Relative Sizes of Diagrams in $B \to \pi\pi, \pi K$ Decays

Maxime Imbeault

Laboratoire René J.-A. Lévesque, Université de Montréal,
C.P. 6128, succ. centre-ville, Montréal, Québec, Canada H3C 3J7

(Dated: March 26, 2022)

We show that the neglect of the $(V - A) \times (V + A)$ pieces of the electroweak penguin (EWP) amplitudes in the effective hamiltonian (the Wilson coefficients are very small) allows one to calculate the relative size of some tree and EWP diagrams in $B \to \pi\pi$ and $B \to \pi K$ decays. For both decay classes, tree and EWP amplitudes are related using only isospin. In $B \to \pi\pi$, the ratio $C/T$ is calculated using isospin alone; in $B \to \pi K$ it is found using flavor SU(3) symmetry. These results are obtained by computing explicitly all Wick contractions of all effective operators. Relations among these contractions are found using Fierz identities and final-state symmetry arguments.

PACS numbers: 13.25.Hw, 11.30.Hv, 12.15.Lk

A very useful way to parametrize $B$ decays is through the use of diagrams\(^{[1]}\). The size of these diagrams is a priori unknown and can only be estimated using theoretical input. In this letter, we show that, in fact, using simple field-theoretical tools, it is possible to compute the relative size of certain diagrams. (Note that the present paper focuses principally on explanations and re-compute the relative size of certain diagrams. (Note that the present paper focuses principally on explanations and results. For the full calculation see Ref. \([2]\).)

The starting point is the effective hamiltonian. The Wilson coefficients of the $(V - A) \times (V + A)$ pieces of the electroweak penguin (EWP) operators are tiny, and can be neglected\(^{[3]}\). Thus, the operator form of tree and EWP amplitudes is identical: both are $(V - A) \times (V - A)$. It is already known that this fact leads to some relations among these contractions\(^{[4]}\), but flavor SU(3) symmetry is always required, especially for $B \to \pi K$ decays. Our approach is different, and we (surprisingly) prove that some relations between tree and EWP diagrams can be obtained with isospin alone, even for $B \to \pi K$ decays. We simply “sandwich” all effective operators of the effective hamiltonian between initial and final states, and apply the basic rules of quantum field theory by summing over all possible Wick contractions. In so doing it is possible to write tree and EWP diagrams in term of these Wick contractions. Then, using Fierz identities and final-state symmetry arguments (isospin is assumed), many Wick contractions, or diagrams, can be related. This allows us to calculate the ratio of tree and EWP diagrams, including their color suppression. Further relations are obtained by adding flavor SU(3) symmetry. We find many new results and reproduce some others, in agreement with those found in published papers\(^{[3]}\). We explain why the new relations, using only isospin, could not be obtained by previous SU(3) analysis.

For $B \to \pi\pi$ decays the effective hamiltonian is\(^{[3]}\):

$$H_{eff} = \frac{G_F}{\sqrt{2}} \left( \sum_{p=u,c} \chi_p^{(d)} (c_1 O_1^p + c_2 O_2^p) - \chi_1^{(d)} \sum_{i=3}^{10} c_i O_i \right),$$

where $\chi_i^{(d)} = V_{ib} V_{id}^*$, and the tree and EWP operators are respectively (we neglect $Q_7$ and $Q_8$, the $(V - A) \times (V + A)$ EWP’s):

$$Q_1^p = (\bar{p}b)_{V-A}(\bar{d}p)_{V-A},$$

$$Q_2^p = (\bar{p}b_j)_{V-A}(\bar{d}_j p_i)_{V-A},$$

$$Q_9 = \frac{1}{2} (\bar{d}b)_{V-A} \sum_q e_q (\bar{q}q)_{V-A},$$

$$Q_{10} = \frac{3}{2} (\bar{d}_i b_j)_{V-A} \sum_q e_q (\bar{q}_j q_i)_{V-A},$$

with $q = u, d, s, c, b$. For $B \to \pi K$, the non-summed $d$ quarks are replaced by an $s$ quark. Factors of $G_F/\sqrt{2}$ are omitted for the remainder of this paper.

