Word Embedding Visualization Via Dictionary Learning

Juexiao Zhang∗ 2 Yubei Chen∗ 1 3 Brian Cheung1 3 Bruno A Olshausen1 3 4
1 Berkeley AI Research (BAIR), UC Berkeley
2 Department of Electrical Engineering, Tsinghua University, Beijing, China
3 Redwood Center for Theoretical Neuroscience, UC Berkeley
4 Helen Wills Neuroscience Institute & School of Optometry, UC Berkeley

Abstract

Co-occurrence statistics based word embedding techniques have proved to be very useful in extracting the semantic and syntactic representation of words as low dimensional continuous vectors. In this work, we discovered that dictionary learning can open up these word vectors as a linear combination of more elementary word factors. We demonstrate many of the learned factors have surprisingly strong semantic or syntactic meaning corresponding to the factors previously identified manually by human inspection. Thus dictionary learning provides a powerful visualization tool for understanding word embedding representations. Furthermore, we show that the word factors can help in identifying key semantic and syntactic differences in word analogy tasks and improve upon the state-of-the-art word embedding techniques in these tasks by a large margin.

1 Introduction

Several recent works [22, 30, 7, 31] show that co-occurrence statistics can be used to efficiently learn high-quality continuous vector representations of words from a large corpus of text. The learned word vectors encode rich semantic and syntactic information, which is demonstrated with word analogies. As shown by [24], vector arithmetic operations such as: ‘king’ - ‘queen’ = ‘man’ - ‘woman’ reflects the semantic analogy of what ‘king’ is to ‘queen’ as what ‘man’ is to ‘woman’. Thanks to the competitive performance of these models, these methods have become fundamental tools used in the study of natural language processing. Allen and Hospedales [3], Ethayarajh et al. [11], Levy and Goldberg [19] explain and understand these word embeddings from a theoretical perspective. Empirically, visualizing these embeddings can be difficult since individual dimensions of most word embeddings are not semantically meaningful.

In the absence of tools to visualize and gain further insight about the learned word vectors, we have little hope of improving the existing performance on downstream tasks like word analogy [22]. There are two major ways for visualizing word vectors:

• Nearest neighbor approach: we can use either Euclidean distance or cosine similarity to search for each word vector’s nearest neighbors to find its relevant words [31, 7, 35]. This method only provides a single scalar number of relatedness information while two words may exhibit more intricate relationships than just a relatedness [2]. For example, man and woman both describe human beings and yet are usually considered opposite in gender.

• t-SNE approach [21, 14]: This approach nonlinearly reduces word vectors to a very low dimensional (most likely 2) space. While such a global method reveals some interesting separation between word groups, it often distorts important word vector linear structures and does not exhibit more delicate components in each word.

• Subset PCA approach [23]: 1) Select a subset of word pairs, which have certain relations, e.g. city-country, currency-country, comparative etc. 2) Perform PCA on the selected subset and visualize the subset with the first two principle components, as shown in Figure 3. The relationship is frequently encoded by a vector in this subspace.
However, performing PCA with all the word vectors makes this information entirely opaque. This method needs manually selected sets of words which requires human intervention. Despite this, such a visualization method does capture important semantic meaning for word vectors.

The key insight of this work is that the relationships visualized in the human selected subsets represent more elementary word factors and a word vector is a linear combination of a sparse subset of these factors. Dictionary learning [21, 28, 6] is a useful tool to extract elementary factors from different modalities. [25, 13, 12, 32] shows sparsity helps to improve the dimension interpretability. Specifically, in [12], the authors apply non-negative sparse coding (NSC) [15, 18] with binary coefficients to word vectors and suggest to use the resulted sparse vectors as word vector representation of words. Then [32] followed the idea and applied k-sparse autoencoder to further improve the sparse word vector representation. In this work, we thoroughly explore this idea from a visualization perspective. Since a word vector may involve a different number of factors with a different strength, neither binary coefficients or a k-sparse setting would be ideal for such a purpose. In section 2 we reformulate the NSC problem and also introduce the spectral clustering algorithm to further handle group sparsity. Once NSC has been train on word vectors from different word embedding methods, in section 3 we demonstrate reliable word factors with very clear semantic meanings, which is consistent with but not limited to the existing prior knowledge. With these reliable word factors, we then open up the word vectors and visualize them in many different ways. Through these visualizations, we show many interesting compositions:

\[
\text{apple} = 0.09 \text{“dessert”} + 0.11 \text{“organism”} + 0.16 \text{“fruit”} + 0.22 \text{“mobile&IT”} + 0.42 \text{“other”}
\]

Many new word analogy tasks can be easily developed based on these learned word factors. Different embedding models and text corpus bias can be diagnosed, i.e. we find that a factor proportion might change depend on which corpus we use to train the word vector embedding.

