Correlation induced resonances in transport through coupled quantum dots

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We investigate the effect of local electron correlations on transport through parallel quantum dots. The linear conductance as a function of gate voltage is strongly affected by the interplay of the interaction $U$ and quantum interference. We find a pair of novel correlation induced resonances separated by an energy scale that depends exponentially on $U$. The effect is robust against a small detuning of the dot energy levels and occurs for arbitrary generic tunnel couplings. It should be observable in experiments on the basis of presently existing double-dot setups.

In the present Letter, we investigate this problem for a specific model of two parallel quantum dots coupled by an electron interaction $U$ (see Fig. 1). The effect of external electrostatic potentials on two-path interference was studied earlier (magneto-electric Aharonov-Bohm effect).

We here investigate the role of interaction induced potentials. This is especially relevant for transport properties, as the interaction induced potentials will be far larger than the external potentials which are difficult to apply to such small structures. We study how the linear conductance $G$ as a function of gate voltage $V_g$ changes with increasing $U$. Considering the entire parameter space we find a very rich generic behavior and predict the appearance of novel correlation induced resonances (CIRs) if $U$ is larger than a critical interaction $U_c$. The effect is robust: It appears for almost arbitrary combinations of the four tunnel couplings and also remains visible for a small detuning of the dot level energies. The separation of the resonances in gate voltage defines an energy scale that depends exponentially on $U$ and on a combination of the tunnel couplings. It is argued that this new correlation effect is unrelated to Kondo physics.

We employ a powerful new method, the functional renormalization group (fRG), to efficiently obtain both numerical and analytical results for this many-body problem, and we have confirmed all the essential features of our results using the numerical renormalization group (NRG).

![FIG. 1: System of coupled quantum dots with common leads.](image-url)

Double-dot geometries that could form the basis to verify our predictions have been experimentally realized in Refs. 8, 9, 10. Our model is equally of relevance for transport through two nearly degenerate levels of a single dot, a subject that has attracted much attention recently in attempts to understand the puzzling behavior of the transmission phase.

We study a system of two quantum dots $j = 1, 2$ each having a single level $\varepsilon_j$ as sketched in Fig. 1. The dots are coupled by a Coulomb interaction $U \geq 0$ and are connected to two common leads $L, R$ via tunnel barriers $t_{jL}$. The dot Hamiltonian is $H_{\text{dot}} = \sum_j \varepsilon_j d_j^{\dagger} d_j + U(n_1 - 1/2)(n_2 - 1/2)$ and the dot-lead coupling is given by $H_{LR} = -\sum_{j=1}^{2} (t_{jL}^{\dagger} c_{jL} + H.c.)$, where $c_{jL}$ denotes the creation operator at the end of the semi-infinite lead $L$. The leads are modeled by $H_{L} = -t \sum_{m=0}^{\infty} (c_{mL}^{\dagger} c_{m+1L} + H.c.)$. The energy scale of the dot level broadening is given by $\Gamma_j = \pi |t_j|^{2} \rho_{l}$, where $\rho_{l}$ denotes the local density of states at the end of lead $L$. As usual we later take $\rho_{l}$ to be energy independent (wide band limit).

The theoretical and experimental research on electronic transport through ultrasmall quantum dots has become a very active field. Various fundamental physical phenomena such as quantum interference, Coulomb blockade, and the Kondo effect strongly affect the transport properties. Currently, the focus is shifting towards few-electron double-dot structures, that are studied as artificial molecules, interferometers, and for phenomena such as quantum interference, Coulomb blockade, and the Kondo effect. We here investigate the role of interactions in systems of two or more coupled quantum dots that are still at the beginning and much remains to be explored. In parallel quantum dots connected to common leads new physics is to be expected due to the interplay of correlations and quantum interference, an issue that is of interest also from a broader perspective.

FIG. 1: System of coupled quantum dots with common leads.
To compute \( G \) and, in addition, the level occupancies \( \langle n_j \rangle \) we mainly use a recently developed fRG scheme. The starting point is an exact hierarchy of differential flow equations for the real-space self-energy matrix \( \Sigma^A \) and higher order vertex functions, where \( \Lambda \in (\infty, 0] \) denotes an infrared energy cutoff which is the flow parameter. We truncate the hierarchy by neglecting the flow of the two-particle vertex only considering \( \Sigma^A \), which is then energy independent. This approximation and variants of it were successfully used to study a variety of transport problems through quasi one-dimensional wires of correlated electrons (Tomonaga-Luttinger liquids). In addition, spectral and transport properties of locally correlated systems were investigated using the fRG.

Although the guiding principle behind the above approximation scheme is perturbation theory, it was shown that depending on the problem studied, the fRG procedure correctly leads to power-laws with \( \Lambda \) depending on the problem studied, the fRG procedure.

