STUDY OF THE DYNAMICS OF THE EXOSKELETON ACTUATING UNIT

Abstract. A person has more than 300 degrees of mobility, but it is practically impossible to recreate such a kinematic scheme. In this article, a kinematic scheme of the exoskeleton is proposed that is most necessary for human movement. A 3D model of the exoskeleton actuating unit with an electrohydraulic drive has been developed in the CAD system and the values of masses, coordinates of mass centers, inertia tensors of the links of the exoskeleton actuating unit have been calculated. A launch file has been developed in the MATLAB environment for modeling the dynamics of the exoskeleton actuating unit. The control laws in the degrees of mobility of the actuating unit of the exoskeleton are selected. As a result of the theoretical study, the ranges of changes in the generalized coordinates for the joints under study are determined. The dependences of the power and the moment in the joints 9, 10 on time are obtained. The conducted studies have shown that lifting the leg will require more energy and this makes it necessary to develop power plants, explore various types of drives and ways to control them energy-efficiently. The obtained data can serve in the development of a medical exoskeleton.

Keywords: exoskeleton, Denavit-Hartenberg parameters, kinematic structure synthesis, exoskeleton dynamics equation, mathematical modeling.

Introduction. An important stage in the creation of robotic systems is the description of the mathematical modeling of their dynamics. In [1–6], many methods are given for describing the mathematical model, the dynamics of various robots. The author of the work [7] noted that there is a significant lag in the development of medical exoskeletons in the CIS.

The development of robotic systems includes a large number of subtasks from various fields of science and technology [4]. The optimal approach to constructing a mathematical model of the dynamics of the executive mechanism of an active exoskeleton is the use of modified coordinate systems proposed by Denavit-Hartenberg and the Dalembert principle, which allow us to determine the movement of the active exoskeleton taking into account the external forces and moments of the exoskeleton actuating unit (AU) applied to it, as well as the forces and moments imposed on the exoskeleton AU, external kinematic connections and calculate the resulting forces and moments of reactions of these connections.

Materials and methods. The kinematic scheme of the active exoskeleton with the assigned Denavit-Hartenberg CS and its 3D model are presented (Figures 1 and 2). The modified Denavit-Hartenberg system coordinates [8, 9, 10] were obtained from a real human prototype. Links from 0 to 5 are a dummy rack. This fictitious chain consists of weightless links and characterizes the position and orientation of the exoskeleton body in the absolute coordinate system. It has 8 degrees of mobility, all rotational kinematic pairs.

The proposed kinematic scheme can be represented by a directed reachability graph, where the vertices of the graph denote the links of the actuator, and the arcs— the joints connecting them [7]. The kinematic scheme of the exoskeleton and its 3D model are shown in Fig. 1 and 2 respectively.

In mathematical description of the kinematic structures of the AU of the robot is represented as a tree of directed graphs, we use the following definitions [7, 8, 9]:

\[ f(i) \] is the number of the link, is a link-father of link \( i \);

\[ s(i,k) \] is the number of the link, which is the \( k \)-th link-son for level \( i \);

\[ dg(i) \] – polystyrene the outcome of the link \( i \), determines the number of links-the sons of link \( i \);

\[ ns(i) \] – determines how the account link in the son is the link \( i \) to link your father;

\[ \sigma_i = \{0,1\} \] – the coefficient that determines the type of joint of the link \( i \) (1– rotational, 0-translational);

\[ \sigma_i = \text{diag}\{\sigma_1, \ldots, \sigma_N\} \] is a diagonal matrix that defines the types of articulation of the links of the tree AU.

For a mathematical description of a tree structure, they need to determine the order of the links relative to each other. It is defined by the reachability matrix \( D \) – a square matrix, each element of which \( dij = 1 \) if the link \( i \)-th vertex of the directed graph describing the kinematic structure AU is reachable from the link \( j \), and \( dij = 0 \) if the link \( i \)-th \( j \)-th not reachable from the link [9].
The values of the modified Denavit – Hartenberg parameters for the main and auxiliary coordinate systems of the exoskeleton actuating unit are presented in Tables 1 and 2.

