On Linear-Sized Farthest-Color Voronoi Diagrams

Sang Won BAE* †, Member

SUMMARY Given a collection of k sets consisting of a total of n points in the plane, the distance from any point in the plane to each of the sets is defined to be the minimum among distances to each point in the set. The farthest-color Voronoi diagram is defined as a generalized Voronoi diagram of the k sets with respect to the distance functions for each of the k sets. The combinatorial complexity of the diagram is known to be $\Theta(\text{kn})$ in the worst case. This paper initiates a study on farthest-color Voronoi diagrams having $O(n)$ complexity. We introduce a realistic model, which defines a certain class of the diagrams with desirable geometric properties observed. We finally show that the farthest-color Voronoi diagrams under the model have linear complexity.

key words: Voronoi diagrams, farthest-color Voronoi diagrams, realistic models, computational geometry

1. Introduction

Voronoi diagrams and their variations has taken constant and intensive attention for last three decades with lots of applications in many different fields of engineering and sciences [1]. Among those this paper discusses the farthest-color Voronoi diagram.

We consider a family $S = \{S_1, \ldots, S_k\}$ of k geometric objects in the plane $\mathbb{R}^2$, with n total complexity. More specifically, in this work, each $S_i$ is assumed to be a finite set of points. Then, the distance from each point $x \in \mathbb{R}^2$ to each $S_i$ is taken to be the minimum distance to its elements;

$$d_i(x) := \min_{s \in S_i} \delta(x, s),$$

where $\delta$ denotes the $L_p$ metric for any $1 \leq p \leq \infty$. Then, bisectors between $S_i$ and $S_j$, nearest and farthest Voronoi regions of each $S_i$, and thus a certain type of generalized nearest and farthest-site Voronoi diagrams for $S$ are intuitively defined in terms of the distance functions $d_i$.

The (nearest) Voronoi diagram of this kind is less interesting since it coincides with the standard Voronoi diagram of the union of all $S_i$ with respect to $\delta$. Thus, it is easy to see its structural properties directly from those of standard Voronoi diagrams; for example, its complexity is $O(n)$ where $n = \sum_{i=1}^{k} |S_i|$. On the other hand, its farthest sibling, which we call the farthest-color Voronoi diagram $\text{FCVD}(S)$, has drastically different and more complicated structure.

Figure 1 illustrates an example of the farthest-color Voronoi diagram under the Euclidean ($L_2$) metric with $k = 3$ and $n = 9$, in which four black dots belong to $S_1$, four gray-filled circles belong to $S_2$, and a single white-filled square in the middle of the figure belongs to $S_3$. Notice that the region for $S_1$ is colored as dark gray, the region for $S_2$ as light gray, and the region for $S_3$ as white.

The farthest-color Voronoi diagram generalizes typical farthest-site Voronoi diagrams into multiple colors; imagine all points in each $S_i$ are colored by color index $i$. It already finds several interesting applications, such as location problems among multiple type of facilities [2], variants of Steiner tree problems [3], [4], sensor deployment problems in a wireless sensor network [5], the Hausdorff Voronoi diagram [6], and so forth. Its first structural study in the literature is, to the best of our knowledge, by Huttenlocher et al. [7] who investigated the upper envelope of k Voronoi surfaces; the projection of the upper envelope coincides with the farthest-color Voronoi diagram. They have shown in turn that the complexity of the corresponding farthest-color Voronoi diagram $\text{FCVD}(S)$ can be as large as $\Omega(nk)$, so a quadratic size since $k$ can be as large as $\Theta(n)$. They also proved an upper bound $O(n\alpha(nk))$ for any $L_p$ metric, where $\alpha(\cdot)$ denotes the functional inverse of the Ackermann function. Nonetheless, the term, farthest-color Voronoi diagram, was coined by Abellanas et al. [2], who showed a matching upper bound $O(nk)$. Consequently, the complexity of $\text{FCVD}(S)$ is $\Theta(nk)$ in the worst case.

This is somewhat surprising because farthest-site
Voronoi diagrams of point sites usually have less complexity than nearest-site Voronoi diagrams do; they have $\Theta(h)$ complexity for the Euclidean ($L_2$) where $h$ is the number of vertices of the convex hull of the given points, [8, Chapter 6.3] or even constant complexity for the $L_1$ or $L_\infty$ metric. For other $L_p$ metrics, one can apply the theory of the furthest-site abstract Voronoi diagrams [9] to achieve $O(n)$ complexity.

