Deflected Wake Interaction of Tandem Flapping Foils

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Flapping foils are known to produce deflected jets at high frequency-amplitude combinations even at a zero mean angle of attack. This reduces the frequency range of useful propulsive configurations without side force. In this study, we analyse numerically the interaction of these deflected jets for tandem flapping foils, undergoing coupled heave to pitch motion. The impact of the flapping Strouhal number, foil spacing and phasing on wake interaction is investigated. Our primary finding is that the back foil is capable of completely cancelling the deflected wake and mean side force of the front foil, even when located up to 5 chord lengths downstream. This is achieved by attracting the incoming dipoles and disturbing their cohesion within the limits of the back foil’s range of flapping motion. We also show that, the impact on cycle averaged thrust varies from high augmentation to drag generation depending on the wake patterns downstream of the back foil. These findings provide new insights towards the design of biomimetic tandem propulsors by expanding their working envelope and ability to rapidly increase or decrease the forward speed by manipulating the size of the shed vortices.

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1. Introduction

Due to their prevalence within the airborne and aquatic wildlife, single flapping foils have caught the interest of scientists and engineers alike, since the early twentieth century (Knoller 1909; Betz 1912). Moreover, tandem configurations e.g. insect wings (Alexander 1984; Thomas et al. 2004), plesiosaur flippers (Muscutt et al. 2017\(^a\)) etc. have shown propulsive benefits under certain wake-to-wake interactions.

The onset of thrust generation is marked by a reverse von Kármán street downstream of a flapper (Von Karman 1935) although a lag between the two conditions exist (Godoy-Diana et al. 2008; Bohl & Koochesfahani 2009; Lagopoulos et al. 2019). This wake pattern is determined by the oscillating trailing edge (T.E.) amplitude \(A\) and frequency \(f\) of the motion which, form together an \textit{amplitude based} Strouhal number \(St_A = (2fA)/U_\infty\) as described by Triantafyllou et al. (1993). An increasing \(St_A\) can lead

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to permanent deflection of the jet and thus side force generation even when both the camber and the mean angle of attack are zero \cite{Godoy-Diana2009, Cleaver2012}. This is the result of dipole formation when shedding vortices become strong enough to attract each other and depart from the centerline \cite{Godoy-Diana2009, Godoy-Diana2008}.

Multiple foil configurations have also been considered by various researchers as potential strategies to improve propulsive performance. Tandem flapping foils are shown to improve thrust generation via wake recapture both numerically \cite{Muscutt2017b, Broering2012, Akhtar2007} and experimentally \cite{Muscutt2017a, Warkentin2007, Usherwood2008}. More specifically, thrust and efficiency augmentation can be achieved when the hind foil is weaving within the incoming vortices shed by the front one, determined by the inter foil spacing and phase lag.

A prominent feature of these studies is the relative lack of influence the downstream foil is said to have on the wake and forces of the upstream foil. However, those studies focus on cases with symmetric reverse von Kármán street where momentum exchange between shedding vortices is minimal. On the other hand, vortices of deflected wakes are often in very close proximity to each other, forming a long chain of well defined and correlated dipoles.

This paper focuses on the interaction of the deflected wakes (figure 2) for two rigid oscillating foils in a tandem configuration (figure 3). Two dimensional simulations are conducted, while the foils undergo harmonic motion. It is revealed that certain phase-spacing combinations neutralise deflection for both foils while significantly increasing the overall thrust of the system. Distinct types of wake to wake interaction are observed and described in terms of their propulsive characteristics. In addition we clarify the mechanism of deflection cancellation and determine its limits in terms of a simple non-dimensional parameter, the spacing based Strouhal number.

