Revealing universal Majorana fractionalization using differential shot noise and conductance in nonequilibrium states controlled by tunneling phases

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(Dated: May 27, 2022)

Universal fractionalization of quantum transport characteristics in Majorana quantum dot devices is expected to emerge for well separated Majorana bound states. We show that the Majorana universality of the differential shot noise $\partial S^> / \partial V$ and conductance $\partial I / \partial V$ at low bias voltages $V$ arises only in ideal setups with only one Majorana mode entangled with the quantum dot. In realistic devices, where both Majorana modes are entangled with the quantum dot, $\partial S^> / \partial V$ and $\partial I / \partial V$ become very sensitive to tunneling phases and their universal fractional values are hard to observe even for well separated Majorana bound states. In contrast, as revealed here, the ratio $(\partial S^> / \partial V) / (\partial I / \partial V)$ weakly depends on the tunneling phases and is fractional when the Majorana bound states are well separated or integer when they significantly overlap. Importantly, for very large $V$ we demonstrate that this ratio becomes fully independent of the tunneling phases and its universal fractional Majorana value may be observed in state-of-the-art experiments.

I. INTRODUCTION

Reliable access to electronic degrees of freedom fractionalized by formation of essentially nonlocal non-Abelian Majorana bound states (MBSs) 1 in a topological superconductor (TS) is an appealing goal which to some extent dates back to searching particles being their own antiparticles 2. At present Majorana fractionalization is of fundamental importance for exploring universal properties of condensed matter systems 3–6, where MBSs appear in topologically nontrivial phases, and for practical implementation of topologically protected qubits involved in anyonic quantum computing 7.

One way to access MBSs is via quantum transport characteristics sensitive to fractionalizations of electronic degrees of freedom. Here average electric, thermoelectric and heat currents 8–22 provide valuable characteristics of MBSs. Also dynamics of average magnetizations 24 provides an alternative approach to study MBSs within quantum transport experiments. Qualitatively distinct and much more detailed quantum transport information about MBSs is encoded in shot and quantum noise which directly scans universal fractional character of excitations in nonequilibrium states 24–32. In particular, Majorana shot noise taking into account continuum quasiparticles in tunnel junctions 33, giant Majorana shot noise in topological trijunctions 34 and full braiding Majorana protocols obtained from weak measurements based on shot noise statistics 35 demonstrate impressively diverse aspects of Majorana physics beyond mean electric currents. Another opportunity which is fundamentally different from quantum transport is to uniquely probe fractionalization via quantum thermodynamic characteristics such as the entropy. Indeed, well separated MBSs in nanoscopic setups result in fractional plateaus 36–40 of the entropy. Ongoing elaboration of experimental techniques and successful measurements of the entropy 41–45 have established a convincing basis for a future thermodynamic access to MBSs.

Due to well established technology to measure, e.g. electric currents, quantum transport in nanoscopic setups is relatively simpler to perform than quantum thermodynamics. In particular, measurements of mean electric currents in setups presumably involving MBSs provide corresponding electric conductances 46. Unfortunately, this most straightforward way to probe MBSs is unreliable 48–50. Thus other approaches, based preferably on directly and simply measurable quantities, are required to access MBSs beyond mean electric currents.

Here we demonstrate how one can access MBSs via directly measurable observables such as differential shot noise and conductance in realistic Majorana quantum dot (QD) devices. To avoid unreliable measurements 48 in ideal setups, involving only one end of a TS, it is necessary to probe MBSs simultaneously at both ends. Fractionalization in such devices is driven by processes entangling a QD with both Majorana modes. Such an entanglement may also be engineered for implementing Majorana qubits 51–52 or even accidentally induced during a technological process used to prepare a setup. We show that under such circumstances the expected universal Majorana properties of the differential shot noise and conductance are washed out by the Majorana tunneling phases even for well separated MBSs. Thus each of these observables alone does not straightforwardly indicate that in fact one deals with MBSs. This creates a general problem because in majority of experimental setups precise values of the tunneling phases are hard to extract. Remarkably, as we demonstrate, in contrast to the differential shot noise and conductance, the ratio of these two observables weakly depends on the tunneling phases. More importantly, we find that it takes fractional values for well separated MBSs and becomes integer when the MBSs merge into a single Dirac fermion. Thus this ratio
provides a straightforward access to fractionalization in Majorana QD devices.

