Quantum modified moduli spaces and field dependent gauge couplings

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ABSTRACT

In this paper we discuss quantum modified moduli spaces in supergravity. We examine a model suggested by Izawa and Yanagida and by Intriligator and Thomas that breaks global supersymmetry by a quantum deformation of the classical moduli space. We determine the minimum of the supergravity potential when the gauge coupling is taken to depend on a dynamical field, typically a modulus of string theory. We find that the only minimum is at the trivial configuration of vanishing coupling constant and unbroken supersymmetry. We also discuss models involving more complicated superpotentials and find that the gauge coupling is only stabilized in a supersymmetric ground state.

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\section{Introduction}

Supersymmetry plays an important role in particle physics as a possible extension of the Standard Model. Supersymmetric field theories also appear naturally as the low energy limit of superstring theories. However, since the physical world at low energies is not supersymmetric, any realistic model of particle physics necessarily has to incorporate a mechanism for supersymmetry breaking. This breakdown cannot be accomplished by perturbative quantum corrections but has to occur either at the tree level or non-perturbatively. The latter situation has been considered more attractive (for a recent review see \cite{1} and references therein) since it also bears the possibility of generating a hierarchy of scales.

Izawa and Yanagida and independently Intriligator and Thomas suggested a non-perturbative mechanism that breaks supersymmetry for a class of $N = 1$ supersymmetric gauge theories \cite{2}. These models have a specific matter content (e.g. $N$ flavors of quark supermultiplets in the fundamental representation of an $SU(N)$ gauge theory) and in the absence of a tree level superpotential a moduli space of vacua which is not lifted by non-perturbative quantum corrections \cite{3, 4, 5}. The moduli space can be parametrized by the vacuum expectation values (VEVs) of gauge invariant operators which satisfy a constraint equation. Non-perturbative quantum corrections do not lift the moduli space but they do modify the constraint equation. As a consequence certain regions (or points) of the classical moduli space are removed from the quantum moduli space. This, in turn, can lead to spontaneous supersymmetry breaking whenever the quantum constraint of the moduli is incompatible with the classical vacua.

This class of gauge theories can potentially arise in the low energy limit of string theory \cite{6, 7, 8}. In this case the gauge coupling $g$ is not merely a parameter but determined by the VEV of a scalar field (a string modulus) $\hat{S}$. In string perturbation theory $\hat{S}$ is a flat direction of the effective potential and hence its VEV $\langle \hat{S} \rangle$ is undetermined. However, non-perturbative effects generically generate a potential for $\hat{S}$ and thus do determine $\langle \hat{S} \rangle$. Indeed, the model of \cite{3} contains a potential for $\hat{S}$ once the gauge coupling is taken to be field dependent. However, as we will show in this paper the only minimum of the potential is found for vanishing coupling constant $g = 0$. (This has also been noticed in \cite{9}.) Furthermore, at this point the quantum constraint coincides with the classical constraint and therefore supersymmetry is restored at the minimum. This situation also occurs for a large class of generalized superpotentials and thus the mechanism for supersymmetry breaking suggested in \cite{2} appears to be problematic when embedded in string theory.

This paper is organized as follows. We first review the mechanism of \cite{2} and indicate the problem of runaway vacua when the gauge coupling is taken to depend on a dynamical field. We argue that a proper treatment of field dependent gauge couplings requires the coupling of the gauge theory to supergravity. This is done in section 3 using the Veneziano–Yankielowicz formalism which includes a ‘glueball’ superfield in the action \cite{10, 11}. Under reasonable assumptions for the Kähler potential for the glueball field we are able to solve the equations of motion in the absence of a tree level superpotential. For non-vanishing tree-level terms the minimization of the supergravity potential is only achieved in two (different) perturbative expansions. For the model of Izawa, Yanagida, Intriligator, and Thomas (IYIT)
we find that the minimum occurs for vanishing gauge coupling.

In section 4 we extend the analysis to more general superpotentials proposed by Dvali and Kakushadze [12] which show non-trivial minima for $\hat{S}$ and therefore stabilize the gauge coupling at finite values. However, the vacuum is supersymmetric in these models and the gauge coupling is not small enough to generate a phenomenologically interesting hierarchy of scales $\Lambda \ll M_{Pl}$. Thus in the class of models considered $\hat{S}$ either runs away to infinity or it is stabilized in a supersymmetric vacuum without a hierarchy.

2 Supersymmetry breaking on quantum moduli spaces

Let us first summarize some well known results on the low energy limit of supersymmetric quantum chromodynamics (SQCD). Consider SQCD with gauge group $SU(N_c)$, and $N_f$ quark flavors, i.e. $2N_f$ chiral matter fields $Q^i_r, \tilde{Q}^i_r, r = 1, \ldots, N_c, i = 1, \ldots, N_f$ which are in the fundamental and antifundamental representation of $SU(N_c)$, respectively. Affleck, Dine, and Seiberg [13] showed that for $N_f < N_c$ the low energy degrees of freedom are given by the mesons $M^i_j = \tilde{Q}^i_r Q^j_r$, and determined the effective (holomorphic) superpotential to be of the form

$$W = (N_f - N_c) \left( \frac{\Lambda^{3N_c - N_f}}{\det M} \right)^{\frac{1}{N_c - N_f}},$$

where $\Lambda$ is the dynamically generated SQCD-scale.

For $N_f = N_c$ no superpotential is generated by the strong coupling dynamics [13, 3] and hence there is a quantum moduli space spanned by the expectation values of the light degrees of freedom. These are most conveniently parametrized by the mesons $M^i_i$ and the baryons $B = \det Q$ and $\tilde{B} = \det \tilde{Q}$, where $Q$ and $\tilde{Q}$ are viewed as $(N_f \times N_c)$-matrices in the flavor–color–space. However not all of these $N_f^2 + 2$ degrees of freedom are independent. Classically the fields satisfy $\det M = \tilde{B}B$; quantum mechanically this is modified [3] to

$$\det M - \tilde{B}B = \Lambda^{2N_c}.$$

Introducing an auxiliary superfield $\mathcal{A}$ the constraint (2) can be implemented in the superpotential via the Lagrange multiplier method,

$$W = \mathcal{A}(\det M - \tilde{B}B - \Lambda^{2N_c}).$$

By adding a large mass term for one of the quark flavors the superpotential (1) of the corresponding $N_f = N_c - 1$ gauge theory is recovered upon integrating out the massive modes. Furthermore, it can be checked that at any point of moduli space the ’t Hooft matching conditions hold for all unbroken symmetries [3]. An important consequence of the constraint (2) is the fact that the chiral symmetry is necessarily broken by the vacuum since the expectation values of the mesons and baryons cannot simultaneously vanish.

Later in this paper we will concentrate on the case $N_c = 2$ since it is technically easier in some respects. The fundamental and the antifundamental representation are equivalent in this case and therefore the $N_f = 2$ gauge theory consists of four fundamental quarks
The confined degrees of freedom are arranged in an antisymmetric Matrix $M_{ij} = Q_i^r \epsilon_{rs} Q_j^s$ and there are no baryons. The constraint corresponding to (2) now reads

$$\text{Pf} M \equiv \frac{1}{8} \epsilon^{ijkl} M_{ij} M_{kl} = \Lambda^4.$$  \hspace{1cm} (4)

