Convergence Analysis of Wireless Remote Iterative Learning Control Systems with Dropout Compensation

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1. Introduction

One feature of the wireless remote control system is that signals are transmitted in wireless network from the sensor to the controller and from the controller to the actuator, and then the controller is separated from the system platform [1–3]. Consequently, the system has such advantages as easy installation, reduced wiring, and maintenance. In wireless remote control systems, designing a remote controller to track the desired trajectory is not an easy task. Fortunately, when systems execute the same task periodically in a fixed time interval, iterative learning control (ILC) is an effective method [4]. This method uses information obtained from the previous operation to improve the control signal for the next trial. If some conditions are met, the output error could monotonically converge to zero from iteration to iteration.

However, the introduction of wireless network makes the tracking of desired trajectory more complicated than traditional point-to-point connection method due to the unreliability of wireless network. The main issue is that some signals would be lost during transmitting, which can be divided into two different types: control signal dropouts and measurement signal dropouts. The first one occurs when the signal is transmitted from the controller to the actuator after updating, which would disturb the learning process indirectly; the second one arises when the signal is transmitted from the sensor to the controller, which would be involved in the learning process directly. The convergence of output error cannot be guaranteed without considering data dropouts.

The research on networked control systems has attracted much attention. In [5–9], authors are concerned with the state estimation problem for networked control systems with various uncertainties of the communication channels. In [10, 11], the author discussed decentralized stabilization of networked control systems with nonlinear perturbations. In [12], the nonfragile state feedback $H_\infty$ control problem for networked control systems with quantized signals is studied. However, most researches related to the case with data dropouts are conducted only in sensor-to-controller side and did not consider the control scheme adopted by controller. In the very recent years, some researches on ILC systems considering the...
problem of data dropout appeared (see [13–21] and references therein). Liu et al. investigated the implementation of ILC in a remote control systems environment and specifically focused on compensation when both random data dropouts and delays occur at the communication network between the plant output and the controller [13]. In [14], the author proposed an averaging ILC algorithm to overcome the random transport delay and data dropout, which guarantees the convergence property of the ensemble average of the output tracking errors along the iteration axis. In [15], Bu and Hou discussed the stability of ILC with data dropouts via asynchronous system and offered the stability condition in the form of linear matrix inequalities. Bu et al. also studied the stability of first and high order ILC with data dropout when the plant is subject to measurement signal dropouts [16]. Ahn et al. presented a mathematical formulation of the problem of robust ILC design when the system is subject to measurement signal dropouts and used the Kalman filtering approach to design a learning gain such that the system eventually converges to a desired trajectory if there are not complete data dropouts [17], but the results were restricted to the case when the network from sensor to controller has dependency. Aiming at this problem, the author considered discrete-time intermittent iterative learning controller with independent data dropouts in his further study [18]. However, all the previously mentioned researches only considered the dropout of measurement signals. In [19], the author considered the problem of ILC for a class of nonlinear systems with control signal dropouts and measurement signal dropouts, but the convergence analysis needs controller and actuator to know the received signal whether lost or not. In [20], a sampled-data ILC approach was proposed for a class of nonlinear networked control systems to deal with the existence of time delays and packet losses in control signal and measurement signal transmissions. In [21], Pan et al. assumed that the controller and the actuator are all event driven and analyzed the effect of packet loss from controller to actuator side and from sensor to controller side on the convergence property of output error. But the convergence is asymptotic because the control error at the actuator cannot converge to zero. To the best of our knowledge, no one has studied the compensation for the data dropouts in both control signals and measurement signals to guarantee the convergence property of output error. This observation is the motivation of the present paper.

