STATISTICAL SIGNIFICANCE OF SMALL-SCALE ANISOTROPY IN ARRIVAL DIRECTIONS OF ULTRA–HIGH-ENERGY COSMIC RAYS

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ABSTRACT

Recently, the High Resolution Fly’s Eye (HiRes) experiment has indicated that there is no small-scale anisotropy in the arrival distribution of ultra–high-energy cosmic rays (UHECRs) above \( E > 10^{19} \) eV, contrary to the Akeno Giant Air Shower Array (AGASA) observation. In this paper we discuss the statistical significance of this discrepancy between the two experiments. We calculate the arrival distribution of UHECRs above the \( 10^{19} \) eV predicted by the source models constructed using the Optical Redshift Survey galaxy sample. We apply a new method developed by us for calculating arrival distribution in the presence of the Galactic magnetic field. The great advantage of this method is that it enables us to calculate the UHECR arrival distribution with lower energy (\( \sim 10^{19} \) eV) than previous studies within a reasonable time by following only the trajectories of UHECRs actually reaching the Earth. It has been realized that the small-scale anisotropy observed by the AGASA can be explained with the source number density \( \sim 10^{-6} \) to \( 10^{-5} \) Mpc\(^{-3} \), assuming a weak extragalactic magnetic field (\( B \leq 1 \) nG). We find that the predicted small-scale anisotropy for this source number density is also consistent with the current HiRes data. We thus conclude that the statement by the HiRes collaboration that they do not find small-scale anisotropy in the UHECR arrival distribution is not statistically significant at present. We also examine the future prospect of determining the source number density with an increasing amount of observed data.

Subject headings: cosmic rays — galaxies: general — ISM: magnetic fields — large-scale structure of universe — methods: numerical

Online material: color figures

1. INTRODUCTION

The small-scale anisotropy in the observed arrival distribution of ultra–high-energy cosmic rays (UHECRs) is key in identifying their sources, which will tell us a great deal about acceleration mechanisms, the composition of UHECRs, and so on. The Akeno Giant Air Shower Array (AGASA) observation reveals the existence of the small-scale clusterings in the isotropic arrival distribution of UHECRs (Takeda et al. 1999, 2001). The current AGASA data set of 57 events above \( 4 \times 10^{19} \) eV contains four doublets and one triplet within a separation angle of 2.5. The chance probability of observing such clusters under an isotropic distribution is only about 1\% (Hayashida et al. 1999; Takeda et al. 2001). On the other hand, the recent finding with the High Resolution Fly’s Eye (HiRes; Wilkinson et al. 1999) experiment is that there is no small-scale anisotropy in the UHECR arrival distribution above \( E > 10^{19} \) eV (162 events), which is observed by a stereo air fluorescence detector (Finley & Westerhoff 2003). The discrepancy between the AGASA and HiRes involves a number of considerations. Among them are the different numbers of the events, the possibility of the measured energies being shifted between the two experiments (De Marco et al. 2003), and the different angular resolutions. The purpose of this paper is to discuss the first possibility. That is, we consider whether the discrepancy between the two experiments is statistically significant or not at present.

Furthermore, there is a controversy about the presence or absence of a GZK cutoff (Greisen 1966; Zatsepin & Kuzmin 1966) in the cosmic-ray energy spectrum due to photopion production with the photons of the cosmic microwave background (CMB). The HiRes spectrum shows a GZK cutoff (Abu-Zayyad et al. 2002), but the AGASA spectrum does not (Takeda et al. 1998). The statistical significance of this discrepancy has already been discussed in De Marco et al. (2003) and has been shown to be low at present. This issue is left for future investigation by new large-aperture detectors under development, such as South and North Auger projects (Capelle et al. 1998), the Extreme Universe Space Observatory (EUSO; Benson & Linsley 1982), and the Orbiting Wide-Angle Light (OWL; Cline & Stecker 2000) experiments.

