Wandering in Color-Space
– why the life of pentaquark is so long? –

Yuu Maezawa1, Toshiki Maruyama2, Naoyuki Itagaki1 and Tetsuo Hatsuda5
1Department of Physics, University of Tokyo, Tokyo 113-0033, Japan
2Advanced Science Research Center, Japan Atomic Energy Research Institute, Tokai, Ibaraki 319-1195, Japan

The problem of the long life time of the pentaquark $\Theta^+$ is investigated on the basis of the color molecular dynamics simulation. We find that it takes a long time (typically of $50 - 100$ fm/$c$) for the initial pentaquark-state to rearrange its color and spatial positions to decay into the nucleon + kaon final state. Structure of the potential surface in the color and position spaces also supports this picture. Pentaquark wanders on the potential surface to find a narrow channel to decay.

PACS numbers: 14.20.-c, 12.39.Jh, 21.45.+v

I. INTRODUCTION

An exotic baryon, pentaquark $\Theta^+$, has been first reported by LEPS collaboration at SPring-8. Later, there are other experiments reporting the confirmation of its existence, which is summarized in refs. [2, 3]. Since the $\Theta^+$ decays into a neutron ($n$), and a kaon ($K^+$), its quark content is considered to be $uudd\bar{s}$. The mass is found to be about 1540 MeV, while there is yet no direct measurement of its spin and isospin, although $I = J = 1/2$ is rather plausible. Its parity is also not known. Its width ($<10$ MeV) is exceptionally narrow as for a hadron resonance located at 110 MeV above the $nK^+$ threshold. There have been many theoretical analyses on the pentaquark before and after the discovery of $\Theta^+$ on the basis of the skyrme model, quark models, QCD sum rules, lattice QCD simulations and so on, which are summarized in recent reviews [4, 5].

The question we address in this paper is the origin of the narrow width of the pentaquark which is the most peculiar feature of this new resonance. Since there is no known selection rule from symmetry to make the width naturally small, the narrow width should have some dynamical origin. In the past, there have been several attempts to explain the narrow width from combinatorial suppression from the spin-flavor and color-factors or from the special spatial structure due to diquark correlations. However, no definite conclusion has been reached yet.

In this paper, we propose a possible dynamical mechanism for the long life time of the pentaquark by treating its hadronic decay on the basis of the constituent quark model and molecular dynamics. Suppose that the colors of the constituent quarks are “well-mixed” inside the pentaquark (the precise meaning of the mixing will be given later), then it takes a long time for the quarks to rearrange their colors, flavors, spins, and spatial positions into two color-white clusters, i.e. the nucleon $N$ and the kaon $K$. The dynamical time scale of this rearrangement is governed by the strong interactions among quarks and is not simply related to the “distance” in color-flavor-spin-space between the pentaquark and the $NK^+$ state. Therefore, it is not trivial from the outset that the life time is long.

The color molecular dynamics (CMD) simulations originally developed in ref. [6] gives us a suitable framework to study the dynamical decay process of the pentaquark. The CMD is a quantum molecular dynamics for constituent quarks, in which single quark wave function is parameterized by a Gaussian wave pocket in coordinate space and by a color coherent state in the $SU(3)$ color space. Time-dependent dynamics of the multi-quark system is then governed by the Hamiltonian commonly used in the standard constituent quark models [7]. The clusterings and decays of the multi-quarks are easily treated by this approach. As a first attempt, we will treat the most essential part of the decay process (color and spatial coordinates) and neglect spin, flavors and antisymmetrization.
II. BASIC FORMULATIONS OF CMD

We express a total wave function of the system \( \Psi \) as a direct product of single-particle quark wave-functions:

\[
\Psi = \prod_{i=1}^{N} \phi_i(\mathbf{r}) \chi_i,
\]

(1)

\[
\phi_i(\mathbf{r}) \equiv \left( \frac{\pi L^2}{2} \right)^{-3/4} \exp \left[ -\frac{(\mathbf{r} - \mathbf{R}_i)^2}{2L^2} - \frac{i}{\hbar} \mathbf{P}_i \cdot \mathbf{r} \right],
\]

