Non Unitary Neutrino Mixing angles Matrix in a Non Associative 331 Gauge Model

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Abstract. A possible non-unitary neutrino-mixing matrix is discussed in the context of a low energy limit of a recently proposed 331 gauge model within the non associative geometry of Wulkenhaar’s approach. Some bounds on certain physical parameters of the model are also obtained

1. Introduction
One of the greatest achievements of the non-commutative geometry (NCG) is the geometrization of the standard model [1,2,3] and many interesting topics have been treated and outstanding problem have been solved and understood within this approach so far. However, Connes’ prescription in NCG is compatible only with linear representations of the matrix group which imposes very stringent constraints on gauge models. Indeed, it was shown that [4,5] the only models which can be constructed in this approach are the standard model, the Pati-Salam [6] and the Pati-Mohapatra models [7,8]. Now, if one takes into account the reality condition of the K-cycle [9,10], the last two models will be ruled out leaving at the end the standard model as the unique model compatible with Connes' prescription. Recently, Wulkenhaar has proposed a modification to the non-commutative geometry where the differential geometry is formulated in terms of graded differential Lie algebras instead of a unital associative algebras as it is the case in Connes' approach [15,16]. Its application to a list of physical models has been successful. Among this list figure out the standard model [17], the flipped $SU(5) \otimes U(1)$ [18], $SO(10)$ models [19] and left-right gauge model [20] etc. The interesting feature of Wulkenhaar's approach or non-associative geometry (NAG) is the use of graded Lie algebra. On the other hand, the number of fermion families in nature and the pattern of fermion masses and mixing angles are one of the most intriguing puzzles in modern particle physics. Over the last decade, the 3-3-1 extension of the standard model (SM) for the strong and electroweak interactions, based on the local gauge group $SU_c(3) \otimes SU_L(3)L \otimes U_X(1)$ have been studied extensively [21,22,23,24,25,26,27,28]. It provides an interesting attempt to answer the question on family replication. In fact, this extension has among its best features that several models can be constructed so that anomaly cancellation is achieved by interplay between the families [21,22,23,24,25,26,27,28]. Moreover, some models based on the 3-3-1 local gauge structure are
suitable to describe some neutrino properties, because they include in a natural way most of the ingredients needed to explain the masses and mixing in the neutrino sector [28,29,30,31,32,33]. Recently, a very interesting 3-3-1 model within the Wulkenhaar’s approach of graded Lie algebra has been constructed. The goal of this paper is to study the non-unitarity of the neutrino-mixing matrix within the non-associative proposed model (NUNMAM) in the low energy limit as a deviation of the standard. Some new bounds on the various physical parameters are also obtained. In section 1, we present the proposed non-associative 331 model. In section 2, we discuss the NUNMAM. Finally, in section 3, we draw our conclusions.

2. Brief description of the model

The 3-3-1 model is based on the gauge group $SU(3)_c \otimes SU(3)_L \otimes U(1)_N$. The price to pay is the introduction of exotic quarks with electric charges $5/3$ and $-4/3$. The main motivations to study this kind of model are: The natural prediction of three generations based on anomaly cancellation. Thus, the family number must be three. Denoting by:

$$L^i = (\nu_e, e^c); \quad Q^i = \sqrt{2}(u,d,J_1^i)$$

(1)

The triplet (resp. singlet) representations for the left ($L$) (resp. right ($R$)) handed fields (leptons $L^i$ and quarks $Q^i$) are:

$$L^i_L \sim (1,3,0); \quad Q^i_L \sim (3,3,\frac{2}{3})$$

(2)

and

$$L^i_{1R} \sim (1,1,0); \quad L^i_{2R} \sim (1,1,-1); \quad L^i_{3R} \sim (1,1,+1)$$

$$Q^i_{1R} \sim (3,1,\frac{2}{3}); \quad Q^i_{2R} \sim (3,1,-\frac{1}{3}); \quad Q^i_{3R} \sim (3,1,\frac{5}{3})$$

(3)

The numbers $0, 2/3$ in eq. (27) and $2/3$, $-1/3$ and $5/3$ in eq. (27) are $U(1)_N$ charges. The normalization factor $\sqrt{2}$ is introduced for practical reasons as it will be clear later. The electric charge operator $Q_e$ is defined in terms of the $N$ charges as:

$$Q_e = \frac{1}{\sqrt{2}}(\lambda_3 - \sqrt{3}\lambda_8) + N. 1_{3 \times 3}$$

(4)

where $\lambda_3$ and $\lambda_8$ are the usual Gell-Mann matrices. The other two leptons and quarks generations

$$L^2 \sim (\nu_\mu, \mu^c); \quad L^3 \sim (\nu_\tau, \tau^c)$$

$$Q^2 = (s.c.,J_2); \quad Q^3 = (b.t.,J_3)$$

(5)

