Investigation of milling stability based on a cutting force segmentation model

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Abstract. The chatter vibration is the major limiting factor for higher metal removal rates in milling process. This paper proposed an integral model of cutting force with helical cutter considering the sectional characteristics of cutting force. Based on the model, the variation law of the upper and lower limit angles of the cutting process under different radial cutting depths is analysed. The results show that the shape of cutting force for each tooth has a characteristic of three sections. Moreover, the process dynamic equation of thin-walled parts cutting is constructed based on the model. And, the stability lobe diagram is obtained by semi-discretization method. The time-domain simulations are carried out in different cutting parameters around the stable island to verify the correctness of the prediction method. The results show that the method has good ability to predict the cutting stability of cutters with helix angle.

1. Introduction
Milling chatter may cause much problems in practice, such as low machining quality, poor accuracy and surface finish, unpleasant noise and sound, and so on. Thus, chatter has always been a concern of scholars. In the past decades, many methods have been proposed to model and analyze this issue. Such as, the zeroth-order approximation method [1], the multi-frequency solution [2], the semi-discretization method (SDM), the temporal finite element analysis [4] and the full-discretization method [5-7].

In this paper, an integral model of cutting force with helical cutter considering the sectional characteristics of cutting force is presented. The emphasis of this paper is to study the influence of segmented cutting force model on the upper and lower limit angles and the cutting stability.

2. Cutting force modeling
Compared with solid parts, the stiffness thin-walled parts is relatively small in the direction of workpiece thickness. For this direction, its dynamic equation can be expressed as

\[ m\ddot{y}(t) + c\dot{y}(t) + k_y(t) = F(t) \]  \hspace{1cm} (1)

where \( m \) is modal mass, \( c \) is modal damping, \( k \) is modal stiffness. These parameters can be obtained by hammering experiment. \( F(t) \) is the cutting force. If the effect of sectional characteristics (see figure 1) of milling with helical cutters on cutting force[6,7], \( F(t) \) can be obtained by

\[ F(t) = \sum_{j=1}^{N} G(\phi_j(t,z)) \frac{R}{\tan \beta} \int_{\phi_j(t,0)}^{\phi_j(t,\pi)} (K_x(t)(y(t) - y(t - \tau)) + K_z(t)) d\phi \]  \hspace{1cm} (2)
where \( K_1 = kscssk^2 \), \( K_2 = kcsss^2 + kscsskc \, c \) \( (c = \cos \phi, s = \sin \phi) \)

\( \phi_d (t) = \max(\phi_j (t, \alpha), \phi_{0d}) \), \( \phi_u (t) = \min(\phi_j (t, 0), \phi_e) \)

The rotation angles of the \( j \)-th cutter teeth at cutting heights \( z \) are as follows

\[
\phi_j (t, z) = \frac{2\pi}{N} \frac{(j-1)-z \tan \beta}{R}
\]

(3)

3. Analysis of upper and lower angle

The change trend of upper and lower angle of the cutting force is illustrated in Figure 2. On the other hand, Figure 3 shows the change trend of upper and lower angle for three different helix angle 0°, 20° and 45°. It can be seen from the figure 3 that as the increase of the helix angle \( \beta \), there are a similar trend for above there helix cases, however, the starting position has changed significantly. Thus, the wave of cutting force will fluctuate and the cutting force will be affected finally.

(1) The change of lower angle: it can be seen from Figure 3(a) that when the helix angle \( \beta =0^\circ \), the lower limit angle increases linearly with time, and \( a=0, \ b=150 \times 10^{-4} \) s; when the helix angle \( \beta =20^\circ, \ a=1.74 \times 10^{-7} \) and \( b=151.74 \times 10^{-4} \) s, it is same to the process ① in Figure 1(b). Since the helix angle \( \beta \) is not 0, the cut in angle is always be 0.8π. With the increase of helix angle \( \beta \), the values of \( a \) and \( b \) increase gradually, consequently, \( a=4.77 \times 10^{-4} \) s, \( b=154.77 \times 10^{-4} \) s for \( \beta =45^\circ \). The minimum value \( c \) of the lower limit angle is always 0.8 π, and the maximum value \( d \) of the lower limit angle is always π. Therefore, the size of the helix angle \( \beta \) does not affect the maximum value of the lower limit angle.

