Dark Matter in $B-L$ Extended MSSM Models

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We analyze the dark matter problem in the context of supersymmetric $U(1)_{B-L}$ model. In this model, the lightest neutralino can be the $B-L$ gaugino $\tilde{Z}_{B-L}$ or the extra Higgsinos $\tilde{\chi}_{1,2}$ dominated. We compute the thermal relic abundance of these particles and show that, unlike the LSP in MSSM, they can account for the observed relic abundance with no conflict with other phenomenological constraints. The prospects for their direct detection, if they are part of our galactic halo, are also discussed.

INTRODUCTION

Nonvanishing neutrino masses and the existence of nonbaryonic dark matter (DM) are the most important evidences of new physics beyond the Standard Model (SM). A simple extension of the SM, based on the gauge group $G_{B-L} = SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$, can account for current experimental results of light neutrino masses and their large mixing [1]. In addition, the extra-gauge boson and extra-Higgs predicted in this model have a rich phenomenology and can be detected at the LHC [2]. It is worth mentioning that several attempts have been proposed to extend the gauge symmetry of the SM via one or more $U(1)$ gauge symmetries beyond the hypercharge gauge symmetry [3, 4].

Within supersymmetric context, it was emphasized that the three relevant physics scales related to the supersymmetry, electroweak, and baryon minus lepton ($B-L$) breakings are linked together and occur at the TeV scale [5]. Indeed, it was shown that radiative corrections may drive the squared mass of extra $B-L$ Higgs from positive initial values at the GUT scale to negative values at the TeV scale. In such a framework, the size of the $B-L$ Higgs vacuum expectation value (VEV), responsible for the $B-L$ breaking, is determined by the size of the right-handed Yukawa coupling and of the soft SUSY breaking terms.

In this paper, we consider the scenario where the extra $B-L$ neutralinos (three extra neutral fermions: $U(1)_{B-L}$ gaugino $\tilde{Z}_{B-L}$ and two extra Higgsinos $\tilde{\chi}_{1,2}$) can be cold DM candidates. It turns out that the experimental measurements for the anomalous magnetic moment impose a lower bound of order 30 GeV on the mass of $U(1)_{B-L}$ gaugino $\tilde{Z}_{B-L}$, while Higgsinos $\tilde{\chi}_{1,2}$ can be very light. We examine the thermal relic abundance of these particles and discuss the prospects for their direct detection if they form part of our galactic halo.

It is worth mentioning that assuming the lightest neutralino in minimal supersymmetric standard model (MSSM) as DM candidate implies sever constraints on the parameter space of this model. Indeed, in the case of universal soft-breaking terms, the MSSM is almost ruled out by combining the collider, astrophysics and rare decay constraints [6]. Therefore, it is important to explore very well motivated extensions of the MSSM, such as SUSY $B-L$ model which provides new DM candidates that may account for the relic density with no conflict with other phenomenological constraints.

The paper is organized as follows. In section 2 we briefly review the supersymmetric $U(1)_{B-L}$ model with a particular emphasis on its extended neutralino sector. Section 3 is devoted for computing the LSP annihilation rate of $\tilde{Z}_{B-L}$ and $\tilde{\chi}_{1,2}$. In section 4 we examine the possible constraints imposed by the experimental limits of muon anomalous magnetic moment on the mass of $\tilde{Z}_{B-L}$. We discuss the relic abundance of these DM candidates in section 5. We show that they can account for the measured relic density without any conflict with other phenomenological constraints. The direct detection rate of $\tilde{Z}_{B-L}$ and $\tilde{\chi}_{1,2}$ is briefly discussed in section 6. Finally we give our conclusions in section 7.

$U(1)_{B-L}$ SUSY MODEL

In $B-L$ extension of the MSSM, the particle content includes the following fields in addition to the MSSM fields: three chiral right-handed superfields ($N_i$), a vector superfield associated to $U(1)_{B-L}$ ($Z_{B-L}$), and two chiral SM singlet Higgs superfields ($\chi_1, \chi_2$). This class of $B-L$ extension of the SM can be obtained from a unified gauge theory, like $SO(10)$, with the following branching rules for symmetry breaking: $SO(10)$ is broken down to Pati-Salam gauge group: $SU(4)_c \times SU(2)_L \times SU(2)_R$ through the vacuum expectation value (vev) of the Higgs: $(1, 1, 1)$ in $54_H$ or $210_H$ representation at GUT scale. Then Pati-Salam can be directly broken down to $B-L$ model: $SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)_{B-L}$ through the vev of the adjoint Higgs: $(15, 1, 3)$ below GUT scale. Finally, the $B-L$ model is broken down to $SU(3)_c \times SU(2)_L \times U(1)_Y$ at TeV scale as mentioned above. In this case, although the $U(1)_Y$ and $U(1)_{B-L}$ are exact symmetries at high scale (larger than TeV), they are non-orthogonal. This can be seen by noticing that the orthogonality condition [7] is not satisfied:
\[ \sum_f Y_f^I Y^I_{B-L} \neq 0, \text{ where } Y_f^I \text{ and } Y^I_{B-L} \text{ are the hypercharge and the } B - L \text{ charge of the fermion particle } (f). \]

In this respect, there is a kinetic mixing between the gauge fields of \( U(1)_Y \) and \( U(1)_{B-L} \). However, LEP results [10] stringently constrain the corresponding mixing angle to be \( \lesssim 10^{-2} \). Therefore, in our analysis, we neglect this small mixing and consider the following superpotential:

\[ W = (Y_U)_{ij} Q_i H_u U^c_j + (Y_D)_{ij} Q_i H_d D^c_j + (Y_L)_{ij} L_i H_d E^c_j + (Y_N)_{ij} N_i H_d N^c_j \chi_1 + \mu(H_u H_d) + \mu' \chi_1 \chi_2. \quad (1) \]

The \( B - L \) charges of superfields appeared in the superpotential \( W \) are given in Table I.

| \( SU(2)_L \times U(1)_Y \) | \( l \) | \( N \) | \( E \) | \( Q \) | \( U \) | \( D \) | \( H_u \) | \( H_d \) | \( \chi_1 \) | \( \chi_2 \) |
|----------------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| \( U(1)_{B-L} \) | \(-1\) | \(-1\) | \(-1\) | \( \dagger \) | \( \dagger \) | \( 0 \) | \( 0 \) | \(-2\) | \( 2\) |

TABLE I: The \( U(1)_{B-L} \) charges of the superfields.

