Loops in the gluon emission amplitude: 
reggeization from the Glauber scattering

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Abstract It is shown that in the Glauber scattering of a fast quark in the external field loop corrections to the gluon emission amplitude due to virtual softer gluon after renormalization coincide with a correction due to reggeization of the exchanged gluon in the BFKL picture.

1 Introduction

In the perturbative QCD the inclusive gluon production is one of the most important observables, which can be directly compared to the experimental data. Applied to the interaction with heavy nuclei it was first studied in [1] using the standard AGK rules and neglecting emission from the triple pomeron vertex. Later Yu. Kovchegov and K.Tuchin derived the single inclusive cross-section from the dipole picture [2]. Their expression contained an extra term as compared to [1] which was shown to correspond to emission from the triple-pomeron vertex [3]. However recently J.Bartels, M.Salvadore and G.-P. Vacca performed a new derivation based on the dispersion approach, which studies discontinuities of amplitudes in various s-channels [4]. Their results are different from the previous ones in [2]. They contain new terms of a complicated structure, which seem to involve the so-called BKP states, higher pomerons composed of 3 and 4 reggeized gluons. Presence of such states will inevitably make calculations of the inclusive cross-section much more difficult (if possible at all), since their wave functions depend on many variables and are unknown at present. So the problem to understand the origin of the difference in the two derivations is quite important.

It may be that after all the initial physical picture in the two approaches is different. In the Bartels picture one is studying discontinuities of the standard Feynman diagrams in the Regge kinematics. In the Kovchegov-Tuchin picture the gluon emission occurs due to Glauber scattering of fast quarks and gluons passing through the nucleus with instantaneous interactions with its components. However one can prove that tree diagrams which describe single and double gluon emission coincide in these two pictures. The inclusive cross-sections also include diagrams with loops formed by the gluon softer than the observed one. In this paper we demonstrate that for single gluon emission such loops are also correctly given by
the Kovchegov-Tuchin technique. In the BFKL approach the mentioned loop generates the Regge trajectory of the exchange gluon and thus is responsible for the gluon reggeization. Our results show that gluon reggeization is in fact also realized by the loop contributions to the emission during Glauber scattering.

As a result it looks as if the physical picture employed in the two different approaches is fully equivalent. Then the difference in the results either is spurious, with additional terms found in [4] actually cancelling, or is a consequence of the difference in the derivation of the inclusive cross-sections one of which should be in error. We do not know the answer to this question and leave it to future studies.

2 The diagrams

We shall study emission of a gluon of momentum $k$ in a collision of two quarks with momenta $p$ and $l$. Quark masses will be taken equal to zero, so that $p_- = p_\perp = l_+ = l_\perp = 0$. We take the axial gauge with the gluon field $G$ satisfying $Gl = 0$. In the BFKL approach, in the lowest order, the amplitude is described by the "Lipatov vertex"

\[ L_{cba}(k, q) = 2\left(\frac{ke}{k^2_\perp} - \frac{(k + q, e)}{(k + q)^2_\perp}\right)f^{cba}, \]

(see Fig. 1). In the next order the exchanged gluon reggeizes and the Born amplitude $A^{(0)}$ is multiplied by the sum of two gluon Regge trajectories

\[ \omega(k + q)(Y - y) + \omega(q)y, \]

where

\[ \omega(q) = N g^2 \int \frac{d^2\kappa_\perp}{16\pi^2} \frac{q_\perp^2}{\kappa_\perp^2(q - \kappa)^2_\perp} \]

and $Y$ and $y$ are the rapidities of the projectile quark and emitted gluon respectively. One can interpret both $Y - y$ and $y$ as the result of integration over the longitudinal momentum of the virtually emitted gluon:

\[ Y - y = \int_{k_+}^{P_+} \frac{d\kappa_+}{\kappa_+}, \quad y = \int_{k_+}^{k_+} \frac{d\kappa_+}{\kappa_+} \]
From these expressions it follows that $\omega(k + q)(Y - y)$ is formed by virtual emissions of gluons harder that the observed one and $\omega(q)y$ is formed by emissions of gluons softer that the observed one. We shall be interested in the latter case, so that the loop correction will be given just by

$$A^{(0)}\omega(q)y,$$

where

$$\omega(q)y = \int \frac{d\kappa_+}{2\kappa_+} \int \frac{d^2\kappa_\perp}{8\pi^3} N g^2 \frac{q_\perp^2}{\kappa_\perp^2(q - \kappa)_\perp}.$$  

Our aim is to compare this expression with the one which is obtained in the Kovchegov-Tuchin picture, when one introduces virtual corrections to the three lowest order diagrams shown describing emission during Glauber rescattering and shown in Fig. 2. It is assumed here that both the fast quark and gluon interact with the target at a certain light-cone "time" $x_+ \equiv t = 0$ with an instantaneous interaction, corresponding to an exchange of a gluon with a purely transverse momentum $q = q_\perp$. In the lowest order it is trivial to show that the sum of the three diagrams shown in Fig. 2 gives the same Born amplitude $A^{(0)}$ in Fig. 1. The problem is to study the next order and compare it to (5).

All diagrams with such corrections can be obviously obtained from the five diagrams for gluon emission without interaction with either the self-mass insertions or nontrivial vertex function shown in Fig. 3. The desired diagrams will follow if we allow any of the 6 particles involved to once interact with the target. This generates 30 diagrams. However, as we shall see, 9 of them with mass insertions into the external lines are to be dropped, which leaves 21 diagrams. Further reduction of their number will follow from the requirement of the logarithmic character of the longitudinal integration and from the color factors in limit $N_c \to \infty$. In particular we find in this limit

$$f^a t^a t^c = 0,$$

which eliminates diagrams like shown in Fig. 4 from the start.

The remaining diagrams are shown in Figs. 5-8. Figs. 5 and 6 collect diagrams (A1)-(A7) proceeding from the quark and gluon self masses. Figs. 7,8 show diagrams (B1)-(B9) which
Figure 3: Diagrams with gluon emission and self-masses and vertex parts.

Figure 4: Diagram with gluon emission from the quark, which vanishes.
come from the two vertex parts. In these diagrams the harder gluon is shown with a wiggly line and the softer one with a dashed line. To the vertex diagrams (B4)-(B9) one should add similar ones with hard and soft gluon interchanged, which we denote as (B41),(B51) etc, and also diagrams in which in these latter diagrams the direction of the soft gluon momentum is reversed, denoted as (B42), (B52) etc.

3 Technical instruments

3.1 Basic elements

We recall that we use the formalism in which interactions with the target occur at given "times" $t$, which are in fact the light-cone coordinate $x_+$. A convenient technique is provided by the old time-ordered perturbation theory in which intermediate particles are on shell and their propagators are integrated out to give standard energy factors, which we denote as $T$-factors.

We consider a case when the transition includes a single instantaneous interaction $V$ acting at $t = 0$. Then the full $S$-matrix is

$$S(\infty, -\infty) = -iU(\infty, 0)VU(0, -\infty).$$

(8)

Its matrix element is given by

$$\langle \alpha |S|\beta \rangle = -i \langle \alpha |U(\infty, 0)||\beta_1 >\langle \beta_1 |V|\alpha_1 >\langle \alpha_1 |U(0, -\infty)||\beta >,$$

(9)

where

$$\langle \alpha |U_n(0, -\infty)|\beta \rangle = (-i)^n \frac{1}{E_\alpha - E_\beta} \langle \alpha |iL_\gamma \gamma_1 > \prod_{k=2}^{n} \frac{\langle \gamma_{k-1} |iL_\gamma \gamma_k >}{E_{\gamma_{k-1}} - E_\beta}$$

(10)
Figure 6: Diagrams (A5)-(A7) for gluon emission proceeding from the gluon self-mass.

