Controlling quantum flux through measurement: An idealised example

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Abstract – Classically, no transfer occurs between two equally filled reservoirs, no matter how one looks at them, but the situation can be different quantum-mechanically. This paradoxically surprising phenomenon rests on the distinctive property of the quantum world that one cannot stare at a system without disturbing it. It was recently discovered that this seemingly annoying feature could be harnessed to control small quantum systems using weak measurements. Here we present one of the simplest models—an idealised double quantum dot—where by toying with the dot measurement strength, i.e. the intensity of the look, it is possible to create a particle flux in an otherwise completely symmetric system. The basic property underlying this phenomena is that measurement disturbances are very different on a system evolving unitarily and a system evolving dissipatively. This effect shows that adaptive measurements can have dramatic effects enabling transport control but possibly inducing biases in the measurement of macroscopic quantities if not handled with care.

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Introduction. – Monitoring a quantum system — i.e. observing it repeatedly and frequently — induces stochastic evolutions, or quantum trajectories [1–3], where the uncertainty of the outcomes is the source of the randomness. Quantum jumps [4], which consist in the sharp transitions of a system from one quantum state to another, are fundamental illustrations of this behaviour. They have been observed for the first time in fluorescent systems [5,6] and nowadays in almost everyday meso- or nano-scale experiments [7–9]. They emerge naturally as a consequence of the interplay between system evolution and continuous measurement and are very abrupt yet not strictly instantaneous. Aside from their use in quantum Monte Carlo methods [10], quantum trajectories are instrumental to control small quantum systems [11–13]. Feedback control consists in using the measurement record to act on the system in order to constrain or control its evolution. The action on the system can be carried out through a Hamiltonian drive — the most common and historical method [14,15] — yet it was underlined recently [16–18] that measurements themselves could be used as they inevitably have a back-action on the system. A minimal version of this second method deals with adjusting the measurement intensity (or rate) but not the measured observables. Quantum Zeno dynamics [19,20], in which the dynamics of the system is frozen in a subspace selected through strong measurement, is an archetypal example. However, control by measurement is multi-faceted because the latter picture is true only for a system evolving unitarily and breaks down for a system evolving dissipatively. Indeed, Hamiltonian and thermally activated quantum jumps behave very differently with respect to a varying measurement intensity. In the first case, the Zeno effect makes the average time between two jumps increase with the measurement strength, whereas in the dissipative case it converges to a constant which is intrinsic to the system-reservoir coupling[1]. In this paper we consider a model system, idealising a double quantum dot in contact with two electron reservoirs, where both a unitary and a dissipative evolution compete. This is the simplest system where the effect we are ultimately interested in can be seen. The tunnelling rate between dots can be controlled by adjusting the intensity of the particle position measurement without impacting the dot-reservoir jump rates. Available pathways can thus be controlled through

1The main physical rationale behind this fact lies on the very large number of degrees of freedom of the reservoir dissipatively coupled to the system.
measurement, and this is the basic principle making flux control possible. By monitoring the particle position, we may choose to measure strongly when the particle is in the right dot and mildly otherwise. As a consequence, when the particle is in the right dot, the jump rate to the right reservoir is not modified but its tunnelling rate to the left dot is frozen and the probability to backtrack is thereby reduced. This creates a non-zero extra average particle flux from the left reservoir to the right, and a macroscopic effect is obtained from what seemed to be a very innocent measurement method.

**A simple model.** — The physical system we have in mind is that of a double quantum dot (DQD), or rather an abstract idealisation of a DQD\(^2\) with an architecture similar to that of fig. 1, coupled to two electron reservoirs at given chemical potentials. We may schematically picture our model system as in fig. 1, in which electrons can either jump to the left/right dots from the left/right reservoirs, back and forth, or tunnel through the potential barrier from one dot to the other. The position of the electron is continuously monitored with a non-specified measurement apparatus, which in the DQD architecture can be implemented through an auxiliary quantum point contact (QPC) as in [8,21,22]. We further assume that the Coulomb repulsion between two electrons in the quantum dot is high enough to be able to neglect double occupancy in our idealised description. We write \(|0\rangle\), \(|L\rangle\), and \(|R\rangle\), the system space vectors where there is no electron in the DQD, one in the left dot, and one in the right dot, respectively. Three processes are at play: tunnelling between the dots; quantum hopping between dots and reservoirs; and back-action of the measurement. They generate the DQD density matrix evolution, called the quantum trajectory:

\[
d\rho_t = d\rho_t^{\text{tunnel}} + d\rho_t^{\text{bath}} + d\rho_t^{\text{measure}},
\]

