Emergence of spacetime dynamics in entropy corrected and braneworld models

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A very interesting new proposal on the origin of the cosmic expansion was recently suggested by Padmanabhan [arXiv:1206.4916]. He argued that the difference between the surface degrees of freedom and the bulk degrees of freedom in a region of space drives the accelerated expansion of the universe, as well as the standard Friedmann equation through relation \( \Delta V = \Delta t (N_{\text{sur}} - N_{\text{bulk}}) \). In this paper, we first present the general expression for the number of degrees of freedom on the holographic surface, \( N_{\text{sur}} \), using the general entropy corrected formula \( S = \frac{A}{4L_p^2} + s(A) \). Then, as two example, by applying the Padmanabhan’s idea we extract the corresponding Friedmann equations in the presence of power-law and logarithmic correction terms in the entropy. We also extend the study to RS II and DGP branworld models and derive successfully the correct form of the Friedmann equations in these theories. Our study further supports the viability of Padmanabhan’s proposal.

I. INTRODUCTION

Among all the fundamental forces of nature, clearly gravity is the most universal. However, the nature and the origin of gravity has not known very well yet. According to the equivalence principle, gravity is just the dynamics of spacetime. This implies that gravity is an emergent phenomenon. Indeed, the idea that gravity and spacetime geometry are emergent is widely accepted nowadays. The universality of gravity also indicates that its emergence should be understood from general principles that are independent of the specific details of the underlying microscopic theory.

An interesting new idea on the origin of gravity was proposed by Verlinde [1] who claimed that gravity may be not a fundamental interaction but should be interpreted as an entropic force caused by changes of entropy associated with the information on the holographic screen. Applying the holographic principle and the equipartition law of energy, Verlinde obtained the Newton’s law of gravitation. A relativistic generalization of this argument directly leads to the Einstein equations.

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Similar discoveries were also made by Padmanabhan [2]. He observed that the equipartition law of energy for the horizon degrees of freedom combining with the thermodynamic relation $S = E/2T$, also leads to the Newton’s law of gravity, where $S$ and $T$ are entropy and temperature associated with the horizon, respectively, and $E$ is the active gravitational mass producing the gravitational acceleration in the spacetime. This may imply that the entropy is to link general relativity with the statistical description of unknown spacetime microscopic structure when the horizon is present. It therefore provides evidence for the fact that gravity can emerge from a microscopic description that doesn’t know about its existence.

The deep connection between thermodynamics and gravity has now well established through a numerous and complementary theoretical investigations [3]. In [4] Jacobson derived the Einstein field equations of general relativity by applying the Clausius relation $\delta Q = T \delta S$ on the horizon of spacetime, here $\delta S$ is the change in the entropy and $\delta Q$ and $T$ are the energy flux across the horizon and the Unruh temperature seen by an accelerating observer just inside the horizon. Also, by applying the Clausius relation to the apparent horizon of the Friedmann-Robertson-Walker (FRW) universe, the corresponding Friedmann equations can be derived in Einstein, Gauss-Bonnet and more general Lovelock gravity [5]. These kinds of studies were also generalized to the brane cosmology, where it was shown that the differential form of the Friedmann equation on the brane can be transformed to the first law of thermodynamics on the apparent horizon [6–8]. Indeed, studies on the connection between gravity and thermodynamics have got a lot of attentions in recent years (see [9–19] and references therein).

It is important to note that in most of these investigations, one considers the gravitational field equations as the equations of emergent phenomenon, but leave the spacetime as a pre-existed background geometric manifold. Obviously, it is difficult to regard the spacetime itself as an emergent structure. Whereas, there are some conceptual difficulties associated with this idea. For example, it is very hard to think that the time used to describe the evolution of dynamical variables is emergent from some pre-geometric variables and the space around finite gravitational systems is emergent. Very recently, Padmanabhan [20] suggested a new proposal for resolving these difficulties by considering the emergence of spacetime in cosmology. This is due to the fact that the cosmic time of a geodesic observer provides a special choice of time variable, to which the observed cosmic microwave background radiation is homogeneous and isotropic, and the spatial expansion of our universe can be regarded as the consequence of emergence of space. According to Padmanabhan’s idea the cosmic space is emergent as the cosmic time progresses. In addition, he argued that the expansion of the universe is due to the difference between the surface degrees
of freedom and the bulk degrees of freedom in a region of emerged space and successfully derived
the dynamical equation of a FRW universe. This approach provides a novel paradigm to study the
emergence of space and cosmology, and has far reaching implications. Following [20], Cai obtained
the Friedmann equation of a higher dimensional FRW universe. By properly modifying the effective
volume and the number of degrees of freedom on the holographic surface from the entropy formulas
of static spherically symmetric black holes, he also derived successfully the corresponding dynamical
equations of the universe in Gauss-Bonnet and more general Lovelock cosmology [21]. In a similar
way, the authors of [22] derived the Friedmann equations of a flat FRW universe in Gauss-Bonnet
and Lovelock cosmology from the generalized law governing the emergence of the space [22]. Instead
of modifying the number of degrees of freedom on the holographic surface of the Hubble sphere and
the volume increase, in [22], it is assumed that the volume increase is proportional to a function
\( f(\Delta N) \). Here \( \Delta N = N_{\text{sur}} - N_{\text{bulk}} \), where \( N_{\text{sur}} \) is the number of degrees of freedom on the boundary
and \( N_{\text{bulk}} \) is the number of degrees of freedom in the bulk. When the volume of the spacetime is
constant, the function \( f(\Delta N) \) is equal to zero.