Since several learned word factors may encode a similar meaning and frequently work together, we can use spectral clustering [26, 34] to group word factors with similar meanings. Each group can provide robust semantic meaning for each factor and identify key semantic differences in a word analogy task. We show in Section 4 that these groups help to improve the word analogy tasks significantly in almost every subcategory irrespective to which word embedding technique we use. Our simple and reliable discovery provides a new venue to understand the elementary factors in existing word embedding models. In Section 5, we discuss a few interesting questions and point out some potential directions for future work.

## 2 Word Factors Learning and Spectral Grouping

We use non-negative overcomplete sparse coding to learn word factors from word vectors. Given a set of word vectors \( \{x_i \in \mathbb{R}^n\} \) \( n = 300 \) is used in this work as a convention, we assume each of them is a sparse and linear superposition of word factors \( \phi_i \):

\[
x = \Phi \alpha + \epsilon, \quad \text{s.t.} \quad \alpha \geq 0
\]  

where \( \Phi \in \mathbb{R}^{n \times d} \) is a matrix with columns \( \Phi_{:,i} \), \( \alpha \in \mathbb{R}^d \) is a sparse vector of coefficients and \( \epsilon \) is a vector containing independent Gaussian noise samples, which are assumed to be small relative to \( x \). Typically \( d > n \) so that the representation is overcomplete. This inverse problem can be efficiently solved by FISTA algorithm [5]. A word vector \( x_i \) is sampled with respect to the responding frequency \( f_i \) of the word \( i \) in the corpus. Once the sparse coefficients is solved, we can then update the word factors to better reconstruct the word vectors. Through this iterative optimization, the word factors can be learned. We provide more details of the algorithm in Appendix [B].

Though overcomplete sparse coding tends to extract more accurate elementary factors and thus approximate signal vectors at better accuracy given a fixed sparsity level, several learned factors may be corresponding to a similar semantic meanings. [8] proposes to model the relationships by using a manifold embedding, which is essentially a spectral method. In this work, we use spectral clustering [26] to group the learned word factors into groups.

Since word factors with similar semantic meaning tends to co-activate to decompose a word vector, we calculate a normalized covariance matrix \( W \) of word factor coefficients with the unit diagonal removed:

\[
W = \sum_i f_i \hat{\alpha}_i \hat{\alpha}_i^T - I
\]  

where \( f_i \) is the frequency of the \( i \)th word in the corpus,
\( \alpha_i \) is the normalized sparse coefficients by each dimensions standard deviation such that \( \alpha_{ij} = \alpha_{ij} / \sigma_j \), and \( \sigma_j = \left[ \sum_i f_i(\alpha_{ij})^2 \right]^{1/2}, \ j = 1 \ldots n \).

This matrix captures the similarity between the word factors. To better perform a spectral clustering, we first make the normalized covariance matrix \( W \) sparse by selecting \( k \) largest values in each row of \( W \):

\[
W_{sp} = f_k(W, \text{dim} = 0)
\]

Where \( f_k \) stands for keeping the \( k \) largest values unchanged in the given dimension, while setting all the other entities to 0. Then we obtain a symmetric sparse matrix:

\[
W_{adj} = W_{sp} + W_{sp}^T
\]

\( W_{adj} \in \mathbb{R}^{d \times d} \) is the adjacency matrix used in spectral clustering [20, 34]. We use the implementation of the algorithm in Scipy [16]. Using the notation from [34], we first compute a normalized Laplacian matrix \( L_{sym} \):

\[
L_{sym} = I - D^{-1/2}W_{adj}D^{-1/2}
\]

where \( D \) is a diagonal matrix with the sum of each row of the symmetric \( W_{adj} \) on its diagonal. Suppose we set the number of clusters to \( k \), then the \( k \) eigenvectors of \( L_{sym} \) form the columns of matrix \( V \in \mathbb{R}^{d \times k} \):

\[
V = [v_1, v_2, ..., v_k]
\]

And we normalize each row of \( V \) to get a new matrix \( U \):

\[
U = [u_1, u_2, ..., u_d]^T
\]

where each row \( u_i = V_i ./ \| V_i .\|_2 \). \( \| V_i .\|_2 \) indicates the L2 norm of the \( i \)th row of \( V \). Finally, a k-means algorithm is performed on the \( d \) rows of \( U \) to get the clusters.

