The thermal effect on dynamic characteristics of composite material pipe conveying fluid

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Abstract. Based on thermoelasticity, the composite material pipe is studied in this paper with
the variation of thermal expansion coefficient and temperature. Considering the thermal effect, a
model called elastic Bernoulli-Euler beams is developed to analyse the buckling and vibration.
The partial differential equations are established by using Hamiltonian principle. The
static post-buckling problem is studied by eigenvalue method after Galerkin method. By
harmonic balance method, the approximate solution of strong nonlinear vibration is obtained.
The effect of thermal effect on vibration is also focused. The simulation shows that the thermal
effects make conveyor pipes more vulnerable to instability, and has obvious influence on
the dynamic characteristics. The research content is more practical than the dynamic analysis of
the conveyor pipe without considering the temperature effect.

1. Introduction
Structures and materials are often affected by thermal effects. In the field of aerospace and aeronautics,
related thermal analysis has been deeply studied such as the thermal post-buckling which is about
composite laminated shell structures. The material property independent of temperature is not enough
to accurately reflect the real heat conduction environment. The thermal stress of the structure changed
when the composition of composites changed [1]. The thermal buckling of shells whose material
coefficients are distributed by thickness and are non-linear functions of temperature at both ends of
which are simply supported [2]. The physical properties of composites are also related to volume
fraction. The power form changes with assumed material properties as thickness coordinate variables,
and functional gradient index effect the buckling temperature difference of cylindrical shells [3]. The
thermal post-buckling behavior of plates with effects of temperature dependence and volume fraction
index got focus [4]. In addition to material properties, thermal expansion coefficient in thermal analysis
is also affected by temperature change. The thermal expansion coefficient can be taken as a random
variable [5]. While the pipeline conveying flow with critical velocity is unstable, it loses stability by
flutter, also the dynamic characteristics of elastic composite pipe conveying fluid has been studied [6].
The non-linear transverse vibrate because of the high-pressure pipelines and variable-speed conveying
fluids with disappeared bending stiffness by multiple scales [7].
A model called Bernoulli-Euler beam is established in the paper, especially considering the thermal
effect. The governing equation mathematical model of filling heating stress is established by changing
temperature and thermal expansion coefficient. The effects of temperature and thermal expansion
efficient on buckling instability and dynamic characteristics are studied.
2. Thermal effect model of composite pipe

This paper mainly studies the dynamic characteristics of composite material conveying pipe, whose bending stiffness is $EI$, the mass per unit length is $m$, the velocity $v_f$ of per unit mass $M$ flow in the pipe, which is shown in figure 1.

![Figure 1. Fluid-conveying pipe diagram](image)

Considering the heat flow temperature in the pipe, according to thermoelastic theory, the axial thermal stress is written as $EA \alpha T$, the cross section area is $A$, the volume of the pipe is $V$, the temperature change is $T$ and the variable thermal expansion coefficient of the material is $\alpha$. Since there is no internal heat source, the heat flow is stable, and variable temperature satisfies differential equation

$$\left(\frac{d^2 T}{dr^2} + \frac{1}{r} \frac{dT}{dr}\right) = 0,$$

According to the temperature boundary condition such as $(T)_{r=d/2} = T_d$ and $(T)_{r=D} = T_D$, the temperature field with the gradient of vector diameter can be obtained.

$$T = T_d \frac{\ln \frac{D}{d}}{\ln \frac{D}{D}} + T_D \frac{\ln \frac{d}{2}}{\ln \frac{D}{D}}$$

The coefficient of thermal expansion of materials $\alpha$ varies with the vector diameter $r$, and follows the exponential distribution form:

$$\alpha(r) = \alpha_0 - \ln[1 - \tanh\left(\frac{r}{2}\right)]$$

Naming $\alpha_0$ to be the average value of the coefficient of thermal expansion function $\alpha(r)$. From equations (1) and (2), both the temperature field and the coefficient are the increasing functions of the vector diameter $r$. According to the thermal stress formula $\sigma_r = EA \alpha T$, it become a function of the vector diameter:

$$\sigma_r = EA \left( T_d \frac{\ln \frac{D}{d}}{\ln \frac{D}{D}} + T_D \frac{\ln \frac{d}{2}}{\ln \frac{D}{D}} \right) \left( \alpha_0 - \ln \left[ 1 - \tanh\left(\frac{r}{2}\right) \right] \right)$$

