Reliability Evaluation of Mechatronic Equipment Based on Evidence Theory

Zhiyuan He, Weimin Lv* and Wenlin Hu
Naval Aviation University, Yantai, Shandong, 264001, China
*Corresponding author’s e-mail: 1340529592@qq.com

Abstract. To solve the correlation problem in evidence theory, this paper proposes an evidence theory model based on Copula function and related reliability assessment methods. First, we establish a performance degradation model based on the Wiener process and calculate the PDF of the lifetime distribution. Secondly, the fuzzy C-means algorithm is used to discretize the probability density function and obtain the marginal basic reliability allocation (BPA). Finally, the fusion of multidimensional evidence variables is carried out using the improved synthesis rules based on Copula function. The example analysis shows that this method solves the evidence-related problems in traditional evidence theory, and it is more efficient than other methods in the literature.

1. Introduction
In engineering practice, product performance often has uncertainties related to material properties, geometric shapes and environmental factors. Cognition, quantification and control of various uncertainties can significantly affect the reliability and comprehensive performance of products. Typical methods for dealing with uncertainty include probability theory, fuzzy sets [1], convex model theory [2], evidence theory [3-5] and so on.

Evidence theory is an analytical method that provides a reasonable description of incomplete, unreliable or conflicting information. The theory of evidence was first applied to the expert system. Its main feature is to satisfy the weaker conditions than Bayesian probability theory. The introduction of the reliability function has the ability to distinguish between “don't know” and “uncertain”. Probability theory is a special case of evidence theory [6]. In different cases, it can also be transformed into classical probability theory, fuzzy set theory and convex model theory. Because evidence theory has strong cognitive uncertainty modeling ability, it provides a natural and human-oriented method for the expression and synthesis of uncertain information, which makes the evidence theory more applicable in the field of reliability analysis. Liu et al. [7] combined the evidence theory with the confidence rule base to propose a transformation and fusion method for different environmental life data. Bai et al. [8] proposed the concept of “evidence moment theory” and applied it to structural static and dynamic uncertainty analysis.

At present, the application of evidence theory in reliability analysis has achieved certain results. In particular, some scholars have solved the evidence theory’s evidence failure, high conflict, 0 (1) reliability, trust offset and the paradox of focal element blur [9] by improving the synthesis rules of evidence theory. In the process of evidence fusion, most scholars assume that the evidences are independent of each other and that there is little consideration of relevance. However, there are related problems in most systems, especially in the process of product performance degradation. If the evidence theory is directly adopted without considering the interaction between the parameters, the accuracy of
the reliability evaluation of the product performance will be greatly reduced. Aiming at this problem, this paper proposes an improved evidence synthesis rule based on the evidence theory model, which uses the Copula function to describe the correlation between the evidences and evaluate the reliability of the performance of a mechatronic device.

2. D-S evidence theory

D-S evidence theory, also known as Dempster-Shafer theory, is a mathematical method to solve the problem of uncertainty reasoning [10]. The main concepts of the evidence theory are as follows.

(1) Frame of discernment

For a reliability assessment problem, there are multiple evaluation results due to the existence of uncertainty. All possible evaluation results is called the frame of discernment (FD) and is represented by the set \( \Theta \). \( \Theta = \{x_1, x_2, \ldots, x_m\} \), where \( x_i \) represents an element in the FD. The FD \( \Theta \) is usually a finite non-empty set, and the set consisting of all subsets of the FD is called the power set, denoted as \( 2^\Theta \), which means any possible set of propositions. When there are \( n \) elements in it, there are \( 2^n \) elements in the power set \( 2^\Theta \).

(2) Basic probability assignment (BPA).

Assuming that \( \Theta \) is the frame of discernment, if the mapping function \( m: 2^\Theta \rightarrow [0,1] \) satisfies the following properties,

\[
\begin{align*}
    m(\emptyset) &= 0 \\
    m(A) &\geq 0, \forall A \in 2^\Theta \\
    \sum_{A \in 2^\Theta} m(A) &= 1
\end{align*}
\]

(1)

\( m \) is named the Basic probability assignment on FD \( \Theta \). \( \forall A \in 2^\Theta, m(A) \) is the Basic credibility of the set \( A \) and reflects the level of evidence support for set \( A \), where set \( A \) with \( m(A) > 0 \) is named the focal element of \( m \).

