Cartan’s Supersymmetry and the Decay of a $H^0(0^+)$

Sadataka Furui
Graduate School of Teikyo University
2-17-12 Toyosatodai, Utsunomiya, 320-0003 Japan
E-mail: furui@umb.teikyo-u.ac.jp

ABSTRACT: We compare the decay of a Higgs boson $H^0(0^+) \rightarrow \gamma\gamma$, $H^0(0^+) \rightarrow WW \rightarrow \ell\ell\ell\ell$, and $H^0(0^+) \rightarrow ZZ \rightarrow \ell\ell\ell\ell$ using Cartan’s supersymmetry, that defines coupling of a vector particle $X$ and Dirac spinors $\psi$ and $\phi$.

Experimentally the ratio of the signal strength, i.e. branching ratio normalized to the standard model value $\sigma(H^0 \rightarrow xx)/B(H^0 \rightarrow xx)_{SM}$ of $WW$ channel and $\gamma\gamma$ channel $\frac{\sigma(H^0 \rightarrow WW)}{\sigma(H^0 \rightarrow \gamma\gamma)} = 0.87 \pm 0.2$ agrees with the ratio of the number of independent diagrams $\frac{9}{16}$ derived from Cartan’s supersymmetric theory of spinors.

The enhancement of the $\sigma(H^0 \rightarrow ZZ) = 1.11 \pm 0.3$ relative to $\sigma(H^0 \rightarrow WW) = 0.87 \pm 0.2$ is expected to be due to $g\bar{g} \rightarrow \ell\ell\ell\ell$ in the final state of $ZZ$ channel.

KEYWORDS: Supersymmetric Effective Theories, Higgs Physics, B physics
1 Introduction

In the book of the theory of spinors\cite{1}, Cartan proposed two types of Dirac spinors $\phi$ and $\psi$ and their charge conjugates $C\phi$ and $C\psi$ and two types of vector particles $E$ and $E'$.

In \cite{2} we studied decays of $H^0(0^+)$ into photons, and in \cite{3} we studied the decay of $\chi_b(nP)$ into $Y(mS)\gamma$ and possible decay of Higgs partner $h^0(0^+)$ into $Y(mS)\gamma$ ($m = 1, 2$).

In this paper, we study decays of $H^0(0^+)$ into $\ell\bar{\ell}\ell\bar{\ell}$ and compare with the decay of $H^0(0^+)$ to other decay modes.

The Higgs boson $H^0(0^+)$ decays into $WW$, $ZZ$ and $\gamma\gamma$ corresponding to the strength in the unit of the standard model $0.89 \pm 0.2$, $1.11 \pm 0.3$ and $1.58 \pm 0.3$, respectively\cite{4, 5}.

The boson $W$ decays in the average $10.86\%$ to $\ell\nu$, and $Z$ decays in the average $3.6\%$ to $\ell\ell$ ($\ell = e, \mu$ or $\tau$).

The decay branching ratio of $H^0(0^+) \to ZZ$ is obtained from $H^0(0^+) \to \ell\ell\bar{\ell}\bar{\ell}$, and the strength parameter in this channel seems to be very sensitive to the mass of $H^0(0^+)$, and whether the final $\ell\ell\bar{\ell}\bar{\ell}$ originates from $ZZ$ is not clear. By inclusion of $gg \to \ell\ell\bar{\ell}\bar{\ell}$, the signal strength of $H^0(0^+) \to ZZ$ calculated as the average of the true contribution $M_Z$ and the contribution from other decays $M_{\ell\ell\bar{\ell}\bar{\ell}} - M_z$ was reduced to $0.99$\cite{6}, and by inclusion of $gg \to ZZ$, the signal strength was reduced to $0.93 \pm 0.3$\cite{7}.

The coupling of the Higgs boson to two leptons is given by\cite{8}

$$-y_{ij}^j\mathcal{E}_i(L_j \circ H_d) = -y_{ij}^j H^0_{ij} \ell_L \ell_L,$$

where $L_i, L_j$ specify left handed $SU(2)$ lepton degrees of freedom. Right chiral leptons are defined as $\ell_i (i = 1, 2, 3)$

$$\mathcal{E}_1 = \tilde{\phi}_e + \theta \cdot \chi_e + \frac{1}{2} \theta \cdot \theta F_e$$

$$\mathcal{E}_2 = \tilde{\phi}_\mu + \theta \cdot \chi_\mu + \frac{1}{2} \theta \cdot \theta F_\mu$$

$$\mathcal{E}_3 = \tilde{\phi}_\tau + \theta \cdot \chi_\tau + \frac{1}{2} \theta \cdot \theta F_\tau$$

Left chiral leptons are defined as

$$\mathcal{L}_1 = \begin{pmatrix} \tilde{\phi}_e \\ \tilde{\phi}_e \end{pmatrix} + \theta \cdot \begin{pmatrix} \chi_{ve} \\ \chi_e \end{pmatrix} + \frac{1}{2} \theta \cdot \theta \begin{pmatrix} F_{ve} \\ F_e \end{pmatrix}$$

and $\mathcal{L}_2$ and $\mathcal{L}_3$ are defined similarly.
2 Analysis using Cartan’s supersymmetry

Following Cartan’s supersymmetry, we express scalar particles by Ψ and Φ, and leptons by ψ, Cφ and antileptons by Cψ, φ. Assignment of Ψ and Φ can express the roles of Higgs bosons i.e. production of mass and chirality very powerfully.

