Theoretical Study on Carbon Isotopes

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Abstract. This work is devoted to develop nuclear matter density distribution form for the heavy ions valid for the stable and halo nuclei. In this study we consider the neutron halo nuclei (\(^{15,17,19}\)C) and (\(^{16,22}\)C). The first group has a 1n–halo while the second has a 2n–halo. Based on the developed densities, the elastic scattering cross section data of C–isotopes on target of \(^{12}\)C, at incident energies 70.8, 78 and 106.9 MeV/N, have been analyzed within the energy density functional theory. A consistent structural study of the neutron drip–line \(^{22}\)C nucleus is done.

1. Introduction

The common issue between theorists and experimentalists is the phenomenon. Theorists are working on developing mathematical models to explain the observations and the experimentalists are helping them by performing insightful experiments to justify the predictions. One of the fruitful attempts is the discovering of the halo structures in Carbon isotopes. Nuclear–Halo is a kind of exotic structures in which the nuclear matter distribution extends to large radius because of the small separation energy (\(\varepsilon_{\text{tail}}\)) of the tail. Due to this small value, the tail of the last one–or two–neutron wave function spreads far out from the core, this phenomenon is known as Halo–Nuclei. Halo nuclei have been extensively investigated both experimentally and theoretically for decades [1]-[11]. However, up to now most of the confirmed halo nuclei are the neutron halo. Studying the loosely bound exotic nuclei in the neutron–halo region was a puzzle for long time. These nuclei have very small neutron (proton) separation energies implying a long tail of the last loosely bound nucleon. An accurate treatment of this tail is crucial for the description of several reaction observations. The motivation of this work is to investigate the reliability of the extracted nuclear size parameters from the new deduced density, and to compare the experimental data with the calculated results of the elastic scattering differential cross sections based upon the deduced density. The structures of Carbon isotopes have recently attracted much attention. Based on the considered nuclear matter densities (\(D1\) and \(D2\)) we applied for describing the halo nuclei in C–isotopes. These densities provide the neutron and proton densities and is suitable for describing any loosely bound system such as the neutron drip–line nuclei. In these two forms, the ground state wave functions of the loosely bound...
nuclei $^{15,16,17,19,22}\text{C}$, are treated as a core plus tail representing the one or two valence neutrons considering the correct asymptotic behavior governed by the separation energies [12]-[16].

Within the EDF theory, the structure properties of Carbon isotopes are systematically studied. The EDF theory indicates the 1n–neutron halo structures in $^{15}\text{C}$, $^{17}\text{C}$ and $^{19}\text{C}$, and the 2n–neutron halo structures in $^{16}\text{C}$ and $^{22}\text{C}$ nuclei. It is also found that close to the neutron drip–line, there is amazing increase in the neutron radii and decrease in the binding energies, which are tightly related with the blocking effect and correspondingly the blocking effect plays a significant role in the shell model configurations. In the present work we show that, elastic scattering cross section data for the $^{12-22}\text{C}+^{12}\text{C}$ systems, at incident energies 70.8, 78 and 106.9 MeV/N, can be well reproduced and interpreted in the framework of the EDF model. Experimental data are taken from Refs.[13, 17].

In this paper, the optical potential parameters are obtained from the experimental data of the elastic scattering angular distributions of the $^{12-22}\text{C}+^{12}\text{C}$ systems, at incident energies 70.8, 78 and 106.9 MeV/N. All the cross sections, angular distributions, RMS radii and total reaction cross sections are calculated and analyzed using theoretical models for the matter densities, effective interactions and the optical potentials. The motivation of this work is to investigate the reliability of the extracted nuclear size parameters from the new deduced density, and to compare the experimental data with the calculated results of the elastic scattering differential cross sections based upon the modified densities.

2. Theoretical models and model parameters of the nuclear matter density distributions

2.1. Nuclear density for the stable nuclei

In this work, we propose the ground state nuclear matter density containing the effect of the center of mass motion. This can be done by using the ground state nucleon wave function $\Psi$ as per below [18, 19, 20],

$$\rho_{D1}(r) = \int |\Psi(R)|^2 \rho(|r - \frac{1}{A_{\text{Ca}}} R|)d\vec{R},$$  \hspace{1cm} (1)

where $A$ is the nucleus mass number, and the single nucleon wave function is represented in the form

$$\Psi(R) = AR^2 \exp(-\alpha R^2),$$  \hspace{1cm} (2)

with $\alpha = 0.83 fm^{-2}$ and $A = 0.53 fm^{-2}$. The nuclear density is taken in a Gaussian form as

$$\rho(r) = a \exp(-\gamma r^2),$$  \hspace{1cm} (3)

with $a$ and $\gamma$ parameters. By inserting Eq.(2) and Eq.(3) in Eq.(1) over the integration we get

$$\rho_{D1}(r) = \int e^{-ER^2} \exp(-\gamma_i r^2) \exp i\vec{q}.\vec{r}d\vec{R},$$  \hspace{1cm} (4)

