MILNOR-WOOD INEQUALITIES FOR PRODUCTS

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Abstract. We prove Milnor-wood inequalities for local products of manifolds. As a consequence, we establish the generalized Chern Conjecture for products $M \times \Sigma^k$ for any product of a manifold $M$ with a product of $k$ copies of a surface $\Sigma$ for $k$ sufficiently large.

1. Introduction

Let $M$ be an $n$-dimensional topological manifold. Consider the Euler class $\varepsilon_n(\xi) \in H^n(M, \mathbb{R})$ and Euler number $\chi(\xi) = \langle \varepsilon_n(\xi), [M] \rangle$ of oriented $\mathbb{R}^n$-vector bundles over $M$. We say that the manifold $M$ satisfies a Milnor-Wood inequality with constant $c$ if for every flat oriented $\mathbb{R}^n$-vector bundles $\xi$ over $M$, the inequality

$$|\chi(\xi)| \leq c \cdot |\chi(M)|$$

holds. Recall that a bundle is flat if it is induced by a representation of the fundamental group $\pi_1(M)$. We denote by $\text{MW}(M) \in \mathbb{R} \cup \{+\infty\}$ the smallest such constant.

If $X$ is a simply connected Riemannian manifold, we denote by $\tilde{\text{MW}}(X) \in \mathbb{R} \cup \{+\infty\}$ the supremum of the values of $\text{MW}(M)$ when $M$ runs over all closed quotients of $X$.

Milnor’s seminal inequality [Mi58] amounts to showing that the Milnor-Wood constant of the hyperbolic plane $\mathcal{H}$ is $\tilde{\text{MW}}(\mathcal{H}) = 1/2$, and in [BuGe11], we showed that $\tilde{\text{MW}}(\mathcal{H}^n) = 1/2^n$.

We prove a product formula for the Milnor-Wood inequality valid for any closed manifolds:

**Theorem 1.** For any pair of compact manifolds $M_1, M_2$

$$\text{MW}(M_1 \times M_2) = \text{MW}(M_1) \cdot \text{MW}(M_2).$$

For the product formula for universal Milnor-Wood constant, we restrict to Hadamard manifolds:

**Theorem 2.** Let $X_1, X_2$ be Hadamard manifolds. Then

$$\tilde{\text{MW}}(X_1 \times X_2) = \tilde{\text{MW}}(X_1)\tilde{\text{MW}}(X_2).$$

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One important application of Milnor-Wood inequalities is to make progress on the generalized Chern Conjecture.

**Conjecture 3** (Generalized Chern Conjecture). Let $M$ be a closed oriented aspherical manifold. If the tangent bundle $TM$ of $M$ admits a flat structure then $\chi(M) = 0$.

As the name indicates, this conjecture implies the classical Chern conjecture for affine manifolds predicting the vanishing of the Euler characteristic of affine manifolds. This is because an affine structure on $M$ induces a flat structure on the tangent bundle $TM$.

As pointed out in [Mi58], if $MW(M) < 1$ then the Generalized Chern Conjecture holds for $M$. Indeed, if $\chi(M) \neq 0$ the inequality

$$|\chi(M)| = |\chi(TM)| \leq MW(M) \cdot |\chi(M)| < |\chi(M)|$$

leads to a contradiction.

One can use Theorem 1 to extend the family of manifolds satisfying the Generalized Chern Conjecture. For instance:

**Corollary 4.** Let $M$ be a manifold with $MW(M) < +\infty$. Then the product $M \times \Sigma^k$, where $\Sigma$ is a surface of genus $\geq 2$ and $k > \log_2(MW(X))$ satisfies the Generalized Chern Conjecture. In particular, if $\chi(M) \neq 0$, then $M \times \Sigma^k$ does not admit an affine structure.

**Remark 5.** 1. One can replace $\Sigma^k$ in Corollary 4 by any $H^k$-manifold.

2. The corollary is somehow dual to a question of Yves Benoist [Be00, Section 3, p. 19] asking whether for every closed manifold $M$ there exists $m$ such that $M \times S^m$ admits an affine structure. For example, for any hyperbolic manifold $M$, the product $M \times S^1$ admits an affine structure, but in general $m = 1$ is not enough. Indeed, $Sp(2,1)$ has no nontrivial 9-dimensional representations and the dimension of the associated symmetric space is 8.

