Conformal Symmetry on
World Volumes of Branes†

Piet Claus\textsuperscript{a}, Renata Kallosh\textsuperscript{b} and Antoine Van Proeyen\textsuperscript{a,∗}

\textsuperscript{a} Instituut voor theoretische fysica,
Katholieke Universiteit Leuven, B-3001 Leuven, Belgium
\textsuperscript{b} Physics Department,
Stanford University, Stanford, CA 94305-4060, USA

Abstract
We show how the anti-de Sitter isometries of a brane solution of supergravity theory produce superconformal invariance of their world-volume action. In this way linear as well as non-linear superconformal actions are obtained in various dimensions. Two particular examples are a conformal action with the antisymmetric tensor in 6 dimensions in Pasti-Sorokin-Tonin formulation, and superconformal mechanics.

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∗ Onderzoeksdirector FWO, Belgium
We show how the anti-de Sitter isometries of a brane solution of supergravity theory produce superconformal invariance of their world-volume action. In this way linear as well as non-linear superconformal actions are obtained in various dimensions. Two particular examples are a conformal action with the antisymmetric tensor in 6 dimensions in Pasti-Sorokin-Tonin formulation, and superconformal mechanics.

1 Introduction

The world-volume theory of certain branes is superconformal invariant. This can be either the case for a brane living in a flat background, or in the background produced by a stack of other branes. In the former case the linearised action is superconformal, why in the latter one, on which we will concentrate for the main part of the talk, this leads to a non-linear superconformal action.

The starting point is a classical solution of supergravity describing N coincident branes. This solution has a limit ‘the near-horizon limit’ in which the space-time metric is an anti-de Sitter (adS) geometry. (The other limit is flat space which leads to the linear conformal action). The Killing vectors and Killing spinors of that geometry define a supergroup \( g \), which by itself could serve as an alternative starting point of the construction. In that construction the geometry of a supercoset having \( g \) as its isometry group delivers all the input for the world-volume action. The isometries determine rigid symmetries of that world-volume action, which has further local symmetries: general coordinate transformations and \( \kappa \)-symmetry. After gauge fixing of these local symmetries, the rigid symmetries change, because the gauge-fixing is invariant under a combination of the former rigid symmetries with some parts of the general coordinate transformations and \( \kappa \)-symmetry. This results in transformations which we recognize as the superconformal transformations in the world-volume theory.

The above mechanism is applicable for D3 branes in 10 dimensional string theory, M2 and M5 branes in 11 dimensions. The latter case leads to a conformal version of the 6-dimensional antisymmetric tensor multiplet. These
are not the only examples. One may also consider intersections of branes or
er other constructions in lower dimensions. One more application which we will
illustrate, is the construction in this way of a ‘relativistic superconformal me-
nanics’. That one is obtained starting from black hole solutions in $d = 4,$
$N = 2$ supergravity.

In Sec. 2 we review the classification of super-anti-de Sitter and conformal
algebras. The $adS$ geometry appears in limits of the geometry of branes (see
Sec. 3) and provides rigid symmetries in the world-volume theory, as shown in
Sec. 4. In that section we show how in the bosonic case conformal symmetry
arises. The same general principles are used in Sec. 5 to obtain ‘relativistic
superconformal mechanics’ near the horizon of a Reissner-Nordström black
hole.

The supersymmetric world-volume theory depends on superspace quanti-
ties. We therefore first give some results on the supergeometry in Sec. 6. Then
we present the action in Sec. 7, where we use mainly the black hole exa-
mple to illustrate the rigid $adS$-supersymmetries, the role of $\kappa$-symmetry, and its
gauge fixing. Sec. 8 explains the construction of the linear conformal theory,
before presenting the conclusions in Sec. 9. This talk summarizes our papers
1, 2, 3 and some unfinished work on the supersymmetrisation of the conformal
mechanics.

2 $adS$ and conformal superalgebras

The anti-de Sitter algebra is the algebra of Lorentz rotations $M_{\mu\nu}$ and trans-
lations $P_{\mu}$, where the latter do not commute:

$$[M_{\mu\nu}, M_{\rho\sigma}] = \eta_{\mu[\rho} M_{\sigma]\nu} - \eta_{\nu[\rho} M_{\sigma]\mu} ; \quad [P_{\mu}, M_{\nu\rho}] = \eta_{\mu[\nu} P_{\rho]} ,$$

$$[P_{\mu}, P_{\nu}] = \frac{1}{2R^2} M_{\mu\nu} . \quad (1)$$

With the opposite sign for the last commutator we would have the de Sitter
algebra. Defining $M_{d\mu} = -M_{\mu d} = R P_{\mu}$ we have generators $M_{\dot{\mu}\dot{\nu}} = -M_{\dot{\nu}\dot{\mu}}$
with $\dot{\mu} = 0, \ldots, d,$ and defining the metric to be $\eta_{\dot{\mu}\dot{\nu}} = \text{diag}(-+\cdots-)$ the
algebra can be concisely written as

$$[M_{\dot{\mu}\dot{\nu}}, M_{\dot{\rho}\dot{\sigma}}] = \eta_{\dot{\mu}|\dot{\rho}} M_{\dot{\sigma}|\dot{\nu}} - \eta_{\dot{\nu}|\dot{\rho}} M_{\dot{\sigma}|\dot{\mu}} , \quad (2)$$

i.e. it is the algebra $SO(d-1,2)$. Note that every point is invariant under the
rotations around this point, while not invariant under the translations. In this
sense we can write

$$adS_d = \frac{SO(d-1,2)}{SO(d-1,1)} . \quad (3)$$
The conformal algebra, is according to the Coleman–Mandula theorem the largest space–time symmetry which we can impose, allowing non-trivial scattering of particles. Its usefulness is most outspoken in 2 dimensions, because then the group is infinite dimensional. Conformal symmetry is defined as the symmetry which preserves angles. Therefore it should contain the transformations which change the metric up to a factor. That implies that the symmetries are determined by the solutions to the ‘conformal Killing equation’

$$\partial_\mu (\xi_\nu) - \frac{1}{4} \eta_{\mu\nu} \partial_\rho \xi_\rho = 0 .$$

(4)

In dimensions $d > 2$ the conformal algebra is finite-dimensional. Indeed, the solutions are

$$\xi_\mu (x) = a_\mu + \lambda_\mu^\nu x_\nu + \lambda_D x^\mu + (x^2 \Lambda_K^\mu - 2x^\mu x \cdot \Lambda_K) .$$

(5)

Corresponding to the parameters $a_\mu$ are the translations $P_\mu$, to $\lambda_\mu^\nu$ correspond the Lorentz rotations $M_\mu^\nu$, to $\lambda_D$ are associated dilatations $D$, and $\Lambda_K^\mu$ are parameters of ‘special conformal transformations’ $K_\mu$. This is expressed as follows for the full set of conformal transformations $\delta_C$:

$$\delta_C = a_\mu P_\mu + \lambda_\mu^\nu M_\mu^\nu + \lambda_D D + \Lambda_K^\mu K_\mu .$$