The authors of [2] realized that this quantum constraint gives rise to a simple supersymmetry breaking mechanism. Consider the $SU(2) \ N_f = 2$ model with six additional singlet fields $X_{ij}, i, j = 1, \ldots, 4$, transforming in the conjugate antisymmetric tensor representation of the $SU(4)$ flavor group. It is easy to see that supersymmetry is broken when a term $\rho M_{ij} X_{ij}$ is added to the superpotential

$$W = \rho \text{Tr}(MX) + \mathcal{A}(\text{Pf} M - \Lambda^4),$$  \hspace{1cm} (5)

where $\rho$ is a dimensionless coupling parameter. The value of the potential $\mathcal{V}$ of the scalar fields at the (non-supersymmetric) minimum is [2]

$$\langle \mathcal{V} \rangle = \langle \sum_i |\partial_i W|^2 \rangle = 2|\rho \Lambda^2|^2,$$  \hspace{1cm} (6)

where $\partial_i$ denotes the derivative with respect to the $i$-th field. Note that supersymmetry is restored in the limit $\Lambda \to 0$ [9] which corresponds to the trivial situation of an infrared free gauge theory. As we will see this generically occurs for field dependent gauge coupling. In string theory, for example, the coupling constant $g$ of the perturbative gauge symmetry is determined by the expectation value of the dilaton superfield $S$. More generally, it is known [14] that at special points of the string moduli space the gauge symmetry is enhanced by non-perturbative effects and the coupling of this non-perturbative gauge theory is fixed by a modulus $T$ other than the dilaton $S$. To treat the two situations simultaneously we denote the modulus determining the gauge coupling by $\hat{S}$. At the Planck scale ($M_{\text{Pl}}$) the inverse gauge coupling is matched to this modulus according to

$$g^{-2} = \text{Re}(\hat{S})$$

which implies

$$\Lambda = M_{\text{Pl}} e^{-\frac{\text{Re}(\hat{S}}{b}},$$  \hspace{1cm} (7)

where $b = 3N_c - N_f$. If this additional superfield $\hat{S}$ is present the value of $\Lambda$ in the vacuum configuration is such that the potential is minimal also with respect to variations of $\hat{S}$. Naively, one could just minimize $\langle \mathcal{V} \rangle$ of (3) to find that the only minimum is at $\hat{S} \to \infty$. Thus also in the IYIT mechanism we encounter the ‘dilaton problem’, that is the generic fact that there is no stable vacuum at finite dilaton expectation values [15].

For a more detailed treatment of this problem we need some information on the Kähler potential $K$ of the theory. As the kinetic term of the dilaton is typically of order $\mathcal{O}(M_{\text{Pl}}^2)$, it is necessary to consider the coupling of this gauge theory to supergravity. The relevant potential of the scalar fields is then given by

$$\mathcal{V} = e^{K M_{\text{Pl}}^2} (D_i W g^{ij} \overline{D_j W} - 3 M_{\text{Pl}}^{-2} |W|^2),$$  \hspace{1cm} (8)

where $D_i W \equiv \partial_i W + M_{\text{Pl}}^{-2} (\partial_i K) W$ and $g^{ij}$ is the inverse Kähler metric, i.e. $g^{ij} \partial_k \partial_j K = \delta_k^i$. 


Another question concerns the role of the auxiliary field $A$ in (3). The Lagrange multiplier technique is a manifestly supersymmetric way to enforce the quantum constraint on the moduli. But if supersymmetry is broken the condition (2) will presumably no longer be satisfied. It seems difficult to determine the deviations from the supersymmetric quantum constraint in this approach because the Lagrange multiplier is an auxiliary field and therefore not dynamical. A different approach to quantum modified moduli spaces can be developed using the Veneziano–Yankielowicz formalism [10] which provides an appropriate framework to couple this class of gauge theories to supergravity.

3 Coupling to supergravity and field dependent gauge couplings

3.1 The glueball superfield in global supersymmetry

Veneziano and Yankielowicz proposed an effective Lagrangian containing the glueball field $U \equiv \frac{1}{32\pi^2} W^a W_a$, where $W^a$ is the field strength chiral superfield. For the case of ‘pure glue’ ($N_f = 0$) they determined the superpotential to be

$$ W = N_c(U \ln \frac{U}{\Lambda^3} - U). \quad (9) $$

The supersymmetric minimum is at $U = \omega_k \Lambda^3$ with $\omega_k = e^{2\pi i k/N_c}$ a $\mathbb{Z}_{N_c}$ phase factor. The non-trivial VEV of $U$ corresponds to gluino condensation and the $N_c$ vacua are in accordance with Witten’s index [16]. It is instructing for what follows to discuss the $N_f = 0$ case in more detail. Subtleties arise when the classical potential of the scalar fields is considered. The authors of [10] determined the leading term of the Kähler potential to be $K \sim (\bar{U}U)^{\frac{2}{3}}$. More generally one can expand the Kähler potential in powers of $1/M_{Pl}$ and on dimensional grounds one has

$$ K = \sum_{n=1}^{\infty} c_n \frac{(\bar{U}U)^{n/3}}{M_{Pl}^{2(n-1)}}, \quad (10) $$

where the $c_n$ are dimensionless real constants. In global supersymmetry ($M_{Pl} \to \infty$) one only retains the first term in this expansion and denoting by $\phi$ the lowest component of the chiral multiplet $U$ yields the kinetic terms

$$ \mathcal{L}_{\text{kin}} = \frac{c_1}{9} |\phi|^{-4/3} |\partial_\mu \phi|^2 + \text{fermionic}. \quad (11) $$

The potential for the scalar field $\phi$ is given by

$$ V = \frac{9}{c_1} |\phi|^{4/3} \left| \ln \frac{\phi^{N_c}}{\Lambda^{3N_c}} \right|^2, \quad (12) $$

---

1. One has to use the equation of motion $\partial W/\partial U = 0 \mod 2\pi i$ because of the different branches of the logarithm. Equivalently this can be viewed as reflecting the $\mathbb{Z}_{N_c}$ symmetry of the Lagrangian.
2. The lowest component $\phi$ of the superfield $U$ is the gluino bilinear, $\phi \equiv (1/32\pi^2) \lambda^a \lambda_a$.
3. By using symmetry arguments it can be shown that this is an expansion in $M_{Pl}$ and not in $\Lambda$ [17].
which shows the $N_c$ minima $\phi = \omega_k \Lambda^3$ and an additional minimum at $\phi = 0$. In [10] the latter solution is excluded because the kinetic term diverges at $\phi = 0$.

The generalisation of (9) to $0 < N_f < N_c$ is found to be [10]

$$W = U \left( \ln \left( \frac{U^{N_c-N_f} \det M}{\Lambda^{3N_c-N_f}} \right) - (N_c - N_f) \right).$$

(13)

This result can also be obtained directly from (9) by ‘integrating in’ quark matter [18]. The vacuum expectation value of $U$ is determined from $\partial W / \partial U = 0$ and found to be

$$\langle U \rangle = \left( \frac{\Lambda^{3N_c-N_f}}{\det M} \right)^{\frac{1}{N_c-N_f}}.$$

(14)

Expanding the superpotential in the fluctuations around the vacuum, $\bar{U} = U - \langle U \rangle$, shows that the glueball superfield is massive as can be seen from the expansion of the superpotential $W = \frac{N_c-N_f}{\langle U \rangle} \bar{U}^2 - (N_c - N_f) \langle U \rangle + O(\bar{U}^3)$. This was the main reason for the authors of [13] not to include $U$ in the effective Lagrangian, since the Wilsonian effective action only contains the light degrees of freedom. It is easy to see that upon integrating out the massive glueball field one recovers the Affleck–Dine–Seiberg superpotential (1).

For $N_f = N_c$ one has to include the baryons into the analysis and obtains [11] (see also [17])

$$W = U \ln \left( \frac{\det M - \bar{B}B}{\Lambda^{2N_c}} \right).$$

(15)

The supersymmetric minimum which satisfies $\partial_i W = 0$ results in the constraint (2) and $\langle U \rangle = 0$. The kinetic terms only depend on $K$ and hence as in the $N_f = 0$ case they diverge at $U = 0$ (cf. eq. (11)). However, in this case, it is not possible to exclude the solution at $U = 0$, else there would be no stable vacuum at all. Furthermore, from (15) it is clear that $U$ is now massless and thus there should be a consistent low energy description containing the fields $U, M_{ij}, B, \bar{B}$ in the case $N_f = N_c$.

