New interpretation to zitterbewegung

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In previous investigations on zitterbewegung (zbw) of electron, it is believed that the zbw results from some internal motion of electron. However, all the analyses are made at relativistic quantum mechanical level. In framework of quantum field theory (QFT), we find that the origin of zbw is different from previous conclusion. Especially, some new interesting conclusions are derived at this level: 1) the zbw arises from the rapid to-and-fro polarization of the vacuum in the range of the Compton wavelength (divided by $4\pi$) of the electron, which offer the four-dimensional (4D) spin and intrinsic electromagnetic-moment tensor to the electron; 2) Any attempt that attributes spin (rather than double the spin) of the electron to some kind of orbital angular momentum would not be successful; 3) the macroscopic classical speed of the Dirac vacuum medium vanish in all inertial systems.

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I. INTRODUCTION

The investigation of phenomenon of zitterbewegung (zbw) has made great process since it was first proposed by Schrödinger [1]: the description and interpretation of the zbw [2,3] are tried; many “classical models” of the Dirac electron basing on the zbw [4,5] are constructed; the zbw of other particles such as neutrino, quark, photon and Anyon [6,7,8] are studied; the theories such as black hole, superstring, cosmology, and gravity, have been constructed by means of the zbw [9,10] also.

As for the origin of the zbw of electron, more and more people believe that it results from some internal motion of electron. However, all the work on the origin of zbw is examined only at the relativistic quantum mechanical level, where the zbw’s interpretations are based on the probability-amplitude interpretation of wave function. Starting from the velocity operator or the current density of the electron, people obtained the zbw term expressed in term of the spinor wave functions and the Dirac matrices, but they have not so far given any further calculation for this zbw term, let alone giving its explicit and exact form, which is exactly the key to a complete and correct understanding of the zbw.

When we work at the QFT level, we find previous understanding to the origin of zbw is ambiguous or misleading. In this paper, we study the origin of zbw and drew some new interesting conclusions, which would do help to our understanding to zbw.

Firstly, starting from the current density of a free electron, we gain a explicit and exact expression to the zbw current vectors in term of the creation and annihilation operators as well as the helicity states of vector field. We find that the zbw currents come from the convection current and the current related to the intrinsic electric-moment, while the current related to the internal magnetic-moment does not contribute to the zbw current. Furthermore, we draw conclusions as follows: 1) the zbw arises from the rapid to-and-fro polarization of the vacuum in the range of the Compton wavelength (divided by 4\(\pi\)) of the electron, which offers an internal dynamic degrees of freedom (i.e. the 4D spin), or the 4D internal electromagnetic-moment tensor, to the electron. 2) The electron-positron pair arising from the vacuum polarization forms a vector triplet with the \(\pi\) of the vacuum in the range of the Compton wavelength (divided by 4\(\pi\)). Then the zbw velocity projections are \(\pm \frac{\mu}{E}\) (in the longitudinal direction) or \(\pm 1\) (in the transversal direction).

In addition, we find a united expression for the 4D total electromagnetic-moment tensor of the electron, which holds in both classical and quantum mechanics. In fact, because of the zbw, we can attribute 2\(s_{\mu\nu}\) (the 4D-spin tensor of the electron) to the difference between the instantaneous and average 4D orbital angular momentum tensor. In other words, all the traditional attempts [3,4,11] that attribute the spin (rather than double the spin) of the electron to some kind of orbital angular momentum, would not be successful.

II. THE HELICITY STATES OF VECTOR FIELD

The helicity states of vector field play such an important role that we discuss it in detail firstly. For a vector field, the operator of the spin projection in the direction of momentum \(\vec{p}\) is \(\frac{1}{|\vec{p}|} \vec{\tau} \cdot \vec{p}\), where

\[
\tau_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -i \\ 0 & i & 0 \end{pmatrix}, \quad \tau_2 = \begin{pmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{pmatrix}, \quad \tau_3 = \begin{pmatrix} 0 & -i & 0 \\ i & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}.
\]

are the spin matrices of vector filed. Let

\[
\frac{1}{|\vec{p}|} \vec{p} \eta_\gamma = \lambda_\gamma \eta_\gamma,
\]

