Response Functions of Two-Coupled Chains of Tomonaga-Luttinger Liquids

Hideo Yoshioka\textsuperscript{1,2} and Yoshikazu Suzumura\textsuperscript{2}

\textsuperscript{1}Department of Applied Physics, Delft University of Technology, Lorentzweg 1, 2628 CJ Delft, The Netherlands
\textsuperscript{2}Department of Physics, Nagoya University, Nagoya 464-8602, Japan

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Properties of fluctuations in two chains of Tomonaga-Luttinger liquids coupled by the interchain hopping have been studied by calculating retarded response functions $\chi_{\sigma}(q_x,q_y;\omega)$ for charge and $\chi_{\pi}(q_x,q_y;\omega)$ for spin where $q_x$ and $q_y(=0$ or $\pi$) denote the longitudinal and transverse wave vector, respectively, and $\omega$ is the frequency. We have found the notable fact that the repulsive intrachain interaction results in the clear enhancement of $\text{Im}\chi_{\sigma}(q_x,\pi;\omega)$ and the suppression of $\text{Im}\chi_{\pi}(q_x,\pi;\omega)$ at low energies. This result indicates the importance of the dynamical effect by the spin fluctuation with $q_y = \pi$ and small $\omega$, which has a possibility to give rise to the attractive interaction for the electron pairing.

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I. INTRODUCTION

Two-chain system of Tomonaga-Luttinger liquids coupled by the interchain hopping and/or the exchange interaction is a basic model which connects one-dimensional interacting electron systems with quasi-one- and two-dimensional systems \cite{1}. When the interchain interaction between conduction electrons is repulsive, the ground state exhibits the superconducting one \cite{2,3}, where the spin gap found in the half-filling case \cite{2,4,5} still survives. The result is of interest for understanding the experimental fact that superconducting states have been observed in the doped ladder systems, Sr$_{1.3}$Ca$_{1.3}$Cu$_{22}$O$_{41.48}$, under the pressure \cite{4}. Further, the phase transition of quasi-one-dimensional organic conductors, (TMTSF)$_2$X, which takes place from spin density wave state to superconducting one with increasing pressure \cite{5,6} could be related with crossover in the two chains \cite{6}.

It has been known that the superconductivity in the two-chain system results from the repulsive intrachain interaction of the backward scattering between electrons with anti-parallel spin \cite{7,8,9} when the interchain hopping is relevant \cite{10}. The study of the spin and charge fluctuations in terms of the response functions clarifies the dynamics of the two-coupled chains, and is useful for understanding the origin of superconductivity not only for two-chain systems but also for quasi-one-dimensional conductors. The response functions have been examined for two-coupled chain systems with only the forward scattering \cite{11}, but those in the presence of the backward scattering are not yet clear. In the present paper, by use of effective Hamiltonian which is based on the renormalization group analysis, we investigate retarded response functions for charge and spin, given by $\chi_{\sigma}(q_x,q_y;\omega)$ and $\chi_{\pi}(q_x,q_y;\omega)$ where $q_x$ and $q_y(=0$ or $\pi$) are the longitudinal and transverse wave vector, respectively, and $\omega$ is the frequency. We demonstrate that the spin fluctuation dominates the low-lying excitation with $q_y = \pi$ from the calculation of the $\omega$-dependence of both $\text{Im}\chi_{\sigma}(q_x,\pi;\omega)$ and $\text{Im}\chi_{\pi}(q_x,\pi;\omega)$.

In section II, the Hamiltonian for the two-coupled chain is represented by the phase variable and is investigated in terms of the new Fermion fields. The results of the calculation of the retarded response functions, $\chi_{\sigma}(q_x,q_y;\omega)$ and $\chi_{\pi}(q_x,q_y;\omega)$, are shown in section III. Section IV is devoted to summary and discussion.