We in this paper focus on milder cases, where the complexity of FCVD(S) is bounded by $O(n)$. The theme is closely related to realistic input models for geometric algorithms [10], where behavior of practical input data is analyzed rather than theoretically worst-case data. To our best knowledge, there was no such try to study realistic input models for farthest-color Voronoi diagrams. We thus introduce a realistic model for configurations of $S$, namely well-clustered, which defines a class of farthest-color Voronoi diagrams with some good behaviors. We will be able to deduce several natural structural properties and then linear complexity of the farthest-color Voronoi diagram under the model. We believe that our model is reasonable and accept many realistic cases from a wide range of research fields, such as bioinformatics, wireless sensor networks, and so on.

Our main goal is to prove that the complexity of the farthest-color Voronoi diagram under the model is indeed linear $O(n)$. For the purpose, the rest of the paper is organized as follows: We introduce preliminaries in Sect. 2, including formal definitions of diagrams. Section 3 is devoted to establish a general bound for bounded faces of the farthest-color Voronoi diagrams, which will be exploited in showing the linearity of the complexity of the diagram in Sect. 4. Finally, Sect. 5 concludes the paper with remarkable notes.

2. Preliminaries

In this section, we give terms and definitions in a more precise form.

For a fixed $1 \leq p \leq \infty$, let $\delta$ be the $L_p$ metric defined on the plane $\mathbb{R}^2$. Throughout the paper, we frequently use the following notations: For a subset $A \subset \mathbb{R}^2$, int$A$ denotes the interior of $A$, $\partial A$ the boundary of $A$, and cl$A$ the closure of $A$ with respect to the standard topology on $\mathbb{R}^2$. Note that the topology induced by the $L_p$ metric on $\mathbb{R}^2$ for any $1 \leq p \leq \infty$ is equivalent to that induced by the Euclidean metric. We denote the straight line segment joining two points $a, b \in \mathbb{R}^2$ by $ab$, and its (Euclidean) length by $|ab|$.

2.1 Voronoi Skeleton of a Set of Points

Let $S \subset \mathbb{R}^2$ be a finite set of points in the plane $\mathbb{R}^2$. The nearest-site Voronoi diagram $VD_0(S)$ of $S$ with respect to the $L_p$ metric $\delta$ is a subdivision of the plane $\mathbb{R}^2$ into Voronoi regions $R_0(s, S)$, which is defined to be

$$R_0(s, S) := \bigcap_{s \in S} \{x \in \mathbb{R}^2 \mid \delta(x, s) \leq \delta(x, s')\}.$$ 

It is well known that $VD_0(S)$ has $O(|S|)$ complexity and can be computed in $O(|S| \log |S|)$ time [11]. The vertices, edges, and faces of $VD_0(S)$ are naturally induced.

We denote by SK(S) the union of all edges and vertices of $VD_0(S)$ as a subset of $\mathbb{R}^2$. We shall call SK(S) the Voronoi skeleton of $S$. Also, we call each edge of SK(S) a skeleton edge of $S$. Since $VD_0(S)$ has linear complexity, we have $O(|S|)$ skeleton edges in SK(S), too. The Voronoi skeleton SK(S) will play a central role in our further discussion in following sections.

2.2 Definition of Farthest-Color Voronoi Diagrams

Let $S := \{S_1, \ldots, S_k\}$ be a collection of $k$ sites. Each site $S_i \subset \mathbb{R}^2$ is a set of $n_i > 0$ points in the plane $\mathbb{R}^2$, called generators. Let $n := \sum n_i$ be the total complexity of the site set $S$. For any $x \in \mathbb{R}^2$ and any generator $s \in S_i$, denote by $\delta(x, s)$ the distance from $x$ to $s$ with respect to the underlying metric $\delta$. Then the distance to each site $S_i$ is defined as follows: for any $x \in \mathbb{R}^2$,

$$d_i(x) := \min_{s \in S_i} \delta(x, s).$$

The farthest-color Voronoi diagram FCVD(S) is defined to be a subdivision of the plane $\mathbb{R}^2$ into regions $FCVR(x; S)$ for each generator $s \in S_i$ and each $S_j \in S$. The region $FCVR(x; S)$ is called the farthest-color Voronoi region of generator $s \in S_i$, and defined to be the set of points $x \in \mathbb{R}^2$ such that $x$ is farther to $S_i$ than to any other $S_j$ in terms of the functions $d_i$ and, in addition, $d_i(x)$ is determined by $s$; that is, $d_i(x) \geq d_j(x)$ for any $j \neq i$ and $d_i(x) = \delta(s, x)$. More formally, we introduce the following definitions.

