Students’ Understanding on Formal Definition of Limit

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Abstract: This paper reports our preliminary findings of students’ understanding on the formal definition of limit - epsilon delta definition. The aim of this study is to develop students understanding of definition of limit by construction learning trajectories in teaching limit. The subjects were five students majoring in mathematics. The methods were test and interview. The test is used to identify how the subjects prove the limit using formal definition. Interview aims to confirm and explore information about the students’ difficulties in proving the limit using the definition. The interviews were then transcribed and analyzed along with the test results to get conclusions about the difficulties of students proving the limit by using formal definition. The result shows that the subjects were still facing difficulties in interpreting the definition of limits.

Keywords: Formal definition of limit, epsilon, delta, calculus.

1. Introduction

Limit is a mathematical concept introduced in high school level. It is being introduced intuitively by using a graph approach and focuses on determining the limit value in polynomial, rational, and trigonometric functions. At the university level in Indonesia, the limit concept especially in mathematics major introduced the limit in calculus course and the further will be discussed more in the analysis courses.

In calculus, the limit presents in a more formal way. The epsilon-delta definition is introduced as a definition that must be understood by students. They no longer only focus on how the limit value of a function, but try to prove the limit value. In calculus textbooks, the discussion of limits begins with various approaches. This approach is used to make it easier to understand the formal definition of a limit. The book Calculus by [1] begins the discussion of limits intuitively with a graphical approach. After that, it is given an intuitive definition and then it goes into the definition of left limit and right limit. Finally, the formal definition of limit is introduced, namely the definition of epsilon delta.

Various studies have shown the difficulty of students in understanding the definition of limits ([2]; [3]; [4]). In addition, the efforts have also been made to understand how limits are understood by students ([3]; [5]; [6]; [7]; [8]). Even, [9] also developed a learning concept in teaching limit in calculus course.

Introducing the definition of limit is not easy. [10] found that most students are able to calculate limits but cannot explain why this method of calculating limits is possible in relation to the concept of limits. Students can prove the limit correctly but do not show a good understanding of the formal definition of limit. Some of them state that the formal definition of limit is a fixed procedure used to
prove the truth of a limit. The students who can properly state the formal definition of limit and use it in proving limits, cannot link the definition of limit with the meaning of the limit notation that has been studied previously.

This article reports little findings of our projects in developing students understanding of the concept of limit. This article focuses on the initial understanding of students dealing with the formal definition of limit especially epsilon-delta definition.

2. Method
This research is a descriptive research that aims to know and describe the student's knowledge about the definition of limit by using $\epsilon$-$\delta$ definition. The formulation of the problem in this research is how the students majoring in mathematics understanding the formal definition of the limit. The subjects are five students majoring in mathematics. The methods are test and interview. The test is used to identify how the subjects proof the limit using formal definition. Interview aims to confirm and explore information about the students’ difficulties in proving the limit using the definition. The interviews were then transcribed and analyzed along with the test results to get conclusions about the difficulties of students proving the limit by using formal definition.

3. Discussion
This study aims to identify the understanding of students in understanding the concept of a formal limit. Five students were selected as subjects in this study. The five students who have been selected have good math and communication skills. The five students were initially given a problem of proving the limit then the students were asked to prove the value of the limit using the formal definition – epsilon delta definition. The given problem was prove the limit $\lim_{x \to 1} 3x + 5 = 8$. After that, they were interviewed in depth about their written answer. Interviews conducted to confirm students’ understanding the concept of a formal limit.

The results of the tests indicate that the five subjects were able to write proof of limit using the epsilon-delta definition (Figure 1). They initially performed a preliminary analysis to find the delta value associated with epsilon. After that, they began to work on the proof limit by using the definition. This shows that the subject were able to prove the limit using the formal definition of limit.

![Figure 1. Subjects’ Works](image)

At the beginning of the interview, the subjects were asked to explain what they had written to answer the question. After that, they were given several questions aimed at digging deeper into the
extent of their understanding of the meaning of the definition of formal limit. In the interview, some students can explain their written answer in a confident way. Explaining how getting delta by performing first a preliminary steps. And then letting epsilon based on their preliminary analysis, and proof that the limit is true. They all described their work in the way that they know what they wrote is correct. However, when we move deeper, some difficulties and misunderstanding appears.

The results of the interviews show that all subjects did not understand the definition of formal limit. Some misconceptions and difficulties in understanding the formal definition of limits appear at the time of the interview. There is even a subject that still does not understand the meaning of the limit itself. As the result, some of the difficulties found are (1) the relationships between epsilon and delta, (2) interpretation of the formal definition into a graph, and (3) the role of the absolute value in the formal definition of limit.

The results indicate that the students are able to demonstrate the ability to write proof of limit using formal definition, but they have not been able to interpret the definition. This is in line with the results of [4] who have reported that few students understand the definition of limits strictly, even these students have difficulty applying the definition of limits to solve problems related to limits. Students tend to use their understanding of intuitive limit notation to explain the limit of a function, that is, \( \lim_{x \to c} f(x) = L \) tends to be defined \( f(x) \) close to \( L \) when \( x \) approaches \( c \). Meanwhile, [11] found many students have difficulty understanding the definition of formal limit, while students who intuitively understand the definition of limits or based on the meaning of limit notation tend to solve problems related to the concept of limit.

Some researchers who have reported the results of their research on student limits ([4]; [11]; [12]; [13]). All the studies reported that the students have potential to perform misunderstanding of the concept of limits. It cannot justifies the students understand the concept of limits by the ability to solve problems, calculate, prove or explain the concepts related to the limits, without realizing the possibility of students cannot integrate their understandings related to the concept of limits that have been studied. It is possible that the lecturer who teaches the limits in the Calculus and Real Analysis courses in college assumes that his students have understood the limits because they have been able to calculate limits, prove limits, or represent limits in the function graph without realizing the possibility of students having those misunderstandings.

4. Conclusion
This study aims to see students' understanding of the concept of limit definition – epsilon delta definition. The study also obtained information on the difficulties and misconceptions experienced by students. The results showed that the students were able to write the limit proof in accordance with the definition of formal limit. They started with a preliminary analysis to determine the value of \( \delta \) associated with \( \epsilon \) and proceed with the proof of limit according to the limit definition. However, when further searches, students show difficulty in explaining the meaning contained in the formal definition of the limit. Some difficulties encountered are (1) the relationship of epsilon and delta in formal definition limit, (2) interpreting in the definition into graph, and (3) the role of absolute value in the definition of formal limit.

This study shows that the definition of formal limit is still one of the mathematical concepts that need to be focused on. In our project, we are developing learning trajectories to develop students understanding of the concept of formal definition of limits. Therefore, it was expected that our effort to teach limit meaningful for students can be succeed by this preliminary report.

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