When we consider the statistical significance of small-scale anisotropy, we need to compare the observations with numerical calculations predicted by some kind of source distributions. In this paper, we use the Optical Redshift Survey (ORS; Santiago et al. 1995) galaxy sample to construct realistic source models of UHECRs. This sample has been adopted in our previous studies (Yoshiguchi et al. 2003a, 2003b, 2003d). It has been realized that the small-scale anisotropy reflects well the number density of UHECR sources (Yoshiguchi et al. 2003a, 2003d; Blasi & Marco 2004). Also, it has been shown that the small-scale anisotropy above \( E = 4 \times 10^{19} \) eV observed by the AGASA can be explained with the source number density \( \sim 10^{-5} \) to \( 10^{-6} \), assuming a weak extragalactic magnetic field (\( B \leq 1 \) nG; EGMF), and that observed ultra–high-energy (UHE) particles are protons (Yoshiguchi et al. 2003a, 2003d; Blasi & Marco 2004). We thus take the source number density as a parameter of our source model and discuss the statistical significance of small-scale anisotropy by considering to what extent we can determine the source number density from the comparison of our model predictions with the HiRes observation.

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As mentioned above, the finding by the HiRes experiment on small-scale anisotropy is based on data on UHECR arrival directions above $10^{19}$ eV. UHE protons at such energies are significantly deflected (by up to $10^\circ$) by the Galactic magnetic field (GMF). In order to accurately calculate the expected UHECR arrival distribution and compare it with the observations, this effect should be taken into account.

In our recent work (Yoshiguchi et al. 2003b), we have presented a new method for calculating the UHECR arrival distribution that can be applied to several source location scenarios and includes modifications by the GMF. A brief explanation of our method is as follows. We numerically calculate the propagation of antiprotons from the Earth toward the outside of the Galaxy (we set a sphere centered around the Galactic center with radius $r_{\text{src}} = 40$ kpc as the boundary condition), considering the deflections due to the GMF. The antiprotons are ejected isotropically from the Earth. By this calculation, we can obtain the trajectories and a sky map of the position of antiprotons that have reached the boundary at radius $r_{\text{src}} = 40$ kpc.

Next, we regard the obtained trajectories as protons coming from outside the Galaxy toward the Earth. We also regard the obtained sky map of the position of antiprotons at the boundary as the relative probability distribution (per steradian) for protons to be able to reach the Earth for the case in which the flux of the UHE protons from the extragalactic region is isotropic (in this study, this flux corresponds to the one at the boundary $r_{\text{src}} = 40$ kpc). The validity of this treatment is supported by the Liouville theorem. When the flux of the UHE protons at the boundary is anisotropic (e.g., the source distribution is not isotropic), this effect can be included by multiplying this effect (that is, by multiplying the probability density of the arrival direction of UHE protons from the extragalactic region at the boundary) to the obtained relative probability density distribution mentioned above.

By adopting this new method, we can consider only the trajectories of protons that arrive at the Earth, which, of course, saves CPU time and makes calculation of the propagation of cosmic rays even with low energies ($\sim 10^{19}$ eV) possible within a reasonable time.

With this method, we calculate the arrival distribution of UHE protons above $10^{19}$ eV for our source models. We also consider the energy-loss processes that occur when UHE protons propagate in intergalactic space. Using two-point correlation functions as statistical quantities for small-scale anisotropy, we compare our model prediction with the HiRes observation. We then discuss to what extent we can determine the source number density of UHECRs using the current HiRes data. We also briefly discuss the future prospect of determining the source number density by means of the event number expected from future experiments.

In § 2 we introduce the GMF model. We explain the method for calculating UHECR arrival distribution in § 3. Results are shown in § 4. In § 5 we summarize the main results.

2. GALACTIC MAGNETIC FIELD

In this study, we adopt the GMF model used in Alvarez-Muniz et al. (2002) and Yoshiguchi et al. (2003b, 2004), which is composed of a spiral and a dipole field. In the following, we briefly introduce this GMF model.

Faraday rotation measurements indicate that the GMF in the disk of the Galaxy has a spiral structure with field reversals at the optical Galactic arms (Beck 2001). We use a bisymmetric spiral field model, which is favored from recent work (Han et al. 1999; Han 2001). The solar system is located at a distance $r_{\text{s}} = 8.5$ kpc from the center of the Galaxy in the Galactic plane. The local regular magnetic field in the vicinity of the solar system is assumed to be $B_{\text{solar}} \sim 1.5 \mu G$ in the direction $l = 90^\circ + p$, where the pitch angle is $p = -10^\circ$ (Han & Qiao 1994).

