Fairness-Driven Energy Efficient Resource Allocation in Uplink MIMO Enabled HetNets

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ABSTRACT Heterogeneous network (HetNet) is a promising solution to meeting the unprecedented demand for higher data rate in the next generation mobile networks. Compared with the traditional single-layer cellular networks, the resource management (e.g., user association and power control problems) is more challenging in the multi-tier HetNets. In addition, due to the new application scenarios and battery-limited nature of the user equipment (UE), improving energy efficiency (EE) of individual while ensuring EE fairness among UEs is one of the key design issues in the uplink HetNets. In this paper, by considering the fairness for EE, joint user association and power control problems are studied in the uplink multiple input multiple output (MIMO) enabled HetNets. Under the quality of service (QoS) and transmit power constraints of UE, two kinds of fairness, namely, proportional fairness and max-min fairness, are investigated. The optimization problems of maximizing the EE are formulated by considering the fairness among UEs. Considering the non-convex characteristic of the problem, the formulated problem is divided into two subproblems, i.e., user association and power control subproblems, then an iterative algorithm is proposed to achieve the fair EE resource allocation for each fairness criterion until convergence. For the proportional fairness problem, the dual decomposition and Newton methods are applied. Then, by using successive convex approximation (SCA) and dual decomposition, the problem of max-min fairness is solved. Through both theoretical analysis and simulations, the convergence and the effectiveness of the proposed algorithms are proved and evaluated. Simulation results show that the proposed algorithms can significantly improve the average EE of users.

INDEX TERMS Heterogeneous networks, fairness, user association, power control, energy efficiency.

I. INTRODUCTION

To come up with the 1000 times growth of mobile traffic requirements and more diverse quality of service requirements of mobile users, heterogeneous networks (HetNets) have recently attracted intensive attention as one of the key enabling technologies for the fifth generation (5G) of mobile communications. The model for HetNets would consist of K tiers, where each tier is characterized by its transmit power, base station (BS) density, and data rate [1], [2].

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Compared with traditional single-tier homogeneous networks, different types of BSs (macro, micro, pico and femto) in multi-tier HetNets will bring lots of additional challenges along with the benefits, such as user association and power control problems. With the fact that macro BS and small BS have different coverage and processing capability [3], the traditional user association scheme based on the maximum received signal strength (max-RSS) cannot apply to HetNets, since they will cause inefficient small BS deployment and increase the energy consumption of the user equipments (UEs). With the continuous emergence of various new application scenarios of UEs [4], [5], the energy consumption
of UE is rising, moreover, excessive power consumption arises from 5G technologies, such as millimeter wave [6] and device-to-device (D2D) communication [7]. These new applications and technologies adversely affect the battery lifetime of the UE as the UE has a limited battery power, which would heavily affect the user experience and satisfaction. Therefore, in order to prolong the battery life and save the power consumption of the UE, how to enhance the highest energy efficiency (EE) of the UE has become a significant task for the wireless communication networks. In addition, due to the competition for limited resources in HetNets and the fact that each individual wants to maximize their own benefits, the fairness among UEs is critically important. Consequently, in order to improve the EE of UE while ensuring the fairness among the UEs, applying appropriate user association and power control schemes is very important in the uplink multiple input multiple output (MIMO) enabled HetNets.

There have been extensive researches on resource management for the HetNets systems. In [8], an extensive overview of user association mechanisms was discussed in the 5G mobile networks. The authors in [9] developed centralized and distributed user association algorithms to maximize the overall transmission rate for massive MIMO HetNets. Under minimum rate requirements, [10] studied the BS operation and user association problem for energy efficient HetNets. There are also works investigating EE by adopting power control. In [11], an uplink power control scheme was proposed by adjusting the power control coefficient under the constraints of outage probability and transmission power. The author evaluated the EE of uplink transmission in HetNets, where a fractional power control (FPC) algorithm was applied at UEs subject to a maximum transmit power constraint [12]. By using alternating direction method and water filling algorithm, the author studied the distributed power control problem in HetNets when carrier aggregation was utilized [13]. The author investigated the interference management and power allocation problem in two-tier HetNets to maximize the sum rate of system [14]. In order to maximize the downlink sum rate of small cell users, a non-cooperative game-based distributed algorithm was proposed to jointly investigate the small cell clustering, precoding design and the transmit power of macro user problems [15]. By jointly considering power and subchannel allocation, the work in [16] investigates the energy efficient resource allocation in two-tier HetNets with limited wireless backhaul links. The aforementioned works in [8]–[16] do not study the interaction between user association and power control. Since the user association and power control interact with each other, both should be solved jointly in order to achieve better performance. Thus, the joint user association and power control problem is investigated in the following works. For instance, to maximize the weighted sum-rate, the authors in [17] studied the problem of jointly optimizing the transmit power, user scheduling and user association in cellular networks based on a distributed interference penalty algorithm. [18] proposed a joint power control and user association algorithm to maximize the weighted sum-rate for the downlink transmission in multi-cell multi-association orthogonal frequency division multiple access (OFDMA) HetNets. The author jointly addressed user association and power control problem for HetNets and an efficient algorithm was applied to maximize the proposed utility-energy efficiency (UEE) metric [19]. In [20], the authors studied the resource allocation problem about power, precoding coefficient, and subcarrier for energy-efficient communication in multi-cell OFDMA systems. By considering the load balancing in HetNets [21], the author proposed a joint user association and power control strategy to maximize the weighted sum of long-term rate. In order to maximize the total rate of UEs in the cell of different links while reducing interference and guaranteeing the predefined quality of service (QoS) requirements, the author developed a joint user association and power allocation algorithm in HetNets using non-cooperative game theory [22]. A sum-rate maximization problem was formulated by jointly designing user association and power allocation in a non-orthogonal multiple access (NOMA) based multi-cell network [23]. In our previous work [24], we also proposed a user association scheme and a power control algorithm to improve the average EE of all the users in HetNets. However, the aforementioned works [17]–[23] focus on the spectral efficiency or power consumption problems, and do not take the fairness issues among UEs into account.

The fairness of resource allocation in HetNets has also been intensively investigated. In [25], distributed energy efficient fair user association for HetNets was proposed and a network-wide logarithmic utility maximization problem was formulated. In the case of imperfect channel state information, the author in [26] designed a user association and power allocation scheme under loads and transmit power constraints by considering the proportional fairness about the spectral efficiency. The author formulated and solved an uplink security-aware proportional fairness power and subchannel allocation problem for cognitive HetNets with inter-network cooperation in [27]. [28] investigated a fair EE for subcarrier and power allocation scheme in the power domain-NOMA (PD-NOMA) based HetNets. A max-min energy efficient enhanced inter-cell interference coordination (elCIC) configuration algorithm was proposed in ultra dense HetNet [29]. In contrast to the previous works which only studied the user association problem for the fairness of EE [25] or lacked research on the EE [27]. Then, the authors only considered the downlink heterogeneous wireless network [26], [28], [29]. In this paper, we investigate an energy efficient joint user association and power control problem with two different fairness criteria in the uplink MIMO enabled HetNets.