(1)
during a time duration \(dt\). The tunnelling evolution is unitary, \(d\rho_t^{\text{tunnel}} = -i[H, \rho_t]dt\) with inter-dots Hamiltonian,

\[
H = u (|R\rangle\langle L| + |L\rangle\langle R|).
\]

(2)
The dot-bath hopping-generated evolution \(d\rho_t^{\text{bath}}\) is dissipative, and we model it through a deterministic evolution of the form \(d\rho_t^{\text{bath}} = \mathcal{L}^{\text{bath}}(\rho_t)dt\) with \(\mathcal{L}^{\text{bath}}(\cdot)\) a Lindblad operator [23] linear in \(\rho_t\). We assume that, in the absence of tunnelling, the left reservoir would thermalise the left dot into a Gibbs steady state of chemical potential \(\mu_l\) (respectively, \(\mu_r\) for the right reservoir and right dot). We denote by \(a\) (respectively, \(ae^{\beta\mu_l}\)) the hopping rate from the left dot to the left reservoir (respectively, from the left reservoir to the left dot) in the absence of tunnelling, and by \(b\) and \(be^{\beta\mu_r}\) the respective right dot- reservoir hopping rates (\(\beta\) being the inverse temperature).

A possible implementation of this prescription is to write \(\mathcal{L}^{\text{bath}}(\rho_t) = \left(L_{\sigma^+_l} + L_{\sigma^-_l} + L_{\sigma^+_r} + L_{\sigma^-_r}\right)(\rho_t)\) where the associated Lindblad generators \(L_{\sigma^+_l}, L_{\sigma^-_l}, L_{\sigma^+_r}, L_{\sigma^-_r}\) take the form

\[
L_{\sigma^+_l}(\rho) = \sigma_l \rho \sigma_l - \frac{1}{2} \{\sigma_l, \rho\} \text{ with } \sigma_l^+ = \sqrt{ae^{\beta\mu_l}/2}|L\rangle\langle 0|, \quad \sigma^-_l = \sqrt{a}\langle 0|\langle L|, \quad \sigma^+_r = \sqrt{be^{\beta\mu_r}/2}|R\rangle\langle 0| \text{ and } \sigma^-_r = \sqrt{b}\langle 0|\langle R|.
\]

The left/right dot occupancy is monitored continuously via a weak-measurement apparatus. We write \(O = h_l|L\rangle\langle L| + h_r|R\rangle\langle R| + h_0|0\rangle\langle 0|\) the measurement operator (in the following we take \(h_0 = 0\) because only the differences matter), the strength of the measurement, i.e. the intensity of the look, increases with the values of \(h_{l,r}\). The measurement result is a random process, which we denote \(X_t\). Physically, \(X_t\) may represent the total charge going through the quantum point contact monitoring the DQD, the state being read from the local average intensity (the instantaneous intensity being, strictly speaking, singular) as was done in [8]. It is known [11,24–26] that quantum mechanical rules for measurement imply that \(X_t\) varies in time according to:

\[
dX_t = 2\text{tr}(O\rho_t)dt + dW_t.
\]

Here, \(W_t\) is a standard Wiener process, i.e. \(dW_t/dt\) is the usual white-noise in-time process, and its randomness is an echo of the random nature of quantum measurements. A given realisation of this Wiener process corresponds to a given realisation of a time series of weak measurements. It is also known [11,24–26] that the back-action of the measurement on the system density matrix reads

\[
d\rho_t^{\text{measure}} = L_O(\rho_t)dt + D_O(\rho_t)dW_t,
\]

where \(L_O(\cdot)\) is the Lindblad generator associated to \(O\) and where \(D_O(\cdot)\) is the stochastic innovation term,

\[
D_O(\rho) = \{O, \rho\} - 2\rho tr(O\rho).
\]

It is worth noticing that any given realisation of a time series of weak measurements, i.e. a time series of observation, corresponds to a realisation of the process \(X_t\), and hence that the process \(X_t\) is the only information our model observer possesses on the DQD.

