An EPR local theory with local correlation

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An EPR local theory with local correlation is proposed to give an explanation for the contradictions in the GHZ-like schemes for Bell’s theorem and the violation to Bell’s inequality. It agrees with the experimental predictions for the GHZ state of three entangled spins, the entangled state of two spins and one spin state. The contradiction in the GHZ-like schemes and the violation to Bell’s inequality can be attributed to the local correlation between the EPR elements of physical reality.

I. INTRODUCTION

Einstein, Podolsky, and Rosen (EPR) proposed a deterministic local theory\textsuperscript{1}, elements of physical reality, to reproduce the predictions of quantum theory. This local theory was thought to be refuted by Bell’s theorem, which demonstrated that the interpretations of quantum theory must be nonlocal by the well known Bell’s inequality\textsuperscript{2}. Greenberger, Horne, and Zeilinger (GHZ)\textsuperscript{3} provided an experimental scheme to test Bell’s theorem without using inequalities. Because this scheme requires at least three observers for an entangled state of three particles with spin-1/2, Hardy gave a proof with an entangled state of only two spins\textsuperscript{4}, but valid only for non-maximally entangled states. Then Cabello provided a proof for two observers using two pairs of maximally entangled states based on Hardy’s criterion\textsuperscript{5} and another one on GHZ’s criterion\textsuperscript{6}. Following Cabello’s latter idea Chen et al proved Bell’s theorem by one pair of two entangled particles with two spin degrees of freedom and two space degrees of freedom\textsuperscript{7}.

The series of proofs of GHZ,\textsuperscript{3} Hardy,\textsuperscript{4} Cabello,\textsuperscript{6} Chen\textsuperscript{7} and the relevant experiments have similar criterion, thus are called the GHZ-like schemes latter in this paper. They found a contradiction between EPR’s elements of physical reality and the predictions of experiments, hence claimed that elements of physical reality do not exist thus local theories do not hold. The contradiction occurs when \(\sigma_x\) and \(\sigma_y\) are assigned real values, \(m_x, m_y\), though which are called elements of physical reality. In quantum theory, however, one has strong evidences that it is impossible to assign values to \(\sigma_x\) and \(\sigma_y\) since they do not commute each other. This implies that a local theory may survive if it works together with a local correlation between \(m_x\) and \(m_y\).

In this paper we first analyze the contradiction revealed by the GHZ scheme. It is found that this contradiction can be explained by the local correlation between EPR’s elements of physical reality. We then demonstrate that the local correlation satisfies all the predictions of GHZ, two-spin entangled state and even one spin state. It is shown that the all GHZ-like schemes can be explained by the local correlation. Finally we point out that Bell’s inequality is violated only in the region of the local correlation and thus is attributed to the local correlation. A conclusion is given in the last section that an EPR local theory with the local correlation survives.

II. GHZ’S PROOF IS TRIVIAL

GHZ\textsuperscript{3} considered an entangled state of three particles, \(A, B\) and \(C\) with spin-\(\frac{1}{2}\)

\[
|\Psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow_A \uparrow_B \uparrow_C\rangle - |\downarrow_A \downarrow_B \downarrow_C\rangle),
\]

where \(|\uparrow\rangle\) and \(|\downarrow\rangle\) stand for spin up and down states respectively. It is easy to check

\[
\sigma_x^A \sigma_y^B \sigma_z^C |\Psi\rangle = |\Psi\rangle,
\]

\[
\sigma_y^A \sigma_z^B \sigma_x^C |\Psi\rangle = |\Psi\rangle,
\]

\[
\sigma_z^A \sigma_x^B \sigma_y^C |\Psi\rangle = |\Psi\rangle,
\]

\[
\sigma_x^A \sigma_y^B \sigma_z^C |\Psi\rangle = -|\Psi\rangle,
\]

where \(\sigma_{x,y}\) are Pauli matrices. The GHZ state is the common eigenstate of the four commuting Hermitian operators \(\sigma_x^A \sigma_y^B \sigma_z^C, \sigma_y^A \sigma_x^B \sigma_z^C, \sigma_z^A \sigma_x^B \sigma_y^C\), and \(\sigma_x^A \sigma_y^B \sigma_z^C\) with eigenvalues 1, 1, 1, \(-1\), respectively. Since these operators commute with each other, they can be observed simultaneously. These quantities can only be locally observed, e.g., \(\sigma_x^A \sigma_y^B \sigma_z^C\) is observed by measuring \(\sigma_x^A\), \(\sigma_y^B\) and \(\sigma_z^C\) independently. Since the product of the three measured values is certainly equal to the eigenvalue of \(\sigma_x^A \sigma_y^B \sigma_z^C\), two of them predict the other. EPR’s criterion tried to explain this nonlocal prediction by means of a local theory, elements of physical reality\textsuperscript{1,3,8}. In the present
They are supposed to have an element of physical reality polarized in the x-axis direction as shown in the figure. The incident electrons are side by side. Due to this contradiction GHZ claimed that the spin of the incident polarized electrons point to the x-axis direction. The magnetic field in the left-hand side is in the x-direction. The magnetic field, however, it again splits into two branches, arriving at windows a and b equally, i.e., the value of \( m_x \) is missing. It is seen that in fact the elements of physical reality, \( m_x \) and \( m_y \), do not exist simultaneously. Therefore, the hypothesis \( (10) \) does not hold. The goal of GHZ’s scheme is achieved by means of such a simple test.

