Squeezed condensate of gluons and $\eta - \eta'$ mass difference

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Abstract

We consider a mechanism to create the $\eta - \eta'$ mass difference by the gluon anomaly in a squeezed vacuum. We find that the mass of the $\eta_0$ governing this mass difference is determined by the magnetic part of the gluon condensate. For the squeezed state this magnetic part in the thermodynamical limit coincides with the total gluon condensate, so that we get a relation between the mass of the $\eta_0$ and the gluon condensate. The value of the gluon condensate obtained through this relation is in good agreement with the standard value by Shifman, Vainshtein and Zakharov.

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1. In this note we point out the possibility to express the \( \eta - \eta' \) meson mass difference through the magnetic part of the gluon condensate. The comparison with the standard value of the total gluon condensate [1] allows us to draw some conclusions about the physical nature of the gluon vacuum.

Recall that the U(1) problem [2] is the question why the \( \eta' \) mass is much larger than that of the other eight pseudoscalar mesons, especially the \( \eta \). The \( \eta - \eta' \) mass difference \( m^2_{\eta'} - m^2_\eta = 0.616 \text{ GeV}^2 \) is governed by the mass splitting between the singlet and the octet pseudoscalars \( \eta_0 \) and \( \eta_8 \), which are related to the physical states \( \eta, \eta' \) via the mixing

\[
\begin{align*}
\eta &= \eta_0 \sin \phi - \eta_8 \cos \phi \\
\eta' &= \eta_0 \cos \phi + \eta_8 \sin \phi .
\end{align*}
\]

(1)

In Ref. [3] the \( \eta_0 - \eta_8 \) mixing to the physical states \( \eta, \eta' \) with appropriate mass splitting has been obtained in the chiral limit with a mass \( m^2_0 = 0.729 \text{ GeV}^2 \) for the \( \eta_0 \) and the mixing angle \( \phi = -18.1^\circ \). This mixing angle fits well to recent analyses of \( \eta \) and \( \eta' \) decays [4,5]. The mass of the \( \eta_8 \) meson as a member of the pseudoscalar flavour octet is well explained by explicit chiral symmetry breaking in accordance with the Goldstone theorem and the Gell-Mann–Oakes–Renner relation. However, explicit chiral symmetry breaking is not sufficient to explain the large mass of \( \eta_0 \) [2].

In the literature [6]-[9] the large mass of \( \eta_0 \) is explained by the gluon anomaly \( F^{\mu\nu a} \tilde{F}^a_{\mu\nu} \equiv \partial_\mu K^\mu \). There are several ways to implement this gluon anomaly. In Ref. [6] t’Hooft relates this term to the instanton density in Euclidean space and introduces an effective quark interaction simulating the anomalous term which breaks \( U_A(1) \) but conserves the chiral \( SU(3)_L \otimes SU(3)_R \) symmetry. This determinant interaction has been widely used within effective quark models such as the NJL model [10,11].

Not using the concept of instantons, other authors [7]-[9] start from an effective hadron Lagrangian which explicitly includes an anomalous meson-gluon interaction term which can be viewed in analogy to the anomalous \( \pi^0 \to 2\gamma \) decay

\[
\mathcal{L}_{\text{hadron}}^{\text{singlet}} = \frac{1}{2} \partial_\mu \eta_0 \partial^\mu \eta_0 - \eta_0 \frac{c}{4} F^{\mu\nu a} \tilde{F}^a_{\mu\nu} ,
\]

(2)

where

\[
c = \sqrt{N_f} \alpha_s/(\pi f_\pi) ,
\]

(3)

with \( f_\pi = 93 \text{ MeV} \) being the pion decay constant and \( \alpha_s = g^2/4\pi \) the strong coupling constant. Furthermore, a kinetic term \( C(\partial_\mu K^\mu)^2 \) is added to (2) and
the additional phenomenological constant $C$ is fitted in order to describe the empirical $\eta - \eta'$ mass difference. For a review, see e.g. [12]. Note that we discuss here the chiral limit where in the Lagrangian (2) the bare mass of the $\eta_0$ is zero.

