Yukawa Interaction from a SUSY Composite Model

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Abstract

We present a composite model that is based on non-perturbative effects of $N = 1$ supersymmetric $SU(N_C)$ gauge theory with $N_f = N_C + 1$ flavors. In this model, we consider $N_C = 7$, where all matter fields in the supersymmetric standard model, that is, quarks, leptons and Higgs particles are bound states of preons and anti-preons. When $SU(7)_H$ hyper-color coupling becomes strong, Yukawa couplings of quarks and leptons are generated dynamically. We show one generation model at first, and next we show models of three generations.

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1 Introduction

The origin of Yukawa couplings is one of the greatest mystery in the Standard Model (SM). This situation is not changed in the supersymmetric standard model (SSM). A search for the origin of the Yukawa couplings might show us the way to get at the origin of the masses and the Kobayashi-Maskawa (KM) flavor mixing matrix.

There has been an idea that quarks and leptons may be composite particles and Yukawa terms are induced by gauge dynamics. We revive this idea using supersymmetric QCD. According to the recent progress of $N=1$ supersymmetric $SU(N_C)$ with $N_f = N_C + 1$ gauge theories, the non-perturbative effects induce the superpotential

$$W_{\text{dyn}} = \frac{1}{\Lambda^{2N_C-1}} \left( \bar{B}MB - \det M \right).$$

In this case, the moduli space is not changed by quantum corrections. This superpotential might suggest the possibility that matter fields are composite states and Yukawa couplings emerge from the strong dynamics when baryon (anti-baryon) states are regarded as quarks and leptons and Higgs particles regard as meson states.

Recently, many authors have considered various models in which SSM particles are composed from more elementary particles and Yukawa couplings are induced strong gauge dynamics eq.(1.1). In ref.[4], right-handed down quarks $\bar{D}_R$ and $SU(2)_L$ doublet leptons $l_L$ are elementary and the other particles are composite. The Yukawa coupling of the top quark is induced by strong gauge dynamics eq.(1.1) and this explains the reason why the Yukawa coupling of top quark ($Y_t$) is of $O(1)$. On the other hand, the Yukawa couplings of bottom quark ($Y_b$) can not be derived by the strong gauge dynamics eq.(1.1). $Y_b$ is put in tree level superpotential by the hand, thus the $Y_b$ is suppressed of $O(\Lambda/M)$ at the confinement phase. It can naturally explain that the $Y_b$ becomes smaller than $O(1)$. In ref.[6], all quarks and leptons are composite states of the some elementary particles. Yukawa couplings of leptons are induced by non-perturbative effects eq.(1.1). However, it does not follow one philosophy that contains a big problem, that is, the Yukawa couplings of the quarks are not induced from eq.(1.1).

In this article, we present a composite model in which all Yukawa couplings in the SSM are induced dynamically. All quarks, leptons and the Higgs particles are confining states of preons and anti-preons. The Yukawa couplings of ordinary quarks and leptons are induced from the strong gauge dynamics eq.(1.1). Thus this model might suggest the answer of the mystery of the origin of the Yukawa couplings. The anomalies in this model are just the same as the SM as shown later. From now on, we assume the supersymmetry (SUSY) breaking will occurred at low energy. And we consider the supersymmetric model which is suitable above the SUSY breaking scale. In this model, there are some unwanted massless fields. We should consider the mechanism to generate their masses which does not contradict experiments.
This article is organized as follows. In section 2, we will focus on a model with one generation, and we will discuss the properties of this model. In section 3, we will try to construct more realistic scenario, three generation model. Section 4 will be devoted to summary and discussion.

2 One Generation Model

Preon Fields

At first, we focus on a model with only one generation. Where quarks and leptons are confining states of preons and anti-preons. We introduce new gauge group “hyper-color” $SU(7)_H$ in addition to the standard gauge group. At low energy, the gauge coupling of $SU(7)_H$ becomes strong, preons and anti-preons confine and Yukawa couplings are dynamically generated by the non-perturbative effects of $SU(7)_H$. We consider the gauge group

$$SU(7)_H \times SU(3)_C \times SU(2)_L \times U(1)_Y \times [U(1)_B \times U(1)_L],$$

(2.1)

where $SU(3)_C \times SU(2)_L \times U(1)_Y$ is the ordinary standard model gauge group, and $U(1)_B$ and $U(1)_L$ are baryon and lepton number global symmetries, respectively. Under the gauge group eq.(2.1), preon and anti-preon fields transform as shown in table(2.1)

|       | $SU(7)_H$ | $SU(3)_C$ | $SU(2)_L$ | $U(1)_Y$ | $U(1)_B$ | $U(1)_L$ |
|-------|-----------|-----------|-----------|----------|----------|----------|
| $P_Q$ | □         | □*        | □         | $-1/3$   | $-1/21$  | $2/7$    |
| $P_t$ | □         | 1         | □         | $1$      | $2/7$    | $-5/7$  |
| $P_U$ | □*        | □         | 1         | $4/3$    | $1/21$   | $-2/7$  |
| $P_D$ | □*        | □         | 1         | $-2/3$   | $1/21$   | $-2/7$  |
| $P_N$ | □*        | 1         | 1         | 0        | $-2/7$   | $5/7$   |
| $P_E$ | □*        | 1         | 1         | $-2$     | $-2/7$   | $5/7$   |

