SUPERSYMMETRY OF M-BRANES AT ANGLES

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Abstract

We determine the possible fractions of supersymmetry preserved by two intersecting M-5-branes. These include the fractions 3/32 and 5/32 which have not occurred previously in intersecting brane configurations. Both occur in non-orthogonal pointlike intersections of M-5-branes but 5/32 supersymmetry is possible only for specific fixed angles.

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1 Introduction

It has become clear over the past few years that many supersymmetric quantum field theories may be realized as worldvolume theories on branes, or on their intersections. One is tempted to conjecture that all anomaly-free interacting supersymmetric quantum field theories (without gravity) may be realized in this way. Although we shall not attempt to establish this conjecture here, it provides the motivation for the work that we shall report on. Interacting supersymmetric field theories without gravity are restricted to spacetime dimensions $D \leq 10$, and the number of supersymmetries for each value of $D$ is also severely restricted. In the context of branes it is convenient to refer to this number as a fraction $\nu$ of the supersymmetry of the M-theory vacuum, which is maximally supersymmetric. For example, M-branes preserve $1/2$ supersymmetry, as do the D-branes of superstring symmetric. Together, these may be considered as the ‘basic’ branes of M-theory and its superstring duals. The worldvolume field theories of these $\nu = 1/2$ branes are either dimensional reductions of $D = 10$ super Maxwell theory or, in the case of the M-5-brane or type II NS-5-branes, the $D = 6$ $(2,0)$-supersymmetric antisymmetric tensor theory. These are non-interacting field theories but the interacting versions are found as theories on coincident parallel branes\footnote{The $D = 10$ super Yang-Mills theory does, strictly speaking, not have a brane interpretation but it is also anomalous.}. There are no other field theories with this fraction of supersymmetry (where by ‘field theories’ it should now be understood that we mean theories without gravity).

There are of course plenty of other supersymmetric field theories with $\nu < 1/2$. Many of these are known to have an interpretation as worldvolume field theories on the intersection of two or more branes. In the case of orthogonal intersections the determination of the fraction of supersymmetry preserved is straightforward: only the fractions $\nu = 1/4, 1/8, 1/16, 1/32$ occur. All these fractions are known to be realized by supersymmetric field theories in various dimensions but various other fractions are also possible when $D \leq 3$. For example, $\nu = 3/16$ is realized in $D = 3$ by topologically-massive super Yang-Mills theory \cite{1,2}. This was recently argued to be the worldvolume field theory on certain non-orthogonal intersections of two IIB 5-branes preserving $3/16$ supersymmetry \cite{3}. As shown in \cite{4}, these configurations are dual to certain non-orthogonal $D = 2$ intersections of two M-5-branes, for which the effective field theory is presumably the $(3,3)$-supersymmetric dimensional reduction of the $D = 3$ $N = 3$ supersymmetric gauge theory. The M(atrix) theory description of these $\nu = 3/16$ configurations...
has also been found recently \cite{4}.

Various other values of $\nu$ can be realized by $D = 2$ supersymmetric sigma-models with $(p, q)$ supersymmetry. For example $(1,2)$ supersymmetry yields $\nu = 3/32$ while $(1,4)$ supersymmetry yields $\nu = 5/32$. Since $p, q = 0, 1, 2, 4$, the fractions $\nu = k/32$ with $k = 1, 2, 3, 4, 5, 6, 8$ can be found in this way\cite{4}. Note that the fraction $\nu = 7/32$ is absent; in fact, there is no known $\nu = 7/32$ supersymmetric field theory. On the other hand, the fractions $\nu = 3/32$ and $\nu = 5/32$, which do correspond to known $D = 2$ (and hence also $D = 1$) supersymmetric field theories, have not yet been found as fractions of supersymmetry preserved by the intersection of two branes. Clearly, they must be found if the conjecture that all (anomaly-free) supersymmetric field theories are realizable as worldvolume intersection field theories is to have a chance of being true. We shall show here that these fractions are realized by a pair of M-5-branes intersecting at certain angles. We accomplish this by an exhaustive analysis of the fractions of supersymmetry preserved by a pair of intersecting M-5-branes. Specifically, we shall show that the fraction of supersymmetry preserved by such configurations necessarily takes one of the following values:

$$\nu = \frac{1}{2}, \frac{1}{4}, \frac{3}{16}, \frac{5}{32}, \frac{1}{8}, \frac{3}{16}, \frac{1}{16}, \frac{1}{32}. \quad (1)$$

