CHFS: Complex hesitant fuzzy sets-their applications to decision making with different and innovative distance measures

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Abstract
The objective of the authors is to establish an innovative concept of the complex hesitant fuzzy set (CHFS), which is the combination of the hesitant fuzzy set and the complex fuzzy set to manage complex and awkward information in the real-decision theory. The structure and the basic properties of the proposed set are studied in detail. Based on the internal structure of the set and to find the degree of the discrimination between the pairs of the CHFSs, the generalized distance measures and modified generalized distance measures are defined. Several properties and their relationship between them are derived in detail. Also, several cases of the proposed measures are exposed which reduce them to the existing studies. Furthermore, based on these proposed measures, a decision-making approach is established under the uncertain environment and several numerical examples are given to examine the feasibility and validity of the explored measures. Finally, the credibility of the modified and parameterized distance measures based on CHFSs is verified by comparing them with some existing measures.

1 | INTRODUCTION
Multiple attribute decision-making (MADM) process includes the examination of a limited arrangement of options and positioning them as far as the fact that they are so trustworthy to decision-maker(s) when all the rules are thought of at the same time. In this procedure, the rating estimations of every option incorporate both exact information and specialists’ subjective data. However, generally, it is expected that the data given by them are fresh in nature. In any case, because of the unpredictability of the framework step by step, the genuine contains numerous MADM issues where the data is either ambiguous, lose or dubious in nature. To manage it, a concept of fuzzy set (FS) [1] has been explored. To access the degree of discrimination between the pairs of the FSs, a concept of distance and similarity measures are a powerful technique. Both concepts are utilized in many places like decision-making [2] problems, pattern recognition [3], and medical diagnosis [4]. Wang [5] utilized the similarity measures based on FS. FS contains the grade of membership limited to [0,1], as an important technique to describe the opinion of a human being in the form of grades. FS has received extensive attention in the last few decades [6–8].

Recently, a fuzzy collaborative approach for examining the suitability of smart health was established by Chen et al. [9]. The fuzzy counterparts of the Fischer diagonal condition in T-diverges space were explored by Jin et al. [10]. Sanchez-Roger et al. [11] investigated the fuzzy logic and its uses in finance. For more work based on FS (refer [12, 13]).

Many researchers raise a question, when the range of FS will be changed, which is the real number instead of the complex number from a unit disc in a complex plane, and what will be the outcome of the effect. Ramot et al. [14] resolved this issue to explore the complex FS (CFS), which contains the grade of membership in the form of a complex number belonging to unit disc in a complex plane. CFS copes with the two-dimension information in a single set. CFS is an important technique to describe the opinion of a human being in the form of grades. CFS has received extensive attention in the last few years. Li and Chiang [15, 16] explored the complex neuro-FS and their function approximation. The systematic review based on CFS and logic was established by Yazdanbakhsh and Dick [17]. The notion of CFS is also established by Nguyen et al. [18], but the idea of Ramot et al. provides a wide range for decision-makers. Many researchers utilized CFS in different fields [19–21].

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In some situations, when a decision-maker gives membership grades in the form of groups, it is very awkward for the FS to describe these types of problems. For coping such types of issues, Torra [22] explored the novelty of hesitant fuzzy set (HFS) containing the grade of membership in the form of a finite subset of [0,1]. HFS is considered a more generalized technique to manage the awkward and critical types of information in FS theory. HFS has received extensive attention in the last few years. Furthermore, Wei et al. [23] investigated the entropy measures based on HFSs and their application in the multi-attribute group decision making (MAGDM) problem. The hesitant fuzzy linguistic preference relation was explored by Wu et al. [24]. Liang et al. [25] pioneered the risk appetite dual hesitant fuzzy three-way decisions with TODIM (an acronym in Portuguese for Interactive Multi-criteria Decision Making) methods. The divergent measures based on HFS was explored by Kobza et al. [26]. Wang et al. [27] established the new distance measures for HFSs and their application to MAGDM. Due to its structure, many researchers utilized HFS in different fields [28, 29].

Keeping in view of the importance of similarity measures (SMs), recently many researchers have chosen different SMs based on FS [30], CFS [31], and HFS [32]. On the other hand, when a decision-maker gives complex-valued membership grades in the form of groups, the existing measures cannot solve exactly. For handling such kinds of issues and keeping the advantages of the SMs, here, the authors establish a complex hesitant fuzzy set (CHFS), which is the combination of the hesitant fuzzy set (HFS) and CFS. In CHFS theory, degrees of membership are complex-valued and are given in polar coordinates. All theories such as FS, CFS, and HFS worked impressively, when a decision-maker faced such type of data which contains two-dimensional information in a single-set. For instance, \( \{0.1e^{2\pi(0.8)}, 0.5e^{2\pi(0.2)}, 0.9e^{2\pi(0.5)}, 0.3e^{2\pi(0.9)}\} \), then the existing all theories are unable to deal with such type of data. To deal with such type of issues, the CHFS is a perfect technique to sort out practical decision issues in the environment of FS theory. CHFS is more powerful and more general than existing theories like FS, CFS, and HFS to cope with awkward and complicated data in real-life decisions. Since all theories are special cases of the interpreted CHFS, the edges of the interpreted CHFS are examined below:

1. If we let the imaginary parts of the CHFS as zero, then the CHFS is converted into HFS which is in the shape of \( \{0.1, 0.5, 0.9, 0.3\} \).
2. If we let the CHFS as a singleton set, then the CHFS is converted into CFS which is in the shape of \( \{0.1e^{2\pi(0.8)}\} \).
3. If we let the CHFS as a singleton set and the imaginary parts as zero, then the CHFS is converted into FS which is in the shape of \( \{0.1\} \).

Further, the generalized distance measures (GDMs) and modified GDMs (MGDMs) based on established approaches are utilized and also expose the special cases of the established approach. After this, the parameterized distance measures are established and their particular cases are discussed. The established measures are utilized in the environment of decision making to examine the feasibility and validity of the explored measures. Moreover, the numerical examples for established measures are solved to express the superiority and integrity of the explored work. Finally, to evaluate the credibility of the modified and parameterized distance measures based on CHFS, they are verified by comparing them with some existing measures.

The main contributions of this article are as follows:

1. Based on existing concepts, firstly we establish the CHFS, which is the combination of the HFS and the CFS, to manage complex and awkward information in real-decision theory.
2. Based on existing distance and SMs called modified distance measures taken from [33], we established (GDMs) and (MGDMs) and discussed their particular cases.
3. After this, based on existing distance and SMs, and parameterized distance measures taken from [34], we established the parameterized distance measures and their particular cases.
4. The established measures are utilized in the environment of decision making to examine the feasibility and validity of the explored measures. Moreover, the numerical examples for established measures are solved to express the superiority and integrity of the explored work. Finally, to evaluate the credibility of the modified and parameterized distance measures based on CHFS, they are verified by comparing with some existing measures.

The article is arranged as follows: In Section 2, we revised the basics notions and their properties are discussed. In Section 3, based on existing concepts, we established the idea of CHFS and discussed its fundamental laws. In Section 4, the modified and parameterized distance measures based on the established concepts are investigated. In Section 5, the established measures are utilized in the environment of decision making to examine the feasibility and validity of the explored measures. Moreover, the numerical examples for established measures are solved to express the superiority and integrity of the explored work. Finally, to evaluate the credibility of the modified and parameterized distance measures based on CHFS, they are verified by comparing with some existing measures. The conclusion of this article is exposed in Section 6.

## 2 PRELIMINARIES

This section discusses some basic theories related to FSs, CFSs, and HFSs and their fundamental properties are derived. Throughout this article \( X \) represents a fix set.

**Definition 1 [1]** A FS \( Q \) is of the form:

\[
Q = \{< x, \mu_Q(x)> | x \in X \}
\]  

(1)

with a condition \( 0 \leq \mu_Q(x) \leq 1 \), where \( \mu_Q(x) \) represents the grade of truth. The pair \( Q = < x, \mu_Q(x)> \) is called the fuzzy
number. Throughout this article, the collection of all FSs on \( X \) is denoted by \( \text{FS}(X) \).

**Definition 2** [1] Let \( P, Q \in \text{FS}(X) \) Then the basic operations are defined as

1. \( P^c = \{ x < 1 - \mu_P(x) \mid x \in X \} \).
2. \( P \cap Q = \{ x < \min\{\mu_P(x), \mu_Q(x)\} \mid x \in X \} \).
3. \( P \cup Q = \{ x \leq \max\{\mu_P(x), \mu_Q(x)\} \mid x \in X \} \).

**Definition 3** [14] A CFS \( Q \) is of the form:

\[
Q = \{ x < \mu_Q(x) > \mid x \in X \} \tag{2}
\]

where \( \mu_Q(x) = \gamma_Q(x).e^{i2\pi(\alpha_Q(x))} \) represents the complex-valued truth grade in the form of polar coordinate, where \( \gamma_Q(x), \alpha_Q(x) \in [0, 1] \). Furthermore, the pair \( Q = \{ x, \gamma_Q(x).e^{i2\pi(\alpha_Q(x))} \} \) is called the complex fuzzy number (CFN).

**Definition 4** [14] For any two CFNs \( P = \{ x, \gamma_P(x).e^{i2\pi(\alpha_P(x))} \} \) and \( Q = \{ x, \gamma_Q(x).e^{i2\pi(\alpha_Q(x))} \} \), then

1. \( P \cup Q = \{ x, \max\{\gamma_P(x), \gamma_Q(x)\}.e^{i2\pi(\min(\alpha_P(x), \alpha_Q(x)))} \} \).
2. \( P \cap Q = \{ x, \min\{\gamma_P(x), \gamma_Q(x)\}.e^{i2\pi(\max(\alpha_P(x), \alpha_Q(x)))} \} \).
3. \( P^c = \{ x \leq \{1 - \gamma_P(x)\}.e^{i2\pi(1 - \alpha_P(x))} \} \).

**Definition 5** [22] A hesitant fuzzy number (HFN) \( Q \) is of the form:

\[
Q = \{ x < \mu_Q(x) > \mid x \in X \} \tag{3}
\]

where \( \mu_Q(x) \) is the set of different finite values in \([0, 1]\) representing the grade of truth for each element \( x \in X \). Further, the pair \( Q = \{ x, \mu_Q(x) \} \) is called HFN.

**Definition 6** [22] Let \( P = \{ x, \mu_P(x) \} \) and \( Q = \{ x, \mu_Q(x) \} \) are two HFNs. Then the basic operations are defined as

1. \( P \cup Q = \{ x, \cup_{\gamma_1, \gamma_2} \{ \max\{\gamma_1, \gamma_2\} \} \} \).
2. \( P \cap Q = \{ x, \cup_{\gamma_1, \gamma_2} \{ \min\{\gamma_1, \gamma_2\} \} \} \).
3. \( P^c = \{ x, \cup_{\gamma_1} \{ 1 - \gamma \} \} \).

3 | COMPLEX HESITANT FUZZY SETS

Here, we explore the notion of CHFSs and their basic operational laws. The established work is also verified with the help of some numerical examples.

**Definition 7** A CHFS \( Q \) is of the form:

\[
Q = \{ x < \mu_Q(x) > \mid x \in X \} \tag{4}
\]

where \( \mu_Q(x) = \gamma_Q(x).e^{i2\pi(\alpha_Q(x))}, j = 1, 2, 3, \ldots, n > \), represented the complex-valued truth grade which is subset of unit disc in complex plane with a condition: \( 0 \leq \max(\alpha_Q(x)) \leq 1 \) and \( 0 \leq \max(\alpha_Q(x)) \leq 1 \), \( \gamma_Q(x), \alpha_Q(x) \in [0, 1] \). Furthermore, \( Q = \{ x, \gamma_Q(x).e^{i2\pi(\alpha_Q(x))} \} \) is called complex HFN (CFHN).

**Definition 8** Let \( P = \{ x, \gamma_P(x).e^{i2\pi(\alpha_P(x))} \} \) and \( Q = \{ x, \gamma_Q(x).e^{i2\pi(\alpha_Q(x))} \} \) be two CHFSs. Then

1. \( c(\gamma_P(x)) = \{ x, \{1 - \gamma_P(x)\}.e^{i2\pi(1 - \alpha_P(x))} \} \).
2. \( P \cup Q = \{ x, \max(\gamma_P(x), \gamma_Q(x)).e^{i2\pi(\min(\alpha_P(x), \alpha_Q(x)))} \} \).
3. \( P \cap Q = \{ x, \min(\gamma_P(x), \gamma_Q(x)).e^{i2\pi(\max(\alpha_P(x), \alpha_Q(x)))} \} \).

The theory of CHFS makes it a wide range effective technique to deal with awkward and complicated data in practical decision problems. The CHFS holds the grade of the membership in the shape of a subset of the unit disc in the complex plane, whose entities are in the shape of polar coordinates. Fundamentally, the CHFS holds two-dimensional data in a single set. The interpreted CHFS is more general than existing theories like FS, CFS, and HFS, whose reasons and explanation are examined as follows. In CHFS, membership degree is complex-valued and is represented in polar coordinates. The amplitude term corresponding to the membership degree gives the extent of belongings of an object in a CHFS, and the phase term associated with the membership degree gives the additional information, generally related with periodicity. The phase terms are novel parameters of the membership degree, and these are the parameters that distinguish the traditional FS, CFS and HFS theory. HFS theory deals with only one dimension at a time, which results in information loss in some instances.

In Definitions (7) and (8), when we assume the imaginary parts as zero, then the explored theory is transformed for HFS, which is described by Torra [22]. Likewise, when we assume the CHFS as a singleton set, then the CHFS is transformed for CFS, which is described by Ramot et al. [14]. Moreover, when we assume the CHFSs as a singleton set and the imaginary part is zero, then the CHFS is transformed for FS, which is defined by Zadeh [1].

**Example 1** Let

\[
P = \left\{ \begin{array}{c}
<\chi_1, \{0.3e^{i2\pi(0.0)}, 0.5e^{i2\pi(0.2)}\}> \\
<\chi_2, \{0.2e^{i2\pi(0.4)}, 0.8e^{i2\pi(0.1)}, 1e^{i2\pi(0.5)}\}> \\
<\chi_3, \{0.7e^{i2\pi(0.9)}\}> \\
<\chi_4, \{0.8e^{i2\pi(0.3)}, 0.5e^{i2\pi(0.9)}, 0.1e^{i2\pi(0.4)}\}>
\end{array} \right\}
\]  and

\[
Q = \left\{ x < \mu_Q(x) > \mid x \in X \right\}
\]
In this portion we defined some distance measures for CHFSs. Then the operational laws of the Definition 8 are stated as follow as:

1.  
2.  
3.  

be two CHFSs. Then the operational laws of the Definition 8 are stated as follow as:

\[ d_g(P, Q) = \left[ \frac{1}{n} \sum_{k=1}^{n} \left( \frac{1}{m} \sum_{j=1}^{m} \left( |y_{P}(x_k) - y_{Q}(x_k) |^\lambda \right) + |\omega_{r_{Pj}}(x_k) - \omega_{r_{Qj}}(x_k) |^\lambda \right) \right]^{\frac{1}{\lambda}} \]  

where \( \lambda > 0 \).

