Smart machines with flexible rotors

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Abstract. The concept of smart machinery is of significant current interest. Several technologies are relevant in this quest including magnetic bearings, shape memory alloys (SMA) and piezo-electric activation. Recently a smart bearing pedestal was proposed based on SMAs and elastomeric O-rings. However, such a device is clearly relevant only for the control of rigid rotors, for flexible rotors there is a need for some modification on the rotor itself. In this paper, rotor actuation by piezo-electric patches on the rotor is studied. A methodology is presented for the calculation of rotor behaviour and an appropriate control strategy is developed.

1. INTRODUCTION

The concept of Smart Machinery is in its infancy and has received significantly less attention than has been devoted to the related study of Smart Structures. A smart machine is one in which there is some facility for automatic diagnosis of faults coupled with a capacity to apply corrective loads to optimize machine duty until such time as corrective maintenance can be undertaken. The precise form which these facilities take will depend significantly on the duty of the machine in question. The development of smart machines is inherently more complex than the associated structural problem owing to the rotor motion and problems associated with the bearings. However, in essence, the idealized machine has three main features:

(a) The facility to infer its own internal state
(b) A capability to diagnose faults
(c) The introduction of corrective forces in the event of faults.

The simplest case to consider is a machine with a rigid rotor, such as occurs in a range of small machines. This is a particularly simple case to consider sing the natural frequencies (or more particularly, critical speeds) are controlled entirely by the stiffness of the bearing pedestals. Lees et al [1] have shown how a controllable bearing pedestal may be designed using Shape Memory Alloys and elastomers, although it is appreciated that there are a number of possible routes to achieve this goal. Other approaches to the introduction of controllable support stiffness have been considered by Zak et al [2] and Cartmell et al [3]. Recently Maslen [4] has given a brief review of progress on Smart Machinery, but this too focuses of bearings, with particular emphasis on magnetic bearings: this is not surprising as this technology offers great promise.

However, looking further ahead one is naturally led to consider the possibility of controlling machines with flexible rotors. This is important for two reasons: in machinery generally there is a
trend towards the use of flexible rotors and secondly, adequate control would enable the operation of much light machines leading to higher efficiencies and better material utilization.

2 PIEZO-ELECTRIC PATCHES

Piezo-electric devices have been used for the control of structures in aero-space applications for a number of years. There are numerous papers on this and an introductory account is given in reference [5]. Control may be exerted by either a modification of natural frequencies or, more usually, by changing the damping properties. The initial part of this study considers the possible effect of such devices on a rotor. Signals may be input by means of slip-rings.

Consider a simple, single degree of freedom rotor system. We suppose there is a vibration amplitude \(a\), and we may assume that the deflection is of the form

\[
w(x) = 4 \frac{x}{L} \left(1 - \frac{x}{L}\right)
\]  

(1)

Using this deflection, the extension of the neutral axis is given by

\[
\Delta L = \frac{1}{2} \int_0^L \left( \frac{dw}{dx} \right)^2 dx
\]  

(2)

However, the situation under consideration here is the introduction of a piezo-electric patch, not on the neutral axis, but on the extremity of the rotor. Assuming this to be displaced by \(r\) from the neutral axis, the extension here will be rather different and will be given by

\[
\Delta S = \frac{1}{2} \int_0^L \left( \frac{dw}{dx} + r \frac{d^2 w}{dx^2} \right)^2 dx
\]  

(3)

The establishing of a numerical methodology is a relatively straightforward task. What is rather more demanding is the determination of the most effective philosophy to use. In any event it is a fairly straightforward matter to supply the requisite electrical signals to the rotor and this can be achieved either using slip-rings or telemetry systems. Whilst the voltages will be high, the currents and consequently the power will be negligible.

There are four possible approaches to use the forces arising in the piezo-electric MFC patches which may be outlined as follows:

a) Use the forces to directly change the natural frequencies. This may be achieved by using opposite sides of the rotor to modify shaft properties. As shown below, this approach appear to be limited to very small machines.

b) Apply an axial force which will modify the rotor’s properties – this is akin to the common changes of natural frequency associated with axial forces.

c) By suitable scaling and timing use the force to control damping rather than the stiffness.

d) Use the patches to induce rotor bends which can counteract the unbalance.

In the following sections, this paper examines which, if any, of these approaches offer any prospects of viability given typical performance parameters of currently available patches. It is clear
that the discussion here must focus on light machine, but part of the aim must be to seek what is feasible. In the calculations which follow the prototype studied has a shaft of diameter 12mm and a span between the bearings of 1metre. The bearings each have symmetrical bearing properties with a stiffness parameter of 100,000 N/m and associated damping parameter of 800 Ns/m. The single central mass gives a resonant frequency of 124 rad/sec. and the system damping coefficient is 1.8%.

