Boosting photonic quantum computation with moderate nonlinearity

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Abstract: We present a new pathway towards fault-tolerant photonic quantum computing by using moderate nonlinearity to improve elementary computation operations. This improvement can lead to a three orders-of-magnitude reduction of the resource overhead in large-scale computations. © 2021 The Author(s)

Photonic measurement-based quantum computation (MBQC) is a promising route towards fault-tolerant universal quantum computing [1]. A central challenge in this effort is the huge overhead in the resources required for the construction of large photonic clusters using probabilistic linear-optics gates. Although strong single-photon nonlinearity ideally enables deterministic construction of such clusters [2], it is challenging to realise in a scalable way. Here we explore the prospects of using moderate nonlinearity (with conditional phase shifts smaller than π) to boost photonic quantum computing and significantly reduce its costs. The key element in our scheme is a nonlinear router that preferentially directs photonic wavepackets to different output ports depending on their intensity.

As a relevant example, we analyze the nonlinearity provided by Rydberg blockade in atomic ensembles, in which the trade-off between the nonlinearity and the accompanying loss is well understood [3]. We present protocols for efficient Bell measurement and GHZ-state preparation – both key elements in the construction of cluster states, as well as for the CNOT gate and quantum factorization. Given the large number of entangling operations involved in fault-tolerant MBQC, the increase in success probability provided by our protocols already at moderate nonlinearities can result in a dramatic reduction in the required resources.

Our nonlinear router is realized by a Mach–Zehnder interferometer that contains a nonlinear atomic medium. The medium is transparent to single photons, but photon pairs acquire conditional phase shifts ±φ when traveling through the interferometer arms. Using the notation of Fig. 1(a–b), single photons that enter the interferometer in mode α always exit in mode u, while photon pairs may reach either mode u, w or both, depending on φ:

\begin{align}
\text{single photons:} & \quad a^\dagger \xrightarrow{\text{BS}} 1/\sqrt{2}(f^\dagger + ig^\dagger) \xrightarrow{\text{BS}} u^\dagger, \\
\text{photon pairs:} & \quad (a^\dagger)^2 \xrightarrow{\text{BS}} 1/2 \left(e^{iπ/2} f^\dagger u^\dagger + e^{-iπ/2} g^\dagger w^\dagger\right) \xrightarrow{\text{BS}} (w^\dagger \sin φ + u^\dagger \cos φ)^2.
\end{align}

Here, \(x^\dagger\) denotes the creation operator of a photon in mode x. The probabilities for different final states for an incident photon pair are shown in Fig. 1(d). When \(φ = π\), a pair is routed into mode w, while \(φ = 0\) sends all photons to mode u. The key advantage of this device is that the probability for an incident pair to produce a click in w (in contrast to incident single photons that always reach u) is nonzero already for small φ.

Next, we utilize the nonlinear router to improve linear BMs. We define photonic qubits in the linear polarization basis, yet our analysis is applicable to any other choice of optical modes. In a traditional linear-optics BM, photons are sent through a balanced beam splitter (BS) and measured [4]. Using the notation of Fig. 1(c) and applying the BS transformation, \(a^\dagger \rightarrow \frac{1}{\sqrt{2}}(d^\dagger + ic^\dagger)\), \(b^\dagger \rightarrow \frac{1}{\sqrt{2}}(c^\dagger + id^\dagger)\), to each of the four Bell states, \(|ψ_±\rangle, |φ_±\rangle\), one finds

\begin{align}
|ψ_−\rangle & \equiv \frac{1}{\sqrt{2}}(a_d^\dagger b_c^\dagger - a_c^\dagger b_d^\dagger)|\text{vac}\rangle \xrightarrow{\text{BS}} \frac{1}{\sqrt{2}}(d_c^\dagger c_d^\dagger - c_c^\dagger d_d^\dagger)|\text{vac}\rangle, \\
|ψ_+\rangle & \equiv \frac{1}{\sqrt{2}}(a_d^\dagger b_c^\dagger + a_c^\dagger b_d^\dagger)|\text{vac}\rangle \xrightarrow{\text{BS}} \frac{1}{\sqrt{2}}(d_c^\dagger c_d^\dagger + c_c^\dagger d_d^\dagger)|\text{vac}\rangle, \\
|φ_±\rangle & \equiv \frac{1}{2\sqrt{2}}(a_i^\dagger b_i^\dagger ± a_i^\dagger b_i^\dagger)|\text{vac}\rangle \xrightarrow{\text{BS}} \frac{1}{2\sqrt{2}}[(d_i^\dagger)^2 ± (c_i^\dagger)^2]|\text{vac}\rangle.
\end{align}

where \(|\text{vac}\rangle\) is the vacuum state. The states \(|ψ_±\rangle\) lead to distinguishable outcomes; While \(|ψ_−\rangle\) produces one photon in c and one in d, the state \(|ψ_+\rangle\) produces an orthogonal pair in either c or d. In contrast, the states \(|φ_±\rangle\) produce a “bunched” pair in one of the four detectors and are, therefore, indistinguishable by the measurement. Hence, when detectors are placed at the exit of the BS, the success probability of the BM is 50%. To increase
the success probability, we place nonlinear routers that help distinguish between $|\phi>_s$ at the output of the BS. By substituting Eq. (1) into Eq. (2), we obtain the final states before reaching detectors (1–8), from which we find

$$P_{BM} = 1 - \frac{1}{8} (\cos \varphi + 1)^2. \quad (3)$$

The success probability grows monotonically with $\varphi$, reaching 1 when $\varphi = \pi$ [Fig. 1(e), solid]. Also shown (dashed) is a nonlinear modification of the linear Ewert-van Loock protocol, that attains a higher success probability at the cost of using four ancillary qubits [5]. Using similar ideas, we develop nonlinearity-enhanced protocols for GHZ-state generation, CNOT gate and quantum factorization (not shown). Additionally, we carefully analyze the effect of noise on our scheme, using the highly relevant example of Rydberg atoms (see Ref. [6] for details).

To summarize, this work examines photonic quantum computation protocols in the intermediate regime between linear optics and strong nonlinearity at the single photon level. We present efficient protocols for key elementary computation operations. Our results demonstrate the potential of moderate nonlinearity, which is achievable in a variety of platforms. As photonic quantum computation, and fault-tolerant MBQC in particular, require very large number of elementary operations, any modest increase in the success probability of each operation is translated to a dramatic reduction in the required resources. For example, a conditional phase shift of $\varphi = \pi/3$, which in our scheme increases the success probability of ancilla-assisted (Ewert-van Loock) BM from 0.75 to 0.86, is translated into two-orders-of-magnitude reduction in resources after 35 entangling operations, and three-orders-of-magnitude improvement at 50 gates. With the recent developments in integrated photonics and microfabricated atomic vapor cells, few-photon nonlinearity on chip-scale devices is becoming feasible, making protocols that rely on moderate nonlinearities a promising new platform for photonic quantum information processing.

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