We first define the farthest-color Voronoi regions of each site $S_i$ as follows: For $S_i, S_j \in S$ with $i < j$, let

$$R(S_i, S_j) := \{x \in \mathbb{R}^2 \mid d_i(x) \leq d_j(x)\},$$

and

$$R(S_j, S_i) := \{x \in \mathbb{R}^2 \mid d_j(x) < d_i(x)\}.$$ 

Then, for any $S_i \in S$, let

$$R(S_i, S) := \bigcap_{S_j \in S, j \neq i} R(S_i, S_j)$$

and

$$FR(S_i, S) := \bigcap_{S_j \in S, j \neq i} R(S_j, S_i)$$

be the nearest and the farthest-color Voronoi regions of site $S_i$ with respect to $S$. Also, we let $B(S_i, S_j) := \partial R(S_i, S_j)$ be the bisector between two sites $S_i$ and $S_j$. Finally, the farthest-color Voronoi region of a generator $s \in S_i$ is defined to be a subset of $FR(S_i, S)$ in which the distance $d_i$ is determined by $s$:

$$FCVR(s; S) := FR(S_i, S) \cap R_0(s, S_i),$$

where $R_0(s, S_i)$ is the nearest Voronoi region of $s$ in the Voronoi diagram $VD_0(S_i)$ of $S_i$ with respect to $\delta$. Then, the farthest-color Voronoi diagram FCVD(S) under $\delta$ is the cell complex induced by $FCVR(s; S)$ over all $s \in S_i$ and $S_i \in S$. 

IEICE TRANS. INF. & SYST., VOL.E95–D, NO.3 MARCH 2012
Note that we have $FR(S, S) = \bigcup_{s \in S} FCVR(s; S)$.

The vertices, edges, and faces of FCVD($S$) are induced in a standard way, for example, as done in [9]: A face of FCVD($S$) is an open connected component of the cells in FCVD($S$); note that FCVR($s; S$) may contain several faces and thus each face of FCVD($S$) is an open subset of FCVR($s; S$) for some $s \in S$. An edge $e$ of FCVD($S$) is a maximal connected subset of points such that every $x \in e$ lies on $\partial$FCVR($s; S$) for exactly two generators $s \in \bigcup S_j$. A vertex $v$ of FCVD($S$) is a point lying on $\partial$FCVR($s; S$) for at least three generators $s \in \bigcup S_j$.

The combinatorial complexity of FCVD($S$) sums up the number of its vertices, edges, and faces. Note that each edge of FCVD($S$) is defined by a unique pair of generators in $\bigcup S_j$ and thus its descriptive complexity is just $O(1)$. (This does not mean that a pair of generators determines a unique edge of FCVD($S$).) Also, each vertex of FCVD($S$) is incident to at least three edges since a vertex is defined by at least three generators in $\bigcup S_j$. Thus, giving an upper bound of the combinatorial complexity of FCVD($S$), it suffices to show an asymptotic upper bound on the number of either vertices, edges, or faces via Euler’s Formula.

### 3. General Upper Bound for Bounded Faces

In this section, we give a general upper bound on the number of bounded faces of the farthest-color Voronoi diagram FCVD($S$). A face of FCVD($S$) is called unbounded if it contains a half-line or a point at infinity; or bounded, otherwise. The number of bounded faces in the worst case is known to be $\Omega(kn)$ by Huttonlocher et al. [7]. Here, we introduce another parameter $m$ describing the configuration of given sites $S$ and then finally show the number of bounded faces is indeed bounded by $O(mn)$.

