Possible evidence for double precessing nozzle structure in QSO 3C345

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ABSTRACT

Context. The precessing jet-nozzle scenario previously proposed has been tentatively applied to interpret the VLBI-kinematics of twenty-seven superluminal components in blazar 3C345 measured during a ∼38-year period.

Aims. The superluminal components could be divided into two groups ascribed to jet-A (13 knots) and jet-B (14 knots), having different kinematic behaviors. They could be ejected from a double jet-nozzle system forming a double-jet structure.

Methods. Through model-simulation of kinematic behavior of the knots, it was tentatively found that both nozzle could precess with same period of ∼7.30 yr (4.58 yr in the source frame) and in the same direction. The precession of jet-A was simulated over four periods, while that of jet-B was simulated over two periods.

Results. For both jets a steady precessing common trajectory could exist along which different superluminal knots moved according to their precession phases and their kinematics could be well interpreted. Most superluminal knots were model simulated to be accelerated with their bulk Lorentz factor in the range of ∼4 to ∼30. The radio light curves of knot C9 was found to be extraordinarily well coincident with its Doppler boosting profile, implying that its Lorentz factor and Doppler factor profiles were correctly derived and that superluminal components could be recognized as relativistic shocks moving along helical trajectories toward us.

Conclusions. The kinematic features observed in 3C345 and interpreted in terms of our precessing-nozzle scenario can be understood in the framework of MHD theory for formation of relativistic jets, although formation of double-jet structure in black hole binaries seems to be a new theoretical field to be investigated. The double precessing nozzle scenario has now been applied to interpret the VLBI-kinematics of superluminal knots for four blazars (3C279, OJ287, 3C454.3 and 3C345). The characteristic parameters of the four putative supermassive binary black hole systems (including hole masses, mass ratio, orbital separation, post-Newtonian parameter, gravitational radiation lifetime, etc.) were tentatively derived and compared, showing that they are in physically reasonable ranges, and well consistent with some theoretical arguments for close black hole binaries. These results seem providing some enlightening clues that keplerian motion of supermassive black hole binaries in blazars could be discovered through VLBI-observations over sufficient long periods.

Key words. galaxies: active – galaxies: jets – galaxies: nucleus – galaxies: individual 3C345

1. Introduction

3C345 (z=0.595) is an archetypal quasar and one of the best-studied blazars (e.g., Biretta et al. 1986; Hardee et al. 1987; Homan et al. 2013; Jorstad et al. 2005, 2013, 2017; Klare 2003; Klare et al. 2005; Lobanov & Roland 2005; Qian et al. 1991a, 1996, 2009; Schinzel et al. 2010a, 2011a; Schinzel 2011a; Steffen et al. 1996; Unwin et al. 1997; Zensus et al. 1997). Its emission spreads over the entire electromagnetic spectrum, from radio, IR, optical, UV, X-ray to high-energy γ-rays. Prominent and complex variabilities in all these wavebands and the spectral energy distributions have been extensively monitored and studied, leading to many important results on the properties of the emitting sources. Studies of correlation among the variabilities at multi-frequencies (from radio to γ-rays) play an important role. The connection between radio flares and high-energy γ-rays was observed (Schinzel et al. 2010a).

It is a remarkable compact flat-spectrum radio source with a relativistic jet, from which superluminal components are emanated. VLBI-observations reveal the parsec structures of its jet and track the motion of its superluminal components ejected from its radio core. It has shown that flaring activities in multi-frequencies (from radio to γ-rays) are closely connected with the jet-activity and ejection of superluminal knots. In addition, VLBI-monitoring observations have shown that its relativistic jet may be precessing with quasi-periods. This phenomenon may be very important for understanding the properties of the central energy engine in its nucleus. The VLBI-kinematics of superluminal components in 3C345 has been analyzed by Schinzel (2011a) for a ∼30 years period, showing no trend for jet-precession. In this paper we investigate the VLBI-kinematics of 3C345 extending to ∼38 yr time-interval, yielding some new significant results.

Since 1991 (Qian et al. 1991a, 2009), we have tried to explain the VLBI-kinematics of superluminal components in 3C345 in terms of a precessing nozzle scenario. Our scenario not only considered the precession of the jet-nozzle to explain the position angle swing of its superluminal components, but also considered the possible existence of a common (helical) trajectory pattern, which could produce the trajectories of the knots ejected at different times through its precession. It was found that model
simulations of the observed trajectories of its superluminal knots by using the precession of the common trajectory pattern could quite effectively find the period of jet-nozzle precession. Helical motion has been widely invoked to interpret the VLBI-kinematics of superluminal components in radio quasars, especially in blazars (referring to Perucho et al. 2012a, 2012b: S5 0836+710, Lister et al. 2013a, 2013b, BL Lacertae) and Qian et al. (2021, 2009). Cohen et al. (2014, 2015) introduced the concept of relativistic Alfven waves to explain jet structure and structural evolution (BL Lacertae). Some authors suggested that jet instabilities (e.g. Kelvin-Helmholtz instability) could play significant role in forming helical trajectories in outer jet regions (e.g., Perucho 2012a, Schinzel et al. 2010a, 2011a). Our precessing nozzle scenario has been previously applied to analyze the VLBI-kinematics of superluminal knots in several QSOs, e.g., 3C279, B1308+328, PG1302+202, NRAO150, 3C454.3 and OJ287 (Qian et al. 2014, 2017, 2018a, 2019a, Qian et al. 2013, 2016, 2018). These studies revealed that jet-nozzle precession may exist in these sources. Through model-fitting of the VLBI-kinematics in terms of the precessing nozzle scenario, possible periods of precession and other kinematic parameters for the superluminal knots (bulk Lorentz factor, viewing angle, apparent velocity and Doppler factor vs time) were derived. In particular, in two cases (3C279 and OJ287) possible evidence has been obtained that double-jet systems might exist in their nuclei, which could be mostly produced by binary black hole/accretion-disk systems. In the case of OJ287, we might speculate that its quasi-periodic optical variability is connected with its double-jet activity (Villata et al. 1998, Qian 2018d, 2019b, 2019c, 2020; also referring to Qian et al. 2007). Search for periodicities in optical and radio light-curves (e.g. Sillanpää et al. 1988, Babadzhanyants et al. 1995, Kudryavtseva et al. 2006, Qian et al. 2007) are important and could provide key information on the nature of the central engine in blazars. The position angle swings of superluminal components on parsec-scales observed by VLBI-monitoring observations could be used to search for periodicities in ejection of superluminal knots (e.g., Britzen et al. 2001, Tateyama & Kingham 2004, Klare 2005, Schinzel et al. 2012a, Qian et al. 2009). In the case of QSO 3C345 and based on the position angle swing of its superluminal components, some authors argued for the existence of a jet precessing period: e.g., ~8-10 yr (Lobanov & Zensus 1999, Klare et al. 2005, Klare 2003, ~9.5 yr (Lobanov & Roland 2005). In earlier studies we already found that the observed tracks of knots C4 and C5 could be reproduced by the rotation of a common helical trajectory (Qian et al. 1991a, 1991b, Qian & Zhang 1999). Qian et al. 2009 analyzed the distribution of the position angles for seven superluminal components (C4 to C10) at different core separations of 0.15 mas, 0.20 mas and 0.25 mas, and found that their inner trajectories (within core separation $r < 0.4$ mas) could be explained in terms of the precession of a common trajectory and a precession period of its jet-nozzle of $\sim 7.36$ yr was derived.\footnote{It is noted that this is an averaged value: according to equations (15), (16) and (17) of that paper, a precession period of 7.44 yr, 7.34 yr and 7.31 yr were obtained for core distance 0.10 mas, 0.15 mas and 0.20 mas, respectively.}

In this paper we further analyze the kinematics of 27 superluminal knots, spreading over a time-range of ~38 years (1980–2018) and show that the kinematics of these superluminal knots could be consistently explained in terms of our precessing-nozzle scenario, if a double-jet system is assumed to be existing in its nucleus. Obviously, if this result is verified QSO 3C345 should host a binary black hole system in its nucleus.

We point out that our precessing jet-nozzle scenario is well consistent with the magnetohydrodynamic theories for the formation and collimation of relativistic jets in AGN (e.g., Blandford & Payne 1982, Blandford & Znajek 1977, Camenzind 1980, 1987, 1990, Li et al. 1992, Lovelace et al. 1986, Meier & Nakamura 2006, Nakamura & Asada 2013, Valkakis & Königl 2003, 2004).

2. Observational data

In this paper we made use of the data collected from the literature: (1) Data presented in Klare (2003) for period 1980.5-2001.9; (2) Data presented in Schinzel (2011a) for period 1980.5-2010.8. Part of the data were re-calculated to make the compact core at the unified origin of coordinates; (3) Data kindly provided by Jorstad (private communication); for period 2011.1-2018.8. This dataset extended the time-interval for our model fitting of the kinematics in 3C345 to ~38 years (about five precession periods).

Considering the core-shift effects we only used 43GHz and 22GHz observational data (except for knot C5, for which only 15GHz data are available). Generally, we would not mark the observational errors for the positions of individual knots in figures obtained from model-fittings for clarity, but one should keep in mind that errors in measurements of knot’s position are in the range of ~0.05–0.1 mas.

We will apply the concordant cosmological model (Spergel et al 2003, Hogg 1999) with $\Omega_{\Lambda}=0.73$ and $\Omega_m=0.27$, and $H_0=71$ km$^{-1}$Mpc$^{-1}$. Thus the luminosity distance of 3C345 is $D_L=3.49$ Gpc, angular-diameter distance $D_A=1.37$ Gpc, 1 mas=6.65 pc, 1 mas/yr=34.6 c. 1 c is equivalent to an apparent angular velocity 0.046 mas/yr.

3. Geometry of the model

We will apply the precessing jet-nozzle model previously proposed by Qian et al. (2019a, 2019b, 2019c, 2020a, 2020b) to investigate the kinematics and distribution of trajectory of superluminal components on parsec scales in the QSO 3C345. We will use a special geometry consisting of two coordinate systems as shown in Figure 1. We assume that the superluminal components move along helical trajectories around the curved jet axis (i.e. axis of the helix). We use coordinate system $(X_{\nu}, Y_{\nu}, Z_{\nu})$ to define the plane of the sky $(X_{\nu}, Z_{\nu})$ and the direction of observer $(Y_{\nu})$, with $X_{\nu}$-axis pointing toward the negative right ascension and $Z_{\nu}$-axis toward the north pole.

We use coordinate system $(X, Y, Z)$ to locate the curved jet-axis in the plane $(X, Z)$, where $\epsilon$ represents the angle between $Z$-axis and $Y_{\nu}$-axis and $\psi$ the angle between $X$-axis and $X_{\nu}$-axis. Thus parameters $\epsilon$ and $\psi$ are used to
Fig. 1. Geometry of the precessing jet-nozzle scenario for 3C345. The jet-axis is defined in the \((X, Z)\)-plane by parameters \((\epsilon, \psi)\) and function \(x_0(z_0)\). The common helical trajectory pattern is defined by functions \(A(Z)\) and \(\phi(Z)\) given in Section 3.

![Diagram of the precessing jet-nozzle scenario](image)

Fig. 2. Knot C4. Model functions describing its helical trajectory: amplitude \(A(Z)\) and phase \(\phi(Z)\).

define the plane where the jet-axis locates relative to the coordinate system \((X_n, Y_n, Z_n)\).

We use coordinate system \((x', y', z')\) along the jet-axis to define the helical trajectory pattern for a knot, introducing parameters \(A(s_0)\) (amplitude) and \(\phi(s_0)\) (phase), where \(s_0\) represents the arc-length along the axis of helix (or curved jet-axis). \(z'\)-axis is along the tangent of the axis of helix. \(y'\)-axis is parallel to the \(Y\)-axis and \(\eta\) is the angle between \(x'\)-axis and \(X\)-axis (see Figure 1).

In general, we assume that the jet-axis can be defined by a function \(x_0(z_0)\) in the \((X, Z)\)-plane as follows.

\[
x_0 = p(z_0)z_0^\epsilon
\]  
(1)

where

\[
p(z_0) = p_1 + p_2[1 + \exp\left(\frac{z_t - z_0}{z_m}\right)]^{-1}
\]  
(2)

\(\epsilon, p_1, p_2, z_t\) and \(z_m\) are constants.

\[
s_0 = \int_0^{z_0} \sqrt{1 + \left(\frac{dx_0}{dz_0}\right)^2} \, dz_0
\]  
(3)

Therefore, the helical trajectory of a knot can be described in the \((X, Y, Z)\) system as follows.

\[
X(s_0) = A(s_0)\cos \phi(s_0)\cos \eta(s_0) + x_0
\]  
(4)

\[
Y(s_0) = A(s_0)\sin \phi(s_0)
\]  
(5)

\[
Z(s_0) = -A(s_0)\cos \phi(s_0)\sin \eta(s_0) + z_0
\]  
(6)

where \(\tan \eta(s_0) = \frac{dx_0}{dz_0}\). The projection of the helical trajectory on the sky-plane (or the apparent trajectory) is represented by

\[
X_n = X_n \cos \psi - Z_n \sin \psi
\]  
(7)
Z_n = X_p \sin \psi + Z_p \cos \psi

where

X_p = X(s_0)
Z_p = Z(s_0) \sin \epsilon - Y(s_0) \cos \epsilon

(All coordinates and amplitude (A) are measured in units of mas). Introducing the functions

\[ \Delta = \arctan\left(\frac{dX}{dZ}\right) \]
\[ \Delta_p = \arctan\left(\frac{dY}{dZ}\right) \]
\[ \Delta_s = \arccos\left[\frac{dX^2}{ds_0^2} + \frac{dY}{ds_0} \frac{dX}{ds_0} + \frac{dZ}{ds_0} \frac{dY}{ds_0} + \frac{dZ^2}{ds_0^2}\right]^{\frac{1}{2}} \]

we can then calculate the viewing angle \( \theta \), apparent transverse velocity \( \beta_s \), Doppler factor \( \delta \) and the elapsed time \( T \), at which the knot reaches distance \( z_0 \) as follows:

\[ \theta = \arccos[\cos \epsilon (\cos \Delta + \sin \epsilon \tan \Delta_p)] \]
\[ \Gamma = \left(1 - \beta^2\right)^{-\frac{1}{2}} \]
\[ \delta = \left[\Gamma(1 - \beta \cos \theta)\right]^{-1} \]
\[ \beta_s = \beta \sin \theta / (1 - \beta \cos \theta) \]
\[ T = \int_{z_0}^{\infty} \frac{1 + z}{\Gamma \delta \cos \Delta_s} ds_0 \]

The amplitude and phase of the helical trajectory for superluminal knots are defined as follows (Figure 2).

\[ A(Z) = A_0 \left| \sin\left(\frac{\pi Z}{Z_1}\right) \right|^{1/2} \exp\left(-\frac{Z}{Z_2}\right) \]
\[ \phi(Z) = \phi_0 - \left(\frac{Z}{Z_3}\right)^{1/2} \]

\( A_0 \) represents the amplitude coefficient of the common helical trajectory pattern and \( \phi_0 \) is the precession phase of an individual knot, which is related to its ejection time (see below).

The aim of our model fitting of the kinematics of the superluminal components observed in 3C345 is to show that most components have their observed trajectories following the precessing common trajectory and their kinematics can be interpreted in terms of our precessing jet nozzle scenario, indicating the possible presence of a supermassive black hole binary in its nucleus.

4. Model-fitting results for jet-A

It was found that 3C345 might comprise a double precessing jet structure: jet-A and jet-B. The former consists of knots C4–C14, C22 and C23 and the latter comprises knots C15, C15a, C16–C21, B5–B8, B11 and B12. Both jets precess with the same period of 7.30 yr and in the same direction: anti-clockwise seen along the line of sight.

