Driving energy to accelerate the adiabatic electron dynamics in quantum Hall system

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Abstract. We study a scheme of accelerated adiabatic quantum dynamics. This scheme was originally proposed by Masuda-Nakamura. The strategy of combining two opposite idea: infinitely-large time-magnification factor (\( \alpha \)) and infinitely-small growth rate (\( \varepsilon \)) of adiabatic parameter was elucidated. We apply the proposed method to the Hall system with electric and magnetic field and obtain regularization term, driving potential \( V_{FF} \) and driving Hamiltonian \( H_{FF} \) to accelerate the system. The driving potential \( V_{FF} \) and the driving Hamiltonian \( H_{FF} \) to be able to accelerate adiabatic electron dynamics in the ground state trapped in the plane \( xy \) and electric field in the \( x \) direction and a constant magnetic field in \( y \) direction.

1. Introduction

In 1879 Edwin H. Hall observed that when an electric current passed through a gold film under a magnetic field, a transverse voltage developed. This effect, known as the Hall effect, forms the basis of fundamental research and practical applications, such as magnetic field measurements and motion detectors. Recent research has found that Hall effects can occur without an external magnetic field [1]. When electrons are confined in two-dimensional materials, quantum-mechanically enhanced transport phenomena such as the quantum Hall effect can be observed. Graphene, consisting of an isolated single atomic layer of graphite, is an ideal realization of such a two-dimensional system. Nowadays we can control individual atoms [2] and macroscopic wave packets in BEC [3-6]. If we try to fabricate massive amount of such nanoscale structure, we should shorten the dynamics of each atom or BEC to get its desired target states in shorter time. A theory to accelerate quantum dynamics is proposed by Masuda and Nakamura with use of additional phase and driving potential [7]. This theory aims to accelerate a known quantum evolution and to obtain the desired target state on shorter time scale. By tuning the electric field we can enhanced the electron’s evolution and increase the conductivity in two dimensional layer such as graphene. The theory of fast-forward can be developed to accelerate the adiabatic quantum dynamics [8], and constitutes one of the promising means to the shortcut of adiabaticity [9-15]. The relationship between the fast-forward and the shortcut to adiabaticity is nowadays clear [16,17]. The adiabaticity occurs when the external parameter of Hamiltonian is very-slowly changed. The quantum adiabatic theorem [18-22] states that, if the system initially in an eigenstate of the instantaneous Hamiltonian, it remains so during the adiabatic process. In section 2, we briefly summarize the theory of fast-forward of quantum adiabatic dynamics [7,8,23-30].
2. Fast-forward of adiabatic quantum dynamics

Masuda and Nakamura proposed a theory to accelerate the adiabatic quantum dynamics by using a combination of opposite ideas of infinitely-large time multiplication ($\alpha$) and infinitesimally-slow evolution rate of adiabatic dynamics ($\varepsilon$), to generate the finite driving potential, by which we obtain the final adiabatic state in any desired short time [8]. To apply the fast-forward theory of standard dynamics, it is necessary to regularize the adiabatic state so that it satisfies the time-dependent Schrödinger equation. A two dimensional regularized potential and wave function is written by

$$V_0^{reg} = V_0 + \varepsilon V(x, y, t),$$  \hspace{1cm} (1)

$$\Psi_0^{reg} (x, y, t) = \phi_n (x, y, R(t)) e^{-i \int_0^t \eta_n(R(t)) dt} e^{i\varepsilon\Phi(x, y, t)}$$  \hspace{1cm} (2)

where $R$ is a time-dependent adiabatic parameter as

$$R(t) = R(0) + \varepsilon t$$  \hspace{1cm} (3)

with $\varepsilon \ll 1$, and $\phi_n$ is the eigenstate of the time-independent Schrödinger equation,

$$H\phi_n = E_n \phi_n.$$  \hspace{1cm} (4)

Formally, $\phi_n$ can be decomposed as

$$\phi_n = \tilde{\phi}_n e^{i\eta(x, y, t)},$$  \hspace{1cm} (5)

where $\tilde{\phi}_n$ is a real positive amplitude and $\eta$ is the phase. The Schrödinger equation for $\Psi_0^{reg}$ is represented as

$$i\hbar \frac{d\Psi_0^{reg}}{dt} = -\frac{\hbar^2}{2m_0} \nabla^2 \Psi_0^{reg} + V_0^{reg} (x, y, t) \Psi_0^{reg}. $$  \hspace{1cm} (6)