### 3 Visualization

The word factors learned for different word embedding models are qualitatively similar. For simplicity, we show the results for the 300 dimensional GloVe word vectors[30] pretrained on CommonCrawl [2]. We shall discuss the difference across different embedding models at the end in this section.

Once word factors have been learned and each word vector’s sparse decomposition \( \alpha \) has been inferred by Equation [8] we can denote them in the matrix form as the following:

\[
X \approx \Phi A, \ s.t. \ A \succeq 0
\]

where \( X_{i,:} = \alpha_i \) and \( A_{i,i} = \alpha_i \), \( X \in \mathbb{R}^{n \times N} \) and \( A \in \mathbb{R}^{d \times k} \). \( N \) is the size of the vocabulary. We do not require the dictionary \( A \) to be non-negative.

We can visualize a word factor \( \Phi_{i,j} \) by examining its corresponding row \( A_{i,:} \) in \( A \) and visualize a word vector \( x_{:,i} \) by examining the corresponding column \( \alpha_i \) of \( A \). Since word factors are learned in an unsupervised fashion, the explicit meaning of each factor is unknown in advance. To help understand the meaning of a specific factor, we print out the words that have high coefficients for this factor, some examples are illustrated in Table [1]. We first present the visualization of word factors and give each factor a semantic name. Then we decompose word vectors into linear combinations of word factors. Furthermore, we demonstrate factor groups and discuss the difference across multiple word embedding models.

#### 3.1 Word Factor Visualization

Word factors can be visualized through the sparse coding coefficients of each word vector. We refer to these coefficients as activations as they describe how much a factor is turned on for a specific word. In Table [1], we demonstrate a set of factors with their top-activation words. Usually the top-activation words for each factor share an obvious semantic or syntactic meaning. Based on the top-activation words, a semantic name can be given to each factor as a guide. For example, for factor 59, we can call it “medical” since most of the words have activation on it are related to medical purpose. In the Appendix [A], we discuss this naming procedure in more detail.

Reliability. The factors we discovered exhibit strong reliability. For instance, a female factor is illustrated in Figure [1]. Clearly, the activations remain all 0s for the male words, but have high values for the female words. Similarly strong word factors are also found to capture syntactic meanings of words. Figure [1] shows activations on a factor representing the superlative information, i.e. the superlative factor, where the superlative forms of adjectives have relatively high coefficients. The significance is obvious from the sharp contrast in the heights of the bars.

The Learned Factors Match the Prior. We empirically find that for each of the 14 word analogy tasks, there are always a few corresponding factors capture the key semantic difference, e.g. the “female”, “superlative”, “country” factors shown in Table [1]. Given the learned word factors closely matched the 14 word analogy tasks chosen based on human priors, we can expect the rest majority of the learned factors may provide an automatic method to select and construct the word analogy tasks. For instance, Figure [2] shows a factor corresponding to professions. For words such as “entertain”, “poem”, “law” and so on, it has 0 or very small activation, whereas for “entertainer”, “poet” and “lawyer”, the activations are clearly large.
Table 1: In this figure we show a set of learned factors with its top-activation words. Based on the common aspect of the first 20% of the top-activation words (usually around 100 words), we can give each of the factors a semantic name.