II. MODEL

We consider a model of two chains coupled by the interchain hopping $t$ where each chain consists of conduction electrons with repulsive intrachain interactions of both the backward scattering and the forward scattering. We note that the single chain in the absence of $t$ leads to Tomonaga-Luttinger liquids \cite{12}. The present system can be expressed by the phase Hamiltonian \cite{13,14}, which is based on the bosonization method. In case that the energy is less than $t$, the present Hamiltonian is obtained by neglecting the non-linear terms including the misfit parameter $2\delta = 4t/v_F$ which is originated from the separation of Fermi wave vector due to the interchain hopping \cite{15,16,17}. Thus our Hamiltonian is given as,
\[
\mathcal{H} = \frac{v_F}{4\pi} \int dx \left\{ \frac{1}{\eta_\phi} (\partial_x \phi_+)^2 + \eta_\phi (\partial_x \phi_-)^2 \right\} + \frac{v_\phi}{4\pi} \int dx \left\{ \frac{1}{\eta_\theta} (\partial_x \theta_+)^2 + \eta_\theta (\partial_x \theta_-)^2 \right\} \\
+ \frac{v_F}{4\pi} \int dx \left\{ (\partial_x \tilde{\theta}_+)^2 + (\partial_x \tilde{\theta}_-)^2 \right\} + \frac{v_F}{\pi \alpha^2} \int dx \left\{ (\partial_x \tilde{\phi}_+)^2 + (\partial_x \tilde{\phi}_-)^2 \right\} \\
+ \frac{v_F}{\pi \alpha^2} \int dx \left[ g_+ \cos \sqrt{2\tilde{\theta}_-} \cos \sqrt{2\tilde{\phi}_+} + g_+ \cos \sqrt{2\tilde{\theta}_-} \cos \sqrt{2\tilde{\phi}_-} + g_+ \cos \sqrt{2\tilde{\theta}_-} \cos \sqrt{2\tilde{\phi}_+} \right. \\
\left. + g_+^* \cos \sqrt{2\tilde{\phi}_+} \left( \cos \sqrt{2\tilde{\phi}_+} + \cos \sqrt{2\tilde{\phi}_-} - \cos \sqrt{2\tilde{\theta}_-} \right) \right],
\]
(1)

where \(\alpha' \sim v_F / t\) and \(v_F\) is the Fermi velocity. The phase variables \(\theta_\pm (\phi_\pm)\) and \(\tilde{\theta}_\pm (\tilde{\phi}_\pm)\) express the fluctuations of the total charge (spin) and the transverse charge (spin) where \([\theta_+(x), \theta_-(x')] = [\tilde{\theta}_+(x), \tilde{\theta}_-(x')] = [\phi_+(x), \phi_-(x')] = i \pi \text{sgn}(x - x')\). The parameter \(g_1 (g_2)\) is a matrix element of the backward (forward) scattering, whose conventional definition is given by \(g_i \rightarrow g_i / (2\pi v_F)\). The coefficients in Eq. (1) are given as \(v_\theta = v_F \sqrt{(1 + 2g_2 - g_1)(1 - 2g_2 + g_1)}\), \(v_\phi = v_F \sqrt{(1 - g_1^2)(1 + g_1^2)}\), \(\eta_\theta = \sqrt{(1 - 2g_2 + g_1) / (1 + 2g_2 - g_1)}\), \(\eta_\phi = \sqrt{(1 + g_1^2) / (1 - g_1^2)}\) and \(g_\pm = g_2 - g_1 / 2 \pm g_1 / 2\). Following such a procedure, the parts describing the gapful excitation in Eq. (1) is rewritten as

\[
\mathcal{H}' = \frac{v_\phi}{4\pi} \int dx \left\{ \frac{1}{\eta_\phi} (\partial_x \phi_+)^2 + \eta_\phi (\partial_x \phi_-)^2 \right\} \\
+ \frac{v_F}{4\pi} \int dx \left\{ (\partial_x \tilde{\theta}_+)^2 + (\partial_x \tilde{\theta}_-)^2 \right\} + \frac{v_F}{4\pi} \int dx \left\{ (\partial_x \tilde{\phi}_+)^2 + (\partial_x \tilde{\phi}_-)^2 \right\} \\
+ \frac{v_F}{\pi \alpha^2} g_+ \int dx \left[ \langle \cos \sqrt{2\tilde{\theta}_-} \cos \sqrt{2\tilde{\phi}_+} + \cos \sqrt{2\tilde{\theta}_-} \langle \cos \sqrt{2\tilde{\phi}_+} \right] \\
+ \frac{v_F}{\pi \alpha^2} g_+^* \int dx \left[ \langle \cos \sqrt{2\tilde{\phi}_+} \cos \sqrt{2\tilde{\phi}_-} + \cos \sqrt{2\tilde{\theta}_-} \langle \cos \sqrt{2\tilde{\phi}_+} \right] \\
- \left\langle \cos \sqrt{2\tilde{\phi}_+} \cos \sqrt{2\tilde{\phi}_-} - \cos \sqrt{2\tilde{\theta}_-} \langle \cos \sqrt{2\tilde{\phi}_+} \right],
\]
(2)

