SEMI-PHENOMENOLOGICAL APPROACH TO THE ESTIMATE OF CP EFFECTS IN $K^\pm \to 3\pi$ DECAYS

Evgeny Shabalin
Institute for Theoretical and Experimental Physics, Moscow, Russia

Abstract

The amplitudes of the $K^\pm \to 3\pi$ and $K \to 2\pi$ decays are expressed in terms of different combinations of one and the same set of CP-conserving and CP-odd parameters. Extracting the magnitudes of these parameters from the data on $K \to 2\pi$ decays, we estimate an expected CP-odd difference between the values of the slope parameters $g^+$ and $g^-$ of the energy distributions of "odd" pions in $K^+ \to \pi^+\pi^+\pi^-$ and $K^- \to \pi^-\pi^-\pi^+$ decays.

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1 e-mail: shabalin@heron.itep.ru
1 Introduction

The observation of CP effects in $K^{\pm} \rightarrow 3\pi$ decays would allow to understand better how the mechanisms of CP violation work.

Now the Collaboration NA48/2 is ready to begin a search for such effect with accuracy $\delta(g^+g^-) \leq 2 \cdot 10^{-4}$.

Contrary to the case of $K_L \rightarrow 2\pi$ decay where CP violates both in $\Delta S = 2$ and $\Delta S = 1$ transitions, in the $K^{\pm} \rightarrow 3\pi$ decays, only the last (so-called “direct”) CP violation takes place. Experimentally, an existance of the direct CP violation in $K_L \rightarrow 2\pi$ decays, predicted by Standard Model (SM) and characterised by the parameter $\varepsilon'$ is established: $\varepsilon'/\varepsilon = (1.66 \pm 0.16)10^{-3}$. But the large uncertainties in the theoretical predictions $\varepsilon'/\varepsilon = (17^{+14}_{-10})10^{-4}$ [1], $\varepsilon'/\varepsilon = (1.5 - 31.6)10^{-4}$ [2] do not allow to affirm that the contributions from the sources of CP violation beyond the Kobayashi-Maskawa phase are excluded.

To avoid the uncertainties in the theoretical calculation of the ingredients of the theory, we use the following procedure. We express the amplitudes of $K_L \rightarrow 2\pi$ and $K^\pm \rightarrow 3\pi$ in terms of one and the same set of parameters, and calculating $g^+ - g^-$ we use the magnitudes of these parameters extracted from data on $K \rightarrow 2\pi$ decays.

2 The scheme of calculation

A theory of $\Delta S = 1$ non-leptonic decays is based on the effective lagrangian

$$L(\Delta S = 1) = \sqrt{2} G_F \sin \theta_C \cos \theta_C \sum c_i O_i$$

where

$$O_1 = \bar{s}_L \gamma^\mu d_L \cdot \bar{u}_L \gamma^\mu u_L - \bar{s}_L \gamma^\mu u_L \cdot \bar{u}_L \gamma^\mu d_L \quad (\{8_f\}, \Delta I = 1/2) \quad (2)$$

$$O_2 = \bar{s}_L \gamma^\mu d_L \cdot \bar{u}_L \gamma^\mu u_L + \bar{s}_L \gamma^\mu u_L \cdot \bar{u}_L \gamma^\mu d_L + 2 \bar{s}_L \gamma^\mu d_L \cdot \bar{d}_L \gamma^\mu d_L + 2 \bar{s}_L \gamma^\mu d_L \cdot \bar{d}_L \gamma^\mu s_L \quad (\{8_d\}, \Delta I = 1/2) \quad (3)$$

