Enhanced Lidov–Kozai migration and the formation of the transiting giant planet WD 1856+534 b

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ABSTRACT
We investigate the possible origin of the transiting giant planet WD 1856+534 b, the first strong exoplanet candidate orbiting a white dwarf, through high-eccentricity migration (HEM) driven by the Lidov–Kozai (LK) effect. The host system’s overall architecture is a hierarchical quadruple in the ‘2+2’ configuration, owing to the presence of a tertiary companion system of two M-dwarfs. We show that a secular inclination resonance in 2+2 systems can significantly broaden the LK window for extreme eccentricity excitation (\(e \gtrsim 0.999\)), allowing the giant planet to migrate for a wide range of initial orbital inclinations. Octupole effects can also contribute to the broadening of this ‘extreme’ LK window. We suggest that WD 1856+534 b likely migrated from a distance of \(\sim 30–60\) au, corresponding to a semi-major axis \(\sim 10–20\) au during the host’s main-sequence phase. We discuss possible difficulties of all flavours of HEM affecting the occurrence rate of short-period giant planets around white dwarfs.

Key words: celestial mechanics – planets and satellites: dynamical evolution and stability – planetary systems – white dwarfs

1 INTRODUCTION
The recent discovery of WD 1856+534 b, a candidate giant planet transiting its white-dwarf (WD) host with a period of 1.4 days (Vanderburg et al. 2020), is a major milestone in the emerging field of WD planetary science. The existence of remnant planetary systems around WDs has been inferred from observations of a disintegrating planetesimal (e.g., Vanderburg et al. 2015), gas accretion from an evaporating ice giant (Gänsicke et al. 2019), infrared emission from debris discs (e.g., Farihi et al. 2009), and atmospheric pollution (Zuckerman et al. 2003, 2010; Koester et al. 2014). These phenomena are thought to be driven by tidal disruption events (Jura 2003), perhaps triggered dynamically by distant planets (Debes & Sigurdsson 2002; Debes et al. 2012; Frewen & Hansen 2014; Pichierri et al. 2017; Mustill et al. 2018) or stellar binary partners (Bonsor & Veras 2015; Hamers & Portegies Zwart 2016b; Petrovich & Muñoz 2017; Stephan et al. 2017).

Planets orbiting within a few AU of their host stars during the main sequence are likely to be engulfed and destroyed when the star ascends the asymptotic giant branch (AGB) on its way to becoming a WD (Villaver & Livio 2007; Mustill & Villaver 2012). The dynamical history of WD 1856+534 b may then be similar to the hypothesized origin of hot Jupiters around main-sequence stars by high-eccentricity migration (HEM; reviewed by Dawson & Johnson 2018). During HEM, a planet’s orbital eccentricity \(e\) is excited to an extreme value (\(1−e \lesssim 10^{-2}\)) via dynamical interactions. The planet then experiences strong tidal dissipation near pericenter, which shrinks and circularizes its orbit. Possible mechanisms for driving HEM are diverse, including secular interactions with other giant planets or binary companions (e.g., Wu & Murray 2003; Fabrycky & Tremaine 2007; Naoz et al. 2012; Wu & Lithwick 2011; Petrovich 2015a,b; Anderson et al. 2016; Hamers et al. 2017; Vick et al. 2019; Teyssandier et al. 2019). Similar mechanisms are plausible in a WD’s planetary system. Importantly, a successful model for the origin of WD 1856+534 b must be able to delay migration until the host star becomes a WD, since a planet migrating sooner than this would presumably have been destroyed.

For WD 1856+534 b, HEM by way of the Lidov–Kozai (LK) effect (Lidov 1962; Kozai 1962) is an inviting hypothesis because its host star belongs to a hierarchical triple system: Vanderburg et al. (2020) identified two bound M-dwarf companions with a projected separation \(\sim 1000\) au from the primary WD and \(\sim 50\) au apart; we summarize the existing constraints on the system’s properties in Table 1. Including the planet, the system’s overall architecture is a hierarchical quadruple system with a ‘2+2’ configuration (see Figure 1). The dynamics of such systems resembles the standard LK effect in some respects but allows for the excitation of ex-
show that this could have occurred if the planet’s semi-major secular interactions with the WD’s stellar companions. We (e.g., Hamers & Lai 2017).
treme eccentricities from a wider range of initial conditions (O'Connor, Liu & Lai 2017).

In this paper, we demonstrate the possible origin of the WD 1856+534 b through HEM by way of such “enhanced” secular interactions with the WD’s stellar companions. We show that this could have occurred if the planet’s semi-major axis prior to migration was \( \sim 30-60 \text{ au} \), corresponding to \( \sim 10-20 \text{ au} \) during the host’s main sequence. In Section 2, we describe the secular dynamics of a 2+2 hierarchical system and the conditions for HEM in such systems. In Section 3, we apply our dynamical model to HEM in the WD 1856+534 system. We examine the system’s early dynamical history in Section 4, discuss potential issues of HEM affecting the occurrence rate of giant planets around WDs in Section 5, and summarize in Section 6.