When sandwiching these operators between initial and final states, all terms have the form

$$\langle \bar{q}_1 q_2 \bar{q}_3 q_4 | \bar{q}_5 b_6 q_7 | \bar{q}_8 b \rangle,$$

where the $q_i$’s are $u, d$ or $s$ quarks, $\bar{q}_1 q_2$ and $\bar{q}_3 q_4$ are $\pi$ or $K$ mesons and $\bar{q}_8 b$ is a $B$ meson (Dirac and color structures are omitted for notational convenience). Applying the basic rules of quantum field theory, we must sum over all possible Wick contractions of all operators. There are 24 possible contractions and we assign them labels from $A$ to $X$ (see Table\(^{[3]}\)).

From here on, our goal is to minimize the number of independent Wick contraction structures. We can do this simply by comparing them two by two and using the following three rules to relate them:

1. **Flavor symmetries:** Under isospin the contraction of two $u$ quarks is equivalent to that of two $d$ quarks; under flavor SU(3) symmetry this is true also for $s$ quarks. In the following, we always assume isospin symmetry. Note that since the effective hamiltonian is at leading order in the electroweak interaction, the addition of gluons does not violate isospin.

2. **Fierz identities:** Since all operators have a $(V - A) \times (V - A)$ structure, the effect of a Fierz transformation is to simply exchange the first and the third quarks of the

3. **Final-state symmetry:** Among certain diagrams\(^{[3, 4]}\), but flavor SU(3) symmetry is always required, especially for $B \to \pi K$ decays. Our approach is different, and we (surprisingly) prove that some relations between tree and EWP diagrams can be obtained with isospin alone, even for $B \to \pi K$ decays. We simply “sandwich” all effective operators of the effective hamiltonian between initial and final states, and apply the basic rules of quantum field theory by summing over all possible Wick contractions. In so doing it is possible to write tree and EWP diagrams in term of these Wick contractions. Then, using Fierz identities and final-state symmetry arguments (isospin is assumed), many Wick contractions, or diagrams, can be related. This allows us to calculate the ratio of tree and EWP diagrams, including their color suppression. Further relations are obtained by adding flavor SU(3) symmetry. We find many new results and reproduce some others, in agreement with those found in published papers\(^{[3]}\). We explain why the new relations, using only isospin, could not be obtained by previous SU(3) analysis.

For $B \to \pi\pi$ decays the effective hamiltonian is\(^{[3]}\):

$$H_{eff} = \frac{G_F}{\sqrt{2}} \left( \sum_{p=u,c} \chi_p^{(d)} (c_1 O_1^p + c_2 O_2^p) - \chi_1^{(d)} \sum_{i=3}^{10} c_i O_i \right),$$

where $\chi_i^{(d)} = V_{ib} V_{id}^*$, and the tree and EWP operators are respectively (we neglect $Q_7$ and $Q_8$, the $(V - A) \times (V + A)$ EWP’s):

$$Q_1^p = (\bar{p}b)_{V-A}(\bar{d}p)_{V-A},$$

$$Q_2^p = (\bar{p}b_j)_{V-A}(\bar{d}_j p_i)_{V-A},$$

$$Q_9 = \frac{1}{2} (\bar{d}b)_{V-A} \sum_q e_q (\bar{q}q)_{V-A},$$

$$Q_{10} = \frac{3}{2} (\bar{d}_i b_j)_{V-A} \sum_q e_q (\bar{q}_j q_i)_{V-A},$$

with $q = u, d, s, c, b$. For $B \to \pi K$, the non-summed $d$ quarks are replaced by an $s$ quark. Factors of $G_F/\sqrt{2}$ are omitted for the remainder of this paper.