(6)

With these transformations, one can obtain the algebra with as non-zero commutators

$$[M_\mu^\nu, M_\rho^\sigma] = -2\delta_\rho^\sigma [\mu, \nu] ; \quad [P_\mu, M_\nu^\rho] = \eta_\mu\nu P_\rho ; \quad [P_\mu, K_\nu] = 2(\eta_\mu\nu D + 2M_\mu^\nu) ; \quad [D, K_\mu] = -K_\mu .$$

(7)

This is the $SO(d, 2)$ algebra. Indeed one can define

$$M^{\hat{\mu}\hat{\nu}} = \begin{pmatrix}
M_\mu^\nu & \frac{1}{4} (P_\mu - K_\mu) & \frac{1}{4} (P_\mu + K_\mu) \\
\frac{1}{4} (P_\nu - K_\nu) & 0 & -\frac{1}{2} D \\
\frac{1}{4} (P_\nu + K_\nu) & -\frac{1}{2} D & 0
\end{pmatrix}$$

(8)

preserving the metric $\eta = \text{diag} (-1, 1, ..., 1, -1)$. Note that this is the same as the anti-de Sitter algebra in $d + 1$ dimensions

$$\text{Conf}_d = \text{adS}_{d+1} ,$$

(9)

which is the basic observation for the adS/CFT correspondence.

In general, fields $\phi^i (x)$ have the following transformations under the conformal group:

$$\delta_C \phi^i (x) = \xi_\mu (x) \partial_\mu \phi^i (x) + \Lambda_\mu^\nu (x) m_{\mu\nu} \phi^i (x) + w_\mu D(x) \phi^i (x) + \Lambda_K^\mu (k_\mu \phi^i) (x) ,$$

(10)
where the $x$-dependent rotation $\Lambda_{M \mu \nu}(x)$ and $x$-dependent dilatation $\Lambda_D(x)$ are given by

$$\Lambda_{M \mu \nu}(x) = \frac{\partial_{[\mu} \xi_{\nu]} = \lambda_{M \mu \nu} - 4x_{[\mu} \Lambda_{K \nu]} ,$$

$$\Lambda_D(x) = \frac{1}{d} \partial_{\rho} \xi^\rho = \lambda_D - 2x \cdot \Lambda_K . \quad (11)$$

The matrix $m_{\mu \nu}$ represent the usual Lorentz rotations, and the number $w_i$ is called the Weyl weight of the field. Any term in the action should have Weyl weight $d$, counting 1 for a derivative. But furthermore there is one more requirement for special conformal transformations:

$$\delta_K S = 2\Lambda^\mu_K \int d^d x \frac{\partial \delta}{\partial (\partial_{\nu} \phi)} (-\eta_{\mu \nu} w_i \phi^i + 2m_{\mu \nu \rho \sigma} \phi^i \phi^j) + \Lambda^\mu_K \frac{\partial \delta}{\partial \phi^i(x)} (k_{\mu \phi})^i(x) . \quad (12)$$

where $\partial_\nu$ indicates a right derivative. The first terms originate from the $K$-transformations contained in Eq. [5] and Eq. [11]. In most cases these are sufficient to find the invariance and no $(k_{\mu \phi})$ are necessary. However, e.g. for the theories which one obtains from $adS$ backgrounds, such extra terms are present which depend on the field $r$, the distance from the brane.

In supersymmetric theories, the conformal symmetry implies the presence of a second supersymmetry $S$, usually denoted as ‘special supersymmetry’. Indeed, the commutator of the special conformal transformations, and the ordinary supersymmetry $Q$ implies this $S$ due to $[K_{\mu}, Q] = \gamma_{\mu} S$.

The anticommutator $\{ Q, S \}$ generates also an extra bosonic algebra (sometimes called $R$ symmetry). The whole superalgebra can be represented in a supermatrix, as e.g. (symbolically)

$$\begin{pmatrix} SO(d,2) & Q + S \\ Q - S & R \end{pmatrix} . \quad (13)$$

We can consider such superalgebras in general. That is what Nahm did in his classification. The requirements for a superconformal algebra in $d$ or a super-$adS$ algebra in $d + 1$ are:

1. $SO(d,2)$ should appear as a factored subgroup of the bosonic part of the superalgebra. For Nahm, this requirement was motivated by the Coleman-Mandula theorem, but here it is imposed in order to have that the bosonic algebra is the isometry algebra of a space which has the $adS$ space as a factor.
2. fermionic generators should sit in a spinorial representation of that group.
One can then consider the list\footnote{Technically speaking this is the list of Lie superalgebras of classical type, i.e. those for which the fermionic generators are in an irreducible or completely reducible representation of the bosonic algebra. For more explanations, see e.g. the review.} of simple Lie superalgebras\footnote{The table has been changed after the first version of this paper, correcting also the table in \cite{article}. These corrections have been found in discussions with S. Ferrara.} in Table\footnote{The conventions which we use for groups is that $Sp(2n) = Sp(2n, \mathbb{H})$ (always even entry), and $USp(2m, 2n) = U(m, n, \mathbb{H})$. $Sl(n)$ is $Sl(n, \mathbb{R})$, and thus $Sp(2) = Sl(2) = SU(1, 1)$. Further, $SU^*(2n) = Sl(n, \mathbb{H})$ and $SO^*(2n) = O(n, \mathbb{H})$. The exceptional group $G_2$ has only 2 real forms, the compact one $G_{2,14}$ and the ‘normal form’ $G_{2,2}$.} in Table\footnote{Note the particular case of $d = 6$, where we use the notation $OSp(8^\ast | N)$ for the superconformal algebra. Often, including previous articles of ourselves, it was written as $OSp(6, 2|4)$, not paying attention to the existing real forms. In fact, in the series $OSp(m, n|2p)$ the algebra $Sp(2p)$ is non-compact. The possibility for a compact $R$-symmetry group $USp(2p)$ exists due to the isomorphism $SO^\ast(8) = SO(6, 2)$, and thus works only for this signature.}\footnote{We mention the superalgebra with compact $R$-symmetry group.}. In this table\footnote{We mention the superalgebra with compact $R$-symmetry group.} ‘defining representation’ gives the fermionic generators as a representation of the bosonic subalgebra. The ‘number of generators’ gives the numbers of (bosonic,fermionic) generators in the superalgebra. We mention first the algebra as an algebra over $\mathbb{C}$, and then give different real forms of these algebras\footnote{We mention the superalgebra with compact $R$-symmetry group.}. The names which we use for the real forms\footnote{We mention the superalgebra with compact $R$-symmetry group.} is for some different from those in the mathematical literature, and chosen such that it is most suggestive of its bosonic content. There are isomorphisms as $SU(2|1) = OSp(2^\ast|2, 0)$, and $SU(1, 1|1) = Sl(2|1) = OSp(2|2)$. In the algebra $D(2, 1, \alpha)$ the three $Sl(2)$ factors of the bosonic group in the anticommutator of the fermionic generators appear with relative weights 1, $\alpha$ and $-1 - \alpha$. The real forms contain respectively $SO(4) = SU(2) \times SU(2)$, $SO(3, 1) = Sl(2, \mathbb{C})$ and $SO(2, 2) = Sl(2) \times Sl(2)$. In the first and last case $\alpha$ should be real, while $\alpha = 1 + ia$ with real $a$ for $p = 1$. In the limit $\alpha = 1$ one has the isomorphisms $D^p(2, 1, 1) = OSp(4 - p, p|2)$.