## 2 SECULAR DYNAMICS OF A 2+2 HIERARCHICAL SYSTEM

Figure 1 shows a schematic depiction of a 2+2 hierarchical quadruple system. The Hamiltonian describing the secular evolution of such systems has been calculated up to the quadrupole order of approximation by Hamers et al. (2015) and Hamers & Portegies Zwart (2016a). Hamers & Lai (2017) subsequently studied the limit where one body is a test particle and identified the key dynamical mechanism for an enhanced LK effect. We adopt their setup in this paper.

A 2+2 system can be described by three quasi-Keplerian orbits: two “inner” binary systems (labelled 1 and 2) orbit their relative barycentres, which in turn follow an “outer” orbit (3) around the system’s total centre of mass. We label the component masses \( m_0 \) and \( m_1 \) for orbit 1 and \( m_2 \) and \( m_3 \) for 2. For each orbit, we denote the eccentricity vector \( \hat{e}_j = \hat{e}_j, \hat{e}_j \) and angular momentum vector \( L_k = \mu_k [GM_k a_k (1 - e_k^2)]^{1/2} \hat{L}_k \), where \( \mu_j, M_k, \) and \( a_k \) are respectively the reduced mass, total mass, and semi-major axis. We also define the dimensionless angular momentum \( j_k = (1 - e_k^2)^{1/2} \hat{L}_k \) and the total angular momentum \( J = \hat{L}_1 + \hat{L}_2 + \hat{L}_3 \equiv \hat{J} \). Finally, we define \( \cos \hat{l} = L_1 \cdot L_3, \cos \hat{I}_2 = L_2 \cdot \hat{L}_3 \), and \( \cos \alpha = L_3 \cdot \hat{z} \) (see Fig. 1).

We consider a hierarchical system with \( m_1 \ll m_{0,2,3} \) such that \( L_1 \ll L_2, J \ll L_2 + L_3 = \text{const.}, \) and \( L_2 \sin \hat{l} \approx L_3 \sin \alpha \). For simplicity, we assume that \( e_2 = 0 \) and that the mutual inclination of orbits 2 and 3 is \( I_2 < 39.2^\circ \) in order to suppress the LK effect for orbit 2. The latter two assumptions may not hold for general 2+2 systems, but they are appropriate for the WD 1856+534 system.

Under these assumptions, the secular evolution is completely described by the equations

\[
\frac{d\hat{j}_j}{dt} = \frac{3}{4\Omega_{\text{LK}}} \left[ \left( \hat{j}_j \cdot \hat{L}_3 \right) \left( \hat{j}_j \times \hat{L}_3 \right) \right] - 5 \left( \hat{e}_j \cdot \hat{L}_3 \right) \left( \hat{e}_j \times \hat{L}_3 \right),
\]

\[
\frac{d\hat{e}_j}{dt} = \frac{3}{4\Omega_{\text{LK}}} \left[ \left( \hat{j}_j \cdot \hat{L}_3 \right) \left( \hat{e}_j \cdot \hat{L}_3 \right) + 2(\hat{j}_j \times \hat{e}_j) \right] - 5 \left( \hat{e}_j \cdot \hat{L}_3 \right) \left( \hat{j}_j \times \hat{L}_3 \right),
\]

\[
\frac{d\hat{L}_3}{dt} = -\frac{\beta}{\Omega_{\text{LK}}} \left( \hat{z} \times \hat{L}_3 \right).
\]

Above we have defined the LK time-scale for orbit 1 as

\[
\frac{1}{\Omega_{\text{LK}}} = \frac{m_0}{m_3} \left( \frac{a_1}{a_{3,\text{eff}}} \right)^3 n_1,
\]

where \( m_{23} = m_2 + m_3, a_{3,\text{eff}} = a_3 (1 - e_3^2)^{1/2}, \) and \( n_1 = (GM_0/a_1)^{1/2} \). We have also defined a dimensionless quantity

\[
\beta = \frac{3}{4} \left( \frac{m_0}{m_{23}} \frac{a_2}{a_{3,\text{eff}}} \right)^{3/2} J \left( \frac{L_3}{L_2} \right) \cos I_2,
\]

which is simply the precession rate of \( L_3 \) about \( \hat{z} \),

\[
\Omega_{3z} = \frac{3}{4} \frac{m_0}{m_{23}} \left( \frac{a_2}{a_{3,\text{eff}}} \right)^3 J \left( \frac{L_3}{L_2} \right) n_2,
\]

in units of \( t_{\text{LK}}^{-1} \), i.e. \( \beta = \Omega_{3z} t_{\text{LK}} \).

For general \( e_2 \) and \( I_2 \), the precession rate of \( L_3 \) around \( J \) and the angle \( \alpha \) between \( L_3 \) and \( J \) both vary due to eccentricity oscillations. All these variations occur on a similar time-scale per Eq. (6). Thus we do not expect these complications to appreciably change the dynamics.

