Lorentz violating inflation and the Swampland

Oem Trivedi

*School of Arts and Sciences, Ahmedabad University, Ahmedabad 380009, India
E-mail: oem.t@ahduni.edu.in

ABSTRACT: The swampland conjectures from String theory have had very interesting implications for cosmology and particularly for Inflation. It has been shown that the single field inflationary models in a GR based cosmology are in unavoidable tensions with these conjectures, while these single field models can still be consistent in certain non-trivial inflationary regimes. So it becomes interesting to see whether there is a way to overcome the issues of the swampland and single field inflation in an essentially GR based cosmology. We show that this can indeed be the case and for this, we consider a certain type of Lorentz violating inflationary scenario. We work out the swampland bounds for Lorentz violating inflationary models after which show that inflationary models which would have had otherwise serious tensions with these conjectures in a usual GR based scenario, can be very tranquil with the criterion in this regime. For this we take examples of Higgs inflation, radion gauge inflation and Spontaneous symmetry breaking inflation and show that the bounds imposed by the swampland on the Lorentz violating parameter can be easily satisfied in these models.
1 Introduction

The concept of cosmic inflation has been extremely effective in explaining different properties of the early universe [1–5]. Various satellite observations have consistently confirmed inflationary estimates for the early universe, and the most recent evidence from the Planck experiment continues this trend [6–9]. Furthermore, observational evidence supports a wide range of inflationary models, all of which are inspired by ranging from modified gravity theories to quantum gravitational realisations, radically different contexts [10–16]. However, traditional single field models (which some refer to as "Supercooled Inflationary Models [13]") have a lot of empirical support and are still widely used in theoretical studies.

In recent years, a lot of effort has gone into developing a “Theory of Everything,” and String Theory is probably the most well-known candidate for such a paradigm. Since String Theory is presented in such a vivid way, it is reasonable to expect it to have far-reaching consequences for cosmology. As a result, there is a large and diverse body of literature that has looked into the cosmological consequences of String theory theories. The extremely high number of possible vacua states that string theory makes, which goes as high as, is one of the many cosmologically interesting aspects of the theory. This is known as the “landscape” of string theory. The question then becomes, which class of low-energy efficient field theories is actually compatible with String theory. In order to address this issue, Vafa coined the word "Swampland," which refers to a class of low-energy effective field theories that defy the String theory paradigm. Furthermore, a number of field theoretic UV completion criterion from string theory known as the "swampland conjectures" have been proposed in recent years to classify whether a given regime is in the swampland or not. Many people consider string theory to be a feasible quantum gravity model, so if a...
low-energy EFT meets these criteria, tt could also get along with a self-consistent quantum gravity theory. Although there are other swampland criterion like the completeness [17] and cobordism [18] conjectures, the distance and the dS conjectures quite clearly have the most striking implications for cosmology. These two conjectures can be described as follows:

1: Swampland Distance Conjecture: This conjecture limits the field space of validity of any effective field theory [19]. This sets a maximum range traversable by the scalar fields in an EFT as

\[ \frac{\Delta \phi}{m_p} \leq d \sim \mathcal{O}(1) \]  

where \( m_p \) is the reduced Planck’s constant, \( d \) is some constant of \( \mathcal{O}(1) \), and \( \phi \) is the Scalar Field of the EFT.

2 Swampland De Sitter Conjecture: This Conjecture states that it is not possible to create dS Vacua in String Theory [20]. The conjecture is a result of the observation that it has been very hard to generate dS Vacua in String Theory [21, 22]( While it has been shown that creating dS Vacua in String Theory is possible in some schemes, like the KKLT Construction [23]). The Conjecture sets a lower bound on the gradient of Scalar Potentials in an EFT,

\[ m_p |V'| \geq c \sim \mathcal{O}(1) \]  

where \( c \) is some constant of \( \mathcal{O}(1) \), and \( V \) is the scalar Field Potential. A "refined" form of the Swampland De Sitter Conjecture places constraints on the hessian of the scalar potential (a finding which first appeared in [24] and later in [25]) and is given by

\[ m_p^2 \frac{V''}{V} \leq -c' \sim \mathcal{O}(1) \]  

These criterion have intriguing implications for cosmology, especially single-field inflation. Considering the data on inflation, it was shown that single field inflation in a GR based cosmology is incompatible with these conjectures [26]. A lot of effort has gone into resolving this dispute [27–29]. However, it has been shown that if the background cosmology for single field inflation is not GR based, then this inflationary regime can still satisfy the swampland criterion [30–34]. Multi field inflationary models are consistent with the conjectures even in GR based paradigms [35]. It’s also worth noting that in both GR and non-GR based cosmologies, the warm inflation paradigm has been shown to be very compatible with the swampland criterion even for single field models [36–40] The recently proposed “Trans Planckian Censorship Conjecture” (TCC) [41] is another swampland conjecture that has sparked a lot of interest in inflationary cosmology. Models with tachyonic scalar fields as the inflaton can also be consistent with the conjectures [42, 43]. Single field GR based inflationary models can only be compatible with the TCC if they are extremely fine tuned [44], which is ironic given that inflation was invented to solve the fine tuning problem of traditional big bang cosmology. Much further work has been done to clarify the TCC’s problems with single field inflation, with the overall picture appearing to be that the TCC
is more comfortable with non-standard inflationary regimes than usual single field models [45–51]. What one can hence understand from the current literature on the swampland and inflation is that one is not well placed to expect conventional (cold) single field models to be on amicable terms with these conjectures. Hence, it would optimistic for one to think that non-trivial modifications to cold single field models based in a GR cosmology could lead to these paradigms being consistent with the conjectures.

