A numerical model of resistive generation of intergalactic magnetic field at cosmic dawn

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Abstract. Miniati & Bell (2011) proposed a mechanism for the generation of magnetic seeds that is based on the resistivity of the low temperature IGM in the high redshift universe. In this model, cosmic-ray protons generated by the first generation of galaxies, escape into the intergalactic medium carrying an electric current that induces return currents, \(j_t\), and associated electric fields, \(\vec{E} = \eta j_t\) there. Because the resistivity, \(\eta\), depends on the IGM temperature, which is highly inhomogeneous due to adiabatic contraction and shocks produced by structure formation, a non-vanishing curl of the electric field exists which sustains the growth of magnetic field. In this contribution we have developed an approximate numerical model for this process by implementing the source terms of the resistive mechanism in the cosmological code CHARM. Our numerical estimates substantiate the earlier analysis in Miniati & Bell (2011) which found magnetic seeds between \(10^{-18}\) and \(10^{-16}\) Gauss throughout cosmic space at redshift \(z \sim 6\), consistent with conservative estimates of magnetic fields in voids at \(z \sim 0\) from recent gamma-ray experiments.

1. Introduction

Magnetic fields are an intrinsic part of astrophysical plasmas, though in many respects their origin and role is still subject of intense investigation. Recent studies combining gamma-ray measurements at different energies indicate the existence of magnetic fields in cosmic voids (Neronov & Ovch 2010, Taylor et al. 2011, Tavecchio et al. 2011). If confirmed (see e.g. Broderick et al. 2011), the origin of such magnetic fields poses a non trivial challenge for astrophysical scenarios: the mechanisms connected to the Weibel instability (Schlickeiser & Shukla 2003; Medvedev et al. 2006) and the Biermann’s battery at shocks (Kulsrud et al. 1997) generate magnetic fields inside collapsed structures, such as filaments and clusters, not in the voids. The Biermann’s battery activated by ionization fronts (Gnedin et al. 2000) generates fields that are too small in the low density regions. Galactic outflows can in principle export high magnetic flux (e.g Bertone et al. 2006), but despite valuable efforts, the hydrodynamic transport that determines its filling factor in the intergalactic space remains highly uncertain (Rees 2006).

Miniati & Bell (2011) proposed a mechanism to generate widespread intergalactic magnetic fields that is associated with escape of high energy particles (hereafter CR for cosmic-ray) from the first generation of galaxies, also responsible for the re-ionization of the intergalactic medium (IGM). Cosmic re-ionization is mostly driven...
by ionizing photons produced by massive OB stars. When these stars come to an end, they explode as supernova whose blast wave accelerates copious CR (Krymsky 1977; Axford et al. 1977; Bell 1978; Blandford & Ostriker 1978). Owing to their much larger diffusive mean free path compared to thermal particles, CR eventually escape from the parent galaxies into the IGM (Schlickeiser 2002). The charged CR protons carry an electric current in the IGM, \( j_c \), which is not balanced by the CR electrons, typically fewer in number by two orders of magnitude (Schlickeiser 2002) and subject to much more severe energy losses. Due to charge imbalance and induction effect a return current, \( j_t \), carried by the cold thermal plasma is generated, which tends to cancel the CR current, i.e. \( j_t \approx -j_c \). Due to the finite resistivity of the IGM plasma, an electric field, \( \vec{E} = \eta \vec{j_t} \), is required to draw the return current. This field opposes the propagation of the CR but is too weak to prevent their escape. The plasma resistivity in an unmagnetized medium is determined by its collisional rate which only depends on temperature, and can be written as \( \eta(T) = \eta_0 (T/K)^{3/2} \) (Spitzer 1956), where \( \eta_0 \) is a constant. Owing to the formation of structure in the universe, driven by gravitational instability, considerable density and temperature inhomogeneities existed in the IGM long before the onset of re-ionization. Thus the electric field is also inhomogeneous and in particular

\[
\nabla \times \vec{E} = \nabla \times \eta \vec{j_t} \approx -\frac{3}{2} \eta \vec{j_t} \times \frac{\nabla T}{T} \neq 0!
\]

where we have neglected the term \( \propto \nabla \times j_t \) and have used Spitzer’s expression for the resistivity. The important point is that, since the current is unrelated to the IGM inhomogeneities, the curl of the electric field is non vanishing. The rotational component of the electric field sustains Faraday’s induction and generates magnetic field. The analysis of Miniati & Bell (2011) used the observed UV luminosity function of high redshift galaxies to estimate the production rate of CR in those galaxies and the ensuing return currents in the IGM, \( j_t \). They also used simulations of structure formation to estimate the temperature gradient scale-length, \( T/\nabla T \), entering the electric field curl. They found that magnetic field is robustly generated throughout intergalactic space at rate of \( 10^{-17} - 10^{-18} \) Gauss/Gyr, until the temperature of the intergalactic medium is raised by cosmic re-ionization when the age of the universe is roughly \( t = 1 \) Gyr. This value is consistent with the conservative values implied by gamma-ray measurements.