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**Table 1: Properties of the WD 1856+534 system.** All values are as reported by Vanderburg et al. (2020) except for the WD and planetary masses, which have not been robustly measured.

| Quantity          | Symbol | Value                      |
|-------------------|--------|----------------------------|
| WD mass           | \( m_0 \) | 0.6 M\(_\odot\) (assumed) |
| Planetary mass    | \( m_1 \) | 1.0 M\(_\odot\) (assumed) |
| Planetary radius  | \( R_1 \) | (0.93 ± 0.09) R\(_\odot\) |
| Companion masses  | \( m_2 \) | (0.346 ± 0.027) M\(_\odot\) |
|                   | \( m_3 \) | (0.331 ± 0.024) M\(_\odot\) |
| Semi-major axes   | \( a_{1,\text{obs}} \) | \( \approx 0.02 \) au |
|                   | \( a_2 \) | 58.9 ± 7.4 au               |
|                   | \( a_3 \) | 1500 ± 240 au               |
| Eccentricities    | \( e_{1,\text{obs}} \) | (assumed) |
|                   | \( e_2 \) | < 0.63                      |
|                   | \( e_3 \) | 0.30 ± 0.19                 |

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**Figure 1:** Schematic cartoon of a hierarchical 2+2 system with \( m_1 \ll m_0, m_2, m_3 \).
The qualitative nature of the dynamics is determined by the value of $\beta$ (Hamers & Lai 2017). There are three regimes:

(i) When $\beta \ll 1$, $L_1$ precesses around $L_3$ more rapidly than $L_3$ around $\hat{z}$. The system’s dynamical evolution approaches that of a ‘2+1’ hierarchical triple in the standard LK problem. If the initial mutual inclination of orbits 1 and 3 satisfies $\cos I_{1,0} < \sqrt{3}/5$, then the eccentricity of $m_1$ can be excited according to

$$e_{1,\max} = \sqrt{1 - \frac{5}{3} \cos^2(\theta_{1,0})},$$

(7)

where $\theta_{1,0}$ is the initial angle between $\hat{L}_1$ and $\hat{z}$. We call this the “quasi-LK” regime.

(ii) When $\beta \gg 1$, $L_1$ precesses around $L_3$ slowly compared to $L_3$ around $\hat{z}$; one can then average over the precession of $L_3$, so that $L_1$ effectively precesses around $\hat{z}$. The dynamics of $m_1$ resembles the LK problem with a modified conservation law:

$$e_{1,\max} = \sqrt{1 - \frac{5}{3} \cos^2(\theta_{1,0})},$$

(8)

where $\theta_{1,0}$ is the initial angle between $\hat{L}_1$ and $\hat{z}$. We call this the “modified LK” regime.

(iii) When $\beta \sim 1$, $L_1$ precesses around $L_3$ at roughly the same rate as $L_3$ around $\hat{z}$. This secular inclination resonance drives chaotic evolution of orbit 1, featuring extreme eccentricities for a broader range of initial inclinations—an “enhanced LK window.” We call this the “resonant” regime; it is here that the quadruple nature of the system has the greatest effect.

In Figure 2, we compare the eccentricity excitation of $m_1$ between the modified LK and resonant regimes for trajectories with identical initial conditions. When $\beta \gg 1$, the maximal eccentricity $e_1 \approx 0.95$ is consistent with Eq. (8). When $\beta \sim 1$, however, extreme eccentricities $1 - e_1 \sim 10^{-4}$ can be achieved even with moderate initial inclinations. We quantify this in the next section.

2.1 Short-Range Forces and the ‘Extreme’ LK Window

For a planet initially at a large distance ($a_1 \geq 10$ au) to migrate to $\sim 0.02$ au through the LK mechanism, extreme eccentricity excitation ($1 - e_1 \leq 10^{-3}$) is required. However, when the planet’s pericenter separation $a_1(1 - e_1)$ from the host star is small, short-range forces (SRFs) can become significant. We implement these additional forces in our model following Liu et al. (2015).

We include SRFs arising from the 1PN correction to the gravitational potential of $m_0$ and from the tidal distortion of $m_1$. These contribute an additional term in Eq. (2):

$$\left(\frac{de_1}{dt}\right)_{\text{SRF}} = (\hat{\omega}_{\text{1PN}} + \hat{\omega}_{\text{tide}}) L_1 \times e_1,$$

(9)

where

$$\hat{\omega}_{\text{1PN}} = \frac{3Gm_0}{c^2 a_1} \frac{n}{1 - e_1^2}$$

(10)

$$\hat{\omega}_{\text{tide}} = \frac{15}{2} k_2 \frac{m_0}{m_1} \frac{R_1}{a_1} 5^{1 + (3/2)e_1^2 + (1/8)e_1^4} n_1,$$

(11)

with $R_1$ and $k_2$ the radius and tidal Love number of $m_1$.