Scanning through that list, one finds those algebras which satisfy the conditions for super-adS or superconformal algebras. The result are algebras with maximal $d = 4^d$. The result is given in Table\footnote{We mention the superalgebra with compact $R$-symmetry group.} except for $d = 2$. For $d = 2$ the finite bosonic adS or conformal algebra is $SO(2, 2) \approx SO(2, 1) \oplus SO(2, 1)$, i.e. the sum of two $d = 1$ algebras. The super-adS or superconformal algebra is then the sum of two $d = 1$ algebras of Table\footnote{The conventions which we use for groups is that $Sp(2n) = Sp(2n, \mathbb{H})$ (always even entry), and $USp(2m, 2n) = U(m, n, \mathbb{H})$. $Sl(n)$ is $Sl(n, \mathbb{R})$, and thus $Sp(2) = Sl(2) = SU(1, 1)$. Further, $SU^*(2n) = Sl(n, \mathbb{H})$ and $SO^*(2n) = O(n, \mathbb{H})$. The exceptional group $G_2$ has only 2 real forms, the compact one $G_{2,14}$ and the ‘normal form’ $G_{2,2}$.} Notice that these are the finite part of infinite dimensional superconformal algebras in 2 dimensions. For a classification of the infinite superconformal algebras, see\footnote{We mention the superalgebra with compact $R$-symmetry group.}. One may also relax the condition that the bosonic algebra contains the algebra $SO(d, 2)$ as a factored subgroup of the whole bosonic algebra, and suffice with having it as some subgroup. Then the other bosonic symmetries are not necessarily scalars and the Coleman-Mandula theorem is violated. However, this may still be relevant where branes are present and has been used e.g. in\footnote{We mention the superalgebra with compact $R$-symmetry group.} to propose
the $OSp(1|32)$ as super $adS_{11}$ or $conf_{10}$. In that case one has

$$[Q,Q] = \Gamma^\mu P_\mu + \Gamma^{\mu \nu} Z^{(2)}_{\mu \nu} + \Gamma^{\mu \nu \rho \sigma \tau} Z^{(5)}_{\mu \nu \rho \sigma \tau} .$$

This algebra is now known as the $M$-theory algebra.$^4$

3 adS geometry from a hypersurface and from branes

To obtain a space with $adS_d$ metric, we start from defining it as a submanifold of a $(d + 1)$-dimensional space with a flat metric of $(d - 1, 2)$ signature (for convenience we taken here $\mu = 0, \ldots, d - 2$)

$$ds^2 = \frac{dX^\mu \eta_{\mu \nu} dX^\nu}{(d - 2, 1)} + \frac{dX^+ dX^-}{(1, 1)} \Rightarrow (d - 1, 2)$$

The $adS$ space is the submanifold determined by the equation (again $SO(d - 1, 2)$ invariant)

$$X^\mu \eta_{\mu \nu} X^\nu - X^+ X^- + R^2 = 0 .$$

On the hypersurface one can take several sets of coordinates. E.g. the horospherical coordinates \{x^\mu, z\} by

$$X^- = z^{-1}, \quad X^\mu = z^{-1} x^\mu, \quad X^+ = \frac{x^2 + R^2 z^2}{z} .$$

The latter being the solution of Eq. 16 given the first two. The induced metric on the hypersurface is

$$ds^2 = \frac{1}{z^2} \left( dx_\mu^2 + R^2 dz^2 \right) .$$

The $SO(d - 1, 2)$ is linearly realized in the embedding $(d + 1)$-dimensional space, and these transformations, $(\hat{\mu} = \mu, +, -$ and $\Lambda^{\hat{\nu} \hat{\mu}} = -\Lambda^{\hat{\mu} \hat{\nu}}$)

$$\delta X^{\hat{\mu}} = \Lambda^{\hat{\nu} \hat{\rho}} M_{\hat{\rho} \hat{\mu}} X^{\hat{\nu}} = -\Lambda^{\hat{\mu} \hat{\nu}} X^{\hat{\nu}} .$$

are on the $adS$ space distorted to

$$\delta_{adS} x^\mu = -\hat{\Lambda}^\mu_{\mu'} - \Lambda^{\mu \nu} x_\nu - \Lambda^+_{\mu} x^\mu,$$

$$-\Lambda^+_{\mu} x^\mu + 2 x^\mu x_\nu \Lambda^{\nu} .$$

$$\delta_{adS} z = -z \left( \Lambda^+_{\mu} - 2 x_\mu \Lambda^{\mu} \right) .$$
On the other hand, the $p$-brane solutions of $D$-dimensional supergravity have a metric of the form

$$
\begin{align*}
    ds^2 &= H_{\text{brane}}^{-\frac{2}{d}} dX_m^2 + H_{\text{brane}}^\frac{2}{\tilde{d}} dX_{m'}^2 ; \quad d = p + 1 ; \quad \tilde{d} \equiv D - p - 3 \\
    H_{\text{brane}} &= 1 + \left( \frac{R}{r^d} \right)^\frac{d}{\tilde{d}} ; \quad r^2 = X^m X^{m'} ,
\end{align*}
$$

where $m = 0, 1, \ldots p$ denotes the directions along the brane, and $m' = 1, \ldots D - p - 1$ those orthogonal to the brane. The solutions describe $N$ coincident branes, and the parameter $R$ is proportional to $N^{1/d}$. When the constant 1 is neglected in the expression for the harmonic function in the transverse coordinates $H$, one obtains an $\text{adS}_{d+1}$ metric. Indeed then

$$
    ds^2_{\text{hor}} = \left( \frac{r^d}{R} \right)^\frac{d}{\tilde{d}} dX_m^2 + \left( \frac{R}{r^d} \right)^2 dr^2 + R^2 d^2 \Omega ,
$$

and identifying $z$ in Eq. 18 with $\left( \frac{R}{r^d} \right)^{\frac{d}{\tilde{d}}}$ we find that $(X^m, r)$ is a $(d + 1)$-dimensional adS space and the remainder is a $(\tilde{d} + 1)$-sphere.

The described limit can be seen in 3 different ways:

- We can see the brane solution as an interpolation between the asymptotically flat region where $H = 1$, and a near horizon anti-de Sitter geometry where $H = \left( \frac{R}{r^d} \right)^d$.
- The limit can also be seen as a large $N$ (many branes solution) limit.
  This will correspond in the field theory to large $N$ for the $SU(N)$ gauge theory.
- There is a special duality transformation that removes the constant.