Now from the above equations, the resulted density $(D1)$ is given by

$$\rho_{D1}(r) = C_0[1 - \frac{q^2}{3E} + \frac{q^4}{60E^2}] \exp(-H\gamma r^2).$$  \hspace{1cm} (5)

where $C_0 = \frac{15A^2_{\text{Ca}}e^{3/2}}{4E^{7/2}}, q = \frac{-2\gamma_i}{A_{\text{Ca}}}, E = 2\alpha + \frac{\gamma_i}{A_{\text{Ca}}}, H = \gamma_i - \frac{\gamma_i^2}{E}.\frac{1}{A_{\text{Ca}}}$.
2.2. Nuclear density for the halo nuclei
Here we suggest an additional term to the nuclear density distribution form to be valid for describing the halo nuclei. This can be done by assuming the ground state wave function $\Psi$ for the neutron drip-line nuclei using the core–tail model [20]-[25]

$$
\rho(r) = \int |\Psi(R)|^2 \left[ \rho_c \left( \left| r - \frac{1}{A} R \right| \right) + \rho_h \left( \left| r + \frac{C}{A} R \right| \right) \right] dR,
$$

where $A_c$ and $C$ refer to the nucleus mass number and its core if any, respectively. The core–halo relative wave function $\Psi(R)$ is expressed in the form

$$
\Psi(R) = AR^2 \exp(-\alpha R^2),
$$

with $\alpha = 0.15 fm^{-2}$ and $A = 0.015 fm^{-2}$. The nuclear density for the halo nuclei is taken as a core (c) and halo tail (h) in a Gaussian form as

$$
\rho_c(r) = c_1 \exp(-a_1 r^2) + c_2 \exp(-a_2 r^2),
$$

and

$$
\rho_h(r) = \left( \frac{1}{a \sqrt{\pi}} \right)^3 \exp \left( -\frac{r^2}{a^2} \right),
$$

From the above equations, the resulted density ($D_1$) is given by

$$
\rho_D(r) = A^2 [I_1 + I_2 + I_3],
$$

where

$$
I_1 = \rho_0 [1 + A_1 r^2 + B_1 r^4] \exp(-H_1 r^2),
$$

$$
I_2 = \rho_0 [1 + A_2 r^2 + 2B_2 r^4] \exp(-H_2 r^2),
$$

$$
I_3 = \rho_0 [1 + g_1 r^2 + 2g_2 r^4] \exp(-H_3 r^2).
$$

and $\rho_0 = \frac{15c_i A^2 \pi^{3/2}}{4E \sqrt{\pi}}$, $E = 2\alpha + \frac{a_i}{a C}$, $A_i = \frac{4a^2}{34\pi C}$, $B_i = \frac{16a^4}{154\pi C}$, $H_i = a_i - \frac{a^2}{E A C}$, $D = 2\alpha + \frac{C^2}{a^2 A C}$, $g_1 = \frac{4C^2}{3a^2 D A C}$, $g_2 = \frac{4C^4}{15a^2 D^2 A C}$, $W = \frac{1}{a^2} - \frac{C}{a^2 D A C}$, $i = 1, 2$.

2.3. The semi–phenomenological nuclear matter density distribution
In this subsection, we show an effective form of the nuclear matter density distributions and we refer as $D_2$. It provides separately the neutron and proton densities. In this form, the ground state wave functions for C–isotopes, having the correct asymptotic behavior governed by the separation energies $\varepsilon$, as [12]

$$
\rho_i(r) = \frac{\rho_i^0}{1 + [(1 + (r/R)^2) \times e^{a_i \varepsilon_1} + e^{-(r+R)/a_i}]} \label{16}
$$

$$
a_i = \frac{\hbar}{2(2M\varepsilon_1)^{1/2}} \label{17}
$$

$$
\alpha_i = \frac{g e^2}{(4\pi \varepsilon_0) \hbar} \left( \frac{m}{2\varepsilon_1} \right)^{1/2} + 1 \label{18}.
$$

3
where $i = p$ or $n$, $\varepsilon_{p(n)}$ is the nucleon separation energy, respectively, $R$ is radius parameter, $\rho^0_{p,n}$ are the nuclear densities for the protons, and neutrons at the region 0 fm, respectively.