(2)

\[
\chi_i \equiv \left( \begin{array}{c} \cos \alpha_i e^{-i\beta_i} \cos \theta_i \\ \sin \alpha_i e^{i\beta_i} \cos \theta_i \\ \sin \theta_i e^{i\phi_i} \end{array} \right).
\]

(3)

Here \( i \) specifies the quarks and anti-quarks. \( N \) is the total number of quarks+anti-quarks in the system \( (N = 2, 3 \) and \( 5 \) for the kaon, the nucleon, and the pentaquark, respectively). \( \phi_i \) is a Gaussian wave packet centered around \( \mathbf{R}_i \) with momentum \( \mathbf{P}_i \) and a fixed width \( L \), and \( \chi_i \) is a coherent state in the color SU(3) space parameterized by four angles, \( \alpha_i, \beta_i, \theta_i, \phi_i \).

Time evolution of the system is given by solving the equations of motion for \( \{ \mathbf{R}_i, \mathbf{P}_i, \alpha_i, \beta_i, \theta_i, \phi_i \} \) obtained from the time-dependent variational principle \( (\delta \mathcal{L} = 0) \) on the expectation value for the Lagrangian:

\[
\mathcal{L} = \langle \Psi | i\hbar \frac{d}{dt} - \hat{H} | \Psi \rangle = \sum_{i} \left[ -\mathbf{P}_i \cdot \mathbf{R}_i + \hbar \beta_i \cos 2\alpha_i \cos^2 \theta_i - \hbar \phi_i \sin^2 \theta_i \right] - H,
\]

(4)

where \( H = \langle \Psi | \hat{H} | \Psi \rangle \). The Hamiltonian of the system is given as:

\[
\hat{H} = \sum_{i} \sqrt{m^2_i + \mathbf{P}^2_i} + \frac{1}{2} \sum_{i,j \neq i} \left[ -\sum_{a=1}^{8} t^a \hat{V}_C(r_{ij}) + V_M(r_{ij}) \right],
\]

(5)

\[
\hat{V}_C(r) = K r - \frac{\alpha_s}{r},
\]

(6)

where \( t^a = \lambda^a/2 \) for quarks and \( t^a = -\lambda^a/2 \) for anti-quarks with \( \lambda^a \) being Gell-Mann matrices. \( \hat{V}_C \) consists of one-gluon exchange and confinement terms with an infrared cutoff at \( r = 3.0 \text{ fm} \). Typical values of the parameters in the quark model for baryons and mesons read \( m_{u,d} = 300 \text{ MeV} \), \( m_s = 500 \text{ MeV} \) (the constituent-quark mass), \( \alpha_s = 1.25 \) (the QCD fine structure constant), and \( K = 0.75 \text{ GeV/fm} \) (the string tension). We take \( L_{u,d} \) (size of the quark wave packet) to be 0.49 fm, so that the root-mean-square (rms) radius of the quark becomes 0.6 fm which also corresponds to the rms radius of the quark core in the ground state of baryons.

Color-dependent potential \( \hat{V}_C(r) \) alone does not lead to the mass difference between \( \Theta^+ \) and \( nK^+ \) state. Since the 110 MeV mass difference is essential for the pentaquark to decay, we introduce \( V_M(r) \) to reproduce this mass difference. We take color independent potential with a combination of the attractive scalar-type and repulsive vector-type Yukawa potentials. We assume that \( V_M(r) \) does not act on the \( \bar{s} \) quark. The effective size of the wave packet \( L_{\text{eff}} \) in the matrix element of \( V_M \) is chosen to be 0.6 fm to reproduce the mass splitting between \( \Theta^+ \) and \( nK \).

