1. Introduction

Novel robotic applications have demanded lighter robots that can be driven using small amounts of energy, for example robotic booms in the aerospace industry, where lightweight manipulators with high performance requirements (high speed operation, better accuracy, high payload/weight ratio) are required (Wang & Gao, 2003). Unfortunately, the flexibility of these robots leads to oscillatory behaviour at the tip of the link, making precise pointing or tip positioning a daunting task that requires complex closed-loop control. In order to address control objectives, such as tip position accuracy and suppression of residual vibration, many control techniques have been applied to flexible robots (see, for instance, the survey (Benosman & Vey, 2004)). There are two main problems that complicate the control design for flexible manipulators viz: (i) the high order of the system, (ii) the no minimum phase dynamics that exists between the tip position and the input (torque applied at the joint). In addition, recently, geometric nonlinearities have been considered in the flexible elements. This chapter gives an overview to the modelling and control of flexible manipulators and focuses in the implementation of the main control techniques for single link flexible manipulators, which is the most studied case in the literature.

2. State of the art

Recently, some reviews in flexible robotics have been published. They divide the previous work attending to some short of classification: control schemes (Benosman & Vey, 2004), modelling (Dwivedy & Eberhard, 2006), overview of main researches (Feliu, 2006), etc. They are usually comprehensive enumerations of the different approaches and/or techniques used in the diverse fields involving flexible manipulators. However, this section intends to give a chronological overview of how flexible manipulators have evolved since visionaries such as Prof. Mark J. Balas or Prof. Wayne J. Book sowed the seeds of this challenging field of robotics. Moreover, some attention is given to main contributions attending to the impact of the work and the goodness of the results.

In the early 70's the necessity of building lighter manipulators able to perform mechanical tasks arises as a part of the USA Space Research. The abusive transportation costs of a gram
of material into orbit and the reduced room and energy available inside an spacecraft cause the imperative need for reducing weight and size as far as possible in any device aboard. Unfortunately, as the manipulator reduces weight, it reduces also accuracy in its manoeuvres due to the appearance of structural flexibility (and hence, vibrations) of the device.

The interest of NASA in creating these manipulators for use in spatial applications motivated the investment for the research of flexible robots and its associated new control problems. In 1974, Prof. Wayne J. Book provided the first known work dealing with this topic explicitly in his Ph. D. Thesis (Book, 1974) entitled as “Modeling, design and control of flexible manipulators arms” and supervised by Prof. Daniel E. Whitney, who was a professor at MIT Mechanical Engineering Department. In the same department than Prof. Book, the very same year Dr. Maizza-Neto also studied the control of flexible manipulator arms but from a modal analysis approach (Maizza-Neto, 1974). Fruits of their joint labour, the first work published in a journal in the field of flexible robotics appeared in 1975, dealing with the feedback control of a two-link-two-joints flexible robot (Book et al., 1975). After this milestone, Dr. Maizza-Neto quitted from study of elastic arms but Prof. Book continued with its theoretical analysis of flexible manipulators, e.g. taking frequency domain and space-state approaches (Book & Majette, 1983), until he finally came up with a recursive, lagrangian, assumed modes formulation for modelling a flexible arm (Book, 1984) that incorporates the approach taken by Denavit and Hartenberg (Denavit & Hartenberg, 1955), to describe in a efficient, complete and straightforward way the kinematics and dynamics of elastic manipulators. Due to the generality and simplicity of the technique applied, this work has become one of the most cited and well-known studies in flexible robotics. This structural flexibility was also intensively studied in satellites and other large spacecraft structures (again spatial purposes and NASA behind the scenes) which generally exhibit low structural damping in the materials used and lack of other forms of damping. A special mention deserves Prof. Mark J. Balas, whose generic studies on the control of flexible structures, mainly between 1978 and 1982, e.g. (Balas, 1978) and (Balas, 1982), established some key concepts such as the influence of high nonmodelled dynamics in the system controllability and performance, which is known as "spillover". In addition, the numerical/analytical examples included in his work dealt with controlling and modelling the elasticity of a pinned or cantilevered Euler-Bernoulli beam with a single actuator and a sensor, which is the typical configuration for a one degree of freedom flexible robot as we will discuss in later sections.

After these promising origins, the theoretical challenge of controlling a flexible arm (while still very open) turned into the technological challenge of building a real platform in which testing those control techniques. And there it was, the first known robot exhibiting notorious flexibility to be controlled was built by Dr. Eric Schmitz (Cannon & Schmitz, 1984) under the supervision of Prof. Robert H. Cannon Jr., founder of the Aerospace Robotics Lab and Professor Emeritus at Stanford University. A single-link flexible manipulator was precisely positioned by sensing its tip position while it was actuated on the other end of the link. In this work appeared another essential concept in flexible robots: a flexible robot it is a noncolocated system and thus of nonminimum phase nature. This work is the most referenced ever in the field of flexible robotics and it is considered unanimously as the breakthrough in this topic.
Point-to-point motion of elastic manipulators had been studied with remarkable success taking a number of different approaches, but it was not until 1989 that the tracking control problem of the end-point of a flexible robot was properly addressed. Prof. Siciliano collaborated with Prof. Alessandro De Luca to tackle the problem from a mixed open-loop control approach (De Luca & Siciliano, 1989) in the line proposed two years before by Prof. Bayo (Bayo, 1987). Also in 1989, another very important concept called passivity was used for the first time in this field. Prof. David Wang finished his Ph.D Thesis (Wang, 1989) under the advisement of Prof. Mathukumalli Vidyasagar, studying this passivity property of flexible links when an appropriate output of the system was chosen (Wang & Vidyasagar, 1991).

In (Book, 1993), a review on the elastic behaviour of manipulators was meticulously performed. In his conclusions, Prof. Book remarks the exponential growth in the number of publications and also the possibility of corroborating simulation results with experiments, what turns a flexible arm into "one test case for the evaluation of control and dynamics algorithms". And so it was. It is shown in (Benosman & Vey 2004) a summary of the main control theory contributions to flexible manipulators, such as PD-PID, feedforward, adaptive, intelligent, robust, strain feedback, energy-based, wave-based and among others.

3. Modelling of flexible manipulators

One of the most studied problems in flexible robotics is its dynamic modelling (Dwivedy & Eberhard, 2006). Differently to conventional rigid robots, the elastic behaviour of flexible robots makes the mathematical deduction of the models, which govern the real physical behaviour, quite difficult. One of the most important characteristic of the flexible manipulator models is that the low vibration modes have more influence in the system dynamics than the high ones, which allows us to use more simple controllers, with less computational costs and control efforts. Nevertheless, this high order dynamics, which is not considered directly in the controller designed, may give rise to the appearance of bad system behaviours, and sometimes, under specific conditions, instabilities. This problem is usually denoted in the literature as spillover (Balas, 1978).

