A Review of Computational Aeroelasticity Of Civil Fan Blades

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ABSTRACT
This paper presents a review of aeroelasticity research concerning fan blades in modern civil aircraft engines. It summarises the research carried out at the Rolls-Royce Vibration University Technology Centre (VUTC) at Imperial College over the past 25 years. The purpose of this paper is to gather information on all the aeroelastic issues observed for civil aero-engine fan blades into one document and provide a useful synopsis for other researchers in the field. The results presented here are based on numerical methods but wherever possible data from experiments are used to verify the numerical findings. For cases where such datasets do not exist fundamental principles, engine observations and engineering judgement are used to support the numerical results. Numerical methods offer a cheaper alternative to rig tests, especially in cases of blade failure, and can also provide more information about the nature of instabilities, which can be useful in the design of future civil aircraft engines. In fact, in cases such as crosswind testing that use smaller rig-scale blades, such results can even be more representative of real engine flows.

KEYWORDS
Unsteady aerodynamics, aeroelasticity, flutter, bird strike, inlet distortion, forced response, crosswind, manufacturing tolerances

NOMENCLATURE
\( m_{\text{ref}} \) non-dimensional mass flow function
\( U^* \) non-dimensional crosswind speed
\( V_\alpha \) axial flow velocity
\( \alpha \) blade mode twist to plunge ratio
OGV outlet guide vanes
ESS engine section stator
ND nodal diameter

INTRODUCTION
There is fierce competition amongst aero-engine manufacturers to supply next generation aircraft with engines that are lighter, quieter and more efficient than the ones used today. One way of achieving these objectives is to use:

- Large diameter fans with higher bypass ratios (hence increasing propulsive efficiency)
- Shorter intakes reducing size, drag and weight of engines

Fan blades in modern high-bypass aero-engines typically produce around 80% of the thrust and the increase in their diameters will increase the influence of this component of the aero-engine even further. To reduce weight, future fan blades will be made of composite materials. The increased span of the blades and the use of light-weight composite materials makes the blades prone to air-induced vibration [1, 2]. Moreover, such fan blades are highly loaded and tend to operate on the part of the constant speed characteristic where the gradient is horizontal (and changes in the mass flow rate do not correspond to changes in pressure ratio), meaning that they are more prone to aerodynamic and aeroelastic instabilities than conventional ones [3, 4]. As a result of shorter intakes, the fan and the intake are more closely coupled and hence the effects of inlet distortions, such as crosswind, also become more important for fan stability [5]. Since aeroelastic engine/rig tests are very expensive (especially in case of blade failure) and time consuming, computational simulations are increasingly used during engine development. Computational modelling also offers advantages for studying the causes and mechanisms of aeroelastic instabilities because they allow independent variation of several parameters, which would often have to be fixed in an experimental investigation. Moreover, in cases such as crosswind testing, small scale rig blades are deployed in large tunnels which results in unrepresentative Reynold numbers. It is believed by the authors that, in such cases, numerical modelling will be more representative of real engine flows.

In this paper, three forms of aeroelastic vibration that can affect fan blades are studied. These vibrations are part-speed stall flutter, aeroelastic issues due to inlet distortions and aeroelastic vibrations as a result of bird strike. In each case, the physical phenomena which cause the vibration are
presented and the possible numerical modelling approaches are discussed.

TEST CASE AND NUMERICAL MODEL
The fan assemblies under investigation are typical wide-chord, modern designs for large-diameter aero-engines. The hub-tip ratio is around 0.3 for all cases and the tip Mach number varies between 1.0 and 1.3.

Flow solver
The CFD code used for this work is AU3D [6], which is a three-dimensional, time-accurate, viscous, compressible URANS solver, based on a cell-vertex finite volume methodology and mixed-element unstructured grid. The current computations use the one-equation Spalart-Allmaras (SA) turbulence model [7]. It is well known that the standard SA model over-predicts the size of separation zones, which leads to blockage of passages, a considerable total pressure loss and premature stall [8-9]. The situation becomes worse for blades which have flat characteristics such as a low-speed fan. In order to suppress unnecessarily large separation zones, the production term is modified based on the pressure gradient and velocity helicity [10-11]. The parameters in the modified SA model are held constant in all the present work. More details on the methodology and validation can be found in Ref. [12]. The resulting CFD code has been used over the past 20 years for flows at off design conditions, with a good degree of success [12-15].

Computational domain and boundary conditions
The domain used for the unsteady aeroelastic computations includes the complete fan assembly with OGV, ESS, a symmetric intake upstream of the fan, and an external volume which contains the rig, as shown in Figs. 1(a) and 1(b). The free stream boundary of the computational domain was placed 13 intake lengths away from the intake (see Fig. 1(a)), which is (numerically) far enough for the flow to remain undisturbed by the fan. The grid used for the intake is unstructured (Fig. 1(c)) and was generated using the commercial package GAMBIT. A no-slip condition is imposed on the intake casing and spinner walls. The grids used for the blading are semi-structured, with hexahedral elements in the boundary layer region around the aerofoil, and prismatic elements in the passage. The radial grid, containing 49 mesh layers, is refined toward the hub and casing to resolve the end-wall boundary layers. The fan is modelled with a typical rig blade tip clearance and is modelled with 6 radial levels in the tip gap. The total number of grid points used in this study contains about 1.7×10^7 nodes. It has been found that the resulting grid is optimum (in size and accuracy) for AU3D (the CFD code used in these studies).