(3) Belief measure \( \text{Bel} \) and plausibility measure \( \text{Pl} \)

Assuming that \( \Theta \) is the frame of discernment, if the function \( \text{Bel}: 2^\Theta \rightarrow [0,1] \) satisfies the following properties,

\[
\text{Bel}(A) = \sum_{B \subseteq A} m(B) \quad (2)
\]

\( \text{Bel} \) is named the belief measure function of the set \( A \), which indicates the extent to which evidence supports \( A \) for true.

The basic probability assignment differs from the Belief measure in that \( m(A) \) only reflects the size of the set's own confidence, does not involve any subset of the set, but \( \text{Bel} \) reflects the sum of the confidence of the set \( A \) and all its subsets.

Assuming that \( \Theta \) is the frame of discernment, if the function \( \text{Pl}: 2^\Theta \rightarrow [0,1] \) satisfies the following properties,

\[
\text{Pl}(A) = \sum_{B \cap A \neq \emptyset} m(B) \quad (3)
\]

\( \text{Pl} \) is named the plausibility measure function, which indicates the extent to which evidence supports \( A \) for non-false.

(4) Dempster's combination rule

Assuming that \( m_1 \) and \( m_2 \) are the BPA functions of the two evidences under the same FD and the corresponding focal elements are \( B_1, B_2, \ldots, B_k \) and \( C_1, C_2, \ldots, C_j \), then the Dempster's combination rule is described as,
where \( K = \sum_{k} m_k(B_i)m_k(C_j) \),

\( K \) represents a measure of the degree of conflict between evidences, called the conflict amount parameter. A large \( K \) value indicates a large conflict.

Therefore, the basic principle of synthesizing data from multiple information sources using D-S evidence theory is shown in Figure 1.

3. Improved evidence theory model

3.1. Evidence theory combination rule based on Copula function

In the D-S evidence theory, most of the D-S combination rules are used, and it is usually assumed that the evidences are independent of each other. Therefore, the construction of the joint BPA is relatively simple. That is, for multidimensional evidence, the joint BPA value can be obtained by directly multiplying the BPA of each focal element. The D-S evidence theory method based on the independence hypothesis may produce large analytical errors because the evidence is interrelated and the correlation will significantly affect the reliability assessment results. In this section, we present an improved evidence theory model based on the Copula function [15]. This model can perform fusion calculations on correlated evidence variables.

For multidimensional evidence variables \( e_1, e_2, \ldots, e_n \), a joint BPA can be obtained using the marginal BPA function and the joint distribution function \( C(F_1(e_1), F_2(e_2), \ldots, F_n(e_n)) \) of each evidence.

First, the two-dimensional problem is taken as an example to illustrate the construction of the joint BPA. Let \( E_1 \) and \( E_2 \) be the two evidences under the frame of discernment \( \Theta \). The corresponding BPA functions are \( e_1 \) and \( e_2 \) and the corresponding focal BPAs are \( e_{11}, e_{12} \) and \( e_{21}, e_{22} \), respectively. \( d_k \) denotes any of the focal elements on the two-dimensional joint BPA and is represented by a rectangular area surrounded by four lines of evidence focal element \( e_{11}, e_{12}, e_{21}, e_{22} \) in geometric depiction, as shown in Figure 2.