$\Psi_0^{reg}$ and $V_0^{reg}$ are assumed to satisfy Eq.(6) up to $O(\varepsilon)$. By substituting Eqs.(1), (2) and (5) into Eq.(6) and decomposing it into real and imaginary parts, we have $\theta$ and $\bar{V}$ as follows

$$\bar{\phi}_n^2 \nabla^2 \theta + 2 \bar{\phi}_n \nabla \tilde{\phi}_n \nabla \theta + \frac{2m_0}{\hbar} \bar{\phi}_n \frac{\partial \tilde{\phi}_n}{\partial R} = 0$$  \hspace{1cm} (7)

and

$$\bar{V} = -h (\partial_R \eta) - \frac{h^2}{m_0} \nabla \eta \nabla \theta.$$  \hspace{1cm} (8)

Here $\theta$ can be solved as

$$\nabla \theta = -\frac{m_0}{\hbar} \frac{1}{\bar{\phi}_n^2(x, y, R)} \frac{\partial}{\partial R} \int x \int y \bar{\phi}_n^2(x', y', R) dx'dy'$$  \hspace{1cm} (9a)

$$V_{FF} = -\hbar \varepsilon \alpha \theta - \hbar \varepsilon \alpha^2 \frac{\partial}{\partial R} (\nabla \theta) - \hbar^2 \frac{m_0}{\varepsilon^2 \alpha^2 (\nabla \theta)^2} - \varepsilon h \partial_R \eta - \varepsilon h^2 \frac{m_0}{\varepsilon^2 \alpha^2 (\nabla \theta)^2} \nabla \eta \nabla \theta + V_0$$  \hspace{1cm} (9b)

The expression for driving electric field can be written as

$$\hat{E}_{FF} \equiv -\nabla (V_{FF} - V_0) = \hbar \varepsilon \alpha \nabla \theta + \hbar \varepsilon \alpha^2 \frac{\partial}{\partial R} (\nabla \theta) + \frac{\hbar^2}{m_0} (\varepsilon \alpha)^2 \nabla^2 \theta + \varepsilon h \partial_R (\nabla \eta) + \alpha \varepsilon \frac{h^2}{m_0} \nabla (\nabla \eta \nabla \theta).$$  \hspace{1cm} (10)

Taking the limit $\varepsilon \to 0$, $\bar{\alpha} \to \infty$, with $\varepsilon \bar{\alpha} = \bar{v}$, we can define $v(t)$ and $\bar{v}(t)$ as

$$\alpha \varepsilon = \bar{v}(1 - \cos \frac{2\pi t}{T_F}) \equiv v(t),$$  \hspace{1cm} (11)
\[ \dot{\varepsilon} = \dot{\nu}(t). \] (12)

Then, \( \Psi_{FF} \) becomes \( \Psi_{FF} = \overline{\phi_n} e^{i(\eta + \nu(t) \theta)} \), and \( \tilde{E}_{FF} \) is given by

\[ \tilde{E}_{FF} = \hbar \dot{\nu}(t) \nabla \theta + \hbar (\nu(t))^2 \partial \nabla \theta + \frac{\hbar^2}{m_0} (\nu(t))^3 \nabla \theta + \nu(t) \hbar \dot{\partial}_R (\nabla \eta) + \nu(t) \frac{\hbar^2}{m_0} \nabla (\nabla \eta \nabla \theta). \] (13)

The magnification factor \( \alpha \) in Eq.(11) is commonly chosen for \( 0 \leq t \leq T_F \), where \( \tilde{\nu} \) is the time average of \( \alpha(t) \varepsilon \) during the fast forwarding. The final time of the fast-forwarding \( T_F \) is related to the standard final time \( T \) as \( T_F = \varepsilon T/\tilde{\nu} \), where \( T \) is taken as \( T = 1/\varepsilon (\gg 1) \) and \( R(T) - R(0) = 1 \).

