\textbf{δ meson effects on neutron stars in the modified quark-meson coupling model}

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Abstract  
The properties of neutron stars are investigated by including δ meson field in the Lagrangian density of modified quark-meson coupling model. The Σ⁻ population with δ meson is larger than that without δ meson at the beginning, but it becomes smaller than that without δ meson as the appearance of Ξ⁻. The δ meson has opposite effects on hadronic matter with or without hyperons: it softens the EOSes of hadronic matter with hyperons, while it stiffens the EOSes of pure nucleonic matter. Furthermore, the leptons and the hyperons have the similar influence on δ meson effects. The δ meson increases the maximum masses of neutron stars. The influence of (σ*,φ) on the δ meson effects are also investigated.

1. INTRODUCTION

δ meson is an isovector scalar meson, its contribution is expected to be neglectable in nuclei with small isospin asymmetry and in nuclear matter at saturation density. However, for strongly isospin-asymmetric matter at high densities in neutron stars the contribution of the δ field should be considered [1]. In the last decade, the effects of coupling to the δ meson like field on nuclear structure properties of the drip-line nuclei, on the dynamic situations of heavy ion collisions and on asymmetric nuclear matter are investigated [2, 3, 4, 5]. Recently, the density dependent coupling constants are introduced additionally to reexamine the δ meson effects on properties of finite nuclei and asymmetric nuclear matter in the Quantum Hadron Dynamics (QHD) model [6, 7]. The δ meson effects are also investigated in other models, such as a chiral SU(3) model [8], a relativistic point coupling model [9], relativistic transport model [10] and so on. But there is no similar work in the quark-meson coupling (QMC) model yet, so we will investigate the δ meson effects by using this model in this paper.

The quark-meson coupling model was proposed by Guichon in 1988 [11] where nuclear matter is described as nonoverlapping MIT bags interacting through the exchange of mesons in the mean-field approximation. The effective nucleon masses in the QMC model are obtained self-consistently at the quark level, which is an important difference from QHD model. The model is refined by including nucleon Fermi motion and center of mass corrections to the bag energy by Fleck [12]. Jin and Jennings introduced the density-dependent bag constant, which is called modified quark-meson coupling (MQMC) model, to get larger scalar and vector potentials compatible with experiments [13]. Furthermore, the MQMC model possibly includes the effects of quark-quark correlations associated with overlapping bags which was missing in the original QMC model, therefore it is applicable at the densities appropriate to neutron stars. The (σ*,φ) meson fields are incorporated to account for the strong attractive ΛΛ interaction observed in hypernuclei which cannot be reproduced by the (σ,ω,ρ) only in MQMC model [14]. The MQMC model gives a satisfactory description of finite nuclei [15] and nuclear matter [16], and it is widely used in nuclear physics. For example, the temperature effects of nuclear matter [17], K condensation [18, 19, 23], trapped neutrinos [20], strong magnetic field [21] and deconfined phenomena [22] in neutron stars are all investigated in the MQMC model.

In this paper, we extend MQMC model to incorporate δ meson field, in which the density-dependent couplings between baryons and scalar mesons are calculated self-consistently. The model parameters are determined by the properties of symmetric nuclear matter and pure neutron matter. Then the influences of leptons, baryons and (σ*,φ) mesons on the δ meson effects are discussed.

2. THE MODEL

The modified quark-meson coupling model is extended to include the δ meson field. δ meson couples only to u and d quarks, because it is built out of nonstrange quarks. σ* and φ mesons are also incorporated which couple only
to the $s$ quark in a hyperon bag. So there are isoscalar scalar mesons $\sigma$ and $\sigma^*$, isoscalar vector mesons $\omega$ and $\phi$, isovector scalar meson $\delta$ and isovector vector meson $\rho$ in our present model.

