Robust PD+ Control Algorithm for Satellite Attitude Tracking for Dynamic Targets

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In this paper, a PD + controller combined with the sliding mode surface is proposed to improve system convergence rate and efficiency on control torque for satellite attitude tracking control. A sliding mode surface with the maneuver stage is constructed by the Euler axis; hence, the constant angular velocity is achieved. The PD + controller with the variable structure and auxiliary term is constructed to track the desired sliding mode surface. The Lyapunov function in PD control analysis is modified to simplify stability analysis. Model uncertainty, external disturbance, control torque constraint, and angular velocity constraint are taken into consideration, and a novel method to reduce the overshoot of angular velocity is proposed. The performance and superiority of the proposed method are demonstrated by numerical simulation results.

1. Introduction

In the rapid development of aerospace science technology and a series of practical processes of science and technology works, researchers have paid extensive attention and worked on the attitude tracking control of satellites. Among these works, PD control algorithm is the industry’s most mature and widely used method. The PD control law for satellite attitude control was proposed by Wie [1, 2] when solving the attitude tracking control issue. At the same time, Wie summarized some related Lyapunov functions and gave the general stability proof methods. And recently, for the satellite attitude control problem, James [3] proposed a passive PD control law for satellite attitude stabilization control, and it was pointed out that the attitude system governed by the PD controller is energy consumed. The directional cosine matrix was treated as an equivalent proportional term in this paper. Generally, standard PD control algorithm is a mature method, and since it is a very mature method, researchers did not focus on this field for almost a decade.

The reason why the PD control method can be deeply developed and widely applied for a long time is its obvious advantages, such as uncomplicated structure, clear physical meaning, and strong robustness. However, it also has the following limitations: (1) the initial control torque is too large, and the control torque drops drastically with the decrease of system state, so the utilization efficiency of the control law is low. (2) The convergence rate is slow, and the rapid declination in attitude angular velocity causes the drop of the convergence rate of the attitude quaternion. (3) The modeled system is not fully utilized, and in reality, we can partially determine the system’s rotational inertia parameters.

For the aforementioned problems, Jovan et al. [4] designed a robust attitude tracking control law. In order to improve the convergence rate of the standard PD controller, Verbin et al. [5–7] applied the backstepping method to design an angular velocity curve with fast convergence characteristics, and the control law designed by Verbin enables the actual state to track the designed reference trajectory. In [5, 6], the problem of satellite control torque limitation is discussed, while in [5, 7], the satellite angular velocity limitation is discussed, but [5–7] lacked the discussion about the uncertainty of system’s moment of inertia. Cao et al. [8] considered the uncertainty of rotational inertia, added external disturbance moments, and designed a
The conversion matrix from the error coordinate system to the inertial system is expressed as follows:

$$\mathbf{R}(\mathbf{q}_e) = \left(\frac{q_{e0}^2 - q_{e3}^2}{4} q_{e4}, q_{e3} q_{e4}, q_{e0} q_{e4} + q_{e3}^2\right).$$  \(3\)

Error angular velocity $\omega_e$ is defined as

$$\omega_e = \omega - \mathbf{R}(\mathbf{q}_e) \omega_d,$$  \(4\)

and the cross-product manipulator $\mathbf{r}^x$ for three-dimensional vector $\mathbf{r}$ is defined as follows:

$$\mathbf{r}^x = \begin{bmatrix} 0 & -r_3 & r_2 \\ r_3 & 0 & -r_1 \\ -r_2 & r_1 & 0 \end{bmatrix},$$  \(5\)

and this manipulator is defined for vector cross-production, i.e., for three-dimensional vectors $\mathbf{a}$ and $\mathbf{b}$, we have $\mathbf{a} \times \mathbf{b} = \mathbf{a}^T \mathbf{b}$.

In equation (1), $\omega$ is the satellite angular velocity, $\mathbf{J}$ is the satellite rotational inertia matrix, and $\mathbf{J}$ is the real positive definite symmetric matrix, $\mathbf{u}$ is the control torque, and $\mathbf{d}$ is the external disturbance torque. In this study, it is assumed that the external disturbance torque is norm-bounded Gaussian white noise, which is expressed as follows:

$$\|\mathbf{d}\| \leq \mathbf{d}.$$  \(6\)

Also, it is worth noticing that the attitude quaternion has a property that $\mathbf{q}$ and $-\mathbf{q}$ describe the same attitude; hence, it is assumed that $q_{e0} \geq 0$ in this paper.

Considering that it is impossible to know the rotational inertia matrix accurately in actual control, it is assumed that

$$\mathbf{J} = \mathbf{J} + \mathbf{J},$$  \(7\)

where $\mathbf{J}$ is the estimated value of the rotational inertia matrix and $\mathbf{J}$ is the error value of the rotational inertia matrix.

The kinematic model of the error quaternion is expressed as follows:

$$\begin{align*}
\dot{q}_{e0} &= -\frac{1}{2} q_{e4}^T \omega_e, \\
\dot{q}_{ev} &= \frac{1}{2} (q_{e0} I_3 + q_{ev}) \omega_e - \frac{1}{2} F_e \omega_e, \\
\dot{\varepsilon}_e &= \frac{1}{2} \varepsilon_e^T \left( I_3 - \cot \left( \frac{\varphi_e}{2} \right) \varepsilon_e \right) \omega_e, \\
\dot{\varphi}_e &= \varepsilon_e^T \omega_e,
\end{align*}$$  \(8\)

where $\varphi_e$ is the Euler angle corresponding to error quaternion $\mathbf{q}_e$ and $\varepsilon_e$ is the Euler axis corresponding to error quaternion $\mathbf{q}_e$.

