Shahane, Shantanu; Vanka, Surya Pratap
A semi-implicit meshless method for incompressible flows in complex geometries. (English)
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Summary: We present an exponentially convergent semi-implicit meshless algorithm for the solution of
Navier-Stokes equations in complex domains. The algorithm discretizes partial derivatives at scattered
points using radial basis functions (RBF) as interpolants. Higher-order polynomials are appended to the
polyharmonic splines (PHS-RBF) and a collocation method is used to derive the interpolation coefficients.
The interpolating kernels are then differentiated and the partial-differential equations are satisfied by
collocation at the scattered points. The PHS-RBF interpolation is shown to be exponentially convergent
with discretization errors decreasing as a high power of a representative distance between points. We
present here a semi-implicit algorithm for time-dependent and steady state fluid flows in complex domains.
At each time step, several iterations are performed to converge the momentum and continuity equations.
A Poisson equation for pressure corrections is formulated by imposing divergence free condition on the
iterated velocity field. At each time step, the momentum and pressure correction equations are repeatedly
solved until the velocities and pressure converge to a pre-specified tolerance. We have demonstrated the
convergence and discretization accuracy of the algorithm for two model problems and simulated four other
complex problems. In all cases, the algorithm is stable for Courant numbers in excess of ten. The algorithm
has the potential to accurately and efficiently solve many fluid flow and heat transfer problems in complex
domains. An open source code Meshless Multi-Physics Software (MeMPhyS) [S. Shahane and S. P.
Vanka, “MeMPhyS: meshless multi-physics software”, https://github.com/shahaneshantanu/memphys]
is available for interested users of the algorithm.

MSC:
76Mxx Basic methods in fluid mechanics
76Dxx Incompressible viscous fluids
65Nxx Numerical methods for partial differential equations, boundary value problems

Keywords:
meshless method; radial basis function based finite difference; polyharmonic spline; semi-implicit method;
incompressible Navier-Stokes equation

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