Differential cross sections in a thick brane world scenario

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Abstract. The elastic differential cross section is calculated at low energies for the elements He and Ne using an effective 4D electromagnetic potential coming from the contribution of the massive Kaluza-Klein modes of the 5D vector field in a thick brane scenario. The length scale is adjusted in the potential to compare with known experimental data and to set bounds for the parameter of the model.

1. Introduction
The possibility that our space has more than three spatial dimensions has been attracting continuing interest for many years. Recently, the emphasis has shifted towards the so-called brane worlds scenarios. In the brane scenarios the ordinary matter are confined to a 3-brane embedded in higher-dimensional space-time [1, 2, 3].

In the brane world context, there are many models of thin branes [4, 5, 6] and thick branes [7]. Here, we use a recently proposed thick brane world configuration [8]. In this scenario the thick brane is modeled by an intriguing relation between the curvatures generated by the 5D and 4D cosmological constant without the inclusion of bulk scalar fields at all. An interesting
feature of the thick brane world models is that singularities are not present at the position of the brane as other models [9, 10].

Within the framework of the brane world models, it has been seen that imply usually possible low energy consequences that should be either suppressed or else checked with experimental data. So the aim of this paper is to investigate some consequences of the extra dimensional theories at low energy. Specifically, we study the scattering of electrons by helium and neon in a thick brane scenario. Thus, for the sake of clarity, we first show the analysis of Ref. [11] to the case of the electromagnetic field. Finally we compare ours results for the scattering of electrons by atoms with known experimental data and to set bound on the scale of the scenarios.

The paper is organized as follows. In Sec. 2 we briefly describe the thick brane world setup, Sec. 3 is dedicated to obtain the static potential for an electromagnetic field. In Sec. 4 we discuss the elastic differential cross section for the scattering of electrons by atoms of different species. Finally Sec. 5 we focus on the discussion of our results.

2. Model

In this section we briefly describe the brane world used in the present work. The 5D action for the thick brane world model is

\[ S = \frac{1}{8 \pi G_5} \int d^5x \sqrt{-g} (R - 2 \Lambda_5), \]

(1)

here \( R \) is the 5D scalar curvature, \( \Lambda_5 \) is the bulk cosmological constant and \( G_5 \) is the 5D Newton constant. The action (1) provides a solution to Einstein equations with a cosmological constant in 5D that has the form

\[ ds^2 = e^{2A(y)} \left( -dt^2 + a^2(t) \left[ dx_1^2 + dx_2^2 + dx_3^2 \right] \right) + dy^2, \]

(2)

where \( e^{2A(y)} \) is the warp factor, \( a(t) \) is the scale factor of the brane, \( t, x_1, x_2, x_3 \) denotes the 4D coordinates and \( y \) stands for the extra-dimensional coordinate.

The warp factor and the scale factor are given by

\[ A(y) = \ln \left[ \frac{H}{b} \cos(by) \right], \]

(3)

\[ a(t) = e^{Ht}. \]

(4)

Here \( H \) is the Hubble parameter and the \( 1/b \) parametrizes the thickness of the 3-brane. \( H \) and \( b \) are related to the 4D and 5D cosmological constant as

\[ 3H^2 = \Lambda_4, \]

\[ \Lambda_5 = 6b^2, \]

(5)

(6)

where \( \Lambda_4 \) is obtained upon integration of the fifth dimension. In this model, the 3-brane is located at \( y = 0 \) and the range of the fifth dimension is \(-|\pi/2b| < y < |\pi/2b|\).

For future purposes, it turns out to be convenient to perform the change of variable \( dz = e^{-A(y)} dy \). One gets the expression for \( z \)

\[ z(y) = \frac{2}{H} \text{arctanh} \left[ \tan \left( \frac{by}{2} \right) \right]. \]

(7)

Due to this transformation, the warp factor adopts the form

\[ A(z) = \ln \left[ \frac{H}{b} \text{sech} (Hz) \right], \]

(8)
and the metric can be written as

$$ds^2 = \frac{H}{b} \text{sech} (Hz) \left[ -dt^2 + e^{2Ht} \left( dx_1^2 + dx_2^2 + dx_3^2 \right) + dz^2 \right]. \quad (9)$$

In this scenario the effective 4D Planck mass is

$$M_{Pl}^2 = \frac{\pi M_p^4 H^2}{b^4}, \quad (10)$$

with $M_p$ the 5D Planck mass. By assigning $H \approx 10^{-60} M_{Pl}$ and setting $M_p \approx 10^{-55} M_{Pl}$, one gets the correct 4D gravitational coupling on the thick brane. On the other hand, in order to generate the desired mass hierarchy, in this model we need that the ratio between the Hubble parameter and the compactification scale is

$$\frac{H}{b} \approx 10^{-5}. \quad (11)$$

Of course this reformulation of the hierarchy problem raises the questions about the stability of the brane separation.

