SU(2N_F) SYMMETRY OF QCD AT HIGH TEMPERATURE AND ITS IMPLICATIONS*

L.YA. GLOZMAN

Institute for Physics, University of Graz
Universitätsplatz 5, 8010 Graz, Austria

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If above a critical temperature not only the SU(N_F)_L × SU(N_F)_R chiral symmetry of QCD but also the U(1)_A symmetry is restored, then the actual symmetry of the QCD correlation functions and observables is SU(2N_F). Such a symmetry prohibits existence of deconfined quarks and gluons. Hence, QCD at high temperature is also in the confining regime and elementary objects are SU(2N_F) symmetric “hadrons” with not yet known properties.

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1. Introduction

Nonperturbatively QCD is defined in terms of its fundamental degrees of freedom, quarks and gluons in the Euclidean space-time. These fundamental degrees of freedom are never observed in the Minkowski space, a property of QCD which is called confinement. Only hadrons are observed. It is believed, however, that at high temperature, QCD is in a deconfinement regime and its fundamental degrees of freedom, quarks and gluons, are liberated. Is it true? Here, we present results of our recent findings [1] that suggest that this is actually not true.

In the Minkowski space-time, the QCD Lagrangian in the chiral limit is invariant under the chiral transformations

\[ \text{SU}(N_F)_L \times \text{SU}(N_F)_R \times \text{U}(1)_A \times \text{U}(1)_V. \]

The axial U(1)_A symmetry is broken by anomaly [2]. The SU(N_F)_L × SU(N_F)_R symmetry is broken spontaneously by the quark condensate in

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the vacuum. According to the Banks–Casher relation [3], the quark condensate in the Minkowski space can be expressed through a density of the near-zero modes of the Euclidean Dirac operator

\[
\lim_{m \to 0} \langle 0 | \bar{\Psi}(x) \Psi(x) | 0 \rangle = -\pi \rho(0).
\] (2)

Consequently, if we remove by hands the near-zero modes of the Dirac operator, we can expect a restoration of the chiral \( \text{SU}(N_F)_L \times \text{SU}(N_F)_R \) symmetry in correlation functions. If hadrons survive this “surgery”, then the chiral partners should become degenerate. The chiral partners of the \( J = 1 \) mesons are shown in Fig. 1.

\[
\begin{align*}
(0, 0) & \quad f_1(0, 1^{++}) & SU(2)_A & \quad \omega(0, 1^{--}) & \Psi(1_F \otimes \gamma^5 \gamma^k) \Psi \\
(1/2, 1/2)_a & \quad b_1(1, 1^{-+}) & U(1)_A & \quad \omega(0, 1^{--}) & \Psi(1_F \otimes \gamma^0 \gamma^k) \Psi \\
(1/2, 1/2)_b & \quad \rho(1, 1^{-+}) & SU(2)_A & \quad h_1(0, 1^{--}) & \Psi(\tau^a \otimes \gamma^0 \gamma^k) \Psi \\
(1, 0) \oplus (0, 1) & \quad \rho(1, 1^{-+}) & SU(2)_A & \quad a_1(1, 1^{++}) & \Psi(\tau^a \otimes \gamma^5 \gamma^k) \Psi
\end{align*}
\]

Fig. 1. \( SU(2)_L \times SU(2)_R \) and \( U(1)_A \) classification of the \( J = 1 \) meson operators.

It was observed in \( N_F = 2 \) dynamical simulations with the overlap Dirac operator that, indeed, hadrons survive this truncation (except for the ground states of \( J = 0 \) mesons) and the chiral partners get degenerate [4–7]. Not only the \( SU(2)_L \times SU(2)_R \) restoration was observed. Mesons that are connected by the \( U(1)_A \) transformation get also degenerate. We conclude that the same low-lying modes of the Dirac operator are responsible for both \( SU(2)_L \times SU(2)_R \) and \( U(1)_A \) breakings, which is consistent with the instanton-induced mechanism for both breakings [8].

Restoration of the full chiral symmetry \( SU(2)_L \times SU(2)_R \times U(1)_A \) of the QCD Lagrangian assumes degeneracies marked by arrows in Fig. 1. However, a larger degeneracy that includes all possible chiral multiplets in Fig. 1 was detected, see Fig. 2.

