Lienard-Wiechert potentials for charged tachyons and several remarks on the tachyon Cherenkov radiation

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Abstract
The Lienard-Wiechert potentials for charged tachyons are deduced in this note. They turn out to be twice as large as compared to the corresponding potentials for ordinary particles. Several remarks are made on tachyon Cherenkov radiation and its intensity. In particular, it follows in a straightforward way from the derivation of the Lienard-Wiechert potentials that the tachyon Cherenkov radiation angle obeys the same formula as that for ordinary particles.
1 Introduction

We continue a discussion of the properties of faster-than-light particles called tachyons, the particles with spacelike momenta, initiated in [1]. But before that we would like to recall several historical facts concerning the subject.

The first theoretical arguments for the possibility of the existence of particles with spacelike momenta can be found in a famous paper by Wigner in which the classification of unitary irreducible representations (UIR’s) of the Poincaré group was done for the first time [2]. In the 1960’s Wigner returned to discuss the UIR’s of the Poincaré group corresponding to particles with spacelike momenta [3]. He has shown that quantum mechanical equations corresponding to these UIR’s describe particles with imaginary rest mass moving faster than light. This coincided in time with the appearance of two pioneering works in which the hypothesis of faster-than-light particles was formulated explicitly, accompanied by a kinematic description of them [4] and by their quantum field theory [5]. The particles were called tachyons, from the Greek word ταχύς meaning swift [5].

These propositions immediately encountered strong objections related to the causality principle. It has been shown in several papers [6, 7, 8], in agreement with an earlier remark by Einstein [9] (see also [10, 11, 12]), that by using tachyons as information carriers one can build a causal loop, making possible information transfer to the past time of an observer. This is deduced from the apparent ability of tachyons to move backward in time, which happens when they have a negative energy provided by a suitable Lorentz transformation, this property of tachyons being a consequence of the spacelikeness of their four-momenta. A consensus was achieved that within the special relativity faster-than-light signals are incompatible with the principle of causality.

Another important problem related to tachyons was their vacuum instability. It is a well-known problem which usually appears when considering theoretical models with a Hamiltonian containing a negative mass-squared term (for an instructive description of the problem see e.g. [13]). Applied in a straightforward manner to consideration of faster-than-light particles it results in a maximum, instead of a minimum, of the Hamiltonian for tachyonic vacuum fields, and leads to the conclusion that the existence of tachyons as free particles is not possible.

Fortunately, both problems turned out to be mutually connected and, having hidden loopholes, they were resolved in the 1970’s - 1980’s, as described in detail in [1, 14].

In brief, the causality problem was resolved by combining the tachyon hypothesis with modern cosmology, which establishes a preferred reference frame: so called comoving frame, in which the distribution of matter in the universe, as well the cosmic background (relic) radiation, are isotropic. This changes the situation with the causality violation by tachyons drastically, since the fast tachyons needed for a construction of a causal loop (they are called transcendent tachyons) are extremely sensitive to the metrics of this frame, which varies in time due to universe expansion. Therefore this frame has to be involved when considering the propagation of tachyon signals through space. These signals turn out to be ordered by retarded causality in the preferred frame, and after causal ordering is established in this frame, no causal loops appear in any other frame.

Furthermore, in parallel with the causal ordering of the tachyon propagation one succeeds in obtaining a stable tachyon vacuum which presents the minimum of the field Hamiltonian and appears, in the preferred frame, to be an ensemble of zero-energy, but finite-momentum, on-mass-shell tachyons propagating isotropically. The boundaries of
This vacuum confine the acausal tachyons.

This note is devoted to the question which arises when discussing properties of electrically charged tachyons, namely, what would be the Lienard-Wiechert potentials for charged tachyons obeying Maxwell equations? It will be shown that the standard approach to the finding of the Lienard-Wiechert potentials for ordinary charged particles, e.g. that presented in [15][16], is applicable to the charged tachyons.

