Investigating full-heavy tetraquarks composed of $cc\bar{c}\bar{b}$ and $bb\bar{c}\bar{c}$

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Abstract The full-heavy tetraquarks $cc\bar{c}\bar{b}$ and $bb\bar{c}\bar{c}$ are systematically investigated within a quark model. The meson-meson structure, diquark-antidiquark structure and K-structure are considered in this work. There is no bound state for $cc\bar{c}\bar{b}$ and $bb\bar{c}\bar{c}$ systems in $IJ^{P} = 0^{+}, 0^{+}$ and $0^{+}$ channels. However, for $cc\bar{c}\bar{b}$ system, three possible resonance states with energy of 10,079 MeV, 10,081 MeV and 10,177 MeV are found in $IJ^{P} = 00^{+}, 01^{+}$ and $02^{+}$, respectively, and their decay widths $\Gamma$ are 6.7–8.4 MeV, 1.4–7.2 MeV and 9.1–11.1 MeV. For $bb\bar{c}\bar{c}$ system, there also exist three possible resonance states with energy of 16,474 MeV, 16,474 MeV and 16,541 MeV in $IJ^{P} = 00^{+}, 01^{+}$ and $02^{+}$, respectively, and the decay widths $\Gamma$ of them are 2.2–6.1 MeV, 2.2–6.9 MeV and 5.3–8.5 MeV. $bcc\bar{c}$ and $cbb\bar{c}$ systems will have the same results as $cc\bar{c}\bar{b}$ and $bb\bar{c}\bar{c}$, respectively. These full-heavy resonance states are worthy to be searched in the future experiments.

1 Introduction

In the past few decades, many discoveries about exotic states inspired extensive interest in probing the structures of the multiquark hadrons. Gell-Man and Zweig have been proposed and invented multiquark states in the early quark models [1,2]. In 2003, Belle collaboration announced the observation of exotic state $X(3872)$ [3], and other experimental groups also found this exotic state [4–6], which make the tetraquark state become a heated research area. After that, plenty of exotic states were observed and investigated, for review articles, please see [7–17]. In 2017, the CMS collaboration implemented a benchmark measurement of the $\Upsilon(1S)$ pair production at $\sqrt{s} = 8$ TeV in $pp$ collision [18]. However, no evidence has been found in the $\Upsilon(1S)\mu^{+}\mu^{-}$ invariant mass spectrum by the LHCb collaboration [19]. Therefore, the fully-bottom tetraquark needs more experiments to confirm its signal. For fully-charm tetraquark systems, $J/\Psi$ pair production and double $c\bar{c}$ production have also been observed in experiments [20,21]. The $J/\Psi J/\Psi$ and $\eta_{c}(1S)\eta_{c}(1S)$ channels are suggested to search for the doubly hidden-charm. In 2020, the LHCb collaboration reported their elementary conclusion on the observations of full-charm states, a narrower structure at 6.9 GeV with significance about $5\sigma$, a broad structure in the range 6.2–6.8 GeV, and there is also a hint for a structure around 7.2 GeV [22]. These experimental results greatly stimulated much interest in full-heavy tetraquark states. In 2022, based on all the proton-proton collisions collected from 2016 to 2018, the CMS collaboration has observed three structures $X(6600)$, $X(6900)$, and $X(7300)$ in the invariant mass spectra of the charm quarks [23].

In 1975, Iwasaki proposed that full-charm tetraquark with the mass of about 6.0 GeV or 6.2 GeV is a sharp resonance state [24]. Lloyd et al. investigated $cc\bar{c}\bar{b}$ states and find several close-lying bound states [25]. Berezhnoy et al. obtained the mass $M_{0}(cc\bar{c}\bar{b}) = 6124$ MeV and $M_{0}(bb\bar{c}\bar{c}) = 18,857$ MeV without hyperfine splitting, which involving charmed and bottom tetraquarks, respectively [26]. Karliner et al. discovered $M(cc\bar{c}\bar{b}) = 6192 \pm 25$ MeV and $M(bb\bar{c}\bar{c}) = 18,826 \pm 25$ MeV for $J^{PC} = 0^{++}$ involving charmed and bottom tetraquarks, respectively [27]. Chen et al. investigated $cc\bar{c}\bar{b}$ and $bb\bar{c}\bar{c}$ states by a moment QCD sum rule method, and concluded that the mass of $\Upsilon \Upsilon$ and $\eta_{b}\eta_{b}$ are below and close to the corresponding thresholds except one current of $J^{PC} = 0^{++}$, while the mass of $cc\bar{c}\bar{b}$ all above the thresholds [28]. Wu et al. discussed the $QQ\bar{Q}\bar{Q}$ configuration, and the result shows that the lowest $J^{P} = 1^{+}$ state of $cc\bar{c}\bar{b}$ system should be less stable than that of $bb\bar{c}\bar{c}$ as the $cc$ interaction is stronger than the $bb$ interaction [29]. Anwar et al. found that the ground state $bb\bar{b}\bar{b}$ tetraquark mass is predicted to be $M(bb\bar{b}\bar{b}) = 18.72 \pm 0.02$ GeV [30]. A. Esposito et al. proposed a model based on the conjecture of...
a short range diquark repulsion in a compact tetraquark to
estimate masses and widths of tetraquarks, and found that
the $bb\bar{b}\bar{b}$ system with the mass of 18.8 GeV [31]. Wang et al.
 systematically calculate the mass spectra of the S-wave fully-
heavy tetraquark states $cc\bar{c}\bar{c}$, $bb\bar{b}\bar{b}$ and $bb\bar{c}\bar{c}$ in two nonrel-
ativistic quark models, and the numerical results shows that
the ground $QQ\bar{Q}\bar{Q}$ tetraquark states are located above the
corresponding scattering states [32]. Liu et al. studied the
all-heavy tetraquark systems, i.e., $cc\bar{c}\bar{c}$, $bb\bar{b}\bar{b}$, $bb\bar{c}\bar{c}\bar{c}$,
$bc\bar{c}\bar{c}\bar{c}$, $bc\bar{b}\bar{b}\bar{b}$, and $bc\bar{c}\bar{b}\bar{b}$ within a potential model by
including the linear confining potential, Coulomb potential,
and spin-spin interactions, and the results showed that all
diquark–antidiquark states are found to have masses above
the corresponding the thresholds of $(Q\bar{Q})-(Q\bar{Q})$ structure
[33]. Moreover, there are also many researches about the nar-
row structure $X(6900)$ reported by LHCb in 2020 [34–44],
and more papers can be found in Ref. [17] and reference there in.
Some of these researches interprets $X(6900)$ as excited state
of diquark–antidiquark structure in the $cc\bar{c}\bar{c}$ system by
various quark model [34,37,39,41–44] and by QCD sum
rules method [35,38,40], while other study suggest that the
component of it contains the excited state of meson–meson
structure mixing with diquark–antiquark structure [36].