Flutter
It should be noted that the work presented in this section is a summary of previous work by the authors [14-23] and for further details the reader is referred to these papers. The aeroelastic instability leading to what is commonly called stall flutter for a fan blade is described here. Although called stall flutter, this phenomenon does not require the stalling of the fan blade and can occur when the slope of the pressure rise characteristic is still negative. This type of flutter typically occurs at part-speed operating conditions, in low nodal diameter, forward traveling waves and in the first flap (1F) mode of blade vibration. Discussion in this paper is restricted to the flutter of fans used on modern aircraft engines, so does not consider geometric features such as part-span shrouds. Flutter can occur when the pressure ratio of the fan is increased beyond its normal operating range, by reducing the mass flow through the fan. In such cases, the onset of flutter, rather than stall and surge, forms the limiting operating condition. For specific and narrow speed ranges, flutter is seen to remove a ‘bite’ from the map of stable operating conditions of the blade - hence the term ‘flutter bite’ [21]. This phenomenon is clearly illustrated in Figure 2. In this plot, pressure ratio (as defined by the ratio of total pressure at trailing edge to that at leading edge) is plotted against inlet mass flow function. Also shown in this figure are lines of constant fan rotation speed (characteristics) and the design working line. It is seen from this plot that the flutter bite can remove a significant part of the stable operating region and moreover, it can bring the stability boundary very close to the working line. Therefore, two situations that should be avoided are:
1) The working line cutting through the flutter bite, as shown by the dashed red line in Fig. 2 (labelled ‘High Working line’).

2) The design speed characteristic cutting through the flutter bite. In the plot of Fig. 2 there is a gap between the flutter bite and the design speed characteristic.

In the remainder of this section, the physical phenomena that contribute to the flutter bite are discussed.

**Fig 2: Demonstration of flutter bite**

In [14], CFD was used to explore the stable and unstable behaviour of a high-speed fan. The fan that was modelled had been operated in a rig and is representative of the type of fan used on recent, large aero-engines. The numerical simulations were used rather like an experimental technique, varying input parameters to give insight into the effects controlling the onset of flutter. Initially, the calculations presented in this paper assume no intake (or even domain boundary) reflections, achieved using ‘infinitely long’ intake and exhaust ducts. This approach will isolate the fan and can be used to determine the flow and structural features which cause flutter. The effects of intake reflections are considered subsequently.

Before proceeding with a description of flutter, some basic wave propagation terms will be introduced. Blade vibration creates unsteady flow perturbations which can be grouped into three types of waves: entropic, vortical, and acoustic [24]. Entropic and vortical waves can only convect with the flow, whereas the acoustic waves may propagate in the upstream as well as downstream directions. An acoustic wave is termed ‘cut-on’ if the wave can propagate axially in the duct without attenuation, whereas an acoustic wave is termed ‘cut-off’ if it decays exponentially from the source. For more information regarding turbomachinery noise and acoustic propagation in ducts the reader is referred to the seminal paper by Tyler and Soffrin [25].

**Fig 3: Constant speed characteristic at flutter speed (left), and damping as a function of mass flow at flutter speed (right)**

**Fig. 3** shows the computed part-speed characteristic and aerodynamic damping plotted as a function of mass flow. The mass flow and pressure ratio are non-dimensionalised by their values at the design condition. It is seen from this figure that the aerodynamic damping becomes negative, i.e. flutter occurs, as the pressure ratio of the fan is increased (mass flow is decreased) beyond its normal operating range. Also, **Fig. 3** shows that, for this test case, the onset of negative damping occurs at mass flow $m_{ref} = 0.95$. As there is no intake in these computations, this type of flutter is referred to as ‘blade only’. In reference [14] the main aerodynamic, acoustic and mechanical features that result in ‘blade only’ flutter were identified. They were:

1) **Aerodynamic effects; characterised by flow on the suction surface of the blade.** It was shown in [14,19] that the part span separation on the suction surface of the blade and the consequent radial migration of flow is one of the main drivers for flutter instability for this blade (as demonstrated in **Fig. 4**). In this plot, the Mach number is shown on the suction surface of the blade when operating at a constant speed, both for a stable mass flow ($m_{ref} = 0.97$) and an unstable mass flow ($m_{ref} = 0.93$). Surface streamlines are also superimposed on these plots. In this plot the flow direction is from right to left, as indicated by $V_r$. For the stable case ($m_{ref} = 0.97$) the flow is smooth and follows the blade profile whereas for the unstable case ($m_{ref} = 0.93$) there is a part span separation at 65% span which causes radial migration of the flow. Also shown on this plot (**Fig. 4c**) are radial profiles of damping. It is clear from this plot that for the stable case ($m_{ref} = 0.97$) the aerodynamic damping is positive (stable) for all the radial sections. For the unstable case ($m_{ref} = 0.93$), most of the negative damping (unstable) region is outboard of 80% blade span which is outboard of the separation region (as shown by the arrow). It was shown in [26] that, the instability outboard of 80% blade span is caused by the vibration and the flow at 65% height. The above finding indicates that 2D models (at representative heights) will not be appropriate for stall flutter computations as this phenomenon is driven by 3D flow features.