As shown in Section 3, in our precessing jet-nozzle scenario the jet-axis is defined in the \((X, Z)\)-plane by parameters \( (\epsilon, \psi) \) and formulas (1) and (2). For jet-A we assume the following parameters:

\( \epsilon = 0.0349 \text{rad} = 0.59^\circ; \quad \psi = 0.125 \text{rad} = 7.16^\circ; \quad \zeta = 2.0; \quad p_1 = 0; \quad p_2 = 1.34 \times 10^{-4}; \quad z_i = 66 \text{ mas}; \quad z_m = 6 \text{ mas}. \)

The amplitude of the helical trajectory is defined by formula (19) in Section 3. For jet-A we assume the following parameters: \( A_0 = 0.605 \text{ mas}, Z_1 = 396 \text{ mas}; Z_2 = 3000 \text{ mas}. \)

The phase of the helical trajectory is defined in Section 3 by formula (20). For jet-A we assume \( Z_3 = 3.58 \text{ mas} \) and the precession phase \( \phi_0 \) is related to the ejection time of a knot as follows:

\[ \phi_0 = 4.28 + \frac{2\pi}{T_0}(t_0 - 1979.00) \]

where \( T_0 = 7.30 \text{ yr} \)–precession period of the jet-A nozzle.
Fig. 4. Model-fitting of the entire kinematic behavior within core separation of ∼8 mas for knot C4: trajectory $Z_n(X_n)$, coordinates $X_n(t)$ and $Z_n(t)$, core separation $r_n(t)$, and modeled apparent velocity $\beta_a(t)$, viewing angle $\theta(t)$, bulk Lorentz factor $\Gamma(t)$ and Doppler factor $\delta(t)$. Due to using appropriate mathematical formulas, functions and model-parameters for describing the curved jet-axis and its helical trajectory, the entire trajectory of knot C4 is well model-fitted till ∼8 mas from the core.

4.1. A note on the model-parameters

In our previous studies of the VLBI-kinematics of superluminal components in blazar 3C345 we found three distinct features: (1) the motion of some knots could be model-simulated as moving along helical trajectories (Qian et al. 1991a); (2) there could exist a common trajectory which precesses with a period of ∼7.4 yr, producing the trajectories for different knots at corresponding precession phases (Qian et al. 2009); (3) there were some clues showing that the knots could possibly be divided into two groups having different kinematic behaviors. Thus we needed some new methods to further investigate the VLBI-kinematics of superluminal knots in 3C345.

In order to interpret the kinematics of superluminal components in 3C345 in terms of our precessing nozzle scenario, we used model-simulation methods to model-fit the observed trajectories of its 27 superluminal components. Thus a large amount of model parameters were involved: e.g., parameters for describing the jet-direction in space, jet-cone size and curved jet-axis; jet precession period, ejection time and trajectory pattern of the 27 superluminal components, etc. Based on the formulation (Section 3) of the precessing jet-nozzle model (more details referring to the recent paper (for 3C454.3) published in

2 Specifically, the model-parameters include $\epsilon, \psi, p_1, p_2, \zeta, t_0$ (ejection time), A and $\phi$ (defining helical trajectory), and so on.
A&A: Qian et al. [2021] and through trial and error\footnote{“Trial and error” method may be the most feasible and effective one to disentangle the two precessing jets and to find appropriate model-parameters to describe their geometric features (jet direction, jet-cone size, precession period, phase distribution and direction of nozzle-precessing) and kinematic properties (helical trajectory pattern (mathematical function) and bulk Lorentz factor, etc.). It seems very difficult to use usual statistical methods to fulfill this complex task.} we found two specific sets of model parameters (one for jet-A and other for jet-B) and associated functions to model-fit (or model-simulate) the kinematics of its superluminal knots in terms of the precessing nozzle scenario (Qian et al. [2019a, 2021]).

In this work we made two assumptions: (1) jets in blazars have the distinct feature: precession; (2) superluminal components move along certain precessing common trajectory. These assumptions greatly decreased the number of model-parameters describing the trajectory patterns and ejection times of the 27 superluminal knots. We would like to indicate that the values selected for the model parameters and associated functions for both jet-A and jet-B are not statistical samples and not unique either. They are specific and physically appropriate and applicable sets of working ingredients. However, as shown in the main text below the model-simulation methods with these specific model-parameter values could be well applied to analyze the distribution of the observed trajectories and kinematics of superluminal components in blazar 3C345 on VLBI-scales, especially discovering the possible separation of its superluminal knots into two groups with different kinematic properties. By using the model-simulation methods we reached our aims: (1) seeking for possible jet-precession; (2) searching for double-jet structure; (3) disentangling the observed superluminal components into two groups ascribed to respective jets; (4) determining precession periods; and (5) studying the properties of the putative supermassive black hole in its nucleus.

Similar methods have also been applied to blazar 3C279 (Qian et al. [2018a], OJ287 (Qian [2018b]) and 3C454.3 (Qian et al. [2021]). Interestingly, we found that the four blazars could all have double-jet structure with their jets precessing with the same period in the same direction.

Since we dealt with model-fittings (or model simulations) of the kinematics of superluminal components involving multiple parameters and functions, mainly involving the model fits of the observed trajectories of the 27 knots, we introduced a new criterion to judge the validity of the model-fitting results. That is, a reasonable and effective model-fit was required to satisfy the condition: its observed trajectory (or the related data-points) had to be fitted to follow the precessing common trajectory predicted by the scenario within ±5% of the precession period. Taking the model-fit of the trajectory of knot C9 as an example, in Figure 3 two plots are shown: the left panel represents that the observed trajectory marked by observational errors was well fitted by the model trajectory. The observed data-points were well concentrated around the model trajectory. The right panel represents that the observed trajectory was well within the region limited by the model trajectories defined by the ±5% of the precession period. It can be seen that since the model fitted the entire trajectory of C9, fitting quality of the entire observed trajectory was determined by the systematic deviations of the data-points from the model trajectory (or the concentration of the data-points relative to the model trajectory), not much depending on the observational errors of individual data-points.\footnote{So the figures showing the model-fitting of the observed trajectories are not marked with observational errors of the data-points. Error estimate of ±5% of precession period is an effective criterion for judging the accuracy of trajectory model-fits.} Thus the new criterion is a simple and quite effective one for assessing the validity of the model-simulation results as a whole. The model-fit of the trajectory for knot C9 is a good example, where almost all the observational data-points were well within the region defined by the common trajectories at ±5% precession period. Obviously, adopting ±3% of period would let quite a number of data-points becoming outliers. Adopting ±10% of period would let the derived precession-period having larger error. In the case of adopting +/−5% of period, more than 85% of the knots had their trajectories well model-simulated. (See the [status column] in Table 1 and Table 2 below). This is a very high probability of success.

Thus, based on our precessing nozzle scenario, possible evidence for double precessing relativistic jets and a putative black-hole binary in 3C345 could be tentatively investigated. The kinematics of the superluminal components of both jets have been well model-fitted and a precession period of 7.3±0.36 yr for both the jet-nozzles has been derived. We emphasize that these results should be tested by VLBI-observations in the future years.

4.2. Entire kinematic behavior’s model-simulation of knot C4

We first discuss the model-fitting of the entire kinematics of knot C4 within the core separation ∼8 mas, including its trajectory, and core separation, coordinates, apparent velocity, viewing angle, bulk Lorentz factor and Doppler factor versus time.

We assume that its ejection time $t_0=1979.00$ and the corresponding precession phase $\phi_0=4.28$ rad.

It is worth-while to note that, due to adopting appropriate mathematical formulas, functions and model-parameters to describe the curved jet-axis and its helical trajectory pattern (see Sec.3 and Fig.2), the entire trajectory of knot C4 extending to core separation of ∼8 mas was well fitted (Figure 4). For the model-fitting of the entire kinematics of knot C4, the exponential factor in the expression of $p(z_0)$ (see equations (1) and (2) of Section 3) plays a significant role, resulting in the modeled trajectory steadily curving northward. Its helical trajectory is described by amplitude $A(z_0)$ and phase $\phi(z_0)$, which are presented in Figure 3.

The mode-fitting results are shown in Figure 4. It can be seen that its trajectory, coordinates, core separation are well fitted. And its apparent velocity, viewing angle, Lorentz factor and Doppler factor are all derived as functions of time along the whole trajectory.

Bulk acceleration is required and its bulk Lorentz factor ($\Gamma$) changes as: for $Z<2.0$ mas: $\Gamma=8$; for $Z=2.0-20$ mas: $\Gamma=8+2(Z/2)/(20-2)$; for $Z=20-30$ mas $\Gamma=10+5(Z/20)/(30-20)$; for $Z=30-40$ mas $\Gamma=15+3(Z/30)/(40-30)$; for $Z>40$ mas $\Gamma=18$.

The fitting results in Figure 4 clearly show that the entire kinematics of knot C4 can be well explained in terms of
our precessing nozzle model. This may be the first time for a superluminal knot being well fitted by a 3-dimensional helical motion to core-separation of \(\sim8.0\) mas, corresponding to a spatial distance \(\sim1.1\) kpc from the galaxy center. Our model-fitting of the kinematics for knot C4 is physical and thus its viewing angle and bulk Lorentz factor/Doppler factor vs time can be derived (or simulated). When the Doppler factor vs time for knot C4 is derived, the characteristic features of its radiation and evolution can then be fully investigated and the physical parameters of its emitting regions can be determined (e.g., Qian et al. 1999a).

Although we have quite successfully performed the model-fitting of the entire kinematics of knot C4, we mostly concern about the model-fit of its innermost trajectory: whether it could be model-fitted by the precessing common trajectory.

4.3. Precessing common trajectory for knot C4

Although the entire kinematics of knot C4 has been model-fitted as described above, we do not know how far its precessing common trajectory extending from the core. Obviously only its inner trajectory within a certain core-separation could join in with the other superluminal components commonly having the precessing trajectory. In comparison with the observed trajectory of knot C9 (having a very long precessing common trajectory) we assumed that for knot C4 its trajectory section within core separation \(r_n \leq 1.8\) mas (equivalent to spatial angular distance \(\sim52\) mas or \(346\) pc from the core; Fig.6) could be ascribed to the precessing common trajectory and model-fitted consistently with other superluminal knots of jet-A in terms of our precessing nozzle scenario.

The distribution of precessing common trajectory for jet-A is shown in Figure 5 (left panel) and the observed trajectories of knots C4, C5 and C13 are shown in right panel for comparison. In Figure 6 the model-fitting results of the kinematics for knot C4 within core separation \(r_n \sim1.8\) mas is presented by using its precession phase \(\phi_0=4.28\) rad and ejection epoch \(t_0=1979.0\). In order to model-fit its core separation more appropriately, we need to slightly adjust its bulk Lorentz factor: for \(Z\leq2\) mas \(\Gamma=8.0\); for \(2-20\) mas \(\Gamma=8+4.4(Z-2)/(20-2); \) for \(Z>20\) mas \(\Gamma=12.4\). It can be seen that its kinematics within core separation \(r_n \leq 1.8\) mas can be well explained in terms of our precessing nozzle scenario. Knot C4 has the second longest precessing common trajectory among the knots of group-A with a spatial extension of \(Z_{c,m} \sim52.0\) mas, corresponding to a spatial distance \(Z_{c,p} \sim346\) pc (see Table 1 below). During the period 1980.1-1987.5 its bulk Lorentz factor \(\Gamma\), Doppler factor \(\delta\), apparent velocity \(\beta_a\) and viewing angle \(\theta\) vary over the respective ranges: [8.3,12.4], [16.1,(21.7)-17.8], [3.0,11.1] and [1.28,2.88]. (Note: as shown in Fig.6 (bottom/left panel) the Doppler factor curve \(\delta(t)\) has a bump structure responding precession phase \(\phi_0\) and \(\delta_0\) to \(t=1980.1-1987.5\). Its observed precessing common trajectory might be ascribed to the precessing common trajectory and viewing angle \(\theta\) vary over the respective ranges: [5.5,15.0], [10.1,(24.8),23.6], [2.9,(10.0),(8.3),12.2] and [3.03,1.98](deg).

4.4. Model-fitting of kinematics for knot C5-C14, C22 and C23

4.4.1. A brief introduction

We would like to note that the observed inner trajectories of knots C5, C7, C10 and C11-C13 are similar, revealing the precession of jet-A nozzle. Moreover, the observed inner trajectories of knots C6, C9, C22 and C23 are similar, also showing the precession of jet-A nozzle. Interestingly, both knot-sets demonstrate the same nozzle-precession period \(\sim7.30\) yr, which was derived about ten years ago in Qian et al. (2009). We had already found that the kinematics of superluminal knots in QSO 3C345 could be understood in terms of our precessing jet nozzle scenario. Now we have found the second jet (jet-B) and its precessing nozzle, ejecting knots C15-C21 and B5-B8, B11 and B12, the kinematics of which could also be explained in terms of the precessing nozzle scenario for jet-B. Therefore, the kinematic behavior of superluminal knots in QSO 3C345 may likely imply that QSO 3C345 has a double relativistic jet system and host a binary black hole in its nucleus. In the following we will first present the model-fitting results for the superluminal knots of jet-A in detail.

4.4.2. Model-fitting results for knot C5

The model-fitting results of the kinematics of knot C5 are shown in Figure A.1. Its ejection epoch \(t_0=1980.80\) and corresponding precession phase \(\phi_0=5.83\) rad.

Bulk acceleration is required and its Lorentz factor is modeled as: for \(Z\leq3.0\) mas \(\Gamma=5.3\); for \(Z=3-20\) mas \(\Gamma=5.3+(Z-3)/(15-5.3)/(20-3); \) for \(Z>20\) mas \(\Gamma=15\). It can be seen in Figure A.1 that the entire kinematic behavior within core separation \(r_n \leq 1.2\) mas can be well fitted, implying that its observed precessing common trajectory may extend to a spatial distance of \(Z_{c,m}=39.0\) mas (or \(Z_{c,p}=259.3\) pc) from the core (see Table 1).

4.4.3. Model-fitting results for knot C6

According to the precessing-nozzle scenario for jet-A, the kinematics of knot C6 is model-fitted with precession phase \(\phi_0=5.74\) rad.\(\pm2\) rad and ejection epoch \(t_0=1987.99\). In this case the observed precessing common trajectory may only extend to \(r_n \approx 0.30\) mas. The kinematics in its outer trajectory has to be explained by introducing changes in parameter \(\psi\) (or rotation of its trajectory): for \(Z\leq6.0\) mas \(\psi=0.125\) rad (just the same as for the precessing common trajectory); for \(Z=6-20\) mas \(\psi=0.125-0.225(Z-6)/(20-6); \) for \(Z>20\) mas \(\psi=-0.1\). Acceleration in its motion is required and its bulk Lorentz factor is modeled as: for \(Z\leq6.6\) mas \(\Gamma=7.9\); for \(Z=6-20\) mas \(\Gamma=9.7+6.3(Z-6)/(20-6); \) for \(Z>20\) mas \(\Gamma=16\). The model-fitting results of kinematic behavior of knot C6 are shown in Figure A.2. Due to lack of observational data-points within \(X_m<0.3\) mas, the model fitting of its kinematic behavior along the precessing common trajectory is only marginal. Its observed precessing common trajectory might be as-
Fig. 5. Left: Distribution of the precessing common trajectory for jet-A. The jet axis is at position angle $\sim 42.5^\circ$ (at core separation 0.5 mas). The opening angle of the jet is $\sim 1.12^\circ$ in space. Right: the observed trajectories of knots C4, C5 and C13 in the jet are shown for comparison.

sumed to extend to $r_n \sim 0.40$ mas, corresponding to a spatial distance $Z_{c,m} = 7.47$ mas (or $Z_{c,p} = 49.7$ pc) from the core. During the period 1988.2–1990.0 its Lorentz factor $\Gamma$, Doppler factor $\delta$, apparent velocity $\beta_a$ and viewing angle $\theta$ vary over the following ranges respectively: [9.7,11.1], [13.5,18.7], [8.9,8.0] and [3.90,2.21](deg).

4.4.4. Model-fitting results for knot C7

According to the precessing jet nozzle scenario for jet-A, the kinematic behavior of knot C7 could be fitted with parameters: precession phase $\phi_0$(rad)=6.14+2$\pi$ and ejection epoch $t_0$=1988.46. The model-fitting results of its kinematic behavior along the precessing common trajectory are shown in Figure A.3. It can be seen that the observed precessing common trajectory may extend to $r_n \sim 0.7$ mas, corresponding to a spatial distance of $Z_{c,p} \sim 99.3$ pc (or corresponding $Z_{c,m} \sim 14.9$ mas) from the core.

Bulk acceleration is required. Its Lorentz factor is modeled as: For $Z \leq 1.6$ mas $\Gamma = 3.2$; for $Z = 1.6–5$ mas $\Gamma = 3.2 + (Z-1.6)/(5-1.6)$; for $Z > 5$ mas $\Gamma = 13.0$.

During the period 1989.0–1994.5 its Lorentz factor $\Gamma$, Doppler factor $\delta$, apparent velocity $\beta_a$ and viewing angle $\theta$ vary over the following ranges: [3.2,13.0], [6.2,21.8], [0.65,(11.9),9.5] and [2.00,1.92](deg).