Figure 7: Diagrams (B1)-(B3) for gluon emission proceeding from the vertex part.
Figure 8: Diagrams (B4)-(B9) for gluon emission proceeding from the vertex part.
Here \( L_I \) is the interaction Lagrangian for the gluon field, \(|\alpha\rangle, |\beta\rangle\) and \(\gamma_k\) are intermediate states over which summation is to be done. Note that in the relativistic normalization the phase volume contains denominators \(2k_+\) for each particle with momentum \(k\). Part of this factors cancels by appropriate relativistic \(\delta\)-functions, so finally there appears exactly one such denominator for each particle propagator in the Feynman picture.

The initial projectile and target quarks with momenta \(p\) and \(l\) respectively move along the \(z\)-axis, with \(p_− = l_+ = 0\) and both \(p_+\) and \(l_-\) are assumed to be equal and large. We shall study emission of a gluon with momentum \(k\) such that \(k_+ << p_+\). The loop correction will proceed from emission of a softer gluon with momentum \(\kappa\) such that \(\kappa << k_+\). One is to integrate over \(\kappa\)

\[
\int \frac{d\kappa_+ d^2\kappa_\perp}{(2\pi)^3 2\kappa_+}
\]

which operation will not be indicated explicitly in the following. For the gluons we use the axial gauge with the numerator in the propagator

\[
h_{\mu\nu}(k) = g_{\mu\nu} - \frac{k_\mu l_\nu + l_\mu k_\nu}{(kl)}.\]

Interaction will be carried by an exchange of a purely transverse gluon with momentum \(q = q_\perp\) and the propagator numerator \(l_\mu p_\alpha/(pl)\), where \(\mu\) refers to the projectile and \(\alpha\) to the target. In accordance with the Glauber picture it is also assumed that the "\(+\"-component of the travelling particle is not changed by the interaction. Due to this form of the interaction the numerator in the gluon propagator with arbitrary number of interactions with the target has a simple form

\[
H_{\mu\nu}(k_1, k_2, ... k_n) = g_{\mu\nu} - \frac{k_\mu l_\nu + l_\mu k_\nu}{(kl)} + \frac{l_\mu l_\nu(k_1 k_n)}{(kl)^2}.
\]

Here \(k_1, k_2, ... k_n\) are the successive gluon momenta during its interactions and it is assumed that \(k_{1+} = k_{2+} = ... = k_{n+}\). The emitted gluon has a polarization vector \(e(k)\) which satisfies

\[
(e(k)k) = (e(k)l) = 0, \text{ so that } e_+ = 0, \ e_- = -\frac{k_\perp^2}{k_+}.
\]

If the emitted gluon interacts with the target, so that successive gluon momenta from its emission until its observation are \(k_1, k_2, ... k_n, k\) with \(k_{1+} = k_{2+} = ... k_+\), then the polarization vector \(e(k)\) is changed to

\[
E(k_1) = e(k) - l_\perp \frac{(k_1 e(k))}{(k_1 l)}
\]

with the properties

\[
(E(k_1)k_1) = (E(k_1)l) = 0, \text{ so that } E_+ = 0, \ E_- = -\frac{k_\perp^2}{k_+}, \ E_\perp = e_\perp.
\]
Note that both (14) and (16) include additional factors $1/2k_+$ which appear as the external field gives rise to new gluon propagators. So in the end exactly one such factor appears for the propagating gluon whether in the external field or not.

In the projectile quark each matrix $\gamma^\mu$ which corresponds to some interaction is changed to $2p^\mu$. In the target quark a similar matrix $\gamma^\alpha$ is changed to $2l^\alpha$.

From these rules one concludes that if the soft gluon is attached to the target quark, its propagation is described by the factor

$$Q(\kappa_1, \kappa_2) = p^\mu p^\nu H_{\mu\nu}(\kappa_1, \kappa_2) = \frac{p^2}{k_+^2}(\kappa_1\kappa_2)_\perp,$$

where $\kappa_1$ and $\kappa_2$ are the gluon momenta with which it interacts with the quark and it is assumed that $\kappa_{1+} = \kappa_{2+} = \kappa_+$.  

As a part of our diagrams the emission vertex with interaction between hard and soft gluon enters shown in Fig. 9, where also Lorentz indexes of the participating gluons are indicated. Coupled to the projectile quark and emitted gluon polarization vertex it is given by a factor

$$V = p_\mu H^{\mu\nu}(\kappa_2, \kappa_1)\Gamma_{\nu\sigma\rho}(\kappa_1, k_1, -k_0)H^{\xi\sigma}(k_2, k_1)p_\xi E^\rho(k_0),$$

where $k_0 = k_1 + \kappa_1$.

Taking into account that $\kappa_+ << k_+$ and retaining only the leading and the next-to-leading terms we find (see a sketch of the derivation in Appendix 1)

$$V = -2\frac{p^2}{\kappa_+^2}(\kappa_1\kappa_2)_\perp (k_2e)_\perp + 2\frac{p^2}{\kappa_+ k_+} \left((k_1\kappa_2)_\perp(k_2e)_\perp - (\kappa_1k_2)_\perp(\kappa_2e)_\perp + (\kappa_2k_2)_\perp(\kappa_1e)_\perp\right).$$

Interchange of the gluons $k \leftrightarrow \kappa$ leads to a change of sign

$$V_1 = V(k \leftrightarrow \kappa) = -V.$$  

The final element we have to know is the momentum part of the diagram with emission of the observed gluon from the gluon polarization operator, formed by the hard and soft gluons shown in Fig. 10, with $k_{i+} = k_+ >> \kappa_{1+} = \kappa_{2+} = \kappa_+$, $i = 0, 1, 2, 3, 4$. It is given by

$$\Pi = p^\alpha H_{\alpha\beta}(k_4, k_3)\Gamma_{\beta\xi\mu}(k_3, k_2, \kappa_2)H_{\mu\nu}(\kappa_2\kappa_1)H_{\xi\sigma}(k_2, k_1)\Gamma_{\nu\sigma\rho}(\kappa_1, k_1, k_0)E^\rho(k_0).$$
Figure 10: Gluon emission from the gluon self-mass.

A sketch of its calculation is given in Appendix 2. The leading and next-to-leading terms in $1/\kappa_+$ of $\Pi$ are found to be

$$\Pi = 4 \frac{p_+ k_+}{\kappa_+^2} (\kappa_1 \kappa_2)_{\perp} (k_4 e)_{\perp} - 4 \frac{p_+}{\kappa_+} (k_4 e)_{\perp} \left( (k_1 \kappa_2)_{\perp} + (k_2 \kappa_1)_{\perp} \right) + 8 \frac{p_+}{\kappa_+} \left( (\kappa_1 k_4)_{\perp} (\kappa_2 e)_{\perp} - (\kappa_2 k_4)_{\perp} (\kappa_1 e)_{\perp} \right).$$

(23)

4 Calculation of the diagrams

In this section we find the final expressions for the contribution of our diagrams. As mentioned, in writing them out we suppress the sign of integration over the soft gluon momentum

$$\int \frac{d\kappa_+}{2\kappa_+} \int \frac{d^2 \kappa_{\perp}}{8\pi^3}$$

and also factor $q_{\perp}^2$ supplied by the interaction and factors $g$. Interaction with the target quark contributes a common factor

$$i t \frac{2(p l)}{p_+}$$

of which we retain only the momentum part $2(p l)/p_+$. Generally the contributions of each diagram splits into three factors: the momentum factor $M$, the colour factor $C$ and the energetic factor $T$ coming from the time integrations. We denote $a$ and $b$ the colors of the emitted gluon and the one interacting with the target quark respectively.

