Gillian: Compositional Symbolic Execution for All

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We present Gillian, a language-independent framework for the development of compositional symbolic analysis tools. Gillian supports three flavours of analysis: whole-program symbolic testing, full verification, and bi-abduction. It comes with fully parametric meta-theoretical results and a modular implementation, designed to minimise the instantiation effort required of the user. We evaluate Gillian by instantiating it to JavaScript and C, and perform its analyses on a set of data-structure libraries, obtaining results that indicate that Gillian is robust enough to reason about real-world programming languages.

1 INTRODUCTION

Developing symbolic execution analyses for modern programming languages is a challenging and time-consuming task. The complexity of the underlying meta-theory and the associated tool development is substantial. However, symbolic execution tools share a number of features independent of the target language: for instance, interaction with first-order solvers [De Moura and Bjørner 2008] and the management of the variable store. Even so, the amount of effort required to transfer an analysis from one programming language to another is often prohibitive. While there has been work on streamlining this process by embedding the target language into a host language with support for symbolic analysis [Bucur et al. 2014; Torlak and Bodík 2013], these approaches do not scale well to fully-fledged real-world languages.

We present Gillian, a general framework for swift development of compositional symbolic analysis tools for real-world programming languages. Gillian is underpinned by GIL, a simple goto language parametric on the memory model of the target language: that is, on a set of actions describing the fundamental ways in which the programs of the target language interact with their respective memories. Gillian comes with a fully parametric meta-theory that unifies the treatment of symbolic execution, compositional program reasoning based on separation logic (SL), and bi-abduction. In particular, it supports three main styles of analysis:

- **whole-program symbolic execution**, where end-users write unit tests with symbolic inputs and outputs and use simple first-order assertions to describe the properties that the outputs must satisfy, while Gillian tries to generate symbolic traces that invalidate those assertions;
- **verification**, where end-users annotate the functions of their programs with SL-style specifications and use Gillian to verify that the functions meet their specifications; and
- **bi-abduction**, where end-users simply provide a program with no specifications or unit tests and are given back a set of SL-style specifications describing the behaviour of every function in the program up to a pre-established bound; if, in this process, a possible bug is found, users are given back the symbolic trace that leads to the bug.

To obtain an instantiation of each type of analysis for their target languages, users need to provide a compiler from the target language to GIL, and then simply instantiate Gillian with a memory model that provides an action-level implementation of the corresponding analysis: whole-program symbolic execution requires a symbolic implementation of the memory model of the target language; verification additionally requires that memory model to expose a set of core predicates describing the memory’s atomic constituents (essentially, SL assertions specific to that memory model); and bi-abduction requires the memory model to provide, on top of that, a mechanism for...
inferring missing resource whenever an action cannot be executed due to missing information. The implementation of a memory model with support for these three types of analyses (in OCaml) is substantially simpler than the implementation of each type of analysis from scratch.

Importantly, the meta-theory of Gillian is fully parametric on the memory model of the target language. Hence, the soundness theorems of whole-program symbolic execution, verification, and bi-abduction are proven in general once and for all, and can be instantiated to each target language provided that the user proves the necessary memory-specific lemmas. Similarly to the implementation, proving the memory-specific lemmas is far easier than creating a new soundness proof for each instantiation and analysis.

We evaluate Gillian by instantiating it to JavaScript and C. Using the obtained tools, we run the three analyses of Gillian on a series of data-structure libraries, purposefully written using the programming idioms specific to the two languages. For instance, our JavaScript examples rely on extensible objects, prototype-based inheritance, and closures, while our C examples use dynamic memory allocation, structures, and pointer arithmetic. The evaluation results indicate that Gillian is robust enough to reason about real-world programming languages.

Outline of the Paper. In §2, we present the parametric symbolic execution of Gillian. In §3, we give a thorough account of our parametric soundness result. The verification and bi-abduction analyses are then presented in detail in §4 and §5, respectively. In §6, we instantiate Gillian to obtain analysis tools for JavaScript and C. We discuss the related work in §7 and conclude in §8.

Throughout the paper, to give the reader a better intuition, we illustrate how to instantiate Gillian to each type of analysis using as an example a simple While language with static objects, reminiscent of that of [Berdine et al. 2005]. We also comment on our implementation choices and demonstrate how the implementation closely follows the theory.

2 PARAMETRIC SYMBOLIC EXECUTION

At the core of Gillian is a parametric interpreter for GIL, the simple intermediate goto language of Gillian. We present the syntax of GIL and give its semantics in terms of state models (§2.1). A state model can be viewed as an interface through which a programming language interacts with its state, and is parametric on the underlying memory model. We introduce concrete and symbolic state and memory models, and demonstrate how to automatically lift memory models to appropriate state models (§2.3). As the running example, we introduce a simple While language: we give its syntax, actions, and compiler to GIL in §2.2, and present its concrete and symbolic memory models in §2.4.

2.1 GIL Syntax and Semantics

GIL is a simple goto language with top-level procedures. It is parametric on a set of actions, A, which provide a general mechanism for interacting with GIL states. This parametricity allows us to maintain GIL states opaque throughout the meta-theoretical development and to provide parametric soundness results, minimising the burden of proof for users of Gillian.

The Syntax of GIL.

\[
\begin{align*}
\forall v & \in \mathcal{V} \quad n \in \mathcal{N} \mid s \in \mathcal{S} \mid b \in \mathcal{B} \mid l, \zeta \in \mathcal{Symb} \mid \tau \in \mathcal{T} \mid f \in \mathcal{F} \mid \overline{\tau} \\
\forall e & \in \mathcal{E} \triangleq v \mid x \in \mathcal{X} \mid \Theta e \mid e_1 \oplus e_2 \\
\forall c & \in \mathcal{C}_A \triangleq x := e \mid \text{id} \mid \text{return} e \mid \text{fail} e \mid \text{vanish} \\
\forall x & \in \mathcal{X} \mid x := \text{fresh}_j \mid \text{return} e \mid \text{fail} e \mid \text{vanish} \\
\forall \alpha & \in \mathcal{A} \\
\forall \text{proc} & \in \mathcal{Proc}_A \triangleq \text{proc } f(x)(\overline{\tau}) \\
\forall p & \in \mathcal{Prog}_A : \mathcal{F} \rightarrow \mathcal{Proc}_A
\end{align*}
\]

where \( i, j \in \mathcal{N} \) and \( \alpha \in \mathcal{A} \).

GIL values, \( v \in \mathcal{V} \), include numbers, strings, booleans, uninterpreted symbols, types, procedure identifiers, and lists of values. Uninterpreted symbols are mostly used to denote memory locations and instantiation-specific constants. Types are standard: they include, for example, the types of
numbers, strings, booleans, and lists. GIL expressions, \( e \in E \), include values, program variables \( x \), and various unary and binary operators.

GIL commands include, first of all, the standard variable assignment, conditional goto, and dynamic procedure call.\(^1\) Next, we have three GIL-specific commands: action execution, \( x := \alpha(e) \), executes the action \( \alpha \in A \) with the argument obtained by evaluating \( e \); and two allocations, \( x := \text{symb}_{j} \) and \( x := \text{fresh}_{j} \), which generate fresh uninterpreted and interpreted symbols, respectively. These allocations are annotated with a jointly unique identifier \( j \in \mathbb{N} \), addressed in §2.1.1 and §2.3. Finally, we have three additional control-flow related commands: return terminates the execution of the current procedure; fail terminates the execution of the entire program with an error; and vanish silently terminates the execution of the entire program without generating a result.

A GIL procedure, \( \text{proc} \in \mathcal{P}_{\text{Proc}} \), is of the form \( \text{proc} \, f(x)(\pi) \), where \( f \) is its identifier, \( x \) is its formal parameter, and its body \( \pi \) is a sequence of GIL commands. A GIL program, \( p \in \mathcal{P}_{\text{Proc}} \), is a finite partial function, mapping procedure identifiers to their corresponding procedures.

**Semantics.** The semantics of GIL is parameterised by a state model, \( S \in \mathcal{S} \), defined as follows.

**Definition 2.1 (State Model).** A state model \( S \in \mathcal{S} \) is a triple \( \langle |S|, V, A \rangle \), consisting of: (1) a set of states on which GIL programs operate, \( |S| \ni S \), (2) a set of values stored in those states, \( V \ni v \), and (3) a set of actions that can be performed on those states, \( A \ni \alpha \). All GIL states contain a variable store, \( \rho : X \rightarrow V \), mapping program variables to values.

A state model defines the following functions for acting on states (\( \equiv_{pp} \) denotes pretty-printing for readability):

- \( \text{setVar} : |S| \rightarrow X \rightarrow V \rightarrow |S| \)  
  \( \begin{align*} \text{setVar}(\sigma, x, v) &\equiv_{pp} \sigma.\text{setVar}(x, v) \end{align*} \)
- \( \text{setStore} : |S| \rightarrow (X \rightarrow V) \rightarrow |S| \)  
  \( \begin{align*} \text{setStore}(\sigma, \rho) &\equiv_{pp} \sigma.\text{setStore}(\rho) \end{align*} \)
- \( \text{getStore} : |S| \rightarrow (X \rightarrow V) \rightarrow |S| \)  
  \( \begin{align*} \text{getStore}(\sigma) &\equiv_{pp} \sigma.\text{getStore}() \end{align*} \)
- \( \text{ee} : |S| \rightarrow E \rightarrow V \)  
  \( \begin{align*} \text{ee}(\sigma, e) &\equiv_{pp} \sigma.\text{ee}(e) \end{align*} \)
- \( \text{ea} : A \rightarrow |S| \rightarrow V \rightarrow \varnothing(|S| \times V) \)  
  \( \begin{align*} (\sigma', v') &\in \text{ea}(\sigma, x, v) \equiv_{pp} \sigma.\alpha(x) \sim (\sigma', v') \end{align*} \)

The intuition behind the state functions is as follows: (1) \( \text{setVar}(x, v) \) sets \( x \) to \( v \) in the store of \( \sigma \); (2) \( \text{setStore}(x, \rho) \) replaces the store of \( \sigma \) with \( \rho \); (3) \( \text{getStore}(\sigma) \) obtains the store of \( \sigma \); and (4) \( \text{ee}(\sigma, e) \) evaluates the expression \( e \) in the store of \( \sigma \); and (5) \( \text{ea}(\alpha, x, v) \) executes the action \( \alpha \) on the state with argument \( v \). Note that, since actions result in sets of state-value pairs, the GIL semantics may be non-deterministic.

A state model \( S = \langle |S|, V, A \rangle \) is said to be proper if and only if it defines the following three distinguished actions: assume, for extending the state with new information; and symb and fresh, for generating new uninterpreted and interpreted symbols, respectively. Onward, we assume to work with proper state models.

**GIL Semantic Domains for \( S = \langle |S|, V, A \rangle \)**

| Call stacks: | \( cs \in CS_{S} \triangleq \langle f \rangle \mid \langle f, x, \rho, i \rangle : cs \) where \( f \in F, x \in X, \rho : X \rightarrow V, i \in \mathbb{N} \) |
|---|---|
| Configurations: | \( cf \in CF_{S} \triangleq \langle p, \sigma, cs, i \rangle \) where \( p \in P_{\text{Prog}} \), \( \sigma \in |S|, cs \in CS_{S}, i \in \mathbb{N} \) |
| Outcomes: | \( o \in O \triangleq \mid \mid N(v) \mid E(v) \) where \( v \in V \) |

The GIL semantics is defined in Figure 1. On each procedure call, it keeps track of the execution context of the caller, so that control can correctly be returned once the execution of the callee finishes. We achieve this by using call stacks, \( cs \in CS_{S} \), which are non-empty lists of stack frames. A top-level stack frame, \( \langle f \rangle \), only contains the identifier of the procedure that started the execution. An inner stack frame, \( \langle f, x, \rho, i \rangle \), contains: (1) the identifier \( f \) of the procedure being executed; (2) the variable \( x \) to which the return value of \( f \) will be assigned; (3) the store \( \rho \) of the caller of \( f \);

\(^{1}\)The procedure call is dynamic in that the identifier is obtained by evaluating the caller expression.
and (4) the index \( i \) to which control is transferred when the execution of \( f \) terminates. Additionally, we denote the argument of a function \( f \) by \( f \).arg.

\[
\begin{align*}
\text{Assign} & \quad \text{Action} \\
\text{cmd}(p, cs, i) = x := e & \quad \text{cmd}(p, cs, i) = x := e(e) & \sigma.e(e) = v \\
\sigma.e(e) = v & \quad \sigma.e(e) = v & \sigma.a(e) \sim (\sigma', v') \\
\text{IFGOTO - TRUE} & \quad \text{IFGOTO - FALSE} \\
\text{cmd}(p, cs, i) = \text{if goto } e & \quad \text{cmd}(p, cs, i) = \text{if goto } e & \sigma.e(e) = v \\
& \quad \sigma.e(e) = v & \sigma.e(e') = v \\
\text{Symb} & \quad \text{Symb} \\
\text{cmd}(p, cs, i) = x \leftarrow \text{symby} & \quad \sigma.e(e) = v & \sigma.symp(j) \sim (\sigma', v) \\
\text{Top Return} & \quad \text{FAIL} \\
\text{cmd}(p, cs, i) = \text{return } e & \quad \text{cmd}(p, cs, i) = \text{fail } e \\
& \quad \sigma.e(e) = v & \sigma.e(e) = v \\
\text{Return} & \quad \text{Return} \\
\text{cmd}(p, cs, i) = \text{return } e & \quad \text{cmd}(p, cs, i) = \text{return } e \\
& \quad \sigma.e(e) = v & \sigma.e(e) = v \\
\end{align*}
\]

\( \sigma \in \mathcal{O} \).

To capture the flow of the execution, we use outcomes, \( o \in \mathcal{O} \). GIL has three possible outcomes: (1) continuation, \( - \), signifying that the execution should proceed; (2) return, \( N(v) \), signifying that there was a top-level return with value \( v \); and (3) error, \( E(v) \), signifying that the execution failed with value \( v \). In the rules, we elide the continuation outcome whenever it is clear from the context.

Semantic transitions for GIL commands are of the form \( p \vdash (\sigma, cs, i) \sim (\sigma', cs', j)^0 \), meaning that, given a program \( p \), the evaluation of the \( i \)-th command of the top procedure of the call stack \( cs \) in the state \( \sigma \) generates the state \( \sigma' \), call stack \( cs' \), and outcome \( o \), and the next command to be evaluated is the \( j \)-th command of the top procedure of \( cs' \).

**Implementation.** In the implementation, procedure calls and actions may have multiple parameters. Further, we have several additional commands: the unconditional goto, goto \( i \); procedure application, \( x := \text{apply}(e, e) \), for modelling functions which take a variable number of arguments; the external procedure call, \( x := \text{extern}(e(\overline{e})) \), for modelling language features that step out of the program, such as the eval command of JavaScript or system calls in C; argument collection, arguments, which returns a GIL list containing the arguments with which the current procedure was called; and the phi-node command, \( x := \phi(x: \overline{x}) \), which allows GIL programs to be written in Single-Static-Assignment (SSA) style [Cytron et al. 1989]. These commands can all be compiled to the GIL of the paper, with the exception of the external procedure call. We do not provide meta-theoretical guarantees for GIL programs that use external procedure calls.

The general GIL interpreter follows the GIL semantics given in Figure 1 and is implemented as an OCaml functor parameterised by an OCaml module with type State. The State module type, whose signature

\begin{verbatim}
module type State = sig
  type t (** Type of GIL states *)
  type vt (** Type of GIL values *)
  type a (** Type of GIL actions *)

  val init : t
  val setVar : t -> Var.t -> vt -> t
  val setStore : t -> Var.t -> Map.t
  val store : t -> (Var.t, vt) Map.t
  val e : t -> Exp.t -> vt
  val fresh : t -> Var.t -> vt
  val symb : t -> Var.t -> vt

end
\end{verbatim}

\begin{verbatim}
module type Allocator = sig
  type t (** Type of allocation records *)
  type vts (** Type of uninterpreted symbols *)
  type vtf (** Type of interpreted symbols *)

  val alloc : t -> int -> t * vts
  val alloc : f : t -> int -> t * vtf

end
\end{verbatim}

Fig. 2. OCaml State/Allocator Signature
We demonstrate how to instantiate Gillian using a simple While language with static objects. Its synchronization from needing to reason about this issue by having built-in fresh-value location sites. Arriving at the set of actions in allocator for generating fresh locations, we do not need a separate action for object allocation, an action to each of those would be a reasonable first attempt. However, since Gillian has a built-in allocator for generating fresh locations, we do not need a separate action for object allocation, arriving at the set of actions $A_W$.

2.1.1 Gill Allocation. Fresh value generation is a common source of technical clutter often omitted or hand-waved in the formal presentation of program analyses. Gillian relieves the user of the framework from needing to reason about this issue by having built-in fresh-value allocators, inspired by the work of Banerjee and Naumann [2002].

Definition 2.2 (Allocator). An allocator $AL \in \mathcal{AL}$ is a pair, $(|AL|, V)$, consisting of: (1) a set $|AL| \ni \xi$ of allocation records$^2$; and (2) the set $V$ of values that may be allocated. It exposes the function alloc : $|AL| \rightarrow \mathbb{N} \rightarrow \phi(V) \rightarrow |AL| \times V$, which satisfies the well-formedness constraint $(\xi', v) = \text{alloc} (\xi, j, Y) \implies v \in Y$, and is pretty printed as $\xi.\text{alloc} (j) \rightarrow_V (\xi', v)$.

Informally, alloc $(\xi, j, Y)$ generates a fresh value $v$ taken from $Y \subseteq V$, associates it with the allocation site$^3$ uniquely identified by the natural number $j$, and returns it together with a updated allocation record. We discuss allocators and their properties in more detail in §3.2, in the context of our soundness results.

Implementation. We implement allocators as shown in Figure 2. As we do not have the expressive power to pass arbitrary subsets in OCaml, we instead require two separate types, vts/vtf used generating fresh uninterpreted/interpreted symbols, each generated by a dedicated allocation function, alloc_s/alloc_f. We show how allocators can be used in practice in §2.3.

2.2 While: Syntax, Actions, and Compilation to GIL

We demonstrate how to instantiate Gillian using a simple While language with static objects. Its syntax includes: the variable assignment; the skip command; sequencing; the if-then-else conditional; the while loop; the static function call; the return statement; assume and assert statements for driving the symbolic analysis; and statements for operating on static objects: object creation, property lookup, property mutation, and object disposal. For simplicity, we assume that the semantics of expressions and the variable store are the same for While and GIL.

The Syntax of While

\[
ws \in Stmt_W \triangleq x := e \mid \text{skip} \mid ws_1 ; ws_2 \mid \text{if}(e)\{ws_1\}\text{else}\{ws_2\} \mid \text{while}(e)\{ws\} \mid x := f(e) \mid \text{return} e \mid \text{assume} e \mid \text{assert} e \mid x := \{p_i : e_i \mid j_{i=1}^n\} \mid x := e.p \mid e.p := e' \mid \text{dispose} e
\]

In order to compile While to GIL, we first have to pick a set of actions $A_W$ for acting on While states. As we have four operations on objects—allocation, lookup, mutation, and disposal—assigning an action to each of those would be a reasonable first attempt. However, since Gillian has a built-in allocator for generating fresh locations, we do not need a separate action for object allocation, arriving at the set of actions $A_W = \{\text{lookup}, \text{mutate}, \text{dispose}\}$.

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$^2$Intuitively, an allocation record maintains information about already allocated values; this approach is complementary to the free set approach of Raza and Gardner [2009], where information is maintained about values that can still be allocated.

$^3$An allocation site $j$ is the program point that is associated with either the symbol, or the fresh$_j$ command.
A part of the While-to-GIL compiler is given in Figure 3 (cf. Appendix A). It is modelled as a function $C_W: \text{Stmt}_W \rightarrow \mathbb{N} \rightarrow C_A$ list $\times \mathbb{N}$, mapping a While statement $ws \in \text{Stmt}_W$ and a natural number $pc$ (read: program counter) to a sequence of GIL commands and the next available program counter, $npc$ (denoted in Figure 3 using the notation $\cdot \leftarrownpc$). For instance, if $C_W(ws, pc) = (\tau, npc)$, then the while statement $ws$ compiles to the sequence of GIL commands given by $\tau$ and that the commands in $\tau$ are labelled with indexes $pc$ to $npc - 1$.

The compilation rules are straightforward; we explain the ones given in Figure 3. We compile the While assignment to a GIL assignment, shallowly embedding While variables to GIL variables. The assume statement, $\text{assume } e$, compiles to a goto statement, $\text{if } goto e (pc + 2)$, which branches on the value of the expression to be assumed, followed by a silent cutting of the branch in which it does not hold by using the GIL command $\text{vanish}$.

The assert statement, $\text{assert } e$, is compiled to the same goto statement that branches on $e$, but this time, if the branch in which $e$ does not hold is reached, the execution will terminate with error by using the GIL command $\text{fail}$. Finally, the lookup of While is compiled as a call to the corresponding action, $\text{lookup}$, whose parameter is a GIL list containing the expression denoting the address of the object, $e$, and the looked-up property, $p$.

### 2.3 Concrete and Symbolic States

Reasoning about programs can, at a high level, be separated into reasoning about the variable store and about the memory model of the programming language in question. Gillian simplifies this process by providing built-in reasoning about the variable store, leaving to the user only to take care of the memory model. In particular, it is possible to lift a given memory to a GIL state by coupling said memory with an appropriate variable store and allocator. In this section, we illustrate this lifting for concrete and symbolic memories, obtaining concrete and symbolic states.

Concrete memories store concrete values, $v \in \mathcal{V}$. Symbolic memories store logical expressions, $\hat{e} \in \hat{\mathcal{E}}$, generated by the grammar $\hat{e} \in \hat{\mathcal{E}} \triangleq v | x \in X | \Theta \hat{e} | \hat{e}_1 \oplus \hat{e}_2$, where $x$ ranges over a set of logical variables, $X$. The formal definitions of these two memory models are as follows:

**Definition 2.3 (Concrete Memory Model).** A concrete memory model $M \in \hat{\mathcal{M}}$ is a pair $\langle |M|, A \rangle$, consisting of a set of concrete memories, $|M| \ni \mu$, and a set of actions $A \ni \alpha$. A concrete memory model additionally defines a function $ea$, for concrete action execution on memories:

$$ea : A \rightarrow |M| \rightarrow \mathcal{V} \rightarrow |M| \times \mathcal{V} \quad (ea(\alpha, \mu, v))$$

**Definition 2.4 (Symbolic Memory Model).** A symbolic memory model $\hat{M} \in \hat{\mathcal{M}}$ is a pair $\langle |\hat{M}|, A \rangle$, consisting of a set of symbolic memories, $|\hat{M}| \ni \hat{\mu}$, and a set of actions $A \ni \alpha$. A symbolic memory model additionally defines a function $\hat{e}a$ for symbolic action execution on memories:

$$\hat{e}a : A \rightarrow |\hat{M}| \rightarrow \hat{\mathcal{E}} \rightarrow \Pi \rightarrow \varphi([\hat{M}] \times \hat{\mathcal{E}} \times \Pi) \quad ((\hat{\mu}', \hat{\epsilon}', \pi') \in \hat{e}a(\alpha, \hat{\mu}, \hat{\epsilon}, \pi))$$

where $\pi \in \Pi \subset \hat{\mathcal{E}}$ denotes a boolean logical expression.

From a concrete memory, $\mu \in |M|$, we construct a concrete state $\sigma$ by coupling $\mu$ with a concrete store, $\rho : X \rightarrow \mathcal{V}$, and a concrete allocation record, $\xi \in |AL|$. Analogously, from a symbolic memory, $\hat{\mu} \in |\hat{M}|$, we construct a symbolic state $\hat{\sigma}$ by coupling $\hat{\mu}$ with a symbolic store, $\hat{\rho} : X \rightarrow \hat{\mathcal{E}}$, and a symbolic allocation record, $\hat{\xi} \in |\hat{AL}|$. Symbolic states also include a boolean logical expression $\pi \in \Pi$, referred to as the path condition of $\hat{\sigma}$. Path conditions [Baldoni et al. 2018] bookkeep the constraints on the symbolic variables that led the execution to the current symbolic state. We formally describe the liftings from concrete and symbolic memories to the appropriate states below, with $A_0 = \{ \text{assume}, \text{fresh}, \text{symb} \}$.
Definition 2.5 (Concrete State Constructor (CST)). Given an allocator $AL = \langle |AL|, V \rangle$, the concrete state constructor $CST : \mathbb{M} \to \mathbb{S}$ is defined as $CST(|M|, A) \triangleq \langle |S|, V, A \cup A_0 \rangle$, where:

- $|S| = |M| \times (X \times V) \times |AL|$
- $setVar(\langle \mu, \rho, \xi, x, v \rangle) \triangleq \langle \mu, \rho[x \mapsto v], \xi \rangle$
- $setStore(\langle \mu, \rho, \xi \rangle, \rho) \triangleq \langle \mu, \rho, \xi \rangle$
- $getStore(\langle \mu, \rho, \xi \rangle) \triangleq \rho$
- $ee(\langle \rho, \xi, e \rangle) \triangleq \|e\|_\rho$
- $ea(\alpha, \langle \mu, \rho, \xi, v \rangle) \triangleq \{(\mu', \rho, \xi, v) | (\mu', v') = ea(\alpha, \mu, v)\}$
- $assume(\sigma, \nu) \triangleq \{(\sigma, v) | v = true\}$
- $symb(\langle \mu, \rho, \xi, j \rangle) \triangleq \{(\mu, \rho, \xi'), \nu) | \xi, \text{alloc}(j) \to \text{symb}(\xi', \nu)\}$
- $fresh(\langle \mu, \rho, \xi, j \rangle) \triangleq \{(\mu, \rho, \xi'), \nu) | \xi, \text{alloc}(j) \to \nu(\xi', \nu)\}$

Definition 2.6 (Symbolic State Constructor (SST)). Given an allocator $\hat{AL} = \langle |\hat{AL}|, \hat{E} \rangle$, the symbolic state constructor $\text{SST} : \hat{\mathbb{M}} \to \hat{\mathbb{S}}$ is defined as $\text{SST}(|\hat{M}|, A) \triangleq \langle \hat{S}, \hat{E}, A \cup A_0 \rangle$, where:

- $|\hat{S}| = |\hat{M}| \times (X \times \hat{E}) \times |\hat{AL}| \times \Pi$
- $setVar(\langle \hat{\mu}, \hat{\rho}, \hat{\xi}, \pi, x, \hat{e} \rangle) \triangleq \langle \hat{\mu}, \hat{\rho}[x \mapsto e], \hat{\xi}, \pi \rangle$
- $setStore(\langle \hat{\mu}, \hat{\rho}, \hat{\xi}, \hat{\pi} \rangle, \hat{\rho}) \triangleq \langle \hat{\mu}, \hat{\rho}, \hat{\xi}, \hat{\pi} \rangle$
- $getStore(\langle \hat{\mu}, \hat{\rho}, \hat{\xi}, \hat{\pi}\rangle) \triangleq \hat{\rho}$
- $ee(\langle \hat{\rho}, \hat{\xi}, e \rangle) \triangleq \hat{\rho}(e)$
- $ea(\alpha, \langle \hat{\mu}, \hat{\rho}, \hat{\xi}, \pi \rangle) \triangleq \{(\mu', \rho, \xi, \pi \land \pi') \land \hat{\pi'}, \hat{\rho}(e) \}$
- $assume(\langle \hat{\mu}, \hat{\rho}, \hat{\xi}, \pi, \pi' \rangle) \triangleq \hat{\rho}(\pi \land \pi') \}$
- $symb(\langle \hat{\mu}, \hat{\rho}, \hat{\xi}, \pi, \nu \rangle) \triangleq \{(\mu, \rho, \xi, \pi), \nu) | \xi, \text{alloc}(j) \to \text{symb}(\xi', \nu)\}$
- $fresh(\langle \hat{\mu}, \hat{\rho}, \hat{\xi}, \pi, \nu \rangle) \triangleq \{(\mu, \rho, \xi', \pi), \nu) | \xi, \text{alloc}(j) \to \hat{\chi}(\xi', \nu)\}$

The concrete/symbolic lifting constructs all of the functions that a state model exposes, with the help of the action execution function of the parameter concrete/symbolic memories and the alloc function of the parameter concrete/symbolic allocator. In both cases, the construction of setVar, setStore, and getStore is straightforward. The remaining cases are described below.

[EvalExpr]. In the concrete case, expression evaluation is performed concretely (we use $\|e\|_\rho$ to denote the standard expression evaluation of $e$ with respect to $\rho$). In the symbolic case, it amounts to substituting all the program variables in $e$ with their associated logical expressions given by the store (we denote this substitution by $\hat{\rho}(e)$). In the implementation, Gillian’s first-order solver applies a number of algebraic identities to simplify the expression resulting from $\hat{\rho}(e)$. It is also in charge of discharging satisfiability/entailment questions (e.g. $\pi \vdash \hat{e} \equiv \hat{e}'$), by encoding them into the Z3 solver of De Moura and Bjørner [2008].

[Action]. Action execution on states amounts to calling action execution on the parameter memory. As symbolic actions, unlike concrete actions, may branch, they additionally generate a logical expression, $\pi'$, describing the conditions under which the chosen branch is taken. Hence, the path condition of the obtained state is a conjunction of $\pi'$ with the path condition $\pi$ of the original state.

[Assume]. The function $assume(\sigma, \nu)$ extends the state $\sigma$ with information that the value $\nu$ holds. In the concrete case, $assume(\sigma, b)$ returns the singleton set containing the original state when $b = true$ and the empty set otherwise. In the symbolic case, $assume(\hat{\sigma}, \pi')$ returns $\hat{\sigma}$ with its path condition strengthened with $\pi'$ if this new path condition is satisfiable, and the empty set otherwise.

[Symb/Fresh]. The functions symb and fresh generate symbols using the parameter allocator. We use the arrow parameter of the allocator to indicate the set from which to pick the freshly generated value: symb picks an uninterpreted and fresh picks an interpreted symbol. As symbolic values are logical expressions, it makes sense to pick a fresh logical variable when generating a fresh interpreted symbol and then later impose constraints on it via the assume function.
Implementation. The concrete and symbolic state constructors are implemented as OCaml functors, respectively parameterised by OCaml modules with types CMemory and SMemory (cf. Figure 4). Both functors are additionally parameterised by a module with type Allocator for the generation of fresh values. The CMemory and SMemory module types precisely follow our formal characterisation of concrete and symbolic state models as per Definitions 2.3 and 2.4. Furthermore, the functors CState and SState also follow the concrete and symbolic liftings described in Definitions 2.5 and 2.6.