This mechanism applies in various situations. We give in Table 3 cases where the manifold is always of the form $\text{adS}_{d+1} \times S^{\tilde{d}+1}$. The isometry groups are thus $SO(d, 2) \times SO(\tilde{d}+2)$. For later convenience we mention also $w \equiv d/\tilde{d}$. E.g. the self-dual string can also be obtained from compactifying a 10-dimensional string theory on $\text{adS}_3 \times S^3 \times E_4$, where $E_4$ denotes an Euclidean space. The Tangerlini black hole has been discussed, together with related rotating black holes in 5 dimensions in $[\text{[5]}]$, where the mentioned superalgebra was obtained (the extra $SU(2)$ rotates the supercharges, it is in fact $D^2(2, 1, 0)$). We are not aware of a similar result for the magnetic string, but we conjecture the appearance of the superalgebra in the table. In many cases other manifolds than simple spheres appear in the compactification, see e.g. $[\text{[6]}]$. 


4 Bosonic world-volume theory

The world-volume theory is in general a sum of a Born-Infeld term, a Wess-Zumino term and an extra one in case of the M5 brane. We first concentrate on the bosonic part.

\[ S_{cl} = S_{BI} + S_{WZ} + S' \]

\[ S_{BI} = - \int d^d\sigma \sqrt{-\text{det} (g_{\mu\nu} + T_{\mu\nu})} \; ; \quad g_{\mu\nu}^\text{ind} = \partial_\mu X^M \partial_\nu X^N G_{MN} , \quad (23) \]

where \( G_{MN} \) is the metric on target space, solution of the supergravity theory. We use for the latter immediately the near-horizon form Eq. 22. Concentrating on the first three and the last case of Table 3,

\[
\begin{align*}
\text{RN or M2} & \quad T_{\mu\nu} = 0 \quad S' = 0 ; \\
\text{D3} & \quad T_{\mu\nu} = F_{\mu\nu} \quad S' = 0 ; \\
\text{M5} & \quad T_{\mu\nu} = i\mathcal{H}^*_{\mu\nu} \quad S' = \int d^6\sigma \mathcal{H}^{*\mu\nu}\mathcal{H}_{\mu\nu} . \quad (24)
\end{align*}
\]

In the case of M5 we use the Pasti-Sorokin-Tonin (PST) formulation [16] for describing self-dual tensors. That means that there is an auxiliary field \( a \) apart from the antisymmetric tensor \( B_{\mu\nu} \), and

\[
\begin{align*}
\mathcal{H}_{\mu\nu} &= \frac{u^\rho}{\sqrt{u^2}} \mathcal{H}_{\mu\nu\rho} ; \\
\mathcal{H}^*_{\mu\nu} &= \frac{u^\rho}{\sqrt{u^2}} \mathcal{H}^{*\mu\nu\rho} . \quad (25)
\end{align*}
\]

The field equations of \( B_{\mu\nu} \) and extra gauge invariances imply that \( a \) is a gauge degree of freedom and \( \mathcal{H}_{\mu\nu\rho} \) is self dual.

The input which we thus need for the world-volume action is \( G_{MN} \) and the Wess-Zumino term. The latter is related to the integral of the \((p+2)\)-field strength form of the p-brane. Its exact form can vary for different cases, see e.g.

\[
\begin{align*}
\text{M5} & : \int \Omega_7 \; ; \quad \Omega_7 = dX_0 \ldots dX_5 dH^{-1} ; \\
\text{D3} & : \int \Omega_5 + \tilde{\Omega}_5 \; ; \quad \Omega_5 = dX_0 \ldots dX_3 dH^{-1} ; \\
\text{BH} & : \int F \; ; \quad F = dX^0 dH^{-1} . \quad (26)
\end{align*}
\]

The isometries of the solution lead to rigid symmetries of the action. These are thus the \( SO(d+2) \) rotations and the adS isometries

\[
\begin{align*}
\delta_{\text{adS}}(\xi)X^m &= -\xi^m(X,r) = -\xi^m(X) - (wR)^2 \left( \frac{R}{r} \right)^{2/w} \Lambda^m_K \quad , \\
\delta_{\text{adS}}(\xi)X^{m'} &= w\Lambda_D(X)X^{m'} = w(\lambda_D - 2X^m\Lambda_K m)X^{m'} , \quad (27)
\end{align*}
\]
In the full theory there is also rigid supersymmetry, determined by the Killing spinors of the metric.

Note that in case of $d = 1$ or 2 infinite dimensional symmetries exist. In $\text{adS}_3$ they have been found as asymptotic geometries by Brown and Henneaux\(^{17}\). That means that these are not symmetries of the action, but rather between different geometries which have $\text{adS}_3$ as near-horizon limit. They form the Virasoro algebra of which $\text{SO}(2, 2) = \text{SU}(1, 1) \times \text{SU}(1, 1)$ is the finite dimensional subgroup. However, it has also been found that the action $\int d^2\sigma \sqrt{-\det(g_{\mu\nu} + F_{\mu\nu})}$ has an infinite symmetry group\(^{18}\). For any isometry with $h_M(X)$ as Killing vector, the action is invariant under

$$\delta X^M = h^M(X) \lambda(\mathcal{F}) ; \quad \mathcal{F} = -\frac{\epsilon^{\mu\nu} F_{\mu\nu}}{2 \sqrt{g}} ;$$

$$\delta V_\mu = -\lambda'(\mathcal{F}) \sqrt{g} (1 + \mathcal{F}^2) \epsilon_{\mu\nu} (\partial^\nu X^M) h_M(X) , \quad (28)$$

where $\lambda(\mathcal{F})$ is an arbitrary function which provides a Kač-Moody extension of the isometry group.

Furthermore there are the local symmetries: world-volume diffeomorphisms and its fermionic partner: the $\kappa$-symmetry, which we will discuss in section 7. Gauge fixing of the diffeomorphisms, e.g. by identifying the first $d$ space-time fields with the coordinates on the brane $X^\mu(\sigma) = \sigma^\mu$ leaves invariant linear combinations of the rigid isometries with local symmetries where the parameters of the latter are determined functions of the rigid parameters of the isometries and local fields.

$$\delta_C(\xi) = \delta_{\text{adS}}(\xi) + \delta_{\text{ld}}(\eta = \hat{\xi}) . \quad (29)$$

The result is that these remaining symmetries take the form of conformal transformations on the world-volume. The remaining scalar fields are $X^m$, which are scalars of Weyl weight $w$ in Table 3. There are extra parts in the special conformal transformations

$$\Lambda^\mu_K K_\mu \phi = \delta_{ld} \left( \eta^\mu = (wR)^2 \left( \frac{R}{r} \right)^{2/w} \Lambda^\mu_K \right) . \quad (30)$$

We will see below how a similar mechanism works for the fermionic sector, and the superconformal algebra appears where the $R$ symmetry group is provided by the isometry group of sphere.