The fraction $1/2$ occurs only for parallel M-5-branes which, strictly speaking, is not an ‘intersecting brane’ configuration but it is convenient to include this case as it will be the starting point for obtaining all the other possibilities by rotation of one of the M-5-branes. The fractions $5/32, 3/32$ and $1/32$ occur only when the tangent vectors to the two 5-branes span the entire ten-dimensional space. Thus, these fractions cannot be realized by, for example, intersecting M-2-branes. This is consistent with the fact that fractions of the form $\nu = (2n + 1)/32$ are possible only for pointlike ($D = 1$) or stringlike ($D = 2$) intersections; for intersecting M-5-branes the intersection is pointlike. It is possible that these fractions are also realized by other intersecting brane configurations with stringlike intersections, but it is unlikely that fractions $\nu$ other than those listed above will be found in this way. The current state of knowledge concerning the conditions imposed by supersymmetry in various dimensions is essentially complete\cite{4}, and there is no known example with a number of supersymmetries other than those implied

\footnote{The fraction $\nu = 3/16$ arises from the possibility of $(2,4)$ supersymmetry. It is not clear at present whether such field theories can be realized as worldvolume field theories on intersecting branes. If so, it must be that the branes are intrinsically intersecting, as against merely overlapping, since if they could be pulled apart all fields would be massive and the intersection field theory could not be chiral.}

\footnote{For example, a complete analysis for $(p, q)$ sigma-models can be found in \cite{1, 2}. A partial analysis of $D = 1$ sigma-models can be found in \cite{3, 4}; it would be desirable to have a complete analysis for this case.}
by (1). Furthermore, it seems likely that any configuration of two intersecting branes will be in the same ‘duality equivalence class’ as one involving only M-5-branes. This is certainly true for orthogonal intersections for which there are just two duality equivalence classes; the M-5-brane representatives are the intersection of M-5-branes over a 3-plane [9, 10] and M-5-branes intersecting on a line [11].

2 M-5-branes at angles

We start from two parallel M-5-branes. We can choose cartesian coordinates such that these M-5-branes lie in the 12349 5-plane. This configuration is summarised by the array

\[
M : 1 \quad 2 \quad 3 \quad 4 \quad - \quad - \quad - \quad 9 \quad - \\
M : 1 \quad 2 \quad 3 \quad 4 \quad - \quad - \quad - \quad 9 \quad - ,
\]

and is associated with the constraint

\[
\Gamma_{091234} \epsilon = \epsilon .
\]

The $SO(1,10)$ spinors $\epsilon$ satisfying this relation can be considered as the asymptotic values of Killing spinor fields of an associated $D = 11$ supergravity solution. We shall therefore refer to them as ‘Killing spinors’. The parallel M-5-branes may be coincident or they may be separated by some distance in the transverse directions. The distinction will not be of relevance to the following discussion, but if the M-5-branes are not coincident the rotation of one will lead to a configuration of ‘overlapping’, rather than intersecting, branes. For convenience we ignore this distinction here, and refer only to ‘intersecting’ branes.

We now fix one M-5-brane, the ‘first’, and rotate the second one. Denoting the spinor representation of the rotation matrix for the second M-5-brane by $R$, we now have an additional constraint [12]

\[
R \Gamma_{091234} \epsilon = \epsilon .
\]

As explained in [3, 13], $R$ effectively depends on five independent angles and can be chosen to take the form

\[
R = e^{\frac{1}{2} [\vartheta \Gamma_{15} + \psi \Gamma_{26} + \varphi \Gamma_{37} + \rho \Gamma_{48} + \zeta \Gamma_{90}]} ,
\]

where we use the symbol $\natural$ for the number 10, and $\vartheta, \psi, \varphi, \rho$ and $\zeta$ are the five angles characterising the rotation. Note that $\Gamma_{091234} R^{-1} = R \Gamma_{091234}$, so that (4) becomes $R^2 \Gamma_{091234} \epsilon = \epsilon$. 