4.4.5. Model fitting results for knot C8 : A particular case

The observed kinematic behavior of knot C8 is exceptional, but instructive revealing the complex structure of its innermost trajectory. Only one data-point obtained at 22/43GHz (1992.4) by Klare [2003], showing its initial trajectory following the precessing common trajectory (precession phase $\phi_0 = 2.13$ rad $+ 4\pi$ and corresponding ejection epoch $t_0 = 1991.10$; see Figure 7). Other observational data-points indicate that its trajectory changed rapidly at core separation of $\sim 0.12$ mas. This observational fact clearly demonstrates that knot C8 might have been regarded as not following the precessing common trajectory if no available observations at core separations $r_n < 0.1$ mas (like that given by Klare). We will further indicate the importance of VLBI-observations at core separations $r_n \leq 0.05$-0.1 mas for our precessing nozzle scenario.

We model-fit its outer trajectory by introducing changes in parameter $\psi$: For $Z \leq 6$ mas $\psi = 0.125$ rad which are the same for knot C4; for $Z = 6-15$ mas $\psi$(rad)=$0.125+0.6(Z-6)/(15-6)$; for $Z = 15-40$ mas $\psi$(rad)=$0.725-0.580(Z-15)/(40-15)$; for $Z > 40$ mas $\psi = 0.145$ rad.

Bulk acceleration is required. For $Z \leq 6$ mas $\Gamma = 6.2$; for $Z = 6-15$ mas $\Gamma = 6.2+7.6(Z-6)/(15-6)$; for $Z > 15$ mas $\Gamma = 13.8$.

Its observed precessing common trajectory might be assumed to extend to $\sim 0.15$ mas, equivalent to a spatial distance $Z_{c,m} = 6.2$ mas (or $Z_{c,p} = 41.2$ pc) from the core. During the period 1992.0–1998.0 its Lorentz factor $\Gamma$, Doppler factor $\delta$, apparent velocity $\beta_a$ and viewing angle $\theta$ vary over the following ranges: [6.2,13.8], [12.0,19.0], [1.9,(12.9),12.8] and [1.48,(2.87),2.80](deg).

We would like to emphasize that rapid changes of trajectory as in knot C8 is a significant ingredient which also occurred in other blazars like 3C279 and 3C454.3 (Qian et al. 2013, 2014). This ingredient makes the fitting of the kinematics on pc-parsec scales more subtle and difficult. The cause of these sudden curvatures of trajectory is unclear.

4.4.6. Model-fitting of kinematics for knot C9

The model-fitting of the kinematics of knot C9 is very important and encouraging for our precessing nozzle scenario for blazar 3C345. Its kinematic behavior can be very well explained in terms of our scenario as shown in Figure 8.

Its ejection epoch is modeled to be $t_0 = 1995.06$, corresponding to precession phase $\phi_0$(rad)=5.54+4$\pi$.

It can be seen from Fig.8 that its observed precessing common trajectory may extend to $r_n \sim 1.8$ mas, corresponding to a spatial distance of $Z_{c,m} = 62.3$ mas (or $Z_{c,p} = 414.5$ pc) from the core.

Bulk acceleration is required and its Lorentz factor is modeled as: For $Z \leq 1.0$ mas $\Gamma = 4$; for $Z = 1-6$ mas $\Gamma = 4+14.5(Z-1)/(6-1)$; for $Z > 6$ mas $\Gamma = 18.5$.

During the period 1996.5–2001.25 its Lorentz factor $\Gamma$, Doppler factor $\delta$, apparent velocity $\beta_a$ and viewing angle $\theta$ vary over the following ranges: [5.0,18.5], [9.0,(32.5),21.3], [2.7,(17.9),(12.0),18.3] and [3.52,(1.14),2.67](deg).

As shown in Figure 8 (bottom left panel) during the period 1997.5-2001.0 the derived apparent velocity is $\sim 14+2$ c,
Fig. 6. Model fitting of the kinematics of knot C4 in the inner jet (within $r_n \approx 1.8$ mas). The precessing common trajectory is shown by the black dashed-line in upper/left panel (with precession phase $\phi_0 = 4.28 \text{ rad}$, corresponding to ejection time $t_0 = 1979.00$). The dashed-lines in magenta and blue show the precessing common trajectories at precession phases $\phi_0 (\text{rad}) = 4.28 \pm 0.31$ and $4.28 - 0.31$, respectively ($0.31 \text{ rad} = 5\%$ of the precession period). Most data-points are distributed near the precessing common trajectory. Note: The Doppler factor curve $\delta(t)$, derived in the model-simulation, had a bump-structure during the period 1980–1985 with a maximum at $\sim 1983$ and produced the Doppler boosting effect which can explain its radio flux evolution (Qian, in preparation).

which is very well consistent with the observed apparent velocity $13.2 \pm 1.2c$ (Schinzel 2013a). The model-fitting results for knot C9 show favor toward our precessing nozzle scenario:

- The modeled ejection epoch 1995.06 is very close to that (1994.75) derived by Klare 2003 using polynomial extrapolation.

- Its entire trajectory observed during $\sim 1996.5$–2001.5 (in a time range of 5 years) can extraordinarily well be fitted by the precessing nozzle model with its precession phase $\phi_0 = 5.54 + 4\pi$.

- The observational data-points are all concentrated around the precessing common trajectory predicted by the scenario, showing its clear regularity, especially considering its curved structure and quite long extension of $\sim 1.80$ mas from the core (equivalent to a spatial distance 414.5 pc).

- Around the precessing common trajectory almost all the observational data-points locates within the region defined by the magenta and blue lines (Figure 8) which represent the criterion of validity with the model-fitting.
accuracy of ±5% of the precession period for its precession phase (or ±0.36 yr for its ejection time). This result for knot C9 is very helpful for verifying the assumption of precessing common trajectory for blazar 3C345.

More important, its precessing common trajectory is very similar to that of knots C5 and C22/C23 (also C7 and C12, see below), showing some recurrent occurrence of periodic ejection of knots, not only in ejection periodicity, but also in similar curved trajectory structure. Such a kind of phenomenon may not be interpreted in terms of instabilities of jets during their propagation through surrounding medium.

All these results were consistently obtained by using physically reasonable assumptions and methods, demonstrating that the model-fitting of the kinematics of knot C9 has provided very encouraging and confident results for justifying our precessing nozzle scenario for blazar 3C345. However, there still remains a question: whether the Lorentz factor and viewing angle (Γ(t) and θ(t) as functions of time) derived in the model-simulation are correct. This would be a (or the last) determinative test for validating our precessing nozzle scenario. Thus we investigated the radio (at 43GHz and 15GHz) flux evolution of knot C9 and its relation to the Doppler boosting effect. As its Doppler factor

5 Γ(t) represents the acceleration of knot C9 and θ(t) represents the variation in viewing angle due to its helical motion.

6 This is the only way to check the correctness of the derived Lorentz factor and viewing angle curves.
Knot C9: Fitting results are given for trajectory $Z_n(X_n)$, coordinates $X_n(t)$ and $Z_n(t)$ and core separation $r_n(t)$. The trajectory can be well fitted by the model within $X_n \sim 1.7$ mas. The core separation and coordinates ($X_n(t)$, $Z_n(t)$) can be well fitted till $\sim 2001$. The upper/left panel shows the precessing common trajectories for precessing phases $5.54 \text{ rad}$ and $5.54 \pm 0.31 \text{ rad}$ (lines in magenta and blue), indicating most data-points are within the area defined by the two lines within $X_n \sim 1.7$ mas. Precession phase $\phi_0 = 5.54 \text{ rad} + 4\pi$ and $t_0 = 1995.06$ approximately equal to 1987.99 ($t_0$ for knot C6) plus one period (7.3yr). Note that the Doppler factor curve $\delta(t)$, which was produced by the accelerated and helical motion of knot C9, has a smooth bump structure during period 1997–2001. Such a pattern of Doppler factor curve can well interpret the radio flux evolution of knot C9 through Doppler boosting effect. See text.

$\delta(t)$ has been derived from the model-fitting of its kinematics, showing a bump-structure during the period 1997–2001 (Fig.8, bottom/right panel), it would be a vital test to see whether its radio light-curves could be interpreted in terms of Doppler boosting effect. In order to investigate the relation between the radio flux evolution and Doppler boosting we used both 15 GHz and 43 GHz light-curves and took spectral effect into consideration. The light curves observed at 15 GHz and 43 GHz are shown in Figure 9 (left panel). They have similar profiles with a similar peaking time at $\sim 1999.5$ and similar rising and decreasing phases. But the 15 GHz light-curve had enough data-points to determine the shape of its rising phase, while the 43 GHz light-curve had enough data-points to determine the shape of its decaying phase. The spectral index between 15GHz and 43GHz is $\sim 0.80$ ($S \propto \nu^{-\alpha}$). Thus we reformed the rising shape of the 43 GHz light-curve by using the 15 GHz flux during the period 1997.62–1999.99 and reformed the decreasing phase of the 15 GHz light-curve by using the 43GHz flux during the period 1999.64–2000.26. Both reformed light-curves are shown in Figure 9 (right panel), which have quite regular patterns. We have also taken the spectral effect into account.

**Fig. 8.** Knot C9: Fitting results are given for trajectory $Z_n(X_n)$, coordinates $X_n(t)$ and $Z_n(t)$ and core separation $r_n(t)$. The trajectory can be well fitted by the model within $X_n \sim 1.7$ mas. The core separation and coordinates ($X_n(t)$, $Z_n(t)$) can be well fitted till $\sim 2001$. The upper/left panel shows the precessing common trajectories for precessing phases $5.54 \text{ rad}$ and $5.54 \pm 0.31 \text{ rad}$ (lines in magenta and blue), indicating most data-points are within the area defined by the two lines within $X_n \sim 1.7$ mas. Precession phase $\phi_0 = 5.54 \text{ rad} + 4\pi$ and $t_0 = 1995.06$ approximately equal to 1987.99 ($t_0$ for knot C6) plus one period (7.3yr). Note that the Doppler factor curve $\delta(t)$, which was produced by the accelerated and helical motion of knot C9, has a smooth bump structure during period 1997–2001. Such a pattern of Doppler factor curve can well interpret the radio flux evolution of knot C9 through Doppler boosting effect. See text.
to derive the Doppler boosting profile: \(S(t) \propto [\delta(t)]^{3+\alpha}\). In Figure 10 are shown the comparison between the reformed 15GHz and 43GHz light-curves and the Doppler boosting profile\(^8\). It can be seen that both the light-curves are extremely well coincident with the Doppler boosting profile during the period of \(\sim 1997.50 - 2000.25\) and thus be fully interpreted in terms of the Doppler boosting effect derived by our precessing nozzle model. This may imply that the relativistic shock producing the radio emission of knot C9 during this period is extremely stable without distinct variations in its intrinsic radiation. Because the Doppler factor curve \(\delta(t)\) has already been determined by the model-fitting of the kinematics of knot C9 (Figure 8, bottom/right panel), thus the reformed 15GHz and 43GHz light-curves for knot C9 being well coincident with the Doppler boosting profile is very important. This coincidence may imply that we have made a vital test justifying our precessing nozzle scenario, demonstrating that it not only can be used to interpret the kinematics of superluminal knots (including trajectory, coordinates, core separation and apparent velocity), but also the Lorentz-factor and Doppler-factor profiles of superluminal components can be correctly derived, thus providing effective and applicable ways to investigate their radio flux evolution and intrinsic variations. The almost perfect interpretations of the VLBI-kinematics, flux evolution and nature of the superluminal component C9 in 3C345\(^7\) certainly imply that our precessing jet-nozzle scenario has stood all the observational tests for blazar 3C345 and could also be effectively applicable to study other blazars.

Moreover, these results certainly justify the traditional scenario (or common viewpoint) for blazars: superluminal knots move relativistically along helical trajectories toward us with acceleration. Obviously, our results may not be favorable to some scenarios which do not take relativistic motion of superluminal components (as entities) toward us into consideration, e.g., the lit-up underlying-pattern scenario, which suggested that the apparent trajectories of superluminal components could result from the jet internal structure lit-up by plasma condensations ejected during nuclear flares (Schinzel et al. 2012a, 2012b). However, except the main flare during \(\sim 1997.50 - 2000.25\) produced by Doppler boosting effect, there remain some flux variations (or sub-flares) on smaller time-scales to be explained: for example, the variations at 1997.2, 1999.17 and 2000.1 and during period 2000.5-2002.6. These flux variations might be intrinsic due to its passages through local standing re-collimation shocks.

4.4.7. Model-fitting of kinematics for knot C10

According to the precessing jet-nozzle scenario for jet-A, the kinematic behavior of knot C10 could be explained by assuming its precession phase \(\phi_0(\text{rad}) = 5.88 + 4\pi\) and ejection epoch \(t_0 = 1995.46\). The model-fitting results are shown in Figure A.4. Its kinematics within \(r_n < 0.8\) mas can be interpreted in terms of the precessing nozzle model. This corresponds to a spatial distance of \(Z_{c,m} \sim 18.0\) mas, equivalent to \(Z_{c,p} \sim 119.7\) pc from the core.

Knot C10 is found to be accelerated. Its bulk Lorentz factor is modeled as: for \(Z \leq 2\) mas \(\Gamma = 4.5\); for \(Z = 2-15\) mas \(\Gamma = 4.5 + 24.5(2-Z)/(15-2)\); for \(Z > 15\) mas \(\Gamma = 29.0\).

During the period 1997.0–2000.0 its Lorentz factor \(\Gamma\), Doppler factor \(\delta\), apparent velocity \(\beta_a\) and viewing angle \(\theta\) vary over the respective ranges: [4.5,29.0], [8.3,34.6], [2.2,29.0], [28.4] and [3.53,1.62](deg.).

4.4.8. Model fitting of kinematics for knot C11

Model fitting results of the kinematic behavior of knot C11 are shown in Figure A.5. Its precession phase and ejection epoch are assumed to be \(\phi_0(\text{rad}) = 5.88 + 4\pi\) and \(t_0 = 1995.46\). It can be seen from Figure A.5 that its observed precessing common trajectory may extend to \(r_n \sim 0.75\) mas, which corresponds to a spatial distance of \(Z_{c,m} \sim 15.7\) mas, equivalent to \(Z_{c,p} \sim 104.2\) pc from the core.

Knot C11 is found to be accelerated and its bulk Lorentz factor is modeled as: for \(Z \leq 1.0\) mas \(\Gamma = 2.5\); for \(Z = 1-3\) mas \(\Gamma = 2.5 + 13.5(Z-1)/3\); for \(Z > 3.0\) mas \(\Gamma = 16.0\).

During the period 1998.0–2000.5 its Lorentz factor \(\Gamma\), Doppler factor \(\delta\), apparent velocity \(\beta_a\) and viewing angle \(\theta\) vary over the respective ranges: [2.7,15.0], [5.1,22.3], [0.8,(14.7),13.1] and [3.70,2.24](deg.).

4.4.9. Model fitting of kinematics for knot C12

According to the precessing jet-nozzle scenario for jet-A, the kinematic behavior of knot C12 can be model-fitted by assuming its precession phase \(\phi_0(\text{rad}) = 6.30 + 4\pi\) and ejection epoch \(t_0 = 1995.95\). The model-fitting results are shown in Figure A.6.

It can be seen from Figure A.6 that its observed precessing common trajectory may extend to \(r_n \sim 0.50\) mas from the core, corresponding to a spatial distance \(Z_{c,m} \sim 9.67\) mas, equivalent to \(Z_{c,p} \sim 64.3\) pc from the core.

The acceleration of its apparent motion can be modeled as: for \(Z \leq 1.2\) mas \(\Gamma = 2.7\); for \(Z = 1.2-3\) mas \(\Gamma = 2.7 + 10.3(Z-1.2)/3\); for \(Z > 3.0\) mas \(\Gamma = 13.0\).

During the period 1999.0–2001.0 its Lorentz factor \(\Gamma\), Doppler factor \(\delta\), apparent velocity \(\beta_a\) and viewing angle \(\theta\) vary over the following ranges respectively: [3.5,13.0], [6.5,19.9], [1.3,10.9] and [3.48,2.43](deg.).

4.4.10. Model-fitting of kinematics for knot C13

The model-fitting results of the kinematic behavior for knot C13 are shown in Figure A.7. Its observed precessing common trajectory may be assumed to extend to \(r_n \sim 0.70\) mas, corresponding to a spatial distance \(Z_{c,m} \sim 14.3\) mas or \(Z_{c,p} \sim 95.3\) pc from the core.

Its precession phase and ejection epoch are modeled as: \(\phi_0 = 6.50 + 4\pi\) and \(t_0 = 1996.18\). Due to plenty of data-points Figure A.7 indicates that the model fitting of its kinematic behavior is very good: most of data-points are located within the region limited by the lines in magenta and blue which are the modeled trajectories for precession phases \(\phi_0 = 6.50+0.31\) rad and 6.50-0.31 rad. This seems strongly justifying the periodicity (7.30 yr) of knot-ejection for jet-
A. The acceleration of knot C13 is modeled as: for $Z \leq 1.6$ mas $\Gamma = 3.5$; for $Z = 1.6–13$ mas $\Gamma = 3.5 + 17.5(Z-1.6)/(13-1.6)$; for $Z > 13$ mas $\Gamma = 21.0$.