This form is suitable for describing the loosely bound systems such as the nuclei in the neutron–halo region. In addition, it provides a simple physical insight into the density in the outer (halo) region. In this form the loosely bound nuclei (e.g. $^{15,16,17,19,22}$C) are treated as a core plus tail corresponding to the last halo neutrons having the correct asymptotic behavior governed by the separation energy of the last 1n–or 2n–halo neutrons [13, 14, 15, 16, 26, 27]. We apply this formula to reproduce the nuclear densities for C–isotopes. For the loosely bound nuclei, we use a density which explicitly contains an additional term describing the tail (halo). So, the matter density for the halo nucleus ($D2$) can be written as

$$\rho_{D2}(r) = \rho_{\text{core}}(r) + \rho_h(r)$$

(19)

$$\rho_{\text{core}}(r) = \rho_n(r) + \rho_p(r).$$

(20)

where $\rho_{\text{core}}(r)$ can be getting by Eq.(20), which is the sum density distributions for the neutrons and protons, respectively. The density for the neutron tail or halo part ($\rho_h$) as follows

$$\rho_h(r) = N_0 \left( \frac{r^2}{(r^2 + R^2)^2} \right) \left[ e^{-r/a_t} + e^{r/a_t} \right],$$

(21)

$$a_t = \frac{\hbar}{2(2M\varepsilon_t)^{1/2}}.$$

(22)

with parameters listed in Refs.[12, 13, 14, 26, 27, 28].

2.4. JLM NN effective interaction

In this part, the Jeukenne, Lejeune and Mahaux (JLM) NN effective interaction [2, 15] is folded with the ground state nuclear matter density of the target nucleus to study the elastic scattering cross sections at a broad band of incident energies. It has been widely used to analyze the measured data of the elastic scattering cross sections [2, 5, 6, 8]. This section is devoted to generate the SF potentials for the p+ Ca–isotopes systems using the JLM NN effective interaction. This interaction is in G–matrix effective obtained from Brueckner–Hartree–Fock (BHF) approximation. The JLM NN effective interaction parameterized explicitly in terms of the local density approximation (LDA) and the nucleon energy, so the Hartree term alone is able to reproduce the entire nucleon–nucleus potential. Such an interaction is therefore designed for the folding calculations without exchange component. The optical potential for the nucleon with energy $E$ traversing nuclear matter density $\rho$ can be obtained from the definition of mass operator $M_\rho(k_F(E), E)$ for the BHF theory, where $k_F$ is the fermi momentum. By applying the LDA, one may obtain

$$M_\rho(k_F(E), E) = M(k(E), E),$$

(23)

where $k$ is the momentum variable. According to the calculations performed in Ref.[2], the real JLM NN effective interaction can be given as

$$v_{NN}(\rho, E) = M_\rho(k(E), E)/\rho.$$  

(24)

The real energy and density–dependent $V(\rho, E)$ are written in the form

$$V(\rho, E) = g_1(R) + g_2(R)\rho(r) + g_3(R)\rho^2(r),$$

(25)
where
\[ g_1(R) = C_1 - V_c d_1, \]
\[ g_2(R) = C_2 - V_c d_2, \]
\[ g_3(R) = C_3 - V_c d_3, \]

and
\[ d_1 = a_{12} + 2a_{13}E, \]
\[ d_2 = a_{22} + 2a_{23}E, \]
\[ d_3 = a_{32} + 2a_{33}E. \]

The coefficients \( a_{ij} \) are tabulated in Refs. [2, 15]. The above equations represent the basis of real JLM NN effective interaction in the final formula which can be used in the SF expression at which
\[ v_{JLM}^{NN}(s, \rho, E) = V[(\rho, E)] h(s), \]
where \( h(s) \) is the Gaussian radial form factor defined by
\[ h(s) = \frac{1}{(t\sqrt{\pi})^3} \exp(-s^2/t^2). \]

The range parameter \( t \) is chosen as 1.2 fm, and \( s = |\vec{R} - \vec{r}| \). The calculations are performed using the computer code HIOPTM–94 for the elastic scattering calculations. Best fits are obtained by minimizing \( \chi^2/N \), where
\[ \chi^2 = \sum_{k=1}^{N} \left[ \frac{\sigma_{th}(\theta_k) - \sigma_{ex}(\theta_k)}{\Delta\sigma_{ex}(\theta_k)} \right]^2, \]
where \( \sigma_{th} (\sigma_{ex}) \) are the theoretical (experimental) cross sections at the angle \( \theta_k \), \( \Delta\sigma_{ex} \) is the experimental error and \( N \) is the number of data points.

### 3. Result and discussion

Two different forms of the nuclear matter density distributions are used for Carbon isotopes as shown in Fig.(1). The elastic scattering cross sections for the \(^{12}\text{C}+^{12}\text{C}\) system, at incident energies 70.8, 78 and 106.9 MeV/N, are obtained with real renormalization parameters \( (N_R) \) closely approach to unity.