5 Relativistic conformal mechanics

One may apply the above mechanism to get conformal symmetry in one dimension. This is the case $p = 0$ or $d = 1$ and $R = M$, the mass of a black hole
solution in $d = 4$, $N = 2$ pure supergravity. The solution near the horizon has the Bertotti-Robinson (BR) metric

$$ds^2 = -\left(\frac{r}{M}\right)^2 dt^2 + \left(\frac{M}{r}\right)^2 dr^2 + M^2 d^2 \Omega ; \quad W = \frac{Q}{M^2} r dt + P \cos \theta d \phi ,$$

(31)

where $(r, \theta, \phi)$ are polar coordinates around the black hole. $W_\mu$ is the graviphoton which has in this solution electric charge $Q$ and magnetic charge $P$, satisfying $P^2 + Q^2 = M^2$. The supergravity algebra of $d = 4$, $N = 2$ contains

$$\{ \bar{e}^2 Q_i, \bar{e}^2 Q_j \} = \bar{e}^2 \gamma^\mu \epsilon_{1i} \left( P_\mu - \omega_{\mu}{}^{ab} M_{ab} - W_\mu G - \bar{\psi}_\mu Q_k \right) + \bar{e}^2 \epsilon_{ij} \left( G + F_{ab} M_{ab} \right) + \bar{e}^2 \gamma_5 \epsilon_{ij} \tilde{F}_{ab} M_{ab} ,$$

(32)

where $G$ is the gauge transformation of the graviphoton, whose field strength is $F$, and $\tilde{F}$ is the dual of $F$. In the solution, the gravitino $\psi_\mu$ vanishes, and the non-zero values of spin-connection $\omega$, and gauge field $W_\mu$ conspire to promote the solution to a BPS state.

In the world line action one can introduce a WZ term with electric charge $q$ and magnetic charge $p$ for the particle in the background with charges $(Q, P)$

$$S = m S_{BI} + S_{WZ}$$

$$= -m \int d\tau \sqrt{-g_{00}^{\text{ind}}} + q \int_{M_2} F + ip \int_{M_2} \tilde{F}$$

$$g_{00}^{\text{ind}} = \left( \frac{2M}{\rho} \right)^4 \left[ - \dot{\tau}^2 + \left( \frac{\rho}{2M} \dot{\rho} \right)^2 \right] + M^2 \left[ \dot{\theta}^2 + \sin^2 \theta \dot{\phi}^2 \right] ,$$

(33)

where $F$ is the pullback of $dW$ on a 2-dimensional manifold $M_2$ which has the worldline as its boundary, and we have used a new radial variable $\rho$ defined by

$$\frac{r}{M} = \left( \frac{2M}{\rho} \right)^2 .$$

(34)

We can identify the space-time $t$ with the world-line parameter $\tau$, as gauge choice of $\tau$-reparametrizations.

For pure electric particles in an electric background, $p = P = 0$, the Hamiltonian gets the interesting form

$$H = \left( \frac{2M}{\rho} \right)^2 \left[ \sqrt{m^2 + \frac{\rho^2 p_\rho^2}{\rho^2} + \frac{4L^2}{4M^2}} - q \right] ,$$

(35)

10
where \( L^2 = p_\theta^2 + \sin^{-2} \theta p_\phi^2 \) is the angular momentum. This Hamiltonian can be seen as \( H = -p_0 \) solving
\[
(p_0 - qW)^2 G^{00} + p_{m'} G^{m' \nu'} p_{\nu'} + m^2 = 0 .
\]
This thus describes a charged particle in the BR background. We may write also
\[
H = \frac{p_\rho^2}{2f} + \frac{mg}{\rho^2 f} ; \quad f = \frac{1}{2} \left[ \sqrt{m^2 + (\rho^2 p_\rho^2 + 4L^2)/4M^2 + q} \right] ; \quad mg = 2M^2(m^2 - q^2) + 2L^2 .
\]
The limit of large black hole mass
\[
M \to \infty ; \quad (m - q) \to 0 ; \quad M^2(m - q) \text{ fixed}
\]
gives \( f \to m \), and is the conformal mechanics of de Alfaro, Fubini and Furlan. We denote this as ‘non-relativistic conformal mechanics’, and the full action with finite black hole mass as relativistic conformal mechanics. Similarly the supersymmetric case generalises, which is a superconformal mechanics model, to a new relativistic superconformal mechanics.

The conformal invariance appears by the general mechanism. Changing the \( adS \) isometries by time-diffeomorphisms due to the gauge choice. The \( adS \) isometries (parameters \( a, b, c \)) and general coordinate transformation (parameter \( \xi(\tau) \)) act as
\[
\delta t = a + b t + c t^2 + c \frac{M^4}{r^2} + \xi(\tau) \dot{t} ; \quad \delta r = -r (b + 2ct) + \xi(\tau) \dot{r} .
\]
After the gauge choice \( t = \tau \) the reparametrization parameter is constrained:
\[
-\xi(\tau) = a + b \tau + c \tau^2 + c \frac{M^4}{r^2} = 0 ,
\]
and \( b \) takes the role of dilatation parameter. \( r \) transforms as a scalar with Weyl weight \( w = 1 \).

\section{Supergeometry}

So far, we neglected the fermionic sector. For world-volume theories of supersymmetric branes in a supersymmetric background, we need the complete supergeometry of the background. The complete interacting world-volume
theory indeed depends on the geometric superfields of the background to all orders in anticommuting coordinates $\theta$. Before discussing the world-volume action in section 7 we first explain the two methods to obtain these quantities in curved space: gauge completion or supergravity superspace and supercoset methods. The method called ‘gauge completion’ works in principle for generic backgrounds. One starts with comparing the component expressions with the general transformations in superspace (the superspace coordinates are $Z^\Lambda = \{X^M, \theta^A\}$, i.e. superdiffeomorphisms, local lorentz transformations and superspace gauge transformations with respectively parameters $\Xi^\Lambda$, $L_{MN}$) and $\Sigma$ (considering the case $p = 0$). They act on the fields as

$$
\Delta E_\Lambda^{\bar{\Lambda}} = \Xi^\Pi \partial_\Pi E_\Lambda^{\bar{\Lambda}} + (\partial_\Lambda \Xi^\Pi) E_\Pi^{\bar{\Lambda}} + E_\Lambda^{\bar{\Lambda}} L_\Sigma \Sigma,
$$

$$
\Delta A_\Lambda = \Xi^\Pi \partial_\Pi A_\Lambda + (\partial_\Lambda \Xi^\Pi) A_\Pi + \partial_\Lambda \Sigma,
$$

where barred indices denote those in flat coordinates. One compares also the algebra, first at the $\theta = 0$ level, defining

$$
E_\Lambda^{\bar{\Lambda}}|_{\theta=0} = \left( \begin{array}{c} \epsilon_M^M \\ \psi_M^{\bar{A}} \\ \bar{A} \end{array} \right); \quad A_M|_{\theta=0} = W_M; \quad A_{\bar{A}}|_{\theta=0} = 0
$$

$$
\Xi^M|_{\theta=0} = \xi^M; \quad \Xi^A|_{\theta=0} = \epsilon^{\bar{A}} A^{-1}_A.
$$

Note the choice of the spinor-spinor part of the supervielbein. The matrix appearing there is related to the Killing spinors of the solution, which in general can be written as

$$
\epsilon^{\bar{A}}(x) = \eta^{\bar{A}} A_\Lambda^{\bar{A}}(x).
$$

Here $\epsilon^{\bar{A}}(x)$ is the supersymmetry parameter, and the equation determining the Killing spinors the requirement that the transformation of the fermions should be zero in this background. This determines the local $\epsilon^{\bar{A}}(x)$ in terms of rigid spinor parameters $\eta^{\bar{A}}$. The solutions which we consider here, preserve as many rigid supersymmetries as there are local supersymmetries. The gauge Eq. 22 has been called the Killing spinor gauge.

