Self-consistent simulations of star cluster formation from gas clouds under the influence of galaxy-scale tidal fields

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ABSTRACT
We present the first results of a project aimed at following the formation and long-term dynamical evolution of star clusters within the potential of a host galaxy. Here, we focus on a model evolved within a simplified potential representing the Large Magellanic Cloud. This demonstrates for the first time the self-consistent formation of a bound star cluster from a giant molecular cloud. The model cluster reproduces the density profiles and structural characteristics of observed star clusters.

Key words: stellar dynamics – methods: N-body simulations – stars: formation – globular clusters: general – open clusters and associations: general – galaxies: star clusters.

1 INTRODUCTION
Star clusters are vital components of galaxies. Indeed, extracting information from the massive globular clusters (GCs) of our Galaxy provides fundamental information on the epoch of galaxy formation. Furthermore, systems of extragalactic GCs are used as key determinants for tracing the dynamical, chemical and gaseous evolution of their host galaxies (see Brodie & Strader 2006, for a review). The smaller open clusters also have a role to play as well, as understanding their destruction within in the Galactic disc impacts the growing field of Galactic archaeology (Freeman & Bland-Hawthorn 2002), for example.

Efforts to model the evolution of individual star clusters have made great advances in the last decade. On the hardware front, the advent of special purpose GRAPE processors for calculating gravitational forces, with the latest incarnation being the GRAPE-6 (Makino 2002), have allowed gravitational forces, with the latest incarnation being the GRAPE-6 (Makino 2002), have allowed $N$-body models of up to $N \sim 100\,000$ stars to be completed in a reasonable time-frame (Baumgardt & Makino 2003). The next generation of GRAPE (available in 2008), the use of massively parallel supercomputers, and even graphics processing technology, will push this $N$ continually higher towards the realm of direct GC models. Complementing this is the push to make the models as realistic as possible by including algorithms to deal with processes such as stellar and binary evolution in concert with the treatment of the gravitational interactions of the stars. This is the case for both $N$-body (Hurley et al. 2005) and statistical (Freytag & Rasio 2007; Giersz, Heggie & Hurley 2008) techniques.

On a grander scale simulations of galaxy formation which identifies the formation sites of star clusters have been performed (Bekki & Chiba 2005). Furthermore, the formation of stars within turbulent giant molecular clouds (GMCs) based at these formation sites can then be followed with smoothed particle hydrodynamics (SPH) simulations (Bekki & Chiba 2007). Our aim is to interface these galactic-scale simulations of star cluster formation with the latest modelling techniques for following the long-term evolution of star clusters. The immediate benefits will be: (i) the first self-consistent simulations of star cluster evolution from formation through to destruction, and (ii) a non-simplistic picture of how GC systems evolve with time and how this impacts the interpretation of observed extragalactic GC systems. This latter advance, in particular, will be important for understanding the connection between extragalactic GCs and galaxy formation by injecting much needed numerical models in to what has become a data-dominated field (Brodie & Strader 2006). We will also be able to explore the origin of observed features of star cluster systems, such as the age and parameter distributions of the Large Magellanic Cloud (LMC) clusters (Mackey & Gilmore 2003).

In this Letter, we provide an overview of the modelling process that will be used to achieve our goals. We illustrate this process by describing the evolution of a prototype model which demonstrates a working interface between the cluster formation and long-term evolution codes. It also entails the first simulation of bound cluster formation from a gas cloud within an external galactic potential. We then discuss future improvements to the model and details of how this fits in to our plans for a full investigation of the long-term evolution of Galactic and extragalactic star cluster systems.

2 A TWO-FOLD MODELLING PROCEDURE
The present model is two-fold as follows. We first determine the spatial distributions of stars that are formed within GMCs gravitationally influenced by their host galaxies. In this first step, we investigate short-term star formation processes in GMCs based on

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GRAPE-SPH simulations of turbulent GMCs. Then, we investigate the long-term dynamical evolution of collisional stellar systems, incorporating processes such as two-body relaxation and mass-loss from stars, for initial distributions of stars derived in the first step. In this second step, we emphasize that we are taking dynamically unrelaxed stellar systems formed from GMCs and placing these within an N-body code for following the evolution of very young SCs gravitationally influenced by the tidal field of their host galaxies.