The flexibility in robotics can appear in the joints (manipulators with flexible joints) or in the links (widely known as flexible link manipulators or simply flexible manipulators). The joint flexibility is due to the twisting of the elements that connect the joint and the link. This twisting appears, for instance, in reduction gears when very fast manoeuvres are involved, and produces changes in the joint angles. The link flexibility is due to its deflection when fast manoeuvres or heavy payloads are involved. From a control point of view, the flexibility link problem is quite more challenging than the joint flexibility.

3.1 Single-link flexible manipulators

Single-link flexible manipulators consist of a rigid part, also denominated as actuator, which produces the spatial movement of the structure; and by a flexible part, which presents distributed elasticity along the whole structure. Fig. 1 shows the parametric representation of a single-link flexible manipulator, which is composed of the following: (a) a motor and a reduction gear of $1:n$, reduction ratio at the base, with total inertia (rotor and hub) $I_b$, dynamic friction coefficient $\nu$ and Coulomb friction torque $\Gamma_f$; (b) a flexible link with uniform linear mass density $\rho$, uniform bending stiffness $EI$ and length $L$; and (c) a payload
of mass $M_P$ and rotational inertia $J_p$. Furthermore, the applied torque is $\Gamma_m$, $\Gamma_{coup}$ denotes the coupling torque between the motor and the link, $\theta_m$ is the joint angle and $\theta_l$ represents the tip angle.

The dynamic behaviour of the system is governed by a differential partial equation which presents infinite vibration modes. The objective is to obtain a simplified model (finite number of vibration modes) of the differential equation that characterizes the dynamics of the link. A number of models can be found in the literatures obtained from methods such as the truncation of the infinite dimensional model (Cannon & Schmitz, 1984); the discretization of the link based on finite elements (Bayo, 1987); or directly from concentrated mass models (Feliu et al., 1992).

The hypothesis of negligible gravity effect and horizontal motion are considered in the deduction of the model equations. In addition, the magnitudes seen from the motor side of the gear will be written with an upper hat, while the magnitudes seen from the link side will be denoted by standard letters. With this notation and these hypotheses, the momentum balance at the output side of the gear is given by the following expression

$$\dot{\Gamma}_m(t) = K_m u(t) = J_0 \ddot{\theta}_m(t) + \nu \dot{\theta}_m(t) + \dot{\Gamma}_f(t) + \dot{\Gamma}_{coup}(t),$$

where $K_m$ is the motor constant that models the electric part of the motor (using a current servoamplifier) and $u$ is the motor input voltage. This equation can be represented in a block diagram as shown in Fig. 2, where $G_c(s)$ and $G_l(s)$ are the transfer functions from $\theta_m$ to $\Gamma_{coup}$ and $\theta_l$ respectively.

![Diagram](image-url)  

Fig. 2. Block diagram of the single-link flexible manipulator system.
The link model is deduced by considering small deformations, which allows us to use a linear beam model to obtain the dynamic equations. Based on this hypothesis, in this chapter we use models derived from the truncation of infinite dimensional model obtained from concentrated mass model and assumed mode method.

3.1.1 Concentrated mass models
In the concentrated mass models, the link mass is concentrated in several points along the whole structure (see Fig. 3), where the inertia produced by the point mass rotations is rejected. An example of this technique can be found in (Feliu et al., 1992). Fig. 3 shows the scheme of the concentrated mass model. The lumped masses are represented by $m_i$, with $1 \leq i \leq n$; the distance between two consecutive masses $i-1$ and $i$ is $l_i$, $l_1$ is the distance between the motor shaft and the first mass; finally, the distance between the mass $m_i$ and the motor shaft is $L_i$. $F_n$ represents the applied external force at the tip of the link. $\Gamma_n$ is the torque applied in the same location. Assuming small deflections and considering that the stiffness $EI$ is constant through each interval of the beam the deflection is given by a third order polynomial:

$$y_i(x) = u_{i,0} + u_{i,1}(x-L_{i-1}) + u_{i,2}(x-L_{i-1})^2 + u_{i,3}(x-L_{i-1})^3,$$  \hspace{1cm} (2)

where $u_{ij}$ are the different coefficients for each interval, and $L_0=0$.

![Fig. 3. Concentrated masses model of a single-link flexible manipulator.](image)

The dynamic model of the flexible link is obtained from some geometric and dynamic equations as follows (see (Feliu et al., 1992) for more details):

$$M \frac{d^2 \Theta}{dt^2} = EI \left[ A \Theta + B \dot{\Theta} \right] + PT_n + QF_n,$$  \hspace{1cm} (3)

where $M=\text{diag}(m_1,m_2,...,m_n)$ represents the masses matrix of the system and $\Theta=[\theta_1,\theta_2,...,\theta_n]^T$. On the other hand, $A \in \mathbb{R}^{n \times n}$ is a constant matrix, $B=-A[1,1,...,1]^T$, $P \in \mathbb{R}^{n \times 1}$ and $Q \in \mathbb{R}^{n \times 1}$ are constant column vectors, which only depend on the link geometry.
Finally, the coupling torque affecting the motor dynamics (see Equation (1)) is defined as 
\[ \Gamma_{\text{coup}} = -2EIu_{1,2} \]. Notice that the coupling torque has the same magnitude and different sign to the joint torque \( 2EIu_{1,2} \). This torque can be expressed as a linear function:

\[ \Gamma_{\text{coup}} = C\Theta - c_{n+1}\theta_m - c_{n+2}\Gamma_n, \tag{4} \]

where \( C = (c_1, c_2, \ldots, c_n) \), \( c_i, 1 \leq i \leq n+2 \), are parameters which do not depend on the concentrated masses along the structure and \( c_{n+1} = -C[1,1,\ldots,1]' \).

For example, the transfer functions \( G_c(s) \) and \( G_t(s) \) for only one point mass located in the tip \( (m_t) \) are as follows:

\[ G_c(s) = \frac{(3EI/L)}{\left(s^2 + \omega_i^2\right)} \quad \text{and} \quad G_t(s) = \left(\frac{\omega_i^2}{s^2 + \omega_i^2}\right), \tag{5} \]

in which \( \omega_i = \sqrt{3EI/Lm_i} \). This model can be used for flexible robots with a high payload/weight ratio.

### 3.1.2 Assumed mode method

The dynamic behaviour of an Euler-Bernoulli beam is governed by the following PDE (see, for example, (Meirovitch, 1996))

\[ Elw''''(x,t) + \rho w''(x,t) = f(x,t), \tag{6} \]

where \( f(x,t) \) is a distributed external force, \( w \) is the elastic deflection measured from the undeformed link. Then, from modal analysis of Equation (6), which considers \( w(x,t) \) as

\[ w(x,t) = \sum_{i=1}^{n} \phi_i(x)\eta_i(t), \tag{7} \]

in which \( \phi_i(x) \) are the eigenfunctions and \( \eta_i(t) \) are the generalized coordinates, the system model can be obtained (see (Belleza et al., 1990) for more details).