For the double-dot \( \Sigma^A \) is a \( 2 \times 2 \) matrix in the dot label \( j \). The diagonal parts are real and \( V_j^A = \Sigma^A_{j,j} + V_g \) can be considered as effective dot level positions. The \( \Lambda \) complex off-diagonal contribution \( t^A_{j,j} = -\Sigma^A_{1,2} \) is a hopping between the two dot states generated by the interaction. The flow equations are

\[
\partial_\Lambda V_j^A = - U \sum_{\omega = \pm \Lambda} G^A_{j,j}(i\omega),
\]

\[
\partial_\Lambda t^A_{j,j} = - U \sum_{\omega = \pm \Lambda} G^A_{1,2}(i\omega),
\]

with \( j \) being the complement of \( j \). The Green function is \( G^A(i\omega) = [i\omega - h^A(i\omega)]^{-1} \) with

\[
h^A(i\omega) = \begin{pmatrix}
V_j^A - i\Gamma_j \, \text{sgn} \omega & -t^A_{j,j} - i\gamma \, \text{sgn} \omega \\
(t^A_{j,j})^* - i\gamma^* \, \text{sgn} \omega & V_j^A - i\Gamma_j \, \text{sgn} \omega
\end{pmatrix},
\]

and \( \Gamma_j = \sum_{l} \Gamma^l_{j,l} = \gamma \sqrt{\Gamma^l_0 \Gamma^l_1 + e^{i\phi} \Gamma^l_0 \Gamma^l_2} \) the initial conditions are \( V_j^{\Lambda=\infty} = V_j^0 + \delta \), \( V_j^{\Lambda=\infty} = V_j^0 \), and \( t^A_{j,j} = 0 \). To obtain an approximation for \( \Sigma \) and the one-particle Green function (using the Dyson equation) one has to solve the system of four real coupled differential equations. This can easily be done numerically and for a specific class of parameters also analytically. The occupancies \( \langle n_j \rangle \) can directly be calculated from the Green function. For the present problem the same holds for \( G \) as current vertex corrections vanish. One can easily derive a lengthy expression for \( G \) in terms of the parameters \( \Gamma^l_j, \phi \) and the renormalized level positions \( V_j = V_j^{\Lambda=0} \) and hopping \( t^A_{j,j} = t^A_{j,j}^{\Lambda=0} \) not presented here. \( V_j \) and \( t^A_{j,j} \) depend on \( \Gamma^l_j, \phi \) as well as on \( V_j^0, \delta, \) and \( U \).

We use the NRG as a nonperturbative method to confirm the essential validity of the physics discovered within the fRG. We allow arbitrary dot-lead couplings by employing the Kubo-formula for the current, which yields the frequency-dependent conductance

\[
\text{Re} G(\omega) = \frac{\pi}{\omega} \sum_f |I_{0,f}|^2 \delta(E_f - E_0 - \omega), \quad \omega > 0. \quad (3)
\]

The current operator \( I = e(\dot{N}_R - \dot{N}_L)/2 \) (with \( N_l = \sum_{m=0}^{\infty} c^t_{m,l} c_{m,l} \) and \( \dot{N}_l = i[H, N_l] \)) is expressed in terms of lead- and dot-operators. Its matrix elements \( I_{0,f} \) (with 0 indicating the ground state and \( f \) the excited states; \( E_0 \) and \( E_f \) are the respective energies) are evaluated in the NRG basis and the \( \omega \to 0 \) limit then provides the conductance \( G \).

We first consider the case of degenerate levels with \( \delta = 0 \) and later discuss how the results change for \( \delta > 0 \). The behavior of \( G(V_g, U) \) can be cast in four classes that can already be identified at \( U = 0 \). (i) If two or more of the \( \Gamma^l_j \) are 0 such that no closed path between the left and right lead exists \( G(V_g, U) \equiv 0 \). (ii) \( \Gamma^l_j \Gamma^l_1 = \Gamma^l_2 \Gamma^l_2 \neq 0 \) and \( \phi = \pi \). In this case one can introduce new fermionic dot states such that one only couples to the left lead and the other only to the right lead, implying \( G(V_g, U) \equiv 0 \). We note that in this case (and only in this case) a conserved pseudo-spin variable (left/right) exists. (iii) A nonvanishing (but nongeneric) conductance is found for \( \Gamma^l_1 \Gamma^l_2 = \Gamma^l_2 \Gamma^l_2 \neq 0 \) and \( \phi = 0 \). In this case \( G(V_g, U = 0) \) is given by a Lorentzian centered around 0. \( U > 0 \) dependence of \( G(V_g, U) \) can most easily be studied in the exactly solvable case of equal \( \Gamma^l_j \). It is characterized by two Coulomb blockade peaks located at \( \pi \pm U/2 \). (iv) For all other \( \Gamma^l_j, \phi \), that is for generic parameters on which we focus in the following, the peak in \( G(V_g, U = 0) \) at \( V_g = 0 \) [as in case (iii)] is replaced by a dip with \( G(V_g, U = 0) \). For equal \( \Gamma^l_1, \phi = 0, \delta > 0 \), and \( U = 0 \) the appearance of such dips was discussed earlier and explained as a destructive interference between path traversing dots 1 and 2 respectively. In the limit of a strong asymmetry of the transmission probability via dots 1 and 2, e.g. for \( \Gamma^l_1 \ll \Gamma^l_2 \), the dip can be viewed as a Fano anti-resonance resulting from the interference of a resonant path and a path with energy independent transmission: For energies at which the transmission via dot 1 shows a resonance, the transmission via dot 2 can be regarded as constant.

