Formation of Common Investment Networks by Project Establishment between Agents

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Abstract

We present an investment model integrated with trust-reputation mechanisms where agents interact with each other to establish investment projects. We investigate the establishment of investment projects, the influence of the interaction between agents in the evolution of the distribution of wealth, as well as the formation of common investment networks and some of their properties. Simulation results show that the wealth distribution presents a power law in its tail. Also, it is shown that the trust and reputation mechanism presented leads to the establishment of networks among agents, which present some of the typical characteristics of real-life networks like a high clustering coefficient and short average path length.

Keywords: agent-based computational economics, trust/reputation dynamics, investment networks

1 Introduction

Recently, different socio-economical problems have been modeled using agent-based simulations, presenting a different perspective (usually more flexible and realistic) for modeling social and economical behavior. Many important contributions to this field are provided by the research group called agent-based computational economics (ACE) \[1\] \[2\]. Different ACE models have been proposed to study for example the relationship between market structure and worker-employer interaction networks \[5\], investors and brokers in financial markets \[6\], among others. A key concern in many of these studies is to understand the loyalty from buyers to sellers by means of repeated business \[5\], as well as the mechanisms for coalition formation between agents which for example may depend on voluntary agreement and payoff of the agents \[3\].

Other interesting contributions have been made to understand the emergence of networks between the agents, for example trading networks among buyers

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and sellers who adaptively select their trade partners by looking at their past experiences [11]. Finally, also of interest is to investigate the topology of the networks emerging from the interaction between agents, usually using methods borrowed from the field of statistical physics [1].

The main goal of this paper is to improve the understanding of two main problems in ACE models: (i) the economical component that describes the dynamics of the wealth distribution among agents; and (ii) the social component that describes the dynamics of loyalty, trust and reputation among agents. For this, we integrate in this article a wealth distribution model based on constant proportional investments and a network formation model where agents interact with each other to establish investment projects.

2 The Model

2.1 Wealth dynamics

Consider an agent-based system populated with $N$ agents, where each agent possesses a budget $x_k(t)$ (measure of its “wealth” or “liquidity”) that evolves over time given the following dynamic:

$$x_k(t+1) = x_k(t) \left[1 + r_{mk}(t) q_k(t) \right] + a(t),$$  

(1)

where $r_{mk}(t)$ denotes the return on investment (RoI) that the agent $k$ receives from its investment $q_k(t)$ in project $m$. $q_k(t)$ denotes a proportion of investment, i.e the fraction or ratio of the budget of agent $k$ that the agent prefers to invest in a market and $a(t)$ denotes an external income.

In this model, agent $k$ invests a portion $q_k(t)x_k(t)$ of its total budget at every time step $t$ yielding a gain or loss in the market $m$, expressed by $r_{mk}(t)$. Similar wealth models have been presented in [9, 4, 7] where the dynamics of the investment model are investigated using some results from the theory of multiplicative stochastic additive processes [2, 10].

Note that this approach assumes that the market, which acts as an environment for the agent, is not influenced by its investments, i.e. the returns are exogenous and the influence of the market on the agent is simply treated as random. This is a crucial assumption which makes this approach different from other attempts to model real market dynamics, e.g. in financial markets [6].

2.2 Trust-reputation mechanisms and project establishment

In order to launch a particular investment project $m$ at time $t$, a certain minimum amount of money $I_{\text{thr}}$ needs to be collected among the agents. The existence of the investment threshold $I_{\text{thr}}$ is included to enforce the interaction between agents, as they need to collaborate until the following condition is reached:

$$I_m(t) = \sum_{k}^{N_m} q_k(t) x_k(t) \geq I_{\text{thr}},$$  

(2)

where $N_m$ is the number of agents collaborating in the particular investment project $m$. There may be different investment projects $m$ at the same time, but
for simplicity, it is assumed that each agent participates in only one investment project at a time.