Note that since there are only finitely many isomorphism classes of oriented $\mathbb{R}^n$-bundles which admit a flat structure, it is immediate that the set

$$\{|\chi(\xi)| \mid \xi \text{ is a flat oriented } \mathbb{R}^n\text{-bundle over } M\}$$

is finite for every $M$. In particular, if $\chi(M) \neq 0$, there exists a finite Milnor-Wood constant $MW(M) < +\infty$.

However, in general, the Milnor-Wood constant can be infinite. Indeed, the implication $\chi(M) = 0 \Rightarrow \chi(\xi) = 0$, for $\xi$ a flat oriented $\mathbb{R}^n$-bundle, does not hold in general. In Section 5 we exhibit a flat bundle $\xi$ with $\chi(\xi) \neq 0$ over a manifold $M$ with $\chi(M) = 0$. This example is inspired by Smillie’s counterexample of the Generalized...
Chern Conjecture [Sm77] for nonaspherical manifolds, and likewise this manifold is nonaspherical.

The following questions are quite natural:

1. Does there exist a finite constant \( c(n) \) depending on \( n \) only such that \( MW(M) \leq c(n) \) for every closed aspherical \( n \)-manifold?
2. Let \( X \) be a contractible Riemannian manifold such that there exists a closed \( X \)-manifold \( M \) with \( MW(M) < \infty \). Is \( \tilde{MW}(X) \) necessarily finite?
3. Does \( \chi(M) = 0 \Rightarrow \chi(\xi) = 0 \) for flat \( t \) oriented \( \mathbb{R}^n \)-bundles \( \xi \) over aspherical manifolds \( M \)?

2. Representations of products

**Lemma 6.** Let \( H_1, H_2 \) be groups and \( \rho : H_1 \times H_2 \to GL_n(\mathbb{R}) \) a representation of the direct product and suppose that \( \rho(H_i) \) is nonamenable for both \( i = 1, 2 \). Then, up to replacing the \( H_i \)'s by finite index subgroups, either

- \( V = \mathbb{R}^n \) decomposes as an invariant direct sum \( V = V' \oplus V'' \) where the restriction \( \rho |_{V'} = \rho_1' \otimes \rho_2' \) is a nontrivial tensor representation, or
- \( V = V_1 \oplus V_2 \) where \( G_i \) is scalar on \( V_i \).

**Proof.** This can be easily deduced from the proof of [BuGe11, Proposition 6.1].

**Proposition 7.** Let \( H = \prod_{i=1}^{k} H_i \) be a direct product of groups and let \( \rho : H \to GL_n^+(\mathbb{R}) \) be an orientable representation, where \( n = \sum_{i=1}^{k} m_i \). Suppose that \( \rho(H_i) \) is nonamenable for every \( i \). Then, up to replacing the \( H_i \)'s by finite index subgroups \( H' = \prod_{i=1}^{k} H_i' \), either

1. there exists \( 1 \leq i_0 < k \) such that \( V = \mathbb{R}^n \) decomposes nontrivially to an invariant direct sum \( V = V' \oplus V'' \) and the restricted representation \( \rho |_{(H'_i \times \prod_{i > i_0} H'_i, V')} \)

\[
H'_{i_0} \times \prod_{i > i_0} H'_i \longrightarrow GL(V')
\]

is a nontrivial tensor, or
2. the representation \( \rho' \) factors through

\[
\rho' : \prod_{i=1}^{k} H'_i \longrightarrow \left( \prod_{i=1}^{k} GL_{m_i}^+(\mathbb{R}) \right)^+ \longrightarrow GL_n^+(\mathbb{R}),
\]

where the latter homomorphism is, up to conjugation, the canonical diagonal embedding, and \( \rho'(H'_i) \) restricts to a scalar representation on each \( GL_{m_j}(\mathbb{R}) \), for \( i \neq j \).
Moreover, if all \( m_i \) are even then either \( m'_i < m_i \) for some \( i \) or one can replace \( GL \) with \( GL^+ \) everywhere.

The notation \( \left( \prod_{i=1}^{k} GL_{m'_i}(\mathbb{R}) \right)^+ \) stands for the intersection of \( \prod_{i=1}^{k} GL_{m'_i}(\mathbb{R}) \) with the positive determinant matrices.

