Seismic displacement of geosynthetic-reinforced slopes subject to cracks

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Abstract. The kinematical approach of limit analysis associated with pseudo static assumption is employed to evaluate the displacement of geosynthetically reinforced soil slopes subject to cracks. According to existing literature, the seismic displacements for soil slopes have been calculated with the effect of possible cracking being neglected, such cracking is likely to emerge due to an earthquake with even moderately large motion. In this paper, a new technique is proposed to estimate the horizontal displacement of the slope toe for geosynthetically reinforced slopes resulting from a given earthquake postulating a rough estimation of real time crack propagation. The effect of crack formation as part of the failure process during the earthquake on the horizontal displacement of the slope toe is specifically tackled. The seismic displacement is estimated by incorporating a stepwise yield acceleration corresponding to postulated crack propagation. Rotational failure mechanisms accounting for either intact reinforced slopes that can show cracks or reinforced slopes with pre-existing cracks are considered. Two types of reinforcement layouts are employed here; uniformly distributed reinforcement along the slope height and linearly increasing distribution (i.e. the spacing between layers decreases linearly with depth). An example illustrating the procedure for a given earthquake is presented. Results show that the horizontal displacement of the slope toe calculated using the stepwise yield acceleration for both uniform distribution of reinforcement and for linearly increasing distribution can provide a reasonable estimation of the slope displacement. Furthermore, in terms of the slope displacement, linearly increasing distribution yields better results than the uniform layout.

1. Introduction
Cracks can cause significant reduction in the stability of unreinforced soil slopes [1, 2], especially if these slopes are subjected to seismic action [3]. Although, geosynthetics have been used successfully and effectively as soil reinforcement material for the last thirty years [4, 5], pre-existing cracks can be detrimental for geosynthetically reinforced slopes [6]. The presence of a vertical crack can reduce the safety factor of the slope depending mainly on its location and depth. Cracks form not only potential parts of the slip surface, but also they form easy flow channels for rainfall water which reduces the soil strength and exerts a lateral stress inducing the failure when these cracks are filled with water. Cracks can be found in soil slopes and embankments due to tensile stresses such as seismic action or external static loading, and/or due to desiccation and cycles of wetting and drying.

Methods for assessing the seismic stability of slopes have been developed during the last century. The Mononobe-Okabe method might be the first published work that addresses the stability of retaining walls and dams during earthquake incorporating dynamic earth pressure [7, 8]. Then, several
limit equilibrium methods were developed for this purpose [9-11] which are still the most commonly used by practitioners. More recently, numerical methods for continuum mechanics such as the finite element method with strength reduction technique [12, 13] and finite element limit analysis [14] have provided the capability to reliably detect the onset of failure in slopes according to the approach of continuum mechanics. However, if cracks are present, a continuum approach no longer works since the onset of instability is ruled by the behaviour of single fractures. In this case, the Discrete Element Method can nowadays be employed for 3D analyses of slopes with cracks [15]. Recent algorithmic advances in terms of contact detection algorithms [16, 17] have substantially reduced the runtime of these analyses. However, when little information on the presence of cracks is known, extensive parametric analyses requiring large computational times are necessary. In this case, an analytical approach is very desirable so that numerical analyses would be run only for the case(s) identified by the analytical approach as the most critical.

Newmark’s method [18] is an analytical method which is very popular among practitioners where a pseudo-static force is used instead of the dynamic excitation to calculate earthquake induced displacements. Analytical solution for earthquake induced displacements undergone by slopes subject to a rotational failure mechanism is presented by [19]. One of the main limitations of Newmark’s method is the neglect of the earthquake induced strength degradation of the soil, i.e. it assumes a constant yield acceleration throughout the analysis [20]. In this paper, earthquake induced crack occurrence and the consequent reduction of yield acceleration will be accounted for geo-reinforced slopes. Seismic induced displacements will be calculated based on a stepwise time varying yield acceleration.