In this contribution, we present results from a cosmological numerical model that also includes the source terms associated with the resistive mechanism described above. The numerical model and results are described below but we anticipate that they validate the quantitative findings of Miniati & Bell (2011). The rest of this manuscript is organized as follows. The numerical model is described in Sec. 2, the results are reported in Sec. 3 and Sec. 4 contains summary and conclusions.

2. Approximate Numerical Model of the Resistive Mechanism

We start from a numerical model of cosmological structure formation. This includes the evolution, in an expanding background space-time, of a collision-less dark matter fluid and a collisional baryonic gas, coupled through self-gravity, and subject to appropriate initial conditions. We then extend the hydrodynamics to include the effects of magnetic field in the MHD approximation and modify the RHS of the governing equations to include the various effects due to the return current \( j_t \) (see below). Because \( j_t \approx -j_c \),
for convenience in the following we express the source terms associated to the return current in terms of \(-j_c\).

The full set of equations solved in a cosmological model is given, e.g., in Miniati & Colella [2007]. Here we only describe the equations of hydrodynamics, which contain the modifications required by the resistive model. As usual, these equations employ comoving density, \(\rho\), and pressure, \(p_g\), and peculiar velocity, \(u\). If \(a(t)\) is the spatial scale factor of the universe and \(\dot{a}\) its rate of change, density and pressure are modified as \(\rho \leftarrow a^3\rho\), \(p_g \leftarrow a^3P\) respectively, to scale out the effect of Hubble expansion. Likewise the peculiar velocity does not include the Hubble flow component (see, e.g., Miniati & Colella [2007] for details). For numerical reasons we find most convenient to rescale the magnetic field as, \(B \leftarrow a^2B\), \(j_c \leftarrow a^2j_c\), even though it does not scale out completely the adiabatic expansion effects. So finally the governing equations read

\[
\frac{\partial \rho}{\partial t} + \frac{4\pi}{a} \nabla \cdot (\rho \mathbf{u}) = 0, \tag{2}
\]

\[
\frac{\partial \rho \mathbf{u}}{\partial t} + \frac{4\pi}{a} \nabla \left( \rho \mathbf{u} + p - \frac{1}{4\pi} \mathbf{B} \cdot \mathbf{B} \right) = -\frac{\dot{a}}{a} \rho \mathbf{u} - \frac{1}{a} \rho \nabla \phi - \frac{\dot{\mathbf{j}}}{c} \times \mathbf{B}, \tag{3}
\]

\[
\frac{\partial \rho e}{\partial t} + \frac{4\pi}{a} \nabla \left[ (\rho e + p) - \frac{1}{4\pi} \mathbf{B} \cdot (\mathbf{B} \cdot \mathbf{u}) \right] = -2\frac{\dot{a}}{a} \rho e - \frac{1}{a} \rho \mathbf{u} \cdot \nabla \phi - \mathbf{u} \cdot \left( \frac{\dot{\mathbf{j}}}{c} \times \mathbf{B} \right) + \eta \mathbf{j}^2. \tag{4}
\]

Here \(p = p_g + \frac{1}{8\pi} \mathbf{B} \cdot \mathbf{B}\) is the total sum of the gas and magnetic pressures, \(e = \frac{1}{2} \mathbf{u} \cdot \mathbf{u} + e_{th} + \frac{1}{8\pi} \mathbf{B} \cdot \mathbf{B}\) is the total specific energy density. The thermal energy is related to the pressure through a \(\gamma\)-law equation of state, \(e_{th} = p_g/p(\gamma - 1)\), \(\phi\) is the gravitational potential, \(-\dot{\mathbf{j}}\) the return current and \(\eta\) the resistivity. The last term on the RHS of the momentum equation describes the interaction between the magnetic field and the return current, which also contributes the term before the last in the energy equation. The last term in the energy equation represents ohmic dissipation of the return current. The electric field, \(\eta \mathbf{j}\), driving this current does not appear in the momentum equation as it is balanced by frictional forces, the same ones that are in fact responsible for the dissipation.

