A shape of invariant differential cross section for hadron production as function of transverse momentum is analysed. The systematic analysis of the available data demonstrates a need for a modification of the parameterization traditionally used to approximate the measured spectra. The properties of the new proposed parameterization are discussed.

I. INTRODUCTION

There exists a large body of experimental data on hadron production in high energy proton-proton, photon-proton, photon-photon and heavy ion collisions [1–15]. The spectra of hadrons produced in these collisions are characterized by an exponential behavior as function of transverse energy ($E_T$) for the bulk of produced hadrons, which populate the low $E_T$ part of the spectra. This behavior resembles the Boltzmann-like spectrum in classical thermodynamics. The exponential shape of the spectra changes to a power law for high $E_T$ hadrons. This change is traditionally interpreted as an onset of the perturbative QCD regime of hadron production. These features of the spectra shape are found to be universal for any type of colliding particles. Therefore, it is tempting to find one universal smooth functional form, which describes the spectra of produced hadrons in the whole available $E_T$ range for different energies and types of colliding particles. The parameters of such universal functional form are expected to vary for different collision energies, types of colliding particles and types of produced hadrons. A study of the variations of these parameters provides a unique information on the hadron production dynamics.

In the present paper the experimentally measured inclusive spectra of long-lived charged particles (mainly charged pions) produced at central rapidities in the colliding particles center of mass system are analyzed. Further on it is assumed all charged particles being pions for simplicity. The present analysis is based on the published hadron production measurements made with $pp$-collisions at ISR [1] and LHC [15], $p\bar{p}$-collisions at $SpS\bar{S}$ [2–4] and Tevatron [5, 6], $AuAu$-collisions with different centralities at RHIC [14], real $\gamma\gamma$-collisions at LEP [9, 11] and $\gamma p$-interactions with different values of photon virtuality ($Q^2$) at HERA. The HERA data additionally allow to consider two different regimes of $\gamma p$-interactions: photoproduction at low values of $Q^2$ [7, 8, 10] and Deep Inelastic Scattering (DIS) for high $Q^2$ values [12, 13]. The data for all these inclusive differential cross section measurements have been taken with a minimum bias trigger conditions and at center of mass energy ($\sqrt{s}$) ranging from 23 to 2360 GeV.

II. SPECTRUM ANALYSIS

A typical charged particle spectrum as function of transverse energy is shown in Fig. 1.

![Graph of charged particle spectrum](image)

FIG. 1. A typical charged particle spectrum fitted using the Tsallis-type function (1).

This spectrum is fitted using the Tsallis-type function [16]

$$\frac{d\sigma}{P_TdP_T} = \frac{A}{(1 + \frac{E_T}{T})^n},$$

(1)

where $P_T$ is transverse momentum of the produced particle, $E_T = \sqrt{P_T^2 + M^2}$ with $M$ equal to the pion mass. The parameterization (1) has only three free parameters: $A$, $T$, and $n$. While $A$ is an overall normalization, the $T$ and $n$ carry important information on the hadron production dynamics. Since for low $E_T$ values the parameterization (1) is reduced to the Boltzmann exponent $\sim \exp(-E_T/T)$, the parameter $T$ is a QCD analogy to a temperature in classical thermodynamics. Note, this analogy isn’t straight forward, however. In perturbative QCD calculations the value of the parameter $n$ depends on the shape of structure functions and partonic content of the colliding particles as well as on the differential cross sections of parton-parton interactions. As it is

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FIG. 2. A ratio of the data to the Tsallis-type (1)(a,b) and the modified (2)(c,d) fit functions in \( p\bar{p} \) collisions at \( \sqrt{s} = 630 \text{ GeV} \) (a,c) and RHIC \( Au - Au \) collisions at \( \sqrt{s} = 200 \text{ GeV/\(N\)} \) (b,d).

III. MODIFICATION OF THE TSALLIS FUNCTION

A. Systematic defects of the fit

However, a closer look at the fit shown in Fig 1 discloses systematic defects of the fit. In order to see these defects it is convenient to plot a ratio of the data to the fit function. Two such ratios are shown in Fig 2(a,b) for the data sets provided by the UA1 experiment with \( p\bar{p} \) collisions at \( \sqrt{s} = 630 \text{ GeV} \) (Fig 2a) and by RHIC \( Au - Au \) collisions at \( \sqrt{s} = 200 \text{ GeV/\(N\)} \) (Fig 2b). On both plots one observes dips and bumps around or above 1 GeV scale.