Figure 1. D-S evidence theory combination rule

Figure 2. Basic probability assignment of \( d_k \)

The shaded part of the figure indicates the joint effect of the two evidences. Since the correlation between the evidence focal elements is assumed, the combination of each evidence coke is realized by the Copula function. For example, the cumulative probability that the focal elements \( e_1 \) and \( e_2 \) and the coordinate axes enclose the area in the figure is denoted as \( C(F_1(e_{11}), F_2(e_{21})) \). After calculating
the cumulative probability of each region by this rule, the focal unit of the joint BPA can be expressed as,

\[ m(d_r) = C(F_1(e_{12}), F_2(e_{22})) - C(F_1(e_{12}), F_2(e_{21})) - C(F_1(e_{11}), F_2(e_{22})) + C(F_1(e_{11}), F_2(e_{21})) \]  

(5)

For each focus element in the evidence based on the Copula function rule, we can use equation (5) to calculate the BPA value and finally obtain the joint BPA function of the two evidence variables. Similarly, this evidence combination rule can be extended to multidimensional evidence.

3.2. Performance degradation modeling

Performance degradation data is an important source of information for the reliability assessment of on-ball components and the basis for obtaining the functions of each evidence body. Due to the complex structure of the mechatronic equipment, the performance degradation process tends to be random, and the trajectory of the degradation amount has the characteristics of uncertainty. Therefore, the method of modeling the performance degradation data by using random processes is Product degradation data analysis and processing has good applicability.

Assuming that the degradation factor of the product performance parameter at time \( t \) is \( X(t) \), the performance degradation model can be expressed as,

\[ X(t) = \mu t + \sigma B(t) \]

(6)

where \( \mu \) represents drift coefficient; \( \sigma \) represents diffusion coefficient; \( B(t) \) represents standard brownian motion.

When the performance degradation of the product reaches a preset critical level, the product is considered to be ineffective at this time. This critical level is called the failure threshold. Assuming that the product has a failure threshold of \( \omega \), the product has a lifetime of \( T \), and the product's performance degradation trajectory is described by the Wiener process of equation (6). Then the life \( T \) of a product can be defined as the time when the performance degradation of the product reaches its failure threshold \( \omega \) for the first time. The life of the product can be expressed as,

\[ T = \{ t : X(t) \geq \omega | X(0) < \omega \} \]  

(7)

Assuming that \( \mu \) and \( \sigma \) are fixed unknown parameters, and reference [16] shows that the product lifetime obeys the inverse Gaussian distribution. Then the probability density function of the product lifetime \( T \) is,

\[ f_T(t) = \frac{\omega}{2\pi \sigma^2 t^3} \exp \left\{ \frac{(\omega - \mu t)^2}{2\sigma^2 t} \right\} \]  

(8)

According to the characteristics of independent increment of Wiener process, the maximum likelihood estimation method is used to estimate the parameters of \( \mu \) and \( \sigma \). We can get,

\[ \hat{\mu} = \frac{\sum_{i=1}^{n} \Delta X_i}{\sum_{i=1}^{n} \Delta t_i} \]

\[ \hat{\sigma}^2 = \frac{1}{n} \sum_{i=1}^{n} \left( \frac{\Delta X_i - \hat{\mu} \Delta t_i}{\Delta t_i} \right)^2 \]

(9)

where \( \Delta X_i = X(t_i) - X(t_{i-1}) \); \( \Delta t_i = t_i - t_{i-1} \).

We integrate the equation (8) to obtain the corresponding product life distribution function,

\[ F_T(t) = \Phi \left( \frac{-\omega + \mu t}{\sigma \sqrt{t}} \right) + \exp \left( \frac{2\omega \mu}{\sigma^2} \right) \Phi \left( \frac{-\omega + \mu t}{\sigma \sqrt{t}} \right) \]

(10)

where \( \Phi(\cdot) \) is a distribution function of the standard normal distribution.

The reliability function can be expressed as:
\[ R(t) = 1 - F_r(t) \]  

(11)

3.3. Basic probability assignment (BPA) acquisition

Combined with the content of the previous section, the life distribution function of the product is taken as the body of evidence. Before the evidence is combined, the basic probability assignment (BPA) of each body of evidence is also required. In this section, the fuzzy C-means algorithm is used to discretize the probability density function of the life distribution, and the marginal BPA of the life distribution under the influence of each performance index is obtained.