(6)

belong to the representations:

$$L^2_L \sim (1,3,0); \quad L^2_R \sim (1,3,0)$$

(7)

$$L^2_{1R} \sim (1,1,0); \quad L^2_{2R} \sim (1,1,-1); \quad L^2_{3R} \sim (1,1,+1)$$

(8)

$$L^3_L \sim (1,1,0); \quad L^3_R \sim (1,1,-1); \quad L^3_{3R} \sim (1,1,+1)$$

(9)

$$Q^2_L \sim (3,3^*,\frac{1}{3}); \quad Q^2_R \sim (3,3^*,-\frac{1}{3})$$

(10)

$$Q^2_{1R} \sim (3,1,-\frac{1}{3}); \quad Q^2_{2R} \sim (3,1,\frac{2}{3}); \quad Q^2_{3R} \sim (3,1,-\frac{4}{3})$$

(11)

$$Q^3_{1R} \sim (3,1,-\frac{1}{3}); \quad Q^3_{2R} \sim (3,1,\frac{2}{3}); \quad Q^3_{3R} \sim (3,1,-\frac{4}{3})$$

(12)
Here $u, d, s, c, b, t$ and $J_1, J_2, J_3$ stand for the wave function of the up, down, strange, charm, bottom, top and exotic quarks respectively. Now, following ref. [30] for the three generations, the total internal Hilbert space is $C^{72}$ labelled by the elements:

$$(Q^i_1, Q^i_2, Q^i_3, Q^i_4, Q^i_5, L^i_1, L^i_2, L^i_3, L^i_4, L^i_5)$$

where $i = 1, 2, 3$, and $Q^i_1, Q^i_2, Q^i_3, Q^i_4, Q^i_5, L^i_1, L^i_2, L^i_3, L^i_4, L^i_5 \in C^3 \otimes C^3$ and $L^i_1, L^i_2, L^i_3, L^i_4, L^i_5 \in C^3$.

$\hat{\alpha}, \hat{\beta}, \hat{\gamma}$ and $\hat{\delta}$ are hermitian operators for which the eigenvalues on the states $Q^i_1$ are denoted by

$$
\hat{\alpha} Q^i_1 = \left(\frac{1}{2} \delta_{ij} + \delta_{ik} + \delta_{jl}\right) \alpha^i Q^j_1, \quad \hat{\beta} Q^i_1 = \left(\frac{1}{2} \delta_{ij} + \delta_{ik} + \delta_{jl}\right) \beta^j Q^i_1
$$

$$
\hat{\gamma} Q^i_1 = \left(\frac{1}{2} \delta_{ij} + \delta_{ik} + \delta_{jl}\right) \gamma^j Q^i_1, \quad \hat{\delta} Q^i_1 = \left(\frac{1}{2} \delta_{ij} + \delta_{ik} + \delta_{jl}\right) \delta^j Q^i_1
$$

The eigenvalues $\alpha^i, \beta^j, \gamma^j$ and $\delta^j$ correspond exactly to the $N$ charges of the commutative model [21].

Similarly for leptons, we assign operators coefficients $\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}$ and $\tilde{\delta}$ arbitrary for which their action (eigenvalues) $\tilde{\alpha}^i, \tilde{\beta}^j, \tilde{\gamma}^j$ and $\tilde{\delta}^j$ such that:

$$
\tilde{\alpha} U^i_{lL} = \alpha^i U^i_{lL}, \quad \tilde{\beta} U^i_{lR} = \beta^j U^j_{lR}, \quad \tilde{\gamma} U^i_{lR} = \gamma^j U^j_{lR}, \quad \tilde{\delta} U^i_{lR} = \delta^j U^j_{lR}
$$

The action of the hermitian operators $\tilde{\alpha}, \tilde{\beta}, \tilde{\gamma}, \tilde{\delta}$, $\tilde{\alpha}^i, \tilde{\beta}^j, \tilde{\gamma}^j$ and $\tilde{\delta}^j$ on the scalar matter and vector fields $\phi_i$ and $A_{i}$ respectively is denoted by:

$$
\tilde{\alpha} \phi_i = \alpha^i \phi_i = a \phi_i, \quad \tilde{\beta} \phi_i = \beta^j \phi_i = b \phi_i, \quad \tilde{\gamma} \phi_i = \gamma^j \phi_i = g \phi_i, \quad \tilde{\delta} \phi_i = \delta^j \phi_i = d \phi_i
$$

$$
\left(3 \alpha + \beta + \gamma + \delta\right) A_0 = \sqrt{x} A_0, \quad \left(3 \tilde{\alpha} + \tilde{\beta} + \tilde{\gamma} + \tilde{\delta}\right) A_0 = 0
$$

