Switched-mode impedance synthesis for electrical tuning of a vibration energy harvester

J A Bowden, S G Burrow, and L R Clare
Department of Aerospace, Faculty of Engineering, University of Bristol, UK.

E-mail: james.bowden@bristol.ac.uk

Abstract. Switched-mode power circuits are able to efficiently synthesise a variable complex load impedance that can tune a vibration energy harvester, whilst also providing rectification and feeding the harvested energy into a DC store. The electrical tuning system presented in this paper is based upon a boost rectifier configured as a variable power factor converter. Its performance is benchmarked against a more basic resistance emulator where it is demonstrated that electrical tuning provides an increase in power bandwidth of over three times. The paper describes the experimental results of electrical tuning in some detail and elucidates the design challenges for these systems.

1.1. Introduction
Tuning of vibration energy harvesters via the input impedance of the electrical load is one method of broadening the frequency response of a harvesting system. This approach has been most widely reported for vibration absorbers [1] where the goal is to suppress structural vibrations and since in this application power autonomy is often not required, there is considerably more freedom in how the impedance synthesis is implemented. However, for an energy harvesting application it is necessary to harvester all the available power and hence energy-efficient switched-mode power circuits must be used. In this paper we report on the latest experimental work toward the development of a self-contained electrical tuning system.

1.2. Theoretical and simulated behavior
The concept of an electrically tuned energy harvester is illustrated by the diagram of figure 1. The mechanical components comprises of a base excited mass/spring system with parasitic damping, and coupling into the electrical domain. In this case electromagnetic coupling is considered. The transducer will introduce parasitic resistance ($R_{coil}$), and inductive reactance ($L_{coil}$). The variable impedance load is shown represented by resistance, capacitance and inductance. The electrical load can be expressed as: $Z_l = R_l + jX_L$ (noting for inductors and capacitors, $X_L = \omega L$ and $X_C = -1/\omega C$, respectively), with values ranging $0 < R_L < \infty$ and $-\infty < X_L < \infty$. Detailed analysis of this system has been undertaken in [2], where it has been shown that the optimum value of load impedance, to maximise power generation is given by (1).
Figure 1. Representation of an electromagnetic energy harvester with variable impedance electrical load. \( y = \) input excitation, \( k = \) spring compliance, \( c = \) mechanical damping, \( m = \) mass, \( bl = \) electro-magnetic coupling coefficient, \( L_{\text{coil}} = \) coil impedance, \( R_{\text{coil}} = \) coil resistance, \( R_l = \) load resistance, \( L_l = \) load inductance, \( C_l = \) load capacitance.

\[
Z_l = R_l + jX_l = R_{\text{coil}} + \frac{bl^2c\omega^2}{(k-m\omega^2) + (c\omega)^2} + j\left( \frac{bl^2(m\omega^2-k)\omega}{(k-m\omega^2) + (c\omega)^2} - \omega L_{\text{coil}} \right)
\]  

(1)

Where, \( \omega \) is the frequency of excitation.

Figure 2 plots the apparent, reactive and true input power into the optimum electrical load impedance. These results are derived from the numerical simulation of the system in figure 1 using the parameters given in table 1, measured on a prototype energy harvester shown in figure 5. The prototype harvester develops a nominal 20mW of power into an optimal resistive load with harmonic base excitation of 70µm. Referring to figure 2 it is evident that when tuning away from the harvester’s original resonant frequency significantly more reactive power flows between the harvester and its electrical load as the optimum load impedance becomes more reactive, where at its peak it is almost four times higher than true power delivered to the electrical load resistance. This results in a very low power factor, P.F., as given by equation (2) and plotted in figure 3 where \( |S| \) is the apparent input power and \( P \) the true input power.

\[
P.F. = \frac{P}{|S|} = \frac{R_{\text{opt}}}{|Z_{\text{opt}}|}
\]  

(2)

Thereby the design requirements of the converter are driven by the apparent power that it must handle rather than the true power that will be delivered to the load. Figure 4 plots the magnitude of the input current the converter must handle which shows additionally that the converter must also handle very high input currents with very little loss to achieve highly efficient operation. This is particularly true above the harvester’s original resonant frequency the input current increases to over five times higher than that at of a unity power factor, this results in the requirement of the converter times higher than that at of a unity power factor, this results in the requirement of the converter to have extremely low conduction losses in order to tune to higher frequencies above mechanical resonance.
Figure 2. Normalised input power to the bridge with the energy harvester optimally loaded versus frequency, apparent power (VA) (.), reactive power (VAR) (•••), true power (W) (...).

Figure 3. Input power factor versus normalised frequency. Power factor falls rapidly from unity away from the harvester’s mechanical resonant frequency.

Figure 4. Normalised rms input current versus frequency.

Figure 5. The physical energy harvester used in the experimental validation.