The observed difference between the shapes of data spectra and function (1) is also typical for other available data sets, not shown here. This observation indicates that the true shape of the measured spectra doesn’t follow exactly the Tsallis-type parameterization (1). Moreover, a larger mismatch between the shapes of the data spectrum and fit function (1) is found for \( J/\Psi \) production at the Tevatron [17]. In the following an attempt is made to find a function which approximates the shape of the inclusive hadron production spectra better than the Tsallis parameterization.

B. Search for the new fit function

To construct a new fit function a combination of two functional forms has been used: the exponential and power-law. The both functional forms must be the functions of scalars \( P_T^2 \) or \( E_{kin}^T = E_T - M \). This choice of these variables is very convenient since the both \( P_T^2 \) and \( E_{kin}^T \) vary from zero to the corresponding kinematical limits. Non-true scalars like \( P_T \) or \( P_T^3 \) have not been considered here. Finally, the parameterization

\[
\frac{d\sigma}{P_T dP_T} = A_c \exp\left(-\frac{E_{kin}^T}{T_c}\right) + \frac{A}{(1 + \frac{P_T^2}{T^2})^n},
\]  

(2)

demonstrated in Fig 1 the economic Tsallis-type parameterization (1) provides a good overall description of the spectrum, thus making the Tsallis-type function broadly used to fit the recent measurements at RHIC and LHC.
turned out to be in a good agreement with the data. The new ratios between data and fit function (2) are shown in Fig 2(c,d) for the considered above UA1 and RHIC data sets correspondingly. Fig 2(c,d) demonstrates a significant improvement of the quality of the spectra shape approximation by function (2) with respect to that using the Tsallis-type function (1). In the following we present the arguments why this improvement is not a trivial result of increasing the number of free parameters in the fit function.

IV. CORRELATION BETWEEN THE PARAMETERS

The most surprising feature of the new parameterization (2) is a strong correlation between the parameters $T_e$ and $T^2$. The dependence of fitted values of the parameter $T^2$ versus $T_e^2$ for charged particles produced in $pp$, $p\bar{p}$ and $Au - Au$ collisions is shown in Fig 3.

This dependence is approximated by a linear function. This provides an additional constraint for the parameterization (2) and therefore reduces a number of free parameters used in the fit to the data. Though the physical origin of the observed correlation is not quite clear, the new constraint helps to minimize uncertainties of the parameter values obtained from the fits. This constraint will be used further on through this paper.

V. A TOY MODEL INTERPRETATION

The form of the parameterization (2) has a simple toy-model interpretation. Within this toy-model a small fraction of hadrons are produced directly in parton-parton interactions of the colliding particles. As required by the perturbative QCD the spectrum of particles produced in parton-parton interactions is described by a power law distribution. The rest bulk of irradiated hadrons represents a quasi-thermolized hadronic gas produced with a characteristic temperature $T_e$. The spectrum of hadrons in this gas has the Boltzmann exponential shape.

A. Contributions of the exponential and power law terms

The contributions of the exponential and power law terms of the parameterization (2) to the typical spectrum of charge particle produced in $pp$ collisions are shown separately on Fig 4. The relative contribution of these terms is characterized by a ratio $R$ of the exponential to power law terms integrated over $P_t^2$:

$$R = \frac{A_e(2m + 2T_e)(n - 1)}{18.11A\cdot n\cdot T_e}$$

For the spectrum in Fig 4 the exponential term dominates, while the power law term contributes at the level of about 20%.

The ratio $R$ for the inclusive charged particle spectra for $p\bar{p}$ and $pp$ collisions as function of $\sqrt{s}$ is shown in Fig 5a. Interestingly, this ratio is almost independent of the collision energy and equals to about 4. In Fig 5b the ratio $R$ is shown for $Au - Au$ interactions at RHIC as function of centrality of the heavy ions collision. As seen on Fig 5b the relative contribution of the exponent term reaches minimum values at medium centralities of heavy ion collisions.