In the mean field approximation the Dirac equation for a quark field of flavor $q \equiv (u, d, s)$ in the bag for the hadron species $B \equiv (p, n, \Lambda, \Sigma^+, \Sigma^0, \Sigma^-, \Xi^0, \Xi^-)$ is then given by

$$\left[i\gamma \cdot \partial - (m_q - g_\sigma^q \sigma - g_\sigma^q \sigma^* - g_\phi^q I_{3q}\delta_3) - \gamma^0 \left(g_\omega^q \omega_0 + g_\phi^q \phi_0 + g_\rho^q I_{3q}\rho_{03}\right)\right] \psi_{qB}(\vec{r}, t) = 0. \quad (1)$$

Here $I_{3q}$ is the isospin projection of quark $q$; $g_\delta^q$ is the coupling constant between quark $q$ and $\delta$ meson, $\delta_3$ denotes expectation value of the isospin 3rd-component of $\delta$ meson field, and the other symbols are the same as in $[23]$. The normalized ground state is solved as

$$\psi_{qB} = N_{qB} \exp \left(-\frac{i\epsilon_{qB}t}{R_B}\right) \begin{pmatrix} j_0 \left(\frac{x_qBr}{R_B}\right) \left|\psi_{qB}\right\rangle \end{pmatrix} \frac{\chi_{qB}}{\sqrt{4\pi}} \quad (2)$$

where

$$\epsilon_{qB} = \Omega_{qB} \pm R_B \left(g_\sigma^q \omega_0 + g_\phi^q I_{3q}\rho_{03} + g_\rho^q \phi_0\right), \quad (3)$$

$$\beta_{qB} = \sqrt{\frac{\Omega_{qB} - R_B m_q^*}{\Omega_{qB} + R_B m_q^*}}, \quad (4)$$

$$\Omega_{qB} = \sqrt{x_{qB}^2 + (R_B m_q^*)^2}, \quad (5)$$

with $R_B$ is the bag radius of baryon $B$ and $x_{qB}$ is the dimensionless quark momentum which can be determined by the linear boundary condition

$$j_0(x_{qB}) = \beta_{qB} j_1(x_{qB}) \quad (6)$$

The effective quark mass is

$$m_q^* = m_q - g_\sigma^q \sigma - g_\sigma^q \sigma^* - g_\phi^q I_{3q}\delta_3 \quad (7)$$

The energy of a MIT bag for baryon $B$ is then given by

$$E_{B}^{\text{bag}} = \sum_q n_{qB} \Omega_{qB} - z_B \frac{4}{3} \pi R_B^3 B_B(\sigma, \sigma^*, \delta_3) \quad (8)$$

where $n_{qB}$ is the number of constituent quark $q$ in baryon $B$, $z_B$ is the zero-point motion parameter and $B_B$ is the medium dependent bag parameter. The ansatz for the coupling of bag parameter to the scalar fields $\sigma$, $\sigma^*$ $[24]$ is extended to $\delta_3$

$$B_B(\sigma, \sigma^*, \delta_3) = B_0 \exp \left(-\frac{4}{M_B} \left(n_{sB} g_{\sigma^s}^{\text{bag}} \sigma^s + \sum_{q=u,d} n_{qB} \left(g_{\sigma}^{\text{bag}} \sigma + g_{\phi}^{\text{bag}} I_{3q}\delta_3\right)\right)\right) \quad (9)$$

where $B_0$ is the bag constant in free space, $M_B$ is the bare mass of the baryon $B$, and $g_{\sigma}^{\text{bag}}$, $g_{\sigma^s}^{\text{bag}}$ and $g_{\phi}^{\text{bag}}$ are real parameters.

After the corrections of spurious center of mass motion, the effective baryon mass is given by

$$M_B^* = \sqrt{\left(E_B^{\text{bag}}\right)^2 - \left\langle p_{c.m.}^2\right\rangle_B} \quad (10)$$

where

$$\left\langle p_{c.m.}^2\right\rangle_B = \frac{1}{R_B^2} \sum_q n_{qB} x_{qB}^2 \quad (11)$$
The bag radius \( R_B \) could be obtained through the minimization of the baryon mass with respect to the bag radius

\[
\frac{\partial M_B}{\partial R_B} = 0 \tag{12}
\]

