Nonlinear vibration behaviors of marine rotor system coupled with floating raft-airbag-displacement restrictor under ship heaving motion

Xuan Xie, Ming Li and Junwei Wang

Abstract
To study the nonlinear vibration behaviors of rotor system coupled with floating raft-airbag-displacement restrictor under ship heaving motion, the dynamic model is established considering the effect of heaving motion, its steady-state responses are numerically obtained using Runge-Kutta method and the results are surveyed by tools such as the spectrum waterfall diagram, time-domain response, frequency-domain response, axis orbit, and Poincaré map. The effects of rotating speed, ship heaving amplitude, and its frequency on the nonlinear dynamic behavior of the system are mainly studied. The results show that the responses of the rotor and raft are of obvious nonlinear behaviors such as amplitude jumping, bifurcation, and chaos due to the effects of nonlinear oil film force and ship heaving motion. With the increase of rotating speed, the motion of rotor and raft presents quasi-periodic and chaotic vibrations. Ship heaving amplitude and its frequency all have great effect on the vibration of rotor and raft; as heaving amplitude or frequency increases, the motion state of rotor and raft changes, and the amplitude of raft increases significantly. The displacement restrictor can effectively limit the vibrating displacements of the raft when ship heaving amplitude or its frequency is large.

Keywords
Ship heaving motion, marine rotor-bearing system, floating raft-airbag-displacement restrictor structure, nonlinear vibration, chaos

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Introduction
Ship heaving motion is a common form of ship motion, which has the characteristics of large motion amplitude and low frequency and has a great effect on the safety and applicability of marine equipment.1,2 The vibration generated by marine rotating machinery seriously threatens the concealment of ships and the comfort of staff; therefore, it is of great importance to study the dynamic behaviors of the core component rotor-bearing system. Moreover, it is also necessary to assemble the floating raft isolation device to realize the vibration control of the rotor-bearing system. However, with ship heaving motion action, the vibration of rotor and raft will be affected by the heaving inertial force, and it is urgent to realize parameter optimization and vibration control of the system. Thus, it is vital to study the nonlinear vibration behaviors of the marine rotor system coupled with floating raft-airbag-displacement restrictor under ship heaving motion.

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In recent years, many scholars have made useful explorations on rotor-bearing system dynamics considering the base motion. For instance, Zhu and Chen\textsuperscript{3,4} established the dynamic model of the rotor-bearing system under arbitrary maneuver flight conditions based on Lagrange’s equation, and they studied the effect of maneuvering flight on the dynamic characteristics of the rotor-bearing system by the numerical calculation. And Hou et al.\textsuperscript{5–7} considered the influence of aircraft maneuvering flight on the vibration of rotor-bearing system, assuming that the support of the bearing on the rotor is Duffing nonlinear support and the maneuvering flight is sinusoidal flight, the dynamic equation is derived and solved by the multiple scale method, and the response and bifurcation characteristics of the system are analyzed. Furthermore, Qiu et al.\textsuperscript{8} and Yi et al.\textsuperscript{9} established a finite element model of the rotor-bearing system under base angular motion and studied its dynamic behavior considering the effect of time base angular motion on the dynamic behaviors of the asymmetric rotor-bearing system. Considering the effect of base motion, a dynamic equation of rotor system supported by squeeze film damper under base excitation was established by Chen et al.,\textsuperscript{10–12} and the dynamic characteristics of the system were studied, and Gao et al.\textsuperscript{13,14} analyzed the dynamic characteristics of aero-engine rotor system under maneuvering flight from theoretical simulation and experimental perspectives. The above researches are all about the dynamic behaviors of the rotor-bearing system considering base motion in the airborne field.

The motion of the ship has the characteristics of large amplitude and period under the effect of wind and wave, which is quite different from the maneuvering flight of the aircraft. More and more studies on the dynamics of the shipboard rotor-bearing system considering the base motion. For example, Soni et al.\textsuperscript{15,16} considered the effect of the ship’s foundation excitation on the rotor system that is supported by the magnetic bearing; then, they used Floquet-Liapunov theory to analyze the stability of the system and studied the dynamic behavior of the rotor system under base motion. Considering the effect of pitching and rolling motion on the dynamic characteristics of a marine rotor-bearing system, a dynamic model of the system was established by Zhang et al.\textsuperscript{17} and the nonlinear dynamic behaviors of the system were studied. Han and Li\textsuperscript{18} and Du and Li\textsuperscript{19} considered the effect of heaving motion on the dynamic characteristics of a rotating machinery system, and the dynamic behavior of rotating machinery system under heaving motion was studied by numerical method and multiple scale method. All of the above studies paid attention to the dynamic behaviors of the rotor-bearing system under base motion.

Because the floating raft-airbag vibration isolation device has the characteristics of low natural frequency, large bearing capacity and flexible structure, it is widely applied in the vibration control of marine rotating machinery.\textsuperscript{20} For instance, Fang et al.,\textsuperscript{21,22} Jiang et al.,\textsuperscript{23} and Lei et al.\textsuperscript{24} studied the vibration characteristics of rotating machinery coupled with floating raft-airbag vibration isolation device. Fang et al.\textsuperscript{21,22} analyzed the vibration transmission characteristics of a floating raft isolation system on the average method to obtain the approximate theoretical solution. And Jiang et al.\textsuperscript{23} took the floating raft active vibration isolation system as the research object and realized effective vibration control based on vibration acceleration and changes in vibration acceleration. In order to control the severe vibration problem of the heavy compressor, Lei et al.\textsuperscript{24} proposed a composite vibration control method, which used the floating raft isolation system and particle dampers to achieve effective vibration control.

There are few studies on rotor-bearing system dynamics considering both base motion and floating raft vibration isolation. With ship heaving motion action, the dynamic behaviors of the rotor system and the damping performance of the floating raft-airbag vibration isolation device need to be studied. Hence, in this study, we consider the effect of ship heaving motion on the dynamic behaviors of the marine rotor-bearing system coupled with floating raft-airbag-displacement restrictor structure. A dynamic model of the system is established and solved by a numerical method. Moreover, we focus on the effects of rotating speed, heaving amplitude and its frequency on the dynamic behaviors of the system.

