A New Quantum Phase Transition in the Coupled Quantum Dots System

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We study two quantum dots in the limit of strong dot-lead coupling and weak dot-dot tunneling. The model maps on Ising-coupled Kondo impurities. We argue that a new quantum critical fixed point exists at an intermediate value of the mutual capacitance, supporting non-Fermi liquid behaviour. We construct the total conductance across the double dot structure. It exhibits a strongly peaked behaviour as a function of the mutual capacitance, gate voltage, and temperature.

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Electron tunneling through quantum dots is fundamentally affected by intriguing many body effects. The Coulomb interaction imposes a prohibitive energy cost $E_C$ on the transfer of electrons, known as Coulomb blockade. Fine tuning of the gate voltage $V_G$ is required to reinstate charge flow, manifesting itself in sharp conductance peaks as a function of $V_G$.

Remarkably, the charge transfer is also accompanied by an orthogonality catastrophe. The analogy to the Kondo problem was recognized early, with an exact formulation due to Matveev. For a single dot the Kondo type slow rearrangement of the electron states leads to a substantial downward renormalization of $E_C$, as well as a smoothing of the conductance peaks. Additional processes, such as the effect of higher order terms, inelastic cotunneling, and mapping to the out of equilibrium Anderson model, were also analyzed.

New effects arise when two such systems are allowed to interact. We argue that a genuinely new and robust quantum phase transition takes place in the coupled dot system when their mutual capacitance is varied. It is driven by a change of the degeneracy of the ground state. Interdot tunneling will be included perturbatively. It turns out to be a relevant operator, manifesting itself in an inverse power law temperature dependence of the total conductance at criticality.

In the related system of two isotropically coupled Kondo impurities, a quantum phase transition was predicted also, as a function of the interaction. The results of numerical renormalization group studies were confirmed by conformal field theoretical methods, and rationalized by phase shift arguments. Unlike our case, the same Fermi sea of electrons interacts with both impurities and Particle-Hole symmetry is required to protect the fixed point. This may render it harder to realize experimentally.

Let us start by considering a structure of two quantum dots, each coupled to their own leads. The lead-dot barriers are assumed to be narrow enough such that the tunneling is correctly modelled as a point contact. Furthermore we assume the presence of a strong enough magnetic field to achieve a fully spin-polarized electron gas. Thus the number of “flavors”, i.e. of the additional quantum numbers of transverse momenta and spin of the electrons is restricted to one. The dot is assumed to be relatively large, thus supporting a degenerate electron gas with small level spacing. This level spacing serves as a low energy cutoff, below which our scaling arguments do not hold. The Hamiltonian of one lead-dot system can then be written:

$$H = \sum_{k\alpha} \epsilon_k c_{k\alpha}^\dagger c_{k\alpha} + J \sum_{kk'\alpha\beta} c_{k\alpha}^\dagger c_{k'\beta} + \hbar c.$$  

where $\epsilon_k$ is the energy of the electrons, $J$ is the tunneling amplitude and the indices $\alpha$ and $\beta$ take the values 1, when referring to the lead and 2 when describing electrons in the dot. In a pseudospin notation the tunneling term is proportional to $\sigma^+_{\alpha\beta} + \sigma^-_{\alpha\beta}$.

Next we recall that for small enough dots the Coulomb repulsion introduces an interaction term between dot-electrons, the scale of which is $E_C = e^2/2C$, the charging energy, where $C$ is the capacitance of the dot. Experimentally it is also possible to tune the overall potential of the system by a gate voltage $V_G$. The electrostatic energy of the dot can then be expressed as $E_Q = (Q - Q_G)^2/2C$, where (essentially) $Q_G = CV_G$ and $Q$ is the charge on the dot. Tuning $Q_G$ beyond $e/2$ makes it energetically favorable to transfer electrons across the barrier, giving rise to the well-known set of parabolae as the “band-structure” of the system. Transport becomes possible when the energies of states with different number of electrons are degenerate. Thus the conductance shows sharp peaks as a function of $Q_G$, with maxima at $Q_G/e = n + 1/2$. In the vicinity of these degeneracy points the energies of the states with $n$ and $n + 1$ electrons are much closer to each other than to any other state. It is then reasonable to truncate the Hilbert space to two states. A second pseudospin of $S = 1/2$ can be introduced to represent this constraint on the allowed states. With this notation $H$ assumes the Kondo type form:

$$H^K = \sum_{k\alpha} \epsilon_k c_{k\alpha}^\dagger c_{k\alpha}.$$  


\[+J \sum_{kk'\alpha\beta} c^{\dagger}_{k\alpha}(\sigma^{+}_{\alpha\beta}S^{-} + \sigma^{-}_{\alpha\beta}S^{+})c_{k'\beta} - \Delta S^z.\]