### 3.2 Multi-link flexible manipulators

For these types of manipulators truncated models are also used. Some examples are: (De Luca & Siciliano, 1991) for planar manipulators, (Pedersen & Pedersen, 1998) for 3 degree of freedom manipulators and (Schwertassek et al., 1999), in which the election of shape functions is discussed.

The deflections are calculated from the following expression:

\[ w_i(x,t) = \Phi_i^T(x) \cdot \Delta_i(t), 1 \leq i \leq n_L, \tag{8} \]

(see for example (Benosman & Vey 2004)), in which \( i \) means the number of the link, \( n_L \) the number of links, \( \Phi_i(x) \) is a column vector with the shape functions of the link (for each considered mode), \( \Delta_i(t) = (\eta_{i1}, \ldots, \eta_{iN})^T \) is a column vector that represents the dynamics of each mode, in which \( N \) is the number of modes considered.
The dynamics equations of the overall system from the Lagrange method are described as follows:
\[
\frac{d}{dt} \frac{\partial L}{\partial \dot{q}_i} - \frac{\partial L}{\partial q_i} + \frac{\partial D_k}{\partial \dot{q}_k} = u_k',
\]
(9)

where \( L \) is the lagrangian defined as \( L=E-P \), being \( E \) the total kinetic energy of the manipulator and \( P \) its potential energy. This expression is similar to the used in rigid robots, but in this case the potential energy is the sum of the gravity and the elastic deformation terms. The term \( D_k \) is the dissipation function of Rayleigh, which allows us to include dissipative terms like frictions, and \( u_k \) is the generalized force applied in \( q_k \). From Equation (9) the robot dynamics can be deduced (see for example Chapter 1 of (Wang & Gao, 2003))
\[
I(Q)\ddot{Q} + b(Q,\dot{Q}) + K(Q)\cdot\dot{Q} + D\cdot\dot{Q} + g(Q) = E\cdot\Gamma,
\]
(10)

were \( Q=(\theta_1,\ldots,\theta_{nL},|\Delta_1|,\ldots,|\Delta_{nL}|)^T \) is the vector of generalized coordinates that includes the first block of joint angles \( \theta_i \) (rigid part of the model) and the elastic deflections of the links \( \Delta_i \), \( \Gamma \) is the vector of motor torques of the joints, \( I \) is the inertias matrix of the links and the payload of the robot, which is positive definite symmetric, \( b \) is the vector that represents the spin and Coriolis forces \( (b=\beta(Q,\dot{Q})\cdot\dot{Q}) \), \( K \) is stiffness matrix, \( D \) is the damping matrix, \( g \) is the gravity vector and \( \Gamma \) is the connection matrix between the joints and the mechanism. Equation (10) presents a similar structure to the dynamics of a rigid robot with the differences of: (i) the elasticity term \( (K(Q)\cdot\dot{Q}) \) and (ii) the vector of generalized coordinates is extended by vectors that include the link flexibility.

### 3.3 Flexible joints

In this sort of systems, differently to the flexible link robots, in which the flexibility was found in the whole structure from the hub with the actuator to the tip position, the flexibility appears as a consequence of a twist in those elements which connect the actuators with the links, and this effect has always rotational nature. Therefore, the reduction gears used to connect the actuators with the links can experiment this effect when they are subject to very fast movements. Such a joint flexibility can be modelled as a linear spring (Spong, 1987) or as a torsion spring (Yuan & Lin, 1990). Surveys devoted to this kind of robots are (Bridges et al., 1995) and (Ozgoli & Taghirad, 2006), in which a comparison between the most used methods in controlling this kind of systems is carried out. Nevertheless, this problem in flexible joints sometimes appears combined with flexible link manipulators. Examples of this problem are studied in (Yang & Donath, 1988) and (Yuan & Lin, 1990).

### 4. Control techniques

This section summarizes the main control techniques for flexible manipulators, which are classified into position and force control.
4.1 Position Control
The benefits and interests jointly with advantages and disadvantages of the most relevant contributions referent to open and closed control schemes for position control of flexible manipulators have been included in the following subsections:

4.1.1 Command generation
A great number of research works have proposed command generation techniques, which can be primarily classified into pre-computed and real-time. An example of pre-computed is (Aspinwall, 1980), where a Fourier expansion was proposed to generate a trajectory that reduces the peaks of the frequency spectrum at discrete points. Another pre-computed alternative uses multi-switch bang-bang functions that produce a time-optimal motion. However, this alternative requires the accurate selection of switching times which depends on the dynamic model of the system (Onsay & Akay, 1991). The main problem of pre-computed command profiles is that the vibration reduction is not guaranteed if a change in the trajectory is produced.
The most used reference command generation is based on filtering the desired trajectory in real time by using an input shaper (IS). An IS is a particular case of a finite impulse response filter that obtains the command reference by convolving the desired trajectory with a sequence of impulses (filter coefficients) ((Smith, 1958) and (Singer & Seering, 1990)). This control is widely extended in the industry and there are many different applications of IS such as spacecraft field (Tuttle & Seering, 1997), cranes and structures like cranes (see applications and performance comparisons in (Huay et al., 2008)) or nanopositioners (Jordan, 2002). One of the main problems of IS design is to deal with system uncertainties. The approaches to solve this main problem can be classified into robust (see the survey of (Vaughan et al., 2008)), learning ((Park & Chang, 2001) and (Park et al., 2006)) or adaptive input shaping (Bodson, 1998).
IS technique has also been combined with joint position control ((Feliu & Rattan 1999) and (Mohamed et al., 2005)), which guarantees trajectory tracking of the joint angle reference and makes the controlled system robust to joint frictions. The main advantages of this control scheme are the simplicity of the control design, since an accurate knowledge of the system is not necessary, and the robustness to unmodelled dynamics (spillover) and changes in the systems parameters (by using the aforementioned robust, adaptive and learning approaches). However, these control schemes are not robust to external disturbance, which has motivated closed loop controllers to be used in active vibration damping.