where \(\langle \cdots \rangle\) expresses the thermal average. In Eq. (2), the terms proportional to \(\langle \cos \sqrt{2\tilde{\theta}_-} \cos \sqrt{2\tilde{\phi}_-}\) and \(\langle \cos \sqrt{2\tilde{\phi}_+} \cos \sqrt{2\tilde{\phi}_-}\) have been discarded because these terms give rise to the finite expectation value of \(\cos \sqrt{2\tilde{\phi}_-}\), which is inconsistent with the results derived from the renormalization group analysis. The quantities, \(\langle \cos \sqrt{2\tilde{\phi}_-}\rangle\), \(\langle \cos \sqrt{2\tilde{\phi}_+}\rangle\) and \(\langle \cos \sqrt{2\tilde{\phi}_+}\rangle\) are related to the gap \(\Delta_s\), \(\hat{\Delta}_c\) and \(\Delta_s\) as

\[
\hat{\Delta}_s = \frac{v_F}{\alpha'} \left\{ g_+ \langle \cos \sqrt{2\tilde{\theta}_-} \rangle + g_1^* \langle \cos \sqrt{2\tilde{\phi}_+} \rangle \right\},
\]
(3)

\[
\hat{\Delta}_c = \frac{v_F}{\alpha'} \left\{ g_+ \langle \cos \sqrt{2\tilde{\phi}_+} \rangle - g_1 \langle \cos \sqrt{2\tilde{\phi}_+} \rangle \right\},
\]
(4)

\[
\Delta_s = \frac{v_F}{\alpha} g_1^* \left\{ \langle \cos \sqrt{2\tilde{\phi}_+} \rangle - \langle \cos \sqrt{2\tilde{\theta}_-} \rangle \right\},
\]
(5)

and are determined self-consistently similar to the previous case \(g_1 = 0\). Now we examine Eq. (2) by introducing the Fermionic representation which has been applied for \(\eta_\phi = 1\). Although the case \(\eta_\phi = 1\) is obtained as a special case in the calculation of the renormalization equation \(10\), it is considered that such a setting does not change qualitatively the behavior of the solutions. The effect of \(\eta_\phi \neq 1\) is discussed in the last section. Thus Eq. (3) is expressed as
\[ \mathcal{H}' = v_F \int dx \left\{ \psi_1^\dagger (-i \partial_x \psi_1) - \psi_2^\dagger (-i \partial_x \psi_2) \right\} + \tilde{\Delta}_s \int dx \left\{ \psi_2^\dagger \psi_1 + \psi_1^\dagger \psi_2 \right\} + v_F \int dx \left\{ \psi_3^\dagger (-i \partial_x \psi_3) - \psi_4^\dagger (-i \partial_x \psi_4) \right\} + \tilde{\Delta}_c \int dx \left\{ i \psi_3 \psi_4 - i \psi_4^\dagger \psi_3^\dagger \right\} + v_\phi \int dx \left\{ \psi_3^\dagger (-i \partial_x \psi_3) - \psi_4^\dagger (-i \partial_x \psi_4) \right\} + \Delta_s \int dx \left\{ \psi_0 \psi_5 + \psi_5^\dagger \psi_0 \right\} , \tag{6} \]

where \( \psi_1 (\psi_2), \psi_3 (\psi_4), \) and \( \psi_5 (\psi_6) \) are the field operators of right going (left going) Fermions corresponding to the transverse spin, the transverse charge and the total spin degree of freedoms, respectively. In terms of \( \phi_\pm, \theta_\pm \) and \( \phi_\pm \), field operators, \( \psi_j \) are defined by

\[
\psi_{1+n} = \frac{1}{\sqrt{2 \pi x}} e^{\pm \frac{\pi}{x}(1)^n \phi_+ + \phi_-} e^{(1)^n \frac{\pi}{x} (\tilde{N}_1 + \tilde{N}_2)} ,
\]

