In linear transport, the fluctuation-dissipation theorem relates equilibrium current correlations to the linear conductance coefficient. For nonlinear transport, there exist fluctuation relations that rely on Onsager’s principle of microscopic reversibility away from equilibrium. However, both theory and experiments have shown deviations from microreversibility in the form of magnetic field asymmetric current-voltage relations. We present novel fluctuation relations for nonlinear transport in the presence of magnetic fields that relate current correlation functions at any order at equilibrium to response coefficients of current cumulants of lower order. We illustrate our results with the example of an electrical Mach-Zehnder interferometer.

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Eq. (2) is the Fourier transform of Eq. (1) and determines the symmetry of the generating function. For convenience, the (anti-)symmetrized generating function

\[ F_{\pm}(i\Lambda, A) = \pm F_{\pm}(-i\Lambda - A, A) \]

(2)

The Mach-Zehnder interferometer (MZI) – As an instructive example, we present a MZI (see Fig. 1) and show how interaction (screening) effects lead to deviations from reversibility. It is a four terminal conductor with two quantum point contacts (QPC) acting as beam splitters as shown in Fig. 1. The two interferometer arms enclose a magnetic flux \( \Phi \), such that interference arises due to the Aharonov-Bohm effect. In experiments [12, 13, 14, 16], the MZI is realized using edge states in the quantum Hall regime, and it is often operated at filling factor 2. Only carriers in the outer edge enter the interferometer and are able to interfere. Here, the inner edge state moves in vicinity in both interferometer arms and carries current from terminal 1 to 3 and from 2 to 4 [20]. Although a four-terminal conductor, the MZI is characterized by only a single transmission probability \( T_{31} \), due to the separation of left- and right movers. \( T_{31} \) is the probability for a particle to be transmitted in the outer edge state from terminal 1 to 3. In the linear transport regime reciprocity means that [21]

\[ T_{31}(+B) = T_{13}(-B) \]

We next demonstrate that already Hartree interactions lead to a violation of Eqs. (1) and (2).

Breakdown – Interactions can lead to magnetic field asymmetry in nonlinear transport, as was shown theoretically [22, 23, 24] as well as experimentally [25, 26, 27, 28, 29]: Every particle is moving in a local potential \( U(\vec{r}) \) generated by all the other particles. The internal potential has to be determined self-consistently and depends on all potentials \( V_\gamma \) applied in the external contacts, \( U(\vec{r}) = U(\vec{r}, \{ V_\gamma \}) \). The scattering matrix depends on the energy of the particle and is a functional of the internal potential, \( S = S(E, U(\vec{r})) \). Indeed the functional dependence of the scattering matrix is required by gauge invariance: The generating function has to be invariant under a shift of all potentials by the same amount, \( V_\gamma \rightarrow V_\gamma + U_0 \). This condition can be expressed as

\[ \frac{dF}{dU_0} = 0 \]

For long times and neglecting interactions, the generating function in the scattering approach is

\[ F(i\Lambda) = \frac{t}{\hbar} \int dE \text{tr} \left[ \ln(1 - \tilde{f}K) \right]. \]

(3)

Here, \( K = (1 - \lambda S^T \lambda S) \) is composed of the scattering matrix \( S \), the unit matrix \( I \) and the matrix \( \lambda \) introducing the counting fields, \( \lambda = \text{diag}(e^{-i\lambda_1}, e^{-i\lambda_2}, \ldots, e^{-i\lambda_M}) \). The diagonal matrix \( \tilde{f} \) contains the Fermi-functions of the different terminals with \( \tilde{f} = \text{diag}(f_1, f_2, \ldots, f_M) \). With this we can show that gauge invariance requires

\[ \sum_\gamma \frac{\partial K}{\partial V_\gamma} + e \frac{\partial K}{\partial E} = 0. \]

(4)

We have used that the derivative of the Fermi-functions with respect to \( U_0 \) can be expressed as an energy-derivative and that \( \partial K/\partial U_0 = \sum_\gamma \partial K/\partial V_\gamma \). Therefore, the scattering matrix entering the expression for \( K \) depends not only on the energy \( E \) of the carriers but also via the internal potential landscape on the external voltages \( S = S(E, \{ V_\gamma \}) \). As mentioned above, the local internal potential has to be determined self-consistently and it is not necessarily an even function of magnetic field [22]. As a consequence, for nonlinear transport, the scattering matrix is not reversible, \( S_{\alpha\beta}(B, \{ V_\gamma \}) \neq S_{\beta\alpha}(-B, \{ V_\gamma \}) \). This implies immediately the breakdown of the fluctuation theorem [11] and [2] for nonlinear transport, since any derivation is based upon reciprocity.