**Theorem 1** The GCHND satisfies the following three properties

1. \( 0 \leq d_g(P, Q) \leq 1 \)
2. \( d_g(P, Q) = 0 \) if and only if \( P = Q \)
3. \( d_g(P, Q) = d_g(Q, P) \).

**Proof 1**

1. Since \( \frac{1}{m} \sum_{j=1}^{m} |y_{P}(x_k) - y_{Q}(x_k) |^\lambda \in [0, 1] \), \( \frac{1}{m} \sum_{j=1}^{m} |\omega_{r_{Pj}}(x_k) - \omega_{r_{Qj}}(x_k) |^\lambda \in [0, 1] \) then for \( k = 1 \)

\[ \frac{1}{2} \left( \frac{1}{m} \sum_{j=1}^{m} \left( |y_{P}(x_1) - y_{Q}(x_1) |^\lambda + |\omega_{r_{Pj}}(x_1) - \omega_{r_{Qj}}(x_1) |^\lambda \right) \right) \in [0, 1] \]

For \( k = 2 \)

\[ \frac{1}{2} \left( \frac{1}{m} \sum_{j=1}^{m} \left( |y_{P}(x_2) - y_{Q}(x_2) |^\lambda + |\omega_{r_{Pj}}(x_2) - \omega_{r_{Qj}}(x_2) |^\lambda \right) \right) \in [0, 1] \]

By continuing this process we get

\[ \sum_{k=1}^{n} \frac{1}{2} \left( \frac{1}{m} \sum_{j=1}^{m} \left( |y_{P}(x_k) - y_{Q}(x_k) |^\lambda + |\omega_{r_{Pj}}(x_k) - \omega_{r_{Qj}}(x_k) |^\lambda \right) \right) \in m[0, 1] \]

\[ \Rightarrow 0 \leq \sum_{k=1}^{n} \frac{1}{2} \left( \frac{1}{m} \sum_{j=1}^{m} \left( |y_{P}(x_k) - y_{Q}(x_k) |^\lambda + |\omega_{r_{Pj}}(x_k) - \omega_{r_{Qj}}(x_k) |^\lambda \right) \right) \leq m \]

**Definition 9** Let \( P \) and \( Q \) be two CHFSs on set \( X \). Then distance measure between \( P \) and \( Q \) is designated by \( d_g(P, Q) \), which satisfies the following properties:

1. \( 0 \leq d_g(P, Q) \leq 1 \)
2. \( d_g(P, Q) = 0 \) if and only if \( P = Q \)
3. \( d_g(P, Q) = d_g(Q, P) \).

**Definition 10** Let \( P \) and \( Q \) be two CHFSs on set \( X \). Then SM between \( P \) and \( Q \) is designated by \( s_g(P, Q) \), which satisfies the following properties:

1. \( 0 \leq s_g(P, Q) \leq 1 \)
2. \( s_g(P, Q) = 1 \) if and only if \( P = Q \)
3. \( s_g(P, Q) = s_g(Q, P) \).

**Remark 1**

1. If \( d_g \) is the distance measure between two CHFSs \( P \) and \( Q \), then \( s_g(P, Q) = 1 - d_g(P, Q) \) is the SM between CHFSs \( P \) and \( Q \).
2. If \( s_g \) is the SM between two CHFSs \( P \) and \( Q \), then \( d_g(P, Q) = 1 - s_g(P, Q) \) is the distance measure between CHFSs \( P \) and \( Q \).

**Definition 11** Let \( P \) and \( Q \) be two CHFSs on set \( X \). Then generalized complex hesitant normalized distance (GCHND) is defined as follows:
\[
\Rightarrow 0 \leq \frac{1}{n} \sum_{k=1}^{n} \frac{1}{2} \left( \frac{1}{m} \sum_{j=1}^{m} \left( |y_{pk}(x_k) - y_{qk}(x_k)|^2 \right) \right)
\]
\[
+ |\omega_{yp}(x_k) - \omega_{yq}(x_k)|^4 \right) \right) \leq 1
\]
\[
\Rightarrow 0 \leq d_c(P, Q) \leq 1
\]

2. By definition we have \(d_c(P, Q) = \left[ \frac{1}{n} \sum_{k=1}^{n} \left( \frac{1}{m} \sum_{j=1}^{m} \left( |y_{pk}(x_k) - y_{qk}(x_k)|^4 + |\omega_{yp}(x_k) - \omega_{yq}(x_k)|^4 \right) \right) \right]^{\frac{1}{2}} \]

Suppose that \(d_c(P, Q) = 0\) then

\[
\Rightarrow \frac{1}{n} \sum_{k=1}^{n} \left( \frac{1}{m} \sum_{j=1}^{m} \left( |y_{pk}(x_k) - y_{qk}(x_k)|^4 + |\omega_{yp}(x_k) - \omega_{yq}(x_k)|^4 \right) \right) = 0
\]
\[
\Rightarrow \frac{1}{m} \sum_{j=1}^{m} \left( |y_{pk}(x_k) - y_{qk}(x_k)|^4 = 0, \right.
\]
\[
\Rightarrow \frac{1}{m} \sum_{j=1}^{m} |\omega_{yp}(x_k) - \omega_{yq}(x_k)|^4 = 0
\]
\[
\Rightarrow |y_{pk}(x_k) - y_{qk}(x_k)| = 0, |\omega_{yp}(x_k) - \omega_{yq}(x_k)| = 0
\]
\[
\Rightarrow y_{pk}(x_k) = y_{qk}(x_k), \omega_{yp}(x_k) = \omega_{yq}(x_k) \forall x_k \in X
\]
\[
\Rightarrow \mu_p(x) = \mu_q(x)
\]
\[
\Rightarrow P = Q
\]

Conversely suppose that \(P = Q\) then

\[
\Rightarrow \frac{1}{n} \sum_{k=1}^{n} \left( \frac{1}{m} \sum_{j=1}^{m} \left( |y_{pk}(x_k) - y_{qk}(x_k)|^4 + |\omega_{yp}(x_k) - \omega_{yq}(x_k)|^4 \right) \right) \right) \]
\[
\Rightarrow d_c(P, Q) = 0.
\]

\textbf{Remark 2}

1. If \(\lambda = 1\), then GCHND become a Hamming complex hesitant normalized distance (HCHND) between \(P\) and \(Q\) i.e.

\[
d_c(P, Q) = \frac{1}{n} \sum_{k=1}^{n} \frac{1}{2} \left( \frac{1}{m} \sum_{j=1}^{m} \left( |y_{pk}(x_k) - y_{qk}(x_k)|^2 \right)
\]
\[
+ |\omega_{yp}(x_k) - \omega_{yq}(x_k)|^4 \right) \right) \]
\[
\Rightarrow d_c(P, Q) = 0.
\]

\textbf{Definition 12} Let \(P\) and \(Q\) be two CHFSs on set \(X\) and \(w_k\) be a weight for each \(x \in X\) such that \(\sum_{k=1}^{n} w_k = 1\). Then, weighted generalized complex hesitant normalized distance (WGCHND) are defined as follows

\[
d_{cw}(P, Q) = \left[ \sum_{k=1}^{n} w_k \frac{1}{n} \left( \frac{1}{m} \sum_{j=1}^{m} \left( |y_{pk}(x_k) - y_{qk}(x_k)|^2 + |\omega_{yp}(x_k) - \omega_{yq}(x_k)|^4 \right) \right)^{\frac{1}{2}} \right]
\]
\[
\Rightarrow \mu_{p}(x) = \mu_{q}(x)
\]
\[
\Rightarrow P = Q.
\]

where \(\lambda > 0\).

\textbf{Remark 3} 1. If \(\lambda = 1\) then WGCHND become the weighted Hamming complex hesitant normalized distance (WHCHND) between \(P\) and \(Q\) that is,

\[
d_{cwp}(P, Q) = \sum_{k=1}^{n} \frac{1}{2} \left( \frac{1}{m} \sum_{j=1}^{m} \left( |y_{pk}(x_k) - y_{qk}(x_k)|^2 + |\omega_{yp}(x_k) - \omega_{yq}(x_k)|^4 \right) \right)^{\frac{1}{2}}
\]
\[
\Rightarrow d_c(P, Q) = 0.
\]
2. If \( \lambda = 2 \) then WGCHND become the weighted Euclidean complex hesitant normalized distance (WECHND) between \( P \) and \( Q \) that is,

\[
d_{cw}(P, Q) = \left[ \sum_{k=1}^{n} w_k \frac{1}{2} \left( \frac{1}{m} \sum_{j=1}^{m} \left( |y_j(x_k) - y_Q(x_k)| \right)^2 + |\omega_{r_j}(x_k) - \omega_{r_Q}(x_k)| \right) \right]^{\frac{1}{2}}
\]

(10)

**Definition 13** Let \( Q \) be a CHFS on \( X = \{x_1, x_2, x_3, \ldots, x_n \} \). Then for any \( x_k \in X \), \( m(\mu_Q(x_k)) \) is the length of \( \mu_Q(x_k) \). We define the hesitant degree of \( \mu_Q(x_k) \) as \( u_c(\mu_Q(x_k)) = 1 - \frac{1}{m(\mu_Q(x_k))} \) and the hesitant degree of \( Q \) is defined as \( u_c(Q) = \frac{1}{n} \sum_{k=1}^{n} u_c(\mu_Q(x_k)) \).

**Definition 14** Let \( P \) and \( Q \) be two CHFSs on set \( X \). Then we defined GCHND including the hesitant degree between \( P \) and \( Q \) as

\[
d_{cgw}(P, Q) = \left[ \frac{1}{2n} \sum_{k=1}^{n} \left( u_c(\mu_P(x_k)) - u_c(\mu_Q(x_k)) \right)^2 \right]^{\frac{1}{2}} + \frac{1}{2} \left( \frac{1}{m} \sum_{j=1}^{m} \left( |y_j(x_k) - y_Q(x_k)| \right)^2 + |\omega_{r_j}(x_k) - \omega_{r_Q}(x_k)| \right) \]

(11)

where \( \lambda > 0 \).

**Remark 4** If \( \lambda = 1 \), then GCHND including hesitant degree become HCHND including the hesitant degree between \( P \) and \( Q \) that is,

\[
d_{chw}(P, Q) = \left[ \frac{1}{2n} \sum_{k=1}^{n} \left( u_c(\mu_P(x_k)) - u_c(\mu_Q(x_k)) \right)^2 \right]^{\frac{1}{2}} + \frac{1}{2} \left( \frac{1}{m} \sum_{j=1}^{m} \left( |y_j(x_k) - y_Q(x_k)| \right)^2 + |\omega_{r_j}(x_k) - \omega_{r_Q}(x_k)| \right) \]

(12)

2. If \( \lambda = 2 \), then GCHND including the hesitant degree become ECHND including the hesitant degree between \( P \) and \( Q \) that is,

\[
d_{cw}(P, Q) = \left[ \frac{1}{2n} \sum_{k=1}^{n} \left( u_c(\mu_P(x_k)) - u_c(\mu_Q(x_k)) \right)^2 \right]^{\frac{1}{2}} + \frac{1}{2} \left( \frac{1}{m} \sum_{j=1}^{m} \left( |y_j(x_k) - y_Q(x_k)| \right)^2 + |\omega_{r_j}(x_k) - \omega_{r_Q}(x_k)| \right) \]

(13)

**Definition 15** Let \( P \) and \( Q \) be two CHFSs on \( X \) and \( w_k \) be a weight for each \( x_k \in X \) such that \( \sum_{k=1}^{n} w_k = 1 \). Then we defined WGCHND including hesitant degree between \( P \) and \( Q \) as

\[
d_{cwg}(P, Q) = \left[ \frac{1}{2n} \sum_{k=1}^{n} w_k \left( |u_c(\mu_P(x_k)) - u_c(\mu_Q(x_k))| \right)^2 \right]^{\frac{1}{2}} + \frac{1}{2} \left( \frac{1}{m} \sum_{j=1}^{m} \left( |y_j(x_k) - y_Q(x_k)| \right)^2 + |\omega_{r_j}(x_k) - \omega_{r_Q}(x_k)| \right) \]

(14)

**Remark 5** If \( \lambda = 1 \), then WGCHND including hesitant degree become WHCHND including hesitant degree between \( P \) and \( Q \) that is,

\[
d_{cw}(P, Q) = \left[ \frac{1}{2n} \sum_{k=1}^{n} w_k \left( |u_c(\mu_P(x_k)) - u_c(\mu_Q(x_k))| \right)^2 \right]^{\frac{1}{2}} + \frac{1}{2} \left( \frac{1}{m} \sum_{j=1}^{m} \left( |y_j(x_k) - y_Q(x_k)| \right)^2 + |\omega_{r_j}(x_k) - \omega_{r_Q}(x_k)| \right) \]

(15)

2. If \( \lambda = 2 \), then WGCHND including hesitant degree become WECHND including hesitant degree between \( P \) and \( Q \) that is,
\[ d_{\text{GCHND}}(P, Q) = \left( \frac{1}{2n} \sum_{k=1}^{n} w_k \left( |u_c(\mu_P(x_k)) - u_c(\mu_Q(x_k))|^2 \right) \right)^{\frac{1}{2}} + \frac{1}{2} \left( \frac{1}{m} \sum_{j=1}^{m} \left( |\gamma_P(x_k) - \gamma_Q(x_k)|^2 + |\omega_{\gamma_P}(x_k) - \omega_{\gamma_Q}(x_k)|^2 \right) \right)^{\frac{1}{2}} \]

(16)

If we consider different preference between the hesitation degrees and the membership values then the distance measures are defined as

**Definition 16** Let \( P \) and \( Q \) be two CHFSs on \( X \). Then we defined GCHND including hesitant degree with preference between \( P \) and \( Q \) as

\[ d_{\text{GCHND}}(P, Q) = \left[ \frac{1}{2n} \sum_{k=1}^{n} \alpha_c \left( |u_c(\mu_P(x_k)) - u_c(\mu_Q(x_k))|^2 \right) \right]^{\frac{1}{2}} + \beta_c \left( \frac{1}{m} \sum_{j=1}^{m} \left( |\gamma_P(x_k) - \gamma_Q(x_k)|^2 + |\omega_{\gamma_P}(x_k) - \omega_{\gamma_Q}(x_k)|^2 \right) \right]^{\frac{1}{2}} \]

(17)

where \( \lambda > 0, 0 \leq \alpha_c, \beta_c \leq 1 \) and \( \alpha_c + \beta_c = 1 \).

