Collective Flow from the Intranuclear Cascade Model

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ABSTRACT

The phenomenon of collective flow in relativistic heavy ion collisions is studied using the hadronic cascade model ARC. Direct comparison is made to data gathered at the Bevalac, for Au+Au at $p = 1 - 2$ GeV/c. In contrast to the standard lore about the cascade model, collective flow is well described quantitatively without the need for explicit mean field terms to simulate the nuclear equation of state. Pion collective flow is in the opposite direction to nucleon flow as is that of anti-nucleons and other produced particles. Pion and nucleon flow are predicted at AGS energies also, where, in light of the higher baryon densities achieved, we speculate that equation of state effects may be observable.

Collective flow in relativistic heavy ion collisions has long been a subject of interest, since it was felt the phenomenon might carry information about the nuclear matter equation of state. In this letter, we will be concerned only with the so-called ‘Sideward Flow’, which is related to an event-by-event azimuthal asymmetry in the distribution of final state particles.

Theories which have been used to describe the heavy ion collisions fall into two broad classes: those based on macroscopic thermodynamical/hydrodynamical considerations, and those which attempt a more microscopic description of the ion–ion collision, for instance by carrying out successive collisions (cascading) of the elementary (hadronic) constituents. Among the microscopic models one can distinguish pure cascade models which include only the elementary 2- or in principle n-body collisions of the constituents. These models take as their main input the experimentally measured cross-sections (and angular distributions) for hadron-hadron $\sigma(hh \rightarrow X)$ in free space, and then carry out the ion–ion collision by Monte-Carlo methods. The model, ARC = A Relativistic Cascade, which we will use to discuss flow, is such a pure hadronic cascade model.

Additionally, there are microscopic models which include mean field, collective, or in-medium effects in some fashion, as well as treating the elementary hadron–hadron collisions. Both classes of microscopic model presumably simulate semi-classical (relativistic) kinetic...
theory of the hadron gas in some limit, and these models would then be equivalent to solving transport equations, either Boltzmann-Vlasov, or Boltzmann-Vlasov-Uehling-Uehlenbeck (BUU), depending on whether or not a mean field is included from the outset. In the BUU case, the mean field enters through the gradient of a potential energy, which may be calculated, e.g., from a phenomenological equation of state for nuclear matter.

Our treatment of flow using ARC will neglect mean fields entirely, and this is based on the assumption that the mean field $U$ satisfies $U \ll T$ where $T$ is the typical kinetic energy involved in a hadron–hadron collision within the cascade. For the initial nucleon–nucleon collisions in Au+Au at $p = 1.7$ GeV/c this is a reasonable assumption, but of course the kinetic energy available in successive collisions cascades down as the ion+ion collision takes place. So, the mean field may not be negligible in late or very soft collisions or for co-moving spectators in the projectile and target. However, a spectator cut is applied to the data and to ARC so as to reject particles having small kinetic energy in the projectile or target frames. We expect therefore that mean field contributions will not be dominant.

Indeed, we shall see that ARC, using un-modified free space cross-sections and no mean field is, at least by direct calculation, adequate to the task of describing sideward flow in Au+Au at Bevalac energies. Given the prior extensive success of ARC in predicting and describing inclusive data for heavy ion collisions at AGS energies, our strong theoretical prejudice would then be that mean fields and/or phenomenological equations of state need not be included over this range of energies (1-15 GeV/c). In-medium effects if and when they do arise ought then, in our point of view, to be included by modifying the elementary interactions. Nevertheless, the high baryon densities apparently achieved during massive ion collisions at AGS energies may still manifest themselves through traditional equation of state effects.

