Mathematical model of pressure distribution in the main gas-transport system

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Abstract. The issues of improving the methods for calculating pressures and material balance of the main gas-transport system based on the energy theory of gas flux-distribution are considered. The matrix method for determining the material balance equations for linearly independent and nonlinearly independent gas pipeline network contours was used as a mathematical model of the process to calculate the pressure distribution at the nodal points and gas flows. On the basis of the energy conservation law, a power balance equation for gas supply sources and its consumption distributed among the elements of the gas-transport network has been proposed. A mathematical model of the gas distribution process has been obtained.

Introduction

New opportunities have appeared for conducting research in many areas of engineering and communication systems with the application of the basic principles of the energy theory of flows and networks which forced to re-examine the general research methods, the individual computational mathematical methods and at the same time the issues of developing theoretical methods for analyzing and synthesizing Gas-Transport Systems (GTS) for gas [1, 2, 3, 4].

The GTS networks have their own specifics and differ from the electrical networks of the power system (except for the non-linear dependencies of the state parameters), in particular the following is true for GTS networks:

- there are only longitudinal elements (pipelines and compressor stations - CS) and there are no transverse elements and contours;
- inputs (gas sources) and outputs (gas consumers) are carried out only by one available clip and it can be considered that an ideal source of gas flow is connected (similar to an ideal current source of an electrical circuit);
- there is a CS element which sharply raises the gas pressure at the plot output during a constant gas flow to the plot.

These distinctive features led to the need to modify the structure of the energy equations, that is: the components of the power sources and consumers of gas, since they do not constitute branches, are derived from the total sum of the components along the branches of the networks and are added as a separate amount for the GTS nodes with appropriate consideration of their signs.
Methodology

With steady gas movement in the gas turbine system occurs, the pressure distribution at the nodal points and gas flows on its branches is described with a certain accuracy by linear and nonlinear functions, which are the equations of state of the GTS and in some specific cases can be used with different forms of writing. The most expedient of them is determined by the nature of the problem being solved [5, 6, 7, 8, 9].

The structure of the network of GTS is determined by the method of connecting elements and does not depend on their type. This structure can be displayed in the form of a graph, which is a set of vertices (nodes) that are connected to each other by edges (branches). A network of GTS of arbitrary structure consisting of \( s \) nodes, \( n \) branches and \( L \) linearly independent contours is considered. The material balance equation at each node of the system is

\[
\sum_{i=1}^{s} A_{ji} Q_i = q_j, \quad \text{where } q = q_j - \text{vector of outflows (tributaries) of gas from the system (to the system) in the } j \text{ node; } A = [A_{ji}] - \text{the full matrix of incidence, that is, the matrix of connections of branches in the nodes. The number of rows of which is equal to the number of vertices of the graph } s \text{ and the number of columns to the number of } n \text{ edges. The elements of this matrix can take one of three values: } A_{ji} = 1, -1, \text{ if node } j, \text{ respectively, is the initial and final for the branch } i; A_{ji} = 0, \text{ if node } j \text{ does not belong to the branch } i. \text{ Since in the chain graph each branch connects two nodes and the flow of this branch comes from one node and enters another node, the gas flow in each branch enters only two equations of the system (1). Therefore in each column of the matrix } A_{ji} \text{ can only have one positive and one negative unit and the other elements are zeros. Therefore the sum of all rows of this matrix (by columns) should give a zero (row) matrix:}
\]

\[
n_i^{(s)} A_{ji} = 0,
\]

where \( n_i \) - unit line dimension \( s \times 1 \).

