Novel BPS Wilson Loops in Quiver Chern-Simons-matter theories

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  - Gravity dual of Wilson loops was studied soon after the correspondence was proposed.
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    - Maldacena 1998

  - The VEV of a circular half-BPS Wilson loop can be computed exactly using localization at finite 't Hooft coupling and finite $N$.
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  - This VEV is a nontrivial function of the coupling constant, interpolating between weak coupling results from perturbative field theory and strong coupling results (in Large $N$ limit) from gravity side.
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ABJM theory

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- This theory is low energy effective theory of $N$ M2-branes put at $C_4/Z_k$.

- It is holographically dual to M-theory on $AdS_4 \ast S^7/Z_k$ or type IIA string theory on $AdS_4 \times CP^3$. 
Wilson loops in ABJM theory - I

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  [Drukker, Plefka, Young (DPY), 08][Chen, JW, 08]
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Such Wilson loop was 1/6 BPS when it is along a line or a circle. It is closely related to 1/2— (1/3-)BPS Wilson loop in generic $\mathcal{N} = 2(3)$ Chern-Simons-matter (CSM) theories in [Gaiotto, Yin (GY), 07].
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- To provide a matrix model calculation for the VEV of this 1/6 BPS Wilson loop was the original motivation for Kapustin, Willett and Yaakov to develop the localization in 3d CSM theories.
  [Marino, lecture notes, 11]
It was proved that such Wilson loop is at most $1/6$ BPS \cite{DPY08,RSY08}. In \cite{Ouyang1511} we gave a proof without some unnecessary assumptions.

However the simplest F-string solution dual to Wilson loop is half-BPS \cite{DPY08,RSY08}. Then the construction of half-BPS Wilson loops in ABJM theory appeared as a big challenge. Such loop was finally constructed by Drukker and Trancanelli (DT) in 2009 by including the fermions in the construction and build a super-connection. This construct was elegantly explained by K. Lee and S. Lee in 2010 via the Brout-Englert-Higgs mechanism.
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- 2/5-BPS DT-type Wilson loops in $N = 5$ CSM theories ([Hosomichi, Lee$^3$, Park, 0806], [ABJ, 08]) were found by K. Lee and S. Lee.

- 1/2-BPS DT-type Wilson loops in $N = 4$ CSM theories [HLLLP, 0805][for $N = 4$ orbifold ABJM theory, Benna etal, 08] were constructed in [Ouyang, JW, Zhang, 1506][Cooke, Drukker, Trancanelli, 1506].
Wilson loops in CSM theories - II

- The previous results may tend to let people feel that DT type Wilson loops are very rare and their existence requires that the theory have a quite large number of supersymmetries.
- DT type Wilson loops also seem to preserve more supersymmetries than the GY type Wilson loops when they are along the same contour.
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Let us consider generic $\mathcal{N} = 2$ quiver SCSM theories with bifundamental matters.

- The Chern-Simons levels are $k_1$ and $k_2$, respectively.
- The vector multiplet for gauge group $U(N_1)$ include $A_{\mu}, \sigma, \chi, D$ and the last three fields are the auxiliary fields.
- Similarly for gauge group $SU(N_2)$ we have the vector multiplet $\hat{A}_{\mu}, \hat{\sigma}, \hat{\chi}, \hat{D}$. 
Let us consider generic $\mathcal{N} = 2$ quiver SCSM theories with bifundamental matters.

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\( \mathcal{N} = 2 \) quiver CSM theories - vector multiplets

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\[ \mathcal{N} = 2 \] quiver CSM theories - chiral multiplets

- The chiral multiplet in the bifundamental representation of \( U(N_1) \times U(N_2) \) includes the scalar \( \phi \), the spinor \( \psi \) and the auxiliary field \( F \).
For the vector multiplet part, we only need the off-shell supersymmetry transformation of $A_\mu, \sigma, \hat{A}_\mu, \hat{\sigma}$ is,

$$
\delta A_\mu = \frac{1}{2}(\bar{\chi}\gamma_\mu \theta + \bar{\theta}\gamma_\mu \chi), \quad \delta \sigma = -\frac{i}{2}(\bar{\chi}\theta + \bar{\theta}\chi), \\
\delta \hat{A}_\mu = \frac{1}{2}(\bar{\chi}\gamma_\mu \theta + \bar{\theta}\gamma_\mu \hat{\chi}), \quad \delta \hat{\sigma} = -\frac{i}{2}(\bar{\chi}\theta + \bar{\theta}\hat{\chi}).
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Supersymmetry transformation