**Proof.** We argue by induction on \( M \). First note that the inequality is immediate from Lemma 6. Suppose \( k > 2 \). If Item (1) does not hold, it follows from Lemma 6 that, up to replacing the \( H_i \)'s by some finite index subgroups, \( V \) decomposes invariantly to \( V = V_1 \oplus V'_1 \) where \( \rho(H_1) \) is scalar on \( V'_1 \) and \( \rho(\prod_{i>1} H_i) \) is scalar on \( V_1 \). We now apply the induction hypothesis for \( \prod_{i>1} H_i \) restricted to \( V'_1 \).

Finally, in Case (2), since \( \sum m_i = n \), either \( m'_i < m_i \) for some \( i \) or equality holds everywhere. In the later case, if all the \( m_i \)'s are even, given \( g \in H_i \), since the restriction of \( \rho(g) \) each \( V'_{j \neq i} \) is scalar, it has positive determinant. We deduce that also \( \rho(g)|_{V_i} \) has positive determinant. \( \square \)

3. **MULTIPLICATIVITY OF THE MILNOR-WOOD CONSTANT FOR PRODUCT MANIFOLDS – A PROOF OF THEOREM 1**

Let \( M_1, M_2 \) be two arbitrary manifolds. We prove that

\[
MW(M_1 \times M_2) = MW(M_1) \cdot MW(M_2).
\]

First note that the inequality \( MW(M_1 \times M_2) \geq MW(M_1) \cdot MW(M_2) \) is trivial. Indeed, let \( \xi_1, \xi_2 \) be flat oriented bundles over \( M_1 \) and \( M_2 \) respectively of the right dimension such that \( |\chi(\xi_i)| = MW(M_i) \cdot |\chi(M_i)| \) for \( i = 1, 2 \). Then \( \xi_1 \times \xi_2 \) is a flat bundle over \( M_1 \times M_2 \) with

\[
|\chi(\xi_1 \times \xi_2)| = |\chi(\xi_1)||\chi(\xi_2)| = MW(M_1) \cdot MW(M_2) \cdot |\chi(M_1 \times M_2)|.
\]

For the other inequality, let \( \xi \) be a flat oriented \( \mathbb{R}^n \)-bundle over \( M_1 \times M_2 \), where \( n = \dim(M_1) + \dim(M_2) \). We need to show that

\[
|\chi(\xi)| \leq MW(X_1) \cdot MW(X_2) \cdot |\chi(M)|.
\]

Observe that if we replace \( M \) by a finite cover, and the bundle \( \xi \) by its pullback to the cover, then both sides of the previous inequality are multiplied by the degree of the covering.

The flat bundle \( \xi \) is induced by a representation

\[
\rho : \pi_1(M_1 \times M_2) \cong \pi_1(M_1) \times \pi_1(M_2) \rightarrow GL_n^+(\mathbb{R}).
\]

If \( \rho(\pi_1(M_i)) \) is amenable for \( i = 1 \) or \( 2 \), then \( \rho^*(\varepsilon_n) = 0 \) [BuGe11, Lemma 4.3] and hence \( \chi(\xi) = 0 \) and there is nothing to prove. Thus, we can without loss of generality suppose that, upon replacing \( \Gamma \) by a finite index subgroup the representation \( \rho \) factors as in Proposition 7.
In case (1) of the proposition, we obtain that $\rho^*(\varepsilon_n) = 0$ by Lemma 10 and [BuGe11, Lemma 4.2]. In case (2) we get that $\rho$ factors through 

$$\rho: \pi_1(M_1) \times \pi_1(M_2) \longrightarrow \left(\text{GL}_{m'_1}(\mathbb{R}) \times \text{GL}_{m'_2}(\mathbb{R})\right)^+ \longrightarrow \text{GL}_n^+(\mathbb{R}),$$

where the latter embedding $i$ is up to conjugation the canonical embedding. Furthermore, up to replacing $\rho$ by a representation in the same connected component of 

$$\text{Rep}(\pi_1(M_1) \times \pi_1(M_2), \left(\text{GL}_{m'_1}(\mathbb{R}) \times \text{GL}_{m'_2}(\mathbb{R})\right)^+),$$

