Adaptive Multi-objective Optimization for Energy Efficient Interference Coordination in Multi-Cell Networks

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Abstract

In this paper, we investigate the distributed power allocation for multi-cell OFDMA networks taking both energy efficiency and inter-cell interference (ICI) mitigation into account. A performance metric termed as throughput contribution is exploited to measure how ICI is effectively coordinated. To achieve a distributed power allocation scheme for each base station (BS), the throughput contribution of each BS to the network is first given based on a pricing mechanism. Different from existing works, a bi-objective problem is formulated based on multi-objective optimization theory, which aims at maximizing the throughput contribution of the BS to the network and minimizing its total power consumption at the same time. Using the method of Pascoletti and Serafini scalarization, the relationship between the varying parameters and minimal solutions is revealed. Furthermore, to exploit the relationship an algorithm is proposed based on which all the solutions on the boundary of the efficient set can be achieved by adaptively adjusting the involved parameters. With the obtained solution set, the decision maker has more choices on power allocation schemes in terms of both energy consumption and throughput. Finally, the performance of the algorithm is assessed by the simulation results.

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1. Introduction

Ever-increasing demand on high data rates inspires and promotes the development of wireless technologies. In order to achieve the desired high information rates, many innovative ideas are introduced into wireless system designs. Referring to cellular networks, interference becomes the main limitation of performance improvement and multi-point coordination is a promising and powerful technology to improve the efficiency and reliability of wireless communications. In the scenario of multi-cell orthogonal frequency division multiple access (OFDMA) networks, a large number of users try to share the same spectrum to carry out wide-band multimedia communications and thus the performance of wireless networks is heavily limited by mutual interference. This fact motivates researchers to design various power control optimization algorithms to effectively coordinate interference.

In a multi-cell network, generally speaking the design of interference coordination is very challenging due to many practical limitations [1]–[3]. One of the difficulties for interference mitigation stems from the competition of utility benefits among different base stations (BSs). Specifically, the interactions between different BSs will greatly affect the whole network performance. Hence, to efficiently schedule the inter-cell interference the interactions among different BSs and the characteristics of the BSs’ behaviors should be carefully exploited. In literature a successful model for the problem of interference coordination is game theory which can effectively analyze the behaviors of wireless nodes [4]–[8].

In general, the game models exploited for wireless designs can be classified into two categories: cooperative and non-cooperative games. For non-cooperative game, it is convenient to devise totally distributed algorithms, however it may suffer from a significant performance loss compared with the optimal centralized solution due to the fact that there is no cooperation among cells. On the other hand, cooperative game usually suffers a cost of high overhead and complexity though it has a better performance.

To overcome the inefficiency of the non-cooperative game and high overhead of the cooperative game, recently pricing mechanism has been proposed [9], which is employed as an effective means to stimulate cooperation among players. Specifically, pricing schemes can guide the players’ behaviors toward efficient Nash equilibrium, by introducing a certain degree of coordination in a non-cooperative game. This approach has been introduced by C. Shi in [10].
An algorithm for allocating power among multiple interfering transmitters in a wireless network using OFDM was presented. The algorithm attempts to maximize the sum over user utilities, where each user’s utility is a function of his total transmission rate. Users exchange interference prices reflecting the marginal cost of interference on each sub-channel, and then update their power allocations given the interference prices and their own channel conditions. Similar works have also been done in [11]–[15] with local interference pricing, and the corresponding algorithms adjusting beamforming vectors or power allocation schemes to maximize the sum transmission rate, respectively. It is interesting that though only limited information is exchanged in these existing algorithms, the performance of the distributed algorithms proved to be close to the centralized optimization under a pricing mechanism [16].

Game theory has achieved a great success in modeling the resource allocation in multi-cell networks. In addition, it has also introduced a series of successful mechanisms such as pricing mechanism. Also inspired by the pricing mechanism, we take a further step to tackle the resource allocation from the viewpoint of multi-objective optimization theory. In our work, the performance metrics of both throughput contribution and power consumption are taken into account. In an interference-dominated network, at high signal-to-noise ratio (SNR) regime, it is well-known that increasing transmit power will be useless to improve system performance. On the other hand, power consumption is also a very important performance metric which should be carefully addressed in wireless network designs, as green communications are of great importance in practical applications. These two design issues are closely related with each other, while for most of the existing works, they are investigated separately. To the best of the authors’ knowledge, there are few works jointly considering energy efficiency and mitigating the inter-cell interference at meantime [17], [18].

In this paper, based on a multi-objective optimization framework an energy-efficient power optimization for multi-cell networks is investigated. Using the pricing mechanism, a bi-objective optimization problem is formulated for each BS, which aims at both maximizing the throughput contribution of the BS to the network and minimizing its total power consumption at the same time. The throughput contribution of the BS is a function of the transmission rate of the BS and his interference cost. To find out the tradeoff between the throughput contribution of the BS and its power consumption, the Pascoletti and Serafini scalarization method [19] instead of the widely used weighted sum method is first applied to transform the bi-objective problem
into an equivalent scalar objective problem considering that Pascoletti and Serafini scalarization method has strengths in overcoming the deficiencies of the weighted sum method. Then a proposed adaptive parameter control algorithm is used to adaptively changing some parameters such that the complete minimal solution set for the bi-objective optimization problem can be derived. The minimal solutions are those on the boundary of the set of feasible solutions. These points have a common distinguishing characteristic. For a given constant power consumption, the throughput contribution of the BS is the maximum one and otherwise for a fixed throughput contribution, the consumed power is the minimum. Finally, the performance advantage of the proposed algorithm is demonstrated by the simulation results and it shows that the proposed algorithm can effectively achieve different tradeoffs among the involved objective functions. In addition several famous solutions can be covered by our solution, such as the solution to capacity maximization problem, the solution to pricing-based utility optimization problem, and the solution to equal power allocation problem.

This paper is organized as follows. In Section 2, the system model is introduced and the corresponding multi-objective optimization problem is formulated. The proposed adaptive parameter control algorithm is detailed discussed in Section 3, in which we study the connection between parameters and minimal solutions. Later the performance of the algorithm is shown by simulations in Section 4. Finally, conclusions are drawn in Section 5.

2. SYSTEM MODEL AND PROBLEM FORMULATION

In this paper, we investigate a downlink OFDMA multi-cell network with $M$ BSs and $N$ subcarriers. The BSs are connected by high-speed fiber. Each user is served by only one BS. In addition, each subcarrier is exclusively assigned to one user. As shown in Fig.1 because of the openness of wireless channels the users in one cell will inevitably receive the interference signals from the neighboring BSs, especially the users at the edge of a cell. This fact becomes the most distinguished factor limiting wireless system performance. As a result, interference coordination is of great importance and attracts a lot of attention. Simply speaking, reducing transmit power will definitely reduce mutual interference and save valuable resources to realize green communications. However, with decreasing transmit power meanwhile the throughput also decreases. It is not surprising that in a complicated interference scenario there is hardly closed-form optimal solutions. Due to the fact that the considered optimization problem is in nature a
multi-objective optimization problem, it is impossible to define optimality. There definitely exists some tradeoff between transmit powers at BSs and the whole system performance within this awkward scenario and this is the focus of our work. In view of this, relying on multiple-objective optimization theory we want to formulate an optimization problem with multiple objective functions and aim to derive a set of the optimal solutions. The set will reflect different design preferences and include several well-known special cases.

In our design for each BS $m$ the following two performance utility functions are considered simultaneously when it optimizes its resource allocation.