When sandwiching these operators between initial and final states, all terms have the form

$$\langle \bar{q}_1 q_2 \bar{q}_3 q_4 | \bar{q}_5 b_6 q_7 | \bar{q}_8 b \rangle,$$

where the $q_i$’s are $u, d$ or $s$ quarks, $\bar{q}_1 q_2$ and $\bar{q}_3 q_4$ are $\pi$ or $K$ mesons and $\bar{q}_8 b$ is a $B$ meson (Dirac and color structures are omitted for notational convenience). Applying the basic rules of quantum field theory, we must sum over all possible Wick contractions of all operators. There are 24 possible contractions and we assign them labels from $A$ to $X$ (see Table\(^{[3]}\)).

From here on, our goal is to minimize the number of independent Wick contraction structures. We can do this simply by comparing them two by two and using the following three rules to relate them:

1. **Flavor symmetries:** Under isospin the contraction of two $u$ quarks is equivalent to that of two $d$ quarks; under flavor SU(3) symmetry this is true also for $s$ quarks. In the following, we always assume isospin symmetry. Note that since the effective hamiltonian is at leading order in the electroweak interaction, the addition of gluons does not violate isospin.

2. **Fierz identities:** Since all operators have a $(V - A) \times (V - A)$ structure, the effect of a Fierz transformation is to simply exchange the first and the third quarks of the
TABLE I: The various Wick contractions for the decay $\bar{q}_8 b \rightarrow \bar{q}_1 q_2 \bar{q}_3 q_4$. Any contraction not listed is symmetric to one listed by the exchange $\bar{q}_1 q_2 \leftrightarrow \bar{q}_3 q_4$.

| Operator | Wick Contractation |
|----------|--------------------|
| $A$      | $\langle \bar{q}_1 q_2 q_4 | \bar{q}_8 b | \bar{q}_8 | b \rangle$ |
| $B$      | $\langle \bar{q}_1 q_2 q_4 | \bar{q}_8 b | q_8 | b \rangle$ |
| $C$      | $\langle \bar{q}_1 q_2 q_4 | \bar{q}_8 b | \bar{q}_8 | q_8 \rangle$ |
| $D$      | $\langle \bar{q}_1 q_2 q_4 | \bar{q}_8 b | q_8 | q_8 \rangle$ |
| $E$      | $\langle \bar{q}_1 q_2 q_4 | \bar{q}_8 b | \bar{q}_8 | q_8 \rangle$ |
| $F$      | $\langle \bar{q}_1 q_2 q_4 | \bar{q}_8 b | q_8 | q_8 \rangle$ |
| $G$      | $\langle \bar{q}_1 q_2 q_4 | \bar{q}_8 b | \bar{q}_8 | q_8 \rangle$ |
| $H$      | $\langle \bar{q}_1 q_2 q_4 | \bar{q}_8 b | q_8 | q_8 \rangle$ |
| $J$      | $\langle \bar{q}_1 q_2 q_4 | \bar{q}_8 b | \bar{q}_8 | q_8 \rangle$ |
| $K$      | $\langle \bar{q}_1 q_2 q_4 | \bar{q}_8 b | q_8 | q_8 \rangle$ |
| $L$      | $\langle \bar{q}_1 q_2 q_4 | \bar{q}_8 b | \bar{q}_8 | q_8 \rangle$ |
| $M$      | $\langle \bar{q}_1 q_2 q_4 | \bar{q}_8 b | q_8 | q_8 \rangle$ |
| $N$      | $\langle \bar{q}_1 q_2 q_4 | \bar{q}_8 b | \bar{q}_8 | q_8 \rangle$ |
| $O$      | $\langle \bar{q}_1 q_2 q_4 | \bar{q}_8 b | q_8 | q_8 \rangle$ |

where the trivial contraction of $b$ fields is always understood. This example shows that $A$-type and $G$-type contractions are related. In general, all contractions are related in pairs.