### 3. Attitude Tracking PD + Controller

The goal of attitude tracking control discussed in this section is to ensure that the system can maneuver along the desired dynamic trajectory, i.e., a controller $\mathbf{u}$ is supposed to be designed to make the system state satisfy the equation.
The attitude tracking PD + controller proposed in this section can be written as follows:

\[ u = \begin{cases} 
\rho_1 u_1 + \tau_1, & \|q_{ev}\| \geq \alpha, \\
\rho_2 u_2 + \tau_2, & \alpha > \|q_{ev}\| \geq \beta, \\
\rho_3 u_3 + \tau_3, & \beta > \|q_{ev}\|,
\end{cases} \tag{10} \]

where \( \rho_i \) (i = 1, 2, and 3) is a control gain factor. The purpose of \( \rho_i \) is to make the control torque not exceed the system upper bound. \( \alpha \) and \( \beta \) are positive scalars to be determined. \( \rho_i \) is defined as follows:

\[ \rho_i = \begin{cases} 
\frac{-u^T \tau_j + \sqrt{(u^T \tau_j)^2 + \|u\| (\|q_{ev}\|^2 - \|\tau_j\|^2)}}{\|u\|^2}, & \|u\| \geq \overline{u}, \\
1, & \|u\| < \overline{u},
\end{cases} \tag{11} \]

where \( \overline{d} \) is a positive scalar which satisfies \( \overline{d} \geq \|d\| \), \( \overline{\zeta} \), and \( \zeta \), and \( \tau_i \) (i = 1, 2, and 3) are defined as follows:

\[ \begin{aligned}
\zeta & = \frac{1}{2} \hat{R} \omega_d - \frac{1}{2} \omega_d \hat{R} \omega_d + (\omega_e + R \omega_d) \left( \omega_e + R \omega_d \right), \\
\tau_i & = 2 k_i \omega_i \omega_i + \omega_i \omega_i + \omega_e + R \omega_d, \\
r_1 & = \frac{1}{2} \frac{s_i}{s_i - \frac{1}{2} \cot \frac{s_i}{2} \omega_d \omega_d - \frac{1}{2} \lambda} \left( 1 + \cot \frac{s_i}{2} \right) \|s_i\| \| \begin{pmatrix} \omega_e \omega_e \omega_e \end{pmatrix} \right), \\
r_2 & = \frac{1}{2} \frac{s_i}{s_i - \frac{1}{2} \|s_i\| \| \begin{pmatrix} \omega_e \omega_e \omega_e \end{pmatrix} \right), \\
r_3 & = \frac{1}{2} \frac{s_i}{s_i - \frac{1}{2} \|s_i\| \| \begin{pmatrix} \omega_e \omega_e \omega_e \end{pmatrix} \right). \tag{13} \end{aligned} \]

where \( \overline{d} \) is the upper bound of control torque. \( \omega_e \) and \( \tau_i \) (i = 1, 2, and 3) are defined as follows:

\[ \begin{aligned}
u_e &= 0, \\
\beta &\leq \|q_{ev}\|, \\
s_e &= \begin{cases} 
\omega_e + k_1 \omega_e - \frac{1}{2} c_3 F_e \omega_e, & \|q_{ev}\| \geq \alpha, \\
\omega_e + k_2 q_{ev}, & \|q_{ev}\| < \alpha, 
\end{cases} \tag{16} \]

where \( c_1, c_2, \) and \( c_3 \) are positive scalars to be determined. In order to avoid the abrupt change of angular velocity at the switching point of the sliding mode surface, \( k_1 \) and \( k_2 \) should be set to satisfy

\[ k_2 = \frac{k_1}{\alpha} \tag{17} \]

Integral term \( \omega_e \) and sliding mode surface \( s_e \) are defined as follows:

In equations (13) and (15), \( \lambda \) is a positive scalar and satisfies
\[ \lambda \geq \lambda (T). \]  
(19)

The design idea of the attitude tracking controller is to only apply PD + control when the system state is far from the equilibrium point to make the system maneuver with constant angular velocity and then decelerate along the sliding mode surface. As the system state approaches the equilibrium point, an integral term is implemented to reduce the oscillation and improve the steady-state accuracy.

Controller (12) has the following properties: under the condition of selecting appropriate control parameters, systems (1) and (8) governed by controller (12) are uniformly asymptotically stable, and the system state can maneuver along sliding mode surface (17). At the same time, by selecting appropriate constant angular velocity and control torque are below the upper bound. Next, the aforementioned properties of controller (12) are demonstrated.

3.1. Controller Stability Proof. In order to prove the stability of systems (1) and (8) governed by controller (12), three lemmas need to be given first.

**Lemma 1.** For any positive scalars \( k_d \) and \( k_e \) and three-dimensional vector \( \mathbf{r} \), systems (1) and (7) governed by the controller with the following structure are uniformly asymptotically stable:

\[ \mathbf{u} = -k_d \omega_e - k_p \mathbf{q}_{ew} + \omega_e^r \mathbf{r} - \mathbf{R} \mathbf{sgn} (\omega_e) - \lambda \mathbf{R} \mathbf{sgn} (\omega_e) + \zeta, \]  
(20)

**Proof.** Select the Lyapunov function as follows:

\[ V_1 = \frac{1}{2} \omega_e^T \mathbf{J} \omega_e + 2k_p (1 - q_{eb}). \]  
(21)

Calculate the derivative of (21), and substitute it into controller (20) to get

\[ \dot{V}_1 = \omega_e^T \mathbf{J} \omega_e - 2k_p q_{eb} \]

\[ = \omega_e^T (\mathbf{u} + d - \mathbf{R} \omega_d + \omega_e^r \mathbf{R} \omega_d) - (\omega_e + \mathbf{R} \omega_d)^T \mathbf{J} (\omega_e + \mathbf{R} \omega_d) + k_p \omega_e^T \mathbf{q}_{ew} \]

\[ \leq - k_d \omega_e^T \omega_e + \omega_e^r \mathbf{R} \omega_d - \lambda \| \omega_e \|_2^2 \]

\[ + \omega_e^T \mathbf{J} \omega_e^r \mathbf{R} \omega_d - \lambda \| \omega_e \|_2^2 \| \omega_d \|_2^2 \]

\[ - \omega_e^T (\omega_e + \mathbf{R} \omega_d)^T \mathbf{J} (\omega_e + \mathbf{R} \omega_d) - \lambda \| \omega_e \|_2 \| \omega_e \|_2^2 \]

\[ \leq - k_d \omega_e^T \omega_e \leq 0. \]

It is obvious that when \( \dot{V}_1 < 0 \), the system is uniformly asymptotically stable, and the next step is to discuss system stability when \( \dot{V}_1 = 0 \). Noting that only when \( \omega_e = 0, V_1 = 0 \) could occur and if the system stays \( \dot{V}_1 = 0 \) only when \( \omega_e \equiv 0 \), under this condition, it could be found that \( \omega_e \equiv 0 \); substitute this into controller (20); it could be easily found that \( \mathbf{q}_{ew} \equiv 0 \) (other terms such as \( \omega_d \) and \( \omega_d \) are eliminated by terms \( \omega_e \) and \( \zeta \), and coupled terms with \( \omega_e \) are equal to zero since error angular velocity is equal to zero). This property means when \( \dot{V}_1 = 0 \) occurs, the system has reached its equilibrium point. Above all, when \( \dot{V}_1 = 0 \) and \( V_1 < 0 \), system states \( \omega_e \) and \( \mathbf{q}_{ew} \), both tend to zero, and the system is uniformly asymptotically stable.