Here \(\Delta\) is the gap between the \(n\) and \(n+1\) electron states on the dot. The introduction of the two types of pseudospin operators allows a complete mapping of a single dot to the Kondo problem in a magnetic field \(\Delta\), as first realized in its entirety by Matveev \[4\]. Note that the Kondo term is not spin-rotationally invariant, it contains only the spin flip terms.

We proceed by including the interaction between the two dots caused by their mutual capacitance \(C_{m}\). The generated dot-dot coupling is proportional to \(n_{L}n_{R}\), where we introduced the L, R notation for the left and right dot respectively. Here \(n_{L,R}\) denote the charge of the left or right dot. In pseudospin notation \(S^z_{L,R} = n_{L,R} - 1/2\). The mutual capacitance in pseudospin notation gives rise to an antiferromagnetic Ising type coupling: \(H^{AF}_{L,R} = I_{z}S^z_{L}S^z_{R}\), where \(I_{z} \sim E_{C_{m}}\). The total Hamiltonian then takes the form: \(H = H^{AF}_{L} + H^{AF}_{R} + H^{AF}_{LR}\), describing two anisotropic Kondo impurities, coupled by an antiferromagnetic Ising term.

The physical content of the model can be analyzed by generalizing the arguments of the theory of the two-impurity Kondo model at \(I_{z} = 0\) we have two decoupled Kondo models. At \(T=0\) in the magnetic language two independent isotropic Kondo singlets are formed with a "binding energy" \(\sim T_{K}\), as the anisotropy of the Kondo coupling is known to be irrelevant around this fixed point. In the charge language, the electrons form strongly hybridized states between the lead and the dot. This hybridization manifests itself by the strong coupling Kondo phase shift, \(\delta = \pi/2\). The key observation is that the ground state is a singlet. At finite but small \(I_{z}\) we follow Nozieres, who showed that all operators around this fixed point break the singlet and thus are of dimension two. In particular the dot-dot interaction involves virtual hopping operators to fourth order in the lead-dot hybridization amplitude. The corresponding diagrams contain a large number of fermionic operators, and thus are irrelevant. Alternatively the large number of fermion operators strongly confine the relevant phase space, leading to a positive exponent for the temperature dependence. This consideration again yields a vanishing contribution at \(T=0\).

In the opposite limit, \(I_{z} = \infty\), the dot spins are aligned antiferromagnetically. The \((\uparrow, \downarrow)\) and \((\downarrow, \uparrow)\) states are degenerate and form a doublet, which is independent of the conduction electrons. In the charge language these states consist of one extra electron being either on the left or on the right dot: \((1,0)\) and \((0,1)\). The energy of forcing on or taking away a dot-electron is \(\sim I_{z}\), and is thus prohibited in this limit. Let us recall that the interaction term between the lead electrons and the pseudospin contains only spin raising and lowering terms. In the allowed Hilbert space the matrix elements of this coupling are zero. Therefore the phase shift of the conduction electrons vanishes. The degeneracy of the ground state extends to large but finite \(I_{z}\) couplings as well, since the flip from one state to another again requires a high power of fermion operators and thus is irrelevant. In other words, whichever dot the electron resides on, it hybridizes only with its corresponding lead. Since the dot-dot coupling does not allow for charge transfer, these fluctuations remain confined to the dot and lead on the same side. Therefore the energies of the doublet’s two states renormalize symmetrically for finite \(I_{z}\), and thus the degeneracy is preserved.

To sum it up, the ground state is a singlet for small values of \(I_{z}\), but changes its symmetry to a doublet at large values of \(I_{z}\). This change cannot be continuous: the two regions are necessarily separated by a phase transition. While this transition seems to be related to the case of two Kondo impurities, it is certainly different and its nature is as yet unexplored. In the remaining of this paper we assume that the transition is of second order.

