Neural Network for Weighted Signal Temporal Logic

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Abstract
In this paper, we propose a neuro-symbolic framework called weighted Temporal Logic Neural Network (wSTL-NN) that combines the characteristics of neural networks and temporal logics. Weighted Signal Temporal Logic (wSTL) formulas are recursively composed of subformulas that are combined using logical and temporal operators. The quantitative semantics of wSTL is defined such that the quantitative satisfaction of subformulas with higher weights has more influence on the quantitative satisfaction of the overall wSTL formula. In the wSTL-NN, each neuron corresponds to a wSTL subformula, and its output corresponds to the quantitative satisfaction of the formula. We use wSTL-NN to represent wSTL formulas as features to classify time series data. STL features are more explainable than those used in classical methods. The wSTL-NN is end-to-end differentiable, which allows learning of wSTL formulas to be done using back-propagation. To reduce the number of weights, we introduce two techniques to sparsify the wSTL-NN. We apply our framework to an occupancy detection time-series dataset to learn a classifier that predicts the occupancy status of an office room.

1 Introduction

Time series classification (TSC) has been considered as one of the most challenging tasks in machine learning (ML). Many supervised ML algorithms have been proposed to solve TSC problems [Qian et al., 2020; Fawaz et al., 2019]. However, these classification models are often not human-readable, for example, hyperplanes in a higher-dimensional space. Temporal logics are formal languages that can express specifications about the temporal properties of systems. Compared with traditional ML models, temporal logic formulas can express temporal and logical properties in a human-readable form. Human-readability is important because it can give non-expert users insights into the model from qualitative and quantitative perspectives. Temporal logics have been widely used to solve formal verification and controller synthesis problems of cyber-physical systems [Carr et al., 2020; Rodrigues da Silva et al., 2020; Kantaros and Zavlanos, 2020; Lindemann and Dimarogonas, 2020; Gundana and Kress-Gazit, 2020].

Signal Temporal Logic (STL), a branch of temporal logic, is used to express specifications over real-time and real-valued data. STL has been a popular tool for analyzing real-time series data by learning STL formulas from data. The learning task is based on a notion of quantitative satisfaction of STL formulas and posed as an optimization problem with the quantitative satisfaction in the objective function [Xu and Topcu, 2019; Kong et al., 2014]. A notion of quantitative satisfaction of STL was proposed in [Donzé and Maler, 2010] that uses max and min functions, which are non-convex and non-smooth. As pointed out in [Mehdipour et al., 2020], the traditional quantitative satisfaction has a limitation that the satisfaction of parts of the formula other than the most "significant" part has no contribution to the overall quantitative satisfaction. Mehdipour et al then proposed a weighted STL (wSTL) that allows the expression of user preference on STL specifications by encoding the preference of satisfaction with weights. However, the quantitative satisfaction proposed in [Mehdipour et al., 2020] has two limitations: 1) the subformulas with zero weights are still influential in the overall quantitative satisfaction; 2) it does not reflect that quantitative satisfaction of subformulas with higher weights has a greater influence on the overall quantitative satisfaction.

Combining neural networks and symbolic logic to perform learning tasks has attracted lots of attention [Riegel et al., 2020; Yang et al., 2017; Serafini and Garces, 2016; Rocktäschel and Riedel, 2017] in recent years. Activation functions corresponding to truth functions of logical operators in first-order logic have been proposed in [Riegel et al., 2020] such that the truth value bounds of formulas can be learned from the neural network. Most of the existing STL learning algorithms solve a non-convex optimization problem to find the parameters in the formula [Kong et al., 2014; Xu et al., 2019; Yan et al., 2019], where the loss function is not differentiable everywhere with respect to the parameters.

The problem of using neural networks to perform temporal logic learning tasks has not been well studied. The contributions of this paper are: 1) we define a novel quantitative satisfaction for wSTL that has the properties of non-influence of zero weights, ordering of influence, and monotonicity (see Section 4 for more details); 2) we construct a novel frame-
work that combines the characteristics of neural network and wSTL such that it can perform wSTL learning tasks, where neurons represent the quantitative satisfaction of temporal and logical operators; 3) we propose two sparsification techniques that can prune the weights and reduce the memory.

2 Preliminaries

A discrete $l$-dimensional time series data is denoted as $s = s(0), s(1), ..., s(l)$, where $l \in \mathbb{Z}_{\geq 0}$, $s(k) \in \mathbb{R}^l$, $k \in \mathbb{Z}_{\geq 0}$. A time interval between $k_1$ and $k_2$ is denoted as $I = [k_1, k_2] = \{k | k_1 \leq k < k_2, k_1, k_2 \in \mathbb{Z}_{\geq 0}\}$, and $k + I$ denotes the time interval $[k + k_1, k + k_2]$.