2.4 While: Concrete and Symbolic Memories

We show how to instantiate Gillian to obtain a concrete/symbolic interpreter for While by combining the While-to-GIL compiler of §2.2 with the concrete/symbolic While memory model.

The first step towards defining a memory model is to pick the carrier set of the model, in this case: the set of While memories. We define a concrete While memory as a partial mapping from symbols (corresponding to object locations) and strings (corresponding to property names) to values; formally: $\mu \in M^W : Symb \times S \rightarrow \mathcal{V}$. Analogously, we define a symbolic memory to be a partial mapping from logical expressions and strings to logical expressions; formally: $\hat{\mu} \in \hat{M}^W : \hat{\mathcal{E}} \times \mathcal{S} \rightarrow \hat{\mathcal{E}}$. Property names are not lifted to logical expressions in the symbolic case, since the While statements that act on object properties do not allow the property name to be resolved dynamically.

Having chosen the concrete and symbolic carrier sets, we can now define the functions $\text{ea}$ and $\text{e}a$, for acting on concrete and symbolic memories, respectively. In particular, these functions must support the actions $\text{A}_W = \{\text{lookup, mutate, dispose}\}$ introduced in §2.2. The definition of the While concrete/symbolic actions is given in Figure 5 (concrete on the left and symbolic on the right), and is straightforward. In the rules, we use several projection operators: $\mu \mid I$ splits the concrete memory $\mu$ into a pair $(\mu', \mu'')$, where $\mu'$ contains the memory cells at location $l$ and $\mu''$ contains the others;

$$\begin{align*}
\text{C-Lookup} & \quad \mu = \_ \cup l.p \mapsto v \\
& \quad \mu.\text{lookup}(\{l, p\}) \rightsquigarrow (\mu, v) \\
\text{C-Mutate-Present} & \quad \mu = \mu' \cup l.p \mapsto \_ \quad \mu'' = \mu' \cup l.p \mapsto v \\
& \quad \mu.\text{mutate}(\{l, p, v\}) \rightsquigarrow (\mu'', v) \\
\text{C-Mutate-Absent} & \quad (l, p) \notin \text{dom}(\mu) \quad \mu' = \mu \cup l.p \mapsto v \\
& \quad \mu.\text{mutate}(\{l, p, v\}) \rightsquigarrow (\mu', v) \\
\text{C-Dispose} & \quad \mu \mid I = (\_, \mu') \\
& \quad \mu.\text{dispose}(l) \rightsquigarrow (\mu', \text{true}) \\
\text{S-Lookup} & \quad \pi \ni \hat{e} = \hat{e}' \quad \hat{\mu} = \_ \cup \hat{e}' . p \mapsto \hat{e}_v \\
& \quad \hat{\mu}.\text{lookup}(\{\hat{e}, p\}, \pi) \rightsquigarrow ((\hat{\mu}, \hat{e}_v, \text{true})) \\
\text{S-Mutate-Present} & \quad \pi \ni \hat{e} = \hat{e}'' \quad \hat{\mu} = \hat{\mu}' \cup \hat{e}'' . p \mapsto \hat{e}' \\
& \quad \hat{\mu}.\text{mutate}(\{\hat{e}, p, \hat{e}'\}, \pi) \rightsquigarrow ((\hat{\mu}'', \text{true}, \text{true})) \\
\text{S-Mutate-Absent} & \quad \hat{\mu} \mid \hat{e}, p, n = \emptyset \quad \hat{\mu}' = \hat{\mu} \cup \hat{e} . p \mapsto \hat{e}' \\
& \quad \hat{\mu}.\text{mutate}(\{\hat{e}, p, \hat{e}'\}, \pi) \rightsquigarrow ((\hat{\mu}'', \text{true}, \text{true})) \\
\text{S-Dispose} & \quad \hat{\mu} \mid \hat{e}, n = (\_, \hat{\mu}') \\
& \quad \hat{\mu}.\text{dispose}(\hat{e}, \pi) \rightsquigarrow ((\hat{\mu}', \text{true}, \text{true}))
\end{align*}$$

Fig. 5. While: Actions in Concrete and Symbolic Memories
\( \mu |_{\hat{e}, \pi} \) splits the symbolic memory \( \hat{\mu} \) into a pair \((\hat{\mu}', \hat{\mu}'')\), where \( \hat{\mu}' \) contains the memory cells at location corresponding to \( \hat{e} \) under \( \pi \) and \( \hat{\mu}'' \) contains the others; and \( \hat{\mu} |_{\hat{e}, \pi} \) returns a set of possible locations in \( \hat{\mu} \) corresponding to \( \hat{e} \) under \( \pi \) that have property \( p \). We describe the rules \([C\text{-Dispose}]\) and \([S\text{-Dispose}]\) in detail.

\[C\text{-Dispose}\]. To dispose of the object at location \( l \), we split the memory \( \mu \) using \( \mu |_{l} \) and return the part of the memory that does not contain the cells at location \( l \), and the value true. The value true is returned to comply with the expected type of \( \text{ea} \); it is not used by the compiled code.

\[S\text{-Dispose}\]. Analogously to \([C\text{-Dispose}]\), we split the symbolic memory \( \hat{\mu} \) using \( \hat{\mu} |_{\hat{e}, \pi} \) and return the part of the memory that does not contain the cells corresponding to \( \hat{e} \) under \( \pi \), the value true, and the path condition true. The returned path condition indicates that the action does not branch and that, therefore, the path condition of the corresponding state will effectively not be updated.

### 3 Parametric Soundness

Proving the soundness of symbolic analyses is a time-consuming task that often requires a considerable number of auxiliary lemmas and definitions. The complexity of such proofs becomes unwieldy as we move towards real-world programming languages with multiple program constructs and intricate runtime environments, which tends to detract from mathematical rigour in favour of less time consuming, but also less trustworthy, informal arguments. Gillian streamlines the development of soundness proofs for the instantiations of the framework, by focussing the user’s proof effort only on the target language memory and the actions that it exposes.

We propose a proof infrastructure, illustrated in Figure 6, consisting of: (1) a class of soundness relations between state models, \( R_S \), which are preserved by the semantics of GIL; (2) a class of soundness relations between memory models, \( R_M \), which, when lifted to states, yield relations in \( R_S \); and (3) the lifting mechanism, \( RT \). With this infrastructure in place, proving the soundness of a given symbolic semantics in terms of a given concrete semantics amounts to proving that the soundness relation between the two corresponding memory models is in \( R_M \).

The section is structured as follows: §3.1 describes a class of soundness relations that are preserved by the semantics of GIL; §3.2 describes the mechanism for lifting a soundness relation between memories to a soundness relation between states; and §3.3 shows how to leverage the proposed proof infrastructure to prove the soundness of the While symbolic analysis.

#### 3.1 Parametric Soundness

The standard approach for defining soundness [Cousot and Cousot 1977] can be coarsely described as in the diagram of Figure 7 (left), where: (1) \( \hat{\sigma}_1 \) is an abstract state over-approximating a concrete state, \( \sigma_1 \); (2) \( \hat{\sigma}_2 \) is the abstract state obtained by abstractly executing a given command on \( \hat{\sigma}_1 \); and (3) \( \sigma_2 \) is the state obtained by concretely executing the same command on \( \sigma_1 \). In this setting, the abstract semantics is sound with respect to the concrete one if \( \hat{\sigma}_2 \) also over-approximates \( \sigma_2 \).

In the context of abstract analyses that may branch, this elegant characterisation of soundness cannot describe what it means for a single abstract trace to be sound. For instance, Figure 7 (mid) shows a scenario with two possible abstract transitions from the original abstract state. The only way to ensure that the final abstract state over-approximates the final concrete state is to merge the two final abstract states, forcing us to reason about all possible abstract traces at the same time.

---

4The union of abstract states as in the standard collecting semantics [Cousot and Cousot 2004] is a form of merging.
We propose a characterisation of soundness that allows us to describe both what it means for a single abstract trace to be sound independently of all the others and also to recover the standard notion of soundness when given the set of all possible abstract traces. This property is inspired by work on symbolic execution [Cadar et al. 2011; Cadar and Sen 2013], where one can talk about the soundness of single symbolic execution trace by strengthening the path condition of the initial symbolic state with the path condition of the final state. This effectively filters out all of the initial concrete states for which the concrete execution diverges from the path of the given symbolic trace.

\[
\begin{array}{c}
\hat{\sigma}_1 \dashv \vdash \mathcal{R}_S \dashv \vdash \sigma_1 \\
\downarrow & \downarrow & \downarrow \\
\hat{\sigma}_2 \dashv \vdash \mathcal{R}_S \dashv \vdash \sigma_2 \\
\end{array}
\begin{array}{c}
\hat{\sigma}_1' \dashv \vdash \mathcal{R}_S \dashv \vdash \sigma_1 \\
\downarrow & \downarrow & \downarrow \\
\hat{\sigma}_2' \dashv \vdash \mathcal{R}_S \dashv \vdash \sigma_2 \\
\end{array}
\begin{array}{c}
\hat{\sigma}_1 \dashv \vdash \mathcal{R}_S \dashv \vdash \sigma_1 \\
\downarrow & \downarrow & \downarrow \\
\hat{\sigma}_2 \dashv \vdash \mathcal{R}_S \dashv \vdash \sigma_2 \\
\end{array}
\]

Fig. 7. Soundness Properties

**Restriction Operators.** As Gillian states are general, we cannot use path conditions as a device to pinpoint the set of concrete traces that need to be modelled by a given abstract trace. Instead, we introduce restriction operators on abstract states. Informally, the restriction of an abstract state \(\hat{\sigma}_1\) with another abstract state \(\hat{\sigma}_2\), written \(\hat{\sigma}_1 \vdash \hat{\sigma}_2\), denotes the state \(\hat{\sigma}_1\) strengthened with some information coming from \(\hat{\sigma}_2\). In Figure 7 (right), we give further intuition on how we will use restriction: for example, if \(\hat{\sigma}_1\) over-approximates \(\sigma_1\) and it also holds that \(\hat{\sigma}_1 = \hat{\sigma}_1 \vdash \hat{\sigma}_2\), meaning that \(\hat{\sigma}_1\) does not learn anything from \(\hat{\sigma}_2\), then \(\hat{\sigma}_2\) will also over-approximate \(\sigma_2\).

We define restriction operators algebraically. A restriction operator \(\vdash: X \rightarrow X \rightarrow X\) on a set \(X\), written \(x_1 \vdash x_2\) for \((x_1, x_2)\), is a binary associative function satisfying the following properties:

- **Idempotence**
  \(x \vdash x = x\)

- **Right Commutativity**
  \((x_1 \vdash x_2) \vdash x_3 = (x_1 \vdash x_3) \vdash x_2\)

- **Weakening**
  \(x_1 \vdash x_2 \vdash x_3 = x_1\)

meaning that self-restriction does not gain information, the order of applied restrictions does not influence the accumulated information gain, and that if \(x_1\) cannot gain information from the combined knowledge of \(x_2\) and \(x_3\), then it cannot gain information from either \(x_2\) or \(x_3\). It is easily verifiable that every restriction operator \(\vdash: X \rightarrow X \rightarrow X\) induces a pre-order \((X, \sqsubseteq)\), given by \(x_1 \sqsubseteq x_2 \iff x_1 \vdash x_2 = x_1\).

A restriction operator on states \(\vdash: |S| \rightarrow |S| \rightarrow |S|\) is said to be preserved by a state model \(S = \langle |S|, V, A \rangle\) if all of the state-generating functions exposed by the state model are monotonic with respect to the pre-order induced by \(\vdash\); put formally:

\[
\begin{align*}
\text{RMono-SetVar} & : \sigma.\text{setVar}(x, v) = \sigma' \implies \sigma' \sqsubseteq \sigma \\
\text{RMono-SetStore} & : \sigma.\text{setStore}(\rho) = \sigma' \implies \sigma' \sqsubseteq \sigma \\
\text{RMono-Action} & : \sigma.\text{act}(v) = \sigma' \implies \sigma' \sqsubseteq \sigma \\
\end{align*}
\]

We say that \(\vdash\) is a restriction operator on a state model \(S = \langle |S|, V, A \rangle\), if \(\vdash\) is a restriction operator on the carrier set \(|S|\) and \(\vdash\) is preserved by \(S\). Restriction operators are extended from states to configurations straightforwardly: \(\langle p, \sigma, cs, i \rangle \vdash \langle p, \sigma', cs, i \rangle\).

**Compatibility.** In the following, we assume a pre-order \(\leq\) on abstract states, writing \(\hat{\sigma}_2 \leq \hat{\sigma}_1\) to mean that the models of \(\hat{\sigma}_2\) are contained in the models of \(\hat{\sigma}_1\) (we say that \(\hat{\sigma}_2\) is more precise than \(\hat{\sigma}_1\)). This pre-order may differ from that induced by the chosen restriction operator on abstract states. Consider the symbolic execution setting: the fact that the path condition of a state \(\hat{\sigma}_2\) implies the path condition of a state \(\hat{\sigma}_1\) does not necessarily mean that all the models of \(\hat{\sigma}_2\) are contained in the models of \(\hat{\sigma}_1\), as these two states may describe different memories. However, the chosen restriction operator must be compatible with the pre-order on states. Formally, we say that a pre-order \((X, \leq)\) is compatible with a restriction operator \(\vdash\) on \(X\) iff the following properties hold:
These properties essentially describe that restriction increases \(\leq\)-precision, that \(\leq\)-precision implies \(\preceq\)-precision, and how \(\leq\)-precision and \(\preceq\) combine under \(\preceq\)-precision. Unsurprisingly, the pre-order \(\preceq\) induced by \(\preceq\) is indeed compatible with \(\preceq\).

**Soundness Relations.** We are now in the position to describe the class of soundness relations that are preserved by the semantics of GIL. This class is formally given in Definition 3.1, which makes use of the notion of induced pre-order. Formally, given a relation \(\sim \in X \times Y\) between two sets \(X\) and \(Y\), the pre-order on \(X\) induced by \(\sim\), written \(\leq\), is defined as follows: \(x_1 \leq x_2\) if and only if \(\{y \mid x_1 \sim y\} \subseteq \{y \mid x_2 \sim y\}\). We elide the \(\sim\) in \(\leq\) when it is clear from the context.

**Definition 3.1 (Soundness Relation - States).** Given two state models, \(\hat{S} = (|\hat{S}|, \hat{V}, A)\) and \(S = (|S|, V, A)\), a soundness relation \(SR\) for \(\hat{S}\) with respect to \(S\) is a triple \((|\cdot|, \sim_\sigma, \sim_\rho)\), consisting of: (1) a restriction operator \(|\cdot|\) on \(\hat{S}\); (2) a binary relation \(\sim_\sigma \subseteq |\hat{S}| \times |S|\); and (3) a ternary relation \(\sim_\rho \subseteq |\hat{S}| \times \hat{V} \times V\), such that \(|\cdot|\) is compatible with the pre-order induced by \(\sim_\sigma\) (denoted by \(\leq\)) and the following constraints hold:

\[
\begin{align*}
\text{Store} & \quad \hat{\sigma} \sim_\sigma \sigma \implies \hat{\sigma} \triangleright \hat{\sigma}.\text{store} \sim_\rho \sigma.\text{store} \\
\text{EvalExpr} & \quad \hat{\sigma} \leq \hat{\sigma}' \land \hat{\sigma} \sim_\sigma \sigma \implies \hat{\sigma}' \triangleright \hat{\sigma}'.\text{ee}(e) \sim_\rho \sigma.\text{ee}(e) \\
\text{SetVar} & \quad \hat{\sigma} \leq \hat{\sigma}' \land \hat{\sigma} \sim_\sigma \sigma \land \hat{\sigma} \triangleright \hat{\sigma} \sim_\rho \rho \implies \hat{\sigma}'.\text{setVar}(x, \hat{\rho}) \sim_\rho \sigma.\text{setVar}(x, \rho) \\
\text{SetStore} & \quad \hat{\sigma} \leq \hat{\sigma}' \land \hat{\sigma} \sim_\sigma \sigma \land \hat{\sigma} \triangleright \hat{\rho} \sim_\rho \rho \implies \hat{\sigma}'.\text{setStore}(\rho) \sim_\rho \sigma.\text{setStore}(\rho) \\
\text{Action} & \quad \hat{\sigma}.\alpha(\hat{\rho}) \sim_\sigma (\hat{\sigma}'', \hat{\rho}') \land \hat{\sigma} \leq \hat{\sigma}' \land \hat{\rho} \sim_\rho \rho \implies \exists \sigma', \hat{\sigma}.\alpha(\hat{\rho}) \sim_\sigma (\sigma', \hat{\rho}') \land \hat{\sigma}' \sim_\rho \sigma' \land \hat{\sigma}' \sim_\rho \hat{\rho}' \sim_\rho \hat{\rho}' \\
\text{Weakening} & \quad \hat{\sigma} \leq \hat{\sigma}' \land \hat{\sigma} \sim_\rho \rho \implies \hat{\sigma}' \sim_\rho \rho \sim_\rho \rho
\end{align*}
\]

where \(\hat{\sigma} \triangleright \hat{\rho} \sim_\rho \rho\) is shorthand for: \(\text{dom}(\hat{\rho}) = \text{dom}(\rho) = X\) and \(\forall x \in X . \hat{\sigma} \triangleright \hat{\rho}(x) \sim_\rho \rho(x)\).

In a nutshell, if \(SR = (|\cdot|, \sim_\sigma, \sim_\rho)\) is a soundness relation for \(\hat{S} = (|\hat{S}|, \hat{V}, A)\) in terms of \(S = (|S|, V, A)\), then: (1) \(\hat{\sigma} \sim_\sigma \sigma\) means that \(\hat{\sigma}\) is an over-approximation of \(\sigma\), and (2) \(\hat{\sigma} \triangleright \sigma_1 \sim_\rho \sigma_2\) means that, considering the information in \(\hat{\sigma}\), \(\sigma_1\) is an over-approximation of \(\sigma_2\). The constraints imposed on the state functions guarantee that \(\sim_\sigma\) and \(\sim_\rho\) are preserved by the GIL interpreter. Below, we discuss the Action and Weakening constraints; the other ones can be understood analogously.

**[Action].** This constraint states that if we have an abstract action execution \(\hat{\sigma}.\alpha(\hat{\rho}) \sim_\sigma (\hat{\sigma}'', \hat{\rho}'')\), then, for any abstract state \(\hat{\sigma}\), concrete state \(\sigma\), and concrete value \(\rho\), such that: (1) \(\hat{\sigma} \leq \hat{\sigma}' \land \hat{\sigma}''\) (meaning that \(\hat{\sigma}\) is more precise than the original state \(\hat{\sigma}'\) restricted to the final state \(\hat{\sigma}''\)), (2) \(\hat{\sigma} \sim_\sigma \sigma\) (meaning that \(\hat{\sigma}\) is an over-approximation of \(\sigma\)), and (3) \(\hat{\sigma} \sim_\rho \rho\) (meaning that, according to \(\hat{\sigma}\), \(\rho\) is an over-approximation of \(\rho\)); then, there must exist a concrete state \(\sigma'\) and a concrete value \(\rho'\), such that: (i) \(\sigma.\alpha(\rho) \sim_\sigma (\sigma', \rho')\), (ii) \(\hat{\sigma}' \land \hat{\rho} \sim_\rho \rho \) (meaning that the restriction of the final state, \(\hat{\sigma}'\) to \(\hat{\sigma}\)) is an over-approximation of the final concrete state, \(\sigma'\), and (iii) \(\hat{\sigma}' \land \hat{\rho} \sim_\rho \rho \) (meaning that \(\hat{\rho}'\) is an over-approximation of \(\rho'\) according to \(\hat{\sigma}'\)).

**[Weakening].** This constraint states that, if \(\hat{\sigma}\) is more \(\preceq\)-precise than \(\hat{\sigma}'\) and, according to \(\hat{\sigma}\), \(\rho\) is an over-approximation of \(\rho\), then \(\hat{\rho}\) is also an over-approximation of \(\rho\) according to \(\hat{\sigma}'\).

Next, Theorem 3.2 states that the semantics of GIL preserves soundness relations. It uses the standard liftings of \(\sim_\sigma\), \(\sim_s\), and \(\leq_s\) to call stacks and configurations (cf. Appendix B).
Theorem 3.2 (Soundness). Let \( SR = \langle \land, \sim, \bowtie \rangle \) be a soundness relation for \( \hat{S} = \langle |\hat{S}|, \hat{V}, A \rangle \) in terms of \( S = \langle |S|, V, A \rangle \) and \( \preceq \) the pre-order induced by \( \sim \). It holds that:

\[
\hat{c}f' \sim \hat{c}f' \land \hat{c}f \preceq \hat{c}f' \land \hat{c}f \sim_s \hat{c}f' \implies \exists \hat{c}f' . \hat{c}f \sim_s \hat{c}f' \land \hat{c}f \sim_{\hat{c}f} \hat{c}f'
\]

From there, by choosing \( \hat{c}f' \equiv \hat{c}f' \land \hat{c}f'' \), we obtain the desired soundness result.

Corollary 3.3 (Soundness). Let \( SR = \langle \land, \sim, \bowtie \rangle \) be a soundness relation for \( \hat{S} = \langle |\hat{S}|, \hat{V}, A \rangle \) in terms of \( S = \langle |S|, V, A \rangle \) and \( \preceq \) the pre-order induced by \( \sim \). It holds that:

\[
\hat{c}f \sim \hat{c}f' \land (\hat{c}f' \bowtie \hat{c}f) \sim_s \hat{c}f \implies \exists \hat{c}f' . \hat{c}f \sim_s \hat{c}f' \land \hat{c}f' \sim_{\hat{c}f'} \hat{c}f'
\]

This corollary states that, if we have an abstract GIL trace and the initial concrete configuration \( \hat{c}f \) is over-approximated by the initial abstract configuration strengthened with the information of the final abstract configuration, then there exists a concrete GIL trace starting from \( \hat{c}f' \) such that the final abstract configuration is an over-approximation of the final concrete configuration.

3.2 Concrete-Symbolic Soundness

We identify a class of relations between symbolic memory models and concrete memory models from which one can construct soundness relations between the corresponding symbolic state models and concrete state models.

Symbolic Memory Interpretation. Intuitively, a symbolic memory model \( \hat{M} = \langle |\hat{M}|, A, \hat{\varepsilon} \rangle \) is related to a concrete memory model \( M = \langle |M|, A, \varepsilon \rangle \), if there is an interpretation function \( I \) mapping the memories in \( |\hat{M}| \) to memories in \( |M| \). Memory interpretation must preserve actions, meaning that: if an action can be executed in a given symbolic memory \( \hat{\mu} \) under a path condition \( \pi \), then it can also be also be executed in any interpretation of \( \hat{\mu} \) that satisfies both the original path condition, \( \pi \), and the path condition generated by the action, \( \pi' \). Furthermore, the output concrete memory must be an interpretation of the output symbolic memory. The notion of interpretation is made precise in Definition 3.4.

Definition 3.4 (Symbolic Memory Interpretation). Given a symbolic memory model \( \hat{M} = \langle |\hat{M}|, A, \hat{\varepsilon} \rangle \) and a concrete memory model \( M = \langle |M|, A, \varepsilon \rangle \), an interpretation of \( \hat{M} \) with respect to \( M \) is a function \( I : |\hat{M}| \rightarrow (\hat{X} \rightarrow V) \rightarrow |M| \) such that:

\[
\hat{\mu}.\alpha(\hat{e}, \pi) \sim (\hat{\mu}', \hat{e}', \pi') \land \mu = I(\hat{\mu}, e) \land \langle \pi \land \pi' \rangle_e = \text{true} \implies \exists \mu'. \mu' = I(\hat{\mu'}, e) \land \mu.\alpha(\hat{e}') = (\mu', \hat{e}')
\]

(1)

Note that the interpretation function is parametric on a logical environment, \( e \), which maps symbolic variables to concrete values. Onward, we write \( \llbracket e \rrbracket_e \) to denote interpretation of logical expressions under \( e \), defined in the standard way.

Allocator Interpretation and Restriction. Before we proceed to the lifting of memories to states, we introduce the notions of interpretation and restriction for allocators. The interpretation is straightforwardly defined as follows.

Definition 3.5 (Symbolic Allocator Interpretation). Given a symbolic allocator \( \hat{AL} = \langle |\hat{A}|, \hat{V} \rangle \) and a concrete allocator \( AL = \langle |AL|, V \rangle \), an interpretation of \( \hat{AL} \) with respect to \( AL \) is a function \( I_{AL} : |\hat{AL}| \rightarrow (\hat{V} \rightarrow V) \rightarrow |AL| \) such that:

\[
\hat{\xi}.\text{alloc}(j) \rightarrow_Y (\hat{\xi}', \hat{\xi}) \land \xi = I_{AL}(\hat{\xi}, e) \implies \exists \xi'. \xi' = I_{AL}(\hat{\xi}', e) \land \xi.\text{alloc}(j) \rightarrow_{\varepsilon(\hat{Y})} (\xi', e(\hat{\xi}))
\]

On the other hand, restriction is more involved. A restriction operator on allocator records, \( |: |AL| \rightarrow |AL| \rightarrow |AL| \), is defined algebraically, in the same way as in §3.1. The intuition behind
it, however, is different. In this setting, it makes sense to talk about $\xi \models \xi'$ only if $\xi'$ has allocated the same values as $\xi$, and possibly more. Then, $\xi \models \xi'$ describes an allocator record that necessarily follows the allocation of $\xi'$ on values already allocated by $\xi'$, but not yet by $\xi$.

A restriction operator is said to be preserved by an allocator $\mathcal{A}L = \langle |\mathcal{A}L|, V \rangle$ if it satisfies the following two properties:

$$\text{RMono-Alloc}$$

$$\xi.\text{alloc}(j) \to_Y (\xi', v) \implies \xi' \subseteq \xi$$

$$\text{FutureToPastAlloc}$$

$$\xi.\text{alloc}(j) \to_Y (\xi', v) \land \xi'' \equiv \xi \models \xi' \implies \xi''.\text{alloc}(j) \to_Y (\xi', \xi'', v)$$

We say that $\models$ is a restriction operator on an allocator $\mathcal{A}L = \langle |\mathcal{A}L|, V \rangle$, if $\models$ is a restriction operator on the carrier set $|\mathcal{A}L|$ and $\models$ is preserved by $\mathcal{A}L$.

We turn our attention now to the [FutureToPastAlloc] property, which merits further discussion. In §3.1, we showed how to use restriction to transfer state-level information from the final abstract state to the initial abstract state in order to filter out initial concrete states for which the concrete execution diverges from the given abstract trace. Here, similarly, restriction allows us to use future allocation information to direct present allocation to the desired traces. This is essential, as both the allocation of locations and symbolic variables can be non-deterministic. We discuss a simplified version of [FutureToPastAlloc] given diagrammatically in Figure 8; the full version is a straightforward generalisation. This property states that, if we allocate using $\xi$ at site $j$ and obtain $\xi'$ and a value $v$, then allocation using $\xi$ restricted to $\xi'$ at the same site must yield the same allocator and the same value. Combined with the definition of the lifting from memories to states, given shortly, this property effectively directs concrete allocation to the appropriate traces.

**Lifting Interpretations.** Given an interpretation $I : |\mathcal{M}| \to (\hat{X} \to \{\top, \bot\}) \to |\mathcal{M}|$ of a symbolic memory model $\hat{M} = \langle |\hat{M}|, A, e_{\hat{a}} \rangle$ in terms of a concrete memory model $M = \langle |M|, A, e_a \rangle$, the candidate soundness relation $RT(I) = \langle |, \sim_s, \sim_v \rangle$ for $\text{SST}(\hat{M})$ in terms of $\text{CST}(M)$ is defined as follows:

$$\langle \hat{\mu}, \hat{\rho}, \hat{\xi}, \pi \rangle \models (\_,\_,\xi',\pi')$$

$$\hat{\sigma} \sim_s \sigma$$

$$\langle \_,\_,\_,\pi \rangle \vdash \hat{e} \sim_v \hat{v}$$

where:

$$\text{Mod}(\langle \hat{\mu}, \hat{\rho}, \pi, \hat{\xi} \rangle) \triangleq \left\{ ((\mu, \rho, \xi) | \pi \models = \text{true} \land \mu = I(\hat{\mu}, \epsilon) \land \rho = I(\hat{\rho}, \epsilon) \land \xi = I_{\mathcal{A}L}(\hat{\xi}, \epsilon)) \right\}$$

Theorem 3.6 states that, given a memory interpretation $I$, $RT(I)$ is a soundness relation as per Definition 3.1. Hence, we conclude, appealing to Theorem 3.2, that $RT(I)$ is preserved by the semantics of GIL. This means that, in order to prove the soundness of their analyses, the users of Gillian only have to provide an interpretation function linking their symbolic memories to their concrete memories and prove that that interpretation preserves the actions exposed by the memories, meaning that they must satisfy Equation 1.

**Theorem 3.6 (Soundness Relation - Lifting).** Let $I$ be an interpretation of a symbolic memory model $\hat{M}$ in terms of a concrete memory model $M$; then, $RT(I) = \langle |, \sim_s, \sim_v \rangle$ is a soundness relation for $\text{SST}(\hat{M})$ in terms of $\text{CST}(M)$.
3.3 While: Sound Symbolic Analysis

The interpretation of While symbolic memories in terms of While concrete memories, $I_{\text{Wh}}$, is inductively defined as follows:

$$I_{\text{Wh}}(\emptyset, \varepsilon) \triangleq \emptyset$$

**CELL**

$$l = [\hat{e}]_{\varepsilon}, \quad v = [\hat{e}']_{\varepsilon}$$

$$I_{\text{Wh}}(\hat{e}.p \mapsto \hat{e}', \varepsilon) \triangleq I_{\text{Wh}}(\hat{e}.p \mapsto v)$$

**UNION**

$$\mu_1 = I_{\text{Wh}}(\hat{\mu}_1, \varepsilon), \quad \mu_2 = I_{\text{Wh}}(\hat{\mu}_2, \varepsilon)$$

$$I_{\text{Wh}}(\hat{\mu}_1 \uplus \hat{\mu}_2, \varepsilon) \triangleq \mu_1 \uplus \mu_2$$

This interpretation is standard: While memories are interpreted piecemeal and the interpretations are composed together using the disjoint union operator.

Lemma 3.7 states that $I_{\text{Wh}}$ preserves the actions of While, $A_{\text{Wh}} = \{\text{lookup, mutate, dispose}\}$. Its proof is straightforward, requiring only a case analysis on the rules given in Figure 5. This is much simpler than the customary inductive proofs on semantic derivations, which underpin standalone soundness proofs.

