\( B_d - \bar{B}_d \) mixing vs. \( B_s - \bar{B}_s \) mixing with the anomalous \( Wtb \) couplings

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We explore the effects of the anomalous \( tbW \) couplings on the \( B_d - \bar{B}_d \) mixing and recently measured \( B_s - \bar{B}_s \) mixing. The combined analysis of mixings via box diagrams with penguin decays provides strong constraints on the anomalous top quark couplings. We find the bound from the \( B_d - \bar{B}_d \) mixing data is stronger than that from the \( B_s - \bar{B}_s \) mixing.

I. INTRODUCTION

Expected is the production of a large number of top quark pairs at the CERN Large Hadron Collider (LHC), which allows us to probe the top quark couplings [1,2]. The \( tbW \) coupling will be directly tested with high precision through the dominant \( t \rightarrow bW \) decays at the LHC and other top decay channels are highly suppressed by small mixing angles. The present value of \( |V_{tb}| \) is determined at the Tevatron to be \(|V_{ts}| > 0.78 \) [3] and \(|V_{tb}| > 0.89 \) [4] at 95\% C. L. with assuming the Cabibbo-Kobayashi-Maskawa (CKM) unitarity. The direct determination of \( |V_{tb}| \) without assuming unitarity is performed through the single top quark production and the CDF [5] and the D0 [6] obtained the limit \(|V_{tb}| > 0.74 \) at 95\% C. L.

The standard model (SM) of electroweak and strong interactions has been successful in describing a wide range of experimental data so far. The only unobserved ingredient of the SM is the Higgs boson and a few coupling constants are not precisely tested yet. The present measurements on \( |V_{ts}, V_{tb}| \) mixing involves the t\( \gamma \) and \( t\mu \) decays at the LHC and other top decay channels are highly suppressed by small mixing angles. The \( B_s - \bar{B}_s \) mixing arises through the box diagrams with internal lines of \( W \) boson and \( u \)-type quarks in the SM. Since the top quark loop dominates, the box diagrams are sensitive to the anomalous top couplings. The current average of the \( B_d - \bar{B}_d \) mixing is found to be [7]

\[
\Delta M_d = (0.507 \pm 0.005) \text{ ps}^{-1}.
\]  

Recently the measurements of the \( B_s - \bar{B}_s \) mixing by the CDF [8] and D0 [9] collaborations are reported to be

\[
\Delta M_s = \begin{cases} 
(17.77 \pm 0.10 \pm 0.07) \text{ ps}^{-1} & \text{(CDF)}, \\
(18.53 \pm 0.93 \pm 0.30) \text{ ps}^{-1} & \text{(D0)},
\end{cases}
\]  

where the first error is statistical and the second is systematic.

Effects of the anomalous top quark couplings have been widely studied in direct and indirect ways [10,11,12,13,14,15,16]. Without specifying the underlying model, we use an effective lagrangian in this work by introducing two complex parameters such that

\[
\mathcal{L} = -\frac{g}{\sqrt{2}} \sum_{q=d,s,b} V_{tq}^{\text{eff}} \bar{t} \gamma^\mu (P_L + \xi_q P_R) q W^\mu_+ + H.c.,
\]