For the purpose, we give another definition on the faces: a face $f \subseteq FCVR(s; S)$ of FCVD($S$) is called skeleton-supported if there is an edge $e$ lying on the boundary of $f$ such that $e$ is defined by two generators $s$, $s'$ in the same site. Note that such an edge $e$ on $\partial$Fr is represented as a subset of a skeleton edge $e'$ of $SK(S_j)$; more precisely, $e$ appears to be a connected component of $e' \cap FR(S_j, S)$. Consider the Voronoi region $R_d(s, S_j)$ for any $s \in S_i$ and $S_j \in S$ and shoot a ray $\gamma$ from any point $x \in R_d(s, S_i)$ in the direction away from $s$. Let $\mu(x)$ be the endpoint of $\gamma \cap clR_d(s, S_j)$ other than $x$ if it is a (bounded) segment, or a point at infinity along $\gamma$, otherwise.

**Lemma 1** For any face $f \subseteq FCVR(s; S)$ of FCVD($S$) and any $x \in clf$, $\mu(x)$ lies in $clf$. Proof. By the definition of $\mu(x)$ and the star-shapedness of $R_d(s, S_j)$, $\mu(x)$ lies in $clR_d(s, S_j)$ where $x \in S_j$. We assume to the contrary that there exists $x \in clf$ such that $\mu(x)$ does not lie inside $clf$. Then, there exists a point $y \in clf$ such that $y \notin clFCVR(s, S)$. Since $\mu(x) \in clR_d(s, S_j)$, we have $y \in clR_d(s, S_j)$. See Fig. 2. This implies that $y \notin clFR(s, S_j)$.

Now, let $m$ be the minimum positive integer such that for every skeleton edge $e$ of $SK(S_j)$, $e \cap clFR(S_j, S)$ consists of at most $m$ connected components. In the following, we show an upper bound on the number of bounded faces, which depends on the parameter $m$.

**Lemma 2** Every bounded face of FCVD($S$) is skeleton-supported. Therefore, every non-skeleton-supported face of FCVD($S$) is unbounded.

**Proof.** Let $s \in S_j$ and $f \subseteq FCVR(s; S)$ be a bounded face of FCVD($S$). For any $x \in clf$, we know from Lemma 1 that $\mu(x)$ lies on $\partial f$. Note that $\mu(x)$ lies on an edge lying on $\partial FR(s, S_j)$ by definition unless $f$ is unbounded, which is a skeleton edge of $SK(S_j)$. Hence, any bounded face of FCVD($S$) is skeleton-supported while the second statement of the lemma follows from the first by contrapositive.

**Lemma 3** Suppose that for any $S_j \in S$ and any skeleton edge $e$ of $SK(S_j)$, $e \cap clFR(S_j, S)$ consists of at most $m$ connected components. Then, the number of skeleton-supported faces of FCVD($S$) is $O(mn)$.

**Proof.** By the assumption, each skeleton edge $e$ of $SK(S_j)$ for any $S_j \in S$ contributes to at most $m$ edges of FCVD($S$). Since the total number of skeleton edges of $SK(S_j)$ over all $S_j \in S$ is bounded by $O(n)$, we have at most $O(mn)$ edges $e'$ of FCVD($S$) such that $e' \subseteq e \cap clFR(S_j, S)$ for some skeleton...
edge $e$ of $SK(S_i)$. On the other hand, since any skeleton-supported face of $FCVD(S)$ has at least one such edge on its boundary by definition, we conclude that $FCVD(S)$ consists of $O(mn)$ bounded faces.

Lemma 3 together with Lemma 2 implies that the number of bounded faces is indeed $O(mn)$. The above arguments provide us a general way of bounding the number of bounded faces, which will be exploited in the following section.

4. A Realistic Model for Farthest-Color Voronoi Diagrams

In this section, we introduce a realistic model for farthest-color Voronoi diagrams, namely well-clustered, and finally give a linear bound on the combinatorial complexity of any farthest-color Voronoi diagram under this model.

**Definition 1** (well-clustered sites). A collection $S$ of $k$ sites is said to be well-clustered if and only if the following hold for any $S_i, S_j \in S$:
1. $R(S_i, S_j)$ is connected.
2. $SK(S_i) \cap R(S_i, S_j)$ is connected and contains at least one vertex of $SK(S_j)$.

