Annihilating Cold Dark Matter

Manoj Kaplinghat,1 Lloyd Knox,1 and Michael S. Turner1,2,3
1 Department of Astronomy and Astrophysics
The University of Chicago, 5640 S. Ellis Ave., Chicago, IL 60637, USA
2 NASA/Fermilab Astrophysics Center
Fermi National Accelerator Laboratory, PO Box 500
Batavia, IL 60510-0500, USA
3 Department of Physics, Enrico Fermi Institute
The University of Chicago, Chicago, Illinois 60637, USA
(March 19, 2002)

Structure formation with cold dark matter (CDM) predicts halos with a central density cusp, which are observationally disfavored. If CDM particles have an annihilation cross section \( \sigma v \sim 10^{-29} (m/\mathrm{GeV}) \, \mathrm{cm}^2 \), then annihilations will soften the cusps. We discuss plausible scenarios for avoiding the early Universe annihilation catastrophe that could result from such a large cross section. The predicted scaling of core density with halo mass depends upon the velocity dependence of \( \sigma v \), and s-wave annihilation leads to a core density nearly independent of halo mass, which seems consistent with observations.

95.35.+d; 95.30.Cq; 98.80.Es; 98.80.Cq

Introduction. The idea that the large-scale structure developed by gravitational instability from initially small-amplitude, adiabatic and nearly scale-invariant fluctuations is compatible with a number of observables across a wide range of length scales (e.g., from the cosmic microwave background anisotropy to the Lyman-\( \alpha \) forest). Essential to this compatibility is the existence of cold dark matter: matter which is non-baryonic, has only very weak interactions with photons and baryons, and (prior to gravitational collapse) is cold.

The greatest challenge to this otherwise successful scenario comes from the apparent discrepancy between predicted dark-matter halo-density profiles and those inferred from observations. Simulations with non-interacting cold dark matter lead to halo density profiles that are singular at the center \( 1 \), whereas observations indicate uniform density cores. In this Letter we explore the possibility that the dark matter today has a large cross-section for annihilation which results in preferential destruction in high-density regions, softening halo cores \( 3 \).

Detecting and determining the properties of the dark matter is a major goal of observational cosmology. If annihilations are indeed altering the properties of dark matter halos, then we have a new means of studying the dark matter. The interactions of the CDM particles determine both the magnitude and velocity dependence of the annihilation cross section. For example, for s-wave annihilation, \( \sigma_A|v| \) is independent of velocity and for p-wave annihilation, \( \sigma_A|v| \) is proportional to \( v^2 \). These two different dependences result in different scaling relations between core density and halo velocity dispersion, which can be tested by current observations.

As we show below, current data for high velocity dispersion systems such as clusters of galaxies to low velocity dispersion systems such as galactic satellites are consistent with the same core density of about 1 GeV/cm\(^3\) (= 0.026 \( M_\odot/pc^3 \) \( 4 \)). This scale invariance can be explained by s-wave annihilation with a cross-section \( \sigma v \sim 10^{-29} (m/\mathrm{GeV}) \, \mathrm{cm}^2 \), although future improvements in both the data and predictions will be necessary before such a statement can be made with confidence. As we shall discuss, the cosmological and astrophysical constraints on annihilating CDM point to a candidate beyond those currently favored (e.g., axion, neutralino).

Halos of Annihilating Dark Matter. Numerical simulations of structure formation in the CDM scenario show that the dark matter halos which form with a wide range of masses are all well-fit with the so-called NFW \( 4 \) form for the density profile. This form has \( \rho \propto r^{-3} \) at large \( r \) and a cuspy inner region with \( \rho \propto r^{-\alpha} \) with \( \alpha = 1 \). More recent higher resolution simulations \( 5 \) predict cusps that are even stronger, with \( \alpha \simeq 1.5 \). Nevertheless, in most of what follows, we use the NFW theory for simplicity.