### Dynamic model

#### Dynamic differential equations

A typical model of marine rotor system coupled with floating raft-airbag-displacement restrictor under heaving motion is shown in Figure 1, in which $O_0 - X_0Y_0Z_0$ is the absolute coordinate system that fixed on the ground, $O_1 - X_1Y_1Z_1$ and $O_2 - X_2Y_2Z_2$ are the ones that fixed in the center of the hull and the center of the bearing, respectively. Due to only considering the effect of ship heaving motion, the motion of the whole system in the $Z$ direction is ignored. Let the translational displacement coordinate of the hull be $(x_0, y_0)$ and the heaving displacement be $x_1$.

For the rotor system coupled with floating raft-airbag-displacement restrictor, here are the following assumptions: (1) The rotor is rigid, and its mass of disk is $m_1$ with the eccentricity $e_1$; rotor speed is $\omega$, and the vertical and horizontal displacements of the rotor from the initial position relative to the hull are $x_1$ and $y_1$. 

respectively. (2) The bearings are infinitely short, and their axial behavior is consistent. (3) The raft frame and bearing support are regarded as a rigid whole, and the mass is $m_2$; the vertical and horizontal displacements of the centroid from the initial position relative to the hull are $x_2$ and $y_2$, respectively. (4) The airbags are arranged symmetrically, which provide cubic nonlinear stiffness and linear damping, the total elastic forces of the airbags in the vertical and horizontal directions are $F_y = k_y x_1 + k_y x_1^3$, $F_h = k_h y_1 + k_h y_1^3$; the total damping factors of the airbags in the vertical and horizontal directions are $c_y$, $c_h$, respectively. (5) The displacement restrictors are limited to install the displacement of the raft. When the displacement of the raft is too large, the displacement restrictors provide linear vertical force $F_{vv}$ and lateral force $F_{hh}$.

According to the Newton Second Law, the dynamic differential equation of rotor system coupled with floating raft-airbag-displacement restrictor under ship heaving motion can be obtained

\[
\begin{align*}
    m_1 \ddot{x}_1 &= F_x(x_1 - x_2, y_1 - y_2, \dot{x}_1 - \dot{x}_2, \dot{y}_1 - \dot{y}_2) \\
    &\quad + m_1 e_1 \omega^2 \cos \omega t + m_1 g - m_1 \ddot{H} \\
    m_1 \ddot{y}_1 &= F_y(x_1 - x_2, y_1 - y_2, \dot{x}_1 - \dot{x}_2, \dot{y}_1 - \dot{y}_2) \\
    &\quad + m_1 e_1 \omega^2 \sin \omega t \\
    m_2 \ddot{x}_2 &= -F_x(x_1 - x_2, y_1 - y_2, \dot{x}_1 - \dot{x}_2, \dot{y}_1 - \dot{y}_2) - k_v x_2 \\
    &\quad - c_v \dot{x}_2 + m_2 g - m_2 \ddot{H} + F_{vv} \\
    m_2 \ddot{y}_2 &= -F_y(x_1 - x_2, y_1 - y_2, \dot{x}_1 - \dot{x}_2, \dot{y}_1 - \dot{y}_2) \\
    &\quad - k_h y_2 - c_h \dot{y}_2 + F_{hh}
\end{align*}
\]

in which $m_1$ is the mass of the rotor, $e_1$ is the mass eccentricity of the rotor, $\omega$ is the rotating speed; $m_2$ is the mass of the raft and bearing support; $k_v$ and $k_h$ are the total vertical linear stiffness coefficient, cubic nonlinear stiffness coefficient of airbags, respectively; $c_v$ and $c_h$ are vertical total damping factors and horizontal total damping factors of airbags, respectively. $F_x$ and $F_y$ are the bearing oil film forces in the $x$ and $y$ directions, respectively. Moreover, $\ddot{H}$ is the acceleration of ship heaving motion, which is

\[
\ddot{H} = -a_{max} \omega_H^2 \sin(\omega_H t)
\]

where $a_{max}$ is ship heaving amplitude, and $\omega_H$ is ship heaving frequency.

Figure 2 shows the schematic diagram floating raft-airbag-displacement restrictor structure. The displacement restrictor will provide the vertical elastic force $F_{vv}$ and the horizontal elastic force $F_{hh}$ when the raft collides with it. And the expressions of the elastic forces are as follows

\[
\begin{align*}
    F_{vv} &= \begin{cases} 
    -k_{vv}(|x_2 - x_{20}| - \Delta) & x_2 - x_{20} > \Delta \\
    0 & |x_2 - x_{20}| \leq \Delta \\
    k_{vv}(|x_2 - x_{20}| - \Delta) & x_2 - x_{20} < - \Delta 
    \end{cases} \\
    F_{hh} &= \begin{cases} 
    -k_{hh}(|y_2| - \Delta) & y_2 > \Delta \\
    0 & |y_2| \leq \Delta \\
    k_{hh}(|y_2| - \Delta) & y_2 < - \Delta 
    \end{cases}
\end{align*}
\]

where $k_{vv}$ and $k_{hh}$ are the vertical and horizontal stiffness of the displacement restrictor, respectively; $\Delta$ is the gap of displacement restrictor; $x_{20}$ is the static compression displacement of the airbags, which satisfies the expression
By simplifying equation (1), we can obtain

\[
\begin{align*}
\dot{x}_1 &= \ell(x_1-x_2,y_1-y_2)-\omega u_0^2 \cos \omega t + g - \ddot{x}_H \\
\dot{y}_1 &= \ell(x_1-x_2,y_1-y_2)-\omega u_0^2 \sin \omega t + g - \ddot{y}_H \\
\dot{x}_2 &= -F(x_1-x_2,y_1-y_2)-\omega \omega x_2^2 - \frac{\omega^2}{\delta_x} x_2^3 \\
\dot{y}_2 &= -F(x_1-x_2,y_1-y_2)-\omega \omega y_2^2 - \frac{\omega^2}{\delta_y} y_2^3
\end{align*}
\]