Consider an many-particle system consisting of the full baryon octet which interact via \( \sigma, \sigma^*, \omega, \phi, \delta, \rho \) meson fields. The Lagrangian density is

\[
\mathcal{L} = \sum_B \bar{\Psi}_B \left[ i \gamma_\mu \partial^\mu - M^*_B (\sigma, \sigma^*, \delta_3) - \left( g_\omega \omega_B \gamma^\mu + g_{\rho B} \frac{2}{\sqrt{2}} \bar{\rho}_\mu \gamma^\mu + g_{\phi B} \phi_\mu \gamma^\mu \right) \right] \Psi_B + \frac{1}{2} (\partial_\mu \sigma \partial^\mu \sigma
\]

\[+ \partial_\mu \delta \cdot \partial^\mu \delta + \partial_\mu \sigma^* \partial^\mu \sigma^* - \frac{1}{4} \left( m_\sigma^2 \sigma^2 + m_\delta^2 \delta \cdot \delta + m_{\sigma^*}^2 \sigma^* \cdot \sigma^* - m_\omega^2 \omega_B \omega^\mu - m_{\phi B}^2 \phi_\mu \cdot \phi^\mu_\mu - m_{\rho B}^2 \rho_\mu \cdot \rho^\mu \right)
\]

\[= \frac{1}{4} \left( W_{\mu \nu} W^{\mu \nu} + \tilde{G}_{\mu \nu} \cdot \tilde{G}^{\mu \nu} + F_{\mu \nu} F^{\mu \nu} \right) \right] + \sum_l \bar{\Psi}_l (i \gamma_\mu \partial^\mu - m_l) \Psi_l \tag{13}
\]

where \( l = (e, \mu) \). Then from Eq. (13) and (14), we can derive the equations of the motion for the meson fields in uniform static matter:

\[
m^2_\sigma \sigma = \frac{1}{\pi^2} \sum_B g_{\sigma B} C_B(\sigma) \int_0^{k_B} M^*_B \left[ \frac{M^*_B}{[k^2 + M^*_B]^1/2} \right] \frac{k^2 \, dk}{2} \tag{14}
\]

\[
m^2_\sigma^* \sigma^* = \frac{1}{\pi^2} \sum_B g_{\sigma^* B} C_B(\sigma^*) \int_0^{k_B} M^*_B \left[ \frac{M^*_B}{[k^2 + M^*_B]^1/2} \right] \frac{k^2 \, dk}{2} \tag{15}
\]

\[
m^2_\delta \delta_3 = \frac{1}{\frac{1}{\pi^2}} \sum_B g_{\delta B} C_B(\delta_3) \int_0^{k_B} M^*_B \left[ \frac{M^*_B}{[k^2 + M^*_B]^1/2} \right] \frac{k^2 \, dk}{2} \tag{16}
\]

\[
m^2_\omega \omega_0 = \frac{1}{\frac{1}{\pi^2}} \sum_B g_{\omega B} k^3_B \tag{17}
\]

\[
m^2_\phi \phi_0 = \frac{1}{\frac{1}{\pi^2}} \sum_B g_{\phi B} k^3_B \tag{18}
\]

\[
m^2_\rho \rho_0 = \frac{1}{\frac{1}{\pi^2}} \sum_B g_{\rho B} k^3_B \tag{19}
\]

Here \( k_B \) is the Fermi momentum of the baryon species \( B \). The factors \( C_B(\sigma), C_B(\sigma^*), C_B(\delta_3) \) are:

\[
g_{\phi B} C_B(\phi) = -\frac{\partial M^*_B}{\partial \phi}, \quad \phi = \sigma, \sigma^*, \delta_3
\]

\[
-\frac{\partial M^*_B}{\partial \sigma} = E^B_B \sum_{q=u,d} n_{qB} \left[ g_\sigma^B \left( S_{qB} \left( 1 - \frac{\Omega_{qB}}{E^B_B R_B} \right) + \frac{m^*_\sigma}{E^B_B} \right) \right] + \frac{16 \pi g_{\sigma B}^{bag} B_B R^2_B}{3M_B} \tag{20}
\]