In general the entropy associated with the holographic surface is a function of its area. Since
the concept of number of degrees of freedom on the holographic surface is closely related to the
entropy, any modification of the entropy expression leads to a particular number of degrees of
freedom on the holographic surface. In this paper, we would like to consider the general form of
the entropy expression in the presence of quantum correction terms and derive the corresponding
Friedmann equations by determining \( N_{\text{sur}} - N_{\text{bulk}} \). We also apply the idea proposed in [20] to
the braneworld scenarios. By calculating the number of degrees of freedom on the Hubble horizon
of the brane we extract successfully the Friedmann equations in Randall-Sundrum II (RS II) and
Dvali, Gabadadze, Porrati (DGP) braneworld models. Our study may indicate that the novel
proposal of Padmanabhan [20] is powerful enough to apply for deriving the dynamical equations
in other gravity theories.

The outline of our paper is as follows. In the next section, we present the general formalism
for deriving the dynamical equation of spacetime by determining the difference between the bulk
and the boundary degrees of freedom. In section III, we derive the power-law and logarithmic
entropy-corrected Friedmann equations of FRW universe by using Padmanabhan new idea. Then,
in section IV, we extend the study to RS II and DGP braneworld models and derive successfully
the corresponding Friedmann equations. The last section is devoted to some closing remarks.
II. THE GENERAL FORMALISM

According to Padmanabhan’s observation, the number of degrees of freedom on the spherical surface of Hubble radius $H^{-1}$ is given by \[ N_{\text{sur}} = 4S = \frac{A}{L_p^2} = \frac{4\pi}{L_p^2H^2}, \] (1)
where $L_p$ is the Planck length, $A = 4\pi H^{-2}$ represents the area of the Hubble horizon and $S$ is the entropy which obeys the area law. The bulk degrees of freedom obey the equipartition law of energy,

\[ N_{\text{bulk}} = \frac{2|E|}{T}. \] (2)

Through this paper we set $k_B = 1 = c = \hbar$ for simplicity. We also assume the temperature associated with the Hubble horizon is the Hawking temperature $T = H/2\pi$, and the energy contained inside the Hubble volume $V = 4\pi/3H^3$ is the Komar energy

\[ E_{\text{Komar}} = |(\rho + 3p)|V. \] (3)

The novel idea of Padmanabhan is that the cosmic expansion, conceptually equivalent to the emergence of space, is being driven towards holographic equipartition, and the basic law governing the emergence of space must relate the emergence of space to the difference between the number of degrees of freedom in the holographic surface and the one in the emerged bulk \[20\]. He proposed that in an infinitesimal interval $dt$ of cosmic time, the increase $dV$ of the cosmic volume is given by

\[ \frac{dV}{dt} = L_p^2(N_{\text{sur}} - N_{\text{bulk}}). \] (4)

In general, one may expect $dV/dt$ to be some function of $(N_{\text{sur}} - N_{\text{bulk}})$ which vanishes when the latter does. In this case one may regard Eq. (1) as a Taylor series expansion of this function truncated at the first order \[20\]. This issue was also investigated in \[22\]. Substituting the cosmic volume $V = 4\pi/3H^3$, the degrees of freedom on the holographic boundary (1), the temperature $T = H/2\pi$, and the bulk degrees of freedom in Eq. (1), after using the Komar energy in the bulk (3), the standard dynamical equation for the Friedmann model can be obtained \[20\]

\[ \frac{\ddot{a}}{a} = -\frac{4\pi L_p^2}{3}(\rho + 3p). \] (5)

Multiplying both hand side of (5) by factor $\dot{a}a$ and using the continuity equation

\[ \dot{\rho} + 3H(\rho + p) = 0, \] (6)
after integrating, one obtains the Friedmann equation
\[ H^2 + \frac{k}{a^2} = \frac{8\pi L_p^2}{3} \rho, \]
where \( k \) is an integration constant, which can be interpreted as the spatial curvature of the FRW universe.