(3) Final-state symmetry: In our notation, the order of mesons in the final state is arbitrary. Thus, a change of this order ($\bar{q}_1 q_2 \leftrightarrow \bar{q}_3 q_4$) has no consequence. For example,

$$\langle \bar{u}d\bar{d}|\bar{u}b\bar{u}|\bar{u}b \rangle = \langle \bar{u}d\bar{d}|\bar{u}b\bar{u}|\bar{u}b \rangle,$$

showing that $A$-type and $Q$-type contractions are related. Again, all contractions can be related in pairs.

Note that, although the above analysis is at the level of quarks instead of mesons, one can prove that our results hold at the level of mesons. The thrust of the proof is that we only compare contractions two by two at the level of quarks, so that the mesons affect both side of any equality identically. Thus, the equality remains true at the level of mesons.

We begin by considering the tree contributions to $B \rightarrow \pi\pi$ decays. For $B^- \rightarrow \pi^-\pi^0$ we have (recall that isospin symmetry is always assumed)

$$T_{\pi^-\pi^0} = \lambda_p^{(d)} c_i \langle \frac{1}{\sqrt{2}} (\pi^-\pi^0 + \pi^0\pi^-) | O_i^1 | B^- \rangle,$$

where $p = u, c$ and a sum over $i = 1, 2$ is understood. Then,

$$T_{\pi^-\pi^0} = \lambda_p^{(d)} c_i \left( \langle \bar{u}\bar{d}\bar{d}\bar{d}|\bar{p}\bar{b}\bar{p}|\bar{u}\bar{b} \rangle - \langle \bar{u}\bar{d}\bar{u}|\bar{p}\bar{b}\bar{p}|\bar{u}\bar{b} \rangle + \langle \bar{d}\bar{d}\bar{d}|\bar{p}\bar{b}\bar{p}|\bar{d}\bar{b} \rangle - \langle \bar{u}\bar{u}\bar{u}|\bar{p}\bar{b}\bar{p}|\bar{u}\bar{b} \rangle \right).$$

The color indices are not written explicitly, but they are understood with $i$ subscripts. When we sum over all Wick contractions and simplify we then get

$$T_{\pi^-\pi^0} = -\frac{\lambda_u^{(d)}}{2} c_i [T_i^u + M_i^u + E_i^u + F_i^u],$$

where the $u$ exponents stand for $p = u$. Using the final-state symmetry [rule (3)] we have $c_i E_i^u = c_i M_i^u$ and $c_i F_i^u = c_i T_i^u$, so that

$$T_{\pi^-\pi^0} = -\lambda_u^{(d)} c_i [E_i^u + F_i^u].$$

A similar procedure can be carried out for $B^0 \rightarrow \pi^-\pi^+$ and $B^0 \rightarrow \pi^0\pi^0$. We find

$$T_{\pi^0\pi^0} = -\lambda_u^{(d)} c_i [A_i^u + H_i^u + S_i^u],$$

$$T_{\pi^-\pi^+} = -\lambda_u^{(d)} c_i [A_i^u + H_i^u + S_i^u - F_i^u] + \lambda_c^{(d)} c_i [A_i^c + S_i^c].$$

Comparing this parametrization in terms of contractions with that of the language of diagrams we can write all tree diagrams in terms of Wick contractions:

$$T = \sqrt{2} c_i E_i^u,$$

$$C = \sqrt{2} c_i F_i^u,$$

$$E = \sqrt{2} c_i H_i^u,$$

$$P_{u,c} = \sqrt{2} c_i A_i^{u,c},$$

$$P A_{u,c} = \sqrt{2} c_i S_i^{u,c},$$

where $T$ and $C$ are respectively the color-allowed and color-suppressed tree diagrams, $E$ is the exchange diagram, and $P_{u,c}$ and $PA_{u,c}$ are the tree parts which renormalize respectively the gluonic penguin and penguin-annihilation amplitudes. A priori there is an ambiguity in finding this one-to-one correspondence, but this is completely removed by using the fact that the exchange and the penguin-annihilation contributions cannot be described by a contraction which contains a spectator quark ($E$-type and $F$-type) and $T$, $C$ and $P_{u,c}$ must involve the spectator quark. Note also that in this notation diagrams do not contain Cabibbo-Kobayashi-Maskawa (CKM) factors.