**Remark 6** 1. If \( \lambda = 1 \), then GCHND including hesitant degree with the preference become HCHND including hesitant degree with the preference between \( P \) and \( Q \) that is,

\[ d_{\text{HCHND}}(P, Q) = \frac{1}{2n} \sum_{k=1}^{n} \alpha_c \left( |u_c(\mu_P(x_k)) - u_c(\mu_Q(x_k))| \right) \]

\[ + \beta_c \left( \frac{1}{m} \sum_{j=1}^{m} \left( |\gamma_P(x_k) - \gamma_Q(x_k)| \right) \right) \]

\[ + |\omega_{\gamma_P}(x_k) - \omega_{\gamma_Q}(x_k)| \]

(18)

2. If \( \lambda = 2 \) then GCHND including hesitant degree with preference become ECHND including hesitant degree with preference between \( P \) and \( Q \) that is,

\[ d_{\text{ECHND}}(P, Q) = \frac{1}{2n} \sum_{k=1}^{n} \alpha_c \left( |u_c(\mu_P(x_k)) - u_c(\mu_Q(x_k))|^2 \right) \]

\[ + \beta_c \left( \frac{1}{m} \sum_{j=1}^{m} \left( |\gamma_P(x_k) - \gamma_Q(x_k)|^2 + |\omega_{\gamma_P}(x_k) - \omega_{\gamma_Q}(x_k)|^2 \right) \right) \]

(19)

when \( \alpha_c = 0 \), it means that we are not considering the hesitation degree and \( d_{\text{GCHND}}, d_{\text{ECHND}}, d_{\text{ECHND}} \) become distance measures \( d_{\text{G}, d_{\text{h}}, d_{\text{eh}}} \), respectively.

**Definition 17** Let \( P \) and \( Q \) be two CHFSs on \( X \) and \( w_k \) be a weight for each \( x_k \in X \) such that \( \sum_{k=1}^{n} w_k = 1 \).

Then we defined WGCHND including hesitant degree with preference between \( P \) and \( Q \) as

\[ d_{\text{WGCHND}}(P, Q) = \left[ \frac{1}{2n} \sum_{k=1}^{n} \alpha_c \left( |u_c(\mu_P(x_k)) - u_c(\mu_Q(x_k))| \right) \right]^{\frac{1}{2}} + \beta_c \left( \frac{1}{m} \sum_{j=1}^{m} \left( |\gamma_P(x_k) - \gamma_Q(x_k)| \right) \right) \]

\[ + |\omega_{\gamma_P}(x_k) - \omega_{\gamma_Q}(x_k)| \]

(20)

**Remark 7** 1. If \( \lambda = 1 \), then WGCHND including hesitant degree with the preference become WHCHND including hesitant degree with the preference between \( P \) and \( Q \) that is,

\[ d_{\text{WHCHND}}(P, Q) = \frac{1}{2n} \sum_{k=1}^{n} \alpha_c \left( |u_c(\mu_P(x_k)) - u_c(\mu_Q(x_k))| \right) \]

\[ + \beta_c \left( \frac{1}{m} \sum_{j=1}^{m} \left( |\gamma_P(x_k) - \gamma_Q(x_k)| \right) \right) \]

\[ + |\omega_{\gamma_P}(x_k) - \omega_{\gamma_Q}(x_k)| \]

(21)

2. If \( \lambda = 2 \), then GCHND including hesitant degree with preference become ECHND including hesitant degree with preference between \( P \) and \( Q \) that is,
\[ d_{c_{eq}}(P, Q) = \left[ \frac{1}{2} \sum_{k=1}^{n} \omega_k \left( \alpha_c \left( |u_c(\mu_P(x_k)) - u_c(\mu_Q(x_k))|^2 \right) \right. \right. \]
\[ \left. \left. + \frac{\beta}{2} \left( \sum_{j=1}^{m} \left( |\gamma_{Pj}(x_k) - \gamma_{Qj}(x_k)|^2 \right) \right) \right]^{\frac{1}{2}} \] (22)

**Definition 18** Let \( P \) and \( Q \) be two CHFSs on \( X \). Then we defined modified generalized complex hesitant normalized distance (MGCHND) between \( P \) and \( Q \) as follows

\[ d_{c_{eq}}(P, Q) = \left[ \frac{1}{n} \sum_{k=1}^{n} \frac{1}{2} \left( \frac{1}{m} \sum_{j=1}^{m} \left( |\gamma_{Pj}(x_k) - \gamma_{Qj}(x_k)|^2 \right) \right. \right. \]
\[ \left. \left. + \frac{\omega_{Pj}(x_k) - \omega_{Qj}(x_k)}{\omega_{Pj}(x_k) + \omega_{Qj}(x_k)} \right) \right]^{\frac{1}{2}} \] (23)

where \( \lambda > 0 \).

**Remark 8.1** If \( \lambda = 1 \), then MGCHND become the modified Hamming complex hesitant normalized distance (MHCHND) between \( P \) and \( Q \) that is,

\[ d_{c_{eq}}(P, Q) = \left[ \frac{1}{n} \sum_{k=1}^{n} \frac{1}{2} \left( \frac{1}{m} \sum_{j=1}^{m} \left( |\gamma_{Pj}(x_k) - \gamma_{Qj}(x_k)|^2 \right) \right. \right. \]
\[ \left. \left. + \frac{\omega_{Pj}(x_k) - \omega_{Qj}(x_k)}{\omega_{Pj}(x_k) + \omega_{Qj}(x_k)} \right) \right]^{\frac{1}{2}} \] (24)

1. If \( \lambda = 2 \), then MGCHND become modified Euclidean complex hesitant normalized distance (MECHND) between \( P \) and \( Q \) i.e.

\[ d_{c_{eq}}(P, Q) = \left[ \frac{1}{n} \sum_{k=1}^{n} \frac{1}{2} \left( \frac{1}{m} \sum_{j=1}^{m} \left( |\gamma_{Pj}(x_k) - \gamma_{Qj}(x_k)|^2 \right) \right. \right. \]
\[ \left. \left. + \frac{\omega_{Pj}(x_k) - \omega_{Qj}(x_k)}{\omega_{Pj}(x_k) + \omega_{Qj}(x_k)} \right) \right]^{\frac{1}{2}} \] (25)

**Definition 19** Let \( P \) and \( Q \) be two CHFSs on \( X \) and \( \omega_k \) be a weight for each \( x_k \in X \) such that \( \sum_{k=1}^{n} \omega_k = 1 \). Then we defined modified generalized complex hesitant normalized distance (MGCHND) between \( P \) and \( Q \) as follows

\[ d_{c_{eq}}(P, Q) = \left[ \sum_{k=1}^{n} \omega_k \left( \frac{1}{2} \left( \sum_{j=1}^{m} \left( \frac{\gamma_{Pj}(x_k) - \gamma_{Qj}(x_k)}{\gamma_{Pj}(x_k) + \gamma_{Qj}(x_k)} \right) \right) \right]^{\frac{1}{2}} \] (26)

where \( \lambda > 0 \).

**Remark 9.1** If \( \lambda = 1 \) then MGCHND become modified weighted Hamming complex hesitant normalized distance (MWHCHND) between \( P \) and \( Q \) that is,

\[ d_{c_{eq}}(P, Q) = \left[ \sum_{k=1}^{n} \omega_k \left( \frac{1}{2} \left( \sum_{j=1}^{m} \left( \frac{\gamma_{Pj}(x_k) - \gamma_{Qj}(x_k)}{\gamma_{Pj}(x_k) + \gamma_{Qj}(x_k)} \right) \right) \right]^{\frac{1}{2}} \] (27)

2. If \( \lambda = 2 \), then MGCHND become modified weighted Euclidean complex hesitant normalized distance (MWECHND) between \( P \) and \( Q \) i.e.

\[ d_{c_{eq}}(P, Q) = \left[ \sum_{k=1}^{n} \omega_k \left( \frac{1}{2} \left( \sum_{j=1}^{m} \left( \frac{\gamma_{Pj}(x_k) - \gamma_{Qj}(x_k)}{\gamma_{Pj}(x_k) + \gamma_{Qj}(x_k)} \right) \right) \right]^{\frac{1}{2}} \] (28)

**Definition 20** Let \( Q \) be a CHFS on \( X = \{x_1, x_2, x_3, \ldots, x_n\} \). Then for each \( x_k \in X \), \( m(\mu_Q(x_k)) \) is the cardinal number of \( \mu_Q(x_k) \). The credibility factor of \( \mu_Q(x_k) \) is given as \( c_c(\mu_Q(x_k)) = (m(\mu_Q(x_k)))^{-1} \). Now let \( P \) and \( Q \) be two CHFS. Then the credibility factor between \( Q \) and \( P \) is defined as
\[ c_c(\mu_p(x_k), \mu_Q(x_k)) = [c_c(\mu_p(x_k))c_c(\mu_Q(x_k))]^{\frac{1}{2}} \]  \quad (29)

and normalized credibility factor is defined as:

\[ c_c^*(\mu_p(x_k), \mu_Q(x_k)) = \frac{[c_c(\mu_p(x_k))c_c(\mu_Q(x_k))]^{\frac{1}{2}}}{\sum_{k=1}^{n}[c_c(\mu_p(x_k))c_c(\mu_Q(x_k))]^{\frac{1}{2}}} \]  \quad (30)

Now we define novel distance measures using credibility factor.

**Definition 21** Let \( P \) and \( Q \) be two CHFSs on \( X \). Then we defined the novel GCHND between \( P \) and \( Q \) as:

\[
d_{c_c}(P, Q) = \left[ \sum_{k=1}^{n} c_c^*(\mu_p(x_k), \mu_Q(x_k)) \left[ \frac{1}{2} \left( \frac{1}{m} \sum_{j=1}^{m} (|y_{P}(x_k) - y_{Q}(x_k)| + |\omega_{r_P}(x_k) - \omega_{r_Q}(x_k)|) \right) \right]^{\frac{1}{2}} \right]^{\frac{1}{2}} \tag{31}
\]

where \( \lambda > 0 \).

**Remark 10** 1. If \( \lambda = 1 \), then the novel GCHND become novel HCHND between \( P \) and \( Q \) as:

\[
d_{c_c}(P, Q) = \sum_{k=1}^{n} (c_c^*(\mu_p(x_k), \mu_Q(x_k))) \left[ \frac{1}{2} \left( \frac{1}{m} \sum_{j=1}^{m} (|y_{P}(x_k) - y_{Q}(x_k)| + |\omega_{r_P}(x_k) - \omega_{r_Q}(x_k)|) \right) \right]^{\frac{1}{2}} \tag{32}
\]

2. If \( \lambda = 2 \), then the novel GCHND become novel ECHND between \( P \) and \( Q \) as:

\[
d_{c_c}(P, Q) = \left[ \sum_{k=1}^{n} c_c^*(\mu_p(x_k), \mu_Q(x_k)) \left[ \frac{1}{2} \left( \frac{1}{m} \sum_{j=1}^{m} (|y_{P}(x_k) - y_{Q}(x_k)| + |\omega_{r_P}(x_k) - \omega_{r_Q}(x_k)|) \right) \right]^{\frac{1}{2}} \right]^{\frac{1}{2}} \tag{33}
\]

**Definition 22** Let \( P \) and \( Q \) be two CHFSs on \( X \) and \( w_k \) be a weight for each \( x_k \in X \) such that \( \sum_{k=1}^{n} w_k = 1 \) and \( 0 \leq w_k \leq 1 \). Then we defined the novel WGCHND as follows:

\[
d_{c_{c_{w}}}(P, Q) = \left[ \sum_{k=1}^{n} w_k \left[ c_c^*(\mu_p(x_k), \mu_Q(x_k)) \left[ \frac{1}{2} \left( \frac{1}{m} \sum_{j=1}^{m} (|y_{P}(x_k) - y_{Q}(x_k)| + |\omega_{r_P}(x_k) - \omega_{r_Q}(x_k)|) \right) \right]^{\frac{1}{2}} \right]^{\frac{1}{2}} \right]^{\frac{1}{2}} \tag{34}
\]

where \( \lambda > 0 \).

**Remark 11** 1. If \( \lambda = 1 \), then the novel WGCHND become novel WHCHND between \( P \) and \( Q \) as:

\[
d_{c_{c_{w}}}(P, Q) = \sum_{k=1}^{n} c_c^*(\mu_p(x_k), \mu_Q(x_k)) \left[ \frac{1}{2} \left( \frac{1}{m} \sum_{j=1}^{m} (|y_{P}(x_k) - y_{Q}(x_k)| + |\omega_{r_P}(x_k) - \omega_{r_Q}(x_k)|) \right) \right]^{\frac{1}{2}} \tag{35}
\]

2. If \( \lambda = 2 \), then the novel WGCHND become novel WECHND between \( P \) and \( Q \) as:

\[
d_{c_{c_{w}}}(P, Q) = \left[ \sum_{k=1}^{n} w_k \left[ c_c^*(\mu_p(x_k), \mu_Q(x_k)) \left[ \frac{1}{2} \left( \frac{1}{m} \sum_{j=1}^{m} (|y_{P}(x_k) - y_{Q}(x_k)| + |\omega_{r_P}(x_k) - \omega_{r_Q}(x_k)|) \right) \right]^{\frac{1}{2}} \right]^{\frac{1}{2}} \right]^{\frac{1}{2}} \tag{36}
\]

For the deep understanding of the relationship between the cardinalities and the values of CHFSs, we consider the conservative factor \( \alpha_{cc} \) and the risk factor \( \beta_{cc} \). By using these factors, the above defined novel weighted distance measures become:

\[
d_{c_{c_{w}}}(P, Q) = \left[ \sum_{k=1}^{n} w_k \left[ (c_c^*(\mu_p(x_k), \mu_Q(x_k)))^{\alpha_{cc}} \left[ \frac{1}{2} \left( \frac{1}{m} \sum_{j=1}^{m} (|y_{P}(x_k) - y_{Q}(x_k)| + |\omega_{r_P}(x_k) - \omega_{r_Q}(x_k)|) \right) \right]^{\frac{1}{2}} \right]^{\frac{1}{2}} \right]^{\frac{1}{2}} \tag{37}
\]

\[
d_{c_{c_{w}}}(P, Q) = \left[ \sum_{k=1}^{n} w_k \left[ c_c^*(\mu_p(x_k), \mu_Q(x_k)) \left[ \frac{1}{2} \left( \frac{1}{m} \sum_{j=1}^{m} (|y_{P}(x_k) - y_{Q}(x_k)| + |\omega_{r_P}(x_k) - \omega_{r_Q}(x_k)|) \right) \right]^{\frac{1}{2}} \right]^{\frac{1}{2}} \right]^{\frac{1}{2}} \tag{38}
\]
where $\alpha_{c}, \beta_{c} \in [0, 1]$ and $\alpha_{c} + \beta_{c} = 1$.

