An Improved Stability Criterion for Discrete-Time Linear Systems With Two Additive Time-Varying Delays

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ABSTRACT In this paper, the stability analysis problem for discrete-time linear systems with additive time-varying delays is further investigated. In the first place, an augmented Lyapunov-Krasovskii functional (LKF) based on delay interval decomposition is designed, where some augmented vectors are selected to supplement the coupling relationships between some system state variables and different delay subintervals. In the second place, based on the augmented LKF, a new delay-dependent stability criterion is derived via a general summation inequality lemma. The stability criterion is derived in the form of linear matrix inequality (LMI), which can be solved quickly by Matlab LMI-Tool. In the end, the effectiveness of the proposed method is illustrated by some common numerical examples.

INDEX TERMS Discrete-time systems, Lyapunov-Krasovskii functional, time-varying delays, time-delayed system.

I. INTRODUCTION

Time delay is a common phenomenon in industrial network system, such as boiler steam temperature control, flight attitude adjustment, flight speed control, network data transmission, etc. Numerous real systems with time delays can be modeled as linear systems with time delays. For example, interconnected power grid based on load frequency control [1], hydraulic systems [2], and so on. It is well known that time delays are often considered an important reason for system performance degradation or even instability. Thus, the research and analysis of time-delayed systems have become increasingly popular in recent years [3]. Especially, the time-delayed practical system based on network controller involves two kinds of delays: a) Only the time-varying delays of communication between controller and actuator are considered (See Figure 1 for single channel communication delay) [4], where the interconnected power grid based on load frequency control was modeled as a time-delayed system with single communication delay; b) Not only the time-varying delays between controller and actuator, but also the time-varying delays between sensor and controller are considered (See Figure 2 for double channel communication delays) [5], where the network system with successive delays of different properties due to the variable network transmission conditions was modeled as a time-delayed system with additive time-varying delay components.

Obviously, the stability of the time-delayed system is the premise of other performances. Therefore, the stability analysis of time-delayed systems has been a hot topic in recent decades, and many research methods and excellent stability-related conclusions have been obtained. In the literature, the main stability criteria of time-delayed systems have two categories: delay-dependent stability criteria and delay-independent ones. The results of the preliminary study of time delay systems are basically delay-independent, i.e., there is no constraint on delay. So, the stability or other performance criteria of the system are valid for any delay, which is called delay-independent criterion [6]. The smaller the time delay is, the more obvious the conservatism of the
delay-independent criterion is. Thus, the influence of time delay on system stability or other performance should be considered, which is called the delay-dependent criterion [7]. It should be pointed out that the delay-dependent stability condition for the time-delayed system is usually sufficient, but not necessary. Thus, it is still conservative to some extent. How to enlarge the upper bound of the time delay to reduce the conservatism of the stability criterion is always a hot issue in control theory [8], [9]. At present, most scholars focus on the delay-reliant stability conditions. The main methods to reduce the conservatism include the following three aspects: a) Constructing an appropriate LKF, which contains as much information as possible about the system state variables, the time delays and some terms contained in the matrix inequality techniques. For example, a discretized LKF combined with the dwell time method [10], [11], which uses the linear interpolation to discretize the LKF and divides the set matrix domains into finite points or intervals; A state decomposition LKF method [12], [13], [14], [15], [16], which can reduce the number of decision variables to decrease the computational complexity; The time delay product class LKFs [17], [18], [19], [20], [21]; LKFs based on Legendre polynomials and membership functions [22], [23], which point that the conservatism of the conditions decreases as the order of the Legendre polynomials increases; ect.. b) Updating the inequality technique to make the upper bound of the LKF derivative tight. Such as, for discrete-time systems, Bessel summation inequality [24], a novel finite sum inequality technique [25], discrete Wirtinger-based inequality technique [26], [27]; for continuous systems, an inequality technique based on nonorthogonal polynomials [28], [29], a generalized multiple

**FIGURE 1.** Single channel communication delay structure diagram.

**FIGURE 2.** Double channel communication delays structure diagram.
integral inequality [30], an integral inequality based on a
generalized reciprocally convex inequality [31], [32], the
quadratic matrix-vector form and Jensen’s inequality [33],
and so on. c) Increasing the degree of freedom of LMIs in the
stability criterion, which further increases the upper bound of
the time-varying delays. For instance, a general free-matrix-
based single integral inequality [34], [35], a free-matrix-
based double integral inequality [36], some state-dependent
zero equations [37], etc..

Recently, a new delay-range-dependent stability criterion
for discrete-time linear systems with interval time-varying
delays was given in [38], where an improved summation
inequality based on free matrices was used to improve the
solution degree of freedom of the LMIs. For discrete-time
linear systems with additive input time-varying delays, the
authors of [39] improved the stability margin of the system
with respect to time delay division and a new summation
inequality technique. However, the summation inequality
used in [39] is a special case of the one proposed in [38].
Moreover, the LKF chosen in [39] ignores some state
information. Based on the above discussions, in order to
increase the accuracy of the delay stability region, how
to derive a stability criterion with less conservatism needs
further investigation. This motivates our research.

In this paper, the stability of the discrete-time linear system
with additive time-varying delays is investigated. Firstly,
a general free-matrix-based summation inequality lemma
is given, which involves additional coupling information
of some state variables by a few free matrices. Secondly,
we construct an improvement LKF by the summation
inequality lemma, and the LKF augments the vectors of
the single summation sign by introducing both difference
and non-difference terms. In this way, some necessary
system state variables in the summation inequality lemma
are supplemented, making the summation inequality lemma
further fully utilized. Thirdly, a modified delay-dependent
stability criterion is derived in terms of LMI via the
augmented LKF and the general summation inequality based
on free matrices lemma application. The stability criterion
in this paper reduces the conservatism of some recent
published results. Finally, in the section 4 of this paper,
some common numerical examples are given. The values
of the maximum allowable delay upper bound (MADUB)
are obtained by using the stability criteria of this paper, and
some comparisons and discussions with the corresponding
results of some recent literatures show that our improvement
is effective and a larger margin of stability is obtained.

The structure of this paper is logically arranged as
follows: In section 2, the system model, research content,
some hypothetical constraints and necessary lemmas are
given. Some new delay-dependent stability criteria, including
theorems and corollaries are derived in Section 3. Some
numerical examples compared with some existing literature
are shown in Section 4. Section 5 gives the conclusion.

Notation: In this paper, the notations are standard. $\mathbb{Z}$ is
the set of integers, $\mathbb{R}^n$, $\mathbb{R}^{n \times m}$ represent the $n$-dimensional vector
and the $n \times m$ matrix space, respectively. $\mathbb{R}_+^{n \times n} (\mathbb{R}_-^{n \times n})$ means
the sets of positive definite (respectively, negative) real matrix
space. $n$-order block diagonal matrix diag $\{S_1, S_2, \cdots, S_n\}$
with diagonal partitioned elements $S_1, S_2, \cdots, S_n$. $x_i (i =
1, \ldots, m)$ are a column block matrix in which only the $i$-th
block is the identity matrix and the others are 0 matrices.

Such as, $e_3 = \col\left\{0, 0, I, 0, \ldots, 0\right\}_{m-3}$. This symbol $*$
in a block symmetric matrix denotes transpose of the
corresponding symmetric element. col$\{\cdot\}$ denotes a column vector.
$\Pi(h_1(k), h_2(k))$ denotes $\Pi$ is the binary function of
$h_1(k)$ and $h_2(k)$. Sym$\{\Xi\} = \Xi + \Xi^T$.