As depicted in the second paragraph, there are two different kinds of data dropouts in wireless remote ILC systems. For the sake of convenience, we only consider the control signal dropouts in this paper, but the results can be extended to the measurement signal dropouts. The data dropout would be described as a binary sequence which obeys a Bernoulli distribution taking the value of one and zero with certain probability. In order to eliminate the effect of data dropouts on the convergence property of output error, we consider using the signal at the same time with the lost one but in the last iteration to compensate the data dropout at the actuator. The convergence performance of the output error with dropout compensation would be analyzed by studying eigenvalues and other elements in the lower triangular of the system transition matrix.

2. Problem Formulations

The discrete-time linear system is considered by the following equation:

\[ x_k(t+1) = Ax_k(t) + Bu_k(t), \]
\[ y_k(t) = Cx_k(t), \]

where \( x_k(t) \), \( u_k(t) \), and \( y_k(t) \) are state, control, and output variables, respectively, and \( A \), \( B \), and \( C \) are the system matrices. The subscript \( k \) = 0, 1, 2, ..., is used to denote iteration and \( t \in [0, T] \) is used to denote time. The system operates repeatedly in the iteration domain with the desired trajectory \( y_d(t) \), which can be of the form

\[ x_d(t+1) = Ax_d(t) + Bu_d(t), \]
\[ y_d(t) = Cx_d(t), \]

where \( u_d(t) \) and \( x_d(t) \) are desired control and state. In order to track \( y_d(t) \) precisely, adopting ILC method at the controller is a useful method and the first order ILC can be expressed as

\[ u_{k+1}(t) = u_k(t) + \Gamma(t) e_k(t+1), \]

where \( \Gamma(t) \) is learning gain, and output error \( e_k(t) = y_d(t) - y_k(t), \) \( t \in [0, T-1] \). The setup of wireless remote ILC systems is illustrated as in Figure 1 [22, 23]. During the measurement signal \( e_k(t) \) transmitting from the sensor to the controller and the control signal \( u_k(t) \) transmitting from the controller to the actuator, data dropouts occur due to the unreliability of wireless network. Taking the effect of data dropouts into account, the system controlled remotely by iterative learning controller can be represented as

\[ x_k(t+1) = Ax_k(t) + B\tilde{u}_k(t), \]
\[ y_k(t) = Cx_k(t), \]
\[ u_{k+1}(t) = u_k(t) + \Gamma(t) \tilde{e}_k(t+1), \]

where control signal \( \tilde{u}_k(t) \) received at the actuator and measurement signal \( \tilde{e}_k(t+1) \) received at the controller can be expressed as

\[ \tilde{u}_k(t) = \xi_k(t) u_k(t), \]
\[ \tilde{e}_k(t+1) = \eta_k(t) e_k(t+1), \]

where \( u_k(t) \) and \( e_k(t+1) \) are signals at the controller and the sensor, respectively. The stochastic parameters \( \xi_k(t) \) and \( \eta_k(t) \) are two scalar Bernoulli distributed random variables taking value 0 or 1 (i.e., \( \xi_k(t), \eta_k(t) \in \{0,1\}, \forall k, t \)). \( \xi_k(t) \) is uncorrelated with \( \eta_k(t) \). In this model, if the variable takes value 0, then the data is lost correspondingly; otherwise there is no data dropout.
Random data dropouts could disturb the learning process of controller and have an effect on the convergence performance of output error. In order to remove the effect of data dropouts, a compensation method would be proposed in Section 3.

3. The Data Dropout Compensation Scheme

In this section, we will derive the relation between output error and control error at the actuator and the relation of control errors at the controller in different iterations first, and then we will propose a compensation scheme based on the relations and attribute of control error. In order to simplify the analysis, the following assumptions are made

Assumption 1. Consider \(\|\Gamma(t)\| \leq \beta_\Gamma, \|A\| \leq \beta_A, \|B\| \leq \beta_B, \|C\| \leq \beta_C\).

Assumption 2. Consider \(x_k(0) = x_k(0)\), for all \(k\).