It is important to note that in the above derivation, the number of degrees of freedom on the holographic boundary (11) as well as the entropy-area relation play a crucial role. However, the entropy-area relation can be modified from the inclusion of quantum effects [23, 24]. To consider the general corrections to area law, we write down the general entropy-corrected relation as [25]
\[ S = \frac{A}{4L_p^2} + s(A), \]
where \( s(A) \) stands for the general correction terms. Applying the general entropy formula (8) to the holographic surface, we assume that the effective area of the holographic surface is
\[ \tilde{A} = A + 4L_p^2 s(A), \]
where \( A = 4\pi H^{-2} \). Using the expression for the volume \( V = 4\pi / 3H^3 \) as well as the area \( A \) of a 3-sphere with radius \( R \), one has
\[ \frac{dV}{dA} = \frac{1}{2H}, \]
and
\[ \frac{dA}{dt} = -8\pi H^{-3} \dot{H}. \]

We also assume that the effective volume enveloped by the effective holographic surface \( \tilde{A} \) increase with time according to
\[ \frac{d\tilde{V}}{dt} = \frac{dV}{dA} \frac{d\tilde{A}}{dt} = \frac{1}{2H} \left( \frac{dA}{dt} + 4L_p^2 \frac{ds(A)}{dA} \frac{dA}{dt} \right), \]
\[ = -4\pi H^{-4} \dot{H} \left( 1 + 4L_p^2 \frac{ds(A)}{dA} \right), \]
\[ = -2\pi H^{-5} \left( 2\dot{H}H + 8L_p^2 \frac{ds(A)}{dA} H \dot{H} \right). \]

The above equation can be further rewritten
\[ \frac{d\tilde{V}}{dt} = -2\pi H^{-5} \frac{d}{dt} \left( H^2 + 8L_p^2 \int \frac{ds(A)}{dA} HdH \right). \]
Inspired by Eq. (15), we propose that the number of degrees of freedom on the holographic surface with general entropy expression (8) is given by

\[ N_{\text{sur}} = \frac{4\pi}{L_p^2 H^4} \left( H^2 + 8L_p^2 \int \frac{ds(A)}{dA} H dH \right). \]  

(16)

The bulk degrees of freedom is still given by (2),

\[ N_{\text{bulk}} = -\frac{4\pi(\rho + 3p) V}{H}, \]

\[ = -\frac{16\pi^2}{3} H^{-4}(\rho + 3p), \]  

(17)

where we have added a minus sign in front of \( E_{\text{Komar}} \) in order to have \( N_{\text{bulk}} > 0 \), which makes sense only in an accelerating universe with \( \rho + 3p < 0 \) \[46\]. Next, motivated by Eq. (4), we assume

\[ \frac{d\tilde{V}}{dt} = L_p^2 (N_{\text{sur}} - N_{\text{bulk}}). \]  

(18)

Substituting Eqs. (13), (16) and (17) in relation (18), after simplification we get

\[ \frac{\dot{a}}{a} + 4L_p^2 \left( \frac{ds(A)}{dA} \dot{H} + 2 \int \frac{ds(A)}{dA} H dH \right) = -\frac{4\pi L_p^2}{3}(\rho + 3p). \]  

(19)

This is nothing, but the general form of the entropy corrected dynamical equation of a FRW universe filled by perfect fluid. To simplify the above equation further we need to identify the functional form of the correction term \( s(A) \). This is the subject of the next section. In the absence of the correction term, \( s(A) = 0 \), and Eq. (19) reduces to the well-known standard dynamical equation (5) of FRW universe.

## III. EMERGENCE OF ENTROPY CORRECTED FRIEDMANN EQUATIONS

In this section we apply the general method developed in the previous section to derive the dynamical Friedmann equations in the presence of the correction terms to the area law of entropy. Two well-known quantum corrections to the area law have been introduced in the literatures, namely, logarithmic and power-law corrections. Logarithmic corrections, arises from the loop quantum gravity due to thermal equilibrium fluctuations and quantum fluctuations \[26\]

\[ S = \frac{A}{4L_p^2} + \alpha \ln \frac{A}{4L_p^2} + \beta \frac{4L_p^2}{A}, \]  

