Longitudinal dynamic force distribution for different hysteretic buffer characteristics

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Abstract. The study of longitudinal dynamics of trains during braking has and will continue to be an important subject, capable of generating numerous research themes and discussions on both national and international levels. Recent studies are centred on identifying certain aspects that can lead to a reduction of the longitudinal efforts that appear in the couplers and buffers when the train is braking. The purpose of the present study is to highlight the significance of the hysteretic characteristic of buffer, on the longitudinal dynamic forces and their distribution in the body of the train. The train composition model features identical vehicles, in order to diminish the effects of other parameters. The study takes into consideration only emergency braking with identical filling characteristics for the brake cylinders so that all vehicles in the train have the same braking power. All vehicles are considered to be equipped with the same buffers used in passenger carriages. Another important aspect of this paper is the modification of key parameters that influence the hysteretic characteristic of a buffer so that different stages of wear are simulated, corresponding to different periods since the vehicle has been in operation.

1. Introduction
The study of dynamic longitudinal phenomena manifested across the length of a train has played a significant role in understanding how to form longer and heavier trains, commonly used for freight transport. In order to understand how and why these phenomena appear and what their influence on train dynamics during breaking and acceleration are, beginning with the 1960s, on a worldwide level, algorithms and simulators have been used to demonstrate the role of longitudinal forces on the integrity and safe operation of a railway vehicle [1].

Appreciating the longitudinal dynamic forces depends on a series of parameters such as: the technical conditions required to operate the vehicle, the mass distribution throughout the whole length of the train, the constructive and functional parameters describing the couplers and buffers, the length of the brake pipe, etc. Therefore, it is especially difficult to study and analyse these dynamic forces, considering the large amount of parameters involved in this appreciation [1].

In spite of the difficulties implied by such a study, a variety of papers regarding the longitudinal dynamics found in a train during braking have been written. The main purpose is to reduce the level of these forces, to decrease the influence of certain factors and, overall, to improve the dynamic behaviour of a train. Theoretical studies dealing with the longitudinal dynamics of trains during braking have been written and presented in famous and important papers, by scientists such as: Cole [2], Belforte, Cheli [3], Nasr, Mohamndi [4], Zobory [5,6].
The succession of mechanical, pneumatic and thermic phenomena appearing during the braking process, developed individually and in different points of each of the vehicles with varying intensities is what makes braking an extremely complex process [7,8,9].

Considering the classical braking system, due to the air’s compressibility and the length of the train, an interval between the moment when the first and last vehicles forming the train start braking will always exist. According to the speed of the braking wave, the air distributors mounted on the vehicles in the train will start operating in a successive manner; this means that, because the first vehicles are already reducing their speed, the ones found towards the end of the train which have not yet started applying the brakes will impact those preceding them, causing longitudinal dynamic phenomena to appear. Usually, this will cause passenger discomfort; the goods being transported can also be damaged and, in serious cases, problems regarding safe operation will appear. The modelling of the braking system, done to make it usable in different simulators and computer programs, is presented by Pugi, Malavezzi and others, being the main theme of the papers [10,11]. Jinghui Wang [12] developed a simulation of these longitudinal forces which can be used in a program, without depending on too many mechanical parameters slowing the calculus and making it more difficult to interpret the results. Such an algorithm can be used to highlight acceleration and deceleration values, depending on the regime the trains are in at a given moment.

The forces appearing between the vehicles forming the train can be studied and evaluated by using a mathematical model adapted for each individual type of couplers and buffers. On a worldwide scale, there are two major categories of systems currently in used: the manual couplers and buffers (with a screw and a traction hook) and the automatic couplers, fulfilling both the function of the buffers and that of a classic coupler. A famous study that highlights the activity carried out by all the researchers studying ways of modelling the couplers and buffers is written by Qing Wu, Colin Cole et. al [13]. In this paper, the authors demonstrate the differences between the equipment used worldwide, the importance of having a correct evaluation of longitudinal forces and that of having complex or simple mathematical models used for the computer programs.

The purpose behind the mathematical modelling of couplers and buffers is to gain the capability to recreate, as accurately as possible and in a realistic manner, the value of the forces and their distribution across the length of the train. Because of this, it is necessary to model each individual system for trains equipped with classical couplers (mechanical, fitted with a hook) and buffers, and only afterwards can they be expressed when operating simultaneously. When discussing the dynamics of a train during braking, the buffers take over the forces caused by the colliding consecutive vehicles and the manual interlock and hook start acting only after the forces have been equalized and the recoil begins throughout the train, preventing the vehicles from distancing each other.

