Limits on the Dipole Moments of the $\tau$-Lepton via the Process $e^+e^- \rightarrow \tau^+\tau^-\gamma$ in a Left-Right Symmetric Model

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Abstract

Limits on the anomalous magnetic moment and the electric dipole moment of the $\tau$ lepton are calculated through the reaction $e^+e^- \rightarrow \tau^+\tau^-\gamma$ at the $Z_1$-pole and in the framework of a left-right symmetric model. The results are based on the recent data reported by the L3 Collaboration at CERN LEP.

Due to the stringent limit of the model mixing angle $\phi$, the effect of this angle on the dipole moments is quite small.

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I. INTRODUCTION

In the Standard Model (SM) [1], the electromagnetic interactions of each of the three charged leptons are identical. However, there is no experimentally verified explanation for the existence of three generations of leptons nor for why they have such different masses. New insight might be forthcoming if the leptons were observed to have a substructure which could manifest itself in deviations from the SM values for the anomalous magnetic or electric dipole moments. The anomalous moments for the electron and muon have been measured with very high precision [2] compared to those of tau for which there are only upper limits [3–7].

In general, a photon may couple to a tau through its electric charge, magnetic dipole moment or electric dipole moment. This coupling may be parametrised using a matrix element in which the usual $\gamma^\mu$ is replaced by a more general Lorentz-invariant form

$$\Gamma^\mu = F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i}{2m_\tau}\sigma^{\mu\nu}q_\nu + F_3(q^2)\sigma^{\mu\nu}\gamma^5q_\nu,$$

where $m_\tau$ is the mass of the $\tau$ lepton and $q = p' - p$ is the momentum transfer. The $q^2$-dependent form-factors, $F_i(q^2)$, have familiar interpretations for $q^2 = 0$: $F_1(0) \equiv Q_\tau$ is the electric charge; $F_2(0) \equiv a_\tau = (g-2)/2$ is the anomalous magnetic moment; and $F_3 \equiv d_\tau/Q_\tau$, where $d_\tau$ is the electric dipole moment.

The analysis of radiative $\tau$ pair production provides a means to determine the anomalous magnetic and electric dipole moments of the $\tau$ lepton at $q^2 = 0$. An anomalous magnetic dipole moment at $q^2 = 0$ ($F_2(0)$) or an electric dipole moment ($F_3(0)$) affects the total cross section for the process $e^+e^- \rightarrow \tau^+\tau^-\gamma$ as well as the shape of energy and angular distributions of the three final state particles [5,8,9]. Previous experimental limits [5–7,10,11] on $F_2(0)$ and $F_3(0)$ have been based on approximate calculations of the $e^+e^- \rightarrow \tau^+\tau^-\gamma$ cross section and photon energy distribution.

The first direct determination of the anomalous magnetic moment of the $\tau$ lepton, i.e. of $F_2(0)$, is due to Grifols and Méndez using L3 data [5]. They derived a limit for $F_2(q^2) =$
0) \leq 0.11 and $F_{EDM}(q^2 = 0) \leq 6 \times 10^{-16}ecm$ at $q^2 = 0$. More recently, Escribano and Massó [4] have used electroweak data to find $d_{\tau} \leq 1.1 \times 10^{-17}ecm$ and $-0.004 \leq a_{\tau} \leq 0.006$ at the 2\sigma confidence level.

On the $Z_1$ peak, where a large number of $Z_1$ events are collected at $e^+e^-$ colliders, one may hope to constrain or eventually measure the anomalous magnetic moment and electric dipole moment of the $\tau$ by selecting $\tau^+\tau^-$ events accompanied by a hard photon. The Feynman diagrams which give the most important contribution to the cross section are shown in Fig. 1.

Our aim in this paper is to analyze the reaction $e^+e^- \rightarrow \tau^+\tau^-\gamma$. We use recent data collected with the L3 and OPAL detector at CERN LEP [6,7] in the $Z_1$ boson resonance. The analysis is carried out in the context of a left-right symmetric model [12–14] and we attribute a magnetic moment and an electric dipole moment to the tau lepton. Processes measured in the resonance serve to set limits on the tau magnetic moment and electric dipole moment. We take advantage of this fact to set limits for $a_{\tau}$ and $d_{\tau}$ for different values of the mixing angle $\phi$ [15–17], which is consistent with other constraints previously reported [4–7].