2. Formulation of the Problem
The kinematical approach of limit analysis will be used to calculate the least upper bound on the yield (critical) acceleration $K_y$ for uniform $c$-$\phi$ geo-reinforced slopes. Two different distributions of reinforcement are considered, namely uniform distribution (UD) and linearly increasing distribution with depth (LID), both illustrated in figure 1. Let us introduce the dimensionless variable $K_r$ as the average tensile strength of reinforcement per unit height of the slope. Where $K_r$ is a function of the tensile strength of the reinforcement layer per unit width, $T$, and of the spacing between reinforcement layers, $S$, and can be written as:

$$K_r = \frac{T}{S} \quad (1)$$

the two cases of reinforcement distributions have been analysed by assigning the corresponding value of $K_r$ for uniform distribution (UD):

$$K_r = K_i \quad (2)$$

and for linearly increasing distribution (LID):

$$K = 2K_i \exp\left[\tan(\theta - \chi)\right] \sin \theta - \sin \chi \quad (3)$$

$$\exp\left[\tan(\nu - \chi)\right] \sin \nu - \sin \chi$$

where $\theta$, $\chi$, and $\nu$ are the angles made by $r$ (or $r_x$), $r_y$ and $r_z$ respectively with a reference axis, (see figure 2), $r$ is the distance between point P and any point on the log-spiral slip surface, $r_x$ is the distance between point P and any point on the crack, $r_y$ and $r_z$ are the lengths of the chords P-F and P-D respectively, and $\phi$ is the angle of internal friction of the soil.
The yield acceleration can be defined as the minimum level of horizontal acceleration (vertical acceleration being proportional to the horizontal acceleration) that brings the slope to failure (i.e. safety factor = 1). According to Newmark’s method [18], slope displacements start to occur whenever the seismic induced acceleration exceeds the yield acceleration. Then, the displacements occurring during the earthquake can be obtained by double integrating the differences between the applied accelerations and the yield one during the time intervals when the ground velocity is larger than zero.

In the following, an initially intact slope subject to the formation of tension cracks as result of the earthquake is considered. In this case, the cracks are formed as part of the failure process the first time the slope yield acceleration is exceeded. Then, in order to calculate the slope displacements generated by the earthquake, a new yield acceleration, accounting for the presence of the cracks formed the first time the yield acceleration of the intact slope was exceeded, needs to be calculated for all the subsequent steps. Four cases are considered in this paper:
a) Slopes made of cohesive soils of unlimited tensile strength, hence not subject to tension cracks.
b) Slopes made of cohesive soils of limited tensile strength.
c) Slopes made of cohesive soils of zero tensile strength.
d) Slopes subject to the most unfavourable crack from a stability point of view pre-existing the onset of the earthquake.

The procedure for calculating the stepwise time varying yield acceleration is outlined as follows:

1. Determine the yield acceleration for an initially intact slope subject to the formation of tension cracks $K_{\text{y}c1}$. Vertical tension cracks are formed as part of the occurring failure mechanism [21] since energy is needed to form any crack. Therefore, the yield acceleration of a slope subject to the formation of tension cracks is lower than the yield acceleration of a slope of unlimited tensile strength $K_{\text{y}c1}^{\text{int}}$, i.e. not subject to tension crack formation: $K_{\text{y}c1} \leq K_{\text{y}c1}^{\text{int}}$. This acceleration is used to calculate the displacements, the first time that the seismic acceleration exceeds the yield acceleration

2. Determine the yield acceleration for the same slope, but accounting for the presence of the crack generated in step 1, treated now as a pre-existing crack, i.e. the crack is already present so that no energy is dissipated for crack formation). This new value of yield acceleration, $K_{\text{y}c2}^{\text{int}}$, is used to calculate the displacements in all subsequent steps.

3. Determine the accumulated wedge displacement $D_i$ with respect to the ground surface, at each time step (i) when the seismic acceleration exceeds $K_{\text{y}c2}^{\text{int}}$.

4. Calculate the dimensionless coefficient $C$ that relates the displacement of the slope toe to the integral of the earthquake acceleration record above the level of yield acceleration.

5. Determine the accumulated horizontal displacement at the slope toe $D_{\text{h}}$, where $D_{\text{h}} = C \times D_i$; and then the total horizontal displacement $D_t$ is to be found.

It should be noted that, although several tension cracks at different locations in the slope may form during the earthquake, only the crack which has the worst detrimental effect on slope stability needs to be considered in the calculation. This is because, according to the kinematic approach of limit analysis, the failure mechanism taking place is the most critical mechanism for the stability of the slope among all the kinematically feasible mechanisms.