| factors | top activation words |
|---------|----------------------|
| 59 "medical" | hospital, medical, physician, physicians, nurse, doctor, hospitals, doctors, nurses, patient, nursing, medicine, care, healthcare, psychiatric, clinic, psychiatry, ambulance, pediatric |
| 116 "vehicle" | vehicles, vehicle, driving, drivers, cars, car, driver, buses, truck, trucks, taxi, parked, automobile, fleet, bus, taxis, passenger, van, automobiles, accidents, motorcycle, mph |
| 193 "ware" | pottery, bowl, bowls, porcelain, ware, vase, teapot, china, saucer, denby, vases, saucers, ceramic, glass, plates, earthenware, pitcher, wedgewood, pots, plate, tureen, jug, pot, jar |
| 296 "mobile&IT" | ipad, iphone, ios, itunes, apple, android, app, ipod, airplay, 3g, 4s, apps, ipads, htc, tablet, galaxy, jailbreak, iphones, netflix, mac, os, touch, nook, skyfire, dock, siri, eris, 4g, tablets |
| 337 "superlative" | strongest, funniest, largest, longest, oldest, fastest, wettest, tallest, heaviest, driest, sexiest, scariest, coldest, hardest, richest, biggest, happiest, smallest, toughest, warmest, most |
| 461 "country" | venezuela, germany, paraguay, uruguay, norway, russia, lithuania, ecuador, netherlands, estonia, korea, brazil, argentina, albania, denmark, poland, europe, sweden, colombia |
| 470 "bedding" | mattress, pillow, bed, mattresses, beds, pillows, queen, ottoman, simmons, cushion, bedding, topper, foam, plush, sleeper, sofa, comforter, couch, futon, seat, bolster, pad, sleeping |
| 493 "royal" | king, royal, throne, prince, monarch, emperor, duke, queen, reign, coronation, kings, empress, regent, dynasty, palace, monarchs, ruler, crown, heir, monarchy, kingdom, sultan, consort |
| 582 "fruit" | fruit, fruits, pears, oranges, apples, peaches, grapes, apple, ripe, plums, bananas, mandarin, grapefruit, peach, berries, tomatoes, kiwi, watermelon, berry, lemons, mango, canning, kiwis |
| 635 "Chinese" | china, fujian, zhejiang, guangdong, hangzhou, shandong, shanghai, qingdao, beijing, chongqing, guangzhou, sichuan, jiangsu, hainan, hebei, luoyang, shenzhen, nanjing, hainan |
| 781 "national" | croatian, american, lithuanian, norwegian, vietnamese, chinese, romanian, bulgarian, indonesian, armenian serbian, turkish, hungarian, korean, malaysian, italian, austrian |
| 886 "female" | her, queen, herself, she, actress, feminist, heroine, princess, empress, sisters, woman, dowager, lady, sister, mother, goddess, women, daughter, diva, maiden, girl, ne, feminism, heroines |

In Appendix C we provide more of such generated word analogy tasks by the learned factors.

**Unclear Factors.** While most of the factors have a strong and clear semantic meaning, there are also about less than 10% of them that we can not identify. Some of these factors have relatively dense activations that they may activate on more than 10% of the whole vocabulary while the activations are relatively low. Some of the factors seem to cluster either high or low frequency words regardless of the semantics, e.g. a factor’s top-activation words are all rare words. We feel that some of these unidentifiable factors might actually have semantic meaning with more
Figure 3: PCA visualization of a new word analogy task: “profession”, which are automatically generated by the “profession” word factor.

summarization effort and the rest might be due to an optimization choice, e.g. we sampled word embedding in proportion to the words’ frequency during optimization, so that high frequency words got more exposure than low frequency ones.

Factor Groups. Different factors may correspond to similar semantic meaning and in a particular word vector, they co-activate or only one of them activate. But in general similar factors tend to have a relatively higher co-activation. Based on the co-activation strength, we can cluster word factors in to groups, each provides a more reliable semantic and syntactic meaning detection. In Appendix D we show the co-activation patterns in more details.

3.2 Word Vector Visualization

As a result of sparse coding, every word vector can now be expressed as a linear combination of a limited number of word factors. This makes it possible for us to open up continuous word vectors and see different aspects of meanings through the component factors. In Figure 5 we show several word vectors as a combinations of highly activated factors.

Polysemy Separation. Words like “apple” contain multiple senses of meanings but are encoded into one continuous vector. By visualization through word factors, different senses are separated. As is shown in figure 5, the vector of “apple” contains 4 major factors: “technology”, “fruit”, “dessert” and “organism”. This combination coincides with our knowledge that “apple” is a fruit, a food ingredient, a living creature and a well-known tech company. Another polysemous example is the word “China”: the presence of factor “ware” makes sense as the training corpus is not case sensitive. We further notice the “country” factors and the “Chinese” factor, which is closely related to specific Chinese nouns such as the names of its provinces and cities. In fact a combination of the “country” factor and a “country-specific” factor shows up as a common combination in the names of countries.

Semantic + Syntactic. Besides polysemy, we also find that words are opened up as combinations of both semantic and syntactic factors: “big” has both “size” and “comparative” factor; “won” has “match”, “award”, “sports” and of course “past tense”.

Unexpected Meaning. Sometimes a word may have an unexpected factor, e.g. in the visualization of “so”, we find a “German” factor, of which all the top activated words are German word pieces like “doch”, “aber”, “voll”, “schn” and “ich”. The possible explanation for this is that the training data of the embedding model covers German corpus, and “so” is actually also used in German.

Word Vectors Manipulations. Prior work [24] has demonstrated that linear operations between continuous word vectors can capture linguistic regularities. Now given such factors with clear and strong semantic meanings, it is natural to think of some manipulations. An interesting question is: if we manually add in or subtract out a certain factor from a word vector, would the new word vector be consistent with the semantic relations entailed by the manipulation? To validate this, we manipulate a vector with some factors, and see what is the nearest word in the embedding space. Examples are listed in Table 2. Since the average norm of the word vectors we use is about 7.2, while the factors are of unit norm, we give a constant coefficient of 4 to the factors so that their lengths become compa-
Figure 5: Word vectors can be decomposed into a sparse linear combination of word factors. Due to a space limit, we only show the major factors and leave the rest as “others”.