5 | APPLICATIONS

Here, we applied the proposed distance measure to the environment of CHFSs to show the usefulness and effectiveness of the proposed distance measures.

**Example 2** For every society, energy is a crucial factor for the socio-economic development. Thus the accurate energy strategy accomplishes economic development and environment, and so, the most suitable energy strategy selection is crucial. Suppose five energy projects represented by $S(j = 1, 2, 3, 4, 5)$ are to be invested. In the meantime, consider four attributes that are given as technological ($x_1$), environmental ($x_2$), socio-political ($x_3$), and economic ($x_4$). Consider the weight for attributes as $W = (0.15, 0.3, 0.2, 0.35)$.

After this, a batch of experts is invited to assess the performance of the five alternatives with the respect to four attributes on the notion of excellence. By using CHFSs, the assessment results are gained as Table 1.

**Table 1** Complex hesitant fuzzy decision matrix

| Alternative | $x_1$ | $x_2$ | $x_3$ | $x_4$ |
|-------------|------|------|------|------|
| $S_1$       | 0.5$r_2x^{(0.25)}$ | 0.4$r_2x^{(0.5)}$ | 0.3$r_2x^{(0.166)}$ | 0.9$r_2x^{(0.125)}$, 0.8$r_2x^{(0.25)}$, 0.7$r_2x^{(0.5)}$, 0.1$r_2x^{(1)}$ |
| $S_2$       | 0.5$r_2x^{(0.125)}$ | 0.3$r_2x^{(0.5)}$ | 0.9$r_2x^{(0.0133)}$, 0.7$r_2x^{(0.1667)}$, 0.6$r_2x^{(0.5)}$, 0.5$r_2x^{(1)}$, 0.2$r_2x^{(1)}$ |
| $S_3$       | 0.7$r_2x^{(0.5)}$ | 0.6$r_2x^{(0.1)}$ | 0.9$r_2x^{(1)}$, 0.6$r_2x^{(0.5)}$ | 0.7$r_2x^{(0.125)}$, 0.5$r_2x^{(0.25)}$, 0.3$r_2x^{(0.5)}$ |
| $S_4$       | 0.8$r_2x^{(0.5)}$, 0.7$r_2x^{(0.1)}$, 0.4$r_2x^{(0.25)}$, 0.3$r_2x^{(0.5)}$ | 0.7$r_2x^{(0.5)}$, 0.4$r_2x^{(0.125)}$, 0.2$r_2x^{(1)}$ | 0.8$r_2x^{(0.1)}$, 0.4$r_2x^{(0.1)}$ |
| $S_5$       | 0.9$r_2x^{(1)}$, 0.7$r_2x^{(0.2)}$, 0.6$r_2x^{(0.1)}$, 0.5$r_2x^{(0.0133)}$, 0.1$r_2x^{(0.5)}$ | 0.8$r_2x^{(0.1)}$, 0.7$r_2x^{(0.5)}$, 0.6$r_2x^{(0.25)}$, 0.4$r_2x^{(0.1)}$ | 0.9$r_2x^{(0.25)}$, 0.8$r_2x^{(0.1)}$, 0.7$r_2x^{(0.1667)}$, 0.6$r_2x^{(0.1667)}$, 0.3$r_2x^{(0.5)}$ |

**Table 2** Deviation between each alternative and ideal alternative and $(\alpha_{c}, \beta_{c}) = (0.9, 0.1)$

| $\lambda$ | $S_1$ | $S_2$ | $S_3$ | $S_4$ | $S_5$ | Ranking |
|-----------|------|------|------|------|------|---------|
| $\lambda = 1$ | 0.2608 | 0.2560 | 0.2650 | 0.2648 | 0.2658 | $S_2 > S_1 > S_4 > S_3 > S_5$ |
| $\lambda = 2$ | 0.2612 | 0.2587 | 0.2658 | 0.2663 | 0.2669 | $S_2 > S_1 > S_4 > S_3 > S_5$ |
| $\lambda = 6$ | 0.2629 | 0.271 | 0.2684 | 0.2731 | 0.2713 | $S_3 > S_1 > S_4 > S_2 > S_5$ |
| $\lambda = 10$ | 0.2647 | 0.2836 | 0.2705 | 0.2803 | 0.2757 | $S_1 > S_2 > S_4 > S_3 > S_5$ |

Note: The graphical interpretation for the values in Table 2 is shown in Figure 1.
decision-makers with different subjective preferences can select certain parameters according to their experiences and point of view. It indicates that the proposed parameterized distance measures are useful for the combination of subjective and objective decision-making information.

**FIGURE 1** Graphical interpretation of the Table 2 results

**TABLE 3** Deviation between each alternative and ideal alternative and \((\alpha_{cr}, \beta_{cr}) = (0.7, 0.5)\)

| \(\lambda\) | \(S_1\) | \(S_2\) | \(S_3\) | \(S_4\) | \(S_5\) | Ranking |
|-----|-----|-----|-----|-----|-----|-----|
| 1   | 0.2966 | 0.2852 | 0.2927 | 0.2961 | 0.2991 | \(S_2 > S_1 > S_4 > S_3 > S_5\) |
| 2   | 0.2968 | 0.2875 | 0.2943 | 0.2978 | 0.3012 | \(S_2 > S_1 > S_4 > S_3 > S_5\) |
| 6   | 0.2975 | 0.3000 | 0.3006 | 0.3060 | 0.3092 | \(S_2 > S_1 > S_4 > S_3 > S_5\) |
| 10  | 0.2983 | 0.3146 | 0.3061 | 0.3155 | 0.3160 | \(S_2 > S_1 > S_4 > S_3 > S_5\) |

Note: The graphical interpretation for the values in Table 3 is shown in Figure 2.

**FIGURE 2** Graphical interpretation of the established work for different values of parameters

**TABLE 4** Deviation between each alternative and ideal alternative and \((\alpha_{cr}, \beta_{cr}) = (0.5, 0.5)\)

| \(\lambda\) | \(S_1\) | \(S_2\) | \(S_3\) | \(S_4\) | \(S_5\) | Ranking |
|-----|-----|-----|-----|-----|-----|-----|
| 1   | 0.3377 | 0.3190 | 0.3244 | 0.3312 | 0.3369 | \(S_2 > S_1 > S_4 > S_3 > S_5\) |
| 2   | 0.3379 | 0.3223 | 0.3279 | 0.3334 | 0.3408 | \(S_2 > S_1 > S_4 > S_3 > S_5\) |
| 6   | 0.3389 | 0.3370 | 0.3419 | 0.3437 | 0.3538 | \(S_2 > S_1 > S_4 > S_3 > S_5\) |
| 10  | 0.3398 | 0.3523 | 0.3582 | 0.3548 | 0.3636 | \(S_2 > S_1 > S_4 > S_3 > S_5\) |

Note: The graphical interpretation for the values in Table 4 is shown in Figure 3.

**FIGURE 3** Graphical interpretation of the established work for different values of parameters

**TABLE 5** Deviation between each alternative and ideal alternative and \((\alpha_{cr}, \beta_{cr}) = (0.3, 0.7)\)

| \(\lambda\) | \(S_1\) | \(S_2\) | \(S_3\) | \(S_4\) | \(S_5\) | Ranking |
|-----|-----|-----|-----|-----|-----|-----|
| 1   | 0.3848 | 0.3585 | 0.3604 | 0.3708 | 0.3801 | \(S_2 > S_1 > S_4 > S_3 > S_5\) |
| 2   | 0.3855 | 0.3640 | 0.3673 | 0.3737 | 0.3864 | \(S_2 > S_1 > S_4 > S_3 > S_5\) |
| 6   | 0.38377 | 0.383737 | 0.3930 | 0.3869 | 0.4059 | \(S_2 > S_1 > S_4 > S_3 > S_5\) |
| 10  | 0.3914 | 0.3988 | 0.4109 | 0.4003 | 0.4191 | \(S_2 > S_1 > S_4 > S_3 > S_5\) |

Note: The graphical interpretation for the values in Table 5 is shown in Figure 4.

**FIGURE 4** Graphical interpretation of the established work for different values of parameters

**TABLE 6** Deviation between each alternative and ideal alternative and \((\alpha_{cr}, \beta_{cr}) = (0.1, 0.9)\)

| \(\lambda\) | \(S_1\) | \(S_2\) | \(S_3\) | \(S_4\) | \(S_5\) | Ranking |
|-----|-----|-----|-----|-----|-----|-----|
| 1   | 0.4390 | 0.4045 | 0.4017 | 0.4153 | 0.4393 | \(S_2 > S_1 > S_4 > S_3 > S_5\) |
| 2   | 0.4406 | 0.4143 | 0.4157 | 0.4194 | 0.4389 | \(S_2 > S_1 > S_4 > S_3 > S_5\) |
| 6   | 0.4478 | 0.442 | 0.4551 | 0.4367 | 0.4668 | \(S_2 > S_1 > S_4 > S_3 > S_5\) |
| 10  | 0.4550 | 0.4573 | 0.4797 | 0.4528 | 0.4837 | \(S_2 > S_1 > S_4 > S_3 > S_5\) |

Note: The graphical interpretation for the values in Table 5 is shown in Figure 5.
TABLE 8 Complex hesitant fuzzy decision matrix

| Alternative | $x_1$       | $x_2$       | $x_3$       | $x_4$       |
|-------------|------------|------------|------------|------------|
| $S_1$       | \{0.5, 0.4, 0.3\} | \{0.9, 0.8, 0.7, 0.1\} | \{0.5, 0.4, 0.2\} | \{0.9, 0.6, 0.5, 0.3\} |
| $S_2$       | \{0.5, 0.3\} | \{0.9, 0.7, 0.6, 0.5, 0.2\} | \{0.8, 0.6, 0.5, 0.1\} | \{0.7, 0.4, 0.3\} |
| $S_3$       | \{0.7, 0.6\} | \{0.9, 0.6\} | \{0.7, 0.5, 0.3\} | \{0.6, 0.4\} |
| $S_4$       | \{0.8, 0.7, 0.4, 0.3\} | \{0.7, 0.4, 0.2\} | \{0.8, 0.1\} | \{0.9, 0.8, 0.6\} |
| $S_5$       | \{0.9, 0.7, 0.6, 0.3, 0.1\} | \{0.8, 0.7, 0.6, 0.4\} | \{0.9, 0.8, 0.7\} | \{0.9, 0.7, 0.6, 0.3\} |

TABLE 9 Comparison between the proposed method and existing methods for Example 3 with parameter $(\alpha_{\text{cut}}, \beta_{\text{cut}}) = (0, 1)$