**LEMMA 3.7 (While: Memory Interpretation).** $I_{\text{Wh}}$ is an interpretation of $\hat{\hat{M}}_{\text{Wh}}$ with respect to $M_{\text{Wh}}$.

We conclude with the instantiation of Theorem 3.2 to While.

**THEOREM 3.8 (While: Soundness).** Given $\langle \downarrow, \sim_s, \sim_v \rangle = RT(I_{\text{Wh}})$, it holds that:

$$\hat{\hat{c}}f \sim^{*}_{\text{Wh}} \hat{\hat{c}}f' \land (\hat{\hat{c}}f | \hat{\hat{c}}f') \sim_s cf \implies \exists cf'. cf \sim^{*}_{\text{Wh}} cf' \land cf' \sim_s cf'$$

4 PARAMETRIC VERIFICATION

We extend the semantics of GIL with support for separation logic (SL) specifications. This allows the general semantics to:

- (i) use SL specifications to jump over procedure calls instead of re-executing a procedure at each call site; and
- (ii) use symbolic execution to verify SL specifications efficiently instead of re-implementing an SL proof system from scratch. To achieve this, we introduce a parametric assertion language, whose semantics is defined in terms of an underlying state model. In this way, users of Gillian gain access to an out-of-the-box verification tool for their target languages, while only having to provide a minimal description of their own specific assertions (onward, core predicates) in terms of the actions of the state model.

The section is structured as follows: §4.1 discusses the type of verification guarantee that can be obtained from the analysis presented in §2 and its main shortcomings; §4.2 presents our parametric assertion language together with its semantics; §4.3 describes the extension of the semantics of GIL to account for the use of SL-specifications and predicates; and, finally, §4.4 shows how to leverage the proposed infrastructure to obtain a verification tool for While.

4.1 Naïve Verification

The symbolic analysis presented in §2 is sound in the following sense: if we pick a concrete execution that follows the same path as that of the given symbolic execution and if the initial symbolic state is an over-approximation of the initial concrete state; then, we are guaranteed to terminate in a concrete state that is also over-approximated by the final symbolic state. The key point here is that we are only referring to concrete executions that follow the same path as that of the given symbolic execution; nothing is said about the others.

Note, however, that if the symbolic execution does not branch, then no restriction needs to be imposed on the corresponding concrete executions. This intuition is captured by Theorem 4.1, below, using the notion of restriction (cf. §3.1): essentially, a symbolic execution is guaranteed not to branch if the initial symbolic state is as $\subseteq$-precise as the final symbolic state.
Theorem 4.1 (Naïve Verification). Let $SR = \langle |, \sim_s, \sim_v \rangle$ be a soundness relation for $\hat{S} = \langle |\hat{S}|, \hat{V}, A \rangle$ in terms of $S = \langle |S|, V, A \rangle$ and $\leq$ the pre-order induced by $\sim_s$. It holds that:

$$\hat{c}\phi \sim \hat{c}\phi' \land \hat{c}\phi \subseteq \hat{c}\phi' \land \hat{c}\phi \sim_s cf \implies \exists cf'. cf \sim^* cf' \land cf' \sim_s cf'$$

Theorem 4.1 gives us a standard verification guarantee for programs that do not branch. Most programs, however, do branch and are, therefore, not in the conditions of the theorem. To account for branching, we need a mechanism for merging symbolic execution traces. With such a mechanism, one can re-interpret Theorem 4.1 as follows: if a concrete execution follows the same path as that of one of the symbolic execution traces that are merged together in the given symbolic trace, then the conclusions of the theorem hold.

In general, merging symbolic traces is not easy, as different traces may describe different structures in memory. In the symbolic execution literature, many techniques address this problem, such as predicate abstraction [Hala and McMillan 2006; Pasareanu et al. 2007] or the combination of guarded unions with carefully crafted merging algorithms [Torlak and Bodík 2014]. Here, we use SL predicates for describing inductive data structures in memory, allowing us to fold symbolic traces that operate on different unfoldings of the same data structure.

4.2 Parametric Assertion Language

Given a set of core predicates $\Delta$ to be provided by the user of the framework, GIL assertions, $P, Q \in A_\Delta$, include: the empty memory assertion $\text{emp}$, boolean logical expressions, $\pi \in \Pi$, core predicate assertions, $\delta(e)$ with $\delta \in \Delta$, user-defined predicate assertions, $\text{pn}(e)$ (where $\text{pn} \in \mathcal{PN} \subset S$, the set of predicate names), and the standard separating conjunction, $P \ast Q$. User-predicate definitions are of the form $\text{pred} \text{pn}(x) := P_0; \ldots; P_n$ where: $\text{pn}$ is the predicate name, $x$ its argument, and $P_0, \ldots, P_n$ are $n$ alternative predicate definitions, each a GIL assertion. This is a presentational simplification, as predicates in general can have more than one argument. To model folded predicate assertions in a state with values $v \in V$, we introduce the notion of value predicates. A value predicate $\omega \in \Omega_V$, is a pair $(pn, v)$, written $\text{pn}(v)$ for legibility, consisting of a predicate name, $pn$, and a value, $v$. Onward, we refer to value predicates simply as predicates.

Predicate States. Unsurprisingly, in order to define the interpretation of assertions, we require that the carrier set of the parameter state model, $|S|$, forms a partial commutative monoid [Calcagno et al. 2007], under a given composition operator, $\circ$, and neutral element $\emptyset$. Furthermore, we require all the state-generating functions to be frame preserving. Put formally:

$$\begin{align*}
\text{FRAME-SETVAR} & \quad \sigma.\text{setVar}(x, v) = \sigma' \\
\text{(} (\sigma \circ \sigma_f).\text{setVar}(x, v) = \sigma' \circ \sigma_f \quad \end{align*}$$

$$\begin{align*}
\text{FRAME-SETSTORE} & \quad \sigma.\text{setStore}(\rho) = \sigma' \\
\text{(} (\sigma \circ \sigma_f).\text{setStore}(\rho) = \sigma' \circ \sigma_f \quad \end{align*}$$

$$\begin{align*}
\text{RMONO-ACTION} & \quad \sigma.\alpha(v) \sim (\sigma', -) \\
\text{(} (\sigma \circ \sigma_f).\alpha(v) \sim ((\sigma' \circ \sigma_f), -) \quad \end{align*}$$

Finally, we extend a given state model with support for predicates. We say that a state model supports predicates if it exposes an action $\text{getP}$ for recovering a given predicate from the state and an action $\text{setP}$ for adding a new predicate to the state. Given an arbitrary state model $S = \langle |S|, V, A \rangle$, we construct a new state model with support for predicates by coupling $|S|$ with a list of predicates $\sigma \in \Omega_V$. There, $\text{getP}$ simply removes the required predicate from $\sigma$, while $\text{setP}$ adds it to $\sigma$. This lifting is formally described below.

Definition 4.2 (Predicate State Constructor (PST)). The predicate state constructor $\text{PST} : S \rightarrow S$ is defined as $\text{PST}((|S|, V, A)) \triangleq (|S'|, V, A \uplus \{\text{setP}, \text{getP}\})$, where:
Core Predicate Interpretation. In order to define the semantics of their assertions, the users of the framework need to describe the meaning of their core predicates in terms of the actions exposed by the parameter state model. Informally, each core predicate \( \delta \in \Delta \) is associated with two actions: (i) the getter action, \( \text{get}_\delta \), for recovering its footprint from the state; and (ii) the setter action, \( \text{set}_\delta \), for extending the state with the footprint of the core predicate.

**Definition 4.3 (Core Predicate Action Interpretation).** A core predicate action interpretation is a 4-tuple \( A I = (\Delta, A, \text{set}, \text{get}) \) consisting of a set of core predicates \( \Delta \), a set of actions \( A \), and two functions \( \text{set}, \text{get} : \Delta \rightarrow A \); we write \( \text{set}_\delta \) for \( \text{set}(\delta) \) and \( \text{get}_\delta \) for \( \text{get}(\delta) \). A core predicate action interpretation \( (\Delta, A, \text{set}, \text{get}) \) is said to be well-formed with respect to a state model \( S = (|S|, V, A) \) if and only if, for all core predicates \( \delta \in \Delta \), it holds that:

\[
\sigma.\text{get}_\delta (v) \rightsquigarrow \sigma' \iff \sigma'.\text{set}_\delta (v) \rightsquigarrow \sigma
\]  

(2)

This well-formedness constraint captures the intuition behind getters and setters and states that they have to be, essentially, each other’s inverses.

Given a state model \( S = (|S|, V, A) \) and a core action interpretation \( (\Delta, A, \text{set}, \text{get}) \), the induced action interpretation of an assertion \( P \in \mathcal{A}_\Delta \) is a pair of functions consisting of the getter and setter of \( P \), respectively, \( \text{get}_P \) and \( \text{set}_P \). Formally, Figure 9 defines two induced functions:

- \( \text{set}_P : \mathcal{A}_\Delta \rightarrow |S| \rightarrow (\hat{X} \cup X \rightarrow V) \rightarrow |S| \) \( \quad (\sigma' = \text{set}_P^S(P, \sigma, \theta) \equiv_{pp} \sigma.\text{set}_P(\theta) \rightsquigarrow \sigma') \)
- \( \text{get}_P : \mathcal{A}_\Delta \rightarrow |S| \rightarrow (\hat{X} \cup X \rightarrow V) \rightarrow |S| \) \( \quad (\sigma' = \text{get}_P^S(P, \sigma, \theta) \equiv_{pp} \sigma.\text{get}_P(\theta) \rightsquigarrow \sigma') \)

mapping each assertion \( P \in \mathcal{A}_\Delta \) to its getter and setter, respectively. The definition makes use of a substitution, \( \theta \), essentially a function mapping variables and logical variables to values.

Lemma 4.4 states that, if we are given a well-formed core predicate interpretation, the induced getter and setter of every assertion \( P \) satisfy the well-formedness constraint stated in Equation 2. Finally, Theorem 4.5 establishes that assertion interpretation preserves soundness relations. Hence, if we execute the getter/setter of a given assertion \( P \) in a given abstract state, and if we are given an initial concrete state that is over-approximated by the initial abstract state strengthened with the information of the final abstract state, then the concrete execution of the getter/setter of \( P \) will yield a final state that is over-approximated by the final abstract state.

**Lemma 4.4 (Assertion Interpretation).** Let \( (\Delta, A, \text{set}, \text{get}) \) be a well-formed core predicate interpretation with respect to a predicate state model \( S = (|S|, V, A) \); then, it holds that: \( \sigma.\text{set}_P(\theta) \rightsquigarrow \sigma' \) if and only if \( \sigma'.\text{get}_P(\theta) \rightsquigarrow \sigma \).

**Theorem 4.5 (Assertion Interpretation - Soundness).** Let \( SR = (\sim, \sim_s, \sim_v) \) be a soundness relation for \( \hat{S} = (|\hat{S}|, \hat{V}, A) \) in terms of \( S = (|S|, V, A) \) and \( \leq \) the pre-order induced by \( \sim_s \); and let
We extend the syntax of GIL commands with logical commands for interacting with predicate assertions and specifications in state models with support for predicates. Besides the original commands \( c \in C_A \), we include: a command fold \( pn(e) \) with \( \langle j; (\hat{x}_i : e_i) \rangle_{i \in \mathbb{N}} \) for folding the predicate denoted by \( pn(e) \)\(^5\); a command unfold \( pn(e) \) for unfolding the predicate denoted by \( pn(e) \); and, a command \( x := e_1(e_2) \) with \( \langle j; (\hat{x}_i : e_i) \rangle_{i \in \mathbb{N}} \) for executing the procedure denoted by \( e_1 \) with the argument denoted by \( e_2 \) using the procedure specification instead of executing the body of the procedure. Observe that the standard procedure call command remains part of the syntax, as it is included in \( c \in C_A \); to avoid confusion, we refer to the logical procedure call as spec call. Finally, we write \( p \) and \( \text{proc} \) to respectively denote GIL programs and procedures that use logical commands.

Extended GIL Syntax

\[ c \in C_A \overset{\text{def}}{=} c \in C_A \mid x := e_1(e_2) \text{ with } \langle j; (\hat{x}_i : e_i) \rangle_{i \in \mathbb{N}} \mid \text{fold } pn(e) \text{ with } \langle j; (\hat{x}_i : e_i) \rangle_{i \in \mathbb{N}} \mid \text{unfold } pn(e) \]

\[ \text{proc } \in \text{Proc}_A \overset{\text{def}}{=} \text{proc } f(x)(y) \]

\[ p \in \text{Prog}_A : F \rightarrow \text{Proc}_A \]

\(^5\)(ignoring the additional symbolic paraphernalia for the moment)
GIL specifications have the form \( \{P\} f(x) \{Q, e\} \), where \( P \) and \( Q \) are the pre- and post-condition of the procedure with identifier \( f \) and parameter \( x \), and \( e \) denotes its return value. Procedures are allowed to have multiple specifications; we use \( p.\text{specs} . f . j \) to refer to the \( j \)-th specification of the procedure with identifier \( f \) in the program \( p \). Note that both specifications and predicate definitions may include logical variables. Accordingly, the fold and spec call commands allow the programmer to provide the appropriate bindings for these variables. At the implementation level, Gillian includes a search algorithm that automatically finds the appropriate bindings. A description of this algorithm, however, is out of the scope of this paper.

An excerpt of the semantics of GIL logical commands is given in Figure 11 (cf. Appendix C). The rules have the same structure as those of GIL commands (given in Figure 1). Below, we give a brief description of their behaviour.

**Fold**

The rule folds the predicate denoted by \( pn(e) \). First, it obtains the \( j \)-th definition of the predicate to be folded, \( P_j \), and uses its getter, \( get_{P_j} \), to remove the corresponding footprint from the current state; then, it uses the action \( set_P \) to extend the current state with the folded predicate.

**Spec Call**

The rule executes the procedure with the identifier denoted by \( e \) with the argument \( e_0 \) using its \( j \)-th specification. In order to “execute” the procedure, it uses the getter of the precondition, \( get_P \), to remove the corresponding footprint from the current state, and the setter of the postcondition, \( set_Q \), to produce the corresponding footprint in the obtained state.

Finally, Theorem 4.6 states that the execution of logical commands preserves correctness relations. At first glance, it coincides with the Naïve Verification Theorem (Theorem 4.1). However, since logical commands offer us a mechanism to merge symbolic traces together, the theorem is applicable to programs which branch and loop.

**Theorem 4.6 (Verification)**. Let \( SR = \langle [\cdot], \sim_s, \sim_v \rangle \) be a soundness relation for \( \hat{S} = \langle [\hat{S}], \hat{V}, A \rangle \) in terms of \( S = \langle [S], V, A \rangle \) and \( \leq \) the pre-ordered induced by \( \sim_s \). It holds that:

\[
\hat{c}_f \sim^* \hat{c}_f' \land \hat{c}_f = \hat{c}_f' \lor \hat{c}_f \sim_s \hat{c}_f' \quad \Longrightarrow \quad \exists \hat{c}_f'. \hat{c}_f \sim^* \hat{c}_f' \land \hat{c}_f' \sim_s \hat{c}_f'
\]

### 4.4 While: Verification

The only core predicate for While is the \( \text{cell}(\langle l, p, v \rangle) \) predicate, whose footprint is a single heap cell and which states that the property \( p \) of the object at location \( l \) has value \( v \). We also introduce the setter and the getter actions for the \( \text{cell} \) core predicate, \( \text{setCell} \) and \( \text{getCell} \). The rules for \( \text{setCell} \) are given below (cf. Appendix C). One can observe that those rules are similar to the rules of \( \text{mutate} \), except that they produce the resource of the \( \text{cell} \) core predicate. In fact, we could have chosen to have only \( \text{setCell} \) and \( \text{getCell} \), but pay the price of complicating While-to-GIL to pre-emptively remove the resource that \( \text{setCell} \) produces and re-produce the resource that \( \text{getCell} \) consumes, cluttering the presentation.
We write $\sigma \mapsto→ \text{Action Fixes}$. The first step toward having automatic error correction is the appropriate modelling of action errors during execution. This approach streamlines the formalism, avoiding redundancy, and leads to a modular implementation, which has no code duplication.

Instead of redesigning a bi-abductive analysis from scratch, we maintain the semantics of GIL unaltered and, instead, instrument the parameter state model with a mechanism for on-the-fly correction of action errors during execution. This approach streamlines the formalism, avoiding redundancy, and leads to a modular implementation, which has no code duplication.

### Action Fixes

The first step toward having automatic error correction is the appropriate modelling of action errors. To this end, we modify the signature of the action execution function, $\text{ea}$, exposed by every state model, in order to account for the cases in which the resource required by the parameter action, $\alpha$, is not part of the given state.

$$\text{ea} : A \rightarrow |S| \rightarrow V \rightarrow \phi(|S| \times V) \cup \phi(A)$$

We write $\sigma,\alpha(v) \rightsquigarrow S(\sigma', v')$ to mean that $\alpha$ terminates successfully when executed on $\sigma$, generating the state $\sigma'$ and the value $v'$, and $\sigma,\alpha (v) \rightsquigarrow F(P)$ to mean that the state model was not able to execute the action $\alpha$ due to not having the required resource. The assertion $P$, which we refer to as the action fix, describes a possible way of completing the original state so that the action can be performed successfully. Unsurprisingly, fixes are assumed to work:\footnote{This is for the user of Gillian to prove.} meaning that, if we extend the original state with the generated fix, it must be possible to execute the action successfully. This constraint is formally described by the equation below:

$$\sigma,\alpha(v) \rightsquigarrow F(P) \land \sigma,\text{set}(P) \rightsquigarrow \sigma'' \quad \implies \quad \exists \sigma', v'.\sigma'', \alpha(v) \rightsquigarrow S(\sigma', v')$$ (5)
We note that, since we model fixes using assertions, the underlying parameter state model must have support for predicates. Additionally, only the fixes that talk about logical variables that existed in the initial state are admissible.

**Bi-abductive Analysis.** In order to define the bi-abductive analysis, we instrument the parameter state model so that every time an action generates a fixable error, the corresponding fix is applied and the execution continues. The bi-abductive execution also keeps track of all the applied fixes to allow for the generation of procedure specifications from from bi-abductive execution traces.

Given a state model with support for predicates $S = (\langle S \rangle, V, A)$, we construct a new state model $\sigma \in |S|$ with the footprint of the corresponding fix and continue with the execution. The lifting is formally described below. We write $\uparrow v$ to denote the upcast of a value to a boolean expression.

**Definition 5.1 (BiState Constructor (BiST)).** The bi-abductive state constructor $\text{BiST} : \mathbb{S} \rightarrow \mathbb{S}$ is defined as $\text{BiST}((\langle S \rangle, V, A)) \triangleq (\langle S' \rangle, V, A)$, where:

- $|S'| = |S| \times \mathcal{A}_\Delta$
- $\text{setVar}_{bi}(⟨⟨\sigma, P⟩, x, v⟩) \triangleq (\text{setVar}(\sigma, x, v), P)$
- $\text{setStore}_{bi}(⟨⟨\sigma, P⟩, \rho⟩) \triangleq (\text{store}(\sigma), P)$
- $\text{store}_{bi}(⟨\sigma, −, −⟩) \triangleq \text{store}(\sigma)$
- $\text{ee}_{bi}(⟨\sigma, −, −⟩, e) \triangleq \text{ee}(\sigma, e)$
- $\text{ea}_{bi}(⟨⟨\sigma, P⟩, v⟩) \triangleq \{((\sigma', P), v') \mid \sigma.\alpha(v) \rightsquigarrow S(\sigma', v')\}$ if $\alpha \neq \text{assume}$
- $\text{ea}_{bi}(\text{assume}, ⟨⟨\sigma, P⟩, v⟩) \triangleq \{((\sigma', P \cdot \pi), v') \mid [\pi] = \pi \land \sigma.\text{assume}(v) \rightsquigarrow S(\sigma', v')\}$
- $\text{ea}_{bi}(⟨⟨\sigma, P⟩, v⟩) \triangleq \{(\sigma'', P \cdot Q), v') \mid [\sigma.\alpha(v) \rightsquigarrow F(Q) \land \sigma.\text{set}(\pi) \rightsquigarrow \sigma' \land \sigma'.\alpha(v) \rightsquigarrow S(\sigma'', v')\}$

Finally, Theorem 5.2 connects successful bi-abductive executions to successful parameter-state executions. Given a bi-abductive execution $p \vdash (⟨⟨\sigma, [ ]⟩, cs, i⟩) \rightsquigarrow_{\text{BiST}(S)}^* (⟨⟨\sigma', cs', j⟩⟩\sigma.\text{set}(P) \rightsquigarrow \sigma'' \land \sigma.\text{set}(P) \rightsquigarrow \sigma'' \implies p \vdash (⟨⟨\sigma'', cs, i⟩⟩\rightsquigarrow_{\text{BiST}(S)}^* (⟨⟨\sigma', cs', j⟩⟩\sigma.\text{set}(P) \rightsquigarrow \sigma'' \land \sigma.\text{set}(P) \rightsquigarrow \sigma''

**Implementation.** The bi-abductive state constructor is implemented as an OCaml functor parametric on an OCaml module of type State together with a module of type CorePred. The BiState functor precisely follows the lifting described in Definition 5.1. It is straightforward to check that the carrier type of the bi-abductive state, $t$, matches the carrier sets of the lifted state, $|S| \times \mathcal{A}_\Delta$. The CorePred module is required for extending the parameter state with the footprint of the generated fixes during the bi-abductive analysis.

### 5.2 While: Bi-abductive Analysis

Let us recall the actions of the While: $A_W = \{\text{lookup, mutate, dispose, setCell, getCell}\}$. Out of those, the two that may fail and can be fixed are $\text{lookup}$ and $\text{getCell}$; $\text{mutate}$ will always succeed, $\text{setCell}$ may fail because the resource to be set could still be in the heap, but this cannot be corrected; and $\text{dispose}$ may fail because the object might not be in the heap, but this also cannot be corrected. For $\text{lookup}$ and $\text{getCell}$, the only cause of error is the absence of the looked-up property in the heap,
and the fix is, simply, to add it. We give the fix for `lookup`; the fix for `getCell` is similar.

\[
\mu \mid \hat{e}, \pi = (\hat{\mu}', -) \\
\hat{\mu}' = \omega_{n=0} (-, p_i \mapsto -) \\
\pi + p \notin \{p_i | i=0\} \\
\hat{x} \text{ fresh} \\
\hat{\mu}.\text{lookup}([\hat{e}, p], \pi) \rightsquigarrow F(\text{cell}([\hat{e}, p, \hat{x}]))
\]

The following lemma is easily shown to hold by case analysis on the fixes.

**Lemma 5.3 (While: Correctness of Fixes).** The fixes of While satisfy Equation (5).

## 6 Gillian in the Real World: JavaScript and C

We instantiate the Gillian framework to obtain analysis tools for JavaScript and C, two real-world, widely-used programming languages. For each language, we present the concrete and symbolic memory models along with one of their actions, and discuss the challenges encountered in the process. We highlight the trustworthiness of the obtained analyses: we compile JavaScript to GIL by linking to JS-2-JSIL [Santos et al. 2018b], a thoroughly tested compiler for JavaScript; and we compile C to GIL by linking to CompCert [Leroy 2009a,b], the first verified C compiler. We evaluate the obtained tools by successfully performing whole-program symbolic testing, full verification, and automatic compositional testing on a series of data-structure libraries, including binary search trees, key-value maps, priority queues, and singly-and doubly-linked lists, purposefully written using the programming idioms specific to the two languages. This demonstrates that Gillian can be used to quickly obtain analysis tools sufficiently robust to reason about complex real-world languages.

### 6.1 Gillian-JS: Trustworthy Analysis of ES5 Strict Programs

**The JavaScript Memory Model.** We inherit the concrete and symbolic memory models of JavaScript from Santos et al. [2019], adapting them slightly to the setting of Gillian.

Concrete JavaScript memories, \(\mu \in M_{JS}\), consist of a concrete heap and a concrete domain table. The concrete heap, \(h: Symb \times S \rightarrow \mathcal{V}\), maps object locations (modelled as symbols) and property names (modelled as strings) to GIL values. This is similar to the While semantics except that, in the values, we designate a distinguished symbol \(\varnothing\) (read: none) to denote property absence: that is, if \(h(l, p) = \varnothing\), then the object at \(l\) does not have property \(p\). This ability to explicitly address the absence of properties is required because JavaScript has extensible objects. The concrete domain table, \(d: Symb \rightarrow \wp(S)\), maps objects to the sets of properties that they may have: that is, if \(p \notin d(l)\), then the object located at \(l\) is guaranteed not to have property \(p\). This, together with the use of \(\varnothing\), ensures that we have a semantics of JavaScript that respects the frame property [Reynolds 2002].

Symbolic JavaScript memories, \(\hat{\mu} \in \hat{M}_{JS}\), consist of a symbolic heap and a symbolic domain table. The symbolic heap, \(\hat{h}: \hat{E} \times \hat{E} \rightarrow \hat{E}\), maps pairs of logical expressions (property names are also modelled as logical expressions, as JavaScript has dynamic property access) to logical expressions. The signature of the symbolic domain table, analogously, is: \(\hat{d}: \hat{E} \rightarrow \hat{E}\).

We note that heaps and domain tables are connected both in the concrete and in the symbolic semantics via the heap-domain invariant: for a given location, if its domain is defined, then all of the properties of that object that are in the heap must also be in its domain.

Below, we present a rule for one of the actions of the memory model, `getProp`, which is meant to receive an object location and a property and retrieve the value of the property. This rule illustrates a branching action, which passes the constraint \(\hat{e}_p = \hat{e}_i\) back into the state after understanding that we have full knowledge about the object in question (meaning that the properties of the object in the heap, collected by \(\hat{h} \mid \hat{e}_i\), coincide with its domain) and that the looked-up property \(\hat{e}_p\) may be equal to one of its properties.
We inherit the concrete memory model of C from CompCert [Leroy e t al. 2012], with minor adaptations related to encoding C values in GIL. We define a symbolic memory model, deriving it from the concrete model and using ideas from CompCertS [Besson et al. 2017], a memory model for CompCert that uses symbolic values to improve precision. For clarity, we elide the details from the CompCert memory model related to concurrency.

Concrete C memories, $\mu \in M_C$, consist of a concrete heap and a concrete permission table. The concrete heap, $h : \text{Symb} \times \mathbb{Z} \to \mathcal{V}$, maps memory locations (modelled as symbols) and offsets (modelled as integers) to concrete C memory values. A C memory value, $mv \in \mathcal{V}$, is either a byte (integer in the range $[0, 255]$) or a three-element GIL list $[l, off, k]$, denoting the $k^{th}$ byte of the pointer to location $l$ with offset $off$. The concrete permission table, $d : \text{Symb} \times \mathbb{Z} \to \mathcal{Z}$, maps memory locations and offsets to their permissions, which indicate the allowed operations on the associated cell. We model permissions as integers from 1 to 4, representing, respectively, Nonempty, Readable, Writable, and Freeable, where Nonempty indicates that the cell only admits pointer comparison.

Symbolic C memories, $\hat{\mu} \in \hat{M}_C$, consist of a symbolic heap and a symbolic permission table, which have the same meaning as in concrete memories. Symbolic heaps, $\hat{h} : (\hat{E} \times \hat{E}) \to \hat{E}$, model locations and offsets using logical expressions, similarly to While and JavaScript. Symbolic memory values, $\hat{mv} \in \hat{E}$, are three-element lists, $[\hat{e}, k, n]$, denoting the $k^{th}$ out of $n$ bytes of the C value represented by $\hat{e}$. Luckily, we always statically know the size of a value, meaning that $k$ and $n$ will always be concrete. The signature of symbolic permission tables, expectedly, is $\hat{p} : \hat{E} \times \hat{E} \to \hat{E}$.

In Figure 12, we present one symbolic rule for the load action, which retrieves a value from the memory. Note that, when loading or storing a value, a memory chunk has to be provided to indicate the size, alignment, and type of what should be read in the memory. For clarity, we present chunks as three-element lists, $mch = [size, alignment, type]$. The load function receives a memory chunk, the location, and the offset. First, it ensures that the value is correctly aligned and that it is allowed to be read. Next, it confirms that the read part of the memory

\[ SGetProp - Branch - Found \]
\[ \hat{\mu} = (\hat{h}, \hat{d}) \]
\[ (\pi + \hat{e}_l = \hat{e}_l' \land \hat{d}(\hat{e}_l') = \hat{h}(\hat{e}_l')) \land \hat{\pi} \land (\hat{\epsilon}_p = \hat{\epsilon}_l) \land SAT \]
\[ \hat{h} = \_\lor (\hat{\epsilon}_l', \hat{\epsilon}_l \rightarrow \hat{\epsilon}) \]

\[ \hat{\mu}.\text{getProp}([\hat{\epsilon}_l, \hat{\epsilon}_p], \pi) \rightsquigarrow (\hat{\mu}, \hat{\epsilon}, \hat{\epsilon}_p = \hat{\epsilon}_l) \]

\[ SLoad - Valid Access \]
\[ \hat{\mu} = (\hat{h}, \hat{pt}) \]
\[ \pi + \hat{e}_l = \hat{e}_l' \land \hat{\epsilon}_o = \hat{e}_o' \]
\[ \pi + \hat{\epsilon}_o \mod\ alignment = 0 \]
\[ (\pi + \hat{pt}(\hat{e}_l', \hat{\epsilon}_o + i) \geq \text{Readable})[\text{size} - 1] \]
\[ (\hat{h}(\hat{e}_l', \hat{\epsilon}_o + i) = [\hat{e}', i, size - 1])[\text{size} - 1] \]
\[ \hat{\epsilon} = \text{decodeSym}(\hat{e}', \text{type}) \]

\[ \hat{\mu}.\text{load}([\text{size}, \text{alignment}, \text{type}, \hat{\epsilon}_l, \hat{\epsilon}_o], \pi) \rightsquigarrow (\hat{\mu}, \hat{\epsilon}, \text{true}) \]

Fig. 12. Symbolic load Action of C (excerpt)
represents the symbolic value \( \hat{e}' \). Finally, it decodes \( \hat{e}' \) using its type. The decoding, for example, understands if the result should be an integer or a floating-point, and of which precision.

**Implementation.** We import the concrete memory model of CompCert into Gillian by directly plugging in the appropriate OCaml module, extracted from the Coq development; the symbolic memory model we implement ourselves. We also implement directly in GIL the functions describing various internals of C, such as, for example, unary and binary operators, `malloc` and `free`. We note that, currently, Gillian-C assumes that, whenever memory is dynamically allocated, the size of the allocated chunk is known. This allows us to focus on making the analyses work and leave the complex layer of reasoning about maps with unknown domains for immediate future work.

