Photon-number-resolving segmented avalanche-photodiode detectors

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(Dated: August 29, 2017)

We investigate the feasibility and performance of photon-number-resolved photodetection employing avalanche photodiodes (APDs) with low dark counts. The main idea is to split \( n \) photons over \( m \) modes such that every mode has no more than one photon, which is detected alongside propagation by an APD. We characterize performance by evaluating the purities of positive-operator-valued measurements (POVMs), in terms of APD number and photon loss.

INTRODUCTION

Quantum measurements are essential to quantum technology. Photon-Number-Resolution (PNR) has become a need in various fields including linear optical quantum computing [1], quantum metrology and sensing [2], quantum cryptography [3], quantum imaging [4], and quantum communication [5] and conditional state preparation [6].

A PNR detector produces a signal proportional to the number of incident photons. Currently, PNR detectors can be realized with silicon photomultipliers [7], superconducting nanowires [8–10], linear mode avalanche photodiodes (APDs), quantum-dot field-effect transistors [11, 12], and transition-edge sensors (TES) [13, 14]. Several methods based on spatial- and time-multiplexing have been proposed for PNR measurements using APDs [10, 15–17].

In this paper we propose a way for PNR using segmented APDs with low dark currents. In the figure shown below we have APDs on waveguide. The set up whittles down \( n \) photons, one at a time. Our segmented photodetector features multiple APDs fed by the same waveguide, as shown in Fig. 1. Photons that are not absorbed in the first APD couple back into the waveguide and will be absorbed in one of the following detectors. The crucial advantage of this configuration is that nonideal quantum efficiency of the APDs doesn’t amount to photon loss, unlike butt-coupled PNR detectors in which temporally or spatially split photons impinge on APDs on the end of their path [15–17]. Hence, this linear array is essentially a long detector divided into \( m \) short detectors, each with an individual read-out. We envision that such a segmented photodetector will become feasible in large-scale integrated photonic platforms using either monolithic or heterogeneous integration of APDs on low-loss waveguides.

We will model the segmented detector using beam splitters as shown in Fig. 2 where waveguide coupling with APDs can be modeled by the reflectivities of the beam splitters, and the transmissivity per APD by \( \eta \). Before we address the model in earnest, we need to iterate that \( \eta \) won’t include quantum efficiency of the APD. This can be seen by a detail model of each APD coupling, pictured in Fig. 3. The APD’s field transmissivity is \( \alpha \) (such that the APD’s quantum efficiency is \( 1 - \alpha^2 \)). In fact, \( \alpha \) cannot be too large because APDs are not PNR detectors and we want the quantum efficiency \( 1 - \alpha^2 \ll 1/n \), for \( n \) incident photons, which will ensure that photons are detected no more than one at a time.

In a design reminiscent of interaction-free measurements proposed by Elitzur and Vaidman [18], we require that

- \( R_j + T_j = r_j^2 + t_j^2 = 1, \forall j \in [1, m] \).

\[
|n\rangle_{(r,t)} \rightarrow |n\rangle_{(r',t')} \quad \text{with} \quad t' + r'\sqrt{1 - \alpha^2} \quad \text{and} \quad tr' - r't'\sqrt{1 - \alpha^2}.
\]

FIG. 1: Segmented detector. Guided optics are used to detect photons alongside propagation by APDs. The design goal is to keep all undetected photons in the waveguide, for further detection.

FIG. 2: Model of a PNR segmented photodetector.

\[
|n\rangle_{(r,t)} \rightarrow |n\rangle_{(r',t')} \quad \text{with} \quad t' + r'\sqrt{1 - \alpha^2} \quad \text{and} \quad tr' - r't'\sqrt{1 - \alpha^2}.
\]

FIG. 3: Model for detection alongside propagation. If this is the \( j^{th} \) APD, then \( t_j = t_j' + r_j'\sqrt{1 - \alpha^2} \), where \( t_j = T_j^{1/2} \) in Fig. 2.
the bottom output of the exit beam splitter by nulled by destructive interference. The condition can be achieved by choosing parameters \((r, t, r', t')\) of the beam splitters, and absorption coefficient \(\alpha\), such that

\[
tr' - rt'\sqrt{1 - \alpha^2} = 0.
\]

If this is the case, then the detection process truly takes place alongside propagation and finite quantum efficiency — necessary here to attain PNR performance — won’t contribute to photon loss.