Here we pause to make connection to previous work by reviewing the band structure. The parabolae now have two indices, representing the charge states of the two dots. For \(I_{z} = 0\) the \((0,0)\) and \((1,1)\) curves are touching \(E_{Q} = 0\), the latter displaced along the gate charge \(Q_{G}\) axis by \(e\). The \((0,1)\) and \((1,0)\) curves are centered at \(Q_{G} = e/2\), and are also shifted upward such that they go through the intersection of the \((0,0)\) and \((1,1)\) curves. Exactly this degeneracy of states with different number of charges allows for transport across the dots, and thus gives rise to the conductance peak. If one now introduces the mutual capacitance \(I_{z}\), the upper parabolae are customarily shifted down by an amount \(\sim I_{z}\). This creates two degeneracy points at \(Q_{G} \sim e/2(1 \pm I_{z}/E_{C})\). Thus the original degeneracy of e.g. the \((0,0)\) and \((1,0)\) states, which allowed for the Kondo effect and underlied much of the above considerations, seems to have been destroyed. One might expect that whatever is left from the above picture, will be observable at the shifted degeneracy points.

In contrast we predict that this new quantum critical point is observable, located precisely at the original value of \(Q_{G} = e/2\). The reason for this is that for small \(I_{z}\) the Kondo energy scale \(T_{K}\) is bigger than \(I_{z}\). Therefore it is incorrect to construct a band structure first and then try to include the Kondo physics. Instead one has to start by accounting for the formation of the Kondo singlet, a deeply non-perturbative effect. The subsequent inclusion of \(I_{z}\) then means only a small perturbation, similar to a fluctuating magnetic field. According to the above reasoning such a field has a vanishing polarizing effect on the Kondo singlet unless its (Zeeman) energy is comparable to \(T_{K}\). Thus for \(I_{z} < T_{K}\) the \((0,1)\) and \((1,0)\) parabolae should not be viewed as shifted from their \(I_{z} = 0\) location, and their degeneracy is preserved. A strongly analogous situation occurs in single dots: as shown first
in Ref. 8, the effect of the Kondo type many body processes is to strongly collapse the band structure, sustaining their degeneracy up to some finite $I_z$. On the other hand, for $I_z > T_K$ it is a reasonable starting point to account for $I_z$ first, and then treat the Kondo coupling, i.e. the tunneling term as a perturbation. The two regimes are separated by the quantum critical point at $I_z^c \sim T_K$.

Next we determine the total conductance of the two-dot structure. The interdot tunneling Hamiltonian takes the form:

$$H_{\text{tun}} = I_{\pm} \sum_{kk'} c_{kL}^\dagger c_{k'R}^\dagger \gamma^z \sigma^z_{L} \sigma^z_{R} + h.c. \quad (3)$$

The pseudospin index 2 appears explicitly, as we are considering dot-dot tunneling. This term breaks time reversal symmetry, and is a relevant perturbation at the quantum critical point. One then expects that the low temperature behaviour of the renormalized tunneling at criticality exhibits a singularity: $I_{\pm}(T) \sim T^{-\gamma}$. The conducting path across the structure involves a lead-dot, dot-dot and dot-lead transitions. In the $I_{\pm} \ll J$ limit the bottleneck, and thus the determining factor in the total conductance $G$ is the dot-dot tunneling:

$$G(I_z = I_z^c, T) \sim (I_{\pm}(T))^2 \sim T^{-2\gamma}. \quad (4)$$

This is only a crossover behaviour. As $T$ is further lowered, $I_{\pm}$ grows large and flows to an attractive fixed point, controlling its asymptotic behavior. Given its analogous structure, it is plausible that the dimension of $I_{\pm}$ is the same as that of the particle-hole symmetry breaking operator. However the actual value of $\gamma$ still needs to be determined [10].

What happens away from criticality? For $I_z < I_z^c$ the Kondo singlets inhibit the transport. At $T = 0$ the binding is complete, thus $G(T = 0) = 0$. Concentrating once again on the bottleneck dot-dot tunneling we compute the scaling dimensions of the involved operators. The fermion operators carry dimension 1/2, the spin raising operator has dimension 1. The current operator is constructed from the $[N, H]$ commutator. Collecting the terms the current-current correlator decays with the sixth power of time. Substituting this into the Kubo formula finally yields $G(T) \sim T^4$. The lead-dot process occurs via the Kondo coupling which scaled to its unitarity limit, thus it does not give rise to additional powers of $T$.