1.1. Geometry and Kinematics

As in the previous work conducted by \cite{Muscutt2017b} our tandem system consists of two rigid NACA0016 with a thickness $D = 0.16 C$ (where $C$ is the chord length) undergoing sinusoidal heave to pitch coupling around the quarter chord, against a uniform free stream velocity $U_\infty$. Pure pitch is simply the harmonic rotation about the pivot point $P = 0.25$ (normalised by the chord $C$) while pure heave is the harmonic vertical translation of the foil. Thus coupling for a single flapping foil is the superposition of these two kinematic components:

$$y_s(t) = y_h(t) + y_\theta(t) \quad \text{where} :$$

$$y_h(t) = h_0 \sin(2f \pi t) , \quad y_\theta(t) = (1 - P)C \sin(\theta(t)) \quad \text{and}:$$

$$\theta(t) = \theta_0 \sin(2f \pi t + \psi)$$

The subscript $s$ refers to the single foil configuration. Furthermore, the instantaneous value of pure pitch is expressed as the harmonic $\theta(t)$ with the $h_0$ and $\theta_0$ being the amplitudes of pure heave and pure pitch. Previous work \cite{Platzer2008} considers an optimal phase difference between pitch and heave of $\psi = 90^\circ$ which is also implemented in this study.

Another important kinematic parameter in coupled motions is the effective angle of attack $\alpha_{eff}(t)$ which is the summation of the instantaneous pitch angle $\theta(t)$ and the heave induced angle of attack. Thus for $\psi = 90^\circ$ the amplitude of $\alpha_{eff}(t)$ is:
\[
\alpha = \arctan \left( \frac{2\pi f h_0}{U_\infty} - \theta_0 \right)
\]

where \(2\pi f h_0\) is the amplitude of \(dy_h/\text{dt}\). Here, we set \(\alpha = 10^\circ\) to achieve high efficiency kinematics \cite{Muscutt2017b}.

Inter foil distance is indicated by \(S\) commonly referred to as spacing and measured in units of \(C\) or the thickness normalised spacing \(S_D = S/D\). Furthermore, the phase lag between the two foils is expressed as \(\phi\) and will be referred to as simply the phase:

\[
y_f(t) = y_h(t) + y_\theta(t)
\]

\[
y_b(t) = y_h(t + \phi) + y_\theta(t + \phi)
\]

where subscripts \(f\) and \(b\) denote the front and back foils respectively.

1.2. Dimensionless parameters

Three non-dimensional parameters are used to describe the interaction between the oscillating foil and the free stream: the Reynolds number based on chord length, \(Re = U_\infty C/\nu = 1173\) (where \(\nu\) is the kinematic viscosity) the thickness based Strouhal number, \(Sr\) and the non-dimensional peak-to-peak T.E. amplitude \((A_D)\):

\[
Sr = \frac{Df}{U_\infty}, \quad A_D = \frac{2A}{D}
\]

where \(A\) is the cumulative T.E. amplitude of the coupled motion (see figure 1). Moreover, \(St_A = Sr \cdot A_D\) and can be understood as the ratio between the speed of the foil tip and \(U_\infty\) \cite{Godoy2009}.

The \(x\) and \(y\) force are normalized by the chord and by the dynamic pressure of the freestream to give the domain’s fixed thrust and lift coefficients. Cycle averaged quantities are presented with an overbar to distinguish them from their instantaneous counterparts.

\[
C_t = \frac{F_x}{\frac{1}{2} \rho U_\infty^2 C}, \quad C_l = \frac{F_y}{\frac{1}{2} \rho U_\infty^2 C}
\]

1.3. Computational Method

The CFD solver utilized in this work is capable of simulating complex geometries and moving boundaries for a variety of Reynolds numbers in 2D and 3D domains, via...
N. S. Lagopoulos, G.D. Weymouth and B. Ganapathisubramani

Figure 2. Normalised vorticity field of a flapping foil undergoing harmonic heave to pitch coupling at (a) $Sr = 0.1$, (b) $Sr = 0.25$ and (c) $Sr = 0.4$. All produced wakes are steadily deflected for at least 40 cycles generating $C_{t,s} \sim 0.4$.

| $Sr$ | $A_D$   | $C_{t,s}$ | $C_{l,s}$ |
|------|---------|-----------|-----------|
| 0.10 | 8.7792  | 0.59      | 0.400     |
| 0.15 | 4.0676  | 0.49      | 0.400     |
| 0.20 | 2.7301  | 0.42      | 0.400     |
| 0.25 | 2.0360  | 0.37      | 0.400     |
| 0.30 | 1.6112  | 0.32      | 0.390     |
| 0.35 | 1.3334  | 0.28      | 0.395     |
| 0.40 | 1.1331  | 0.33      | 0.405     |

