A fraction of dark matter faded with early dark energy?

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Abstract

In pre-recombination early dark energy (EDE) resolutions of the Hubble tension, the rise of Hubble constant value $H_0$ is usually accompanied with the exacerbation of so-called $S_8$ tension. Inspired by the swampland conjecture, we investigate what if a fraction $f_\ast$ of dark matter is coupled to EDE, $m_{cdm} \sim \exp\left(-c|\Delta \phi_{ede}|/M_{pl}\right)$ with $c \sim O(1)$. We perform the MCMC analysis for the relevant EDE models with PlanckCMB, BAO, Pantheon and SH0ES dataset, as well as DES-Y1 data, and find that such a fraction helps to alleviate the $S_8$ tension. However, though $c \gtrsim 0.1$ is allowed for a very small $f_\ast$, which suggests that a small fraction of dark matter has ever faded with EDE, $c \sim 0$ is also consistent.

PACS numbers:
I. Introduction

There is a $5\sigma$ conflict between the Hubble constant $H_0 \sim 67\text{km/s/Mpc}$ inferred by Planck collaboration [1] using cosmological microwave background (CMB) data based on $\Lambda$CDM model and that obtained by SH0ES in light of Cepheid-calibrated SN data, $H_0 \sim 73\text{km/s/Mpc}$ [2], which is so-called Hubble tension [3, 4], see [5–7] for reviews. Currently, it seems impossible to explain this conflict by systematic errors, thus it has been widely thought that this tension signals new physics beyond $\Lambda$CDM.

The Hubble tension is possibly resolved with early dark energy (EDE) [8–10]. Here, the EDE is non-negligible only before recombination, which suppressed the sound horizon and so naturally brings a high $H_0$ without spoiling fit to CMB and baryon acoustic oscillations (BAO) data. In particular, if an Anti-de Sitter (AdS) phase existed around recombination (AdS-EDE model [11, 12]), we would have the bestfit $H_0 \simeq 73\text{km/s/Mpc}$, which is $1\sigma$ consistent with local $H_0$ measurement. Recently, besides Planck data, combined analysis of CMB data have been also performed for EDE, such as Planck+SPT [13–15], Planck+ACT
In such pre-recombination EDE resolutions, the rise of $H_0$ is usually accompanied with the exacerbation of so-called $S_8$ tension [23–25], see also [26, 27], where

$$S_8 = \sigma_8 \left( \frac{\Omega_m}{0.3} \right)^{1/2},$$

and $\sigma_8$ is the amplitude of matter perturbations at $8h^{-1}\text{Mpc}$ scale. It is well-known that $S_8 \sim 0.82$ for $\Lambda$CDM and $S_8 \gtrsim 0.84$ for EDE, while the local large-scale structure observations [28–30] have reported lower $S_8 \sim 0.76$. Recently, the resolution of $S_8$ tension has been intensively studied, which might be completely independent of EDE, such as Dark matter (DM)-DE drag [31] at low redshifts, ultra-light axion as a little part of DM [32–34], Decaying DM [35–37], massive neutrino [38], and also [39, 40].

However, the DM physics responsible for the $S_8$ tension might also be relevant with EDE, e.g. [41–43]. The conformal coupling of DM to EDE has been considered in Ref.[41], see also [44, 45] for neutrino-assisted EDE. The coupled EDE and the impact of massive neutrinos also has been studied in Ref.[46]. The evolution of our Universe must be implemented in a UV-complete effective field theory (EFT). It has been argued in Ref.[47] that such EFTs must satisfy the *swampland distance conjecture* (SDC): the excursion of any field must comply with $|\Delta \phi| \lesssim M_{pl}$, or it will cause the exponential suppression of the mass of other fields in EFT. Thus it is possible that DM might be exponentially lightened (called “fading dark matter” [48]) with the evolution of EDE. However, in such an early dark sector [42], the results favored by current datasets seem conflicted with SDC.