In the next section, we recall basic properties of the quantum formalism needed to evaluate PNR performance. We then return to the segmented detector model, present our theoretical characterization results, and conclude.

**DETECTOR CHARACTERIZATION USING POSITIVE-OPERATOR-VALUED-MEASUREMENTS (POVMS)**

The most general measurements in quantum physics are known as POVMs [19, 20]. Each measurement outcome \(k\) is given by an Hermitian operator, POVM element \(\hat{\Pi}_k\), with nonnegative eigenvalues such that the probability of an outcome for a quantum state \(\hat{\rho}\) is given by

\[
p_k = Tr(\hat{\rho}\hat{\Pi}_k). \tag{2}
\]

These operators satisfy the completeness property,

\[
\sum_k \hat{\Pi}_k = I
\]

which makes the sum of probabilities \(\sum_k p_k = 1\) for different outcomes to be unity. Therefore, these measurements completely describe all possible outcomes for any quantum measurement. For a phase insensitive detector the POVM element for \(k\) clicks is given as

\[
\hat{\Pi}_k = \sum_{n=0}^\infty P(k|n)|n\rangle\langle n|, \tag{3}
\]

where \(P(k|n)\) represents the conditional probability of getting \(k\) clicks given \(n\)-photon input state. The POVMs are more general from the projective measurements in couple of ways. First, unlike the projective measurements the POVMs are not orthogonal measurements. For outcomes \(k\) and \(k'\), the pairwise POVMs need not satisfy,

\[
\hat{\Pi}_k\hat{\Pi}_{k'} \neq \delta_{kk'}\hat{\Pi}_k. \tag{4}
\]

Note that the orthogonal POVMs are essentially the projective measurements, therefore we can define the purity of the POVM for outcome \(k\) as

\[
Purity(\hat{\Pi}_k) = \frac{[Tr(\hat{\Pi}_k^2)]}{[Tr(\hat{\Pi}_k)]}. \tag{5}
\]

The purity satisfies \(0 \leq Purity(\hat{\Pi}_k) \leq 1\), where a value of 1 denotes a pure, i.e., projective POVM.

**MODEL**

In order to study the PNR performance of the segmented detector we consider 2 cases: first, the beam splitters all have the same reflectivity and, second, the reflectivity of the \(j\)th beam splitter is given by \(R_j = \frac{1}{m+j+1}\) such that the last beam splitter in Fig. 2 is balanced. It can be shown that the field splitting generated by this is identical to that of a symmetric beam splitter tree. We study each case in turn in the lossless case before investigating the symmetric case in the presence of losses.

**SEGMENTED DETECTOR WITH IDENTICAL BEAM SPLITTERS**

In Fig. 2 the quantum input mode has annihilation operator \(a_1\) and is in the input Fock state \(|n\rangle\), and the other \(m-1\) input modes, of \(a_2, a_3, ..., a_m\), are vacuum ones. We consider \(m-1\) identical beam splitters (T, R) and \(\eta = 1\) (no losses) for all modes. The input quantum state is

\[
|n\rangle = \frac{a_1^n}{\sqrt{n!}}|0\rangle \otimes |m\rangle. \tag{6}
\]

In the Heisenberg picture, after \(m-1\) beam splitters \(a_1^\dagger\) evolves to

\[
\left(U_{m-1} U_2 U_1 a_1^\dagger U_{m-1} U_2 U_1 \ldots U_1 a_1^\dagger U_{m-1} U_2 \ldots U_1 a_1^\dagger\right) \tag{7}
\]

\[
= (t^{m-1}a_1^\dagger + tr^{m-2}a_2^\dagger + \cdots + ra_m^\dagger).
\]

It is easy to see that the output quantum state is

\[
|\psi\rangle_{out} = \frac{1}{\sqrt{n!}} (Ua_1^\dagger U^\dagger)^n|0\rangle \otimes |m\rangle \tag{8}
\]

\[
= \frac{1}{\sqrt{n!}} (t^{m-1}a_1^\dagger + tr^{m-2}a_2^\dagger + \cdots + ra_m^\dagger)^n|0\rangle \otimes |m\rangle. \tag{9}
\]

