Simulation of gas release from pipelines using a new numerical method based on the Godunov approach

V A Shargatov¹, S I Sumskoi¹ and A S Pecherkin²

¹National Research Nuclear University MEPhI (Moscow Engineering Physics Institute), Kashirskoye shosse 31, Moscow 115409, Russia
²CJSC Scientific technical center of industrial safety problems research, Block 14, Perevedenovsky Pereulok 13, Moscow 105082, Russia

E-mail: shargatov@mail.ru

Abstract. We propose a numerical method for calculating the flows arising in a pipeline during its rupture, including calculation of the release rate. Test calculations are performed and a comparison is made with the available experimental data. A satisfactory agreement between the experimentally determined flow rate and flow rate calculated by the proposed method is shown. It is also shown that the proposed method allows one to calculate with much higher accuracy than the currently used parametric formulas.

1. Introduction

Modern pipeline systems for the transportation of natural gas are characterized by high pressures, high flow rates and huge quantities of pumped hydrocarbons, including those that are simultaneously in the equipment. In some systems, the gas pressure in the pipeline may exceed 200 atmospheres. After the emergency destruction of the gas pipeline, intensive natural gas release can occur at a velocity of several hundred meters per second. In case of fire, such releases are fraught with serious consequences: the length of fire flares can be tens or even hundreds of meters. At the same time, for the gas pipelines the destruction almost always occurs at the full cross section (the guillotine rupture), which leads to the release with the maximum flow rate.

Another feature of the release following pipeline rupture is the long extension of the pipeline, the length of individual sections can reach several hundred kilometers. This means that the involvement of the gas into motion will occur sequentially due to the propagation of the unloading waves from the place of failure, with a gradual decrease in pressure in the pipe. Accordingly, the gas discharge rate in the release will gradually decrease with time. Correct calculations of such a weakening allow us not only to estimate the time variation of the outflow intensity, but also can serve as initial data for calculating processes such as dispersion, including the stage of an intensive initial release, when the starting jet is formed, which, when ignited, will lead to the formation of a fireball.

Thus, the assessment of the consequences of accidental releases from a ruptured gas pipeline is an important and urgent task, and the simulation of the outflow of gas from a destroyed pipeline is basic in modeling the entire chain of emergency consequences.

At the same time, for all the importance of obtaining information on the intensity of the emergency flow rate from the gas pipeline, a simplified approach is practically universally applied in the calculation of this process today - Bell's parametric formula [1], designed to calculate the release rate.
from a pipeline with a compressed single-component gas without additional gas injection. In this approach, it is assumed that the flow rate decrease is determined by two time scales:

- "fast" – at the initial stage of release; the flow rate depends on the initial pressure at the pipe;
- "slow" – when the gas is released after the pressure is reduced at the point of the rupture.

Each of these decreases is described by an exponent. In accordance with this approach, Bell proposed the first formula for the gas flow rate from a long pipeline [1, 2]. In subsequent years, several variants of the two-exponential formula were proposed for calculating the gas flow rate [3–8].

The use of two-exponential formulas is extremely attractive because of their simplicity. However, when using them, an important problem is the a priori specification of a number of parameters, for example, such parameter as the flow rate at the initial instant of time.

In addition to the parametric formulas for calculating the discharge rate from the gas pipeline during the accident (and at the same time, the entire process of gas flow in the pipeline) in recent decades the methods of computational fluid dynamics (CFD methods) based on the solution of the complete system of equations of motion expressing the laws of conservation of mass, momentum and energy.

Using the CFD methods, one can fully take into account the influence of such factors as non-isothermal flow, friction loss, heat transfer through the walls, and also changes in the specific heat of the gas and phase transitions in the gas (primarily due to the condensation of heavier natural gas fractions).

Initially, for such a simulation, a method of characteristics was used, based on the so-called Hartee's hybrid algorithm [8, 9]. The results of calculations using this approach are given in [9–12].

In addition to the method of characteristics, other numerical methods, such as the finite difference method [14–16] and the finite element method [17–19], are also used to calculate the flow rate from the destroyed gas pipeline.

One of the most effective and physically justified methods for calculating gas-dynamic flows is Godunov's method [20]. Previously, it was used to calculate the flows in oil pipelines [21–23]. In this paper Godunov's approach is proposed to be applied to the calculation of emergency flow rate from gas pipelines.