In this paper, we investigate the possible resonance states
of $cc\bar{b}\bar{b}$ and $bb\bar{b}\bar{c}$ systems in $I\bar{J}^P = 0^+, 0^+, 1^+$, and $0^+$
channels by using a quark model. The four-body configurations,
which refers to meson–meson structure, diquark–antidiquark
structure, and K-structure, as well as their couplings, are con-
sidered in the calculation. In Sect. 2, we give the introduction
of the construction of wave functions. The numerical results
and discussions are shown in Sect. 3. The summary is given
in Sect. 4.

## 2 The quark model and wave functions of $cc\bar{b}\bar{b}$ and $bb\bar{b}\bar{c}$ systems

Quantum chromodynamics(QCD) is recognized as a funda-
mental theory of strong interaction and the basic theory of
multiquark states. However, with non-perturbative properties
of QCD in the low energy region, it is inappropriate to
directly use QCD theory to solve specific problems, such as
hadron-hadron interactions and multiquark states. Therefore,
some researchers have proposed and developed some quark
models to solve these problems in low-energy regions.

For full-heavy tetraquark systems, the potential is com-
posed of the confinement and the one-gluon-exchange. The
gluonic potential have various forms, such as linear confine-
ment [36,39,41–45], square confinement [34,46], and expon-
ential confinement [36,37]. The mass spectrum of these
models is all in consistent with experimental results, and they
all explained $X(6900)$ [34,36,37,39,41–44]. In other words,
the forms of the confinement and the one-gluon-exchange in
full-heavy tetraquark systems need more experimental data
to be determined. Here we use one of these forms [36,47] to
study possible resonance state in $cc\bar{b}\bar{b}$ and $bb\bar{b}\bar{c}$ systems.

The Hamiltonian in this model for the present study is
written as:

$$ H = \sum_{i=1}^{4} \left( m_i + \frac{p_i^2}{2m_i} \right) - T_{CM} + \sum_{j>i=1}^{4} \left( V_{ij}^{CON} + V_{ij}^{OGE} \right), $$

(1)

where $m_i$, $p_i$ refer to the mass and the kinetic of $i$-th
quark (antiquark), $T_{CM}$ is the kinetic energy of the center
of mass in tetraquark system; $V_{ij}^{CON}$ and $V_{ij}^{OGE}$ are the
interactions of the confinement and the one-gluon-exchange
between the $i$-th and $j$-th quark, respectively. The $V_{ij}^{CON}$ is
written as

$$ V_{ij}^{CON} = [-\alpha_c \left( 1 - e^{-\nu_{ij}} \right) + \Delta] \lambda^c_i \cdot \lambda^c_j, $$

(2)

and the $V_{ij}^{OGE}$ express as

$$ V_{ij}^{OGE} = \frac{\alpha_s}{4} \lambda^c_i \cdot \lambda^c_j \left[ \frac{1}{r_{ij}} - \frac{1}{6m_im_j} \left( \frac{e^{-r_{ij}/r_\sigma}}{r_{ij}^2 r_\sigma^2} \right) \right], $$

(3)

$$ \alpha_s = \frac{\alpha_0}{\ln(\mu^2 + \mu_c^2)}, \quad r_\sigma = \frac{\tilde{r}_\sigma}{\mu}. $$

(4)

where $r_{ij}$ stands for the distance between the two quarks/
antiquarks, and $\sigma$ indicates the SU(2) Pauli matrices, while
$\lambda^c$ is SU(3) color Gell-Mann matrices, respectively. $\alpha_s$ is an
effective scale-dependent running coupling. $\mu$ is the reduced
mass of two quarks, and it written as $\mu_{ij} = \frac{m_i m_j}{m_i + m_j}$. The
$\alpha_c$, $\Delta$, $\mu_0$, $\Lambda_0$, $\mu_c$, and $\tilde{r}_\sigma$ are parameters listed in Table 1,
and these parameters are taken from Ref. [47]. The mass of
mesons that related to the present work are shown in Table 2.