(5)

in which \( \omega u_0 = \sqrt{\frac{k_f}{m_f}} \), \( \omega u_0 = \sqrt{\frac{k_h}{m_h}} \), \( \delta_x = \sqrt{\frac{k_x}{m_x}} \), \( \delta_h = \sqrt{\frac{k_h}{m_h}} \). Among them, \( \omega u_0 \) and \( \omega u_0 \) are the vertical and horizontal natural frequencies of raft and airbags derived system; \( \zeta \) and \( \zeta_h \) are vertical and horizontal damping ratios; \( \delta_x \) and \( \delta_h \) are the vertical and horizontal characteristic lengths, respectively. Moreover, the elastic force \( F_v \) and \( F_h \) can also be written as

\[
F_v = \begin{cases} 
-\gamma h, \omega^2 \omega x_2^2 & (|x_2 - x_2| - \Delta) \\
0 & |x_2 - x_2| \leq \Delta \\
-\gamma h, \omega^2 \omega y_2^2 & (|y_2| - \Delta) \\
0 & |y_2| \leq \Delta \\
-\gamma h, \omega^2 \omega x_2^2 & (|y_2| - \Delta) \\
0 & |y_2| \leq \Delta 
\end{cases}
\]

(6)

where \( \gamma \) and \( \gamma_h \) are proportional coefficients, which respectively represent the ratio of the vertical stiffness of the displacement restrictor to the vertical linear stiffness of the airbags, the ratio of the horizontal stiffness of the displacement restrictor to the horizontal linear stiffness of the airbags. Their expressions are \( \gamma = k_v/k_v \) and \( \gamma_h = k_{hh}/k_h \).

**Bearing oil film force**

The structure diagram of the journal bearing, see in Figure 3. The transient oil film pressure distribution \( p(\theta, z) \) satisfies the Reynolds equation

\[
\frac{1}{R^2} \frac{\partial}{\partial \theta} \left( \frac{h^3}{12 \mu} \frac{\partial p}{\partial \theta} \right) + \frac{\partial}{\partial z} \left( \frac{h^3}{12 \mu} \frac{\partial p}{\partial z} \right) = \frac{1}{2} \left( \omega - 2 \phi \right) \frac{\partial h}{\partial \theta} + \dot{e} \cos \theta
\]

(7)

in which \( R \) is radius of the journal bearing; \( \mu \) is dynamic viscosity of lubricating oil; \( \epsilon \) and \( \phi \) are eccentricity and attitude angle of the bearing, respectively; \( \dot{e} = \sqrt{x^2 + y^2}, \dot{e} = \frac{de}{dt} \) and \( \phi = \frac{df}{dt} \). \( h \) is thickness of oil film, and \( h = c + e \cos \theta = c(1 + e \cos \theta) \), where \( c \) is oil gap of the journal bearing, \( \epsilon \) is eccentricity ratio of the journal bearing, and \( \dot{e} = e/c \).

According to the short bearing assumption and semi-Sommerfeld boundary condition, the oil film force expressions of bearing with the radial and tangential directions

\[
\begin{align*}
F_r &= \mu \frac{\dot{h}}{\dot{z}} \left( \omega - 2 \phi \right) G_1 + 2 \dot{e} G_2 \\
F_r &= \mu \frac{\dot{h}}{\dot{z}} \left( \omega - 2 \phi \right) G_3 + 2 \dot{e} G_4
\end{align*}
\]

(8)

in which \( L \) is axial length of bearing, \( \epsilon = \frac{de}{dt} \), \( G_1, G_2, G_3 \), and \( G_4 \) are Sommerfeld integrals whose expressions are as follows

\[
\begin{align*}
G_1 &= \frac{4c^2}{(1 - c^2)^2} \\
G_2 &= \frac{4c^2}{(1 - c^2)^{5/2}} \\
G_3 &= \frac{4c^2}{(1 - c^2)^2} \\
G_4 &= \frac{4c^2}{(1 - c^2)^{5/2}}
\end{align*}
\]

(9)

The expression of oil film force in the \( x \) and \( y \) directions can be obtained by the coordinate transformation

\[
\begin{align*}
F_x &= -F_r \cos \phi - F_r \sin \phi \\
F_y &= -F_r \sin \phi + F_r \cos \phi
\end{align*}
\]

(10)

**Dimensionless equation**

In order to eliminate the influence of physical units, the time scale \( \tau \) and the bearing oil film clearance \( c \) are introduced, and the motion differential equation is dimensionless. The specific dimensionless parameters are shown in Table 1.

Dimensionless to equation (5), and the dimensionless equation of the system under ship heaving motion can be obtained

\[
\begin{align*}
X_1'' &= \frac{f(X_1 - X_2, Y_1 - Y_2, X_1', Y_1')}{\alpha c} + \alpha \cos \tau \\
+ \frac{1}{\alpha c} + A_{max} \frac{\dot{h} \sin \theta}{\alpha c} \sin \frac{\theta}{\alpha c} \\
Y_1'' &= \frac{f(X_1 - X_2, Y_1 - Y_2, X_1', Y_1')}{\alpha c} + \alpha \sin \tau \\
X_2'' &= \frac{f(X_1 - X_2, Y_1 - Y_2, X_1', Y_1')}{\alpha c} + \alpha \cos \tau \\
- \frac{\dot{h} \sin \theta}{\alpha c} \sin \frac{\theta}{\alpha c} \\
Y_2'' &= \frac{f(X_1 - X_2, Y_1 - Y_2, X_1', Y_1')}{\alpha c} + \alpha \sin \tau \\
- \frac{\dot{h} \sin \theta}{\alpha c} \sin \frac{\theta}{\alpha c}
\end{align*}
\]

(11)

in which \( f_r \) and \( f_r \) are dimensionless nonlinear oil film forces, and there are expressions

\[
\begin{align*}
f_r &= -f_r \cos \phi - f_r \sin \phi \\
f_y &= -f_r \sin \phi + f_r \cos \phi
\end{align*}
\]