The calculated angular distributions of the differential cross sections based upon the derived density distributions of the projectiles (\( D1 \) and \( D2 \)) produce in general reasonable agreement with the experimental data (see Fig.2). Similar behavior has been obtained by Awad et al. [17] using the DF model and Badawy et al. [18] using the phenomenological real and imaginary optical potentials. From the above discussion one can easily notice that, the obtained \( \sigma_T \) using the EDF potentials are in a good agreement with the measured data. Also, one can notice that, the values of \( N_R \) vary from 0.92 to 1.10 for \( D1 \) density and from 0.76 to 1.32 for \( D2 \) density, all over the three considered energies. These values come close to the smaller values of the fitting.
Figure 1. Calculated elastic scattering angular distributions in the Rutherford ratio compared to the experimental data for the $^{12}\text{C}+^{12}\text{C}$ system, at incident energies $E$ (MeV/N), using the density distributions $D_1$ and $D_2$. The results are offset by the factors $10^2$

Due to the unavailable experimental data for the rest of Carbon isotopes, we conducted a predicted calculations of the elastic scattering cross sections and angular distributions for the $^{13-20}\text{C}+^{12}\text{C}$ systems, at the same incident energies (70.8, 78 and 106.9 MeV/N) based upon the derived real EDF and imaginary WS potential parameters. The obtained real ($J_R/A_P A_T$) and imaginary ($J_I/A_P A_T$) volume integrals are plotted versus the incident energies compared with the quoted values obtained by Grama et al. [17], as shown in Fig.(2). The obtained total reaction cross sections $\sigma_T$, are plotted versus the incident energies as shown in Figs.(3,4). In general, for both $D_1$ and $D_2$ densities, $\sigma_T$ increases till a specific energy then decreases. Compared to the experimental $RMS$ radii and BE for Carbon isotopes, our calculations based on the considered densities, give too large radii and total reaction cross sections for the Halo–Nuclei $^{15,16,17,19,22}\text{C}$, and accordingly small BE values with increasing the mass number ($A$). The obtained $RMS$ radii are plotted versus the projectile mass number $A$ (see Fig.5), compared with the recent published values [13, 15, 18, 32]. It is clear that, the $RMS$ values related to the density $D_1$ are more consistent with the considered data than $D_2$. One can obviously see that, the considered densities ($D_1$ and $D_2$) resemble each other in reproducing the matter distributions for Carbon isotopes.

The resulted $RMS$ matter radius of the one neutron drip–line $^{15}\text{C}$ is 3.23 fm, compared to the previous value of the stable isotope $^{14}\text{C}$, 2.67 fm, which based on the $^{14}\text{C}+n$ model, is
**Figure 3.** Energy–dependence of the total reaction cross section $\sigma_T$, for the $^{12-17}C+^{12}C$ systems.

**Figure 4.** Energy–dependence of the total reaction cross section $\sigma_T$, for the $^{18-22}C+^{12}C$ systems.

**Figure 5.** Calculated RMS radii for Carbon isotopes, compared to the recent published values.
probably very realistic. The deviation of the resulted binding energy from the experimental one is due to the blocking effect for the odd nucleus $^{15}$C. Same interpretation should be valid for the one neutron drip–line nuclei $^{17,19}$C. Also, the resulted RMS radius of the two neutrons drip–line nucleus $^{16}$C, is 3.44 fm, which works well with the $^{14}$C+n+n model. Same interpretation should be valid for two neutrons drip–line nucleus $^{22}$C. The $0d_{5/2}$ orbit is fully occupied in the nucleus $^{20}$C. As one neutron is added into $^{20}$C, it will become no particle–bound system, and if further more neutron is added it will become a bound system of $^{22}$C (see Fig.5).

4. Concluding remarks
In this study, the structural properties of the neutron drip–line nuclei ($^{15,16,17,19,22}$C ) are discussed. The ground state wave functions of C–isotopes are calculated using the core plus halo based on a phenomenological mean–field MF potential. The deduced wave functions are applied to provide the nuclear matter densities which are necessary in the calculations of the total reaction cross sections. The deduced densities $D_1$ and $D_2$ confirm the halo structure of the($^{15,16,17,19,22}$C ) nuclei. It is found that with the semi–phenomenological nuclear density, the EDF formalism can properly describe the structural properties of C–isotopes, e.g., reproducing the BE and RMS radii by certain quantitative precision. In addition, due to the limit of the present experimental platform, we only performed a comparison study for the $^{12}$C+$^{12}$C system, within the EDF theory. Based on the considered C–isotopes densities, one may conclude that the EDF theory has successfully described the experimental RMS radii and BE.

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6. References
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