During the period 1999.0–2002.0 its Lorentz factor $\Gamma$, Doppler factor $\delta$, apparent velocity $\beta_a$ and viewing angle $\theta$ vary over the respective ranges: [4.1, 21.0], [7.7, 25.7], [1.8, (20.9), 20.5] and [3.30, 2.19] (deg.).

4.4.11. Model fitting of kinematics for knot C14

The kinematic behavior of knot C14 seems exceptional and instructive. Its precession phase and ejection epoch are modeled as: $\phi_0(\text{rad}) = 3.16 + 6\pi$ and $t_0 = 1999.61$. The model-fitting results are shown in Figure A.8. It can be seen that the modeled initial trajectory has a complex pattern of curvature: firstly curved southward and then northward. Interestingly, the data-points within $X_n \sim 0.3$ mas are well located in the region limited by the lines in magenta and blue defined by precession phases $\phi_0(\text{rad}) = 3.16\pm 0.33$, respectively.

Its observed precessing common trajectory might be assumed to extend to $r_n \sim 0.5$ mas, corresponding to spatial distance $Z_{c,m} \sim 18.8$ mas, equivalent to $Z_{c,p} \sim 125.0$ pc. Its motion has been modeled as accelerated: for $Z \leq 0.3$ mas $\Gamma = 9.0$; for $Z = 0.3–10$ mas $\Gamma = 9+1.8(Z-0.3)/(10-0.3)$; For $Z > 10$ mas $\Gamma = 10.8$.

During the period 2000.0–2003.5 its Lorentz factor $\Gamma$, Doppler factor $\delta$, apparent velocity $\beta_a$ and viewing angle $\theta$ vary over the following ranges respectively: [9.3, 10.8], [18.2, 17.6], [2.2, 8.1] and [0.74, 2.40] (deg.).

4.4.12. Model fitting of kinematics for knot C22

The model-fitting of the kinematics for knots C22 and C23 is very important for justifying the precessing nozzle scenario of jet-A, because the VLBI-observations of knot C22 and C23 have extended the periodic behavior of jet-A to 4 precession periods relative to ejection of knot C4 (from ~1979 to 2009, about 30 years). In the following we present the fitting results of the kinematic behavior for knots C22 and C23.

According to the precessing nozzle scenario the kinematics of C22 may be modeled by using precession phase $\phi_0(\text{rad}) = 5.28 + 8\pi$ and ejection epoch $t_0 = 2009.36$. The model-fitting results are shown in Figure 11. Its observed precessing common trajectory may be assumed to extend to core separation $r_n \sim 0.4$ mas, corresponding to spatial distance $Z_{c,m} \sim 9.67$ mas, equivalent to $Z_{c,p} \sim 64.3$ pc from the core.
The model-fitting of its trajectory is very good, because most of the data-points within $X_n \sim 0.4$ mas are closely concentrated around the precessing common trajectory ($\phi_0 = 5.28 \text{ rad}$) predicted by the precessing nozzle scenario. The motion of knot C22 was accelerated and its Lorentz factor was modeled as: for $Z \leq 1.0$ mas $\Gamma = 7.5$; for $Z = 1-4$ mas $\Gamma = 7.5 + 23.5 (Z-1)/(4-1)$; for $Z > 4$ mas $\Gamma = 31.0$.

During the period 2009.5–2010.2 its Lorentz factor $\Gamma$, Doppler factor $\delta$, apparent velocity $\beta_a$ and viewing angle $\theta$ vary over the following ranges respectively: $[7.5,31.0]$, $[14.0,37.4]$, $[3.6,30.3]$ and $[2.00,1.95]$(deg.).

4.4.13. Model fitting of kinematics for knot C23

The kinematic behavior of knot C23 has also been model-fitted in terms of our precessing nozzle model for jet-A. The model-fitting results are shown in Figure 12. Although the data-points only showed its initial trajectory within $X_n \sim 0.15$ mas, they are all closely concentrated around the modeled precessing common trajectory, strongly justifying our precessing nozzle scenario for jet-A with a precession period of 7.30 yr.

Its precession phase $\phi_0(\text{rad}) = 5.20 + 8\pi$ and ejection epoch $t_0 = 2009.27$. This ejection time was a bit earlier than that of knot C22 by $\sim 0.09$ yr (about one month), but it is a more accurate fit to its observed trajectory.

It can be seen from Figure 12 that its observed precessing common trajectory may be assumed to extend to core separation $r_n \sim 0.20$ mas, corresponding to spatial distance $Z_{c,m} \sim 3.4$ mas or $Z_{c,p} \sim 22.6$ pc from the core. The motion of knot C23 was also modeled as accelerated. Its bulk Lorentz factor was modeled as: for $Z \leq 0.5$ mas $\Gamma = 4.3$; for $Z = 0.5-2$ mas $\Gamma = 4.3 + 12.7(1-0.5)/(2-0.5)$; for $Z > 2$ mas $\Gamma = 17.0$.

During the period 2009.8–2010.2 its Lorentz factor $\Gamma$,
We would like to point out the characteristic features of kinematic behavior of knots C22 and C23 as follows.

- Their ejection times (2009.36 and 2009.27) are close to those determined by using linear extrapolation methods in Schinzel (2011a): 2009.544±0.018 and 2009.651±0.042.
- The trajectories observed for knots C22 and C23 are extraordinarily well fitted by the predicted precessing common trajectories, strongly favoring our precessing nozzle scenario.
- Most importantly, their curved trajectories are very similar to those observed for knots C5 and C9, not only implying the existence of a precession period of 7.30 yr, but also exhibiting the recurrence of similar curved trajectory structures.

All these observational facts are very helpful to justify our precessing nozzle scenario for jet-A.

5. A brief summary for jet-A

The model-fitting results of the kinematic behavior of the thirteen knots (C4–C14, C22 and C23) of jet-A are summarized in Table 1. The main features are:

- The kinematics of the 13 knots can be consistently explained in terms of our precessing nozzle scenario for jet-A.
- The precession period of the jet-nozzle may be 7.30 yr (or 4.60 yr in the rest frame), which has been observed during a time-interval of ~30 years (or in four periods).
We have taken the model parameters as follows. 

\[ \psi = 0.209 \text{ rad} = 12^\circ \] 
\[ r_{n,c} \approx 1 \text{ mas} \] 
\[ Z_{c,m} \approx 5 \text{ mas} \] 
\[ Z_{c,p} \approx 15 \text{ mas} \] 
\[ Z_0 \approx 300 \text{ mas} \] 
\[ Z_\delta \approx 3581 \text{ mas} \] 
\[ z_n = 66 \text{ mas} \] 
\[ z_\phi = 6 \times 10^4 \text{ mas} \] 
\[ x_0 \] and \( p(i) \) are similarly defined by equations (1) and (2), but taking parameter \( \zeta = 1.0 \).

The amplitude and phase of the helical trajectory are defined as:

\[ A(Z) = A_0 \sin[\pi z_0/Z_1] \] (22)
\[ \phi(Z) = \text{const.} = \phi_0 = 5.70 + \frac{2\pi}{T_0}(t_0 - 2002.12) \] (23)

\[ A_0 = 1.09 \text{ mas} \] and \( \phi_0 \) also represents the precession phase of individual knots. \( \phi \) is assumed to be independent of \( Z \) and the motion of the knots is approximately ballistic. \( T_0 \approx 7.30 \text{ yr} \). The modeled distribution of trajectory of superluminal components produced by the jet-nozzle precession of jet-B is shown in Figure 13. We found that most of the superluminal knots followed the precessing common trajectories predicted by the precessing nozzle scenario for jet-B.

We would like to point out that the distributions of the precessing common trajectory for jet-B (Figure 13) and jet-A (Figure 5) are similar: their jet-axes are at respective position angles \( \sim 102.0^\circ \) and \( \sim 97.2^\circ \), and their cone apertures are \( \sim 36.8^\circ \) and \( \sim 42.5^\circ \), respectively. However, this does not imply that jet-A and jet-B overlap in space. Using the values given for the parameters defining the helical trajectories for jet-A in Section 4 and jet-B (this Section), we can calculate the angle between their jet-axes and their jet-cones in space and obtain the following results (at core separations \( r_{n,c} \approx 1 \text{ mas} \), or corresponding spatial distances \( Z \approx 30 \text{ mas} \)):

1. The angle between their jet-axes is \( \sim 6.8^\circ \) in space; (2) their jet-cone-apertures in space are \( \sim 1.12^\circ \) (jet-A) and \( \sim 0.98^\circ \), respectively. Thus the two jets (jet-A and jet-B) are widely separated in space. The observed overlap of the projections of jet-A and jet-B in the sky-plane should not be misled into thinking that they are overlapped in space.

As for the case of jet-A, the values selected for the model parameters of jet-B and the associated functions are not statistical samples and not unique. They are specific and physically applicable set of working ingredients, which have been obtained through trial and error over the past few years.

\[ \psi \] Their difference \( 4.8^\circ \) is similar to the difference in their parameters \( \psi \).

6. Model fitting results for jet-B

Our double jet scenario assumes that fourteen superluminal knots (C15–C21, B5-B8, B11 and B12) were ejected from jet-B. This is because we find that the ejection epochs of these knots can not be explained in terms of the precessing nozzle model for jet-A. Moreover, most of the trajectories of the knots seem ballistic by visual inspection, obviously different from the curved trajectories of superluminal knots of group-A. Thus we had to take a different set of parameters to describe the properties of jet-B.

We have taken the model parameters as follows.

| Knot | \( t_0 \) | \( \phi_0 \) | \( r_{n,c} \) | \( Z_{c,m} \) | \( Z_{c,p} \) | \( \Gamma \) | \( \delta \) | \( \beta_\delta \) | \( \theta \) | status |
|------|---------|---------|-----------|-----------|-----------|-------|-------|----------|-------|--------|
| C1   | 1979.90 | 4.28    | 1.80      | 50.9      | 44.8      | 8.3-12.4 | 16.1-17.1-18.8 | 3.0-11.1 | 1.28-2.88 | +    |
| C5   | 1980.80 | 5.83    | 1.20      | 39.9      | 259.3     | 5.5-15.0 | 10.1-24.8-23.6 | 2.9-(10.0)-(8.3)-12.2 | 3.03-1.98 | +    |
| C6   | 1987.99 | 5.74+2\( \pi \) | 0.40     | 7.5       | 49.7      | 9.7-11.1 | 13.5-18.7 | 8.9-8.0 | 3.90-2.21 | +    |
| C7   | 1988.46 | 6.14+2\( \pi \) | 0.70     | 14.9      | 99.3      | 3.2-13.0 | 6.2-21.8 | 0.65-(11.9)-9.5 | 2.00-1.92 | +    |
| C8   | 1991.10 | 2.13+4\( \pi \) | 0.15     | 6.2       | 41.2      | 6.2-13.8 | 12.0-19.0 | 1.9-(12.9)-12.8 | 1.48-(2.87)-2.80 | +    |
| C9   | 1995.06 | 5.54+4\( \pi \) | 1.80     | 62.3      | 414.5     | 5.0-18.5 | 2.7-(17.9)-(12.0)-18.3 | 3.52-(1.14)-2.67 | +    |
| C10  | 1995.76 | 6.14+4\( \pi \) | 1.80     | 18.0      | 119.7     | 4.5-29   | 8.3-34.6 | 2.2-(29.0)-28.4 | 3.53-1.62 | +    |
| C11  | 1995.46 | 5.88+4\( \pi \) | 0.75     | 15.7      | 104.2     | 2.7-15.0 | 5.1-22.3 | 0.82-(14.7)-13.1 | 3.70-2.24 | +    |
| C12  | 1995.95 | 6.30+4\( \pi \) | 0.50     | 9.7       | 64.3      | 3.5-13.0 | 6.5-19.9 | 1.3-10.9 | 3.48-2.43 | +    |
| C13  | 1996.18 | 6.50+4\( \pi \) | 0.70     | 14.3      | 95.3      | 4.1-21.0 | 7.7-25.7 | 1.8-(20.9)-20.5 | 3.30-2.19 | +    |
| C14  | 1999.61 | 3.16+6\( \pi \) | 0.50     | 18.8      | 125.0     | 9.3-10.8 | 18.2-17.6 | 2.2-8.1 | 0.74-2.40 | +    |
| C22  | 2009.36 | 5.28+8\( \pi \) | 0.40     | 9.7       | 64.3      | 7.5-31.0 | 14.0-37.4 | 3.6-30.3 | 2.00-1.95 | +    |
| C23  | 2009.27 | 5.20+8\( \pi \) | 0.20     | 3.4       | 22.6      | 4.3-17.0 | 8.3-20.9 | 1.2-16.5 | 2.00-2.66 | +    |
years. Our aim was to search for evidence of jet precession in 3C345 and we found that they could be applied to analyze the distribution of the observed trajectories and kinematics of superluminal components in 3C345 on VLBI-scales and disentangled its possibly existing double jets. In this paper we used the methods of multi-parameter model simulations, being not able to give statistical errors to the model parameters. But we provided a new criterion to judge the validity of the entire model-fitting by showing that the observed trajectories of most knots followed their precessing common trajectories predicted in terms of the scenario within ±5% of the precession period.

6.1. Model fitting results for knot C15

According to the precessing nozzle scenario for jet-B the precession phase and ejection epoch are assumed as: \( \phi_0 = 5.7 \) rad and \( t_0 = 2002.12 \).

The model fitting results of its kinematic behavior are shown in Figure 14. Obviously, its kinematic behavior (including trajectory, core separation and coordinates vs time) can be very well explained in terms of the precessing nozzle scenario for jet-B. The apparent velocity and bulk Lorentz factor of its motion as functions of time have been derived. Its accelerated motion could be modeled as: for \( Z \leq 4 \) mas \( \Gamma = 1.09 \) mas; for \( Z = 4-10 \) mas \( A_0 = 1.09-0.87(Z-4)/(10-4) \); for \( Z > 10 \) mas \( A_0 = 0.218 \) mas. And \( \psi \) was assumed as: for \( Z \leq 5 \) mas \( \psi = 0.209 \) rad (same as for the axis of jet-B); for \( Z = 5-10 \) mas \( \psi = 0.209+0.146(Z-5)/(10-5) \); for \( Z > 10 \) mas \( \psi = 0.355 \) rad.

Its accelerated motion could be modeled as: for \( Z \leq 5 \) mas \( \Gamma = 8.5 \); for \( Z = 5-10 \) mas \( \Gamma = 8.5+1.5(Z-5)/(10-5) \); for \( Z > 10 \) mas \( \Gamma = 10.0 \)\(^{10}\).

During the period 2003-2005.5 its Lorentz factor \( \Gamma \), Doppler factor \( \delta \), apparent velocity \( \beta_a \) and viewing angle \( \theta \) vary over the respective ranges as: [8.5-10.0], [15.8-18.6], [4.2-5.0] and [1.80-1.55](deg).

6.3. Model-fitting results for knot C16

The model-fitting results for knot C16 are shown in Figure A.10. Its precession phase and ejection epoch are assumed to be \( \phi_0 = 5.80 \) rad and \( t_0 = 2002.24 \).

It can be seen that its kinematic behavior has been well modeled. Its observed precessing common trajectory may extend to core separation \( r_n \sim 0.8 \) mas, corresponding to spatial distance \( Z_{c,m} \sim 24.4 \) mas or \( Z_{c,p} \sim 37.2 \) pc from the core.

Its accelerated motion is modeled by its bulk Lorentz factor \( \Gamma \), Doppler factor \( \delta \), apparent velocity \( \beta_a \) and viewing angle \( \theta \) vary over the respective ranges as: [8.0-24.0], [14.9-29.4], [3.9-23.4] and [1.91-1.90].

6.4. Model fitting results for knot C17

According to the precessing nozzle scenario the kinematic behavior of knot C17 could be modeled by assuming

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\(^{10}\) Due to lack of data-points within core separation \( r_n \sim 0.15 \) mas, the model fitting of its observed precessing common trajectory is marginal.
Fig. 14. Model fitting of kinematics for knot C15: precession phase $\phi_0=5.70 \text{ rad}$, $t_0=2002.12$.

its precession phase $\phi_0(\text{rad})=1.57+2\pi$ and ejection epoch $t_0=2004.62$. The model fitting results are shown in Figure A.11. It can be seen that its kinematic behavior has been well fitted and its observed precessing common trajectory may extend to core separation $r_n \sim 0.8 \text{ mas}$, equivalent to spatial distance $Z_{c,m} \sim 44.8 \text{ mas}$ or $Z_{c,p} \sim 297.9 \text{ pc}$ from the core.