For the following the numerical coefficients and signs are of importance. We take the vertexes as they are given by Feynman rules from $i L_I$ for each gluon emission. According to (10) and (11) this brings in an overall factor $(-i)$ for each such interaction. The interaction with the target $V = i L_0$ consists of three factors: two vertexes for the gluon emission and absorption and a gluon propagator in the momentum space, the latter depending only on the transverse momentum. Each vertex comes with factor $i$ and the gluon propagator carries factor $(-i)$. Of these we preserve only the $i$ for the gluon emission from the projectile, the other two $i$’s cancelling each other. So the total overall factor is given by $(-i)^n$ where $n$ is the total number of interactions excluding the one with the target, with all interactions including the one with the target give by $i L$. For all the diagrams with a loop it gives $(-i)^3 = i$. For the tree diagrams it gives $-i$. 
For a particular diagram each interaction with the quark supplies factor $i$. Each gluon line supplies factor $-1$. The interaction with the target involves the 3-gluon vertex with all lines outgoing:

$$-f^{abc} \Gamma_{\mu\nu\rho}(k_1, k_2, k_3),$$

where the gluons $(k_1, \mu, a)$ $(k_2, \nu, b)$ and $(k_3, \rho, c)$ are counted anti-clockwise. So effectively each 3-gluon vertex accompanying the interaction with the target supplies $-1$, so that interactions with the target do not produce additional factors $(-1)$. The 3-gluon vertexes for gluon emission are taken with all lines incoming. They have the opposite sign with respect to (24) and so do not give any new factors $(-1)$. The two 3-gluon vertexes in the gluon self mass supply factor $-1$, since one of them includes reflected momenta.

### 4.1 Lowest order

The corresponding diagrams, which we denote as (1), (2) and (3) are shown in Fig. 2.

1. The initial expression for the momentum part is

$$M = i\bar{u}\hat{e}\hat{p}\hat{l}u \cdot \frac{1}{2p_+l_-} \cdot \frac{2(pl)}{p_+}.$$  

The last factor comes from the target. Calculating the matrix element gives

$$\bar{u}\hat{e}\hat{p}\hat{l}u = 4(ep)(pl) = -4(pl)p_+ \frac{(ke)_\perp}{k_+},$$

so in the end

$$M = -4i(pl)\frac{(ke)_\perp}{k_+}.$$  

The energetic factor is

$$T = \frac{1}{k_-} = \frac{2k_+}{k_\perp^2}.$$  

The colour factor is $t^a t^b$, so we find

$$(1) = -8i t^a t^b (pl) \frac{(ke)_\perp}{k_\perp^2}. $$

2. The $M$ factor is the same, the $T$ factor has the opposite sign and the in the $C$-factor the order of $t$’s is reversed. So

$$(2) = +8i t^b t^a (pl) \frac{(ke)_\perp}{k_\perp^2}.$$  

3. Now

$$M = -\bar{u}\hat{e}(k_1)u \cdot \frac{2(pl)}{p_+} = -4(pl)(pe) = 4(pl)\frac{(k_1e)_\perp}{k_+}.$$  

The $T$-factor is

$$T = \frac{1}{k_1^-} = \frac{2k_+}{k_\perp^2}.$$
The colour factor is
\[ C = f^{acb}t^c = -f^{abc}t^c. \]

So finally
\[ (3) = -8(pl)f^{abc}t^c\frac{(k_1e)_\perp}{k_1^2}. \] (27)

Summing all three contributions we find the standard Lipatov expression graphically shown in Fig. 1
\[ (1) + (2) + (3) = 8(pl)f^{abc}t^c\left(\frac{(ke)_\perp}{k_2^2} - \frac{(k_1e)_\perp}{k_1^2}\right). \] (28)

4.2 Diagrams generated by the self-masses (Figs. 5 and 6)

A1.

\[ M = -i\bar{u}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma p_{\perp}h_{\mu\nu}(\kappa)\frac{1}{(2p_+)^3} \frac{2(pl)}{p_+}. \]

Calculating the matrix element and using (18) we get
\[ M = -i4(pl)(pe)\frac{1}{p_+^2}\frac{p_+^2}{\kappa_+^2} = 4(pl)(ke)\frac{\kappa_+^2}{\kappa_+^2}. \]

The colour factor is
\[ C = t^at^bt^c = \frac{N}{2}t^at^b. \]

The \( T \) factor can be readily read from the diagram:
\[ T = \frac{1}{k_+^2(\kappa_+ - k_-)} = \frac{1}{k_+^2k_-} + \frac{1}{k_-\kappa_+^2}. \] (29)

The first term has order \( \kappa_+ \) and does not lead to a logarithmic contribution. So for our purpose
\[ T = -\frac{8\kappa_+^2k_+}{\kappa_+^2k_+^2}. \]

Combining all factors we get
\[ (A1) = -16Ni(pl)t^at^b(ke)\frac{1}{k_+^2}\frac{1}{\kappa_+^2}. \] (30)

A2. Similar calculations give
\[ M = -i^4\bar{u}\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma p_{\perp}H_{\mu\nu}(\kappa_1, \kappa_2)\frac{1}{(2p_+)^2} \frac{2(pl)}{p_+} \]
\[ -4(pe)\frac{p_+^2}{\kappa_+^2} = 4(pl)(ke)\frac{\kappa_1\kappa_2}{\kappa_+^2}. \]

The colour factor is
\[ C = -t^at^bt^c = \frac{1}{2}iNt^b. \]
The time factor, read from the diagram, is

\[ T = 8 \frac{\kappa^2 k_+}{\kappa_{1+}^2 \kappa_{2+}^2 k_{1+}^2}. \]

Combing all factors we get

\[ (A2) = 16iN t^a t^b (pl) \frac{\langle ke \rangle_{k_+} (\kappa_1 \kappa_2)_{\perp}}{k_{1+}^2 \kappa_{1+}^2 k_{2+}^2} = 16iN t^a t^b (pl) \frac{\langle ke \rangle_{k_+} [\kappa(\kappa + q)]_{\perp}}{k_{1+}^2 \kappa_{1+}^2 (\kappa + q)_{\perp}^2}. \] (31)

**A3.** The \( M \)-factor is the same as in (A1). The \( T \)-factor has the opposite sign. In the colour factor the order of \( t^a \) and \( t^b \) will be reversed. So we find

\[ (A3) = 16iN (pl) t^b t^a \frac{\langle ke \rangle_{k_+} 1}{k_{1+}^2 \kappa_{1+}^2}. \] (32)

**A4.** The \( M \) factor is the same as in (A2). The \( T \)-factor has the opposite sign. In the colour factor the order of \( t^a \) and \( t^b \) will be reversed. So we find

\[ (A4) = -16iN t^b t^a (pl) \frac{\langle ke \rangle_{k_+} [\kappa(\kappa + q)]_{\perp}}{k_{1+}^2 \kappa_{1+}^2 (\kappa + q)_{\perp}^2}. \] (33)

**A5.** Now the momentum part is given via the gluon self-mass insertion \( \Pi \) shown in Fig 10 with \( k_4 = k_3 \rightarrow k_2 \):

\[ M = -2\Pi \frac{1}{(2k_+)^2} \cdot \frac{2(pl)}{p_+}. \]

The time-factor given has order \( \kappa_{1+}^2 \), so that it is sufficient to take \( \Pi \) from (23) in the leading order

\[ M = -4(pl) \frac{\langle ke \rangle_{k_+} \kappa_{1+}^2 (\kappa_1 \kappa_2)_{\perp}}{k_{1+}^2 k_{2+}^2}. \]

The colour factor is

\[ t^c f^{abc} f^{cde} f^{c2ad} = -\frac{1}{2} N f^{bac} f^{c}. \]

The \( T \)-factor is

\[ T = -8 \frac{\kappa_{1+}^2 k_+}{\kappa_{1+}^2 \kappa_{2+}^2 k_{2+}^2 k_{1+}^2}. \]