1) Power consumption of the BS;
2) Throughput contribution of the BS to the whole network.

It should be highlighted that the second performance utility function consists of two parts. One is its own throughput which is naturally a positive contribution to the whole network. The other is the interference it caused, which is a negative contribution as interference will decrease other terminal’s throughput. In the following, we will discuss the formulation of the throughput contribution in detail.

2.1 Throughput contribution of the BS to the network

To measure the throughput contribution of the BS $m$ to the network, pricing mechanism [10], [12]–[15] is adopted in our work. Specifically, the pricing-based multi-cell power allocation game can be formulated as

$$ G = \{\mathcal{M}, \{p_m\}_{m \in \mathcal{M}}, \{\bar{U}_m\}_{m \in \mathcal{M}}\}, \quad (1) $$

where the involved elements are defined as follows

- **Player set**: The set of BSs is denoted by $\mathcal{M} = \{1, 2, \ldots, M\}$.
- **Strategy set**: The allocated power vector $\{p_1, \ldots, p_M\}$ is defined as
  $p_m = \left\{ [p_m^{[1]}, \ldots, p_m^{[N]}]^T : \sum_{n \in \mathcal{N}} p_m^{[n]} \leq P_{max} \right\}.$ \quad (2)

In the previous formulation, $p_m^{[n]}$ is the power allocated to subcarrier $n$. $\mathcal{N} = \{1, 2, \ldots, N\}$ is the set of subcarriers with the total number $N$, and $P_{max}$ is the maximum transmit power of each BS.
- **Payoff function set**: The payoff function set is denoted by $\{\bar{U}_1, \ldots, \bar{U}_M\}$, where $\bar{U}_m$ is a
function of the power allocation of all involved BSs. To highlight this fact, in the following, we can write \( \tilde{U}_m = \tilde{U}_m(p_m, p_{-m}) \) where \( p_m \) is the power allocation of BS \( m \) and \( p_{-m} \) is the power allocations of the other BSs except BS \( m \) with \( p_{-m} = \{p_1, \ldots, p_{m-1}, p_{m+1}, \ldots, p_M\} \).

Additionally all the power allocation vectors are assumed to be known by all BSs.

In order to theoretically analyze the throughput contributions to the whole network, we first investigate the throughput of BS \( m \) on subchannel \( n \), which can be expressed as

\[
U_{m[n]} = \log_2 \left( 1 + \frac{|h_{m,n}[n]|^2}{\sigma^2 + \sum_{j \in M \setminus m} |h_{j,n,m}[n]|^2 p_{j[n]}} \right)
\]

\[
= \log_2 \left( 1 + \frac{|h_{m,n,m}[n]|^2 p_{m[n]}}{\sigma^2 + I_{m[n]}} \right)
\]

where \( h_{m,j,n} \), \( \forall j, n \) denotes the complex channel gain between BS \( j \) and the user who is served by BS \( m \) on subchannel \( n \), \( \sigma^2 \) is the noise variance and \( I_{m[n]} \) denotes the interference term.

From (3) it can be seen that the relationship between \( U_{m[n]} \) and \( I_{m[n]} \) is nonlinear. To overcome this problem a linear model called pricing mechanism [11] is exploited to represent the total cost BS \( m \) needs to pay to the system when it applies \( p_{m[n]} \) on subchannel \( n \).

\[
\sum_{j \in M \setminus m} \pi_{m,j}[n] |h_{m,j,m}[n]|^2 p_{m[n]}
\]

where \( \pi_{m,j}[n] \) is the interference pricing rate and is defined by [11]

\[
\pi_{m,j}[n] = -\frac{\partial U_{j[n]}[n]}{\partial I_{j[n]}[n]}. \tag{5}
\]

Need to notice that the pricing rate requires to be updated after a BS updates its power. Because only the updated pricing rate makes sense for the neighboring BSs.

Summing up the costs in (4) across all the subchannels \( n \in \mathcal{N} \), the first utility function is formulated as

\[
\tilde{U}_m(p_m, p_{-m}) = \sum_{n \in \mathcal{N}} U_{m[n]} - \sum_{n \in \mathcal{N}} \sum_{j \in M \setminus m} \pi_{m,j}[n] |h_{m,j,m}[n]|^2 p_{m[n]}
\]

\[
\triangleq C(p_m, p_{-m}) \tag{6}
\]

where the term \( C(p_m, p_{-m}) \) can be interpreted to be the cost of BS \( m \) that should be paid to the system. As a result, \( \tilde{U}_m(p_m, p_{-m}) \) is exploited to measure the throughput contribution of the BS \( m \) to the network.
**Remark:** The BSs receive channel state information (CSI) from their serving users and exchange interference price as well as a small portion of CSI information with neighboring BSs. Different from the widely used non-linear Gauss-Seidel algorithm, the BSs can make their decisions simultaneously with the pricing mechanism.

### 2.2 Problem Formulation

According to the previous discussions, for the resource optimization the following two objective functions should be minimized simultaneously

\[
 f_1(p_m) = -\sum_{n \in \mathcal{N}} U_m^{[n]} + \sum_{n \in \mathcal{N} \setminus \mathcal{M}} \sum_{j \in \mathcal{M} \setminus \mathcal{m}} \pi_j^{[n]} | h_{m,j}^{[n]} |^2 p_{m}^{[n]}, \tag{7}
\]

\[
 f_2(p_m) = \sum_{n \in \mathcal{N}} p_{m}^{[n]} \tag{8}
\]

based on which the multi-objective optimization problem (MOP) for each BS \( m \) can be formulated as

\[
 \text{MOP:} \quad \min_{p_m} \quad f(p_m) = \begin{bmatrix} f_1(p_m) \\ f_2(p_m) \end{bmatrix} \tag{9}
\]

\[
 \text{s.t.} \quad g_j(p_m) \geq 0, \quad j = 1, \ldots, (N + 1),
\]

where the power constraints are defined as

\[
 g_j(p_m) = p_{m}^{[j]}, \quad j = 1, \ldots, N,
\]

\[
 g_j(p_m) = P_{\text{max}} - \sum_{n \in \mathcal{N}} p_{m}^{[n]}, \quad j = N + 1. \tag{10}
\]

As each BS solves MOP individually, in the following \( p_m \) is replaced by \( p \) for notational simplicity.

Generally speaking, MOPs are usually much more difficult to solve than its single objective counterpart. The difficulty in solving MOP \([9]\) comes from its multi-objective functions and a common logic to remove this difficulty relies on scalarization \([20]\). It should be pointed out that in the existing wireless research, a common logic for scalarization is to replace the original multiple objectives by their linear weighted sum. Unfortunately, this kind of scalarization has the following drawbacks.

(1) If the considered optimization problem is nonconvex, the traditional scalarization will incur some losses. In specific, the set of final solutions may not be the Pareto optimal set \([19]\).
(2) When searching the optimal points, in the traditional scalarization method the adjustment of weighting factors may make the objective value change nonuniformly. As the optimal solutions are unknown, if the objective function changes sharply some important points are not obtained [21].

(3) The following operations after the traditional scalarization are complicated for nonlinear MOP. There are two reasons. First, linear weighted operation cannot change the nonlinear nature of the objective function. Second, the adaptive updating strategy for the weighting factors are largely open. In most of the works, the weighting vectors are determined by the importance among these objectives [22] [23] [24]. However, it is difficult to find reasonable weighting vectors because there are usually no direct relationship between weighting vectors and objective functions, especially the objectives have different physical meanings.