To be precise, the NFW profile is

\[ \rho(r = x r_s) = \rho_s x^{-1}(1 + x)^{-2}, \]

where the value of \( \rho_s \) is determined by the mean density of the Universe at the time the halo collapsed. In CDM theory, small objects collapse first, followed later by larger ones. Thus, there is an inverse relationship between \( \rho_s \) and halo size. In Fig. 1 we show this scaling relation with halo size represented by velocity dispersion for the halo, estimated as \( \sigma_{\text{vir}} = \sqrt{GM_{\text{vir}}/2r_{\text{vir}}} \). (The virial radius, \( r_{\text{vir}} \), is defined such that the mean density inside the \( r_{\text{vir}} \) sphere is 200 times the present mean density of the Universe, and \( M_{\text{vir}} \) is the mass contained within \( r_{\text{vir}} \).)

Annihilations will alter the halo profiles near the core where the density of the dark matter particles is the highest. The annihilation rate (per particle) \( \Gamma = n\langle |\sigma|v| \rangle \) depends on the velocity dispersion. We parameterize the
velocity dependence as \( \Gamma = (\rho/m)\sigma_A v^n \) (\( n = 0 \) for s-wave; \( n = 2 \) for p-wave), where \( v \) is the velocity dispersion and \( m \) is the CDM particle mass.

![Graph showing halo density at different r values](image)

**FIG. 1.** The halo density at \( r = 0.1r_s \) (where the cusp problem becomes prominent) in structures of different size according to NFW \( \bullet \) (solid curve). The objects are characterized by their virial velocity dispersion as indicated. Annihilation lines (dashed curves), normalized to LSB galaxies, are shown for the cases \( n = 0 \) and 2. Above the line, annihilations are very important (at \( r = 0.1r_s \)) and below the line they are unimportant. For \( n = 0 \), LSB and smaller objects have their cores softened significantly, while clusters do not, consistent with observations. For \( n = 2 \), clusters would be adversely affected.

Fig. \( \bullet \) also shows in a qualitative way how annihilations affect the core structure of different objects. The annihilation lines drawn show whether or not annihilations are important at a NFW halo radius of 0.1\( r_s \) in different kinds of objects. (Note, since halo densities diverge, for any object annihilations become significant deep enough into the core.) The annihilation lines are normalized to soften the cores of low surface-brightness (LSB) spiral galaxies. Because of how annihilations scale with velocity, for \( n = 0 \) clusters remain unaffected at \( r \gtrsim 0.1r_s \), while the cores of LSBs and smaller objects are dramatically softened. For \( n = 2 \), the opposite is true, which contradicts observations that indicate the NFW profile works well for clusters. We expect any \( n > 1 \) to be inconsistent with observations. The case of \( n = 0.5 \) is interesting since the annihilation line runs parallel to the structure line (for \( \sigma_{\text{vir}} \lesssim 100 \text{ km/s} \)), implying that all the systems will be smoothed off at the same value of \( r/r_s \).

**Model Building Constraints.** For the annihilations to be effective in galaxy cores today, the annihilation rate must satisfy the (approximate) constraint:

\[
\Gamma \sim (\rho/\rho_{\text{LSB}}) (v/v_{\text{LSB}})^n H_0 ,
\]

where the subscript LSB denotes the appropriate values for a typical LSB and \( H_0 = 100h \text{ km s}^{-1} \text{ Mpc}^{-1} \) is the present expansion rate of the Universe. Outside collapsed objects today, the density of CDM is much lower and annihilations will be unimportant for \( n \geq 0 \). The early Universe is another matter as densities were much higher, \( \rho \propto T^3 \), where \( T \) is the cosmic background radiation temperature.