One compares the transformations of the fields to obtain $E$ at order $\theta$. Then comparing the algebra leads to $\Xi$ at order $\theta$. Afterwards one can obtain expressions at order $\theta^2$, ... . Equivalent to gauge completion is to solve the supergravity torsions and curvature constraints to all orders in $\theta$, which can in principle be done also order by order in $\theta$. It should be clear that this is a tedious task for theories with high numbers of supersymmetry. Fortunately, for certain backgrounds there is a shortcut and one can solve the supergravity constraints to all orders in $\theta$ in closed form. The conditions on the background are that the gravitino vanishes and that the forms and dilaton are covariantly
constant. The near-horizon limit of the brane solutions given in Table 3 satisfy these conditions \(^{23}\). The supergravity superspace has been derived explicitly in \(^{24}\) for the M-branes and has been shown to be completely equivalent to the coset superspace results.

This brings us to the second method to obtain the geometric data of the background, i.e. the ‘supercoset approach’\(^ {25},^{26}\). One constructs the geometric connections for a supercoset \(G/H\). In a \(G\)-covariant construction one decomposes the generators of the superalgebra into bosonic (\(B_a\)) and fermionic (\(F_{\bar{A}}\)) ones. With this decomposition the covariant derivative becomes

\[
D = d + L^{\bar{a}} B_{\bar{a}} + L^{\bar{A}} F_{\bar{A}}. \tag{44}
\]

\(L^{\bar{a}}\) and \(L^{\bar{A}}\) are the bosonic and fermionic Cartan-forms and are given in terms of the supercoset representative \(G(Z)\)

\[
G(Z)^{-1} dG(Z) = L^{\bar{a}} B_{\bar{a}} + L^{\bar{A}} F_{\bar{A}}. \tag{45}
\]

The nilpotency \(D^2 = 0\) leads to Maurer–Cartan equations and determines \(dL\). In the following, we restrict the coset representative to the form

\[
G(Z) = g(X) e^{\Theta(X)^{\bar{A}} f_{\bar{A}}^{\bar{a}} B_{\bar{a}}}, \tag{47}
\]

where \(f\) are the structure constants of \(G\). The general solution reads in Killing spinor gauge, i.e. \(L^{\bar{A}}_{\bar{a}}|_{\theta=0} = A_{\bar{a}} B_{\bar{a}}\) and \(\Theta^{\bar{A}}(X) = \theta^{\bar{A}} A_{\bar{a}} B_{\bar{a}}(X)\),

\[
L^{\bar{A}} = \left( d\theta A_{\bar{a}} \frac{\sinh \mathcal{M}}{\mathcal{M}} \right)^{\bar{a}}, \quad L^{\bar{a}} = L^{\bar{a}}_0 + 2 \Theta_{\bar{AB}} \frac{\sinh^2 \mathcal{M}/2}{\mathcal{M}^2}, \tag{47}
\]

where \(L^{\bar{a}}_0\) is independent of \(\theta\) and \(d\theta\).

Then one goes about identifying vielbeine and spin connections by splitting the generators into ‘coset generators’ \((K_{\bar{M}}, K_{\bar{A}})\) and stability group generators, which generate \(H\). The Cartan-forms in the \(K\) directions are the supervielbeins \((E_{\bar{M}}, E_{\bar{A}})\), and the super spin connections are the Cartan-forms in the \(H\)-directions. For maximally supersymmetric backgrounds the fermionic generators are all contained in \(K\). In the supercoset approach the superforms \(\mathcal{F}\) are constructed by trial and error from the supervielbeins, by demanding that they are closed.
7 Supersymmetric world-volume theory

The world volume actions are again given by Eq. 23. In the Dirac-Born-Infeld term we use the induced metric
\[ g_{\mu\nu}^{\text{ind}} = \left( \partial_\mu Z^A E_A^M \right) \left( \partial_\nu Z^\Sigma E_\Sigma^\bar{N} \right) \eta_{MN}. \] (48)

Furthermore there are the superforms which define the Wess-Zumino term. For most of this section we will concentrate on the black hole solutions of \( d = 4, N = 2 \) supergravity leading to superconformal mechanics. In that case there is the super two-form related to the graviphoton \( F = dA(Z) \), and the Wess-Zumino term for pure electrically charged particles is
\[ S_{WZ} = q \int d\tau \dot{Z}^A A_A. \] (49)

E.g. in flat superspace we have in that case
\[ E_M^\bar{M} = \delta_M^\bar{M}; \quad E_{\alpha i}^\bar{M} = \frac{1}{2} \left( \gamma^M \theta_i \right)_\alpha; \quad A_M = 0; \quad A_{\alpha i} = \frac{1}{2} \epsilon_{ij} \theta^i_\alpha. \] (50)

Remark that the general spinor index \( A \) is here \( A = (\alpha i) \), where \( \alpha \) is a 4-dimensional spinor index, and \( i = 1, 2 \) are the \( N = 2 \) indices. The value of the supervielbein corresponds to the second part of Eq. 47 with
\[ L_0^\bar{M} = dX^\bar{M}; \quad M = 0; \quad A = 1; \quad f_{(\alpha i)(\beta j)}^\bar{M} = \delta_{ij} \left( C_{\gamma}^\bar{M} \right)^{\alpha \bar{\beta}}. \] (51)

This leads to
\[ g_{\tau \bar{\tau}}^{\text{ind}} = \left( \dot{X}^M - \frac{i}{2} \bar{\theta} \Gamma^M \dot{\theta} \right) \left( \dot{X}^N - \frac{i}{2} \bar{\theta} \Gamma^N \dot{\theta} \right) \eta_{MN} \]
\[ S_{WZ} = -\frac{Q}{2} \int d\tau \theta \dot{\epsilon}_{ij} \theta^j. \] (52)

