Solutions for the amplitudes that give accurate description of pp and p\bar{p} scattering at high energies are investigated, with particular attention given to the properties of their zeros and slopes, whose determination is required for the study of the Coulomb interference region. Proper extrapolations of these quantities to the LHC energies are important for the analysis of the forthcoming experiments.
I.  INTRODUCTION

The detailed knowledge of t-dependence of the real and imaginary amplitudes in pp and \( \bar{p}p \) scattering is important for the study of the Coulomb interference and the determination of the collision parameters. In general, the behavior of the real amplitude is oversimplified, with assumption that real and imaginary slopes are identical and that the ratio \( \rho \) is independent of \( t \). However, the present work, based on our analysis of the ISR, SPS and Fermilab data, shows that the variation of the real amplitude in the Coulomb interference region is very fast, with a slope about twice that of the imaginary part. The forthcoming experiments at RHIC and LHC, respectively at 200-500 GeV and 14 TeV, make urgent the investigation of the structure of the amplitudes in the low \(|t|\) region.

The real forward slope is obviously connected with the value of \( t \) at which this amplitude passes through zero. We find this connection, fulfilling the expectation from the theorem by A. Martin that the real part has a zero that approaches the forward direction as the energy increases.

Our work covers the full \( t \)-range, obtaining also the zeros of imaginary amplitudes and the shapes of the dips in the cross sections of pp and \( \bar{p}p \) scattering.

II.  AMPLITUDES, SLOPES AND ZEROS

We have constructed an accurate description of the t-dependence of the amplitudes, with smooth energy dependence in the parameters, with particular attention to the determination of slopes and zeros.

The amplitudes have peculiar shapes as functions of \(|t|\), with approximate exponential behaviour only for very small \(|t|\), and then turning fast towards zero. The results for some energies (52.8 GeV in pp, and 541 and 1800 GeV for \( \bar{p}p \) ) are shown in Fig. 1. Theoretical arguments about the existence of a real zero close to \(|t| = 0\) at high energies are confirmed by our construction. The real pp amplitude has a second zero, while in \( \bar{p}p \) only the first zero at small \(|t|\) is observed. The pp case is drawn in Fig. 2, where the real amplitude at \( \sqrt{s} = 52.8 \) GeV data is shown for small and for large \(|t|\), so that we can see where the second zero in the pp channel occurs. This second zero, being close to the zero of the imaginary amplitude, stresses the dips observed in the pp differential cross sections, but not those of
FIG. 1: a-Left: Real and imaginary amplitudes in forward directions for pp scattering at 52.8 GeV, and \( \bar{p}p \) at 541 and 1800 GeV, obtained through analysis of the data on differential cross sections. b-Right: Detail of the pp amplitudes at 52.8 GeV, normalized to one at \( t=0 \), with determination of the exponential slopes. The imaginary amplitudes show upwards curvature before they start decreasing to zero.

The \( \bar{p}p \) channel, due to a difference in the sign of the real tail.

FIG. 2: Real amplitude of pp scattering at 52.8 GeV. The two separate figures point to different parts of the \( |t| \) range where the first and second zeros are located.

With real and imaginary amplitudes described at low \( |t| \) by exponential slopes, we write for each channel, \( \text{Re}F(s,t) = \text{Re}F(s,0)\exp(-B^R|t|/2) \), \( \text{Im}F(s,t) = \text{Im}F(s,0)\exp(-B^I|t|/2) \) and the experimental slope \( B \) is given by \( B(s) = (\rho^2(s)B^R + \)
We determine separately $B^R$ and $B^I$ for each channel, and observe the approximate relation $B^R \approx 2B^I$ above 30 GeV.

The exponential slopes at $|t| = 0$ for 52.8 GeV are shown in Fig. 1, right hand side, where we plot the pp amplitudes normalized to one at $|t| = 0$. We observe that the concept of exponential slope is limited to a small $|t|$ range, with strong characteristic curvatures of the real and imaginary amplitudes appearing very soon in the $|t|$ scale. It is remarkable that the imaginary amplitude has a positive (going upwards) curvature at the beginning, before it starts running fast to zero.

These results for the slopes are shown in Fig. 3. For $B^I$ we have a simple parametrization

$$B^I_{pp} = B^I_{\bar{p}p} = B^I(s) = 8.6255 + 1.0372 \log \sqrt{s} \text{ in GeV}^{-2},$$

valid for both pp and $\bar{p}p$ channels, with a prediction $B^I = 18.53 \text{ GeV}^{-2}$ for the LHC energy $\sqrt{s} = 14$ GeV. The description is not so simple for the real slope. Here the pp and $\bar{p}p$ cases are different at the lower energies, and an extrapolation to higher energies is not neat for pp. For $\bar{p}p$, using the 541 and 1800 GeV points, we guess a linear log $s$ behavior

$$B^R_{\bar{p}p} = 19.4314 + 1.7569 \log \sqrt{s} \text{ in GeV}^{-2}.$$ 

The role of $B^R$ in the determination of the collision parameters can be seen in the work of Gauron, Nicolescu and Selyugin, who find a point $t_{\text{min}}$ where the real amplitude is compared to the Coulomb part. They study pp scattering at 52.8 GeV, suggesting a lower value for $\rho$ than 0.077 determined experimentally. However, to write the real amplitude at the matching point $t_{\text{min}}$, they use the exponential factor $\exp -B|t_{\text{min}}|/2$ with the experimental
slope \( B = 12.87 \text{ GeV}^{-2} \). However, according to our work, they should use the faster slope \( B^R = 26 \text{ GeV}^{-2} \) of the real amplitude. Then the new value that they suggest for \( \rho \) is not properly evaluated.

III. THEOREM ON THE FIRST REAL ZEROS

A theorem by A. Martin\[4\] predicts that in pp and \( \bar{p}p \) scattering the real part has a change of sign at a point \( |t_0| \) which moves to the forward direction as the energy increases. The paper suggests real zeros of the form \( |t_0| = \frac{1}{A + B \log s} \), as we find with our description of amplitudes, as shown in Fig. 4.

\[ |t_0| \text{ (GeV}^2) \]

\[ \text{energy } \sqrt{s} \text{ (GeV)} \]

\( * \) predicted for LHC \( |t_0|=1/(A+B \log \sqrt{s}) \)

\( * \) pp  \( * \) p-ap

\( \bullet \)  \( \circ \)

FIG. 4: Values \( |t_0| \) of the zeros of the real amplitude in pp and \( \bar{p}p \) scattering.

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