Dynamic Response Spectrum of Multi-Story Shear Frame Subjected to Moderate Ground Motion

Assal Hussein*

Department of Civil Engineering, College of Engineering, University of Diyala, 32001 Diyala, Iraq

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ABSTRACT

Design structures to resist natural hazards is a vital issue to mitigate the impact of such threats. Earthquakes could lead to numerous injuries and infrastructure destruction. Specifically, when structures are not designed to resist seismic load. This article presents dynamic analysis of four-story shear frame under moderate ground motion to determine the dynamic response. The proposed location of the considered frame is near Iraq-Iran border due to increase of seismic activities in last years. Structures in the considered seismic zone are essentially either residential or commercial buildings and not designed to resist seismic load, therefore structural system failure is probable. Simplified model is considered to determine response spectrum according to the International Building Code requirements. The algorithm of the analysis is developed using MATLAB® code to get mode shapes, response spectrum acceleration, maximum displacement, maximum shear forces, and modal participation mass at each story. The article develops design response spectra curve of Erbil city. Furthermore, the analysis results showed that first mode shape has more contribution than the other modes because higher percentage of the mass of the shear frame responds to the ground motion, and 88.53% of shear-frame mass is participating responds to the ground motion in the same mode.

1. Introduction and background

Earthquakes have been recorded beginning of the last century. Effect of the hazards comprises structural collapse, life-loss, and earth-layers movement [1]. The intensity of seismic ground motions measured on seismometers depends on how far from the earthquake epicentre and magnitude of earthquake. Seismic activity has been noticed north and northeast of Iraq in last years. The tectonic boundaries of Iraq, north of the Arabian Plate and east of the Bitlis-Zgros Thrust and Fold Belt make it vulnerable to intense earthquake [2]. Most earthquakes have been recorded close to Taurus and Zagros mountains [2]. For instance, on November 12, 2017, according to the U.S. Geological Survey (USGS), an earthquake recorded 29 km south of Halabjah near Iraq-Iran border (34.911°N 45.959). The intensity is 7.3 on the magnitude scale caused by movement on a thrust fault dipping at a shallow angle to the northeast (see Figure 1).

Second Earthquakes data sets have been normally presented in two magnitude scales, surface–wave magnitudes ($M_s$), and body magnitude wave ($M_b$). How-ever, other magnitude scales, moment magnitudes ($M_w$), and local magnitudes ($M_L$), and duration magnitudes, ($M_D$), are used but not often [4]. The epicenter map of the earthquakes of the magnitude scale in Iraq from 1900-2010 is shown in Figure 2 [4].

Design structures to attenuate energy of seismic wave is ever pressing when there is a...
possibility of having an earthquake. Structural system behavior under seismic load is complex, therefore simplified system could be useful to predict response of the structure. Numbers of independent single-degree-of-freedom (SDOF) is the main factor of specified the complexity of the system. Modal analysis can be considered for each mode as an independent DOF system. This approach is restricted to linearly elastic systems and all applied forces should have the same time change [5]. Modal equations are uncoupled, and superposition method can be applied, but it is possible to apply for non-classical damping by using alternative method to obtain uncoupled modal equations [6]. However, modal analysis can not be used for non-linear analysis for all periods, only possible to apply it for a specified time step and calculate displacements and forces at each step and same process for other steps [6].

Response spectrum analysis is a standard computational approach for linear seismic analysis before developing computers [7]. Computational procedures of peak response of a structure known as response spectrum analysis (RSA). The framework of the analysis can predict response spectrum without need to run response history analysis (RHA). The limitations of
response spectrum analysis are, it is limited for linear elastic analysis, so it is not used for inelastic analysis, there is no wide range for considering damping properties, and accuracy is questionable. However, a good arithmetical or computational advantage of applying response spectrum method for seismic analysis to predict response of systems.

According to International Building Code (IBC), the mapped of spectral response acceleration factors should be specified from seismic maps published by the USGS collaboration with FEMA for short and long periods, then these factors should adjust for the site conditions since seismic maps are considered for site class B. Farman and Said [2] developed design response spectrum curve of three Iraqi cities, Baghdad, Basra, and Erbil for returning period 2475 years, as shown in Figure 3. The authors stated the necessity to update seismic maps of Iraq as the probability of having seismic hazard is increased in the last years.