The difficulties with singular kinetic terms could be avoided if in the general expansion for the Kähler potential (10) the first two coefficients $c_1$ and $c_2$ vanish for $N_f = N_c$. This has the additional property that the kinetic term of the glueball field vanishes in the limit of global supersymmetry ($M_{Pl} \to \infty$). The field $U$ can therefore be identified with the Lagrange multiplier $\mathcal{A}$ of eq. (3). More precisely, expanding the logarithm in (15) in powers of $\epsilon = \frac{\det M - \bar{B}B}{\Lambda^{2N_c}} - 1$ and comparing with (3), one sees that $U = \Lambda^{2N_c} \mathcal{A}$ up to corrections of order $O(\epsilon^2)$. Therefore the approach to quantum modified moduli spaces containing the glueball superfield $U$ seems a natural candidate to elevate the Lagrange multiplier in (3) to a dynamical field [3, 4]. In the following we will assume $c_1 = c_2 = 0$ in (10) although we were not able to prove this statement more rigorously.

Before turning to a concrete application of this method to supergravity, we would like to comment on a suggestion of Kovner and Shifman [19]. They propose that the $U = 0$ solution

\footnote{Alternatively this can be seen by a field redefinition $\tilde{U} = U^{\frac{1}{2}}$, such that the kinetic term has the canonical form. Now $\partial W / \partial \tilde{U}$ shows the additional zero at $\tilde{U} = 0$ but at $\tilde{U} = 0$ this field redefinition is not invertible.}
of the $N_f = 0$ theory could be physical, in contrast to the discussion of eq. (12) above. They argued that the divergence of the kinetic term simply reflects the fact that one cannot trust the Veneziano–Yankielowicz Lagrangian for dynamical issues. A possible vacuum of $SU(N_c)$ gluodynamics with $\langle U \rangle = 0$ corresponds to a vanishing gluino condensate, $\langle \lambda \lambda \rangle = 0$. This has drastic consequences for the vacuum structure of SQCD ($N_f > 0$). The authors of [13] found that the effective superpotential (I) is generated by gluino condensation (if $N_f < N_c - 1$). The gluino condensate $\langle \lambda \lambda \rangle = \Lambda^3$ appears in the strongly coupled unbroken $SU(N_c - N_f)$ pure gauge theory on the Higgs branch of $SU(N_c)$ SQCD, where all mesons acquire large vacuum expectation values. If the chirally symmetric phase exists at $N_f = 0$, then at $N_f > 0$ an additional branch with vanishing effective potential must be present. Consequently, in this case, the quantum moduli space at $N_f = N_c$ should contain a point at the origin $M = B = \tilde{B} = 0$ in addition to values of $M, B, \tilde{B}$ obeying (2). Otherwise one would not recover a vanishing superpotential upon integrating out the massive modes in the limit that all quark flavors get very heavy. This scenario predicts a stable ground state of SQCD for all $N_f \leq N_c + 1$, with the vacuum expectation values of all gauge invariant composites vanishing, in contrast to what was expected from the work of [13]. On the assumed additional branch of the moduli space the full global symmetry $SU(N_f)_L \times SU(N_f)_R \times U(1)_B \times U(1)_R$ is unbroken. As a consequence the IYIT model presented in the previous section would not break supersymmetry but have a supersymmetric ground state at the origin of the moduli space.

The possible existence of this chirally symmetric phase was also discussed by the authors of [20, 21]. They pointed out that in the $N_f = 0$ case a vanishing gluino condensate does not seem to be compatible with the global symmetries. A similar problem is found when $N_f > 0$; there is no obvious spectrum of effective degrees of freedom such that the 't Hooft anomalies are matched between the microscopic and the macroscopic theory. The effective (macroscopic) degrees of freedom are gauge invariant holomorphic polynomials in the elementary fields $Q, \bar{Q}, W^\alpha$. For the minimal spectrum containing only the fields $M^i, B, \tilde{B}, U$ the matching conditions are not satisfied at the origin of the moduli space (e.g. the $SU(N_f)^2 U(1)_B$ anomaly is proportional to $N_c$ and non-zero in the microscopic theory but vanishes in the macroscopic theory). The only possible additional effective degrees of freedom are exotics of the form

$$E_B^{(k)} \sim \epsilon_{r_1 \cdots r_{N_c}} Q^{r_1i_1} \cdots Q^{r_{N_c-k}i_{N_c-k}} (W^{\alpha_1})^{r_{N_c-k}i_{N_c-k+1}} \cdots (W^{\alpha_k})^{r_{N_c}i_{N_c}}$$

$$E_{M,1}^{(k)} \sim \epsilon_{r_1 \cdots r_{N_c}} s^{s_1 \cdots s_{N_c}} Q^{r_1i_1} \cdots Q^{r_{N_c-k}i_{N_c-k}} \bar{Q}_{s_1j_1} \cdots \bar{Q}_{s_{N_c-k-j}j_{N_c-k}} (W^{\alpha_1})^{r_{N_c-k}i_{N_c-k+1}} \cdots (W^{\alpha_k})^{r_{N_c}i_{N_c}}$$

$$E_{M,2}^{(k)} \sim Q^{r_i}(W^{\alpha_1} \cdots W^{\alpha_k})^k \tilde{Q}_{s_j}.$$

For $N_f < N_c$ it seems difficult to find a subset of these that satisfy all of the matching conditions. For $N_f = N_c$ the argument can be made more precise. Since the non-anomalous R-charge of the quarks vanishes in this case, the R-charge of the fermionic component of $E_B^{(k)}$, $E_{M,1}^{(k)}$ is $\geq 1$. However the $U(1)_R$ anomaly is matched between the microscopic and the macroscopic theory when only the the effective fields $M^i, B, \tilde{B}, U$ are present. This matching is no longer possible when any of the exotics is added, because there are no (holomorphic) exotics with zero or negative R-charge. We will therefore assume that at least for $N_f = N_c$ there is no chirally symmetric phase.
3.2 Quantum moduli spaces in supergravity

As an application of the ideas presented above let us now discuss the embedding of quantum modified moduli spaces in supergravity. We start with the situation where no tree-level superpotential is present. For an $SU(2)$ gauge theory with $N_f = 2$ quark flavors the superpotential is given by

$$W = U \ln \frac{\text{Pf}M}{\Lambda^4}.$$  \hfill (16)

As we do not know the Kähler potential of the theory for $M$ and $U$, we make an Ansatz as a power series in the fields:

$$K = \sum_{i<j} \sum_{n=1}^{\infty} a_n \left( \frac{M_{ij} M_{ij}}{M_{Pl}^{2(n-1)}} \right)^n + \sum_{n=3}^{\infty} c_n \left( \frac{U^n}{M_{Pl}^{2(n-1)}} \right) - d \ln(\hat{S} + \hat{S}), \quad (17)$$

where the coefficients $a_n, c_n, d$ are real dimensionless constants. Positivity of the Kähler metric $\partial_i \partial_j K$ requires that generically at least $a_1, c_3, d > 0$. If $\hat{S}$ is the dilaton, then $d = 1$ holds.

The supersymmetric vacuum is determined by the equations

$$D_{M_{ij}} W = \frac{U}{\text{Pf}M} \frac{1}{M_{Pl}^2} e^{ijkl} M_{kl} + K_{M_{ij}} M_{kl} U \ln \frac{\text{Pf}M}{\Lambda^4} \stackrel{!}{=} 0,$$

$$D_U W = \ln \frac{\text{Pf}M}{\Lambda^4} + K_{\text{U}} M_{Pl} U \ln \frac{\text{Pf}M}{\Lambda^4} \stackrel{!}{=} 0,$$

$$D_S W = 8\pi^2 U - d \frac{1}{2 \text{Re}(\hat{S})} U \ln \frac{\text{Pf}M}{\Lambda^4} \stackrel{!}{=} 0,$$  \hfill (18)

where $K_{M_{ij}} \equiv \partial_i \partial_j M_{ij}, K_{U} \equiv \partial_i \partial U$, and we have used the relation $\Lambda^4 = M_{Pl}^4 e^{-8\pi^2 \hat{S}}$. The eqs. (18) are satisfied for $\text{Pf}M = \Lambda^4, U = 0$ and $\hat{S}$ arbitrary which determines the supersymmetric vacuum configurations. The supergravity potential $\mathcal{V}$, defined in (5) vanishes at this configuration, since $D_i W = W = 0$. To see if there are other minima than the supersymmetric one, we need the explicit expression of $\mathcal{V}$. It is given by \footnote{For simplicity of notation we use the same symbols for chiral superfields and their lowest components.}