The first essential feature to be noticed for the formation of common investment networks is the establishment of preferences between agents. It is assumed that the decision of an agent to collaborate in a project will mainly depend on the previous history it has gained with other agents. Consider an agent \( k \) which accepts to collaborate in the common investment project \( m \) initiated by agent \( j \). Thus, agent \( k \) receives the following payoff at time \( t \):

\[
p_{kj}(t) = x_k(t) q_k(t) r_m(t).
\]

(3)

Reiterated interactions between agent \( k \) and agent \( j \) lead to different payoffs over time that are saved in a decision weight:

\[
w_{kj}(t + 1) = p_{kj}(t) + w_{kj}(t) e^{-\gamma},
\]

(4)

where \( \gamma \) represents the memory of the agent with initial condition \( w_{kj}(0) = 0 \).

The payoffs obtained from previous time steps \( t \) may have resulted from the collaborative action of different agents, however, these are unknown to agent \( k \), i.e agent \( k \) only realizes the initiator of the project, agent \( j \). Furthermore, in order to mirror reality, it is assumed that there are more investors than initiators of projects. For this, we consider that from the population of \( N \) agents only a small number \( J \) are initiators, i.e. \( J \ll N \), where the reputation of an initiator \( j \) can be calculated as follows (for more on trust and reputation models see [8]):

\[
W_j(t + 1) = \sum_{k=0}^{N} w_{kj}(t); \quad W_k(t + 1) = \sum_{j=0}^{J} w_{kj}(t).
\]

(5)

At every time step \( t \) an initiator is chosen randomly from the population and assigned with an investment project. The initiator randomly tries to convince other agents to invest in the project until an amount larger than the threshold \( I_{\text{thr}} \) has been collected. For this, we use a Gibbs or Boltzmann distribution to determine the probability that the contacted agent \( k \) may accept the offer of agent \( j \):

\[
\tau_{kj}(t) = \frac{e^{\beta w_{kj}(t)}}{\sum_{i=1}^{J} e^{\beta w_{ki}(t)}},
\]

(6)

where in terms of the weight \( w_{kj} \), the probability \( \tau_{kj}(t) \) considers the good or bad previous experience with agent \( j \) with respect to the experience obtained with other initiators; and \( \beta \) denotes the greediness of the agent, i.e. how much importance does the agent give to the decision weight \( w_{kj} \). In order to take a decision, agent \( k \) uses a technique analogous to a roulette wheel where each slice is proportional in size to the probability value \( \tau_{kj}(t) \). Thus, agent \( k \) draws a random number in the interval \((0, 1)\) and accepts to invest in the project of agent \( j \) if the segment of agent \( j \) in the roulette spans the random number. Finally, an initiator \( j \) stops to contact other agents if either the investment project has reached the threshold \( I_{\text{thr}} \) or if all agents in the population have been asked for collaboration. If the project could be launched it has to be evaluated. The evaluation should in general involve certain “economic” criteria that also reflects the nature of the project. However, for simplicity we assume that the failure or success of an investment project \( I_m \) is randomly drawn from a uniform distribution, i.e. \( r(t) \sim U(-1, 1) \).
3 Results of Computer Simulations

We performed some simulations using the parameter values in Table 1, for simplicity, we assume that the initial budget is the same for all agents. Moreover, the proportion of investment is assumed to be constant and the same for all agents i.e. $q_k(t) = q = \text{const.}$.

Table 1: Parameter values of the computer experiments for the investment networks formation model.

| Parameter                  | Value          |
|----------------------------|----------------|
| Num. of agents             | $N = 10^4$     |
| Num. of initiators         | $J = 100$      |
| Num. of time steps         | $t = 10^5$     |
| Investment threshold       | $I_{thr} = 9$  |
| Return on Investment       | $r \sim U(-1,1)$ |

Fig. 1 (left) shows the evolution of the budget distribution over time for $q = 0.5$. Note that the probability distribution of the budget converges to a stationary distribution with a power law in the tail, a property of investment models based on multiplicative processes repelled from zero [10, 7].

Fig. 1 (right) shows the distribution of the budget at time step $t = 10^5$ for different proportion of investment $q_k = q$. Note that even for a large number of time steps, the budget distribution for agents with a proportion of investment of $q = 0.1$ has not yet converged to a stationary distribution, whereas for $q = 0.5$ and $q = 0.9$, the distribution reached a stationary state after $t = 70000$ and $t = 50000$ time steps, respectively.