One such non trivial modification to single field inflationary models can be found by investigating such regimes in a Lorentz violating background. Several inflationary regimes in Lorentz-violating cosmological scenarios have been studied in the recent times [52, 53, 53–59]. Gasperini [52] proposed that the primordial period of rapid expansion of the Universe may be achieved if the gravitational interactions were characterised by a non locally Lorentz invariant theory at some extremely early epoch. The authors in [58] examined a Lorentz-violating inflationary theory based on an Einstein-aether theory and a scalar field Lagrangian. Such scenarios have also been used to understand dark energy, where it has been demonstrated that violating the Lorentz invariance creates Lagrangians capable of driving the universe’s current acceleration [60, 61]. The former and latter theories, as a result, deal with Lorentz symmetry violation at small and large distances, respectively [62]. In [59], it was shown that a time-like Lorentz violating background can produce sufficient inflation whilst also providing an explanation for the current dark energy epoch. Lorentz violation during inflation generates some non standard changes in the usual inflationary dynamics and could hence be an interesting regime with regards to the swampland. In the next section, we briefly describe a particular kind of Lorentz violating cosmology, while in section III we discuss the swampland status quo of (cold) single field inflation in that regime. In section IV, we will consider 3 different inflationary models and discuss which of them could be consistent with the swampland on the basis of the groundwork made in Section III. We summarize our work in section V with some concluding remarks about the overall picture of the early universe painted by the swampland so far.

2 Basics of the Lorentz violating cosmology

We will be working in a time-like Lorentz violating cosmological background here. A purely time-like background can be defined by a tiny subset of Lorentz invariant violating operators that preserve rotational invariance, which is why one would use a time-like Lorentz operator here [63]. Furthermore as Kostelecky and Mewes describe in [64], the Cosmic Microwave Background becomes a natural choice of a preferred frame in this scenario. We start off with a Lagrangian in a 3+1 dimensional background describing a scalar field (which will eventually be the inflaton) coupled to a Lorentz violating background (similar to the one here [59])

$$\mathcal{L} = \left( \frac{R}{2\kappa^2} - \frac{1}{2} \left( g_{\mu\nu} + \xi_{i\mu} \xi_{\nu} \right) \partial_{\mu} \phi \partial^{\nu} \phi - V(\phi) \right) \sqrt{-g}$$

(2.1)

where $\kappa^2 = 8\pi G$ and $V(\phi)$ is the inflaton potential while $k_{i\mu}$ are time-like tensors which can even couple to other fields in a more general lagrangian. It does so in a way such that
the only non-zero component is \( k_{i0}^{i} \), which means

\[
k_{\mu\nu}^{i} = \begin{pmatrix}
-\beta_{i} & 0 \\
0 & 0
\end{pmatrix}
\]  

(2.2)

where the tensor couples to the scalar field via the coupling \( \xi_{1} > 0 \). This kind of a theory has been touted to be able to explain both the current expansion of the universe as well the inflationary phase [59]. It can do so if \( \beta_{1} \to 0 \) for short distances, which allows for all the inflationary dynamics to be controlled only by the inflaton field while if \( \beta_{1} \to -1 \) for large distances, the Lorentz violating regime can also explain dark energy. We will, however, just be discussing about the inflationary phase in our analysis in the next section.

For studying the cosmological properties of the Lagrangian described above, we can take the metric to be of the FLRW form

\[
d\mathbf{s}^{2} = -dt^{2} + a(t)^{2}dx^{2} = g_{\mu\nu}dx^{\mu}dx^{\nu}
\]  

(2.3)

The background fields responsible for the Lorentz violating factor would change the metric as

\[
d\mathbf{s}^{2} = \mathcal{g}_{\mu\nu}dx^{\mu}dx^{\nu} = (g_{\mu\nu} + \xi_{1}k_{\mu\nu})dx^{\mu}dx^{\nu} = -(1 + \xi_{1}\beta_{1})dt^{2} + a(t)^{2}dx^{2}
\]  

(2.4)

here one can then go on to write an effective velocity for the scalar field as

\[
v = \sqrt{1 + \xi_{1}\beta_{1}c}
\]  

(2.5)

where \( c \) is the usual speed of light. One can then also define a relationship between the Lorentz violating parameter and the redshift \( z \) as [59]

\[
\xi_{1}\beta_{1} = -\frac{C_{1}}{z(z + 2) + C_{1}}
\]  

(2.6)

where \( C_{1} \) is a constant. This relation becomes very important when one wants to realize both the inflationary phase and the dark energy phase with the same inflaton field but as we are currently interested only in the former phase we will not explore the redshift relation further.

If one considers a General Relativity based cosmological scenario, then the Friedmann equation will remain the same as they are in the usual GR sense. Hence, the Friedmann equation remains as

\[
H^{2} = \frac{\rho}{3m_{p}^{2}}
\]  

(2.7)

where we are working in \( m_{p} = \sqrt{\frac{1}{8\pi\kappa}} \) units. However, the energy and pressure densities will not remain as they usually are in GR. The energy-momentum tensor can be written here as

\[
T_{\mu\nu} = \partial_{\mu}\phi\partial_{\nu}\phi + g_{\mu\nu}\mathcal{L}_{\phi}
\]  

(2.8)
where \( \mathcal{L}_\phi = -\frac{1}{2} (g_{\mu\nu} + \xi_1 h_{\mu\nu}^1) \partial^\mu \phi \partial^\nu \phi - V(\phi) \). Now we can write down the pressure and energy densities, given that our field configurations are homogenous and so \( \phi = \phi(t) \), we have the energy and pressure densities as

\[
\rho_\phi = \frac{1}{2} (1 - \kappa) \dot{\phi}^2 + V(\phi) \quad (2.9)
\]

\[
p_\phi = \frac{1}{2} (1 - \kappa) \dot{\phi}^2 - V(\phi) \quad (2.10)
\]

Finally, the equation of motion for the scalar field is found out to be

\[
\ddot{\phi} + 3H \dot{\phi} + \frac{V'(\phi)}{1 + \xi_1 \beta_1} = 0 
\]

(2.11)

This completes our groundwork for the formulation of inflation in this scenario. In the next section we will be discussing about the different inflationary parameters in this scenario and consider the possibility of this regime being swampland consistent.

