Efficient FDTD-PE Method for GNSS Propagation in Airport to Support DCPS

Jing Liu*, Jiaquan Ye, Ping Yang, Xiaojia Yang and Zhengbo Yang
ATC Lab, CAACSRI, Chengdu, China

*Corresponding author

Abstract. To support Differentially Corrected Positioning Service (DCPS) in airport, the Global Navigation Satellite System (GNSS) receiver’s anti-multipath ability should be validated carefully to airport environment. This paper discusses the quantification of GNSS signal characteristics in airport based on Finite Difference Time Domain-Parabolic Equation (FDTD-PE) method. The proposed method can deal with complex environment with different objects and large scale clear space of airport efficiently. An equivalent process is introduced to deal with the problem of non-uniform sampling grid. By comparing with FDTD method, the validation of this method is proved. The simulation results demonstrate the effectiveness and accuracy of this method for predicting the GNSS signal propagation in airport environment.

1. Introduction
The increasing trend of civil air transport has placed a growing stress on airport to manage more traffic. The committee of FANS (Future Air Navigation Systems) of ICAO has recommended satellite technology to deal with this problem [1]. After augmented, GNSS with high integrity can provide Differentially Corrected Positioning Service (DCPS) for aircrafts or vehicles moving in apron. Ground Based Augmentation System (GBAS) is one of such systems, which correction data and other information is broadcasted through VHF data broadcasting datalink to augment GNSS signal. In apron, the reflections from buildings or ground as well as all scattering from bodies near the GNSS receiver antenna will affect the performance of GNSS receiver’s code tracking or range measuring. In order to conquer this challenging, different technologies have been studied. But a universal one has not been developed. To verify the abilities of these technologies in DCPS application, it is important to provide a precise model to quantify potential mechanisms of multipath path geometry, signal characteristics as well as their coupling with antenna or receiver design to GNSS signal reception in apron.
In the past years, based on the multipath effects on Global Positioning System (GPS) derived from [2], much endeavor has been paid in modeling GNSS multipath propagation characteristics. Such as in [3], the problem is studied geometrically, but the feature of reflected signal is not considered. The terrain induced multipath is discussed in [4]. The statistical model for urban and suburban is studied in [5] based on field data collected. For DCPS application in airport, limited data is collected to verify the effectiveness of these models. The standardized multipath model in civil aviation is only valid for airborne [6]. In apron, to keep the position integrity and accuracy in required level, multipath effect on GNSS signal reception from objects and ground should be analyzed.
To quantitative the effect of multipath on GNSS signal reception, it is convenient to construct the model based on radio wave propagation theory. The FDTD is a high accuracy method for modeling wave propagation in time-domain which using direct form of Maxwell’s equations. But for long distance, it will be more calculation demanding than ray-tracing method. After Hardin proposed the
application of Split-Step Fourier (SSPE) to solve PE for wave propagation problem [7], the PE method is used to analyze radio wave propagation over terrain [8]. Despite its less accurate, the PE method is faster and less computation consuming for wide area wave propagation approximation. Considering the merits of both methods, we present a FDTD-PE method to model the GNSS signal propagation in apron. The FDTD method is employed to approximate the effect of multipath due to obstacles, and construct the initial conditions for PE method. The PE method is employed to deal with the clear propagation environment approximation, and produce excitant field for FDTD. The description of other part of this work is as follows: section 2 describes the problem of GNSS signal propagation in airport; section 3 gives an overview of FDTD method and the PE method briefly; section 4 describes the FDTD-PE method; section 5 demonstrates the effectiveness of this method; section 6 summaries the results.

2. The Problem Formulation
In apron, the signals radiated directly from GNSS satellites will be acquired by GNSS receiver, and the ones from objects around the receiver will be either. As satellites of a GNSS constellation running along an orbit high above ground, the signal radiated from a satellite can be seen as parallel rays with an elevation angle arrived at a plane perpendicular to the propagation direction. The signal sent from jth satellite can be expressed as [9]

$$s^j(t) = \sum_{n=-\infty}^{+\infty} d(n)[g(t) + \sum_{l=0}^{L-1} c(l) \delta(t - kT_c)] \times \sqrt{A}\cos(2\pi ft)$$  \hspace{1cm} (1)

where, $d(n)$ denotes the navigation data bit; $c(i)$ denotes the spreading; $L$ denotes the length of pseudorandom sequence; $T_c$ denotes the duration time of each chip; $f$ denotes the carrier frequency; $g(t)$ denotes the modulation waveform.