To overcome such weakness, from the perspective of operational mathematics, another famous scalarization technique named Pascoletti and Serafini Scalarization is much preferred. In the following, a resource allocation algorithm is proposed based on Pascoletti and Serafini Scalarization. To the best of our knowledge, it is the first attempt to take advantage of Pascoletti and Serafini Scalarization to design resource allocation algorithm for multi-cell networks.

3. **THE PROPOSED ALGORITHM BASED ON MULTI-OBJECTIVE OPTIMIZATION THEORY**

As there are two objective functions, the first question is how to define the optimality or what kind of solutions we want to achieve. Referring to the optimal solution, the following properties are desired to be met. For a given constant power consumption, the throughput contribution of the BS is the maximum one and on the other hand for a fixed throughput contribution, the consumed power is the minimum. From the viewpoint of multi-objective optimization, this kind of solutions are named as K-minimal points [19].

In Pascoletti and Serafini method, a new set of control parameters equivalent to the weighting vectors are introduced. By varying the parameters, the whole solution set of MOP [9] can be found. Thus in the following, we will first discuss the relationship between MOP and the Pascoletti and Serafini scalarization problem and then study how the new set of parameters affect minimal solutions in subsection 3.1. And further in subsection 3.2, we will study how to adaptively control the parameters such that the whole solution set can be obtained.
3.1 Pascoletti and Serafini Scalarization

As discussed previously, we will choose a scalarization scheme named Pascoletti and Serafini scalarization to transform the considered multi-objective optimization problem into a single-objective optimization problem (SP). Using the Pascoletti and Serafini method, MOP (9) is equivalent to the following SP,

$$
\text{SP:} \quad \min_{t, p \in D} t
$$

$$\text{s.t. } \quad a + tr - f(p) \in K,
$$

\begin{align*}
g_j(p) &\geq 0, \quad j = 1, \ldots, (N + 1),
\end{align*}

where the parameters are defined as $a = [a_1, a_2]^T \in \mathbb{R}^2$ and $r = [r_1, r_2]^T \in \mathbb{R}_+^2$. In addition, $K = \mathbb{R}^2_+$ is the closed pointed convex cone. The parameter $a$ can be interpreted as a reference point and the parameter $r$ as a direction [25] as shown in Fig. 2. For a reasonable $r$ and $a$ in the coordinates, it is always possible to find a minimal $t (< 0)$ and then a K-minimal solution $\bar{p}$ corresponding to this $(a, r)$.

It has been proved in [19] that SP has all key properties a scalarization method should have for determining minimal solutions of MOP. If $(\bar{t}, \bar{p})$ is a minimal solution of SP, then $\bar{p}$ is a weakly K-minimal solution of MOP at least. Besides, by varying $(a, r)$ all K-minimal points of MOP can be found which are also the solutions of SP. It is obvious that $(a, r)$ are the key parameters to SP and different $(a, r)$ leads to different solutions to SP. As there is more than one parameter in the transformation procedure, the motivation is how to reduce the number of parameters and simplify the computation for the optimal solutions.

**Conclusion 1:** When the parameter $r$ is fixed, i.e., $r = r^0$, all K-minimal points can still be found by varying the parameter $a$.

**Proof:** The proof has been given in Section 6.1.

As a result, our attention is restricted to the relationships between parameter $a$ and the minimal solution $p$ as well as $f(p)$ assuming $r$ is constant. Regarding this, we denote the SP w.r.t. the parameter $a$ by $\text{SP}(a)$ and regard $p$ as a function of $a$, denoted by $p(a)$. Considering the perturbed parameters $a$ as

$$
a \approx a^0 + sv
$$

where $s \in \mathbb{R}_+$ is the distance between $a$ and $a^0$, and $v \in \mathbb{R}^2$ is the direction in which $a$ changes.
Substituting (12) into (11), the resulting SP\((a)\) becomes
\[
\text{SP}(a) : \min_{t,p} t \\
\text{s.t. } (a^0 + sv) + tr^0 - f(p) \geq 0, \\
\quad g(p) \geq 0_{(N+1)}.
\] (13)

To find the whole solution set, we embark on a reference problem \(\text{SP}(a^0)\) and assume its minimal solution \((t^0(a^0), p^0(a^0))\) as well as the Lagrange multipliers \((\mu^0, \beta^0)\) have already been obtained, where \(\mu^0\) is the Lagrange multipliers to the first constraint of (13) and \(\beta^0\) to the second. Based on the reference point, we can find the next solution \(p(a)\) and thus \(f(p(a))\) in the neighborhood of \(f(p^0)\) with the help of directional derivatives. Then iteratively implementing the process until the whole solution set is achieved. To realize this, the relationship between \(a\) and \(p(a)\) as well as \(f(p(a))\) should be investigated first.

Assume that a variation of the parameters \(a\) in one direction only, i.e., \(v\) is fixed and the solutions of (13) only depend on the parameter \(s\). Then, we can regard all the terms \((t, p, \mu, \beta)\) as a function of \(s\), i.e., \((t(s), p(s), \mu(s), \beta(s))\). For clarity, we use \((t, p, \mu, \beta)\) in the following.

The Lagrange function of (13) is first given by:
\[
\mathcal{L}(t, p, \mu, \beta) = t - \sum_{i=1}^{M} \mu_i \left[ (a^0_i + sv_i) + tr^0_i - f_i(p) \right] - \sum_{j=1}^{N+1} \beta_j g_j(p)
\] (14)

For a solution in nonlinear programming \(\text{SP}(s)\) (13) to be optimal, Karush-Kuhn-Tucker (KKT) conditions are necessary:
\[
\nabla_t \mathcal{L}(t, p, \mu, \beta) = 1 - \sum_{i=1}^{M} \mu_i r^0_i = 0 \\
\nabla_p \mathcal{L}(t, p, \mu, \beta) = \sum_{i=1}^{M} \mu_i \nabla_p f_i(p) - \sum_{j=1}^{N+1} \beta_j \nabla_p g_j(p) = 0
\]
\[
\mu_i \left( (a^0_i + sv_i) + tr^0_i - f_i(p) \right) = 0, \quad \mu_i \geq 0, \quad \forall i \\
\beta_j g_j(p) = 0, \quad \beta_j \geq 0, \quad \forall j
\] (15)

These nonlinear equations can be solved with the help of directional derivatives, which is derived in Section 6.2.
With the solution \((\bar{t}, \bar{p}, \bar{\mu}, \bar{\beta})\) obtained by solving (29), the minimal solution of SP(s) can be attained by

\[
\begin{pmatrix}
  t \\
  p \\
  \mu \\
  \beta
\end{pmatrix} \approx
\begin{pmatrix}
  t^0 \\
  p^0 \\
  \mu^0 \\
  \beta^0
\end{pmatrix} + s \cdot
\begin{pmatrix}
  \bar{p} \\
  \bar{\mu} \\
  \bar{\beta}
\end{pmatrix}
\]  

(16)

So far, the relationship between the minimal solution \((t, p, \mu, \beta)\) and \(s\) is obtained. And in Conclusion 2, we will show how \(s\) affect the minimal points \(f(p)\).

**Conclusion 2:** Since \(s\) in (16) can be defined very small \((s \to 0^+)\), the minimal points \(f(p)\) can be approximated using the first-order Taylor approximation

\[
f(p) \approx f(p^0) + s \cdot v + s(\nabla_\alpha \bar{\tau}^{\delta}(a^0)^T v)r,
\]

where \(\nabla_\alpha \bar{\tau}^{\delta}(a^0) = -\mu^0\) is the derivative of the local minimal value function \(\bar{\tau}^{\delta}\) in the point \(a^0\).