It might be also possible that not all but only a fraction of DM coupled EDE. In this paper, we will investigate such a coupling in Axion-like EDE and AdS-EDE models, respectively. In section-II, we comment the correlation of $S_8 - H_0$, and outline our setup in section-III. In sections-IV and V, we perform the Markov Chain Monte Carlo (MCMC) analysis with PlanckCMB, BAO, Pantheon and SH0ES dataset, as well as full DES-Y1 dataset, and report our results. We conclude in sections-VI.

II. $S_8 - H_0$ IN EDE

It is well-known that Planck dataset strictly set the angular scale

$$\theta_{CMB} = \frac{r_s}{D_A} \sim r_s H_0.$$  

(2)
where $D_A = \int_0^{z_*} \frac{dz}{H(z)}$ is the angular radius to last scattering surface, $r_s = \int_{z_*}^{\infty} \frac{dz}{H(z)}$ is the sound horizon, and $z_*$ is the redshift of last scattering. In pre-recombination EDE setup, $r_s$ is suppressed (see [49] for recent result) so that we have a high $H_0$ in light of (2). Recently, the relevant EDE models have widely studied e.g.[10–12, 50–61], and [62–67], see also its effects on cosmic birefringence [68, 69] and gravitational waves background [70, 71].

It has been showed in Ref.[72] that even if the state equation $w(z)$ of DE after recombination evolved with the redshift $z$, its rise for the bestfit $H_0$ is also negligible. However, though the post-recombination beyond-ΛCDM modification seems difficult to resolve the Hubble tension, it is still worth exploring, e.g.[73–86], see also [87–89] (the physics of our Universe might be abruptly interrupted at redshifts $z = 0.01$ in the past), or it might be also possible that flat ΛCDM model is breaking down [90–92].

In pre-recombination EDE resolutions, the cosmological parameters must shift with $\delta H_0$, particularly the shift of $\omega_m = \Omega_m h^2$ scales approximately [12]

$$\delta \omega_m \simeq 2 \frac{\delta H_0}{H_0} \omega_m, \quad (3)$$

since PlanckCMB+BAO dataset required $\omega_m \sim H_0^2$ (or $\Omega_m \sim const.$). The dust-like matter will cluster in the matter-dominated era, so higher $H_0$ will proportionally bring a higher $S_8$. Thus the EDE will inevitably suffer from the exacerbation of $S_8$ tension, see Fig.1.

In the standard EDE+ΛCDM model, all DM not only participated in the background evolution of the Universe but also is responsible for perturbation growth, which naturally results in (3) and so suggests exacerbated $S_8$ tension for EDE. There might, however, other possible matter forms or coupling which can break the correlation between $S_8$ and $\omega_m$. In this sense, the $S_8$ tension is actually an opportunity of understanding CDM physics.

### III. DARK MATTER FRACTIONALLY COUPLED TO EDE

The SDC suggests [47] that any EFT is only valid in field space bounded by the Planck scale, $|\Delta \phi| < M_{pl}$, and its breakdown that occurs at Planckian field excursions is encoded in the mass spectrum of other fields, $m \sim \exp (-c|\Delta \phi|/M_{pl})$, where $c \simeq O(1)$, see [93, 94] for reviews.

Inspired by the SDC, we consider the couple of DM with EDE. The dark matter is
FIG. 1: The $S_8 - H_0$ contour for $\Lambda$CDM, Axion-like EDE and AdS-EDE models. The shadows correspond to the 1$\sigma$ and 2$\sigma$ regions of $H_0$ in light of recent SH0ES $H_0 = 73.04 \pm 1.04$km/s/Mpc [2] and $S_8$ in light of KiDS+VIKING-450+DES-Y1 constraint $S_8 = 0.755 \pm 0.02$ [28], respectively. It is clearly seen that the EDE also proportionally lift $S_8$ while lift $H_0$.

