Neutrino spin oscillations in gravitational fields in noncommutative spaces

S A Alavi and S Nodeh

Department of Physics, Hakim Sabzevari University, PO Box 397, Sabzevar, Iran

E-mail: s.alavi@hsu.ac.ir and alaviag@gmail.com

Received 30 May 2014, revised 6 October 2014
Accepted for publication 3 December 2014
Published 4 February 2015

Abstract

We study neutrino spin oscillations in gravitational fields in noncommutative spaces. For the Schwarzschild metric the maximum frequency decreases as the noncommutativity parameter increases. In the case of the Reissner–Nordstrom (RN or Re–No) metric, the maximum frequency of oscillation is a monotonically increasing function of the noncommutativity parameter. In both cases, the frequency of spin oscillations decreases as the distance from the gravitational source grows. We present a phenomenological application of our results. It is also shown that the noncommutativity parameter is bounded as \( \theta > 0.1 l_p \).

Keywords: neutrino spin oscillation, noncommutative spaces, Schwarzschild metric, Reissner–Nordstrom (RN) metric

Introduction

As is well known, gravitation is one of the most studied phenomena in physics. Investigations over the last century have led to important insights. Interaction of particles with a gravitational field may allow us to probe physics at the Planck scale and increase our knowledge about the underlying yet-to-be-found theory. It can also teach us to find the missing link between general relativity and quantum theory.

Recently there have been notable studies on the formulation and possible experimental consequences of extensions of theories in noncommutative spaces. There has also been growing interest in possible cosmological consequences of space noncommutativity; see, e.g., [1].

For a manifold parameterized by the coordinates \( x_i, x_j \), the noncommutative relations can be written as \( [\hat{x}_i, \hat{x}_j] = i\theta_{ij} \), \( [\hat{p}_i, \hat{p}_j] = i\delta_{ij} \), \( [\hat{p}_i, \hat{x}_j] = 0 \), where \( \theta_{ij} \) is an antisymmetric tensor. (See [2] for a review.) Coordinate noncommutativity implies the existence of a finite minimal length \( \sqrt{\theta} \) below which the concept of distance becomes physically meaningless, so in noncommutative spaces there cannot be point-like objects. To study a gravitational field in noncommutative spaces, it is not necessary to change Einstein’s tensor part of the field equations, and noncommutative effects act only on the matter source. So one can modify the distribution of a point-like source in favor of smeared objects. Noncommutativity replaces pointlike structures with smeared objects and so may eliminate the divergences that normally appear in general relativity. The effect of smearing is mathematically implemented as follows: the position Dirac–delta function is replaced everywhere with a Gaussian distribution of minimum width \( \sqrt{\theta} \). So we choose the mass density of a static, spherically symmetric, smeared particle-like gravitational source as \([3, 4]::

\[
\rho_p(r) = \frac{M}{(4\pi\theta)^2} \exp\left(-\frac{r^2}{4\theta}\right).
\]

A particle of mass \( M \), instead of being perfectly localized at a point, is diffused throughout a region of line size \( \sqrt{\theta} \).

At the same time, neutrino physics is an active area of research with important implications for particle physics, cosmology, and astrophysics [5]. The physics associated with neutrino oscillation is an interesting subject to study from both the theoretical and the experimental point of view. The phenomenon of neutrino oscillations can explain solar and atmospheric neutrino problems. It also provides the first experimental evidence for physics beyond the standard model since it requires nonzero mass for neutrinos. Interaction of neutrinos with an external field provides one of the factors required for a transition between helicity states. In this paper, we study neutrino spin oscillations in the Schwarzschild and RN metrics in noncommutative spaces. We recall that the
effect of a quantum gravity-induced minimal length on neutrino oscillations has been studied in [6].

