A GaAs/AlAs superlattice as an electrically pumped THz acoustic phonon amplifier

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Abstract. In a semiconductor superlattice (SL), phonon-assisted electron transitions can occur under a quasi-population inversion, brought about by electrical biasing. This paper demonstrates the amplification of an optically excited quasi-monochromatic phonon beam by stimulated emission of phonons. Coherent phonons are generated by ultrafast optical excitation of a generator SL and passed once through a dc biased, GaAs/AlAs gain SL. A 20\% increase in the phonon flux is detected when pumping is applied to the gain superlattice, which corresponds to an acoustic gain coefficient of 3600 cm\textsuperscript{−1}. A theoretical model of the phonon amplification is presented that also includes the effects of disorder in the SL. It is found that the amplification process is robust in the presence of disorder and good agreement is obtained with the main features of the experimental observations.
1. Introduction

There has been considerable interest in the generation and detection of coherent terahertz (THz) acoustic phonons in recent years [1–4]. Primarily, this has been driven by the potential applicability of high frequency sound in such areas as: the non-destructive probing of nanostructures [5], high frequency modulation of light [6], coherent control of electron transport in heterostructures [7, 8] and THz spectroscopy [9]. Consequently, THz sound has been studied in a number of semiconductor materials and heterostructures, such as bulk Si, bulk GaAs [10] and GaAs/AlAs superlattices (SLs) [2, 11].

Traditional methods used to generate and detect coherent high frequency sound waves rely on expensive pulsed laser systems and complex optical setups, which confine most potential applications to the laboratory. There has been an effort over the last few years to create acoustic analogues of active optical devices [12, 13], such as lasers and optical amplifiers, which may provide a highly efficient source of THz sound. As well as investigations into the active components of such structures, it has also been shown [14] that high Q THz acoustic cavities can provide efficient confinement of sound at selected frequencies.

Different methods of coherent THz phonon generation (and amplification) have been suggested [15, 16], but no turn-key THz acoustic analogue of the laser has been developed to date. However, previous studies of acoustic gain in semiconductor SLs have been made both theoretically [17, 18] and experimentally [12], which have shown great promise for THz phonon amplification and oscillation in these systems. In particular, coherent measurements of GaAs/AlAs SLs have shown spectral line narrowing in SLs under electrical bias [19].

The mechanism of sound amplification in the SL structures is via the stimulated emission of phonons, under ‘hopping’ electron transport between quantum wells (QWs), in an electrically biased SL [12, 19]. If the SL is biased such that the electron system forms a Stark-ladder of states [20], in which the inter-well coupling is weak, electrons are able to tunnel from one QW to the neighbouring QW. Importantly, these transitions can be stimulated by an incident phonon. When the energy separation, Δ, between the Fermi levels of the QWs is close to the incident phonon energy, a quasi-population inversion can produce single mode amplification via stimulated emission. Such a structure is the acoustic analogue of a semiconductor optical amplifier and could easily be incorporated into an acoustic cavity as a gain medium, to make a saser (for sound amplification via stimulated emission of radiation).

The aim of the current study is to demonstrate single pass amplification of THz acoustic phonons by an electrically biased GaAs/AlAs SL, and to subsequently model this in the context of realistic, non-ideal, devices. We present experimental results obtained using a
Figure 1. (a) A schematic of the device structure. (b) The superconducting transition for an Al bolometer. (c) A photograph of the optical mesa with contact and bond wire. (d) The device current–voltage characteristics.

superconducting transition edge bolometer to detect the phonons propagated through the SL at low temperatures, and a theoretical model which accounts for the disordered nature of real structures.

2. Experiments

The basic principle of the experiment was to generate a quasi-monochromatic beam of phonons, pass them through an amplifier SL, and detect the power in the emergent phonon beam with the pumping to the amplifier on and off. This was done using a structure with two SLs, the top for the generation (the seed phonons) and the bottom for the amplification (figure 1(a)). Detection was made using a superconducting bolometer on the back surface of the substrate.

As a result of the difference in acoustic impedance of GaAs and AlAs, the SL phonon dispersion becomes folded into a mini-zone, which is defined by the super-period of the structure. Narrow stop gaps open up at phonon wave vectors, $q$, corresponding to the phonon Bragg reflections at the mini-zone centre and mini-zone edge. By design the SLs in our experimental structure had different periods, ensuring the stop gaps in the folded dispersion did not overlap. Critically, this allowed the optically excited phonons, generated near the first mini-zone centre stop gap of the generation SL, to pass through the amplifier (active) SL unimpeded (with approximately the same velocity as in the surrounding bulk material).