Signal Temporal Logic (STL)

In this paper, we consider a fragment of Signal Temporal Logic (STL) proposed in [Maler and Nickovic, 2004], whose syntax is defined recursively as follows:

$$\phi := \top | \neg \phi_1 \land \phi_2 | \phi_1 \lor \phi_2 \square \phi \diamond \phi,$$  (1)

where $\phi_1, \phi_2$ are STL formulas, $\top$ is Boolean True, $\pi := f(s) = a^T s < c$ is a predicate defined over $s$, $a \in \mathbb{R}^l$, $c \in \mathbb{R}$, $\neg, \land, \lor, \square, \diamond$ are STL formulas, $\neg, \land, \lor$ are logical negation, conjunction, and disjunction operators. Temporal operators $\square, \diamond$ read as "always, eventually", respectively. $I$ is a time interval and $\square_1 \phi$ is satisfied at $k$ if $\phi$ is satisfied at all $k' \in k + I$, $\diamond_1 \phi$ is satisfied at $k + I$ if $\phi$ is satisfied at some $k' \in k + I$.

The Boolean semantics of STL measures whether $s$ satisfies $\phi$ at $k$ quantitatively, for example, $(s, k) \models \phi$ as "$s$ satisfies $\phi$ at $k$" and $(s, k) \not\models \phi$ as "$s$ violates $\phi$ at $k$". The quantitative semantics of STL measures the degree of satisfaction or violation of $\phi$ over $s$ at $k$.

**Definition 1.** The quantitative semantics (satisfaction) of STL is defined as [Donzé and Maler, 2010]:

$$r(s, \pi, k) = c - a^T s(k),$$

$$r(s, \neg \phi, k) = -r(s, \phi, k),$$

$$r(s, \phi_1 \land \phi_2, k) = \min\{r(s, \phi_1, k), r(s, \phi_2, k)\},$$

$$r(s, \phi_1 \lor \phi_2, k) = \max\{r(s, \phi_1, k), r(s, \phi_2, k)\},$$

$$r(s, \square_1 \phi_1, k) = \min_{k' \in [k_1, k_2]} r(s, \phi_1, k'),$$

$$r(s, \diamond_1 \phi_1, k) = \max_{k' \in [k_1, k_2]} r(s, \phi_1, k').$$

The quantitative satisfaction at $k = 0$ is simplified as $r(s, \phi)$.

3 Weighted Signal Temporal Logic (wSTL)

The quantitative satisfaction of the $\land, \lor, \forall, \exists$ operators in **Definition 1** is the minimum or maximum of the quantitative satisfaction of subformulas. This means the overall quantitative satisfaction is determined by the quantitative satisfaction of a single subformula. Also, traditional quantitative satisfaction cannot express importance over different subformulas. In many cases, we require the quantitative satisfaction function capable of reflecting the influence of different subformulas.

**Example 1.** A mobile robot needs to transport building materials from $Q$ to a construction site $G$ and it has to go to either store $A$ or $B$ for materials before arriving at $G$ (See Figure 1 as an illustration). We can use an STL formula $\phi = \left(\lnot \exists [1,5](\pi_1 \lor \pi_2) \land \lnot \exists [8,10](\pi_3 \lor \pi_4)\right)$ to express this requirement, where $\pi_1, \pi_2, \pi_3, \pi_4$ are predicates describing $A$, $B$, $G$, respectively. Formula $\phi$ reads as "The robot needs to arrive at $A$ or $B$ before arriving at $G$ between 1 s and 5 s and arrive at $G$ between 8 s and 10 s." The problem of planning the movement of the robot is posed as an optimization problem by minimizing the energy and maximizing the quantitative satisfaction simultaneously. Even though the distance of going to $A$ is longer, it is favorable for the robot to go to $A$ because the price is cheaper. In this way, the quantitative satisfaction of going $A$ should have a greater influence on the quantitative satisfaction of $\phi$ than the quantitative satisfaction of going $B$.

We introduce the notion of importance weights into STL, where quantitative satisfaction of subformulas with larger weights has a greater influence on the overall quantitative satisfaction. The novel STL is called weighted STL (wSTL).