Table 1. Kinematics and cycle averaged force coefficients of single flapping foils undergoing heave to pitch coupling for a variety of $Sr$, $\alpha = 10^\circ$ and $\psi = 90^\circ$. All cases result in steady wake deflection.

the boundary data immersion method BDIM, [Schlanderer et al. 2017]. BDIM solves the viscous time-dependent Navier-Stokes equations and simulates the entire domain by combining the moving body and the ambient fluid through a kernel function. This technique has quadratic convergence and has been validated for flapping foil simulations over a wide span of kinematics ([Maertens & Weymouth 2015; Polet et al. 2015]).

The mesh profile is a rectangular Cartesian grid. A dense uniform grid is used near the body and in the near wake while an exponentially stretched mesh is used in the far-field. The boundary conditions consist of a uniform inflow, zero-gradient outflow and free-slip conditions on the upper and lower boundaries. Moreover, no slip conditions are imposed on the surface of the oscillating foil. Grid density is indicated by the number of grid points per chord. A uniform grid of $\delta x = \delta y = C/192$ is used for the numerical experiments, which showed that this resolution resulted in force predictions with less than 3% error compared to the converged values ([Lagopoulos et al. 2019]).

2. Results and discussion

2.1. Single foil analysis

Single foil arrangements undergoing harmonic heave to pitch coupling are tested for $Sr \sim [0.1, 0.4]$. The $St_A$ is chosen so that the wake deflection converges to a consistent $C_{t,s} \sim 0.4$ across the $Sr$ range. All single foil simulations ran for at least 40 cycles and the chosen $A_D$ values and resulting $C_{t,s}$ are shown in table 1.

The resultant wakes under the coupled motion (see figure 2) are similar to those reported in literature for pure pitch ([Godoy-Diana et al. 2009, 2008; He et al. 2012] and
Figure 3. Snapshots of normalised vorticity for tandem configurations at cycle increments of 1/4. Three distinct wake patterns of lift cancellation can be seen namely, (a) type I, (b) type II and (c) type III. The first mode is generated for $Sr = 0.1$, $S = 5$ and $\phi = 1.6\pi$. The second for $Sr = 0.1$, $S = 2$ and $\phi = 1.375\pi$ while the third case is derived for $Sr = 0.25$, $S = 1$ and $\phi = 0.875\pi$.

pure heave (Cleaver et al. 2012; Kozlowski & Kudela 2014). While the first vortex is independently shed away from the centerline, the distance between the second and the third subsequent vortices is noticeably smaller. This results in the formation of a dipole as shorter distances lead to stronger synergy among vortices according to the Biot – Savart vortex induction law (Zheng & Wei 2012). This initial dipole departs from the centerline, breaking the symmetry of the mean jet and imposing its path to the subsequent dipole (Godoy-Diana et al. 2008). Furthermore, the three cases shown in figure 3 indicate that $f$ is proportional to the vortex circulation $\Gamma$ and inversely proportional the distance between consecutive vortices in agreement with Godoy-Diana et al. (2008).

2.2. Tandem foil analysis

Tandem foil configurations are tested for the same kinematics and frequencies as the single foils. The system is tested for a wide range of spacings $S \sim [1C - 6C]$ and $\phi \sim [0\pi - 1.75\pi]$ in increments of 0.125$\pi$. Remarkably, for certain $S$ - $\phi$ combinations the presence of the downstream foil results in a stable symmetric wake and zero net lift, even for the upstream foil (see figure 3). Specifically, we define lift cancellation as the condition when:
Figure 4. Cycle averaged Lift coefficient versus phase lag for a tandem foil configuration at $Sr = 0.2$ and (a) $S = 3$, (b) $S = 4$. The red cross in plot (a) marks lift cancellation.

\[ |Cl,f| > \epsilon , \quad |Cl,b| > \epsilon , \quad |Cl,f + Cl,b| > \epsilon \]

where $\epsilon = 0.05 Cl,s$ (2.1)

This lift cancellation condition is illustrated for two spacing at $Sr = 0.2$ in figure 4. At $S = 3$ and $\phi = 1.5\pi$ both $Cl$ curves go to zero simultaneously, demonstrating complete lift cancellation on the front and back foil due to wake interaction. However, at $S = 4$, while the back foil does significantly adjust the lift force on the front foil, there is no phase shift $\phi$ which causes lift cancellation on both foils.

