Formation of the $Q$ ball in the thermal logarithmic potential and its properties

Shinta Kasuya

Department of Information Sciences, Kanagawa University, Kanagawa 259-1293, Japan

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We investigate the $Q$-ball formation in the thermal logarithmic potential by means of the lattice simulation, and reconfirm qualitatively the relation between $Q$-ball charge and the amplitude of the Affleck-Dine field at the onset of its oscillation. We find time dependence of some properties of the $Q$ ball, such as its size and the field value at its center. Since the thermal logarithmic potential decreases as the temperature falls down, the gravity-mediation potential will affect the properties of the $Q$ ball. Even in the case when the gravity-mediation potential alone does not allow $Q$-ball solution, we find the transformation from the thick-wall type of the $Q$ ball to the thin-wall type, contrary to the naive expectation that the $Q$ balls will be destroyed immediately when the gravity-mediation potential becomes dominant at the center of the $Q$ ball.

I. INTRODUCTION

$Q$-ball formation is ubiquitous in the Affleck-Dine mechanism for baryogenesis\cite{Affleck-Dine}. Soon after the Affleck-Dine field begins rotation in the potential which allows the $Q$-ball solution, the homogeneous field starts to fluctuate and transforms into lumps. The actual formation was investigated numerically on the lattices for the gauge- and gravity-mediated supersymmetry (SUSY) breaking scenarios\cite{Affleck-Dine}.

On the other hand, the properties of the $Q$ ball are quite different among the different forms of the potential. For example, for the flat potential, such as in the gauge-mediated SUSY breaking scenario, the $Q$-ball size, the field value at its center, and the field rotation speed depend on the charge $Q$ of the $Q$ ball nontrivially\cite{Affleck-Dine}.

The potential may be dominated by thermal effects after inflation. In the Affleck-Dine scenario, the field amplitude at the onset of the rotation is very large, and the two-loop thermal effects on the potential are crucial\cite{Affleck-Dine}. This potential is given by

$$V_T \sim T^4 \log \left( \frac{|\Phi|^2}{T^2} \right),$$

(1)

for large field values\cite{Affleck-Dine}. We called it the thermal logarithmic potential\cite{Affleck-Dine}. The energy density of the universe is dominated by the oscillation of the inflaton after inflation, but there exists dilute plasma\cite{Affleck-Dine}, which would build up the thermal logarithmic potential. We considered in Ref.\cite{Affleck-Dine} that the $Q$-ball formation in the thermal logarithmic potential would be more or less similar to the time-independent logarithmic potential because of the fast growth of the fluctuations of the field in spite of the time dependence of the temperature $T$. We thus borrowed the results from the time-independent logarithmic potential case. For example, the charge of the produced $Q$ ball is given by\cite{Affleck-Dine}

$$Q = \beta \left( \frac{|\Phi|}{T} \right)^4_{\text{osc}},$$

(2)

where $\beta \approx 6 \times 10^{-4}$, and the subscript “osc” denotes the values of the variables at the onset of the oscillation of the field.

In this article we actually perform lattice simulations of the $Q$-ball formation in the thermal logarithmic potential, and see the evolution of the field and the distribution of the produced $Q$ balls. We also study of the properties of the formed $Q$ balls, especially about the evolution of the field amplitude at the center of the $Q$ ball.

In addition to the thermal logarithmic term, there would also be a mass term due to the gravity-mediated SUSY breaking effects for both the gauge- and gravity-mediation scenarios. The mass term alone (including one-loop potential) does not allow the $Q$-ball solution in some cases. In such cases, one may naively consider those $Q$ balls, created when the thermal logarithmic potential dominates, to disappear when the mass term begins to dominate the potential at the field value of the $Q$-ball center. As shown below, however, the $Q$-ball solution still exists later. Actually, the thick-wall type (gauge-mediation type) $Q$ ball transforms to the thin-wall type.