In polar coordinates $(r, \phi)$, the strength of the spiral field in the Galactic plane is given by

$$B(r, \phi) = B_0 \left( \frac{r_{\text{s}}}{r} \right) \cos \left( \phi - \beta \ln \frac{r}{r_{\text{s}}} \right),$$

where $B_0 = 4.4 \mu G$, $r_{\text{s}} = 10.55$ kpc and $\beta = 1/\tan p = 5.67$. The field decreases with Galactocentric distance as $1/r_{\text{s}}$, and it is zero for $r_{\text{s}} > 20$ kpc. In the region around the Galactic center ($r_{\text{s}} < 4$ kpc) the field is highly uncertain, and it is thus assumed to be constant and equal to its value at $r_{\text{s}} = 4$ kpc.

The spiral field strengths above and below the Galactic plane are taken to decrease exponentially with two scale heights (Stanev 1997):

$$|B(r_{\text{s}}, \phi, z)| = |B(r_{\text{s}}, \phi)| \begin{cases} \exp (-z) & |z| \leq 0.5 \text{ kpc}, \\ \exp \left( -\frac{3}{8}z \right) \exp \left( -\frac{z}{4} \right) & |z| > 0.5 \text{ kpc}, \end{cases}$$

where the factor $\exp (-\frac{z}{8})$ makes the field continuous in $z$. The bisymmetric spiral field spiral field we use is of even parity, that is, the field direction is preserved at disk crossing.

Observations show that the field in the Galactic halo is much weaker than that in the disk. In this work we assume that the regular field corresponds to a $A_0$ dipole field, as suggested in Han (2002). In spherical coordinates $(r, \theta, \phi)$, the $(x, y, z)$ components of the halo field are given by

$$B_x = -3 \mu G \sin \theta \cos \theta \cos \varphi / r^3,$$
$$B_y = -3 \mu G \sin \theta \cos \theta \sin \varphi / r^3,$$
$$B_z = \mu G \left( 1 - 3 \cos^2 \theta / r^3 \right),$$

where $\mu G \sim 184.2 \mu G$ kpc$^3$ is the magnetic moment of the Galactic dipole. The dipole field is very strong in the central region of the Galaxy but is only 0.3 $\mu G$ in the vicinity of the solar system, directed toward the north Galactic pole.

There may be a significant turbulent component, $B_{\text{tan}}$, of the Galactic magnetic field. Its field strength is difficult to measure, and results found in literature are in the range of $B_{\text{tan}} = 0.5 \ldots 2B_{\text{reg}}$ (Beck 2001). However, we neglect the random field throughout the paper for simplicity. Possible dependence of the results on this assumption is discussed in § 5.

3. NUMERICAL METHOD

3.1. Propagation of UHE Protons in Intergalactic Space

The energy spectrum of UHECRs injected from extragalactic sources is modified by energy-loss processes when they propagate in intergalactic space. In this subsection, we explain the method of using Monte Carlo simulations for propagation of UHE protons in intergalactic space.

We assume that the composition of UHECRs is protons that are injected with a power-law spectrum in the range of $10^{19} - 10^{22}$ eV. We inject 10,000 protons in each of 31 energy bins (10 bins per decade of energy). Then, UHE protons are propagated, including the energy-loss processes (explained below) over 3 Gpc for 15 Gyr. We use a power-law index of 2.6 to fit
the predicted energy spectrum to the one observed by HiRes (De Marco et al. 2003).

Below $\sim 8 \times 10^{19}$ eV UHE protons lose their energies mainly by pair creation and adiabatic energy losses, and above $8 \times 10^{19}$ eV by photopion production (Berezinsky & Grigorieva 1988; Yoshida & Teshima 1993) in collisions with photons of the CMB. We treat the adiabatic loss as a continuous loss process. We calculate the redshift \( z \) of a source at a given distance using the cosmological parameters \( H_0 = 71 \text{ km s}^{-1} \text{ Mpc}^{-1} \), \( \Omega_m = 0.27 \), and \( \Omega_{\Lambda} = 0.73 \). Similarly, pair production can be treated as a continuous loss process, considering its small inelasticity ($\sim 10^{-5}$). We adopt the analytical fit functions given by Chodorowski et al. (1992) to calculate the energy-loss rate for the pair production of isotropic photons.

