The study of charmless hadronic $B_s$ decays

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The perturbative QCD approach has achieved great success in the study of hadronic $B$ decays. Utilizing the constrained parameters in these well measured decay channels, we study most of the possible charmless $B_s \to PP, PV$ and $VV$ decay channels in the perturbative QCD approach. In addition to the branching ratios and CP asymmetries, we also give predictions to the polarization fractions of the vector meson final states. The size of SU(3) breaking effect is also discussed. All of these predictions can be tested by the future LHCb experiment.

Keywords: $B_s$ decays, factorization, pQCD

1 Introduction

There is a continuous progress in the study of hadronic $B$ decays since the so called naive factorization approach. In recent years, the QCD factorization approach (QCDF) and perturbative QCD factorization (pQCD) approach together with the soft-collinear effective theory solved a lot of problems in the non-leptonic decays. Although most of the branching ratios measured by the B factory experiments can be explained by any of the theories, the direct CP asymmetries measured by the experiments are ever predicted with the right sign only by the pQCD approach. The LHCb experiment will soon run in the end of 2007. With a very large luminosity, it will accumulate a lot of $B_s$ events. The progress in both theory and experiment encourages us to apply the pQCD approach to the charmless $B_s$ decays in this work.

In the hadronic $B(B_s)$ decays, there are various energy scales involved. The factorization theorem allows us to calculate them separately. First, the physics from the electroweak scale down to b quark mass scale is described by the renormalization group running of the Wilson coefficients of effective four quark operators. Secondly, the hard scale from b quark mass scale to the factorization scale $\sqrt{\Lambda m_B}$ are calculated by the hard part calculation in the perturbative QCD approach. When doing the integration of the momentum fraction $x$ of the light quark, end point singularity will appear in the collinear factorization (QCDF and SCET) which breaks down the factorization theorem. In the pQCD approach, we do not neglect the transverse momentum $k_T$ of the light quarks in meson. Therefore the endpoint singularity disappears. The inclusion of transverse momentum will also give large double logarithms $\ln^2 k_T$ and $\ln^2 x$ in the hard part calculations. Using the renormalization group equation, we can resum them for all loops to the leading order resulting Sudakov factors. The Sudakov factors suppress the endpoint contributions to make the calculation consistent.

The physics below the factorization scale is non-perturbative in nature, which is described by the hadronic wave functions of mesons. They are not perturbatively calculable, but universal for all the decay processes. Since many of the hadronic and semi-leptonic $B$ decays have been measured well in the two B factory experiments, the light wave functions are strictly constrained. Therefore, it is useful to use the same wave functions in our $B_s$ decays determined from the hadronic $B$ decays. The uncertainty of the hadronic wave functions will come mainly from the SU(3) breaking effect between the $B_s$ wave function
and $B$ wave function. In practice, we use a little larger $\omega_0$ parameter for the $B_s$ meson than the $B_d$ meson, which characterize the fact that the light $s$ quark in $B_s$ meson carries a littler larger momentum fraction that the $d$ quark in the $B_d$ meson.

## 2 Results and Discussion

For $B_s$ meson decays with two light mesons in the final states, the light mesons obtain large momentum of 2.6GeV in the $B_s$ meson rest frame. All the quarks inside the light mesons are therefore energetic and collinear like. Since the heavy $b$ quark in $B_s$ meson carry most of the energy of $B_s$ meson, the light $s$ quark in $B_s$ meson is soft. In the usual emission diagram of $B_s$ decays, this quark goes to the final state meson without electroweak interaction with other quarks, which is called a spectator quark. Therefore there must be a connecting hard gluon to make it from soft like to collinear like. The hard part of the interaction becomes six quark operator rather than four. The soft dynamics here is included in the meson wave functions. The decay amplitude is infrared safe and can be factorized as the following formalism:

$$ C(t) \times H(t) \times \Phi(x) \times \exp \left[ -s(P, b) - 2 \int_{1/b}^{t} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_0 (\alpha_s (\bar{\mu})) \right], $$

(1)

where $C(t)$ are the corresponding Wilson coefficients of four quark operators, $\Phi(x)$ are the meson wave functions and the variable $t$ denotes the largest energy scale of hard process $H$, which is the typical energy scale in PQCD approach and the Wilson coefficients are evolved to this scale. The exponential of $S$ function is the so-called Sudakov form factor resulting from the resummation of double logarithms occurred in the QCD loop corrections, which can suppress the contribution from the non-perturbative region. Since logarithm corrections have been summed by renormalization group equations, the above factorization formula does not depend on the renormalization scale $\mu$ explicitly.