The lack of reversibility can be shown explicitly for the Mach-Zehnder interferometer. Coulomb-interactions between the two edge states moving through the interferometer lead to internal potentials \( U_o, U_i \) in the outer and inner edge. In this respect the inner edge acts as a gate on the outer edge. For the interference, this gives rise to an additional phase difference \( \varphi(B) = e\Delta U_o \tau /\hbar \) between the two interferometer arms. Here, \( \tau \) is the time an electron needs to traverse the interferometer, and \( \Delta U_o \) is the difference of the internal potential in the outer edge between upper and lower arm.

It is easy to see that the internal screening potential \( U_o \) is not an even function of magnetic field: For positive magnetic field as shown in Fig. 11 only processes from left to right contribute, and the internal potential will depend on the reflection \( R_A = 1 - T_A \) of the left beam splitter and on the voltages \( V_1 \) and \( V_2 \). For inverted magnetic field, processes from right to left are important, which depend on \( R_B = 1 - T_B \) and voltages \( V_3 \) and \( V_4 \). To be explicit, we determine the local internal potential self-consistently within a Hartree approximation [22, 24, 30]. The average charges \( q_o \) and \( q_i \) in the edges of the upper interferometer arm are on the one hand expressed as the difference between injected and screened charge, and are on the other hand determined by Coulomb interaction. For positive magnetic field, this determines the internal...
The number of vertical lines mean the order of the derivative with respect to quantities of different terminals. The summations go over all possible permutations, where the number of permutations is given by \( \binom{n}{k} \) with \( k \) the number of circles and \( l \) the number of vertical lines. Higher order correlations follow the same rules and can easily be constructed.

\[ q_i = e^2 D(V_1 - U_i) = C(U_i - U_o) \]  
\[ q_o = e^2 D(R_A V_1 + T_A V_2 - U_o) = C(U_o - U_i). \]

Here, \( C \) is the geometric capacitance between the two edges, and \( D \) is the density of states of an edge state. Similar equations hold for the lower interferometer arm and for reversed magnetic field. To first order in external voltage, the potential difference \( \Delta U_o = \sum u_o V_o \) is determined by the characteristic potentials \( u_o = [\partial \Delta U_o / \partial V_o]_{eq} \). We find \( u_3(B) = u_1(-B) = 0 \) and \( u_1(B) = R_A - e^2 DT_A/(2C + e^2 D) \), \( u_3(-B) = R_B - e^2 DT_B/(2C + e^2 D) \), as well as \( u_2(B) = -u_1(B) \) and \( u_4(-B) = -u_3(B) \).

Using the characteristic potentials, the self-consistent transmission probability \( T_{31} = T_{31}(+B, V_1 - V_2) \) for a particle in the interfering edge to transmit from terminal 1 to 3 for positive magnetic field is \( T_{31} = R_AR_B + T_AT_B - 2\sqrt{R_AR_BT_AT_B} \cos(\Phi - \varphi) \) with \( \varphi(+B) = eu_1(+B)\tau(V_1 - V_2)/h \). For \( T_{13} = T_{13}(-B, V_3 - V_4) \) at negative magnetic field, the additional phase is \( \varphi(-B) = eu_3(-B)\tau(V_3 - V_4)/h \). The lack of reversibility out of equilibrium is evident:

\[ T_{31}(+B, V_1 - V_2) \neq T_{13}(-B, V_3 - V_4). \]

This means that the fluctuation relation \([2]\) is strictly speaking valid only at equilibrium but has corrections for finite voltages. In general, taking into account interactions beyond the Hartree-level will not reestablish reversibility.