II. PROBLEM FORMULATION AND PRELIMINARY

Consider the following discrete-time linear system with
additive time-varying delays as

$$
\begin{align}
  x(k+1) &= A_1x(k) + A_2x(k - h_1(k) - h_2(k)), \\
  x(k) &= \varphi(k), \quad k = -h_2, -h_2 + 1, \cdots, 0,
\end{align}
$$

(1)

where $x(k) \in \mathbb{R}^n$, $\varphi(k)$ are the state vector, the initial
condition, respectively. $A_1$ and $A_2 \in \mathbb{R}^{n \times n}$ are constant
matrices, respectively, the delays $h_1(k)$ and $h_2(k)$ are positive
integer sequence of time-varying functions satisfying the
following upper and lower bound constraints.

$$
h_{11} \leq h_1(k) \leq h_{12}, \quad h_{21} \leq h_2(k) \leq h_{22},
$$

(2)

where $h_{11}$, $h_{12}$, $h_{21}$ and $h_{22}$ are positive integers and $h_k =
h_1(k) + h_2(k)$, $h_1 = h_{11} + h_{12}$, $h_2 = h_{12} + h_{22}$, $s_1(\sigma) = \sigma + 1,

s_2(\sigma) = (\sigma+1)(\sigma+2)$. Based on the above linear system, a new stability
criterion will be derived in this paper. The reduction of the
conservatism of this stability criterion lies in the technique
of augmented LKF and tight summation inequality. Several
necessary lemmas are given below.

Lemma 1 [40]: Give a vector function $y(k) \in \mathbb{R}^n, \mu_1, \mu_2 \in
\mathbb{Z}$ with $\mu_{12} = \mu_2 - \mu_1$. $P \in \mathbb{R}^{n \times n}$. For any vectors $\delta, \gamma$, we have

$$
\begin{align}
  \sum_{\mu_2-1}^{\mu_1-1} y^T(r) P y(r) &\geq \frac{1}{\mu_{12}} \delta^T \Lambda_1^T \Xi_1^T \Xi_2 \Lambda_1 \delta, \\
  \sum_{\mu_2}^{\mu_1-1} \Delta y^T(r) P \Delta y(r) &\geq \frac{1}{\mu_{12}} \gamma^T \Xi_1^T \Xi_2 \Lambda_1 \gamma,
\end{align}
$$

(3) (4)

where

$$
\delta = \col\left\{y(\mu_2), \sum_{r=\mu_1}^{\mu_2} y(r), \sum_{r=\mu_1}^{\mu_2} \sum_{r_1=\mu_1}^{\mu_2} y(r_1)\right\},
$$

$$
\gamma = \col\left\{y(\mu_2), y(\mu_1), \sum_{r=\mu_1}^{\mu_2} y(r), \sum_{r=\mu_1}^{\mu_2} \sum_{r_1=\mu_1}^{\mu_2} y(r_1)\right\},
$$

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\[ \tilde{P} = (P, 3P, 5P) \], \[ \Xi = \begin{bmatrix} -I & I & 0 & 0 \\ -I & -I & 2I & 0 \\ -I & I & -6I & 6I \end{bmatrix}, \]
\[ \lambda = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & \frac{1}{s_1(\tau_{12})} & 0 \\ 0 & 0 & \frac{1}{s_2(\mu_{12})} & I \end{bmatrix}, \Delta y(r) = y(r+1) - y(r). \]

**Lemma 2 [38]:** Give a vector function \( \gamma(k) \in \mathbb{R}^n, \mu_1, \mu_2 \in \mathbb{E} \) with \( \mu_{12} = \mu_2 - \mu_1 \), \( P \in \mathbb{R}^{n \times n} \), arbitrary matrices \( M, N, H, K \), a time-varying positive integer sequence \( \tau(k) \) satisfying \( \tau(k) \in [\mu_1, \mu_2] \), the following inequalities hold for any vectors \( \theta_1, \theta_2 \).

\[
\begin{align*}
\sum_{r=\mu_1}^{\mu_2-1} y^T(r)P y(r) & \geq \text{Sym} \{ \theta_1^T [MN \Xi \eta_1(r) \alpha_1] \} \\
& + \theta_1^T \left\{ (\mu_1 - \tau(k))M \tilde{P}^{-1} M^T + (\tau(k) - \mu_2)N \tilde{P}^{-1} N^T \right\} \theta_1, \quad (5) \\
\sum_{r=\mu_1}^{\mu_2-1} \Delta y^T(r)P \Delta y(r) & \geq \text{Sym} \{ \theta_2^T [H \Xi K \Xi \eta_2(r) \alpha_2] \} \\
& + \theta_2^T \left\{ (\mu_1 - \tau(k))H \tilde{P}^{-1} H^T + (\tau(k) - \mu_2)K \tilde{P}^{-1} K^T \right\} \theta_2, \quad (6)
\end{align*}
\]

where \( \alpha_1 = \text{col}\{A_1 \delta_1, A_2 \delta_2\} \) and \( \alpha_2 = \text{col}\{A_1 \gamma_1, A_2 \gamma_2\} \).

\[
\begin{align*}
A_1 & = \text{diag}[I, \frac{1}{s_1(\tau(k) - \mu_1)}, \frac{1}{s_2(\tau(k) - \mu_1)}], \\
A_2 & = \text{diag}[I, \frac{1}{s_1(\mu_2 - \tau(k))}, \frac{1}{s_2(\mu_2 - \tau(k))}], \\
\delta_1 & = \text{col}\left\{ y(\tau(k)), \sum_{r=\mu_1}^{\tau(k)} y(r), \sum_{r=\mu_1}^{\tau(k)} \sum_{r_2=\mu_1}^{\tau(k)} y(r_1), \sum_{r_3=\mu_1}^{\tau(k)} \sum_{r_2=\mu_1}^{\tau(k)} \sum_{r_1=\mu_1}^{\tau(k)} y(r_1) \right\}, \\
\delta_2 & = \text{col}\left\{ y(\mu_2), \sum_{r=\mu_2}^{\mu_2} y(r), \sum_{r=\mu_2}^{\mu_2} y(r_1), \sum_{r_3=\mu_2}^{\mu_2} \sum_{r_2=\mu_2}^{\mu_2} \sum_{r_1=\mu_2}^{\mu_2} y(r_1) \right\}, \\
\gamma_1 & = \text{col}\left\{ y(\tau(k)), y(\mu_1), \sum_{r=\mu_1}^{\tau(k)} y(r), \sum_{r_2=\mu_1}^{\tau(k)} \sum_{r_1=\mu_1}^{\tau(k)} y(r_1) \right\}, \\
\gamma_2 & = \text{col}\left\{ y(\mu_2), y(\tau(k)), \sum_{r=\mu_2}^{\mu_2} y(r), \sum_{r_2=\mu_2}^{\mu_2} \sum_{r_1=\mu_2}^{\mu_2} y(r_1) \right\}.
\end{align*}
\]

**Remark 1:** Lemma 2 is introduced to involve coupling information for additional state variables by additional free matrices, which can relax the conditions of derived criteria. From (5), two free matrices \( M \) and \( N \) are employed to make the vectors in \( \tilde{E} \lambda_1 \delta_1 \) and \( \tilde{E} \lambda_2 \delta_2 \) not only connect to each other but also to themselves. The inequality (3) is a special case of the inequality (5) because of the introduction of the two free matrices \( M \) and \( N \). Indeed, letting \( M = \frac{\sqrt{2-1}}{\mu_1-\tau(k)} \text{col}(P, 0), N = \frac{\sqrt{2-1}}{\mu_2-\tau(k)} \text{col}(0, \tilde{P}) \) and \( \theta_1 = \text{col}(\tilde{E} \lambda_1 \delta_1, \tilde{E} \lambda_2 \delta_2) \), the inequality (5) reduces to the inequality (3). When letting \( \theta_1 = \text{col}(\omega_{01}, \omega_{11}, \omega_{12}, \omega_{20}, \omega_{21}, \omega_{22}) \), \( M = \text{col}(\text{diag}(N_0, N_1, N_2, 0) \text{ and } N = \text{col}(0, \text{diag}(N_2, N_2, N_2)) \), the inequality (5) reduces to the lemma 1 in [40] and the corollary 1 in [41]. Thus, the Lemma 2 can reduce the conservatism of the stability criterion since matrices \( M \) and \( N \) are with more freedom than the diagonal matrices \( \text{diag}(N_0, N_1, N_2) \) and \( \text{diag}(N_2, N_2, N_2) \).

