Quickest Detection of COVID-19 Pandemic Onset

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Abstract—When should restrictive measures be taken to contain the COVID-19 pandemic explosion? An important role for interpreting the pandemic data in a fully rational manner, and thus proactively supporting the political decision makers, is played by signal processing tools. In this paper, we exploit quickest-detection theory applied to pandemic time-series to predict the explosion of COVID-19 infection on a large scale. We develop a version of the celebrated Page’s CUSUM test, specifically tailored to pandemic data. Its application to the publicly-available sequence of new positive individuals per day, from different countries, is detailed in [1].

I. INTRODUCTION

The outbreak of the COVID-19 infection is certainly one of the most serious global crises of the last two decades. To contain the “first wave” of the COVID-19 pandemic, in the spring of 2020 strict lockdown was imposed in many countries, with huge societal and economic costs [2]–[7]. In the fall of 2020, a “second pandemic wave” seems to be growing in many regions of the world, and governments and authorities are again faced with the dilemma of if and when to impose social restrictions. Signal processing tools, and more specifically quickest-detection theory [8]–[10], may provide valuable support to make informed and rational decisions. In this article, we develop a version of the Page’s CUSUM quickest-detection procedure [11]–[14], specifically tailored to pandemic COVID-19 timeseries. The proposed test is called MAST, an acronym for mean-agnostic sequential test. Application of MAST to pandemic data is discussed in [1], while in this document we focus on the derivation of the test.

Starting from the landmark SIR model developed in [15], a multitude of sophisticated epidemiological models has been proposed, based, e.g., on stochastic evolution of epidemic compartments [16]–[20], or metapopulation networks. [21], [22], just to cite two examples. The trend in the topical literature is to conceive increasingly complex models, often suitable to be analyzed by big-data techniques. The main goal of these models is to predict mid/long-term evolution of the infection. Our focus, instead, is to quickly detect the onset of the pandemic explosion. To this aim, we consider an abbreviated observation model, built on the basic concept that the pandemic evolution is essentially a multiplicative phenomenon.

We model the number of new positive individuals on day \( n \), say \( p_n \), as the number \( p_{n-1} \) of new positive individuals on day \( n-1 \), multiplied by a random variable \( x_n \). Further including a “noise” term \( w_n \), yields the iterative scheme

\[
p_n = p_{n-1} x_n + w_n, \quad n \geq 1, \tag{1}
\]

for some \( p_0 \). Model (1), under various assumptions for the sequences \( \{(x_n, w_n)\} \), is known as a perpetuity and appears in many disciplines [23]–[24]. We assume that the noise term in (1) is negligible, yielding

\[
p_n = p_{n-1} x_n \quad \Rightarrow \quad p_n = p_0 \prod_{k=1}^{n} x_k, \tag{2}
\]

for some \( p_0 > 0 \). In this article, we refer to model (2), in which \( x_1, x_2, \ldots \) are independent random variables. This is akin to the popular random walk model, with the difference that the independence of the increments of the random walk is replaced with the independence of the ratios \( p_n/p_{n-1} \). Model (2) has been derived from SIR-like models and validated on COVID-19 pandemic data in [1], where it is also shown that the \( x_k \)’s closely follow a Gaussian distribution with (unknown) time-varying expected value \( \mathbb{E}x_k \), and a common standard deviation \( \sigma \).

As long as \( \mathbb{E}x_k < 1 \), the sequence \( \{p_n\} \) tends to decrease exponentially fast, while, for \( \mathbb{E}x_k > 1 \), \( \{p_n\} \) tends to increase exponentially fast. We are interested in quickly detecting the passage from the former situation (a controlled regime) to the latter (critical). Detecting this change of regime can be cast in terms of a binary decision problem between two hypotheses, referred to as the null and the alternative, as discussed next.

II. QUICKEST DETECTION OF PANDEMIC ONSET

Along the same lines of the derivations of Page’s test, see e.g., [12] Sec. 2.2.3] or [20] Sec. 8.2], we consider the following decision problem involving two statistical hypotheses with independent data:

\[
\begin{align*}
\text{null} & : \quad x_k \sim \mathcal{N}(\mu_0, \sigma), \quad k = 1, \ldots, n, \\
\text{alternative} & : \quad \begin{cases} 
\{x_k \sim \mathcal{N}(\mu_0, \sigma), \quad k = 1, \ldots, j - 1, \\
\{x_k \sim \mathcal{N}(\mu_1, \sigma), \quad k = j, \ldots, n.
\end{cases}
\end{align*} \tag{3}
\]

The same multiplicative structure shown in (2) applies to, other than \( p_n \), different timeseries related to the pandemic evolution, e.g., the number of new hospitalization per day [11].

