A new quantum theory of gravity in the framework of general relativity

Chang-Yu Zhu

1Department of Physics, Zhengzhou University, Zhengzhou, Henan 450052, China

Heng Fan

2Institute of Physics, Chinese Academy of Sciences, Beijing 100190, China

(Dated: November 10, 2009)

Observed physical phenomena can be described well by quantum mechanics or general relativity [1, 2]. People may try to find an unified fundamental theory [3] which mainly aims to merge gravity with quantum theory. However, difficulty in merging those theories self-consistently still exists, and no such theory is generally accepted. Here we try to propose a quantum theory with space and time in symmetrical positions in the framework of general relativity. In this theory, Dirac matter fields, gauge fields and gravity field are formulated in an unified way which satisfies Dirac equation, Yang-Mills equation and Einstein equation in operator form. This combines the quantum mechanics and general relativity.

PACS numbers: 00., 12.10.-g, 03.65.-w, 04.60.-m, 11.15.-q, 95.36.+x

I. INTRODUCTION

Two great discoveries of science in 20th century are the theory of relativity [1, 2] and quantum mechanics. At first it looked as if non-relativistic quantum mechanics would be enough to explain the spectrum of the hydrogen atom. However, to explain a finer structure in the spectrum due to the spin of the electron, Dirac [4] introduced his equation, the Dirac equation, by adding spin corrections to the Schrödinger equation [3]. Dirac equation satisfies invariance under special relativity, which can describe a single-particle obeying both relativity and quantum mechanics. This merges the theory of special relativity to quantum mechanics.

Dirac equation is also a field theory describing “field-like” objects. Later, the quantum field theory provides a unified framework describing both fields and particles [5, 6, 7, 8, 9]. In 1950s, Yang and Mills [10] introduced a class of quantum field theories known as gauge theories. In 1960s and 1970s, based on gauge theories, the Standard Model [11, 12, 13, 14, 15, 16] and related theories, see [17] for complete references, were proposed for elementary particles and the interactions between them. Up to then, the electromagnetic force, weak and strong nuclear forces are merged into an unified model. While only gravity which is one of the four basic forces in Nature remained outside of this unified framework.

In the past decades, much effort has been put into studying how to combine quantum mechanics with general relativity into a quantum theory of gravity. In the Standard Model and field theory, the motion of point-like particles is described by a graph of its position against time which forms a world-line.

In this work, we start directly from the general relativity, particles and fields are set of “events” which are quantum states of quantum field. An event is like a world-point in 4D space-time, a concept first coined by Einstein in which there is no absolute space and time. We propose a quantum theory with general relativity, an event or a quantum state is described by not only a complete set of canonical coordinates or canonical momentums of 4D space in operator form $\hat{x}^\mu, \hat{p}_\mu$, but also spin operators $\hat{s}_{\mu\nu}$ and gauge charge operators $\hat{T}_A$. These operators are covariance and constitute a Lie algebra. We then define a covariance derivative operator. In contrast with the derivative operator in Standard Model or in conventional quantum field theory, this covariance derivative operator has frame and connections of general relativity. Then the Einstein equation of general relativity, Dirac equation and Yang-Mill equation are all reformulated by this covariance derivative operator. This provides an unified framework of quantum mechanics and general relativity.

II. ELEMENTARY PARTICLES AND QUANTUM FIELDS

What we study is the all elementary particles and the interactions between them. As usual, they are all named as fields in quantum field theory. The elementary particles are divided into three families as matter particles (Ψ), gauge particles (A) and graviton (g). They satisfy Dirac equation, Yang-Mills equation and Einstein equation, respectively. In this work, we will present an unified quantum theory for those equations.

Matter particles (Ψ) include quarks and leptons each class have 6 types depending on mass and electro-charge. Each type of lepton has isospin singlet state and isospin doublet state, so leptons have 12 classes, altogether. Each flavor of quark has three color states, each color has isospin singlet state and isospin doublet state. So quarks have 36 types. Thus matter particles have 48 types. Note that the spin of matter particles is 1/2 described by a 4D Dirac spinor.

Gauge particles (A) are divide into photons, three types of weak gauge bosons and eight types of gluons.
Photons have no mass, while gluons are generally assumed to be massless, however, our theory allows the massive gluons. The masses of weak gauge bosons are $m_Z, m_W, m_{W'}$ corresponding to three types of bosons $Z_0, W_+, W_-$, respectively. We also have $m_W = m_{W'} = m_Z \cos \theta_W$, where $\theta_W$ is Weinberg angle. The spin of gauge particles is 1 and thus are bosons. The mass of gluons are all equal.