2) **Aeroacoustics effects; characterised by the nature of the acoustic wave generated by vibration.** It was shown in [14] that, as a pre-condition for flutter, the acoustic disturbances produced by the blade vibration must be ‘cut-on’ (i.e. propagating) upstream and ‘cut-off’ (i.e. the acoustic wave is evanescent) downstream of the blade. This is demonstrated in the contour plot in **Fig. 5** which depicts...
the instantaneous unsteady pressure at 90% blade height, at the flutter condition. Therefore, the blade can only flutter in a certain frequency range at each fan speed. In Fig. 5 aero-
damping against frequency for the 1F/2ND mode at \( m_{\text{ref}} = 0.93 \) is also plotted. It clearly shows that aerodynamic damping is only negative when the reduced frequency is between 0.065 and 0.105.

In the computations shown above, AU3D, which is a non-
linear time domain model, was used to obtain the aero-
damping of the blade. However, the authors believe (also shown in [17]) that a linear frequency domain analysis would produce similar results and will reduce computational time significantly.

Fig 5: Aero-damping as a function of frequency (left) and instantaneous variation of unsteady pressure at 90% blade height (right) at the flutter condition

3) Mechanical effects; characterised by the amount of twist in the 1F mode. Fig. 6 shows the mechanical mode shape for blade vibration, represented by contours of magnitude of displacement. Fig. 6(a) shows the total motion. The displacement normal to the chord line near the tip, which is referred to here as plunging motion, is shown in Fig. 6(b). The plunging component is obtained by averaging the displacement along each radial section. The twisting motion about an axis near the middle of the blade, obtained by subtracting the plunge of Fig. 6(b) from the total displacement of Fig. 6(a), is shown in Fig. 6(c). As the figure shows, the 1F mode shape can be decomposed into a pure plunging and a pure twist motion. The twist to plunge ratio parameter \( \alpha \) is defined as:

\[
\alpha = \frac{\alpha_T c_{0.5}}{\alpha_p}
\]

where \( \alpha_T \) is the twist angle amplitude (in radians), \( c_{0.5} \) is the semi-chord of the blade at the tip and \( \alpha_p \) is the plunging amplitude at the tip of the blade. Both \( \alpha_T \) and \( \alpha_p \) can be computed from ratio of LE to TE displacements [14]. Fig. 7 shows the variation of 1F/2ND damping at various mass flows as a function of \( \alpha \). The datum value of \( \alpha \) for this blade is 0.3. The numbers next to each curve denote the mass flow at that operating condition and corresponds to the operating points depicted in Fig. 3. It is seen that as \( \alpha \) increases the blade becomes more unstable (negative aero-damping increases). Therefore, it can be concluded that the design of flutter free blades would require blades with low \( \alpha \) values. It is also seen from this plot that the importance of \( \alpha \) decreases as the flow coefficient increases. In fact, for a flow coefficient of 1.0 (near the working line), the aero-
damping of the blade is almost independent of value of \( \alpha \). Moreover, it is seen from this plot that as the value of \( \alpha \) decreases (to nearly zero), the aero damping curves for different mass flows converge to a point, indicating that for small values of \( \alpha \) the aero-damping becomes independent of the flow. The above conclusions indicate that flow/mode shape interaction is highly non-linear and for flutter one requires both elements.

Intake Effects. It was shown in previous work [19-21] that acoustic reflections from the intake play an important part in fan flutter and should therefore be considered during the design of new engines. Fig. 8 illustrates this interaction mechanism, showing that outgoing acoustic waves are reflected at the intake highlight. Fig. 9 shows the aero-
damping plotted against fan speed (tip Mach number). Also shown in this plot is the aero-damping for the case without an intake. The behaviour of the case without intake (shown in black) is only dependent on the flow on the blade and mechanical properties of the blade, as described in the preceding section. It can be seen from this plot that the reflections from intake can play an important role in flutter, changing the flutter stability of the blade significantly. This contribution to flutter is referred to as ‘acoustic flutter’ in this paper.

At the flutter condition, the intake length and mean Mach number determine the propagation time and thus the phasing of the reflected acoustic wave, which can be beneficial or detrimental to the overall stability of the fan system. It was shown in [20] that the most destabilizing case occurs when the upstream wave lags the reflected wave by 90° and the
The most beneficial condition occurs when the upstream wave leads by $90^\circ$. In [21] a simple model that can be used to study the effects of the intake on flutter was introduced. The results of the simple model were compared against those of AU3D and show a good agreement.

It can be inferred from the above that it would be possible to increase the flutter margin of the blade by introducing acoustic liners in the intake. Acoustic liners are frequently used in the intake to reduce the level of noise emitted from the inlet of turbofan-engines, especially during landing and take-off [28, 29]. The properties of such liners (such as depth and resistance) are usually optimised for fan noise, which have much higher frequencies than flutter tones. However, it is also known that acoustic liners with an appropriate depth can stabilise flutter by absorbing the flutter wave energy [15]. Unfortunately, the relatively low frequency of these disturbances means a simple acoustic liner design would have to be sufficiently deep to attenuate the pressure waves and ultimately have any impact on fan stability [15, 27]. Such a liner would be impossible to use in a typical civil aero-engine intake. It may be possible to use more advanced designs, such as folded-cavity liners [30].