For EW’s, the principle is exactly the same, but the operators are slightly different. The result is

$$P_{\pi^-\pi^0}^{EW} = \frac{3}{2} \lambda_u^{(d)} c_i [E_i + F_i],$$

$$P_{\pi^-\pi^+}^{EW} = \frac{3}{2} \lambda_u^{(d)} \sqrt{2} c_i \left( \frac{2}{3} F_i + \frac{1}{3} (B_i + G_i + N_i) - \frac{1}{3} (A_i + H_i + S_i) \right),$$

$$P_{\pi^0\pi^0}^{EW} = \frac{3}{2} \lambda_u^{(d)} c_i \left[ E_i + \frac{1}{3} F_i - \frac{1}{3} (B_i + G_i + N_i) + \frac{1}{3} (A_i + H_i + S_i) \right].$$
where $x^a_i = x^d_i = x_i$ (with $x = A, B, C, ...$) by isospin. Contractions are of the form $\langle q\bar{q}\bar{q}\bar{q}|d\bar{b}g_{j\bar{y}}|q\bar{b}\rangle$ where the $q$’s are $u$ or $d$ independently, and $y = u, d$ (pieces with $y = s, c, b$ are absorbed into gluonic penguin operators by isospin). A sum over $i = 9, 10$ is understood. Again comparing with diagrams [11] we must have

$$P_{EW} = -\frac{3}{2} \sqrt{2} c_i E_i ,$$
$$P_{EW}^c = -\frac{3}{2} \sqrt{2} c_i F_i ,$$

where we have omitted the other EWP’s because they are not interesting for our purpose. Comparing Eqs. [1] and [12], it is clear that $T$, $C$ and the EWP’s are expressed in terms of the same types of contractions. The order of quarks in their operators is slightly different, but these are related by isospin [rule (1)].

We now have to explicitly compute the effect of the color indices. For example,

$$\sum_{i=1,2} c_i E_i = c_1 \langle \bar{q}_1 \bar{q}_2 q_3 q_4 |\bar{q}_5 b_6 \bar{q}_7 | q_8 b_9 \rangle + c_2 \langle \bar{q}_1 \bar{q}_2 q_3 q_4 |\bar{q}_5 b_6 | q_7 (q_8 b_9) \rangle .$$

In the above, the first contraction is color-allowed, while the second one is color-suppressed. Thus,

$$\sum_{i=1,2} c_i E_i = c_1 \delta_{x_2} \delta_{x_1} \delta_{y_2} \delta_{y_1} \delta_{z_2} \delta_{z_1} \langle \bar{q}_1 \bar{q}_2 \bar{q}_3 q_4 |\bar{q}_5 b_6 \bar{q}_7 | q_8 b_9 \rangle + c_2 \delta_{x_2} \delta_{x_1} \delta_{y_2} \delta_{y_1} \delta_{z_2} \delta_{z_1} \langle \bar{q}_1 \bar{q}_2 \bar{q}_3 q_4 |\bar{q}_5 b_6 \bar{q}_7 | q_8 b_9 \rangle$$

$$= c_1 N_c^2 \vec{E} + c_2 N_c \vec{E} ,$$

which is exactly Eq. (23) of Ref. [4] by Gronau, Pirjol and Yan (GPY). However, note that with our approach the heavy formalism of Clebsch-Gordan coefficients is not required. This result also represents a cross-check of our calculation.