Table 2.1: Representation of the preons and anti-preons for the relevant gauge group.

where □(□*) denote the fundamental (anti-fundamental) representation of the gauge group. Assuming that the SM gauge couplings are weaker than the hyper-color couplings, this theory is regarded as $SU(7)_H$ supersymmetric QCD with 8 flavors.

Bellow the $SU(7)_H$ confinement scale $\Lambda$, composite states of “baryon” and “anti-baryon” appear as table(2.2), and states of composite “meson” appear as table(2.3).

In addition to the ordinary one generation quarks and leptons in the SSM ($Q$, $\bar{U}$, $\bar{D}$, $\ell$ and $E$), there exist right-handed neutrino ($\bar{N}$) and two set of the Higgs doublet ($\Phi_u$, $\Phi_d$ and $\bar{\phi}_u$, $\bar{\phi}_d$), color octet fields ($G_u$ and $G_d$) and two sets of the leptoquarks ($X$, $\bar{X}$ and $Y$, $\bar{Y}$) as the $SU(7)_H$ “meson”. This model also predicts the existences of right-handed neutrino ($\bar{N}$), two set of the Higgs doublet ($\Phi_u$, $\Phi_d$ and $\bar{\phi}_u$, $\bar{\phi}_d$).
Table 2.2: Transformation of “baryon” and “anti-baryon” under relevant gauge group

|   | $SU(7)_H$ | $SU(3)_C$ | $SU(2)_L$ | $U(1)_Y$ | $U(1)_B$ | $U(1)_L$ | baryon ($\times 1/\Lambda^6$) |
|---|---|---|---|---|---|---|---|
| $Q$ | 1 | ☐ | ☐ | 1/3 | 1/3 | 0 | $P^5_Q P^6_l$ |
| $l$ | 1 | 1 | ☐ | -1 | 0 | 1 | $P^8_Q P^9_l$ |
| $U$ | 1 | ☐* | 1 | -4/3 | -1/3 | 0 | $P^2_U P^3_D P^6_E P^9_N$ |
| $\bar D$ | 1 | ☐* | 1 | 2/3 | -1/3 | 0 | $P^3_U P^2_D P^6_E P^9_N$ |
| $N$ | 1 | 1 | 1 | 0 | 0 | -1 | $P^3_U P^3_D P^6_E$ |
| $\bar E$ | 1 | 1 | 1 | 2 | 0 | -1 | $P^3_U P^3_D P^9_N$ |

Table 2.3: Transformation of “meson” under relevant gauge group

|   | $SU(7)_H$ | $SU(3)_C$ | $SU(2)_L$ | $U(1)_Y$ | $U(1)_B$ | $U(1)_L$ | meson ($\times 1/\Lambda$) |
|---|---|---|---|---|---|---|---|
| $\Phi_u$ | 1 | 1 | ☐ | 1 | 0 | 0 | $P_Q P_U$ |
| $\Phi_d$ | 1 | 1 | ☐ | -1 | 0 | 0 | $P_Q P_D$ |
| $G_u$ | 1 | Ad. | ☐ | 1 | 0 | 0 | $P_Q P_U$ |
| $G_d$ | 1 | Ad. | ☐ | -1 | 0 | 0 | $P_Q P_D$ |
| $X$ | 1 | ☐* | ☐ | -1/3 | -1/3 | 1 | $P_Q P_N$ |
| $\bar X$ | 1 | ☐ | ☐ | 1/3 | 1/3 | -1 | $P_l P_D$ |
| $Y$ | 1 | ☐* | ☐ | -7/3 | -1/3 | 1 | $P_Q P_E$ |
| $\bar Y$ | 1 | ☐ | ☐ | 7/3 | 1/3 | -1 | $P_l P_U$ |
| $\phi_u$ | 1 | 1 | ☐ | 1 | 0 | 0 | $P_l P_N$ |
| $\phi_d$ | 1 | 1 | ☐ | -1 | 0 | 0 | $P_l P_E$ |

Bellow the confinement scale $\Lambda$, quarks, leptons and Higgs particles appear as “baryon (anti-baryon)” and “meson”, respectively. The dynamical superpotential eq.(1.1) becomes

$$W_{\text{dyn}} = y_u \bar U (\Phi_u + \bar \phi_u) Q + y_d \bar D (\Phi_d + \bar \phi_d) Q + y_n \bar N (\Phi_u + \bar \phi_u) \ell + y_e \bar E (\Phi_d + \bar \phi_d) \ell + \alpha_u \bar U G_u Q + \alpha_d \bar D G_d Q + \beta_u \bar U Y \ell + \beta_d \bar D \bar X \ell + \beta_n \bar N X Q + \beta_e \bar E Y Q$$