Regarding the mass matrix $M$ of the $L$-cycle, it has the form:

$$
M = \begin{pmatrix}
0 & 0 \\
0 & M_L
\end{pmatrix}
$$

where

$$
M_Q = \begin{pmatrix}
0 & 0 & 0 & M_Q^1 & 0 & 0 \\
0 & 0 & 0 & 0 & M_Q^2 & 0 \\
0 & 0 & 0 & 0 & 0 & M_Q^3 \\
0 & 0 & 0 & M_Q^1 & 0 & 0 \\
0 & 0 & 0 & M_Q^2 & 0 & 0 \\
0 & 0 & 0 & M_Q^3 & 0 & 0
\end{pmatrix}
$$

and

$$
M_L = \begin{pmatrix}
0 & 0 & 0 & M_L^1 & 0 & 0 \\
0 & 0 & 0 & 0 & M_L^2 & 0 \\
0 & 0 & 0 & 0 & 0 & M_L^3 \\
0 & 0 & 0 & M_L^1 & 0 & 0 \\
0 & 0 & 0 & M_L^2 & 0 & 0 \\
0 & 0 & 0 & M_L^3 & 0 & 0
\end{pmatrix}
$$
here \( M_{Q'} \) and \( M_{L'} \in M_3(C) \) are the mass matrices of the fermions (quarks and leptons) such that:

\[
M_{Q'} = \begin{pmatrix}
m_u & 0 & 0 \\
0 & m_s & 0 \\
0 & 0 & m_b
\end{pmatrix}, \quad M_{Q''} = \begin{pmatrix}
m_d & 0 & 0 \\
0 & m_e & 0 \\
0 & 0 & m_{\tau}
\end{pmatrix}, \quad M_{Q'} = \begin{pmatrix}
m_{\mu} & 0 & 0 \\
0 & m_{\mu} & 0 \\
0 & 0 & m_{\mu}
\end{pmatrix}
\]

and

\[
M_{L'} = \begin{pmatrix}
m_{e'} & 0 & 0 \\
0 & m_{e'} & 0 \\
0 & 0 & m_{e'}
\end{pmatrix}, \quad M_{L'} = \begin{pmatrix}
m_{\mu} & 0 & 0 \\
0 & m_{\tau} & 0 \\
0 & 0 & m_{\tau}
\end{pmatrix}
\]

Now, if we denote by:

\[
\overline{M}_{Q\phi} = M_{Q'}M_{Q''} - \frac{1}{3}\text{tr}M_{Q'}M_{Q''}, \quad \overline{M}_{L\phi} = M_{L'}M_{L'}^* - \frac{1}{3}\text{tr}M_{L'}M_{L'}^*
\]

the bosonic action \( S_B \) containing the scalar fields and bosons-scalar interactions:

\[
S_B = \frac{1}{36g_0^2} \int dx tr(e(\theta)^2) = \frac{1}{36g_0^2} \int dx (\mathcal{L}_0 + \mathcal{L}_1)
\]

here \( g_0 \) is the \( U(1)_N \) gauge couplings constants. After straightforward but tedious simplifications, we obtain:

\[
(\mathcal{L}_0) = \frac{1}{18g_0^2} (\text{Tr}(\overline{M}_{Q\phi})^2 + (\overline{M}_{L\phi})^2)[\overline{\phi}_1\phi_1 + \overline{\phi}_2\phi_2 + (\overline{\phi}_3 - 1)(\phi_3 - 1)]^2
\]

\[
+ 2\text{Tr}(\overline{M}_{Q\phi}\overline{M}_{Q\phi} + \overline{M}_{L\phi}\overline{M}_{L\phi})[(\phi_4 - \overline{\phi}_5)\phi_1 - \overline{\phi}_2\phi_3]^2
\]

\[
+ \text{Tr}(\overline{M}_{Q\phi})^2)[\overline{\phi}_1\phi_1 + \overline{\phi}_2\phi_2 + (\overline{\phi}_3 - 1)(\phi_3 - 1)]^2
\]

\[
+ 2\text{Tr}(\overline{M}_{Q\phi}\overline{M}_{Q\phi} + \overline{M}_{L\phi}\overline{M}_{L\phi})[(\phi_5 - \overline{\phi}_6)\phi_1 - \overline{\phi}_2\phi_3]^2
\]

\[
+ \text{Tr}(\overline{M}_{Q\phi})^2)[(\overline{\phi}_2\phi_2 + \overline{\phi}_3\phi_3 + (\overline{\phi}_5 - 1)(\phi_5 - 1)]^2
\]

\[
+ 2\text{Tr}(\overline{M}_{Q\phi}\overline{M}_{Q\phi} + \overline{M}_{L\phi}\overline{M}_{L\phi})[(\phi_6 - \overline{\phi}_4)\phi_2 - \overline{\phi}_3\phi_1]^2
\]