A. Criterion for color-whiteness

To make a color-white hadron from arbitrary initial configurations, we use a cooling technique in color space. Whether the system of five quarks (four quarks labeled by \( i = 1 \sim 4 \) and one anti-quark labeled by \( i = 5 \)) become color-white or not is decided by solving the following criterion:

\[
\sum_{i=1}^{4} \langle \chi_i | \lambda^a | \chi_i \rangle - \langle \chi_5 | \lambda^{a*} | \chi_5 \rangle = 0 \quad (a = 1, \cdots, 8).
\]

(7)

Criterion of color whiteness for the \( q \bar{q} \) and \( qgg \) systems are similarly defined. The color-white pentaquark satisfying eq. (7) is not necessarily composed of two color-white subclusters. This is why dynamical rearrangement of internal colors is required for the pentaquark to decay into the \( nK \) state.
III. DECAY OF THE PENTAQUARK

A. Color mixing rate ($\alpha$) and effective spatial distance ($D$)

The decay of the pentaquark to the $NK$ state is a dynamical process in which all the coordinate values (10 for each quark and 50 for the five quarks) defined in the previous section change in time. Although CMD simulations trace time-development of all these coordinates, it is convenient to define some particular combinations which can qualitatively characterize the decay process. $\alpha$ and $D$ defined below are such key parameters.

$\alpha$ is a measure how well the colors are mixed among five quarks. We take notice of an anti-quark in the system and calculate a distance between the anti-quark and one of the other quarks in color-space. The minimum value within four possible distances is called $\alpha$. The definition of $\alpha$ in color space can be seen from the following formula:

$$\alpha = \min_{t=1,\ldots,4} \left\{ \sum_{n=1}^{8} \left| \langle \chi_{i} | \lambda^{n} | \chi_{i} \rangle - \langle \chi_{5} | \lambda^{*n} | \chi_{5} \rangle \right|^{2} \right\}. \tag{8}$$

For $\alpha = 0$, not only the five quarks are color-white as a whole, but also there are color-white $\bar{q}q$ and $qqq$ subclusters. As $\alpha$ increases, the mixture of color in the five quarks increases. The system cannot be separated into two white clusters any more for large $\alpha$. Note that this parameter is defined only in the color space. Therefore, even if subclusters formed at small $\alpha$ are color-white, they do not have to be clusters in coordinate space.

Next, we define an effective distance $D$ which is a measure the rate of clustering of the five quarks in the coordinate space. We consider the anti-quark. A quark closest to the anti-quark is supposed to form a subcluster with the anti-quark in the coordinate space. Then, we define a center of mass coordinate as $R_{\bar{q}q}$. Together with the center of mass coordinate of the remaining three quarks $R_{qqq}$, $D$ is defined by

$$D \equiv |R_{\bar{q}q} - R_{qqq}|. \tag{9}$$

For small $D$, the five quarks are in one unit, while for large $D$, they split into two spatially separated subclusters. Since $D$ is defined only in the coordinate space, each subcluster is not necessarily color-white.

By using these parameters, the pentaquark $\Theta^{+}$ is characterized as a state with large $\alpha$ and small $D$, while the $NK$ scattering state is characterized by small $\alpha$ and large $D$. Both parameters are time-dependent and their initial values at $t = 0$ are defined as

$$\alpha_{\text{init}} \equiv \alpha(t = 0), \quad D_{\text{init}} \equiv D(t = 0). \tag{10}$$

We simulate the time development of the five quarks from initial conditions with various color and spatial configurations ($\alpha_{\text{init}}$ and $D_{\text{init}}$). Then we estimate the lifetime of the system until its decay into the $NK$ state.

B. Correlation between the color mixing and the life time

Let us first investigate the correlation between the initial color mixing rate and the lifetime. We start with initial variables $\{R_{i}, \alpha_{i}, \beta_{i}, \theta_{i}, \varphi_{i}\}$ randomly chosen under the constraint that the five quarks are color white as a whole. We choose the initial momentum $P_{i} = 0$. Then, the equation of motion for 50 ($5 \times 10$) coordinates are solved until the system decays into color-white $\bar{q}q$ ($K$) + color-white $qqq$ ($N$) system. The life time is defined when $\alpha$ becomes sufficiently small and $D$ becomes sufficiently large so that the system becomes the $NK$ state. Actual conditions we use are

$$\alpha < 0.05 \quad \text{and} \quad D > 3.0 \text{ fm}. \tag{11}$$

Figure II(a) shows a relation between $\alpha_{\text{init}}$ and the lifetime $T$. The solid point for a given $T$ is obtained by averaging over randomly distributed values of $\alpha_{\text{init}}$. The error-bars show the 1σ variance of $\alpha_{\text{init}}$. This figure clearly shows a positive correlation between $T$ and $\alpha_{\text{init}}$. The result can be easily understood: it takes more time for the state with large color mixing to rearrange their colors to white and white, which decays into the $NK$ final state.