Using multinomial expansion we get

\[
|\psi\rangle_{out} = \sum_{n_1=0}^{n} \ldots \sum_{n_m=0}^{n} \left\{ \frac{\sqrt{n!}}{n_1!n_2!\ldots n_m!} \cdot \prod_{j=1}^{m} (a_j^\dagger)^{n_j} \right\} |0\rangle \otimes |m\rangle, \tag{10}
\]

where each \(n_i\) can take any value from 0 to \(n\). In order to have PNR each \(n_i\) must have at most one photon. The probability of splitting \(n\) photons over \(m\) modes is

\[
P(\{0,1\}^m, n, K) = n!\sum_{j=1}^{m} 2^{(m-j)n_{m-j+1}}r^{2(n-n_m)}, \tag{11}
\]
where \( K = (n_1, n_2, \ldots, n_m)^T \) is an \( m \)-dimensional vector. The total probability of photon-number-resolution is

\[
P(\{0,1\}^m, n) = n! \sum_{n_1=0}^{n} \cdots \sum_{n_m=0}^{n} t^{2(\sum_{j=1}^{m} (m-j)n_{m-j+1})} \frac{1}{2^n} \cdot \frac{1}{n!} \cdot X,
\]

(12)

\[
= n! \sum_{n_1=0}^{n} \cdots \sum_{n_m=0}^{n} T^{\sum_{j=1}^{m} (m-j)n_{m-j+1}} R^{(n-n_m)}.
\]

(13)

Note that our goal is to split the \( n \) input photons over \( m \) modes such that each more have either zero or one photon, no matter what modes get the photons. When all beam splitters are identical, the most constrained is the first one as its reflectivity must be much less than \( 1/n \). However, this approach isn’t optimal since the subsequent modes will gradually see fewer photons and can therefore afford larger reflectivities without running the risk of detecting more than one photon. Also, at the end of the device, the last beam splitter should clearly be balanced since the constraint is symmetric for both its output ports. Bearing all this in mind, a symmetrized device appears to be the optimal choice. We investigate it next.

**SYMMETRIC SEGMENTED DETECTOR**

*Lossless case*

Now we consider the case where the beam splitters have reflectivity \( R_j = \frac{1}{m-j+1} \), still in the lossless case (\( \eta = 1 \)). It is easy to see that the probability of getting \( k \) APD clicks (where each click may result from one or several simultaneous photons) from \( m \) modes, given \( n \) input photons, \( P_m(k|n) \) is

\[
P_m(k|n) = \binom{m}{k} \frac{n!}{\prod_{i=1}^{k} n_i!} X,
\]

(14)

where \( X \) is given by

\[
X = [(R_1)^{n_1}(T_1)^{n-n_1}] [(R_2)^{n_2}(T_2)^{n-n_1-n_2}] \cdots
\]

\[
[(R_{m-1})^{n_{m-1}}(T_{m-1})^{n-n_1-n_2-\cdots-n_{m-1}}].
\]

(15)

Using \( R_j = \frac{1}{m-j+1} \), and \( R_j + T_j = 1 \) yields the simplification

\[
X = \frac{1}{m^n}.
\]

(16)

Considering the number of possible configurations, we get

\[
P_m(k|n) = \frac{n!}{m^n} \binom{m}{k} \sum_{n_1=1}^{n} \cdots \sum_{n_m=1}^{n} \frac{1}{\prod_{i=1}^{k} n_i!}.
\]

(17)

These conditional probabilities are plotted in Fig.4 for \( m = 100 \) APDs and \( \eta = 1 \).

