Natural inflation at the GUT scale

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Natural inflation driven by pseudo-Nambu-Goldstone bosons have a problem that the nearly scale invariant spectrum of density perturbations is attained only when the symmetry breaking scale is of the order of Planck scale. We show here that if one couples the PNGB to a thermal bath as in warm inflation models, the amplitude and spectral index which agrees with the Wilkinson Microwave Anisotropy Probe (WMAP) data is obtained with the symmetry breaking in the GUT scale. We give a GUT model of PNGB arising out of spontaneously broken lepton number at the GUT scale which gives rise to heavy Majorana masses for the right handed neutrinos which is needed in seesaw models. This model also generates a lepton asymmetry because of the derivative coupling of the PNGB to the lepton current. A characteristic feature of this model is the prediction of large non-gaussianity which may be observed in the forthcoming PLANCK experiment.

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I. INTRODUCTION

Inflation \[ \text{(1)} \] was introduced to solve the horizon and curvature problems of cosmology and in addition it predicted a scale invariant spectrum of density perturbations which was verified by the Cosmic Background Explorer (COBE), WMAP and other CMBR anisotropy experiments. The successful model of inflation requires a flat potential and a natural candidate for such a potential is the Pseudo-Nambu-Goldstone potential as first pointed out in [2, 3, 4].

One limitation of natural inflation models is that the symmetry breaking scale \( f \) is related to the spectral index \( n_s = 1 - M_P^2/(8\pi f^2) \) and observations of microwave anisotropy constrain the symmetry breaking scale to be close to the Planck scale. As discussed in Banks et al [5], a symmetry breaking scale larger than \( M_P \) makes the theory susceptible to large quantum corrections which can destabilize the flat PNGB potential. There have been several attempts at solving this large \( f \) problem in natural inflation. Arkadi-Hamed et al [6] invoke extra dimensions with the Wilson loop of a gauge field in the extra dimension to explain why \( f \sim M_P \). Similar arguments are also given by Kaplan and Weiner [7]. Kim et al [8] invoke two field natural inflation to bring down the symmetry breaking scale below Planck scale. Kinney and Mahanthappa [9] show that in some special symmetry breaking schemes the quadratic term in the PNGB field is subdominant compared to the higher order terms and in these models the symmetry breaking scales can be lower than the Planck scale.

In this paper we show that if the PNGB inflaton of the natural inflation model is coupled to a radiation bath (with a sub-dominant energy density) as in warm inflation models [10] the symmetry breaking scale \( f \) can be in the GUT scale and be consistent with the observations of the temperature anisotropy spectrum observed by WMAP [11]. In this model the dissipative coupling of the PNGB inflaton makes it roll slowly even in a steep potential which results when \( f \) is lowered from \( M_P \) to \( M_{GUT} \sim 10^{16} \text{GeV} \).

As a specific model let us consider the SU(5) model where the right handed neutrino \( N \) is a singlet. In the seesaw mechanism [12] one generates a heavy Majorana mass by coupling this right handed neutrino to a SU(5) singlet Higgs,

\[ -\mathcal{L}_\nu = g H N N^C. \]  \( \text{(1)} \)

In order to break lepton number spontaneously we have a potential for the Higgs

\[ -\mathcal{L}_H = \frac{\lambda}{8}(H^1 H - \frac{f^2}{2})^2. \]  \( \text{(2)} \)

Here \( f \) is the spontaneous symmetry breaking scale.

At the minima of the potential the Higgs is given by \( H = \frac{1}{\sqrt{2}} f e^{i\phi} \). Here angular variable \( \phi \) is the Goldstone boson of the spontaneously broken lepton number symmetry. Quantum gravity effects are expected to break global symmetries at the Planck scale. If there is an explicit symmetry breaking due to gravity the Goldstone boson acquires mass. The explicit symmetry breaking term can be of the form

\[ -\mathcal{L} = \frac{M_P^2}{M_P} N N^C + O\left(\frac{1}{M_P^2}\right). \]  \( \text{(3)} \)

Because of this explicit symmetry breaking the potential of PNGB is given by [13]

\[ V(\phi) = A^4 \left(1 + \cos\left(\frac{\phi}{f}\right)\right). \]  \( \text{(4)} \)
The mass of the PNGB is given by $m_\phi = \frac{\mu^2}{M_p^2}$. This implies that $\Lambda = \mu = \frac{\mu^2}{M_p^2}$. Now if we take $M \sim M_{\text{GUT}} \sim 10^{16} - 10^{17}\text{GeV}$ then we have $\Lambda \sim 10^{13} - 10^{14}\text{GeV}$ which is the allowed range by WMAP data.