As a further simplification we take the DQD density matrix \(\rho_t\) to be of the following form:

\[
\rho = \begin{bmatrix}
Q_0 & 0 & 0 \\
0 & Q_l & iK/2 \\
0 & -iK/2 & Q_r
\end{bmatrix}.
\]

(3)

This choice can actually be made with no lack of generality as a diagonal density matrix is naturally prepared and as the form (3) is preserved by the evolution (1). This parametrisation of the density matrix simply shows the probabilities \(Q_0, Q_l\) and \(Q_r\) for the DQD to be in the basis.
states $|0\rangle$, $|L\rangle$, and $|R\rangle$, plus a (real) phase $K$ encoding the delocalisation due to the tunnel coupling between the dots. We now have everything needed to expand eq. (1) and get explicit evolution equations for $Q_0$, $Q_l$, $Q_r$ and $K$,

\[
\begin{align*}
\frac{dQ_0}{dt} &= (aQ_l + bQ_r - (am_l + bm_r)Q_0)\ dt - 2(h_lQ_l + h_rQ_r)Q_0\ dW_t,
\frac{dQ_l}{dt} &= (-uK + am_lQ_0 - aQ_l)\ dt + 2(h_l(1 - Q_l) - h_rQ_l)\ dt + W_t,
\frac{dQ_r}{dt} &= (uK - bQ_r + bm_rQ_0)\ dt + 2(h_r(1 - Q_r) - h_lQ_r)\ dt + dW_t,
\frac{dK}{dt} &= 2(-\nu_h K + u(Q_l - Q_r))\ dt + (h_l(1 - 2Q_l) + h_r(1 - 2Q_r))K\ dW_t.
\end{align*}
\] (4)

Fig. 2: A sample trajectory with $a = b = 0.02$, $\beta\mu_l = -\beta\mu_r = 1.0$, $h_r = -h_l = 7.0$ and $u = 1.0$.

Quantum jumps. — Equation (1) is a set of stochastic differential equations (SDEs) for the DQD density matrix coefficients. It is very complicated, there is no hope to solve it exactly, and the description may look a bit complicated at this point. However it is obvious from fig. 2 that the behaviour of the system is very peculiar in the large $h_l$ and $h_r$ limit. Indeed, if the measurement strength is strong enough, the system has a tendency to collapse most of the time onto one of the three states $|0\rangle$, $|L\rangle$, and $|R\rangle$, and do random jumps between them. These are the quantum jumps, and we can approximately describe the DQD evolution as a time series of random jumps between those three states. This limiting description, which is often assumed from the start and which is admittedly much simpler but only valid in an appropriate regime, is actually not trivial to derive from the full description we have provided. Proving rigorously that it does so will not be done in this paper, rather we will invoke heuristic arguments and numerical simulations. Because $t(\rho) = 1$, the diagonal coefficients of the density matrix evolve inside a triangle. When the monitoring is not strong enough, their random trajectory explores the whole bulk but when it is intense, they spend most of their time near the corners.

From a physical point of view this is completely expected: indirectly measuring the particle position has a tendency to make the wave packet collapse onto one of the position eigenstates.

We thus have an approximated effective Markov chain description of our system in the strong-measurement limit (see fig. 3), in which the DQD system is described by the probabilities $\Pi_{0,l,r}$ for the DQD density matrix to be near one of those triangle vertices, that is, the probabilities for the DQD state to be close to $|0\rangle$, $|L\rangle$, and $|R\rangle$. The time evolution of these probabilities is fully characterised by the transition rates between the different states. Their direct computation from eq. (4) is technical, and will be done properly in [27]. As suggested by the numerics, let us assume that the $0 \leftrightarrow L$ transition process is dominated by trajectories close to the $0L$ boundary of the simplex for which $Q_r \approx 0$ (and hence $K \approx 0$ and $Q_l \approx 1 - Q_0$). Under these approximations, eq. (4) then becomes $dQ_0 = \lambda(p - Q_0) dt + 2h(1 - Q_0)Q_0 dW_t$ with $\lambda = a(m_l + 1) + bm_r$ and $p = a/\lambda$. This evolution encodes the competition between the bath coupling and the measurement back-action, see, e.g., [28]. The sharp transitions can be understood analogously with Kramers’ theory where a particle excited by white-noise fluctuations shows a jumpy behaviour because it evolves in a bimodal effective potential. The jump statistics from $Q_0 = 0$ to $Q_0 = 1$ (or the reversed) becomes Poissonian in the large $h$ limit, the transition rates, defined as the mean time between two jumps, were shown to converge to $\lambda_{0L} = am_l$, $\lambda_{L0} = a$ (and similarly $\lambda_{0R} = bm_r$, $\lambda_{R0} = b$) for large $h$. Similar methods along the lines of [29,30] enable one to estimate the transition rates in the $R \leftrightarrow L$ region. However, these arguments assume a decoupling between the different processes which has to be proved [27]. In what follows, we will
only use the fact (see [27]) that in the strong-measurement limit, these transition rates are \( \lambda_{LR} = \lambda_{RL} \approx u^2 / \hbar^2 \), \( \lambda_{LO} \approx am \), \( \lambda_{OR} \approx a \), and \( \lambda_{ROI} \approx b \). The point to notice is that the rates \( \lambda_{LR} \) and \( \lambda_{RL} \), induced by the tunnelling transition \( R \leftrightarrow L \), vanish as \( \hbar^{-2} \) for strong measurement in accordance with the quantum Zeno effect, while the rates \( \lambda_{OR} (\lambda_{LO}) \) and \( \lambda_{ROI} \), activated by the dissipative contacts \( 0 \leftrightarrow R(L) \), are not renormalised either by the tunnelling or the measurement and remain finite in the strong-measurement limit.