(12)
where

\[
\begin{align*}
f_v &= \frac{\Omega_{\text{in}}^2}{2}[(\Omega - 2\varphi')G_1 + 2\varphi'G_2] \\
f_h &= \frac{\Omega_{\text{in}}^2}{3}[(\Omega - 2\varphi')G_3 + 2\varphi'G_4]
\end{align*}
\]

moreover, \(f_v\) and \(f_h\) are the dimensionless elastic force provided by the displacement restrictor, their expressions are

\[
\begin{align*}
f_v &= \begin{cases} 
- \gamma_v \Omega_{\text{in}}^2(|X_2 - X_{20}| - \Delta) & X_2 - X_{20} > \Delta \\
0 & |X_2 - X_{20}| \leq \Delta \\
\gamma_v \Omega_{\text{in}}^2(|X_2 - X_{20}| - \Delta) & X_2 - X_{20} < - \Delta 
\end{cases} \\
f_h &= \begin{cases} 
- \gamma_h \Omega_{\text{in}}^2(|Y_2| - \Delta) & Y_2 > \Delta \\
0 & |Y_2| \leq \Delta \\
\gamma_h \Omega_{\text{in}}^2(|Y_2| - \Delta) & Y_2 < - \Delta
\end{cases}
\end{align*}
\]

where \(\Delta\) is the dimensionless gap of the displacement restrictor; \(X_{20}\) is the dimensionless static compression displacement of airbags, which satisfies the expression

\[
- \Omega_{\text{in}}^2 X_{20} - \frac{\Omega_{\text{in}}^2}{\delta_v} X_{20}^3 + 1 + \frac{1}{n} = 0
\]

where \(n\) is the mass ratio of the raft to the rotor.

### Numerical solution and simulation

#### Numerical simulation

The dimensionless dynamic equation of the rotor system coupled with floating raft-airbag displacement restrictor under heaving motion, which has four degrees of freedom, is obtained in Section 2. It is extremely difficult to obtain the analytical expression of the system solution because the bearing oil film force has strong nonlinear characteristics and the displacement restrictor provides the piecewise linear elastic force. In this study, we use the Runge-Kutta method to solve the dimensionless differential equation, and the steady-state response of the system can be obtained. The coordinates \(X = X_1 - X_2, Y = Y_1 - Y_2, X_2\), and \(Y_2\) in the figure represent the displacement of the rotor relative to the bearing in the \(x\) direction, the displacement of the rotor relative to the bearing in the \(y\) direction, the displacement of the raft in the \(x\) direction and the displacement of the raft in the \(y\) direction, respectively. The specific parameter values of the rotor system coupled with floating raft-airbag-displacement restrictor are shown in Table 2, and the value range of the dimensionless parameters of ship heaving motion are shown in Table 3.

Figure 4 shows the steady-state response of the system when \(\Omega = 1.55, A_{\text{max}} = 0, \text{ and } \Omega_H = 0, \text{ and }\) the initial values of the system are \(X_1 = 4.2, X_1' = 0, Y_1 = 0.1, Y_1' = 0, X_2 = 4, X_2' = 0, Y_2 = 0.05, \text{ and } Y_2' = 0.\) As shown in Figure 4(a), the rotor orbit is an elliptical line, and there is one point on the Poincaré map. From Figure 4(c), the spectrum of the rotor response presents a large amplitude of the rotating frequency \(f_1 = 1/(2\pi)\), accompanied by a slight amplitude of super-harmonic component \(2f_1\). From Figure 4(d), the trajectory of the raft is a “fishbone” shape, and there is one point on the Poincaré map. As shown in Figure 4(e), the displacement of the raft changes little with time, and it can be found that the displacement is also undergoing periodic vibration by local amplification. Figure 4(f) shows the frequency domain response of raft, and the spectrum of
the initial values of the system are
heaving frequency
the raft response presents a large amplitude of the
lanced inertial force. From Figure 5(f), the spectrum of
the displacement of the raft is also affected by unba-
amplification of Figure 5(e), it can also be found that
ally with time, and the swing is larger. After local
the raft displacement in Figure 5(e) changes sinusoid-
map. Comparing Figures 4(e) and 5(e), we can find that
the parameter
Symbol Value range
\( \lambda \)
0.2
\( \sigma \)
3
\( \alpha \)
0.05
\( n \)
60
\( \Omega_{\text{rn}} \)
0.5
\( \Omega_{\text{hn}} \)
0.5
\( \delta_{\text{x}} \)
20
\( \delta_{\text{y}} \)
20
\( \Delta \)
0.5
\( \gamma_{\text{v}} \)
22
\( \gamma_{\text{h}} \)
22

Table 2. The dimensionless parameter values of the rotor system coupled with floating raft-airbag- displacement restrictor.

| The description of the parameters | Symbol | Values |
|----------------------------------|--------|--------|
| Length diameter ratio of bearing | \( \lambda \) | 0.2 |
| Sommerfeld number                | \( \sigma \) | 3 |
| Rotor eccentricity ratio         | \( \alpha \) | 0.05 |
| Mass ratio                       | \( n \) | 60 |
| Linear natural frequency of floating raft and airbag in vertical direction | \( \Omega_{\text{rn}} \) | 0.5 |
| Linear natural frequency of floating raft and airbag in horizontal direction | \( \Omega_{\text{hn}} \) | 0.5 |
| Characteristic length in vertical direction | \( \delta_{\text{x}} \) | 20 |
| Characteristic length in horizontal direction | \( \delta_{\text{y}} \) | 20 |
| Gap of the displacement restrictor | \( \Delta \) | 0.5 |
| Proportional coefficient in vertical direction | \( \gamma_{\text{v}} \) | 22 |
| Proportional coefficient in horizontal direction | \( \gamma_{\text{h}} \) | 22 |

Table 3. The values range of dimensionless parameters for ship heaving motion.

| The description of the parameter | Symbol | Value range |
|----------------------------------|--------|-------------|
| Heaving amplitude                | \( A_{\text{max}} \) | 0–3000 |
| Heaving frequency                | \( \Omega_{\text{H}} \) | 0–0.03 |

the raft response presents a large amplitude of the rotating frequency \( f_1 \), accompanied by a small amplitude of super-harmonic component \( 2f_1 \). It can be concluded that under this set of parameters, the vibration of the rotor and the raft is mainly affected by the unbalanced inertial force, and the motion of the rotor and the raft is in period 1.