Also, it is worth noticing that, in the stability proof, the constant proportional term \( k_p \) is discussed, and it is pointed out that, for any positive scalar \( k_p \) and \( k_d \), the system is stable. By implementing this property, the PD controller with variable control parameters could also be proved. When a new parameter is generated, it could be treated as a new Lyapunov function and the system state still tends to its equilibrium point (the key is not to prove that by implementing some fixed Lyapunov function, system stability could be proved, but for any time-varying parameter \( k_p \), there exist a positive definite \( V \) function and a negative definite \( \dot{V} \) function; hence, the system could have expected stability, and this method has been used in [16]). This property would be used in the later text.

**Lemma 2.** By selecting appropriate control parameters, systems (1) and (8) are uniformly asymptotically stable governed by a controller with the following structure:

\[ \mathbf{u} = -k_d \omega_e - k_p \mathbf{q}_{ew} + \omega_e^r \mathbf{r} - \mathbf{R} \mathbf{sgn} (\omega_e) - \lambda \mathbf{R} \mathbf{sgn} (\omega_e) + \zeta, \]

\[ \mathbf{s}_e = \omega_e + k \mathbf{q}_{ew}, \]

where the definition of \( \bar{\zeta} \) and \( \zeta \) is given in (13). \( k_d, k_e \), and \( k \) are all positive scalars, while \( \mathbf{r} \) is a norm-bounded three-dimensional vector.

**Proof.** Select the Lyapunov function as follows:

\[ V_2 = \frac{1}{2} \omega_e^T \mathbf{J} \omega_e + 2( k_p + k_k_d) (1 - q_{eb}) + k \mathbf{q}_{ew}^T \mathbf{J} \omega_e. \]  
(25)

Considering that

\[ 1 - q_{eb} \geq \frac{1}{2} (1 - q_{eb}^2) = \frac{1}{2} \mathbf{q}_{ew}^T \mathbf{q}_{ew}, \quad \forall q_{eb} \in [-1, 1], \]  
(26)

it can be obtained that
\[ V_2 \geq \frac{1}{2} \omega_e^T J \omega_e + (k_p + k k_d) \dot{q}_{e,0}^T q_{e,0} + k k_e \dot{q}_{cv}^T J \omega_e \]

\begin{align*}
\geq & \left[ \omega_e^T \ q_{cv}^T \right] \begin{bmatrix}
\frac{1}{2} \lambda_m (J) & \frac{1}{2} k \lambda_M (J) \\
\frac{1}{2} k \lambda_M (J) & k_p + k k_d 
\end{bmatrix} \begin{bmatrix}
\omega_e \\
q_{cv}
\end{bmatrix}.
\end{align*} \tag{27}

Therefore, as long as the following inequality is satisfied, \( V_2 \)'s positive definiteness can be guaranteed.

\[ \lambda_m (J) (k_p + k k_d) > \frac{1}{2} k^2 \lambda_M^2 (J). \tag{28} \]

Calculate the derivative of \( V_2 \), and substitute it into controller (23) to get

\[ V_2 = \omega_e^T J \dot{\omega}_e - 2 (k_p + k k_d) \dot{q}_{e,0}^T q_{e,0} + k \omega_e^T J \dot{q}_{cv} + k \omega_e^T J \dot{q}_{cv} \]

\[ = (\omega_e + k q_{cv})^T J \dot{\omega}_e + (k_p + k k_d) \dot{q}_{e,0}^T q_{e,0} + k \omega_e^T J F \dot{\omega}_e \\
\leq - \omega_e^T (k_d I_3 - k J) \omega_e - k k_p q_{e,0}^T q_{e,0} + s_t^T d - \beta \| s_t \| \| \omega_e \| + s_e^T J R \dot{w}_d - \lambda \| s_e \| \| \omega_d \| \\
- s_t^T J \omega_e^T R \omega_d - \lambda \| s_t \| \| \omega_e \| \| \omega_d \| + s_e^T (\omega_e + R \omega_d)^T (\omega_e + R \omega_d) \\
- \lambda \| s_t \| \| \omega_e \| + R \omega_d \| ^2 + (\omega_e + k q_{cv})^T \omega_e
\]

\[ \leq - k_d (k_d I_3 - k J) \omega_e - k k_p q_{e,0}^T q_{e,0} - k \omega_e^T \omega_e \\
\leq - \left[ \omega_e^T \ q_{cv}^T \right] \begin{bmatrix}
k_d - k \lambda_m (J) & \frac{1}{2} k \| \| \\
\frac{1}{2} k \| \| & k k_p
\end{bmatrix} \begin{bmatrix}
\omega_e \\
q_{cv}
\end{bmatrix},
\]

where the definition of \( \bar{\zeta} \) and \( \zeta \) is given in (14). \( k, k_p, k, k_d, c_1, \text{ and } c_3 \) are all positive scalars.

**Proof.** Select the Lyapunov function as follows:

\[ V_3 = 2 l_1 (1 - q_{e,0}) + \frac{1}{2} \omega_e^T J \omega_e + k q_{e,0}^T q_{e,0} + l_2 (\omega_e + c_1 q_{e,0})^T (\omega_e + c_1 q_{e,0}), \tag{33} \]

where \( l_1 \) and \( l_2 \) are positive scalars.

Considering that

\[ 1 - q_{e,0} \geq \frac{1}{2} (1 - q_{e,0}^2) = \frac{1}{2} q_{e,0}^T q_{e,0}, \forall q_{e,0} \in [-1, 1], \tag{34} \]

it can be obtained that

\[ V_3 \geq l_1 q_{e,0}^T q_{e,0} + \frac{1}{2} \omega_e^T J \omega_e + k q_{e,0}^T J \omega_e \geq \left[ \omega_e^T \ q_{cv}^T \right] \begin{bmatrix}
\frac{1}{2} \lambda_m (J) & \frac{1}{2} k \lambda_M (J) \\
\frac{1}{2} k \lambda_M (J) & l_1
\end{bmatrix} \begin{bmatrix}
\omega_e \\
q_{cv}
\end{bmatrix}.
\] \tag{35}
So, if there is
\[ l_1 \lambda_m (J) > \frac{1}{2} k^2 \lambda_M^2 (J), \tag{36} \]
it can be guaranteed that \( V_3 \) is positive definite.