C. Correlation between spatial clustering and the life time

Next we investigate the relation between the initial spatial distance $D_{\text{init}}$ and the lifetime $T$. Figure II(b) shows the relation between $D_{\text{init}}$ and the lifetime $T$. The samples are the same as in the case of Sec.3.2. The solid point for
FIG. 1: (a) Relation between the initial color mixing rate $\alpha_{\text{init}}$ and the lifetime $T$. (b) Relation between the initial spatial distance $D_{\text{init}}$ and the lifetime $T$.

a given $T$ is obtained by averaging over randomly distributed values of $D_{\text{init}}$. The error-bars show the $1\sigma$ variance of the $D_{\text{init}}$. The figure shows a negative correlation between $T$ and $D_{\text{init}}$. The result can be easily understood: it takes more time for the state with small spatial distance between subclusters to rearrange their coordinates to the $NK$ final state.

If we choose $\alpha_{\text{init}} = 0.5$ and $D_{\text{init}} = 0$ as a typical parameter set for the pentaquark state $\Theta^+$, we have the lifetime (decay width) of about 100 fm/c (2 MeV). It is too hasty at the moment to compare this number with the experimental upper bound of 10 MeV, but the result is suggestive.

D. Potential surface as a function of $\alpha$ and $D$

To study the effect of color mixing rate $\alpha$ and the effective spatial distance $D$ on the dynamical decay process in more details, we calculate an effective potential $V$ between the $q\bar{q}$ and $qqq$ subclusters as functions of $\alpha$ and $D$.

Shown in Fig. 2(a) is the result of such calculation using the potential part of the Hamiltonian in eq.(5). In practice, $V(\alpha, D)$ is calculated as follows: first, we prepare a $q\bar{q}$ state with quark and anti-quark sitting on top of each other in coordinate space, and $qqq$ is prepared in the same way. Then, we calculate $\alpha$, $D$ and the expectation value of the potential energy of this state. Then, this process is repeated to cover the variety of points in two-dimensional $\alpha$−$D$ plane. With this procedure, $V$ is adjusted to be zero for the $NK$ final state at large $D$ and small $\alpha$. Also $V=110$ MeV for small $D$ and large $\alpha$ corresponds to the ideal excitation energy of the pentaquark state $\Theta^+$.

In general, $V$ increases as $D$ increases due to the effect of color confinement potential. Namely, it takes more and more energy when one tries to separate the five quarks into color non-white subclusters in the coordinate space. The only exception is $\alpha = 0$ where the potential energy decreases as $D$ increases. This is because there is no resistance from the confinement force in this case. Note also that the potential surface is flat in the $\alpha$ direction for small $D$. (One can prove that $V$ is exactly $\alpha$-independent for $D = 0$). This implies that the color-dependent potential become important only when quarks are separated in space.

Now, suppose that the pentaquark state $\Theta^+$ is located at small $D$ and large $\alpha$ region as indicated by the solid circle in Fig. 2(a). For this state to decay into the $NK$ state indicated by the open circle in Fig. 2(a), it has to rearrange color to find a narrow channel near $\alpha = 0$. This takes a long time since the potential surface is flat in $\alpha$ direction and thus the system goes back and forth before reaching the channel. Once it reaches the region around $\alpha = 0$, it quickly decays into the $NK$ state along the narrow channel. Namely, the flat potential along the $\alpha$-axis near $D = 0$ and the narrow channel along the $D$-axis near $\alpha = 0$ are two essential sources of the long life time of the pentaquark.