On the other hand, protons lose a large fraction of their energy in the photopion production. For this reason, its treatment is very important. We use the interaction length and the energy distribution of the final protons as a function of initial proton energy, which is calculated by simulating the photopion production event with the event generator SOPHIA (Mucke et al. 2000). The same approach has been adopted in our previous studies (Ide et al. 2001; Yoshiguchi et al. 2003b, 2003c, 2003d).

In this study, we neglect the effect of the EGMF for two reasons. First, numerical simulations of UHECR propagation in the EGMF that include lower energy ($\sim 10^{19}$ eV) ones take a long CPU time. Second, we have shown in a previous study that the small-scale clustering observed by the AGASA can be well reproduced in the case of a weak EGMF ($\leq 1 \text{ nG}$) (Yoshiguchi et al. 2003a, 2003d). The purpose of this paper is to discuss whether the finding by the HiRes experiment, with statistical significance, is contrary to the AGASA result. We should calculate the arrival distribution of UHECRs using the source model that explains the AGASA experiment.

Of course, the effects of a strong EGMF (say, $\sim 100$ nG) on UHECR propagation have been investigated intensively (Sigl et al. 2003a, 2003b, 2004; Aloisio & Berezinsky 2004) and can hardly be excluded considering the limited amount of observational data on the EGMF. We should also comment that the EGMF affects not only the small-scale anisotropy but also the energy spectrum of UHECRs in the Galaxy because of the time delays in propagation. Accounting for the EGMF reduces the volume of the region from which UHECRs above a given detection energy (in this study, $10^{19}$ eV) can reach the Earth within the age of the universe. This effect makes the energy spectrum at the Earth smaller at lower energies. However, we assume an extremely weak EGMF and neglect it throughout this paper for simplicity.

### 3.2. Source Distribution

In this study, we apply a method for calculating the UHECR arrival distribution with modifications by the GMF (§ 3.3) to our source location scenario, which is constructed by using the ORS (Santiago et al. 1995) galaxy catalog.

In order to calculate the distribution of the arrival directions of UHECRs realistically, two key elements of the galaxy sample must be corrected. First, galaxies in a given magnitude-limited sample are biased tracers of matter distribution because of the flux limit (Yoshiguchi et al. 2003c). Although the sample of galaxies more luminous than $-20.5$ mag is complete within $80$ h$^{-1}$ Mpc (where h is the Hubble constant divided by 100 km s$^{-1}$ and we use $h = 0.71$), it does not contain galaxies outside it because of the selection effect. We distribute the sources of UHECRs outside $80$ h$^{-1}$ Mpc homogeneously. Their number density is set to be equal to that inside $80$ h$^{-1}$ Mpc.

Second, our ORS sample does not include galaxies in the zone of avoidance ($|b| < 20^\circ$). In the same way, we distribute UHECR sources in this region homogeneously and calculate the number density from the number of galaxies in the observed region.

As mentioned above, we take the number density of UHECR sources as a model parameter when we construct the source models. For a given number density, we randomly select galaxies from the above sample with probability proportional to the absolute luminosity of each galaxy. In this paper, the model parameters are taken to be $10^{-3}$, $10^{-4}$, $10^{-5}$, and $10^{-6}$ Mpc$^{-3}$.

### 3.3. Calculation of the UHECR Arrival Distribution with Modifications by the GMF

This subsection provides a method of calculating the UHECR arrival distribution with modifications by the GMF. The explanation largely follows our recent paper (Yoshiguchi et al. 2003b). We start by injecting antiprotons from the Earth isotropically and following each trajectory until (1) the antiprotons reach a sphere of radius $40$ kpc centered at the Galactic center, or (2) the total path length traveled by the antiprotons is larger than $200$ kpc, by integrating the equations of motion in the magnetic field. We regard these antiprotons as protons injected from outside the Galaxy toward the Earth. The number of propagated antiprotons is $2,000,000$. We have checked that the number of trajectories that are stopped by limit (2) is smaller than 0.1% of the total number. The energy loss of protons can be neglected for these distances. Accordingly, we inject antiprotons with an injection spectrum similar to the one observed, $\sim E^{-2.7}$. Note that this injection spectrum at $40$ kpc from the Galactic center is tentative. When we calculate the arrival distribution of UHECRs, the effects of propagation in intergalactic space are properly taken into account, as we explain below.

