Contact tracing Inspired Efficient Computation by Energy Tracing

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Received XX Month XXXX; revised XX Month, XXXX; accepted XX Month XXXX; posted XX Month XXXX (Doc. ID XXXXX); published XX Month XXXX

Inspired by the epidemic contact tracing technique, we propose a method to efficiently solve electromagnetics by tracing the energy distribution. The computational domain is adaptively decomposed, and the available computational resources are focused on those energy-active (infections) and their adjacent (exposed) domains, while avoiding the unnecessary computation of energy-null (unexposed) domains. As an example, we employ this method to solve several optics problems. The proposed method shows high efficiency while maintaining a good accuracy. The energy tracing method is based on the causality principle, and therefore is potentially transformative into other computational physics and associated algorithms.

1. Introduction

The Covid-19 pandemic outbreak has greatly challenged the capacity of healthcare systems worldwide [1]. Epidemic contact tracing is a classical method to efficiently classify susceptibles according to their risks [2,3]. Besides infected patients (red), the current contact tracing technique further differentiates susceptibles as either exposed (yellow) or unexposed (green) according to their contact history, as illustrated in Fig. 1 (a). Exposed susceptibles are monitored for an average incubation period to accurately trace dynamic infection spread. Thus, the distribution of limited medical resources can be appropriately prioritized away from those unexposed and toward those with higher infection risks. Computational physics also faces the same kind of challenge in allocating the limited computational resources. As only energy-active domains are of physical interest, if the energy distribution can be adaptively traced and predicted, then the computational resources can be arranged in a more efficient manner.

In this work, we observed that the electromagnetic energy propagation can be accurately traced in a way similar to the tracing of contractual diseases as they both conform to the causality principle. Specifically, such a tracing method can be easily deployed for a numerical solution to hyperbolic equations that conform to the Courant-Friedrichs-Lewy (CFL) conditions, which is usually regarded as the numerical expression of causality [4,5]. The efficient computation of hyperbolic equations is a topic of profound importance and has potential wide ranging applications in fields such as electromagnetics [6], general relativity [7], fluid dynamics [8], spectral theory [9], dynamical systems [10] and far beyond.

For validation, we apply this adaptive energy tracing method to discontinuous Galerkin time domain (DGTD) method to solve electromagnetic problems. With numerical flux, DGTD computes fields in each mesh cell separately. This flexibility makes it ideal candidate for the proposed dynamic element-wised simplification [11,12].

2. Formulation

Courant, Friedrichs, and Lewy pointed out that the convergence of the numerical solution of hyperbolic equations is dependent on the ratio of space and time discretization [4]. For the one-dimensional convection equation, the CFL condition requires that for stability:

\[ C = \frac{u|\Delta t}{\Delta x} \leq 1, \]  \hspace{1cm} (1)

where \( u \) represents the magnitude of the velocity, and the dimensionless number \( C \) is called the Courant number. In practice of numerical simulation, this theory is often applied as an upper bound restriction on the timestep size for most time explicit iterative algorithms [5]:

\[ \Delta t \leq \frac{\Delta x}{|u|}. \]  \hspace{1cm} (2)
With that, it numerically limits the velocity of energy propagation. Therefore, CFL condition is usually viewed as the causality principle in numerical expression.

In this paper, however, we use the CFL condition in a reverse manner: In other words, for all time explicit iterative algorithms conforming to the CFL condition, it ensures that in every time step the energy in one mesh cell (infection) cannot affect other cells except for the adjacent ones (exposed). Therefore, we can treat the energy propagation as an analogy to epidemic spreading, and simplify the computation of those mesh cells that far away from the energy active ones.

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![Diagram](image1)

**Fig. 1.** The energy tracing method. (a) Illustrative map of epidemic spreading. (b) the wave energy distribution in the spatial domain at a specific time. (c) the flowchart of the adaptive energy tracing algorithm and its associated solver.

Fig. 1 (b) demonstrates the spatial energy distribution at a specific time step $t_n$. With the CFL condition, we can predict that in the next time step $t_{n+1}$, the energy can only affect the adjacent cells. Other mesh elements far beyond are unexposed to the energy and thus will have no energy distribution at $t_{n+1}$. Therefore, at $t_{n+1}$ their unknowns can be efficiently simplified as zero without computation.

![Diagram](image2)

**Fig. 2.** Example in a segment of coaxial line. (a) The electric field at a probe node. (b) The simplification ratio of the predictive method. (c) The convergence plots of the DGTD methods with and without simplification.

