A MATHEMATICAL MODEL FOR ARCH FINGERPRINT

A PREPRINT

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ABSTRACT

In this paper, different categories of the arch fingerprint are set up in a general dynamical system model using ordinary differential equations. We study its global dynamics and analyze the existence and stability of equilibria. Numerical simulations using Maple show the matching between real images of categories of arch fingerprint and phase portraits of the considered dynamical system.

Keywords arch fingerprint · Numerical simulations of categories of arch fingerprint.

1 Introduction

Fingerprints are patterns, made by friction ridges of a human finger, which appear on the pads of the fingers and thumbs. The fingerprints is the most accurate and reliable for identifying person, since fingerprints are most unique biometric characteristics to any person; therefore it is used in forensic divisions worldwide for security, and in criminal cases where even the twins have non-identical fingerprints, see [4, 5]. Early study about fingerprints appeared in 1892 by Sir Francis Galton in his book, finger prints [1]. The patterns of fingerprints are classified into three general types; loop, whorl, and arch, many studies indicate that arch patterns are observed in about 5% of all fingerprints see [3, 2, 1].

The arch patterns display a relatively horizontal ridges run from the left to the right side of the fingerprint with growth in the center. This paper discusses three categories of arch fingerprint.

• Plain arch in which the ridges flow relatively horizontally with a little rise in the middle, see the left picture in figure 1.
• Tented arch is similar to the plain arch with a bigger rise, and at least one ridge with short length is vertically oriented in the middle, see the middle picture in figure 1.
• Strong arch is similar to the tented arch with many relatively longer ridges are vertically oriented, this category looks like Christmas tree, see the right picture in figure 1.

To generate a system for arch fingerprint, we think about a deformation of the phase portrait of straight flow. We proceeded by trial and error to approximate the following dynamical system that represents the general map of the above categories of the arch.

\[
\frac{dx}{dt} = y^2, \\
\frac{dy}{dt} = -\theta x, \quad \theta > 0.
\]

(1)

This paper is organized as follows: In the next section, we study the stability of the equilibria of system (1). In section 3 we go over different values of \( \theta \) in the dynamical system (1) to simulate the categories of the arch fingerprint.
2 Steady states and their stability

To study the stability of the system (1), we find the equilibria first, which are the solutions of the following equations:

\[ 0 = y^2, \]
\[ 0 = -\theta x, \]

and are given by \( E_0 = (0, 0) \). The Jacobian matrix of (1) takes the form

\[
J = \begin{bmatrix}
0 & 2y \\
-\theta & 0
\end{bmatrix}.
\]

The Jacobian matrix evaluated at the equilibrium point \( E_0 = (0, 0) \) is

\[
J(E_0) = \begin{bmatrix}
0 & 0 \\
-\theta & 0
\end{bmatrix}.
\]

From the Jacobian matrix (5), the eigenvalues of \( J(E_0) \) are \( \lambda_{1,2} = 0 \) which means degenerate nonhyperbolic equilibrium point, the phase portrait for this system is shown in section 3. In all categories of arch fingerprints, we see that a deleted neighborhood of the origin consists of upper and lower hyperbolic sectors, also it consists of left and right separatrices. This type of critical point is called a cusp. But the flow which is located above the origin makes different types of the angle depending on the value of \( \theta \), for example we have obtuse angle in plain category, around right angle in tented category, and acute angle in strong category.

3 Discussion

In this section, we study the system (1) with different values of \( \theta \) and display its simulations and their matching images of the above categories of the arch fingerprint. We use bold red lines to represent the separatrices, the green lines at different initial conditions above x-axis to show the above hyperbolic sector, and the brown line at different initial conditions below x-axis to display the below hyperbolic sector. When \( \theta \) is very small and closed to zero in (1), the flow of phase portrait is relatively horizontally with a slight growth in the middle as in figure 2. For example consider system (1) with \( \theta = 0.001 \).

Example 3.1.

\[
\begin{align*}
\frac{dx}{dt} &= y^2, \\
\frac{dy}{dt} &= -0.001x.
\end{align*}
\]

Figure 2 shows the simulation of the example (3.1) using Maple. Figure 3 explains a closer look at the neighborhood of the origin, we have explain the separatrices in the phase portrait, and we use a thin red line to determine the region of hyperbolic sectors, and hence the origin is cusp fixed point.

If we increase \( \theta \) reaching around 0.5 in (1), the flow rises more in the middle with a smaller angle above the origin than the case of plain type. In this case we get phase portrait looks like tented arch, for example consider system (1) with \( \theta = 0.5 \).
Example 3.2.

\[
\begin{align*}
\frac{dx}{dt} &= y^2, \\
\frac{dy}{dt} &= 0.5x.
\end{align*}
\] (7)

The phase portrait of the example (3.2) is shown in figure 4, and figure 5 illustrates a focused attention around the center.

Finally, if \(\theta\) grows up enough more than one, the flow stretched longer vertically in the middle with a cusp at the origin making acute angle. This flow agrees with the general shape of the strong arch, for example consider system (1) with \(\theta = 5\).
Example 3.3.

\[
\frac{dx}{dt} = y^2, \\
\frac{dy}{dt} = -5x.
\]  

(8)

In figure 6 we see the matching between the image of strong arch and the phase portrait of example (3.3), and figure 7 shows the identification between the center regions in both image of strong arch and phase portrait with acute angle.

References

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