In this paper, we take one step further from [24] to provide a comprehensive study on the optimal user association design and resource allocation by considering the fairness about EE in MIMO enabled HetNets. The main contributions of this paper are summarized as follows:
In the uplink MIMO enabled HetNets, we studied two kinds of fairness criteria (i.e., proportional fairness and max-min fairness) in EE resource allocation by jointly considering the user association and power control problem.

The proportional fairness criterion is modeled as the logarithmic maximization of UE’s EE optimization problem. Then, for the max-min fairness criterion, the optimization problem is modeled as the maximization of the worst UE’s EE. The UE’s QoS and transmit power constraints in the two fairness criteria are also guaranteed simultaneously.

Since the optimization variables are highly correlated and the characteristics of the optimization problems are non-convex and mixed-integer, it is very hard to obtain the optimal solution. In order to get a tractable solution, each problem is separated into two subproblems and a two-layer iterative suboptimal joint user association and power control scheme is proposed. Furthermore, the convergence of the two proposed fairness iteration algorithms have been proven.

For the proportional fairness optimization problem, the dual decomposition and Newton methods are applied to maximize the log utility of EE with QoS and transmit power constraints of UE. For the max-min fairness optimization problem, the dual decomposition and successive convex approximation (SCA) methods are utilized to solve the user association and power control subproblems, respectively. Simulation results show that the proposed suboptimal algorithms have better EE performance than other schemes.

The remainder of this paper is organized as follows. Section II presents the system model. In Section III, problem formulations for different fair resource allocation are presented. In Section IV, solutions and algorithms to solve the proposed optimization problems are investigated. The computational complexity of the proposed algorithms for the various solution methods are analyzed in Section V. Simulations results are presented in Section VI for performance evaluation. Finally, Section VII concludes the whole paper.

II. SYSTEM MODEL

A. NETWORK MODEL

A two-tier uplink wireless HetNet consisting of macro and small cell, as shown in Fig.1, is studied. We denote the BSs and UEs set as \( B_{BS} = \{1, \ldots, m, \ldots, M\} \) and \( B_{U} = \{1, \ldots, n, \ldots, N\} \), respectively. In order to avoid cross-tier interference, the different tier’s BSs operate in orthogonal frequency bands. The bandwidths of macro and small cells are assumed to be equal, i.e., \( B_{m} = B_{s} = B \). The location of BSs and UEs are deployed following Poisson Point Process (PPP) distribution with density \( \lambda_{k} \) for \( k \in \{1, 2\} \) and \( \lambda_{u} \), respectively, where \( k = 1 \) denotes the macro cell tire and \( k = 2 \) denotes the small cell tire and the index \( u \) denotes UEs who can connect any BS for communication. In the HetNet, the \( k \)th tier BSs have \( N_{k} (k \in \{1, 2\}) \) antennas and \( N_{1} > N_{2} \), and all UEs are with single antenna.

B. SIGNAL MODEL

Assuming that a UE \( n_{0} \) is associated with the \( k \)th tier BS with \( N_{k} \) antennas, the received signal vector \( Y_{n_{0}} \) at a tagged BS is given by

\[
Y_{n_{0}} = \sqrt{\frac{P_{T}}{d_{m,n_{0}}} h_{m,n_{0}} x_{n_{0}}} + \sum_{i \neq n_{0}} \sqrt{\frac{P_{T}}{d_{m,i}^{\alpha}} h_{m,i} x_{i}} + n, \tag{1}
\]

where the symbol \( x_{n_{0}} \) is the signal transmitted by the UE \( n_{0} \) to the \( m \)th BS, \( E[|x_{n_{0}}|^{2}] = 0 \) and \( E[|x_{i}|^{2}] = 1 \). The symbol \( d_{m,n} \) represents the distance between the UE \( n_{0} \) and the \( m \)th BS. Index \( \alpha \) denotes the path loss exponent, where \( \alpha > 2 \). \( h_{m,i} = [h_{m,i_{1}}, h_{m,i_{2}}, \ldots, h_{m,i_{N_{k}}} ]^T \) is the complex channel gain and \( h_{m,i_{l_{k}}} \) follows Rayleigh distribution, where \( h_{m,i_{l_{k}}} \sim \mathcal{CN}(0, 1) \). \( x_{i} \) is the signal transmitted by the \( i \)th UE. \( n = [n_{1}, n_{2}, \ldots, n_{N_{k}}]^{T} \) is the vector of complex additive white Gaussian noise at the \( m \)th tagged BS.

During the uplink information transfer phase, all UEs transmit their mutually orthogonal pilot sequences simultaneously for channel estimation. By using maximal ratio combining (MRC) on the BS, \( g_{n_{0}} = h_{m,n_{0}}^{H} \), then the corresponding received signal at the \( m \)th BS from the UE \( n_{0} \) can be expressed as

\[
Z_{n_{0}} = h_{m,n_{0}}^{H} Y_{n_{0}} = \sqrt{\frac{P_{T}}{d_{m,n_{0}}} h_{m,n_{0}}^{H} \| h_{m,n_{0}} \|^{2}} x_{n_{0}} \\
+ \sum_{i \neq n_{0}} \sqrt{\frac{P_{T}}{d_{m,i}^{\alpha}} h_{m,i}^{H} h_{m,i} x_{i}} + h_{m,n_{0}}^{H} n. \tag{2}
\]

Then, the signal to interference plus noise ratio (SINR) of the \( n_{0} \)th UE connected to the \( m \)th BS can be expressed as

\[
\gamma_{m,n_{0}} = \frac{P_{T} \frac{d_{m,n_{0}}^{-\alpha}}{h_{m,n_{0}}^{H} h_{m,n_{0}}}}{I_{m,n_{0}} + \sigma^{2}}, \tag{3}
\]

where \( I_{m,n_{0}} = \sum_{i \neq n_{0}} P_{T} \frac{d_{m,i}^{-\alpha}}{h_{m,i}^{H} h_{m,i}^{H}} \) represents the sum of interference from the other UEs who select the BSs from the same layer, and \( |\mathcal{V}| \) means the number of users served by all the macro BSs or small BSs.
C. POWER CONSUMPTION MODEL

A conventional linear power consumption model [30] is adopted for UEs. The power consumption of each UE is composed of two parts, i.e., static power consumption and dynamic power consumption. The static power is consumed in running the circuit components, such as converters, mixers, filters and so on [31]. For the dynamic power consumption, it is deemed as the transmit power consumption. By assuming that the static power consumption of UE is $P_{CU}$ and the transmit power consumed by the $n$th UE is $P_n^T$, the total power consumption $P_{sum}^n$ of the $n$th UE can be expressed as

$$P_{sum}^n = P_{CU} + \xi P_n^T,$$  \hspace{1cm} (4)

where $\xi$ represents the power consumption coefficient.