The output error \(e_k(t + 1)\) can be represented as

\[
e_k(t + 1) = y_d(t + 1) - y_k(t + 1) = C\delta x_k(t + 1).
\]

From (2) and (4), it is easy to find the nonrecursive solution to \(\delta x_k(t + 1)\) as

\[
\delta x_k(t + 1) = A\delta x_k(t) + B\delta u_k(t) = A^{t+1}\delta x_k(0) + \sum_{i=0}^{t} A^{t-i}B\delta u_k(i), \tag{8}
\]

where \(\delta u_k(i)\) denotes the control error at the actuator. If Assumption 2 is satisfied, substitute (8) into (7) to get

\[
e_k(t + 1) = \sum_{i=0}^{t} CA^{t-i}B\delta u_k(i). \tag{9}
\]

Take norms on both sides of (9) to yield

\[
\|e_k(t + 1)\| = \sum_{i=0}^{t} \beta_C\beta_B^{t-i}\|\delta u_k(i)\|. \tag{10}
\]

From (10), the relation between output error \(e_k(i), i \in [1, T]\), and control error \(\delta u_k(i), i \in [0, T - 1]\), at the actuator can be expressed in the matrix form

\[
\begin{bmatrix}
\|e_k(1)\| \\
\|e_k(2)\| \\
\vdots \\
\|e_k(T)\|
\end{bmatrix} =
\begin{bmatrix}
\rho'(0) & \cdots & 0 \\
\vdots & \ddots & \vdots \\
\rho'(T - 1) & \cdots & \rho'(0)
\end{bmatrix}
\times
\begin{bmatrix}
\|\delta u_k(0)\| \\
\|\delta u_k(1)\| \\
\vdots \\
\|\delta u_k(T - 1)\|
\end{bmatrix}, \tag{11}
\]

where \(\rho'(i) = \beta_C\beta_B^{t-i}\beta_B, i \in [0, T - 1]\).

On the other hand, the relation of control errors at the controller in different iterations can be get from (5) as follows:

\[
\delta u_{k+1}(t) = u_k(t) - u_{k+1}(t) = u_k(t) - u_k(t) - \Gamma(t)\delta\hat{x}_k(t + 1)
\]

\[
= \delta u_k(t) - \Gamma(t)\delta\hat{x}_k(t + 1). \tag{12}
\]

If there are no signals happen data dropouts, \(\delta\hat{x}_k(t + 1) = e_k(t + 1)\) and \(\tilde{u}_k(t) = u_k(t)\). Substituting (9) into (12), we have

\[
\delta u_{k+1}(t) = \delta u_k(t) - \Gamma(t)\sum_{i=0}^{t} CA^{t-i}B\delta u_k(i). \tag{13}
\]

Taking norms of both sides of (13), the relation can be rewritten as

\[
\|\delta u_{k+1}(t)\| \leq H_{k+1,k}(t)H_{k,k-1}(t)\psi_{k-1}(t), \tag{14}
\]

where the vector of control error \(\psi_{k-1}, H_{k+1,k}(t), \) and transition matrix \(H_{k,k-1}(t)\) can be represented as (15), (16), and (17), respectively

\[
\psi_{k-1}(t) = \begin{bmatrix} \|\delta u_{k-1}(0)\|, \|\delta u_{k-1}(1)\|, \ldots, \|\delta u_{k-1}(t)\| \end{bmatrix}^T, \tag{15}
\]

\[
H_{k+1,k}(t) = \begin{bmatrix} \rho_1(0) & \cdots & \rho_1(1) & \rho_B(0) \\
\vdots & \ddots & \vdots & \vdots \\
\rho_1(t) & \cdots & \rho_1(1) & \rho_0(t) \end{bmatrix}, \tag{16}
\]

\[
H_{k,k-1}(t) = \begin{bmatrix} \rho_1(0) & 0 & \cdots & 0 \\
\vdots & \ddots & \ddots & \vdots \\
\rho_1(t) & \cdots & \rho_1(1) & \rho_0(t) \end{bmatrix}, \tag{17}
\]