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Hence the condition (3) becomes

\[ [R^2 - 1] \epsilon = 0, \]  

(6)

with \( R \) given by (3).

We wish to determine the number of simultaneous solutions (equal to 32\( \nu \)) of (3) and (4) as a function of the five angles characterising the rotation matrix \( R \). The previously known partial solutions to this problem were summarised in [E]. Here we present the general solution. We first note that

\[
R^2 - 1 = 2R \Gamma_{15} \left[ \sin \frac{\vartheta}{2} \cos \frac{\psi}{2} \cos \frac{\varphi}{2} \cos \frac{\rho}{2} \cos \frac{\zeta}{2} - \Gamma_{1526} \cos \frac{\vartheta}{2} \sin \frac{\psi}{2} \cos \frac{\varphi}{2} \cos \frac{\rho}{2} \cos \frac{\zeta}{2} \right. \\
- \Gamma_{1537} \cos \frac{\vartheta}{2} \cos \frac{\psi}{2} \sin \frac{\varphi}{2} \cos \frac{\rho}{2} \cos \frac{\zeta}{2} - \Gamma_{1548} \cos \frac{\vartheta}{2} \cos \frac{\psi}{2} \cos \frac{\varphi}{2} \sin \frac{\rho}{2} \cos \frac{\zeta}{2} \\
- \Gamma_{1592} \cos \frac{\vartheta}{2} \cos \frac{\psi}{2} \cos \frac{\varphi}{2} \sin \frac{\rho}{2} \sin \frac{\zeta}{2} - \Gamma_{15374892} \cos \frac{\vartheta}{2} \sin \frac{\psi}{2} \cos \frac{\varphi}{2} \sin \frac{\rho}{2} \sin \frac{\zeta}{2} \\
- \Gamma_{1526374892} \cos \frac{\vartheta}{2} \sin \frac{\psi}{2} \sin \frac{\varphi}{2} \sin \frac{\rho}{2} \sin \frac{\zeta}{2} + \Gamma_{4892} \sin \frac{\vartheta}{2} \cos \frac{\psi}{2} \sin \frac{\varphi}{2} \sin \frac{\rho}{2} \sin \frac{\zeta}{2} \\
+ \Gamma_{3792} \sin \frac{\vartheta}{2} \cos \frac{\psi}{2} \sin \frac{\varphi}{2} \sin \frac{\rho}{2} \sin \frac{\zeta}{2} + \Gamma_{3748} \sin \frac{\vartheta}{2} \cos \frac{\psi}{2} \sin \frac{\varphi}{2} \cos \frac{\rho}{2} \sin \frac{\zeta}{2} \\
+ \Gamma_{2692} \sin \frac{\vartheta}{2} \sin \frac{\psi}{2} \cos \frac{\varphi}{2} \cos \frac{\rho}{2} \sin \frac{\zeta}{2} + \Gamma_{2648} \sin \frac{\vartheta}{2} \sin \frac{\psi}{2} \cos \frac{\varphi}{2} \cos \frac{\rho}{2} \sin \frac{\zeta}{2} \\
+ \Gamma_{2637} \sin \frac{\vartheta}{2} \sin \frac{\psi}{2} \sin \frac{\varphi}{2} \cos \frac{\rho}{2} \cos \frac{\zeta}{2} + \Gamma_{26374892} \sin \frac{\vartheta}{2} \sin \frac{\psi}{2} \sin \frac{\varphi}{2} \sin \frac{\rho}{2} \sin \frac{\zeta}{2} \right]. \]  

(7)