Based on the previous description, the EE (bits/Joules) of the $n$th UE who selects the $m$th BS can be written as

$$\eta_{m,n} = \frac{r_{m,n}}{P_{sum}^n},$$  \hspace{1cm} (5)

where $r_{m,n}$ is the achievable data rate of the $n$th UE associated with the $m$th BS. According to Shannon equation, $r_{m,n}$ can be expressed as

$$r_{m,n} = B \log_2 (1 + \gamma_{m,n}).$$  \hspace{1cm} (6)

III. PROBLEM FORMULATION

In this section, we formulate the joint power control and user association problems to achieve fair energy efficient resource management by considering proportional fairness and max-min fairness, respectively. For the user association problem, the user-associated variables are defined by the integer binary $x_{m,n}$ as follows

$$x_{m,n} = \begin{cases} 1, & \text{if the } n\text{th UE selects the } m\text{th BS}, \\ 0, & \text{otherwise}. \end{cases}$$  \hspace{1cm} (7)

Then, the user association matrix for all UEs is defined as $X$. For the power control problem, we denote the transmit power vector of UE $p = [P_1^T, P_2^T, \ldots, P_N^T]$. Based on the description of $x_{m,n}$, the data rate of the $n$th UE is

$$R_{m,n}(X, p) = \sum_{m=1}^{M} x_{m,n} r_{m,n}.$$  \hspace{1cm} (8)

A. PROPORTIONAL FAIRNESS PROBLEM FORMULATION

Maximizing the proportional fairness utility of EE by jointly designing the user association and power control problems is investigated in this subsection. Since the logarithm function is concave [32], it is widely used to construct the utility function when considering fairness. The utility of the $n$th UE is defined as the logarithm of its achievable EE and it can be given as $\ln(R_{m,n}/P_{sum}^n)$. The optimization object is the sum of users’ utilities and it can be expressed as

$$\sum_{n=1}^{N} \ln \left( \sum_{m=1}^{M} x_{m,n} r_{m,n} / P_{sum}^n \right) = \sum_{n=1}^{N} \sum_{m=1}^{M} x_{m,n} \ln(r_{m,n}/P_{sum}^n).$$  \hspace{1cm} (8)

So, by setting $\mu_{m,n} = \ln(r_{m,n}/P_{sum}^n)$, the optimization problem can be formulated as

$$\mathcal{P}1: \max_{X,p} \sum_{n=1}^{N} \sum_{m=1}^{M} x_{m,n} \mu_{m,n},$$  \hspace{1cm} s. t. C1: $0 \leq P_n^T \leq P_{MAX}$, $n = 1, 2, \ldots, N$, C2: $\sum_{m} x_{m,n} \gamma_{m,n} \geq \gamma_{th}$, $m = 1, 2, \ldots, M, \hspace{0.5cm} n = 1, 2, \ldots, N$, C3: $x_{m,n} \in \{0, 1\}$, C4: $\sum_{m} x_{m,n} = 1$, $n = 1, 2, \ldots, N.$  \hspace{1cm} (9)

In the above formulated problem $\mathcal{P}1$, the constraint C1 means that the transmit power of each UE cannot exceed the given maximum transmit power. C2 ensures that the QoS requirement for each UE can be satisfied, i.e., the predefined minimum SINR $\gamma_{th}$ is guaranteed. C3-C4 help to make sure that each UE can associate with only one BS.

B. MAX-MIN FAIRNESS PROBLEM FORMULATION

In this subsection, we aim to study the max-min fairness by maximizing the minimum EE among all users during the uplink communication through user association and power control. Max-min fairness can sufficiently improve the performance of users in the worst case and thus lead to a high level of fairness [33], [34]. The corresponding max-min fairness problem is formulated as follows

$$\mathcal{P}2: \max_{X,p} \min_{n} \frac{R_{m,n}(X, p)}{P_{sum}^n(p)},$$  \hspace{1cm} s. t. C1 - C4,  \hspace{1cm} (10)

where all constraints are the same as shown in $\mathcal{P}1$.

IV. SOLUTION OF RESOURCE ALLOCATION PROBLEMS

In the following, the solution process of the formulated EE proportional fairness and max-min fairness optimization problems are presented, respectively.

A. THE SOLUTION FOR PROPORTIONAL FAIRNESS PROBLEM

From $\mathcal{P}1$, we can find that the user association and power control problems are interacted with each other, and the problem is a mixed-integer and non-convex problem. It is difficult to obtain the global optimal solution. In order to find a suboptimal solution, an alternating method is applied. The main idea of the method is that $\mathcal{P}1$ is divided into two subproblems. For the user association subproblem, a dual decomposition method is applied to obtain the user association matrix under the fixed transmit power of each UE, then the optimized transmit power is obtained by the Newton method with fixed user association matrix for the power control subproblem. Finally, through iterations, the user association matrix $X$ and transmit power $p$ are constantly updated until convergence.
According to the above analysis, \( \mathcal{P}1 \) can be decomposed into two subproblems, i.e., user association subproblem \( \mathcal{P}1.1 \) and power control subproblem \( \mathcal{P}1.2 \), as follows

\[
\mathcal{P}1.1: \quad \max_{x} \sum_{n=1}^{N} \sum_{m=1}^{M} x_{m,n} h_{m,n}^{*} \quad \text{s. t.} \quad C2 - C4, \quad (11)
\]

\[
\mathcal{P}1.2: \quad \max_{p} \sum_{n=1}^{N} \sum_{m=1}^{M} x_{m,n} h_{m,n}^{*} \quad \text{s. t.} \quad C1. \quad (12)
\]

For \( \mathcal{P}1.1 \), in order to maximize the logarithmic utility of user EE under fixed transmit power, the dual decomposition method is adopted. According to the nature of the log function, we can get the following formula

\[
\sum_{n=1}^{N} \sum_{m=1}^{M} x_{m,n} \ln(r_{m,n}) - \sum_{n=1}^{N} \sum_{m=1}^{M} x_{m,n} \ln(P_{n}^{\text{sum}}). \quad (13)
\]

Then the Lagrangian function can be obtained as below

\[
L_{1} = (\{x_{m,n}\}, \{P_{n}^{\text{sum}}\}, \alpha) = \sum_{n=1}^{N} \sum_{m=1}^{M} x_{m,n} \ln(r_{m,n}) - \sum_{n=1}^{N} \sum_{m=1}^{M} x_{m,n} \ln(P_{n}^{\text{sum}}) + \sum_{n=1}^{N} a_{n}(x_{m,n} - \gamma_{m,n}), \quad (14)
\]

where \( \alpha = [a_{1}, a_{2}, \ldots, a_{n}, \ldots, a_{N}] \) and \( a_{n} \) is the nonnegative lagrange multiplier. Then, we can get the Lagrangian dual function and the Lagrangian dual problem as follows

\[
g_{1}(\alpha) = \max_{x} L_{1}(\{x_{m,n}\}, \{P_{n}^{\text{sum}}\}, \alpha), \quad (15)
\]

and

\[
\min g_{1}(\alpha), \quad \text{s. t.} \quad \alpha \geq 0. \quad (16)
\]

By solving the dual problem, the best BS associated with the \( n \)th UE as follows

\[
m^{*} = \max_{m} (\ln(r_{m,n}) + a_{n}(t_{1})y_{m,n} - \ln(P_{n}^{\text{sum}})). \quad (17)
\]

It means that the \( n \)th UE will associate with the \( m \)th BS, i.e.

\[
x_{m,n} = \begin{cases} 1, & \text{if } m = m^{*}, \\ 0, & \text{if } m \neq m^{*}. \end{cases} \quad (18)
\]

Moreover, we can not get a closed-form expression of the variable \( \alpha \), because the Lagrangian dual function is not a differentiable function. In order to update the nonnegative Lagrangian multiplier \( \alpha = [a_{1}, a_{2}, \ldots, a_{n}, \ldots, a_{N}] \), the subgradient approach is used, i.e.

\[
a_{n}(t_{1} + 1) = a_{n}(t_{1}) - \delta_{a}(t_{1})(\sum_{m} x_{m,n}(t_{1})y_{m,n} - y_{m,n}). \quad \forall n, \quad (19)
\]

where \( \delta_{a}(t_{1}) \) is the step size of subgradient method.

