Screening in Supersymmetric Gauge Theories in Two Dimensions

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Abstract

We show that the string tension in $\mathcal{N} = 1$ two-dimensional super Yang-Mills theory vanishes independently of the representation of the quark anti-quark external source. We argue that this result persists in $SQCD_2$ and in two-dimensional gauge theories with extended supersymmetry or in chiral invariant models with at least one massless dynamical fermion.

We also compute the string tension for the massive Schwinger model, as a demonstration of the method of the calculation.

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1 Introduction

In a paper by Gross et al.\cite{1} it was conjectured that two dimensional $\mathcal{N} = 1$ Super Yang-Mills exhibits a screening nature for any representation of the external sources. In particular the adjoint vector multiplet can screen completely an external source which transforms in the fundamental representation.

$SYM_2$ is an interesting theory, since it is probably the simplest non-trivial supersymmetric model in $1+1$ dimensions. It’s dynamics is similar to the adjoint fermions model, but SUSY simplifies its behavior. The model was analyzed lately in\cite{2, 3}.

In this letter we present a short proof that the theory is indeed screening. We show that that the string tension vanishes in this model (though we don’t exclude less than linear confining potential). An evidence to this phenomenon was given in\cite{4}. The proof follows an idea presented in\cite{5} which was generalized to the non-Abelian case in\cite{6}. We use a local chiral rotation to eliminate the external source from the action. The chiral rotation affects terms which are not chiral invariant. In the case of $SYM_2$ it is the interaction term of the gluino and the pseudo-scalar. The string tension is then computed using the change in the Hamiltonian density.

As a demonstration of the technique, we compute the string tension in the massive Schwinger model. A different derivation in the fermionic basis was given in\cite{7}.

The expression for the string tension in the massive Schwinger model was calculated by using bosonization\cite{5}

$$\sigma_{QED} = m\mu \left( 1 - \cos \left( 2\pi \frac{q_{ext}}{q_{dyn}} \right) \right) + O(m^2),$$

where $m$ is the electron mass, $\mu = g \frac{\exp(\gamma)}{4\pi \sqrt{2}}$ ($g$ is the gauge coupling and $\gamma$ is the Euler number) and $q_{ext}, q_{dyn}$ are the external and dynamical charges respectively.

The expression in massive $QCD_2$ is\cite{6}

$$\sigma_{QCD} = \mu R \sum_i \left( 1 - \cos \left( 4\pi \lambda_i \frac{k_{ext}}{k_{dyn}} \right) \right) + O(m^2)$$

where $\mu R \sim g$, $\lambda_i$ are the isospin eigenvalues of the dynamical representation, $k_{ext}$ and $k_{dyn}$ are the affine current algebra levels of the external and
dynamical representations, respectively. This expression is valid only for the fundamental and the adjoint representations. Other representations were also discussed in [6].

Note that when $m = 0$ the string tensions (1) and (2) vanish. In this case the theories are screening due to chiral invariance of the actions. The $O(m^2)$ term in (1) and (2) indicates that these expressions are only the leading terms in mass perturbation theory and are valid when $m \ll g$. The next to leading order correction, in the Abelian case, was derived in [7].

The derivation of the string tension in the massive Schwinger model is as follows. Consider the partition function of two dimensional massive QED

$$Z = \int DA_\mu D\bar{\Psi} D\Psi \exp \left( i \int d^2 x \left( -\frac{1}{4g^2} F^2_{\mu\nu} + \bar{\Psi} i \partial \Psi - m \bar{\Psi} \Psi - q_{\text{dyn}} A_\mu \bar{\Psi} \gamma^\mu \Psi \right) \right),$$

where $q_{\text{dyn}}$ is the charge of the dynamical fermions. Let us add an external electron-positron source with charge $q_{\text{ext}}$ at $\pm L$, namely $j^\mu_{\text{ext}} = q_{\text{ext}} (\delta(x + L) - \delta(x - L))$, so that the change of $L$ is $-j^\mu_{\text{ext}} A^\mu(x)$. Note that by choosing $j^\mu_{\text{ext}}$ which is conserved, $\partial^\mu j^\mu_{\text{ext}} = 0$, the action including the coupling to the external current is gauge invariant.