| Method       | Value | Score function | Ranking |
|--------------|-------|---------------|---------|
| Xu et al. [32] | $\lambda = 1$ | $d_{\text{gub}}(S, S_1) = 0.4799$, $d_{\text{gub}}(S, S_3) = 0.5027$; $d_{\text{gub}}(S, S_2) = 0.4292$ | $S_1 > S_3 > S_4 > S_1 > S_2$ |
|              |       | $d_{\text{gub}}(S, S_1) = 0.4025$, $d_{\text{gub}}(S, S_3) = 0.4292$ |         |
|              |       | $d_{\text{gub}}(S, S_1) = 0.3558$ |         |
| Singha et al. [33] | $\lambda = 1$ | $d_{\text{marg}}(S, S_1) = 0.3499$, $d_{\text{marg}}(S, S_2) = 0.3630$ | $S_1 > S_3 > S_4 > S_1 > S_2$ |
|              |       | $d_{\text{marg}}(S, S_1) = 0.2661$, $d_{\text{marg}}(S, S_3) = 0.3155$ |         |
|              |       | $d_{\text{marg}}(S, S_1) = 0.2393$ |         |
| Li et al. [35] | $\lambda = 1$ | $d_{\text{upbg}}(S, S_1) = 0.4779$, $d_{\text{upbg}}(S, S_2) = 0.5027$ | $S_1 > S_3 > S_4 > S_1 > S_2$ |
|              |       | $d_{\text{upbg}}(S, S_1) = 0.4025$, $d_{\text{upbg}}(S, S_3) = 0.4292$ |         |
|              |       | $d_{\text{upbg}}(S, S_1) = 0.3558$ |         |
| Li et al. [34] | $\lambda = 1$ | $d_{\text{uwbg}}(S, S_1) = 0.4799$, $d_{\text{uwbg}}(S, S_2) = 0.5027$ | $S_1 > S_3 > S_4 > S_1 > S_2$ |
|              |       | $d_{\text{uwbg}}(S, S_1) = 0.4025$, $d_{\text{uwbg}}(S, S_3) = 0.4292$ |         |
|              |       | $d_{\text{uwbg}}(S, S_1) = 0.3558$ |         |
| Method            | Value | Score function                     | Ranking          |
|-------------------|-------|-------------------------------------|------------------|
| Proposed method   | \( \lambda = 1 \) | \( d_{rav}(S, S_1) = 0.1960, d_{rav}(S, S_2) = 0.1863 \) | \( S_4 > S_2 > S_1 > S_5 \) |
|                   |       | \( d_{rav}(S, S_3) = 0.1863, d_{rav}(S, S_4) = 0.1775 \) |                  |
|                   |       | \( d_{rav}(S, S_5) = 0.226 \) |                  |
| Xu et al. [32]    | \( \lambda = 2 \) | \( d_{gbch}(S, S_1) = 0.5378, d_{gbch}(S, S_2) = 0.5451 \) | \( S_1 > S_5 > S_4 > S_3 > S_2 \) |
|                   |       | \( d_{gbch}(S, S_3) = 0.4366, d_{gbch}(S, S_4) = 0.5052 \) |                  |
|                   |       | \( d_{gbch}(S, S_5) = 0.4139 \) |                  |
| Singha et al. [33] | \( \lambda = 2 \) | \( d_{mavg}(S, S_1) = 0.4159, d_{mavg}(S, S_2) = 0.3630 \) | \( S_1 > S_5 > S_3 > S_2 > S_4 \) |
|                   |       | \( d_{mavg}(S, S_3) = 0.2978, d_{mavg}(S, S_4) = 0.3179 \) |                  |
|                   |       | \( d_{mavg}(S, S_5) = 0.3005 \) |                  |
| Li et al. [35]    | \( \lambda = 2 \) | \( d_{upfg}(S, S_1) = 0.5378, d_{upfg}(S, S_2) = 0.5451 \) | \( S_1 > S_5 > S_3 > S_2 > S_4 \) |
|                   |       | \( d_{upfg}(S, S_3) = 0.4366, d_{upfg}(S, S_4) = 0.5052 \) |                  |
|                   |       | \( d_{upfg}(S, S_5) = 0.4129 \) |                  |
| Li et al. [34]    | \( \lambda = 2 \) | \( d_{wchg}(S, S_1) = 0.5378, d_{wchg}(S, S_2) = 0.5451 \) | \( S_1 > S_5 > S_3 > S_2 > S_4 \) |
|                   |       | \( d_{wchg}(S, S_3) = 0.4366, d_{wchg}(S, S_4) = 0.5052 \) |                  |
|                   |       | \( d_{wchg}(S, S_5) = 0.4139 \) |                  |
| Proposed method   | \( \lambda = 2 \) | \( d_{rav}(S, S_1) = 0.1964, d_{rav}(S, S_2) = 0.1873 \) | \( S_2 > S_4 > S_1 > S_5 \) |
|                   |       | \( d_{rav}(S, S_3) = 0.1937, d_{rav}(S, S_4) = 0.1891 \) |                  |
|                   |       | \( d_{rav}(S, S_5) = 0.2276 \) |                  |
| Xu et al. [32]    | \( \lambda = 6 \) | \( d_{gbch}(S, S_1) = 0.6604, d_{gbch}(S, S_2) = 0.6561 \) | \( S_1 > S_4 > S_5 > S_3 > S_2 \) |
|                   |       | \( d_{gbch}(S, S_3) = 0.5156, d_{gbch}(S, S_4) = 0.6704 \) |                  |
|                   |       | \( d_{gbch}(S, S_5) = 0.5699 \) |                  |
| Singha et al. [33] | \( \lambda = 6 \) | \( d_{mavg}(S, S_1) = 0.5658, d_{mavg}(S, S_2) = 0.4714 \) | \( S_2 > S_3 > S_5 > S_4 \) |
|                   |       | \( d_{mavg}(S, S_3) = 0.3778, d_{mavg}(S, S_4) = 0.5859 \) |                  |
|                   |       | \( d_{mavg}(S, S_5) = 0.4807 \) |                  |
| Li et al. [35]    | \( \lambda = 6 \) | \( d_{upfg}(S, S_1) = 0.6599, d_{upfg}(S, S_2) = 0.6476 \) | \( S_1 > S_4 > S_5 > S_3 > S_2 \) |
|                   |       | \( d_{upfg}(S, S_3) = 0.5156, d_{upfg}(S, S_4) = 0.6704 \) |                  |
|                   |       | \( d_{upfg}(S, S_5) = 0.5699 \) |                  |
| Li et al. [34]    | \( \lambda = 6 \) | \( d_{wchg}(S, S_1) = 0.6604, d_{wchg}(S, S_2) = 0.6561 \) | \( S_1 > S_5 > S_3 > S_2 > S_4 \) |
|                   |       | \( d_{wchg}(S, S_3) = 0.5156, d_{wchg}(S, S_4) = 0.6704 \) |                  |
|                   |       | \( d_{wchg}(S, S_5) = 0.5699 \) |                  |
| Proposed method   | \( \lambda = 6 \) | \( d_{rav}(S, S_1) = 0.1979, d_{rav}(S, S_2) = 0.191 \) | \( S_2 > S_3 > S_5 > S_4 \) |
|                   |       | \( d_{rav}(S, S_3) = 0.1968, d_{rav}(S, S_4) = 0.2149 \) |                  |
|                   |       | \( d_{rav}(S, S_5) = 0.2337 \) |                  |
| Xu et al. [32]    | \( \lambda = 10 \) | \( d_{gbch}(S, S_1) = 0.7160, d_{gbch}(S, S_2) = 0.7140 \) | \( S_1 > S_5 > S_4 > S_3 > S_2 \) |
|                   |       | \( d_{gbch}(S, S_3) = 0.5639, d_{gbch}(S, S_4) = 0.7492 \) |                  |
|                   |       | \( d_{gbch}(S, S_5) = 0.6771 \) |                  |
| Singha et al. [33] | \( \lambda = 10 \) | \( d_{mavg}(S, S_1) = 0.6400, d_{mavg}(S, S_2) = 0.5207 \) | \( S_1 > S_2 > S_5 > S_3 > S_4 \) |
|                   |       | \( d_{mavg}(S, S_3) = 0.4210, d_{mavg}(S, S_4) = 0.6583 \) |                  |
|                   |       | \( d_{mavg}(S, S_5) = 0.5796 \) |                  |

(Continues)
| Method                  | Value | Score function | Ranking       |
|------------------------|-------|----------------|---------------|
| Li et al. [35]         | λ = 10| \(d_{wphg}(S, S_1) = 0.7213, d_{wphg}(S, S_2) = 0.7046\) | \(S_1 > S_3 > S_4 > S_2 > S_5\) |
|                        |       | \(d_{wphg}(S, S_3) = 0.5607, d_{wphg}(S, S_4) = 0.7373\) |               |
|                        |       | \(d_{wphg}(S, S_5) = 0.6537\) |               |
| Li et al. [34]         | λ = 10| \(d_{wchg}(S, S_1) = 0.7160, d_{wchg}(S, S_2) = 0.7140\) | \(S_1 > S_3 > S_2 > S_4 > S_5\) |
|                        |       | \(d_{wchg}(S, S_3) = 0.5639, d_{wchg}(S, S_4) = 0.7492\) |               |
|                        |       | \(d_{wchg}(S, S_5) = 0.6771\) |               |
| Proposed method        | λ = 10| \(d_{wcm}(S, S_1) = 0.1993, d_{wcm}(S, S_2) = 0.1943\) | \(S_1 > S_3 > S_4 > S_2 > S_5\) |
|                        |       | \(d_{wcm}(S, S_3) = 0.2262\) |               |
|                        |       | \(d_{wcm}(S, S_4) = 0.2389\) |               |

| Method                  | Value | Score function | Ranking       |
|------------------------|-------|----------------|---------------|
| Xu et al. [32]         | λ = 1 | Cannot be calculated | Cannot be calculated |
| Singha et al. [33]     | λ = 1 | Cannot be calculated | Cannot be calculated |
| Li et al. [35]         | λ = 1 | \(d_{wphg}(S, S_1) = 0.6965, d_{wphg}(S, S_2) = 0.6788\) | \(S_3 > S_4 > S_2 > S_1 > S_5\) |
|                        |       | \(d_{wphg}(S, S_3) = 0.5203, d_{wphg}(S, S_4) = 0.6242\) |               |
|                        |       | \(d_{wphg}(S, S_5) = 0.7023\) |               |
| Li et al. [34]         | λ = 1 | \(d_{wchg}(S, S_1) = 0.26126, d_{wchg}(S, S_2) = 0.26127\) | \(S_1 > S_3 > S_2 > S_4 > S_5\) |
|                        |       | \(d_{wchg}(S, S_3) = 0.2631, d_{wchg}(S, S_4) = 0.2628\) |               |
|                        |       | \(d_{wchg}(S, S_5) = 0.2582\) |               |
| Proposed method        | λ = 1 | \(d_{wcm}(S, S_1) = 0.2391, d_{wcm}(S, S_2) = 0.2362\) | \(S_2 > S_1 > S_3 > S_2 > S_5\) |
|                        |       | \(d_{wcm}(S, S_3) = 0.2456, d_{wcm}(S, S_4) = 0.2393\) |               |
|                        |       | \(d_{wcm}(S, S_5) = 0.2481\) |               |
| Xu et al. [32]         | λ = 2 | Cannot be calculated | Cannot be calculated |
| Singha et al. [33]     | λ = 2 | Cannot be calculated | Cannot be calculated |
| Li et al. [35]         | λ = 2 | \(d_{wphg}(S, S_1) = 0.7057, d_{wphg}(S, S_2) = 0.691\) | \(S_1 > S_4 > S_2 > S_1 > S_5\) |
|                        |       | \(d_{wphg}(S, S_3) = 0.5283, d_{wphg}(S, S_4) = 0.6375\) |               |
|                        |       | \(d_{wphg}(S, S_5) = 0.7159\) |               |
| Li et al. [34]         | λ = 2 | \(d_{wchg}(S, S_1) = 0.2622, d_{wchg}(S, S_2) = 0.2645\) | \(S_3 > S_1 > S_2 > S_3 > S_4\) |
|                        |       | \(d_{wchg}(S, S_3) = 0.2636, d_{wchg}(S, S_4) = 0.2646\) |               |
|                        |       | \(d_{wchg}(S, S_5) = 0.2585\) |               |
| Proposed method        | λ = 2 | \(d_{wcm}(S, S_1) = 0.2396, d_{wcm}(S, S_2) = 0.2385\) | \(S_2 > S_1 > S_3 > S_2 > S_5\) |
|                        |       | \(d_{wcm}(S, S_3) = 0.2462, d_{wcm}(S, S_4) = 0.2398\) |               |
|                        |       | \(d_{wcm}(S, S_5) = 0.2489\) |               |
| Xu et al. [32]         | λ = 6 | Cannot be calculated | Cannot be calculated |
| Singha et al. [33]     | λ = 6 | Cannot be calculated | Cannot be calculated |
| Li et al. [35]         | λ = 6 | \(d_{wphg}(S, S_1) = 0.7206, d_{wphg}(S, S_2) = 0.7205\) | \(S_3 > S_4 > S_2 > S_1 > S_5\) |
|                        |       | \(d_{wphg}(S, S_3) = 0.5541, d_{wphg}(S, S_4) = 0.666\) |               |
|                        |       | \(d_{wphg}(S, S_5) = 0.7358\) |               |
Comparison between the proposed method and the existing methods for Example 3 with parameter $(\alpha_s, \beta_s) = (0.7, 0.3)$

| Method          | Value | Score function                                                                 | Ranking          |
|-----------------|-------|--------------------------------------------------------------------------------|------------------|
| Xu et al. [32]  | $\lambda = 1$ | $d_{\text{ucbg}}(S, S_1) = 0.3021$, $d_{\text{ucbg}}(S, S_2) = 0.2777$ | $S_1 > S_2 > S_3 > S_4 > S_5$ |
| Li et al. [34]  | $\lambda = 1$ | $d_{\text{ucbg}}(S, S_1) = 0.648$, $d_{\text{ucbg}}(S, S_2) = 0.6396$ | $S_1 > S_2 > S_3 > S_4 > S_5$ |
| Li et al. [35]  | $\lambda = 1$ | $d_{\text{ucbg}}(S, S_1) = 0.4941$, $d_{\text{ucbg}}(S, S_2) = 0.5808$ | $S_1 > S_2 > S_3 > S_4 > S_5$ |
| Li et al. [34]  | $\lambda = 1$ | $d_{\text{ucbg}}(S, S_1) = 0.2869$, $d_{\text{ucbg}}(S, S_2) = 0.2904$ | $S_1 > S_2 > S_3 > S_4 > S_5$ |
| Proposed method | $\lambda = 1$ | $d_{\text{ucbg}}(S, S_1) = 0.2286$, $d_{\text{ucbg}}(S, S_2) = 0.2233$ | $S_1 > S_2 > S_3 > S_4 > S_5$ |
| Xu et al. [32]  | $\lambda = 2$ | Cannot be calculated                                                               | Cannot be calculated |
| Li et al. [35]  | $\lambda = 2$ | $d_{\text{ucbg}}(S, S_1) = 0.672$, $d_{\text{ucbg}}(S, S_2) = 0.6614$ | $S_1 > S_2 > S_3 > S_4 > S_5$ |
| Method                  | Value | Score function                                      | Ranking         |
|------------------------|-------|-----------------------------------------------------|-----------------|
| Li et al. [34]         | \( \lambda = 2 \) | \( d_{\text{wdg}}(S, S_1) = 0.3003, d_{\text{wdg}}(S, S_2) = 0.3021 \) | \( S_1 > S_2 > S_1 > S_2 \) |
|                        |       | \( d_{\text{wdg}}(S, S_1) = 0.2880, d_{\text{wdg}}(S, S_2) = 0.2940 \) |                 |
|                        |       | \( d_{\text{wdg}}(S, S_1) = 0.2752 \) |                 |
| Proposed method        | \( \lambda = 2 \) | \( d_{\text{wdg}}(S, S_1) = 0.2289, d_{\text{wdg}}(S, S_2) = 0.2248 \) | \( S_4 > S_2 > S_1 > S_2 \) |
|                        |       | \( d_{\text{wdg}}(S, S_1) = 0.2331, d_{\text{wdg}}(S, S_1) = 0.2211 \) |                 |
|                        |       | \( d_{\text{wdg}}(S, S_1) = 0.2439 \) |                 |
| Xu et al. [32]         | \( \lambda = 6 \) | Cannot be calculated | Cannot be calculated |
| Singha et al. [33]     | \( \lambda = 6 \) | Cannot be calculated | Cannot be calculated |
| Li et al. [35]         | \( \lambda = 6 \) | \( d_{\text{apth}}(S, S_1) = 0.7092, d_{\text{apth}}(S, S_2) = 0.7073 \) | \( S_1 > S_2 > S_1 > S_2 \) |
|                        |       | \( d_{\text{apth}}(S, S_1) = 0.5466, d_{\text{apth}}(S, S_1) = 0.667 \) |                 |
|                        |       | \( d_{\text{apth}}(S, S_1) = 0.7172 \) |                 |
| Li et al. [34]         | \( \lambda = 6 \) | \( d_{\text{wdg}}(S, S_1) = 0.3083, d_{\text{wdg}}(S, S_2) = 0.3186 \) | \( S_3 > S_1 > S_1 > S_2 \) |
|                        |       | \( d_{\text{wdg}}(S, S_1) = 0.2021, d_{\text{wdg}}(S, S_1) = 0.3073 \) |                 |
|                        |       | \( d_{\text{wdg}}(S, S_1) = 0.2738 \) |                 |
| Proposed method        | \( \lambda = 6 \) | \( d_{\text{wdg}}(S, S_1) = 0.2298, d_{\text{wdg}}(S, S_1) = 0.2308 \) | \( S_4 > S_1 > S_1 > S_3 \) |
|                        |       | \( d_{\text{wdg}}(S, S_1) = 0.2349, d_{\text{wdg}}(S, S_1) = 0.2246 \) |                 |
|                        |       | \( d_{\text{wdg}}(S, S_1) = 0.2478 \) |                 |
| Xu et al. [32]         | \( \lambda = 10 \) | Cannot be calculated | Cannot be calculated |
| Singha et al. [33]     | \( \lambda = 10 \) | Cannot be calculated | Cannot be calculated |
| Li et al. [35]         | \( \lambda = 10 \) | \( d_{\text{apth}}(S, S_1) = 0.7271, d_{\text{apth}}(S, S_2) = 0.7315 \) | \( S_1 > S_4 > S_1 > S_1 \) |
|                        |       | \( d_{\text{apth}}(S, S_1) = 0.5741, d_{\text{apth}}(S, S_1) = 0.6999 \) |                 |
|                        |       | \( d_{\text{apth}}(S, S_1) = 0.7311 \) |                 |
| Li et al. [34]         | \( \lambda = 10 \) | \( d_{\text{wdg}}(S, S_1) = 0.3156, d_{\text{wdg}}(S, S_2) = 0.3304 \) | \( S_3 > S_2 > S_1 > S_2 \) |
|                        |       | \( d_{\text{wdg}}(S, S_1) = 0.2960, d_{\text{wdg}}(S, S_1) = 0.3178 \) |                 |
|                        |       | \( d_{\text{wdg}}(S, S_1) = 0.2763 \) |                 |
| Proposed method        | \( \lambda = 10 \) | \( d_{\text{wdg}}(S, S_1) = 0.2308, d_{\text{wdg}}(S, S_1) = 0.2377 \) | \( S_4 > S_1 > S_1 > S_2 \) |
|                        |       | \( d_{\text{wdg}}(S, S_1) = 0.2363, d_{\text{wdg}}(S, S_1) = 0.228 \) |                 |
|                        |       | \( d_{\text{wdg}}(S, S_1) = 0.2516 \) |                 |