where \(\rho_1(i) = \beta_T\beta_C\beta_B^{t-i}\beta_B\) and \(\rho_0(i) = \|I - \Gamma(i)CB\|, i \in [0, t]\).
If $\tilde{e}_k(t + 1) = e_k(t + 1)$ and $\tilde{u}_k(t) = u_k(t)$ are satisfied, the convergence of $\|e_k(t + 1)\|$ is determined by $\|\tilde{u}_k(i)\|$, $i \in [0, t]$, and the convergence of $\|\tilde{u}_k(t)\|$ is determined by the eigenvalues of transition matrix. From (14), we have $\|\tilde{u}_k(i)\| \leq H_{k+1}\|\psi(t)\|$. Since $H_{k+1}(t)$ is a lower triangular matrix, all of its eigenvalues are the diagonal elements $\rho_0(i)$. Selecting the learning gain $\Gamma(t)$ such that all eigenvalues of $H_{k+1}(t)$ are within unit circle; that is, $\rho_0(i) = \|\mathbf{I} - \Gamma(i)\mathbf{CB}\| < 1$, and $\lim_{k \to \infty}\|\tilde{u}_k(i)\|=0$, $t \in [0, T - 1]$, and then $\lim_{k \to \infty}\|e_k(t)\| = 0$, for all $t \in [1, T]$, is guaranteed. However, with the introduction of random data dropouts in control signals, $\tilde{u}_k(t) \neq u_k(t)$, the element values of $H_{k+1}(t)$ and $H_{k+1}(t)$ would be changed, so the convergence of $\|\tilde{u}_k(i)\|$ cannot be guaranteed and then they have an effect on the convergence performance of output error $\|e_k(t + 1)\|$.

In order to remove the effect, we propose to use the control signal at the same time with the lost one but in the last iteration to compensate the data dropout at the actuator based on the attribute that control signal converges in the iteration domain, which can be represented by

$$
\tilde{u}_k(t) = \xi_k(t) u_k(t) + (1 - \xi_k(t)) u_{k-1}(t).
$$

The convergence of output error with the proposed compensation scheme would be analyzed in the next section.

4. Convergence Analysis of Output Error with Data Dropout Compensation

In the following part, we start by assuming that there is one control signal $u_k(t)$ loss during the $k$th iteration and that $u_{k-1}(t)$ is applied to compensate the lost $u_k(t)$ at the actuator, and then we extend it to the condition with multiple control signals loss. The convergence property of output error with data dropout compensation can be concluded in the following theorem.

**Theorem 1.** With the compensation method presented in (18), if the condition $0 < \rho(\Gamma(t)\mathbf{CB}) < 1$ is satisfied, then

$$
\lim_{k \to \infty}\|e_k(t)\| = 0, \quad \forall t \in [1, T].
$$

**Proof.** If the control signal $u_k(t)$ is lost, $u_{k-1}(t)$ would be applied at the actuator to compensate the dropout of $u_k(t)$. In this condition, output errors $e_k(i)$, $(0 \leq i \leq t)$ are not affected but $e(i)$, $(t + 1 \leq i \leq T)$ are all affected according to (11). Consequently, the proof can be divided into the following two parts due to changing $e_k(t + 1), e_k(t + 2)$ and afterwards.

1. (1) Convergence Analysis of $\|e_k(t + 1)\|$ with Dropout Compensation.