Now, to eliminate this charge we perform a local, space-dependent left-handed rotation

$$\Psi \rightarrow e^{i\alpha(x)\frac{1}{2}(1-\gamma_5)} \Psi$$
$$\bar{\Psi} \rightarrow \bar{\Psi} e^{-i\alpha(x)\frac{1}{2}(1+\gamma_5)},$$

where $\gamma^5 = \gamma^0 \gamma^1$. The rotation introduce a change in the action, due to the chiral anomaly

$$\delta S = \int d^2 x \frac{\alpha(x)q_{\text{dyn}}}{4\pi} F,$$

where $F$ is the dual of the electric field $F = \frac{1}{2} \epsilon^{\mu\nu} F_{\mu\nu}$.

The new action of the original fields is

$$S = \int d^2 x \left( -\frac{1}{4g^2} F^2_{\mu\nu} + \bar{\Psi} i \partial \Psi - \bar{\Psi} \partial_\mu \alpha(x) \gamma^\mu \frac{1}{2} (1 - \gamma_5) \Psi \
-m \bar{\Psi} e^{-i\alpha(x)\gamma_5} \Psi - q_{\text{dyn}} A_\mu \bar{\Psi} \gamma^\mu \Psi - q_{\text{ext}} (\delta(x + L) - \delta(x - L)) A_0 + \frac{\alpha(x)q_{\text{dyn}}}{4\pi} F \right)$$

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The external source and the anomaly term are similar, both being linear in the gauge potential. This is the reason that the $\theta$-vacuum and electron-positron pair at the boundaries are the same in two-dimensions\textsuperscript{[5]}. In the following we assume $\theta = 0$, as otherwise we absorb it in $\alpha$. Choosing the $A_1 = 0$ gauge and integrating by parts the anomaly term looks like an external source

$$\frac{q_{\text{dyn}}}{2\pi} A_0 \partial_1 \alpha(x)$$

This term can cancel the external source by the choice

$$\alpha(x) = 2\pi \frac{q_{\text{ext}}}{q_{\text{dyn}}} (\theta(x + L) - \theta(x - L)).$$

Let us take the limit $L \rightarrow \infty$. The form of the action, in the region $B$ of $-L < x < L$ is

$$S_B = \int_B d^2 x \left(-\frac{1}{4g^2} F_{\mu\nu}^2 + \bar{\Psi} \partial^\mu \Psi - m \bar{\Psi} e^{-i2\pi \frac{q_{\text{ext}}}{q_{\text{dyn}}} \gamma_5} - q_{\text{dyn}} A_\mu \bar{\Psi} \gamma^\mu \Psi \right)$$

Thus the total impact of the external electron-positron pair is a chiral rotation of the mass term. This term can be written as

$$\bar{\Psi} e^{-i2\pi \frac{q_{\text{ext}}}{q_{\text{dyn}}} \gamma_5} \Psi = \cos(2\pi \frac{q_{\text{ext}}}{q_{\text{dyn}}}) \bar{\Psi} \Psi - i \sin(2\pi \frac{q_{\text{ext}}}{q_{\text{dyn}}}) \bar{\Psi} \gamma_5 \Psi$$

The string tension is the vacuum expectation value (v.e.v.) of the Hamiltonian density in the presence of the external source relative to the v.e.v. of the Hamiltonian density without the external source, in the $L \rightarrow \infty$ limit.

$$\sigma = \langle \mathcal{H} \rangle - \langle \mathcal{H}_0 \rangle$$

The change in the vacuum energy is due to the mass term. The change in the kinetic term which appears in (7) does not contribute to the vacuum energy\textsuperscript{[3]}. Thus

$$\sigma = m \cos(2\pi \frac{q_{\text{ext}}}{q_{\text{dyn}}}) \langle \bar{\Psi} \Psi \rangle - m \sin(2\pi \frac{q_{\text{ext}}}{q_{\text{dyn}}}) \langle \bar{\Psi} i\gamma_5 \Psi \rangle - m \langle \bar{\Psi} \Psi \rangle$$