If outlining (2) to the line corresponding to the balancing node and accept the number of it last then the expression (2) will be written as follows:

\[
[n_i : 1]^{(s)} \frac{A_{ji}}{A_{ji}} = 0, \text{ from which follows } A_{ji}^{(s)} = -n_i^{(s)} A_{ji},
\]

where \( A, A_{ji}^{(s)} \) - respectively, the matrix of incidents for schemes without balancing and with balancing nodes; \( n_i \) - unit line dimension \((s-1)x1\). As a result of the deduction from (1) from the last equation a system \((s-1)\) of linearly independent equations will be obtained:

\[
\sum_{i=1}^{s} A_{ji} Q_i = q_j; \quad j = 1, 2, \cdots, (s - 1).
\]

System (4) in matrix form can be represented as

\[
[A_{ji}] Q_i = q_j \text{ or } A Q = q, \quad j = 1, 2, \cdots, (s - 1) \quad i = 1, 2, \cdots, n.
\]

Nonlinear equations from [3, 5]
for a given structure GTS can be jointly represented as
\[
\sum_{j=1}^{i} \overline{A}_{ij}P_j^2 = \sum_{j=1}^{i} \overline{A}_{ij}P_j^2 = \overline{K}Q_j^2,
\]  

(8)

where \( \overline{A}_{ij} \) are elements of a complete rectangular matrix of connections of nodes and branches for the GTS CS scheme which definitely describes the structure of this scheme and the orientation of its branches. Here \( \overline{A}_{ij} = 1 \), where node \( j \) is the starting for the branch \( i \) without CS; \( \overline{A}_{ij} = 0 \),

if node \( j \) is the starting for the branch \( i \) with CS; \( \overline{A}_{ij} = -1 \), if the node \( j \) is a final for the branches \( i \) with the CS and without the CS (i.e., the branch with the CS and without the CS is oriented in the direction to the node \( j \));

\( \overline{A}_{ij} = 0 \), if the node \( j \) does not belong to the branch \( i \).

System (8) in matrix form is represented as
\[
\overline{A}'P^2 = \begin{bmatrix} E_n \overline{K} \end{bmatrix} \begin{bmatrix} Q^2 \end{bmatrix} \text{ or } \overline{A}'_jP^2 = \begin{bmatrix} E_n \overline{K} \end{bmatrix} \begin{bmatrix} Q^2 \end{bmatrix}; \quad j = 1,2,\cdots,s; \quad i = 1,2,\cdots,n,
\]  

(9)

where \( E_n \) - unit matrix of dimension \( n^2 \); \( \overline{A}' \) - transposed matrix with respect to \( \overline{A}_{ij} \).

The form of writing the CS equation (7), which has an identical structure with the expression of the gas pipeline section (6) is convenient for solving them together and analyzing the GTS modes. However the presence of equivalent parameters \( a_i \) and \( b_i \) in expression (7) makes it very difficult to represent equations in the compact canonical form of a writing. In many papers, in particular [3], these expressions in the form of the balance equation for pressure drops along the contour are represented using implicit functions in which all state variables of the GTS are not explicitly participating. To overcome this difficulty, the equation of CS (7) is represented in another form of writing:
\[
P_j^2 - P_k^2 = b_i Q_j^2 - h_i,
\]  

(10)

where \( h_i = (a_i - 1)P_j^2 \) - additional term of the equation (a.t.e.) of CS \( i \) section \( i \in I_z \). Then for the GTS network of arbitrary structure the equation (10) can be represented by the matrix equation
\[
A'_iP^2 = \begin{bmatrix} E_n \overline{K} \end{bmatrix} \begin{bmatrix} Q^2 \end{bmatrix} - h \quad \text{or} \quad A'_jP^2 = \begin{bmatrix} E_n \overline{K} \end{bmatrix} \begin{bmatrix} Q^2 \end{bmatrix} - h_i,
\]  

(11)

where \( h = h_i = (a_i - 1)P_j^2 \) - vector of a.t.e. C.S. \( 1 \times n \); \( E_n \) - dimension unit matrix \( n^2 \).

The equations of state for linearly independent contours had been composed, for which we introduce the branch connection matrix into independent contours, which is a rectangular matrix
\[
B = [B_{ij}], \quad i = 1,2,\ldots,L; \quad i = 1,2,\ldots,n,
\]  

(12)

the number of rows of which is equal to the number of independent contours of the \( L \) graph network, and the number of columns - to the number of branches \( n \).