For universal SUSY soft breaking terms at grand unification scale, \( M_X \), the soft breaking Lagrangian is given by

\[ -\mathcal{L}_{soft} = m_0^2 \left[ |\tilde{Q}_i|^2 + |\tilde{U}_i|^2 + |\tilde{D}_i|^2 + |\tilde{L}_i|^2 + |\tilde{E}_i|^2 + |\tilde{N}_i|^2 + |H_u|^2 + |H_d|^2 + |\chi_1|^2 + |\chi_2|^2 \right] 
+ A_0 \left[ Y_U \tilde{Q}^c H_u + Y_D \tilde{Q} \tilde{D}^c H_d + Y_E \tilde{L} \tilde{E}^c H_d + Y_N \tilde{N} \tilde{N}^c H_u + Y_N \tilde{N} \tilde{N}^c \chi_1 \right] 
+ [B(\mu H_u H_d + \mu' \chi_1 \chi_2) + h.c.] + \frac{1}{2} M_{1/2} \left[ \tilde{g}^a \tilde{g}^a + \tilde{W}^a \tilde{W}^a + \tilde{B} \tilde{B} + \tilde{Z}_{B-L} \tilde{Z}_{B-L} + h.c. \right], \quad (2) \]

where the tilde denotes the scalar components of the chiral matter superfields and fermionic components of the vector superfields. The scalar components of the Higgs superfields \( H_{u,d} \) and \( \chi_{1,2} \) are denoted as \( H_{u,d} \) and \( \chi_{1,2} \), respectively.

As shown in Ref. [5], both \( B - L \) and electroweak (EW) symmetries can be broken radiatively in the supersymmetric theories. In this class of models, the EW, \( B - L \) and soft SUSY breaking are related and occur at the TeV scale. The conditions for the EW symmetry breaking are given by

\[ \mu^2 = \frac{m_{H_u}^2 - m_{H_d}^2 \tan^2 \beta}{\tan^2 \beta - 1} - M_Z^2/2, \quad \sin 2\beta = \frac{-2m_3^2}{m_1^2 + m_2^2}, \quad (3) \]

where

\[ m_i^2 = m_0^2 + \mu_i^2, \quad i = 1, 2, \quad m_3^2 = -B\mu, \quad \tan \beta = \frac{v_u}{v_d}, \quad <H_u> = v_u/\sqrt{2}, \quad <H_d> = v_d/\sqrt{2}. \quad (4) \]

Here \( m_{H_u} \) and \( m_{H_d} \) are the SM-like Higgs masses at the EW scale. \( M_Z \) is a neutral gauge boson in the SM. It is worth noting that the breaking \( SU(2)_L \times U(1)_Y \) occurs at the correct scale of the charged gauge boson (\( M_W \sim 80 \text{ GeV} \)). Similarly, the conditions for the \( B - L \) radiative symmetry breaking are given by [8]

\[ \mu' = \frac{\mu_1^2 - \mu_2^2 \tan^2 \theta}{\tan^2 \theta - 1} - M_{Z_{B-L}}^2/2, \quad \sin 2\theta = \frac{-2\mu_3^2}{\mu_1^2 + \mu_2^2}, \quad (5) \]
where

\[ \mu_i^2 = m_i^2 + \mu_i^2, \quad i = 1, 2 \quad \mu_3^2 = -B\mu', \quad \tan \theta = \frac{v_1'}{v_2'}, \]

\[ < \chi_1 >= v_1' / \sqrt{2}, \quad < \chi_2 >= v_2' / \sqrt{2}. \quad (6) \]

Here \( m_{\chi_1} \) and \( m_{\chi_2} \) are the \( U(1)_{B-L} \)-like Higgs masses at the TeV scale. The key point for implementing the radiative \( B-L \) symmetry breaking is that the scalar potential for \( \chi_1 \) and \( \chi_2 \) receives substantial radiative corrections. In particular, a negative squared mass would trigger the \( B-L \) symmetry breaking of \( U(1)_{B-L} \). It was shown that the masses of Higgs singlets \( \chi_1 \) and \( \chi_2 \) run differently in the way that \( m_{\chi_1}^2 \) can be negative whereas \( m_{\chi_2}^2 \) remains positive. The renormalization group equation (RGE) for the \( B-L \) couplings and mass parameters can be derived from the general results for SUSY RGEs of Ref. [9]. After \( B-L \) symmetry breaking, the \( U(1)_{B-L} \) gauge boson acquires a mass \( m^2_{Z_{B-L}} = 4g_{B-L}^2v^2 \). The high energy experimental searches for an extra neutral gauge boson impose lower bounds on this mass. The most stringent constraint on \( U(1)_{B-L} \) obtained from LEP II result, which implies [10]

\[ \frac{M^2_{Z_{B-L}}}{g_{B-L}} > 6 \text{ TeV}. \quad (7) \]