SRFs impose an upper limit $e_{\text{lim}}$ on the eccentricity that can be achieved through secular dynamical excitation; this limit is given by (Liu et al. 2015)

$$\left(\hat{\omega}_{\text{1PN}} + \hat{\omega}_{\text{tide}}\right) L_1 \times e_{\text{lim}} = \frac{9}{8},$$

(12)

where

$$\hat{\omega}_{\text{LK}} = \frac{1}{L_1(1 - e_1^2)^{1/2}}.$$  

(13)

Eq. (12) was derived for the standard ‘2+1’ quadrupole LK problem with SRFs, but it is also valid when octupole effects are included (Liu et al. 2015; see also Section 2.2). Our numerical calculations show that it holds for the ‘2+2’ problem as well (Fig. 3).

In the general problem of HEM via the LK effect, additional SRFs can arise from the rotational distortion of $m_0$ and $m_1$. However, rotational effects are negligible compared to the 1PN and tidal effects for a giant planet migrating around a WD (e.g., Anderson et al. 2016). The limiting eccentricity for our purposes is determined mainly by the tidal perturbation; thus:

$$1 - e_{\text{lim}} \sim 5 \times 10^{-5} \left(\frac{m_0}{0.6 M_\odot}\right)^{4/9} \left(\frac{m_{23}}{0.66 M_\odot}\right)^{-2/9} \times \left(\frac{k_{2,1}}{0.37}\right)^{2/9} \left(\frac{R_1}{R_1}\right)^{10/9} \left(\frac{m_1}{M_1}\right)^{-2/9} \times \left(\frac{a_{1,\text{eff}}}{1500 \text{ au}}\right)^{2/3} \left(\frac{a_1}{50 \text{ au}}\right)^{-16/9},$$

(14)

where we have assumed planetary properties analogous to Jupiter.

To produce the observed giant planet (at $a_{1,\text{obs}} \approx 0.02$ au) around WD 1856+534 via HEM, we require the pericenter distance $a_1(1 - e_1)$ to reach below 0.01 au (recall that tidal dissipation conserves angular momentum). Using Eq. (14), we find that the semi-major axis must satisfy

$$a_1 \gtrsim 8.4 \text{ au} \left(\frac{m_0}{0.6 M_\odot}\right)^{4/7} \left(\frac{m_{23}}{0.66 M_\odot}\right)^{-2/7} \times \left(\frac{k_{2,1}}{0.37}\right)^{2/7} \left(\frac{R_1}{R_1}\right)^{10/7} \left(\frac{m_1}{M_1}\right)^{-2/7} \times \left(\frac{a_{1,\text{obs}}}{1500 \text{ au}}\right)^{6/7} \left(\frac{a_1}{50 \text{ au}}\right)^{-9/7},$$

(15)

in order for external perturbations to overcome SRFs and push the planet to sufficiently high eccentricity.

Even when Eq. (15) is satisfied, the standard ‘2+1’ LK effect can produce extreme eccentricity excitation only when $I_{1,0}$ is very close to 90 degrees (top-left panel of Fig. 3, cyan points). This is where the resonant LK effect in a 2+2 system becomes important: when $\beta \sim 1$, the inclination resonance broadens the LK window for extreme eccentricities (Hamers & Lai 2017). For typical properties of the WD 1856+534 system (from Table 1: $m_0 = 0.6 M_\odot$, $m_2 = m_3 = 0.33 M_\odot$, $a_2 = 60$ au, $a_3 = 1500$ au, $I_2 = 30^\circ$; note that $\alpha \approx 2^\circ$ and $J \approx L_3$), Eq. (5) provides the value of $a_1$ corresponding to a given $\beta$:

$$a_1 = 49.5 \text{ au} \left(\frac{m_0}{0.6 M_\odot}\right)^{4/7} \left(\frac{m_{23}}{0.66 M_\odot}\right)^{-2/7} \times \left(\frac{a_2}{60 \text{ au}}\right)^{2/3} \left(\frac{\cos I_2}{\sqrt{3}/2}\right)^{2/3}.$$  

(16)
Figure 2: Eccentricity (as $1 - e_1$; upper panels) and inclination (lower) of the test particle as a function of time for trajectories in the resonant regime ($\beta \sim 1$, left panels) and modified LK ($\beta \gg 1$, right) regime. In these examples, we use identical initial conditions $I_{1,0} = 75^\circ$, $\Omega_{1,0} = 135^\circ$, $e_{1,0} = 0.01$, $\omega_{1,0} = 344^\circ$.