3. Calculation of Yield Acceleration

The upper bound theorem of limit analysis will be here employed to calculate the yield acceleration for both intact and cracked slopes. The analytical expressions for the calculation of the external work done by soil masses sliding along composite log-spiral failure surfaces, which requires the use of fictitious wedges bordered by a log-spiral, was first presented in [22, 23] for the case of slopes with horizontal upper part subject to a sequence of landslides, and in [24] for the more general case of slopes with an inclined upper part. Note that these calculations apply to slopes made of bonded granulates [25, 26] as well. In [27], the calculation of the work done by a wedge enclosed by two log-spirals was first presented. The analytical solution is derived here for the case of a horizontal upper slope surface and vertical pre-existing cracks from the upper slope (see figure 1). However, the solution can be straightforwardly extended to the case of a non-horizontal upper slope and the case of cracks departing from the slope face. Such an extension is reported in [2] for the static case.

Let us consider the failing wedge E-D-C-B which is about to rotate rigidly around the centre of rotation P, as yet undefined, with the ground lying on the right of the log-spiral piece D-C and of the vertical crack C-B remaining at rest. The equation of log-spiral D-C is:

$$ r = r_2 \exp \left[ \tan \phi (\nu - \chi) \right] $$

(4).
The upper bound on the yield acceleration $K_y$ will be derived imposing energy balance for the failing wedge E-D-C-B:

$$
\dot{D} = \dot{W}
$$

(5)

where $\dot{D}$ and $\dot{W}$ are the rate of dissipated energy and of external work respectively. In this paper $\dot{D}$ has four terms as follow:

$$
\dot{D} = \dot{D}_{C-D} + \dot{D}_{B-C} + \dot{D}_{r_{C-D}} + \dot{D}_{r_{B-C}}
$$

(6)

where $\dot{D}_{C-D}$ and $\dot{D}_{B-C}$ are the rates of dissipated energy within the soil along the log-spiral segment (D-C) and along the crack (B-C) respectively. While $\dot{D}_{r_{C-D}}$ and $\dot{D}_{r_{B-C}}$ are the rates of dissipated energy within the geosynthetic reinforcement along the log-spiral segment (D-C) and the crack (B-C), respectively. The energy dissipated within the soil, $\dot{D}_{C-D}$, along the log-spiral segment (D-C) can be written as:

$$
\dot{D}_{C-D} = c\theta r_f^2 (\chi, \nu, \zeta, \phi)
$$

(7)

$$
\dot{D}_{B-C} = c\theta r_f^2 \exp[2\tan(\zeta - \chi)]\frac{\exp[2\tan(\nu - \zeta)] - 1}{2\tan}\,(8)
$$

the dissipated energy within the soil $\dot{D}_{B-C}$ along the crack, as it forms as part of the failure [21], can be written as:

$$
\dot{D}_{B-C} = \theta r_f^2 \left[ \frac{\sin \chi}{\tan \theta} \right] \left[ \frac{f_c}{2} \int \frac{1 - \sin \theta}{\cos^2 \theta} d\theta + \frac{f_t}{1 - \sin \phi} \int \frac{\sin \theta - \sin \phi}{\cos^2 \theta} d\theta \right]
$$

(9)

with $\theta$ is the angle made by the segment P-B with the horizontal (see figure 2), $f_c$ and $f_t$ are the unconfined compressive and tensile strength of the geo-material, respectively. According to the Mohr-Coulomb failure criteria, they can be expressed as:

$$
f_c = 2c \frac{\cos \phi}{1 - \sin \phi}
$$

(10)

$$
f_t = 2\alpha c \frac{\cos \phi}{1 + \sin \phi}
$$

(11)

where $\alpha$ is a dimensionless coefficient introduced here to express the amount of tensile strength, with $0 \leq \alpha \leq 1$. Now substituting equations (10 and 11) into equation (9), the following expression is obtained:

$$
\dot{D}_{B-C} = c\theta r_f^2 f_c (\chi, \nu, \zeta, \phi, \alpha)
$$

(12)

$$
\dot{D}_{B-C} = c\theta r_f^2 \left[ \frac{\sin \chi}{\tan \theta} \right] \left[ \frac{\cos \phi}{1 - \sin \phi} \int \frac{1 - \sin \theta}{\cos^2 \theta} d\theta + \frac{2\alpha \cos \phi}{1 - \sin^2 \phi} \int \frac{\sin \theta - \sin \phi}{\cos^2 \theta} d\theta \right]
$$