Previous work along these lines that bears mention starts with the simulations of galaxy formation reported by van Albada (1982, and see associated references within) and McGlynn (1984). These dealt with similar physical processes – the evolution of an initially irregular mass distribution – although the context and approach were quite different. Importantly, these were collision-free simulations performed without the influence of an external potential. Aarseth, Lin & Papaloizou (1988) examined the collapse of protogalactic clusters through N-body models. They took an initial distribution of fragments (representing low-mass pre-stellar subcondensations) and documented the ensuing gravitational relaxation phase leading to the creation of a core-halo density structure. These models also did not include an external potential. Furthermore, each of these studies did not model the fragmentation process leading to the formation of gas clumps or the subsequent creation of stars from these clumps. These prior studies do, however, provide a good basis to which we can compare the results of our models.

2.1 Turbulent GMCs
Radial mass density distributions, \( \rho (r) \), of GMCs with the sizes, \( r_g \), and masses, \( m_g \), are assumed to have homogeneous spherical distributions. Here, we set-up the initial velocity fields due to turbulent flows within GMCs in the same way as Mac Low et al. (1998). We therefore assume that a turbulent velocity field within a GMC is a Gaussian random field with a power spectrum for \( 0 \leq k \leq k_{\text{max}} \) as follows

\[
P(k) = P_0 k^\alpha, \tag{1}
\]

where \( \alpha \) is set to be 2.0 for most models and \( P_0 \) is a parameter controlling the total kinematical energy due to the turbulent flow in the GMC. We mainly use \( k_{\text{max}} = 8 \) in our models, however, the final dynamical and chemical properties of the simulated GCs do not depend strongly on \( k_{\text{max}} \).

The virial ratio, \( t_v \), is a free parameter described as

\[
t_v = 2T_k/W = f(P_0), \tag{2}
\]

where \( T_k \) and \( W \) are the total kinematical energy and the absolute magnitude of the total potential energy for a GMC, respectively. As shown in equation (2), \( t_v (0 \leq t_v \leq 1) \) is determined by \( P_0 \) and thus can control the initial random kinematical energy of gas particles in this study. We typically take \( t_v = 0.25 \) in our models. Since an isothermal equation of state is suggested to be appropriate for star-forming interstellar clouds of molecular gas (e.g. Mac Low et al. 1998; Klessen, Heitsch & Mac Low 2000), we adopt the equation with the initial temperature of 10 K.

2.2 Star formation and stellar feedback effects
Gas particles with initial masses of \( m_i \) are assumed to be converted into new stellar particles if (i) local dynamical time-scales are shorter than the local sound crossing time, and (ii) local gas densities exceed a threshold gas density, \( \rho_{\text{th}} \), of star formation (e.g. Nakasato, Mori & Nomoto 2000). Since the gravitational softening length, \( \epsilon \), for gas particles of GMCs in a model is fixed during the evolution of the GMC (\( \epsilon \sim 10^{-2} r_g \)), there is a maximum density, \( \rho_{\text{max}} \), which the gas densities of individual SPH particles, \( \rho_i \), cannot exceed in the adopted GRAPE-SPH method: \( \rho_{\text{max}} \sim N_i m_i/\epsilon^3 \), where \( N_i \) is the total number of ‘neighbor particles’ surrounding an \( i \)th SPH particle. Here, \( \rho_{\text{max}} \) is estimated to be of the order of 10\(^3\) atom cm\(^{-3}\) for models with \( m_g \sim 10^4 M_\odot \). We thus assume that \( \rho_i = \rho_{\text{max}} \) in this study. It should be noted here that the above \( \rho_{\text{max}} \) is significantly lower than the threshold gas density (\( \sim 10^5 \) atom cm\(^{-3}\)) of individual stars suggested by Elmegreen (2004).

2.3 External gravitational potential
As star clusters evolve they lose stars to their host galaxy at a rate that depends on the strength of the galactic potential and the orbit within this potential. Our plan is to improve the treatment of the external gravitational potential used in simulations of star cluster evolution by using the results of the galaxy-scale calculations to provide a ‘live’ model of the potential. However, to start with we adopt a smooth and static potential that is the current standard for N-body star cluster models. We tailor this to an external gravitational potential reasonable for the LMC in our initial study as one of our goals is to compare our results with the observed properties of young star clusters in the LMC.

Star-forming GMCs are thus assumed to be gravitationally influenced by the LMC represented by a point-mass, \( M_{\text{gal}} \), of \( 0.9 \times 10^{10} M_\odot \). Within this simplified potential, the GMCs are assumed to have circular velocities determined by their locations and the mass of the LMC.