Furthermore, a detailed numerical analysis of the dependences of the differential shot noise and conductance on the tunneling phases and bias voltage reveals a fine structure of their resonances. Using these resonances one can extract all the Majorana tunneling parameters including the tunneling phases. The dependence on these parameters also suggests a way to shift the Majorana resonances to higher bias voltages where it will be simpler to detect them in experiments.

The paper is organized as follows. In Section II we present the physical model of a feasible experimental setup which admits measurements of quantum transport observables such as its differential shot noise and conductance. Numerical results obtained for these observables are explored in Section III. We conclude the paper with Section IV where it is shown that the Majorana universal fractionalization at high bias voltages may be accessed in experiments performed at high temperatures achievable in modern labs.

II. MAJORANA QUANTUM DOT SETUP, ITS DIFFERENTIAL SHOT NOISE AND CONDUCTANCE

To quantitatively analyze how an entanglement of a QD with both Majorana modes is revealed in fluctuations and the mean value of the electric current, flowing through the QD, we consider the setup in Fig. 1. The schematic setup in Fig. 1 is fully sufficient for our main goal, that is for the theoretical analysis presented below. However, it is important to note that an experimental realization of this theoretical model is feasible. For example, it can be implemented using an InAs nanowire with an epitaxial Al layer grown on a part of the nanowire’s surface 53, 54. Here, the Al shell is etched on one end of the nanowire to prepare a bare InAs segment where afterwards a QD is formed. Thus in this technological process the TS is located with respect to the QD in an essentially asymmetric way: one end of the TS is much closer to the QD than the other one. As a result, the first Majorana mode $\gamma_1$ is coupled to the QD stronger than the second Majorana mode $\gamma_2$, that is $|\eta_1| > |\eta_2|$ (see below for more details). The setup in Ref. 54 may be adapted to our theoretical model by forming near the QD two normal metallic contacts whose coupling strength $\Gamma$ (see below for more details) may be varied by the voltage on gates located between the contacts and QD. An alternative location of the TS with respect to the QD is used in Ref. 8 and assumes a curved shape of the TS. In such a curved setup both $|\eta_1|$ and $|\eta_2|$ are assumed to be controllable and thus one can have both situations $|\eta_1| > |\eta_2|$ and $|\eta_1| \approx |\eta_2|$. In particular, the regime $|\eta_1| > |\eta_2|$ is used to implement driven dissipative Majorana qubits 51. Below we will assume $|\eta_1| > |\eta_2|$ since this situation is more relevant technologically as well as for practical purposes. We also note that while in the setup of Ref. 54 the phases $\phi_{1,2}$ may have arbitrary fixed values induced during the technological process, in Ref. 8 the phases $\phi_{1,2}$ are assumed to be externally controlled by a magnetic flux. Such an external control of $\phi_{1,2}$ is also assumed for practical implementations of Majorana qubits 51. Our theoretical analysis below is applicable to both of these situations and may be used to interpret experiments where the phases $\phi_{1,2}$ are fixed or externally controlled.

To perform a theoretical analysis, we assume that the physical system includes a QD,

$$\hat{H}_d = \epsilon_d \hat{d}^\dagger \hat{d},$$

(1)

with one nondegenerate energy level $\epsilon_d$, whose location may be tuned by a gate voltage. Although here the QD is spinless and thus Kondo correlations are absent, in the following we assume $\epsilon_d \geq 0$ (for example, empty dot, see also Ref. 9) in order to describe a universal regime induced solely by MBs, that is excluding even for spin-degenerate QDs a possible interplay between universal Majorana fractionalizations and the universality induced by Kondo correlations 55, 61 which would have emerged in the spin-degenerate case for $\epsilon_d < 0$. In practice the spinless model in Eq. (1) is realized, for example, in strong magnetic fields used to bring the TS into its topological phase. Under such circumstances interactions in the QD do not play a role and the non-interacting spinless model in Eq. (1) is an adequate tool to analyze Majorana signatures 62. This has also been confirmed, for example in Ref. 63, using numerical renormalization group calculations showing a transition of the linear conductance from the low magnetic field plateau $3e^2/2h$ to the high magnetic field plateau $e^2/2h$ induced solely by the MBs after the magnetic field has switched the QD into the spinless regime and thus made it non-interacting via eliminating the Kondo correlations. In spin-degenerate QDs interactions play an important role 64, 65. However, numerical renormalization group calculations indi-
cater (see, for example, Ref. [69]) that even in interacting spin-degenerate QDs Majorana induced fractionalizations may decouple from Kondo correlations. Thus interacting spin-degenerate QDs coupled to MBSs may behave as their non-interacting counterparts. In particular, the low-temperature entropies in interacting and non-interacting spin-degenerate QDs coupled to MBSs are identical and may be obtained using non-interacting spin-degenerate QDs coupled to MBSs [69]. Thus even for spin-degenerate cases one may consider non-interacting QDs coupled to MBSs to explore Majorana induced fractionalization (see, for example, Ref. [69]) that even in interacting spin-degenerate QDs Majorana induced fractionalizations may decouple from Kondo correlations.

