Quality Control of Labelstock Using Fuzzy Exponentially Weighted Moving Average (FEWMA) Control

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Abstract. Production development in the industrial era sues the company to continue creating the best product for consumer society. Product quality is the main consideration for the consumer in selecting products. This is certainly driving the company to maintain the quality of its products. Therefore, companies need to control the quality of production so their products can fulfill the standards set. One of the ways that can be used to control quality is using the control chart. The most frequently used control chart is the Shewhart type control chart. However, the Shewhart control chart can only be used for crisp data, while for data containing ambiguity, a fuzzy control chart can be applied. In this research, the thickness of the label stock glue quality on HVS P 60 is monitored using the fuzzy control chart. The type of fuzzy control chart used in this study is the FEWMA (Fuzzy Exponentially Weighted Moving Average) control chart. This control chart was chosen because it can monitor data that contains ambiguity and data with small process shifts. In this research, rigorous inspection (α-cuts) was used to detect shifting data. Based on the analysis, it was found that the weight (λ) 0.1 is the optimum weight value in detecting a process shift in the thickness of the label stock glue. Through the cause and effect diagram, four factors that cause the out of control data was found, namely machine, method, man, and material.

1. Introduction
Currently, the goods and services industry is experiencing rapid development. Head of the Badan Penelitian Pengembangan Industri (BPPI) of the Ministry of Industry, revealed that until June 2019 the development of the Indonesian industry continued to grow. Examples of industries that experienced an increase were the textile apparel industry which increased by 20.71% and the food and beverage industry which increased by 7.99%. Of course, this is not far from the company's role in maintaining the quality of its products. Therefore, every company needs to control quality.

A statistical tools used for quality control is a control chart. The control chart’s advantage is to show abnormal conditions and detect process movements in the production process. The Shewhart control chart is often used in detecting process movements. However, in practice, Shewhart chart is less precise at revealing small process movements [1]. This control chart also requires normality assumptions that must be met, while product quality characteristics are not always normally distributed. Also, Shewhart control charts cannot detect a movements in the process if there is uncertainty or vagueness in the measurement.

PT "X" is a company that started operations in 1992 with the direction of movement in the manufacturing of label-stock, release liner, and adhesive tape. This product is used in the general
industry, hygiene products, food-grade, printing, and packaging industries. The production process begins with the installation of a multipurpose coating machine. The coating process carried out at this company is to apply wet glue that is placed on the release paper then passed through the dryer to let the glue dry. The process is measured through 3 measurement points, namely left, middle, and right. This measurement occurs during the coating process. Therefore the thickness of the glue is necessary quality control. So far, PT "X" controls the quality of the glue thickness based on a certain limit calculation, where there are 3 thickness levels of glue on the label stock, namely S + 1, S + 2, and S + 3. Level S + 1 is the thickness of standard glue ± 7.5%, level S + 2 is the thickness of standard glue ± 15%, and level S + 3 is the thickness of standard glue ±> 15%. If the thickness of the glue is at the S + 3 level, then a hold will be done on the label stock. PT "X" measured the thickness of the HVS P 60 paper type label stock glue through three observation sides, allowing for vagueness or uncertainty of measurement.

In 1965, Zadeh introduced the fuzzy theory used for data containing uncertainties and was developed by Bradshaw in 1983. Then in 1990, Raz and Wang proposed the fuzzy control chart. Then in 2014, Senturk et. al. combining the fuzzy theory and EWMA control chart. The EWMA control chart can detect small movements in the mean for both data are normally distributed or non-normally by maintaining the expected ARL value to minimize the occurrence of a false alarm [2]. Therefore, in this study, the Fuzzy Exponentially Weighted Moving Average (FEWMA) control chart was used to detect process shifts in the thickness of the label stock glue.

2. Research Methods

This section explains the literature related to the method used for monitoring the thickness of label stock glue, which is as follows.

2.1. Normality Test with Kolmogorov Smirnov

The one sample normal distribution test is used to determine whether the distribution of the observed sample values is normally distributed. Kolmogorov Smirnov has the advantage of being simple and does not cause a difference in perception between one observer and another, which often occurs in normality tests using graphs [3].

Through equation (1), it can be seen the pdf (probability density function) of the normal distribution.

\[
f(X) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}
\]  

(1)

The \( f(x) \) is the density of normal random variable \( x \), with mean \( \mu \) and variance \( \sigma^2 \). Normal distribution testing can be done with the Kolmogorov Smirnov test as follows.