we have

\[
\lambda_1 = 1, \eta_1 = \frac{1}{\sqrt{2|\vec{p}|}} \begin{pmatrix} p_{1p_3 - ip_2|}\vec{p}\rangle \\ \frac{p_{1p_3 + ip_2|}\vec{p}}{p_{2p_3 + ip_2|}\vec{p}} \\ \frac{p_{1p_3 + ip_2|}\vec{p}}{-(p_{1} + ip_{2})} \end{pmatrix}, \quad \lambda_2 = -1, \eta_2 = \eta_1^*, \lambda_3 = 0, \eta_3 = \frac{1}{|\vec{p}|} \begin{pmatrix} p_1 \\ p_2 \\ p_3 \end{pmatrix}.
\]
where $\eta^*_B$ is the complex conjugate of $\eta_B$. Obviously, $\{\eta_B\}$ form a complete orthonormal basis. On the other hand, in a coordinate system formed by a orthonormal basis $\{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$, where $\vec{e}_3 = \vec{e}_1 \times \vec{e}_2 = \frac{1}{m}\vec{p}$, is transformed into the following vector form

$$\begin{align*}
\vec{n}_1 &= \frac{1}{\sqrt{2}}(\vec{e}_1 + i\vec{e}_2) \\
\vec{n}_2 &= \frac{1}{\sqrt{2}}(\vec{e}_1 - i\vec{e}_2) \\
\vec{n}_3 &= \vec{e}_3.
\end{align*}$$

Clearly, $\vec{p} \cdot \vec{n}_1 = \vec{p} \cdot \vec{n}_2 = 0$ and $\vec{n}_3 \parallel \vec{p}$. That is, $\{\vec{n}_1, \vec{n}_2, \vec{n}_3\}$ is the spinor representation of the vector basis $\{\vec{e}_1, \vec{e}_2, \vec{e}_3\}$. Correspondingly, the right-hand and left-hand circular polarization vectors are denoted by $\eta_1$ and $\eta_2$, respectively, while the longitudinal polarization one by $\eta_3$.

### III. THE ZBW FROM THE VIEWPOINT OF QFT

In free field’s case, the Gordon decomposition of the Dirac current is [24]:

$$j^\mu = \bar{\psi}\gamma^\mu \psi = \frac{1}{2m} \big[ \bar{\psi} \gamma^\mu \gamma^0 \psi - (\gamma^\mu \bar{\psi})\psi \big] - \frac{i}{2m} \vec{p} \cdot \bar{\psi} \Sigma^{\mu
u} \psi,$$

where $\bar{\psi}$ is the adjoint of the spinor wave operator $\psi$ ($\bar{\psi} = \psi^\dagger \gamma^0$), $\gamma^\mu$ is the usual Dirac matrices. $\frac{1}{2} \Sigma^{\mu
u} = \frac{i}{4} \epsilon^{\mu
u\alpha\beta} \gamma^\alpha \gamma^\beta$ is the 4D spin tensor, corresponds to which, we define $G^{\mu\nu} = \frac{2m}{i} \bar{\psi} \Sigma^{\mu\nu} \psi$ as the internal electromagnetic moment density tensor, where $e$ denotes the electron charge. Let $\epsilon^{ijk}$ denotes the totally antisymmetric tensor with $\epsilon^{123} = 1(i, j, k = 1, 2, 3)$, then

$$m^i = \frac{1}{2} \epsilon^{ijk} G_{jk}, n^i = G^{i0}$$

are the magnetic moment and the electric moment, respectively.

Traditionally, when we study the conserved Noether’s charges corresponding to the Lorentz invariance of quantum field, our interest is only focused on the spatial components of the 4D angular momentum tensor (i.e. the 3D angular momentum), whereas for the 0 – $i(i = 1, 2, 3)$ components we haven’t so far given any attention to them. However, the electron possesses not only an internal magnetic moment but also an internal electric moment. That is to say, the relevant components of 4D spin do have observable effects. As for the components of 4D orbital angular momentum tensor, we have to be confronted with the notion of time operator, which will be discussed in our later paper.