Within this formalism the second quantized fermionic operators are replaced by the Grassmann fields, $\psi(t), \phi_{k}(t)$, $\zeta(t)$, corresponding to the QD, contacts and TS, respectively. The electric current operator in the $l$-th contact is expressed through the Grassmann fields on the forward ($q = +$) or backward ($q = -$) branch of the Keldysh contour,

$$I_{lq}(t) = \frac{(i e/\hbar)}{\sum_{k}} [\mathcal{T}_{l} \bar{\psi}_{lkq}(t) \psi_{q}(t) - \text{G.c.}], \quad (6)$$

where G.c. denotes the Grassmann conjugation. The Hamiltonians in Eqs. (4–6) are replaced by the corresponding Keldysh actions of the isolated QD, contacts and TS, $S_{d}$, $S_{c}$ and $S_{ts}$, which are of conventional $2 \times 2$ matrix form in the retarded-advanced space. The Hamiltonians in Eq. (6) are replaced by the following actions:

$$S_{d+e} = - \int_{-\infty}^{\infty} dt \sum_{l=L,R} \sum_{k} \{ \mathcal{T}_{l} \bar{\phi}_{lk+}(t) \psi_{-}(t) - \mathcal{T}_{l} \bar{\phi}_{lk-}(t) \psi_{+}(t) \} - \tilde{\phi}_{lk-}(t) \psi_{-}(t) + \text{G.c.}, \quad (7)$$

$$S_{d+ts} = - \int_{-\infty}^{\infty} dt \{ \eta_{1}^{*} \tilde{\psi}_{+}(t) \zeta_{+}(t) + \tilde{\psi}_{-}(t) \tilde{\zeta}_{+}(t) \} - \tilde{\psi}_{-}(t) \zeta_{-}(t) - \tilde{\psi}_{-}(t) \tilde{\zeta}_{-}(t) + \text{G.c.}, \quad (8)$$

To generate the mean electric current and various current-current correlators one also adds the current action,

$$S_{l}[J_{lq}(t)] = - \int_{-\infty}^{\infty} dt \sum_{l=L,R} \sum_{q=+,-} J_{lq}(t) I_{lq}(t), \quad (8)$$

with the source fields $J_{lq}(t)$. The full Keldysh action, $S_{K}[J_{lq}(t)] = S_{d} + S_{c} + S_{ts} + S_{d+e} + S_{d+ts} + S_{l}[J_{lq}(t)]$, determines the Keldysh generating functional,

$$Z[J_{lq}(t)] = \int \mathcal{D}[\tilde{\psi}_{q}, \psi_{q}, \tilde{\phi}_{lkq}, \phi_{lkq}, \tilde{\zeta}_{q}, \zeta_{q}] e^{\int_{-\infty}^{\infty} S_{K}[J_{lq}(t)]}, \quad (9)$$

from which one, e.g., generates

$$\langle I_{lq}(t) \rangle_{S_{K}} = i \hbar \frac{\delta Z[J_{lq}(t)]}{\delta J_{lq}(t)} \bigg|_{J_{lq}(t)=0}, \quad (10)$$

$$\langle I_{lq}(t) I_{l'q'}(t') \rangle_{S_{K}} = (i \hbar)^{2} \frac{\delta^{2} Z[J_{lq}(t)]}{\delta J_{lq}(t) \delta J_{l'q'}(t')} \bigg|_{J_{lq}(t)=0}, \quad (11)$$

where $\langle \ldots \rangle_{S_{K}}$ means averaging at $J_{lq}(t) = 0$ that is with respect to the action $S_{K} = S_{K}[J_{lq}(t) = 0]$.