Thus, the values of the condensates $\langle \bar{\Psi} \Psi \rangle$ and $\langle \bar{\Psi} \gamma_5 \Psi \rangle$ are needed. The easiest way to compute these condensates is Bosonization\textsuperscript{[3]}, but it can also be computed directly in the fermionic language\textsuperscript{[3]}

$$\langle \bar{\Psi} \Psi \rangle = -g \frac{\exp(\gamma)}{2\pi^{3/2}}$$

$$\langle \bar{\Psi} \gamma_5 \Psi \rangle = 0.$$
The condensates (14) and (15) were computed in the massless Schwinger model. However, the corrections to these expressions will affect the string tension only by terms higher in $m/g$. Eq.(15) is due to parity invariance (with our choice $\theta = 0$). The resulting string tension is eq.(1).

Though eq.(15) gives only the leading term in a $m/g$ expansion and might be corrected[7], when $q_{ext}$ is an integer multiple of $q_{dyn}$ the string tension is exactly zero, since in this case the rotated action(10) is the same as(3).

2 Super Yang-Mills

The same technique can be used to prove screening in $SYM_2$. In this case the action is

$$S = \int d^2x \; tr \left( -\frac{1}{4g^2} F_{\mu\nu}^2 + i \bar{\lambda} \gamma \phi \lambda + \frac{1}{2} (D_{\mu} \phi)^2 - 2i g \phi \gamma_5 \lambda \right), \quad (16)$$

where $A_\mu$ is the gluon field, $\lambda$ the gluino (a Majorana fermion) and $\phi$ a pseudoscalar, are the components of the vector supermultiplet and transform as the adjoint representation of $SU(N_c)$. Also $D_{\mu} = \partial_{\mu} - i[A_{\mu_1},]$. The action (16) is invariant under SUSY

$$\delta A_{\mu} = -ig \bar{\epsilon} \gamma_5 \gamma_{\mu} \sqrt{2} \lambda$$
$$\delta \phi = -\bar{\epsilon} \sqrt{2} \lambda$$
$$\delta \lambda = \frac{1}{2\sqrt{2}g} \epsilon F_{\mu\nu} + \frac{i}{\sqrt{2}} \gamma^\mu \epsilon D_{\mu} \phi$$

We now introduce an external current. The external source breaks explicitly supersymmetry. However, this breaking does not affect our derivation. We assume a semi-classical quark anti-quark pair which points in some direction in the algebra. Without loss of generality this direction can be chosen as the '3' direction ('isospin'). The additional part in the Lagrangian is $-tr j^{ext}_\mu A^\mu$ where $j^{a \text{ ext}} \equiv [C(R_{ext})] 3^a \delta(x + L) - \delta(x - L)$ and $[C(R_{ext})]$ is a c-number which depends on the representation of the external source (see ref.[6]). The interaction term can be eliminated by a left-handed rotation in the '3' direction, of the gluino field (we are using a spherical basis, and so we can perform appropriate complex transformation also for real fermions)
\[ \lambda \rightarrow \tilde{\lambda} = e^{i\alpha(x) \frac{1}{2}(1-\gamma_5)T^3} \lambda \]

(17)

\[ \bar{\lambda} \rightarrow \tilde{\bar{\lambda}} = \bar{\lambda}e^{-i\alpha(x) \frac{1}{2}(1+\gamma_5)T^3} \]

(18)

\(T^3\) is in the 3 direction of the adjoint representation

\[
T^3 = \text{diag}(\mu_1, \mu_2, ..., \mu_{N_c^2-1})
= \text{diag}(1, 0, -1, \frac{1}{2}, -\frac{1}{2}, \frac{1}{2}, ..., 0, \underbrace{0, 0, ..., 0}_2)_{\text{doublets}}^{(N_c-2)^2}
\]

The chiral rotation introduces an anomaly term \(\text{tr} \frac{\alpha(x)T^3}{4\pi} F\), which is used to cancel the external charges. Note that the chiral rotation introduces additional terms. However, these terms involve more derivatives and therefore do not affect the string tension. This situation is very similar to the Abelian case and QCD2 \cite{6}.