which will have no influence on the pullback of the Euler class, we can without loss of generality suppose that the scalar representations of $\pi_1(M_1)$ on $\text{GL}_{m'_2}(\mathbb{R})$ and $\pi_1(M_2)$ on $\text{GL}_{m'_1}(\mathbb{R})$ are trivial, so that $\rho$ is a product representation. If $m'_1$ or $m'_2$ is odd, then $i^*(\varepsilon_n) = 0 \in H^c_n((\text{GL}_{m'_1}(\mathbb{R}) \times \text{GL}_{m'_2}(\mathbb{R}))^+)$. If $m'_1$ and $m'_2$ are both even then Proposition 7 further tells us that either $m'_i < m_i$ for $i = 1$ or 2, or the image of $\rho$ lies in $\text{GL}_{m'_1}(\mathbb{R}) \times \text{GL}_{m'_2}(\mathbb{R})$. In the first case, the Euler class vanishes [BuGe11, Lemma 4.2], while in the second case, we immediately obtain the desired inequality. This finishes the proof of Theorem 1.

4. Multiplicativity of the universal Milnor-Wood constant for Hadamard manifolds - a proof of Theorem 2

Theorem 2 can be reformulated as follows:

**Theorem 8.** Let $X$ be a Hadamard manifold with de-Rham decomposition $X = \prod_{i=1}^k X_i$, then $\tilde{MW}(X) = \prod_{i=1}^k \tilde{MW}(X_i)$.

We shall now prove Theorem 8. Note that the inequality "$\geq" is obvious. Let $M = \Gamma \backslash X$ be a compact $X$-manifold. We must show that $\text{MW}(M) \leq \prod_{i=1}^k \tilde{MW}(X_i)$. Note that $\Gamma$ is torsion free. Let us also assume that $k \geq 2$. If $M$ is reducible one can argue by induction using Theorem 1. Thus we may assume that $M$ is irreducible. Observe that this implies that Isom($X$) is not discrete. If $\Gamma$ admits a nontrivial normal abelian subgroup then by the flat torus theorem (see [BH99, Ch. 7]) $X$ admits an Euclidian factor which implies the vanishing of the Euler class. Assuming that this is not the case we apply the Farb–Weinberger theorem [FaWe08, Theorem 1.3] to deduce that $X$ is a symmetric space of non-compact type. Thus, up to replacing $M$ by a finite cover (equivalently, replace $\Gamma$ by a finite index subgroup), we may assume that $\Gamma$ lies in $G = \text{Isom}(X) = \prod_{i=1}^k \text{Isom}(X_i) = \prod_{i=1}^k G_i$ and $G$ is an adjoint semisimple Lie group without compact factors and $\Gamma \leq G$ is irreducible in the sense that its projection to each factor is dense.
Denote by $\tilde{G}_i$ the universal cover of $G_i$, and by $\tilde{\Gamma} \leq \prod_{i=1}^k \tilde{G}_i$ the pullback of $\Gamma$.

Let $\rho : \Gamma \to \text{GL}_n^+(\mathbb{R})$ be a representation inducing a flat oriented vector bundle $\xi$ over $M$. Up to replacing $\Gamma$ by a finite index subgroup, we may suppose that $\rho(\Gamma)$ is Zariski connected. Let $S \leq \text{GL}_n^+(\mathbb{R})$ be the semisimple part of the Zariski closure of $\rho(\Gamma)$, and let $\rho' : \Gamma \to S$ be the quotient representation. By superrigidity, the map $\text{Ad} \circ \rho' : \Gamma \to \text{Ad}(S)$ extends to $\phi : \Gamma \to \text{Ad}(S)$ (see [Ma91, Mo06, GKM08]). This map can be pulled to $\tilde{\phi} : \tilde{\Gamma} \to S$.

Recall also that $\prod_{i=1}^k \tilde{G}_i$ is a central discrete extension of $\prod_{i=1}^k G_i$ and, likewise, $\tilde{\Gamma}$ is a central extension of $\Gamma$. If $n_i = \dim X_i$ and $n = \sum_{i=1}^k n_i$ we deduce from Proposition 7 and Lemma 10 that either the Euler class vanishes or the image of $\tilde{\phi}$ lies (up to decomposing the vector space $\mathbb{R}^n$ properly) in $(\prod_{i=1}^k \text{GL}_{n_i})^+$.