This unexpected degeneracy implies a symmetry that is larger than the chiral symmetry of the QCD Lagrangian. This not yet known symmetry was reconstructed in Refs. [9,10] and turned out to be

\[
SU(2N_F) \supset SU(N_F)_L \times SU(N_F)_R \times U(1)_A.
\] (3)
Fig. 2. $J = 1$ meson mass evolution as a function of the number $k$ of truncated lowest-lying Dirac modes. $\sigma$ shows energy gap in the Dirac spectrum.

This group includes as a subgroup the SU(2)$_{CS}$ (chiral spin) invariance. The SU(2)$_{CS}$ chiral spin generators are

$$\Sigma = \{ \gamma^0, i\gamma^5\gamma^0, -\gamma^5 \} , \quad [\Sigma^i, \Sigma^j] = 2ie^{ijk} \Sigma^k .$$

The Dirac spinor transforms under a global or local SU(2)$_{CS}$ transformation as

$$\Psi \rightarrow \psi' = e^{ie\Sigma/2} \psi .$$

2. SU(2$N_F$) as a hidden classical symmetry of QCD [11]

The SU(4) symmetry of $N_F = 2$ Euclidean QCD was obtained in lattice simulations. This means that this symmetry must be encoded in the nonperturbative Euclidean formulation of QCD. Obviously, the Euclidean Lagrangian for $N_F$ degenerate quarks in a given gauge background $A_\mu(x)$

$$\mathcal{L} = \Psi^\dagger(x)(\gamma_\mu D_\mu + m)\Psi(x)$$

is not SU(2)$_{CS}$ and SU(2$N_F$)-symmetric, because the Dirac operator does not commute with the generators of SU(2)$_{CS}$. A fundamental dynamical
reason for the absence of these symmetries are zero modes of the Dirac operator, $\gamma_{\mu}D_{\mu}\Psi_{0}(x) = 0$. The zero modes are chiral, L or R. With a gauge configuration of a nonzero global topological charge, the number of the left-handed and right-handed zero modes is, according to the Atiyah–Singer theorem, not equal. Consequently, there is no one-to-one correspondence of the left- and right-handed zero modes. The SU(2)$_{\text{CS}}$ chiral spin rotations mix the left- and right-handed Dirac spinors. Such a mixing can be defined only if there is a one-to-one mapping of the left- and right-handed spinors: The zero modes break the SU(2)$_{\text{CS}}$ invariance.

We can expand independent fields $\Psi(x)$ and $\Psi^\dagger(x)$ over a complete and orthonormal set $\Psi_n(x)$ of the eigenvalue problem

$$i\gamma_{\mu}D_{\mu}\Psi_n(x) = \lambda_n \Psi_n(x), \quad (6)$$

$$\Psi(x) = \sum_n c_n \Psi_n(x), \quad \Psi^\dagger(x) = \sum_k \bar{c}_k \Psi_k^\dagger(x), \quad (7)$$

where $\bar{c}_k, c_n$ are Grassmann numbers. The fermionic part of the QCD partition function takes the following form:

$$Z = \int \prod_{k,n} d\bar{c}_k dc_n e^{\sum_{k,n} \int d^4x \bar{c}_k c_n (\lambda_n + im) \Psi_k^\dagger(x) \Psi_n(x)}. \quad (8)$$

In a finite volume, the eigenmodes of the Dirac operator can be separated into two classes. The exact zero modes, $\lambda = 0$, and nonzero eigenmodes, $\lambda_n \neq 0$. It is well-understood that the exact zero modes are irrelevant since their contributions to the Green functions and observables vanish in the thermodynamic limit $V \to \infty$ as $1/V$ [12–14]. Consequently, in the finite-volume calculations, we can ignore the exact zero-modes.