Figure 5 depicts the steady-state response of the system when \( \Omega = 1.55 \), \( A_{\text{max}} = 800 \), and \( \Omega_{\text{H}} = 0.01 \), and the initial values of the system are \( X_1 = 4.2 \), \( X'_1 = 0 \), \( Y_1 = 0.1 \), \( Y'_1 = 0 \), \( X_2 = 4 \), \( X'_2 = 0 \), \( Y_2 = 0.05 \), and \( Y'_2 = 0 \). Comparing Figures 4(a) and 5(a), we find that the rotor axis trajectory is complex, and there is no longer one point but a closed curve composed of points on the Poincaré map. From Figure 5(c), the spectrum of the rotor response presents a large amplitude of the rotating frequency \( f_1 \), accompanied by slight amplitudes of components \( 2f_1 \) and heaving frequency \( f_{\text{H}} \). Comparing Figures 4(d) and 5(d), it is easy to find that the displacement of the raft increases obviously in the \( x \) direction, and there is a closed curve on the Poincaré map. Comparing Figures 4(e) and 5(e), we can find that the raft displacement in Figure 5(e) changes sinusoidally with time, and the swing is larger. After local amplification of Figure 5(e), it can also be found that the displacement of the raft is also affected by unbalanced inertial force. From Figure 5(f), the spectrum of the raft response presents a large amplitude of the heaving frequency \( f_{\text{H}} \), accompanied by a slight amplitude of component \( f_1 \). It can be seen that the vibration of the rotor is affected by the unbalanced inertial force and heaving motion, and the vibration of the raft frame is mainly affected by the heaving motion. Under this set of parameters, the motion of rotor and raft is quasi-periodic. The marine rotor-bearing system coupled with floating raft-airbag-displacement restrictor, with heaving motion action, the amplitude of rotor and raft is larger than that without the above motion, and the motion of rotor and raft is more complex than that without the above motion.

Effect of rotating speed

The rotating speed directly affects the nonlinear characteristics of oil film force, and its effect on system dynamics needs to be specifically analyzed, especially when the system under ship heaving motion. In order to reflect the effect of the rotating speed on the dynamic behaviors of the system, we take the parameter values \( A_{\text{max}} = 300 \) and \( \Omega_{\text{H}} = 0.01 \), and the steady-state response of the system can be obtained by changing the rotating speed.

The relationship between the amplitude of the system and the rotating speed during the acceleration and deceleration process is shown in Figure 6. In order to save the calculation time, we take the last set of values obtained from the previous rotating speed calculation as the initial value of the next rotating speed calculation. From Figure 6(a), with the change of the rotating speed, the amplitude of the rotor has three resonance peaks, and the first resonance peak where rotating speed \( \Omega = 1.13 \) coincides with the linear natural frequency of the rotor and oil film; moreover, the amplitude of the rotor has an obvious jumping phenomenon when \( \Omega = 2.3 \) and \( \Omega = 3.16 \). From Figure 6(b), with the change of the rotating speed, the amplitude of the raft has four resonance peaks, and the first resonance peak corresponds to the rotating speed \( \Omega = 0.52 \), which is close to the value of the vertical linear natural frequency of the floating raft and airbags \( \Omega_{\text{vn}} \) in Table 2.
Figure 4. The steady-state response of the system when $\Omega = 1.55$, $A_{\text{max}} = 0$ and $\Omega_A = 0$: (a) rotor orbit and its Poincaré map, (b) time domain response of rotor, (c) frequency domain response of rotor, (d) raft trajectory and its Poincaré map, (e) time domain response of raft, and (f) frequency domain response of raft.

Figure 5. The steady-state response of the system when $\Omega = 1.55$, $A_{\text{max}} = 800$ and $\Omega_A = 0.01$: (a) rotor orbit and its Poincaré map, (b) time domain response of rotor, (c) frequency domain response of rotor, (d) raft trajectory and its Poincaré map, (e) time domain response of raft, and (f) frequency domain response of raft.
Similarly, the amplitude of the raft also appears apparent amplitude jumping phenomenon when $\Omega = 2.3$ and $\Omega = 3.16$.

Figure 7 illustrates the spectral waterfall diagram of the system when the rotating speed changes. From Figure 7(a), when the rotating speed is low, the spectrum of the rotor response presents a large amplitude of the rotating frequency $f_1$, accompanied by slight amplitudes of components $2f_1$ and heaving frequency $f_H$. With the increase of rotating speed, the spectrum of the rotor response presents large amplitudes of frequency $f_1/2$ and rotating frequency $f_1$, accompanied by small amplitudes of components $3f_1/2$, $2f_1$, and heaving frequency $f_H$. With the continuous increase of rotating speed, the spectrum of the rotor response is the same as that at low rotating speed. When the rotating speed is high, the spectrum of the rotor response presents a continuous spectrum. From Figure 7(b), with the increase of rotating speed, the spectrum of the raft response always presents a large amplitude of the heaving frequency $f_H$, accompanied by a slight amplitude of the rotating frequency $f_1$.

Figures 8 to 11 describe the steady-state responses of the rotor when the rotating speeds $\Omega$ are 0.89, 2.31, 2.87, and 3.48, respectively. From Figure 8, the rotor orbit is an “elliptical ring” surrounded by multiple elliptical lines, and there is a closed curve on the Poincaré map. The spectrum of the rotor response presents a large amplitude of the rotating frequency $f_1$, accompanied by slight amplitudes of components $2f_1$ and heaving frequency $f_H$. The motion of the rotor and the raft is quasi-periodic at this rotating speed.

Comparing Figure 9(a) with Figure 8(a), we can find that the amplitude of the rotor in Figure 9(a) is larger...
than that in Figure 8(a). As shown in Figure 9(a), the rotor orbit is “banana shape,” and there are two closed curves on the Poincaré map which means there is the bifurcation phenomenon before $\Omega = 2.31$, which corresponds to the jump phenomenon in Figure 6(a) when $\Omega = 2.30$. In Figure 9(c), the spectrum of the rotor response presents large amplitudes of rotating frequency $f_1$ and frequency $f_1/2$, accompanied by slight amplitudes of components $3f_1/2$, $2f_1$, and heaving frequency $f_H$. The nonlinear characteristics of the oil film force are reflected when $\Omega = 2.31$. Moreover, the vibration of rotor and raft is quasi-periodic.