This approach is based on the consideration of a point-like particle. Indeed, as was argued in [1], tachyons, if they exist, are most probably realizations of the infinite-dimensional unitary irreducible representations of the Poincaré group and hence can be considered to be string-like objects, i.e. possess a non-zero length \( l \). However, due the linearity of the Maxwell equations one can argue that the applied approach is also valid for extended tachyons by using a superposition of solutions found for the point-like ones. Remarks concerning the tachyon finite length are given below, in footnote 3 and Sect. 3.

To simplify the derivation, it will be carried out in the preferred reference frame, mentioned above, in which the tachyon behaviour is governed by retarded causality [1][14].

2 Lienard-Wiechert potentials for tachyons

Guided by [15] we start with the equations for the vector and scalar potentials, \( A \) and \( \Phi \)

\[
\nabla A - \frac{1}{c^2} \frac{\partial^2 A}{\partial t^2} = -4\pi j,
\]

\[
\nabla \Phi - \frac{1}{c^2} \frac{\partial^2 \Phi}{\partial t^2} = -4\pi \rho,
\]

where \( j = ev\delta(R(t)), \rho = e\delta(R(t)) \) for a charged tachyon of unit charge \( e \) moving with the (constant) velocity \( v \) and separated by the distance \( R(t) \) from the observation point (positioned at the origin of the coordinate frame) at the moment \( t \). Thus, in particular, for the scalar potential \( \Phi \) we have

\[
\nabla \Phi - \frac{1}{c^2} \frac{\partial^2 \Phi}{\partial t^2} = -4\pi e\delta(R(t)).
\]

Everywhere, except the origin of the coordinate frame, \( \delta(R) = 0 \), and we have the equation

\[
\nabla \Phi - \frac{1}{c^2} \frac{\partial^2 \Phi}{\partial t^2} = 0,
\]

whose retarded solution has the form

\[
\Phi = \frac{\chi(t - R(t)/c)}{R(t)},
\]

1Within the anzatz that elementary particles should be realizations of unitary irreducible representations of the Poincaré group, the only alternative to the infinite-dimensional representations suitable for tachyons is a zero-spin representation corresponding to a scalar tachyon. But scalar tachyon models have several serious deficiencies discussed in [1][17] and are unlikely to be realistic models for tachyons.

2We omit, as usual, the advanced solution of (2.4) having the form \( \Phi = \frac{\psi(t + R/c)}{R} \), in accordance with the remark made in the Introduction about validity of retarded causality for tachyons in the preferred reference frame.
where \( \chi \) is an arbitrary function. Its choice is dictated by the need to obtain the correct value of the potential near \( R = 0 \). This can be done by noting that as \( R \to 0 \), the potential tends to infinity and therefore its derivatives with respect to the coordinates increase more rapidly than its time derivative. Consequently as \( R \to 0 \), we can neglect the term \( (1/c^2)/(\partial^2 \Phi/\partial t^2) \) compared to \( \Delta \Phi \) in equation (2.3)\(^3\). Then (2.3) goes into the Poisson equation for a point charge

\[
\Delta \Phi = -4\pi e\delta(R), \quad R \to 0.
\]

Thus, near \( R = 0 \), relation (2.5) must go to the Coulomb law, from which it follows that

\[
\Phi = \frac{e\delta(t - R(t)/c)}{R(t)},
\]

where we have introduced the additional delta function in order to eliminate the implicit arguments in the function \( \chi \).

From this it is easy to get the solution of equation (2.2) for an arbitrary 4-dimensional position of an observer, \( P(r, t) \). Thus, the solution for the scalar potential of a tachyon moving in a trajectory \( r_0(t) \), has the form:

\[
\Phi(r, t) = \int \int \frac{e\delta(r' - r_0(\tau))}{|r - r'|} \delta(\tau - t + \frac{1}{c}|r - r'|) \, d\tau \, dV',
\]

where \( r' = (x', y', z') \), the volume element \( dV' = dx', dy', dz' \), and \( |r - r'| \) is the distance from the volume element \( dV' \) to the observation point. Integrating over \( dV' \), we get

\[
\Phi(r, t) = e \int \frac{d\tau}{|r - r_0(\tau)|} \delta(\tau - t + \frac{1}{c}|r - r_0(\tau)|).
\]

