Quantum thermodynamic properties of a nanoscopic system offer a rich source of information about its physical states. For instance, the system’s entropy at low temperatures provides the physical nature of the quantum ground states which also determine quantum transport characteristics such as the linear conductance. Thus getting access to thermodynamics of a nanoscopic system is an extremely challenging task for modern experiments to reveal both the system’s quantum ground states and how they govern quantum transport response of the system.

This is particularly important for nanoscopic systems involving one-dimensional topological superconductors in their topologically nontrivial phases replicating [1–5] the prototypical phase of the Kitaev model [6] realized, e.g., on the basis of semiconductors with strong spin-orbit interactions [7, 8]. This phase is characterized by non-Abelian Majorana bound states (MBS) localized at the ends of a topological superconductor appearing after a topological quantum phase transition as a consequence of a qualitative change of the quantum ground state from a trivial one to a nontrivial Majorana state.

The Majorana quantum ground state dictates specific features of quantum transport response such as the linear conductance observed in experiments [9, 10], in particular, its universality [11]. In fact, quantum transport predicts many specific Majorana induced signatures in various characteristics such as thermoelectric currents [12–16], shot noise [17, 22], quantum noise [23], thermoelectric shot noise [24], thermoelectric quantum noise [25], tunnel magnetoresistances in ferromagnetic systems [26], linear conductances in quantum dissipative systems [27], quantum transmission in photon-assisted transport [28], pumped heat and charge statistics [29]. Transport properties may also be combined with thermodynamic ones [30] to observe dual Majorana universality.

Revealing many consequences of the Majorana quantum ground states, quantum transport is, however, unable to directly access the Majorana ground states themselves. To uniquely detect MBS within quantum transport experiments one should be able to adapt them to mimic Majorana braiding protocols, e.g., by means of nonequilibrium noise measurements [31].

Nevertheless, it is highly appealing to uniquely reveal MBS directly from the system’s quantum ground state avoiding braiding protocols or their conceivable counterparts. Successful experimental entropy measurements [32, 33] provide an exceptional opportunity to access the Majorana entropy in nanoscopic systems. In practice, however, this is not a straightforward task. Indeed, to measure the Majorana entropy one could consider an ideal setup presumably involving a pair of highly nonlocal non-Abelian MBS with a finite coupling to only one Majorana mode ignoring completely any coupling to the other Majorana mode. However, this is the result of emergence of additional degrees of freedom, if left uncontrolled, ruin the Majorana entropy to a much smaller value. Remarkably, it turns out that exactly the Majorana tunneling phases provide a revival of the Majorana entropy that is the result of emergence of additional degrees of freedom, namely the Majorana tunneling phases.

Here we demonstrate that the above idealization will likely fail to guide experimental observations of the Majorana entropy \( S_M = k_B \log(2^j) \) because of inevitable finite coupling to the second Majorana mode in a realistic setup. As discussed below, even if the tunneling amplitude of the second Majorana mode is several orders of magnitude smaller than the tunneling amplitude of the first Majorana mode, straightforward experiments will be essentially brought in a regime with entropy \( S \ll S_M \). We reveal that such a strong sensitivity of the system’s entropy to the coupling of the second Majorana mode is the result of emergence of additional degrees of freedom, namely the Majorana tunneling phases. Exactly these new degrees of freedom, if left uncontrolled, ruin the Majorana entropy to a much smaller value. Remarkably, these new degrees of freedom provide a revival of the Majorana entropy that is a return of the system to the Majorana quantum ground state with entropy \( S = S_M \). As we show, one may revive Majorana quantum ground states via tunneling phases in setups with experimentally realizable parameters.