Calculating the derivative of \( V_3 \) gives
\[
\dot{V}_3 = -2l_1 \dot{q}_{e0} + \omega_v^T J \omega_v + k q_v^T J \dot{q}_{v} + k \omega_v^T J q_v + 2l_2 (v_c + c_3 q_{cv})^T (v_c + c_3 q_{cv}) \\
= (\omega_v + k q_v)^T J \omega_v + l_1 \omega_v^T q_v + k \omega_v^T J F_v \omega_v \\
+ 2l_2 (v_c + c_3 q_{cv})^T (c_1 \omega_v + c_2 q_{cv} - \frac{1}{2} c_3 F_v \omega_v + \frac{1}{2} c_3 F_v \omega_v) \\
= s_v^T (u + d - J R \dot{\omega}_d + J \omega_v^T R \omega_d - (\omega_v + R \omega_d)^T J (\omega_v + R \omega_d)) \\
+ l_1 \omega_v^T q_v + k \omega_v^T J F_v \omega_v + 2l_2 (v_c + c_3 q_{cv})^T (c_1 \omega_v + c_2 q_{cv}). \tag{37} 
\]

Substituting controller (23) into it gives
\[
\dot{V}_3 \leq s_v^T d - \tilde{r} \omega_v - \lambda \omega_v - \lambda \omega_v + \lambda \omega_v - (k_3 J - k J F_v) \omega_v \\
+ (2l_2 c_1 c_3 + l_1 - k_p - k k_d) q_v^T q_v + (2l_2 c_1 - k_1) \omega_v^T v_e + (2l_2 c_2 - k k_d) q_v^T q_v \\
- (k k_p - 2l_2 c_1) q_v^T q_v + s_v^T \omega_v^T r \\
\leq - \omega_v^T (k_3 J - k J F_v) \omega_v - (k k_p - 2l_2 c_1 c_3) q_v^T q_v - k r q_v^T q_v \\
+ (2l_2 c_1 c_3 + l_1 - k_p - k k_d) q_v^T q_v + (2l_2 c_1 - k_1) \omega_v^T v_e + (2l_2 c_2 - k k_d) q_v^T q_v. \tag{38} 
\]

Selecting parameters to satisfy
\[
2l_2 c_1 c_3 + l_1 - k_p - k k_d = 2l_2 c_1 - k_1 = 2l_2 c_2 - k k_d = 0, \tag{39} 
\]
equation (38) can be transformed into
\[
\dot{V}_3 \leq - \omega_v^T (k_3 J - k J F_v) \omega_v - (k k_p - 2l_2 c_1 c_3) q_v^T q_v - k r q_v^T q_v \\
\leq - \begin{bmatrix} \omega_v^T & q_v^T \end{bmatrix} \begin{bmatrix} k_d - k \lambda_m (J) & \frac{1}{2} k \| r \| \\ \frac{1}{2} k \| r \| & k k_p - 2l_2 c_1 c_3 \end{bmatrix} \begin{bmatrix} \omega_v \\ q_v \end{bmatrix}. \tag{40} 
\]

Based on (40), control parameters can be chosen to satisfy
\[
(k_d - k \lambda_m (J))(k k_p - 2l_2 c_1 c_3) \frac{4}{k^2} > 0. \tag{41} 
\]

Then, it can be guaranteed that \( \dot{V}_3 \leq 0 \) so that systems (1) and (8) are uniformly asymptotically stable governed by controller (30). Lemma 3 is proved.

From the proofs of Lemmas 1–3, it can be concluded that Lemmas 1 and 2 give the stability proof of the PD controller with additional terms in the attitude tracking control (Lemma 1 corresponds to the first stage of controller (10), i.e., when \( \| q_{cv} \| \geq \alpha \), and the controller has totally the same structure as that in Lemma 1, and Lemma 2 corresponds to the second stage of the controller, i.e., when \( \alpha > \| q_{cv} \| \geq \beta \), while Lemma 3 gives the PD controller stability proof in the attitude tracking control (which corresponds to the third stage of controller (10), i.e., when \( \beta > \| q_{cv} \| \)). Lemmas 1 and 2 demonstrate the stability of the standard PD controller with a cross-product term of error angular velocity and a sign function term. In stability analysis, it could be found that the product term does not affect system stability (since it could eliminate the velocity term in the derivative of the \( V \) function), and its function is to ensure that the system could converge along the desired trajectory. The main difference between Lemmas 1 and 2 is the sign function term. The goal of the function term is to offset the affection of disturbance and model uncertainty, and the difference between the chasing trajectory of stages 1 and 2 causes the difference of controllers in Lemmas 1 and 2. And this is why we use two lemmas to demonstrate the stability of the PD + controller. Lemma 3 mainly describes the stability when approaching the equilibrium point, and under this condition, the integral term is added in the controller to improve system robustness; and this is why the \( V \) function and stability analysis
differ to Lemmas 1 and 2. These three lemmas correspond to the three stages of controller (12), respectively.

The structure of the first stage ($\|q_m\| \geq \alpha$) in controller (10) is exactly the same as the structure of controller (20) in Lemma 1 while satisfying all the constraints in Lemma 1. Thus, the system is uniformly asymptotically stable under the first stage of controller (10).

The structure of the second stage ($\alpha > \|q_m\| \geq \beta$) in controller (10) is similar to that in controller (23) in Lemma 2. The difference is that the first item is implemented to $r_2$ in controller (10). Consider

$$\left\| \frac{k_2}{2} q_\omega \right\| \leq \frac{\sqrt{3}}{2} k_2 \lambda \|w\|.$$  (42)

Therefore, the differential term of controller (10) in the second stage can be regarded as

$$k'_d = k_d + \frac{k_2}{2} \alpha \lambda_m (J) - \frac{\sqrt{3}}{2} k_2 \lambda,$$  (43)

Consider the constraints in Lemma 2. Note that

$$\|r_2\| = \left\| \frac{1}{2} J \dot{\theta} - \frac{1}{2} \dot{\omega} \|s\| \text{sgn} \left( \frac{q_r}{\|q_r\|} \right) \right\| \leq \frac{1}{2} \left( \sqrt{3} \lambda + \lambda_m (J) \right) \|s\| \leq \frac{1}{2} \left( \sqrt{3} \lambda + \lambda_m (\bar{l}) \right) (\bar{w} + k_2).$$  (44)

where $\bar{w}$ is the upper bound of the system angular velocity. Then, the system parameters are selected to satisfy the following inequalities:

$$\lambda_m (J) (k_p + k_2 k_d) > \frac{1}{2} k_2^2 \lambda_m (J),$$  (45)

$$(k'_d - k_2 \lambda_m (J)) k_p > \frac{1}{16} k_2^2 \left( \sqrt{3} \lambda + \lambda_m (\bar{l}) \right) (\bar{w} + k_2)^2.$$  (46)

Then, the second stage of controller (10) can satisfy the constraint conditions in Lemma 2 so that the system is uniformly asymptotically stable governed by the second stage of controller (10).