| Method                  | Value | Score function                                      | Ranking         |
|------------------------|-------|-----------------------------------------------------|-----------------|
| Xu et al. [32]         | \( \lambda = 1 \) | Cannot be calculated | Cannot be calculated |
| Singha et al. [33]     | \( \lambda = 1 \) | Cannot be calculated | Cannot be calculated |
| Li et al. [35]         | \( \lambda = 1 \) | \( d_{\text{apth}}(S, S_1) = 0.5994, d_{\text{apth}}(S, S_2) = 0.6005 \) | \( S_1 > S_4 > S_1 > S_2 \) |
|                        |       | \( d_{\text{apth}}(S, S_1) = 0.4679, d_{\text{apth}}(S, S_1) = 0.5375 \) |                 |
|                        |       | \( d_{\text{apth}}(S, S_1) = 0.5483 \) |                 |
| Li et al. [34]         | \( \lambda = 1 \) | \( d_{\text{wdg}}(S, S_1) = 0.3409, d_{\text{wdg}}(S, S_2) = 0.3494 \) | \( S_3 > S_1 > S_2 > S_1 \) |
|                        |       | \( d_{\text{wdg}}(S, S_1) = 0.3143, d_{\text{wdg}}(S, S_1) = 0.3327 \) |                 |
|                        |       | \( d_{\text{wdg}}(S, S_1) = 0.2944 \) |                 |
| Method               | Value | Score function | Ranking            |
|---------------------|-------|----------------|--------------------|
| Proposed method     | λ = 1 | $d_{cm}(S, S_1) = 0.2187, d_{cm}(S, S_2) = 0.2116$ | $S_2 > S_1 > S_3 > S_4 > S_5$ |
|                     |       | $d_{cm}(S, S_1) = 0.2204, d_{cm}(S, S_4) = 0.2048$ |                     |
|                     |       | $d_{cm}(S, S_1) = 0.2379$ |                     |
| Xu et al. [32]      | λ = 2 | Cannot be calculated | Cannot be calculated |
| Singha et al. [33]  | λ = 2 | Cannot be calculated | Cannot be calculated |
| Li et al. [35]      | λ = 2 | $d_{uphg}(S, S_1) = 0.6365, d_{uphg}(S, S_2) = 0.6304$ | $S_1 > S_4 > S_1 > S_2 > S_3$ |
|                     |       | $d_{uphg}(S, S_1) = 0.4896, d_{uphg}(S, S_4) = 0.5825$ |                     |
|                     |       | $d_{uphg}(S, S_1) = 0.6004$ |                     |
| Li et al. [34]      | λ = 2 | $d_{wchg}(S, S_1) = 0.3445, d_{wchg}(S, S_2) = 0.3528$ | $S_1 > S_3 > S_4 > S_2 > S_1$ |
|                     |       | $d_{wchg}(S, S_1) = 0.3137, d_{wchg}(S, S_4) = 0.3298$ |                     |
|                     |       | $d_{wchg}(S, S_1) = 0.2958$ |                     |
| Proposed method     | λ = 2 | $d_{cm}(S, S_1) = 0.2188, d_{cm}(S, S_2) = 0.2123$ | $S_2 > S_1 > S_3 > S_4 > S_5$ |
|                     |       | $d_{cm}(S, S_1) = 0.2209, d_{cm}(S, S_4) = 0.2079$ |                     |
|                     |       | $d_{cm}(S, S_1) = 0.239$ |                     |
| Xu et al. [32]      | λ = 6 | Cannot be calculated | Cannot be calculated |
| Singha et al. [33]  | λ = 6 | Cannot be calculated | Cannot be calculated |
| Li et al. [35]      | λ = 6 | $d_{uphg}(S, S_1) = 0.6968, d_{uphg}(S, S_2) = 0.6928$ | $S_1 > S_4 > S_2 > S_4 > S_1$ |
|                     |       | $d_{uphg}(S, S_1) = 0.5386, d_{uphg}(S, S_4) = 0.668$ |                     |
|                     |       | $d_{uphg}(S, S_1) = 0.6851$ |                     |
| Li et al. [34]      | λ = 6 | $d_{wchg}(S, S_1) = 0.3588, d_{wchg}(S, S_2) = 0.2662$ | $S_1 > S_2 > S_4 > S_3 > S_1$ |
|                     |       | $d_{wchg}(S, S_1) = 0.3274, d_{wchg}(S, S_4) = 0.3515$ |                     |
|                     |       | $d_{wchg}(S, S_1) = 0.3002$ |                     |
| Proposed method     | λ = 6 | $d_{cm}(S, S_1) = 0.2194, d_{cm}(S, S_2) = 0.2157$ | $S_2 > S_4 > S_1 > S_3 > S_5$ |
|                     |       | $d_{cm}(S, S_1) = 0.2227, d_{cm}(S, S_4) = 0.2183$ |                     |
|                     |       | $d_{cm}(S, S_1) = 0.2435$ |                     |
| Xu et al. [32]      | λ = 10| Cannot be calculated | Cannot be calculated |
| Singha et al. [33]  | λ = 10| Cannot be calculated | Cannot be calculated |
| Li et al. [35]      | λ = 10| $d_{uphg}(S, S_1) = 0.7255, d_{uphg}(S, S_2) = 0.7247$ | $S_1 > S_4 > S_1 > S_4 > S_1$ |
|                     |       | $d_{uphg}(S, S_1) = 0.5705, d_{uphg}(S, S_4) = 0.7125$ |                     |
|                     |       | $d_{uphg}(S, S_1) = 0.7156$ |                     |
| Li et al. [34]      | λ = 10| $d_{wchg}(S, S_1) = 0.3707, d_{wchg}(S, S_2) = 0.3780$ | $S_1 > S_3 > S_4 > S_1 > S_2$ |
|                     |       | $d_{wchg}(S, S_1) = 0.3347, d_{wchg}(S, S_4) = 0.3641$ |                     |
|                     |       | $d_{wchg}(S, S_1) = 0.3032$ |                     |
| Proposed method     | λ = 10| $d_{cm}(S, S_1) = 0.22, d_{cm}(S, S_2) = 0.2198$ | $S_1 > S_2 > S_1 > S_4 > S_5$ |
|                     |       | $d_{cm}(S, S_1) = 0.2245, d_{cm}(S, S_4) = 0.2255$ |                     |
|                     |       | $d_{cm}(S, S_1) = 0.2477$ |                     |
| Method                  | Value | Score function                  | Ranking |
|------------------------|-------|---------------------------------|---------|
| Xu et al. [32]         | $\lambda = 1$ | Cannot be calculated | Cannot be calculated |
| Singha et al. [33]     | $\lambda = 1$ | Cannot be calculated | Cannot be calculated |
| Li et al. [35]         | $\lambda = 1$ | $d_{uphg}(S, S_1) = 0.5508$, $d_{uphg}(S, S_2) = 0.5614$, $S_1 > S_1 > S_1 > S_2$ | |
| Li et al. [35]         | $\lambda = 1$ | $d_{uphg}(S, S_1) = 0.4418$, $d_{uphg}(S, S_2) = 0.4942$, $d_{uphg}(S, S_3) = 0.4713$ | |
| Li et al. [34]         | $\lambda = 1$ | $d_{wch}(S, S_1) = 0.3900$, $d_{wch}(S, S_2) = 0.4041$, $d_{wch}(S, S_1) = 0.3459$, $d_{wch}(S, S_2) = 0.3604$, $d_{wch}(S, S_3) = 0.3166$ | |
| Proposed method        | $\lambda = 1$ | $d_{wch}(S, S_1) = 0.3959$, $d_{wch}(S, S_2) = 0.4076$, $d_{wch}(S, S_1) = 0.4691$, $d_{wch}(S, S_2) = 0.5529$, $d_{wch}(S, S_3) = 0.5334$ | |
| Li et al. [35]         | $\lambda = 2$ | $d_{wch}(S, S_1) = 0.3959$, $d_{wch}(S, S_2) = 0.4076$, $d_{wch}(S, S_1) = 0.3524$, $d_{wch}(S, S_2) = 0.3729$, $d_{wch}(S, S_3) = 0.3205$ | $S_1 > S_1 > S_1 > S_2$ |
| Proposed method        | $\lambda = 2$ | $d_{wch}(S, S_1) = 0.20942$, $d_{wch}(S, S_2) = 0.2013$, $d_{wch}(S, S_1) = 0.20944$, $d_{wch}(S, S_2) = 0.1985$, $d_{wch}(S, S_3) = 0.2344$ | $S_1 > S_1 > S_1 > S_2$ |
| Xu et al. [32]         | $\lambda = 6$ | Cannot be calculated | Cannot be calculated |
| Singha et al. [33]     | $\lambda = 6$ | Cannot be calculated | Cannot be calculated |
| Li et al. [35]         | $\lambda = 6$ | $d_{uphg}(S, S_1) = 0.6832$, $d_{uphg}(S, S_2) = 0.6765$, $d_{uphg}(S, S_1) = 0.53$, $d_{uphg}(S, S_2) = 0.6689$, $d_{uphg}(S, S_3) = 0.6505$ | |
| Li et al. [34]         | $\lambda = 6$ | $d_{wch}(S, S_1) = 0.4192$, $d_{wch}(S, S_2) = 0.4211$, $d_{wch}(S, S_1) = 0.3715$, $d_{wch}(S, S_2) = 0.4051$, $d_{wch}(S, S_3) = 0.3316$ | |
| Proposed method        | $\lambda = 6$ | $d_{wch}(S, S_1) = 0.2101$, $d_{wch}(S, S_2) = 0.2036$, $d_{wch}(S, S_1) = 0.2116$, $d_{wch}(S, S_2) = 0.2159$, $d_{wch}(S, S_3) = 0.2395$ | $S_1 > S_1 > S_1 > S_2$ |
| Xu et al. [32]         | $\lambda = 10$ | Cannot be calculated | Cannot be calculated |
| Singha et al. [33]     | $\lambda = 10$ | Cannot be calculated | Cannot be calculated |
| Li et al. [35]         | $\lambda = 10$ | $d_{uphg}(S, S_1) = 0.7238$, $d_{uphg}(S, S_2) = 0.7173$, $d_{uphg}(S, S_1) = 0.5666$, $d_{uphg}(S, S_2) = 0.7234$, $d_{uphg}(S, S_3) = 0.6964$ | |

TABLE 13 Comparison between the proposed method and the existing methods for Example 3 with parameter $\alpha_c$, $\beta$, $\gamma = (0.3, 0.7)$
TABLE 13 (Continued)

| Method            | Value | Score function | Ranking      |
|-------------------|-------|----------------|--------------|
| Li et al. [34]    | $\lambda = 10$ | $d_{\text{wchg}}(S, S_1) = 0.4368$, $d_{\text{wchg}}(S, S_2) = 0.4329$ | $S_1 > S_3 > S_4 > S_2 > S_1$ |
|                   |       | $d_{\text{wchg}}(S, S_3) = 0.3822$, $d_{\text{wchg}}(S, S_4) = 0.4197$ |              |
|                   |       | $d_{\text{wchg}}(S, S_5) = 0.3385$ |              |
| Proposed method   | $\lambda = 10$ | $d_{\text{cchg}}(S, S_1) = 0.2107$, $d_{\text{cchg}}(S, S_2) = 0.206$ | $S_2 > S_1 > S_4 > S_5$ |
|                   |       | $d_{\text{cchg}}(S, S_3) = 0.2139$, $d_{\text{cchg}}(S, S_4) = 0.2254$ |              |
|                   |       | $d_{\text{cchg}}(S, S_5) = 0.2441$ |              |

**FIGURE 6** Graphical interpretation of the comparison between the established work and the existing work for different values of the parameters.

**FIGURE 7** Graphical interpretation of the comparison between the established work and the existing work for different values of the parameters.

**FIGURE 8** Graphical interpretation of the comparison between the established work and the existing work for different values of the parameters.
5.1 | Comparison

This section shows the superiority and integrity of the explored work by comparing it with some existing work.

Example 3 For every society, energy is a crucial factor for socio-economic development. Thus the accurate energy strategy accomplishes economic development and environment, and so, the most suitable energy strategy selection is crucial. Suppose five energy projects $S_j (j = 1, 2, 3, 4, 5)$ are to be invested. In the meantime, consider four attributes that are given as technological ($x_1$), environmental ($x_2$), socio-political ($x_3$) and economic ($x_4$). Consider the weight for attributes as $W = (0.15, 0.3, 0.2, 0.35)$. After this, a batch of experts is invited to assess the performance of the five alternatives with the respect to four attributes on the notion of excellence. Using HFSs, the assessment results obtained are presented in Table 7.

