A Mathematical Model of the Color Preference Scale Construction in Quality Management at the Machine-Building Enterprise

V I Averchenkov¹, S V Kondratenko¹, L A Potapov², V V Spasennikov³

¹Department of Computer Technologies and Systems, Bryansk State Technical University, Bryansk, Russia
²Department of Industrial Electronics and Electrical Engineering, Bryansk State Technical University, Bryansk, Russia
³Department of Economics and Management, Bryansk State Technical University, Bryansk, Russia

Abstract. In this article, the author consider the basic features of color preferences. The famous scientists’ works confirm their identity and independence of subjective factors. The article examines the method of constructing the respondent’s color preference individual scale on the basis of L Thurstone’s pair election method. The practical example of applying this technique for constructing the respondent’s color preference individual scale is given. The result of this method application is the color preference individual scale with the weight value of each color. The authors also developed and presented the algorithm of applying this method within the program complex to determine the respondents’ attitude to the issues under investigation based on their color preferences. Also, the article considers the possibility of using the software at the industrial enterprises to improve the quality of the consumer quality products.

1. Introduction

Emotional processes and definition of the person’s relationship to some subject is one of the most difficult problems for psychological researches. First of all, it is due to the labour-intensive methods of the emotional sphere diagnostics. They differ, as a rule, by a large number of questions in the questionnaire, or they require a lot of time to carry out a survey and to interpret it, or in general, they have a high degree of subjectivity, since they are basically based on projective drawing methods. [1]

For this reason, lately there has been a question of searching and developing new approaches for identifying the respondent’s relation to a particular subject. Application of color estimates for the purpose of emotional diagnostics is the basic for the offered method. The color range allows presenting the person’s emotion through a large number of intermediate shades and states. [2,3]

Color preferences play an important role in carrying out the work to improve the output ergonomic qualities, because they allow taking into account the influence of a particular color on the person.

2. A general overview of research issues

In the works of the Russian researchers under Izmailov Ch A’s leadership, it is claimed that the color palette can be represented in the form of the uninterrupted continuum on which it is possible to arrange the majority of human emotions. Most of our emotions are correlated with a color palette from...
blue to blue-red or purple. The rest of the emotions are related to the spectrum of the green and green-blue color and very few emotions are related to the spectrum of light green tones.

While working, G Fliring and K Auer [6] came to the conclusion that all people had their individual relations to color. Man feels sympathy for one color, and antipathy towards others, so we can say that each person has a rating of favourite and least favourite colors. However, it should be borne in mind that the person's predisposition to a particular color changes due to temporal or cultural factors. [1]

Constructing a color preference individual scale is an important stage while creating the system of testing on the basis of respondents’ color preferences. The following article presents the implementation algorithm of such a process, which is based on the pair election method.

3. The analysis of the existing approaches to defining individual color preferences

There are many ways to construct an individual color preference scale. At the moment, the subjective methods of scaling performance incentives that do not have direct physical correlates, which are based on Thurstone’s model of scaling, are most popular [8]. The gist of it consists in presenting simultaneously a pair of incentives causes by two processes of distinguishing $d_i$ and $d_j$. The difference $(d_j - d_i)$ is called a distinctive difference. If there is a large number of presentations of two incentives, the distinguishing differences also form the normal distribution on the psychological continuum. Therefore, the average distinguishing difference distribution $(d_j - d_i)$ is equal to the difference of the average distributions between the difference processes themselves - $(S_j- S_i)$. In this case, distribution dispersion of distinguishing differences will be equal to

$$s(d_j - d_i) = \left( s_i^2 + s_j^2 - 2rs_{ij}s_is_j \right)^{\frac{1}{2}}, \tag{1}$$

where $s_i$ and $s_j$ – dispersions of distinguishing processes of i-th and j-th of incentives respectively, and $r_{i,j}$ – correlation between the instantaneous values of distinguishing processes of i and j incentives.

Let us consider the following situation. Let couple of colors i and j be shown to the observer, and he is required to choose the most preferable color incentive from the offered two.

It is assumed that if a distinctive process for the j incentive turns out to be higher than the i incentive on the psychological continuum, that is if the distinctive difference is $(d_j - d_i)>0$, then there follows the judgment that color j is favoured more than color i. And, accordingly, if $(d_j - d_i)<0$ – the reverse judgment will take place.

L Thurstone allows zero correlation between two processes of distinguishing $(r = 0)$ and equalities of distinctive dispersions of these processes $(s_j = s_i = s)$ as one of the widest-spread variants of the law of comparative assessments. As a result, expression (1) will be transformed in

$$S_j - S_i = z_{j,i}s, \tag{2}$$

where $z_{j,i}$ – designates the required distinction between the offered incentives on the psychological continuum.