4.1.2 Classic control techniques
In this chapter, the term “classic control techniques” for flexible manipulators refers to control laws derived from the classic control theory, such as proportional, derivative and/or integral action, or phase-lag controllers. Thus, classic control techniques, like Proportional-Derivative (PD) control (De Luca & Siciliano, 1993) or Lead-Lag control (Feliu et al., 1993) among others, have been proposed in order to control the joint and tip position (angle) of a lightweight flexible manipulator. The main advantage of these techniques is the simplicity of its design, which makes this control very attractive from an industrial point of view. However, in situations of changes in the system, its performance is worse (slow time...
response, worse accuracy in the control task...) than other control techniques such as robust, adaptive or learning approaches among others. Nevertheless, they can be used in combination with more modern and robust techniques (e.g. passive and robust control theories) to obtain a controller more adequate and versatile to do a determined control task, as a consequence of its easy implementation. Classic control techniques are more convenient when minimum phase systems are used (see discussions of (Wang et al., 1989)), which can be obtained by choosing an appropriate output (Korolov & Chen, 1989), taking them into account in the design of the control law or by using some analysis techniques to make the system robust to any or several of these uncertainties. The output/input linearization added to Linear Quadratic Regulator (LQR) was applied in (Singh & Schy, 1985). Nevertheless, LQR regulators are avoided to be applied in practical setups because of the well-known spillover problems. The Linear Quadratic Gaussian (LQG) was investigated in (Cannon & Schmitz, 1984) and (Balas, 1982). However, these LQG regulators do not guarantee general stability margins (Banavar & Dominic, 1995). Nonlinear robust control method has been proposed by using singular perturbation approach (Morita et al., 1997). To design robust controllers, Lyapunov’s second method is widely used (Gutman, 1999). Nevertheless the design is not that simple, because the main difficulty is the non trivial finding of a Lyapunov function for control design. Some examples in using this technique to control the end-effector of a flexible manipulator are (Theodore & Ghosal, 2003) and (Jiang, 2004).

Another robust control technique which has been used by many researchers is the optimal H∞ control, which is derived from the L2-gain analysis (Yim et al., 2006). Applications of this technique to control of flexible manipulators can be found in (Moser, 1993), (Landau et al., 1996), (Wang et al., 2002) and (Lizarraga & Etxebarria, 2003) among others. Major research effort has been devoted to the development of the robust control based on Sliding Mode Control. This control is based on a nonlinear control law, which alters the dynamics of the system to be controlled by applying a high frequency switching control. One of the relevant characteristics of this sort of controllers is the augmented state feedback, which is not a continuous function of time. The goal of these controllers is to catch up with the designed sliding surface, which insures asymptotic stability. Some relevant publications in flexible robots are the following: (Choi et al., 1995), (Moallem et al., 1998), (Chen & Hsu, 2001) and (Thomas & Mija, 2008).

4.1.4 Adaptive control
Adaptive control arises as a solution for systems in which some of their parameters are unknown or change in time (Åström & Wittenmark, 1995). The answer to such a problem consists in developing a control system capable of monitoring his behaviour and adjusting
the controller parameters in order to increase the working accuracy. Thus, adaptive control is a combination of both control theory, which solves the problem of obtaining a desired system response to a given system input, and system identification theory, which deals with the problem of unknown parameters.

For obvious reasons, robotics has been a platinum client of adaptive control since first robot was foreseen. Manipulators are general purpose mechanisms designed to perform arbitrary tasks with arbitrary movements. That broad definition leaves the door open for changes in the system, some of which noticeably modify the dynamics of the system, e.g. payload changes (Bai et al., 1998). Let us use a simple classification for adaptive control techniques, which groups them in (Åström & Wittenmark, 1995):

• Direct Adaptive Control, also called Control with Implicit Identification (CII): the system parameters are not identified. Instead, the controller parameters are adjusted directly depending on the behaviour of the system. CII reduces the computational complexity and has a good performance in experimental applications. This reduction is mainly due to the controller parameters are adjusted only when an accurate estimation of the uncertainties is obtained, which requires, in addition to aforementioned accuracy, a fast estimation.

• Indirect Adaptive Control, also called Control with Explicit Identification (CEI): the system parameters estimations are obtained on line and the controller parameters are adjusted or updated depending on such estimations. CEI presents good performance but they are not extendedly implemented in practical applications due to their complexity, high computational costs and insufficient control performance at start-up of the controllers.

First works on adaptive control applied to flexible robots were carried out in second half of 80’s (Siciliano et al., 1986), (Rovner & Cannon, 1987) and (Koivo & Lee, 1989), but its study has been constant along the time up to date, with application to real projects such as the Canadian SRMS (Damaren, 1996). Works based on the direct adaptive control approach can be found: (Siciliano et al., 1986), (Christoforou & Damaren 2000) and (Damaren, 1996); and on the indirect adaptive control idea: (Rovner & Cannon, 1987) and (Feliu et al., 1990). In this last paper a camera was used as a sensorial system to close the control loop and track the tip position of the flexible robot. In other later work (Feliu et al., 1999), an accelerometer was used to carry out with the same objective, but presented some inaccuracies due to the inclusion of the actuator and its strong non-linearities (Coulomb friction) in the estimation process. Recently, new indirect approaches have appeared due to improvements in sensorial system (Ramos & Feliu, 2008) or in estimation methods (Becedas et al., 2009), which reduce substantially the estimation time without reducing its accuracy. In both last works strain gauges located in the coupling between the flexible link and the actuator were used to estimate the tip position of the flexible robot.

4.1.5 Intelligent control

Ideally, an autonomous system must have the ability of learning what to do when there are changes in the plant or in the environment, ability that conventional control systems totally lack of. Intelligent control provides some techniques to obtain this learning and to apply it appropriately to achieve a good system performance. Learning control (as known in its
begins (some surveys of this period are (Tsypkin, 1968) and (Fu, 1970)), and its popularity and applications have increased continuously since, being applied in almost all spheres of science and technology. Within these techniques, we can highlight machine learning, fuzzy logic and neural networks.

Due to the property of adaptability, inherent to any learning process, all of these schemes have been widely applied to control of robotic manipulators (see e.g. (Ge et al., 1998)), which are systems subjected to substantial and habitual changes in its dynamics (as commented before). In flexible robots, because of the undesired vibration in the structure due to elasticity, this ability becomes even more interesting. For instance, neural networks can be trained for attaining good responses without having an accurate model or any model at all. The drawbacks are: the need for being trained might take a considerable amount of time at the preparation stage; and their inherent nonlinear nature makes this systems quite demanding computationally. On the other hand, fuzzy logic is an empirical rules method that uses human experience in the control law. Again, model is not important to fuzzy logic as much as these rules implemented in the controller, which rely mainly on the experience of the designer when dealing with a particular system. This means that the controller can take into account not only numbers but also human knowledge. However, the performance of the controller depends strongly on the rules introduced, hence needing to take special care in the design-preparation stage, and the oversight of a certain conduct might lead to an unexpected behaviour. Some examples of these approaches are described in (Su & Khorasani, 2001), (Tian et al., 2004) and (Talebi et al., 2009) using neural networks; (Moudgal et al., 1995), (Green, & Sasiadek, 2002) and (Renno, 2007) using fuzzy logic; or (Caswar & Unbehauen, 2002) and (Subudhi & Morris, 2009) presenting hybrid neuro-fuzzy proposals.

4.2 Force control

Manipulator robots are designed to help to humans in their daily work, carrying out repetitive, precise or dangerous tasks. These tasks can be grouped into two categories: unconstrained tasks, in which the manipulator moves freely, and constrained task, in which the manipulator interacts with the environment, e.g. cutting, assembly, gripping, polishing or drilling.

