Scalar glueball decay

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We evaluate the coupling constant for the lightest scalar glueball to decay to pseudoscalar meson pairs. The calculation is done in the valence approximation on a $16^3 \times 24$ lattice at $\beta = 5.70$ for two different values of pseudoscalar meson mass.

A valence approximation calculation [1] using between 25000 and 30000 gauge configurations at each of several different $\beta$ gives a value of $1740 \pm 71$ MeV for the infinite volume continuum limit of the mass of the lightest scalar glueball. An independent calculation [2] with between 1000 and 3000 configurations at several different $\beta$, when interpreted [3] following Ref. [1], yields $1625 \pm 94$ MeV for the lightest scalar glueball mass. Finally, an analysis in Ref. [3] suggests that the error in the valence approximation to the scalar glueball mass is probably less than 60 MeV. The mass calculation with larger statistics then appears to favor $f_J(1710)$ as a scalar glueball, with $f_0(1590)$ a possible but less likely candidate. A third alternative is that the scalar glueball is a linear combination of the two states with significant contributions from each. No other well established resonance consistent with scalar glueball quantum numbers lies within 200 MeV of the established resonance. Second, since the scalar glueball is a flavor singlet, one might expect its coupling to a pair of pseudoscalars. Our preliminary results strongly suggest this particle has a width significantly below 200 MeV. Thus we consider it quite likely that the scalar glueball has already been found in experiment. In addition, the coupling constants we obtain show a variation with the mass of the decay products consistent with the variation in both the $f_J(1710)$ and the $f_0(1590)$ couplings.

To evaluate the glueball decay couplings, consider a euclidean lattice gauge theory, on a lattice $L^3 \times T$, with the plaquette action for the gauge field, the Wilson action for quarks, and the gauge field transformed to lattice Coulomb gauge. Let $U_i^r(x)$ for a space direction $i = 1, 2, 3$, and integer radius $r \geq 1$, be a smeared link field found by averaging the $r^2$ links running in direction $i$ from the sites of the $(r-1) \times (r-1)$ square oriented in the two positive space directions orthogonal to $i$ starting at site $x$. Let $V_{ij}^r(x)$ be the trace of the product of $U_i^r(y)$ and $U_j^r(y)$ around the boundary of an $(r-1) \times (r-1)$ square in the $ij$ plane beginning at $x$. Let the zero-momentum scalar glueball operator $g^r(t)$ be the sum of the $V_{ij}^r(x)$ for all $i,j$ and $x$ in the time $t$ lattice hyperplane. Define the quark and antiquark fields $\overline{\Psi}^r(x)$ and $\Psi^r(x)$ to be Wilson quark and antiquark fields smeared by convoluting the local Wilson fields with a space direction gaussian with root-mean-square radius $r'$. Define the smeared pseudoscalar field $\pi_i^r(x)$ with flavor index $i$ to be $\overline{\Psi}^r(x)\gamma^5\lambda_i\Psi^r(x)$, where $\lambda_i$ is a Gell-Mann flavor matrix. Let $\hat{\pi}_i^r(k,t)$ be the Fourier transform of $\pi_i^r(x)$ on the time $t$ lat-
tice hyperplane. We now fix \( r \) and \( r' \) to the values 3 and \( \sqrt{6} \), respectively, used in the summary of results presented below, and omit the radius superscripts.

At large time separations \( t \), the vacuum expectation values \( \langle \hat{\pi}^i_1(0,t)\hat{\pi}^i_0(0,0) \rangle > \) and \( \langle \hat{\pi}^i_1(\vec{k},t)\hat{\pi}^i_0(\vec{k},0) \rangle > \) approach \( \eta^2_1L^3\exp(-E_\pi t) \) and \( \eta^2_2L^3\exp(-E_\sigma t) \), respectively. Here \( \vec{k} \) is any momentum with norm \( 2\pi L^{-1} \), \( E_\pi \) is the mass of a pion, and \( E_\pi \) is the energy of a pion with momentum \( 2\pi L^{-1} \). In addition, in the valence approximation, \( \langle g(t)g(0) \rangle > \) approaches \( \eta^2_2L^3\exp(-\tilde{E}_g t) \), where \( E_g \) is the mass of the lightest scalar glueball. At this point it is convenient to redefine \( g(t) \), \( \hat{\pi}^i_0(0,t) \) and \( \hat{\pi}^i_0(\vec{k},t) \) by dividing each by the corresponding \( \eta \).