The fuzzy C-means (FCM) algorithm obtains the membership degree of each sample point to all class centers by optimizing the objective function, and determines the generic of the sample points \[ [17] \], which is the most widely used cluster analysis algorithm. It is also an effective way to discretize continuous data. The algorithm divides \( n \)-dim vectors (sample points) \( x_i \in R^l, (i=1,2,\cdots,n) \) into \( C \) partition classes. It uses the degree of membership between values \([0,1]\) to determine the extent to which each sample point belongs to each classification class and finds each partition class. At the center point, the cluster center point satisfies the minimum objective function value.

The objective function \([18] \) of the FCM algorithm is,

\[
J(U, V) = \sum_{k=1}^{N} \sum_{i=1}^{C} (u_{ik})^m (d_{ik})^2
\]

(12)

where \( U \) represents membership matrix; \( V \) represents cluster center matrix; \( u_{ik} \) represents that the \( k \)-th sample point belongs to the membership degree of the \( i \)-th partition class; \( d_{ik} \) represents the distance from the \( k \)-th sample point to the center of the \( i \)-th division, usually, \( (d_{ik})^2 = \|x_i - v_i\| = \|(x_i - v_i)^T(x_i - v_i)\) ; \( m \) represents the weighted index, \( m \in [1, \infty) \).

Since the columns in the matrix \( U \) are independent of each other, to make \( J(U,V) \) take the minimum value, there is,

\[
\text{min} \{ J(U,V) \} = \text{min} \left\{ \sum_{k=1}^{N} \sum_{i=1}^{C} (u_{ik})^m (d_{ik})^2 \right\} = \sum_{k=1}^{N} \min \left\{ \sum_{i=1}^{C} (u_{ik})^m (d_{ik})^2 \right\}
\]

(13)

The constraint for obtaining the extreme value of the above equation is \( \sum_{i=1}^{C} u_{ik} = 1, k = 1,2,\cdots,n \). The necessary conditions for obtaining the minimum value of the above equation by Lagrangian multiplication are,

\[
v_j = \frac{\sum_{k=1}^{N} (u_{ik})^m x_k}{\sum_{k=1}^{N} (u_{ik})^m}
\]

\[
u_{ik} = \frac{1}{\sum_{j=1}^{C} \left( \frac{d_{ik}}{d_{jk}} \right)^{m-1}}
\]

(14)

Given the sample data \( X \), the number of clusters \( C \) and the weight index \( m \), the optimal clustering center of the life distribution data can be obtained. Furthermore, we can divide the time interval \([T_{\text{min}}, T_{\text{max}}]\) of the life distribution to meet the following conditions,
ch. According to equation (9), the maximum thresholds of the three performance
ure thresholds of the three performance
\( T_{\min} = \frac{1}{2}(T_{c_{i-1}} + T_{c}) \)
\( T_{\max} = \frac{1}{2}(T_{c_i} + T_{c_{i+1}}) \)
where \( i \) represents the clustering interval of the \( i \)-th life distribution.
Combining equation (8), we can find the basic probability assignment \( P(t_i) \) of the \( i \)-th information source in interval \( [T_{\min}, T_{\max}] \) as:
\[ P(t_i) = \int_{t_{\min}}^{t_{\max}} f_T(t) dt \]

4. Application and analysis
Through the analysis of the failure mode, mechanism and influence of a mechatronic equipment, it is found that the stresses such as temperature, humidity and vibration will have a greater impact on the performance of the component, and the temperature stress has the greatest influence. Therefore, we select three samples in this paper and design the performance degradation test based on temperature stress. The test temperature was set to 60 °C, and the sample was fixed in the incubator. The test selected zero-bias, zero-bias stability and scale factor as the measurement indicators. Each performance index of each sample was measured and recorded every 240h. A total of 20 measurements were made. The total test duration was 4560h. The data is shown in Table 1.