Figure 3: Maximum eccentricity of orbit 1 (as $1 - e$) versus initial inclination $I_0 \equiv I_{1,0}$ for different values of $\beta$ (Eq. 5). In all cases, we choose $m_0 = 0.6M_J$, $m_1 = 1.0M_J$, $m_2 = m_3 = 0.33M_J$, $a_2 = 60$ au (except the top-left panel where $a_2 = 1$ au), $e_2 = 0$, $e_3 = 0.3$, $a_3 = 1500$ au, and $I_2 = 30^\circ$ ($\alpha \approx 2^\circ$). The value of $a_1$ in each panel is 52 au ($\beta = 0$ and $\beta = 0.7$), 38.5 au ($\beta = 1.1$), 31.3 au ($\beta = 1.5$), 25.8 au ($\beta = 2$), and 19.7 au ($\beta = 3$). The planet’s initial $e_{1,0} = 0.01$. The argument of pericentre $\omega_{1,0}$ and longitude of the node $\Omega_{1,0}$ are randomly chosen in $[0, 2\pi)$. We integrate the vectorial secular equations for $500 t_{LK}$ and plot the maximal eccentricity achieved by $m_1$ in that time. We include SRFs due to the 1PN correction and tidal distortion of $m_1$, assuming $R_1 = 1.0R_J$ and $k_{2,1} = 0.37$. The cyan dots represent the results including quadrupole-order perturbations and SRFs whilst the purple dots also include the octupole-order terms from $m_{23}$ perturbing $m_1$. The dotted horizontal line is the limiting eccentricity $e_{\text{lim}}$ of Eq. (12).
To evaluate the ‘extreme’ LK window for a given $\beta$, we numerically integrate the quadrupole-order secular equations (including SRFs) for a duration of 500$t_{\text{LK}}$ using initial conditions over the full range of $I_{1,0}$ with the angles of the node ($\Omega_{1,0}$) and pericentre ($\omega_{1,0}$) randomly distributed. Fig. 3 shows the result of this calculation as cyan points. For $\beta \ll 1$ (the quasi-LK regime), the second binary behaves like a point mass $m_{23}$, the relation between $I_{1,0}$ and $e_{1,\text{max}}$ is analytical (Eq. 7), and the limiting eccentricity is achieved only if the initial inclination is extremely close to 90°. For $\beta$ around unity, we see a substantial widening of the extreme LK window. The window appears to be widest around $\beta = 1.5$, spanning the range $70° \lesssim I_{1,0} \lesssim 110°$. For progressively larger $\beta$, the window shrinks again to a narrow range around $I_{1,0} = 90°$ as the system enters the modified LK regime.

Importantly, Fig. 3 confirms that despite the complex LK evolution for $\beta \sim 1$, the limiting eccentricity of Eqs. (12, 14) still sets the floor for $1 - e_1$. Note that the “double-valued” features seen in Fig. 3 (e.g., in the lower-left panel) are a consequence of chaotic evolution for $\beta \sim 1$: the time required to achieve large eccentricities (beyond the standard ‘2+1’ LK effect) is highly variable (i.e. is sensitive to initial conditions) and may exceed 500$t_{\text{LK}}$ (the duration of each integration).

### 2.2 Octupole-Order Effects

So far, we have discussed the dynamics of a test particle in a 2+2 system at the quadrupole order of approximation. However, octupole-order effects may be necessary for an accurate treatment for nonzero $e_1$ and $a_1$ sufficiently large. The strength of the octupole perturbation of $m_1$ by $m_{22}$ relative to the quadrupole is measured by the dimensionless quantity

$$
\varepsilon_{\text{oct}} = \frac{a_1}{a_{\text{lim}}} \frac{e_1}{1 - e_1}.
$$

Liu et al. (2015) showed that including the octupole terms in the 2+1 LK problem with SRFs preserves the limiting eccentricity (Eq. 12) and that octupole effects can enhance the extreme LK window. Orbit 1 can always achieve $e_1 = e_{\text{lim}}$ when the initial inclination $I_{1,0}$ exceeds a critical value $I_{1,cr}$ (i.e., $I_{1,cr} < I_{1,0} < 180° - I_{1,cr}$; see the top-left panel of Fig. 3). An analytical fit for $I_{1,cr}$ is

$$
\cos^2(I_{1,cr}) \approx 0.26 \left( \frac{\varepsilon_{\text{oct}}}{0.1} \right)^2 - 0.536 \left( \frac{\varepsilon_{\text{oct}}}{0.1} \right)^2 + 12.05 \left( \frac{\varepsilon_{\text{oct}}}{0.1} \right)^3 - 16.78 \left( \frac{\varepsilon_{\text{oct}}}{0.1} \right)^4
$$

for $\varepsilon_{\text{oct}} \lesssim 0.05$ and $\cos^2(I_{1,cr}) \approx 0.45$ for $\varepsilon_{\text{oct}} \gtrsim 0.05$ (Muñoz et al. 2016). This form of enhancement is independent of the secular resonance effect in a 2+2 system and therefore can persist when $\beta \ll 1$. For fiducial properties of the WD 1856+534 system ($a_3 = 1500$ au, $e_3 = 0.3$), we have

$$
\varepsilon_{\text{oct}} \approx 0.01 \left( \frac{30a_1}{a_3} \right) \left( \frac{e_3}{0.3(1 - e_3)} \right),
$$

$$
|I_{1,cr} - 90°| \approx 10^3 \left( \frac{30a_1}{a_3} \right)^{1/2} \left( \frac{e_3}{0.3(1 - e_3)} \right)^{1/2}.
$$