\[
-\frac{\partial M^*_B}{\partial \sigma^*} = E^B_B \sum_{q=u,d} n_{qB} \left[ g_{\sigma^*}^B \left( S_{qB} \left( 1 - \frac{\Omega_{qB}}{E^B_B R_B} \right) + \frac{m^*_\sigma}{E^B_B} \right) \right] + \frac{16 \pi g_{\sigma^* B}^{bag} B_B R^2_B}{3M_B} \tag{21}
\]

\[
-\frac{\partial M^*_B}{\partial \delta_3} = E^B_B \sum_{q=u,d} n_{qB} \left[ g_\delta^B \left( S_{qB} \left( 1 - \frac{\Omega_{qB}}{E^B_B R_B} \right) + \frac{m^*_\sigma}{E^B_B} \right) \right] + \frac{16 \pi g_{\delta B}^{bag} B_B R^2_B}{3M_B} \tag{22}
\]

The scalar density of quark \( q \) in the bag \( B \) are:

\[
S_{qB} = \frac{\Omega_{qB}}{2} + \frac{R_B m_q^2 \left( \Omega_{qB} - 1 \right)}{\Omega_{qB} \left( \Omega_{qB} - 1 \right) + R_B m_q^2 / 2}, \quad q = (u, d, s), \tag{23}
\]

At last, there are two conditions left:

\[
\text{charge neutrality:} \quad \sum_B q_B k^3_B = \sum_l k^3_l, \tag{24}
\]

\[
\beta \text{ equilibrium:} \quad \mu_B = \mu_n - q_B \mu_e, \quad \mu_\mu = \mu_e. \tag{25}
\]
where $q_B$ and $\mu_B$ correspond to the electric charge and chemical potential of baryon $B$, respectively. The energy eigenvalue of Dirac equation for baryon $B$ and lepton $l$ are:

$$
\epsilon_B = \sqrt{k_B^2 + M_B^2} + g_{\omega B} \omega_0 + g_{\phi B} \phi_0 + g_{\rho B} \rho_0,
$$

$$
\epsilon_l = \sqrt{k_l^2 + m_l^2}.
$$

(26) (27)

Then the Fermi momentum can be obtained from the equations

$$
\epsilon_B(k_B) = \mu_B, \quad \epsilon_l(k_l) = \mu_l
$$

(28) (29)

After the meson fields ($\sigma, \sigma^*, \omega, \phi, \delta_3, \rho_0$), Fermi momenta ($k_B, k_l$) and effective masses $M_B^*$ are obtained by solving the Eqs. (17) - (19), (28) - (29) and (12) self-consistently at a given baryon number density

$$
\rho = \frac{1}{3\pi^2} \sum_B b_B k_B^2
$$

(30)

where $b_B$ is the baryon number of baryon $B$, we can obtain the total energy density and pressure:

$$
\varepsilon = \frac{1}{2} (m_\sigma^2 \sigma^2 + m_\omega^2 \omega_0^2 + m_\phi^2 \phi_0^2 + m_\delta_3^2 \delta_3^2 + m_\rho_0^2 \rho_0^2)
$$

$$+ \frac{1}{\pi^2} \sum_B \int_0^{k_B} \left[ k^2 + M_B^2 \right]^{1/2} k^2 dk + \frac{1}{\pi^2} \sum_l \int_0^{k_l} \left[ k^2 + m_l^2 \right]^{1/2} k^2 dk
$$

(31)

$$
P = \frac{1}{2} (m_\omega^2 \omega_0^2 - m_\phi^2 \phi_0^2 + m_\rho_0^2 \rho_0^2 - m_\sigma^2 \sigma_0^2 - m_\delta_3^2 \delta_3^2 + m_\rho_0^2 \rho_0^2)
$$

$$+ \frac{1}{3\pi^2} \sum_B \int_0^{k_B} \frac{k^4 dk}{[k^2 + M_B^2]^{1/2}} + \frac{1}{3\pi^2} \sum_l \int_0^{k_l} \frac{k^4 dk}{[k^2 + m_l^2]^{1/2}}
$$

(32)

3. PARAMETERS AND CALCULATION DETAILS

Take the current quark mass to be $m_u = m_d = 0$ and $m_s = 150$ MeV. Small current quark mass for the non-strange flavors or other values for the strange flavor lead only to small numerical refinements (19). The meson masses are

$m_\sigma = 550$ MeV, $m_\omega^* = 980$ MeV, $m_\rho = 775$ MeV, $m_\phi = 985$ MeV, $m_\omega = 783$ MeV, $m_\phi = 1020$ MeV, respectively.