II. WARM NATURAL INFLATION

In warm inflation the equation of motion of inflaton field is given by

$$\ddot\phi + (3H + \Gamma)\dot\phi + V'(\phi, T) = 0.$$  \hfill (5)

Here $V'$ denotes differentiation of $V$ with respect to $\phi$, $\Gamma$ is the damping term and $V(\phi, T)$ is thermodynamic potential. In slow roll approximation we neglect $\ddot\phi$ in the Eq. (5). During inflation the potential energy of the inflaton field dominates over radiation density. So the dynamics of $\phi$ field is governed by

$$\dot\phi = -\frac{V'}{3H + \Gamma},$$  \hfill (6)

$$H^2 = \frac{8\pi}{3M_p^2} V.$$  \hfill (7)

The slow role parameters are defined as

$$\epsilon = \frac{M_p^2}{16\pi} \left(\frac{V'}{V}\right)^2, \quad \eta = \frac{M_p^2}{8\pi} \frac{V''}{V},$$

$$\beta = \frac{M_p^2}{8\pi} \frac{\Gamma V'}{V}, \quad \delta = \frac{M_p^2}{8\pi} \frac{TV'}{V}.$$  \hfill (8)

Here two extra slow roll parameters appear because of $\phi$ dependence of damping term and temperature dependence of the potential.

The density perturbations during warm inflation are generated by thermal fluctuations. The power spectrum for the density perturbations given in [14] is

$$P_R = \left(\frac{\pi}{4}\right)^{1/2} \frac{H^2 \Gamma^{1/2} T}{\phi^2}$$  \hfill (9)

which can be written in terms of potential and its derivative using Eq. (4) and Eq. (6) as

$$P_R = \left(\frac{\pi}{4}\right)^{1/2} \frac{8\pi}{3M_p^2} \frac{V^{5/4} \Gamma^{5/2} T}{V^2}.$$  \hfill (10)

Using the natural inflation potential [14] we get for the power spectrum,

$$P_R = \left(\frac{\pi}{4}\right)^{1/2} \frac{8\pi}{3M_p^2} \frac{\Gamma^{5/2} T f^2}{\Lambda^3} \frac{(1 + \cos \phi \frac{3}{2})^{5/4}}{\sin^2 \frac{\phi}{T}}.$$  \hfill (11)

The spectral index can be defined as

$$n_s - 1 = \frac{\partial \ln P_R}{\partial \ln k}.$$  \hfill (12)

In terms of the slow roll parameters this can be written as

$$n_s - 1 = \frac{3H}{\Gamma} \left(-\frac{9}{4} \epsilon + \frac{3}{2} \eta - \frac{9}{4} \beta\right).$$  \hfill (13)

For the given potential [14] the spectral index will be

$$n_s - 1 = \frac{3H}{\Gamma} \frac{3M_p^2}{64\pi f^2} \frac{\left(3 + \cos \frac{\phi}{T}\right)}{\left(1 + \cos \frac{\phi}{T}\right)}. $$  \hfill (14)

The observational constraint on $n_s$ from WMAP 5-year data [11] is $0.948 < n_s < 0.977$. So it is obvious from above Eq. that if we take warm inflation in strong dissipative regime i.e $\Gamma$ is very large compared to $H$, we can have small value of $f$ (fig. 1). In the cold natural inflation models on the other hand the spectral index $n_s = 1 - M_p^2/(8\pi f^2)$. This implies that in the cold natural inflation models WMAP data gives a strong constrain $f > 0.7 M_p$ [13].