The experimental device was grown by molecular beam epitaxy on a 0.35 mm GaAs substrate. An $n^+$ ($2 \times 10^{18}$ cm$^{-3}$) bottom contact was grown first, which was followed by a 50
period, uniformly $n$-doped ($2 \times 10^{16}$ cm$^{-3}$), GaAs/AlAs SL. The GaAs wells were nominally 5.9 nm thick and the AlAs barriers were 3.9 nm thick. The top $n^+$ contact was grown above the doped SL to enable the application of an electric field across this lower (active) SL. The contact layers were separated from the SL by 20 nm GaAs buffers, which had a ramped doping. Above the top contact a 20 nm undoped GaAs buffer separated the $n^+$ layer from the upper GaAs/AlAs (generation) SL. In the undoped, 40 period, top SL the GaAs well layers were 4.9 nm thick and the AlAs barrier layers were 3.3 nm thick. Finally, the structure was capped with a 20 nm GaAs layer.

The experimental device was a 400 $\mu$m diameter optical mesa fabricated by photolithography and wet etching (figure 1(c)). A bottom contact was made to the ‘floor’ of the chip, and the top contact was made by etching a ring-shaped hole in the mesa, through the upper SL, to the top contact layer. The contact metallization was made using a 1 : 1 (by mass) alloy of In:Ge, capped with gold, and annealed at a temperature of 350°C for 30 seconds.

For a control experiment it was necessary to generate a broadband heat pulse to pass through the active SL. This was achieved by completely etching away the top (generator) SL and depositing the metal contact over the full area of the top contact layer. Optically exciting the In/Ge/Au film provided the heat pulse.

Directly opposite to the experimental device (on the back surface of the substrate) a superconducting aluminium bolometer was fabricated. The detection region of the bolometer was $40 \times 40$ $\mu$m$^2$, which consisted of a ‘u’-shaped Al strip with a length-to-width ratio of 10 : 1 and thickness $\sim$30 nm. The bolometer and contact pads were formed by vacuum evaporation from an alumina crucible, creating a granular film in the lithographically defined shape. The bolometer was characterized by its superconducting transition, which is shown in figure 1(b). All measurements were made at $\approx$2 K (on the transition), where the bolometer resistance was $\sim$50 $\Omega$.

Figure 1(d) shows the I–V characteristic for the SL device mesa. The switch on threshold occurs at a bias voltage, $V_{\text{bias}}$, of 250 mV, which probably corresponds to the field required to align the Fermi energy of the emitter with the first quantum well. Once the threshold is reached the current increases monotonically until electric field domains begin to form, at a bias of about 660 mV. In the monotonic region the Stark splitting is given approximately by

$$\Delta \approx \frac{V_{\text{bias}} - 250}{50} \text{meV},$$

where $V_{\text{bias}}$ is given in mV.

The top SL was optically excited, with light pulses of $\sim$120 fs duration and energy density $\sim$10 $\mu$J cm$^{-2}$, from a tuneable, mode-locked, Ti:sapphire pulsed laser, thus generating quasi-monochromatic phonons at the mini-zone centre of the SL phonon dispersion. It has been previously shown that these modes leak out of the SL [21] and may propagate through the sample [22]. Calculation of the generation SL phonon dispersion, using the Rytov model [23], gives a phonon frequency of 620 GHz at the mini-zone centre, which corresponds to a phonon energy, $h\omega$, of 2.6 meV. This narrow band of modes passed through the phonon amplifier and the substrate to be incident on the bolometer.

It is important that no significant fraction of the excitation light pulse is absorbed by the active SL, resulting in the direct generation of phonons there. In practice, there are two factors that ensure the near total absorption of light in the upper layers of the sample. Firstly, for efficient generation in the top SL the optical excitation energy was tuned just above the heavy hole exciton resonance, resulting in high absorption [22]. Secondly, in the event of a small amount of pump light passing through the generation SL, it must also pass through the 500 nm thick GaAs top contact layer before reaching the active SL. The laser wavelength is centred on $\lambda = 760$ nm,
and the corresponding photon energy (1.64 eV) is greater than the GaAs band gap energy at 2 K (1.52 eV), ensuring further optical absorption. Even allowing for the finite bandwidth of the transform-limited laser pulses, which is, in wavelength terms, about 5 nm, little light is able to penetrate through the top, phonon generator, SL and reach the active SL. Similarly, the skin depth of the metal film (≈13 nm for light of wavelength 760 nm) was significantly exceeded by the thickness (130 ± 5 nm) of the contact metallization, again ensuring no light was incident on the active SL in the control experiment. The fact that the optical seed pulse is fully absorbed before reaching the gain SL is reflected in the I–V characteristic; there is no significant difference in the I–V characteristic with the sample under illumination compared to in the dark.

A typical bolometer signal as a function of time, t, is shown in figure 2. The background signal, at \( t = 0 \), due to electromagnetic radiation has been removed to leave only the acoustic component of the signal. The phonon signal is derived from the change in resistance of the bolometer, which is strongly dependent on its temperature. The plot of figure 2, therefore, represents the power in the phonon flux incident on the bolometer as a function of time. Zero time represents the incidence of the optical pulse on the top surface of the mesa. The arrival times of the longitudinal acoustic (LA) and the transverse acoustic (TA) phonon modes are 75 ± 1 ns and 105 ± 1 ns, respectively, and are marked. This is in excellent agreement with time-of-flight estimates for the sample. The acoustic wave speeds for these modes are 4730 ms\(^{-1}\) for the LA and 3340 ms\(^{-1}\) for the TA [24]. Given the 350 µm thickness of the substrate, the estimates for the arrival times are 75 ns and 105 ns for the LA and TA, respectively.