To specify our discussion let us consider a concrete setup shown in Fig. [1]. The system is composed of a quantum dot, two normal massive metals and a...
grounded topological superconductor with MBS localized at its ends. The quantum dot Hamiltonian is \( H_d = \epsilon_d d^\dagger d \). The location of the energy level \( \epsilon_d \) with respect to the chemical potential \( \mu \) is tuned by a gate voltage. The quantum dot interacts with the normal metals and topological superconductor through tunneling mechanisms. The Hamiltonian of the normal metals, \( H_c = \sum_{i=L,R} \sum_k \epsilon_k t_{kL} c^\dagger_{kL} + \text{H.c.} \) is characterized by continuous spectra \( \epsilon_k \) resulting in a density of states \( \nu(\epsilon) \) assumed energy independent in the vicinity of the chemical potential, \( \nu(\epsilon) \approx \nu_c/2 \). The normal metals are assumed to be in equilibrium specified by the Fermi-Dirac distribution with the chemical potential \( \mu \) and temperature \( T \), \( f_c(\epsilon) = \{\exp[(\epsilon - \mu)/k_BT] + 1\}^{-1} \). The tunneling interaction of the quantum dot with the left and right normal metals is described by the Hamiltonian \( H_{d-c} = \sum_{i=L,R} \sum_k \epsilon_k t_{ki} c^\dagger_{ki} + \text{H.c.} \) bringing the energy scale \( \Gamma \equiv \Gamma_L + \Gamma_R, \Gamma_i \equiv \pi\nu_C|\Gamma_i|^2. \) The highly nonlocal Majorana modes localized at the ends of the topological superconductor, \( \gamma_1 \) and \( \gamma_2 \), are both linked with the quantum dot and may also have a finite overlap between each other. The links of \( \gamma_1 \) and \( \gamma_2 \) with the quantum dot have, respectively, tunneling amplitudes \( |\eta_1| \) and \( |\eta_2| \) as well as tunneling phases \( \phi_1 \) and \( \phi_2 \). The corresponding Hamiltonian is \( H_{d-ts} = \eta_1^2 \gamma_1 + \eta_2^2 \gamma_2 + \text{H.c.} \), where \( \gamma_1 = \gamma_1, \{\eta_1, \eta_2\} = 2\delta_{ij}, \eta_{1,2} = |\eta_{1,2}| \exp(i\phi_{1,2}) \). The Majorana mode \( \gamma_1 \) is linked to the quantum dot stronger than the Majorana mode \( \gamma_2 \), i.e. \( |\eta_1| > |\eta_2| \). The Majorana’s overlap, shown by the dashed arrows, is modeled by the Hamiltonian \( H_{ts} = i\xi \gamma_2 \gamma_1/2 \) with the overlap energy scale \( \xi \). We note that physical observables, in particular the system’s entropy, cannot depend separately on the two phases \( \phi_1 \) and \( \phi_2 \) but they must depend only on their difference \( \Delta \phi \equiv \phi_1 - \phi_2 \).

As in Ref. [34], replacing the second quantized operators in the above Hamiltonians with the corresponding Grassmann fields on the imaginary time axis, the thermodynamic partition function \( Z \) of the system is represented by a field integral in imaginary time with the Grassmann fields subject to the antiperiodic boundary conditions [30]. The system’s entropy is then obtained from the thermodynamic potential \( \Omega = -k_B T \log Z \) as the first derivative over the temperature, \( S = -\partial \Omega / \partial T \). It has the form:

\[
S = k_B \log \left[ \cosh \left( \frac{\xi}{2k_BT} \right) \right] - \frac{\xi}{2T} \tanh \left( \frac{\xi}{2k_BT} \right) + k_B \log(2) + \frac{1}{8\pi k_BT^2} \int_{-\infty}^{\infty} d\epsilon \frac{\phi(\epsilon)}{\cosh^2(\xi/2k_BT)},
\]

where \( \phi(\epsilon) \) represents the phase of a complex expression involving the retarded and advanced hole-particle, hole-hole and particle-particle Green’s functions,

\[
G_{hp}(-\epsilon)G_{hp}^{R}(\epsilon) - G_{hh}(-\epsilon)G_{pp}^{R}(\epsilon) = \rho(\epsilon) \exp[i\phi(\epsilon)],
\]

where \( iG_{hp}^{R}(t)(t') \equiv \pm \Theta(\pm t + t') \{(d_1(t), d_2(t'))\}, j = p, h \) and \( d_p \equiv d^\dagger_1, d_h \equiv d \). The Green’s functions depend on the parameters of the above setup, in particular on the tunneling phase difference \( \Delta \phi \), and are found from a field integral in real time, the Keldysh field integral [30].

