Towards a background independent approach to $\mathcal{M}$ theory

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ABSTRACT

Work in progress is described which aims to construct a background independent formulation of M theory by extending results about background independent states and observables from quantum general relativity and supergravity to string theory. A list of principles for such a theory is proposed which is drawn from results of both string theory and non-perturbative approaches to quantum gravity. Progress is reported on a background independent membrane field theory and on a realization of the holographic principle based on finite surfaces.

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1 Introduction

Up till this day, the different approaches to quantum gravity can be divided into two groups, which are the background dependent and background independent approaches. The background dependent approaches are those in which the definitions of the states, operators and inner product of the theory require the specification of a classical metric geometry. The quantum theory then describes quanta moving on this background. The theory may allow the description of quanta fluctuating around a large class of backgrounds, but nevertheless, some classical background must be specified before any physical situation can be described or any calculation can be done. All weak coupling perturbative approaches are background dependent, as are a number of non-perturbative developments. In particular, up to this point, all successful formulations of string theory are background dependent.

The background independent approaches are those in which no classical metric appears in the definition of the states, operators and inner product of the theory. A classical spacetime geometry can only appear in such a formulation in an appropriately defined continuum or classical limit. Background independent approaches include loop quantum gravity, dynamical triangulations and non-commutative geometry.

One quick way to describe the present state of research in quantum gravity is that the biggest problem faced by the background dependent approaches is in getting rid of the background, while the biggest problem faced by the background independent approaches is restoring the background through the discovery of a good classical limit. In spite of this situation, it is still true that almost all work in quantum gravity nowadays is carried out strictly within the context of one research program or another. One is working on strings, or loop quantum gravity or non-commutative geometry or twister theory, or perhaps something else. The main message I want to convey in this essay is that this is counterproductive, and that progress from this point on will be faster if more people can think in terms of a single, “quantum theory of gravity under construction”, which will have elements of more than one of these programs. If we do this we will force ourselves to discover and bridge the link between the background dependent and background independent approaches.

There is a good reason to believe that this is the moment to attempt to bridge this gap. This is that both string theory and the background independent approaches to quantum gravity have produced results which, by their robustness, generality and simplicity, may be considered predictions
about the real world. At the same time, there is evidence within each of these research programs that it is not itself the whole story. This, together with the striking fact that in several cases results of more than one research program point to the same conclusion, suggests strongly that the complete theory must involve elements of both string theory and the background independent approaches.

One example of this is the fact that a large number of results in string theory point to the existence of a new theory, called \( \mathcal{M} \) theory, which unifies all the perturbative string theories such that all of those theories turn out to describe expansions around different vacua or phases of it\( \textsuperscript{1} \). Almost by definition, that theory requires a description which is background independent, as its different vacua or phases involve different background manifolds. Even more striking, there are results such as mirror manifolds that indicate that different manifolds may be equivalent in this theory\( \textsuperscript{2} \), or that there are processes that allow transitions between different manifolds\( \textsuperscript{3} \).

A theory that encompasses these phenomena cannot be based on an expansion around a manifold, and must therefore be background independent. A key problem in string theory then must be to construct a background independent formulation of \( \mathcal{M} \) theory.

It is then striking that there are very few ideas for how to approach the construction of a background independent string or \( \mathcal{M} \) theory. String field theory is a natural possibility but, so far, there is no completely satisfactory background independent formulation of closed string field theory. Matrix models\( \textsuperscript{5} \) are so far (but see\( \textsuperscript{6} \)) not only background dependent but restricted to light cone gauge. The wonderful new AdS/CFT correspondence\( \textsuperscript{7} \),\( \textsuperscript{8} \),\( \textsuperscript{9} \), for all its attractive features, also seems dependent on the choice of a fixed manifold. There is an interesting proposal of Horava\( \textsuperscript{10} \), for a diffeomorphism invariant version of \( \mathcal{M} \) theory based on an 11 dimensional Chern-Simons theory. That theory has a large number of local degrees of freedom, and even at the classical level the sorting out of gauge and physical degrees of freedom is a difficult job that has only just been started\( \textsuperscript{11} \).