**Proof:** The proof can be found in Section 6.3.

Thus far, the function relationship among the minimal solution \(p\), the minimal point \(f(p)\) and \(a\) can be clearly seen. Based on the results in (16) and (17), we then care how to adaptively control the parameter \(a\) such that all the minimal solution \(p\) and the minimal point \(f(p)\) can be obtained instead of artificially modifying \(a\).

### 3.2 Adaptive control of parameter \(a\)

In the following, a procedure is developed to achieve the whole minimal solution set of the MOP by controlling the choice of the parameter \(a\). First, it is necessary to give the set from which the parameter \(a\) is chosen.

**Theorem 1:** Define a hyperplane

\[
H = \{y \in \mathbb{R}^2 | b^T y = \beta\},
\]

with \(b \in \mathbb{R}^2\), \(\beta \in \{0, 1\}\), and \(b^T r \neq 0\). It is sufficient to get the efficient set by varying the parameter \(a \in H\). Further, it is shown that a subset \(a \in H^a \subset H\) for

\[
H^a = \{y \in \mathbb{R}^2 | y = \lambda a^1 + (1 - \lambda) a^2, \ \lambda \in [0, 1]\},
\]

is also sufficient. Where \(\bar{a}^1 \in H\) and \(\bar{a}^2 \in H\) are given by

\[
\bar{a}^i := f(\bar{p}^i) - \bar{t}^i r \quad \text{with} \quad \bar{t}^i := \frac{b^T f(\bar{p}^i) - \beta}{b^T r}, \quad i = 1, 2.
\]

(20)
with \( \bar{p}^1 \) the minimal solution of the scalar-valued problem \( \min_{p \in \mathcal{D}} f_1(p) \) and \( \bar{p}^2 \) the minimal solution of \( \min_{p \in \mathcal{D}} f_2(p) \). Without loss of generality, we assume \( \bar{a}^1 \) is smaller than \( \bar{a}^2 \) on the first dimension, i.e., \( \bar{a}^1 < \bar{a}^2 \).

**Proof:** The proof can be found in our work [27].

With the stricter set \( \mathcal{H}^a \) in (19) from which \( a \) should be chosen, we now want to determine the parameters \( a^0, a^1, a^2, \ldots \) adaptively (starting with \( a^0 = \bar{a}^1 \)) such that the related minimal points \( f(p(a^i)) \), \( i = 0, 1, 2, \ldots \), gained by solving SP\((a^i)\) for \( i = 0, 1, 2, \ldots \), have the equal distance \( \alpha > 0 \), i.e.,

\[
\|f(p(a^0)) - f(p(a^1))\| = \alpha, \tag{21}
\]

should be satisfied for any neighboring \( a^i \) and \( a^{i+1} \). This metric is applied to avoid the loss of some important points caused by the sharply change of an objective function. In the following, we replace \( f(p(a^0)) \) with \( f(p^0) \) for clarity. The advantage of choosing a predefined distance \( \alpha > 0 \) between two neighbor points can be seen from Fig2. With the evenly distributed points, more accurate information about the relationship between the objectives would be known.

Then, we aim at finding a direction \( v \) and a scalar \( s \) such that \( a^1 := a^0 + sv \) satisfies (21). Based on (17), we have

\[
\alpha = \|f(p^0) - f(p^1)\| \\
\approx \|f(p^0) - (f(p^0) + sv + s(-\mu^0)^Tv)r)\| \\
= |s| \|v + (-\mu^0)^Tv)r)\|. \tag{22}
\]

Then, the stepsize \( s \) and direction \( v \) of parameter \( a \) can be chosen by

\[
s^0 = \frac{\alpha}{\|v + (-\mu^0)^Tv)r)\|}, \tag{23}
\]

\[v = a^1 - a^0.\]

which leads to the generalized formula of \( a \),

\[
a^{i+1} = a^i + s^i v = a^i + \frac{\alpha}{\|v + (-\mu^i)^Tv)r)\|} v. \tag{24}
\]

Based on (24), an algorithm is developed to adaptively solve SP\((a^i)\) with \( \bar{a}^1 \leq a^i \leq \bar{a}^2 \), whose solutions constitute the solution set. This algorithm is referred to as adaptive parameter control (APC) algorithm and is shown in Algorithm 1.
Algorithm 1 Adaptive Parameter Control Algorithm

Input: Choose \( r = (r_1, r_2)^T \in \mathbb{R}^2_+ \) with \( r_1 > 0 \), and predefine \( \alpha > 0 \) between the neighboring two points. The hyperplane is chosen as

\[ H = \{ y \in \mathbb{R}^2 \mid b^T y = \beta \}, \]

where \( b \in \mathbb{R}^2 \) and \( b^T r \neq 0 \), \( \beta \in \{0, 1\} \). Given \( M^1 \in \mathbb{R} \) with

\[ M^1 > \max_{p \in D} f_2(p) - \min_{p \in D} f_1(p) \frac{r_2}{r_1}. \]

Step 1: Finding the minimal solution \((\tilde{t}^1, p^1)\) and Lagrange multiplier \( \mu^1 \in \mathbb{R}^2_+ \) of \( \text{SP}(\tilde{a}^1) \) with \( \tilde{a}^1 = (0, M^1)^T \). Calculate

\[ t^1 := \frac{b^T f(p^1) - \beta}{b^T r} \quad \text{and} \quad a^1 := f(p^1) - t^1 r. \]

Set \( l := 1 \).

Step 2: Finding the minimal solution \( p^E \) of \( \min_{p \in D} f_2(p) \) and set

\[ t^E := \frac{b^T f(p^E) - \beta}{b^T r} \quad \text{and} \quad a^E := f(p^E) - t^E r. \]

Let \( v := a^E - a^1 \).

Step 3: Update \( a^{l+1} \) by

\[ a^{l+1} := a^l + \frac{\alpha}{\|v + (-\mu^l)^T v\|} \cdot v. \]

Step 4: Set \( l := l + 1 \).

- If \( a^l = a^1 + \rho v \) for a \( \rho \in [0, 1] \), find minimal solution \((t^l, p^l)\) and Lagrange multiplier \( \mu^l \) by solving \( \text{SP}(a^l) \), and go to step 3.
- Else stop.

Output: Set \( A = \{p^1, \ldots, p^{l-1}, p^E\} \) is the minimal solution set of MOP. Set \( B = \{f(p^1), \ldots, f(p^{l-1}), f(p^E)\} \) is an approximation of the minimal points.

In the Input Step, we arbitrarily choose a \( \tilde{a}^1 = (0, M^1)^T \) with \( M^1 > \max_{p \in D} f_2(p) - \min_{p \in D} f_1(p) \frac{r_2}{r_1} \) at the initial for the purpose of engineering practice. Actually, the optimization problem \( \text{SP}(\tilde{a}^1, r) \)
with such a \( \tilde{a}^1 = (0, M^1)^T \) in Step 1 is

\[
\min_{p \in D} t \\
\text{s.t.} \quad tr_1 - f_1(p) \geq 0, \\
M^1 + tr_2 - f_2(p) \geq 0,
\]

\[t \in \mathbb{R}. \tag{25}\]

Therefore \( t \geq \frac{f_1(p)}{r_1} \) holds for any feasible point \((t, p)\), and \( M^1 + tr_2 - f_2(p) > (f_2(p) - f_1(p))\frac{r_2}{r_1} \) is also satisfied. Therefore, (25) can be replaced by \( \min_{p \in D} \frac{f_1(p)}{r_1} \) which equals to \( \min_{p \in D} f_1(p) \). However, there is no need to find the exact solution of \( \min_{p \in D} f_1(p) \) in practice, because we can always find the starting point \( a^1 \) even if the initial \( \tilde{a}^1 \) is roughly given.