$$O_3 = \bar{s}_L \gamma^\mu d_L \cdot \bar{u}_L \gamma^\mu u_L + \bar{s}_L \gamma^\mu u_L \cdot \bar{u}_L \gamma^\mu d_L + 2 \bar{s}_L \gamma^\mu d_L \cdot \bar{d}_L \gamma^\mu d_L - 3 \bar{s}_L \gamma^\mu d_L \cdot \bar{s}_L \gamma^\mu s_L \quad (\{27\}, \Delta I = 1/2) \quad (4)$$
\begin{align}
O_4 &= \bar{s}_L \gamma_\mu d_L \cdot \bar{u} \gamma_\mu u_L + \bar{s}_L \gamma_\mu u_L \cdot \bar{d} \gamma_\mu d_L - \\
&\quad - \bar{s}_L \gamma_\mu d_L \cdot \bar{d} \gamma_\mu d_L \quad \{27\}, \Delta I = 3/2 \\
O_5 &= \bar{s}_L \gamma_\mu \lambda^a d_L (\sum_{q=u,d,s} \bar{q}_R \gamma_\mu \lambda^a q_R) \quad \{8\}, \Delta I = 1/2 \\
O_6 &= \bar{s}_L \gamma_\mu d_L (\sum_{q=u,d,s} \bar{q}_R \gamma_\mu q_R) \quad \{8\}, \Delta I = 1/2
\end{align}

This set is sufficient for calculation of the CP-even parts of the amplitudes under consideration. To calculate the CP-odd parts, it is necessary to add the so-called electroweak contributions originated by the operators \(O_7, O_8\):

\begin{align}
O_7 &= \frac{3}{2} \bar{s} \gamma_\mu (1 + \gamma_5) d \cdot \left( \sum_{q=u,d,s} e_q \bar{q} \gamma_\mu (1 - \gamma_5) q \right) \quad (\Delta I = 1/2, 3/2) \\
O_8 &= -12 \sum_{q=u,d,s} e_q (\bar{s}_L q_R)(\bar{q}_R d_L), \quad e_q = \left( \frac{2}{3}, \frac{1}{3}, \frac{1}{3} \right), \quad (\Delta I = 1/2, 3/2)
\end{align}

The coefficients \(c_{5-8}\) have the imaginary parts necessary for CP violation.

The bosonization of these operators can be done using the relations

\begin{align}
\bar{q}_j (1 + \gamma_5) q_k &= -\frac{1}{\sqrt{2}} F_\pi r \left( U - \frac{1}{\Lambda^2} \partial^2 U \right)_{kj} \\
\bar{q}_j \gamma_\mu (1 + \gamma_5) q_k &= i[(\partial_\mu U)^\dagger - U (\partial_\mu U^\dagger)] - \frac{r F_\pi}{\sqrt{2} \Lambda^2} \left[ m (\partial_\mu U^\dagger) - (\partial_\mu U) m \right]_{kj}
\end{align}

if the non-linear realization of chiral symmetry is used:

\begin{align}
U &= \frac{F_\pi}{\sqrt{2}} \left( 1 + \frac{i \sqrt{2} \hat{\pi}}{F_\pi} - \hat{\pi}^2 + a_3 \left( \frac{i \hat{\pi}}{\sqrt{2} F_\pi} \right)^3 + 2(a_3 - 1) \left( \frac{i \hat{\pi}}{\sqrt{2} F_\pi} \right)^4 + \ldots \right) \\
\text{where} \\
\hat{\pi} &= \begin{pmatrix}
\frac{\pi_0}{\sqrt{3}} + \frac{\pi_8}{\sqrt{6}} + \frac{\pi_3}{\sqrt{2}} \\
\pi^- \\
K^-
\end{pmatrix}
\end{align}