The quasi-population inversion, necessary for amplification, is only realized for LA phonons propagating near-parallel to the SL axis. Therefore, the focus of the following discussion will be the LA region of the curve in figure 2, marked by the hatched region. The signal intensity in this region is proportional to the power in the LA phonon flux.

To characterize the gain profile as a function of pump bias, we swept through a range of bias (Stark-splitting) values. The normalized plot of LA power as a function of Stark splitting is shown in figure 3 (red line). As the bias is increased and the device threshold is passed, the
Stark-splitting increases and so the power in the phonon flux also increases. A peak is observed at a Stark-splitting energy of 3 meV and at high Stark-splitting values the curve shows a region of phonon absorption in the SL.

As the current is passed through the active SL, a steady power is dissipated in the device. It is possible that this results in a constant background heating of the bolometer, which shifts the working point and hence changes its sensitivity to the LA pulse. In order to check that the increase in the LA signal was due to amplification in the SL and not a change in the bolometer sensitivity, we also measured the bias dependence of the signal due to a broadband heat pulse using the control sample. Under optical excitation with the 120 fs pulses, the metal film produces a broadband heat pulse. The bolometer signal for the heat pulse shows no features at any Stark-splitting (figure 3 (black line)). There are two reasons for this behaviour. Importantly, only a very small fraction of the power in the pulse is resonant with the Stark-splitting, due to its broadband nature, and any amplification would be small compared to the total power in the pulse. Secondly, again due to the broad band nature of the pulse, there exists within it a phonon energy resonant with all measured Stark-splitting values. Most importantly, the lack of features in the signal shows that the sensitivity of the bolometer is not affected by the dissipation in the active SL.

We conclude, therefore, that the resonant peak is due to the amplification of the 2.6 meV phonon beam. We can now make a more quantitative assessment of the gain properties. The normalized curve (figure 3 (red line)) is 20% higher at 3 meV than at small Stark-splitting values, which represents a large gain for a single pass of the amplifier SL. When characterizing laser gain media it is common practice to obtain the gain coefficient $g(\nu)$ per unit length; defined as

$$I = I_0 e^{g(\nu)l},$$
where $I$ and $I_0$ are the final and initial intensities, respectively, $\nu$ is the frequency of the amplified mode and $l$ is the length of the gain medium. We may substitute the initial and final intensities for power given the constant excitation geometry in our experiments. For our structure the gain coefficient is measured as 3600 cm$^{-1}$.

Whilst this value for the gain coefficient is large, compared to typical laser gain media, for an accurate comparison it must be normalized to the wavelength of the radiation involved. THz acoustic phonons in GaAs have a wavelength of the order 10 nm and in many lasers the light has a wavelength of the order 1 $\mu$m (Nd : YAG, for example). Even with this factor of 100 the gain coefficient is still large, and this must be the case due to the small size of the device. Indeed, it is expected to be; from the gain coefficient we can estimate the phonon increment (the difference in phonon emission and absorption probabilities per unit time) to be, $s_l g(\nu) = 2.5 \times 10^9$ s$^{-1}$, where $s_l$ is the longitudinal sound velocity. This is in order of magnitude agreement with the theory for ideal SLs [17]. However, the width of the peak in the experimental curve is broad; this is not predicted by the theory for ideal SLs and maybe due to broadening of the quantum well levels as a result of scattering. More importantly, the bias dependence of the increment, predicted by the ideal SL theory, does not match the experimental data: according to that theory, amplification occurs if the Stark splitting exceeds the phonon energy, and attenuation is expected in the opposite case. The following discussion will illustrate the role of disorder in the amplification process and show how including such disorder in its theory reproduces the experimental observations.

3. Theory

Previously, a theory of acoustic phonon amplification in electrically biased weakly-coupled SLs was developed for the case of perfect structures [17]. To address the applicability of actual structures for phonon amplification and generation, it is essential to analyse the influence of disorder on the magnitude and spectral dependence of the phonon amplification coefficient. Note that in terahertz quantum cascade optical lasers, which employ a similar principle of operation, disorder plays a negative role, and realization of lasing requires the use of high-quality growth techniques. As we show below, for sasers the situation is crucially different. In fact, disorder makes phonon amplification a more robust phenomenon, possible for a wide range of the SL parameters. In addition, we find that disorder can change qualitatively the bias dependence of the phonon amplification coefficient in comparison to that inherent for ideal SLs. Physically, this happens because disorder gives rise to a redistribution of electrons among the quantum levels (QWs) of the SL, strongly enhancing the inversion for some QW pairs. Increasing the bias, we bring different portions of the SL to the state of such an enhanced inversion. As a result, a large amplification coefficient can be realized in a broad range of Stark-splitting values. Naturally, a similar consideration applies to the phonon attenuation, realized for a distinct range of Stark splitting.