A similar exercise can be carried out for $B \to \pi K$ decays. For simplicity, we use the same notation as for $B \to \pi \pi$, but it is clear that, for example, an $A$-type contraction in $B \to \pi \pi$ is not the same as that in $B \to \pi K$ because of the $s$-quark fields, unless we assume flavor SU(3) symmetry. To keep track of different flavors, primed contractions have the form $\langle \bar{q}\bar{q}\bar{q}\bar{q}|\bar{s}bq|q\bar{b}\rangle$ and non-primed contractions have the form $\langle \bar{q}\bar{q}\bar{q}\bar{q}|\bar{b}sq|q\bar{b}\rangle$ ($q = u, d$ independently). Note also that, contrary to $B \to \pi \pi$, we have no symmetry in the final state and this leads to some differences. Again we can express the graphical amplitudes of Refs. [11] in terms of our contractions:

$$T' = \sqrt{2} (c_1 N_c^2 + c_2 N_c) \vec{E} ,$$
$$C' = \sqrt{2} (c_1 N_c + c_2 N_c^2) \vec{F} ,$$
$$A' = \sqrt{2} (c_1 N_c^2 + c_2 N_c) \vec{F} ,$$
$$P'_{EW} = -\frac{3}{2} \sqrt{2} (c_9 N_c^2 + c_{10} N_c) \vec{E} ,$$
$$P'_{EW}^c = -\frac{3}{2} \sqrt{2} (c_9 N_c + c_{10} N_c^2) \vec{F} ,$$

which imply that for some specific ratios, the long-distance parts (matrix elements) cancel:

$$\frac{P_{EW}}{T} = -\frac{3}{2} \frac{c_9 + \frac{c_{10}}{N_c}}{c_1 + \frac{c_2}{N_c}} \approx 0.013 ,$$
$$\frac{P_{EW}^c}{C} = -\frac{3}{2} \frac{c_9 + \frac{c_{10}}{N_c}}{c_1 + \frac{c_2}{N_c}} \approx 0.013 .$$

These relations are new. For the Wilson coefficients, we have used values given in Ref. [3] evaluated at NLO with $\mu = m_b$.

But there is more. Using the final-state symmetry ($\vec{E} = M$ and $\vec{F} = \tilde{F}$), and adding Fierz transformations ($\vec{E} = \tilde{I}$ and $\vec{F} = \tilde{F}$), we have $\vec{E} = \tilde{F}$. This implies

$$\frac{P_{EW}}{C} = \frac{-\frac{c_9}{N_c} + \frac{c_{10}}{N_c}}{c_1 + \frac{c_2}{N_c}} \approx 0.17 ,$$
$$\frac{P_{EW}^c}{P_{EW}} = \frac{-\frac{c_9}{N_c} + \frac{c_{10}}{N_c}}{c_1 + \frac{c_2}{N_c}} \approx 0.16 .$$

Again, these relations are new and confirm naive estimations of color suppression [3]. Finally, Eqs. [1] also imply

$$\frac{P_{EW} + P_{EW}}{C + T} = 3 \frac{c_9 + c_{10} + c_9 + \frac{c_{10}}{N_c}}{c_1 + \frac{c_2}{N_c}} = \frac{3}{2} \frac{c_9 + c_{10}}{c_1 + \frac{c_2}{N_c}} \approx 0.013 ,$$

which is exactly Eq. (23) of Ref. [4] by Gronau, Pirjol and Yan (GPY). However, note that with our approach the heavy formalism of Clebsch-Gordan coefficients is not required. This result also represents a cross-check of our calculation.
Again, these two relations are new. The difference between these equations and those of Eqs. (17) is due to the absence of symmetry in the final state. These two equations are important since it is generally believed that it is impossible to relate tree diagrams and EWPs without SU(3) symmetry (for example, see Ref. [1]). We have stressed on several occasions that all previous relations do not require flavor SU(3). However, if we add this symmetry, the position of the $s$ quark is no longer important, so that there is a final-state symmetry in $B \rightarrow \pi K$. Indeed, this decay can be related to $B \rightarrow \pi \pi$. Under SU(3) we have $\bar{E} = \bar{F} = \bar{M} = \bar{I}$. It is then easy to show that Eqs. (18) are valid also for $B \rightarrow \pi K$. In addition, using the final-state symmetry in Eqs. (20), one can derive

$$\frac{P_{EW}^c + P_{EW}^u}{C^c + T^u} = -\frac{3}{2} \frac{c_9 + c_{10}}{c_1 + c_2} \approx 0.013,$$