There is only one type of graviton ($g$), its mass and gauge charge are zeroes, the spin is 2 in tensor representation.

There are 12 gauge charges, including hypercharge $Y$, isospin charges $I_i, (i = 1, 2, 3)$ and color charges $\lambda_p, (p = 1, 2, ..., 8)$. They satisfy the commutation relations $[I_i, I_j] = i\epsilon_{ijk} I_k$, $[\lambda_p, \lambda_q] = i\epsilon_{pq} \lambda_r$, $[Y, I_i] = [Y, \lambda_p] = 0$, where $\epsilon_{ijk}$ is Levi-Civita symbol, $\epsilon_{pq} \lambda_r$ are structure constants of $su(3)$ group. We use notations $t_a, (a = 1, 2, ..., 12)$ to represent those 12 gauge charges as $\hat{t}_1 = g_1 \hat{Y}$, $\hat{t}_{i+1} = g_2 \hat{I}_i, (i = 1, 2, 3)$, $\hat{t}_{i+p} = g_3 \hat{\lambda}_p, (p = 1, 2, ..., 8)$, where $g_1, g_2$ and $g_3$ are coefficients in Standard Model. So the commutation relations take a concise form as $[t_a, t_b] = i\epsilon_{abc} t_c$, where coefficients $\epsilon_{abc}$ are defined as $\epsilon_{1+1+1,3} = g_2\epsilon_{ijk} \; \epsilon_{1+3+4,3} = g_3\epsilon_{ijk} \; \epsilon_{1+4+5,3} = g_4\epsilon_{ijk}$ and $\epsilon_{i+5+6,3} = 0$ elsewhere. Those gauge charges $t_a$ can be denoted as gauge bosons in Cartan-Weyl basis $\hat{T}_a$, $(a=1, 2, ..., 12)$ and satisfy the relation $[T_a, T_b] = C_{abc} T_c$.

In 4D space-time, we use notations $\hat{s}_{\alpha\beta}, (\alpha, \beta = 0, 1, 2, 3)$ to denote the spin operators which are antisymmetric $\hat{s}_{\alpha\beta} = -\hat{s}_{\beta\alpha}$ and satisfy the commutation relation $[s_{\alpha\beta}, s_{\rho\sigma}] = -i(\eta_{\alpha\rho}s_{\beta\sigma} - \eta_{\alpha\sigma}s_{\beta\rho} + \eta_{\beta\rho}s_{\alpha\sigma} - \eta_{\beta\sigma}s_{\alpha\rho})$, where $\eta_{\alpha\beta}$ is the Minkowski metric defined as $\eta_{00} = 1, \eta_{ij} = -\delta_{ij}, \eta_{0i} = \eta_{i0} = 0, \eta_{ij} = \eta_{ij}$.

Similar as in 3D quantum mechanics, in 4D space-time, the coordinate and momentum are denoted as $\hat{x}^\mu$ and $\hat{p}_\mu$ which satisfy the relations $[\hat{x}^\mu, \hat{p}_\nu] = -i\delta^\mu_\nu$, $[\hat{x}^\mu, \hat{x}^\nu] = [\hat{p}_\mu, \hat{p}_\nu] = 0$. Here we let the coordinates and momentums be general covariant so that coordinates and momentums transform and inverse transformation take the forms $\hat{x}^\mu = \tilde{x}^\mu(\hat{x}), \hat{x}^\mu = \hat{x}^\mu(\tilde{x})$ and $\hat{p}_\mu = \frac{\partial \tilde{x}^\mu}{\partial \hat{x}^\nu}\hat{p}_\nu, \hat{p}_\mu = \frac{\partial \hat{x}^\mu}{\partial \tilde{x}^\nu}\tilde{p}_\nu$. One can check that commutation relations remain invariant under general coordinates transformation $[\hat{x}^\mu, \hat{p}_\nu] = -i\delta^\mu_\nu, [\hat{x}^\mu, \hat{x}^\nu] = [\hat{p}_\mu, \hat{p}_\nu] = 0$. In this sense, we mean those commutation relations are quantum general covariance. In our theory, all physical equations should be quantum general covariance. This can be considered as the generalization of the general covariance from classical case to quantum case and thus can realize the unification between general relativity and quantum mechanics. Under the condition of quantum general covariance, we will present a quantum theory for gravity.