We do our analysis on the $Z_1$ peak ($s = M_{Z_1}^2$). Our results are therefore independent of the mass of the additional heavy $Z_2$ gauge boson which appears in these kind of models. Thus, we have the mixing angle $\phi$ between the left and the right bosons as the only additional parameter apart from the SM parameters.

This paper is organized as follows: In Sect. II we describe the model with the Higgs sector having two doublets and one bidoublet. In Sect. III we present the calculus of the process $e^+e^- \rightarrow \tau^+\tau^-\gamma$. In Sect. IV we make the numerical computations. Finally, we summarize our results in Sect. V.

II. THE LEFT-RIGHT SYMMETRIC MODEL (LRSM)

We consider a Left-Right Symmetric Model (LRSM) consisting of one bidoublet $\Phi$ and two doublets $\chi_L, \chi_R$. The vacuum expectation values of $\chi_L, \chi_R$ break the gauge symmetry to
give mass to the left and right heavy gauge bosons. This is the origin of the parity violation at low energies \[12\] \(i.e.,\) at energies produced in actual accelerators. The Lagrangian for the Higgs sector of the LRSM is \[13\]

\[
\mathcal{L}_{LRSM} = (D_{\mu}\chi_L)^\dagger(D^{\mu}\chi_L) + (D_{\mu}\chi_R)^\dagger(D^{\mu}\chi_R) + Tr(D_{\mu}\Phi)^\dagger(D^{\mu}\Phi) .
\] (1)

The covariant derivatives are written as

\[
D_{\mu}\chi_L = \partial_{\mu}\chi_L - \frac{1}{2}ig\tau \cdot W_{L}\chi_L - \frac{1}{2}ig' B\chi_L ,
\]
\[
D_{\mu}\chi_R = \partial_{\mu}\chi_R - \frac{1}{2}ig\tau \cdot W_{R}\chi_R - \frac{1}{2}ig' B\chi_R ,
\] (2)
\[
D_{\mu}\Phi = \partial_{\mu}\Phi - \frac{1}{2}ig(\tau \cdot W_{L}\Phi - \Phi \tau \cdot W_{R}) .
\]

There are seven gauge bosons in this model: the charged \(W_{L,R}^1, W_{L,R}^2\) and the neutral \(W_{L,R}^3, B\). The gauge couplings constants \(g_L\) and \(g_R\) of the \(SU(2)_L\) and \(SU(2)_R\) subgroups, respectively, are equal: \(g_L = g_R = g\), since manifest left-right symmetry is assumed \[18\]. \(g'\) is the gauge coupling for the \(U(1)\) group.

The transformation properties of the Higgs bosons under the group \(SU(2)_L \times SU(2)_R \times U(1)\) are \(\chi_L \sim (1/2, 0, 1)\), \(\chi_R \sim (0, 1/2, 1)\) and \(\Phi \sim (1/2, 1/2^*, 0)\). After spontaneous symmetry breaking, the ground states are of the form

\[
\langle \chi_L \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_L \end{pmatrix} , \quad \langle \chi_R \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v_R \end{pmatrix} , \quad \langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} k & 0 \\ 0 & k' \end{pmatrix} ,
\] (3)

which break the symmetry group to form the \(U(1)_{em}\), giving mass to the gauge bosons and fermions with the photon remaining massless. In Eq. (3), \(v_L, v_R, k\) and \(k'\) are the vacuum expectation values. The part of the Lagrangian that contains the mass terms for the charged boson is

\[
\mathcal{L}_{mass}^C = (W_L^+ W_R^+) M^C \begin{pmatrix} W_L^- \\ W_R^- \end{pmatrix} ,
\] (4)

where \(W^\pm = \frac{1}{\sqrt{2}}(W^1 \mp W^2)\).

The mass matrix \(M^C\) is
\[ M^C = \frac{g^2}{4} \begin{pmatrix} v_L^2 + k^2 + k'^2 & -2kk' \\ -2kk' & v_R^2 + k^2 + k'^2 \end{pmatrix}. \] (5)

This matrix is diagonalized by an orthogonal transformation parametrized \([18]\) by the angle \(\zeta\). This angle has a very small value because of the hyperon \(\beta\) decay data \([19]\).