The \( \tau \) integration will be done using the formula

\[
\delta(F(\tau)) = \sum \frac{\delta(\tau - t_{em})}{|F'(t_{em})|},
\]

where the sum extends over \( t_{em} \) solutions of the equation

\[
F(t_{em}) = t_{em} - t + \frac{1}{c}|r - r_0(t_{em})| = 0.
\]

In order to find these solutions we first consider the 2-dimensional Minkowsky diagram in the coordinate plane \( x, t \), i.e. the simplified case of 4-dimensional \( r_0 = (x, 0, 0, t) \) (with \( x = vt \)), see Fig. 1, with the observer position at the origin of the coordinate frame. As can be seen from this figure, the tachyon world line intersects the past light cone of the observer (symbolized by red lines in Fig. 1) at two world points, which results in two solutions of (2.11) for \( t_{em} \). Let us denote these solutions \( t_1 \) and \( t_2 \), as indicated in Fig. 1.\(^3\)

\(^3\)The neglect of the term \( (1/c^2)/(\partial^2 \Phi/\partial t^2) \) in the case of fast tachyons can be justified by our final consideration of a tachyon to be an extended (string-like) object with the length \( l = l_0 \sqrt{v^2/c^2 - 1} \), where \( l_0 \) is the tachyon intrinsic length [1]. Then the characteristic time of any process involving fast tachyons cannot be shorter than about \( l_0/c \), which leads, in particular, to final values of \( \partial^2 \Phi/\partial t^2 \).
The relationship between \( t_1 \) and \( t_2 \) can be obtained from the equation of proportionality of space and time intervals in Fig. 1:

\[
\frac{ct_2 + vt_2}{t_0} = \frac{vt_1}{t_1},
\]  

(2.12)

where \( t_0 \) is the time of tachyon passage through the origin of the \( x \) axis. It can be determined from the definition of the tachyon velocity, e.g. via equation

\[
v = \frac{ct_1}{t_1 - t_0}
\]

(2.13)

from which

\[
t_0 = \frac{v - c}{v}t_1
\]

(2.14)

Substituting this expression into (2.12) we get

\[
\frac{t_2(v + c)v}{t_1(v - c)} = v,
\]

(2.15)

from which we obtain the relationship between \( t_1 \) and \( t_2 \):

\[
t_2 = \frac{v - c}{v + c} t_1.
\]

(2.16)

Fig. 1. The world line of a charged tachyon.

Let us consider now the problem in four dimensions, with the coordinates of point of the field observation at \( x, y, z, t \). Without loss of generality we can hold the position of
the observer at the origin of the coordinate frame, \( x = 0, y = 0, z = 0 \) and \( t = 0 \), with
the tachyon velocity directed along the \( x \) axis. Let \( b = \sqrt{y^2 + z^2} \) (the tachyon impact parameter). Then the equation (2.11) in our coordinate frame can be rewritten as

\[
c^2 t_{em}^2 = (x_0 + vt_{em})^2 + b^2,
\]

where \( t_{em} \) can be either \( t_1 \) or \( t_2 \) (note, they are both negative in our construction). This gives the quadratic equation

\[
t_{em}^2 (v^2 - c^2) + 2x_0 vt_{em} + x_0^2 + b^2 = 0,
\]
whose solutions are

\[
t_{em} = \frac{-vx_0 \pm \sqrt{c^2 x_0^2 - b^2 (v^2 - c^2)}}{v^2 - c^2}
\]

with the condition required for \( t_{em} \) to be negative. We need to consider two cases:

1. \( v < c \). The expression under the square root is always positive. The root value is greater than \( cx_0 \) and therefore greater than \( vx_0 \). We have only one solution for \( t_{em} \) (with sign + in front of the square root) keeping the \( t_{em} \) negative. This is in accordance with statements about the single solution of equation (2.11) in the derivation of the Lienard-Wiechert potentials for ordinary charged particles contained in textbooks on the classical electrodynamics, e.g. in [16, 18]. The field advances the moving charge.