Hypothesis:

\( H_0: F_0(x) = F(x) \) for all values (normally distributed data)

\( H_1: F_0(x) \neq F(x) \) for at least one (data not normally distributed)

Test statistics:

\[
D = \sup \left| F_0(x) - F_0(x) \right| 
\]  

(2)

Where \( D \) is Kolmogorov Smirnov statistic test for supremum all of \( x \) from \( \left| F_0(x) - F_0(x) \right| \) value with \( F_0(x) \) is cumulative distributions function of the data sample and \( F_0(x) \) cumulative normal distribution function. The test based on \( D \) rejects \( H_0 \) at \( |D| > D_{(1-\alpha, n)} \) or \( p-value < \alpha \).

2.2. Fuzzy Sets Theory

An uncertainty and vagueness arise due to randomness, an inaccuracy, or a lack of the knowledge in making measurements. This can be overcome by using fuzzy methods [4]. In a study of data containing vagueness, compared to the non-fuzzy method, the fuzzy methods show a better ability to detect abnormal changes, especially in a small sample size [5].
A fuzzy sets class is $A$ in $x$ is characterized by a membership function $f_A(x)$ related to each point in $x$ a true number within the interval $[0,1]$, with the worth of $f_A(x)$ at $x$ representing the “grade of membership” of $x$ during $A$. Thus, $X = \{x\}$. When $A$ is a set in the ordinary sense of the term, its membership function can combat only two values 0 and 1, with $f_A(x) = 1$ or 0 take as $x$ does or doesn’t belong to $A$ [6]. It means in control charts it will be assumed that the product is neither completely bad nor completely good, nor it can be a “gray” area [7].

In the fuzzy function theory, the main component that is very influential is fuzzy membership. Here are some fuzzy memberships.

a. Linear representation,
b. triangular representation,
c. trapezoidal representation curve,
d. S-curve representation,
e. bell shape representation,
f. shoulder shape representation.

In this study, triangular representation was used.

2.3. Fuzzy Exponentially Weighted Moving Average (FEWMA) Control Chart

To overcome data that contains uncertainty or vagueness in the measurements process, fuzzy control chart is used, cause it can provide flexibility to prevent false alarm [7]. One of control chart methods for small shifts that can be used is exponentially weighted moving average (EWMA) control chart. The EWMA control chart was first proposed by Hunters in 1986. According to equation (3), the EWMA control chart was built.

$$z_t = \lambda \overline{X}_t + (1-\lambda)z_{t-1}$$

where $z_t$ is the $t^{th}$ EWMA, $\overline{X}_t$ denotes $t^{th}$ sample average, $0 < \lambda \leq 1$ is a constant, $m$ is the sample number and $t = 1, 2, \ldots, m$.

In its use, a traditional EWMA can detect process shifts only in crips data, when there are an ambiguity it can be used a combination of fuzzy theory and EWMA control chart, namely Fuzzy Exponentially Weighted Moving Average (FEWMA) control chart. The FEWMA control chart plot is given as follows.

$$\widetilde{z}_t = \lambda (\overline{X}_{a,t}, \overline{X}_{b,t}, \overline{X}_{c,t}) + (1-\lambda)\widetilde{z}_{t-1},$$

where $\widetilde{z}_0 = \overline{X}_a, \overline{X}_b, \overline{X}_c$ dan $\overline{X}_{a,t}$ is the average observation side $a$ in sample $t^{th}$ [8].

There are two calculations of FEWMA control chart control limit namely FEWMA control chart if $(\sigma_a, \sigma_b, \sigma_c)$ is known and FEWMA control chart if $(\sigma_a, \sigma_b, \sigma_c)$ is unknown. In this study, we used FEWMA control chart if $(\sigma_a, \sigma_b, \sigma_c)$ unknown.

Fuzzy observations $(X_{a,1}, X_{b,1}, X_{c,1})$ are obtained from the fuzzy observation process shown by a triangular membership function with a sample size of $n$. $(\overline{X}_{a,t}, \overline{X}_{b,t}, \overline{X}_{c,t})$ is the average of the sample $t$ at points $a$, $b$, and $c$, where $\widetilde{z}_0 = (z_{a0}, z_{b0}, z_{c0}) = (\overline{X}_a, \overline{X}_b, \overline{X}_c)$ and $(\overline{R}_a, \overline{R}_b, \overline{R}_c)$ is the fuzzy range [8].

