Modern and future schemes of very-high cycle fatigue tests

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Abstract. The development of a new piezoelectric fatigue machine operating in pure torsion is presented in this paper. The proposal for implementation of a testing machine to research on very-high cycle fatigue at biaxial bending is studied. Results of pure torsion tests on titanium alloy VT3-1 processed by forging and extrusion are shown. Evaluations of specimens shapes made of aluminum alloy D16T and titanium alloy VT3-1 were conducted as well as the possibility to conduct VHCF bi-axial bending tests was confirmed.

1. Introduction

As Japanese researchers [1,2] showed in the mid-1980s, structural materials may be fractured even at stress levels below the classical fatigue limit while subjected to $10^8$ cycles or more. In the following years the French researcher C. Bathias and his colleagues experimentally approved the concept of very-high cycle fatigue [3]. Using of traditional fatigue testing methods such as servo hydraulic (maximum loading frequency ~ 35 Hz), electrodynamic or electromagnetic test sets (maximum frequency ~ 100 Hz) is not suitable for very-high cycle fatigue (VHCF), as the test time required to conduct research on $10^9$-$10^{10}$ cycles is too long (from 1 to 3 years). Thus, the study of fatigue failure of structural materials in the VHCF mode requires using of quicker testing methods, such as ultrasonic fatigue tests [4].

Subsurface crack initiation is one of the VHCF features. This property allows identifying the cases of fatigue failure of various structural elements subjected to high-frequency loads as failed by the

Figure 1. a) broken disc; b) fatigue crack nucleation zone; c) example of load calculation with definition of crack initiation zone.
mechanism of VHCF [5]. Disks and blades of gas turbine engines and gearboxes are good examples of such elements. The elements of the destroyed gas turbine engine compressor disc and the results of mathematical modeling of the high-frequency loading process with the durability evaluation according to the generalized criterion of multiaxial failure [6-7] are shown at figure 1.

Various loading conditions for real structural elements require the development of ultrasonic fatigue testing machines capable of operating in such modes as tension-compression, bending, fretting-corrosion and torsion [8]. Some of these testing machines are already on the market (e.g. tensile-compression testing systems, figure 2). Others are still under development (torsion, bending, biaxial loading). This article is devoted to new VHCF test systems.

2. Pure torsion

2.1. Pure torsion fatigue machine

There are 2 approaches to conduct VHCF tests – either in pulse or continuous mode. In pulse mode there are fewer concerns about temperature of a specimen due to pauses between loading cycles; but because of the pause between the pulses and the rise and fall of amplitude, the loading conditions are close to variable amplitude, which requires the use of fatigue damage accumulation. Moreover, the necessity to estimate total amount of loading cycles raises questions about whether pauses and cycles at the beginning and end of a pulse, when the amplitude changes, contribute to the final damage and fatigue life. For this reason, the fatigue machine discussed in this article operates in continuous mode.

All loading fatigue machines to study the VHCF mode are based on a single principle – the use of standing elastic waves to form deformation fields [9]. For a long period of time, this principle limited the available modes of loading by such machines to a uniaxial tension-compression regime with the coefficient of asymmetry of the cycle equal to -1.

The frequency of ultrasonic fatigue tests varies from 15 kHz to 30 kHz with the standard frequency of 20 kHz [9]. A specimen is designed to resonate at this frequency. In the case of torsion, the ultrasonic vibration is rotational, and the resonant frequency corresponds to the first mod of torsional oscillations. All mechanical parts of the ultrasonic machine for fatigue tests, including the specimen, resonate at the same loading frequency.

Load parameters are defined and continuously controlled using special software developed by the authors. Angular oscillations are generated by a piezoelectric element. It converts an ultrasonic sinusoidal electrical signal into mechanical angular oscillation at the same frequency. These oscillations form a standing elastic wave inside the mechanical components of the machine (i.e. in the horn, amplifier and specimen).

