Fabrication of the impedance-matched parametric amplifier and the study of the gain profile

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Abstract—We designed and fabricated an impedance matched Josephson junction parametric amplifier (JPA) working in the flux-pump mode for the broadband amplification of microwave signals. We developed a very simple fabrication method suitable for a small lab. We studied the phase response, as well as the gain, as a function of frequency and pump power at various pump frequencies. The phase response can be explained with the behavior of the non-linear Duffing oscillator. The observed decrease of the resonance frequency as the pump power increases, as well as the emergency of an unstable bifurcation zone, are the characteristic non-linear behavior of the Duffing oscillator. The gain profile in the stable zone can be explained with a model adapted from the theoretical model for the two-dimensional gain profile of an impedance-matched current-pumped JPA. With an appropriate environmental impedance, the theoretical model captures the features and morphology of the gain profile, such as the emergence of a gain hot zone with two branches around the resonance frequency of the JPA. Based on the gain profile, we propose that the best working zone is the merging point of the two branches of the gain hot zone before the emergence of the bifurcation zone, which gives a large bandwidth and a good gain. Over 17dB gain with a bandwidth larger than 300MHz was observed. The impedance matched JPA is used in our superconducting quantum computers for improving the fast readout fidelity of the transmon qubits.

Index Terms—Josephson junction parametric amplifier, quantum computer, nanofabrication, gain profile

I. INTRODUCTION

Quantum computer holds the promise for solving many problems that can not be solved efficiently with conventional computers[1], [2], [3], [4]. It can provide an efficient way to simulate quantum mechanical systems[1]. Quantum algorithms also show great advantage over conventional algorithms[2], [3], [4]. The proof of concept demonstration of quantum computation and quantum simulation on the small scale quantum computers has been realized[5], [6], [7], [8], [9], [10], [11]. The promising prospect of building a large quantum computer of practical use draws the attention of many institutions. Many countries join the race towards a practical quantum computer. Within the last several years, progress in the study of superconducting quantum computers is impressive. For example, Google built a 72 qubits quantum computer, IBM built quantum processors up to 50 qubits and put several quantum processors on the cloud. Intel also built quantum processors with more than 50 qubits. A key issue in the development of a practical multi-qubits quantum computer is the high fidelity readout of the signals from multiple qubits. A high distinguish-ability between the qubits is crucial for various operations on the quantum computer, such as the quantum algorithm, error correction and quantum feedback experiments[12], [13], [14], [15], [16]. The microwave signal carrying information of the qubits is very weak, amplification is thus required. The commercial microwave HEMT amplifier adds lots of noise photons during amplification, the equivalent noise temperature is in several Kelvins[17], [18], thus degrading the distinguish-ability between quantum states. To improve the readout fidelity, an amplifier with lowest possible added noise is favored. Josephson junction parametric amplifier is the known system that can provide such low noise level[18], [19], [20]. To read multi-qubits simultaneously, a large bandwidth for the JPA is also required. The simplest Josephson junction parametric amplifier is a non-linear resonator composed of a superconducting quantum interference device (SQUID) loop shunted by a big capacitor (a large capacitance can give rise to a large Kerr non-linearity of the resonator, which is the foundation for a large parametric amplification effect in the cavity-QED[21]). However, the bandwidth of such a resonant structure is quite small, thus making it a narrow band JPA. A way to increase the bandwidth of this type of JPA is to incorporate a gradual impedance transformer into the microwave feedline, thus lowering the quality factor of the resonant structure without introducing too much reflections[22]. The gain profile of such an impedance matched JPA differs from that of a narrow band JPA. The characterization and understanding of this gain profile can help us choose the best working zone for the qubit readout task.

In this work, we first designed and fabricated an impedance matched Josephson junction parametric amplifier (JPA) working in the flux-pump mode. The fabrication process we developed simplifies the fabrication of the impedance matching part, thus suitable for a small lab without etching facilities for the dielectrics. We studied the phase and amplitude of the microwave scattering parameter, as well as the gain, as a function of frequency and pump power, at various pump frequencies. The phase response reflects the behavior of the non-linear Duffing oscillator. The observed decrease of the resonance frequency as a function of the pump power, as well as the emergency of an unstable bifurcation zone (which also has an impact on the amplitude response), are characteristic for a non-linear Duffing oscillator. The observed amplitude...
or gain response has two zones. When the pump power is below some critical level, the system is in the stable zone. When the pump power is larger than a critical value thus the system entering the bifurcation zone, the amplitude or gain response is unstable and a sudden change usually occurs. We adapted a theoretical model for the two-dimensional (2d) gain profile of an impedance-matched current-pumped JPA to explain the observed 2d gain profile in the stable zone. With an appropriate environmental impedance, the theoretical model captures the features and morphology of the data, such as the emergence of a gain hot zone with two branches around the resonance frequency of the JPA. Based on the gain profile, we propose that the best working zone is the merging point of the gain hot zone before the emergence of the bifurcation zone, which gives large bandwidth and good gain. Over 17 dB gain with a bandwidth larger than 300 MHz was observed. The impedance matched JPA is used in our superconducting quantum computers for improving the fast readout fidelity of the transmon qubits.

