Acoustoelectric current and pumping in a ballistic quantum point contact

Y. Levinson
Department of Condensed Matter Physics, The Weizmann Institute of Science, Rehovot 76100, Israel

O. Entin-Wohlman
School of Physics and Astronomy, Raymond and Beverly Sackler Faculty of Exact Sciences,
Tel Aviv University, Tel Aviv 69978, Israel

P. Wölfle
Institut für Theorie der Kondensierten Materie, Universität Karlsruhe, 76128 Karlsruhe, Germany

(March 21, 2022)

The acoustoelectric current induced by a surface acoustic wave (SAW) in a ballistic quantum point contact is considered using a quantum approach. We find that the current is of the “pumping” type and is not related to drag, i.e., to the momentum transfer from the wave to the electron gas. At gate voltages corresponding to the plateaus of the quantized conductance the current is small. It is peaked at the conductance step voltages. The peak current oscillates and decays with increasing SAW wavelength for short wavelengths. These results contradict previous calculations, based on the classical Boltzmann equation.

The interaction of surface acoustic waves (SAW) with electrons in a two dimensional electron gas (2DEG) has recently attracted much attention. In particular the acoustoelectric effect (d.c. current driven by the SAW) was investigated experimentally in a point contact (PC) defined in GaAs/AlGaAs heterostructure by a split gate [1,3]. Most of the theoretical considerations of this effect were classical, based on the Boltzmann equation for electrons in a 1D channel, with the SAW considered as a classical force [1,2] or as a flux of monochromatic surface phonons [3,4]. Such an approach is valid only when the channel length is much longer than the electron Fermi wavelength and when the electron diffraction at the channel ends can be neglected. In this picture the acoustoelectric current results from the drag of electrons by the SAW. Its value is determined by the competition between the momentum transfer from the SAW to the 2DEG and the momentum relaxation due to impurity scattering [3,4] or due to electron escape from the PC [4,5], for a ballistic PC.

A quantum approach was used in [4], but only PC’s of length short compared to the SAW wavelength were considered for the experimentally relevant low frequencies. We present here a quantum description of the problem, based on a different formalism, which allows a more general consideration and leads to results qualitatively different from those given by the classical approach. In particular, we find that the drag mechanism is not valid for the quantum acoustoelectric current, and that the reflection of the electrons within the PC is crucial for producing the SAW effect. Unfortunately the results of the experiments do not allow to reach a definite conclusion about the mechanism of the acoustoelectric current.

Consider a nanostructure (NS) of arbitrary geometry (e.g., a PC) where the 2DEG is confined by a potential $U(r)$ and is attached to terminals $\alpha$ (with no voltage bias). The NS is exposed to a random a.c. potential $\delta U(r, t)$, produced by a gate or by radiation, infrared or acoustic, and localized within the NS. This a.c. potential induces a current through the NS, the d.c. component of which is the acoustoelectric current or the photovoltaic current, depending on the nature of the radiation. Using time-dependent scattering states (see below for details), we find that the d.c. current $J_\beta$ entering terminal $\beta$ is given by ($e < 0, \hbar = 1$)

$$J_\beta = \sum_\alpha J_{\beta \alpha}, \quad J_{\beta \alpha} = -e \int d\mathbf{r}_1 d\mathbf{r}_2 P(\mathbf{r}_1, \mathbf{r}_2) \times \int \frac{dE}{2\pi} \left( -\frac{\partial f(E)}{\partial E} \right) g_\beta(E | \mathbf{r}_1, \mathbf{r}_2) g_\alpha(E | \mathbf{r}_1, \mathbf{r}_2).$$

(1)

The properties of the a.c. potential are condensed in the pumping factor

$$P(\mathbf{r}_1, \mathbf{r}_2) = \int d\omega \omega \delta U(\mathbf{r}_1) \delta U(\mathbf{r}_2),$$