Now we analyze mass-spectrums which have some deviations from MSSM-spectrums in particular, SM singlet Higgs bosons, the right-handed sneutrinos, and the neutralinos. The Higgs sector in the SUSY \( B-L \) extension of the SM consists of two Higgs doublets and two Higgs singlets with no mixing. However, after the \( B-L \) symmetry breaking, one of the four degrees of freedom contained in the two complex singlet \( \chi_1 \) and \( \chi_2 \) are swallowed by the \( Z_{B-L}^0 \) to become massive. Therefore, in addition to the usual five MSSM Higgs bosons: neutral pseudoscalar Higgs bosons \( A \), two neutral scalars \( h \) and \( H \) and a charged Higgs boson \( H^\pm \), three new physical degrees of freedom remain [5]. They form a neutral pseudoscalar Higgs boson \( A' \) and two neutral scalars \( h' \) and \( H' \). Their masses at tree level are given by

\[ m_{A'}^2 = \mu_1^2 + \mu_2^2, \quad m_{H', h'}^2 = \frac{1}{2} \left( m_A^2 + m_{Z_{B-L}}^2 \pm \sqrt{\left(m_A^2 + m_{Z_{B-L}}^2\right)^2 - 4m_A^2m_{Z_{B-L}}^2 \cos 2\theta} \right). \quad (8) \]

The physical CP-even extra-Higgs bosons are obtained from the rotation of angle \( \alpha \):

\[ \left( \begin{array}{c} h' \\ H' \end{array} \right) = \left( \begin{array}{cc} \cos \alpha & \sin \alpha \\ -\sin \alpha & \cos \alpha \end{array} \right) \left( \begin{array}{c} \chi_1 \\ \chi_2 \end{array} \right), \quad (9) \]

where the mixing angle \( \alpha \) is given by

\[ \alpha = \frac{1}{2} \tan^{-1} \left[ \frac{\tan 2\theta \left( M_A^2 + M_Z^2 \right)}{M_A^2 - M_Z^2} \right]. \quad (10) \]

For \( v_1' >> v_2' \), one finds the mixing angle \( \alpha \) is very small, hence the above diagonalizing matrix is close to the identity. In this case, to a good approximation, one can assume that \( h' \equiv \chi_1 \) and \( H' \equiv \chi_2 \). We are going to adopt this assumption here.

Now we turn to the right-handed sneutrinos, in the basis of \( (\phi_{\nu_L}, \phi_{\nu_R}) \) with \( \phi_{\nu_L} = (\bar{\nu}_L, \bar{\nu}_L^*) \) and \( \phi_{\nu_R} = (\bar{\nu}_R, \bar{\nu}_R^*) \), the sneutrino mass matrix is given by the following 12 \( \times \) 12 hermitian matrix:

\[ \mathcal{M}^2 = \left( \begin{array}{cc} \mathcal{M}^{2}_{\nu_{L}e} & \mathcal{M}^{2}_{\nu_{R}e} \\ \mathcal{M}^{2}_{\nu_{R}e}^* & \mathcal{M}^{2}_{\nu_{R}e} \end{array} \right), \quad (11) \]

where \( \mathcal{M}^{2}_{\nu_{A}B}(A, B = L, R) \) can be written as

\[ \mathcal{M}^{2}_{\nu_{A}B} = \left( \begin{array}{cc} M_A^{2} & M_A^{2} \nonumber \\ M_A^{2} & M_A^{2} \end{array} \right), \quad (12) \]

with

\[ M_{\nu_{L}e}^{2} = U_{MNS}^{\dagger}m_{0}^2U_{MNS} + \frac{M_Z^2}{2} \cos 2\beta + v^2 \sin^2 \beta U_{MNS}^{\dagger}(Y_{e}^\dagger Y_{e})U_{MNS}, \]

\[ M_{\nu_{L}e}^{2} = m_{0}^2 + M_N^2, \]

\[ M_{\nu_{R}e}^{2} = v \sin \beta U_{MNS}^{\dagger}A_0(Y_{N})^\dagger + v \cos \beta U_{MNS}^{\dagger} \]

\[ M_{\nu_{R}e}^{2} = v' \sin \theta A_0(Y_{N})^\dagger, \]

\[ M_{\nu_{R}e}^{2} = \sin \beta A_0(Y_{e})M_N, \]
where \( v' \sin \theta = <\chi_1> \), \( M_N = Y_N v' \) and \( U_{MNS} \) is \( 3 \times 3 \) unitary matrix termed the Maki-Nakagawa-Sakata lepton mixing matrix \([12]\). Therefore, in general the order of magnitude of the sneutrino mass matrix is as follows:

\[
\mathcal{M}^2 \sim \begin{pmatrix}
O(v^2) & O(vv') \\
O(vv') & O(v'^2)
\end{pmatrix}.
\]  \hspace{1cm} (14)

Since \( v' \sim \text{TeV} \), the sneutrino matrix elements are of the same order and there is no seesaw type behavior as usually found in MSSM extended with heavy right-handed neutrinos. Therefore a significant mixing among the left- and right-handed sneutrinos is obtained. The phenomenological consequences for such mixing have been studied in \([13]\).

