A Brief Study of In-Domain Transfer and Learning from Fewer Samples using A Few Simple Priors

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Abstract

Domain knowledge can often be encoded in the structure of a network, such as convolutional layers for vision, which has been shown to increase generalization and decrease sample complexity, or the number of samples required for successful learning. In this study, we ask whether sample complexity can be reduced for systems where the structure of the domain is unknown beforehand, and the structure and parameters must both be learned from the data. We show that sample complexity reduction through learning structure is possible for at least two simple cases. In studying these cases, we also gain insight into how this might be done for more complex domains.

1. Introduction

Many domains are constrained by data availability. This includes domains, such as YouTube recommendations, which ostensibly have a large amount of data, but which have a long tail of instances that each have only a handful of data points. This also includes domains for which humans require several orders of magnitude less training data than state-of-the-art approaches, such as motor manipulation, playing Atari, and understanding low-frequency words. The No Free Lunch Theorem (Wolpert & Macready, 1997) suggests that a domain prior is needed to offset sample complexity for these cases.

For domains such as images and audio, convolutions are commonly used as one such prior. In our view, convolutions allow for in-domain transfer by sharing weights among otherwise weakly-connected areas of the domain, effectively multiplying the number of samples. For example, if a convolutional layer takes a 28 by 28 MNIST image and uses filters of size 5x5 with stride 2, this gives 144 5x5 windows per image, which means that for every image, we have 144 training points for the filter, which effectively decreases our sample complexity by 144 times for this filter. The convolutions allow us to internally transfer information from the top-left 5x5 window to the bottom-right 5x5 window (and to the other 142 windows).

However, there are domains where the structure might not be known beforehand, such as robot joint angle trajectories or car traffic speed sensor data. The question we address in this paper is for domains where we aren’t given the structure a priori, is there a weaker prior that will both allow a system to learn a structure and leverage the learned structure to still have a net decrease in sample complexity over a baseline without the prior?

We present a simple probability density estimation problem and examine how three priors affect sample complexity. We show, at least for one simple domain, the structure can be recovered merely given the prior that there is a repeated structure, with the number of filters and size of windows. Using this prior, we show how a system may automatically transfer from parts of the domain where samples are relatively plentiful to parts where samples are more rare.

2. The Four Urns Problem

To help illustrate how a simple prior might help reduce sample complexity, consider the setup where we are given four urns \( U_1, U_2, U_3, U_4 \), each filled with balls of eight different colors \( c_1 \cdots c_8 \). We are given samples drawn from the urns with replacement, and we assume that the urns are independent of each other. In this setup, we are given samples from the urns one at a time. We do not get to choose which urn we sample, but we are told which urn was sampled. For example, we might get a sequence \((U_1, c_5), (U_2, c_3), (U_1, c_7)\). Our samples aren’t uniform among the four urns: we sample \( U_1 \) with probability .025, while \( U_2, U_3, \) and \( U_4 \) are each sampled with probability .325. Our goal is to model the urns’ distributions, minimizing the KL divergence \( D_{KL}(P||Q) \) from the estimated distribution \( Q_i = Q(c_j|U_i) \) to the unseen true distribution \( P_i = P(c_j|U_i) \). If we assume a uniform prior for each

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urn’s distribution, we can’t do better than tallying the outcomes and taking the the expected values from a Dirichlet distribution. In this case, it takes thousands of samples to get a strong estimate for the distribution of Urn 1 because it’s sampled so infrequently, as shown in Figure 1(b).

If we are given prior knowledge that there are actually only two distributions instead of four, then our sample complexity can be cut significantly. That is, while draws from each of the four urns are still independent of the other urns, we are told that each urn was filled with balls sampled from one of two much larger urns, though we aren’t told the distributions of the balls in the larger urns, nor with which of the larger urns each of the four urns is filled. Formally, we assume larger urns $a$ and $b$ such that $V_{i} \in \{ 1, 2, 3, 4 \}$ $P_{i} \in \{ P_{a}, P_{b} \}$. In this case, we use EM to alternatively update our estimates of the classification of the distributions, then use these classification probabilities to update the estimates of the underlying distributions. Specifically, we estimate the probability that each distribution is drawn from $a$ or $b$, $V_{i} \in \{ 1, 2, 3, 4 \}$, $j \in \{ a, b \}$ $P_{i} = P_{j} \mid D, Q_{a}, Q_{b}$, where $D$ is the data seen thus far, then use these estimates to compute new estimates for $Q_{a}$ and $Q_{b}$. We then use these values for $Q_{a}, Q_{b}$ to update the classification probabilities $P_{i} = P_{j} \mid D, Q_{a}, Q_{b}$, and so on until convergence.

We show the results of this process in Figures 1, plotting $D_{KL}(P||Q)$ as a function of number of samples for estimates $Q$ for a single run. In Figure 1(a), we see a significantly faster convergence for the case where we make use of our priors over the “raw” or uniform prior. We break this down into the error for the estimates for the four urns in Figure 1(b), where we see two sources for this difference in the estimates. The first source is that the model quickly concludes (correctly) that Urns 4 and 2 have identical distributions. Thus, it uses samples from Urn 4 to inform the probability distribution of Urn 2 and vice versa. In effect, it doubles its samples for these urns, halving the number of samples needed to create its probability estimates for them.