**Measuring the flux without feedback.** – By monitoring the particle position continuously in time we have access to the electron flux. However, the only information our model observer has on the DQD is the measurement output \( X_t \), and thus she/he has to read out the flux from this process\(^3\). Since for strong enough measurement, the particle position is approximately well defined, it is possible to rely on a classically inspired characterisation of the flux. One can simply define the electron transfer during a time duration \( t \) as the difference between the number \( N_{LR}(t) \) of quantum jumps from state \( |L \rangle \) to state \( |R \rangle \) and the number \( N_{RL}(t) \) of quantum jumps from \( |R \rangle \) to \( |L \rangle \) during this time period. This can be read out from the numbers of break points in the time slope of \( X_t \), see fig. 2. This is a random number whose statistics is induced from that of the DQD density matrix components. Hence, the mean flux that the observer is measuring is the “statistical mean” of the electron transfer (and not a quantum average in the usual sense, see below). By ergodicity, averaging in time yields the mean, and the “statistically computed” mean flux is naturally defined as\(^4\)

\[
\langle J \rangle_{\text{stat}} = \lim_{t \to +\infty} \frac{1}{t} (N_{LR}(t) - N_{RL}(t)).
\]  

(5)

We numerically solved the system of SDEs (1) for the DQD quantum trajectory —and not the approximate Markov chain— and computed the measured average flux we described in (5). As the equations are strongly non-linear, we used an adaptive Runge-Kutta discretisation method which is described in more detail in [27]. The results are shown in fig. 4. The flux decreases as a function of the measurement strength: strong measurements tend to Zeno-freeze the tunnelling transition between \( |L \rangle \) and \( |R \rangle \) thus making it harder for the electrons to go through the DQD.

Within the Markov chain approximation this mean flux can be easily computed. Standard results from Probability Theory [31] allow the flux to be expressed as \( \lambda_{LR} \Pi_{\text{stat}} - \lambda_{RL} \Pi_{\text{stat}} \) with \( \Pi_{\text{stat}} \) (respectively, \( \Pi_{\text{stat}}^\ast \)) the Markov chain stationary probability to be in \( L \) (respectively, \( R \)). The stationary probabilities can be easily computed from the Markov master equation implied by the transitions rates (see [27] for more details) which gives

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Feedback. – The peculiarity of the present situation which allows for feedback is that our model observer gets information on the flux continuously in time and realisation, i.e. not as an ensemble average. The main idea behind the feedback rests on the fact that the transition rates for the Hamiltonian junction or the dissipative coupling do not scale in the same way as the measurement strength increases. By measuring more strongly when the particle is on the right we increase its probability to jump in the right reservoir rather than backtrack to the left dot. If we continuously monitor the particle position we can increase the measurement intensity only when it is on the right thus breaking the symmetry of the system and creating a flux from left to right.

In order to control the system, the first thing to do is to infer the value of the density matrix ρt from the measurement records X_t. Since the time evolution of the DQD is formally driven5 by X_t, one could theoretically extract exactly ρ_t from X_t. However, it is more practical [15] and more robust to estimate it using the fact that ρ_t is given by the slope of the measurement records in the strong-measurement limit. Indeed, the drift term in the time evolution of X_t, i.e. dX_t = 2 tr(ρ dρ_t) dt + dW_t, overcomes the noise for strong measurement. It is equal to 0 when Q_0 ≈ 1, 2h_t when Q_t ≈ 1 and 2h_r when Q_t ≈ 1. So when the density matrix is concentrated on one of the natural basis states and if h_t and h_r are big enough, i.e. the measurement is strong enough, the DQD density matrix ρ_t can be approximately read out from the time slope of the measurement results. Besides, this instantaneous slope is (formally) infinite because of white-noise properties, and it consequently needs to be averaged over a small time window τ_int. We thus write s_t = \int_{-\infty}^\infty e^{(t-\tau)/\tau_{int}} dX_t. Further, we take a measurement operator that discriminates equally L and R from 0, i.e. we choose h_r = -h_l = h > 0. We then measure as strongly as we can when the electron is likely to be in the right dot and keep measuring mildly

