Optical measurement of a fundamental constant with the dimension of time

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We consider the concept of a rotating reference frame with the axis of rotation at each point and the applicability of this concept to different areas of physics. The transformation for the transition from the resting to rotating frame is assumed to be non-Galilean. This transformation must contain a constant with dimension of time. We analyze different possibilities of experimental testing this constant in optics, as most suitable field for measurements presently, and also in general relativity and quantum mechanics.

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INTRODUCTION

The concept of a "point rotation reference frame", i.e., the frame with the axis of rotation at every point, arises in optics. However, this concept is also applicable to other areas of physics. An example of such a frame is the optical indicatrix (index ellipsoid) [1]. Any rotating field, including spinor and gravitational, is the object of the point rotation.

Coordinates of the frame are the angle, time and axis of rotation. The radial coordinate is not used in manipulations with the frames. Centrifugal forces don’t exist in such frames. Optically, the rotating half-wave plate is an equivalent to the resting electrooptical crystal with the rotating indicatrix [2] but, physically, they are different because the plate has only one axis of rotation. The frames are not compatible with Cartesian’s frames.

The main question in such a concept: what is the transformation for point rotating reference frames? Is the transformation Galilean or not?

From the viewpoint of contemporary physics a non-Galilean transformation, with different time for frames rotating at different frequencies, is much more preferable in comparison with the Galilean, where time is the same. Moreover, such a transformation must contain a constant with the dimension of time similarly to the Lorentz transformation and the speed of light. This constant should define limits of applicability of basic physical laws.

In contrast to mechanics, where the relativity principle is used to deduce the transformation for the rectilinear motion, such a general principle does not exist for the point rotation. Therefore, this transformation cannot be explicitly determined.

It is known that an electric field, rotating perpendicular to the optical axis of a 3-fold electrooptic crystal, causes rotation of the optical indicatrix at a frequency equal to half the frequency of the field. It means that the optical indicatrix of such a crystal possesses some properties of two-component spinor.

The sense of rotation of the circularly polarized optical wave propagating through this crystal, is reversed, and the frequency is shifted, if the amplitude of the applied electric field is equal to the half-wave value. The device for such a shifting is the electrooptical single-sideband modulator [3].

The use of the transformation makes the description of the light propagation in the electrooptical single-sideband modulator simpler and comprehensible.

For the description of the phenomenon transit to a rotating reference frame associated with axes of the indicatrix. As result of such a transition, the frequency of the wave is shifted by half frequency of the modulating electric field. This shift is doubled at the modulator output due to the polarization reversal and transition to the initial reference frame [3].

In this paper we study the general form of two-dimensional non-Galilean transformation and the possibility of its experimental verification. Emphasize, experiment always involves the direct and reverse transformation because an observer rotating at each point does not exist.

THE TRANSFORMATION

The general form of the normalized non-Galilean transformation may be written as follows

\[ \tilde{\varphi} = \varphi - \Omega t, \quad \tilde{t} = -\tau \varphi + t, \]  

where the tilde corresponds to the rotating frame, \( \varphi, \) \( \dot{t} \) and \( \dot{t}, \) \( \Omega \) are the normalized angle and time, \( \Omega \) is the frequency of the rotating frame (the modulating frequency is 2\( \Omega \)), \( \tau(\Omega) \) is a parameter with the dimension of time.

The reverse transformation follows from

\[ (1 - \Omega \tau) \varphi = \tilde{\varphi} + \Omega \tilde{t}, \quad (1 - \Omega \tau) t = \tau \tilde{\varphi} + \tilde{t}. \]  

Consider a plane circularly polarized light wave propagating through the modulator. Transit into the rotating frame. The optical frequency in this frame is

\[ \tilde{\omega} = \frac{\omega - \Omega}{-\tau \omega + 1}, \]  

where the frequency in the resting and rotating frame is defined as \( \omega = \varphi/t \) and \( \tilde{\omega} = \tilde{\varphi}/\tilde{t} \) respectively.
If the half-wave condition is fulfilled, the reversal of rotation occurs at the modulator output. For the circularly polarized wave, the negative sign of the frequency corresponds to the opposite rotation. Making transition into the resting frame with changing the sign of $\omega$, obtain the output frequency as a function of $\omega$ and $\Omega$

$$\omega' = \frac{-\omega (1 + \tau \Omega) + 2\Omega}{-2\tau \omega + \tau \Omega + 1}. \quad (4)$$

In fact, we consider the single-sideband modulator in this approach as a black box. This box changes the sense of rotation of the circularly polarized light wave and shifts its frequency.

**OPTICS**

For the evaluation of the parameter $\tau$ we use results of optical measurements from the work [4]. In this work the principle of the single-sideband modulation was checked and the Galilean transformation was used for the theoretical description of the process.

Circularly polarized light from Helium-Neon laser was modulated by a Lithium Niobate single-sideband modulator at the frequency 110 MHz. The experiment showed an asymmetry of the frequency shift for two opposite polarizations. The extra shifts was of the order of few MHz.

Proof. W. H. Steier, one of the authors of the work [4], kindly answered my question about the origin of this asymmetry: "Your are correct about the apparent asymmetry. We never noticed it earlier. I do not know if this is a property of the scanning mirror interferometer. It has been many many years since we did that work and all of the equipment has now been replaced. It would not be possible for us to redo any work or start the experiments again”.

Possibly, the origin of this extra shift is a defect of the equipment. In any case this shift can be used for approximate estimates of the upper boundary of the parameter $\tau$. From this the important conclusion follows. The parameter $\tau$ is very small.

For small $\tau$ and $|\Omega| \ll |\omega|$ the output frequency [4] may be written as

$$\omega' \approx -\omega + 2\Omega + 2\tau \omega^2. \quad (5)$$

The extra shift equals $2\tau \omega^2$.