### 3. Effective four-dimensional electrostatic potential

In this section, we show the analysis to calculate corrections to Coulomb’s law developed in Ref. [11]. It is known that the charged fermions in quantum electrodynamics interact with one another by exchanging a photon. Hence the interaction between fermions and a gauge field can be written as

$$S_I = \int d^4x dz \sqrt{-g} (-e_5) \tilde{\Psi}(x, z) \Gamma^M \Psi(x, z) A_M(x, z), \quad (12)$$

where the $e_5$ is a 5D coupling constant. After the dimensional reduction (see [11] for more details), we obtain the following coupling of fermion zero mode and photon Kaluza-Klein modes

$$S_I = (-e_5) \sum_{n \neq 0} \int dz e^{-A/2} \rho_n(z) L_0^2(z) \int d^4x \sqrt{-\tilde{g}} \bar{\psi}_0(x) \gamma^\mu a^{(n)}_\mu(x) \psi_0(x),$$

$$= \int d^4x \sqrt{-\tilde{g}} \left\{ -e \bar{\psi}_0(x) \gamma^\mu a^{(0)}_\mu(x) \psi_0(x) - \sum_{n \neq 0} \epsilon_n \bar{\psi}_0(x) \gamma^\mu a^{(n)}_\mu(x) \psi_0(x) \right\}. \quad (13)$$

Here, $e$ is the usual 4D charge of fermion zero mode, given by

$$e = e_5 \int dz e^{-A/2} \rho_0(z) L_0^2(z) = e_5 \sqrt{\frac{b}{\pi}}, \quad (14)$$

and $\epsilon_n$’s are 4D effective couplings

$$\epsilon_n = e_5 \int dz e^{-A/2} \rho_n(z) L_0^2(z) = e \sqrt{\frac{\pi}{b}} \int dz e^{-A/2} \rho_n(z) L_0^2(z). \quad (15)$$

$L_0(z)$ is the zero mode for the 5D fermion, $\rho_0(z)$ and $\rho_n(z)$ are the Kaluza-Klein modes for the 5D vector field, whose wave functions are given by

$$L_0(z) = \left[ \frac{H \Gamma \left( \frac{b+2M}{2b} \right)}{\sqrt{\pi} \Gamma \left( \frac{M}{b} \right)} \right] \text{sech}^M (Hz), \quad (16)$$

$$\rho_0(z) = \sqrt{\frac{H}{\pi}} \text{sech} (Hz), \quad (17)$$

$$\rho_n(z) = C_+ P_n^{+ij} \left[ \text{tanh} (Hz) \right] + C_- P_n^{-ij} \left[ \text{tanh} (Hz) \right], \quad (18)$$

where $C_+$ and $C_-$ are constants.
with $\Gamma(z)$ the Gamma function and $P_{1/2}^{\mu}(z)$, $Q_{1/2}^{\mu}(z)$ are associated Legendre functions of first and second kind, respectively, degree $\nu = 1/2$ and purely imaginary order $\mu = i\beta = i\sqrt{m^2/H^2} - 1/4$. The constants $C_+ \text{ and } C_-$ are expressed as follows

$$C_\pm(\beta) = \frac{\Gamma(1 \mp i\beta)}{\sqrt{2\pi}} = \frac{\Gamma(1 + i\beta)}{\sqrt{2\pi}}.\quad (19)$$

$\Sigma$ implies summations or integrations (or both) with respect to $n$, depending on the respective discrete or continuous character of the $a_{\mu}^{(n)}(x)$ and $c_n(z)$, where latter is determined by $\rho_n(z)$.