(2) The change of upper limit angle. It can be seen from Figure 3(b) that the value of \( a \) keeps to 150. With the increase of helix angle \( \beta \), upper limit angle reaches the maximum value, \( \phi_u \), and then keep constant for a long time. This time is determined by the corresponding \( \phi_{0u} \) and spindle speed \( n \).

When the rotation speed is fixed, \( \phi_{0u} \) increases with the bigger helix angle \( \beta \). It is easy to see that the size of the helix angle \( \beta \) does not affect the maximum value of the lower and upper limit angles.

Figure 1. Schematic diagram of milling process.

Figure 2. Schematic diagram of the upper and lower limit angle of cutting force.
Figure 3. Change of upper and lower limit angle of cutting force in milling process

(a) lower limit angle  
(b) upper limit angle

Figure 4. Stability lobe diagram and the selected cutting points
Figure 5. The stability conditions under different $a_p$. (a) $a_p = 5$ mm, (b) $a_p = 10$ mm; (c) $a_p = 15$ mm.
3.1 Method validation

Figures 4 shows the stability lobe diagram based on the SDM \cite{3} and the proposed cutting force. The simulation parameter are: tool radius 9.525mm, the number of teeth 3, cutter helix angle 30°, radial depth ratio 5.25 %, feed per tooth 1.78×10-4 m/s. Then, 54 points around the stability boundaries are selected, and their cutting stability is investigated by time-domain method in Ref.[1]. In figures 4, the star with four corners is stable, and the circle with a solid core is unstable. Obviously, it can be seen from figures 4 that the stability chart by the proposed method is inconsistent with that by time-domain ones.

Figures 5 shows the cutting time history and cycle points (left graphic), Poincare map (middle graphic) and spectrum map (right graphic) corresponding to the axial depth of cut \( a_p = 5, 10 \) and 15 mm respectively. It can be seen that when the axial cutting depth increases from 5 to 10 mm, the amplitude increases significantly, and the most important factors affecting the amplitude are the maximum effective cutting depth and the maximum effective cutting thickness, i.e., \( d_{max} = (\phi_c - \phi_s) R \tan^{-1} \beta \approx 8.91 \text{mm} \). In the three cutting conditions, the periodic points tend to be a straight line, while the Poincare mapping tends to be a point. In the spectrum diagram, only the passing frequency of the cutter tooth and its frequency multiplication exist, so the three milling processes are stable.

Figures 6 shows the cutting time history and cycle points, Poincare map and spectrum map corresponding to the axial depth of cut \( a_p = 20, 25 \) and 30mm respectively. It can be seen that the periodic points of three cutting conditions tend to two straight lines, and the Poincare mapping tends to two points, so flip bifurcation occurs. Meanwhile, the chatter frequencies occur, so these milling processes are unstable.

![Graphs showing stability conditions](image)

Figure 6. The stability conditions under \( a_p \). (a) \( a_p = 20 \text{mm} \); (b) \( a_p = 25 \text{ mm} \); (c) \( a_p = 30 \text{ mm} \).

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**Vibration history**

**Poincare mapping**

**Spectrogram**
4. Conclusion
This paper presents an integral model of cutting force with helical cutter considering the sectional characteristics of cutting force. Based on the model, the change rule of the upper and lower limit angles of the cutting process under different radial cutting depth is analyzed. The results show that the shape of cutting force for each tooth has a characteristic of three sections. Moreover, the process dynamic equation of thin-walled parts cutting is constructed based on the model. And, the stability lobe diagram is obtained by SDM. The stability simulation is carried out in different cutting parameters around the stable island to verify the correctness of the prediction model. The results show that the method has good ability to predict the cutting stability of cutters with helix angle.

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