**Trustworthiness.** We compile C to GIL via C#m (read: C-sharp-minor), one of the intermediate representations of CompCert. Therefore, similarly to Gillian-JS, the correctness of compilation for Gillian-C rests on the correctness of C-to-C#m and our C#m-to-GIL compiler. However, we have a stronger correctness guarantee for C-to-C#m than for JS-2-JSIL, as this compilation step has been already verified in Coq as part of CompCert. Moreover, as C#m and Gillian-C share the same memory model by design, we could formalise GIL in Coq and re-use the proof techniques of CompCert to obtain a fully certified C-to-GIL compiler.

### 6.3 Evaluation: Data-Structure Libraries

We evaluate Gillian-JS and Gillian-C on data-structure libraries: binary search trees, key-value maps, priority queues, singly- and doubly-linked lists, and sorted lists. To demonstrate that Gillian can handle the complexity of the memory models of JavaScript and C, we write these libraries using the programming idioms specific to the two languages: for JavaScript, we use prototype inheritance and function closures; for C, we use pointers and structures.

#### Whole-Program Symbolic Testing

We write symbolic tests with the goal of achieving full line coverage for each of the data structures. The results are given in Table 1 and they include, for each data structure: the number of tests, the number of executed GIL commands, and the obtained times.

The results indicate that whole-program symbolic testing can scale to larger code-bases. The obtained times for Gillian-JS are comparable to those of JaVerT 2.0 [Santos et al. 2019], with the number of commands run being higher, as GIL is more low-level than JSIL. The analysis of Gillian-C demonstrates the difference in complexity between the JavaScript and the C semantics: the large number of executed commands for Gillian-JS stems from the repeated execution of numerous internal functions, of which C has far fewer. It also shows that the core of the obtained time does not lie with the number of executed commands, but rather with the time spent in the first-order solver, as highlighted by the BST example, which yields more complex entailments than the other examples.

**Verification.** We specify and verify the data structures. For JS, we manually write the required predicates, using the specification techniques of JaVerT [Santos et al. 2018b]. For C, we use the type information to automatically generate recursive predicates describing structures in memory, which we then extend with required information about the values. The results are shown in Table 2. For each data structure, we give: the number of verified functions, the total number of specifications for those functions, the number of executed GIL commands, and the verification time.

#### Table 1. Whole-program Symbolic Testing: Gillian-JS (left); Gillian-C (right)

| Name | #T | GIL cmds | Time  | Name | #T | GIL cmds | Time  |
|------|----|----------|-------|------|----|----------|-------|
| BST  | 6  | 137,379  | 1.36s | BST  | 5  | 15,256   | 0.33s |
| KVMap| 3  | 67,216   | 0.35s | KVMap| 1  | 1,165    | 0.13s |
| PriQ | 6  | 61,338   | 0.42s | PriQ | 4  | 1,008    | 0.13s |
| SLL  | 6  | 28,389   | 0.21s | SLL  | 3  | 4,218    | 0.18s |
| DLL  | 6  | 30,929   | 0.21s | DLL  | 5  | 7,000    | 0.22s |
| SL   | 3  | 37,251   | 0.42s | SL   | 3  | 2,688    | 0.11s |
We can observe that, when compared to whole-program symbolic testing, the number of executed commands is dramatically lower, but the obtained times are longer. This is in line with the facts that whole-program symbolic testing does not use procedure summaries during execution and that verification involves considerable predicate manipulation. We also note that obtained times for Gillian-JS are marginally slower than those of JaVerT 2.0. This is expected, given the more general nature of Gillian. The verification times for Gillian-C are fast across the board, with the exception of binary search trees, which, as mentioned earlier, yield complex entailments that Z3 has to solve.

**Bi-abduction.** We use bi-abduction to automatically create specifications for the data structures. We note that these specifications describe the behaviour of the up to a given bound. The results are presented in Table 1. For both languages, we give the number of success and bug specs found (S/B), and the time required to find them. For JavaScript, we additionally include the number of error specs (as JavaScript programs can terminate with a user-thrown error). It is important to clarify that, for JavaScript, a bug spec means that the program terminated with a native error (e.g. a TypeError or ReferenceError); for C, a bug spec means that the program has thrown an exception (e.g. a segmentation fault). To obtain bug specs for C, we introduce errors into the code; for JavaScript, the lack of typing information in the language is sufficient on its own. Finally, we note that the obtained specifications may describe behaviour outside of the use cases of the function and could contain false positive bug reports that need to be treated afterwards either automatically or by the developer.

As for verification, the obtained times for Gillian-JS are slightly slower than those of JaVerT 2.0 due to Gillian being a general framework. We note that they are still quite long, as we have not yet solved the bi-abduction issues of JaVerT 2.0 related to the internal branching of the JavaScript semantics, and view this to be beyond the scope of this paper. The bi-abduction of Gillian-C, on the other hand, yields promising results, as it is able to detect the introduced bugs in very quick times.

### Table 2. Verification: Gillian-JS (left); Gillian-C (right)

| Name | #F | #S | GIL cmds | Time  |
|------|----|----|----------|-------|
| BST  | 5  | 5  | 9,620    | 2.26s |
| KVMap| 4  | 9  | 11,284   | 0.75s |
| PriQ | 6  | 9  | 12,823   | 1.38s |
| SLL  | 3  | 3  | 2,978    | 0.61s |
| DLL  | 3  | 3  | 4,220    | 0.82s |
| SL   | 2  | 2  | 3,152    | 0.42s |

### Table 3. Bi-abduction: Gillian-JS (left); Gillian-C (right)

| Name | S/E/B specs | Time  |
|------|-------------|-------|
| BST  | 26/0/5      | 3.98s |
| KVMap| 9/12/12     | 1.57s |
| PriQ | 15/1/12     | 3.23s |
| SLL  | 6/0/2       | 0.78s |
| DLL  | 9/0/3       | 1.04s |
| SL   | 8/0/2       | 1.09s |

### 7 RELATED WORK

There is a wide range of works on symbolic analyses for C and JavaScript, such as [Botincan et al. 2009; Jensen et al. 2009; Jourdan et al. 2015; Kashyap et al. 2014; Park and Ryu 2015]. As this paper focusses on the design of Gillian and its associated meta-theory, we centre our discussion on parametric frameworks for obtaining modular symbolic analyses in general, covering the fields of symbolic execution, abstract interpretation, and logic-based analysis and verification.

**Symbolic Execution.** Building and maintaining new symbolic execution engines for real-world programming languages is known to be a daunting task [Cadar et al. 2011; Cadar and Sen 2013]. Researchers in the field have, therefore, tried to automate this process, with tools such as ROSETTE.
and CHEF [Bucur et al. 2014], which propose two different mechanisms to automatically lift a user-provided vanilla concrete interpreter for a given target language to a fully-fledged symbolic execution engine for the same language.

More concretely, ROSETTE is an extension of RACKET [Racket 2017] which additionally provides a set of solver-aided facilities for creating symbolic values and expressing constraints on those values. With ROSETTE, the concrete interpreter of the target language is written directly in RACKET and is then symbolically interpreted using ROSETTE’s core symbolic execution engine. In contrast, CHEF takes a specially-packaged interpreter as input and executes the target language programs symbolically by symbolically executing the interpreter’s binary.

Importantly, neither ROSETTE nor CHEF can scale to real-world programming languages. ROSETTE is aimed at domain-specific languages with restricted expressive power, while CHEF is limited to languages with "moderately-sized" interpreters. Interestingly, by implementing a JavaScript symbolic execution engine natively in OCaml [Santos et al. 2019], instead of running a concrete interpreter on top of ROSETTE [Santos et al. 2018a], Fragoso Santos et al. gained a performance speed-up of two orders of magnitude.

Abstract Interpretation. When it comes to general abstract interpreters, we identify two main strands of work: those based on small-step semantics and those based on big-step semantics.

Small-Step Abstract Interpreters. Might [Might 2010] proposes a methodology for automatically deriving a family of sound, computable abstract interpreters from a given concrete interpreter written in small-step style. This methodology assumes an initial Galois connection [Cousot and Cousot 1977] between the concrete and abstract domains in order to guarantee the optimality of the derived abstract interpreters. Later, Might and others [Horn and Might 2010, 2012] apply the proposed methodology to a family of abstract machines, obtaining the so-called abstract abstract machines, which include expressive programming language features, such as: first-class control, exception handling, and state. This technique is further refined in [Sergey et al. 2013], where the authors show how to derive monadically-parameterised abstract interpreters from concrete interpreters. The additional level of parameterisation is used to capture in a single unified formalism a number of different styles of program analyses, making it easy to instrument an analysis with parameterisable strategies for improving precision and performance. In [Darais et al. 2015], the authors propose a library of Galois transformers to streamline the construction of such strategies. The appeal of this approach is that each Galois transformer can be proved sound once and for all, making it much easier to prove the soundness of the obtained analysis.

We observe that, despite offering a general methodology for designing abstract interpreters, none of the works mentioned above automates that methodology, in that one always has to manually follow it to obtain an abstract interpreter for a given language. We further note that none of these techniques has been applied to real world programming languages, such as JavaScript and C.

Big-Step Abstract Interpreters. In [Schmidt 1995], Schmidt presents a general approach for designing abstract interpreters based on co-inductively defined big-step semantics. Following similar ideas, Bodin et al. [2019] establish a general framework, the skeletal semantics, for developing concrete and abstract big-step semantics, connected with a general consistency result, leaving the user to prove a number of simple language-dependent lemmas. This work, however, has only been applied to a simple While language with no heap, making its broader applicability difficult to assess.

Logic-based Analysis and Verification. K [G. Roşu and T. Florin Şerbânuţă 2010] is a language-independent verification infrastructure instantiated to several real-world languages, such as Java, JavaScript, and C [Bogdanas and Rosu 2015; Hathhorn et al. 2015; Park et al. 2015; Stefanescu et al. 2016], and evaluated on data-structure libraries similar to those of Gillian. As in Gillian, the user of K gets the verification guarantee for free, by construction. Gillian, however, has several advantages
over $\mathbb{K}$: Gillian specifications are language-tailored; it supports compositional analyses; and it is faster. For example, the functions of the BST example are verified in $\mathbb{K}$ for C in 88.5 seconds and for JavaScript in 25.5 seconds; we verify the same functions in approx. 8 seconds for both languages.

CoreStar [Botinčan et al. 2011], was envisioned as a general back-end for tools based on separation logic, where the user would encode the assertions of their language as abstract predicates and provide CoreStar with a separation algebra describing the entailments specific to those predicates. However, CoreStar does not appear to be fit for reasoning about highly complex real-world dynamic languages, such as JavaScript. For instance, in JaVerT [Santos et al. 2018b], the authors obtained prohibitive performance even for simple examples.

Viper [Müller et al. 2016, 2017] is also a verification framework designed to serve as a back-end for tools for permission-based verification, such as separation-logic-based tools. However, Silver, the intermediate language of Viper, is not parameterisable. Hence, in order to use Viper, users have to encode their memory models in the memory model of Silver. We believe that this approach does not scale well when there is a big mismatch between the memory model to encode and that of the host language, in this case, Silver.

Iris [Jung et al. 2018, 2015] is a Coq-based framework for reasoning about the safety of concurrent programs. It provides a general methodology for designing new sound concurrent program logics, as users can encode their own program logics into Iris and leverage Iris’s soundness result to prove the soundness of their logics. Iris is, however, mainly aimed at the mechanisation of meta-theory, while the main goal of Gillian is to streamline the development of analysis tools.

JaVerT 2.0 [Santos et al. 2019] is a tool for compositional analysis of JavaScript programs, similarly to Gillian, it supports whole-program symbolic testing, full verification, and bi-abduction. While the analysis of JaVerT 2.0 is structured modularly, all of its meta-theoretical results, as well as its implementation, are specific to its intermediate language and rely on the specific concrete and symbolic memory models of JavaScript. Gillian takes the highly non-trivial step of generalising both the theory and the implementation of JaVerT 2.0 to a fully language-independent setting.

8 CONCLUSIONS AND FURTHER WORK

We have presented Gillian, a language-independent framework for the development of compositional symbolic analysis tools, and demonstrated that it can be used to reason about real-world programming languages. Thanks to its parametric meta-theory and modular implementation, Gillian can readily be used by developers to create analysis tools for their language of choice, be it toy, domain-specific, or real-world. We believe that Gillian will be of interest to a broad range of users wishing to obtain correctness guarantees for their code.

The avenues for further work on Gillian are numerous. First of all, our immediate next steps are to streamline the bi-abduction of Gillian-JS and extend the reasoning about the symbolic memory model of Gillian-C to include arbitrary dynamic memory allocation, taking inspiration for the latter from the work of [Kirchner et al. 2015] done for Frama-C.

We are also investigating ways of extending Gillian with support for reasoning about complex language features, such as events and concurrency, as well as with additional forms of analysis, such as concolic execution. The modular design of Gillian lends itself well to these purposes.

Moreover, the infrastructure of Gillian can be extended to support the analysis of systems running code written in multiple programming languages that inter-operate with each other. A noteworthy goal in this vein of research would be a joint analysis of JavaScript and WebAssembly [Haas et al. 2017], the emerging low-level language for the Web.

Finally, we plan to improve Gillian’s error reporting mechanisms and develop interactive tools, such as a trace visualiser and a code-stepper, for the debugging of programs analysed by Gillian, in order to make Gillian more accessible to a wider audience of interested users.
REFERENCES

R. Baldoni, E. Coppa, D. Cono D’Elia, C. Demetrescu, and I. Finocchi. 2018. A Survey of Symbolic Execution Techniques. ACM Computing Surveys 51, 3 (2018), 50:1–50:39.

A. Banerjee and D. A. Naumann. 2002. Secure Information Flow and Pointer Confinement in a Java-like Language. In CSFW.

J. Berdine, C. Calcagno, and P. W. O’Hearn. 2005. Symbolic Execution with Separation Logic. In APLAS. 52–68.

Frédéric Besson, Sandrine Blazy, and Pierre Wilke. 2017. CompCertS: A Memory-Aware Verified C Compiler Using Pointer as Integer Semantics. In ITP.

M. Bodin, P. Gardner, T. Jensen, and A. Schmitt. 2019. Skeletal Semantics and their Interpretations. PACMPL 3, POPL (2019).

B. Bogdanas and G. Rosu. 2015. K-Java: A Complete Semantics of Java. In POPL. 445–456.

M. Botincan, M. J. Parkinson, and W. Schulte. 2009. Separation Logic Verification of C Programs with an SMT Solver. Electr. Notes Theor. Comput. Sci. 254 (2009), 5–23.

M. Botínčan, D. Distefano, M. Dodds, R. Grigore, D. Naudžiūnienė, and M. J. Parkinson. 2011. coreStar: The Core of jStar. In Boogie.

S. Bucur, J. Kinder, and G. Candea. 2014. Prototyping Symbolic Execution Engines for Interpreted Languages. In ASPLOS.

C. Cadar, P. Godefroid, S. Khurshid, C. S. Păsăreanu, K. Sen, N. Tillmann, and W. Visser. 2011. Symbolic Execution for Software Testing in Practice: Preliminary Assessment. In ICSE. 1066–1071.

C. Cadar and K. Sen. 2013. Symbolic Execution for Software Testing: Three Decades Later. Commun. ACM 56 (2013), 82–90.

C. Calcagno, D. Distefano, J. Dubreil, D. Gabi, P. Hoismeijer, M. Luca, P. W. O’Hearn, I. Papakonstantinou, J. Purbrick, and D. Rodriguez. 2015. Moving Fast with Software Verification. In NASA Formal Methods Symposium. 3–11.

C. Calcagno, P. W. O’Hearn, and H. Yang. 2007. Local Action and Abstract Separation Logic. In LICS.

Patrick Cousot and Radhia Cousot. 1977. Abstract Interpretation: A Unified Lattice Model for Static Analysis of Programs by Construction or Approximation of Fixpoints. In POPL. ACM Press, 238–252.

P. Cousot and R. Cousot. 2004. Basic Concepts of Abstract Interpretation. In IFIP.

Ron Cytron, Jeanne Ferrante, Barry K. Rosen, Mark N. Wegman, and F. Kenneth Zadeck. 1989. An Efficient Method of Computing Static Single Assignment Form. In Conference Record of the Sixteenth Annual ACM Symposium on Principles of Programming Languages, Austin, Texas, USA, January 11-13, 1989. 25–35.

David Darais, Matthew Might, and David Van Horn. 2015. Galois transformers and modular abstract interpreters: reusable metatheory for program analysis. In OOPSLA.

Leonardo De Moura and Nikolaj Bjørner. 2008. Z3: An Efficient SMT Solver. In TACAS.

ECMA TC39. 2017. Test262 Test Suite. https://github.com/tc39/test262.

G. Roșu and T. Florin Șerbănuța . 2010. An Overview of the K Semantic Framework. Journal of Logic and Algebraic Programming 79, 6 (2010), 397–434.

A. Haas, A. Rossberg, D. L. Schuff, B. L. Titzer, M. Holman, D. Gohman, L. Wagner, A. Zakai, and JF Bastien. 2017. Bringing the Web Up to Speed with WebAssembly. In PLDI (PLDI 2017).

C. Hathorn, C. Ellison, and G. Rosu. 2015. Defining the undefinability of C. In PLDI.

D. Van Horn and M. Might. 2010. Abstracting Abstract Machines. In ICFP.

D. Van Horn and Matthew Might. 2012. Systematic Abstraction of Abstract Machines. J. Funct. Program. 22, 4-5 (2012), 705–746.

S. H. Jensen, A. Møller, and P. Thiemann. 2009. Type Analysis for JavaScript. In SAS (Lecture Notes in Computer Science), Vol. 5673. Springer, 238–255.

R. Jhala and R. L. McMillan. 2006. A Practical and Complete Approach to Predicate Refinement. In TACAS.

J-H. Jourdan, V. Laporte, X. Leroy, and D. Pichardie. 2015. A Formally-Verified C Static Analyzer. In ICFP.

J. Funct. Program.

R. Jung, R. Krebbers, J. Jourdan, A. Bizjak, L. Birkedal, and D. Dreyer. 2018. Iris from the ground up: A modular foundation for higher-order concurrent separation logic. J. Funct. Program. 28 (2018).

R. Jung, D. Swasey, F. Siewczkowski, K. Svendsen, A. Turon, L. Birkedal, and D. Dreyer. 2015. Iris: Monoids and Invariants as an Orthogonal Basis for Concurrent Reasoning. In POPL.

Vineth Kashyap, Kyle Dewey, Ethan A Kuefner, John Wagner, Kevin Gibbons, John Sarracino, Ben Wiedermann, and Ben Hardekopf. 2014. JSAI: a static analysis platform for JavaScript. In FSE. ACM.

F. Kirchner, N. Kosmatov, V. Prevost, J. Signoles, and B. Yakobowski. 2015. Frama-C: A software analysis perspective. Formal Aspects of Computing 27, 3 (2015), 573–609.

X. Leroy. 2009a. Formal verification of a realistic compiler. Commun. ACM 52, 7 (2009), 107–115.

X. Leroy. 2009b. A formally verified compiler back-end. Journal of Automated Reasoning 43, 4 (2009), 363–446.

X. Leroy, A. W. Appel, S. Blazy, and G. Stewart. 2012. The CompCert Memory Model, Version 2. Research Report RR-7987. INRIA. 26 pages.

M. Might. 2010. Abstract Interpreters for Free. In SAS.

P. Müller, M. Schwerhoff, and A. J. Summers. 2016. Viper: A Verification Infrastructure for Permission-Based Reasoning. In VMCAI.
P. Müller, M. Schwerhoff, and A. J. Summers. 2017. Viper: A Verification Infrastructure for Permission-Based Reasoning. In Dependable Software Systems Engineering.

P. W. O’Hearn. 2018. Continuous Reasoning: Scaling the Impact of Formal Methods. In LICS. 13–25.

Changhee Park and S. Ryu. 2015. Scalable and Precise Static Analysis of JavaScript Applications via Loop-Sensitivity. In ECOOP. 735.

D. Park, A. Stefanescu, and G. Rosu. 2015. KJS: a Complete Formal Semantics of JavaScript. In PLDI.

C. S. Pasareanu, R. Pelánek, and W. Visser. 2007. Predicate Abstraction with Under-Approximation Refinement. Logical Methods in Computer Science 3, 1 (2007).

Racket. 2017. The Racket Programming Language. racket-lang.org.

Mohammad Raza and Philippa Gardner. 2009. Footprints in Local Reasoning. Logical Methods in Computer Science 5, 2 (2009).

J. C. Reynolds. 2002. Separation Logic: A Logic for Shared Mutable Data Structures. In LICS. 55–74.

J. Fragoso Santos, P. Maksimovic, T. Grohens, J. Dolby, and P. Gardner. 2018a. Symbolic Execution for JavaScript. In PPDP. 11:1–11:14.

J. Fragoso Santos, P. Maksimovic, D. Naudziuniene, T. Wood, and P. Gardner. 2018b. JaVerT: JavaScript Verification Toolchain. PACMPL 2, POPL (2018), 50:1–50:33.

J. Fragoso Santos, P. Maksimovic, G. Sampaio, and P. Gardner. 2019. JaVerT 2.0: Compositional Symbolic Execution for JavaScript. PACMPL 3, POPL (2019), 66:1–66:31.

D. A. Schmidt. 1995. Natural-Semantics-Based Abstract Interpretation (Preliminary Version). In SAS. 1–18.

I. Sergey, D. Devriese, M. Might, J. Midtgaard, D. Darais, D. Clarke, and F. Piessens. 2013. Monadic Abstract Interpreters. In PLDI.

A. Stefanescu, D. Park, S. Yuwen, Y. Li, and G. Rosu. 2016. Semantics-based Program Verifiers for All Languages. In OOPSLA.

E. Torlak and R. Bodík. 2013. Growing Solver-aided Languages with Rosette. In Onward!

E. Torlak and R. Bodík. 2014. A Lightweight Symbolic Virtual Machine for Solver-Aided Host Languages. In PLDI.
A SECTION 2: PARAMETRIC EXECUTION

The Syntax of GIL:

\[
v \in V \triangleq n \in N \mid i \in \mathbb{Z} \mid s \in S \mid b \in B \mid \zeta \in \text{Symb} \mid \tau \in \mathcal{T} \mid f \in \mathcal{F} \mid \mathbb{V}
\]
\[
e \in \mathcal{E} \triangleq v \in X \mid \text{true} \mid e_1 \oplus e_2
\]
\[
c \in C_A \triangleq x : e \mid \text{ifgoto } e \mid i \mid x := e'(e) \mid x := a(e)
\]
\[
x := \text{symb} \mid x := \text{fresh} \mid \text{return } e \mid \text{fail } e \mid \text{vanish}
\]
\[
p \in \text{Proc}_A \triangleq \text{proc } f(x)(\mathbb{F})
\]

Definition A.1 (State Model). A state model \( S \in \mathbb{S} \) is a triple \( (|S|, V, A) \), consisting of: (1) a set of states on which GIL programs operate, \(|S|\), (2) a set of values stored in those states, \( V \), and (3) a set of actions that can be performed on those states, \( A \). A state model defines the following functions for acting on states (\( \equiv_{pp} \) denotes pretty-printing for readability):

- \( \text{setVar} : |S| \rightarrow X \rightarrow V \rightarrow |S| \)
- \( \text{setStore} : |S| \rightarrow (X \rightarrow V) \rightarrow |S| \)
- \( \text{store} : |S| \rightarrow (X \rightarrow V) \rightarrow |S| \)
- \( \text{ee} : |S| \rightarrow \mathcal{E} \rightarrow V \)
- \( \text{ea} : A \rightarrow |S| \rightarrow V \rightarrow \wp(|S| \times V) \)

A state model \( S = (|S|, V, A) \) is said to be proper iff it defines the following three distinguished actions: assume, symb, and fresh.

GIL Semantic Domains for \( S = (|S|, V, A) \):

- Call stacks: \( cs \in \mathbb{C}_S \triangleq |f| \mid |f, x, p, i| : cs \)
- Configurations: \( cf \in \mathbb{C}_S \triangleq (p, \sigma, cs, i) \)
- Outcomes: \( o \in \mathbb{O} \triangleq |N(v) \mid E(v) \)

GIL semantic transitions: \( \leadsto_{S} : |S| \times \mathbb{C}_S \times \mathbb{N} \times \mathbb{O} \rightarrow \wp(|S| \times \mathbb{C}_S \times \mathbb{N} \times \mathbb{O}) \).

Semantics of GIL:

ASSIGNMENT

\[
p \vdash (\sigma, cs, i) \leadsto (\sigma', cs', j)_{o'}
\]

ACTION

\[
p \vdash (\sigma, cs, i) \leadsto (\sigma'.\text{setVar}(x, v'), cs, i+1)
\]

IfGoto - True

\[
s.e.e(e) = v
\]
\[
p \vdash (\sigma, cs, i) \leadsto (\sigma', cs, j)
\]

IfGoto - False

\[
s.e.e(-e) = v
\]
\[
p \vdash (\sigma, cs, i) \leadsto (\sigma'.\text{assume}(v) \leadsto \sigma')
\]

Symb

\[
s.e.e(f) = v
\]
\[
p \vdash (\sigma, cs, i) \leadsto (\sigma'.\text{fresh}(f) \leadsto (\sigma', v'))
\]

Call

\[
s.e.e(e') = v
\]
\[
p \vdash (\sigma, cs, i) \leadsto (\sigma'.\text{setStore}(f, x, \text{store}, i+1) | cs)
\]

Return

\[
\text{cmd}(p, cs, i) = \text{return } e
\]
\[
\text{cmd}(p, cs, i) = \text{fail } e
\]

Top Return

\[
\text{cmd}(p, cs, i) = \text{return } e
\]
\[
\text{cmd}(p, cs, i) = \text{fail } e
\]

Fail

\[
\text{cmd}(p, cs, i) = \text{return } e
\]
\[
\text{cmd}(p, cs, i) = \text{fail } e
\]

\[
p \vdash (\sigma, [f], i) \leadsto (\sigma', [f], i^{(v)})
\]

Definition A.2 (Allocator Model). An allocator model \( AL \in \mathcal{A}L \) is a pair \( (|AL|, V) \) consisting of: (1) a set \( |AL| \ni \xi \) of allocators; and (2) a set \( V \) of values to allocate. It exposes the following function:

\[
\text{alloc} : |AL| \rightarrow \mathbb{N} \rightarrow \wp(V) \rightarrow |AL| \times V
\]
which satisfies the well-formedness constraint:
\[ (\xi', v) = \text{alloc} (\xi, j, Y) \implies v \in Y. \]

Logical variables: \( \hat{x} \in \hat{X} \).
Logical expressions: \( \hat{e} \in \hat{E} \triangleq v | \hat{x} \in \hat{X} | \hat{\varnothing} \hat{e} | \hat{e}_1 \oplus \hat{e}_2 \).

Additionally, we write \( \pi \in \Pi \) to denote logical expressions that that be statically typed as boolean (for example, true, false, \( \hat{x} \) and \( \hat{y} \), etc.).