If the amplitude and phase of signal is stable during the sample time, when the signal of the jth GNSS satellite arrives at the receiver’s antenna, electrical field formed by Line-Of-Sight (LOS) and indirect multipath signal will be expressed as:

$$r^j(t) = \sqrt{A}\left[\sum_{n=-\infty}^{+\infty} \beta_0 d(n)[g(t - t_0) + \sum_{l=0}^{L-1} c(l) \delta(t - t_0 - kT_c)] \cos(2\pi ft) + \sum_{i=1}^{k} \beta_i \sum_{n=-\infty}^{+\infty} d(n)[g(t - t_j) + \sum_{l=0}^{L-1} c(l) \delta(t - t_j - kT_c)] \cos(2\pi ft + \epsilon_j)\right]$$  \hspace{1cm} (2)

where, $k$ denotes the number of multipath; $\beta_i$, $0 \leq i \leq k$ denotes the amplitude effect on different path signal. In (2), if we only pay more attention to carrier, we can simplify it into the sum of a LOS signal and the multipath ones with different amplitude decreasing, propagation time delay and extra phase change:

$$r^j(t) = A_{LOE}e^{j[2\pi f(t-t_0)+\phi_0]} + \sum_{k=1}^{K} A_{MK}e^{j[2\pi f(t-t_k)+\phi_0+\phi_k]}$$  \hspace{1cm} (3)

To quantify the actual effect of multipath, it is necessary to derive the amplitude, phase and delay etc. from a model. Also, the received signal processed by GNSS receiver is a narrow band signal. So, we can simply the electric field into direct part and scatter part, which add up to the total electric field:

$$\vec{E}(t) = \vec{E}_d(t) + \vec{E}_s(t)$$  \hspace{1cm} (4)

where, $\vec{E}_d(t)$ and $\vec{E}_s(t)$ denote electric field produced by direct signal and scatter signal separately. Both ones can be represented by two orthogonal electric field calculated through approximation of electromagnetic radiation propagation.

3. Three Dimension FDTD Method and PE Method

3.1. 3D FDTD Method
Three dimension (3D) FDTD is a method to approximate the Maxwell’s equations evolution in time domain through discretion of time and space. This method can deal with the electric field in 3D environment with arbitrary materials and various boundary conditions, and also describe the situation of wave propagation. To calculate the solution, with the help of time derivative of electromagnetic equations and gridding of time and space, the electric field and the magnetic field can be updated.
iteratively. The details can be found in [10]. To acquire the stable solution, not only the wave frequency should be less than the Nyquist sampling frequency, but also the discretion in space and time should maintain following relationship:

$$
\Delta t \leq \left( \frac{c}{\sqrt{\frac{1}{\Delta x^2} + \frac{1}{\Delta y^2} + \frac{1}{\Delta z^2}}} \right)^{-1}
$$

(5)

3.2. PE Method

The PE is a method to approximate the wave equation incorporating one way propagation along a direction in Cartesian coordinate. The electric field and magnetic field of the electromagnetic wave can be expressed as a time convention factor dependent:

$$
\begin{align*}
\vec{E}(x, y, z, t) &= \vec{E}(x, y, z)e^{-i2\pi ft} \\
\vec{H}(x, y, z, t) &= \vec{H}(x, y, z)e^{-i2\pi ft}
\end{align*}
$$

(6)

So, for the Laplace’s equations, the solution for electric field or magnetic field can be expressed as a function of wave vector in the direction of positive and negative:

$$
\begin{align*}
S^+_{k_x, k_y, k_z}(x, y, z, t) &= s_p e^{i(k_x x + k_y y + k_z z - 2\pi f t)} \\
S^-_{k_x, k_y, k_z}(x, y, z, t) &= s_n e^{i(k_x x + k_y y + k_z z + 2\pi f t)}
\end{align*}
$$

(7)

(8)

where, $c$ denotes the speed of light in vacuum; $k_x, k_y$ and $k_z$ denote the arguments of wave vector; $s_p$ and $s_n$ denote the amplitude of positive and negative direction wave respectively.