3 Swampland implications for Lorentz violating inflation

In the inflationary phase, the energy density \( \rho \) is dominated by the inflaton energy density \( \rho_\phi \) and so one can take \( \rho \sim \rho_\phi \) during inflation. Hence, the Friedmann equation (10) during inflation takes the form

\[
H^2 = \frac{1}{3m_p^2} \left( \frac{1}{2} (1 - \xi_1 \beta_1) \dot{\phi}^2 + V(\phi) \right) \quad (3.1)
\]

From now onwards in the paper, we define the Lorentz violating parameter to be \( \kappa = \xi_1 \beta_1 \).

Now during inflation, \( \dot{\phi}^2 \ll V(\phi) \) which corresponds to the slow roll condition. Applying this criterion here, we find the Friedmann equation to be

\[
H^2 = \frac{V(\phi)}{3m_p^2} \quad (3.2)
\]

One can note that this is the same form that the Friedmann equation has for all sorts of inflationary models in a GR based cosmology, which are Lorentz-abiding regimes. Hence, the Friedmann equation remains unchanged even after factoring in Lorentz violations in this case. Now, we cast our attention to the field equation for the inflaton (14). Another slow roll criterion during inflation is that \( \ddot{\phi} \ll 3H \dot{\phi} \). Considering this, we can write the field equation to be

\[
3H \dot{\phi} + \frac{V'(\phi)}{1 + \kappa} \approx 0 \quad (3.3)
\]

Now we begin to see effects of Lorentz violation in the inflationary dynamics, as the Lorentz violating parameter plays an important role in the field equation during inflation. Now let's find expressions for an important part of the inflationary setup, the slow roll parameters. The \( \epsilon \) slow roll parameter is given by

\[
\epsilon = -\frac{\dot{H}}{H^2} \quad (3.4)
\]
Using the Friedmann equation (16) and the Field equation (17), we can write the $\epsilon$ potential slow roll parameter from its usual definition as

$$\epsilon = \frac{m_p^2}{2(1 + \kappa)} \left( \frac{V'}{V} \right)^2$$

(3.5)

And further, we can find the $\eta$ potential slow roll parameter similarly as

$$\eta = \frac{m_p^2}{1 + \kappa} \frac{V''(\phi)}{V(\phi)}$$

(3.6)

After this, one can then find out the e-fold number during Inflation as

$$N = \int_{t_i}^{t_f} H dt$$

(3.7)

This can then be written in terms of $\phi$ as

$$N = \int_{\phi(t_i)}^{\phi(t_f)} \frac{(1 + \kappa)}{m_p^2 \frac{V'(\phi)}{V(\phi)}} d\phi$$

(3.8)

where $\phi(t_i)$ and $\phi(t_f)$ are the values of the inflaton field at the time of Horizon crossing and at the end of inflation, respectively. At this point, we can start to discuss the implications of the swampland conjectures on this inflationary regime. A point to note is that for the dS conjecture is that if a low energy EFT scenario is consistent with either the original dS conjecture (2) or it’s refined form (3), then it can have a consistent UV completion with regards to the dS criterion. Hence one can use only one of these conjectures in this regard if they like to, and we will be using the original dS conjecture (2) instead of the refined form in our analysis. The issues of the swampland conjectures with single field inflation were firstly discussed in [26], where the background cosmology was also general relativistic. Two of the very strong disagreements between the conjectures and inflation concerned the bounds on the $\epsilon$ parameter during inflation and the number of e-folds. In their work, they showed that the dS conjecture forbids the $\epsilon$ parameter to satisfy it’s usual bound needed for sufficient inflation, which is $\epsilon \ll 1$. It is easy to see how this is the case, as this $\epsilon$ parameter bound for single field inflation in a GR based cosmology is given as

$$\epsilon = \frac{m_p^2}{2} \left( \frac{V'}{V} \right)^2 \geq \frac{c^2}{2}$$

(3.9)

where $c$ is an $O(1)$ parameter in the original dS conjecture (2). The above equation makes it clear that it would be problematic to achieve $\epsilon \ll 1$ during inflation considering $c \sim O(1)$. From this definition, they were further able to work out some bounds on the scalar spectral index and they then found out that there is a disagreement between what the data on single field inflation and string theory itself predicts about the order estimates of the $c$ parameter. Namely, the $c$ parameter should be an $O(0.1)$ term for it to be consistent with the data on inflation instead of $O(1)$ as estimated from String theory. The next serious issue is concerns the e-fold number during Inflation. If one considers both the distance (1) and the
dS conjectures (2) to hold true, then one finds out that these conjectures induce a fatal bound on the number of e-folds during inflation. This can also be readily shown here, as the number of e-folds for single field (in a GR based cosmology) can be roughly written as

\[ N \simeq \frac{\Delta \phi}{m_p m_p \frac{V'}{V}} \]  

(3.10)

Applying both the distance \( (\Delta \phi / m_p \leq d \sim \mathcal{O}(1)) \) and dS conjecture \( m_p |V'| \geq c \sim \mathcal{O}(1) \) here, one finds out that the e-fold number is constrained to be less than unity by these criterion. This is an incredibly distressing prediction, as the latest observational bounds [7] require atleast around 60 e-folds in order for Inflation to explain the problems of big bang cosmology which it was originally brought in to rectify. Hence these explorations seem to suggest that all kinds of single field inflationary models in a GR based cosmology lie in the swampland and so, possibly, would not be viable with quantum gravity eventually.