Since \( \sigma \ll 1 \) and \( \mathbb{E}x_k \approx 1, \) \( \mathbb{P}(x_k < 0) \) is negligible, for all \( n \). Thus, one can safely assume that \( \{x_n\} \) is a sequence of independent nonnegative random variables.
In [3], \( \{x_k\}_{k=1}^n \) are the data available to the decision maker, \( 1 \leq j \leq n + 1 \) is an unknown deterministic change time and the standard deviation \( \sigma \) is assumed known. Note in [3] that in the case \( j = n + 1 \), the alternative hypothesis is equivalent to the null one, i.e., there is no change of regime. Different from the classical assumption of Page’s test, in our problem, the expected values before and after the change are unavailable. Accordingly, we model \( \{\mu_{0,k}\}_{k=1}^n \) and \( \{\mu_{1,k}\}_{k=1}^n \) as unknown deterministic sequences and we assume that they satisfy the following constraints:

\[
\mu_{0,k} \leq \delta_t, \quad \mu_{1,k} > \delta_u, \quad 0 < \delta_t \leq \delta_u < \infty. \tag{4}
\]

Thus, model [3] contains \( 2n+1 \) unknown parameters: the index of change \( j \) and the two sequences of the expected values. In [4], the most natural choice is \( \delta_t = \delta_u = 1 \), but it is convenient to consider the general case having an implied hysteresis. For example, \( \delta_u \) may be specified based on tolerable time to reach hospital capacity, while \( \delta_t \) may be specified based on time people can endure restrictions before reopening the economy or tolerable level of positive cases.

One might also consider

\[
\mu_0 \leq \delta_t, \quad \mu_1 > \delta_u, \quad 0 < \delta_t \leq \delta_u < \infty, \tag{5}
\]

in place of (4). In some sense this might be more natural, since the mean levels before and after the change remain assumed unknown, but are merely constant. However, formulation (3) does not admit a recursive Page-like procedure whereas MAST that results from (4) does.

According to the Generalized Likelihood Ratio Test (GLRT) principle [8, 27], the decision statistic for problem [3] is

\[
T_n(\delta_t, \delta_u) = \max_{1 \leq j \leq n+1} \mathcal{T}_{j:n}(\delta_t, \delta_u),
\]

where

\[
\mathcal{T}_{j:n}(\delta_t, \delta_u) = \max_{1 \leq j \leq n+1} \mathcal{T}_{j:n}(\delta_t, \delta_u),
\]

or, equivalently, taking the logarithm:

\[
T_n(\delta_t, \delta_u) = \max_{1 \leq j \leq n+1} \mathcal{T}_{j:n}(\delta_t, \delta_u),
\]

The passage from the controlled to the critical regime is declared at the smallest \( n \) such that

\[
T_n(\delta_t, \delta_u) > \gamma,
\]

where the threshold level \( \gamma \) is selected to trade-off decision delay and risk, two quantities that will be defined in Sec. III

The test in (12) will be referred to as MAST(\( \delta_t, \delta_u \)) — an acronym for mean-agnostic sequential test — with boundaries \( \delta_t \) and \( \delta_u \). The subscript \( n \) appended to \( T_n(\delta_t, \delta_u) \) denotes its dependence on the stream of data \( x_1, \ldots, x_n \), and the subscript \( j \) appended to \( \mathcal{T}_{j:n}(\delta_t, \delta_u) \) denotes its dependence on \( x_j, \ldots, x_n \). Finally, by introducing the non-linearity

\[
g(x_k; \delta_t, \delta_u) = \begin{cases} 
\frac{(x_k - \delta_t)^2}{2\sigma^2}, & x_k \leq \delta_t, \\
\frac{\delta_t - x_k}{2\sigma^2}, & \delta_t < x_k \leq \delta_u, \\
\frac{(x_k - \delta_u)^2}{2\sigma^2}, & x_k > \delta_u,
\end{cases}
\]

we have

\[
\mathcal{T}_{j:n}(\delta_t, \delta_u) = \sum_{k=j}^n g(x_k; \delta_t, \delta_u).
\]

