A New Electrostatic mode in Two-Fluid (ENe-Ion) Formalism of a Partially Ionized Plasma

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Abstract
It is economical to devise ways and means to simplify a multi-species particle system. A plasma consisting of electrons and ions can be described by a single fluid in the magnetohydrodynamic (MHD) limit. The MHD description is sufficiently adequate to study phenomena on large spatial and time scales. A plasma containing multi-species of particles can also be described by representing each particle species by the corresponding fluid. A partially ionized plasma consisting of electrons, ions and the neutral particles can be described as a single fluid by appropriately combining the dynamics of each of the species. The three component plasma can also be reduced to a two-fluid system. The most obvious way is to combine the electron and the ion fluids to obtain a single fluid in the MHD limit and the neutral particles to form the second fluid. The other two possibilities are 1) to combine electron fluid and the neutral fluid into one fluid, here, christened as ENe fluid, and treat the ions as the second fluid and 2) to combine the ion fluid and the neutral fluid into one fluid, here, christened as INe fluid, and treat the electrons as the second fluid. It is found that the process of combining the electrons and the neutrals endows the neutrals with a negative electric charge. So, here is a new plasma with heavy (with mass of the neutral particle assuming the electron mass to be negligible) negatively charged particles along with the positively charged ions. One could in principle, similarly treat the second possibility. However, since there can be a huge variation between the relative masses of the neutrals and the ions, the system turns out to be more complex and its discussion is deferred until later. Here, after establishing the framework for the two-fluid ENe-Ion system, the characteristic electrostatic wave mode
of this novel two-fluid plasma is determined. The wave frequency is found to be a function of the collision frequencies in addition to the dependence on usual parameters.

Keywords: partially ionized plasma, electrostatic waves, hydromagnetic fluids

1. Introduction

A multi-species particle system is hard to crack. Often, simplifications are sought which can reduce the effective degrees of freedom of the system. Statistical averaging is a powerful technique by which a collection of a large number of particles can be transformed into a single fluid with its characteristic properties of mass density, flow velocity, pressure and viscosity. It is like getting water from water molecules. The simplest plasma is a fully ionized plasma consisting of electrons and ions. This plasma can be described by a single conducting fluid by appropriately combining the dynamics of electrons and ions (Alfve’n and Falthammer, 1963). The magnetohydrodynamic (MHD) fluid description so obtained has served well in accounting for a variety of phenomena in the area of plasma physics. The simplest partially ionized plasma consists of three species of particles viz. electrons, ions and neutrals. Such a system can be modeled in diverse ways. One can describe it as a three-fluid system consisting of an electron fluid, an ion fluid and a neutral fluid. The three fluids can again be appropriately combined into a single conducting fluid in the manner of the MHD. If one desires more information about the system, the two-fluid description can be accessed by combining the electron and the ion fluids to obtain the MHD conducting fluid and the second the neutral fluid (Krishan, V. 2014 and references therein). This system has been investigated variously, e.g. Krishan and Gangadhara, (2008); Hiryaki et al. (2010); Krishan and Varghese (2009); Ganadhar et al (2014)., Paradhkar et al (2019). The other two possibilities of getting a two-fluid representation of a partially ionized plasma are 1) one fluid by combining the electron and the neutral fluids and the second the ion fluid and 2) one fluid by combining the ion and the neutral fluids and the second the electron fluid. These two possibilities, to the best of my knowledge, have remained unexplored to-date. Here, in section 2, the dynamics of the electron and the neutral fluids will be combined to generate a single fluid which is christened as the ENe fluid. In section 3, the dynamics of the ion fluid is given. In
section 4, the Poisson equation along with the electric charge conservation is discussed. The dispersion relation of a new electrostatic mode in the two ENe - ion fluids is derived in section 5. The paper is concluded with the possible extensions of this preliminary piece of work.

2. The ENe Fluid

We begin with the equation of motion of the electron fluid:

\[
\begin{align*}
\rho_e \left[ \frac{\partial V_e}{\partial t} + (V_e \cdot \nabla) V_e \right] &= \\
- e n_e \left[ E + \frac{V_e \times B}{c} \right] - \nabla P_e - \rho_e v_{en} (V_e - V_n) - \rho_e v_{ei} (V_e - V_i)
\end{align*}
\]

Neglecting electron inertia i.e. putting the LHS = 0 begets the difference between the electron fluid velocity and the neutral fluid velocity:

\[
(V_e - V_n) = - \frac{e}{v_{en} m_e} \left[ E + \frac{V_e \times B}{c} \right] - \frac{\nabla P_e}{\rho_e v_{en}} - \frac{v_{ei}}{v_{en}} (V_e - V_i)
\]