**Neutrino spin oscillations in the noncommutative Schwarzschild metric**

By solving Einstein’s equations with \( \rho_\theta(r) \) as a matter source and setting \( \hbar = c = 1 \), we have [7]:

\[
\begin{align*}
\frac{d^2x}{dt^2} &= \left(1 - \frac{4M}{r \sqrt{r^2 - \frac{3}{2} r^2}} \right) dt^2 \\
&- \left(1 - \frac{4M}{r \sqrt{r^2 - \frac{3}{2} r^2}} \right)^{-1} dr^2 \\
&- r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right),
\end{align*}
\]

where

\[
\gamma \left( \frac{3}{2}, \frac{r^2}{4 \theta} \right) = \int_0^{\frac{2}{r^2}} dt t^{1/2} e^{-t}.
\]

is the lower incomplete gamma function.

Here the mass \( M \) is diffused throughout a region of line size \( \sqrt{\theta} \). The components of vierbein four velocity are as follows [8]:

\[
w^a = (aA, U, r, \theta, \phi). \tag{4}
\]

where

\[
U^a = (U^0, U, r, \theta, \phi). \quad \alpha = U^0 = \frac{dr}{d\tau}. \tag{5}
\]

is the four velocity of a particle in its geodesic path, which is related to vierbein four velocity through \( u^a = e_i^a U^\mu \). The four velocity of a particle in the relevant metric \( U^\mu \) is related to the world velocity of the particle through \( \tilde{U} = a \tilde{V} \), where \( a = \frac{dr}{d\tau} \) and \( \tau \) is the proper time. The nonzero vierbein vectors are presented in the appendix. To study the spin evolution of a particle in a gravitational field, we calculate \( G_{ab} = (\tilde{E}, \tilde{B}) \), which is the analog of the electromagnetic field tensor. It is defined as follows:

\[
G_{ab} = e_{abc} e^b_{\mu} U^\nu. \tag{6}
\]

where \( e_{abc} \) are the covariant derivatives of vierbein vectors which are defined through the following expression:

\[
e_{abc} = \frac{\partial e_{ab}}{\partial x^c} - \Gamma_{ac}^d e_{db}. \tag{7}
\]

Using this equation one can calculate the covariant derivatives of vierbein vectors. (See the appendix.)

Using the fact that any anti-symmetric tensor in four-dimensional Minkowskian space-time can be stated in terms of two three-dimensional vectors (such as electric and magnetic fields), we have:

\[
G_{ab} = (\tilde{E}, \tilde{B}). \quad G_{00} = E, \quad G_{ij} = -\epsilon_{ijk}B_k. \tag{8}
\]

Using equations (5), (6), and (8), we have the following forms for the analogs of the electric and magnetic fields:

\[
\tilde{E} = \left( -\frac{1}{2} \left( \frac{4M}{r^2 \sqrt{r^2 - \frac{3}{2} r^2}} \right) X^{3/2} \right) \alpha \phi^0, \tag{9}
\]

\[
\tilde{B} = \left( \alpha \cos \theta \phi^0, -\sin \theta \left( \frac{1}{r} \sqrt{3} X^{1/2} \right) \phi^0 \right),
\]

where \( \alpha = u^\tau = \frac{dr}{d\tau} \).

The geodesic equation of a particle in a gravitational field is as follows [9]:

\[
\frac{d^2x^\mu}{dq^2} + \Gamma_{\mu}^{\nu\sigma} \frac{dx^\nu}{dq} \frac{dx^\sigma}{dq} = 0. \tag{10}
\]

where the variable \( q \) parameterizes the particle’s world line. For a circular orbit with constant radius \( r \), \( \left( U_\tau = \frac{dr}{d\tau} = 0 \right) \).

From equations (2) and (10), we can calculate the values of \( \nu_\phi \) and \( \alpha^{-1} \):

\[
\alpha^{-1} = \frac{d\tau}{dt} = \sqrt{A - r^2}, \tag{11}
\]

\[

\nu_\phi = \frac{\dot{A}^{1/2}}{r^2}. \tag{12}
\]

Neutrino spin precession is given by the expression \( \vec{\Omega} = \frac{\vec{G}}{A} \), where vector \( \vec{G} \) is defined as follows [8]:

\[
\vec{G} = \frac{1}{2} \left[ \vec{B} + \frac{1}{1 + u^0} \left( \vec{E} \times \vec{u} \right) \right]. \tag{13}
\]