2. In difference to [3] and [7]-[9] we start from a Lagrangian which includes in addition to (2) the standard gluon kinetic term

$$\mathcal{L}_{\text{singlet}} = \mathcal{L}_{\text{hadron}}^{\text{singlet}} - \frac{1}{4} F^{\mu\nu a} F_{\mu\nu}^a . \quad (4)$$

From the Lagrangian (4) it should be possible to obtain the mass of the singlet pseudoscalar $\eta_0$ as a consequence of the coupling to the gluon field. To calculate the mass of the $\eta_0$ let us construct the corresponding Hamiltonian. We have

$$F^{\mu\nu a} F_{\mu\nu}^a = -2(F^a_{0i})^2 + 2(B^a_i)^2 , \quad F^{\mu\nu a} F_{\mu\nu}^a = -4F^a_{0i} B^a_i , \quad (5)$$

where $B^a_i$ are the components of the magnetic field strength. For the quantization of the physical gluon fields $A_0$ has to be eliminated, we use here the convention $A_0 = 0$. Introducing the canonical momenta

$$\Pi_{\eta_0} = \frac{\partial \mathcal{L}_{\text{singlet}}}{\partial \dot{\eta}_0} = \dot{\eta}_0 ,$$

$$E^a_i = \frac{\partial \mathcal{L}_{\text{singlet}}}{\partial \dot{A}^a_i} = F^a_{0i} + c B^a_i \eta_0 , \quad (6)$$

the Hamiltonian density reads

$$\mathcal{H}_{\text{singlet}} = \dot{\eta}_0 \Pi_{\eta_0} + \dot{A}^a_i E^a_i - \mathcal{L}_{\text{singlet}}$$

$$= \frac{1}{2} \Pi_{\eta_0}^2 + \frac{1}{2} (\partial_i \eta_0)^2 + \frac{1}{2} (E^a_i - c B^a_i \eta_0)^2 + \frac{1}{2} (B^a_i)^2 . \quad (7)$$

Taking the expectation value of the corresponding Hamiltonian $H = \int d^3 x \mathcal{H}$ with respect to the nonperturbative physical gluon vacuum $|0_{\text{gluon}}>$ where $<0_{\text{gluon}}|B^a_i|0_{\text{gluon}}> = 0$, the effective Hamiltonian reads

$$H = \int d^3 x \left[ \frac{1}{2} \Pi_{\eta_0}^2 + \frac{1}{2} (\partial_i \eta_0)^2 + \frac{1}{2} m_{\eta_0}^2 \eta_0^2 \right] + \text{const} . \quad (8)$$

We thus obtain the $\eta_0$ mass formula

$$m_{\eta_0}^2 = \frac{3\alpha_s}{2\pi f_\pi^2} G_B^2 , \quad (9)$$
where $G_B^2$ is the magnetic part of the condensate

$$G_B^2 \equiv \frac{2\alpha_s}{\pi} <0_{\text{gluon}}|(B^a_i)^2|0_{\text{gluon}}> \ .$$

(10)

In order to obtain a numerical estimate, we take $\alpha_s \sim 1$ and the phenomenological result $m_0^2 = 0.729 \text{ GeV}^2$ [3] to get for the magnetic condensate the value

$$G_B^2 = 0.013 \text{ GeV}^4 .$$

(11)

A comparison of this estimate for the magnetic condensate with the standard value for the total gluon condensate [1],

$$G^2 = \frac{\alpha_s}{\pi} <0_{\text{gluon}}|F^{\mu\nu a}F^{a}_{\mu\nu}|0_{\text{gluon}}> = G_B^2 - G_E^2 = 0.012 \text{ GeV}^4,$$

(12)

indicates that the value of the electric part might be negligible when compared with the magnetic part. Below we discuss that such a conclusion is quite plausible if we assume that the gluon vacuum is in a squeezed state.

3. The squeezed condensate of gluons has been investigated recently [13]-[17] in order to construct a Lorentz and gauge invariant stable QCD vacuum in Minkowski space. Different alternative approaches have not solved this problem. For instance the simple perturbative vacuum is unstable [18], and there is no stable (gauge invariant) coherent vacuum in Minkowski space [19].