According to different coupling methods, $cc\bar{b}\bar{b}$ and $bb\bar{b}\bar{c}$
systems have three structures are considered, i.e., meson–
meson structure, diquark–antidiquark structure and
K-structure, and are shown in Figs. 1, 2 and 3. The hollow
circles and black discs refer to quark and antiquark, respectively.
The total wave functions of each structure are constructed by four parts: orbit, spin, flavor and color wave functions. The meson–meson structure and diquark–antidiquark structure are constructed by coupling two sub-clusters wave functions, while the K-structure is coupling a quark on the basis of quark–antiquark system and then coupling another antiquarks. In the following discussions, the spin, flavor and color wave functions of meson–meson structure and K-structure are written in order of $N_1N_2N_3N_4$, while that of diquark–antidiquark structure is written in order of $N_1N_3N_2N_4$.

### 2.1 Gaussian expansion method (GEM)

This work focuses on radial excitation state of these tetraquark systems. The GEM has been successfully used in the calculation of few-body systems due to it just demands a small number of Gaussians to stabilize the system [49]. Therefore, this work calculates the orbit wave function by GEM. In GEM, the radial part of orbit wave function of meson–meson structure, diquark–antidiquark structure and K-structure are all expanded by Gaussians.
\[ \Phi_n = \sum_{n=1}^{n_{\max}} c_n \psi_n^G (r), \]  
(9)

\[ \psi_n^G (r) = \frac{1}{\sqrt{4 \pi}} N_n e^{-v_n r^2}, \]  
(10)

\[ N_n = \left[ \frac{4(2v_n)^{1/2}}{\sqrt{\pi}} \right]^{1/2}. \]  
(11)

\( N_n \) and \( c_n \) means normalization constants and the variational parameters, respectively. \( n_{\max} \) is the Gaussian number, while \( r_{\max} \) and \( r_{\min} \) are parameters of the Gaussian size \( v_n \), and the formulas of \( v_n \) are chosen according to the following geometric progression:

\[ a = \left( \frac{r_{\max}}{r_{\min}} \right) \frac{1}{n_{\max} - 1}, \quad r_n = r_{\min} a^{n-1}, \quad v_n = \frac{1}{r_n^2}. \]  
(12)

For \( c\bar{c}\bar{b} \) systems, the internal parameters clusters are chosen as \( r_{\min} = 0.01 \) (fm), \( r_{\max} = 2 \) (fm), \( n_{\max} = 12 \), and the parameters between the clusters are chosen as \( r_{\min} = 0.01 \) (fm), \( r_{\max} = 6 \) (fm), \( n_{\max} = 16 \). For \( b\bar{b}\bar{c}\bar{c} \) systems, the internal parameters clusters are chosen as \( r_{\min} = 0.008 \) (fm), \( r_{\max} = 2 \) (fm), \( n_{\max} = 15 \), and the parameters between the clusters are chosen as \( r_{\min} = 0.008 \) (fm), \( r_{\max} = 6 \) (fm), \( n_{\max} = 16 \). To test the stability, a second calculation was performed, which takes \( r_{\min} = 0.01 \) (fm), \( r_{\max} = 2 \) (fm), \( n_{\max} = 15 \) for internal clusters and \( r_{\min} = 0.01 \) (fm), \( r_{\max} = 6 \) (fm), \( n_{\max} = 20 \) between the clusters. The result of the energy for ground states and excited states are the same as the first calculation. Hence, the energy calculated by the Gauss number used for the first time is enough to stabilize the present study.

### 2.2 Spin wave function

The spin wave functions of meson–meson structure, diquark–antidiquark structure and K-structure are denoted by the subscript “\( \chi_{\Delta S} \)”. The \( S \) and \( \Delta \) refers to spin angular momentum and its third component, and \( i \) means the number of spin wave functions. \( \alpha \) and \( \beta \) represent spin wave functions of spin-up (1 0) and spin-down (0 1) wave functions. Tables 3 and 4 lists the spin wave function of quark, meson and baryon, and Tables 5 and 6 lists all spin wave function of meson–meson structure, diquark–antidiquark structure and K-structure.

### 2.3 Flavor wave function

The total flavor wave functions are

\[ \chi_m^{f_1} = Q \bar{Q} \bar{O} \bar{O}, \]  
(13)

\[ \chi_m^{f_2} = Q \bar{O} \bar{O} \bar{Q}. \]  
(14)

\[ \chi_{K}^{f_5} \] is same as \( \chi_m^{f_4} \) and represents flavor of K-structure.
Table 6 The spin wave function of K-structure

| Number   | Spin wave function |
|----------|--------------------|
| $\chi_{00}^{(4)}$ | $\frac{1}{\sqrt{2}}(\chi_{1/2}^{(3)}(3)\chi_{-1/2}^{(3)} - \chi_{-1/2}^{(1)}(3)\chi_{1/2}^{(1)})$ |
| $\chi_{01}^{(4)}$ | $\frac{1}{\sqrt{2}}(\chi_{1/2}^{(3)}(3)\chi_{-1/2}^{(3)} - \chi_{-1/2}^{(1)}(3)\chi_{1/2}^{(1)})$ |
| $\chi_{10}^{(4)}$ | $\chi_{1/2}^{(3)}(3)\chi_{-1/2}^{(3)}(1)$ |
| $\chi_{11}^{(4)}$ | $\chi_{1/2}^{(3)}(3)\chi_{-1/2}^{(3)}(1)$ |
| $\chi_{20}^{(4)}$ | $\chi_{1/2}^{(3)}(3)\chi_{-1/2}^{(3)}(1)$ |

