The universal freeze-out criterion at SPS and RHIC

Boris Tomášik and Urs Achim Wiedemann

CERN, Theory Division, CH-1211 Geneva 23, Switzerland

We formulate a freeze-out criterion for ultra-relativistic heavy ion collisions in terms of the pion escape probability from the collision region. We find that the increase in pion phase-space density from SPS to RHIC reported at this conference has a small influence on particle freeze-out because of the small $\pi\pi$ cross-section. Our treatment takes into account dynamical expansion, chemical composition and momentum of the escaping particle for particle freeze-out. It supports a freeze-out at rather low temperature—below 100 MeV—and earlier decoupling of high $p_\perp$ particles.

A precise formulation of the condition under which particles decouple (“freeze-out”) from a heavy ion collision is important for understanding the dynamical origin of the final state. The observation of STAR [1] that the pion phase-space density increases at RHIC indicates that freeze-out is not characterised solely by a universal value of pion phase-space density, in contrast to a recent suggestion [2]. Also, freeze-out is not characterised solely by a universal value of the total particle density since the volume from which particles decouple shows a non-monotonous dependence on collision energy [3].

It has been argued [4] and demonstrated [3] that the chemical composition of the system must be taken into account. For pion freeze-out, the nucleon and anti-nucleon density is more important than that of pions since the pion-nucleon cross-section is larger than the pion-pion one. With increasing collision energy the pion abundance and their contribution to scattering grows, but it is argued that the momentum-averaged mean free path of a pion at freeze-out stays constant [3].

This argumentation still ignores the influence of global dynamics on the freeze-out criterion. A strong expansion makes the particle density decrease fast and if the characteristic time scale for density decrease $\tau_{\exp} = (\partial_u u^\mu)^{-1}$ is smaller than the mean free time between collisions $\tau_{\text{scatt}}$, a particle is likely to seize interacting, i.e., to freeze-out [3].

In our work we study—in addition to these two aspects—how the probability for a particle to decouple depends on its momentum [3]. We formulate freeze-out as condition for an individual particle: a particle freezes-out at the time of its last scattering. This is in contrast to the “fireball freeze-out” which is understood as a boundary in space-time between interacting matter and free-streaming particles. In the extreme case of a hydrodynamic model the “fireball freeze-out” is usually treated by the Cooper-Frye prescription [7] in which all particles decouple along the same three-dimensional hyper-surface when the medium fulfils certain condition. Our treatment goes beyond Cooper-Frye: particle freeze-out depends on its type, position, and momentum in addition to the characteristics of the medium. This corresponds to freeze-out as described by a cascade
The freeze-out criterion which we explore is based on the escape probability
\[ \mathcal{P}(x, p, \tau) = \exp \left( - \int_{\tau}^{\infty} d\bar{\tau} \mathcal{R}(x + u\bar{\tau}, p) \right), \]
which determines the probability that a pion emitted with momentum \( p \) from the position \((x, \tau)\) escapes from the medium without further interaction. Here, the scattering rate \( \mathcal{R}(x, p) \) is integrated along the trajectory of the pion. Freeze-out is assumed to occur when \( \mathcal{P}(x, p, \tau) \) reaches a universal value of order one.

The role of collective expansion: To illustrate how collective expansion affects particle freeze-out, we consider the case of particles with vanishing momentum in the centre of a fireball with longitudinally boost-invariant and transversely linear expansion velocity profile of the form
\[ u^\mu = (\cosh \eta \cosh \eta_t, \cos \phi \sinh \eta_t, \sin \phi \sinh \eta_t, \sinh \eta \cosh \eta_t). \]
Here, \( \eta = \text{Arctanh}(z/t) \) and \( \eta_t = \xi r \) where \( \xi = 0.08 \text{ fm}^{-1} \) is a typical value for the transverse expansion gradient. At the time \( \tau_{\text{em}} \) at which the pion is emitted, the density of scattering centres decreases by the rate
\[ \frac{1}{\rho} \frac{\partial \rho}{\partial \tau} \bigg|_{\tau=\tau_{\text{em}}} = \partial_\mu u^\mu = \frac{1}{\tau_{\text{em}}} + 2\xi. \]
A power-law decrease of density with time is consistent with (3) if
\[ \rho(\tau) = \rho_{\text{em}} \left( \frac{\tau_{\text{em}}}{\tau} \right)^\alpha, \quad \alpha = 1 + 2\xi \tau_{\text{em}}. \]
Under the assumption \( \mathcal{R} \propto \rho \) we obtain for the opacity integral of a particle of zero momentum
\[ \int_{\tau_{\text{em}}}^{\infty} d\tau \mathcal{R}(\tau) = \frac{\mathcal{R}_{\text{em}}}{\alpha - 1} \tau_{\text{em}} = \frac{\mathcal{R}_{\text{em}}}{2\xi}, \]
where \( \mathcal{R}_{\text{em}} \) is the scattering rate at \( \tau_{\text{em}} \). Technically, the calculation of this opacity integral for particles of non-zero momentum \( p \) is more complicated, since one has to follow the propagation of the particle through layers of different density. For a quantitative statement, this requires a model of the space-time evolution of the fireball. Qualitatively, however, the general result will be consistent with the main feature of the case \( p = 0 \) considered here: as seen from (3), freeze-out from a denser fireball (larger \( \mathcal{R}_{\text{em}} \)) is possible if it is compensated by a stronger expansion gradient such that the value of opacity integral stays constant.