The result of the velocity directions of antiprotons at a sphere of radius of $40$ kpc is shown in the Galactic coordinates in Figure 1 (right). From Liouville’s theorem, if the cosmic-ray flux outside the Galaxy is isotropic, one expects an isotropic flux at the Earth even in the presence of the GMF. This theorem is confirmed by the numerical calculations shown in Figure 6 of Alvarez-Muniz et al. (2002), which is the same figure as our Figure 1 except for the threshold energy. Thus, the mapping of the velocity directions in Figure 1 (right) corresponds to the sources that actually give rise to the flux at the Earth when the sources (including ones that do not actually give rise to the flux at the Earth) are distributed uniformly.

We calculate the UHECR arrival distribution for our source scenario using the numerical data of the propagation of UHE antiprotons in the Galaxy. The detailed method is as follows. First, we divide the sky into a number of bins with the same solid angle. The number of bins is taken to be $360(l) \times 200(b)$. We then distribute all the trajectories into each bin according to the direction of their velocities (source directions) at the sphere of radius, $40$ kpc. Finally, we randomly select trajectories from each bin with probability \( P_{\text{select}}(j, k, E) \), defined as

$$P_{\text{select}}(j, k, E) \propto \frac{\sum_i dN/dE(d_i, E)}{E^{-2.7}}.$$

Here the subscripts \( j \) and \( k \) distinguish each cell of the sky, \( d_i \) is the distance of each galaxy within the cell of \((j, k)\), and the summation runs over all of the galaxies within it. \( E \) is the proton energy, and \( dN/dE(d_i, E) \) is the energy spectrum of protons at our galaxy injected from a source at distance \( d_i \).
The normalization of \( P_{\text{selc}}(j, k, E) \) is determined so as to set the total number of events equal to a given number, for example, the event number of the current HiRes data. When \( P_{\text{selc}} > 1 \), we generate an event number of \( (P_{\text{selc}} - 1) \times N(j, k) \), where \( N(j, k) \) is the number of trajectories within a sky cell of \((j, k)\). The arrival angle of the newly generated protons (equivalently, injection angle of antiproton) at the Earth is calculated by adding a normally distributed deviation with zero mean and variance equal to the experimental resolution of \( 2^{\circ} \times (1.8) \) for \( E > 10^{19} \) eV (\( 4 \times 10^{19} \) eV) to the original arrival angle.

### 3.4. Statistical Methods

A standard tool for searching for small-scale anisotropy is the two-point correlation function. This subsection provides an explanation of this function.

We start from a set of simulated events. For each event, we divide the sphere into concentric bins of angular size \( \Delta \theta \) and count the number of events falling into each bin. We then divide it by the solid angle of the corresponding bin, that is,

\[
N(\theta) = \frac{1}{2\pi \cos \theta - \cos (\theta + \Delta \theta)} \sum_{\theta \leq \phi \leq \phi + \Delta \theta} 1 \text{ (sr}^{-1}),
\]

where \( \phi \) denotes the separation angle of the two events; \( \Delta \theta \) is taken to be \( 2^\circ \) in this analysis. We also evaluate \( N_{\text{sim}}(\theta) \) for an equal number of events in a Monte Carlo simulation for a uniform source distribution. The estimator for the correlation function is then defined as \( w(\theta) = N(\theta)/N_{\text{sim}}(\theta) - 1 \).