With the fixed user association matrix, the optimal transmit power of each UE will be obtained by solving the problem \( \mathcal{P}1.2 \). In the problem \( \mathcal{P}1.2 \), it is difficult to obtain the exact solution for the variable \( p \). In order to solve the transmit power optimization problem, we use the Newton method to search the suboptimal points [35]. Rewriting the objective function as \( f(P_{n}^{T}) \), i.e.

\[
f(P_{n}^{T}) = \sum_{n=1}^{N} \sum_{m=1}^{M} x_{m,n} \ln \left( \frac{1 + \frac{P_{n}^{T}||h_{m,n}||2 - a}{P_{n}^{\text{CU}} + \sigma^{2}P_{n}^{T}}}{P_{n}^{\text{CU}} + \sigma^{2}P_{n}^{T}} \right), \quad (20)
\]

Before taking the derivative of \( f(P_{n}^{T}) \), the non-target user’s SINR \( \gamma_{m,n^{'}} \) is expressed as

\[
\gamma_{m,n^{'}} = \frac{P_{n}^{T}}{I_{m,n'} + \sigma^{2}} ||h_{m,n^{'}}||^{2}, \quad (21)
\]

where

\[
I_{m,n'} = P_{n}^{T} ||h_{m,n'}||^{2} d_{m,n'}^{a} + \sum_{i \neq n', m} P_{n}^{T} ||h_{m,n}||^{2} d_{m,i}^{a}. \quad (22)
\]

Then, in order to obtain the searching direction of transmit power, the first-order and the second-order partial derivatives of \( f \) with respect to variable \( P_{n}^{T} \) are derived in (23) and (24), as shown at the bottom of the next page, respectively.

In equation (24), the variables \( C \) and \( \gamma_{m,n^{'}} \) are defined as

\[
C = (\frac{h_{m,n}^{H}h_{m,n}}{||h_{m,n}||^{2}})^{2} d_{m,n}^{a}/(P_{n}^{T}||h_{m,n}||^{2})^{2} \quad \text{and} \quad \gamma_{m,n^{'}} = -\gamma_{m,n^{'}}^{2} \cdot C, \quad \text{respectively}. \]

A line search method is adopted to update the variable \( P_{n}^{T} = [P_{1}^{T}, P_{2}^{T}, \ldots, P_{n}^{T}] \) and it depends on the direction \( \Delta P_{n}^{T} \) and the step length \( \delta_{P}(t) \), i.e.

\[
P_{n}^{T}(t + 1) = P_{n}^{T}(t) + \delta_{P}(t)\Delta P_{n}^{T}. \quad (25)
\]

A back-tracking approach is applied to find the step size \( \delta_{P}(t) \) and the incremental updating direction Newton step is \( \Delta P_{n}^{T} = \frac{g_{n}^{T}}{||g_{n}||^{2}} \). Therefore, by seeking the step length and direction, the power control subproblem can be solved.

In conclusion, to solve the proportional fairness optimization problem, the alternating iterative method for joint user association and power control is summarized in Algorithm 1. By applying the dual decomposition and Newton method, the two subproblems \( \mathcal{P}1.1 \) and \( \mathcal{P}1.2 \) can be solved alternately until convergence.

\textit{Proposition 1:} With a given fixed transmit power vector, the subgradient method will converge to the optimum of its dual problem \( g_{1}(\cdot) \).
Proof: The first derivative of $g_1(\cdot)$ subject to $a_n$ can be expressed as

$$
\frac{\partial g_1(a)}{\partial a_n} = \sum_{n=1}^{N} \sum_{m=1}^{M} x_{m,n}(a_n) \gamma_{m,n} - \gamma_b, \tag{26}
$$

where $\sum_{n=1}^{N} \sum_{m=1}^{M} x_{m,n}(a_n)$ is bounded because $x_{m,n} \in \{0, 1\}$. Then we can get the following formula

$$
\sup(\frac{\partial g_1(a)}{\partial a_n}) \leq s, \tag{27}
$$

where $s$ is a scalar. Through the above analysis, the necessary conditions of the convergence for the problem can be proofed as in [36].

Algorithm 1

Iterative Gradient Algorithm With Proportional Fairness

1: Initialize $P_{n}^{\text{MAX}}$.
2: Initialize the Lagrange multipliers $a_n$ to zero.
3: Set $t = 0$.
4: Initialize $P_n^T$.
5: repeat
6: User association
7: for $n = 1 : M$ do
8: for $m = 1 : N$ do
9: Calculate the power consumption $P_{n}^{\text{sum}}$.
10: Calculate $m^* = \arg \max_m (\ln(r_{m,n}) + a_n(t) \gamma_{m,n} - \ln(P_{n}^{\text{sum}}))$.
11: Use the $m^*$ to update $x_{m,n}$.
12: Update $a_n$.
13: end for
14: end for
15: Power control
16: Update $P_n^T(t + 1)$ with fixed $x_{m,n}$ until $P_n^T(t + 1)$ converges.
17: $t = t + 1$.
18: until Convergence.

In Algorithm 1, the convergence of user association with fixed transmit power $p$ is proved in proposition 1, while for the power control subproblem, the convergence is proved in [35]. Since $\mathcal{P}1.1$ and $\mathcal{P}1.2$ are maximizing the same objective function, i.e., $\sum_{n=1}^{N} \sum_{m=1}^{M} x_{m,n} \alpha_{m,n}$, Algorithm 1 is guaranteed to converge as in [37].

B. THE SOLUTION FOR MAX-MIN FAIRNESS PROBLEM

In this section, the formulated optimization problem for max-min fairness is solved by utilizing the fractional programming and the lagrangian dual decomposition methods.

Through the analysis of $\mathcal{P}2$, similar to problem $\mathcal{P}1$, it is a mixed-integer nonlinear programming problem and it is difficult to obtain the global optimal solution. In order to solve it, nonlinear fractional programming theory [38] is applied and a suboptimal solution is obtained. For notational simplicity, we denote the feasible region of $C1 \sim C4$ by $\bar{\mathcal{G}}$ and define $\varphi^*$ as the optimal value of EE which is given by

$$
\varphi^* = \max_{(X,p) \in \bar{\mathcal{G}}} \min_n R_{m,n}(X, p) = \min_n R_{m,n}(X^*, p^*), \tag{28}
$$

where $\{X^*, p^*\}$ is the optimal solution of the problem $\mathcal{P}2$. In the formula (28), $R_{m,n}(X^*, p^*) \geq 0$ and $P_{n}^{\text{sum}}(p^*) > 0$. Then, we have the following proposition.