According to $e_k(t) = y_d(t) - y_k(t)$, the output error $e_k(t + 1)$ can be represented as

$$
e_k(t + 1) = y_d(t + 1) - y_k(t + 1) = C\delta x_k(t + 1).
$$

Using (2), (4), and Assumption 2, the relation between state error $\delta x_k(t)$ and control error $\delta u_k(t)$ can be expressed as

$$
\begin{align*}
\delta x_k(t + 1) &= x_d(t + 1) - x_k(t + 1) \\
&= A\delta x_k(t) + B(\xi_k(t) \delta u_k(t) + (1 - \xi_k(t)) \delta u_{k-1}(t)) \\
&= \Gamma(t) \sum_{i=0}^{t-1} A^{i+1} B (\xi_k(i) \delta u_k(i) + (1 - \xi_k(i)) \delta u_{k-1}(i)).
\end{align*}
$$

(21)

From (5), (20), (21), and $\tilde{e}_k(t + 1) = e_k(t + 1)$, the relation of control errors in different iterations can be represented as

$$
\begin{align*}
\delta u_{k+1}(t) &= u_d(t) - u_{k+1}(t) \\
&= u_d(t) - u_k(t) - \Gamma(t) \tilde{e}_k(t + 1) \\
&= \delta u_k(t) - \Gamma(t) C\delta x_k(t + 1) \\
&= \delta u_k(t) - \Gamma(t) \\
&\quad \times \sum_{i=0}^{t} C A^{i+1} B (\xi_k(i) \delta u_k(i) + (1 - \xi_k(i)) \delta u_{k-1}(i)).
\end{align*}
$$

(22)

If the control signal $u_k(t)$ is lost, the random parameter $\xi_k(t) = 0$ correspondingly, then (22) could be changed to

$$
\begin{align*}
\delta u_{k+1}(t) &= \delta u_k(t) - \Gamma(t) C\delta x_k(t + 1) \\
&\quad - \Gamma(t) \sum_{i=0}^{t-1} C A^{i+1} B \delta u_k(i).
\end{align*}
$$

(23)

We can show $\delta u_{k+1}(t)$ using control error in $(k - 1)$st iteration further, which is given by

$$
\begin{align*}
\delta u_{k+1}(t) &= (I - 2\delta(t) \mathbf{CB}) \delta u_{k-1}(t) \\
&\quad - \Gamma(t) \sum_{i=0}^{t-1} C A^{i+1} B \delta u_k(i) \\
&\quad - \Gamma(t) \sum_{i=0}^{t-1} C A^{i+1} B \delta u_{k-1}(i).
\end{align*}
$$

(24)

Furthermore, (24) could be rewritten in the form of transition matrix.
Taking norms of both sides of (25), we have

$$
\|\delta u_{k+1}(t)\| \leq H'_{k+1,k}(t) H'_{k,k-1}(t) \psi_{k-1}(t),
$$

where $H'_{k+1,k}(t)$ is given by

$$
H'_{k+1,k}(t) = \begin{bmatrix}
\rho_0(t) & 0 & \cdots & \cdots & 0 \\
\rho_1(t) & \ddots & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
\rho_1(t) & \cdots & \rho_1(t) & \rho_0(t) & 0
\end{bmatrix},
$$

and $H'_{k,k-1}(t)$ is given by

$$
H'_{k,k-1}(t) = \begin{bmatrix}
\rho_0(0) & 0 & \cdots & \cdots & 0 \\
\rho_1(1) & \ddots & \ddots & \ddots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \vdots \\
\rho_1(t-1) & \cdots & \rho_1(t-1) & \rho_0(t-1) & 0 \\
\rho_1(t) & \cdots & \rho_1(t) & \rho_0(t) & \rho_2
\end{bmatrix}
$$

(26)

Convergence Analysis of $\|e_k(t+1+i)\|$, $1 \leq i \leq T - t - 1$ with Dropout Compensation.

In this part, we continue to analyze the convergence of $\|e_k(t+1+i)\|$, $1 \leq i \leq T - t - 1$. Due to the similarity in the analysis process, we would prove the convergence of $\|e_k(t+1+i)\|$ as the previous part and prove the convergence of $\|e_k(t+1+i)\|$, $2 \leq i \leq T - t - 1$ by analogy.