In the following we will focus on the left contact, $l = L$, and explore the mean electric current $I(V, \Delta \phi) = \langle I_{lq}(t) \rangle_{S_{K}}$ and the greater current-current correlator $S^{2}(t, t'; V, \Delta \phi) = \langle \delta I_{L-}(t) \delta I_{L+}(t') \rangle_{S_{K}}, \delta I_{Lq}(t) = \frac{(i e/\hbar)}{\sum_{k}} [\mathcal{T}_{L} \bar{\psi}_{lkq}(t) \psi_{q}(t) - \text{G.c.}]$.
\[ I_{L0}(t) - I(V, \Delta \phi). \] The Fourier transform of the correlator \( S^>(t, t'; V, \Delta \phi) = S^>(t - t'; V, \Delta \phi), \]

\[
S^>(\omega; V, \Delta \phi) = \int_{-\infty}^{\infty} dt \, e^{i\omega t} S^>(t; V, \Delta \phi),
\]

is an important quantity because at \( \omega = 0 \) it specifies the shot noise \( S^>(V, \Delta \phi) \equiv S^>(\omega = 0; V, \Delta \phi). \)

In experiments one directly measures the differential shot noise and conductance, \( \partial S^>/\partial V, \partial I(V, \Delta \phi)/\partial V \), and below we focus on these partial derivatives. We obtain \( S^>(V, \Delta \phi) \) and \( I(V, \Delta \phi) \) via numerical integrations and then perform numerical differentiations. The regime where \( |\eta| \) prevails, \( |\eta| > \max\{|\epsilon_d|, |eV|, k_B T, \Gamma, |\eta_2|, \xi\} \), is of particular interest for observing Majorana universality in experiments and below we explore it explicitly. As discussed above, the physical setup assumes that \( \Gamma \) may be enhanced or suppressed by the voltage on gates located between the QD and contacts. Our motivation for suppressing \( \Gamma \) below \( |\eta| \) is that for \( \Gamma < |\eta| \) the properties of the QD will be dominated by Majorana fractional character in a wide energy range. Physically this is expected because the tunneling of the MBSs into the QD is much stronger than the tunneling from the contacts whose non-fractional degrees of freedom could significantly wash out various Majorana fractionalizations in the opposite regime, that is for \( \Gamma > |\eta| \). We also note that to get the numerical results presented below we specify all the energies in units of \( \Gamma. \) In particular, the bias voltage \( |eV| \) is varied in a wide range, from small to large values in units of \( \Gamma. \) However, since \( \Gamma \) is made very small using the gates mentioned above, \( |eV| \) will also be small even if it takes large values in units of \( \Gamma. \) This means that in our numerical results the bias voltage always remains small enough so that other levels of the QD are not relevant and our theoretical model provides a reasonable description of experiments.

### III. NUMERICAL RESULTS

In the upper panel of Fig. 2 we show \( \partial S^>/\partial V \) and \( \partial I/\partial V \) as functions of \( \Delta \phi \) for \(|eV|/\Gamma \ll 1.\) Both quantities exhibit resonances around \( \Delta \phi_{\text{res}} = 0, \pi, 2\pi \) where they reach their universal fractional values, \( \partial S^>/\partial V = e^3/4h \) and \( \partial I/\partial V = e^2/2h. \) However, due to strong dependences of \( \partial S^>/\partial V \) and \( \partial I/\partial V \) on \( \Delta \phi \) they quickly deviate from these fractional values which, thus, can be observed only in very narrow ranges of \( \Delta \phi. \) The left inset shows the fine structures of the resonances of \( \partial S^>/\partial V \) and \( \partial I/\partial V \) in the vicinity of \( \Delta \phi = \pi. \) As can be seen, these resonances are split into two peaks. We find numerically that the distances \( \Delta \phi_{1, \text{max}} \) and \( \Delta \phi_{2, \text{max}} \) between the peaks of \( \partial I/\partial V \) and \( \partial S^>/\partial V, \) respectively, depend on both \( |\eta_2| \) and \( |eV|, \)

\[
\Delta \phi_{1, \text{max}} = \frac{1}{2} |eV| |\eta_2|, \quad \Delta \phi_{2, \text{max}} \sim \sqrt{\frac{\Gamma |eV|}{|\eta_2|}}.
\]