II. DESIGN AND DEVICE FABRICATION

The impedance matched JPA was designed and fabricated in our lab. The impedance transformer is a compact Klopfenstein filter which gradually changes the impedance of the transmission line from 50 Ohm to a smaller value (about 15 Ohm, limited mainly by the dielectric constant of the crossover capacitors and the size of the JPA), thus increasing the bandwidth as well as the saturation power of the JPA to the desired values, since both the bandwidth and the saturation power are inversely proportional to the impedance seen by the nonlinear resonator composed of the Josephson junction and the shunt capacitor [22]. The large shunt capacitor is designed to be around 4 pF, the inductance of the josephson junction is designed to be around 70 pH, the corresponding room temperature junction resistance is around 62 Ohm. The design of the impedance transformer and the circuit illustration can be found in Fig. 1.

The device fabrication involves many processes. Firstly, a thin Al film 100 nm thick was deposited on a cleaned silicon wafer (with a 1 micron thick SiO$_2$ layer). Then, a layer of photoresist S1805 was spin-coated and baked. Then, lithography was performed by a laser writer (Heidelberg DWL66) to form the etch mask. Then, wet etch was performed to define the co-planar wave-guide (CPW) by removing some Al from the designed area. Then, photoresist LOR5A and S1805 were spin-coated and lithography by a laser writer was performed again to define the regions for the dielectrics. Then, a layer of 200 nm-thick CaF$_2$ was deposited and liftoff was performed to form the dielectric layers. Then, LOR5A and S1805 were spin-coated again and lithography by a laser writer was performed again to define the regions for the top electrodes of the shunt capacitor and the crossovers on the impedance transformer. Then, a layer of Al was deposited again to form the top electrodes. There is no etching for the dielectrics in our fabrication steps. This simplified fabrication process saves the steps of etching dielectrics. The Josephson junction region is defined by the electron beam lithography on a MAA/PMMA ebeam resist mask. The Josephson junctions were deposited in a Plassys machine with the double angle evaporation method [23].

![Fig. 1. (a) The design of the impedance transformer (the lavender-colored winding line represents the transmission line, the magenta-colored structures represent the small crossovers). The density of the small crossovers modified the impedance along the line. The denser the crossovers are, the smaller the impedance of the segment of transmission line will be. (b) The working circuit of the JPA: The circle with two crosses representing the SQUID loop with two Josephson junctions, shunted by a capacitor. The line with $I_{bias}$ passing through is the flux bias line, which tunes the flux penetrating the SQUID loop. $\omega_p$ represents the pump tone applied on the flux bias line. $\omega_s$ represents the signal to be amplified, $\omega_i$ represents the idle tone generated during the amplification.

III. MODULATION CURVES

The phase angle of the microwave reflected from the JPA can be modulated by the flux penetrating the SQUID loop. Fig. 2 shows the phase of the reflected microwave as a function of the microwave frequency and flux bias. The observed phase modulation can be modelled by this formula: $\text{angle} = \arctan \left( \frac{2Z_0 I_c (J_j(\phi, I_c))}{\phi_0 \phi (1 - \omega^2 L_j(\phi, I_c) C_j)^2 - Z_0^2} \right)$, with $L_j(\phi, I_c) = \frac{\phi_0}{2 \pi I_c (\cos(\pi \phi / \phi_0))}$, where $\phi_0$ is the flux quantum (about $2.07 \times 10^{-15}$ Wb), $Z_0 \approx 50$ Ohm is the characteristic impedance of the transmission line, $I_c$ is the zero-field critical current, $\phi$ is the flux penetrating the SQUID loop, $C$ is the shunt capacitance [24]. The zero-field critical
current $I_c$ and shunt capacitance can be obtained by fitting theory to the observed phase modulation curves of the JPA. The extracted $I_c$ is about 4.5 µA, the extracted $C_p$ is about 3.5 pF. These values are close to the designed values.

The extracted theory to the observed phase modulation curves of the JPA.

The bifurcation zone, this is because the cavity photon number usually occurs in the region not far from the onset of the bifurcation zone, this is where we should focus to search for the optimal stable gain hot zone.