Units of scale values can always be chosen so that constant "s" is equal to 1. Then

$$S_j - S_i = z_{j,i}. \tag{3}$$

In case of lacking mistakes, the required distinction will be equal to observable (denoted $z'_{j,i}$). But as a result of errors between $z_{j,i}$ and $z'_{j,i}$, there will be some discrepancy of a. The task consists of receiving such a set of incentive scale value estimates for which the sum of squares of all divergences is minimum that is it is necessary to minimize the value of

$$w_{ij} = \sum_{l=1}^{n} \sum_{j=1}^{n} (z'_{ij} - z_{ij})^2 \tag{4}$$

Substituting $z_{i,j}$ for scale values, we get
Differentiating elements of each column on $S_i$, we will get
\begin{equation}
\alpha_{ij} = \sum_{i=1}^{n} \sum_{j=1}^{n} (z_{ij} - s_i + s_j)^2.
\end{equation}

(5)

Let us equate the partial derivative to zero, and after transferring, we will obtain
\begin{equation}
\sum_{j=1}^{n} z_{ij} + \sum_{j=1}^{n} s_j = \sum_{j=1}^{n} s_i.
\end{equation}

(6)

Let us divide expressions (7) by $n$ and take the initial value of the scale equal to $\frac{1}{n} \sum_{j=1}^{n} z_{ij}$. The result is a quantitative expression of the respondent’s attitude towards incentive $S_i$:
\begin{equation}
S_i = \frac{1}{n} \sum_{j=1}^{n} z_{ij}.
\end{equation}

(8)

where $i = 1,2 \ldots, n$.

Thus, for minimizing mistakes, it is necessary to take the arithmetic average of the column of matrix $Z$, and we will get the optimal value of scale values $S_i$.

The considered procedure gives the chance for each of the $S_i$ incentives to get its value on the scale of intervals.

For matrixes with zero and units (they are called inexact matrixes), there are special analysis algorithms. The most widespread of them was described by B. Torgerson [14].

From expression (3) for the $j$ incentive, it follows that incentive $j + 1$ will be described by the following equation:
\begin{equation}
S_{j+1} = S_j = z_{j+1} + e.
\end{equation}

(9)

Subtracting equation (9), from equation (3), we will get $d_{j+1} = -d_j$ - the comparative distinction for the incentive interesting for us in an indirect way. In terms of the minimized mistake, this value may be calculated from the following expression:
\begin{equation}
d_{ij+1} = S_{j+1} - S_j = \frac{1}{n} \sum_{j=1}^{n} (z_{ij+1} + z_{ij}),
\end{equation}

(10)

where $n_i$ - a toting index.

For practical convenience, the $Z$ matrix should be reconstructed so that the columns were arranged according to magnitude. For such an ordered $Z$ matrix, the distinction ($S_{j+1} - S_j$) can be directly calculated from expression (10). If we equate the scale value of first incentive ($S_1$) to zero, then the scale value of any incentive is the sum of scale values of the incentives and the distance between this incentive and the previous one is
\begin{equation}
S_1 = 0, \quad S_2 = d_{1,2}, S_3 = S_2 + d_{2,3}, \quad S_n = S_{n-1} + d_{n-1,n}.
\end{equation}

(11)

4. The practical example of constructing the color preference scale
Let us consider a practical example of solving the fifth version of the comparative estimate law by the method of least squares. [4] The respondent is randomly given 8-color cards from the Luscher’s test.
small set and is asked to choose the most preferable in each pair. Each pair is shown 6 times, each card is shown 3 times on the left and 3 times on the right. As a result, the following matrix of color choice frequencies $F_i$ for one of the examinees (Table 1) was formulated:

**Table 1. Frequency matrix, $[F]$**

| Colors, $f_{ij}$ | Grey | Blue | Green | Red | Yellow | Purple | Brown | Black |
|------------------|------|------|-------|-----|--------|--------|-------|-------|
| Grey             | -    | 2    | 1     | 3   | 4      | 3      | 6     | 2     |
| Blue             | 4    | -    | 4     | 4   | 3      | 1      | 2     | 2     |
| Green            | 5    | 2    | -     | 5   | 3      | 2      | 1     | -     |
| Red              | 3    | 2    | 1     | -   | 2      | 0      | 3     | 1     |
| Yellow           | 2    | 3    | 3     | 4   | -      | 2      | 5     | 6     |
| Purple           | 3    | 5    | 4     | 6   | 4      | -      | 3     | 4     |
| Brown            | 0    | 4    | 5     | 3   | 1      | 3      | -     | 1     |
| Black            | 4    | 4    | 1     | 5   | 0      | 2      | 5     | -     |

The element of matrix $[F]$ is the frequency with which in pair $j,i$, the $i$ incentive is estimated more preferably than the $j$ incentive.