(13).
The third term of the dissipated energy is the one that occurs within the geosynthetic reinforcement along the log-spiral part, $D_{(C-D)}$. This can be calculated by integrating the product of the infinitesimal increment of strain rate undergone by the reinforcement and the tensile strength of the reinforcement $T$ averaged over the spacing $S$ between consecutive layers of reinforcement [28]. For the sake of space, the calculations here are only for uniform distribution (UD) of reinforcement (i.e. $K = K_r$):

$$dr = \int_0^{\sin \eta} K \sin \eta \dot{\epsilon}_x \, dx = K \sin \eta \dot{u} \cos(\eta - \phi)$$  \hspace{1cm} (14)

with $\dot{\epsilon}_x$ : strain rate in the direction of reinforcement, $t$ : thickness of the discontinuity layer, and $\eta$ : angle made by the reinforcement layer with discontinuity surface, which can be written as:

$$\eta = \frac{\pi}{2} - \theta + \phi$$  \hspace{1cm} (15)

now, by integrating equation (14) over the log-spiral part (C-D),

$$\dot{D}_{(C-D)} = \int_{(C-D)} K \sin \eta \dot{u} \cos(\eta - \phi) \frac{r \, d\theta}{\cos \phi}$$  \hspace{1cm} (16)

after substituting and simplifying, the following expression is obtained:

$$\dot{D}_{(C-D)} = r^2 \dot{\theta} \int_{\zeta}^\theta K \left( \exp[2 \tan \phi(\theta - \chi)] \sin \theta \cos \theta + \sin^2 \theta \tan \phi \exp[2 \tan \phi(\theta - \chi)] \right) d\theta$$  \hspace{1cm} (17)

for uniform distribution of reinforcement (UD), $K = K_r$, then:

$$\dot{D}_{(C-D)} = K_r r^2 \dot{\theta} g_2(\chi, \nu, \zeta, \phi)$$  \hspace{1cm} (18)

with $g_2 = \frac{1}{2} \left[ \exp[2 \tan \phi(\nu - \chi)] \sin^2 \nu - \exp[2 \tan \phi(\zeta - \chi)] \sin^2 \zeta \right]$  \hspace{1cm} (19)

the dissipated energy by the reinforcement along the crack B-C has been reported by [6]. An analytical formula similar to that one presented by [28] to calculate the energy dissipated along the log-spiral part C-D is employed. Note that here the angle made by the velocity vector of the ground mass slipping away and the crack, $\phi$, is different from the soil friction angle, $\phi$, (see figure 2). Now using equation (14), but with vertical crack (i.e. $\eta = \frac{\pi}{2}$) the following expression can be obtained:

$$\dot{D}_{(B-C)} = \int_{(B-C)} K \dot{u} \sin \phi \, d\delta$$  \hspace{1cm} (20)

with $\dot{u} = r \dot{\theta} = \left( \frac{r \cos \zeta}{\cos \theta} \right) \dot{\theta}$  \hspace{1cm} (21)

$$d\delta = \frac{r \, d\theta}{\cos \theta}$$  \hspace{1cm} (22)
with \( \dot{u}_i \) the velocity vector along the crack B-C, \( r_i \) the distance between point P and any point along the crack B-C, then,

\[
\dot{D}_r(B-C) = \theta \int_{\mu} \frac{K r_i \sin \theta}{\cos \theta} \, d\theta \tag{23}
\]

with \( \mu \) is the angle made by the line P-B and a horizontal reference.

\[
\dot{D}_r(B-C) = r_h^2 \theta \exp\left[2 \tan \phi (\zeta - \chi)\right] \cos^2 \zeta \int_{\mu} \frac{K \sin \theta}{\cos \theta} \, d\theta \tag{24}
\]

for uniform distribution of reinforcement (UD), \( K = K_r \), then, integration leads to

\[
\dot{D}_r(B-C) = K_r r_h^2 \theta g_3(\chi, \psi, \zeta, \phi) \tag{25}
\]