(20)

where \( \alpha \) and \( \beta \) are dimensionless constants of order unity. The exact values of these constants are not yet determined and still is an open issue in loop quantum cosmology (see \[17\] and references therein). These corrections arise in the black hole entropy in loop quantum gravity due to thermal
equilibrium fluctuations and quantum fluctuations \[27, 28\]. The logarithmic term also appears in a model of entropic cosmology which unifies the inflation and late time acceleration \[29\]. Another form of correction to area law, namely the power-law correction, appears in dealing with the entanglement of quantum fields inside and outside the horizon. The entanglement entropy of the ground state obeys the well-known area law. Only the excited state contributes to the correction, and more excitations produce more deviation from the area law \[30\]. For a recent review on the origin of black hole entropy through entanglement, see \[31\]. The power-law corrected entropy can be written as \[32, 33\]

\[
S = \frac{A}{4L^2} \left[ 1 - K_\alpha A^{1-\alpha/2} \right],
\]  

(21)

where \(\alpha\) is a dimensionless constant whose value is currently under debate, and

\[
K_\alpha = \frac{\alpha (4\pi)^{\alpha/2-1}}{(4 - \alpha)r_c^{2-\alpha}},
\]  

(22)

where \(r_c\) is the crossover scale. The second term in Eq. (21) can be regarded as a power-law correction to the area law, resulting from entanglement, when the wave-function of the field is chosen to be a superposition of ground state and exited state \[32\].

Let us first consider the logarithmic corrected entropy \[20\]. We define the effective area of the holographic surface corresponding to entropy \[20\] as

\[
\tilde{A} = 4L^2 S
\]  

(23)

\[
= 4L^2 \left[ \frac{A}{4L^2} + \alpha \ln \frac{A}{4L^2} + \beta \frac{4L^2}{A} \right].
\]  

(24)

Now we calculate the increasing in the effective volume,

\[
\frac{d\tilde{V}}{dt} = \frac{1}{2H} \frac{d\tilde{A}}{dt} = \frac{1}{2H} \left[ 1 + \alpha \frac{4L^2}{A} - \beta \frac{16L^4}{A^2} \right] \frac{dA}{dt}.
\]  

(25)

Using relation \(A = 4\pi H^{-2}\), after some simplification we get

\[
\frac{d\tilde{V}}{dt} = 4\pi \dot{H} H^{-4} \left( -1 - \frac{\alpha L^2}{\pi} H^2 + \frac{\beta L^4}{\pi^2} H^4 \right),
\]  

(26)

Next we suppose from (26) that the number of degrees of freedom on the holographic surface is given by

\[
N_{\text{sur}} = \frac{4\pi}{L^2_H H^4} \left[ H^2 + \frac{\alpha L^2}{2\pi} H^4 - \frac{\beta L^4}{3\pi^2} H^6 \right].
\]  

(27)
Substituting Eqs. (17), (25) and (27) in relation (18), one obtains
\[
\dot{H} \left( 1 + \frac{\alpha L^2}{\pi} H^2 - \beta \frac{L^4}{\pi^2} H^4 \right) + H^2 + \frac{\alpha L^2}{2\pi} H^4 - \beta \frac{L^4}{3\pi^2} H^6 = -\frac{4\pi L^2}{3} (\rho + 3p). \tag{28}
\]
Using the continuity equation (6) and multiplying the both hand side by factor 2\dot{a}a, the above equation can be written
\[
\frac{d}{dt} \left( H^2 a^2 \right) + \frac{\alpha L^2}{2\pi} \frac{d}{dt} \left( H^4 a^2 \right) - \beta \frac{L^4}{3\pi^2} \frac{d}{dt} \left( H^6 a^2 \right) = \frac{8\pi L^2}{3} \frac{d}{dt} \left( \rho a^2 \right). \tag{29}
\]
Integrating (29), we arrive at
\[
H^2 + \frac{\alpha L^2}{2\pi} H^4 - \beta \frac{L^4}{3\pi^2} H^6 = \frac{8\pi L^2}{3} \rho, \tag{30}
\]
where we have put the integration constant equal to zero. This is exactly the result of [17, 18] for the entropy corrected Friedmann equation derived by applying the first law of thermodynamics on the apparent horizon of FRW universe. Here we arrived at the same result by using completely different approach. This indicates that, given the entropy expression at hand, one is able to reproduce the corresponding dynamical equation using the novel Padmanabhan’s idea on the emergence nature of spacetime dynamics due to the difference between the surface degrees of freedom and the bulk degrees of freedom in a region of space. Note that, unlike the case of Einstein’s gravity, one cannot interpret the constant of integration as the spatial curvature of the universe if one does not set it equal to zero [21].

