A calculation of the thermal quark propagator is presented taking the gluon condensate above the critical temperature into account. The quark dispersion relation and the dilepton production following from this propagator are derived.

As an alternative method to lattice and perturbative QCD we suggest to include the gluon condensate into the parton propagators [1]. In this way non-perturbative effects are taken into account within the Green functions technique.

In the case of a pure gluon gas with energy density $\epsilon$ and pressure $p$ the gluon condensate can be related to the interaction measure $\Delta = (\epsilon - 3p)/T^4$ via [2]

$$\langle G^2 \rangle_T = \langle G^2 \rangle_0 - \Delta T^4,$$

where $G^2 \equiv (11\alpha_s/8\pi) : G^a_{\mu\nu} G^a_{\mu\nu} :$ and $\langle G^2 \rangle_0 = (2.5 \pm 1.0) T_c^4$ is the zero temperature condensate. Here $G^a_{\mu\nu}$ is the field strength tensor and $T_c$ the critical temperature of the phase transition to the quark-gluon plasma (QGP).

At zero temperature the quark propagator containing the gluon condensate has been constructed already [3]. Here we will extend these calculations to finite temperature.

The full quark propagator in the QGP can be written by decomposing it according to its helicity eigenstates [4] ($P = (p_0, \vec{p}), \ p = |\vec{p}|$)

$$S(P) = \frac{\gamma_0 - \hat{p} \cdot \vec{\gamma}}{2D_+(P)} + \frac{\gamma_0 + \hat{p} \cdot \vec{\gamma}}{2D_-(P)},$$

where

$$D_\pm(P) = (-p_0 \pm p) (1 + a) - b$$

and

$$a = \frac{1}{4p^2} \left[ tr (P \Sigma) - p_0 tr (\gamma_0 \Sigma) \right].$$
Figure 1. Quark self energy containing a gluon condensate.

\[ b = \frac{1}{4p^2} \left[ P^2 \text{tr} \left( \gamma_0 \Sigma \right) - p_0 \text{tr} \left( P \Sigma \right) \right]. \quad (4) \]

Using the imaginary time formalism and expanding the quark propagator in Fig.1 for small loop momenta \[ k \ll p \text{ and } k_0 = 2\pi i n T = 0, \]
we obtain \[ a = -\frac{4}{3} g^2 T \int \frac{d^3k}{(2\pi)^3} \left[ \left( \frac{1}{3} p^2 - \frac{5}{3} p_0^2 \right) k^2 \tilde{D}_l(0, k) + \left( \frac{2}{3} p^2 - 2p_0^2 \right) k^2 \tilde{D}_t(0, k) \right], \]
\[ b = -\frac{4}{3} g^2 T \int \frac{d^3k}{(2\pi)^3} \left[ \frac{8}{3} p_0^2 k^2 \tilde{D}_l(0, k) + \frac{16}{15} p_0^2 k^2 \tilde{D}_t(0, k) \right], \quad (5) \]

where \( \tilde{D}_{l,t} \) are the longitudinal and transverse parts of the non-perturbative gluon propagator at finite temperature in Fig.1.

The moments of the longitudinal and transverse gluon propagator in (5) are related to the chromoelectric and chromomagnetic condensates via

\[ \langle E^2 \rangle_T = \frac{\alpha_s}{\pi} \left( \langle G^a_{0i} G^a_{0i} \rangle_T \right) = 8T \int \frac{d^3k}{(2\pi)^3} k^2 \tilde{D}_l(0, k), \]
\[ \langle B^2 \rangle_T = \frac{1}{2} \left( \langle G^a_{ij} G^a_{ij} \rangle_T \right) = -16T \int \frac{d^3k}{(2\pi)^3} k^2 \tilde{D}_t(0, k). \quad (6) \]

These condensates can be extracted from the expectation values of the space- and timelike plaquettes \( \Delta_{\sigma,\tau} \) computed on the lattice \[ b \], using

\[ \frac{\alpha_s}{\pi} \langle E^2 \rangle_T = \frac{4}{11} \Delta_{\tau} T^4 - \frac{2}{11} \langle G^2 \rangle_0, \]
\[ \frac{\alpha_s}{\pi} \langle B^2 \rangle_T = -\frac{4}{11} \Delta_{\sigma} T^4 + \frac{2}{11} \langle G^2 \rangle_0. \quad (7) \]

The quark dispersion relation \[ a \], describing collective quark modes in the QGP in the presence of a gluon condensate, follows from \( D_{\pm}(P) = 0 \). Using the lattice results for the plaquette expectation values they have been determined numerically and are shown in Fig.2 for various temperatures.