The driving electric field \( (\tilde{E}_{FF}) \) guarantees the fast-forward of adiabatic quantum dynamics [23]. The present scheme also introduces a current density \( j_{FF} \). We see

\[ j_{FF}(x, y, t) = Re[\Psi_{FF} \frac{\hbar}{im} \nabla \Psi_{FF}] = \frac{\hbar}{m} \overline{\phi_n^2}(x, y, \Lambda(t)) [\nabla \eta(x, y, \Lambda(t)) + \nu(t) \nabla \theta(x, y, \Lambda(t))] \] (14)

With

\[ j(x, y, t) = \frac{\hbar}{m} \overline{\phi_n^2}(x, y, t) \nabla \eta(x, y, t). \] (15)

and find the relationship:

\[ j_{FF}(x, y, t) = j(x, y, \Lambda(t)) + \frac{\hbar}{m} \nu(t) \overline{\phi_n^2}(x, y, \Lambda(t)) \nabla \theta(x, y, \Lambda(t)). \] (16)

### 3. The quantum dynamics of hall systems

In this section we will discuss the concept of Hall effects.

#### 3.1. Drude model

One model that can explain Hall effects is the Drude model. In this model, electrons are thought to be able to move freely in a material such as gas. Electrons are assumed (if not affected by the external field) to move in a straight path and the interaction between electrons is only when the collision occurs. The probability of collisions is expressed as a magnitude inversely proportional to the average time between collisions. From Ohm formula, current density can be written as

\[ J = \sigma E \] (17)

where \( \sigma \) is conductivity. In the form of tensor rank 2, can be written as

\[ \begin{pmatrix} J_x \\ J_y \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ -\sigma_{xy} & \sigma_{xx} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix} \] (18)

The conductivity matrix describes how the current flow responds to the electric field given. This matrix has the same diagonal component, and the off-diagonal components are equal but opposite signs.

\[ \sigma = \frac{\sigma_{DC}}{1 + \omega_B^2 \tau^2} \begin{pmatrix} 1 & -\omega_B \tau \\ \omega_B \tau & 1 \end{pmatrix} \] (19)

With \( \sigma_{DC} = ne^2 \tau / m \) is direct current conductivity if there is no magnetic field. The part that contribute to the Hall effect are off-diagonal term.

#### 3.2. Quantum dynamics

In this discussion, we will ignore the effect of electron spin, arguing that in a \( B \) magnetic field, there is a Zeeman separation phenomenon between the energy of the up spin electron and down spin as \( \Delta = \)
$$2\mu_B B$$, where $$\mu_B = e\hbar/2m$$. Consider electron that can only move in $$xy$$ direction, with mass $$m$$ and in the magnetic field $$B = \nabla \times A$$. Lagrangian of the electron can be written as

$$L = \frac{1}{2}m x^2 - ex \cdot A$$

(20)

with $$x = x\hat{x} + y\hat{y}$$, and $$x$$ is first derivation of time. From Lagrangian, canonical momentum can be written as

$$p = \frac{\partial L}{\partial x} = mx - eA$$

(21)

Then Hamiltonian of the system written as

$$H = x \cdot p - L = \frac{1}{2m} (p + eA)^2$$

(22)

### 3.2.1. Landau gauge

We want to give a magnetic field $$z$$ direction. In other words, the selected $$A$$ must fulfill $$\nabla \times A = B\hat{z}$$

This $$A$$ selection is not unique. We will choose $$A$$ as

$$A = xB \hat{y}$$

(23)

This selection is called Landau gauge. A potential only has translation simmetry in $$y$$ direction (magnetic field $$B$$ still invariant of translation and rotation on $$x$$ and $$y$$ axis). Hamiltonian in Eq.(22) become

$$H = \frac{1}{2m} (p_x^2 + (p_y + exB)^2)$$

(24)

The choice to maintain translation symmetry in $$y$$ direction means we can look for the eigen state which is also the eigen state of the momentum operator $$p_y$$. Momentum operator in $$y$$ direction, $$p_y = -i\hbar \partial / \partial y$$ has an eigen state in the form of flat waves in the direction of $$y$$. Thus we can use variable separation for wave functions

$$\psi_k(x, y) = e^{ik_y f_k(x)}$$

(25)

If we apply the Hamiltonian to this wave function, the operator $$p_y$$ can be replaced by its eigenvalue, which is $$\hbar k$$.

$$H \psi_k(x, y) = \frac{1}{2m} (p_x^2 + (\hbar k + exB)^2) \psi_k(x, y) \equiv H_k \psi_k(x, y)$$