$-\text{determinant}, \quad (2.2)$

where determinant denotes 8 powers interactions of composite meson fields. And all of coefficient, $y_u, y_d, y_n, y_e, \alpha_u, \alpha_d, \beta_u, \beta_d, \beta_n$ and $\beta_d$, are of $O(1)$ and equal if $SU(3)_C \times SU(2)_L \times U(1)_Y$ is absent. Above the $SU(7)_H$ confinement scale $\Lambda$, this model do not have any Yukawa couplings, however when the $SU(7)_H$ gauge coupling becomes strong, all Yukawa couplings in the SSM emerge from the confining dynamics.

Anomalies and Property
In this model anomalies are absent except those of $SU^2(2)_L \times U(1)_{B+L}$, $U^2(1)_Y \times U(1)_{B+L}$. (2.3)

These anomalies are the exactly same as those of the SSM.

If we combine $(\bar{P}_U$ and $\bar{P}_D$) and $(\bar{P}_N$ and $\bar{P}_E$) into $SU(2)_R$ doublet, this model can be embedded into a Left-Right symmetric model. And, if preon fields ($P_Q$ and $P_L$) and anti-preon fields ($\bar{P}_U$, $\bar{P}_D$, $\bar{P}_E$ and $\bar{P}_N$) are combined into $SU(4) \times SU(2)_L \times SU(2)_R$ fields, this model can be extended to the model with Pati-Salam type gauge symmetry. [11] .

Higher order interaction

The Yukawa interaction eq.(2.2) does not give masses to color octet fields $G_u$ and $G_d$, leptoquarks $X$, $\bar{X}$, $Y$ and $\bar{Y}$, and Higgs particles $\Phi_d$, $\Phi_u$, $\phi_d$ and $\phi_u$. Thus, we must consider the mechanism to generate their masses. For this, we introduce 4 dimensional higher order operators in the tree level superpotential. The general gauge invariant dimension 4 operators are

$$W_{tree} = g_G \frac{P_Q P_U \bar{P}_D}{M} + g_\mu \frac{P_L P_L \bar{P}_E \bar{P}_N}{M} + g_x \frac{P_Q P_L \bar{P}_D \bar{P}_N}{M} + g_y \frac{P_Q P_L \bar{P}_U \bar{P}_E}{M},$$

where $M$ is a mass parameter and we assume that coefficient $g_\mu$, $g_G$, $g_x$ and $g_y$ are of $O(1)$. Below the energy scale $\Lambda$, eq.(2.4) becomes

$$W_{tree}^{eff} = m(g_G G_u G_d + g_G \Phi_u \Phi_d + g_\mu \phi_u \phi_d + g_x \bar{X} X + g_x \bar{\Phi}_d \Phi_d + g_y \bar{Y} Y + g_y \bar{\Phi}_u \Phi_d),$$

where mass parameter $m$ define as

$$m \equiv \frac{\Lambda^2}{M}. \quad (2.6)$$

Therefore, leptoquarks and color-octet particles have mass of $O(m)$. The mass matrix of Higgs particles becomes

$$\begin{pmatrix} \phi_u & \Phi_u \end{pmatrix} \begin{pmatrix} mg_\mu & mg_x \\ mg_y & mg_G \end{pmatrix} \begin{pmatrix} \phi_d \\ \Phi_d \end{pmatrix}. \quad (2.7)$$

If $g_\mu g_G \simeq g_x g_y$, one of the Higgs masses become small.

3 Three Generation Model

Now, let us try to extend a previous model to the three generation model. We may consider following three possibilities:

*The former one is consider to be useful for electro-weak baryogenesis [10].
i) Family Symmetry

We add the global family symmetry $SU(3)_F$ to eq. (2.1). We adapt the gauge symmetry, $SU(23)_H \times SU(3)_C \times SU(2)_L \times U(1)_Y \times [SU(3)_F \times U(1)_B \times U(1)_L]$. Under the gauge symmetry preons and anti-preons transform as table (A.1). Baryon (anti-baryon) states are fundamental (anti-fundamental) representation of the family symmetry $SU(3)_F$ (see table (A.2)). It is worth noting that $SU(3)_F$ octet “meson” fields are added to the one generation model (see table (A.3)). Below the composite scale $\Lambda$, dynamically induced superpotential eq. (1.1) is generated:

$$W_{dyn} = W_{dyn}^{\text{singlet}} + W_{dyn}^{\text{octet}},$$

where

$$W_{dyn}^{\text{singlet}} = \begin{align*}
y_{u1} U(\tilde{\Phi}^s_u + \tilde{\phi}^s_u)Q + y_{d1} D(\tilde{\Phi}^s_d + \tilde{\phi}^s_d)Q \\
y_{n1} N(\tilde{\Phi}^s_u + \tilde{\phi}^s_u)\ell + y_{e1} E(\tilde{\Phi}^s_d + \tilde{\phi}^s_d)\ell \\
+ \alpha_{u1} UG_uQ + \alpha_{d1} DG_dQ \\
+ \beta_{u1} UY^s\ell + \beta_{d1} D\bar{X}^s\ell + \beta_{n1} N\bar{X}^sQ + \beta_{e1} E\bar{Y}^sQ,
\end{align*}$$

and

$$W_{dyn}^{\text{octet}} = \begin{align*}
y_{us} U(\tilde{\Phi}^o_u + \tilde{\phi}^o_u)Q + y_{ds} D(\tilde{\Phi}^o_d + \tilde{\phi}^o_d)Q \\
y_{ns} N(\tilde{\Phi}^o_u + \tilde{\phi}^o_u)\ell + y_{es} E(\tilde{\Phi}^o_d + \tilde{\phi}^o_d)\ell \\
+ \alpha_{us} UG_uQ + \alpha_{ds} DG_dQ \\
+ \beta_{us} UY^o\ell + \beta_{ds} D\bar{X}^o\ell + \beta_{ns} N\bar{X}^oQ + \beta_{es} E\bar{Y}^oQ.
\end{align*}$$

Here we neglect the determinant term for mesons. All coefficient are considered be of $O(1)$, and suffix of the particles “s” and “o” denote the singlet and the octet representation of the $SU(3)_F$ family symmetry, respectively.

This model has the family symmetry which is troublesome. For example, flavor changing neutral current (FCNC) exists. Also Nambu-Goldstone (NG) bosons appears when family symmetry is broken. When the global family symmetry is spontaneously broken, many massless NG bosons appear. We need to consider how to break the flavor symmetry to make realistic scenario.

ii) Three Copies of One Generation

The second way is to consider three copies of one generation model. The gauge group is extended to, $SU(7)_{H1} \times SU(7)_{H2} \times SU(7)_{H3} \times SU(3)_C \times SU(2)_L \times U(1)_Y \times [SU(3)_F \times U(1)_B \times U(1)_L \times \mathbb{Z}_8]$, where three $SU(7)_{H_i}$ are “hyper-color” gauge groups, by which dynamics preons and anti-preons of each generation are confined. Bellow the confinement scale of the all hyper-color dynamics, the composite fields appear as shown in table (A.3) and table (A.6), and the superpotential is derived by the non-perturbative effects as

$$W_{dyn} = \sum_{a=1}^{3} \left\{ y_{ua} U_a(\tilde{\Phi}_u + \tilde{\phi}_u)Q_a + y_{da} D_a(\tilde{\Phi}_d + \tilde{\phi}_d)Q_a \right\}$$
This model predicts that each generation has two sets of the Higgs doublets, and the Yukawa couplings of the first, the second and the third generations are of the $O(1)$. Since $W_{\text{dyn}}$, eq.(3.4) does not include flavor mixing interactions, we introduce the higher dimensional operators in the tree level superpotential.

$$W_{\text{tree}} = \sum_{i,j,k=1}^{3} \left\{ y_{ijk}^u \left( \frac{\Lambda}{M} \right)^{13} \bar{U}_i (\Phi_u + \bar{\phi}_u)_k Q_j + y_{ijk}^d \left( \frac{\Lambda}{M} \right)^{13} \bar{D}_i (\Phi_d + \bar{\phi}_d)_k Q_j 
+ y_{ijk}^e \left( \frac{\Lambda}{M} \right)^{13} \bar{N}_i (\Phi_u + \bar{\phi}_u)_k \ell_j + y_{ijk}^\nu \left( \frac{\Lambda}{M} \right)^{13} \bar{E}_i (\Phi_d + \bar{\phi}_d)_k \ell_j \right\}$$

(3.5)

in order to obtain the KM matrix. Where $y_{ijk}^a = \alpha_{ijk}^a \Lambda_i^6 \Lambda_j^6 \Lambda_k^6 / \Lambda_{13}^6$ ($\alpha_{ijk}^a \simeq O(1)$) and $\Lambda_i$ is the confinement scale of $SU(7)_{H_i}$. The $\mathbb{Z}_8$ symmetry is introduce in order to avoid large FCNC. Here preons and anti-preons are assigned to have charge

$$P_1 : 1, \quad P_2 : 3, \quad P_3 : 5, \quad \bar{P}_1 : 6, \quad \bar{P}_2 : 5, \quad \bar{P}_3 : 4,$$

(3.6)

under the $\mathbb{Z}_8$ symmetry. Here $P_i (\bar{P}_i)$ is denote $i$th generation preons (anti-preons) (see table(A.4)). Texture of the Yukawa couplings are

$$Y^{u,d,e,n} \sim \left( \begin{array}{ccc} O(1) & 0 & 0 \\ \varepsilon^{13} & O(1) & \varepsilon^{13} \\ 0 & 0 & O(1) \end{array} \right),$$

(3.7)

in the flavor space, where $\varepsilon \sim \frac{\Lambda}{M}$.