These expressions are valid only when \(|eV| \gg k_B T. \) For \(|eV| \ll k_B T \) the resonance of the linear conductance also splits into two resonances with the distance between them depending on \( |\epsilon_d| \) as also confirmed by the entropy analysis [35]. In particular, for \( \epsilon_d = 0 \) it is equal to \( \pi \) [3]. The right inset shows \( \partial S^>/\partial V \) and \( \partial I/\partial V \) for \( \xi/\Gamma \approx 1 \) when the MBSs strongly overlap and form a single Dirac fermion. Specifically, in the inset \( \xi/\Gamma = 10^2. \) The two physical quantities, \( \partial S^>/\partial V \) and \( \partial I/\partial V, \) are strongly suppressed and coincide with each other leading to a single curve in the inset. Moreover, as the inset explicitly demonstrates, the strong overlap of the MBSs doubles the period of \( \partial S^>/\partial V \) and \( \partial I/\partial V \) from \( \pi \) to \( 2\pi. \) The lower panel shows the ratio \( (\partial S^>/\partial V)/(\partial I/\partial V) \) corresponding to the black and red curves in the upper panel. In contrast to \( \partial S^>/\partial V \) and \( \partial I/\partial V \) their ratio is almost independent of \( \Delta \phi. \) Indeed, it has wide plateaus, on which it is equal to \( 3e/2, \) and narrow antiresonances, corresponding to the resonances in the upper panel. The minimum of these antiresonances is also fractional, equal to \( e/2, \) while their width is determined by both the bias voltage \( |eV| \) and tunneling amplitude \( |\eta_2| \). The left inset shows the ratio \( (\partial S^>/\partial V)/(\partial I/\partial V) \) as a function of \( \xi \) for the case \(|eV|/\Gamma = 10^{-3} \) and \( \Delta \phi = 0 \) (dashed curve), \( \Delta \phi = \pi/4 \) (solid curve). We see that both fractional values, \( 3e/2 \) and \( e/2, \) are observed for well separated MBSs, when \( \xi/\Gamma \ll 1. \) For strongly overlapping MBSs, when \( \xi/\Gamma \gg 1, \) the ratio takes the integer Dirac value, \( (\partial S^>/\partial V)/(\partial I/\partial V) = e, \) and becomes independent of \( \Delta \phi. \) The right inset shows for the case \(|eV|/\Gamma = 10^{-2} \) the universality of the two nontrivial Majorana fractional values, \( 3e/2 \) (solid curve, \( \Delta \phi = \pi/2 \)) and \( e/2 \) (dashed
curve, $\Delta \phi = 0$), that is their independence of the gate voltage controlling the value of $\epsilon_d$.

The differential shot noise and conductance for large bias voltages are shown as functions of $\Delta \phi$ in the upper panel of Fig. 3. As in Fig. 2, the curves exhibit resonances with $\partial S^\gamma / \partial V = e^2/4h$ and $\partial I / \partial V = e^2/2h$. However, now $\Delta \phi_{\text{res}} \neq 0, \pi, 2\pi$. The new values of $\Delta \phi_{\text{res}}$ are determined by $|\eta|/|\eta_2|$ and $\|\eta_2\|$ in correspondence with $|\eta|$. Here, the value of $|\eta_2|$ is increased in comparison with Fig. 2 in order to clearly show that for two different values of the tunneling phase difference $\Delta \phi$ and $\partial I / \partial V$ are determined by $\Delta \phi_{\text{res},1}$ (and not by $\Delta \phi_{\text{res},2}$) in Eq. (13). The left inset shows that at large bias voltages there happens a qualitative change in the structure of the resonances: it is split into two peaks. From our numerical calculations we find that the distance between these two peaks depends on $|\eta_2|$ and almost independent of $|\epsilon|$.

