Nuclear Superfluidity and Specific Heat in the Inner Crust of Neutron Stars

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We analyse the temperature dependence of pairing correlations in the inner crust matter of neutron stars. The study is done in a finite-temperature HFB approach and by using a zero range pairing force adjusted to the pairing properties of infinite neutron matter. Within the same approach we investigate how the specific heat of the inner crust depends on temperature, matter inhomogeneity, and the assumption used for the pairing force. It is shown that in a physical relevant range of densities the pairing properties of inner crust matter depend significantly on temperature. The finite-temperature HFB calculations show also that the specific heat is rather sensitive to the presence of nuclear clusters inside the inner crust. However, the most dramatic change of the specific heat is determined by the scenario used for the neutron matter superfluidity.

The inner crust of neutron stars consists of a lattice of neutron-rich nuclei immersed in a sea of unbound neutrons and relativistic electrons. Down to the inner edge of the crust, the crystal lattice is most probably formed by spherical nuclei. More inside the star, before the nuclei dissolve completely into the liquid of the core, the nuclear matter can develop other exotic configurations as well, i.e., rods, plates, tubes, and bubbles (for a discussion of inner crust matter see [1] and references therein). The thickness of the inner crust is rather small, of the order of one kilometer, and its mass is only about 1% of the neutron star mass. However, in spite of its small size, the properties of inner crust matter, especially its superfluid properties, have important consequences for the dynamics and the thermodynamics of neutron stars.

The superfluid properties of the inner crust have been considered long ago in connection with post-glitch timing observations and colling processes [2, 3, 4]. Through the neutron star matter superfluidity has been intensively studied in the last decades [5], so far only a few microscopic calculations have been done for the superfluidity of inner crust matter. The existing calculations are done in the framework of the Hartree-Fock-Bogoliubov (HFB) approach and at zero temperature [6, 7, 8, 9]. As it is well-known, the limitation to zero temperature is justified only if the pairing gap of the superfluid is much larger than its temperature. This is not generally the case for the inner crust matter, where this condition is fulfilled only for some limited density regions.

The purpose of this paper is to study how the pairing properties of inner crust matter are changing with the temperature and what are the consequences of these changes on the specific heat. The present investigation is done in a finite-temperature HFB approach. In the calculations both the mean field and the pairing field are determined self-consistently by using a Skyrme-type interaction and an effective pairing force adjusted to the superfluid properties of infinite neutron matter. The setting of the calculations follows Ref. [9], where the same two-body forces have been used for describing the pairing properties of inner crust matter at zero temperature.

The finite-temperature HFB (FT-HFB) equations have a structure similar to the HFB equations at zero temperature [10]. For zero range forces and spherical symmetry the radial FT-HFB equations can be written as:

\[
\left( \frac{h_T(r) - \lambda}{\Delta_T(r)} \right) \begin{pmatrix} U_i(r) \\ V_i(r) \end{pmatrix} = E_i \begin{pmatrix} U_i(r) \\ V_i(r) \end{pmatrix},
\]

where \(E_i\) is the quasiparticle energy, \(U_i, V_i\) are the components of the radial FT-HFB wave function and \(\lambda\) is the chemical potential. The quantity \(h_T(r)\) is the thermal averaged mean field Hamiltonian and \(\Delta_T(r)\) is the thermal averaged pairing field. In a self-consistent calculation based on a Skyrme-type force as used in this study, \(h_T(r)\) is expressed in terms of thermal averaged densities, i.e., kinetic energy density \(\tau_T(r)\), particle density \(\rho_T(r)\) and spin density \(J_T(r)\), in the same way as in the Skyrme-HF approach [11]. The thermal averaged densities are given by:

\[
\rho_T(r) = \frac{1}{4\pi} \sum_i (2j_i + 1)\left[ V_i^*(r)V_i(r)(1-f_i) + U_i^*(r)U_i(r)f_i \right]
\]

