Elusive exotic states

Hyun-Chul Kim *

Institut für Theoretische Physik II,
Postfach 102148, Ruhr-Universität Bochum,
D-44780 Bochum, Germany

Mikhail Shmatikov†

Russian Research Center "Kurchatov Institute", 123182 Moscow, Russia

(July 1995)

Abstract

The existence of flavor exotic $QQar{q}ar{q}$ molecular-type states is investigated. An attractive force between two pseudoscalar $H = (Qar{q})$ heavy meson is generated by (correlated) two-pion exchange. The emergence of a (loosely) bound state depends crucially on the value of the coupling constant $g$ of the $H^*H\pi$ vertex. For a $g$ value calculated from the experimental upper limit on the width of the $D^*$ meson the considered mechanism alone is strong enough to generate a bound state in the $BB$ system while the $DD$ system is very close to become bound. Such states, if exist, are stable with respect to strong interactions. They may be observed as stable scalar particles with the mass $M \approx 2m_H$ and flavor quantum number $\pm 2$.

* e-mail address: kim@hadron.tp2.ruhr-uni-bochum.de

† e-mail address: msh@ofpnp.kiae.su
At present basically all the hadrons, both baryons and mesons, can be safely classified as $qqq$ and $\bar{q}q$ quark states respectively. No reliable evidence for multiquark states but nuclei, which can be considered as weakly bound states of the baryons, has been found. One of the most clear-cut signals for such multiquark states would be an observation of hadronic states with exotic flavor quantum numbers. A promising hunting ground for such an exotics is the domain of heavy flavors, or more precisely, hadrons with the $QQ\bar{q}\bar{q}$ structure where $Q$ is a heavy quark. The sought after hadronic states are known to exist in the $m_Q \to \infty$ limit \[1–3\]. Quantitatively the heavy-mass limit can be recast in the form of an inequality

\[ \alpha_s^2(m_Q)m_Q \gtrsim \Lambda_{QCD}. \]  

However, this inequality is satisfied for the $t$-quark only which, on the other hand, decays before the hadronization. For $Q = c, b$ quarks $QQ\bar{q}\bar{q}$ hadrons may exist as weakly-bound systems of two $Q\bar{q}$ mesons.

A couple of $Q\bar{q}$ mesons may be bound, for the large enough mass of a heavy quark, by a comparatively weak force generated by one-pion exchange. The attractive feature of such a long-range potential is that it is calculable in a chiral perturbation theory. A molecule-type $H - H$ ($H = Q\bar{q}$) hadron exists in the limit of the infinitely heavy mass $m_Q$ \[3\]. Corrections $\sim 1/m_Q$, in the realistic case of $b$- or $c$- quarks, prove to be of importance. The investigation of such systems in \[3–5\] yielded rather controversial results. According to \[3\] the $D$-mesons are too light to be bound by the one-pion-exchange force, whereas a loosely bound state was found in the system of two $B$-mesons consisting of $BB^*$ and $B^*B^*$ with equal weights. At the same time the one-pion-exchange force between two heavy mesons was found in \[4,5\] to be too weak (or even repulsive) to produce a bound state. In contrast, systems with non-exotic quantum numbers were shown to have a rich spectrum of bound states: $DD$ system is expected to have a bound state and the $B\bar{B}$ one possesses several such states with various spin-parity quantum numbers. It should be stressed that conclusions as to the existence of a bound state(s) rely heavily on the specific value of the strength constant $g$ of the $\pi$-meson coupling to a heavy meson which sets the overall scale of the interaction
strength (see below).

In the limit \( m_Q \to \infty \), mesons containing a single heavy quark come in degenerate doublets of pseudoscalar \((H)\) and vector \((H^*)\) mesons. Mass corrections being taken into account, the degeneracy in the total momentum is lifted off and the \( H \) and \( H^* \) mesons are to be treated separately. For this reason one-pion exchange is operative in the \( H^*H^* \) and \( HH^* \) (or \( \bar{H}^*H^* \) and \( \bar{H}H^* \)) systems only and all the conclusions of [3–5] refer just to such systems.

In the present paper we investigate coupling and possible bound states of two pseudoscalar heavy mesons \( HH \). There are two reasons arousing interest to such a system. First, since a pseudoscalar \( H \)-meson is lighter than its vector counterpart \( H^* \), a bound \( H-H \) state will be stable with respect to a strong decay. An \( H^*-H^* \) system, spin-parity allowing, may decay into the \( HH \) pair. Open decay channel brings additional repulsion which may destroy a bound state [6]. Another reason for investigating a system of two pseudoscalar meson has a dynamical nature. The vertex of pion coupling to a pair of the \( H \)-mesons apparently vanishing, the next-in-range force operative in this system is (correlated) two-pion exchange (CTPX). This force is known to provide the bulk of the attraction in the \( NN \) system [7]. Thus it is of interest to investigate its effects in heavy-meson systems, and the \( HH \) pair provides a testing ground where the CTPX forces are not obscured by the one-pion exchange.