The well-clustered model postulates the Voronoi region $R(S_i, S_j)$ and the Voronoi skeleton $SK(S_i)$ in the region $R(S_i, S_j)$ to be connected, which means that, intuitively speaking, each $S_i$ is not properly pierced or interfered by another. Figure 3 illustrates a typical example of a collection of well-clustered sites and its corresponding farthest-color Voronoi diagram. In many realistic inputs, each site $S_i$ consisting of point generators preserves its medoidal structure or skeleton partially when considering several number of sites of the same kind. This naturally motivates the above model.

Notice that the conditions in the definitions of the model are described in terms of the relations among the Voronoi skeletons $SK(S_i)$ and the nearest Voronoi regions $R(S_i, S_j)$ or $R(S_i, S_j)$ of each site $S_i \in S$.

A possible concrete example could be found in protein configurations. Consider several protein molecules consisting of a number of atoms. When they interact each other, the corresponding interaction surface is often described by Voronoi surfaces that are defined by two atoms from different molecules. In this case, the atoms in a molecule act as the generators in a site $S_i$, and such a collection of $k$ molecules appear to be well-clustered or skeleton-preserved since any two molecules would not get too close to each other so that one pierces or stabs the other. See also Ban et al. [12].

In the following, we investigate the farthest-color Voronoi diagram under the realistic model, and show that the diagram has indeed $O(n)$ complexity.

4.1 Unbounded Faces

We first give a bound on the number of unbounded faces of $FCVD(S)$. For our purpose, we will need the following observations.

**Lemma 4** Suppose that $S$ is well-clustered. Then, the bisector $B(S_i, S_j)$ between two sites $S_i, S_j \in S$ is a simple curve.

**Proof.** Since we suppose $S$ to be well-clustered, for any two $S_i, S_j \in S$, the nearest Voronoi regions $R(S_i, S_j)$ and $R(S_j, S_i)$ are connected. Also, by definition, $R(S_i, S_j) \cup R(S_j, S_i) = \mathbb{R}^2$. The bisector $B(S_i, S_j)$ is the boundary of $R(S_i, S_j)$ and equally of $R(S_j, S_i)$. Thus, the lemma follows.

The above lemma implies that the bisector $B(S_i, S_j)$ can be either open or closed, and that $B(S_i, S_j)$ has at most two points at infinity in either case. Now, we are ready to show the number of unbounded faces.

**Lemma 5** Suppose that $S$ is well-clustered. Then, the number of unbounded faces of $FCVD(S)$ is at most $2n - 2$.

**Proof.** Let $\Gamma$ be a closed curve at infinity. $\Gamma$ intersects every unbounded face in $FCVD(S)$ at points at infinity. Walking around along $\Gamma$ until reaching our starting point, we check which farthest-color Voronoi region $FCVR(s; S)$ we are in and record our trajectory as a sequence $\sigma$ of symbols $\bigcup S_i$ which we have passed by. In this proof, we consider each generator $s \in \bigcup S_i$ as a symbol by abuse of notation. In the following, we show that $\sigma$ is a Davenport-Schinzel sequence of order 2 with alphabet $\bigcup S_i$ so that the length of $\sigma$ is at most $2n - 1$ [13], which implies that the number of unbounded faces is at most $2n - 2$.

Suppose to the contrary that $\sigma$ is not a Davenport-Schinzel sequence of order 2. Then, $\sigma$ contains a subsequence $s_t s_t$ for $s, t \in \bigcup S_i$ with $s \neq t$. Here, we have two possibilities: (i) both $s$ and $t$ belong to a common site $S_i$, or (ii) $s \in S_i$ and $t \in S_j$ with $i \neq j$. For the former case, we have $FCVR(s; S) \subseteq FCVR(s; S_i) = R(s, S_i)$. Analogously, $FCVR(t; S) \subseteq FCVR(t; S_i) = R(t, S_i)$. This implies that we again get a subsequence $s_t s_t$ when we walk along $\Gamma$ on $FCVD(S_i)$, which coincides with the Voronoi diagram $VD_d(S_i)$ of $S_i$ with respect to $\delta$. This, however, leads to a contradiction because each region of $VD_d(S_i)$ is connected and thus any generator $s' \in S_i$ cannot have two unbounded faces in $R_d(s', S_i)$.