From the physical point of view, the squeezed state differs from the coherent one by the condensation of colour singlet gluon pairs rather than of single gluons. In analogy to the Bogoliubov model [20] we consider the case of a homogeneous condensate, but in a squeezed instead of a coherent state. We assume

$$|0_{\text{gluon}}> = |0_{sq}[f_0]> ,$$

(13)

where the squeezed state $|0_{sq}[f_0]>$ as a candidate for a homogeneous colourless gluon vacuum is constructed from the reference vacuum $|0> = |0_{sq}[f_0 = 0]>$, further specified below, according to

$$|0_{sq}[f_0]> = U_{sq}^{-1}[f_0]|0> .$$

(14)

The squeezing operator

$$U_{sq}[f_0] = \exp \left[ \frac{i}{2} \sum_{a,i} (A^a_i(0)E^a_\mu(0) + E^a_\mu(0)A^a_i(0)) \right] $$

(15)
with the zero momentum components \( A_i^a(0) \) and \( E_i^a(0) \) of the fields and their canonical momenta contains the parameter \( f_0 \) given below. This special transformation for the homogeneous condensate does not violate Lorentz invariance, since the gauge fields are massless [21]. The question of gauge invariance is very difficult but as in Ref. [22] we suppose the gauge invariance of all the spatial zero momentum components of the gauge fields. The multiplicative transformations of fields corresponding to (14) and (15) are

\[
U_{sq}[f_0] \ A_i^a(0) \ U_{sq}^{-1}[f_0] = e^{f_0} A_i^a(0) ,
\]

\[
U_{sq}[f_0] \ E_i^a(0) \ U_{sq}^{-1}[f_0] = e^{-f_0} E_i^a(0) .
\]

After this canonical transformation the squeezed expectation values as functions of the squeezing parameter \( f_0 \) behave like

\[
< 0_{sq}[f_0] | (B_i^a(A_i^b(0)))^2 | 0_{sq}[f_0] > = e^{4f_0} < 0 | (B_i^a(A_i^b(0)))^2 | 0 > ,
\]

\[
< 0_{sq}[f_0] | (E_i^a(0))^2 | 0_{sq}[f_0] > = e^{-2f_0} < 0 | (E_i^a(0))^2 | 0 > ,
\]

which follows from (16), noting that \( B_i^a(0) = f^{abc} \epsilon_{ijk} A_j^b(0) A_k^c(0) \). Let the reference vacuum \( | 0 > \) be such that the expectation values \( < 0 | (B_i^a(0))^2 | 0 > \) and \( < 0 | (E_i^a(0))^2 | 0 > \) behave in the large volume limit \( (V \rightarrow \infty) \) like \( V^{-4/3} \) in accordance with dimensional analysis. The parameter of the squeezing transformation \( f_0 \) can be chosen so that the magnetic condensate density (10) remains finite in the large volume limit \( (e^{4f_0} \sim V^{4/3}) \) and agrees with our above estimate (11). This entails that the electric condensate (18) vanishes in this limit

\[
\lim_{V \rightarrow \infty} < 0_{sq}[f_0] | (E_i^a(0))^2 | 0_{sq}[f_0] > = O[1/V^2] .
\]

Hence we conclude that in the squeezed vacuum the electric condensate does not contribute.

4. In conclusion we have considered a mechanism to create a large mass for the \( \eta_0 \) due to its anomalous interaction with the gluons of the squeezed vacuum. We have found that the mass of the \( \eta_0 \) is determined by the magnetic part of the gluon condensate. For the squeezed state this magnetic part coincides with the total gluon condensate, \( G^2 = G_B^2 \), and hence

\[
m_0^2 = \frac{3\alpha_s}{2\pi f_\pi^2} G^2 .
\]
stein and Zakharov [1]. In the present work we have pointed out a special interesting possibility to resolve the U(1) problem in the framework of a squeezed gluon vacuum. The above exploratory calculations are based on the chiral limit and the assumption $\alpha_s \sim 1$ which should be relaxed in a more elaborated approach.

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