(26)

Note that the above Hamiltonian similar to a Hamiltonian on a harmonic oscillator. We can rewrite the Hamiltonian as follows.

$$H_k = \frac{1}{2m} p_x^2 + \frac{\hbar \omega_B^2}{2} (x + kl_B^2)^2$$

(27)

with $$\omega_B = eB/m$$ is cyclotron frequency, and $$l_B = \sqrt{\hbar/eB}$$ is magnetic length. The thing that is different from the usual harmonic oscillator is that the center of the oscillation shifts to $$-kl_B^2$$. With this, we can explicitly write the wave function of the general solution of the harmonic oscillator as

$$\psi_{n,k}(x, y) \sim e^{ik_y H_n(x + kl_B^2)} e^{-\left(x + kl_B^2\right)^2/2}$$

(28)

where $$H_n$$ is the Hermite polynomial. The first exponential factor is the factor that describes the flat wave in the $$y$$ direction, while the next factor is the general solution of the harmonic oscillator. Note that the argument in the general solution of the harmonic oscillator shows that the center of the oscillation shift
shifts to $x = -kl_B^2$. The above equation is the expression of the unnormalized wave function (can be seen from the $\sim$ sign).

3.2.2. Electric field. Add another electric field to $x$ direction. By knowing that $E = -\nabla \phi$, we choose electric potential $\phi = -Ex$. The Hamiltonian now become

$$H = \frac{1}{2m} \left(p_x^2 + (p_y + exB)^2\right) + eEx \quad (29)$$

We can see that the Hamiltonian is similar to the Hamiltonian when we only give a magnetic field, as in the Eq.(27) because it has the same type as the Hamiltonian in harmonic oscillator. The difference is that in the case of this electric field, the center of the oscillation is shifted again by $mE/eB^2$ from the previous case where there was only a magnetic field without being given an electric field. If we compare the wave function in both cases, we will get the following.

$$\psi_E(x, y) = \psi_{n,k}(x + mE/eB^2, y) \quad (30)$$

with energy from Hamiltonian in Eq.(29) that can be written as

$$\xi_{n,k} = \hbar \omega_B \left( n + \frac{1}{2} \right) - eE \left( kl_B^2 + \frac{eE}{m\omega_B^2} \right) + \frac{mE^2}{2B^2} \quad (31)$$

The addition of an electric field to the system changes the energy. If we add the shift in the center of oscillation, we can see that the Eq.(30) is a wave with momentum $k$ localized at $x = -kl_B^2 - eE/m\omega_B^2$.

3.3. Integer of quantum hall effect

For cases where the temperature is low and the magnetic field is large, there is a unique phenomenon that occurs in resistivity. This phenomenon is in the form of quantization of material resistivity. The experimental results show that the transverse resistivity will form a plain and will remain the same value (even with the increase in the magnetic field) until it will rise to the next plane when the magnetic field exceeds a certain limit. On the other hand, longitudinal resistivity will be zero, but it rises sharply when there is a change in the transverse resistivity.

In this phenomenon, the observed resistivity of the Hall follows the equation

$$\rho_{xy} = \frac{2\pi \hbar}{e^2 \nu}$$

where $\nu$ is integer. This integer is the number of Landau levels that are filled. When $\nu$ landau level is filled, then there is no empty state with the same energy. The existing empty state will have an energy gap of $\hbar \omega_B$. If we give a small electric field, the electrons will be trapped in place so that the conductivity remains.

4. Accelerated adiabatic dynamics in hall systems

The system that we are consider is the dynamics of electron with mass $m$ and charge $-e$ moving at $xy$ direction, in the magnetic field $B$ in $z$ direction and electric field $E$ in $z$ direction.