The coordinates and momentums are independent with spin and gauge charges $[\hat{x}^\mu, \hat{s}_{\alpha\beta}] = [\hat{x}^\mu, \hat{T}_a] = [\hat{p}_\mu, \hat{s}_{\alpha\beta}] = [\hat{p}_\mu, \hat{T}_a] = [\hat{s}_{\alpha\beta}, \hat{T}_a] = 0$.

### III. Representation Theory and Covariance Derivative Operator

In our theory, field is described by spin, gauge charges and coordinates-momentum. It is represented as a vector for matter fields or operator for force fields in space tensor product by coordinate-momentum space, spin space and gauge space, $V(M) = V_s \otimes V_S(M) \otimes V_G(M)$. The quantum state $|e_{st}\rangle$ of a matter particle is defined as the tensor product in coordinate-basis, spin-basis and gauge-basis, $|e_{st}(x)\rangle = |x\rangle \otimes |e_s\rangle \otimes |e_t\rangle$, where $x \in R^4, s = 1, 2, 3, 4$ and $t = 1, 2, ..., 48$. It is the common eigenstate of 10 operators $\hat{x}^0, \hat{x}^1, \hat{x}^2, \hat{x}^3, \hat{\gamma}_5, \hat{s}_{12}, Y, \hat{I}_3, \hat{\lambda}_3$ and $\hat{s}_\alpha$. In this representation, coordinate state can be changed to momentum state $|x\rangle \rightarrow |p\rangle$ and we have $|e_{st}(p)\rangle = |p\rangle \otimes |e_s\rangle \otimes |e_t\rangle$. The quantum state of matter-particle $|\psi\rangle$ can then be expanded by either coordinate state $|e_{st}(x)\rangle$ or momentum state $|e_{st}(p)\rangle$.

$$|\Psi\rangle = \int_{R^4} \Psi^{st}(x)|e_{st}(x)\rangle d^4x = \int_{R^4} \tilde{\Psi}^{st}(p)|e_{st}(p)\rangle d^4p, \quad (1)$$

where coefficients $\Psi^{st}(x)$ in the expansion are defined as $\Psi^{st}(x) = \langle e^{st}(x)|\Psi\rangle = (2\pi)^{-2} \int_{R^4} \tilde{\Psi}^{st}(p) \exp(-ipx) d^4p$, similar for case $\Psi^{st}(p)$.

In our representation, spin, gauge and general coordinate-momentum are dealt in the symmetric positions. In quantum mechanics, the time evolution of a quantum state or an operator are described by Schödinger representation or Heisenberg representation, respectively. We may notice that time and space are not symmetric. In comparison for our work, there is no absolute time and space in general relativity, thus four coordinates are dealt symmetrically. The state $|e_{st}\rangle$ is an event. The spin frame formalism takes the form $\hat{\theta} = \hat{\theta}_\alpha \otimes \hat{s}_\alpha \otimes \hat{p}_\mu, \hat{\theta}_\alpha = \int_{R^4} \theta_\alpha(x)\hat{\Phi}(x) d^4x = \int_{R^4} \theta_\alpha(p)\hat{\Phi}(p) d^4p$, where $\hat{\Phi}(x)$ are Dirac matrices, $\hat{\Phi}(p)$ is the coefficient, $\hat{\theta}_\alpha(x)$ and $\hat{\theta}_\alpha(p)$ are coordinate functions and momentum functions in spin frame formalism, they satisfy relations $\theta_\alpha(p) = (2\pi)^{-2} \int_{R^4} \theta_\alpha(p) \exp(-ipx) d^4x, \theta_\alpha(p) = (2\pi)^{-2} \int_{R^4} \theta_\alpha(p) \exp(-ipx) d^4p$. The spin frame formalism $\hat{\theta}_\alpha = \hat{\theta}_\alpha \otimes \hat{p}_\mu$ satisfy the commutation relations $[\hat{\theta}_\alpha, \hat{\theta}_\beta] = i\hat{f}_{\alpha\beta}\hat{\gamma}_7$, where $\hat{f}_{\alpha\beta}$ are the structure coefficients in spin frame formalism and is represented as $\hat{f}_{\alpha\beta} = (\hat{\theta}_\beta\partial_\alpha - \hat{\theta}_\alpha\partial_\beta - \hat{\theta}_\alpha\hat{\theta}_\beta)\hat{\gamma}_7$. The gravity connection is defined as $\hat{\Gamma} = \hat{\Gamma}_\alpha^\mu \otimes \hat{\gamma}_\alpha \otimes \hat{s}_\rho$, where $\hat{\Gamma}_\alpha^\mu = \frac{1}{2}(\hat{f}_{\rho\sigma} + \hat{f}_{\sigma\rho} - \hat{f}_{\rho\sigma})$, and we have used the notations $\hat{f}_{\rho\sigma} = \eta_{\sigma\rho} \eta_{\alpha\beta} \hat{f}_{\alpha\beta}, \hat{f}_{\rho\sigma} = \eta_{\rho\sigma} \hat{f}_{\rho\sigma}$. The gauge connections is defined as $\hat{A} = \hat{A}_\alpha^\mu \otimes \hat{\gamma}_\alpha \otimes \hat{T}_\alpha$, similar as for gravity field we have $\hat{A}_\alpha = \int_{R^4} A_\alpha^a(x)\hat{\Phi}(x) d^4x = \int_{R^4} A_\alpha^a(p)\hat{\Phi}(p) d^4p$, where $A_\alpha^a(x)$ and $A_\alpha^a(x)$ are coordinate and momentum functions, respectively.
Now we define the covariance derivative operator as
\[ D_\alpha = -i\delta^\mu_\alpha \otimes \hat{\gamma}_\mu + \frac{i}{2} \hat{\Gamma}_\alpha^{\rho\sigma} \otimes s_\rho s_\sigma - i\hat{A}_\alpha^{\rho} \otimes \hat{T}_\alpha \]  
(2)