Actually, disorder influences the phonon amplification in the SL in two ways. First of all, as a result of electron scattering, it gives rise to the uniform broadening of the quantum levels of the QWs in the SL. Such broadening is expected to suppress phonon amplification. Indeed, important phonon-induced inter-well transitions occur for electrons close to the quasi-Fermi levels of distinct QWs. ‘Broadening’—both statistical due to finite temperature, and dynamical due to scattering—gives rise to a decrease of the population inversion in the system. In addition, spatial fluctuations of the SL parameters lead to deviation of the Stark splitting of particular
Figure 4. The scheme of the QW pair in disordered SL. The dashed lines show position of the quantum levels in ideal SL, while the thick solid lines indicate actual levels shifted by $\delta E^{(l)}$ and $\delta E^{(l+1)}$ in $l$-th and $(l + 1)$-th QWs. Also indicated are the mismatched quasi-Fermi energies, $E_F^{(l)}$ and $E_F^{(l+1)}$, and the Stark splitting, $\Delta^{(l)}$.

QW pairs from its average value, $\Delta$. In ideal structures strong amplification is predicted in a relatively narrow range of $\Delta$. Moreover, amplification is realized for $\Delta > \hbar \omega$, while for $\Delta < \hbar \omega$ phonons are attenuated. Intuitively, these properties of phonon amplification are expected to suppress the magnitude of the phonon amplification in actual disordered structures. However, as we show below, disorder also results in a redistribution of electrons among the QWs. Such a redistribution increases the population inversion for some pairs of QWs, and on the other hand, electrostatics brings some order into the random distribution of $\Delta$, giving rise to large values of average phonon increment.

Let us first determine the phonon amplification characteristics for a pair of QWs, numbered $l$ and $l + 1$, assuming some Stark splitting, $\Delta^{(l)}$, between them, and distinct quasi-Fermi levels, $E_F^{(l)}$ and $E_F^{(l+1)}$ (figure 4). In the case of strong enough intra-well electron scattering, it is more reasonable to treat inter-well transport within the so-called sequential tunnelling approach, rather than Wannier–Stark hopping model [25]. For the probability of the phonon emission under the electron transition from $l$th to $(l + 1)$th QW, $P_{\text{em}}^{(\text{down})}$ in this case we obtain:

$$P_{\text{em}}^{(\text{down})} = \frac{2\pi}{\hbar} \frac{\hbar E_F^2 J \omega}{2 \rho S N_L d} \sum_k f_l(E_k) (1 - f_{l+1}(E_{k-q})) \delta \Gamma(E_k - E_{k-q} - \hbar \omega + \Delta^{(l)}).$$

(1)

Here we assume standard deformation potential electron-longitudinal acoustic phonon coupling with deformation potential constant $E_1$, and use the following notations: $J$ is the inter-well electron–phonon form-factor, $\omega$ is the phonon frequency, $\rho$ is the material density, $s_l$ is the longitudinal sound velocity, $d$ is the SL period, $S$ is the SL cross-section, $N_L$ is number of QWs in the SL, $k$ is two-dimensional (2D) electron wavevector, and $f_l(E_k)$ is electron distribution function in $l$th QW. To account phenomenologically for the level broadening, instead of the Dirac $\delta$-function in equation (1) we put $\delta \Gamma(E) = \frac{\Gamma}{\pi (E^2 + \Gamma^2)}$, where $\Gamma$ is the level broadening, and $\lim_{\Gamma \to 0} \delta \Gamma(E) = \delta(E)$. We are focused on consideration of phonons propagating close to the axis of the SL, and having a small in-plane component of wavevector, $q_{\parallel}$, since, in the case of an ideal SL, their amplification is strong. If the change of in-plane kinetic energy of electrons under the phonon-induced transition is less than $\Gamma$, i.e. $\Gamma^2 > 2\hbar^2 q_{\parallel}^2 E_F/m$, where $m$ is
Figure 5. The illustration of the population inversion in disordered SL. The two parabolas correspond to the electron subbands of the adjacent QWs and the arrows indicate the inter-well electron transitions assisted by the phonon emission. One can see that mismatch of the quasi-Fermi energies reduces the population of the final states and, thus, enhances the inversion in the system.

the electron effective mass, from equation (1), we obtain

\[ P^{(\text{down})}_{\text{em}} = \frac{1}{N_L} \alpha_0 \frac{2\Gamma}{(\hbar\omega - \Delta^{(l)})^2 + \Gamma^2} \int dE f_l(E)(1 - f_{l+1}(E)), \]  

(2)

where \( \alpha_0 = \frac{E_{\text{F}0} J_m}{2\pi \rho \xi \hbar^2} \) is some characteristic phonon emission rate. For actual SL structures this can exceed 10^8 s^{-1}. Similarly, we can obtain expressions for the phonon emission rate under the transitions from the right to the left QW, as well as phonon absorption rates. The phonon amplification coefficient is \( \alpha^{(l)} = P^{(\text{down})}_{\text{em}} + P^{(\text{up})}_{\text{em}} - P^{(\text{up})}_{\text{ab}} - P^{(\text{down})}_{\text{ab}} \), where 'up'-probability is that corresponding to the transitions from \( l+1 \) to \( l \) QW. For \( \alpha^{(l)} \) we obtain

\[ \alpha^{(l)} = \frac{2\Gamma}{N_L} \alpha_0 \left( \frac{E_F^{(l+1)} - E_F^{(l)}}{(\hbar\omega - \Delta^{(l)})^2 + \Gamma^2} - \frac{E_F^{(l+1)} - E_F^{(l)}}{(\hbar\omega + \Delta^{(l)})^2 + \Gamma^2} \right). \]  