The structure of the third stage ($\beta > \|q_m\|$) of controller (10) is similar to the structure of controller (30) in Lemma 3, except that the first item is implemented to $r_3$ in controller (10). Similar to the previous circumstances, considering property (42), the differential term of controller (10) in the third stage can also be regarded as

$$k'_d = k_d + \frac{k_2}{2} \alpha \lambda_m (J) - \frac{\sqrt{3}}{2} k_2 \lambda.$$  (47)

Noting the constraints on the stability of the system in Lemma 3, taking into account that $r_3$ and $r_2$ are exactly the same, appropriate control parameters can be selected to satisfy

$$l_1 \lambda_m (J) > \frac{1}{2} k_2^2 \lambda_m (J),$$  (48)

$$2 l_2 c_1 c_3 + l_1 - k_p - k_2 k_d' = 2 l_2 c_1 - k_1 = 2 l_2 c_2 - k_2 k_1 = 0,$$  (49)

Then, the third stage of controller (10) satisfies the constraint conditions in Lemma 3 so that the system is uniformly asymptotically stable governed by the third stage of controller (10).

Based on the above discussion, systems (1) and (8) are uniformly asymptotically stable governed by controller (10). System stability has been proved.

3.2. Controller Performance Analysis. As mentioned above, controller (10) has the following properties: (1) control torque norm does not exceed the upper bound of the system; (2) the system can maneuver along sliding mode surface (16); (3) by selecting appropriate control parameters, the satellite attitude angular velocity is norm upper bounded. These properties will be explained and proved in this section.

Property (1) is fairly obvious, which can be obtained from the definition of the gain factor $\rho_i$. In fact, the function of $\rho_i$ is to reduce the PID terms proportionally when control torque saturation occurs, and based on the conclusion in [18], this method does not change system stability. When saturation occurs, the PID terms are reduced to a relatively small value, and based on the definition of $\rho_i$, it could be found that, in this condition, the control output is locked to the system norm upper bound, and this is how the control saturation issue is solved in this paper. Also, it is assumed that the upper bound of the system control torque is $\bar{T}$, and a suitable control parameter is selected to ensure $\|r_i\| \leq \bar{T}$. The essence of $\rho_i$ is to maintain the original controller unchanged when control torque $u$’s norm does not exceed the upper bound of the system. When $u$’s norm exceeds the upper bound of the system, the differential term, proportional term, and integral term in controller (10) will be reduced proportionally to let $\|u\| = \bar{T}$. It is worth noting that term $r_i$ in controller (10) is complicated, so the selection of parameters should be careful.

Next, property (2) is demonstrated. Similar to the foregoing, a Lyapunov function is selected as follows:

$$V_4 = \frac{1}{2} k_2^T J s.$$  (50)

Differentiate $V_4$ at the $\|q_m\| \geq \alpha$ stage of controller (10). Consider that when the system state is far away from the equilibrium point, the proportional terms and differential terms are much larger than the interference term and term $\bar{T}$.
which describes the uncertainty of the moment of inertia in the controller. Ignoring the aforementioned two items and substituting into controller (10), it can be obtained that

\[
V_4 = s_\epsilon^T J \dot{s}_\epsilon = s_\epsilon^T J \dot{\omega}_e + k_1 s_\epsilon^T J e_{\omega_e} = s_\epsilon^T \left( u + d - J R \omega_d + J \omega_e^r R \omega_d - (\omega_e + R \omega_d)^T J (\omega_e + R \omega_d) \right) + \frac{1}{2} k_1 s_\epsilon^T J \left( I_3 - \cot \frac{\phi_\omega}{2} e_{\omega_e}^T e_{\omega_e} \right) e_{\omega_e} \\
\approx -k_\delta s_\epsilon^T s_\epsilon + s_\epsilon^T \omega_e^r r_1 + \frac{1}{2} k_1 s_\epsilon^T J e_{\omega_e} \omega_e - \frac{1}{2} k_1 \cot \frac{\phi_\omega}{2} s_\epsilon^T J e_{\omega_e} e_{\omega_e} \\
= -k_\delta s_\epsilon^T s_\epsilon + k_1 e_{\omega_e}^T \omega_e^r r_1 + \frac{1}{2} k_1 e_{\omega_e}^T \omega_e^r J s_\epsilon + \frac{1}{2} k_1 \cot \frac{\phi_\omega}{2} e_{\omega_e}^T e_{\omega_e} e_{\omega_e} J s_\epsilon \\
\leq -k_\delta s_\epsilon^T s_\epsilon + \frac{1}{2} k_1 e_{\omega_e}^T \omega_e^r J s_\epsilon - \frac{1}{2} k_1 \lambda \| s_\epsilon \| \| e_{\omega_e}^T \omega_e \| + \frac{1}{2} k_1 \cot \frac{\phi_\omega}{2} e_{\omega_e}^T e_{\omega_e} e_{\omega_e} J s_\epsilon \\
- \frac{1}{2} k_1 \lambda \cot \frac{\phi_\omega}{2} s_\epsilon^T s_\epsilon \| e_{\omega_e}^T \omega_e \| \\
\leq -k_\delta s_\epsilon^T s_\epsilon \leq 0.
\]  

Differentiating \( V_4 \) at the \( \alpha > \| q_{\omega_e} \| \geq \beta \) stage of controller (10) and substituting it into controller (10), it can be obtained that

\[
V_4 = s_\epsilon^T J \dot{\omega}_e + k_2 s_\epsilon^T J q_{\omega_e} = s_\epsilon^T \left( u + d - J R \omega_d + J \omega_e^r R \omega_d - (\omega_e + R \omega_d)^T J (\omega_e + R \omega_d) \right) + \frac{1}{2} k_2 s_\epsilon^T J \left( q_{\omega_e} I_3 + q_{\omega_e} \right) \omega_e \\
= -k_\delta s_\epsilon^T s_\epsilon + s_\epsilon^T d - \alpha s_\epsilon^T s_\epsilon \text{sgn}(s_\epsilon) + \frac{1}{2} k_2 s_\epsilon^T J q_{\omega_e} - \frac{1}{2} k_2 \lambda \| q_{\omega_e} \| \| s_\epsilon \| \text{sgn}(s_\epsilon) \\
+ s_\epsilon^T \left( -J R \omega_d + \tilde{J} \omega_e^r R \omega_d - (\omega_e + R \omega_d)^T J (\omega_e + R \omega_d) \right) - \tilde{\alpha} s_\epsilon^T \text{sgn}(s_\epsilon) \\
+ \frac{1}{2} k_2 s_\epsilon^T J q_{\omega_e} + s_\epsilon^T \omega_e b_\omega \\
\leq -k_\delta s_\epsilon^T s_\epsilon + \frac{1}{2} k_2 q_{\omega_e}^T \omega_e^r J s_\epsilon + k_2 q_{\omega_e}^T \omega_e^r b_\omega \\
\leq -k_\delta s_\epsilon^T s_\epsilon + \frac{1}{2} k_2 q_{\omega_e}^T \omega_e^r J s_\epsilon - \frac{1}{2} k_2 \lambda \| s_\epsilon \| \| q_{\omega_e} \| \omega_e \\
\leq -k_\delta s_\epsilon^T s_\epsilon \leq 0.
\]  