The slow roll parameter $\epsilon$ for this model is

$$\epsilon = \frac{M_p^2}{16\pi f^2} \frac{\sin^2 \frac{\phi}{T}}{(1 + \cos \frac{\phi}{T})^2}. $$  \hfill (15)

At the end of inflation $\epsilon = 1 + r$, where $r = \frac{\Gamma}{\pi T}$. This
will give $\phi_f$ as
\[
\cos \frac{\phi_f}{f} = \left( \frac{1 - (1 + r) \frac{16\pi^2 f^2}{M_p^2}}{1 + (1 + r) \frac{16\pi^2 f^2}{M_p^2}} \right).
\]

Putting $r = 3.9 \times 10^4$ and $f = 8 \times 10^{16}$ GeV we get $\phi_f = 2.9 f$. The e-foldings may be calculated as
\[
N = \int_{\phi_i}^{\phi_f} \frac{H d\phi}{\phi} = \frac{8\pi \Gamma}{3H M_p^2} \int_{\phi_i}^{\phi_f} V d\phi
= \frac{16\pi f^2}{3H M_p^2} \left( \log \frac{\sin \left( \frac{\phi_f}{f} \right)}{\sin \left( \frac{\phi_i}{f} \right)} \right).
\]

The scalar field lies between $\pi f$ and 0. For $N = 60$ we get $\phi_i = 1.02 f$. The value of the scalar field remains in the GUT regime and still gives adequate e-foldings to solve the horizon and curvature problems.

### III. MICROPHYSICAL MODEL FOR LARGE DISSIPATION

To get large dissipation the inflaton can be coupled to another scalar field $\chi$ by another explicit symmetry breaking term
\[
\mathcal{L}_\chi = 2g^2 \phi^2 \chi^2
\]
which in turn is coupled to the radiation field $\sigma$ as
\[
\mathcal{L}_{\chi\sigma} = \frac{1}{\sqrt{2}} hf \left( \phi^2 \chi^* + \chi^2 \sigma^* \right).
\]

This two step coupling is necessary in order to generate a large dissipation without destabilizing the inflaton potential by loop corrections [16]. The dissipation coefficient $\Gamma$ for this model has been calculated by Berera et al [16],
\[
\Gamma = \frac{16 g^2}{\pi \hbar^2} T \ln \frac{T}{m_\chi}
\]

The interaction terms in the Lagrangian [18] can generate one loop corrections to the inflaton mass that can destabilize the flatness of the potential [14]. For the potential to remain flat the mass correction $g^2 f^2$ should be smaller than $\frac{\Lambda^2}{T^2}$. If we take $\Lambda \sim 10^{13}$ GeV and $f \sim 10^{16}$ GeV then $g \leq 10^{-6}$. For the validity of above expression [20], the mass of $\chi$ field should be smaller than $T$. So if we take one loop correction to the mass of $\chi$ field ($T \sim 10^{12}$ GeV) because of $\sigma$ field $h$ should be smaller than $10^{-4}$. If we take $g$ and $h$ of the same order we can have $\Gamma \sim 10^{12}$ Gev.

### IV. PREDICTIONS FOR NON-GAUSSIANITY

Non-gaussianity is a very important characteristic of the model of inflation. Its magnitude is conventionally defined by the parameter called $f_{NL}$, which is the ratio of the three point correlation to the two point correlation. In standard inflation non-gaussianity parameter $f_{NL}$ is proportional to the slow roll parameter and are therefore small [17, 18]. In warm inflation models non-gaussianity arises because of non-linear coupling between inflaton and radiation. The $f_{NL}$ for warm inflation models has been calculated in [19]. It is given by
\[
f_{NL} = -15 \ln \left( 1 + \frac{\Gamma}{42 H} \right) - \frac{5}{2}.
\]

Taking the allowed range of $\Gamma$ from the fig 1 i.e $1 \times 10^{12} < \Gamma < 3 \times 10^{12}$ we get $-122.6 < f_{NL} < -106.2$ which is allowed by WMAP-5 data [11] ($-151 < f_{NL} < 253$).