Similarly, the part of the Lagrangian that contains the mass terms for the neutral bosons is

\[
L_{\text{mass}}^N = \frac{1}{8}(W_3^L W_3^R B)M^N \begin{pmatrix} W_3^L \\ W_3^R \\ B \end{pmatrix},
\] (6)

where the matrix \(M^N\) is given by

\[
M^N = \frac{1}{4} \begin{pmatrix} g^2(v_L^2 + k^2 + k'^2) & -g^2(k^2 + k'^2) & -gg'v_L^2 \\ -g^2(k^2 + k'^2) & g^2(v_R^2 + k^2 + k'^2) & -gg'v_R^2 \\ -gg'v_L^2 & -gg'v_R^2 & g'^2(v_L^2 + v_R^2) \end{pmatrix}. \] (7)

Since the process \(e^+e^- \rightarrow \nu\bar{\nu}\gamma\) is neutral, we center our attention on the mass terms of the Lagrangian for the neutral sector as shown in Eq. (6).

The matrix \(M^N\) for the neutral gauge bosons is diagonalized by an orthogonal transformation which can be written in terms of the angles \(\theta_W\) and \(\phi\) \([20]\)

\[
U^N = \begin{pmatrix} c_Wc_\phi - s_Wt_Wc_\phi - r_Ws_\phi/c_W & t_W(s_\phi - r_Wc_\phi) \\ c_Ws_\phi - s_Wt_Ws_\phi + r_Wc_\phi/c_W & -t_W(c_\phi + r_Ws_\phi) \\ s_W & s_W & r_W \end{pmatrix}, \] (8)

where \(c_W = \cos \theta_W\), \(s_W = \sin \theta_W\), \(t_W = \tan \theta_W\) and \(r_W = \sqrt{\cos 2\theta_W}\), and \(\theta_W\) is the electroweak mixing angle. Here, \(c_\phi = \cos \phi\) and \(s_\phi = \sin \phi\). The angle \(\phi\) is considered as the angle that mixes the left and right handed neutral gauge bosons \(W_3^L,R\). The expression that relates the left and right handed neutral gauge bosons \(W_3^L,R\) and \(B\) with the physical bosons \(Z_1, Z_2\) and the photon is:
The diagonalization of (5) and (7) gives the mass of the charged \( W_{1,2}^{\pm} \) and neutral \( Z_{1,2} \) physical fields:

\[
M_{W_{1,2}}^2 = \frac{g^2}{8} [v_L^2 + v_R^2 + 2(k^2 + k^{'2}) \mp \sqrt{(v_R^2 - v_L^2)^2 + 16(kk')^2}],
\]

(10)

\[
M_{Z_{1,2}}^2 = B \mp \sqrt{B^2 - 4C},
\]

(11)

respectively, with

\[
B = \frac{1}{8} [(g^2 + g'^2)(v_L^2 + v_R^2) + 2g^2(k^2 + k'^2)],
\]

\[
C = \frac{1}{64} g^2(g^2 + 2g'^2)[v_L^2 v_R^2 + (k^2 + k'^2)(v_L^2 + v_R^2)].
\]

Taking into account that \( M_{W_2}^2 \gg M_{W_1}^2 \), we conclude from the expressions for the masses of \( M_{Z_1} \) and \( M_{Z_2} \), that the relation \( M_{W_1}^2 = M_{Z_1}^2 \cos^2 \theta_W \) still holds in this model.

From the Lagrangian of the LRSM, we extract the terms for the neutral interaction of a fermion with the gauge bosons \( W_{3L,R} \) and \( B \):

\[
\mathcal{L}_{int}^N = g (J_{3L}^3 W_3^3 + J_{3R}^3 W_3^3) + \frac{g'}{2} J_Y B.
\]

(12)

Specifically, the Lagrangian interaction for \( Z_1 \rightarrow f \bar{f} \) [21] is

\[
\mathcal{L}_{int}^N = \frac{g}{c_W} Z_1 [(c_\phi - \frac{s^2_W}{r_W}s_\phi)J_L - \frac{c^2_W}{r_W}s_\phi J_R],
\]

(13)

where the left (right) current for the fermions are

\[
J_{L,R} = J_{3L,R}^3 - \sin^2 \theta_W J_{em}
\]

and
\[ J_{em} = J_L^3 + J_R^3 + \frac{1}{2} J_Y \]

is the electromagnetic current. From (13), we find that the amplitude \( \mathcal{M} \) for the decay of the \( Z_1 \) boson with polarization \( \epsilon^\lambda \) into a fermion-antifermion pair is:

\[ \mathcal{M} = \frac{g}{c_W} [\bar{u} \gamma_\mu \frac{1}{2} (ag_Y - bg_A \gamma_5) v] \epsilon^\lambda, \quad (14) \]

with

\[ a = c_\phi - \frac{s_\phi}{r_W} \quad \text{and} \quad b = c_\phi + r_W s_\phi, \quad (15) \]

where \( \phi \) is the mixing parameter of the LRSM [15,16].