Differentiating \( V_4 \) at the \( \beta > \| q_{\omega_e} \| \) stage of controller (10) and substituting it into controller (10), it can be obtained that
\[ \dot{V}_4 = s_e^T J \dot{\omega}_e + k_d s_e^T \tilde{q}_{ev} \]
\[ = s_e^T (u + d - J \dot{\omega}_d + J \omega_e^T R \omega_d - (\omega_e + R \omega_d)^T J (\omega_e + R \omega_d)) \]
\[ + \frac{1}{2} k_d s_e^T J (q_{\omega 0} I_3 + q_{\omega 0}^T) \omega_e \]
\[ = -k_d s_e^T \dot{s}_e - k_i s_e^T \nu_e + \frac{1}{2} k_d s_e^T J (q_{\omega 0} I_3 + q_{\omega 0}^T) \omega_e \]
\[ + s_e^T (-J \dot{\omega}_d + \tilde{J} \omega_e^T R \omega_d - (\omega_e + R \omega_d)^T \tilde{J} (\omega_e + R \omega_d)) - \tilde{q}_e s_e^T \text{sgn}(s_e) \]
\[ + \frac{1}{2} k_d s_e^T J q_{\omega 0}^T \omega_e + s_e^T \omega_e^T r_2 - \frac{1}{2} k_d \lambda q_{\omega 0} \| \omega_e \| s_e^T \text{sgn}(s_e) \]
\[ \leq -k_d s_e^T \dot{s}_e - k_i s_e^T \nu_e + \frac{1}{2} k_d q_{\omega 0}^T \omega_e J s_e + k_d q_{\omega 0} J \omega_e r_2 \]
\[ \leq -k_d s_e^T \dot{s}_e + \frac{1}{2} k_d q_{\omega 0}^T \omega_e J s_e - \frac{1}{2} k_d \lambda s_e^T q_{\omega 0} \| \omega_e \| \]
\[ \leq -k_d s_e^T \dot{s}_e - k_i s_e^T \nu_e, \quad (53) \]

Consider that, as the state converges, it will be obtained that \( \nu_e \rightarrow 0 \). So, it is assumed that
\[ \| \dot{s}_e \| \leq \rho \| s_e \|. \quad (54) \]
Selecting control parameters to satisfy
\[ k_d \geq \rho k_i, \quad (55) \]
\( \dot{V}_4 \leq 0 \) can be guaranteed.

From (50)–(52), it can be concluded that the sliding mode \( s_e \) is uniformly asymptotically stable. Thus, property (2) is proved.

Then, property (3) is proved. A lemma needs to be given first.

**Lemma 4.** For the system,
\[ J \dot{\omega}_e + J \dot{\omega}_d - J \omega_e^T R \omega_d + (\omega_e + R \omega_d)^T J (\omega_e + R \omega_d) = u + d, \quad (56) \]
its controller structure is

\[ \dot{\rho}_e = 2 \omega_e^T J \dot{\omega}_e \]
\[ = 2 \omega_e^T \left( -k_d \omega_e - k_d q_{\omega 0} - \lambda \text{sgn}(\omega_e) + d + \omega_e^T r - J \dot{\omega}_d + \tilde{J} \omega_e^T R \omega_d \right) \]
\[ \leq -2 k_d \omega_e^T \omega_e - 2 k_d \omega_e^T q_{\omega 0} \]
\[ \leq -2 k_d \frac{\rho_{\omega 0}^1}{\lambda_{\omega 0}(J)} + 2 k_d \frac{\rho_{\omega 0}^{1/2}}{\sqrt{\lambda_{\omega 0}(J)}}, \quad (61) \]

Where \( \rho_{\omega 0} \geq \| d \| \). The definition of \( \lambda \) and \( \zeta \) is given in (14). \( r \) is an arbitrary three-dimensional vector. The initial value of error angular velocity satisfies \( \| \omega_e \| \leq (k_p \lambda_{\omega 0}(J))/(k_d \lambda_{\omega 0}(J)) \); then, the error angular velocity norm always satisfies
\[ \| \omega_e \| \leq \frac{k_p \lambda_{\omega 0}(J)}{k_d \lambda_{\omega 0}(J)}, \quad (58) \]

**Proof.** Define \( \rho_e \) as follows
\[ \rho_e = \omega_e^T \tilde{J} \omega_e. \quad (59) \]
Then, it can be obtained that
\[ \sqrt{\lambda_{\omega 0}(J)} \| \omega_e \| \leq \rho_e^{1/2} \leq \sqrt{\lambda_{\omega 0}(J)} \| \omega_e \|. \quad (60) \]

Differentiate (59) and substitute it into controller (57) to get
Consider the system

$$\dot{\rho}_e = -2k_d \hat{\rho}_e \lambda_m(J) + 2k_p \frac{\rho_e^{1/2}}{\sqrt{\lambda_m(J)}}$$

(62)

Its equilibrium point is

$$\rho_e^{1/2} = \frac{k_d}{k_p} \frac{\lambda_M(J)}{\sqrt{\lambda_m(J)}}$$

(63)

Furthermore, if the initial state of system (62) is less than its equilibrium point, then the following inequality holds:

$$\|\rho_e^{1/2} \leq \frac{k_p}{k_d} \frac{\lambda_M(J)}{\sqrt{\lambda_m(J)}}$$

(64)

Noting property (60), there is

$$\|\omega \| \leq \frac{k_p}{k_d} \frac{\lambda_M(J)}{\sqrt{\lambda_m(J)}}$$

(65)

Thus, the proof of Lemma 4 is completed.