$$\mathcal{V} = e^{K M_{Pl}^{-2}} \left( \ln \frac{\text{Pf}M}{\Lambda^4} \right)^2 g^U M_{Pl}^4 + |U|^2 \left( \sum_{i<j} \left| \frac{1}{\text{Pf}M} e^{ijkl} M_{kl} \right| + \frac{K_{M_{ij}}}{M_{Pl}^2} \ln \frac{\text{Pf}M}{\Lambda^4} \right)^2 g^{M_{ij}M_{ij}}$$

$$+ \frac{d}{M_{Pl}^2} \left[ \frac{16\pi^2}{d} \text{Re}(\hat{S}) - \ln \frac{\text{Pf}M}{\Lambda^4} \right] + \frac{1}{M_{Pl}^2} \left( \left| U \right|^2 M_{Pl}^4 (\tilde{K}_U)^2 \hat{g}^U + 2 \tilde{K}_U \hat{g}^U - 3 \right) \left( \ln \frac{\text{Pf}M}{\Lambda^4} \right)^2 \right),$$  \hfill (19)

where $\tilde{K}_U = \sum_{n=3}^{\infty} c_n \left( \frac{|U|^{2/3}}{M_{Pl}} \right)^{n-3}$ and $\hat{g}^U = M_{Pl}^{-4} g^U = \sum_{n=1}^{\infty} \gamma_n \left( \frac{|U|^{2/3}}{M_{Pl}} \right)^{n-1}$ are dimensionless real functions of $U$, and $\hat{g}^U > 0$. An exact minimization of this potential is difficult but one can show that $\mathcal{V} > 0$ holds if $\text{Pf}M \neq \Lambda^4$ when reasonable assumptions on the Kähler
potential are made. We split the potential into two parts, one containing all the terms proportional to $|\ln(PfM/\Lambda^4)|^2$ and another one consisting of the remaining terms. For the first part one has

$$
\frac{1}{\tilde{K}_U} \left( \frac{|U|^2 \tilde{K}_U}{M_{Pl}^8} - \frac{|U|^2 \tilde{K}_U}{M_{Pl}^4} (2 \tilde{K}_U \tilde{g}^U - 3) + \tilde{K}_U \tilde{g}^U M_{Pl}^4 \right) > 0 \quad \text{if } \tilde{K}_U \tilde{g}^U > \frac{3}{4}.
$$

The second part is always positive and therefore $\mathcal{V}$ is positive whenever $\tilde{K}_U \tilde{g}^U > \frac{3}{4}$. We do not know the coefficients $c_n$ in the general expansion of $\tilde{K}_U$ but when we assume that the coefficients $\tilde{c}_n$ of the Kähler metric $\partial_U \partial_U K = \sum_n \tilde{c}_n (|U|^{2/3}/M_{Pl}^2)^n$ are at most of order one, i.e. do not grow with $n$, (else it would diverge for $U \approx M_{Pl}^3$), then $c_n = (3/n)^2 \tilde{c}_n = \mathcal{O}(9/n^2)$. From the power series expansion of $\tilde{K}_U$ and $\tilde{g}^U$ one finds that the condition $\tilde{K}_U \tilde{g}^U > \frac{3}{4}$ holds if $|U| < 0.5 M_{Pl}^3$. In addition, a numerical analysis shows that the left hand side of (20) is always $> -|U|^2$ (when the Kähler metric has no singularities for $0 < |U| < 1$). In addition, one has for the second part of the potential

$$
\sum_{i<j} \frac{1}{\beta_{ijkl} \Lambda^4} \ln \left( \frac{PfM}{\Lambda^4} \right) ^2 \bar{g}_{ij} \tilde{M}_{ij} + \frac{d}{\Lambda^4} \left( \frac{16 \pi^2}{d} \text{Re}(\tilde{S}) - \ln \left( \frac{PfM}{\Lambda^4} \right) ^2 \right) > \ln \left( \frac{PfM}{\Lambda^4} \right) ^2
$$

and therefore $\mathcal{V} > 0$. This implies that the supersymmetric configuration is indeed the absolute minimum of $\mathcal{V}$. Thus, as could be expected, the quantum moduli space of global supersymmetry is not altered by supergravity.

### 3.3 Local supersymmetry breaking with fixed $\Lambda$

Now we are ready to couple the IYIT model to supergravity. Before discussing the question of a field dependent scale $\Lambda$ let us determine the effects of supergravity when $\Lambda$ is a fixed parameter of the theory. We first calculate the correction for the constraint (1) when supersymmetry is broken. In the limit $M_{Pl} \to \infty$ this constraint is not modified because $U$ is an auxiliary Lagrange multiplier field in this case. The corrections to the ratio $PfM/\Lambda^4$ will therefore be suppressed by some power of $|\Lambda|/M_{Pl}$. We assume that the expectation values of all fields are of order $\mathcal{O}(\Lambda)$ and $|\Lambda| \ll M_{Pl}$. The superpotential (0) now reads

$$
W = \rho \text{Tr}(MX) + U \ln \left( \frac{PfM}{\Lambda^4} \right).
$$

At the leading order in $|\Lambda|/M_{Pl}$ the Kähler potential is

$$
K = \sum_{i<j} a_{i,j} (\tilde{M}_{ij} \tilde{M}_{ij})^{1/2} + \sum_{i<j} b_{i,j} (\tilde{X}_{ij} \tilde{X}_{ij}) + \mathcal{O} \left( \frac{|\Lambda|^2}{M_{Pl}^2} \right).
$$

Next, the potential (0) is expanded in powers of $|\Lambda|/M_{Pl}:

$$
\mathcal{V} = \alpha_0 \sum_{i<j} \rho X_{ij} + \frac{U}{PfM} \frac{1}{2} \epsilon_{ijkl} M_{kl} \left| M_{ij} \right|^2 + \beta_1 \rho^2 \sum_{i<j} |M_{ij}|^2
$$

$$
+ \left( \ln \left( \frac{PfM}{\Lambda^4} \right) \right) ^2 \left( \gamma_1 (M_{Pl}^4 + K M_{Pl}^2 + \frac{1}{2} K^2) + \gamma_2 (M_{Pl}^2 + K) |U|^{2/3} + \gamma_3 |U|^{4/3} \right) + \mathcal{O} \left( \frac{|\Lambda|^6}{M_{Pl}^4} \right),
$$

$$
\theta \to \theta + \frac{1}{\sqrt{2}} \beta (\Lambda^2)^{1/2} \left( \frac{1}{\Lambda^2} \right) ^2 \left( \frac{1}{\Lambda^4} \right) ^2
$$

$$
\frac{1}{\sqrt{2}} \beta (\Lambda^2)^{1/2} \left( \frac{1}{\Lambda^2} \right) ^2 \left( \frac{1}{\Lambda^4} \right) ^2
$$

$$
\frac{1}{\sqrt{2}} \beta (\Lambda^2)^{1/2} \left( \frac{1}{\Lambda^2} \right) ^2 \left( \frac{1}{\Lambda^4} \right) ^2
$$

$$
\frac{1}{\sqrt{2}} \beta (\Lambda^2)^{1/2} \left( \frac{1}{\Lambda^2} \right) ^2 \left( \frac{1}{\Lambda^4} \right) ^2
$$
where \( \alpha_1 = \frac{1}{a_1}, \beta_1 = \frac{1}{b_1}, \gamma_1 = \frac{1}{c_1}, \gamma_2 = -\frac{16c_4}{9c_3}, \gamma_3 = \frac{256c_2}{81c_3} - \frac{25c_5}{9c_3}. \) The minimum in the directions \( X^{ij} \) and \( U \) is at \( X^{ij} = U = 0. \) Then, minimizing with respect to \( M_{12} \) yields

\[
\beta_1 |\rho|^2 M_{12} + \frac{a_1}{2} \gamma_1 \frac{M_{12}^2}{|M_{21}|} + K \left| \ln \frac{\text{Pf} M}{\Lambda^4} \right|^2 + \gamma_1 \frac{M_{34}}{\text{Pf} M} (M_{12}^4 + K M_{12}^2 + \frac{1}{2} K^2) \ln \frac{\text{Pf} M}{\Lambda^4} + O \left( \frac{|\Lambda^6|}{M_{12}^4} \right) = 0.
\]