Let us focus on the stable gain hot zone in the stable region, we can see that the gain hot zone generally tracks the curve representing the pump power dependence of the resonance frequency. Thus the gain hot zone for a higher pump power locates at a smaller pump power, because the resonance frequency increases at a lower pump power. The gain hot zone usually has two branches, sometimes they merge into a broad plateau. A theoretical model for the 2d gain profile of an impedance-matched current-pumped JPA can be adapted to describe the observed 2d gain profile of our JPA. The gain profile of an impedance matched current-pumped JPA can be calculated based on the differential equation of the circuit system or the Hamiltonian of the circuit system.

IV. TWO DIMENSIONAL GAIN PROFILE

After studying the phase modulation of the JPA devices by the external flux bias, we turned on the flux pump and characterized the 2d gain profile (gain as a function of frequency and pump power, at various pump frequencies and flux biases) in detail (Fig. 3(a)). The example of the measured 2d gain profile can be found in Fig. 3(c).

This 2d gain profile actually shows a phase diagram of the JPA, if we look at the phase information. Take figure Fig. 3(b) as an example, at low pump power, the boundary indicated by the black curve representing the pump-power dependence of the resonance frequency of the JPA. We can see that, the resonance frequency shifts to lower values as the pump power increases, this phenomenon can be explained by the non-linearity of the JPA oscillator. This pump power dependence is a characteristic behavior of a non-linear Duffing oscillator. Above a threshold pump power, the phase changes suddenly, this phenomenon could also be explained by the physics of the non-linear Duffing oscillator, see Fig. 3(d).

Above some critical value of the pump power, the Duffing oscillator enters a bifurcation zone, the resonance amplitude will become multi-valued in this zone (see Fig. 3(d)), leading to the instability of the system. The bifurcation zone also manifests itself as a sudden change of the gain in the corresponding amplitude or gain plot (Fig. 3(c)). Comparing the amplitude or gain plot (Fig. 3(c)) with the phase plot, we can see that, there are some gain hot zones in the different regions defined by the phase response. Further experiments show that the gain hot zone lies in the region between the black curve and the bifurcation zone is stable (useful for readout purpose of qubit states). The stable gain hot zone usually occurs in the region not far from the onset of the bifurcation zone, this is because the cavity photon number changes abruptly at the bifurcation point, thus having the maximum parametric conversion effect. Therefore, the region between the black curve and the bifurcation zone is where we should focus to search for the optimal stable gain hot zone.

Comparing it with the differential equation of the flux-pump case:

$$\frac{d^2 \delta_s(t)}{dt^2} + \frac{d \delta_s(t)}{dt} + \Omega_p^2 (1 - \epsilon)(1 - \frac{\epsilon}{\gamma}) \sin(2 \Omega_p \ell - 2 \psi) \delta_s(t) = A_s(t)$$

where

$$\Omega_p^2 = \frac{2 \delta_c^2}{C_p \phi_0^2} \cos \left( \frac{\phi}{\phi_0} \right), \eta = \tan \left( \frac{\phi}{\phi_0} \right) \Delta \Phi$$

These two equations are basically identical, with $\Omega_p$ = $2 \Omega_0$, $\Omega_p^2 (1 - \epsilon) = \frac{2 \delta_c^2}{C_p \phi_0^2} \cos \left( \frac{\phi}{\phi_0} \right)$ and $\epsilon = 1 - \frac{\epsilon}{\gamma}$.

Therefore, the differential equation for the flux-pump case is similar to that of the current-pump case. We just need to do some substitution, replacing some terms in the parametric oscillating term with their counterparts in the flux pump mode. In other words, there is some one to one correspondence between the terms of the differential equation in the current-pump mode and the flux-pump mode. Therefore, we can calculate the gain profile for the flux-pump mode with the current-pump mode. The calculated gain profile is the same as that of the current-pump mode, except the substitutions of some variables. A key observation is that, we can directly apply the gain formula to the flux-pump mode, by simply converting the pump strength in the original current-pump JPA to the flux pump power in the flux-pump JPA.

The resulted gain is

$$g = \left[ 1 - \kappa_1 \chi_{11} \right]^2$$

with $\kappa_1$ related to the real part of the environmental admittance and $\chi_{11}$ is an element of the susceptibility matrix, details can be found in Roy’s paper

With the critical current and shunt capacitance extracted from the phase modulation curves and an appropriate environmental impedance as the input, the theoretical model gives a 2d gain profile qualitatively matches with the data. The calculated gain profile can be found in Fig. 3(c). The model captures the features and morphology of the data, such as the emergence of a gain hot zone with two branches around the resonance frequency of the JPA. The branching behavior in the gain hot zone depends on the environmental impedance.