The rate of external work for the sliding wedge E-B-C-D, (i.e \( \dot{W} \)), is calculated as the work of block E-D-F minus the work of block B-C-F [29]. The work of block E-D-F will be calculated by algebraic summation of the work of blocks P-D-F, P-E-F and P-D-E called here \( \dot{W}_1 \), \( \dot{W}_2 \) and \( \dot{W}_3 \) respectively. The work of block B-C-F will be calculated by algebraic summation of the work of blocks P-C-F, P-B-F and P-C-B called here \( \dot{W}_4 \), \( \dot{W}_5 \) and \( \dot{W}_6 \) respectively. So, \( \dot{W} \) can be calculated from the following summation:

\[
\dot{W} = \dot{W}_1 - \dot{W}_2 - \dot{W}_3 - (\dot{W}_4 - \dot{W}_5 - \dot{W}_6) = \dot{W}_1 - \dot{W}_2 - \dot{W}_3 + \dot{W}_4 + \dot{W}_5 - \dot{W}_6 \tag{26}
\]

The expressions for \( \dot{W}_i \) are derived for each block by calculation of the vectorial product of the displacement rate, \( u_i \), of the block (see figure 2) times its weight force. Here instead, in addition to the weight force, a horizontal pseudo-static force, \( h = mK_s g = \gamma K_s A \), with \( g \) being the gravitational acceleration and \( m \) the mass of the wedge, and a vertical one, \( F_v = mK_v g = \gamma K_v A \), are added to account for seismic action. For the sake of space, only the final expressions are reported here:

\[
\dot{W}_i = \dot{\theta} r_h^3 \left[ (1 + K_s) f_{iv}(\chi, \psi, \phi) + K_h f_{ih}(\chi, \psi, \phi) \right] \tag{27}
\]

\[
= \dot{\theta} r_h^3 \left[ (1 + K_v) \exp\left[3 \tan \phi (\psi - \chi)\right] \left[ \frac{3 \tan \phi \cos \psi + \sin \psi - 3 \tan \phi \cos \chi - \sin \chi}{3 (1 + 9 \tan \phi^2)} \right] \right] + K_h \left[ \exp\left[3 \tan \phi (\psi - \chi)\right] \left[ \frac{3 \tan \phi \sin \psi - \cos \psi - 3 \tan \phi \sin \chi + \cos \chi}{3 (1 + 9 \tan \phi^2)} \right] \right]
\]

with \( \dot{\theta} \) being the rate of angular displacement of the failing wedge E-B-C-D. For block P-E-F instead:

\[
\dot{W}_2 = \dot{\theta} r_h^3 \left[ (1 + K_v) f_{iv}(\chi, \psi, \phi) + K_h f_{ih}(\chi, \psi, \phi) \right] \tag{28}
\]
\[
\dot{W}_s = \dot{\theta} r_x \left[ (1 + K_r) \frac{L_z}{6r_x} \sin \chi \left( 2 \cos \chi - \frac{L_z}{r_x} \right) + K_h \frac{L_z}{3r_x} \sin^2 \chi \right]
\]

for block P-D-E:

\[
\dot{W}_s = \dot{\theta} r_x \left[ (1 + K_r) f_{sv} (\chi, \nu, \phi) + K_h f_{sh} (\chi, \nu, \phi) \right]
\]

\[
= \dot{\theta} r_x \left[ \frac{(1 + K_r)}{6} \exp \left[ \tan \phi (\nu - \chi) \right] \left( \sin (\nu - \chi) - \frac{L_z}{r_x} \sin \nu \right) \left( \cos \chi - \frac{L_z}{r_x} + \exp \left[ \tan \phi (\nu - \chi) \right] \cos \nu \right) + \right]
\]

\[
\frac{K_h}{6} \exp \left[ \tan \phi (\nu - \chi) \right] \left( \sin (\nu - \chi) - \frac{L_z}{r_x} \sin \nu \right) \left( \sin \chi + \exp \left[ \tan \phi (\nu - \chi) \right] \sin \nu \right)
\]

for block P-C-F:

\[
\dot{W}_s = \dot{\theta} r_x \left[ (1 + K_r) f_{sv} (\chi, \nu, \phi) + K_h f_{sh} (\chi, \nu, \phi) \right]
\]

\[
= \dot{\theta} r_x \left[ \frac{(1 + K_r)}{6} \exp \left[ 3 \tan \phi (\zeta - \chi) \right] \left( 3 \tan \phi \cos \zeta + \sin \zeta \right) - 3 \tan \phi \cos \chi - \sin \chi \right] + \frac{3 (1 + 9 \tan^2 \phi)}{1 + 9 \tan^2 \phi}
\]

\[
\frac{K_h}{6} \exp \left[ 3 \tan \phi (\zeta - \chi) \right] \left( 3 \tan \phi \sin \zeta - \cos \zeta \right) - 3 \tan \phi \sin \chi + \cos \chi
\]

for block P-B-F:

\[
\dot{W}_s = \dot{\theta} r_x \left[ (1 + K_r) f_{sv} (\chi, \nu, \phi) + K_h f_{sh} (\chi, \nu, \phi) \right]
\]

\[
= \dot{\theta} r_x \left[ (1 + K_r) \frac{L_z}{6r_x} \sin \chi \left( 2 \cos \chi - \frac{L_z}{r_x} \right) + K_h \frac{L_z}{3r_x} \sin^2 \chi \right]
\]

for block P-C-B:

\[
\dot{W}_s = \dot{\theta} r_x \left[ (1 + K_r) f_{sv} (\chi, \nu, \phi) + K_h f_{sh} (\chi, \nu, \phi) \right]
\]

\[
= \dot{\theta} r_x \left[ \frac{(1 + K_r)}{3} \exp \left[ 2 \tan \phi (\zeta - \chi) \right] \cos^2 \zeta \left( \exp \left[ \tan \phi (\zeta - \chi) \right] \sin \zeta - \sin \chi \right) + \right]
\]

\[
\frac{K_h}{3} \exp \left[ \tan \phi (\zeta - \chi) \right] \cos \zeta \left( \exp \left[ 2 \tan \phi (\zeta - \chi) \right] \sin^2 \zeta - \sin^2 \chi \right)
\]

substituting equation (6) and equation (26) with their components into equation (5), the following expression is obtained:

\[
e\dot{\theta} r_x \left( f_{b+c} + f_{c-d} \right) + K_r^2 \dot{\theta} \left( g_2 + g_3 \right) = \dot{\theta} r_x \left[ (1 + K_r) (f_{sv} - f_{2v} - f_{3v} - f_{4v} + f_{sv} + f_{sv}) + K_h (f_{1h} - f_{2h} - f_{3h} - f_{4h} + f_{sh} + f_{sh}) \right]
\]

Now, let us introduce the ratio of vertical to horizontal acceleration, \( \lambda = K_r / K_h \). Consistently with figure 2, the + sign indicates vertical downward acceleration, while the – sign indicates vertical
upward acceleration. An upper bound on the coefficient of yield acceleration $K_y$ is obtained by solving equation (33) with respect to $K_h$:

$$
K_y = \left[ \frac{c}{\gamma H} (f_{b,C} + f_{C,D}) + \frac{K_h}{\gamma H} (g_5 + g_3) \right] - \frac{r_y}{H} \left( \frac{f_{1v} - f_{2v} - f_{3v} - f_{4v} + f_{5v} + f_{6v}}{f_{2h} - f_{1h} - f_{3h} - f_{4h} + f_{5h} + f_{6h}} \right)
$$

(34)

$$
K_y = f_y \left( \chi, \nu, \zeta, \phi, c, \gamma H, \beta, \lambda, \alpha \right)
$$

(35)