![Figure 3. Design response spectrum of specified Iraqi cities [2]](image)

In this article, dynamic response of four-story shear frame subjected to moderate earthquake is investigated. of multi-story shear frame located in northern of Iraq near borders with Iran due to significant activities in last years. There is few published research to investigate the dynamic response of structures in this seismic zone. While the topic is well known in the structural dynamics filed, but the application of the current study is ever pressing since most of structures have not been designed to resist natural hazards such as earthquakes and casualties and losses is probable. A modal analysis is considered to calculate the frequency and corresponding mode shapes. The total peak response of the system has been obtained using SRSS method. Convention method has been considered to calculate response spectrum acceleration as an approximate method in order to estimate peak displacements and forces according to the International Building Code requirements.

2. Earthquake’s history in Iraq

According to the seismic maps, north and northeast of Iraq fail in an active seismic zone northeast boundary of Arabian plate and Zagros-Taurus Belts [4,9]. Decades of seismic inactivity was noticed since subduction stopped millions of years ago [9]. However, seismologists recorded more than one thousand earthquakes between 1900 and 2010 [1,4], [9-14].

The magnitude range of these events between 3.4 - 7.4 are reported by Alsinawi and Al-Qarsani (2003) [10]. Higher magnitude of ground motion activities had noticed in the north and northeast of Iraq. Specified seismic regions have been considered for different time intervals. For instance, Ameer et al. (2005) [14] summarized earthquakes events in Iraq for the con-fined area between latitude (29–38.5° N) and longitude (39–50° E) including more than one thousands of earthquake above 4.0 in magnitude [4,14]. Figure 4 shows the seismicity map of Iraq of the 95 years (1905-2000) for earthquakes above 3.0 Ms [14]. These records were used to con-duct probabilistic risk analysis for Iraq. The out-comes of the statistical analysis showed the percentage of earthquakes ranged
between 4.0-5.4 $M_b$ equal to 90.95%, while events between 5.5-7.4 $M_b$ magnitude represents 6.03% of the total records [9]. With a unique event above 6.0 magnitude and it is recorded inside Iraqi region records [9]. This was the seismic activities beginning of the current century. Thereafter, in last ten years seismological centers noticed higher seismic activities. Epicenters of these events were within Iran territory near the borders with Iraq. Iraqi residents in the border cities with Iran felt the power of the earthquakes specially cities in mid and north-east of Iraq.

3. Natural frequency and mode shapes

The solution of eigenvalue problem gives the natural frequencies and mode shapes of a system. Response of the system can be represented as follow

$$U(t) = q_n \phi_n$$

where, $\phi_n$, is the deflected shape. The variation of the response with time can be described as a simple harmonic function by considering harmonic function for $q_n(t)$, gives

$$[-\omega_n^2 m \phi_n + K \phi_n] q_n(t) = 0$$

This equation known as the matrix eigenvalue problem, and formal solution which has nontrivial solution can be written as

$$\text{det} [K - \omega_n^2 m \phi_n] = 0$$

Equation 3, known as the characteristic equation or frequency equation. The roots of $\omega_n^2$, called eigenvalue while roots known $\phi_n$ as eigenvectors [6].

4. Response spectrum analysis

Uncertainty in the materials properties, and earthquake hazards intensity [14]. Time history analysis (THA) of joint response and member forces of a seismic loading can be obtained by applying superposition method [6,7,13]. This approach is limited to apply for linear elastic systems and requires computationally inefficiency [7]. However, response spectra method estimates peak response and member forces with minimum bias of exact solution [6, 7]. The framework analysis of response spectra method calculates displacement and member forces for each mode of set of earthquake records. These individual modal responses combine to get the peak response either by considering complete quadratic combination (CQC) or Square-Root-of-Sum-of-Squares (SRSS). In this method peak modal responses assumed to be occurred at the same joint for all modes, while SRSS method presumes all peak responses are statically independent [7].

