The temperature dependence of rates of electron impact ionization and two electrons recombination are calculated using Wannier cross section of electron impact ionization of neutral hydrogen atom. Entropy production and power dissipation are derived for the case when ionization degree deviates from its equilibrium value. Damping rate of the sound waves is calculated and the conditions when ionization processes dominate are considered. A WKB approximation for the heating mechanism of solar chromosphere is proposed. Several analogous phenomena for damping rates in liquids and crystals are shortly discussed, for example, deaf sound of a glass of beer or English salt solution. An explicit expression for the second or bulk viscosity of hydrogen plasma is calculated from firsts principles.

I. INTRODUCTION

In many cases the absorption of sound waves cannot be described by hydrodynamic equations. The oscillations of the pressure of the longitudinal sound waves create oscillations of the temperature. Further temperature oscillations change the equilibrium densities participating in the chemical equilibrium constants. In such a way the propagation of sound waves induces small oscillations of the chemical composition of the medium. The forced chemical reactions and oscillations of the compound however are related with average increase of the entropy at almost constant temperature. This increase of the entropy gives an irreversible energy dissipation and this dissipation creates an additional damping rate of the sound waves.

Within the hydrodynamic approach, the sound absorption is proportional to the square of the frequency $\omega^2$ while the new mechanism proposed in the present article gives a constant damping rate. Definitely at some low frequencies the considered mechanism dominates.

The hydrogen gas is actually a very simple object and in this model case all steps in the listed above chain of reasoning can be performed analytically.

Starting from the classical consideration by Wannier on near threshold electron impact ionization of the hydrogen (H) atom in ground state we calculate analytically the chemical rates of electron ionization and corresponding to the inverse process of two electron recombination.

Further we derive the kinetic equations for the densities of electrons $n_e$, protons $n_p$, neutral atoms $n_0$, time dependence of the temperature $T(t)$ and the time derivative of entropy which is one of the central results.

The energy dissipation related to entropy production gives the explicit expression for the extinction of sound waves and all details of the derived formula can be calculated from firsts principles and compared with the experiment for a contemporary review see Ref. 6. In the WKB approximation the derived sound absorption reveals the mechanism of heating of hydrogen plasma in the partially ionized solar chromosphere. Physics of some related phenomena in liquids and crystals are shortly analyzed.

The paper is organized as follows: in the next section hydrogen atom ionization by electron impact is considered, then in Sec. II electron density kinetics is analyzed. The kinetic equation for the entropy is derived in Sec. III. The derived result are applied for the calculation of sound absorption in Sec. IV. Finally in Sec. V are discussed several analogous phenomena and it is made conclusion how the present research can be continued.

II. HYDROGEN ATOM IONIZATION BY ELECTRON IMPACT

For electron energies $\varepsilon$ slightly above the ionization threshold $I$ correlated motion of two electrons in the final state is almost classical and for this difficult problem Wannier obtained the well-known result for the ioniza-
where \( \omega \approx 1.18 \), \( C_W \) is a dimensionless constant and
\[
a_B = \frac{\hbar c}{e^2 mc} = \frac{\hbar}{me^2}, \quad \frac{e^2}{\hbar c} \approx \frac{1}{137}
\]
is the Bohr radius.
For the rate of the ionization reaction \( H + e \rightarrow p + e + e \)
\[
\beta = (v \sigma (\varepsilon)), \quad v = \sqrt{2\varepsilon/m},
\]
the integration on the Maxwell velocity \( v \) distribution
\[
\beta = 2\sqrt{\frac{2}{\pi}} C_W a_B^2 v_T e \int (\varepsilon - i)^\omega e^{1/2} e^{-\varepsilon} d\varepsilon,
\]
\[
v_T e \equiv \sqrt{T/m}, \quad \varepsilon \equiv \varepsilon/T, \quad \iota \equiv I/T \gg 1
\]
for low temperatures \( T \ll I \) gives
\[
\beta \approx C_\beta \beta_B e^{-\iota}, \quad C_\beta \equiv 2\Gamma(\omega + 1)C_W/\sqrt{\pi},
\]
\[
\beta_B \equiv v_B a_B, \quad v_B = (e^2/\hbar c) \iota \equiv e^2/\hbar.
\]
In other words the ionization potential \( I \) is the activation energy of the rate or the chemical reaction parameterized by the Bohr velocity and natural for the atomic physics rate unit \( v_B a_B^2 \). According experimental study by McGowan and Clarke they found \( \omega \approx 1 \) and the slope of the almost linear dependence depicted in their Fig. 6 gives \( C_W \approx 2.7 \) whence according Eq. (6) we calculate \( C_\beta \approx 3.0 \approx 1 \).

