Formation of robust and completely tunable resonant photonic band gaps

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We identify different types of the photonic band gaps (PBGs) of two dimensional magnetic photonic crystals (MPCs) consisting of arrays of magnetic cylinders and study the different tunability (by an external static magnetic field) of these PBGs. One type of the band gaps comes from infinitely degenerate flat bands and is closely related to those in the study of plasmonics. In addition, such PBGs are magnetically tunable and robust against position disorder. We calculate the transmission of the PBG’s and found excellent agreement with the results of the photonic band structure calculation. Positional disorder of the lattice structure affects the different types of PBGs differently.

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The unique characteristic of photonic crystals (PCs) is the existence of photonic band gaps (PBGs), the frequency ranges over which electromagnetic (EM) modes are forbidden. For applications in optical devices, it is important to obtain tunable PCs so that the photonic band structures or the PBGs can be modulated and, preferably, robust. Magnetic photonic crystals (MPCs) have attracted interest because of the potential tunability of the PBGs by an external static magnetic field (ESMF) and the fast switching time of magnetic systems. Very recently Chen et al found experimentally that the ESMF can vary the position of the PBGs, but by only a small amount. This raises doubt on the practical tunability of the MPCs.

In this paper we demonstrate for realistic experimental parameters that even though not all PBGs are easily tunable, there are PBGs that can easily be controlled by ESMFs. We found three kinds of PBGs: (1) from the Bragg scattering; (2) from the Mie scattering resonance of magnetic cylinders, and (3) from the spin wave resonance. The responses of these to ESMFs differ considerably. Both the 2nd and the 3rd types of bands are very sensitive to the ESMF and are magnetically tunable whereas the first type is not. We calculate the transmission of the PBG’s under different ESMFs and found excellent agreement with the results from the photonic band structure. The transmission as a function of lattice position disorder is also studied. Positional disorder only affects the Bragg type of PBGs significantly but not the other two, implying the robust tunable PBGs. We now describe our results in detail.

For bands with flat dispersion the plane wave expansion method does not work well, in our work the multiple scattering method is employed. The two dimensional (2D) MPCs are composed of hexagonal lattices of soft Mg-Mn ferrite rods in nonmagnetic plexiglas, with lattice constant \( a \). The axis of the cylinder and the ESMF \( H_{ext} = H_{ext} \hat{e}_z \) are along the \( z \) direction, the wave vector is in the \( xy \) plane. For ferrite rods, the relative permeability tensor of the ferrite and its inverse are

\[
\hat{\mu} = \mu_s \begin{pmatrix} \mu_r & -i\mu_k & 0 \\ i\mu_k & \mu_r & 0 \\ 0 & 0 & 1 \end{pmatrix}, \mu_s \hat{\mu}^{-1} = \begin{pmatrix} \mu'_r & -i\mu'_k & 0 \\ i\mu'_k & \mu'_r & 0 \\ 0 & 0 & 1 \end{pmatrix},
\]

where \( \mu_s = 1, \mu_r \mu_r = 1 + \omega_m(\omega_0 - i\alpha\omega)/[\omega_0 - i\alpha\omega]^2 - \omega^2], \mu_k \mu_k = \omega_m \omega/[\omega_0 - i\alpha\omega]^2 - \omega^2], \omega_m = \gamma 4\pi M_s \) with \( M_s \) the saturation magnetization, \( \omega_0 = \gamma H_0 \) is the ferromagnetic resonance frequency with \( \gamma \) the gyromagnetic ratio, \( H_0 \) is the sum of the external field and the shape anisotropy field, \( \alpha \) is the damping coefficient of the ferrite. \( \mu'_r = \mu_r/(\mu_r^2 - \mu_k^2), \mu'_k = -\mu_k/(\mu_r^2 - \mu_k^2). \)

The time dependence of the field is assumed to be \( e^{-i\omega t} \) and suppressed. In 2D EM systems both the transverse electric (TE) mode and the transverse magnetic (TM) mode are the eigen-modes. Only the TM mode is tunable by the ESMF and will be considered here. To illustrate the physics, we consider a concrete example where the lattice constant is taken as \( a = 8mm \), the radius of the ferrite cylinder is taken as \( r_s = \frac{1}{4}a = 2mm \). The relative permittivity and permeability of the nonmagnetic background medium (plexiglas) are \( \epsilon_m = 2.6 \) and \( \mu_m = 1 \). For the ferrite cylinder \( \epsilon_s = 12.3, \) the saturation magnetization \( 4\pi M_s = 1700G \). We have neglected the absorption of the ferrite cylinder, the imaginary part of \( \hat{\mu} \) of the ferrite in the microwave region is very weak.