(3)

In this equation we neglect the contribution that describes very weak phonon amplification or attenuation in a SL with no Fermi energy mismatch; the contribution of the intra-well phonon-assisted transitions is also very small and can be neglected. As we see, \( \alpha^{(l)} \) can be large if the Fermi-energy mismatch considerably exceeds the level broadening (the condition for the small in-plane energy transfer is still valid, since \( 2\hbar^2 q_{\parallel}^2 / m \) is typically about few \( \mu \text{eV} \)). This result can be qualitatively understood from figure 5, where the electron transitions are shown for the case of Fermi energy mismatch. As we see, in this case the number of empty states for the phonon-assisted transitions is large, which enhances the inversion. One can also see that the increment does not depend explicitly on temperature, as well as on the lateral phonon wavevector, until the condition \( \Gamma^2 \gg 2\hbar^2 q_{\parallel}^2 E_F / m \) is satisfied.

To calculate the average value of the increment, we have to average expression (3) by the actual distribution of \( E_F^{(l+1)} \) and \( \Delta^{(l)} \). These values, however, are not those imposed independently by the disorder of the SL. In fact, the fluctuating value is the magnitude of the quantization energy, \( \delta E^{(l)} \) (figure 4). Physically, the reason for fluctuation of the quantization energy could
be fluctuation of the SL layer thickness, or the Coulomb potential of the randomly distributed charged impurities. Using the distribution of $\delta E^{(l)}$ as an input, we have to find the distributions of $E_{F}^{(l)}$ and $\Delta^{(l)}$ to calculate the average value of the phonon increment. This task can be accomplished by a self-consistent solution of the Poisson and transport equations. In the typical case where the characteristic spatial scale of $\delta E^{(l)}$ exceeds considerably both the SL period and characteristic electron Fermi wavelength, a 1D local solution of Poisson’s equation is possible, and a quasi-classical approach to the lateral transport of confined electrons in the QWs is valid. From the Poisson equation we obtain

$$\varepsilon^{(l+1)} = \varepsilon^{(l)} + \frac{e^{2}n(E_{F}^{(l)})d}{\kappa \kappa_{0}} - \frac{e^{2}Nd^{2}}{\kappa \kappa_{0}}, \tag{4}$$

where $\varepsilon^{(l)}$ is an electric field at the left edge of the $l$th QW multiplied by $d$, $n(E_{F})$ is the dependence of the electron density on the quasi-Fermi energy, $N$ is the doping density, and $\kappa$, $\kappa_{0}$ are the dielectric permittivity and absolute dielectric constant, respectively. From the Poisson equation we also obtain the following expression for $\Delta^{(l)}$:

$$\Delta^{(l)} = \varepsilon^{(l)} + \frac{e^{2}n(E_{F}^{(l)})d}{\kappa \kappa_{0}} - \frac{e^{2}Nd(d + d_{QW})}{2 \kappa \kappa_{0}} - \frac{e^{2}d_{QW}}{2 \kappa \kappa_{0}} \left( \frac{1}{4} - \frac{1}{\pi^{2}} \right) \left( n(E_{F}^{(l)}) - n(E_{F}^{(l+1)}) \right) + \delta E^{(l)} - \delta E^{(l+1)} \tag{5}.$$

Here, $d_{QW}$ is the QW thickness. In the derivation of equation (5), we assumed that the magnitude of the electric field in the system is small, and the electron wavefunctions are the same as they are in zero electric field. This is reasonable since the quantization energy in our structure exceeds 100 meV, while $\Delta^{(l)}$ are about few meV.

The transport equations in the steady-state follow from the continuity condition:

$$j_{\text{tun}}^{(l-1)} - j_{\text{tun}}^{(l)} + \nabla \cdot j_{\text{lat}}^{(l)} = 0 \tag{6}.$$

Here, $\nabla = (\partial/\partial x, \partial/\partial y)$ is the 2D nabla operator, $j_{\text{tun}}^{(l)}$ is tunnelling electron flux density from $l$th to $(l + 1)$th QW, and $j_{\text{lat}}^{(l)}$ is lateral flux in the $l$th QW. For $j_{\text{tun}}^{(l)}$ we use the following expression obtained within the sequential tunnelling model [25]:

$$j_{\text{tun}}^{(l)} = \frac{2mTt^{2} \Gamma}{\pi \hbar^{3}} \frac{1}{\Gamma^{2}} \ln \left( \frac{1 + \exp(E_{F}^{(l)}/T)}{1 + \exp((E_{F}^{(l+1)} - \Delta^{(l)})/T)} \right). \tag{7}$$