modelled as a population of non-relativistic Dirac fermions $\psi$,

$$L_{int} \sim -m_{cdm}(\phi)\bar{\psi}\psi,$$

$$m_{cdm}(\phi) = f_s m(\phi) + (1 - f_s)m_i, \quad \text{with} \quad 0 \leq f_s \leq 1,$$

$$m(\phi) = m_i e^{-c\left(\frac{|\Delta \phi| - \phi_*}{M_{pl}}\right)}, \quad \text{for} \quad |\Delta \phi| \geq \phi_*.$$

where $m_i = \text{const}$ is the initial mass of DM, $|\Delta \phi| = |\phi - \phi_i|$ (see Fig.2), $\phi_*$ signals the insensitivity of DM on a shift of $\phi$ within $|\Delta \phi| < \phi_*$, and $c$ is the coupling intensity. Here, when $c = 0$, we have $m_{cdm} = m_i$ (the standard EDE+$\Lambda$CDM model is recovered).

In non-relativistic limit, we have $\rho_{cdm} = nm_{cdm}(\phi)$, so

$$\rho_{cdm} = nf_s m(\phi) + n(1 - f_s)m_i,$$
with \( n \) being the number density, which suggests that \( f_* \) is actually equivalent to the fraction of DM coupled to EDE.

Here, we follow Ref.[42]. The evolution of EDE is rewritten as \( \phi'' + 2H\phi' + a^2 V_{\phi} = -a^2 \frac{d\rho_{cdm}}{d\phi} \), while the continuity equation for DM is

\[
\rho_{cdm}' + 3H\rho_{cdm} = \phi' \frac{d\rho_{cdm}}{d\phi}, \tag{8}
\]

where the prime is the derivative with respect to \( d\eta = dt/a \), and \( H = a'/a \). Integrating Eq.(8), we have

\[
\rho_{cdm}(a) = \frac{3M_{pl}^2 H_0^2 \Omega_{cdm}}{a^3} \left[ 1 - f(\phi_0) + \frac{m(\phi)}{m(\phi_0)} f(\phi_0) \right], \tag{9}
\]

where \( f(\phi) = \frac{m(\phi)f_*}{m(\phi)f_* + m_i (1 - f_*)} \), and \( \phi_0 \) is the present-day value of \( \phi \). In axion-like EDE model, see Fig.2(a), \( \phi - \phi_i < 0 \) for \( \phi_i > 0 \), so \( d\rho_{cdm}/d\phi = c\rho_{cdm} f(\phi)/M_{pl} \). This suggests that \( \phi_0 \) must be obtained by solving the equation of motion.

FIG. 2: A sketch of the EDE potential \( V(\phi) \) and \( m(\phi) \) in Axion-like EDE and AdS-EDE models, respectively. Initially the field sits at \( \phi_i \), after its excursion \( |\Delta \phi| > \phi_* \), the mass of DM will be exponentially lightened with the evolution of \( \phi \).

In the synchronous gauge, with \( \rho_{cdm} \) in Eq.(9), the perturbations equations have been derived fully in Ref.[42]. However, in AdS-EDE model \( \phi - \phi_i > 0 \), see Fig.2(b), so \( d\rho_{cdm}/d\phi = -c\rho_{cdm} f(\phi)/M_{pl} \). This suggests that \( \phi_0 \) must be obtained by solving the equation of motion,
so it is not convenient to use Eq.(9). Integrating Eq.(8), instead we have

\[
\rho_{\text{cdm}}^{(\text{AdS})}(a) = \frac{3M_p^2 H_0^2 \Omega_{\text{cdm}}}{a^3} [1 - f(\phi_0) + (1 - f(\phi_0)) \frac{f_*}{1 - f_*} \frac{m(\phi)}{m(\phi_i)}]
\]

\[
= \frac{3M_p^2 H_0^2 \tilde{\Omega}_{\text{cdm}}}{a^3} [1 + f^{(\text{AdS})}_* \frac{m(\phi)}{m(\phi_i)}],
\]

(10)

where \(\tilde{\Omega}_{\text{cdm}} = \Omega_{\text{cdm}} (1 - f(\phi_0))\) is defined to absorb \(\phi_0\) and \(f^{(\text{AdS})}_* = f_*/(1 - f_*)\).