In Fig.1 we show the photonic band structure with an ESMF \( H_0 = 900Oe \). The frequency is in units of \( c/a \) with \( c \) the speed of light in vacuum. The PBGs are located near 0.108c/a, 0.125c/a, 0.135c/a and 0.275c/a, respectively. In conventional dielectric PCs, the PBGs could be understood as a result of Bragg scattering. The location of a PBG due to Bragg scattering scales with the inverse lattice constant. In Fig.2 we show the photonic band structure near the top PBG for lattice constants \( a = 5mm, r = \frac{1}{4}a = 1.25mm \) and \( b = 8mm, r = \frac{1}{4}a = 2mm \); the other parameters are the same as those in Fig.1. When expressed in units of \( c/a \), the PBGs appeared in almost the same position, in addition the photonic band structures are nearly the same on both sides of the PBGs. This provides convincing evidence that scale invariance is satisfied by this PBG. We conclude that this PBG is dominated by Bragg scattering.

The photonic band structures near the top PBG for different ESMFs of \( H_0 = 700Oe \) and \( H_0 = 900Oe \) are nearly the same as each other. This explains why there is only a tiny change in the positions of the PBGs when different ESMFs are applied in the experiments of Chen et al, which is performed in this frequency range. We next turn our attention to the other PBGs.

The "Mie" scattering resonance of cylinders for all angular momenta \( n > 0 \) occurs when \( \mu_{eff} = \mu_r + \mu_k = \mu'_r + \mu'_k = -1 \) \( (\epsilon = -1) \) for TM (TE) modes in the long wavelength limit. We found analytically that this corresponds to infinite number of degenerate flat bands in 2D when the cylinder radius is small. For scattering from spheres in 3D, the resonance conditions are different for different angular momenta and the 3D bands are nondegenerate. This kind of flat bands for TE modes have been discussed in the context of plasmonics where the focus is on the fact that \( \epsilon = -1 \) \( (\epsilon = -2 \) in 3D for s wave) corresponds to the resonance condition of surface plasmons. \( \mu_{eff} = -1 \) at a frequency \( f_s = \frac{1}{2\pi}[\omega_0 + \frac{1}{2}\omega_m] \). For our parameters this corresponds to frequencies of 0.1158c/a and 0.1308c/a for ESMFs of \( H_0 = 700Oe \) and \( H_0 = 900Oe \).

In Fig.3 we plotted the photonic band structures near the second and the third PBG with the eigenfrequencies divided by \( f_s \) for different ESMFs. The scaling of the frequencies of these bands with \( f_s \) is obvious. ESMFs change \( f_s \)
through $\omega_0$ and hence the positions of the second and the third PBGs. The position of the PBG can be switched to a new position without a PBG originally. A propagating wave can become totally reflected and vice versa by changing the ESMF. This phenomenon can be useful in the design of optical devices. The PBG near these flat bands are not associated with the Bragg scattering dominating the conventional dielectric PBG. We performed calculations of the photonic band structures near these flat bands for different lattice constants $c/a = 5 \text{mm}$ and $8 \text{mm}$ and found that the flat band between the second and the third PBGs are located at the same position and does not scale with $1/a$. Our understanding suggests that if disorder is introduced in the periodicity the first PBG will be affected seriously while the second and the third will persist.

There exists another flat band in Fig. 1 lying between the third and the fourth PBG. The position of this band can also be varied by an ESMF. A "spin wave" resonance occurs at a frequency $f_m = \frac{1}{2\pi} \sqrt{\omega_0^2 + \omega_m^2}$ when $\mu_r = \mu_s^r = 0$ and the wavevector inside the cylinder, which is proportional to $1/\sqrt{\mu^r}$, becomes infinite. For $H_0 = 9000 \text{Oe}$ and $4\pi M_s = 17000 \text{G}$, $f_m = 0.114 c/a$ corresponding to the bottom of the flat band in Fig. 4. This fourth PBG can be manipulated by an ESMF, which may just as useful as the second and the third PBGs.