Typically, the control techniques used for unconstrained tasks are focused to the motion control of the manipulator, in particular, so that the end-effector of the manipulator follows a planned trajectory. On the other hand, the control techniques used for constrained tasks can be grouped into two categories: indirect force control and direct force control (Siciliano & Villani, 1999). In the first case, the contact force control is achieved via motion control, without feeding back the contact force. In the second case, the contact force control is achieved thanks to a force feedback control scheme. In the indirect force control the position error is related to the contact force through a mechanical stiffness or impedance of adjustable parameters. Two control strategies which belong to this category are: compliance (or stiffness) control and impedance control. The direct force control can be used when a force sensor is available and therefore, the force measurements are considered in a closed loop control law. A control strategy belonging to this category is the hybrid position/force control, which performs a position control along the unconstrained task directions and a force control along the constrained task directions. Other strategy used in the direct force control is the inner/outer motion/force control, in which an outer closed loop force control works on an inner closed loop motion control.
There are also other advanced force controls that can work in combination with the previous techniques mentioned, e.g. adaptive, robust or intelligent control. A wide overview of the all above force control strategies can be found in the following works: (Whitney, 1987), (Zeng & Hemami, 1997) and (Siciliano & Villani, 1999). All these force control strategies are commonly used in rigid industrial manipulators but this kind of robots has some problems in interaction tasks because their high weight and inertia and their lack of touch senses in the structure. This becomes complicated any interaction task with any kind of surface because rigid robots do not absorb a great amount of energy in the impact, being any interaction between rigid robots and objects or humans quite dangerous.

The force control in flexible robots arises to solve these problems in interaction tasks in which the rigid robots are not appropriated. A comparative study between rigid and flexible robots performing constrained tasks in contact with a deformable environment is carried out in (Latornell et al., 1998). In these cases, a carefully analysis of the contact forces between the manipulator and the environment must be done. A literature survey of contact dynamics modelling is shown in (Gilardi & Sharf, 2002).

Some robotic applications demand manipulators with elastic links, like robotic arms mounted on other vehicles such a wheelchairs for handicapped people; minimally invasive surgery carried out with thin flexible instruments, and manipulation of fragile objects with elastic robotic fingers among others. The use of deformable flexible robotic fingers improves the limited capabilities of robotic rigid fingers, as is shown in survey (Shimoga, 1996). A review of robotic grasping and contact, for rigid and flexible fingers, can be also found in (Bicchi & Kumar, 2000).

Flexible robots are able to absorb a great amount of energy in the impact with any kind of surface, principally, those quite rigid, which can damage the robot, and those tender, like human parts, which can be damaged easily in an impact with any rigid object. Nevertheless, despite these favourable characteristics, an important aspect must be considered when a flexible robot is used: the appearance of vibrations because of the high structural flexibility. Thus, a greater control effort is required to deal with structural vibrations, which also requires more complex designs, because of the more complex dynamics models, to achieve a good control of these robots. Some of the published works on force control for flexible robots subject, by using different techniques, are, as e.g., (Chiou & Shahinpoor, 1988), (Yoshikawa et al., 1996), (Yamano et al., 2004) and (Palejiya & Tanner, 2006), where a hybrid position/force control was performed; in (Chapnik, et al., 1993) an open-loop control system using 2 frequency-domain techniques was designed; in (Matsumo & Kasai, 1998) and (Morita et al., 2001) an optimal control was used in experiments; in (Becedas et al., 2008) a force control based on a flatness technique was proposed; in (Tian et al., 2004) and (Shi & Trabia, 2005) neural networks and fuzzy logic techniques were respectively used; in (Siciliano & Villani, 2000) and (Vossoughi & Karimzadeh, 2006), the singular perturbation method was used to control, in both, a two degree-of-freedom planar flexible link manipulator; and finally in (Garcia et al., 2003) a force control is carried out for a robot of three degree-of-freedom.

Unlike the works before mentioned control, which only analyze the constrained motion of the robot, there are models and control laws designed to properly work on the force control, for free and constrained manipulator motions. The pre-impact (free motion) and post-impact (constrained motion) were analyzed in (Payo et al., 2009), where a modified PID controller was proposed to work properly for unconstrained and constrained tasks. The
authors only used measurements of the bending moment at the root of the arm in a closed loop control law. This same force control technique for flexible robots was also used in (Becedas et al., 2008) to design a flexible finger gripper, but in this case the implemented controller was a GPI controller that presents the characteristics described in Section 0

5. Design and implementation of the main control techniques for single-link flexible manipulators

Control of single link flexible manipulators is the most studied case in the literature (85% of the published works related to this field (Feliu, 2006)), but even nowadays, new control approaches are still being applied to this problem. Therefore, the examples presented in this section implement some recent control approaches of this kind of flexible manipulators.

5.1 Experimental platforms

5.1.1 Single link flexible manipulator with one significant vibration mode

In this case, the flexible arm is driven by a Harmonic Drive mini servo DC motor RH-8D-6006-E050A-SP(N), supported by a three-legged metallic structure, which has a gear with a reduction ratio of 1:50. The arm is made of a very lightweight carbon fibre rod and supports a load (several times the weight of the arm) at the tip. This load slides over an air table, which provides a friction-free tip planar motion. The load is a disc mass that can freely spin (thanks to a bearing) without producing a torque at the tip. The sensor system is integrated by an encoder embedded in the motor and a couple of strain gauges placed on to both sides of the root of the arm to measure the torque. The physical characteristics of the platform are specified in Table 1. Equation (5) is used for modelling the link of this flexible manipulator, in which the value of \( m_1 \) is equal to \( M_p \). For a better understanding of the setup, the following references can be consulted (Payo et al., 2009) and (Becedas et al., 2009). Fig. 4a shows a picture of the experimental platform.