The received frequency matrix $[F]$ will be transformed to a matrix of probabilities $[P]$ by division of the $f_{i,j}$ frequency by the number of presentations ($N = 6$).

**Table 2. Probability matrix, $[P]$**

| Colors, $f_{ij}$ | Grey | Blue | Green | Red | Yellow | Purple | Brown | Black |
|------------------|------|------|-------|-----|--------|--------|-------|-------|
| Grey             | -    | 0.33 | 0.17  | 0.50| 0.67   | 0.50   | 1.00  | 0.33  |
| Blue             | 0.66 | -    | 0.67  | 0.67| 0.50   | 0.17   | 0.33  | 0.33  |
| Green            | 0.83 | 0.33 | -     | 0.83| 0.50   | 0.33   | 0.17  | 0.50  |
| Red              | 0.5  | 0.33 | 0.17  | -   | 0.33   | 0.00   | 0.50  | 0.17  |
| Yellow           | 0.33 | 0.50 | 0.50  | 0.67| -      | 0.33   | 0.83  | 1.00  |
| Purple           | 0.5  | 0.83 | 0.67  | 1.00| 0.67   | -      | 0.50  | 0.67  |
| Brown            | 0    | 0.67 | 0.83  | 0.50| 0.17   | 0.50   | -     | 0.17  |
| Black            | 0.66 | 0.67 | 0.17  | 0.83| 0.00   | 0.33   | 0.83  | -     |
| $\sum_{P_{ij}}$ | 3.5  | 3.66 | 3.16  | 5   | 2.83   | 2.16   | 4.16  | 3.16  |

Elements of the matrix $[P]$ is the probability with which the $i$ stimulus in pair $j,i$ is assessed more preferably than the $j$ incentive.

Each value is probability $p_{i,j}$ of matrix $[P]$, which is transferred further by means of the probabilities conversion table into the units of a standard deviation of the normal curve - $z_{i,j}$, on which scale values $S_i$ on each incentive are calculated. [1]

**Table 3. Rating matrix $Z_i$**

| Colors, $f_{ij}$ | Grey | Blue | Green | Red | Yellow | Purple | Brown | Black |
|------------------|------|------|-------|-----|--------|--------|-------|-------|
| Grey             | 0.00 | -0.44| -0.95 | 0.00| 0.44   | 0.00   | -     | -0.44 |
| Blue             | 0.44 | 0.00 | 0.44  | 0.44| 0.00   | -0.95  | -0.44 | -0.44 |
| Green            | 0.95 | -0.44| 0.00  | 0.95| 0.00   | -0.44  | -0.95 | 0.00  |
| Red              | 0.00 | -0.44| -0.95 | 0.00| -0.44  | -      | 0.00  | -0.95 |
| Yellow           | -0.44| 0.00 | 0.00  | 0.44| 0.00   | -0.44  | 0.95  | -     |
| Purple           | 0.00 | 0.95 | 0.44  | -   | 0.44   | 0.00   | 0.44  | -     |
| Brown            | -    | 0.44 | 0.95  | 0.00| -0.95  | 0.00   | 0.00  | -0.95 |
| Black            | 0.44 | 0.44 | -0.95 | 0.95| -      | -0.44  | 0.95  | 0.00  |
| $\sum_{Z_{ij}}$ | 1.39 | 0.51 | -1.02 | 2.78| -0.51  | -2.27  | 0.51  | -2.34 |

The element of the $Z'_{i,j}$ matrix is the probability of $p'_{i,j}$, which is converted into the units of standard deviation. Let us rearrange the columns in matrix $Z$ so that the first column has the least sum of elements, and the last - the greatest.
Table 4. Sum of estimates $Z'_i$

| Incentive, $Z'_ij$ | Black (8) | Purple (6) | Green (3) | Yellow (5) | Blue (2) | Brown (7) | Grey (1) | Red (4) |
|-------------------|-----------|------------|-----------|------------|----------|----------|---------|--------|
| $\sum Z'_ij$     | -2.34     | -2.27      | -1.02     | -0.51      | 0.51     | 0.51     | 1.39    | 2.78   |

From the values of sums $Z_i$, there can be received a matrix of distinctions between the adjacent pairs of columns. Subtracting them element-wise one from another. In each j-th line, the element of this matrix will be equal to $(z_{j,i+1} - z_{j,i})$.