Similarly, the output error $e_k(t+2)$ can be represented as

$$
e_k(t+2) = y^d(t+2) - y_k(t+2)
$$

$$= C\delta x_k(t+2)
$$

$$= C\sum_{i=0}^{t+1} A^{t+1-i} B (\xi_k(i) \delta u_k(i) + (1 - \xi_k(i)) \delta u_{k-1}(i)).
$$

(27)

According to (5), (27), and $\xi_k(t+2) = e_k(t+2)$, $\delta u_{k+1}(t+1)$ can be expressed as

$$
\delta u_{k+1}(t+1) = u_{d}(t+1) - u_{k+1}(t+1)
$$

$$= u_d(t+1) - u_k(t+1) - \Gamma(t+1) \bar{e}_k(t+2)
$$

$$= \delta u_k(t) - \Gamma(t+1) C\delta x_k(t+2)
$$

$$= \delta u_k(t) - \Gamma(t+1) - \Gamma(t+1) \sum_{i=0}^{t+1} C A^{t+1-i} B (\xi_k(i) \delta u_k(i) + (1 - \xi_k(i)) \delta u_{k-1}(i)).
$$

(28)

If the control signal $u_k(t)$ is lost, $\xi_k(t) = 0$ correspondingly,
then (28) can be rewritten as
\[
\delta u_{k+1}(t+1) = (I - \Gamma(t+1)CB) \delta u_k(t+1) - \Gamma(t)CA\delta u_{k-1}(t) - \Gamma(t)\sum_{i=0}^{t-1}CA^{t-i-1}B\delta u_k(i).
\] (29)

Expressing \(\delta u_{k+1}(t)\) by means of control error in \((k-1)\)st iteration, we have
\[
\delta u_{k+1}(t+1) = \begin{bmatrix}
\Gamma(t+1)CA^{t+1}B & \cdots & \cdots & \cdots & \cdots & 0 \\
\vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\
\Gamma(t+1)CA^2B & \cdots & \cdots & \cdots & \cdots & 0 \\
\Gamma(t+1)CAB & \cdots & \cdots & \cdots & \cdots & 0 \\
I - \Gamma(t+1)CB
\end{bmatrix}
\times
\begin{bmatrix}
I - \Gamma(0)CB & \cdots & \cdots & \cdots & \cdots & 0 \\
\Gamma(1)CAB & \cdots & \cdots & \cdots & \cdots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\
\Gamma(t-1)CA^{t-1}B & \cdots & \cdots & \cdots & \cdots & 0 \\
\Gamma(t+1)CA^{t+1}B & \cdots & \cdots & \cdots & \cdots & \Gamma(t+1)CAB \\
\end{bmatrix}
\times
\begin{bmatrix}
\delta u_{k-1}(0) \\
\delta u_{k-1}(1) \\
\vdots \\
\delta u_{k-1}(t-2) \\
\delta u_{k-1}(t-1) \\
\delta u_{k-1}(t+1)
\end{bmatrix}.
\] (30)

Furthermore, (30) could be rewritten in the form of transition matrix
\[
H_{k+1,k}'(t+1) = \begin{bmatrix}
\rho_0(0) & 0 & \cdots & \cdots & \cdots & 0 \\
\rho_1(1) & \cdots & \cdots & \cdots & \cdots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\
\rho_1(t-1) & \cdots & \rho_1(1) & \rho_0(t-1) & \cdots & \cdots \\
0 & \cdots & 0 & 1 & 0 \\
\rho_1(t+1) & \cdots & \cdots & \cdots & \cdots & \rho_1(1) \rho_0(t+1)
\end{bmatrix}.
\] (31)