Suppose an ideal alternative is $S^* = \{1 e^{2x(1)}\}$. As $\epsilon = 1$ the above data transformed into the CHFSs is shown as follows
### Table 14
Comparison between proposed method and existing methods for Example 3 with parameter \((\alpha_c, \beta_w) = (0.1, 0.9)\)

| Method            | Value | Score function | Ranking  |
|-------------------|-------|----------------|----------|
| Xu et al. [32]    | \(\lambda = 1\) | Cannot be calculated | Cannot be calculated |
| Singh et al. [33] | \(\lambda = 1\) | Cannot be calculated | Cannot be calculated |
| Li et al. [35]    | \(\lambda = 1\) | \(d_{\text{uphg}}(S, S_1) = 0.5022, d_{\text{uphg}}(S, S_2) = 0.5222\) \(S_3 > S_1 > S_2 > S_1 > S_2\) |
| Li et al. [34]    | \(\lambda = 1\) | \(d_{\text{wchg}}(S, S_1) = 0.4465, d_{\text{wchg}}(S, S_2) = 0.4711\) \(S_3 > S_1 > S_2 > S_1 > S_2\) |
| Proposed method   | \(\lambda = 1\) | \(d_{\text{wchg}}(S, S_1) = 0.2003, d_{\text{wchg}}(S, S_2) = 0.1909\) \(S_3 > S_2 > S_1 > S_1 > S_5\) |
| Xu et al. [32]    | \(\lambda = 2\) | Cannot be calculated | Cannot be calculated |
| Singh et al. [33] | \(\lambda = 2\) | Cannot be calculated | Cannot be calculated |
| Li et al. [35]    | \(\lambda = 2\) | \(d_{\text{uphg}}(S, S_1) = 0.5589, d_{\text{uphg}}(S, S_2) = 0.5632\) \(S_1 > S_1 > S_2 > S_1 > S_2\) |
| Li et al. [34]    | \(\lambda = 2\) | \(d_{\text{wchg}}(S, S_1) = 0.4814, d_{\text{wchg}}(S, S_2) = 0.4848\) \(S_3 > S_1 > S_2 > S_1 > S_2\) |
| Proposed method   | \(\lambda = 2\) | \(d_{\text{wchg}}(S, S_1) = 0.2006, d_{\text{wchg}}(S, S_2) = 0.1917\) \(S_3 > S_4 > S_1 > S_1 > S_5\) |
| Xu et al. [32]    | \(\lambda = 6\) | Cannot be calculated | Cannot be calculated |
| Singh et al. [33] | \(\lambda = 6\) | Cannot be calculated | Cannot be calculated |
| Li et al. [35]    | \(\lambda = 6\) | \(d_{\text{uphg}}(S, S_1) = 0.6681, d_{\text{uphg}}(S, S_2) = 0.6579\) \(S_3 > S_1 > S_2 > S_1 > S_4\) |
| Li et al. [34]    | \(\lambda = 6\) | \(d_{\text{wchg}}(S, S_1) = 0.4914, d_{\text{wchg}}(S, S_2) = 0.4825\) \(S_3 > S_1 > S_2 > S_1 > S_3\) |
| Proposed method   | \(\lambda = 6\) | \(d_{\text{wchg}}(S, S_1) = 0.2017, d_{\text{wchg}}(S, S_2) = 0.1945\) \(S_2 > S_1 > S_4 > S_5\) |
| Xu et al. [32]    | \(\lambda = 10\) | Cannot be calculated | Cannot be calculated |
| Singh et al. [33] | \(\lambda = 10\) | Cannot be calculated | Cannot be calculated |
| Li et al. [35]    | \(\lambda = 10\) | \(d_{\text{uphg}}(S, S_1) = 0.7221, d_{\text{uphg}}(S, S_2) = 0.7091\) \(S_3 > S_1 > S_2 > S_1 > S_4\) |

(Continues)
| Method          | Value | Score function                                      | Ranking         |
|-----------------|-------|-----------------------------------------------------|-----------------|
| Li et al. [34]  | \( \lambda = 10 \) | \( d_{\text{wshg}}(S, S_1) = 0.5157, d_{\text{wshg}}(S, S_2) = 0.4963 \) | \( S_1 > S_3 > S_4 > S_2 > S_5 \) |
|                 |       | \( d_{\text{wshg}}(S, S_1) = 0.4390, d_{\text{wshg}}(S, S_2) = 0.4864 \) |                 |
|                 |       | \( d_{\text{wshg}}(S, S_1) = 0.3833 \) |                 |
| Proposed method | \( \lambda = 10 \) | \( d_{\text{cm}}(S, S_1) = 0.2028, d_{\text{cm}}(S, S_2) = 0.197 \) | \( S_2 > S_1 > S_3 > S_4 > S_5 \) |
|                 |       | \( d_{\text{cm}}(S, S_1) = 0.2044, d_{\text{cm}}(S, S_2) = 0.2259 \) |                 |
|                 |       | \( d_{\text{cm}}(S, S_1) = 0.2406 \) |                 |

| Method          | Value | Score function                                      | Ranking         |
|-----------------|-------|-----------------------------------------------------|-----------------|
| Xu et al. [32]  | \( \lambda = 1 \) | Cannot be calculated                                  | Cannot be calculated |
|                 |       |                                                     |                 |
| Singh et al. [33]| \( \lambda = 1 \) | Cannot be calculated                                  | Cannot be calculated |
|                 |       |                                                     |                 |
| Li et al. [35]  | \( \lambda = 1 \) | Cannot be calculated                                  | Cannot be calculated |
|                 |       |                                                     |                 |
| Li et al. [34]  | \( \lambda = 1 \) | Cannot be calculated                                  | Cannot be calculated |
| Proposed method | \( \lambda = 1 \) | \( d_{\text{cm}}(S, S_1) = 0.469, d_{\text{cm}}(S, S_2) = 0.4304 \) | \( S_1 > S_2 > S_3 > S_4 > S_5 \) |
|                 |       | \( d_{\text{cm}}(S, S_1) = 0.4244, d_{\text{cm}}(S, S_2) = 0.4397 \) |                 |
|                 |       | \( d_{\text{cm}}(S, S_1) = 0.4565 \) |                 |
| Xu et al. [32]  | \( \lambda = 2 \) | Cannot be calculated                                  | Cannot be calculated |
|                 |       |                                                     |                 |
| Singh et al. [33]| \( \lambda = 2 \) | Cannot be calculated                                  | Cannot be calculated |
|                 |       |                                                     |                 |
| Li et al. [35]  | \( \lambda = 2 \) | Cannot be calculated                                  | Cannot be calculated |
|                 |       |                                                     |                 |
| Li et al. [34]  | \( \lambda = 2 \) | Cannot be calculated                                  | Cannot be calculated |
| Proposed method | \( \lambda = 2 \) | \( d_{\text{cm}}(S, S_1) = 0.4715, d_{\text{cm}}(S, S_2) = 0.4431 \) | \( S_1 > S_2 > S_3 > S_4 > S_5 \) |
|                 |       | \( d_{\text{cm}}(S, S_1) = 0.4398, d_{\text{cm}}(S, S_2) = 0.4445 \) |                 |
|                 |       | \( d_{\text{cm}}(S, S_1) = 0.4681 \) |                 |
| Xu et al. [32]  | \( \lambda = 6 \) | Cannot be calculated                                  | Cannot be calculated |
|                 |       |                                                     |                 |
| Singh et al. [33]| \( \lambda = 6 \) | Cannot be calculated                                  | Cannot be calculated |
|                 |       |                                                     |                 |
| Li et al. [35]  | \( \lambda = 6 \) | Cannot be calculated                                  | Cannot be calculated |
|                 |       |                                                     |                 |
| Li et al. [34]  | \( \lambda = 6 \) | Cannot be calculated                                  | Cannot be calculated |
| Proposed method | \( \lambda = 6 \) | \( d_{\text{cm}}(S, S_1) = 0.4819, d_{\text{cm}}(S, S_2) = 0.4766 \) | \( S_1 > S_2 > S_3 > S_4 > S_5 \) |
|                 |       | \( d_{\text{cm}}(S, S_1) = 0.4905, d_{\text{cm}}(S, S_2) = 0.4645 \) |                 |
|                 |       | \( d_{\text{cm}}(S, S_1) = 0.5009 \) |                 |
| Xu et al. [32]  | \( \lambda = 10 \) | Cannot be calculated                                 | Cannot be calculated |
|                 |       |                                                     |                 |
| Singh et al. [33]| \( \lambda = 10 \) | Cannot be calculated                                 | Cannot be calculated |
|                 |       |                                                     |                 |
| Li et al. [35]  | \( \lambda = 10 \) | Cannot be calculated                                 | Cannot be calculated |
|                 |       |                                                     |                 |
| Li et al. [34]  | \( \lambda = 10 \) | Cannot be calculated                                 | Cannot be calculated |
| Proposed method | \( \lambda = 10 \) | \( d_{\text{cm}}(S, S_1) = 0.4919, d_{\text{cm}}(S, S_2) = 0.4924 \) | \( S_1 > S_2 > S_3 > S_4 > S_5 \) |
|                 |       | \( d_{\text{cm}}(S, S_1) = 0.5186, d_{\text{cm}}(S, S_2) = 0.4821 \) |                 |
|                 |       | \( d_{\text{cm}}(S, S_1) = 0.5198 \) |                 |
**Table 16** Comparison between the proposed method and the existing methods for Example 2 with parameter \((\alpha_0, \beta_0) = (0.9, 0.1)\)

| Method               | Value | Score function | Ranking          |
|----------------------|-------|----------------|------------------|
| Xu et al. [32]       | \(\lambda = 1\) | Cannot be calculated | Cannot be calculated |
| Singha et al. [33]   | \(\lambda = 1\) | Cannot be calculated | Cannot be calculated |
| Li et al. [35]       | \(\lambda = 1\) | Cannot be calculated | Cannot be calculated |
| Li et al. [34]       | \(\lambda = 1\) | Cannot be calculated | Cannot be calculated |
| Proposed method      | \(\lambda = 1\) | \(d_{eq}(S, S_i) = 0.2608, \ d_{eq}(S, S_j) = 0.256\) | \(S_2 > S_1 > S_3 > S_5\) |
|                      |       | \(d_{eq}(S, S_k) = 0.265, \ d_{eq}(S, S_l) = 0.2648\) | \(d_{eq}(S, S_m) = 0.2698\) |
| Xu et al. [32]       | \(\lambda = 2\) | Cannot be calculated | Cannot be calculated |
| Singha et al. [33]   | \(\lambda = 2\) | Cannot be calculated | Cannot be calculated |
| Li et al. [35]       | \(\lambda = 2\) | Cannot be calculated | Cannot be calculated |
| Li et al. [34]       | \(\lambda = 2\) | Cannot be calculated | Cannot be calculated |
| Proposed method      | \(\lambda = 2\) | \(d_{eq}(S, S_i) = 0.2612, \ d_{eq}(S, S_j) = 0.2587\) | \(S_2 > S_1 > S_3 > S_5\) |
|                      |       | \(d_{eq}(S, S_k) = 0.2658, \ d_{eq}(S, S_l) = 0.2663\) | \(d_{eq}(S, S_m) = 0.2699\) |
| Xu et al. [32]       | \(\lambda = 6\) | Cannot be calculated | Cannot be calculated |
| Singha et al. [33]   | \(\lambda = 6\) | Cannot be calculated | Cannot be calculated |
| Li et al. [35]       | \(\lambda = 6\) | Cannot be calculated | Cannot be calculated |
| Li et al. [34]       | \(\lambda = 6\) | Cannot be calculated | Cannot be calculated |
| Proposed method      | \(\lambda = 6\) | \(d_{eq}(S, S_i) = 0.2629, \ d_{eq}(S, S_j) = 0.271\) | \(S_1 > S_2 > S_3 > S_4\) |
|                      |       | \(d_{eq}(S, S_k) = 0.2684, \ d_{eq}(S, S_l) = 0.2731\) | \(d_{eq}(S, S_m) = 0.2713\) |
| Xu et al. [32]       | \(\lambda = 10\) | Cannot be calculated | Cannot be calculated |
| Singha et al. [33]   | \(\lambda = 10\) | Cannot be calculated | Cannot be calculated |
| Li et al. [35]       | \(\lambda = 10\) | Cannot be calculated | Cannot be calculated |
| Li et al. [34]       | \(\lambda = 10\) | Cannot be calculated | Cannot be calculated |
| Proposed method      | \(\lambda = 10\) | \(d_{eq}(S, S_i) = 0.2647, \ d_{eq}(S, S_j) = 0.2836\) | \(S_1 > S_2 > S_3 > S_4\) |
|                      |       | \(d_{eq}(S, S_k) = 0.2705, \ d_{eq}(S, S_l) = 0.2803\) | \(d_{eq}(S, S_m) = 0.2757\) |

**Table 17** Comparison between the proposed method and the existing methods for Example 2 with parameter \((\alpha_0, \beta_0) = (0.7, 0.3)\)

| Method               | Value | Score function | Ranking          |
|----------------------|-------|----------------|------------------|
| Xu et al. [32]       | \(\lambda = 1\) | Cannot be calculated | Cannot be calculated |
| Singha et al. [33]   | \(\lambda = 1\) | Cannot be calculated | Cannot be calculated |
| Li et al. [35]       | \(\lambda = 1\) | Cannot be calculated | Cannot be calculated |
| Li et al. [34]       | \(\lambda = 1\) | Cannot be calculated | Cannot be calculated |
| Proposed method      | \(\lambda = 1\) | \(d_{eq}(S, S_i) = 0.2966, \ d_{eq}(S, S_j) = 0.2852\) | \(S_2 > S_3 > S_2 > S_1\) |
|                      |       | \(d_{eq}(S, S_k) = 0.2927, \ d_{eq}(S, S_l) = 0.2961\) | \(d_{eq}(S, S_m) = 0.2999\) |

(Continues)
| Method          | Value | Score function                           | Ranking |
|-----------------|-------|------------------------------------------|---------|
| Xu et al. [32]  | $\lambda = 2$ | Cannot be calculated                     | Cannot be calculated |
| Singha et al. [33] | $\lambda = 2$ | Cannot be calculated                     | Cannot be calculated |
| Li et al. [35]  | $\lambda = 2$ | Cannot be calculated                     | Cannot be calculated |
| Li et al. [34]  | $\lambda = 2$ | Cannot be calculated                     | Cannot be calculated |
| Proposed method | $\lambda = 2$ | $d_{\text{cc}}(S, S_1) = 0.2968$, $d_{\text{cc}}(S, S_2) = 0.2875$; $d_{\text{cc}}(S, S_3) = 0.2943$, $d_{\text{cc}}(S, S_4) = 0.2978$; $d_{\text{cc}}(S, S_5) = 0.3012$ | $S_4 > S_3 > S_1 > S_2 > S_5$ |
| Xu et al. [32]  | $\lambda = 6$ | Cannot be calculated                     | Cannot be calculated |
| Singha et al. [33] | $\lambda = 6$ | Cannot be calculated                     | Cannot be calculated |
| Li et al. [35]  | $\lambda = 6$ | Cannot be calculated                     | Cannot be calculated |
| Li et al. [34]  | $\lambda = 6$ | Cannot be calculated                     | Cannot be calculated |
| Proposed method | $\lambda = 6$ | $d_{\text{cc}}(S, S_1) = 0.2975$, $d_{\text{cc}}(S, S_2) = 0.3001$; $d_{\text{cc}}(S, S_3) = 0.3006$, $d_{\text{cc}}(S, S_4) = 0.306$; $d_{\text{cc}}(S, S_5) = 0.3092$ | $S_4 > S_3 > S_1 > S_2 > S_5$ |
| Xu et al. [32]  | $\lambda = 10$ | Cannot be calculated                      | Cannot be calculated |
| Singha et al. [33] | $\lambda = 10$ | Cannot be calculated                      | Cannot be calculated |
| Li et al. [35]  | $\lambda = 10$ | Cannot be calculated                      | Cannot be calculated |
| Li et al. [34]  | $\lambda = 10$ | Cannot be calculated                      | Cannot be calculated |
| Proposed method | $\lambda = 10$ | $d_{\text{cc}}(S, S_1) = 0.2983$, $d_{\text{cc}}(S, S_2) = 0.3146$; $d_{\text{cc}}(S, S_3) = 0.3061$, $d_{\text{cc}}(S, S_4) = 0.315$; $d_{\text{cc}}(S, S_5) = 0.3161$ | $S_4 > S_3 > S_1 > S_2 > S_5$ |