The other source of difference is the model’s estimate for the probabilities of the rarely seen Urn 1. Of the first 1000 samples, only 16 are from Urn 1. With 8 different ball colors, the uninformed estimate is nowhere near convergence, having seen an average of only two samples per color. Conversely, after only two samples, our system correctly concludes that Urns 1 and 3 are drawn from the same distribution and “transfers” its knowledge about Urn 3 to Urn 1. Note that the “knowledge transfer” goes both ways: our green line for Urn 3 dips slightly below the red line. This is because once our system concludes that Urns 1 and 3 have identical distributions, it adds the paltry samples from Urn 1 to the tallies for the distribution shared by both urns. Another interpretation is that the system is primarily creating its model of the probability distribution with samples from Urn 3, whereas it uses the few samples from Urn 1 to classify Urn 1 as the same type as Urn 3. An analogy might be made to the scenario where knowing that “Donald is a duck” tells us much about Donald, but it also informs us a little of what it means to be a duck. Finally, note that the averaged error for “Ours” in Figure 1(a) briefly increases before decreasing. Some insight might be gained to explain this by looking at the breakdowns in Figure 1(b). We suspect that our model initially erroneously assigns Urn 3 to be the same distribution as Urns 2 and 4, thus negatively transferring their tallies to Urn 3 for about 30 samples.

3. Weaker Priors

In this section, we give a simple example of how we might generalize these priors to domains in which we must simultaneously learn the structure and the probability distributions. We present a vastly simplified version of searching for convolutional structure in images. Conceptually, we would like to discover “convolutions” in images without prior knowledge of which pixels (or vector indices) are next to which, or even that we’re dealing with 2D grid, but just given the prior knowledge that there is repeated structure. In real images, we might be given real-valued vectors of size 3,072. We grossly simplify this to bit-vectors of size 12 to see if we can discover the repeated structure for this case. We are given one bit-vector at a time, and we can no longer assume independence among the elements of the bit vectors. As in the previous task, our task is to model the 12-way joint distribution. If we assume a uniform distribution for each outcome, then without other priors, the best we can do is model the outcomes as a Dirichlet distribution, with one bin for each of the $2^{12}$ possible outcomes.

We consider the reduction in sample complexity given by the following priors:

1. That the 12 variables form 4 independent distributions, each of 3 variables (though we are not told what the groups are).
2. That the 4 distributions are of only 2 types. (This is the same as the prior in the previous section, with the exception that we are no longer given the groupings of the variables beforehand.)
3. We are given which of the 12 variables form the 4 groups, and the variables’ order within the group. This is equivalent to being told which of the 8 colors a ball is, and from which urn. With this knowledge, a vector of length 12 is equivalent to a sample from each of the four urns. For example, if each sample consists of Boolean variables $\{ V_{1}, \ldots, V_{12} \}$, this prior will break these into 4 ordered triples such as $(V_{5}, V_{1}, V_{11}), (V_{2}, V_{8}, V_{11}), (V_{4}, V_{2}, V_{3}), (V_{10}, V_{6}, V_{5})$. So if $V_{5} = 1$, $V_{1} = 1$, and $V_{11} = 0$, this is equivalent to Urn 1 being color 6 or $(1, 1, 0)$.
With these priors, we have the following cases, the plots of which are shown in Figure 2(a):

**Case 0** We assume (incorrectly) that the 12 elements of the bit-vector are independent of each other. Here, we model each variable using a beta distribution with a uniform prior. This model converges quickly, but is not expressive enough to represent the true distribution.

**Case 0’** We assume a uniform prior over each of the $2^{12}$ outcomes. This model will eventually converge to the correct distribution, but takes many more than 200 samples to do so.

**Case 1,2,3** We assume priors 1 and 3 (we know that there are four distributions and we are told which variables comprise each distribution). This is equivalent to the “Raw tallies” plot in Figure 1(a), except that we sample all four urns at every timestep.

**Case 1,2** We assume prior 1, that there are 4 independent distributions of 3 variables, but we don’t know which variables comprise each distribution.

**Case 1,2,3** This is the most interesting case for our purposes. We assume that there are four distributions (each of 3 variables), and that these four distributions are really only of two distinct types, though we’re not told which variables are grouped together.

For Case 1,2 and Case 1, we do an exhaustive search over the possible groupings of the variables and compute the most likely ordering, using similar techniques to the previous section. For example, one grouping of the 12 indices might be $(V_1, V_3, V_2), (V_5, V_{12}, V_4), (V_7, V_{11}, V_6)$, and $(V_{10}, V_9, V_8)$. Given a grouping, the problem is equivalent to the Four Urns problem. E.g., using the grouping above, if $(1, 1, 0)$ is color 6, and $V_{10}, V_9,$ and $V_8$ are 1, 1, and 0, respectively, this would be equivalent to drawing a ball of color 6 from Urn 4. Given a grouping, we can explore the different assignments. Since there are only 2 latent variables $a$ and $b$, each of the 4 distribution gets assigned to exactly one of the latent variables (a or b). E.g., we may say that “Urn” $(V_{10}, V_9, V_8)$ is assigned as coming from either distribution $a$ or $b$.