The \( \text{adS} \) solutions preserve rigid supersymmetry. The Killing spinors (solutions of \( \delta \psi_\mu = 0 \) in this background) are
\[ \epsilon^i = M \left( \frac{r}{M} \right)^{1/2} \eta^i_+ + \left( \frac{r}{M} \right)^{1/2} \left( \eta^i_+ - t \gamma_0 \eta^i_- \right), \] (53)
where spinors are split in \( 2 \times 4 \) real spinors as
\[ \eta^i_\pm = P_{\pm} \eta^i = \frac{1}{2} \left( \eta^i \pm \frac{1}{M} (Q + i \gamma_5 P) \varepsilon^{ij} \gamma_0 \eta^j \right), \] (54)
\( \eta^i_{\pm} \) depend only on \( X^\hat{m} = (X^\theta, X^\phi) \) and are Killing spinors of sphere

\[
\nabla_{\hat{m}} \eta^i_{\pm}(\theta, \phi) = \pm \frac{1}{2M} \gamma_{r} \gamma_{\hat{m}} \eta^i_{\pm}(\theta, \phi)
\]

(55)

There are 4 solutions for each sign \( \pm \). The commutators of these give the two translations and rotation in \( adS_2 \) (in \( (t,r) \)) and \( SO(3) \) transformations of the 2-sphere \( (\theta, \phi) \). These killing spinors provide us with the matrix \( A \) defined in Eq. 43. The relevant supercoset to construct is

\[
\frac{SU(1,1|2)}{U(1) \times U(1)}
\]

(56)

and yields the supergeometry.

Apart from rigid supersymmetries, i.e. the superisometries of the background, there is the counterpart of world-volume diffeomorphisms, which is the (local) \( \kappa \)-symmetry. It acts on the fermionic components of superspace as

\[
\delta_\kappa \theta = (1 + \Gamma) \kappa ,
\]

(57)

where \( \Gamma \) is a complicated matrix in spinor space, function of the world-volume fields, such that \( \Gamma^2 = \mathbf{1} \) and \( \text{Tr} \Gamma = 0 \). For a particular value, e.g. on a classical solution, the operation in Eq. 57 is thus a projection matrix. In the black hole case \( \kappa \)-symmetry requires \( q^2 + p^2 = m^2 \) and

\[
\Gamma = \frac{1}{\sqrt{-g_{00}}} \varepsilon^{ij} \gamma_{kl} E^M .
\]

(58)

At 'classical values', \( t = \tau \) and \( r, \theta, \phi \) constant and vanishing fermions, and for only electrically charged black hole \((P = 0 \text{ and } Q = M)\)

\[
\Gamma_{cl} = \varepsilon^{ij} \gamma_0 ; \quad 1 + \Gamma_{cl} = 2P_-.\]

(59)

\( \kappa \) is a reducible symmetry, in the sense that if \( \kappa = (1 - \Gamma) \kappa' \) in Eq. 57, it does not contribute to the transformations, i.e. \( \kappa' \) is a zero mode. One can choose an irreducible \( \kappa \) symmetry: in our case, this is obtained if we demand \( P_+ \kappa = 0 \). E.g. for the M5 brane, the analogon is a chiral projection on \( \kappa \). Correspondingly gauge fixing can be done by a chiral condition on \( \theta \). Indeed the expansion of Eq. 57 around the classical solution is \( \delta_\kappa \theta = \mathcal{P}_- \kappa + \ldots \), such that we can gauge fix the \( \kappa \) symmetry by imposing

\[
\mathcal{P}_- \theta = 0 .
\]

(60)
The remaining symmetry is determined by \((\delta_\kappa + \delta_\eta)P - \theta = 0\), which can be solved for \(\kappa\) in terms of \(\eta\). This leads to a modification of the supersymmetry transformations, similar to what happened in the bosonic case.

The result is a conformal supersymmetry: where \(\eta_+\) takes role of \(Q\)-supersymmetry and \(\eta_-\) of \(S\)-supersymmetry. A similar mechanism works also for other brane backgrounds and world-volumes. At the end the super-\(adS_{d+1}\) is thus deformed to a super\(Conf_d\).

8 Linearised action with superconformal symmetry

The world-volume action can also be considered in a flat background. As above, after gauge fixing \(X^m = \sigma^m\), the remaining bosonic fields are \(X'^m\) and possibly gauge fields. In line with the theme of this workshop, we take as an example the \(M5\) theory, arising from \(D = 11\) dimensions. The remaining scalars are thus the coordinates of the 5 dimensions perpendicular to the brane, and the corresponding rotations are \(SO(5) = Sp(4)\), which will be the \(R\)-symmetry group. There is further an antisymmetric tensor \(B_{\mu\nu}\) on the world-volume. On the fermionic side half of the fermionic superspace coordinates disappear in the \(\kappa\) gauge fixing, the other half remains as fermionic fields on the world-volume. In the \(M5\) example we start from 11 dimensions, with a 32 component spinor \(\theta\). This is split in chiral and anti-chiral spinors in 6 dimensions. One chirality is put to zero by the gauge condition, and the other chirality are 4 \(\times\) 4 spinors \(\lambda_i\): 4-components spinors of the 6-dimensional theory, in a 4 of \(SO(5) = Sp(4)\). This gives the content of a \((0,2)\) tensor multiplet in 6 dimensions, and we have used again the auxiliary field \(a\) of the PST formulation as in Eq. 25. If we linearise the action in \(X'^m\), \(B_{\mu\nu}\) and \(\lambda_i\),

\[
S_{\text{lin}} = \int d^6x \left[ -\frac{1}{2} H_{\mu
u\rho} H^{\mu\nu\rho} - \frac{1}{2} \partial_\mu X'^m \cdot \partial^\mu X'^m + 2 \bar{\lambda} \gamma^i \lambda_i \right], \tag{61}
\]

it turns out that there is again conformal symmetry. We find ordinary supersymmetries with left-handed parameter \(\epsilon\) and special supersymmetries with right-handed \(\eta\)

\[
\begin{align*}
\delta X'^m & = - 2 \epsilon(x) \gamma^m \lambda_i, \\
\delta \lambda & = \frac{1}{2} (\bar{\lambda} X'^m) \gamma_m \epsilon(x) - \frac{1}{2} h^+_{\mu\nu\rho} \gamma^{\mu\nu\rho} \epsilon(x) + 2 X'^m \gamma_m \eta \\
\delta B_{\mu\nu} & = -2 \epsilon(x) \gamma_{\mu\nu} \lambda \; ; \; \delta a = 0 \\
\h^+_{\mu\nu\rho} & = \frac{1}{4} H_{\mu\nu\rho} - \frac{3}{2} \epsilon^{[\mu} H_{\nu\rho]} \; ,
\end{align*}
\]

with \(\epsilon(x) = \epsilon + \gamma_\mu x^\mu \eta\). The fields have Weyl weights

\[
\begin{align*}
w(X'^m) & = w(B_{\mu\nu}) = 2 \; ; \; w(a) = 0 \; ; \; w(\lambda) = \frac{5}{2} \; . \tag{62}
\end{align*}
\]
The first one is in accordance with Table 3. The algebra is the $N = 4$ superconformal algebra $OSp(8^*|4)$ as in Table 3.