(24)

This quadratic equation in \( \ln (\text{Pf} M/\Lambda^4) \) is solved by

\[
\ln \frac{\text{Pf} M}{\Lambda^4} = -|\rho|^2 \beta_1 \frac{M_{12} \text{Pf} M}{\gamma_1 M_{34}^4 M_{12}^4} + O \left( \frac{|\Lambda^6|}{M_{12}^6} \right)
= -|\rho|^2 \beta_1 \frac{1}{\gamma_1} \frac{1}{2} \sum_{i<j} |M_{ij}|^2 M_{12}^4 + O \left( \frac{|\Lambda^6|}{M_{12}^4} \right).
\]

(25)

The second equality follows from the other equations \( \partial V / \partial M_{ij} = 0; \) they imply \( |M_{12}| = |M_{34}|, |M_{13}| = |M_{24}|, |M_{14}| = |M_{23}| \) and the phases are such that \( e^{-i\varphi} \text{Pf} M = \frac{1}{2} \sum_{i<j} |M_{ij}|^2, \) where \( \varphi \) is defined by \( M_{12} M_{34} = e^{i\varphi} |M_{12} M_{34}|. \) Therefore the leading correction to the constraint (4) is given by

\[
\frac{\text{Pf} M}{\Lambda^4} = 1 - |\rho|^2 e^{-i\varphi} \frac{\Lambda^4}{M_{12}^4} + O \left( \frac{|\Lambda^6|}{M_{12}^6} \right),
\]

(26)

where we have assumed \( \beta_1 = \gamma_1 = 1 \) for simplicity.

At the first order in \( \sqrt{|\Lambda^2|/M_{12}^2} \) the result obtained in global supersymmetry for the vacuum energy \( \langle V \rangle, \) eq. (6), is only modified by the prefactor \( e^{K M_{12}^2}, \)

\[
\langle V \rangle = 2 |\rho \Lambda^2|^2 \left( 1 + 2a_1 \frac{|\Lambda^2|}{M_{12}^2} \right) + O \left( \frac{|\Lambda^4|^2}{M_{12}^4} \right).
\]

(27)

The correction (26) causes a shift of order \( O(\frac{|\Lambda^4|^2}{M_{12}^4}) \) in \( V \) but there are other corrections of the same order due to the fact that the vacuum configuration is no longer at \( X^{ij} = U = 0 \) when higher powers of \( |\Lambda|/M_{12} \) are taken into account in (23). To determine these would take more effort. Instead let us now turn to the case of a field-dependent scale \( \Lambda. \)

### 3.4 Field dependent gauge coupling

In the previous section we discussed quantum modified moduli in supergravity when either the tree-level superpotential vanishes or the scale \( \Lambda \) is a fixed parameter. Let us now treat in more detail the model of [2] with \( \Lambda \) being a field dependent scale. In particular we are interested if there is a stable vacuum at finite values of \( \hat{S} \) or not.
We need to locate the minimum of the potential \( \mathcal{V} \) associated to the superpotential (21). Since the analytical expression of \( \mathcal{V} \) is again rather involved it is difficult to minimize the supergravity potential exactly. When discussing the case of fixed \( \Lambda \) we expanded the potential in powers of \( |\Lambda|/M_{P_1} \). The same expansion procedure now produces at leading order

\[
\langle \mathcal{V} \rangle = |\rho\Lambda^2|^d \operatorname{Re}(\tilde{S})^{-d} \left( 1 + 2a_1 |\Lambda|^2/M_{P_1}^2 \right) + \mathcal{O}\left( |\Lambda^4|^2/M_{P_1}^4 \right),
\]

where \( \Lambda^4 = M_{P_1}^4 e^{-8\pi^2 S} \). The additional term \( g^{S\bar{S}}|D_S^2 W|^2 \) in the potential \( \mathcal{V} \) is of order \( \mathcal{O}(|\Lambda^4|^2/M_{P_1}^4) \) because \( X^{ij} \) and \( U \) vanish at leading order and the additional term \(-d \ln(\tilde{S} + \tilde{S})\) in the Kähler potential gives the overall factor \( 1/2 \operatorname{Re}(\tilde{S})^d \) in (22). Positivity of the Kähler metric in the limit \( M_{P_1} \to \infty \) requires \( a_1 > 0 \). Thus we recover the problem of the runaway vacuum indicated above: the only minimum is at \( \tilde{S} \to \infty \) which is again a supersymmetric solution.

It is interesting to note that, at the leading order, this result is not altered when an arbitrary superpotential \( W_N(\tilde{S}) \) is added to (21), which only depends on \( \tilde{S} \) and is at most of order \( \mathcal{O}(\Lambda^3) \). Since \( g^{S\bar{S}}|D_S^2(W + W_N)|^2 \) and \( M_{P_1}^2|W_{NTT} + W_N|^2 \) are of order \( \mathcal{O}(|\Lambda^6|/M_{P_1}^2) \) we obtain \( \langle \mathcal{V} \rangle = |\rho\Lambda^2|^d \operatorname{Re}(\tilde{S})^{-d} + \mathcal{O}(|\Lambda^6|/M_{P_1}^2) \). This implies that the string modulus \( \tilde{S} \) is still driven to infinity even in the presence of an arbitrary superpotential \( W_N(\tilde{S}) \).

An expansion in \( |\Lambda|/M_{P_1} \) is slightly more problematic when the gauge coupling depends on the dynamical field \( \tilde{S} \), since the ratio \( \Lambda/M_{P_1} = e^{-2\pi^2(\tilde{S})} \) is fixed by the equations of motion and not apriori a small parameter. Thus within a self-consistent treatment we can only state that for large (but finite) \( \langle \tilde{S} \rangle \) there is no stable vacuum. The domain which is not covered by this approximation corresponds to a gauge theory which is strongly coupled at \( M_{P_1} \). For example, demanding \( 4\pi/g^2 > 1 \) results in \( |\Lambda^2|/M_{P_1}^2 < 0.04 \) which does justify the above expansion procedure.

An alternative approximation which does not use the smallness of \( |\Lambda^2|/M_{P_1}^2 \) is an expansion of \( \mathcal{V} \) in powers of \( \rho \). In section 3.2 we found the exact solution for the \( \rho = 0 \) case. Let us now calculate the perturbations around the \( \rho = 0 \) solution. For simplicity we place ourselves at the point \( M_{ij}^{(0)} \) of the moduli space where \( M_{12}^{(0)} = M_{34}^{(0)} = \Lambda^2, M_{13}^{(0)} = M_{14}^{(0)} = M_{23}^{(0)} = M_{24}^{(0)} = 0 \). The Ansatz

\[
M_{ij} = M_{ij}^{(0)} + \rho \tilde{M}_{ij}, \quad U = \rho \tilde{U},
\]

yields the superpotential

\[
W = \rho \Lambda^2(X^{12} + X^{34}) + \mathcal{O}(\rho^2).
\]