Here, $T$ is temperature in energy units and $t$ is inter-well tunnelling matrix element ($t$ is equal to one fourth of the SL miniband width in zero electric field). Note that the tunnelling flux in equation (7) is due to disorder-induced inter-well transitions. Phonon-assisted transitions, important for phonon amplification, usually provide a minor contribution to the total current. The lateral flux can be written down in the drift-diffusion approximation:

$$j_{\text{lat}}^{(l)} = -n(E_{F}^{(l)})(\mu/e) \nabla (\phi^{(l)} + \delta E^{(l)}) - D \nabla n(E_{F}^{(l)}), \tag{8}$$

where $\phi^{(l)}$ is 2D electron electrostatic energy averaged over the confined electron wavefunction, $\mu$ is lateral electron mobility, and $D$ is electron diffusion coefficient. Using the Einstein relation, we obtain

$$j_{\text{lat}}^{(l)} = -n(E_{F}^{(l)})(\mu/e) \nabla (\phi^{(l)} + \delta E^{(l)} + E_{F}^{(l)}). \tag{9}$$

In general, the formulated 2D problem is quite complicated, requiring the use of sophisticated numerical techniques. We restrict ourselves by consideration of two extreme cases that allow
reduction to 1D equations. They rely on the properties of the lateral transport of electrons. If \( \mu \) is high, which corresponds to efficient lateral transport, to fulfil the continuity equation (6) one has to assume that \( \phi^{(l)} + \delta E^{(l)} + E_F^{(l)} = C^{(l)} = \text{const} \). As a result, for \( \Delta^{(l)} = \phi^{(l)} + \delta E^{(l)} - \phi^{(l+1)} - \delta E^{(l+1)} \) we have \( \Delta^{(l)} = C^{(l)} - C^{(l+1)} - (E_F^{(l)} - E_F^{(l+1)}) \). In the following we assume that the system allows an average uniform configuration, i.e. the average electron density (and, correspondingly, quasi-Fermi energy) does not change from one QW layer to another. Under this assumption, we obtain
\[
\Delta^{(l)} = \Delta - (E_F^{(l)} - E_F^{(l+1)}). \tag{10}
\]
In the opposite case of low \( \mu \), in equation (6) one can neglect the lateral flux and the continuity equation requires local continuity of \( j_{\text{lin}}^{(l)} \) (its independence on \( l \)). As a result, from equation (7) it is possible, in general, to link
\[
\Delta^{(l)} = F(E_F^{(l)}, E_F^{(l+1)}), \tag{11}
\]
where \( F \) is some function. One can see that in both cases of low and high electron mobility, using equations (4) and (10) or (11) along with expression (5) for \( \Delta^{(l)} \), mathematically the problem can be formulated by the following system
\[
E_F^{(l+1)} = f(E_F^{(l)}, E_F^{(l)}, \delta E^{(l)}, \delta E^{(l+1)}), \quad \varepsilon^{(l+1)} = g(E_F^{(l)}, \varepsilon^{(l)}), \tag{12}
\]
which must be supplemented by the proper boundary conditions describing, for example, the properties of the contacts. Even in this 1D form, the problem is complicated. In this paper we address its solution in the linear response case, where \( \delta E^{(l)} \) are assumed to be small. In addition, we consider an infinite SL. In this case, the boundary conditions are the finiteness of \( \delta E_F^{(l)} \) and \( \delta \varepsilon^{(l)} \) for \( l \to \pm \infty \). In both extreme cases of the lateral transport efficiency, the problem allows a mathematically similar solution, we may now proceed with its consideration, leaving estimates of the transport characteristics of the actual SL for the end of the section.

We introduce the deviations of \( E_F^{(l)} \) and \( \varepsilon^{(l)} \) from their values for an ideal SL (with \( \delta E^{(l)} = 0 \), \( \delta E_F^{(l)} \) and \( \delta \varepsilon^{(l)} \), and for vector \( \chi^{(l)} = (\delta E_F^{(l)}, \delta \varepsilon^{(l)}) \), from equation (12) we obtain
\[
\chi^{(l+1)} = \hat{A}\chi^{(l)} + x^{(l)}, \tag{13}
\]
where \( \hat{A} \) is \( 2 \times 2 \) matrix whose elements are determined by the linear expansion of equation (12),
\[
x^{(l)} = \begin{pmatrix} \xi (\delta E^{(l)} - \delta E^{(l+1)}) \\ 0 \end{pmatrix},
\]
where \( \xi \) is some coefficient. Naturally, the general form of the solution of equation (13) is the following:
\[
\delta E_F^{(l)} = \sum_{l'=\pm\infty}^{\infty} B_{ll'} \delta E^{(l')}, \quad \varepsilon^{(l)} = \sum_{l'=\pm\infty}^{\infty} A_{ll'} \delta E^{(l')}, \tag{14}
\]
where \( A_{ll'} \) and \( B_{ll'} \) are to be obtained from (13). The details of this solution are described in the appendix. For the calculation of the average increment, we need to link the values \( \delta E_F^{(l)} - \delta E_F^{(l+1)} \) and \( \Delta_l - \Delta \) with \( \delta E \). From (14) it is straightforward to obtain a similar expansion for these values:
\[
\delta E_F^{(l)} - \delta E_F^{(l+1)} = \sum_{l'=\pm\infty}^{\infty} B_{ll'} \delta E^{(l')}, \quad \Delta^{(l)} - \Delta = \sum_{l'=\pm\infty}^{\infty} A_{ll'} \delta E^{(l')}. \tag{15}
\]
To arrive at the following approximate form, we make the assumption that $\delta E^{(l)}$ obeys a Gaussian distribution, with dispersion $\Delta E$, and that there is no correlation of $\delta E^{(l)}$ for different $l$. In this case, for the average value of the increment, $\bar{\alpha}$, we obtain