Taking the cross product with B fetches

\[
V_{e\perp} \ \text{a} = - \frac{e}{v_{en} m_e} E_{\perp} - \frac{\nabla P_e}{\rho_e v_{en}} + \frac{v_{ei}}{v_{en}} V_{i\perp} + V_{n\perp}
\]

where

\[
a = (1 + \frac{\omega_{ec}}{v_{en}} + \frac{v_{ei}}{v_{en}}),
\]

\[
V_{e\perp}, \ V_{i\perp}, \ V_{n\perp}
\]

are respectively the components of the electron, ion and the neutral velocities perpendicular to the magnetic field and \(\omega_{ec}\) is the electron cyclotron frequency. The parallel component of the electron velocity is found to be:

\[
V_{e\parallel} (1 + \frac{v_{ei}}{v_{en}}) = - \frac{e}{v_{en} m_e} E_{\parallel} \\
- \frac{\nabla P_e}{\rho_e v_{en}} + \frac{v_{ei}}{v_{en}} V_{i\parallel} + V_{n\parallel}
\]

(4)

The momentum conservation law for the neutral fluid is

\[
\rho_n D_n = - \nabla P_n - \rho_n v_{ne} (V_n - V_e) - \rho_n v_{ni} (V_n - V_i)
\]

where

\[
D_n = [ \frac{\partial V_n}{\partial t} + (V_n \cdot \nabla) V_n ]
\]

Substitute for \((V_n - V_e)\) from Eq.(2) and for \(V_{e\perp}\) from Eq. (3) to get

\[
\rho_n D_n = - (q_n n_n) \left[ E + \frac{V_n \times B}{ac} \right] - \nabla P - \rho_n v_{ni} (V_n - V_i) + (q_n n_n) \frac{\omega_{en}}{v_{en} ac} E_{\perp}
\]

\[
- (q_n n_n) \frac{v_{ei} V_n \times B}{v_{en} ac} \frac{\nabla P_{ne} \times B}{\omega_{en} v_{en} \rho_e}
\]

\[
- \rho_e v_{ei} (V_e - V_i)
\]

(6)

where

\[
q_n = \frac{n_e}{n_n} e
\]

and

\[
P = P_n + P_e
\]

Equations (6) and (7) describe the motion of the ENe fluid. The parallel component of the Eq.(6), is then found to be

\[
\rho_n \left[ \frac{\partial V_{e\parallel}}{\partial t} + (V_n \cdot \nabla) V_{n\parallel} \right] = -(q_n n_n) E_{\parallel}
\]

\[
- (\nabla P_{\parallel}) - \rho_n v_{ni} (V_{n\parallel} - V_{i\parallel})
\]

\[
- \rho_e v_{ei} (V_{e\parallel} - V_{i\parallel})
\]

(8)
The perpendicular component of the equation of motion of the ENe fluid, Eq.(6), is given by
\[
\rho_n \left[ \frac{\partial V_{ni}}{\partial t} + (V_n \cdot \nabla) V_{ni} \right] = \\
\rho_e V_{ei} (V_e - V_i) - (q_n n_n) \left[ E_{n} (1 - \frac{\omega_{ec}}{\alpha v_{en}}) + \frac{V_n \times B}{ac} \right] - \nabla P_n 
\]
Equations (8) and (9) describe the momentum conservation law of the new ENe fluid with the charge density \((q_n n_n)\) in the limit \(\rho_e \to 0\).
The mass conservation laws for the electron and the neutral fluids can be combined as:
\[
\frac{\partial (\rho_n + \rho_e)}{\partial t} + \nabla \cdot (\rho_n V_n + \rho_e V_e) = 0 
\]
which in the limit \(\rho_e \to 0\), becomes
\[
\frac{\partial \rho_n}{\partial t} + \nabla \cdot (\rho_n V_n) = 0 
\]
which can be taken to represent the mass conservation law of the ENe fluid.

3. The Ion Fluid

The equation of motion of the ion fluid is:
\[
\rho_i \left[ \frac{\partial V_i}{\partial t} + (V_i \cdot \nabla) V_i \right] = \\
en_i \left[ E + \frac{V_i \times B}{c} \right] - \nabla P_i - \\
\rho_i V_{in} (V_i - V_n) - \rho_e V_{ei} (V_i - V_e) 
\]
The mass conservation law for the ion fluid is:
\[
\frac{\partial \rho_i}{\partial t} + \nabla \cdot (\rho_i V_i) = 0 
\]