(2)

where $\delta U(\mathbf{r}_1) \delta U(\mathbf{r}_2)$ is the Fourier component of the random field correlator $\delta U(\mathbf{r}_1, t_1) \delta U(\mathbf{r}_2, t_2)$ and the overbar denotes statistical averaging. The properties of the NS are embodied in

$$g_\alpha(E | \mathbf{r}_1, \mathbf{r}_2) = \sum_n \chi_{\alpha n}(E | \mathbf{r}_1) \chi_{\alpha n}(E | \mathbf{r}_2)^*,$$

(3)

where $\chi_{\alpha n}(E | \mathbf{r})$ is a scattering state excited by an incoming wave $w_{\alpha n}(E | \mathbf{r})$ (normalized to unit incoming flux) of an electron with energy $E$ entering the NS from channel $n$ of terminal $\alpha$. $f(E)$ is the Fermi distribution in the terminals.
Equation (1) is valid in the weak field adiabatic approximation, when the a.c. perturbation $\delta U$ is smaller than the Fermi energy $E_F$ and when the relevant frequencies of this perturbation are smaller than all scales which determine the energy dependence of the scattering states. The statistical averaging replaces the temporal averaging, which is unavoidable when measuring a d.c. current induced by an a.c. perturbation. Below we assume zero temperature, which reduces the energy integration in (1) to $E = E_F$.

The a.c. potential created by a SAW propagating in the $x$ direction is $\delta U(r,t) = A(t) \exp(\pm iqx - \omega_0 t) + c.c.$, where the amplitude $A(t)$ is a stationary, slowly varying, random function. This potential is screened by the electrons of the 2DEG [8]. In the wide part of the PC the screening strongly reduces the potential (by the factor $qa_B$, where $a_B$ is the Bohr radius), while in the narrow part screening is weak. To account for the screening effect we multiply $\delta U(r,t)$ by a screening factor $S(x) = S(-x)$. For this screened a.c. potential the pumping factor is $P(t_1, t_2) = 2i\omega_0/q^2 \sin(q(x_1 - x_2))S(x_1)S(x_2)$.

Let us first consider a PC attached to two ideal 1D leads at $x \to \pm \infty$, and assume that the PC is represented by a repulsive delta function potential, $U(x) = V \delta(x)$. The scattering states excited from the left terminal $\alpha = l$ at $x = -\infty$ and the right terminal $\alpha = r$ at $x = +\infty$ are $\chi_\alpha(E|x) = \frac{1}{\sqrt{2}}[\exp(\pm ikx) + \exp(ikx)]$, where $\pm$ denotes $\alpha = l$ and $\alpha = r$, respectively, and $k_E$ and $v_E$ are the electron momentum and velocity at energy $E$. The transmission and reflection amplitudes are $t_E = 1 + iV/v_E$ and $r_E = (1 - iv/v_E)^{-1}$. For the screening factor we choose $S(x) = \exp(-|x|/L_s)$. Eq. (1) then yields that the partial currents $J_{lr} = J_{r,l} = 0$, while $J_{lr} = -J_{r,l} = J$, where $J$ is the d.c. current through the PC in the $x$-direction. For $q \ll k_F$ we have

$$J = \frac{\omega_0}{2\pi} \frac{|A|^2}{2E_F} \frac{qk_F}{q^2 + L_s^2} |t_F|^2 |r_F|^2,$$

where the index $F$ means $E = E_F$. This result shows that (i) the current is proportional to $\omega_0$, and hence is of the pumping type; (ii) the current increases with $q$ for small wavevectors, and decays for large ones; (iii) a finite reflection is necessary for producing the effect.