However, now we make the case that single field inflation can be consistent with the swampland conjectures even when the background cosmology is described by GR, albeit with some Lorentz violations. We can now tackle both the prime concerns raised in [26] and find bounds on the parameter \( \kappa \) due to Lorentz violation which would allow us to rectify these issues. The \( \epsilon \) parameter in the Lorentz violating background is given by (19)

\[ \epsilon = \frac{m_p^2}{2(1 + \kappa)} \left( \frac{V'}{V} \right)^2 \]

Considering the dS conjecture (2), one can write the following bound for this parameter

\[ \epsilon \geq \frac{c^2}{2(1 + \kappa)} \]  

(3.11)

One can clearly notice that the dS conjecture bound for this \( \epsilon \) parameter is similar to the one encounter for the usual single field model encountered in (23), with the difference between both being the \( (1 + \kappa) \) term in the denominator here. As \( c \sim \mathcal{O}(1) \), in order for \( \epsilon \ll 1 \) one requires that

\[ 2(1 + \kappa) >> 1 \implies \kappa >> -\frac{1}{2} \]  

(3.12)

This provides us a lower bound on the Lorentz violating parameter in order for the inflationary regime to be consistent with the swampland. Furthermore, we can roughly write the e-fold number(22) during inflation in this case as

\[ N \simeq (1 + \kappa) \frac{\Delta \phi}{m_p m_p \frac{V'}{V}} \]  

(3.13)

Again applying the dS and distance conjectures together here, we find that the term \( \frac{\Delta \phi}{m_p m_p \frac{V'}{V}} \) has to be less than 1. Hence, in order to have sufficient inflation one requires

\[ (1 + \kappa) >> 1 \implies \kappa >> 0 \]  

(3.14)
The last inequality is consistent and in fact even more general the requirement for satisfying the $\epsilon$ parameter bounds in (26). Hence in order for inflation to be consistent with the swampland conjectures in this regime, the minimum requirement that needs to be satisfied is $\kappa > 0$. This will get us rid of the issues of both the $\epsilon$ parameter and the e-fold number and hence this lower limit on $\kappa$ is what we need in order to have swampland consistent inflation in this regime.

A question one might wonder about now is, how would we actually get to know whether or not this inequality is satisfied in a particular inflationary model? The answer to that lies in the perturbation parameters for inflation, in particular the scalar spectral index. The definition of the scalar spectral index remains the same here as it is for usual inflationary models [59, 65]

$$n_s - 1 = 2\eta - 6\epsilon$$

(3.15)

One can even further work out other perturbation parameters like the tensor-to-scalar ratio, the tensor spectral index etc. but the scalar spectral index is all we need here to constrain $\kappa$ in this case using the latest observational data from the Planck experiment [7]. In fact, the quadratic inflation model $V(\phi) = m^2\phi^2/2$ was worked out in this Lorentz violating regime in [59]. There the authors used the Planck value for the scalar spectral index to find that $\kappa \simeq -10^{-2}$. This shows us that the quadratic inflation model is in fact inconsistent with the swampland conjectures even in a Lorentz violating scenario (although this model already has a lot of issues with the latest Planck data itself, even though there have been some attempts to reconcile it with the data [66]).

In the next section we will now constrain the Lorentz violating parameter in 3 different inflationary models and try to ascertain whether the constraints from the swampland conjectures can be satisfied by a significant amount of potentials.

4 Analysing the swampland consistency of some models in the Lorentz violating regime

In this section, we will work out the Lorentz violating parameter in the Higgs, Radion Gauge and Spontaneous Symmetry breaking inflationary models and check whether these models are swampland consistent in a Lorentz violating regime.

4.1 Higgs Inflation

Higgs Inflation is an inflationary regime of particular phenomenological interest [67–70]. As the name suggests, here the well known Higgs field $h$ itself is considered to be playing the role of the inflaton field. The starting point of achieving this lies in the quantum corrected Standard model Lagrangian

$$S = \frac{M^2}{2} \int \left[ F(h)\bar{R} - Z(h)\bar{g}^{\mu\nu}\partial_\mu h\partial_\nu h - 2U(h) \right] \sqrt{-\bar{g}} d^4x$$

(4.1)
where bars over certain quantities represent the fact that the above action is in Jordan frame and \( \bar{M} \) here is considered to be a mass scale. Choosing some particular forms for the functions \( F(h) \), \( Z(h) \) and \( U(h) \) and after doing a conformal transform to the Einstein frame, one finds the Inflaton potential in this case to be [10, 67, 68, 70]

\[
V(\phi) = M^4(1 - e^{-\sqrt{2/3}\phi/m_p})^2
\]  

(4.2)

where \( M \) is a mass scale (in fact this will remain the convention for mass scale depiction for the later potentials in this work as well). One interesting thing to note here is that the Higgs inflation potential is an example of an inflationary model with no free parameters. In order to obtain the \( \kappa \) values allowed by this model, we first need to find the slow roll parameters \( \epsilon \) and \( \eta \) (as defined in (19) and (20) for the potential (31)). Although this model has been shown to be consistent with observational data, it has been shown that even the higgs potential can be in tensions with the swampland conjectures in the usual GR inflationary regime [71]. Hence it would be interesting to see whether considering a Lorentz violating regime for inflation would help alleviate these issues.

For this potential, one can find these parameters to be

\[
\epsilon = \frac{4}{3(1+\kappa)} \left( e^{\sqrt{2/3}\phi/m_p} - 1 \right)^2
\]  

(4.3)

\[
\eta = \frac{4 \left( 2 - e^{\sqrt{2/3}\phi/m_p} \right)}{3(1+\kappa) \left( e^{\sqrt{2/3}\phi/m_p} - 1 \right)^2}
\]  

(4.4)

After being powered with the knowledge of these parameters, we can now find the scalar spectral index \( n_s \) after some algebra as

\[
n_s - 1 = \frac{-4(-1 + \coth[\phi/(\sqrt{6}m_p)]) \coth[\phi/(\sqrt{6}m_p)]}{3(1+\kappa)}
\]  

(4.5)

The latest observational constraints [7] on the scalar spectral index estimate that \( n_s = 0.9665 \pm 0.0038 \) at the time of Horizon crossing, hence for our purposes here we can take \( n_s \approx 0.9665 \) and be assured of considerable precision in our analysis. Using this value of the scalar spectral index, we can arrive at the following value for the Lorentz violating parameter

\[
\kappa = 37.986 \left( \coth \left( \frac{\phi}{\sqrt{6}m} \right) - 1 \right) \coth \left( \frac{\phi}{\sqrt{6}m} \right) - 1
\]  

(4.6)

The \( \phi \) value at the time of horizon crossing \( \phi = \phi(t_i) \) can be reasonably considered to be to roughly be the reduced Planck mass

\[
\phi \approx m_p
\]  

(4.7)

Using this field value, we remain in line with the distance conjecture (1) \( \Delta \phi/m_p \leq O(1) \) throughout inflation (even when one considers the field value at the end of inflation \( \phi_f \) to
be nearly zero). Inserting this into the above equation, we finally find the Lorentz violating parameter to be

$$\kappa \simeq 161.92$$  \hspace{1cm} (4.8)

This is well above the minimum bound implied by the swampland conjectures on inflation in the Lorentz violating regime, which is $\kappa > 0$(28). Hence Higgs Inflation is consistent with the swampland conjectures even in a GR based cosmology when one considers Lorentz violations in the cosmological background. It is also interesting to note that this is a zero parameter model (in the sense that the potential has no free parameters to tune accordingly) but is still consistent with the swampland conjectures.