Therefore, the $3 \otimes 3$ has 1/3 of $1 \otimes 1$ and 2/3 of $8 \otimes 8$, while $6 \otimes 6$ has 2/3 of $1 \otimes 1$ and 1/3 of $8 \otimes 8$.

Classify all color wave functions to $1 \otimes 1$ and $8 \otimes 8$ have physical sense. We can separate color configurations $1 \otimes 1$ to infinitely far, because the expectation value of the operator $\lambda_i^c \cdot \lambda_j^c$ is zero, when $i$ and $j$ belongs to different cluster. However, if we try to separate color octet-octet $8 \otimes 8$ structure, the operator $\lambda_i^c \cdot \lambda_j^c$ provide attractive force, when $i$ and $j$ belongs to different cluster. As confinement potential exist in infinite far, the energy of $8 \otimes 8$ goes to an infinite value due to the color confinement. In this sense, the physical channel with the color configurations $3 \otimes 3$, $6 \otimes 6$ and $8 \otimes 8$ are “compact state”.

Because $|\psi_1^{(1)}\rangle (\psi_1^{(1)} | + |\psi_8^{(8)}\rangle |\psi_8^{(8)}\rangle = 1$, here $\psi_1^{(1)}$ and $\psi_8^{(8)}$ corresponds to color $1 \otimes 1$ and $8 \otimes 8$ configurations, respectively. We insert it to each physical channel as:

$$\langle \psi_i | \langle \psi_i | \psi_i^{(1)} \rangle | \psi_i^{(1)} \rangle | \psi_i \rangle + \langle \psi_i | \psi_8^{(8)} \rangle | \psi_8^{(8)} \rangle | \psi_i \rangle,$$

where $\psi_i$ is a total wave function of a physical channel expressed in Eqs. (5)–(7). So, the component of physical channel with $1 \otimes 1$ is calculated by:

$$P_{1 \otimes 1} = \frac{\sum_i |\psi_i | \psi_i^{(1)} \rangle | \psi_i^{(1)} \rangle | \psi_i \rangle}{\sum_i |\psi_i | \psi_i \rangle},$$

and the component of physical channel with $8 \otimes 8$ is:

$$P_{8 \otimes 8} = \frac{\sum_i |\psi_i | \psi_8^{(8)} \rangle | \psi_8^{(8)} \rangle | \psi_i \rangle}{\sum_i |\psi_i | \psi_i \rangle},$$

One can easily show that $P_{1 \otimes 1} + P_{8 \otimes 8} = 1$.

3 Results and discussions

We investigate full-heavy tetraquarks $cc\bar{c}\bar{b}$ and $bb\bar{b}\bar{c}$ systems in three kind of quark structures, i.e., meson–meson structure, diquark–antidiquark structure and K-structure, and construct all possible physical channels according to the quantum numbers $IJ^P = 00^+, 01^+$, and $02^+$. The physical channel and the mass are shown in Tables 8 and 9. The first column of these tables shows the wave function of orbit, spin, flavor and color for each channel. We denote the total function by the subscript $[i, j, k]$ type, “$i, j, k$” indicates spin, flavor and color, and “type” denotes meson–meson structure, diquark–antidiquark structure and K-structure, i.e., “m”, “di” and “K”, respectively. The second columns “channels” enumerates the subscript $[\chi_m^{(1)} S(N_1, N_2) \otimes S(N_3, N_4)]$ in meson–meson structure, the subscript $[\chi_{di}^{(1)} S(N_1, N_2) \otimes S(N_3, N_4)]$ in diquark–antidiquark structure, and the subscript $[\chi_K^{(1)} S(N_1, N_2) \otimes S(N_3, N_4)]$ in K-structure.
structure, respectively. The columns headed with $E_{ih}$ means the theoretical thresholds for two meson systems, which is the energy when separate two color-singlet mesons infinitely far from each other.

For a given $IJF^p$, the $E_{sc}$ refers to the energy of each single-channel, and the $E_{cc}$ denotes the lowest energies of the coupling of all channels, respectively [39]. Which are written as:

\[
\langle \Psi_i | H | \Psi_i \rangle (c_i) = E_{sc} \langle \Psi_i | \Psi_i \rangle (c_i) ,
\]

and,

\[
\left( \langle \Psi_i | H | \Psi_i \rangle \ldots \langle \Psi_i | H | \Psi_n \rangle \right) (c_1) \\
\ldots \\
\left( \langle \Psi_n | H | \Psi_i \rangle \ldots \langle \Psi_n | H | \Psi_n \rangle \right) (c_n)
\]

\[
= E_{cc} \left( \langle \Psi_i | \Psi_i \rangle \ldots \langle \Psi_i | \Psi_n \rangle \right) (c_1) \\
\ldots \\
\left( \langle \Psi_n | \Psi_i \rangle \ldots \langle \Psi_n | \Psi_n \rangle \right) (c_n)
\]

where $c_i$ is the eigenvector of corresponding energy.