Proposition 2: The optimal solution $\{X^*, p^*\}$ is achieved if and only if

$$
\max_{(X,p) \in \bar{\mathcal{G}}} \min_n [R_{m,n}(X, p) - \varphi^* P_{n}^{\text{sum}}(p)]
= \min_n [R_{m,n}(X^*, p^*) - \varphi^* P_{n}^{\text{sum}}(p^*)] = 0. \tag{29}
$$

Proof: The necessary and sufficient condition for $\varphi^*$ being the optimal solution of the fractional programming problem has been proved in [39]. There exists an equivalent objective function, e.g., $\min_n [R_{m,n}(X^*, p^*) - \varphi^* P_{n}^{\text{sum}}(p^*)]$. As a result, we focus on solving the equivalent problem.

An alternating method is applied to solve the proposed optimization problem $\mathcal{P}2$. Similar to the proportional fairness, problem $\mathcal{P}2$ is decomposed into two subproblems, i.e., power control subproblem and user association subproblem. For the power control subproblem, SCA method is applied. And for the user association subproblem, the convexity of the subproblem is analyzed and a Lagrangian dual method is employed to solve it. The main solution process is summarized in Algorithm 3.
The power control subproblem can be formulated as

$$P2.1: \max_{p_n} \min_{y_n} \left[ R_{m,n}(p) - \phi P_{\text{sum}}(p) \right],$$

s.t. $C1: 0 \leq P_n^T \leq P_{n,T}^{\text{MAX}}, \ n = 1, 2, \ldots, N$

$$C2: \gamma_{m,n} \geq \gamma_{th}, \ n = 1, 2, \ldots, N, \ C3: \alpha_{m,n} \log_2(\gamma_{m,n}) + \beta_{m,n} - \varphi(P_{CU} + \xi P_n^T) \geq y.$$  

(30)

Through the relaxation of the subproblem, SCA approach is applied to achieve a lower bound of the user's rate, i.e.

$$\log_2(1 + \gamma_{m,n}) \geq \alpha_{m,n} \log_2(\gamma_{m,n}) + \beta_{m,n}.$$  

(31)

where $\alpha_{m,n} = \gamma_{m,n}/(1 + \gamma_{m,n})$ and $\beta_{m,n} = \log_2(1 + \gamma_{m,n}) - (\gamma_{m,n}/(1 + \gamma_{m,n})) \log_2 \gamma_{m,n}$. $\gamma_{m,n}$ is the SINR of the $n$th user over $m$th BS in the previous iteration.

By using an auxiliary variable $y$, we have the following optimization problem

$$P2.1.1: \max_{y, p_n} y,$$

s.t. $C1: 0 \leq P_n^T \leq P_{n,T}^{\text{MAX}}, \ n = 1, 2, \ldots, N$

$$C2: \gamma_{m,n} \geq \gamma_{th}, \ m = 1, 2, \ldots, M, \ n = 1, 2, \ldots, N, \ C3: \alpha_{m,n} \log_2(\gamma_{m,n}) + \beta_{m,n} - \varphi(P_{CU} + \xi P_n^T) \geq y.$$  

(32)

For $P2.1.1$, it is a non-convex problem for the two variables $y$ and $p$. By adopting the transformation $P_n^T = \exp(P_n^T)$, it is converted into the following form

$$P2.1.2: \max_{y, p_n} y,$$

s.t. $C1: \exp(P_n^T) \leq P_{n,T}^{\text{MAX}}, \ n = 1, 2, \ldots, N$

$$C2: \tilde{\gamma}_{m,n} \geq \gamma_{th}, \ m = 1, 2, \ldots, M, \ n = 1, 2, \ldots, N, \ C3: \varphi(P_{CU} + \xi \exp(P_n^T)) - \alpha_{m,n} \log_2(\tilde{\gamma}_{m,n}) + \beta_{m,n} + y \leq 0, \ n = 1, 2, \ldots, N.$$  

(33)

where $\tilde{\gamma}_{m,n} = \sum_{\substack{i \in \mathcal{N} \setminus \{i\}}} \exp(P_i^T) d_{m,i}^{-\alpha} ||h_{m,i}||^2 / \sum_{i = 1}^{\mathcal{N}} \exp(P_i^T) d_{m,i}^{-\alpha} + \sigma^2$.

**Proposition 3:** The problem of $P2.1.2$ is convex.

**Proof:** It can be easily derived that the objective function $y$ is linear, and the constraints $C1$ is convex.

- Convexity illustration of $C2$

  Taking the logarithms of both sides of $C2$ with natural base

  $$\ln(\gamma_{th}) - \ln(\tilde{\gamma}_{m,n}) = \ln(\gamma_{th}) - \tilde{P}_n^T - \ln(d_{m,n}^{-\alpha} ||h_{m,n}||^2)$$

  $$+ \ln(\sigma^2) + \sum_{i = 1, i \neq n}^{\mathcal{N}} \sum_{\mathcal{N} \setminus \{i\}} \exp(P_i^T) ||h_{m,i}||^2 d_{m,i}^{-\alpha}.$$  

(34)

Since the second term is linear and the forth term is convex (the log-sum-exp is convex as in [35]), $C2$ is convex.

- Convexity illustration of $C3$

  The left hand side of $C3$ can be rewritten as

  $$f_1(P_n^T) = \varphi(P_{CU} + \xi \exp(P_n^T)) - (\alpha_{m,n} \log_2(\tilde{\gamma}_{m,n}) + \beta_{m,n} + y).$$  

(35)

where

$$\log_2(\tilde{\gamma}_{m,n}) = \left[ \tilde{P}_n^T + \ln(d_{m,n}^{-\alpha} ||h_{m,n}||^2) \right] - \ln(\sigma^2) - \ln(\sum_{i = 1, i \neq n}^{\mathcal{N}} \exp(P_i^T) ||h_{m,i}||^2 d_{m,i}^{-\alpha}) / \ln(2).$$  

(36)

Then the second-order derivative of $f_1$ with respect to $P_n^T$ can be expressed as

$$\frac{\partial^2 f_1(P_n^T)}{\partial P_n^T} = \varphi \exp(P_n^T) > 0.$$  

(37)

Since the second-order derivative of $f_1(P_n^T)$ is greater than zero, $C3$ is convex.

Finally, standard convex maximization problem of the new variable $P_n^T$ is obtained.