4.2 Radion Gauge Inflation

Radion Gauge inflation was first studied in [72] and is an extension of the gauge inflation scenario in which the radius modulus field around which the Wilson loop is wrapped assists inflation as it shrinks [73]. The potential in this scenario can be written as [10]

$$V(\phi) = \frac{M^4(\phi^2/m_p^2)}{\alpha + (\phi^2/m_p^2)}$$  \hspace{1cm} (4.9)

where $M$ is again a mass scale and $\alpha$ is a positive dimensionless parameter, which would act as the only free parameter of the model. This potential can also be found in S-dual superstring models [74] Although Radion gauge inflation has never been studied in the context the swampland for GR based single field inflation, the issues pointed out for GR based inflation in [26] are so strong that they would generally rule all kinds of GR based potentials in the swampland and hence, it would be interesting to see how this model stands with these criterion in a Lorentz violating regime.

We proceed similar to the way we did for Higgs Inflation in the previous subsection, by firstly finding out the slow roll parameters in this regime. One can find that the parameters, after some algebra, take the forms

$$\epsilon = \frac{2\alpha^2m_p^6}{(1 + \kappa)(\alpha m_p^2\phi + \phi^3)^2}$$  \hspace{1cm} (4.10)

$$\eta = \frac{2\alpha m_p^4(\alpha m_p^2 - 3\phi^2)}{(1 + \kappa)(\alpha m_p^2\phi + \phi^3)^2}$$  \hspace{1cm} (4.11)

After finding the slow roll parameters, it is straightforward to write the scalar spectral index using its definition (29) and one finds the index to be

$$n_s - 1 = -\frac{4\alpha m_p^4(2\alpha m_p^2 + 3\phi^2)}{(1 + \kappa)(\alpha m_p^2\phi + \phi^3)^2}$$  \hspace{1cm} (4.12)

Again using the estimate $n_s \simeq 0.9665$ and the field value at Horizon crossing to be $\phi \simeq m_p$, one finds the following relation for the Lorentz violating parameter in terms of $\alpha$ as

$$\kappa = \frac{\alpha(238.8\alpha + 358.2)}{(\alpha + 1)^2} - 1$$  \hspace{1cm} (4.13)
In order to be consistent with the swampland conjectures, one requires the minimum bound \( \kappa > 0 \) on the Lorentz violating parameter and this implies

\[
\kappa = \frac{\alpha(238.8\alpha + 358.2)}{(\alpha + 1)^2} - 1 > 0 \implies \alpha > 0.0028
\]

(4.14)

The requirement imposed on the free parameter \( \alpha \) in order for this model to be swampland consistent is quite minimal and extremely in line with the basic view that \( \alpha \) is a positive parameter. Hence, Radion gauge inflation can be easily viable with the swampland conjectures in a Lorentz violating regime even when the background cosmology is general relativistic.

### 4.3 Spontaneous Symmetry Breaking Inflation

Spontaneous symmetry breaking inflation, in the sense that we are going to be discussing here, was firstly worked on by Moss in [75]. Moss considered the forthcoming potential in the framework of models with spontaneous symmetry breaking where \( \phi \) represents one of the components of a Higgs field. The potential in this inflationary regime takes the form

\[
V(\phi) = M^4 \left( 1 + \frac{\alpha \phi^2}{m_p^2} + \frac{\gamma \phi^4}{m_p^4} \right)
\]

(4.15)

where \( \alpha \) and \( \gamma \) are constant dimensionless parameters and \( M \) is again a mass scale. This model is different from the others we have discussed so far in that it has 2 free parameters in \( \alpha \) and \( \gamma \) instead of just one (like the radion gauge model) or none (like Higgs Inflation). It is interesting to mention that besides spontaneous symmetry breaking, this type of potential also finds place in certain SUSY and Spontaneous D-Parity Breaking inflationary scenarios[76–78]. It also becomes interesting to see the consistency of this model with the swampland as spontaneous symmetry breaking is the way in which Lorentz violating effects are introduced in the Standard Model extension [79], which is an EFT framework which contains the Standard Model, General Relativity and other possible operators which can break Lorentz symmetry.

The slow roll parameters for this potential take the form

\[
\epsilon = \frac{2 (\alpha m_p^2 \phi^2 + 2 \gamma m_p \phi^3)}{(1 + \kappa) (\alpha m_p^2 \phi^2 + \gamma \phi^4 + m_p^4)^2}
\]

(4.16)

\[
\eta = \frac{2m_p^2 (\alpha m_p^2 + 6 \gamma \phi^2)}{(1 + \kappa) (\alpha m_p^2 \phi^2 + \gamma \phi^4 + m_p^4)}
\]

(4.17)

This allows us to find the scalar spectral index (29) to be

\[
n_s - 1 = \frac{4m_p^2 (-2m_p^2 \phi^2 (\alpha^2 - 3\gamma)) - 5\alpha \gamma m_p^2 \phi^4 + \alpha m_p^6 - 6\gamma^2 \phi^6}{(1 + \kappa) (\alpha m_p^2 \phi^2 + \gamma \phi^4 + m_p^4)^2}
\]

(4.18)