Taking norms of both sides of (31), we have
\[
\|\delta u_{k+1}(t+1)\| \\
\leq H_{k+1,k}'(t+1)H_{k,k-1}'(t+1)\psi_{k-1}(t+1),
\]
\[
H_{k+1,k}'(t+1) \\
= \begin{bmatrix}
\rho_0(0) & 0 & \cdots & \cdots & \cdots & 0 \\
\rho_1(1) & \cdots & \cdots & \cdots & \cdots & \vdots \\
\vdots & \ddots & \ddots & \ddots & \ddots & \vdots \\
\rho_1(t-1) & \cdots & \rho_1(1) & \rho_0(t-1) & \cdots & \cdots \\
0 & \cdots & 0 & 1 & 0 \\
\rho_1(t+1) & \cdots & \cdots & \cdots & \cdots & \rho_1(1) \rho_0(t+1)
\end{bmatrix}.
\] (32)

The analysis shows that \(H_{k+1,k}'(t+1)\) is equal to \(H_{k,k-1}'(t+1)\) in (16). \(H_{k,k-1}'(t+1)\) in (17) becomes \(H_{k,k-1}'(t+1)\) in which the eigenvalue in \((t+1)\)st row is changed from \(\rho_0(t)\) to 1, and elements in the left of the eigenvalue are all changed to 0. Since \(H_{k,k-1}'(t+1)\) is a lower triangular matrix, and all of its eigenvalues are the diagonal elements with one element being “1” at \((t+1, t+1)\) and other elements \(\rho_0(i) < 1, i \in [0, t+1)\cup(t+1, t+2]\). Similar to the proof in the previous part, we can have
\[
\lambda_m((\prod_{i=0}^{k-2}H_{i+1,i}(t+1))H_{k,k-1}'(t+1)(\prod_{i=k+1}^{t+1}H_{i+1,i}(t+1))) \to 0,
\]
\[
m = 0, 1, \ldots, t+2,
\]
and \(\lim_{k \to \infty}\|\delta u_{k+1}(t+1)\| = 0\) can be guaranteed.

With regard to \(\|\delta u_{k+1}(t+i)\|, 2 \leq i \leq T-t-1\), according to the convergence analysis of \(\|\delta u_{k+1}(t+i)\|\), we may know that the eigenvalue in \((t+i)\)th row of \(H_{k,k-1}'(t+i)\) would be changed to 1, and elements in the left of the eigenvalue would be all changed to 0. For this condition, we can also have
\[
\lambda_m((\prod_{i=0}^{k-2}H_{i+1,i}(j))H_{k,k-1}'(j)(\prod_{i=k+1}^{t+1}H_{i+1,i}(j))) \to 0,
\]
\[
m = 0, j \in [t+2, T-1],
\]
and \(\lim_{k \to \infty}\|\delta u_{k+1}(i)\| = 0, i \in [t+2, T-1]\), can also be guaranteed.
By analogy, if multiple control signals are lost at time $t$ in each iteration, which will result in more “1” in the diagonal elements of transition matrix, we also have $\lambda_m(\prod_{i=0}^{m} H_{L1}(j)) \to 0$, $m \in \{0, j\}$, $j \in \{t + 1, T - 1\}$, and then $\lim_{k \to \infty} \| \delta u_{k+1}(i) \| = 0$, $i \in \{t + 1, T - 1\}$.

Similarly, we can conclude that $\lim_{k \to \infty} \| \delta \tilde{u}_{k+1}(i) \| = \lim_{k \to \infty} \| u_{k+1}(i) - u_k(i) \| = 0$, $i \in \{t + 2, T - 1\}$ because $\lim_{k \to \infty} \| u_{k+1}(i) - u_k(i) \| = \lim_{k \to \infty} \| u_{k-1}(i) \|$. Because the output error $\| e_t(i+1) \|$, $i \in \{0, T - t - 1\}$ is a function of the control error $\| \delta \tilde{u}_k(j) \|$, $j \in \{0, t+i\}$, according to (11), the convergence of $\| \delta \tilde{u}_k(j) \|$, $j \in \{0, t+i\}$, indicates $\lim_{k \to \infty} \| e_k(t+1+i) \|$, $i \in \{0, T - t - 1\}$.