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$$

For the matter part we only need the off-shell supersymmetry transformation of $\phi$ and $\psi$

$$
\delta \phi = i\bar{\theta}\psi, \quad \delta \bar{\phi} = i\bar{\psi}\theta,
$$
$$
\delta \psi = (-\gamma^{\mu}D_{\mu}\phi - \sigma\phi + \phi\hat{\sigma})\theta + i\bar{\theta}F, \quad (2)
$$
$$
\delta \bar{\psi} = \bar{\theta}(\gamma^{\mu}D_{\mu}\bar{\phi} + \hat{\sigma}\bar{\phi} - \bar{\phi}\sigma) - i\theta F,
$$
GY type BPS Wilson loops

In Minkowski spacetime, one can construct a GY type 1/2 BPS Wilson loop along an infinite straight line $x^\mu = \tau \delta^\mu_0$ as

$$W_{GY} = \mathcal{P} \exp \left( -i \int d\tau L_{GY}(\tau) \right),$$

$$L_{GY} = \begin{pmatrix} A_\mu \dot{x}^\mu + \sigma |\dot{x}| & \hat{A}_\mu \dot{x}^\mu + \hat{\sigma} |\dot{x}| \\ \dot{A}_\mu \dot{x}^\mu + \dot{\sigma} |\dot{x}| & \hat{A}_\mu \dot{x}^\mu + \hat{\sigma} |\dot{x}| \end{pmatrix}. \quad (3)$$
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▶ The preserved SUSY can be denoted as

$$\gamma_0 \theta = i \theta, \quad \bar{\theta} \gamma_0 = i \bar{\theta}. \quad (4)$$
We can also construct the DT type Wilson loop

\[ W_{DT} = \mathcal{P} \exp \left( -i \int d\tau L_{DT}(\tau) \right), \]

\[ L_{DT} = \begin{pmatrix} A & \bar{f}_1 \\ f_2 & \hat{A} \end{pmatrix}, \]

\[ A = A_\mu \dot{x}^\mu + \sigma |\dot{x}| + m\phi\bar{\phi}|\dot{x}|, \quad \bar{f}_1 = \bar{\zeta}\psi|\dot{x}|, \]

\[ \hat{A} = \hat{A}_\mu \dot{x}^\mu + \hat{\sigma} |\dot{x}| + n\phi\bar{\phi}|\dot{x}|, \quad f_2 = \bar{\psi}\eta|\dot{x}|. \]
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- To make it preserve the SUSY in (4) at least classically, it is enough to require that [K. Lee, S. Lee, 2010]

\[ \delta L_{DT} = \partial_\tau G + i[L_{DT}, G], \]

for some Grassmann odd matrix

\[ G = \begin{pmatrix} \bar{g}_1 \\ g_2 \end{pmatrix}. \]
We find that the necessary and sufficient conditions for the existence of such $\bar{g}_1$ and $g_2$ are

$$\bar{\zeta}_\alpha = \bar{\alpha}(1, i), \quad \eta_\alpha = (1, -i)\beta,$$

$$m = n = 2i\bar{\alpha}\beta. \quad (8)$$

Such DT type Wilson loop is $1/2$ BPS, and the preserved SUSY is the same as (4). Note that there are two free complex parameters $\bar{\alpha}$ and $\beta$ in the Wilson loop, and they can be any complex constants. When $\bar{\alpha} = \beta = 0$, it becomes the GY type Wilson loop.
Generalizations

- There could be other matters couple to these two gauge fields. They will change the on-shell values of $\sigma$ and $\hat{\sigma}$ in the Wilson loops we will construct. The structure of these Wilson loops will not be changed.

- We also constructed half-BPS circular Wilson loops for $\mathcal{N} = 2$ superconformal quiver Chern-Simons theory in Euclidean space.

- This construction can be also applied to the case when $U(N)$ is replaced by $SO(N)$ or $USp(N)$, and the case when there are matter fields in the adjoint representation. For the last case, one just simply let $\hat{A}_\mu \equiv A_\mu$ and $\hat{\sigma} \equiv \sigma$.