The coupling of a vector field X to Dirac spinors,

\[ \psi = \xi_1 i + \xi_2 j + \xi_3 k + \xi_4 I = \left( \xi_1 + i \xi_3 \frac{3}{2} \xi_1 - \xi_2 \right) \]

\[ C\psi = -\xi_{234} i - \xi_{314} j - \xi_{124} k + \xi_{123} I = \left( \xi_{123} - i \xi_{124} - i \xi_{314} + \xi_{314} \right) \]

(2.1)

and the spinor operator

\[ \phi = \xi_{14} i + \xi_{24} j + \xi_{34} k + \xi_0 I = \left( \xi_0 + i \xi_{34} i \xi_{14} - i \xi_{24} \right) \]

\[ C\phi = -\xi_{23} i - \xi_{31} j - \xi_{12} k + \xi_{1234} I = \left( \xi_{1234} - i \xi_{122} - i \xi_{31} + \xi_{31} \right) \]

(2.2)

are specified by[1, 10]

\[ F_H = \left( C\phi \right) \gamma_0 x^i \gamma_i \psi + \left( C\phi \right) \gamma_0 x^i \gamma_i \psi \]

\[ = x^1 \left( \xi_{12} \xi_{314} - \xi_{31} \xi_{124} - \xi_{14} \xi_{123} + \xi_{1234} \xi_1 \right) + x^2 \left( \xi_{23} \xi_{124} - \xi_{12} \xi_{234} - \xi_{24} \xi_{123} + \xi_{1234} \xi_2 \right) + x^3 \left( \xi_{31} \xi_{234} - \xi_{23} \xi_{314} - \xi_{34} \xi_{123} + \xi_{1234} \xi_3 \right) + x^4 \left( -\xi_{14} \xi_{234} - \xi_{24} \xi_{314} - \xi_{34} \xi_{124} + \xi_{1234} \xi_4 \right) + x^5 \left( -\xi_0 \xi_{234} + \xi_{23} \xi_4 - \xi_{24} \xi_3 + \xi_{34} \xi_2 \right) + x^6 \left( -\xi_0 \xi_{314} + \xi_{31} \xi_4 - \xi_{34} \xi_1 + \xi_{14} \xi_3 \right) + x^7 \left( -\xi_0 \xi_{124} + \xi_{12} \xi_4 - \xi_{14} \xi_2 + \xi_{24} \xi_1 \right) + x^8 \left( \xi_0 \xi_{1234} - \xi_{23} \xi_1 - \xi_{31} \xi_2 - \xi_{12} \xi_3 \right) \]

(2.3)

In the expression of \(-y^i_j H_d^0 \tilde{\ell}_L \cdot \tilde{\ell}_j\) coupling,

\[ \ell_{L_j} = \left( \begin{array}{c} \psi \\ C\psi \end{array} \right) \text{ and } \tilde{\ell}_{i} = (C\phi, \phi)_i \]

In the case of coupling of γ to Higgs bosons, we considered 16 diagrams two of which are shown in Fig.1 and 2.

The Higgs boson \(H^0(0^+)\) has a partner \(h^0(0^+)\), and in [3] we discussed possibility of its decay into \(b\bar{b} \ell \bar{\ell}\). We compared the amplitude of production of \(\Upsilon(m_S)\ell \bar{\ell}\) near the threshold, in which \(\ell \bar{\ell}\) becomes a γ. In the case of \(h^0(0^+)\) decay, in addition to the coupling \(y^{ij}_d\), another coupling \(y'^{ij}_d\) defined by[8]

\[ -y'^{ij}_d D_i \left( Q_j \circ H_d \right) = -y^{ij}_d \nu_d \bar{b}_i b_j \]
appear. Here, $\nu_d$ is defined by the minimum of the potential $H_d = t(\nu_d, 0)$, and after diagonalization $m_{d,s,b} = \nu_d y_{d,s,b}$.