Turning now to a more realistic description of the PC, we model it by the 2D saddle-point potential $U(x,y) = (1/2md^2)[-(x/L)^2 + (y/d)^2]$, where $m$ is the electron mass, $L$ is the length of the PC and $d$ is its width. For $L \gg d$ this potential corresponds to a waveguide in the $x$-direction with parabolic walls (at $|x| \lesssim L$) adjusted to horns (at $|x| \gtrsim L$) with opening angle $d/L$. These horns represent the left and right terminals at $x = \mp \infty$. The scattering states are given by $\chi_{\alpha\pi}(E|x) = \Phi_n(y) \chi^\pm(x_\alpha|x)$. Here $\Phi_n$ is a normalized harmonic oscillator wave function with energy $E_n = \Delta(n + 1/2)$, where $\Delta = 1/md^2$, and $n = 0, 1, 2, ...$ labels the channels in both terminals. (There is no channel mixing in a saddle-point potential.) $\chi^\pm(x_\alpha|x)$ is given by the complex Weber (parabolic cylinder) function $E(a,x)$, as defined in [10]

$$\chi^\pm(x_\alpha|x) = -i\sqrt{m(Ld/2)^{1/4}} t(x_\alpha) E(-\varepsilon_n, \pm \xi).$$

Here $\xi = (2/Ld)^{1/2} x$ and $\varepsilon_n = (E - E_n)/\delta$ with $\delta = 1/mLd$. In Eq. (3) $t(x_\alpha) = (1 + e^{-2\pi \xi})^{-1/2}$ is the transmission amplitude of the barrier created by the saddle-point potential. Again, $\pm$ denote $\alpha = r, l$, respectively.

The Landauer conductance (at zero temperature, in units of $e^2/h$) of such a PC is $G = \sum \alpha t(\varepsilon_n)^2$, where $\varepsilon_n = (E_F - E_n)/\delta$. When $L \gg d$ the conductance as a function of $E_F$ is a step like function, with plateaus of width $\Delta$ and steps of width $\delta$. The steps occur at energies $E_n$, where $E_F$ equals the bottom of the transverse quantization mode $n$; for $E > E_n$ this mode is propagating, while at $E < E_n$ it is evanescent.

The current in the saddle-point PC, as obtained from Eq. (4), consists of a sum over the separate mode contributions

$$J = J_0 \sum_n F(\varepsilon_n, p), \quad p = q(Ld/2)^{1/2},$$

with the nominal value $J_0 = 2e((\omega_0/2\pi)|A|^2/\delta^2)$. The function $F(\varepsilon, p)$ [see Eq. (4) below] is positive for $p > 0$, and is odd in $p$, i.e. the electron flux is along the direction of the SAW propagation. We find that $F(\varepsilon, p)$ is exponentially small when $|\varepsilon| \gg 1$, that is, modes whose energies are far from the threshold $E_n$ do not contribute to the current. This is expected for the evanescent modes; for the propagating ones it means that in a free channel with no reflection the acoustoelectric current is zero. The crucial role of reflection in producing the current can be seen also from Eq. (4).

Near the threshold, for $|\varepsilon| \lesssim 1$, where the current is not small, we have (for $p > 0$)

$$F(\varepsilon, p) = 2pe^{-\pi^2 t(\varepsilon)^3 c(\varepsilon) \text{erf} \left( \frac{p}{\sqrt{2}\sigma} \right)}, \quad p \ll 1,$$

$$F(\varepsilon, p) = 4\pi t(\varepsilon)^2 \cos^2 \left( \frac{p^2}{2} - \frac{\pi}{4} - \varepsilon \right) e^{-\pi p^2}, \quad p \gg 1.$$
contribution to the current are depicted in the figure, for \( L/d = 10 \) and \( \varepsilon = 0, \pm 0.5 \). The intermediate region in which \( F \) is independent of \( q \) is not distinguished for the not very small \( d/L \) ratios. One can see from this figure that below the threshold (\( \varepsilon < 0 \)) the current is much weaker then above it (\( \varepsilon > 0 \)).

![Graph](image)

**Fig. 1** Contribution of a single channel to the current (see Eq. (6)) as a function of the dimensionless SAW wavenumber \( p \), for different values of Fermi energy close to the step of the conductance.