By substituting (4), (9), (11), and (13) in \( \vec{\Omega} = \frac{\vec{G}}{A} \), the only nonzero component of frequency, i.e., \( \Omega_2 \), is obtained:

\[
\Omega_2 = \frac{\nu_\phi}{2} \left[ -\left( 1 - \frac{4M}{r \sqrt{r^2 - \frac{3}{2} r^2}} \right)^{1/2} \right]
\]

\[
+ \frac{\alpha}{1 + u^0} \left( \frac{1}{r^2} - \frac{4M}{r \sqrt{r^2 - \frac{3}{2} r^2}} \right) \frac{3}{2} \left( \frac{X}{2} \right)
\]

\[
- \frac{4M}{r \sqrt{r^2 - \frac{3}{2} r^2}} \frac{d}{dr} \left( \frac{3}{2} \right) \left( \frac{X}{2} \right) \right] = \frac{\nu_\phi}{2} \alpha^{-1}. \tag{14}
\]

If \( X \rightarrow \infty \) we recover the results of the commutative case [8]: Using equation (14), we plot \( 2 \left| \Omega_2 \right| M \) versus \( \frac{r}{2M} \) for different values of \( \eta = \frac{\nu_0}{M} \).

We observe that the maximum frequency decreases as the noncommutativity parameter increases.
Neutrino spin oscillations in the noncommutative Reissner–Nordstrom metric

In this section we investigate the effects of charge on neutrino spin oscillations in a gravitational field, so we start with the RN metric. In comparison with the earlier calculations, it is interesting to study whether this approach produces new results.

The RN metric in a noncommutative space is given by [10, 11]:

\[
dx^2 = g_{00} dr^2 - g_{\alpha\beta} d\theta^\alpha d\theta^\beta - r^2 \sin^2 \theta d\phi^2,
\]

where:

\[
g_{00} = A^2 = 1 - \frac{4M}{r} \sqrt{\frac{3}{2} X} \left( \frac{3}{2} \right) + \frac{Q^2}{4\pi^2 r^4} \left( \frac{1}{2} \right)^2.
\]

Here \( Q \) and \( M \) are the charge and the mass, respectively, of the gravitational source.

The nonzero vierbein vectors and their covariant derivatives are presented in the appendix.

Using equations (5), (6), and (8) we have the following forms for the gravi-electric and gravi-magnetic fields:

\[
E_1 = \frac{\alpha}{2} \left[ \frac{4M}{r} \sqrt{\frac{3}{2} X} \right] + \frac{Q^2}{4\pi^2 r^4} \left( \frac{1}{2} \right)^2
\]

\[
- \frac{2Q^2}{\pi^2 r^4} \left( \frac{1}{2} \right)^2 + \frac{Q^2}{4\pi^2 r^4} \left( \frac{1}{2} \right)^2
\]

\[
+ \frac{Q^2}{\pi^2 r^4} \left( \frac{1}{2} \right)^2 - \frac{2Q^2}{\pi^2 r^4} \left( \frac{1}{2} \right)^2.
\]

\[
E_2 = 0, \quad E_3 = 0,
\]

\[
B_1 = -\alpha \cos \theta, \quad B_2 = \alpha A \sin \theta, \quad B_3 = \alpha A \theta.
\]

From equations (10) and (15) we can calculate the values of \( \nu_{\psi} \) and \( \alpha^{-1} \):

\[
\alpha^{-1} = \sqrt{A - \frac{A^2}{2}}, \quad \nu_{\psi} = \frac{A'}{2\nu}.
\]

where

\[
A' = \left( 1 - \frac{4M}{r} \sqrt{\frac{3}{2} X} \right) + \frac{Q^2}{4\pi^2 r^4} \left( \frac{1}{2} \right)^2
\]

\[
- \frac{2Q^2}{\pi^2 r^4} \left( \frac{1}{2} \right)^2 + \frac{Q^2}{4\pi^2 r^4} \left( \frac{1}{2} \right)^2.
\]