Similarly, for the latter case where $s \in S_i$ and $t \in S_j$ with $i \neq j$, we have $FCVR(s; S) \subseteq FCVR(s; S_i, S_j)$ and $FCVR(t; S) \subseteq FCVR(t; S_i, S_j)$. We thus get a subsequence $s_t s_t$ when we walk along $\Gamma$ on $FCVD(S_i, S_j)$. Furthermore, we have $FCVR(s; S_i, S_j) \subseteq FR(S_i, S_j) = R(S_i, S_j)$ and $FCVR(t; S_i, S_j) \subseteq FR(S_j, S_i) = R(S_j, S_i)$. This implies that we must have crossed the bisector $B(S_i, S_j)$ three or more times when walking along $\Gamma$. But $B(S_i, S_j)$ is shown to be simple in Lemma 4 and thus $B(S_i, S_j)$ has at most two points at infinity. Consequently, we get a contradiction again, and finally show the lemma.
4.2 Bounded Faces

It is relatively easy to see that the farthest-color Voronoi diagram of a well-clustered set of sites has linear complexity, based on the general observations built in Sect. 3.

**Lemma 6** Suppose that $S$ is well-clustered and let $S_i, S_j \in S$ with $i \neq j$. For each skeleton edge $e$ of $\text{SK}(S_i)$, $e \cap \text{clFR}(S_j, S_i)$ is connected.

**Proof.** By the well-clustered property of $S$, we have that for any $S_i, S_j \in S$ with $i \neq j$, $\text{SK}(S_i) \cap R(S_i, S_j)$ is connected and includes at least one vertex of $\text{SK}(S_i)$. Thus, for each skeleton edge $e$ of $\text{SK}(S_i)$, unless $e \cap R(S_i, S_j)$ is empty, $e \cap R(S_i, S_j)$ is connected and contains at least one endpoint of $e$; or it consists of two connected components and contains both endpoints of $e$. Observe that $e \cap \text{clFR}(S_j, S_i) = e \setminus \text{intR}(S_j, S_i)$ is a connected subset in any case, implying the lemma.

**Lemma 7** Suppose that $S$ is well-clustered. Then, for any skeleton edge $e$ of $\text{SK}(S_i)$, $e \cap \text{clFR}(S_j, S_i)$ is connected.

**Proof.** Recall that $\text{FR}(S_i, S) = \bigcap_j R(S_j, S_i)$, and thus we have $e \cap \text{clFR}(S_j, S_i) = \bigcap_j (e \cap \text{clR}(S_j, S_i))$. Since each $e \cap \text{clR}(S_j, S_i)$ is connected by Lemma 6, their intersection is also connected.

Thus, if $S$ is well-clustered, then a subedge of any skeleton edge $e$ appears at most once in $\text{FCVD}(S)$.

**Lemma 8** Suppose that $S$ is well-clustered. The number of bounded faces of $\text{FCVD}(S)$ is at most $O(n)$.

**Proof.** This directly follows from Lemmas 7, 3 and 2.

4.3 Complexity Bound

Putting the above arguments altogether concludes the main theorem.

**Theorem 1** Given a well-clustered set of $k$ sites $S$ consisting of a total of $n$ point generators, the farthest-color Voronoi diagram $\text{FCVD}(S)$ under any $L_p$ metric has complexity $O(n)$.

**Proof.** The number of faces of $\text{FCVD}(S)$ is already shown to be $O(n)$ by Lemmas 5 and 8, provided that $S$ is well-clustered. On the other hand, as discussed above, each vertex of $\text{FCVD}(S)$ is incident to at least three edges. This implies by Euler’s Formula that the number of vertices and edges of $\text{FCVD}(S)$ is asymptotically linear to the number of its faces. Hence, we conclude the theorem.
5. Concluding Remarks

The currently best algorithm to compute FCVD(S) for any $L_p$ metric requires $O(kn \log n)$ time in general due to Huttenlocher et al. [7]. A natural question is: can one compute the farthest-color Voronoi diagram $FCVD(S)$ much faster, provided that $S$ is well-clustered? We believe that computing $FCVD(S)$ can be done in $O(n \text{polylog } n)$ time if $S$ is well-clustered; one possible approach which must be examined is to use the parametric search technique by Cheong et al. [14], which was used to merge two farthest-polygon Voronoi diagrams.