**III. MAIN RESULTS**

A stability criterion which is less conservative than some existing results is given in this section. For the convenience of symbol representation in the derivation of stability criterion, the following symbol definitions are given.

\[
\begin{align*}
h_{k1} & = h_{11}(k), h_{2k} = h_{22}(k), h_{k1} = h_{1k} - h_{11}, \\
h_{k1} & = h_{12} - h_{1k}, h_{k2} = h_{2k} - h_{21}, \\
h_{k2} & = h_{22} - h_{2k}, h_{10} = h_{1k} - h_{11}, \\
h_{20} & = h_{22} - h_{21}, h_k = h_{1k} + h_{2k}, h_1 = h_{11} + h_{21}, \\
h_2 & = h_{12} + h_{22}, \\
\eta_1(k) & = \text{col}(x(k), s_1(h_{11})^a_3(k), s_1(h_{11})^a_1(k) + s_1(h_{12})^a_2(k), s_1(h_{22})^a_3(k), s_1(h_{22})^a_1(k), s_2(h_{11})^a_1(k), s_2(h_{21})^a_1(k)), \\
\eta_2(k) & = \text{col}(x(k), \Delta x(k)), \\
\xi(k) & = \text{col}(x(k), x(k) - (k - h_{11}), x(k) - h_{1k}), \\
x(k - h_{12}), x(k - h_{21}), x(k - h_{2k}), \\
x(k - h_{22}), x(k - h_{1k}), x(k - h_{2k}), \\
\mu_1(k), \mu_2(k), \mu_3(k), v_1(k), v_2(k), v_3(k), v_1(k), v_2(k), v_3(k), \\
\alpha_1(k), \alpha_2(k), \alpha_3(k), \theta_1(k), \theta_2(k), \theta_3(k), \theta_4(k), \\
u_1(k) = \sum_{i=k-h_{11}}^{k-h_{12}} x(i)s_1(h_{11}), u_2(k) = \sum_{i=k-h_{12}}^{k-h_{11}} x(i)s_1(h_{11}), \\
u_3(k) = \sum_{i=k-h_{11}}^{k-h_{21}} x(i)s_1(h_{11}), v_1(k) = \sum_{i=k-h_{21}}^{k-h_{11}} x(i)s_1(h_{11}), \\
v_2(k) = \sum_{i=k-h_{21}}^{k-h_{22}} x(i)s_1(h_{11}), v_3(k) = \sum_{i=k-h_{22}}^{k-h_{21}} x(i)s_1(h_{11}), \\
o_1(k) = \sum_{i=k-h_{11}}^{k-h_{11}} x(i)s_2(h_{11}), o_2(k) = \sum_{i=k-h_{11}}^{k-h_{11}} x(i)s_2(h_{11}).
\end{align*}
\]

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\[ \omega_5(k) = \sum_{i=k-h_1}^{k-h_{12}} \sum_{j=i}^{k-h_{12}} \frac{x(j)}{s_2(h_1)}, \quad \alpha_1(k) = \sum_{i=k-h_2}^{k-h_{22}} \sum_{j=i}^{k-h_{22}} \frac{x(j)}{s_2(h_2)}, \]
\[ \alpha_2(k) = \sum_{i=k-h_1}^{k-h_{12}} \sum_{j=i}^{k-h_{12}} \frac{x(j)}{s_2(h_1)}, \quad \alpha_3(k) = \sum_{i=k-h_2}^{k-h_{22}} \sum_{j=i}^{k-h_{22}} \frac{x(j)}{s_2(h_2)}, \]
\[ \theta_1(k) = \sum_{i=k-h_1}^{k-h_{12}} \frac{x(i)}{s_1(h_1)}, \quad \theta_2(k) = \sum_{i=k-h_2}^{k-h_{22}} \frac{x(i)}{s_1(h_2)}, \]
\[ \theta_3(k) = \sum_{i=k-h_1}^{k-h_{12}} \sum_{j=i}^{k-h_{12}} \frac{x(j)}{s_2(h_1)}, \quad \theta_4(k) = \sum_{i=k-h_2}^{k-h_{22}} \sum_{j=i}^{k-h_{22}} \frac{x(j)}{s_2(h_2)}, \]

**Theorem 1:** For given \( h_{11}, h_{12}, h_{21}, h_{22}, M \in \mathbb{R}^{n \times n}, Q_{i}, R_i \in \mathbb{R}^{n \times n}, \) arbitrary matrices \( M, T, N, H \in \mathbb{R}^{n \times n}, (i \in \{1, 2, \ldots, 6\}) \), and vectors \( e_s, e_r \in \mathbb{R}^m \), (1) has a positive solution \( (h_{11}, h_{21}, h_{12}, h_{22}) \) if and only if the following inequalities hold:

\[ \Pi(h_{11}, h_{21}) h_{10} \beta_1^T T h_{10} \beta_1 U h_{20} \beta_2^T H h_{20} \beta_2^T J < 0, \]
\[ \Pi(h_{12}, h_{21}) h_{10} \beta_1^T M h_{10} \beta_1 T h_{20} \beta_2^T H h_{20} \beta_2^T J < 0, \]
\[ \Pi(h_{11}, h_{22}) h_{10} \beta_1^T T h_{10} \beta_1 U h_{20} \beta_2^T N h_{20} \beta_2^T G < 0, \]
\[ \Pi(h_{12}, h_{22}) h_{10} \beta_1^T M h_{10} \beta_1 T h_{20} \beta_2^T N h_{20} \beta_2^T G < 0, \]