The structure of the article is as follows. In the next section, we show the results of lattice simulations for both time-independent and thermal logarithmic potentials. Some properties of the $Q$ ball with thermal logarithmic potential are shown in Sec.III, while, in Sec. IV, we focus on the transition from the gauge-mediation type to the thin-wall type $Q$ balls when the mass term in the potential gradually dominates over the thermal logarithmic potential. We finally conclude in Sec.V.

II. $Q$-BALL FORMATION

We investigate the $Q$-ball formation by means of three-dimensional lattice simulations. Interpolating the thermal mass term at smaller field amplitudes and the thermal two-loop potential growing logarithmically at larger...
field values, we take the following form of the potential:

\[ V_T(\Phi) = T^4 \log \left( 1 + \frac{\Phi^2}{T^2} \right). \]  

(3)

In addition, we restudy the time-independent case where \( T \) is replaced by the constant mass \( M_F \) for comparison, which is nothing but the potential of the gauge-mediated SUSY breaking effects. Here we consider the inflaton-oscillation dominated universe after inflation before reheating, but a similar argument applies also to the radiation dominated era. The temperature decreases as the universe expands as \( T \propto a^{-3/8} \propto t^{-1/4} \), where \( a(t) \) is the scale factor of the universe. Since we are interested in the period after the Affleck-Dine field starts the oscillation (rotation) when \( H^2 \approx |V''| \), we take initial conditions as

\[
\begin{align*}
\phi_1(0) &= \varphi_0 (1 + \delta_1), \quad \phi_1'(0) = \delta_2, \\
\phi_2(0) &= \delta_3, \quad \phi_2'(0) = \sqrt{2}(1 + \delta_4), \\
\tau(0) &= \frac{2}{3h} = \frac{\sqrt{2}}{3} \rho_0,
\end{align*}
\]

(4)

where all the variables are normalized by the temperature at the onset of the oscillation, \( T_{osc} \), such that \( \varphi = \Phi/\sqrt{2}T_{osc}, \ h = H/T_{osc}, \ \xi = T_{osc}^{-1/2} \) and \( \tau = T_{osc} \). Here we decompose the field into real and imaginary parts as \( \varphi = \varphi_1 + i\varphi_2. \) \( \delta \)'s represent the fluctuations of \( O(10^{-7}) \). We mostly use 256\(^3\) lattices, but in order to see any box size effects we also perform on 350\(^3\) lattices in some cases, but find no crucial differences between them.

Figure 1 shows the initial amplitude dependence of the largest charge of the Q ball produced for the time-independent (lower) and thermal (upper) logarithmic potentials. Here we average the charge over five realizations of the initial fluctuations except for the smallest initial amplitude case (\( \varphi_0 = 300 \)). The lower line corresponds to the relation obtained in Ref. \[3\], \( Q = \beta \varphi_0^4 \) where \( \beta \approx 6 \times 10^{-4} \), thus we reconfirmed the previous results.

On the other hand, in the thermal logarithmic potential, we have a similar relation \( Q = \beta' \varphi_0^4 \) with \( \beta' \approx 2 \times 10^{-3} \), although it might have a little tilt. Thus, qualitative features for the Q balls in the thermal logarithmic potential can be captured by the case with the time-independent logarithmic potential with \( M_F \) being replaced by \( T_{osc} \).

The difference between the time-independent and thermal logarithmic potentials can be qualitatively considered as follows. Q-ball charge can be estimated as \( Q \sim q^2 T_H^3 \) at the formation time, where \( q \) is the charge density and \( r_H \sim t \) is the horizon scale. From Fig. 2, the formation times are \( a_0 \sim 4.6 \) and \( a_T \sim 9.3 \), respectively, for the time-independent and thermal logarithmic potential. Thus, the ratio of the charges would be \( \approx (a_T/a_0)^{-3} (T_T/t_0)^3 \sim (a_T/a_0)^{3/2} \sim 3 \). Notice that the later rise and the slower growth of the amplitude of the fluctuations are due to the shrinking instability band \( 0 < k/a < 2T^2/\varphi_0 \) and the lowering growth rate \( T^2/(\sqrt{2}\varphi_0) \) due to the decreasing temperature \( T \).