Since the gamma matrix products appearing in (7) commute with each other and with \( \Gamma_{091234} \), we can simultaneously diagonalize all these matrices, and since each of them squares to the identity their eigenvalues are all \( \pm 1 \). Moreover, the traces of these matrices, and the traces of products of pairs of them, vanish. We can therefore arrange for these matrices to take the form

\[
\Gamma_{09123} = \text{diag}((1, \cdots, 1, -1, \cdots, -1)), \\
\Gamma_{1526} = \text{diag}((1, \cdots, 1, -1, \cdots, -1, \cdots)), \\
\Gamma_{1537} = \text{diag}((1, \cdots, 1, -1, \cdots, -1, \cdots, 1, -1, \cdots, -1, \cdots)), \\
\Gamma_{1548} = \text{diag}((1, 1, -1, -1, 1, -1, -1, 1, -1, -1, 1, -1, -1, \cdots)), \\
\Gamma_{1592} = \text{diag}((1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, -1, \cdots)), \\
\Gamma_{15374892} = \text{diag}((1, \cdots, 1, -1, \cdots, -1, \cdots, 1, -1, \cdots, -1, \cdots)), \\
\Gamma_{1526374892} = \text{diag}((1, \cdots, 1, -1, \cdots, -1, \cdots, 1, -1, \cdots, -1, \cdots)), \\
\Gamma_{4892} = \text{diag}((1, \cdots, 1, -1, \cdots, -1, \cdots, 1, -1, \cdots, -1, \cdots)). 
\]  

(8)

the rest being determined by the products of those given. Note that in this basis the first condition (3) projects out the second 16 components of the Killing spinor \( \epsilon \), leaving just the
first 16 components. Our task is therefore to determine the consequences of the second condition (3) for the first 16 components.

To proceed with the analysis we use (8) in (7) to derive the following result:

\[
R^2 - 1 = 2R \Gamma_{15} \times \text{diag} \left( \frac{\vartheta - \psi - \varphi - \rho - \zeta}{2}, \frac{\vartheta - \psi - \varphi - \rho + \zeta}{2}, \frac{\vartheta - \psi + \varphi - \rho - \zeta}{2}, \frac{\vartheta - \psi + \varphi + \rho - \zeta}{2}, \frac{\vartheta + \psi - \varphi + \rho - \zeta}{2}, \frac{\vartheta + \psi - \varphi + \rho + \zeta}{2}, \frac{\vartheta + \psi + \varphi - \rho - \zeta}{2}, \frac{\vartheta + \psi + \varphi + \rho - \zeta}{2}, \frac{\vartheta + \psi + \varphi + \rho + \zeta}{2}, \cdots \right),
\]

where the last 16 components are omitted because, for the reason just given, they are not needed for the determination of the fraction \(\nu\) of unbroken supersymmetry. We shall now use this result to provide a systematic analysis of the possible values of \(\nu\).

### 2.1 One angle

We begin with the simplest case of a rotation by single angle \(\vartheta\). Setting the other angles to zero, we have

\[
R^2 - 1 = 2R \Gamma_{15} \times \text{diag} \left( \frac{\vartheta - \psi - \varphi - \rho - \zeta}{2}, \frac{\vartheta - \psi - \varphi - \rho + \zeta}{2}, \frac{\vartheta - \psi + \varphi - \rho - \zeta}{2}, \frac{\vartheta - \psi + \varphi + \rho - \zeta}{2}, \frac{\vartheta + \psi - \varphi + \rho - \zeta}{2}, \frac{\vartheta + \psi - \varphi + \rho + \zeta}{2}, \frac{\vartheta + \psi + \varphi - \rho - \zeta}{2}, \frac{\vartheta + \psi + \varphi + \rho - \zeta}{2}, \frac{\vartheta + \psi + \varphi + \rho + \zeta}{2}, \cdots \right),
\]

where \(\vartheta\) is the 16 \(\times\) 16 identity matrix. Supersymmetry is completely broken unless \(\sin \frac{\vartheta}{2} = 0\), i.e. \(\vartheta = 0\) (mod 2\(\pi\)).