where \( \xi_q \) are complex parameters measuring effects of the anomalous right-handed couplings while \( V_{tq}^{\text{eff}} \) measures the SM-like left-handed couplings. The \( B_d - \bar{B}_d \) mixing involves the \( tdW \) and \( tbW \) couplings while the \( B_s - \bar{B}_s \) mixing involves the \( tsW \) and \( tbW \) couplings. On the other hand, the radiative \( B \rightarrow X_s \gamma \) decay also provides strict constraints on the \( tbW \) and \( tsW \) couplings. If we consider all possible anomalous top quark couplings, there are too many parameters, \( 3(d,s,b) \times 2(L - R) \times 2(\text{complex}) = 12 \), and it is hard to get meaningful informations. Thus we
concentrate on the couplings for only one flavour by keeping the other couplings to be zero. In the Ref. [11], we have probed the $tsW$ couplings through $B_s - B_d$ mixing and $B \rightarrow X_s \gamma$ decay. We probe the anomalous $tbW$ couplings in this work, and the $B_d - \bar{B}_d$ mixing should be incorporated since $tbW$ couplings are common to the $B_d - \bar{B}_d$ and $B_s - \bar{B}_s$ mixings. Actually the effects of the anomalous right-handed coupling $\xi_b$ in $B \rightarrow X_s \gamma$ decay are enhanced by $m_t/m_b$ due to the structure of the penguin diagram in the presence of the right-handed couplings, but no such enhancements exist for the box diagram. Consequently the $\Delta M_q$ constrain only the anomalous left-handed coupling $V_{tb}^{\text{eff}}$, while the penguin diagrams constrain both of $V_{tb}^{\text{eff}}$ and $\xi_b$. Thus the combined analysis of $B - \bar{B}$ mixing and $B \rightarrow X_s \gamma$ decay provides a synergy in probing the anomalous top couplings. This paper is organized as follows: In section II, the $B \rightarrow X_s \gamma$ constraints on the anomalous $tbW$ couplings is given. In section III, the analysis of the $B_d - \bar{B}_d$ mixing and the $B_s - \bar{B}_s$ mixing with anomalous $tbW$ couplings is presented. Finally we conclude in section IV.

II. $B \rightarrow X_s \gamma$ DECAYS

The $\Delta B = 1$ effective Hamiltonian for $b \rightarrow s \gamma$ process is given by

$$\mathcal{H}_{e,f}^{\Delta B=1} = -\frac{4G_F}{\sqrt{2}} V_{ts}V_{tb} \sum_{i=1}^{8} C_i(\mu)O_i(\mu),$$

where the dimension 6 operators $O_i$ constructed in the SM are given in the Ref. [17]. Matching the effective Hamiltonian and our model given in Eq. (3) at $\mu = m_W$ scale, we obtain the Wilson coefficients $C_i(\mu = m_W)$

$$C_2(m_W) = C_2^{\text{SM}}(m_W),$$

$$C_7(m_W) = C_7^{\text{SM}}(m_W) + \xi_b \frac{m_t}{m_b} F_R(x_t),$$

$$C_8(m_W) = C_8^{\text{SM}}(m_W) + \xi_b \frac{m_t}{m_b} G_R(x_t),$$

and otherwise coefficients are zeros, where $C_2(m_W) = -1$, $C_7(m_W) = F(x_t)$, and $C_8(m_W) = G(x_t)$ with the well-known Inami-Lim loop functions $F(x)$ and $G(x)$ found in [17, 18] and the new loop functions

$$F_R(x) = -\frac{20 + 31x - 5x^2}{12(x - 1)^2} + \frac{x(2 - 3x)}{2(x - 1)^3} \ln x,$$

$$G_R(x) = -\frac{4 + x + x^2}{4(x - 1)^2} + \frac{3x}{2(x - 1)^3} \ln x,$$

agree with those in Ref. [19]. We note that the anomalous right-handed coupling $\xi_b$ involves an enhancement factor $m_t/m_b$.

We obtain the branching ratio of $B \rightarrow X_s \gamma$ process at next-leading-order (NLO) in terms of $\xi_b$ as

$$\text{Br}(B \rightarrow X_s \gamma) = \text{Br}^{\text{SM}}(B \rightarrow X_s \gamma) \left( \frac{|V_{ts}^*V_{tb}^{\text{eff}}|}{0.0404} \right)^2 \left[ 1 + \text{Re}(\xi_b) \frac{m_t}{m_b} \left( 0.68 \frac{F_R(x_t)}{F(x_t)} + 0.07 \frac{G_R(x_t)}{G(x_t)} \right) + \frac{|\xi_b|^2 m_t^2}{m_b^2} \left( 0.112 \frac{F_R^2(x_t)}{F^2(x_t)} + 0.002 \frac{G_R^2(x_t)}{G^2(x_t)} + 0.025 \frac{F_R(x_t)G_R(x_t)}{F(x_t)G(x_t)} \right) \right],$$

of which numerical coefficients depends on the kinematic cut of the photon energy spectrum. We take the cut $E_\gamma > 1.6$ GeV and the numerical values are obtained in the Ref. [20]. The SM branching ratio is predicted to be \begin{equation}
\text{Br}(B \rightarrow X_s \gamma) = (3.15 \pm 0.23) \times 10^{-4}
\end{equation}
with the same photon energy cut at next-to-next-to-leading order (NNLO) [21].