FIG. 1: Charge of the Q ball depending on the amplitude of the Affleck-Dine field at the onset of the oscillation. The lower (upper) points and line correspond to the time-independent (thermal) logarithmic potential.

FIG. 2: Evolution of the fluctuations of the Affleck-Dine in the time-independent (green, the earlier rise) and thermal (red, the later rise) logarithmic potentials for \( \varphi_0 = 10^3 \). Notice that all the parameters including initial conditions are taken to be the same.

The distribution of the Q balls in example cases are shown in Table III for the thermal logarithmic potential and in Table IV for the time-independent logarithmic potential, where we identified Q balls with \( Q > 10^3 \) and \( Q > 10^5 \), respectively, in these cases. As one can see, the charge is dominated by those Q balls with the charge of the largest magnitude. Therefore, it is fairly reasonable to estimate any relations among the Q-ball parameters by using the largest Q ball, as we derived the charge and

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2 It is implicitly assumed that the helical motion is dynamically achieved by so-called A terms.
the initial field amplitude above. Since it is beyond the scope of the present paper to provide an analytical estimate of the distribution, we just provide a fitting formula of the form

\[ N_i(\tilde{Q}) = \alpha_i \varphi_0 \tilde{Q}^{-\eta} e^{-\left(\frac{\tilde{Q}}{Q_{\text{max}}}ight)^2} \quad (i = 0, T), \]

where \( \tilde{Q} \) denotes the charge in terms of the order of magnitude. Here \( \eta \approx 0.3 \), \( Q_{\text{max}} = \beta_0 \varphi_0^2 \) with \( \beta_0 \approx 6 \times 10^{-4} \) and \( \beta_T = 2 \times 10^{-3} \) for the time-independent and thermal logarithmic potentials, respectively. \( \alpha_0 \approx 1.6 \) and \( \alpha_T \approx 1.1 \).

### TABLE I: Distribution of Q balls in the thermal logarithmic potential

| Charge Numbers | Sum of the charge | Fraction |
|----------------|------------------|----------|
| \( O(10^9) \) | 2                | 3.77 \times 10^9 | 0.7358 |
| \( O(10^9) \) | 2                | 6.33 \times 10^9 | 0.1235 |
| \( O(10^7) \) | 15               | 4.39 \times 10^8 | 0.0857 |
| \( O(10^6) \) | 15               | 5.82 \times 10^7 | 0.0114 |
| \( O(10^5) \) | 12               | 7.21 \times 10^6 | 0.0014 |
| \( O(10^4) \) | 51               | 1.05 \times 10^6 | 0.0002 |
| \( O(10^3) \) | 195              | 8.94 \times 10^5 | 0.0002 |

### TABLE II: Distribution of Q balls in the time independent logarithmic potential

| Charge Numbers | Sum of the charge | Fraction |
|----------------|------------------|----------|
| \( O(10^{10}) \) | 5                | 1.42 \times 10^{11} | 0.7807 |
| \( O(10^9) \) | 10               | 2.33 \times 10^{10} | 0.1280 |
| \( O(10^8) \) | 20               | 5.92 \times 10^9 | 0.0326 |
| \( O(10^7) \) | 44               | 1.56 \times 10^9 | 0.0086 |
| \( O(10^6) \) | 56               | 2.27 \times 10^8 | 0.0012 |
| \( O(10^5) \) | 163              | 6.46 \times 10^7 | 0.0004 |

