Non-Thermal Production of WIMPs and the Sub-Galactic Structure of the Universe

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There is increasing evidence that conventional cold dark matter (CDM) models lead to conflicts between observations and numerical simulations of dark matter halos on sub-galactic scales. Spergel and Steinhardt showed that if the CDM is strongly self-interacting, then the conflicts disappear. However, the assumption of strong self-interaction would rule out the favored candidates for CDM, namely weakly interacting massive particles (WIMPs), such as the neutralino. In this paper we propose a mechanism of non-thermal production of WIMPs and study its implications on the power spectrum. We find that the non-vanishing velocity of the WIMPs suppresses the power spectrum on small scales compared to what it obtained in the conventional CDM model. Our results show that, in this context, WIMPs as candidates for dark matter can work well both on large scales and on sub-galactic scales.

There is strong evidence for the existence of a substantial amount of cold dark matter (CDM). The leading candidates for CDM are weakly interacting massive particles (WIMPs), such as the neutralino. The neutralino is the lightest supersymmetric particle. In models with R parity it is stable, and its mass density in the universe is generally assumed to be a relic of an initially thermal distribution in the hot early universe. Assuming, in addition, the presence of a small cosmological constant, the CDM scenario is consistent with both the observations of the large scale structure of the universe (\(\gg 1\)Mpc) and the fluctuations of the cosmic microwave background \([1]\). Many experiments searching for neutralino dark matter particles are under way.

The collisionless CDM scenario, however, predicts too much power on small scales, such as a large excess of dwarf galaxies \([2,3]\), the over-concentration of dark matter in dwarf galaxies \([4]\) and in large galaxies \([5]\). Recently Spergel and Steinhardt proposed a new concept of dark matter with strong self interaction \([6]\). This puts WIMPs as candidates for dark matter in considerable jeopardy \([7,8]\).

In this paper we propose a scenario with non-thermal production of WIMPs. These WIMPs could be relativistic when generated. Their comoving free-streaming scales could be as large as of the order 0.1 Mpc or larger. The density fluctuations on scales less than the free-streaming scale would then be severely suppressed. Consequently the discrepancies between the observations of dark matter halos on the sub-galactic scales and the predictions of the standard WIMPs dark matter picture could be resolved.

To begin with, we consider a general case of non-thermal production of the neutralinos by the decay of topological defects such as cosmic string \([9]\), by the decay of an unstable heavy particle, or produced non-thermally by the reheating process in a scenario of inflation at low energy scale \([10]\) (see also \([11]\)). The momentum distribution function of the neutralinos is for simplicity assumed to be Gaussian:

\[
f(p) = \frac{A}{\sqrt{2\pi} \sigma} \exp \left( -\frac{(p - p_c)^2}{2\sigma^2} \right),
\]

where \(p_c\) is the central value and \(\sigma\) describes the width of the distribution.

Given a model, the parameters \(p_c\) and \(\sigma\) can be determined. For instance, in the supersymmetric version of the \(U_{B-L}(1)\) model, the (Higgsino-like) neutralinos arise directly from the decay of the right-handed neutrinos and their superpartners \([12]\). In this model, \(p_c\) is about a half of the mass of the mother particles. For a two-body decay, if the mother particle is at rest, the distribution function \(f(p)\) is a \(\delta\)-function. The value of \(\sigma\) characterizes the average non-vanishing velocity of the mother particles (when \(\sigma \to 0\), \(f(p)\) approaches a \(\delta\)-function). In the model of Ref. \([11]\), since the mother particles when released from the cosmic string loop are non-relativistic, \(\sigma\) is small compared with \(p_c\) and one has \(p_c \approx <p> > \approx <p^2>^{1/2}\).

In Eq.(0.2), \(A\) is a normalization factor determined by the energy density of the non-thermal component

\[
\rho_{NT} = 4\pi \int E(p)f(p)p^2dp,
\]

where \(E(p) = (p^2 + m^2)^{1/2}\) and \(m\) is the rest mass of the dark matter particle. Given that the physical momentum \(p(t)\) scales as the inverse of the cosmic scale factor \(a(t)\), we define \(r \equiv a(t)p(t)/m\). During cosmic evolution \(r\) is a constant. Throughout this paper we set \(a(t_0) = 1\), so \(r\) can be understood as the velocity of the particles at the present time (note that the dark matter particles are non-relativistic now even though they are relativistic when generated).
The comoving free-streaming scale $R_f$ for the non-thermal particles can be calculated as follows\cite{14,15}: 