By substituting (4), (13), (16), and (17) in \( \Omega = \frac{\nu_{\psi} \nu_{\phi}}{\nu_{\psi} + \nu_{\phi}} \), the only nonzero component of frequency, i.e., \( \Omega_2 \), is obtained:

\[
\Omega_2 = -\frac{1}{2} \sqrt{A - \frac{A^2}{2}} \left( \frac{1}{4} \right) \approx -\frac{1}{2} \alpha^{-1} \nu_{\psi}.
\]

Using equation (18), we have plotted 2 \( |\Omega_2| \) \( M \) versus \( r < 2M \) for different values of \( \eta = \sqrt{\theta / M} \) and the commutative case in figure 2. It is seen that the maximum frequency of oscillation is a monotonically increasing function of the noncommutativity parameter.

One can recover the results of the Schwarzschild metric (equation (14)) as a special case by taking \( Q = 0 \). It can also be shown that in the limit \( X \to \infty \), we recover the results of the commutative case presented in [12].

Phenomenological application

We consider a simple bipolar neutrino system, which helps us understand many qualitative features of collective neutrino oscillations in supernovae. The system is composed of a homogeneous and isotropic gas that initially consists of mono-energetic \( \nu_e \) and \( \bar{\nu}_e \) and is described by the flavor pendulum [13]. We introduce \( \epsilon \) as the fractional excess of neutrinos over antineutrinos: \( \epsilon = \frac{\nu_e}{\bar{\nu}_e} \). Figure 1 shows that the period of transition probability is higher for the case of the Schwarzschild metric in noncommutative spaces:

\[
T_{\Omega=0} < T_{\Omega 
eq 0}.
\]

So at a later time \( t > 0 \), we have:

\[
\epsilon_{\Omega=0} < \epsilon_{\Omega 
eq 0}.
\]

This implies that the precession frequency \( \Omega \) of the flavor pendulum as a function of the neutrino number density for the case of the Schwarzschild metric will be higher in commutative spaces.

We have interpreted the neutrino spin precession as neutrino–antineutrino oscillations, which is true for Majorana neutrinos.

Similarly, it is seen from figure 2 for the RN metric that:

\[
T_{\Omega 
eq 0} < T_{\Omega=0},
\]

which results in:

\[
\epsilon_{\Omega 
eq 0} < \epsilon_{\Omega=0}.
\]

This implies that for the RN metric the precession frequency \( \Omega \) of the flavor pendulum as a function of the neutrino number density will be higher in noncommutative spaces.

Bound on noncommutativity parameter

An interesting point is that by analyzing the diagrams of neutrino spin oscillations in noncommutative metrics in the preceding sections, we obtain the following bound for the parameter \( \eta \):

\[
\eta = \sqrt{\theta / M} > 0.1.
\]

For instance, the diagram corresponding to \( \eta = 0.1 \) is plotted in figure 3, which is not a correct physical representation of the process.
Figure 1. Neutrino spin oscillation frequency versus radius of the neutrino orbit for different values of \( \eta \) (a), and the commutative case \( X \to \infty \) (b).

Figure 2. Neutrino spin oscillation frequency versus radius of the neutrino orbit for (a) different values of \( \eta \) and (b) the commutative case \( X \to \infty \).
As we know, the most important property of a function is that each input is related to exactly one output. The diagram in figure 3 does not have this property. For one value of \( \frac{r}{2M} \) there is more than one value for 2M \( |\Omega| \); in particular, for 2M \( |\Omega| < 10 \) (as the data shows), 2M \( |\Omega| \) is parallel to the vertical axis. Therefore the diagram violates the definition of a function and cannot represent a function.

Using the fact that for the objects described by the metrics studied here we have \( m \geq m_p \), where \( m_p \) is the Planck mass, we arrive at the following bound for the noncommutativity parameter:

\[
\sqrt{\theta} > 0.1 \, l_p, \tag{24}
\]

which is consistent with the results reported in [7, 10, 14]. Here \( l_p \) is the Planck length. It is worth mentioning that an upper bound for the noncommutativity parameter was obtained in [15].

