Process to assess rheological characteristics of rocks by underground geodesic data in terms of a viscoelastic model

AV Panov*, LA Nazarov** and NA Miroshnichenko***

Chinakal Institute of Mining, Siberian Branch, Russian Academy of Sciences, Novosibirsk, Russia

E-mail: *anton-700@yandex.ru; **naz@misd.ru; ***mna@misd.ru

Abstract. Under consideration is the process for evaluation of rheological properties of construction components in the underground chamber-and-pillar system to mine solid mineral reserves. The process is based on solving the inverse mixed-type problems in terms of a 2D viscoelastic geomedium model. Feasibility is demonstrated to evaluate in-situ state parameters characterizing rheological deformation of rocks. Numerical experiments with the use of synthetic input data (viz. relative displacements of the worked-out space contour points recorded by underground geodesic monitoring) for the prescribed absolute precision of the instruments made it possible to establish a required population of measurement data to provide an inverse problem-solving procedure.

1. Introduction
Most rocks under prolonged loading exhibit rheological properties. To model the mineral mining processes, land surface subsidence, convergence of mine face contour, pillar stability with a target to describe rheological behavior of rocks involves the use of different state equations [1-5], which parameters can be determined under laboratory conditions. However, a number of hereto related problems such as sensibility of some parameters to a stress state, time length of rock creep tests in the case of poorly exhibited rheological properties, spatial variability of rheological properties imply the necessity of in-situ determination of the state equation parameters [6].

The present paper tackles the problem on evaluation of the rheological rock property by measurement data on relative displacements of a mine working contour.

2. Problem statement and solution
Let consider a fragment of an underground space configuration typical of the chamber-and-pillar mining for shallow bedded sub-horizontal deposits with chain pillars (Figure 1), specific for potassium mines.

Dimensions of a computation domain in Cartesian coordinates \((x, z)\) are \(L_x = L_z = 200\) m; upper boundary is at depth \(H = 100\) m. At distance of \(100\) m from the upper edge of the computation domain a subhorizontal bed of \(10\) m in thickness occurs and exhibits viscoelastic properties. In the central area of the bed two long workings are tunneled in direction normal to the cross-section, so in the first approximation it is reasonable to suppose that the study area is in the plain deformation state [7]. The protection pillar is preserved between two workings. Geometric dimensions of the workings and the
pillar are 10×10 m. The computation domain is within the field of vertical \( \sigma_v \) and horizontal \( \sigma_h \) compressive stresses. The vertical stress corresponds to weight of overlying rocks while the horizontal stress is specified with lateral thrust coefficient \( q \).

\[ \sigma_v = \rho g (H + z) \]

Figure 1. Fragment of computation domain discretization into final elements and boundary conditions.

Equations to determine the viscoelastic medium deformation versus time are selected as [8]:

\[
\begin{align*}
\varepsilon(t) &= \frac{1}{K_0} \left[ \sigma(t) + \delta_v \int_0^t (t-s)^{-\alpha} \sigma(s) \, ds \right], \\
\gamma(t) &= \frac{1}{\mu_0} \left[ \tau(t) + \delta_s \int_0^t (t-s)^{-\alpha} \tau(s) \, ds \right],
\end{align*}
\]

where \( \varepsilon \) – volumetric strain, \( \sigma \) – average stress, \( \gamma \) – main shear, \( \tau \) – maximum tangential stress, \( K_0 = \lambda_0 + 2\mu_0 / 3 \), \( \lambda_0 \) and \( \mu_0 \) – «instantaneous» values of Lame constants, corresponding to time moment \( t = 0 \); \( \alpha \), \( \delta_v \), \( \delta_s \) – parameters characterizing rheological properties of rocks.

Inversion (1) results in:

\[
\begin{align*}
\sigma(t) &= \frac{K_0}{D_v} \varepsilon(t), \\
\tau(t) &= \frac{\mu_0}{D_s} \gamma(t),
\end{align*}
\]

where \( D_v = 1 + \delta_v^{1-\alpha} / (1 - \alpha) \), \( D_s = 1 + \delta_s^{1-\alpha} / (1 - \alpha) \).

Then Lame parameters \( \lambda_1(t) \) and \( \mu_1(t) \) for a viscoelastic problem can be presented as:

\[
\mu_1(t) = \mu_0 / D_s,
\]

\[
\lambda_1(t) = \frac{\lambda_0 + 2\mu_0 / 3 \cdot 2\mu_0 / 3}{D_v / D_s}
\]

To describe deformation of a rock mass we make use an equation set for the linear viscosity-and elasticity theory, including:
equilibrium equations
\[ \sigma_{ij,j} + \rho g \delta_{ijz} = 0, \]  \hspace{1cm} (4)

Hooke law
\[ \sigma_{ij} = \lambda_1 \varepsilon_{ij} + 2 \mu \varepsilon_{ij}, \]  \hspace{1cm} (5)

Cauchy relation
\[ \varepsilon_{ij} = 0.5(u_{i,j} + u_{j,i}), \]  \hspace{1cm} (6)

where \( \sigma_{ij} \) and \( \varepsilon_{ij} \) — components of stress and strain tensors \( (i,j = x, z) \), \( u_i \) — shears, \( \rho \) — rock density, \( g \) — gravity factor, \( \delta_{ij} \) — Kronecker symbol.