Suppose that $\text{MW}(X_i)$ is finite for all $i = 1, \ldots, k$ and let $M_i$ be closed $X_i$-manifolds. Let $\xi'$ be the flat vector bundle on $\prod_{i=1}^k M_i$ coming from $\tilde{\rho}$ reduced to $\prod_{i=1}^k M_i$, and let $\xi'_i$ be the vector bundle on $M_i$ induced by $\tilde{\rho}_i$, $i = 1, \ldots, k$. By Lemma 9, we have

$$\frac{\chi(\xi)}{\text{vol}(M)} = \frac{\chi(\xi'_i)}{\text{vol}(M_i)} = \prod_{i=1}^k \frac{\chi(\xi'_i)}{\text{vol}(M_i)} \leq \prod_{i=1}^k \text{MW}(X_i),$$

which finishes the proof of Theorem 8. \hfill $\square$

5. Example: a flat bundle with nonzero Euler number over a manifold with zero Euler characteristic

Recall that given two closed manifolds of even dimension, the Euler characteristic of connected sums behaves as

$$\chi(M_1 \# M_2) = \chi(M_1) + \chi(M_2) - 2.$$

The idea is to find $M = M_1 \# M_2$ such that $M_1$ admits a flat bundle with nontrivial Euler number in turn inducing such a bundle on the connected sum, and to choose then $M_2$ in such a way that the Euler characteristic of the connected sum vanishes. Take thus

$$M_1 = \Sigma_2 \times \Sigma_2, \quad M_2 = (S^1 \times S^3) \# (S^1 \times S^3) \quad \text{and} \quad M = M_1 \# M_2.$$

These manifolds have the following Euler characteristics:

$$\begin{align*}
\chi(M_1) &= 4, \\
\chi(M_2) &= 2\chi(S^1 \times S^3) - 2 = -2, \\
\chi(M) &= 0.
\end{align*}$$

Let $\eta$ be a flat bundle over $\Sigma_2$ with Euler number $\chi(\eta) = 1$. (Note that we know that such a bundle exists by [Mi58].) Let $f : M \to M_1$
be a degree 1 map obtained by sending $M_2$ to a point, and consider
\[
\xi = f^*(\eta \times \eta).
\]
Obviously, since $\eta$ is flat, so is the product $\eta \times \eta$ and its pullback by $f$. Moreover, the Euler number of $\xi$ is
\[
\chi(\xi) = \chi(\eta \times \eta) = 1.
\]
Indeed, the Euler number of $\eta \times \eta$ is the index of a generic section of the bundle, which we can choose to be nonzero on $f(M_2)$, so that we can pull it back to a generic section of $\xi$ which will clearly have the same index as the initial section on $\eta \times \eta$.

6. Proportionality principles and vanishing of the Euler class of tensor products

**Lemma 9.** Let $X$ be a simply connected Riemannian manifold, $G = \text{Isom}(M)$ and $\rho : G \to GL^+_{\mathbb{R}}(\mathbb{R})$ a representation. Then $\chi(\xi)$, where $M = \Gamma \backslash X$ is a closed $X$-manifold and $\xi$ is the flat vector bundle induced on $M$ by $\rho$ restricted to $\Gamma$, is a constant independent of $M$.

**Proof.** There is a canonical isomorphism $H^*_c(G) \cong H^*(\Omega^*(X)^G)$ between the continuous cohomology of $G$ and the cohomology of the cocomplex of $G$-invariant differential forms $\Omega^*(X)^G$ on $X$ equipped with its standard differential. (For $G$ a semisimple Lie group, every $G$-invariant form is closed, hence one further has $H^*(\Omega^*(X)^G) \cong \Omega^*(X)^G$.) In particular, in top dimension $n = \dim(X)$, the cohomology groups are 1-dimensional $H^n_c(G) \cong H^n(\Omega^*(X)^G) \cong \mathbb{R}$ and contain the cohomology class given by the volume form $\omega_X$.