The FEWMA control chart used for moderately large sample $t$ is according to equation (5).
When obtaining the formulation of FEWMA control chart, the \( \alpha - \text{cuts} \) approximation is employed. The \( \alpha - \text{cuts} \) FEWMA control chart for unknown \( (\sigma_a, \sigma_b, \sigma_c) \) is given by equation (6).

\[
\begin{align*}
UCL_{\alpha-\text{cuts}} & = \left( \bar{X}_a, \bar{X}_b, \bar{X}_c \right) + A_2 \left( \bar{R}_a, \bar{R}_b, \bar{R}_c \right) \sqrt{\lambda \over (2 - \lambda)}, \\
& = \bar{X}_a + A_2 \bar{R}_a \sqrt{\lambda \over (2 - \lambda)} \cdot \bar{X}_b + A_2 \bar{R}_b \sqrt{\lambda \over (2 - \lambda)} \cdot \bar{X}_c + A_2 \bar{R}_c \sqrt{\lambda \over (2 - \lambda)}, \\
CL_{\alpha-\text{cuts}} & = \left( \bar{X}_a, \bar{X}_b, \bar{X}_c \right), \\
LCL_{\alpha-\text{cuts}} & = \left( \bar{X}_a, \bar{X}_b, \bar{X}_c \right) - A_2 \left( \bar{R}_a, \bar{R}_b, \bar{R}_c \right) \sqrt{\lambda \over (2 - \lambda)} \\
& = \left( \bar{X}_a - A_2 \bar{R}_a \sqrt{\lambda \over (2 - \lambda)} \right) \left( \bar{X}_b - A_2 \bar{R}_b \sqrt{\lambda \over (2 - \lambda)} \right) \left( \bar{X}_c - A_2 \bar{R}_c \sqrt{\lambda \over (2 - \lambda)} \right).
\end{align*}
\]

In fuzzy theory, there are fuzzy transformation techniques that are useful in transforming fuzzy numbers into crisp (emphatic) data, namely \( \alpha \)-level fuzzy midrange, fuzzy average, fuzzy median, and fuzzy mode [9]. In this study, an analysis of fuzzy transformation with \( \alpha \)-level fuzzy midrange was carried out. The control limit used is to determine a process out of control or in control [10].

The fuzzy midrange transformation technique is integrated to the \( \alpha \) - level fuzzy midrange for \( \alpha - \text{cuts} \) FEWMA for unknown \( (\sigma_a, \sigma_b, \sigma_c) \) is given as follows,

\[
\begin{align*}
UCL_{mu-\text{EWMA}} & = CL_{mu-\text{EWMA}} + {1 \over 3} A_2 \left( \bar{R}_a + \bar{R}_b + \bar{R}_c \right) \sqrt{\lambda \over (2 - \lambda)}, \\
CL_{mu-\text{EWMA}} & = \left( \bar{X}_a + \bar{X}_b + \bar{X}_c \right), \\
LCL_{mu-\text{EWMA}} & = CL_{mu-\text{EWMA}} \frac{1}{3} A_2 \left( \bar{R}_a + \bar{R}_b + \bar{R}_c \right) \sqrt{\lambda \over (2 - \lambda)}.
\end{align*}
\]

where,

\[
\begin{align*}
\bar{R}_a & = \bar{R}_a + \alpha - \text{cuts} \left( \bar{R}_a - \bar{R}_a \right), \\
\bar{R}_c & = \bar{R}_a - \alpha - \text{cuts} \left( \bar{R}_c - \bar{R}_a \right).
\end{align*}
\]
and,
\[
\overline{X}_a^{\alpha-cuts} = \overline{X}_a + \alpha - cuts \left( \overline{X}_b - \overline{X}_a \right),
\]
\[
\overline{X}_c^{\alpha-cuts} = \overline{X}_c - \alpha - cuts \left( \overline{X}_c - \overline{X}_a \right).
\]