Figure 2. a) the machine overview; b) example of FEM sample form calculation for high-frequency resonance loading.
The ultrasonic torsion method is based on the theory of transverse wave propagation. In pure torsion equation (1) describes behavior of a cylindrical bar oriented along $z$ axis [9], where $\rho$ stands for density, $G$ – shear modulus, $J_p(z)$ – polar moment of inertia, $J_T(z)$ – torsion stiffness, $\varphi(z)$ – rotation angle.

$$\rho J_p(z) \frac{\partial^2 \varphi(z,t)}{\partial t^2} = G \frac{\partial}{\partial z} \left( J_T(z) \frac{\partial \varphi(z,t)}{\partial z} \right)$$ \hspace{1cm} (1)

Based on equation (1), an analytical solution can be obtained for the oscillations of all mechanical parts of an ultrasonic torsional system. The analytical solution for the movement of elastic waves in the horn can be determined by "cross-linking" the analytical solutions for the different parts shown in figure 3. Since all parts of the ultrasonic waveguide have axial symmetry in the direction $z$, their polar moments of inertia are equal to their torsional stiffness.

The general solution for the stationary wave in parts 1-2 of the ultrasonic waveguide (figure 3) can be found using (3), where $\xi(z)$ stands for amplitude value of $\varphi(z,t) = \xi(z) e^{i\omega t}$.

$$\frac{\partial^2 \varphi(z,t)}{\partial t^2} - \frac{G}{\rho} S^2(z) \frac{\partial \varphi(z,t)}{\partial z} - \frac{G}{\rho} \frac{\partial^2 \varphi(z,t)}{\partial z^2} = 0$$ \hspace{1cm} (2)

$$\xi_1(z) = C_1 \cos(kz) + C_2 \sin(kz), \quad \xi_2(z) = C_4 e^{i\beta} + C_5 e^{-i\beta} \frac{\cosh(\alpha z)}{\cosh(\beta L_2)}$$ \hspace{1cm} (3)

The constants $C_i$ ($i=1-4$) are found from the boundary conditions:

$$C_1 = A_0 \cos(kL_1), \quad C_2 = A_0 \sin(kL_1), \quad C_3 = -C_4 = \frac{A_0 \cos(kL_2) \cosh(\alpha L_2)}{2 \sinh(\beta L_2)}$$

where the resonant length $L_1 = \arctan \left[ \beta \coth(\beta L_2) - \alpha \tanh(\alpha L_2) / k \right] / k$, $L = L_1 + L_2$, $\beta = \sqrt{\alpha^2 - k^2}$, wave vector $k = \omega / b$, $b = \sqrt{G / \rho}$. The shape of the specimen is defined by the function:

Figure 3. Estimated loading scheme for torsion tests.
\( f(z) = R_2 \) while \( L_2 < |z| \leq L \)

\( f(z) = R_1 \cosh(\alpha z) \) while \( |z| \leq L_2 \)

where \( \alpha = \arccosh\left(\frac{R_2}{R_1}\right)/L_2 \), \( R_1 \) is the smallest radius of the reduced part of the specimen, \( R_2 \) is the outer radius of the specimen’s cylindrical part.

The machine was designed in two steps. Firstly, an analytical approach was used for preliminary measurement of the horn geometry. Second, finite element analysis was carried out to optimize the geometry. In the first step, the following main dimensions of the horn were calculated (figure 1).

![Figure 4. Results of torsion experiments of smooth samples; ■ – forged sample, □ – extruded sample.](image)

In the second step, the dimensions of the horn are optimized to obtain a frequency close to that required (in this case 20 kHz). In addition, since the amplitude of the torsional angle generated by the piezoelectric element does not exceed a few milliradians, it has to be amplified by changing the diameter of the horn and the resonance parts. The piezoelectric element is powered by a high-frequency electrical sinusoidal signal with amplitude ranging from 0 to 10 V. The maximum rotation angle of the piezoelectric element reaches 0.25 mrad at the amplitude of 10 V. A computer equipped with a high-speed data acquisition and control card, as well as specialized software, was used to monitor the load parameters (i.e., the amplitude of the rotation angle and frequency of the load) and calculate the number of cycles. The machine is shown in figure 2. The compressed air cooling system allows for a continuous torsional VHCF test with complete change of direction of rotation while keeping a specimen’s temperature close to room temperature throughout the experiment, except for the last few seconds, when the crack is actively growing.

The specimen is calculated in the same way as for the horn described above. The diameter of the cylindrical part with a constant cross section is equal to 10 mm to leave free space for the M5 thread. After fixing the working area, the resonance length of the specimen can be adjusted so that the specimen has the natural frequency of 20 kHz under torsion loading.