2. Basic equations
To describe the gas flow in a pipeline (including a case of its rupture and depressurization), it is proposed to use a system of gas dynamics equations describing non-stationary one-dimensional motion in a pipe of constant cross section.

\[
\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} = 0, \quad (1)
\]

\[
\frac{\partial (\rho u)}{\partial t} + \frac{\partial (\rho u^2 + p)}{\partial x} = -\lambda \frac{1}{D_0} \frac{\rho u |u|}{2} - \rho g \frac{dz}{dx}, \quad (2)
\]

\[
\frac{\partial \left( \rho \left( \varepsilon + \frac{u^2}{2} \right) \right)}{\partial t} + \frac{\partial \left( \rho u \varepsilon + \rho u \frac{u^2}{2} + pu \right)}{\partial x} = \pi D_0 q_s - \rho u g \frac{dz}{dx}. \quad (3)
\]

here \( x \) – coordinate along the axis of the pipeline, \( t \) – time, \( \rho \) – density, \( u \) – velocity, \( p \) – pressure, \( z \) – height of the pipeline line, \( g \) – acceleration of gravity, \( D_0 \) – diameter of the pipeline, \( \lambda \) – friction factor, taking into account the effect of friction on the walls of the pipe (in this work, it is assumed equal to 0.0086), \( \alpha_k \) – is a coefficient that takes into account the presence of the velocity pulsation component that is complementary to the mainstream flow velocity (in this work, it is assumed equal to
1.036 [24]), \( e \) – is the specific internal energy, \( q_s \) – is the heat flux through the pipeline walls from the environment.

The energy conservation equation, in addition to the standard terms, includes a special term for describing heat exchange with the environment \( (q_s) \). The following relation is used to calculate \( q_s \):

\[
q_s = -K_T (T - T_i)
\]

(4)

here \( T \) – the gas temperature in the pipeline, \( T_i \) – the temperature outside the pipeline, \( K_T \) – the heat transfer coefficient (in this work, it is assumed equal to 1.2 kcal/m²/h/K).

To calculate the pressure in the pipeline, we propose to use the equation of state of an ideal gas.

As the boundary conditions, the condition of constant pressure at the inlet and outlet of the pipeline is used. The initial condition for the pipeline is the stationary pumping mode.

Only the case of the total rupture is considered in this paper when modeling the gas release. In this case, to simulate the flow, it is proposed to set at the end of each branch of the broken pipeline a velocity equal to the speed of sound in the critical flow regime, or a pressure equal to the surrounding (1 atm.) at subsonic release mode.

3. Numerical method of solution

The system of equations (1) – (4) was solved within the framework of the Gudunov approach [20]. To organize the computational procedure, the pipeline section was divided into cells of size \( \Delta x \), the state in which at any time \( n \Delta t \) was described using linear profiles of the corresponding parameters (pressure \( p_i^n(x) \), density \( \rho_i^n(x) \) and fluid velocity \( u_i^n(x) \)) in each \( i \)-th cell \( (x_{i-1/2} \leq x \leq x_{i+1/2}) \):

\[
p_i^n(x) = \bar{p}_i^n + \Delta \bar{p}_i^n(x-x_i), \quad \rho_i^n(x) = \bar{\rho}_i^n + \Delta \bar{\rho}_i^n(x-x_i), \quad u_i^n(x) = \bar{u}_i^n + \Delta \bar{u}_i^n(x-x_i),
\]

(5)

here \( x_i \) – the coordinate of the center of the \( i \)-th cell, \( x_{i+1/2} \) – the coordinate of the boundary between the \( i \)-th and \( i-1 \) cells, \( \bar{p}_i^n, \bar{\rho}_i^n, \bar{u}_i^n \) – pressure, density and velocity in the center of the \( i \)-th cell at the \( n \)-th time step.

The values \( \Delta \bar{p}_i^n, \Delta \bar{\rho}_i^n \) and \( \Delta \bar{u}_i^n \) are calculated according to the TVD-principle proposed in [25].

Simulation consists of two stages: the time step is split into physical processes in the same way as it was realized in [21–23].

First, at stage I, the gas flow is modeled without taking friction and gravity into account, and stage II takes into account the effect of frictional forces and gravity.