It may seem that the construction of a background independent string theory should be very difficult, given that all the versions of string theory

\( \textsuperscript{1} \)It should be stressed that by background independent I mean something much stronger than results that show that the theory is well defined as an expansion around any background, or any background which solves some equation. In truly background independent formulations the metric and connection enter the theory only as operators, and no classical metric appears in the definition of the state space, dynamics or gauge symmetries.
so far formulated are background dependent\(^2\). However, the problem may seem difficult because one is limiting the perspective to methods developed in string theory itself. If one is willing to bring into string theory structures and methods discovered in research programs whose very purpose has been to develop background independent methods for quantum theories of gravity, the problem may be easier than anticipated. Chief among these has been the program to quantize diffeomorphism invariant theories of gravity, commonly known as “loop quantum gravity”\(^3\).

Coming to that program, it is helpful to divide the results which have been achieved into two sets which we may call “kinematic” and “dynamic”. In the first class are all those results that require only that we are studying a gauge theory, based on a connection valued in some algebra or superalgebra, \(A\). Such a theory is defined on a spatial manifold \(\Sigma\) of some dimension \(d\), in which the gauge symmetries include both ordinary gauge transformations valued in \(A\) and diffeomorphisms of \(\Sigma\). These include general relativity and supergravity, coupled to arbitrary matter fields, plus a large number of other theories, both topological and with local degrees of freedom. Here the results include a complete characterization of the space of gauge invariant states, which turn out to have an elegant description in terms of combinatorics and representation theory\(^15, 17\).

Related to the characterization of the gauge invariant states has been the development of techniques to construct large classes of gauge invariant operators\(^15, 17\). Because of the diffeomorphism invariance, when these constructions succeed they produce finite operators\(^15\). Amongst those so constructed are, in general relativity and supergravity, operators that correspond to the areas of surfaces and the volumes of regions they bound\(^13, 16, 17, 22\), as well as dynamical operators such as the hamiltonian constraint\(^12, 13, 24, 25\) and the hamiltonian in fixed gauges\(^26\).

This whole class of results has also been found to be consequences of a general and rigorous formulation of diffeomorphism invariant quantum field theory. Results such as the characterization of the gauge invariant states in terms of diffeomorphism classes of spin networks and the discreteness of area and volume have thus been elevated from results of calculations\(^13, 16, 17, 22\) to rigorous mathematical theorems that depend on rather general assumptions\(^27, 28, 29, 25\). These express only the fact

\(^2\)There are of course non-perturbative results in string theory, but these are, so far, background dependent.

\(^3\)The extension of loop quantum gravity to supergravity is described in a number of papers\(^19, 20, 21\).
that the theory is based on a connection valued in a Lie algebra $A$, that the
gauge invariance includes diffeomorphisms and $A$ valued gauge transforma-
tions and that the frame fields are constructed from the canonical momenta
of the connection (which is the case for a large class of gravitational theo-
ries, including supergravity.) Beyond these, no assumption is made about
the Planck scale dynamics of the theory.

In contrast to these kinematical results, the dynamical results are those
that concern a particular theory, such as general relativity, and so depend on
the form of its hamiltonian or hamiltonian constraint. It is very striking in
light of recent results in string theory that here there are two very different
kinds of results, distinguished by whether or not the cosmological constant,
$\Lambda$, vanishes. In the case of $\Lambda \neq 0$ there is a completely holographic\cite{34, 35} formulation of at least one sector of the theory\cite{32, 33}, which seems to
have a good classical limit\cite{34, 35}. This is based on the construction of
a set of physical states in a spacetime with a timelike boundary on which
certain boundary conditions have been imposed. The Bekenstein bound\cite{36}
is satisfied explicitly as the dimension of the physical state space is finite
and grows like the exponential of the area of the boundary. Moreover, the
classical limit is precisely deSitter or Anti-deSitter spacetime\cite{34, 35}. It is
thus likely relevant to the AdS/CFT correspondence, as I shall discuss in
section 4.