The magnetic field evolution is described by Faraday’s equation, with the electric field given by Ohm’s law. The only electric fields are those induced by motions of the magnetized fluid as well as the return current, i.e., \(\mathbf{E} = -(\mathbf{u}/c) \times \mathbf{B} - \eta \mathbf{j}\). So the modified induction equation (again in comoving quantities) reads

\[
\frac{\partial \mathbf{B}}{\partial t} = \frac{1}{a} \nabla \times (\mathbf{u} \times \mathbf{B}) + \frac{1}{a} \nabla \times (\eta \mathbf{j}) - \frac{1}{2a} \frac{\dot{a}}{a} \mathbf{B}. \tag{5}
\]

The full set of cosmological equations is solved with the code CHARM [Miniati & Colella 2007]. The MHD solver is based on the constrained-transport scheme and is described in details in Miniati & Martin [2011]. The source terms associated with the resistive process have been implemented using a second order accurate time-centered scheme.

Concerning the CR current, \(j_c\), as discussed in Miniati & Bell (2011), it is typically dominated by the most luminous nearby galaxy. For a bright L* galaxy at redshift \(z = 6\), with star formation rate of \(6.5 \, M_{\odot} \,\text{yr}^{-1}\) [Bouwens et al. 2007, 2010], assuming a Salpeter initial-mass-function and a 30% conversion efficiency of supernova energy into CR, it is found

\[
j_c \approx 5.3 \times 10^{-20} \left( \frac{L}{L_*} \right) \left( \frac{d}{\text{Mpc}} \right)^{-2} \left( \frac{p_{\text{min}}}{m_p c} \right)^{-0.3} \left( 1 + \frac{p_{\text{min}}}{m_p c} \right)^{-1} \text{Amp m}^{-2}, \tag{6}
\]
where \(d\) is the physical distance from the parent galaxy and \(p_{\text{min}}\) is the minimum energy of the escaping CR. In the following we consider a distance \(d \sim 1\)Mpc, which typically separates bright \(L^*\) galaxies at \(z \sim 6\), and a volume of linear size \(L \ll d\) so that the distance dependence of the CR current can be neglected within the simulated volume. In addition, the spatial inhomogeneities of \(j_c\) are neglected, so \(j_c\) is effectively a constant, whose value is set in accordance with Eq. 6. We neglect the effects due to the generated magnetic field on the CR propagation. Though they introduce some degree of diffusivity in the CR propagation, the net effect on the current should remain negligible for a field strength below \(10^{-15}\) G.

3. Results

Using the model described in the previous section we have run a model of cosmic structure formation that includes the resistive generation of magnetic field. For the purpose, we have adopted a concordance Λ-CDM based with cosmological parameters given by the results of the WMAP experiment Komatsu et al. (2009). We use a computational box with \(512^3\) resolution elements and as many collision-less particles to describe the dark matter component. For the physical size of the computational domain we consider two cases. In the first one the simulated volume corresponds to a cube of side \(L = 140\) comoving kpc. At redshift \(z \sim 10\), corresponding to the epoch of the results presented below, this translates into a physical size of 12.7 kpc and a spatial resolution of 25 pc, sufficient to resolve temperature structures on physical scales of a few kpc, identified in Miniati & Bell (2011) as the most relevant for the resistive process. Fig. 1 shows a snapshot of the distribution of the baryonic gas in units of the total matter density (left) and magnetic field strength in Gauss (right), on a two-dimensional slice passing through the simulated volume at redshift \(z = 10\). Gas density is in units of the average total (baryonic plus dark matter) matter density, and magnetic field is in Gauss. The physical length each panel’s side is 12.7 kpc.

Figure 1. Simulation snapshots for the distribution of baryonic gas (left) and magnetic field strength (right) on a slice through the simulated volume at redshift \(z = 10\).
through the simulated volume at redshift $z \sim 10$. As anticipated above, the matter density distribution shows considerable structure, which translates into a corresponding temperature structure due either to adiabatic compression or shocks. Notice that the magnetic field ranges from $10^{-18}$ to several times $10^{-17}$ Gauss. An important feature is that the magnetic field with strength in this range fills is generated throughout the simulated volume. This is better illustrated in the histogram on the left hand side of Fig. 2, which shows the distribution of magnetic field as a function of magnetic field strength. According to the histogram, all space is magnetized and the average magnetic field value is of order $10^{-17}$ Gauss.