In the regime $I_z > I_z^c$ electrons have to break an Ising bond. Thus at zero temperature again $G(0) = 0$, and at finite $T$ the temperature dependence takes an activated form, $G(T) \sim \exp(-W/T)$, where $W \sim I_z$. To sum it up, the conductance as a function of $I_z$ at zero temperature is zero nearly everywhere, and exhibits a resolution limited peak at $I_z = I_z^c$. At finite but low temperatures the peak persists in $G(T)$ as a function of $I_z$. On the two sides of the peak $G$ assumes non-zero values. These wings are asymmetric, with $T$ dependent values. The different regimes are shown in Fig.1.

Finally we examine the effect of changing the gate voltage, which tunes the gate charge away from the special point $Q_G = e/2$, considered so far. In the Kondo language this gives a finite value to the magnetic field $\Delta$. For $I_z < I_z^c$ we utilize the same observation as before: for the unperturbed state is the Kondo singlet. Thus for $\Delta = 0$, i.e. at $Q_G = e/2$ transport is still impossible at $T = 0$. For $0 < \Delta < T_K$ the singlet is somewhat polarized, and weak transport is possible. This manifests itself in two small-amplitude “shadow-bands” in a V shape determined by $|\Delta| = I_z$. This is the location of the crossing of the typically constructed “shifted parabola”. An important transport channel in this region is co-tunneling, which only virtually breaks the Kondo singlet. For $I_z = I_z^c$ the pronounced conductance peak of the quantum critical point is present at $\Delta = 0$. This peak continues out to finite $\Delta$, forming a parabola-like ridge, which smoothly connects to the usual split conductance peaks at $|\Delta| = I_z$. In this region $I_z$ is larger of the energy scales and constructing the band structure first is appropriate.

Constructing the picture from the large $I_z$ side, the magnetic field $\Delta$ is trying to induce a spin-flip transition in the antiferromagnetic singlet. It is competing with the singlet binding energy, so the spin-flip can only occur when the binding energy equals the Zeeman energy: $|\Delta| \sim I_z$, forming the usual V locus for the split peaks. Approaching the quantum critical point however the binding energy collapses to zero, hence the V becomes rounded, and closes at $I_z^c$, as shown in Fig.2.

To summarize, the key experimental predictions of our calculations are:

(i) If the gate voltage is fixed so that $Q_G = e/2$, then by tuning $I_z$ a pronounced new conductance peak has to be observed at some critical value $I_z = I_z^c \sim T_K$.

(ii) Staying at this point $Q_G = e/2$, $I_z = I_z^c$, the conductance $G(T)$ should exhibit a power-law singularity in its temperature dependence.

(iii) The amplitude of the split conductance peaks at
$Q \neq e/2$ should exhibit a marked collapse as a function of $I_z$ when $I_z$ approaches $I_z^0$ from above.

We know of no experimental observation of these predictions yet. This maybe due to the fact that the above theory applies only under the following conditions: (i) Typical experiments [17] and the corresponding theory [14] considered the case of fixed $I_z$, and described the evolution of the peak structure with tuning of $I_z$, the dot-dot tunneling. The present theory addresses the case of fixed and small $I_z$, and tuning with $I_z$ instead. (ii) The number of tunneling channels should be small.

The above theory strictly applies only for the case of a single channel. This requires a narrow, long constriction between the leads and the dot, similar to the case considered in [14]. We expect important changes when the number of flavors of the electrons is increased. Switching off the magnetic field increases the number of channels to two. It can be shown [15] that the Ising term is marginal around the “decoupled” fixed point, leading to a line of fixed points which terminates at some intermediate value. The fixed point structure of the related two Kondo impurities model changes analogously. The Kondo singlet and the antiferromagnetic singlet phases remain intact, but the quantum critical fixed point expands into a very unusual fixed point area, a whole region of the parameter space consisting of fixed points [14]. If such a structure emerges in our case, then a broadened conductance peak will form as a function of $I_z$ at $T = 0$, and the finite temperature conductance should exhibit singular temperature dependence with $I_z$ dependent exponents. The case of even larger number of channels has been investigated for single scatterers in relation to the physics of two level systems [20]. It has been shown that a two dimensional subspace of the flavor indices emerges to dominate exponentially over the others in the course of scaling. Therefore we expect the basic picture of two distinct phases and a well defined quantum phase transition in between to carry over, but obviously further calculations are needed on this point.

In sum we studied the system of two coupled quantum dots. We established the existence of an intriguing new quantum critical point. Several experimentally accessible predictions were reached: a new conductance peak at $Q_G = e/2$, an inverse power law dependence of the conductivity at this same point and a marked collapse of the split conductance peaks, when the experimental parameters are in the suitable range.

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