**Fluctuation relations for correlation functions** – Including interactions, the fluctuation relation \([2]\) for the counting statistics is not valid anymore, as shown above explicitly within a Hartree-model. Nevertheless, we can derive fluctuation relations for current correlation functions. We emphasize that the following section is general, no specific model for interactions is needed. It is useful to expand the first few cumulants for \( eV \ll k_B T \),

\[ I_\alpha = \sum_\beta G^{(1)}_{\alpha\beta} V_\beta + \sum_\beta G^{(2)}_{\alpha\beta\gamma} V_\beta V_\gamma + O(V^3) \]

\[ S_{\alpha\beta} = S^{(0)}_{\alpha\beta} + \sum_\gamma S^{(1)}_{\alpha\beta\gamma} V_\gamma + O(V^2) \]

\[ C_{\alpha\beta\gamma} = C^{(0)}_{\alpha\beta\gamma} + O(V) \]

Up to second order in voltage, the mean current \( I_\alpha \) in terminal \( \alpha \) is determined by the linear and nonlinear conductance coefficients, \( G^{(1)}_{\alpha\beta} \) and \( G^{(2)}_{\alpha\beta\gamma} \). The zero-frequency current correlations \( S_{\alpha\beta} = \langle \Delta I_\alpha \Delta I_\beta \rangle \) contain equilibrium Nyquist noise \( S^{(0)}_{\alpha\beta} \) and the noise susceptibility \( S^{(1)}_{\alpha\beta\gamma} \) which includes the emergent shot noise. Of the third cumulant \( C_{\alpha\beta\gamma} = \langle \Delta I_\alpha \Delta I_\beta \Delta I_\gamma \rangle \), only the equilibrium value \( C^{(0)}_{\alpha\beta\gamma} \) is used in the following. All response coefficients are obtained from the generating function, e.g. \( G^{(2)}_{\alpha\beta\gamma} = -ie\partial^2 F/\partial \phi_\alpha \partial \phi_\beta \partial \phi_\gamma / t \), where the index 0 means setting \( \Lambda \) and \( \Lambda \) to zero.

(Anti-)symmetrizing the above definitions, both the fluctuation-dissipation theorem (for +), and the Onsager-Casimir relations (for −), can be formulated concisely as

\[ S^{(0)}_{\alpha\beta\pm} = k_B T (G^{(1)}_{\alpha\beta\pm} + G^{(1)}_{\beta\alpha\pm}) = \pm S^{(0)}_{\alpha\beta\pm} \]

The next order fluctuation relation connects the third cumulant at equilibrium which is odd in magnetic field with combinations of the noise susceptibility and nonlinear conductance coefficients,

\[ C^{(0)}_{\alpha\beta\gamma, \pm} = k_B T \left( S^{(1)}_{\alpha\beta\gamma\pm} + S^{(1)}_{\alpha\gamma\beta\pm} + S^{(1)}_{\beta\alpha\gamma\pm} \right) - k_B T (G^{(2)}_{\alpha\beta\gamma\pm} + G^{(2)}_{\beta\alpha\gamma\pm} + G^{(2)}_{\gamma\alpha\beta\pm}) = \mp C^{(0)}_{\alpha\beta\gamma, \pm} \]

These universal fluctuation relations can be extended to any order: A current correlation function at equilibrium is expressed by combinations of successive response coefficients of lower order current cumulants. They are graphically represented in figure \([2]\). The two first lines of the figure correspond to Eqs. \([11]\) and \([12]\), higher order relations can easily be constructed.

The derivation of the fluctuation relations is based on the following properties of the generating function:

\[ F_\pm(-\Lambda, \Lambda) = F_\pm(0, \Lambda) = 0 \]

\[ F_\pm(i\Lambda, 0) = \pm F_\pm(-i\Lambda, 0) \]

The first equation defines a special symmetry point at \( i\Lambda = -\Lambda \) for which the generating function vanishes, just as for \( \Lambda = 0 \) which originates from probability conservation. To demonstrate it, for the case of a non-interacting system, we expand Eq. \([2]\) in terms of multi-particle scattering events \([11]\) and use the detailed balance for Fermi
functions \( f_\alpha(1 - f_\beta) = \exp(A_\alpha - A_\beta)f_\beta(1 - f_\alpha) \). Neither magnetic field symmetry nor microreversibility are needed. For a system with electron-electron interactions we start from the definition of the generating function \( F(i\Lambda) = \ln \left\langle e^{-i\Lambda Q_0} e^{i\Lambda Q_t} \right\rangle_0 \). Here, \( Q_0 \) and \( Q_t \) denote the charge operators at time 0 and time \( t \), and the expectation value is taken with respect to the initial state, described by a grand-canonical density matrix. At time 0 the conductor is decoupled from the reservoirs, and the initial Hamiltonian \( H_0 \) commutes with the charge \( Q_0 \).