Up to now, by compensating the data dropout with the control signal at the same time with the lost one but in the last iteration, the convergence property of output error has been proved. The effect of data dropout was also discussed in [21] by assuming that the actuator is event driven. That is, when there is one signal loss at $t$ during the $k$th iteration, the signal sent at $t - 1$ will continue to be applied in the system before the signal sent at $t + 1$ arrives. If $u_k(t)$ is lost, this method can guarantee the control error $\| \delta \tilde{u}_k(t) \| = 0$, $t \in \{0, T - 1\}$, at the controller converge, but the control error $\delta \tilde{u}_k(t)$ at the actuator cannot converge as iteration goes on because $\lim_{k \to \infty} \| \delta \tilde{u}_k(t) \| = \lim_{k \to \infty} \| u_{k+1}(t) - u_{k-1}(t) \| \neq 0$. According to (11), the output error $\| e_k(t) \|$ cannot converge to zero, and the effect of random data dropouts increases with the increment of random data dropout probability.

5. Simulation Results

In this section, some simulation results are provided to illustrate the validity of the proposed method. Let us consider the following discrete-time system:

$$
x_k(t + 1) = \begin{bmatrix}
-0.5 & 0 & 0 \\
1 & 1.24 & -0.87 \\
0 & 0.87 & 0
\end{bmatrix} x_k(t) + \begin{bmatrix}
1 \\
0 \\
0
\end{bmatrix} \tilde{u}_k(t),
$$

$$
y_k(t) = \begin{bmatrix}
2 & 2.6 & -2.8
\end{bmatrix} x_k(t).
$$

The desired trajectory is

$$
y_d(t) = 5 \sin \left( \frac{8(t - 1)}{T} \right).$$

ILC method is described in (5). Initial state error $\delta x_k(0)$ and initial control error $\delta u_k(t)$ are 0, $T = 200$, $\Gamma(t) = 0.2$, so $0 < \rho(\Gamma(t)CB) = 0.4 < 1$ is satisfied. The mean of output errors $\sum_{i=1}^{T} \| e_k(t) \|$ in each iteration is selected to show the convergence of output error. In each iteration, $e_k(i)$ and $\eta_k(i)$ take value 0 or 1 according to the data dropout rate correspondingly.

The mean of output errors without any compensation versus iterations is shown in Figure 2. From Figure 2, it is easy to find that the effect of control signal dropouts on the mean of output error increases as random data dropout rate increases. The mean of output errors compensated by the control signal in the same iteration with the lost one but at the last time is shown in Figure 3. The comparison of the simulation results in Figures 3 and 2 shows that although the effect of data dropouts is reduced obviously, the mean of output errors does not converge to zero because this method cannot guarantee control error at the actuator converge to zero. And what is more, the effect of data dropouts increases with the increment of dropout rate. The mean of output errors compensated by the control signal at the same time with the lost one but in the last iteration is shown in Figure 4. By comparing Figure 4 with Figures 2 and 3, it is clear that even though the dropout rate would affect the convergence speed of the output error mean, which can still converge to zero from iteration to iteration with different dropout rates.
6. Conclusions

In order to remove the effect of data dropouts on the wireless remote ILC systems, a compensation method using the signal at the same time with the lost one but in last iteration is proposed to guarantee the convergence property of output error. The method can not only guarantee the convergence of control error at the controller, but also guarantee the convergence of control error at the actuator. By modeling the random data dropouts as a Bernoulli distributed sequence, the convergence property of control error at the controller is proved through studying the element values of system transition matrix, and then the convergence of control error at the actuator and output error is analyzed. Finally, the method is supported by some simulation results.

In wireless remote ILC systems, channel noises and time delays are other two main issues. In the future, we should consider the effects of data dropouts, channel noises, and time delays simultaneously. Besides, in order to improve the tracking performance further, we may select the learning gain adaptively according to such optimization criterions as least mean square and so on.

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