This occurrence of the superconformal symmetry can be understood from expanding the one in $adS$ background. The latter is for small $R$ (or big $r$):

$$S_{M5} = S_{lin} + O \left( \frac{R}{r} \right)^3 \text{ (higher derivative terms)}.$$  \hfill (63)

This mechanism works as well e.g. for $M2$, where it gives the $N = 8$ conformal scalar multiplet in 3 dimensions, as for $D3$ where it leads to the $N = 4$ conformal vector multiplet in 4 dimensions.

9 Conclusions

We spelled out the procedure establishing superconformal symmetry of the gauge-fixed brane actions, starting from a background with $adS_{d+1} \times S^{\tilde{d}+1}$ geometry. The symmetries appear in the classical brane actions before gauge-fixing local diffeomorphism and $\kappa$-symmetry, and in the process of gauge-fixing of the latter, these rigid symmetries become superconformal transformations.

This symmetry also appears in a flat background, i.e. $ISO(D - 1,1)$ invariant, after gauge fixing and linearising, which is equivalent to the leading terms of an expansion in $R/r$ or dropping higher derivative terms of the action in $adS$ background. For $M5$ one obtains a superconformal tensor multiplet in 6 dimensions, including a conformal realization of the PST action.

We applied the same procedure in $adS$ background to generalize the ‘non-relativistic’ superconformal mechanics of Akulov and Pashnev, and Fubini and Rabinovici, to a ‘relativistic superconformal mechanics’, having the former as limit $M \to \infty$.

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Table 1: Lie superalgebras of classical type. For the real forms of $SU(m|n)$, the one-dimensional subalgebra of the bosonic algebra is not part of the irreducible algebra. Furthermore, in that case there are subalgebras obtained from projection of those mentioned here with only one factor $SU(n)$, $S\ell(n)$, $SU^*(n)$ or $SU(n-p,p)$ as bosonic algebra.

| Name | Range | Bosonic algebra | Defining repres. | Number of generators |
|------|-------|----------------|-----------------|---------------------|
| $SU(m|n)$ | $m \geq 2$ | $SU(m) \oplus SU(n)$ | $(m,\bar{n})$ | $m^2 + n^2 - 1$, $2mn$ |
| $m \neq n$ | | $\oplus U(1)$ | $(\bar{m},n)$ | $2(m^2 - 1), 2m^2$ |
| $m = n$ | | no $U(1)$ | | |
| $S\ell(m|n)$ | | $S\ell(m) \oplus S\ell(n)$ | | |
| $SU(m-p,p|n-q,q)$ | | $SU(m-p,p) \oplus SU(n-q,q) \oplus U(1)$ | | |
| $SU^*(2m|2n)$ | | $SU^*(2m) \oplus SU^*(2n)$ | | |
| $S\ell'(n|n)$ | | $S\ell(n,\bar{1})$ | | |
| $OSp(m|n)$ | $m \geq 1$ | $SO(m) \oplus Sp(n)$ | $(m,n)$ | $\frac{1}{2}(m^2 - m + n^2 + n), mn$ |
| $n = 2, 4, \ldots$ | | | | |
| $OSp(m-p,p|n)$ | | $SO(m-p,p) \oplus Sp(n)$ | | |
| $OSp(m^*|n-q,q)$ | | $SO^*(m) \oplus USp(n+q,q)$ | | |
| $D(2,1,\alpha)$ | $0 < \alpha \leq 1$ | $SO(4) \oplus S\ell(2)$ | $(2,2,2)$ | 9, 8 |
| $D^p(2,1,\alpha)$ | | $SO(4-p,p) \oplus S\ell(2)$ | | |
| $F(4)$ | | $SO(7) \oplus S\ell(2)$ | $(8,2)$ | 21, 16 |
| $F^p(4)$ | | $SO(7-p,p) \oplus S\ell(2)$ | | |
| $G^p(3)$ | | $SO(7-p,2) \oplus SU(2)$ | | |
| $G_{p}(3)$ | | $G_{2p} \oplus S\ell(2)$ | | |
| $P(m-1)$ | $m \geq 3$ | $S\ell(m)$ | $(m \otimes m)$ | $m^2 - 1, m^2$ |
| $Q(m-1)$ | $m \geq 3$ | $SU(m)$ | Adjoint | $m^2 - 1, m^2 - 1$ |
| $Q((m-1)^*)$ | | $SU^*(m)$ | | |
| $UQ(p,m-1-p)$ | | $SU(p,m-p)$ | | |
Table 2: Super \( \text{adS}_{d+1} \) or \( \text{conf}_d \) algebras.

| \( d \) | superalgebra | \( R \) | number of fermionic |
|---|---|---|---|
| 1 | \( \text{OSp}(N|2) \) | \( \text{O}(N) \) | 2N |
| | \( \text{SU}(N|1,1) \) | \( \text{SU}(N) \times U(1) \) for \( N \neq 2 \) | 4N |
| | \( \text{SU}(2|1,1) \) | \( \text{SU}(2) \) | 8 |
| | \( \text{OSp}(4^*|2N) \) | \( \text{SU}(2) \times USp(2N) \) | 8N |
| | \( G(3) \) | \( G_2 \) | 14 |
| | \( F^0(4) \) | \( \text{SO}(7) \) | 16 |
| | \( D^0(2,1,\alpha) \) | \( \text{SU}(2) \times SU(2) \) | 8 |
| 3 | \( \text{OSp}(N|4) \) | \( \text{SO}(N) \) | 4N |
| 4 | \( \text{SU}(2,2|2N) \) | \( \text{SU}(N) \times U(1) \) for \( N \neq 4 \) | 8N |
| | \( \text{SU}(2,2|4) \) | \( \text{SU}(4) \) | 32 |
| 5 | \( F^2(4) \) | \( \text{SU}(2) \) | 16 |
| 6 | \( \text{OSp}(8^*|N) \) | \( USp(N) \) (\( N \) even) | 8N |

Table 3: Brane solutions with horizon geometry \( \text{adS}_{d+1} \times S^{d+1} \), where \( d = p + 1 \). The number \( w = d/d \) is the Weyl weight of the scalars in the conformal theory.

| \( D \) | \( d \) | \( d \) | \( w \) | superalgebra |
|---|---|---|---|---|
| M5 | 11 | 6 | 3 | 2 | \( \text{OSp}(8^*|4) \) |
| M2 | 11 | 3 | 6 | 1 \( \frac{1}{2} \) | \( \text{OSp}(8|4) \) |
| D3 | 10 | 4 | 4 | 1 | \( \text{SU}(2,2|4) \) |
| Self-dual string (D1+D5) | 6 | 2 | 2 | 1 | \( \text{SU}(1,1|2) \oplus SU(1,1|2) \) |
| Magnetic string | 5 | 2 | 1 | 2 | \( \text{SU}(1,1|2) \oplus SU(1,1|1) \) |
| Tangerlini black hole | 5 | 1 | 2 | 1 | \( \text{SU}(1,1|2) \oplus SU(2) \) |
| Reissner-Nordström black hole | 4 | 1 | 1 | 1 | \( \text{SU}(1,1|2) \) |