Obviously, the solution can be treated as a product of several variables respectively according to split-step algorithm [11]:

$$
\begin{align*}
\psi_{n+1}|_{n+1}(\zeta + \Delta\zeta) &= e^{i\frac{\Delta\zeta}{2}}\left(\frac{4\pi^2f^2c^{-2} - k^2_{\zeta} - 2\pi f c^{-1}}{4\pi^2f^2c^{-2} - k^2_{\zeta} + 0.5\Delta\zeta - 2\pi f c^{-1}}\right)\psi_{n+1}|_{n+1}(\zeta + \Delta\zeta) \\
\psi_{n}|_{n+1}(\zeta) &= \mathcal{F}^{-1}\left\{e^{i\frac{\Delta\zeta}{2}}\left(\frac{4\pi^2f^2c^{-2} - k^2_{\zeta} - 2\pi f c^{-1}}{4\pi^2f^2c^{-2} - k^2_{\zeta} + 0.5\Delta\zeta - 2\pi f c^{-1}}\right)\psi_{n}|_{n}(\zeta)\right\}
\end{align*}
$$

(10)

(11)

where, $\psi_{n+1}|_{n+1}(\zeta)$ denotes the Fourier transform of field components, and $\zeta$ may be one component in x, y, z direction; $\Delta\zeta$ denotes discrete step in $\zeta$ direction; $\mathcal{F}$ denotes the Fourier transform; $\mathcal{F}^{-1}$ denotes the inverse Fourier transform. After solving the Fourier transform, one component of the final field will be expressed in time and space domain.

To achieve a stable calculation, the discretization in space and wavenumber domain should preserve following relationship [11]:

$$
\begin{align*}
\Delta k_x &= \frac{\pi}{L_x} |x| \leq L_x \\
\Delta k_y &= \frac{\pi}{L_y} |y| \leq L_y \\
\Delta k_z &= \frac{\pi}{L_z} |y| \leq L_z
\end{align*}
$$

(12)

4. FDTD-PE Method

Using FDTD-PE method to approximate GNSS signal propagation in apron can deal with situation with complex objects or large clear space with reasonable resources [12]. In this paper, the FDTD method is applied to calculate the effects in complex environment including multiple reflections and
diffractions; the PE method is applied to do with that in clear space. The transition between the area boundaries of these two methods is based on spatial filter.

4.1. Transition between FDTD and PE

As indicated in (5), the discretization in time limits the maximum frequency which the FDTD method can approximate. While corresponding to (12), the maximum wavenumber along one of the axis and the discrete space should satisfy the Nyquist theorem. Therefore, there is need to transform between time domain and frequency domain, and match the different grid size [12]. To deal with first one, the results from FDTD method are changed by Fourier transform at the interface of two areas and taken as the initial values of PE method. To the second one, an equivalent process method as stated in [13] is introduced. So, the grid size of different computation areas is only determined by the approximation method used, and can consider the boundary condition simply.

As stated in [14], equivalent currents are used as sources of fields which radiate waves between different media. Then to construct the equivalent current, only the nodes around the interested cell should be considered as the following way:

\[
\mathbf{\delta E}_{x}^{1}(1,0,0) = -\frac{\mu g_{1}}{\epsilon \Delta x_{g_{1}}} \mathbf{\delta S}_{y}(1,0,0) + \frac{\mu g_{1}}{\epsilon \Delta x_{g_{1}}} \mathbf{\delta S}_{y}(0,0,1) - \frac{\epsilon g_{1}}{\mu} \mathbf{J}_{x}(0,0,0) \tag{13}
\]