Table 2: Factor-vector manipulations and factor-factor manipulations. The word vectors’ average norm is 7.2. The learned word factors all have unit norm.

| Manipulations                      | Nearest Neighbors |
|-----------------------------------|-------------------|
| $v_{good} + 4f_{superlative}$     | $v_{best}$        |
| $v_{king} + 4f_{female}$          | $v_{queen}$       |
| $v_{italy} + 4f_{national}$       | $v_{italian}$     |
| $v_{unwise} + 4f_{negative prefix}$ | $v_{wise}$      |
| $v_{hospital} + 4f_{vehicle}$     | $v_{ambulance}$   |
| $v_{soldier} - 4f_{military}$     | $v_{man}$         |
| $v_{dancers} - 4f_{military}$     | $v_{man}$         |
| $v_{dancers} - 4f_{profession}$   | $v_{dance}$       |
| $v_{bed} - 4f_{bed}$              | $v_{pill}$        |

rable but still shorter than the word vectors, therefore can be appropriately regarded as components of word vectors. Results show that both syntactic and semantic meanings including part of speech, gender, sentiment and so on are successfully modified in the desired way. This interesting experiment shows the potential of word factors. Given their explicit meanings, now we are no longer limited to operations between word vectors, but can also conduct operations between word vectors and factors.

3.3 Comparison Across Different Models

We conducted experiments with several mainstream word embedding models, including GloVe, Fasttext, Word2vec CBow and Word2vec skipgram, all of 300 dimension. For GloVe, we download model pretrained on CommonCrawl [2]. For fasttext, we download model pretrained on Wikipedia 2017, UMBC web-base corpus and statmt.org news dataset [1]. And for Word2vec models, we trained them on 3B token wikipedia dump [36]. Although the results are similar between different embedding models, we also notice some interesting differences that can provide understanding of the models. In the fastText embeddings, word “sing” has a abnormally high activation on the factor “present tense”, as is shown in Figure 6. This is because the algorithm trains word embeddings based on subword n-grams [7, 17], in this way word “sing” is considered as if a word in present tense because it contains a three-gram “ing”. This shows that despite the advantages of using subword n-grams to embed words, such as tackling out-of-vocabulary issue and encode strong syntactic information, it may also lead to problems. Such a visualization can be used to diagnose and provide insights to improve the existing methods.

Figure 6: The word vector “sing” learned by fastText has a high activation on the “present tense” factor, due to the subword structure ‘ing’.

There are also differences that reflect bias in different corpus. For example, we compared the pretrained GloVe and a GloVe model trained only on Wikipedia, and refer to them as “GloVe Crawl” and “GloVe Wiki”. As a result, we discovered the factor “bedding” in the visualization of vector “king” in the “GloVe Crawl”, while it is missing in “GloVe Wiki”. The only
Figure 7: Difference between embedding models. The visualization of “king” has significant “bedding” factor in “GloVe Crawl”, but it is not found in “GloVe Wiki”.

difference between the two models is the training data. The presence of factor “bedding” actually makes sense because “king” is frequently used to describe the size of beds and bedclothes. The reason why it is missing in “GloVe Wiki” is likely due to the difference in training data. Which is to say such usage of “king” is much less frequent in Wikipedia than in CommonCrawl, so it is possible that this aspect of meaning is not significant on Wikipedia. In order to verify this, we examined the average co-occurrence statistics in Wikipedia between “king” and top 100 activated words of factor “royal” and “bedding”. The fact that the former is more than 30 times larger than the latter supports the assumption that “king” appears rather rarely in the “bedding” context. Thus a factor of “bedding” is hard to get given the corpus from Wikipedia. This comparison shows that the difference in data is captured by embedding models and displayed by our factors.

4 Improvement in Semantic and Syntactic Tasks with Word Factors

Word analogy task is a classic task for evaluating the quality of word embeddings. Proposed first by [22], it measures the accuracy of answering the question: A is to B as C is to D, such as ‘king’ is to ‘queen’ as ‘man’ is to ‘woman’ and ‘slow’ is to ‘slower’ as ‘good’ is to ‘better’. Given word A, B and C, a model must try to predict D. The conventional approach taken by the word embeddings [22, 30, 7] is a vector arithmetic: calculate $B - A + C$, and find its nearest neighbor in the embedding space as the predicted word. While this approach leads to good results, several failure modes are shown in Table 5. Although vector arithmetic returns a word very close to the ground truth in terms of meaning, it fails to identify the key semantic relationship implied in the question.