Combining all factors we get

\[ (A5) = -16N (pl) f^{bac} f^{c} \langle ke \rangle_{k_+} (\kappa_1 \kappa_2)_{\perp} \frac{(k_2 e)_{\perp}}{k_{2+}^2 \kappa_{2+}^2 k_{1+}^2} = -16N (pl) f^{bac} f^{c} \langle [k + q e]_{\perp} [\kappa(\kappa + q)]_{\perp} \frac{(k + q)_{\perp}^2}{(k + q)_{\perp}^2 \kappa_{1+}^2 (\kappa + q)_{\perp}^2}. \] (34)

**A6.** The momentum part is given via the gluon self-mass insertion \( \Pi \) shown in Fig 10 with \( k_4 = k_3 \) and \( \kappa_1 = \kappa_2 \):

\[ M = -2\Pi \frac{1}{(2k_+)^2} \cdot \frac{2(pl)}{p_+} = -4(pl) \frac{\langle ke \rangle_{k_+} \kappa_{1+}^2}{k_{1+}^2 \kappa_{1+}^2}. \]

The colour factor is found to be the same as in (A5). The \( T \)-factor is

\[ T = -8 \frac{\kappa_{1+}^2 k_+}{\kappa_{1+}^2 \kappa_{2+}^2 k_{2+}^2}. \]
So we finally get
\[(A6) = -16N(pl) f^{bac} e \frac{(k_3 e)_{\perp}}{k_3^2} \frac{1}{\kappa_+^2} = -16N(pl) f^{bac} e \frac{[(k + q) e]_{\perp}}{(k + q)^2} \frac{1}{\kappa_+^2}. \quad (35)\]

**A7.** In this case the $M$ factor is given by the self-mass insertion $\Pi$ with $k_2 \to k_1$, and $k_4 = k_3 = k_0 \to k_2$.

\[M = - (pl) \Pi \frac{1}{p_+ k^2_+}.\]

The $T$-factor has a leading term of the order $\kappa_+$:
\[T = -8 \frac{\kappa_+ k^2_+}{\kappa_+^2 k^4_+} + 8 \frac{\kappa^2_+ k^2_+ k^2_{\perp}}{\kappa_+^4 k^4_{\perp}},\]

so that we have to retain terms of the order $1/\kappa_+$ in (23). We find
\[\Pi T = 32\mu_+ k^2_+ \frac{(k_2 e)_{\perp}}{k^2_{\perp}} \left( k^2_{\perp} + 2(k_1 \kappa)_{\perp} \right) = 32\mu_+ k^2_+ \frac{(k_2 e)_{\perp}}{k^2_{\perp}} \left( 1 - \frac{\kappa^2_+}{k^2_{\perp}} \right)\]

The colour factor is twice greater than in (A5) and (A6):
\[C = f^{cc_1 c_2} f^{c_1 c_2 d} f^{dabc} e = -N f^{bac} e.\]

So in the end
\[(A7) = 32N(pl) f^{bac} e \frac{[(k + q) e]_{\perp}}{(k + q)^2} \frac{1}{\kappa_+^2} \left( 1 - \frac{\kappa^2_+}{(k + q)^2} \right). \quad (36)\]

The second term in this expression
\[32N(pl) f^{bac} e \frac{[(k + q) e]_{\perp}}{(k + q)^2} \]

corresponds to a contribution $\propto 1/(k + q)^4$ in the 4-dimensional language, since $(k + q)_- = (p - p'_- \simeq 0$. This contribution is eliminated by the gluon self-mass renormalization which requires it to vanish on the mass shell.

### 4.3 Diagrams generated by vertexes (Figs. 7 and 8)

Diagrams (B1)-(B3) have the $T$-factor $\propto \kappa_+^3$, so that they do not lead to the logarithmic integration in $\kappa_+$ and can be neglected. To the rest of the diagrams shown in Figs. 7 and 8 and denoted as (B4), (B5),... new ones should be added with soft and hard gluon interchanged, denoted correspondingly as (B41), (B51),.... Furthermore to these latter diagrams one should add ones with the direction of the soft gluon momentum reversed: $\kappa \to -\kappa$. These last diagrams will be denoted as (B42), (B52),...

**B4.** The momentum part is
\[M = \bar{\psi} \gamma^\xi \gamma^\mu \psi u h_{\mu \nu}(\kappa) h_{\nu \sigma}(k_1) e^{\rho} \Gamma_{\rho \sigma}(k; \kappa, k_1) \frac{1}{l^2 (2\kappa + 2p_+)^2} \frac{2(pl)}{p_+} . \]
Calculating the matrix element and convoluting $p^\xi p^\mu$ with the rest factors to form the effective vertex $V$ (see (21) we get

$$M = 2V \frac{1}{p^2 k_+}.$$ 

Since the time-factor contains only the term proportional to $\kappa_+^2$ we need only the leading term from $V$: The $T$ factor read from the diagram is

$$T = \frac{8\kappa_+^2 k_+}{\kappa_+^4 k_+^2}.$$ 

The colour factor is

$$C = t^c t^d t^f f^{dc} = -\frac{1}{2}iN t^e t^b$$

and thus

$$(B4) = 16iN(p) t^a t^b \frac{(k_1 e)_\bot}{k_\bot^2} \frac{1}{\kappa_+^2}.$$ 

Integration over the angles of $\kappa$ transforms this into

$$(B4) = 16iN(p) t^a t^b \frac{(k e)_\bot}{k_\bot^2} \frac{1}{\kappa_+^2}. \quad (38)$$

**B41.** The $M$-factor is the same with the substitution $V \to V_1$ (see Eq. (20). The $T$-factor is

$$T = \frac{8\kappa_+^2 k_+}{\kappa_1^2 k_1^2 (k_1^2 - k_\bot^2)} - \frac{8\kappa_+^2 k_+}{\kappa_+^4 k_+^2}.$$ 

Since it also contains a term proportional to $\kappa_+$ we have to take into account all terms in $V_1$. The colour factor is the same as in (B4) So our final answer is

$$(B41) = 16iN(p) t^a t^b \frac{(k_1 e)_\bot}{k_\bot^2} \frac{1}{\kappa_1^2} \left( 1 + \frac{(k_1 \kappa)_\bot}{k_\bot^2 - k_\bot^2} \right). \quad (39)$$

**B42.** The $M$-factor is the same with the substitution $\kappa \to -\kappa$ in $V_1$ (20). Now the $T$ factor contains only a term proportional to $\kappa_+$ and we have to retain only the subleading terms in the vertex. The colour factor is the same as in (B4). So in the end we find

$$(B42) = 16iN(p) t^a t^b \frac{(k e)_\bot}{k_\bot^2} \frac{(k_1 \kappa)}{k_1^2 - k_\bot^2}. \quad (40)$$

**B8.** The momentum part of diagrams (B8), (B81) and (B82) is the same as in (B4), (B41) and (B42) respectively. In the colour factor $t^a t^b \to t^b t^a$. So the difference is only related with the $T$-factors. For (B8) the $T$ factor has the opposite sign as compared to (B4). As a result we immediately get

$$(B8) = -16iN(p) t^b t^a \frac{(k e)_\bot}{k_\bot^2} \frac{1}{\kappa_+^2}. \quad (41)$$

**B81.**

$$(B81) = -16iN(p) t^b t^a \frac{(k_1 e)_\bot}{\kappa_1^2 k_1^2 k_\bot^2} \left( \frac{k_\bot^2}{k_\bot^2 - k_\bot^2 + (k_1 \kappa)_\bot} \right). \quad (42)$$
\( B82 \).

\[
(B82) = -16iN(pl)_t^b t^a \frac{(k_1^e)_\perp}{\kappa_+^2 k_\perp^2} \left( 1 + \frac{\kappa k_1^e}{k_\perp^2} \right).
\]

**B5**  
Here we start with the colour factor.

\[
C = f^{acd} t^c t^d = f^{acd} t^b t^d' + if^{acd} f^{cbe} t^d = f^{acd} t^b t^d + if^{acd} f^{cbe} t^d t^e.
\]