To obtain an explicit expression for the scalar potential we need an additional assumption on the form of the Kähler potential. In analogy to (L7) we make the Ansatz

\[
K = \sum_{i<j} \sum_{n=1}^{\infty} a_n \left( \frac{M_{ij} M_{ij}}{M_{P_1}^{2(n-1)}} \right)^{n/2} + \sum_{i<j} \sum_{n=1}^{\infty} b_n \left( \frac{X^{ij} X^{ij}}{M_{P_1}^{2(n-1)}} \right)^n + \sum_{n=3}^{\infty} c_n \left( \frac{U}{M_{P_1}^{2(n-1)}} \right)^{n/3} - d \ln(\tilde{S} + \tilde{S}). \tag{31}
\]
For $V$ of (3) this gives to lowest order in $\rho$

$$V = e^{KM_{M_2}^2} |\rho|^2 \left( \left| X^{12} + \frac{\bar{U}}{\Lambda^2} + (X^{12} + X^{34})\tilde{K}_M \right|^2 
+ |X^{34} + \frac{\bar{U}}{\Lambda^2} + (X^{12} + X^{34})\tilde{K}_M|^2 \right) \bar{g}^M M_{P_1}^2 + \left| \tilde{M}_{12} + \tilde{M}_{34} \right|^2 g^{U\bar{U}} (32)
$$

$$+ |\Lambda|^2 \left( \left| 1 + (X^{12} + X^{34})\frac{K_{X^{12}}}{M_{P_1}^2} \right|^2 g^{X^{12}X^{12}} + \left| 1 + (X^{12} + X^{34})\frac{K_{X^{34}}}{M_{P_1}^2} \right|^2 g^{X^{34}X^{34}} \right)
$$

$$+ \frac{d}{M_{P_1}^2} \left| \frac{16\pi^2}{d} \bar{U} \text{Re}(\hat{S}) - \Lambda^2 (X^{12} + X^{34}) \right|^2 - 3 \frac{|\Lambda|^2 |X^{12} + X^{34}|^2}{M_{P_1}^2} + O\left(|\rho|^4\right),$$

where $\tilde{K}_M = \sum_{n=1}^{\infty} a_n n \left( \frac{\Lambda^2}{M_{P_1}^2} \right)^n$ and $\tilde{g}^M = M_{P_1}^2 g^{M_{12}M_{12}} |_{M_{12} = \Lambda} = \sum_{n=1}^{\infty} a_n \left( \frac{|\Lambda|^2}{M_{P_1}^2} \right)^n$.

The minimum with respect to $M_{ij}$ and $U$ is at

$$\bar{M}_{12} = -\tilde{M}_{34}, \quad \bar{U} = -(X^{12} + X^{34})\Lambda^2 h(\hat{S}) (33)$$

with $$h(\hat{S}) = \frac{\bar{g}^M (\frac{1}{2} + \tilde{K}_M) - 8\pi^2 \text{Re}(\hat{S})|\Lambda|^2}{\bar{g}^M + \frac{2}{d}(8\pi^2 \text{Re}(\hat{S}))^2 |\Lambda|^2 M_{P_1}^2}.$$ 

To further simplify the expression of $V$ we will assume canonical Kähler potential for the $X^{ij}$, i.e $K_{X^{12}} = \bar{X}^{12}$, $K_{X^{34}} = \bar{X}^{34}$. This is justified because the $X^{ij}$ are elementary fields and not composite and the one-loop correction to the canonical Kähler potential is of order $O(|\rho|^2)$ [22]. Now, inserting (33) in (32) gives

$$V = e^{KM_{M_2}^2} |\rho|^2 \left( \left| X^{12} + (X^{12} + X^{34})(\tilde{K}_M - h(\hat{S})) \right|^2 
+ |X^{34} + (X^{12} + X^{34})(\tilde{K}_M - h(\hat{S}))|^2 \right) \bar{g}^M M_{P_1}^2
$$

$$+ \left( 2 + \frac{2}{M_{P_1}^2} |X^{12} + X^{34}|^2 + \frac{1}{M_{P_1}^2} |X^{12} + X^{34}|^2 \left( |X^{12}|^2 + |X^{34}|^2 \right) \right) |\Lambda|^2
$$

$$+ \left( d \left( \frac{16\pi^2}{d} h(\hat{S}) \text{Re}(\hat{S}) + 1 \right)^2 - 3 \right) |X^{12} + X^{34}|^2 \frac{|\Lambda|^2}{M_{P_1}^2} + O\left(|\rho|^4\right). (34)$$

If $h(\hat{S}) \geq 0$ then the minimum is at is at $X^{12} = X^{34} = 0$ (we assume $d \geq 1$). This is indeed the case if the coefficients $a_n$ of the Kähler potential are at most of order $O(4/n^2)$, which is necessary for a regular behavior of the Kähler metric $\partial_{M_{ij}} \partial_{M_{ij}} K$. This result is obtained from a numerical analysis of the function $h(\hat{S})$ defined in (33).
On the subspace of field configurations where all fields except $\hat{S}$ are at the minimum values $V$ has the form

$$V|_{\text{min}} = |\rho|^2 M_{\text{Pl}}^4 \exp\left(2 \sum_{n=1}^{\infty} a_n e^{-4\pi^2 \text{Re}(\hat{S})^n} - 8\pi^2 \text{Re}(\hat{S})\right) \frac{\text{Re}(\hat{S})^d}{\text{Re}(\hat{S})^d} + O(|\rho|^4).$$

We find again that, at least to order $O(|\rho|^2)$ and for $\tilde{K}_M \geq 0$, there is no stable ground state except the trivial one at $\hat{S} \to \infty$. To summarize, the supersymmetry breaking mechanism proposed in [2] only works when the scale $\Lambda$ is a fixed parameter. When this mechanism is embedded in string theory, the gauge coupling and therefore the SQCD-scale depend on a modulus field $\hat{S}$ and the vacuum is driven to the trivial case $\Lambda = 0$.

4 Other models and conclusion

An interesting question to ask is whether the behavior of the simple IYIT model is generic or if there are more general models that break supersymmetry on quantum moduli spaces and at the same time stabilize the string modulus $S$ at finite expectation values. We start by considering more general tree-level superpotentials and then show how this can be generalized to other gauge groups.

Let us first mention a simple mechanism to fix the VEV of $\hat{S}$ while preserving supersymmetry which was proposed by Dvali and Kakushadze [12]. Consider an $SU(2)$ gauge theory with $N_f = 2$ and one additional singlet field $X$ and superpotential

$$W = (\xi - X) \text{Pf}M + \frac{\eta}{k+1} X^{k+1} + U \ln \frac{\text{Pf}M}{\Lambda^4}. \quad (36)$$

It results in a supersymmetric ground state at

$$\Lambda^4 = \text{Pf}M = \eta \xi^k \to \hat{S} = -(k/8\pi^2) \log(\xi \eta^k), \quad (37)$$

provided $|\eta \xi^k| < 1$, which is equivalent to $1/g^2 = \text{Re}(\hat{S}) > 0$. The solution (37) is obtained by demanding $\partial_i W = 0$ [12]. A numerical analysis shows that when supergravity is switched on there is still a supersymmetric solution, i.e. $D_i W = 0$. For example, for $\eta = 1$, $\xi = 0.3$, $k = 2$ one finds $\Lambda^4 \approx \text{Pf}M \approx 0.16 M_{\text{Pl}}^4 \to 8\pi^2 \hat{S} \approx 1.8$. The supergravity corrections to the quantum constraint are very small $1 - \text{Pf}M/\Lambda^4 \approx 7 \cdot 10^{-6}$, but the result for $\text{Pf}M$ differs considerably from the one obtained in global supersymmetry which is given by $\text{Pf}M = \eta \xi^2 = 0.09 M_{\text{Pl}}^4$. As the superpotential $W$ does not vanish at the minimum ($W \approx 0.005 M_{\text{Pl}}^3$), the potential $V \approx -9 \cdot 10^{-4} M_{\text{Pl}}^4$ is negative for this non-trivial ground state, whereas $V = 0$ in the limit $\hat{S} \to \infty$.

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6This should be a good approximation because in the IYIT model global supersymmetry is broken for arbitrarily small $\rho$.