IV. DATASET AND RESULTS

Here, our baseline dataset consists of:

1. **CMB**: Planck 2018 low-l and high-l TT, TE, EE spectra, and reconstructed CMB lensing spectrum [1, 95, 96].

2. **BAO**: The BOSS DR12 [97] with its full covariant matrix for BAO as well as the 6dFGS [98] and MGS of SDSS [99].

3. **Supernovae**: The Pantheon dataset [100].

4. **SH0ES**: To avoid the prior volume effect \(^1\) [101–103], which will compel the EDE models prefer a low \(f_{\text{ede}}\), we take \(H_0 = 73.04 \pm 1.04\) km/s/Mpc reported by the SH0ES [2] as the Gaussian prior, see also [104, 105].

We modified the MontePython-3.3 sampler [106, 107] and CLASS codes [108, 109]\(^2\) to perform the MCMC analysis for axion-like EDE and AdS-EDE, respectively, with baseline dataset and baseline+DES-Y1 dataset, see [110] for DES-Y1 data. The Gelman-Rubin criterion for all chains is converged to \(R - 1 < 0.05\).

A. Axion-like EDE

The original EDE model is: axion-like EDE [9]. An axion field with \(V(\phi) \propto (1 - \cos[\phi/f])^3\) is responsible for EDE (see recent [111, 112] for models in string theory), which starts to oscillate at the redshift \(z_c \sim 3000\). It is usually parameterized by \(\phi_i\), \(a_c\) and \(f_{\text{ede}}\) [9, 10]. In Table.I, we present the MCMC results for axion-like EDE with the baseline dataset

\(^1\) In AdS-EDE model, the prior volume effect is actually removed by AdS bound, as explained in [15, 22].
\(^2\) The corresponding cosmological codes are available at: axion-like EDE (https://github.com/PoulinV/AxiCLASS) and AdS-EDE (https://github.com/genye00/class_multiscf).
and baseline+DES-Y1 dataset. In Fig.3, we show the 1σ and 2σ marginalized posterior distributions of parameter set \{ω_b, ω_{cdm}, H_0, \ln(10^{10} A_s), n_s, τ_{reio}, \log_{10} a_c, f_{ede}, ϕ_i, c, ϕ_s, f_s\).

Though with the baseline dataset, we have the bestfit \(S_8 = 0.8438\), which is larger than local \(S_8\) measurements, the baseline+DES-Y1 dataset prefers a lower \(S_8\) (the bestfit is \(S_8 = 0.8186\), which almost equals to \(S_8 = 0.8156\) in \(ΛCDM\)), than that with only baseline dataset. However, the cost is that the bestfit \(H_0 = 70.14\) is lowered.

In Table.I, we see that with baseline dataset, \(c \sim 0\) at 1σ region, consistent with the result in Ref.[42], which suggests that such a coupling (4) is not favored, while the case is not altered with baseline+DES-Y1 dataset. The bestfit of \(c\) is negative and inconsistent with SDC, but the possibility of \(c > 0\) is not ruled out.