$$\Delta \phi_{\text{res},2} \sim \frac{\Gamma}{|\eta_2|}. \quad (14)$$

The right inset shows $\partial S^\gamma / \partial V$ (solid curve) and $\partial I / \partial V$ (dashed curve) as functions of $\Delta \phi$ for very large bias voltage, $|\epsilon|/\Gamma = 10^2$. As can be seen, at very large bias voltages both the differential shot noise and conductance are strongly suppressed, $\partial S^\gamma / \partial V \ll e^2/4h$, $\partial I / \partial V \ll e^2/2h$, and vary relatively weakly as functions of $\Delta \phi$. The curves in the lower panel demonstrate the ratio of $\partial S^\gamma / \partial V$ / $\partial I / \partial V$ corresponding to the two bias voltages in the upper panel. As in Fig. 2 whereas $\partial S^\gamma / \partial V$ and $\partial I / \partial V$ change relatively strongly (several orders of magnitude), their ratio exhibits weak dependence on $\Delta \phi$ characterized by a narrow resonance and at the minima of the antiresonances the ratio is fractional, equal to $3e/2$ and $e/2$, respectively. The left inset shows the ratio $\partial S^\gamma / \partial V$ / $\partial I / \partial V$ obtained from the curves in the right inset of the upper panel. We see that although at very large bias voltages ($|\epsilon|/\Gamma = 10^2$) both $\partial S^\gamma / \partial V$ and $\partial I / \partial V$ are strongly suppressed, their ratio takes the Majorana fractional value $3e/2$ in the whole range of $\Delta \phi$. The right inset demonstrates the universality (independence of $\epsilon_d$) of the Majorana fractional value $3e/2$ at very large bias voltages ($|\epsilon|/\Gamma = 10^2$). The curve in this inset is also independent of $\Delta \phi$. As expected, the Majorana universality takes place only at $|\epsilon_d| \ll |\eta_1|$. The upper panel of Fig. 4 shows $\partial S^\gamma / \partial V$ and $\partial I / \partial V$ as functions of $V$ for two different values of $\Delta \phi$. Both $\partial S^\gamma / \partial V$ and $\partial I / \partial V$ exhibit two resonances at $|\epsilon|_{\text{res},1}$ and $|\epsilon|_{\text{res},2}$, where they reach their universal fractional values, $\partial S^\gamma / \partial V = e^3/4h$ and $\partial I / \partial V = e^2/2h$. We find numerically that $|\epsilon|_{\text{res},1}$ depends only on $|\eta_1|$ whereas $|\epsilon|_{\text{res},2}$ depends on both $|\eta_2|$ and $\Delta \phi$.

$$|\epsilon|_{\text{res},1} = 4|\eta_1|, \quad |\epsilon|_{\text{res},2} = 4|\eta_2| \sin(\Delta \phi). \quad (15)$$

Away from these resonances both $\partial S^\gamma / \partial V$ and $\partial I / \partial V$ quickly deviate from their universal fractional values. The left and middle insets show the detailed profiles of the resonances at $|\epsilon|_{\text{res},2}$ for $\Delta \phi = \pi/4$ and $\Delta \phi = \pi/2$. **FIG. 3.** Upper panel: Differential shot noise $\partial S^\gamma / \partial V$ (solid lines) and differential conductance $\partial I / \partial V$ (dashed lines) as functions of the tunneling phase difference $\Delta \phi$ for large bias voltages, $|\epsilon|/\Gamma = 10$. The black curves correspond to the case $|\epsilon|/\Gamma = 10$ while the red curves correspond to the case $|\epsilon|/\Gamma = 20$. The other parameters are the same for the two cases: $\epsilon_d/\Gamma = 10$, $\eta_1/\Gamma = 10^3$, $|\eta_2|/\Gamma = 10$, $\xi/\Gamma = 10^{-4}$. Lower panel: Black and red curves show the ratio $\partial S^\gamma / \partial V$ / $\partial I / \partial V$ obtained from the corresponding curves in the upper panel.