For comparison with the experimental result it is necessary to perform the simulation of transmission coefficients. This can be expressed as the forward scattering amplitude of the MPC [7]. For this calculation, the small imaginary part of the realistic susceptibilities are included: $\epsilon_b = 2.6 + i 0.005$, $\epsilon_s = 12.3 + i 0.0006$, $\alpha = 0.007$. In Fig. 4 we show the transmission for different ESMFs for a 7-layer structure normal to the $x$ direction with each layer consisting of 13 ferrite cylinder along $y$ direction $a = 8 \text{mm}$, $r = \frac{1}{4}a = 2 \text{mm}$. For convenience, the frequency are given in units of GHz (bottom) and $c/a$ (top). The locations of the second and the third PBGs are labeled with arrows in the figure. With the increase of the ESMF these PBG rise from the low frequency up to high frequency significantly. However, for the first PBG only tiny change occurs with the variation of ESMFs from $3000 \text{Oe}$ to $9000 \text{Oe}$. This can be seen from the vertical line denoting the low frequency edge of the first PBG. The positions of the PBGs of the transmission coefficients can be compared with those in the photonic band structures. For example, in Fig. 4(d) the middle gap frequency of the first, the second and the third PBGs are at about $0.275c/a$, $0.135c/a$ and $0.125c/a$, respectively, the same as those shown in the photonic band structures in Fig. 1. In addition, the width of the PBGs in transmission coefficients and photonic band structures are almost the same. In Fig. 4 we find some oscillation of the transmission away from the PBG. This is due to the finite size of the sample, as is pointed by Stefanou et al. [8]. The same kind of results are obtained for $a = 5 \text{mm}$, $r = a/4$.

In our approach, randomness of magnitude $\xi$ can be introduced for the positions of the ferrite cylinder. Along $x$ and $y$ directions the maximum variations of the position of the ferrite cylinder are $d_{x,0} = \sqrt{\xi}a - 2r$ and $d_{y,0} = a - 2r$ respectively, so that the displacement of each cylinder is $d_i = d_{0i} \xi$ with $i = x$ or $y$ and $\nu_i$ are random numbers between 0 and 1. In Fig. 5 the results of the transmission coefficients with different randomness $\xi$ are presented. There exists no obvious change in the transmission coefficient and the PBGs when the disorder is very weak. The oscillation of the transmission coefficient off the PBG becomes irregular. With the increase of the randomness ($\xi = 0.5$) the first PBG disappears due to the breakdown of the Bragg scattering; the "magnetic" PBG remain relatively unchanged. The larger the randomness, the less the transmission coefficient becomes.

In summary, we have performed photonic band structure calculations and simulations of the transmission coefficients with and without randomness. Four PBGs are found. The top PBG originates from the Bragg scattering and is sensitive to position disorder, the second and the third PBGs, from the magnetic Mie scattering of the cylinder and the fourth PBG, from the "spin wave" resonance. Even though the Bragg PBG is insensitive to an external field, the magnetic PBGs are easily tunable, are robust against position disorder and thus are suitable for application in designing optical devices.

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FIG. 1: The photonic band structure of a two dimensional MPC. \( a = 8 \text{mm} \) and \( r = 2 \text{mm} \), \( H_0 = 900 \text{Oe} \). The reduced Brillouin zone (RBZ) is illustrated in the upper right hand corner. The coordinates of the high symmetry points appearing in the RBZ are \( \Gamma = \frac{2\pi}{a}(0, 0) \), \( M = \frac{2\pi}{a}(0, \frac{1}{\sqrt{3}}) \) and \( K = \frac{2\pi}{a}(\frac{1}{2}, \frac{1}{\sqrt{3}}) \).

FIG. 2: The photonic band structure near the top PBG for (a) \( a = 5 \text{mm} \) and \( r_s = 1.25 \text{mm} \); (b) \( a = 8 \text{mm} \) and \( r_s = 2 \text{mm} \). The other parameters are the same as those in Fig. 1.

FIG. 3: Photonic band structure near the second and the third PBGs with the frequency divided by \( f_s \). The parameters are the same as those in Fig. 1 except that (a) \( H_0 = 700 \text{Oe} \); (b) \( H_0 = 900 \text{Oe} \).

FIG. 4: Transmission coefficients of the sample exerted with different ESMFs. The lattice constant is \( 8 \text{mm} \) and the radius of the ferrite cylinder is \( r_s = \frac{1}{4}a = 2 \text{mm} \). (a) \( H_0 = 300 \text{Oe} \); (b) \( H_0 = 500 \text{Oe} \); (c) \( H_0 = 700 \text{Oe} \); (d) \( H_0 = 900 \text{Oe} \).
FIG. 5: Transmission coefficients of the sample with the introduction of disorder. The ESMF exerted is $H_0 = 900\text{Oe}$. The lattice constant is $8\text{mm}$ and the radius of the ferrite cylinder is $r_s = \frac{1}{4}a = 2\text{mm}$.