Voltage rectification in two dimensional Josephson junction arrays

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Abstract

We study numerically the directed motion of vortices (antivortices) under an applied ac bias in two-dimensional Josephson junction arrays (JJA) with an asymmetrically modulated periodic vortex pinning potential. We find that the ratchet effect in large 2D JJA can be obtained using the RSJ model for the overdamped vortex dynamics. The rectification effect shows a strong dependence on vortex density as well as an inversion of the vortex flow direction with the ac amplitude, for a wide range of high magnetic field around $f = 1/2$ ($f$ being the vortex density). Our results are in good agreement with very recent experiments by D.E. Shalom and H. Pastoriza [Phys. Rev. Lett. 94, 177001, (2005)].

Key words: Josephson junction arrays, ratchet effect, periodic pinning
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1. INTRODUCTION

The study of ratchet effects in superconductors has attracted great interest in recent years due to the possibility of controlling vortex motion [1]. However, only very recently nano-micro engineering has allowed to successfully fabricate different types of asymmetric pinning potentials for the motion of flux quanta. Voltage rectification was spectacularly observed in Nb films with a triangular dots array of pinning potential [2] and in Al films with square arrays of hole pinning sites [3].

Ratchet effect has also been analyzed theoretically and experimentally in Josephson Junctions arrays (JJA), both in the classical and quantum regime, particularly for the cases of 1D parallel JJA, SQUID’s or circular arrays [4]. Less attention has been paid however to larger two dimensional JJA, where collective effects are expected to play an important role. Indeed, this collective behavior was very recently observed in experiments with large square JJA [5]. In this case, improved e-lithography techniques were used to modulate the gap between the superconducting islands and thus an asymmetric and periodic sawtooth potential for the vortex-antivortex motion was generated. Voltage rectification was then clearly observed under an applied ac current (rocking ratchet), and analyzed as a function of the vortex density. In this paper we show that the main experimental results reported in [5] can be successfully reproduced and analyzed in detail by numerical simulations of the overdamped RSJ model for a large 2D JJA with asymmetrically modulated critical currents.
2. MODEL

We study the dynamics of asymmetrically modulated 2D JJA using the resistively shunted junction (RSJ) model for the SNS (superconductor-normal-superconductor) junctions in square two dimensional arrays [6]. The current flowing in the junction between two superconducting islands is the sum of the Josephson supercurrent and the normal current (see details of the model in [7]):

\[ I_\mu(n) = I_0 \sin \theta_\mu(n) + \frac{\Phi_0}{2\pi c R_N} \frac{\partial \theta_\mu(n)}{\partial t} + \eta_\mu(n, t) \tag{1} \]

where \( I_0 \) is the critical current of the junction between the sites \( n \) and \( n + \mu \) in a square lattice \( \{n = (n_x, n_y), \mu = \vec{x}, \vec{y}\} \), \( R_N \) is the normal state resistance and \( \theta_\mu(n) = \theta(n + \mu) - \theta(n) - A_\mu(n) = \Delta_\mu \theta(n) - A_\mu(n) \) is the gauge invariant phase difference with \( A_\mu(n) = \frac{2\pi}{\Phi_0} \int_{n_{min}}^{n} A \cdot dl \) being \( \Phi_0 \) the flux quantum. The Langevin noise term \( \eta_\mu \) models the contact with a thermal bath at temperature \( T \) and satisfies \( \langle \eta_{\mu}(n, t) \eta_{\nu}(n', t') \rangle = \frac{2kT}{\Phi_0} \delta_{\mu, \nu} \delta_{n, n'} \delta(t - t') \).

The asymmetry in the underlying periodic pinning potential is introduced by modulating the critical currents \( I_0 \), and therefore the Josephson coupling energy between islands \( E_J = \Phi_0 I_0 / 2\pi \). Experimentally, the ratchet potential is generated by modulating the gap between the superconducting islands, \( d \), as a sawtooth with a given period, \( p \) [5]. For SNS junctions, the coupling energy decrease exponentially with \( d \): \( E_J \approx \exp[-d/\xi_N] \), \( \xi_N \) being the coherence length in the normal metal. In our model we make a simplification in order to modulate the energy couplings through the critical currents. We consider a sawtooth modulation of \( I_0 \) where \( I_0 = f(n_x) \) which increases linearly from \( I_{0_{min}} \) to \( I_{0_{max}} \) with each period \( p \).

An external magnetic field \( H \) perpendicular to the sample is applied, such that \( \Delta_\mu \times A_\mu(n) = 2\pi f \), where \( f = Ha^2/\Phi_0 \) is the vortex density and \( a \) is the array lattice spacing. In addition, we apply an external ac driving, \( I = I_{ac} \sin(2\pi \omega t) \) in the \( y \) direction. We take periodic boundary conditions in both directions.

The dynamical equations for the superconducting phases are obtained after considering conservation of the current in each node (see details in [7]). The resulting set of Langevin equations are solved using a second-order Runge-Kutta algorithm with time step \( \Delta t = 0.1 \tau_J \) (\( \tau_J = 2\pi c R_N I_0 / \Phi_0 \)) and time integration ranges \( 2 \times 10^6 \tau_J - 1.1 \times 10^7 \tau_J \) after the transient. Under ac external currents applied perpendicularly to the sawtooth modulation, the current-voltages characteristics are calculated as well as the vortex structure. The normalization used is the following: currents by \( I_0 \), voltages by \( R_N I_0 \), temperature by \( I_0 \Phi_0 / 2\pi k_B \) and frequencies by \( (\tau_J)^{-1} \).