However, if semantic meaning captured by the factors are reliable enough, they should be able to identify the semantic difference and therefore correct such mistakes very easily. To do this, we propose a simple factor group selection approach, where we require the predicted word to not only be the closest in the embedding space, but also have an higher activation on the specific factor groups than that of the words in the other category. For example, to be selected as an answer, “queen”‘s activation on the “female” factor group must be higher than those male words in the subtask. By simply applying this factor group selection approach, we achieve consistent improvement for every embedding algorithm on almost every subtask. For many subtasks, the improvement is quite significant. Three experiments in Table 3 get a decreased performance, most likely due to the correct factors are not grouped together during spectral clustering.

Besides validating the previously identified relationships, as mentioned in Section 3.1, we are also able to find many new ones using the factors, such as “ideology” (Figure C.2), “profession” (Figure 3) and “adj-to-verb” (Figure C.1). The questions are constructed in the same way as word analogy tasks. Here are examples from each new task:

\[ V_{collectivist} - V_{collectivism} = V_{liberal} - V_{liberalism} \]
\[ V_{entertain} - V_{entertainer} = V_{poem} - V_{poet} \]
\[ V_{sensational} - V_{sensationalize} = V_{marginal} - V_{marginalize} \]

Performance of each embedding method on the new tasks is shown in Table 4, which has the same behavior as shown in Table 3. Consistent improvements are obtained once we apply the simple factor group selection method. This further demonstrates that word factors can capture reliable semantic meanings and the phenomenon is not only constraint to the previously proposed ones in the word analogy task.

5 Discussion

In this work, we show that dictionary learning can extract elementary factors from continuous word vectors. By using the learned factors, we can meaningfully decompose word vectors and visualize them in many interesting ways. Further, we demonstrate with these factors, we can consistently improve many word embedding methods in word analogy tasks. The word factors may provide an convenient mechanism to develop new word analogy tasks beyond the existing 14 ones. Further examination of existing word embedding models may leads to further improvements.

A fundamental question that remains to be answered is why word factors can be combined in such linear fashion? [19] provides one possible explanation: with sparse word representation, which explicitly encode each word’s context statistics, one can also construct equally good word vectors. If we see a word vector

Table 3: Performance on word analogy task for different word embedding models

|        | W2V sg ori | W2V sg group | Fasttext ori | Fasttext group | W2V cb ori | W2V cb group | GloVe ori | GloVe group |
|--------|------------|--------------|--------------|----------------|------------|--------------|-----------|-------------|
| 0      | 93.87      | 93.87        | 56.92        | 58.10          | 88.34      | 89.13        | 80.04     | 83.99       |
| 1      | 89.86      | 90.52        | 40.01        | 40.79          | 87.48      | 87.97        | 79.86     | 85.52       |
| 2      | 11.06      | 12.02        | 36.54        | 37.26          | 16.35      | 18.27        | 20.19     | 22.84       |
| 3      | 72.15      | 75.76        | 23.89        | 24.11          | 68.22      | 68.79        | 65.46     | 66.11       |
| 4      | 86.17      | 87.15        | 89.13        | 89.92          | 89.33      | 90.12        | 96.44     | 98.02       |
| 5      | 32.16      | 44.56        | 75.00        | 76.61          | 34.38      | 38.81        | 43.15     | 42.24       |
| 6      | 44.46      | 47.91        | 68.97        | 71.18          | 40.39      | 43.47        | 35.10     | 42.61       |
| 7      | 83.93      | 86.71        | 97.15        | 97.15          | 80.04      | 83.99        | 87.69     | 91.67       |
| 8      | 62.59      | 73.86        | 99.15        | 100.00         | 69.98      | 86.74        | 82.77     | 96.12       |
| 9      | 66.38      | 87.41        | 97.73        | 100.00         | 69.98      | 86.74        | 82.77     | 96.12       |
| 10     | 90.31      | 90.43        | 85.05        | 85.74          | 88.12      | 88.68        | 69.36     | 72.67       |
| 11     | 61.99      | 63.08        | 84.94        | 90.71          | 67.05      | 71.60        | 64.10     | 83.72       |
| 12     | 76.43      | 71.55        | 96.70        | 97.15          | 80.26      | 82.36        | 95.95     | 91.44       |
| 13     | 71.26      | 82.99        | 95.63        | 98.74          | 63.79      | 72.07        | 67.24     | 88.51       |
| Sem    | 79.53      | 81.16        | 40.81        | 41.42          | 77.22      | 77.86        | 72.84     | 75.31       |
| Syn    | 67.95      | 73.47        | 89.38        | 91.19          | 69.32      | 74.01        | 72.59     | 79.80       |
| Tot    | 72.70      | 76.63        | 74.42        | 75.86          | 72.56      | 75.59        | 72.69     | 77.96       |

