Fluxon collider for multiple fluxon–antifluxon collisions

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Abstract. We describe a device for generation and trapping of fluxon–antifluxon (FA) pairs in a long annular Josephson junction. The trapped fluxons and anti-fluxons experience multiple collisions and eventually decay into plasmons. We analyse the energy dissipation in the system and find the criteria for trapping of the FA pair. We describe a possible experiment for realization of multiple FA trapping and formation of a FA plasma. Similar to vortex liquids discussed recently by Anderson it might be a good candidate for another vortex liquid. The proposed device can be made of niobium or high temperature superconductors.
1. Introduction

Josephson vortices or fluxons represent a remarkable class of elementary excitations which are topological defects and solitons at the same time. Although Josephson vortices are different from conventional Abrikosov vortices arising in superconductors in a magnetic field, in many situations they have a similar behaviour. Like Abrikosov vortices, fluxons may form stable lattices, can be driven by external current and propagate over a large distance [1, 2]. On collision they pass through each other. The behaviour of a fluxon and an antifluxon at the collision is very non-trivial. In particular, fluxon–antifluxon (FA) pairs are proposed to be responsible for the flux–flow oscillations observed in a ring made of bismuth strontium calcium copper oxide (BSCCO) [3]. Recently, we have suggested that the FA collision and their subsequent annihilation can be observed in an annular Josephson junction (AJJ) [4]. In the present paper, we propose a device for realization of multiple collisions of any number of FA pairs. In particular, with such a device not only collision of a single fluxon with an antifluxon can be investigated, but a liquid or gas consisting of a large number of mixed fluxons and antifluxons can be experimentally realized. Here we refer to such states as a FA plasma.

The term ‘vortex liquid’ was originally introduced by Feigelman and Ioffe [5]. They mapped the vortex liquid into a two-dimensional (2D) liquid of bosons. This way the Abrikosov lattice was identified as a boson crystal while the vortex liquid as a liquid of bosons. Recently, vortex liquids have been discussed in the literature by Anderson [6] in connection with the anomalous Nernst effect in cuprate superconductors [7] and non-classical rotational inertia in solid He [8]. According to Anderson the state of matter of the ‘vortex liquid’ is distinct from a conventional liquid because its properties are associated with conserved supercurrents flowing around the vortices. One may notice that this also applies to the mix of fluxon and antifluxons. Models of soliton gases similar to the FA plasma have a crucial importance in quantum field theory [9]. For instance, quantum tunnelling between two local minima of a double well potential is theoretically described by an instanton gas model very similar to the FA plasma.

2. Geometry of the device and model

The device for FA collisions consists of an AJJ attached to a Josephson transmission line (JTL) by means of a Y junction, figure 1. The original fluxon is created by a current pulse at the end
Figure 1. Geometrical configuration of the fluxon collider (top view in the $XY$-plane). The geometry is formed by the JTL connected to an AJJ by means of the Y junction. The dashed arrow represents the parental fluxon approaching to the Y junction. The plain arrows represent the FA pair induced by the split parental fluxon. Value $x$ is the coordinate along the ring, $0 \leq x \leq L$, where $L = 2\pi R$ is its circumference. Function $W(x)$ describes the width of the AJJ. Value $d$ is a displacement of the centre of internal circle with respect to the external one. The position of the fluxon inside the ring is denoted by $x_0$.

of this long junction (that we call here parental fluxon) and then moves in the JTL towards the AJJ. For certain velocities of the incident fluxon, it may pass the Y junction without reflection and split into two solitons, similar to the flux cloning effect observed in a T junction [10]. The two new solitons born in the process have opposite vorticities, i.e. form a FA pair. In the ideal case of zero damping the fluxon and antifluxon propagate in the AJJ in different directions, pass through each other on the far side of the ring and merge again at the Y junction. The combined ‘giant’ antifluxon leaves the system and starts to propagate in the direction opposite to the parental fluxon. Therefore, in order to trap the pair in the ring some minimal damping is needed to suppress the formation of the ‘giant’ antifluxon. In this case, after being injected into the ring, the fluxon and antifluxon have no energy to leave the system and experience multiple collisions.