$$
\bar{\alpha} = a_0 \sqrt{\frac{2}{\pi}} \frac{K \Gamma}{\gamma^3 \Delta E} \left( \int_{-\infty}^{\infty} dx \exp \left( -\frac{x^2 \Gamma^2}{2\gamma^2 \Delta E^2} \right) \frac{1}{(x - (\Delta - \hbar \omega) / \Gamma)^2 + 1} \right) 
$$

$$
+ \int_{-\infty}^{\infty} dx \exp \left( -\frac{x^2 \Gamma^2}{2\gamma^2 \Delta E^2} \right) \frac{1}{(x - (\Delta + \hbar \omega) / \Gamma)^2 + 1},
$$

(16)

where

$$
\gamma^2 = \sum_{l'=\pm \infty} A_{ll'}, \quad K = - \sum_{l'=\pm \infty} A_{ll'} B_{ll'}.
$$

(17)

Note that the sums in (17) for an infinite system actually do not depend on $l$. In the limit $\gamma \Delta E \gg \Gamma$ we obtain from (16)

$$
\bar{\alpha} = a_0 \sqrt{\frac{2\pi K}{\gamma^3 \Delta E}} \left( (\Delta - \hbar \omega) \exp \left( -\frac{(\Delta - \hbar \omega)^2}{2\gamma^2 \Delta E^2} \right) + (\Delta + \hbar \omega) \exp \left( -\frac{(\Delta + \hbar \omega)^2}{2\gamma^2 \Delta E^2} \right) \right).
$$

(18)

If $\hbar \omega \gg \gamma \Delta E$, one can neglect the second term in the brackets. As we see, in this case, as for an ideal SL, the increment changes its sign at $\Delta = \hbar \omega$. However, the qualitative behaviour of the increment under variation of $\Delta$ depends on the sign of the coefficient $K$. For the case of high $\mu$ from equation (10) one finds that $A_{ll'} = -B_{ll'}$, and $K > 0$. In this case, as for ideal SLs, the increment is positive at $\Delta > \hbar \omega$. In contrast, for typical SL parameters and low $\mu$, $K < 0$ and, roughly speaking, $\bar{\alpha} > 0$ for $\Delta > \hbar \omega$ (see figure 6). It is worth stressing here that the change in sign of $\bar{\alpha}$ in disordered SLs occurs for a different reason than in ideal SLs. In ideal SLs, variation of $\Delta$ changes the sign of the increment for a phonon of particular energy in each SL period. In disordered SLs the main contribution to the increment is provided by the QW pairs for which Stark splitting is close to the phonon energy. The sign and magnitude of this contribution depend on the electron density mismatch in these QWs. Variation of $\Delta$ changes the most probable value of this mismatch for QW pairs having such a Stark splitting.

The dependences of $\bar{\alpha}$ on $\Delta$ obtained by numerical evaluation of equation (16) for $\hbar \omega = 2.6$ meV and two extreme types of lateral transport are shown in figures 7(a) and (b). The structure parameters were selected to be equal to those in the experimental sample, $T = 10K$, and the values of $\Delta E$ were 0.25 and 0.5 meV. In comparison to the case of an ideal SL, the magnitude of increment is large in a much wider range of $\Delta$. To illustrate that, we put the corresponding dependence obtained for an ideal SL in the insert of figure 7(a). One can also notice that the dependences of $\bar{\alpha}$ have a different shape in figures 7(a) and (b). This is because for the case of high $\mu$ the parameters $\gamma$ and $K$ are independent of $\Delta$, while in the opposite case they depend on $\Delta$ (figure 6).