The considered process gives for the time derivative of the electron density
\[
d_t n_e \approx \beta n_0 n_e,
\]
where \( n_0 \) is the volume density of neutral atoms. In the next section we will consider other processes in the hydrogen plasmas.

### III. ELECTRON DENSITY KINETICS

In general case we have to take into account the rate of many processes
\[
\beta_\varphi : \quad H + \gamma \rightarrow p + e,
\]
\[
\beta : \quad H + e \rightarrow p + e + e,
\]
\[
\beta_\nu : \quad H + H \rightarrow p + e + H,
\]
\[
\gamma_\varphi : \quad p + e \rightarrow H + \gamma,
\]
\[
\gamma : \quad p + e + e \rightarrow H + e,
\]
\[
\gamma_\nu : \quad (p + e)^* + H \rightarrow H + H,
\]
and more complete set of kinetic equations for the densities of electrons \( n_e \), protons \( n_p \) and neutral atoms \( n_0 \) looks like
\[
\dot{n}_e = n_e^{(+)} - n_e^{(-)}, \quad \dot{n}_e = \dot{n}_p = -\dot{n}_0,
\]
\[
\dot{n}_e^{(+)} = \beta_\varphi n_0 + \beta_\nu n_e + \beta_\nu n_0^2,
\]
\[
\dot{n}_e^{(-)} = \gamma_\varphi n_p n_e + \gamma_\nu n_p^2 + \gamma_\nu n_e n_p n_0.
\]
Let us give for reference the rate of direct process of radiation recombination of the free electrons directly to the ground state of the H atom
\[
\gamma_\varphi = C_\varphi \left( \frac{\hbar}{me} \right)^3 \frac{I}{\sqrt{T}}, \quad C_\varphi = \frac{210\pi^3/2}{3e^4} \approx 34.8,
\]
\[
\sigma_\varphi = C_\varphi \frac{(e^2/\hbar c)a_B^4 T}{m^2 c^2 v^2}, \quad \frac{h}{mc} = \frac{e^2}{\hbar c} a_B.
\]
In the derivation the principle of the detailed balance is used and the result for the low energy cross-section of photo-ionization \( \sigma_\varphi \).

Another process is the neutral atom catalyzed rate of radiationless recombination at low temperatures considered by Pitaevskii\textsuperscript{9,10}
\[
\gamma_\varphi = C_\varphi \left( \frac{\hbar}{mc} \right)^3 I \frac{\sqrt{T}}{\sqrt{T}}, \quad C_\varphi = \frac{32\sqrt{2\pi}}{3} \approx 26.7, \quad T \ll I \frac{m}{M},
\]
where \( M \) is the proton mass; see also article\textsuperscript{11} and references therein.

The rates \( \dot{n}_p \bar{n}_e \gamma_\varphi = \bar{n}_0 \beta_\varphi \) are related by the principle of the detailed balance\textsuperscript{7,10} followed by thermal averaging
\[
\frac{\bar{n}_p \bar{n}_e}{\bar{n}_0} = \frac{\beta}{\gamma} = \frac{\beta_\varphi}{\gamma_\varphi} = \frac{\beta_\nu}{\gamma_\nu},
\]
where \( \bar{n}_e, \bar{n}_p, \) and \( \bar{n}_0 \) are the equilibrium densities for electrons, protons and neutral atoms.