Because the system state in attitude tracking control is error angular velocity, the error angular velocity norm needs to be discussed to ensure that system’s attitude angular velocity does not exceed its upper bound. If the maximum angular velocity of the system is \(\bar{\omega}\), consider

$$\omega = \omega_e + R\omega_d, \quad \lambda(R) = 1.$$  

(66)

So, it is just needed to guarantee

$$\|\omega \| \leq \bar{\omega} - \|\omega_e\| = \bar{\omega}'$$

(67)

Then, angular velocity of the attitude will not exceed the upper bound of the system.

Considering that the system state converges along sliding mode surface (16) and when \(\|\omega_d\| < \alpha\), the system enters the deceleration phase of the sliding mode surface; meanwhile, the error angular velocity norm starts to decrease. Therefore, it can be considered that, under the premise of selecting appropriate control parameters, the situation that angular velocity exceeds the upper bound of the system will not occur at this stage. What needs to be emphasized is the state of the system not entering sliding mode surface (16) and the constant speed stage of sliding mode surface (16), that is, the first stage of controller (10). At this stage, the structure of controller (10) is exactly the same as the structure of controller (57) in Lemma 4, so control parameters are just selected to satisfy

$$\frac{k_1 \lambda_M(J)}{\alpha \lambda_m(J)} \leq \bar{\omega}'.$$

(68)

Then, the attitude angular velocity of the satellite will not exceed its upper bound, so property (3) is proved.

Equation (68) gives strict constraints on the parameters of the controller. Its essence is to reduce the proportional term to achieve the constraint of angular velocity, but this is not consistent with the goal of controller (12) to improve convergence rate. Considering that the decrease of \(\rho_e\) can be approximately regarded as the decrease of \(\|\omega\|\), it can be set that \(\rho_e \leq 0\) when error angular velocity approaches its upper bound \(\bar{\omega}'\), and it can be approximated that the system attitude angular velocity does not exceed its upper bound. Based on the above discussion, considering system (62), it is only needed to satisfy

$$\rho_e^{1/2} \leq \frac{k_p}{k_d} \frac{\lambda_m(J)}{\sqrt{\lambda_M(J)}}$$

(69)

Then, it can be obtained that \(\dot{\rho}_e \leq 0\). Note that

$$\rho_e^{1/2} \leq \sqrt{\lambda_M(J)} \|\omega_e\|$$

(70)

Substituting it into controller (12), it can be obtained that

$$\rho_e^{1/2} \leq \frac{k_p}{k_d} \sqrt{\lambda_m(J)} \|\omega_e\|$$

(71)

Thereby, it is only needed to satisfy

$$k_1 \leq \bar{\omega}'$$.

(72)

Remark 1. There are many control parameters in the proposed controller, and it is necessary to demonstrate how these parameters affect the system performance. The effect of \(k_d\), \(k_p\), and \(k_i\) is basically the same as the standard PID controller; proportional term \(k_p\) determines the system steady accuracy, differential term \(k_d\) determines the system convergence rate and larger \(k_d\) could bring better stability, and integral term \(k_i\) could enhance system robustness to high-frequency vibration. Coefficient parameters \(\alpha\) and \(\beta\) are determined when the system starts deceleration, and coefficients \(k_1\) and \(k_2\) determine the system convergence rate. Generally, larger \(\alpha\) and smaller \(\beta\) could extend the maneuver stage with constant angular velocity, and \(k_1\) and \(k_2\) could determine the angular velocity during the control process; larger \(k_1\), \(k_2\), and \(\alpha\) could improve system convergence rate; however, the larger control torque is also required. These parameters are most critical to system performance, and other parameters are not such important; the main function is to simplify system stability analysis.

4. Simulation

Set system parameters as follows:
\[ J = \text{diag}([20\ 18\ 15]) \text{kg} \cdot \text{m}^2, \]
\[ \bar{J} = \text{diag}([21\ 17\ 14]) \text{kg} \cdot \text{m}^2, \]
\[ \lambda = 3, \]
\[ t_{\text{sample}} = 0.5 \text{s}, \]
\[ d = 0.001 \text{N} \cdot \text{m}, \]
\[ \bar{u} = 0.5 \text{N} \cdot \text{m}, \]
\[ \bar{w} = 0.1 \text{rad/s} \] (73)
\[ \omega(t_0) = [0\ 0\ -0.08]^T \text{rad/s}, \]
\[ q(t_0) = \left[ \frac{\sqrt{2}}{3}\ \frac{\sqrt{3}}{3}\ \frac{\sqrt{6}}{3} \right]^T \]
\[ \omega_d(t_0) = [0.005\ 0\ 0]^T \text{rad/s}, \]
\[ \dot{\omega}_d(t_0) = 0_{3 \times 1}, \]
\[ q_d(t_0) = [1\ 0\ 0\ 0]^T. \]

In order to demonstrate the superiority of the controller proposed in this paper, PID controller (74) proposed in [18] is compared. In [18], the authors proposed a PID controller with better robustness to disturbance and angular velocity constraint.

\[ u = -k_d \omega_e - k_p q_{nev} - k_l \omega_e - d \text{sgn}(\beta \omega_e + l_2 q_{nev}), \]
\[ \dot{\omega}_e = k_q q_{nev}. \] (74)

Set control parameters as follows:
\[ k_d = 20, \]
\[ k_p = 5, \]
\[ k_l = 1, \]
\[ k = 0.05, \]
\[ \beta = 0.05, \]
\[ l_2 = 0.01. \] (75)

The simulation results for controller (74) are shown as follows.

Based on Figures 1 and 2, it is obvious that the system converges to its equilibrium point, and this proves the stability of the standard PD controller. Also, it could be found that the convergence time is about 300 s, and system accuracy at the steady stage is about \(2 \times 10^{-5}\) rad/s and \(8 \times 10^{-4}\) of angular velocity and attitude quaternion. Based on Figures 3 and 4, it could be found that, during the initial 20 s, system control torque exceeds its upper bound by a large margin, and its maximum value is about 6 Nm which is hard to achieve for small satellites with such configuration. And after that, system control torque descends to zero drastically with system state still far away from its equilibrium point, and this means system capability is not fully utilized by implementing the standard PD controller. Also, it could be found that the angular velocity exceeds the system upper bond with a 10-second period; this is caused by the overshoot of the two-stage system. Generally, the PD controller is a mature controller and has such advantages; however, the system performance could still be improved in convergence time and angular velocity overshoot.