| Parameters | ACDM baseline | ACDM baseline+DES-Y1 |
|------------|---------------|----------------------|
| 100ω_b     | 2.252(2.249) ± 0.013 | 2.284(2.286) ± 0.020 |
| ω_{cdm}    | 0.1182(0.1184) ± 0.0008 | 0.1306(0.1290) ± 0.0020 |
| H_0        | 68.21(68.16) ± 0.39 | 71.66(70.87) ± 0.63 |
| \ln(10^{10} A_s) | 3.052(3.052) ± 0.015 | 3.060(3.051) ± 0.013 |
| n_s        | 0.9691(0.9686) ± 0.0035 | 0.9889(0.9834) ± 0.0056 |
| τ_{reio}   | 0.0595(0.0594) ± 0.0073 | 0.0576(0.0571) ± 0.0064 |
| f_{ede}    | -              | 0.116(0.101) ± 0.017 |
| \log_{10} a_c | -              | -3.748(-3.841) ± 0.137 |
| c          | -              | 0.289(-0.129) ± 0.472 |
| ϕ_s        | -              | 0.305(0.361) ± 0.147 |
| f_s        | -              | 0.183(0.222) ± 0.229 |
| S_8        | 0.8140(0.8156) ± 0.0098 | 0.8451(0.8438) ± 0.0112 |

TABLE I: Mean(best-fit) values of \(ΛCDM\) and Axion-like EDE with coupling (6) in fit to the baseline and the baseline+DES-Y1 datasets, respectively.

B. AdS-EDE

In AdS-EDE model [11], we have \(V(ϕ) = V_0(ϕ/M_p)^4 - V_{ads}\), see Fig.2(b), which is glued to a cosmological constant \(V(ϕ) = Λ\) by interpolation \((V_{ads} > 0\) is the AdS depth). Here, the scalar field starts to roll at the redshift \(z_c \sim 3000\), and then rolls over an AdS minimum like
FIG. 3: Posterior distributions for Axion-like EDE with coupling (6) in fit to the baseline and the baseline dataset+DES-Y1 datasets, respectively. The shadows correspond to the 1σ and 2σ regions of $H_0$ in light of recent SH0ES [2].

a fluid with $w > 1$. It climbs up to the $\Lambda > 0$ region shortly after recombination, hereafter the Universe will be effectively described by the $\Lambda$CDM model. It is well-known that AdS vacua are ubiquitous in string landscape, so the AdS-EDE model can be well-motivated, see also [113–120] for other studies on the implications of AdS vacua for our Universe.

The AdS-EDE model is usually parameterized by $V_{ads}$, $z_c$ and $f_{ede}$. In order to have a significant AdS phase while make the field able to climb out of the AdS well, we fixed $V_{ads}$ by setting $V_{ads} = 0.26 \times 10^4 (\rho_m(z_c) + \rho_r(z_c))$, as in Ref.[11]. In Table.II, we present the MCMC results for AdS-EDE with the baseline dataset and baseline+DES-Y1 dataset. In Fig.4, we show the 1σ and 2σ marginalized posterior distributions of parameter set
\{\omega_b, \omega_{cdm}, H_0, \ln(10^{10}A_s), n_s, \tau_{reio}, \ln(1+z_c), f_{ede}, c, \phi_s, f_s}\}.

Though the baseline+DES-Y1 dataset prefers a lower \(S_8\) (the bestfit is \(S_8 = 0.8433\)) than that with only baseline dataset, it is still larger than that in \(\Lambda\)CDM. However, unlike in axion-like EDE, \(f_{ede}\) is not suppressed by the coupling (4), since \(f_s \sim 0.02\) is fairly small. It is also noted that with baseline+DES-Y1 dataset, we have the bestfit \(H_0 = 73.33\), which is slightly higher than that in AdS-EDE [11].