The $(1, 2), (1, 3), (3, 1)$ and $(3, 2)$ elements of all Yukawa coupling matrices vanish because of $\mathbb{Z}_8$ symmetry. Eq.(3.7) is the texture of the Yukawa couplings and not the mass matrices. It is necessary to determine the SUSY breaking terms to deduce the VEV’s of Higgs particles in each generation.

**iii) 2-1 Generation**

The last one is that only third generation and Higgs particles to be composite, and quarks and leptons of the first and the second generations are elementally particles. This model include two set of Higgs doublets, which is different from models $i$) and $ii$). As for the first and the second generations, the quarks and leptons are singlet of the $SU(7)_{H_i}$. On the other hand, the preons and anti-preons of the third generation that have an opposite charge of the first and the second generations for the SM charge confine to be composite states. Fields with an opposite charge, so-called “anti-generation(miller field)”, is naturally included in the String Inspired Model [12].
When $SU(7)_H$ gauge coupling becomes strong, dynamical superpotential eq. (2.2) are induced. Only the Yukawa couplings of third generations are induced. Thus, at the tree level we should introduce the Yukawa couplings of the first and second generations, and the eq. (2.4) gives masses of Higgs particles, leptoquarks and color-octet Higgs. We introduce the tree level superpotential as

$$W_{\text{tree}} = y^{ij}_u \bar{u}_i \left( P_Q P_U + P_t P_N \right) q_j + y^{ij}_d \bar{d}_i \left( P_Q P_D + P_t P_E \right) q_j + y^{ij}_u \bar{u}_i \left( P_Q P_U + P_t P_N \right) P^5_{Q} P^2_{\ell} \frac{P^5_{Q}}{M^7} + y^{ij}_d \bar{d}_i \left( P_Q P_D + P_t P_E \right) P^5_{Q} P^2_{\ell} \frac{P^5_{Q}}{M^7} + y^{ij}_u \bar{u}_i \left( P_Q P_U + P_t P_N \right) \frac{P^6_{Q}}{M^7} + y^{ij}_d \bar{d}_i \left( P_Q P_D + P_t P_E \right) \frac{P^6_{Q}}{M^7} + g \frac{P_Q P_U \bar{P}_D + P_t \bar{P}_N}{M} + g_\mu \frac{P_t \bar{P}_E \bar{P}_N}{M} + g_\nu \frac{P_Q P_U \bar{P}_D + P_t \bar{P}_N}{M} + g_y \frac{P_Q P_U \bar{P}_D + P_t \bar{P}_N}{M},$$

(3.8)

where $\bar{u}_i, \bar{d}_i, q_i, \bar{e}_i, \bar{n}_i$ and $\ell_i$ are $SU(7)_H$ singlet up-part, down-part, doublet quark, charged lepton, neutral lepton and doublet lepton of $i$th generation ($i, j = 1, 2$), respectively. $M$ is the arbitrary mass scale of this model.

Below the confinement scale $\Lambda$, the effective superpotential $W^{(\text{eff})}$ becomes

$$W^{(\text{eff})} = W_{\text{tree}}^{(\text{eff})} + W_{\text{dyn}}$$

where $u_i, d_i, Q_i, \bar{e}_i, \bar{n}_i$ and $\ell_i$ are $SU(7)_H$ singlet up-part, down-part, doublet quark, charged lepton, neutral lepton and doublet lepton of $i$th generation ($i, j = 1, 2$), respectively. $M$ is the arbitrary mass scale of this model.
\[ +m(g_G G_u G_d + g_G \Phi_u \Phi_d + g_\mu \phi_u \phi_d + g_x \bar{X} X + g_x \phi_u \Phi_d + g_y \bar{Y} Y + g_y \Phi_u \bar{\phi}_d) \]

\(-\text{determinant of mesons},\)

\[ (3.9) \]

where \( \varepsilon \equiv \frac{\Lambda}{M} \) and \( m \equiv \frac{\Lambda^2}{M} \). When \( \bar{\Phi}_u + \bar{\phi}_u \) and \( \bar{\Phi}_d + \bar{\phi}_d \) have vacuum expectation values (VEVs), mass matrices in the flavor space become

\[ M_{u,d,e,n} \sim \begin{pmatrix} \varepsilon & \varepsilon & \varepsilon^7 \\ \varepsilon & \varepsilon & \varepsilon^7 \\ \varepsilon^7 & \varepsilon^7 & 1 \end{pmatrix}, \]

\[ (3.10) \]

where each element of mass matrices has coefficient \( y_a^{ij} \), \((i,j = 1 \sim 3 \text{ and } a = u,d,n,e)\).