By using (6), the spatial components of $j^\mu$ can be written in form [3]:

$$\vec{j} = \psi^\dagger \vec{\alpha} \psi = \frac{1}{2m} \big[ \bar{\psi} \vec{p} \psi - (\vec{p} \psi) \big] + \frac{1}{2m} \vec{\nabla} \times (\bar{\psi} \Sigma \psi) - \frac{i}{2m} \frac{\partial}{\partial t} (\bar{\psi} \vec{\alpha} \psi),$$

where

$$\Sigma = \begin{pmatrix} \sigma & 0 \\ 0 & \sigma \end{pmatrix}, \quad \vec{\alpha} = \begin{pmatrix} 0 & \sigma \\ -\sigma & 0 \end{pmatrix}$$

with $\sigma$ being the Pauli matrices. Clearly,

$$\vec{m} = \frac{1}{2m} \bar{\psi} \Sigma \psi, \quad \vec{n} = -\frac{i}{2m} \bar{\psi} \vec{\alpha} \psi.$$  

That is, the first term of (10) is the so-called convection current, while the remaining two terms are the contributions coming from the magnetic moment $\vec{m}$ and the electric moment $\vec{n}$, respectively.

We write $\psi$ as

$$\psi = \sum_{p,s} \sqrt{\frac{m}{VE}} \left[ c(p, s) u(p, s) e^{-ipx} + d^\dagger(p, s) v(p, s) e^{ipx} \right],$$

(10)
where \( s = 1, 2 \), correspond to the spin \( \pm \frac{1}{2} \), respectively.

Let
\[
c(\vec{p}, s) \equiv c(p_0, \vec{p}, s) = c(p, s), c(-\vec{p}, s) \equiv c(p_0, -\vec{p}, s), \text{etc},
\]
each term of (13) is integrated to yield
\[
\int d^3x \frac{1}{2m} \left[ \bar{\psi} \vec{p} \psi - (\vec{p} \bar{\psi}) \psi \right] = \sum_{\vec{p}, s} \frac{\vec{p}}{E} [c^\dagger (p, s)c(p, s) - d^\dagger (p, s)d(p, s)]
\]
\[
+ \sum_{\vec{p}, s} \left( \frac{m}{E} - \frac{E}{m} \right) \lambda_s \bar{\eta}_s [c^\dagger (\vec{p}, s)d^\dagger (-\vec{p}, s)e^{i2Et} - c(-\vec{p}, s)d(\vec{p}, s)e^{-i2Et}],
\]
\[
\int d^3x \frac{1}{2m} \nabla \times (\bar{\psi} \sum \psi) = 0,
\]
\[
\int d^3x \left[ -\frac{i}{2m} \frac{\partial}{\partial t} \bar{\psi} \vec{\alpha} \psi \right] = \sum_{\vec{p}, s} \frac{E}{m} \lambda_s \bar{\eta}_s [c^\dagger (\vec{p}, s)d^\dagger (-\vec{p}, s)e^{i2Et} - c(-\vec{p}, s)d(\vec{p}, s)e^{-i2Et}]
\]
\[
+ \sum_{\vec{p}, s} \sum_{s \neq s'} \sqrt{2}\bar{\eta}_s \bar{\eta}_{s'} [c^\dagger (\vec{p}, s)d^\dagger (-\vec{p}, s')e^{i2Et} - c(-\vec{p}, s')d(\vec{p}, s)e^{-i2Et}],
\]
where \( s, s' = 1, 2, \lambda_1 = 1, \lambda_2 = -1 \) and \( \bar{\eta}_1, \bar{\eta}_2, \bar{\eta}_3 \) are given by (3).

It can be found from (12)-(14), the zbw current comes from the convection current and the current is related to the internal electric-moment, while the current related to the intrinsic magnetic-moment does not contribute to the zbw current.

That is to say, the traditional conclusion that the Gordon decomposition of \( \vec{j} \) can be regarded as the separation of \( \vec{j} \) into the convection current, originating from the moving charge only, and the current associated with the internal magnetic-moment of the electron, is incorrect!