In Table.I, we see that with baseline dataset, \(c \sim 0.4\) at 1\(\sigma\) region, which is different from that in axion-like EDE (see section-III.A) and consistent with SDC, and with baseline+DES-Y1 dataset \(c \gtrsim 0.1\) at 1\(\sigma\) region. However, in both case \(c \sim 0\) is still 1\(\sigma\) consistent. In AdS-EDE, \(f_s \sim 0.03\) is smaller than that in axion-like EDE, and \(f_s = 1\) is ruled out at 2\(\sigma\).

| Parameters          | \(\Lambda\)CDM          | AdS-EDE            |
|---------------------|--------------------------|--------------------|
|                     | baseline                 | baseline+DES-Y1    |
| \(100\omega_b\)     | 2.252(2.249) ± 0.013     | 2.328(2.327) ± 0.014 |
| \(\omega_{cdm}\)    | 0.1182(0.1184) ± 0.0008  | 0.1298(0.1299) ± 0.0035 |
| \(H_0\)             | 68.21(68.16) ± 0.39      | 72.01(72.05) ± 0.51 |
| \(\ln(10^{10}A_s)\) | 3.052(3.052) ± 0.015     | 3.076(3.088) ± 0.014 |
| \(n_s\)             | 0.9691(0.9686) ± 0.0035  | 0.9963(0.9973) ± 0.0035 |
| \(\tau_{reio}\)     | 0.0595(0.0594) ± 0.0073  | 0.0579(0.0601) ± 0.0075 |
| \(f_{ede}\)         | -                        | 0.1061(0.1002) ± 0.0076 |
| \(\ln(1+z_c)\)      | -                        | 8.2697(8.2138) ± 0.0958 |
| \(c\)               | -                        | 0.367(0.302) ± 0.434 |
| \(\phi_s\)          | -                        | 0.333(0.323) ± 0.136 |
| \(f_s\)             | -                        | 0.030(0.008) ± 0.025 |
| \(S_8\)             | 0.8140(0.8156) ± 0.0098  | 0.8554(0.8610) ± 0.0097 |

TABLE II: Mean(best-fit) values of \(\Lambda\)CDM and AdS-EDE with coupling (6) in fit to the baseline and the baseline+DES-Y1 datasets, respectively.

We list the \(\chi^2\) of bestfit points for axion-like EDE and AdS-EDE models in Tables.III and IV, respectively. In Tables.III, only with baseline dataset, we see that both models have improvements over the bestfit \(\Lambda\)CDM by \(\Delta\chi^2 \sim -19\), where the \(\chi^2\) of Planck low-l TT, EE and \(H_0\) are significantly improved while the \(\chi^2\) of BAO is slightly exacerbated. In Table.IV with baseline+DES-Y1 dataset, we see that both models have improvements over the bestfit \(\Lambda\)CDM but by only \(\Delta\chi^2 \sim -6\). In both axion-like EDE and AdS-EDE models, compared
FIG. 4: Posterior distributions for AdS-EDE with coupling (6) in fit to the baseline and the baseline+DES-Y1 datasets, respectively. The shadows correspond to the 1σ and 2σ regions of $H_0$ in light of recent SH0ES [2].

with $\Lambda$CDM, the $\chi^2$ of DES-Y1 is exacerbated with $\Delta \chi^2 \sim 3$ and $\Delta \chi^2 \sim 8$, respectively.

We also plot the TT, EE and TE residuals $\Delta C_l = C_{l,model} - C_{l,\Lambda}$ of both models in units of the cosmic variance per multipole

$$\sigma_{CV} = \begin{cases} 
\sqrt{2/(2l+1)}C_{l,TT}^T, & TT \\
\sqrt{1/(2l+1)}\sqrt{C_{l,TT}^T C_{l,EE} + (C_{l,TE}^T)^2}, & TE \\
\sqrt{2/(2l+1)}C_{l,EE}^T, & EE 
\end{cases}$$