Before we proceed further, let us compare the result obtained here with the modified Friedmann equation in loop quantum cosmology. Applying the techniques of loop quantum gravity to homologous and isotropic spacetime leads to the so-called loop quantum cosmology. Due to quantum correction, the Friedmann equations get modified. The big bang singularity is resolved and replaced by a quantum bounce [34]. For a brief summary on loop quantum cosmology, see [35]. Considering quantum correction, the modified Friedmann equation for spatially flat universe turns out to be
\[
H^2 = \frac{8\pi L^2}{3} \rho \left( 1 - \frac{\rho}{\rho_{\text{crit}}} \right), \tag{31}
\]
where
\[
\rho_{\text{crit}} = \frac{\sqrt{3}}{(32\pi L^4 \gamma^3)}, \tag{32}
\]
is the critical energy density. Here \(\gamma\) is Barbero-Immirzi parameter which could be fixed as 0.2375 in order to give the area formula of black hole entropy in loop quantum gravity [36]. Due to the
corrected term in (31), the big bang singularity is replaced by a quantum bounce happening at \( \rho = \rho_{\text{crit}} \).

Let us stress here that although in the literature there is doubt about the second correction term in entropy-corrected relation (20), it is widely believed that the first correction term originates from loop quantum gravity is the logarithmic term. Considering the case with \( \beta = 0 \), the modified Friedmann equation (30) reduces to

\[
H^2 + \frac{\alpha L_p^2}{2\pi} H^4 = \frac{8\pi L_p^2}{3} \rho. \tag{33}
\]

Solving for \( H^2 \) we have

\[
H^2 = \frac{\pi}{\alpha L_p^2} \left( -1 + \sqrt{1 + \frac{16\alpha L_p^4}{3} \rho} \right). \tag{34}
\]

If we regard \( \alpha \) as a small quantity, we can expand the right hand side of (34), up to the linear order of \( \alpha \) and get

\[
H^2 \approx \frac{8\pi L_p^2}{3} \rho \left( 1 - \frac{4\alpha L_p^4}{3} \rho \right). \tag{35}
\]

It is apparent that Eq. (35) is quite similar to the modified Friedmann equation (31) in loop quantum cosmology, provided we define

\[
\rho_{\text{crit}} = \frac{3}{4\alpha L_p^4}, \tag{36}
\]

which is in agreement with (32) if we define \( \alpha = 8\sqrt{3}\pi \gamma^3 \). For \( \gamma = 0.2375 \), one gets \( \alpha \approx 0.58 \).

Next, we consider the power-low correction to the entropy relation, namely (21). In this case, by using the same procedure, it is easy to show that the effective volume varies with time according to

\[
\frac{d\tilde{V}}{dt} = -2\pi H^{-5} \frac{d}{dt} \left( H^2 - \frac{H^\alpha}{r_c^{-\alpha}} \right), \tag{37}
\]

where we have also used relation (22). Therefore, inspired by (37), we assume the number of degrees of freedom on the holographic surface with radius \( H^{-1} \) is given by

\[
N_{\text{sur}} = \frac{4\pi}{L_p^2 H^4} \left( H^2 - \frac{H^\alpha}{r_c^{-\alpha}} \right). \tag{38}
\]

The number of degrees of freedom inside the bulk is still given by relation (17). Combining Eqs. (17), (37) and (38) with relation (18), after using continuity equation (19), we arrive at

\[
\frac{d}{dt} \left( H^2 a^2 \right) - r_c^{\alpha - 2} \frac{d}{dt} \left( H^\alpha a^2 \right) = \frac{8\pi L_p^2}{3} \frac{d}{dt} \left( \rho a^2 \right). \tag{39}
\]
Integrating and dividing by $a^2$ we reach

$$H^2 - r_c^{\alpha - 2} H^\alpha = \frac{8\pi L_p^2}{3} \rho,$$

(40)

where we have again set the integration constant equal to zero. This equation is indeed the corresponding Friedmann equation of a flat FRW universe in the presence of power-law corrected entropy derived in [24]. Note that in [24] one of the present author and Hendi derived Eq. (40) by applying the first law of thermodynamics on the apparent horizon (see Eq. (47) of [24]), however our approach here is quite different. Here we arrived at (40) by assuming that the difference between the number of degrees of freedom in the bulk and on the boundary is proportional to the volume change of the spacetime. This is a remarkable result and shows that the approach presented here is powerful enough to derive the correct form of the gravitational field equations.