Comparing Figure 10(a) with Figure 9(a), it is easy to find that the amplitude of rotor in Figure 10(a) is smaller than that in Figure 9(a). As shown in Figure 10(a), the rotor orbit is an “elliptical ring” composed of multiple elliptical lines, and there is a closed curve on the Poincaré map. In Figure 10(c), the spectrum of the rotor response presents a large amplitude of the rotating frequency $f_1$, accompanied by small amplitudes of components $2f_1$ and heaving frequency $f_H$. The rotor and raft perform quasi-periodic vibration.

From Figure 11(a), the rotor orbit seems regular but never repeats, and there are many scattered points on the Poincaré map. From Figure 11(b), the change of the displacement of the rotor with time is irregular. As shown in Figure 11(c), the spectrum of the rotor response is continuous and has multiple spectrum components. At this rotating speed, the vibration of the rotor and the raft appears as chaotic vibration.

In short, the system under ship heaving motion, with the change of rotating speed, the response of rotor and raft show obvious nonlinear characteristics such as amplitude jump, bifurcation, and chaos; and the nonlinear characteristics of oil film force are reflected when $2.30 \leq \Omega \leq 2.55$ and $3.13 \leq \Omega \leq 3.5$; moreover, the response of the system undergoes quasi-periodic and chaotic vibration.

**Effect of ship heaving parameters**

Heaving amplitude and frequency are essential characterization parameters of ship heaving motion, and their changes will affect the vibration of the whole system. In
this section, we mainly study the dynamic behaviors of the marine rotor-bearing system coupled with floating raft-airbag-displacement restrictor under ship heaving motion when the heaving amplitudes and frequencies change.

**Effect of heaving amplitude**

To reflect the effect of heaving amplitude on the dynamic behaviors of the system, we take the parameter value heaving frequency $\Omega_H = 0.01$, and the steady-state response of the system can be obtained by changing the heaving amplitude.

Figure 12 depicts the relation of system amplitude with rotating speed and heaving amplitude. As shown in Figure 12(a), the rotor amplitude and rotor speed curves at different heaving amplitudes have similar topological structures when the heaving amplitude $A_{\text{max}} \leq 1500$. However, the topological structures of the rotor amplitude and rotor speed curves change greatly when $A_{\text{max}} > 1500$, especially the rotor amplitude and rotor speed curves when $2.30 \leq \Omega \leq 2.55$. In Figure 12(b), the raft amplitude and rotor speed curves have the similar topological structures when $A_{\text{max}} \leq 1500$. However, the raft collides with the displacement restrictor when $A_{\text{max}} > 1500$, and the topological structures of the raft amplitude and rotor speed curves are greatly changed.

Figure 13 indicates the system amplitude and heaving amplitude curves at different rotating speeds. From Figure 13(a), when the rotating speed $\Omega = 1.05$, as heaving amplitude increases, the rotor amplitude increases almost linearly. When the rotating speed $\Omega = 2.18$ and $\Omega = 2.75$, the rotor amplitude increases almost linearly with the increase of $A_{\text{max}}$ when $A_{\text{max}} < 1500$, and there is a sudden increase in rotor amplitude when $A_{\text{max}} > 1500$. However, when the rotating speed $\Omega = 2.41$, the rotor amplitude remains the same or slightly decreases with the increase of $A_{\text{max}}$ when $A_{\text{max}} < 1500$, and there is a sudden jump in rotor amplitude when $1500 < A_{\text{max}} < 1750$; moreover, the rotor amplitude increases with the increase of $A_{\text{max}}$ when $A_{\text{max}} > 1750$. As shown in Figure 13(b), before the raft collides with the displacement restrictor, as the heaving amplitude increases, the raft amplitude increases almost linearly; however, the raft amplitude...
increases slightly with the increase of heaving amplitude when $A_{\text{max}} > 1500$, and the raft amplitude and heaving frequency curves at different speeds are obviously different.

Figure 14 shows the amplitude of the system and the ship heaving amplitude curve when $\Omega = 2.41$. As shown in Figure 14(a), the amplitude of the rotor is basically unchanged with the increase of the heaving amplitude when the heaving amplitude $A_{\text{max}} < 1500$; and the amplitude of the rotor decreases significantly with the increase of the heaving amplitude when $1500 \leq A_{\text{max}} \leq 1750$; when $A_{\text{max}} > 1750$, the amplitude of the rotor increases with the increase of the heaving amplitude. Moreover, when $A_{\text{max}} < 1350$, the displacement curve of rotor with displacement restrictor action coincides with that without displacement restrictor action, which means that the raft does not touch the displacement restrictor; when $A_{\text{max}} > 1350$, the displacement curve of rotor with displacement restrictor action is always below that without displacement restrictor action, which means that the displacement restrictor reduces the amplitude of rotor. From Figure 14(b), the displacement of the raft increases linearly with the increase of the heaving amplitude, and the displacement restrictor restricts the displacement of the raft when $A_{\text{max}} > 1350$. Combining Figure 14(a) and (b), we can find that the displacement restrictor can effectively control the vibration of the rotor and the raft when the ship heaving amplitude is large.

Figure 15 indicates the amplitude of the system and heaving amplitude curve during processes of heaving amplitude increasing and decreasing when $\Omega = 2.41$. In order to save the calculation time, we take the last set of values obtained from the previous heaving amplitude varying with rotating speed $\Omega$ and heaving amplitude $A_{\text{max}}$: (a) Rotor; and (b) Raft.
amplitude calculation as the initial value of the next heaving amplitude calculation. From Figure 15(a), with the change of the heaving amplitude, the amplitude of the rotor appears obvious jumping phenomenon when $1500 \leq A_{\text{max}} \leq 1750$. As shown in Figure 15(b), there is no obvious amplitude jumping phenomenon in the response of the raft with the change of heaving amplitude.