\[
\psi_{3+n} = \frac{1}{\sqrt{2 \pi x}} e^{\pm \frac{\pi}{x}(1)^n \theta_+ + \theta_-} e^{(1)^n \frac{\pi}{x} (\tilde{N}_3 + \tilde{N}_4)} ,
\]

\[
\psi_{5+n} = \frac{1}{\sqrt{2 \pi x}} e^{\pm \frac{\pi}{x}(1)^n \phi_+ + \phi_-} e^{(1)^n \frac{\pi}{x} (\tilde{N}_5 + \tilde{N}_6)} ,
\tag{7} \]

where \( n = 0 \) and \( 1, \tilde{N}_i \) \((i = 1 \sim 6)\) is a number operator of the \( i \)-th Fermion. The Hilbert space is taken so that the numbers, \( N_1 + N_2, N_3 + N_4 \) and \( N_5 + N_6 \), are even integers \((N_i \ (i = 1 \sim 6) : \) eigenvalue of \( \tilde{N}_i \) \) in deriving Eq.(8). Note that phase variables, \( \phi_\pm, \theta_\pm \) and \( \phi_\pm \) can also be expressed as,

\[
\hat{\phi}_\pm (x) = - \sum_{q \neq 0} \frac{\sqrt{2 \pi}}{qL_1} e^{(-\alpha'|q|/2+iqx)} \left\{ D_1(-q) \pm D_2(-q) \right\} ,
\tag{8} \]

\[
\hat{\theta}_\pm (x) = - \sum_{q \neq 0} \frac{\sqrt{2 \pi}}{qL_1} e^{(-\alpha'|q|/2+iqx)} \left\{ D_3(-q) \pm D_4(-q) \right\} ,
\tag{9} \]

\[
\hat{\phi}_\pm (x) = - \sum_{q \neq 0} \frac{\sqrt{2 \pi}}{qL_1} e^{(-\alpha'|q|/2+iqx)} \left\{ D_5(-q) \pm D_6(-q) \right\} ,
\tag{10} \]

where \( D_j(-q) = \int \psi_j^\dagger \psi_j e^{-i qx} dx \).

In Eq.(11), the excitation spectra of the total spin, the transverse charge and the transverse spin degrees of freedom are respectively calculated as \( E_{k,s} = \sqrt{\xi_k^2 + \Delta_s^2} \), \( \tilde{E}_{k,c} = \sqrt{\xi_k^2 + \Delta_c^2} \) and \( \tilde{E}_{k,s} = \sqrt{\xi_k^2 + \Delta_s^2} \), where \( \xi_{k,s} = v_\phi k \) and \( \xi_k = v_F k \). By use of Eq.(8), the self-consistent equations Eqs.(11) - (13) for \( \Delta_s, \Delta_c \) and \( \Delta_s \) are rewritten as

\[
\frac{\tilde{\Delta}_s}{\pi v_F} = g_+ \left( i \langle \psi_3 \psi_4 \rangle - i \langle \psi_1^\dagger \psi_2^\dagger \rangle \right) + g_1^* \left( \langle \psi_1^\dagger \psi_5 \rangle + \langle \psi_3^\dagger \psi_6 \rangle \right)
\]

\[
= - g_+ \left( \frac{\tilde{\Delta}_s}{L} \sum_k \frac{1}{E_{k,c}} - g_1^* \frac{\tilde{\Delta}_c}{L} \sum_k \frac{1}{E_{k,s}} \right) ,
\tag{11} \]

\[
\frac{\tilde{\Delta}_c}{\pi v_F} = g_+ \left( \langle \psi_2^\dagger \psi_1 \rangle + \langle \psi_1^\dagger \psi_2 \rangle \right) - g_1 \left( \langle \psi_6^\dagger \psi_5 \rangle + \langle \psi_2^\dagger \psi_3 \rangle \right)
\]

\[
= - g_+ \left( \frac{\tilde{\Delta}_c}{L} \sum_k \frac{1}{E_{k,s}} + g_1^* \frac{\tilde{\Delta}_s}{L} \sum_k \frac{1}{E_{k,s}} \right) ,
\tag{12} \]

\[
\frac{\tilde{\Delta}_s}{\pi v_F} = g_1 \left( \langle \psi_2^\dagger \psi_1 \rangle + \langle \psi_1^\dagger \psi_2 \rangle - i \langle \psi_3 \psi_4 \rangle + i \langle \psi_4^\dagger \psi_3^\dagger \rangle \right)
\]

\[
= g_1 \left\{ - \frac{\tilde{\Delta}_c}{L} \sum_k \frac{1}{E_{k,s}} + \frac{\tilde{\Delta}_c}{L} \sum_k \frac{1}{E_{k,c}} \right\} .
\tag{13} \]