A two-point correlation function of the HiRes data with 164 events above \( 10^{19} \) eV does not show significant small-scale anisotropy and is consistent with the isotropic source distribution within a 1 \( \sigma \) confidence level. We thus compare the two-point correlation functions predicted by our source models with that expected for an isotropic source distribution rather than with the HiRes data itself. When we consider the extent to which we can determine the source number density of UHECRs, we have to quantify deviations of the predictions of our source models from isotropic sources. To do so, we define the variable \( \chi^2 \) as

\[
\chi^2 = \frac{1}{N_{\text{bin}}} \sum_{i=1}^{N_{\text{bin}}} \frac{(w_{\text{th}}(\theta_i) - w_{\text{sim}}(\theta_i))^2}{(\sigma_{\text{th}}^i)^2 + (\sigma_{\text{sim}}^i)^2},
\]

where \( \theta_i \) is the angle at the \( i \)th bin and \( \sigma_{\text{th}}^i \) and \( \sigma_{\text{sim}}^i \) are the statistical fluctuations of \( w_{\text{th}} \) and \( w_{\text{sim}} \) at the angle \( \theta_i \). The summation is taken over the angular bins, whose number is denoted by \( N_{\text{bin}} \).

Even if we specify the source number density, the source distribution itself cannot be determined because of randomness when we select galaxies from the ORS sample. Therefore, we first evaluate \( \chi^2 \) for a source distribution in the case of a given number density of UHECR sources. We then repeat such calculations for a number of realizations of the source selection and obtain the \( \chi^2 \) probability distribution for a source number density. The source selections are performed 100 times for all the source number densities.

### 4. RESULTS

#### 4.1. Arrival Distribution of UHECRs

In this subsection, we present the results of the arrival distribution of UHECRs above \( 10^{19} \) eV obtained by using the method explained in § 3.3. Figures 2 and 3 show realizations of the event generations in the cases of source number densities of \( 10^{-3} \) and \( 10^{-5} \) Mpc\(^{-3} \), respectively. The events are shown by color according to their energies. Note that the event number of the HiRes data is 162 (\( E > 10^{19} \) eV). This roughly corresponds to the 300 events of Figures 2 and 3, considering the difference of the range \( \delta \) (declination) between the observation and the numerical calculation.

As is evident from Figures 2 and 3, the arrival distributions of UHECRs are quite isotropic. Harmonic analysis of the right ascension distribution of events is the conventional method of searching for the global anisotropy of cosmic-ray arrival distribution. Using this analysis, we have checked that the large-scale isotropy predicted by our source models is consistent with the isotropic source distribution within a 90\% confidence level, with the event number equal to the current HiRes and AGASA observations. On the other hand, the arrival distributions observed by HiRes and AGASA do not show any significant global anisotropy either. There is no discrepancy between the two experiments. Thus we do not discuss large-scale isotropy in the following.

Comparing Figures 2 and 3, we easily find that the arrival distribution expected for smaller source number density exhibits stronger anisotropy on small angle scales. This is simply because contributions from each source to the total events are larger for smaller source number density. Furthermore, the clustered events are aligned in the sky according to the order of their energies, reflecting the direction of the GMF at each direction. This interesting feature becomes more evident as the event number increases.

#### 4.2. Statistical Significance of the Small-Scale Anisotropy

Next, we discuss the statistics on arrival distribution of UHECRs. Figures 4 and 5 show the two-point correlation function expected in the case of source distributions of Figures 2 and 3, respectively. We calculate the two-point correlation function for the simulated events within only \( 0^\circ \leq \delta \leq 90^\circ \)
in accordance with the exposure time of the HiRes observation. For this reason, the event numbers written in Figures 4 and 5 differ from the ones in Figure 2 and 3. The error bars represent 1 \( \sigma \) statistical fluctuations of our numerical calculations due to the finite number of events. The shaded regions represent 1 \( \sigma \) fluctuations for uniform source distribution. The event selections are performed 1000, 100, and 30 times for the event number 150, 1500, and 5000, respectively. If we compare our numerical results with the HiRes data itself, we have to calculate the arrival distribution of UHECRs taking the exposure time of the HiRes observation into account. However, we compare our results with not the HiRes data but the prediction of uniform source distribution as mentioned above. Thus, we calculate the arrival distributions predicted by our source models and uniform sources without considering the HiRes exposure, and compare these two results.