In the following section, we make the calculations for the reaction \( e^+e^- \rightarrow \tau^+\tau^-\gamma \) by using the expression (14) for the transition amplitude.

**III. THE TOTAL CROSS SECTION**

We calculate the total cross section of the process \( e^+e^- \rightarrow \tau^+\tau^-\gamma \) using the Breit-Wigner resonance form [22,23]:

\[ \sigma(e^+e^- \rightarrow \tau^+\tau^-\gamma) = \frac{4\pi(2J + 1) \Gamma_{e^+e^-} \Gamma_{\tau^+\tau^-\gamma}}{(s - M_{Z_1}^2)^2 + M_{Z_1}^2 \Gamma_{Z_1}^2}, \quad (16) \]

where \( \Gamma_{e^+e^-} \) is the decay rate of \( Z_1 \) to the channel \( Z_1 \rightarrow e^+e^- \) and \( \Gamma_{\tau^+\tau^-\gamma} \) is the decay rate of \( Z_1 \) to the channel \( Z_1 \rightarrow \tau^+\tau^-\gamma \).

In the next subsection, we calculate the widths of Eq. (16).

**A. Width of \( Z_1 \rightarrow e^+e^- \)**

We now calculate the total width of the reaction

\[ Z_1 \rightarrow e^+e^-, \quad (17) \]

in the context of the left-right symmetric model which is described in Section II.
The expression for the total width of the process \( Z_1 \to e^+ e^- \), according to the diagrams depicted in Fig. 1 and using the expression for the amplitude given in Eq. (14), is:

\[
\Gamma(Z_1 \to e^+ e^-) = \frac{G_F M_{Z_1}^2}{6\pi \sqrt{2}} \sqrt{1 - 4\eta[a^2 (g_V^e)^2 (1 + 2\eta) + b^2 (g_A^e)^2 (1 - 4\eta)]},
\]

where \( \eta = m_e^2 / M_{Z_1}^2 \).

We take \( g_V^e = -\frac{1}{2} + 2\sin^2 \theta_W \) and \( g_A^e = -\frac{1}{2} \), from the experimental data [22] and \( m_e = 0 \) so that the total width is

\[
\Gamma(Z_1 \to e^+ e^-) = \frac{\alpha M_{Z_1}}{24} \left[ \frac{\frac{1}{2} (a^2 + b^2) - 4a^2 x_W + 8a^2 x_W}{x_W (1 - x_W)} \right],
\]

where \( x_W = \sin^2 \theta_W \) and \( \alpha = e^2 / 4\pi \) is the fine structure constant.

**B. Width of \( Z_1 \to \tau^+ \tau^- \gamma \)**

The expression for the Feynman amplitude \( \mathcal{M} \) of the process \( Z_1 \to \tau^+ \tau^- \gamma \) is due only to the \( Z_1 \) boson exchange, as shown in the diagrams in Fig. 1. We use the expression for the amplitude given in Eq. (14) and assume that a tau lepton is characterized by the following phenomenological parameters: a charge radius \( \langle r^2 \rangle \), a magnetic moment \( a_\tau \), and an electric dipole moment \( d_\tau \). Therefore, the expression for the Feynman amplitude \( \mathcal{M} \) of the process \( Z_1 \to \tau^+ \tau^- \gamma \) is given by

\[
\mathcal{M}_1 = \left[ \bar{u}(p_{\tau^-}) \right] \Gamma^\alpha \frac{i}{(\not{\ell} - m_\tau)} \gamma^\beta (ag_V^{\tau} - bg_A^{\tau} \gamma_5) v(p_{\tau^+})] \epsilon^\alpha_\beta (Z_1)
\]

and

\[
\mathcal{M}_2 = \left[ \bar{u}(p_{\tau^-}) \right] \frac{i g}{2 \cos \theta_W} \gamma^\beta (ag_V^{\tau} - bg_A^{\tau} \gamma_5) \frac{i}{(\not{k} - m_\tau)} \Gamma^\alpha v(p_{\tau^+})] \epsilon^\alpha_\beta (Z_1),
\]

where

\[
\Gamma^\alpha = e F_1 (q^2) \gamma^\alpha + \frac{ie}{2m_\tau} F_2 (q^2) \sigma^{\alpha \mu} q_\mu + e F_3 (q^2) \gamma_5 \sigma^{\alpha \mu} q_\mu
\]