(2)

\[
J_T(r) = \frac{1}{4\pi} \sum_i (2j_i + 1)\left[ j_i(j_i + 1) - l_i(l_i + 1) - \frac{3}{4} \right] V_i^2(1-f_i) + U_i^2 f_i
\]

(3)

\[
\tau_T(r) = \frac{1}{4\pi} \sum_i (2j_i + 1)\left[ \left( \frac{dV_i}{dr} - \frac{V_i}{r} \right)^2 + \frac{l_i(l_i + 1)}{r^2} V_i^2 \right] \times (1-f_i) + \left[ \left( \frac{dU_i}{dr} - \frac{U_i}{r} \right)^2 + \frac{l_i(l_i + 1)}{r^2} U_i^2 \right] f_i,
\]

(4)

where \(f_i = [1 + exp(E_i/k_BT)]^{-1}\) is the Fermi distribution, \(k_B\) is the Boltzmann constant and \(T\) is the temperature. The summations in the equations above are over the whole positive energy quasiparticle spectrum, including the continuum states. In the latter case the summations should be replaced by an integral over the energy [12].
The thermal averaged pairing field is calculated with a density dependent contact force of the following form [13]:

\[ V(r' - r) = V_0 [1 - \eta \left( \frac{\rho(r)}{\rho_0} \right) \delta(r' - r)] \equiv V_{eff}(\rho(r)) \delta(r' - r), \]  

(5)

where \( \rho(r) \) is the baryonic density. With this force the thermal averaged pairing field is local and is given by:

\[ \Delta_T(r) = V_{eff}(\rho(r)) \frac{1}{4\pi} \sum_i (2j_i + 1) U_i^*(r) V_i(r) (1 - 2f_i) \equiv V_{eff}(\rho(r)) \kappa_T(r), \]  

(6)

where \( \kappa_T(r) \) is the thermal averaged pairing tensor. Due to the density dependence of the pairing force, the thermal averaged mean field hamiltonian \( h_T(r) \) depends also on \( \kappa_T \).

The inner crust matter is usually divided into independent Wigner-Seitz cells, each cell containing in its center a neutron-rich nucleus surrounded by unbound neutrons and permeated by a relativistic electron gas uniformly distributed inside the cell. In this study the FT-HFB equations are solved for some representative spherical cells determined in Ref. [14]. To generate far from the nucleus a constant density corresponding to the neutron gas, the FT-HFB calculations are done up to a maximum temperature \( T=0.5 \) MeV, which is covering the temperature range of physical interest. According to numerical simulations [21], the temperature in the inner crust is increasing from about \( T=0.1 \) MeV in the low-density region to about \( T=0.3 \) MeV at the inner edge of the crust. This temperature profile corresponds to the state of matter before the cooling wave is passing through the inner crust from the core region to the star surface.

In the FT-HFB calculations we use for the particle-hole channel the Skyrme effective interaction SLy4 [14], which has been adjusted to describe properly the mean field properties of neutron-rich nuclei and infinite neutron matter. For the effective pairing force the choice is more problematic. A realistic FT-HFB calculation of crust superfluidity should be based on a pairing force able to describe reasonably well at least the pairing properties of neutron matter, which forms the main part of the inner crust. However, the magnitude of pairing correlations in neutron matter is still a subject of debate. On one hand, BCS calculations based on a Gogny force commonly used in finite nuclei [14], give for the pairing gap of infinite neutron matter a maximum value of about 3.2 MeV at a Fermi momentum \( k_F \approx 0.9 \) fm\(^{-1}\) [17]. On the other hand, the microscopic calculations based on induced interactions predict for the maximum value of the gap much smaller values, of around 1 MeV [17,18,19]. These two different scenarios for the neutron matter superfluidity are used here to determine the parameters of the pairing force (5) employed in the FT-HFB calculations. Thus, for each Wigner-Seitz cell we perform two FT-HFB calculations, with two sets of parameters for the pairing force. For the first calculation we use the parameters: \( V_0 = 430 \) MeV fm\(^3\), \( \eta = 0.7 \), and \( \alpha = 0.45 \). With these parameters and with a cut-off energy for the quasiparticle spectrum equal to 60 MeV one obtains approximately the pairing gap given by the Gogny force in nuclear matter [12,20]. In the second calculation we reduce the strength of the force to the value \( V_0 = -330 \) MeV fm\(^3\). With this value of the strength we simulate the second scenario for the neutron matter superfluidity, in which the maximum gap in neutron matter is around 1 MeV. In principle, the other two parameters of the pairing force should be adjusted to the density dependence of the gap in neutron matter corresponding to the calculations based on induced interactions. However, since at present this dependence is not well-established, we keep for the parameters \( \eta \) and \( \alpha \) the same values as in the first calculation.