To derive Eq. (13), we use that the total energy in the system “conductor+reservoirs” is conserved at all times. To this last term leads to

\[
\langle -h^{2k-l} \partial V_{\alpha}^{k-l} \rangle^2 \uparrow \downarrow \equiv \langle -h^{2k-l} \partial V_{\alpha}^{k-l} \rangle^2
\]

valid even if Eq. (1) is not true. The second equation, Eq. (14) represents the fluctuation relation (2) at equilibrium to response coefficients of correlations of internal potential, and vanish for \( B \). Similar arguments hold for response coefficients with 1 ↔ 3. Using \( dg/\mathcal{U} \equiv (4e^{2}\tau / h^{2}) \sqrt{R_B T_B R_A T_A} \sin \Phi \), Eq. (12) simplifies for the MZI to

\[
2S_{31,3}^{(1)} = k_B T_0 T_{G}^{(2)} = \pm k_B T_{u3}(-B) dg/\mathcal{U} \tag{18}
\]

\[
2S_{31,1}^{(1)} = k_B T_0 T_{G}^{(2)} = k_B T_{u1}(B) dg/\mathcal{U}. \tag{19}
\]

The fluctuation relation (2) which does not account for magnetic field asymmetry in screening effects, would require that the anti-symmetrized part (−) of the above equations is identically zero \[16\]. Measuring a nonlinear conductance coefficient \( G_{33}^{(2)} \) or noise susceptibility \( S_{31,3}^{(1)} \) which is asymmetric in magnetic field proofs Eq. (2) wrong. The fluctuation relations Eqs. (15) and (19) are linear in temperature, periodic with the magnetic flux \( \Phi \) and depend on the reflection of the beam splitters; they can be experimentally verified.

**Conclusion** – We have shown that electron-electron interactions lead to a breakdown of the usual fluctuation relations for the full counting statistics in the presence of a magnetic field. The reason is, that interactions can induce effective deviations from microreversibility of scattering processes out of equilibrium. Instead fluctuation relations can be derived which relate correlation functions at equilibrium to response coefficients of correlations of lower order. These fluctuation relations are valid even in presence of magnetic field asymmetry.

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**Appendix A: Derivation of the fluctuation relation** – The fluctuation relation, Eq. (1), which holds in the presence of microreversibility, was derived in various systems \[12, 13, 14, 16\], here we demonstrate it for a non-interacting system described by a scattering matrix.

Most directly, the symmetry relation Eq. (2) in its non-symmetrized form is derived, \( F(i\Lambda, B) = F(-i\Lambda - A, -B) \). The generating function Eq. (4) is \( F(i\Lambda) = t \int \frac{dg}{\mathcal{U}} H(i\Lambda) \) with the integrand

\[
H(i\Lambda) = \ln \det \left[ 1 + f \left( \lambda \mathcal{A} \lambda S - 1 \right) \right]. \tag{20}
\]

The determinant can be expanded in terms of multi-particle scattering events \[11, 32\],

\[
H(i\Lambda, B) = \ln \sum_{\{a\}, \{b\}} \left| \det \left( S_{\{a\}\{b\}} \right) \right|^2 \tag{21}
\]

\[
\exp \left( i \sum_{\alpha \in \{a\}} \lambda_\alpha - i \sum_{\alpha \in \{b\}} \lambda_\alpha \right) \prod_{\gamma \in \{a\}} f_\gamma \prod_{\gamma \notin \{a\}} (1 - f_\gamma).
\]
For conductors with a single transport mode, \{a\} denotes a set of reservoirs from which particles are injected, and \{b\} is the set of reservoirs into which particles are emitted. The first sum in Eq. (21) runs over all possible sets \{a\} and \{b\}, and represents all possible, distinct ways of scattering a number \(m\) of particles, with \(m\) ranging from 0 to \(M\). The probability that \(m\) particles are scattered from \{a\} to \{b\} is given by \(|\det S_{\{a\}}^{(b)}|^2\), where the matrix \(S_{\{b\}}^{(a)}\) is formed by taking the intersecting matrix elements of the columns corresponding to the elements in \{a\} and the rows corresponding to the elements in \{b\} from the scattering matrix \(S\). The products over the occupation functions of the different terminals in Eq. (21) determine the probability that exactly \(m\) particles from set \{a\} are injected. The exponent contains all counting fields of set \{a\} and \{b\}.