**Definition 2.** The syntax of wSTL is defined over $s$ as:

$$\tilde{\phi} := \top | \neg \tilde{\phi}_1 \land \tilde{\phi}_2 | \tilde{\phi}_1 \lor \tilde{\phi}_2 \tilde{\square}_1 \tilde{\phi} \tilde{\diamond}_1 \tilde{\phi},$$  (3)

where $\tilde{\top}$, $\tilde{\pi}$, and the logical and temporal operators are the same as the ones in STL, $\tilde{\pi}_1$ and $\tilde{\pi}_2$ are positive weights on the subformulas $\tilde{\phi}_1$ and $\tilde{\phi}_2$ correspondingly, $w = [w_{k_1}, w_{k_1} + 1, ..., w_{k_2}]^T \in \mathbb{R}_{\geq 0}^{k_2 - k_1 + 1}$ assigns a positive weight $w_{k'}$ to $k' \in [k_1, k_2]$ in the temporal operators.

With importance weights, wSTL can express more complicated specifications. The Boolean semantics of wSTL formulas are the same as the corresponding STL formulas. Throughout the paper, we use $w$ to denote weights associated with an operator and $w^\phi$ to denote weights associated with a wSTL formula $\phi$ that may have multiple operators.

**Definition 3.** The quantitative semantics (satisfaction) of wSTL is defined as:

$$r^w(s, \pi, k) = c - a^T s(k),$$

$$r^w(s, \neg \tilde{\phi}, k) = -r^w(s, \tilde{\phi}, k),$$

$$r^w(s, \tilde{\phi}_1 \land \tilde{\phi}_2, k) = \omega^w([w_1, r^w(s, \tilde{\phi}_1, k)]_{i=1,2}),$$

$$r^w(s, \tilde{\phi}_1 \lor \tilde{\phi}_2, k) = \gamma^w([w_1, r^w(s, \tilde{\phi}_1, k)]_{i=1,2}),$$

$$r^w(s, \tilde{\square}_1 \tilde{\phi}, k) = \omega^w([w_1, r^w(s, \tilde{\phi}, k + k')]_{k' \in [k_1, k_2]}),$$

$$r^w(s, \tilde{\diamond}_1 \tilde{\phi}, k) = \gamma^w([w_1, r^w(s, \tilde{\phi}, k + k')]_{k' \in [k_1, k_2]}),$$  (4)

where $\omega^w, \gamma^w : \mathbb{R}_{>0} \times \mathbb{R} \rightarrow \mathbb{R}$, $\omega^w, \gamma^w : \mathbb{R}_{>0}^{k_2 - k_1 + 1} \times \mathbb{R} \rightarrow \mathbb{R}$ are activation functions corresponding to the $\land, \lor, \forall, \exists$ operators, respectively.
Remark 1. The double negation property holds for the activation functions in (4) because \( r^w(s, \neg \bar{\phi}, k) = -r^w(s, \neg \bar{\phi}, k) = r^w(s, \bar{\phi}, k) \).

Remark 2. The quantitative satisfaction in (4) satisfies Demorgan’s law if and only if the activation functions for the \( \land, \lor, \square, \Diamond \) operators satisfy Demorgan’s law. For example, we can show that Demorgan’s law holds for the \( \land \) and \( \lor \) operators, and the validity of Demorgan’s law for the \( \square \) and \( \Diamond \) operators can be proved similarly. It follows that \( r(s, \neg((w_1 \land w_2) \land \bar{\phi}_1 \land \bar{\phi}_2), k) = -\bar{\land}((|w_1|, -r^w(s, \bar{\phi}_1), |w_2|, -r^w(s, \bar{\phi}_2), k)) = r(s, w_1 \land w_2 \land \bar{\phi}_1 \land \bar{\phi}_2, k) \).

4 Neural Network for wSTL (wSTL-NN)

Structurally, a wSTL-NN is a graph composed of neurons representing signals and operators that are connected in the way determined by the formula. Neurons representing predicates accept \( s \) at single time points as inputs and have activation functions corresponding to \( r^w(s, \pi, k) \), whose outputs are quantitative satisfaction of predicates. Neurons representing logical or temporal operators accept quantitative satisfaction of subformulas as inputs and have activation functions corresponding to the operators.