Figure 1: (left) Evolution of the budget distribution over time for $q = 0.5$; (right) budget distribution at time step $t = 10^5$ for different proportion of investment $q$. Additional parameters as in Table 1.

Now, in order to understand the role that initiators play in the dynamics of the investment model, we examine the evolution of their budget and reputation over time. For the sake of clarity, we show the rank-size distribution of the budget instead of the probability distribution of the budget. Fig. 2 (left) shows the rank-size distribution of the budget of the initiators, note that the slope
of the distribution increases over time. It is also interesting to examine the evolution of the budget of the initiator with the largest and the smallest budget at the end of the simulation. This is shown in the inset of Fig. 2 (left), note that the budget of the best agent was not always increasing over time. Fig. 2 (right) shows the rank-size distribution of the reputation of the initiators, Eq. (5), note that the distribution does not change over time and only for a small number of agents there is a significant increase or decrease on reputation over time. Moreover, it can be shown that the average value of the reputation has a shift to larger positive values over the course of time. This occurs due to aggregation over time of the external incomes $a(t)$ in Eq. (1) into the dynamics of the decision weights in Eq. (4). Moreover, the inset in Fig. 2 (right) shows the reputation of the best and the worst initiator indicating the presence of no symmetrical positive/negative reputation values.

Figure 2: Evolution of the rank-size distribution of initiators for: (left) budget; (right) reputation. Insets show the budget and reputation, respectively, of the best and the worst initiator. Additional parameters as in Table 1.

The influence of the other parameters in the dynamics of the model was also analyzed, however, for the sake of brevity, we discuss only the role of the number of initiators $J$ in the dynamics. It can be shown that if a less number of initiators is considered, then more investors will be willing to invest in their projects, leading to a larger amount of investment that can be collected by the initiators. It was mentioned before that the tail of the wealth distribution has a power law distribution and it can be shown that the larger $J$ the larger the slope of the power law. The reason for this is that a small number of initiators collect more money from the investors leading to larger profits and losses which over time lead to wider distributions than for a large number of initiators.

4 Structure of Common Investment Networks

In this section, we analyze the topology of the networks for different constant proportion of investment. For this, we run different computer experiments for a small population of agents $N = 1000$ ($N$ is also the number of nodes in the network) and the other parameter values as in Table 1. The first experiment investigates the influence of the proportion of investment in the properties of the network. Fig. 3 shows the networks emerging from the investment and
trust-reputation models for different proportion of investment $q$ at time step $t = 1000$. Note that these networks have two types of nodes, the red(bold) nodes represent investors and the blue(gray) nodes represent initiators. Based on visual impression, the density of the network decreases with respect to the proportion of investment. This occurs because agents investing more also tend to loose more, which leads to more mistrust.

However, from the visual representation of the network it is not possible to draw many conclusions from the dynamics of the networks. Thus, we obtained the following typical properties of the networks:

- Number of links $V$ and maximal degree $k_{\text{max}}$ (the degree of the highest degree vertex of the network).
- Average path length $l$: the average shortest distance between any pair of nodes in the network. Small world networks have a small average path length which scales logarithmically with the size of the network, i.e. $l \sim \log N$ [13].
- Clustering coefficient $C$: measures the transitivity of the network. It has been shown that in real social networks the clustering coefficient is usually much larger than the clustering coefficient in a random network with the same number of nodes and links [13].

For the sake of brevity, we present on the following the most important results for our analysis on the previous listed properties of the networks. First, it can be shown that the number of links $V$ over time fits a power law where the slope of the power law decreases if the proportion of investment increases. The maximal degree $k_{\text{max}}$ of the network also increases over time with a power-law behavior. It can be shown that the clustering coefficient $C$ is larger for small proportions of investment. This occurs because a small proportion of investment leads to a higher clustering in the network due to the mistrust that large losses generate in the investors.