**Definition A.3 (Concrete Memory Model).** A concrete memory model \( M \in \hat{M} \) is a pair \( \langle |M|, A \rangle \),
consisting of a set of concrete memories, \( |M| \ni \mu \), and a set of actions \( A \).
A concrete memory model additionally defines a function \( \text{ea} \) for acting on memories:
\[
\text{ea} : A \rightarrow |M| \rightarrow \mathcal{V} \rightarrow |M| \times \mathcal{V} \quad \text{where } (\text{ea} (\alpha, \mu, v)) = \mu.\alpha (v)
\]

**Definition A.4 (Symbolic Memory Model).** A symbolic memory model \( \hat{M} \in \hat{M} \) is a pair \( \langle |\hat{M}|, A \rangle \),
consisting of a set of symbolic memories, \( |\hat{M}| \ni \hat{\mu} \), and a set of actions \( A \).
A symbolic memory model additionally defines a function \( \hat{\text{ea}} \) for acting on memories:
\[
\hat{\text{ea}} : A \rightarrow |\hat{M}| \rightarrow \hat{E} \rightarrow \Pi \rightarrow \varnothing (|\hat{M}| \times \hat{E} \times \Pi) \quad (\hat{\mu}', \hat{\varnothing} \hat{e}', \pi') \in \hat{\text{ea}} (\alpha, \hat{\mu}, \hat{\varnothing} \hat{e}, \pi) \equiv \mu.\alpha (\hat{\mu}, \hat{\varnothing} \hat{e}, \pi) \sim (\hat{\mu}', \hat{\varnothing} \hat{e}', \pi')
\]

Concrete stores: \( \rho : X \rightarrow \mathcal{V} \) 
Symbolic stores: \( \hat{\rho} : X \rightarrow \hat{E} \)
Concrete allocators: \( \xi \in |AL|_{\mathcal{V}} \) 
Symbolic allocators: \( \hat{\xi} \in |AL|_{\hat{E}} \)
Mandatory actions: \( A_0 \triangleq \{ \text{assume, fresh, symb} \} \)

**Definition A.5 (Concrete State Constructor (CST)).** Given an allocator \( AL = \langle |AL|, \mathcal{V} \rangle \), the concrete state constructor \( \text{CST} : \hat{M} \rightarrow \hat{S} \) is defined as \( \text{CST} (\langle |M|, A \rangle) \triangleq \langle |S|, \mathcal{V}, A \cup A_0 \rangle \), where:
- \( |S| = |M| \times (X \rightarrow \mathcal{V}) \times |\mathcal{V}|_{\mathcal{V}} \)
- \( \text{setVar}(\mu, \rho, \xi, x, v) \triangleq (\mu, \rho [x \mapsto v], \xi) \)
- \( \text{setStore}(\mu, \xi, \rho) \triangleq (\mu, \rho, \xi) \)
- \( \text{store}(\_, \rho, \_) \triangleq \rho \)
- \( \text{ee}(\_, \rho, v) \triangleq \{ \mu \} \)
- \( \text{ea}(\alpha, \mu, \rho, \xi, v) \triangleq \{ (\mu', \rho, \xi, \omega') | (\mu', \omega') = \text{ea} (\alpha, \mu, v) \} \)
- \( \text{assume}(\sigma, v) \triangleq \{(\sigma, v) | v = \text{true} \} \)
- \( \text{fresh}(\mu, \rho, \xi, i) \triangleq \{ (\mu, \rho, \xi, \omega) | \xi \in \text{alloc} (i) \rightarrow \text{Symb} (\xi, \omega) \} \)
- \( \text{symb}(\mu, \rho, \xi, i) \triangleq \{ (\mu, \rho, \xi, \omega) | \xi \in \text{alloc} (i) \rightarrow \mathcal{V} (\xi, \omega) \} \)

**Definition A.6 (Symbolic State Constructor (SST)).** Given an allocator \( \hat{AL} = \langle |\hat{AL}|, \hat{E} \rangle \), the symbolic state constructor \( \text{SST} : \hat{M} \rightarrow \hat{S} \) is defined as \( \text{SST} (\langle |\hat{M}|, A \rangle) \triangleq \langle \hat{S}, \hat{E}, A \cup A_0 \rangle \), where:
- \( \hat{S} = |\hat{M}| \times (X \rightarrow \hat{E}) \times |\mathcal{V}|_{\mathcal{V}} \times \Pi \)
- \( \text{setVar}(\hat{\mu}, \rho, \xi, x, \hat{\varnothing} \hat{e}) \triangleq (\hat{\mu}, \rho [x \mapsto \hat{\varnothing} \hat{e}], \xi, \hat{\varnothing} \hat{e}) \)
- \( \text{setStore}(\hat{\mu}, \xi, \rho, \hat{\varnothing} \hat{e}) \triangleq (\hat{\mu}, \rho, \xi, \hat{\varnothing} \hat{e}) \)
- \( \text{store}(\_, \rho, \_) \triangleq \rho \)
- \( \text{ee}(\_, \rho, \hat{\varnothing} \hat{e}) \triangleq \hat{\rho} (\hat{\varnothing} \hat{e}) \)
- \( \text{ea}(\alpha, \hat{\mu}, \rho, \xi, \hat{\varnothing} \hat{e}) \triangleq \{ (\hat{\mu}', \hat{\varnothing} \hat{e}', \xi, \hat{\varnothing} \hat{e}') | (\hat{\mu}', \hat{\varnothing} \hat{e}', \hat{\varnothing} \hat{e}') = \text{ea} (\alpha, \hat{\mu}, \hat{\varnothing} \hat{e}) \} \)
- \( \text{assume}(\hat{\mu}, \rho, \xi, \pi, \hat{\varnothing} \hat{e}) \triangleq \{ (\hat{\mu}, \rho, \xi, \pi, \hat{\varnothing} \hat{e}) | \pi \wedge \hat{\varnothing} \hat{e} \text{ not UNSAT} \} \)
- \( \text{fresh}(\hat{\mu}, \rho, \xi, i) \triangleq \{ (\hat{\mu}, \rho, \xi, \omega) | \xi \in \text{alloc} (i) \rightarrow \text{Symb} (\xi, \omega) \} \)
- \( \text{symb}(\hat{\mu}, \rho, \xi, i) \triangleq \{ (\hat{\mu}, \rho, \xi, \omega) | \xi \in \text{alloc} (i) \rightarrow \hat{X} (\xi, \hat{x}) \} \)
# A.1 While: Syntax, Actions, Compilation to GIL, Concrete/Symbolic memories

## While: Syntax, Actions, Memories

Let \( ws \in Stmt_W \) denote an action in the Concrete Memory. The action of assignment is defined as

\[
  ws = x := e | ws_1; ws_2 | \text{if}(e)\{ws_1\}\text{else}\{ws_2\} | \text{while}(e)\{ws\} | \text{return}\ e \mid x := f(e)
\]

\( W \)

### Assume

\[
  \begin{array}{l}
    \text{assume } e \mid \text{assert } e \mid x := \{p_1 : e_1 | n_1\} \mid x := e \cdot p \mid e \cdot p := e' \mid \text{dispose } e
  \end{array}
\]

\( A_W \)

\[
  \begin{array}{l}
    \mu \in M_W : \text{Symb}\times S \rightarrow \mathcal{V}
  \end{array}
\]

\( \hat{\mu} \in M_W : \hat{\mathcal{E}} \times S \rightarrow \hat{\mathcal{E}}
\]

## While-to-GIL Compiler: \( C_W : Stmt_W \rightarrow \mathbb{Z} \rightarrow [C_{A_W}] \times \mathbb{Z} \)

### Assignments

- \( C_W(x := e, pc) \)
  - \( pc : x := e \)
  - \( \cdot \leftarrow pc + 1 \)

### Skip

- \( C_W(\text{skip}, pc) \)
  - \( pc : \text{ifgoto true (pc + 1)} \)
  - \( \cdot \leftarrow pc + 1 \)

### Return

- \( C_W(\text{return } e, pc) \)
  - \( pc : \text{return } e \)
  - \( \cdot \leftarrow pc + 1 \)

### Call

- \( C_W(x := f(e), pc) \)
  - \( pc : x := f e \)
  - \( \cdot \leftarrow pc + 1 \)

### If

- \( C_W(\text{ifgoto } (pc + 1)) \)
  - \( pc : \text{ifgoto (pc + 1)} \)
  - \( \cdot \leftarrow pc + 1 \)

### While

- \( C_W(\text{while } e | ws, pc) \)
  - \( pc : \text{ifgoto (not e) (pc'} + 1) \)
  - \( pc' : \text{ifgoto true } pc \)
  - \( \cdot \leftarrow pc' + 1 \)

### Assume

- \( C_W(\text{assume } e, pc) \)
  - \( pc : \text{ifgoto } e (pc + 2) \)
  - \( pc + 1 : \text{vanish} \)
  - \( \cdot \leftarrow pc + 2 \)

### Assert

- \( C_W(\text{assert } e, pc) \)
  - \( pc : \text{ifgoto } e (pc + 2) \)
  - \( pc + 1 : \text{fail } e \)
  - \( \cdot \leftarrow pc + 2 \)

### Lookup

- \( C_W(x := \text{lookup}(x, p), pc) \)
  - \( pc : x := \text{lookup}(x, p) \)
  - \( \cdot \leftarrow pc + 1 \)

### Mutate

- \( C_W(e_1, p) \rightarrow e_2, pc) \)
  - \( pc : \_ := \text{mutate}(e_1, p, e_2) \)
  - \( \cdot \leftarrow pc + 1 \)

### Dispose

- \( C_W(\text{dispose } e, pc) \)
  - \( pc : \_ := \text{dispose } e \)
  - \( \cdot \leftarrow pc + 1 \)

### While: Actions in Concrete and Symbolic Memories

- **C-Lookup**
  - \( \mu \mid_1 \{l,p\} \rightarrow (\mu, v) \)

- **S-Lookup**
  - \( \pi \vdash \hat{e} = e' \)
  - \( \hat{\mu} \mid_1 \{\hat{e}, \hat{p}, \hat{v}\} \rightarrow (\hat{\mu}, \hat{v}) \)

- **C-Mutate-Present**
  - \( \mu = \mu' \cup l \cdot p \leftarrow_\mu \mu'' = \mu' \cup l \cdot p \rightarrow v \)

- **S-Mutate-Present**
  - \( \pi \vdash \hat{e} = e'' \)
  - \( \hat{\mu} = \hat{\mu}' \cup \hat{e}'' \cdot p \leftarrow_\hat{\mu} \hat{\mu}'' = \hat{\mu}' \cup \hat{e}'' \cdot p \rightarrow \hat{e}' \)

- **C-Mutate-Absent**
  - \( (l,p) \notin \text{dom}(\mu) \)

- **S-Mutate-Absent**
  - \( \hat{\mu} \mid_1 \{\hat{e}, \hat{p}, \hat{v}\} \rightarrow (\hat{\mu}, \hat{v}, \text{true}) \)

- **C-Dispose**
  - \( \mu \mid_1 = (\_ \cup \mu') \)

- **S-Dispose**
  - \( \hat{\mu} \mid_1 \{\hat{e}, \hat{p}\} \rightarrow (\hat{\mu}', \text{true}) \)
B \ SECTION 3: PARAMETRIC SOUNDNESS

B.1 Parametric Soundness

Restriction Operators. A restriction operator $|: X \to X \to X$ on a set $X$, written $x_1 \vdash x_2$ for $(x_1, x_2)$, is a binary associative function satisfying the following properties:

\[
\begin{align*}
\text{Idempotence} & \quad x \vdash x = x \\
\text{Right Commutativity} & \quad (x_1 \vdash x_2) \vdash (x_1 \vdash x_2) = x_1 \\
\text{Weakening} & \quad x_1 \vdash x_2 x_3 = x_1 \vdash x_2 x_3 = x_1
\end{align*}
\]

A restriction operator on states $|: |S| \to |S| \to |S|$ is said to be preserved by a state model $S = \langle |S|, V, A \rangle$ if all of the state-generating functions exposed by the state model are monotonic with respect to the pre-order induced by $|$; put formally:

\[
\begin{align*}
\text{RMono-SetVar} & \quad \sigma.\text{SetVar}(x, v) = \sigma' \Rightarrow \sigma' \subseteq \sigma \\
\text{RMono-SetStore} & \quad \sigma.\text{SetStore}(\rho) = \sigma' \Rightarrow \sigma' \subseteq \sigma \\
\text{RMono-Action} & \quad \sigma.\text{Action}(v) \sim (\sigma', -) \Rightarrow \sigma' \subseteq \sigma
\end{align*}
\]

We say that $|$ is a restriction operator on a state model $S = \langle |S|, V, A \rangle$, if $|$ is a restriction operator on the carrier set $|S|$ and $|$ is preserved by $S$. Restriction operators are extended from states to configurations in the standard way: $(p, \sigma, cs, i) \vdash (p, \sigma', cs, i) \triangleq (p, \sigma' \sim c, cs, i)$.

Compatibility. A pre-order $(X, \leq)$ is compatible with a restriction operator $|$ on $X$ iff the following properties hold:

\[
\begin{align*}
|\leq \text{ Compatibility} & \quad x_1 \leq x_2 \leq x_1 \\
\leq | \text{ Compatibility} & \quad x_1 \leq x_2 \leq x_1 \\
\text{Strengthening} & \quad x_1 \leq x_1' \leq x_2 \leq x_2' \leq x_1
\end{align*}
\]

Soundness Relations. Given a relation $\sim \in X \times Y$ between two sets $X$ and $Y$, the pre-order on $X$ induced by $\sim$, written $\leq_{\sim}$, is defined as follows: $x_1 \leq_{\sim} x_2$ if and only if $\{y \mid x_1 \sim y \} \subseteq \{y \mid x_2 \sim y \}$.

Definition B.1 (Soundness Relation - States). Given two state models, $\hat{S} = \langle |\hat{S}|, \hat{V}, \hat{A} \rangle$ and $S = \langle |S|, V, A \rangle$, a soundness relation $SR$ for $\hat{S}$ in terms of $S$ is a triple $\langle |, \sim_{\hat{S}}, \sim_{\hat{V}} \rangle$, consisting of: (1) a restriction operator $|$ on $\hat{S}$; (2) a binary relation $\sim_{\hat{S}} \subseteq |\hat{S}| \times |\hat{S}|$ between $|\hat{S}|$ and $|S|$; and (3) a ternary relation $\sim_{\hat{V}} \subseteq |\hat{S}| \times \hat{V} \times \hat{V}$ between $|\hat{S}|$, $\hat{V}$, and $V$, such that $|$ is compatible with the pre-order induced by $\sim_{\hat{S}}$ (denoted by $\leq$) and the following constraints hold:

\[
\begin{align*}
\text{Store} & \quad \hat{\sigma} \sim_{\hat{S}} \sigma \Rightarrow \hat{\sigma} \vdash \hat{\sigma}.\text{store} \sim_{\hat{V}} \sigma.\text{store} \\
\text{EvalExpr} & \quad \hat{\sigma} \leq \hat{\sigma}' \land \hat{\sigma} \sim_{\hat{S}} \sigma \Rightarrow \hat{\sigma}' \vdash \hat{\sigma}'.\text{ee}(e) \sim_{\hat{V}} \sigma.\text{ee}(e) \\
\text{SetVar} & \quad \hat{\sigma} \leq \hat{\sigma}' \land \hat{\sigma} \sim_{\hat{S}} \sigma \land \hat{\sigma} \vdash \hat{\sigma} \sim_{\hat{V}} v \\
& \quad \Rightarrow \hat{\sigma}'.\text{SetVar}(x, \hat{v}) \sim_{\hat{S}} \sigma.\text{SetVar}(x, v) \\
\text{SetStore} & \quad \hat{\sigma} \leq \hat{\sigma}' \land \hat{\sigma} \sim_{\hat{S}} \sigma \land \hat{\sigma} \vdash \hat{\sigma} \sim_{\hat{V}} \rho \\
& \quad \Rightarrow \hat{\sigma}'.\text{SetStore}(\hat{\rho}) \sim_{\hat{S}} \sigma.\text{setStore}(\rho) \\
\text{Action} & \quad \hat{\sigma} \vdash \hat{\sigma} \sim_{\hat{V}} \rho \\
& \quad \Rightarrow \exists \sigma', \sigma.\text{Action}(v) \sim (\sigma', v') \land \hat{\sigma}' \sim_{\hat{S}} \sigma' \land \hat{\sigma}' \vdash \hat{\sigma}' \sim_{\hat{V}} v' \\
\text{Weakening} & \quad \hat{\sigma} \leq \hat{\sigma} \vdash \hat{\sigma} \sim_{\hat{V}} \rho \\
& \quad \Rightarrow \hat{\sigma}' \vdash \hat{\rho} \sim_{\hat{V}} \rho
\end{align*}
\]

where $\hat{\sigma} \vdash \hat{\rho} \sim_{\hat{V}} \rho$ is shorthand for: dom($\hat{\rho}$) = dom($\rho$) = $X$ and $\forall x \in X \cdot \hat{\sigma} \vdash \hat{\rho}(x) \sim_{\hat{V}} \rho(x)$.

Soundness Relations - Call stacks and Configurations

\[
\begin{array}{|c|c|c|c|}
\hline
\text{Top Frame} & \text{Inner Frame} & \text{~S-Configuration} & \text{~L-Configuration} \\
\hline
\hat{\sigma} \vdash \hat{f} \sim_{\hat{V}} f & \hat{\sigma} \vdash \hat{\rho} \sim_{\hat{V}} \rho & \hat{\sigma} \vdash \hat{\rho} \sim_{\hat{V}} \rho & \hat{\sigma} \sim_{\hat{S}} \sigma \\
\hline
\hat{\sigma} \vdash \hat{f}, x, \hat{\rho}, i : \hat{c} \sim_{\hat{V}} f, x, \rho, i : c & \hat{\sigma} \vdash \hat{\rho} \sim_{\hat{V}} \rho & \hat{\sigma} \vdash \hat{\rho} \sim_{\hat{V}} \rho & \hat{\sigma} \sim_{\hat{S}} \sigma \\
\hline
\hline
\end{array}
\]
Theorem B.2 (One-Step Soundness). Let $SR = \langle \|, \sim_s, \sim_v \rangle$ be a soundness criterion for $\hat{S} = \langle \hat{S}, \hat{V}, \hat{A} \rangle$ in terms of $S = \langle |S|, V, A \rangle$ and $\leq$ the pre-order induced by $\sim_s$. It holds that:

$$\hat{c} \preceq \hat{c}'' \wedge \hat{c} \leq \hat{c}' \mid \hat{c}_\nu' \wedge \hat{c} \sim_s \hat{c} \implies \exists \hat{c}'. \hat{c}' \preceq \hat{c}' \wedge \hat{c}' \mid \hat{c}_\nu' \sim_s \hat{c}'$$

Proof:
We proceed by case analysis on the rule that produced $\hat{c} \preceq \hat{c}''$. We only provide the proof for the variable assignment and action cases. The others cases are analogous.

1. Assume: 1. (H1) $\hat{c}' \preceq \hat{c}''$
   2. (H2) $\hat{c} \leq \hat{c}' \mid \hat{c}_\nu''$
   3. (H3) $\hat{c} \sim_s \hat{c}$

2. Case: Assignment
   2.1. $\hat{c}' = \langle \hat{σ}', \hat{c}_s, i \rangle$ [Destructing on H1 - assignment case]
   2.2. $cmd(p, \hat{c}_s, i) = x := \hat{e}$
   2.3. $\hat{σ}.ee(\hat{e}) \preceq (\hat{σ}_1, \hat{v})$
   2.4. $\hat{σ}_1.setVar(x, \hat{v}_1) = \hat{σ}''$
   2.5. $\hat{c}''' = (\hat{σ}'', \hat{c}_s, i + 1)$
   2.6. $\hat{c}' = (\hat{σ}, \hat{c}_s, i)$ [From H2 + definition of $\leq_{cf}$]
   2.7. $\hat{σ} \leq \hat{σ}' \mid \hat{c}_\nu''$
   2.8. $\hat{c}' = (\hat{σ}, \hat{c}_s, i)$ [From H3 + 2.7 + definition of $\sim_{cf}$]
   2.9. $\hat{v} \sim_s \hat{σ}$
   2.10. $\hat{σ} + \hat{c}_s \sim_v, cs$
   2.11. $cmd(p, \hat{c}_s, i) = x := e$ [From 2.2 + 2.11]
   2.12. $\hat{σ} \leq \hat{σ}' \mid \hat{c}_\nu$ [From 2.4 we know that $\hat{σ}'' \leq \hat{σ}_1$ and from 2.7 we know that $\hat{σ} \leq \hat{σ}' \mid \hat{c}_\nu''$. From $\hat{σ}'' \subseteq \hat{σ}_1$, it follows that $\hat{σ}' \mid \hat{c}_\nu \leq \hat{σ} \mid \hat{σ}_1$, from which it follows, by transitivity that $\hat{σ} \leq \hat{σ}' \mid \hat{σ}_1$.]

2.13. $\exists \sigma_1, \nu$.
   1. $σ.ee(σ) \preceq (σ_1, ν)$ [From 2.3 + 2.9 + 2.12 + Correctness relation]
   2. $σ_1 \mid \hat{σ} \sim_s σ_1$
   3. $σ_1 \mid \hat{σ} + \hat{v} \sim_v, ν$

2.14. $\exists \hat{σ}_2$: [From 2.5 + SM(Trans. strenghtening)]
   1. $(\hat{σ}_1 \mid \hat{σ}).setVar(x, \hat{v}) = \hat{σ}_2$
   2. $\hat{σ}_2 \leq \hat{σ}''$

2.15. $\hat{σ}'$: [From 2.14.1 + 2.13.2 + 2.13.3 + CR(setVar )]
   1. $σ_3 \mid \hat{σ} \sim_s σ''$
   2. $σ_1.setVar(x, ν) = \hat{σ}''$

2.16. $σ'' \mid \hat{σ} \sim_s σ'$ [From 2.14.2 + 2.15.1 + CR-Orders]

2.17. $\hat{c}' \sim_s (σ', \hat{c}_s, i + 1) = \hat{c}'$ [From 2.8 + 2.11 + 2.13.1 + 2.15.2]

2.18. $\hat{σ}'' \mid \hat{σ} + \hat{c}_s \sim_v, cs$ [From 2.10 + noting that $\hat{σ}'' \mid \hat{σ} \subseteq \hat{σ}$]

2.19. $\hat{c}' \mid \hat{σ} \sim \hat{c}'$

3. Case: Action
   3.1. $\hat{c}' = \langle \hat{σ}', \hat{c}_s, i \rangle$ [Destructing on (H1) - action case]
   3.2. $cmd(p, \hat{c}_s, i) = x := σ(\hat{e})$
   3.3. $\hat{σ}'.ee(\hat{e}) \preceq (\hat{σ}_1, \hat{v}_1)$
   3.4. $\hat{σ}_1, \hat{c}_s(\hat{v}) \preceq (\hat{σ}_2, \hat{v}_2)$
   3.5. $\hat{σ}_2.setVar(x, \hat{v}_2) = \hat{σ}''$
   3.6. $\hat{c}''' = (\hat{σ}'', \hat{c}_s, i + 1)$
3.7. \( \hat{cf} = (\hat{s}, i) \) [From H2 + definition of \( \leq \)]
3.8. \( \hat{s} \leq \hat{s}' | \hat{\alpha}'' \)
3.9. \( cf = (s, cs, i) \) [From H3 + 3.6 + definition of \( \sim \) cf]
3.10. \( \hat{s} \sim_s \sigma \)
3.11. \( \sigma \vdash \hat{c} \sim_v cs \)
3.12. \( cmd(p, cs, i) = x := a(\hat{e}) \) [From 3.2 + 3.11]
3.13. \( \hat{s} \leq \hat{s}' | \hat{\alpha}_1 \) [From 3.4 we know that \( \hat{s}_2 \subseteq \hat{s}_1 \) and from 3.5 we know that \( \hat{s}'' \subseteq \hat{s}_2 \). If follows by transitivity that \( \hat{s}'' \subseteq \hat{s}_1 \), from which it follows (together with \( \hat{s}'' \subseteq \hat{s}' \)) that \( \hat{s} \leq \hat{s}'' | \hat{\alpha}_1 \).]
3.14. \( \exists \sigma_1, v_1 : \) [From 3.5 + 3.11 + 3.13 + CR(\( ee \))]
  1. \( \sigma.e(e) \sim (\sigma_1, v_1) \)
  2. \( \sigma_1 |_{\hat{\sigma}} \sim_s \sigma_1 \)
  3. \( \sigma_1 |_{\hat{\sigma}} \vdash \sigma_1 \sim_v v_1 \)
3.15. \( \hat{s}' \subseteq \hat{s}_2 \) [From 3.4 + SM(monotonicity)]
3.16. \( \hat{s} \subseteq \hat{s}'' \) [From 3.8 + 3.15]
3.17. \( \sigma_1 |_{\hat{\theta}} \vdash \sigma_1 |_{\hat{\theta}_2} \) [From 3.16]
3.18. \( \exists \sigma_2, v_2 : \) [From 3.4 + 3.18 + 3.14.2 + 3.14.3 + CR(\( ea \))]
  1. \( \sigma_2, c(a)(v_2) \sim (\sigma_3, v_2) \)
  2. \( \sigma_2 |_{\hat{\sigma}} \sim v_2 \)
  3. \( \sigma_2 |_{(\hat{\sigma} | \hat{\alpha}_3)} \sim v_2 \vdash v_2 \)
3.19. \( \hat{s}'' \subseteq \hat{s}' \) [From 3.4 + SM(Monotonicity)]
3.20. \( \hat{s}'' | _{\hat{\sigma}} \sim \sigma_3 \) [From 3.14.1 + 3.14.3 + 3.15, noting that: \( \hat{s}_3 |_{(\hat{\alpha} | \hat{\alpha}_3)} = (\hat{s}'' | _{\hat{\sigma}}) |_{\hat{\sigma}} = \hat{s}'' | _{\hat{\sigma}} \) (since \( \hat{s}_2 \subseteq \hat{s}_1 \))]
3.21. \( \exists \sigma_4 : \) [From 3.5 + SM(Trans. strenghtening)]
  1. \( \hat{s}'' | _{\hat{\sigma}} . setVar(x, v_1) = \sigma_4 \)
  2. \( \sigma_4 \leq \hat{s}'' \)
3.22. \( \exists \sigma' : \) [From 3.14 + 3.15 + 3.21.1 + CR(setVar)]
  1. \( \hat{s}'' | _{\hat{\sigma}} \sim_s \sigma' \)
  2. \( \sigma_1 |_{\hat{\sigma}} . setVar(x, v_1) = \sigma' \)
3.23. \( \hat{s}'' | _{\hat{\sigma}} \sim_s \sigma' \) [From 3.21.2 + 3.22.1 + CR - Orders]
3.24. \( \hat{s}'' | _{\hat{\sigma}} \sim_v cs \sim_v cs \) [From 3.11 + noting that \( \hat{s}'' | _{\hat{\sigma}} \subseteq \hat{s} \)]
3.25. \( \hat{cf}'' | _{\hat{\sigma}} \sim \hat{cf}' \) [From 3.6 + 3.7 + 3.23 + 3.24]

**Theorem B.3 (Soundness - General).** Let \( SR = \langle |, \sim_s, \sim_v \rangle \) be a soundness relation for \( \hat{S} = \langle |\hat{S}|, \hat{V}, A \rangle \) in terms of \( S = \langle |S|, V, A \rangle \) and \( \leq \) the pre-order induced by \( \sim_s \). It holds that:

\[
\hat{cf}'' \sim_s \hat{cf}'' \wedge \hat{cf} \leq \hat{cf}'' | _{\hat{\sigma}} |_{\hat{\sigma}''} \wedge \hat{cf} \sim_s cf' \implies \exists cf'. cf \sim_s cf' \wedge \hat{cf}'' | _{\hat{\sigma}} \sim_s cf'
\]

**Proof:** We proceed by induction on \( n \).

1. **Base Case:** \( n = 0 \)
   1.1. \( (H1) \ \hat{cf}' \sim_0 \hat{cf}'' \)
   1.2. \( (H2) \ \hat{cf} \leq \hat{cf}' | _{\hat{\sigma}}'' \)
   1.3. \( (H3) \ \hat{cf} \sim_s cf' \)
1.4. **To prove:** \( \hat{cf}'' | _{\hat{\sigma}} \sim_s cf' \)
   1.4.1. \( \hat{cf} \leq \hat{cf}' | _{\hat{\sigma}'} [H2 + 1.2] \)
   1.4.2. \( \hat{cf} \leq \hat{cf}' [1.4.1] \)
1.4.3. $\hat{cf} \leq \hat{cf}$
1.4.4. $\hat{cf} \subseteq \hat{cf}$ [1.4.3]
1.4.5. $\hat{cf} \leq \hat{cf}' | \hat{cf}''$ [1.4.2 + 1.4.4]
1.4.6. $\hat{cf}' | \hat{cf} \sim_s cf$ [1.4.5 + (H3)]

2. Inductive Step: $n = k + 1$
2.1. (H1) $\hat{cf}' \rightarrow_k \hat{cf}'_1$
2.2. (H2) $\hat{cf} \leq \hat{cf}' | \hat{cf}'_2$
2.3. (H3) $\hat{cf} \sim_s cf$ [From H1]
2.4. $\hat{cf}' \leq \hat{cf}'$
2.5. $\hat{cf}' | \hat{cf}'' \leq \hat{cf}' | \hat{cf}'_1$ [From 2.3 + 2.4]
2.6. $\hat{cf} \leq \hat{cf}' | \hat{cf}'_1$ [From H2 + 2.5]
2.7. $\exists cf'$: [Applying the IH on H3 + 2.2.1 + 2.6]
   1. $cf \rightarrow_k cf'$
   2. $cf'_1 | \hat{cf} \sim_s cf''$
2.8. $\hat{cf}_1 \leq \hat{cf}'_1$ [-]
2.9. $\hat{cf}' \subseteq \hat{cf}''$ [From H2]
2.10. $\hat{cf}'_1 | \hat{cf} \leq \hat{cf}'_1 | \hat{cf}''$ [From 2.8 + 2.9]
2.11. $\exists cf$: [Applying Theorem 3.2 to 2.2.1 + 2.7.1 + 2.10]
   1. $cf'' \rightarrow cf$
   2. $cf'' | \hat{cf} \sim_s cf'$
2.12. $cf \rightarrow_k cf'$ [From 2.7.1 + 2.11.1]

From there, by choosing $\hat{cf}' \equiv \hat{cf}' | \hat{cf}''$, we obtain the desired soundness result.

**Corollary B.4 (Soundness).** Let $SR = (\|, \sim_s, \sim_o)$ be a soundness relation for $\hat{S} = (|\hat{S}|, \hat{V}, A)$ in terms of $S = (|S|, V, A)$ and $\leq$ the pre-order induced by $\sim_s$. It holds that:

$$\hat{cf} \rightarrow^* \hat{cf}' \land (\hat{cf} | \hat{cf}') \sim_s cf \implies \exists cf'. cf \rightarrow^* cf' \land \hat{cf}' \sim_s cf'$$

### B.2 Concrete-Symbolic Soundness

**Definition B.5 (Symbolic Memory Interpretation).** Given a symbolic memory model $\hat{M} = (|\hat{M}|, A, e\hat{a})$ and a concrete memory model $M = (|M|, A, e_a)$, an interpretation of $\hat{M}$ with respect to $M$ is a function $I : |\hat{M}| \rightarrow (\hat{X} \rightarrow \hat{V}) \rightarrow |M|$ such that:

$$\hat{\mu}.\alpha(\hat{e}, \pi) \rightarrow (\hat{\mu}', \hat{e}', \pi') \land \mu = I(\hat{\mu}, \hat{e}) \land [\pi \land \pi'] = true$$
$$\implies \exists \mu'. \mu' = I(\hat{\mu}', \hat{e}) \land \mu.\alpha([\hat{e}']) = (\mu', [\hat{e}'])$$

(6)

**Allocator Interpretation and Restriction.**
Definition B.6 (Symbolic Allocator Interpretation). Given a symbolic allocator model $\hat{AL} = \langle AL, V \rangle$ and a concrete allocator model $AL = \langle AL, V \rangle$, an interpretation of $\hat{AL}$ with respect to $AL$ is a function $I_{\hat{AL}} : |\hat{AL}| \to (V \to V) \to |AL|$ such that:

$$\hat{\xi}.\text{alloc}(j) \to_Y (\hat{\xi}', \hat{\nu}) \land \xi = I_{\hat{AL}}(\hat{\xi}, \epsilon) \implies \exists \xi'. \xi' = I_{\hat{AL}}(\hat{\xi}', \epsilon) \land \xi.\text{alloc}(j) \to_{\epsilon(Y)} (\xi', \epsilon(\hat{\nu}))$$

A restriction operator is said to be preserved by an allocator model $AL = \langle |AL|, V \rangle$ if it satisfies the following two properties:

$$\begin{align*}
\text{RMono-Alloc} & : \xi.\text{alloc}(j) \to_Y (\xi', \nu) \implies \xi' \subseteq \xi \\
\text{FutureToPastAlloc} & : \xi.\text{alloc}(j) \to_Y (\xi', \nu) \land \xi'' \subseteq \xi \land \xi' \implies \xi''.\text{alloc}(j) \to_Y (\xi' \land \xi'', \nu)
\end{align*}$$

We say that $|$ is a restriction operator on an allocator model $AL = \langle |AL|, V \rangle$, if $|$ is a restriction operator on the carrier set $|AL|$ and $|$ is preserved by $AL$.