Its accelerated motion can be modeled by the increase in its bulk Lorentz factor as follows: for $Z \leq 2 \text{ mas}$ $\Gamma=6.8$; for $Z=2-20 \text{ mas}$ $\Gamma=6.8+13.2(Z-2)/(20-2)$; for $Z>20 \text{ mas}$ $\Gamma=20.0$.

During the observed period 2006.5–2009.0 its Lorentz factor $\Gamma$, Doppler factor $\delta$, apparent velocity $\beta_a$ and viewing angle $\theta$ vary over the respective ranges: [9.9,20.1], [19.1,35.1], [3.5,13.1] and [1.01,1.07](deg).

6.5. Model fitting results for knot C18

The kinematics of knot C18 can be explained in terms of the precessing nozzle scenario for jet-B by assuming its precession phase $\phi_0(\text{rad})=1.62+2\pi$ and ejection epoch $t_0=2004.68$. The model-fitting results of its kinematic behavior are shown in Figure A.12. It can be seen that the data-points are well concentrated around the predicted precessing common trajectory.

Its observed precessing common trajectory may extend to core separation $r_n \sim 0.6 \text{ mas}$, corresponding to spatial distance $Z_{c,m} \sim 33.8 \text{ mas}$ or $Z_{c,p} \sim 224.8 \text{ pc}$ from the core.

The accelerated motion of knot C18 can be modeled by assuming its bulk Lorentz factor as follows: for $Z \leq 2 \text{ mas}$ $\Gamma=6.5$; for $Z=2-20 \text{ mas}$ $\Gamma=6.5+9.5(Z-2)/(20-2)$; for $Z>20 \text{ mas}$ $\Gamma=16.0$.

During the period 2007–2009.5 its Lorentz factor $\Gamma$, Doppler factor $\delta$, apparent velocity $\beta_a$ and viewing angle $\theta$ vary over
the respective ranges: [9.1,16.0], [17.8,29.5], [2.8,8.5] and [1.01,1.03](deg).

6.6. Model fitting results for knot C19

According to the precessing nozzle scenario for jet-B, the kinematics of knot C19 can be explained by assuming its precession phase $\phi_0(\text{rad})=1.77+2\pi$ and ejection epoch $t_0=2004.85$.

The model-fitting results of its kinematic behavior are shown in Figure 15. Due to plenty of data-points the fitting results are satisfying and all the data-points are located within the region defined by precession phases $\phi_0=1.77\pm0.31$ rad (corresponding $t_0=2004.85\pm0.36$ yr).

Its observed precessing common trajectory may extend to core separation $r_n=0.6$ mas, equivalent to spatial distance $Z_{c,m}\sim33.4$ mas or $Z_{c,p}\sim222.1$ pc from the core. Its accelerated motion can be explained by assuming its Lorentz factor as follows: for $Z\leq 3.2$ its Lorentz factor $\Gamma=5.2+15.8(Z-2)/(20-2)$; for $Z>20$ $\Gamma=21.0$.

During period 2007.5–2010.0 its Lorentz factor $\Gamma$, Doppler factor $\delta$, apparent velocity $\beta_a$ and viewing angle $\theta$ vary over the respective ranges: $[8.0,21.0]$, $[15.6,36.5]$, $[2.2,14.1]$ and $[1.02,1.05]$(deg).

We emphasize that the difference between the ejection epochs for knot C19 and knot B6 (see below) is $\sim 7.53$ yr, very close to one precession period.

6.7. Model fitting results for knot C20

The model-fitting results of the kinematic behavior of knot C20 are shown in Figure 16. It can be seen that its kinematics can be very well explained by assuming its precession phase $\phi_0(\text{rad})=4.20+2\pi$ and ejection epoch $t_0=2007.68$. It observed precessing common trajectory may extend to core separation $r_n=0.45$ mas, corresponding to spatial distance $Z_{c,m}\sim13.2$ mas or $Z_{c,p}\sim87.8$ pc from the core. Its accelerated motion can be modeled by assuming its bulk Lorentz factor as follows: for $Z\leq 3.5$ mas $\Gamma=7.4$; for $Z=3.5–11$ mas $\Gamma=7.4+13.6(Z-3.5)/(11-3.5)$; for $Z=11$ mas $\Gamma=21.0$.

During period 2008.4–2010.2 its Lorentz factor $\Gamma$, Doppler factor $\delta$, apparent velocity $\beta_a$ and viewing angle $\theta$ vary over the respective ranges: $[7.4,21.0]$, $[13.9,27.9]$, $[3.5,19.8]$ and $[1.95,1.94]$(deg).

Here we would like to point out that the ejection epoch has been determined by linear and polynomial extrapolation methods to be 2007.83 and 2006.91 respectively (Schinzel 2012a). Thus the ejection epoch 2007.68 derived for knot C20 by our precessing nozzle scenario is closely equal to the averaged epoch obtained by extrapolation methods.

6.8. Model fitting results for knot C21

The model-fitting results for knot C21 are shown in Figure 17. It can be seen that its kinematic behavior could be very well fitted by assuming its precession phase $\phi_0(\text{rad})=3.90+2\pi$ and ejection epoch $t_0=2007.32$.

Its observed precessing common trajectory may extend to core separation $r_n=0.45$ mas, corresponding to spatial distance $Z_{c,m}\sim14.6$ mas or $Z_{c,p}\sim97.1$ pc.

Its accelerated motion could be modeled by the increase in its Lorentz factor: for $Z\leq 3$ mas $\Gamma=5.4$; for $Z=3$–8 mas $\Gamma=5.4+19.6(Z-3)/(8-3)$; for $Z>8$ mas $\Gamma=25.0$.

During the period 2009.0–2014.0 its Lorentz factor $\Gamma$, Doppler factor $\delta$, apparent velocity $\beta_a$ and viewing angle $\theta$ vary over the following ranges: $[5.4,25.0]$, $[10.4,31.4]$, $[1.7,24.1]$ and $[1.77,1.76]$(deg).

6.9. Model fitting results for knot B5

In the following we discuss the model-fitting of VLBI-kinematics for superluminal components B5–B8, B11 and B12. Data on these knots are kindly provided by S.G. Jorstad (private communication).

Knot B5 is an interesting case, because its trajectory was observed nearly at the upper edge of the projected jet-cone. Obviously, it has been curved away northward at $X_{n}^{B}<0.2$ mas. Although its initial trajectory within $X_{n}<0.2$ mas was not observed, it still could be explained in terms of the precessing nozzle model as shown in Figure A.13 by assuming its precession phase $\phi_0(\text{rad})=0.33+4\pi$ and ejection epoch $t_0=2010.48$. However, in this case introduction of change in amplitude parameter is required to fit its outer trajectory: for $Z\leq 0$ mas $A_0=1.09$ mas; for $Z=6$–15 mas $A_0=1.09+0.82(Z-6)/(15-6)$; for $Z=15$ mas $A_0=1.91$. Bulk acceleration is also needed: for $Z\leq 15$ mas $\Gamma=12.0$; for $Z=15$–20 mas $\Gamma=12+1.5(Z-15)/(20-15)$; for $Z>20$ mas $\Gamma=26.5$.

Its precessing common trajectory might be assumed to extend to core separation $r_n\leq 0.15$ mas, corresponding to a spatial distance $Z_{c,m}\sim5.40$ mas or $Z_{c,p}\sim35.9$ pc.

During the period 2011.0–2013.2 its Lorentz factor $\Gamma$, Doppler factor $\delta$, apparent velocity $\beta_a$ and viewing angle $\theta$ vary over the respective ranges: $[12.0,26.5]$, $[21.6,32.7]$, $[7.1,25.7]$ and $[1.57,1.70]$(deg).

6.10. Model fitting results for knot B6

The model fitting of the kinematics of knot B6 in terms of the precessing nozzle scenario for jet-B is a key ingredient for justifying the double-jet scenario for 3C345, because the observational data extends the monitoring time-interval to $\sim 1.4$ times the precession period. Its kinematic behavior can be very well fitted as shown in Figure 18: most of the data-points almost exactly locate on the predicted precessing common trajectory within $X_{n}\sim1.2$ mas, strongly justifying our precessing nozzle scenario for jet-B.

Its precession phase is modeled as $\phi_0(\text{rad})=1.97+4\pi$, corresponding to ejection epoch $t_0=2012.38$. In fact the model fitting of the kinematics for knot B6 may be regarded as an posterior test of our double-jet scenario, because the double precessing jet scenario had been constructed before the observational data on knot B6 was collected.

Its observed precessing common trajectory may extend to core separation $r_n\sim 1.2$ mas, corresponding to a spatial distance $Z_{c,m}\sim 67.7$ mas or $Z_{c,p}\sim 420.3$ pc from the core, the second longest extension of precessing common trajectory in group-B (Table 2).

Its motion is modeled as uniform: Lorentz factor is assumed to be $\Gamma=26.0$. During the period 2012.70–2014.25 its Doppler factor $\delta$, apparent velocity $\beta_a$ and viewing angle $\theta$ vary over the following ranges respectively: $[42.2,41.3]$, $[20.3,21.0]$ and $[1.06,1.13]$(deg). The successful model-fitting of the kinematic behavior of knot B6 is very
6.11. Model fitting results for knot B7

We would like to note that the following model fitting results for knots B7, B8, B11 and B12 were obtained after our finishing the works for the thirteen knots (C4 to C23) of jet-A and the ten knots (C5 to B6) of jet-B, thus their model-fittings played a role of follow-up or afterward verification of the double-jet scenario. The successful model fitting of kinematics for the four knots proved our scenario being valid and effective, and extend the model-fitting of the jet-B to two precession periods.

In our precessing nozzle scenario, the observed trajectories of knots B7 and B8 were distributed earlier than that of knot B6, implying their ejection times earlier than that of knot B6 \((t_0=2012.38, \phi_0=1.97 \text{ rad}+4\pi)\). We assumed \(t_0=2011.60\) for knot B7 (equivalent to the precession phase \(\phi_0=1.30 \text{ rad}+4\pi\)) and \(t_0=2011.95\) for knot B8 (equivalent to \(\phi_0=1.60 \text{ rad}+4\pi\)).

The model fitting results for knot B7 are shown in Figure A.14. It can be seen that its kinematics can be explained well in terms of our precessing nozzle model. No acceleration in its motion was observed and its bulk Lorentz factor was assumed to be \(\Gamma=10.5=\text{const.}\) During the period of 2013.0–2017.0, its Doppler factor \(\delta\), apparent velocity \(\beta_a\), and viewing angle \(\theta\) vary over the following respective ranges: [20.2-20.2], [3.8-3.9], and [1.04-1.06](deg), almost staying constant. Its precessing common trajectory may be regarded as extending to the core separation of \(r_{n,c}=0.80\text{ mas}\), equivalent to a spatial distance \(Z_{c,m}=43.7\text{ mas}\) (or \(Z_{c,p}=290.8\text{ pc}\)).
6.12. Model fitting results for knot B8

The model fitting results for knot B8 are shown in Figure A.15. Obviously, its kinematics can be interpreted well in terms of our precessing nozzle scenario. Acceleration in its motion was observed and its bulk Lorentz factor was assumed to be as follows: for $Z \leq 20$ mas, $\Gamma = 17$; for $Z = 20-30$ mas, $\Gamma = 17 + (Z-20)(23-17)/(30-20)$; and for $Z > 30$ mas, $\Gamma = 23.0$. During the period of 2013.0–2015.5, its bulk Lorentz factor $\Gamma$, Doppler factor $\delta$, apparent velocity $\beta_a$, and viewing angle $\theta$ vary over the following respective ranges: $[17.0-23.0]$, $[31.2-39.3-38.3]$, $[9.4-17.1]$, and $[1.02-1.11]$. Its precessing common trajectory may extend to core separation of $r_{n,c} = 1.20$ mas, equivalent to a spatial distance $Z_{c,p} = 439.8$ pc (or $Z_{c,m} = 66.1$ mas).

6.13. Model fitting results for knot B11

Knot B11 is an uncertain case. In our scenario, the trajectory observed for knot B11 was distributed later than that of knot B12, but its core separation versus time showed that its ejection time should be earlier than B12. Thus for explaining the kinematics of B11 some changes in its kinematics were needed to take into consideration. We assumed its ejection time $t_0 = 2015.67$ ($\phi_0 = 4.80 + 4\pi$). Its bulk Lorentz factor was assumed to follow as: for $Z \leq 10$ mas, $\Gamma = 14.0$; for $Z = 10-15$ mas, $\Gamma = 14 + (Z-10)(23-14)/(15-10)$; and for $Z > 15$ mas, $\Gamma = 23.0$. The changes in its trajectory was described by the change in parameter $\psi$ as follows: for $Z \leq 4$ mas, $\psi = 0.209$ (rad); for $Z = 4-8$ mas, $\psi (\text{rad}) = 0.209 - (Z-4)(0.05-0.209)/(8-4)$; and for $Z > 8$ mas, $\psi = 0.05$ rad. Its precessing common trajectory might be assumed to extend to $r_{n,c} = 0.12$ mas, equivalent to a spatial distance $Z_{c,m} = 3.40$ mas (or $Z_{c,p} = 22.6$ pc). The model fitting results
are shown in Figure A.16. It can be seen that the rotation of its trajectory in the range of Z=4 mas to 8 mas made the kinematics of B11 being well explained within the scenario for jet-B. Unfortunately, no observational data available to confirm this rotation. (in comparison with the case of knot C8).

During the period of 2015.5–2018.0 its bulk Lorentz factor $\Gamma$, Doppler factor $\delta$, apparent velocity $\beta_a$, and viewing angle $\theta$ vary over the following respective ranges: [14.0-23.0], [22.6-28.4], [11.0-22.3], and [1.99-1.96](deg).

6.14. Model fitting results for knot B12

Knot B12 is a significant knot for the model fitting of jet-B because it regularly followed the precessing common path as the scenario predicted, extending the model fitting time-interval for jet-B to approximately two times the precession period. The model fitting results are shown in Figure 19. Its ejection time was assumed to be $t_0=2016.08$ (or $\phi_0=5.15$ rad+4$\pi$). The observational data-points are closely distributed around the predicted trajectory within the regions defined by lines in magenta and blue.

Its motion was nearly ballistic and no acceleration needs to be modeled. Its bulk Lorentz factor is modeled as $\Gamma=16.5=\text{const.}$ During the period of 2016.5–2019.0, its Doppler factor $\delta$, apparent velocity $\beta_a$, and viewing angle $\theta$ vary over the following respective ranges: [25.0-25.2], [14.1-14.0], and [1.96-1.93](deg). They almost stay constant. Its precessing common trajectory may be regarded as extending to core separation $r_n \sim 1.20$ mas, corresponding to a spatial distance $Z_{c,m}=35.2$ mas (or $Z_{c,p}=234.1$ pc).

In order to show that knot B12 should be ascribed to jet-B (not to jet-A), its observed trajectory has also been modeled in the framework of the scenario for jet-A (taking $t_0=2016.08$ and corresponding precession phase
Fig. 18. Model fitting of kinematics for knot B6: precession phase $\phi_0 = 1.97\,\text{rad} + 4\pi$, $t_0 = 2012.38$. Most of the data-points almost exactly locate on the predicted precessing common trajectory, justifying our precessing nozzle scenario for jet-B.

$\phi_0 = 4.78 + 10\pi$, which is shown in Figure 20 (left panel). It can be seen that the trajectory of B12 is very different from that predicted by the jet-A modeling scenario, but is well fitted by the jet-B modeling scenario, favoring it associated with jet-B. In Figure 20 (right panel) we present the modeled relation between position angle and viewing angle at core separation 0.3 mas for jet-A and jet-B, which also demonstrates jet-A having a different relation from that of jet-B, favoring a double jet structure.

We would like to note that in this paper the superluminal knots observed in 3C345 were separated into two groups (group-A and group-B) according to the characteristic features of their kinematic behaviors: (1) Knots of group-A moved along curved trajectories while knots of group-B moved approximately along straight trajectories; (2) Knots of group-A and group-B construct different distributions of precessing common trajectories (i.e., different forms of the function $\phi_0(t_0)$) although they had the same precession period of 7.3 yr; (3) The axis of jet-A and jet-B have different directions in space: their spatial viewing angles differ by $\sim 6.8^\circ$, although their projections in the sky-plane have closely similar position angles; (4) Jet-A and jet-B have different degrees of activity during their respective time-intervals.

6.15. A brief summary for jet-B

We briefly summarize the model-fitting results for jet-B.