**FIG. 4.** Upper panel: Differential shot noise $\partial S^\gamma / \partial V$ (solid lines) and differential conductance $\partial I / \partial V$ (dashed lines) as functions of the bias voltage $|\epsilon|$ for two values of the tunneling phase difference $\Delta \phi$. The black curves correspond to the case $\Delta \phi = \pi/2$ while the red curves correspond to the case $\Delta \phi = \pi/4$. The other parameters are the same for the two cases: $\epsilon_d/\Gamma = 10$, $\eta_1/\Gamma = 10^3$, $|\eta_2|/\Gamma = 10$, $\xi/\Gamma = 10^{-4}$. Lower panel: Black and red curves show the ratio $\partial S^\gamma / \partial V$ / $\partial I / \partial V$ obtained from the corresponding curves in the upper panel.
respectively, while the right inset shows the detailed profile of the resonances at $|eV_{\text{res},1}|$. These three insets all demonstrate that the resonances of $\partial I/\partial V$ have a simple structure of a single peak while the resonances of $\partial S^+/\partial V$ have a fine structure composed of two peaks. From numerical calculations we find that the distance between these two peaks is approximately equal to $\Gamma$ and does not depend on the other parameters. In the lower panel we show the ratio $(\partial S^+ / \partial V)/(\partial I / \partial V)$ corresponding to the two curves in the upper panel. Note, while the variations of the $\partial S^+ / \partial V$ and $\partial I / \partial V$ reach several orders of magnitude, their ratio changes relatively weakly and shows a number of characteristic features. Specifically, in the Majorana regime, that is for $|eV| < |\eta_1|$, this ratio is characterized by plateaus separated by an antiresonance located at $|eV_{\text{res},2}|$. At these plateaus the ratio is equal to $3e/2$ and at the minimum of the antiresonance it is equal to $e/2$. Also at $|eV_{\text{res},1}|$ there exists an antiresonance with the same minimum $e/2$. For very large bias voltages, $|eV| \gg |\eta_1|$, when the Majorana tunneling is ineffective, the two Majorana modes behave as a single Dirac fermion and there forms a trivial plateau with the Dirac value $(\partial S^+ / \partial V)/(\partial I / \partial V) = e$.

**IV. CONCLUSION**

We have demonstrated how to reveal Majorana fractionalization using quantum transport characteristics such as differential shot noise and conductance in Majorana QD setups where both Majorana modes are entangled with a strongly nonequilibrium QD. Our numerical results show that the tunneling phases wash out the Majorana universal values of each of these two observables. At the same time it has been found that the ratio of the differential shot noise and conductance weakly depends on the tunneling phases and provides a reliable access to Majorana fractionalization in realistic strongly nonequilibrium setups. In particular, it has been explicitly demonstrated that this ratio takes fractional values when MBSs in a TS are well separated whereas the ratio becomes integer for strongly overlapping MBSs merging into a single Dirac fermion. Additionally, we have demonstrated the universality of the Majorana fractionalization that is its independence not only of the tunneling phase difference $\Delta \phi$ but also its independence of the QD gate voltage $\epsilon_d$ and bias voltage $V$. Regarding the behavior of the differential shot noise and conductance we have found that each of these observables has a resonant structure in its dependence on both the tunneling phase difference and bias voltage. Fine structures of these resonances have been explicitly shown and explored.

Let us in conclusion probe the Majorana universality at high temperatures achievable in state-of-the-art experiments. The insets in the lower panel of Fig. 4 illustrate results obtained for $k_B T / \Gamma = 10$. At both of the insets $\Delta \phi = \pi/512$ and the other parameters remain unchanged. The two curves in the right inset show $\partial S^+ / \partial V$ (solid) and $\partial I / \partial V$ (dashed). The curve in the left inset shows the ratio $(\partial S^+ / \partial V)/(\partial I / \partial V)$ obtained from the two curves in the right inset. As one can see in the left inset, high temperatures destroy the low-energy part of the universal fractional Majorana plateau. However, for bias voltages in the range $k_B T \ll |eV| < |\eta_1|$ the high-energy part of this plateau is still present and provides an experimental access to the universal fractional Majorana ratio $3e/2$.

Finally, we note that our results enable to extract all the Majorana tunneling parameters. Indeed, according to Eq. (13), one obtains $|\eta_2|$ from measurements of $\Delta \phi_{\text{th}}$ since the bias voltage $V$ is known in experiments. Measuring then $|eV_{\text{res},1}|$ and $|eV_{\text{res},2}|$ one gets $|\eta_1|$ and $\Delta \phi$ using Eq. (15). Besides, the expression for $\partial S^+ / \partial V$ in Eq. (15) suggests that one may increase $|\eta_2|$ as necessary to shift the corresponding Majorana resonances of $\partial S^+ / \partial V$ and $\partial I / \partial V$ to higher bias voltages where it is easier to measure them at higher temperatures.

**ACKNOWLEDGMENTS**

The author thanks Reinhold Egger, Andreas K. Hütten and Wataru Izumida for useful discussions.

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