At the boundary of the computation domain the following terms are set:
\[ u_x(0,z) = 0, \quad \sigma_{xz}(0,z) = 0; \]  \hspace{1cm} (7)
\[ \sigma_{xx}(L_x,z) = q \sigma_{y} (z), \quad \sigma_{xz}(L_x,z) = 0; \]  \hspace{1cm} (8)
\[ \sigma_{xx}(x,0) = \rho g H, \quad \sigma_{xz}(x,0) = 0; \]  \hspace{1cm} (9)
\[ u_z(x,H + L_z) = 0, \quad \sigma_{xz}(x,H + L_z) = 0, \]  \hspace{1cm} (10)

Boundaries of mine workings are free from stresses.

The system of equations (4) – (6) was solved with the original code [9], realizing the final element method for structurally heterogeneous media with discontinuities. A final element net is built with step of 2 m in space and contains 10201 nodes. Calculations were performed under terms that host rock is an elastic medium: \( \rho = 2500 \text{ kg/m}^3 \), Young modulus \( E = 2 \text{ GPa} \), Poisson ratio \( \nu = 0.3 \) [10], and the bed is a salt rock with viscoelastic properties \( \lambda_0 = \mu_0 = 0.67 \text{ GPa}, \quad \alpha = 0.7, \quad \delta_\tau = 0.0001 \nu^\alpha - 1, \quad \delta_\varepsilon = 0.0018 \nu^\alpha - 1 \) [10,11].

External stress field, which can be determined by the measurement hydrofracturing under mine conditions [12], is composed of overlying rock weight and the lateral thrust coefficient \( q = 0.8 \). Mine workings nos. 1 and 2 are assumed to be driven simultaneously.

Figures 2a, b present convergence of contour of working no. 1 by the data obtained with different observation intervals since the time of this mine working drivage.

![Figure 2. Convergence of (a) mine working walls and (b) roof and floor for different time intervals: 1 – day, 2 – month, 3 – 12 months, 4 – 48 months.](image)

The data were recorded in 24 measurement points, uniformly located along walls, roof and floor of the mine working. It is worth noting that with time the convergence rate tends to fade because of higher consolidation of a pillar and stress redistribution. This peculiarity should be considered in planning of field measurements.
Numerical experiments made it possible to find that an increment of contour displacements amounted to 10-12 µm for first 10 days since the workings were driven, but with time this parameter lowered down to 5 µm for next 50 days. Thus, this is a strong justification that it is imperative to provide higher precision of measurement procedure and metering instruments.

The inverse problem is stated as follows: to assess rheological parameters of the geomechanical model $\alpha, \delta_s, \delta_v$ for an extended pillar specified with homogeneous physical properties by using the data on displacement of an adjacent working contour. The displacement of working contour can be controlled by means of benchmarks, high-precision leveling devices [13] or downhole strain meters of resolution up to 5 µm [14].

To determine rheological parameters $\alpha, \delta_s, \delta_v$ we introduce functional $\Phi$, its minimum delivers a solution to the problem:

$$\Phi(\alpha, \delta_s, \delta_v) = \sum_{i} [\psi S U_x + (1-\psi) S U_z],$$

$$S U_x = \sum_{n} [\Delta U(z_n, \alpha, \delta_s, \delta_v, t) - \Delta U_{real}(z_n, t)],$$

$$S U_z = \sum_{m} [\Delta V(x_m, \alpha, \delta_s, \delta_v, t) - \Delta V_{real}(x_m, t)],$$

where $x_m, z_n$—coordinates governing locations of benchmarks for measurement of relative vertical $\Delta V$ and horizontal $\Delta U$ displacements of the mine working contour; $\Delta U(z_n, \alpha, \delta_s, \delta_v, t)$ and $\Delta V(x_m, \alpha, \delta_s, \delta_v, t)$—calculated displacement values, $\psi$—weight coefficient, the proper selection of this coefficient provides uni-modality of objective functional $\Phi$, $\Delta U_{real}$ and $\Delta V_{real}$—field measurement results. In the numerical modeling the synthetic data were used for the last of the above parameters:

$$\Delta U_{real}(z_n, t) = (1+\xi)\Delta U(z_n, \alpha^*, \delta^*_s, \delta^*_v, t),$$

$$\Delta V_{real}(x_m, t) = (1+\xi)\Delta V(x_m, \alpha^*, \delta^*_s, \delta^*_v, t),$$

where $\alpha^*, \delta^*_s, \delta^*_v$—target values of the parameters (accurate solution), $\xi$—random magnitude, uniformly distributed at section $[-A_{err}; A_{err}]$ to imitate multiplicative noise.