IV. EMERGENCE OF FRIEDMANN EQUATIONS IN BRANEWORLD

In the previous section we showed that using the Padmanabhan’s new idea one is able to re-produce the dynamical equations of FRW universe by calculating the number of degrees of freedom on the Hubble horizon and inside the universe. Is the obtained relation between the degrees of freedom and the gravitational field equation just an accident? Does this connection hold in all dynamical spacetimes and even beyond Einstein gravity? Does it imply something in deep? Very recently, Cai showed that the above procedure can be applied to extract the corresponding Friedmann equation of flat FRW universe in Gauss-Bonnet and more general Lovelock gravity [21]. In this section we will address the question on the connection between the degrees of freedom and spacetime dynamics by investigating whether and how the relation can be found in braneworld models. Gravity on the brane does not obey Einstein theory, thus the usual area formula for the holographic boundary get modified on the brane [6, 7].

There are two main pictures in the braneworld scenario. In the first picture which we refer as the RS II model, a positive tension 3-brane embedded in an 5-dimensional AdS bulk and the cross over between 4D and 5D gravity is set by the AdS radius [37, 38]. In this case, the extra dimension has a finite size and the localization of gravity on the brane occurs due to the negative cosmological constant in the bulk. In another picture which is based on the work of DGP model [39, 40], a 3-brane is embedded in a spacetime with an infinite-size extra dimension, with the hope that this picture could shed new light on the standing problem of the cosmological constant as well as on supersymmetry breaking [39, 41]. The recovery of the usual gravitational laws in this
picture is obtained by adding to the action of the brane an Einstein-Hilbert term computed with the brane intrinsic curvature.

Let us begin by RS II model in which no intrinsic curvature term on the brane contributes in the action. The entropy associated with the apparent horizon for an \((n - 1)\)-brane embedded in an \((n + 1)\)-dimensional bulk in RS II brane scenario is given by

\[
S = \frac{2\Omega_{n-1}\bar{r}_A^{n-1}}{4G_{n+1}} \times {}_2F_1 \left( \frac{n-1}{2}, \frac{1}{2}; \frac{n+1}{2}, -\frac{\bar{r}_A^2}{\ell^2} \right),
\]

where \( {}_2F_1(a, b, c, z) \) is a hypergeometric function, and \( \ell \) is the bulk AdS radius,

\[
\ell^2 = -\frac{n(n-1)}{16\pi G_{n+1}\Lambda_{n+1}}, \quad \Omega_{n-1} = \frac{\pi^{(n-1)/2}}{\Gamma((n+1)/2)}.
\]

Here \( \Lambda_{n+1} \) is the \((n + 1)\)-dimensional bulk cosmological constant and \( \bar{r}_A \) is the apparent horizon radius which can be extended into the bulk. The explicit evaluation of the apparent horizon for the FRW universe gives

\[
\bar{r}_A = \frac{1}{\sqrt{H^2 + k/a^2}}.
\]

It is worth noticing when \( \bar{r}_A \ll \ell \), which physically means that the size of the extra dimension is very large if compared with the apparent horizon radius, one recovers the \((n + 1)\)-dimensional area formula for the entropy on the brane, \( S = 2\Omega_{n-1}\bar{r}_A^{n-1}/4G_{n+1} \). The factor 2 comes from the \( \mathbb{Z}_2 \) symmetry in the bulk. This is due to the fact that in this case \( \Lambda_{n+1} \to 0 \) and thus in the absence of the negative cosmological constant in the bulk, no localization of gravity happens on the brane. As a result, the gravity on the brane is still \((n + 1)\)-dimensional.

We further rewrite the entropy expression in the form

\[
S = \frac{(n-1)\ell\Omega_{n-1}}{2G_{n+1}} \int_0^{\bar{r}_A} \frac{\bar{r}_A^{n-2}}{\sqrt{\bar{r}_A^2 + \ell^2}} d\bar{r}_A.
\]

Now we consider the flat 4-dimensional universe (the generalization of this study to arbitrary dimensions is quite straightforward). In this case we have \( n = 4 \) and \( \bar{r}_A = H^{-1} \) and relation can be written as

\[
S = \frac{3\ell\Omega_3}{2G_5} \int \frac{H^{-2}}{\sqrt{H^{-2} + \ell^2}} d(H^{-1}).
\]

We define the effective area as

\[
\bar{A} = 4G_5 S = 6\ell\Omega_3 \int \frac{H^{-2}}{\sqrt{H^{-2} + \ell^2}} d(H^{-1}).
\]
Taking the time derivative, we get
\[
\frac{d\tilde{A}}{dt} = 6\ell\Omega_3 \frac{d}{d(H^{-1})} \left( \int \frac{H^{-2}}{\sqrt{H^{-2} + \ell^2}} d(H^{-1}) \right) \frac{d(H^{-1})}{dt}.
\] (47)