As mentioned before, it is evident that smaller source number density predicts stronger correlation between events at small angle scales. The small-scale anisotropy reflects well the number density of UHECR sources. It has already been shown that the small-scale anisotropy above \( E = 4 \times 10^{19} \text{ eV} \) observed by the AGASA can be explained with the source number density \( \sim 10^{-6} \) to \( 10^{-5} \text{ Mpc}^{-3} \) assuming weak extragalactic magnetic field (\( B \leq 1 \text{ nG; EGMF} \)) (Yoshiguchi et al. 2003a, 2003d; Blasi & Marco 2004). We thus discuss the statistical significance of the discrepancy between the HiRes and AGASA by considering to what extent we can determine the UHECR source number density from the current HiRes data.

Even if we specify the source number density, source distribution itself cannot be determined because of randomness when we select galaxies from the ORS sample. Therefore, we first evaluate the statistical significance of deviation of the two-point correlation function predicted by a source distribution for a given number density from that expected for uniform source distribution. We then repeat such calculations for a number of realizations of the source selection and obtain the probability distribution of \( \chi^2 \) for a given source number density.

The results of the probability distribution of \( \chi^2 \) for \( N_{\text{bin}} = 1 \) and 5 are shown as histograms in Figures 6 and 7, respectively. The source selection is performed 100 times for all the source...

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Fig. 2.—Arrival directions of protons with \( E > 10^{19} \text{ eV} \) predicted by a source distribution for the source number density \( 10^{-3} \text{ Mpc}^{-3} \) in Galactic coordinates. Events are shown by color according to their energies. [See the electronic edition of the Journal for a color version of this figure.]

Fig. 3.—Same as Fig. 2, but for the source number density \( 10^{-5} \text{ Mpc}^{-3} \). [See the electronic edition of the Journal for a color version of this figure.]
number densities. As we can see from the figures, differences between probability distributions of $\chi^2$ for different source number densities are clearer when $N_{\text{bin}} = 1$. This is related to the fact that the two-point correlation function is most sensitive to the source number density at the smallest angular bin (see Figs. 4 and 5). We thus focus our attention to the result for $N_{\text{bin}} = 1$ in the following.

As mentioned, we know that the small-scale anisotropy observed by the AGASA can be explained with the number density $\sim 10^{-6}$ to $10^{-5}$ Mpc$^{-3}$. From Figure 6, we find that the small-scale anisotropy predicted by this source number density is also consistent with the prediction of the isotropic source distribution when the event number is equal to the HiRes data ($\sim 150$). It is noted that the AGASA data have about 1000 events above $10^{19}$ eV. The possibility that the number density is about $\sim 10^{-6}$ to $10^{-5}$ Mpc$^{-3}$ cannot be ruled out, although the HiRes result seems to be in agreement with the isotropic source distribution. We thus conclude that the statement by the HiRes collaboration that they do not find small-scale anisotropy in...
UHECR arrival distribution is not statistically significant at present.

We now turn to the future prospects of determining the UHECR source number density. The probability distributions of \( \chi^2 \) with larger event numbers than the current observations are also shown in Figure 6. Of course, there is no observational data to be compared. We thus compare the model predictions with the case of isotropic source distribution. The results should be interpreted as quantifying deviations of our calculations from the prediction of isotropic sources in the unit of statistical fluctuations.

As is evident from Figure 6, the differences between the probability distribution of \( \chi^2 \) for different source number densities becomes clear when the number of data increases. We will be able to determine the source number density by future observations of the small-scale anisotropy. In order to discuss to what accuracy we can know the source number density, we need to calculate the probability distribution of \( \chi^2 \) more precisely with a large number of simulations of the source selection. This issue is beyond the scope of the present paper. We will study it in a future publication.

5. SUMMARY AND DISCUSSION

In this paper, we considered the statistical significance of the discrepancy between the HiRes and the AGASA experiment on the small-scale anisotropy in arrival distribution of UHECRs. We calculated arrival distribution of UHECRs above \( 10^{19} \text{ eV} \) predicted by our source models constructed using the ORS galaxy sample. We applied the new method developed by us (Yoshiguchi et al. 2003b) for calculating the arrival distribution with the modifications by the GMF. The great advantage of this method is that it enables us to calculate UHECR arrival distribution with lower energy (\( \sim 10^{19} \text{ eV} \)) than previous studies by following only the trajectories actually reaching the Earth.