This paper continues the activity of the many authors who addressed this topic, underlining the importance of using the characteristics of buffers in longitudinal dynamics. Therefore, in this study, as many parameters as possible will be eliminated, such as draft, forward resistance, different types of vehicles with different braking patterns etc. to observe how the forces are influenced and which is their overall distribution throughout the length of the train.

2. Theoretical aspects

2.1. Mechanical model of the train
In order to study the longitudinal dynamic forces and reactions acting over the length of a train during braking, the general model has been established (figure 1); it resembles an elastic system with energy dampening, made with \( n \) individual, rigid masses \( m_i \) – these represent each individual vehicle forming the train. These are connected with buffers and couplers [8,14-20]. The latter have the elastic constants symbolized as \( k_i \) and the damping coefficients \( c_i \) which are well defined, corresponding to the constructive type and their characteristic function diagram.

For each individual vehicle, the forces applied on it have been represented in figure 1, as follows: the inertial forces \( I_i \), the braking forces \( F_{bi}(t) \) and those contained within the couplers and buffers,
The hypothesis that the first vehicle in the train (which can be a locomotive) does not have any force applied to the frontal buffers because there is no frontal load is taken into account; this aspect is also applicable for the coupling and dampening equipment installed on the last vehicle of the train.

![Figure 1](image.png)

**Figure 1.** The forces applied throughout the train [7,18].

Based on the model presented in figure 1, the equilibrium equations for each vehicle can be written as follows:

\[
\begin{align*}
    m_1 \cdot \ddot{x}_1 &= -F_1(\Delta x_1, \Delta \dot{x}_1) - F_{b1}(t) \\
    m_2 \cdot \ddot{x}_2 &= -F_2(\Delta x_2, \Delta \dot{x}_2) + F_1(\Delta x_1, \Delta \dot{x}_1) - F_{b2}(t) \\
    m_3 \cdot \ddot{x}_3 &= -F_3(\Delta x_3, \Delta \dot{x}_3) + F_2(\Delta x_2, \Delta \dot{x}_2) - F_{b3}(t) \\
    \vdots \\
    m_{i-1} \cdot \ddot{x}_{i-1} &= -F_{i-1}(\Delta x_{i-1}, \Delta \dot{x}_{i-1}) + F_{i-2}(\Delta x_{i-2}, \Delta \dot{x}_{i-2}) - F_{b_{i-1}}(t) \\
    m_i \cdot \ddot{x}_i &= F_{i-1}(x_{i-1}, \dot{x}_{i-1}) - F_{b_i}(t)
\end{align*}
\]

Therefore, by applying the laws of classical mechanics, \(n-1\) non-linear equations are obtained, each describing the movement between two consecutive vehicles. For a given vehicle \(I\) we will obtain [18,20]:

\[
\ddot{y}_i = \frac{F_i(y_i, \dot{y}_i) - F_{i+1}(y_{i+1}, \dot{y}_{i+1}) + F_{b_{i+1}}(t)}{m_{i+1}} + \frac{F_i(y_i, \dot{y}_i) - F_{i-1}(y_{i-1}, \dot{y}_{i-1}) - F_{b_i}(t)}{m_i}
\]

It is specified that, in order to simplify the above equations, the notation \(y_i = \Delta x_i\) was introduced.

2.2. Mathematical model of couplers

Vehicles fitted with classical buffers and screw couplers have two lateral buffers, mounted on the frontal beam and equipped with elastic and friction elements capable of transforming the kinetical energy in elastic displacement work.

Obtaining a mathematical formula which would help characterize the evolution of the net force applied to the buffers is possible by using the characteristic diagram of a buffer found on most passenger cars; built in Romania by ICPVA-SA, it has a RINGFEDER (with metallic rings) type elastic and absorbing element. The characteristic diagram, as seen in [8,14-17], is featured in figure 2.

The traction device must have the same kind of elastic elements as buffers, so it uses metallic rings. In figure 2, the characteristic diagram of a discontinuous coupler with RINGFEDER type elements is presented.

To preserve the hysteretic allure of the diagram, the mathematical model portraying the evolution of the forces applied to the equipment must depend on the elastic characteristic, on the friction forces...
appearing in the buffer, on its maximum stroke and also on the positive or negative value of the speed
which appears during bumping.