In order to evaluate the relative contributions of the octupole and 2+2 “resonant” effects to the enhancement of LK window, we repeat the exercise of Section 2.1, this time including additional terms in Eqs. (1)-(3) to describe the octupole-order perturbation of orbit 1 by orbit 3 following Liu et al. (2015) (see also Liu & Lai 2019). The results are displayed as purple points in Fig. 3. As predicted, the octupole-order results feature an extreme LK window even for $\beta \ll 1$, consistent with Eq. (20).

For $\beta = 0.7$, the inclusion of octupole effects significantly widens the LK window relative to 2+2 resonant quadrupole effects alone. This suggests that the resonant-quadrupole and octupole effects can interfere constructively for $0.1 \lesssim \beta \lesssim 1$. For $\beta \gtrsim 1$, $a_1$ and $e_{\text{oct}}$ are smaller and the “resonant” quadrupole effect dominates the width of the extreme LK window; in these cases, octupole effects introduce a modest scatter in $e_{1,\text{max}}$ about the quadrupole result but do not otherwise affect the outcome.

### 3 HIGH-ECCENTRICITY MIGRATION AROUND WD 1856+534

We now apply our model to the WD 1856+534 system in order to demonstrate the feasibility of HEM through the enhanced LK effect in the resonant regime. For simplicity, we do not simulate the system’s dynamical evolution prior to the WD phase (but see Section 4). We also neglect octupole effects in this section.

We assume the same stellar and planetary parameters as in the previous section. In addition to the quadrupole secular perturbations and SRFs used previously, we now include dissipation of the equilibrium tide raised on the planet by the WD. We adopt the weak-friction model, where the planet’s internal dissipation is parametrized by a constant lag-time $\Delta t_{\text{inf}}$ (e.g., Alexander 1973; Hut 1981). The additional terms in Eqs. (1, 2) due to weak friction are given by Anderson et al. (2016). We assume that the planet rotates pseudosynchronously during HEM.

In Figure 4, we display an example of successful HEM of WD 1856+534 b from an initial configuration with $a_1 = 40$ au (corresponding to $\beta = 1.11$) and $I_1 = 75°$. For this example, we adopt $\Delta t_{\text{inf}} = 10$ s, corresponding to a factor of 100 enhancement relative to Jupiter’s dissipation; this is necessary to ensure timely orbital circularization of the planet and avoid tidal disruption (see Section 5). The planet’s minimal pericentre distance is just larger than the tidal disruption limit,

$$
R_{p,\text{dis}} = \eta R_1 \left( \frac{m_p}{m_1} \right)^{1/3}
$$

where $\eta \approx 2 - 3$ for a giant planet (e.g., Guillouëchon et al. 2011). The planet’s final semi-major axis is approximately twice this value, close to WD 1856+534 b’s orbit at $\approx 0.02$ au. The entire evolution takes place in slightly less than 332 Myr, well within the $\sim 6$ Gyr cooling age of WD 1856+534 (Vanderburg et al. 2020).

We note in the bottom panels of Fig. 4 that the planet’s final orbit is nearly perpendicular to the outer orbit. This is consistent with the actual viewing geometry for this system, with orbit 3 lying roughly in the celestial plane and orbit 1 along the line of sight (Vanderburg et al. 2020).
4 PRE-WD DYNAMICAL EVOLUTION

Let us consider the possible dynamical evolution of the WD 1856+534 system prior to the WD phase. As we noted previously, the transiting planet could not have migrated to its current location until after the host star had evolved into a WD; if an enhanced LK effect was responsible for this migration, then it is desirable that the effect be suppressed prior to the WD phase. The two most likely ways to accomplish this involve stellar evolution or the presence of additional planets in this system prior to the WD phase.

Most known WDs, including those showing evidence for planetary systems, evolved from main-sequence (MS) stars somewhat more massive than the Sun, typically \(\sim 1.5-3.0M_\odot\). Such stars become WDs of \(\sim 0.5-0.8M_\odot\), losing most of their mass during the AGB phase. AGB mass loss occurs over several Myr, a long time-scale compared to the orbital period of planets or stellar companions closer than \(\sim 10^4\) au; thus its effect on a 2+2 system can be treated using the principle of adiabatic invariance.