Balance of the fall (loss) of the square of pressure \( \Delta P_i = P_j^2 - P_k^2 \) on the closed contours of
the network and is formulated as follows: algebraic sum of falls \( \Delta P_i \) on the branches of the contour is zero, i.e. \( B \cdot \Delta P = 0 \) or

\[
[B]_i \cdot \Delta P = 0, \quad i = 1, \ldots, L, \; \Delta P = 0.
\]

According to the nodal pressures using the complete matrix of incidents it’s possible to determine the fall of \( \Delta P_i \) on the branches of the scheme. Since each column of the matrix \( A_n \) has one positive unit in place of the initial vertex and one negative on the final vertex of the branch, it is sufficient to multiply the matrix of compounds \( A'_n = A_{ij} \) transposed to the right of the column of the square of the nodal pressures \( P^2 = P_j^2 \), to get the column of the differences of the square of pressures at the ends of each branch, i.e. the fall \( \Delta P \) on the branches:

\[
\Delta P = A'_n P^2 \quad \text{or} \quad \Delta P = [A_n] P_i^2.
\]

Considering that \( \Delta P = \Delta P_i \), jointly solving (11), (13) and (14) we get the expression

\[
BA'^2 = B[E_n \bar{K}]Q^2 - h_i = 0 \quad \text{or} \quad [B]_i [A_n] P_j^2 = [B]_i [E_n \bar{K}] Q^2 - h_i = 0,
\]

where \( h_i = [B]_i h_i \) - contour column a.t.e. CS, representing algebraic sums a.t.e. of CS \( P^i \) branches included in each independent contour.

The components of the vector \( P \) in (15) are the nodal pressures of all the \( s \) nodes of the GTS scheme (including the balancing one). Often it is advisable to determine them relatively to the balancing node as the difference of squares from each of the independent nodes of the GTS scheme with respect to the pressure square of the balancing node. These values differ from the squares of the nodal pressures \( P^i_j (j = 1, \ldots, s) \), included in (15), by the same value - the square pressure of the balancing node \( P^i_\delta \) (at the same time the balancing node is supposed to be the last by the number, i.e. \( P_j = P^i_\delta \)). For this, the complete incident matrix included in (15) and the nodal pressure vector can be divided into blocks and sub vectors as follows:

\[
A_n = \begin{bmatrix} A \\ -nA \end{bmatrix}; \quad P = \begin{bmatrix} P_j \\ P_\delta \end{bmatrix}, \quad j = 1, \ldots, (s-1).
\]

Then the expression (14) for the difference of the squares of pressure on the branches of the GTS scheme can be represented as

\[
\Delta P = A'_n P = A' - A' n \begin{bmatrix} p_j^2 \\ p_\delta^2 \end{bmatrix} = A' \left( p_j^2 - n p_\delta^2 \right),
\]

\( j = 1, \ldots, (s-1), \)

where \( n \) is the unit column of dimension \( 1 \times (s-1) \).

Solving (13) and (16) together, we obtain the matrix equation in another form of writing

\[
BA'(P_j^2 - n P_\delta^2) = B[E_n \bar{K}]Q^2 - h_i = 0 \quad \text{or} \quad [B]_i [A_n] (P_j^2 - P_\delta^2) = [B]_i [E_n \bar{K}] Q^2 - h_i = 0.
\]

To form the equation of state for a GTS, according to (5) and (17) it is necessary to preliminarily determine the matrixes of the compounds \( A \) and \( B \) which in an analytical form, definitely reflect the configuration scheme of the network. The matrix \( A \) contains comprehensive information about the network of the scheme and it is definitely possible to recover the configuration of the corresponding
The matrix $B$ in the general case does not contain complete information about the structure of the considered GTS scheme since the open parts of the circuit are not reflected in it. Difficulties associated with the formation of the matrix $B$ can be avoided if we take into account the fact that the matrix $A$ contains comprehensive information about the structure of the GTS scheme, including the necessary for the preparation of the matrix $B$. To realize this possibility, it is necessary to establish an analytical dependence connecting matrixes $A$ and $B$.