Hence either in the aspect of quantum theory or experiment GHZ’s conclusion is quite trivial.

### III. AN EPR LOCAL THEORY WITH LOCAL CORRELATION

The co-existence hypothesis \( (10) \), however, is not a necessity of a local theory. A local theory denies only non-local correlations between different particles apart but it may admit a local correlation between the elements of physical reality of one particle. For example, in the case of the GHZ scheme the following local correlation for each particle can be accepted by a local theory,

\[
m_{xy} \equiv m_x m_y = \pm i. \tag{12}
\]

It assumes that when \( m_x \) is determined, e.g. \( m_x = 1 \), then \( m_y \) will be stochastic or illy defined, and vise versa. In fact, \( (11) \) has told that there must be a local correlation between \( m_x \) and \( m_y \) if they did exist.

Surprisingly, the local correlation \( (12) \) satisfies the prediction equations in the GHZ scheme. For example, multiplying \( (6)-(9) \) one obtains

\[
(m_x^A m_y^B m_C^C)^2(m_x^B m_y^B m_C^C)^2 = -1. \tag{13}
\]

Substituting \( (12) \) into the above equation gives \(-1 = -1\). GHZ’s contradiction disappears! The local correlation gives an explanation to the contradiction in GHZ’s proof. It revives EPR’s local theory.

One should not take the elements of physical reality, \( m_x \) and \( m_y \), as really-measured values. From the point of view of experiment one never tested Eqs. \( (2) \) on one group of entangled particles. When one of them, e.g., \( (2) \), is realized through local measurements, the state \( |\psi> \) collapses to the eigenstate of \( \sigma_x^A, \sigma_y^B \) and \( \sigma_y^C \), e.g., \(|\psi' \rangle \), here \(|\psi \rangle \) denotes the eigen-states of \( \sigma_x, \sigma_y \). Thus one has to pick new groups of entangled particles in the GHZ state to realize other equations. Different groups, however, give different measured values. Therefore, it is impossible to represent different measured values on different groups of particles by the same group of numbers \( (m_x, m_y) \). This is why \( (m_x, m_y) \) are named only as elements of physical reality.
Nevertheless, any form of elements of physical reality, even though including the above local correlation, is definitely contradicting to quantum theory. It is impossible to make two independent numbers, either real or complex, to satisfy the anti-commuting relation \((11)\). Therefore, a real experimental scheme to test local theories should be able to refute the elements of physical reality, with such \((12)\), but not the independent existence \((10)\). Unfortunately, neither Bell’s inequality \((2)\), GHZ’s \((3)\), Hardy’s \((4, 10)\), 5Cabello’s \((6)\), and Chen’s \((7)\) schemes, nor the relevant experimental demonstrations did.

IV. LOCAL CORRELATION IN HARDY’S PROOF

In a general understanding to entanglement a maximally entangled state should be most nonlocal. It is surprising that Hardy’s proof is valid only for two non-maximally entangled particles \((11)\). Up to date nobody has provided a proof for a maximally entangled state of two particles. Why is this the case? The local correlation gives an answer.

Goldstein provided a simpler version \((10)\) For Hardy’s proof. He considered an entangled state of two particles,

\[
|\psi\rangle = a|\uparrow_1\downarrow_2\rangle + b|\downarrow_1\uparrow_2\rangle + c|\downarrow_1\downarrow_2\rangle,
\]

where \(|\uparrow\rangle\) and \(|\downarrow\rangle\) are two orthogonal and normalized basis vectors. The four Hermitian operators, \(\{\hat{U}_i = |\uparrow_i\rangle\langle\uparrow_i|, \hat{W}_i = |\beta_i\rangle\langle\beta_i|, i = 1, 2\}\) are measured, where

\[
|\beta_i\rangle = \frac{a|\downarrow_i\rangle + x|\uparrow_i\rangle}{\sqrt{|a|^2 + |x|^2}}, \quad x = \begin{cases} b & \text{for } i = 1, \\ c & \text{for } i = 2. \end{cases}
\]

It was proved that the elements of physical reality, \(U_i, W_i\), corresponding to \(\hat{U}_i, \hat{W}_i\), contradict each other under the condition \(abc \neq 0\). This condition makes \(|\psi\rangle\) non-maximally entangled.