In order to recall the equations for thermal equilibrium let us introduce convenient notations\textsuperscript{12}
\[
n_Q \equiv \left( \frac{MT}{2\pi\hbar^2} \right)^{3/2}, \quad n_q \equiv \left( \frac{mT}{2\pi\hbar^2} \right)^{3/2},
\]
\[
n_g \equiv n_q e^{-\iota} \equiv \left( \frac{mT}{2\pi\hbar^2} \right)^{3/2} e^{-1/T}, \quad p_g \equiv T n_g.
\]
In this case the chemical balance of the reaction \( H \leftrightarrow p + e \) leads to relation between the chemical potentials and densities\textsuperscript{13}
\[
\mu_0 = \mu_e + \mu_p, \quad \mu_e = I - T \ln \frac{\bar{n}_e n_g}{n_e},
\]
\[
\mu_p = -T \ln \frac{\bar{n}_p n_g}{n_p}, \quad \mu_0 = -T \ln \frac{\bar{n}_p n_g}{n_0},
\]
\[
\frac{\bar{n}_p \bar{n}_e}{\bar{n}_0} = n_g(T).
\]
where \( g_0 = 4, g_e = g_p = 2 \) are the statistical weights of the particles. The equilibrium degree of ionization \(^{13,14}\)

\[
\alpha \equiv \frac{n_e}{n_p}, \quad n_p = n_0 + n_p = \rho_m/M, \quad n = n_0 + n_p + n_e,
\]

\[
n_e = n_p = \alpha n_p, \quad n_0 = (1 - \alpha) n_p \tag{25}
\]

can be expressed by the mass density of the gas \( \rho_m = M n_p \) or by the pressure \( p = n T \). The substitution of these \( n_e, n_p \) and \( n_0 \) in Eq. (24) gives \(^{13}\)

\[
\beta = \frac{1}{1 + \frac{p}{p_s(T)}} = -1 + \frac{1}{2} \frac{n_p}{n_s(T)}, \quad (0, 1), \tag{26}
\]

where \( g_0/g_0 g_e = 1 \) is taken into account. Using so introduced notations for plasma with constant density \( n_p = \text{const} \) we obtain the dynamic equation for the degree of ionization

\[
\frac{d\alpha}{d\theta} = -\frac{[(1 - \alpha) - \alpha^2 (1 - \alpha)\alpha]}{\alpha^2}, \tag{27}
\]

\[
\theta = \int_0^t \nu_\beta (t') \, dt' \approx \nu_\beta t, \quad \nu_\beta = \beta n_p, \tag{28}
\]

\[
\frac{\alpha + (1 - \alpha)\alpha}{\alpha + (1 - \alpha)\alpha} \left[ \frac{(1 - \alpha)\alpha}{\alpha + (1 - \alpha)\alpha} \right]^{1/(2 - \alpha)} \frac{\alpha - \alpha}{\alpha_0 - \alpha} = \frac{\alpha}{\alpha_0} e^{-\theta},
\]

which (in the model case of constant temperature) even have the analytical solution \( \alpha(t) \) describing the kinetics between initial condition \( \alpha(0) = \alpha_0 \) and final asymptotic equilibrium value \( \alpha(\infty) = \alpha_T \). Let us analyze the relaxation close to equilibrium when \( |\alpha - \alpha_T| \ll 1 \). Simple differentiation of the right side of Eq. (27) gives approximate equation and its solution

\[
\frac{d\alpha}{dt} = -\frac{1}{\tau(\alpha - \alpha_T)}, \quad \frac{1}{\tau} = (2 - \alpha)\beta n_p, \tag{29}
\]

\[
(\alpha - \alpha_T) = (\alpha_0 - \alpha_T) e^{-t/\tau}.
\]

This approximation corresponds to Mandelstam-Leontiev\(^{21}\) consideration of the second viscosity\(^{15}\). In the present paper for illustration we focus on the high-frequency case for which \( \omega \gg \nu_\beta = \beta n_p \) which gives frequency independent absorption. For these high frequencies for one wave period ionization-recombination process is negligible.