2.4 Dynamical evolution of SCs just after their birth
The long-term dynamical evolution of the young SCs emerging from the GMCs is then followed using the \textsc{nbody4} code (Aarseth 1999, 2003). This code employs the fourth-order Hermite integration scheme, without softening, and is designed to exploit the fast evaluation of the gravitational force and its time derivative by the GRAPE-6. The use of the GRAPE-6 allows models of up to \( N = 100,000 \) to be completed although for \( N < 10,000 \) it is possible to make progress even when performing the full force calculation on the host computer. \textsc{nbody4} includes algorithms for stellar and binary evolution as described in Hurley et al. (2001). It allows for the full range of possible interactions within binary stars as well as dealing directly with the effects of close dynamical encounters: perturbations to binary orbits, collisions and mergers, formation of three- and four-body subsystems, exchange interactions, tidal capture and binary disruption. The tidal field of the host galaxy is modelled as described in Section 2.3 by placing the model cluster on a circular orbit at a specified radial distance, \( R_g \), from the centre of a point-mass galaxy. As such \textsc{nbody4} facilitates an investigation of the combined effects of internal (stellar, binary and dynamical) and external (galactic tide) influences on the evolution history of a star cluster.

The typical starting point for models of star cluster evolution is to assume that the star formation process is complete. An initial model is designed by first drawing the positions of the stars from some density distribution: the \( n = 5 \) polytrope proposed by Plummer (1911) and the family of King models (King 1962, 1966) derived from observations of mature star clusters are common choices, the former primarily because of its mathematical simplicity. Next, the stellar velocities are determined by assuming the model cluster is...
3 RESULTS OF A PROTOTYPE MODEL

In this Letter, we highlight our procedure by describing the evolution of a protocluster of 8420 equal-mass stars emerging from a GMC. We set $m_g = 10^4 \, M_\odot$ and $r_g = 10 \, pc$ for the GMC, and take the number of gas particles to be $2 \times 10^4$. Thus, the mass of the stars formed is $0.5 \, M_\odot$ per star. The time-scale of star formation in this step is $\sim 3 \times 10^6 \, yr$ and the star formation efficiency is 42 per cent. Full details of the formation process will be supplied in an upcoming paper. The spatial distribution of the protocluster is shown in the top panels of Fig. 1. This is the initial model for the Nbody4 step of the evolution and as such is given an age of 0 Myr. In this initial model, there are no gas particles remaining but residual gas can be communicated to the N-body code, and its effect accounted for, in any future models. The stars in the initial model do not constitute a bound system (the total energy is positive) and are definitely not in virial equilibrium: the ratio of kinetic to potential energy is 1.84 (this is $Q_0$ from equation 3 with $E_t = 0$). The half-mass radius, $r_{hm}$, is 5 pc, the velocity dispersion is 2.8 km s$^{-1}$ and the corresponding crossing-time for this initial model is approximately 7 Myr.

The cluster was placed on a circular orbit within our simplified LMC potential at a distance of $R_{gc} = 1.5 \, kpc$ from the galaxy centre. This matches the location of the parent GMC and corresponds to the scalelength of the stellar disc of the LMC. The choice of tidal field gives an initial tidal radius of 8 pc. We note that as the model cluster evolves, stars are denoted as having escaped from the cluster (and are removed from the simulation) when their distance from the cluster centre exceeds twice the tidal radius. This prototype model was evolved to an age of 550 Myr using a single processor of the Swinburne supercomputer. At this point, a bound cluster comprising 10 per cent of the original stars remains.

The total energy of the system first becomes negative after 5 Myr have elapsed. Fig. 1 shows the spatial distribution of the stars at ages of 10, 30 and 300 Myr. Already at 10 Myr, the makings of a star cluster are evident and certainly by 300 Myr, the system has reached approximate virial equilibrium. Certainly by an age of 55 Myr, when a two-body relaxation time (as measured at the half-mass radius) has passed, the cluster appears to be dynamically relaxed. When the model cluster is 100 Myr old, the escape rate of stars from the cluster is 14 Myr$^{-1}$. This is down from an average of 80 Myr$^{-1}$ during the first 55 Myr of evolution but matches the escape rate from a comparison cluster started in virial equilibrium and evolved on the same LMC orbit. The model at 100 Myr contains 2817 bound stars. It has spatial parameters of $r_1 = 5.6 \, pc$, $r_b = 1.2 \, pc$ and core-radius, $r_c = 0.36 \, pc$. The half-mass relaxation time-scale has decreased to 28 Myr and the velocity dispersion is 1.5 km s$^{-1}$.