7To do this calculation we assumed canonical Kähler potential for $X$ and chose the coefficients for $M_{ij}$ and $U$ in (31) to be $c_3 = 1$, $a_1 = 4$, $a_2 = 1$, $a_3 = 4/9$, $a_4 = 1/4$. Higher orders do not significantly change the results. Different values for the coefficients $a_n$, which still decrease like $1/n^2$ to guarantee the regularity of the Kähler metric, may change the results by 10–20%.
This can easily be generalized to a large class of models achieving the same results \[\text{[12]}\]. To get dimensionless couplings in more general superpotentials it is convenient to scale out factors of $M_{P1}$ and define $\hat{M}_{ij} = M_{ij}/M_{P1}$. Let $\mathcal{P}(\text{Pf}\, M)$ be an arbitrary polynomial of Pf $M$ and $X$ a $SU(2)$ singlet. Then consider

$$W = \rho X \mathcal{P}(\text{Pf}\, \hat{M}) M_{P1}^2 + U \ln \frac{\text{Pf} M}{\Lambda^4}. \quad (38)$$

As there is no potential for the singlet field $X$, it acts like a Lagrange multiplier enforcing another constraint on the moduli (in addition to the quantum constraint). The global equations of motion set $X = U = 0 = \mathcal{P}(\text{Pf}\, \hat{M})$ and $\text{Pf} M = \Lambda^4$. Therefore $\Lambda^4$ is fixed to be one of the zeroes of $\mathcal{P}$. If $\mathcal{P}(x)$ has zeroes at values $|x| = e^{-8\pi^2 \text{Re}(\hat{S})} < 1$ then $\hat{S}$ and therefore also the gauge coupling are stabilized at finite values, or else supersymmetry is broken. In the former case, because of $\langle W \rangle = 0$ the globally supersymmetric solution, $\partial_i W = 0$, is still a solution of $D_i W = 0$ in supergravity. To see if there are additional non-supersymmetric minima of the potential, we have to write down the explicit expression for $\mathcal{V}$. We expand in powers of $\rho$ and make the Ansatz \[\text{[20]}\]. At lowest order in $\rho$ this yields the superpotential

$$W = \rho X \mathcal{P} \left( \frac{\Lambda^4}{M_{P1}} \right) M_{P1}^2 + \mathcal{O} \left( \rho^2 \right) \quad (39)$$

and the scalar potential (we assume $K_X = \bar{X}$)

$$\mathcal{V} = |\rho|^2 e^{K M_{P1}^2} \left( 2 \left| X \left( \frac{\mathcal{P}'}{M_{P1}} \right) + \frac{\bar{U}}{M_{P1}} \right|^2 M_{P1}^2 g^M \frac{M_{P1}^4}{|\Lambda^2|^2} + |\mathcal{P}|^2 \left( M_{P1}^2 + 2 |X|^2 M_{P1}^2 + |X|^4 \right) + \left| \frac{\bar{M}_{12} + \bar{M}_{34}}{\Lambda^2} \right|^2 g^{UU} \right. \right.$$  

$$\left. + M_{P1} d \left( \frac{16 \pi^2}{d} \text{Re}(\hat{S}) - X \mathcal{P} \right)^2 - 3 M_{P1}^2 |X \mathcal{P}|^2 \right) + \mathcal{O} \left( |\rho|^4 \right), \quad (40)$$

where $\mathcal{P}'(x) = d\mathcal{P}(x)/dx$. An analogous discussion as in section 4 shows that the minimum is at

$$\bar{M}_{12} = -\bar{M}_{34}, \quad \bar{U} = -X \mathcal{P} M_{P1}^2 h(\hat{S})$$

with $h(\hat{S}) = \frac{\bar{g}^M \left( \frac{\mathcal{P}'(M_{P1})}{M_{P1}} + \bar{K}_M \right) - 8 \pi^2 \text{Re}(\hat{S}) \frac{|\Lambda^2|^2}{M_{P1}^4}}{\bar{g}^M + \frac{2}{3} (8 \pi^2 \text{Re}(\hat{S}))^2 \frac{|\Lambda^2|^2}{M_{P1}^4}}$. 

Generically the function $h(\hat{S})$ is positive and, as a consequence, the minimum is at $X = 0$. Thus we obtain

$$\mathcal{V}|_{\text{min}} = |\rho|^2 M_{P1}^4 |\mathcal{P}(e^{-8\pi^2 \hat{S}})|^2 \exp \left( \frac{2 \sum_{n=1}^{\infty} a_n e^{-4\pi^2 \text{Re}(\hat{S}) n}}{2 \text{Re}(\hat{S}) d} \right) + \mathcal{O}(|\rho|^4). \quad (42)$$
This result remains true in the limit $\mathcal{P} \to 0$, although $h(\hat{S})$ is not well defined in this limit. Thus the supersymmetric configuration $\mathcal{P} = 0 = X = U$ is the absolute minimum of $\mathcal{V}$ with vanishing vacuum energy. For large $\text{Re}(\hat{S})$ there are no additional non-supersymmetric relative minima if $\mathcal{P}$ contains no constant terms (i.e. $\mathcal{P}(0) = 0$). The minimum requires $\tilde{K}_M + 1/(8\pi^2\text{Re}(\hat{S})^d) + \mathcal{P}'/\mathcal{P} = 0$, but since for large $\text{Re}(\hat{S})$ the quotient $\mathcal{P}'/\mathcal{P}$ goes like $e^{+8\pi^2\hat{S}}$, this condition can never be satisfied.

The following class of models seems to be interesting, because they allow to stabilize the gauge coupling without introducing additional singlet fields at all. Let $\mathcal{P} (\text{Pf} \hat{M})$ again be an arbitrary polynomial of Pf $\hat{M}$ and consider the superpotential

$$ W = \rho \mathcal{P} (\text{Pf} \hat{M}) M_{Pl}^3 + U \ln \frac{\text{Pf} \hat{M}}{\Lambda^4}. \quad (43) $$

In global supersymmetry and fixed $\Lambda$ this has a supersymmetric minimum at $\text{Pf} \hat{M} = \Lambda^4$ and $U = -\rho \mathcal{P}' \Lambda^4/\text{Pf} M_{Pl}$. Varying with respect to $\hat{S}$ demands $U = 0$ and therefore fixes $\Lambda^4/\text{Pf} M_{Pl}$ to be a zero of $\mathcal{P}'$. If $\mathcal{P}'(x)$ has no zero at $|x| < 1$, then only the trivial solution, $\hat{S} \to \infty$ and Pf $\hat{M} = 0$, remains. Of course a correct treatment of this problem requires to solve the supergravity equations of motion. Numerically one finds that there is a supersymmetric solution, i.e. $D_i W = 0$, and at least for small $\rho$, $\Lambda^4/\text{Pf} M_{Pl}^4$ lies near a zero of $\mathcal{P}'$. E.g. for $\mathcal{P}(x) = 2x^2 - x$ and $\rho = 0.1$ one has $e^{-8\pi^2\hat{S}} \approx 0.254$. The quantum constraint gets only small corrections, $1 - \text{Pf} M/\Lambda^4 \approx 6 \cdot 10^{-5}$, and the vacuum energy is negative $\langle \mathcal{V} \rangle \approx -0.02 \text{M}_{Pl}^4$.

The analytic expression of the potential at the minimum with respect to $U$ and $M_{ij}$ is given by

$$ \mathcal{V}|_{\text{min}} = |\rho|^2 e^{K M_{Pl}^2} M_{Pl}^4 \left( 2 \left| \mathcal{P}' \frac{\Lambda^4}{M_{Pl}^4} + \mathcal{P} \tilde{K}_M - \mathcal{P} h(\hat{S}) \right| \frac{M_{Pl}^4}{|\Lambda^2|^2} \hat{g}^M \right. + \left. d \left| (16\pi^2/d) \text{Re}(\hat{S}) \mathcal{P} h(\hat{S}) + \mathcal{P} \right|^2 - 3 |\mathcal{P}|^2 \right) + \mathcal{O} \left( |\rho|^4 \right). \quad (44) $$

In general it is rather hard to see if there are additional minima which possibly stabilize $\hat{S}$ while breaking supersymmetry. However the simplest case $\mathcal{P}(\text{Pf} \hat{M}) = \text{Pf} \hat{M}$ can be treated at least numerically. As $\mathcal{P}'$ has no zeroes we expect that there is no non-trivial vacuum configuration. The supergravity potential now reads (for simplicity we assume $d = 1$)

$$ \mathcal{V}|_{\text{min}} = |\rho|^2 e^{K M_{Pl}^2} \frac{|\Lambda^4|^2}{M_{Pl}^4} \left( 2 \left( 1 + \tilde{K}_M - h(\hat{S}) \right) \frac{M_{Pl}^4}{|\Lambda^2|^2} \hat{g}^M + |16\pi^2 \text{Re}(\hat{S}) h(\hat{S}) + 1|^2 - 3 \right) + \mathcal{O} \left( |\rho|^4 \right). \quad (45) $$

We find that $\mathcal{V}$ is positive and monotonously decreasing. This means that indeed the only minimum is at $\hat{S} \to \infty$.