It was shown in [20] that the contribution to aero-damping due to the blade motion (for the isolated rotor in an infinitely long duct) and due to intake reflection seem to be independent and so can be analysed separately. Therefore, the design of a new intake can be assessed on its own, independent of the fan design. Moreover, worse flutter instability occurs when the ‘blade only’ flutter is at the same speed as 'acoustic flutter' due to the intake. It is therefore possible to gain a significant improvement in flutter margin by designing a fan and intake system that separates the speed at which these two phenomena occur.

4) Effects of manufacturing tolerances on flutter stability

In this section of the paper the effects of mistuning (variation of blade frequency around the annulus) and mis-stagger (variation of blade stagger around the annulus) are discussed.

Mistuning Effects. The frequency mistuned patterns used in this study are shown in Fig 10. The patterns have been obtained using a random number generator. The mistuning amplitude for blade $i$ is here measured as deviation of the blade frequency ($F_i$) from the tuned system frequency ($\Delta F_i = F_i - F_{tuned}$) and normalized by the maximum deviation: $\Delta F_i^* = \Delta F_i/|\Delta F_{i,\text{max}}|$. In the following, the maximum deviation $|\Delta F_{i,\text{max}}|$ is referred to as the system mistuning level. The mode shape of the blades is assumed to remain the same.

![Fig 10: Fan blade mistuning pattern](image)

Fig. 10 shows the computed flutter boundary for different levels of mistuning. It is seen from this figure that by increasing the mistuning level the blade flutter stability improves, and for a sufficient level of mistuning the flutter boundary moves outside the stall boundary.

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![Fig 11: Computed flutter boundary for different levels of mistuning](image)

It has been shown in [31-33] that mistuning increases the flutter stability of the blade by distributing the energy from the unstable circumferential mode over many circumferential modes, most of which dissipate energy due to positive aero damping. This is clearly demonstrated in Fig. 12. The computations start by exciting the assembly in a 2ND pattern only (see the red curve on Fig 12). The computations are performed at $m_{ref} = 0.93$ (a flutter condition presented Fig. 3) for which the blade is unstable.
in a 2ND mode. For a tuned case, the initial pattern of the excitation would remain unchanged (as the tuned assembly structural modes are circumferential and orthogonal) however its amplitude would increase at this unstable condition. It is seen from Fig. 12 that this is not the situation for a mistuned case. After only 15 cycles of vibration, the 2ND pattern has scattered into many circumferential modes and, from the spectra plot in Fig. 12, it is seen that 1ND, 3ND and 4ND are also present in the assembly response. After, 30 cycles, the 3ND shows the biggest response levels, and after 60 cycles, the circumferential modes from 1 to 7 are present in the response. This plot clearly shows that, although the system is excited in such a way that the initial energy is only in a 2ND mode, the introduction of mistuning distributes this energy over many circumferential modes. This is because, for the mistuned system, the assembly modes are no longer simply the set of circumferential modes; the assembly modes are still orthogonal but more complicated in structure and, in general, no longer rotationally symmetric. However, one could still circumferentially deconstruct the new modal response to the original 2ND excitation. Since most of these circumferential components are positively damped, the overall response is to dissipate the excitation, resulting in a stable system. This observation is similar to the one found in [31] for compressor blades.

**Mis-Staggering Effects.** There are further parameters which influence the flutter stability of fan blades and which would vary due to changes in environmental conditions or manufacturing tolerances. One of these parameters is aerodynamic mistuning, i.e., a deviation of individual blades from the (tuned) design intent, which affects the blade aerodynamics. This section studies the effects of aerodynamic mistuning, in the form of mis-staggering, on the flutter stability of a fan blade. The term mis-staggering implies that each blade around the annulus has a different stagger. Mis-staggering in an aero-engine exists on all blade rows and is a result of manufacturing tolerances. The computations were performed with a “random” mis-staggering pattern as shown in Fig. 13. In this plot, the variations of tip stagger about the mean stagger are plotted against the blade number and a negative value denotes a blade that is more closed compared to the mean. In the following, different mis-stagger conditions are compared. To achieve this, the stagger pattern itself is kept constant but the mis-stagger angles of all the blades are scaled. Each level is identified by its maximum mis-stagger angle, referred to here as mis-stagger amplitude. All values are normalized by the maximum mis-stagger amplitude tested in this study.

