Fig. 2d and Fig. 3f)-(j)). As a result, for the AO modes existing at $\theta = 70^\circ$ (see Fig. 3g)-(i)), the evolution of $f_0$, $\Delta f$, and $P$ of the AO modes with increasing $I_{DC}$ are more complex than for 65°: the E1 and E2 modes split at 7 mA, and there can be various mode-mode couplings with different strengths of dipolar and exchange interactions depending on the spatial distance between the two interacting modes (e.g., edge-edge, edge-bulk and bulk-bulk). This information can be valuable in developing strategies that employ spectral and spatial mode separation to reduce mode couplings and thus enhance the performance of auto-oscillators. We note that these mode couplings should be distinguishable from the nonlinear bullet mode that employ spectral and spatial mode separation to reduce mode couplings and enhance the performance of auto-oscillators. We thank Denis V. Pelekhov for helpful discussions. This work was primarily supported by the Center for Emergent Materials, an NSF MRSEC, grant DMR-2011876.

Appendix A: Angle Dependence of auto-oscillation spectrum over extended angle range

Fig. 5 shows the dependence of the auto-oscillation spectrum on angle, similar to Fig. 2a), but over an extended angular range: $\theta = 50^\circ - 85^\circ$. At $\theta \leq 65^\circ$, the lowest frequency mode has the largest amplitude and shifts slightly towards lower frequencies as $\theta$ increases. The mode splits and its amplitude decreases above 70° while the frequency shifts monotonically lower with increasing $\theta$. On the other hand, the high frequency modes hardly shift with increasing $\theta$, and at $\theta \geq 70^\circ$, one of them has the largest amplitude among all excited AO modes in the spectrum instead of the lowest frequency mode. Our micromagnetic simulations in the next section show that edge modes at low frequencies shift monotonically to lower frequencies, while bulk modes at high frequencies hardly shift; this characteristic behavior allows edge and bulk modes to be differentiated for $\theta \geq 70^\circ$.

Appendix B: Micromagnetic Simulations

We perform micromagnetic simulations using MuMax3 [34] to understand complex AO spectra and identify their relevant spatial mode profiles. In simulations, the computational dimension $2.6 \mu m \times 9 \mu m \times 5 \mu m$ is subdi-
FIG. 5. Angle dependence of the auto-oscillation spectrum in Fig. 2(a) of the main text extended to a wide angle range. It is measured at $I_{DC} = 7$ mA, $T = 77$ K and $H_0 = 570$ Oe.

vided into $5$ nm $\times$ $17.5$ nm $\times$ $5$ nm cells. As magnetic parameters, we use the gyromagnetic ratio of $\gamma/2\pi = 2.8$ MHz/G and effective magnetization $4\pi M_{\text{eff}} = 6502$ G obtained from the ST-FMR data (see Appendix D) measured at $I_{DC} = 0$ via Kittel equation.

$$f_0 = \frac{\gamma}{2\pi} \left[ H_0 (H_0 + 4\pi M_{\text{eff}}) \right]^{1/2}$$  \hspace{1cm} (B1)

The standard values of exchange stiffness $A_{\text{ex}} = 1.3 \times 10^{-11}$ J/m and the Gilbert damping constant $\alpha = 0.01$ for permalloy are used.

1. Linear Spin Wave Eigenmode

First, we perform micromagnetic simulations in the linear regime of magnetodynamics with no bias current. Initially, the magnetic system is excited by a sinc rf field with an amplitude of $10$ mT and a cutoff frequency of $40$ GHz. Then Gilbert damping is turned off by making $\alpha = 0$, allowing the magnetic dynamic system to proceed freely for $187$ ns. We obtain the spectral and spatial profiles of the modes by performing Fourier transform with dynamic motion after $62$ ns to avoid initial transients.

Fig. 6(b)-(f) show the spatial profile of the linear spin wave eigenmodes corresponding to each spectral peak indicated by the number in Fig. 6(a) for each angle $\theta$. The edge and bulk modes are combined for $\theta \leq 65^\circ$, whereas the edge and bulk modes are spatially and spectrally separated for $\theta \geq 70^\circ$ due to the significant difference in the demagnetizing field between the edge and center regions. Edge modes at low frequencies shift monotonically to lower frequencies, and bulk modes at high frequencies shift little.