when it is not to keep track of its position. This can be enforced by taking h(t) = h_{min} 1_{s_t≤\delta} + h_{max} 1_{s_t>\delta}, where δ is an arbitrary threshold that can be optimised numerically and 1_{s_t<\delta} is the indicator function, equal to 1 if the inequality is satisfied and 0 otherwise. A snapshot of a trajectory is given in fig. 5. The evolution of the system now obeys an even more complicated stochastic differential equation. However, assuming that the measurement stays strong enough for the Markovian approximation to be faithful, we can estimate the maximum flux that would be obtained with a perfect feedback by simply changing the jump rates, i.e. computing λ_{LR}, λ_{L0}, λ_{0R}, λ_{0L} with h = h_{min} and λ_{LR}, λ_{0L} with h = h_{max}. Actually, only λ_{LR} and λ_{LR} depend on h, so we only have to change two rates. This gives

\langle J \rangle_{stat} \simeq \frac{u^2}{1 + m_l + m_r} \left[ \frac{m_l}{h_{min}^2} - \frac{m_r}{h_{max}^2} \right]. \quad (8)

That is, as expected, a non-zero flux even when the two chemical potentials are equal. This result is valid only for a perfect feedback scheme, that is if the system state is perfectly known at all times. As a result it is unfortunately just an upper bound for practical feedback schemes. However, this feedback scheme is robust enough for the existence of an induced current to persist away from the strong-measurement regime.

Using the same numerical method as before, we estimate the electron flux going through the idealised DQD for various feedback strengths, i.e. for various maximum measurement rates. We compared the numerical flux with the upper bounds we have just found. The numerical computations were done with complete SDEs, including feedback, and not with the Markovian approximation.

Fig. 5: A snapshot of a trajectory with measurement feedback $a = b = 0.02$, $\beta \mu_l = \beta \mu_r = 0.0$, $u = 1.0$, $\delta = 2.0$, $\tau_{int} = 1$, $h_{min} = 5.0$ and $h_{max} = 15.0$.

Fig. 6: (Colour on-line) Flux with feedback. (a) Average electron flux as a function of the feedback strength $h_{max}$. The analytic upper bound previously found is shown by a dashed line. Here $h_{min} = 15$, $a = b = 0.02$, $u = 1.0, \delta = 4.0$, $\tau_{int} = 0.5$ and $\mu_l = \mu_r = 0$ that is no potential difference between the two baths. The feedback scheme creates the flux in the DQD as expected. (b) Average electron flux as a function of the difference of chemical potential between the two electron bath. The analytic expressions in the Markovian limit are shown by dotted lines. The feedback scheme indeed increases the flux in the DQD and is even able to counter a small difference of potential. ($a = b = 0.02$, $u = 1.0$, $\delta = 2.0$, $\tau_{int} = 1.$)

5Indeed, $d\rho_t = [L_{tot}(\rho_t) - 2\tau(D(\rho)D_{\sigma}(\rho_t))] dt + D_{\sigma}(\rho_t)dX_t$, where the deterministic part of $\rho$ is collected in $L_{tot}(\rho_t)$. 
The results are shown in fig. 6. In particular, they clearly show that this feedback procedure is able to generate a flux in the absence of a chemical potential difference. The disagreement between the upper bound and the practical feedback scheme for strong feedback is a consequence of the finite window of integration of the measurement records: as the jumps get sharper it becomes increasingly difficult to know the system state in time and to switch the measurement strength fast enough.

Discussion. – Using a simple feedback scheme, in the sense that it relies on the measurement strength of the apparatus only, we have shown that it is possible to create, control or reverse, a particle flux in a quantum system. Even if our study has been done in a simple setting, the effect we have discovered is undoubtedly very general.

The way we monitor a quantum system not only modifies its perceived fluctuations or correlations but also changes its non-equilibrium properties in a dramatic way (changing the sign of a flux in our example). This remains true even if the measurement itself has a completely symmetric back-action on the whole system. In this respect, the effect we described bears similarities with the Parrondo paradox [32] or flashing ratchets [33–35] because it amounts to the production of an asymmetric output by oscillating between different yet symmetric situations. This effect could be used to engineer out-of-equilibrium quantities in systems where the measurement intensity is the only control parameter. It needs to be taken into account in the numerous experiments that use adaptive measurement schemes to measure fluxes or other macroscopic quantities as it could introduce substantial biases in the results.

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