Stage I. At this stage, the mass, momentum and energy fluxes are calculated in each \( i \)-th cell, both due to convective motion, and due to the action of pressure. The fluxes are calculated by solving the problem of the discontinuity decay both on the left (index \( l \)) and right (index \( r \)) cell boundaries.

To solve the problem of a discontinuity disintegration at the boundaries of two neighboring cells, the values \( p_i^n(x), \rho_i^n(x) \) and \( u_i^n(x) \) are calculated from formulas (5) on the right and left boundaries of each \( i \)-th difference cell. Accordingly, for \( i \)-th cell - to the right of its boundary \( i-1/2 \), i.e. for \( x=x_{i+1/2} \), these quantities are denoted as \( p_i^{n-1/2}, \rho_i^{n-1/2}, u_i^{n-1/2} \), and to the left of its boundary \( i + 1/2, x=x_{i-1/2} \) they are denoted by \( p_i^{n+1/2}, \rho_i^{n+1/2}, u_i^{n+1/2} \).

For the boundary between the \( i-1 \) and \( i \)-th cells, the Riemann problem is solved with the initial data in the form \( p_i^{n-1/2}, \rho_i^{n-1/2}, u_i^{n-1/2} \) and \( p_i^{n+1/2}, \rho_i^{n+1/2}, u_i^{n+1/2} \).

In this paper, for each \( i \)-th cell, the standard procedure for solving the Riemann problem [20] is used on each of its boundaries. The only difference between the Riemann problem in this paper and
the solution algorithm in [20] is the use of quantities $\alpha_k \frac{u_i^2}{2}$ in the expression for the kinetic energy instead of the standard kinetic energy $\frac{u_i^2}{2}$.

As a result of solving Riemann problems, the pressure $p_i^n$, density $\rho_i^n$, and velocity $u_i^n$ are determined on the right boundary of the $i$-th cell ($x=x_{i+1/2}$), and the pressure $p_{i-1}^n$, density $\rho_{i-1}^n$, and velocity $u_{i-1}^n$ on the left-hand boundary of the $i$-th cell ($x=x_{i-1/2}$).

Using the obtained pressure values $p_i^n$, $p_{i-1}^n$ and density values $\rho_i^n$, $\rho_{i-1}^n$, the values of the specific internal energy $e_i^n$, $e_{i-1}^n$, the solution algorithm in $[20]$ is the use of quantities $\rho_i^n$, $e_i^n$ are calculated at the boundaries of the $i$-th cell.

Knowing $p_i^n$, $\rho_i^n$, $u_i^n$, $e_i^n$ and $p_{i-1}^n$, $\rho_{i-1}^n$, $u_{i-1}^n$, $e_{i-1}^n$ one can calculate the values of the mass $\Phi_{mi}^n$, momentum $\Phi_{pi}^n$ and energy $\Phi_{ei}^n$ fluxes:

$$
\Phi_{mi}^n = u_i^n \cdot \rho_i^n - u_{i-1}^n \cdot \rho_{i-1}^n,
$$

$$
\Phi_{pi}^n = u_i^n \cdot \rho_i^n + p_i^n - u_{i-1}^n \cdot \rho_{i-1}^n - p_{i-1}^n,
$$

$$
\Phi_{ei}^n = \rho_i^n \cdot u_i^n \left( e_i^n + \alpha_k \frac{u_i^2}{2} \right) + u_{i-1}^n \cdot p_{i-1}^n - \rho_{i-1}^n \cdot u_{i-1}^n \left( e_{i-1}^n + \alpha_k \frac{u_{i-1}^2}{2} \right) - u_{i-1}^n \cdot p_{i-1}^n.
$$