Let us also recall the expressions for the volume densities of the internal energy

\[
\mathcal{E} = c_v (n_e + n_p + n_0) T + n_e I, \quad c_v = 3/2 \quad (30)
\]

and entropy

\[
\mathcal{S} = \mathcal{S}_e + \mathcal{S}_p + \mathcal{S}_0, \tag{31}
\]

\[
\mathcal{S}_e = n_e \left( \ln \left[ \frac{g_e n_e}{n_e} \right] + c_v + 1 \right), \tag{32}
\]

\[
\mathcal{S}_p = n_p \left( \ln \left[ \frac{g_p n_p}{n_p} \right] + c_v + 1 \right), \tag{33}
\]

\[
\mathcal{S}_0 = n_0 \left( \ln \left[ \frac{g_p n_p}{n_0} \right] + c_v + 1 \right). \tag{34}
\]

In the next section we derive their kinetics.

**IV. KINETIC EQUATION FOR THE ENTROPY**

Radiation processes are essential only for very low densities. As was pointed out by Pitaevskii\(^{10}\) for dense enough but still cool \( T \ll I \) plasma, the main role can come to recombination with participation of a second electron as a third body. In this case the chemical equilibrium relation Eq. (22) and Saha ionization equation Eq. (24) give the rate of the two electron recombination

\[
\gamma = \frac{\beta}{n_s(T)} = C_0 \beta \left( \frac{2\pi h^2}{mT} \right)^{3/2}, \tag{35}
\]

which together with the ionization rate Eq. (6) is one of the results of the present paper. In this section we use the approximate kinetic equation

\[
\dot{n}_e \approx \beta n_0 n_e - \gamma n_p n_e^2. \tag{36}
\]

Let in the beginning consider spatial homogeneous plasma with constant energy density \( \mathcal{E} = \text{const} \), which is not in chemical equilibrium. Differentiation of Eq. (36) gives for the time derivative of the temperature

\[
c_v n_0 \dot{T} = -(1 + c_v) \dot{n}_e, \quad \dot{T} = \frac{dT}{dt}. \tag{37}
\]

This time derivative we have to substitute in the time derivative of the entropy

\[
\dot{\mathcal{S}} = \frac{\partial \mathcal{S}}{\partial \dot{\mathcal{T}}} \dot{\mathcal{T}} + \frac{\partial \mathcal{S}_e}{\partial n_e} \dot{n}_e + \frac{\partial \mathcal{S}_p}{\partial n_p} \dot{n}_p + \frac{\partial \mathcal{S}_0}{\partial n_0} \dot{n}_0. \tag{38}
\]

The elementary substitution after some algebra gives

\[
\dot{\mathcal{S}} = (\beta n_e n_0) \ln \left[ \frac{n_e n_p}{n_0 n_s} \right] \left( \frac{n_e n_p}{n_0 n_s} - 1 \right). \tag{39}
\]

Introducing a dimensionless variable which describes the deviation from the chemical equilibrium

\[
\chi \equiv \frac{n_e n_p}{n_0 n_s} - 1 = \frac{n_e n_p}{n_0} \frac{\tilde{n}_0}{n_e n_p} - 1 \tag{40}
\]

and positive income rate in the kinetic equation

\[
\dot{n}_e^+ \approx \beta n_0 n_e > 0 \tag{41}
\]
the entropy production reads as
\[ \dot{S} = \dot{n}_\varepsilon^{(+)} \eta(\chi), \quad \eta(\chi) \equiv \chi \ln(1 + \chi) > 0, \] (42)
which is actually the general Boltzmann Eta-theorem applied to our current problem. At thermodynamic equilibrium \( \dot{\chi} = 0 \), \( \eta(\dot{\chi}) = 0 \) and entropy is constant \( \dot{S} = 0 \).