in Figs.5 and 6. The TT residual becomes comparable to $\sigma_{CV}$ at $l \sim 700$ for axion-like EDE (but is suppressed by DES-Y1 dataset), while DES-Y1 significant impacts the TT, EE and TE residuals of AdS-EDE.
| Dataset            | $\Lambda$CDM  | Axion-EDE uncoupled | AdS-EDE uncoupled |
|--------------------|---------------|---------------------|------------------|
| Planck high-l TT,TE,EE | 2347.50       | 2344.27             | 2349.29          |
| Planck low-l EE    | 398.2         | 398.19              | 396.06           |
| Planck low-l TT    | 23.9          | 20.56               | 20.83            |
| Planck lensing     | 9.10          | 10.12               | 9.46             |
| BAO BOSS DR12      | 1.8           | 3.46                | 3.42             |
| BAO smallz 2014    | 2.2           | 2.06                | 1.92             |
| Pantheon           | 1026.9        | 1026.68             | 1026.87          |
| SH0ES              | 15.40         | 1.38                | 3.08             |
| Total              | 3825          | 3811.79             | 3806.04          |
| $\Delta \chi^2$    | 0             | -13.21              | -18.96           |

**TABLE III:** $\chi^2$ of both Axion-like EDE and AdS-EDE for the baseline dataset, where “uncoupled” corresponds to the models without coupling (6).

| Dataset            | $\Lambda$CDM  | Axion-EDE | AdS-EDE |
|--------------------|---------------|-----------|---------|
| Planck high-l TT,TE,EE | 2354.43       | 2354.31   | 2357.54 |
| Planck low-l EE    | 398.09        | 395.80    | 395.79  |
| Planck low-l TT    | 21.94         | 20.36     | 20.19   |
| Planck lensing     | 9.10          | 10.26     | 10.58   |
| BAO BOSS DR12      | 4.56          | 3.49      | 3.99    |
| BAO smallz 2014    | 3.01          | 2.14      | 2.67    |
| Pantheon           | 1027.17       | 1026.98   | 1026.93 |
| SH0ES              | 13.20         | 7.72      | 0.45    |
| DSE-Y1             | 517.73        | 520.82    | 525.28  |
| Total              | 4349.27       | 4342.94   | 4343.46 |
| $\Delta \chi^2$    | 0             | -6.33     | -5.81   |

**TABLE IV:** $\chi^2$ of both Axion-like EDE and AdS-EDE with coupling (6) for baseline+DES-Y1 dataset.
FIG. 5: The TT, EE and TE residuals $\Delta C_l/\sigma_{CV}$ for Axion-EDE model with coupling (6) in fit to the baseline and baseline+DES-Y1 datasets, respectively. The reference model is $\Lambda$CDM.

FIG. 6: The TT, EE and TE residuals $\Delta C_l/\sigma_{CV}$ for AdS-EDE model with coupling (6) in fit to the baseline and baseline+DES-Y1 datasets, respectively. The reference model is $\Lambda$CDM.
V. HAS DM EVER FADED ?

In Fig.7, we see that the smaller $S_8$ caused by DES-Y1 dataset brought with a lower bestfit $\omega_{cdm} = 0.1236$ and $H_0 = 70.14$ for axion-like EDE, but a higher bestfit $\omega_{cdm} = 0.1302$ and $H_0 = 73.33$ for AdS-EDE. Thus though (3) is still right for both models with the coupling (6), such a coupling actually impairs the correlation between $\omega_m$ and $S_8$, so that the rise of $H_0$ must not be accompanied with the exacerbation of $S_8$ tension.

![Fig. 7: The $S_8 - H_0$ contour of Axion-like EDE and AdS-EDE with the coupling (6) in fit to the baseline and baseline+DES-Y1 datasets, respectively. The shadows correspond to the 1σ and 2σ regions of $H_0$ in light of recent SH0ES $H_0 = 73.04 \pm 1.04$km/s/Mpc [2] and $S_8$ in light of KiDS+VIKING-450+DES-Y1 constraint $S_8 = 0.755 \pm 0.02$ [28].](image)

Though in axion-like EDE and AdS-EDE models $c = 0$ is 1σ consistent, however, the 1σ contour of $c$ is wide so that $c \gtrsim 0.1$ is also allowed due to a small $f_*$, see Table.1. In Fig.8, we plot the evolution of the scalar field, $f_{ede}$, and the mass $m_{cdm}(\phi)$ of DM in both models with their bestfit values. The baseline dataset allows a higher $c \gtrsim 0.1$, so a larger shift of $m_{cdm}(\phi)$, which suggests that a small fraction of DM ($f_* \sim 0.2$ and $f_* \sim 0.03$ for both models) will fade with EDE. However, it should be mentioned that in axion-like EDE the bestfit of $c \sim 0$ is a negative value, consistent with the result in Ref.[42].

