A General Investigation on the Differential Leakage Factor for Symmetrical and Asymmetrical Multiphase Winding Design

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Abstract: This work provides an investigation based on a fast estimation of the degree of unbalance (D.U.%) and the differential leakage factor (\(\sigma_0\)) of multiphase electrical machine windings. This analysis is carried out by exploring almost 5000 combinations in terms of number of slots, pole pairs, phases and layers. The variability of the leakage factor is examined for each condition, defining an optimal region for its minimization. As a result, an extended mapping is carried out for both the degree of unbalance and the leakage factor, providing a useful tool during the early design stage of winding configurations for multiphase electric machines, even with slight asymmetries. The results obtained from this investigation are validated through finite element analysis and demonstrate that the differential leakage factor can be significantly reduced by adopting winding configurations with slight asymmetries, which also represent a valuable alternative in the electrical machine design.

Keywords: electrical machines; differential leakage factor; symmetrical windings; asymmetrical windings; unbalance degree; winding design

1. Introduction

In recent years, scientific research regarding electric machines has been strongly directed towards the adoption of multi-phase winding configurations, thanks to the many advantages over conventional three-phase topologies, such as fault-tolerance, lower harmonic currents and torque ripple [1–4]. Moreover, adopting slightly asymmetrical winding configurations represents a valuable element during the early stage of the electric machine design, that can bring several advantages in different application fields [5–8]. It is well known that, historically, winding configurations with slight asymmetries were only used for pole-changing machines [9,10] or for motors with Pole Amplitude Modulation (PAM) [11]. Nevertheless, recent advances have demonstrated that asymmetrical winding configurations may offer a significant performance improvement by increasing the number of possible slot/pole combinations, while keeping comparable performances to traditional machines. It is worthwhile noting here that even symmetrical windings, once fabricated, present slight asymmetries due to manufacturing tolerances. It has also been demonstrated that a careful use of asymmetries may even serve to improve motor performances by reducing the effects of slot harmonics [12]. Moreover, the exact knowledge of the differential leakage factor, namely \(\sigma_0\), is a crucial aspect for the adequate characterization of an electrical machine during the design stage, because it represents a valuable
parameter influencing some of the performances of the machine itself. In this context, we suggest extended mapping in terms of the unbalance degree and the leakage factor for both single-layer and double-layer multiphase symmetrical and asymmetrical winding configurations of rotating electrical machines. This mapping can be a useful tool for the designer with the aim of choosing the optimal combination between the slot/poles ratio and the number of phases during the design stage of the related winding, even with a slightly asymmetrical configuration. More in detail, the proposed investigation is extended to electrical machines whose layer number $y$ ranges from 1 (single-layer) to 2 (double-layer), the slots number $N$ from 20 to 60, the pole pairs number $p$ is from 1 to 15 and the number of phases $m$ from 3 to 6, exploring almost 10,000 values of unbalance degree and differential leakage factor. This paper is organized as follows: the definition of the symmetry winding conditions, of the unbalance degree and the investigation of its variability is reported in Section 2. Section 3 describes the general formula for the determination of the differential leakage factor in $m$-phase motors. Section 4 reports and discusses the obtained mapping for all the analyzed winding configurations; finally, the finite element validation is proposed in Section 5.

2. Symmetry Conditions and Degree of Unbalance in $m$-Phase Machine Windings

Considering a generic $m$-phase electrical machine winding with $N$ slots, $y$-layers and $p$ pole pairs, a set of general symmetry conditions can be easily introduced [13,14].

Let $t$, the number of repetitions of the elementary winding:

$$t = \gcd(N, p),$$

and $\eta$ the number of empty slots. The number of slots per phase per repetition can be defined as:

$$g = \begin{cases} \frac{N}{mt} & \text{for normal and non-reduced systems} \\ \frac{N}{2mt} & \text{for reduced systems.} \end{cases}$$

The first symmetry condition follows:

$$g \in \mathbb{N} \quad \text{for symmetrical windings}$$

$$g \notin \mathbb{N} \quad \text{for asymmetrical windings.}$$

Moreover, naming $\gamma$ as

$$\gamma = y \frac{N - \eta}{2m}$$

where

$$\gamma \in \mathbb{N} \quad \text{for coil windings}$$

$$\gamma \in \{\mathbb{N}, \mathbb{N}/2\} \quad \text{for bar windings.}$$

$\mathbb{N}$ is the set of natural numbers, whereas $\eta$ represents the number of empty (unfilled) slots. The winding is defined as symmetrical if, and only if, conditions (5) and the first of Equation (3) are satisfied. On the contrary, the winding is defined as asymmetrical.