5.1.2 Single link flexible manipulator with three significant vibration modes

The setup consists of a DC motor with a reduction gear 1:50 (HFUC-32-50-20H); a slender arm made of aluminium flexible beam with rectangular section, which is attached to the motor hub in such way that it rotates only in the horizontal plane, so that the effect of gravity can be ignored; and a mass at the end of the arm. In addition, two sensors are used: an encoder is mounted at the joint of the manipulator to measure the motor angle, and a strain-gauge bridge, placed at the base of the beam to measure the coupling torque. The physical characteristics of the system are shown in Table 1. The flexible arm is approximated by a truncated model of Equation (7) with the first three vibration modes to carry out the simulations (Bellezza et al., 1990). The natural frequencies of the one end clamped link model obtained from this approximate model, almost exactly reproduce the real frequencies of the system, which where determined experimentally. More information about this experimental setup can be found in (Feliu et al., 2006). Fig. 4b shows a picture of the experimental platform.
Fig. 4. Experimental platforms: (a) Single link flexible arm with one significant vibration mode; (b) Single link flexible arm with three significant vibration modes.

| PARAMETER | DESCRIPTION | PLATFORM 1 VALUE | PLATFORM 2 VALUE |
|-----------|-------------|------------------|------------------|
| EI        | Stiffness   | 0.37 Nm²         | 2.40 Nm²         |
| l         | Length      | 0.7 m            | 1.26 m           |
| d         | Diameter    | 2.80·10⁻³ m      | -                |
| h         | Width       | -                | 5·10⁻² m         |
| b         | Thickness   | -                | 2·10⁻³ m         |
| MP        | Mass in the tip | 0.03 kg   | 0-0.30 kg       |
| JP        | Inertia in the tip | -          | 0-5.88·10⁻⁴ kgm² |

Data of the motor-gear set

| PARAMETER | DESCRIPTION | PLATFORM 1 VALUE | PLATFORM 2 VALUE |
|-----------|-------------|------------------|------------------|
| J₀        | Inertia     | 6.87·10⁻⁵ kgm²   | 3.16·10⁻⁴ kgm²   |
| v         | Viscous friction | 1.04·10⁻⁴ kgm²/s | 1.39·10⁻³ kgm²/s |
| n₀        | Reduction ratio of the motor gear | 50          | 50               |
| Km        | Motor constant | 2.10·10⁻¹ Nm/V  | 4.74·10⁻¹ Nm/V   |
| uₘₐₓ      | Saturation voltage of the servo amplifier | ± 10 V    | ± 3.3 V          |

Table 1. Physical characteristics of the utilized experimental platforms.

5.2 Actuator position control.

Control scheme shown in Fig. 5 is used to position the joint angle. This controller makes the system less sensible to unknown bounded disturbances ($\Gamma$ in Equation (1)) and minimizes the effects of joint frictions (see, for instance (Felius et al., 1993)). Thus, the joint angle can be controlled without considering the link dynamics by using a PD, PID or a Generalized Proportional Integral (GPI) controller, generically denoted as $C_{a(s)}$. In addition, this controller, as we will show below, can be combined with other control techniques, such as command generation, passivity based control, adaptive control or force control.
5.2 Actuator position control.

Control scheme shown in Fig. 5 is used to position the joint angle. This controller makes the system less sensible to unknown bounded disturbances ($\Gamma_{\text{coup}}$ in Equation (1)) and minimizes the effects of joint frictions (see, for instance (Felius et al., 1993)). Thus, the joint angle can be controlled without considering the link dynamics by using a PD, PID or a Generalized Proportional Integral (GPI) controller, generically denoted as $Ca(s)$. In addition, this controller, as we will show below, can be combined with other control techniques, such as command generation, passivity based control, adaptive control or force control.

5.3 Command generation

The implementation of the IS technique as an example of command generation is described herein. It is usually accompanied by the feedback controller like the one shows in Fig. 5. Thus, the general control scheme showed in Fig. 6 is used, which has previously utilized with success for example in (Felius & Rattan, 1999) or (Mohamed et al., 2005). The actuator controller is decided to be a PD with the following control law:

$$u(t) = \Gamma_{\text{coup}}(t)/n_{m}K_{m} + K_{p}(\theta'_{m}(t) - \theta_{m}(t)) - K_{v}\dot{\theta}_{m}(t),$$

where $\Gamma_{\text{coup}}/n_{m}K_{m}$ (decoupling term) makes the design of the PD constants ($K_{p}$, $K_{v}$) independent of the link dynamics. Thus, if the tuning of the parameters of the PD controller ($K_{p}$, $K_{v}$) is carried out to achieve a critically damped second-order system, the dynamics of the inner control loop ($G_{m}(s)$) can be approximated by

$$\theta_{m}(s) \simeq G_{m}(s)(\theta'_{m}(s))/\left(1 + \alpha s\right)^{2},$$

where $\alpha$ is the constant time of $G_{m}(s)$. From Equations (11) and (12) the values of $K_{p}$ and $K_{v}$ are obtained as

$$K_{p} = J_{0}n_{m}/K_{m}\alpha^{2}, \quad K_{v} = n_{r}(2J_{0} - \nu\alpha)/K_{m}\alpha.$$  

As it was commented in Section 0, the IS ($C(s)$) can be a robust, learning or adaptive input shaper. In this section, a robust input shaper (RIS) for each vibration mode obtained by the so-called derivative method (Vaughan et al., 2008) is implemented. This multi-mode RIS is obtained as follows:

$$C(s) = \prod_{i=1}^{N}C_{i}(s) = \prod_{i=1}^{N}\left((1 + z_{i}e^{-d_{i}s})/(1 + z_{i})\right)^{p_{i}},$$

in which

$$z_{i} = e^{\omega_{i}d_{i}/\sqrt{1 - \xi_{i}^{2}}}, \quad d_{i} = \pi/(\omega_{i}\sqrt{1 - \xi_{i}^{2}}),$$

$p_{i}$ is a positive integer used to increase the robustness of each $C_{i}(s)$ and $\omega_{i}$ and $\xi_{i}$ denote the natural frequencies and damping ratio of each considered vibration mode.
This example illustrates the design for the experimental platform of Fig. 4b of the multimode RIS of Equation (14) for a payload range $M_p \in [0.02, 0.12]$kg and $J_p \in [0.0, 5.88 \times 10^{-4}]$kgm$^2$. Each of one $C_i(s)$ is designed for the centre of three first frequency intervals, which has the next values: $\omega_1=5.16$, $\omega_2=35.34$ and $\omega_3=100.59$rad/s. If the damping is neglected ($\xi_1$, $\xi_2$ and $\xi_3$ equal to zero), the parameters of $C(s)$ are $z_1=z_2=z_3=1$, $d_1=0.61$, $d_2=0.089$ and $d_3=0.031$s. In addition, if the maximum residual vibration is kept under 5% for all vibration modes, the value of each $p_i$ is: $p_1=3$, $p_2=2$ and $p_3=2$. The dynamics of $G_m(s)$ is designed for $\alpha=0.01$. Then from Table 1 and Equations (12) and (13), the values of $K_p$ and $K_v$ were 350.9 and 6.9. This value of $\alpha$ makes the transfer function $G_m(s)$ robust to Coulomb friction and does not saturate the DC motor if the motor angle reference is ramp a reference with slope and final value equal to 2 and 0.2rad, respectively. Fig. 7 shows the experimental results for the multimode RIS design above. The residual vibration for the nominal payload ($M_p=0.07$ kg and $J_p=3 \times 10^{-4}$ kgm$^2$) is approximately zero whereas one of the payload limits ($M_p=0.12$ kg and $J_p=5.88 \times 10^{-4}$ kgm$^2$) has a residual vibration less than 5%.