Assume $\sigma, \omega, \rho, \delta$ mesons couple only to the $u, d$ quarks and $\sigma^*, \phi$ mesons couple only to the $s$ quark, we have

$$
g_\sigma^* = g_\omega^* = g_\rho = g_\sigma^* = g_\delta^* = g_\phi = g_\delta^* = g_\phi = 0
$$

(33)

By assuming the SU(6) symmetry of the simple quark model (14)

$$
g_\sigma^* = g_\omega^* = \sqrt{2} g_\sigma^*, \quad g_\delta^* = g_\delta^*, \quad g_\phi^* = g_\phi^*, \quad g_\rho^* = g_\rho^*,
$$

(34)

we can get the relations

$$
\frac{1}{3} g_{\omega N} = \frac{1}{2} g_{\omega A} = \frac{1}{2} g_{\omega \Sigma} = g_{\omega \Xi} = g_\sigma^*
$$

(35)

$$
g_{\phi A} = g_{\phi \Sigma} = \frac{1}{2} g_{\phi \Xi} = \sqrt{2} g_\sigma^*, \quad g_{\phi N} = 0
$$

(36)

$$
g_{\rho N} = g_{\rho A} = g_{\rho \Sigma} = g_{\rho \Xi} = g_\rho^*
$$

(37)

To reduce parameters we set

$$
\frac{g_\delta^b}{g_\delta^*} = \frac{g_\sigma^b}{g_\sigma^*} = \frac{g_\phi^b}{g_\phi^*}
$$

(38)

The free nucleon zero-point motion parameter $\varepsilon_{\Delta N}$ and the free bag constant $B_0$ are fixed to reproduce the free mass of nucleon $m_N = 939$ MeV with the minimization condition (12) at a free bag radius $R_{N0} = 0.6$ fm. Then the free
TABLE I: The zero-point motion parameters $z_{B0}$ and bag radii $R_{B0}$ in free space are obtained to reproduce the free space mass spectrum after the parameters $B^{1/4}_0 = 188.102$ MeV and $z_{N0} = 2.030$ have been fixed by the properties of nucleon.

| $M_B$ (MeV) | $z_{B0}$ | $R_{B0}$ (fm) |
|-------------|----------|--------------|
| $\Lambda$   | 1115.68  | 1.815        |
| $\Sigma^+$  | 1189.37  | 1.638        |
| $\Sigma^0$  | 1192.64  | 1.630        |
| $\Sigma^-$  | 1197.45  | 1.612        |
| $\Xi^0$     | 1314.83  | 1.501        |
| $\Xi^-$     | 1321.31  | 1.483        |

TABLE II: Four independent coupling constants are fixed to reproduce the symmetric nuclear matter binding energy $B/A = 16$ MeV, symmetry energy $a_{\text{sym}} = 32.5$ MeV and compressibility $K = 289$ MeV at saturation density $\rho_0 = 0.17$ fm$^{-3}$. The $\delta$ meson coupling constant is set 0 and 4.2.

| $g_0^\delta$ | $g_u^\sigma$ | $g_u^\omega$ | $g_u^\rho$ | $g_u^{\text{bag}}$ |
|--------------|-------------|-------------|-----------|-----------------|
| 0            | 0.980      | 2.705      | 7.948    | 2.278           |
| 4.2          | 0.980      | 2.705      | 10.217   | 2.278           |

zero-point motion parameters $z_{B0}$ and free radii $R_{B0}$ of other baryons are obtained by reproducing the free baryon mass $M_B$ with the minimization condition (12). They are all listed in the Table I.