### 2.2. VHCF pure torsion experimental results

A series of tests on samples from titanium alloy VT3-1 produced by forging or extrusion was carried out on the machine for pure torsion. Fatigue tests were carried out until a specimen failed or until a fatigue life of \( 10^7 \) cycles was reached. The results are shown in figure 4. It is shown that fatigue failure can occur at more than \( 10^7 \) cycles for both materials at stress levels significantly lower than the classical fatigue limit at \( 10^6 - 10^7 \) cycles.
The crack surfaces were studied using scanning electron microscope. In forged as well as in extruded specimens crack initiation sites were both on and under the surfaces.

3. Biaxial bending

3.1. General information about the machine

Nowadays, with the development of numerical methods and 3D modeling, it has become possible to calculate and implement new loading modes. When modeling new modes, it is often necessary to optimize the forms of loading and loading elements (horns and specimen). Resonant lengths of such specimens can be estimated both analytically and numerically. The specimen is supported at the points of standing wave nodes formation, while the loading punch should be located in the central part of the specimen. In the case of a one-dimensional three-point bend, a rectangular bar acts as the specimen. To obtain a two-dimensional bend, a disk or plate is used, which is tested in the displacement nodes, figure 5. Loading is performed by small amplitude shifts using piezoelectric oscillators.

3.2. Specimen designing method

To determine the geometry of a specimen, the stress-strain state and evaluate durability in the biaxial loading one has to solve the problem of resonant bending vibrations of the circular plate with intermediate support.

According to [10], the equation of dynamic axisymmetric bending of the circular plate may be written:

$$
\Delta^2 \zeta + \frac{12\rho (1-\nu^2)}{Eh^2} \frac{\partial^2 \zeta}{\partial r^2} = \frac{q}{D}
$$

where \( \zeta(r,t) \) stands for flexural displacement of the centerline of the plate, \( \Delta = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} \) – axisymmetric Laplace operator in polar coordinates, \( \rho \) – density, \( E \) – Young’s modulus, \( \nu \) – Poisson’s ratio, \( h \) – plate thickness, \( q \) – distributed vertical force, \( D = Eh^3 \left[ 12(1-\nu^2) \right]^{-1} \) – plate stiffness.

Figure 5. a) bi-axial bending machine overview; b) schematic view.
The flexural displacement to describe harmonic oscillation is \( \zeta(r,t) = w(r)e^{i\omega t} \), where \( \omega = 2\pi f \), \( f \) – oscillation frequency, \( w(0) = W_0 \) – amplitude value. Let’s denote \( k^4 = \omega^2 \frac{12\rho(1-v^2)}{EH^2} \). To achieve high displacements we have to find resonance regime of a plate where \( q = 0 \).

Then the equation of dynamic axisymmetric bending of the circular plate may be written with bi-harmonic operator may be written

\[ \Delta^2 w - k^4 w = 0 \]

This equation may be divided into two equations with harmonic operator:

\[ \Delta w + k^2 w = 0, \quad \Delta w - k^2 w = 0 \]

Using Bessel functions their solution may be written:

\[ w(r) = C_1 J_0(kr) + C_2 N_0(kr) + C_3 I_0(kr) + C_4 K_0(kr) \]

While \( r \to 0 \) Neumann and Macdonald functions \( N_0, K_0 \to \infty \) hence \( C_2 = 0 \) and \( C_4 = 0 \):

\[ w(r) = C_1 J_0(kr) + C_3 I_0(kr) \quad (4) \]

In this solution \( J_0(kr) \) is a Bessel function of the first kind and the solution for \( \Delta w = -k^2 w \); \( I_0(kr) \) is a modified Bessel function of the first kind and the solution for \( \Delta w = k^2 w \).

The loading scheme is shown at figure 6. \( R \) is the external radius of a plate and \( R_0 \) is the radius of intermediate support of a plate, located at one of the resonance nodes of a specimen.

Boundary conditions for displacement \( w \), torque \( M_\theta \) and cutting force \( Q \), expressed by (5). Equation for torque \( M_\theta \) is (6).
\[
\begin{align*}
\frac{w}{r} &= W_0, \\
\frac{w}{r} &= 0, \\
M_r &= -D \left( \frac{\partial^2 w}{\partial r^2} + \frac{\nu \partial w}{r \partial r} \right) = -D \left( \Delta w - \frac{(1-\nu) \partial w}{r \partial r} \right) = 0, \quad \text{at} \quad r = R \\
Q_r &= -D \left( \frac{\partial^2 w}{\partial r^2} + \frac{1 \partial w}{r \partial r} \right) = -D \left( \Delta w \right) = 0, \quad \text{at} \quad r = R
\end{align*}
\]

