WEAK FORM FACTORS FOR HEAVY MESON DECAYS

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We calculate the form factors for weak decays of $B(s)$ and $D(s)$ mesons to light pseudoscalar and vector mesons within a relativistic dispersion approach based on the constituent quark picture. This approach gives the form factors as relativistic double spectral representations in terms of the wave functions of the initial and final mesons. The form factors have the correct analytic properties and satisfy general requirements of nonperturbative QCD in the heavy quark limit. The effective quark masses and meson wave functions are determined by fitting the quark model parameters to lattice QCD results for the $B \to \rho$ transition form factors at large momentum transfers and to the measured $D \to (K, K^*)\ell\nu$ decay rates. This allows us to predict numerous form factors for all kinematically accessible $q^2$ values.

The knowledge of the weak transition form factors of heavy mesons is crucial for a proper extraction of the quark mixing parameters, for the analysis of non-leptonic decays and CP violating effects and for a search of New Physics.

Theoretical approaches for calculating these form factors are quark models, QCD sum rules, and lattice QCD (a detailed list of references can be found in \cite{1}). Although in recent years considerable progress has been made, the theoretical uncertainties are still uncomfortably large. An accuracy better than 15% has not been attained. Moreover each of the above methods has only a limited range of applicability: QCD sum rules are suitable for describing the low $q^2$ region of the form factors; lattice QCD gives good predictions for high $q^2$. As a result these methods do not provide for a full picture of the form factors and, more important, for the relations between the various decay channels.

Quark models do provide such relations and give the form factors in the full $q^2$-range. However, quark models are not closely related to QCD and therefore have input parameters which may not be of fundamental significance.

Clearly, a combination of various methods can be fruitful for obtaining reliable predictions for many decay form factors in their full $q^2$-ranges. To achieve this goal, one needs a general frame for the description of a large variety of processes. This can only be a suitable quark model, since only a quark model connects different processes through the meson soft wave functions and describes the full $q^2$-range of the form factors. This program has been implemented in our recent work \cite{1} where the predictions of the quark model

\textsuperscript{a}Talk presented by D. Melikhov
have been considerably improved by incorporating the results from lattice QCD and the available experimental data.

1. The physical picture
The constituent quark picture is based on the following phenomena:

- the chiral symmetry breaking in the low-energy region which provides for the masses of the constituent quarks;
- a strong peaking of the nonperturbative meson wave functions in terms of the quark momenta with a width of the order of the confinement scale;
- a $q\bar{q}$ composition of mesons in terms of the constituent quarks.

2. The formalism
For the description of the transition form factors in their full $q^2$-range and for various initial and final mesons, a fully relativistic treatment is necessary. We make use of the dispersion formulation of the quark model which guarantees the correct spectral and analytic properties of the form factors.

The transition form factors in the decay region are given by the relativistic double spectral representations through the wave functions of the initial and final mesons. These spectral representations obey rigorous constraints from QCD on the structure of the long-distance corrections in the heavy quark limit: the form factors of the dispersion quark model have the correct heavy-quark expansion at leading and next-to-leading $1/m_Q$ orders in accordance with QCD for transitions between heavy quarks. For the heavy-to-light transition the dispersion quark model satisfies the relations between the form factors of vector, axial-vector, and tensor currents valid at small recoil. In the limit of the heavy-to-light transitions at small $q^2$ the form factors obey the lowest order $1/m_Q$ and $1/E$ relations of the Large Energy Effective Theory.

3. Parameters of the model
A possible way to control quark masses and the meson wave functions is to use the lattice results for the $B \to \rho$ form factors at large $q^2$ as 'experimental' inputs. In we have included into consideration also charm and strange mesons and fixed their wave functions and the effective masses $m_c$ and $m_s$ by fitting the measured rates for the decays $D \to (K, K^*)l\nu$.

With these few inputs we gave in Ref numerous predictions for the form factors for the $D(s)$ and $B(s)$ decays into light mesons which nicely agree at places where data are available.
4. Results

The main results of our analysis are as follows:

1. In spite of the rather different masses and properties of mesons involved in weak transitions, all existing data on the form factors can be understood in the quark picture, i.e. all form factors can be described by the few degrees of freedom of constituent quarks. Details of the soft wave functions are not crucial; only the spatial extension of these wave functions of order of the confinement scale is important. In other words, only the meson radii are essential.

2. The calculated transition form factors are in good agreement with the results from lattice QCD and from sum rules in their regions of validity. The only exception is a disagreement with the sum rules\(^5\) for the \(B \rightarrow K^*\) transition. This disagreement is caused by a different way of taking into account the SU(3) violating effects when going from \(B \rightarrow \rho\) to \(B_s \rightarrow K^*\) and is not related to specific details of the model. We suspect that the sum rules\(^5\) overestimate the SU(3) breaking in the long-distance region, but this problem deserves further clarification.

3. We have estimated the products of the meson weak and strong coupling constants by extrapolating the form factors to the meson pole. The value of each coupling constant can be obtained independently from the residues of several form factors. In all cases the values extracted from the different form factors agree with each other within the 5-10% accuracy. This gives additional argument in support of the reliability of our estimates for the form factors.

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