The PCAC condition demands \(a_3 = 0\) and we adopt this condition, bearing in mind that, on mass shell, the values of the mesonic amplitudes are independent of \(a_3\).
Using also the relations between matrices in the colour space
\[
\delta^\gamma_3 \delta^\gamma_3 = \frac{1}{4} \delta^\gamma_3 \delta^\gamma_3 + \frac{1}{2} \lambda^\alpha_3 \lambda^\gamma_3
\]
\[
\lambda^\alpha_3 \lambda^\gamma_3 = \frac{16}{9} \delta^\gamma_3 \delta^\gamma_3 - \frac{1}{4} \lambda^\gamma_3 \lambda^\gamma_3
\]
and the Fierz transformation relation
\[
\bar{s} \gamma_\mu (1 + \gamma_5) d \cdot \bar{q} \gamma_\mu (1 - \gamma_5) q = -2 \bar{s} (1 - \gamma_5) q \cdot \bar{q} (1 + \gamma_5) d
\]
and representing \( M(K \rightarrow 2\pi) \) in the form
\[
M(K^0 \rightarrow \pi^+ \pi^-) = A_0 e^{i\delta_0} - A_2 e^{i\delta_2}
\]
\[
M(K^0 \rightarrow \pi^0 \pi^0) = A_0 e^{i\delta_0} + 2A_2 e^{i\delta_2}
\]
\[
M(K^+ \rightarrow \pi^+ \pi^0) = -\frac{3}{2} A_2 e^{i\delta_2}
\]
we obtain
\[
A_0 = G_F F_\pi \sin \theta_C \cos \theta_C \frac{m_K^2 - m_\pi^2}{\sqrt{2}} \cdot [c_1 - c_2 - c_3 + \frac{32}{9} \beta (Re\tilde{c}_5 + iIm\tilde{c}_5)]
\]
\[
A_2 = G_F F_\pi \sin \theta_C \cos \theta_C \frac{m_K^2 - m_\pi^2}{\sqrt{2}} \cdot [c_4 + i\frac{2}{3} \beta \Lambda^2 Im\tilde{c}_7 (m_K^2 - m_\pi^2)^{-1}]
\]
where
\[
\tilde{c}_5 = c_5 + \frac{3}{16} c_6; \quad \tilde{c}_7 = c_7 + 3c_8; \quad \beta = \frac{2m_\pi^4}{\Lambda^2 (m_u + m_d)^2}.
\]
The contributions from \( \tilde{c}_7 O_7 \) into \( ReA_0 \) and \( ImA_0 \) are small because \( \tilde{c}_7 / \tilde{c}_5 \sim \alpha_{em} \) and we neglected these corrections. From data on widths of \( K \rightarrow 2\pi \) decays we obtain
\[
c_4 = 0.328; \quad c_1 - c_3 - c_3 + \frac{32}{9} \beta Re\tilde{c}_5 = -10.13.
\]
At \( c_1 - c_2 - c_3 = -2.89 \) \cite{3}, \cite{6} and \( \beta = 6.68 \) we obtain
\[
\tilde{c}_5 = -0.305.
\]
From the expression for \( A_2 \), it is seen that the contribution of the operators \( O_{7,8} \) is enlarged by the factor \( \Lambda^2 / m_K^2 \) in comparison with the rest operators contribution.
Using the general relation
\[ \varepsilon' = i e^{i(\delta_2 - \delta_0)} \left[ -\frac{ImA_0}{ReA_0} + \frac{ImA_2}{ReA_2} \right] \cdot \left| \frac{A_2}{A_0} \right| \] (21)

and the experimental value \( \varepsilon' = (3.4 \pm 0.45) \times 10^{-6} \), we come to the relation
\[ -\frac{Im\tilde{c}_5}{Re\tilde{c}_5} \left( 1 - \Omega_{\eta,\eta'} + 20.66 \frac{Im\tilde{c}_7}{Im\tilde{c}_5} \right) = 1.48 \cdot 10^{-4}. \] (22)

where \( \Omega_{\eta,\eta'} \) takes into account the effects of \( K^0 \to \pi^0 \eta(\eta') \to \pi^0 \pi^0 \) transitions.

The naive estimate gives
\[ -\frac{Im\tilde{c}_5}{Re\tilde{c}_5} \approx 1.7s_2s_3 \sin \delta \] (23)

where \( s_2, s_3 \) and \( \delta \) are the parameters of CKM matrix. At
\[ 4.6 \times 10^{-4} \leq s_2s_3 \leq 6.7 \times 10^{-4} \quad \text{(Landsberg'2002)} \]

\[ \frac{Im\tilde{c}_5}{Re\tilde{c}_5} = (-9.6 \pm 1.8) \times 10^{-4} \sin \delta \] (24)

and
\[ \frac{Im\tilde{c}_7}{Im\tilde{c}_5} = \begin{cases} 
-0.026 & \text{for } \Omega_{\eta,\eta'} = 0.3 \\
-0.041 & \text{for } \Omega_{\eta,\eta'} = 0 
\end{cases} \] (25)