**Discussion**

In this section we discuss some interesting issues. We start with a discussion about the possible effects of noncommutativity on the geodesic equations.

In reference [16], the authors extended the properties of noncommutativity in flat space-time to curved space-time. They derived the geodesic equation, or in other words, the force on a test particle in a noncommutative curved space-time, as:

\[
\frac{d^2 x^i}{dt^2} = \left[ n^i + 3G \left( \theta^2 - \left( \bar{\theta} \cdot \bar{\theta} \right) \right) n^i + 2G \left( \bar{\theta} \cdot \bar{\theta} \right) \theta^i \right] \frac{h}{2r^2}
\]

where \( h = \frac{-2GM}{r} \) is the Newtonian potential and \( n^i = \frac{x^i}{r} \) is the radial unit vector. All the corrections are second order in \( \theta \).

The important point is that in this paper the authors considered space–time noncommutativity, so their results are different from the space–space noncommutativity studied by many physicists. For instance, it has been shown by many researchers—see, e.g., [17, 18]—that the corrections due to space–space noncommutativity on a central force potential (including gravitational and Coulomb potentials) is first order in \( \theta \) but that the corrections obtained by the authors on Newtonian potential \( -\frac{2GM}{r} \) are totally second order. (See equation (5.14) of the paper.) It is also shown in the paper that the time–space components of \( \theta^{\mu \nu} \) are proportional to the space–space components (see equation (5.13)):

\[
\tilde{\theta} = \pm \theta, \text{ where } \theta = \theta^0, \quad i = 1,2,3.
\]

So if one considers only space–space noncommutativity, i.e., \( \theta^0 = 0 \), this implies that \( \tilde{\theta} = 0 \), and the noncommutativity corrections of the geodesic equation vanish. As mentioned earlier, most of the papers devoted to the study of noncommutativity have considered only space–space noncommutativity. In this paper we have also applied only space–space noncommutativity, so there are no corrections to the geodesic equations.

It is also interesting to determine whether the neutrino spin can be directly connected to the noncommutativity parameter. In [19], the authors studied the actions of the relativistic spinless and spinning particles in the framework of noncommutative field theory and extracted \( \theta \)-modified actions using path integral representations of particle propagators in noncommutative field theory. They considered particles that interact with an external electromagnetic field. Their calculations show that the noncommutative corrections to the actions of the scalar (spin zero) and spinning particles are different. We mention the following points:

The quantum field theory corrections obtained in this paper are for particles which interact with a real external electromagnetic field \( A_\mu(x) \), with coupling \( g \), but as mentioned in the text, the fields in our paper are not real external electric and magnetic fields (the coupling \( g = 0 \) in our case) but rather classical gravi-electric and magnetic fields.

Our study and also other works on neutrino spin oscillations use general relativity, which is not a quantum field theory. But the main point is that even if we work in the framework of QFT, the corrections obtained in the mentioned paper for spin zero (scalar) and spinning particles are the same (note that in our case \( g = 0 \)), which can be verified by comparing equations (17) and (29a) in [19]. Even for \( g \neq 0 \), the corrections are more or less the same; in particular, the essential term \( p_\mu \theta^{\mu \nu} p^\nu \) is the same for spin zero and spinning particles. This means that the noncommutative corrections obtained in this paper are not due to the spin of the particles but come from the charge of the particles (which is zero for...
neutrinos), which interact with an external electromagnetic field in a noncommutative background.

Now let us comment about the collective oscillations. Collective oscillations are known to happen for neutrinos that leave, e.g., a supernova mainly along radial trajectories, but an important point is involved here.

In neither commutative nor noncommutative spaces are there spin oscillations for radial neutrinos in the Schwarzschild and RN backgrounds. As is observed from equations (14) and (18) of this paper, the only nonzero component of the frequency, i.e., \( \Omega_2 \), is in the \( \hat{\theta} \) direction. We get the same results for Schwarzschild and RN metrics in the commutative spaces by looking at equations (4-9)–(4-11) and equation (15) in reference [8] and reference [12], respectively.

