Optical vibrations in a closed chain of coupled waveguides or cavities

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Abstract. We study the dynamics of light field in tunnel-coupled oscillators of two types such as cavities and waveguides. The introduction of linear and nonlinear defects leads to localization and screening fields. Rotating modes and discrete vortices are considered by analytical solutions and numerical simulation.

1. Introduction
Systems of tunnel-coupled nonlinear optical waveguides and cavities are of interest in connection with the potential applications such as optical couplers, elements of optical logical devices, etc. [1, 2, 3, 4, 5, 6]. Therefore detailed understanding of the basic characteristics of such structures is a very important component for new effects research in nonlinear and coherent optics.

2. Basic equations of the model
We study the spatial dynamics of the three oscillators using three waveguides model. Obtained results are easily transferred to the description of the temporal dynamics of coupled cavities. In general, one of the oscillators has a different eigenfrequency or refractive index. External radiation is introduced into one of the oscillators. Then the light propagation dependence on the basic parameters of the system with and without the defect is considered. The effects of radiation localization and waveguide screening are investigated in detail.

Figure 1. Cross-section of the system of three (a) and four (b) tunnel-coupled optical waveguides.
We consider discrete systems shown in figure 1a. The second waveguide is defective. The system under consideration consists of three tunnel-coupled waveguides, i.e. light beam energy will penetrate from one of the waveguides into the neighboring, thus exciting them. Coupling coefficient describes the rate of energy transfer between the waveguides. It depends on the distance between the waveguides and dielectric permittivity of substrate.

Amplitudes of the light in the waveguides can be found from these equations:

\[
\frac{\partial C_j}{\partial z} = -i\alpha (C_j + C_\perp)
\]

where \( C_j = |C_j| e^{i\phi} \) is the complex amplitude of the wave in the \( j \)-th waveguide, \( \beta \) is the coefficient of nonlinearity. In the theory of coupled cavities spatial coordinate \( x \) is replaced by the time coordinate \( t \). From the system (1) we can get the following equations describing the variation of energy and phase of the light wave in the waveguides:

\[
\begin{align*}
\frac{d |C_1|^2}{dz} &= -2\alpha |C_1| |C_2| \sin(\phi_1 - \phi_2) - 2\alpha |C_1| |C_3| \sin(\phi_1 - \phi_3) \\
\frac{d |C_2|^2}{dz} &= -2\alpha |C_2| |C_1| \sin(\phi_2 - \phi_1) - 2\alpha |C_2| |C_3| \sin(\phi_2 - \phi_3) \\
\frac{d |C_3|^2}{dz} &= -2\alpha |C_3| |C_1| \sin(\phi_3 - \phi_1) - 2\alpha |C_3| |C_2| \sin(\phi_3 - \phi_2)
\end{align*}
\]

The analysis of the equations (2) and (3) shows that the direction of energy transfer between the waveguides is determined by the phase difference of the light wave in the adjacent waveguides. For \( \beta = 0 \) equations (1) can be solved analytically:

\[
\begin{align*}
C_1 &= \frac{E_1 + E_2 + E_3}{3} + e^{i2\alpha} \frac{2E_1 - E_2 - E_3}{3} e^{-i\alpha} \\
C_2 &= \frac{E_1 + E_2 + E_3}{3} e^{i2\alpha} + \frac{2E_2 - E_1 - E_3}{3} e^{-i\alpha} \\
C_3 &= \frac{E_1 + E_2 + E_3}{3} + e^{i2\alpha} \frac{2E_3 - E_2 - E_1}{3} e^{-i\alpha}
\end{align*}
\]
energy transfer from one waveguide to another occurs, so-called rotating mode. The essence of this phenomenon is that the amplitude of the radiation in the waveguides remains constant, but nevertheless energy is transferred between the waveguides due to fixed phase mismatch. How much energy is transferred per time unit from the first to the second waveguide the same is pumped from the third to the first waveguide, etc. So if we put fourth waveguide in the centre of our system (figure 1b) the energy will not penetrate into this waveguide. Therefore if the initial light amplitude at the input of the central waveguide is equal to zero, it will be zero along the whole waveguide. This effect can be called «discrete optical vortex». If the chain contains 6 oscillators, then it is possible to excite vortex with double charge: \( \varphi_\alpha = (m - 1) \pi / 6 \). Discrete double-charge vortices were thoroughly studied in [6].