The variation of aero-damping with mis-stagger angle is shown in Fig. 14 for operating point $m_{ref} = 0.93$ (see Fig. 3). It is seen from this plot that, unlike mistuning, mis-staggering of the blade can have both beneficial and deteriorating effects on flutter stability of the blade [22]. The results from the baseline flutter analyses presented at the beginning of this paper clearly showed that for this blade, the onset of flutter is related to a three-dimensional separation and radial migration of flow on the suction surface of the blade. The result in Ref. [32] showed that the introduction of mistuning distributes the energy of the initial disturbance over many circumferential modes, and as most of the other circumferential modes are stable, they dissipate this energy resulting in an increase of aero-damping. The main difference between mistuning and mis-staggering is the fact that, for the mistuning the mean flow remains unchanged and the same for all the blades, whereas, for the mis-staggering the overall mean flow changes but also the flow will differ for individual blades. Fig. 15 shows the variation of Mach number with superimposed streamlines on the suction surface for $m_{ref} = 0.93$ with mis-staggering of 0.2. Comparing this figure with Fig. 4 and considering the stagger pattern of Fig. 12, it is seen that the introduction of mis-stagger increases the radial migration for blades that are opened (blades 1, 2, 19), and decreases it for the blades that are closed (blades 3, 5, 10). Moreover, as a result of the increase of the stagger angle the flow pattern on some blades resembles that of the tuned assembly at a lower mass flow and it also follows that, due to a decrease in stagger angle, some blades resemble those of the tuned assembly at higher mass flow. The damping plot of Fig. 14 suggests that, at low levels of mis-stagger, the dominant behaviour of the assembly in terms of flutter is similar to that of the tuned assembly but with a shifted mean flow condition and that, consequently (based on Fig. 14), the aero-damping may decrease. However at large levels of mis-stagger the asymmetry due to mis-stagger becomes the dominant factor.
AEROELASTICITY UNDER CROSSWIND

As previously mentioned, the next-generation of turbofan engine designs are moving towards fans with larger diameters. As the fan (and hence intake) diameter increases, shorter intakes are required to reduce the overall weight and drag of the aircraft [21–22]. A result of shorter intakes is that the fan and the intake will become closely coupled and hence the effects of inlet distortions, such as crosswind, will become more important for fan stability [23]. In this section the effects of crosswind on forced response, flutter and stall driven vibrations will be discussed. In the presented analyses it is assumed that the aircraft is at sea-level-static (SLS) conditions and the wind is blowing at 90° angle to the engine. The same principles also apply to aircraft climb where the engine experiences flow angles of attack.

To investigate this further, the nodal diameter content of the unsteady forcing around the annulus is analysed. Fig. 16 shows the forces obtained from a full annulus computation for $m_{\text{ref}} = 0.93$ with a mis-stagger amplitude of 0.2 and for $m_{\text{ref}} = 0.93$ with 0.6 mis-stagger amplitude. The amplitude of unsteady forcing is normalized with the amplitude of forcing obtained from the tuned assembly. For the case with zero mis-stagger (i.e. a tuned system), only the second nodal diameter (2ND) would exist in the spectrum. It is seen that, for a 0.2 mis-stagger amplitude, most of the energy remains within 2ND but for 0.6 this energy has been distributed over all nodal diameters, and hence has a similar outcome on aero-damping as mistuning. The mis-stagger study shows that the relationship between mis-stagger level and aero-damping is complex. There is a trade-off between the aerodynamic mistuning effect, which is stabilizing, and the effect of the change in the flow on individual blades which, depending on the blade stagger angle, could be stabilizing or destabilizing. For large values of mis-stagger the aerodynamic mistuning is the dominant factor and hence blade stability improves in such cases.

Forced response

The problem of the vibration of bladed-disks due to forced response is commonly investigated during the development phase of new aero-engines. A primary mechanism of blade failure is high-cycle fatigue (HCF) which is caused by vibrations at levels that exceed material endurance limits. In the context of the forced response of a civil aero-engine fan, downstream obstacles such as OGVs give rise to blade passing frequency (BPF) forced response, and flow distortions due to non-axisymmetric intake geometries, pylons, environmental conditions and angle of attack (during climb) give rise to low-engine order (LEO) forced response. In both cases, the actual vibration levels depend on three quantities: unsteady aerodynamic pressure, correlation of unsteady pressure with the mode shape and total damping (aero + mechanical) for the mode of interest. For previous research by the authors on distortion driven force response the reader is referred to [34,35].

Crosswind can lead to boundary layer separation at the inlet of the engine resulting in a non-homogeneous flow field upstream of the fan. Fig. 17 is a schematic of flow past an intake during crosswind, with a streamline visualized near the intake lip. For high levels of crosswind, the flow can separate on the intake lip and the level of distortion at the fan face is determined by the size of the separation. Fig. 18 shows the variation of total pressure contours at the fan-face for different magnitudes of crosswind. It is seen from this plot that as the amplitude of crosswind increases the size of
the distorted region increases. As a result of this distortion, a fan blade would experience different upstream conditions as it rotates around the circumference and hence flow on the blade becomes dependent on its circumferential position. The corrected mass flow and total pressure ratio of a fan blade as it rotates around the circumference is shown by the dashed line in Fig. 19. Also shown in this figure are the steady (undistorted) constant speed characteristics. It can be seen from Fig. 19 that the distortion:

a) moves the blade to a higher/lower speed line, as in part A and C
b) moves the blade along the constant speed characteristic, as in part B and D.

The change in corrected mass flow is due to the presence of the distortion, which reduces the axial velocity in the distorted sector of the azimuth and can be explained with classical parallel compressor theory [36].