Let us present a sentence from the conclusion of reference [8]:

‘According to equations (4-9)–(4-11), there is no neutrino spin flip in the case of radial motion in the Schwarzschild metric’.

For the Kerr metric the situation is different, and as mentioned in [12] and shown in [20], there are also spin oscillations for radial neutrinos. But in the current paper we have studied only Schwarzschild and RN metrics, for which there are no spin oscillations for radial neutrinos. So although the ratio of circular orbit neutrinos to radial neutrinos is small, there is no spin oscillation for radial neutrinos in Schwarzschild and RN backgrounds. Therefore spin oscillations are a relevant effect for trapped neutrinos in noncommutative spaces.

The last issue we will address is the effects of noncommutativity on the stability of circular orbits. To study the stability of the circular orbits of particles in a central force potential (including the potential of Schwarzschild and RN black holes) in commutative and noncommutative spaces, we need the noncommutative effective potential. The effects of noncommutativity of space on the circular orbits of the particles in a central force potential is studied in [18]. The authors derived the effective potential in a noncommutative background and showed that for a particle in a central force potential the noncommutative correction to the first order in \( \theta \) is given by:

\[
-\frac{1}{2r} (\hat{\theta}, \hat{l}) \frac{\partial V}{\partial r} = -\frac{1}{2r} \theta \cos \psi \frac{\partial V}{\partial r}
\]

where \( \psi \) is the angle between \( \hat{\theta} \) and \( \hat{l} \).

So the noncommutativity of space manifests itself through an angular momentum-dependent term. As discussed in [18], for not very large values of angular momentum the difference between commutative and noncommutative effective potentials is not considerable. Even for very large values of ‘\( l \)’ for which \( \cos \psi = 0 \), the contribution of the noncommutativity of space to the effective potential is zero.

It is worth mentioning that even in commutative spaces not all the circular orbits with arbitrary radius are stable. For instance, not all the orbits studied in [8] are stable. In this paper we also study both stable and unstable orbits.

**Conclusion**

In this paper we have studied neutrino spin oscillations in gravitational fields in noncommutative spaces. We have also analyzed the dependence of the frequency of neutrino spin oscillations on the radius of the orbit. For the case of the RN metric, the maximum frequency of oscillation is a monotonically increasing function of the noncommutativity parameter. For the Schwarzschild metric, the maximum frequency decreases as the noncommutativity parameter increases. We have also briefly studied the effects of noncommutativity of space on a bipolar neutrino system, which reveals many qualitative features of collective neutrino oscillations in supernovae. Finally we have obtained the bound \( \sqrt{\theta} > 10^{-1} l_p \) for the noncommutativity parameter. If any noncommutativity exists in nature, as seems to emerge from different theories and arguments, its implications should appear in neutrino oscillations in gravitational fields and in collective neutrino oscillation in supernovae.

**Acknowledgments**

S A Alavi would like to thank the Department of Physics of the University of Torino, where some of this work was done, for their hospitality. He is also very grateful to Samoil Bilenkey (JINR Dubna) and Carlo Giunti for useful discussions.

**Appendix**

The nonzero vierbein vectors and their covariant derivatives in the Schwarzschild metric are as follows:

\[
e_\mu = \begin{cases}
\left( 1 - \frac{4M}{r\sqrt{\theta}} \right)^{-\frac{1}{2}} \hat{\theta}, & 0, 0, 0, 0,
\end{cases}
\]

\[
e_\nu = \begin{cases}
\left( 1 - \frac{4M}{r\sqrt{\theta}} \right)^{-\frac{1}{2}} \hat{\phi}, & 0, 0, 0, 0,
\end{cases}
\]

\[
e_\rho = \begin{cases}
\left( 1 - \frac{4M}{r\sqrt{\theta}} \right)^{-\frac{1}{2}} \hat{r}, & 0, 0, 0, 0,
\end{cases}
\]

\[
e_\mu' = \begin{cases}
\left( 1 - \frac{4M}{r\sqrt{\theta}} \right)^{-\frac{1}{2}} (r \sin \theta) \hat{\theta}, & 0, 0, 0, 0,
\end{cases}
\]