Our specific concern will be with the proton-like sideward flow measured at the Bevalac in Au+Au collisions at lab momentum $p = 0.96 - 1.9$ GeV/c. Such data have already been measured using the Plastic Ball spectrometer, and new experiments with better immunity to detector distortions were recently carried out using the EOS time projection chamber; flow results from the EOS collaboration are expected to be available later this year. We anticipate the EOS data by considering forward rapidity data only in our comparisons, as is reasonable given the downstream location of the EOS detector. By proton-like, we mean to say that protons contained in identified outgoing nuclear fragments are counted towards the flow, together with outgoing free protons. We consider proton-like flow, because ARC does not as yet dynamically include the production of nuclear fragments larger than single nucleons. Coalescence calculations for deuterons and tritons from ARC have been carried out at AGS energies, and are in good agreement with data. There is in principle no obstacle to carrying these calculations to lower energy.

We calculate sideward flow à la Danielewicz and Odyniec, by first defining a reaction plane for the ion+ion collision, neglecting pions, using the beam direction and a vector defined as a weighted average of outgoing transverse momenta:

$$Q = \sum_i w(y_i)p_{iT}^i,$$

with $w(y)$ selected as in Ref[12]. The essential point is that $w$ is an odd function of $y_{cm}$. The sideward flow curve is then defined by projecting proton-like momenta into the reaction
plane, and averaging:

\[ \langle P_x(y) \rangle = \frac{\sum_i (\mathbf{p}_i \cdot \mathbf{Q}_i)/|\mathbf{Q}_i|}{N(y)} \]  

(2)

Here \( N(y) \) is the number of protons detected in a bin of width \( dy \) around \( y \). Kinematic cuts are placed on the ARC calculation. These are:

1. **Spectator Cut:** \( p_{\text{proj}} > 0.25 \) MeV.

2. **Forward Rapidity Cut:** \( y_{\text{cm}} > 0 \)

The cuts are chosen with the yet to be published EOS data in mind. The target in the EOS detector is located upstream of the main tracking chamber, which optimises acceptance for \( y_{\text{cm}} > 0 \) but compromises it for \( y_{\text{cm}} < 0 \). This combination of cuts ensures that our flow calculation is made in a region where detector distortions should be minimal. The charge multiplicity \( M \) is then defined as the number of protons surviving the cuts.

ARC was designed, of course, for much higher energy heavy-ion collisions at AGS, so it was not certain from the outset how well lower energy Bevalac data would be described. However, we felt it was of interest to compare flow from the Plastic Ball and the forthcoming EOS data, especially the recent Au+Au runs at maximum Bevalac energy, with an unmodified version of ARC. We do not include comparisons of inclusive spectra, because such data have not been published for the Plastic Ball, and are not yet published in a journal in the case of EOS. We simply comment that the agreement with mid-rapidity spectra in Ref. seems very good considering that the present beam energy is a factor of ten below that for which ARC was designed. Differences between theoretical and experimental spectra can be expected to show up at forward rapidities due to a simplified treatment of fermi motion which does not completely respect conservation of energy. This is of course a negligible effect at the AGS energies.

Our flow comparison is based on the slope of the \( \langle P_x \rangle \) curve near mid-rapidity. This observable is minimally distorted by the Plastic Ball acceptance, as judged by simulations. The measured slope for Plastic Ball events with a multiplicity selection comparable to that placed on ARC (the upper 50\% of events in the multiplicity spectrum are accepted) is shown as a dotted line in Fig(1). The Plastic Ball slope is corrected for dispersion in the estimated reaction plane; for the calculation of ARC \( \langle P_x \rangle \) we use the ideal reaction plane.

It is commonly asserted that cascade models produce little or no collective flow. ARC, like any cascade model, generates \( \langle P_x \rangle \) through successive collisions, and the details of the treatment of these individual collisions may increase or decrease the amount of flow produced. Two salient features of ARC (as well as of other cascade models) with respect to flow are:

1. **ARC treats two body collisions in the center of momentum frame, where the impact parameter at closest approach and the initial momentum together span a plane: the two body reaction plane. For those collisions also having two bodies in the final state cascades often generate the azimuthal angle (relative to the reaction plane) of the outgoing momentum randomly.**

2. **After final state momenta are chosen, there still exist distinct possibilities for the virtual spatial path of the particles through the collision, corresponding semi-
classically to repulsive and attractive orbits. Cascade models typically allow either type of orbit, choosing at random between the two.