Replacing in Eq. (21) the magnetic field \(B\) by \(-B\) and all counting fields \(i\lambda_o\) by \(-i\lambda_o - A_o\) leads to additional exponential factors. It is useful to define the set \{c\} as the contacts from which particles are injected but not into which they are emitted, the set \{d\} as those into which particles are transmitted but not from which they are injected, the set \{e\} as the contacts from which they are injected and into which they are transmitted, and the set \{f\} as the set not at all touched. Then, with \(\{a\} = \{c, e\}, \{b\} = \{d, e\}\) and \(\not\{a\} = \{d, f\}, \not\{b\} = \{c, f\}\), the summations and products over elements of the different sets can be split up, for example \(\sum_{\gamma \in \{a\}} = \prod_{\gamma \in \{c\}} \prod_{\gamma \in \{e\}} \sum_{\alpha \in \{d\}} = \sum_{\alpha \in \{c\}} + \sum_{\alpha \in \{e\}}\). With the help of the detailed balance relation for Fermi functions, \(f_\alpha(1-f_\beta) = e^{A_\alpha-A_\beta}f_\beta(1-f_\alpha)\), we find for products concerning the sets \{c\} and \{d\}

\[
\exp \left( \sum_{\alpha \in \{d\}} A_\alpha - \sum_{\alpha \in \{e\}} A_\alpha \right) \prod_{\gamma \in \{c\}} f_\gamma \prod_{\gamma \in \{d\}} (1-f_\gamma) = \prod_{\gamma \in \{c\}} (1-f_\gamma) \prod_{\gamma \in \{d\}} f_\gamma. \tag{22}
\]

The second and crucial point of the derivation is the use of reciprocity \(S_{\alpha,\beta}(B) = S_{\beta,\alpha}(-B)\) which implies \(S_{\{b\}}^{(a)}(-B) = S_{\{a\}}^{(b)}(B)\). With this, and recombining the sets of Fermi functions one obtains finally

\[
H(-A - i\Lambda, B) = \ln \sum_{\{a\}, \{b\}} \left|\det \left( S_{\{a\}}^{(b)}(B) \right) \right|^2 \tag{23}
\]

\[
\exp \left( i \sum_{\alpha \in \{b\}} \lambda_\alpha - i \sum_{\alpha \in \{a\}} \lambda_\alpha \right) \prod_{\gamma \in \{b\}} f_\gamma \prod_{\gamma \not\in \{b\}} (1-f_\gamma),
\]

which is indeed equal to \(H(i\Lambda, B)\) in Eq. (21), since the sum runs over all possible sets \{a\} and \{b\}.

As discussed in the core of the paper, screening effects lead to an internal potential which can be asymmetric in magnetic field. Then, away from equilibrium, the reciprocity relation is not valid, \(S_{\alpha,\beta}(E, U(B)) \neq S_{\beta,\alpha}(E, U(-B))\), and with this the symmetry relation (24) breaks down.

Appendix B: Symmetry point of the generating function

- Eq. (13) defines a symmetry point for the generating function which is valid for any value of the magnetic field, \(F(-A, B) = F(0, B) = 0\). It can be derived without the use of microreversibility. For a system, described by a scattering matrix, we start with Eq. (21) at \(i\Lambda = -A\)

\[
H(-A) = \ln \sum_{\{a\}, \{b\}} \left|\det \left( S_{\{a\}}^{(b)}(B) \right) \right|^2 \tag{24}
\]

\[
\exp \left( - \sum_{\alpha \in \{a\}} A_\alpha + \sum_{\alpha \in \{b\}} A_\alpha \right) \prod_{\gamma \in \{a\}} f_\gamma \prod_{\gamma \not\in \{a\}} (1-f_\gamma).
\]

Reformulating the sets of contacts \{a\} and \{b\} in terms of \(\{c\} - \{f\}\) as introduced above and inserting the property (22), the exponential factors will be absorbed into the Fermi functions and we find

\[
H(-A) = \ln \sum_{\{a\}, \{b\}} \left|\det \left( S_{\{a\}}^{(b)}(B) \right) \right|^2 \prod_{\gamma \in \{b\}} f_\gamma \prod_{\gamma \not\in \{b\}} (1-f_\gamma) = 0 \tag{25}
\]

In the first line, the sum over \{a\} can be performed and equals one because of probability conservation. In the scattering picture it is thus easy to see that the identity \(F_\pm(0) = F_\pm(-A) = 0\) is a consequence of both probability conservation and the detailed balance for Fermi functions.