Now let us consider the phenomenology. We set the confinement scale at \( \Lambda \approx O(10^{17}) \text{ GeV} \) because the \( O(1) \) Yukawa coupling of third generations can be realized in \( SO(10) \) GUT model \([13]\). We consider \( M \) as the Plank scale, then, it becomes \( \varepsilon \approx O(10^{-1}) \), and \( m \) can be estimated:

\[ m \approx \frac{\Lambda^2}{M} \approx 10^{16}\text{GeV}. \]

\[ (3.11) \]

Therefore leptoquark \( X, \bar{X}, Y \) and \( \bar{Y} \) and \( SU(3)_C \) octet fields \( G_d, G_u \) have large mass of \( O(m) \). However, masses of each element of the Higgs mass matrix, eq.(2.7), are also of \( O(m) \). In order to obtain the realistic \( \mu \)-term of \( O(M_Z) \), we have to fine tune couplings of eq.(2.5). For example, when we fine tune the \( g_\mu g_G = g_x g_y \) in eq.(2.7), the determinant of the eq.(2.7) becomes to be zero. In this case, by introducing another gauge group, we could obtain the suitable \( \mu \)-term \([14]\).

When Yukawa couplings of the third generation are united at the confinement scale \( \Lambda \approx 10^{17} \text{ GeV} \) \([1]\),

\[ y_b \approx y_\tau \approx y_t \approx y_n, \]

\[ (3.12) \]

the relationships between the Yukawa couplings of the third generation become

\[ y_b \approx 3y_\tau \approx y_t \approx 3y_n \]

\[ (3.13) \]

at the weak scale \( O(M_Z) \), where we tune the value of \( \tan \beta \) to obtain the difference between the top quark and bottom quark mass. In this case, however, \( \nu_\tau \) becomes heavy because

\[ y_b, y_\tau, y_t \text{ and } y_n \text{ are Yukawa coupling of the b quark, } \tau \text{ lepton, t quark and } \nu_\tau, \text{ respectively.} \]
neutrino and top quark couple the same Higgs particles and there is the relation eq.(3.13). Mass of the neutrino ($m_\nu$) become about 1/3 of the top mass, that is

$$m_\nu \simeq 60 \text{ GeV}. \quad (3.14)$$

It is not in agreement with the nature. Thus, we must introduce a Majorana type mass matrix for the right handed neutrinos. Mass of neutrino becomes small by the See-Saw mechanism \[13\] and the mass of $\nu_\tau$ does not contradict then experiments. We introduce new elementary gauge singlet fields $S$ with $-2$ lepton number. We also add discrete $Z_3$ symmetry, and we assign for the neutrinos and singlet $S$ as

$$Z_3(n_1) = 1, \quad Z_3(n_2) = 2, \quad Z_3(S) = 1, \quad (3.15)$$

where $n_i$ is $i$th generation neutrino. The other elementary particles do not have this discrete $Z_3$ charge. Thus, the new additional superpotential is

$$W_\nu = Sn_1n_1 + Sn_2N \left( \frac{A}{M} \right)^6. \quad (3.16)$$

Then the mass matrix of right-hand neutrino becomes

$$M_N = \begin{pmatrix} S & 0 & 0 \\ 0 & 0 & S\varepsilon^6 \\ 0 & S\varepsilon^6 & 0 \end{pmatrix}. \quad (3.17)$$

When the singlet particle $S$ has vacuum expectation value $< S > \sim 10^{16}$ GeV, all neutrino mass become light enough to be consistent with experiments.

### 4 Summary

We present a composite model that is based on non-perturbative effects of $N = 1$ supersymmetric $SU(N_C)$ gauge theory with $N_f = N_C + 1$ flavors. In this model, we consider $N_C = 7$, where all particles in the SSM are composite states of elementary preons and anti-preons. When $SU(7)_H$ hyper-color gauge couplings become strong, preons and anti-preons are confined to be quarks, leptons and Higgs particles. At the same time, Yukawa interactions in the SSM emerge from the non-perturbative dynamics of the $SU(7)_H$ hyper-color.

At first, we consider one generation model. This model predicts the existence of two sets of Higgs doublets. However, unwanted massless fields also appear, which are lepto-quarks and color-octet particles. We introduce 4 dimensional higher order operators in the superpotential to generate the masses of these unwanted particles.

We then generalize a model to three generations in three ways. In the model $i)$, we introduce global family symmetry $SU(3)_F$. In this model, many Higgs particles are induced
by the strong gauge dynamics. Especially, this model include octet Higgs particles of the $SU(3)_F$ family symmetry. Therefor, it is necessary to think about more complex and newer scenario to make this model more realistic. In Model $ii)$, we introduce three sets of hyper-color gauge symmetry which confine each generations. This model predicts that each generation has two sets of the Higgs doublets, and the Yukawa couplings of the all generations are of $O(1)$. Since flavor mixing matrix is not induced by the gauge dynamics, we introduce tree level the higher dimensional operators. At this time we introduce the $\mathbb{Z}_8$ symmetry in order to avoid large FCNC. In model $iii)$, the first and the second generations are hyper-color singlet elementary particles and only third generations and Higgs Particles are composite states of the preons and anti-preons. We should adjust confinement scale to $10^{17}$ GeV because the all Yukawa couplings of the third generation are of $O(1)$. Then, we have to fine tune the couplings in the eq.(3.7) to obtain a realistic $\mu$-term. And we introduce the new elementary gauge singlet fields $S$ in order to obtain small masses of neutrinos.