By noting that the gap equations lead to \( \tilde{\Delta}_s = -\tilde{\Delta}_c = \tilde{\Delta} \) due to \( g_+ > 0 \), we use \( \tilde{E}_k = \sqrt{\xi_k^2 + \Delta^2} = \tilde{E}_{k,s} = \tilde{E}_{k,c} \) in the following. In Fig.1, the numerical results of the gap equations are shown with a choice of \( g_1 = 0.45 \) and \( t \alpha / v_F = 0.1 \).
where \( g_+ > g_1^* \). The result of \(|\Delta_+| = |\Delta_0| > |\Delta|\) obtained in Fig. 1 is consistent with that of the renormalization analysis showing the fact that the term proportional to \( \cos \theta \) scaled to the strong coupling regime faster than the other relevant terms.

\[
\chi_1(q_x, q_y; \omega) = \frac{1}{2} \int_0^\beta \! d\tau \int \! d(x - x') e^{i\omega_n \tau} e^{-iq_y(x - x')} \times \langle T_\tau \{ \nu(x, 1; \tau) + e^{i\nu_0} \nu(x, 2; \tau) \} \{ \nu(x', 1; 0) + e^{i\nu_0} \nu(x', 2; 0) \} \rangle_{\omega_n + \omega + i\delta},
\]

We calculate the response functions defined by \( \nu = \rho \) and \( \sigma \)

\[
\frac{\Delta_+}{\xi_c} = -\frac{\Delta_0}{\xi_c} = -\frac{\Delta}{\xi_c},
\]

**FIG. 1.** Solutions of the gap equations of Eqs. (11)–(13) as a function of \( g_+ - g_1^* \), for \( g_1 = 0.45 \) and \( t\alpha / v_F = 0.1 \). The solid line and the dotted one express \( \Delta_+ / \xi_c \) and \( -\Delta / \xi_c \), respectively, where \( \xi_c \) is the cut-off energy of the order of \( t \).

### III. RESPONSE FUNCTIONS

We calculate the response functions defined by \( \nu = \rho \) and \( \sigma \)

\[
\chi_\rho^R(q_x, 0; \omega) = \frac{\eta_0 q_x}{\pi} \left( \frac{1}{v_\theta q_x + \omega + i\delta} + \frac{1}{v_\theta q_x - \omega - i\delta} \right),
\]

\[
\chi_\sigma^R(q_x, 0; \omega) = \frac{1}{L} \sum_k \left( \frac{1}{E_{k,s} + E_{k+q_{x,s}} + \omega + i\delta} + \frac{1}{E_{k,s} + E_{k+q_{x,s}} - \omega - i\delta} \right) \times \left( 1 - \frac{\Delta_s^2}{E_{k,s}E_{k+q_{x,s}}} \right),
\]

\[
\chi_\mu(\pi; \omega) = \frac{1}{2L} \sum_{\nu = \pm} \left( \frac{1}{E_k + E_{k'} - \omega - i\delta} + \frac{1}{E_k + E_{k'} + \omega + i\delta} \right).
\]
lated as follows,
\[
\frac{\xi_k}{E_k E_{k'}} + (-\frac{\Delta_v}{E_k E_{k'}})_{k'=\nu q_0 + q_0 - k}.
\]

Note that Eqs. (18) and (17) have been derived in terms of Eqs.(18) and Eqs.(17), respectively.

