Fatigue strength of S355JC steel under harmonic and random bending-torsion loading by a tri-axis shaker: Preliminary experimental results

Davide Zanellati\textsuperscript{1}, Denis Benasciutti\textsuperscript{1}, Roberto Tovo\textsuperscript{1}

\textsuperscript{1}Department of Engineering, University of Ferrara, 44122 Ferrara, Italy

Abstract. A new testing system for applying coupled/uncoupled bending-torsion loading in vibratory tests by a tri-axis shaker has been recently developed at the Department of Engineering of University of Ferrara. The system is composed of a cylindrical specimen with eccentric tip masses, excited by horizontal and/or vertical base accelerations. A special design of the gripping system for specimen constraint allows torsional and bending deformations to be produced and controlled independently, when vertical and horizontal base accelerations are applied simultaneously by the shaker. The values of accelerations and strains in the tested specimens can be monitored continuously. This paper presents a first set of experimental results under harmonic bending and torsion, and under narrow-band combined random loading. Estimations from two frequency-domain approaches are also discussed.

1 Introduction

In structural durability of mechanical structures, one of the most challenging goals is durability assessment under random multiaxial loading. Within this complex research field, several numerical and theoretical contributions are present in the literature and, specifically, the research group at the University of Ferrara carried out several activities \cite{1-5} particularly in the frequency domain approach to fatigue damage assessment.

Normally, it is possible to carry out experiments to test the fatigue strength under multiaxial random loading by means of servo-hydraulic machines or shaker tables. The first solution is more frequently used for constant amplitude loading at relative low frequency; however, in multiaxial testing devices, tensile and torsional loading can be applied independently.

On the other hand, in the literature, several testing methodologies were proposed for multiaxial random testing through shakers tables \cite{6-10}. Unfortunately, some of the suggested testing layouts simply use a uniaxial shaker for the random excitation of specimens. By so doing, combined loading – usually bending and torsion – cannot be controlled independently, so that at critical locations different stress components result to be generally linearly correlated, leading to a loading condition that is usually called
“proportional”. Hence, in such testing systems a specific simple uniaxial case from the general multiaxial condition cannot be obtained.

Over the last two years, a research activity [11,12] has been pursued with the aim to design an innovative testing system to carry out bending-torsion random loading tests by a tri-axis shaker. In this testing device, three independently controlled shakers act on the same table by offering the possibility of controlling the three Cartesian accelerations independently.

At this stage of the research, a preliminary testing layout has been developed by considering only two out of three available degrees of freedom, in order to generate independent bending and torsion loading on tested specimens.

Within the framework of this research activity, the aims of this contribution are the following:
- to provide an update on the current state of development of the testing device;
- to discuss the capability, the control problems and the obtained accuracy;
- to present preliminary results under tensile and torsional loading for harmonic and narrow-band random loading.

The fatigue test results have been obtained on a standard structural steel S355JC which the specimen is made of. Tests first considered harmonic loading, and then random loading with different stress Power Spectral Density (PSD) functions and various correlation degrees between bending and torsion. Harmonic test results were compared with constant amplitude data obtained by a MTS servo-hydraulic testing machine. Random bending-torsion test results were compared with theoretical estimations by the ‘Projection-by-Projection’ fatigue criterion, showing a good agreement.

2 Testing layout

2.1 Overall geometry and main properties

The experimental set-up (see Fig. 1) is mainly based on a Dongling tri-axis electrodynamic shaker (3ES-10-HF-500 model) capable of exciting simultaneously 3 orthogonal directions in the space, from 5 to 2000 Hz with a maximum rated force of 10 kN and a maximum rated velocity of 1.2 m/s. The controller and acquisition unit is a LMS SCADAS Mobile, driven by LMS TestLab software.

Fig. 1. Testing layout
On the system, a cylindrical notched specimen is fixed to a T-clamp and it mounts a couple of eccentric tip masses. When vertical and/or horizontal accelerations are applied to the shaker table, the specimen undergoes bending and/or torsion loading (either harmonic or random, depending on the type of input acceleration).

A thin and flexible plate, fixed to the specimen tip, impedes any bending when the specimen is excited horizontally, but permits specimen torsional rotations. As a result, bending and torsional deformations (and thus loadings) can be controlled independently, with any intensity and correlation.

2.2 Numerical simulations

Numerical simulations and experiments have been used in [11,12] to verify that the system behaves exactly as expected and it can thus be used to perform fatigue tests under harmonic or random bending-torsion loading. The finite element model is shown in Fig. 2.

![Finite element model of the testing layout](image1)

Fig. 2. Finite element model of the testing layout

The obtained Frequency Response Function (FRF) of the specimen to 1g accelerations of the shaker table is displayed in Fig. 3. In this case, the figure depicts the response acceleration of the specimen extremity. Similar trends are obtained if the stress at notch or other stress components are considered. Obviously, at lower frequency the response is close to 1 and increases approaching the resonance frequencies, which are respectively 75 Hz and 108 Hz for the accelerations that induce bending and torsion in the specimens. Higher frequency resonances are present at specific parts of the fixture, but they are irrelevant for fatigue tests.