For \( t \)th subgroup, \( \alpha \) -level fuzzy midrange value \( S_{mr-EWMA,J}^{\alpha-cuts} \) is according to equation (10),
\[
S_{mr-EWMA,J}^{\alpha-cuts} = \frac{1}{3} \left( \overline{X}_{a,t}^{\alpha-cuts} + \overline{X}_{b,t}^{\alpha-cuts} + \overline{X}_{c,t}^{\alpha-cuts} \right).
\]

For each sample in control process is defined as follows.
\[
\text{process control} = \begin{cases} \text{in control, } LCL_{mr-EWMA}^{\alpha-cuts} \leq S_{mr-EWMA,J}^{\alpha-cuts} \leq UCL_{mr-EWMA}^{\alpha-cuts} , & \text{Out of control, for otherwise.} \end{cases}
\]

2.4. Cause and Effect Diagram
In situations where a cause is not obvious (sometimes it is), a tool that can used to identifying potential causes is causeand effect diagram. It is an effective tool that can help to know/identify the cause of a specific problem that my occur. The causes of problems are often caused by five elements, namely man, method, machine, material, and environment [1].

3. Result and Discussion
This section explains the result of the thickness of the label stock glue monitoring process.

3.1. Normality Test Result
Testing the data normality assumption used the Kolmogorov Smirnov normal test with a significance level \( (\alpha) \) of 5\%. A data is said to be normally distributed if or the \( p \)-value exceeds the specified significance limit. Through equation (1), the results of the normality test for the thickness of the label stock glue are shown in Table 1 below.

| No | The Thickness of the Glue | \( |D| \) | \( P \)-value |
|----|---------------------------|---------|-------------|
| 1  | Left                      | 0.14105 | 2.5446e-7   |
| 2  | Middle                    | 0.14907 | 3.9754e-8   |
| 3  | Right                     | 0.14811 | 4.9968e-8   |

It is known that the value of \( D_{0.95,395} \) is 0.06833; then based on Table 1, it is found that the calculated statistic (D) on the glue thickness data from the left, middle, and right sides has a value greater than the value of \( D_{0.95,395} \). This means that a decision is made to fail to reject \( H_0 \) so that the glue thickness data from the left, middle, and right sides are not normally distributed. Besides, to determine normally distributed data, it can be seen from the \( p \)-value, from the three sides data on the thickness of the label stock glue, the same decision was obtained with the test statistic \( |D| \) because the \( p \)-value is less than \( \alpha (0.05) \).

3.2. FEWMA Control Chart Analysis
In monitoring small process shifts, a FEWMA control charts can be used. The specification limit of the thickness of the glue is around 18-20 gsm (gram square meters). So when there is a point that is outside the control limit then it is out of control.

After doing the normal test with Kolmogorov Smirnov, the next step is to analyze the FEWMA control chart. The initial step of the analysis is to form a TFN (Triangular Fuzzy Number) by sorting the data in each observation where \( X_a < X_b < X_c \). Then forming a subgroup, where the subgroup is formed based on the change in thickness of the release, where when the release changes, the machine is reset.
The overall mean score of $X\bar{r}$ for the first (a), second (b), and third (c) sides is 18.613; 19.469; and 20.140. Meanwhile, the average value of the range $\bar{R}_r$ for each side is 0.955; 1.933; and 3.517. Because the number of samples for each subgroup is different, the average sample value for each subgroup is calculated to obtain the constants $A_2$, $D_3$, and $D_4$. Obtained the average sample $n$ is 4, then the value of $A_2 = 0.729$; $D_3 = 0$, and $D_4 = 2.283$. With a weighting ($\lambda$) between 0 and 1 with a weighting distance of 0.1, the FEWMA control limit calculation is performed with a value $\alpha\text{-cuts} = 0.65$. Because in the manufacturing process some literature chose a 0.65 value [8]. The control limit transformations and transformation statistical values for the FEWMA control chart can be seen in Table 2 and Table 3.

| No | $\lambda$ | LCL  | CL   | UCL  |
|----|---------|------|------|------|
| 1  | 0.1     | 19.112 | 19.447 | 19.782 |
| 2  | 0.2     | 18.961 | 19.447 | 19.934 |
| 3  | 0.3     | 18.834 | 19.447 | 20.061 |
| 4  | 0.4     | 18.718 | 19.447 | 20.177 |
| 5  | 0.5     | 18.605 | 19.447 | 20.290 |
| 6  | 0.6     | 18.492 | 19.447 | 20.403 |
| 7  | 0.7     | 18.376 | 19.447 | 20.518 |
| 8  | 0.8     | 18.256 | 19.447 | 20.639 |
| 9  | 0.9     | 18.127 | 19.447 | 20.768 |