- The case with multi matter fields in the bifundamental and anti-bifundamental representations were also considered. The DT type Wilson loops can be divided into four classes.
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GY-type Wilson loops in ABJM theory - I

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A general GY type Wilson loop along the timelike infinite straight line $x^\mu = \tau \delta^\mu_0$ takes the form

$$W_{GY} = \mathcal{P} \exp \left( -i \int d\tau L_{GY}(\tau) \right),$$

$$L_{GY} = \begin{pmatrix} \mathcal{A}_{GY} & \hat{\mathcal{A}}_{GY} \\ \hat{\mathcal{A}}_{GY} & \mathcal{A}_{GY} \end{pmatrix},$$

$$\mathcal{A}_{GY} = A_\mu \dot{x}^\mu + \frac{2\pi}{k} R^I J \phi_I \phi^J |\dot{x}|,$$

$$\hat{\mathcal{A}}_{GY} = \hat{A}_\mu \dot{x}^\mu + \frac{2\pi}{k} S_I J \phi^I \phi J |\dot{x}|.$$  \hspace{1cm} (9)
GY-type Wilson loops in ABJM theory - 1

Up to some $SU(4)$ transformation, the only GY-type Wilson line preserving Poincare supercharges are the ones with $R^I_J = S^I_J = \text{diag}(-1, -1, 1, 1)$. They are $1/6$-BPS preserving the supersymmetries

$$\gamma_0 \theta^{12} = i \theta^{12}, \quad \gamma_0 \theta^{34} = -i \theta^{34},$$

$$\theta^{13} = \theta^{14} = \theta^{23} = \theta^{24} = 0.$$  \hspace{1cm} (10)
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- This is just the Wilson loop that was constructed in [DPY][Chen JW][RSY]. Here we show that this is the only form of GY type $1/6$ BPS Wilson loops up to some $SU(4)$ transformation. Especially, we find that we do not need to require that $R^I_J$ or $S_J^I$ is a hermitian matrix \textit{a priori}, and we show that it is the result of supersymmetry.
We turn to constructing a DT type Wilson loop along a straight line that preserves at least the supersymmetries (10). A general DT type Wilson loop is

\[ W_{\text{DT}} = \mathcal{P} \exp \left( -i \int d\tau L_{\text{DT}}(\tau) \right), \]

\[ L_{\text{DT}} = \begin{pmatrix} \mathcal{A} & \bar{f}_1 \\ f_2 & \hat{\mathcal{A}} \end{pmatrix}, \]

\[ \mathcal{A} = \mathcal{A}_{\text{GY}} + \frac{2\pi}{k} M_I^J \phi_I \bar{\phi}^J |\hat{x}|, \]

\[ \hat{\mathcal{A}} = \hat{\mathcal{A}}_{\text{GY}} + \frac{2\pi}{k} N_I^J \bar{\phi}^I \phi_J |\hat{x}|, \]

\[ \bar{f}_1 = \sqrt{\frac{2\pi}{k}} \bar{\zeta}_I \psi^I |\hat{x}|, \quad f_2 = \sqrt{\frac{2\pi}{k}} \bar{\psi}_I \eta^I |\hat{x}|. \]
The supersymmetry conditions give that

\[
\begin{align*}
\bar{\zeta}_{1,2} &= \bar{\alpha}_{1,2} \zeta, & \bar{\zeta}^\alpha &= (1, i), \\
\bar{\zeta}_{3,4} &= \bar{\gamma}_{3,4} \bar{\mu}, & \bar{\mu}^\alpha &= (1, -i), \\
\eta^{1,2} &= \eta^{1,2}, & \eta^\alpha &= (1, -i), \\
\eta^{3,4} &= \nu^{3,4}, & \nu^\alpha &= (1, i),
\end{align*}
\]

(13)

\[
M^I_J = N_J^I = 2i \begin{pmatrix}
\bar{\alpha}_2 \beta^2 & -\bar{\alpha}_2 \beta^1 \\
-\bar{\alpha}_1 \beta^2 & \bar{\alpha}_1 \beta^1 \\
\bar{\gamma}_4 \delta^4 & -\bar{\gamma}_4 \delta^3 \\
-\bar{\gamma}_3 \delta^4 & \bar{\gamma}_3 \delta^3
\end{pmatrix}.
\]

(14)
We found four class of solutions,

- Class I: $\bar{\gamma}_{3,4} = \delta_{3,4} = 0$.