(22)

which is the well known Neubert-Rosner relation [3, 4]. The fact that we reproduce known results [3, 4] is an important cross-check to our calculation. Indeed, the reproduction of Eq. (22) above is particularly important since this equation is quite complicated. At every step our calculation is consistent with all known SU(3) relations. Note that it supports the fact that we are working at the level of mesons since GPY are clearly working at this level.

Finally, there is an interesting relation involving $A$ and $E$ diagrams. From Eqs. (16) and (20), we have

$$\frac{E}{A} = \frac{c_1 + c_2}{c_1 + 5c_2} \approx 0.17,$$

(24)

Again, this relation is new.

Before concluding, there is an important issue we must address. We have derived several new relations among diagrams. Why could these new relations not be obtained from standard systematic isospin and SU(3) analysis? The answer is simple. Consider tree diagrams for example. There are six tree topologies: $T$, $C$, $A$, $E$, $P_u$ and $PA_u$. However, under the SU(3) formalism, only five linear combinations of these six topologies can appear in any amplitude: $P_u + T$, $P_u + A$, $C - P_u$, $P_u + PA_u$ and $C - E$. Consequently, it is possible to find relations among these five linear combinations of diagrams. However, it is impossible to find a relation between $T$ and $C$ alone, for example. This is because $T$ and $C$ alone do not exist in this formalism. On the other hand in our approach, isolated diagrams are well-defined contractions.

Relations among contractions automatically imply relations among diagrams.

To summarize, the neglect of the $(V - A) \times (V + A)$ pieces of the electroweak penguin (EWP) amplitudes in the effective hamiltonian allows us to describe both tree and EWPs operators sandwiched between initial and final states, we are able to make the connection between diagrams in $B$ decays and these contractions. The ratios of the sizes of various diagrams can then be expressed as a ratio of contractions. Note that these contractions include (uncalculable) matrix elements. However, the key point is that, in certain ratios, the matrix elements cancel due to symmetry arguments, so that the ratio of sizes of diagrams is expressible purely in terms of (calculable) Wilson coefficients. For the case where this symmetry is purely isospin, we have presented a variety of new results in $B \rightarrow \pi \pi$ and $B \rightarrow \pi K$ decays. These are rigorous, and are consistent with naive estimates [1]. If the symmetry is extended to flavor SU(3), we get additional results, all in agreement with published papers (especially Refs. [2, 4]).

The potential for applications of this method is great in $B$ and $D$ decays. For example, methods for extracting CKM weak phases from $B \rightarrow \pi \pi$ and $B \rightarrow \pi K$ decays can be greatly improved. The standard model can be tested in $B \rightarrow \pi \pi$ measurements and $\gamma$ can be extracted from $B \rightarrow \pi K$ decays without using SU(3) approximations. Also, estimates of SU(3) breaking should be facilitated since we can avoid the heavy formalism of Clebsch-Gordan coefficients in our approach. Applications, as well as explicit calculations, are discussed in more detail in Ref. [2].

I wish to thank D. London, Th. Mannel and Th. Feldmann for useful discussions. I especially thank D. London for his help with the manuscript. Finally, i thank everyone from LAPTH in France and Universit¨ at Siegen in Germany for their great hospitality. This work was financially supported by NSERC of Canada.

* Electronic address: maxime.imbeault@umontreal.ca