Table 3. Statistic value of transformation $\alpha$ – level FEWMA

| Subgroup | Statistic Value |
|----------|----------------|
| 1        | 19.700         |
| 2        | 19.450         |
| 3        | 19.936         |
| 4        | 20.733         |
| 5        | 20.790         |
| 6        | 20.400         |
| 7        | 19.994         |
| 8        | 19.521         |
| 9        | 20.217         |
| 10       | 20.550         |
| ...      | ...            |
| 86       | 20.328         |
| 87       | 20.019         |
| 88       | 19.883         |
| 89       | 20.267         |

After obtaining the statistical values for each value of the weight, then the FEWMA chart is shown in Figure 1.
Figure 1. (a) FEWMA control chart with \( \lambda = 0.1 \); (b) fewma control chart with \( \lambda = 0.3 \); (c) fewma control chart with \( \lambda = 0.7 \); (d) fewma control chart with \( \lambda = 0.9 \)

The Upper Control Limit (UCL) value calculated in Figure 1. (a) is 19.782 and the Lower Control Limit (LCL) is 19.112. In Figure 1 it is known that many observations are outside the control limit, namely 66 observation points. The farthest point from the control limit in Figure 1. (a) is the 21st observation with a value of 16.772. Based on this, it can be concluded that the label stock quality process for the HVS P 60 paper type is not statistically controlled because of the many out of control observations.

Figure 1 (b) shows that the label stock quality process for the HVS P 60 paper type has not been controlled statistically. This is because there are still many observations that are outside the control limits values (UCL and LCL). The number of out of control observations in Figure 1. (b) is 44 observations.

Based on Figure 1. (c) it can be seen that the observations have not been controlled statistically. This can be seen from the number of observations that are outside the control limit as in Figure 1. (a) and 1. (b). The number of out of control observations in Figure 3. (c) is 28 observations.

Figure 1. (d) shows the number of out of control is less than in Figure 1 (a) / 1 (b) / 1 (c). The number of out of control observations for the FEWMA control chart with \( \lambda = 0.9 \) is 13 points, where LCL = 18.127 and UCL = 20.768. This shows that the observations have not been controlled statistically. To determine the optimum weight from the FEWMA control chart, it can be seen from the number of observations that are out of control. The more observations that are outside the control limit indicate that the more sensitive the weight are. For more details in choosing the optimum weight, see Table 4.
Table 4. Comparison of the number of out of control observations

| No | $\lambda$ | OOC |
|----|-----------|-----|
| 1  | 0.1       | 66  |
| 2  | 0.2       | 59  |
| 3  | 0.3       | 44  |
| 4  | 0.4       | 40  |
| 5  | 0.5       | 34  |
| 6  | 0.6       | 30  |
| 7  | 0.7       | 28  |
| 8  | 0.8       | 21  |
| 9  | 0.9       | 13  |

The FEWMA control chart for monitoring the thickness of the glue shows that the greater the value of $\lambda$, the smaller the number of out of control. It means that $\lambda = 0.1$ for FEWMA control chart is the optimum $\lambda$ for detecting the shift in the process of a thickness of the label stock glue.

3.3. Cause and Effect Diagram

Based on tracing the causes of statistically uncontrolled processes, it can be seen in Figure 4.6. The cause of the thickness of the glue that is outside the limits set by the company can be seen from 4 factors, namely humans, machines, raw materials used, and methods.

**Figure 2.** Cause and effect diagram of the thickness of the labelstock glue

4. Conclusion

Based on normal testing by Kolmogorov Smirnov, it shows that the glue thickness data from the right, middle, and left sides shows the same results, which is not normally distributed. Then for analysis with the FEWMA control chart, it was found that many observations were outside the limits of control. So that the thickness of the label stock glue is not statistically controlled. The optimum weight of the FEWMA control chart to detect the movements in the glue thickness process is $\lambda = 0.1$. This shows that $\lambda = 0.1$ is the most sensitive weighting, that is, the number of out of control observations is the most compared to other weighting values. Then after a discussion, the cause of the out of control data was found in 4 categories, namely man, material, machine, and methods. The FEWMA control chart to be considered in the industry for future work.
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