### 2.2 Two angles

We now have

\[
R^2 - 1 = 2R \Gamma_{15} \times \text{diag} \left( \frac{\vartheta - \psi}{2}, \frac{\vartheta + \psi}{2} \right), \quad 18 \times 18
\]

where \(18\) is the 8 \(\times\) 8 identity matrix. Supersymmetry is completely broken unless \(\vartheta \pm \psi = 0\) (mod 2\(\pi\)). When this condition is satisfied the fraction of unbroken supersymmetry is \(1/4\). This includes as a special case the orthogonal intersection over a 3-plane of two M-5-branes. We thus recover the result of [12] that rotations away from orthogonality can preserve 1/4 supersymmetry.
2.3 Three angles

We now have

\[ R^2 - 1 = 2R_\Gamma 15 \times \text{diag.} \left( \sin \frac{\vartheta - \psi - \varphi}{2} \mathbf{1}_4, \sin \frac{\vartheta - \psi + \varphi}{2} \mathbf{1}_4, \sin \frac{\psi - \varphi}{2} \mathbf{1}_4, \sin \frac{\psi + \varphi}{2} \mathbf{1}_4, \cdots \right), \]

(12)

where \( \mathbf{1}_4 \) is the 4×4 identity matrix. Supersymmetry is completely broken unless \( \vartheta \pm \psi \pm \varphi = 0 \) (mod \( 2\pi \)). When this condition is satisfied the fraction of unbroken supersymmetry is 1/8; an example involving 6-branes was given in [12]. Note that the condition on the three angles imposed by supersymmetry does not allow a rotation to a configuration of orthogonally intersecting branes. This shows that rotations away from orthogonality do not yield all possible supersymmetric configurations of branes intersecting at angles; one must instead consider rotations away from parallel branes, as we are doing here.

2.4 Four angles

We have

\[ R^2 - 1 = 2R_\Gamma 15 \times \text{diag.} \left( \sin \frac{\vartheta - \psi - \varphi - \rho}{2} \mathbf{1}_2, \sin \frac{\vartheta - \psi - \varphi + \rho}{2} \mathbf{1}_2, \sin \frac{\vartheta + \psi - \varphi - \rho}{2} \mathbf{1}_2, \sin \frac{\vartheta + \psi + \varphi - \rho}{2} \mathbf{1}_2, \sin \frac{\psi + \varphi + \rho}{2} \mathbf{1}_2, \sin \frac{\psi + \varphi - \rho}{2} \mathbf{1}_2, \cdots \right), \]

(13)

where \( \mathbf{1}_2 \) is the 2×2 identity matrix.

Supersymmetry is completely broken unless \( \vartheta \pm \psi \pm \varphi \pm \rho = 0 \) (mod \( 2\pi \)). We are free to change the signs of the angles, so without loss of generality we may set

\[ \rho = \vartheta - \psi + \varphi. \]

(14)

The generic configuration of this type preserves 1/16 supersymmetry, a fraction found previously only in orthogonal intersections of four branes. Thus, these are new supersymmetric intersecting brane configurations. In special cases the supersymmetry is enhanced. For example we have (generically) 1/8 supersymmetry for the following special values considered in [13]:

\[
\begin{aligned}
& \vartheta = \psi \neq \varphi = \rho , \\
& \text{or} \quad \vartheta = \rho \neq \psi = \varphi , \\
& \text{or} \quad \vartheta = -\varphi \neq \rho = -\psi .
\end{aligned}
\]

(15)
The further special cases in which the inequalities in (15) are replaced by equalities up to sign, i.e.

\[
\begin{align*}
&\vartheta = \psi = \pm \varphi = \pm \rho , \\
&\text{or}\quad \vartheta = \rho = \pm \psi = \pm \varphi , \\
&\text{or}\quad \vartheta = -\varphi = \pm \rho = \mp \psi ,
\end{align*}
\]

preserve $3/16$ supersymmetry. This includes the case discussed in ref. [3] in which all four angles are equal. Finally if the equal four angles in eq. (16) take the special values $\pm \frac{\pi}{2}$, then we have $1/4$ supersymmetry.