For \( \omega = 0 \), the real parts of Eqs.(17)-(17) can be calculated analytically. The quantities, \( \text{Re} \chi(q_x, 0; 0) \) and \( \text{Re} \chi(q_x, \pi; 0) \), are given by 2\( \pi v_F \) and 2\( \pi v_F \), respectively, while \( \text{Re} \chi(q_x, 0; 0) \) and \( \text{Re} \chi(q_x, \pi; 0) \) are calculated as follows,
\[
\text{Re} \chi(q_x, 0; 0) = \frac{2}{\pi v_F} \left\{ 1 - \frac{2\Delta_v^2}{v_0 q_x \sqrt{(v_0 q_x)^2 + 4\Delta_v^2}} \ln \left| \frac{v_0 q_x + \sqrt{(v_0 q_x)^2 + 4\Delta_v^2}}{v_0 q_x - \sqrt{(v_0 q_x)^2 + 4\Delta_v^2}} \right| \right\},
\]
\[
\text{Re} \chi(q_x, \pi; 0) = \frac{2}{\pi v_F} \sum_{\nu = \pm} \left\{ 1 - \frac{\Delta_v^2}{\xi_{\nu} \sqrt{\xi_{\nu}^2 + 4\Delta_v^2}} \ln \left| \frac{\xi_{\nu} + \sqrt{\xi_{\nu}^2 + 4\Delta_v^2}}{\xi_{\nu} - \sqrt{\xi_{\nu}^2 + 4\Delta_v^2}} \right| \right\}.
\]

In Fig. 2, we show the real parts of the response functions which are normalized by 2\( \pi v_F \). For \( q_y = 0 \), \( \text{Re} \chi(q_x, 0; 0) \) as a function of \( q_x \) remains constant. The quantity \( \text{Re} \chi(q_x, 0; 0) \) is reduced around \( q_x = 0 \) owing to the spin gap where the limiting behavior of \( \text{Re} \chi(q_x, 0; 0) \) for \( |q_x| \ll |\Delta_v|/v_0 \) is given by 2\( /\pi v_F \)\times \xi^2 q_x^2 / (6\Delta_v^2). For \( q_y = \pi \), the gaps of the transverse fluctuations, \( \Delta_\perp \) and \( \Delta_\parallel \), lead to the suppression of \( \text{Re} \chi(q_x, \pi; 0) \) around \( q_x = \pm q_0 \). The minimum of \( \text{Re} \chi(q_x, \pi; 0) \) near \( q_x = \pm q_0 \) comes from the separation of Fermi wave vector which is caused by the interchain hopping. The quantity, \( \text{Re} \chi(q_x, \pi; 0) \), for \( |q_x| \ll |\Delta_v|/v_0 \) is expressed as 2\( /\pi v_F \)\times \{1/2 - (\Delta_v/4t)^2 \ln(4t/\Delta_v^2) + v_F^2 (q_x \pm q_0)^2 / 12\Delta_v^2 \}. The quantity \( \text{Re} \chi(q_x, \pi; 0) \) as a function of \( q_x \) becomes flat due to the fact that the effect of \( \Delta_\perp \) and \( \Delta_\parallel \) compensate each other in Eq.(17).

Now we examine the \( \omega \)-dependence of the imaginary parts of Eq.(17), which represents the spectral weight for charge and spin fluctuations with \( q_y = \pi \). In Fig. 3, the quantities \( \text{Im} \chi^R(q_x, \pi; \omega) \) and \( \text{Im} \chi^\sigma^R(q_x, \pi; \omega) \), which are normalized by 2\( \pi v_F \), are shown with the fixed \( q_x/q_0 = 1.25 \) ( Fig. 3(a) ) and 1.0 ( Fig. 3(b) ). These quantities diverge at two locations given by \( \omega_{\perp} (\perp) / (2t) = \sqrt{1 - (\pm q_0/2t)^2 + 4(\Delta_0/2t)^2} \), which are ascribed to the separation of the Fermi wave vector by the interchain hopping. These singularities are also found in the absence of the interaction, where the corresponding response function becomes \( \chi^R_{\nu(\sigma)}(q_x, \pi; \omega) \rightarrow \chi^R(q_x, \pi; \omega) \) with
\[
\chi^R(q_x, \pi; \omega) = \frac{1}{2\pi} \sum_{\nu = \pm} \left\{ \frac{\nu q_x + q_0}{v_F(v_0 q_x + q_0) - \omega - i\delta} + \frac{\nu q_x + q_0}{v_F(v_0 q_x + q_0) + \omega + i\delta} \right\}.
\]
The weight of Eq. (17) appears in the small region of the energy higher than \( \omega_L(H) \), while Eq. (20) shows the weight with a delta function in the absence of the interaction. The location of \( \omega \) corresponding to the singularity depends on the magnitude of the gap. It is worthy to note that the lower peak of the response function shows the dominant behavior of the spin fluctuation compared with the charge fluctuation. For \( q_x = q_0 \), the weight at the lower energy is given only by the spin fluctuation, i.e., the charge fluctuation is absent as is shown in Fig. 3(b).