This operator has connections of gravity, connections of gauge and spin frame. Thus it can describe all force fields. As in quantum mechanics, when acting on operators, it is represented in the form of commutating calculation, when acting on matter fields, it is represented as an operator acting on quantum states. The covariance differential can take the form \( D = \hat{\gamma}_\alpha \otimes D_\alpha \).

We then define the interaction curvature as
\[ \hat{\Omega}_{\alpha\beta} = i[D_\alpha, D_\beta] - i D_{[\alpha} \hat{\gamma}_{\beta]} \tag{3} \]
and \( \hat{\Omega} = \hat{\Omega}_{\alpha\beta} \otimes \hat{s}_{\alpha\beta} \). It is the summation of gravity curvature and gauge curvature \( \Omega_{\alpha\beta} = \frac{1}{2} R^\rho_{\alpha\beta\gamma} \otimes s_{\rho\sigma} + F^\rho_{\alpha\beta} \otimes \hat{T}_\rho \). The gravity curvature takes the form \( \hat{R}^\sigma_{\alpha\beta} = e^\rho_\alpha \partial_\rho \hat{\gamma}_\beta - e^\rho_\beta \partial_\rho \hat{\gamma}_\alpha + \hat{\Gamma}_\sigma^{\rho\beta} \hat{\gamma}_\rho - \hat{\Gamma}_\sigma^{\rho\alpha} \hat{\gamma}_\beta - \hat{\Gamma}_\sigma^{\rho\beta} \hat{\gamma}_\alpha \), and the gauge curvature takes the form \( \hat{F}^\alpha_{\alpha\beta} = e^\rho_\alpha \partial_\rho A^\beta_\beta - e^\rho_\beta \partial_\rho A^\alpha_\beta + C_{\alpha\beta}^\rho \hat{A}_\rho - \hat{\gamma}_{\alpha\beta} \hat{A}^\rho_\rho \).

The covariance derivative operator and the interaction curvature satisfy the Bianchi identity, \( D_\alpha \hat{\Omega}_{\beta\gamma} + D_\beta \hat{\Omega}_{\gamma\alpha} + D_\gamma \hat{\Omega}_{\alpha\beta} = 0 \). It turns out be Bianchi identities for gravity curvature and gauge curvature, respectively. \( D_\alpha \hat{F}^\alpha_{\rho\sigma} + D_\beta \hat{F}^\sigma_{\rho\alpha} + D_\gamma \hat{F}^\rho_{\sigma\alpha} = 0 \) and \( D_\alpha \hat{F}^\alpha_{\beta\gamma} + D_\beta \hat{F}^\beta_{\gamma\alpha} + D_\gamma \hat{F}^\gamma_{\alpha\beta} = 0 \).