Four independent coupling constants $g_u^\sigma, g_u^\omega, g_u^\rho$ and $g_u^{\text{bag}}$ can be adjusted by reproducing the symmetric nuclear matter binding energy $B/A = 16$ MeV, symmetry energy $a_{\text{sym}} = 32.5$ MeV and compressibility $K = 289$ MeV at saturation density $\rho_0 = 0.17$ fm$^{-3}$, as listed in Table II. The $\delta$ meson-quark coupling constant is constrained in a range of $0 \leq g_0^\delta \leq 4.2$ so that the pure neutron matter EOS is consistent with the experimental flow data in heavy-ion collision [25], which is shown in the upper panel of Figure. 1, the EOS for symmetric nuclear matter is also shown in the lower panel and we can see that it is also consistent with the experimental flow data in heavy-ion collision.

The equilibrium properties of neutron stars are obtained by solving Tolman-Oppenheimer-Volkoff (TOV) equations

$$\frac{dP(r)}{dr} = - \frac{G [\varepsilon(r) + P(r)] [M(r) + 4\pi r^3 P(r)]}{r^2 [1 - 2G M(r)/r]}$$

$$\frac{dM(r)}{dr} = 4\pi r^2 \varepsilon(r).$$

The Baym-Pethick-Sutherland model [27] is used to describe the EOS at subnuclear densities.

4. RESULTS AND DISCUSSIONS

Four cases in Table III are investigated: (1) pure neutron matter denoted by $nn$; (2) $\beta$-equilibrium nucleonic matter denoted by $np$; (3) $\beta$-equilibrium hadronic matter composed of baryon octet without $(\sigma^*, \phi)$ meson fields, denoted by $npH$; (4) The same as in Case (3) with two additional meson fields $(\sigma^*, \phi)$, denoted by $npH^*$.

The meson fields for $npH^*$ are shown in the left panel of Figure. 2. We can see that $\delta$ meson field decreases $(\sigma, \omega)$ fields while increases $(\sigma^*, \phi)$ fields. This is because the $\delta$ meson increases the strange number in nuclear matter, which is shown in the right panel of Figure. 2, and $(\sigma^*, \phi)$ couple only to $s$ quark. The $\delta$ meson increases $\rho_{03}$ meson field, and the effect becomes smaller when the $\delta$ meson field decreases as baryon density increases.

TABLE III: The cases we study in the paper. $H$ represents hyperons ($\Lambda, \Sigma^+, \Sigma^0, \Sigma^-, \Xi^0, \Xi^-$).

| notation | $nn$ | $np$ | $npH$ | $npH^*$ |
|----------|------|------|-------|---------|
| baryons  | $n$  | $n, p$ | $n, p, H$ | $n, p, H$ |
| leptons  | $e$  | $e, \mu$ | $e, \mu$ | $e, \mu$ |
| mesons   | $\sigma, \omega, \rho(\delta)$ | $\sigma, \omega, \rho(\delta)$ | $\sigma, \omega, \rho(\delta)$ | $\sigma, \omega, \rho(\delta)$ | $\sigma^*, \phi$ |
FIG. 1: (Color online) The EOSes obtained in MQMC model for pure neutron matter and symmetric nuclear matter. The upper magenta hatched area and the lower orange hatched area correspond to the pressure regions for neutron matter after inclusion of the pressure from asymmetry term with strong density dependence and for symmetric nuclear matter consistent with the experimental flow data, respectively [25].

FIG. 2: (Color online) The left panel are meson fields as functions of baryon density for $npH^*$. The right panel is the ratio of $s$ quark to total quark in nuclear matter versus baryon density. The upper two curves are for $npH$ and the lower two are for $npH^*$.

Let’s look at the compositions of nuclear matter for $npH^*$ in Figure. $\delta$ meson decreases the neutron fraction while increases the proton and lepton fractions when $\rho \gtrapprox \rho_0$. From the right panel of Figure, we see that $\delta$ meson decreases the effective mass of neutron, which makes the neutron fraction fall when the density exceed some critical density which is approximately nuclear matter density $\rho_0$ as shown in Figure. The proton fraction goes up because the similar reason, and the charge neutrality condition requires larger lepton fractions.