3. **Decay** \( K^± \to \pi^± \pi^± \pi^∓ \)

In the leading \( p^2 \) approximation
\[ M(K^+ \to \pi^+(p_1)\pi^+(p_2)\pi^−(p_3)) = k[1 + ia_{KM} + \frac{1}{2}gY(1 + ib_{KM}) + \ldots], \] (26)

where
\[ k = G_F \sin \theta_C \cos \theta_C m_K^2 c_0 (3\sqrt{2})^{-1} \] (27)
\[ a_{KM} = \left[ \frac{32}{9} \beta Im\tilde{c}_5 + 4\beta Im\tilde{c}_7 \left( \frac{3\Lambda^2}{2m_K^2} + 2 \right) \right] / c_0 \] (28)
\[ b_{KM} = \left[ \frac{32}{9} \beta Im\tilde{c}_5 + 8\beta Im\tilde{c}_7 \right] / (c_0 + 9c_4) \] (29)
\[ g = -\frac{3m^2_\pi}{2m^2_K}(1 + 9c_4/c_0), \quad Y = (s_3 - s_0)/m^2_\pi \] (30)

\[ c_0 = c_1 - c_2 - c_3 - c_4 + \frac{32}{9}\beta Re\tilde{c}_5 = -10.46 \] (31)

As the field \( K^+ \) is the complex one and its phase is arbitrary, we can replace \( K^+ \) by \( K^+(1 + ia_{KM})(\sqrt{1 + a_{KM}^2})^{-1} \). Then

\[ M(K^+ \to \pi^+\pi^+\pi^- (p_3)) = k[1 + \frac{1}{2}gY(1 + i(b_{KM} - a_{KM}) + ...] \] (32)

Though this expression contains the imaginary CP-odd part, it does not lead to observable CP effects. Such effects arise due to interference between CP-odd imaginary part with the CP-even imaginary part produced by rescattering of the final pions. Then

\[ M(K^+ \to \pi^+\pi^+\pi^-) = k[1 + ia + \frac{1}{2}gY(1 + ib + i(b_{KM} - a_{KM}) + ...] \] (33)

where \( a \) and \( b \) are corresponding CP-even imaginary parts of the amplitude. These parts can be estimated (in \( p^2 \)) approximation calculating the imaginary part of the two-pion loop diagrams with

\[ M(\pi^+(r_2)\pi^+(r_3) \to \pi^+(p_2)\pi^-(p_3)) = F^{-2}_\pi[(p_2 + p_3)^2 + (r_2 - p_2)^2 - 2m^2_\pi] \]
\[ M(\pi^0(r_2)\pi^0(r_3) \to \pi^+(p_2)\pi^-(p_3)) = F^{-2}_\pi[(p_2 + p_3)^2 - m^2_\pi] \]
\[ M(\pi^+(r_1)\pi^+(r_2) \to \pi^+(p_1)\pi^+(p_2)) = F^{-2}_\pi[(r_1 - r_2)^2 + (r_1 - p_2)^2 - 2m^2_\pi] \]

Then we find:

\[ a = 0.12065; \quad b = 0.714 \] (34)

Using the definition of the slope parameter

\[ |M(K^+ \to \pi^+\pi^+\pi^+(p_3))|^2 \sim 1 + \frac{g}{1 + a^2}Y(1 + ab \pm a(b_{KM} - a_{KM}) + ... \] (35)

we find

\[ R_g \equiv \frac{g^+ - g^-}{g^+ + g^-} = \frac{a(b_{KM} - a_{KM})}{1 + ab} \] (36)

At the fixed above numerical values of the parameters we obtain

\[ (R_g)_{p^2} = 0.030 \frac{Im\tilde{c}_5}{Re\tilde{c}_5}(1 - 14.9 \frac{Im\tilde{c}_7}{Imc_5}) = -(4 \pm 0.75) \cdot 10^{-5} \sin \delta \] (37)

This numerical result is obtained for \( \Omega_{\eta,\eta'} = 0.3 \). For zero magnitude of this parameter, a value would be \((-4.6 \pm 0.86)10^{-5}\).
4 The role of $p^4$ and other corrections