![Numerical and experimental FRF](image2)

Fig. 3. Numerical and experimental FRF under vertical (bending) and horizontal (torsion) excitation
2.3 Instrumentation, control and accuracy

The actual mechanical behaviour of the testing device has been experimentally verified, too. A set of sensors has been applied, see Fig. 4. Accelerometers at the shaker table and at the cantilever tip have been used to measure the actual shaker excitation and the overall response of both specimens and fixture accessories. In addition, by taking advantage of the particular geometry of the testing system, the two bending moments acting on the T-clamp stem were measured by two couples of opposite strain gages glued on the opposite faces of the T-clamp stem. Due to the system geometry, the bending moments $M_x$ and $M_y$ acting on the stem are directly related, respectively, to the bending and torsion moments at the notched specimen section.

The overall comparison with numerical result is possible by analysing the numerical FRFs with the experimental ones shown in Fig. 3. The agreement is substantial by considering that the numerical model does not consider some minor features (like bolts) present in the real device.

![Fig. 4. Used sensors along with the adopted reference frame and measured directions](image)

Within the research, one of the tackled critical aspects of the developed equipment is the variability of the response, among and within the fatigue tests.

During a set of preliminary investigations under harmonic, narrow-band and broad-band loading, all experimental signals have been recorded and compared.

Just for a first check, the Root Mean Square (RMS) values of the obtained time signals have been measured and compared.

The correlation between the RMS value at Accelerometer 1 and the measured signals at Accelerometer 2 (that is, between Bending and Torsion) is equal to 0.98 by changing specimen or by disassembling and reassembling the testing device. Hence, the repeatability and reproducibility seems to be sufficiently verified.

On the other hand, by taking a series of short signals (30 seconds in length) in long run tests, the same correlation between input and output of the mechanical system behaviour decreases to 0.95 and 0.87, respectively, for harmonic and random loading (both narrow-band and wide-band loading). A portion of this variability is due to the intrinsic scatter of sampled random signals. The remaining variability could be the actual errors or instability of the overall control system. This phenomenon is currently under investigation and, for the time being, the error is estimated to be close to $\pm 3\%$. In any case, this error is randomly distributed with mean value equal to zero and it does not introduce any bias in experimental
tests. At most it could possibly affect the scatter of experimental data in middle cycle fatigue range (close to $10^5$ cycles to failure).

3 Preliminary results

A preliminary set of tests, carried out up to specimens failure, have been completed under harmonic loading and under narrow-band random loading.

Different acceleration spectra have been applied at the shaker table and verified by the Accelerometer 1. Fatigue failures arise at the specimen notch. Fatigue damage due to crack propagation at the net section of the notch causes a substantial and fast increase of the acceleration before failure occurs. This increment can be measured by the FRF of Accelerometer 2 and it is evident in the final 5% portion of the total fatigue life. Failure criteria has been defined as complete net section failure or, preferably, by an increment of 10% of the FRF.

3.1 Harmonic tests results

The simplest tests carried out were the harmonic tests under bending or torsional loading. Harmonic (constant amplitude) sinusoidal loading was applied to specimens at 50 Hz for bending loading and at 89 Hz for torsional loading.

Fig. 5 shows the obtained number of cycles to failure as a function of the applied peak stress amplitude at the notch. As usual, the SN curve has been obtained by linear regression in double logarithmic diagram. Reference strength amplitudes have been estimated at $2 \times 10^6$ cycles. These SN curves for harmonic loading are actually comparable with conventional SN curves obtained under constant amplitude loading by a servo-hydraulic MTS machine. Therefore, in the following SN curves for harmonic loading will be used as reference for fatigue strength assessment.

3.2 Narrow-band tests results

The focus of the research is to investigate the fatigue strength under random multiaxial loading condition. The first step towards this complex target has been the application of a narrow-band loading. The PSD of the acceleration applied to the shaker table had a central frequency close to 50 Hz, with an expected number of peaks per second equal $v_p = 49.5$ and a regularity index $\alpha_2 = 0.994$. The real responses of the stress in tested specimens, estimated.
through the measurements provided by the strain gages at T-clamp, had the aforementioned scatter and turned out to be $\nu_p = 50.4\pm0.4$ and $\alpha_2 = 0.992\pm0.001$.

By increasing the PSD values keeping its shape fixed led to a decrease in the number of cycles to failure. The RMS of the applied stress at notch is used as a first index to quantify the intensity of applied stress. Until now, the following tests have been carried out:
- 8 tests under narrow-band random bending;
- 2 tests under multiaxial narrow-band loading with correlated tensile and shear stress;
- 2 tests under multiaxial narrow-band loading with un-correlated tensile and shear stress.