The FT-HFB results are analysed here for two representative Wigner-Seitz cells chosen from Ref. [14]. These cells contain \( Z=50 \) protons and have rather different baryonic densities, i.e., 0.0204 fm\(^{-3}\) and 0.00373 fm\(^{-3}\). The cells, which contain \( N=1750 \) and \( N=900 \) neutrons, respectively, are denoted below as a nucleus with \( Z \) protons and \( N \) neutrons, i.e., \( ^{180}\text{Sn} \) and \( ^{950}\text{Sn} \). For the baryonic densities and the nucleonic numbers listed above, the radii of the cells are equal to 27.6 fm and 39.3 fm, respectively. In the calculations we integrate the FT-HFB equations up to 27.6 fm for both cells. Up to this distance in the second cell there are \( N=279 \) neutrons. Since at 27.6 fm we are already far out in the region of the neutron gas, the pairing field of the second cell is not influenced by this choice of the integration radius.

As expected, the dependence on temperature of the pairing field is weak when the gap values are much larger than the temperature. This is the case for the high-density cell \( ^{180}\text{Sn} \) and the first pairing force. For all other cases the temperature dependence of the pairing field is significant. This is clearly seen in the low-density cell \( ^{950}\text{Sn} \). From Fig.1 we can see that for the second pairing force even in the high-density cell \( ^{180}\text{Sn} \) the pairing field is changing significantly with the temperature.
To analyse how the pairing correlations are globally changing in the cell as a function of temperature, in Fig.3 we plot the pairing energy per neutron for the cases in which its temperature dependence is significant. In Fig.3 are shown also the results obtained when in the corresponding cell are distributed uniformly only the neutrons. One can notice that the pairing energy of the non-uniform system, i.e., cluster plus the neutron gas, has a rather different temperature dependence compared to the uniform neutron matter. From Fig.3 we can eventually extract a critical temperature for the superfluid-normal phase transition in the non-uniform nuclear matter of the cell.

However, as seen for the cell $^{950}$Sn, we should be aware of the fact that close to this averaged critical temperature the non-uniform system may still have regions in which the pairing field has non-negligible values.

![Graph](image1)

**FIG. 1:** Neutron pairing fields for the cell $^{1800}$Sn calculated at various temperatures. The numbers 1 and 2 which follow the cell symbol (see the inset) indicate the variant of the pairing force used in the calculations. The full and the dashed lines corresponds (from bottom upwards) to the set of temperatures $T=\{0.0, 0.5\}$ MeV and $T=\{0.0, 0.1, 0.3, 0.5\}$ MeV, respectively.

![Graph](image2)

**FIG. 2:** The same as in Fig.1, but for the cell $^{950}$Sn. The full and the dashed lines corresponds (from bottom upwards) to the set of temperatures $T=\{0.0, 0.1, 0.3, 0.5\}$ and $T=\{0.0, 0.1\}$ MeV, respectively.

![Graph](image3)

**FIG. 3:** Pairing energy per neutron as a function of temperature. The calculated values, done in step of $\Delta T=0.1$ MeV, are joined by straight lines. The symbol "n" following the cell symbol (see the inset) refers to the results obtained when in the corresponding cell are distributed uniformly only the neutrons. The rest of the notations are the same as in Fig.1.