Here,

\[ \Pi(h_{1k}, h_{2k}) = \Pi_1(h_{1k}, h_{2k}) + \Pi_2 + \text{Sym} \{ \Pi_3(h_{1k}, h_{2k}) \}, \]
\[ \Pi_1(h_{1k}, h_{2k}) = \Theta \text{Pos}^T + \text{Sym} \{ \Phi(h_{1k}, h_{2k}) \text{Pos}^T \}, \]
\[ \Pi_2 = e_1 Q_1 e_1^T + e_2(Q_2 - Q_1) e_2^T + e_1 Q_5 e_1^T + e_4 Q_6 e_4^T + e_5 Q_7 e_5^T + e_6 Q_8 e_6^T + e_7 Q_9 e_7^T + e_8 Q_{10} e_8^T + e_9 Q_{11} e_9^T + e_{10} Q_{12} e_{10}^T + e_1 K_1 e_1^T + e_2(K_2 - K_1) e_2^T + e_3(K_3 - K_2) e_3^T + e_4 K_4 e_4^T + e_5(K_5 - K_4) e_5^T + e_6(K_6 - K_5) e_6^T + e_7 K_6 e_7^T + e_8 e_1 h_{11} R_1 + h_{10} R_2 + h_{21} R_3 + h_{20} R_4 + h_{12} R_5 + h_{11} R_6 e_1 e_1^T + e_1 h_{11} Z_1 + h_{10} Z_2 + h_{21} Z_3 + h_{20} Z_4 + h_{12} Z_5 + h_{11} Z_6 e_s \]
\[ \frac{1}{h_{11}} X_{10} \bar{R}_1 X_{10}^T \frac{1}{h_{11}} \gamma_1 \bar{Z}_1 X_{10}^T \frac{1}{h_{22}} X_{30} \bar{R}_X X_{30} \frac{1}{h_{12}} X_{31} \bar{R}_X X_{31} \frac{1}{h_{22}} X_{32} \bar{Z}_X X_{32} \frac{1}{h_{12}} X_{33} \bar{Z}_X X_{33} \]
\[ \Theta = \{ e_s, e_1 e_2, e_2 e_3, e_1 e_5, (e_5 - e_7) s_1(h_{11})(e_1 - e_7) s_1(h_{21})(e_1 - e_15) \}, \]
\[ \Phi(h_{1k}, h_{2k}) = \{ e_1 s_1(h_{11}) e_12 - e_1, (s_1(h_{11})) e_11 + s_1(h_{11}) e_11 - e_12 - e_13, s_1(h_{21}) e_15 - e_11, s_1(h_{21}) e_15 - e_12 - e_13, \}
\[ \chi_{10} = \{ s_1(h_{11}) e_10 - e_12 - e_1, s_1(h_{11}) e_10 - e_12 - e_13, s_1(h_{21}) e_15 - e_12 - e_13, s_1(h_{21}) e_15 - e_12 - e_13, \}
\[ \chi_{11} = \{ s_1(h_{11}) e_11 - e_13, s_1(h_{11}) e_11 - e_13, s_1(h_{21}) e_15 - e_12 - e_13, s_1(h_{21}) e_15 - e_12 - e_13, \}
\[ \chi_{12} = \{ s_1(h_{11}) e_11 - e_13, s_1(h_{11}) e_11 - e_13, s_1(h_{21}) e_15 - e_12 - e_13, s_1(h_{21}) e_15 - e_12 - e_13, \}
\[ \chi_{20} = \{ s_1(h_{11}) e_10 - e_12 - e_13, s_1(h_{11}) e_10 - e_12 - e_13, s_1(h_{21}) e_15 - e_12 - e_13, s_1(h_{21}) e_15 - e_12 - e_13, \}
\[ \chi_{21} = \{ s_1(h_{11}) e_11 - e_13, s_1(h_{11}) e_11 - e_13, s_1(h_{21}) e_15 - e_12 - e_13, s_1(h_{21}) e_15 - e_12 - e_13, \}
\[ \chi_{22} = \{ s_1(h_{11}) e_11 - e_13, s_1(h_{11}) e_11 - e_13, s_1(h_{21}) e_15 - e_12 - e_13, s_1(h_{21}) e_15 - e_12 - e_13, \}
\[ \gamma_33 = \{ e_7 e_9 e_2 e_10, e_7 e_9 e_2 e_10, e_7 e_9 e_2 e_10, e_7 e_9 e_2 e_10, \}
\[ \gamma_70 = \{ e_1 e_2 e_3 e_6, e_1 e_2 e_3 e_6, e_1 e_2 e_3 e_6, e_1 e_2 e_3 e_6, \}
\[ \gamma_71 = \{ e_3 e_4 e_5 e_6, e_3 e_4 e_5 e_6, e_3 e_4 e_5 e_6, e_3 e_4 e_5 e_6, \}
\[ \gamma_21 = \{ e_6 e_7 e_8 e_2, e_6 e_7 e_8 e_2, e_6 e_7 e_8 e_2, e_6 e_7 e_8 e_2, \}
\[ \gamma_22 = \{ e_6 e_7 e_8 e_2, e_6 e_7 e_8 e_2, e_6 e_7 e_8 e_2, e_6 e_7 e_8 e_2, \}
\[ \beta_1 = \{ e_2 e_3 e_4 e_5 e_6, e_2 e_3 e_4 e_5 e_6, e_2 e_3 e_4 e_5 e_6, e_2 e_3 e_4 e_5 e_6, \}
\[ \beta_2 = \{ e_5 e_6 e_7 e_8 e_9, e_5 e_6 e_7 e_8 e_9, e_5 e_6 e_7 e_8 e_9, e_5 e_6 e_7 e_8 e_9, \}
\[ \mathcal{R}_i = \left[ \begin{array}{c} 0 \ K_i \ K_i \ K_i \ K_i \end{array} \right], \quad (i = 1, 2, 3, 4), \]
\[ V(k) = \sum_{i=1}^{5} V_i(k) \] (11)

with

\[ V_1(k) = \eta_1^T(k)P_1\eta_1(k), \]
\[ V_2(k) = \sum_{r=k-h_1}^{k-h_2-1} x^T(r)Q_1x(r) + \sum_{r=k-h_2}^{k-1} x^T(r)Q_3x(r) + \sum_{r=k-h_2}^{k-1} x^T(r)Q_4x(r), \]
\[ V_3(k) = \sum_{r=-h_1}^{-1} \sum_{j=k+r}^{k-h_2-1} \eta_2^T(j)R_1\eta_2(j) + \sum_{r=-h_2}^{-1} \sum_{j=k+r}^{k-h_2-1} \eta_2^T(j)R_3\eta_2(j) + \sum_{r=-h_2}^{-1} \sum_{j=k+r}^{k-h_2-1} \eta_2^T(j)R_4\eta_2(j), \]
\[ V_4(k) = \sum_{r=-h_1}^{-1} \sum_{j=k+r}^{k-h_2-1} \Delta x^T(j)Z_1\Delta x(j) + \sum_{r=-h_2}^{-1} \sum_{j=k+r}^{k-h_2-1} \Delta x^T(j)Z_2\Delta x(j) + \sum_{r=-h_2}^{-1} \sum_{j=k+r}^{k-h_2-1} \Delta x^T(j)Z_4\Delta x(j), \]
\[ V_5(k) = \sum_{r=k-h_2}^{k-h_2-1} x^T(r)Q_5x(r) + \sum_{r=-h_2}^{-1} \sum_{j=k+r}^{k-h_2-1} \eta_5^T(j)R_5\eta_2(j). \]

The forward differences of \( V(k) \) along the trajectory of the system (1) are, respectively, computed as

\[ \Delta V_1(k) = V_1(k+1) - V_1(k) = \xi^T(k) \left( \Theta P\Theta^T + \Phi(h_k)P\Theta^T + \Theta P\Phi(h_k)^T \right) \xi(k), \]
\[ \Delta V_2(k) = \xi^T(k) \left( e_1Q_1e_1^T + e_2Q_2 - Q_1e_2^T + e_1Q_3e_1^T \right) + es_4(Q_4 - Q_3)e_3^T - e_4Q_2e_4^T - e_7Q_4e_7^T \]
\[ \Delta V_3(k) = \eta_2^T(k)(h_1R_1 + h_0R_2 + h_2R_3 + h_0R_1)\eta_2(k), \]
\[ \Delta V_4(k) = \Delta x^T(k)(h_1Z_1 + h_0Z_2 + h_2Z_3 + h_0Z_4)\Delta x(k) \]
\[ \Delta V_5(k) = \xi^T(k) \left( e_4Q_5e_4^T + e_7Q_6e_7^T \right) - es_4(Q_5 + Q_6)e_5^T \]
\[ + \Delta x^T(k)(h_2Z_5 + h_1Z_6)\Delta x(k) \]
\[ + \Delta x^T(k)(h_2Z_5 + h_1Z_6)\Delta x(k) \]
\[ + \sum_{r=k-h_2}^{k-h_2-1} x^T(r)Q_5x(r) + \sum_{r=-h_2}^{-1} \sum_{j=k+r}^{k-h_2-1} \eta_5^T(j)R_5\eta_2(j). \]
The following equations are obvious for symmetric matrices $K_i\ (i = 1, \cdots, 6)$.

\begin{align}
0 = x^T(k)K_1x(k) - x^T(k - h_{11})K_1x(k - h_{11}) \\
- \sum_{r = k - h_{11}}^{k - 1} \left[ \Delta x^T(r)K_1\Delta x(r) + 2\Delta x^T(r)K_1x(r) \right], \quad (17)
\end{align}

\begin{align}
0 = x^T(k - h_{11})K_2x(k - h_{11}) - x^T(k - h_{11})K_2x(k - h_{11}) \\
- \sum_{r = k - h_{11}}^{k - 1} \left[ \Delta x^T(r)K_2\Delta x(r) + 2\Delta x^T(r)K_2x(r) \right], \quad (18)
\end{align}

\begin{align}
0 = x^T(k - h_{11})K_3x(k - h_{11}) - x^T(k - h_{11})K_3x(k - h_{11}) \\
- \sum_{r = k - h_{11}}^{k - 1} \left[ \Delta x^T(r)K_3\Delta x(r) + 2\Delta x^T(r)K_3x(r) \right], \quad (19)
\end{align}

\begin{align}
0 = x^T(k)K_4x(k) - x^T(k - h_{21})K_4x(k - h_{21}) \\
- \sum_{r = k - h_{21}}^{k - 1} \left[ \Delta x^T(r)K_4\Delta x(r) + 2\Delta x^T(r)K_4x(r) \right], \quad (20)
\end{align}

\begin{align}
0 = x^T(k - h_{21})K_5x(k - h_{21}) - x^T(k - h_{21})K_5x(k - h_{21}) \\
- \sum_{r = k - h_{21}}^{k - 1} \left[ \Delta x^T(r)K_5\Delta x(r) + 2\Delta x^T(r)K_5x(r) \right], \quad (21)
\end{align}

\begin{align}
0 = x^T(k - h_{21})K_6x(k - h_{21}) - x^T(k - h_{21})K_6x(k - h_{21}) \\
- \sum_{r = k - h_{21}}^{k - 1} \left[ \Delta x^T(r)K_6\Delta x(r) + 2\Delta x^T(r)K_6x(r) \right]. \quad (22)
\end{align}