Define the two pion zero momentum operator \( \tilde{\Pi}(0,t) \) to be \( 6^{-1/2}\sum_i\hat{\pi}^i(0,t)\hat{\pi}^i(0,0) \). Let the two pion operator \( \tilde{\Pi}(k,t) \) for nonzero \( k \) be \( 6^{-1}\sum_i\hat{\pi}^i(\vec{k},t)\hat{\pi}^i(-\vec{k},0) \) where the sum is over the six ways of placing a vector \( \vec{k} \) of length \( k \) in a positive or negative lattice coordinate direction. Define the vacuum subtracted operators \( \Pi_s(k,t) \) to be \( \tilde{\Pi}(k,t) - \langle \Omega|\tilde{\Pi}(k,t)|\Omega \rangle > \). Let \( |1> \) be the lowest energy eigenstate in \( \Pi_s(0,0)|\Omega \rangle > \), and let \( |2> \) be the lowest energy eigenstate in \( \Pi_s(2\pi L^{-1},t)|\Omega \rangle > \). The states \( |1> \) and \( |2> \) are both normalized to 1. Let \( E_{\pi \pi 1} \) and \( E_{\pi \pi 2} \) be the energies of these two states, respectively. Define the amplitudes \( \eta_{ij}(t) \)

\[
\eta_{11}(t) = L^{-3} \langle \Omega|\Pi_s(0,t)|\Omega \rangle >, \\
\eta_{22}(t) = L^{-3} \langle \Omega|\Pi_s(2\pi L^{-1},t)|\Omega \rangle >.
\]

For large \( t \), \( \eta_{ij}(t) \) has the asymptotic form \( \eta_{ij}(t)\exp(-E_{\pi \pi} t) \). Connected three-point functions from which coupling constants can be extracted are now given by

\[
T_1(t_g, t_\pi) = \langle g(t_g)\Pi_s(0, t_\pi) \rangle >, \\
T_2(t_g, t_\pi) = \langle g(t_g)\Pi_s(2\pi L^{-1}, t_\pi) \rangle >.
\]

If the quark mass, and thus the pion mass, is chosen so that \( E_{\pi \pi 1} \) is equal to \( E_g \), the lightest intermediate state which can appear between the glueball and pions in a transfer matrix expression for \( T_1(t_g, t_\pi) \) is \( |1> \). Thus for large enough \( t_g \) with \( t_\pi \) fixed, \( T_1(t_g, t_\pi) \) will be proportional to the coupling constant of a glueball to two pions at rest. If the quark mass is chosen so that \( E_{\pi \pi 2} \) is equal to \( E_g \), however, the lightest intermediate state which can appear between the glueball and pions in a transfer matrix expression for \( T_2(t_g, t_\pi) \) is still \( |1> \), not \( |2> \). This holds since \( \eta_{12}(t) \) in Eq. (2) has no reason to vanish. To obtain from \( T_2(t_g, t_\pi) \) the coupling of a glueball to two pions with momentum \( 2\pi L^{-1} \), the contribution to \( T_2(t_g, t_\pi) \) arising from the \( |1> \) intermediate state must be cancelled off. From the three-point functions defined in Eqs. (3) and (4) we therefore define the amplitudes

\[
S_1(t_g, t_\pi) = T_1(t_g, t_\pi) - \frac{\eta_{11}(t_\pi)}{\eta_{12}(t_\pi)}T_2(t_g, t_\pi), \\
S_2(t_g, t_\pi) = T_2(t_g, t_\pi) - \frac{\eta_{12}(t_\pi)}{\eta_{11}(t_\pi)}T_1(t_g, t_\pi).
\]

In \( S_2(t_g, t_\pi) \) the contribution of the undesirable \( |1> \) intermediate state has been cancelled. In \( S_1(t_g, t_\pi) \) a contribution from the intermediate state \( |2> \) has been cancelled. Although the subtraction in \( S_1(t_g, t_\pi) \) is irrelevant for large enough \( t_g \), we expect that as a result of this subtraction \( S_1(t_g, t_\pi) \) will approach its large \( t_g \) behavior more rapidly than does \( T_1(t_g, t_\pi) \).

An additional intermediate state which can also appear in a transfer matrix expression for either \( T_1(t_g, t_\pi) \) is the isosinglet scalar bound state of a quark and an antiquark. The relative contribution of this state to either three-point function, however, is suppressed by the factor \( \mu(t_g - t_\pi) \), where \( \mu \) is the coupling constant of this state to the glueball. Phenomenological arguments following Ref. [3] strongly suggest that \( \mu \) is less than 60 MeV. In our main results below, removing the additional intermediate state would therefore give at most a 4% correction.