The exact form of the dependency $\tau(\Omega)$ is unknown. Therefore, assume that $\tau$ may be expanded in power series in $\Omega$

$$\tau = \tau_0 + \tau_1^2 \Omega + \tau_2^2 \Omega^2 + \ldots \quad (6)$$

In such a form all the coefficients $\tau_n$ have the dimension of time.

Since $\tau$ is very small and, usually, $\Omega \ll \omega$, we can restrict ourselves only the first non-zero term in the expansion [4].

If $\tau_0$ is exactly equal to zero, the accuracy should be increased by $1/(\tau_1 \Omega)$ times. Accordingly to results of [4], the upper boundary of $\tau_1$ is $\sim 10^{-16}$. Using the optical range for the modulation is connected with the problem of phase matching [3].

Below briefly summarized results of the analysis and possibilities of measurements in other areas of physics.

**GENERAL RELATIVITY**

Consider the case $\tau_0 = 0$ in application to general relativity. Now we restrict ourselves only by the second term of $\tau(\Omega)$ and consider [4] as the Lorentz transformation. Usually this name relates to the rectilinear motion in mechanics. Here the role of the coordinate and velocity is played by the angle and frequency respectively.

After a normalization

$$\hat{\varphi}, \hat{t} \rightarrow \frac{\hat{\varphi}, \hat{t}}{\sqrt{1 + \tau_1^2 \Omega^2}}, \quad (\varphi, t) \rightarrow (\varphi, t) \sqrt{1 - \tau_1 \Omega}, \quad (7)$$

obtain

$$\hat{\varphi} = \frac{\varphi - \Omega t}{\sqrt{1 - \tau_1^2 \Omega^2}}, \quad \hat{t} = \frac{-\tau_1 \Omega \varphi + t}{\sqrt{1 - \tau_1^2 \Omega^2}}. \quad (8)$$

Analogously mechanics, $\tau_1$ can be regarded as the minimum possible time interval and $1/\tau_1$ as the maximum possible frequency.

The form $(\tau_1^2 \varphi^2 - t^2)$ is invariant under the transformation [3].

Despite the fact that the Cartesian reference frames are not compatible with the point rotation reference frames,
there exists a solution of Einstein’s equation invariant under the transformation \( T \). Consider an exact solution with cylindrical symmetry \( T \)

\[
  ds^2 = A r^{a+b} dt^2 + r^2 d\varphi^2 + b dz^2 + C r^a dt^2,
\]

where \( A, C, a, b \) are constants. This solutions is an invariant under the transformation \( T \) provided \( a = 2 \). Moreover, at \( a = b = 2 \) and a normalization of \( r \) and \( t \) the metric can be reduced to the form

\[
  ds^2 = (1 + \frac{1}{L}r)[dr^2 + l^2 d\varphi^2 + dz^2 - c^2 dt^2],
\]

where \( l \equiv cr^2 \) and \( L \) are constants with the dimension of length. For a "center", at \( r = 0 \), this metric looks like "Euclidean metric" for the point rotations.

Non-stationary solutions of Einstein’s equation, invariant under the transformation \( T \), also exists.

The existence of such metrics opens the way for applying the concept of the point rotation reference frames to general relativity. In this sense suitable solutions of Einstein’s equation are possible but searching for consequences of such solutions applicable for measurements of \( \tau_1 \) or \( l \) is not simple problem.

QUANTUM MECHANICS

Initially, quantum mechanics was considered as the most suitable area of physics for the measurement of the parameter \( \tau \). However, the hope to find in quantum mechanics a consequence of the transformation \( T \) applicable to measurements, proved to be illusory.

Quantum states in rotating magnetic or electromagnetic fields are not stationary. The problem becomes stationary by the transition to the rotating frame. Main role in the transition plays the phase transformation of the spinor, which is defined by the first equation in \( T \). The second equation, containing the parameter \( \tau \), plays a minor role.

The transition was used for finding a new class of exact localized solutions of the Dirac equation \( T \) in the rotating electromagnetic field.

However, the further study showed that the parameter \( \tau \) vanishes from final results due to the reverse transition into the resting frame. It allows also to conclude that the non-Galilean transformation is not related to the problem of anomalous magnetic moment in any case for the above exact localized solutions.

CONCLUSION

We have considered the concept of the point rotating frame and the non-Galilean transformation for such frames. The concept is applicable to optics, general relativity and quantum mechanics. The parameter \( \tau \) with the dimension of time is a distinguishing feature of the non-Galilean transformation. This parameter is very small.

Presently optics can be considered as the main area of physics for measurements of the parameter \( \tau \).

The nonzero term \( \tau_0 \) in the expansion \( T \) is the most favorable case for optical measurements. However, in the case \( \tau_0 = 0 \) measurements are also possible. The experiment would be similar to \( T \), but on the basis of modern technology. Best accuracy may be achieved in a ring schematic similarly measurements of the anomalous magnetic moment of electron. This schematic, regardless of results of experiments (positive or negative), can also be used for high-precise manipulations with the laser frequency in variety applications, in particular, for standards of length and time.

A fundamental constant with dimension time must be on the list of basic physical constants. However, this constant is absent in this list. The parameter \( \tau \) contains this constant and it should be the basic physical constant because it is determined by such a basic physical process as rotation.

The investigation of this problem is very important since the constant defines the limits of the applicability of the basic physical laws for very small intervals of time and length. Moreover, this constant might determine the minimum possible values of such intervals as well as the minimum possible value of energy.

The above opinion of prof. W. H. Steier about the origin of the asymmetry is an argument against funding the high-precise measurements. Nevertheless, the problem of "to be or not to be" (in the sense of Galilean or not) must be solved.

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