To tackle the problem of $P2.1.2$, the Lagrangian function is defined as

$$L_2 = \langle \tilde{p}, y, b, c, d \rangle$$

$$= y + \sum_{n = 1}^{\mathcal{N}} b_n R_{m,n}(\tilde{p}) - \varphi P_{\text{sum}}(\tilde{p}) - y$$

$$+ \sum_{n = 1}^{\mathcal{N}} c_n [P_{n,T}^{\text{MAX}} - \tilde{p}_n^T] + \sum_{n = 1}^{\mathcal{N}} d_n (\tilde{\gamma}_{m,n} - \gamma_{th})$$

$$= y(1 - \sum_{n = 1}^{\mathcal{N}} b_n) + \sum_{n = 1}^{\mathcal{N}} (c_n P_{n,T}^{\text{MAX}} - b_n \varphi P_{CU} - d_n \gamma_{th})$$

$$+ \sum_{n = 1}^{\mathcal{N}} (b_n R_{m,n}(\tilde{p}) - \tilde{p}_n^T) (c_n + b_n \xi \varphi + d_n \tilde{\gamma}_{m,n}).$$  

(38)

where $b, c, d$ are the nonnegative Lagrangian multipliers and $\tilde{p} = [\tilde{p}_1^T, \tilde{p}_2^T, \ldots, \tilde{p}_n^T]$. Then, the dual function and dual problem can be written as follows

$$g_2(b, c, d) = \max_{\tilde{p}} L_2(\tilde{p}, y, b, c, d),$$  

(39)

and

$$\min_{b, c, d} g_2(b, c, d),$$  

s.t. $b, c, d \geq 0.$  

(40)

It is difficult to directly obtain the optimal variable $\tilde{p}_n^T$. Then a gradient-based adaption method is applied to get the suboptimal solution of $\tilde{p}_n^T$ with the known Lagrangian multipliers. The update formula for $\tilde{p}_n^T$ and $y$ are as follows

$$\tilde{p}_n^T(l_i) = \tilde{p}_n^T(l_i - 1) - \frac{\partial L_2}{\partial \tilde{p}_n^T},$$  

(41)

$$y(l_i) = y(l_i - 1) - \frac{\partial L_2}{\partial y},$$  

(42)
where $t$ is the small step size.

$$\frac{\partial L_2}{\partial y} = 1 - \sum_{n=1}^{N} b_n, \quad (43)$$

$$\frac{\partial L_2}{\partial F_n} = b_n \alpha_{m,n} - e^{\tilde{b}_n}(c_n + b_0 \xi \phi) + d_n e^{\tilde{b}_n} \frac{d_{m,n}^\alpha \|h_{m,n}\|^2}{I_{m,n} + \sigma^2}, \quad (44)$$

where

$$\tilde{I}_{m,n} = \sum_{i=1, i \neq n}^{[N]} \text{exp}(\tilde{P}_i) \frac{\|h^H_{m,n} h_{m,n} \|^2}{\|m_n\|^2} d_{m,i}. \quad (45)$$

Once the optimal $\tilde{P}_n^T \in \tilde{P}$ are obtained, the subgradient method is used to update the Lagrangian multipliers, i.e.

$$b_n(l_1 + 1) = b_n(l_1) - \delta b_n ((\alpha_{m,n} \log_2(\gamma_{m,n})) + \beta_{m,n})$$

$$-\phi(P_{CU} + \xi e^{\tilde{b}_n} - y), \quad \forall n, \quad (46)$$

$$c_n(l_1 + 1) = c_n(l_1) - \delta c_n ((P_n \max - e^{\tilde{b}_n}), \quad \forall n, \quad (47)$$

$$d_n(l_1 + 1) = d_n(l_1) - \delta d_n ((\gamma_{m,n} - \gamma_h), \quad \forall n, \quad (48)$$

where $\delta b_n(l_1) = \delta c_n(l_1) = \delta d_n(l_1) = \frac{0.01}{\sqrt{n}}$.

**Algorithm 2 Iterative Power Control Algorithm Procedure**

1. Initialize iteration index $l_1 = 1$, the maximum number of iterations $l_{\max}$ and a small $\epsilon > 0$.
2. Initialize the variables $\tilde{p}(1), y(1), b(1), c(1), d(1)$.
3. repeat
   4. Set $\tilde{p}(1) = \tilde{p}(1), y(1) = y(1)$.
   5. Set $l_1 = l_1 + 1$.
   6. Update $\tilde{p}(l_1)$ and $y(l_1)$ with Lagrangian multipliers $b(l_1 - 1), c(l_1 - 1), d(l_1 - 1)$ as in (41) and (42) until convergence.
   7. Update Lagrangian multipliers $b, c, d$ according to (46)-(48).
   8. if $|b(l_1) - b(l_1 - 1)| < \epsilon, |c(l_1) - c(l_1 - 1)| < \epsilon$ and $|d(l_1) - d(l_1 - 1)| < \epsilon$ then
      9. Return $\tilde{p} = \tilde{p}(l_1), y = y(l_1)$.
     10. break
     11. end if
   12. until $l_1 > l_{\max}$.
13. Output the final $\tilde{p}$.

**Proposition 4**: For the problem $\mathcal{P}_2.1$, under a fixed user association matrix, the SCA method produces a monotonically increasing objective and converges to the optimal solution $\tilde{p}$.

**Proof**: After a certain number of iterations $l_1$, a feasible solution of the $n$th user’s transmit power $\tilde{p}^T_{m,l_1}$ can be obtained. For the iterations $l_1$ and $l_1 + 1$, there is the following relationship

$$R_{m,n}(\tilde{p}^{(l_1)}) \leq R_{m,n}(\tilde{p}^{(l_1 + 1)}),$$

where (a) is a consequence of the tightening in $(\tilde{p}^{(l_1)}, \alpha^{(l_1 + 1)}, \beta^{(l_1 + 1)})$ about SCA. The first inequality (b) follows from the optimal objective function value at $\tilde{p}^{l_1 + 1}$, while inequality (c) is the approximate characteristic of logarithmic functions. Consequently, after each iteration, the value of $R_{m,n}(\tilde{p})$ either improves or stays unaltered as the previous iteration value until convergence.

As shown in Algorithm 2, the transmit power of each UE can be obtained by the lagrangian dual decomposition method. Given the optimal transmit power, the subproblem of finding the optimal user association $X$ is given by

$$\mathcal{P}_2.2 : \max \min R_{m,n}(X),$$

$$X \in \mathbb{R}^n,$$

$$s. t. C_2 - C_4. \quad (50)$$

Since the variable $P_n^T$ is given, the $p_{\text{sum}}$ can be obtained and the denominator of formula (50) is a constant. By setting $R_{m,n}(X) = \sum_{m=1}^{M} x_m \log_2(1 + \gamma_{m,n})$, the problem of $\mathcal{P}_2.2$ can be turned into the following problem

$$\mathcal{P}_2.2.1 : \max \min R_{m,n}(X),$$

$$X \in \mathbb{R}^n,$$

$$s. t. C_2 - C_4. \quad (51)$$

In order to solve the $\mathcal{P}_2.2.1$, the variable $x_{m,n}$ is relaxed to $[0, 1]$ and $x_{m,n}$ is equivalently transformed as $e^{\tilde{x}_{m,n}}$, i.e., $x_{m,n} \triangleq e^{\tilde{x}_{m,n}}, n = 1, \ldots, N$. In addition, a constraint $\hat{C}_4$ is added to guarantee the rate requirement of each user and an auxiliary variable $z$ is introduced. Then the detailed description is as follows