\[
\mathbf{\delta E}_{x}^{1}(1,0,0) = -\frac{\mu g_{1}}{\epsilon \Delta y_{g_{1}}} \mathbf{\delta S}_{x}(1,0,0) + \frac{\mu g_{1}}{\epsilon \Delta y_{g_{1}}} \mathbf{\delta S}_{x}(0,0,1) - \frac{\epsilon g_{1}}{\mu} \mathbf{J}_{y}(0,0,0) \tag{14}
\]

\[
\mathbf{\delta E}_{z}^{1}(1,0,0) = -\frac{\mu g_{1}}{\epsilon \Delta z_{g_{1}}} \mathbf{\delta S}_{x}(1,0,0) + \frac{\mu g_{1}}{\epsilon \Delta z_{g_{1}}} \mathbf{\delta S}_{x}(0,0,1) - \frac{\epsilon g_{1}}{\mu} \mathbf{J}_{z}(0,0,0) \tag{15}
\]

\[
\mathbf{\delta S}_{x}^{1}(0,0,0) = \frac{\mu g_{1}}{\epsilon \Delta x_{g_{1}}} \mathbf{\delta E}_{y}(0,1,0) - \frac{\mu g_{1}}{\epsilon \Delta x_{g_{1}}} \mathbf{\delta E}_{y}(1,0,0) - \frac{\mu g_{1}}{\mu} \mathbf{K}_{x}(1,0,0) \tag{16}
\]

\[
\mathbf{\delta S}_{y}^{1}(0,1,0) = \frac{\mu g_{1}}{\epsilon \Delta y_{g_{1}}} \mathbf{\delta E}_{x}(0,1,0) - \frac{\mu g_{1}}{\epsilon \Delta y_{g_{1}}} \mathbf{\delta E}_{x}(1,0,0) - \frac{\mu g_{1}}{\mu} \mathbf{K}_{y}(1,0,0) \tag{17}
\]

\[
\mathbf{\delta S}_{z}^{1}(0,1,0) = \frac{\mu g_{1}}{\epsilon \Delta z_{g_{1}}} \mathbf{\delta E}_{y}(0,1,0) - \frac{\mu g_{1}}{\epsilon \Delta z_{g_{1}}} \mathbf{\delta E}_{y}(1,0,0) - \frac{\mu g_{1}}{\mu} \mathbf{K}_{z}(1,0,0) \tag{18}
\]

where, \(\mathbf{\delta S}_{x}^{1}\) denotes the unit vector normal to the surface, sub-index \(g_{1}\) denotes the grid of next area which the wave is going to, \(\mathbf{\delta E}_{x}\) denotes the electric current sheets, \(\mathbf{\delta E}_{y}\) denotes the magnetic current sheets, \(\mathbf{\delta E}_{z}\) denotes the difference between neighbor grid cells, the superscript index denotes the grid cell of source, and the subscript indexes denote the axis and neighbor grid cell. In this two equivalent transform process, (13) to (20) are used to interpolate and distribute the radio wave electromagnetic field on the boundary of areas with different grid size.

4.2. Algorithm

The proposed FDTD-PE method is implemented as following:
1) The satellite parameters (such as frequency, initial elevation and azimuth angle), the receiver parameters (such as position and height) and the default values for PE calculation are set.
2) The terrain and object data are loaded, then sampling grid and their property are created.
3) For every sample grid points, the electromagnetic field is calculated by FDTD method, PE method or the boundary equivalent transform process.
4) If the fields of all points are calculated, the process is ended; else the process is iterated until the fields of all points field are calculated.

5. Numerical Results and Comparison

In the validation, the GNSS signal which radiates at the power of -159.5 dBw is assumed to work at the frequency of 1.575 GHz in elevation 15 degree from azimuth 275 degree. Firstly, the calculated
electric field based on FDTD method and FDTD-PE method along one vertical profiles with objects are shown in Fig. 1 and Fig. 2 respectively. According to these figures, near the objects, the maximum difference is 9.6679 dB, but the decreasing of accuracy is reasonable accounting the saving of calculation resource consumption. In free space, the maximum difference is 0.2181 dB, but the gridding cell size can be much bigger, so the proposed method can calculate with less time. Secondly, in a scenario described as in Fig. 3, the calculated electric field based on FDTD-PE method at the height of 2.7 meter is shown in Fig. 4, which demonstrates the proposed method can deal with horizontal plane efficiently.