3 INITIAL CALCULATIONS

Figure 1 shows the basic layout for a simple rotor. Piezo-electric multi-fibre composite patches are positioned at the centre of the rotor span, but for a system involving higher modes, further patches would be required. Depending on the phasing of the applied voltages, either forces or bending moments may be applied. By using two pairs of patches around the periphery of the rotor, resultant forces/moments can be applied at any phase angle with respect to the rotor.

For the light machine considered, Figure 2 shows the response as a function of speed for two distinct cases. The first is an unbalance at the central disc of magnitude 0.0001 kg.m, whilst the second trace shows the response due to a bend of magnitude 0.1mm at the central point. Both excitations have been given zero phase for the sake of convenience. Note that although the response is synchronous in both cases, the response curve is rather different. For the unbalance case, the response for a steady shaft rotation rate $\Omega$ is given by

$$K_y + M\ddot{y} = me\Omega^2 \exp(j\Omega t) \quad (4)$$

The response due to the bend is given by

$$K_y + C\ddot{y} + M\ddot{y} = K_{y_{\text{bend}}} \exp(j\Omega t) \quad (5)$$

Simple calculations based on Rayleigh methods using (1) as a trial function that is is difficult to affect a substantial change in natural frequency by the application of signal to the piezo-electric patches. They can be used to impose an axial force on the rotor, but calculations reveal that the influence on critical speeds of credible forces is fairly minor.

4. FORCED RESPONSE

It is clear from both the form of equations 4 and 5, that by adjusting phase, an imbalance may be used to counteract a rotor bend, but only at some selected speed (see [6] for a more detailed discussion). This is often done in practice, but in the present study we examine the converse situation, namely inducing a bend to counteract an unbalance. Figure 3 shows the response of the system in which a bend has been imposed to exactly cancel the imbalance response.
In the rotor fitted with a set of offset piezo-electric patches, a bend can be induced by effectively exerting bending moments about two nodal positions. In the current arrangement (shown in Figure 4) moments of $P$ and $-P$ were applied about neighbouring nodes and Figure 5 shows the steady state bends induce by these moments.

However, because in the case of this study, $P$ can be time dependent, it is clear that the bend formation cannot occur instantaneously and there is inertia in the system. For the rotor, spinning at $\Omega$, when a force $P(t)$ is applied, the development of the bend will be described by the equation

$$K_{\text{bend}} y_{\text{bend}} - \Omega^2 M_{y_{\text{bend}}} + M_{\ddot{y}_{\text{bend}}} = P(t)$$

(6)

The objective is now to solve the equations simultaneously during a machine run up or rundown. The forcing function $P$ may be chosen. The equations are integrated using a Runge-Kutta scheme in Matlab. To do this, the equations are transformed to a state-space formulation using the variables $[y_{\text{bend}} \quad \dot{y}_{\text{bend}} \quad \ddot{y}_{\text{bend}}]^T$. In this formulation, the equation of motion becomes

$$Aq + B\dot{q} = F$$

(7)

To examine this in more detail we need to distinguish between the stationary and rotating frame. Let $\dot{y}$ denote deflection in the rotating frame (i.e. stationary with respect to the rotor) then $y = R\dot{y}$ where the transformation $R$ takes the diagonal block form.
\[ R = \begin{bmatrix} r & 0 & 0 \\ 0 & r & 0 \\ \cdots & \cdots & \cdots \\ 0 & 0 & 0 \end{bmatrix} \quad \text{and} \quad r = \begin{bmatrix} \cos \theta & \sin \theta & 0 & 0 \\ -\sin \theta & \cos \theta & 0 & 0 \\ 0 & 0 & \cos \theta & -\sin \theta \\ 0 & 0 & \sin \theta & \cos \theta \end{bmatrix} \] (8 & 9)

\( \theta \) being the instantaneous position of the rotor. Clearly for the case of rotation at constant speed \( \Omega, \theta = \Omega t \).