Finally, we consider the neutralino sector. The neutral gaugino-higgsino mass matrix can be written as:

\[
\mathcal{M}_7(\tilde{B}, \tilde{W}^3, \tilde{H}^0_d, \tilde{H}^0_u, \tilde{\chi}_1, \tilde{\chi}_2, \tilde{Z}_{B-L}) \equiv \begin{pmatrix}
\mathcal{M}_4 & \mathcal{O} \\
\mathcal{O}^T & \mathcal{M}_3
\end{pmatrix},
\]  \hspace{1cm} (15)

where the \( \mathcal{M}_4 \) is the MSSM-type neutralino mass matrix and \( \mathcal{M}_3 \) is \( 3 \times 3 \) additional neutralino mass matrix, which is given by:

\[
\mathcal{M}_3 = \begin{pmatrix}
0 & -\mu' & -2g_{B-L}v' \sin \theta \\
-\mu' & 0 & 2g_{B-L}v' \cos \theta \\
-2g_{B-L}v' \sin \theta & 2g_{B-L}v' \cos \theta & M_{1/2}
\end{pmatrix}.
\]  \hspace{1cm} (16)

As a feature of the orthogonality of \( U(1)_Y \) and \( U(1)_{B-L} \) in this class of models, there is no mixing between \( \mathcal{M}_4 \) and \( \mathcal{M}_3 \) at tree level. Note that in extra \( U(1) \) gauged models, which are proposed to provide an explanation for the TeV scale of \( \mu \)-term through the vev of a singlet scalar, the neutralino mass matrix is given by \( 6 \times 6 \) matrix. If the extra singlet fermion is the lightest neutralino, then it can be an interesting candidate for dark matter, as shown in Ref.\([14]\). In our case, one diagonalizes the real matrix \( \mathcal{M}_7 \) with a symmetric mixing matrix \( V \) such as

\[
VM_7V^T = \text{diag}(m_{\chi_i}^0), \hspace{0.5cm} k = 1, \ldots, 7.
\]  \hspace{1cm} (17)

In this aspect, the lightest neutralino (LSP) has the following decomposition

\[
\chi_1^0 = V_{11}\tilde{B} + V_{12}\tilde{W}^3 + V_{13}\tilde{H}^0_d + V_{14}\tilde{H}^0_u + V_{15}\tilde{\chi}_1 + V_{16}\tilde{\chi}_2 + V_{17}\tilde{Z}_{B-L}.
\]  \hspace{1cm} (18)

The LSP is called pure \( \tilde{Z}_{B-L} \) if \( V_{17} \sim 1 \) and \( V_{1i} \sim 0, \ i = 1, \ldots, 6 \) and pure \( \tilde{\chi}_{1(2)} \) if \( V_{15(6)} \sim 1 \) and all the other coefficients are close to zero. In our analysis, we will focus on these two types of LSP and analyze their potential contributions to DM in the universe. The mass eigenstates of the matrix \( \mathcal{M}_3 \) are in general non-trivial mixtures of the fermions \( (\tilde{\chi}_1, \tilde{\chi}_2, \tilde{Z}_{B-L}) \). The limit of pure \( \tilde{Z}_{B-L} \) that we consider can be obtained if \( v' << \mu \) and the limit of pure \( \tilde{\chi}_{1(2)} \) can be obtained if \( \mu' \), \( v' \sin(cos) \theta << v' \cos(sin) \theta \).

**LSP ANNIHILATION CROSS SECTION IN \( U(1)_{B-L} \) SUSY MODEL**

As advocated in the previous section, we focus on the cases where LSP is pure \( \tilde{Z}_{B-L} \) or \( \tilde{\chi}_{1(2)} \). In this case, the relevant Lagrangian is given by

\[
-L_{\tilde{Z}_{B-L}} \simeq i\sqrt{2}g_{B-L}Y_{B-L}^f \tilde{Z}_{B-L}P_L f_L + i\sqrt{2}g_{B-L}Y_{B-L}^f \tilde{Z}_{B-L}P_L f_R + c.c.,
\]  \hspace{1cm} (19)

\[
-L_{\tilde{\chi}_1} \simeq i\sqrt{2}g_{B-L}Y_{B-L}^\chi \tilde{\chi}_1 \tilde{Z}_{B-L} \gamma_1 \tilde{\chi}_1 + i\sqrt{2}g_{B-L}Y_{B-L}^\chi \tilde{Z}_{B-L} \chi_1 + (Y_N)_{ij} \tilde{\chi}_1 N^c_i N^c_j + c.c.,
\]  \hspace{1cm} (20)

\[
-L_{\tilde{\chi}_2} \simeq i\sqrt{2}g_{B-L}Y_{B-L}^\chi \tilde{\chi}_2 \tilde{Z}_{B-L} \gamma_2 \tilde{\chi}_2 + i\sqrt{2}g_{B-L}Y_{B-L}^\chi \tilde{Z}_{B-L} \chi_2 + c.c.,
\]  \hspace{1cm} (21)
where $f$ refers to all the SM fermions, including the right-handed neutrinos. $f_L$ and $f_R$ are the left-handed and right-handed sfermions mass eigenstates respectively. $Y^f_{B-L}$ is the $B-L$ charge defined in Table I. We assume the first right-handed neutrino $N_1$ is of order $\mathcal{O}(100)$ GeV, therefore the annihilation channel of the LSP into $N_1N_1$ is also considered.

From Eq. (19), one finds that the dominant annihilation processes of $\chi_1^0 \equiv \tilde{Z}_{B-L}$ are given in Figure 1. Our computation for the annihilation cross section leads to the following $a_{\tilde{Z}_{B-L}}$ and $b_{\tilde{Z}_{B-L}}$, where the approximation $\langle \sigma_{\text{ann}}v \rangle \simeq a + bv^2$, with $v$ is the velocity of the incoming LSP, is assumed:

\begin{align}
a_{\tilde{Z}_{B-L}} &= \frac{g^4_{B-L}}{54\pi m_{\chi_1^0}^2} \left[ \beta'_{\chi_1^0} \frac{r^2}{2} \right] \frac{27\beta'_{N_1} r^2 N_1}{z_N^2}, \\
b_{\tilde{Z}_{B-L}} &= \frac{167g^4_{B-L}}{16\pi m_{\chi_1^0}^2} \left[ (1 - 2r_f + 2r_f^2) + \frac{g^4_{B-L}}{4\pi m_{\chi_1^0}^2} \right] \left[ \beta'_{\chi_1^0} \frac{r^2}{2} \right] \left\{ (a_1 + r_1 + z^2(a_4 + r_4) \right\} + \frac{4g^4_{B-L}}{\pi m_{\chi_1^0}^2} |O_m O^T_{im}|^2 |V_{2i+4}^T V_{2j+4}|^2 \frac{27\beta'_{\chi_1^0} r^2}{2} (1 + w_2^2) \beta^2 - 1 \right\} + (i = 1 \text{ or } 2) \\
&= \frac{2}{3} + \frac{1}{4} x_a^2, a_4 = \frac{x_a^2 - 3}{4}, r_1 = \frac{r_a}{3} \left\{ -4 + 3z_a - r_a(4 - 3z_a - z_4^2) \right\} , r_4 = \frac{r_a}{3} (3 + 2z_a^2 + 5r_a/3), \\
z_a &= m_a/m_{\chi_1^0}, w_a = m_a/m_{\chi_1^0}, r_a = (1 - z_a^2 + w_a^2)^{-1}, \beta^2_a = 1 - z_a^2, x_a^2 = \frac{z_a^2}{2(1 - z_a^2)}.
\end{align}

where $m_a$ is a final-state mass, $m_\alpha$ is a mediated-particle mass, $O$ is the extra Higgs mixing matrix, as defined in Eq. (9) and Eq. (10). In our approximation, $O_m$ is given by $O_m = \delta_{im}$, and we set $m_\tilde{f} \equiv m_\tilde{f}_L \approx m_\tilde{f}_R$. Moreover, $V_{2i}$ is the coefficient of Next LSP (NLSP). We assume that $\tilde{\chi}_1(2)$ is our NLSP, therefore $V_{2i+4} \simeq 1(i = 1 \text{ or } 2), V_{2j} \simeq 0(j \neq 5 \text{ or } 6)$. In the range of parameter space that we consider, the values of $a_{\tilde{Z}_{B-L}}$ and $b_{\tilde{Z}_{B-L}}$ are typically $\lesssim 10^{-8}$. For $m_{\tilde{Z}_{B-L}} \gtrsim 100 \text{ GeV}$, the annihilation channels into extra-Higgs $\chi_1$ and $\chi_2$ may give the dominant contributions to $b_{\tilde{Z}_{B-L}}$.

Now we turn to the Higgsino contributions. From Eq. (20), one finds that the dominant annihilation processes of $\chi_1^0 \equiv \tilde{\chi}_1$ are given in Figure 2. The computation of the cross section leads to the following results for $a_{\tilde{\chi}_1}$ and $b_{\tilde{\chi}_1}$:

\begin{align}
a_{\tilde{\chi}_1} &= \frac{\beta'_{\chi_1^0}}{\pi} \left[ \frac{(Y_{N,1m})^4 r_N^2}{32 m_{\chi_1^0}^2} + \frac{g^4_{B-L} m_{\chi_1^0}^2}{m_{\tilde{Z}_{B-L}}^2} \left( 1 - 4 \frac{m_{\chi_1^0}^2}{m_{\tilde{Z}_{B-L}}^2} \right)^2 \right] - \frac{\sqrt{2} (Y_{N,1m})^2 g^2_{B-L}}{4 m_{\tilde{Z}_{B-L}}^2} \left( 1 - 4 \frac{m_{\chi_1^0}^2}{m_{\tilde{Z}_{B-L}}^2} \right)^{-1}, \\
b_{\tilde{\chi}_1} &= \frac{4g^4_{B-L}}{3\pi} \frac{m_{\chi_1^0}^2}{M_{\tilde{Z}_{B-L}}^2} \left[ \frac{23}{3} + \frac{1}{2} (1 - z_l^2) \left( \frac{2}{3} + \frac{1}{4} x_l^2 + \frac{5}{12} \right) \right] \left( 1 - 4 \frac{m_{\chi_1^0}^2}{M_{\tilde{Z}_{B-L}}^2} \right)^2 + \frac{\beta'_{\chi_1^0}}{\pi} \left[ \frac{(Y_{N,1m})^4 r_N^2}{32 m_{\chi_1^0}^2} (a_1 + r_1) N + \frac{g^4_{B-L} m_{\chi_1^0}^2}{m_{\tilde{Z}_{B-L}}^2} \left( 1 - 4 \frac{m_{\chi_1^0}^2}{m_{\tilde{Z}_{B-L}}^2} \right)^2 \right] a_{1N}
\end{align}
have very similar annihilation cross section values. Therefore, in this region of parameter space both \( \tilde{\chi}\) are dominated by extra-Higgs channel. There-

\[
\begin{align*}
1 \chi
\chi_1(2) \quad N \quad N \quad \tilde{\chi}_1(2) \quad \tilde{\chi}_1
\end{align*}
\]

Here \( V_{27} \) is the coefficient of NLSP. We assume that \( \tilde{Z}_{B-L} \) is our NLSP, therefore \( V_{27} \simeq 1 \), \( V_{27} \simeq 0(i \neq 7) \). We also assume \( (Y_N)_{1m} \simeq (Y_N)_{11} \).