- Class II: $\bar{\alpha}_{1,2} = \beta_{1,2} = 0$.

- Class III: $\beta_{1,2} = \delta_{3,4} = 0$.

- Class IV: $\bar{\alpha}_{1,2} = \bar{\gamma}_{3,4} = 0$.

Generically these Wilson loops preserving the same SUSY as GY type Wilson loops, i.e., they are $\frac{1}{6}$-BPS. In class I and II, for special parameters in the constructions, the Wilson loops become the half-BPS ones found by Drukker and Trancanelli. Our novel DT type Wilson loops include $\frac{1}{6}$-BPS GY type and half-BPS DT type ones as special cases.
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- **Class II**
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- **Class III**
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- **Class IV**
  \[ \bar{\alpha}_{1,2} = \bar{\gamma}_{3,4} = 0. \]  
  \[ (18) \]

Generically these Wilson loops preserving the same SUSY as GY type Wilson loops, i.e., they are 1/6-BPS.
Novel DT-type Wilson loops in ABJM theory - III

- We found four class of solutions,
- Class I

\[ \bar{\gamma}_{3,4} = \delta^{3,4} = 0. \]  (15)

- Class II

\[ \bar{\alpha}_{1,2} = \beta^{1,2} = 0. \]  (16)

- Class III

\[ \beta^{1,2} = \delta^{3,4} = 0. \]  (17)

- Class IV

\[ \bar{\alpha}_{1,2} = \bar{\gamma}_{3,4} = 0. \]  (18)

- Generically these Wilson loops preserving the same SUSY as GY type Wilson loops, i.e., they are 1/6-BPS.
- In class I and II, for special parameters in the constructions, the Wilson loops become the half-BPS ones found by Drukker and Trancanelli. Our novel DT type Wilson loops include 1/6-BPS GY type and half-BPS DT type ones as special cases.
DT type Wilson loops in $\mathcal{N} = 3, 4$ Chern-Simons-matter theories

- We found similar pattern in $\mathcal{N} = 4$ CSM theories: generally the DT type BPS Wilson loop along a straight line/circle is $1/4$ BPS, the same as GY type Wilson loops. For special parameters, supersymmetries preserved by the loops are enhanced to half-BPS.

For $\mathcal{N} = 3$ CSM theories, the DT type BPS Wilson loop along a straight line/circle is $1/3$ BPS. There is no supersymmetry enhancement here. This is consistent with the results from the dual M-theory side [Chen, JW, Zhu, 14].
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Conclusion

▶ We constructed DT-type Wilson loops in general $\mathcal{N} = 2$ quiver CSM theories.
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- Along a straight line or a circle, the generic DT-type Wilson loops in $ABJM$ theory are $1/6$-BPS, which include $1/6$-BPS GY-type Wilson loops and half-BPS DT-type Wilson loops as special case.
Conclusion

- We constructed DT-type Wilson loops in general $\mathcal{N} = 2$ quiver CSM theories.

- Along a straight line or a circle, the generic DT-type Wilson loops in $ABJM$ theory are $\frac{1}{6}$-BPS, which include $\frac{1}{6}$-BPS GY-type Wilson loops and half-BPS DT-type Wilson loops as special case.

- Generically, these $\frac{1}{6}$-BPS DT-type Wilson loops are not locally $SU(3)$ invariant. This is different from the Wilson loops constructed in [Cardinali, Griguolo, Martelloni, Seminara, 12].
Discussions

- Cooke, Drukker and Trancanelli argued that classically half-BPS DT-type Wilson loops in $\mathcal{N} = 4$ CSM theories may not be truly BPS at the quantum level. Only special linear combination of these loops will be BPS at the quantum level.
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Currently they are performing three-loop computations.
Discussions

- If the argument of \textit{Cooke, Drukker and Trancanelli} is correct, we expect that this should apply as well to our Wilson loops in general $\mathcal{N} = 2$ CSM theories.
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- **Which linear combination is truly BPS at the quantum level?**
Discussions

- If the argument of Cooke, Drukker and Trancanelli is correct, we expect that this should apply as well to our Wilson loops in general $\mathcal{N} = 2$ CSM theories.
- This leads to the following two questions:
  - Which linear combination is truly BPS at the quantum level?
  - How to construct the holographical dual of the above Wilson loops?
Thanks for Your Attention!