Figure 1. Spatial distributions in the XZ– and XY–planes for the prototype model at ages of 0, 10, 30 and 300 Myr. The numbers of single stars, $N_s$, and binaries, $N_b$, bound to the cluster at each age are denoted in the figure.
cluster ($E_t = 0.0$ at all times). This model approaches virial equilibrium on a time-scale of ∼40 Myr. Thus, we see a weak trend for the virial time-scale to increase for models with a decreased influence from the external tidal force. At the same time, the half-mass relaxation time-scale is greater in models with a weaker tidal field, owing to larger $N$ and $r_h$ at any particular time. We note that in equation (3) we have neglected a term that represents the rotation of the cluster (the Coriolis force) as this is negligible for clusters in dynamical equilibrium (Fukushige & Heggie 1995). However, its absence explains the slight deviation from $Q_e = 0.5$ for our relaxed models, as does the fact that a cluster can approach a state of dynamical equilibrium but will never actually reach it in practice. The Milky Way model at an age of 100 Myr contains 6724 stars, and has $r_t = 21$, $r_s = 3.5$ and $r_c = 0.36$ pc and a velocity dispersion of 1.4 km s$^{-1}$.

In Fig. 3, we show the two-dimensional radial density profiles of the prototype cluster models (in the LMC tidal field) at 0, 30 and 300 Myr. The profiles are compared to two families of observationally determined density profiles: empirical King (1962) models based on Milky Way globular clusters and Elson, Fall & Freeman (EFF; 1987) models based on young LMC clusters. The former is described by

$$\sigma(r) = \sigma_0 \left\{ \frac{1}{1 + (r/r_c)^2} \right\}^{3/2} \left\{ 1 + \left( r/r_s \right)^2 \right\}^{1/2},$$

and the latter by

$$\sigma(r) = \sigma_0 \left( 1 + \frac{r^2}{a^2} \right)^{-\gamma/2},$$

where

$$r_c = a(2^\gamma - 1)^{1/2}.$$

The main difference between the model profiles is the introduction of $\gamma$ in the EFF profiles which allows greater flexibility to fit the slope of the distribution exterior to the core for clusters that do not show significant tidal truncation. As noted by Mackey & Gilmore (2003), the two families overlap if we set $\gamma = 2$ in equation (6) and assume $r_t \rightarrow \infty$ in equation (5). In fact, the profile of our prototype cluster at 30 Myr is best fitted by such a scenario: an EFF model with $r_c = 0.28$ pc, $\gamma = 2$ and a central surface density of 2500 stars pc$^{-2}$. The tidal radius at this time is 7 pc but using this in equation (4) gives a profile that is too truncated to fit the data for $r > 1$ pc. It is interesting to note that the functional form of the EFF profile was originally suggested by McGlynn (1984) to represent the equilibrium profile arising from an initially irregular distribution of objects but in the context of dissipationless collapse of protogalaxies. The profile at 300 Myr is taken as representative of the density profile at late stages in the evolution of the bound cluster. At more advanced times, the profile becomes progressively noisier as the number of bound stars decreases. This profile is well fitted by an EFF model with parameters of $r_c = 0.16$ pc, $\sigma_0 = 5500$ stars pc$^{-2}$ and $\gamma = 2.6$, noting that this is the median slope determined by both EFF and Mackey & Gilmore (2003) from samples of young and intermediate age LMC star clusters. However, the data at 300 Myr are even better fitted by a King model with the same $r_c$ and $\sigma_0$ as well as the addition of $r_t = 5$ pc (see Fig. 3). This is to be expected as the model cluster orbiting close to the centre of the LMC is strongly truncated by this stage. In comparison, the density profiles of the cluster evolved in the standard MW tidal field are adequately fitted across the 30–100 Myr time-frame by King models with $r_c = 0.26$ pc and $\sigma_0 = 3500$ stars pc$^{-2}$.

We note that the prototype LMC cluster reaches a minimum in $r_t$ at an age of approximately 400 Myr which is associated with the end of the core-collapse phase of evolution. The $r_c/r_h$ value at this point is 0.06 which is a typical core-collapse value for standard models of star cluster evolution (see Hurley 2007). There are two
binaries in the cluster at this age, both residing in the core. The maximum number of binaries in the simulation is recorded shortly after the cluster forms (5 Myr) when 36 binaries are present.