4.1 Activation Functions for Logical and Temporal Operators

Gradient-based neural learning of importance weights \( w^\phi \) requires the activation functions to be differentiable with respect to \( w^\phi \), where \( w^\phi \) is all the weights in \( \bar{\phi} \). Activation functions for the \( \lor, \land, \Diamond, \square \) operators can be derived from the activation function for the \( \land \) operator using DeMorgan’s law. A weighted version of the quantitative satisfaction in (2) was proposed in [Mehdipour et al., 2020], where the activation function for the \( \land \) operator is expressed as

\[
\vec{\land}^N([w_i, r_i]_{i=1:N}) = \min_{i=1:N} \left\{ \left( \frac{1}{2} - \bar{w}_i \right) \cdot \text{sign}(r_i) + \frac{1}{2} \right\}, \tag{5}
\]

where \( r_i = r^w(s, \bar{\phi}_i, k) \), and \( \bar{w}_i = w_i / \sum_{j=1}^{N} w_j \) is the normalized weight. The activation function in (5) has two limitations: 1) If \( w_i = 0 \), then \( r_i \) is still possible to be the one deciding the quantitative satisfaction, for example, if \( N = 3, r_1 = 1, r_2 = r_3 = 3, \) and \( \bar{w}_1 = 0, \bar{w}_2 = 0.5, \bar{w}_3 = 0.5 \), then the overall quantitative satisfaction is \( \vec{\land}^N([w_i, r_i]_{i=1:N}) = r_1 \). This means even if the importance weight associated with \( \bar{\phi}_1 \) is 0, it still can determine the overall quantitative satisfaction. 2) The quantitative satisfaction of \( \bar{\phi} \) is still determined by the signal at a single time point. To address these limitations, we propose several principles of the activation function for the \( \land \) operator such that subformulas with higher weights have a greater influence on the overall quantitative satisfaction and the weights can be learned by a neuro-symbolic network.

The choice of the activation function for the \( \land \) operator should obey the following principles:

- **Non-influence of zero weights:** \( r^w(s, \bar{\phi}_1, k) \) has no influence on \( r^w(s, \bar{\phi}_1 \land w_2 \bar{\phi}_2, k) \) if \( w_1 = 0 \).
- **Ordering of influence:** if \( r^w(s, \bar{\phi}_1, k) = r^w(s, \bar{\phi}_2, k) \) and \( w_1 > w_2 \), then we have
  \[
  \left| \frac{\partial r^w(s, w_1 \bar{\phi}_1 \land w_2 \bar{\phi}_2, k)}{\partial r^w(s, \bar{\phi}_1, k)} \right| > \left| \frac{\partial r^w(s, w_1 \bar{\phi}_1 \land w_2 \bar{\phi}_2, k)}{\partial r^w(s, \bar{\phi}_2, k)} \right| ;
  \]
- **Monotonicity:** \( r^w(s, \bar{\phi}_1 \land w_2 \bar{\phi}_2, k) \) increases monotonically over \( r^w(s, \bar{\phi}_1, k) \), i.e.
  \[
  \vec{\land}^N([w_i, r^w(s, \bar{\phi}_1, k)]_{i=1:N}) \leq \vec{\land}^N([w_i, r^w(s, \bar{\phi}_1, k) + d]_{i=1:N}), \text{where } d \geq 0.
  \]

This paper comes up with a concrete activation function for the \( \land \) operator by introducing another variable \( \sigma \), which promotes the activation functions for the other operators.

**Definition 4.** The activation functions for the \( \land, \lor, \square, \Diamond \) operators in (4) are defined as

\[
\begin{align*}
\vec{\land}^N([w_i, r^w(s, \bar{\phi}_1, k)]_{i=1:N}, \sigma) &= \sum_{i=1}^{N} \frac{\bar{w}_i s_i r_i^w}{\sum_{i=1}^{N} \bar{w}_i s_i}, \\
\vec{\lor}^N([w_i, r^w(s, \bar{\phi}_1, k)]_{i=1:N}, \sigma) &= -\sum_{i=1}^{N} \frac{\bar{w}_i s_i r_i^w}{\sum_{i=1}^{N} \bar{w}_i s_i}, \\
\vec{\square}(w, [r^w(s, \bar{\phi}_1, k + i)]_{i \in k + I}, \sigma) &= \frac{\sum_{i=1}^{N} \bar{w}_i s_i}{\sum_{i=1}^{N} \bar{w}_i s_i}, \\
\vec{\Diamond}(w, [r^w(s, \bar{\phi}_1, k + i)]_{i \in k + I}, \sigma) &= \frac{\sum_{i=1}^{N} \bar{w}_i s_i}{\sum_{i=1}^{N} \bar{w}_i s_i},
\end{align*}
\]

where \( \bar{w}_i \) is the normalized weight \( \bar{w}_i = w_i / \sum_{j=1}^{N} w_j \) in the \( \land, \lor \) operators, \( \bar{w}_i = w_i / \sum_{j=1}^{N} w_j \) in the \( \square \) operators, \( I = [k_1, k_2], r_i = r^w(s, \bar{\phi}_i, k) \) in the \( \land \) operator, \( r_i = -r^w(s, \bar{\phi}_i, k) \) in the \( \lor \) operator, \( r_i = r^w(s, \bar{\phi}_i, k + i) \) in the \( \square \) operator, \( r_i = -r^w(s, \bar{\phi}_i, k + i) \) in the \( \Diamond \) operator, \( s_i = e^{-\phi} / \sum_{j=1}^{N} e^{-\phi_j} \) in the \( \land \) and \( \lor \) operators, \( s_i = e^{-\phi} / \sum_{j=1}^{N} e^{-\phi_j} \) in the \( \square \) and \( \Diamond \) operators, \( \sigma > 0 \). To clarify the notation, we use \( w \) to denote weights of an operator before normalization, and \( \bar{w} \) to denote weights of the same operator after normalization.