For a non-optimal environmental impedance, the gain profile has two branches, related to the normal modes of the JPA and the environment. For the optimal environmental impedance,
the two branches merge, giving rise to a broad plateau. The gain hot zone usually gets sculptured/cut by the emergence of the bifurcation zone, thus has a distorted shape extended into the bifurcation zone.

Fig. 3. (a) The cube shows the phase response as a function of pump power, frequency and flux bias. (b) A slice of the phase response, the black curve indicates the decrease of the resonance frequency as a function of pump power, the green dashed line indicates the beginning of the bifurcation zone (c) The corresponding amplitude response of (b), the entering of the bifurcation zone also manifested itself here, the abrupt change when pump power is about 5 dBm indicates the beginning of the bifurcation zone (d) Comparison between a normal oscillator and a Duffing oscillator, the bifurcation zone shown in (b) and (c) corresponds to the Duffing oscillator behavior.

Fig. 4. Calculated 2d gain profile: The calculated gain as a function of pump strength and frequency qualitatively matches with the experiments.

Based on the 2d gain profile, we propose that the best working zone is near the merging point of the two branches of the gain hot zone before the emergence of the bifurcation zone, which gives a large bandwidth and good gain. In our JPA, >17dB gain with a bandwidth larger than 300MHz was observed, see Fig. 5. The large bandwidth with a decent gain is ideal for the readout task of the multi-qubits quantum processors.

Fig. 5. Bandwidth of the JPA: This figure shows the gain as a function of the frequency. We can see that over 17dB gain can be reached in a frequency range of around 300MHz.

V. SATURATION POWER AND NOISE TEMPERATURE

Besides the gain profile, we also characterized the saturation power and noise temperature of our JPA. The gain decreases at higher signal power. The saturation power can be characterized by the 1dB compression point, the value of the signal power where the gain decreases by 1dB. The 1dB compression power in the middle of our best gain hot zones is about -110dBm. The noise temperature measures how many noise photons are added during the amplification process. With gain information, it can be easily estimated with the method in Yamamoto’s paper [27]. The formula for estimating the noise temperature is: 

$$T_{\text{HEMT noise}} > \left( \frac{hf_s}{2k_b} + T_{\text{JPA noise}} \right) \times 10^{G/10}/10^{L/10},$$

where $H_{\text{HEMT noise}}$ (about 3K) is the noise temperature of the HEMT amplifier, $f_s$ (about 6.5GHz) is the signal frequency (thus $\frac{hf_s}{2k_b}$ is the thermal noise, $k_b$ is the Boltzmann constant, $h$ is the Planck’s constant), the G is the JPA gain in dBs, L is the loss (in dBs) caused by the cable between the sample and the JPA. In our setup, for gain larger than 15dB, the quantum limit of the noise temperature will be reached. More details about the noise temperature of the JPA made in our lab can be found in a previous paper [28].

VI. QUBIT READOUT

Our impedance matched JPA is used in our superconducting quantum computers for improving the fast readout fidelity of the transmon qubits. Fig. 6 shows the improvement in the readout fidelity with our JPA. Fig. 6a shows the fidelity without JPA in action, Fig. 6b shows the fidelity with our JPA turned on, we can see that our JPA improved the readout fidelity a lot. In the experiments on our superconducting computers, up to 6 qubits can be read simultaneously with our JPA [29]. The large bandwidth with decent gain and saturation power is ideal for the readout task of our multi-qubits quantum processors.

VII. CONCLUSIONS

In summary, we designed and fabricated an impedance matched Josephson junction parametric amplifier (JPA) working in the flux-pump mode. The simple fabrication process
is suitable for a small lab without the etching facilities for dielectrics. We studied the 2d gain profile in detail. We adapted a theoretical model for the 2d gain profile of an impedance-matched current-pumped JPA to describe the observed 2d gain profile of our JPA. With an appropriate environmental impedance, the theoretical model captures the features of the data, such as the emergence of a gain hot zone with two branches around the resonance frequency of the JPA. Nonlinear behaviour was also observed, such as the shift of the resonance frequency at a higher pump power (Duffing shift), as well as the emergence of the bifurcation zones (unstable regions with a sudden drop of the gain) at a higher pump power. Based on the gain profile, we propose that the best working zone is the branching point of the gain hot zone in the stable zone right before the emergence of the bifurcation zone, which gives a large bandwidth and good gain. Over 17dB gain with a bandwidth larger than 300MHz was observed. The impedance matched JPA is used in our superconducting quantum computers for improving the fast readout fidelity of the transmon qubits.

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