For many practical problems it is important to know the entropy production of a system close to equilibrium. For small deviations of the concentrations from the Saha equilibrium equation
\[ \eta(\chi) = \chi \ln(1 + \chi) \approx \chi^2, \quad \text{for } |\chi| \ll 1 \] (43)
we can take constant income rate \( \nu \equiv \beta n_0 \bar{n}_e \) and the volume density of entropy production takes the form
\[ \dot{S} \approx \nu \chi^2, \quad Q = T \dot{S} = Q_\chi \chi^2, \quad Q_i \equiv T \nu. \] (44)
This small increase of the entropy at almost constant temperature is related to the power density of the irreversible processes \( Q \) parameterized by a constant \( Q_i \) describing the rate of the chemical reactions per unit volume.

In the next section we apply average volume density of dissipated power
\[ \overline{Q} = Q_i \langle \chi^2 \rangle \] (45)
for the problem of damping of sound waves in partially ionized hydrogen plasma. This averaging of energy production on the period of oscillations is a good approximation only for high enough frequencies \( \omega \gg \nu_\chi = (1-\alpha) \alpha \beta n_\rho \) for which the change of the ionization degree is small for the period of oscillations.

V. SOUND ABSORPTION

Let us consider a plane sound wave with frequency \( \omega \) and \( x \)-axis chosen along the wave-vector \( \mathbf{k} \) and the direction of the longitudinal oscillation of the velocity \( \nu_\chi \)
\[ v_x(x,t) = v_0 \cos(kx - \omega t), \quad \omega = c_s k, \quad v_0 \ll c_s \] (46)
with amplitude \( v_0 \) much smaller than the sound speed \( c_s \). The sound wave has the averaged volume density of mechanical energy
\[ \overline{E} = 2 \left\langle \frac{1}{2} \rho v^2 \right\rangle = \frac{\rho v_0^2}{2}, \] (47)
twice time the averaged density of the kinetic energy density \( \rho v^2 / 2 \) and \( \langle \cos^2 \rangle = 1/2 \). The calculated in the former section dissipation power is just the damping power of the sound wave
\[ \overline{Q} = -\overline{\dot{E}}. \] (48)
In WKB approximation we suppose that amplitude of the velocity \( v_0(t) \propto e^{-\gamma t} \) is exponentially decaying with small rate \( \gamma \ll \omega \). For the energy density the rate is doubled \( \overline{E} \propto e^{-\gamma t} \). The damping rate is given by the logarithmic derivative
\[ 2 \gamma = -\frac{\overline{\dot{E}}}{\overline{E}} = \frac{\overline{Q}}{\overline{E}}. \] (49)

For sound waves which have negligible dispersion \( c_s = \omega/k \) is more convenient to follow a wave packet propagating along the \( x \)-axis with coordinate of the packet (in WKB approximation) \( x = c_s t \). This means that we consider spatial dependence of the amplitude of the velocity \( v_0 \propto e^{-\gamma t} \) and for the space damping rate we have
\[ \gamma_{\text{ion}} = \frac{\overline{Q}}{2 c_s \langle E \rangle}. \] (50)

In order to calculate this ionization induced damping we have to calculate the numerator \( \overline{Q} \) considering plane sound wave.

Oscillations of the velocity \( \nu(x,t) \) create oscillations of the mass density \( \rho = M n_\rho \), temperature \( T \), electron \( n_e \), proton \( n_p \) and atom \( n_0 \) densities, and the density of Saha \( n_s \) given by definition Eq. (23):
\[ \rho = \rho_0 + \rho', \quad n_p = n_p(0) + n_p', \quad n_0 = n_0 + n_0', \] (51)
\[ n_e = \bar{n}_e + n_e', \quad n_p = \bar{n}_p + n_p', \quad T = T_0 + T', \] (52)
\[ n_s(T) = n_s(T_0) + \frac{dn_n}{dT} T', \] (53)
\[ \frac{dn_n}{dT} = \frac{3}{2T} + \frac{I}{T^2}, \quad \text{for } I / T \gg 1, \] (54)
where the 0 subscript is omitted from now on in the linearized approximation for all oscillating variables.\( ^{46} \)

The mass conservation equation \( \partial_t \rho = -\text{div}(\rho \mathbf{v}) \) for small amplitude waves gives
\[ \frac{\rho'}{\rho} = \frac{v_0}{c_s} \cos(kx - \omega t). \] (55)