Table 4: Word analogy performance on new tasks for different word embedding models

|        | W2V sg ori | W2V sg group | Fasttext ori | Fasttext group | W2V cb ori | W2V cb group | GloVe ori | GloVe group |
|--------|------------|--------------|--------------|----------------|------------|--------------|-----------|-------------|
| Ideology | 56.00      | 56.00        | 93.33        | 93.33          | 56.50      | 58.67        | 82.00     | 85.50       |
| Profession | 27.78      | 38.30        | 65.79        | 77.19          | 36.55      | 45.03        | 36.26     | 53.80       |
| Adj-to-verb | 8.97       | 13.46        | 62.82        | 65.38          | 16.03      | 16.67        | 24.36     | 26.28       |
| Total    | 40.53      | 44.44        | 80.42        | 84.34          | 44.54      | 48.45        | 59.56     | 67.21       |

Table 5: A few examples to show the typical errors in word analogy tasks using word vectors.

| Vector arithmetic | Ground truth | Prediction |
|-------------------|--------------|------------|
| unreasonable - reasonable + competitive | unreasonable | competitive |
| worse - bad + cheap | cheaper | cheapest |
| greece - athens + beijing | china | shanghai |
| danced - dancing + enhancing | enhanced | enhance |
| wife - husband + policeman | policewoman | policemen |

as an explicit statistic based on a word’s surrounding context, then this context may fall into sub-categories of words. Our guess is that the learned word factors reflect each of these sub-context categories. This suggests an interesting future direction of our work. A limit of this work is that all the word vectors we visualize are trained from methods which ignore the context of a word used in a specific instance. Applying dictionary learning to attention-based methods [33, 9] is another interesting future direction. Finally, the existence of more elementary meaning than words is a debatable argument in linguistic study. The learned word factors may also provide insights and verification to the sememe theory [4, 27, 37]. We leave this to the future work.

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Appendix

A The Word Factor Naming Procedure

In this section we illustrate how the factors are named. A factor is named based on the common aspect of its top-activation words. Specifically, for every factor, we use the word frequency to weight the factor’s activation on each word, and take the top words that totally contributing 20% of the total weighted activation. The idea is that a factor should be better represented by words that have strong and obvious activation and show up frequently as well. Usually we get up to 200 words but the number varies from factor to factor. We demonstrate four factors: “national”, “mobile&IT”, “superlative”, “ideology”, among which the first two are semantic factors and the latter two are syntactic factors.

“national” Factor. The top 20% activation of the No.781 factor contains about 80 words. They are enumerated as the following:

croatian, american, lithuanian, norwegian, vietnamese, chinese, romanian, bulgarian, indonesian, armenian, serbian, turkish, hungarian, korean, malaysian, italian, australian, portuguese, mexican, macedonian, german, scottish, albanian, cambodian, bosnian, rican, filipino, lebanese, swedish, estonian, irish, venezuelan, dutch, pakistani, haitian, iranian, peruvian, argentine, malay, colombian, danish, ethiopian, australian, european, chilean, brazilian, israeli, japanese, indian, finnish, singaporean, african, british, nigerian, argentinian, belgian, hispanic, french, cypriot, guatemalan, latvian, russian, welsh, algerian, bolivian, egyptian, moroccan, belarussian, jamaican, icelandic, samoan, uruguayan, georgian, ukrainian, jordanian, flemish, muslim, yugoslav, greek, jewish

By looking into these words, we can easily identify that almost every one of them is related to mobile devices and IT technology, such as apps, brands and etc. Thus we name factor No.296 as “mobile&IT”.

Such naming procedure is less subjective if a factor captures syntactic meaning.