Since equation (17) is valid for any vector of nodal pressures and $P_i \neq 0$, $P_j \neq 0$, therefore it should always be

$$BA' = [B_i] [A_j] = 0. \quad (18)$$

Expression (18) displays the general topological property of the GTS network graph. Knowing the matrix $A$, we can find the matrix $B$, using relation (18) and elements of graph theory.

Summary
In a complex network of $s$ nodes and $n$ branches, nodal pressures $P_j (j=1,2,\ldots,s)$, gas flows in the branches $Q_i (i=1,2,\ldots,n)$ and external flows (tributaries and withdrawals) of gas $q_j (j=1,2,\ldots,s)$ are interconnected by equations (4), (17). However, for the same network it’s only possible to make a single equation connecting these parameters together. Considering the distinctive features of GTS networks based on the energy conservation law for a given GTS scheme, we can write the balance of power supply sources of gas, gas consumers and power loss (energy dissipation) in the GTS network in the form of

$$\sum_{j=1}^{s} P_j q_j - \sum_{i=1}^{n} \Pi_i Q_i = 0. \quad (19)$$

This equation shows that the sum of the products of pressures $P_j$ and external gas $q_j$ flows of all $s$ nodes of the directional graph of the GTS circuit minus the sum of the products of the pressure $\Pi_i$ drop of all $n$ branches to the corresponding gas flows $Q_i$, is zero [10].

Expression (19) is the energy equation for the power balance of the GTS circuit and expresses the energy conservation law for the same network at the same time. The physical meaning of this equation is explained as follows: at each moment of time, power (energy) which is introduced into the GTS network through its inputs and through the CS is consumed by gas consumers through its outputs and is distributed among network elements (gas pipelines) in the form of scattering, i.e. power loss.

References
[1] Volskiy E, Garlyauskas A, Gerchikov S 1980 Reliability and optimal reservation of gas fields and gas pipelines (Depths, Moscow).
[2] Merenkov A, Khosilev V 1985 Theory of hydraulic network (Science, Moscow).
[3] Pankratov V, Dubinskiy A, Sipershtein B 1988 Information and Computing Systems in the Pipeline Dispatcher Control (Depths, Leningrad).
[4] Ionin A 1989 Gas supply, Central scientific and technical publishing house.
[5] Pirumyan N, Kazaryan A 2018 Improving the methods of calculation and design of the gas-transport system of Armenia (Materials of international conference "Logistics, transport, ecology – 2018) 126-132.
[6] Pirumyan N, Ghazaryan H, Stakyan M 2018 Study of work reliability of gas subsurface storage by the method of finite elements (Proceedings of the 10th International Conference on Contemporary Problems of Architecture and Construction) 76-78.
[7] Ghazaryan H, Chibukhchyan S, Stakyan M 2016 Increasing reliability of gas-transportation system (GTS) by constructions and vibration stability criteria (Proceedings of the 8th International Conference on Contemporary Problems of Architecture and Construction) 168-170.

[8] Stakyan M, Ghazaryan A, Ghazaryan Yu 2016 Issues of optimizations of reliability of integrated system of power supply (Bulletin of NUACA) 2 (51) 74-77.

[9] Pirumyan N, Ghazaryan H, Stakyan M 2017 Changes in the stress-strain state of a rock mass in a zone of an underground gas storage (Proceedings of NUACA) 1 (64) 130-137.

[10] Pirumyan N, Stakyan M 2019 Bearing capacity of elements of a gas transportation system (E3S Web of Conferences, Form-2019, XXII International Scientific Conference “Construction the Formation of Living Environment”) 97 04027.