As a sanity check, let us assume that values of \( x_k \) closer to \( \delta_t \) are confused with \( \delta_t \) and, likewise, values of \( x_k \) closer to \( \delta_u \) are confused with \( \delta_u \). Then, we see from (11) that the contribution to \( \mathcal{T}_{j:n}(\delta_t, \delta_u) \) provided by the sample \( x_k \) is \( \pm (\delta_u - \delta_t)^2 / (2\sigma^2) \), where the negative sign applies to the former case and the positive one to the latter. In the actual operation of \( \mathcal{T}_{j:n}(\delta_t, \delta_u) \), the contribution given by the sample \( x_k \) is regulated by its distance to the boundaries, as shown in (13):

- values \( x_k \leq \delta_t \) give a negative contribution proportional to the square of the distance of \( x_k \) from the upper boundary \( \delta_u \);
- values \( \delta_t < x_k < \delta_u \) give a linear contribution, whose sign depends on which boundary \( x_k \) is closest to;
- values \( x_k > \delta_u \) give a positive contribution proportional to the square of the distance of \( x_k \) from the lower boundary \( \delta_t \).
Using the non-linearity of (13) in (10), one gets

\[ T_n(\delta_t, \delta_u) = \max_{1 \leq j \leq n+1} \sum_{k=j}^{n} g(x_k; \delta_t, \delta_u) \]

\[ = \max \left[ 0, \max_{1 \leq j \leq n} \sum_{k=j}^{n} g(x_k; \delta_t, \delta_u) \right] , \quad (14) \]

where we have used \( \sum_{j}^{n} g(x_k; \delta_t, \delta_u) = 0 \).

The MAST \((\delta_t, \delta_u)\) decision statistic (14) can be expressed in recursive form. To see this, let us define \( S_m = \max_{1 \leq j \leq m} G_j^m \), with \( G_j^m = \sum_{k=j}^{m} g(x_k; \delta_t, \delta_u), \) \( m = 1, \ldots, n \). By using the notation \( (x)^+ = \max(0, x) \), we see that (14) can be written as \( T_n(\delta_t, \delta_u) = (S_n)^+ \). Then,

\[ T_n(\delta_t, \delta_u) = (S_n)^+ = \max \left[ 0, \max_{0} \left( g(x_n; \delta_t, \delta_u) + \sum_{j=1}^{n} g(x_j; \delta_t, \delta_u) \right) \right] \]

\[ = \max \left[ 0, g(x_n; \delta_t, \delta_u) + \max \left( 0, \max_{1 \leq j \leq n} \left( g(x_j; \delta_t, \delta_u) + \sum_{k=j}^{n} g(x_k; \delta_t, \delta_u) \right) \right) \right] \]

\[ = \max \left[ 0, g(x_n; \delta_t, \delta_u) + \max \left( 0, \sum_{s=1}^{n} g(x_s; \delta_t, \delta_u) \right) \right] \]

\[ = \max \left[ 0, g(x_n; \delta_t, \delta_u) + \max \left( 0, \sum_{s=1}^{n} g(x_s; \delta_t, \delta_u) \right) \right] \]

\[ = \max \left[ 0, T_{n-1}(\delta_t, \delta_u) \right] + (S_n)^+ \]

\[ = \max \left[ 0, T_{n-1}(\delta_t, \delta_u) + g(x_n; \delta_t, \delta_u) \right] . \quad (15) \]

We have thus arrived at the recursive expression for the decision statistic: \( T_0(\delta_t, \delta_u) = 0 \) and, for \( n \geq 1 \), \( T_n(\delta_t, \delta_u) = \max \left[ 0, T_{n-1}(\delta_t, \delta_u) + g(x_n; \delta_t, \delta_u) \right] \).

The second special case is when \( \delta_t = \delta_u = \delta \), a case referred to as the MAST \((\delta)\) detector, with decision statistic \( T_0(\delta) = 0 \) and, for \( n \geq 1 \), \( T_n(\delta) = \max \left[ 0, T_{n-1}(\delta) + \frac{(x_n - \delta)^2}{2\sigma^2} \right] . \quad (17) \]

Further assuming \( \delta = 1 \) in (17), yields a decision procedure that we simply call MAST, whose decision statistic \( T_n(1) \) is denoted by \( T_n : T_0 = 0 \) and, for \( n \geq 1 \), \( T_n = \max \left[ 0, T_{n-1} + \frac{(x_n - 1)^2}{2\sigma^2} \right] . \quad (18) \]