### III. Q-BALL PROPERTIES

Let us investigate the evolution of the \( D \)-dimensional Q ball formed in the thermal logarithmic potential, where for \( D = 1 \) and 2 it is wall- and stringlike objects, respectively [3]. The charge of the Q ball is given by

\[ Q \sim a^3 R^D q \sim \text{const.}, \]

where \( q \) is the charge density and \( R \) is the Q-ball size. Charge conservation implies that \( Q \) is constant. If we write \( \Phi(x, t) = \varphi(x) e^{i\omega t}/\sqrt{2} \), the energy of the Q ball is written as

\[ E = \int d^3 x \left[ \frac{1}{2} (\nabla \varphi)^2 + V(\varphi) - \frac{1}{2} \omega^2 \varphi^2 \right] + \omega Q \]

\[ = \int d^3 x [E_{\text{grad}} + V_1 + V_2] + \omega Q, \]

where

\[ E_{\text{grad}} \sim \frac{\varphi^2}{a^2 R^2}, \]

\[ V_1 \sim T^4 \log \left( 1 + \frac{\varphi^2}{2T^2} \right) \sim T^4, \]

\[ V_2 \sim \omega^2 \varphi^2. \]

When the energy takes the minimum value, the equipartition is achieved for gauge-mediation type Q balls [3]. From \( E_{\text{grad}} \sim V_1 \sim V_2 \), together with the charge conservation (6), we obtain the evolution of the (comoving) Q-ball size \( R \), the rotation speed of the field \( \omega \), and the field amplitude at the center of the Q ball \( \phi_c \), respectively as

\[ R \propto a^{-(4-\gamma)/(D+1)}, \]

\[ \omega \propto a^{-(D-3+\gamma)/(D+1)}, \]

\[ \phi_c \propto a^{(D-3-D^{-1})/(D+1)}, \]

where we define \( \gamma = 2 \) for the time-independent and thermal logarithmic potentials, respectively. These properties are observed in the lattice simulations which we perform for the \( D = 3 \) case.

### IV. TRANSFORMATION OF Q-BALL TYPES

In addition to the thermal logarithmic term in the potential, there is a mass term which stems from the gravity-mediated SUSY breaking effects. This potential can be written as

\[ V_m = m_{3/2}^2 |\Phi|^2 \left[ 1 + K \log \left( \frac{|\Phi|^2}{M^2} \right) \right], \]

where \( m_{3/2} \) is the gravitino mass, and one-loop effects are included. \( K \) is either a positive or negative constant of \( O(0.01 - 0.1) \), and \( M_c \) is a normalization scale. This potential alone allows the Q-ball solution only if \( K < 0 \) [2, 12]. In the opposite case \( K > 0 \), Q-ball formation is prohibited. One may thus be apt to consider that the Q ball created in the thermal logarithmic potential will disappear once the mass term with \( K > 0 \) dominates over the thermal one at the field value of the Q-ball center, \( V_T(\phi_c) < V_m(\phi_c) \). This condition can be written as

\[ \phi_c > \phi_{eq} \sim \frac{T^2}{m_{3/2}}. \]

Since \( \phi_{eq} \propto a^{-3/4} \) and \( \phi_c \propto a^{-3/8} \) for \( D = 3 \) in the inflaton-oscillation domination, and \( \phi_{eq} \propto a^{-2} \) and \( \phi_c \propto a^{-1} \) for any \( D \) in the radiation domination, it is true that \( V_m(\phi_c) \) will eventually overcome \( V_T(\phi_c) \). However, the Q-ball solution does exist for \( V = V_T + V_m \) with \( K > 0 \) for \( T > m_{3/2} \). Therefore, Q balls are not destroyed, but the metamorphosis will take place in such
situations. It might be best to simulate on the lattices to verify this phenomenon, but it is very time-consuming to perform. Here, instead, we take another approach, and leave the lattice simulations for future work.

We seek the Q-ball solution for \( V = V_T + V_m \) at some time snapshots. In order to obtain the solution, we just have to solve the equation

\[
\frac{d^2 \phi}{dr^2} + \frac{2}{r} \frac{d \phi}{dr} + \left( \omega^2 \phi - \frac{dV}{d\phi} \right) = 0, \tag{16}
\]

with boundary conditions \( \phi(\infty) = 0 \) and \( \phi'(0) = 0 \) \[14\,13\].