“superlative” Factor. For instance, the top 20% activation of the No.337 factor contains about 70 words:

strongest, funniest, largest, longest, oldest, fastest, wettest, tallest, heaviest, driest, sexiest, scarcest, coldest, hardest, richest, biggest, happiest, smallest, toughest, warmest, most, brightest, loudest, shortest, costliest, coolest, smartest, darkest, slowest, weakest, greatest, lightest, deadliest, thickest, craziest, sunniest, deepest, quickest, busiest, best, cleanest, saddest, worst, ugliest, densest, sweetest, nicest, wealthiest, hottest, weirdest, dumbest, dullest, poorest, highest, bloodiest, prettiest, grandest, safest, meanest, bravest, strangest, catchiest, dirtiest, proudest, cleverest, purest, quietest, fairest, youngest, shapest

It’s clear that this factor captures the “superlative” form of different words.

“ideology” Factor. Finally, we demonstrate the top 20% activating about 120 words of the No.674 factor:

nationalism, liberalism, socialism, individualism, capitalism, communism, fascism, anarchism, materialism, humanism, secularism, feudalism, republicanism, modernism, conservatism, rationalism, imperialism, totalitarianism, militarism, multiculturalism, feminism, marxism, racism, ideology, consumerism, pacifism, modernity, romanticism, utilitarianism, fundamentalism, positivism, democracy, authoritarianism, patriotism, unionism, politics, environmentalism, internationalism, paganism, absolutism, nazism, radicalism, commercialism, pluralism, naturalism, colonialism, protestantism, relativism, idealism, egalitarianism, patriarchy, sexism, spiritualism, libertarianism, regionalism, atheism, mysticism, populism, collectivism, ideologies, pragmatism, universalism, isolationism, anarchy, paternalism, antisemitism, pro-
tectionism, federalism, transcendentalism, deism, religiosity, elitism, determinism, neoclassicism, postmodernism, centralism, orthodoxy, empiricism, industrialization, catholicism, puritanism, monasticism, separatism, promoted, realism, classicism, altruism, zionism, nihilism, bolshevism, globalization, sectarianism, progressivism, expressionism, orientalism, morality, modernization, barbarism, christianity, occultism, expansionism, slavery, interventionism, traditionalism, tyranny, monogamy, surrealism, abolitionism, primitivism, hedonism, vegetarianism, historicism, chauvinism, humanitarianism, asceticism, dualism, doctrine, unitarianism, misogyny, extremism

The idea that it reflects ideology forms of different concepts is quite obvious once we see the words. So the factor summarized as “ideology”.

B The Details of the Non-negative Sparse Coding Optimization

As a convention, all the word vectors used in this word is 300 dimensional and we choose our dictionary to have 1000 word factors. To learn the word factors, we use a typical iterative optimization procedure:

\[
\min_A \frac{1}{2} \|X - \Phi A\|_F^2 + \lambda \sum_i \|\alpha_i\|_1, \text{ s.t. } \alpha_i \geq 0, \tag{9}
\]

\[
\min_{\Phi} \frac{1}{2} \|X - \Phi A\|_F^2, \|\Phi_j\|_2 \leq 1. \tag{10}
\]

These two optimizations are both convex, we solve them iteratively to learn the word factors: In practice, we use minibatches contains 100 word vectors as \(X\). Optimization \(9\) can converge in 500 steps using the FISTA algorithm. We experimented with different \(\lambda\) values from 0.3 to 1, and choose \(\lambda = 0.5\) to give results presented in this paper. Once the sparse coefficients have been inferred, we update our dictionary \(\Phi\) based on Optimization \(10\) by one step using an approximate second-order method, where the Hessian is approximated by its diagonal to achieve an efficient inverse. The second-order parameter update method usually leads to much faster convergence of the word factors. Empirically, we train 200k steps and it takes about 2-3 hours on a Nvidia 1080 Ti GPU.

C The New Word Analogy Tasks Generated

In this section, we would like to demonstrate further that the word factors are more elementary structures than word vectors.

\[\text{Figure C.1: PCA of the generated adj-to-verb examples.}\]

\[\text{Figure C.2: PCA visualization of a new word analogy task: “ideology”, which are automatically generated by the “ideology” word factor.}\]

D Factor Group Co-activation

In Figure \[\text{D.1}\] and \[\text{D.2}\] we use heat maps to visualize the activations of factors within a group. A heat map shows a fraction of the activation matrix \(A\) in Equation \(8\) with each row corresponds to a factor, each column to a word. Therefore, a bright block indicates a high activation on the given word and the dark background means 0 values. It is very clear that factors within a group are often activated together on the same words, forming parallel bright bands across the heat maps.

\[\text{We also experimented other settings and they all lead to qualitatively similar result and discussing the difference is beyond the scope of this work.}\]
Figure D.1: This figure shows the co-activation pattern of factors in the “past tense” factor group.

Figure D.2: This figure shows the co-activation pattern of factors in the “singular form” factor group.