Finally we consider the annihilation process of \( \chi_2^0 \equiv \tilde{\chi}_2 \). From Eq. (21), one finds that \( \tilde{\chi}_2 \chi_2 \) annihilation is dominated by the diagrams in Figure 3. The computation to the cross section of \( \tilde{\chi}_2 \) leads to \( a_{\tilde{\chi}_2}^\mu = 0 \) and \( b_{\tilde{\chi}_2} \) is given by

\[
b_{\tilde{\chi}_2} = \frac{g_{B-L}^2}{\pi m_{\chi_2}^2} \left[ \frac{23}{3} + \frac{1}{2} (1 - z_t^2) \left( \frac{2}{3} + \frac{1}{4} t_z^2 - \frac{5}{12} s_t^2 \right) \right] \left( 1 - 4 \frac{m_{\chi_2}^2}{M_{Z_{B-L}}^2} \right)^{-2} + \frac{g_{B-L}^2}{\pi m_{\chi_2}^2} |O_{2n}O_{2m}^T| |V_{27}V_{27}^T| \beta_{\chi_2}^2 \left[ \frac{4}{3} (1 + w_{\chi_2}^2) \beta_{\chi_2}^2 - 1 \right].
\]

It is remarkable that for \( m_{\tilde{\chi}_1,2} \gtrsim 100 \text{ GeV} \), their anni-
hilations are dominated by extra-Higgs channel. Therefore, \( b_{\tilde{\chi}_1} \) is very close to \( b_{\tilde{\chi}_2} \) and \( a_{\tilde{\chi}_2} \) is quite suppressed. Thus, in this region of parameter space both \( \tilde{\chi}_1 \) and \( \tilde{\chi}_2 \) have very similar annihilation cross section values.

**CONSTRAINTS FROM MUON ANOMALOUS MAGNETIC MOMENT**

In the case of \( \tilde{Z}_{B-L} \)-like LSP, a significant contribution to muon anomalous magnetic moment \( (a_\mu) \) may be obtained due to the 1-loop diagram mediated by \( \tilde{Z}_{B-L} \) and smuon, as shown in Figure 4. Note that \( \tilde{\chi}_{1,2} \) have no direct couplings with the SM fermions, thus they do not contribute to \( a_\mu \). The recent experimental value has been determined with a very high precision by the E821 Collaboration at the National laboratory [16]

\[
a_\mu^{\exp.} = (116592080 \pm 60) \times 10^{-11}. \tag{28}
\]

This value differs from the SM prediccion by the following:

\[
\Delta a_\mu = a_\mu^{\exp.} - a_\mu^{SM} = (278 \pm 82) \times 10^{-11}. \tag{29}
\]

Therefore, \( \tilde{Z}_{B-L} \) contribution to \( a_\mu \) should satisfy the following constraints:

\[
1.96 \times 10^{-7} \leq \Delta a_\mu^{\tilde{Z}_{B-L}} \leq 3.6 \times 10^{-9}. \tag{30}
\]
Our computation for $\Delta a_{\mu}^{B-L}$ contribution to $a_{\mu}$ leads to the following result:

\[
\Delta a_{\mu}^{B-L} = \frac{g_{B-L}^2}{8\pi^2} \sum_{i=1,2} \frac{m_{\mu}}{6m_{\tilde{\mu}_i}^2 (1-s_i)} \left[ \sqrt{s_i} (1-s_i)(U_{\tilde{\mu}})(U_{\tilde{\mu}})^{\dagger} Y_{B-L}^E (1-s_i^2 + 2s_i \ln s_i) + \frac{m_{\mu}}{m_{\tilde{\mu}_i}} \left| (U_{\tilde{\mu}})_{21} Y_{B-L}^E \right|^2 + |(U_{\tilde{\mu}})_{11} Y_{B-L}^E|^2 \right] (1 - 6s_i + 3s_i^2 + 2s_i^3 - 6s_i^2 \ln s_i),
\]

(31)

where $U_{\tilde{\mu}}$ is a diagonalized unitary matrix of the slepton sector, $s_i = (m_{\chi_1^0}/m_{\tilde{\mu}_i})^2$ and $Y_{B-L}^{(E)}$ is the $U(1)_{B-L}$ charge in the Table I. This result is consistent with the derivation of the new contribution to $a_{\mu}$ in supersymmetric $U(1)'$ model [17].

Here few comments are in order: (i) The second term in $\Delta a_{\mu}^{B-L}$ is suppressed by $m_{\mu}/m_{\tilde{\mu}_i} \sim O(10^{-3})$, while the first term is proportional to the off-diagonal elements of the diagonalized matrix $U_{\tilde{\mu}}$ which are typically of order $O(10^{-2})$. Therefore the first term is Eq. (31) gives the dominant contribution to $\Delta a_{\mu}$. (ii) From the Eq. (30), the sign of $\Delta a_{\mu}$ should be positive. Thus $[(U_{\tilde{\mu}})_{11}(U_{\tilde{\mu}})^{\dagger}]_{21}$ and $[(U_{\tilde{\mu}})_{12}(U_{\tilde{\mu}})^{\dagger}]_{22}$ of $Y_{B-L}^E$ must be negative. Note that $s_i < 1$, hence the function $f(s_i) = \sqrt{s_i} (1-s_i)(1-s_i^2 + 2s_i \ln s_i)$ is always positive. The elements of $U_{\tilde{\mu}}$ have a sign difference that helps in satisfying this requirement and allows for positive contribution to $\Delta a_{\mu}$. For example, in case $m_{\tilde{\mu}L} = m_{\tilde{\mu}R} = A \simeq 300$ GeV, $\mu \simeq 500$ GeV and $\tan \beta = 10$, the corresponding $U_{\tilde{\mu}}$ matrix is given by

\[
U_{\tilde{\mu}} \sim \begin{pmatrix} -1/\sqrt{2} & 1/\sqrt{2} \\ 1/\sqrt{2} & -1/\sqrt{2} \end{pmatrix}.
\]

(32)

(iii) Large values of $\tan \beta$ enhance the off-diagonal elements of $U_{\tilde{\mu}}$. Hence $\Delta a_{\mu}^{B-L}$ are enhanced by large values of $\tan \beta$.

