Recurrent swelling of horizontally shaken granular material

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This Letter reports an experimental study of the dynamics of granular materials when vibrated horizontally. Convective motion is observed over wide range of parameters, similarly to the known effect observed under vertical vibration. However, under horizontal vibration we observe a striking novel effect on the oscillating height of the surface of the material: the frequency of the surface oscillations are totally decoupled from the frequency of the drive. We explain the effect as resulting from an interplay between Reynolds dilatancy due to convective motion and mechanical stability of the material.

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When a rectangular container with granular material is shaken either vertically or horizontally one observes convection. For the case of vertical shaking this effect has been intensively studied experimentally, analytically and by means of computer simulations (s. e.g. [123]). Many results on vertical shaking are known and there is a certain degree of understanding of these phenomena. For horizontal shaking convection cells have been reported only recently [4,5] and presently one is far from understanding these complex phenomena.

In the present letter we report a new effect experimentally observed in horizontally shaken granular material. As far as we know this is the first dynamical effect where the frequency of forcing decouples from the frequency of the effect, i.e. the system reveals an inherent time scale [9].

The new effect, which we want to focus here, is recurrent swelling of horizontally shaken material: For a certain range of amplitude \( A \) and frequency \( f \) of the sinusoidal motion of the container \( x(t) = A \cos(2\pi f t) \) and for a certain region of filling level we observe a time dependent variation of the volume of the material. We will show that the fluctuations of the height of the material, the intensity of the convection flow and the energy dissipation rate in the flowing material are closely related effects.

There are several experimental investigations of exciting time-dependent phenomena in vertically vibrated containers [23]. While the time scales of these effects are comparable with the period of forcing, for horizontal vibration we find a main difference: The effect reported in this letter has a period which is up to several hundred times larger than the period of oscillation \( f^{-1} \). Hence we do not observe a period doubling scenario as e.g. in [3].

Fig. 1 shows the experimental setup which we have used. The probe carrier was mounted on a precisely balanced horizontal linear bearing (left in Fig. 1). A second bearing (middle) was driven by a stepping motor via a crankshaft with adjustable eccentricity. Both bearings were connected by a piezoelectric force sensor which allows to measure the driving force with 2 kHz time resolution. The motor was computer controlled, i.e. in precisely 2,000 impulses the motor axis revolves once. This high angular resolution provides quasi steady motion. We checked, that the finite step size per computer signal does not influence the convective behavior. The entire mechanical device was fixed on an oscillation damping table. The container of size 6 cm \( \times \) 10 cm, consists of transparent plastic material. It was illuminated from the top and was monitored by a videocamera, which was located in a distance of 4 m in a direction perpendicular to the axis of oscillation. With this camera we observed the dynamical processes in the container. For that purpose the camera was levelled exactly at the vertical height of the granular material at rest.

The lower part of Fig. 1 displays snapshots of the surface region of width 7 cm. The container is horizontally shaken with amplitude \( A = 0.15 \) cm (the same used in the investigations described below) and frequency \( f = 25 \) sec\(^{-1} \). As discussed in [4], we observe a small heap in the center. The heap-height fluctuates between about 2.3 and 3.7 mm with respect to the level at rest. The dashed line is drawn 3 mm above the level at rest. The
time dependence of the height of the surface on forcing parameters is subject of our interest. As shown below, the height fluctuates periodically with a period of about 2.2 sec, which is 55 times the period of the driving oscillation.

The swelling oscillation of the material has been studied qualitatively for various combinations of amplitude $A$, frequency $f$ and filling height and for a number of different materials and grain sizes ($20 \mu m \leq r \leq 100 \mu m$). To analyze the effect we recorded time series of the height of the material surface in the middle of the container. Fig. 2 displays the height over a period of 40 sec for different driving frequencies.

The height of the displayed regions is 0.4 cm each. The material filling height (at rest) was 2.5 cm, and the average grain size was 100 $\mu m$. For all frequencies in Fig. 2 one sees that the material height varies with time. For small driving frequency ($f \lesssim 22 \text{ sec}^{-1}$) and large driving frequency ($f \gtrsim 30 \text{ sec}^{-1}$) the height changes irregularly with time, whereas for $23 \text{ sec}^{-1} \lesssim f \lesssim 29 \text{ sec}^{-1}$ we observe regular, almost periodic oscillation of the material height. This behavior can be characterized by the Fourier transforms of the full series of total lengths 120 sec (Fig. 3). For $f < 23 \text{ sec}^{-1}$ no characteristic frequency can be observed, i.e. the Fourier spectrum contains many frequencies.

At $f = 23 \text{ sec}^{-1}$ there seems to be a sharp transition into another regime, where we find a characteristic frequency of the height oscillation as indicated by a peak in the Fourier spectrum. For high frequency $f \gtrsim 29 \text{ sec}^{-1}$ the amplitude of swelling becomes very small and the effect vanishes. Hysteresis of the transition point has not been observed.

As seen from Fig. 3 in the region of periodic swelling ($23 \text{ sec}^{-1} \lesssim f \lesssim 29 \text{ sec}^{-1}$) the swelling frequency $F$ is a function of the driving frequency $f$.