We again take note of the fact that the index is being evaluated at horizon crossing, and consider \( \phi = m_p \). Using the estimate \( n_s \simeq 0.9665 \) again and imposing the minimum
requirement for swampland consistency for Lorentz violating inflation, $\kappa > 0$, we get the following inequality

$$\kappa = \frac{\alpha(119.4 - 597.01\gamma) + \gamma(716.4 - 716.4\gamma) - 238.8\alpha^2}{(\alpha + \gamma + 1)^2} - 1 > 0$$  \hspace{1cm} (4.19)

One can check with the help of any mathematical computation tool that this inequality can be satisfied for a huge range of $\alpha$ and $\gamma$, where both of them can be either positive or negative. Just for an example, we highlight one possible solution of the above inequality which puts the following bounds on $\alpha$

$$0.008671 < \alpha \leq 0.480901$$  \hspace{1cm} (4.20)

And the corresponding bound on $\gamma$ in terms of $\alpha$ as

$$\gamma < 0.497877 - 0.417453\alpha - 6.969 \times 10^{-7}\sqrt{5.0752 \times 10^{11} - 5.189 \times 10^{11}\alpha - 3.293 \times 10^{11}\alpha^2}$$  \hspace{1cm} (4.21)

This finally allows us to write the bounds on $\gamma$ to as

$$0.594288 < \gamma < 0.988514$$  \hspace{1cm} (4.22)

These restrictions on $\alpha$ and $\gamma$ do not represent a great deal of fine tuning and are just one of the many possible range of values which can be attained by these parameters in order for satisfy the swampland constraint for Lorentz violating inflation. Hence, Spontaneous symmetry breaking inflation can also be readily consistent with the swampland conjectures in a Lorentz violating regime.

5 Concluding remarks and discussion

To summarize, in this work we attempted to find a way in which single field inflationary models can be consistent with the swampland conjectures even when the cosmological background in essentially general relativistic. Although there has been significant work on this in recent times, the novelty of our current work lies in the fact that we achieve this goal in quite a trouble-free way by considering a time-like Lorentz violating background. We start off our work by firstly reflecting on the problems between the swampland conjectures and single field inflation, which are particularly in unavoidable loggerheads in a general relativistic cosmology. We then discuss some crucial aspects of the Lorentz violating cosmology we have considered in our work, showing how the inflationary dynamics is affected by considering the Lorentz violations. We then show what is the requirement for Lorentz violating single field models to be consistent with the swampland conjectures, which eventually turns out to be a minimum bound on the value of the Lorentz violating. We then briefly touch upon the fact that quadratic inflation is not consistent with the swampland bound on the Lorentz violating parameter, building on previous work on the same model in this regime. After this we consider 3 inflationary models of deep phenomenological interest in the form of Higgs inflation, Radion gauge inflation and Spontaneous symmetry breaking inflation and show that all three of these models(which would otherwise have faced the same difficulties
with the swampland in a simple GR based cosmology as their other compatriots) are quite easily consistent with the swampland criterion in our Lorentz violating GR based cosmology.

The main takeaway from this work is that there can still be ways for (cold) single field inflation to be consistent with the swampland conjectures even when the background cosmology is essentially GR based. Another interesting outcome idea that can be pondered upon from our work here concerns the significance of Lorentz violations in the Early Universe. Lorentz symmetry is a cornerstone idea of Relativity and if essentially GR based inflationary regimes, which is in quite unavoidable tension with the swampland criterion, becomes rather easily consistent with these conjectures only by considering a certain form of Lorentz violation in the background cosmology then it could possibly have interesting implications from a quantum gravity point of view. The premise of the swampland conjectures is that these criterion are supposedly necessary conditions in order for low energy EFT’s to have consistent UV completion, so the fact that a Lorentz violating cosmology makes it more tranquil for inflationary regimes to be consistent with the swampland could really give heed to the notion that quantum gravity points towards Lorentz violations being more significant in the early universe than what one persumes till now. Also, it would be interesting to see in future works about the implications of a Lorentz violating scenario on warm inflation, non-GR based inflation and multi-field inflation in the context of the swampland and any corresponding implications it could have on more late universe scenarios like dark energy, dark matter and the Hubble tension.

Acknowledgments

The author would like to thank the Prof. Ralf Lehnert and all the organizers of the 4th IUCSS Summer School on Lorentz- and CPT- Violating Standard Model extensions, which was hosted by Indiana University, Bloomington. The ideas for this work developed during the summer school and the author would like to express his deepest gratitude to the school organizers for putting together such an intellectually enriching event.

References

[1] AA Starobinskii. Spectrum of relict gravitational radiation and the early state of the universe. *JETP Letters*, 30(11):682–685, 1979.

[2] Katsuhiko Sato. First-order phase transition of a vacuum and the expansion of the universe. *Monthly Notices of the Royal Astronomical Society*, 195(3):467–479, 1981.

[3] Alan H Guth. Inflationary universe: A possible solution to the horizon and flatness problems. *Physical Review D*, 23(2):347, 1981.

[4] Andrei D Linde. Chaotic inflation, 1983.

[5] Andrei Linde. Quantum cosmology and the structure of inflationary universe. *arXiv preprint gr-qc/9508019*, 1995.
[6] Nabila Aghanim, Y Akrami, F Arroja, M Ashdown, J Aumont, C Baccigalupi, M Ballardini, AJ Banday, RB Barreiro, N Bartolo, et al. Planck 2018 results. *Astronomy and Astrophysics - A&A*, 641:A1, 2020.

[7] Yashar Akrami, Frederico Arroja, M Ashdown, J Aumont, C Baccigalupi, M Ballardini, AJ Banday, RB Barreiro, N Bartolo, S Basak, et al. Planck 2018 results-x. constraints on inflation. *Astronomy & Astrophysics*, 641:A10, 2020.

[8] N Aghanim, Yashar Akrami, M Ashdown, J Aumont, C Baccigalupi, M Ballardini, AJ Banday, RB Barreiro, N Bartolo, S Basak, et al. Planck 2018 results. vi. cosmological parameters. *arXiv preprint arXiv:1807.06209*, 2018.

[9] Y Akrami, F Arroja, M Ashdown, J Aumont, C Baccigalupi, M Ballardini, AJ Banday, RB Barreiro, N Bartolo, S Basak, et al. Planck 2018 results. i. overview and the cosmological legacy of planck. *arXiv preprint arXiv:1807.06205*, 2018.