2. \( v > c \). The expression under the square root can be positive or negative. The latter case will be considered later, in Sect. 3. In the former case the square root is real and is smaller than \( cx_0 \), and hence smaller than \( vx_0 \). Thus both solutions for \( t_{em} \) are valid, being negative, and have to be accepted (naturally, if \( b = 0 \) the relation (2.16) between them is recovered). They have a non-trivial common property of the following nature.

In the final derivation of the Lienard-Wiechert potentials we will need the expression for \( |R - vR/c|, R = c\tau \). Let us express it in terms of \( b \) and \( x_0 = v\tau \), considering the process in the impact parameter plane, i.e. omitting the time axis, as drawn in Fig. 2.

Noting that \( vR/c \) is the distance \( D \) in Fig. 2 and \( |vR/c - R| \) is the distance \( d \) in that figure, we write first \( d^2 = c^2 - a^2 \), where \( a = \beta R \sin \theta \), while \( \sin \theta = b/R \), so \( a = \beta b \). Finally, with \( c^2 = b^2 + x_0^2 \), we obtain

\[
d^2 = b^2 + x_0^2 - \beta^2 b^2 = x_0^2 - b^2 (\beta^2 - 1),
\]

which is (up to a factor of \( 1/c^2 \)) just the expression under the square root in (2.19). Since this square root is common for both solutions, \( t_1 \) and \( t_2 \), the values of \( |R - vR/c| \) are equal one to another in both cases. Now we are prepared to integrate (2.9) obtaining

\[
\Phi(r, t) = \frac{e}{|R_1 - vR_1/c|} + \frac{e}{|R_2 - vR_2/c|} = \frac{2e}{|R - vR/c|},
\]

5
where any of two solutions $R_1$, $R_2$ can be used in the last fraction.

![Diagram showing the geometry of retarded potentials with labels](image)

**Fig. 2.** The geometry of retarded potentials. The observer position is at the top of line $b$.

Analogously, for the tachyonic vector potential $A$ we obtain

$$A(r, t) = \frac{2e\nu}{c|R - \nu R/c|}$$

These are Lienard-Wiechert potentials for tachyons considered to be point-like charge carriers. They are twice as large as compared to the corresponding potentials for ordinary particles, $[16, 18]$

$$\Phi(r, t) = \frac{e}{R - \nu R/c}, \quad A(r, t) = \frac{e\nu}{c(R - \nu R/c)}.$$  

Note that in both cases the vector potentials $A$ have a single component $A_x$.

### 3 Remarks on the tachyon Cherenkov radiation in vacuum and the characteristic radiation angle

Let us return to the consideration of the expression under the square root in (2.19). When $c^2 x_0^2 < b^2 (\nu^2 - c^2)$ the square root is imaginary which means the absence of the field in this region. The boundary of this region, free of the field, with the region where the field is present, is given by the equation

$$c^2 x_0^2 - b^2 (\nu^2 - c^2) = 0.$$  

(3.1)
This boundary is denoted by the cone $Rx_0S$ in Fig. 3, which is the Cherenkov cone (this figure reproduces famous figures with wavelets existing in each description of the Cherenkov effect in a transparent media, e.g. in [19]). The cone angle of the Cherenkov radiation in the vacuum from a superluminal particle can be easily deduced from Fig. 3:

$$\theta_c = \cos^{-1}(c/v).$$  \hspace{1cm} (3.2)

(analogous formula for the tachyon Cherenkov radiation angle in transparent media is obtained in Appendix A on the base of a kinematic consideration of the corresponding reaction of the tachyon Cherenkov radiation).

At the boundary (3.1) the Lienard-Wiechert potentials become infinite as can be seen from formulae (2.21), (2.22) taking into account the equation (3.1). This property of the shock wave from a charged particle moving with faster-than-light velocity in the vacuum was noticed a long time ago by A. Sommerfeld who considered, before special relativity appeared, the radiation from an electron moving in vacuum with the superluminal speed [20, 21].

