Nonlinear dynamics of closed cylindrical shells under the action of longitudinal and transversal loads

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Abstract. The nonlinear dynamics of a closed cylindrical shell, described by the kinematic model of the first approximation (Kirchhoff-Love) under the action of external alternating longitudinal and transverse loads, is studied. Geometric nonlinearity is described by the T. von Karman model. The resulting system of nonlinear partial differential equations is reduced to an ODE system using the Bubnov-Galerkin method in higher approximations, the Cauchy problem is solved by the 4th and 6th orders of accuracy with the Runge-Kutta method. Numerical results are compared with experimental data for longitudinal loads obtained by Zippo A., Barbieri M., Pellicano F. A qualitative agreement of the results is shown.

1. Introduction
Closed cylindrical shells are widely used in various engineering structures, in aircraft and rocket science, instrument making, and medicine. In this regard, the work on theoretical studies of such structures in the world literature is presented very widely [1-3]. However, there are relatively few works on experimental studies of closed cylindrical shells, we note among them [4-6].

The aim of this work is to study the nonlinear dynamics of a closed cylindrical shell under the action of external loads of various types. The paper compares the results of a numerical experiment conducted by the authors on the basis of the developed mathematical model of nonlinear oscillations of a closed cylindrical shell, which is under the action of external loads with the results of full-scale experiments given in [4, 5].

2. Formulation of the problem
The mathematical model of a closed cylindrical shell is based on the kinematic assumptions of Kirchhoff-Love taking into account geometric nonlinearity according to the model of T. von Karman. The system of nonlinear partial differential equations, boundary and initial conditions are obtained from the Hamilton-Ostrogradskiy energy principle [7].

The system of partial differential equations for a closed cylindrical shell after reduction to a dimensionless form is given in (1):
We study the dependence of the critical load \( q_c \) on the pressure bandwidth, which is characterized by \( \varphi_0 \). To do this, we construct for \( \forall \varphi_0 \in [0; 2\pi] \) the set \( \{q_i, w_i\} \) by which the critical load \( q_c \) is determined. The study will be carried out for different approximations. Since the load is applied along the entire length of the cylindrical shell, the number of members of the series along the \( x \) coordinate does not play a role, and one member of the series can be kept in expansion (2), i.e. \( N_1 = 1 \). We study the dependence of the obtained results on the number of members of the series along the \( y \)-coordinate, from \( N_2 \). In figure 1 we give dependency graphs \( q_c(\varphi_0) \) for \( N_2 = 9; 10; 11; 12; 13; 14 \). The dependence \( q_c(\varphi_0) \) is nonmonotonic and oscillatory; an increase in the number \( N_2 \) leads to noticeable refinement of the results. This suggests that under inhomogeneous loading, the use of a small number of series members in expansion (2) leads to large errors, i.e., the results largely depend on the number of members of the series taken into account when solving by the Bubnov-Galerkin method. Starting from \( N_2 = 13 \),
the dynamic properties of the system are stabilized so that an increase in the number of equations does not bring anything new to the results and we obtain a convergent sequence. All subsequent results will be presented for $\omega = 13$.

![Figure 1. Dependence of critical loads on pressure bandwidth](image)

We study the change in the spatio-temporal characteristics of the shell when the type of loads changes. Under the action of a transverse alternating load $q_0 = 0.25, \omega_p = 19.5$, we study the change in the main characteristics (signal $w(t,0,0;\tau)$, phase portrait $u(\dot{w})$, power spectrum $S(\omega)$ and Poincare section $u_1,w_{s,T}$) and waveforms before (figure 2) and after applying an additional longitudinal distributed load. Each of the waveforms is recorded at time points $\tau$, which are indicated by the numbers 1, 2, 3, 4, 5. On the signal, the system is in chaos under the action of a transverse load.

In Fig. 2 there is a change in the number of half-waves in time. Maximum deflections spread over the entire surface of the shell.

Under the combined action of transverse $q(t) = q_0 \sin(\omega_p t)$ and longitudinal $p_3(t) = 10 \sin(\omega_p t)$ loads, shell vibrations pass from chaotic to harmonic (figure 3), which affects the forms of wave formation of the shell (figure 3 e-i).
The number of half-waves along the circumferential coordinate is kept constant, the maximum deflections are concentrated on the boundary of the local load application.

**Figure 2.** Spatial-temporal characteristics of the system under the action of a transverse load

**Figure 3.** Spatial-temporal characteristics of the system under the action of longitudinal and transverse loads
Forced synchronization of external influences leads to significant changes in the spatio-temporal characteristics of a closed cylindrical shell. The paper studies the scenario of the transition of oscillations from harmonic to chaotic. As a result, it was concluded that the vibrations of a closed cylindrical shell go over to chaotic ones according to the Feigenbaum scenario. A series of bifurcations of doubling the period of shell vibrations was recorded. The experimental value of the Feigenbaum constituent is calculated. The difference between the experimental value of the Feigenbaum constant and the theoretical value is 0.05%.

In [4], the results of full-scale experiments for closed cylindrical shells under the action of longitudinal constant and sinusoidal loads are presented. Among non-linear phenomena, such types of oscillations as subharmonic and quasiperiodic oscillations were experimentally obtained.

4. Concluding remarks
The nonlinear dynamics of a closed cylindrical shell under the influence of longitudinal and transverse loads is investigated. When longitudinal and transverse loads are exposed to the shell, the shell oscillates from chaotic to harmonic. The results obtained numerically and experimentally qualitatively coincide.

Acknowledgement
This work has been supported by the Grant RSF № 16-11-10138

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