The role of chemical composition: To determine the value of \( \mathcal{R}_{\text{em}} \) for collisions at \( \sqrt{s} = 17 \text{ AGeV} \) (SPS) and \( \sqrt{s} = 130 \text{ AGeV} \) (RHIC), we calculate the scattering rate
\[ \mathcal{R}(p) = \sum_i \int d^3k \rho_i(k) \sigma_i'(s) |v_{\pi} - v_i|, \]
where \( \sigma_i' \) is the cross-section for pion scattering on species \( i \). For the collinear cross-section we use the parametrisation
\[ \sigma_i(s) = \sum_r \langle j_i, m_i, j_\pi, m_\pi | J_r, M_r \rangle \frac{2S_r + 1}{(2S_i + 1)(2S_\pi + 1)} \frac{\pi}{p_{\text{CMS}}^2} \frac{\Gamma_{r \rightarrow \pi i} \Gamma_{i,\text{tot}}}{(M_r - \sqrt{s})^2 + \Gamma_{i,\text{tot}}^2/4}. \]
Figure 1. The pion scattering rate as a function of pion momentum with respect to medium calculated at $T = 100$ MeV and the highest estimates of chemical potentials allowed by data from SPS (left column) and RHIC (right column). Contributions to the total scattering rate from scattering on nucleons, anti-nucleons and pions are indicated. The lower row shows the baryonic and mesonic relative contributions.

where the sum runs over all resonances in the $\pi i$ scattering. Particle densities $\rho_i(k)$ are assumed to be thermal. For realistic temperatures, only low-lying resonances are relevant and we include $i = \pi, N, \bar{N}, K, \rho, \Delta, \bar{\Delta}$ when summing in (6) over scattering partners. Since the freeze-out temperature is not known a priori, we scan three values: $T = 90, 100, 120$ MeV. Chemical potentials for pions are estimated from data on phase-space density, those for other species from measured ratios of $dN/dy$ at mid-rapidity (see [6] for more details on the estimates).

Figure 1 shows the scattering rates as a function of the pion momentum relative to the heat bath calculated for SPS and RHIC, as well as the most important contributions to $\mathcal{R}$. In spite of the strong increase of the pion phase-space density at RHIC [1], the relative meson contribution to the total scattering rate does not grow proportionally since the $\pi\pi$ cross-section is much smaller than the one for $\pi N$. The total baryonic contribution changes very little since the smaller amount of baryons at RHIC is roughly compensated by anti-baryons. In summary, there is about 10% of the relative contribution shifted from baryons to mesons when going from SPS to RHIC.

As seen in Figure 1, the scattering rate depends strongly on the assumed freeze-out temperature since the pion-nucleon scattering is typically dominated by total CMS energy below the $\Delta$ resonance peak. At higher temperature, average CMS energies move closer to the peak value thus leading to increased scattering cross-sections. Estimating the pion escape probability \( \mathcal{R} \) from eq. (5) and $\xi = 0.08\text{fm}^{-1}$, one finds that a pion only has a
reasonable chance to escape (of order 50%) when the temperature is 100 MeV at most.

We finally point out that the scattering rates in Figure 2 generically decreases with increasing pion momentum. This leads us to conjecture that high $p_\perp$ pions may decouple earlier, from hotter and smaller system. This may provide a novel as yet unexplored contribution to the observed $M_\perp$ dependence of HBT radii.

The freeze-out criterion studied here differs from the Cooper-Frye formalism typically employed in hydrodynamical model studies. It will be interesting to explore to what extent this affects the predictions of hydrodynamical simulations for the collision evolution.

REFERENCES

1. L. Ray for the STAR collaboration, these proceedings.
2. D. Ferenc et al., Phys. Lett. B 457 (1999) 347.
3. D. Adamová et al. [CERES collab.], arXiv:nucl-ex/0207008 and these proceedings.
4. S. Nagamiya, Phys. Rev. Lett. 49 (1982) 1383.
5. J. P. Bondorf, S. I. Garpman and J. Zimányi, Nucl. Phys. A 296 (1978) 320.
6. B. Tomášik and U. A. Wiedemann, arXiv:nucl-th/0207074.
7. F. Cooper and G. Frye, Phys. Rev. D 10 (1974) 186.
8. Y. M. Sinyukov, S. V. Akkelin and Y. Hama, Phys. Rev. Lett. 89 (2002) 052301.
9. B. Tomášik, U. A. Wiedemann and U. W. Heinz, arXiv:nucl-th/9907090.

Figure 2. The pion scattering rate as a function of pion momentum with respect to the medium calculated for different temperatures and a range of chemical potentials allowed by data (see [4]). New results of STAR collaboration [1] correspond rather to the “high $\mu$’s” estimates.