THREE QUARK CLUSTERS IN HOT AND DENSE NUCLEAR MATTER

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Abstract

We present a relativistic in-medium three-body equation to study correlations in hot and dense quark matter. The equation is solved for a zero-range force for parameters close to the phase transition of QCD.

1 Introduction

Models to study $q\bar{q}$ bound states, correlations, or condensates in a hot and dense medium such as nuclear/quark matter are rather elaborate. These models are used to explore the phase structure of QCD in particular in areas presently not accessible to lattice calculations. Little attention, however, has been paid to $qqq$ correlations in this context \[1, 2, 3, 4\]. As three valence quarks are a dominant Fock component in any quark model of baryons those should be relevant in the confinement-deconfinement region from nucleons to quarks.

Another reason to study three-quark correlations is their relevance namely $q^3$ correlations to color superconductivity (see e.g. \[5\]). It has been shown in a different context (Hubbard model) that the inclusion of three-particle correlations lowers the critical temperature $T_c$ compared to the bare two-particle Thouless instability \[6\]. The existence and the area of the QCD phase diagram where one might find color superconductivity might depend on dynamical details and in particular on the influence of three-quark correlations which have not been investigated so far.

Presently, our aim is to provide a proper framework to tackle the question of three-body correlations at finite temperatures and densities for a relativistic many-body system, i.e. soluble effective Alt-Grassberger-Sandhas (AGS) or Faddeev equations that include medium effects as well as special relativity. To do so, several problems have to be overcome and in what we present here one might consider an approach towards a systematic inclusion of medium effects and relativity in effective three-body equations. In the past years utilizing
the Dyson approach we have derived suitable in-medium equations to treat three-body correlations as well as four-body bound states and condensates in a nuclear medium of finite temperatures and densities, see e.g. [7]. These equations are generalized here to include special relativity. To this end we use the light front approach [8]. Except for details in the treatment of the medium the use of the light front at finite densities is similar to that suggested in Ref. [9] for $T = 0$.

2 Theory

For the time being we consider a zero-range interaction. A simple effective theory of zero range is provided by the Nambu Jona-Lasinio model [10] that has also been extended to finite temperatures and densities, see, e.g. [11]. Note that due to screening effects the effective potential between quarks may lose the confining property above the critical temperature [12]. However, a closer connection to QCD including confinement is certainly desired, see e.g. [13, 14]. However, presently, the main focus of our approach is on the structure of the relativistic in-medium equation.

2.1 Self energy-correction, gap equation

In Hartree-Fock approximation the self-energy correction induced by the medium is given by the gap equation. The gap equation leads to effective masses $m(\mu, T)$ that depend on the temperature $T$ and the chemical potential $\mu$ of the medium. To proceed we use the light front formalism. The four-dimensional gap equation is projected onto the light front using the Lepage-Brodsky regularization scheme [15].

$$m = m_0 + 2i\lambda N_c N_f \int_{LB} \frac{d^4k}{(2\pi)^4} \mathrm{Tr}\{S(k)\} \ (1 - f(k)) \quad (1)$$

The Fermi-Dirac distribution function

$$f(k^+, \vec{k}_\perp) = \left( \exp \left[ \frac{1}{k_B T} \left( \frac{k_{\text{on}}^+ + k^+}{2} - \mu \right) \right] + 1 \right)^{-1} \quad (2)$$

is expressed in terms of light front form momenta that are defined by $\vec{k}_\perp = (k_x, k_y)$ and $k^\pm = k_0 \pm k_z$. This treatment leads to the same definition of blocking factors in the limit $T \to 0$ as given in Ref. [9]. Note that the on-shell light front energy $k_{\text{on}} = (\vec{k}_\perp^2 + m(\mu, T)^2)/k^+$ depends on $\mu$ and $T$. The (familiar) mass dependence is shown in Figure [4].
2.2 Two-body case

The technical difficulties including angular momentum in relativistic many-body systems are well known. For the time being we average over the spin projections which means that the spin degrees of freedom are washed out in the medium. This will be improved while the investigation proceeds along the lines suggested in Ref. [16]. Also we expect antiparticle degrees of freedom to be of minor importance for a zero range interaction on the light front [17].

The solution for the two-body propagator $\tau(M_2)$ for a zero-range interaction is given by [17]

$$\tau(M_2) = \left(i\lambda^{-1} - B(M_2)\right)^{-1},$$

where the expression for $B(M_2)$ in the rest system of the two-body system is

$$B(M_2) = -\frac{i}{(2\pi)^3} \int \frac{dx d^2k_{\perp}}{x(1-x)} \frac{1 - f(x, k_{\perp}^2) - f(1 - x, k_{\perp}^2)}{M_2^2 - M_{20}^2},$$

and $M_{20}^2 = (k_{\perp}^2 + m^2)/x(1-x)$ and $x = k^+/P^+$. 