For convenience of representation, the concrete function $\sigma_r$ varying with the vector diameter $r$ is only substituted in the final calculation result. Assuming that the vibration of the elastic tube only considers the transverse vibration, the dynamics of the macroscopic pipe is established according to Bernoulli-Euler theory[8], the axial strain of the pipe is written like $\varepsilon(x) = -z \frac{\partial^2 w}{\partial x^2}$. In the direction $x$ and direction $z$, $u$ and $w$ denote the pipe’s displacement. The stress-strain formula is obtained, and then the formula is introduced into the Hamiltonian principle. In the case of linearity, the governing partial differential equation related to temperature can be derived[9].
\[ (EI) \frac{\partial^4 w}{\partial x^4} + (M + m) \frac{\partial^2 w}{\partial t^2} + 2Mv_f \frac{\partial^2 w}{\partial x \partial t} + [MV_f^2 + \sigma_T - \frac{EA}{2L} \int_0^L \left( \frac{\partial w}{\partial x} \right)^2 dx] \frac{\partial^2 w}{\partial x^2} = 0 \] (4)

\( S \) is the static moment, \( E \) is composite pipe’s the Young’s modulus, the area inertia moment of the cross section is \( I \), and the additional axial force related to the average axial extension of the tube is \( EA / 2L \int_0^L \left( \frac{\partial w}{\partial x} \right)^2 dx \). The boundary conditions of the model are as follows:

\[ \frac{\partial^2 w(0,t)}{\partial x^2} = w(0,t) = \frac{\partial^2 w(L,t)}{\partial x^2} = w(L,t) = 0 \] (5)

The mechanical properties of the composites, such as mass density and Young’s modulus, were determined by modified mixing rules and Halpin-Tsai equation [10].

\[ E = \begin{pmatrix} \frac{1}{8} & \frac{2L_{GPL}}{\mu L_{GPL}} \eta_l V_G \\ \frac{5}{8} & \frac{2wh_{GPL}}{\mu L_{GPL}} \eta_w V_G \end{pmatrix} \times E_0, \rho = \rho_{GPL} V_G + \rho_0 (1 - V_G) \] (6)

Where \( V_G \) is the volume fraction about GPL, which changing linearly with the mass fraction; \( E_{GPL} = 1.01 \) TPa and \( E_0 = 3 \) GPa are young’s modulus describing the GPL and polymer matrix; \( \rho_{GPL} = 1.06 \) g/cm\(^3\) and \( \rho_0 = 1.2 \) g/cm\(^3\) are GPL and matrix polymer density; \( L_{GPL} = 2.5 \) μm, \( t_{GPL} = 1.5 \) μm, \( w_{GPL} = 1.5 \) μm respective the length, thickness and width of the GPL.

3. Thermal effect on post-buckling

If the fluid velocity is close to the critical value, the simply supported pipe will diverge. Based on the governing equation (4), the static post-buckling problem are discussed. The critical velocity can be derived by deleting the time correlation in the equation. The results are as follows:

\[ \frac{\partial^4 w}{\partial x^4} + \left( \frac{MV_f^2}{EI} + \frac{S\rho T}{I} \right) \frac{\partial^2 w}{\partial x^2} - \frac{A}{2IL} \int_0^L \left( \frac{\partial w}{\partial x} \right)^2 dx \frac{\partial^2 w}{\partial x^2} = 0 \] (7)

Since \( \int_0^L \left( \frac{\partial w}{\partial x} \right)^2 dx \) is integral constant in equation (8), we suppose:

\[ \lambda^2 = \frac{MV_f^2}{EI} + \frac{S\rho T}{I} - \frac{A}{2IL} \int_0^L \left( \frac{\partial w}{\partial x} \right)^2 dx \] (8)