In the general case it is possible to solve equations (1) only numerically, but for linear defect (i.e. one of waveguides has different refractive index) they transform to the equations (5), which can be solved and analyzed analytically:

\[
\begin{align*}
\frac{\partial C_1}{\partial z} &= -i \alpha (C_2 + C_3) \\
\frac{\partial C_2}{\partial z} &= -i \alpha (C_1 + C_3) + i \Delta k C_2 \\
\frac{\partial C_3}{\partial z} &= -i \alpha (C_2 + C_1)
\end{align*}
\]

where \( k_j \) is the wave number for \( j \)-th waveguide, \( k_1 = k_3 \), and \( \Delta k = k_2 - k_1 \) - wave detuning. Solution:

\[
\begin{align*}
C_1 &= \frac{E_1 - E_2}{2} e^{i \alpha} + \frac{1}{2} (E_1 + E_2) e^{-\left(\frac{\alpha - \Delta k + i}{2}\right)} \\
C_2 &= \frac{1}{4} (E_1 + 4E_2 + E_3) e^{-\left(\frac{\alpha - \Delta k + i}{2}\right)} + \frac{1}{4} (E_1 + E_3) e^{-\left(\frac{\alpha - \Delta k + i}{2}\right)} \\
C_3 &= \frac{E_1 - E_2}{2} e^{i \alpha} + \frac{1}{2} (E_1 + E_3) e^{-\left(\frac{\alpha - \Delta k + i}{2}\right)}
\end{align*}
\]  

(6)

From equations (6) we can see that localization occurs if the initial amplitude \( E_2 \) in the defective waveguide is much higher than the amplitudes in other waveguides, and conversely, if the initial amplitude \( E_2 \) is much lower than \( E_1 \) or \( E_3 \) we can observe the screening effect. As shown by the simulation results (see section 3) these conditions are also valid for nonlinear defect.

All the above considerations can be applied to the closed chain of coupled oscillators or cavities. Parameters will have different meaning, but the solution remains the same.

3. The simulation results
Curves on the following graphs, describing the dependence of light intensity in the waveguides on the distance, are marked by the number of the relating waveguide.

Figure 2. Localization of the radiation in the defective waveguide. \( \alpha = 1, \beta = 0.5 \).

Figure 3. Defective waveguide screening. \( \alpha = 1, \beta = 10 \).
In this paper we show that almost all energy of the system can be concentrated either in a single waveguide (figure 2), or in two (figure 3), without going into the third, or in all three uniformly, cyclically flowing from one waveguide to another. Our system is closed, i.e. after supplying the initial radiation into the system its energy remains constant, but can be freely redistributed within the system, except cases in which it is not allowed by the physics of the process.

By analyzing numerical simulations of light propagation in waveguides with different parameters we have shown that after the excitation of the defect waveguide the radiation can be localized in it (figure 2), and the degree of localization increases with increasing nonlinearity coefficient (figure 4a) or input light intensity. If we excite not the defective waveguide, but any other, then the defective waveguide will be screened, i.e. the radiation will redistribute between two linear waveguides and almost will not pass through defective one. The screening level is also increased with increasing nonlinearity coefficient (figure 4b).

**Figure 4.** Dependence of the minimum and maximum of light intensity in the waveguides on the nonlinearity coefficient. Solid lines correspond to the defective waveguide. a) The localization effect. b) The screening effect.

4. Conclusions
We studied the localization of waves in nonlinear defective waveguide. It was shown that the degree of localization increases with increasing nonlinearity coefficient or intensity of input emission in the defective waveguide. An effect of defective waveguide screening from neighboring zero-defect waveguides was found. Conditions for excitation of discrete optical vortex in the coupled waveguides were found.

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