The figure of merit for the effectiveness of annihilations in the early Universe is measured by annihilation rate divided by the expansion rate: when \( \Gamma/H > 1 \) annihilations are effective (and vice versa). Assuming that the velocity dispersion of the CDM particles can be characterized by the background radiation temperature and normalizing the cross section to the desired value today, the temperature dependence of \( \Gamma/H \) is

\[
\frac{\Gamma}{H} \sim 10^9 \left( \frac{T}{\text{GeV}} \right) \left( \frac{T}{10^{-3}\text{m}} \right)^n \sqrt{\frac{T}{T + T_{\text{eq}}}},
\]

where \( T_{\text{eq}} \sim 1 \text{ eV} \) is the temperature at matter – radiation equality. There are three important things to note: (1) the large coefficient in front of this expression – annihilations in the early Universe are a significant consideration; (2) for \( n = -1 \), the effectiveness of annihilations is epoch independent and disastrous; and (3) for \( n > -1 \) annihilations were more important in the past.

Observational data suggest that if halos are made of annihilating CDM particles, their annihilation cross section is characterized by \( n \lesssim 1 \). Thus we will focus on \( n > -1 \), where the danger of annihilations is in the past: \( \Gamma/H > 1 \) for

\[
T > T_A \sim 10^{-3(3+n)/(1+n)} \text{GeV (m/GeV)}^{n/(1+n)} ,
\]

or 1 eV for \( n = 0 \). To ensure that early annihilations do not reduce CDM particles to negligible numbers, they must be protected against annihilation in the early Universe. We suggest two mechanisms; doubtless, there are other possibilities.

First, CDM particles could be produced late \((T < T_A)\) by the decays of another massive particle. Note that this requires a long lifetime, \( \tau > t(T_A) \sim 10^9 \text{ yrs} \), and the mass difference between the two particles should be small enough to ensure that the relativistic decay products do not make the Universe radiation dominated.

The second way of avoiding the early-Universe annihilation catastrophe is to make the mass of the annihilation product be dynamical. For example, a phase transition that takes place at \( T < T_A \) could change annihilation from being kinematically impossible to possible if the mass of the annihilation product dropped below threshold after the phase transition (or if the mass of the CDM particle rose above threshold). A variation on this theme is coupling the annihilation produced particle to a scalar field, \( \phi \), with \( \langle \phi \rangle \neq 0 \). As \( \langle \phi \rangle \) decreases, either quickly to zero as a result of a symmetry-restoring phase transition, or slowly as \( \langle \phi \rangle \) rolls to the minimum of its potential, the product particle’s mass may drop below
threshold, opening up the new annihilation channel, at $T < T_A$.

Finally, the CDM annihilation products must not include photons because their $\gamma$-ray flux would far exceed observational limits. For example for 1 GeV CDM particles, the flux would be around $10^5 \text{cm}^{-2}\text{sr}^{-1}\text{s}^{-1}$, some ten orders of magnitude above the observed diffuse $\gamma$-ray flux at 1 GeV.

**Observational Constraints.** We henceforth restrict ourselves to $n = 0$. The contribution of annihilations to the evolution of the density profile is given by

$$d[\rho(r)/\rho_A]/dt = -[\rho(r)/\rho_A]^2 t_0^{-1}, \quad (5)$$

where $\rho_A \equiv m/(\sigma A t_0)$ and $t_0$ is the age of the Universe today. Assuming the initial density profile to be NFW, the resulting density profile is

$$\rho(r) = \rho_s [x(1 + x)^2 + \rho_s/\rho_{\text{core}}]^{-1}, \quad (6)$$

where $x \equiv r/r_s$, and a core of constant density, $\rho_{\text{core}} = \rho_A$, is clearly evident. However, the mass loss due to annihilations results in adiabatic expansion of the core, such that the quantity $M(r)r$ is left invariant. This expansion results in a lower core density and one can estimate that the ratio $\rho_{\text{core}}/\rho_A$ ranges from about 0.1 (dwarf galaxies) to about 0.3 (clusters). We have verified this by more detailed numerical work which allows us to determine $\rho_{\text{core}}/\rho_A$ as a function of halo mass.