One interesting result of this work is that the cosmological constant
induces a quantum deformation of the gauge theory, so that the states and
operators must be described in terms of the representation theory of the
quantum group $SU(2)_q$\cite{32, 33}. This makes the case of $\Lambda \neq 0$ very different
from the theory with vanishing cosmological constant. In the $\Lambda = 0$ case, an
infinite dimensional space of exact solutions to the hamiltonian constraint is
known explicitly\cite{12, 13, 25], but the solutions have a very different character
than for the case $\Lambda \neq 0$. In contrast to that case, it appears to be the case
that, for reasons argued in\cite{37}, which are reinforced by recent results\cite{38}, the
theory very likely does not have a classical limit in which massless particles
propagate. This is true both of the Euclidian theory in the form studied in
\cite{12, 24} and in the Lorentzian theory in the form studied by Thiemann\cite{25}.
At present no way is known out of these difficulties and, while there is no
theorem, it appears likely that at least for $\Lambda = 0$, quantum general relativity
is a theory that exists, and describes a world with a dynamical discrete
quantum geometry, but lacks a good classical limit.

Largely because of this result, for the last two years effort in this area
has gone into attempts to modify the theory to arrive at a theory which does
have a good classical limit. Most of this work has involved formulating the
theory in terms of sums over histories, rather than in a strictly canonical
language. There are in fact two distinct approaches to the path integral for
spin network states, which are intrinsically Euclidian\textsuperscript{[39, 40, 41, 42, 43, 44, 45]} and Lorentzian \textsuperscript{[46, 47, 48, 49, 50, 51, 52, 53]}. Some of this has also
involved extending the degrees of freedom, particularly by the addition of
supersymmetry\textsuperscript{[54]}.

I believe that if this program is to succeed, it must become the same as
the search for a background independent form of $\mathcal{M}$ theory. The argument
for this is very simple. Whatever quantum gravity is, if it succeeds it must
have a classical limit. If it is to reproduce ordinary quantum field theory
it must also have a sensible expansion around the classical limit, which is
by definition a perturbative theory of quantum gravity. The only good
perturbative quantum theories of gravity that we know of are perturbative
string theories. In fact, there seem to be a large number of these, but what
is important is that no successful perturbative quantum theory of gravity
has ever been found that was not a string theory.

There are in fact good reasons to believe that any successful perturbative
theory of quantum gravity must involve extended objects, whose high energy
behavior is that of string theory. This is because any quantum theory of
gravity must be finite, which means that there is a fixed length scale, $l_{Pl}$
which marks the transition between phenomena well described by classical
general relativity and those described by quantum gravity. However, the
theory we want must also have a classical limit corresponding to Minkowski
spacetime. The perturbation theory around that limit must have Poincare
invariance.

This brings us face to face with a problem, which is in whose frame is
$l_{Pl}$ to mark the boundary between the classical and quantum description of
geometry? It seems that there can be phenomena which are on one side of
the line for me that are at the same time well on the other side for you,
if our relative velocity is close enough to $c$. Thus, it seems that there is a
contradiction between the requirement that our theory be Poincare invariant
around one classical limit and that the theory has a physical length scale
that marks the boundary of the classical domain.

We encountered this problem in loop quantum gravity\textsuperscript{[55]}, trying to
extend the results on the existence of gravitons in the long wavelength
limit\textsuperscript{[56, 57]} (in the frame of one observer) to a Lorentz covariant result.
On the other hand, it is resolved in string theory, and in a very beautiful
way discovered by Thorne\textsuperscript{[58]} and Klebanov and Susskind\textsuperscript{[59, 61]}. Their
arguments also constitute part of the evidence that string theory, which is based initially on the assumption that spacetime is continuous, actually points to a discrete picture of quantum geometry. Their work shows that the only way the apparent paradox can be resolved is if the excitations of the gravitational field are extended objects, which scale in energy as strings do. I believe that this constitutes a very strong argument that any quantum theory of gravity that succeeds will have weakly coupled excitations that behave as strings, even if strings are not among the fundamental degrees of freedom.