- The kinematics of the fourteen superluminal knots (C15–C21, B5–B8, B11 and B12) can be consistently and well modeled in the precessing nozzle scenario for jet-B.
Fig. 19. Model fitting results for knot B12. Precession phase $\phi_0$ (rad) = 5.15 + 4$\pi$ and ejection time $t_0$ = 2016.08.

- Its precession period is just the same with that for jet-A (7.30 yr) and the sense of precession is also similar to that for jet-A: counterclockwise seen along the line of sight.
- As shown in Table 2 the time-interval for our model-fitting is $\sim$14 yr, corresponding to $\sim$1.5 times the precession period.
- Combined with the model-fitting results for the 13 superluminal knots of jet-A, we would be able to tentatively conclude that QSO 3C345 could contain a double jet system formed by a putative binary supermassive black hole.
- The jet cone described by the precessing nozzle of jet-B was distinctly different from that of jet-A and this could be tentatively regarded as a significant clue to the double-jet structure in 3C345.
- As argued above, the knots of jet-B approximately moved along straight-lines (ballistically), while the knots of jet-A along curved trajectories. This might imply that the knots of jet-B were observed at larger core distances. In fact, there were seven knots of jet-B were observed till spatial core distances greater than 200 pc (see parameter $Z_{c,p}$ in Table 2), while only three knots for jet-A (see Table 1). This might imply that the knots of jet-B could move along helical trajectories with much smaller pitch angles.

7. Discussion

We have investigated and explained the kinematic behavior on pc-scales of twenty-seven superluminal components in QSO 3C345. It was tentatively found that they could be divided into two groups (group-A and group-B; see Table 1 and Table 2, respectively) and the superluminal components could be ejected from a double-jet system consisting of jet-A and jet-B, which produce the superluminal compo-
Table 2. Model parameters for the fourteen superluminal knots of jet-B: ejection epoch $t_0$, precession phase $\phi_0$, extension of precessing common trajectory $r_{n,c}$ from the core, corresponding spatial distance $Z_{c,m}$ (mas) and $Z_{c,p}$ (pc), bulk Lorentz factor $\Gamma$, Doppler factor $\delta$, apparent velocity $\beta_{\nu}$, viewing angle (deg) and status. The superluminal components were ejected during a time-interval of about two precession periods. Column 'status' denotes the quality of trajectory model-fits: + for well-fitted cases and – for not fitted cases.

| Knot | $t_0$ | $\phi_0$ | $r_{n,c}$ | $Z_{c,m}$ | $Z_{c,p}$ | $\Gamma'$ | $\delta$ | $\beta_{\nu}$ | $\theta$ | status |
|------|------|--------|---------|---------|---------|--------|--------|----------|-------|--------|
| C15  | 2002.12 | 5.70  | 0.70    | 21.0    | 139.7   | 8-24   | 14.9-29.4 | 3.9-23.4 | 1.91-1.90 | +      |
| C15a | 2002.18 | 5.75  | 0.17    | 5.60    | 37.2    | 8.5-10 | 15.8-18.6 | 4.2-5.0  | 1.80-1.55 | +      |
| C16  | 2002.24 | 5.80  | 0.80    | 24.4    | 162.2   | 6.0-18.0 | 11.5-26.7 | 2.2-15.7 | 1.88-1.87 | +      |
| C17  | 2004.62 | 1.57+2\pi | 0.80 | 44.8    | 297.9   | 9.9-20.1 | 19.1-35.1 | 3.5-13.1 | 1.01-1.07 | +      |
| C18  | 2004.68 | 1.62+2\pi | 0.60 | 33.8    | 224.8   | 9.1-16.0 | 17.8-29.5 | 2.8-8.5  | 1.01-1.03 | +      |
| C19  | 2004.85 | 1.77+2\pi | 0.60 | 33.4    | 222.1   | 8.0-21.0 | 15.6-36.5 | 2.2-14.1 | 1.02-1.05 | +      |
| C20  | 2007.68 | 4.20+2\pi | 0.45 | 3.2    | 87.8    | 7.4-21.0 | 13.9-27.9 | 3.5-19.8 | 1.95-1.94 | +      |
| C21  | 2007.32 | 3.90+2\pi | 0.45 | 14.6    | 97.1    | 5.4-25.0 | 10.4-31.4 | 1.7-24.1 | 1.77-1.76 | +      |
| B5   | 2010.48 | 0.33+4\pi | 0.15  | 5.40    | 35.9    | 12.0-26.5 | 21.6-32.7 | 7.1-25.7 | 1.57-1.70 | –      |
| B6   | 2012.38 | 1.97+4\pi | 1.20  | 67.7    | 420.3   | 26.0=const. | 42.2-41.3 | 20.3-21.0 | 1.06-1.13 | +      |
| B7   | 2011.60 | 1.30+4\pi | 0.80  | 43.7    | 290.8   | 10.5=const. | 20.2=const. | 3.8-3.9  | 1.04-1.06 | +      |
| B8   | 2011.95 | 1.60+4\pi | 1.20  | 66.1    | 439.8   | 17.0-23.0 | 31.2-(39.3)-38.3 | 9.4-17.1 | 1.02-1.11 | +      |
| B11  | 2015.67 | 4.80+4\pi | 0.12  | 3.40    | 22.6    | 14.0-23.0 | 22.6-28.4 | 11.0-22.3 | 1.99-1.96 | –      |
| B12  | 2016.08 | 5.15+4\pi | 1.20  | 35.2    | 234.1   | 16.5=const. | 25.0-25.2 | 14.1-14.0 | 1.96-1.93 | +      |

Fig. 20. Left panel: Model fitting of the observed trajectory of knot B12 in the framework of the scenario for jet-A, showing it unlikely being ascribed to jet-A (dashed line represents the predicted trajectory for B12 in the jet-A framework). Right panel: The modeled jet-cone patterns (relation between position angle and viewing angle for jet-A (in black) and jet-B (in magenta)) at core separation 0.3 mas. Symbol + represents the position of the jet-axis at core separation 0.3 mas.
The long-extended acceleration/collimation of relativistic jets might also be helpful to explain why jets in giant radio galaxies could extend to Mpc-scales (e.g., in giant radio galaxy DA240, see Tsien (1982a), Tsien & Saunders (1982a), Willis et al. (1974)). As Qian et al. (2017) suggested that except precession of jet-nozzles, other mechanisms may also cause jet precession on different time scales, e.g., geodetic precession, Newtonian-driven precession, Lense-Thirring effect (Begelman et al. 1980; Katz 1997; Lense & Thirring 1918). Especially, geodetic precession may occur on timescales of \( \sim 10^{−5} \) years. This kind of jet precession has been observed in the inverse symmetric distribution of lobes and hotspots in FRII radio galaxies (e.g., in Cygnus A and NGC326, see Ekers et al. 1981 and Ekers 1982) also referring to Hargrave & Ryle 1974; Perley et al. 1984; Oort 1982; Fanaroff & Riley 1974; Tsien 1982a. Tsien & Saunders (1982b). In blazars different jet-precession mechanisms may cause different periodicities in their optical/radio light-curves.

- Previously through model-fitting of the kinematics of superluminal components in blazars in terms of the precessing nozzle scenario we have tentatively found three blazars, where double-jet systems might exist in their nucleus: 3C279 (Qian et al. 2019a), OJ287 (Qian 2018a), 3C454.3 (Qian et al. 2021). Here we add a new one: 3C345. These double-jet systems may be produced by binary supermassive black hole systems and thus our precessing nozzle scenario would be useful to investigate the characteristics of binary black holes, e.g., determining orbital period, mass of black holes, precession of jet-nozzle, mechanism of precession, etc. It might be expected that more blazars could be found to possibly house binary supermassive black hole systems, if observational data are plenty enough to make model-simulation of their VLBI-kinematics.

- As previously suggested, in our precessing nozzle scenario for 3C345 and other blazars we assumed that precessing nozzles not only eject superluminal knots (relativistic shocks or plasmons), but also eject rotating magnetized plasmas. Thus the whole jets (or jet-bodies) comprise multiple superluminal knots (ejected at different times, distributing at different positions and moving along different helical trajectories) plus the rotating plasmas associated with the superluminal knots. The accelerated motion and the increase in Lorentz factors along helical trajectories are one of the most important features of these knots, which can well be interpreted in terms of MHD theories for jet-formation/collimation/acceleration mechanisms (e.g., Vlahakis & Königl 2003, 2004; Blandford & Znajek 1977; Meier & Nakamura 2006). The apparently superluminal components can be interpreted as relativistic shocks traveling along helical trajectories toward us in the work-frame of electromagnetic mechanisms.

The results obtained in this work for 3C345, especially those for its knot C9, firmly support these viewpoints on the nature of superluminal components (Jorstad et al. 2005, 2013). In contrast, some authors suggested that the apparent trajectories of superluminal components could result from the underlying jet structure pattern (formed by Kelvin-Helmholtz instabilities) lit up by passages of plasma condensations ejected during nuclear flares. The close correlation between flux evolution and Doppler boosting effect (found for knot C9 in this paper) \(^{11}\) obviously do not support such interpretations.

Since 3-dimensional models were used for simulating the trajectory and kinematic behavior of knots in 3C345, we could separate their intrinsic bulk acceleration from the effects of trajectory curvature. Thus their kinematic parameters (ejection time; bulk Lorentz factor, viewing angle, Doppler factor and apparent velocity vs time) and the location of bulk acceleration zone could be consistently modeled. Generally, the modeled parameters are consistent with those derived in other works by using different methods (e.g., Jorstad et al. 2005, 2013, 2017; Klare 2003; Schinzel 2011a): for example, our modeled Doppler factors could be compared with those derived from variability time scales of optical/radio outbursts.

Our model-fitting results for 3C345 demonstrate that within the collimation/acceleration zone its superluminal knots ejected from the precessing nozzle could have a common inner trajectory pattern, which precesses to produce the observed inner trajectories of the knots. However, beyond this zone their outer motion would follow different individual trajectories. This precessing common inner trajectory could extend to different distances for different knots (see Table 1 and Table 2). We have made use of the precessing nozzle scenario to model fitting of the inner-jet kinematics of superluminal knots in 3C345. The concept of precessing common trajectory may be essential for doing this study. Obviously, the strong magnetic fields of the magnetosphere in its nucleus may play determinative role to form the steady common trajectory pattern, which precesses to produce the trajectories of the knots ejected at different times. The model-fitting of the inner trajectories of the knots in terms of the precessing nozzle scenario naturally explain the position angle swings observed in 3C345. In other blazars and QSOs (e.g., 3C279, OJ287, 3C454.3, B1308+326, PG1302-102, NRAO 150) similar phenomena were discovered and model-fitted (Qian et al. 2015a, 2015b; Qian 2018a). We would like to point out that the precessing nozzle scenarios for these blazars and QSOs are not only based on the model-fittings of the available observational data-sets, but also are confirmed by other observations. For example, Hodgson et al. (2017) have found some evidence for the possible presence of double ejection directions. Particularly, in the case of 3C279 one of the double jet (jet B) which was predicted and searched for quite a long time had already been observed in much earlier years (Pauliny-Toth et al. 1987, 1998; Pauliny-Toth et al. 1998b; de Pater & Perley 1983; Cheung 2002). For blazar OJ287 the possibility of double jet structure has been suggested (Villata et al. 1998, Qian 2018a). Thus the assumptions in our precessing nozzle scenarios seem

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\(^{11}\) Such kind of close correlation between flux evolution and Doppler boosting will be presented elsewhere for more superluminal components in 3C345 (Qian, in preparation).
In the process of model-simulations bulk Lorentz factor $\Gamma$ as function of time was derived for each superluminal components. Interestingly, most of the components (for both jet-A and jet-B) are modeled as accelerated and their Lorentz factors varied over quite large ranges, e.g. knot C10 (jet-A) in [4.5, 29] (Table 1). These intrinsic acceleration should be produced by the strong torsion of toroidal magnetic fields in the collimation/acceleration zones (e.g., Vlahakis & Königl 2004), which could extend as far as ~400 pc from the radio core (e.g. for knot C9; Table 1). Generally, when the Doppler factor versus time of a knot was derived as in this paper, one can investigate the intrinsic radio light-curves of the knot and model-simulating the intrinsic evolution of its electron density/magnetic field (e.g., Qian et al. 1996). However, the 15GHz and 43GHz light-curves of knot C9 could almost completely be interpreted in terms of its Doppler boosting effect during the main flare (during ~1997.50–2000.25), implying that the intrinsic radio emission of this superluminal knot was very stable or the relativistic shock responsible for producing the knot having very stable physical properties (see Sec.4.4.6). This result is important, because it is for the first time to find the radio light-curves at both 15GHz and 43GHz being closely coincident with the Doppler boosting profile and strongly justify our precessing nozzle scenario. The common viewpoint (or scenario) for superluminal knots participating relativistic motion and magnetohydrodynamic acceleration mechanisms are firmly supported.

Due to both jet nozzles (nozzle-A and nozzle-B of 3C345) precessing with the same period (4.6 yr in source frame), the precession of the nozzles could be caused by the orbital motion of the binary holes, rather than other mechanisms of precession (for example, geodetic precession and Newtonian-driven precession; referring to Qian et al. 2017, 2018a, Britzen et al. 2017). If this interpretation is valid, the mass-ratio $q=m/M$ between the primary and secondary holes could be approximately estimated to be equal to the ratio between the jet apertures: $q=0.87$. The orbital period and mass ratio obtained for 3C345 here could tentatively provide some useful constraints on the total mass and gravitational radiation lifetime of the putative supermassive black hole binary in 3C345. Some results are presented in Table 3 to show the relation between the mass of the primary black hole and the parameters of the binary (total mass, orbital separation, post-Newtonian parameter and gravitational radiation lifetime), showing that the total mass should be less than $9.5 \times 10^8 M_\odot$ if its gravitational radiation lifetime $t_{gr}>10^5$ yr.

Interestingly and to our own surprise, the VLBI-kinematics of the four blazars (3C345, 3C454.3, 3C279 and OJ287) could have been interpreted in terms of the precessing jet-nozzle scenario with binary black hole systems, and their precession periods and mass ratios could have been tentatively derived (Qian 2018b, Qian et al. 2018a, 2021 and this paper). Thus we would have to investigate whether there are physically reasonable parameters to describe these binary black hole systems which have the orbital period $P_{orb}$ and mass ratio $q$ as listed in Table 4. For the four blazars (or the four putative binary black hole systems), their parameters (mass $M_8$ and $m_8$, post-Newtonian parameter $e_8$, gravitational radiation lifetime $t_{gr}$, ratios $r/R_\text{r}$ and $r/R_\text{g}$ between orbital separation and gravitational radius calculated for different orbital separations $r$ (0.01 pc–0.05 pc) are given in Table 5, taking the $P_{orb}$ and q listed in Table 4 into account. Based on Table 5, we can search for appropriate ranges for the masses and separations $r$ for the four binary systems, if requiring their gravitational radiation lifetime limited in the range $10^5$–$10^6$ yr.

It was found that: (a) For 3C279, the ranges $[r=0.03–0.04 \text{pc}]$ and $[t_{gr}=5.21 \times 10^6–1.24 \times 10^6 \text{yr}]$ correspond to mass ranges $[M_8=4.91, m_8=2.45]$ and $[M_8=11.6, m_8=5.80]$, respectively. These masses are well consistent with the values (3–8 $10^8 M_\odot$) given in Woo & Urry (2002), Wang et al. (2004), Gu et al. (2001) and Nilsson et al. (2009). This is a very good case, indicating that the related parameters of the binary system derived for 3C279 by our precessing jet-nozzle scenario seem to be physically sound; (b) For 3C454.3, the ranges $[r=0.015–0.02 \text{pc}]$ and $[t_{gr}=3.41 \times 10^6–8.22 \times 10^6 \text{yr}]$ correspond to the mass ranges $[M_8=6.01, m_8=1.80]$ and $[M_8=14.2, m_8=4.26]$. These masses are well consistent with the values of $1.3–1.5 \times 10^9 M_\odot$ given in Woo & Urry (2002) and Wang et al. (2018a). (c) For 3C345, as mentioned in the previous item, its total mass should be less than $9.5 \times 10^8 M_\odot$ if requiring $t_{gr}>10^5$ yr. According to Woo & Urry its black hole mass $=2.63 \times 10^9 M_\odot$, but Lobanov & Roland (2005) adopted a mass $1.42 \times 10^8 M_\odot$ for their binary black hole model. Here we found that the ranges $[r=0.01–0.014 \text{pc}]$ and $[t_{gr}=5.73 \times 10^6–1.06 \times 10^7 \text{yr}]$ correspond to the mass ranges of $[M_8=1.83–5.02]$ and $[m_8=1.59–4.37]$, which seem to be reasonable values. According to Wang et al. (2018a), 3C345 is distinctly different from 3C279 and 3C454.3 in the relation between its kinematic luminosity ($L_{Bir}$) and broad-line region luminosity ($L_{BLR}$), possibly implying some jet-disk connection which could result in its measured mass higher than that of the black hole itself. In addition, Xie et al. (2005) measured a mass of $2.57 \times 10^8 M_\odot$, similar to that

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12 Only within our precessing nozzle scenario, where Doppler factor curve $\delta(t)$ (and Doppler boosting profile $[\delta(t)]^{\alpha+}$) can be obtained for interpreting flux evolution of superluminal knots. Usually analysis of VLBI-observations obtains only averaged values for bulk Lorentz factor or Doppler factor, which could not be used to study flux evolution of superluminal knots.