Consider the adiabatic reduction of the primary mass from \(m_{0,\text{MS}}\) (the MS value) to \(m_0 = f m_{0,\text{MS}}\) (the final WD mass, with \(f < 1\)). Assuming that mass is expelled isotropically and that none is captured by the other bodies, adiabatic invariance implies that orbits 1 and 3 expand according to

\[
a_{1,\text{MS}} \rightarrow a_1 = \frac{a_{1,\text{MS}}}{f},
\]

\[
a_{3,\text{MS}} \rightarrow a_3 = \frac{m_0,\text{MS} + m_{23}}{m_0 + m_{23}} a_{3,\text{MS}}
\]

whilst all other orbital elements are unchanged. It follows that the LK time-scale and the parameter \(\beta\) change according to

\[
t_{\text{LK,MS}} \rightarrow t_{\text{LK}} = f^2 \left( \frac{m_0,\text{MS} + m_{23}}{m_0 + m_{23}} \right)^3 t_{\text{LK,MS}},
\]

\[
\beta_{\text{MS}} \rightarrow \beta = f^3 \beta_{\text{MS}},
\]

where we have used \(J \sim L_3\) and \(I_2 = \text{constant}\) during the evolution. We see that adiabatic mass loss reduces \(\beta\) by a large factor. Thus, orbit 1 can transition from the modified LK regime (\(\beta_{\text{MS}} \gg 1\)) to the resonant regime (\(\beta \sim 1\)) as a result of stellar mass loss.

For typical WD 1856+534 system parameters, significant enhancement of the extreme LK window requires \(2 \gtrsim \beta \gtrsim 0.7\) (Fig. 3), corresponding to the planet’s semi-major axis at the end of the AGB phase in the range 28 au \(\lesssim a_1 \lesssim 57\) au (Eq. 16). If we adopt \(m_{0,\text{MS}} = 2.0M_\odot\)
for the progenitor mass and $m_0 = 0.6 M_\odot$ for the WD (e.g., Koester et al. 2014), i.e., $f = 0.3$, we find that this “resonant” $a_1$ range translates to 8 au $\lesssim a_{1,MS} \lesssim 17$ au. This is consistent with WD 1856+534 b having been a typical “cold Jupiter” during the MS.

As discussed in Section 2.2, octupole-order effects can also enhance the window for extreme eccentricity excitation through the LK effect. During adiabatic mass loss, $\varepsilon_{\text{oct}}$ (Eq. 17) evolves according to

$$\varepsilon_{\text{oct},\text{MS}} \to \varepsilon_{\text{oct}} = \frac{1}{f} \left( \frac{m_0 + m_{23}}{m_{0,\text{MS}} + m_{23}} \right) \varepsilon_{\text{oct},\text{MS}}.$$  \hspace{1cm} (26)

For the canonical masses in the WD 1856+534 system and $f = 0.3$, we have $\varepsilon_{\text{oct,MS}} \approx 0.63 \varepsilon_{\text{oct}}$, meaning octupole effects are less important during the MS. From Eqs. (19, 20), octupole effects can induce an appreciable extreme LK window (e.g., wider than 10$^\circ$) only if the planet’s semi-major axis is larger than $\sim 1/30$ of the semi-major axis of orbit 3, or $a_{1,MS} \gtrsim 24$ au for $a_{1,MS} = 700$ au.

In principle, HEM through LK effects could occur during the MS. Equation (14) with the above MS system parameters implies that a planet at $a_{1,MS} \approx 15$ au could have reached a pericentre distance $a_{1,MS} (1 - \cos \omega) \approx 0.007$ au. However, the inclination window to reach such extreme eccentricities would have been narrow since $\beta_{\text{MS}} \gg 1$ and $\varepsilon_{\text{oct,MS}} \ll 1$. AGB mass loss expands the extreme LK window, acting as a “trigger” for HEM during the WD phase.

In addition to the effects of stellar evolution, another planet orbiting closer to host star could have suppressed the LK effect during the MS by augmenting the free precession rate of WD 1856+534 b (see Holman et al. 1997; Petrovich & Muñoz 2017). A planet of mass $m'$ at a distance $a' < a_{1,MS}$ from the host star would completely suppress LK oscillations of $m_1$ if

$$m' \gtrsim m_{23} \frac{a_{1,MS}^5}{a_{3,MS}^3 (a')^2} \approx 60 M_\odot \left( \frac{a_{1,MS}}{10 \text{ au}} \right)^5 \left( \frac{a_{3,MS}}{700 \text{ au}} \right)^{-3} \left( \frac{a'}{1 \text{ au}} \right)^{-2}.$$  \hspace{1cm} (27)

A smaller $m'$ would still be sufficient to suppress the high-eccentricity excursion of $m_1$. If this planet had $a' \lesssim 5$ au during the MS, it would have been engulfed during the AGB stage (Mustill & Villaver 2012), thereby “switching on” the LK effect during the WD phase. The architecture envisioned in this scenario (e.g., two planets between $\sim 1$ and 15 au) is plausible given current knowledge of the population of extrasolar planets. For instance, Bryan et al. (2016) found that $\sim 50\%$ of giant planets orbiting between 1 and 5 au have companions of comparable mass within a distance of 20 au.