The choice \(\alpha(x) = 2\pi \frac{C(R_{ext})}{N_c}(\theta(x+L) - \theta(x-L))\) leads to an action which is similar to the original (16), but has a chiral rotated term. The information of the external source is now transformed into a rotation angle.

The terms which are relevant to the computation of the string tension are those which appear in the interaction Lagrangian. In this case, it is the gluino pseudo-scalar term

\[ \text{tr} \ 2i\phi \bar{\lambda} \gamma_5 \lambda \rightarrow \text{tr} \ 2i\phi \bar{\tilde{\lambda}} \gamma_5 \tilde{\lambda} \]

(19)

Let us see how this change influences the Hamiltonian vacuum energy. In the original theory, without the external source, \(< H_0 >= 0\) since the theory is supersymmetric and \(H_0 \sim Q^2\) (where \(Q\) is the supercharge). In particular it means that there is no \(< \text{tr} \ \phi \bar{\lambda} \gamma_5 \lambda >\) condensate. Here we assume that SUSY is not broken dynamically. The numerical analysis of\cite{3} indicates that this is indeed the case.

Let us compute the Hamiltonian density of the rotated theory. In the regime \(-L < x < L\)

\[ < \mathcal{H} >= 2ig < \text{tr} \ \phi \bar{\tilde{\lambda}} \gamma_5 \tilde{\lambda} > \]

(20)
By using the fact that $T^3$ is diagonal, and the vacuum state is color symmetric, we get

$$< tr \phi \bar{\lambda} \gamma_5 \lambda > =$$

$$\frac{1}{N_c^2 - 1} \sum_a \cos(\alpha \mu_a) < tr \phi \bar{\lambda} \gamma_5 \lambda > - i \frac{1}{N_c^2 - 1} \sum_a \sin(\alpha \mu_a) < tr \phi \bar{\lambda} \lambda >,$$

where $\alpha = \lim_{L \to \infty} \alpha(x)$. The first term on the right hand side vanishes since as argued before $< tr \phi \bar{\lambda} \gamma_5 \lambda > = 0$, and the second term vanishes since the isospin eigenvalues, $\mu_a$, come in pairs of opposite signs.

Thus $<H> = 0$ and the string tension is zero.

Note that though we used the classical expression for the external current and the effective Hamiltonian may include other terms, these terms cannot change the value of the string tension. It is so because this theory contains only one dimension-full parameter, the gauge coupling $g$, and therefore the string tension is some number times $g^2$. We showed that this number is zero and higher terms in $g$ which may appear in the effective Hamiltonian cannot affect the string tension.

The meaning of the last result is that a quark anti-quark pair located at $x = \pm \infty$ does not generate a linear potential. In the non supersymmetric case, it is a consequence of infinitely many adjoint fermions which are produced from the vacuum, as there is no mass gap, that are attracted to the external source, form a soliton in the fundamental representation and result in screening it. We do not have a construction of the soliton, but such must occur as a result of the situation implied by the equivalence in [10]. We believe that similar mechanism occurs in the supersymmetric case too. A complementary argument [11, 4] is that due to loop effects, the intermediate gauge boson acquires a mass $M^2 \sim g^2 N_c$, which leads to a Yukawa potential between the external quark anti-quark pair.

The above result can be generalized to theories with extended supersymmetry and additional massive or massless matter content.

We argue that any supersymmetric gauge theory in two dimensions is screening. Technically, the reason is that the gluino is coupled to other fields in such a way that $<H> = 0$ (guaranteed if SUSY is not broken dynamically) and therefore there are no non-trivial chiral condensates. However, since the string tension is proportional to chiral condensates, SUSY leads to zero string tension. Physically, it follows from the fact that the gluino is an
adjoint *massless* fermion. Since it does not acquire mass, external sources are screened, as in the non-supersymmetric massless model.

In fact, the essential requirement for a screening nature of the type argued above, is to have among the charged particles at least one massless particle whose masslessness is protected by an unbroken symmetry. The symmetry can be gauge symmetry combined with supersymmetry or chiral symmetry.
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