We now turn to the observable constraints on annihilating CDM. A robust prediction of the s-wave annihilation scenario is that the cores are more evident in smaller mass halos, as can be seen in Fig. 2. So we first turn to the galactic satellites in the Milky Way group, of which there are 11 known.

![FIG. 2. The solid curve is the NFW density profile. Annihilation-modified profiles are labeled by the virial mass of the halo: $10^{16} M_\odot$ (cluster) and $10^{12} M_\odot$ (galaxy).](image)

For a $10^8 M_\odot$ galactic satellite, the core radius produced by annihilations is about 1 kpc, which is about the same as the cut-off radius induced by tidal forces. Most of the galactic satellites have large velocity dispersions ($\sim 10 \text{ km/s}$) for their stellar content, which suggests that they are CDM dominated. If so, their internal velocity dispersions indicate that $\rho_{\text{core}} = \mathcal{O}(1 \text{ GeV/cm}^3)$.

We also looked at dwarf spiral galaxies and LSBs. One must use these with caution since van den Bosch et al. have recently claimed that most of the HI rotation curve data do not have sufficient spatial resolution to put meaningful constraints on the halo cusps. They do identify three nearby galaxies which have sufficient spatial resolution – NGC 247, DDO 154 and NGC 3109. van den Bosch et al. find that $0.55 < \alpha < 1.26$ for the LSB (NGC 247), and $\alpha < 0.5$ for the two dwarfs, at the 99.73% confidence level which at face value, argues for soft cores in low-mass systems. The annihilation scenario naturally explains this since the cores are more evident in low-mass systems (see Fig. 3). However, it should be noted that the error bars on the rotation velocity data are probably not a complete description of the total uncertainty and that a critical reevaluation might lead to a less stringent bound on $\alpha$, thus alleviating the discrepancy between the observed dwarf rotation curves and CDM predictions.

To estimate the cross-section required to achieve consistency with observations, we fit to the two dwarf galaxies identified above with the halo profile in Eq. 6, a thin stellar disk and the observed gas. We have included the effect of finite resolution. We find that $\rho_A \simeq 0.2 M_\odot/\text{pc}^3$ results in a good fit to both (see Fig. 3). In both cases, the outer parts of the halo (determined by $\rho_s$ and $r_s$) are consistent with NFW theory.

![FIG. 3. Rotation curve fits with $\rho_A \simeq 0.2 M_\odot/\text{pc}^3$.](image)
terpreting these results since the evidence for soft cores in clusters is largely based on just one cluster (CL 0024). A value of $\rho_A = 0.2 M_\odot / pc^3$ would produce a core density of about $0.06 M_\odot / pc^3$ in a cluster-sized object. We find that this core density is consistent with the surface density reconstruction of CL 0024 by Tyson et al. [15]. The implied CDM mass within the arc radius (of $10^7 h^{-1} kpc$) is in agreement with the quoted value of about $1.66 \times 10^{14} h^{-1} M_\odot$ for the total mass within the arc radius [14].

Discussion. The $s$-wave annihilation scenario with a cross-section of $\langle \sigma v \rangle = 10^{-29} (m/ GeV) cm^2$ produces a core density of about $1 GeV/cm^3$ over widely different scales. Intriguingly, this seems to be consistent with observations.

Apart from cuspy cores [17], simulations of non-interacting CDM also predict a much larger number of sub-halos for a galactic size halo than the observed number of galactic satellites [18]. Certainly, the $s$-wave annihilation scenario has a dramatic effect on the smallest halos, and this could contribute to their destruction. However, further study is required to test this hypothesis.

Another particle-physics solution in which CDM particles have a large cross section for self interaction ($\sigma \sim 10^{-23} cm^2$) has been discussed [19]. This possibility is being tested by numerical simulations [20]. However, there are indications that self interactions lead to halos that are inconsistent with observations [21]. The jury is still out.

The requirements on a model for annihilating CDM are stringent, but by no means impossible [24]. They point to a particle beyond those currently being considered, and therefore, to new physics. While it is possible that the solution to the CDM cusp problem will involve the interpretation of the observations or less exotic astrophysics, it is appealing to think that the properties of halo cores may teach us about the fundamental properties of the CDM particle.