[10] Jerome Martin, Christophe Ringeval, and Vincent Vennin. Encyclopædia inflationaris. *Physics of the Dark Universe*, 5:75–235, 2014.

[11] Jérôme Martin, Christophe Ringeval, Roberto Trotta, and Vincent Vennin. The best inflationary models after planck. *Journal of Cosmology and Astroparticle Physics*, 2014(03):039, 2014.

[12] David Wands. Multiple field inflation. In *Inflationary cosmology*, pages 275–304. Springer, 2008.

[13] Arjun Berera. Warm inflation. *Physical Review Letters*, 75(18):3218, 1995.

[14] Gia Dvali and S-H Henry Tye. Brane inflation. *Physics Letters B*, 450(1-3):72–82, 1999.

[15] Stephon Alexander, Antonino Marciano, and David Spergel. Chern-simons inflation and baryogenesis. *Journal of Cosmology and Astroparticle Physics*, 2013(04):046, 2013.

[16] Panagiota Kanti, Radouane Gannouji, and Naresh Dadhich. Gauss-bonnet inflation. *Physical Review D*, 92(4):041302, 2015.

[17] Joe Polchinski. Monopoles, duality, and string theory. *International Journal of Modern Physics A*, 19(supp01):145–154, 2004.

[18] Jacob McNamara and Cumrun Vafa. Cobordism classes and the swampland. *arXiv preprint arXiv:1909.10355*, 2019.

[19] Hiroshi Ooguri and Cumrun Vafa. Non-supersymmetric ads and the swampland. *arXiv preprint arXiv:1610.01533*, 2016.

[20] Keshav Dasgupta, Maxim Emelin, Mir Mehdi Faruk, and Radu Tatar. De sitter vacua in the string landscape. *arXiv preprint arXiv:1806.08362*, 2018.

[21] Shamit Kachru, Renata Kallosh, Andrei Linde, and Sandip P Trivedi. De sitter space and the swampland. *arXiv preprint arXiv:1908.05288*, 2019.

[22] Sumit K Garg and Chethan Krishnan. Bounds on slow roll and the de sitter swampland. *Journal of High Energy Physics*, 2019(11):75, 2019.
[25] Hirosi Ooguri, Eran Palti, Gary Shiu, and Cumrun Vafa. Distance and de sitter conjectures on the swampland. *Physics Letters B*, 788:180–184, 2019.

[26] William H Kinney, Sunny Vagnozzi, and Luca Visinelli. The zoo plot meets the swampland: mutual (in) consistency of single-field inflation, string conjectures, and cosmological data. *Classical and quantum gravity*, 36(11):117001, 2019.

[27] Hao Geng. A potential mechanism for inflation from swampland conjectures. *Physics Letters B*, page 135430, 2020.

[28] Marco Scalisi and Irene Valenzuela. Swampland distance conjecture, inflation and $\alpha$-attractors. *Journal of High Energy Physics*, 2019(8):160, 2019.

[29] Amjad Ashoorioon. Rescuing single field inflation from the swampland. *Physics Letters B*, 790:568–573, 2019.

[30] Chia-Min Lin, Kin-Wang Ng, and Kingman Cheung. Chaotic inflation on the brane and the swampland criteria. *Physical Review D*, 100(2):023545, 2019.

[31] SD Odintsov and VK Oikonomou. Swampland implications of gw170817-compatible einstein-gauss-bonnet gravity. *Physics Letters B*, page 135437, 2020.

[32] Zhu Yi and Yungui Gong. Gauss–bonnet inflation and the string swampland. *Universe*, 5(9):200, 2019.

[33] Oem Trivedi. Swampland conjectures and single field inflation in modified cosmological scenarios. *arXiv preprint arXiv:2008.05474*, 2020.

[34] V. K. Oikonomou, Ilgenezia Giannakoudi, Achilles Gitsis, and Konstantinos-Rafail Revis. Rescaled Einstein-Hilbert Gravity: Inflation and the Swampland Criteria. 5 2021.

[35] Rafael Bravo, Gonzalo A Palma, and M Simón Riquelme. A tip for landscape riders: multi-field inflation can fulfill the swampland distance conjecture. *Journal of Cosmology and Astroparticle Physics*, 2020(02):004, 2020.

[36] Meysam Motaharfar, Vahid Kamali, and Rudnei O Ramos. Warm way out of the swampland. *arXiv preprint arXiv:1810.02816*.

[37] Meysam Motaharfar and Hamid Reza Sepangi. Warm-tachyon gauss–bonnet inflation in the light of planck 2015 data. *The European Physical Journal C*, 76(11):646, 2016.

[38] Meysam Motaharfar, Vahid Kamali, and Rudnei O Ramos. Warm inflation as a way out of the swampland. *Physical Review D*, 99(6):063513, 2019.

[39] Suratna Das. Warm inflation in the light of swampland criteria. *Physical Review D*, 99(6):063514, 2019.

[40] Suratna Das and Rudnei O Ramos. Runaway potentials in warm inflation satisfying the swampland conjectures. *Physical Review D*, 102(10):103522, 2020.

[41] Alek Bedroya and Cumrun Vafa. Trans-planckian censorship and the swampland. *arXiv preprint arXiv:1909.11063*, 2019.

[42] Abolhassan Mohammadi, Tayeb Golanbari, Haidar Sheikhamadi, Kosar Sayar, Lila Akhtari, MA Rasheed, and Khaled Saeedi. Warm tachyon inflation and swampland criteria. *Chinese Physics C*, 44(9):095101, 2020.

[43] Oem Trivedi. Rejuvenating the hope of a swampland consistent inflated multiverse with tachyonic inflation in the high energy RS-II Braneworld. 1 2021.
[44] Alek Bedroya, Robert Brandenberger, Marilena Loverde, and Cumrun Vafa. Trans-planckian censorship and inflationary cosmology. Physical Review D, 101(10):103502, 2020.