\[
R_f = \int_{t_s}^{t_{EQ}} \frac{v(t')}{a(t')} dt' \simeq \int_0^{t_{EQ}} \frac{v(t')}{a(t')} dt' \\
\simeq 2r_c t_{EQ}(1 + z_{EQ})^2 \ln \left( \frac{1 + \frac{1}{r_c^2(1 + z_{EQ})^2} + \frac{1}{r_c(1 + z_{EQ})}}{r_c(1 + z_{EQ})} \right),
\] (0.3)

where $r_c \equiv a(t)p_c(t)/m = p_c(t_0)/m$ and the subscript 'EQ' denotes radiation-matter equality. Below the free-streaming scale, the power spectrum will be severely damped. To account for the lack of substructure in the Local Group, N-body simulations study show that the free-streaming scale of the dark matter should be $\sim 0.1 h^{-1} \text{Mpc}$\cite{16}. This corresponds to $r_c \sim 10^{-7}$, which gives rise to a constraint on the parameters of our model.

To calculate the power spectra for our models, we adapt the CMBFAST code\cite{17} so that it applies to a thermal as well as a non-thermal dark matter distribution. To check the program we replaced the non-thermal distribution function in our adapted code by the standard thermal distributions for hot dark matter and cold dark matter respectively, and calculated the power spectra, obtaining the same results as with the standard CMBFAST code.

In the numerical calculations, we took the presently favored values $\Omega_m = 0.4$, $\Omega_\Lambda = 0.6$, $h = 0.65$ and $\Omega_b h^2 = 0.02$ of the relevant cosmological parameters, where $\Omega_m$, $\Omega_\Lambda$, and $\Omega_b$ are the ratios of the contributions from total mass, vacuum energy, and baryons to the total density of the universe. In general, there are two sources of WIMPs which contribute to $\Omega_m$, one is from thermal production, the other is from non-thermal production. For simplicity, in our study we neglected the contribution of WIMPs from the thermal production mechanism.

In Fig. 1, we show the power spectrum of our model with $r_c = 1.5 \times 10^{-7}$. For comparison, we also plot the power spectra for the conventional CDM model and the warm dark matter (WDM) model with $m_W = 1 \text{keV}$ (For this mass the free-streaming scale is $0.11 h^{-1} \text{Mpc}$\cite{18}). Throughout this paper, we normalize all power spectra by the COBE observations\cite{19}.

We have varied the parameters of the non-thermally produced WIMPs dark matter model in the numerical calculations. Different values of $r_c$ gives rise to different power spectra. However the spectrum is independent of the rest mass $m$ of the dark matter particle (for fixed $r_c$), and is very insensitive to $\sigma$ for $\sigma \ll p_c$. This can be understood easily from Eq. (0.3) which shows that the comoving free-streaming scale $R_f$ only depends on $r_c$.

One can see from Fig. 1 that on large scales, the three power spectra are the same, which shows that our model retains all the merits of the conventional CDM model on large scales, while on sub-galactic scales, the power spectrum of non-thermal dark matter (NTDM) model is damped severely relative to that of the CDM model, and is close to that of the WDM model with $m_W = 1 \text{keV}$. Ref.\cite{16} argued that such a warm dark matter scenario provides a solution to the substructure problem.

The model we present in this paper with non-thermal production of WIMPs provides a promising scenario for large-scale structure formation of the universe. However, we need to check its consistency with observations on small scales, especially on scales of the Lyman-\(\alpha\) forest. In Fig. 2 we plot the power spectrum of our model and the observed power spectrum of the Lyman-\(\alpha\) system at $z = 2.5$ shown as the filled circles with error bars\cite{20,21}. For comparison we also give the power spectra for the conventional CDM and WDM models. In fitting to the observed data, we choose the primordial spectral index $n = 0.97$ for all models. The mass for the WDM particles is chosen as $750 \text{eV}$, and the parameters for the NTDM models are $r_c = (1.3, 1.4, 1.5) \times 10^{-7}$, respectively.

To obtain a quantitative constraint on $r_c$, we closely follow the study by Narayanan et al in Ref.\cite{15}. They studied explicitly the power spectrum of WDM models and came to the conclusion that any mechanism which suppresses the conventional CDM linear power spectrum more severely than a $750 \text{eV}$ WDM particle will be inconsistent with Lyman-\(\alpha\) forest observations in the scale range $-2.5 \leq \log(k\text{[km}^{-1}\text{s}]) \leq -1$. This requires that the linear theory power spectrum of any substitute to the conventional CDM models be higher than that of a WDM model with $m_W = 750 \text{eV}$\cite{15}. For a flat universe with $\Omega_\Lambda = 0.6$ the range of these scales corresponds to $0.4 h^{-1} \text{Mpc} \leq k \leq 12.8 h^{-1} \text{Mpc}$. From Fig. 2 one can see that the larger the value of $r_c$ is, the lower the small-scale power spectrum becomes. By comparing the values of the power spectra at the upper limit of the above range in $k$ with the power spectrum of the WDM model with $m_W = 750 \text{eV}$, we obtain an upper limit on $r_c$ of $r_c \leq 1.5 \times 10^{-7}$.