13 Our works have shown that the double-jet structures revealed in the four blazars (3C279, 3C454.3, 3C345 and OJ287) have similar (common) features: both jets precess in the same direction with the same period. Such kind of jet precession could not be caused by hydrodynamical instabilities induced by the interactions between the jets and the surrounding media.

14 This value is estimated at core separation ~0.5 mas. In the case of knots moving along curved trajectories the jet-cone apertures critically depend on core separations and thus the value $q=0.87$ is only a rough estimation.

15 Formulas for calculating these parameters can be found in Qian et al. (2017). Values for $r=0.014$ and $0.015 \text{pc}$ were also used, but not listed in Table 5. For the four blazars 0.01 pc corresponds to angular distance of ~1-2 $\mu$as.
Table 3. Parameters of the binary black holes suggested in blazar 3C345: total mass M+m (in units of $10^8 M_\odot$), separation between the holes $r$ (in units of parsecs), post-Newtonian parameter ($\epsilon_n$) and gravitational radiation lifetime $t_{gr}$ (yr) calculated for different masses M of the primary black hole.

| M   | M+m | $r_{pc}$ | $\epsilon_n$ | $t_{gr}$ |
|-----|-----|---------|-------------|---------|
| 1.0 | 1.87| 8.2×10^{-3} | 1.09×10^{-3} | 1.58×10^9 |
| 1.5 | 2.81| 9.37×10^{-3} | 1.43×10^{-3} | 7.96×10^5 |
| 2.0 | 3.74| 1.03×10^{-2} | 1.74×10^{-3} | 4.93×10^5 |
| 3.0 | 5.60| 1.18×10^{-2} | 2.28×10^{-3} | 2.53×10^5 |
| 4.0 | 7.48| 1.30×10^{-2} | 2.76×10^{-3} | 1.56×10^5 |
| 5.0 | 9.35| 1.40×10^{-2} | 3.21×10^{-3} | 1.08×10^5 |

Table 4. Orbital period $P_{orb}$ (yr, in source frame) and mass ratio q for the four blazars: 3C345, 3C454.3, OJ287 and 3C279. z-redshift.

| source | z    | $P_{orb}$ | q   |
|--------|------|-----------|-----|
| 3C345  | 0.595| 4.6       | 0.87|
| 3C454.3| 0.859| 9.6       | 0.30|
| OJ287  | 0.306| 9.2       | 0.30|
| 3C279  | 0.538| 16.3      | 0.50|

Table 5. Parameters of the binary black holes suggested in blazars 3C345, 3C454.3, OJ287 and 3C279: Mass M and m of the primary and secondary holes (in units of $10^8 M_\odot$), post-Newtonian parameter $\epsilon_n$, gravitational radiation lifetime $t_{gr}$ (yr), $r/R_g$ ($R_g$—gravitational radius of the primary hole) and $r/r_g$ ($r_g$—gravitational radius of the secondary hole) calculated for different orbital separations r in circular motion (in units of parsecs).

| Source parameter | $r_{pc} = 0.01$ | $r_{pc} = 0.02$ | $r_{pc} = 0.03$ | $r_{pc} = 0.04$ | $r_{pc} = 0.05$ |
|------------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 3C345 M_s        | 1.83            | 14.7            | 49.5            | 1.17×10^2       | 2.29×10^2       |
| M_s              | 1.59            | 12.8            | 43.0            | 1.02×10^2       | 1.99×10^2       |
| $\epsilon_n$     | 1.64×10^{-3}    | 6.58×10^{-3}    | 4.14×10^{-2}    | 2.63×10^{-2}    | 4.11×10^{-2}    |
| $t_{gr}$         | 5.73×10^5       | 1.78×10^4       | 2.34×10^3       | 5.56×10^3       | 1.83×10^2       |
| $r/R_g$          | 1140            | 284             | 127             | 71.6            | 45.7            |
| $r/r_g$          | 1320            | 328             | 146             | 122             | 52.6            |

| 3C454.3 M_s      | 1.78            | 14.2            | 48.0            | 1.14×10^2       | 2.23×10^2       |
| M_s              | 0.53            | 4.26            | 14.4            | 34.1            | 66.7            |
| $\epsilon_n$     | 1.1×10^{-3}     | 4.43×10^{-3}    | 9.99×10^{-3}    | 1.78×10^{-2}    | 2.78×10^{-2}    |
| $t_{gr}$         | 2.59×10^6       | 8.22×10^4       | 1.07×10^4       | 2.54×10^3       | 8.27×10^2       |
| $r/R_g$          | 1180            | 295             | 131             | 73.5            | 47.0            |
| $r/r_g$          | 3950            | 984             | 437             | 246             | 157             |

| OJ287 M_s        | 0.66            | 5.57            | 17.8            | 42.2            | 82.5            |
| M_s              | 0.20            | 1.58            | 5.34            | 12.7            | 24.8            |
| $\epsilon_n$     | 4.13×10^{-4}    | 1.64×10^{-3}    | 3.70×10^{-3}    | 6.59×10^{-3}    | 1.03×10^{-2}    |
| $t_{gr}$         | 5.02×10^7       | 1.60×10^6       | 2.10×10^5       | 4.96×10^4       | 1.62×10^4       |
| $r/R_g$          | 3170            | 795             | 354             | 198             | 127             |
| $r/r_g$          | 10500           | 2650            | 1180            | 660             | 422             |

| 3C279 M_s        | 0.18            | 1.35            | 4.91            | 11.6            | 22.7            |
| M_s              | 0.09            | 0.73            | 2.45            | 5.80            | 11.4            |
| $\epsilon_n$     | 1.3×10^{-4}     | 5.23×10^{-4}    | 1.18×10^{-3}    | 2.09×10^{-3}    | 3.27×10^{-3}    |
| $t_{gr}$         | 1.27×10^9       | 3.95×10^7       | 5.21×10^6       | 1.24×10^6       | 4.04×10^5       |
| $r/R_g$          | 11650           | 2880            | 1280            | 721             | 461             |
| $r/r_g$          | 23300           | 5740            | 2560            | 1450            | 917             |

we derived;
(d) For OJ287, the ranges $[r=0.02–0.03]$pc and $[t_{gr}=1.60\times10^6–2.10\times10^7]$yr correspond to the mass ranges $[M_s=5.27–17.8]$ and $[m_s=1.58–5.34]$. These values are broadly consistent with the values (mostly $\lesssim10^8 M_\odot$) adopted in several works (e.g., Valtaoja et al. 2000; Liu & Wu 2002; Wang et al. 2004; Gupta et al. 2012; Katz 1997). However, in the disk-impact scenario proposed by Lehto & Valtonen (1996) also Dey et al. (2019) Valtonen et al. (2012), the masses of the binary holes are modeled to be $\sim1.5\times10^{10} M_\odot$ and $\sim1.4\times10^9 M_\odot$ with a mass-ratio $q\sim0.008$. Obviously, the issue on the properties of the black hole binary in OJ287 should be further investigated and clarified (Qian 2013; 2014; 2015; 2016; Britten et al. 2018; Villata et al. 1998; Sillanpää et al. 1988, 1996).
In Table 5 are also given the ratios \((r/R_g)\) between orbital separation and gravitational radius for the primary and secondary holes. It can be seen that the reasonable ranges of masses described above correspond to reasonable ranges of the ratios for the primary and secondary holes: approximately a few hundreds to a few thousands (for example, for 3C279 \(r/R_g=1280-721\) and \(r/r_g=2560-1450\), indicating that the formation of accretion disks and jets could proceed without destructive influences by the gravitational interaction between the two holes. However, the formation of double-jet and disks and their instabilities need to be investigated for understanding the complex phenomena observed in blazars (including the VLBI-kinematics and the multi-wavelength radiation and the connection between low-energy and high-energy radiations). In binary black hole systems, cavity accretion might restrain the formation of large-scale accretion disks, but spin of the holes might play more significant roles in formation of relativistic jets. Perhaps the binary black holes in blazars might be rapidly rotating with spin parameter \(j>0.5\) \((j=3/J_{max})\), \(J_{max}=GM^2/c^2\) maximum spin angular momentum of a black hole). In addition, for the reasonable ranges of the black hole masses, the post-Newtonian parameter \(e_0\) given in Table 5 has its values in the range of \([1-4]\times10^{-3}\), implying that the keplerian motion of the black hole binaries in the four blazars is still non-relativistic. That is, the black hole binaries of the four blazars have not been entering the stages of orbital evolution dominated by the general relativity effects (Einstein 1916–1918). The results listed in Table 5 for the physical parameters of the black hole binaries could only describe the characteristic features of their initial-in-spiraling processes.

We would like to point out that our results derived for the putative black hole binaries in the four blazars (3C279, 3C454.3, OJ287 ans 3C345) were well consistent with the theoretical arguments about close binary systems by Begelman et al. (1980). This was unexpected and confirmed posteriorly. They have shown that, when the orbital separation of binaries approaches \(R_g\) at which gravitational radiation becomes to dominate their orbital evolution, their keplerian motion will have an orbital period of \(T_{orb}(R_g)=48.4\) yr for a specific model (assuming \(q=0.3\), \(M=10^9\)\(M_\odot\), and corresponding \(r_{gr}=0.067\) pc and \(t_{gr}(R_g)=1.8\times10^8\) yr; referring to Begelman et al.). If the orbital separation \(r < R_g\), for example, \(r = 0.32R_g (0.021\) pc\) and \(r = 0.18R_g (0.012\) pc\), corresponding \(t_{gr}=1.8 \times 10^6\) yr, \(T_{orb}=8.6\) yr\) and \(t_{gr}=1.8 \times 10^7\) yr, \(T_{orb}=3.7\) yr\), respectively. These values are closely similar to the gravitational radiation lifetimes and orbital periods derived by our works for the four blazars (especially for 3C279; Tables 3–5). The double jet-nozzle precession we tentatively found through analysis of the VLBI-kinematics of the four blazars could be the direct consequences of their orbital motion. Therefore both theoretical investigations and VLBI-observations seem consistently to approach the same conclusion: our investigations and analyzes of the VLBI-kinematics of superluminal components in the four blazars (3C345, 3C454.3, 3C279 and OJ287) might have revealed the keplerian motion of the black hole binaries putatively suggested to be existing in their nucleus.

In fact, only in blazars one could possibly discern (or detect) the precession of double jets caused by the keplerian motion through analyzing their VLBI-kinematic behaviors, since blazars are observed at very small viewing angles (\(\sim 1 - 5^\circ\) for the four blazars discussed here), which assure the precession cones of the double jets projected in the plane of the sky being sufficiently wide and separated to be resolved by VLBI-observations, and the superluminal components can radiate strongly to be measured on VLBI-scales due to relativistic beaming effects. Search for evidence of possible keplerian orbital motion of black hole binaries in blazars may be a severe challenge for future VLBI-observations.

Relativistic jets in blazars (and generally in AGNs) are believed to be formed through MHD processes in the magnetosphere of black-hole/accretion disk systems in the nuclei of host galaxies (e.g., Blandford & Payne 1982, Blandford & Znajek 1977, Camenzind 1980, 1987, 1990, Lovelace et al. 1986, Li et al. 1992, Vlahakis & Königl 2003, 2004). In these scenarios magnetic fields dominate the processes: magnetic pressure gradient accelerate the jets and magnetic pinch effects of the toroidal fields collimate the plasma flows. MHD theoretical scenarios can also explain the extended (parsec-scale) acceleration observed in blazars. It is shown by Blandford& Znajek (1977), Li et al. (1992) and Vlahakis & Königl (2001) that there are radically self-similar solutions for stationary axisymmetric MHD flows in the magnetosphere of black-hole/accretion-disk systems which allow extended acceleration after the flow passes the classical fast-magnetosonic point approaching the modified fast-magnetosonic point through "magnetic nozzle mechanism". Vlahakis & Königl (2004) nicely explained the accelerating component C7 in blazar 3C345 (observed by Unwin et al. 1997) in terms of their radically self-similar MHD solution for a proton-electron jet. The acceleration zone \((30-300\pc)\) and the range of bulk Lorentz factor \(\sim 4-20\) derived in our model fittings can be understood within the Vlahakis-Königl’s radially self-similar MHD model. Thus available MHD jet-formation theories are already effective to explain the extended acceleration observed in blazars.

We would like to point out that the cavity-accretion models may be very helpful to understand the accretion process, jet-formation and ejection of superluminal components and alternative quasi-periodic activity in blazars with double-jets, hosting binary black holes (e.g., Shi et al. 2012, Shi & Krolik 2015, Artymovicz & Lubow 1996, Artymovicz 1998). Specifically, Shi & Krolik (2015) argued that in MHD scenarios cavity-accretion rates can be raised to the level that both black holes can produce a jet, forming a double jet system. One distinct feature of our results is the precession of a common trajectory pattern around the jet axis to produce the observed inner trajectories of the knots and the 7.30 yr precession period for 3C345. It is worth emphasizing that for jet-A its nozzle-precession has been

\(^{16}\) Double-jet structure in blazar OJ287 has tentatively been suggested by Qian (2018b) through model-simulation of its superluminal components in terms of our precessing nozzle scenario. The secondary jet ejected by its secondary black hole could be observed in near future (Dey et al. 2021).
observed over about four precession periods (~1979–2009) and for jet-B over about two precession periods (~2002–2016), implying that jet-A and jet-B have been active during respective periods with respective levels of activity. Precessing nozzles could possibly exist in the putative binary black hole system in 3C345, which needs to be tested by more observations.

– Based on the assumptions about the possible existence of a precessing jet-nozzle and precessing common trajectory in 3C345, we have analyzed the kinematic behavior of 27 superluminal components in terms of the precessing nozzle scenario and model-fitted their trajectory $Z_n(X_n)$, core separation $r_n(t)$, coordinates $X_n(t)$ and $Z_n(t)$ and apparent velocity $\beta_n(t)$ versus time during a time-interval of ~38 years (1979–2016). The double jet structure was disentangled and their precession periods (7.30 yr) were derived. The derived bulk Lorentz factor $\Gamma$, Doppler factor $\delta$ and viewing angle $\theta$ as functions of time may be very useful for studying the intrinsic emission and physical properties of these knots, and the connection between the radio, optical and $\gamma$-ray emitting regions. The double-jet structure and their precession might be useful for investigating the putative binary black hole system, providing some constraints on its physical properties.

We have used a new method, which is different from the ordinary methods for analyzing the observational data obtained on VLBI-scales. The assumptions we suggested should be tested by future observations of the kinematics for superluminal components within core separations $r_n < 0.05–0.1$ mas, where the superluminal knots might follow helical trajectories with large pitch angles (or strong toroidal field-lines; referring to Qian (2019c)) and might be more difficult to determine their precessing behavior.\textsuperscript{14} The assumption of precessing common trajectory would be confronted with future VLBI-observations with higher-resolutions in order of ~10 mas.

In fact, there have been different results by analyzing the kinematic behaviors of superluminal components for 3C345 and quite a lot of different suggestions were proposed in literature. For example, Klares (2003) claimed the presence of jet precession of ~8-10 yr, but Schinzel (2011a) claimed no clear evidence for periodic trends in the kinematic behavior of superluminal components; Lobanov & Zensus (1999) suggested a ~8-10 yr period; Lobanov & Roland (2005) suggested a binary hole scenario and 9.5 yr precession period, and Qian et al. (2009) suggested a 7.36 yr precession period, etc.\textsuperscript{15} Our results in this paper may be regarded as one of the alternative explanations. Using our new methods for analyzing the VLBI-kinematics in 3C345 much more information on its kinematic properties and physical implications could be obtained.

However, this work was established on the assumption that jets in blazars should precess with certain regular periods. At present it still remains unsettled as a question: whether blazar 3C345 has a single-jet structure or double-jet structure with or without jet-nozzle precession. More observations and investigations are needed to solve this issue.