According to $\Delta V_3(k)$, the equations (17)-(22), we have

\begin{align}
\Delta V_3(k) = \eta_2^T(k)(h_1R_1 + h_{10}R_2 + h_{21}R_3 + h_{20}R_4)\eta_2(k) \\
+ \xi^T(k) \left( e_1(K_1 + K_4)e_1^T \\
e_3(K_3 - K_2)e_3^T - e_4K_3e_4^T + e_5(K_5 - K_4)e_5^T \\
e_6(K_6 - K_5)e_6^T - e_7K_6e_7^T \right) \xi(k) \\
- \sum_{r = k - h_{11}}^{k - 1} \eta_2^T(r)R_1\eta_2(r) - \sum_{r = k - h_{11}}^{k - 1} \eta_2^T(r)R_2\eta_2(r) \\
- \sum_{r = k - h_{12}}^{k - 1} \eta_2^T(r)R_2\eta_2(r) - \sum_{r = k - h_{12}}^{k - 1} \eta_2^T(r)R_3\eta_2(r) \\
- \sum_{r = k - h_{21}}^{k - 1} \eta_2^T(r)R_4\eta_2(r) - \sum_{r = k - h_{21}}^{k - 1} \eta_2^T(r)R_4\eta_2(r). \quad (23)
\end{align}

The following $R_1, R_2, Z_1, Z_3, Z_4, Z_5, Z_6, \text{ and } Z_7$ dependent summation inequalities in $\Delta V_3(k), \Delta V_4(k)$ and $\Delta V_5(k)$ can be obtained according to lemma 1:

\begin{align}
- \sum_{r = k - h_{11}}^{k - 1} \eta_2^T(r)R_1\eta_2(r) \leq - \frac{1}{h_{11}} \xi^T(k)\chi_{10}\tilde{R}_1\chi_{10}\xi(k), \quad (24)
\end{align}

\begin{align}
- \sum_{r = k - h_{21}}^{k - 1} \eta_2^T(r)R_3\eta_2(r) \leq - \frac{1}{h_{21}} \xi^T(k)\chi_{20}\tilde{R}_3\chi_{20}\xi(k), \quad (25)
\end{align}

\begin{align}
- \sum_{r = k - h_{11}}^{k - 1} \Delta x^T(r)Z_1\Delta x(r) \leq - \frac{1}{h_{11}} \xi^T(k)\gamma_{10}\tilde{Z}_1\chi_{10}\xi(k), \quad (26)
\end{align}

\begin{align}
- \sum_{r = k - h_{21}}^{k - 1} \Delta x^T(r)Z_3\Delta x(r) \leq - \frac{1}{h_{21}} \xi^T(k)\gamma_{20}\tilde{Z}_3\chi_{20}\xi(k), \quad (27)
\end{align}

\begin{align}
- \sum_{r = k - h_{11}}^{k - 1} \eta_2^T(r)R_5\eta_2(r) \leq - \frac{1}{h_{11}} \xi^T(k)\chi_{30}\tilde{R}_5\chi_{30}\xi(k), \quad (28)
\end{align}

\begin{align}
- \sum_{r = k - h_{21}}^{k - 1} \eta_2^T(r)R_6\eta_2(r) \leq - \frac{1}{h_{12}} \xi^T(k)\chi_{31}\tilde{R}_6\chi_{31}\xi(k), \quad (29)
\end{align}

\begin{align}
- \sum_{r = k - h_{11}}^{k - 1} \Delta x^T(r)Z_5\Delta x(r) \leq - \frac{1}{h_{12}} \xi^T(k)\chi_{32}\tilde{Z}_5\chi_{32}\xi(k), \quad (30)
\end{align}

\begin{align}
- \sum_{r = k - h_{21}}^{k - 1} \Delta x^T(r)Z_7\Delta x(r) \leq - \frac{1}{h_{12}} \xi^T(k)\chi_{33}\tilde{Z}_7\chi_{33}\xi(k). \quad (31)
\end{align}
From Eqs. (12)-(15) and (24)-(35), we have
\[ \Delta V(k) \leq \xi^T(k) \left[ \Pi(h_{1k}, h_{2k}) + h_{1k} \beta_1^T \left[ M \bar{R}_2^{-1} M^T \right] \right. \]
\[ + Y \bar{Z}_2^{-1} Y^T \left[ \beta_1 + h_{2k} \beta_2^T \left[ T \bar{R}_4^{-1} T^T + U \bar{Z}_2^{-1} U^T \right] \beta_1 \right. \]
\[ + h_{2k} \beta_2^T \left[ N \bar{R}_4^{-1} N^T + G \bar{Z}_4^{-1} G^T \right] \beta_2 \]
\[ + \left. \left. \bar{h}_{2k} \beta_2^T \left[ H \bar{R}_4^{-1} H^T + J \bar{Z}_4^{-1} J^T \right] \beta_2 \right] \xi(k), \]
which together with Schur complement and (7)-(10) imply that \( \Delta V(k) < 0 \). Therefore, by Lyapunov stability theorem, it can be guaranteed that the linear system (1) is asymptotically stable.

Remark 2: In the derivation of Theorem 1, the summation inequalities of Lemma 2 are mainly used to estimate the upper bounds of the difference of the modified LKF. In different delay intervals \([h_{11}, h_{12}], [h_{21}, h_{22}], [h_{11}, h_2]\) and \([h_{21}, h_2]\), in order to make the state variables contained in the Lemma 2 inequality not only isolated from the estimation of the inequality, some additional summation terms are introduced into the augmented LKF. However, these additional summation terms are just required in the inequalities of Lemma 2 and ignored in the LKFs of [13], [39], and [42]. Thus, an improved LKF is proposed and the main improvements are summarized as: some double summation terms \( \sum_{i=1}^{h_{11}} \sum_{j=1}^{h_{12}} x(i) \) and \( \sum_{i=1}^{h_{21}} \sum_{j=1}^{h_{22}} x(i) \) are augmented in the vectors of \( V_1(k) \); \( \Delta x(k) \) is augmented in the vectors of \( V_3(k) \) and \( V_4(k) \). Therefore, the Theorem 1 reduces the conservativeness of the existing stability criteria, which will be illustrated in numerical examples of section IV.

Remark 3: The novel stability criterion can also be naturally extended to discrete-time Markovian jump systems with additive time-varying delays systems. Consider the following discrete-time Markovian jump system [14]
\[ \{ \lambda(k) \} \text{ denotes the finite state Markov chains. The following formula represents the transition probability matrix } \Pi \{ \pi_{ij} \}_{i \times s} \]
\[ \Pr[\lambda(k + 1) = m | \lambda(k) = r] = \pi_{rm}, \]
\[ \forall r, m \in \mathcal{S} = \{ 1, 2, \ldots, s \}, \]
\[ \sum_{m=1}^{s} \pi_{rm} = 1, \ 0 \leq \pi_{rm} \leq 1. \]  

The system coefficient matrices \( A(\lambda(k)), A_d(\lambda(k)) \) can be replaced with \( A_r \) and \( A_{dr} \), for each \( \lambda(k) = r \in \mathcal{S} \). Here, rewrite system (37) as the following system model.
\[ \{ y(k + 1) = A_r y(k) + A_{dr}(k - h_1(k) - h_2(k)), \]
\[ y(k) = \varphi(k), \ k = -h_2, -h_2 + 1, \ldots, 0; r \in \mathcal{S}. \]  

Similar to Theorem 1, the stochastic stability problem for system (37) can be naturally solved.