2.3 Three-body case

The solution for the two-body propagator $\tau(M_2)$ is the input for the relativistic three-body equation. The inclusion of finite temperature and chemical potential is determined by the Dyson equations as explained, e.g. in Ref. [7]. With the introduction of the vertex function $\Gamma$ the equation becomes

$$\Gamma(y, \vec{q}_{\perp}) = \frac{i}{(2\pi)^3} \tau(M_2) \int_{M_2^2/M_3^2}^{1-y} \frac{dx}{x(1-y-x)} \int_{k_{\perp}^{\text{max}}}^{\bar{k}_{\perp}^2} d^2k_{\perp} \frac{1 - f(x, \vec{k}_{\perp}^2) - f(1 - x - y, (\vec{k} + \vec{q}_{\perp})^2)}{M_3^2 - M_{03}^2} \Gamma(x, \vec{k}_{\perp}),$$

where $m = m(\mu, T)$,

$$k_{\perp}^{\text{max}} = \sqrt{(1-x)(xM_3^2 - m^2)},$$

and the mass of the virtual three-particle state in the rest system is given by

$$M_{03}^2 = \frac{\bar{k}_{\perp}^2 + m^2}{x} + \frac{\bar{q}_{\perp}^2 + m^2}{y} + \frac{(\vec{k} + \vec{q}_{\perp})_1^2 + m^2}{1 - x - y}.$$

The blocking factors, $1 - f - f$ that appear in (3) can be rewritten as $\bar{f} \bar{f} - f f$, where $\bar{f} = 1 - f$ to exhibit the particle and the hole blocking. For $T \to 0$, $\bar{f} \to \theta(k - k_F)$ that cuts the integrals below the Fermi momentum.
Figure 1: Dependence of the mass on the (quark) chemical potential and temperature.

Figure 2: Masses of two-quark $M_2$ vs. three-quark $M_3$ bound states at $T = 10$ MeV for different $\mu$. For further explanations see text.

3 Results

For the time being we assume a bound state $M_{2B}$ in the two-body subsystem. This can be relaxed while the investigation proceeds and more realistic models are implemented. Our main focus is on effects the medium has on the competition between two and three-quark states. This is particularly relevant in the vicinity of the critical temperatures. A first step towards this aim here is the investigation of the Mott transition from the three-body bound state to the two body (2+1) channel that differs from the three-nucleon case [18]. To this end, we vary $M_{2B}$ implicitly choosing a particular model strength $\lambda$ in (3) for each value of $M_{2B}$. This is shown in Figure 2. For a given $M_{2B}$ the solid line reflects the corresponding three-body bound state for the isolated system. For a temperature of $T = 10$ MeV the various dashed-dotted lines correspond to increasing chemical potential (see Figure 3 for the corresponding values of $\mu$). The long dashed line is the two-body threshold. In a simple chemical picture, the equilibrium composition of the system is dominated by three-body bound states below the dotted line and by two-body states above this line (law of mass action). In Figure 3 we show the binding energies

$$B_3(\mu, T) = m(\mu, T) + M_{2B}(\mu, T) - M_{3B}(\mu, T)$$

$$B_2(\mu, T) = 2m(\mu, T) - M_{2B}(\mu, T).$$
Figure 3: Correlations between two-quark and three-quark binding energies in units of quark mass at $T = 10$ MeV for different chemical potentials $\mu$, as indicated. Dashed with triangles [1], further explanations see text.

Figure 4: Mott lines for the three-body system at rest in the medium. For values of $T$ and $\mu$ below the Mott lines three-body bound states can be formed. Solid line for $M_2 = m$, dashed line for $M_2 = 15m/8$.

The solid line refers to the isolated case and the various dashed-dotted lines again represent the in-medium results for different chemical potentials at $T = 10$ MeV. In addition, we refer to the NJL results (dashed line with triangles) given in Ref. [1] for $m = 450$ MeV and $T = 0$: The triangles are for values of $\mu(T = 0) \approx 1.0, 1.05, 1.08, 1.12, 1.22$ (top to bottom) in units of the respective $m(\mu, 0)$. The dashed vertical lines reflect specific values for $M_{2B} = m$ and $M_{2B} \approx 1.88m$ in our model covering a wide range of $M_{2B}$ and for $T = 10$ MeV. For a given two-body binding energy and increasing chemical potential we find weaker three-body binding. This leads to the disappearance of the three-quark bound state for a certain value of the chemical potential which is known as Mott transition, $B_3(\mu_{\text{Mott}}, T_{\text{Mott}}) = 0$. The values of $T$ and $\mu$ for which this transition occurs is plotted in Fig. 4 for the two different models given above. Clearly the behavior qualitatively reflects the confinement deconfinement phase transition.

In conclusion, we have given a consistent equation for a relativistic three-quark system in a medium of finite temperature and density. This equation includes the dominant effects of the medium, viz. Pauli blocking and self energy corrections. As many details need to be improved, this first calculation
shows that for a large range of models the Mott transition agrees qualitatively with the phase transition expected from other sources. Based on this approach it is now possible to systematically investigate the influence of three-quark correlations on the critical temperature and the onset of color superconductivity at high density.

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