Using the fact that \( d(H^{-1})/dt = -\dot{H}H^{-2} \), we arrive at
\[
\frac{d\tilde{A}}{dt} = -6\ell\Omega_3 \frac{H^{-4}\dot{H}}{\sqrt{H^{-2} + \ell^2}}.
\] (49)

Therefore, the increase in the effective volume is obtained as
\[
\frac{d\tilde{V}}{dt} = -3\Omega_3 \frac{H^{-4}\dot{H}}{\sqrt{H^{-2} + \ell^2}}.
\] (50)

Next, we assume the number of degrees of freedom on the apparent horizon is given by
\[
N_{\text{sur}} = \frac{6\Omega_3}{G_5} H^{-4} \sqrt{H^2 + \frac{1}{\ell^2}}.
\] (51)

We also replace \( L_p^2 \) in relation (15), with \( G_5 \),
\[
\frac{d\tilde{V}}{dt} = G_5 (N_{\text{sur}} - N_{\text{bulk}}).
\] (52)

Inserting Eqs. (17), (50) and (51) in relation (52), one gets
\[
\frac{\dot{H}}{\sqrt{H^2 + 1/\ell^2}} + 2\sqrt{H^2 + \frac{1}{\ell^2}} = -\frac{4\pi G_5}{3} (\rho + 3p).
\] (53)

Using the continuity equation (6) and multiplying the both hand side by \( \dot{a}a \), we arrive at
\[
\frac{d}{dt} \left( a^2 \sqrt{H^2 + \frac{1}{\ell^2}} \right) = \frac{4\pi G_5}{3} \frac{d}{dt} (\rho a^2).
\] (54)

Integrating and dividing by \( a^2 \) we obtain
\[
\sqrt{H^2 + \frac{1}{\ell^2}} = \frac{4\pi G_5}{3} \rho,
\] (55)

where we set the integration constant equal to zero. In this way we derive the Friedmann equation of flat FRW universe in RS II by determining the difference between the number of degrees of freedom on the boundary and in the bulk (see Eq. (34) in [6]). The main characteristic of Friedmann equation in RS II braneworld model, namely \( H^2 \propto \rho^2 \) can be seen in obtained equation.

If one invokes the standard assumption that the energy density on the brane can be separated into two contributions, the ordinary matter component, \( \rho_b \), and the brane tension, \( \lambda > 0 \), such that
\[ \rho = \rho_b + \lambda \], (after fine tuning between the brane tension and the bulk cosmological constant), then one can recover another form of the Friedmann equation in RS II braneworld [8].

Finally, we extend the discussion to the case in which the intrinsic curvature term of the brane is included in the action, namely DGP braneworld. In this case, the entropy of the apparent horizon for an \((n-1)\)-brane embedded in an \((n+1)\)-dimensional Minkowski bulk is given by [6]

\[ S = \frac{(n-1)\Omega_{n-1}r_A^{n-2}}{4G_n} - \epsilon \frac{2\Omega_{n-1}r_A^{n-1}}{4G_{n+1}} = S_n - \epsilon S_{n+1}. \]  

(56)

where \(\epsilon = \pm 1\) corresponds to the two branch of the DGP braneworld [42]. It is interesting to note that in this case the entropy can be regarded as a sum of two area formulas; one (the first term) corresponds to the 4-dimensional gravity on the brane and the other (the second term) to the 5-dimensional gravity in the bulk. The factor 2 in the second term comes from the \(\mathbb{Z}_2\) symmetry in the bulk [39]. This indeed reflects the fact that there are two gravity terms in the action of DGP model. Consider flat 4-dimensional FRW universe on the brane, with \(n = 4\) and \(\tilde{r}_A = H^{-1}\), we have

\[ S = \frac{3\Omega_3}{4G_4} H^{-2} - \epsilon \frac{2\Omega_3}{4G_5} H^{-3}. \]  

(57)

Defining the effective surface, we have

\[ \tilde{A} = 4G_5 S = 2\Omega_3 \left( \frac{3G_5}{2G_4} H^{-2} - \epsilon H^{-3} \right), \]  

(58)

which varies with time according to

\[ \frac{d\tilde{A}}{dt} = -6\Omega_3 \dot{H} H^{-3} \left( \frac{G_5}{G_4} - \epsilon H^{-1} \right). \]  

(59)

Hence, the effective volume increase is obtained as

\[ \frac{d\tilde{V}}{dt} = \frac{1}{2H} \frac{d\tilde{A}}{dt} = -3\dot{H} H^{-4} \Omega_3 \left( \frac{G_5}{G_4} - \epsilon H^{-1} \right) = 3\Omega_3 \frac{G_5}{3G_4} H^{-3} - \epsilon H^{-4}. \]  