Given the small box size, at later time the formation of structure quickly saturates and the numerical model becomes inadequate to describe the intergalactic medium. We have thus run a second model which employs a box ten times as large as for the previous case, $L = 1.4$ comoving Mpc. We run this model down to redshift $z \sim 6$, when cosmic re-ionization raises the IGM temperature, effectively shutting down the resistive mechanism. Since we employ the same numerical resources as before, so the resolution is at least 390 kpc physical, which is still sufficient to resolve the desired temperature structures. The histogram on right hand side of Fig. 2 shows the distribution of the magnetic field strength in this larger simulated volume at redshift $z \sim 6$. It shows that the magnetic field continues to grow in strength from redshift 10 to 6 and at this later time it fills the intergalactic volume with a strength between $10^{-18}$ and $10^{-16}$. The histogram peaks at a value of several times $10^{-17}$, which is quantitative consistent with the estimates in Miniati & Bell (2011).

Finally, although the governing equations include the ingredients necessary to drive non resonant amplification of magnetic field (Bell 2004, 2005), there is no appreciable effect is our simulations. This is because for the tiny resistive magnetic field the growth timescales of the non-resonant mechanism is too slow on the spatial scales resolved by our simulation. It remains unclear whether or not this mechanism may efficiently kick in at later times.
4. Summary and Conclusions

Miniati & Bell (2011) proposed a mechanism for the generation of magnetic seeds that is based on the finite resistivity of the low temperature IGM in the high redshift universe. In that model, CR escaping from the first generation of galaxies, induce a return current and an electric field in the IGM plasma. The electric field, \( \mathbf{E} = \eta(T) \mathbf{j} \), depends on the IGM temperature, which is highly inhomogeneous due to adiabatic contraction and shocks produced by structure formation. It is easy to show that this electric field has a non vanishing curl which sustains the growth of magnetic field. The analytic results in Miniati & Bell (2011) were consistent with conservative values of magnetic fields in voids inferred cosmic-voids by gamma-ray experiments. Here we have implemented the source terms describing the resistive process in the cosmological code CHARM. To keep the problem tractable we made a number of reasonable approximations. Nevertheless, the numerical model allows a more self-consistent estimate of the magnetic field generation by the resistive process. We have carried out numerical experiments using two different computational boxes to best estimate the generated magnetic fields at different cosmic epochs of interest. In conclusion the numerical estimates substantiate the earlier analysis in Miniati & Bell (2011), showing that most of the made approximations and assumptions were essentially correct.

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References

Axford W. I., Leer E., Skadron G., 1977, in Int. Cosmic Ray Conf. Vol. 11, The acceleration of cosmic rays by shock waves. Plovdiv: Bulgaria, pp 132–137
Bell A. R., 1978, MNRAS, 182, 147
Bell A. R., 2004, MNRAS, 353, 550
Bell A. R., 2005, MNRAS, 358, 181
Bertone S., Vogt C., Enßlin T., 2006, MNRAS, 370, 319
Blandford R. D., Ostriker J. P., 1978, ApJ, 221, L29
Bouwens R. J., Illingworth G. D., Franx M., Ford H., 2007, ApJ, 670, 928
Bouwens R. J., et al. 2010, ApJ, 709, L133
Broderick A. E., Chang P., Pfrommer C., 2011, ArXiv e-prints 1106.5494
Gnedin N. Y., Ferrara A., Zweibel E. G., 2000, ApJ, 539, 505
Komatsu E., et al. 2009, ApJS, 180, 330
Krymsky G. F., 1977, Dokl. Akad. Nauk SSSR, 234, 1306
Kulsrud R. M., Cen R., Ostriker J. P., Ryu D., 1997, ApJ, 480, 481
Medvedev M. V., Silva L. O., Kamionkowski M., 2006, ApJ, 642, L1
Miniati F., Bell A. R., 2011, ApJ, 729, 73
Miniati F., Colella P., 2007, Journal of Computational Physics, 227, 400
Miniati F., Martin D. F., 2011, ApJS, 195, 5
Neronov A., Vovk I., 2010, Science, 328, 73
Rees M. J., 2006, Astronomische Nachrichten, 327, 395
Schlickeiser R., 2002, Cosmic Ray Astrophysics
Schlickeiser R., Shukla P. K., 2003, ApJ, 599, L57
Tavecchio F., Ghisellini G., Bonnoli G., Foschini L., 2011, MNRAS, 414, 3566
Taylor A. M., Vovk I., Neronov A., 2011, A&A, 529, A144+