In the non-relativistic limit, the potential between two charged fermion zero modes can be found by the Kaluza-Klein photon exchange process as

$$v(r) = \frac{e^2}{4\pi r} + \int_{H/2}^{\infty} \frac{d\rho}{4\pi r} e^{-mr},$$

$$= \frac{e^2}{4\pi r} \left[ 1 + \frac{\pi}{b} \int_{H/2}^{\infty} dm e^{-mr} \left( \int dz e^{-A/2} \rho_n(z) L_0^2(z) \right)^2 \right],$$

where $r$ is the spatial separation in 4D between charged fermions. The first term of (20) represents the contribution of massless photon, while remaining terms come from its massive Kaluza-Klein modes.

Substituting the expressions (16)-(18) in Eq. (21), we obtain

$$v(r) = \frac{e^2}{4\pi r} \left[ 1 + \frac{\pi}{H} \left( \frac{2M+b}{2} \right) \frac{\Gamma(4M-b)}{\Gamma(M/2) \Gamma(4M-b)} \right] \int_{H/2}^{\infty} dm e^{-mr} \left| \sum_{\pm} C_\pm(\beta) P_{1/2}^{\pm,i\beta}(0) \right|^2,$$

where we have used the following relation

$$\delta(z) = \lim_{H \to \infty} \frac{H\Gamma(4M+b)}{\sqrt{\pi} \Gamma(4M-b)} \text{sech} \frac{4M-b}{2r} (Hz), \quad 4M > b.\quad (23)$$

Taking into account Eq. (19) as well as the following reaction

$$P_{1/2}^{\mu}(0) = \frac{2^\mu}{\Gamma(1 - \nu - \mu/2) \Gamma(1 + \nu - \mu/2)}\quad (24)$$

The expression (22) can be rewritten as

$$v(r) = \frac{e^2}{4\pi r} \left[ 1 + \frac{2\pi}{H} \left( \frac{2M+b}{2} \right) \frac{\Gamma(4M-b)}{\Gamma(M/2) \Gamma(4M-b)} \right] \int_{H/2}^{\infty} dm e^{-mr} \left| \frac{\Gamma(1 + i\beta)}{\Gamma(1 - i\mu/2) \Gamma(5/4 - i\beta/2)} \right|^2.$$

(25)

To perform the integral in Eq. (25), we do an expansion in the integrand with respect to $H/2$, i.e. around $\beta = 0$. So, the leading-order contribution for the potential $v(r)$ is

$$v(r) \approx \frac{e^2}{4\pi r} \left[ 1 + 2\pi \left( \frac{2M+b}{2} \right) \frac{\Gamma(4M-b)}{\Gamma(M/2) \Gamma(4M-b)} \right] \frac{1}{\Gamma(1) \Gamma(5)} \frac{e^{-Hr/2}}{Hr} + \ldots.\quad (26)$$

The first term in (26) is the usual Coulomb interaction potential which coming from the exchange of the zero mode of the photon and the second term has a Yukawa potential form because it is mediated by an infinite tower of massive Kaluza-Klein vector modes. Also the corresponding correction to 4D effective potential depends on the 5D paramters of the massless zero mode fermion field $M$ and $b$.

In the following section we study the scattering using this effective potential.
4. Scattering by helium and neon atoms

In this section we study the collision of a particle of charge $ze$ and mass $m$ with a neutral atom of atomic number $Z$. Within the Born approximation the differential scattering cross section for elastic processes is given by

$$\frac{d\sigma}{d\Omega} = -\frac{m}{2\pi\hbar^2} \int e^{i\vec{q}\cdot\vec{r}} V(\vec{r}) d^3\vec{r},$$

(27)

where $\vec{r}$ is the position vector of the incident particle and the momentum transfer $\vec{q}$ is given in terms of $\theta$ and the incident particle momentum $\vec{k}$ as

$$|\vec{q}| = 2|\vec{k}| \sin(\theta/2).$$

(28)

The electrostatic potential $V(r)$ considers the effective interaction of the incident particle with the constituents of the neutral atom. Considering the nucleus as a point charge, $V(r)$ can be calculated in terms of the atomic electron density $\rho(\vec{r})$ as

$$V(r) = ze\left[Z\epsilon_v(\vec{r}) + e\int \rho(\vec{r}) v(|\vec{r} - \vec{r}'|) d^3\vec{r}'\right].$$

(29)