2. Self-Oscillatory Mode

The auto-oscillation mode of the system is obtained by solving the Landau-Lifshitz-Gilbert equation with an anti-damping spin torque applied to the active region of the AO device. In the simulations, we try two different current density distributions shown in Fig. 7 and Fig. 10 considered for a $1$ mA bias current.

As an initial state in the simulation, the magnetic system is allowed to relax to a state close to the ground state. The anti-damping torque proportional to the current density in Fig. 7 and 10 scaled by the current value is activated at $0$ ns. If this anti-damping torque is smaller than the Gilbert damping, the magnetization oscillations decay, whereas if the anti-damping torque is larger than the Gilbert damping, the magnetization oscillations grow until their amplitude saturates. We define a threshold current as that at which anti-damping is balanced with Gilbert damping, where the magnetization oscillates with a constant amplitude over time. The scale factor for the current in the simulations is chosen so that the threshold currents in the simulations are as close as possible to the threshold currents in the actual experimental data. The spectral and spatial profiles of the excitation modes are obtained by performing Fourier transforms of the time dependence of the magnetization dynamics.

a. Micromagnetic simulation using current density distribution calculated in COMSOL

Fig. 7 shows the current density distribution of the AO system for a $1$ mA current calculated using COMSOL and the Oersted field produced by it. These determine the anti-damping torque in micromagnetic simulations, and the corresponding results are shown in Fig. 8 and Fig. 9.

Figs. 8(a) and 8(b) show the evolution of power spectral density (PSD) with increasing bias current $I_{DC}$ for $\theta = 65^\circ$ and $\theta = 70^\circ$, respectively, and their representative linecuts are shown in Fig. 8(c) and 8(d), respectively. These simulation data show the key features of the experimental data in Fig. 3 of the main text, such as monotonic redshift of the resonant frequency, mode amplitude growing and mode amplitude saturation with increasing $I_{DC}$. In the simulation, the strong mode broadening starts at a relatively lower $I_{DC}$ compared to the experimental data, so Fig. 8 shows the simulation result applicable only to the low current region of the experimental data Fig. 3. This indicates that the nonlinear magnonic effect occurring at high currents, which can cause stronger self-excitation of the AO mode with high coherence, seems to be lacking in the simulations. Since thermal effects play no role in the simulations, the mode...
b. Micromagnetic simulation using current density distribution of 2D Gaussian model

In order to test the effect of the inhomogeneous current density distribution on the excitation of the AO modes in the simulation, we try a micromagnetic simulation using the different current density distribution in Fig. 10(a), where the current density $J_z$ has a 2D Gaussian distribution with a maximum at the center and a broader distribution as function of position in the device compared to Fig. 7(a), and $J_y = J_z = 0$. Indeed, the spectral shapes in this simulation are considerably different as shown in Fig. 2(b) of the main text and Fig. 11(c) and 11(d). In contrast to Fig. 9, bulk modes excited around the central region are unsuppressed for $\theta = 70^\circ$ and $75^\circ$ and can have larger mode amplitudes than edge modes, as shown in Fig. 2. These bulk modes exhibit improved agreement with experimentally observed AO modes, in particular small frequency shift with increasing $\theta$ as shown in Fig. 2(a) and Fig. 5. Also, the overall spectral shapes associated with the number of the excited AO modes, their relative frequencies and amplitudes, their frequency shift behaviors for varying $\theta$ in Fig. 2(b) better match the experimental data in Fig. 2(a) compared to the other AO simulations in Fig. 6(a) and Fig. 9(a). Therefore, we conclude that the actual current density distribution is closer to the 2D Gaussian function in Fig. 10 than that calculated using COMSOL in Fig. 7. We note that unidentified structural defects around the edges of the AO active region of the sample can significantly alter the spectral shapes, especially for $\theta = 70^\circ$ and $75^\circ$, where the mode constriction effect is stronger.