There are no bound states in Tables 8 and 9. However, some resonance state could exist in the higher energy region. The ground and excited states of “compact state”, i.e., color configurations $3 \otimes 3, 6 \otimes 6$ and $8 \otimes 8$, could be the candidate for resonance state. To assess the stability of these resonance states, we use the real-scaling method (RSM) [50].

The RSM is a way to find the possible resonance state. In this method, the Gaussian size parameters $r_n$ between two color-singlet sub-clusters are scaled by multiplying a factor $\alpha$, i.e., $r \to \alpha r$. The resonance state might exist if the avoid-crossing structure appears repeatedly with the increasing $\alpha$. The repeated avoid-crossing structures are caused by the interaction between the interaction of scattering state with higher energy and resonance state at larger distances. Then, the decay widths could be calculated by the following formula [50]:

\[
\Gamma = 4|V(\alpha_r)|\sqrt{|S_r||S_c|} / |S_c - S_r|.
\]

Here $V(\alpha_r)$ stands for half of the minimal energy difference between resonance state and scattering state, while $S_r$ and $S_c$ represent the slope of the resonance state and the scattering state, respectively.

The avoid-crossing structures could be caused by two ways: (i) the interaction between the scattering and resonance states with the increasing $\alpha$, and these avoid-crossing structures would be possible resonance states, (ii) the interaction between two scattering states with different decay rates, which means its dominated component of the structure are scattering states, so these avoid-crossing structures couldn’t regard as resonance states.

To determine a state is a true resonance or not is whether it have resonance mechanism. For present study, because there is no bound state of meson–meson structure, the resonance

| $IJF^p$ | $|i, j, k\rangle_{type}$ | Channel | $E_{ih}$ | $E_{sc}$ | $E_{cc}$ |
|---------|---------------------|---------|---------|---------|---------|
| $01^+$  | $[1, 1, 1]\_{lm}$   | $|\eta, B^*_c\rangle_{\mathcal{Q}^0}$ | 1930    | 1930    | 1930    |
|         | $[2, 1, 1]\_{lm}$   | $|\psi B_c\rangle_{\mathcal{Q}^0}$ | 1942    | 1942    | 1942    |
| $02^+$  | $[1, 1, 2]\_{lm}$   | $|\eta, B^*_c\rangle_{\mathcal{Q}^0}$ | 9673    | 9673    | 9673    |
|         | $[2, 1, 2]\_{lm}$   | $|\psi B_c\rangle_{\mathcal{Q}^0}$ | 9663    | 9663    | 9663    |
|         | $[1, 2, 3]\_{di}$   | $|\bar{cc}\bar{c}\bar{b}\rangle_{\mathcal{Q}^0}$ | 9660    | 9660    | 9660    |
|         | $[2, 2, 4]\_{di}$   | $|\bar{cc}\bar{c}\bar{b}\rangle_{\mathcal{Q}^0}$ | 9685    | 9685    | 9685    |
|         | $[7, 3, 5]\_{K}$    | $|\bar{c}\bar{c}\bar{c}\bar{c}\rangle_{\mathcal{Q}^0}$ | 9572    | 9572    | 9572    |
|         | $[8, 3, 5]\_{K}$    | $|\bar{c}\bar{c}\bar{c}\bar{c}\rangle_{\mathcal{Q}^0}$ | 9662    | 9662    | 9662    |
|         | $[7, 3, 6]\_{K}$    | $|\bar{c}\bar{c}\bar{c}\bar{c}\rangle_{\mathcal{Q}^0}$ | 9689    | 9689    | 9689    |
|         | $[8, 3, 6]\_{K}$    | $|\bar{c}\bar{c}\bar{c}\bar{c}\rangle_{\mathcal{Q}^0}$ | 9665    | 9665    | 9665    |
| $00^+$  | $[6, 1, 1]\_{lm}$   | $|\psi B^*_c\rangle_{\mathcal{Q}^0}$ | 9427    | 9427    | 9427    |
|         | $[6, 1, 2]\_{lm}$   | $|\psi B_c\rangle_{\mathcal{Q}^0}$ | 9642    | 9642    | 9642    |
|         | $[6, 2, 4]\_{di}$   | $|\bar{c}\bar{c}\bar{c}\bar{c}\rangle_{\mathcal{Q}^0}$ | 9701    | 9701    | 9701    |
|         | $[12, 3, 5]\_{K}$   | $|\bar{c}\bar{c}\bar{c}\bar{c}\rangle_{\mathcal{Q}^0}$ | 9679    | 9679    | 9679    |
|         | $[12, 3, 6]\_{K}$   | $|\bar{c}\bar{c}\bar{c}\bar{c}\rangle_{\mathcal{Q}^0}$ | 9687    | 9687    | 9687    |
Table 9 The energies (in MeV) of the meson–meson structure for tetraquarks $bbbc$