For systems with arbitrary interactions, the generating function for charge transfer is defined as an expectation value with respect to the initial state \(\tilde{\rho} = \langle e^{-iA\tilde{Q}_b}e^{iA\tilde{Q}_b}\rangle_0\).

\[
F(i\Lambda) = \ln \langle e^{-i\Lambda\tilde{Q}_b}e^{i\Lambda\tilde{Q}_b}\rangle_0. \tag{26}
\]

Here, the vector quantities \(\tilde{Q}_b\) and \(\tilde{Q}_l\) denote the charge operators in the different terminals at time 0 and time \(t\). The initial state is described by the grand-canonical density matrix \(\rho_0 = e^{-\beta\tilde{H}_0 + \Lambda\tilde{Q}_b}/Z_0\), with the partition sum \(Z_0 = tr[e^{-\beta\tilde{H}_0 + \Lambda\tilde{Q}_b}]\). The Hamiltonian \(\tilde{H}_0\) is composed of the Hamiltonians of all reservoirs and of the scatterer, which at time 0 are supposed to be decoupled. Importantly, the time evolution operator \(\tilde{U}(t)\) contains in addition the coupling to the reservoir and interaction terms and does not commute with \(\tilde{H}_0\). Inserting the initial density matrix into the definition above and using the cyclic property of the trace as well as the fact that
For the last step we used that $\hat{H}_0$ and $\hat{Q}_0$ commute, we obtain
\begin{equation}
F(-A) = \ln \langle e^{\hat{A}\hat{Q}_0} e^{-\hat{A}\hat{Q}_0} \rangle = \frac{1}{Z_0} tr \left[ e^{-\beta \hat{H}_0 + \hat{A} \hat{Q}_0 \hat{U}^\dagger(t)} e^{\hat{A} \hat{Q}_0 \hat{U}(t)} e^{-\hat{A} \hat{Q}_0} \right]
\end{equation}

\begin{align}
&= \frac{1}{Z_0} tr \left[ e^{-\beta \hat{H}_0 \hat{U}(t)} e^{\hat{A} \hat{Q}_0 \hat{U}(t)} \right] \\
&= \frac{1}{Z_0} tr \left[ e^{-\beta \hat{H}_0 \hat{U}(t)} e^{\hat{A} \hat{Q}_0} e^{\beta \hat{H}_0 \hat{U}(t)} \right].
\end{align}

The trace is evaluated by $tr[...] = \sum_n \langle n | ... | n \rangle$, where we choose the eigenbasis of the initial Hamiltonian, $\hat{H}_0 | n \rangle = \epsilon_n(0) | n \rangle$. The state $| n \rangle$ is characterized by a configuration of numbers of particles in all reservoirs and the scatterer. We introduce the total energy $\epsilon_n(0)$ of this particular configuration. The important point we can make is that scattering processes through the conductor leave this energy invariant. To proceed, we consider a matrix element of the last four operators in the last line of Eqs. (27), insert $1 = \sum_k | k \rangle | k \rangle$ in the middle and find
\begin{equation}
\langle m | e^{-\beta \hat{H}_0 \hat{U}(t)} e^{-\beta \hat{H}_0 \hat{U}^\dagger(t)} | n \rangle = \sum_k \langle m | e^{\beta \hat{H}_0 | k \rangle} e^{-\beta \epsilon_k(0)} \langle k(t) | m \rangle \\
= \sum_k \langle m | \langle k(t) | n \rangle = \delta_{nm}.
\end{equation}

For the last step we used that $\hat{H}_0 | k \rangle = \epsilon_k(0) | k \rangle$. This is justified, since the time evolution of state $| k \rangle$ simply means a change of the numbers of particles in each reservoir. Assuming that scattering processes are instantaneous, it will at any time form a basis for $\hat{H}_0$. Then, using that the total energy is conserved, $\epsilon_k(t) = \epsilon_k(0)$, the matrix element (28) is diagonal. This result leads to
\begin{equation}
F(-A) = \ln \left[ \frac{1}{Z_0} tr \left[ e^{-\beta \hat{H}_0 + \hat{A} \hat{Q}_0} \right] \right] = 0 \text{ as in Eq. (13).}
\end{equation}

We emphasize again that the identity Eq. (13) is valid for arbitrary electron-electron interactions and without the use of microreversibility. Special care should be taken of the case when (i) the problem is time-dependent, (ii) the temperature is not equal in all reservoirs, and (iii) a bath allows energy exchange, e.g. via electron-phonon interactions. Then, we would have to consider energy currents as well and introduce additional counting fields that account for the transferred energy. In this case, a similar relation can be derived.