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8This was obtained by a numerical analysis taking for $\tilde{K}_M$, $\hat{g}^M$ the power series expansion given in the line below eq. (12), expanding $\mathcal{V}$ in powers of $|\Lambda^2|/M_{Pl}^2$ up to $(|\Lambda^2|/M_{Pl}^2)^8$ and assuming for the coefficients of the Kähler potential $a_n = 4/n^2$. If the coefficients are smaller, e.g. $a_n = 1/n^2$, this result is not modified for $8\pi^2 \text{Re}(\hat{S}) > 0.3$. For very small $\hat{S}$ we can make no statement because even higher orders of the power series expansion become important.
These models show that dynamical stabilization of the gauge coupling constant is possible in supersymmetric vacua. This mechanism determines \( 8\pi^2 \langle \hat{S} \rangle \) at values of order one, but \( \langle \hat{S} \rangle \gg (8\pi^2)^{-1} \) is only possible when some of the coefficients of the polynomial \( P \) (or \( \eta \xi^k \) for the model (34)) are exponentially small, which is not natural. Thus for moduli which determine the gauge coupling of a non-perturbative gauge group this mechanism is viable but it is not so attractive for stabilizing the dilaton of perturbative string theory.

Finally, let us observe that the results obtained for the \( SU(2) \) gauge theory can easily be generalized to other (symplectic or unitary) gauge groups. First consider an \( Sp(2N_c) \) gauge theory with \( N_f = N_c + 1 \) quark flavors, i.e. \( 2(N_c + 1) \) quarks \( Q_i \) in the fundamental representation of the gauge group. The low energy theory is described by a quantum moduli space spanned by the VEVs of the ‘mesons’ \( M_{ij} = Q_r^i J_{rs} Q_s^j \), where \( J \) is the \( Sp(2N_c) \) invariant skew-symmetric form. They satisfy the constraint \[ PfM = \frac{1}{2^{N_f} N_f!} \epsilon^{i_1 \ldots i_{2N_f}} M_{i_1i_2} \cdots M_{i_{2N_f}i_{12N_f}} = 2^{N_c-1} \Lambda^{2(N_c+1)}. \] (46)

The IYIT superpotential has the same form as in eq. (21),

\[ W = \rho \text{Tr}(MX) + U \ln \frac{PfM}{\Lambda^{2(N_c+1)}}, \] (47)

where the factor \( 2^{N_c-1} \) has been absorbed in a redefinition of the mesons. The minimization of the supergravity potential is very analogous to the \( SU(2) \) case and yields

\[ V|_{\text{min}} = (N_c + 1)|\rho|^2 M_{Pl}^2 \exp \left( \left( \frac{N_c + 1}{2} \sum_{n=1}^{\infty} a_n e^{-8\pi^2 \text{Re}(\hat{S}) n/(N_c+1)} - \frac{16\pi^2}{N_c+1} \text{Re}(\hat{S}) \right) \right) / 2 \text{Re}(\hat{S})^d + O(|\rho|^4), \] (48)

where we have used \( \Lambda^{2(N_c+1)} = M_{Pl}^{2(N_c+1)} e^{-8\pi^2 \hat{S}} \).

A generalization to \( SU(N_c) \) gauge theories is technically slightly more difficult because of the appearance of baryons, but this should not change the conclusions. The superpotential is (we need two additional singlets \( Y \) and \( \tilde{Y} \))

\[ W = \rho(\text{Tr}(MX) + YB + \tilde{Y}\tilde{B}) + U \ln \frac{\det M - \tilde{B}B}{\Lambda^{2N_c}}. \] (49)

We expect that the supergravity potential will be qualitatively very similar to (48). Thus, the dilaton runaway problem is present in all IYIT type models with unitary or symplectic gauge groups.

Let us summarize. In this paper we investigated the IYIT mechanism for the case of a dynamical, that is field dependent gauge coupling. We argued that the presence of a dilaton-like modulus \( \hat{S} \) requires the coupling of the IYIT model to supergravity. This can be achieved by employing the Veneziano–Yankielowicz formalism. The minimization of the corresponding supergravity potential shows that the vacuum is destabilized and runs to the supersymmetric configuration at \( \Lambda = 0 \). This result holds for any symplectic (and presumably also unitary) gauge theory with only fundamental matter. Some more general tree-level superpotentials
for the mesons show non-trivial minima for \( \hat{S} \) but these are supersymmetric ground states. Thus, it seems that for a gauge theory whose low energy dynamics is described by a quantum modified moduli space only two situations generically occur when a non-vanishing tree-level potential is present: Either in global supersymmetry there is a stable ground state which leads to a supersymmetric minimum for all fields including \( \hat{S} \) in supergravity or global supersymmetry is broken by the ground state but restored when the gauge coupling gets dynamical because \( \langle \hat{S} \rangle \) is driven to infinity.

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**References**

[1] E. Poppitz and S. P. Trivedi, hep-th/9803107.

[2] K.-I. Izawa and T. Yanagida, *Prog. Theor. Phys.* 95 (1996) 829, hep-th/9602180; K. Intriligator and S. Thomas, *Nucl. Phys.* B473 (1996) 121, hep-th/9603158.

[3] N. Seiberg, *Phys. Rev.* D49 (1994) 6857, hep-th/9402044.

[4] C. Csáki, M. Schmaltz and W. Skiba, *Phys. Rev.* D55 (1997) 7840, hep-th/9612207.

[5] B. Grinstein and D.R. Nolte, hep-th/9710001.

[6] S. Kachru, N. Seiberg and E. Silverstein, *Nucl. Phys.* B480 (1996) 170, hep-th/9605036.

[7] S. Kachru and E. Silverstein, *Nucl. Phys.* B482 (1996) 92, hep-th/9608194; S. Kachru, *Nucl. Phys. Proc. Suppl.* 61A (1998) 42, hep-th/9705173.

[8] M. Klein and J. Louis, *Nucl. Phys.* B511 (1998) 197, hep-th/9707212.

[9] I. Maksymyk, C. P. Burgess, A. de la Macorra and F. Quevedo, hep-th/9712178.

[10] G. Veneziano and S. Yankielowicz, *Phys. Lett.* 113B (1982) 231; T.R. Taylor, G. Veneziano and S. Yankielowicz, *Nucl. Phys.* B218 (1983) 493.

[11] D. Amati, K. Konishi, Y. Meurice, G. C. Rossi and G. Veneziano, *Phys. Rep.* 162 (1988) 169.

[12] G. Dvali and Z. Kakushadze, *Phys. Lett.* 417B (1998) 50, hep-th/9709093.
[13] I. Affleck, M. Dine and N. Seiberg, *Nucl. Phys.* B241 (1984) 493; *Nucl. Phys.* B256 (1985) 557.

[14] E. Witten, *Nucl. Phys.* B460 (1996) 541, hep-th/9511030.

[15] M. Dine and N. Seiberg, *Phys. Lett.* 162B (1985) 299.

[16] E. Witten, *Nucl. Phys.* B202 (1982) 253.

[17] V. Kaplunovsky and J. Louis, *Nucl. Phys.* B444 (1995) 191, hep-th/9502077.

[18] K. Intriligator, *Phys. Lett.* 336B (1994) 409, hep-th/9407106.

[19] A. Kovner and M. Shifman, *Phys. Rev.* D56 (1997) 2396, hep-th/9702174.

[20] C. Csáki and H. Murayama, hep-th/9710105.

[21] M. Schwetz and M. Zabzine, hep-th/9710125.

[22] J. de Boer, K. Hori, H. Ooguri and Y. Oz, hep-th/9801060.

[23] K. Intriligator and P. Pouliot, *Phys. Lett.* 353B (1995) 471, hep-th/9505006.