**Proposition 1.** The activation functions defined in (6) satisfy the principles of non-influence of zero weights, ordering of influence, and monotonicity.

**Remark 3.** For sufficiently small \( \sigma \) and \( w_1 = w_2 \), \( r^w(s, w_1 \bar{\phi}_1 \land w_2 \bar{\phi}_2, k) \) can be arbitrarily close to the traditional STL quantitative satisfaction, i.e., \( r^w(s, w_1 \bar{\phi}_1 \land w_2 \bar{\phi}_2, k) \approx \min\{r^w(s, \bar{\phi}_1, k), r^w(s, \bar{\phi}_2, k)\} \).

With the activation function defined in (6), we can design neural networks for wSTL formulas. For example, the wSTL-NN for \( \phi = \Diamond_{[1,2]} [w_1^2 \bar{w}^2 (\bar{w}^2 - \bar{w}^2)] \) is shown in Figure 2.

4.2 Learning of wSTL Formulas with wSTL-NN

In this paper, wSTL learning refers to the process of finding a wSTL formula that can classify two sets of time series data
from a system. The two sets of data refer to a set \((D_P)\) of positive data with label 1 and a set \((D_N)\) of negative data with label -1. The inferred formula is satisfied by the data in \(D_P\) and violated by the data in \(D_N\).

Suppose \(|D_P| = \bar{p}, |D_N| = \bar{n}\), and \(D_C = D_P \cup D_N\) that has \(M = \bar{p} + \bar{n}\) data. The loss function should satisfy the requirements that the loss is small when the inferred formula is satisfied by the positive data or violated by the negative data, and the loss is large when these two conditions are not satisfied. A candidate loss function that satisfies the above requirements could be [Yan et al., 2019]

\[
J(\tilde{\phi}) = \sum_{j=1}^{M} h(l_j, r^w(D^j_C, \tilde{\phi})),
\]

where \(D^j_C\) is the \(j\)-th data in \(D_C\), \(l_j\) is the label of \(D^j_C\), and

\[
h(l_j, r^w(D^j_C, \tilde{\phi})) = \begin{cases} \zeta l_j r^w(D^j_C, \tilde{\phi}) & \text{if } l_j r^w(D^j_C, \tilde{\phi}) > 0, \\ \gamma & \text{else}, \end{cases}
\]

where \(\zeta > 0\) is a tuning parameter, and \(\gamma\) is a positive large number that penalizes the cases when \(\tilde{\phi}\) is violated by positive data or satisfied by negative data. However, this loss function is not differentiable everywhere with respect to \(w, a, \text{and } c\). An alternative loss function that is differentiable could be

\[
J(\tilde{\phi}) = \sum_{j=1}^{M} \exp(-\zeta l_j r^w(D^j_C, \tilde{\phi})).
\]

\(J(\tilde{\phi})\) is small when \(l_j r^w(D^j_C, \tilde{\phi}) > 0\) and grows exponentially when \(l_j r^w(D^j_C, \tilde{\phi}) < 0\). Notice that the loss is proportional to \(l_j r^w(D^j_C, \tilde{\phi})\) in (8) and inverse proportional to \(l_j r^w(D^j_C, \tilde{\phi})\) in (9), however, the loss in (9) still works because in the binary classification problem, positive and negative data have contradictory effects. We use Pytorch to build the wSTL-NN and perform the back-propagation to learn the formula. The wSTL learning algorithm with wSTL-NN is shown in Algorithm 1.