In general, the equation of motion of a system having $N$ degrees of freedom subjected to earthquake excitation can be written in the following form

$$m \ddot{u} + c \dot{u} + ku = -m_i \ddot{u}_d(t)$$

Figure 4. Epicenter of earthquake events in Iraq from 1900-2010 [14]
where, \( m, c, \) and \( k \) are mass, damping, and stiffness matrices of the system respectively. The vector \( u \) is the nodal displacement vector, and \( \ddot{u}_g(t) \) is ground acceleration. Influence vector represents the displacements of the masses from the static analysis of a unit ground displacement \( u_g = 1 \).

Dynamic response of linear system governs by the equation of motion which is derived based on the Newton’s second law of motion shown in Equation 5.

\[
m\ddot{u} + c\dot{u} + ku = P(t)
\]  

(5)

For structural systems excited by non-harmonic excitation, dynamic displacement vector (\( u \)) of a Multi-degree freedom system (MDOF) in terms of modal contributions [6].

The simultaneous solutions of Equation 5, is not effective for MDOF systems, hence, extend response of such systems to modal coordinates is essential and dynamic response can be written as

\[
u(t) = \sum_r^n \phi_r q_r(t) = \bar{\phi} q(t)
\]  

(6)

Equation (6) can transform to a set of uncoupled equations with modal coordinates \( q_r(t) \). Substituting Eq. (6) in Eq. (5), gives

\[
\sum_r^n \phi_r^T n \phi_r \ddot{q}_r(t) + \sum_r^n \phi_r^T c \phi_r \dot{q}_r(t) + \sum_r^n \phi_r^T k \phi_r q_r(t) = \phi^T n P(t)
\]  

(7)

Because of orthogonality (\( r = n \)), all summation terms will vanish, and the equation of motion will be equal to

\[
M_n \ddot{q}_n(t) + \sum_r^n C_{nr} \dot{q}_r(t) + K_n q_n(t) = P_n(t)
\]  

(8)

where, \( M_n = \phi^T n m \phi_n \), \( C_{nr} = \phi^T n c \phi_r \), \( K_n = \phi^T n k \phi_n \), \( P_n(t) = \phi^T n p(t) \).

5. Numerical analysis of four-story shear frame

5.1 Building description and analysis assumptions

Dynamic response of four-story shear frame is calculated using MATLAB® (RSM) code. The shear frame topology is shown in Figure 5. The proposed frame is assumed to be in Erbil city seismic zone. The mass of the shear frame is lumped at the level of each story.

This assumption simplified the dynamic analysis of the shear frame to four DOF with respect to lumped masses. The beam-connection is assumed to be rigid. The materials and structure properties are listed in Table 1. The computational algorithm of the analysis is shown in Figure 6.

| Properties          | Value             |
|---------------------|-------------------|
| Story height (h)    | 3(m)              |
| Floor width (b)     | 4(m)              |
| (E)                 | 200 (GPa)         |
| Story mass          | 130 kN, sec²/m    |
| Column stiffness    | 18500 kN/m        |
5.2 Design response spectrum

To apply the response spectrum analysis of MDOF shear frame subjected to ground motion excitation according to the conventional method which was published in International Building Code [8] for Erbil seismic region (site class B). According to IBC code, the mapped of spectral response acceleration factors should be specified from seismic maps prepared by the U.S. Geological Survey for short and long periods. These factors should adjust for the site conditions because seismic maps are for site class B. The response spectral accelerations for short and long periods $S_s$ and $S_1$ respectively, the maximum spectral acceleration for short period ($T_n = 0.2$ sec), $S_{ms}$, and the maximum spectral acceleration for long period ($T_n = 1.0$ sec), $S_{m1}$, should compute for site class effects as follow

$$S_{ms} = F_a S_s$$  \hspace{1cm} (9)

$$S_{m1} = F_v S_1$$  \hspace{1cm} (10)