$$\mathcal{P}_2.2.2 : \max \ z,$$

$$X, z,$$

$$s. t. \hat{C}_1 : \sum_{m} e^{\tilde{x}_{m,n}} \tilde{y}_{m,n} \geq \gamma_h,$$

$$\hat{C}_2 : e^{\tilde{x}_{m,n}} \in [0, 1], \quad m = 1, 2, \ldots, M,$$

$$n = 1, 2, \ldots, N,$$

$$\hat{C}_3 : \sum_{m} e^{\tilde{x}_{m,n}} \leq 1, \quad n = 1, 2, \ldots, N,$$

$$\hat{C}_4 : \sum_{m=1}^{M} e^{\tilde{x}_{m,n}} \log_2(1 + \tilde{y}_{m,n}) \geq e^{z},$$

$$m = 1, 2, \ldots, M, \quad n = 1, 2, \ldots, N. \quad (52)$$

where $\tilde{y}_{m,n} = \frac{\tilde{P}_n^{T_{m,l_1}} \|h_{m,n}\|^2}{\sum_{m=1, m \neq n}^{[N]} \|h_{m,n}\|^2 \|h_{m,n}\|^2 I_{m,n} + \sigma^2}.$

**Proposition 5**: The problem of $\mathcal{P}_2.2.2$ is convex.

**Proof**: See Appendix.
Since the transformed problem $\mathcal{P}2.2.2$ is convex, the Lagrangian dual method is used to solve it and we do not repeat the specific solution process of the user association subproblem by the reason that it is similar to the power control process.

**Algorithm 3 Iterative User Association and Power Control Algorithm With Max-Min Fairness**

1. Initialize $\phi(l)$, and a small $\varepsilon > 0$.
2. Set $l = 1$.
3. Initialize any suitable user association matrix.
4. repeat
   5. repeat
      6. Solve power control subproblem $\mathcal{P}2.1$ with fixed user association matrix.
      7. Solve user association subproblem $\mathcal{P}2.2$ with fixed power control determinant.
   8. until Convergence
   9. if $|\min_{n}[R_{m,n}(X^*, P^*)] - \phi^*P_{\text{sum}}(p^*)| < \varepsilon$ then
      10. $X_{\text{output}}^* = \{X^*(l), P^*(l)\}$,
      11. $\phi_{\text{output}}^* = \min_n R_{m,n}(X^*, P^*)$.
      12. break
   13. else
      14. Calculate $\phi^*(l + 1) = \min_n R_{m,n}(X^*, P^*)$.
      15. $l = l + 1$.
   16. end if
   17: until $l > l_{\text{max}}$.
18. Output the final $\{X^* \text{ and } P^*\}$.

**Proposition 6:** After each iteration, the Algorithm 3 improves the value of objective function in (10) until convergence.

**Proof:** Firstly, for the iterative algorithm presented in Algorithm 2, i.e., the power control subproblem, we obtain the transmit power of iteration $l + 1$, i.e., $p(l + 1)$ under the user association matrix $X(l)$. Through the Proposition 5, we will have $R_{m,n}(X(l), p(l)) \leq R_{m,n}(X(l), p(l + 1))$. Furthermore, by fixing the transmit power $p(l + 1)$ and obtaining the user association of iteration $X(l)$, we will have $R_{m,n}(X(l), p(l + 1)) \leq R_{m,n}(X(l + 1), p(l + 1))$ for the nature of the exponential function in the function of $R_{m,n}(X)$. After each iteration, the user association problem improves the objective function $R_{m,n}(X)$. Therefore, at the last iteration, the suboptimal solution of $X^*$ and $P^*$ are obtained. Thus, the duality gap $\text{Gap} = R_{m,n}(X^*, P^*) - R_{m,n}(X(l), p(l))$ will gradually decrease after each iteration. Then, the duality gap will be minimal which ensures the convergence at the last iteration. Therefore, the objective function is bounded above due to the finite set of the total power consumption. It proves the convergence of the proposed Algorithm 3.

**V. COMPLEXITY ANALYSIS**

In the following, the computational complexity for the proposed Algorithm 1 and Algorithm 3 are studied, respectively. In Algorithm 1, the user association stage needs $\mathcal{O}(M \times 2N)$ operations to establish the associated relationship between users and BSs at each iteration, so the total computation complexity of user association stage is $\mathcal{O}(t_1(M \times 2N))$, where $t_1$ is the number of iteration taken by the dual decomposition method. Then in the power control stage, the computational complexity in Newton’s method mainly focuses on computing the updating direction Newton step, so the complexity of the proposed power control stage needs $\mathcal{O}(MN)$ operations at each iteration. The computational complexity of Algorithm 1 is $\mathcal{O}(t_2(2MNt_1 + MN))$, where $t_2$ is the number of iterations when Algorithm 1 converges. In Algorithm 3, for the power control subproblem, the computational complexity of the SCA method needs $\mathcal{O}(4Nl_2)$, where $l_2$ is the number of iterations when subproblem $\mathcal{P}2.2$ converges. Therefore, the computational complexities of the max-min fairness is $\mathcal{O}(l(4Nl_1 + 4MNl_2))$, where $l$ is the number of iterations when Algorithm 3 converges. Through the above analysis, the computational complexity of Algorithm 3 is higher than the Algorithm 1.

**VI. SIMULATION RESULTS AND ANALYSIS**

In this section, the EE performance of the proposed algorithms is evaluated through simulations. A two-tier HetNet is considered, where the spatial distribution of the macro and micro BSs follows PPP with different densities. The performance of the proportional fairness and max-min fairness are compared. The detailed simulation parameters are shown in Table 1.

| Parameter | Description | Value |
|-----------|-------------|-------|
| $R$       | Cell radius | 500 m |
| $\sigma^2$| Additive noise power | $-174 \text{ dBm}$ |
| $\alpha$  | Path-loss exponent | 4 |
| $P_{\text{GU}}$ | The static power of UE | $0.01W$ |
| $\gamma_{\text{th}}$ | SINR requirement of UEs | $-10\text{dB}$ |
| $P_{\text{MAX}}^n$ | Maximum transmit power of UEs | $23 \text{ dBm}$ |

We first evaluate the convergence of the proposed algorithms. As shown in Fig. 2, the Algorithm 1 has a satisfactory convergence to solve the proportional fairness problem. We can see that the achievable average EE gradually increases with the number of iterations, and finally stabilizes and reaches the optimal point. In addition, the convergence of Algorithm 3 is shown in Fig. 2b. From the Fig. 2b, we can see that the EE decreases in the first iteration. The reason is that the initial condition of the iterative optimization algorithm is not in the feasible domain and all variables enter the feasible domain after the first iteration. Then the average EE begins to increase monotonically until convergence. The maximum average EE is achieved within about 6 iterations.
In order to distinguish between the two fairness criteria, we use the letter P to denote the proportional fairness and use the letter M to represent max-min fairness. To evaluate the performance of the proposed Algorithm 1 and Algorithm 3, another three resource allocation schemes, i.e., TUA with PPC, PUA with MP, TUA with MP are included as reference. For TUA with PPC scheme, a user chooses the traditional user association scheme (i.e., the nearest BS) and adopts the proposed power control algorithm. For PUA with MP, users adopt the proposed user association scheme and transmit the maximum power. For TUA with MP scheme, users choose their nearest BS and transmit the maximum power. Then, the letter P or M is added to each scheme to differentiate the Algorithm 1 and Algorithm 3.