The performance of MAST \((\delta_t, \delta_u)\) is expressed in terms of mean delay time \( \Delta \) and false alarm probability \( P_f \). The mean delay \( \Delta \) is the difference between the time at which the MAST \((\delta_t, \delta_u)\) statistic \( T_n(\delta_t, \delta_u) \) crosses a preassigned threshold level \( \gamma \), see (12), and the time of passage from the controlled to the critical regime. In the critical regime, the pandemic grows exponentially fast and it is therefore important to ensure that \( \Delta \) is as small as possible. This requirement is in contrast with the requirement \( P_f \ll 1 \). The false alarm probability \( P_f \) is defined as the reciprocal of the mean time between two false alarms\(^3\). In turn, the mean time between false alarms is the mean time between two threshold crossings, assuming that the decision statistic is reset to zero at any threshold crossing event, occurring in the controlled regime. Because of the huge social and economic impact of the measures presumably taken by the authorities when passage into the critical regime is detected, it is evident that \( P_f \) must be extremely small. The same performance indices \( \Delta \) and \( P_f \) used to characterize MAST \((\delta_t, \delta_u)\) are used for the Page’s test.

We now investigate the performance of MAST \((\delta_t, \delta_u)\) by computer experiments, limiting the analysis to the case \( \delta_t = \delta_u = 1 \), i.e., the simple MAST. The performance of the Page’s test is used as a benchmark. Let us consider the following “scenario 1”. Fix \( \alpha > 0 \). Suppose that the state of nature (mean value of the \( x_n \)’s) is \( \mu_{0,n} = 1 - \alpha \) for all \( n \) in the controlled regime; likewise, suppose \( \mu_{1,n} = 1 + \alpha \) for all \( n \) in the critical regime. By standard Monte Carlo counting, for MAST we found that the delay \( \Delta \) varies almost linearly with the threshold level \( \gamma \), and that \( P_f \) varies almost linearly with \( \gamma \). The same approximate behavior is found, again by standard Monte Carlo counting, for the clairvoyant Page’s test. The performance of the Page’s test, at least when the threshold \( \gamma \) is sufficiently large, in view of the Wald’s approximation, see, e.g., (12) Eq. 5.2.44. In

\(^3\)Note that in a quick detection application the concept of a “false alarm” is different from that in a fixed-block test.
the present Gaussian case, more accurate formulas — known as Siegmund’s approximations — are also available for the performance of the Page’s test [12, Eqs. 5.2.64, 5.2.65].

We assume that the aforementioned linear mappings observed for MAST and Page’s test hold true for any value of the threshold, and this assumption allows us to consider values of the mean delay and (especially) of false alarm probability to be constant, i.e., $\mu_{0,n}$ and $\mu_{1,n}$ are time-varying, as happens in the COVID-19 pandemic.

The numerical analysis has been conducted for “scenario 2”, also shown in Fig. 1. In scenario 2, we suppose that in the controlled regime, any $\mu_{0,n}$ is an instantiation of a uniform random variable with support $(1 - \alpha, 1)$, while in the critical regime any $\mu_{1,n}$ is an instantiation of a uniform random variable with support $(1, 1 + 10\alpha)$. To implement the Page’s test, it is assumed that the mean values are constant, i.e., $\mu_{0,n} = 1 - \alpha$ and $\mu_{1,n} = 1 + \alpha$, as in scenario 1. Of course, no assumption about the mean values is instead needed for implementing the MAST test, except that they are bounded by one. We see that MAST outperforms Page’s test, confirming its effectiveness when the mean values are unknown, except for being bounded as shown in (4), namely $\mu_{0,n} \leq 1$ and $\mu_{1,n} > 1$, for all $n$.

IV. CONCLUSION

This article derived a sequential test called MAST, which is used in [1], to detect passage from the controlled regime in which the COVID-19 pandemic is restrained, to the critical regime in which the infection spreads exponentially fast. MAST is a variation of the celebrated Page’s test based on the CUSUM statistic, designed for cases in which the expected values of the data are bounded below a lower barrier $\delta_0$ in the controlled regime, and above an upper barrier $\delta_1$ in the critical one, but are otherwise unknown. We show that MAST admits a recursive form and in the simplest case $\delta_1 = \delta_0 = 1$, is formally obtained from the Page’s test with nominal expected values $1 \pm \alpha$, by replacing $\alpha$ with an estimate thereof. The performance of MAST is investigated by computer experiments. If the the expected values of the data are constant and known, the performance loss of MAST with respect to the optimal Page’s test is moderate. In pandemic scenarios, lacking knowledge of the expected values of the data, MAST can well overcome the Page’s test designed with nominal values of the unknowns. Finally, we would like to stress that the domain of application of MAST goes beyond the specific example of COVID-19 pandemic addressed here, to cover different change-detection applications in the presence of data uncertainty.

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