Our quantum theory predicts that the current is strong only when \( E_F \) is at the vicinity of the transverse quantization channel bottom. This finding is in agreement with that of the classical approach, of giant oscillations in the acoustoelectric current. However, in the quantum theory the width of the peak, \( \delta \), is determined by the diffraction at the opening angle of the channel. Indeed, the width-to-spacing ratio of the oscillations is \( \delta/\Delta = d/L \). On the other hand, the peak width in the classical theory is determined (at zero temperature) by the scattering or by the escape time in the case where the SAW is described as a force, or by the energy and momentum conservation when the SAW is described as a phonon flux. Experiment shows the oscillations, however unfortunately the peaks are not very pronounced, and this is why one cannot make statements about the nature of their width.

Another important difference between the quantum and the classical approaches concerns the behavior of the current at short waves. According to the (classical) drag picture, the current should increase with the current at short waves. According to the (classical) drag picture, the current should increase with the current at short waves. According to the (classical) drag picture, the current should increase with the current at short waves. According to the (classical) drag picture, the current should increase with the current at short waves. According to the (classical) drag picture, the current should increase with the current at short waves. According to the (classical) drag picture, the current should increase with the current at short waves. According to the (classical) drag picture, the current should increase with the current at short waves. According to the (classical) drag picture, the current should increase with the current at short waves.

L-independent, the predictions of both theories agree regarding the dependences on SAW frequency and contact length.

We now outline the derivation of Eq. (1). This is accomplished using the concept of *time-dependent scattering states* [12]. Let the NS under the a.c. field be described by the Hamiltonian \( H = \int dV \Psi^\dagger(r)H(r,t)\Psi(r) \), where \( H(r,t) = H_0(r) + \delta U(r,t) \) and \( H_0(r) = (1/2m)(-i\nabla)^2 + U(r) \). Here \( \Psi(r) \) is the electron field operator. The time-dependent scattering state \( \chi_{\alpha n}(E|r,t) \) is defined as the solution of the equation

\[
i(\partial/\partial t)\chi_{\alpha n}(E|r,t) = H(r,t)\chi_{\alpha n}(E|r,t),
\]

which is excited by an incoming wave \( w_{\alpha n}^-(E|r)\exp(-iEt) \) in the presence of the a.c. potential. The state \( \chi_{\alpha n}(E|r,t) \) is labeled according to the energy of the incoming wave, but it contains components with energies \( E' \neq E \), since due to the time dependent perturbation \( \delta U(r,t) \) the transmission and the reflection of the incoming wave are inelastic. For a weak time-dependent potential, \( \delta U \ll E_F \), this equation can be solved by iterations,

\[
\chi_{\alpha n}(E|r,t) = e^{-iEt}[\chi_{\alpha n}(E|r) + \chi_{\alpha n}^{(1)}(E|r,t) + \chi_{\alpha n}^{(2)}(E|r,t) + \ldots].
\]

The first term here is the (time-independent) scattering solution of \( H_0 \), and the subsequent terms contain only outgoing waves. The latter can be found in terms of the retarded Green’s function of \( H_0 \),

\[
(i\partial/\partial t - H_0(r))\tilde{\Psi}(r,r',t) = \delta(r-r')\delta(t).
\]

The time-dependent field operator, required for the calculation of the current density operator \( \langle \mathbf{j}(r,t) = -(ie/2m)\tilde{\Psi}(r,t)^+\nabla\tilde{\Psi}(r,t) + h.c. \rangle \), can be written in terms of the scattering states,

\[
\tilde{\Psi}(r,t) = \int dE/2\pi \sum_{\alpha n} a_{\alpha n}(E)\chi_{\alpha n}(E|r,t).
\]

Here \( a_{\alpha n}^+(E) \) is an operator creating an incoming electron in channel \( n \) of lead \( \alpha \) with energy \( E \). The averages of these operators are determined by the temperatures and the chemical potentials of the terminals connected to the leads. For the scattering states which are normalized to unit incoming flux

\[
\langle n_{\alpha n}^+(E)a_{\alpha' n'}(E') \rangle = 2\pi\delta(E-E')\delta_{\alpha\alpha'}f_{\alpha}(E),
\]

where \( f_{\alpha}(E) \) is the Fermi distribution in terminal \( \alpha \).