The numerical results of the $B_s$ decays branching ratios and CP asymmetry parameters are displayed in ref.\(^7\). In all the decay channels for charmless $B_s$ decays, only several are measured by the CDF collaboration.\(^9\) We show those channels together with results by QCDF\(^10\) and SCET approaches\(^11\) in table \(\text{Table 1}\). From those comparison, we notice that the measured branching ratios are still consistent with the theoretical calculations. Like the case in $B$ decays, the calculated branching ratios from the three kinds of methods overlap with each other, considering the still large theoretical uncertainties. A global fit is useful when we have enough measured channels.

In table \(\text{Table 1}\), the only measured CP asymmetry in $B_s \to K^-\pi^+$ decay prefer our pQCD approach rather than QCDF approach. This is similar with the situation in $B$ decays. The direct CP asymmetry is proportional to the sine of the strong phase difference of two decay topologies.\(^7\) The strong phase in our pQCD approach is mainly from the chirally enhanced space-like penguin diagram, while in the QCDF approach, the strong phase mainly comes from the virtual charm quark loop diagrams. The different origin of strong phases gives different sign to the direct CP asymmetry imply a fact that the dominant strong phase in the charmless decays should come from the annihilation diagrams. It should be noted that the SCET approach can not predict the direct CP asymmetry of $B$ decays directly, since they need more experimental measurements as input. However, it also gives the right CP asymmetry for $B_s$ decay if with the input of experimental CP asymmetries of $B$ decays, which means good SU(3) symmetry here.

For the $B_s \to VV$ decays, we also give the polarization fractions in addition to the branching ratios and CP asymmetry parameters\(^7\). Similar to the $B \to VV$ decay channels, we also have large transverse polarization fractions for the penguin dominant processes, such as $B_s \to \phi\phi$, $B_s \to K^{*+}K^{*-}$, $K^{*0}\bar{K}^{*0}$ decays, whose transverse polarization fraction can reach 40-50%.

| $B(B_s \to K^-\pi^+)(10^{-6})$ | SCET | QCDF | PQCD | EXP |
|-------------------------------|------|------|------|-----|
| $B(B_s \to K^-K^+)(10^{-6})$ | 18 ± 7 | 23 ± 27 | 17 ± 9 | 24 ± 5 |
| $B(B_s \to K^-\pi^+)(10^{-6})$ | 20 ± 26 | −6.7 ± 16 | 30 ± 6 | 39 ± 15 ± 8 |

### Table 1: The branching ratios and CP asymmetry calculated in PQCD approach, QCDF and SCET approaches together with Experimental Data.
3 SU(3) breaking effect

The SU(3) breaking effects comes mainly from the $B_s(B_d)$ meson decay constant and distribution amplitude parameter, light meson decay constant and wave function difference, and various decay topology differences. As an example we mainly focus on the decays $B \rightarrow \pi\pi$, $B \rightarrow K\pi$, $B_s \rightarrow K\pi$ and $B_s \rightarrow KK$, as they can be related by SU(3)-symmetry. A question of considerable interest is the amount of SU(3)-breaking in various topologies (diagrams) contributing to these decays. For this purpose, we present in Table 2 the magnitude of the decay amplitudes (squared, in units of GeV$^2$) involving the distinct topologies for the four decays modes. The first two decays in this table are related by U-spin symmetry ($d \rightarrow s$) (likewise the two decays in the lower half). We note that the assumption of U-spin symmetry for the (dominant) tree ($T$) and penguin ($P$) amplitudes in the emission diagrams is quite good, it is less so in the other topologies, including the contributions from the W-exchange diagrams, denoted by $E$ for which there are non-zero contributions for the flavor-diagonal states $\pi^+\pi^-$ and $K^+K^-$ only. The U-spin breakings are large in the electroweak penguin induced amplitudes $P_{EW}$, and in the penguin annihilation amplitudes $P_A$ relating the decays $B_d \rightarrow K^+\pi^-$ and $B_s \rightarrow K^+K^-$. In the SM, however, the amplitudes $P_{EW}$ are negligibly small.