IV. DIRAC EQUATION, YANG-MILLS EQUATION AND EINSTEIN EQUATION

The Dirac equation for matter fields can be written as:
\[ (i\hat{\gamma}_\alpha \hat{D}_\alpha - m)\Psi = 0, \tag{4} \]
where \( m \) is the mass matrix in gauge in square, its eigenvalues are masses of the corresponding elementary particles.

The Yang-Mills equation takes the form
\[ \hat{D}_\alpha \hat{F}^\alpha_{\beta\gamma} = \hat{j}^\beta_\gamma + M^b_\alpha A^\beta_\gamma, \tag{5} \]
where the l.h.s is \( \hat{D}_\alpha \hat{F}^\alpha_{\beta\gamma} = i\theta^\rho_\alpha \partial_\rho \hat{\psi} + \hat{\psi} \Gamma_\rho_\alpha \hat{F}^{\rho\beta} - \hat{\psi} \Gamma_\rho_\beta \hat{F}^{\rho\alpha} + C_{\alpha\beta}^\rho \hat{A}_\rho \), the r.h.s is the total current density, \( j^\beta_\gamma \) is gauge current density, \( M^b_\alpha \) are mass tensor of gauge bosons with \( M^2_a = m_\alpha^2, M^3_a = M^4_a = m^2_{a\perp}, \) and \( M^{l+}_{a} = m_{a}, l = 1, 2, \ldots, 8, \) \( M^{l-}_{a} = 0 \) elsewhere, the mass of gluon is \( m \).

The Einstein equation takes the form
\[ \hat{R}^\beta_\beta - \frac{1}{2} \hat{s}^2_\beta \hat{R} = -8\pi G T^\beta_\beta, \tag{6} \]
where \( G \) is gravity constant, \( T^\beta_\beta \) is energy-momentum tensor, \( R^\beta_\beta = \hat{R}^\beta_\beta \), and \( \hat{R} = \hat{R}^\beta_\beta \). The Einstein equation and the contracted tensor of Bianchi identity of gravity \( \hat{D}_\alpha (R^\beta_\beta - \frac{1}{2} \hat{s}^2_\beta \hat{R}) = 0 \) can lead to the energy-momentum conservation law, \( \hat{D}_\alpha \hat{R}^\beta_\beta = 0 \).

V. PHYSICAL QUANTITIES AND ENERGY-MOMENTUM CONSERVATION LAW

Current density of particles is defined as \( \rho^\alpha(x) = \langle \hat{\psi}(x) \hat{\gamma}^\alpha \hat{\psi} \rangle \). Due to Dirac equation, the conservation of the number of particles can be found to be \( \hat{D}_\alpha \rho^\alpha = 0 \). For each \( t \) in gauge basis, the current density of the particle takes the form \( \hat{p}^\alpha_\beta(x) = \langle \hat{\psi}(x) \hat{\gamma}^\alpha \hat{\psi} \rangle \).

Gauge current density of the matter fields is defined as \( \hat{j}^\alpha_\beta(x) = \langle \hat{\psi}(x) \hat{\gamma}^\alpha \hat{J}_\beta \rangle \), gauge charge density of production rate of matter fields takes the form \( \hat{u}_\alpha(x) = -i \langle \hat{\psi}(x) \hat{J}_\alpha \hat{T}_\alpha \hat{\psi} \rangle \), by Dirac equation we have \( \hat{D}_\alpha \hat{j}^\alpha_\beta = \hat{u}_\beta \). For electric-charge and color charges \( \hat{u}_\alpha = 0 \), they are conserved quantities. For three type weak charges \( \hat{u}_a \neq 0 \).

The spin angular-momentum current density of the matter fields take the form \( S^{\alpha\beta\gamma}(x) = \frac{1}{2} \langle \hat{\psi}(x) (\hat{\gamma}^\alpha \hat{\gamma}^\beta \hat{\gamma}^\gamma) \hat{\psi} \rangle \), they are anti-symmetric. And one may check \( \hat{D}_\alpha S^{\beta\gamma\alpha} = 0 \).