By substituting equation (8) in equation (7), it can get:

\[ w^{(4)} + \lambda^2 \frac{w^{(2)}}{x} = 0 \] (9)

According to the literature [11], the general solution of the four order ordinary differential equation (9) including constant coefficients is assumed as:

\[ w(x) = c_1 + c_2x + c_3 \cos(\lambda x) + c_4 \sin(\lambda x) \] (10)

Where \( c_i (i = 1, 2, 3, 4) \) is constant. By substituting (10) in (7) to satisfy the boundary conditions, such equations will be obtained:

\[ c_1 + c_2 = 0, c_4 \lambda^2 = 0 \]

\[ \lambda^2 c_1 \cos \lambda \lambda' + \lambda^2 c_4 \sin \lambda \lambda' = 0, c_1 + c_2 + c_3 \cos \lambda \lambda' + c_4 \sin \lambda \lambda'=0 \] (11)

After the coefficient matrix is expanded, the following characteristic equations are obtained:
\[ \lambda = \frac{m\pi}{L}, m = 1, 2, 3 \]  \hspace{1cm} (12)

the coefficients can be obtained by substituting equation (12) into equation (8), the exact expression of the static post-buckling structure are obtained by substituting coefficients in equation (10) which:

\[ w(x) = \pm 2L \left( \frac{Mv^2}{e^2} + \frac{ST\alpha}{e^2} - \frac{I}{A} \right)^{1/2} \sin \left( \frac{m\pi}{L} x \right) \]  \hspace{1cm} (13)

The critical velocity can be calculated from equation (13) and when the temperature varies with the vector diameter, the coefficient of thermal expansion should be substituted into equation (14).

\[ v_f = \frac{1}{M} E \left( \frac{\lambda^2 I - S}{M} \left[ T_d + \frac{d}{D} \ln \frac{D}{d} \right] - T_0 \ln \left( 1 - \tanh \left( \frac{r}{2} \right) \right) \right)^{1/2} \]  \hspace{1cm} (14)

Numerical simulation is carried out according to equation (14). The simulation results are shown in figures 2 to 4. Among them, \( V_G \) is 0.05%, 0.1%, 0.15% and 0.2% respectively, and the vector diameter \( r \) equals \( (D + d)/4, D/2 \) and \( d/2 \).

![Figure 2. buckling behavior in \( r = D/2 \)](image)

![Figure 3. buckling behavior in \( r = (D + d)/4 \)](image)

![Figure 4. Bifurcation diagram of buckling behavior in \( r = d/2 \)](image)

Figure 2-4 shows bifurcation diagrams of buckling behavior of composite pipes with different volume fraction \( V_G \) and vector diameter \( r \), which indicates that the thermal effect. It means that the thermal expansion coefficient \( \alpha \) and temperature \( T \) of the material increase with radius vector \( r \) increasing. In addition, with the volume fraction increasing linearly, the post-buckling displacement of the composite pipe increased. The increase of temperature and thermal expansion coefficient makes the conveyor pipe more instability.
4. Harmonic Equilibrium Method to Analyze the Effect of Thermal Effect on Vibration

This paper analyzes non-linear free vibration with simple support at both ends by using Galerkin truncation method to separate variables. The transverse displacement \( w(x) \) is assumed to be the product of the first mode \( \phi(x) \) and the generalized coordinates \( q(t) \), and the vibration mode of the beam satisfies the same boundary condition. The displacement expression is substituted into equation (4) to obtain that:

\[
\ddot{q} + (\kappa_1 + \kappa_2)q + \kappa_3q^3 = 0
\]  

(15)

where \( \kappa_1 = \frac{EI}{M + \rho V} \int_0^L \frac{\partial^2 \phi(x)}{\partial x^2} \phi(x) \, dx \), \( \kappa_3 = \frac{-EA}{2L(M + \rho V)} \int_0^L \frac{\partial^2 \phi(x)}{\partial x^2} \left[ \int_0^L \phi(x) \, dx \right]^2 \, dx \)

\[
\kappa_2 = \frac{1}{M + \rho V} \left[ \frac{\pi^2 EI}{L^4 (M + \rho V)} \left\{ T_a \ln \frac{D}{2r} + T_a \ln \frac{d}{D} - \frac{2r}{D} \frac{d}{D} \alpha_0 \right\} \right] + \frac{\pi^4 EA}{4L^4 (M + \rho V)} \int_0^L \frac{\partial^2 \phi(x)}{\partial x^2} \phi(x) \, dx \int_0^L \phi(x)^2 \, dx
\]