Table 1. Harvester parameters.

| Property              | Symbol | Value (unit) |
|-----------------------|--------|--------------|
| Inertial mass         | m      | 22g          |
| Spring compliance     | k      | 3371 Nm⁻¹    |
| Mechanical damping    | c      | 0.345 Nsm⁻¹  |
| Coil inductance       | Lcoil  | 250 µH       |
| Coil resistance       | Rcoil  | 1.6 Ω        |
| Coupling coefficient  | bl     | 5.64 Vsm⁻¹   |

1.3. Experimental setup
A four quadrant boost rectifier configured in average current mode control is used to form the power system, figure 6 and 7 respectively. Current demand is derived from the output voltage of the harvester, modified by the reciprocal of the desired impedance to be synthesised. Careful choice of power devices, micro-power current sensing and gate driver circuits are required to achieve operation at this very low power level. A compensated feedback control, implemented using micro power
analogue circuitry, forces the converter’s input current to the required value. Referring to figure 7 a dSPACE system is used to derive the current demand from the measured harvester voltage (admittance block in figure 7), this allows for ease of modifying the parametric values of load impedance in real-time during experimentation.

**Figure 6.** Four quadrant boost rectifier power stage. The 3.3V DC link is maintained by a power supply that is both capable of sinking and sourcing power. The MOSFET’s drive electronics is not shown.

**Figure 7.** The controller structure for the impedance synthesiser. Blocks are implemented in dSPACE, except for error amplifier which is constructed from discrete components.

1.4. Results

To contrast the performance of the electrical tuning technique at providing useable DC power with conventional power conditioning circuits, a resistance emulator, based on the flyback topology, similar to that reported in [3]. Experimental results are given in figure 8. With reference to figure 8, there are two groups of three curves representing the theoretical power output, measured power output of the harvester (AC side) and the useful DC side power, for the resistance emulator circuit and the electrical tuning circuit. It can be seen that electrical tuning extends the frequency bandwidth of usable power over the resistance emulator.

**Figure 8.** Comparison of power bandwidth of tuning converter and resistance emulator

1.5 Losses in the switched-mode converter

Close to the resonant frequency of the harvester the converter is able to provide most of the input power at its DC output: at its peak an efficiency of 91.6% was achieved. However, moving away from resonance, the percentage of useful power on the DC side drops. This is due to the fact that as the harvester is excited further away from its resonant frequency current magnitude increase and thus
conduction losses. Figure 9 shows the AC current magnitude, showing the large increase in magnitude away from resonance, and made up of: 1) conduction losses from fundamental current component; 2a) conduction and magnetic losses from switching frequency components; 2b) switching loss in the MOSFETs, measured experimentally. Details of the parameters and values used in the loss analysis are given in table 2. It can be seen that around resonance, the quiescent losses dominate, but as the current magnitude increases the conduction loss at the fundamental frequency starts to dominate. The values calculated for loss by this method show good agreement with the experimental data in figure 8 around resonance; however for higher current levels found at higher frequencies they predict only 75% of the losses seen in practice.

**Table 2. Parameters and measured values used in loss analysis.**

| Property                  | Symbol  | Value (unit) |
|---------------------------|---------|--------------|
| Resistance of Rsense      | R_{sense} | 0.2Ω         |
| DC resistance of L₁       | R_{LDC} | 0.278 Ω      |
| Voice coil resistance     | R_{VC}  | 1.6 Ω        |
| MOSFET R_{DSON}           | R_{DSON}| 77mΩ         |
| Switching loss            | P_{Lsw} | 470µW        |
| Switching ripple current losses | P_{Lrip} | 440µW    |

1.5. Conclusion

Implementation of electrical tuning with switching converters offers the ability to extend the bandwidth of an energy harvester. The system presented in this paper demonstrate a switched-mode converter operating as a variable impedance synthesiser can increase the half-power bandwidth of the harvester to over three times that of a unity power factor converter operating as a fixed load resistance. However, the constraint is that the design of the converter must be based upon the apparent power rather than the real power that is delivered to the DC link to achieve efficient operation whilst operating with very low power factors. In the case described in this paper the converter needed to handle currents in excess of five times higher than a converter designed just to operate with the harvester just at mechanical resonance. This is made worse when tuning harvesters with high quality factors. This is in direct contrast to the rather more relaxed power handling capabilities of the unity power factor converter. To achieve tuning over wide bandwidths the design of converter topologies with very high efficiencies is required.

[1] D Niederberger, S Behrens, A J. Fleming, S. O. Reza Moheimani, and M Morari ‘Adaptive Electromagnetic Shunt Damping’. IEEE/ASME Transactions on Mechatronics, vol.11, No.1,February 2006 pp 103-104.

[2] Cammarano A, Burrow S G, Barton D A W, Carrella A, Clare L R. “Tuning a resonant energy harvester using a generalized electrical load” Journal of smart materials and structures, vol. 19, 055003, March 2010.

[3] S Burrow, L Clare ‘Open-loop power conditioning for vibration energy harvesting’ Electronic Letters, vol.45, Issue 19, pp999-1000. September 2009.