Using the above results one performs the quantum and statistical averaging to obtain the current density \( \langle \mathbf{j}(r) \rangle \). Evaluating \( \langle \mathbf{j}(r) \rangle \) far away in terminal \( \beta \) and integrating over the cross section of this terminal gives \( J_{\beta} \) of Eq. (4). The asymptotic behavior of the current density is derived using the following relation [13] for the Fourier transform of the Green function defined by Eq. (14),
where \( w_{\beta m}^{+}(E|\mathbf{r}) \) is an outgoing wave in channel \( m \) of terminal \( \beta \) (normalized to unit flux).

The acoustoelectric current for the saddle-point confining potential, Eq. (6), is obtained using for the complex Weber functions the representation $E(a, x) = k^{-1/2}W(a, x) + ik^{1/2}W(a, -x)$, where \( W(a, \pm x) \) are the real Weber functions and \( k = (1 + e^{2\pi\alpha})^{1/2} - e^{\pi\alpha} \). We find

\[
F(\varepsilon, p) = t(\varepsilon)^3G(\varepsilon, p)H(\varepsilon, p),
\]

where

\[
G(\varepsilon, p) = \int_{-\infty}^{+\infty} d(\xi) \exp(-\sigma\xi^2/2) \sin p\xi g(\varepsilon, \xi),
\]

\[
H(\varepsilon, p) = \int_{-\infty}^{+\infty} d(\xi) \exp(-\sigma\xi^2/2) \cos p\xi h(\varepsilon, \xi),
\]

with

\[
g(\varepsilon, \xi) = W(-\varepsilon, -\xi)^2 - W(-\varepsilon, \xi)^2 = -g(\varepsilon, -\xi),
\]

\[
h(\varepsilon, \xi) = W(-\varepsilon, -\xi)W(-\varepsilon, \xi) = h(\varepsilon, -\xi).
\]

The appearance of the transmission amplitude \( t(\varepsilon) \) in Eq. (4) ensures the exponential smallness of the function \( F(\varepsilon, p) \) for evanescent modes. To show that it is also exponentially small for propagating modes belonging to \( \varepsilon \gg 1 \) we use the Darwin representation of the Weber functions [10] \( (\xi > 0) \) and obtain

\[
g(\varepsilon, \xi) = \left[ \sqrt{\varepsilon} \cosh(s/2) \right]^{-1} \left[ e^{-\pi\xi} + \sin \theta \right],
\]

\[
h(\varepsilon, \xi) = \left[ 2\sqrt{\varepsilon} \cosh(s/2) \right]^{-1} \cos \theta,
\]

\[
\theta = \varepsilon(s + \sinh s), \quad \xi = 2\sqrt{\varepsilon} \sinh(s/2).
\]

The functions \( g \) and \( h \) then contain exponentially small or fast oscillating terms. The integrals \( G \) and \( H \) can then be calculated near the saddle-point \( s = i\pi \) and are found to be \( \sim e^{-\pi\xi} \).

We now turn to the case \( |\varepsilon| \lesssim 1 \). For large \( p \), \( F(\varepsilon, p) \) is determined by the singular points of \( g \) and \( h \). These functions are regular on the real \( \xi \) axis; at \( \xi \to \infty \) they are given by

\[
g(\varepsilon, \xi) = 2\xi^{-1}[e^{-\pi\xi} + t(\varepsilon)^{-1} \sin \vartheta],
\]

\[
h(\varepsilon, \xi) = \xi^{-1} \cos \vartheta,
\]

\[
\vartheta = \xi^2/2 + 2\varepsilon \ln \xi + \arg \Gamma(1/2 - i\varepsilon).
\]

Thus, the main contribution to \( H \) and \( G \) comes from large \( \xi \), as both \( g \) and \( h \) have a singular point \( \xi = \infty \). Using Eq. (18) one can check that the saddle points of \( \cos \vartheta \cos p\xi \) and \( \sin \vartheta \sin p\xi \) are \( \xi = \pm p \). Calculating \( G \) and \( H \) near these points yields the second of Eqs. (7).