Assuming \( \phi(x) = \sin \left( \frac{\pi}{L} x \right) \), the relationship between linear frequency and coefficient is

\[
\omega_L = \kappa_1 + \kappa_2 = \frac{\pi^2 EI}{L^4 (M + \rho V)} [\pi^2 \frac{r}{L} \left( \frac{r}{D} - \frac{D}{L} \right) - \frac{2r}{D} \frac{d}{D} \alpha_0 \right] \]

\[
\kappa_3 = \frac{\pi^4 EA}{4L^4 (M + \rho V)} \]

(16)

Equation (16) is a strongly nonlinear differential equation. Therefore, for beam-like structures subjected to strong non-linear vibration, the second-order approximate response is obtained by using the harmonic balance method [12]:

\[
q(t) = a_0 + a_1 \cos(\omega_{nl}t) + a_2 \cos(2\omega_{nl}t)
\]

(17)

Supposing \( a_2 = a_0 - a \) for the convenience, where \( \omega_{nl} \) is the nonlinear natural frequency given by the equation:

\[
a_1 = \frac{83\kappa_0 a_t^4 + 5[1(\kappa_1 + \kappa_2) a_t^2 + 6a_t^3 + 8(\kappa_1 + \kappa_2) a_t^2 + 2(\kappa_1 + \kappa_2)]/ \kappa_3 - a_0 (11 \kappa_0 a_t^3 + 3(\kappa_1 + \kappa_2) a_t)}{44 \kappa_0 a_t^3 - 12 \kappa_0 a_t^2 + 12(\kappa_1 + \kappa_2) a_t - 4(\kappa_1 + \kappa_2) - 1} 
\]

\[
a_0 = \frac{11 \kappa_0 a_t^3 + 3(\kappa_1 + \kappa_2) a_t}{12 \kappa_0 a_t^3 - 4(\kappa_1 + \kappa_2) + 1} \cdot \omega_{nl} = \frac{[3 \kappa_0 a_t^2 + (\kappa_1 + \kappa_2)]^\frac{1}{2}}{83 \kappa_0 a_t^4 + 5(1 + \kappa_1 + \kappa_2) a_t^2 + 8(\kappa_1 + \kappa_2) a_t^2 + 2(\kappa_1 + \kappa_2)}
\]

(18)

The specific expression of thermal stress is substituted. Fig. 5 compares the ratio of linear to non-linear frequencies of composite pipes. The result decreases with the aspect ratio increasing, also decreases when vector diameter \( r \) increasing with the same aspect ratio, which means, the ratio decreases with the increase of temperature and thermal expansion coefficient.
Figure 5. Ratio of linear to nonlinear natural frequency following the change of aspect ratio

Figure 6. Bifurcation diagram of buckling behavior in $r = D/2$

Figure 7. Bifurcation diagram of buckling behavior in $r = d/2$

The vibration characteristics of composite pipes are discussed based on the Euler-Bernoulli beam theory. Fig. 6-7 shows that with the decrease of the vector $r$, the cycle of the image decreases and the vibration frequency increases. That means the vibration frequency increases with the temperature and thermal expansion coefficient increasing, which indicates that the thermal effect accelerated the vibration.

5. Conclusion
The temperature and thermal expansion coefficient effecting on the instability and dynamics of composite pipes are discussed. The results show that the thermal effect also is significant on the buckling behavior of material physical properties coefficient; Thus the thermal effect is enhanced, the flow tube is easier to lose stability. The temperature and thermal expansion coefficient increasing factors also affects vibration, the thermal has a greater impact on the non-linear natural frequency, and the vibration frequency increases, which indicates that the thermal effect accelerates the vibration.

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