\[ \text{FIG. 3. The } \omega/(2t)-\text{dependence of } \text{Im} \chi^{R}_{\rho}(q_x, \pi; \omega) \text{ and } \text{Im} \chi^{R}_{\sigma}(q_x, \pi; \omega) \text{ for } q_x/q_0 = 1.25 \text{ (a) and } q_x/q_0 = 1.0 \text{ (b), which are normalized by } 2/\pi v_F. \text{ The solid (dashed) curve represents } \text{Im} \chi^{R}_{\rho}(q_x, \pi; \omega) \left( \text{Im} \chi^{R}_{\sigma}(q_x, \pi; \omega) \right). \text{ The parameters are the same as those in Fig. 2.} \]

\[ \text{FIG. 4. The total weights, } S^{L}_{\rho(\sigma)} \text{ and } S^{H}_{\rho(\sigma)} \text{ as a function of } q_x/q_0. \text{ The dotted lines, } S^{L} \text{ and } S^{H} \text{ are those in the absence of the interaction. The parameters are the same as those in Fig. 2.} \]

In order to examine such a notable fact, we evaluate the weights by dividing Eq. (17) into two parts, i.e.,

\[ \text{Im} \chi^{R}_{\rho(\sigma)}(q_x, \pi; \omega) = \text{Im} \chi^{R,L}_{\rho(\sigma)}(q_x, \pi; \omega) + \text{Im} \chi^{R,H}_{\rho(\sigma)}(q_x, \pi; \omega), \]

where \( \text{Im} \chi^{R,L}_{\rho(\sigma)}(q_x, \pi; \omega) \) ( \( \text{Im} \chi^{R,H}_{\rho(\sigma)}(q_x, \pi; \omega) \) ) comes from the part with \( \nu = -(+) \), and leads to the singularity near \( \omega = \omega_L \left( \omega_H \right) \). In Fig. 4, we show the total weight of \( \text{Im} \chi^{R,L}_{\rho(\sigma)}(q_x, \pi; \omega) \) and \( \text{Im} \chi^{R,H}_{\rho(\sigma)}(q_x, \pi; \omega) \) which are defined by

\[ S^{L}_{\rho(\sigma)} = \frac{\pi v_F}{4t} \int_{0}^{\infty} d\omega \text{Im} \chi^{R,L}_{\rho(\sigma)}(q_x, \pi; \omega), \]  

\[ S^{H}_{\rho(\sigma)} = \frac{\pi v_F}{4t} \int_{0}^{\infty} d\omega \text{Im} \chi^{R,H}_{\rho(\sigma)}(q_x, \pi; \omega). \]

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Solid and dashed curves denote $S_{\sigma}^{L,H}$ and $S_{\sigma}^{L,H}$, respectively while $S^L$ and $S^H$ shown by the dotted lines are the total weights in the absence of the interaction. The deviation of $S_{\sigma}^{L,H}$ from $S^{L,H}$ is opposite to that of $S_{\sigma}^{L,H}$. The fact that $S_{\sigma}^{L} \gg S_{\sigma}^{L}$ around $q_x \sim q_0$ shows the clear evidence that the spin fluctuation with low energy dominates the charge fluctuation. The results indicate that the low-lying excitation with $q_y = \pi$ in case of the repulsive intrachain interaction is determined mainly by the spin degree of freedom.

IV. SUMMARY AND DISCUSSION

In the present paper, we investigated the response functions for charge and spin densities in the system of the two chains of the electron system with the repulsive intrachain interaction by using the method of the effective Hamiltonian based on the renormalization group analysis.