The energy-momentum tensor of the matter fields is defined as \( \hat{T}^{\alpha\beta}(x) = \frac{1}{2} \langle \hat{\psi}(x) (\hat{\gamma}^\alpha \hat{\gamma}^\beta) \hat{\psi} \rangle \) and \( \langle \hat{\psi}(x) \hat{\gamma}^\alpha \hat{\psi} \rangle \). According to Dirac equation, the divergence equation for energy-momentum tensor can be proved to be \( \hat{D}_\alpha \hat{t}^{\alpha\beta} = F^{\alpha\beta}_a A^a_\beta + A^\alpha_\beta \hat{u}_\beta + \hat{u}_\alpha + \hat{u}_\alpha \). While the dark energy solution will, on the other hand, predict that the elementary particle gluon are massive though its mass is very small. We will also see

VI. OUTLOOK

A combined and unified description of quantum mechanics and general relativity is very important for physics. It is not only for a fundamental understanding of nature but also it can provide a theory for systems where both quantum mechanics and general relativity is important. We will see in Ref.[18] that with this unified theory we can provide a solution of dark energy of cosmos which is one of the most mysterious problem presently since the dark energy is the main part (around 72%) of our universe. While the dark energy solution will, on the other hand, predict that the elementary particle gluon are massive though its mass is very small. We will also see
that this theory will lead to some exciting results such as: mass problem is solved for gauge theory, we can explain the color confinement of quarks, the parity violation for weak interactions, we can find that gravity can cause the CPT violation. Conceptually, we will find that no Higgs mechanism is necessary, while all of our results agree with the basic facts of physics. The detailed presentation of the whole theory is in Ref. [18].

We believe that this theory can provide a foundation of quantum physics. Thus a lot of problems should be clarified, in particular, our results should agree with all well-established results both theoretically and experimentally.

Acknowledgments

We thank Mr. Shi-Ping Ding for consistent supporting and discussions. This work is supported by grants of National Natural Science Foundation of China (NSFC) Nos.(10674162,10974247), “973” program (2010CB922904) of Ministry of Science and Technology (MOST), China, and Hundred-Talent Project of Chinese Academy of Sciences (CAS), China.

[1] Eistein, A. “Zur Elektrodynamik bewegter Körper,” Annalen der Physik und Chemie 17, 891 (1905).
[2] Eistein, A. “Die Grundlage der allgemeinen Relativitätstheorie,” Annalen der Phys. 49, 769 (1916).
[3] Weinberg, S. “Dreams of a Final Theory: The Search for the Fundamental Laws of Nature,” (Hutchinson Radius, London, 1993).
[4] Dirac, P. A. M. “The Quantum Theory of the Electron,” Proc. Roy. Soc. A 117, 610 (1928).
[5] Schrödinger, E. “Quantisierung als Eigenwertproblem,” Annalen de Physik 79, 361 (1926).
[6] Feynman, R. P. “Space-Time approach to Quantum Electrodynamics,” Phys. Rev. 76, 769 (1949).
[7] Schwinger, J. S. “Quantum Electrodynamics, III: The Electromagnetic Properties of the Electron: Radiative Corrections to Scattering,” Phys. Rev. 76, 790 (1949).
[8] Tomonaga, S. “On a Relativistically Invariant Formulation of the Quantum Theory of Wave Fields,” Prog. Theor. Phys. 1, 27 (1946).
[9] Dyson, F. J. “The Radiation Theories of Tomonaga, Schwinger and Feynman,” Phys. Rev. 75, 486 (1949).
[10] Yang, C. N. and Mills, R. L. “Conservation of Isotopic Spin and Isotopic Gauge Invariance,” Phys. Rev. 96, 191 (1954).
[11] Weinberg, S. “A Model of Leptons,” Phys. Rev. Lett. 19, 1264 (1967).
[12] Salam, A. “Weak and Electromagnetic Interactions,” Proc. Nobel Sym. 1968 at Lerum, Sweden, 267 (1968).
[13] Glashow, S. L. “Partial Symmetries of Weak Interactions,” Nucl. Phys. 22, 579 (1961).
[14] Gross, D. J. and Wilczek, F. “Ultraviolet Behavior of Non-Abelian Gauge Theories,” Phys. Rev. Lett. 30, 1343 (1973).
[15] Kobayashi, M. and Maskawa, K. “CP violation in the renormalizable theory of weak interactions,” Progr. Theor. Phys. 49, 652 (1964).
[16] Politzer, H. D. “Reliable Perturbative Results for Strong Interactions?,” Phys. Rev. Lett. 30, 1346 (1973).
[17] Scientific Background of the Nobel Prize in Physics 2008, Broken Symmetries, available at http://nobelprize.org/
[18] Zhu, C. Y. and Fan, H. “Quantum gravity and mass of gauge field: a four-dimensional quantum theory”, e-print arXiv:0911.1402.