\[
e_\nu' = \begin{cases}
\left( 1 - \frac{4M}{r\sqrt{\theta}} \right)^{-\frac{1}{2}} (r \sin \theta) \hat{\phi}, & 0, 0, 0, 0,
\end{cases}
\]

\[
e_\rho' = \begin{cases}
\left( 1 - \frac{4M}{r\sqrt{\theta}} \right)^{-\frac{1}{2}} (r \sin \theta) \hat{r}, & 0, 0, 0, 0,
\end{cases}
\]

\[
e_\mu'' = \begin{cases}
\left( 1 - \frac{4M}{r\sqrt{\theta}} \right)^{-1} \hat{\theta}, & 0, 0, 0, 0,
\end{cases}
\]

\[
e_\nu'' = \begin{cases}
\left( 1 - \frac{4M}{r\sqrt{\theta}} \right)^{-1} \hat{\phi}, & 0, 0, 0, 0,
\end{cases}
\]

\[
e_\rho'' = \begin{cases}
\left( 1 - \frac{4M}{r\sqrt{\theta}} \right)^{-1} \hat{r}, & 0, 0, 0, 0,
\end{cases}
\]

\[
e_\mu''' = \begin{cases}
\left( 1 - \frac{4M}{r\sqrt{\theta}} \right)^{-1} (r \sin \theta) \hat{\theta}, & 0, 0, 0, 0,
\end{cases}
\]

\[
e_\nu''' = \begin{cases}
\left( 1 - \frac{4M}{r\sqrt{\theta}} \right)^{-1} (r \sin \theta) \hat{\phi}, & 0, 0, 0, 0,
\end{cases}
\]

\[
e_\rho''' = \begin{cases}
\left( 1 - \frac{4M}{r\sqrt{\theta}} \right)^{-1} (r \sin \theta) \hat{r}, & 0, 0, 0, 0,
\end{cases}
\]
The nonzero vierbein vectors and their covariant derivatives in the RN metric are given by the following expressions:

\[ e_{0;r} = \frac{\delta e_{0r}}{\delta t} - \Gamma^r_{\mu\nu} e_{0\nu} = \frac{1}{2} \left( 1 - \frac{4M}{r \sqrt{\pi}} \left( \frac{3}{2}, X \right) \right)^{\frac{1}{2}} \]

\[ \times \left( \frac{4M}{r^2 \sqrt{\pi}} \frac{2}{3} X^{3/2} - \frac{4M}{r \sqrt{\pi}} X^{1/2} e^{-X} \right), \]

\[ e_{1;r} = \frac{1}{2} \left( 1 - \frac{4M}{r \sqrt{\pi}} \left( \frac{3}{2}, X \right) \right)^{\frac{1}{2}} \]

\[ \times \left( \frac{4M}{r^2 \sqrt{\pi}} \frac{2}{3} X^{3/2} - \frac{4M}{r \sqrt{\pi}} X^{1/2} e^{-X} \right), \]

\[ e_{\theta,\theta} = r \left( 1 - \frac{4M}{r \sqrt{\pi}} \left( \frac{3}{2}, X \right) \right)^{\frac{1}{2}}, \]

\[ e_{\theta,\phi} = r \left( 1 - \frac{4M}{r \sqrt{\pi}} \left( \frac{3}{2}, X \right) \right)^{\frac{1}{2}} \sin^2 \theta, \]

\[ e_{2;r} = 1, \quad e_{2,\phi} = -r \sin \theta \cos \theta, \]

\[ e_{3;r} = \sin \theta, \quad e_{3,\phi} = r \cos \theta. \]

where \( X = \frac{r^2}{\sqrt{\pi}} \).