The number of possible lift cancellation instances for each $\phi - S_D$ combination investigated at $Sr \sim [0.1, 0.4]$, is shown in Figure 7. It is clear that smaller spacings lead to more chances of lift cancellation and a similar trend exists for the frequencies as stronger wakes (for $Sr > 0.25$ and above), are difficult to control. Although most published work supports the idea that downstream flow has no impact on the propulsive characteristics of the fore foil, two lift cancellation instances are reported for $Sr = 0.1$ at $S \sim 5$.

2.3. Discussion

The mechanism for lift cancellation is depicted in figure 5a. The initial dipole shed from the leading foil advects downstream and splits in to two after colliding with the vertically moving aft foil. Consequently, the advection speed of the clockwise vortex (red) decreases, which reduces its distance from the subsequent dipole shed by the front foil. This affects the cohesion of the second dipole and its clockwise vortex (blue) is now under the influence of two counter-clockwise vortices (one its own and the other from the first dipole). This situation propagates upstream, affecting every upstream dipole in the same manner, resulting in wake convergence towards the centreline forming a classic reverse von Kármán street.

As the collision of the first dipole with the rear foil is the necessary condition for the lift cancellation, we need to examine the circumstances that would lead to this interception. By definition, a collision between two entities is only possible if their paths intersect. Assuming that the first dipole advects downstream along an inclined path set by the initial advection speed, it can be shown that the dipole travels downstream at an angle of $\tan \theta_d \propto fA/U_\infty$ (or $\tan \theta_d = pfA/U_\infty$). This is based on assuming that the horizontal
In line wake interaction

Figure 5. Captions of normalised vorticity at $Sr = 0.1$ show the mechanics of lift cancellation. Wake deflection is halted after a successful collision of the first shed upstream dipole with the leading edge (L.E.) of the aft foil (a) which can be achieved at an angle greater or equal to $\theta_d$ (b).

Advection speed is proportional to the freestream speed ($U_\infty$) and the vertical advection speed is proportional to the vertical speed of the trailing edge of the front foil ($fA$). Note that the actual path of the dipole is not a straight line as it follows a more complex elliptic path. However, the factors leading to the overall angle are sufficient for this discussion. To achieve lift cancellation, this angle, $\tan \theta_d$, must be smaller than the largest angle between the trailing edge of the front foil and the L.E. of the back foil, $\tan \theta_g = A/S$ so that:

$$\tan \theta_d \leq \tan \theta_g \rightarrow pfA/U_\infty \leq A/S$$

$$\therefore fS/U_\infty \leq 1/p$$

This suggests that there is a spacing based Strouhal number that will act as a clear boundary between areas where lift cancellation is possible and areas where wake deflection is maintained. This geometric relationship accounts for all possible phase differences between the fore and aft foils and could even be independent of the frequency of the aft foil. This Strouhal number only depends on the ratio between the horizontal and vertical advection speeds of the dipole shed by the front foil, which probably varies with the kinematics of the front foil.

The number of occurrences of lift cancellation are plotted on the map of Figure 7 as a function of non-dimensional spacing ($S_D$) and non-dimensional frequency ($fD/U$). It can be clearly seen that lift cancellation instances (for different phase differences) reduce to zero above a certain region marked with dashed solid black curve. This curve has the form, $Sr * S_D = fS/U_\infty = constant$. Fitting this equation to the data in figure 7 gives a spacing based Strouhal number that determines the lift cancellation border at $St_S = fS/U_\infty \sim 4$. Taking into account 2.2 this means that $p = 1/4$. In other words, for a given spacing $S = 1C$ lift cancellation is only possible when the vertical speed of the foil is approximately less than (or equal to) a quarter of the horizontal advection speed of the first shed dipole.