Corollary 1: For given \( h_{11}, h_{12}, h_2, h_{21}, h_{22} \) matrices \( P_i \in \mathbb{R}^{n \times n}, R_i \in \mathbb{R}^{n \times n}, Q_i, Q_r, Q_2, Z_1, K_1 \in \mathbb{R}^{n \times n}, \quad (r = 1, \ldots, 4; l = 1, \ldots, 6; i \in \mathcal{S}), \) and any matrices \( M, T, N, H \in \mathbb{R}^{7n \times 4n} \) and \( Y, U, G, J \in \mathbb{R}^{5n \times 3m} \) that satisfy the following inequalities, the discrete-time Markovian jump system (37) with additive time-varying delays \( h_{11}, h_2 \) satisfying (2) is stochastically stable.

\[ \sum_{j=1}^{4} \pi_{ij}(Q_j - Q_i - \bar{Q}_r) < 0, \ r = 1, \ldots, 4. \]

\[ \Pi_i(h_{11}, h_{12}) h_{10} \beta_1^T T \ h_{10} \beta_1^T U \ h_{20} \beta_2^T H \ h_{20} \beta_2^T J \]
\[ \begin{bmatrix} * & * & * & -h_{10} \bar{R}_2 & 0 & 0 & 0 \\ * & * & * & -h_{10} \bar{Z}_2 & 0 & 0 & 0 \\ * & * & * & -h_{10} \bar{R}_4 & 0 & 0 & 0 \\ * & * & * & -h_{10} \bar{Z}_4 & 0 & 0 & 0 \end{bmatrix} < 0, \]  

\[ \Pi_i(h_{12}, h_{21}) h_{10} \beta_1^T M \ h_{10} \beta_1^T Y \ h_{20} \beta_2^T H \ h_{20} \beta_2^T G \]
\[ \begin{bmatrix} * & * & * & -h_{10} \bar{R}_2 & 0 & 0 & 0 \\ * & * & * & -h_{10} \bar{Z}_2 & 0 & 0 & 0 \\ * & * & * & -h_{20} \bar{R}_4 & 0 & 0 & 0 \\ * & * & * & -h_{20} \bar{Z}_4 & 0 & 0 & 0 \end{bmatrix} < 0, \]  

\[ \Pi_i(h_{12}, h_{22}) h_{10} \beta_1^T M \ h_{10} \beta_1^T Y \ h_{20} \beta_2^T N \ h_{20} \beta_2^T G \]
\[ \begin{bmatrix} * & * & * & -h_{10} \bar{R}_2 & 0 & 0 & 0 \\ * & * & * & -h_{10} \bar{Z}_2 & 0 & 0 & 0 \\ * & * & * & -h_{20} \bar{R}_4 & 0 & 0 & 0 \\ * & * & * & -h_{20} \bar{Z}_4 & 0 & 0 & 0 \end{bmatrix} < 0, \]  

\[ \Pi_i(h_{12}, h_{22}) h_{10} \beta_1^T M \ h_{10} \beta_1^T Y \ h_{20} \beta_2^T N \ h_{20} \beta_2^T G \]
\[ \begin{bmatrix} * & * & * & -h_{10} \bar{R}_2 & 0 & 0 & 0 \\ * & * & * & -h_{10} \bar{Z}_2 & 0 & 0 & 0 \\ * & * & * & -h_{20} \bar{R}_4 & 0 & 0 & 0 \\ * & * & * & -h_{20} \bar{Z}_4 & 0 & 0 & 0 \end{bmatrix} < 0, \]

Here,
\[ \Pi_i(h_{1k}, h_{2k}) = \Pi_i \left( \sum_{j=1}^{s} \pi_{ij} P_j \Pi_{ij}^T - \Pi_{0} \Pi_{0}^T + \Pi_{2i} \right) \]
\[ + \text{Sym} \{ \Pi_3(h_{1k}, h_{2k}) \}, \]
\[ \Pi_{1i} = [e_{ji} s_{1}(h_{1i}) e_{12} - e_{2i} s_{1}(h_{2i}) e_{15} - e_{5} s_{2}(h_{1i}) e_{16} - s_{1}(h_{1i}) e_{12} s_{2}(h_{2i}) e_{19} - s_{1}(h_{2i}) e_{15}], \]
\[ \Pi_{0} = [e_{1} s_{1}(h_{1i}) e_{12} - e_{1} s_{1}(h_{2i}) e_{15} - e_{1} s_{2}(h_{1i}) e_{16} - s_{1}(h_{1i}) e_{12} s_{2}(h_{2i}) e_{19} - s_{1}(h_{2i}) e_{15}], \]
\[ \Pi_{2i} = \bar{P}_{2} - \left[ e_{1} Q_{1} e_{1}^T + e_{2} (Q_{2} - Q_{1}) e_{1}^T + e_{3} Q_{3} e_{1}^T \right. \]
\[ + e_{5} (Q_{4} - Q_{3}) e_{1}^T - e_{4} Q_{2} e_{1}^T - e_{7} Q_{4} e_{1}^T \]
Define the trajectory of the system (37) are
\[
\Delta \tilde{V}_i(k, i) = \eta_i^T(k + 1) \sum_{j \in \mathcal{G}} \pi_{ij} \eta_j(k + 1) - \eta_i^T(k) P_i \eta_j(k)
\]
\[
\Delta \tilde{V}_2(k, i) \leq \xi^T(k) \left( \Pi_{ii}(\sum_{j \in \mathcal{G}} \pi_{ij} P_j) \Pi_i^T - \Pi_0 P_i \Pi_0^T \right) \xi(k),
\]

The rest of the proof is similar to Theorem 1 and has been omitted for brevity.

Remark 4: When the communication network has only one delay, we naturally extend the stability criterion of linear system with additive delays in Theorem 1 to a discrete Markov jump system [43] with single delay, as shown below:
\[
\begin{cases}
  y(k + 1) = A_r y(k) + A_{dr} y(k - h_1(k)), \\
  y(k) = \varphi(k), \quad k = -h_{12} - h_{12} + 1, \cdots, 0; \quad r \in \mathcal{G}.
\end{cases}
\]

From Theorem 1 and Corollary 1, the stochastic stability problem for system (48) can be naturally solved.