$\Sigma^-$, $\Xi^-$ (negative isospin projection) and $\Lambda$, $\Sigma^0$ (zero isospin projection) appear earlier when $\delta$ meson are included, but the appearance of $\Sigma^+$ (positive isospin projection) is postponed. From equations (25), (26) and (28), we know
that the fraction for baryon $B$ is determined by $(\mu_n - q_{B} \mu_{e})$, $(g_{\omega B} \omega_0 + g_{\phi B} \phi_0 + g_{\rho B} I_{3B} \rho_{03})$ and $M^*_{B}$, which are all shown in the Figure. 4. We see that the changes of $g_{\omega B}^{\alpha} \omega_0$ and $g_{\phi B}^{\alpha} \phi$ are proximately offseted; $M^*_{B}$ and $g_{\rho B}^{\alpha} \rho_{03}$ (Compare Figure. 4 with Figure. 2) we can see that the change of $g_{\rho B}^{\alpha} \rho_{03}$ mainly origins in the large change of quark-$\rho$ meson coupling constant $g_{\rho B}^{\alpha}$ change obviously. The changes of $g_{\rho B} I_{3B} \rho_{03}$ and $M^*_{B}$ are isospin-dependent, so whether the hyperon appears earlier is determined by its isospin projection. The critical density of its appearance only shifts a little except $\Xi^-$, since the changes of $g_{\rho B} I_{3B} \rho_{03}$ and $M^*_{B}$ are almost the same.

The $\Sigma^-$ population with $\delta$ meson is larger than that without $\delta$ meson at the beginning, but it becomes smaller than that without $\delta$ meson because of the appearance of $\Xi^-$. The reasons are that charge neutrality can be kept more economically by the larger mass particles with the same charge, and the $\delta$ meson decreases $M^*_{\Xi^-}$ more than $M^*_{\Sigma^-}$ (right panel of Figure[4] since the isospin projection of $\Xi^-$ is $-1/2$ and $\Sigma^-$ is $-1$. $\delta$ meson increases $\Xi^-$ population obviously larger than other hyperons since it decreases $\Sigma^-$ population. The appearance of $\Sigma^-$ makes the lepton fraction begins to fall, which can also be explained by charge neutrality condition. There is another interesting phenomenon that the $\Sigma^+$ may not appear in neutron stars with $\delta$ meson while its fraction could exceed 1% for neutron stars at the maximum masses without $\delta$ meson.

The EOSes for $nn, np, npH, npH^*$ are plotted in Figure. 5. The effects of $\delta$ meson can be seen clearly from this figure: The $\delta$ meson makes the EOS of $nn$ stiffer similar as in QHD model [28]. For $np$, the $\delta$ meson stiffens the EOS.
at low density while softens at high density. The density-dependent coupling constants are introduced additionally in QHD model \cite{7} to get the similar results, but the density-dependence of couplings between scalar mesons and baryons are obtained self-consistently in our paper. If hyperons are taken into account, the EOSes with $\delta$ meson suffer a transition to nucleon-hyperon phase at some density and become softer, this can be seen from the EOSes of $npH$ and $npH^*$ in Figure. 5 clearly. $(\sigma^*, \phi)$ mesons obviously stiffen the EOSes as in Ref.\cite{14}, but their influences on the $\delta$ meson effect could be neglected. Since $(\sigma^*, \phi)$ meson fields couple only to $s$ quark and $\delta$ meson couple only to $(u, d)$ quarks, $\delta$ meson has no direct influence on $(\sigma^*, \phi)$ meson fields, which can also be seen from the lower panel in Figure. 2 as mentioned above.

We find that no matter whether hyperons are positive, negative or neutral, their inclusions can make the EOSes with $\delta$ meson become softer. This result probably reveals that it is the strange quark makes the EOSes with $\delta$ meson become softer. That is to say that it is the strange quarks in hyperons results in reversed direction changes of EOSes if compared with a nucleonic star.