(60)

(61)

In this case we assume the number of degrees of freedom to be

\[ N_{\text{sur}} = \frac{3\Omega_3 H}{G_5} \left( \frac{G_5}{G_4} H^{-3} - 2\epsilon H^{-4} \right). \]  

(62)

Substituting (60) and (62) into (52) together with the bulk degrees of freedom (17), we arrive at

\[ \frac{G_5}{2G_4} \left( 2\dot{H} + 2H^2 \right) - \epsilon H^{-1} \left( \dot{H} + 2H^2 \right) = -\frac{4\pi G_5}{3} (\rho + 3p). \]  

(63)
Integrating this equation with help of the continuity equation yields

\[ \frac{G_5}{2G_4} H^2 - \epsilon H = \frac{4\pi G_5}{3} \rho, \tag{64} \]

where we have assumed the integration constant to be zero. We define the crossover length scale between the small and large distances in DGP braneworld as

\[ r_c = \frac{G_5}{2G_4}. \tag{65} \]

Thus Eq. (64) can be rewritten

\[ H^2 - \frac{\epsilon}{r_c} H = \frac{8\pi G_4}{3} \rho. \tag{66} \]

The above equation can be further rewritten in the following form

\[ H^2 = \left[ \frac{\epsilon}{2r_c} + \sqrt{\frac{8\pi G_4}{3} \rho + \frac{1}{4r_c^2}} \right]^2. \tag{67} \]

This equation is indeed the corresponding Friedmann equation of flat FRW universe in DGP brane scenario derived in \[42\] from the field equations. When \( r_c \to \infty \), one recovers the standard Friedmann equation in flat FRW universe. Physically, this means that the apparent horizon is not extended in the bulk and located totally on the brane. As a result, the effect of the extra dimension, does not appear in the Friedmann equation. Equation (66) with \( \epsilon = 1 \) and \( \rho = 0 \) has an interesting self-accelerating solution with a Hubble parameter given by the inverse of the crossover scale \( r_c \) \[43\]. This is due to the fact that the intrinsic curvature term on the DGP brane appears as a source for the bulk gravity, so that with appropriate initial conditions this term can cause an expansion of the braneworld without the need of matter or a cosmological constant on the brane \[43\].

V. CLOSING REMARKS

In summary, we have investigated the novel idea recently proposed by Padmanabhan \[20\] which argues that the acceleration of the cosmic expansion is due to the difference between the number of degrees of freedom on the Hubble horizon of the universe and the one in the emerged bulk. This new proposal also leads to derive the dynamical equation governing the evolution of the universe. Since in general, the entropy associated with the horizon is a function of its area and depends on the underlying theory of gravity, therefore any modification of the entropy expression leads to a particular number of degrees of freedom on the Hubble horizon.
In this paper, we developed the method of [20] by assuming that the entropy associated with the horizon is a general function of its area. In this case, we derived successfully the general expression for the number of degrees of freedom on the Hubble horizon as well as the general expression for the effective volume increase. We considered two special corrections to the entropy expression namely, power-law and logarithmic corrections and extracted the corresponding modified Friedmann equations in flat FRW universe. We also applied our general formalism to the braneworld scenarios. Using the entropy expression associated with the apparent horizon on the brane, we calculated the volume increase and the number of degrees of freedom on the apparent horizon on the brane. Then, we obtained the Friedmann equations in RS II and DGP braneworld models. Our study may indicate that the novel proposal of Padmanabhan [20] is powerful enough to apply for deriving the dynamical equations in other gravity theories.

It is important to note that in this work, we only modified the number of degrees of freedom on the Hubble horizon, \( N_{\text{sur}} \), while we keep the number of degrees of freedom in the bulk, \( N_{\text{bulk}} \), as in case of standard cosmology. Indeed, when the underlying theory of gravity is modified, or the entropy expression get quantum correction terms, then \( N_{\text{sur}} \) get modified as well. However, we proposed the bulk degrees of freedom, \( N_{\text{bulk}} \), has the same expression as in the absence of correction terms. This is due to the fact that we have assumed \( N_{\text{bulk}} \) depends only on the matter degrees of freedom.

Finally, we would like to mention that unlike the case of Einstein gravity, we cannot interpret the integration constant as the spatial curvature of the FRW universe. In other words, we could only derive the Friedmann equations in flat universe. Thus, it is of great importance to extend the method developed in this paper for the spatially non-flat FRW universe and derive the corresponding Friedmann equations. This is quite an interesting subject, which deserves further investigation.

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