In order to create a trapping potential for a FA pair, the ring is made narrower on one side opposite to the Y junction. Such a geometry is constrained by two circles of radii $R_e$ and $R_i$ with centres shifted by distance $d$ with respect to each other. The width of the AJJ depends on the coordinate $x$ along the ring and is $W(x) \simeq \Delta R + d \cos(x/R)$, where $\Delta R = R_e - R_i$ is the average width and $R = (R_e + R_i)/2$ is the average radius. The width of the attached JTL is denoted by $W_0$ (here and below we work with normalized units with coordinates and distances normalized to the Josephson penetration length $\lambda_J$, velocity normalized to the Swihart velocity $\tilde{c}$, time scaled by $\omega_p^{-1}$ where $\omega_p$ is the plasma frequency, the energy normalized to $j_c \lambda^2 J / \Phi_0/2\pi$, where $\Phi_0 = \hbar/2e$ is the flux quantum and $j_c$ is the critical current density).
Figure 2. (a) Numerical simulations of the dynamics of the superconducting phase in the presence of damping $\alpha = 0.01$ and initial velocity of the parental fluxon $u_0 = 0.8$ (in units of Swihart velocity $\bar{c}$). When the parental fluxon reaches the Y junction, it splits into two solitons. The injected fluxon and antifluxon pass through each other, merge at the Y junction and leave the system as a ‘giant’ antifluxon. (b) Numerical simulations of the dynamics of superconducting phase damping $\alpha = 0.01$ and initial velocity of the parental fluxon $u_0 = 0.4$. In this case, the injected fluxon and antifluxon have not enough energy to combine and get trapped. The colour scale represents the superconducting phase difference $\varphi$. Fluxon and antifluxon are represented by the borders between the light (green) and dark (red and blue) domains. The driving current is absent. Time $t$ is scaled by $\omega_p^{-1}$ where $\omega_p$ is the plasma frequency.

We have simulated the dynamics of the superconducting phase in the system figure 1 with the use of the dissipative 2D sine-Gordon equation written in orthogonal coordinates $X$ and $Y$ as

$$\varphi_{tt} - \nabla^2 \varphi + \alpha \dot{\varphi} + \sin \varphi = 0,$$

and with homogeneous Neumann boundary conditions $\mathbf{n} \cdot \nabla \varphi|_{\partial \Omega} = 0$ where $\mathbf{n}$ is the normal vector defined on the boundary $\partial \Omega$. To solve it we use the finite element program package Comsol Multiphysics. The parameters of the system were $W_0 = 3$, $R_e = 5$, $R_i = 4$, $d = 0.25$. The initial conditions for the phase and its time derivative were of the soliton type $\varphi(X, t) = 4 \arctan \left( \frac{X-X_0}{\sqrt{1-u^2}} \right)$. At the initial moment the parental fluxon is positioned on the JTL at $X_0 = -7.5$ (see figure 1) and has velocity $u_0$ towards the ring. First, we have varied the velocity $u_0$ at fixed damping $\alpha = 0.01$. Two types of behaviour have been observed in this case. If the kinetic energy of the parental fluxon is high enough, the injected fluxon and antifluxon pass through each other and merge again at the Y junction into a ‘giant’ antifluxon propagating away from the ring (figure 2(a)). On the other hand, if the velocity or kinetic energy of the parental fluxon is smaller than critical one, the fluxon and antifluxon are trapped in the AJJ and experience multiple collisions (figure 2(b)). There is a certain critical velocity for trapping of a FA pair by the fluxon collider. We have carried out simulations for different values of damping constant $\alpha$ and have reconstructed the numerical dependence of the critical velocity on $\alpha$ (figure 3).
3. Trapping threshold energy and critical velocity

We have made analytical estimations of the critical velocity needed to trap a FA pair. We assume that the main energy losses are due to the two non-interacting solitons propagating in the ring. In this case, the contribution of each soliton can be evaluated separately. The energy of each fluxon and antifluxon injected into the ring is $H_0 = (E_0 - \Delta E_Y)/2$ where $E_0 = 8 W_0/\sqrt{1 - u_0^2}$ and $u_0$ are the energy and the velocity of the parental fluxon before striking the ring, respectively and $\Delta E_Y$ is the energy dissipated on the Y junction. $\Delta E_Y$ depends on the specific configuration of the Y junction, mainly on the interior angle between the outgoing transmission lines. For the configuration presented on figure 1 where the interior angle of the Y junction is very small, the energy loss $\Delta E_Y$ can be neglected.

In the absence of dissipation and external magnetic fields we may describe a JTL of a varying width $W(x)$ by the effective Hamiltonian

$$H = \int_0^L dx W(x) \left[ \frac{\varphi_x^2}{2} + \frac{\varphi_x^2}{2} + 1 - \cos \varphi \right],$$