We have

\[
f^{acd} t^d = -f^{acd} t^c t^d = -f^{acd} t^c = -i \frac{1}{2} N t^a,
\]

so that

\[
C = \frac{1}{2} \left[ -i \frac{1}{2} N \{ t^a, t^b \} + if^{acd} f^{cbe} t^d t^e \right]
\]

We use

\[
\{ t^a, t^b \} = \frac{1}{N} \delta_{ab} + d^{abc} t^d
\]

to obtain

\[
C = \frac{1}{2} \left[ -i \frac{1}{2} N \left( \frac{1}{N} \delta_{ab} + d^{abc} t^d \right) + if^{acd} f^{cbe} \left( \frac{1}{N} \delta_{de} + d^{deh} t^h \right) \right].
\]

The identity

\[
f^{acd} f^{cbe} d^{ehd} = -f^{dac} f^{cbe} d^{ehd} = \frac{1}{2} N d^{abh}
\]

brings us to the final expression

\[
C = \frac{1}{2} \left[ -i \frac{1}{2} \delta_{ab} - i \frac{1}{2} N d^{abcd} t^d + i \delta_{ab} + i \frac{1}{2} N d^{abcd} t^d \right] = \frac{1}{2} \delta_{ab}.
\]

So factor \( C \) has a subdominant order in \( N \) (the dominant factor has order \( N \)). This means that we can neglect all diagrams (B5), (B51) and (B51).

**B6.** The \( T \) factor has order \( \kappa_+^2 \), so that \( (B6) \) can be neglected.

**B61.** We find the \( M \)-factor as

\[
M = -2i(p_l) V_1 \frac{1}{\bar{p}_+^2 k_+}.
\]

The \( T \) factor has order \( \kappa_+^2 \) so only the main term from \( V_1 \) contributes. The colour factor

\[
C = f^{acd} f^{c_1 d} t^c t^b
\]

**B62.** We have the same \( M \) and \( T \)-factors. The colour factor

\[
C = f^{acd} f^{c_1 d} t^c t^b
\]

has the opposite sign as compared to \( (B61) \).

So \( (B6) + (B61) + (B62) = 0 \)

**B7.** The \( T \) factor has order \( \kappa_+^3 \) so that \( (B7) = 0 \)

**B71.** As before

\[
M = -2i(p_l) V_1 \frac{1}{\bar{p}_+^2 k_+}.
\]
The $T$ factor has order $\kappa_+$ so that only the subdominant term in $V_1$ contributes. The colour factor is

\[ C = f^{ac_1} f^{c_1 db} t^c t^d. \]

It can be simplified:

\[ C = -[t^a, t^{c_1}][t^b, t^{c_1}] = t^{c_1} t^a t^b t^{c_1} + t^a t^{c_1} t^b - t^a t^{c_1} t^b t^{c_1} - t^a t^b t^{c_1} t^{c_1}. \]

The last two terms are zero. The second term is equal to $(1/2) N t^a t^b$ The first can be calculated using

\[ t^a t^b = \frac{1}{2N} \delta_{ab} + F_{c_d t_c t^d}. \]

with some numerical coefficients $F_{c_d t_c t^d}$. So the first term is $(1/4) \delta_{ab}$ and is negligible. Thus

\[ C = \frac{1}{2} N t^a t^b. \]

**B72.** Both the $M$ and colour factors are the same. So in the sum $(B71)+(B72)$ the contribution doubles

\[ (B71) + (B72) = -32iN(p_l) t^a t^b \frac{(k_1 \kappa_\perp (k_2 e_\perp)}{\kappa_\perp^2 k_{2 \perp} (k_{1 \perp} - k_{1 \perp}^2).} \]

**B9.** The $M$ factor is the same

\[ M = -2i(p_l) V \frac{1}{p_1 \perp k_+}. \]

The $T$-factor is $\propto \kappa_+^2$ so only the first part of $V$ contributes. The colour factor is

\[ C = f^{ac_1} f^{c_1 db} t^c t^d = -\frac{1}{2} N f^{bac} t^c. \]

So

\[ (B9) = -16(p_l) N f^{bac} t^c \frac{(k_1 e_\perp)}{\kappa_\perp^2 k_{0 \perp}^2}. \]

**B91.** Again

\[ M = -2i(p_l) V_1 \frac{1}{p_1 \perp k_+}. \]

Now $T$ contains terms of the orders $\kappa_+$ and $\kappa_+^2$. So both leading and subleading terms in $V_1$ contribute. The colour factor is the same as in (B9). We find

\[ (B91) = -16(p_l) N f^{bac} t^c \frac{(k_1 e_\perp)}{\kappa_\perp^2 k_{0 \perp}^2} \left( 1 + \frac{(k_1 \kappa_\perp)}{k_{1 \perp}^2} \right). \]

**B92.** Again

\[ M = -2i(p_l) V_2 \frac{1}{p_2 \perp k_+}. \]

The $T$ factor contains only the term of the order $\kappa_+$. So the subleading term in $V_2$ contributes. The colour factor is the same as in (B9) and we find

\[ (B92) = -16(p_l) N f^{bac} t^c \frac{(k_1 \kappa_\perp (k_1 e_\perp)}{\kappa_\perp^2 k_{0 \perp}^2 k_{1 \perp}^2}. \]

This ends the calculation of vertex part diagrams.
4.4 Summary of the vertex diagrams

We have obtained the following non-zero contributions from the vertex diagrams. Omitting the trivial common factor $-16iN(p)l$

\[ (B4) = -t^a t^b \frac{(k_1 e)_\perp}{k_2^2 \kappa_\perp}, \]

\[ (B41) = -t^a t^b \frac{(k_1 e)_\perp}{k_2^2 \kappa_\perp} \left(1 + \frac{(k_1 \kappa)_\perp}{k_2^2 - k_\perp^2}\right), \]

\[ (B42) = -t^a t^b \frac{(k_1 e)_\perp}{k_2^2 \kappa_\perp}(k_1 \kappa)_\perp \]

\[ (B8) = t^b t^a \frac{(k_1 e)_\perp}{k_2^2 \kappa_\perp}, \]

\[ (B81) = t^b t^a \frac{(k_1 e)_\perp}{k_2^2 \kappa_\perp \kappa_\perp} \left(k_2^2 - k_\perp^2 + (k_1 \kappa)_\perp\right), \]

\[ (B82) = t^b t^a \frac{(k_1 e)_\perp}{k_2^2 \kappa_\perp \kappa_\perp} \left(1 + \frac{(k_1 \kappa)_\perp}{k_2^2 - k_\perp^2}\right), \]

\[ (B71) = t^a t^b \frac{(k_2 e)_\perp (k_1 \kappa)_\perp}{k_2^2 \kappa_\perp \kappa_\perp \kappa_\perp}, \]

\[ (B72) = t^a t^b \frac{(k_2 e)_\perp (k_1 \kappa)_\perp}{k_2^2 \kappa_\perp \kappa_\perp \kappa_\perp}, \]

\[ (B9) = -i f^{bac} \frac{(k_1 e)_\perp}{k_2^2 \kappa_\perp \kappa_\perp}, \]

\[ (B91) = -i f^{bac} \frac{(k_1 e)_\perp}{k_2^2 \kappa_\perp \kappa_\perp \kappa_\perp} \left(1 + \frac{(k_1 \kappa)_\perp}{k_2^2 - k_\perp^2}\right), \]

\[ (B92) = -i f^{bac} \frac{(k_1 e)_\perp (k_1 \kappa)_\perp}{k_2^2 \kappa_\perp \kappa_\perp \kappa_\perp}. \]