The global minimum of $f_y (\chi, \nu, \zeta, \phi, c, \gamma H, \beta, \lambda, \alpha)$ over the three geometrical variables $\chi, \nu, \zeta$ provides the least upper bound on the coefficient of yield acceleration, assuming the most unfavourable crack to be present. Results obtained using equation (34) are presented in figure 3, providing the two terms needed to find the proposed stepwise yield acceleration for soil slopes with either zero tensile strength (i.e. $\alpha = 0$) or half the full tensile strength of the Mohr-Coulomb criterion (i.e. $\alpha = 0.5$). The solid lines in figure 3 refer to slopes that are initially intact but they can exhibit cracks forming as part of the incipient failure mechanism, while the dashed lines are for slopes with earthquake induced cracks (i.e. the formed crack is treated here as an open crack). The stepwise yield acceleration, proposed in this paper, can be found using these two lines (i.e. solid and dashed), where for a given soil slope properties, two values of yield acceleration are obtained. The one obtained from the solid line represents the starting value of the yield acceleration which steps down to the value obtained from the dashed line as soon as it is exceeded for the first time by the applied acceleration, given by the earthquake record. It can be observed that the two lines (solid and dashed) in figure 3 are close to each other. This is because the depth of the formed crack within reinforced soil is relatively shallow, especially for gentle slopes. The definition of the stepwise yield acceleration is detailed by an illustrative example later in this paper. It also should be noted that failure passes below the slope toe were not permitted during the calculations for these charts. This type of failure might occur for gentle slope with a low angle of internal friction [30].

4. Seismic Displacement

Based on Newmark’s method, and following [19, 30], the maximum horizontal displacement of the slope face that occurs at the slope toe, denoted here as $\delta u$, (see figure 4), is calculated as follow:

$$
\delta u = r_1 \sin \nu \delta \theta = r_1 \sin \nu \int \hat{\theta} \, dt\, dt = C \int \left( K_i - K_y \right) g \, dt\, dt
$$

(36)

where $\delta \theta$ is the angular displacement, $\hat{\theta}$ is the angular acceleration, $K_i$ is the applied horizontal acceleration at step $i$, and $C$ is a dimensionless coefficient relates the displacement of the slope toe to the integral of the earthquake acceleration record above the level of yield acceleration. This coefficient depends on the slope geometrical features and the ground strength parameters and can be expressed as:

$$
C = \frac{\gamma r_1^4 \sin \nu \exp \left[ \tan \phi (\nu - \chi) \right]}{G l^2}
$$

(37)

with $G$ being the weight of the potential sliding mass and $l$ is the distance from point P to the centre of gravity of that mass. The calculations for $G$ and $l$ are listed in Appendix A. To this end, the seismic induced displacements can be calculated using equation (36) by assigning an earthquake record and calculating the yield acceleration for the slope of interest.
Figure 3. Yield horizontal acceleration for uniform distribution of reinforcement with $\phi = 20^\circ$, $c/\gamma H = 0.05$ and $\lambda = 0$. (a and b) for uniform distribution of reinforcement and (c and d) for linearly increasing distribution. Left hand side charts are for soil slopes with zero tensile strength while the right hand side are for soil slopes with limited tensile strength (i.e. half of Mohr-Coulomb’s tensile strength).

5. Illustrative Example

The two types of reinforcement distributions, used in this paper, are compared here along with this illustrative example. A soil slope with $\beta = 75^\circ$, $\phi = 20^\circ$, $c/\gamma H = 0.1$, $\lambda = 0$, and $K_r/\gamma H = 0.1$, is considered and assumed to be subjected to the Northridge earthquake (1994), whose main characteristics are listed in table 1. Four cases are analysed: case (a) soil slope with full tensile strength, (i.e. $\alpha = 0$), therefore not subject to tension cracks, case (b) soil slope of limited tensile strength, in this case $\alpha = 0.5$, case (c) soil slope of zero tensile strength, (i.e. $\alpha = 0$), and case (d) soil slope subjected to the most adverse pre-existing crack.

Now, according to the procedure mentioned earlier in this paper, the stepwise yield acceleration for soil with limited or zero tensile strength is illustrated in figure 5(a). It can be noticed that the yield acceleration for a soil slope with limited or zero tensile strength is reduced significantly when it is
exceeded for the first time by the applied acceleration, because once the crack is formed as part of the failure at that instance, it is then treated as an earthquake-induced (pre-existing) crack. Consequently, this increases the estimated displacement as shown in figure 5(b). It can be seen that the displacement corresponding to a slope with the most detrimental pre-existing crack seems over conservative, at the same time, assuming an intact slope that remains intact during the earthquake may underestimate the slope displacement. Hence, assuming a limited tensile strength for the soil slope seems to reasonably bridge the gap between the conservatism, corresponding to a slope with the most detrimental pre-existing crack, and the underestimation of the displacement when ignoring the crack formation (i.e. intact slope). Figure 5(c) provides an insight as to the way the limited tensile strength can change the crack properties and the orientation of the failure mechanism.