Moreover, the definition of the degree of unbalance $D.U.\%$ is given by the following formula:

$$D.U.\% = \frac{k_{wpd}}{k_{wpi}} \cdot 100$$

where $k_{wpd}$ is the direct component of the winding factor at the $p$th harmonic, whereas $k_{wpi}$ is the inverse component of the winding factor at the $p$th harmonic. These components and the related phases for the $h$th phase can be computed as follows [13,15,16]:
\[ k_{wp,v,h} = \frac{1}{2pqy} \sqrt{\left( \sum_{n=1}^{2pqy} C_{p,v,h,n} \right)^2 + \left( \sum_{n=1}^{2pqy} S_{p,v,h,n} \right)^2} \]  

(7)

and

\[ \text{arg} \left( k_{wp,v,h} \right) = \text{atan2} \left( \sum_{n=1}^{2pqy} S_{p,v,h,n}, \sum_{n=1}^{2pqy} C_{p,v,h,n} \right), \]

(8)

where

\[ C_{p,v,h,n} = k \cdot \cos \left( 2\pi pv \frac{|a_{h,n}|}{N} \right), \]

(9)

\[ S_{p,v,h,n} = k \cdot \sin \left( 2\pi pv \frac{|a_{h,n}|}{N} \right), \]

(10)

\[ k = \begin{cases} 
+1 & \text{if } a_{h,n} > 0 \\
-1 & \text{if } a_{h,n} < 0 
\end{cases} \]

(11)

where \( v \) is the generic harmonic order, \( h = 1 \ldots m \) is the \( h \)th winding phase and the \( a_{h,n} \) are the slot emf coefficients, which can be computed by referring to [13]. If the winding is symmetrical, the computation of harmonic winding factors can be limited to only one phase (for example, the 1st phase—i.e., \( h = 1 \)). If D.U.% is less than 5%, the winding can be classified as slightly asymmetrical.

The variability of the unbalance degree in polyphase machines can be investigated by applying Equation (6), computed through the ACWIND Software [16], for different winding configurations in terms of number of slots, pole pairs, phases and number of layers. More in detail, these parameters have been varied according to Table 1, reaching 4800 combinations, which are graphically summarized in Figure 1.

This range of variability has been selected in order to cover the most common base winding configurations, both symmetrical and asymmetrical. The values of D.U.% are also classified as a function of the number of phases and number of layers. From these graphs, it appears clear that the adoption of slightly asymmetrical windings allows a higher possibility of choices with regard to the slot-poles combinations. Indeed, almost 96% of the examined combinations can be theoretically considered between symmetrical (18%) and slightly asymmetrical (76%) configurations. Moreover, the range of symmetrical combinations decreases for increased values of \( m \), while, as \( m \) increases, the possibility of asymmetrical configurations increases up to 500 combinations. The results regarding the four-phase configuration, even if reported in the previous graphs, should not be considered due to their low practical utility.

| Parameter | Range of Variability |
|-----------|----------------------|
| \( N \)   | 20 ÷ 60              |
| \( p \)   | 1 ÷ 15               |
| \( m \)   | 3 ÷ 6                |
| \( y \)   | 1 ÷ 2                |
3. The Differential Leakage Factor in \( m \)-Phase Machine Windings

It is well known that the differential leakage factor, namely \( \sigma_0 \), is a valid parameter for the characterization of an electrical machine during its early design stage. However, the exact determination of \( \sigma_0 \), especially for multiphase asymmetrical machines, could be very troublesome \[11,17,18\].Recently, the authors have proposed a general procedure that simplifies its exact computation by adopting the Goerges polygon \[19,20\]. Generally, \( \sigma_0 \) is defined as follows \[11,18,20–24\]:

\[
\sigma_0 = \frac{W_\delta - W_p}{W_p} = \frac{W_\delta}{W_p} - 1 \tag{12}
\]

where \( W_p \) is defined as the magnetic energy at the working harmonic, whereas \( W_\delta \) represents the magnetic energy in the air-gap, which can be expressed by the following formula:

\[
W_\delta = \mu_0 l_i^2 \int_0^{2p\tau} v_p^2(\xi) d\xi = \frac{\mu_0 l_i^2 p\tau}{2\delta''} \cdot V_p^2 \tag{13}
\]

where \( \mu_0 \) is the permeability of vacuum, \( l_i \) is the ideal axial length of the machine, \( \tau \) is its pole pitch, \( v_i \) is the instantaneous value of air-gap MMF at a generic \( i \)th slot and \( \delta'' \) is the fictive air-gap length, given by:

\[
\delta'' = k_C k_{sat} \delta, \tag{14}
\]

where \( k_C \) is the Carter factor, \( k_{sat} \) is the saturation factor and \( \delta \) is the air-gap length.

The energy \( W_p \) can be expressed as follows:

\[
W_p = \frac{\mu_0 l_i^2}{2\delta''} \cdot \frac{1}{N} \sum_{i=1}^{N} v_i^2 \tag{15}
\]

where

\[
v_p(\xi) = V_p \sin \left( \frac{\pi \xi}{\tau} \right).
\]

Finally, \( \sigma_0 \) is given by:

\[
\sigma_0 = \frac{W_\delta}{W_p} - 1 = \frac{2}{V_p^2} \cdot \frac{1}{N} \sum_{i=1}^{N} v_i^2 - 1 \tag{16}
\]

\( V_p \) is defined as follows:
where \( w \) is the number of series-connected coil turns per phase and \( k_{wp} \) is the winding factor at the working harmonic \((v = p)\). Finally, \( I \) and \( I_s \) are the \( rms \) and peak values of the sinusoidal phase current, respectively.

It must be underlined that symmetrical Goerges polygons lead to constant leakage factor, independently from the slot number taken into account. On the contrary, asymmetrical polygons (to which correspond, in the majority of cases, asymmetrical winding configurations) lead to temporal variability of the leakage factor. For the latest case, the mean leakage factor all over the electric period is considered. In addition, this analysis is carried out by imposing a symmetrical system of supply currents.

4. Results and Discussion

This section presents an extended mapping for the fast estimation of both \( D.U.% \) and \( \sigma_0 \) in a \( N - p \) plane, providing a useful tool for the designers in the early design stage of electric machine windings.

A first result to be discussed is reported in Figure 2a, which provides the mapping in a \( N - p \) plane of all the possible double-layer symmetrical configurations for \( m \) ranging between 3 and 6. As expected, it can be noticed that several \( N - p \) combinations are symmetrical for any number of phases (e.g., \( N60 - p1, N60 - p7, \) etc.), whereas other slot-poles combinations are symmetrical for only a specific \( m \) \((N21 - p1\) only for \( m = 3, N20 - p2 \) only for \( m = 5 \) and so on). In addition, Figure 2b summarizes the distribution in the \( N - p \) plane of asymmetrical configurations with \( D.U. > 5\% \).

It can be noticed that the intensification of these configurations is detected for a low number of slots. The latest mapping could be useful for the designer in order to avoid the \( N - p - m \) combinations that provide high unbalances in the winding. It appears evident that the solutions not included in Figure 2a,b correspond to the \( N - p - m \) combinations of slightly asymmetrical windings. Therefore, the simultaneous analysis of the two mappings can help the designer to choose the optimal solution in terms of slot/poles ratio.

Another significant result is in Figure 3, comparing the mapping, in the \( N - p \) plane, of the leakage factors for all proposed double-layer and single-layer combinations. In this case, the mapping is limited to values of leakage factor confined within 20\%. Firstly, for all graphics, a region with a low value of \( \sigma_0 \) can be identified, mainly corresponding to areas with a low number of pole pairs. This region appears more confined and less uniform for single-layer winding configurations. Secondly, for each graph, some intensification regions can be detected, as shown by the red areas plotted in the same Figure, corresponding to the \( N - p - m - y \) winding configurations with a value of \( \sigma_0 \) close to 20\%. Furthermore, the white region corresponds to all the \( N - p \) combinations with \( \sigma_0 > 20\% \), which should be avoided by the designer. All the plotted graphics are also integrated with the possible symmetrical configurations of the specific phase taken into account (red dots).