However, after the DES-Y1 dataset included, we have $c \sim 0$ further, particularly for
axion-like EDE with smaller excursion of scalar field and a lower $f_{ede}$. Thus in both the axion-like EDE and AdS-EDE models the fading of DM is actually not be favored by the baseline+DEY-Y1 dataset, but it can not be ruled out at present.

![Graph of scalar field evolutions](image)

**FIG. 8:** The evolutions of the scalar field, $f_{ede}$ and $m_{cdm}(\phi)$ in Axion-like EDE and AdS-EDE models with their bestfit values.

VI. CONCLUSIONS

Inspired by SDC, we investigated the impact of DM fractionally coupled to EDE, specially the possibility of resolving $S_8$ tension. We performed the MCMC analysis for Axion-like EDE and AdS-EDE, respectively, with PlanckCMB, BAO, Pantheon and SH0ES dataset (our baseline dataset), as well as DES-Y1 dataset.

It is found that the $S_8$ tension can be alleviated with such a fractional coupling. In axion-like EDE model, the baseline+DES-Y1 dataset prefers a lower $S_8$ (the bestfit is $S_8 \simeq 0.82$, which almost equals to $S_8 = 0.8156$ in $\Lambda$CDM), however, the cost is a lower bestfit $H_0 = 70.14$, while in AdS-EDE model, though the baseline+DES-Y1 dataset prefers a lower $S_8$ (the bestfit is $S_8 = 0.8433$), it is still larger than that in $\Lambda$CDM.

The baseline+DES-Y1 dataset allows $c \gtrsim 0.1$ in both EDE models due to a small $f_* \ll 1$ (bestfit $f \sim 0.5$ for axion-like EDE while a smaller $f_* \sim 0.02$ for AdS-EDE), which so is compatible with SDC and suggests that a small fraction of DM has ever faded with EDE. However, $c \sim 0$ is still at 1$\sigma$ region, particularly in axion-like EDE the bestfit of $c$ is a negative value, consistent with the result in Ref.[42].

It is worth mentioning that in EDE models with fullPlanck+BAO+Pantheon dataset, the shift of primordial scalar spectral index scales as $\delta n_s \simeq 0.4 \frac{\Delta H_0}{H_0}$ [121], which suggests a scale-
invariant Harrison-Zeldovich spectrum \((n_s = 1)\) for \(H_0 \sim 73\text{km/s/Mpc}^3\). In Refs.[19, 20], with Planck+ACT+SPT+BAO+Pantheon dataset, similar results have also been found. Here, we observed that the preference for \(n_s = 1\) is not altered by the inclusion of large-scale structure DES-Y1 dataset (see recent [122] for BOSS dataset), see Table-I,II. In this sense, the Hubble tension seems to hint that we might live in a scale-free Universe, so it is interesting to think about how \(n_s = 1\) would dramatically impact our understanding on the primordial Universe and inflation [121, 123–128].

Acknowledgments

We thank Gen Ye, Jun-Qian Jiang for helpful discussions. HW is supported by UCAS Undergraduate Innovative Practice Project. YSP is supported by the NSFC, No.12075246 and by the Fundamental Research Funds for the Central Universities. We acknowledge the use of publicly available codes AxiCLASS (https://github.com/PoulinV/AxiCLASS) and classmultiscf (https://github.com/genye00/class_multiscf.git).

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