In the several non-linear terms of the bosonized Hamiltonian, we replaced the terms with the finite expectation value by the averaged one and utilized the mean-field approximation in order to estimate the gap. Such an approximation seems effective in the sense that the method gives the results consistent with the ones derived from the renormalization group analysis. In addition, we used the approximation $\eta_{\phi} \to 1$ to express the mean-field Hamiltonian by the new Fermion field. Such a setting does not change qualitatively the behavior of the total spin fluctuation, i.e., the spin gap appears. We note that such a treatment of $\eta_{\phi} = 1$ breaks $SU(2)$ symmetry which is preserved in Eq.(1). In fact, the formulas for the spin susceptibilities, $\chi^{R}_{xx}(q_x;0;\omega)$ and $\chi^{R}_{yy}(q_x,\pi;\omega)$, which are the response functions for spin with the $x$-direction, are different from Eqs.(16) and (17) in the sense that $\chi^{R}_{xx}(q_x;0;\omega)$ and $\chi^{R}_{yy}(q_x,\pi;\omega)$ contains the gaps of both $\Delta_s$ and $\Delta_s$ ($\Delta_s$ and $\Delta_c$). However, the behaviors of $\chi^{R}_{xx}(q_x;0;\omega)$ and $\chi^{R}_{yy}(q_x,\pi;\omega)$ are approximately the same as those of Eqs.(16) and (17), respectively. For the real part of the susceptibility, $\text{Re}\chi^{R}_{xx}(q_x;0;0)$ is strongly suppressed near $q_x = 0$ ($\text{Re}\chi^{R}_{xx}(q_x;0;0) \sim 0.05$ in unit of $2/\pi v_F$ for the parameters in Fig.2) and $\text{Re}\chi^{R}_{xx}(q_x,\pi;0)$ does not show the remarkable suppression near $q_x = q_0$. In addition, there are the two peaks in $\text{Im}\chi^{R}_{xx}(q_x,\pi;\omega)$ as a function of $\omega$ even at $q_x = q_0$ and the total weight of the lower peak is larger than that of the non-interacting case. Thus the present treatment leads to results qualitatively reasonable for spin fluctuations though $SU(2)$ symmetry is broken.

From the imaginary part of the response functions with $q_y = \pi$, we concluded that among the fluctuations with $q_y = \pi$, the spin degree of freedom is dominant compared to the charge one. The excitation of spin degree of freedom with $q_y = \pi$ and low energy seems to be closely related to superconductivity of the two chains. In the present superconducting state, the pair of the electrons is formed between two chains with in phase. This corresponds to the intraband pairing with out of phase in the momentum space $(k_x,k_y)$ where the Fermi points are given by $(\pm(k_F + q_0/2),0)$ for the lower band and $(\pm(k_F - q_0/2),0)$ for the upper band, respectively. The superconducting

![Diagram](image-url)
state is explained in Fig. 5 where the pairing of the electrons around the Fermi points are expressed by the enclosed-dashed curves. By noting that

$$\chi_{\rho}(q_x, \pi; i\omega_n) = \frac{-1/\pi}{\Gamma} \int_{-\infty}^{\infty} d\omega \text{Im} \chi_{\rho}(q_x, \pi; \omega)/(i\omega_n - \omega),$$

it is considered that the fluctuations connecting these two kinds of pairs, $$\chi_{\rho}(q_x, \pi; i\omega_n)$$ with $$q_x \approx \pm q_0$$ act as the attractive force \[23\]. Since the spin degree of freedom in such fluctuation is dominant compared to the charge fluctuation, it is possible that the spin fluctuation with $$q_y = \pi$$ results in the superconductivity of the two chains. Also in the organic conductors, \((\text{TMTSF})_2X\), it is argued that the pair is formed between the chains from the theoretical analysis \[24\] of the NMR relaxation rate of \((\text{TMTSF})_2\text{ClO}_4\) \[25\]. Therefore the origin of the superconductivity of the organic conductor seems to be the spin fluctuation with $$q_y = \pi$$.

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$$\tilde{\theta}_+ = \sqrt{2}\tilde{\phi}_{\rho+}, \tilde{\theta}_- = -\sqrt{2}\tilde{\phi}_{\rho-}, \tilde{\phi}_+ = \sqrt{2}\tilde{\phi}_{\rho+}, \tilde{\phi}_- = -\sqrt{2}\tilde{\phi}_{\rho-}.$$
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