The change in operating condition of the fan (Fig. 19) as it rotates around the annulus creates high amplitude of unsteady forcing on the blade. The forcing on the blade has many harmonics which correspond to multiples of the shaft speed frequency (also known as Engine Order). The amplitude of unsteady forcing for mode 1F/1EO and mode 1F/3EO are shown in Fig. 20a. The amplitude of crosswind is measured here by the crosswind velocity ratio $U^* = U_i / U_\infty$ where $U_i$ is the intake speed and $U_\infty$ is the crosswind speed. The unsteady forcing for all the harmonics increases as the amplitude of crosswind increases. Fig. 20b shows the computed response level of the fan for mode 1F/3EO under the maximum crosswind condition shown in Fig. 20a. Both mechanical and aero-damping are included for computing the response levels in this figure. The response levels for both bladed disks and blisks are shown in this plot. It should be noted that the use of blisks is becoming more common in aero-engine designs. Therefore, it is very important to predict aero-damping accurately as air becomes the only source of damping for such structures. Aero-damping may be computed using techniques such as those presented in the initial sections of this paper concerning flutter.

The crossing of EO forcing and blade natural frequency can be determined from the well-known Campbell diagram, as shown in Fig. 21. In the case of aero-engine fan blades, which operate at very diverse conditions, it will not be possible to avoid all crossings of EO forcing and blade natural frequencies in the running range. It should be noted that at cases with high levels of crosswind (as demonstrated in Fig. 19) the amplitude of unsteady forcing can become extremely large which may result in significant off-response levels.

Fig. 17 Schematic diagram of flow around an intake during crosswind

Fig. 18 Total pressure variations at fan-face

Fig. 19: Operating point for a single fan blade at different positions around the azimuth, in distorted flow (NASA rotor 67)

Fig 20: (a) Unsteady forcing on the blade and (b) resulting response 1F/3EO during fan speed acceleration

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1 Blisks are a turbomachine component comprising of both a rotor disk and blades in a single structure, crucially with a very low mechanical damping.
The blade-to-blade differences, which exist due to manufacturing tolerances, may cause a non-uniform vibration response among the blades [37]. The response of the worst blade may be several times higher than that of the best blade. The standard practice is to base all predictions on a tuned assembly and to use a scaling factor for estimating the mistuning effects [37].

**Stall driven vibration**

Fig. 23 shows the computed characteristic for a fan at two operating speeds and for zero crosswind speed (referred to as 0XW) and a significant value of crosswind speed (here referred to as 3XW). These crosswind conditions correspond to values shown in Fig 20a. The values of $U^*$ at each operating condition are also shown in Fig. 23. Although, the same magnitude and direction of crosswind is used for all the crosswind computations shown in Fig. 23, the value of $U^*$ changes according to the mass flow into the intake (i.e. fan operating point). The pressure ratio shown here is calculated from fan inlet to fan exit and does not include the intake losses. Fig. 23 shows that as the fan speed decreases the loss in stall margin of the blade increases, which is partially due to a decrease in the value of $U^*$ at the lower fan speed. At the lower fan speed B and under 3XW there is a significant loss in stall margin, and the slope of the characteristic becomes positive at a much higher flow coefficient than the clean flow case ($m_{ref} = 1.0$ instead of $m_{ref} = 0.85$).

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**Flutter**

In this section the effects of crosswind on the flutter stability of a blade is discussed. Fig. 22 shows the change in flutter margin of the blade as a function of crosswind speed. It is seen from this figure that relationship between crosswind speed and aero-damping is complex; for low values of crosswind the flutter margin of the blade decreases but as the crosswind speed increases the flutter stability of the blade improves. The relationship here is similar to mis-staggering effects on flutter, shown earlier in the paper. The presence of crosswind breaks the symmetric pattern of the flow on the fan and as the wind speed increases the magnitude of asymmetry increases. Small values of crosswind result in an increase in incidence to the blade (due to the distortion pattern that arises) which moves the fan towards stall and reduces aero-damping. However, small values of crosswind cannot create sufficient asymmetry in the flow and hence the level of aerodynamic mistuning remains low. Consequently, the blade loses flutter margin. However, as the crosswind increases the level of aerodynamic mistuning increases and the blade becomes stable, from the perspective of flutter. Therefore, there is a trade-off between the aerodynamic mistuning, which is stabilizing, and the change in the flow on individual blades which is destabilizing.

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**Fig. 21: A typical Campbell diagram**

**Fig. 22: Change in flutter margin of a blade as a function of crosswind speed**

**Fig. 23: Comparison of characteristic map between 0XW and 3XW at two rotor speeds**

It was shown in [25] that pre-stall cells start to appear when the slope of constant speed characteristic approaches zero. The results from both measured and CFD pressure transducers mounted on the casing of the fan revealed the appearance of non-synchronous frequencies in the pressure time histories as the flow coefficient decreased and the slope of constant speed characteristic approached zero. Furthermore, it was observed that the amplitude of the non-synchronous signals increased as the flow coefficient decreased. Fig. 24 shows the formation of stall under two different inflow conditions. Fig. 24(a) and 24(b) show the stalling process for 0XW and 3XW respectively. In these plots, the circumferential profiles of axial velocity downstream of the fan (in a stationary frame of reference) at 99% radial height together with instantaneous variation of axial velocity at mid-chord (in a rotating frame of reference) are used to track the stall process.
Bird strike during both take-off and landing, an incident which can have extremely serious consequences. Bird strike is a major consideration during the design of fan blades for such large-diameter aero-engines. Current methods rely on impact tests and structural optimisation, but it is highly desirable to have predictive numerical models to assess the aerodynamic and aeroelastic stability of bird-damaged fan assemblies. Experimental evidence suggests that bird damage may reduce the existing flutter margins and/or give rise to rotating stall cells, depending on the radial position of the damage, the number of blades that are affected and the actual operating condition.