$\phi = -Ex$

$A = xB \hat{y}$

$E = \nabla \phi = E \hat{x}$

$B = \nabla \times A = B \hat{z}$

Hamiltonian of the system is written by.
\[ H = \frac{1}{2m} (p_x^2 + (p_y + eB)^2) + eEx \]
\[ = \frac{1}{2m} p_x^2 + \frac{m\omega_B^2}{2} (k l_B^2) + (x + \frac{mE}{eB})^2 + \frac{m\omega_B}{B} xE \]  

where \( l_B = \sqrt{\hbar/eB} \) dan \( \omega_B = eB/m \). We write the energy as \( \xi \) which is obtained from Eq.31. The wave function can be written as

\[ \psi_{n,k}(x,y) \sim e^{iky} H_n \left( x + \frac{mE}{eB^2} + kl_B^2 \right) e^{-\frac{(x + \frac{mE}{eB^2} + kl_B^2)^2}{2l_B^2}} \]  

Comparing with the general formula of wave function for the normalized harmonic oscillator.

\[ \psi_n(y) = \left( \frac{\alpha}{\pi} \right)^{\frac{1}{4}} \frac{1}{\sqrt{2^n n!}} H_n(y) e^{-\alpha y^2/2} \]  

then for this case,

\[ y = x + \frac{mE}{eB^2} + kl_B^2 \]
\[ \alpha = \frac{1}{l_B^2} = \frac{eB}{\hbar} = \frac{m\omega_B}{\hbar} \]

Remember that our goal is to be able to maintain the conductivity of the system even if there is an electric field that is not small. For this reason, the specific case to be discussed is the ground \((n = 0)\) and the constant magnetic field \( (dB/dt = 0) \), and the electric field changes adiabatically \( (E = E(R(t))) \). This slowly changing electric field means that the interference we give is only small and does not change the conductivity of the system.

The expression of the wave function for ground state is written as

\[ \psi_{0,k}(x,y,t) = \left( \frac{m\omega_B}{\pi \hbar} \right)^{\frac{1}{4}} e^{iky} e^{-\frac{(x + \frac{mE}{eB^2} + kl_B^2)^2}{2l_B^2}} e^{-\frac{i}{\hbar} \int_{t_0}^t \xi_{0,k}} \]  

4.1. Assumptions used

It should be noted that, by changing the electric field, the induction of magnetic field will actually appear described by the following equation.

\[ \nabla \times B_{\text{ind}} = \mu_0 \varepsilon_0 \frac{dE}{dt} \]  

we can use

\[ \nabla \times B_{\text{ind}} = \hat{\rho} \left( \frac{\partial B_{\text{ind},x}}{\partial \rho} - \frac{\partial B_{\text{ind},\phi}}{\partial x} \right) + \hat{\phi} \left( \frac{\partial B_{\text{ind},\rho}}{\partial \phi} - \frac{\partial B_{\text{ind},x}}{\partial \rho} \right) + \hat{x} \frac{1}{\rho} \left( \frac{\partial (\rho B_{\text{ind},\phi})}{\partial \rho} - \frac{\partial B_{\text{ind},\rho}}{\partial \phi} \right) \]

We know that the right hand side in Eq.(37) only has a component in \( x \) direction, so we can write

\[ \frac{1}{\rho} \left( \frac{\partial (\rho B_{\text{ind},\phi})}{\partial \rho} - \frac{\partial B_{\text{ind},\rho}}{\partial \phi} \right) = \mu_0 \varepsilon_0 \varepsilon \frac{dE}{dR} \]  

In the form of integral, Eq.(37) become

\[ \oint B \cdot d\mathbf{l} = \mu_0 \varepsilon_0 \frac{dE}{dt} \int \mathbf{E} \cdot d\mathbf{A} \]  

Thus can be obtained

\[ B_{\text{ind},\phi} = \frac{\mu_0 \varepsilon_0}{2} \varepsilon \rho \frac{dE}{dR} \]
Our system is a plate that has no height, so we can see only for $\varphi = 0$ and $\varphi = \pi$, and also $0 \leq \rho \leq \bar{Y}$, where in the cartesian coordinates, $-Y \leq y \leq Y$. With this, we can write it down

$$B_{\text{ind}} = \frac{\mu_0 e_0}{2} \frac{dE}{dR} \hat{y} \hat{z}$$

(41)

We can choose a potential that produces the field $B_{\text{ind}}$ as above, and if we combine it with the potential of the initial magnetic field, it will get

$$A = \left( xB + \frac{\mu_0 e_0}{2} \frac{dE}{dR} xy \right) \hat{y}$$

(42)