This conclusion is further supported by the evolution of the power spectral density (PSD) with increasing bias current $I_{DC}$ for $\theta = 65^\circ$ and $70^\circ$ shown in Fig. 11(a) and 11(b), respectively. In Fig. 11(a) and 11(c) for $\theta = 65^\circ$, the edge mode at the lowest frequency has the largest amplitude at any $I_{DC}$ whereas in Fig. 11(b) and 11(d)

broadening seen at high $I_{DC}$ arises purely from dipole or exchange spin-spin interactions of dynamic magnetization occurring at relatively large cone angles. This directly demonstrates nonlinear damping due to mode couplings that occur via dipole or exchange interactions, supporting the main conclusion of the paper.

Fig. 9(a) shows the spectrum for various $\theta$ and Fig. 9(b)-(f) show the spatial eigenmode profiles corresponding to the spectral peaks indicated by the numbers in Fig. 9(a). Most of the AO modes excited by the anti-damping torque are edge modes that shift only to lower frequencies with increasing $\theta$, while the barely shifted bulk modes shown in the linear spin wave modes in Fig. 6 are mostly suppressed. Compared with the spatial mode profiles of the linear spin wave modes in Fig. 6, the edge modes in this simulation seem to be selectively excited and evolved from the linear spin wave modes by the inhomogeneous current density distribution.
FIG. 7. COMSOL [40] calculation of current densities in the AO structure for a 1 mA current: (a) \(x\)-component of current density distribution \(J_x\), (b) \(y\)-component of current density distribution \(J_y\), (c) \(y\)-component of Oersted field \(H_y\), and (d) \(z\)-component of Oersted field \(H_z\). Anti-damping spin torque in the micromagnetic simulations is calculated based on these maps. The corresponding results are shown in Fig. 8 and Fig. 9.

for \(\theta = 70^\circ\), the bulk mode at \(\sim 5.6\) GHz grows with increasing \(I_{DC}\) and eventually has the larger amplitude than any edge modes located at the lower frequencies at high \(I_{DC}\), which agrees well with our experimental data in Fig. 3.

Appendix C: Quantifying Nonlinear Damping with the Coefficient \(Q\)

The nonlinear single-mode auto-oscillator can be described with a universal oscillator model, another form of the Landau-Lifshitz equation, derived by Slavin and Tiberkevich [39],

\[
\frac{dc}{dt} + i\omega(p)c + \Gamma_+(p)c - \Gamma_-(p)c = f_n(t) \tag{C1}
\]

where \(c\) is the complex dimensionless dynamic magnetization amplitude, \(p (= |c|^2)\) is the oscillation power, \(\omega(p)\) is the power-dependent nonlinear frequency,

\[
\begin{align*}
\Gamma_+(p) &\approx \Gamma_G (1 + Qp) \tag{C2} \\
\Gamma_-(p) &\approx \sigma I (1 - p) \tag{C3}
\end{align*}
\]

are the positive and negative damping constant, respectively, \(f_n(t)\) is the thermal fluctuation noise, \(\Gamma_G (= \alpha \omega)\) is the Gilbert damping, \(Q\) is the nonlinear damping coefficient, \(\sigma\) is the spin-current efficiency, and \(I\) is the drive current. On the left side of Eq. (C1), the second term describes precession, the third term describes damping, and the fourth term describes anti-damping. Note that \(\omega(p), \Gamma_+(p), \) and \(\Gamma_-(p)\) are auto-oscillation power \(p\) dependent.

In order to describe the nonlinear damping of the multimode AO system in a simple and quantitative way, we calculate \(Q\) from the relationship with \(P_{total} = N_0p\), where for \(I_{DC} < I_{MI}\),

\[
p = \frac{Q\eta}{Q + \eta} \left[ 1 + \frac{\exp(-\zeta)}{E_\beta((\zeta + Q)/Q^2\eta)} \right] \quad + \frac{\zeta - 1}{\zeta + Q} \tag{C4}
\]

for \(I_{DC} \geq I_{MI}\),

\[
p = \frac{\zeta - 1}{\zeta + Q} \tag{C5}
\]