Table 5. The matrix of distinctions between columns

| St/di,j | Purple (6) | Green (3) | Yellow (5) | Blue (2) | Brown (7) | Grey (1) | Red (4) |
|---------|------------|-----------|------------|----------|-----------|---------|--------|
| $\sum$ | 0.07       | 1.25      | 4.31       | 1.02     | 0.00      | 0.88    | 0.51   |
| Number of elements | 8.00 | 8.00      | 5.00       | 5.00     | 8.00      | 4.00    | 7.00   |
| 1/n    | 0.0087     | 0.1562    | 0.862      | 0.204    | 0         | 0.22    | 0.0728 |

Using expression (11), calculate scale values of incentives from the received distinctions, where $S_8 = 0$:

- $S_8 = 0$;
- $S_6 = 0 + 0.0087 = 0.0087$;
- $S_3 = 0.0087 + 0.1562 = 0.165$;
- $S_5 = 0.165 + 0.862 = 1.027$;
- $S_2 = 1.027 + 0.204 = 1.231$;
- $S_7 = 1.231 + 0 = 1.231$;
- $S_1 = 1.231 + 0.22 = 1.451$;
- $S_4 = 1.451 + 0.0728 = 1.523$.

The resulting range of individual color preferences of the investigated respondent is presented in Table 6.

Table 6. The individual range of the respondent’s color preferences

| Black | Purple | Green | Yellow | Blue | Brown | Grey | Red |
|-------|--------|-------|--------|------|-------|------|-----|
| 0     | 0.0087 | 0.165 | 1.027  | 1.231| 1.231 | 1.451| 1.523|

5. Applying a technique within a program complex

This method described above was chosen for constructing the module of the preliminary survey in the testing system on the basis of the respondent’s color preference. Figure one shows the general scheme of this software package.

The system, in addition to the main survey module, includes a pre-testing module, which is needed to determine the relationship of the respondent to the colors. The answers to the questions of the basic testing, correlate with respondent’s color preferences individual scale and, as a result of processing, the conclusion about the respondent’s emotional attitude to the tested object is made.

In the software package, M Lusher’s set of colors is used. These colors cover completely the most fundamental human emotions. Just considering the psychological characteristics of a person, 7-8 colors for assessment are the best estimates of the volume scale, since if there is a greater number of options, the respondent is lost, and is not always able to give an informed answer [7].

The preliminary survey process is as follows: pairs of colored rectangles in the window of the program are presented to the respondent, the respondent’s task is to evaluate which of the 2 colors are most preferable for him. As soon as the respondent gives an answer, the next pair of incentives appears on the screen. All colors are shown 6 times. Each color is presented on the left side three times and it is presented on the right side three times as well. Each time, the serial number of the sample is displayed in the upper right corner of the screen. [4]
While carrying out the experiments using the developed software package, it took the majority of respondents not more than five minutes to construct a color preference individual scale.

Once the respondent has answered all color pairs, his color preference individual scale is made up, which enables us to further use color assessments as an alternative to traditional quantitative methods of discrete estimates by transferring the color response into traditional scoring. This eliminates the subjective factor in the respondents’ answers related to the traditions of the society, the desire to answer the question as it is accepted in the society.

Research respondents’ attitude towards different objects and phenomena with the use of color estimates can be attributed to projective testing methods. Since the study primarily addresses respondent’s relations to the side object or subject, and does not determine its psychological state, we do not have a clear reference to how the colors will be displayed on a particular respondent’s screen. Here, the necessary conditions will be to provide the contrast of displayed colors so that the respondent could clearly distinguish all 8 colors, also the same display settings should remain unchanged throughout the test. The respondent should operate with the same colors both at the stage of preliminary testing in constructing the color preference individual scale, and in response to the main testing survey. These specific conditions related to the particular display devices do not have a significant impact on the test results.

The proposed method can be used in industry while introducing new products to the market, assessing the product quality characteristics by consumers.

The same software package can be used while implementing the enterprise staff policy. The proposed method allows us to estimate the loyalty of an employee, his aspirations, attitude towards colleagues and management.

6. Conclusion

Using color evaluation, as opposed to score ones allows one to yield much more accurate results, which do not depend on external factors (social norms, established imperatives).

When using the computer, the time required for testing based on the color evaluation and for results processing is comparable with traditional scoring methods.

All these qualities make it possible to use the method of color assessments in a variety of areas. First of all, these are the areas where it is difficult to give an unambiguous qualitative assessment and as a rule qualitative assessments are given, which are hardly amenable to automated mass analysis.

The spheres of software package application are different, they are marketing research, testing while employment, group formation. In such cases, the surveys based on color preferences allow one to carry out easily formalized, amenable to computer automation researches.

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