Next we examine the constraints on our model by the recent studies of the phase-space density\cite{22,24}. The particle phase-space density $Q$ is defined as $Q \equiv \rho / \langle v^2 \rangle^{3/2}$, where $\rho$ is the energy density and $\langle v^2 \rangle$ is the mean square value of the particle velocity. The astrophysically observable quantity is the mean coarse-grained phase-space density. In the absence of dissipation, the coarse-grained phase space density can only decrease from its primordial value. Thus, one can use the observed maximum phase-space density to set a lower limit on the phase-space density for the dark matter particles. The highest observed phase-space density is obtained from dwarf spheroidal galaxies: $Q_{\text{obs}} \sim 10^{-4} M_\odot \text{pc}^3 \text{/(km/s)}^3$\cite{22}. For our models, $\langle v^2 \rangle \simeq r_c^2$ at the present time, and therefore

\[
Q_0 \simeq \rho_{NT0} / r_c^3.
\] (0.4)
Because the primordial phase space density decreases with time when the particles are relativistic and becomes a constant after the particles become non-relativistic, one requires \( Q_0 > Q_{\text{obs}} \), which can be translated into a constraint on \( r_c \):

\[
r_c < \left( \frac{\rho_{NT0}}{Q_{\text{obs}}} \right)^{1/3} \sim 2.5 \times 10^{-7}.
\]

(0.5)

This limit is of the same order of magnitude but slightly weaker than that from the observations of the Lyman-\( \alpha \) forest.

In conclusion, we have demonstrated that our model with non-thermally produced WIMPs is consistent with observations on scales ranging from the Lyman-\( \alpha \) systems to the CMB. Compared to the conventional CDM model, the power spectrum of our model on sub-galactic scales is severely damped, which will account both for the lack of substructure in the Local Group and for the observed smooth inner halos\[1\]. In addition, during the structure formation, the non-vanishing velocity of the WIMPs may further reconcile the discrepancies between the theoretical predictions and observations on sub-galactic scales \[24,40\]. However, the detailed effect of the dark matter velocity on the formation of halos needs further study with high resolution N-body simulations.

To solve the sub-galactic-scale problem, our study in this paper requires the value \( r_c \sim 1.5 \times 10^{-7} \). This has implications on the models of non-thermal production of WIMPs. In the scenario of Ref. \[11\] where neutralinos are released by cosmic string decay, \( r_c \) can be approximately expressed as

\[
r_c \sim \frac{T_0}{2m}(\frac{m}{M/T_d}),
\]

where \( T_0 \) is the present temperature, \( T_d \) is the temperature when the string decay first leads to nonthermal neutralinos, and \( M \) is the characteristic mass of the mother particle and the scale of the defect which determine the initial momentum. For \( m \) in the range of 50Gev to 100Gev, \( M \) is required to be in the range of \( 1.6 \times 10^8 \text{Gev} \) to \( 6.4 \times 10^8 \text{Gev} \). This is consistent with the results given in Ref. \[11\] and shows that the value which we used in the text is quite realistic.

In summary, the discrepancies between theory and observations on sub-galactic scales disfavors the conventional WIMPs CDM model. In this paper we show that if the dark matter particles have a non-thermal origin, these discrepancies can be resolved and the WIMPs remain good candidates for the dark matter particle.

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There are other alternatives to the Spergel-Steinhardt solution to the discrepancies, for instance, the warm dark matter models \[14,22,28\], the proposal of damping of the primordial spectrum on sub-galactic scales in some broken-scale-invariance (BSI) inflationary scenarios \[10\], among others \[29,30\].
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FIG. 1. Comparison of the power spectra of the CDM model (long dashed curve), the WDM model with $m_W = 1$ keV (short dashed curve) and the NTDM model with $r_c = 1.5 \times 10^{-7}$ (solid curve).
FIG. 2. The power spectra of the CDM model (long dashed curve), the WDM model with $m_W = 750\text{eV}$ (short dashed curve) and the NTDM models with $r_c = (1.3, 1.4, 1.5) \times 10^{-7}$ (solid curves, from top down), compared to the observed Lyman-α $P(k)$ at $z = 2.5$ (filled circles with error bars).