![Image](image1.png)

**Figure 2.** Quasi-static characteristic of Ringfeder type lateral buffer in use on passenger vehicles [19].

Considering the $k_e$, constant which depends on the elasticity of those elements inside the buffer and
another constant, $k_f$, depending on the friction between the metallic rings, the net force in the buffers
can be evaluated with the formula [11-19]:

$$F_{b,f,i}(x_i, \dot{x}_i) = k_e(x_i) + k_f \left| \dot{x}_i \right| \tanh(u \cdot \dot{x}_i)$$

(3)

where $x$ is the buffer’s stroke, $\dot{x}$ is the speed relative to the buffer, and $u$ is the scalar used to eliminate
the problems appearing when the $sign$ function is integrated. For the net force applied to the, traction
devices an identical formula will be used, with the $k_{ec}$ elastic and $k_{fc}$ friction constants. Taking into
account that during braking the longitudinal dynamic forces appear randomly in the hooks and buffers
as well, the net longitudinal dynamic force developed in the classical buffers and screw coupler can be
determined by using the formula [14-18]:

$$F_{dl,f}(x_i, \dot{x}_i) = \begin{cases} 
 k_e(x_i) + k_f \left| \dot{x}_i \right| \tanh(u \cdot \dot{x}_i) & \text{if } x_i < 0, \\
 0 & \text{if } x_i = 0, \\
 k_{ec}x_i + k_{fc} \left| x_i \right| \tanh(u \cdot \dot{x}_i) & \text{if } x_i > 0 
\end{cases}$$

(4)

By using the relation (4), the moment when each of the two devices starts operating can be pointed:
when the relative distance between two consecutive vehicles is decreasing, the buffers will start
operating, and when it increases, the mechanical hook coupler will begin acting.

2.3. Braking forces
The main brake used by railway vehicles is pneumatic. Therefore, it is necessary to know and
understand the evolution over time of the air pressure in the brake cylinder. It has been experimentally
determined by using the computerised experimental stand for testing railway braking equipment,
found in the Railway Rolling Stock department within the University POLITEHNICA of Bucharest.
The evolution was further used to determine the braking force developed by each of the vehicles
forming the train. In doing so, the condition that prohibits the maximum braking force from exceeding
the available grip is respected [8-9, 14-18]:

$$F_{b,i} \leq F_{u,i} = \mu_u \cdot m_i \cdot g$$

(5)
where $\mu_a$ is the wheel-rail grip coefficient and $m_i$ is the mass of each vehicle forming the train. The evolution of the braking forces can be determined with the following formula [7,17,19,21]:

$$F_{h,i}(t) = \frac{\mu_a \cdot m_i \cdot g}{p_{cf\,\text{max}}} \cdot p_{cf,i}(t)$$

(6)

where $p_{cf\,\text{max}}$ is the maximum pressure measured in the brake cylinders, $p_{cf,i}(t)$ is the instantaneous pressure measured in the brake cylinders (determined in an experimental way), and $g$ is the standard gravity value.

3. Numerical application

The simulation of the longitudinal dynamic forces’ evolution over time, measured between consecutive vehicles during the period when the train is braking, has been done by using a Matlab algorithm. It was based on the aforementioned formulas and mechanical model and the results obtained are presented in the following figures. In order to check which the modifications brought to the force distribution when the parameters describing the buffers are different from those on the characteristic diagram, the simulator used a passenger train composed of one locomotive and 7 identical cars. The locomotive has a total mass of 120 tonnes, and each wagon is considered as weighing 40 tonnes, together with the payload. An emergency braking is simulated, initiated at 160 km/h, leading to identical braking times for each individual vehicle, given that the brake cylinders have identical filling characteristics.

The couplers and buffers work accordingly to the diagrams shown in figures 2 and 3. Applying the relationship (4) was only possible after they were determined on the basis of the diagrams, the constants that depend on the elasticity and friction of the elements inside these devices having the following nominal values: $k_e = 2.8 \times 10^6$ N/m and $k_f = 1.4 \times 10^6$ N/m in the case of buffers, and for the traction device $k_{ec} = 5.46 \times 10^6$ N/m and $k_{fc} = 2.43 \times 10^6$ N/m. Studies take into consideration only the modifications appearing within the buffers; the traction devices have the same constants without any differences from the previous state:

- The coefficient depending on element elasticity ($k_e$) drops between 50% and 75% in value,
- The coefficient depending on the friction between inner components in the buffer ($k_f$) drops between 50% and 75% in value,
- The coefficient depending on the friction between inner components in the buffer ($k_f$) increases anywhere between 50% and 100%.

Figure 4. Characteristics of buffer in the case of the decrease of the coefficient $k_e$.
In order to highlight the changes in the hysteretic aspect in the aforementioned situations, a Matlab program was developed to simulate the buffer’s operation on a stand, by imposing the following displacement:

\[ x = x_0 \cdot \sin(\omega \cdot t) \]  \hspace{1cm} (7)

with a compression and extension speed as follows:

\[ \dot{x} = x_0 \cdot \omega \cdot \cos(\omega \cdot t) \]  \hspace{1cm} (8)

Considering \( \omega = 2 \cdot \pi \cdot v \) and \( v = 1 \) Hz, the results shown in figures 4, 5 and 6 are obtained. For these, the following mentions must be made: case 1 represents the nominal aspect of the buffer, modelled with formula (5), and cases 2 and 3 represent the increases or decreases in the previously mentioned coefficients.