2.5 Five angles

For the general case of five independent angles we must return to consider (9). Supersymmetry is completely broken unless $\vartheta \pm \psi \pm \varphi \pm \rho \pm \zeta = 0$ (mod $2\pi$). To investigate the various possibilities that this condition allows, we set $\zeta = -\vartheta - \psi - \varphi - \rho$ (mod $2\pi$). Eq. (9) then becomes

\[
R^2 - 1 = 2R\Gamma_{15} \times \text{diag.} (\sin \vartheta, -\sin(\psi + \varphi + \rho), \sin(\vartheta + \rho), -\sin(\psi + \varphi), \sin(\vartheta + \varphi), -\sin(\psi + \rho), \sin(\vartheta + \varphi + \rho), -\sin \varphi, \sin(\vartheta + \psi + \varphi), -\sin \rho, \sin(\vartheta + \psi + \varphi + \rho), 0, \cdots),
\]

showing that, generically, $1/32$ supersymmetry is preserved. However, there are various special cases to consider when one or more of the arguments of the sine functions vanish. Let $\rho + \psi + \varphi = 0$ (so $\zeta = -\vartheta$); one can show that other choices give essentially the same results. Then eq. (17) becomes

\[
R^2 - 1 = 2R\Gamma_{15} \times \text{diag.} (\sin \vartheta, 0, \sin(\vartheta - \psi - \varphi), -\sin(\psi + \varphi), \sin(\vartheta + \varphi), -\sin \varphi, \sin(\vartheta - \psi), -\sin \psi, \sin(\vartheta + \psi), \sin \psi, \sin(\vartheta - \varphi), -\sin \varphi, \sin(\vartheta + \psi + \varphi), \sin(\psi + \varphi), \sin \vartheta, 0, \cdots),
\]

which yields $1/16$ supersymmetry, generically, although there are now various subcases to consider with enhanced supersymmetry. Some of these reduce the problem to one already considered. For example, if (in addition to the restrictions already being considered) we set $\varphi + \psi = 0$ then $\rho = 0$ and the problem is reduced to the four-angle case. However, the choice $\varphi = \vartheta - \psi$ yields

\[
R^2 - 1 = 2R\Gamma_{15} \times \text{diag.} (\sin \vartheta, 0, 0, -\sin \vartheta, \sin(2\vartheta - \psi), \sin(\vartheta - \psi), -\sin \psi, \sin(\vartheta + \psi), \sin \vartheta, -\sin(\vartheta - \psi), \sin(2\vartheta), \sin \vartheta, 0, \cdots),
\]
which gives $3/32$ supersymmetry, a fraction not previously seen. In this case we have $\rho = \zeta = -\vartheta$. There are now several subcases in which supersymmetry is further enhanced. These are as follows:

1. $\psi = 2\vartheta$: $1/8$ supersymmetry. (Here we have $\varphi = \rho = \zeta = -\vartheta$.)
   
   When $\vartheta = \pm \frac{\pi}{3}$ supersymmetry is further enhanced to $5/32$, again a new fraction not previously seen.
   
   When, instead, $\vartheta = \pm \frac{\pi}{2}$ we have $1/4$ supersymmetry.

2. $\psi = \vartheta$: $3/16$ supersymmetry. (Here we have $\varphi = 0, \rho = \zeta = -\vartheta$.)
   
   When $\vartheta = \pm \frac{\pi}{2}$ supersymmetry is again enhanced to $1/4$.

3. $\psi = -\vartheta$: $1/8$ supersymmetry. (Here we have $\varphi = 2\vartheta, \rho = \zeta = -\vartheta$.)
   
   Again supersymmetry is enhanced to $5/32$ for $\vartheta = \pm \frac{\pi}{3}$.
   
   When, instead, $\psi = -\vartheta = \rho = \zeta = \pm \frac{\pi}{2}, \varphi = \mp \pi$, we have $1/4$ supersymmetry.

Other special choices do not produce anything new. For example, setting $\varphi = -\vartheta$ (so that $\rho = -\psi + \vartheta, \zeta = -\vartheta$) we have

$$R^2 - 1 = 2R\Gamma_{15} \times \text{diag.} \left( \sin \vartheta, 0, \sin(2\vartheta - \psi), \sin(\vartheta - \psi), 0, -\sin \vartheta, \sin(\vartheta - \psi), -\sin \psi, \sin(\vartheta + \psi), \sin \psi, \sin(2\vartheta), \sin \vartheta, \sin \psi, \sin(\psi - \vartheta), \sin \vartheta, 0, \cdots \right).$$

But this differs from (19) only by a permutation of the entries. In summary the possible fractions of supersymmetry that can be preserved by rotations with five independent angles are $1/32$, $1/16$, $3/32$, $1/8$ and $5/32$ (which occurs only for fixed angles). The fractions $3/16$ and $1/4$ are also possible but only if at least one of the angles vanishes (or equals $\pm \pi$ for the latter case).