Finally, Figure 4 reports the mapping of all the possible valuable configurations with a leakage factor < 20\% and a \( D.U.% < 5\% \) for all proposed phase numbers. This comparison leads to the fact that the five-phase configuration provides a higher choice of possibilities in terms of \( N - p \) combination.

In conclusion, it can be stated that the graph of Figure 4 can be used by the designer as an aid for choosing a suitable combination of slots, poles and phases minimizing the related leakage factor. On the contrary, it appears clear that the \( N - p - m \) combinations corresponding to relevant peaks of \( \sigma_0 \) should be avoided by the designers.

In order to provide a practical example of the proposed approach, a five-phase, double-layer winding located into 45 slots of a 10 poles machine is taken into account. The mapping of Figure 3c and the overview of Figure 4 demonstrate the feasibility of its design in terms of both degree of unbalance and leakage factor, allowing for their fast estimation and avoiding complex computations. Indeed, according to (6) and (16), the configuration has a slight asymmetry (\( D.U.% \) is equal to 0.4\%)
and $\sigma_0$ is equal to 11.44%, obtained by means of the Goerges polygon plotted in Figure 5 [25,26]. The related winding scheme and the spatial distribution of the Magneto Motive Force (MMF) are plotted in Figures 6 and 7, respectively.

![Symmetrical configurations](image1)

![Asymmetrical configurations with D.U.% > 5%](image2)

**Figure 2.** Mapping, for $m = 3 \div 6$, of the (a) symmetrical configurations and (b) asymmetrical configurations with D.U.% > 5%.

A dual three-phase, double-layer winding located into 36 slots of a 4 poles machine is considered as a second practical example. As well as for the previous case, according to the mapping of Figures 3e and 4, this winding configuration, whose layout scheme is depicted in Figure 8, can be assumed as feasible. According to (6) and (16), the winding configuration is slightly asymmetrical and $\sigma_0$ is equal to 1%. The corresponding Goerges polygon is plotted in Figure 9, whereas the spatial distribution of the Magneto Motive Force is depicted in Figure 10. As previously reported, the proposed mapping of
both D.U.% and $\sigma_0$ can help the designer for the fast estimation of crucial parameters involved during the design stage of electrical machines.

Figure 3. Mapping of the leakage factors for all the proposed combinations: (a) $m = 3$, $y = 2$, (b) $m = 3$, $y = 1$, (c) $m = 5$, $y = 2$, (d) $m = 5$, $y = 1$, (e) $m = 6$, $y = 2$ and (f) $m = 6$, $y = 1$. 
Figure 4. Mapping of the $N - p$ configurations with $D.U. < 5\%$ and $\sigma_0 < 20\%$ ($m = 3 \div 6$).

Figure 5. Goerges polygon for a five-phase machine with $N = 45$, $p = 5$ and $y = 2$.

Figure 6. Winding scheme for a five-phase machine with $N = 45$, $p = 5$ and $y = 2$. 
Figure 7. Spatial MMF distribution for a five-phase machine with $N = 45$, $p = 5$ and $y = 2$.

Figure 8. Winding scheme for a six-phase machine with $N = 36$, $p = 2$ and $y = 2$.

Figure 9. Goerges polygon for a six-phase machine with $N = 36$, $p = 2$ and $y = 2$. 
5. Finite-Element Validation

The Finite-Element Method (FEM) is used to demonstrate the validity of the obtained values of the mean leakage factors for the winding configurations taken into account in the previous sections. More in detail, the FEMM4.2 software is adopted in order to carry out the finite element models of four Interior Permanent Magnet Synchronous Motors (IPMSMs), which mainly differ for the number of slots (equal to 24, 27, 30 and 33). For each of the 6-poles model, the number of phases has been varied from 3 to 6, reaching 16 models with different combinations between $N$ and $m$.

Afterwards, the angular position of the rotor has been varied from $0^\circ$ to $360^\circ$ with steps of $10^\circ$, obtaining 37 simulations for each model and almost 600 overall simulations. Figure 11 shows an example of the simulation results, in terms of flux density plot, achieved from the FEM analysis. Moreover, the variability over an entire electrical cycle of the leakage factor, computed both with FEM and with (16), for configurations with asymmetrical and symmetrical Goerges polygons, are depicted in Figures 12 and 13, respectively. In any of the proposed cases, the comparison between the computed leakage factors and the FEM results confirms the validity of the reported $\sigma_0$ values.