![Fig. 3. Cherenkov cone of a charged tachyon.](image)

The singularities of potentials disappear when one considers the faster-than-light particle of a finite size, which leads to the final depth (spread) of the shock wave front $Rx_0S$ in Fig. 3. This can be concluded also from the Sommerfeld formula for energy loss by a superluminal electron due to the radiation in vacuum which contains a characteristic size of the electron, $a_0$:

$$\frac{dE}{dx} = \frac{9e^2(1 - c^2/v^2)}{4a_0^2},$$  \hspace{1cm} (3.3)
It is interesting that the Sommerfeld formula predicts the rate of the energy loss due to superluminal electron vacuum radiation which is close to the results of modern calculations for the Cherenkov radiation by a finite-size, “spherically symmetric” charged tachyon, see [22, 23, 24]. Note, it is rather high, but one can expect that such a radiation will be strongly suppressed in a high energy range (for the photon energies comparable with the energy of the tachyon) after introduction of a longitudinal tachyon form-factor [25], and due to quantum selection rules. Therefore in our derivation of the Lienard-Wiechert potentials we assumed (implicitly) that the tachyon Cherenkov radiation energy loss is small as compared to the tachyon energy in order to treat the tachyon velocity $v$ as constant.

4 Conclusion

The Lienard-Wiechert potentials for charged tachyons and some aspects of the tachyon Cherenkov radiation were considered in this note. It has been shown that the tachyon Lienard-Wiechert potentials are twice as large as compared to the corresponding potentials for ordinary particles. The cone angle of the tachyon Cherenkov radiation has been shown to obey the same formula as that for ordinary particles, as was expected in [4].

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Appendix A. Tachyon Cherenkov radiation angle in transparent media

The cone angle of the Cherenkov radiation by tachyons $\theta_c$ in transparent media is related to the tachyon velocity $v$ and to the radiator refraction index $n$ in the same way as for ordinary particles:

$$\cos \theta_c = \frac{c}{nv}.$$  \hspace{1cm} (A.1)

The validity of the formula (A.1) for tachyons follows from the fact that this formula can be considered as having purely kinematic origin. It can be obtained from the kinematics of the reaction

$$t \rightarrow t' + \gamma,$$  \hspace{1cm} (A.2)

where $t$ designates a charged tachyon, by use, for example, of the equation of four-momentum conservation:

$$P = P' + K,$$  \hspace{1cm} (A.3)

where $P, P'$ are tachyon four-momenta before and after emission of a Cherenkov photon, respectively, and $K$ is a four-momentum of the photon. Moving $K$ to the left side of the equation (A.3) and squaring both sides of it we get

$$(P - K)^2 = (P')^2,$$  \hspace{1cm} (A.4)

which reduces to

$$(PK) = E \ h \omega - \hbar c^2 (p k) = 0,$$  \hspace{1cm} (A.5)

where $E$ and $h \omega$ are energies of the initial tachyon and the Cherenkov photon, respectively, and $p, \hbar k$ are their 3-momenta. For photons of optical and near-optical frequencies, propagating in medium, the relation between $\omega$ and $k$ is given by [26]

$$\omega \ n(\omega) = c \ k(\omega),$$  \hspace{1cm} (A.6)

where $n(\omega)$ is the refraction index of the medium. Then (A.5) transforms to

$$E - p c \cos \theta_c \ n(\omega) = 0,$$  \hspace{1cm} (A.7)

from which (A.1) follows, taking into account that tachyon velocity $v$ equals to $pc^2/E$. Naturally, (A.1) reduces to (3.2) for the radiation in vacuum when $n = 1$.