In Fig. 2 we show the results obtained for the system’s entropy as a function of the tunneling phase difference \( \Delta \phi \) in polar coordinates. Specifically, the distance from the center (the origin of coordinates) to a point on a curve is equal to \( S \) while the polar angle is equal to \( \Delta \phi \). Different curves correspond to different values of a gate voltage controlling \( \epsilon_d \). The solid red, blue, green, orange and magenta curves show \( S \) for positive values of \( \epsilon_d \) while the dashed blue, green and magenta curves show \( S \) for the corresponding negative values of \( \epsilon_d \). The solid black curve is for \( \epsilon_d = 0 \). Here \( k_B T / \Gamma = 10^{-8}, |\eta_1| / \Gamma = 4 \cdot 10^{-2}, |\eta_2| / \Gamma = 10^{-4}, \xi / \Gamma = 10^{-2} \). As can be seen, although \( |\eta_1| \) is more than six orders of magnitude larger than \( |\eta_2| \), all the curves are highly anisotropic showing a very strong dependence of the system’s entropy \( S \) on the tunneling phase difference \( \Delta \phi \). The points on the curves with \( \epsilon_d \geq 0 \) and the circles on the curves with \( \epsilon_d < 0 \) indicate where \( S \) reaches its maximal value. The radius of the largest polar circle is equal to the Majorana entropy \( S_M \). We find numerically that all the curves with \( |\epsilon_d| < 4|\eta_1||\eta_2|/\xi \) touch the largest polar circle \( S_M \) at two points while the curves with \( |\epsilon_d| = 4|\eta_1||\eta_2|/\xi \) touch it only at one point, at \( \Delta \phi = \pi/2 \) for \( \epsilon_d < 0 \) and at \( \Delta \phi = 3\pi/2 \) for \( \epsilon_d > 0 \). For \( |\epsilon_d| > 4|\eta_1||\eta_2|/\xi \) the maximal values of \( S \) start to depend on \( \epsilon_d \) but do not rotate any more and remain at the two fixed polar angles, \( \Delta \phi = \pi/2 \) (\( \epsilon_d < 0 \)) and \( \Delta \phi = 3\pi/2 \) (\( \epsilon_d > 0 \)). When \( |\epsilon_d| \to \infty \), the maximal values of \( S \) at these two polar angles decrease and go from the universal Majorana value \( S_M \) to zero. This situation is demonstrated by the black arrows representing a flow of the entropy maximum in the direction of increasing values of \( \epsilon_d \). For large negative values of \( \epsilon_d \) one starts at the center and moves up along the \( y \)-axis when \( \epsilon_d \) increases up to \( \epsilon_d = -4|\eta_1||\eta_2|/\xi \) where the maximal
value of $S$ is equal to the Majorana entropy $S_M$. After this point, when $\epsilon_d$ increases further, the maximal value of $S$ does not depend on $\epsilon_d$ and remains equal to the universal Majorana entropy $S_M$. However, the polar angle at which it is observed splits from $\Delta \phi = \pi/2$ into two polar angles which rotate, upon increasing $\epsilon_d$, on the largest polar circle $S_M$ in the opposite directions, anticlockwise and clockwise, and merge again at the polar angle $\Delta \phi = 3\pi/2$ when $\epsilon_d = 4|\eta_1||\eta_2|/\xi$. For $\epsilon_d > 4|\eta_1||\eta_2|/\xi$ the maximal value of $S$ again starts to depend on $\epsilon_d$ and goes from the universal Majorana value $S_M$ to zero when $\epsilon_d$ goes to large positive values, that is one moves up along the $y$-axis from the largest polar circle $S_M$ to the center where the flow returns to its starting point and finally stops.

In Fig. 3 we demonstrate the universality of the Majorana ground state. The parameters have the same values as in Fig. 2. Upper panel: The differences of the tunneling phases $\Delta \phi$ corresponding to maximal values of the system’s entropy $S$ are shown as functions of $\epsilon_d$. As in Fig. 2 the black arrows indicate a flow of the maximal value of $S$ in the direction of increasing values of $\epsilon_d$. However, in comparison with Fig. 2 in the present representation this flow is not closed: its initial point at $\epsilon_d = -\infty$ and its final point at $\epsilon_d = \infty$ do not coincide. The flow splits at the point $\epsilon_d = -4|\eta_1||\eta_2|/\xi$ into two flows which merge again at the point $\epsilon_d = 4|\eta_1||\eta_2|/\xi$. For $\Delta \phi$ on the two flows inside the window $|\epsilon_d| < 4|\eta_1||\eta_2|/\xi$ the system’s entropy is equal to the Majorana value, $S = S_M$, and it does not depend on $\epsilon_d$ revealing universal thermodynamic behavior induced by MBS. Outside that universal window, that is for $|\epsilon_d| > 4|\eta_1||\eta_2|/\xi$, the system’s entropy on the flow is no longer universal, that is it depends on $\epsilon_d$ and its value always remains below the Majorana value, $S < S_M$. Lower panel: The black curve is the entropy maximum reached at $\Delta \phi$ from the upper panel. In other words, this curve shows the system’s entropy on the flow shown at the upper panel as a function of $\epsilon_d$. It has the Majorana plateau $S = S_M$ for $|\epsilon_d| < 4|\eta_1||\eta_2|/\xi$ and decreases when moving away from this universal window. The red curve shows the behavior of the maximal value of the linear conductance. The maximum of the linear conductance is reached also at $\Delta \phi$ from the upper panel. Therefore, the linear conductance is also taken on the flow from the upper panel. The linear conductance has the Majorana plateau $G = G_M \equiv e^2/2h$ in the same range of $\epsilon_d$ where $S = S_M$. This clearly shows how the nontrivial Majorana ground state, keeping inside the nonlocality of the MBS, uniquely determines the quantum dot linear response.