The construction of the CTPX potential is described in the plethora of works (see [7–9] just to mention a few). The generic form of two-pion exchange is depicted in fig.1. A pair of pions in the \( t \)-channel may have various orbital momenta and isospin. We focus our attention on the \( J^\pi(T) = 0^+(0) \) channel, which is the most relevant for the system under consideration, since it is responsible for the strong attraction. We will postpone the discussion of \( J \geq 1 \) contributions to the conclusion. As is well known from the works on the \( NN \) interaction, the static potential for the scalar-isoscalar channel can be written by

\[
V_{2\pi}(r) = -\frac{1}{\pi} \int_{4m^2_\pi}^{t_{\text{max}}} dt \rho(t) \frac{e^{-\sqrt{t}r}}{4\pi r},
\]

(2)
Fig. 1: Two-pion intermediate state in the $HH \to HH$ scattering.

The dynamics of the intercation is controlled by the behavior of the spectral function $\rho(t)$. It vanishes at the two-pion threshold in the $t$-channel, i.e. at $t = 4m^2_\pi$. The upper integration limit $t_{\text{max}}$ is usually chosen $t_{\text{max}} \simeq 50m^2_\pi$. Thus the CTPX potential is a superposition of Yukawa forces corresponding to the exchange by a meson with the $\sqrt{t}$ mass and the weight of this configuration is given by the corresponding value of the $\rho(t)$ spectral function.

The spectral function is in turn expressed, by means of the unitarity condition, in terms of the amplitude $A$ of the $\bar{H}H \to \pi\pi$ process:

$$\rho(t) = \frac{1}{32\pi} \sqrt{\frac{t - 4m^2_\pi}{t}} |A(t)|^2.$$  \hspace{1cm} (3)

Note that in contrast to the entirely familiar $\bar{N}N \to \pi\pi$ case there is no iterated Born term to be subtracted. In case of the $\bar{N}N \to \pi\pi$ process, the quasirempirical data in the pseudophysical region ($4m^2_\pi < t < 50m^2_\pi$) can be obtained by making an analytic continuation of the $\pi N$ scattering data. However, for lack of the quasirempirical information on the $\pi$-meson scattering off a heavy meson we are forced to apply a dynamical model.

Such a model developed in [10] for the $NN$ interaction was shown to reproduce with good accuracy available experimental data on scattering phase shifts in a wide energy region. The dynamical model tailored for the case of $\bar{H}H \to \pi\pi$ amplitude is shown in fig.2.
Fig. 2: Dynamical model of the $\bar{H}H \rightarrow \pi\pi$ annihilation amplitude. Empty circle denotes the T-matrix of the $\pi\pi \rightarrow \pi\pi$ rescattering.

It consists of two components: the Born term where outgoing pions do not interact with each other and the one involving pion rescattering. The amplitude of the (half-off-shell) $\pi\pi \rightarrow \pi\pi$ interaction can be evaluated by using the meson-exchange model of $\pi\pi$, $\pi K$ and $KK$ processes [11]. As usual, a nonvanishing range of the strong interaction is taken into consideration by introducing a formfactor $F$ in the $HH^*\pi$ vertex. We choose the exponential form [12] for it:

$$F(k^2) = \exp\left\{-\frac{k^2 + M^2}{2\Lambda}\right\}$$  \hspace{1cm} (4)

where $M$ is the mass of the exchange particle and $\Lambda$ denotes the cut-off parameter.

The overall strength scale of the (2) potential is established by the constant $g$ of the $\pi H^*H$ coupling. It is defined as follows [13]

$$\mathcal{L} = \frac{i}{2}g \text{Tr} \left[ H\gamma_\mu\gamma_5 \left( \xi^\dagger\partial_\mu\xi - \xi\partial_\mu\xi^\dagger \right) \bar{H} \right]$$  \hspace{1cm} (5)

In principle, the value of $g$ could be determined from the width of the $H^* \rightarrow H\pi$ decay channel. However, because of small mass difference $\Delta m_B$ between $B$ and $B^*$ mesons ($\Delta m_B \approx 46$ MeV) this channel is closed for the $B^*$-meson. As to the system of (vector and pseudoscalar) $D$-mesons, at present the upper limit for the total width of the $D^*$-meson and partial width of the $D^* \rightarrow D\pi$ decay are measured [14]. Theoretical estimates of the $g$ value are controversial. The upper limit in both the chiral and the heavy quark limits in the leading $1/N_c$ order reads [13]:
\[ g \leq \frac{1}{\sqrt{2}} \left[ 1 + \mathcal{O}(1/m_H) + \mathcal{O}(1/N_c) + \mathcal{O}(m_\pi m_H) \right] \] (6)