At large \( t_g \) and \( t_\pi \), the three-point functions become

\[
S_1(t_g, t_\pi) \rightarrow \frac{\sqrt{3}\lambda_1\eta_{11}(1 - r_{12}r_{21})L^3}{\sqrt{16E_g E_{\pi 1}}}s_1(t_g, t_\pi), \\
S_2(t_g, t_\pi) \rightarrow \frac{3\lambda_2\eta_{22}(1 - r_{12}r_{21})L^3}{\sqrt{8E_g E_{\pi 2}}}s_2(t_g, t_\pi).
\]
Here $\lambda_1$ and $\lambda_2$ are the glueball coupling constants to pseudoscalar pairs at rest or with momentum $2\pi L^{-1}$, respectively. Up to a factor of $-i$ these coupling constants are invariant decay amplitudes with the standard normalization convention. The factors $\eta_{ij}$ are given by the large $t$ behavior of $\eta_{ij}(t)$ as discussed earlier, and $\tau_{ij}$ is $\eta_{ij}/\eta_{ij}$. The factors $s_i(t_g, t_\pi)$ are

$$s_i(t_g, t_\pi) = \sum_t \exp(-E_\pi|t - t_g| - E_\pi|t|).$$

In Eq. (9), we make the simplifying assumption that $E_{\pi\pi}$ is $2E_{\pi i}$. In our results to be presented below we actually use a slightly more complicated expression for $s_i(t_g, t_\pi)$ which takes into account small differences between $E_{\pi\pi}$ and $2E_{\pi i}$.

To obtain values of $\lambda_i$ from Eqs. (10) and (11) we need the amplitudes $\eta_{ij}(t)$. These we determine from propagators for two pion states. Define the two pion operator $\Pi(t_1, t_2)$ from smeared pion fields, not Fourier transformed and not renormalized, to be $6^{-1/2}\sum_i \pi_i(0, t_1)\pi_i(0, t_2)$. Define two pion propagators by

$$C_i(t_1, t_2) = \Pi(t_1 + 2t_2, t_1 + t_2)\Pi_s(0, t_2),$$

$$C_{ij}(t_1, t_2) = \Pi(t_1 + 2t_2, t_1 + t_2)\Pi_s(2\pi/L, t_2).$$

For large values of $t_1$, these amplitudes approach

$$C_i(t_1, t_2) = C_i(t_1, t_2) = C_1(t_1, t_2) + C_2(t_1, t_2) + C_3(t_1, t_2) + C_4(t_1, t_2),$$

$$C_{ij} = \eta_{ij}(t_2)\eta_{ji}(t_2) + \sqrt{6}\eta_{ij}(t_2)\eta_{ji}(t_2),$$

where the $\eta_i$ are determined from the asymptotic behavior of single pion propagators as discussed earlier.

At $\beta = 5.7$, we found that $k_0 = 0.1650$ nearly gives $E_{\pi\pi}$ equal to $E_\pi$, and $k = 0.1675$ nearly gives $E_{\pi\pi}$ equal to $E_\pi$. For $k = 0.1650$ and $t$ from 0 to 5, we determined $\eta_{ij}(t)$ with Eqs. (10) and (11) applied to $C_i(t_1, t_2)$ from an ensemble of 100 independent gauge configurations on a $16^3 \times 40$ lattice. For $k = 0.1675$ and $t$ from 0 to 5, we determined $\eta_{ij}(t)$ from $C_i(t_1, t_2)$ using 870 independent gauge configurations. The $\eta_{ij}(t)$ turn out to be nearly independent of $t$. In each case $\eta_{90}$ is statistically consistent with 1.0, $\eta_{11}$ is between 1.0 and 1.1 and $\eta_{12}$ and $-\eta_{21}$ are positive and less than 0.1.

We then evaluated $\lambda_1$ for $k = 0.1650$ and $\lambda_2$ for $k = 0.1675$ using Eqs. (10) and (11) applied to $S_1(t_g, t_\pi)$ and $S_2(t_g, t_\pi)$ found with an ensemble of 7200 independent gauge configurations on a lattice $16^3 \times 24$. We obtained statistically significant results for $t_g - t_\pi$ of 0, 1 and 2 and $t_\pi$ of 1, 2, 3, 4, and 5. The $\lambda_i$ are nearly constant, within statistical errors, across these 15 points. Figure 1 shows predicted coupling constants fitted to the data with $t_g - t_\pi$ of 1 in comparison to observed decay couplings for decays of $f_J(1710)$ to pairs of $\eta'$s, $K'$s and $\pi'$s. The horizontal axis gives the squared mass of the decay products. Masses and decay constants are shown in units of the $\rho$ mass. If couplings to $\eta'$s, $K'$s and $\pi'$s are determined from our predicted couplings by linear interpolation in the squared mass of the decay products, we predict a total width for glueball decay to pseudoscalar pairs of $52^{+45}_{-14}$ MeV, in comparison to 84 ± 23 MeV for $f_J(1710)$.

We would like to thank Steve Sharpe for finding an error in an earlier version of this work.

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