For adiabatic in initial approximation compressions the chemical compound is not changed and all particles have proportional oscillations of the density.
\[ n_e' = \bar{n}_e, \quad n_p' = n_p, \quad T' = T, \quad n_0' = n_0, \quad n_s' = n_s \] (56)

For adiabatic \( S = \text{const} \) compression of an ideal gas with constant heat capacity per particle \( c_v = 3/2 \) we have the well known low\( ^{41} \) \( TV^{1/c_v} = \text{const} \) whence
\[ \frac{T'}{T} = \frac{1}{c_v} \frac{\rho'}{\rho}, \] (57)

This oscillation of the temperature describes the oscillations of the temperature density determining the ionization rate
\[ \frac{n_s'}{n_s} \approx \frac{I T'}{I T} \frac{\rho'}{\rho} = \frac{I}{c_v} \frac{\rho'}{\rho}. \] (58)
Now we have all ingredients to calculate the variable \( \chi \) from Eq. (49) describing the deviation from equilibrium
\[
\chi \approx \frac{n_0' + n_p'}{n_e} - \frac{n_0}{n_p} - \frac{n_0'}{n_s} \approx \frac{\nu_0}{c_v n_s c_s} = \frac{\nu_0}{c_v c_s} \cos(kx - \omega t).
\]
For low temperatures \( T \ll kT \), the dimensionless ratio \( \epsilon = T/T \gg 1 \) and only variations of the Saha density \( n_s \) are essentials. Averaging \( \langle \cos^2 \rangle = 1/2 \) gives
\[
\langle \chi^2 \rangle = \frac{1}{2} \left( \frac{1}{c_\gamma} T \right)^2 \ll 1, \tag{59}
\]
Now we can calculate the power according Eq. (50)
\[
\langle \mathcal{Q} \rangle = \frac{1}{2} \left( \frac{\nu_0}{c_v c_s} \right)^2 Q_0. \tag{60}
\]
Substitution of this power density together with energy density Eq. (47) in the definition for the extinction Eq. (50) finally gives
\[
\gamma_{\text{ion}} = \frac{Q_0 t^2}{2 c_v^2 c_s^2} = \frac{\nu_0^2}{2 c_v^2 c_s^2} (1 - \alpha) \alpha n_p T^2 e^{-T/T}, \tag{63}
\]
\[
\gamma_{\eta} = \frac{(4 \eta + \zeta)}{2 \rho c_s^3} \left[ \frac{1}{\gamma_s} - \frac{1}{\epsilon_p} \right] e = a_\eta \omega^2, \tag{64}
\]
where we have introduced the thermal velocity of protons \( v_{\text{TrP}} \equiv \sqrt{T/M} \) and for comparison we give the hydrodynamic extinction \( \gamma_{\eta} \) created by absorption of the sound waves by the first \( \eta \) and second \( \zeta \) viscosity coefficients, and heat conductivity \( \kappa \). For mono-atomic ideal gas cocktail \( c_p = c_v + 1 = 5/2 \). For frequencies
\[
\omega < \omega_{\text{ion}} \equiv \sqrt{\gamma_{\text{ion}}/a_\eta} \tag{62}
\]
the ionization mechanisms definitely dominates. And for every physical problem we have to check whether this inequality is satisfied or not.

VI. DISCUSSION AND CONCLUSION

Here we wish to emphasize the difference between the hydrodynamic sound damping by viscosity coefficients and heat capacity and energy dissipation due to fluctuations of the chemical concentrations and related to them irreversible increasing of entropy. On one hand, the energy dissipation in hydrodynamics is quadratic with respect to the gradients of the velocity and temperature. That is why the hydrodynamic damping is proportional to the square of the wave-vector and the square of the frequency
\[
\gamma_{\eta} = \frac{k^2}{2 c_s} \left[ \frac{4 \eta}{3 \beta} + \ldots \right] \propto k^2 \propto \omega^2. \tag{72}
\]
Roughly speaking fluids are low-pass filters. That is why from a distant thunderstorm we hear only the low frequency roar of the lightning. On the other side the entropy increase due to fluctuations of the chemical composition does not require spatial homogeneity. That is why the ionization sound damping is frequency independent; even the basses are absorbed for \( \omega \gg \omega_{\text{ion}} \).