The numerical values utilized in [4,5] and [3], which ensured the existence of a (loosely) bound state, are equal to \( g \approx 0.6 \) and 0.7 respectively. These values were obtained by assuming that the upper experimental limit on the width of the \( D^* \) meson is just the realistic value of the latter. Direct calculations of \( g \) based on the QCD sum rules [16] yielded much smaller value: \( g \approx 0.2 \). Note that the \( H^*H\pi \) vertex enters in the spectral function \( \rho(t) \) four times, which implies that the CTPX potential \( V_{2\pi} (2) \) contains the \( g^4 \) factor. Thus the specific value of \( g \) proves to be crucial for the (non)existence of a bound state of two pseudoscalar \( H - H \) mesons.

Under these circumstances we have investigated the existence of the \( H - H \) bound state and, if present, calculated the binding energy \( E_b \) in a wide range of \( g \)'s: \( 0 \leq g \leq 1 \). In the numerical calculations of the CTPX potential (2) the value of \( t_{\text{max}} \) was chosen, as it is customary in the \( NN \) system, equal to \( t_{\text{max}} = 50 m_\pi^2 \). The spectral function (3) being positive-defined, a decrease of the \( t_{\text{max}} \) may be compensated by the corresponding increase of the \( g \) coupling constant. The cut-off parameter \( \Lambda \), entering into the formfactor (3), was varied in the \( \Lambda = 1.5 \div 2.1 \) GeV interval.

**Fig.3**: Dependence of the binding energy \( E_b \) of two pseudoscalar \( D \)-mesons on the \( D^*D\pi \) coupling constant \( g \). Curves labeled 1, 2 and 3 correspond to the cut-off paremeter \( \Lambda = 2.1, 1.8 \) and 1.5 GeV respectively.
Results of calculations for the $DD$ system ($S$-wave) are presented in fig.3. Note the (anticipated) sharp dependence of the binding energy on the $g$ value and its rather weak spread with the variation of $\Lambda$. A conclusion, which can be drawn from the investigation of the $g$ dependence, is that the bound state emerges at the value of $g \approx 0.75 \div 0.85$, i.e. at somewhat larger $g$’s than the molecular states of refs. [3,4]. In the case of more heavy $B$-mesons situation proves to be more favorable (see fig.4). Here the bound state emerges, depending upon the cut-off parameter, in the $g \approx 0.58 \div 0.78$ range.

![Fig.4: Dependence of the binding energy $E_b$ of two pseudoscalar $B$-mesons on the $B^*B\pi$ coupling constant $g$. Curves labeled 1, 2 and 3 correspond to the cut-off parameter $\Lambda = 2.1$, 1.8 and 1.5 GeV respectively.](image)

Hence, one can conclude that flavor exotic $DD$ (molecular) states are rather non-existing. Two pseudoscalar $B$ mesons become bound at about the same value of the $g$ coupling constant when the bound state in the $\bar{B}^*B^*$ and $\bar{B}^*B$ [1, 3] or $B^*B^*$ [3] systems emerges. For small $g \approx 0.2$ [10] molecular-type states with any meson content do not exist. It should be noted, that in contrast to the case of one-pion exchange which has the opposite (because of G-parity) sign in the $H^*H^*$ and $\bar{H}^*H^*$ systems, CTPX remains attractive in both meson-meson and meson-antimeson systems. It implies that obtained results hold true also for the $\bar{D}D$ and $\bar{B}B$ (non-exotic) systems. Stated differently, provided the latter molecular states are observed, existence of their flavor exotic counterparts ($BB$ and $DD$) is highly...
plausible. Considered mechanism is operative as well in similar system containing vector heavy mesons. The main difference is that one-pion exchange is operative in such systems. Then the attraction provided by the CTPX can either in combination with the one-pion-exchange induced attraction or overwhelming repulsion, generate otherwise unobtainable molecular-type states.

We have considered the possible formation of molecular-type bound states of two pseudoscalar heavy mesons generated by the CTPX with the \( t \)-channel quantum numbers \( J^\pi = 0^+ \). The effective potential \( V_{2\pi} \) is contributed as well by the CTPX with other quantum numbers \( J^\pi = 1^- \) and \( 2^+ \). However, the \( \pi\pi \) interaction with such quantum numbers involves additional, as compared to the considered case, momenta of pions. The integration over the pionic loop (fig.2) will result then in a spectral function peaked at larger values of \( t \) or, in the coordinate space, to the CTPX potential with smaller interaction range. Bar-ring accidental emergence of a state with almost vanishing binding energy such states are expected to be compact ones. Their dynamics will be governed by the combined action of CTPX and QCD quark-quark interaction. Existence of bound states in the situation when both meson exchanges and quark coupling come into play will be considered elsewhere.