Corollary 2: For given $h_{11}, h_{12}$, matrices $P_i \in \mathbb{R}^{3n \times 3n}$, $R_i \in \mathbb{R}^{2n \times 2n}$, $Q_i, \bar{Q}_r, \bar{Z}_r, \bar{K}_i \in \mathbb{R}^{m \times n}$, $(r \in \{1, 2, 3\}; \quad i \in \mathcal{G})$, arbitrary matrices $Y, U \in \mathbb{R}^{m \times 3n}$ and $M, T \in \mathbb{R}^{n \times 4n}$ that satisfy the following inequalities, the system (48) with $h_1(k)$ satisfying (2) is stochastically stable.
\[
\sum_{j=1}^{s} \pi_{ij} (Q_{ij} - Q_{ii} - \bar{Q}_r) < 0, \quad r = 1, 2
\]
\[
\left[ \begin{array}{c}
\bar{P}_{i}(h_{11}) h_{10} \bar{P}_{i}^T T_{h_{10}} \bar{P}_{i}^T U \\
* & -h_{10} \bar{R}_3 & 0 \\
* & * & -h_{10} \bar{Z}_2
\end{array} \right] < 0,
\]
\[
\left[ \begin{array}{c}
\bar{P}_{i}(h_{12}) h_{10} \bar{P}_{i}^T M_{h_{10}} \bar{P}_{i}^T Y \\
* & -h_{10} \bar{R}_2 & 0 \\
* & * & -h_{10} \bar{Z}_2
\end{array} \right] < 0
\]
with
\[
\bar{P}_{i}(h_{1k}) = \bar{P}_{i}(h_{1k}) + \text{Sym}(\{\Pi_3(h_{1k})\}),
\]
\[
\bar{P}_{i1} = [\bar{e}_{ni} s_1(h_{11}) \bar{e}_{12} - \bar{e}_2 s_2(h_{11}) \bar{e}_{16} - s_1(h_{11}) \bar{e}_{12}],
\]
\[
\bar{P}_{i0} = [\bar{e}_1 s_1(h_{11}) \bar{e}_{12} - \bar{e}_1 s_2(h_{11}) \bar{e}_{16} - s_1(h_{11}) \bar{e}_{12}],
\]
\[
\Pi_2i = e_1[\sum_{j\in S} \pi_{ij}Q_{ij} - (h_{11} - 1)\breve{Q}_1\breve{e}_1^T - \breve{e}_4Q_{2i}\breve{e}_4^T \\
+ e_2[\sum_{j\in S} \pi_{ij}Q_{ij} - Q_{ii} - (h_{11} - 1)\breve{Q}_2\breve{e}_2^T \\
+ e_1K_1e_1^T + \breve{e}_2(K_2 - K_1)\breve{e}_2^T + \breve{e}_3(K_3 - K_2)\breve{e}_3^T \\
- \breve{e}_4K_3\breve{e}_4^T + [e_1 \breve{e}_u][h_{11}R_1 + h_{10}R_2][e_1 \breve{e}_u]^T \\
+ e_5(h_{11}Z_1 + h_{10}Z_2)e_5^T \\
- \frac{1}{h_{11}}\tilde{X}_{10}\tilde{X}_{10}^T - \frac{1}{h_{11}}\tilde{Y}_{10}\tilde{Y}_{10}^T], \\
\Pi_3(h_{1k}) = \tilde{\beta}_l'[M\ T][\tilde{X}_{11}\ \tilde{X}_{12}]^T + \tilde{\beta}_l'[Z\ U\ Z][\tilde{Y}_{11}\ \tilde{Y}_{12}]^T, \\
\tilde{e}_{hi} = A_i\breve{e}_1 + A_i\breve{e}_3, \tilde{e}_{si} = (A_i - I)\breve{e}_1 + A_i\breve{e}_3, \\
\tilde{\chi}_{10} = [s_1(h_{11})\tilde{e}_1 - \breve{e}_1(\tilde{e}_1 - \breve{e}_2) \\
(h_{11} + 2)\breve{e}_8 - s_1(h_{11})\tilde{e}_7 - \breve{e}_1 + \breve{e}_2 - 2\breve{e}_7], \\
\tilde{\chi}_{11} = [s_1(h_{11})\tilde{e}_6 - \breve{e}_2(\tilde{e}_2 - \breve{e}_3) \\
(h_{11} + 2)\breve{e}_9 - s_1(h_{11})\tilde{e}_8 - \breve{e}_2 + \breve{e}_3 - 2\breve{e}_5], \\
\tilde{\chi}_{12} = [s_1(h_{11})\tilde{e}_6 - \breve{e}_3(\tilde{e}_3 - \breve{e}_4) \\
(h_{11} + 2)\breve{e}_{10} - s_1(h_{11})\tilde{e}_7 - \breve{e}_3 + \breve{e}_4 - 2\breve{e}_6], \\
\tilde{\gamma}_{10} = [\breve{e}_1^T \breve{e}_2 \breve{e}_7 \tilde{e}_8], \tilde{\gamma}_{11} = [\breve{e}_2^T \breve{e}_3 \tilde{e}_9 \tilde{e}_{10}], \\
\tilde{\gamma}_{12} = [\tilde{e}_3^T \tilde{e}_4 \tilde{e}_6 \tilde{e}_{10}], \\
\tilde{\beta}_l = \text{col} \left\{ \tilde{e}_{10}^T, \tilde{e}_{3}^T, \tilde{e}_{5}^T, \tilde{e}_{7}^T, \tilde{e}_{9}^T, \tilde{e}_{10}^T \right\}^T, \\
\mathcal{R}_r = R_r + \begin{bmatrix} 0 & K_r \\ K_r & K_r \end{bmatrix}, \ (r = 1, 2), \\
\mathcal{R}_3 = R_2 + \begin{bmatrix} 0 & K_3 \\ K_3 & K_3 \end{bmatrix}, \\
\mathcal{R}_l = \{\mathcal{R}_l, 3\mathcal{R}_l\}, \ (l = 1, 2, 3). \\
\]

Proof: For each \( i \in S \), construct the following augmented LKF

\[
\tilde{V}(k) = \tilde{V}_1(k, i) + \tilde{V}_2(k, i) + \tilde{V}_3(k) + \tilde{V}_4(k), \tag{52}
\]

where

\[
\tilde{V}_1(k, i) = \tilde{\eta}_{1i}^T(k)P(\lambda(k))\tilde{\eta}_{1i}(k), \\
\tilde{V}_2(k, i) = \sum_{r=k-h_{11}}^{k-1} y^T(r)Q_1(\lambda(k))y(r) \\
+ \sum_{r=k-h_{12}}^{k-1} y^T(r)Q_2(\lambda(k))y(r) \\
+ \sum_{r=k-h_{11}}^{k-1} y^T(r)(r - k + h_{11})Q_1y(r) \\
+ \sum_{r=k-h_{12}}^{k-1} y^T(r)(r - k + h_{12})Q_2y(r), \\
\tilde{V}_3(k) = \sum_{r=-h_{11}j=k+r}^{k-h_{11}-1} \eta_{1j}^T(r)R_1\eta_{1j}(r) \\
+ \sum_{r=-h_{11}j=k+r}^{k-h_{11}-1} \eta_{2j}^T(r)R_2\eta_{2j}(r), \\
\tilde{V}_4(k) = \sum_{r=-h_{11}j=k+r}^{k-1} \Delta y^T(j)Z_1\Delta y(j) \\
+ \sum_{r=-h_{12}j=k+r}^{k-1} \Delta y^T(j)Z_2\Delta y(j)
\]

with

\[
\tilde{\eta}_{1i}(k) = \text{col} \left\{ y(k), s_1(h_{11})u_3(s_2(h_{11})\omega_1)(k) \right\}, \\
\tilde{\eta}_{2i}(k) = \text{col} \left\{ y(k), \gamma(k - h_{11}), y(k - h_{12}), \gamma(k - h_{12}), \mu_1(k), \mu_2(k), \mu_3(k), \omega_1(k), \omega_2(k), \omega_3(k) \right\}.
\]

The rest of the proof is similar to Corollary 1 and has been omitted for brevity.

Remark 5: If the double summation terms and \( \Delta x(k) \) are removed from \( V_1(k) \), \( V_2(k) \) and \( V_5(k) \), respectively, the LKFs (11), (45) and (52) in this paper reduce to those in the literature [39], [46]. To highlight the advantages of the augmented LKFs, the corresponding LKFs by removing the augmented terms are given in the following forms. The corresponding stability condition forms are similar to the above theorems and corollaries and are omitted for brevity. The relevant calculative results are added to Tables 1-3 of Section IV.

- The simplified LKF (11)

\[
V'(k) = V'_1(k) + V'_2(k) + V'_3(k) + V'_4(k) + V'_5(k) \tag{53}
\]

with

\[
V'_1(k) = \tilde{\eta}_{1i}^T(k)P_0\tilde{\eta}_{1i}(k), \\
V'_2(k) = \sum_{r=-h_{11}j=k+r}^{k-h_{11}-1} x^T(r)R_1x(j) \\
+ \sum_{r=-h_{12}j=k+r}^{k-h_{12}-1} x^T(r)R_2x(j) \\
+ \sum_{r=-h_{21}j=k+r}^{k-h_{21}-1} x^T(r)R_3x(j) \\
+ \sum_{r=-h_{22}j=k+r}^{k-h_{22}-1} x^T(r)R_4x(j), \\
V'_3(k) = \sum_{r=k-h_{2}}^{k-h_{2}-1} x^T(r)Q_5x(r) \\
+ \sum_{r=-h_{2}j=k+r}^{k-h_{2}-1} x^T(r)Q_6x(j) \\
+ \sum_{r=-h_{2}j=k+r}^{k-h_{2}-1} x^T(r)Q_7x(j) \\
+ \sum_{r=-h_{2}j=k+r}^{k-h_{2}-1} x^T(r)Q_8x(j)
\]
IV. NUMERICAL EXAMPLES

This section gives some examples frequently found in some literature to verify the feasibility of the proposed method. ‘NoV’ is the number of decision variables.