**Table 1**

Values of Denavit – Hartenberg parameters of the main coordinate systems of the exoskeleton actuating unit

| № CS | \(\theta, \text{rad}\) | \(d, \text{m}\) | \(a, \text{m}\) | \(a, \text{rad}\) | \(f (i)\) | \(ns (i)\) |
|------|----------------|-------------|--------------|-------------|---------|---------|
| 1    | 0              | 0           | 0            | 0           | 0       | 1       |
| 2    | 0              | 0           | 0            | 0           | 0       | 1       |
| 3    | 0              | 0           | 0            | 0           | 0       | 2       |
| 4    | 0              | 0           | 0            | 0           | 0       | 3       |
| 5    | 0              | 0           | 0            | 0           | 0       | 4       |
| 6    | - \(\pi/2\)    | 0           | 0,238        | - \(\pi/2\) | 5       | 1       |
| 7    | 0              | 0,198       | 0,167        | \(\pi/2\)   | 6       | 1       |
| 8    | \(\pi/2\)      | -0,107      | 0            | - \(\pi/2\) | 7       | 1       |
| 9    | \(\pi/2\)      | 0           | 0,476        | 0           | 8       | 1       |
| 10   | 0              | 0           | 0,674        | \(\pi/2\)   | 9       | 1       |
| 11   | \(\pi\)        | 0,198       | -0,094       | \(\pi/2\)   | 6       | 2       |
| 12   | \(\pi/2\)      | -0,107      | 0            | - \(\pi/2\) | 11      | 1       |
| 13   | \(\pi/2\)      | 0           | 0,476        | 0           | 12      | 1       |
| 14   | 0              | 0           | 0,674        | \(\pi/2\)   | 13      | 1       |
Table 2

| № CS | $\theta$, rad | $d$, m   | $a$, m  | $\alpha$, rad | $f(i)$ | $ns(i)$ |
|------|--------------|---------|--------|---------------|--------|---------|
| 6,2  | $\pi$        | 0       | 0,476  | 0             | 6      | 2       |

| Table 2
Values of Denavit–Hartenberg parameters of the auxiliary coordinate systems of the exoskeleton actuating unit

| № CS | $\theta$, rad | $d$, m | $a$, m | $\alpha$, rad | $f(i)$ | $ns(i)$ |
|------|--------------|--------|--------|---------------|--------|---------|
| 6,2  | $\pi$        | 0      | 0,476  | 0             | 6      | 2       |

Initial data for calculating the dynamics of the exoskeleton AU [8, 9]:

- $d_i$, $a_i$, $\alpha_i$ [rad] – the Denavit–Hartenberg parameters of the main and auxiliary CS of the exoskeleton, as well as the parameters $f(i)$ and $ns(i)$, where $f(i)$ – the number of the parent link of link $i$ and $ns(i)$ – a parameter showing which son is link $i$ for link $f(i)$;
- parameters of AU links: mass, tensors of inertia, coordinates of the centers of mass of links, obtained from the results of 3D–modeling of the structure;
- coordinates of points of application of external forces;
- coordinates of points of overlapping of external connections.

1. Let us express the efforts developed by the drives in terms of generalized coordinates and their derivatives [11]. By grouping the factors at $q^\dot{}$ and $q^{\ddot{}}$, an equation for the dynamics of the robots AU with tree-like CS determined by the reachability matrix of $D$ links, a block vector $0z$ and a diagonal matrix $\sigma$ is obtained:

$$
A(q) \cdot q^\dot{} + B(q, q^\dot{}) - C(q) \cdot 0f - H(q) \cdot 0n_s \cdot z = \tau,
$$

$$
A(q) = \sigma \cdot (0z)^T \cdot (-(\Lambda(0c_{fD}))^T \cdot m^d \cdot (D - \sigma(0z) \cdot (E - \sigma) + \Lambda(0c_{fD}) \cdot 0z \cdot \sigma) +
+D^T \cdot 0J^d \cdot D \cdot 0Z^d \cdot \sigma) + (E - \sigma) \cdot (0z)^T \cdot D^T \cdot m^d \cdot (D - \sigma(0z) \cdot (E - \sigma) + \Lambda(0c_{fD}) \cdot 0z \cdot \sigma);$$