A. Mandelstam-Leontiev result for the dispersion of second viscosity

In order to derive the second viscosity coefficient of the hydrogen plasma we use the Mandelstam-Leontiev results as they are presented by Landau-Lifshitz. The main details are the derivatives of the pressure with respect to the mass density. There are two limit cases: fast adiabatic compression of mono-atomic gases at constant
degree of ionization $\alpha$ and extremely slow compression at which in every moment the ionization degree follows the equilibrium Saha equation corresponding to the density

$$c_\infty^2 \equiv c_\alpha^2 = \left(\frac{\partial p}{\partial \rho}\right)_\alpha, \quad c_0^2 \equiv \left(\frac{\partial p}{\partial \rho}\right)_\text{eq}, \quad c_0 < c_\infty. \quad (73)$$

In both the cases indexed with “$\infty$” and “eq” there is no external heating applied to the gases. Supposing that the frequency $\omega$ is real the wave-vector is complex

$$k = k_1 + ik_2 = \sqrt{\frac{1 - i\omega\tau}{c_0^2 - c_\infty^2 i\omega\tau}}, \quad k_1 = \Re(k), \quad (74)$$

Wave propagation requires the imaginary part of the wave-vector to be always small $k_2 \equiv \gamma_{\text{ion}} = 3(k) \ll k_1$. The Mandelstam-Leontiev dispersion has two limit cases

$$k \approx \frac{\omega}{c_0} + i \frac{\omega^2\tau}{2c_0^2(c_\infty^2 - c_0^2)}, \quad \omega\tau \ll 1 \quad (75)$$

and

$$k \approx \frac{\omega}{c_\infty} + i \frac{c_\alpha^2 - c_0^2}{2\tau c_\infty^3}, \quad \omega\tau \gg 1. \quad (76)$$

For arbitrary frequencies, introducing frequency dependent second viscosity

$$\zeta(\omega) \equiv \frac{\zeta_0}{1 - i\omega\tau}, \quad \zeta_0 \equiv (c_{\infty}^2 - c_0^2)\tau \rho \quad (77)$$

one can use the usual hydrodynamic relation between the second viscosity and extinction

$$c_\omega = \frac{\omega}{k_1}; \quad \gamma_{\text{ion}} = \frac{\omega^2\Re(\zeta)}{2\rho c_\omega^2} \quad (78)$$

This general formula gives: quadratic dependence of the extinction $\gamma(\omega)$ at low frequencies, and the frequency independent extinction $\gamma_{\infty}$ for high frequencies

$$\gamma_0 = \frac{\omega^2\zeta_0}{2\rho c_0^2}, \quad \gamma_{\infty} = \frac{c_\alpha^2 - c_0^2}{2\tau c_\infty^3} = \frac{\zeta_0}{2\tau^2\rho c_\infty^2}. \quad (79)$$

For high frequencies $\omega \gg 1/\tau$ the comparison between the last Mandelstam-Leontiev expression and our former result Eq. (61) using Eq. (29) gives after some algebra the final result for the second viscosity of the hydrogen plasma

$$\zeta_0 = 2\rho\tau^2 c_\infty^3 \gamma_{\infty} = \frac{(1 - \alpha)\alpha I^2}{(2 - \alpha)^2 T \beta} \frac{I^2}{e^{u/T}} \approx \frac{(1 - \alpha)\alpha I^2 e^{u/T}}{(2 - \alpha)^2 C_\beta \beta \delta B T}. \quad (80)$$

Perhaps only for this simplest gas it is possible to calculate the bulk viscosity from first principles. This result has fundamental importance for calculation of heating of solar chromosphere by sound and slow-magnetosonic waves.