Comparing the left hand side of figure 4 which is related to a slope with UD of reinforcement, with the right hand side of the same figure related to the same slope and with the same amount and strength of reinforcement but with the LID of reinforcement. It is noticed that slope with LID has better performance than the same slope with UD. The yield acceleration seems less affected by the presence of crack, whether earthquake-induced or pre-existing crack. Consequently, the total horizontal displacement of the slope reinforced with LID is reduced.

![Figure 4. Illustration of the horizontal displacement at the slope toe δυ, and the angular displacement δθ.](image)

Table 1. Main characteristics of the earthquakes considered in the example case.

| Date         | 17/1/1994 |
|--------------|-----------|
| Station      | 24283 Moorpark - Fire Sta. |
| Magnitude    | 6.7       |
| Direction    | 180°      |
| Peak acceleration (g) | 0.292 |
| Epicentre distance (km) | 23  |
Figure 5. Yield accelerations, horizontal displacement ($\delta u_x$) and failure mechanisms for a slope with $\beta=75$, $\phi=20$, $c/\gamma H=0.1$, $K_t/\gamma H=0.1$. Left hand side is for uniform distribution of reinforcement while the right hand side is for linearly increasing distribution of reinforcement. (a and a') Yield accelerations corresponding to the four cases explained earlier employing Northridge earthquake (1994). (b and b') Comparison of the accumulated horizontal displacement of the slope toe. (c and c') Failure mechanisms related to the yield accelerations illustrated in (a and a').
6. Conclusions
The upper bound theorem of limit analysis associated with pseudo static approach is employed to evaluate the displacement of soil slopes subject to cracks. A new technique is proposed to estimate the horizontal displacement of geosynthetic reinforced soil slopes at the slope toe, due to a given earthquake postulating a rough estimation of real time crack propagation. It should be pointed out that, in reality, during a moderate or strong earthquake, the soil slopes whether reinforced or not, are likely to show cracks even when assuming that the soil has full tensile strength. So, it is reasonable to assume that the soil has limited tensile strength when designing soil slopes subject to seismic actions.

The procedure for the new technique is described herein with an illustrative example, according to which, soil slopes with LID have better performance than the same slope with UD. Where using LID, the yield acceleration seems less affected by the presence of crack, whether earthquake-induced or pre-existing crack. Consequently, the total horizontal displacement of the slope reinforced with LID is less than the UD’s one.

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Appendix A
The calculations of the weight of the sliding mass B-C-E-D, called $G$ in the manuscript is detailed below.

$$ G = \gamma A $$

with $A = A_1 - A_2 - A_3 - A_4 + A_5 + A_6$  \hspace{1cm} (A.1)

$$ A_1 = \frac{1}{2} r_x \left[ \frac{\exp \left[ 2 \tan \phi \left( \nu - \chi \right) \right] - 1}{2 \tan \phi} \right] $$

$$ A_2 = \frac{1}{2} r_x L_1 \sin \chi $$

$$ A_3 = \frac{1}{2} r_x L_2 \sin \chi $$

$$ A_4 = \frac{1}{2} L_x \left[ \frac{\exp \left[ 2 \tan \phi \left( \zeta - \chi \right) \right] - 1}{2 \tan \phi} \right] $$

$$ A_5 = \frac{1}{2} r_x L_2 \sin \chi $$

$$ A_6 = \frac{1}{2} \delta r_x \cos \zeta = \frac{1}{2} r_x \delta \left[ \exp \left[ \tan \phi \left( \zeta - \chi \right) \right] \cos \zeta \right] $$

The arm length of the weight, $l$, is given by:

$$ l = \sqrt{\frac{\left( \gamma r_x \left( f_1 - f_2 - f_3 - f_4 + f_5 + f_6 \right) \right)^2 + \left( \gamma r_x \left( f_{h1} - f_{h2} - f_{h3} - f_{h4} + f_{h5} + f_{h6} \right) \right)^2}{G}} $$

(A.9)
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