Lifting Interpretations. Given an interpretation $I : |M| \to (\hat{X} \to V) \to |M|$ of a symbolic memory model $\hat{M} = \langle |M|, A, e, \hepsilon \rangle$ in terms of a concrete memory model $M = \langle |M|, A, e \rangle$, the candidate soundness relation $RT(I) = \langle I, \sim_s, \sim_v \rangle$ for SST($\hat{M}$) in terms of CST($M$) is defined as follows:

$$\langle \hat{\mu}, \hat{\rho}, \hat{\xi}, \pi \rangle \downarrow_{\langle\_, \_, \xi, \pi\rangle} \triangleq \langle \hat{\mu}, \hat{\rho}, \xi \\
\hepsilon \downarrow_s \sigma \\
\langle\_, \_, \_, \xi, \pi\rangle \vdash \hepsilon \downarrow_v \nu \triangleq \exists \pi \cdot \|\pi\|_\epsilon = \true \land \hat{\mu} = I(\hat{\mu}, \epsilon) \land \hat{\rho} = \|\rho\|_\epsilon \land \hat{\xi} = I_{\hat{AL}}(\hat{\xi}, \|\xi\|_\epsilon)$$

Definition B.7 (Env). The function $\text{Env} : \Pi \to \mathcal{E}nv$ is defined as follow:

$$\text{Env}(\pi) \triangleq \{ \epsilon \cdot \|\pi\|_\epsilon = \true \}$$

If $\hat{\sigma} = \langle \hat{\mu}, \hat{\rho}, \hat{\xi}, \pi \rangle$, we also note

$$\text{Env}(\hat{\sigma}) = \text{Env}(\pi)$$

Definition B.8 (Restriction of Interpretations with Path Conditions). Given a path condition $\pi \in \Pi$, a concrete memory model $M \in \mathcal{M}$, a symbolic memory model $\hat{M} \in \hat{\mathcal{M}}$ and an interpretation $I :: |M| \to \mathcal{E}nv \to |M|$ of $\hat{M}$ with respect to $M$, we define $I_{\hat{\sigma}} :: |M| \to \text{Env}(\pi) \to |M|$, the restriction of $I$ to $\text{Env}(\pi)$:

$$I_{\hat{\sigma}} = I|_{\text{Env}(\pi)}$$

Lemma B.9. Let $M \in \mathcal{M}$ and $\hat{M} \in \hat{\mathcal{M}}$ be a concrete and a symbolic memory model and $I :: |\hat{M}| \to \mathcal{E}nv \to |M|$ an interpretation of $\hat{M}$ with respect to $M$, and let $\text{SR} = \langle I, \sim_s, \sim_v \rangle$ be the candidate soundness relation between CST($M$) and SST($\hat{M}$) induced by $I$. It holds that:

$$\hat{\sigma} \subseteq \hat{\sigma}' \iff \text{Env}(\hat{\sigma}) \subseteq \text{Env}(\hat{\sigma}') \land \hat{\sigma}' \hat{\xi} \subseteq \hat{\sigma} \hat{\xi}$$

Proof:

First, we need to observe that $\hat{\sigma} \subseteq \hat{\sigma}'$ is just a shorthand for $\hat{\sigma} \downarrow \hat{\sigma}' = \hat{\sigma}$. We rewrite the equivalence we want to prove as

$$\hat{\sigma} \downarrow \hat{\sigma}' = \hat{\sigma} \iff \text{Env}(\hat{\sigma}) \subseteq \text{Env}(\hat{\sigma}') \land \hat{\sigma}' \hat{\xi} \subseteq \hat{\sigma} \hat{\xi}$$

We analyse one direction of the equivalence at a time, first destructuring on $\hat{\sigma}$ and $\hat{\sigma}'$.

1. Let $\hat{\sigma} = \langle \hat{\mu}, \hat{\rho}, \hat{\xi}, \pi \rangle$ and $\hat{\sigma}' = \langle \hat{\mu}', \hat{\rho}', \hat{\xi}', \pi' \rangle$
2. Prove: $\hat{\sigma} \downarrow \hat{\sigma}' = \hat{\sigma} \implies \text{Env}(\hat{\sigma}) \subseteq \text{Env}(\hat{\sigma}') \land \hat{\sigma}' \hat{\xi} \subseteq \hat{\sigma} \hat{\xi}$
   2.1. Assume: $\hat{\sigma} \downarrow \hat{\sigma} = \hat{\sigma}$
   2.2. $\hat{\sigma} \downarrow \hat{\sigma}' = \langle \hat{\mu}, \hat{\rho}, \hat{\xi} \land \pi, \pi' \rangle$ [From 1 and 2.1]
2.3. \[ \pi \implies \pi' \] [From 2.1 and 2.2]
2.4. \[ \xi \mid \xi = \xi \] [From 2.1 and 2.2]
2.5. \[ \text{Env}(\hat{\sigma}) \subseteq \text{Env}(\hat{\sigma}') \] [From 2.3]
2.6. \[ \xi' \subseteq \xi \] [From 2.4]

3. **Prove:** \[ \hat{\sigma} \mid_{\hat{\sigma}'} = \hat{\sigma} \iff \text{Env}(\hat{\sigma}) \subseteq \text{Env}(\hat{\sigma}') \land \hat{\sigma}' \subseteq \hat{\sigma} \]

3.1. **Assume:** 1. \[ \text{Env}(\hat{\sigma}) \subseteq \text{Env}(\hat{\sigma}') \]
2. \[ \hat{\sigma}' \subseteq \hat{\sigma} \]
3.2. \[ \pi \implies \pi' \] [From 3.1.1]
3.3. \[ \hat{\sigma} \mid_{\hat{\sigma}'} = \hat{\sigma} \] [From 3.1.2 and 3.2]

**Lemma B.10.** Let \( \hat{\sigma} = \langle \hat{\mu}, \hat{\rho}, \hat{\xi}, \pi \rangle \) and \( \hat{\sigma}' = \langle \hat{\mu}', \hat{\rho}', \hat{\xi}', \pi' \rangle \). Then it holds that
\[
\hat{\sigma} \subseteq \hat{\sigma}' \iff I_\pi(\hat{\mu}) = I_\pi(\hat{\mu}') \\
\quad \land \quad I_\pi(\hat{\rho}) = I_\pi(\hat{\rho}') \\
\quad \land \quad \pi \implies \pi'
\]

**Lemma B.11.** Let \( \hat{\sigma} = \langle \hat{\mu}, \hat{\rho}, \hat{\xi}, \pi \rangle \) and \( \hat{\sigma}' = \langle \hat{\mu}', \hat{\rho}', \hat{\xi}', \pi' \rangle \). Then it holds that
\[
\hat{\sigma} \subseteq \hat{\sigma}' \iff (\pi \implies \pi') \land I_\pi(\hat{\xi}') \subseteq I_\pi(\hat{\xi})
\]

**Lemma B.12 (Lifted Restriction Operator).** Let \( I \) be an interpretation of a symbolic memory model \( \hat{M} \) in terms of a concrete memory model \( M \), and \( \langle \cdot, \sim_s, \sim_v \rangle = RT(I) \). Then \( \mid \) is a restriction operator on \( \text{SST}(\hat{M}) \).

**Proof:**
Let \( M \in \mathbb{M}, \hat{M} \in \mathbb{\hat{M}}, I \) an interpretation of \( \hat{M} \) in term of \( M \), \( \langle \cdot, \sim_s, \sim_v \rangle = RT(I) \). We need to prove that \( \mid \) is a restriction operator on the carrier set \( \text{SST}(\hat{M}) \), and that it is preserved by \( \text{SST}(\hat{M}) \).

1. **Prove:** \( \mid \) is a restriction operator
   In order to prove that \( \mid \) is a restriction operator, we need to prove its associativity, idempotence, right-commutativity and that it has the Weakening property. We only provide the proof for associativity, the other proofs are analogous.

   **Prove:** \( (\hat{\sigma} \mid \hat{\sigma}') \mid \hat{\sigma}'' = \hat{\sigma} \mid (\hat{\sigma} \circ \hat{\sigma}' \circ \hat{\sigma}'') \)
   1.1. **Let:** 1. \( \hat{\sigma} = \langle \hat{\mu}, \hat{\rho}, \hat{\xi}, \pi \rangle \)
   2. \( \hat{\sigma}' = \langle \hat{\mu}', \hat{\rho}', \hat{\xi}', \pi' \rangle \)
   3. \( \hat{\sigma}'' = \langle \hat{\mu}'', \hat{\rho}'', \hat{\xi}'', \pi''' \rangle \)
   1.2. \( \hat{\sigma} \mid \hat{\sigma}' = \langle \hat{\mu}, \hat{\rho}, \hat{\xi}, \pi \rangle \mid \hat{\xi} \mid \pi' \) [From 1.1.1 and 1.1.2]
   1.3. \( (\hat{\sigma} \mid \hat{\sigma}') \mid \hat{\sigma}'' = \langle \hat{\mu}, \hat{\rho}, \hat{\xi}, \pi \rangle \mid \hat{\xi} \mid \pi' \mid \pi'' \) [From 1.1.3 and 1.2]
   1.4. \( \hat{\sigma}' \mid \hat{\sigma}'' = \langle \hat{\mu}', \hat{\rho}', \hat{\xi}', \pi' \mid \pi'' \rangle \) [From 1.1.2 and 1.1.3]
   1.5. \( \hat{\sigma} \mid (\hat{\sigma} \circ \hat{\sigma}' \circ \hat{\sigma}'') = \langle \hat{\mu}, \hat{\rho}, \hat{\xi}, \pi \rangle \mid \hat{\xi} \mid \pi' \mid \pi'' \) [From 1.1.1 and 1.4]
   1.6. \( \hat{\sigma} \mid (\hat{\sigma} \circ \hat{\sigma}' \circ \hat{\sigma}'') = \hat{\sigma} \mid (\hat{\sigma} \circ \hat{\sigma}' \circ \hat{\sigma}'') \) [From 1.3, 1.5 and associativity of allocator restriction operator]

2. **Prove:** \( \mid \) is preserved by \( \text{SST}(\hat{M}) \)
   In order to prove that \( \mid \) is preserved by \( \text{SST}(\hat{M}) \), we need to prove that all of the state-generating functions exposed by the state model are monotonic with respect to the pre-order \( \subseteq \) induced by \( \mid \). We only provide the proof for the monotonicity with respect to action executions, the other cases are analogous.

   **Case:** Actions
   **Prove:** \( \hat{\sigma} \cdot \alpha(v) \cdot (\hat{\sigma}', -) \implies \hat{\sigma}' \subseteq \hat{\sigma} \)
   2.1. **Let:** \( \hat{\sigma} = \langle \hat{\mu}, \hat{\rho}, \hat{\xi}, \pi \rangle \in \text{SST}(\hat{M}) \)
2.2. Assume: $\hat{\sigma}.\alpha(v) \rightsquigarrow (\sigma',-)$

2.3. $\exists \hat{\mu}', \pi'$.

1. $(\hat{\mu}', \hat{\epsilon}', \pi') \in e_\chi(\alpha, \hat{\mu}, \hat{\epsilon})$
2. $\hat{\sigma}' = \langle \hat{\mu}', \hat{\rho}, \hat{\xi}, \pi \land \pi' \rangle$

[From 2.2 and 2.3]

2.4. $\text{Env}(\hat{\sigma}') \subseteq \text{Env}(\hat{\sigma})$ [From 2.3.2]

2.5. $\hat{\sigma} \cdot \hat{\xi} \leq \hat{\sigma}' \cdot \hat{\xi}$ [From 2.3.2]

2.6. $\hat{\sigma}' \subseteq \hat{\sigma}$ [From 2.4, 2.5 and Lemma B.9]

**Theorem B.13 (Soundness Relation - Lifting).** Let $I$ be an interpretation of a symbolic memory model $\hat{M}$ in terms of a concrete memory model $M$; then, $RT(I) = \langle l, \sim_s, \sim_v \rangle$ is a soundness relation for SST($\hat{M}$) in terms of CST($M$).

**Proof:**

In order to establish that $RT(I) = \langle l, \sim_s, \sim_v \rangle$ is a soundness relation for SST($\hat{M}$) in terms of CST($M$), we need to prove that

1. $|$ is a restriction order on SST($\hat{M}$)
2. $\sim_s$ and $\sim_v$ are monotonic with respect to the functions exposed by the state model
3. $|$ is compatible with $\leq$

The first property is the result of Lemma B.12. Let us prove property 2 and 3.

1. **Prove:** $\sim_s$ and $\sim_v$ are monotonic with respect to the functions exposed by the state model

   We need to prove that the property holds for store, ee, setVar, setStore and actions. We only provide the proof for the action cases. The other cases are analogous.

   **Case:** Actions

   **Prove:** $\hat{\sigma}.\alpha(\hat{v}) \rightsquigarrow (\hat{\sigma}', \hat{\epsilon}') \land \hat{\sigma}'' \leq \hat{\sigma} \land \hat{\sigma}'' \sim_s \sigma \land \hat{\sigma}'' \vdash \hat{\epsilon} \sim_v v \implies \exists \sigma', \hat{\epsilon}', \sigma.\alpha(v) \rightsquigarrow (\sigma', \hat{\epsilon}') \land \hat{\sigma}' \mid \hat{\sigma}'' \sim_s \sigma \land \hat{\sigma}' \vdash \hat{\epsilon}' \sim_v v$

   1.1. **Assume:** 1. (H1) $\hat{\sigma} = \langle \hat{\mu}, \hat{\rho}, \hat{\xi}, \pi \rangle$
   2. (H2) $\hat{\sigma}'' = \langle \hat{\mu}'', \hat{\rho}'', \hat{\xi}'', \pi'' \rangle$
   3. (H3) $\sigma = \langle \mu, \rho, \xi \rangle$
   4. (H4) $\hat{\sigma}.\alpha(\hat{v}) \rightsquigarrow (\hat{\sigma}', \hat{\epsilon}')$
   5. (H5) $\hat{\sigma}'' \leq \hat{\sigma} \mid \hat{\sigma}'$
   6. (H6) $\hat{\sigma} \sim_s \sigma$
   7. (H7) $\hat{\epsilon} \vdash \hat{\sigma} \sim_v v$

   1.2. $\exists \hat{\mu}', \hat{\epsilon}', \pi'$.
   1. $\hat{\mu}.\alpha(\hat{v}, \pi) \rightsquigarrow (\hat{\mu}', \hat{\epsilon}', \pi')$
   2. $\hat{\sigma}.\alpha(\hat{v}) \rightsquigarrow (\hat{\mu}, \hat{\rho}, \hat{\xi}, \pi \land \pi'), (\hat{\epsilon}')$
   3. $\hat{\sigma}' = \langle \hat{\mu}', \hat{\rho}, \hat{\xi}, \pi \land \pi' \rangle$

   [From H1 and H4]

   1.3. $\exists \epsilon$.
   1. $[\pi'']_\epsilon = \text{true}$
   2. $[\hat{\epsilon}]_\epsilon = v$

   [From H2 and H7]

   1.4. $\exists \epsilon'$.
   1. $\epsilon \leq \epsilon'$
   2. $[\pi'']_{\epsilon'} = \text{true}$
   3. $I(\hat{\mu}'', \epsilon') = \mu$
   4. $I(\hat{\rho}'', \epsilon') = \rho$
   5. $I(\hat{\xi}'') \subseteq \xi$
2.6. \hat{\pi} \subseteq \hat{\pi}''

2.7. \hat{I}(\hat{\mu}, \hat{\rho}) = \hat{I}(\hat{\mu}''', \hat{\rho}''')

[From H2, H5, 1.5]

1.7. \|\hat{v}\|_{\hat{\pi}'} = \nu[From 1.3.2 and 1.4.1]

1.8. \|\pi \land \pi'\|_{\nu} = \text{true} [From 1.4.2 and 1.6.1]

1.9. \hat{I}(\hat{\mu}, \hat{\rho}) = \mu [From 1.4.2, 1.4.3 and 1.6.3]

1.10. \exists \hat{\pi}', \hat{\rho}':

\begin{align*}
1. & \hat{\pi}' = \hat{I}(\hat{\mu}', \hat{\rho}') \\
2. & \hat{\rho}' \leq \hat{\rho}'' \\
3. & \mu, \alpha \ (v) \sim (\hat{\mu}', \hat{\rho}', \hat{\xi}', \hat{\pi}'') [From 1.2.1, 1.7, 1.8, 1.9]
\end{align*}

1.11. LET: \(\sigma' = \langle \mu', \rho, \xi \rangle\)

1.12. \(\sigma, \alpha \ (v) \sim (\sigma', \hat{v}', \hat{\nu}') [From H3, 1.10.3 and 1.11]

1.13. \(\hat{\sigma}' \hat{\pi}'\) = \(\hat{\sigma}', \hat{\rho}', \hat{\xi}', \hat{\pi}'') [From 1.2.3 and H2]

1.14. PROVE: \(\sigma', \hat{\pi}'\) = \(\hat{\sigma}', \hat{\rho}', \hat{\xi}', \hat{\pi}'') \in \text{Mod}(\hat{\sigma}' \hat{\pi}'\rangle [From 1.14.1 to 1.14.4]

1.15. \(\|\hat{\pi}'\|_{\hat{\pi}''} = \text{true}

1.16. \(\hat{\sigma}' \hat{\pi}''\rangle \sim \nu \|\hat{\pi}'\|_{\hat{\pi}''} [From 1.13 and 1.15]

2. PROVE: \(\langle \rangle\) is compatible with \(\leq\)

In order to prove that \(\langle \rangle\) is compatible with \(\leq\), we need to prove the \(\langle \rangle \sim \langle \rangle\) compatibility, the \(\leq \sim \langle \rangle\) compatibility and the strengthening property. We only provide the proof for the strengthening property. The other cases are analogous.

2.1. LET: 1. (H1) \(\hat{\sigma}_1 = \langle \hat{\mu}_1, \hat{\rho}_1, \hat{\xi}_1, \pi_1 \rangle\)

2. (H2) \(\hat{\sigma}_2 = \langle \hat{\mu}_2, \hat{\rho}_2, \hat{\xi}_2, \pi_2 \rangle\)

3. (H3) \(\hat{\sigma}'_1 = \langle \hat{\mu}'_1, \hat{\rho}'_1, \hat{\xi}'_1, \pi'_1 \rangle\)

4. (H4) \(\hat{\sigma}'_2 = \langle \hat{\mu}'_2, \hat{\rho}'_2, \hat{\xi}'_2, \pi'_2 \rangle\)

be 4 symbolic states

2.2. ASSUME: 1. (H5) \(\text{sst}_1 \leq \hat{\sigma}'_1\)

2. (H6) \(\hat{\sigma}_2 \subseteq \hat{\sigma}'_2\)

PROVE: \(\hat{\sigma}_1 \hat{\sigma}_2 \leq \hat{\sigma}'_1 \hat{\sigma}'_2\)

2.3. \(\text{Mod}(\hat{\mu}_1, \hat{\rho}_1, \hat{\xi}_1, \pi_1) \subseteq \text{Mod}(\hat{\mu}'_1, \hat{\rho}'_1, \hat{\xi}'_1, \pi'_1) [From H1, H2 and H5]

2.4. \(\pi_2 \subseteq \hat{\pi}_2 \subseteq \hat{\pi}''\)

2.5. \(\hat{\sigma}_1 \hat{\sigma}_2 = \langle \hat{\mu}_1, \hat{\rho}_1, \hat{\xi}_1 \rangle \hat{\pi}_1 \hat{\pi}_2\) [From H1 and H2]

2.6. \(\hat{\sigma}'_1 \hat{\sigma}'_2 = \langle \hat{\mu}'_1, \hat{\rho}'_1, \hat{\xi}'_1 \rangle \hat{\pi}'_1 \hat{\pi}'_2\) [From H3 and H4]

2.7. PROVE: \(\text{Mod}(\hat{\mu}_1, \hat{\rho}_1, \hat{\xi}_1 \rangle \hat{\pi}_1 \hat{\pi}_2 \rangle \subseteq \text{Mod}(\hat{\mu}'_1, \hat{\rho}'_1, \hat{\xi}'_1 \rangle \hat{\pi}'_1 \hat{\pi}'_2 \rangle \land \rho\pi'_2\)

2.7.1. LET: \((\mu, \rho, \xi, \nu) \in \text{Mod}(\hat{\mu}_1, \hat{\rho}_1, \hat{\xi}_1 \rangle \hat{\pi}_1 \hat{\pi}_2 \rangle\)

2.7.2. 1. \(\|\pi_1 \land \pi_2\| = \text{true} \)
For every action \( \alpha \)

1. \( \bar{\iota} \cdot \alpha \cdot (\bar{e}, \pi) \leadsto (\bar{e}', \bar{e}', \bar{\pi}') \quad \land \quad \bar{\mu} = \bar{\iota} \) (\( \bar{\mu} \in \bar{\iota} \))

2. \( \bar{\mu} = \bar{\iota} \cdot \alpha \Rightarrow \bar{\mu} = \bar{\iota} \)

3. \( \bar{\mu} = \bar{\iota} \cdot \alpha \Rightarrow \bar{\mu} = \bar{\iota} \)

4. \( \bar{\mu} = \bar{\iota} \cdot \alpha \Rightarrow \bar{\mu} = \bar{\iota} \)

2. \( \bar{\mu} = \bar{\iota} \cdot \alpha \Rightarrow \bar{\mu} = \bar{\iota} \)

1. \( \bar{\iota} \cdot \alpha \Rightarrow \bar{\iota} \)

2. \( \bar{\iota} \cdot \alpha \Rightarrow \bar{\iota} \)

3. \( \bar{\iota} \cdot \alpha \Rightarrow \bar{\iota} \)

4. \( \bar{\iota} \cdot \alpha \Rightarrow \bar{\iota} \)

While: Sound Symbolic Analysis

The interpretation of While symbolic memories in terms of While concrete memories, \( \bar{\iota}_W \), is inductively defined as follows:

| Empty | Cell | Union |
|-------|------|-------|
| \( \bar{\iota}_W(\emptyset, \epsilon) \triangleq \emptyset \) | \( l = \llbracket \bar{e} \rrbracket _{\epsilon} \Rightarrow \llbracket \bar{e}' \rrbracket _{\epsilon} \triangleq l \Rightarrow \llbracket \bar{e}' \rrbracket _{\epsilon} \) | \( \mu_1 = \bar{\iota}_W(\bar{\mu}_1, \epsilon) \quad \mu_2 = \bar{\iota}_W(\bar{\mu}_2, \epsilon) \) |
| \( \bar{\iota}_W(\emptyset, \epsilon) \triangleq \emptyset \) | \( \bar{\iota}_W(\emptyset, \epsilon) \triangleq \emptyset \) | \( \bar{\iota}_W(\bar{\mu}_1 \cup \bar{\mu}_2, \epsilon) \triangleq \bar{\mu}_1 \cup \bar{\mu}_2 \) |

Lemma B.14 (While: Memory Interpretation). \( \bar{\iota}_W \) is an interpretation of \( \bar{\iota}_W \) with respect to \( M_W \).

Proof:

For every action \( \alpha \in A \), we have to prove that:

\[ \bar{\mu}(\bar{e}, \pi) \leadsto (\bar{e}', \bar{e}', \bar{\pi}') \quad \land \quad \bar{\mu} = \bar{\iota} \]

\[ \Rightarrow \exists \bar{\mu}' \cdot \bar{\mu}' = \bar{\iota} \cdot \alpha \Rightarrow \bar{\mu}' \land \bar{\mu}' = \bar{\iota} \cdot \alpha \;
\]

We proceed by case analysis on the rule that was used to derive \( \bar{\mu}(\bar{e}, \pi) \leadsto (\bar{e}', \bar{e}', \bar{\pi}') \).

1. Case: lookup

   1. Assume: \( (H1) \ \bar{\mu} \cdot \cdot \cdot \leadsto (\bar{e}', \bar{e}', \bar{\pi}') \)

   2. (H2) \( \llbracket \bar{e}' \rrbracket _{\epsilon} = \llbracket \bar{e}' \rrbracket _{\epsilon} \)

   3. (H3) \( \mu = \bar{\iota}_W(\bar{\mu}, \epsilon) \)

2. \( \exists \bar{e}'' \cdot \bar{e}'' \)

   1. \( \pi + \bar{e}'' = \bar{e} \)

   2. \( \bar{\mu} = \_ \cup (\bar{e}'', \bar{e}) \leadsto \bar{e}'' \)
3. $\pi' = \pi$
4. $\hat{\mu}' = \hat{\mu}$

[From H1]

1.3. $\mu = \_ \cup I_{\hat{\mathcal{W}}}(\hat{\epsilon}'', p) \mapsto \hat{\epsilon}', \epsilon$ [From H3 + 1.2.2]
1.4. $\mu = \_ \cup (\llbracket \hat{\epsilon}'', \epsilon \rrbracket_e, p) \mapsto \llbracket \hat{\epsilon}' \rrbracket_e$ [From 1.3]
1.5. $\llbracket \hat{\epsilon}' \rrbracket_e = \llbracket \hat{\epsilon} \rrbracket_e$ [From H4 + 1.2]
1.6. $\mu = \_ \cup (\llbracket \hat{\epsilon} \rrbracket_e, p) \mapsto \llbracket \hat{\epsilon}' \rrbracket_e$ [From 1.4 and 1.5]
1.7. $\mu.\text{lookup} (\llbracket \hat{\epsilon} \rrbracket_e, p) \rightsquigarrow (\mu, \llbracket \hat{\epsilon}' \rrbracket_e)$ [From 1.6]

2. Case: mutate

2.1. Assume:
   1. (H1) $\hat{\mu}.\text{mutate} ([\hat{\epsilon}, p, \hat{\epsilon}'], \pi) \rightsquigarrow (\hat{\mu}', \hat{\epsilon}'', \pi')$
   2. (H2) $\llbracket \hat{\epsilon} \rrbracket_e = v$ and $\llbracket \hat{\epsilon}' \rrbracket_e = v'$
   3. (H3) $\mu = I_{\hat{\mathcal{W}}} (\hat{\mu}, \epsilon)$

2.2. $\exists \hat{\epsilon}'', \hat{\mu}''$.
   1. $\hat{\mu}' = \hat{\mu}'' \cup (\hat{\epsilon}'', p) \mapsto _-$
   2. $\pi' \vdash \hat{\epsilon}''$
   3. $\hat{\mu}' = \hat{\mu}'' \cup (\hat{\epsilon}'', p) \mapsto \hat{\epsilon}'$

[From H1]

2.3. $\mu = I_{\hat{\mathcal{W}}} (\hat{\mu}'', \epsilon) \cup (\llbracket \hat{\epsilon}'', \epsilon \rrbracket_e, p) \mapsto _-$ [From H3 + 2.2.3]
2.4. $\llbracket \hat{\epsilon}' \rrbracket_e = \llbracket \hat{\epsilon} \rrbracket_e = v$ [From H2 + H4 + 2.2.2]
2.5. $\exists \cdot v = 1$ [We assume $\epsilon$ is "well-formed"]
2.6. $\mu = I_{\hat{\mathcal{W}}} (\hat{\mu}'', \epsilon) \cup (l, p) \mapsto _-$ [From 2.3 + 2.5]
2.7. $\mu.\text{mutate} ([l, p, \hat{\epsilon}']) \rightsquigarrow (I_{\hat{\mathcal{W}}} (\hat{\mu}'', \epsilon) \cup (l, p) \mapsto v', v')$

2.8. $I_{\hat{\mathcal{W}}} (\hat{\mu}''', \epsilon) = I_{\hat{\mathcal{W}}} (\hat{\mu}'', \epsilon) \cup I_{\hat{\mathcal{W}}} (\llbracket \hat{\epsilon}'', \epsilon \rrbracket_e, p) \mapsto \hat{\epsilon}', \epsilon)
   = I_{\hat{\mathcal{W}}} (\hat{\mu}'', \epsilon) \cup (\llbracket \hat{\epsilon}' \rrbracket_e, p) \mapsto \llbracket \hat{\epsilon}' \rrbracket_e
   = I_{\hat{\mathcal{W}}} (\hat{\mu}'', \epsilon) \cup (l, p) \mapsto v'$ [From H2 + 2.4 + 2.5]

3. Case: dispose

3.1. Assume:
   1. (H1) $\hat{\mu}.\text{dispose} (\hat{\epsilon}, \pi) \rightsquigarrow (\hat{\mu}', \text{true}, \pi)$
   2. (H2) $\llbracket \hat{\epsilon} \rrbracket_e = \nu$
   3. (H3) $\mu = I_{\hat{\mathcal{W}}} (\hat{\mu}, \epsilon)$

3.2. $\hat{\mu} |_{\hat{\epsilon}, \pi} = (\_ , \hat{\mu}')$ [From H1]
3.3. $I_{\hat{\mathcal{W}}} (\hat{\mu} |_{\hat{\epsilon}, \pi}, \epsilon) = (\_ , I_{\hat{\mathcal{W}}} (\hat{\mu}', \epsilon))$ [From 3.2]
3.4. $I_{\hat{\mathcal{W}}} (\hat{\mu} |_{\hat{\epsilon}, \pi}, \epsilon) = I_{\hat{\mathcal{W}}} (\hat{\mu}, \epsilon) |_{\hat{\epsilon}^L}$
3.5. $I_{\hat{\mathcal{W}}} (\hat{\mu}, \epsilon) |_{\hat{\epsilon}^L} = \mu |_{\hat{\epsilon}^L}$ [From H3]
3.6. $\mu |_{\hat{\epsilon}^L} = (\_ , I_{\hat{\mathcal{W}}} (\hat{\mu}', \epsilon))$ [From 3.3 - 3.5]
3.7. $\mu.\text{dispose} (\llbracket \hat{\epsilon} \rrbracket_e) \rightsquigarrow (I_{\hat{\mathcal{W}}} (\hat{\mu}'', \epsilon), \text{true})$

Theorem B.15 (While: Soundness). Given $\langle \_ , \sim, \sim \rangle = RT (I_{\mathcal{W}})$, it holds that:
$$\hat{\epsilon}' \sim_{\mathcal{W}} \hat{\epsilon}' \land (\hat{\epsilon} |_{\hat{\epsilon}^L} \sim_{\mathcal{W}} \hat{\epsilon} \Rightarrow \exists \hat{\epsilon}''. \epsilon \sim_{\mathcal{W}} \hat{\epsilon}'' \land \hat{\epsilon}'' \sim_{\mathcal{W}} \hat{\epsilon}''')$$
C SECTION 4: PARAMETRIC VERIFICATION

C.1 Parametric Assertion Language

GIL Parametric Assertions

\[ P, Q \in \mathcal{A}_\Delta \triangleq \{ \pi \mid \delta(e) \mid prn(e) \mid P \ast Q \} \]

\[ \text{pred} \in \mathcal{Pred} \triangleq \text{pred} \text{prn}(x) \Rightarrow P_0; \ldots; P_n \]

\[ \omega \in \Omega_V \triangleq \text{prn}(v), \text{where } v \in V \]

Definition C.1 (Predicate State Constructor (PST)). The predicate state constructor \( \text{PST} : \mathbb{S} \rightarrow \mathbb{S} \) is defined as \( \text{PST} (\langle |S|, V, A \rangle) \triangleq \langle |S'|, V, A \cup \{ \text{setP}, \text{getP} \} \rangle \), where:

- \[ |S'| \triangleq |S| \times \Omega_V \]
- \[ \text{setVar}_p((\sigma, \overline{\omega}), x, v) \triangleq (\text{setVar}(\sigma, x, v), \overline{\omega}) \]
- \[ \text{setStore}_p((\sigma, \overline{\omega}), \rho) \triangleq (\text{setStore}(\sigma, \rho), \overline{\omega}) \]
- \[ \text{store}_p((\sigma, -)) \triangleq \text{store}(\sigma) \]
- \[ \text{ee}_p(\sigma, e) \triangleq \text{ee}(\sigma, e) \]
- \[ \text{ea}_p(\alpha, (\sigma, \overline{\omega}), v) \triangleq \{(\sigma', \overline{\omega'}, v') \mid (\sigma', v') \in \text{ea}(\alpha, \sigma, v), \text{if } \alpha \notin \{\text{getP}, \text{setP}\}\}
- \[ \text{ea}_p(\text{setP}((\sigma, \overline{\omega}), [pn, v])) \triangleq \{(\sigma, (pn(v) : \overline{\omega})), -\}
- \[ \text{ea}_p(\text{getP}((\sigma, \overline{\omega}), [pn, v])) \triangleq \{(\sigma, (\omega_1 + \omega_2), -), \text{where } \overline{\omega} = \overline{\omega}_1 + [pn(v)] + \overline{\omega}_2\}

Definition C.2 (Core Predicate Action Interpretation). A core predicate action interpretation is a 4-tuple \( \langle A, \lambda, \text{set}, \text{get} \rangle \) consisting of a set of core predicates \( \Delta \), a set of actions \( A \), and two functions \( \text{set} : \lambda \rightarrow \lambda \) and \( \text{get} : \lambda \rightarrow \lambda \). A core predicate action interpretation \( \langle \Delta, A, \text{set}, \text{get} \rangle \) is said to be well-formed with respect to a state model \( S = \langle |S|, V, A \rangle \) in and only if, for all core predicates \( \delta \in \Delta \), it holds that:

\[ \sigma.\text{get}_\delta(v) \rightsquigarrow \sigma' \iff \sigma'.\text{set}_\delta(v) \rightsquigarrow \sigma \]  

(8)

Given a state model \( S = \langle |S|, V, A \rangle \) and a core action interpretation \( \langle \Delta, A, \text{set}, \text{get} \rangle \), the induced action interpretation of an assertion \( P \in \mathcal{A}_\Delta \) is a pair of functions consisting of the getter and setter of \( P \), respectively, \( \text{get}_P \) and \( \text{set}_P \). Formally, we define two induced functions:

- \[ \text{set}_P : \mathcal{A}_\Delta \rightarrow |S| \rightarrow (\hat{X} \cup X \rightarrow V) \rightarrow |S| \]
- \[ \text{get}_P : \mathcal{A}_\Delta \rightarrow |S| \rightarrow (\hat{X} \cup X \rightarrow V) \rightarrow |S| \]

mapping each assertion \( P \in \mathcal{A}_\Delta \) to its getter and setter, respectively, as follows:

**Assertion Interpretation:** \( \sigma.\text{set}_P(\theta) \rightsquigarrow \sigma' \) and \( \sigma.\text{get}_P(\theta) \rightsquigarrow \sigma' \)

**Lemma C.3 (Assertion Interpretation).** Let \( \langle \Delta, A, \text{set}, \text{get} \rangle \) be a well-formed core predicate interpretation with respect to a predicate state model \( S = \langle |S|, V, A \rangle \); then, it holds that: \( \sigma.\text{set}_P(\theta) \rightsquigarrow \sigma' \) if and only if \( \sigma'.\text{get}_P(\theta) \rightsquigarrow \sigma \).