Let us discuss the applicability of high-$\mu$ and low-$\mu$ models to actual SL structures. Using the standard expression for $\mu$, we can roughly estimate the $\nabla \cdot j_{\text{lat}}$ term as $n \delta E_{\text{ch}} \tau_{\text{sc}} / (ml^2)$, where $\delta E_{\text{ch}}$ is the characteristic magnitude of $\delta E$, $l_c$ is the characteristic length of lateral variation of $\delta E$, and $\tau_{\text{sc}}$ is the intra-well electron scattering time. The tunnelling terms in (6) can be estimated as $n / \tau_{\text{tun}}$, where $\tau_{\text{tun}}$ is the characteristic tunnelling time. Thus, for the ratio of lateral and tunnelling terms we obtain $\delta E_{\text{ch}} \tau_{\text{sc}} \tau_{\text{tun}} / (ml^2)$. If this factor is large in comparison
to unity, the high-µ model is valid, and if it is small the low-µ approach can be used. The parameters $\tau_{sc}$ and $\tau_{tun}$ can be estimated to be $10^{-12}$ and $10^{-8}$ s, respectively, from the mobility data of similar systems and the steady-state current–voltage characteristics. Unfortunately, we are not aware of the other important parameters entering into this ratio for our particular system ($\delta E_{ch}$ and $l_{ch}$). In fact, both limiting cases are realistic, but the experimentally observed dependence of the amplification coefficient on $\Delta$ is similar to that of the low-µ case.

As we see, the results of our calculations can qualitatively explain the experimentally measured phonon amplification. In particular, this concerns the observation of the strong amplification and attenuation in a wide range of applied biases. They provide an idea of why the bias dependence of the amplification is reversed with respect to that expected in the theory for a perfect SL. We have to note, however, the model developed here is probably not refined enough to quantitatively account for experimental results. Most importantly, it is limited by the linear-response approach and cannot account for the large fluctuations that are very likely to be present in the experiments. Indeed, even one-monolayer fluctuation of the QW width gives rise to $\delta E$ about few meV, which is comparable to the characteristic Stark splitting and quasi-Fermi energy. Furthermore, we restricted ourselves to the case of an infinite SL, while in the actual SL the near-contact periods of the SL could be strongly non-uniform, providing an essential

Figure 6. The Stark-splitting dependence of $\gamma$, (a), and $K$, (b), for high and low lateral mobility.
Figure 7. The dependence of the 2 meV-phonon increment on the average Stark splitting for the cases of (a) high and (b) low lateral mobility. The calculations are performed for the parameters of the SL studied experimentally, temperature of 10 K, and two values of the dispersion of $\delta E^{(j)}$ distribution, 0.25 and 0.5 meV. The insert in (a) shows the normalized increment for the ideal SL and in-plane phonon wavevector $10^6$ m$^{-1}$. As we see, in the ideal SL amplification and attenuation are possible in a much smaller range of Stark-splitting values.

contribution to the increment. We believe these effects may be responsible for the fact that the calculated increment is less than the measured value. In addition, one has to keep in mind that not only the lateral transport characteristics, but also the aforementioned factors can influence the bias dependence of amplification. To clarify these points more accurate simulations are required, which will be the subject of further investigations.

4. Conclusions

In conclusion, the application of bias to a lightly doped GaAs/AlAs superlattice can amplify a phonon beam incident on it. When the energy drop per period of the SL is close to the phonon energy the conditions for amplification are satisfied. We observed gain in a quasi-monochromatic LA mode that was optically excited in a generation SL. The gain was of the order of 20%, which relates to a gain coefficient of 3600 cm$^{-1}$. Furthermore, we have presented a
theoretical model of phonon amplification in structures containing disorder, and obtained a good qualitative agreement with the experiments. This single pass phonon amplifier is analogous to a semiconductor optical amplifier and could easily be incorporated into an acoustic cavity formed from Bragg mirrors.

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Appendix

To solve equation (12) it is convenient to perform a linear transformation which diagonalizes matrix $A$:

$$
y''(l) \equiv \begin{pmatrix} y''_1(l) \\ y''_2(l) \end{pmatrix} = U^{-1}y, \quad x''(l) \equiv \begin{pmatrix} x''_1(l) \\ x''_2(l) \end{pmatrix} = Ux, \quad U^{-1}AU = \begin{pmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{pmatrix},
$$

(A.1)

where $\lambda_1, 2$ are the eigennumbers of $A$. In this case equations for $y''_1(l)$ and $y''_2(l)$ are decoupled:

$$
y''_1(l+1) = \lambda_1 y''_1(l) + x''_1(l), \quad y''_2(l+1) = \lambda_2 y''_2(l) + x''_2(l).
$$

(A.2)

We will seek the solution of (A.2) for $l = 0$ through $l = N_L \gg 1$. As the boundary condition, we will require that $y''_{1,N_L}$ and $y''_{2,N_L}$ do not diverge with increasing $N_L$. In other words, we will seek a solution for an infinite system with finite value average parameters in each QW layer. The following step depends on the value of the eigennumbers. For $|\lambda| < 1$ such a solution is

$$
y''(l) = y''(0)\lambda^l + \sum_{l'=0}^{l-1} x''(l')\lambda^{l-l'-1},
$$

(A.3)

and for $|\lambda| > 1$

$$
y''(l) = y''(N_L)\lambda^{l-N_L} - \sum_{l'=l}^{N_L-1} x''(l')\lambda^{l-l'-1}.
$$

(A.4)

Naturally, the first terms in (A.3) and (A.4) vanish for $l \gg 1$ and $N_L - l \gg 1$. Performing the reverse transformation, we obtain solution of the problem in the form of equation (14).

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