| sample | zero-bias (°)/h | zero-bias stability (°)/h | scale factor (°)/h |
|--------|----------------|--------------------------|-----------------|
| 1      | 0.0000         | 0.0000                   | 0.0000          |
| 2      | 0.0021         | 0.0011                   | 0.0044          |
| 3      | 0.0078         | 0.0151                   | 0.0078          |
| 4      | 0.0156         | 0.0072                   | 0.0076          |
| 5      | 0.0259         | 0.0123                   | 0.0195          |
| 6      | 0.0286         | 0.0133                   | 0.0197          |
| 7      | 0.0488         | 0.0306                   | 0.0311          |
| 8      | 0.0506         | 0.0307                   | 0.0333          |
| 9      | 0.0531         | 0.0301                   | 0.0374          |
| 10     | 0.0558         | 0.0353                   | 0.0297          |
| 11     | 0.0644         | 0.0340                   | 0.0437          |
| 12     | 0.0657         | 0.0416                   | 0.0491          |
| 13     | 0.0603         | 0.0447                   | 0.0585          |
| 14     | 0.0660         | 0.0449                   | 0.0571          |
| 15     | 0.0690         | 0.0602                   | 0.0544          |
| 16     | 0.0686         | 0.0602                   | 0.0550          |
| 17     | 0.0713         | 0.0633                   | 0.0582          |
| 18     | 0.0773         | 0.0689                   | 0.0604          |
| 19     | 0.0792         | 0.0908                   | 0.0594          |
| 20     | 0.0936         | 0.0960                   | 0.0623          |

As can be seen from Table 1, the three performance indicators have obvious degradation trends, in line with the performance degradation model established in Section 3.2. According to the experience of the mechatronic equipment in actual equipment, the failure thresholds of the three performance indicators are set to 0.6 (°)/h, 0.4 (°)/h and 2.5 (°)/h. According to equation (9), the maximum likelihood estimates of the unknown parameters \( \mu \) and \( \sigma^2 \) of each degradation model are shown in Table 2.
Table 2. Degenerate model unknown parameters

| parameters | zero-bias | zero-bias stability | scale factor |
|------------|-----------|---------------------|--------------|
| $\mu$      | 0.00186   | 0.00125             | 0.00799      |
| $\sigma^2$ | 1.54×10^{-5} | 5.96×10^{-7}       | 8.47×10^{-5} |

We bring the unknown parameters into equation (8), and we can obtain the probability density function of the life distribution under the influence of each performance index, that is, the probability density function of each evidence body. According to the method of Section 3.2, the evidence body is assigned the reliability, and 1000 sample points are randomly selected on the probability density function of the life distribution. Set the number of clusters $C = 200$, weight index $m = 2$. The evidence combination is started after the basic reliability distribution of each evidence is obtained, and the combination rule is improved based on the Copula function. Combined with the performance degradation model established in this paper and reference [19], we select Frank Copula as the connection function and correlation coefficient $\theta = 128.76$. Finally, according to Section 3.1, the above function is used to determine the joint BPA, and the evidence is synthesized, and the reliability curve under the joint influence of multiple performance indicators is obtained. The reliability of life distribution is shown in Figure 3.

![Figure 3. Equipment reliability curve](image)

We can see from the figure that the improved evidence theory method proposed in this paper is more conservative than the traditional D-S evidence theory method, which shows that the method fully considers the correlation between the evidences. In the life prediction, the life of the mechatronic equipment is calculated to be about 8400h, and the joint probability density method [20] is used to obtain a life of about 8550h. Compared with the joint probability density method, under the same evaluation accuracy, the calculation method used in this paper is simpler, less restrictive and wider.

5. Conclusion

This paper proposes a method to improve the reliability based on evidence theory. The Copula function is used to improve the traditional evidence theory model, which solves the correlation between evidence in traditional evidence theory methods and greatly improves the accuracy of the reliability evaluation. Compared with the joint probability density method, the proposed method is more suitable for the more complex reliability evaluation model. Under the condition of ensuring the same accuracy, the method improves the computational efficiency to a large extent. In this paper, the fuzzy C-means algorithm is used to discretize the life distribution probability density function, and finally the marginal BPA of the evidence body is obtained. However, the cluster number of the method needs to be determined multiple times, and the process is cumbersome. In response to this problem, it is worthy of further study to use a more efficient method to discretize the continuous function.

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