We find that this contradiction is just the requirement of the local correlation, because

\[
[\hat{U}_i, \hat{W}_i] = \frac{a^*x|\downarrow_i\rangle\langle\downarrow_i| - ax^*|\uparrow_i\rangle\langle\uparrow_i|}{\sqrt{|a|^2 + |x|^2}}.
\]

When \(a \neq 0, x \neq 0, \text{ i.e. } b \neq 0, c \neq 0\) the operators \(\hat{U}_i\) and \(\hat{W}_i\) do not commute each other, thus the local correlation between \(U_i\) and \(W_i\) exists just under the condition \(abc \neq 0\). This is why Hardy’s proof does not fit the maximally entangled state. In the point of view of experiment it is impossible to measure \(U_i\) and \(W_i\) simultaneously when \(abc \neq 0\). In this case \(U_i\) and \(W_i\) do not exist independently for a local theory, i.e., there is a local correlation between them. Hence Hardy’s contradiction does not exist.

V. MAXIMALLY ENTANGLED STATE OF TWO PARTICLES

Although until now nobody refuted elements of physical reality by a two-particle maximally entangled state, we find that the local correlation fit this case. As an example, we consider a Bell-basis state of two spin-\(\frac{1}{2}\) particles

\[
|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow_A\downarrow_B\rangle - |\downarrow_A\uparrow_B\rangle).
\]

One has

\[
\sigma_x^A\sigma_x^B|\Phi^-\rangle = -|\Phi^-\rangle,
\]

\[
\sigma_y^A\sigma_y^B|\Phi^-\rangle = |\Phi^-\rangle,
\]

where \(\sigma_x^A\sigma_x^B\) and \(\sigma_y^A\sigma_y^B\) commute each other and can be experimentally determined simultaneously. In the same way as the GHZ scheme the elements of physical reality corresponding to these operators obey

\[
m_x^A m_x^B = -1.
\]

\[
m_y^A m_y^B = 1.
\]

Multiplying these two equations one obtains

\[
(m_x^A m_x^B)(m_x^A m_x^B) = -1.
\]

This equation agrees the local correlation \((12)\). If fact the local correlation is the only choice for a local theory. Consider the following Hermitian operator

\[
-2i\sigma_x \sigma_y = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.
\]

It is an observable quantity with eigenvalues \(\pm 1\). If elements of physical reality do exist one has to assume

\[
-2im_x m_y = \pm 1.
\]

This equation is just the local correlation \((12)\).

VI. LOCAL CORRELATION IN BELL’S INEQUALITY

Finally we consider Bell’s inequality \((9)\), which is given by

\[
f(b, c) = |P(a, b) - P(a, c)| + P(b, c) \leq 1,
\]

where \(P(x, y)\) are correlation functions between the spin components of electrons in directions \(x, y = a, b, c\). \(a\) belongs to electron A, and \(b, c\) belong to electron B.

According to quantum theory \(P(x, y)\) are given by

\[
\langle\psi|\sigma_x^A\sigma_y^B|\psi\rangle = \cos(x, y),
\]

where \(x, y\) denote the angle between the two vectors \(x, y\). Then one has

\[
f(b, c) = |\cos(a, b) - \cos(a, c)| + \cos(b, c).
\]
FIG. 2: $f(b,c)$ in quantum theory. The x axis represents the angle between $b$ and $a$, and the y axis the angle between $c$ and $a$. Violation to Bell’s inequality occurs inside the four peak regions, labeled as 1,2,3,4, where $f(b,c) > 1$.

This function $f(b,c)$ is plotted in FIG.2. It is seen that (25) is violated inside the four peak regions. It was claimed that the local theory is refuted by this violation.

In these four peak regions, however, we have $[\sigma_b, \sigma_c] \neq 0$, i.e., there is a local correlation between the measured quantities in $P(b,c)$. In particular, it is seen from the figure that when $\theta \to 0$ or $\pi$, here $\theta$ is the angle between $b$ and $c$, the local correlation vanishes and thus the violation disappears. A violation without a local correlation was never found. Therefore, the violation to Bell’s inequality can be attributed to the local correlation of one particle but not necessarily the non-locality of the two entangled particles.

It is seen that in all the GHZ-like schemes and Bell’s inequality a common point is the local correlation between the measured quantities on one particle. A EPR local theory can survive if the local correlation is included between the elements of physical reality. A future scheme to refute such local theories should be built upon measurements to uncorrelated physical quantities.

VII. CONCLUSIONS

In this work an EPR local theory with local correlation is proposed, giving an explanation to the contradictions in the GHZ-like schemes for Bell’s theorem and the violation to Bell’s inequality. It agrees with the predictions for the GHZ state of three spins, two-particle entangled state and even one spin state. It is shown that Bell’s inequality is violated only in the region of the local correlation between the measured quantities. It is concluded that the contradictions found in the GHZ-like schemes and the violation to Bell’s inequality can be explained by the EPR local theory with local correlation. This local correlation, however, disobeys with quantum theory. More stringent experiments are required to test this EPR local theory with local correlation.

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