Note that in contrast to the Landau Gauge in the 23 equation, this new potential has a term with $y$, so it does not have a translation in the $y$ direction. Without translation, the method of separation variables cannot be used. For that, we assume that $B \gg \frac{\mu_0 e_0}{2} \frac{dE}{dR} y$. With this assumption, the potential $A$ will return as in the Eq. (23)

4.2. Regularization

The regularization wave function is obtained by the regularization exponential factor $\theta(x, y, t)$ as in the Eq. (2). For the case that we consider, we can write it down

$$\psi_k^{\text{reg}}(x, y, t) = \left( \frac{m \omega_B}{\pi \hbar} \right)^{1/4} e^{ik y} e^{-\left( x + \frac{m E(R(t))}{e B^2} k l_B^2 \right)^2 / 2l_B^2} \times e^{-i \frac{t}{l_B^2} \int_0^t d \tau \xi_k(R(\tau))} e^{i e \theta(x, y, t)}$$

(43)

Note that in order for this wave function has a form as in the Eq. (2), we can write it as

$$\psi_k^{\text{reg}}(x, y, t) = \phi(x, R(t)) e^{ik y} e^{-i \frac{t}{l_B^2} \int_0^t d \tau \xi_k(R(\tau))} e^{i e \theta(x, y, t)}$$

(44)

where

$$\phi(x, E(R(t))) = \left( \frac{m \omega_B}{\pi \hbar} \right)^{1/4} e^{-\left( x + \frac{m E(R(t))}{e B^2} k l_B^2 \right)^2 / 2l_B^2}$$

(45)

It can be seen that from the wave function in 44, we can consider the $y$ position variable as an additional phase and from the Hamiltonian in the Eq. (33) that the momentum operator $p_y$ has been changed to eigen $k l_B^2$. In other words, the position of $y$ will have no effect on the system, and therefore, the $\theta$ regularization phase can be written only as a function of the position of $x$ and time $t$, in other words $\theta = \theta(x, t)$.

The Hamiltonian in the Eq. (33) can be rewritten and separated between potential and kinetic energy terms, and becomes

$$H = -\frac{\hbar}{2m} \frac{\partial^2}{\partial x^2} + V_0(x, R(t))$$

(46)

By grouping the potential terms of the electric and magnetic fields, we can get $V_0$ as the following.

$$V_0(x, R(t)) = \left( \frac{m \omega_B^2}{2} (k l_B^2 + x)^2 + \frac{m \omega_B}{B} x E(R(t)) \right)$$

(47)

Then we can use regularization Hamiltonian for the case that we consider. Thus can be written

$$H^{\text{reg}} = \frac{1}{2m} p_x^2 + \frac{m \omega_B^2}{2} (k l_B^2 + x)^2 + \frac{m \omega_B}{B} x E(R(t)) + \varepsilon V(x, R(t))$$

(48)
4.3. Accelerated adiabatic dynamics

We will find the regularization phase $\theta$. From Eq.(9), we can write the differential equation that must be fulfilled by $\theta$.

$$\frac{\partial \theta}{\partial x} = - \frac{m}{\hbar} \frac{1}{\phi_n^2(x, R)} \frac{\partial}{\partial R} \int x \phi_n^2(x', R) dx'. \tag{49}$$

Here $\phi_n(x, R)$ is the real positive amplitude of the $x$-dependent part of the eigenfunction as in Eq.(45), and its $x$ and $R$ dependence appear only through a single variable $X$ as $\phi_n(x, R) \equiv G(X)$, where

$$X = x + \frac{mE(R(t))}{eB^2}. \tag{50}$$

Then the integral in Eq.(49) is rewritten as

$$\frac{\partial \theta}{\partial x} = - \frac{m}{\hbar} \frac{1}{G^2(X)} \frac{\partial}{\partial R} \int x G^2(X') dx'. \tag{51}$$

Noting $\frac{\partial}{\partial R} = \frac{\partial x}{\partial R} \frac{\partial}{\partial x}$, Eq.(51) reduces to

$$\frac{\partial \theta}{\partial x} = - \frac{m^2}{\hbar eB^2} \frac{dE}{dR} G^2(X) = - \frac{m^2}{\hbar eB^2} \frac{dE}{dR} \tag{52}$$