### Algorithm 1 wSTL Learning Algorithm

**Input:** Time-series data \(D_C\), wSTL formula with specified structure \(\tilde{\phi}\), number of iterations \(K\), number of subformulas \(J\)

**Output:** Learned wSTL formula \(\tilde{\phi}\)

1. Construct a wSTL-NN based on the structure of \(\tilde{\phi}\), and initialize \(\hat{w}, a, c\).
2. for \(k = 1, 2, ..., K\) do
   3. Select a mini-batch data \(D^k_C\) from \(D_C\).
   4. for \(j = 1, 2, ..., J\) do
      5. Perform forward-propagation to compute the quantitative satisfaction of the \(j\)-th subformula using (6).
   6. end for
7. Compute the loss at the current iteration using (9).
8. Perform back-propagation to update the parameters in the wSTL-NN.
9. end for
10. return \(\tilde{\phi}\)

As a result, the memory needed for back-propagation will be intensive. Sparsifying the weights of the wSTL-NN is one way to reduce the memory footprint of the network.

**Example 2.** Consider we want to infer a formula to describe a mobile robot performing a package delivery task. Suppose the formula that we learn is \(\tilde{\phi} = \Box [0,4] 3\), and the normalized weight is \(\tilde{w} = [10^{-4}, 10^{-3}, 5 \times 10^{-4}, 0.5, 0.4984]\) which indicates the data at time point \(k = 3\) and \(k = 4\) have greater influence on deciding whether a package delivery task is performed than the data at time points \(k = 0, 1, 2\). Therefore, for this case, we could constrain the first three weights to 0 and learn the last two weights, which reduces the memory required. Furthermore, if we want to use the inferred formula to monitor whether the robot has delivered a package, we could just check the satisfaction at \(k = 3\) and \(k = 4\). With weight sparsification techniques, we can set many weights that are less important to 0.

### 5.1 Sparsification with Weight Thresholding

During the back-propagation process, we can add additional constraints on the weights to set the less important weights...
to zero. Specifically, there are two perspectives to add constraints. The first one is by adding a threshold on the weights:

$$
\tilde{w}_i = \begin{cases} 
0 & \text{if } \tilde{w}_i \leq \tau, \\
\tilde{w}_i & \text{otherwise},
\end{cases} 
$$

(10)

where $\tau$ is a specified threshold, and $\tilde{w}_i$ is the $i$-th weight after sparsification. The second perspective is by thresholding the number ($\bar{s}$) of weights to keep, which is called top-$\bar{s}$ sparsification, i.e.

$$
\tilde{w}_i = \begin{cases} 
\tilde{w}_i & \text{if } \tilde{w}_i \text{ is one of the top-$\bar{s}$ weights}, \\
0 & \text{otherwise}.
\end{cases} 
$$

(11)

Weight Thresholding Analysis

In this part, we will analyze how much of the weights can be set to 0 such that the classification accuracy is not affected. For simplicity, we analyze the $\Box$ operator, and the results of other operators are also straightforward to obtain. In general, $r^w(s, \Box^w_k, k)$ can be written as

$$
r^w(s, \Box^w_k, k) = \sum_{i=1}^{I_+} \tilde{w}_i^+ z_i^+ + \sum_{i=1}^{I_-} \tilde{w}_i^- z_i^-, 
$$

(12)

where $I_+$ ($I_-$) is the number of $r_i$ that is positive (negative),

$$
z_i^+ = \frac{e^{-r_i} r_i}{\sum_{j=1}^{I_+} e^{-r_j} r_j} (r_i > 0), z_i^- = \frac{e^{-r_i} r_i}{\sum_{j=1}^{I_-} e^{-r_j} r_j} (r_i < 0),
$$

and $\tilde{w}_i^+$ ($\tilde{w}_i^-$) is the weight associated with $z_i^+$ ($z_i^-$). If we want to keep the same classification accuracy, then the sign of the quantitative satisfaction should not change. We analyze how much $\tilde{w}_i$ can change from the following perspectives:

- The quantitative satisfaction $r^w(s, \Box^w_k, k) > 0$, which means

$$
\sum_{i=1}^{I_+} \tilde{w}_i^+ z_i^+ + \sum_{i=1}^{I_-} \tilde{w}_i^- z_i^- > 0.
$$

(13)

If we set $\tilde{w}_i^- = 0$, then the quantitative satisfaction will not change sign. Hence we need to compute how much $\tilde{w}_i^+$ can change such that $r^w(s, \Box^w_k, k)$ will not change sign. Suppose $\min\{z_i^+\} = \delta$, $\max\{z_i^+\} = \gamma$, $\tilde{w}_s^+ = \sum_{i=1}^{I_+} \tilde{w}_i^+$, and the fraction of $\tilde{w}_s^+$ set to 0 is $\bar{f}$, we have

$$
\gamma (1 - \bar{f}) \tilde{w}_s^+ + \delta (1 - \bar{f}) \tilde{w}_s^+ = 0,
\bar{f} = 1 - \frac{\delta (1 - \tilde{w}_s^+)}{\gamma \tilde{w}_s^+}, 
$$