For small \( p \) the behavior of \( G \) and \( H \) is different. In \( H \) one can put \( \cos p\xi = 1 \) and \( \sigma = 0 \). In \( G \), however, the limit \( p \to 0 \) should be taken with care: for \( \sigma = 0 \), \( G \) has a singularity of the form \( \text{sgn}(p) \) coming from the non-oscillating term in \( g \). The factor \( \text{erf}(p/\sqrt{2\pi}) \) in the first of Eqs. (7) results from the smoothing of this singularity by the screening factor.

Being proportional to the frequency \( \omega_0 \) of the SAW, the acoustoelectric current is of the pumping type: independent of the value of \( \omega_0 \), a given fraction of the electron charge is transferred through the PC during each period of the SAW. Therefore, the current can be compared with the pumping current produced by two gates with phase shifted a.c. potentials, \( \delta U(x, t) = A_1(t)u_1(x) + A_2(t)u_2(x) \). Let the gates be symmetric, \( u_1(x) = u(x), u_2(x) = u(-x) \) and take \( A_1(t) = A(t) \cos(\omega_0 t - \varphi(t) + \phi), A_2(t) = A(t) \cos(\omega_0 t + \varphi(t)) \), where the amplitude \( A(t) \) and the phase \( \varphi(t) \) are slowly varying, random functions, but the phase shift \( \phi \) is fixed.

One can compare now the pumping factors, Eq. (4), for the SAW and the two gates and see that they are equal if one replaces \( \exp(iqx) \) by \( \exp(i\varphi(qx)) \) and \( \cos(\omega_0 t) \) by \( \cos(\omega_0 t - \varphi(t)) \) and \( \cos(\omega_0 t + \varphi(t)) \).

This work was supported by the Alexander von Humboldt Foundation, the Israel Academy of Sciences (Y.L), the German-Israeli Foundation and the Deutsche Forschungsgemeinschaft (P.W).

[1] J. M. Shilton, D. R. Mace, V. I. Talyanskii, Yu. Galperin, M. Y. Simmons, M. Pepper, and D. A. Ritchie, J. Phys.: Condens. Matter 8, L337 (1996).
[2] J. M. Shilton, V. I. Talyanskii, M. Pepper, D. A. Ritchie, J. E. F. Frost, C. J. Ford, C. G. Smith, and G. A. C. Jones, J. Phys.: Condens. Matter 8, L531 (1996).
[3] V. I. Talyanski, J. M. Shilton, M. Pepper, C. G. Smith, C. J. Ford, E. H. Linfield, D. A. Ritchie, and G. A. C. Jones, Phys. Rev. B56, 15180 (1997).
[4] H. Toland and Yu. M. Galperin, Phys. Rev. B54, 8814 (1996).
[5] V. L. Gurevich, V. B. Pevzner, and G. J. Iaftrade, Phys. Rev. Lett. 77, 3881 (1996).
[6] V. L. Gurevich, V. I. Kozub, and V. B. Pevzner, Phys. Rev. B58, 13088 (1998).
[7] F. A. Maas and Yu. M. Galperin, Phys. Rev. B56, 4028 (1997).
[8] A. Knöbchen, Y. Levinson, and O. Entin-Wohlman, Phys. Rev. B55, 5352 (1997).
[9] Y. B. Levinson, M. I. Lubin, and E. V. Sukhorukov, Phys. Rev. B45, 11936 (1992).
[10] Handbook of Mathematical Functions, edited by M. Abramowitz and A. Stegun, (National Bureau of Standards, Washington, D.C., 1964). Note that in the right hand side of 19.25.4 the sign ± has to be replaced by ±.
[12] Y. Levinson and P. Wölfle, Phys. Rev. Lett 83, 1399 (1999).
[13] Y. Levinson, Phys. Rev. B, to be published (2000).
[14] F. Hekking and Yu. V. Nazarov, Phys. Rev. B44, 9110 (1991).
[15] P. W. Brouwer, Phys. Rev. B58, R10135 (1998).