4. Poisson Equation and Charge Conservation

The electric field is described through the Poisson equation as:
\[
\nabla \cdot E = 4\pi (en_i - en_e) = 4\pi (q_i n_i - q_e n_e) 
\]
The electric charge density conservation of the electron fluid is given by
\[
\frac{\partial (en_e)}{\partial t} + \nabla \cdot (en_e V_e) = 0 \\
\frac{\partial (en_e)}{\partial t} + \nabla \cdot (en_e V_e) = 0 \\
\frac{\partial (q_n n_n)}{\partial t} + \nabla \cdot (q_n n_n (V_n + V_e - V_n)) = 0 \\
q_n n_n V_n + \nabla \cdot (-e^2 n_e \frac{E}{v_{en}kT}) = 0 
\]
Using
\[
\frac{\partial (n_n)}{\partial t} + \nabla \cdot (n_n V_n) = 0 
\]
we get the charge variation equation
\[
n_n \frac{\partial (q_n)}{\partial t} + n_n V_n \cdot \nabla q_n + \nabla \cdot (-\frac{E}{\eta}) = 0 \\
\eta = (\frac{\omega_{ep}^2}{4\pi v_{en}})^{-1} \\
\]
or
\[
\frac{d(q_n)}{dt} + \frac{1}{n_n} \nabla \cdot (-\frac{E}{\eta}) = 0 
\]
So there is a role of collisions in fluctuating charge of the neutrals. The first order charge variation would appear only if there is a zeroth order electric field.
We now have the complete mathematical formulation of the dynamics of the two fluids, the ENe and the ion fluids in Eqs.(8)-(14) along with the Poisson equation.

5. Linear Electrostatic Waves, $E \parallel B$

We can linearize Eqs. (8) - (14) in order to determine the dispersion relation of the electrostatic waves in the ENe and the ion fluids for $E \parallel B$. The linear form of Eq. (8) in the limit $\rho_e \rightarrow 0$ and neglecting the pressure gradient term is found to be

$$\rho_n \left[ \frac{\partial V_{n\|}}{\partial t} + (V_n \cdot \nabla) V_{n\|} \right] = -q_{n0} n_{n0} E_{\|} - \rho_n V_{n\|} (V_{n\|} - V_{i\|})$$

(15)

Where the subscript 0 and 1 represent respectively the equilibrium and the perturbed quantities. For a one dimensional plane wave variation, $e^{i k x - i \omega t}$, of the perturbed quantities, Eq. (15) becomes

$$\left( \omega + i v_{ni} \right) V_{n\|} = -\frac{i q_{n0}}{m_e} E_{\|} + i v_{ni} (V_{i\|})$$

(16)

The linearized form of the continuity Eq. (10) becomes

$$\omega \rho_{n1} - k \rho_n V_{n\|} = 0$$

(17)

The linear form of the ion fluid momentum Eq. (11) in the limit $\rho_e \rightarrow 0$ and neglecting the pressure gradient term becomes

$$(\omega + iv_{in}) V_{i\|} = \frac{i e}{m_i} E_{\|} + iv_{in} V_{n\|}$$

(18)

The linearized Poisson Equation gets the form

$$i k E_{\|} = 4 \pi ( e n_{i1} - e n_{e1} )$$

(20)

Substituting from the continuity equation, one gets

$$i E_{\|} = \frac{4 \pi e n_{e0}}{\omega} ( V_{i\|} - V_{e\|} )$$

(21)

The perturbed electron velocity can be determined from Eq. as

$$V_{e\|} = \frac{V_{n\|}}{b} - \frac{e}{m_e v_{Be} b} E_{\|} + \frac{v_{ei}}{v_{Be}} V_{i\|}$$

(22)

using which, Eq. (20) becomes

$$E_{\|} \left( i - \frac{\omega_{el}^2}{\omega_{Be} b} \right) = \frac{4 \pi e n_{e0}}{\omega b} ( V_{i\|} - V_{n\|} )$$

(23)

We can now substitute for the electric field in Eq. (16) to get

$$\left[ b \left( \omega + i v_{ni} \right) \left( i \omega - \frac{\omega_{el}^2}{v_{Be}} \right) - i \omega_{el}^2 \right] V_{n\|}$$

(24)

and in Eq.(18) to get

$$\left[ b \left( \omega + i v_{in} \right) \left( i \omega - \frac{\omega_{el}^2}{v_{Be}} \right) - i \omega_{el}^2 \right] V_{n\|} =$$

$$\left[ b \left( \omega + i v_{in} \right) \left( i \omega - \frac{\omega_{el}^2}{v_{Be}} \right) - i \omega_{el}^2 \right] V_{i\|}$$

(25)

The dispersion relation of the linear electrostatic waves is then given by
existence of other wave modes in this system such as the counterparts of the ion-acoustic wave, the hybrid waves and the electromagnetic waves. I shall be extremely happy to see the further development of this subject of investigation.

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