Specifically, in this set of data the ratio between the shear stress RMS and the tensile stress RMS is close to 0.16, whereas the correlation between shear and tensile stress in correlated and uncorrelated tests is 0.95 and 0.05, respectively. Results are collected in Fig. 6.

The figure makes clear that fatigue tests cannot be compared in terms of RMS. In other words, despite the RMS of stress (or load) is a useful parameter to compare the frequency content of signals, it has little if no significance in fatigue strength assessment, as – even considering only uniaxial loading – harmonic and narrow-band loadings lead to a different fatigue behaviour even under the same RMS content of stress.

![Experimental results under harmonic and narrow band random loading](image)

In addition to the change of the results from harmonic to random loading, the multiaxiality of the problem introduces a further variation. The observed fatigue resistance under torsional loading is obviously lower. The ratio of the reference strength is equal to 1.5 and the two SN curves has rather different slopes ($k_\sigma = 7.29$ and $k_\tau = 14.23$).

Unfortunately, in the multiaxial tests performed till now, the intensity of shear stress is so small that its influence, and maybe also the influence of the correlation between shear and normal stress, on the fatigue life is not noticeable.

### 3.3 First comparison with random multiaxial criteria

For a proper assessment of fatigue strength, both the randomness and the multiaxiality of the load need to be considered through appropriate tools and theoretical frameworks. Several possibilities are available in the literature. In this note, only a couple of possible approaches are considered.

One of the most simple and frequently used approaches to deal with multiaxial loadings is the uniaxial-equivalent stress by Pitoiset-Premont [13]. In this approach, the multiaxial stress variability is reduced to an equivalent uniaxial time-variable stress component. This transformation is related to the Von Mises equivalent stress used for static loading and,
when applied to the experimental data here presented, it produces harmonic and narrow-band uniaxial time-histories.

In order to compare harmonic constant amplitude loadings and narrow-band loadings, the simple “narrow-band approximation” can be used. For instance, it is possible to introduce a harmonic loading corresponding to a narrow-band loading. Such a harmonic loading must have the same number of peaks per seconds as the narrow-band loading, and have amplitude $S_a$ so that it yields the same fatigue damage. Such corresponding amplitude turns out to be:

$$S_a = \sqrt[k]{\frac{\sigma^2}{2} \Gamma \left(1 + \frac{k}{2}\right)} \sqrt{2 \lambda_0}$$

(1)

where $k$ is the inverse slope of the SN curve (in this case $k=7.29$), $\Gamma(-)$ is the conventional gamma function, $\lambda_0$ is the “zero moment” of the PSD which, according to fundamental properties of random signals, it is related to RMS and to signal variance as:

$$\lambda_0 = \sqrt{\sigma_{\tilde{x}}^2}$$

(2)

By using these definitions, the uniaxial amplitude corresponding to a multiaxial random loading can be computed and it is possible to represent the experimental time to failure in a SN diagram as a function of the corresponding amplitude stress. Results are given in Fig. 7.

Fig. 7. Comparison of experimental data as a function of corresponding amplitude

Obtained results are promising. A possible problem of this approach is its limited capability to consider the influence of torsional loading properly. This problem is here hidden as only few tests with significant shear test amplitude have been carried out till now.

Other methodologies can be considered. For instance, the Projection-by-Projection (PbP) method [14,15] has been specifically developed for random multiaxial loading. In this case, a corresponding constant amplitude loading cannot be defined and the comparison between the estimations and experimental data is only possible in an N-N (life-life) diagram. Fig. 8 shows the N-N comparisons of experimental versus estimated number of cycles to failure for both methodologies: the Pitoiset-Premont narrow-band approximation and the PbP approach. Both show a satisfactory agreement. The PbP has specifically been developed to properly account for differences in tensile and shear SN curves, hence it seems to yield a higher accuracy. However, this conclusion cannot be generalised, as a
larger number of experimental data is indeed necessary to get more meaningful comparisons.

Conclusions

The paper presented a testing device that has been recently developed for multiaxial random fatigue testing on a multiaxial shaker. The overall geometry, as well as a brief discussion on accuracy and repeatability of tests, was given.

A first set of experimental test results under harmonic and narrow-band loading was presented. A preliminary comparison between the experimentally obtained life and estimations from some assessment procedures available from the literature was also discussed.

The obtained results confirm that the fatigue strength under random multiaxial loading cannot be addressed by simple spectral or loading indexes, as for example the RMS of stress components. On the other hand, some specific procedure provides far better estimations. Obviously, by increasing the complexity of theoretical models, the accuracy of assessments is consequently improved, as it turns out by a first comparison of Pitoiset-Premont model with the Projection-by-Projection method.

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