Interestingly the highest concentration of lift cancellation instances is observed at $S_D = 6.25$ for $Sr = 0.1$ and $Sr = 0.25$. This $S_D$ coincides with a spacing of $1C$ and thus any differences among various $Sr'$s are related to the flapping $f$ at the early stages of wake development. At the beginning of the foil’s oscillation a temporary dipole is formed by the first two shed vortices and then moves upwards. As the third subsequent vortex reaches the dipole it attracts one of the vortices forming a new, permanent dipole which then travels elliptically (see figure 5).

As seen in figure 6 for $Sr \leq 0.2$ and $S_D = 6.25$ the temporary dipole is still present.
Yet higher frequencies bring the vortices closer making it harder for the back foil to interfere; this is why at this Strouhal range more lift cancellation instances are present at $Sr = 0.1$. On the other hand, for $Sr \geq 0.2$ the first permanent dipole has been formed and at $S_D = 6.25$ it is moving towards the centerline (as opposed to the upward motion of the temporary dipole). However, once again higher Strouhals result in tighter dipoles and thus harder to dissolve by the back foil. This is essentially a Goldilocks case where the $Sr = 0.25, S_D = 6.25$ is the ideal combination for lift cancellation throughout the tested parameter space.

Also noted that, there are more than one ways of dipole decompositions and this is linked to the type of initial dipole - back foil interaction. The resultant pattern affects both the front and the back wake and has a crucial effect on the systems overall thrust. In figure 3 we see three possible modes of lift cancellation:

- **Type I**, where the back foil slides within the channel between the two vortices that form the incoming dipole (see figure 3a). This leads to a remarkable thrust enhancement as the aft foil performs in an accelerated field [Muscett et al. (2017)](2017b) e.g. the configuration of figure 3a generated a $C_{t,f} \sim 0.97$ and a $C_{t,b} \sim 3.08$.

- **Type II**, where the back foil collides with one of the two vortical components of the dipole (see figure 3b). The back foil - vortex collision usually involves a significant drag penalty as the impact increases the friction and slows the incoming flow.

- **Type III**, which is effectively an intermediate condition between the previous two modes. More specifically, at $S < 2$ or $Sr > 0.15$ even at proper $\phi$ the incoming vortices slide upon the boundary layer of the aft foil. For fast convected vortices of small size (at high Sr’s) two symmetrical deflected wakes are developed at the L.E. of the aft foil (see figure 3c). As the vortex - foil interaction varies from minimal (towards Type I mode) to significant (towards Type II mode), Type III modes do not present a clear thrust or drag enhancing behaviour.

### 3. Conclusions

Flapping foils generating fully deflected wakes, are analysed in both single and tandem configurations undergoing coupled heave and pitch kinematics. The characteristics of single foil deflected wakes are similar to those reported for pure heave or pitch motions with the dipole formation being the driving mechanism of wake symmetry breaking.

Tandem configurations are able to reorder deflected jets into symmetric wakes with $C_{t,f} = C_{t,b} \sim 0$. Certain $\phi$, $S_D$, $Sr$ combinations are shown to direct both wakes back to the centerline even for inter foil distances up to 5 chord lengths. To achieve lift cancellation the back foil has to dissolve the first shed dipole of the front wake and this is achieved when the angle between the T.E. of the front foil and the L.E. of the back
is greater than the dipole's convection angle. This is further expressed via a maximum spacing based Strouhal, $St_S = 4$, above which any lift cancellation is impossible.

When the total lift of the tandem configuration is neutralised symmetrical patterns appear downstream of the back foil. Three main modes are reported. Type I, occurs when the back foil separates the two vortical components of the incoming dipole by weaving between them which leads to a remarkable increase in $C_t$. Type II mode occurs when the L.E. directly collides on one of the dipole's vortices which introduces a significant drag penalty. In addition, an intermediate mode Type III is reported, whose behaviour varies according to the intensity of the vortex-foil collision.

This study is the first to provide evidence of the significant impact of the downstream field to the front foil. In addition, it is demonstrated that the wake deflection can be diminished with a subsequent remarkable thrust enhancement. These findings can support the design of high performance biomimetic propulsors as a simple change of the back foil’s phase enables high thrust generation without side force at high frequencies previously considered impossible.

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