In Figure 5, we plot $\Delta a_{\mu}^{B-L}$ as a function of the $\tilde{\mu}_{B-L}$-like LSP mass, $m_{\chi_1^0}$, for $\tan \beta = 10$, 20, and 30. Other SUSY parameters are fixed as above. From this figure, it can be easily seen that a significant $B - L$ contribution to $\Delta a_{\mu}$ can be obtained for $\tan \beta > 10$. For $\tan \beta = 30$, the LSP mass is constrained within the region 30 GeV < $m_{\chi_1^0} < 100$ GeV. While for $\tan \beta = 20$, the allowed region of $m_{\chi_1^0}$ is rather wider: $m_{\chi_1^0} \gtrsim 60$ GeV.

**LSP RELIC ABUNDANCE IN $U(1)_{B-L}$ SUSY MODEL**

In this section, we compute the LSP relic abundance in $U(1)_{B-L}$ SUSY Model. We adopt the standard computation of the cosmological abundance, where the LSP is assumed to be in thermal equilibrium with the SM particles in the early universe and decoupled when it was non-relativistic. Therefore, the LSP density can be obtained by solving the Boltzmann equation [15]:

\[
\frac{dn_{\chi_1^0}}{dt} + 3H n_{\chi_1^0} = - < \sigma v > \Omega_{\chi_1^0} \left[ (n_{\chi_1^0}^2 - (n_{\chi_1^0}^{eq})^2) \right],
\]

(33)

where $n_{\chi_1^0}$ is LSP number density with $m_{\chi_1^0} = \rho_{\chi_1^0} a_{\chi_1^0}$. One defines $\Omega_{\chi_1^0} = \rho_{\chi_1^0} / \rho_c$, where $\rho_c$ is the critical mass.
FIG. 6: $\Omega h^2$ versus $\tilde{Z}_{B-L}$–like LSP mass for $g_{B-L} \in [0, 0.5]$. The horizontal lines are experimentally allowed regions from the Wilkinson Microwave Anisotropy Probe (WMAP) [18] results for cold dark matter relic density. Here, we have imposed the constraint on the mass of $\tilde{Z}_{B-L}$: $m_{\tilde{Z}_{B-L}} > \sim 30$ GeV, due to the experimental limits on the muon anomalous magnetic moment.

From this figure, one notes that since the sfermion mass is fixed at 200 GeV, the annihilation channel due to its exchange produces a resonance at the LSP mass of order 100 GeV. For $g_{B-L} = 0.2$, the allowed region is reduced to around 120 GeV. Finally, for $g_{B-L} > \sim 0.4$, a lighter LSP ($m_{\tilde{Z}_{B-L}} < \sim 100$ GeV) is favored.

Now we turn to the Higgsino $\tilde{\chi}_{1,2}$ LSP. In Figure 7, we plot the LSP relic density $\Omega h^2$ as a function of $\tilde{\chi}_1$ or $\tilde{\chi}_2$ mass. As expected the relic abundance of $\tilde{\chi}_1$ or $\tilde{\chi}_2$ are quite similar since they have very close annihilation cross section. From this figure, one notes that for $g_{B-L} \leq 0.1$, there is no essentially any allowed region due to the fact that the relic abundance becomes quite large. While for $g_{B-L} = 0.2, 0.3, 0.4, 0.5$, the allowed regions for $m_{\tilde{\chi}_{1,2}}$ are given by $[150 - 190]$ GeV, $[130 - 135]$ GeV, $[125 - 130]$ GeV, and $[115 - 120]$ GeV, respectively.

FIG. 7: $\Omega h^2$ versus $\tilde{\chi}_{1,2}$-like LSP mass for $g_{B-L} \in [0.1, 0.5]$. 

In our numerical calculation for the LSP annihilation cross section, we consider the following values of masses for the particles contributing in the process (extra-light Higgses ($\chi_1^0, \chi_2^0$), sfermions ($\tilde{f}$), the lightest right-handed neutrino ($N_1$) and the NLSP ($\chi_{1,2}^0$)): $m_{\chi_1^0} = 100$ GeV, $\tilde{f} = 200$ GeV, $N_1 = 100$ GeV, $m_{\chi_2^0} = m_{\chi_1^0} + 30$ GeV.

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LSP DETECTION RATE IN U(1)$_{B-L}$ SUSY MODEL

In this section we analyze the effect of the event rates of our relic neutralinos ($\tilde{Z}_{B-L}, \tilde{\chi}_{1(2)}$) scattering off nuclei in terrestrial detectors. The direct detection experiments provide the most natural way of searching for the neutralino dark matters. The differential cross section rate is given by [19]

$$\frac{dR}{dQ} = \frac{\sigma \rho \chi}{2 m_{\chi_1} m_f} |F(Q)|^2 \int_{v_{\min}}^{\infty} \frac{f_1(v)}{v} dv,$$

(35)

where $f_1(v)$ is the distribution of speeds relative to the detector. The reduced mass is $m_r = \frac{m_{\chi_1} m_f}{m_{\chi_1} + m_f}$, where $m_N$ is the mass of the nucleus, $v_{\min} = \left( \frac{Q m_N}{2 m_f^2} \right)^{1/2}$, $Q$ is the energy deposited in the detector, and $\rho\chi^2$ is the density of the neutralino near the Earth. It is common to fix $\rho\chi^2$ to be $0.3 GeV/cm^3$. The quantity $\sigma$ is the elastic-scattering cross section of the LSP with a given nucleus. In our model, $\sigma$ has two contributions: spin-independent (scalar) contribution due to the squark exchange diagrams for $\tilde{Z}_{B-L}$-like LSP, and spin-dependent contribution arising from $Z_{B-L}$ gauge boson exchange diagrams for $\tilde{\chi}_{1(2)}$-like LSP. For $^{76}Ge$ detector, where the total spin of $^{76}Ge$ is equal to zero, we have a contribution from the scalar part only, which is given by