This technique of searching all possible groupings is clearly intractable, taking exponential time in the length of the vectors, but is feasible for our tiny vectors. More explicitly, given data $D_1, \ldots, D_t$, where $D_i \in \{0, 1\}^{12}$, we search over each permutation $perm$ of the ordered set $(1, \cdots, 12)$, and each possible group assignment function $A(x) \rightarrow \{a, b\}$ to maximize $P(A, perm|D)$ using equations 1 and 2. (Taking into account symmetries, we can reduce the number of permutation and assignment pairs from $2^4 \cdot 12!$ to $2^{4}12! / 2^{3}3! = 106, 444, 800$.)

Let $D_i^p$ be the result of applying the permutation $perm$ to $D_i$, and for $j \in \{1, 2, 3, 4\}$ let $D_{i,j}^p$ be the 3j to 3(j + 1) elements of $D_{i,j}^p$. If we assume an even prior of all permutations and assignments, we get:

$$P(perm, A|D) \propto P(D|perm, A) = P(D^p|A) = \prod_{i,j} P(D_{i,j}^p | A_j) \approx \prod_{i,j} Q(D_{i,j}^p | A_j) = \prod_{i,j} Q_{A_j}(D_{i,j}^p)$$

Here we compute the estimated probability $Q_A$ using a Dirichlet distribution with $2^3$ outcomes with tallies from our observations. Since there are only two actual distribu-

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**Figure 1.** Error vs samples seen. (a) The total error for estimates of the four urns for a single run (solid lines) and averaged over 1000 runs (light-colored lines). (b) The breakdown of the KL-error over the estimates for the four urns using just the raw tallies and using the prior that there are only two types of distributions. Individual samples for Urn 1 are shown by markers at each sample.
tions, we sum our tallies for instances that have the same assignment (where $\delta(x, y) = 1$ if $x = y$ else 0).

$$Q_{A,i}(D_p^j) = \frac{1 + \sum_{k,l} \delta(D_k^p, D_l^p) \delta(A_l, A_i)}{2^j + 4|D|}$$ (2)

Figure 2. Error vs. sample complexity for different priors. (a) A single run showing that Cases 1, and 1,2 converge to Case 1,3 and 1,2,3, respectively. (b) The average over 100 runs for all but Case 1,2, which takes a week on 40 cores for a single run!

As indicated in Figure 2(a), this search takes about 125 vector samples to converge on the correct grouping, and then follows the same patterns as the curves in Figure 1(a). This shows that, at least in this case, sample complexity can be reduced using only the first two priors. Figure 2(b) shows averages over 100 runs.

4. Related Work

The Four Urns Problem can be framed as a constrained instance of latent Dirichlet allocation (LDA) (Blei et al., 2003), where there are two latent classes and each “document” or urn is a “mixture” of exactly one latent class. To our knowledge, this special case isn’t directly addressed in the topic analysis literature, because the latent classes are not allowed to mix. It is more difficult to phrase the 12-bit vector problem as an instance of topic analysis, because we are not given the documents/urns, but only parts of documents and we must deduce how to put them together.

There is also some overlap between our setup and contextual bandits (Dudik et al., 2011; Zhou & Brunskill, 2016) in that both systems are motivated to make estimates of the processes true distributions efficiently in terms of number of samples. Our system differs in that its utility is directly tied to the accuracy of its estimate (instead of the payoffs of bandits) and in that it has no choice about which observation it will see next.

The main motivation for this work came from transfer learning (Pan & Yang, 2010; Taylor et al., 2016; Rusu et al., 2016), continual learning (Ring, 1994; Pickett et al., 2016; Kirkpatrick et al., 2017), lifelong learning (Ruvolo & Eaton, 2013; Ammar et al., 2015), learning to learn (Thrun & Pratt, 2012; Andrychowicz et al., 2016), and multitask learning (Luong et al., 2015), which all share the idea that knowledge learned from one area can be leveraged to learn from another area with fewer samples. This paper contributes to these areas by investigating a simple case and offering the insight that a system can transfer knowledge between areas if it has some estimate of its respective certainty about those areas.

5. Conclusions and Future Work

We have shown an example where a few assumptions will allow a decrease in sample complexity. This can be thought of as a simple example of in-domain transfer (e.g., transferring knowledge between Urns 1 and 3). We hypothesize that to transfer knowledge, one needs to have some measure of certainty of our parameters, something that Bayesian approaches handle naturally. This measure can also be more implicit, such as by freezing weights that have been trained until convergence (Rusu et al., 2016).

This work is a preliminary exploration that this can be done. There are two main directions we’d like to explore in future research. The first is generalizing Priors 1 and 2. We conjecture that both of these can be wrapped into a description length prior, where it is cheaper to inherit from existing models of distributions than to create a new distribution from scratch. This would allow our model to search over both the number of distributions and distribution types. The second direction is to find heuristics to make the search for structure tractable.
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