is the tau electromagnetic vertex, \( e \) is the charge of the electron, \( q^\mu \) is the photon momentum and \( F_{1,2,3}(q^2) \) are the electromagnetic form factors of the tau which correspond to charge
radius, magnetic moment and electric dipole moment, respectively, at \( q^2 = 0 \) [4,5]. \( \epsilon_\alpha^\lambda \) and \( \epsilon_\beta^\lambda \) are the polarization vectors of photon and of the boson \( Z_1 \), respectively. \( l \) (\( k \)) stands by
the momentum of the virtual tau (antitau), and the coupling constants \( a \) and \( b \) are given in
the Eq. (15).

After applying some of the trace theorems of the Dirac matrices and of sum and average
over the initial and final spin, the square of the matrix elements becomes

\[
\sum_s |M_T|^2 = \frac{g^2}{\cos^2 \theta_W} \left[ \frac{e^2 a^2}{4m^2_T} + d_T^2 \right] \left[ (a^2(g^r_V)^2 + b^2(g^r_A)^2)(s - 2\sqrt{s}E_{\gamma}) + b^2(g^r_A)^2 E_{\gamma}^2 \sin^2 \theta_{\gamma} \right]. \tag{23}
\]

Now that we know the square of the Eq. (23) transition amplitude, our final step is to
calculate the total width of \( Z_1 \rightarrow \tau^+ \tau^- \gamma \):

\[
\Gamma_{(Z_1 \rightarrow \tau^+ \tau^- \gamma)} = \int \frac{\alpha}{12\pi^2 m_{Z_1} x_W (1 - x_W)} \left[ \frac{e^2 a^2}{4m^2_T} + d_T^2 \right] \left[ (a^2(g^r_V)^2 + b^2(g^r_A)^2)(s - 2\sqrt{s}E_{\gamma}) + b^2(g^r_A)^2 E_{\gamma}^2 \sin^2 \theta_{\gamma} \right] E_{\gamma} dE_{\gamma} d \cos \theta_{\gamma}, \tag{24}
\]

where \( E_{\gamma} \) and \( \cos \theta_{\gamma} \) are the energy and scattering angle of the photon.

The substitution of (19) and (24) in (16) gives

\[
\sigma(e^+e^- \rightarrow \tau^+ \tau^- \gamma) = \int \frac{\alpha^2}{48\pi} \left[ \frac{e^2 a^2}{4m^2_T} + d_T^2 \right] \left[ \frac{x_W (1 - x_W)^2}{(s - M^2_{Z_1} + M^2_{Z_1} \Gamma^2_{Z_1})^2} \left( \frac{1}{2}(a^2 + b^2) - 4a^2 x_W + 8a^2 x_W^2 \right) (s - 2\sqrt{s}E_{\gamma}) + \frac{1}{2}b^2 E_{\gamma}^2 \sin^2 \theta_{\gamma} \right] E_{\gamma} dE_{\gamma} d \cos \theta_{\gamma}, \tag{25}
\]

where \( x_W \equiv \sin^2 \theta_W \).

After evaluating the limit when the mixing angle is \( \phi = 0 \), the expression for \( a \) and \( b \) is
reduced to \( a = b = 1 \) and Eq. (25) is reduced to the expression (4) given in Ref. [5].

IV. RESULTS

In practice, detector geometry imposes a cut on the photon polar angle with respect to
the electron direction, and further cuts must be applied on the photon energy and minimum
opening angle between the photon and tau in order to suppress background from tau decay
products. Therefore, to evaluate the integral of the total cross section as a function of mixing angle $\phi$, we require cuts on the photon angle and energy to avoid divergences when the integral is evaluated at the important intervals of each experiment. We integrate over $\cos \theta_\gamma$ from $-0.74$ to $0.74$ and $E_\gamma$ from $5$ GeV to $45.5$ GeV for various fixed values of the mixing angle $\phi = -0.009, -0.005, 0, 0.004$. Using the numerical values: $\sin^2 \theta_W = 0.2314$, $m_\tau = 1.776$ GeV, $M_{Z_1} = 91.187$ GeV, and $\Gamma_{Z_1} = 2.49$ GeV, we obtain the cross section $\sigma = \sigma(\phi, a_\tau, d_\tau)$.