As the energy distribution is dynamic over time, the simplification ratio defined in Eq. 3 is also time dependent. Fig. 1 (c) shows the flowchart of the adaptive energy tracing algorithm. At each timestep, the energy tracer decomposed the computational domain based on the field distribution prediction. Accordingly, the solver only computes those mesh cells that potentially energy-active, while avoids computation of those energy-null areas. The domain decomposition is updated adaptively at every timestep.

**3. Numerical Examples**
To validate the accuracy and efficiency improvement of the proposed method, we present several numerical examples. Although this energy tracing method can be applied to other time domain methods (S1), it shows best performance on DGTD methods.

Besides, the proposed method is advantageous for applications with multiple pulses as many areas between those temporal pulse are energy-nulls and hence can be simplified. Multiple pulse input scenarios are important in applications such as radar, EMC/EMI, and space-time modulation problems. In our examples, we excite two sinusoidally modulated Gaussian pulses. Here we assume that the amplitude of the incident pulses is normalized.

As the amplitude of input pulses are normalized, field components less than 1% are considered as numerically zero. Mesh cells with such negligible field strength are labelled as energy-null.

Therefore, the first example is chosen as a multi-pulse wave propagates in a coaxial line.

The coaxial line has inner and outer radius of 0.254 mm and 1.008 mm, respectively. It has length of 7.62 mm, filled with dielectric whose relative permittivity $\varepsilon_r = 2.2$. Fig. 2 (a) shows the magnitude of the electric field at a sample node in the coaxial line with normalized incidence. The incident excitation contains two separate pulses. Fig. 2 (b) shows the simplification ratio of the proposed algorithm. As the wave energy propagates, the simplification ratio varies over time. When pulse energy remains in the computational area, many mesh elements are energy active. Therefore, the simplification ratio reaches its minimum of 0.3, which means that about 30% of mesh elements are predicted as energy-null areas and thus can be set to zero without computation. When the electromagnetic wave passes through and no energy remains, more than 95% of mesh cells are simplified. It cannot be 100% because we treat the wave port as potential energy active as it relates with unpredictable external energy source. The two minimum of the simplification ratio corresponds with the energy of the two pulses, which indicates that the algorithm can adaptively conduct domain decomposition according to the energy distribution.

The computational times of the DGTD solver with or without prediction are 26 s and 65 s, respectively, showing the great efficiency improvement of this predictive method. The computed temporal response in Fig 2 (a) matches very well with the analytical result. Fig. 2 (c) shows the convergence plot of the proposed algorithm with and without prediction. The proposed algorithm has similar accuracy performance comparing with the conventional one.

A. Convergence analysis of a segment of coaxial line

Coaxial line is an important type of guided transmission devices used in microwave engineering. Besides, it provides an analytical result for the easy evaluation of the proposed method's accuracy.

![Fig. 3. Example in a segment of stepped coaxial line. (a) The geometry of the dielectric inside the stepped coaxial line. The conductors are not shown in this figure. (b) The electric field at a probe node. (c) The simplification ratio of the predictive method.](image)
But it should be noticed that Fig. 2.b shows the electric field only in one sample mesh cell, while the simplification ratio in Fig. 2.d conveys the overall energy distribution in the entire area of interest. When the pulse energy enters the area of interest, the simplification ratio reaches its minimum. Except for that, more than 90% of mesh elements are predicted as energy-null areas, and thus are simplified (i.e., set to zero) without computation. Note that all of the mesh cells on the incident port are always fully computed, because they might have external energy input and thus cannot be predicted. This limits the upper bound of the simplification ratio. Nevertheless, this adaptive domain decomposition and simplification can save significant computational resources while maintaining a high degree of accuracy. The computational times of the DGTD solver with and without prediction are 1830 s and 3117 s, respectively.

4. Conclusion

In this letter, we enhance the computational efficiency of electromagnetics by tracing the energy propagation in a way that is analogous to the process of epidemic contact tracing. The method is based on the CFL condition, which is a general rule for solving hyperbolic equations. In addition to the electromagnetics example presented, this efficiency boosting technique may be extended to other areas of the computational sciences that are governed by hyperbolic difference equations.

Funding. Penn State MRSEC Center for Nanoscale Science (NSF DMR-1420620), and DARPA/DSO Extreme Optics and Imaging (EXTREME) Program (HR00111720032).

Disclosures. The authors declare no conflicts of interest.

Data availability. Data underlying the results presented in this paper are not publicly available at this time but may be obtained from the authors upon reasonable request.

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