(14)

- The quantitative satisfaction $r^w(s, \Box^w_k, k) < 0$, which means

$$
\sum_{i=1}^{I_+} \tilde{w}_i^+ z_i^+ + \sum_{i=1}^{I_-} \tilde{w}_i^- z_i^- < 0.
$$

(15)

Using the similar approach as above, we have

$$
\delta (1 - \bar{f}) \tilde{w}_s^- + \gamma (1 - \bar{f}) \tilde{w}_s^- = 0,
\bar{f} = 1 - \frac{\gamma (1 - \tilde{w}_s^-)}{\delta \tilde{w}_s^-}, 
$$

(16)

5.2 Sparsification with Gate Variables

Apart from thresholding the weights, weight sparsification can also be achieved by introducing another variable called the gate variable.

Gate Variables

Assume each weight variable $\tilde{w}_i^\phi$ is associated with a sparsification variable $g_i^\phi \in \{0, 1\}$, then the sparsified weight is $\tilde{w}_i^\phi = \tilde{w}_i^\phi g_i^\phi$, where $\tilde{w}_i^\phi$ is the $i$-th weight in $\phi$. This sparsification technique results in many weights being 0 if many $g_i^\phi$ are zero. To learn $g_i^\phi$, we adopt the method in [Srinivas et al., 2017] that considers $g_i^\phi$ as a sample drawn from a Bernoulli random variable $g_i$. Hence we can learn the gate variables $g_i$ instead of $g_i^\phi$. To clarify the notation, we use $g$ to denote the real-valued gate variables, and $g^\phi$ to denote samples drawn from the Bernoulli distribution specified by $g$. The learning process becomes learning the weight variables, variables in predicates, and gate variables simultaneously.

Sparsification Realization

A critical point to obtain sparse weights is that the gate variables $g$ are small, i.e., most $g_i$ is less than 0.5. To facilitate this, we incorporate a so-called bi-modal regularizer that ensures most gate variables are small while only few gate variables are large [Murray and Ng, 2010]. The bi-modal regularizer is expressed as $g_i (1 - g_i)$. Also, we incorporate the $L_1$ regularizer into the loss function, which leads to

$$
J(\phi, g) = \sum_{j=1}^{M} \exp(-\zeta l_j r(D_{\phi_j}^\phi, \omega)) + \lambda_1 \sum_{i=1}^{m} g_i (1 - g_i) + \lambda_2 \sum_{i=1}^{m} g_i,
$$

where $m$ is the number of weights in $\phi$.

6 Case Study

In this section, we use an experiment to evaluate the performance of the wSTL learning algorithm. The dataset we use is the occupancy detection dataset from the UCI Machine Learning Repository [Dua and Graff, 2019; Candanedo and Feldheim, 2016]. The dataset is a time-series dataset that records the information including temperature ($s_1$), relative humidity ($s_2$), light intensity ($s_3$), CO2 concentration ($s_4$), humidity radio ($s_5$), and occupancy of an office room from 2015-02-02 to 2015-02-18. The data is recorded every minute and the target variable is the occupancy (-1 for not occupied and 1 for occupied).

Learning wSTL Formulas without Sparsification

Since the dataset was published in 2016, many ML algorithms have been applied to this dataset to predict the occupancy status. These ML models are less interpretable and readable than the wSTL-NN classifier. The weights in these ML models represent how much each feature in the model contributes to the prediction, however, the model cannot be written in a human-readable form. The wSTL-NN classifier can not only reflect the contribution of features, but also express the model as a wSTL formula that is human-readable. The learned wSTL formula can tell us what kind of temporal property determines the room is occupied or not occupied.
To facilitate the learning of wSTL formulas, we combine \( K_f \) consecutive instances of the original data that have the same label as one instance of data in our experiment. This means each sample in \( D_P \) is a trajectory segment associated to a period of constant occupation for \( K_f \) minutes. Conversely, each sample in \( D_N \) is a trajectory segment associated to a period of constant vacancy for \( K_f \) minutes. In the experiment, we set \( K_f = 16 \), \( K = 10 \), \( \sigma = 1 \), \( \zeta = 1 \), which results in 744 training data and 186 testing data. The formula structure is specified as \( \tilde{\phi} = \square_{[0,15]} \pi \). The classification accuracy of wSTL-NN and other ML models is shown in Table 1. We also evaluate the performance of the wSTL-NN using the evaluation measures of sensitivity, specificity, positive predictive value (PPV), and negative predictive value (NPV) [Fawcett, 2006] as these are standard measures to evaluate a binary classification model. The results of these evaluation measures are shown in Table 2. The wSTL formula we learn is