In the second equality of Eq.(52), we prescribed $\lim_{x \to x_c} \frac{G^2(x)}{G^2(x)}$ will be zero at $X = X_c$. From Eq.(52), one finds

$$\theta = - \frac{m^2}{\hbar eB^2} \frac{dE}{dR} x, \tag{53}$$

$$\frac{\partial \theta}{\partial R} = - \frac{m^2}{\hbar eB^2} \frac{d^2E}{dR^2} x. \tag{54}$$

By knowing that the regularization phase is valued as in Eq.(8) we get

$$\bar{V} = 0. \tag{55}$$

By using Eq.(9b), fast-forward potential can be written be written

$$V_{FF} = V_0 + \hbar \frac{d\alpha}{dt} e \frac{m^2}{\hbar eB^2} \frac{dE}{dR} x - \hbar \nu(t)^2 \frac{m^2}{\hbar eB^2} \frac{d^2E}{dR^2} x - \frac{\hbar^2}{2m} \nu(t)^2 \left( \frac{m^2}{\hbar eB^2} \frac{dE}{dR} \right)^2 \tag{56}$$

The expression for driving electric field can be written as

$$\bar{E}_{FF} \equiv \frac{\alpha e}{\hbar} \frac{m^2}{eB^2} \left( \frac{\nabla}{\partial R} \frac{dE}{dR} x + \frac{dE}{\partial R} \right) - \nu(t)^2 \frac{m^2}{\hbar eB^2} \frac{d^2E}{dR^2} x - \frac{1}{2m} \nu(t)^2 \frac{m^4}{e^2B^2} \nabla \left( \frac{dE}{\partial R} \right)^2 \tag{57}$$

Hamiltonian can be written as

$$H_{FF} = \frac{1}{2m} p_x^2 + V_0 + \hbar \frac{d\alpha}{dt} e \frac{m^2}{\hbar eB^2} \frac{dE}{dR} x - \hbar \nu(t)^2 \frac{m^2}{\hbar eB^2} \frac{d^2E}{dR^2} x - \frac{\hbar^2}{2m} \nu(t)^2 \left( \frac{m^2}{\hbar eB^2} \frac{dE}{dR} \right)^2 \tag{58}$$

Next is determine the accelerated wave function. Additional phase $f$ is determined by

$$f = \bar{v} \left( 1 - \cos \frac{2\pi t}{T_F} \right) \left( - \frac{m^2}{\hbar eB^2} \frac{dE}{dR} x \right) \tag{59}$$

Then, the accelerated wave function can be written as follows.
\[ \psi_{FF}(x, y, t) = \exp\left(-i \frac{m^2}{\hbar e B_0^2} \frac{\partial E}{\partial R} x \vec{v} \left(1 - \cos \frac{2\pi t}{T_F}\right) \right) \psi_{k \text{reg}}^\epsilon(x, y, \Lambda(t)) \]  

where the regularization wave function \( \psi_{k \text{reg}}^\epsilon(x, y, t) \) corresponds to the Eq.(2). We have derived the driving potential \( V_{FF} \) as in Eq.(56) we must be added to Hamiltonian regularization so that the dynamics of wave function \( \psi_{FF} \) in Eq.(60) can be accelerated adiabatically, and reaches the final state within \( T_F \) compared to the time of \( T \) if the system is not accelerated.

In this paper, the calculation for a system with a magnetic field that is not constant is not done because of the complexity that arises when calculating derivatives of all terms containing \( l_B \) and \( \omega_B \).

5. Conclusion

The driving potential \( V_{FF} \) in Eq.(56) and the driving Hamiltonian \( H_{FF} \) in Eq.(58) to be able to accelerate adiabatic electron dynamics in the ground state trapped in the plane \( xy \) and electric field in the \( x \) direction and a constant magnetic field in \( y \) direction. With electrons not moving state, it can be ascertained also that conductivity will still have the same value. Another thing that can be seen is, the greater the value of the \( B \) magnetic field given, then the potential drive \( V_{FF} \) will get closer to the initial potential of \( V_0 \), so does the Hamiltonian. That is, the greater the magnetic field we provide, the easier the system will be to maintain its state, and at the same time its conductivity. The two driving terms also indicate dependence on electric field derivatives on adiabatic parameters \( R \) to second order. If the electric field does not depend on \( R \), then the last three terms in the Eq.(58) will disappear, and the Hamiltonian returns to its original state, namely the harmonic oscillator with the center shifted.

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