Amplitude of random error $A_{err}$ varied within 1–20%, $\psi$—ranged within an interval [0; 1] with step of 0.2. The target parameters were selected with reference to [10, 15]: $\alpha^* = 0.7$, $\delta_s^* = 0.0018v^{-1}$, $\delta_v^* = 0.0001v^{-(1-\alpha)}$. Duration of observations varied in a wide range from 1 day to 10 years with data recording intervals of 1 day and one month.

Isolines of functional $\Phi(\alpha, \delta_s, \delta_v)$ with 20% error in input data are shown in Figure 3. It is evident that in Figure 3a, b the functional exhibits explicit ravine structure; the conjugate gradients method can be applied to find its minimum [16, 17].
Figure 3. Isolines of objective function $\Phi$ in cross-sections: (a) $\Phi(\alpha = \text{const})$; (b) $\Phi(\delta_s = \text{const})$; (c) $\Phi(\delta_v = \text{const})$.

The input data error resulted from instrumental imperfection, or related to inadequate assessment of pillar state affects considerably an observation time consumed for establishment of rheological parameter values. Let the input data error is 10%, then the required measurement data can be obtained based on 36 measurements of displacements in benchmarks, but in the case when the error is 20% the required number of measurements increases up to 84; thereto, the data recording periodicity does not influence the numerical calculation results. According to the modeling test results the rheological parameters $\alpha, \delta_s$ can be really established with high precision even at notable interference ($\xi = 0.2$). The objective function appears less “sensitive” to parameter $\delta_v$ ( $\delta_v$ value is as a rule by an order less than $\delta_s$ value), and greater data population is demanded for its search.

3. Conclusion
The new-proposed process for evaluation of rheological rock properties is based on solving the mixed-type inverse problem in terms of the 2D viscoelastic geomeedium model and can be realized by the underground geodesic data. The calculated parameters of state equations enable to predict stability of carrying components of mining techniques by employing forward calculations.

Acknowledgements
The study was supported by the Russian Science Foundation, Project No. AAAA-A17-11712209002-5.

References
[1] Erzhanov ZhS 1964 Rock Creep Theory and its Appendices Alma-Ata: Nauka Vol 175 (in Russian)
[2] Baryakh AA, Konstantinova SA, and Asanov VA 1996 Saliferous Rock Deformation Ekaterinburg: UrB RAS (in Russian)
[3] Baryakh AA, et al 2005 Prediction of the intensive surface subsidences in mining potash series J Min Sci Vol 41 No 4 pp 312–319
[4] Dyad’kov PG, Nazarov LA, and Nazarova LA 2004 3D Viscoelastic model of Central Asia lithosphere: the design procedure and numerical experiment Fizicheskaya Mezomekhanika Vol 7 No 1 pp 91-101
[5] Nazarova LA, Nazarov LA, and Kozlova MP 2009 Dilatancy and the formation and evolution of disintegration zones in the vicinity of heterogeneities in a rock mass J Min Sci Vol 43 No 5 pp 411–419
[6] Laptev BV, Komkov VF, and Azanova NS 1986 Determination of rheological parameteres of
silvinit creep by field data publ in Saliferous Deposit Mining Perm: PPI (in Russian)

[7] Novatsky V 1975 Elasticity Theory Moscow: Mir (in Russian)

[8] Nazarova LA and Nazarov LA 2005 Estimation of pillar stability based on viscoelastic model of
rock mass J Min Sci Vol 41 No 5 pp 399–406

[9] Nazarova LA 1995 Modeling three-dimentional stress fields in crust fractures Doklady AN Vol
342 No 6 pp 804–808

[10] Baryakh AA, Konstantinova SA, and Asanov VA 1996 Saliferous Rock Deformation
Ekaterinburg: UrO RAN (in Russian)

[11] Permyakov RS, et al 1986 Saliferous Rock Deposit Mining Manual Moscow:Nedra

[12] Skulkin AA 2017 Experimental evaluation of stress field parameters at Solikamsk saliferous
rock mine Proc IX Int Conf “Modern Technics and Technology in Scientific Research
Bishkek Kyrgyzstan March 27-28 2017

[13] GOST R 53340–2009 2010 Geodesic Instruments Specifications (in Russian)

[14] Oparin VN et al 2007 Methods and measurement instruments to model and to conduct field
investigation into non-linear strain-wave processes in rock blocks

[15] Permyakov RS, et al 1986 Saliferous Rock Deposit Mining Manual Moscow: Nedra

[16] Nazarov LA et al 2012 Evaluation of stresses and deformation properties of rock masses based
on inverse problem solution by measurement data on displacements at free boundaries
Sibirsky Zhurnal Industrialnoi Matematiki Vol 15 No 4 pp 102–109.

[17] Vasiliev FP 1988 Numerical Methods to Solve Extremal Problems Moscow: Nauka (in Russian)