For the mixing angle $\phi$ between $Z_1$ and $Z_2$, we use the reported data of M. Maya et al. [17]:

$$-9 \times 10^{-3} \leq \phi \leq 4 \times 10^{-3},$$

(26)

with a 90 % C. L. Other limits on the mixing angle $\phi$ reported in the literature are given in the Refs. [18,19].

As was discussed in Refs. [6,7], $N \approx \sigma(\phi, a_\tau, d_\tau)\mathcal{L}$. Using the Poisson statistic [6,24], we require that $N \approx \sigma(\phi, a_\tau, d_\tau)\mathcal{L}$ be less than 1559, with $\mathcal{L} = 100$ $pb^{-1}$, according to the data reported by the L3 Collaboration Ref. [6]. Taking this into consideration, we put a bound for the tau lepton magnetic moment as a function of the $\phi$ mixing parameter with $d_\tau = 0$. We show the value of this bound for values of the $\phi$ parameter in Tables 1 and 2.

| $\phi$ | $a_\tau$ | $d_\tau (10^{-16}$e$cm)$ |
|--------|---------|-----------------|
| -0.009 | 0.084   | 4.61            |
| -0.005 | 0.083   | 4.59            |
| 0      | 0.082   | 4.55            |
| 0.004  | 0.081   | 4.53            |

Table 1. Limits on the $a_\tau$ magnetic moment and $d_\tau$ electric dipole moment of the $\tau$-lepton for different values of the mixing angle $\phi$ before the $Z_1$ resonance, i.e., $s \approx M_{Z_1}^2$.

These results differ from the limits obtained in the references [6,7]. However, the derived limits in Table 1 could be improved by including data from the entire $Z_1$ resonance as shown...
in Table 2.

| $\phi$ | $a_\tau$ | $d_\tau(10^{-16}\text{ecm})$ |
|--------|---------|------------------------------|
| -0.009 | 0.06    | 3.27                         |
| -0.005 | 0.059   | 3.25                         |
| 0      | 0.058   | 3.22                         |
| 0.004  | 0.057   | 3.21                         |

Table 2. Limits on the $a_\tau$ magnetic moment and $d_\tau$ electric dipole moment of the $\tau$-lepton for different values of the mixing angle $\phi$ in the $Z_1$ resonance, $i.e.$, $s = M_{Z_1}^2$.

The above analysis and comments can readily be translated to the electric dipole moment of the $\tau$-lepton. The resulting limit for the electric dipole moment as a function of the $\phi$ mixing parameter is shown in Table 1.

The results in Table 2 for the electric dipole moment are in agreement with those found by the L3 Collaboration [6].

Fig. 2 shows the total cross section as a function of the mixing angle $\phi$ for the limits of the magnetic moment given in Tables 1, 2. We observe in Fig. 2 that for $\phi = 0$, we reproduce the data previously reported in the literature. Also, we observe that the total cross section increases constantly and reaches its maximum value for $\phi = 0.004$.

We end this section by plotting the differential cross section in Fig. 3 as a function of the photon energy versus the cosine of the opening angle between the photon and the beam direction ($\theta_\gamma$), for $\phi = 0.004$ and $a_\tau = 0.057$. We observe in this figure that the energy and angular distributions are consistent with those reported in the literature. In addition, the form of the distributions does not change significantly for the values $\phi$ and $a_\tau$ because $\phi$ and $a_\tau$ are very small in value, as shown in Table 2.
V. CONCLUSIONS

We have determined a limit on the magnetic moment and the electric dipole moment of the tau lepton in the framework of a left-right symmetric model as a function of the mixing angle $\phi$, as shown in Table 1 and Table 2.

In summary, we conclude that the estimated limit for the tau lepton magnetic moment and the electric dipole moment are almost independent of the experimental allowed values of the $\phi$ parameter of the model. In the limit $\phi = 0$, our bound takes the value previously reported in the literature [6,7].

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FIGURE CAPTIONS

Fig. 1 The Feynman diagrams contributing to the process $e^+e^- \rightarrow \tau^+\tau^-\gamma$ in a left-right symmetric model.

Fig. 2 The total cross section for $e^+e^- \rightarrow \tau^+\tau^-\gamma$ as a function of $\phi$ and $a_\tau$ (Tables 1, 2).

Fig. 3 The differential cross section for $e^+e^- \rightarrow \tau^+\tau^-\gamma$ as a function of $E_\gamma$ and $\cos\theta_\gamma$ for $\phi = 0.004$ and $a_\tau = 0.057$. 
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