ACKNOWLEDGMENTS

We gratefully acknowledge support by the DOE, NASA and NSF at Chicago, and by the DOE and NASA grant NAG 5-7092 at Fermilab. We thank M. Valluri for useful discussions and pointers. We also thank S. Carroll, D. Eisenstein, S. Hannestad, J. Mohr, D. Spergel and L. Widrow.

[1] J.F. Navarro, C.S. Frenk & S.D.M. White, Astrophys. J. 490, 493 (1997).
[2] J. Dubinski & R.G. Carlberg, Astrophys. J. 378, 496 (1991); S.W. Warren et al., Astrophys. J. 399, 405 (1992).
[3] B. Moore et al., Mon. Not. R. astron. Soc. 310, 1147 (1999) have mentioned that annihilating CDM could alleviate the cusp crisis.
[4] The invariance of the core density with respect to the mass of the halo has also recently been noted by C. Firmani et al., astro-ph/0002374.
[5] B. Moore et al., Astrophys. J. Lett. 499, 5 (1998); Y.P. Jing & Y. Suto, Astrophys. J. Lett. 529, 69 (2000).
[6] For simplicity of exposition, we have assumed that the velocity dispersion is independent of radius.
[7] D.O. Richstone & M.D. Potter, Astrophys. J. 254, 451 (1982).
[8] For simplicity, we have neglected the fact that the initial time (when the structure collapses) will depend on halo mass.
[9] M. Mateo, Annu. Rev. Astron. Astrophys. 36, 435 (1998).
[10] Tidal disruption makes the DM content of these systems uncertain; see e.g., K.S. Oh, D.N.C. Lin & S.J. Aarseth, Astrophys. J. 442, 142 (1995).
[11] C. Pryor & J. Kormendy, Astron. J. 100, 127 (1990); M. Irwin & D. Hatzidimitriou, Mon. Not. R. astron. Soc. 277, 1354 (1995).
[12] F.C. van den Bosch et al., Astron. J. 119, 1579 (2000).
[13] F.C. van den Bosch & R.A. Swaters, astro-ph/0006044.
[14] B. Moore, Nature 370, 629 (1994) has also used the two dwarf galaxies to argue against cuspy cores.
[15] J.A. Tyson, G.P. Kochanski & I.P. Dell’Antonio, Astrophys. J. 498, L107 (1998); I. Smail et al., Astrophys. J. 469, 508 (1996); A. Kassiola, I. Kovner & B. Fort, Astrophys. J. 400, 41 (1992); T. Broadhurst et al., astro-ph/9902316.
[16] This is higher than the estimates of Smail et al. [13] and Kassiola et al. [12] who calculate a mass of about $1 \times 10^{14} h^{-1} M_\odot$, in agreement with the recent work by Broadhurst et al. [13].
[17] The observation of bars in high surface brightness galaxies (including the Milky Way) have also been used to argue against cuspy cores by V.P. Debattista & J.A. Sellwood, Astrophys. J. Lett. 493, 5 (1998).
[18] G. Kaufmann, S.D.M. White & B. Guiderdoni, Mon. Not. R. astron. Soc. 264, 201 (1993); A. Klypin et al., Astrophys. J. 522, 82 (1999); B. Moore et al., Astrophys. J. 524, L19 (1999).
[19] D.N. Spergel & P.J. Steinhardt, Phys. Rev. Lett. 84, 3760 (2000).
[20] N. Yoshida et al., astro-ph/0002362; C.S. Kochanek & M. White, astro-ph/0003480; R. Davé et al., astro-ph/0006218.
[21] J. Miralda-Escude, astro-ph/0002056.
[22] A. Riotto & I. Tkachev (astro-ph/0003388) have recently proposed a model where the halo is made of a coherent, self-interacting scalar field.