**Table 18** Comparison between proposed method and existing methods for Example 2 with parameter $(\alpha_{cc}, \beta_{cc}) = (0.5, 0.5)$

| Method          | Value | Score function                           | Ranking |
|-----------------|-------|------------------------------------------|---------|
| Xu et al. [32]  | $\lambda = 1$ | Cannot be calculated                     | Cannot be calculated |
| Singha et al. [33] | $\lambda = 1$ | Cannot be calculated                     | Cannot be calculated |
| Li et al. [35]  | $\lambda = 1$ | Cannot be calculated                     | Cannot be calculated |
| Li et al. [34]  | $\lambda = 1$ | Cannot be calculated                     | Cannot be calculated |
| Proposed method | $\lambda = 1$ | $d_{\text{cc}}(S, S_1) = 0.3377$, $d_{\text{cc}}(S, S_2) = 0.319$; $d_{\text{cc}}(S, S_3) = 0.3244$, $d_{\text{cc}}(S, S_4) = 0.3312$; $d_{\text{cc}}(S, S_5) = 0.3369$ | $S_3 > S_2 > S_1 > S_4 > S_5$ |
| Xu et al. [32]  | $\lambda = 2$ | Cannot be calculated                     | Cannot be calculated |
| Singha et al. [33] | $\lambda = 2$ | Cannot be calculated                     | Cannot be calculated |
| Li et al. [35]  | $\lambda = 2$ | Cannot be calculated                     | Cannot be calculated |
| Li et al. [34]  | $\lambda = 2$ | Cannot be calculated                     | Cannot be calculated |
### Table 18 (Continued)

| Method          | Value | Score function | Ranking               |
|-----------------|-------|----------------|-----------------------|
| Proposed method | $\lambda = 2$ | $d_{c\text{-}cc}(S, S_i) = 0.3379$, $d_{c\text{-}cc}(S, S_2) = 0.3223$ | $S_2 > S_3 > S_4 > S_1 > S_5$ |
|                 |       | $d_{c\text{-}cc}(S, S_i) = 0.3279$, $d_{c\text{-}cc}(S, S_4) = 0.3334$ |                           |
|                 |       | $d_{c\text{-}cc}(S, S_1) = 0.3408$ |                           |
| Xu et al. [32]  | $\lambda = 6$ | Cannot be calculated | Cannot be calculated |
| Singha et al. [33] | $\lambda = 6$ | Cannot be calculated | Cannot be calculated |
| Li et al. [35]  | $\lambda = 6$ | Cannot be calculated | Cannot be calculated |
| Li et al. [34]  | $\lambda = 6$ | Cannot be calculated | Cannot be calculated |
| Proposed method | $\lambda = 6$ | $d_{c\text{-}cc}(S, S_i) = 0.3389$, $d_{c\text{-}cc}(S, S_2) = 0.3371$ | $S_2 > S_3 > S_4 > S_1 > S_5$ |
|                 |       | $d_{c\text{-}cc}(S, S_1) = 0.3419$, $d_{c\text{-}cc}(S, S_4) = 0.3437$ |                           |
|                 |       | $d_{c\text{-}cc}(S, S_1) = 0.3538$ |                           |
| Xu et al. [32]  | $\lambda = 10$ | Cannot be calculated | Cannot be calculated |
| Singha et al. [33] | $\lambda = 10$ | Cannot be calculated | Cannot be calculated |
| Li et al. [35]  | $\lambda = 10$ | Cannot be calculated | Cannot be calculated |
| Li et al. [34]  | $\lambda = 10$ | Cannot be calculated | Cannot be calculated |
| Proposed method | $\lambda = 10$ | $d_{c\text{-}cc}(S, S_i) = 0.3398$, $d_{c\text{-}cc}(S, S_2) = 0.3523$ | $S_1 > S_2 > S_3 > S_4 > S_5$ |
|                 |       | $d_{c\text{-}cc}(S, S_1) = 0.3532$, $d_{c\text{-}cc}(S, S_4) = 0.3548$ |                           |
|                 |       | $d_{c\text{-}cc}(S, S_1) = 0.3636$ |                           |

### Table 19
Comparison between the proposed method and the existing methods for Example 2 with parameter $(\alpha_c, \beta_c) = (0.3, 0.7)$

| Method          | Value | Score function | Ranking               |
|-----------------|-------|----------------|-----------------------|
| Xu et al. [32]  | $\lambda = 1$ | Cannot be calculated | Cannot be calculated |
| Singha et al. [33] | $\lambda = 1$ | Cannot be calculated | Cannot be calculated |
| Li et al. [35]  | $\lambda = 1$ | Cannot be calculated | Cannot be calculated |
| Li et al. [34]  | $\lambda = 1$ | Cannot be calculated | Cannot be calculated |
| Proposed method | $\lambda = 1$ | $d_{c\text{-}cc}(S, S_i) = 0.3848$, $d_{c\text{-}cc}(S, S_1) = 0.3585$ | $S_2 > S_3 > S_4 > S_5 > S_1$ |
|                 |       | $d_{c\text{-}cc}(S, S_1) = 0.3604$, $d_{c\text{-}cc}(S, S_1) = 0.3798$ |                           |
|                 |       | $d_{c\text{-}cc}(S, S_1) = 0.3801$ |                           |
| Xu et al. [32]  | $\lambda = 2$ | Cannot be calculated | Cannot be calculated |
| Singha et al. [33] | $\lambda = 2$ | Cannot be calculated | Cannot be calculated |
| Li et al. [35]  | $\lambda = 2$ | Cannot be calculated | Cannot be calculated |
| Li et al. [34]  | $\lambda = 2$ | Cannot be calculated | Cannot be calculated |
| Proposed method | $\lambda = 2$ | $d_{c\text{-}cc}(S, S_i) = 0.3855$, $d_{c\text{-}cc}(S, S_1) = 0.364$ | $S_2 > S_3 > S_4 > S_5 > S_1$ |
|                 |       | $d_{c\text{-}cc}(S, S_1) = 0.3673$, $d_{c\text{-}cc}(S, S_1) = 0.3737$ |                           |
|                 |       | $d_{c\text{-}cc}(S, S_1) = 0.3864$ |                           |
| Xu et al. [32]  | $\lambda = 6$ | Cannot be calculated | Cannot be calculated |
| Singha et al. [33] | $\lambda = 6$ | Cannot be calculated | Cannot be calculated |
| Li et al. [35]  | $\lambda = 6$ | Cannot be calculated | Cannot be calculated |

(Continues)
\textbf{TABLE 19} (Continued)

| Method            | Value | Score function | Ranking                      |
|-------------------|-------|----------------|------------------------------|
| Li et al. [34]    | \(\lambda = 6\) | Cannot be calculated | Cannot be calculated |
| Proposed method   | \(\lambda = 6\) | \(d_{nc}(S, S_1) = 0.3884\), \(d_{nc}(S, S_2) = 0.3837\) | \(S_2 > S_4 > S_1 > S_3\) |
|                   |       | \(d_{nc}(S, S_3) = 0.3869\) |                              |
|                   |       | \(d_{nc}(S, S_4) = 0.4069\) |                              |
| Xu et al. [32]    | \(\lambda = 10\) | Cannot be calculated | Cannot be calculated |
| Proposed method   | \(\lambda = 10\) | \(d_{nc}(S, S_1) = 0.3914\), \(d_{nc}(S, S_2) = 0.3988\) | \(S_1 > S_2 > S_4 > S_3\) |
|                   |       | \(d_{nc}(S, S_3) = 0.4192\) |                              |
|                   |       | \(d_{nc}(S, S_4) = 0.4193\) |                              |

\textbf{TABLE 20} Comparison between the proposed method and the existing methods for Example 2 with parameter \((\alpha_{0c}, \beta_{0c}) = (0.1, 0.9)\)

| Method            | Value | Score function | Ranking                      |
|-------------------|-------|----------------|------------------------------|
| Xu et al. [32]    | \(\lambda = 1\) | Cannot be calculated | Cannot be calculated |
| Singha et al. [33]| \(\lambda = 1\) | Cannot be calculated | Cannot be calculated |
| Li et al. [35]    | \(\lambda = 1\) | Cannot be calculated | Cannot be calculated |
| Li et al. [34]    | \(\lambda = 1\) | Cannot be calculated | Cannot be calculated |
| Proposed method   | \(\lambda = 1\) | \(d_{nc}(S, S_1) = 0.439\), \(d_{nc}(S, S_2) = 0.4045\) | \(S_1 > S_2 > S_4 > S_3\) |
|                   |       | \(d_{nc}(S, S_3) = 4.017\), \(d_{nc}(S, S_4) = 0.4153\) |                              |
|                   |       | \(d_{nc}(S, S_5) = 0.4293\) |                              |
| Xu et al. [32]    | \(\lambda = 2\) | Cannot be calculated | Cannot be calculated |
| Singha et al. [33]| \(\lambda = 2\) | Cannot be calculated | Cannot be calculated |
| Li et al. [35]    | \(\lambda = 2\) | Cannot be calculated | Cannot be calculated |
| Li et al. [34]    | \(\lambda = 2\) | Cannot be calculated | Cannot be calculated |
| Proposed method   | \(\lambda = 2\) | \(d_{nc}(S, S_1) = 0.4406\), \(d_{nc}(S, S_2) = 0.4143\) | \(S_1 > S_2 > S_3 > S_4\) |
|                   |       | \(d_{nc}(S, S_3) = 0.4137\), \(d_{nc}(S, S_4) = 0.4194\) |                              |
|                   |       | \(d_{nc}(S, S_5) = 0.4389\) |                              |
| Xu et al. [32]    | \(\lambda = 6\) | Cannot be calculated | Cannot be calculated |
| Singha et al. [33]| \(\lambda = 6\) | Cannot be calculated | Cannot be calculated |
| Li et al. [35]    | \(\lambda = 6\) | Cannot be calculated | Cannot be calculated |
| Li et al. [34]    | \(\lambda = 6\) | Cannot be calculated | Cannot be calculated |
| Proposed method   | \(\lambda = 6\) | \(d_{nc}(S, S_1) = 0.4478\), \(d_{nc}(S, S_2) = 0.4421\) | \(S_1 > S_2 > S_3 > S_4\) |
|                   |       | \(d_{nc}(S, S_3) = 0.4551\), \(d_{nc}(S, S_4) = 0.4367\) |                              |
|                   |       | \(d_{nc}(S, S_5) = 0.4668\) |                              |
| Xu et al. [32]    | \(\lambda = 10\) | Cannot be calculated | Cannot be calculated |
### TABLE 20  (Continued)

| Method          | Value | Score function | Ranking     |
|-----------------|-------|----------------|-------------|
| Singha et al. [33] | \( \lambda = 10 \) | Cannot be calculated | Cannot be calculated |
| Li et al. [35]   | \( \lambda = 10 \) | Cannot be calculated | Cannot be calculated |
| Li et al. [34]   | \( \lambda = 10 \) | Cannot be calculated | Cannot be calculated |
| Proposed method  | \( \lambda = 10 \) | \( d_{ccw}(S, S_1) = 0.455, \ d_{ccw}(S, S_2) = 0.4573 \) | \( S_1 > S_2 > S_3 > S_4 > S_5 \) |

\[
\begin{align*}
    d_{ccw}(S, S_1) &= 0.455, \\
    d_{ccw}(S, S_2) &= 0.4573, \\
    &\vdots
\end{align*}
\]

\[
\begin{align*}
    d_{ccw}(S, S_4) &= 0.4797, \\
    d_{ccw}(S, S_5) &= 0.4528 \\
    &\vdots
\end{align*}
\]

\[
\begin{align*}
    d_{ccw}(S, S_5) &= 0.4837
\end{align*}
\]

### FIGURE 12  Graphical interpretation of the comparison between the established work and the existing work for different values of parameters

### FIGURE 13  Graphical interpretation of the comparison between established work and existing work for different values of parameters

### FIGURE 14  Graphical interpretation of the comparison between established work and existing work for different values of parameters
The comparison outcomes between the proposed distance measures and existing distance measures for Example 3 are given in Tables 8–13. We observe that the data of Example 3 is in the form HFSs and through the existing distance measures for HFSs, the distance between $S$ and $S_j$ ($j = 1, 2, 3, 4, 5$) is found. As $e^{i} = 1$, then the given data for Example 3 is transformed to the CHFSs and by the proposed method we find the similarity between $S$ and $S_j$ ($j = 1, 2, 3, 4, 5$) as shown in Tables 8–14. The graphical representation of such results are shown in Figures 6–11. The comparison outcomes for Example 2 are given in Tables 14–20 and their graphical representation is shown in Figures 12–17. We noted that the data of Example 2 is in the form of CHFSs and the existing methods are incapable achieving this type of data. Through the existing methods, we solve the data in the form of FS, CFS, and HFS.

If we assume the imaginary part as zero, then the explored distance measures are reduced for the HFS. Likewise, if we assume the CHFS as a singleton set then the explored methods are reduced for CFS. Furthermore, if we assume the CHFS as the singleton set and the imaginary part as zero, then the explored methods are reduced for FS. This type of structure
makes the explored methods proficient and more general than the existing methods.

6 | CONCLUSION

Similarity and distance measures are used to examine the difference between two objects. The objective of the authors is to establish the CHFS, which is the combination of the HFS and CFS to manage complex and awkward information in real-decision theory. Furthermore, the GDMs and MGDMs based on established approaches are utilized and also expose the special cases of the established approach. After this, we established the parameterized distance measures and their particular cases are discussed. The established measures are utilized in the environment of decision-making to examine the feasibility and validity of the explored measures. Moreover, the numerical examples for established measures are solved to express the superiority and integrity/reliability of the explored work. Finally, to evaluate the credibility of the modified and parameterized distance measures based on CHFS, they are verified by comparing with some existing measures. Our future work is to explore the application of CHFNs in many other applications and under different environments [36–42].

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