As aforementioned, in realistic cases, the farthest-color Voronoi diagram hardly has complexity $\Omega(kn)$. Thus, it can be said that this paper initiates research on realistic inputs for farthest-color Voronoi diagrams. On the other hand, the model we introduced defines a class of diagrams having linear complexity. An ultimate goal of this work would be to find as large a class of linear-sized farthest-color Voronoi diagram as possible. The research in this direction is also related to the axiomatic study on Voronoi diagram having linear complexity, such as the abstract Voronoi diagram [15] and the furthest-site abstract Voronoi diagram [9].

References

[1] F. Aurenhammer and R. Klein, “Voronoi diagrams,” in Handbook of Computational Geometry, ed. J.R. Sack and J. Urrutia, Elsevier, 2000.
[2] M. Abellanas, F. Hurtado, C. Icking, R. Klein, E. Langetepe, L. Ma, B. Palop, and V. Sacristán, “The farthest color Voronoi diagram and related problems,” Technical Report 002, Rheinische Friedrich-Wilhelms-Universität, Bonn, 2006.
[3] S.W. Bae, S. Choi, C. Lee, and S. Tanigawa, “Exact algorithms for the bottleneck Steiner tree problem,” Proc. 20th Int. Sympos. Algo. Comput., pp.24–33, 2009.
[4] S.W. Bae, C. Lee, and S. Choi, “On exact solutions to the Euclidean bottleneck Steiner tree problem,” Inf. Process. Lett., vol.110, no.16, pp.672–678, 2010.
[5] C. Lee, D. Shin, S.W. Bae, and S. Choi, “Best and worst-case coverage problems for arbitrary paths in wireless sensor networks,” Proc. IEEE 7th Int. Conf. Mobile Adhoc and Sensor Systems (MASS 2010), pp.127–136, 2010.
[6] E. Papadopoulou, “The Hausdorff Voronoi diagram of point clusters in the plane,” Algorithmica, vol.4, no.2, pp.63–82, 2004.
[7] D.P. Huttenlocher, K. Kedem, and M. Sharir, “The upper envelope of Voronoi surfaces and its applications,” Discrete Comput. Geom., vol.9, pp.267–291, 1993.
[8] F. Preparata and M. Shamos, Computational Geometry: An Introduction, Springer Verlag, 1985.
[9] K. Mehlhorn, S. Meiser, and R. Rasch, “Farthest site abstract Voronoi diagrams,” in Proc. 11th Annu. Sympos. Comput. Geom., pp.583–616, 2001.
[10] M. de Berg, M.J. Katz, F. van der Stappen, and J. Vleugels, “Realistic input models for geometric algorithms,” Algorithmica, vol.34, pp.81–97, 2002.
[11] D.T. Lee, “Two-dimensional Voronoi diagrams in the $L_p$-metric,” J. ACM, vol.27, pp.604–618, 1980.
[12] Y.E.A. Ban, H. Edelsbrunner, and J. Rudolph, “Interface surfaces for protein-protein complexes,” Proc. 8th Annual International Conference on Research in Computational Molecular Biology (RECOMB 2004), pp.205–212, 2004.
[13] M. Sharir and P.K. Agarwal, Davenport-Schinzel Sequences and Their Geometric Applications, Cambridge University Press, New York, 1995.
[14] O. Cheong, H. Everett, M. Glisse, J. Gudmundsson, S. Hormus, S. Lazard, M. Lee, and H.S. Na, “Farthest-polygon Voronoi diagrams,” Proc. 15th Annu. Euro. Sympos. on Algorithms, LNCS, vol.4698, pp.407–418, 2007.
[15] R. Klein, “Concrete and abstract Voronoi diagrams,” Lect. Notes Comput. Sci., vol.400, Springer-Verlag, Berlin, Germany, 1989.

Sang Won Bae was born in 1980 and got a Ph.D. in 2008 at Dept. Computer Science, Korea Advanced Institute of Science and Technology, Daejeon, Korea. At present, he is working as an assistant professor at Dept. Computer Science, Kyonggi University, Suwon, Korea.