The corrections to the result obtained in the conventional chiral theory up to leading $p^2$ approximation are of two kinds. The first kind corrections are connected with a necessity to take into account the observed enlargement of $S$-wave $I = 0$ $\pi\pi$ amplitude having no explanation in conventional chiral theory. The corrections of the second kind are the $p^4$ corrections. As it was argued in [7], both kinds corrections can be properly estimated in the framework of special linear $U(3)_L \otimes U(3)_R$ $\sigma$ model with broken chiral symmetry. The above mentioned enlargement of $S$ wave in this model is originated by mixing between the $\bar{q}q$ and $(G^a_{\mu\nu})^2$ states. In such a model

$$U = \hat{\sigma} + i\hat{\pi}$$

where $\hat{\sigma}$ is $3 \times 3$ matrix of scalar partners of the mesons of pseudoscalar nonet. The relations between diquark combinations and spinless fields are as given by eqs.(10), (11), but without the terms proportional to $\Lambda^{-2}$. Such contributions in $\sigma$ model appear from an expansion of the intermediate scalar mesons propagators. The parameter $\Lambda^2$ acquires a sense of difference $m_{a_0(980)}^2 - m_\pi^2$. The strength of mixing between the isosinglet $\sigma$ meson and corresponding gluonic state is characterised by the parameter $\xi$.

If the $p^2$ approximation gives

$$(k)_{p^2} = 1.495 \cdot 10^{-4}, \quad (g)_{p^2} = -0.172$$

instead of

$$(k)_{exp} = 1.72 \cdot 10^{-4}, \quad (g)_{exp} = -0.2154 \pm 0.0035,$$

the corrected values of these CP-even parameters of $K^+ \to \pi^+\pi^+\pi^-$ amplitude practically coincide with the experimental ones [7]:

$$(k)_{(p^2+p^4; \xi=-0.225)} = 1.72 \cdot 10^{-4}, \quad (g)_{(p^2+p^4; \xi=-0.225)} = -0.21.$$  

(40)

The meaning of the parameter $\xi$ is explained in [7]. The expressions for the corrected $\pi\pi \to \pi\pi$ amplitudes are presented in [8].

Calculating the CP-even imaginary part of the $K^\pm \to \pi^\pm\pi^\pm\pi^\mp$ amplitude originated by two-pion intermediate states, we obtain

$$a(p^2 + p^4; \xi = -0.225) = 0.16265$$

(41)
\[ b(p^2 + p^4; \xi = -0.225) = 0.762 \]  

An estimate of the parameter \( a \) can be obtained also without any calculations using the circumstance that at \( \sqrt{s} = \sqrt{s_0} \), the only significant phase shift is \( \delta_0 \). The rest phase shifts are very small: \( |\delta_2(s_0)| < 1.8^\circ \) and \( \delta_1(s_0) < 0.3^\circ \) [9]. Then, according to eq.\((33)\), \( a \approx \tan \delta_0(s_0) \), or \( a = 0.13 \pm 0.05 \), if \( \delta_0(s_0) = (7.50 \pm 2.85)^\circ \) [10] and \( a = 0.148 \pm 0.018 \), if \( \delta_0(s_0) = (8.4 \pm 1.0)^\circ \) [11]. These results coincide inside the error bars with the result \((41)\). The corrected magnitude of \( R_g \) is

\[ (R_g)_{(p^2 + p^4; \xi = -0.225)} = 0.039 \frac{Im \tilde{c}_5}{Re \tilde{c}_5} \left( 1 - 11.95 \frac{Im \tilde{c}_7}{Im \tilde{c}_5} \right) \sin \delta = -(4.9 \pm 0.9) \times 10^{-5} \sin \delta. \]

This result is by 22\% larger in absolute magnitude than that calculated in the leading approximation. Therefore, we come to conclusion that the corrections to the result obtained in the framework of conventional chiral theory to the leading approximation are not negligible (20-30 \%), but not so large, as it was declared in [12].

5 Conclusion

If the arguments against the correctness of eq.\((23)\) will not be found, an expected value of \( R_g \) in the Standard Model is not larger in absolute magnitude than \( 6 \cdot 10^{-5} \sin \delta \).

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