The References section follows:

References

[1] Alavi S A and Nasseris F 2005 Running of the spectral index in noncommutative inflation Int. J. Mod. Phys. A20 4941–50
Nasseris F and Alavi S A 2005 Noncommutative decoupling inflation and running of the spectral index Int. J. Mod. Phys. D14 621–34
Khosravi N, Jalalzadeh S and Sepangi H R 2007 Quantum noncommutative multidimensional cosmology Gen. Rel. Grav. 39 899–911
Miao Y-G, Xue Z and Zhang S-J 2012 Tunneling of massive particles from noncommutative inspired schwarzschild black hole Gen. Rel. Grav. 44 555–66
Ding C and Jing J 1974 Probing spacetime noncommutative constant via charged astrophysical black hole lensing J. High Energy Phys. JHEP10(2011)052
[2] Szabo R J 2010 Quantum gravity, field theory and signatures of noncommutative space time Gen. Rel. Grav. 42 1–29
Szabo R J 2003 Quantum field theory on noncommutative spaces Phys. Rept. 378 207–99
[3] Smalagic A and Spallucci E 2003 Feynman path integral on the noncommutative plane J. Phys. A36 L467
Smalagic A and Spallucci E 2003 UV divergence-free QFT on noncommutative plane J. Phys. A36 L157
[4] Rinaldi M 2011 A New approach to non-commutative inflation Class. Quant. Grav. 28 105022
[5] Mavromatos N E 2013 Neutrinos and the universe (Naufel 11, CERN and University of Geneva 1–6 August 2011) J. Phys. Conf. Ser. 408 012003
[6] Sprenger M, Bleicher M and Nicoli P 2011 Neutrino oscillations as a novel probe for a minimal length Class. Quant. Grav. 28 235019
Sprenger M, Nicoli P and Bleicher M 2011 Quantum gravity signals in neutrino oscillations Int. J. Mod. Phys. A 26 1229–308
[7] Nicoli P 2009 Noncommutative black holes, the final appeal to quantum gravity Int. J. Mod. Phys. A 24 1229–308
Nicoli P, Smalagic A and Spallucci E 2006 Noncommutative geometry inspired Schwarzschild black hole Phys. Lett. B 632 547–51
[8] Dvornikov M 2006 Neutrino spin oscillations in gravitational fields Int. J. Mod. Phys. D 15 1017–33
[9] Weinberg S 1972 *Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity* (New York: Wiley)

[10] Alavi S A 2009 Reissner-nordstrom black hole in noncommutative spaces Acta Phys. Polon. B 40 2679–87

[11] Ansoldi S, Nicolini P, Smailagic A and Spallucci E 2007 Noncommutative geometry inspired charged black holes Phys. Lett. B 645 261–6

[12] Alavi S A and Hosseini S F 2013 Neutrino spin oscillations in gravitational fields Grav. Cosmol. 19 129–33

[13] Duan H, Fuller G M, Carlson J and Qian Y-Z 2007 Analyses of collective neutrino flavor transformation in supernovae Phys. Rev. D 75 125005

Duan H, Fuller G M and Qian Y-Z 2010 Collective neutrino oscillations Ann. Rev. Nucl. Part. Sci. 60 569–94

[14] Kim W and Lee D 2010 Bound of noncommutativity parameter based on black hole entropy Mod. Phys. Lett. A 25 3213–8

[15] Alavi S A 2008 Hyperfine splitting in noncommutative spaces Phys. Scr. 78 015005

[16] Harikumar E and Rivelles V O 2006 Noncommutative gravity Class. Quantum Grav. 23 7551

[17] Chaichian M, Sheikh-Jabbari M M and Tureanu A 2001 Hydrogen atom spectrum and the Lamb shift in noncommutative QED Phys. Rev. Lett. 86 2716

[18] Nozari K, Akhshabi S and Sadeghnezhad N 2008 Stability of circular orbits in noncommutative Schwarzschild spacetime Acta. Physica Polonica B 39 2867–77

Nozari K and Akhshabi S 2008 On the stability of circular orbits in noncommutative spaces Chaos Soliton & Fractals 37 324

[19] Gitman D M and Kupriyanov V G 2008 Path integral representation in noncommutative quantum mechanics and noncommutative version of Berezin-Marinov action Eur. Phys. J. 54 325

[20] Dvornikov M 2013 Neutrino spin oscillations in matter under the influence of gravitational and electromagnetic fields JCAP 06 015