On the other hand, the QCD partonic interaction must describe any hard scattering process like the heavy quark production, or high $Q^2$ DIS. Indeed, as shown in Fig 6 the spectrum of heavy $J/\Psi$ quarkonium produced in high energy $p\bar{p}$ collisions at Tevatron [17] follow the pure power law distribution and has no room for an exponential term of the parameterization (2). In addition, in the high en-
energy DIS, photoproduction and $\gamma\gamma$ collisions the power law term of the new proposed parameterization dominates the produced particle spectra as shown in Fig. 7. Thus, only the inclusive spectra of charged particles produced in pure baryonic collisions require a substantial contribution of the Boltzmann-like exponential term. We found it to be particularly interesting that the interactions of real photons and protons with real photons (high energy photoproduction) have practically no Boltzmann-like ther-molized hadronic final states.

VI. MAP OF PARAMETERS

Finally, a map of the parameters $T$ and $n$ for proton-(anti)proton, heavy ion, gamma-proton and gamma-gamma collision at different energies is displayed in Fig. 8. There are two clearly distinct trends seen in Fig. 8. The $pp$ and $p\bar{p}$ collision data show an increase of the $T$-parameter and decrease of the $n$-parameter with collision energy $\sqrt{s}$ increasing. The second trend, where the values of both parameters the $T$ and $n$ increase, is defined mainly by the RHIC $Au - Au$ collision data at $\sqrt{s} = 200$ GeV per nucleon. In this case a simultaneous increase of the $T$ and $n$ values corresponds to an increase of the centrality (or charged multiplicity) of heavy ion collisions. Surprisingly, the both trends cross each other at medium centralities corresponding to the minimum bias $Au - Au$ collisions and $p\bar{p}$ interactions with energy of $\sqrt{s} = 200$ GeV. Naively one could expect the single $p\bar{p}$ interaction has more similarity to the very peripheral single nucleon-nucleon interactions. Contrary to that naive expectation, the Deep Inelastic Scattering (DIS), $\gamma p$ and $\gamma\gamma$ interaction data with $\sqrt{s}$ ranging approximately from $100$ GeV to $200$ GeV belong to the second trend shown in Fig. 8 and are located on the parameter map (Fig. 8) nearby very peripheral heavy ion interactions at about

![FIG. 5. The ratio $R$ of the exponential to power law contributions to the parameterization for $p\bar{p}$ and $pp$ collisions as function of $\sqrt{s}$ (a) and for $Au - Au$ interactions at RHIC as function of centrality (b).](image)

![FIG. 6. The spectrum of $J/\Psi$ produced in high energy $p\bar{p}$ collisions at Tevatron.](image)

![FIG. 7. The ratio $R$ of the exponential to power law contributions to the parameterization in the high energy DIS, photoproduction and $\gamma\gamma$ collisions.](image)
The proposed new parameterization (2) describes the data well at low and medium values of the transverse momentum of the produced particles. However, as it was recently shown by the CDF measurements, in $p\bar{p}$ interaction the inclusive particle spectrum has a significant excess over the simple power law shape for particles produced with $P_T > 10 \text{ GeV/c}$ [6]. In Fig 9 it is demonstrated, that observed excess is well described by adding a second power law function with somewhat lower value of the exponent $n$. This phenomenon, which has no consistent explanation by now, seems to be found earlier in $\gamma p$ and $\gamma \gamma$ interactions (shown in Fig 10).

Though, in these types of collisions with at least one photon involved the excess of the data spectra over a single power law function is visible already for particles with $P_T > 3\text{GeV/c}$.

**VIII. CONCLUSION**

In conclusion we have proposed a new parameterization of the spectrum shape of inclusive charged particles produced in high energy collisions. This new pa-
rameterization describes the available experimental data significantly better than the broadly used Tsallis-type parameterization. The proposed parameterization is a sum of an exponential (Boltzman-like) and a power law (Tsallis-like) terms. The parameters of these two terms turned out to be strongly correlated. We observe, that the shapes of the power law terms in minimum bias heavy ion collisions and in proton-(anti)proton interactions at the same collision energy are practically the same. Additionally, the shapes of the power law terms in the spectra measured with very peripheral heavy ion collisions and interactions of high energy particles with photons show very close similarity. The difference in size of the exponential Boltzman-like contribution to these spectra is mainly responsible for the difference in the spectra shapes observed in these experiments.

ACKNOWLEDGMENTS

The authors thank professor M.Ryskin for helpful discussions. This work was partially supported by Russian Foundation for Basic Research and the Grant of Helmholtz Association HRJRG-02.

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