$$
B(q, q^\dot{}) = \sigma \cdot (0z)^T \cdot \left\{-\left((\Lambda(0c_{fD}))^T \cdot m^d \cdot \Lambda(0c_{fD}) \cdot \Lambda(0z \cdot \sigma \cdot 0\dot{q}) \cdot (D - E) +
+\Lambda \left((\Lambda(0c_{fD}))^T \cdot m^d \cdot \Lambda(0c_{fD}) \cdot \Lambda(0z \cdot \sigma \cdot 0\dot{q}) \right)^T
+2 \cdot D^T \cdot \Lambda^T \left(0z \cdot (E - \sigma) \cdot 0\dot{q}\right) \cdot (D - E)\right\} +
+D^T \cdot 0J^d \cdot D \cdot \sigma \cdot 0\dot{q} \cdot \Lambda^T(0z) \cdot (D - E) + D^T \cdot \Lambda \left(D \cdot 0z \cdot \sigma \cdot 0\dot{q} \right)^T \cdot 0J^d \cdot D;$$

$$
C(q) = \sigma \cdot (0z)^T \cdot \left\{(D - E) \cdot \Lambda(0z) \cdot 0\dot{q} \right)^T + D^T \cdot \Lambda(0z) \right) \cdot (D - E) +
+2 \cdot D^T \cdot \Lambda^T(0z \cdot (E - \sigma) \cdot 0\dot{q}) \cdot (D - E) +
+\Lambda^T(\Lambda(0c_{fD})) \cdot \sigma \cdot 0\dot{q} \cdot \left(D - E\right) \cdot \sigma \cdot 0\dot{q}\right) \cdot (D - E) +
+\Lambda^T(\Lambda(0c_{fD})) \cdot \sigma \cdot 0\dot{q} \cdot \left(D - E\right) \cdot \sigma \cdot 0\dot{q} \right)^T \cdot 0z \cdot \sigma \cdot 0\dot{q};$$

$$
H(q) = \sigma \cdot (0z)^T \cdot D^T. $$

In these expressions:

- $m = (m_1, m_2 \ldots m_n)^T$ – matrix of masses of AU links;
- $J_c = (J_{c_1}, J_{c_2}, \ldots J_{c_n})^T$ – block matrix of inertia tensors of links;
- $0J = diag(0T_{J_1}, 0T_{J_2}, \ldots 0T_{J_n})$ – block diagonal matrix of vectors connecting the origin of coordinate systems of links $f(i)$, $ns(i)$ with points through which the resultant external forces applied to links pass.

In the process, the movement of the active exoskeleton of its foot interacts with the supporting surface. Then the equation of the dynamics of the exoskeleton AU, taking into account the influence of external forces and moments, as well as the imposed
external kinematic connections, can be written in the following form:

\[
\begin{pmatrix}
A(q) & -J^T_{r,s}(q) \\
J_r(q) & 0
\end{pmatrix}
\begin{pmatrix}
\dot{q} \\
\dot{R}_r
\end{pmatrix}
+ \begin{pmatrix}
B(q,\dot{q}) \\
P(q)
\end{pmatrix}
= \begin{pmatrix}
L(q) \cdot F_r \\
0
\end{pmatrix}
+ \begin{pmatrix}
\tau \\
0
\end{pmatrix},
\]

where \(q\) – the vector of generalized coordinates of the AU;
\(
\tau
\) – a column vector of moments developed by AU drives.

The values of the remaining block vectors and matrices of equation (2) are determined in accordance with [9, 11].

Determine the following parameters of the links:
- mass;
- coordinates of the centers of mass of the links in the connected main CS of these links;
- tensors of inertia of the links relative to the CS, the axes of which are parallel to the axes of the main connected CS of these links, and the origin of coordinates is in the AU of the links;
- coordinates of the points of application of external forces to the links of the actuating unit in the connected main CS of these links;
- coordinates of the points of application of reaction forces to the links of the mechanism in the connected main CS of these links.

The mass-inertial parameters of the exoskeleton AU were obtained from the results of 3D modeling in the SolidWorks system.