**Theorem C.4 (Assertion Interpretation - Soundness).** Let \( SR = \langle |S|, \sim_s, \sim_v \rangle \) be a soundness relation for \( \hat{S} = \langle |\hat{S}|, \hat{V}, A \rangle \) in terms of \( S = \langle |S|, V, A \rangle \) and \( \leq \) the pre-order induced by \( \sim_s \); and let
\(\langle \Delta, A, \text{set, get} \rangle\) be a well-formed core predicate action interpretation for \(\hat{S}\) and \(S\); then, it holds that:

\[
\hat{\sigma}.\text{set}_p(\hat{\theta}) \leadsto \hat{\sigma}' \land \hat{\sigma}'' \leq \hat{\sigma}_1 \land \hat{\sigma}'' \sim_s \sigma \land \hat{\sigma}'' \vdash \hat{\theta} \sim_v \theta \tag{9}
\]

\[
\hat{\sigma}.\text{get}_p(\hat{\theta}) \leadsto \hat{\sigma}' \land \hat{\sigma}'' \leq \hat{\sigma}_1 \land \hat{\sigma}'' \sim_s \sigma \land \hat{\sigma}'' \vdash \hat{\theta} \sim_v \theta \tag{10}
\]

**Proof:**
We are going to prove equations the two equations by induction on the structure of \(P\), considering only the core predicate and separating conjunction cases for both proofs. The other cases are analogous.

1. **Prove:**
   \[
   \hat{\sigma}.\text{set}_p(\hat{\theta}) \leadsto \hat{\sigma}' \land \hat{\sigma}'' \leq \hat{\sigma}_1 \land \hat{\sigma}'' \sim_s \sigma \land \hat{\sigma}'' \vdash \hat{\theta} \sim_v \theta \tag{9}
   \]

   1.1. **Case:** \(\delta(e)\)
   1.1.1. **Assume:**
   1. \((H1)\) \(
   \hat{\sigma}.\text{set}_\delta(e)(\hat{\theta}) \leadsto \hat{\sigma}'
   \)
   2. \((H2)\) \(
   \hat{\sigma}'' \leq \hat{\sigma}_1 \land \hat{\sigma}'' \sim_s \sigma
   \)
   3. \((H3)\) \(
   \hat{\sigma}'' \sim_s \sigma
   \)
   4. \((H4)\) \(
   \hat{\sigma}'' \vdash \hat{\theta} \sim_v \theta
   \)

   **Prove:** \(\exists \sigma'. \hat{\sigma}.\text{set}_\delta(e)(\hat{\theta}) \leadsto \sigma' \land \hat{\sigma}' \land \hat{\sigma}'' \sim_s \sigma\)

   1.1.2. \(\exists \hat{\sigma}e\).
   1. \(\hat{\sigma}.\text{ee}(\hat{\theta}(e)) = \hat{\sigma}\)
   2. \(\hat{\sigma}.\text{set}_\delta(\hat{\sigma}) \leadsto \hat{\sigma}'\)
   [From \(H1\)]

   1.1.3. \(\exists \sigma.\)
   1. \(\sigma.\text{ee}((\hat{\theta}(e)) = \nu\)
   2. \(\hat{\sigma}'' \vdash \hat{\sigma} \sim_v \nu\)
   [From \(H2, H3, H4\) and 1.1.2.1]

   1.1.4. \(\exists \sigma'.\)
   1. \(\sigma.\text{set}_\delta(\nu) \leadsto \sigma'\)
   2. \(\hat{\sigma}' \land \hat{\sigma}'' \sim_s \sigma\)
   [From \(H2, H3 1.1.2.2\) and 1.1.3.2]

   1.1.5. \(\hat{\sigma}.\text{set}_\delta(e)(\hat{\theta}) \leadsto \sigma'\)

   \(\exists \hat{\sigma}e[From 1.1.3.1 \text{ and } 1.1.4.1]\)

1.2. **Case:** \(P \ast Q\)

1.2.1. **Assume:**
1. \((H1)\) \(
   \hat{\sigma}.\text{set}_P(\hat{\theta}) \leadsto \hat{\sigma}'
   \)
2. \((H2)\) \(
   \hat{\sigma}'' \leq \hat{\sigma}_1 \land \hat{\sigma}'' \sim_s \sigma
   \)
3. \((H3)\) \(
   \hat{\sigma}'' \sim_s \sigma
   \)
4. \((H4)\) \(
   \hat{\sigma}'' \vdash \hat{\theta} \sim_v \theta
   \)

   **Prove:** \(\exists \sigma'. \hat{\sigma}.\text{set}_P(\theta) \leadsto \sigma' \land \hat{\sigma}' \land \hat{\sigma}'' \sim_s \sigma\)

   1.2.2. \(\exists \hat{\sigma}_1.\)
   1. \(\hat{\sigma}.\text{set}_P(\hat{\theta}) \leadsto \hat{\sigma}_1\)
   2. \(\hat{\sigma}_1.\text{set}_Q(\hat{\theta}) \leadsto \hat{\sigma}'\)
   1.2.3. \(\hat{\sigma}' \subseteq \hat{\sigma}_1\) [From 1.2.2.2]

   1.2.4. \(\sigma \leq \hat{\sigma}\)

   1.2.5. \(\hat{\sigma} \mid \hat{\sigma} \leq \hat{\sigma}_1\) [From 1.2.3 and 1.2.4]

   1.2.6. \(\hat{\sigma}'' \leq \hat{\sigma}_1\) [From \((H2)\) and 1.2.5]

   1.2.7. \(\exists \sigma_1.\)
   1. \(\sigma.\text{set}_P(\theta) \leadsto \sigma_1\)
2. \[ \hat{\sigma}_1 \models \hat{\sigma}'' \sim \sigma_1 \]  
[From H3, H4, 1.2.2.1 and IH]

1.2.8. \[ \hat{\sigma}_1 \leq \hat{\sigma}_1 \]

1.2.9. \[ \hat{\sigma}'' \subseteq \hat{\sigma}'' [\text{From H2}] \]

1.2.10. \[ \hat{\sigma}'' \subseteq \hat{\sigma}'' [\text{From 1.2.8 and 1.2.9}] \]

1.2.11. \[ \hat{\sigma}'' \subseteq \hat{\sigma}_1 [\text{From 1.2.6}] \]

1.2.12. \[ \hat{\sigma}'' \subseteq \hat{\sigma}'' \]

1.2.13. \[ \hat{\sigma}'' \subseteq \hat{\sigma}_1 \mid \hat{\sigma}'' [\text{From 1.2.11 and 1.2.12}] \]

1.2.14. \[ \hat{\sigma}_1 \mid \hat{\sigma}'' \vdash \theta \sim \theta [\text{From H4 and 1.2.13}] \]

1.2.15. \[ \exists \sigma' \]

\[ \begin{align*}
1. &. \sigma_1. \text{set}_Q (\theta) \sim \sigma' \\
2. &. \hat{\sigma}' \mid (\hat{\sigma}'' \models s) \sim \sigma' \\
\end{align*} \]  
[From 1.2.2.2 and 1.2.7.2]

1.2.16. \[ \hat{\sigma}' \subseteq \hat{\sigma}_1 [\text{From 1.2.2.2}] \]

1.2.17. \[ \hat{\sigma}' \mid \hat{\sigma}'' = (\hat{\sigma}' \mid \hat{\sigma}_1) \mid \hat{\sigma}'' [\text{From 1.2.16}] \]

1.2.18. \[ \hat{\sigma}' \mid \hat{\sigma}'' \models s \sigma' [\text{From 1.2.17 and 1.2.15.2}] \]

1.2.19. \[ \sigma . \text{set}_P (\theta) \models \sigma' [\text{From 1.2.7.1 and 1.2.15.2}] \]

2. **PROVE:** \[ \hat{\sigma} . \text{get}_p (\hat{\theta}) \models \hat{\sigma}'' \wedge \hat{\sigma}'' \models s \sigma' \wedge \hat{\sigma}'' \vdash \hat{\theta} \sim \theta \]

\[ \Rightarrow \exists \sigma'. \sigma . \text{get}_p (\theta) \sim \sigma' \wedge \hat{\sigma}' \mid \hat{\sigma}'' \models s \sigma' \]

2.1. **CASE:** \[ \delta (\check{e}) \]

2.1.1. **ASSUME:**

1. (H1) \[ \hat{\sigma} . \text{get}_{\delta (e)} (\hat{\theta}) \models \hat{\sigma}' \]

2. (H2) \[ \hat{\sigma}'' \leq \hat{\sigma} \mid \hat{\sigma}' \]

3. (H3) \[ \hat{\sigma}'' \models s \sigma \]

4. (H4) \[ \hat{\sigma}'' \vdash \hat{\theta} \sim \theta \]

**PROVE:** \[ \exists \sigma'. \sigma . \text{get}_{\delta (e)} (\theta) \models \sigma' \wedge \hat{\sigma}' \mid \hat{\sigma}'' \models s \sigma' \]

2.1.2. \[ \exists \hat{\delta} \]

\[ \begin{align*}
1. &. \hat{\sigma} . \text{ee} \hat{\theta (e)} = \hat{\theta} \\
2. &. \hat{\sigma} . \text{get}_{\hat{\delta}} (\hat{v}) \models \hat{\sigma}' \\
\end{align*} \]  
[From H1]

2.1.3. \[ \exists \hat{v} \]

\[ \begin{align*}
1. &. \sigma . \text{ee} (\theta (e)) = v \\
2. &. \hat{\sigma}' \vdash \hat{v} \sim v \theta \\
\end{align*} \]  
[From H2, H3, H4 and 2.1.2.1]

2.1.4. \[ \exists \hat{\delta}' \]

\[ \begin{align*}
1. &. \sigma . \text{get}_{\delta} (v) \models \sigma' \\
2. &. \hat{\sigma}' \mid \hat{\sigma}'' \models s \sigma' \\
\end{align*} \]  
[From H2, H3 2.1.2.2 and 2.1.3.2]

2.1.5. \[ \sigma . \text{get}_{\delta (e)} (\theta) \models \sigma' \]

[ÅÅ[From 2.1.3.1 and 2.1.4.1]]

2.2. **CASE:** \[ P \ast Q \]

2.2.1. **ASSUME:**

1. (H1) \[ \hat{\sigma} . \text{get}_P Q (\hat{\theta}) \models \hat{\sigma}' \]

2. (H2) \[ \hat{\sigma}'' \leq \hat{\sigma} \mid \hat{\sigma}' \]

3. (H3) \[ \hat{\sigma}'' \sim s \sigma \]

4. (H4) \[ \hat{\sigma}'' \vdash \hat{\theta} \sim \theta \]

**PROVE:** \[ \exists \sigma'. \sigma . \text{get}_P Q (\theta) \models \sigma' \wedge \hat{\sigma}' \mid \hat{\sigma}'' \models s \sigma' \]

2.2.2. \[ \exists \hat{\sigma}_1 \]


2.2.3. Prove:  \( \hat{\sigma}'' \leq \hat{\sigma} \mid_{\hat{\alpha}} \)
2.2.3.1. \( \hat{\sigma}'' \leq \hat{\sigma} \) [From H2]
2.2.3.2. \( \hat{\sigma}' \subseteq \hat{\alpha} \) [From 2.2.2.2]
2.2.3.3. \( \hat{\sigma}'' \leq \hat{\sigma}'' \mid_{\hat{\alpha}} \) [From 2.2.3.1 and 2.2.3.2]
2.2.3.4. \( \hat{\sigma}'' \leq \hat{\sigma}'' \mid_{\hat{\alpha}} \) [From H2 and 2.2.3.3]

2.2.4. Θσ1.
1. \( \sigma \cdot \text{get}_Q(\hat{\theta}) \leadsto \sigma' \)
2. \( \hat{\sigma}_1 \mid_{\hat{\sigma}'' \leadsto_s} \sigma_1 \)

2.2.5. Prove:  \( \hat{\sigma}_1 \mid_{\hat{\sigma}''} \leq \hat{\sigma}_1 \mid_{\hat{\sigma}''} \)
2.2.5.1. \( \hat{\sigma}_1 \leq \hat{\sigma}_1 \) [From H2]
2.2.5.2. \( \hat{\sigma}'' \subseteq \hat{\sigma}'' \) [From H2]
2.2.5.3. \( \hat{\sigma}_1 \mid_{\hat{\sigma}''} \leq \hat{\sigma}_1 \mid_{\hat{\sigma}''} \) [From 2.2.5.1 and 2.2.5.2]

2.2.6. Prove:  \( \hat{\sigma}_1 \mid_{\hat{\sigma}''} \theta \leadsto \theta \)
2.2.6.1. \( \hat{\sigma}_1 \mid_{\hat{\sigma}''} \subseteq \hat{\sigma}_1 \) [From 2.2.2.2]
2.2.6.2. \( \hat{\sigma}'' \subseteq \hat{\sigma}'' \) [From H2]
2.2.6.3. \( \hat{\sigma}'' \subseteq \hat{\sigma}_1 \) [From 2.2.6.1 and 2.2.6.2]
2.2.6.4. \( \hat{\sigma}'' \subseteq \hat{\sigma}_1 \mid_{\hat{\sigma}''} \) [From 2.2.6.3]
2.2.6.5. \( \hat{\sigma}_1 \mid_{\hat{\sigma}''} \theta \leadsto \theta \) [From H4 and 2.2.6.4]

2.2.7. Θσ'.
1. \( \sigma_1 \cdot \text{get}_p(\theta) \leadsto \sigma' \)
2. \( \hat{\sigma}' \mid_{\hat{\sigma}'' \leadsto_s} \sigma' \)
[From 2.2.2.2, 2.2.4.2, 2.2.5, 2.2.6 and IH]

2.2.8. Prove:  \( \hat{\sigma}' \mid_{\hat{\sigma}'' \leadsto_s} \hat{\sigma}' \mid_{\hat{\sigma}''} \)
2.2.8.1. \( \hat{\sigma}' \subseteq \hat{\sigma}_1 \) [From 2.2.2.2]
2.2.8.2. \( \hat{\sigma}' \mid_{\hat{\sigma}'' \leadsto_s} \hat{\sigma}' \mid_{\hat{\sigma}''} \) [From 2.2.8.1 and 2.2.8.2]
2.2.8.3. \( \hat{\sigma}' \mid_{\hat{\sigma}'' \leadsto_s} \sigma' \)
2.2.8.10. \( \sigma \cdot \text{get}_p(\hat{\theta}) \leadsto \sigma' \) [From 2.2.4.1 and 2.2.7.1]

C.2 Parametric Verification Semantics

Extended GIL Syntax

\[ \xi \in C_A \triangleq c \in C_A \mid x := e_1(e_2) \text{ with } [j; (\hat{x}_i : e_i)]_{i=0}^{n} \mid \text{ fold } p(n(e)) \text{ with } [j; (\hat{x}_i : e_i)]_{i=0}^{n} \mid \text{ unfold } p(n(e)) \]

\[ \text{proc} \in \text{Proc}_A \triangleq \text{proc } f(x)(\overline{\xi}) \]

\[ p \in \text{Proc}_A : F \rightarrow \text{Proc}_A \]

Verification Semantics of GIL: \( p \vdash (\sigma, cs, i)^n \leadsto (\sigma', cs', j)^{\alpha'} \)

| Non-Logical Cmd | Unfold | UnFOLD |
|-----------------|--------|--------|
| \( [p] \vdash (\sigma, cs, i) \leadsto (\sigma', cs', j) \) | \( \text{cmd}(p, cs, i) = \text{unfold } p(n(e)) \) | \( v = \sigma \cdot \text{ce} (e) \) |
| \( p \cdot \text{preds}.pm = \text{pred } p(n(x)) \rightarrow P_0; \ldots ; P_n \) | \( 0 \leq j \leq n \) | \( \sigma \cdot \text{get}_p([p(n, v)]) \leadsto \sigma' \) |
| \( \sigma' \cdot \text{set}_p([x \mapsto v]) \leadsto (\sigma'', -) \) | \( \vdash (\sigma'', cs, i+1) \) | \( p \vdash (\sigma, cs, i) \leadsto (\sigma'', cs, i+1) \) |
Verification Semantics of GIL: $p \vdash (\sigma, cs, i)^{a} \rightsquigarrow (\sigma', cs', j)^{a'}$ (continued)

**FOLD**

$\begin{align*}
\text{cmd}(p, cs, i) &= \text{fold } pn(e_{i}) \text{ with } \{j; (\hat{x}_{j} : e_{j}) \mid [n]_{i=1}^{n}\} \\
\text{preds} \cdot pn &= \text{pred } pn(x) := P_{0}; \ldots; P_{n} \\
\theta &= \{x \mapsto \hat{v}_{0}, \hat{x}_{1} \mapsto \hat{v}_{1}, \ldots, \hat{x}_{n} \mapsto \hat{v}_{n}\} \\
\sigma \cdot \text{get}_{P}(\theta) &= \sigma' \quad \sigma' \cdot \text{set}_{P}(pn, v) \rightsquigarrow \sigma'' \\
\text{cmd}(p, cs, i) &= x := e(e_{i}) \text{ with } \{j; (\hat{x}_{j} : e_{j}) \mid [n]_{i=1}^{n}\}
\end{align*}$

**SPEC CALL**

$\begin{align*}
\text{cmd}(p, cs, i) &= x := e(e_{i}) \text{ with } \{j; (\hat{x}_{j} : e_{j}) \mid [n]_{i=1}^{n}\} \\
f &= \sigma \cdot ee(\epsilon) \\
\theta &= \{x_{0} \mapsto \hat{v}_{0}, \ldots, \hat{x}_{n} \mapsto \hat{v}_{n}\} \\
\sigma \cdot \text{get}_{P}(\theta) &= \sigma' \quad \sigma' \cdot \set_{Q}(\theta) \rightsquigarrow \sigma'' \\
\text{cmd}(p, cs, i) &= x := e(e_{i}) \text{ with } \{j; (\hat{x}_{j} : e_{j}) \mid [n]_{i=1}^{n}\}
\end{align*}$

**Theorem C.5 (Verification).** Let $SR = (\langle |, \sim_{s}, \sim_{o} \rangle)$ be a soundness relation for $\hat{S} = (|S|, \hat{V}, A)$ in terms of $S = (|S|, V, A)$ and $\leq$ the pre-order induced by $\sim_{s}$. It holds that:

$\hat{c}f \rightsquigarrow \hat{c}f' \wedge \hat{c}f'' \rightleftharpoons \hat{c}f | \hat{c}f' \wedge \hat{c}f'' \rightsquigarrow \sim_{s} \hat{c}f' \implies \exists \hat{c}f''. \hat{c}f \rightsquigarrow \hat{c}f' \wedge \hat{c}f'' \rightsquigarrow \sim_{s} \hat{c}f''$

**Proof:**

In order to prove this theorem, we prove the following more general claim, from which the theorem immediately follows:

$\hat{c}f \rightsquigarrow \hat{c}f' \wedge \hat{c}f'' \rightleftharpoons \hat{c}f | \hat{c}f' \wedge \hat{c}f'' \rightsquigarrow \sim_{s} \hat{c}f' \implies \exists \hat{c}f''. \hat{c}f \rightsquigarrow \hat{c}f' \wedge \hat{c}f'' | \hat{c}f'' \rightleftharpoons \sim_{s} \hat{c}f''$

We proceed by use analysis on the rule used to derive $\hat{c}f \rightsquigarrow \hat{c}f'$. We only cover the non-logic command, unfold and spec call cases, the other cases being analogous.

1. **Case: Non-logic commands**
   1.1. **Assume:** 1. (H1) $\hat{c}f \rightsquigarrow \hat{c}f'$
   2. (H2) $\hat{c}f'' \rightleftharpoons \hat{c}f | \hat{c}f'$
   3. (H3) $\hat{c}f'' \rightleftharpoons \sim_{s} \hat{c}f$
   **Prove:** $\exists \hat{c}f''. \hat{c}f \rightsquigarrow \hat{c}f' \wedge \hat{c}f'' \rightsquigarrow \sim_{s} \hat{c}f''$
   1.2. $\hat{c}f \rightsquigarrow \hat{c}f'$ [From H1]
   1.3. $\exists \hat{c}f''$. 1. $\hat{c}f \rightsquigarrow \hat{c}f'$
   2. $\hat{c}f' | \hat{c}f'' \rightleftharpoons \sim_{s} \hat{c}f''$ [From H2, H3, 1.2 and Theorem 4.1]
   1.4. $\hat{c}f \rightsquigarrow \hat{c}f''$ [From 1.3 and execution of non-logic command.]
2. **Case: Unfold**
   2.1. **Assume:** 1. (H1) $\hat{c}f \rightsquigarrow \hat{c}f'$
   2. (H2) $\hat{c}f'' \rightleftharpoons \hat{c}f | \hat{c}f'$
   3. (H3) $\hat{c}f'' \rightleftharpoons \sim_{s} \hat{c}f$
   **Prove:** $\exists \hat{c}f'. \hat{c}f \rightsquigarrow \hat{c}f' \wedge \hat{c}f'' \rightsquigarrow \sim_{s} \hat{c}f''$
   2.2. $\exists \hat{p}, \hat{\sigma}, \hat{\sigma}_{1}, \hat{\sigma}', cs_{1}, i, j, pn, e, x, P_{1} \mid \sigma, \hat{\sigma}$. 1. $\hat{c}f = \langle \hat{\sigma}, cs_{1}, i \rangle$
   2. $\text{cmd}(p, cs_{1}, i) = \text{unfold } pn(e)$
   3. $p \cdot \text{preds} \cdot pn = \text{pred } pn(x) := P_{0}; \ldots; P_{n}$
   4. $\sigma \cdot ee(\epsilon) = \hat{\sigma}$
   5. $\hat{\sigma} \cdot \text{get}_{P}((pn, \hat{v})) \rightsquigarrow \hat{\sigma}_{1}$
   6. $\hat{\sigma}_{1} \cdot \text{set}_{P}((x \mapsto \hat{v})) \rightsquigarrow (\hat{\sigma}', -)$
   7. $\hat{c}f' = \langle \hat{\sigma}', cs_{1}, i + 1 \rangle$
   2.3. $\exists \hat{\sigma}''$. 1. $\hat{c}f'' = \langle \hat{\sigma}'', cs_{1}, i \rangle$
2. \( \hat{\sigma}'' \leq \hat{\sigma} | \hat{\sigma}' \)  
[From 2.2.1 and H2]

2.4. \( \exists \sigma, cs_2, \)
1. \( \tilde{c}'' = (\sigma, cs_2, i) \)
2. \( s\hat{\sigma}'' \sigma \)
3. \( \hat{\sigma}'' + cs_1 \sim_v cs_2 \)
[From 2.3.1 and H3]

2.5. \( \text{cmd}_{p}, cs_2, i = \text{unfold } pn(e) \) [From 2.4.3] and 2.2.2

2.6. \( \hat{\sigma}'' \leq \hat{\sigma} \) [From 2.3.2]

2.7. \( \exists v. \)
1. \( \sigma.ee(e) = v \)
2. \( \hat{\sigma}'' + \hat{\sigma} \sim_v v \)

2.8. PROVE: \( \hat{\sigma}'' \leq \hat{\sigma} | \hat{\sigma}_1 \)

2.8.1. \( \hat{\sigma} \leq \hat{\sigma} \)

2.8.2. \( \hat{\sigma}' \subseteq \hat{\sigma}_1 \) [From 2.2.6]

2.8.3. \( \hat{\sigma} | \hat{\sigma}' \leq \hat{\sigma} | \hat{\sigma}_1 \) [From 2.8.1 and 2.8.2]

2.8.4. \( \hat{\sigma}'' \leq \hat{\sigma} | \hat{\sigma}_1 \) [From 2.3.2 and 2.8.3]

2.9. \( \exists \sigma_1. \)
1. \( \sigma.gem \{ [pn, v] \} \rightsquigarrow \sigma_1 \)
2. \( \hat{\sigma}_1 | \hat{\sigma}_n \sim_\sigma \sigma_1 \)
[From 2.2.5, 2.4.2, 2.7.2, 2.8]

2.10. PROVE: \( \hat{\sigma}_1 | \hat{\sigma}_n \leq \hat{\sigma}_1 | \hat{\sigma}' \)

2.10.1. \( \hat{\sigma}_1 \leq \hat{\sigma}_1 \)

2.10.2. \( \hat{\sigma}'' \subseteq \hat{\sigma}' \) [From 2.3.2]

2.10.3. \( \hat{\sigma}_1 | \hat{\sigma}_n \leq \hat{\sigma}_1 | \hat{\sigma}' \) [From 2.10.1 and 2.10.2]

2.11. PROVE: \( \hat{\sigma}'' \subseteq \hat{\sigma}_1 | \hat{\sigma}_n \)

2.11.1. \( \hat{\sigma}'' \subseteq \hat{\sigma}' \) [From 2.3.2]

2.11.2. \( \hat{\sigma}' \subseteq \hat{\sigma}_2 \) [From 2.2.6]

2.11.3. \( \hat{\sigma}'' \subseteq \hat{\sigma}_1 \) [From 2.11.1 and 2.11.2]

2.11.4. \( \hat{\sigma}'' \subseteq \hat{\sigma}_1 | \hat{\sigma}_n \) [From 2.11.3]

\[
\hat{\sigma}'' | (\hat{\sigma}_1 | \hat{\sigma}_n) = (\hat{\sigma}'' | \hat{\sigma}_1) | \hat{\sigma}_n
\]

\[
\hat{\sigma}'' | \hat{\sigma}_n = \hat{\sigma}''
\]

2.12. \( \hat{\sigma}_1 | \hat{\sigma}_n \Rightarrow \hat{\sigma} \sim_v v \) [From 2.7.2 and 2.11]

2.13. \( \exists \sigma'. \)
1. \( \sigma_1.set_{p_j}([x \mapsto v]) \rightsquigarrow (\sigma', -) \)
2. \( \hat{\sigma}' | (\hat{\sigma}_1 | \hat{\sigma}_n) \sim s \sigma' \)
[From 2.2.6, 2.9.2, 2.10, 2.12 and Theorem 4.5]

2.14. PROVE: \( \sigma' | (\hat{\sigma}_1 | \hat{\sigma}_n) = \hat{\sigma}' | \hat{\sigma}_n \)

2.14.1. \( \hat{\sigma}' \subseteq \hat{\sigma}_1 \) [From 2.2.6]

2.14.2. \( \hat{\sigma}'' | (\hat{\sigma}_1 | \hat{\sigma}_n) = (\hat{\sigma}' | \hat{\sigma}_1) | \hat{\sigma}_n \)

2.14.3. \( (\hat{\sigma}'' | \hat{\sigma}_1) | \hat{\sigma}_n = \hat{\sigma}' | \hat{\sigma}_n \) [From 2.14.1]

2.14.4. \( \hat{\sigma}'' | (\hat{\sigma}_1 | \hat{\sigma}_n) = \hat{\sigma}'' | \hat{\sigma}_n \) [From 2.14.2 and 2.14.3]

2.15. \( \hat{\sigma}' | \hat{\sigma}_n \sim_s \sigma' \) [From 2.13.2 and 2.14.3]