– Similar issues are present for other blazars, for example, for blazar 3C279. Recently, performing VLBI-observations at mm-wavelengths by using Event-Horizon-Telescope, Kim et al. (2020) found similar position angles in 2011 and 2017, suggesting a precession period of ~6 yr and claiming to exclude the 25 yr precession period which was derived through analyzing the kinematic behavior of ~30 superluminal knots observed during ~30 years (Qian et al. 2019a).\textsuperscript{16} Here we would like to propose an alternative interpretation: Kim’s finding might not necessarily be contradictory to the 25 yr period. It could possibly be compatible with the precession period of ~25 yr derived in Qian et al. (2019a) because the 25 yr precession period permits similar position angles appearing at two epochs different within ~12.5 yr (a half of the precession period of 25 yr). Such kind of similar position angles at two times could be observed at the either side of the projected jet boundaries (or the "jet edges"). An appropriate example may be: through model-fitting of the VLBI-kinematics for OJ287 (Qian 2018b), it was found that its knots C11 and C12 were observed at similar position angles with a difference in their ejection times of ~3.8 yr. The two knots were ejected at the either side of the southern edge of its southern jet with a difference in their precession phases of ~2.0 rad, corresponding to a time-interval smaller than ~6 yr (half of the precession period 12 yr). The presence of double-jet structure, jet-nozzle precession and periodicity in ejection of superluminal components in 3C345 and other blazars are still issues in debate and need to be further investigated. Recently, Event-Horizon-Telescope (EHT) Collaboration has begun monitoring observations for a few blazars at 230 GHz with resolutions of ~20 mas (EHT-Collaboration et al. 2019). As suggested by Qian (2018b, 2020) that the non-thermal radiation emitted by blazar OJ287 might originate from its double jets with precessing jet-nozzles. EHT-monitoring observations in the near future would be helpful to reveal its double jet structure, searching for the second jet ejected from the secondary black hole in OJ287 (Qian 2018b, Villata et al. 1998, Dey et al. 2021, 2019). Similar search for double-jet structure in blazars 3C279, 3C454.3 and 3C345 should be tried, and EHT-monitoring observations will be very helpful. All kinds of scenarios proposed to interpret the kinematic phenomena in blazars would experience severe tests.

\textsuperscript{17} Thus it seems uncertain whether the cores of jet-A and jet-B could be concurrently observed.\textsuperscript{18} At large core-separations helical magnetic fields may have much smaller pitch angles and thus the motion of superluminal knots becomes to be approximately ballistic.

\textsuperscript{19} In fact, all these determinations of precession period were derived by using the observational data on the knots of jet-A only. One could not find any precession by using observational data on the knots of both jet-A and jet-B.

\textsuperscript{20} Similar position angles observed at two different epochs is inadequate to determine the precession period, because it still needs to confirm that the two epochs correspond to a difference in precession phase of $2\pi$. In our precessing jet-nozzle scenarios suggested for the blazars (3C345, OJ287, 3C454 and 3C454.3) recurrences of the curved trajectories observed at different epochs corresponding to differences in precession phases of $\sim2\pi$ and the distribution of the modeled precessing common trajectory were very helpful for determining their precession periods.
8. Concluding remarks

In this work the kinematics of 27 superluminal components in blazar 3C345 were model-simulated in terms of our precessing nozzle scenario. Double jet-nozzle structure was tentatively suggested consisting of jet-A and jet-B. Interestingly, taking the model-simulation of kinematic behavior of knot C9 as an example, its trajectory was extremely well fitted by the precessing common trajectory at precession phase $5.44 \pm 4\pi$ corresponding to its ejection time 1995.06, with a very high accuracy of $\pm 5\%$ precession period (Fig.3). Its observed kinematic properties (trajectory $Z_n(X_n)$, core separation $r_n(t)$, coordinates $(X_n(t)$ and $Z_n(t)$ and apparent velocity $(\beta_{app}(t)$) were well fitted within core separation $\sim 1.8$ mas, equivalent to spatial distance from the core of $\sim 400$ pc (Fig.8). Moreover, its radio light curves were confirmed to be extremely well coincident with its Doppler factor profile (Figs.9 and 10), thus demonstrating its bulk Lorentz factor and viewing angle as functions of time being correctly derived and superluminal knots in 3C345 being a relativistic shock moving toward us with acceleration. This was unexpected and for the first time both the kinematic properties and Doppler boosting effect in the flux evolution of a superluminal knot can be studied in detail. Thus the model-simulation for the kinematic and physical properties in terms of our precessing nozzle scenario was almost perfect.

Model-simulation results of both kinematic behavior and Doppler boosting effect in flux evolution for more superluminal knots in 3C345 are in preparation, showing their kinematic and physical properties similar to that for knot C9. All these results certainly indicate that our precessing nozzle scenario may be an effective one and appropriate to study the kinematic and physical properties of blazars. As generally suggested, the swing of ejection position angle observed for superluminal components in QSOs or blazars has been assumed to be related to the jet precession in the sources. For blazar 3C345 precession periods of $\sim 7-10$ yr were really identified (e.g., Lobanov & Zensus 1999, Klare 2003, Qian et al. 2010). Interestingly, these studies only used the earlier VLBI-observational data (equivalent to the data for jet-A identified in this paper). However, by using the entire dataset till recent years (e.g., to $\sim 2010$, including the dataset for jet-A and jet-B both) no trend of jet precession was identified (this work, Schinzel et al. 2013). As shown by the detailed analysis of the kinematics of all superluminal components in 3C345, the source could possibly comprise of two groups of superluminal components, ascribed to two jets (jet-A and jet-B). In this case both jets could be modeled as having their own precession cone pattern, but having similar periods and in the same direction. This kind of precession of double-jet could be produced in close black hole binaries through the modulation of jet flow by keplerian orbital motion (Artymowicz 1998, Qian et al. 2021, 2017, 2018, Roos et al. 1993). Thus in this paper we tentatively adopted the following assumptions: (1) 3C345 has double-jet structure with double precessing nozzles for jet-A and jet-B; (2) both jet-A and jet-B precess with the same period $\sim 7.30$ yr; (3) there are similar patterns. Based on these assumption the kinematic behavior of superluminal components ascribed to jet-A and jet-B were successfully model-simulated and the relation between the flux evolution and Doppler-boosting effect for some superluminal components were studied and interpreted.

3C345 is the fourth blazar which was model-simulated in terms of our precessing nozzle scenario within the framework of double jet-nozzle structure. Interestingly, it was found consistently that for all the four blazars, only under the assumption of double nozzle structures, the precession of both jet-nozzles could be model-simulated. These double jet structures naturally led to black hole binaries putatively existing in the nuclei of the four blazars. Double jet-nozzle scenario may not necessarily imply concurrent ejection of superluminal components: the two nozzles might be active alternatively or one jet could exist temporarily. At present, few theoretical investigations of the formation of double relativistic jets in black hole binaries. Obviously, formation of double jets, jet precession, their stability and short orbital period, etc. involve new theoretical fields in binary black hole physics and should be intensively investigated for providing deeper understanding of blazar phenomena.

Although under the assumption of double-jet structure for 3C345, significant and unprecedented results and physical information about its kinematic behavior and physical nature were derived, single-jet structure scenario can not be excluded. The existence of a double-jet structure or single-jet structure in blazar 3C345 is still a matter in debate. However blazar OJ287 seems likely to have a double-jet structure in its nucleus (Qian 2018b, Villata et al. 1998, Dey et al. 2021). At present, for blazars 3C345, 3C454.3 and 3C279, double jet-nozzle scenarios may be regarded as some hypothesized working-frames which need to be tested by future observations.

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Fig. A.1. Knot C5: Precession phase $\phi_0=5.83$ rad and ejection time $t_0=1980.80$. Model-fitting results: trajectory $Z_n(X_n)$, coordinates $X_n(t)$ and $Z_n(t)$, core separation $r_n(t)$, modeled apparent velocity $\beta_n(t)$ and viewing angle $\theta(t)$, bulk Lorentz factor $\Gamma(t)$ and Doppler factor $\delta(t)$.
Fig. A.2. Knot C6: precession phase $\phi_0(\text{rad}) = 5.74 + 2\pi$ and ejection time $t_0 = 1987.99$. Mode-fitting results: trajectory $Z_n(X_n)$, coordinates $X_n(t)$ and $Z_n(t)$, core separation $r_n(t)$, modeled apparent velocity $\beta_a(t)$ and viewing angle $\theta(t)$, bulk Lorentz factor $\Gamma(t)$ and Doppler factor $\delta(t)$. Kinematics for $r_n > 0.4$ mas has been fitted by introducing changes in parameter $\psi$ (see text).
Fig. A.3. Knot C7: Precession phase $\phi_0(\text{rad}) = 6.14 + 2\pi$ and ejection time $t_0 = 1988.46$. Model-fitting results: trajectory $Z_n(X_n)$, coordinates $X_n(t)$ and $Z_n(t)$, core separation $r_n(t)$, modeled apparent velocity $\beta_n(t)$ and viewing angle $\theta(t)$, bulk Lorentz factor $\Gamma(t)$ and Doppler factor $\delta(t)$. Its kinematics within $r_n\sim 0.7$ mas can be well fitted in terms of the precessing nozzle scenario, or its observed precessing common trajectory extends to core separation $\sim 0.7$ mas.
Fig. A.4. Knot C10: precession phase $\phi_0(\text{rad})=6.14+4\pi$ and ejection time $t_0=1995.76$. Model-fitting results: trajectory $Z_n(X_n,)$, coordinates $X_n(t)$ and $Z_n(t)$, core separation $r_n(t)$, modeled apparent velocity $\beta_n(t)$ and viewing angle $\theta(t)$, bulk Lorentz factor $\Gamma(t)$ and Doppler factor $\delta(t)$. Its observed precessing common trajectory is assumed to extend to $r_n \sim 0.8$ mas.
Fig. A.5. Knot C11: precession phase $\phi_0(\text{rad})=5.88+4\pi$ and ejection time $t_0=1995.46$. Model-fitting results: trajectory $Z_n(X_n)$, coordinates $X_n(t)$ and $Z_n(t)$, core separation $r_n(t)$, modeled apparent velocity $\beta_n(t)$ and viewing angle $\theta(t)$, bulk Lorentz factor $\Gamma(t)$ and Doppler factor $\delta(t)$. Its kinematics within $r_n\sim0.75$ mas can be well fitted in terms of the precessing nozzle scenario.
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Fig. A.6. Knot C12: precession phase $\phi_0(\text{rad})=6.30+4\pi$ and ejection time $t_0=1995.95$. Model-fitting results: trajectory $Z_n(X_n)$, coordinates $X_n(t)$ and $Z_n(t)$, core separation $r_n(t)$, modeled apparent velocity $\beta_a(t)$ and viewing angle $\theta(t)$, bulk Lorentz factor $\Gamma(t)$ and Doppler factor $\delta(t)$. Its kinematics within $r_n \sim 0.5$ mas could be well fitted by the precessing nozzle model.
Fig. A.7. Knot C13: precession phase $\phi_0(\text{rad})=6.50+4\pi$ and ejection time $t_0=1996.18$. Mode-fitting results: trajectory $Z_n(X_n)$, coordinates $X_n(t)$ and $Z_n(t)$, core separation ($r_n(t)$), modeled apparent velocity $\beta_a(t)$ and viewing angle $\theta(t)$, bulk Lorentz factor $\Gamma(t)$ and Doppler factor $\delta(t)$. Its kinematics within $r_n \sim 0.70$ mas could be very well model-simulated in terms of the precessing nozzle scenario.
Fig. A.8. Knot C14: precession phase $\phi_0(\text{rad}) = 3.16 + 6\pi$ and ejection time $t_0 = 1999.61$. Model-fitting results: trajectory $Z_n(X_n)$, coordinates $X_n(t)$ and $Z_n(t)$, core separation $r_n(t)$, modeled apparent velocity $\beta_n(t)$ and viewing angle $\theta(t)$, bulk Lorentz factor $\Gamma(t)$ and Doppler factor $\delta(t)$. Its kinematics within $r_n \sim 0.50$ mas could be well fitted in terms of the precessing nozzle model.
Fig. A.9. Knot C15a: precession phase $\phi_0=5.75$ rad and ejection time $t_0=2002.18$. Model-fitting results: trajectory $Z_n(X_n)$, coordinates $X_n(t)$ and $Z_n(t)$, coordinates $X_n(t)$ and $Z_n(t)$, core separation $r_n(t)$, modeled apparent velocity $\beta_n(t)$ and viewing angle $\theta(t)$, bulk Lorentz factor $\Gamma(t)$ and Doppler factor $\delta(t)$. Its observed precessing common trajectory was assumed to extend to core separation $r_n\sim0.17$ mas.
Fig. A.10. Knot C16: precession phase $\phi_0=5.80$ rad and ejection time $t_0=2002.24$. Model-fitting results: trajectory $Z_n(t)$, coordinates $X_n(t)$ and $Z_n(t)$, core separation $r_n(t)$, modeled apparent velocity $\beta_n(t)$ and viewing angle $\theta(t)$, bulk Lorentz factor $\Gamma(t)$ and Doppler factor $\delta(t)$. Its kinematics within $r_n \sim 0.8$ mas could be well model-simulated in terms of the precessing nozzle scenario.
Fig. A.11. Knot C17: precession phase $\phi_0$(rad)=$1.57+2\pi$ and ejection time $t_0=2004.62$. Model-fitting results: trajectory $Z_n(X_n)$, coordinates $X_n(t)$ and $Z_n(t)$, core separation $r_n(t)$, modeled apparent velocity $\beta_a(t)$ and viewing angle $\theta(t)$, bulk Lorentz factor $\Gamma(t)$ and Doppler factor $\delta(t)$. Its kinematics within $r_n\sim0.8$ mas could be well model-simulated in terms of the precessing nozzle scenario.
Fig. A.12. Knot C18: precession phase $\phi_0 (\text{rad})=1.62+2\pi$ and ejection time $t_0=2004.68$. Model-fitting results: trajectory $Z_n(X_n)$, coordinates $X_n(t)$ and $Z_n(t)$, core separation $r_n(t)$, modeled apparent velocity $\beta_a(t)$ and viewing angle $\theta(t)$. Its kinematics within core separation $r_n \sim 0.60\text{ mas}$ could be well simulated by the precessing nozzle scenario.
Fig. A.13. Knot B5: precession phase $\phi_0(\text{rad})=0.33+4\pi$ and ejection time $t_0=2010.48$. Its trajectory was observed near the edge of the jet-cone and the data-points within $X_n\sim0.2$ mas are near the modeled trajectories defined by phases $0.33\pm0.31$ rad, which are almost overlapped with each other in this case. Its precessing common trajectory might be assumed to extend to $r_n\sim0.15$ mas.
Fig. A.14. Knot B7: precession phase $\phi_0(\text{rad})=1.30+4\pi$ and ejection time $t_0=2011.60$. Model-fitting results: trajectory $Z_n(X_n)$, coordinates $X_n(t)$ and $Z_n(t)$, core separation $r_n(t)$, modeled apparent velocity $\beta_n(t)$ and viewing angle $\theta(t)$, bulk Lorentz factor $\Gamma(t)$ and Doppler factor $\delta(t)$. Its kinematics within core separation $r_n\sim0.80\text{ mas}$ could be well model-simulated in terms of the precessing nozzle scenario.
Fig. A.15. Knot B8: precession phase $\phi_0(\text{rad})=1.60+4\pi$ and ejection time $t_0=2011.95$. Model-fitting results: trajectory $Z_n(X_n)$, coordinates $X_n(t)$ and $Z_n(t)$, core separation $r_n(t)$, modeled apparent velocity $\beta_a(t)$ and viewing angle $\theta(t)$, bulk Lorentz factor $\Gamma(t)$ and Doppler factor $\delta(t)$. Its kinematics within core separation $r_n \sim 1.20$ mas could be well model-simulated in terms of the precessing nozzle scenario.
Fig. A.16. Knot B11: precession phase $\phi_0$(rad)=$4.80+4\pi$ and ejection time $t_0=2015.67$. Model-fitting results: trajectory $Z_n(X_n)$, coordinates $X_n(t)$ and $Z_n(t)$, core separation $r_n(t)$, modeled apparent velocity $\beta_a(t)$ and viewing angle $\theta(t)$, bulk Lorentz factor $\Gamma(t)$ and Doppler factor $\delta(t)$. Its innermost trajectory within core separation $r_n$$\sim$0.12 mas was coincident with the precessing common trajectory, but no observational data available there. Beyond $r_n$$\sim$0.12 mas the trajectory was modeled by taking its rotation into account.
Epoch (year)

Flux and Doppler boosting (normalized)

43GHz flux
Doppler boosting