5 PROBLEMS WITH MIGRATION AND PLANETARY OCCURRENCE RATES FOR WHITE DWARFS

The challenges of HEM through secular dynamics have been studied in the context of hot-Jupiter formation around MS stars. A major issue is that it is easy for a migrating planet to be tidally disrupted. Population-synthesis models (Petrovich 2015a; Anderson et al. 2016; Hamers et al. 2017; Vick et al. 2019; Teyssandier et al. 2019) and analytical calculations (Muñoz et al. 2016) show that tidal disruption can limit the efficiency of hot-Jupiter formation through secular HEM to a few percent. In particular, although octupole effects broaden the ‘extreme’ LK window and thus increase the migration fraction, most migrating planets are tidally disrupted (Anderson et al. 2016; Muñoz et al. 2016). A similar situation occurs for the secular-chaos scenario (Wu & Lithwick 2011; Teyssandier et al. 2019). In Section 3, although we have not systematically surveyed a large parameter space, we also find that a large faction of migrating planets are disrupted for HEM in 2+2 systems. In fact, we had to use a relatively large tidal lag-time ($\Delta t_{L1} = 10$ s) in order to find a reasonable number of surviving cases.

The problem of tidal disruption during HEM can be somewhat alleviated by invoking forms of tidal dissipation other than weak friction, such as chaotic dynamical tides with non-linear dissipation (Wu 2018; Vick & Lai 2018; Vick et al. 2019; Teyssandier et al. 2019). Vick et al. (2019) show that the strong dissipation associated with chaotic tides can shepherd to safety some planets that are otherwise destined for tidal disruption by rapidly decreasing their eccentricities; they find $\sim 20\%$ of migrating planets survive as hot Jupiters. Strong planet–planet scattering may also give rise to HEM. Two giant planets that are dynamically stable during the MS can become unstable after AGB stellar mass loss (Debes & Sigurdsson 2002). However, strong scatterings of unstable giant planets mostly result in a planetary ejection or a two-planet merger; the ‘branching ratio’ of one planet being injected into a low-pericentre orbit suitable for HEM is less than 1\% (e.g., Anderson et al. 2020; Li et al. 2020). We consider this an even less promising HEM mechanism.

The observed configuration of WD 1856+534 b is improbable: the planet only obscures half of the WD in transit despite having $\sim 10$ times its radius (Vanderburg et al. 2020). The probability of observing such ‘grazing’ transits is $\sim R_\odot/a_{1,obs} = 0.2\%$ (for $a_{1,obs} = 0.02$ au and WD radius $R_\odot \approx R_\odot$), whilst the probability of a ‘full’ transit is $\sim R_\odot/a_{1,obs} = 2\%$. Hence, the detection of WD 1856+534 b with a ‘grazing’ transit suggests there should be $\sim 10$ times as many systems where the WD is completely blocked. Given that TESS has observed $\sim 1700$ isolated WDs thus far (Vanderburg et al. 2020), this further implies that short-period giant planets could orbit $\sim 30\%$ of them (although this inference is based on a single observation and therefore highly uncertain). Such an abundance seemingly defies the difficulties of all flavours of HEM discussed above and motivates further study.

6 CONCLUSION

In this paper, we have demonstrated the possible HEM of WD 1856+534 b through an enhanced LK effect from the WD’s distant M-dwarf companions. We show that the inclination resonance in hierarchical 2+2 systems and the octupole effect can both significantly broaden the LK window for extreme eccentricity excitation (see Fig. 3), allowing HEM to operate in the WD 1856+534 system for a wide range of initial planetary orbital inclinations. We find that the planet could have migrated through the resonant 2+2 effect from a distance of $\sim 30$–60 au from the WD, corre-
sponding to ~10–20 au during the host’s MS. Importantly, extreme eccentricity excitation can be delayed until the WD phase.

Although WD 1856+534 b is so far the only intact short-period giant planet known to orbit a WD, HEM and tidal disruption of giant planets may have occurred in other WD systems. For instance, WD J0914+1914 possesses a gaseous accretion disc and a pollution signature rich in hydrogen, oxygen, and sulfur (Gänsicke et al. 2019). This has been interpreted as evidence for an ice giant’s ongoing or previous disruption by the WD. As noted in Section 5, any flavor of HEM will more likely lead to tidal disruption than survival of the planet. Although WD J0914+1914’s cooling age is quite short (~13 Myr), secular interactions can deliver a planet to the WD in time given a suitable companion (e.g., Stephan et al. 2017).

Atmospheric pollution may be a more prevalent signature of remnant planetary systems around WDs than the occurrence of short-period giant planets. To advance understanding of the occurrence of planetary systems in hierarchical star systems, we recommend further observational efforts to determine the fraction of polluted WDs that belong to wide binaries or hierarchical triples. The Gaia mission and LSST will aid in the identification of previously unknown stellar or substellar companions of WDs (e.g., Gentile Fusillo et al. 2019).

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DATA AVAILABILITY

The data underlying this article will be shared on reasonable request to the corresponding author.

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