Table 2: Contributions from the various topologies to the decay amplitudes (squared) for the four indicated decays. Here, $T$ is the contribution from the color favored emission diagrams; $P$ is the penguin contribution from the emission diagrams; $E$ is the contribution from the W-exchange diagrams; $P_A$ is the contribution from the penguin annihilation amplitudes; and $P_{EW}$ is the contribution from the electro-weak penguin induced amplitude.

| mode (GeV$^2$) | $|T|^2$ | $|P|^2$ | $|E|^2$ | $|P_A|^2$ | $|P_{EW}|^2$ |
|----------------|--------|--------|--------|----------|----------|
| $B_d \rightarrow \pi^+\pi^-$ | 1.5 | $9.2 \times 10^{-3}$ | $6.4 \times 10^{-3}$ | $7.5 \times 10^{-4}$ | $2.7 \times 10^{-6}$ |
| $B_s \rightarrow \pi^+K^-$ | 1.4 | $7.4 \times 10^{-3}$ | 0 | $7.0 \times 10^{-3}$ | $5.4 \times 10^{-6}$ |
| $B_d \rightarrow K^+\pi^-$ | 2.2 | $18.8 \times 10^{-3}$ | 0 | $4.7 \times 10^{-3}$ | $7.4 \times 10^{-6}$ |
| $B_s \rightarrow K^+K^-$ | 2.0 | $14.7 \times 10^{-3}$ | $4.6 \times 10^{-3}$ | $9.8 \times 10^{-3}$ | $3.1 \times 10^{-6}$ |

In $B_d^0 \rightarrow K^-\pi^+$ and $B_d^\mp \rightarrow K^+\pi^-$, the branching ratios are very different from each other due to the differing strong and weak phases entering in the tree and penguin amplitudes. However, as shown by Gronau[12] the two relevant products of the CKM matrix elements entering in the expressions for the direct CP asymmetries in these decays are equal, and, as stressed by Lipkin[13] subsequently, the final states in these decays are charge conjugates, and the strong interactions being charge-conjugation invariant, the direct CP asymmetry in $B_d^\mp \rightarrow K^-\pi^+$ can be related to the well-measured CP asymmetry in the decay $B_d^\mp \rightarrow K^+\pi^-$ using U-spin symmetry.

Following the suggestions in the literature, we can define the following two parameters:

$$R_3 = \frac{|A(B_s \rightarrow \pi^+K^-)|^2 - |A(\bar{B}_s \rightarrow \pi^-K^+)|^2}{|A(B_d \rightarrow \pi^-K^+)|^2 - |A(B_d \rightarrow \pi^+K^-)|^2}$$

(2)

$$\Delta = \frac{\alpha_{dir}(B_d \rightarrow \pi^+K^-)}{\alpha_{dir}(B_s \rightarrow \pi^-K^+)} + \frac{BR(B_s \rightarrow \pi^-K^+)}{BR(B_d \rightarrow \pi^+K^-)} \frac{\tau(B_d)}{\tau(B_s)}$$

(3)