In addition, \( \text{SP}(\tilde{a}^1) \) and \( \text{SP}(a^1) \) have the same minimal solution \( p^1 \) and the same Lagrange multiplier \( \mu^1 \), which will be demonstrated in Theorem 2.

**Theorem 2:** Assume \((\tilde{t}, \tilde{p})\) is a minimal solution of \( \text{SP}(a, r) \) with Lagrange multiplier \( \mu \) for arbitrary \( a \in \mathbb{R}^2 \) and \( r \in \mathbb{R}^2 \) with \( b^Tr \neq 0 \). Surely there could be a \( \bar{k} \in K \) with

\[
a + \tilde{t}r - f(\tilde{p}) = \bar{k}. \tag{26}\]

Further there could be a \( a' \in H \) and \( t' \in \mathbb{R} \) such that \((t', \bar{p})\) is a minimal solution of \( \text{SP}(a', r) \) with

\[
a' + t'r - f(\bar{p}) = 0_2. \tag{27}\]

In addition \( \mu \) is also Lagrange multiplier to the point \((t', \bar{p})\) for \( \text{SP}(a', r) \).

**Proof:** The proof can be found in our work [27].

In Step 1 and Step 2, the subset \( H^c \) of the hyperplane and a direction \( v \) with \( a^1 + sv \in H \) are determined. And then in Step 3 and Step 4, the APC algorithm producing an approximation of the efficient set is done.

4. Simulation Results

In this section, the performance the proposed APC algorithm is assessed. A multi-carrier multi-user OFDM-based system is employed with \( M = 19 \) BSs as shown in Fig. 1. There are 64 users in the simulation model and each user is served using a randomly chosen subcarrier. The number of subcarriers is \( N = 64 \) with the system bandwidth 10 MHz. The distance between
adjacent BSs is 1000 m. Rayleigh fast fading is considered, and large scale path loss is modeled as $PL = 128.1 + 37.6 \log(d)$ [28], where $d$ is the distance in kilometers.

As our work is studied based on the function in (6), we choose it as a comparison to the proposed APC algorithm. This algorithm applies pricing mechanism, and aims at only maximizing the transmission rate contribution of the BS to the network, thus we denote it by “pricing mechanism” in the simulations. The solutions of the equal power allocation and utility maximization are also simulated as the reference points. Here, utility maximization is that each BS tries to maximize his own transmission rate ignoring the interference it may cause to other cells and does not care how much power they will use to achieve the maximum transmission rate. While the equal power allocation scheme is that the total power needed in the APC method is uniformly allocated to all subcarriers. For the initialization, we assume that the total 30 watts energy is evenly distributed across all the subcarriers.

Fig.3 shows the relationship between power consumption and the throughput contribution of BS 1 to the network, which is obtained by the APC method. This curve is the boundary of the efficient set. For each given power consumption, the corresponding throughput contribution of BS 1 to the network is the maximum. And for the contribution of BS 1 to the network, the corresponding power consumption is the minimum be required. When the consumed power is greater than 20 watts, the increase of the network throughput brought by the BS is small. Thus the extra 10 watts power can be viewed as inefficient.

Fig.4 gives a parabola-like relationship between energy efficiency and throughput of the system. Energy efficiency is defined by the fraction where the numerator is the throughput of the system and the denominator is the total power consumption. As our goal is to find an energy-efficient power allocation scheme which can also guarantee a relatively high system throughput, we pay close attention to the interesting local area. If the system agrees to reduce its transmission rate from 8.98 Mbps to 8.68 Mbps, energy efficiency will increase sharply to 47 kbps/watt, which is much greater than the energy efficiency at 8.98 Mbps.

Fig.5 shows the comparison between different power allocation schemes. All the curves are obtained by varying the power consumption of BS 1 and fixing other BSs’ power. It can be seen that the solutions obtained by solving the pricing mechanism-based problem are always on the APC curves, no matter whether the interference pricing rate is updated or not. This result verifies the comprehensiveness of the proposed APC method. Though it seems that the solution to the
utility maximization is better than the solutions to pricing mechanism-based problem as well as APC problem before update the pricing rate, its actual outcome is not so. Because when we recalculate the system throughput after updating the pricing rates, the solutions obtained by the pricing mechanism-based scheme and APC method enjoy much more benefits than the greedy utility maximization scheme. The reason is that the negative contribution of BS 1 under utility maximization is great for it introduces more interference to users in neighboring cells. A final note about this figure is that all the curves are obtained by varying the power consumption of BS 1 and fixing other BSs’ power, and the power allocation scheme adopted by the neighboring BSs is EPA. As a result, there are no significant differences among these considered schemes since only one player changed his strategy during this process.

5. CONCLUSIONS

In this paper, from the perspective of multi-objective optimization theory an energy-efficient power allocation scheme was developed for interference-limited multi-cell network. A bi-objective optimization problem was first formulated based on the pricing mechanism. Then using the method of Pascoletti and Serafini scalarization, the relationship between the varying parameters and minimal solutions has been discovered, and an adaptive algorithm was developed to achieve tradeoff between the two objectives. As a result, all the solutions on the boundary of the efficient solution set can be computed which are best in terms of both energy efficiency and inter-cell interference mitigation. Finally, the performance of the algorithm was demonstrated in the simulations.

6. ACKNOWLEDGMENTS

This work was supported in part by the National Natural Science Foundation of China under Grant 61101130, and the Excellent Young Scholar Research Funding of Beijing Institute of Technology under Grant 2013CX04038.

7. APPENDICES

6.1 Proof of Conclusion 1

Let \( \bar{\rho} \) be a K-minimal solution of the MOP, and set \( a = f(\bar{\rho}) \) and choose \( r \in K \setminus \{0_2\} \) arbitrarily. Then the point \((0, \bar{\rho})\) is a feasible and also minimal solution for SP\((a, r)\). Otherwise
there will be a \( t' < 0 \) and \( p' \in \mathcal{D} \) with \((t', p')\) feasible for \( \text{SP}(a, r) \), and a \( k' \in K \) with \( a + t' r - f(p') = k' \). With these observations, we have

\[
f(p) = f(p') + k' - t' r \in f(p') + K.
\]

Because \( \bar{p} \) is a \( K \)-minimal solution, it can be concluded that \( f(\bar{p}) = f(p') \) and thus \( k' = t' r \).

Since the cone \( K \) is pointed, \( k' \in K \) and \( t' r \in -K \), which leads to \( k' = t' r = 0 \). However, it is contradict to \( t' < 0 \) and \( r \neq 0 \). Thus it can be concluded that if \( \bar{p} \) is a minimal solution of the MOP, then \((0, \bar{p})\) is a minimal solution of \( \text{SP}(a, r) \) for the parameter \( a := f(\bar{p}) \) and for arbitrarily \( r \in K \setminus \{0\} \). Further, it can be inferred that all \( K \)-minimal points can be found by varying the parameter \( a \) only.