The spectral waterfall diagram of the system when the heaving amplitude changes (when $\Omega = 2.41$) is shown in Figure 16. As shown in Figure 16(a), when the heaving amplitude is small, the spectrum of the rotor response presents large amplitudes of frequency $f_1/2$ and rotating frequency $f_1$, accompanied by slight amplitudes of components $3f_1/2$, $2f_1$, and heaving frequency $f_H$. With the increase of heaving amplitude, the amplitude corresponding to frequency $f_1/2$ and the amplitude corresponding to frequency $3f_1/2$ in the spectrum of the rotor response gradually decrease; the amplitude corresponding to frequency $f_1$ and the amplitude corresponding to frequency $2f_1$ in the spectrum of the rotor response remain unchanged; and the amplitude corresponding to heaving frequency $f_H$ in the spectrum of the rotor response gradually increase. When the heaving amplitude is large, the spectrum of the rotor response presents large amplitudes of the rotating frequency $f_1$ and heaving frequency $f_H$, accompanied by a slight amplitude of component $2f_1$. From Figure

**Figure 14.** The relationship between the amplitude of the system and heaving amplitude: (a) the amplitude of the rotor and (b) the amplitude of the raft.

**Figure 15.** The relationship between the amplitude of the system and heaving amplitude during processes of heaving amplitude increasing and decreasing: (a) the amplitude of the rotor and (b) the amplitude of the raft.
When the heaving amplitude is small, the spectrum of the raft response presents a large amplitude of heaving frequency $f_H$, accompanied by a slight amplitude of frequency $f_1/2$; with the increase of heaving amplitude, the amplitude corresponding to heaving frequency $f_H$ in the spectrum of the raft response gradually increases; when the heaving amplitude is large, the spectrum of the raft response presents a large amplitude of heaving frequency $f_H$, accompanied by a slight amplitude of frequency $f_1$. Figure 17 describes the steady-state response of the system when $A_{\text{max}} = 900$, $\Omega = 2.41$, and $\Omega_H = 0.01$, and the initial values of the system are $X_1 = 4.2$, $X'_1 = 0$, $Y_1 = 0.1$, $Y'_1 = 0$, $X_2 = 4$, $X'_2 = 0$, $Y_2 = 0.05$, and $Y'_2 = 0$. From Figure 17(a), the rotor orbit is like a “banana” shape, and there are two closed curves on the...
Poincaré map. As shown in Figure 17(c), the spectrum of the rotor response presents large amplitudes of frequency $f_1/2$ and rotating frequency $f_1$, accompanied by small amplitudes of components $3f_1/2$, $2f_1$, and heaving frequency $f_H$. From Figure 17(d), there are two closed curves on the Poincaré map. From Figure 17(e), the displacement of raft with time is an approximate sine curve. As shown in Figure 17(f), the spectrum of the raft response presents a large amplitude of heaving frequency $f_H$, accompanied by a slight amplitude of frequency $f_1/2$. Under this group of parameters, the rotor and raft are in quasi-periodic vibration.

The steady-state response of the system when $A_{\text{max}} = 1950$, $\Omega = 2.41$, and $\Omega_H = 0.01$ is shown in Figure 18, in which the initial values of the system are $X_1 = 4.2$, $X_1' = 0$, $Y_1 = 0.1$, $Y_1' = 0$, $X_2 = 4$, $X_2' = 0$, $Y_2 = 0.05$, and $Y_2' = 0$. Comparing Figure 18(a) with Figure 17(a), it is obvious that the amplitude of the rotor is significantly reduced. From Figure 18(a), there is a closed curve on the Poincaré map. As shown in Figure 18(e), the displacement curve of the raft is no longer a sine curve under the action of the displacement restrictor. In Figure 18(f), the spectrum of the raft response presents a large amplitude of heaving frequency $f_H$, accompanied by a small amplitude of frequency $f_1$. Under this set of parameters, the vibration of rotor and raft is quasi-periodic.

**Effect of heaving frequency**

In order to study the effect of heaving frequency on the dynamic behaviors of the system, we take the parameter value $A_{\text{max}} = 430$, and the steady-state response of the system can be obtained by changing the heaving frequency.

Figure 19 indicates the relation of system amplitude with rotating speed and heaving frequency. As shown in Figure 19(a), the rotor amplitude and rotor speed curves at different heaving frequencies have similar topological structures when the heaving frequency $\Omega_H \leq 0.0175$. However, the topological structures of the rotor amplitude and rotor speed curves change greatly when $\Omega_H > 0.0175$, especially the rotor amplitude and rotor speed curves when $2 \leq \Omega \leq 3.5$. As shown in Figure 19(b), the raft amplitude and rotor speed curves have the similar topological structures when $\Omega_H = 0.0175$. However, the raft collides with the displacement restrictor when $\Omega_H > 0.0175$, and the
topological structures of the raft amplitude and rotor speed curves are greatly changed.

Figure 20 shows the system amplitude and heaving frequency curves at different rotating speeds. From Figure 20(a), when the rotating speed $\Omega = 1.05$ and $\Omega = 2.18$, as heaving frequency increases, the rotor amplitude increases nonlinearly. When the rotating speed $\Omega = 2.41$, the rotor amplitude is basically unchanged with the increase of $\Omega_H$ when $\Omega_H < 0.015$, and there is a sudden decrease in rotor amplitude when $0.015 < \Omega_H < 0.02$; moreover, the rotor amplitude increases nonlinearly with the increase of $\Omega_H$ when $\Omega_H > 0.02$. However, when the rotating speed $\Omega = 2.75$, the rotor amplitude increases nonlinearly with the increase of $\Omega_H$ when $\Omega_H < 0.024$, and there is a sudden increase in rotor amplitude when $\Omega_H > 0.024$, which needs to be further studied.

As shown in Figure 20(b), before the raft collides with the displacement restrictor, as the heaving frequency increases, the raft amplitude increases nonlinearly; however, the raft amplitude increases slightly with the increase of heaving frequency when $\Omega_H > 0.0175$, and the raft amplitude and heaving frequency curves at different speeds are obviously different.