Mass-inertial parameters of the AU links of the exoskeleton:

| Mass-inertial parameters of links 9, 10 | Link 9 | Link 10 |
|----------------------------------------|--------|---------|
| Mass, [kg]                             | 7.60822700 | 8.36217328 |
| Coordinates of mass centers, [m]       | 0.06150354 | -0.00434194 |
|                                         | 0.05504941 | -0.03884998 |
|                                         | 0.12844091 | -0.00029488 |
| Tensors of inertia, \( [kg \cdot m^2] \) | 0.07701307 | 0.71946350 |
|                                         | 0.08658967 | 0.05400681 |
|                                         | 0.01058238 | 0.76869996 |
|                                         | -0.00017268 | -0.14927912 |
|                                         | 0.00349863 | -0.00021547 |
|                                         | -0.00093428 | -0.00020624 |

In order to determine the laws of change of generalized coordinates in the degrees of mobility of the executive mechanism of the exoskeleton equipped with drives, the characteristic typical movements of the human operator performed by changing the generalized coordinate \( (\phi(i)) \) (angle) were considered in time in the corresponding joint in the previously defined range according to the harmonic law of the form [12, 13]:

\[
\phi(t) = \phi_1 \cdot \sin(\omega \cdot t) + \phi_2
\]

\(\phi_1\) - the amplitude of the angle change in the joint, [degree]; \(\phi_2\) – the initial value of the angle in the joint [deg]; \((t)\) – the rate of change of the angle in the joint [deg/c].

Write down the laws and ranges of change of generalized coordinates for joints equipped with drives:

1) \(q_9(t) = q_{13}(t) = -45^\circ \cdot \sin(90^\circ \cdot t)\) – femoral joint;

2) \(q_{10}(t) = q_{14}(t) = 45^\circ \cdot \sin(90^\circ \cdot t)\) – knee joint.

Main part. The dependence of the power in the joint on time was built, taking into account the reaction of the support in the MATLAB bundled software. The desire to reduce power consumption forces designers to develop power units, explore various types of drives and ways of energy efficient control of them [3, 9].

The proposed method for calculating the energy parameters of the exoskeleton, used in [14, 15, 16] by the method of CS synthesis of the exoskeleton AU, allows for the entire process of product development.
Conclusions. One of the critical parameters of the exoskeleton is the power consumption, which, in turn, determines the degree of its autonomy. The desire to reduce power consumption leads developers to design compact power plants, explore different types of drives and ways to manage them energy-efficiently. There is an obvious need to use methods that allow us to quickly determine the energy parameters of the AU as a function of many factors. The proposed method of calculation of power parameters of the exoskeleton used in conjunction with described in [17, 18] method of synthesis of the kinematic scheme AU exoskeleton allows for the entire process of product development, starting from the stage conceptual design, when possible operational changes to the decisions, with a minimum of effort to predict the energy characteristics of actuators, to track the impact of changes in AU, and to evaluate different design options. The results can be used in the development of medical exoskeletons for the rehabilitation of the lower extremities of patients.
Fig. 5. Graphs of the $10^{th}$ joint

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Исследование динамики исполнительного механизма экзоскелета

**Аннотация.** Человек имеет больше 300 степеней подвижности, но воссоздать такую кинематическую схему практически невозможно. В данной статье предложена кинематическая схема экзоскелета самыми необходимыми для движения человека. В системе CAD разработана 3D модель исполнительного механизма экзоскелета с электрогидравлическим приводом и произведен расчет значений масс, координат центров масс, тензоров инерции звеньев исполнительного механизма экзоскелета. Разработан пусковой файл в среде MATLAB для моделирования динамики исполнительного механизма экзоскелета. Выбраны законы управления в степенях подвижности исполнительного механизма экзоскелета. В результате проведенного теоретического исследования определены диапазоны изменения обобщенных координат для исследуемых сочленений. Получены зависимости мощности, момента в сочленениях 9, 10 от времени. Проведенные исследования показали, что подъёма ноги потребуется больше энергии и это заставит разработать силовые установки, исследовать различные виды приводов и способы энергоэффективного управления ими. Полученные данные могут служить в разработке экзоскелета медицинского назначения.

**Ключевые слова:** экзосkeleton, параметры Денавита-Хартенберга, синтез кинематической структуры, уравнение динамики экзоскелета, математическая модельирование.
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