\[
(\mathcal{L}_1) = \frac{1}{18g_0^2} \hat{\mathcal{L}}_1
\]

where

\[
(\hat{\mathcal{L}}_1) = [\text{tr}(d\phi + (-i((\beta - \alpha)A_0 - A_{38}))\phi_1 + iA_{e\gamma}\phi_2 + iA_{\mu\nu}\phi_3 + iA_{\mu\nu} (\phi_3 + 1]^2)
\]

\[
+ \text{tr}(d\phi_2 + (-i(2A_0 + (\beta - \alpha)A_0))\phi_2 + iA_{e\gamma}\phi_1 + iA_{\mu\nu} (\phi_3 + 1]^2)
\]

\[
+ \text{tr}(d\phi_3 + iA_{\mu\nu} \phi_2 + iA_{\mu\nu}\phi_1 + (-iA_{38} + (\beta - \alpha)A_0) (\phi_2 + 1]^2)\text{Tr}(M_{Q\phi\phi} + M_{L\phi\phi})
\]
+ tr \left( d \phi_1 + (-i(\gamma - \alpha)A_0 - A_{38})\phi_1 + \tilde{\phi}_1 (-iA_{45}) + iA_{12}(\tilde{\phi}_5 + 1) \right)^2 \\
+ tr \left( d \phi_3 - iA_{45}\tilde{\phi}_3 + (-i(2A_0 + (\gamma - \alpha)A_0)\phi_3 - iA_{67}(\tilde{\phi}_5 + 1) \right)^2 \\
+ tr \left( d \phi_5 + iA_{45}\tilde{\phi}_5 + iA_{67}\phi_5 + (i(\delta - \alpha)A_0 - 2A_4)\phi_5 + 1 \right)^2 \right] Tr \left( M_{(0,0_1)} + M_{(0,0_1)} \right)
(27) \\
+ tr \left( d \phi_2 + (i(\delta - \alpha)A_0 + A_{38})\phi_2 + iA_{45}\tilde{\phi}_2 + iA_{45}(\tilde{\phi}_5 + 1) \right)^2 \right] Tr \left( M_{(0,0_1)} + M_{(0,0_1)} \right)
with \\
x = (3\alpha^j + \beta^j + \gamma^j + \delta^j)^2 = \sum (N charges)^2, \quad \forall j = 1,3
(28)

2.1. Fermionic action
Concerning the fermionic action, and if the leptons wave functions $\Psi_L$ are represented by $\Psi_L \left( L_{1L}, L_{2L}, L_{3L}, L_{1R}, L_{2R}, L_{3R} \right)$ than, making the gauge fields redefinitions:

\[
A_0 = \frac{ig_0}{\sqrt{x}} \gamma^\mu W_{\mu}^0 \ast A_{12} = A_1 \pm A_2 = ig\gamma^\mu (W_{\mu}^1 \mp iW_{\mu}^2) = ig\gamma^\mu W_{\mu}^\pm \\
A_\pm = ig\gamma^\mu W_{\mu}^\pm \ast A_{67} = A_6 \pm A_7 = ig\gamma^\mu (-W_{\mu}^6 \mp iW_{\mu}^7) = ig\gamma^\mu U_{\mu}^{\pm \pm} \\
A_3 = ig\gamma^\mu W_{\mu}^3 \ast A_{45} = A_4 \pm iA_5 = ig\gamma^\mu (W_{\mu}^4 \mp iW_{\mu}^5) = ig\gamma^\mu V_{\mu}^\pm
(29) \\

we get the following fermionic interactions where the vector gauge bosons and scalar fields (Yukawa terms) are denoted by $L_{\text{int}}^l$ and $L_{\text{Yukawa}}$ respectively:

\[
L_{\text{int}}^l = \sum_{j=1}^{3} \left[ \frac{-ig_0}{\sqrt{x}} \left\{ L_{12}^{j\mu} W_{\mu}^0 L_{12}^{j} + L_{33}^{j\mu} W_{\mu}^0 L_{12}^{j} \right\} + \frac{1}{2} \left\{ L_{12}^{j\mu} U_{\mu}^{j} \right\} \right] + \frac{1}{2} \left\{ L_{12}^{j\mu} V_{\mu}^{j} \right\}
(30) \\