Figure 2(b) shows the actual path in the simulation of the pentaquark state decaying into the $NK$ state. The initial conditions are taken to be $\alpha_{\text{init}} = 0.4$ and $D_{\text{init}} = 0.25$ fm, and it takes 25 fm/c to decay. Wandering in color space of the pentaquark before the decay can be seen explicitly from this figure.
IV. SUMMARY

In order to study the narrow decay width of the pentaquark suggested by experiments, we have carried out color molecular dynamics (CMD) simulation for five quarks. In CMD, the spatial and color parts of the wave function for each quark is expressed as the Gaussian wave packet and SU(3) coherent state respectively. An advantage of this approach is that one can trace the rearrangement process of spatial and color coordinates as a function of time during the decay.

To characterize the essential part of the decay process, we have introduced two key parameters, the color mixing ratio $\alpha$ and the effective spatial distance $D$. The results of the simulation show that there is a positive (negative) correlation between $\alpha(t = 0)$ ($D(t = 0)$) and the life time $T$ of the five-quark state. $T$ can reach to even 100 fm/c if $\alpha(t = 0)$ is enough large and $D(t = 0)$ is enough small. Narrow channel in the effective potential surface $V(\alpha, D)$ is found to be the physical origin to cause the long life time. The pentaquark wanders around the potential surface.

In the present paper, we have not considered the spin and flavor of the quarks, and hence the spin and flavor dependent interactions in the Hamiltonian. The antisymmetrization of the quark wave functions are also neglected for simplicity. Detailed study of the decay process taking into account these ingredients is an urgent problem to be examined.

In this paper, we have simply assumed that the pentaquark $\Theta^+$ corresponds to a state with a large $\alpha$ and small $D$. Whether one can indeed reach such parameter region by the process $N + K \to \Theta^+ \to N + K$ can be studied by the scattering simulations in CMD, which will be reported elsewhere. Applications of the present study to the decay processes of other possible narrow resonances such as charmed pentaquark are also the interesting problems to be investigated.

Acknowledgments

We thank S. Sasaki, A. Hosaka, Y. Akimura and S. Chiba for fruitful discussions on the physics of pentaquarks. This work was partially supported by the Grants-in-Aid of the Japanese Ministry of Education, Culture, Sports, Science, and Technology (No. 15540254).

[1] LEPS Collaboration, T. Nakano et al., Phys. Rev. Lett. 91 (2003) 012002.
[2] T. Nakano and K. Hicks, Mod. Phys. Lett. A19 (2004) 645.
[3] S. Eidelman et al., The Review of Particle Physics, Phys. Lett. B592 (2004) 1.
[4] M. Oka, *Theoretical Overview of the Pentaquark Baryons*, Prog. Theor. Phys. 112 (2004) 1.

[5] S. Sasaki, *Pentaquarks*, plenary talk at Lattice 2004, *The XXII International Symposium on Lattice Field Theory* (FNAL, June 21-26, 2004); [http://lqcd.fnal.gov/lattice04/](http://lqcd.fnal.gov/lattice04/).

[6] T. Maruyama and T. Hatsuda, Phys. Rev C 61 (2000) 062201.

[7] See, e.g. M. Oka and K. Yazaki, in *Quarks and Nuclei*, ed. W. Weise (World Scientific, Singapore, 1984), p. 489.

[8] Y. Kanada-Enyo, O. Morimatsu, and T. Nishikawa, hep-ph/0404144.

[9] Y. Maezawa, T. Maruyama, N. Itagaki and T. Hatsuda, in preparation.

[10] In general, the pentaquark can decay to the $nK^+$ state and the $pK^0$ state. However, in the present model, we do not distinguish $u$ and $d$ quarks. Therefore, hereafter we use a notation “NK” to describe the final decay product.

[11] Since we use a coherent state in color space, color-white does not necessarily imply color-singlet. The color-singlet part should be projected out from the color-white wave function given in the present paper. This is a future problem to be done.

[12] Spin, flavor and antisymmetrization were recently taken into account in the antisymmetric molecular dynamics (AMD) study of a static pentaquark state in ref. [8]. However, the decay process is considered only in a static and approximate way in that work.