### B. Returning to high frequencies $\omega\tau \gg 1$

It is extremely interesting to check whether the height temperature profile in the partially ionized solar chromosphere can be described by absorption of the longitudinal sound waves coming from turbulent convection zone. We suppose that frequencies much higher than Saha ionization relaxation rate $1/\tau$ will give the main contribution. A calculation to show whether the optically transparent chromosphere is acoustically opaque with small transmissibility (sound “tunneling”) coefficient

$$T_{\text{TR}} = \exp \left(-\int_0^{\hbar_{\text{TR}}(x)} 2\gamma_{\text{ion}}(x) \, dx\right) \ll 1? \quad (81)$$

The integration here is across the whole chromosphere from the photosphere to the solar transition region $\hbar_{\text{TR}}$. No doubts the launching of the Parker Solar Probe satellite and its G$\Sigma$ financing gives a new impetus on solar physics and physics of plasmas in general. Here we present a problem of H plasma which can be extended in many directions.

It is interesting to mention that the damping of mechanic oscillations by chemical or ionization reaction can be observed in many common situations. Pour beer or champagne into identical glasses and compare their deaf sound with the ringing of the of glasses full of wine or cognac produced by hitting them with a teaspoon. What’s the difference? Carbon dioxide in small bubbles.

For those whom some Lord forbids C$_2$H$_5$OH let, they fill their glass with water and add a tablespoon of English salt (Epsom salt, epsomite MgSO$_4$ · 7H$_2$O); cheers. What is the difference between English salt and Himalayan salt? Partial salvation

$$\alpha = \frac{[\text{Mg}^{2+}]}{[\text{MgSO}_4] + [\text{Mg}^{2+}]} \quad (82)$$

versus almost complete one

$$\alpha = \frac{[\text{Na}^+]}{[\text{Na}^+] + [\text{NaCl}]} \quad (83)$$

The deaf sound of a glass water with English salt is related to the big rate $\nu$ of the reaction

$$\text{MgSO}_4 \leftrightarrow \text{Mg}^{2+} + \text{SO}_4^{2-} \quad (84)$$

and the significant value of $(1 - \alpha)\alpha < 0.25$. However the physico-chemistry of water solutions has no the transparent simplicity of the hydrogen plasma and the analogy with bulk viscosity damping is rather qualitative.\text{Fig. 2}

The last example of damping is related to ionization reactions give quartz resonators. Q-factors of those resonators can be significantly decreased due to Al impurities and changes of the ionization charge in Al ion by the strength of piezo-vibrations.

After this short excursion let us return to the H plasma and repeat the results derived in the present article:
1) using the Wannier ionization cross-section we calculate the rate $\beta$ of electron impact ionization Eq. (4) of the neutral atom, and 2) simultaneously as inverse process the rate of two electron recombination $\gamma$ of the proton Eq. (25). 3) The entropy production $\dot{S}$ Eq. (44) and the dissipation power $Q_{\text{ion}}$ due to deviation $\chi$ Eq. (40) from the chemical equilibrium is a good basis for further consideration. 4) Our main result is the damping rate from the chemical equilibrium is a good basis for further consideration. 5) We propose a scenario for heating mechanisms of the solar chromosphere and the explicit formula can be applied for existing models of the solar corona. Up to now the heating mechanism of the solar chromosphere is an open problem and the height dependence of the temperature $T(h)$ has not been calculated by first principle hydrodynamic calculations. 6) In small magnetic fields Alfvén waves (AW) and Slow Magnetosonic Waves (SMW) have common dispersion $\omega = V_A k |\cos(\theta)|$, where $\rho V_A^2 = B^2/\mu_0$ and $\theta$ is the angle between the constant external magnetic field and the wave vector. However the damping rates can be completely different due to the pressure and temperature oscillations accompanying SMW. We expect $\gamma_{\text{SMW}} \gg \gamma_{\text{AW}}$. 7) The strong absorption of SMW in partially ionized plasmas explains why magneto-hydrodynamic oscillations of the magnetic field are mainly orthogonal to the static magnetic field which is in qualitative agreement with the observations by different space missions.

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* E-mail: mishonov@bgp.physics.eu
† E-mail: varonov@isp.bas.bg

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