(5)

We can substitute the solution (4) into torque expressions from (5) and (6) to obtain more explicit formula for torques:

\[
M_r = Dk^2 \left( C_1 \left[ J_0(kr) - (1-\nu)J_1(kr) / (kr) \right] - C_3 \left[ I_0(kr) - (1-\nu)I_1(kr) / (kr) \right] \right)
\]

\[
M_q = Dk^2 \left( C_1 \left[ \nu J_0(kr) + (1-\nu)J_1(kr) / (kr) \right] - C_3 \left[ \nu I_0(kr) + (1-\nu)I_1(kr) / (kr) \right] \right)
\]

\[
Q_r = -Dk^3 \left( C_1 J_1(kr) + C_3 I_1(kr) \right)
\]

Let's study the resonance regime of the plate. In this regime frequency of one of the oscillation modes is equal to loading frequency. Boundary conditions (5) transform into the system:

\[
C_1 \left[ J_0(kR) - (1-\nu)J_1(kR) / (kR) \right] - C_3 \left[ I_0(kR) - (1-\nu)I_1(kR) / (kR) \right] = 0
\]

\[
C_1 J_1(kR) + C_3 I_1(kR) = 0
\]

The determinant of this system must be zero:

\[
\begin{vmatrix}
J_0(kR) - (1-\nu)J_1(kR) / (kR) \\
I_0(kR) - (1-\nu)I_1(kR) / (kR)
\end{vmatrix} J_1(kR) = 0
\]

Here are two independent values, thereby either \( R \) can be found at given \( k \) or vice versa. From the system of equations for \( C_1 \) and \( C_3 \)

\[
C_1 + C_3 = W_0, \quad C_1 J_1(kR) + C_3 I_1(kR) = 0
\]

we can find this constants:

\[
C_1 = W_0 \left( \frac{I_1(kR)}{I_1(kR) - J_1(kR)} \right), \quad C_3 = -W_0 \left( \frac{J_1(kR)}{I_1(kR) - J_1(kR)} \right)
\]

The equation for \( R_0 \) has the form:

\[
C_1 J_0(kR_0) + C_3 I_0(kR_0) = 0
\]

Equations for \( R \) and \( R_0 \) may be solved numerically, but while value \( x = kR / 2 \) is within \( 0 < x < 2 \) range it is rather easy to use a series to find an approximate solution:

\[
\begin{align*}
J_0(2x) &= 1 - x^2 + x^4 / 4 - x^6 / 36 + x^8 / 576 + \ldots \\
I_0(2x) &= 1 + x^2 + x^4 / 4 + x^6 / 36 + x^8 / 576 + \ldots \\
J_1(2x) &= x(1-x^2 / 2 + x^4 / 12 - x^6 / 144 + x^8 / 2880 + \ldots \\
I_1(2x) &= x(1 + x^2 / 2 + x^4 / 12 + x^6 / 144 + x^8 / 2880 + \ldots \\
\end{align*}
\]

Using the given series, equation for \( R \) transforms into simple equation for \( x = kR / 2 \):
Its approximate solution has the form:

\[ x^4 = 12(1 + \nu) / (3 + \nu), \quad R = 2\sqrt{12(1 + \nu) / (3 + \nu) / k} \]

To estimate an error let’s take \( \nu = 0.2 \) and \( x^4 = 4.5 \), what leads us to an insignificant value \( (5 + \nu) x^8 / 1440 \approx 0.073 \). Also using the given series, equation for \( R_0 \) gives the result \( y = x / \sqrt{2} \) or \( R_0 = R / \sqrt{2} \), where \( y = kr_0 / 2 \).

Approximate equations for torque values and cutting force are expressed as following (\( \xi = kr / 2 \)):

\[
M_r = \frac{Dk^2W_0}{(I_1(kR) - J_1(kR))} \left( f_r(\xi)I_1(kR) + f_s(\xi)J_1(kR) \right)
\]

\[
f_r(\xi) = \frac{1 + \nu}{2} + \frac{3 + \nu}{4} \xi^2 + \frac{5 + \nu}{24} \xi^4 + \frac{7 + \nu}{288} \xi^6
\]

\[
M_g = \frac{Dk^2W_0}{(I_1(kR) - J_1(kR))} \left( g_r(\xi)I_1(kR) + g_s(\xi)J_1(kR) \right)
\]

\[
g_r(\xi) = \frac{1 + \nu}{2} + \frac{3 + \nu}{4} \xi^2 + \frac{5 + \nu}{24} \xi^4 + \frac{7 + \nu}{288} \xi^6
\]

\[
Q_r = -\frac{Dk^2W_0}{(I_1(kR) - J_1(kR))} \xi \left( s_r(\xi)I_1(kR) - s_s(\xi)J_1(kR) \right)
\]

\[
s_s(\xi) = \frac{1}{12} \xi^2 / 2 + \xi^4 / 12 + \xi^6 / 144
\]