$v(\vec{r})$ is given by (26). Here we are ignoring all effects of symmetry and spin for the time being. For neutral atoms, the density satisfies

$$\int \rho(\vec{r}) d^3\vec{r} = Z.$$

(30)

Thus introducing Eq. (29) in (27) and making the following change of variable $\vec{R} = \vec{r} - \vec{r}'$, thus we have

$$\frac{d\sigma}{d\Omega} = -\frac{m e^2}{2\pi\hbar^2} \left[ Z e^2 \int e^{i\vec{q}\cdot\vec{r}} V(\vec{r}) d^3\vec{r} - z \int e^{i\vec{q}\cdot\vec{R}} V(\vec{R}) d^3\vec{R} \int \rho(\vec{r}') e^{i\vec{q}\cdot\vec{r}'} d^3\vec{r}' \right]^2,$$

(31)

where $F(\vec{q})$ is called the form factor and it is defined as

$$F(\vec{q}) = \int \rho(\vec{r}) e^{i\vec{q}\cdot\vec{r}} d^3\vec{r}.$$
For many-electrons neutral atoms, exact solutions for the atomic form factor are not obtainable, so a variety of approximations in the literature have been used. Among these approaches, an analytical approximation developed in [3] for the form factor was proposed by using the statistical method Thomas-Fermi. Thus the form factor is given by

$$F(q) = Z \sum_{i=1}^{3} \frac{A_i \alpha_i^2}{\alpha_i^2 + a_0^2 q^2}, \quad (36)$$

with $a_0 = \frac{\hbar^2}{me^2}$ being the Bohr radius and the $A_i$ and $\alpha_i$ parameters are tabulated in [13] for $Z$ values from 1 to 92. The parameters in (36) for helium and neon atoms are given in Table 1.

Table 1. Parameters of the form factor for helium and neon atoms.

| Element | $A_1$  | $A_2$  | $A_3$  | $\alpha_1$ | $\alpha_2$ | $\alpha_3$ |
|---------|--------|--------|--------|-------------|-------------|-------------|
| He      | -0.2259| 1.2259 | 0      | 5.5272      | 2.3992      | 0           |
| Ne      | 0.0188 | 0.9812 | 0      | 34.999      | 2.5662      | 0           |

For incident electrons, we set $z = -1$ and for helium and neon atoms, we have $Z = 2$ and $Z = 10$ respectively. To complete the analysis, we evaluate numerically (37) and we compare the theoretical results with experimental data [14] for differential cross section for elastic scattering of electrons by helium and neon atoms. This comparison is made explicit in Figs. 1 and 2.

Thus, in Figure 1 the elastic differential cross section is analyzed for electrons by helium. We evaluate the elastic cross section (37) at 2000 eV and 3000 eV, varying the parameter $M$ to adjust the experimental points. A best agreement between theoretical differential cross section and experimental data is attained when $M \sim 10^{-43}$ eV.

In the Fig. 2 it is shown that there are discrepancies between theoretical and experimental results for scattering of electrons off neon at small angles for the point line, while for the dashed line, the qualitative agreement is not good.

5. Conclusions
In this work we have explored the corrections to Coulomb’s law coming from massive Kaluza-Klein vector modes in a thick brane scenario. For this purpose, we included the analyses accomplished in reference for the static potential in the low-energy regime. The corresponding corrections to Coulomb’s law decays exponentially and depend on the 5D parameters of the zero massless fermion field $M$ and $b$.

To this end, we have studied numerically this problem making a comparison between the total cross section for electrons scattered by different atoms modified by incorporation of the corrections to Coulomb’s law in a thick brane scenario and the curve experimental data for each case (see Figures 1, 2). This comparison yields the bound of $M = 3 \times 10^{-45}$ eV.

Finally, we mention that in the low energy regime we focused on in this work there are other possible directions which can be pursued, so we wonder whether the same results or behavior holds for the gravitational case.
Figure 1. Comparison of theoretical and experimental elastic differential cross section for electrons off helium. We set $H = 10^{-60} M_{Pl}$ and $b = 10^{-55} M_{Pl}$.

Figure 2. Comparison of theoretical and experimental elastic differential cross section for electrons off neon. We set $H = 10^{-60} M_{Pl}$ and $b = 10^{-55} M_{Pl}$ again.

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