The combined aerodynamic and structural modelling of a bird-damaged fan assembly is fraught with many difficulties. The flow representation must be time-accurate since the flow becomes locally unsteady in some passages. The turbulence model must be able to cope with flow separation and re-attachment. The unsteadiness due to the blade vibration must be included but the structural analysis is not straightforward because of gross mistuning. Clearly then the computational requirement for such a calculation is very considerable since a time-accurate, nonlinear viscous analysis must be undertaken for a whole assembly because of the loss of symmetry in the flow.

The aim of this section is to present such a methodology and to study a representative case. More details about past research on this type instability can be found in [38–41]. The fan assembly under investigation contained two consecutive blades with unequal impact damage, the so-called heavy-damage (HD) and medium-damage (MD) blades. Further details are given in [39]. As shown in Fig. 25, the MD blade is leading the HD blade, the ordering being the reverse of that studied in [39]. Although a bird strike is likely to occur during take-off, the aeroelastic analysis will be conducted at 70% engine speed. This replicates the situation where the pilot would reduce engine power in order to minimise vibration due to the rotating imbalance, while still retaining enough forward thrust for flight.

**Fig. 24:** Comparison of transient and instantaneous solutions for (a) 0XW and (b) 3XW conditions.

The plots show that, in both cases, initially small disturbances, which occupy only the tip region of the blade, are present. As the downstream throttle is closed and the flow coefficient decreases (to 0.75 at 0XW and 0.85 for 3XW) the disturbances expand and begin to merge but remain irregular. When the throttle is closed further and the flow coefficient decreases (to 0.70 for 0XW and 0.8 for 3XW), the irregular disturbances in the flow field begin to merge and eventually a regular pattern is formed. The plots in Fig. 24 clearly show that for the case without crosswind a 2 stall cells pattern is formed whereas for the case with crosswind there are 3 stall cells present. The change in number of stall cells can be quite important on vibration characteristics of the fan blade. If the frequency of unsteady flow becomes close to one of the blades natural frequencies, the interaction between the unsteady pressure and blade vibration mode can result in a ‘lock-in’ which can cause large amplitude vibration levels. Although this type instability can occur for cases without crosswind, the appearance of crosswind moves the occurrence of this phenomena to a higher mass flow and nearer to the design working line of the fan (see Fig. 23).

**BIRD STRIKE**

Large-diameter turbofan engines are particularly susceptible to bird strike during both take-off and landing, an incident which can have extremely serious consequences. Bird strike is a major consideration during the design of fan assemblies, as they are not expected to have a significant effect on the fan's performance. The combined aerodynamic and structural modelling of a bird-damaged fan assembly is fraught with many difficulties. The flow representation must be time-accurate since the flow becomes locally unsteady in some passages. The turbulence model must be able to cope with flow separation and re-attachment. The unsteadiness due to the blade vibration must be included but the structural analysis is not straightforward because of gross mistuning. Clearly then the computational requirement for such a calculation is very considerable since a time-accurate, nonlinear viscous analysis must be undertaken for a whole assembly because of the loss of symmetry in the flow.

The aim of this section is to present such a methodology and to study a representative case. More details about past research on this type instability can be found in [38–41]. The fan assembly under investigation contained two consecutive blades with unequal impact damage, the so-called heavy-damage (HD) and medium-damage (MD) blades. Further details are given in [39]. As shown in Fig. 25, the MD blade is leading the HD blade, the ordering being the reverse of that studied in [39]. Although a bird strike is likely to occur during take-off, the aeroelastic analysis will be conducted at 70% engine speed. This replicates the situation where the pilot would reduce engine power in order to minimise vibration due to the rotating imbalance, while still retaining enough forward thrust for flight.
on the nature of the outcome. On the other hand, disk and shaft flexibilities were included in the model as it was thought that such features were needed to capture the dynamics of the grossly mistuned assembly. This approach is in contrast with single sector analyses which are conducted for a single cantilevered blade. Even undamaged full-assembly analyses use a single cantilevered blade mode shape, albeit in expanded form. A view of the typical computed mode shapes for the test case is shown in Fig. 26. A close inspection of the 1T family revealed the existence of three type of modes:

1) exhibiting a nodal diameter pattern similar to an undamaged assembly (Fig. 26a).

2) exhibiting a non-nodal diameter pattern (Fig. 26b)

3) Only the damaged blades vibrate, the other blades remaining stationary (Fig. 26c).