All terms involving the product $(k_1 \kappa)_\perp$ cancel between contributions of the diagrams (1) and (2). This is nearly obvious, since in these diagrams the direction of $\kappa_\perp$ is opposite. Indeed compare (B41) and (B42) as an example. In (B41) $k_1 + \kappa = k$, so $(k_1 \kappa)_\perp = (1/2)(k_2^2 - k_1^2 - k_\perp^2)$ The momentum part of the term with $(k_1 \kappa)_\perp$ is

\[ \frac{1}{2} \frac{(k_1 e)_\perp}{k_2^2 \kappa_\perp \kappa_\perp} \left(1 - \frac{\kappa_\perp^2}{k_2^2 - k_\perp^2}\right). \]

The second term goes after the angular integration. In terms of the integration variable $k_1 \perp$ we have $\kappa_\perp^2 = (k - k_1)_\perp^2$. In (B42) we have $k_1 - \kappa = k$ so that $(k_1 \kappa)_\perp = -(1/2)(k_2^2 - k_1^2 - k_\perp^2)$ with $\kappa_\perp^2 = (k_1 - k)_\perp^2$. So the momentum part of the term with $(k_1 \kappa)_\perp$ will be the same (49) with the opposite sign. Thus the terms with $(k_1 \kappa)_\perp$ cancel between (A41) and (A42). The same is true for all vertex contributions.
After cancellation of these terms we find

\[(B4) + (B41) + (B42) = 2(B4) = 32iN(p)\frac{t^a t^b (k_1 e)_{\perp}}{k_1^2 \kappa_1^2},\]  

where we have added the suppressed overall factor,

\[(B8) + (B81) + (B82) = 2(B8) = -32iN(p)\frac{t^a (k_1 e)_{\perp}}{k_0^2 \kappa_2^2},\]  

\[(B71) + (B72) = 0\]

and

\[(B9) + (B91) + (B92) = 2(B9) = -32iN(p)\frac{f^{abc} t^c (k_1 e)_{\perp}}{k_0^2 \kappa_2^2}.\]

Since \(k_1 = k_0 \pm \kappa\) and \(k_0 = k + q\) integration over the angles will change the last expression into

\[(B9) + (B91) + (B92) = 2(B9) = -32iN(p)\frac{f^{abc} t^c [(k + q)e]_{\perp}}{(k + q)^2 \kappa^2_2}.\]

Summing (50), (51) and (54) we obtain our final result for the contribution of vertex diagrams \(S_v\)

\[S_v = (B4) + (B41) + (B42) + (B8) + (B81) + (B82) + (B9) + (B91) + (B92)\]

\[= -32N(p)\frac{f^{abc} t^c}{\kappa^2} \left( \frac{(k e)_{\perp}}{k_1^2} - \frac{[(k + q)e]_{\perp}}{(k + q)^2_{\perp}} \right).\]

Remarkably this contribution seems to reverse the sign of the similar contribution from the mass-terms. However we shall see that in fact all vertex contributions are cancelled by renormalization.

## 5 Renormalization

The perturbation theory we use is the unrenormalized one, where we write out all the diagrams as they appear, including the self-masses in external lines. The only renormalization done is that of the quark mass supposed to be zero, so that its self mass is also supposed to be zero on mass-shell. No subtraction is initially done from the vertex part.

As a result, due to self-mass insertions, the Born term is to be multiplied by the product of factors \((Z - 1)^{1/2}\) for each external line. So the lowest order diagrams, studied in Sec. 3.1 are to be multiplied by \((Z_q - 1)(Z_g - 1)^{1/2}\) where \(Z_{q(g)}\) are the wave function renormalization constant for the quark (gluon). In the following we denote

\[Z_{q(g)} - 1 = \xi_{q(g)}\]

So we get an additional contribution

\[[(1) + (2) + (3)](\xi_q + \frac{1}{2}\xi_g).\]
where (1), (2) and (3) are contributions of the lowest order presented in Sec. 3.3. Note that the contribution (56) is of the third order and, limiting to this order, we can substitute the unrenormalized QCD coupling constant $g_0$ by the renormalized one $g$.

However this is not the whole story. In our unrenormalized perturbation theory the Born terms carry the unrenormalized coupling constant $g_0$. Leaving aside the constant associated with the external instantaneous interaction $L_0$, we have just one $g_0$ for gluon emission. Recalling the relation

$$g_0 = gZ_1Z_q^{-1}Z^2 - 1/2 = g(1 + \xi_1 - \xi_q - \frac{1}{2}\xi_a).$$

where $Z_1 = 1 + \xi_1$ is the vertex renormalization constant, we further have to add a term

$$[(1) + (2) + (3)](\xi_1 - \xi_q - \frac{1}{2}\xi_a).$$

In the sum of (56) and (58) contributions from self-masses cancel so that in the end the additional contribution due to renormalization is just the Born term multiplied by the vertex renormalization constant $\xi_1$

$$\xi_1[(1) + (2) + (3)].$$

This means that we have to calculate the vertex part renormalization constant.

The definition of the latter is standard. Let $\Lambda^a_\rho(p, k)$ be the nontrivial vertex part with the initial quark of momentum $p$, emitted gluon with momentum $k$ and Lorentz and colour indexes $\rho$ and $a$ respectively. The vertex renormalization constant $\xi_1$ is defined by the value of $\Lambda$ on mass shell:

$$\left(\Lambda^a_\rho(p, k)\right)_0 = \xi_1\gamma^\rho.$$  

The mass-shell can be defined in different ways depending on what is considered to be the renormalized coupling constant. In our case the most natural definition is to choose the vertex mass shell by putting all the three external particles on their mass shell

$$p^2 = k^2 = (p - k)^2 = 0.$$  

In the light cone variables with $p_0 = p_\perp = 0$, to satisfy (61) we have to require $pk = p+k_+ = 0$, that is

$$k_- = -\frac{k^2}{2k_+} = 0,$$

which obviously requires $k_- = k_\perp = 0$. So on the mass shell all three momenta $p, k$ and $p - k$ have only non-zero plus components $p_+, k_+$ and $p_+ - k_+$. In accordance with our kinematics we assume $k_+ << p_+$.

The simplest possibility to calculate $\xi_1$ is to use our previous results for the calculation of the emission amplitude for diagrams with vertex insertions. Take diagrams (B8), (B81) and (B82). In the Feynman language they correspond to the contribution

$$(B8) + (B81) + (B82) = \frac{1}{l_-} \frac{\lambda}{(p - k)^2} \Lambda^a_\rho(p, k) u\epsilon(p) \cdot \frac{2(p_l)}{p_+},$$
the last factor coming from the target. The vertex part $\Lambda$ enters with two momenta on mass shell: $p^2 = k^2 = 0$, but the third momentum is off mass shell: $(p - k)^2 = -2p_+ k_- = k_\perp^2 (p_+/k_+)$. Putting $k_\perp^2 \to 0$ in the vertex part we get the corresponding Born term multiplied by $\xi_1$:

$$\xi_1 t^b t^a \bar{u} l \frac{\hat{p} - \hat{k}}{l_- (p - k)^2} \hat{u} \cdot \frac{2(pl)}{p_+} = \xi_1 \cdot (2),$$

(63)

where (2) is the contribution from the lowest order diagram (2). Note that the denominator $(p - k)^2$ vanishes as $k_\perp^2 \to 0$, so that this limit can conveniently be taken with (62) multiplied by $(p - k)^2$

$$k_\perp^2 \frac{p_+}{k_+} [(B8) + (B81) + (B82)]_{k_\perp \to 0} = \xi_1 k_\perp^2 \frac{p_+}{k_+} \cdot (2).$$

(64)

The l.h.s of this equality can be read off Eq.(103):

$$l.h.s = \frac{p_+}{k_+} 32N(pl)t^b t^a \frac{(ke)\perp}{k_\perp^2},$$

(65)

where as always we suppressed integration over $\kappa_+$ and $\kappa_\perp$ with the usual weight. The r.h.s can be read off Eq. (70);

$$r.h.s = \xi_1 \cdot 8t^b t^a (pl)(ke)\perp,$$

(66)

wherefrom we conclude

$$\xi_1 = 4N \frac{1}{k_\perp^2}. $$

(67)

This result is confirmed by direct calculations using the Feynman diagram approach.