Figure 10. Spatial MMF distribution for a six-phase machine with $N = 36$, $p = 2$ and $y = 2$.

Figure 11. Flux-density distribution from FEM simulation
Figure 12. FEM validation for asymmetrical Goerges polygons.

Figure 13. FEM validation for symmetrical Goerges polygons.

6. Conclusions

This paper has presented an investigation on the \( D.U.\% \) and \( \sigma_0 \) in \( m \)-phase machines, providing an extended mapping that could help the designer during the early winding design stage. The fast estimation and analysis of these parameters have been carried out through almost 10,000 values among different \( D.U.\% \) and \( \sigma_0 \). An optimal region has been identified for each configuration, with respect to \( m \) and \( y \). The results obtained from this investigation, which have been successfully validated by means of over 600 simulations carried out from the finite-element analysis of several models of electric machines, demonstrate that the use of slightly asymmetrical windings can both provide a higher choice of \( N - p \) combinations and considerably reduce the leakage factor.
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References
1. Barrero, F.; Duran, M.J. Recent Advances in the Design, Modeling, and Control of Multiphase Machines—Part I. IEEE Trans. Ind. Electron. 2016, 63, 449–458. [CrossRef]
2. Duran, M.J.; Barrero, F. Recent Advances in the Design, Modeling, and Control of Multiphase Machines—Part II. IEEE Trans. Ind. Electron. 2016, 63, 459–468. [CrossRef]
3. Liu, Y.; Niu, S.; Fu, W.N. A Novel Multiphase Brushless Power-Split Transmission System for Wind Power Generation. IEEE Trans. Magn. 2016, 52, 1–7. [CrossRef]
4. Levi, E. Multiphase Electric Machines for Variable-Speed Applications. IEEE Trans. Ind. Electron. 2008, 55, 1893–1909. [CrossRef]
5. Demir, Y.; Aydin, M. A Novel Asymmetric and Unconventional Stator Winding Configuration and Placement for a Dual Three-Phase Surface PM Motor. IEEE Trans. Magn. 2017, 53, 1–5. [CrossRef]
6. Demir, Y.; Aydin, M. A Novel Dual Three-Phase Permanent Magnet Synchronous Motor With Asymmetric Stator Winding. IEEE Trans. Magn. 2016, 52, 1–5. [CrossRef]
7. Caruso, M.; Di Tommaso, A.O.; Miceli, R.; Rizzo, R. The use of slightly asymmetrical windings for rotating electrical machines. Int. Trans. Electr. Energy Syst. 2018, 28, e2569. [CrossRef]
8. Caruso, M.; Di Tommaso, A.O.; Ferraris, L.; Miceli, R.; Viola, F. Finite-element performance comparison of IPMSMs with unsymmetrical double-layer windings. In Proceedings of the 2017 Twelfth International Conference on Ecological Vehicles and Renewable Energies (EVER), Monte Carlo, Monaco, 11–13 April 2017; pp. 1–6. [CrossRef]
9. Cistelecan, M.V.; Cosan, B.; Popescu, M.D. Analysis and Design Criteria for Fractional Unbalanced Windings of Three-phase Motors. In Proceedings of the 6th International Symposium on Advanced Electromechanical Motion Systems—ELECTROMOTION 2005, Lausanne, Switzerland, 27–29 September 2005; pp. 1–5. [CrossRef]
10. Melcescu, L.; Cistelecan, M.; Craiu, O.; Cosan, H. A new 4/6 pole-changing double layer winding for three phase electrical machines. In Proceedings of the 2010 XIX International Conference on Electrical Machines (ICEM), Rome, Italy, 6–8 September 2010; pp. 1–6. [CrossRef]
11. Müller, G.; Vogt, K.; Ponick, B. Berechnung Elektrischer Maschinen, 6th ed.; Elektrische Maschinen, Wiley-VCH Verlag GmbH & Co. KGaA: Weinheim, Germany, 2008.
12. Kouchih, D.; Boumalha, N.; Tadjine, M.; Boucherit, M.S. New approach for the modeling of induction machines operating under unbalanced power system. Int. Trans. Electr. Energy Syst. 2016, 26, 1832–1846. [CrossRef]
13. Caruso, M.; Di Tommaso, A.O.; Marignetti, F.; Miceli, R.; Ricco Galluzzo, G. A General Mathematical Formulation for Winding Layout Arrangement of Electrical Machines. Energies 2018, 11, 446. [CrossRef]
14. Caruso, M.; Tommaso, A.O.; Giangrasso, L.; Marignetti, F.; Miceli, R.; Rizzo, R. Differential Leakage Factor in Electrical Machines Equipped with Asymmetrical Multiphase Windings: A General Investigation. In Proceedings of the 2019 Fourteenth International Conference on Ecological Vehicles and Renewable Energies (EVER), Monte-Carlo, Monaco, 8–10 May 2019; pp. 1–7. [CrossRef]
15. Tommaso, A.O.D.; Genduso, F.; Miceli, R. A New Software Tool for Design, Optimization, and Complete Analysis of Rotating Electrical Machines Windings. IEEE Trans. Magn. 2015, 51, 1–10. [CrossRef]
16. Caruso, M.; Di Tommaso, A.O.; Miceli, R.; Rizzo, R. Computer-aided analysis and design procedure for rotating induction machine magnetic circuits and windings. *IET Electr. Power Appl.* 2018, 12, 885–893. [CrossRef]