and

\[
L_{\text{Yukawa}} = \frac{1}{\sqrt{2}} \sum_{j=1}^{3} \left[ \frac{1}{\sqrt{2}} \left\{ \frac{1}{\sqrt{2}} \left\{ L_{12}^{j\mu} \tilde{\phi}_2 L_{12}^{j} - L_{21}^{j\mu} \tilde{\phi}_2 L_{12}^{j} - L_{31}^{j\mu} \tilde{\phi}_2 L_{12}^{j} + c.c \right\} \right\} \\
+ \frac{1}{\sqrt{2}} \left\{ \frac{1}{\sqrt{2}} \left\{ L_{12}^{j\mu} \tilde{\phi}_3 L_{12}^{j} - L_{21}^{j\mu} \tilde{\phi}_3 L_{12}^{j} - L_{31}^{j\mu} \tilde{\phi}_3 L_{12}^{j} + c.c \right\} \right\} \\
+ \frac{1}{\sqrt{2}} \left\{ \frac{1}{\sqrt{2}} \left\{ \frac{1}{\sqrt{2}} \left\{ L_{12}^{j\mu} \tilde{\phi}_5 L_{12}^{j} - L_{21}^{j\mu} \tilde{\phi}_5 L_{12}^{j} + c.c \right\} \\
+ \frac{1}{\sqrt{2}} \left\{ \frac{1}{\sqrt{2}} \left\{ \frac{1}{\sqrt{2}} \left\{ L_{12}^{j\mu} \tilde{\phi}_5 L_{12}^{j} + c.c \right\} \right\} \right\}
(31) \\

If we redefine the scalar fields:

\[
\Phi_i = \frac{3}{\sqrt{2}} g_0 \Omega_\phi \Phi_i, \quad i=1,3, \Phi_i -1 = \frac{3}{\sqrt{2}} g_0 \Omega_\phi \Phi_i, \quad i=4,5
(32)
and set:
\[
\Omega_{\Phi_1} = \frac{1}{2} \left[ \text{Tr} \left( M_{Q \Phi_1} + M_{L \Phi_1} \right) \right]^2
\]
\[
\Omega_{\Phi_2} = \frac{1}{2} \left[ \text{Tr} \left( M_{Q \Phi_2} + M_{L \Phi_2} \right) \right]^2
\]
\[
\Omega_{\Phi_3} = \frac{1}{2} \left[ \text{Tr} \left( M_{Q \Phi_3} + M_{L \Phi_3} \right) \right]^2
\]
\[
\Omega_{\Phi_4} = \frac{1}{2} \left[ \text{Tr} \left( M_{Q \Phi_4} + M_{L \Phi_4} \right) \right]^2
\]
\[
\Omega_{\Phi_5} = \frac{1}{2} \left[ \text{Tr} \left( M_{Q \Phi_5} + M_{L \Phi_5} \right) \right]^2
\]
\[
\Omega_{\Phi_6} = \frac{1}{2} \left[ \text{Tr} \left( M_{Q \Phi_6} + M_{L \Phi_6} \right) \right]^2
\]
The lagrangian density \( L_0 \) becomes:
\[
L_0 = \frac{9}{8} g^2 \sum_{j=1}^6 \Theta_j
\]
where
\[
\Theta_1 = \sigma_1 \left( \chi(\Phi_1, \Phi_1) + \frac{1}{2} \chi(\Phi_2, \Phi_2) + \chi(\Phi_3, \Phi_3) \right)^2
\]
\[
\Theta_2 = 2 \sigma_2 \left[ \chi(\Phi_1, \Phi_1) - \chi(\Phi_2, \Phi_2) - \chi(\Phi_3, \Phi_3) \right]^2
\]
\[
\Theta_3 = \sigma_3 \left( \chi(\Phi_2, \Phi_2) + \chi(\Phi_3, \Phi_3) \right)^2
\]
\[
\Theta_4 = 2 \sigma_4 \left[ \chi(\Phi_1, \Phi_1) - \chi(\Phi_2, \Phi_2) - \chi(\Phi_3, \Phi_3) \right]^2
\]
\[
\Theta_5 = \sigma_5 \left( \chi(\Phi_2, \Phi_2) + \chi(\Phi_3, \Phi_3) \right)^2
\]
\[
\Theta_6 = 2 \sigma_6 \left[ \chi(\Phi_2, \Phi_2) + \chi(\Phi_3, \Phi_3) \right]^2
\]
\[
\chi(A, B) = \Omega_A \Omega_B A B
\]
and
\[
\sigma_j = \text{Tr} \left( M_{Q \Phi_j} M_{Q \Phi_j} + M_{L \Phi_j} M_{L \Phi_j} \right)
\]
If we choose the vacuum expectation values of the dynamical scalar fields as:
\[
\left\langle \Phi_j \right\rangle = 0, \quad j = 1, 3, \quad \left\langle \Phi_j \right\rangle = v_j, \quad j = 4, 6
\]
the usual analysis shows that this set of \( VeV^\dagger \) breaks the symmetry in one single step:
\[
SU(3)_c \otimes SU(3)_L \otimes U(1)_Y \rightarrow SU(3)_c \otimes U(1)_Y
\]
For the particular value \( v_3 = 0 \) or \( v_6 = 0 \), the symmetry breaking chain becomes:
\[
SU(3)_c \otimes SU(3)_L \otimes U(1)_Y \rightarrow SU(3)_c \otimes SU(3)_L \otimes U(1)_Y \rightarrow SU(3)_c \otimes U(1)_Y
\]
After the Higgs mechanism, the scalar bosons masses take the following forms:
\[
M_{\phi_1}^2 = \frac{9}{4} g^2 \Omega_{\phi_1}^2 \left( \Omega_{\phi_1}^2 v_1^2 \sigma_1 - 2 \Omega_{\phi_2}^2 v_1 v_2 \sigma_2 + \Omega_{\phi_2}^2 v_2^2 \sigma_3 \right)
\]
\[
M_{\phi_2}^2 = \frac{9}{4} g^2 \Omega_{\phi_2}^2 \left( \Omega_{\phi_2}^2 v_1^2 \sigma_1 - 2 \Omega_{\phi_3}^2 v_1 v_3 \sigma_6 + \Omega_{\phi_3}^2 v_3^2 \sigma_5 \right)
\]
\[
M_{\phi_3}^2 = \frac{9}{4} g^2 \Omega_{\phi_3}^2 \left( \Omega_{\phi_3}^2 v_2^2 \sigma_3 - 2 \Omega_{\phi_4}^2 v_2 v_4 \sigma_4 + 2 \Omega_{\phi_4}^2 v_4^2 \sigma_3 \right)
\]
\[
M_{\phi_4}^2 = \frac{27}{4} g^2 \Omega_{\phi_4}^2 v_1^2 \Omega_{\phi_4}^2, \quad M_{\phi_5}^2 = \frac{27}{4} g^2 v_2^2 \Omega_{\phi_2}^2, \quad M_{\phi_6}^2 = \frac{27}{4} g^2 v_3^2 \Omega_{\phi_3}^2
\]
Now, for the charged gauge bosons and in the basis \( \{ W^\pm, W^0, U^\pm \} \), and after the one step spontaneous symmetry breaking (SSB) and Higgs mechanism, the charged gauge bosons masses are:

\[
M_{W^\pm}^\pm = g^2 (v_1^2 + v_2^2), M_{W^0}^0 = g^2 (v_1^2 + v_3^2), M_{U^\pm}^\pm = g^2 (v_2^2 + v_3^2)
\]  

(44)

### 2.2 Neutral gauge bosons

For the neutral gauge bosons, one gets (after SSB) in the basis \( \{ W^\mu_\mu, W^\mu_3, W^\mu_0 \} \) the following symmetric non-diagonal mass matrix \( M_{NGB}^\mu \):

\[
M_{NGB}^\mu = \begin{pmatrix}
M_{11} & M_{12} & M_{13}
M_{12} & M_{22} & M_{23}
M_{13} & M_{23} & M_{33}
\end{pmatrix}
\]  

(45)

where

\[
M_{11} = \frac{g^2}{x} \left[ (\beta - \alpha)^2 v_1^2 + (\gamma - \alpha)^2 v_2^2 + (\delta - \alpha)^2 v_3^2 \right] M_{22} = g^2 \left[ v_1^2 + v_2^2 \right]
\]  

(46)

\[
M_{12} = \frac{g^2 g_0}{\sqrt{x}} \left[ (\gamma - \alpha) v_2 + (\alpha - \beta) v_1^2 \right] M_{23} = g^2 \left[ v_1^2 - v_2^2 \right]
\]  

(47)

\[
M_{13} = -\frac{g^2 g_0}{\sqrt{x}} \left[ (\beta - \alpha) v_1^2 + (\gamma - \alpha) v_2 + 2(\delta - \alpha) v_3 \right] M_{33} = g^2 \left[ v_1^2 + v_2^2 + v_3^2 \right]
\]  

(48)

If we set \( v_1 = v_2 \) and in order to have a one vanishing eigenvalue (representing the rest mass square of the photon \( M_Z^0 \)), one has to have the constraint:

\[-M_{12}^2 M_{33} - M_{13}^2 M_{22} + M_{11} M_{22} M_{33} = 0
\]  

(49)

Consequently we deduce that:

\[\delta - \alpha = \beta - \gamma, \alpha = \beta
\]  

(50)

The remaining non vanishing eigenvalues denoted by \( M_1^2 \) and \( M_2^2 \) which can be identified with the mass square of the \( Z' \) and \( Z^0 \) bosons respectively are given by:

\[
M_1^2 = M_Z^0 = \left[ \frac{g_0^2}{x} (\gamma - \alpha)^2 + 4g^2 \right] v_3^2
\]  

(51)

and

\[
M_2^2 = M_Z^{\pm} = \frac{4g^2 \left[ \frac{3v_1^2}{\sqrt{x}} (\gamma - \alpha)^2 + 2g^2 \right]}{\left[ \frac{3v_1^2}{\sqrt{x}} (\gamma - \alpha)^2 + 4g^2 \right]} v_1^2
\]  