Point (1) is not consistent, classically, with the conservation of angular momentum, which requires initial and final momenta to lie in the same plane. As for point (2), it is well known that the nucleon–nucleon interaction is repulsive at high energies, where one expects to see only the hard core. Moreover, at the energy of first collisions here, (\( T_{cm} = 450 \text{ MeV} \)), and in most ensuing collisions, there should not be a large attractive component to the scattering. Therefore two possible sources of flow suggest themselves: preserving the 2-body reaction plane, and eliminating attractive orbits.

Results from ARC, for a beam momentum of 1.7 GeV/c, are plotted in Fig(1). One can see that accounting for the final state plane adds \( \sim 20\% \) to the average flow, while eliminating attractive orbits produces an additional \( \sim 10\% \), or so. We have also made calculations for beam momenta of 0.96 and 1.9 GeV/c in order to explore the dependence of these results on collision energy. The agreement between theory and experiment remains equally good at these higher and lower energies.

Some further theoretical discussion is perhaps in order here, since the cascade model mixes classical propagation between collisions, with empirical scattering cross-sections. If justification can be found in quantum mechanics for the cascade model, it is in part from the eikonal approximation to potential scattering, valid when \( U \ll T \), and for small angle scattering. This is a generalisation of the Born approximation, which assumes that scattering particles move through the potential along a straight line path or classical orbit (WKB); thus one obtains an impact-parameter-dependent phase shift for the outgoing wave. A generalisation of the eikonal approximation to the \( n \)-body system with pairwise interactions would retain the notion of an impact parameter for the 2-body collisions. Hence a 2-body reaction plane exists, and should be preserved for two-body final states. Quantum mechanically the angular momentum must still be conserved, and so a 2-body plane is reasonably well-defined, as long as the scattering is not in an \( s \)-wave. We expect a spread in the angle of the plane: \( \Delta \theta \sim (2l + 1)^{-1} \), and a simple estimate can be made of the number of partial waves present, for the beam momentum and a typical cross-section: \( l_{\text{max}} = kb \sim 5 \), \( (b = (\sigma/\pi)^{1/2}) \), for the first collision. In this way one could allow for quantum mechanical smearing of the orientation of the two body plane according to the angular momentum present, but a first approximation is just to preserve the plane exactly. We expect that not much of the scattering is \( s \)-wave at \( p_L \sim 1.7 \text{ GeV/c} \), even for ensuing cascading collisions.

One further issue of concern is whether fermi motion, which as we pointed out above could be handled more correctly, is in fact somehow responsible for the flow exhibited by the cascade. This question can be dispensed with immediately, by examining Fig(1). One sees that, if anything, the flow increases when fermi motion is turned off.

We expect that some small adjustment of the theory may result when consistent fermi motion, smearing of the 2-body plane, and impact-parameter-dependent choice of repulsive or attractive orbits are introduced, but certainly there would not appear to be any need at this point for large mean field terms, and collective flow at these energies can, it seems, come from a pure cascade model.

It is relevant to ask what the pions do in relation to the nucleons. The direction and size of flow in the pion sector might give a hint about the underlying mechanism generating
Fig. 1: FLOW: PLASTIC BALL vs. ARC (Au+Au at 1.7 GeV/c). Dotted line indicates Plastic Ball slope at mid-rapidity, corrected for dispersion in the estimated reaction plane. The line is not extended to forward rapidity since the Plastic Ball acceptance filter has a significant effect there. ARC $\langle P_x \rangle$ is calculated in the ideal reaction plane.
\langle P_x \rangle in the nucleons. Overall momentum conservation, involving both positive and negative rapidity, does not constrain the direction, or size, of any pion flow. Pursuing this question, we calculated flow at AGS energies, however without using specific experimental cuts. For the pions we define a sideward flow, based on the reaction plane already determined from the protons. A graph of the pion and proton flow curves, obtained in Au+Au at \( p = 11.6 \text{ GeV/c} \) is shown in Fig(2). It is seen that the pion flow is in the opposite direction to the proton flow at these energies, which suggests strongly that pion production enhances the nucleon flow.