4 SUMMARY

We have demonstrated the self-consistent formation of a bound star cluster that exhibits all the markings of a regular relaxed star cluster: dynamical equilibrium, core collapse and an approximately spherical spatial distribution of the stars. This initially unrelaxed model, emerging from its progenitor GMC, approaches virial equilibrium on the order of a few crossing times and before a half-mass relaxation time has passed – a result that is free of the influence of the external tidal field. We have also found that the density profile of the model cluster resembles the distributions derived from observations – King (1962) and Elson, Fall & Freeman (1987) – from the time that virial equilibrium is reached and onwards. Idealized models that start in virial equilibrium can comfortably adopt these density distributions when setting the initial positions of the stars. Our findings are distinct from previous simulations of the collapse of protogalactic clusters and protogalaxies – which also observed the emergence of a core-halo structure at equilibrium – primarily in that the initial process of fragmentation is explicitly followed using a three-dimensional hydrodynamic scheme and the influence of a galaxy-scale tidal field is modelled throughout the process.

This Letter describes a model that demonstrates a working interface between simulations of star cluster formation and long-term star cluster dynamical evolution. In this model, we have included several aspects, such as the use of equal-mass stars and a point-mass galaxy that are commonly utilized in models of star cluster evolution but are not the most realistic approach. Future models will expand our study into a full investigation including:

(i) the effect of a spectrum of stellar masses on the formation and evolution of bound clusters – associated processes such as stellar evolution and mass segregation will affect the binding energy and the evolution time-scales,
(ii) a variety of cluster sizes, in terms of physical size and also a greater number of stars,
(iii) an exploration of how the SFE assumed in the formation step affects the final outcome, and also a greater understanding of the conditions required for substantial binary formation, and
(iv) a variety of GMC locations within the host galaxy and an extension to model the live gravitational potential of the host galaxy in concert with the internal star cluster evolution, with dwarf, spiral and elliptical hosts considered.

This will allow a much more detailed study of the effects of internal and external dynamical processes on the formation and evolution of star clusters in a galactic context.

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REFERENCES

Aarseth S. J., 1999, PASP, 111, 1333
Aarseth S. J., 2003, Gravitational N-body Simulations: Tools and Algorithms (Cambridge Monographs on Mathematical Physics). Cambridge Univ. Press, Cambridge
Aarseth S. J., Lin D. N. C., Papaloizou J. C. B., 1988, ApJ, 324, 288
Baumgardt H., Makino J., 2003, MNRAS, 340, 227
Bekki K., Chiba M., 2005, MNRAS, 356, 680
Bekki K., Chiba M., 2007, ApJ, 665, 1164
Brodie J. P., Strader J., 2006, ARAA, 44, 193
Elmegreen B. G., 2004, MNRAS, 354, 367
Elson R. A. W., Fall S. M., Freeman K. C., 1987, ApJ, 323, 54
Fregeau J. M., Rasio F. A., 2007, ApJ, 658, 1047
Freeman K., Bland-Hawthorn J., 2002, ARAA, 40, 487
Fukushige T., Heggie D. C., 1995, MNRAS, 276, 206
Giersz M., Heggie D. C., 1997, MNRAS, 286, 709
Giersz M., Heggie D. C., Hurley J. R., 2008, MNRAS, 388, 429
Hurley J. R., 2007, MNRAS, 379, 93
Hurley J. R., Tout C. A., Aarseth S. J., Pols O. R., 2001, MNRAS, 323, 630
Hurley J. R., Pols O. R., Aarseth S. J., Tout C. A., 2005, MNRAS, 363, 293
King I. R., 1962, AJ, 67, 471
King I. R., 1966, AJ, 71, 64
Klessen R. S., Heitsch F., Mac Low M.-M., 2000, ApJ, 535, 887
Mackey A. D., Gilmore G. F., 2003, MNRAS, 338, 85
Mac Low M.-M., Smith M. D., Klessen R. S., Burkert A., 1998, Ap&SS, 261, 195
McGlynn T. A., 1984, ApJ, 281, 13
Makino J., 2002, in Shara M. M., ed, ASP Conf. Ser. Vol. 263, Stellar Collisions, Mergers and their Consequences. ASP, San Francisco, p. 389
Nakasato N., Mori M., Nomoto K., 2000, ApJ, 535, 776
Plummer H. C., 1911, MNRAS, 71, 460
van Albada T. S., 1982, MNRAS, 201, 939

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