The mass-radius relation of neutron stars are shown in Figure. 6. We see that the $\delta$ meson increases the maximum masses of neutron stars for all cases we studied in this paper. This is different from QHD model in which the maximum mass decreases for $np$ with density-dependent couplings \cite{7} and $npH$ when $\delta$ meson are included \cite{28}. The $\delta$ meson...
TABLE IV: The maximum masses of neutron stars and the corresponding radii $R_{\text{max}}$, central baryon density $\rho_c$, central energy density $\varepsilon_c$, central pressure $P_c$ for different EOSes.

|     | $M_{\text{max}}/M_\odot$ | $R_{\text{max}}$ (km) | $\rho_c/\rho_0$ | $\varepsilon_c$ (MeV fm$^{-3}$) | $P_c$ (MeV fm$^{-3}$) |
|-----|---------------------------|------------------------|-----------------|-------------------------------|---------------------|
| $nn$ | 4.2                       | 2.275                  | 12.07           | 5.1                           | 1.093               | 4.18               |
| $nn$ | 0                         | 2.147                  | 11.50           | 5.6                           | 1.213               | 4.59               |
| $np$ | 4.2                       | 2.045                  | 11.30           | 6.0                           | 1.274               | 4.46               |
| $np$ | 0                         | 2.012                  | 10.95           | 6.3                           | 1.352               | 4.93               |
| $npH^*$ | 4.2                    | 1.556                  | 11.56           | 6.0                           | 1.157               | 2.01               |
| $npH^*$ | 0                        | 1.543                  | 11.10           | 6.5                           | 1.274               | 2.36               |
| $npH$ | 4.2                       | 1.509                  | 12.03           | 5.2                           | 9.86                | 1.40               |
| $npH$ | 0                         | 1.491                  | 11.54           | 5.8                           | 11.06               | 1.66               |

enlarge the radii of neutron stars about 0.5 km for stars with $M > M_s$, this is an obvious change considering the same EOS at low density are used for all cases. Another conclusion is that the central density of neutron star becomes about 0.5 $\rho_0$ smaller when $\delta$ meson is included. These can be seen from Table IV clearly. Some observation values are also displayed in Figure. We can see that all cases are compatible with the observations from PSR 1913+16 and 4U 0614+09, but $npH^*$ and $npH$ neutron stars might be ruled out by neutron star 4U 1636-536 or EXO 0748-676. To show quantitatively the $\delta$ meson effects on neutron stars properties, the maximum mass $M_{\text{max}}$ and the corresponding radii $R_{\text{max}}$, central baryon density $\rho_c$, central energy density $\varepsilon_c$, central pressure $P_c$ are listed in Table IV for all cases.

5. CONCLUSIONS

We have investigated the $\delta$ meson effects on neutron star properties within the modified quark-meson coupling model. We sum up the conclusions in four aspects:

1. $\delta \leftrightarrow$ strangeness: the $\delta$ meson can make the pure nucleonic matter EOSes stiffer, while make the hyperon matter EOSes softer and this could be explained by the appearance of strange quarks in hyperons.

2. $\delta \leftrightarrow$ leptons: the $\delta$ meson results in opposite effects on the EOSes of $\beta$-equilibrium nuclear matter such as $np$, $npH$ and $npH^*$ compared with the EOS of $nn$, which is similar to the influence of hyperons, but the effect is smaller. This is because of density-dependence of the couplings between baryons and scalar mesons.

3. $\delta \leftrightarrow (\sigma^*, \phi)$: $\delta$ meson has no direct influence on $s$ quark, so it has little effect on $(\sigma^*, \phi)$ meson fields. As a result, $(\sigma^*, \phi)$ mesons have no obvious influence to the $\delta$ meson effect on EOSes, although $(\sigma^*, \phi)$ stiffen the EOSes obviously.

4. $\delta \leftrightarrow$ neutron star properties: the $\delta$ meson can increase the maximum masses of neutron stars, decrease the corresponding both baryon density and energy density. The radii become about 0.5 km larger for stars with $M > M_s$. It changes compositions of neutron stars: decrease the neutron fraction and increase the proton and leptons fractions when $\rho \gtrsim \rho_0$; make the abundance of $\Xi^-$, $\Xi^0$ and $\Sigma^+$ larger, while the abundance of $\Sigma^-$ smaller; and increase the strange number of neutron stars. It can also make the isospin dependent physical quantities splitting, such as effective baryons masses.

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