Table 2 shows some of the most important characteristics for different number of investors $N$ and initiators $J$ for a large number of time steps, i.e. $t = 10^5$. For each network we indicate the average degree $\langle k \rangle$ (the first moment of the degree distribution), the average path length $l$ and the clustering coefficient $C$. For comparison reasons we include the average path length $l_{\text{rand}} = \log (N) / \log (\langle k \rangle)$ and the clustering coefficient $C_{\text{rand}} = \langle k \rangle / N$ that can be obtained from a random network with the same average degree $\langle k \rangle$ of the investment networks.

| $N$   | $J$ | $V$  | $k_{\text{max}}$ | $\langle k \rangle$ | $l$    | $C$    | $l_{\text{rand}}$ | $C_{\text{rand}}$ |
|------|-----|------|------------------|---------------------|--------|--------|-------------------|------------------|
| 1000 | 10  | 4847 | 517              | 0.9694              | 2.05766| 0.74557| -                 | 0.0009694       |
| 2000 | 20  | 19972| 1050             | 3.9944              | 1.99365| 0.71337| 5.488            | 0.0019972       |
| 3000 | 30  | 41073| 1475             | 8.2146              | 1.99314| 0.71130| 3.8018           | 0.0027382       |
| 10000| 100 | 134279| 1477         | 26.86               | 2.1563 | 0.24136| 2.7989           | 0.002686        |
It can be seen that the average degree $\langle k \rangle$ increases with respect to the system size. Note that for the parameters: $N = 1000; J = 10$, the average degree of the network is less than one, which means that the network has either trees or clusters containing exactly one link. In general, the networks show a small average path length $l \approx 2$, meaning that any investor or initiator in the network is in average connected to each other by two links. Moreover, for a large number of nodes, the average path of the networks is approximately equal to that from a random graph generated with same average degree of the investment network. On the other hand, the clustering coefficient of the investment networks is larger than the clustering coefficient of a random network, this indicates the presence of transitivity in our networks. This occurs mainly because of the large number of investors connected to initiators. Note that the values of $C$ in our networks are similar to the clustering coefficient obtained for real bipartite networks, for example it has been reported that the clustering coefficient for the network of movie actors is $C = 0.79$ [13]. Note that a property of random networks is that the clustering coefficient decreases with respect to the size of the network. Finally, note that the clustering coefficient of the networks decreases with respect to $N$, this is in qualitative agreement with properties of small-world networks [13].

5 Conclusions and Further Work

The most important conclusions that can be drawn from the model here presented are that the budget of the agents reaches a stationary distribution after some time steps and presents a power law distribution on the tail, property discussed in other investment models [10, 9, 7]. It was shown that the topology of the investment networks emerging from the model was analyzed showing that the networks present some of the typical characteristics of real-life networks like a high clustering coefficient and short average path length. It was also observed that the evolution over time of the number of links $V$, the maximal degree of the network $k_{\text{max}}$ and the clustering coefficient $C$ can be described by a power-law.

We focused our investigations on the feedback describing the establishment and reinforcement of relations among agents and initiators, which dynamic is mainly driven by the decision weights $w_{kj}(t)$, Eq. (4). This is considered a “social component” of the agents’ interaction and it was shown how this feedback process based on positive or negative experience may lead to the establishment of networks among agents.

For simplicity, we have just assumed a random selection of failure or success, but we note that more elaborated economic assumptions, such as market dynamics based on supply and demand, can be taken into account as well. Furthermore, we noted that the external income sources play an important role on the dynamics of reputation and trust among agents. The results presented indicate that an extra mechanism or behavioral component needs to be added to the model in order to obtain networks with a stationary power-law degree distribution, property which is usually found in real-world networks.

We note also that further experiments are needed for different memory $\gamma$ and greediness $\beta$ values to understand the influence of these parameters in the dynamics of the networks.
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Figure 3: Common investment networks for different proportions of investment at time step $t = 1000$: (left) $q = 0.1$, (right) $q = 0.5$ and (bottom) $q = 0.9$. A link between agents represents a positive decision weight, i.e. $w_{kj} > 0$. For $N = 1000$ investors (blue-gray nodes) and $J = 10$ initiators (red-bold nodes). Additional parameters as in Table 1.