On the other hand, \( \vec{j} = \psi^\dagger \vec{\alpha} \psi \) can be directly integrated to yield
\[
\int \vec{j} d^3x = \int d^3x \psi^\dagger \vec{\alpha} \psi = \vec{v} + \vec{z}_\parallel + \vec{z}_\perp,
\]
where
\[
\vec{v} = \sum_{\vec{p}, s} \frac{\vec{p}}{E} [c^\dagger (\vec{p}, s)c(p, s) - d^\dagger (\vec{p}, s)d(p, s)],
\]
\[
\vec{z}_\parallel = \sum_{\vec{p}, s} \frac{m}{E} \lambda_s \bar{\eta}_s [c^\dagger (\vec{p}, s)d^\dagger (-\vec{p}, s)e^{i2Et} - c(-\vec{p}, s)d(\vec{p}, s)e^{-i2Et}],
\]
\[
\vec{z}_\perp = \sum_{\vec{p}, s \neq s'} \sqrt{2} \bar{\eta}_s \bar{\eta}_{s'} [c^\dagger (\vec{p}, s)d^\dagger (-\vec{p}, s')e^{i2Et} - c(-\vec{p}, s')d(\vec{p}, s)e^{-i2Et}] + \sqrt{2} \bar{\eta}_2 \bar{\eta}_1 [c^\dagger (\vec{p}, 1)d^\dagger (-\vec{p}, 2)e^{i2Et} - c(-\vec{p}, 2)d(\vec{p}, 1)e^{-i2Et}].
\]

Then (15) agrees with (13) – (14). The only difference between (13) and (12)-(14) is that the latter hold only when \( m \neq 0 \) (which, of course, is a consequence of the fact that the Gordon decomposition holds only when \( m \neq 0 \)).

In correspondence to the particles number operator \( \hat{N} = c^\dagger (p, s)c(p, s) \) and \( \hat{N}' = d^\dagger (p, s)d(p, s) \), the operators...
\[ \hat{O} \equiv c^1(\vec{p},s)d^3(-\vec{p},s'), \hat{O}' \equiv c(-\vec{p},s')d(\vec{p},s), \text{etc} \] (19)

in (22)-(28) (s, s' = 1, 2) can be called as the vacuum-polarization operator, and \( \hat{O}(0) \) or \( \langle 0|\hat{O}|0 \rangle \) are the Dirac vacuum-polarization state, which can be regarded as the macroscopic-motion carrier of the Dirac vacuum medium. In view of the fact that the momentum \( \vec{p} \) is arbitrary and the total momentum of \( \hat{O}(0) \) or \( \langle 0|\hat{O}'|0 \rangle \) vanish, the macroscopic average velocity of the Dirac vacuum medium vanishes in all inertial systems (note that \( \vec{j} \) in (13) is free).

\( \vec{n}_1 \) emerging in \( \vec{z}_\parallel \) is parallel to \( \vec{p} \), while \( \vec{n}_1 \) and \( \vec{n}_2 \) emerging in \( \vec{z}_\perp \) are perpendicular to \( \vec{p} \); On the other hand, the spin of the states \( \vec{z}_\parallel[0|0 \rangle \) or \( \langle 0\vec{z}_\perp \) is 0, while the spin of the states \( \vec{z}_\perp[0|0 \rangle \) or \( \langle 0\vec{z}_\parallel \) is \( \pm 1 \). Then, \( \vec{z}_\parallel \) corresponds to a longitudinal-polarization vector current (note that \( m = 0, \vec{z}_\parallel = 0 \), the mass is always related to the longitudinal component of vector); and \( \vec{z}_\perp \) is a transversal-polarization vector current, which includes the right-hand (related to \( \vec{n}_1 \) and \( \vec{n}_2 \)) circular-polarization vector currents. In other words, the electron-positron pair arising from the vacuum polarization forms a vector triplet with the total momentum vanishing.

It can be seen from \( \int \vec{z}_\parallel dt \) and \( \int \vec{z}_\perp dt \), the spatial magnitudes of the zbw are \( \frac{m}{E} \) (for the longitudinal component) or \( \frac{1}{2E} \) (for the right- and left-hand circular components), and the directions of the zbw current are rotating with the angular velocity \( 2E \). Then, the zbw velocity projections are \( \pm \frac{m}{E} \) (in the longitudinal direction) or \( \pm 1 \) (in the transversal direction).