Thus, if there is an extension of loop quantum gravity that has a well defined classical limit, it must have a regime which is described in terms of a perturbative string theory. Ergo, it must be a background independent formulation of string theory. This argument holds for any approach to quantum gravity, including non-commutative geometry.

Non-commutative geometry is a third approach to quantum gravity that has progressed greatly in the last decade. It is by definition a background independent approach, as the basic idea is to replace the background manifold by algebraic generalizations of a certain set of diffeomorphism invariant observables, which are the spectrum of the Dirac operator. Thus, the three approaches also share the emphasis on spinorial and fermionic structures, which was anticipated in much earlier work of Penrose, Finkelstein and others.

Interestingly enough, in the last year, non-commutative geometrical structures have turned out to be fundamental for both string theory and loop quantum gravity.

It is then possible that a background independent theory is at hand which is a synthesis of string theory, loop quantum gravity and non-commutative geometry. In the following section I will outline briefly the picture of space and time at the Planck scale that comes form combining the results of these different approaches. In the conclusion I mention briefly some work in progress directed towards realizing the picture presented here.

2 Principles for background independent $\mathcal{M}$ theory

If we assume that the robust results of string theory and the background independent approaches are all true, we arrive at a picture of quantum spacetime that may be summarized by a small number of statements. These
may be taken to be principles that a theory that unified these different approaches would have to satisfy. Given what we know presently, these are likely to characterize any successful background independent quantum theory of gravity.

1. **The holographic principle** The basic idea of the holographic principle[30, 31] is that in quantum gravity states and observables should be associated only with boundaries of regions of spacetime. This idea has actually emerged in two different contexts, first in work by Louis Crane on the relationship of topological quantum field theory (TQFT) to loop quantum gravity[63] and then in the papers of t Hooft[30] and Susskind[31]. The latter proposal has been developed primarily in string theory, while the proposal of Crane has inspired several developments on the background independent side[32, 64, 65, 66, 75, 76, 41, 42, 43].

There have been so far constructed three explicit realizations of the holographic principle. In historical order these are,

1) Quantum general relativity with finite boundaries and a cosmological constant[32, 64, 33].

2) The matrix models[5].

3) The AdS/CFT correspondence[7, 8, 9]

These are sufficient to show that the idea, surprising as it may seem at first, is completely realizable within the theory we are attempting to construct. Furthermore, given these different realizations, one way to search for a link between the background independent and dependent approaches is to investigate relationships between the different versions of the holographic hypothesis they give rise to.

2. **Quantum spatial geometry is discrete and non-commutative.**

String theory, loop quantum gravity and non-commutative geometry all point to the conclusion that the geometry of space is discrete at Planck scales. These realize earlier speculations by many pioneers of the field such as Penrose’s spin networks[61].

The basic kinematic result of non-perturbative diffeomorphism invariant quantum field theory is that, while the metric at a point cannot be well defined, operators can be constructed that correspond to the areas of surfaces and the volumes they contain[15]. These are finite
and diffeomorphism invariant and have discrete, computable spectra in a large class of theories including general relativity and supergravity, with arbitrary matter couplings [15, 16, 17, 22, 29].

The corresponding basis of diffeomorphism invariant states correspond to diffeomorphism classes of spin networks for the kinematical gauge group \( H \), which for gravity and supergravity is \( SU(2/N) \). These are graphs whose edges are labeled by representations and nodes by intertwiners. The result is a picture of quantum geometry that is discrete, based on representation theory and combinatorics.

Results from string theory that point to a discrete quantum geometry are described in [58, 59, 67, 68].

Evidence that the discrete quantum geometry is also non-commutative has emerged in both non-perturbative quantum gravity [69, 70] and string theory [71]. Conversely, as it makes perturbative divergences finite, noncommutative geometry also points to the discreteness of the quantum geometry [60, 72].