Appendix B. The energy loss due to Cherenkov radiation of a string-like tachyon (the classical approximation)

In this Appendix we consider the problem of the intensity of the tachyon Cherenkov radiation in vacuum for a string-like tachyon of unit electrical charge in the classical approximation. This approximation is restricted to the consideration of the tachyon Cherenkov energy loss which is small as compared to the tachyon energy. In the opposite case a quantum theory of tachyon Cherenkov radiation should be developed, with the quantum selection rules expected to suppress strongly the radiation. In any case, the maximal energy of a single Cherenkov photon is restricted in the preferred reference
frame (see [1]) by the quantum limit $\hbar \omega \leq E_t$, where $E_t$ is the tachyon energy in that frame.

Consider the tachyon as a straight string of a vanishing diameter and the Lorentz-contracted (extended) length of $b = b_0 \sqrt{\frac{v^2}{c^2} - 1}$, as suggested in [1]. Then the distributions of the tachyon current and charge entering the Maxwell equations will be defined as

$$j = \frac{e v}{b} \theta(x - vt + \frac{b}{2}) \theta(x - vt - \frac{b}{2}) \delta^2(r),$$

$$\rho = \frac{e}{b} \theta(x - vt + \frac{b}{2}) \theta(x - vt - \frac{b}{2}) \delta^2(r),$$

where $v$ is a (constant) velocity of a tachyon moving along the $x$ axis, and $\theta$’s are Heaviside step functions.

Similarly to Sect. 2, we start again with the equations for the vector and scalar potentials, $\mathbf{A}$ and $\Phi$:

$$\Delta \mathbf{A} - \frac{1}{c^2} \frac{\partial^2 \mathbf{A}}{\partial t^2} = \frac{-4\pi e}{bc} \theta(x - vt + \frac{b}{2}) \theta(x - vt - \frac{b}{2}) \delta^2(r),$$

$$\Delta \Phi - \frac{1}{c^2} \frac{\partial^2 \Phi}{\partial t^2} = \frac{-4\pi}{b} \theta(x - vt + \frac{b}{2}) \theta(x - vt - \frac{b}{2}) \delta^2(r)$$

(cf. the expression 114.6 in [27]). The corresponding equations for the Fourier components of the potentials are

$$k^2 \mathbf{A}_k(t) + \frac{1}{c^2} \frac{\partial^2 \mathbf{A}_k(t)}{\partial t^2} = \frac{4\pi e}{c} \frac{\mathbf{v} \sin (k_x b/2)}{k_x b/2} \exp (-ik_x vt),$$

$$k^2 \phi_k(t) + \frac{1}{c^2} \frac{\partial^2 \phi_k(t)}{\partial t^2} = \frac{4\pi e}{c} \frac{\sin (k_x b/2)}{k_x b/2} \exp (-ik_x vt)$$

Here $k^2 = k_x^2 + q^2$. Let us introduce the $\omega = k_x v$. One can see from equations (B.5), (B.6) that $A_k(t) = A_k \exp (-i\omega t)$, $\phi_k(t) = \phi_k \exp (-i\omega t)$. With this the solutions of these equations are

$$\mathbf{A}_k(t) = \frac{4\pi e}{c} \frac{\mathbf{v} \sin (\omega b/2v)}{\omega b/2v} \exp (-i\omega t) \frac{\exp (-i\omega t)}{k^2 - \omega^2/c^2},$$

$$\phi_k(t) = 4\pi e \frac{\sin (\omega b/2v)}{\omega b/2v} \exp (-i\omega t) \frac{\exp (-i\omega t)}{k^2 - \omega^2/c^2}$$

The Fourier component of the electric field $\mathbf{E}$ is

$$\mathbf{E} = 4\pi e \frac{v}{c} \left(\frac{\omega v}{c^2} - k \right) \frac{\sin (\omega b/2v) \exp (-i\omega t)}{\omega b/2v} \frac{\exp (-i\omega t)}{k^2 - \omega^2/c^2}$$

The deceleration force acting on an extended tachyon by its field is

$$F = \int \mathbf{E}(x, r) \rho(x, r) \, dx \, dr,$$

where $\mathbf{E}(x, r)$ is the electric field obtained from (B.9) by an inverse Fourier transform.