Fig 26: The various types of mode shape in damaged assemblies

Due to the complicated nature of flow in the damaged passage, it is not always possible to obtain a genuinely steady solution for the damaged assembly. Depending on the operating point of the fan, the flow can be either locally unsteady or induce rotating stall, as is now demonstrated. The computed fan characteristic at part-speed for an undamaged assembly and the above damaged assembly is compared in Fig. 27. It is seen that as a result of the damaged blades there is a considerable loss in performance of the fan. Moreover, it can be seen that the damaged assembly characteristic can be viewed as an undamaged characteristic at a lower speed and the damaged and undamaged characteristics diverge at higher working lines. Fig. 28 shows the instantaneous Mach number profiles and reversed flow region at 90% height for operating point C1 on Fig. 27. Due to the relatively low exit pressure at point C1, the flow separation is confined to the damaged passages and the global flow is assumed to be steady. The global steady flow refers to the fact that the overall mass flow and pressure ratio converge, however, as will be shown later the flow in the passage between HD and MD remains unsteady. Some relevant flow features can be identified from the Mach number plots of Fig. 28a and negative axial velocity contours of Fig. 28b, these being plotted at the radial section of maximum damage. An inspection of Fig. 28a reveals two regions of flow separation. A very small one is situated at the leading edge of the MD blade while a much larger one blocks most of the passage for the HD blade. The blockage causes the flow arriving at the blade trailing the HD blade to have a large incidence angle. This causes a strong Prandtl–Meyer type expansion at the leading edge of that blade as the flow tries to turn a corner. The additional acceleration causes a larger and stronger shock on the trailing blade, thus decelerating the flow to such an extent that the shock on the blade trailing that one to be much weaker.

Fig 27: Comparison of fan performance between damaged and undamaged assemblies

Fig. 29 shows the negative axial velocity contours at 90% blade span during the time-accurate computations at Point C1. It is clearly seen from this plot the shape and the magnitude of stall region is a function of time. It is also apparent from this plot the unsteadiness is larger towards the trailing edge of the pressure surface of the trailing blade. Therefore, at lower working lines, the flow in the passage between the damaged blade and the trailing blade becomes unsteady, causing buffeting to occur on the trailing blade. The unsteady pressures produced by buffeting on the trailing blade correlate with the 1T mode shape very well as can be seen in Fig. 30a. It can be seen from this figure that the amplitude of the 1T forcing is much higher on the trailing blade than other blades and in general 1T forcing is higher than the 1F forcing on all the blade. Moreover, the buffeting has a range of frequencies, some of which are very close to the blade 1T mode as shown in Fig 30b. Hence, the interaction between the unsteady pressure and blade vibration mode can result in a ‘lock-in’ which can cause large amplitude vibration levels, and for mode shapes of the type shown in Fig. 26c it will cause local blade failure. In such cases, it is not the damaged blade which fails but the blade trailing the blade with maximum damage (although undamaged). This phenomenon has also been observed on engine tests.

Fig 28: Instantaneous Mach number profiles at 90% height at operating point C1 (left) and reversed flow region (right)
As shown in Fig. 21, the situation is different for Point C2. A stall cell is formed at the leading edge of the HD blade and it propagates in the direction opposite to assembly rotation relative to the blades. It is somewhat difficult to define the number of stall cells since these are seen to disappear, split into smaller cells or merge into larger cells by ‘catching up’ with other cells. In any case, the stall cells do not cover more than 70% of the annulus at any time. Such a flow instability will clearly cause significant blade vibration. A closer inspection reveals that, due to the stall cells, the blades undergo a torsional motion by pivoting about their leading edge. This observation is in broad agreement with the traditional rotating stall argument of Emmons et al. [42] which is based on critical incidence. Because of the complex behaviour, it is difficult to determine the speed of rotating stall. An approximate value may be obtained by considering the ‘advancing front’ of the rotating stall event, which shows they are moving away from the damaged blades at roughly 30% shaft speed.

CONCLUSIONS

The commercial aerospace industry has committed to halving aviation-related greenhouse gas emissions from 2005 to 2050 and progress towards this target can only be achieved by the advent of more fuel-efficient aircraft engines designs. A detailed synopsis of the scope of aeroelastic challenges in fan design have been outlined in this paper. The need to address such challenges is as critical as it has ever been, especially when one considers the ever-rising demand for air travel and the drive for efficiency improvements that are encroaching on the physical limits of fan blade stability. This paper has presented a review of research that has used computational methods for predicting various phenomena associated with civil aero-engine fan blade aeroelasticity. Specifically, key mechanisms leading to isolated and installed fan blade flutter stability have been addressed and the impact of inlet distortion on fan stability has been highlighted. Aeroelastic simulations for a fan geometry damaged by bird strike have also been briefly reported. The simulations require large amounts of computational power, but such simulations are still relatively very cheap when compared to the experimental testing of such blade failure events.

In summary:

- Current designs in aero-engines have reached their limit in efficiency and new designs are required
- Aeroelastic engine or rig tests are very expensive (specially in case of failure)
- The use of blisks (with low mechanical damping) is becoming common in aero-engine designs, making it important to accurately predict aero-damping
- The increase in computing power has enabled researchers and developers to use large scale CFD models for aeroelastic analysis
- It is envisaged that research in computational aeroelasticity will continue to grow and become more influential in aero-engine design and operation.
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