We see that in our approximation the vertex part does not depend on $k_\perp^2$ at all. This means that renormalization will eliminate all vertex contributions to our emission amplitude. As a result, the only contribution which remain come from the self-mass insertions.

6 Reggeization of the interaction

After cancellation of all contributions from the vertex part insertions we are left with the contributions from the self-mass insertions, which we study here. We first combine terms (A1)-(A4) Summing (A1)+(A3) we find

$$(A1) + (A3) = 16N(pl)f^{abc} \epsilon^c (ke)\perp \frac{1}{k_\perp^2 k_\perp^2}.$$  

(68)

Summing (A2)+(A4)

$$(A2) + (A4) = -16N(pl)f^{abc} \epsilon^c (ke)\perp \frac{[\kappa(\kappa + q)]}{k_\perp^2 (\kappa + q)\perp}. $$

(69)

Thus

$$S_1 \equiv \frac{4}{\sum_{j=1}} (A_j) = 16N(pl)f^{abc} \epsilon^c (ke)\perp \left(\frac{1}{k_\perp^2} - \frac{[\kappa(\kappa + q)]}{k_\perp^2 (\kappa + q)\perp}\right).$$  

(70)
Under the sign of integration over $\kappa$ we can rewrite this as
\[
S_1 = 8N(pl) f^{abc} \ell e \frac{(ke)_{\perp}}{k^2_{\perp}} \left( \frac{1}{\kappa^2_{\perp}} + \frac{1}{(\kappa + q)^2_{\perp}} - 2 \frac{[\kappa(\kappa + q)]_{\perp}}{\kappa^2_{\perp}(\kappa + q)^2_{\perp}} \right).
\]
\[
= 8(pl) f^{abc} \ell e \frac{(ke)_{\perp}}{k^2_{\perp}} \frac{Nq^2}{\kappa^2_{\perp}(\kappa + q)^2_{\perp}}.
\] (71)
Integration over $\kappa$ with the additional factor $g^2$ converts the last factor into $y\omega(q)$ where $y$ is the harder gluon rapidity and $\omega(q)$ is the gluon trajectory Regge (see Eq. (6)). So we have found
\[
S_1 = 8(pl) f^{abc} \ell e \frac{(ke)_{\perp}}{k^2_{\perp}} \cdot y\omega(q) \tag{72}
\]
Now we turn to the rest three diagrams (A5), (A6) and (A7). We find
\[
(A6) + (A7) = -(A6)
\]
and as a result
\[
S_2 = \sum_{j=5}^{7} (A_j) = 16N(pl) f^{abc} \ell e \frac{[(k + q)e]_{\perp}}{(k + q)^2_{\perp}} \left( \frac{[\kappa(\kappa + q)]_{\perp}}{\kappa^2_{\perp}(\kappa + q)^2_{\perp}} - \frac{1}{\kappa^2_{\perp}} \right).
\] (73)
Under the sign of integration over $\kappa$ this can be rewritten as
\[
S_2 = 8N(pl) f^{abc} \ell e \frac{[(k + q)e]_{\perp}}{(k + q)^2_{\perp}} \left( \frac{2[\kappa(\kappa + q)]_{\perp}}{\kappa^2_{\perp}(\kappa + q)^2_{\perp}} - \frac{1}{\kappa^2_{\perp}} - \frac{1}{(\kappa + q)^2_{\perp}} \right)
\]
\[
= -8(pl) f^{abc} \ell e \frac{[(k + q)e]_{\perp}}{(k + q)^2_{\perp}} \frac{Nq^2}{\kappa^2_{\perp}(\kappa + q)^2_{\perp}}.
\] (74)
Integration over $\kappa$ wit factor $g^2$ then gives
\[
S_2 = -8(pl) f^{abc} \ell e \frac{[(k + q)e]_{\perp}}{(k + q)^2_{\perp}} \cdot y\omega(q) \tag{75}
\]
In conclusion the sum of all self-mass diagrams except the last term in (36) gives the contribution
\[
\sum_{j=1}^{7} (A_j) = 8(pl) f^{abc} \ell e \frac{(ke)_{\perp}}{k^2_{\perp}} \left( \frac{[k + q)e]_{\perp}}{(k + q)^2_{\perp}} - \frac{[(k + q)e]_{\perp}}{(k + q)^2_{\perp}} \right) \cdot y\omega(q). \tag{76}
\]
This is just the expected expression implying reggeization of the interaction with the target.

7 Conclusions

We have studied corrections to the lowest order emission amplitudes for the quark moving in the external instantaneous potential and emitting a soft gluon. Only corrections due to virtual gluons softer than the emitted one have been taken into account. The reason is that they play a decisive role in the study of the evolution of the inclusive cross-section for
gluon production at rapidities smaller than that of the observed gluon, where the discrepancy between the results of [2] and [4] originates. We have found that these corrections are fully equivalent to the corresponding corrections in the BFKL language, where they come from the gluon reggeization (and in fact from very different Feynman graphs). Renormalization has played a decisive role in obtaining this result. We have to stress that in principle this result is insufficient to establish full equivalence of the physical picture used in [2] and [4]. The difference between their results starts from the next-to-leading order, where a huge number of more complicated diagrams appear with a greater number of interactions (two or three). The study of loops in such diagrams, although straightforward, requires an extraordinary effort, for which we are not ready at present. However results obtained in this work lead us to believe that also in this more complicated case the results of both approaches coincide.

As mentioned in the introduction, the equivalence of loop diagrams, if fully established, together with the already proved equivalence of tree diagrams shows that the physical foundations on which derivations in [2] and [4] are based are the same. The difference in their results can therefore either be in fact absent, if the new terms in [4] cancel, or originate from the difference in the derivations themselves. The study of the latter difference is not simple, since the technique used is completely different. The final answer therefore requires a detailed analysis of the procedures used, which is postponed for future studies. The immediate and simpler task is to check if the terms with the BKP states found in [4] are actually present and not cancelled. Our preliminary studies indicate that they are not. After due verification this result will be presented for a future publication.

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9 Appendix 1. Calculation of the vertex insertion

Consider the diagram Fig. 9. At the first step we convolute the 3-gluon vertex with the polarization vector $E$. We take into account that $\kappa_{1\nu}H^{\mu\nu}(\kappa_2\kappa_1) = 0$, so that in the vertex we may drop terms proportional to $\kappa_{1\nu}$ and $k_{1\sigma}$ to find

$$\Gamma_{\nu\sigma} \equiv \Gamma_{\nu\sigma\rho}(\kappa_1, k_1, k_0)E^\rho(k_0) = 2k_{1\nu}E_\sigma - 2\kappa_{1\sigma}E_\nu + g_{\nu\sigma}(\kappa_1 - k_1, E).$$  \hspace{1cm} (77)

Next we convolute $H^{\mu\nu}(\kappa_2\kappa_1)$ with $p^\mu k_1^\nu$

$$A_1 \equiv p_\mu k_{1\nu}H^{\mu\nu}(\kappa_2, \kappa_1) = \frac{p_+ k_+}{\kappa_+^2}(\kappa_1\kappa_2)_\perp - \frac{p_+}{\kappa_+}(k_1\kappa_2)_\perp$$ \hspace{1cm} (78)
and $p_\mu E_\nu$

$$A_2 \equiv p_\mu E_\nu H^{\mu\nu}(\kappa_2, \kappa_1) = \frac{p_+}{k_+}(\kappa_2 e)_\perp.$$  \hspace{1cm} (79)

Interchanging $\kappa_{1,2} \leftrightarrow k_{1,2}$ we also find

$$A_3 \equiv \kappa_\sigma H^{\sigma}(k_2, k_1) = \frac{p_+}{k_+} (k_1 k_2)_\perp - \frac{p_+}{k_+} (\kappa_1 k_2)_\perp$$  \hspace{1cm} (80)

and

$$A_4 \equiv E_\sigma H^{\sigma}(k_2, k_1) = \frac{p_+}{k_+} (k_2 e)_\perp.$$  \hspace{1cm} (81)

We have

$$V = 2A_1 A_4 - 2A_2 A_3 + \tilde{V},$$  \hspace{1cm} (82)

where

$$\tilde{V} = (\kappa_1 - k_1, E)H_\nu(\kappa_2, \kappa_1)H^{\nu}(k_2, k_1).$$  \hspace{1cm} (83)

After some calculations we find

$$\tilde{V} = 2\frac{p_1^2}{\kappa^+ + k_+} (k_2 k_2)_\perp \left( (\kappa_1 e)_\perp - \frac{k_1 e)_\perp}{k_1^+} \right).$$  \hspace{1cm} (84)

Summing terms in (82) we take into account that $\kappa_+ << k_+$ and retain only the leading and the next-to-leading terms to find (21).