3. **Excitations are extended objects.** The basic evidence for this is that, as just mentioned, the only good perturbative theories of quantum gravity we know of are string theories. However, it is also the case that perturbations of background independent histories in a large class of theories are associated with \( 1 + 1 \) dimensional worldsheets, that must reproduce perturbative string theory if the classical limit exists [73].

In recent years it has been understood that the extended objects of string theory include D-branes of various dimensionalities. These results point to the conclusion that strings and branes may be equally elementary. However this does not mean that in the final, background independent theory there will not be fundamental degrees of freedom which can be identified. These are likely to be connected with a purely quantum description of the background geometry, whose excitations will be then strings and branes.

4. **Consistency requires supersymmetry.** All good perturbative theories of quantum gravity so far constructed are supersymmetric\(^4\). Thus, any sensible background independent quantum theory of gravity must either incorporate supersymmetry fundamentally or it must have a

\(^4\)For a possible counterexample, see [74].
mechanism whereby supersymmetry spontaneously emerges in the perturbative limit. At present no such mechanism is known. Unless one is discovered we must build supersymmetry into the background independent theory.

5. **Spacetime is relational.** Observables associated with classical general relativity with cosmological boundary conditions measure relations between physical fields. Points have no intrinsic meaning and are only identified through the coincidence of field values. The diffeomorphism invariance of the classical theory is thus an expression that that theory is background independent (up to the specification of the topology of the manifold.)

Any background independent form of quantum gravity must be able to reproduce general relativity as a classical limit, which means it must incorporate (if indeed extend) diffeomorphism invariance. This means that the interpretation of any such theory must be based, as is the interpretation of classical general relativity, on relational concepts of space and time\cite{36, 55}. It is this fundamental point that makes it inconceivable that the final form of $\mathcal{M}$ theory could be expressed in terms of any particular classical background.

6. **Histories have dynamical causal structure.** A corollary of the last point is that the causal structure of spacetime is a dynamical variable, which evolves dynamically. That this is the case in general relativity is a direct consequence of diffeomorphism invariance. This principle must then extend to any background independent form of $\mathcal{M}$ theory.

The evolution of a discrete quantum spatial geometry must then give rise to a discrete dynamical causal structure. For the case of spin network states, the study of the structures that arise has been initiated recently by Markopoulou\cite{46, 75}. The extension of this structure to a form suitable to $\mathcal{M}$ theory is under development\cite{48, 49, 76}.

Once a background independent quantum theory of gravity is formulated in terms of histories with dynamical causal structure, the question arises as to how to study the continuum limits of such systems. This is necessary to understand the existence and properties of the different classical limits of the theory.

It is clear that since the causal structure is fluctuating, the continuum
limits cannot be studied in the usual context of equilibrium second order critical phenomena, as the relevance of this phenomena to quantum field theory depends on the possibility of making a global Euclidian rotation, common to all histories. The question then arises as to what kind of critical phenomena might characterize the continuum limit of theories with dynamical causal structure.

A natural conjecture, discussed in [47], is that the answer is non-equilibrium critical phenomena. As noted in [47, 46, 48, 77] directed percolation, studies of the growth of soap bubbles and other non-equilibrium critical phenomena offer models which may be interpreted as dynamical causal structures. The important difference is that the histories are weighed by complex amplitudes rather than probabilities. What is needed is then a study of critical phenomena associated with what might be called quantum directed percolation problems, which are directed percolation problems in which the weight of a history is complex.

Another advantage of non-equilibrium critical phenomena as a paradigm for the continuum limit of background independent quantum theory of gravity is that there are cases in which no fine tuning is required. This “self-organized criticality [78] is a good feature for theories of quantum cosmology to have as it may resolve the embarrassing situation in which the existence of the classical world requires fine tuning of parameters. Preliminary studies of the classical limit of theories of fluctuating causal structure through the use of the analogy with non-equilibrium critical phenomena are described in [77, 52, 53].