![FIG. 4: Conditional probabilities \( P_m(k|n) \) versus \( n \), for \( m = 100 \) APDs and \( \eta = 1 \).](image)

![FIG. 5: POVM purity, versus click number \( k \), for different numbers \( m \) of APDs.](image)

Considering the number of possible configurations, we get

\[
P_m(k|n) = \frac{n!}{m^n} \binom{m}{k} \sum_{n_1=1}^{n} \cdots \sum_{n_m=1}^{n} \frac{1}{\prod_{i=1}^{k} n_i!}.
\]

(17)

These conditional probabilities are plotted in Fig.4 for \( m = 100 \). Unsurprisingly, they increase for larger photon numbers as \( m \) increases (see supplemental material, for \( m \) up to 2000). This translates directly into the POVM purities, plotted in Fig.5. As can be seen in that figure, reasonably good PNR performance can be reached for \( n \sim 10 \) with \( m \sim 1000 \).

*Lossy case*

We now consider the effect of photon losses in each detection mode, i.e., \( \eta < 1 \). Again, \( \eta \) should not be misconstrued to be the APD quantum efficiency, which
plays no role in photon losses. (An instance of photon loss would be if a photon went undetected and exited the waveguide, losing its chance for further detection.) We assume that the parameter $\eta$ is independent of the photon number. The probability to get zero clicks in one mode is

$$P_1(0|n, \eta) = (1 - \eta)^n.$$  \hspace{1cm} (18)

Likewise, the probability to get one click in one mode is

$$P_1(1|n, \eta) = \sum_{k=1}^{n} \binom{n}{k} \eta^k (1 - \eta)^{n-k} = 1 - (1 - \eta)^n.$$ \hspace{1cm} (19)

It is important to note that in the sum over $k$ starts with 1 here because we neglected dark counts. Therefore, Eq. (17) can be generalized to

$$P_m(k|n, \eta) = n! \left( \frac{1 - \eta}{m} \right)^n \binom{n}{k} \sum_{n_1=0}^{n} \cdots \sum_{n_m=0}^{n} \left\{ \frac{1}{\prod_{j=1}^{m} n_j!} \right\} \prod_{l=n_1}^{n_k} \left[ \left( \frac{1}{1 - \eta} \right)^l - 1 \right].$$  \hspace{1cm} (20)

It is worth noting that the probability of getting zero clicks is still the same as the case of one detector, i.e., Eq. 18. For $\eta < 1$ the computer simulations run extremely slowly for higher values of $m$ (reminiscent of the boson sampling problem). We calculated the conditional probabilities at $m = 50$ for $\eta = 0.9, 0.99, 0.999$, and plot the highest losses for illustration in Fig.7. Plots for other values of $\eta$’s are included in the supplementary material. The degradation of the count probability with photon loss is evident, compared to Fig.4. Also recall that $\eta = 0.9$ means 10% loss per mode which is a very poor performance. The purity calculation, displayed in

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig6}
\caption{Conditional probabilities $P_m(k|n)$ versus $n$, for $m = 50$ APDs and $\eta = 0.9$.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig7}
\caption{POVM purity, versus click number $k$, for several values of $\eta$ at $m = 50$.}
\end{figure}

Fig.7, is particularly illuminating. Indeed, it is clear that, as $\eta$ increases beyond the low eta $= 0.9$ level, the photon losses have decreasing to negligible ($\eta = 0.999$) effect on purity, which is essentially limited by $\eta$, as per Fig.5. This is an interesting result. It is likely that the same level of photon loss may have a more detrimental effect as $m$ increases, however, the exact scaling of this effect is not yet known, due to the long computation times for the nonideal case.

\section*{CONCLUSION}

We carried out the theoretical evaluation of the photon-count POVM for the segmented detector. Results show that PNR detection is indeed achievable in the ideal case. This opens a new path to PNR devices that operate at room temperature and can be manufactured with available integrated photonic technology. The number of integrated APDs appears to be, to a large extent, the dominant factor toward high-quality PNR detection. While photon losses must also be taken in to account, of course, it is important to note that they do not include the quantum efficiency of the APDs, by design of the segmented detector. The reduction of photon losses will therefore only involve passive optical design considerations, a notable difference with butt-coupled tree-splitting detectors.

\section*{ACKNOWLEDGEMENTS}

We are grateful to Joe Campbell, Seth Bank, Aye L. Win, Rafael Alexander, Ben Godek, Sharon S. Philip, Bargav Jayaraman, and Oshin Jakhete for stimulating discussions. This work was supported by the U.S. Defense Advanced Research Projects Agency (DARPA).
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