Fig. 2 shows the generic evolution of \( G(V_g) \) for increasing \( U \) at \( \delta = 0 \). Because of particle-hole symmetry \( G \) is symmetric around \( V_g = 0 \). Energies are given in units of \( \Gamma = \sum_j \Gamma_j \). Increasing \( U \) the height of the two peaks resulting from the dip at \( V_g = 0 \) increases and the maximum flattens. At a critical \( U = \tilde{U}_c \) each of the peaks splits into two. For the present example the FRG approximation is \( U_c/\Gamma \approx 4.69 \). Further increasing \( U \) the two outer most peaks move towards larger \( |V_g| \) and become the Coulomb blockade peaks located at \( V_g \approx \pm U/2 \).

The other two peaks at \( \pm V_{\text{CIR}} \) are the novel CIRs, where \( V_{\text{CIR}} > 0 \) decreases with increasing \( U \). Associated with \( G(V_g = 0) \) at \( U = 0 \) is a jump of the transmis-
FIG. 2: (Color online) Generic results for \(G(V_g) / (e^2 / h)\) (solid lines), \(\langle n_1 \rangle\) (dashed lines), and \(\langle n_2 \rangle\) (dashed-dotted lines) at different \(U\) obtained from the fRG with \(\Gamma_j^L = 0.27 \Gamma\), \(\Gamma_j^R = 0.33 \Gamma\), \(\Gamma_j^L = 0.16 \Gamma\), \(\Gamma_j^R = 0.24 \Gamma\), \(\phi = \pi\), and \(\delta = 0\). The two novel correlation induced resonances are visible in the lower panels (large \(U\)), near \(V_g = 0\).

FIG. 3: (Color online) Comparison of fRG (lines) and NRG (symbols) results for the same parameters as in Fig. 2. Left: \(G(V_g)\) for different \(U = 3.5 \Gamma\): solid line and circles; \(U = 7 \Gamma\): dashed line and squares; \(U = 14 \Gamma\): dashed-dotted line and diamonds). Right: The resonance position \(V_{\text{CIR}}\) as a function of \(U\).

Fig. 2. In particular, for increasing \(U > U_c\), \(V_{\text{CIR}}\) becomes small quickly. The right part of Fig. 4 shows the \(U\) dependence of \(V_{\text{CIR}}\) extracted from the numerical data. For \(U\) sufficiently larger than \(U_c\),

\[
V_{\text{CIR}} / \Gamma \propto \exp \left[ -C \left( \{ \Gamma_j^L \}, \phi \right) U / \Gamma \right],
\]

with \(C > 0\). By construction the fRG based approximation scheme works particularly well for small to intermediate \(U\). At larger \(U\) and for all \(\Gamma_j^L, \phi\) we tested the fRG overestimates the positions of the Coulomb blockade peaks and the CIRs. As will be shown in an upcoming publication this can systematically be improved using a more elaborate fRG truncation scheme. For a specific class of \(\Gamma_j^L, \phi\) we next analytically confirm the exponential dependence of \(V_{\text{CIR}}\) on \(U\) and derive an explicit expression for \(C\) using the fRG. In the most general case the dependence of \(C\) (and \(U_c\)) on \(\Gamma_j^L\) is complex and requires further investigation. Roughly speaking \(C\) increases (\(U_c\) decreases) with increasing asymmetry of the \(\Gamma_j^L\) [see also Eq. 5 below]. For fixed \(\Gamma_j^L\) and increasing \(0 \leq \phi \leq \pi\), \(C\) decreases while \(U_c\) increases.

We now consider \(\Gamma_j^L = \Gamma_1^R\), \(\Gamma_j^L = \Gamma_2^R\), but \(\Gamma_1^L \neq \Gamma_2^L\),
This explains the observed potential dependence of the effective level positions. Within this approach the Hartree-Fock approximation is used to compute the effective wave function of the double-dot interferometer are strongly affected by electron correlations. We discovered a novel pair of correlation-induced resonances that should be measurable in double-dots of appropriate geometry in the presence of strong Zeeman splitting. Varying the tunnel barriers (and thus $U/\Gamma$) it should be possible to study the entire scenario discussed above. Apparently this correlation effect is unrelated to both spin and orbital Kondo physics. Rather, it follows from the interplay of local correlations and quantum interference. It is thus likely that similar effects will appear in transport through more complex systems as e.g. ring-like molecules studied in the context of molecular electronics. Besides revealing interesting new physics, we showed that the fRG method is a very promising tool to investigate problems with local Coulomb correlations. In comparison to NRG the fRG is far superior in terms of the numerical effort required, e.g. enabling efficient analysis of parameter dependencies. Furthermore, the fRG can easily be extended to more complex systems with local electron correlations.

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