### V. LEPTOGENESIS

This model automatically generates lepton asymmetry at the end of inflation. The PNGB coupling to lepton current is obtained from [1] as
\[
\mathcal{L}_{int} = \frac{1}{f} \partial_\mu \phi J^\mu_L
\]

For the homogenous inflaton this will be
\[
\mathcal{L}_{int} = \frac{\dot{\phi}}{f} n_L
\]

here $n_L$ is lepton number. Therefore $\frac{\dot{\phi}}{f}$ is like a chemical potential for the lepton number, $\mu_L = \frac{\dot{\phi}}{f}$. At equilibrium the lepton number is given by
\[
n_L = \frac{g_\nu}{6} \frac{T^3}{\dot{\phi} T^2} \left( \frac{\mu_L}{T} \right).
\]

So the lepton to entropy ratio will be
\[
\frac{n_L}{s} = \frac{15}{4\pi^2 g_\nu f^2} \frac{\dot{\phi}}{T}
\]

Using slow roll approximation $\dot{\phi} = -\frac{\dot{\phi}}{T}$. For this model we get
\[
\frac{n_L}{s} = \frac{15}{4\pi^2 g_\nu \Lambda^4} \frac{\dot{\phi}}{f^2 GT}.
\]
If we take $\Lambda \sim 10^{13}$GeV, $f \sim 10^{17}$GeV, $\Gamma \sim 10^{12}$ GeV and $T \sim 10^{12}$GeV, we get from (26) $\eta_L \sim 10^{-10}$ (fig. 2).

If the lepton number is violated spontaneously at scale $f$ then there is an effective lepton number violating dimension five operator [20]

$$\mathcal{L}_L = \frac{2}{f} hhll + hc$$

(27)

where $l$ is the lepton doublet and $h$ is the Higgs doublet of the standard model. When the electroweak symmetry is broken by the Higgs acquiring a vev $v$ then it generates a light neutrino mass $m_{\nu} = 4v^2$. The operator [21] can wipe out any generated lepton number at high temperature by the lepton number violating interactions $l + h \rightarrow l^c + h^1$. The interaction rate of this lepton number violating reaction is [21]

$$\Gamma_L = 0.04 \frac{T^3}{f^2}.$$ 

(28)

These lepton number violating interactions will decouple at a temperature

$$T_d = 4.16 \left(\frac{f^4 \Lambda^4}{M_P^2}\right)^{\frac{1}{2}}.$$ 

(29)

For $\Lambda \sim 10^{13}$GeV and $f \sim 10^{17}$GeV this temperature is $T \sim 10^{14}$GeV. Since the temperature of the radiation bath is $T < 10^{13}$GeV the lepton asymmetry generated by the rolling PNGB field will not be washed out by lepton number violating interactions with the light Higgs.

The fact that PNGB’s coupling to the lepton/baryon current is of the derivative coupling form which gives rise to spontaneous leptogenesis of Cohen and Kaplan [22] was first recognized by Dolgov et al [23]. In [23] a natural inflation without damping was examined for generation of baryon/lepton number. It was found that oscillations of the inflaton at the end of inflation wipes out the baryon/lepton asymmetry so the PNGB model of creating B/L asymmetry during natural inflation was considered unfeasible [23]. In [24] it was shown that if one assumes the chaotic inflation potential $m^2 \phi^2$ and couples the inflaton to radiation as in warm inflation and in addition assumes a $\partial_\mu j_{\mu L}$ coupling of the inflaton then one can get the required baryon asymmetry with a suitable choice of parameters.

VI. CONCLUSIONS

There has been a long standing problem with utilizing the flat potential of PNGB’s for inflation as the nearly scale invariant power spectrum which is consistent with observations generated only when the symmetry breaking scale $f \sim M_P$ [2, 3, 4, 5, 6, 7, 8]. In this paper we show that by coupling the inflaton to a radiation bath (as in warm inflation models [10]) can reduce $f$ to the GUT scale. The value of the inflaton field $\phi \sim f \sim M_{GUT}$ which makes the inflaton potential stable against Planck scale radiative corrections. We give a model of inflation where the inflaton is the PNGB arising from spontaneous breaking of lepton number which also gives a large Majorana mass for the right handed neutrinos as required in see-saw models [12]. Since the PNGB’s have a derivative coupling to the lepton current this model also generates a lepton asymmetry spontaneously [22] during inflation. We show that with the parameters of the inflation model which give the correct amplitude and spectral index of CMBR also give the required lepton asymmetry of $\eta_L \sim 10^{-10}$ which can be converted to a baryon asymmetry of the same order by sphaleron processes in the electro-weak era [25].
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