(52)

Now, if we introduce a mixing angle \( \theta \) such that:

\[
\tan \theta = \frac{g_0}{g \sqrt{x}}
\]  

(53)

and if we want that the \( Z \) and \( W^\pm \) gauge bosons masses are the same as the ones of the standard model, then, one can show easily that:

\[
\cos \theta_w = \sqrt{1 + \frac{3}{4} (\gamma - \alpha)^2 \tan^2 \theta}
\]  

(54)
where $\theta_w$ is the Weinberg mixing angle. Regarding the eigenstates related to $M^2_y$, $M^2_{Z'}$ and $M^2_Z$ eigenvalues, one can show that they are given by the following expressions:

$$B_\mu = \mathcal{F}_B \left[ \xi_{11} W^0_\mu + \xi_{12} W^3_\mu + \xi_{13} W^8_\mu \right]$$  \hfill (55)

$$Z'_\mu = \mathcal{F}_{Z'} \left[ \xi_{21} W^0_\mu + \xi_{22} W^3_\mu + \xi_{23} W^8_\mu \right]$$  \hfill (56)

$$Z_\mu = \mathcal{F}_Z \left[ \xi_{31} W^0_\mu + \xi_{32} W^3_\mu + \xi_{33} W^8_\mu \right]$$  \hfill (57)

where

$$\xi_{11} = \xi_{21} = \xi_{31} = 1, \xi_{12} = \xi_{32} = \xi_{33} = -\frac{(\gamma - \alpha)}{2} \tan \theta$$  \hfill (58)

$$\xi_{32} = \frac{2 \cot \theta}{(\gamma - \alpha)} + \frac{\gamma - \alpha}{10}$$  \hfill (59)

$$\mathcal{F}_B = \left( \xi_{11}^2 + \xi_{12}^2 + \xi_{13}^2 \right)^{-1/2}$$  \hfill (60)

$$\mathcal{F}_{Z'} = \left( \xi_{21}^2 + \xi_{22}^2 + \xi_{23}^2 \right)^{-1/2}$$  \hfill (61)

and

$$\mathcal{F}_Z = \left( \xi_{31}^2 + \xi_{32}^2 + \xi_{33}^2 \right)^{-1/2}$$  \hfill (62)

### 2.3. Neutral current

If we denote by $\mathcal{L}^{NC}_L$ the leptonic neutral currents lagrangian density coupled to both $Z^0$ and $Z'$ massive vector bosons such as:

$$\mathcal{L}^{NC}_L = \frac{-g}{2 \cos \theta_w} \sum \sum \left[ L_i \gamma^\mu \left( g_{ij,i} - g_{ij,X} \gamma_5 \right) L_i \mu + L_i \gamma^\mu \left( g_{ij,i} - g_{ij,X} \gamma_5 \right) L_i \mu \right]$$  \hfill (63)

where

$$g_{ij,i} = -D^i \left( t_{33}^i + t_{31}^i \right) g_{ij,Z'} = D \left[ t_{32}^i + t_{33}^i \right] g_{ij,i} = g_{ij,i} = -D^i t_{33}$$  \hfill (64)

and

$$g_{ij,i} = D \left[ -t_{32}^i + t_{31}^i \right] g_{ij,Z} = g_{ij,Z} = D \left[ -2t_{33} - t_{31}^i \right] g_{ij,i} = g_{ij,i} = D \left[ -2t_{33} - t_{21}^i \right]$$  \hfill (65)

with

$$D^i = \frac{\cos \theta_w}{\chi}$$  \hfill (66)

$$c_{ij}^i = (\alpha^i + \delta^i) \tan \theta, s_{ij}^i = (\alpha^i - \delta^i) \tan \theta$$  \hfill (67)

$$g_{ij,i} = (\alpha^i + \gamma^i) \tan \theta, c_{ij}^i = (\alpha^i - \gamma^i) \tan \theta$$  \hfill (68)

$$t_{31} = \mathcal{F}_B \mathcal{F}_{Z'} (\xi_{12} - \xi_{23})$$  \hfill (69)

$$t_{32} = \mathcal{F}_B \mathcal{F}_{Z'} (\xi_{32} - \xi_{12})$$  \hfill (70)

and

$$\chi = \mathcal{F}_B \mathcal{F}_{Z'} ( -\xi_{23}^2 - \xi_{12}^2 + \xi_{12}^2 + \xi_{12}^2 )$$  \hfill (71)

with

$$\delta^i - \alpha^i = \alpha^i, \beta^i = \alpha^i, \gamma^i = \gamma^i, \delta^i = -1, \beta^i = 1, \forall j = 1, 3$$
3. Non-unitary neutrino mixing mass matrix