2.16. \( cf^{\hat{\sigma}''}(\sigma', cs_2, i + 1) = cf' \) [From 2.2.3, 2.4.1, 2.5, 2.7.1, 2.9.1 and 2.13.1]

2.17. PROVE: \( \hat{\sigma}'' \subseteq \hat{\sigma}' \) [From 2.3.2]
2.17.2. \( \hat{\sigma}'' \subseteq \hat{\sigma}' \mid \hat{\sigma}'' \) \[From 2.17.1\]
\( \hat{\sigma}' \mid (\hat{\sigma}'' | \hat{\sigma}''') = (\hat{\sigma}''' | \hat{\sigma}''') \)
\( \hat{\sigma}'' | \hat{\sigma}'' = \hat{\sigma}'' \)

2.17.3. \( \hat{\sigma}' \mid \hat{\sigma}'' + c_{s_1} \sim_v c_{s_2} \) \[From 2.4.3 and 2.17.2\]

2.17.4. \( \hat{c}_f \mid \hat{\sigma}'' \sim_c \hat{c}' \) \[From 2.2.7, 2.3.1, 2.15, 2.16 and 2.17.3\]

3. Case: Spec Call

3.1. Assume: 1. (H1) \( \hat{c}_f \overset{*}{\rightarrow} \hat{c}' \)
2. (H2) \( \hat{c}_f'' \leq \hat{c}_f \mid \hat{\sigma}' \)
3. (H3) \( \hat{c}_f'' \sim_s \hat{c}_f \)

Prove: \( \exists \hat{c}_f'. \hat{c}_f'' \sim_s \hat{c}_f' \land \hat{c}_f' \sim_s \hat{c}_f'' \)

3.2. \( \exists p, \hat{\sigma}, \hat{\sigma}_1, \hat{\sigma}_2, \hat{\sigma}', c_{s_1}, i, j, \hat{e}_R, \hat{e}_L, f, x_0, \hat{x}_i | i = 1 \wedge f = \hat{\sigma}.ee(e_f) \)
1. \( \hat{c}_f'' = (\hat{\sigma}, c_{s_1}, i) \)
2. \( \hat{c}_f''(p, c_{s_1}, i) = x := e_f(e_0) \) with \( \{ j; (\hat{x}_i : e_i) | i = 1 \} \wedge f = \hat{\sigma}.ee(e_f) \)
3. \( \hat{p}.specs.f.j = \{ P \} f(x_0) \{ Q, e_R \} \)
4. \( \hat{\sigma}.ee(e_0) = \hat{v}_0 \)
5. \( \hat{\sigma}.ee(e_i) = \hat{v}_i | i = 1 \)
6. \( \hat{\sigma}.ee(e_R) = \hat{v}_R \)
7. \( \hat{\theta} = [x_0 \mapsto \hat{v}_0, ..., \hat{x}_n \mapsto \hat{v}_n] \)
8. \( \hat{\sigma}.getp(\hat{\theta}) \sim \hat{\sigma}_1 \)
9. \( \hat{\sigma}_1.setQ(\hat{\theta}) \sim \hat{\sigma}_2 \)
10. \( \hat{\sigma}_2.setVar(\hat{x}_R, \hat{v}_R) \sim (\hat{\sigma}', c_{s_1}, i + 1) \)
11. \( \hat{c}_f' = (\hat{\sigma}', c_{s_1}, i + 1) \)

3.3. \( \exists \hat{\sigma}'' \)
1. \( \hat{c}_f'' = (\hat{\sigma}'' | c_{s_1}, i) \)
2. \( \hat{\sigma}'' \leq \hat{\sigma} \mid \hat{\sigma}' \)

[From 3.2.1, 3.2.11 and H2]

3.4. \( \exists \sigma, c_{s_2} \)
1. \( \hat{c}_f'' = (\sigma, c_{s_2}, i) \)
2. \( \hat{\sigma}'' \sim_s \hat{\sigma} \mid \hat{\sigma}' \)
3. \( \hat{\sigma}'' + c_{s_1} \sim_v c_{s_2} \)

[From 3.3.1 and H3]

3.5. \( \text{cmd}(p, c_{s_2}, i) = x := e_f(e_0) \) with \( [ j; (\hat{x}_i : e_i) | i = 1 \} \wedge f = \sigma.\text{ee}(e_f) \)

[From 3.2.2 and 3.4.3]

3.6. \( \hat{\sigma}'' \leq \hat{\sigma} \)

[From 3.3.2]

3.7. \( \exists \hat{v}_0 \)
1. \( \sigma.\text{ee}(e_0) = \hat{v}_0 \)
2. \( \hat{\sigma}'' \sim_v \hat{v}_0 \)

[From 3.2.4, 3.4.2 and 3.6]

3.8. \( \exists \hat{v}_i | i = 1 \wedge 1 \leq i \leq n \)
1. \( \sigma.\text{ee}(e_i) = \hat{v}_i \)
2. \( \hat{\sigma}'' \sim_v \hat{v}_i \)

[From 3.2.5, 3.4.2 and 3.6]

3.9. \( \exists \hat{v}_R \)
1. \( \sigma.\text{ee}(e_R) = \hat{v}_R \)
2. \( \hat{\sigma}'' \sim_v \hat{v}_R \)
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3.10. Let: \( \theta = [x_0 \mapsto \nu_0, \ldots, x_n \mapsto \nu_n] \)

3.11. \( \hat{\sigma} \vdash \hat{\theta} \sim_\nu \theta \) [From 3.2.7, 3.7.2, 3.8.2, 3.9.2, 3.10]

3.12. Prove: \( \hat{\sigma}'' \leq \hat{\sigma} |_{\hat{\alpha}_i} \)

3.12.1. \( \hat{\sigma} \leq \hat{\sigma} \)

3.12.2. \( \hat{\sigma}' \sqsubseteq \hat{\sigma}_1 \) [From 3.2.9]

3.12.3. \( \hat{\sigma} |_{\hat{\alpha}_i} \leq \hat{\sigma} |_{\hat{\alpha}_i} \) [From 3.12.2 and 3.12.3]

3.12.4. \( \hat{\sigma}'' \leq \hat{\sigma} |_{\hat{\alpha}_i} \) [From 3.3.2 and 3.12.3]

3.13. \( \exists \sigma_1. \)

1. \( \sigma \_getP(\theta) \sim \sigma_1 \)

2. \( \hat{\sigma}_1 |_{\hat{\alpha}^n} \sim_\theta \sigma_1 \)

[From 3.2.8, 3.4.2, 3.11, 3.12 and Theorem 4.5]

3.14. Prove: \( \hat{\sigma}_1 |_{\hat{\alpha}^n} \leq \hat{\sigma}_2 |_{\hat{\alpha}^n} \)

3.14.1. \( \hat{\sigma}_1 \leq \hat{\sigma}_2 \)

3.14.2. \( \hat{\sigma}'' \subseteq \hat{\sigma}' \) [From 3.3.2]

3.14.3. \( \hat{\sigma} \sqsubseteq \hat{\sigma}_2 \) [From 3.2.10]

3.14.4. \( \hat{\sigma}'' \sqsubseteq \hat{\sigma}_2 \) [From 3.14.2 and 3.14.3]

3.14.5. \( \hat{\sigma}_1 |_{\hat{\alpha}^n} \leq \hat{\sigma}_1 |_{\hat{\alpha}^n} \) [From 3.14.1 and 3.14.4]

3.15. Prove: \( \hat{\sigma}'' \subseteq \hat{\sigma}_1 |_{\hat{\alpha}^n} \)

3.15.1. \( \hat{\sigma}'' \subseteq \hat{\sigma}' \) [From 3.3.2]

3.15.2. \( \hat{\sigma}' \sqsubseteq \hat{\sigma}_1 \) [From 3.2.9, 3.2.10 and transitivity]

3.15.3. \( \hat{\sigma}'' \subseteq \hat{\sigma}_1 \) [From 3.15.1 and 3.15.2]

3.15.4. \( \hat{\sigma}'' \sqsubseteq \hat{\sigma}_1 |_{\hat{\alpha}^n} \) [From 3.15.3]

\( \hat{\sigma}'' |_{\hat{\alpha}_i} = (\hat{\sigma}'' |_{\hat{\alpha}_i}) |_{\hat{\alpha}''} = \hat{\sigma}'' |_{\hat{\alpha}''} \)

3.16. \( \hat{\sigma}_1 |_{\hat{\alpha}^n} \vdash \hat{\theta} \sim_\nu \theta \) [From 3.11 and 3.15]

3.17. \( \exists \sigma_2. \)

1. \( \sigma_2 \_getQ(\theta) \sim \sigma_2 \)

2. \( \hat{\sigma}_2 |_{\hat{\alpha}^n} \sim_\theta \sigma_2 \)

[From 3.2.2, 3.13.2, 3.14, 3.16 and Theorem 4.5]

3.18. Prove: \( \hat{\sigma}_2 |_{\hat{\alpha}^n} \leq \hat{\sigma}_2 |_{\hat{\alpha}^n} \)

3.18.1. \( \hat{\sigma}_2 \leq \hat{\sigma}_2 \)

3.18.2. \( \hat{\sigma}'' \subseteq \hat{\sigma}' \) [From 3.3.2]

3.18.3. \( \hat{\sigma}_2 |_{\hat{\alpha}^n} \leq \hat{\sigma}_2 |_{\hat{\alpha}^n} \) [From 3.18.1 and 3.18.2]

3.19. Prove: \( \hat{\sigma}'' \subseteq \hat{\sigma}_2 |_{\hat{\alpha}^n} \)

3.19.1. \( \hat{\sigma}'' \subseteq \hat{\sigma}' \subseteq \hat{\sigma}_1 \) [From 3.3.2 and 3.2.10]

3.19.2. \( \hat{\sigma}'' |_{\hat{\alpha}_i} = (\hat{\sigma}'' |_{\hat{\alpha}_i}) |_{\hat{\alpha}''} = \hat{\sigma}'' |_{\hat{\alpha}''} \) [From 3.19.1]

3.20. \( \hat{\sigma}_2 |_{\hat{\alpha}^n} \vdash \hat{\theta}_R \sim_\nu \nu_R \) [From 3.9.2 and 3.19]

3.21. \( \exists \sigma'. \)

1. \( \sigma_2 \_setVar(x_R, \nu_R) \sim \sigma' \)

2. \( \hat{\sigma}' |_{\hat{\alpha}^n} \sim_\theta \sigma' \)

3.22. Prove: \( \hat{\sigma}' |_{\hat{\alpha}_i} = \hat{\sigma}' |_{\hat{\alpha}''} \)

3.22.1. \( \hat{\sigma}' |_{\hat{\alpha}_i} = (\hat{\sigma}' |_{\hat{\alpha}_i}) |_{\hat{\alpha}''} = \hat{\sigma}' |_{\hat{\alpha}''} \) [Given that \( \hat{\sigma}' \subseteq \hat{\sigma}_2 \)]

3.23. \( \hat{\sigma}' |_{\hat{\alpha}^n} \sim_\theta \sigma' \)

3.24. Prove: \( \hat{\sigma}'' \subseteq \hat{\sigma}' |_{\hat{\alpha}''} \)
3.24.1. $\dot{\sigma}'' \subseteq \dot{\sigma}'$ [From 3.3.2]
3.24.2. $\dot{\sigma}'' \mid_{\dot{\sigma}'[\dot{\sigma}'']} = (\dot{\sigma}'' \mid_{\dot{\sigma}'}) \mid_{\dot{\sigma}''} = \dot{\sigma}''$

3.25. $\dot{\sigma}' \mid_{\dot{\sigma}''} \models c_s_1 \sim \nu c_s_2$ [From 3.4.3 and 3.23]
3.26. $\mathcal{C} = \langle \sigma, c_s_2, i \rangle \leadsto \langle \sigma', c_s_2, i + 1 \rangle = \mathcal{C}'$
3.27. $\mathcal{C} \mid_{\dot{\sigma}' \sim \nu} \models \mathcal{C}'$

C.3 While: Verification

- $\Delta_W = \{\text{cell}\}$
- $A_W = \{\text{lookup, mutate, dispose, setCell, getCell}\}$

While: Concrete and Symbolic Memories (Continued)

\[
\begin{array}{ll}
\text{C-SetCell} & \text{S-SetCell} \\
(l, p) \notin \text{dom(}\mu\text{)} & \mu' = \mu \cup l.p \mapsto v \\
\mu.\text{setCell}(\{l, p, v\}) \leadsto (\mu', \text{true}) & \mu.\text{setCell}(\{l, p, v\}, \pi) \leadsto \{\mu', \text{true}, \text{true}\} \\
\text{C-GetCell} & \text{S-GetCell} \\
\mu = \mu' \cup l.p \mapsto v & \pi \vdash (\dot{\mu} = \dot{l}.p) \mapsto \dot{v} \\
\mu.\text{getCell}(\{l, p, v\}) \leadsto (\mu', \text{true}) & \mu.\text{getCell}(\{l, p, v\}, \pi) \leadsto \{\mu', \text{true}, \text{true}\}
\end{array}
\]

**Lemma C.6** (While: Concrete Core Predicate Interpretation). The core predicate action interpretation $\langle \Delta_W, A_W, \{\text{cell} \mapsto \text{setCell}, \text{cell} \mapsto \text{getCell}\} \rangle$ is well-formed with respect to CST($M_W$).

**Proof:**

We need to prove that the following equivalent holds for any concrete memories $\mu, \mu' \in M_W$, and any triple $[l, p, v]$:

$$
\sigma.\text{getCell}(\{l, p, v\}) \leadsto \sigma' \iff \sigma'.\text{setCell}(\{l, p, v\}) \leadsto \sigma
$$

We start by proving the left to right implication, before proving the right to left implication.

1. **Prove:** $\sigma.\text{getCell}(\{l, p, v\}) \leadsto \sigma' \implies \sigma'.\text{setCell}(\{l, p, v\}) \leadsto \sigma$

   **1.1. Assume:** (H1) $\sigma.\text{getCell}(\{l, p, v\}) \leadsto \sigma'$

   **Prove:** $\sigma'.\text{setCell}(\{l, p, v\}) \leadsto \sigma$

   **1.2. $\exists \mu'. \mu = \mu' \cup (l, p) \mapsto v$ [From (H1)]

   **1.3. $\exists \mu'. p_i^n \mid_{i=1}.$**

   1. $\mu' \mid_{l} = \mu'' \cup _-
   2. $\mu'' = \cup^n_{i=1}(l, p_i) \mapsto _-
   3. $(p_i \neq p) \mid_{i=1} \quad \text{[From 1.2]}

   **1.4. $\sigma'.\text{setCell}(\{l, p, v\}) \leadsto \sigma$ [From 1.3]

2. **Prove:** $\sigma.\text{getCell}(\{l, p, v\}) \leadsto \sigma' \iff \sigma'.\text{setCell}(\{l, p, v\}) \leadsto \sigma$

   **2.1. Assume:** (H1) $\sigma.\text{getCell}(\{l, p, v\}) \leadsto \sigma'$

   **Prove:** $\sigma'.\text{getCell}(\{l, p, v\}) \leadsto \sigma$

   **2.2. $\exists \mu'. p_i^n \mid_{i=1}.$**

   1. $\mu' \mid_{l} = \mu'' \cup _-
   2. $\mu'' = \cup^n_{i=1}(l, p_i) \mapsto _-
   3. $p \in \{p_1, ..., p_n\}$

   **2.2. $\mu' = \mu \cup (l, p) \mapsto v$ [From 2.2]
2.3. $\alpha'.getCell([l,p,v]) \leadsto \sigma$ [From 2.2]

**Lemma C.7 (While: Symbolic Core Predicate Interpretation).** The core predicate action interpretation $\langle \Delta_W,A_W, [\text{cell} \mapsto \text{setCell}], [\text{cell} \mapsto \text{getCell}] \rangle$ is well-formed with respect to SST($\hat{M}_W$).

**Proof:** The proof is analogous to the proof of Lemma C.6.

**Lemma C.8 (Extended While: Memory Interpretation).** $I_W$ is an interpretation of While symbolic memories $M_W$ with respect to the concrete memories $M_W$, for While extended with specifications.

**Proof:** For every action $\alpha \in A_W$, we have to prove that:

$$\forall \hat{\mu}, \hat{\mu}' \in \hat{M}_W; \mu \in M_W; \hat{e}, \hat{e}' \in \hat{E}; \epsilon \in \mathcal{E}_{nu}; \pi, \pi' \in \Pi.
\hat{\mu}.\alpha(\hat{e}, \pi) \leadsto (\hat{\mu}', \hat{e}', \pi') \land \mu = I_W(\hat{\mu}, \epsilon) \land \llbracket \pi \land \pi' \rrbracket_\epsilon = \text{true} \implies \exists \mu'. \mu' = I_W(\hat{\mu}', \epsilon) \land \mu.\alpha(\llbracket \hat{e} \rrbracket_\epsilon) = (\mu', \llbracket \hat{e}' \rrbracket_\epsilon)$$

(11)

We proceed by case analysis on the rule that was used to derive $\hat{\mu}.\alpha(\hat{e}, \pi) \leadsto (\hat{\mu}', \hat{e}', \pi')$. For the lookup, mutate and dispose actions we refer to the Lemma 3.7. Only the setCell and getCell cases remain. We prove the setCell case, the getCell case being analogous.

1. **Case: setCell**

1.1. **Assume:** 1. (H1) $\hat{\mu}.\text{setCell}([\hat{e}_t,p,\hat{e}_v], \pi) \leadsto (\hat{\mu}', \hat{e}', \pi')$

2. (H2)
   a. $\llbracket \hat{e}_t \rrbracket_\epsilon = l$
   b. $\llbracket \hat{e}_v \rrbracket_\epsilon = v$

3. (H3) $\mu = I_W(\hat{\mu}, \epsilon)$

4. (H4) $\llbracket \pi \land \pi' \rrbracket_\epsilon = \text{true}$

**Prove:** $\exists \mu'. \mu' = I_W(\hat{\mu}', \epsilon) \land \mu.\alpha(\llbracket \hat{e} \rrbracket_\epsilon) = (\mu', \llbracket \hat{e}' \rrbracket_\epsilon)$

1.2. $\exists \hat{\mu}_1, \hat{\mu}_2, p_1 \mid n \mid i=1$
   1. $\hat{\mu} \vdash \hat{e}_t, \pi = \hat{\mu}_1 \uplus _$
   2. $\hat{\mu}_1 = \cup_{i=1}^n(\hat{e}_t, p_i) \mapsto _$
   3. $(p \neq p_1) \mid i=1$
   4. $\hat{\mu}' = \hat{\mu} \uplus (\hat{e}_t, p) \mapsto \hat{e}_v$
   5. $\hat{e}' = \text{true}$
   6. $\pi' = \text{true}$

[From (H1)]

1.3. $I_W(\hat{\mu}, \epsilon) \mid \llbracket \hat{e}_t \rrbracket_\epsilon = I_W(\hat{\mu}, \epsilon) \uplus _$ [From 1.2.1 and (H4)]

1.4. $I_W(\hat{\mu}_1, \epsilon) = \cup_{i=1}^n(\llbracket \hat{e}_t \rrbracket_\epsilon, p_i) \mapsto _$ [From 1.2.2]

1.5. $\mu \mid = I_W(\hat{\mu}_1, \epsilon) \uplus _$ [From (H2.a), (H3) and 1.3]

1.6. $I_W(\hat{\mu}_1, \epsilon) = \cup_{i=1}^n(l, p_i) \mapsto _$ [From (H2.a) 1.4]

1.7. $\mu.\text{setCell}([l, p, v]) \leadsto (\mu \uplus (l, p) \mapsto v, \text{true})$ [From 1.2.3, 1.5 and 1.6]

1.8. $I_W(\hat{\mu}', \epsilon) = I_W(\hat{\mu} \uplus (\hat{e}_t, p) \mapsto \hat{e}_v, \epsilon)$
   = $I_W(\hat{\mu}, \epsilon) \uplus I_W((\hat{e}_t, p) \mapsto \hat{e}_v, \epsilon)$
   = $\hat{\mu} \uplus (\llbracket \hat{e}_t \rrbracket_\epsilon, p) \mapsto \llbracket \hat{e}_v \rrbracket_\epsilon$ [From (H3)]
   = $\hat{\mu} \uplus (l, p) \mapsto v$ [From (H2)]
D  SECTION 5: PARAMETRIC BI-ABDUCTIVE ANALYSIS

D.1  Parametric Bi-abductive Analysis

Action Fixes

\[ \text{ea} : A \to |S| \to V \to \varphi(|S| \times V) \cup \varphi(\mathcal{A}_\Delta) \]

\[ ((\sigma', \nu') \in \text{ea}(\alpha, \sigma, v) \equiv_{PP} \sigma.\alpha(v) \leadsto S(\sigma', \nu')) \]

\[ (P \in \text{ea}(\alpha, \sigma, v) \equiv_{PP} \sigma.\alpha(v) \leadsto F(P)) \]

**Definition D.1 (Well-formedness of Fixes).** If the original state is extended with a generated fix, it must be possible to execute the action successfully.

\[ \sigma.\alpha(v) \leadsto F(P) \land \sigma.set_P([]) \leadsto \sigma'' \implies \exists \sigma', \nu', \sigma''.\alpha(v) \leadsto S(\sigma', \nu') \quad (12) \]

**Bi-abductive Analysis.**

**Definition D.2 (BiState Constructor (BiST)).** The bi-abductive state constructor BiST : |S| \to |S| is defined as BiST(|S|, V, A), where:

- \(|S'|| \triangleq |S| \times \mathcal{A}_\Delta |
- \text{setVar}_{\text{bi}}((\sigma, P), x, v) \triangleq \langle \text{setVar}(\sigma, x, v), P \rangle |
- \text{setStore}_{\text{bi}}((\sigma, P), \rho) \triangleq \langle \text{setStore}(\sigma, \rho), P \rangle |
- \text{store}_{\text{bi}}((\sigma, \rho, -)) \triangleq \text{store}(\sigma) |
- \text{ee}_{\text{bi}}((\sigma, \rho, -), e) \triangleq \text{ee}(\sigma, e) |
- \text{ea}_{\text{bi}}(\alpha, (\sigma, P), v) \triangleq \{ ((\sigma', P), (\nu')) | \sigma.\alpha(v) \leadsto S(\sigma', \nu') \} \text{ if } \alpha \neq \text{assume} |
- \text{ea}_{\text{bi}}(\text{assume}, (\sigma, P), v) \triangleq \{ ((\sigma', P * \pi), (\nu')) | [\nu'] = \pi \land \sigma.\text{assume}(v) \leadsto S(\sigma', \nu') \} |
- \text{ea}_{\text{bi}}(\alpha, (\sigma, P), v) \triangleq \{ \langle \sigma''', P * Q), (\nu') \rangle \}

\[ \text{LEMMA D.3 (One-Step).} \]

\[ \mathcal{P} \vdash \langle \sigma, P, cs, i \rangle \overset{\text{BiST}(\mathcal{S})}{\leadsto} \langle \sigma', Q, cs', j \rangle^o \]

\[ \implies \exists P'. Q = P * P' \land \sigma.set_P([]) \leadsto \sigma'' \land \sigma'' \leq \sigma' \vdash \mathcal{P} \vdash \langle \sigma'', \sigma, cs, i \rangle \overset{\text{BiST}(\mathcal{S})}{\leadsto} \langle \sigma', cs', j \rangle^o \]

**Proof:**

We proceed by case analysis on the rule used to produce \( \mathcal{P} \vdash \langle \sigma, P, cs, i \rangle \overset{\text{BiST}(\mathcal{S})}{\leadsto} \langle \sigma', Q, cs', j \rangle^o \).

We only provide the proof for the execute action case. The other cases are analogous.

There are two rules concerning the execute action case:

1. The case where the action does not generate an error
2. The case where the action generates an error

We are providing the proof for both cases.

1. **Case: [No Error]**
   1.1. **Assume:** \( \mathcal{P} \vdash \langle \sigma, P, cs, i \rangle \overset{\text{BiST}(\mathcal{S})}{\leadsto} \langle \sigma', Q, cs', j \rangle^o \)
   1.2. 1. \( \text{cmd}(\mathcal{P}, \sigma, cs, i) = x := \alpha(\text{exp}) \)
   2. \( \sigma.\text{ee}(\text{exp}) = v \)
   3. \( \sigma.\alpha(v) \leadsto S(\sigma'', \nu') \)
   4. \( \sigma' = \sigma''.\text{setVar}(x, \nu') \)

   [From 1.1]

   1.3. \( \mathcal{P} \vdash \langle \sigma, cs, i \rangle \overset{\text{S}}{\leadsto} \langle \sigma', cs, i + 1 \rangle^o \) [From 1.2]
   1.4. \( \mathcal{P} \vdash P * \text{emp} \)
   1.5. \( \sigma.\text{setemp}([]) \leadsto \sigma \)

2. **Case: [Action Error]**
   2.1. **Assume:** \( \mathcal{P} \vdash \langle \sigma, P, cs, i \rangle \overset{\text{BiST}(\mathcal{S})}{\leadsto} \langle \sigma', Q, cs', j \rangle^o \)
   2.2. 1. \( \text{cmd}(\mathcal{P}, cs, i) = x := \alpha(\text{exp}) \)
2. \( \sigma.\text{ee} (\text{exp}) = \nu \)
3. \( \sigma.\alpha (\nu) \leadsto F(Q) \)
4. \( \sigma' = \sigma''.\text{setVar}(x, \nu') \)
5. \( \sigma.\text{setP}([]) \leadsto \sigma'' \)
6. \( \sigma''.\alpha (\nu) \leadsto S(\sigma_1, v_1) \)
7. \( \sigma' = \sigma_1.\text{setVar}(x, v_1) \)
8. \( Q = P \ast P' \)

[From 2.1]

2.3. \( \sigma''.\text{ee} (\text{exp}) = \nu \) [From 2.2.2 and 2.2.4]

2.4. \( \overline{p} \vdash (\sigma'', \text{cs}, i) \leadsto^0 (\sigma', \text{cs}, i + 1)^0 \) [From 2.2.1, 2.3 and 2.2.3]

THEOREM D.4 (MULTIPLE-STEP).

\( \overline{p} \vdash (\langle \sigma, P \rangle, \text{cs}, i) \leadsto^*_{\text{BIST}(S)} (\langle \sigma', Q \rangle, \text{cs}', j)^0 \)

\[ \implies \exists P'. \overline{Q} \vdash P \ast P' \land \sigma.\text{setP}([]) \leadsto \sigma'' \land \sigma'' \leq \sigma \vdash \overline{p} \vdash (\sigma'', \text{cs}, i) \leadsto^* (\sigma', \text{cs}', j)^0 \]

PROOF:

We proceed by induction on the length of the derivation \( \overline{p} \vdash (\langle \sigma, P \rangle, \text{cs}, i) \leadsto^*_{\text{BIST}(S)} (\langle \sigma', Q \rangle, \text{cs}', j)^0 \).

1. CASE: \( n = 0 \)

1.1. ASSUME: \( \overline{p} \vdash (\langle \sigma, P \rangle, \text{cs}, i) \leadsto^0_{\text{BIST}(S)} (\langle \sigma', Q \rangle, \text{cs}', j)^0 \)

1.2. 1. \( \sigma' = \sigma \)

2. \( Q = P \)

3. \( \text{cs}' = \text{cs} \)

4. \( j = i \)

1.3. \( \overline{p} \vdash (\sigma, \text{cs}, i) \leadsto^0 (\sigma, \text{cs}, i)^0 \)

1.4. \( \overline{P} \vdash P \ast \text{emp} \)

1.5. \( \sigma.\text{setemp}([]) \leadsto \sigma \)

2. CASE: \( n = k + 1 \)

2.1. ASSUME: \( \overline{p} \vdash (\langle \sigma, P \rangle, \text{cs}, i) \leadsto^{k+1}_{\text{BIST}(S)} (\langle \sigma', Q \rangle, \text{cs}', j)^0 \)

2.2. \( \exists \sigma_1, Q_1, \text{cs}_1, j_1 \).

1. \( \overline{p} \vdash (\langle \sigma, P \rangle, \text{cs}, i) \leadsto^k_{\text{BIST}(S)} (\langle \sigma_1, Q_1 \rangle, \text{cs}_1, j_1)^0 \)

2. \( \overline{p} \vdash (\langle \sigma_1, Q_1 \rangle, \text{cs}_1, j_1) \leadsto^*_{\text{BIST}(S)} (\langle \sigma', Q \rangle, \text{cs}', j)^0 \)

[From 2.2]

2.3. \( \exists \sigma_2, P_2 \).

1. \( Q_1 \vdash P \ast P_2 \)

2. \( \sigma.\text{setP}([]) \leadsto \sigma_2 \)

3. \( \overline{J} \vdash \langle \sigma_2, \text{cs}, j_1 \rangle \leadsto^k ([])^{\sigma_1, \text{cs}_1,j_1,\text{Spo}} \)

[From IH and 2.2.1]

2.4. \( \exists P_3, \sigma_3 \).

1. \( Q_1 \vdash Q_3 \ast P_3 \)

2. \( \sigma.\text{setP}([]) \leadsto \sigma_3 \)

3. \( \overline{p} \vdash (\sigma_3, \text{cs}, i) \leadsto^0 (\sigma', \text{cs}', j)^0 \)

[From previous Lemma]

2.5. \( \exists \sigma_4 \).

1. \( \sigma_2.\text{setP}([]) \leadsto \sigma_4 \)

2. \( \overline{p} \vdash (\sigma_4, \text{cs}, i) \leadsto^k (\sigma_3, \text{cs}_1, j_1)^0 \)

[From Frame rule, 2.4.2 and 2.3.3]

2.6. \( \overline{p} \vdash (\sigma_4, \text{cs}, i) \leadsto^{k+1} (\sigma', \text{cs}', j)^0 \) [From 2.4.3 and 2.5.2]

2.7. \( \sigma.\text{setP}(P_2) \leadsto \sigma_5 \) [From 2.3.2 and 2.5.1]
2.8. \( Q \vdash P \cdot P_2 P_3 \) [From 2.3.1 and 2.4.1]

**Theorem D.5 (Bi-Abduction).**

\[
\begin{align*}
    &p \vdash \langle \langle \sigma, \text{emp} \rangle, cs, i \rangle \xrightarrow{\text{BiST}(S)} \langle \langle \sigma', P \rangle, cs', j \rangle^o \land \sigma \cdot \text{set}_P([ ]) \xrightarrow{\cdot} \sigma'' \\
\Rightarrow &p \vdash \langle \sigma'', cs, i \rangle \xrightarrow{\cdot} \langle \sigma', cs', j \rangle^o
\end{align*}
\]

**Proof:** Immediate corollary from Theorem D.4