As shown in Fig. 4 a strong anisotropy of the system’s entropy is observed also after an essential increase of the temperature at experimentally relevant values of the parameters. Here $k_B T/\Gamma = 10^{-2}$, $|\eta_1|/\Gamma = 1$, $|\eta_2|/\Gamma = 10^{-1}$, $\xi/\Gamma = 10^{-1}$. The solid curve is for $\epsilon_d/\Gamma = 1$ and the dashed curve is for $\epsilon_d/\Gamma = -1$. The red arrows correspond to $\Delta \phi$ where $S = S_M$. The red angular sectors display vicinities of $\Delta \phi$ where $S$ is close to $S_M$. As can be seen, the anisotropic character and the maximal value $S_M$ of the system’s entropy are revealed even when the temperature has been raised up six orders of magnitude in comparison with Fig. 2.

Universal Majorana behavior of $S$ and $G$ is also observed at high temperatures as demonstrated in Fig. 5. All the parameters have the same values as in Fig. 4. Upper panel: The flow of $\Delta \phi$ on which $S$ and $G$ reach their maximal values. The flow is parameterized by $\epsilon_d$. Lower panel: The black and red curves show, respectively, $S$ and $G$ on the flow from the upper panel. As can be seen, the universal Majorana thermodynamic and transport behavior is robust and retains all its specific features even at very high temperatures. In particular, the universal Majorana plateaus $S = S_M$ and $G = G_M$ are simultaneously developed inside the window $|\epsilon_d| < \epsilon_M = 4|\eta_1||\eta_2|/\xi$. In other words, this curve shows the system’s entropy on the flow shown at the upper panel as a function of $\epsilon_d$. It has the Majorana plateau $S = S_M$ for $|\epsilon_d| < 4|\eta_1||\eta_2|/\xi$ and decreases when moving away from this universal window. The red curve shows the behavior of the maximal value of the linear conductance. The maximum of the linear conductance is reached also at $\Delta \phi$ from the upper panel. Therefore, the linear conductance is also taken on the flow from the upper panel. The linear conductance has the Majorana plateau $G = G_M \equiv e^2/2h$ in the same range of $\epsilon_d$ where $S = S_M$. This clearly shows how the nontrivial Majorana ground state, keeping inside the nonlocality of the MBS, uniquely determines the quantum dot linear response.

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FIG. 4. A polar plot demonstrating persistence of the Majorana anisotropic character of the system’s entropy at high temperatures.

Here, for the given set of parameters, this universal window is narrowed four times in comparison with Fig. 3.

In conclusion, we note that, as can be seen from Figs. 2 and 3 for a given gate voltage the Majorana entropy is ruined, \( S < S_M \), everywhere except for two values of the phase difference \( \Delta \phi_{\text{max}} \), where \( S = S_M \) and two narrow angular sectors around \( \Delta \phi_{\text{max}} \) where \( S \approx S_M \). Thus, it is highly probable that experiments which do not control the Majorana tunneling phases will reveal the system’s entropy \( S < S_M \). However, in experiments tuning the Majorana tunneling phases one will detect a maximum, \( S_{\text{max}} \), of the system’s entropy as a function of \( \Delta \phi \). Observing \( S_{\text{max}} = S_M \) will be a fully conclusive signature of the topologically nontrivial Majorana quantum ground state which has been revived via the corresponding tunneling phases. Let us estimate the experimental relevance of the parameters used for Figs. 2 and 3. Obviously, \( |\epsilon_d| \) should not exceed the induced superconducting gap \( \Delta \). For the largest energy scale \( \Gamma \) we take \( \Gamma \approx \Delta \). Thus one should expect to observe the Majorana universality in the window \( |\epsilon_d| \ll \Gamma \). Concerning the temperature, for \( \Delta \approx 250 \mu eV \) (see Ref. 9) one obtains \( T \approx 30 \text{ mK} \) which is easily achievable in modern experiments. Another practical aspect of our results is that in general Majorana tunneling phases may also be used to control qubits 37, 38 where Majorana dark states and spaces are stabilized and manipulated by means of special driven dissipative protocols. One may consider those protocols as a natural subsequent stage following the Majorana tunneling phase tuning proposed here to set up a proper initial Majorana equilibrium state of the qubit.

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