The equations of motion for the complete system, whose speed varies with time, in the presence of both imbalance and activation producing a bend, can now be written as

\[
\begin{align*}
K_y + C\ddot{y} + M\ddot{y} & = KR(t)y_{\text{bend}} + ma\Omega^2 e^{i\omega t} - jma \frac{d\Omega}{dt} e^{i\omega t} \\
K_{y_{\text{bend}}} - \Omega^2 M_{y_{\text{bend}}} + M\ddot{y}_{\text{bend}} & = P(t)
\end{align*}
\] (10)

These equations assume shaft symmetry. They may be written in state-space form as

\[
\begin{bmatrix}
0 & 0 & -K & 0 \\
0 & 0 & 0 & -K \\
-K & -KR & -C & 0 \\
0 & -\Omega^2 M - K & 0 & -C
\end{bmatrix}
\begin{bmatrix}
y \\
\dot{y}_{\text{bend}} \\
\ddot{y} \\
\ddot{y}_{\text{bend}}
\end{bmatrix}

= \begin{bmatrix}
0 \\
0 \\
ma\Omega^2 e^{i\omega t} + jma \dot{\Omega} e^{i\omega t} \\
P(t)
\end{bmatrix}
\] (11)

Of course, the effective phases of the deflection arising from an unbalance and a bend is somewhat involved and due recognition must be made of this. It can be shown that the combined deflection due to the two may be limited to the extent of the bend and figure 3 shows the case in which the magnitude and phase of the bend have been chosen to exactly balance at a set rotation speed.

It is easily shown that a bend and a discrete unbalance can exactly compensate at a single rotational speed, and in this case the natural frequency has been chosen as the speed at which the response is minimized. However, in the case under study here, because actuators are on the rotor, the response may, in principle be minimized at all speed simply by varying the voltages to the piezo patches. As this involves not only magnitude but phase, the control of this is non-trivial and a full discussion is beyong the scope of the present paper. However we consider now taking a fixed unbalance by varying the amplitude of \( P_x \), the applied torque between 0 and 200 Nm (forces of the corresponding magnitudes may realistically be obtained from commercially available patches, albeit with the application of considerable voltages. The phase angle between the unbalance and the bend is varied between 0 and 360 in steps of 10 degrees. The response is shown in figure 4.
The point to note here is that although the unbalance used in this simulation is very high, the vibration can be reduced to negligible levels by the application of a torque of 10Nm. Since individual patches can apply 500N and more, such levels of torque are readily obtainable.

It is now important to establish the dynamic response of such a controlling parameter. The results of a preliminary calculation are shown in figure 5. The piezo-electric patches are activated after 2 seconds and a moment of 5Nm is applied to the central portion of the shaft. Note that this comfortably within the range of realistic parameters. The change in the vibration response is almost immediate but in this case, the response has not been reduced to zero. This is because the phase computation has not yet been refined. Note that a substantial change in the vibration level has resulted.

5. DISCUSSION

Interest in Mechatronics and the concept of ‘Smart Machines’ has developed of the past few years. But it has so far focused on the properties of bearings and support structures. Whilst this is appropriate for machines with rigid rotors, the overall trend in machinery is the enhanced use of designs involving flexible rotors. Such rotors require some controlling element to be mounted on the shaft. The voltage applied to piezo-patches can be very considerable, of order hundreds of volts (dc) albeit at negligible current levels. One might envisage that the signal would be fed to the rotor via a slip-ring arrangement: the question as to whether the signal would be at full voltage or amplified on the rotor is a question for future development.
The work presented in this paper is very exploratory. Nevertheless a potential means of controlling flexible rotors has been outlined. In essence the control is effected by imposing a bend to compensate for the imposed imbalance. It is well known that a bend can only be balanced out at a single rotational speed, but in the present case the extent of the bend can be varied by the voltage applied to the piezo-electric patch.

The sample machine taken here is by no means typical: it is extremely flexible but if a machine can be controlled in such a manner, it becomes realistic to operate these devices. This paper has outlined some basic considerations and formulated a method of analysis. Furthermore the possibility of actively controlling a flexible rotor in this way has been established. To develop this idea further the following steps need to be undertaken:

a) A review of the range of machine dimensions which would be feasible for this type of technology.
b) Derivation of an appropriate control algorithm
c) The construction and testing of a device

6. CONCLUSIONS

1. It is more realistic to control the forcing on a rotor rather than the dynamic properties. Calculations show that it is difficult to introduce substantial changes in natural frequency by the use of piezo-electric patches. It has been shown that is feasible to counteract an unbalance with an induced bend.
2. Some simple cases have been examined and shown to be feasible. Whilst this is primarily of interest in small machines, the full range of applicability has yet to be established. Note, however, that if the constraint of avoiding critical speeds can be removed then machines can be made much smaller and lighter.

3. An analysis approach has been presented.

7. REFERENCES

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