Fig. 3 compares the average EE of all users in the Algorithm 1 and Algorithm 3 with user density for different resource allocation methods. Simulation results show that the proposed Algorithm 1 and Algorithm 3 obtain the best performance regarding the average EE of all the users through comparison with the other three schemes. This is because the proposed algorithms optimize not only the user association but also the transmit power. Furthermore, we can find that the max-min fairness has lower EE than the proportional fairness, this is because max-min fairness considers maximizing the minimum EE of users. At last, as the density of users increases gradually, the trend in each algorithm decreases. This is because high user’s density will incur severe interference among UEs.

Furthermore, we also investigate the average EE of all the users for different micro BS density. In the simulation, the density of the user is \(2 \times 10^{-5} \text{m}^{-2}\) and the results are shown in Fig. 4. From the simulation results, it can be seen that as the density of micro BSs increases, the average EE of all the users first increases and then tends to be smooth when the BS density is \(4 \times 10^{-6} \text{m}^{-2}\), and the growth trend is gradually slowing down. This is because the number of users is certain, and there is not much room to increase the average EE of all the users for more BSs. Instead, the resources are wasted because of the increase in the construction of the BSs. Therefore, in the HetNets, the number of deployed BSs should choose a suitable range.
In this paper, the user’s minimum EE is defined as a criterion for evaluating fairness. As shown in Fig. 5, Algorithm 3 has much higher EE than Algorithm 1. This is because the max-min fairness has more fairness in the HetNets system and may allocate the considerable part of resources to the UE who has the weak performance on EE. As the number of micro BS increases, the user’s EE gradually stabilizes, which is consistent with the conclusion of Fig. 4.

In addition, the number of users served by the BSs for different fairness is studied in Fig. 6. The density of user and macro BS are $10 \times 10^{-5} \text{m}^{-2}$ and $2 \times 10^{-6} \text{m}^{-2}$, respectively. From the simulation results, it can be seen that as the ratio of the density of the micro BS to the macro BS increases, the number of users served by micro BS gradually increases. The micro BS can effectively offload the traffic requirement from the macro BS. Through the comparison of the two kinds of fairness, the micro BS serves more users by the proportional fairness. This is because that in order to maximize the minimum EE of users, more users choose to connect macro BS for communication.

VII. CONCLUSION

In this paper, two kinds of fairness criterion named proportional and max-min fairness have been studied in the uplink MIMO enabled HetNets. The goal of the two criteria is to obtain fair and energy efficient resource management, i.e., power control and user association. The formulated optimization problems are solved through dividing the original problem into two subproblems (i.e., user association subproblem and power control subproblem). For the proportional fairness problem, the dual decomposition and Newton methods have been utilized to solve the user association subproblem and power control subproblem, respectively. Through SCA and dual decomposition methods, the power control subproblem has been solved based on max-min fairness. For the user association subproblem, by the relaxation and equivalent change of the variable $x_{m,n}$, the original subproblem has been transformed to a convex problem and a dual method has been used to solve it. The convergence and complexity of the two algorithms were also analyzed. The simulation results have shown that the proposed algorithms offer better EE than other three resource allocation algorithms. In addition, as the numerical results showed, the proposed Algorithm 3 with better fairness has less average EE, and the proportional fairness method has better performance on the average EE.

APPENDIX

PROOF OF PROPOSITION 4

It can be easily derived that the objective function of problem $P_2.2.2$ is linear and we can find that the constraint $C1$ and $C3$ are convex. Next, the convexity of constraint $C4$ is analyzed.

Convexity illustration of $C4$.

Since the sum of convex function is convex function, so we consider the following inequation

$$e^{\hat{t}_{m,n}} \log_2(1 + \tilde{\gamma}_{m,n}) \geq e^{\hat{t}(o_1/M_o)}.$$

where $o_1 + o_2 + \ldots + o_m + \ldots + o_M = M_o$ and $o_m$ is a constant. For the convenience of analysis, let $o_m = 1$. The formula (53)
where $\ln 2$ is a constant. Then, taking the natural logarithm of the left side of formula (54)

$$\ln (1 + \frac{\gamma_{in}^m}{M_o}) \geq \frac{\ln 2 (e^{-\tilde{x}_{mn}} - 1)}{M_o},$$

(55)

where $\ln 2 / M_o$ is a constant, for the convenience of calculation, we only consider $\ln 2$. Through simplification, the formula (55) can be obtained as follows

$$\ln (e^{\ln 2 (e^{-\tilde{x}_{mn}} - 1)} - 1) + \ln \left( \sum_{m=1}^{M} \sum_{n=1}^{N} e^{\tilde{x}_{in}} A_{m,n} + (B_{m,n}) \right) \leq 0,$$

(56)

where $A_{m,n} = \frac{p_i m \alpha_i}{P_{m,n} |h_m|^2}$, $B_{m,n} = \frac{p_i^2 |h_m|^2}{P_{m,n}}$.

The second term in the left-hand-side of function (56) is a convex log-sum-exp function. So, in order to validate the convexity of the first term on the left-hand-side of function (56), an intermediate variable is defined as $g(z, \tilde{x}_{mn})$, where $g(z, \tilde{x}_{mn}) = \ln (e^{\ln 2 (e^{-\tilde{x}_{mn}} - 1)} - 1)$. The second-order derivative of $\frac{\partial^2 g}{\partial (\tilde{x}_{mn})^2}$ is computed as $E(\tilde{x}) F(\tilde{x})$, where $E(\tilde{x})$ and $F(\tilde{x})$ are given by

$$E(\tilde{x}) = \frac{\ln 2 e^{-\tilde{x}_{mn}}}{(\ln 2 e^{-\tilde{x}_{mn}} - 1)^2},$$

$$F(\tilde{x}) = e^{\ln 2 e^{-\tilde{x}_{mn}} - 1} - 1.$$

(57)

(58)

From $E(\tilde{x})$, the numerator and denominator are both greater than zero, so $E(\tilde{x})$ is non-negative. For $F(\tilde{x})$, when $j = \ln 2 e^{-\tilde{x}_{mn}}$, the formula (58) is similar to $f(j) = e^j - j - 1$.

The function of $f(j)$ is an increasing function for $j \geq 0$. Since $f(0) = 0$, we have $f(j) > f(0) = 0$ for all $j > 0$. Through the above analysis, we can have $F(\tilde{x}) > 0$. That is to say, the first term of (56) is convex. Consequently, the constraint $C_4$ is convex.

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