6.2 Derivation of the solution to (15)

For the solution \((t, p, \mu, \beta)\) of \( \text{SP}(s) \) which is differentiable, the righthanded derivatives \((\bar{t}, \bar{p}, \bar{\mu}, \bar{\beta})\) can be written as

\[
\lim_{\alpha \to 0^+} \left( \begin{array}{c}
\frac{t-t^0}{\alpha} \\
\frac{p-p^0}{\alpha} \\
\frac{\mu-\mu^0}{\alpha} \\
\frac{\beta-\beta^0}{\alpha}
\end{array} \right) = \left( \begin{array}{c}
\bar{t} \\
\bar{p} \\
\bar{\mu} \\
\bar{\beta}
\end{array} \right),
\]

based on which it can also be inferred from Theorem 3 of [26] that for sufficiently small \( \alpha \), there exists a unique continuous function \((t, p, \mu, \beta)\) as the minimal solution of \( \text{SP}(s) \). Then we will find the solution \((t, p, \mu, \beta)\) with the help of \((\bar{t}, \bar{p}, \bar{\mu}, \bar{\beta})\) in (28). Because the KKT conditions (15) is always satisfied, it follows that the derivatives \((\bar{t}, \bar{p}, \bar{\mu}, \bar{\beta})\) can be easily obtained by
solving the following system of inequalities and equations,

\[
\begin{align*}
&- \sum_{i=1}^{M} \bar{\mu}_i r_i^0 = 0, \\
&\sum_{i=1}^{M} \bar{\mu}_i \nabla_p^2 f_i(p^0)p - \sum_{i=1}^{N+1} \bar{\beta}_i \nabla_p^2 g_i(p^0)p \\
&+ \sum_{i=1}^{M} \bar{\mu}_i \nabla_p f_i(p^0) - \sum_{i=1}^{N+1} \bar{\beta}_i \nabla_p g_i(p^0) = 0_N, \\
& r_i^0 \bar{t} - \nabla_p f_i(p^0)^T \bar{p} = -v, \quad \forall i \in I^+, \\
& r_i^0 \bar{t} - \nabla_p f_i(p^0)^T \bar{p} \geq -v, \quad \forall i \in I^0, \\
& \bar{\mu}_i \geq 0, \quad \forall i \in I^0, \\
& \bar{\mu}_i (r_i^0 \bar{t} - \nabla_p f_i(p^0)^T \bar{p} + v) = 0, \quad \forall i \in I^0, \\
& \bar{\mu}_i = 0, \quad \forall i \in I^-.
\end{align*}
\]

(29)

\[
\begin{align*}
\nabla_p g_j(p^0)^T \bar{p} &= 0, \quad \forall j \in J^+, \\
\nabla_p g_j(p^0)^T \bar{p} &\geq 0, \quad \forall j \in J^0, \\
\bar{\beta}_j &\geq 0, \quad \forall j \in J^0, \\
\bar{\beta}_j (\nabla_p g_j(p^0)^T \bar{p}) &= 0, \quad \forall j \in J^0, \\
\bar{\beta}_j &= 0, \quad \forall j \in J^-.
\end{align*}
\]

(30)

where \(I^+, I^0, I^-\) are the active, non-degenerate and degenerate constraint sets, respectively

\[
I^+ := \{i \in I \mid a_i^0 + r_i^0 \bar{t} - f_i(p^0) = 0, \quad \mu_i^0 > 0\},
\]

\[
I^0 := \{i \in I \mid a_i^0 + r_i^0 \bar{t} - f_i(p^0) = 0, \quad \mu_i^0 = 0\},
\]

(31)

\[
I^- := \{i \in I \mid a_i^0 + r_i^0 \bar{t} - f_i(p^0) > 0, \quad \mu_i^0 = 0\},
\]

and \(J^+, J^0, J^-\) are another disjoint sets:

\[
J^+ := \{j \in J \mid g_j(p^0) = 0, \quad \beta_j^0 > 0\},
\]

\[
J^0 := \{j \in J \mid g_j(p^0) = 0, \quad \beta_j^0 = 0\},
\]

(32)

\[
J^- := \{j \in J \mid g_j(p^0) > 0, \quad \beta_j^0 = 0\}.
\]
With the derivatives \((\bar{t}, \bar{p}, \bar{\mu}, \bar{\beta})\) obtained by solving (29) and (30), the minimal solution of \(\text{SP}(s)\) can be attained from (28), which equals

\[
\begin{pmatrix}
  t \\
  p \\
  \mu \\
  \beta \\
\end{pmatrix}
\approx
\begin{pmatrix}
  t^0 \\
  p^0 \\
  \mu^0 \\
  \beta^0 \\
\end{pmatrix}
+ s \cdot
\begin{pmatrix}
  \bar{t} \\
  \bar{p} \\
  \bar{\mu} \\
  \bar{\beta} \\
\end{pmatrix},
\]

(33)

6.3 Proof of Conclusion 2

In order to prove the following equation

\[
f(p) \approx f(p^0) + s \cdot v + s(\nabla_a \tau^\delta(a^0)^T v)r,
\]

(34)

the definition of the local minimal value function \(\tau^\delta\) should be given first, and then the derivative of the local minimal value function in the point \(a^0\) should be provided as well. The local minimal value function \(\tau^\delta : \mathbb{R}^2 \to \mathbb{R}\) is defined by

\[
\tau^\delta(a) := \inf\{t \in \mathbb{R} | (t, p) \in \Sigma a \cap B_\delta(t^0, p^0)\},
\]

(35)

where \(B_\delta(t^0, p^0)\) is the closed ball with radius \(\delta > 0\) around \(p^0\) and \(\Sigma a\) is the constraint set of \(\text{SP}\) depending on \(a\):

\[
\Sigma a := \{(t, p) \in \mathbb{R}^{N+1} | a + tr - f(p) \geq 0\}.
\]

(36)

Then the derivative can be achieved by

\[
\nabla_a \tau^\delta(a) = -\mu(a)
\]

\[
- \sum_{i=1}^2 \nabla_a \mu_i(a)(a_i + t(a)r_i - f_i(p(a))).
\]

(37)

Then replace \(\mu(a^0)\) by \(\mu^0\), we obtain the derivative in the point \(a^0\)

\[
\nabla_a \tau^\delta(a^0) = -\mu^0
\]

\[
- \sum_{i \in I^+ \cup I^0} \nabla_a \mu_i(a^0)(a_i^0 + t_i^0 r_i^0 - f_i(p^0))
\]

\[
- \sum_{i \in I^-} \nabla_a \mu_i(a^0)(a_i^0 + t_i^0 r_i^0 - f_i(p^0)) = 0.
\]

(38)
Due to the definition of $I^-$, $a_i^0 + t_i^0r_i^0 - f_i(p^0) > 0$ for $i \in I^-$. Since $a_i + t(a) r_i - f_i(p(a))$ is continuous in $a$, there exists a neighborhood $N(a^0)$ of $a^0$ so that for all $a \in N(a^0)$ it holds

$$a_i + t(a) r_i - f_i(p(a)) > 0 \quad \text{for} \quad i \in I^-.$$  

(39)

Then $\mu_i(a) = 0$ for all $a \in N(a^0)$ and $\nabla_a \mu_i(a^0) = 0$ for $i \in I^-$ can be obtained. Thus, we get

$$\nabla_a \tau^\delta(a^0) = -\mu^0.$$  

(40)

Already obtained the derivative of the local minimal value function $\nabla_a \tau^\delta(a^0)$, we then try to obtain the result in (34) with the help of (40).