Expressions for stress components, the distribution of the maximum tangential stresses and tangential component \( \sigma_r \) depending on \( Q_r \) are the following:

\[
\sigma_r = 12M_r \frac{z}{h^3} = \frac{12Dk^2W_0}{(I_1(kR) - J_1(kR))} \frac{z}{h^3} \left( f_r(\xi)I_1(kR) + f_s(\xi)J_1(kR) \right)
\]

\[
\sigma_s = 12M_g \frac{z}{h^3} = \frac{12Dk^2W_0}{(I_1(kR) - J_1(kR))} \frac{z}{h^3} \left( g_r(\xi)I_1(kR) + g_s(\xi)J_1(kR) \right)
\]

\[
\frac{\sigma_r - \sigma_s}{2} = -\frac{3Dk^2W_0(1 - \nu)}{(I_1(kR) - J_1(kR))} \frac{z^2}{h^3} \left[ I_1(kR) - \left( 1 + \frac{\nu^2}{3} + \frac{\nu^4}{24} \right) J_1(kR) \right]
\]

\[
\sigma_{\beta} = -\frac{6Q_r}{h^3} \left( \frac{h^2}{4} - \xi^2 \right) = \frac{6Dk^2W_0}{(I_1(kR) - J_1(kR))} \frac{z}{h^3} \left( \frac{h^2}{4} - \xi^2 \right) \left( s_r(\xi)I_1(kR) - s_s(\xi)J_1(kR) \right)
\]

Thus, derived formulas at a given eigenfrequency determine the plate radius, intermediate support radius and stress components required to evaluate fatigue strength and durability in the adopted scheme of VHCF tests.

3.3. **Theoretical durability assessments**

We choose the generalized Sines criterion [11] to evaluate durability of the plate. The criterion has the form:
\[
\Delta \tau / 2 + \alpha_s \sigma_m = S_0 + A_s N^\beta, \quad (7)
\]
\[
\Delta \tau = \sqrt{(\Delta \sigma_{11} - \Delta \sigma_{22})^2 + (\Delta \sigma_{11} - \Delta \sigma_{33})^2 + (\Delta \sigma_{22} - \Delta \sigma_{33})^2 + 6\Delta \sigma_{12}^2 + 6\Delta \sigma_{13}^2 + 6\Delta \sigma_{23}^2} / 3
\]
\[
\sigma_m = (\sigma_{11} + \sigma_{22} + \sigma_{33} \text{mean})
\]

Its parameters were calculated in [7]:
\[
S_0 = \sqrt{2} \sigma_u / 3, \quad A_u = 10^{-8}\beta \sqrt{2}(\sigma_u - \sigma_u) / 3, \quad \alpha_u = \sqrt{2}(2k_{-1} - 1) / 3, \quad k_{-1} = \sigma_u / 2\sigma_{u0}
\]
where \( \sigma_u \) – classical fatigue limit for cycle asymmetry -1, \( \sigma_u \) – VHCF fatigue limit for cycle asymmetry -1, \( \sigma_{u0} \) – VHCF fatigue limit for zero cycle asymmetry (figure 7).

For the fully reverse cyclic loading range of any stress components in the cycle is equal to the doubled amplitude, therefore amplitude of the shear stress is equal to
\[
\Delta \tau / 2 = (\sigma_{r} - \sigma_{b})^2 + \sigma_{r}^2 + \sigma_{b}^2 + 6\sigma_{\tau c}^2 / 3, \quad \sigma_m = 0
\]

The stress components are defined by formulas above. Durability \( N \) of a specimen is estimated by criterion (7). The generalization method of the multi-axial fatigue criterion for the VHCF regime and determination of its parameters is based on the modern concepts of the bimodal fatigue curve, figure 7,

![Figure 7. Bimodal fatigue curve with shaded bifurcation zone when the fatigue failure mechanism is changing.](image)

which is discussed in detail in [12].