\[
\tilde{\phi} = \square_{[0,15]} \pi, \tag{17}
\]

where the predicate \( \pi \) is learned as

\[
\pi := (6.4 \times 10^{-6} s_1 + 5.5 \times 10^{-7} s_2 - 0.0075 s_3 - 6.9 \times 10^{-6} s_4 + 0.02 s_5 \leq -0.9854),
\]

and the normalized weight is learned as

\[
\tilde{\omega} = [0.0186, 0.0768, 0.0988, 0.1019, 0.0240, 0.0503, 0.0298, 0.0616, 0.0887, 0.0243, 0.1172, 0.0662, 0.0126, 0.035, 0.1154, 0.0695].
\]

Formula \( \tilde{\phi} \) reads as “\( \pi \) is always satisfied between 0 minute and 15 minute with importance weights \( \tilde{\omega} \) at the corresponding time points”. Compared with the classical ML models that are hyperplanes in an 80 = 16 x 5 dimensional space, the predicate \( \pi \) in the wSTL formula \( \tilde{\phi} \) is only in a 5-dimensional space and \( \phi \) can be written in a natural-language form. This means the wSTL formula is more readable than the other ML models. Apart from comparing with the ML models, we also compare the wSTL formula learned by the wSTL-NN with the traditional STL formula learned by solving a non-convex optimization problem [Yan et al., 2019]. The traditional STL formula that we learn is

\[
\phi = \square_{[1,9]} (0.0032 s_1 + 0.0044 s_2 - 0.0013 s_3 - 0.0003 s_4 + 0.0057 s_5 \leq -0.4096). \tag{19}
\]

Compared with the learning of STL formulas, another advantage of the wSTL learning algorithm is that it accelerates the learning process. The learning of the wSTL formula takes 0.385 s, but the learning of the STL formula takes 269.7 s. The experiments in this paper are implemented on a Windows laptop with a 1.9GHz Intel i7-8665U CPU and an 8 GB RAM.

### Learning wSTL Formulas with Sparsification

In this subsection, we perform the learning of wSTL formulas with the sparsification techniques discussed in Section 5. To save space, we only show the results for the techniques of top-\( s \) sparsification and sparsification with gate variables. The performance of these two techniques is shown in Table 3. From Table 3 we could see for the same sparsification level \( s = 4 \), the accuracy for the top-\( s \) sparsification is lower than the sparsification with gate variables. In reality, if the sparsification level obtained from the sparsification with gate variables is not satisfied, we could use top-\( s \) sparsification to further prune the weights, which may lower the accuracy.

#### Table 1: Classification accuracy on the occupancy dataset.

| Method                                | Accuracy (%) |
|---------------------------------------|--------------|
| Linear Discriminate Analysis (LDA)    | 100          |
| K-Nearest Neighbors (KNN)             | 100          |
| Logistic Regression                   | 100          |
| Supported Vector Machine (SVM)        | 100          |
| Naive Bayes                           | 99.46        |
| Random Forest                         | 99.46        |
| Decision Tree                         | 98.38        |
| wSTL-NN (this paper)                  | 99.46        |

#### Table 2: Performance evaluation measures for wSTL-NN.

| Evaluation measure | wSTL-NN | Decision Tree |
|--------------------|---------|---------------|
| Sensitivity        | 1       | 0.959         |
| Specificity        | 0.991   | 1             |
| PPV                | 0.986   | 1             |
| NPV                | 1       | 0.974         |

#### Table 3: Results of wSTL-NN with sparsification.

| Sparsification technique | \( s \) | Accuracy (%) |
|--------------------------|--------|--------------|
| Top-\( s \) sparsification| 8      | 99.46        |
|                          | 10     | 99.46        |
|                          | 12     | 99.46        |
|                          | 14     | 99.46        |
| Sparsification with gate variables | 4 | 98.92        |

#### 7 Conclusion and Future Work

We propose an extension of STL that is called wSTL by incorporating weights into the specifications, where the proposed quantitative satisfaction has three properties: non-influence of zero weights, ordering of influence, and monotonicity. A novel framework combining the characteristics of neural networks and wSTL is presented, which is called wSTL-NN. Due to the differentiable property of the quantitative satisfaction of wSTL, the parameters of wSTL can be learned from the wSTL-NN and the task of learning wSTL formulas can be accomplished. Two sparsification techniques of wSTL-NN are given such that wSTL-NN will be sparse even if the wSTL formula has a complex structure. Future work will explore extending wSTL-NN to learn wSTL formulas without specified time intervals and more computationally efficient quantitative satisfaction functions.
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