[45] Heliudson Bernardo. Trans-planckian censorship conjecture in holographic cosmology. Physical Review D, 101(6):066002, 2020.

[46] Shuntaro Mizuno, Shinji Mukohyama, Shi Pi, Yun-Long Zhang, et al. Universal upper bound on the inflationary energy scale from the trans-planckian censorship conjecture. Physical Review D, 102(2):021301, 2020.

[47] Suddhasattwa Brahma. Trans-planckian censorship, inflation, and excited initial states for perturbations. Physical Review D, 101(2):023526, 2020.

[48] Mansi Dhuria and Gaurav Goswami. Trans-planckian censorship conjecture and nonthermal post-inflationary history. Physical Review D, 100(12):123518, 2019.

[49] Robert Brandenberger and Edward Wilson-Ewing. Strengthening the tcc bound on inflationary cosmology. Journal of Cosmology and Astroparticle Physics, 2020(03):047, 2020.

[50] Kai Schmitz. Trans-planckian censorship and inflation in grand unified theories. Physics Letters B, 803:135317, 2020.

[51] Omer Guleryuz. On the Trans-Planckian Censorship Conjecture and the Generalized Non-Minimal Coupling. 5 2021.

[52] M Gasperini. Inflation and broken lorentz symmetry in the very early universe. Physics Letters B, 163(1-4):84–86, 1985.

[53] Eugene A Lim. Can we see lorentz-violating vector fields in the cmb? Physical Review D, 71(6):063504, 2005.

[54] Joseph A Zuntz, PG Ferreira, and TG Zlosnik. Constraining lorentz violation with cosmology. Physical review letters, 101(26):261102, 2008.

[55] Cristian Armendariz-Picon, Noela Farina Sierra, and Jaume Garriga. Primordial perturbations in einstein-aether and bpsh theories. Journal of Cosmology and Astroparticle Physics, 2010(07):010, 2010.

[56] Sugumi Kanno and Jiro Soda. Lorentz violating inflation. Physical Review D, 74(6):063505, 2006.

[57] PP Avelino, D Bazeia, L Losano, R Menezes, and JJ Rodrigues. Impact of lorentz violation on the dynamics of inflation. Physical Review D, 79(12):123503, 2009.

[58] William Donnelly and Ted Jacobson. Coupling the inflaton to an expanding aether. Physical Review D, 82(6):064032, 2010.

[59] CAG Almeida, MA Anacleto, FA Brito, E Passos, and JRL Santos. Cosmology in the universe with distance dependent lorentz-violating background. Advances in High Energy Physics, 2017, 2017.

[60] Diego Blas and S Sibiryakov. Technically natural dark energy from lorentz breaking. Journal of Cosmology and Astroparticle Physics, 2011(07):026, 2011.

[61] B Audren, D Blas, J Lesgourgues, and S Sibiryakov. Cosmological constraints on lorentz violating dark energy. Journal of Cosmology and Astroparticle Physics, 2013(08):039, 2013.

[62] Ted Jacobson. Einstein-aether gravity: a status report. arXiv preprint arXiv:0801.1547, 2008.
[63] E Passos, MA Anacleto, FA Brito, O Holanda, GB Souza, and CAD Zarro. Lorentz invariance violation and simultaneous emission of electromagnetic and gravitational waves. *Physics Letters B*, 772:870–876, 2017.

[64] V Alan Kostelecký and Matthew Mewes. Electrodynamics with lorentz-violating operators of arbitrary dimension. *Physical Review D*, 80(1):015020, 2009.

[65] Daniel Baumann. Tasi lectures on inflation. *arXiv preprint arXiv:0907.5424*, 2009.

[66] John Ellis, Malcolm Fairbairn, and Maria Sueiro. Rescuing quadratic inflation. *Journal of Cosmology and Astroparticle Physics*, 2014(02):044, 2014.

[67] Fedor Bezrukov and Mikhail Shaposhnikov. The standard model higgs boson as the inflaton. *Physics Letters B*, 659(3):703–706, 2008.

[68] Fedor Bezrukov and Mikhail Shaposhnikov. Standard model higgs boson mass from inflation: Two loop analysis. *Journal of High Energy Physics*, 2009(07):089, 2009.

[69] Fedor L Bezrukov, Amaury Magnin, and Mikhail Shaposhnikov. Standard model higgs boson mass from inflation. *Physics Letters B*, 675(1):88–92, 2009.

[70] Juan Garcia-Bellido, Javier Rubio, Mikhail Shaposhnikov, and Daniel Zenhäusern. Higgs-dilaton cosmology: from the early to the late universe. *Physical Review D*, 84(12):123504, 2011.

[71] Frederik Denef, Arthur Hebecker, and Timm Wrase. de sitter swampland conjecture and the higgs potential. *Physical Review D*, 98(8):086004, 2018.

[72] Malcolm Fairbairn, L Lopez Honorez, and MHG Tytgat. Radion assisted gauge inflation. *Physical Review D*, 67(10):101302, 2003.

[73] Katherine Freese, Joshua A Frieman, and Angela V Olinto. Natural inflation with pseudo nambu-goldstone bosons. *Physical Review Letters*, 65(26):3233, 1990.

[74] A De la Macorra and S Lola. Inflation in s-dual superstring models. *Physics Letters B*, 373(4):299–305, 1996.

[75] Ian G Moss. Primordial inflation with spontaneous symmetry breaking. *Physics Letters B*, 154(2-3):120–124, 1985.

[76] Micheal Dine and Antonio Riotto. An inflaton candidate in gauge mediated supersymmetry breaking. *Physical review letters*, 79(14):2632, 1997.

[77] Antonio Riotto. Inflation and the nature of supersymmetry breaking. *Nuclear Physics B*, 515(1-2):413–435, 1998.

[78] Jimm-Ouk Gong and Narendra Sahu. Inflation in minimal left-right symmetric model with spontaneous d-parity breaking. *Physical Review D*, 77(2):023517, 2008.

[79] V Alan Kostelecký and Stuart Samuel. Spontaneous breaking of lorentz symmetry in string theory. *Physical Review D*, 39(2):683, 1989.