Figure 5. Characteristics of buffer in the case of the decrease of the coefficient \( k_f \).

Figure 6. Characteristics of buffer in the case of the increase of the coefficient \( k_f \).

From the results shown in figure 4, it can be observed that, in order to reduce the coefficient depending on elasticity, the buffer’s characteristic shows a deviation from the nominal aspect, in the
anti-trigonometric sense, emphasizing the reduction in the maximum force correspondent to both compression and expansion. By forcing a reduction of the friction coefficient, a family of hysteretic characteristics contained within the normal hysteretic curve of the buffer can be obtained.

In this situation, even though the maximum forces have obviously changed – both for the compression and expansion curves – the limits enforced by the real diagrams are not exceeded.

When a 50% and then a 100% increase in the friction coefficient is adopted, while maintaining the elasticity coefficients at a constant value, a symmetrical increase in the buffer characteristic is observed, greatly influencing the maximal values obtained on the compression and extension curves. For the 100% increase of this coefficient, the equipment turns rigid, and on the extension curve the force is extremely close to having a value equal to zero. A conclusion can be drawn: modelling the force-displacement aspect of the buffers, in order to use it in longitudinal dynamic force simulators, must be as accurate as possible, to obtain realistic and precise results.

By introducing the previously modified buffer characteristics in the longitudinal dynamic forces simulation program, the following diagrams displaying the results have been obtained: the distribution of the maximum longitudinal forces during the buffer compression is presented in figures 7, 9 and 11, and the distribution of the maximum expansion forces during expansion is shown in figures 8, 10 and 12.
At a first glance, the distribution of the compression forces suggests the fact that, regardless of the changes in the buffers’ characteristics, these still keep a relatively near-Gaussian allure, with the highest values recorded on the fourth coupler. Modifying the elasticity coefficient will cause an increase in the forces measured throughout the first half of the train’s length (couplers 1-3) relative to the distribution obtained with the real characteristic, and a decrease in those forces through the latter half of the train (couplers 5-7). The distribution has been basically reversed when the friction coefficient is reduced, maintaining the other one constantly at the nominal value (figure 9). If the \( k_f \) coefficient is increased, a behavior similar to that witnessed when the elasticity coefficient was decreased is observed. These results lead to the conclusion that, when a characteristic is being modeled for use in calculus programs, the coefficients must be determined or adopted to obtain a simulated curve as near as possible to the real one.

Stretching forces appear throughout the length of the train due to a recoil phenomenon taking place when all braking forces have been equalized and the buffers start expanding. The way these operate through the expansion stroke can influence the reaction obtained from the couplers. If the friction coefficient is lowered, these forces will stick to a distribution similar to that of the nominal curve characteristic (figure 10), a feature that cannot be confirmed in any of the other mentioned cases. The similarities between the results obtained when the \( k_e \) coefficient is reduced and when the \( k_f \) coefficient is increased is emphasized by the stretch force distribution presented in figures 8 and 12. It can be noticed that a 50% variation in these coefficients leads to obtaining the maximum value for the stretching forces on the first couplers, while the next devices will show a basically linear drop, an aspect no longer valid for the nominal characteristic, when the maximum force is applied to the fourth coupler, situated at the half of the train.

4. Conclusions
The main conclusions can be briefly summarized as follows:

- adopting mathematical models which accurately replicate the nominal characteristic of every type of coupler and buffer plays a key role in obtaining correct results, regarding not only the numerical value of the longitudinal dynamic forces but also, and with greater importance, their distribution in the length of the trains;
- the results show that, for the same functional characteristic of a buffer, modifying the parameters – changes which in operation can be caused by wear and tear – does not lead to substantial differences regarding the value of the compression forces, or their distribution across the train;
- the expansion forces appearing in the hooks as a result of buffer expansion after the braking forces have been equalized are significantly changed, both in terms of numeric value and of
distribution, the latter being an aspect of significant importance. It is considered extremely important because this will cause a supplementary increase in the efforts appearing in the couplers and hooks, in other ways than expected, and equipment failure leading to the train breaking apart might occur in places others than those already known, for example, at the middle of the train.

The present paper opens the way for adapting the longitudinal dynamic forces applied to a train during braking simulation programs, to make significant changes in the numerical value and distribution of these forces more visible and easier to observe, especially in the case when the vehicles have buffers, hooks and couplers with different functional characteristics and parameters.

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