Now, we use the non-standard neutrino interactions (NSI) approach which consists to describe the neutrino new interactions by parameterizing the effects of the new physics in neutrinos oscillation. The NSI effective lagrangian density is:

\[
\mathcal{L}_{\text{NSI}} = -2\sqrt{2} G_F \sum_{i,p} \varepsilon_{aip}^{(i,p)} [\bar{\nu}_a \gamma^\mu P L] [\nu_\mu \gamma^\mu P R] \mathcal{P} = L, R
\]

where \( R = (1 + \gamma^5)/2 \) and \( L = (1 - \gamma^5)/2 \). Here, \( l = e, \mu, \tau \) and \( \varepsilon_{aip}^{(i,p)} \) encodes the deviation from the standard model interactions between the neutrino of flavor \( \alpha \) with component P-handed of leptons, resulting in a neutrinos of flavour \( \beta \). It is very important to mention that some of the parameters \( \varepsilon_{aip}^{(i,p)} \) can be extracted for example from the expressions of the differential cross sections for the elastic scattering processes \( \nu_e + e \rightarrow \nu_e + e \). In fact, in the context of the non associative 331 model of ref. [30] described in the previous section, and after direct but tedious calculations, one gets the following expression of the differential cross section \( \frac{d\sigma}{dy} \):

\[
\frac{d\sigma}{dy} = \frac{m_e E_v}{4 \pi} \left[ \xi_{ij}^\mu + \xi_{ij}^\nu (1 - y)^2 - \frac{m_\nu}{E_v} \xi_{ij}^\nu y \right]
\]

where \( m_e \) is the electron rest mass, \( E_v \) is the incident neutrino energy, \( y = T_e / E_v \) is the rapidity (\( T_e \) is the electron recoil energy at the final state) and

\[
\xi_{ij}^\mu = (\Omega_{ij}^{\mu} + \Omega_{ij}^{\nu})/(M_{\nu_i}^2 M_{\nu_j}^2), \quad \xi_{ij}^\nu = (\Omega_{ij}^{\mu} \Omega_{ij}^{\nu} - \Omega_{ij}^{\nu})/(M_{\nu_i}^2 M_{\nu_j}^2)
\]

with

\[
\Omega_{ij}^{\mu} = A_i A_j + B_i B_j, \quad \tilde{\Omega}_{ij}^{\mu} = \tilde{A}_i \tilde{A}_j + \tilde{B}_i \tilde{B}_j, \quad \Omega_{ij}^{\nu} = B_i A_j - A_i B_j
\]

and

\[
\tilde{\Omega}_{ij}^{\mu} = \tilde{B}_i \tilde{A}_j - \tilde{A}_i \tilde{B}_j, \quad \tilde{\Omega}_{ij}^{\nu} = \tilde{A}_i \tilde{A}_j - \tilde{B}_i \tilde{B}_j
\]

Now, it is easy to show that:

\[
|1 + \xi_{ee}^{(L,R)}| = \frac{4 M_{\nu_i}^2}{g^2 (c_s^2 + c_A^2 + 2)} \left( \sum_{i,j=1}^3 \left| \Omega_{ij}^{\mu} + \Omega_{ij}^{\nu} \tilde{\Omega}_{ij}^{\mu} \right| / M_{\nu_i}^2 M_{\nu_j}^2 \right)
\]

and

\[
|1 + \xi_{ee}^{(L,R)}| = \frac{4 M_{\nu_i}^2}{g^2 (c_s^2 - c_A^2)} \left( \sum_{i,j=1}^3 \left| \Omega_{ij}^{\mu} - \Omega_{ij}^{\nu} \tilde{\Omega}_{ij}^{\mu} \right| / M_{\nu_i}^2 M_{\nu_j}^2 \right)
\]

where

\[
C_s^e = -1/2 + 2 s_w s_w = \sin \theta_w
\]
\[ M^z_{\bar{v}_1} = M^z_{\bar{v}_2} = M^z_{\bar{v}_3} = M^z_{\bar{v}} \]  
(83)

and

\[ \bar{e}^{el}_{ee} = \frac{\bar{e}^{el}_{ee}}{g_L}, \quad \bar{e}^{er}_{ee} = \frac{\bar{e}^{er}_{ee}}{g_R} \]  
(84)

are the relative left and right-handed non-unitary parameters (\( g_L \) and \( g_R \) are the total left and right handed couplings in the standard model charged and neutral currents).

4. Conclusions

Through this paper, we have derived some of the parameters encoding the deviation from the unitarity of the Pontecorvo-Maki-Nakagawa-Sakata modified neutrino mixing matrix (PMNS) in the context of the low energy limit of the non associative geometry 331 model developed in ref. [34] (more developments of the subject and other phenomenological study are under investigation).

Acknowledgement

We are very grateful to the Algerian Ministry of education and research and DGRSDT for the financial support.

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