Now, we can read off the symmetry properties of the partition function (8). For any SU(2)$_{\text{CS}}$ and SU($2N_F$) rotation, the $\Psi_n$ and $\Psi_k^\dagger$ Dirac spinors transform as

$$\Psi_n \to U \Psi_n, \quad \Psi_k^\dagger \to (U \Psi_k)^\dagger, \quad (9)$$

where $U$ is any transformation from the groups SU(2)$_{\text{CS}}$ and SU($2N_F$), $U^\dagger = U^{-1}$. It is then clear that the exponential part of the partition function is invariant under global and local SU(2)$_{\text{CS}}$ and SU($2N_F$) transformations, because

$$(U \Psi_k(x))^\dagger U \Psi_n(x) = \Psi_k^\dagger(x) \Psi_n(x). \quad (10)$$

The exact zero modes contributions

$$\Psi_0^\dagger(x) \Psi_n(x), \Psi_k^\dagger(x) \Psi_0(x), \Psi_0^\dagger(x) \Psi_0(x),$$
for which equation (10) is not defined, are irrelevant in the thermodynamic limit and can be ignored. In other words, QCD classically without the irrelevant exact zero modes has in a finite volume $V$ local $SU(2)_{CS}$ and $SU(2N_F)$ symmetries. These are hidden classical symmetries of QCD.

The integration measure in the partition function is not invariant under a local $U(1)_A$ transformation [2], which is a source of the $U(1)_A$ anomaly. The $U(1)_A$ is a subgroup of $SU(2)_{CS}$. Hence, the axial anomaly breaks $SU(2)_{CS}$ and $SU(2N_F) \to SU(N_F)_L \times SU(N_F)_R$.

In the limit $V \to \infty$ the otherwise finite lowest eigenvalues $\lambda$ condense around zero and provide according to the Banks–Casher relation at $m \to 0$ a nonvanishing quark condensate in the Minkowski space. The quark condensate in the Minkowski space-time breaks all $U(1)_A$, $SU(N_F)_L \times SU(N_F)_R$, $SU(2)_{CS}$ and $SU(2N_F)$ symmetries to $SU(N_F)_V$. In other words, the hidden classical $SU(2)_{CS}$ and $SU(2N_F)$ symmetries are broken both by the anomaly and spontaneously.

3. Restoration of $SU(2)_{CS}$ and $SU(2N_F)$ at high temperature [1]

Above the chiral restoration phase transition, the quark condensate vanishes. If, in addition, the $U(1)_A$ symmetry is also restored [15–17] and a gap opens in the Dirac spectrum, then above the critical temperature, the $SU(2)_{CS}$ and $SU(2N_F)$ symmetries are manifest. The precise meaning of this statement is that the correlation functions and observables are $SU(2)_{CS}$ and $SU(2N_F)$ symmetric.

These $SU(2)_{CS}$ and $SU(2N_F)$ symmetries of QCD imply that there cannot be deconfined free quarks and gluons at any finite temperature in the Minkowski space-time. Indeed, the Green functions and observables calculated in terms of unconfined quarks and gluons in the Minkowski space (i.e. within the perturbation theory) cannot be $SU(2)_{CS}$ and $SU(2N_F)$ symmetric, because the chromo-magnetic interaction necessarily breaks both symmetries. Then it follows that above $T_c$, QCD is in a confining regime. In contrast, color-singlet $SU(2N_F)$-symmetric “hadrons” (with not yet known properties) are not prohibited by the $SU(2N_F)$ symmetry and can freely propagate. “Hadrons” with such a symmetry in the Minkowski space can be constructed [18].

4. Predictions

Restoration of the $SU(2)_{CS}$ and of $SU(2N_F)$ symmetries at high temperatures can be tested on the lattice.

Transformation properties of hadron operators under $SU(2)_{CS}$ and $SU(2N_F)$ groups are given in Refs. [7,10]. For example, the isovector $J = 1$ operators $\bar{\Psi} \tau^i \gamma^\mu \gamma^5 \gamma^a \gamma^b \Psi, (1^{--})$; $\bar{\Psi} \tau^i \gamma^\mu \gamma^5 \gamma^a \gamma^b \Psi, (1^{--})$; $\bar{\Psi} \tau^i \gamma^\mu \gamma^5 \gamma^a \gamma^b \Psi, (1^{+-})$ form an irre-
ducible representation of SU(2)$_{\text{CS}}$. One expects that below $T_c$, all three diagonal correlators will be different and the off-diagonal cross-correlator of $(1^{--})$ operators will not be zero. Above $T_c$, an SU(2)$_{\text{CS}}$ restoration requires that all diagonal correlators should become identical and the off-diagonal correlator of $(1^{--})$ currents should vanish. A restoration of SU(2)$_{\text{CS}}$ and of SU($N_F$)$_L \times$ SU($N_F$)$_R$ implies a restoration of SU($2N_F$).

A similar prediction can be made with the baryon operators.

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