| $i, j, k$ type | Channel | $E_{th}$ | $E_{sc}$ | $E_{cc}$ |
|----------------|---------|---------|---------|---------|
| $IJ^P = 00^+$  |         |         |         |         |
| $[1, 1, 1]$lm  | $[\eta B]_I^{000}$ | 15,729 | 15,730 | 15,730 |
| $[2, 1, 1]$lm  | $[Y B]_I^{100}$ | 15,835 | 15,835 | 15,835 |
| $[1, 1, 2]$lm  | $[\eta B]_I^{000}$ | 16,070 |         |         |
| $[2, 1, 2]$lm  | $[Y B]_I^{100}$ | 16,071 |         |         |
| $[1, 2, 3]$ld  | $[bbbc]_I^{000}$ | 16,059 |         |         |
| $[2, 2, 4]$ld  | $[bbbc]_I^{000}$ | 16,089 |         |         |
| $[7, 3, 5]$K   | $[bbbc]_I^{110}$ | 15,982 |         |         |
| $[8, 3, 5]$K   | $[bbbc]_I^{110}$ | 16,019 |         |         |
| $[7, 3, 6]$K   | $[bbbc]_I^{110}$ | 16,077 |         |         |
| $[8, 3, 6]$K   | $[bbbc]_I^{110}$ | 16,069 |         |         |
| $IJ^P = 01^+$  |         |         |         |         |
| $[4, 1, 1]$lm  | $[T B]_I^{100}$ | 15,781 | 15,781 | 15,781 |
| $[3, 1, 1]$lm  | $[\eta B]_I^{100}$ | 15,784 | 15,785 | 15,785 |
| $[5, 1, 1]$lm  | $[T B]_I^{100}$ | 15,835 | 15,835 | 15,835 |
| $[3, 1, 2]$lm  | $[\eta B]_I^{100}$ | 16,065 |         |         |
| $[4, 1, 2]$lm  | $[T B]_I^{100}$ | 16,066 |         |         |
| $[5, 1, 2]$lm  | $[T B]_I^{100}$ | 16,065 |         |         |
| $[3, 2, 3]$ld  | $[bbbc]_I^{000}$ | 16,056 |         |         |
| $[4, 2, 4]$ld  | $[bbbc]_I^{000}$ | 16,081 |         |         |
| $[5, 2, 4]$ld  | $[bbbc]_I^{000}$ | 16,092 |         |         |
| $[9, 3, 5]$K   | $[bbbc]_I^{110}$ | 15,991 |         |         |
| $[10, 3, 5]$K  | $[bbbc]_I^{110}$ | 16,011 |         |         |
| $[11, 3, 5]$K  | $[bbbc]_I^{110}$ | 16,040 |         |         |
| $[9, 3, 6]$K   | $[bbbc]_I^{110}$ | 16,071 |         |         |
| $[10, 3, 6]$K  | $[bbbc]_I^{110}$ | 16,078 |         |         |
| $[11, 3, 6]$K  | $[bbbc]_I^{110}$ | 16,060 |         |         |
| $IJ^P = 02^+$  |         |         |         |         |
| $[6, 1, 1]$lm  | $[T B]_I^{100}$ | 15,835 | 15,835 | 15,835 |
| $[6, 1, 2]$lm  | $[T B]_I^{100}$ | 16,059 |         |         |
| $[6, 2, 4]$ld  | $[bbbc]_I^{000}$ | 16,098 |         |         |
| $[12, 3, 5]$K  | $[bbbc]_I^{110}$ | 16,050 |         |         |
| $[12, 3, 6]$K  | $[bbbc]_I^{110}$ | 16,071 |         |         |

Fig. 4 Energy spectrum of $IJ^P = 00^+$ in $cc\bar{c}b$

Fig. 5 Energy spectrum of $IJ^P = 01^+$ in $cc\bar{c}b$

Fig. 6 Energy spectrum of $IJ^P = 02^+$ in $cc\bar{c}b$

mechanism is whether the avoid-crossing structures correspond to an excited state of “compact state”.

Figures 4, 5 and 6 are the energy spectrum of $cc\bar{c}b$ system, while Figs. 7, 8 and 9 are the energy spectrum of $bb\bar{b}c$ system. Every point shown in figures represent the energy calculated from channels coupling. The red dash-dot line is the threshold, and the corresponding physical channel is marked on the right of figure. The green dashed line refers to the mass of possible resonance state. The color of each point shows the value of $P_1\otimes 1$. The legend on the right represents the value of $P_1\otimes 1$, i.e., yellow and dark representing high
and low percentage of \( P_1 \otimes 1 \). The percentage of \( P_8 \otimes 8 \) is obtained by \( 1 - P_1 \otimes 1 \).

From Figs. 4, 5, 6, 7, 8, and 9 we found that, lots of avoid-crossing structures appears in \( IJ^P = 00^+ \), \( 01^+ \) and \( 02^+ \) channels. However, only six states have resonance mechanism, i.e., they correspond to the excited states of their “compact state”. We found 3 resonant states in \( cc\bar{c}b \) channel, i.e., \( R(10,079) \) in \( IJ^P = 00^+ \), \( R(10,081) \) in \( IJ^P = 01^+ \) and \( R(10,177) \) in \( IJ^P = 02^+ \), and 3 resonant states in \( bb\bar{c}c \) channel, i.e., \( R(16,474) \) in \( IJ^P = 00^+ \), \( R(16,474) \) in \( IJ^P = 01^+ \) and \( R(16,541) \) in \( IJ^P = 02^+ \). Each of these resonance states has more than 30% of compact states. Table 10 lists the decay width of the resonances states of \( cc\bar{c}b \) and \( bb\bar{c}c \) systems. There are many avoid-crossing structures on the resonant line, so we calculate the maximum decay width from a single avoid-crossing structure as the minimum total decay width of the system, and sum over all decay widths as a maximal total decay width.