Couplings of quarks $u, c, t$ to Higgs bosons are defined by

$$y_{ij}^u U_i(Q_j \circ H_u) = y_{ij}^u \nu_u \bar{u}_i u_j$$

and the coupling of quarks to electromagnetic field is defined as

$$\mathcal{F}_L = \psi^k_L(\gamma_L(i\partial_\mu - eA_\mu) - m)\psi^k_L + \phi^k_L(\gamma_L(i\partial_\mu - eA_\mu) - m)\phi^k_L$$

where

$$\psi^k_L = \begin{pmatrix} \psi \\ C\psi \end{pmatrix}_k \quad \text{and} \quad \phi^k_L = \begin{pmatrix} C\phi \\ \phi \end{pmatrix}_k.$$ 

In electromagnetic interactions $\psi$ v.s. $C\phi$ and $\phi$ v.s. $C\psi$ have no differences, but we expect that our electronic detector is sensitive only to $\psi^k_L$.

In the weak interactions there are difference in $\phi^k_L$ and $\psi^k_L$, as the final states of supersymmetric transformation[2]. When the supersymmetric transformation is $G_{13}$, transformations $E \rightarrow C\psi$ and $E' \rightarrow \psi$ in the lepton sector, and the spinor $\phi^k_L \rightarrow \tilde{\phi}^k_L$ appears in the quark sector, where $\tilde{\phi}^k_L$ means that particle-antiparticle transformation is done on $\phi^k_L$. When the transformation is $G_{132}$, transformations $E \rightarrow \tilde{\phi}$ and $E' \rightarrow C\phi$ occurs in the lepton sector and the spinor $\phi^k_L \rightarrow \tilde{\psi}^k_L$ appears in the quark sector.

When the supersymmetric transformation is $G_{12}$, transformations $E \rightarrow \tilde{\phi}$ in the lepton sector and $\psi^k_L \rightarrow \tilde{\psi}^k_L$ appears in the quark sector. When the transformation is $G_{123}$, transformations of $E \rightarrow C\psi$ and $E' \rightarrow \tilde{\psi}$ occurs in the lepton sector and $\psi^k_L \rightarrow \tilde{\phi}^k_L$ in the quark sector.

In the coupling of $H_d^0 \ell \ell$, $i,j$ in the coupling $y^ij_{\ell\ell}$ specifies generations of left-handed $\ell$ and $\ell$, respectively, and in the coupling of $H_d^0 b b$, $i,j$ in the coupling $y^ij_{b\ell}$ specifies generations of left-handed $b$ and $b$, respectively.

In the case of $H^0(0^+) \rightarrow \ell \ell$, the effective $y^ij_{\ell\ell}$ from final four lepton states which are symmetric on the upper semicircle and on the lower semicircle, when all the $y^ij_{e\ell}$ and $y^ij_{\mu\ell}$ are taken into account in the present analysis. There 3 diagrams of $H(0^+, 11) \rightarrow$
Figure 3. The $h(0^+, 11) \rightarrow \Upsilon(1S)\gamma(k)$ with a vector particle $x2$ exchange.

Figure 4. The $h(0^+, 11) \rightarrow \Upsilon(1S)\gamma(i)$ with a vector particle $x2$ exchange.

$\gamma\gamma$, 2 diagrams of $H(0^+, ii) \rightarrow \gamma\gamma$, 2 diagrams of $H(0^+, jj) \rightarrow \gamma\gamma$, and 2 diagrams of $H(0^+, kk) \rightarrow \gamma\gamma$, in which $\ell$ and $\bar{\ell}$ have parallel momenta. i.e. altogether 9 diagrams that define the coupling of a Higgs boson to $\ell\ell\bar{\ell}\bar{\ell}$ with parallel momenta.

Figure 5. The $H(0^+, 11) \rightarrow \ell\ell(i)\bar{\ell}\bar{\ell}(-i)$ with a vector particle $X$ exchange.

Figure 6. The $H(0^+, 11) \rightarrow \ell\ell(j)\bar{\ell}\bar{\ell}(-j)$ with a vector particle $X$ exchange.

Figure 7. The $H(0^+, 11) \rightarrow \ell\ell(k)\bar{\ell}\bar{\ell}(-k)$ with a vector particle $X$ exchange.
3 Discussion and conclusion

We studied the decay of Higgs boson $H^0(0^+)$ to $\gamma\gamma$ via $\ell\ell$ and $\ell'\ell'$ states, which have strong correlations, in contrast to $h^0(0^+)$ decay to $b\bar{b}\gamma$ via $b\bar{b}e\bar{e}$. 
In the standard model, the vector boson $Z$ and the vector field $A$ are expressed as

$$Z_\mu = \cos \theta_W \cdot W^0_\mu + \sin \theta_W \cdot B_\mu$$
$$A_\mu = - \sin \theta_W \cdot W^0_\mu + \cos \theta_W \cdot B_\mu$$

and $\sin^2 \theta_W \simeq 0.22$ is the Weinberg angle. Relative strength of $B_\mu$ near the energy of the mass of Higgs boson is not so clear.