Figure 1. The vertical profile field distribution modelled by FDTD method (unit in decibel).

Figure 2. The vertical profile field distribution modelled by FDTD-PE method (unit in decibel).

Figure 3. The scenario for approximation.

Figure 4. The electric field calculated by FDTD-PE method in horizontal plane at the height of 2.7 meters (unit in decibel).

6. Conclusion
To study the GNSS signal’s characteristics in airport, the FDTD-PE method is employed and some simulation work has been carried out. It can be seen from the results, the electric fields calculated by FDTD-PE method are almost consistent with those by FDTD method. The proposed FDTD-PE method is efficient to model the influence of GNSS signal in complex environment, which can approximate both large free space and complex objects effects with reasonable accuracy and resource. Obviously, in view of the characters of navigation satellite, further improve the FDTD-PE method can provide more benefits for supporting DCPS in airport.

Acknowledgement
The authors thank the suggestions and constructive comments from the reviewers. This work was supported by the National Key Research Program of China (No.2017YFB0503402).
References

[1] Doc. 9635, Facilities and Services Implementation Document (FASID) for the North Atlantic Region, ICAO, 1995.

[2] L. Hagerman, Effects on Multipath of Coherent and Non Coherent PRN Ranging Receiver. The Aerospace Corporation, 1973.

[3] L. Lau and P. Cross, “Development and testing of a new ray-tracing approach to GNSS carrier-phase multipath modeling,” Journal of Geodesy, vol. 81, 2007, pp. 713-732.

[4] V. U. Zavorotny, K. M. Larson, and J. J. Braun, et al., “A physical model for GPS multipath caused by land reflections: toward bare soil moisture retrievals,” IEEE J. Sel. Top. Appl. Earth, vol. 3, 2010, pp. 100-110.

[5] A. Steingass, and A. Lehner, “Measuring the navigation multipath channel – a statistical analysis,” Proc. ION GNSS 2004, Long Beach, CA, Sept. 2004, pp. 1157-1164, doi: 10.1.1.455.6292.

[6] T. Murphy, M. Harris, and P. Geren, et al., “More results from the investigation of airborne multipath errors,” Proc. ION GNSS 2005, Long Beach, CA, Sept. 2005, pp. 2670-2687.

[7] M. Fournier, “Analysis of propagation in an inhomogeneous atmosphere in the horizontal and the vertical direction using the parabolic equation method,” AGARD CP, no. 453, 1989, pp. 21.1-21.12.

[8] M. F. Levy, “Parabolic equation modeling of propagation over irregular terrain,” Electron. Lett., vol. 26, 1990, pp. 1153-1155.

[9] K. Borre, D. M. Akos, N. Bertelsen, P. Rinder, and S. H. Jensen, A Software Defined GPS and Galileo Receiver – a Single-frequency Approach, Berlin: Birkhäuser, 2007.

[10] A. Taflove, and S. C. Hagness, Computational Electrodynamics : the Finite-Difference Time-Domain Method, 3rd ed., Artech House, 2005.

[11] Y. T. Lin, T. F. Duda, and A. E. Newhall, “Three-dimensional sound propagation models using the parabolic-equation approximation and the split-step fourier method,” Journal of Computation Acoustics, vol. 21, no. 01, 2013, pp. 1-24.

[12] T. V. Renterghem, E. Salomons, and D. Botteldooren, “Parameter study of sound propagation between city canyons with a coupled FDTD-PE model,” Applied Acoustics, vol. 67, 2006, pp. 487-510.

[13] J. P. Bérenger, “The huygens subgridding for the numerical solution of the Maxwell equations,” Journal of Computational Physics, vol. 230, 2011, pp. 5635-5659.

[14] C. J. Railton, “Rotated sub-grids in the FDTD method,” IEEE Transactions on Antennas and Propagation, vol. 64, no. 7, 2016, pp. 3047-3054.