Since the bundle $\xi$ over $M$ is induced by $\rho$, its Euler class $\varepsilon_n(\xi)$ is the image of $\varepsilon_n \in H^n_c(GL^+_{\mathbb{R}}(\mathbb{R}, n)$ under
\[
H^n_c(GL^+_{\mathbb{R}}(\mathbb{R}, n) \longrightarrow H^n_c(G) \longrightarrow H^n(\Gamma) \cong H^n(M),
\]
where the middle map is induced by the inclusion $\Gamma \hookrightarrow G$. In particular, $\rho^*(\varepsilon_n) = \lambda \cdot [\omega_X] \in H^n_c(G)$ for some $\lambda \in \mathbb{R}$ independent of $M$. It follows that $\chi(\varepsilon_n)/\text{Vol}(M) = \lambda$. \hfill $\square$

**Lemma 10.** Let $\rho : GL^+(n, \mathbb{R}) \times GL^+(m, \mathbb{R}) \to GL^+(nm, \mathbb{R})$ denote the tensor representation. If $n, m \geq 2$, then
\[
\rho^*(\varepsilon_{nm}) = 0 \in H^{nm}_c(GL(n, \mathbb{R}) \times GL(m, \mathbb{R})).
\]

**Proof.** The case $n = m = 2$ was proven in [BuGe11, Lemma 4.1], based on the simple observation that interchanging the two $GL^+(2, \mathbb{R})$ factors does not change the sign of the top dimensional cohomology class in $H^4_c(GL(2, \mathbb{R}) \times GL(2, \mathbb{R})) \cong \mathbb{R}$, but it changes the orientation on the tensor product, and hence the sign of the Euler class in $H^4_c(GL^+(4, \mathbb{R}))$. 


Let us now suppose that at least one of $n, m$ is strictly greater than 2, or equivalently, that $n + m < nm$. The Euler class is in the image of the natural map

$$H^{nm}(B\text{GL}(nm, \mathbb{R})) \rightarrow H_c^{nm}(\text{GL}(nm, \mathbb{R})).$$

By naturality, we have a commutative diagram

$$
\begin{array}{ccc}
H^{nm}(B\text{GL}^+(nm, \mathbb{R})) & \rightarrow & H_c^{nm}(\text{GL}^+(nm, \mathbb{R})) \\
\rho^* & & \rho^* \\
H^{nm}(B(\text{GL}^+(n, \mathbb{R}) \times \text{GL}^+(m, \mathbb{R}))) & \rightarrow & H_c^{nm}(\text{GL}^+(n, \mathbb{R}) \times \text{GL}^+(m, \mathbb{R}))).
\end{array}
$$

Since the image of the lower horizontal arrow is contained in degree $\leq n + m$, it follows that $\rho^*_\otimes(\varepsilon_{nm}) = 0$. □

References

[Be00] Y. Benoist, *Towers affines in Crystallographic groups and their generalizations (Kortrijk, 1999)*, Contemp. Math. 262 (2000) 1–37.

[BH99] R. Bridson, A. Haefliger, *Metric Spaces of Non-Positive Curvature*, Springer, 1999.

[BuGe08] M. Bucher, T. Gelander, *Milnor-Wood inequalities for locally $(\mathbb{H}^2)^n$-manifolds*, C. R. Acad. Sci. Paris 346 (2008), no. 11-12, 661-666.

[BuGe11] M. Bucher, T. Gelander, *The generalized Chern conjecture for manifolds that are locally a product of surfaces*, Advances of Math., in press.

[FaWe08] B. Farb, S. Weinberger, *Isometries, rigidity and universal covers*, Ann. of Math. (2) 168 (2008), no. 3, 915D940.

[GKM08] T. Gelander, A. Karlsson, G.A. Margulis, *Superrigidity, generalized harmonic maps and uniformly convex spaces*, Geom. Funct. Anal. 17 (2008), no. 5, 1524Ð1550.

[Ma91] G.A. Margulis, *Discrete Subgroups of Semisimple Lie Groups*, Ergeb. Math., Springer-Verlag, 1991.

[Mi58] J. Milnor, *On the existence of a connection with curvature zero*. Comment. Math. Helv. 32, 1958, 215-223.

[Mo06] N. Monod, *Superrigidity for irreducible lattices and geometric splitting*, J. Amer. Math. Soc. 19 (2006), no. 4, 781D814.

[Sm77] J. Smillie, *Flat manifolds with non-zero Euler characteristics*, Comment. Math. Helv. 52 (1977) 453–355.