5.4 Classic control techniques

This subsection implements the new passivity methodology expounded in (Pereira et al., 2007) in the experimental platform of Fig. 4b, whose general control scheme is shown in Fig. 8. This control uses two control loops. The first one consists of the actuator control shown in Section 5.2, which allows us to employ an integral action or a high proportional gain. Thus, the system is robust to joint frictions. The outer controller is based on the passivity property of $\Gamma_{\text{coup}}(s)/s \theta_c(s)$, which is independent of the link and payload parameters. Thus, if $sC(s)G_m(s)$ is passive, the controller system is stable. The used outer controller is as following:

$$C(s) = K_c \left( \lambda s + 1 \right)/s,$$  \hspace{1cm} (16)

in which the parameter $K_c$ imparts damping to the controlled system and $\lambda$ must be chosen together with $G_m(s)$ to guarantee the stability. For example, if $G_m(s)$ is equal to Equation (12),
the necessary and sufficient stability condition is $0<\alpha/2<\lambda$ (see Pereira et al., 2007) for more details.

![Diagram](image)

Fig. 8. General control scheme proposed in (Pereira, et al., 2007).

![Graph](image)

Fig. 9. Tip angle $\theta$: (—). Simulation with $M_P = 0$; (●) Experiment with $M_P = 0$; (—) Simulation with $M_P = 0.3$; (●) Experiment with $M_P = 0.3$; (—) the reference.

Taking into account the maximum motor torque (i.e., $u_{sat}$ in Table 1), the constant time of the inner loop is set to be $\alpha = 0.02$. Then, the parameters of the PD controller are obtained: $K_p = 83.72$ and $K_v = 3.35$. Next, the nominal condition is taken for $M_P = 0$ and $C'(s)$ is designed ($\lambda = 0.05$ and $K_c = 1.8$) in such a way that the poles corresponding to the first vibration mode are placed at $-3.8$. Notice that $\lambda$ fulfils the condition $0<\alpha/2<\lambda$ and is independent of the payload. Once the parameters of the control scheme are set, we carry out simulations and experiments for $M_P = 0$ and $M_P = 0.3$ kg (approximately the weight of the beam) and $J_p \approx 0$ kgm²). Figure 9 shows the tip angle, in which can be seen that the response for the two mass values without changing the control parameters is acceptable for both simulations and experiments. Notice that the experimental tip position response is estimated by a fully observer since it is not measured directly, which is not used for control purpose. Finally, a steady state error in the vicinity of 1% compared with the reference command arises for in the tip and motor angle for experimental results. This error is due to Coulomb friction and can be minimized using a PD with higher gains in the actuator control.

5.5 Adaptive control

Adaptive controller described in this section is based on the flatness characteristic of a flexible robotic system (see Becedas, et al., 2009). The control system is based on two
nested loops with two controllers designed for both motor and flexible link dynamics. The controller is called Generalized Proportional Integral (GPI). This presents robustness with respect to constant perturbations and does not require computation of derivatives of the system output signals. Therefore, the output signals are directly feedbacked in the control loops, then the usual delays produced by the computation of derivatives and the high computational costs that require the use of observers do not appear. In addition, due to the fact that one of the most changeable parameter in robotics is the payload, a fast algebraic continuous time estimator (see (Fliess & Sira-Ramírez, 2003)) is designed to on-line estimate the natural frequency of vibration in real time. The estimator calculates the real value of the natural frequency when the payload changes and updates the gains of the controllers. Therefore, this control scheme is an Indirect Adaptive Control. A scheme of the adaptive control system is depicted in Fig. 10, where \( \omega_n \) represents the estimation of the vibration natural frequency of the flexible arm, used to update the system controller parameters.

\[
\dot{\theta} = \frac{1}{\omega_n^2} \left[ (\alpha_2 s^2 + \alpha_1 s + \alpha_0) \left( \frac{\omega_n^2}{s(s + \alpha_0)} \right) (\theta_m - \theta) \right],
\]

(17)

where \( \omega_n \) is now an auxiliary ideal open loop control for the outer loop, \( \theta_r \) represents the reference trajectory for the payload, and \( \gamma_i \), \( i = 0, 1, 2 \), are the outer loop controller gains, which are updated each time that the estimator estimates the real values of the system natural frequency.

The system dynamics is obtained by the simplification to one vibration mode of the concentrated mass model (see Section 0). Adding the decoupling term defined in Section 5.2 to the voltage control signal \( u_c \) allows us to decouple both motor and link dynamics. Thus, the design of the controllers, one for each dynamics, is widely simplified. By using the flatness characteristic of the system, the two nested GPI controllers are designed as follows: Outer control law (\( C_d(s) \)):

\[
(\theta_m - \theta) = \left[ (\gamma_r s + \gamma_0)/(s + \gamma_2) \right](\theta_r - \theta) ,
\]

(18)

Inner control law (\( C_s(s) \)):

\[
(u - u_c) = \left[ (\alpha_2 s^2 + \alpha_1 s + \alpha_0)/(s(s + \alpha_0)) \right](\theta_m - \theta) ,
\]
where $u^c_t$ represents the ideal open loop control for the inner loop, $\theta^*_{mr}$ represents the reference trajectory for the motor angle, and $\alpha_i$, $i=0, 1, 2, 3$ are the inner loop controller gains. The algebraic estimator for the natural frequency is given by the following equation:

$$
a^2 \begin{cases} 
\text{arbitrary, } t \in [0, \Delta) \\
\lim_{t \to \infty} n_i(t)/d_i(t), \quad t \in [\Delta, +\infty)
\end{cases}
$$

(19)

where

\[
\begin{align*}
n_i(t) &= t^2 \theta_i(t) + z_1 \\
d_i(t) &= z \\
\dot{z}_1 &= z_2 - 4t \theta_i(t) \\
\dot{z}_2 &= 2 \theta_i(t) \\
\dot{z}_3 &= t^2 (\theta_n(t) - \theta_i(t))
\end{align*}
\]

(20)

Then, this control technique is implemented in the experimental platform of Figure 4a. The value of the tip angle is approximated by $\theta = \theta_n L/(3EI) \Gamma_{coup}$, where $\theta_n$ and $\Gamma_{coup}$ are obtained from the encoder and strain gauges measurements respectively. The desired reference trajectory to be tracked by the flexible robotic system is a two seconds Bezier eighth order trajectory with 1rad of amplitude. The control system starts working with an arbitrary computation of the tip mass, which is represented by a natural frequency $\omega_n = 9rad/s$, very different from the real value $\omega_n = 15.2 rad/s$. In a small time interval $\Delta = 0.5s$ (dashed line), the algebraic fast estimator estimates the real value $\omega_n$, and updates the inner ($u^c_t$) and outer (\theta^*_{mr}, $\alpha_2$, $\alpha_1$ and $\alpha_0$) loop controllers (see details in (Becedas et al., 2009)). After the updating the control system perfectly tracks the desired trajectory (see Fig. 11).