In these expressions, \(N_0\) is the power conversion factor for the normalized \(p\), \(\zeta = I/I_{th}, I_{th}\) is the threshold current, \(\eta\) is the effective noise power, \(E_\beta(x) = \int_1^\infty e^{-xt}/t^\beta dt\).
FIG. 9. (a) Spectra for various $\theta$ obtained from micromagnetic simulations using Mumax3 performed with the current density distribution and Oersted field in Fig. 7. The corresponding $I_{DC}$ are 8.28 mA for $\theta = 55^\circ$, 7.42 mA for $\theta = 60^\circ$, 6.72 mA for $\theta = 65^\circ$, 6.02 mA for $\theta = 70^\circ$, and 5.58 mA for $\theta = 75^\circ$, 0.2 mA larger than the respective threshold current at each $\theta$. Each spectrum is vertically offset for clarity. (b)-(f) Spatial eigenmode profiles corresponding to the spectral peaks indicated by numbers in (a) at various $\theta$. The dynamic magnetization amplitude $m$ is normalized on the color scale of each image. At $\theta = 70^\circ$ and $75^\circ$, the spatial separation of edge (1-4) and bulk (5) modes occurs as shown in (b) and (c) as well as their spectral separation as shown in (a). Compared to the linear spin wave modes in Fig. 6, the bulk modes are mostly suppressed in this simulation.

FIG. 10. (a) x-component of current density distribution $J_x$, (b) y-component of Oersted field $H_y$, and (c) z-component of the Oersted field $H_z$ in the AO system for 1 mA current used to apply anti-damping spin torque in the micromagnetic simulations. $J_y = J_z = 0$ and $H_x = 0$. The corresponding results are shown in Fig. 2 and Fig. 8.

and $\beta = -(1 + Q)\zeta/Q^2\eta$ [39]. Note that the Slavin-Tiberkevich model was originally intended to describe single-mode auto-oscillators, so $Q$ in the multimode excitation regime of $I_{DC} \geq I_{MI}$ is an “effective” nonlinear damping parameter of the entire AO system. We also obtain $\eta$ in Eq. (C4) along with other AO parameters presented in Fig. 4 by fitting, but Fig. 12 reveals no credible temperature dependence to $\eta$.

Appendix D: Spin Torque Ferromagnetic Resonance

Fig. 13 (a) shows a typical spin torque ferromagnetic resonance (ST-FMR) spectrum measured at various DC currents $I_{DC}$ ranging from -8 mA to 8 mA. As the absolute value of $I_{DC}$ < 0 increases, the ST-FMR peak broadens due to the enhanced damping, while as $I_{DC}$ > 0 increases, the mode becomes narrower due to anti-damping and eventually splits into a couple of modes. In order to see the overall shift of the ST-FMR peak, we normalize the signal $V_{\text{mix}}$ to the maximum at each $I_{DC}$, which is shown in Fig. 13 (b). With increasing $I_{DC}$ > 0, the ST-FMR peak shifts to the higher field because as the amplitudes of self-localized AO modes grow, the static magnetization decreases. On the other hand, as the absolute value of $I_{DC}$ < 0 increases up to -8 mA, the ST-FMR peak shifts monotonically to the lower field and does not show a shift to the higher field that might occur due to the decrease in saturation magnetization by Joule heating. This means that even at $I_{DC} = 8$ mA regardless of its sign, our device’s magnetic system has not reached the current regime where the saturation magnet-
FIG. 11. The evolution of power spectral density (PSD) with increasing bias current $I_{DC}$ for (a) $\theta = 65^\circ$ and (b) $\theta = 70^\circ$ obtained from micromagnetic simulations performed with the current density distribution and Oersted field in Fig. 10. (c) Representative spectrum linecuts in (a) for various $I_{DC}$ at $\theta = 65^\circ$. (d) Representative spectrum linecuts in (b) for various $I_{DC}$ at $\theta = 70^\circ$.

FIG. 12. Noise level $\eta$ vs. temperature $T$ for (a) $\theta = 65^\circ$ and (b) $\theta = 70^\circ$ obtained from the fitting curves of solid colored lines in Fig. 10(a) and 10(d).

FIG. 13. (a) Spin torque FMR (ST-FMR) spectrum at various $I_{DC}$ ranging from -8 mA to 8 mA. (b) The DC current evolution of ST-FMR where the signal $V_{mix}$ is normalized to the maximum at each $I_{DC}$ to see the shift of ST-FMR peak. Obviously, the monotonic ST-FMR peak shift to the lower field with no change in the shift to the higher field for the intensity of $I_{DC} < 0$ increasing up to -8 mA indicates no reduction in saturation magnetization that can be caused by Joule heating at large intensity of $I_{DC}$.