It has been realized that the small-scale anisotropy in the UHECR arrival distribution reflects well the number density of UHECR sources (Yoshiguchi et al. 2003a, 2003d; Blasi & Marco 2004). We also show that the small-scale anisotropy above \( E = 4 \times 10^{19} \text{ eV} \) observed by the AGASA can be explained with the source number density \( \sim 10^{-6} \) to \( 10^{-5} \), assuming a weak extragalactic magnetic field (\( B \leq 1 \text{ nG} \); EGMF) and that observed UHE particles are protons (Yoshiguchi et al. 2003a, 2003d; Blasi & Marco 2004). We thus took the source number density as a parameter of our source model and discussed the statistical significance of the small-scale anisotropy by considering to what extent we can determine the source number density from the comparison of the model predictions with the HiRes observation.

When we consider the extent to which we can determine the source number density of UHECRs, we have to quantify deviations of predictions by our source models from the observation. Further, even if we specify the source number density of our model, the source distribution itself cannot be determined because of randomness when we select galaxies from the ORS sample. Therefore, we first evaluated the statistical significance of the deviation (\( \chi^2 \)) of the two-point correlation function predicted by a source distribution for a given number density on the basis of that expected for uniform source distribution. We then repeated such calculations for a number of realizations of the source selection. This gave the probability distributions of \( \chi^2 \) for various number densities of UHECR sources.

The results for \( N_{\text{bin}} = 1 \) and 5 show that differences between the probability distributions of \( \chi^2 \) for different source number densities are clearer when \( N_{\text{bin}} = 1 \). This is related to the fact that the two-point correlation function is most sensitive to the source number density at the smallest angular bin. We thus focused our attention to the result of \( N_{\text{bin}} = 1 \). As mentioned above, we know that the small-scale anisotropy observed by the AGASA can be explained with the number density \( \sim 10^{-6} \) to \( 10^{-5} \text{ Mpc}^{-3} \). It is found that the small-scale anisotropy predicted by this source number density is also consistent with the prediction of isotropic source distribution when the event number is equal to the HiRes data (\( \sim 150 \)). Note that the data number of AGASA is about 1000 above \( 10^{19} \text{ eV} \). The possibility that the number density is about \( \sim 10^{-6} \) to \( 10^{-5} \text{ Mpc}^{-3} \) cannot be ruled out, although the HiRes result seems to be in agreement with the isotropic source distribution. We thus concluded that the statement by the HiRes experiment that they do not find small-scale anisotropy in UHECR arrival distribution is not statistically significant at present.

We also discussed the future prospects of determining the UHECR source number density. As is evident from Figure 6, difference between the probability distribution of \( \chi^2 \) for different source number densities become clear when the number of data increases. We will be able to determine the source number density by future observations of the small-scale anisotropy. In order to discuss to what accuracy we can know the source number density, we need to calculate the probability distribution of \( \chi^2 \) more precisely with a large number of simulations of the source selection. This issue deserves further investigation.

Finally, we mention the assumptions made in this paper. We neglected the effects of the extragalactic magnetic field and the random component of the GMF. If we take these effects into account, the small-scale anisotropy obtained by numerical calculations becomes less obvious. In this case, our statement that the small-scale anisotropy for \( \sim 10^{-6} \) to \( 10^{-5} \text{ Mpc}^{-3} \) is also consistent with the prediction of isotropic sources when the event number is equal to the HiRes data (\( \sim 150 \)) remains valid. Hence, the assumptions are not so important for our conclusion.

*Note added in manuscript.—*While we were finishing this study, a paper by the HiRes collaboration (Abbasi et al. 2004), in which they present the results of a search for small-scale anisotropy in the observed arrival distribution of UHECRs, appeared on the Internet. They find no small-scale anisotropy in 271 events above \( 10^{19} \text{ eV} \), which is slightly larger than the event number considered in the present study (164). However, this increase of the event number does not affect our conclusion very much, as seen from Figure 6.

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