In this article we have suggested one of the possibilities for the origin of Yukawa coupling. Further work is necessary to understand to determine the mechanism of SUSY breaking and the electroweak symmetry breaking in order to discuss the mass matrices and flavor mixing. We hope that these models should shed some light on the origin of Yukawa couplings.

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A  Gauge representations of the each three generation model

|              | $SU(23)_H$ | $SU(3)_C$ | $SU(2)_L$ | $U(1)_Y$ | $SU(3)_F$ | $U(1)_B$ | $U(1)_L$ |
|--------------|------------|------------|------------|-----------|------------|-----------|-----------|
| $P_Q$        | $\Box$     | $\Box^*$   | $\Box$     | $-1/3$    | $\Box$     | $-5/69$   | $6/23$    |
| $P_t$        | $\Box$     | 1          | $\Box$     | $1$       | $\Box$     | $6/23$    | $-7/23$   |
| $P_U$        | $\Box^*$   | $\Box$     | 1          | $4/3$     | $\Box^*$   | $5/69$    | $-6/23$   |
| $P_D$        | $\Box^*$   | $\Box$     | 1          | $-2/3$    | $\Box^*$   | $5/69$    | $-6/23$   |
| $P_N$        | $\Box^*$   | 1          | 1          | 0         | $\Box^*$   | $-6/23$   | $7/23$    |
| $P_E$        | $\Box^*$   | 1          | 1          | $-2$      | $\Box^*$   | $-6/23$   | $7/23$    |

Table A.1: Preon and anti-preon fields in Model $i$)

|              | $SU(23)_H$ | $SU(3)_C$ | $SU(2)_L$ | $U(1)_Y$ | $SU(3)_F$ | $U(1)_B$ | $U(1)_L$ | baryon states ($\times 1/\Lambda^{22}$) |
|--------------|------------|------------|------------|-----------|------------|-----------|-----------|----------------------------------------|
| $Q$          | 1          | $\Box$     | $\Box$     | $1/3$     | $\Box$     | $1/3$     | 0         | $P_Q^1 P_Q^5$                           |
| $l$          | 1          | 1          | $\Box$     | $-1$      | 1          | 0         | 1         | $P_l^0 P_l^5$                           |
| $U$          | 1          | $\Box^*$   | 1          | $-4/3$    | 1          | $-1/3$    | 0         | $P_U^0 P_U^5 P_U^1 P_U^3$               |
| $D$          | 1          | $\Box^*$   | 1          | $2/3$     | 1          | $-1/3$    | 0         | $P_D^0 P_D^5 P_D^1 P_D^3$               |
| $\bar{N}$    | 1          | 1          | 1          | 0         | 1          | 0         | $-1$      | $P_N^0 P_N^5 P_N^1 P_N^2$               |
| $\bar{E}$    | 1          | 1          | 1          | 2         | 1          | 0         | $-1$      | $P_E^0 P_E^5 P_E^1 P_E^3$               |

Table A.2: “Baryon” and “anti-baryon” fields in Model $i$)
| \( \Phi_u^{*} \) | \( \Phi_d^{*} \) | \( G_u^{*} \) | \( G_d^{*} \) | \( X^{*} \) | \( Y^{*} \) | \( \tilde{\phi}_d^{*} \) | \( \tilde{\phi}_d \) |
|---|---|---|---|---|---|---|---|
| 1 | 1 | □ | 1 | 1 | □ | 0 | 0 |
| 1 | 1 | □ | -1 | 1 | 0 | 0 | 0 |
| 1 | Ad | □ | 1 | 1 | 0 | 0 | 0 |
| 1 | Ad | □ | -1 | 1 | 0 | 0 | 0 |
| 1 | □* | □ | -1/3 | 1 | -1/3 | 1 | 0 |
| 1 | □* | □ | -7/3 | 1 | -1/3 | 1 | 0 |
| 1 | □ | □ | 1/3 | 1 | 1/3 | -1 | 0 |
| 1 | □ | □ | 7/3 | 1 | 1/3 | -1 | 0 |
| 1 | 1 | □ | 1 | 1 | 0 | 0 | 0 |
| 1 | 1 | □ | -1 | 1 | 0 | 0 | 0 |