$$\sigma^{SI} = \frac{4 m_f^2}{\pi} |Z f_p + (A - Z) f_n|^2,$$

(36)

where $Z$ is the nuclear charge, and $A - Z$ is the number of neutrons. The expressions for the effective couplings to proton and neutron, $f_p$ and $f_n$, can be found in Ref. [20]. Finally, the form factor $F(Q)$, in this case is given by [19]

$$F^{SI}(Q) = \frac{3 j_1(q R_1)}{q R_1} e^{-q R_1},$$

(37)

where $q = \sqrt{2 m_N Q}$ is the momentum transferred and $R_1$ is given by $R_1 = (R^2 - 5 x^2)^{1/2}$ with $R = 1.2 fm A^{1/2}$ and $A$ is the mass number of $^{76}Ge$. $j_1$ is the spherical Bessel function and $s \approx 1 fm$.

For $^{73}Ge$ detector, where the total spin of $^{73}Ge$ is equal to $J = \frac{9}{2}$, we have a contribution from spin-dependent part only, which can be written as

$$\sigma^{SD}|F^{SD}(Q)|^2 = \frac{4 m_f^2}{2J + 1} |(f_p^n)^2 S_{pp}(q) + (f_n^n)^2 S_{nn}(q) + f_p^n f_n^n S_{pn}(q)|,$$

(38)

where $S_{pp}(q) = S_{00}(q) + S_{11}(q) + S_{01}(q)$, $S_{nn}(q) = S_{00}(q) + S_{11}(q) - S_{01}(q)$ and $S_{pn}(q) = 2[ S_{00}(q) - S_{11}(q) ]$, and the expressions for $f_p^n$ and $f_n^n$ can be found in Ref. [21].

The values of the spin structure functions $S_{00}(q)$, $S_{11}(q)$ and $S_{01}(q)$ are given in [20].

In case of $\tilde{Z}_{B-L}$-like LSP, the effective couplings to proton and neutron are very similar i.e. $f_p \approx f_n$. Therefore, the cross section, $\sigma^{SI} = \sigma^{SI}_{\tilde{Z}_{B-L}}$, is given by

$$\sigma^{SI}_{\tilde{Z}_{B-L}} \simeq \frac{4 m_f^2}{\pi} \left| \sum_q \frac{1}{2} N|\bar{q}q|N > \sum_{k=1}^{6} g_{\tilde{q} \bar{q}} k g_{\bar{q} \bar{q}} k^2 \right|^2,$$

(39)

where $q$ refers to $u, d, s, c, b, t$. The hadronic matrix elements are given by $< N|\bar{q}q|N > = f_{\pi m_q}. m_q$. The values of the parameters $f_{\pi m}$ can be found in Ref. [21]. From Eq. (19), one finds that $\tilde{Z}_{B-L}$ couples universary to all type of quarks, i.e. $g_{\tilde{q} \bar{q}} = g_{\bar{q} \bar{q}} \simeq i \sqrt{2} G_{B-L} Y^B_{B-L}$.

In Figure 8, we present our numerical results for the event rate $R$ as a function of $\tilde{Z}_{B-L}$-like LSP mass for $m_\tilde{q} = 200 GeV$ and $g_{\tilde{B}-L} \in [0.1, 0.5]$. As in previous figures $m_\tilde{q} = 200 GeV$ is assumed.

![FIG. 8: Detection rate versus $\tilde{Z}_{B-L}$-like LSP mass for $g_{\tilde{B}-L} \in [0.1, 0.5]$. As in previous figures $m_\tilde{q} = 200 GeV$ is assumed.](image-url)

Now we turn to the case of $\tilde{\chi}_{1(2)}$-like LSP. As mentioned above, in this case the scattering cross section is given by the spin-dependent part: $\sigma^{SD} = \sigma^{SD}_{\tilde{\chi}_{1(2)}}$, which is given by Eq. (38) with
Here we have used the lower limit on the ratio: $M_{\tilde{Z}_{B-L}}/g_{B-L}$ reported in Eq. (7). The numerical values of $S_{00}(q)$, $S_{11}(q)$, $S_{01}(q)$ and $(\Delta q)_N$ can be found in Ref. [21]. From this expression, it is clear that the detection rates of the extra Higgsinos-like LSP are extremely small. They are typically less than $10^{-16}$ (events/kg/day). This result is consistent with the spin-dependent contribution for the singlino in SUSY models with $U(1)'$ [22, 23]. However, in this class of model, unlike our $U(1)_{B-L}$ model, the singlino dominated LSP may imply large detection rates, due to the spin independent contributions.

CONCLUSIONS

We have studied the DM problem in supersymmetric $B - L$ extension of the SM. We showed that the extra $B - L$ neutralinos (three extra neutral fermions: $U(1)_{B-L}$ gaugino $\tilde{Z}_{B-L}$ and two Higgsinos $\tilde{\chi}^{0}_{1,2}$) are interesting candidates for cold DM. We provided analytic expressions for their annihilation cross sections. We also computed the $\tilde{Z}_{B-L}$ contribution to muon anomalous magnetic moment and showed that the current experimental limits impose a lower bound of order 30 GeV on $\tilde{Z}_{B-L}$ mass. We analyzed the thermal relic abundance of both $\tilde{Z}_{B-L}$ and $\tilde{\chi}^{0}_{1,2}$. We showed that unlike the LSP in MSSM, these particles can account for the measured relic abundance with no conflict with other phenomenological constraints. Finally, we discussed their direct detection rates and showed that they are beyond the reach of our near future experiments.

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