10 Appendix 2. Calculation of the gluon-self mass insertion

We start calculating $A^{(1)}_{\mu\sigma} \equiv H_{\mu\nu} \Gamma_{\nu\sigma}$, where $\Gamma_{\nu\sigma}$ was defined in (77). We define

$$b_{1\mu} \equiv H_{\mu\nu}(\kappa_2, \kappa_1)k_1^n = k_1 \mu - \frac{l_\mu (k_1 k_2 + \kappa_1 k_2)}{(kl)} + l_\mu \frac{(k_1 l)(k_2 k_1)}{(kl)^2}$$  \hspace{1cm} (85)

and

$$b_{2\mu} \equiv H_{\mu\nu}(\kappa_2, \kappa_1)E^n = E_\mu - l_\mu \frac{(\kappa_2 E)}{(kl)},$$  \hspace{1cm} (86)

which gives

$$A^{(1)}_{\mu\sigma} = 2E_\sigma b_{1\mu} - 2\kappa_1 b_{2\mu} + [(\kappa_1 - k_1)E]H_{\mu\sigma}(\kappa_2 \kappa_1).$$

Next we multiply this by $H_{\xi\sigma}(k_2, k_1)$:

$$A^{(2)}_{\xi\mu} \equiv H_{\xi\sigma}(k_2, k_1)A^{(1)}_{\mu\sigma}.$$  \hspace{1cm} (87)

We define

$$a_{1\xi} \equiv H_{\xi\sigma}(k_2, k_1)E^n = E_\xi - l_\xi \frac{k_2 E}{(kl)},$$  \hspace{1cm} (88)

$$a_{2\xi} \equiv H_{\xi\sigma}(k_2, k_1)k_1^n = \kappa_1 \xi - \frac{l_\xi (k_2 k_1) + k_1 \xi (k_1 l)}{(kl)} + l_\xi \frac{(k_1 l)(k_2 k_1)}{(kl)^2}$$  \hspace{1cm} (89)
and
\[ \bar{H}_{\xi\mu}(k_2, \kappa_2) \equiv H_{\xi\sigma}(k_2, k_1)H_{\mu\sigma}(\kappa_2, \kappa_1) = g_{\xi\mu} - \frac{l_\xi k_{2\mu}}{(kl)} - \frac{l_{\mu}\kappa_2}{(kl)} + \frac{l_{\mu}l_\xi(\kappa_2 k_2)}{(kl)(kl)}. \] (90)

This gives
\[ A^{(2)}_{\xi\mu} = 2a_{1\xi}b_{1\mu} - 2a_{2\xi}b_{2\mu} + \bar{A}^{(2)}_{\xi\mu}, \] (91)

where
\[ \bar{A}^{(2)}_{\xi\mu} = [(\kappa_1 - k_1)E] \bar{H}_{\xi\mu}(k_2, \kappa_2). \] (92)

At the next step we calculate
\[ A^{(3)}_{\xi\mu} \equiv H^\beta(k_4, k_3)\Gamma_{\beta\xi\mu}(k_3, k_2, \kappa_2), \] (93)

where
\[ H^\beta(k_4, k_3) = p_\alpha H^{\alpha\beta}(k_4, k_3). \] (94)

We find
\[ A^{(3)}_{\xi\mu} = 2\kappa_2 \xi H_{\mu}(k_4, k_3) - 2k_{2\mu}H_\xi(k_4, k_3) + \bar{A}^{(3)}_{\xi\mu}, \] (95)

where
\[ \bar{A}^{(3)}_{\xi\mu} = g_{\mu\xi}(k_2 - \kappa_2)\beta H^\beta(k_4, k_3) = g_{\xi\mu}\left(\frac{p_+}{k^+_\beta}(k_2 k_4) + \frac{p_+\kappa_2}{k_4^2}(k_3 k_4)\right). \] (96)

Multiplying \( A^{(2)} \) by \( A^{(3)} \) we obtain \( \Pi \) as a sum of 9 terms:
\[ \Pi = 4(a_1 \kappa_2)(b_1 H) - 4(a_2 \kappa_2)(b_2 H) + 2\kappa_2 \xi \bar{A}^{(2)}_{\xi\mu} H_\mu \]
\[ -4(a_1 H)(k_2 b_1) + 4(a_2 H)(k_2 b_2) - 2H_\xi \bar{A}^{(2)}_{\xi\mu} k_{2\mu} + 2a_{1\xi} \bar{A}^{(3)}_{\xi\mu} b_{1\mu} \]
\[-2a_{2\xi} \bar{A}^{(3)}_{\xi\mu} b_{2\mu} + \bar{A}^{(2)}_{\xi\mu} \bar{A}^{(3)}_{\xi\mu}. \] (97)

Since \((a_1 l) = (a_2 l) = (b_1 l) = (b_2 l) = 0\) we can drop terms proportional to \( l \) in \( H \), so that for our purpose
\[ H_\beta = p_\beta - k_4\beta\frac{(pl)}{(kl)}. \] (98)

Next we study the 9 terms in (97) successively, denoting them as (1), (2) and so on. We find in orders in \( 1/\kappa_+^2 \) and \( 1/\kappa_+ \)
\[ (1) = 4(a_1 \kappa_2)(b_1 H) = 4\frac{p_+}{\kappa_+}(\kappa_1 k_4)_{\perp}(\kappa_2 e)_{\perp}, \]
(2) = \(-4(a_2 \kappa_2)(b_2 H) = 0, \]
(3) = \(2\kappa_2 \xi \bar{A}^{(2)}_{\xi\mu} H_\mu = 0, \]
(4) = \(-4(a_1 H)(b_1 k_2) = 4\frac{p_+ k_+}{\kappa_+}(\kappa_1 k_2)_{\perp}(k_4 e)_{\perp} - 4\frac{p_+}{\kappa_+}(k_4 e)_{\perp}\left((k_1 k_2)_{\perp} + (k_2 k_1)_{\perp}\right), \]
(5) = \(4(a_2 H)(b_2 k_2) = 4\frac{p_+}{\kappa_+}(k_4 k_1)_{\perp}(\kappa_2 e)_{\perp}, \)
(6) = -2H_ξ \ddot{A}_{ξμ}^{(2)} k_{2μ} = -4 \frac{P^+}{κ_+} (κ_2 k_4)_⊥ (κ_1 e)_⊥,

(7) = 2a_1 ξ \ddot{A}_{ξμ}^{(3)} b_{1μ} = (6),

(8) = -2a_2 ξ \ddot{A}_{ξμ}^{(3)} b_{2μ} = 0,

(9) = \ddot{A}_{ξμ}^{(2)} A_{ξμ}^{(3)} = 0 \quad (99)

Terms which are written as equal to zero in fact are finite at κ_+ = 0 and so can be neglected.

Summing all terms we finally obtain (23).

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