Table A.3: "Meson" fields in Model \( \imath \)

| \( P_{Q1} \) | \( P_{T1} \) | \( P_{\bar{Q}1} \) | \( P_{\bar{T}1} \) | \( P_{N1} \) | \( P_{\bar{N}1} \) | \( P_{E1} \) | \( \bar{P}_{E1} \) |
|---|---|---|---|---|---|---|---|
| □ | 1 | □* | □ | □* | □ | □* | □* |
| □ | 1 | 1 | □ | 1 | □ | 1 | □ |
| □* | 1 | 1 | □ | 1 | □ | 1 | □ |
| □* | 1 | 1 | □ | 1 | □ | 1 | □ |
| □* | 1 | 1 | □ | 1 | □ | 1 | □ |
| □* | 1 | 1 | □ | 1 | □ | 1 | □ |
| □* | 1 | 1 | □ | 1 | □ | 1 | □ |
| □* | 1 | 1 | □ | 1 | □ | 1 | □ |

Table A.4: Preon and anti-preon fields in Model \( \bar{\imath} \)
Table A.5: "Baryon" and "anti-baryon" fields in Model $ii$
| Field | SU(3)_C | SU(2)_L | U(1)_Y | U(1)_B | U(1)_L | Z_8 | Meson (×/Λ) |
|-------|---------|---------|---------|---------|---------|------|--------------|
| u_1   | 1       |         | 1       | 0       | 0       | 7    | P_{Q1}P_{U1} |
| d_1   | 1       | -1      |         | 0       | 0       | 7    | P_{Q1}P_{D1} |
| u_2   | Ad      |         | 1       | 0       | 0       | 7    | P_{Q1}P_{U1} |
| d_2   | Ad      |         | -1      | 0       | 0       | 7    | P_{Q1}P_{D1} |
| u_3   | □       | -1/3    | -1/3    | 1       | 7       | P_{Q1}P_{D1} |
| d_3   | □       | 1/3     | 1/3     | -1      | 7       | P_{Q1}P_{D1} |

| Field | SU(3)_C | SU(2)_L | U(1)_Y | U(1)_B | U(1)_L | Z_8 | Meson (×/Λ) |
|-------|---------|---------|---------|---------|---------|------|--------------|
| u_1   | 1       |         | 1       | 0       | 0       | 7    | P_{Q1}P_{U1} |
| d_1   | 1       | -1      |         | 0       | 0       | 7    | P_{Q1}P_{D1} |
| u_2   | Ad      |         | 1       | 0       | 0       | 7    | P_{Q1}P_{U1} |
| d_2   | Ad      |         | -1      | 0       | 0       | 7    | P_{Q1}P_{D1} |
| u_3   | □       | -1/3    | -1/3    | 1       | 7       | P_{Q1}P_{D1} |
| d_3   | □       | 1/3     | 1/3     | -1      | 7       | P_{Q1}P_{D1} |

Table A.6: “Meson” fields in Model ii)
Table A.7: Preon and anti-preon fields in Model \( \text{iii} \)

| \( P_{Q1} \) | \( P_{t1} \) | \( P_{u1} \) | \( P_{d1} \) | \( P_{N1} \) | \( P_{E1} \) | \( P_{Q2} \) | \( P_{t2} \) | \( P_{u2} \) | \( P_{d2} \) | \( P_{N2} \) | \( P_{E2} \) | \( P_{Q3} \) | \( P_{t3} \) | \( P_{u3} \) | \( P_{d3} \) | \( P_{N3} \) | \( P_{E3} \) |
|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|----------------|
| \( 1 \) | \( 1 \) | \( 1 \) | \( 0 \) | \( 0 \) | \( 0 \) | \( 1 \) | \( 1 \) | \( 1 \) | \( 1 \) | \( 1 \) | \( 1 \) | \( 1 \) | \( 1 \) | \( 1 \) | \( 1 \) | \( 1 \) | \( 1 \) |
| \( 1 \) | \( 1 \) | \( 0 \) | \( 0 \) | \( 0 \) | \( 0 \) | \( −1 \) | \( −1 \) | \( −1 \) | \( −1 \) | \( −1 \) | \( −1 \) | \( −2 \) | \( −2 \) | \( −2 \) | \( −2 \) | \( −2 \) | \( −2 \) |

Table A.8: “Baryon” and “anti-baryon” fields in Model \( \text{iii} \)

| \( Q_3 \) | \( l_3 \) | \( U_3 \) | \( D_3 \) | \( N_3 \) | \( E_3 \) |
|----------------|----------------|----------------|----------------|----------------|----------------|
| \( 1 \) | \( 1 \) | \( 1 \) | \( 1 \) | \( 1 \) | \( 1 \) |
| \( 1 \) | \( 1 \) | \( 1 \) | \( 1 \) | \( 1 \) | \( 1 \) |
| \( 1 \) | \( 1 \) | \( 1 \) | \( 1 \) | \( 1 \) | \( 1 \) |

Table A.9: “Meson” fields in Model \( \text{iii} \)