Here, $F_a$, is site adjustment coefficient for short period, $F_v$, is site coefficient for long period. The value of site coefficient $F_a$, is listed in table 1613.3(1) and $F_v$ in table 1613.3(2), and linear interpolation can be used in order to get the intermediate values of mapped spectral acceleration at both short and long periods. The maximum design spectral acceleration $S_{DS}$ and $S_{D1}$ which are determined for %5 damping can calculate from equations (11) and (12)[8].

$$S_{DS} = \frac{2}{3} S_{ms}$$  \hspace{1cm} (11)

$$S_{D1} = \frac{2}{3} S_{m1}$$  \hspace{1cm} (12)

where, $S_{DS}$, design spectral response acceleration at short period ($T_n = 0.2$ sec), and $S_{D1}$, design spectral response acceleration at long period ($T_n = 1.0$ sec). The predicted response spectrum design parameters are shown in Table 2 which are used to construct design spectral of Erbil city.

Table 2: Design response spectrum parameters
6. Numerical analysis results

The analysis results of the modal analysis upon the framework of the current study shows a higher contribution of first and second mode shapes in the response of the shear frame. The natural modes of vibration of the four-story shear building is shown in Figure 7. The fundamentals period of the considered shear-frame is equal 1.516 second. Figure 8 shows the period of the structure of each mode shapes.

![Figure 7. Natural modes of vibration of the four-story shear frame](image)

| Parameter | value |
|-----------|-------|
| $S_{mS}$ (g) | 1.0 |
| $S_{m1}$ (g) | 0.4 |
| $F_a$ | 1.0 |
| $F_v$ | 1.0 |
| $S_{DS}$ (g) | 0.67 |
| $S_{DIV}$ (g) | 0.27 |
| $T_o$ (sec) | 0.09 |
| $T_i$ (sec) | 0.4 |

There is a relation between structure response and stiffness to mass ratio which could affect the response of the structure base on the peak of acceleration. Therefore, it is necessary to adjust stiffness to the mass of the structure. Otherwise, structural system will vibrate with a period less than the characteristic period and creating maximum inertia force.

To determine the response spectrum acceleration, design response spectra parameters are used to...
calculate spectral acceleration which is required in the modal analysis framework to predict the maximum acceleration of the shear frame. Design response spectra of Erbil city is determined according to IBC code is shown in Figure 9. The mass of structure acceleration subjected to considered ground motion based on the generated design response spectrum is shown in Figure 10.

![Figure 9. Design response spectrum of Erbil city](image)

![Figure 10. Response spectrum acceleration of each mode shape of four-story shear frame](image)

The displacement matrix of each mode is shown in Eq. 13.

$$ U = \begin{bmatrix}
0.0624 & 0.0182 & -0.0042 & -0.013 \\
0.0693 & -0.0154 & 0.002 & 0.0026 \\
0.038 & 0.0146 & 0.003 & -0.0005
\end{bmatrix} $$(13)

The participation ratio of shear-frame mass is calculated depend on the modal participation factor which is simulating the relation between the maximum modal displacement and modal excitation action. This parameter shows the percentage of the responding mass to the ground motion and to identify the required mode shapes. The responding mass to the ground motion in the first mode shape is the most effective one since 88.53% of mass is participating as shown in Figure 11.
7. Conclusions

The current study computed dynamic response spectrum of multi-story shear frame located in northern of Iraq near borders with Iran due to significant activities in last years. While the topic is well known in the structural dynamics filed, but the application of the current study is ever pressing since most of structures have not been designed to resist earthquakes and casualties and losses is probable.

Numerical evaluation has been performed of four-story shear frame subjected to ground motion using MATLAB®. The design response spectrum is predicted for Erbil city which is located north of Iraq since moderate-scale earthquakes have been noticed in last years. Design structures to resist seismic load is essential to protect occupants and reduce losses. The framework of the study consists of running modal analysis to calculate the natural frequency, period and mode shapes of the shear-frame. The response spectrum matrix is calculated according to IBC code requirements. Furthermore, peak modal displacement, total shear at each story level are determined using square root of the sum of the squares (SRSS) combination method. Finally, the results of the analysis showed 88.53% of shear-frame mass is participating responds to ground motion in the first mode.

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