Assume that we have already solved the problem $\text{SP}(a^0)$ for the parameters $a^0$ with a minimal solution $(t^0, p^0)$ and Lagrange multiplier $\mu^0$. Then by using $\tau^\delta(a^0) = t(a^0) = t^0$, a first order Taylor approximation of the local minimal value function to the optimization problem $\text{SP}(a)$ is derived,

$$t(a) \approx t^0 + \nabla_a \tau^\delta(a^0)^T(a - a^0)$$  

(41)

Then the approximation for the K-minimal points of MOP dependent on the parameter $a$ is launched:

$$f(p(a)) = a + t(a)r$$

$$\approx a^0 + (a - a^0) + (t^0 + \nabla_a \tau^\delta(a^0)^T(a - a^0))r$$

$$= f(p^0) + (a - a^0) + (\nabla_a \tau^\delta(a^0)^T(a - a^0))r$$

$$= f(p^0) + sv + s(\nabla_a \tau^\delta(a^0)^Tv)r.$$  

(42)

6.4 Proof of Theorem 1

First, we will prove that it is sufficient to vary the parameter $a$ on the hyperplane $H = \{y \in \mathbb{R}^2 | b^Ty = \beta\}$. Assume $\bar{p}$ is K-minimal for MOP. For the case that

$$\bar{t} = \frac{b^Tf(\bar{p}) - \beta}{b^Tr}$$

and $a = f(\bar{p}) - \bar{t}r$  

(43)

with arbitrarily $r \in K$ and $b^Tr \neq 0$, we have $a \in H$ and $(\bar{t}, \bar{p})$ is feasible for $\text{SP}(a)$. If $(\bar{t}, \bar{p})$ is not a minimal solution of $\text{SP}(a)$, then there could be another $t' < \bar{t}$, $p' \in D$ and $k' \in K$ with

$$a + t'r - f(p') = k'.$$  

(44)
Replace \( a \) with \((43)\),
\[
f(\bar{p}) = f(p') + k' + (\bar{t} - t')r
\]
then it can be concluded that \( f(\bar{p}) \in f(p') + K \) for \( p' \in D \), which is contradict to the definition of \( \bar{p} \) \( K \)-minimal. Thus \((\bar{t}, \bar{p})\) is a minimal solution of \( \text{SP}(a) \). So far, we have proved that it is sufficient to vary the parameter \( a \) on the hyperplane \( H \). Further, we will show that a subset \( H^a \subset H \) is also sufficient to get the efficient set.

Assume \( \bar{p}^1 \) is a minimal solution of \((46)\)
\[
\min_{p \in D} l_1^T f(p)
\]
and \( \bar{p}^2 \) is a minimal solution of \((47)\)
\[
\min_{p \in D} l_2^T f(p)
\]
where \( l_1 = (1, 0) \) and \( l_2 = (0, 1) \). The parameters \( \bar{a}^1 \in H \) and \( \bar{a}^2 \in H \) are given by
\[
\bar{a}^i := f(\bar{p}^i) - \bar{t}^i r \quad \text{with} \quad \bar{t}^i := \frac{b^T f(\bar{p}^i) - \beta}{b^T r}, \quad i = 1, 2.
\]

Then, we consider the parameters \( a \in H^a \) with the set \( H^a \) given by
\[
H^a = \{ y \in H | y = \lambda \bar{a}^1 + (1 - \lambda)\bar{a}^2, \quad \lambda \in [0, 1] \}.
\]
It can be inferred from the assumption that \( \bar{a}^1, \bar{a}^2 \in H^a \subset H \). For simplicity, we assume \( \bar{a}^1 \) is smaller than \( \bar{a}^2 \) on the first dimension, i.e., \( \bar{a}^1_1 < \bar{a}^2_1 \).

For any feasible \( \bar{p} \), there exists a parameter \( a \in H \) and a \( \bar{t} \in \mathbb{R} \) given by
\[
\bar{t} = \frac{b^T f(\bar{p}) - \beta}{b^T r} \quad \text{and} \quad a = f(\bar{p}) - \bar{t} r
\]
so that \((\bar{t}, \bar{p})\) is a minimal solution of \( \text{SP}(a) \). As \( \bar{p}^1 \) and \( \bar{p}^2 \) are minimal solutions of \((46)\) and \((47)\), we have for any feasible \( \bar{p} \),
\[
l_1^T f(\bar{p}) \geq l_1^T f(\bar{p}^1) \quad \text{and} \quad l_2^T f(\bar{p}) \geq l_2^T f(\bar{p}^2).
\]
(51)
Suppose that \( l_1^T f(\bar{p}) \geq l_1^T f(\bar{p}^2) \), as \((51)\) always holds, it can be concluded that \( f(\bar{p}) - f(\bar{p}^2) \in K \), which is contradict to \( \bar{p} \) \( K \)-minimal. Thus, it can be shown that
\[
l_1^T f(\bar{p}^1) \leq l_1^T f(\bar{p}) \leq l_1^T f(\bar{p}^2).
\]
(52)
In the same way, the following relation can also be achieved
\[ l^2T f(\hat{p}^2) \leq l^2T f(\hat{p}) \leq l^2T f(\hat{p}^1). \] (53)

With the obtained relations (52) and (53), we then demonstrate that the parameter \( a \) lies on the segment between the point \( \hat{a}^1 \) and \( \hat{a}^2 \), i.e., \( a = \lambda \hat{a}^1 + (1 - \lambda) \hat{a}^2 \) for \( \lambda \in [0, 1] \). Using the definition of \( a \), \( \hat{a}^1 \) and \( \hat{a}^2 \), the following can be obtained:
\[ a = f(\hat{p}) - \bar{t}r = \lambda(f(\hat{p}^1) - \bar{t}^1 r) + (1 - \lambda)(f(\hat{p}^2) - \bar{t}^2 r). \] (54)

reformulate (54) as
\[ f(\hat{p}) = \lambda f(\hat{p}^1) + (1 - \lambda)f(\hat{p}^2) + (\bar{t} - \lambda \bar{t}^1 - (1 - \lambda)\bar{t}^2)r. \] (55)

Then we do a case differentiation for \( \bar{t} - \lambda \bar{t}^1 - (1 - \lambda)\bar{t}^2 \geq 0 \) and \( \bar{t} - \lambda \bar{t}^1 - (1 - \lambda)\bar{t}^2 < 0 \) respectively.