FIG. 2. Height of the granular material in the middle of the container over time for driving frequencies between 17 sec$^{-1}$ and 32 sec$^{-1}$.

FIG. 3. Fourier transforms of the oscillation of the material height for different driving frequencies corresponding to Fig. 2.

FIG. 4. Swelling frequency $F$ as a function of the frequency of forcing for various filling heights.
Fig. 4 shows the characteristic swelling frequency \( F \) over the driving frequency \( f \) for different filling heights. The data shown in Fig. 4 can be reproduced with good accuracy, although it seems not to be possible to collapse the data using dimensionless numbers for many combinations of the system parameters.

In the shaken container one observes convection: the material flows downwards close to the walls and upwards in the center of the container. In the interval of driving parameters considered here, there is an intensive convective material flow. Viewing the container from top one observes that the oscillation of material height (Figs. 2, 3) corresponds to a varying particle velocity at the upper surface. When the material is swelt there is an intensive flow, whereas the flow in the center of the surface comes almost to rest when the material is collapsed. A simple way to qualitatively visualize this effect is to take a series of snapshots of the container viewing from top using a digital camera.

In this way one produces a series of difference pictures by subtracting the gray scale values of the pixels of consecutive snapshots. The average value \( \Delta G \) of the gray scale of the pixels of the difference pictures provides a measure for the material flow at the surface. Fig. 5 shows the Fourier transform of \( \Delta G \) for a driving frequency of \( f = 25 \text{ sec}^{-1} \). We find that the variation of the surface flow has the same characteristic frequency as the height oscillation (c.f. Fig. 3). For this reason we believe that convective motion in the horizontally shaken container and swelling are closely related effects.

The data shown in Fig. 4 can be reproduced with good accuracy, although it seems not to be possible to collapse the data using dimensionless numbers for many combinations of the system parameters.

From the force data \( F(t) \) measured by the sensor we calculated the dissipated driving energy during the \( n \)th shaking period:

\[
E_n = 2\pi f A \int \frac{(n+1)/f}{n/f} F(t) \cos(2\pi ft) \, dt.
\]

Figure 6 shows the Fourier transform of the dissipated energy, again for amplitude \( A = 0.15 \text{ cm} \) and a range of driving frequencies \( f \).

FIG. 5. Material flow at the surface (central area of size 6.5 cm x 2 cm) over time for \( f = 25 \text{ sec}^{-1} \).

The influence of air has to be discussed. One could imagine that the convective motion of sand “pumps” air into the bulk of the material which leads to swelling. When the amount of air reaches a certain extend a “bubble” escapes and the material collapses. This mechanism would explain the swelling effect as well as the saw tooth shape of the height over time curves (Fig. 2). But reducing the pressure to \( p = 50 \text{ Pa} \), which corresponds to a mean free path of \( \lambda_{\text{air}} \approx 130 \mu \text{m} \), the swelling effect does not disappear. Since \( \lambda_{\text{air}} \approx 130 \mu \text{m} \) is smaller than the typical free path of sand grains we can exclude air as playing the major role in recurrent swelling.

The origin of the new effect is not completely clear yet,
we suggest a tentative explanation based on the measurement of dissipated mechanical energy and surface flow: In the horizontally shaken container one observes convection \cite{4}, i.e. motion of the particles with respect to each other. To allow for macroscopic motion in a granular material the material has to be diluted below a certain density $\rho_R$ before (Reynolds dilatancy \cite{9}). When the local density of the material is below $\rho_R$ the material can start to flow. The convection in the container implies shear flow, which causes further dilution and fluidization (e.g. \cite{10}). When the shaken material swells according to Reynolds dilatancy, the material becomes diluted and becomes at the same time less stable mechanically. At a certain moment the material becomes diluted to an extent that it loses mechanical stability and collapses. Then the material starts again to swell.

Therefore, we believe that there are two competing effects. First, due to shear the material tends to dilute. Second, the material has to remain mechanically stable, i.e. the grains on top have to be supported by the grains below them and these have to be stabilized by the grains of the next layer etc.

Our explanation is supported by the measurements of the dissipation of mechanical energy in the system and of the surface material flow. If the material is collapsed, i.e. if a large part of the granular material has a density above $\rho_R$, the grains in these regions cannot move with respect to each other due to the Reynolds dilatancy effect. Hence, in these regions there is no shear motion and, therefore, none or just low dissipation of mechanical energy. From this consideration we conclude, that the collapsed material should dissipate less energy per time than the swelled material. This behavior coincides with what we found in the measurement of the dissipated energy. Moreover our results show, that the intensity of the surface flow oscillates with the same swelling frequency $F$. Therefore, the experimental results on energy dissipation and surface flow support our hypothesis on the origin of the swelling effect.

Other interesting questions concern the dependence of the swelling effect on container shape and size as well as on the grain material and the amplitude of shaking. We did experiments with several types of containers, with different grain material and with various amplitudes. While the properties of swelling depend strongly on the details of the experiment we observed the effect for a wide range of parameters. A more detailed description of the properties of the swelling effect will be subject of a forthcoming paper.

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