$$F = \frac{4\pi e^2}{(2\pi)^3 b} \int \frac{\sin (\omega b/2v)}{\omega b/2v} \frac{\exp (-i\omega t)}{k^2 - \omega^2/c^2} \frac{(\omega v - k)dk_x d^2q}{d^2 r} \int_{vt-b/2}^{vt+b/2} \exp (ik_x x) \exp (iqr) \delta^2(r) \, dx \, d^2 r$$

$$= \frac{i e^2}{2\pi^2} \int \left(\frac{\sin (\omega b/2v)}{\omega b/2v}\right)^2 \frac{\omega v - k}{k^2 - \omega^2/c^2} \, dk_x d^2q.$$
Integrating (B.11) over the azimuthal angle of the radiation and accounting for \(dk_x = d\omega/v, k^2 = k^2_2 + q^2\), and that the deceleration force \(\mathbf{F}\) is directed along the \(x\) axis, one obtains the following expression for this force:

\[
F = \frac{ie^2}{\pi} \int \left( \frac{\sin(\omega b/2v)}{\omega b/2v} \right)^2 \left( \frac{1}{c^2} - \frac{1}{v^2} \right) \frac{q \ dq \ \omega \ d\omega}{q^2 - \omega^2(1/c^2 - 1/v^2)}.
\] (B.12)

Replacing the variable \(q\) in this expression by the variable \(\xi\) defined as

\[
\xi = q^2 - \omega^2(1/c^2 - 1/v^2)
\]

one gets

\[
F = \frac{ie^2}{2\pi} \left( \frac{1}{c^2} - \frac{1}{v^2} \right) \int_{\omega=-\infty}^{\infty} \int_{\xi=-\infty}^{\infty} \left( \frac{\sin(\omega b/2v)}{\omega b/2v} \right)^2 \frac{d\xi \ \omega \ d\omega}{\xi}.
\] (B.13)

The integrand of (B.13) contains a pole at \(\xi = 0\), corresponding to a singularity of the potentials on the shock wave front related to the Cherenkov radiation condition (3.2). Recalling that this singularity can be avoided by spreading the origin of the shock wave over the tachyon length we displace the integration path in the vicinity of the pole to the lower half of the \(\xi\) complex plane at \(\omega > 0\) and to the upper half of the plane at \(\omega < 0\). Then the integrals over the real axis of \(\xi\) cancel since the integration over \(\omega\) is carried out within symmetric limits while the \(\omega\) integrand is an odd function. Finally, only branching points of the logarithmic function will give yields to this integral being equal, in sum, to \(2\pi i\). The result is

\[
F = -\frac{e^2}{c^2} \left( 1 - \frac{c^2}{v^2} \right) \int_{\omega=0}^{\infty} \left( \frac{\sin(\omega b/2v)}{\omega b/2v} \right)^2 \omega \ d\omega.
\] (B.14)

In what follows we will use the term of energy loss instead of the deceleration force, \(dE_t/dx = |F|\).

To remain within the classical approximation we restrict the integration in (B.14) to the \(\omega\) values such that \(\hbar \omega_{\max} << E_t\). Then (B.14) can be integrated to

\[
\frac{dE_t}{dx} = \frac{2e^2}{b_0^2} \left[ C + \ln \left( \frac{\omega_{\max} b}{v} \right) - ci \left( \frac{\omega_{\max} b}{v} \right) \right],
\] (B.15)

where \(C \approx 0.577\) is the Euler constant, and \(ci\) denotes the integral cosine. One can see that the velocity dependence (and, due to (3.2), the radiation angle dependence) is rather weak in the final expression for the stringlike tachyon Cherenkov energy loss.

The analysis of characteristics of the string-like tachyon Cherenkov radiation spectrum and the consideration of experimental applications of the obtained results are given in detail in [25], together with the derivation of the Cherenkov energy loss by tachyons with other axially-symmetric form-factors. Here we only note the asymptotic behaviour of the energy loss at the tachyon velocity tending to infinity, i.e. at the tachyon energy \(E_t \to 0\). The analysis of formula (B.15) shows that at this condition the tachyon energy loss tends to zero as \(E_t^2\).
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