FIG. 14. Magnetic field dependence of resonance frequency obtained in (1) the main mode of ST-FMR measured at $I_{DC} = 0$ mA (blue inverted triangle), (2) auto-oscillation (AO) modes of $n = 1$ (red circle), $n = 2$ (square orange), and $n = 3$ (green triangle) measured at $I_{DC} = -7$ mA, and (3) linear spin wave (SW) eigenmodes (red, orange, and green solid lines for $n = 1, 2, 3$ respectively) obtained in the micromagnetic simulations using Mumax3 with $I_{DC} = 0$ mA and $4\pi M_s = 6.5$ kG. The frequency discrepancy $\sim 0.5$ GHz between the linear spin wave modes and the AO mode can be explained by the static magnetization reduction and the frequency jump near the onset $I_{th}$ that has been observed as a feature of self-localization in the nonlinear “bullet” modes.

tization decreases by Joule heating [37]. This provides crucial evidence that the contribution of Joule heating to the nonlinear damping of the AO system described in the main text is not significant at high $I_{DC}$ up to 8 mA.

Fig. 14 shows the resonance frequency $f_0$ of the auto-oscillation as a function of the applied magnetic field $H_0$.
FIG. 15. Temperature dependence of (a) Gilbert damping constant $\alpha$, (b) saturation magnetization $M_S \left(\approx M_{eff}\right)$, (c) spin Hall angle $\theta_{SH}$, obtained from ST-FMR, (d) Temperature dependence of the intrinsic threshold current $I_{th,0}$ induced by the spin Hall effect estimated by Eq. (E3) with parameters obtained from the ST-FMR in (a)-(c) in comparison with that estimated from the AO data in Fig. 4(c) of the main text.

Appendix E: Temperature dependence of the intrinsic threshold current

The threshold current $I_{th}$ we obtain from measurement of the AO power has two contributions as expressed in Eq. (1) of the main text: i) that due to thermal fluctuation, $\kappa T$, and ii) an intrinsic threshold current $I_{th,0}$ arising from the anti-damping torque due to the applied spin current. Here we estimate the temperature dependence of the intrinsic threshold current $I_{th,0}(T)$ from the ST-FMR data. According to the Slavin-Tiberkevich theory described in Appendix C, the intrinsic $I_{th,0}$ arising from the spin Hall effect is given by

$$I_{th,0} = \frac{\alpha \omega \left(\theta_{SH}(T)\right)}{M_S(T)}$$  \hspace{1cm} (E1)

The spin-current efficiency $\sigma$ is proportional to the temperature-dependent $\theta_{SH}/M_S$ in our AO system. Therefore,

$$\sigma(T) \propto \frac{\theta_{SH}(T)}{M_S(T)}$$  \hspace{1cm} (E2)

From Eq. (E1) and (E2), the intrinsic threshold is given as

$$I_{th,0}(T) = C \frac{\alpha(T) M_S(T)}{\theta_{SH}(T)}$$  \hspace{1cm} (E3)

where $C$ is a temperature independent constant. $\theta_{SH}$ has been found to be temperature independent with a value of 0.068 for 13 - 300 K [32], and $\alpha$ for Py film with same thickness 5 nm is also almost constant in the temperature range of 5 - 300 K regardless of capping materials even though there is a slight temperature dependence of surface damping around 50 K [43]. Also, $M_S$ can be considered to be almost constant at $T \ll T_C$. Therefore, $I_{th,0}$ can be taken to be almost constant for varying temperature. Based on this, we express $I_{th}(T)$ determined from our AO power in Fig. 4 as a simple sum of the temperature independent $I_{th,0}$ and additional thermal fluctuation contribution linearly decreasing with temperature as Eq. (1) in the main text. We also measured almost constant $\alpha$, $M_S$, and $\theta_{SH}$ from the ST-FMR in the temperature range of 77 - 200 K as shown in Fig. 15 (a)-(c), and Fig. 15(d) shows our estimated temperature dependence of intrinsic threshold $I_{th,0}(T)$, which is almost constant over the temperature within our estimation uncertainty.

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