The 1S of “compact states” in \( cc\bar{c}b \) and \( bb\bar{c}c \) could not form a resonance through RSM. The reason would be that these 1S states are strongly coupled with scattering state and decay to threshold quickly. The 2S of “compact states” of \( IJ^P = 02^+ \) in \( cc\bar{c}b \) and \( bb\bar{c}c \) systems are very close to the threshold of \( J/\Psi(2S)B_1^+(1S) \) and \( \Upsilon(1S)B_1^+(2S) \), so they couple to threshold very strong, and hard to form a resonance line.

### 4 Summary

We investigate the full-heavy tetraquarks \( cc\bar{c}b \) and \( bb\bar{c}c \) in meson–meson structure, diquark–antidiquark structure and K-structure within the framework of the quark model, and consider the combination of all possible color, flavor, spin configurations. In both of \( cc\bar{c}b \) and \( bb\bar{c}c \) systems, we couple all channels in quantum numbers of \( IJ^P = 00^+, 01^+ \) and \( 02^+ \), and found that there is no bound state exists. However, through RSM, we found three possible resonance states \( R(10,079) \), \( R(10,081) \) and \( R(10,177) \) in \( cc\bar{c}b \) system, and three possible resonance states \( R(16,474) \), \( R(16,474) \) and \( R(16,541) \) in \( bb\bar{c}c \) system. The decay width of \( R(10,079) \), \( R(10,081) \) and \( R(10,177) \) are 6.7–8.4 MeV, 1.4–7.2 MeV and 9.1–11.1 MeV, while the decay width of \( R(16,474) \), \( R(16,474) \) and \( R(16,541) \) are 2.2–6.1 MeV, 2.2–6.9 MeV and 5.3–8.5 MeV, \( bb\bar{c}c \) and \( cc\bar{c}b \) systems will have the same results as \( cc\bar{c}b \) and \( bb\bar{c}c \), respectively.

For the present study, the annihilation interaction is inversely proportional to the masses of the interacting quarks, and the one-gluon-annihilation process must be
very weak for heavy quarks [51]. Therefore, the rearrangement decay is the mainly decay behaviors if the energy below its reference thresholds of $D^+ D^- B_c(10,014\text{MeV})$, $D^+ B^+ \eta_c(10,131\text{MeV})$ for $cc\bar{c}\bar{b}$ system and $B^+ B^- B_c(16,834\text{MeV})$, $D^- B^- \eta_b(16,548\text{MeV})$ for $bb\bar{b}\bar{c}$ system. In this means, $R(10,079)$, $R(10,081)$ and $R(10,177)$ could be found in $D^+ D^- B_c(10,014\text{MeV})$ channel. The full consideration of 4–6 mixing could be calculated in the future works.

Liu et al. [33] and Yang et al. [39] also studied similar system. To evaluate the parameter dependence of each model, we calculate the error $m_{err}$ by:

$$m_{err} = \sqrt{\frac{\sum_i (\bar{m}_i - m_i)^2}{(N - 1)}}. \quad (22)$$

Here $m_i$ refers to the energy of meson with each parameter has changed $\pm 1\%$, while $\bar{m}$ represents the average energy value of each parameter (except $m_c$ and $m_b$) changed by $1\%$, and “N” means the number of changed parameters from three models. The value of “N” in present study, Ref. [33] and Ref. [39] is 14, 14 and 12, respectively.

Table 11 lists theoretical and experimental mass of twelve mesons of present work and Refs. [33,39], we found that three model can explain experiment with a parameter error about 3–5 MeV.

The main difference of present work and Refs. [33,39] are:

1. Liu et al. [33] have studied the meson–meson structure and diquark–antidiquark structure in $cc\bar{c}\bar{b}$ and $bb\bar{b}\bar{c}$ systems by a quark model. However, there are two main differences between our study and theirs: (1) they use different potential of $V_{ij}^{CON}$ and $V_{ij}^{OGE}$, where $V_{ij}^{CON}(r_{ij}) = -\frac{3}{16} \lambda_i^c \cdot \lambda_j^c \cdot b r_{ij}$, $V_{ij}^{OGE}(r_{ij}) = \frac{\alpha_{ij}}{4} \lambda_i^c \cdot \lambda_j^c \frac{1}{r_{ij}} - \frac{3}{2} \frac{\sigma_{ij}^c - \sigma_{ij}^b}{\pi r_{ij}^2} - \frac{4}{3m_{ij}} (\sigma_i \cdot \sigma_j)$. Here $\alpha_{ij}$ and $\sigma_{ij}$ are parameter related to the types of two quarks, and $b$ is the strength of confinement. (2) Their results reveals that the 1S of diquark–antidiquark structure with sextet–antisextet and triplet–antitriplet from single channel calculation could be possible resonance state, but in present study 1S of diquark–antidiquark structure couldn’t form a resonance through $\text{RS}$. The reason would be that these 1S channels of diquark–antidiquark structure are strongly coupled with scattering state and decay to threshold quickly.