We studied two materials: aluminum alloy D16T and titanium alloy VT3-1. Both of them are commonly used in aeronautical applications and prone to work in VHCF regime.

Physical properties of aluminum alloy D16T are: density \( \rho = 2700 \text{ kg/m}^3 \), Young's modulus \( E = 70 \cdot 10^6 \text{ Pa} \), Poisson's ratio \( \nu = 0.3 \), classic fatigue limit \( \sigma_u = 120 \cdot 10^6 \text{ Pa} \), VHCF limit \( \bar{\sigma}_u = 85 \cdot 10^6 \text{ Pa} \), exponent coefficient \( \beta = -0.3 \). Additional parameters: loading frequency \( f = 20 \cdot 10^3 \text{ Hz} \), the amplitude of displacement \( W_0 = 38.5 \cdot 10^{-6} \text{ m} \), thickness of the plate \( h = 5 \cdot 10^{-3} \text{ m} \).
For a given frequency geometrical properties are $R = 23.5 \times 10^{-3}$ m and $R_0 = 16 \times 10^{-3}$ m; durability of the plate is $N = 8 \times 10^6$ cycles, which at given loading frequency is about 110 hours of testing. Stress distributions are shown on figure 8.

Physical properties of titanium alloy VT3-1 are: density $\rho = 4500$ kg/m$^3$, Young's modulus $E = 115 \times 10^9$ Pa, Poisson's ratio $\nu = 0.32$, classic fatigue limit $\sigma_u = 360 \times 10^6$ Pa, VHCF limit $\sigma_u = 250 \times 10^6$ Pa, exponent coefficient $\beta = -0.3$. Additional parameters: loading frequency $f = 20 \times 10^3$ Hz, the amplitude of displacement $W_0 = 75 \times 10^{-6}$ m, thickness of the plate $h = 5 \times 10^{-3}$ m.

For a given frequency geometrical properties are $R = 23.5 \times 10^{-3}$ m and $R_0 = 16 \times 10^{-3}$ m; durability of the plate is $N = 4.1 \times 10^9$ cycles, which at given loading frequency is about 57 hours of testing. Stress distributions are shown on figure 9.

The results of calculations of resonant modes for aluminum and titanium samples have shown that the considered scheme is within the limits of possibilities of experimental machine on carrying out of ultrasonic tests in a mode of VHCF both on parameters of geometry of samples at frequency of tests $\sim 20$ kHz and on estimated time of tests at realistic values of amplitude.

Figure 8. a) $\sigma_r$ and b) $\Delta \tau$ distribution within the aluminum plate.

Figure 9. a) $\sigma_r$ and b) $\Delta \tau$ distribution within the titanium plate.

Figure 10. Stress distribution in the sample for the nominal amplitude of the harmonic wave of 1 µm.
It is to mention that in the current state our testing machine is able to produce as much as near \( W_0 = 5 \times 10^{-6} \) m of displacement. At this level the titanium specimen is going to withstand loading much more time and by now we have to figure out a solution for it.

Also at a known frequency, the specimen geometry was calculated using finite element method (FEM), for which the resonance frequency coincides with the carrier frequency. At a given possible punch displacement – up to 60 µm – the stresses in the specimen and its fatigue life were calculated, figure 10. Numerical and analytical calculations of specimen geometry and stress distribution for resonance mode gave results close to those obtained using FEM.

4. Conclusions
A new ultrasonic fatigue machine has been developed for continuous fatigue testing for pure torsion in the VHCF mode (cycle asymmetry coefficient equal to -1). The first results of tests in VHCF for both forged and extruded aircraft titanium alloys VT3-1 were obtained.

The fatigue limit in full reverse mode in case of torsion is higher for the extruded titanium alloy than for the forged one. For two titanium alloy VT3-1 manufacturing techniques (forging and extrusion), crack initiation did not always occur on the surface of the specimens.

The perspective scheme of VHCF bi-axial bending tests with cycle asymmetry coefficient equal to -1 was studied. The problem of resonant bending vibrations of a circular plate with intermediate support was solved.

Calculations of resonance parameters of the specimens was carried out, the stress fields and estimation of fatigue durability were obtained according to the generalized multiaxial criterion applied to the case of VHCF bi-axial bending tests.

The obtained results of calculations of resonance modes for aluminum and titanium specimens have shown that the considered scheme lies within the limits of possibilities of experimental machine on carrying out of ultrasonic tests in VHCF mode.

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