The standard model predicts $R_3 = -1$ and $\Delta = 0$ if we assume U-spin symmetry. Since we have a detailed dynamical theory to study the SU(3) (and U-spin) symmetry violation, we can check in pQCD approach how good quantitatively this symmetry is in the ratios $R_3$ and $\Delta$. We get $R_3 = -0.96^{+0.11}_{-0.09}$ and $\Delta = -0.03 \pm 0.08$. Thus, we find that these quantities are quite reliably calculable, as anticipated on theoretical grounds. SU(3) breaking and theoretical uncertainties are very small here, because most of the breaking effects and uncertainties are canceled due to the definition of $R_3$ and $\Delta$. On the experimental side, the results for $R_3$ and $\Delta$ are[9]

$$R_3 = -0.84 \pm 0.42 \pm 0.15, \Delta = 0.04 \pm 0.11 \pm 0.08.$$  (4)

We conclude that SM is in good agreement with the data, as can also be seen in Fig. 11 where we plot theoretical predictions for $R_3$ vs. $\Delta$ and compare them with the current measurements of the same. The measurements of these quantities are rather imprecise at present, a situation which we hope will greatly improve at the LHCh experiment.
4 Summary

Based on the $k_T$ factorization, pQCD approach is infrared safe. Its predictions on the branching ratios and CP asymmetries of the $B^0(B^{±})$ decays are tested well by the B factory experiments. Using those tested parameters from these decays, we calculate a number of charmless decay channels $B_s \rightarrow PP, PV$ and $VV$ in the perturbative QCD approach. The experimental measurements of the three $B_s$ decay channels are consistent with our numerical results. Especially the measured direct CP asymmetry of $B_s \rightarrow \pi^-K^+$ agree with our calculations. We also discuss the SU(3) breaking effect in these decays, which is at least around 20-30%. We also show that the Gronau-Lipkin sum rule works quite well in the standard model, where the SU(3) breaking effects mainly cancel.

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References

1. M. Wirbel, B. Stech, M. Bauer, Z. Phys. C29, 637 (1985); M. Bauer, B. Stech, M. Wirbel, Z. Phys. C34, 103 (1987).
2. A. Ali, G. Kramer, and C. D. Lü, Phys. Rev. D58, 094009 (1998) hep-ph/9804363; Phys. Rev. D59, 014005 (1999) hep-ph/9805403; Y. H. Chen, H. Y. Cheng, B. Tseng, and K. C. Yang, Phys. Rev. D60, 094014 (1999) hep-ph/9903453.
3. M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Phys. Rev. Lett. 83, 1914 (1999) hep-ph/9905312; Nucl. Phys. B591, 313 (2000) hep-ph/0006124.
4. Y. Y. Keum, H. n. Li, and A. I. Sanda, Phys. Lett. B504, 6 (2001) hep-ph/0004004; Phys. Rev. D63, 054008 (2001) hep-ph/0004173; C. D. Lü, K. Ukai and M. Z. Yang, Phys. Rev. D63, 074009 (2001) hep-ph/0004213; C. D. Lü and M. Z. Yang, Eur. Phys. J.C23, 275 (2002) hep-ph/0011238.
5. C. W. Bauer, D. Pirjol, I. W. Stewart, Phys. Rev. Lett. 87 (2001) 201806 hep-ph/0107002; Phys. Rev. D65, 054022 (2002) hep-ph/0109045.
6. B. H. Hong, C. D. Lü, Sci. China G49, 357-366 (2006) hep-ph/0505020.
7. X.Q. Yu, Y. Li, C.D. Lu, Phys. Rev. D71, 074026 (2005), Erratum-ibid. D72, 119903 (2005) hep-ph/0501152; A. Ali et al., arXiv: hep-ph/0703162.
8. H.-n. Li, Prog. Part. Nucl. Phys. 51, 85 (2003) hep-ph/0303116, and reference therein.
9. M. Morello [CDF Collaboration], FERMILAB-PUB-06-471-E (2006) hep-ex/0612018.
10. M. Beneke and M. Neubert, Nucl. Phys. B675, 333 (2003) hep-ph/0308039.
11. A. Williamson and J. Zupan, Phys. Rev. D74, 014003 (2006); Erratum-ibid. D 74, 039901 (2006) hep-ph/0601214.
12. M. Gronau, Phys. Lett. B492, 297 (2000) hep-ph/0008292.
13. H. J. Lipkin, Phys. Lett. B621, 126 (2005) hep-ph/0503022.