The dispersions exhibit two collective quark modes. The upper branch comes from the solution of \( D_+ = 0 \) and the lower one from \( D_- = 0 \). The lower branch, showing a minimum, corresponds to a so-called plasmino, possessing a negative ratio of helicity to chirality, and is absent in the vacuum.

At \( p = 0 \) both modes start from a common effective quark mass, which is given by \( m_{\text{eff}} = \left[ \frac{(2\pi\alpha_s/3) \left( \langle E^2 \rangle_T + \langle B^2 \rangle_T \right)}{11/4} \right]^{1/4} \). In the temperature range \( 1.1T_c < T < 4T_c \) we found approximately \( m_{\text{eff}} = 1.15 T \).
Figure 2. Quark dispersion relations at $T = 1.1 T_c$ (a), $T = 2 T_c$ (b), $T = 4 T_c$ (c) and dispersion relation of a non-interacting massless quark (dashed line s).

The qualitative picture of this quark dispersion relation is very similar to the one found perturbatively in the hard thermal loop (HTL) limit \[4\]. The main difference is the different effective mass, which is given by $m_{\text{eff}} = gT/\sqrt{6}$ in the HTL approximation.

As a possible application of this effective quark propagator we compute the dilepton production rate from the QGP. The dilepton production rate follows from the imaginary part of the photon self energy in the case of two massless lepton flavors according to \[6\]

$$
\frac{dR}{d^4x d^4p} = \frac{1}{6\pi^4} \frac{\alpha}{M^2} \frac{1}{e^{E/T} - 1} \text{Im}\Pi^\mu_\mu(P),
$$

where $E = \sqrt{p^2 + M^2}$ is the energy of the virtual photon with invariant mass $M$ and momentum $p$.

Using the one-loop approximation for the photon self energy and replacing the bare quark propagators by the effective propagators containing the gluon condensate the dilepton production rate can be derived numerically \[7\]. The result is shown in Fig.3 for $p = 0$. As in the case of soft dileptons calculated within the HTL approximation \[4\] we find peaks and gaps in the dilepton rate. The peaks (Van Hove singularities) are caused by the presence of the minimum in the plasmino dispersion. The contribution at small $M$ ending with a Van Hove singularity comes from an electromagnetic transition from the upper branch to the lower branch of the quark dispersion relation. After this peak there is a gap before the channel for plasmino annihilation ($q_- \bar{q}_-$) opens up with another singularity. This contribution drops quickly but at $M = 2 m_{\text{eff}}$ the contribution from the annihilation of collective quarks ($q_+ \bar{q}_+$) sets in. This contribution dominates and approaches the one coming from the annihilation of bare quarks (Born term \[8\]) at large $M$. It should be noted that there are no smooth cut contributions in contrast to the HTL dilepton rate \[4\], because the quark self energy containing the gluon condensate has no imaginary part.

Collisions and higher order effects (bremsstrahlung) will smear out and cover these structures to some degree. Also contributions from finite momenta and the space-time evolution of the fireball will wash out these sharp structures somewhat. After all it might be interesting to look at low mass dileptons ($M \lesssim 1$ GeV) at RHIC. Whereas at SPS the
contribution to the dilepton spectrum from the quark phase is suppressed by one or two orders of magnitude compared to the hadronic contribution due to the small lifetime of the quark phase \[9\], the quark phase is expected to dominate at RHIC. Hence these structures coming from the presence of collective quark modes in the QGP might be observable and could serve as an unique signature for the QGP formation.

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