Summarizing we have investigated a possible existence of molecular-type bound states in flavor exotic \( HH \) systems. In a sense it is an extension of the results obtained in \([3]\) which predicted, in the \( m_Q \to \infty \) limit, the existence of a bound heavy-meson state. However, \( 1/m_Q \) corrections are of importance, necessitating the different approach to the vector \( H^* \) and pseudoscalar \( H \) heavy mesons. All the heavy-meson systems differ drastically from the one (\( HH \)) considered in the present paper. First, their dynamics is controlled by the combined action of the one- and (correlated) two-pion exchange. It implies that besides the \( H^*H\pi \) coupling constant one more strength coupling constant (in the \( H^*H^*\pi \) vertex) plays an important role. Second, many of the states involving the vector \( H^* \) heavy meson may decay due to the \( H^* \to H \) transformation. The presence of an open decay channel may affect strongly the properties and the very existence of the sought after bound state \([6]\). At the same time coupling of the considered \( HH \) system to (closed) channels containing vector
heavy mesons makes the binding of two pseudoscalar mesons more strong. Thus, the search of a possible molecular-type state in any heavy-meson system but the one (\(HH\)) considered in the present paper requires much more extensive analysis, making possible conclusions substantially less unambiguous.

The existence of a flavor-exotic \(HH\) tetraquark depends crucially on the value \(g\) of the \(H^*H\pi\) coupling constant. The latter is related to the width of the \(H^* \to H + \pi\) decay and, since corresponding partial width is measured, on the total width \(\Gamma\) of the \(H^*\)-meson. Provided \(\Gamma\) is taken equal to the experimental upper limit, the CTPX induced forces alone are strong enough to produce a bound state of \(BB\) mesons, while the \(DD\) system is very close to support a bound state. Such bound states will manifest themselves as stable scalar mesons with the mass \(M \approx 2m_H\) and the flavor quantum number equal to \(\pm 2\) (note that existence of a \(HH\) bound state implies that the \(\bar{H}\bar{H}\) system is also bound). The same conclusion holds true for the flavor-hidden \(\bar{h}H\) states which have the same binding energy. It is relevant to note that CTPX induced flavor exotic molecular-type states emerge at about the same values of \(g\) which ensure boundedness of flavor-hidden \(\bar{H}^*H^*\) and \(\bar{H}^*H\) states generated by one-pion exchange \([4,5,3]\). At smaller values of \(g\) molecular-type states with any heavy meson content are not expected to emerge. More complicated systems of heavy mesons where both one- and (correlated) two-pion exchanges are operative will be considered elsewhere.
REFERENCES

[1] J.-P.Ader, J.-M.Richard and P.Taxil, Phys.Rev. D25 (1982) 2370

[2] H.J.Lipkin, Phys.Lett. B172 (1986) 242

[3] A.V.Manohar and M.B.Wise, Nucl.Phys. B399 (1993) 17

[4] N.A.Tönnqvist, Phys.Rev.Lett. 67 (1991) 556

[5] N.A.Tönnqvist, Zs.Phys. C61 (1994) 525

[6] M.Shmatikov, Nucl.Phys. A (in press)

[7] T.Ericson and W.Weise, Pions and nuclei, Oxford University Press, 1988

[8] G.E.Brown, In: Mesons in nuclei, vol.I, Eds. M.Rho and D.Wilkinson, North-Holland Publishing Company, Amsterdam 1976, p.329

[9] G.E.Brown and A.D. Jackson, The Nucleon–Nucleon Interaction, North-Holland Publishing Company, Amsterdam 1976, p.137

[10] H.-C.Kim, J.W.Durso and K.Holinde, Phys.Rev. C49 (1994) 2355

[11] D.Lohse, J.W.Durso, K.Holinde and J.Speth, Nucl.Phys. A516 (1990) 513

[12] B.Moussallam, Nucl.Phys. A429 (1984) 429

[13] M.B.Wise, Phys.Rev. D45 (1992) R2188

[14] ACCMOR collaboration, Phys.Lett. B278 (1992) 480

[15] A.A.Bolokhov, A.N.Manashov, V.V.Vereshagin and M.V.Polyakov, Phys.Rev. D50 (1994) 4713

[16] P.Colangelo, G.Nardulli, A.Deandrea, N. Di Bartolomeo, R.Gatto and F.Feruglio, Phys.Lett. B339 (1994) 151