If the Higgs boson $H^0(0^+)$ decay as $h^0(0^+)$ to $b\bar{b}e\bar{e}$, and finally to $\gamma\gamma$, we can consider the Higgs boson decay into $\gamma\gamma$ given by 4 types of diagrams

$$\Psi \rightarrow C\phi C\psi, C\phi C\psi, \quad \Phi \rightarrow C\psi C\phi, C\psi C\phi$$
$$\Psi \rightarrow \phi\psi, \phi\psi, \quad \Phi \rightarrow \psi\phi, \psi\phi$$

each having 4 diagrams with different polarizations, i.e. altogether 16 independent diagrams.

As the decay modes of Higgs boson $H^0(0^+) \rightarrow \ell\ell\ell\ell$ to $\gamma\gamma$, we obtain 9 types of diagrams. In the energy region of Higgs boson mass, the decay to $\gamma\gamma$ occurs through $\ell\ell\ell\ell$ in the final state of $ZZ$. The boson $Z$ has a partner of a vector field $A$ in this energy region. The field $A$ is assigned as a photon, and the $Z_\mu$ and $A_\mu$ contribute to the production of $\ell\ell\ell\ell$ final state of $ZZ$, but the contamination of $g\bar{g} \rightarrow \ell\ell\ell\ell$ in the final state of $ZZ$ reduces the signal strength of $ZZ$ final state. $1.11 \pm 0.3$ observed in [5] to $0.93 \pm 0.3$.

In contrast, the decay of $H^0(0^+)$ to $\ell\nu\ell\nu$ via intermediate state of $WW$ is well known[5, 9]. Using the same reasoning as $H^0(0^+)$ decay to $\ell\ell\ell\ell$, we expect there are 9 independent diagrams in $H(0^+) \rightarrow \ell\nu\ell\nu$, which are detected as $H(0^+) \rightarrow WW$. The signal strength of $WW$ final state $0.87 \pm 0.2$ is consistent with the new signal strength of $ZZ$ state.

Experimentally the signal strength of $H^0 \rightarrow \gamma\gamma$ is $1.58^{+0.27}_{-0.23}$ and that of $H^0 \rightarrow WW$ is $0.87^{+0.24}_{-0.22}$. The ratio $\frac{\sigma(H^0 \rightarrow WW)}{\sigma(H^0 \rightarrow \gamma\gamma)}$ is $0.87 \pm 0.2$, which agrees with the ratio of the number of independent diagrams 9/16, which roughly agrees with the ratio of the number of independent diagrams derived from Cartan’s supersymmetric theory of spinors.

References

[1] É. Cartan, The theory of Spinors, Dover Pub. (1966)
[2] S. Furui, Triality selection rules of Octonion and Quantum Mechanics, [hep-ph/1409.3761].
[3] S. Furui, Cartan’s Supersymmetry and the Decay of a $h^0$ with the mass $m_{h^0} \simeq 11$GeV to $\Upsilon(1S)\gamma$ and to $\Upsilon(2S)\gamma$, [hep-ph/1502.07011].
[4] K.A. Olive et al, (Particle Data Group), Review of Particle Physics, Chinese Physics C38, (2014) 090001.
[5] ATLAS Collaboration, Measurement of Higgs boson and couplings in diboson final states with the ATLAS detector at the LHC, Physics Letters B 726 (2013) 88-119. [Erratum ibid B 734 (2014) 406].
[6] J.S. Gainer, J. Lykken, K.T. Matchev, S. Mrenna and M. Park, Beyond Geolocating: Constraining Higher Dimensional Operators in $H \rightarrow 4\ell$ with Off-Shell Production and More, Phys. Rev. D 91 (2014) 035011.

[7] L. Finco, L., Recent Results of the Higgs Boson Properties in the $H \rightarrow ZZ \rightarrow 4\ell$ decay channel at CMS, in Proceedings of the Second Annual Conference on Large Hadron Collider Physics, Columbia University, New York, U.S.A. June 2014, [hep-ex/1409.2646v2].

[8] P. Labelle, Supersymmetry Demystified, McGraw Hill (2010).

[9] The CMS Collaboration, Evidence for the direct decay of the 125 GeV Higgs boson to fermions, nature Physics, DOI:10.1038/NPHYS3005 (2014).

[10] S. Furui, Fermion Flavors in Quaternion Basis and Infrared QCD, Few Body Syst. 52, (2012) 171-187, [hep-ph/1104.1225].

[11] S. Furui, The flavor symmetry in the standard model and the triality symmetry, Int. J. Mod. Phys. A27 (2012) 1250158, [hep-ph/1203.5213].