The current world average value of the measured branching ratio is given by [22]

$$\text{Br}(B \rightarrow X_s \gamma) = (3.55 \pm 0.24^{+0.09}_{-0.10} \pm 0.03) \times 10^{-4},$$

with the same $E_\gamma$ cut.

III. $B - \bar{B}$ MIXING
A neutral $B^0_q$ meson can oscillate into its antiparticle $\bar{B}^0_q$ via flavour-changing processes of $B_q - \bar{B}_q$ mixing. The oscillation is described by a Schrödinger equation,

$$i \frac{d}{dt} \begin{pmatrix} B_q(t) \\ \bar{B}_q(t) \end{pmatrix} = \begin{pmatrix} M - \frac{i}{2} \Gamma \end{pmatrix} \begin{pmatrix} B_q(t) \\ \bar{B}_q(t) \end{pmatrix},$$

(9)

where $M$ is the mass matrix and $\Gamma$ the decay matrix. The $\Delta B = 2$ transition amplitudes given by

$$\langle B^0_q | H^{\Delta B=2}_{\text{eff}} | \bar{B}^0_q \rangle = M^q_{12},$$

(10)

is related to the mass difference between the heavy and the light mass eigenstates,

$$\Delta M_q \equiv M^q_H - M^q_L = 2|M^q_{12}|,$$

(11)

where $M^q_H$ and $M^q_L$ are the mass eigenvalues for the heavy and the light eigenstates respectively. Correspondingly the total decay width difference is defined by

$$\Delta \Gamma_q \equiv \Gamma^q_L - \Gamma^q_H.$$

(12)

The SM predicts the small $\Delta \Gamma_d/\Gamma_d < 1\%$ and the relatively large $\Delta \Gamma_s/\Gamma_s \sim 10\%$. Since the decay matrix elements $\Gamma^q_{12}$ is derived from the SM decays $b \to c\bar{c}q$ at tree level, it is hardly affected by the new physics. We ignore the new effects of the anomalous top couplings on $\Delta \Gamma_q$ and just consider the mass differences in this analysis.

The box diagrams are calculated to obtain the transition amplitudes $M^q_{12}$. Inclusion of the odd number of right-handed couplings in the box diagram vanishes due to vanishing the loop integrals of the odd number of momentum. Thus the leading contribution of the anomalous right-handed top couplings to the $B_s - \bar{B}_s$ mixing is quadratic order of $\xi_b$.

$$M^q_{12} = \frac{G_F m_b^2}{12\pi} m_{B_s} \bar{B}_s B_{s} f^2_{B_s} S_0(x_t) \left( V^*_{tb} V_{tb}^{\text{eff}} \right)^2 \left( 1 + \frac{S_2(x_t) \xi_b^2}{S_0(x_t)} \right) \frac{\langle B^0_q | (\bar{b}P_L q)(\bar{b}P_L q) | B^0_q \rangle}{4 \langle B^0_q | (b\gamma P_L q)(b\gamma P_L q) | B^0_q \rangle},$$

(13)
FIG. 2: The bounds on the complex $V_{td}V_{tb}^{\text{eff}}$ plane. The yellow (light gray) region is allowed by $B \to X_s \gamma$ and $\Delta M_s$ and the green (dark gray) region by $B \to X_s \gamma$ and $\Delta M_d$. The sin 2$\beta$ measurements constrain the phase of the $V_{td}V_{tb}^{\text{eff}}$ with the two-fold ambiguity and the allowed regions are denoted by black regions.