For \( \bar{t} - \lambda \bar{t}^1 - (1 - \lambda)\bar{t}^2 \geq 0 \), we first divide the set of \( \lambda \) into three parts, i.e., \( \lambda < 0, 0 < \lambda < 1 \) and \( \lambda > 1 \), then start by considering the case that \( \lambda < 0 \).
\[
l^1T f(\hat{p})
\]
\[
= \lambda l^1T f(\hat{p}^1) + (1 - \lambda)l^1T f(\hat{p}^2) + (\bar{t} - \lambda \bar{t}^1 - (1 - \lambda)\bar{t}^2) l^1T r
\]
\[
\geq \lambda l^1T f(\hat{p}^1) + (1 - \lambda)l^1T f(\hat{p}^2) \geq 0
\]
\[
> \lambda l^1T f(\hat{p}^2) + (1 - \lambda)l^1T f(\hat{p}^2)
\]
\[
= l^1T f(\hat{p}^2).
\] (56)

If \( l^1T f(\hat{p}) > l^1T f(\hat{p}^2) \) is satisfied together with \( l^2T f(\hat{p}) > l^2T f(\hat{p}^2) \), it can be concluded that \( f(\hat{p}) - f(\hat{p}^2) \in K \), which is contradicted to \( \hat{p} \) K-minimal. Therefore, (55) is not satisfied for \( \lambda < 0 \). Then we consider the case that \( \lambda > 1 \).
\[
l^2T f(\hat{p})
\]
\[
= \lambda l^2T f(\hat{p}^1) + (1 - \lambda)l^2T f(\hat{p}^2) + (\bar{t} - \lambda \bar{t}^1 - (1 - \lambda)\bar{t}^2) l^2T r
\]
\[
\geq \lambda l^2T f(\hat{p}^1) + (1 - \lambda)l^2T f(\hat{p}^2) \geq 0
\]
\[
> \lambda l^2T f(\hat{p}^1) + (1 - \lambda)l^2T f(\hat{p}^1)
\]
\[
= l^2T f(\hat{p}^1).
\] (57)
If \( l_2^T f(\bar{p}) > l_1^T f(\bar{p}) \) is satisfied together with \( l_1^T f(\bar{p}) > l_1^T f(\bar{p}_1) \), it can be concluded that \( f(\bar{p}) - f(\bar{p}_1) \in K \), which is contradicted to \( \bar{p} \) K-minimal. Thus, (55) is not satisfied for \( \lambda > 1 \). Therefore, it can be concluded that (55) for the case \( \bar{t} - \lambda \bar{t}_1 - (1 - \lambda)\bar{t}_2 \geq 0 \) can only be satisfied for \( \lambda \in [0, 1] \).

Then we consider the case \( \bar{t} - \lambda \bar{t}_1 - (1 - \lambda)\bar{t}_2 < 0 \). We first consider the case \( \lambda > 1 \),

\[
l_1^T f(\bar{p})
= \lambda l_1^T f(\bar{p}_1) + (1 - \lambda)l_1^T f(\bar{p})\underbrace{+ (\bar{t}_2 - \lambda \bar{t}_1 - (1 - \lambda)\bar{t}_2)}_{< 0} l_1^T r 
\geq 0
\tag{58}
\]

As \( l_1^T f(\bar{p}) > l_1^T f(\bar{p}_1) \), (58) can be reformulated as

\[
(\lambda - 1) (l_1^T f(\bar{p}_1) - l_1^T f(\bar{p}_2)) > 0,
\tag{59}
\]

which is contradict to (51).

For the case \( \lambda < 0 \), it can be obtained in the same way that

\[
\lambda l_2^T f(\bar{p}_1) + (1 - \lambda)l_2^T f(\bar{p}_2) \geq l_2^T f(\bar{p}) > l_2^T f(\bar{p}_2)
\tag{60}
\]

and further we have

\[
(-\lambda) (l_2^T f(\bar{p}_2) - l_2^T f(\bar{p}_1)) > 0,
\tag{61}
\]

which is also contradict to (51). Therefore, it can be concluded that (55) for the case \( \bar{t} - \lambda \bar{t}_1 - (1 - \lambda)\bar{t}_2 < 0 \) can only be satisfied for \( \lambda \in [0, 1] \).

Based on the previous results, the following conclusion can be drawn: For any K-minimal solution \( \bar{p} \) of MOP, there exists a parameter \( a \in H^a \) and some \( \bar{t} \in \mathbb{R} \) so that \( (\bar{t}, \bar{p}) \) is a minimal solution of SP(\( a \)).

6.5 Proof of Theorem 2

Defining the following auxiliary variables

\[
t' := \frac{b^T f(\bar{p}) - \beta}{b^T r} \tag{62}
\]

\[
a' := a + (\bar{t} - t')r - \bar{k} = f(\bar{p}) - t'r, \tag{63}
\]
it is straightforward that $a' \in H$ and $a' + t'r - f(\bar{p}) = 0_2$. The point $(t', \bar{p})$ is feasible for $\text{SP}(a')$ and it is also a minimal solution, because otherwise there exists a feasible point $(\hat{t}, \hat{p})$ of $\text{SP}(a')$ with $\hat{t} < t'$ and some $\hat{k} \in K$ with

$$a' + \hat{t}r - f(\hat{p}) = \hat{k}$$

(64)

together with the definition of $a'$, it can be concluded that

$$a + (\hat{t} - t' + \hat{t})r - f(\hat{p}) = \hat{k} + \bar{k} \in K.$$  \hfill (65)

Hence, $(\hat{t} - t' + \hat{t}, \hat{p})$ is feasible for $\text{SP}(a)$ with $\hat{t} - t' + \hat{t} < \hat{t}$, which is in contradiction to the minimality of $(\hat{t}, \hat{p})$ for $\text{SP}(a)$. Thus, $(t', \bar{p})$ is also a minimal solution of $\text{SP}(a')$.

Then we demonstrate that the two scalar problems have the same Lagrange multiplier $\mu$. The Lagrange function $L$ to the scalar optimization problem $\text{SP}(a)$ related to the MOP is given by

$$L(t, p, \mu, \beta, a) = t - \mu^T(a + tr - f(p)) - \beta^T g(p).$$

(66)

If $\mu$ is Lagrange multiplier to the point $(\hat{t}, \hat{p})$, then it follows

$$\nabla_{(t,p)} L(\hat{t}, \hat{p}, \mu, \beta, a)^T \begin{pmatrix} t - \hat{t} \\ p - \hat{p} \end{pmatrix} = \left[ \begin{pmatrix} 1 \\ 0 \end{pmatrix} - \sum_{i=1}^2 \mu_i \begin{pmatrix} r_i \\ -p_i \end{pmatrix} \right] - \sum_{i=1}^{N+1} \beta_i \begin{pmatrix} 0 \\ \nabla_{p_i}(\bar{p}) \end{pmatrix} \right]^T \begin{pmatrix} t - \hat{t} \\ p - \hat{p} \end{pmatrix}.$$  \hfill (67)

Hence $1 - \mu^T r = 0$ and $(\mu^T \nabla_p f(\hat{p}))(p - \hat{p}) \geq 0$. Further we have $\mu^T(a + tr - f(p)) = 0$. For the minimal solution $(t', \bar{p})$ of the problem $\text{SP}(a')$, it is

$$a' + t'r - f(\bar{p}) = 0_2,$$  \hfill (68)

and thus $\mu^T(a' + t'r - f(\bar{p})) = 0$. Together with the following equality

$$\nabla_{(t,p)} L(t', \bar{p}, \mu, \beta, a') = \nabla_{(t,p)} L(\hat{t}, \hat{p}, \mu, \beta, a),$$

(69)

we also have

$$\nabla_{(t,p)} L(t', \bar{p}, \mu, \beta, a')^T \begin{pmatrix} t - t' \\ p - \bar{p} \end{pmatrix} \geq 0.$$  \hfill (70)

Therefore, $\mu$ is also Lagrange multiplier to the point $(t', \bar{p})$ for the problem $\text{SP}(a')$. 

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Fig. 1. System model

Interference to cell 1
Interference from BS 1 to neighbor cells
Interference to cell 2

Fig. 2. Explanation of scalarization

Minimal points to \((a, r)\)
Feasible Region

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Fig. 3. Relationship between power consumption and throughput contribution of the BS to the network

Fig. 4. Tradeoff between energy efficiency and throughput of the system
Fig. 5. Comparison between different schemes in view of throughput of the system and total power consumption