### 3 Discussion

We have found two new fractions of supersymmetry preserved by intersecting M-brane configurations, $\nu = 3/32$ and $\nu = 5/32$. These fractions are realized by particular configurations of two M-5-branes intersecting non-orthogonally on a point. In the $5/32$ case, the relative orientation of the two M-5-branes is completely fixed. We have arrived at these conclusions by a purely algebraic analysis of the conditions imposed on Killing spinors by the presence of M-5-branes with given orientations. It would be of interest to find solutions of $D = 11$ supergravity corresponding to these new configurations. In principle, the new fractions might also be realized
on stringlike intersections of other branes, e.g. of IIA 5-branes with 6-branes. It would be of interest to determine whether such configurations exist.

It is known, in some cases, that there is a close relation between the number of supersymmetries preserved by intersecting brane configurations and reduced holonomy in Kaluza-Klein compactifications \[12\]. This is especially clear in the case of 3/16 supersymmetry. As shown in \[3\], and confirmed here, this fraction arises in a particular class of intersecting M-5-brane configurations, but it also arises on compactification of \(D = 11\) supergravity on (hyper-Kähler) 8-manifolds of \(Sp(2)\) holonomy. That this is not a coincidence is shown by the fact that the \(D = 11\) supergravity solution corresponding to the intersecting M-5-brane configuration is dual to a ‘compactification’ of \(D = 11\) supergravity on a particular class of (non-compact) hyper-Kähler 8-manifolds \[3\]. In the latter case, supersymmetry is preserved because the holonomy is reduced from \(SO(8)\) to the subgroup \(Sp(2)\). In view of this it seems plausible that the new 1/16 supersymmetric ‘4-angle’ M-5-brane configurations found above are related to 8-manifolds of \(Spin(7)\) holonomy. When considering ‘five-angle’ M-5-brane configurations it is therefore natural to wonder whether there might be a connection with reduced holonomy subgroups of \(SO(10)\). The only candidate subgroups that are not also subgroups of \(SO(8)\) are \(SU(3) \times SU(2)\) and \(SU(5)\), both of which lead to \(\nu = 1/16\) in the context of Kaluza-Klein compactifications of \(D = 11\) supergravity. There are no subgroups of \(SO(10)\) that yield the new fractions found here, \(\nu = 3/32, 5/32\). Thus, while it is possible that the configurations of intersecting M-5-branes with \(\nu = 1/16\) are related by duality to Kaluza-Klein ‘compactifications’ of \(D = 11\) supergravity, this is not possible for the intersecting brane configurations preserving 3/32 or 5/32 supersymmetry.

The fraction \(\nu = 3/32\) can be obtained by compactification of the heterotic string on hyper-Kähler 8-manifolds because this preserves 3/16 of the supersymmetry of the heterotic string vacuum, which itself breaks the supersymmetry of the M-theory vacuum. In fact, by a modification of arguments presented in \[3\], it is not difficult to see that such compactifications lead to \(D = 2\) theories with (3,0) supersymmetry. This might seem surprising since (3,0) supersymmetric field theories are normally automatically (4,0) supersymmetric. However, for genuine compactifications (i.e. on compact manifolds) the lower dimensional field theory includes gravity and there certainly do exist (3,0) \(D = 2\) supergravity theories. It is of course possible to consider the same non-compact hyper-Kähler 8-manifolds in the context of the heterotic string as were considered in \[3\] in the context of M-theory but there is no guarantee that these field configurations will be dual to intersecting brane configurations with \(D = 2\) intersections. In-
deed, the above considerations based on holonomy appear to exclude it. The ν = 5/32 case is rather simpler to analyse. There are no Kaluza-Klein compactifications of any supergravity theory that can preserve this fraction of (D = 11) supersymmetry. Thus, the intersecting brane interpretation is the only way in which D = 1 N = 5 supersymmetric or D = 2 (1,4) supersymmetric field theories can be obtained from M-theory.

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