In summary, the zbw arises from the rapid to-and-fro polarization of the vacuum with the spatial range of \( \frac{m}{E} \) or \( \frac{1}{2E} \), which, just as we will discuss below, offers an internal dynamic degree of freedom (i.e. the 4D-spin) to the electron. As a result, the zbw motion is an internal one independent of the macroscopic classic motion. Accordingly, the zbw current in (13) doesn’t vanish as \( \vec{p} = 0 \).

**IV. THE 4D TOTAL ELECTROMAGNETIC-MOMENT TENSOR OF THE ELECTRON**

In view of \( x^0 = t \) and \( \dot{x}^\mu = \frac{\partial x^\mu}{\partial t} = (1, \vec{\alpha}) \), we have

\[ \bar{\psi}^\dagger \dot{x}^\mu \psi = \bar{\psi} \gamma^\mu \psi = j^\mu. \] (20)

Then, the 4D total electromagnetic-moment tensor of the electron can be defined as

\[ M^{\mu\nu} \equiv \frac{\gamma^\mu \gamma^\nu}{2}(x^\rho \dot{x}^\sigma - x^\sigma \dot{x}^\rho), \] (21)

whose components are the total electric- and magnetic-dipole moment of the electron. The definition (21) holds in both classic and quantum cases. It is easy to show that \( \gamma^0 m \) and \( V^\mu \equiv \gamma^0 \dot{x}^\mu = \gamma^\mu \) are the relativistic mass and 4D-velocity operator of the electron, respectively. Due to the zbw, \( V^\mu \) is different from the usual 4D-velocity. We define

\[ p^\mu_{ins} \equiv mV^\mu, L^{\mu\nu}_{ins} \equiv x^\mu p^\nu_{ins} - x^\nu p^\mu_{ins} \] (22)

as the 4D instantaneous momentum and the 4D instantaneous orbital angular momentum, respectively. Clearly,

\[ \bar{\psi}^\dagger M^{\mu\nu} \psi = \frac{e}{2m} \bar{\psi}(x^\mu p^\nu_{ins} - x^\nu p^\mu_{ins})\psi = \frac{e}{2m} \bar{\psi}L^{\mu\nu}_{ins}\psi. \] (23)

By applying \( \gamma^\mu \gamma^\nu = g^{\mu\nu} - i\Sigma^{\mu\nu} \) (where \( g^{\mu\nu} \) is the metric tensor with diag(1, -1, -1, -1)), we have

\[ \bar{\psi}^\dagger M^{\mu\nu} \psi = \frac{e}{2m} \bar{\psi}(x^\mu \gamma^\nu - x^\nu \gamma^\mu)\psi = \frac{e}{2m} \bar{\psi}(L^{\mu\nu} + 2S^{\mu\nu})\psi, \] (24)

where \( L^{\mu\nu} = x^\mu p^\nu - x^\nu p^\mu \) is the usual 4D momentum operator, \( S^{\mu\nu} = \frac{i}{2}\Sigma^{\mu\nu} \) is the 4D spin. By using (23) and (24), we have

\[ \bar{\psi}(2S^{\mu\nu})\psi = \bar{\psi}(L^{\mu\nu}_{ins} - L^{\mu\nu})\psi. \] (25)

That is, \( 2S^{\mu\nu} \) (rather than \( S^{\mu\nu} \)), the difference between the instantaneous and the average orbital angular momentum, has the property of orbital angular momentum, which implies that one cannot attribute the 4D
spin $S^{\mu\nu}$ of the electron to any kind of orbital angular momentum. On the other hand, it is the zbw that results in the differences between all above instantaneous quantities and the corresponding average ones. Then the zbw is the sole origin of the 4D spin of the electron, while the origin of the zbw lies in the rapid to-and-fro polarization of the vacuum within the spatial range $\frac{m}{E^1}$ or $\frac{1}{E^2}$ of the electron.

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