We study the vortex rectification on ratchet-like square \((L \times L)\) JJA, with period \( p = 8 \) and linear modulated critical currents between \( I_{0_{min}} = 0.5 I_0 \) and \( I_{0_{max}} = 1.5 I_0 \). A wide range of magnetic field was explored, from the one corresponding to only one vortex in the system \( (f = 1/L^2) \) to very high vortex densities. We use system sizes from \( L = 32 \) to \( L = 128 \).

3. RESULTS

It is well known that the velocity of an overdamped particle under an applied ac force can be rectified by introducing a periodic potential with broken reflection symmetry. We can thus ask whether this effect, known as rocking ratchet, can be still observed for the motion of interacting vortices in a modulated 2D JJA. Since vortex velocity is proportional to voltage, this kind of ratchet effect would be observable as a voltage rectification. In Fig.1 we show clear evidence of the latter effect, i.e. a net directional vortex motion under the applied ac drive of zero mean. We see that mean dc voltage, \( \langle V_{dc} \rangle \), as a function of \( I_{ac} \) presents a maximum for an optimal value of \( I_{ac} \) and decreases slowly with increasing \( I_{ac} \). This is the qualitative behavior expected for the one particle rocking ratchet. To study the role of the vortex interactions in Fig.1(a) examples for different vortex densities are presented, \( f = 1/8, 1/16, 1/32 \). A decrease in the maximum rectification is seen while the vortex density increases as well as a shift to higher optimal ac amplitude. An important feature is worthy of being mentioned here (see in-
Fig. 1. Rectified vortex motion: (a) $I_{ac} - V$ characteristics for low vortex densities. Inset: Comparison with one single vortex. (b) $I_{ac} - V$ characteristics for higher vortex densities. Simulations performed in $32 \times 32$ system size, at $T=0.05$ and $\omega = 10^{-4}$.

set in Fig.1(a)): the voltage rectification is highly negligible when only one single vortex is present in the system, corresponding to a vortex density $f = 1/(32 \times 32)$ in this case. A more intriguing feature appears when the vortex density is further increased. Examples are shown in Fig.1(b). For instance, an inversion of the vortex flow direction is observed. These observations provide evidence for the relevant role of vortex interactions. Also notice the expected reflection symmetry around $f = 1/2$ due to the symmetries with the magnetic field of the Hamiltonian representing JJA [8]. From $f = 0$ to $f = 1/2$ the dissipation arises from the motion of “positive” vortices. The opposite occurs for $f$ between $f = 1/2$ to $f = 1$ where a flux of negative vortices (antivortices) gives a voltage response of opposite sign. In addition to reproducing the experiments of Ref. [5], we have observed that the instantaneous vortex lattice dynamics behind this phenomenon is very complex [9] compared with the vortex motion observed in Ref.[2].

It is important to accurately determine the range of parameters where vortex motion rectification is observable. In agreement with experiment,

Fig. 2. Rectified dc voltage response dependences: (a) On frequency, $\omega$, for $f = 1/8$ and $I_{ac} = 0.25$. (b) On temperature, $T$, for $I_{ac} = 0.1$ and $I_{ac} = 0.25$, both examples at $\omega = 10^{-4}$.

Fig. 3. Size analysis: (a) Current-voltage characteristics for a high vortex density ($f = 3/4$) and different system sizes $L = 32, 64, 128$. (b) Maximum and minimum optimum mean voltages vs system size for $f = 3/4$ and $f = 1/8$. 

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we obtain almost zero voltage rectification for all ac amplitudes if the frequencies are larger than 0.001τJ−1 due to the fast changes in potential slope not allowing vortices to have time to explore the asymmetric underlying potential during a cycle. A plateau of saturation is obtained for very low frequencies (see Fig. 2(a) where an example is shown for f = 1/8 and Iac = 0.25). Most of the results shown here were also obtained at zero temperature as expected for rocking ratchets. It is also useful to explore the range of temperatures where rectification occurs. An analysis is shown in Fig. 2(b) for two values of the ac amplitude which are representative of the behavior of almost all the vortex densities studied (see also Fig.1): Iac = 0.1 and Iac = 0.2.

In Fig. 3(a) we show that our main results at high vortex density (f = 3/4) are independent of system size. The key features, such as vortex rectification and current reversal effects do not depend strongly on system sizes. An analysis for the maximum and minimum values of the rectification vs L is shown in Fig. 3(b) for two vortex densities, f = 1/8 and f = 3/4 (for more detailed explanation on the dynamics at intermediate ac amplitudes see [9]).

Finally, in Fig. 4, contour plots show the occurrence of voltage rectification in the Iac − f plane. The plot was generated from ⟨V⟩dc vs Iac curves obtained for different magnetic fields as those shown in Fig. 1 and Fig. 3(b). A qualitatively good agreement with experiments [5] is obtained. A more detailed inspection shows some differences comparing both numerical-experimental data. The denser and softer experimental contours are obtained by continuously changing the magnetic field while the simulations were performed for a discrete number of vortex densities.

In conclusion, we find rocking ratchet effects in asymmetric modulated 2D JJA using the RSJ model for overdamped vortex dynamics. A strong dependence on vortex density as well as an inversion of the vortex flow is found, in good agreement with experiments.

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