where $L = 2\pi R$ is the circumference of the ring. If the width $W(x)$ is a slowly varying function of $x$ compared to the Josephson penetration length, a single fluxon can be described by $\varphi(x, t) = 4 \arctan \left( \frac{x - x_0}{\sqrt{1 - u^2}} \right)$ where $x_0 = x_0(t)$ and $u = x_0(t)$ are the position and velocity of the fluxon at the time $t$, correspondingly. Substituting this into (1), we obtain the energy of the fluxon $H = 8 W(x_0)/\sqrt{1 - u^2}$. The energy of the antifluxon is obviously the same. We employ the energy balance considerations [11] generalized to the case of a long Josephson junction with varying width. The energy dissipation rate is given by the expression $\dot{H} = -\alpha u^2 H$. Therefore,
\[ \frac{d}{dx_0} H = -\alpha u H. \]
Substituting into this equation the expression for velocity
\[ u \simeq \sqrt{1 - \left( \frac{8\Delta R}{H} \right)^2} \]
as a function of \( H \), by a straightforward integration we obtain
\[ \log \left( \frac{H + \sqrt{H^2 - (8\Delta R)^2}}{H_0 + \sqrt{H_0^2 - (8\Delta R)^2}} \right) = -\alpha x_0, \]
where \( H_0 = E_0/2 = 4W_0/\sqrt{1 - u_0^2} \) is the initial energy of a fluxon/antifluxon injected into the ring. The energy of a FA pair is twice the energy of each soliton \( H \). The FA pair can be trapped only when its energy is smaller than the minimal energy required to form a ‘giant’ antifluxon. Thus, the criterion for trapping is \( 2H|_{x_0=L} < 8W_0 \). From this inequality follows a formula for the initial critical velocity,
\[ u_c = \sqrt{1 - \left( \frac{8W_0}{E_c} \right)^2}, \quad \text{(2)} \]
where \( E_c \) is the critical initial energy of the parental fluxon before striking the ring
\[ E_c = 8W_0 \cosh(\alpha L) + 8\sqrt{W_0^2 - (2\Delta R)^2} \sinh(\alpha L). \quad \text{(3)} \]

In figure 3, we have compared these theoretical estimations with our numerical results. The solid line represents the analytical calculations (formulae (2) and (3)), while circles represent numerical data. We have found very close agreement between these results.

4. Proposal for experiments

A possible experiment can be realized as follows. Experimental samples can be implemented with Nb technology or using high temperature superconductors (HTSC) such as BSCCO. In order to create the parental fluxon a current pulse is generated on one side of the JTL [12]. The velocity of a fluxon can be controlled by the external current. The relation between the velocity of a fluxon and the applied external current can be estimated by formula (2.11) in [11]. For typical parameters of Nb junctions and HTSC, oscillations of the trapped FA pair fall into the GHz frequency range. Therefore microwave radiation can be used to detect and measure the FA pair oscillations. By means of microwaves one may also support these oscillations for a long time. Stabilization of oscillatory excitations in long Josephson junctions by a/c drive was studied in [13].

5. FA plasma and fluxon liquid

Now consider the general situation when not one but many parental fluxons are sent to a fluxon collider of a larger size, which may accumulate very many fluxons and antifluxons. This pumping
of parental fluxons initiates a neutral gas of fluxons and antifluxons mixed together. These fluxons and antifluxons will chaotically move around the trap area colliding and penetrating through each other. The final state of such a system consisting of the chaotically moving fluxons and antifluxons will depend on their density and the geometry of the trap area. Such a state of fluxons and antifluxons has never been discussed in the literature and can be called a FA plasma. Note that the total number of confined fluxons will be always equal to the number of antifluxons in the gas. Although some fluxons and antifluxons may combine together at the Y junction and leave the system, the overall neutrality will not be affected. The analysis of physical properties of the FA plasma under more general conditions deserves a separate publication.

6. Fluxon interferometer

The proposed device appears to be sensitive to an external magnetic field and has the potential for application as a sensor. The application of a magnetic field shifts the critical velocity up (dashed line on figure 3). The impact of a magnetic field on the collider depends on the magnetic field orientation. The X component of the field modulates the velocities of a fluxon and an antifluxon and thus may only indirectly lead to additional dissipative losses. The influence of the Y component of the magnetic field is more interesting. As the Y component breaks the symmetry of the system, different potentials arise in which a fluxon and an antifluxon move. Thus, besides the modulation of their velocities \( u \), the Y component of the magnetic field causes a lag between the two solitons. A fluxon and an antifluxon no longer come simultaneously to the Y junction and therefore do not have energy to combine into a ‘giant’ antifluxon and to leave the system. Thus, the interference of a fluxon with an antifluxon due to the external magnetic field influences their subsequent evolution, in particular, whether a ‘giant’ antifluxon will be created or not. Because the fluxon and the antifluxon can move with velocity close to the speed of light, a single sensing event can be very quick, of the order \( \tau \sim 2\pi R/\bar{c} \sim 10^{-11} \) s where \( \bar{c} \) is the Swihart velocity and \( R \) is the circumference of the ring. Although the sensitivity of such sensors may not supersede that of conventional SQUIDs, these devices can be used when the rapidity of measurements is particularly essential. A different realization of a fluxon interferometer has been recently discussed in [14].

7. Summary

To summarize, we have described a device for realization of multiple collisions of a fluxon and an antifluxon. The fluxon collider consists of an AJJ attached to a thick JTL by means of a Y junction that splits an incident fluxon into two. We analytically formulate the criteria for trapping of a FA in the fluxon collider and present numerical simulations made with the dissipative 2D sine-Gordon equation. The analytical and numerical results reveal a very close agreement. The device is sensitive to microwave radiation and externally applied magnetic fields that prompts possible applications of the device as a radiation detector or magnetic field sensor. The fluxon collider could become important for fundamental studies. In particular, the fluxon collider can be used to realize a FA plasma—a good candidate for another state of matter.
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