Since we would like to compare the results also to the solution of the pure logarithmic potential case, we change the value of the mass \( m_{3/2} \) in \( V_m \), leaving the logarithmic potential time independent. Thus we use the following potential with various values for \( m_{3/2} \):

\[
V = M_F^4 \log \left( 1 + \frac{\phi^2}{2M_F^2} \right) + \frac{1}{2} m_{3/2}^2 \phi^2 \left( 1 + K \log \frac{\phi^2}{2M_F^2} \right). \tag{17}
\]

Notice that it is practically the same if one varies \( M_F \) while \( m_{3/2} \) is fixed, since the results are derived and shown by variables normalized with respect to \( M_F \). The profiles are shown in Fig. 3 for \( m_{3/2}/M_F = (0, 0.5, 1, 2, 3, 4, 5, 10, 20) \times 10^{-7} \) from the top to the bottom. Here we set \( K = 0.1 \) and \( M_s/M_F = 10^6 \). It mimics the time evolution that the mass term eventually dominates over the logarithmic potential. Since the charge should be conserved, the angular velocity \( \omega \) increases: \( \omega/M_F = (2.0, 2.1, 2.4, 3.2, 4.1, 5.2, 6.2, 11, 21) \times 10^{-8} \) from the top to the bottom. One can see that the thick-wall type (gauge-mediation type) of Q ball transforms into the thin-wall-like type as time goes on. Notice that \( \phi_c > \phi_{eq} \) takes place for \( m/M_F \geq 10^{-7} \).

In the gravity-mediation, Q-ball solutions may exist until \( T \sim m_{3/2} \) for large enough \( \omega \). \[3\] After that, Q balls will disappear quickly, but the Affleck-Dine field can no longer be regarded as homogeneous because the field is localized near the place where destroyed Q balls had existed; they were very much separated from each other. Typical separation length is estimated as \( \ell_{H,\text{formation}}(\ell_{\text{destruction}}/\ell_{\text{formation}}) \). On the other hand, in the gauge-mediation, Q balls will remain intact for \( m_\phi > m_{3/2} \), where \( m_\phi = \sqrt{V'/(0)} \) is the curvature of the gauge-mediation potential at \( \phi = 0 \), since the Q-ball solution will exist irrespective of the temperature.

V. CONCLUSIONS

We have investigated the Q-ball formation in the thermal logarithmic potential by means of three dimensional lattice simulations. First of all, Q balls are actually formed. This is because the growth of the field fluctuations is fast enough to create Q balls, in spite of the shrinking instability band due to the decreasing temperature, and so on. We have found that the charge of the Q ball in the thermal logarithmic potential has almost the same dependence on the initial amplitude of the Affleck-Dine field,

\[
Q = \beta' \left( \frac{\phi_0}{T_{\text{init}}} \right)^4, \tag{18}
\]

with \( \beta \approx 2 \times 10^{-3} \), which is a factor of 3 larger than that in the time-independent logarithmic potential case.

We have also estimated the evolutions of parameters, such as the Q-ball size \( R \), the field rotation velocity \( \omega \), and the field value at the center of the Q ball \( \phi_c \). Since the thermal logarithmic potential decreases as the temperature drops, the mass term would dominate over the thermal logarithmic one at the field value \( \phi = \phi_c \). Even if the mass term \( V_m \) with positive \( K \) alone does not allow any Q-ball solution, the total potential \( V = V_T + V_m \) does allow the solution. In fact, we have found that the thick-wall type Q ball will eventually transform into the thin-wall type. Finally, the Q balls will disappear when \( T \sim m_{3/2} \), with an inhomogeneous Affleck-Dine field being left afterward in the gravity-mediation scenario.

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The fate of the $Q$ ball will depend on the cosmological situation. For thorough analysis, see Ref. [16].

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