- The simplified LKF (45)
  \[
  \bar{V}'(k) = \bar{V}'_1(k,i) + \bar{V}_2(k,i) + \bar{V}_3(k) + \bar{V}_4(k) + \bar{V}_5(k),
  \]
  \( (54) \)
  where
  \[
  \bar{V}'_1(k,i) = \tilde{\eta}_{1}'^T(k)P(\lambda(k))\tilde{\eta}_1(k).
  \]

- The simplified LKF (52)
  \[
  \bar{V}(k) = \bar{V}'_1(k,i) + \bar{V}_2(k,i) + \bar{V}_3(k) + \bar{V}_4(k),
  \]
  \( (55) \)
  where
  \[
  \bar{V}'_1(k,i) = \tilde{\eta}_{1}'^T(k)P(\lambda(k))\tilde{\eta}_1(k),
  \]
  \[
  \bar{V}_3(k) = \sum_{r=-h_{11}}^{k-1} \sum_{j=k+r} x^T(j)R_1 x(j)
  \]
  \[
  + \sum_{r=-h_{12}}^{k-1} \sum_{j=k+r} x^T(j)R_2 x(j).
  \]

Here, \( \tilde{\eta}_1(k) = \text{col} \{x(k), s_1(h_{11})u_3(k), s_1(h_{12})u_1(k) + s_1(h_{12})u_2(k), s_1(h_{21})v_3(k), s_1(h_{22})v_1(k) + s_1(h_{22})v_2(k) \} \), \( \tilde{\eta}_1(k) = \text{col} \{y(k), \sum_{r=k-h_{11}}^{k-1}y(r), \sum_{r=k-h_{12}}^{k-1}y(r) \} \), \( \bar{\lambda}_1(k) = \text{col} \{y(k), s_1(h_{11})u_3(k) \} \).

Examples 1: For linear discrete time-delay system (1), the system matrices commonly used in [44] and [47] are given as follows.

\[
A_1 = \begin{bmatrix} 0.8 & 0 \\ 0.05 & 0.9 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -0.1 & 0 \\ -0.2 & -0.1 \end{bmatrix}.
\]

By solving the corresponding LMIs in theorem 1 of this paper, the MADUBs (maximal admissible delay upper bounds) can be obtained to ensure the stability of the system (1). The results can be directly compared with the corresponding values obtained in some reference. The MADUBs \( h_{22} \) for different \( h_1 \) and \( h_{12} = 2 \) are listed in Table 1, and the MADUBs \( h_2 \) for given \( h_1 = 2 \) are listed in Table 2. It can be seen from the tables that the upper bounds of delay obtained by theorem 1 are significantly larger than those in some literature. Especially, the conservatism decreases obviously when the lower bounds of time delay are small. However, the Novs in the theorem 1 are larger than those in the
literatures. Thus, the theorem 1 reduces the conservatism at the cost of increasing the solution complexity. Moreover, the corresponding stability condition removing the augmented terms is more conservative than Theorem 1, as mentioned in Remark 5. To illustrate the existence of these theoretical MAUBs, we give some simple system state response graphs according to Tables 1 and 2. It follows from Fig. 3 that the system in Example 1 is stable with the simulation conditions of

\[ h_1(k) = \text{int} \left[ 1 + |\sin(\frac{k\pi}{4})| \right], \quad h_2(k) = \text{int} \left[ 9 + 13|\sin(\frac{k\pi}{4})| \right] \]

and \( x(0) = \text{col}[0.1, -0.2] \). Here, ‘int’ indicates integer, ‘-’ indicates the corresponding results are not given.

**Examples 2:** The following system (48) in some literature [14], [48], [49] is given, where system coefficient matrices have the following expression.

\[
A_1 = \begin{bmatrix} 0.8 & 0 \\ 0.05 & 0.9 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.97 \end{bmatrix},
\]

\[
A_{d1} = \begin{bmatrix} -0.1 & 0 \\ -0.2 & -0.1 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} -0.1 & 0 \\ -0.1 & -0.1 \end{bmatrix}, \quad \Pi = \begin{bmatrix} 0.5 & 0.5 \\ 0.1 & 0.9 \end{bmatrix}.
\]

The MAUBs of \( h_k \) can be calculated by the Corollary 2 and other theorems or corollaries in literatures. The MAUBs \( h_{12} \) for different \( h_{11} \) are listed in Table 3. From the table, it is obvious that the results of Corollary 2 are less conservative than the ones in literatures. Especially, the conservatism decreases obviously when the lower bounds of time delay are small. Moreover, the corresponding stability condition removing the augmented terms is more conservative than Corollary 2, as mentioned in Remark 5. The state response curve of the system (48) should be stable according to Table 3. The possibility of a Markov chain \( \lambda(k) \) jumping between two modes is shown in Fig. 4. It follows from Fig. 5 that the two-modes Markovian jump system in Example 1 is stable.
2 is stable with the simulation conditions of $h_1(k) = \text{int} [4 + 16|\sin \left( \frac{k\pi}{4} \right)|]$ and $x(0) = \text{col}(0.2, -0.4)$.

**Examples 3:** For the Markovian jump system (39) with two-modes and additive time-varying delays, the parameters are described as follows.

$$A_1 = \begin{bmatrix} 0.8 & 0 \\ 0.05 & 0.9 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0.8 & 0 \\ 0 & 0.97 \end{bmatrix},$$

$$A_{d1} = \begin{bmatrix} -0.1 & 0 \\ -0.2 & -0.1 \end{bmatrix}, \quad A_{d2} = \begin{bmatrix} 0.1 & -0.2 \\ 0.2 & -0.1 \end{bmatrix},$$

$$\Pi = \begin{bmatrix} 0.8 & 0.2 \\ 0.3 & 0.7 \end{bmatrix}.$$
increases for a fixed $h_{12}$ or $h_{22}$, and the MADUBs decrease as $h_{12}$ or $h_{22}$ increases for a fixed $h_{1}$. The state response curves of the system (39) should be stable according to Tables 4 and 5. The possibility of a Markov chain $\lambda(k)$ jumping between two modes is shown in Fig. 6. It follows from Fig. 7 that the system in Example 3 is stable with $h$ increasing for a fixed $h_{1}$.

The simulation conditions of from Fig. 7 that the system in Example 3 is stable with jumping between two modes is shown in Fig. 6. It follows from Tables 4 and 5. The possibility of a Markov chain $\lambda(k)$ curves of the system (39) should be stable according to $\lambda(k) = int \left[1 + 2|\sin \left(\frac{kT}{4}\right)|\right]$ and $x(0) = col\{0.3, -0.4\}$.

V. CONCLUSION

This paper deals with the stability problem of discrete-time systems with additive time-varying delays. The relevant LKFs are augmented with some additional state variables with time delay information. Based on a general free weighting matrix summation inequality technique, a less conservative stability criterion than some existing literatures is obtained. To apply the obtained stability criteria, some corollaries are given for the Markovian jump system with single or two additive time-varying delays. Finally, in the numerical simulation examples, the Matlab LMI toolbox is used to solve the LMIs in theorems and corollaries, and the MADUBs to guarantee the stability of the system is obtained. The results are compared directly with those in some references. The comparison results show that the stability criterion in this paper is less conservative than those in some previous literatures. However, the Novs in the result of this paper are larger than those in some literatures. Thus, the result of this paper reduces the conservatism at the cost of increasing the solution complexity.

The novel stability condition can also be generalized to some other discrete-time time-delayed control systems, for example, time-delayed linear systems, time-delayed neutral-type systems, real time dynamic systems, and so on. However, there is still some distances from practical application because of the complexity of the control theory, which can be something that we will continue to study in the future.

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