![Fig. 11. Trajectory tracking of the reference trajectory with the GPI adaptive controller.](image)

5.6 Force control

The special feature of the force control for flexible robots described here (Payo et al., 2009) is that the control law is designed to control the force exerted by a robot on the environment, which surrounds for both free and constrained motion tasks. The flexible robot of one
degree of freedom used is described in Section 0. The system dynamics of the arm is obtained by the simplification to one vibration mode of the concentrated mass model (see Section 0, specifically Equation (5)). The tracking of the desired force is obtained by using a feedback control loop of the torque at the root of the arm. This control law is based on a modified PID controller (I-PD controller (Ogata, 1998)), and it is demonstrated the effectiveness of the proposed controller for both free and constrained motion tasks. The sensor system used in this control law is constituted by a sole sensor very lightweight (two strain gauges placed at the root of the arm) to measure the torque, neither the contact force sensor nor the angular position sensor of the motor are used in the control method, unlike others methods described in Section 4.2. The controlled system presents robust stability conditions to changes in the tip mass, viscous friction and environment elasticity. It is also important to mention the good performance of the system response in spite of the nonlinear Coulomb friction term of the motor which was considered to be a perturbation. Fig. 12 shows the control scheme used to implement this force control technique, where the control law is given by the following equation:

\[ u = a_0 \int_0^t \left( \Gamma'_{coup} - \Gamma_{coup} \right) dt - a_1 \Gamma_{coup} - a_2 \frac{d\Gamma_{coup}}{dt}, \]  

where \( a_0, a_1 \) and \( a_2 \) are the design parameters of the I-PD and \( \Gamma'_{coup} \) is the reference signal. The environment impedance is represented by the well known spring-dashpot model (Latornell et al., 1998) and (Erickson et al., 2003):

\[ F_n = k_e x_e + b_e \dot{x}_e, \]

where \( k_e, b_e \) are the stiffness and damping characteristics of the environment and \( x_e \) is the local deformation of the environment. The plant dynamics for free and constrained motion tasks are given respectively by the following equations:

\[ \Gamma_{coup}(s) = \frac{sK_e / J_0n_s}{s(s^2 + sv / J_0 + c / J_0n_s^2) + a_0^2(s + v / J_0)}, \]

\[ \Gamma_{coup}(s) = \frac{(K_e / J_0n_s)(s^2 + sb_e / m + k_e / m)}{(s^2 + sv / J_0)(s^2 + sb_e / m + k_e / m + a_0^2) + c / J_0n_s^2(s^2 + sb_e / m + k_e / m)}. \]

Fig. 12. Force control scheme.
The proposed strategy needs an online collision detection mechanism in order to switch between a command trajectory for free motion torque and a contact torque reference for the case of constrained motion. The collision was detected when the torque exceeded a threshold ($\gamma$) that depends on the amplitude of the reference signal, the Coulomb friction of the motor ($\Gamma_C$) and the noise in the measured signal ($\gamma_3$) according to the following equation (a detailed explication of this can be found in (Payo, et al., 2009)):

$$\gamma = \gamma_1 \Gamma_{coup} + \gamma_2 \Gamma_f + \gamma_3,$$

where $\gamma_1$ and $\gamma_2$ are normalized maximum deviations of the measured signal.

Fig. 13 and Fig. 14 show the results obtained in two experimental tests where the robot carried out both free and constrained motion tasks. The controlled torque is displayed before and after collision. A small value for the torque in free motion was used to prevent possible damages to the arm or to the object at the moment of collision. The chosen torque in these tests for free motion was equal to 0.07Nm. The constrained environment used in these tests was a rigid object with high impedance. Once the collision was detected, the Control law changed the reference value of the torque for constrained motion depending on the particular task carried out. For example, the first experiment matches a case in which the force exerted on the object was increased; and in the second experiment the force exerted on the object was decreased to avoid possible damages on the contact surfaces (case of fragile objects, for instance).
6. Future of flexible manipulators

After the huge amount of literature published on this topic during the last thirty years, flexible robotics is a deeply studied field of autonomous systems. Even complete books have been already devoted to the subject (Tokhi & Azad, 2008) and (Wang & Gao, 2003). Still, new control techniques can be studied due to simplicity of the physical platform, but, as discussed in (Benosman & Vey, 2004), most of the topics regarding modelling or controllability have been satisfactorily addressed in the previous literature.

However, some topics are still open and leave a considerable margin for improvement. Some manipulators with a small rigid arm attached to a large flexible base (called macro-micro manipulators, see (George & Book, 2003) for instance) have been developed for precision tasks, but the technological issue of building flexible robots with similar features to those of actual industrial robots has not been completely solved. While there exists a real prototype of a 3 dof flexible robot (Somolinos et al., 2002) achieving three dimensional positioning of the tip, a mechanical wrist still needs to be coupled for giving the manipulator the ability of reaching a particular position with a particular orientation. On the control side, the search for the perfect controller is still open and, probably, never to be closed. All the robust, adaptive, intelligent techniques have their limitations and drawbacks. Many new controllers have been proposed but there is no standard measurement of the performance and, hence, no objective classification can be performed. The creation of a family of ‘benchmark’ problems would provide some objectivity to the results analysis.

One of the most potential aspects of flexible robots is their recently evolution in the position and force control. Such a combination provides of touch sensibility to the robotic system. Thus, the robot does not only have accuracy in the different positioning tasks, but also has the possibility of detecting whatever interaction with the environment that surrounds it. This characteristic allows the system to detect any collision with an object or surface, and to limit the actuating force in order not to damage the robotic arm nor the impact object or surface. Applications in this sense can be developed for robots involved in grasping, polishing, surface and shape recognition, and many other tasks (Becedas et al., 2008).

Nonlinear behaviour of flexible manipulators has been poorly accounted for in literature. A few works dealing with modelling of geometrical nonlinearities due to large displacements in the links have been published in (Payo et al., 2005) and (Lee, 2005) and a solution for achieving precise point-to-point motion of these systems has also been reported in (O’Connor et al., 2009). But these works are based on single link manipulators, and the multiple link case still has to be addressed. If we think of applications in which the robot is interacting with humans, these large displacements structures increase the safety of the subjects because the system is able to both absorb a great amount of energy in the impact and control effectively the contact force almost instantaneously (hybrid position/force controls). Thus, the development of human-machine interfaces becomes a potential application field for this kind of systems (Zinn, 2004).

Another interesting and not very studied approach to the flexibility of manipulators consists of taking advantage of it for specific purposes. Flexibility is considered as a potential benefit instead of a disadvantage, showing some examples with margin of improvement in assembling (Whitney, 1982), collision (García et al., 2003), sensors (Ueno et al., 1998) or mobile robots (Kitagawa et al., 2002).
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7. References

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