Figure 21 illustrates the amplitude of the system and ship heaving frequency curve when $\Omega = 2.75$. In Figure 21(a), when the heaving frequency $\Omega_H < 0.024$, the amplitude of the rotor increases nonlinearly with the increase of the heaving frequency and its increase rate is increasing, and the amplitude of the rotor is still tiny; when $\Omega_H > 0.024$, the amplitude of the rotor increases sharply with the increase of the heaving frequency. Moreover, the displacement restrictor reduces the
amplitude of the rotor when $0.0235 \leq \Omega_H \leq 0.0255$. From Figure 21(b), the amplitude of raft increases non-linearly with the increase of heaving frequency, and its increase rate increases. And the displacement of the raft is limited by the displacement restrictor when $\Omega_H \geq 0.0175$. Combining Figure 21(a) and (b), we can find that installing the displacement restrictor can effectively reduce the vibration of rotor and raft when the heaving frequency is within a specific range.

Figure 22 shows the amplitude of the system and heaving frequency curve during processes of heaving frequency increasing and decreasing (when $\Omega = 2.75$). In order to save the calculation time, we take the last set of values obtained from the previous heaving frequency calculation as the initial value of the next heaving frequency calculation. From Figure 22(a), with the change of the heaving frequency, the amplitude of the rotor appears apparent jumping phenomenon when the heaving frequency $\Omega_H = 0.0245$ and $0.0275 \leq \Omega_H \leq 0.03$. From Figure 22(b), there is no obvious amplitude jumping phenomenon in the response of the raft with the increase and decrease of the heaving frequency.

The spectral waterfall diagram of the system when the heaving frequency changes (when $\Omega = 2.75$) is shown in Figure 23. As shown in Figure 23(a), when the heaving frequency is low, the spectrum of the rotor response presents a large amplitude of the rotating frequency.
frequency $f_1$, accompanied by slight amplitudes of component $2f_1$ and heaving frequency $f_H$. With the increase of heaving frequency, the amplitude corresponding to frequency $f_1$ and the amplitude corresponding to frequency $2f_1$ in the spectrum of the rotor response remain unchanged; and the amplitude corresponding to heaving frequency $f_H$ in the spectrum of the rotor response gradually increases. When the heaving frequency is high, the spectrum of the rotor response presents a continuous spectrum. From Figure 23(b), the spectrum of the raft response presents a large amplitude of heaving frequency $f_H$, accompanied by a small amplitude of frequency $2f_H$. With the increase of heaving frequency, the amplitude corresponding to heaving frequency $f_H$ in the spectrum of the raft response gradually increases.

The steady-state response of the system when $\Omega_H = 0.0125$, $\Omega = 2.75$ and $A_{\text{max}} = 430$ is shown in Figure 24, in which the initial values of the system are $X_1 = 4.2$, $X'_1 = 0$, $Y_1 = 0.1$, $Y'_1 = 0$, $X_2 = 4$, $X'_2 = 0$, $Y_2 = 0.05$ and $Y'_2 = 0$. From Figure 24(a), the rotor orbit is an “elliptical ring” composed of multiple elliptical lines, and there is a closed curve on the Poincaré map. In Figure 24(c), the spectrum of the rotor response presents a large amplitude of the rotating frequency $f_1$, accompanied by slight amplitudes of component $2f_1$ and heaving frequency $f_H$. From Figure 24(d), there is a closed curve on the Poincaré map. As shown in Figure 24(f), the spectrum of the raft response presents a large amplitude of the heaving frequency $f_H$. Under this set of parameters, the motion of rotor and raft is quasi-periodic.

Figure 25 shows the steady-state response of the system when $\Omega_H = 0.0275$, $\Omega = 2.75$, and $A_{\text{max}} = 430$, in which the initial values of the system are $X_1 = 4.2$, $X'_1 = 0$, $Y_1 = 0.1$, $Y'_1 = 0$, $X_2 = 4$, $X'_2 = 0$, $Y_2 = 0.05$, and $Y'_2 = 0$. As shown in Figure 25(a), the rotor orbit seems regular but never repeats, and there are many scattered points on the Poincaré map. Comparing Figure 25(a) with Figure 24(a), it is evident that the amplitude of the rotor increases a lot. From Figure 25(c), the spectrum of the rotor response is continuous and has multiple spectrum components. As shown in Figure 25(d), the trajectory of the raft seems to be regular but never repeats, and there are many scattered points on the Poincaré map. From Figure 25(f), the spectrum of the raft response is continuous. Under this group of parameters, the vibration of rotor and raft is chaotic.

Conclusions

The dynamic characteristics of the rotor system coupled with floating raft-airbag-displacement restrictor under ship heaving motion are studied. We established the dynamic model of the system under ship heaving motion, and its nonlinear dynamic behaviors are mainly investigated. The conclusions are drawn as follows:

1. The response of rotor and raft under the ship heaving motion, the amplitude is larger than that without above motion, and the motion state becomes complex; for example, there exists the jumping phenomenon, bifurcation behavior and chaotic oscillation.

2. With the increase of heaving amplitude, the rotor amplitude and heaving amplitude curves corresponding to different rotating speeds are different, and there exists the amplitude jumping phenomenon in the response of the rotor;
Figure 24. The steady-state response of the system when $\Omega_{\delta} = 0.0125$, $\Omega = 2.75$, and $A_{\text{max}} = 430$: (a) rotor orbit and its Poincaré map, (b) time domain response of rotor, (c) frequency domain response of rotor, (d) raft trajectory and its Poincaré map, (e) time domain response of raft, and (f) frequency domain response of raft.

Figure 25. The steady-state response of the system when $\Omega_{\delta} = 0.0275$, $\Omega = 2.75$, and $A_{\text{max}} = 430$: (a) rotor orbit and its Poincaré map, (b) time domain response of rotor, (c) frequency domain response of rotor, (d) raft trajectory and its Poincaré map, (e) time domain response of raft, and (f) frequency domain response of raft.
the amplitude of the raft is almost linearly increasing before it collides with the displacement restrictor.

(3) As the heaving frequency increases, the variation of rotor amplitude with heaving frequency at different rotating speeds is different, and there is a jump phenomenon in the rotor amplitude; moreover, the amplitude of the raft increases nonlinearly before the collision with the displacement restrictor.

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