where $\eta_q$ are the perturbative QCD corrections to the $B_q - B_{\bar{q}}$ mixings [22]. The Inami-Lim loop functions are given by

$$S_0(x) = \frac{4x - 11x^2 + x^3}{4(1 - x)^2} - \frac{3x^3}{2(1 - x)^3}\log x,$$

$$S_3(x) = 4x^2 \left( \frac{2}{(1 - x)^2} + \frac{1 + x}{(1 - x)^3}\log x \right).$$

Vacuum insertions to the hadronic matrix elements lead to

$$\frac{\langle B_q^0 | (\bar{b}P_L q)(\bar{b}P_L q) | \bar{B}_q^0 \rangle}{\langle B_q^0 | (\bar{b}\gamma^\mu P_L q)(\bar{b}\gamma^\mu_P L q) | \bar{B}_q^0 \rangle} = \frac{5}{8} \left( \frac{m_{B_q}}{m_b + m_q} \right)^2,$$

and

$$\langle B_q^0 | (\bar{b}\gamma^\mu P_L q)(\bar{b}\gamma^\mu_P L q) | \bar{B}_q^0 \rangle = \frac{8}{3} m_{B_q}^2 \hat{B}_{B_q} f_{B_q}^2,$$

where $\hat{B}_{B_q}$ are the Bag parameters and $f_{B_q}$ the decay constants. The SM predictions of the $B - \bar{B}$ mixings are given by $\Delta M_s = 19.3 \pm 6.74 \text{ ps}^{-1}$ and $\Delta M_d = 0.53 \pm 0.02 \text{ ps}^{-1}$ [23].

We show the allowed parameter sets ($|\xi_b|, |V_{tb}^{\text{eff}}|$) at 95% C. L. in Fig. 1. The black region is allowed by $\text{Br}(B \to X_s \gamma)$ and $\Delta M_s$ while the green (gray) region allowed by $\text{Br}(B \to X_s \gamma)$ and $\Delta M_s$. We have the conservative bounds $|V_{tb}| > 0.93$ and $|\xi_b| < 0.027$ in Fig. 1. Since $\xi_b$ and $V_{tb}^{\text{eff}}$ are complex parameters, the new physics effects arise in both magnitude and phase of $M_{12}^s$ in general. Effects of the phase and CP violation in $M_{12}^s$ have been measured [24], although not very accurately, and discussed in several literatures [23, 25]. The CP phase of the $B_d - \bar{B}_d$ mixing is measured through the $B \to J/\psi K_s$ and has been tested in many $B$ decay processes [26]. The recent world average value of the weak phase defined by

$$\sin 2\beta = -\frac{V_{td}V_{tb}^*}{V_{td}V_{tb}}$$

is given by

$$\sin 2\beta = 0.680 \pm 0.025$$

(17)
through the time-dependent CP asymmetries into all charmonium states. Figure 2 shows the allowed values of $V_{td}^\ast V_{tb}^{\text{eff}}$ on the complex plane. The yellow (light gray) region denotes the allowed region by $B \to X_s \gamma$ and $\Delta M_s$, and the green (dark gray) region by $B \to X_c \gamma$ and $\Delta M_d$. The allowed region by the sin2$\beta$ measurements has the two-fold ambiguity on the complex $V_{td}^\ast V_{tb}^{\text{eff}}$ plane. The black region denotes the allowed regions additionally by the world average values of sin2$\beta$ measurements.

IV. CONCLUDING REMARKS

The neutral $B_d^0$ meson systems are of great use for search for the new physics effects in top quark couplings. We consider the anomalous $tbW$ couplings parametrized by $V_{tb}^{\text{eff}}$ and $\xi_b$. Combined analysis of $B_s - \bar{B}_s$ mixing, $B_d - \bar{B}_d$ mixing and $B \to X_s \gamma$ penguin decay provides strong constraints on the parameters of $V_{tb}^{\text{eff}}$ and $\xi_b$. We find that the bounds from $B_d - \bar{B}_d$ mixing is better than that from $B_s - \bar{B}_s$ mixing. It is because the SM prediction of $\Delta M_d$ is more precise than that of $\Delta M_s$.

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