The new values of density $\tilde{\rho}^{n+1}_i$, velocity $\tilde{u}^{n+1}_i$ and internal energy $\tilde{e}^{n+1}_i$ in the $i$-th cell at $n + 1$ time step at the end of the first stage are calculated as follows:

$$
\tilde{\rho}^{n+1}_i = \frac{\tilde{m}^{n+1}_i}{\Delta x} = \frac{m_i^n + \Phi_{mi}^n \cdot \Delta t}{\Delta x},
$$

$$
\tilde{u}^{n+1}_i = \frac{\tilde{R}^{n+1}_i}{\tilde{m}^{n+1}_i} = \frac{R_i^n + \Phi_{pi}^n \cdot \Delta t}{m_i^{n+1}},
$$

$$
\tilde{e}^{n+1}_i = \frac{\tilde{E}_i^{n+1}}{\tilde{m}^{n+1}_i} - \alpha_k \frac{\tilde{u}^{n+12}_i}{2} = \frac{E_i^n + \Phi_{ei}^n \cdot \Delta t}{m_i^{n+1}} - \alpha_k \frac{\tilde{u}^{n+12}_i}{2},
$$

here $\tilde{m}^{n+1}_i$ and $\tilde{m}^{n}_i$ is the masses, $\tilde{R}^{n+1}_i$ and $R_i^n$ is the pulses and $\tilde{E}^{n+1}_i$ and $E_i^n$ is the total energies in the $i$-th cell at $n + 1$ (after stage I) and $n$ time layers.

Stage II. The effect of friction forces on the walls, gravity and heat exchange through the walls are taken into account. Velocity and energy in the $i$-th cell are recalculated in accordance with formulas:

$$
u^{n+1}_i = \tilde{u}^{n+1}_i - \lambda \left( \tilde{E}_i^{n+1} \right) \left( \frac{\tilde{u}^{n+1}_i}{2D^{n+1}_i} \right) \Delta t - g \beta \Delta t
$$

$$
\varepsilon^{n+1}_i = \tilde{e}^{n+1}_i + \alpha_k \frac{\tilde{u}^{n+12}_i}{2} - \alpha_k \frac{u^{n+12}_i}{2} + \frac{\pi D \bar{q}^{n+1}_i}{\tilde{\rho}_i^{n+1}} \Delta t - u g \frac{dz}{dx} \Delta t,
$$

Since the gravity and friction affects only velocity and internal energy on the new time layer, the pressure is also recalculated, and the other variables in stage II remain unchanged.
Thus, the required values are found on the new time step $u_{i+1}^{n+1}$, $p_{i+1}^{n+1}$, $\rho_{i+1}^{n+1}$, $\varepsilon_{i+1}^{n+1}$, after which the calculation procedure for stages I-II is repeated again to find the values on the subsequent time layers.

4. Results and discussion

The experimental data, described in [26], were taken as initial data on release gas modeling. A 160-km-long pipe with a diameter of 0.768 km is filled with methane at a pressure of 3.2 MPa. The gas is at rest. A full rupture occurs in the middle. Since the initial temperature in the pipe in [26] is not reported, in the calculations for a value of 300 K was chosen.

The results of the experimental measurements and the calculated values are shown in Fig. 1. The values obtained by calculation are given:
- according to the original Bell formula [1] without taking into account the inertia of the gas and taking into account the inertia (with different initial flow rates);
- using the numerical method proposed in this paper.

![Figure 1](image_url)

**Figure 1.** Experimental and calculated time dependence of the flow rate for the experiment [26]

As the boundary conditions at the release point, the atmospheric pressure for the subsonic flow regime and the sound velocity of the outflow for the supersonic regime were set. At the closed end of the pipe, the condition of a wall was specified.

As one can see from Fig. 1, the best agreement with the experimental data is given by the calculation of the complete system of gasdynamic equations. In this case, the flow rate decrease profile is reproduced throughout the interval of the times considered. In this case, the calculated values slightly exceed the observed values, which can be explained by some inaccuracies in the model (inaccuracies in calculating the friction factor, the heat transfer coefficient, neglecting the temperature dependence of the specific heat, etc.). This discrepancy in results can be decreased in subsequent works and the cause of it can be eliminated.

Calculations using the Bell formula [1] give a significantly overestimated flow at the initial instant of time and, with a rapid decay in time, reach released levels below the realistic values observed in the experiment.

Bell’s formula, taking into account the inertia of the gas, gives a more realistic initial flow, but in general, a faster decline in release rate is also obtained, with an appropriate overestimation of the intensity at the initial stage and an underestimation at later stages.
5. Conclusions
In this paper, we propose a new numerical method for calculating flows in gas pipelines following pipeline rupture. The method was developed within the framework of the Godunov approach. The proposed method makes it possible to obtain solutions that agree well with the available experimental data. Simulation is carried out from the "first principles" without additional assumptions, as in the case of widely used parametric formulas.

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