17. Richter, R. *Die Induktionsmaschinen*, 2nd ed.; Elektrische Maschinen, Birkhäuser: Basel, Switzerland; Stuttgart, Germany, 1954; Volume 4, p. 691.

18. Huang, X.; Du, Q.; Hu, M. A novel exact and universal approach for calculating the differential leakage related to harmonic waves in AC electric motors. *IEEE Trans. Energy Convers.* 2004, 19, 1–6. [CrossRef]

19. Di Tommaso, A.O.; Genduso, F.; Miceli, R.; Ricco Galluzzo, G. An Exact Method for the Determination of Differential Leakage Factors in Electrical Machines with Non-Symmetrical Windings. *IEEE Trans. Magn.* 2016, 52, 1–9. [CrossRef]

20. Caruso, M.; Di Tommaso, A.O.; Genduso, F.; Miceli, R.; Ricco Galluzzo, G. A General Mathematical Formulation for the Determination of Differential Leakage Factors in Electrical Machines with Symmetrical and Asymmetrical Full or Dead-Coil Multiphase Windings. *IEEE Trans. Ind. Appl.* 2018, 54, 5930–5940. [CrossRef]

21. Müller, G.; Ponick, B. *Theorie Elektrischer Maschinen*, 6th ed.; Elektrische Maschinen, Wiley-VCH Verlag GmbH & Co. KGaA: Weinheim, Germany, 2009.

22. Hruska, K.; Sokol, M. Advanced numeric tooth winding analyses. In Proceedings of the 2012 15th International Symposium Mechatronika, Prague, Czech Republic, 5–7 December 2012; pp. 1–7.

23. Kron, A.W.; Bopp, K. Beitrag zur praktischen Berechnung des Koeffizienten der doppeltverketteten Streuung. *Arch. Elektrotechnik* 1953, 41, 136–142. [CrossRef]

24. Cistelecan, M.; Melcescu, L.; Cosan, H.; Popescu, M. Induction motors with changeable pole windings in the ratio 1:4. In Proceedings of the 2011 International Aegean Conference on Electrical Machines and Power Electronics and 2011 Electromotion Joint Conference (ACEMP), Istanbul, Turkey, 8–10 September 2011; pp. 781–786. [CrossRef]

25. Di Tommaso, A.O.; Genduso, F.; Miceli, R. A novel improved matlab-based software for the electric and magnetic analysis and design of rotating electrical machines. In Proceedings of the 2015 Tenth International Conference on Ecological Vehicles and Renewable Energies (EVER), Monte-Carlo, Monaco, 31 March–2 April 2015; pp. 1–7.

26. Caruso, M.; Di Tommaso, A.O.; Marignetti, F.; Miceli, R.; Galluzzo, G.R. A general procedure for the construction of Gorges polygons for multi-phase windings of electrical machines. In Proceedings of the 2018 Thirteenth International Conference on Ecological Vehicles and Renewable Energies (EVER), Monte Carlo, Monaco, 10–12 April 2018; pp. 1–7.

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