2. Yang et al. [39] employs a potential model inspired by the Lattice-QCD investigation of Ref. [52] to study $cc\bar{c}\bar{b}$ and $bb\bar{b}\bar{c}$ systems, and the $K$-structure are also considered in their calculation. In their model, the gluonic potential is $V_{ij}^{CON+OGE}(r_{ij}) = -\frac{3}{16} (\lambda_i^c \cdot \lambda_j^c) r_{ij} - \frac{\alpha_{ij}}{4} + \sigma_{ij} + \frac{\beta}{\gamma} r_{ij} (\sigma_i \cdot \sigma_j)$, where the model parameters $\alpha$, $\beta$, $\gamma$, and $\sigma$ can be determined via a calculation of the mass spectrum of the $S$-wave $QQ$ mesons [39], while the $V_{ij}^{CON}$ and $V_{ij}^{OGE}$ in our model are described as Eq. (2) and Eq. (3). According to their results, they found 7 possible resonances in $cc\bar{c}\bar{b}$ system and 3 possible resonances in $bb\bar{b}\bar{c}$ system [39]. According to the data in Table 11 and compared with their conclusions, the difference between our study and theirs mainly reflected in the following: (1) Firstly, they employ a linear confinement potential, while we use a confinement potential with screen effects, and the magnitude of attractive force comes from linear confinement are more than ours in medium and long range. Because the size of excited meson is larger than ground state meson, there model have more deviation between theoretical prediction and experiment result in excited state. (2) In addition, their conclusions points that all the solutions are possible resonance states, but does not analyze the resonance mechanism. We found that the excited states of “compact states” could be candidate of

### Table 11 Theoretical and experimental mass of twelve mesons of present work and Refs. [33,39]

| Mesons | This paper | Ref. [33] | Ref. [39] | Exp |
|--------|------------|-----------|-----------|-----|
| $\eta_c(1s)$ | 2989.2 ± 4.3 | 2983.4 ± 2.7 | 2968.1 ± 2.0 | 2983.9 ± 0.4 |
| $\eta_c(2s)$ | 3626.8 ± 5.0 | 3634.5 ± 2.2 | 3655.1 ± 2.9 | 3637.5 ± 1.1 |
| $J/\psi(1s)$ | 3096.6 ± 4.1 | 3097.1 ± 2.0 | 3102.8 ± 1.8 | 3096.900 ± 0.006 |
| $\psi(2s)$ | 3685.0 ± 5.1 | 3678.5 ± 2.2 | 3721.5 ± 3.0 | 3686.10 ± 0.06 |
| $B_c(1s)$ | 6272.6 ± 2.0 | 6270.5 ± 2.7 | 6275.3 ± 1.8 | 6274.47 ± 0.32 |
| $B_c(2s)$ | 6856.9 ± 4.8 | 6870.7 ± 2.0 | 6883.3 ± 2.5 | 6871.2 ± 1.0 |
| $\eta_b(1s)$ | 9453.6 ± 4.0 | 9389.5 ± 3.6 | 9401.4 ± 2.2 | 9398.7 ± 2.0 |
| $\eta_b(2s)$ | 9985.4 ± 4.4 | 10004.7 ± 1.8 | 9961.1 ± 2.0 | 9990.9 ± 4.3 |
| $\Upsilon(1s)$ | 9504.5 ± 3.8 | 9459.4 ± 3.0 | 9463.1 ± 2.1 | 9460.30 ± 0.26 |
| $\Upsilon(2s)$ | 10,012.6 ± 4.4 | 10,024.0 ± 1.8 | 9981.7 ± 2.0 | 10,023.26 ± 0.31 |
| $\Upsilon(3s)$ | 10335.0 ± 5.0 | 10335.8 ± 2.0 | 10330.1 ± 2.6 | 10355.2 ± 0.5 |
| $\Upsilon(4s)$ | 10557.3 ± 5.4 | 10582.2 ± 2.4 | 10618.3 ± 3.2 | 10579.4 ± 1.2 |
resonance state, and found the corresponding resonant line by RSM.

Since current experimental data are not sufficient to determine which form of confinement potential is better, we expected that more experimental observation data in the future could help.

To evaluate the stability of possible resonance states, we calculate the energy $E$ of $R(10,081)$ and the error $E(1\%)$ calculated by change $\pm 1\%$ of each parameter (except $m_c$ and $m_b$), which is list in Table 12. According to Tables 11 and 12 we conclude that the result of present model has little effects on the changing of parameters.

In addition, all six possible resonance states are below $P$-wave meson composed of $c\bar{c}$ and $b\bar{b}$. In higher energy region, $P$-wave meson might play a significant role, which demands careful consideration. We leave it in the future works.

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Table 12 The energy $E$ of $R(10,081)$ and the error $E(1\%)$ calculated by change $\pm 1\%$ of each parameter (except $m_c$ and $m_b$).

| Resonance state | $E$      | $E(1\%)$          |
|-----------------|---------|-------------------|
| $R(10,081)$     | 10.080.5| 10.079.5 $\pm$ 10.7|
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