We observe that the flow momentum for all produced particles is in the opposite direction to that for nucleons; flow curves for anti-nucleons, kaons, and pions in Fig(2) make this evident. Though more study is needed, it seems that the magnitude of anti-nucleon flow is similar to that for protons, while kaon and pion flows are somewhat less.

In peripheral collisions, that is to say, in collisions having appreciable sideward flow, particle production will take place initially in the dense central region where target and projectile nuclei overlap. Anti-nucleons trying to emerge from the interaction region in the direction of the sideward flow will encounter more target or projectile nucleons than those emerging in the opposite direction. Given the large annihilation cross-section, it is natural that anti-nucleons should anti-flow. Of course, anti-nucleons are a small perturbation to the overall dynamics, but a similar mechanism could function in the case of kaons and pions, where absorption on nucleons is still a strong effect. However, many low energy pions, from \( \Delta \) decay, follow the nucleon flow.

Nucleon flow may also arise from nucleons scattering inelastically at the edge of and away from the dense central region of the collision, giving an average transverse momentum to the produced pions, in the opposite direction. At the kinetic energy \( (T_{cm} \sim 3.5 \text{ GeV}) \) of first \( pp \) collisions, the cross-section is more inelastic than at lower energies. So, production plays an important role at the higher energies. Even at the beam momentum of 1.7 GeV/c \( (T_{cm} = 450 \text{ MeV}) \), considered for the Bevalac Au+Au data, production is relevant. However, a new dynamical region may be entered below the threshold for pion production. In \( pp \) the inelastic cross-section has fallen essentially to zero when the center of mass kinetic energy is below 200 MeV. Then, the mechanism for flow production will involve only elastic processes and mean fields may perhaps make significant contributions.

That a pure cascade model can produce sufficient collective flow is a surprising and interesting result. The question might be asked: why did early cascade models fail to produce enough flow? From the outset, it should be stated that the literature is somewhat contradictory on this question. Early cascade model calculations can be found displaying up to 50% of experimental flow in various systems. It is crucial to know, when evaluating results, whether experimental cuts have been correctly applied to the simulations, since numerical results for flow are quite sensitive to these cuts. For instance, the magnitude of the flow calculated here would be significantly larger for a spectator cut of 0.35 GeV/c. Nevertheless, the general conclusion seems to have been arrived at, that the pure cascade generates little or no collective flow. We would argue that this conclusion is incorrect at least at Bevalac energies, and above, and that a correctly constructed cascade does in fact produce enough flow without mean fields. As we have noted above, mean fields presumably still play some role as the collision energy is reduced.

One may also ask: if the equation of state need not be included explicitly in the modeling to generate flow, then is any role played by the equation of state in producing flow? Certainly
Flow at $p=11.6$ GeV/c

Minimum Bias Sample

Produced particles all have flow momentum in the opposite direction to the nucleons.

Fig. 2: FLOW at AGS ENERGY: Pion and nucleon flows are shown, as well as kaon and anti-nucleon flows. Produced particles all have flow momentum in the opposite direction to the nucleons.
the cascade model, including as it does classical, relativistic kinetic processes, produces at least the thermal pressure and equation of state of an ideal relativistic gas. This thermal pressure may be all that is needed for flow. At the very least the present calculation suggests an insensitivity to off-shell aspects of the equation of state.

At AGS energies, flow is predicted in both pions and nucleons, and it is a not inconsiderable effect. The direction of sideward flow is opposite in pions and nucleons. The success of ARC in describing flow at Bevalac energies, and inclusive data at AGS energies emboldens us to use the code to study flow at AGS energies. High energy flow predictions hopefully will be testable when the EOS detector is moved to BNL and the E895 collaboration begins to take data. Since the densities achieved are higher than at the Bevalac, we can still hope to see explicit equation of state effects, and flow may be a good observable to study in this connection. One may look for deviations from the pure cascade picture, in rarer, very high multiplicity events.

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