Next, the simulation results for the controller proposed in this paper are shown. Select control parameters as follows:
\[ \alpha = 0.15, \]
\[ \beta = 0.05, \]
\[ k_1 = 0.09, \]
\[ k_2 = 0.6, \]
\[ k_d = 20, \]
\[ k_p = 5, \]
\[ k_l = 1, \]
\[ c = 1, \]
\[ l_2 = 0.01, \]
\[ c_1 = 0.25, \]
\[ c_2 = 0.05, \]
\[ c_3 = 1. \] (76)

Based on (48), we have
\[ l_3 = \frac{ck_l}{2c_1} = 2, \]
\[ l_1 = \frac{ck_p + l_2 k_d - 2c_1 c_3 l_3}{2c_1} = 4.2. \] (77)

Substituting (76) and (77) into (47), (48), and (49), it could be found that the stability constraints are all satisfied. Also, it could be found that loose angular velocity constraint (72) is satisfied, but strict constraint (68) is not.

The simulation results are shown as follows.

Based on Figures 5 and 6, it could be found that the convergence time is about 50 s which is largely improved about 80% comparing with the standard PD controller. System accuracy at the steady stage is about \(1 \times 10^{-5}\) rad/s and \(8 \times 10^{-4}\) of angular velocity and attitude quaternion. The steady accuracy is at the same level comparing with the standard PD controller at 400 s. Hence, it could be concluded that the proposed controller could largely improve system converge time; meanwhile, the accuracy is maintained. Based on Figures 5 and 7, it could be found that the system is at the acceleration stage during the first 10 seconds and enters into the maneuver stage with constant angular velocity during 10–30 s. This is the key to improve system convergence rate since with larger norm of angular velocity, attitude quaternion has better convergence rate. Also, it could be found that, during this period, the control torque is at a low level, and this proves that the method proposed in this paper has a better efficiency on control torque capability.
Based on Figures 7 and 8, it could be found that the control torque and angular velocity do not exceed the system upper bound during the whole control process. This proves the effectiveness of the method proposed in this paper. However, it should also be noticed that the cost of performance improvement is the complexity of the controller comparing with the standard PID controller, and the structure should be simplified in later work.

In order to demonstrate the performance of the proposed controller, existing methods such as the finite-time controller in [15], robust PD controller in [16], standard and modified sliding mode controller in [17], and PID controller in [18] under totally the same configuration are compared as given in Table 1.

Based on the comparison, it could be found that the proposed method in this paper has almost the same convergence rate as the finite-time controller and dynamic sliding mode controller (at least at the same level); also, the robustness is relatively high (the proposed method could resist unknown disturbance and model uncertainty and could solve the control torque and angular velocity saturation issue). The fast convergence rate proves that, by designing the angular velocity trajectory properly, the linear feedback controller could have almost the same convergence rate as the fractional-order feedback controller. The key to improve system convergence rate is to maintain the reverse of angular velocity and attitude quaternion and to maintain the angular velocity at a high level as much as possible.

The main drawback of the proposed method is its complexity and large number of control parameters to decide. The former drawback is caused by the sliding mode surface combined with the PD controller; hence, the system has two control goals to achieve at the same time: to stabilize the system state and to stabilize the sliding mode state, and this property improves system convergence rate but makes the controller such complex. The latter drawback is caused by the stability analysis and saturation issue. Since the $V$ function is modified and the saturation issue is taken into
Figure 4: Curve of the control torque norm and angular velocity norm.

Figure 5: Curve of error angular velocity.

Figure 6: Curve of error quaternion.

Figure 7: Curve of the control torque norm and angular velocity norm.
consideration in this paper, the selection of parameters needs to be constrained.

Based on the discussion above, it could be found that the proposed method in this paper could largely improve system convergence rate with good robustness and solve the torque and velocity issue, but the condition is to select control parameters properly.

5. Conclusion

In this paper, a PD+ controller for satellite attitude tracking control is proposed. A sliding mode with the maneuver stage of constant angular velocity is combined with PD control algorithm; hence, system convergence rate is largely improved. The Lyapunov function with coupled terms is also proposed in this paper to simplify the stability analysis process. Strict system stability is proved by the Lyapunov method, and the superiority of the method proposed in this paper is demonstrated by numerical simulation results.

It is pointed out that the low convergence rate and low efficiency on control torque are one of the main drawbacks of the standard PD controller. By designing the trajectory of angular velocity properly, these aforementioned drawbacks could be largely improved. The key to achieve this property is the maneuver stage with angular velocity: (1) in this stage, angular velocity vector is reversed to attitude quaternion vector, and the convergence rate of the attitude quaternion is maintained at a high level which does not shrink with the convergence of the attitude quaternion; (2) during this stage, the system would not accelerate nor decelerate, and the demand control torque is relatively small. Also, the controller proposed in this paper is a combination of the sliding mode surface and standard PD controller; this proves that, by designing the sliding mode surface and attitude controller properly, the system governed by the PD controller could maneuver along the desired trajectory. The angular velocity constraint is also discussed; by implementing the variable parameter method and enlarging the differential term, system angular velocity would not exceed the system upper

Table 1: Comparison of the proposed controller with existing controllers.

| Method                          | Convergence rate (s) | Maximum control torque (Nm) | Maximum angular velocity (rad/s) | Complexity | Number of control parameters | Robustness |
|---------------------------------|----------------------|-----------------------------|---------------------------------|------------|-----------------------------|------------|
| Proposed method in this paper   | 50                   | 0.5                         | 0.08                            | Relatively high | About 10 parameters to decide | High       |
| PID controller                  | 300                  | 5                           | 0.2                             | Low        | 3                           | High       |
| Robust PD controller            | 70                   | 0.5                         | 0.08                            | Medium     | About 15 parameters         | High       |
| Standard sliding mode controller| 200                  | 3                           | 0.2                             | Low        | 3                           | High       |
| Dynamic sliding mode controller  | 40                   | 0.5                         | 0.1                             | Medium     | 8                           | Vulnerable to measurement error and large model uncertainty |
| Finite-time controller          | 35                   | 0.5                         | 0.1                             | High       | About 15 parameters         | Vulnerable to measurement error and large model uncertainty |

Figure 8: Curve of control torque.

Table 1: Comparison of the proposed controller with existing controllers.
bound, while system stability is maintained. These properties mentioned above are insightful for future work on PD controller design.

6. Further Recommendations

In this paper, a theoretical method is proposed for satellite attitude control, and in order to apply the theoretical method to engineering practice, the following improvements should be made: (1) the structure of the proposed method is relatively complex and hard for engineering practice; hence, later work should focus on the controller simplification; (2) more extreme conditions should be considered to avoid the singularity issue since the proposed method uses the Euler axis parameter.

Data Availability

We declare that some or all data, models, or codes generated or used during this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no conflicts of interest.

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