3 Current directions

The basic hypothesis of this essay is that the results and conjectures we have just outlined may all describe nature, in spite of having emerged from different research programs. In fact, there is no reason the results of string theory and the background independent approaches may not be compatible as these programs cover different domains of quantum gravitational phenomena. The question is then whether these can be combined to give one theory of quantum gravity that describes all domains. As I have argued, this must be the same as the question of constructing a background independent form of string, or $\mathcal{M}$ theory.
Several projects are underway, which aim towards this goal. I mention two of them very briefly.

3.1 Background independent membrane dynamics

The matrix models describe a background dependent form of membrane dynamics, in a fixed gauge. It is unfortunately, difficult to extend the matrix models even to a lorentz covariant form, for reasons described in [5]. Thus it seems unlikely they will yield a background independent theory. A background independent form of membrane dynamics was then proposed using an extension of the background independent form of dynamics that has been studied recently for spin network states of quantum gravity [46]. The theory was applied to ($p,q$) string networks in [49] and the application to $\mathcal{M}$ theory is studied in [50]. The theory is purely quantum mechanical, and the two dimensional surfaces which may be considered the constant time slices of the membranes are constructed purely algebraically. The theory involves first of all the choice of an algebra $\mathcal{A}$ whose representation theory allows the construction of a finite dimensional space of conformal blocks associated to every two dimensional manifold with genus $g$, $\mathcal{V}_{A,g}$. To define a form of $\mathcal{M}$ theory, $\mathcal{A}$ may be taken to be a superalgebra with 32 fermionic charges [50]. The hilbert space of the whole theory is taken to be,

$$\mathcal{H} = \sum_g \mathcal{V}_{A,g}$$

These two surfaces are background independent membranes. As there is to begin with no background there are no embedding coordinates, but there are states associated with the representation theory of $\mathcal{A}$. These may also be considered to be states of Chern-Simons theory in a three manifold in the interior of the two surface.

Time evolution is defined by an operator that generates local changes in the topology of the surface and the state. The result may be called a background independent membrane field theory. The rules given in [48, 50], which extend those proposed for quantum gravity in terms of spin networks [16], result in the construction of purely quantum histories, which however have both causal structure and many-fingered time. The time slices, which are defined algebraically, are associated with a basis of states in $\mathcal{H}$. Each history has an amplitude, which is the product of an amplitude for the local moves. The dynamics is given by specifying the forms of these amplitudes as described in [48, 50].
A key problem for such a theory is the classical limit, which is, as I’ve argued above, a problem in non-equilibrium critical phenomena. Some studies of this problem are underway\cite{77, 53}.

Some general results about the perturbation theory in such a framework are known. In particular, it can be argued that perturbations of these abstract histories are given by a discrete field theory defined on a timelike two surface embedded in the history\cite{73}. When there is a classical limit, the two dimensional theory must contain the massless modes, hence it must define in the continuum limit a consistent perturbative string theory.

How is the physical interpretation of such a theory to be given, in the absence of any background? As described in \cite{48} this is done in terms of information projected on two surfaces embedded in the two surfaces on which the states are defined. Thus the theory is holographic, by construction. The relationship between the distribution of information on these surfaces and the causal structure is rather intricate, and may be described using a mathematical formulation developed in \cite{75}.

### 3.2 Holography on finite surfaces

As stressed in the last section, in a background independent theory there will be no asymptotic classical region, and hence, if there is to be a holographic formulation, it must be defined on finite surfaces inside the universe. One way to approach the construction of such a theory is to extend the holographic formulation of quantum general relativity given in \cite{12} to a candidate for a form of $\mathcal{M}$ theory. This may be done by extending the algebra of observables and states of the boundary theory from one suitable for general relativity to one suitable for $N = 8$ supergravity. Such a formulation will be described in \cite{80}, based on a general form for quantum theories of gravity as constrained topological quantum field theories developed in \cite{33}. Of course, these are not the only possible approaches to a background independent form of string theory. They may for example be criticized in that, while supersymmetry can be easily included, it seems optional from the point of view of the background independent formulation. It is possible that the main role of supersymmetry in such formulations is that it guarantees the existence of the classical limit. However, it is also possible that supersymmetry plays an even more fundamental role, which is yet to be revealed.
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