We turn now to the specific heat provided by the FT-HFB calculations. The specific heat of a given cell of volume $V$ is defined by:

$$C_V = \frac{1}{V} \frac{\partial E(T)}{\partial T},$$

(7)

where $E(T)$ is the total energy of the baryonic matter inside the cell, i.e.,

$$E(T) = \sum_i f_i E_i.$$

(8)

Due to the energy gap in the excitation spectrum, the specific heat of a superfluid system is dramatically reduced compared to its value in the normal phase. Since the specific heat depends exponentially on the energy gap, its value for a Wigner-Seitz cell is very sensitive to the local variations of the pairing field induced by the nuclear clusters. This can be clearly seen in Fig.4, where the specific heat is plotted for the cell $^{1800}$Sn and for the neutrons uniformly distributed in the same cell. One can notice that at $T=0.1$ MeV and for the first pairing force the presence of the cluster increases the specific heat by about 6 times compared to the value for the uniform neutron gas. A comparable effect of the nuclear cluster on the specific heat of the cell $^{1800}$Sn was found earlier by using in Eq.(8) the quasiparticle spectrum provided by a zero-temperature HFB calculation [7, 8]. This approximation is justified only when the temperature variation...
of the pairing field in the Wigner-Seitz cells is small, as in the case of high density cells and a Gogny type force.

The behaviour of the specific heat for the low- density cell $^{950}$Sn is shown in Fig.5. For the first pairing force we can also see that at $T=0.1$ MeV the cluster increases the specific heat by about the same factor as in the cell $^{1800}$Sn. However, for the second pairing force the situation is opposite: the presence of the nucleus decreases the specific heat instead of increasing it. From Fig.5 we can also see that the specific heat predicted by the two pairing forces are not very much different (notice the expanded scale) for this low-density cell. Due to the fast decreasing of the pairing correlations with the temperature, at $T=0.3$ MeV all calculations provide almost the same specific heat.

We have also performed FT-HFB calculations for two other cells with 40 protons and having the densities equal to 0.0475 fm$^{-3}$ and 0.00159 fm$^{-3}$. We found that the temperature dependence of the pairing field and specific heat in the first and in the second cell with $Z=40$ is very similar to the one in the cells $^{1800}$Sn and $^{950}$Sn, respectively.

In conclusion, we have shown that the superfluid properties and the specific heat of inner crust matter are affected rather strongly and in a non-trivial way by the temperature of the crust. Therefore, the temperature dependence should be included in the microscopic calculations aiming at describing accurately the superfluid properties of inner crust matter. However, the most serious problem in microscopic calculations remains the pairing interaction. As seen above, shifting to the second pairing force adjusted to the nuclear matter properties calculated with induced interactions, the specific heat of high density cells increases by several orders of magnitude. Consequently, for the second pairing force the neutron and the electron contributions to the total specific heat of high-density cells are becoming competitive, at variance with the results based on a Gogny force scenario [8]. To analyse the implications of these two different scenarios for the pairing force on physical properties as the cooling time one needs to calculate within the FT-HFB approach the heat diffusion along the whole inner crust. The successive step would be to include also into the microscopic calculations the ground state correlations of the inner crust matter, which can modify significantly the neutron single-particle properties related to the heat diffusion (e.g., effective mass and the mean free path) [22]. It is to be expected that such microscopic calculations could be eventually used to determine accurately the consequences of inner crust superfluidity on the cooling time of neutron stars.

FIG. 4: Specific heat for the cell $^{1800}$Sn as a function of temperature. The notations used in the inset and the representation of the calculated values are the same as in Figs.1-3.

FIG. 5: The same as in Fig.4, but for the cell $^{950}$Sn.

The most striking fact seen in Fig.4 is the huge difference between the predictions of the two pairing forces. Thus, for $T=0.1$ MeV this difference amounts to about 7 orders of magnitude. This discrepancy between the predicted values of specific heat, generated by the changes in the pairing gaps, is decreasing rapidly with the temperature, but it remains significant up to $T=0.5$ MeV. From Fig.4 we can see also that for the cell $^{1800}$Sn the effect of the cluster on specific heat is similar for both pairing forces.

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