Study of Two-Frequency Mutual Coherence Function From Two-Dimensional Rough Surfaces

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Abstract. Study of pulse scattering from two-dimensional rough surfaces is of great theoretical and engineering significance. In this paper the analytical expression of two-frequency mutual coherence function of the scattered wave from two-dimensional rough surfaces is derived with the integral equation method, which includes both the single scattering and the multiple scattering and therefore is applicable to surfaces with small or large-scale roughness. Then the numerical simulation of the two-frequency mutual coherence function from two-dimensional surfaces with Gaussian distribution is performed, and the results are compared with those with Kirchhoff approximation.

1. Introduction

Recently with the wide application of pulse techniques in the fields of communication, radar and remote sensing, research on pulse scattering has attracted more and more attention. For pulse scattering from rough surfaces, most rough surfaces in nature such as ocean and terrain are two-dimensional and electrically large in the microwave region, therefore it is difficult to solve the scattering by time domain methods such as Time Domain Moment (TDM) method and Finite-Difference Time-Domain (FDTD) method[1,2] due to the limit of computer memory and running time. By investigating the propagation of pulse waves in random media, Bello and Ishimaru found that the propagation characteristics of pulse waves can be described by the two-frequency mutual coherence function[3-5]. Based on this idea Ishimaru first studied the pulse scattering from one-dimensional rough surfaces utilizing first-order Kirchhoff approximation (KA)[6]. However the first-order Kirchhoff approximation is only applicable to surfaces with small-scale roughness since it does not include the multiple scattering. Later Ishimaru et al. developed the modified second-order Kirchhoff approximation and utilized it to study the pulse scattering[7]. However, only scattering from one-dimensional rough surfaces is discussed.

Since Fung first proposed the integral equation method (IEM)[8], it has been proved to be an effective method for calculating the scattering from surfaces with small or large-scale roughness since it includes both single scattering and multiple scattering[9,10]. In this paper the Integral equation method is adopted to derive the two-frequency mutual coherence function of scattered wave from two-dimensional rough surfaces, and the numerical simulation is performed and discussed.

2. Two-frequency mutual coherence function from two-dimensional rough surfaces
2.1 Derivation of two-frequency mutual coherence function with the integral equation method

Let two plane waves with different frequency impinge upon an \( L \times L \) two-dimensional rough surface from the same incident direction, the incident fields are defined as

\[
\tilde{E}_{i,p} = \hat{p} E_0 \exp\left(-j\vec{k}_i \cdot \vec{r}\right)
\]

\[
\tilde{E}_{i,p}' = \hat{p} E_0 \exp\left(-j\vec{k}_i' \cdot \vec{r}'\right)
\]

where a time factor of the form \( \exp(j\omega t) \) is understood, \( \hat{p} \) and \( E_0 \) are the unit polarization vector and amplitude of the incident electric field, \( \vec{k}_i \) and \( \vec{k}_i' \) are the propagation vector of the two incident waves with different frequency, which can be expressed as

\[
\vec{k}_i = k \sin \theta \cos \phi \hat{x} + k \sin \theta \sin \phi \hat{y} - k \cos \theta \hat{z}
\]

\[
\vec{k}_i' = k' \sin \theta \cos \phi \hat{x} + k \sin \theta \sin \phi \hat{y} - k' \cos \theta \hat{z}
\]

In accordance with the IEM [8], the far-zone scattered field consists of the Kirchhoff component and its complementary component, that is

\[
E_{s,qp} = E_{s,qp}^h + E_{s,qp}^v
\]

\[
E_{s,qp}' = E_{s,qp}^{h'} + E_{s,qp}^{v'}
\]

where

\[
E_{s,qp}^h = CE_0 \int F_{qp} \exp\left[j\left(\vec{k}_s \cdot \vec{r}_s\right)\right] \mathrm{d}x \mathrm{d}y
\]

\[
E_{s,qp}^v = CE_0 \int F_{qp} \exp\left[j\left(\vec{k}_s \cdot \vec{r}_s\right)\right] \mathrm{d}x \mathrm{d}y \mathrm{d}x' \mathrm{d}y' \mathrm{d}x'' \mathrm{d}y''
\]

In the above equations, the incident polarization is denoted by \( p \) and the receiving polarization by \( q \), the expression of \( f_{qp} \) and \( F_{qp} \) can be found in reference [8], \( C = -jk \exp(-jkR)/4\pi R \), and \( R \) is the range from the center of the illuminated area to the point of observation. \( \vec{k}_s \) and \( \vec{k}_s' \) are the propagation vector of the scattered waves, which can be represented by

\[
\vec{k}_s = k \sin \theta \cos \phi \hat{x} + k \sin \theta \sin \phi \hat{y} + k \cos \theta \hat{z}
\]

\[
\vec{k}_s' = k' \sin \theta \cos \phi \hat{x} + k \sin \theta \sin \phi \hat{y} + k' \cos \theta \hat{z}
\]

Similar to the scattering coefficient, the two-frequency mutual coherence function (MCF) can be defined as

\[
\Gamma_{qp}(\vec{k}_i, \vec{k}_i'; \vec{k}_i, \vec{k}_i') = \frac{4\pi R^2}{A_0 E_0^2} \frac{P_{qp}}{E_{s,qp}^h E_{s,qp}^v} \left(\langle E_{s,qp}^h E_{s,qp}^{v*}\rangle - \langle E_{s,qp}^h \rangle \langle E_{s,qp}^{v*} \rangle\right)
\]

where \( \langle \rangle \) is the ensemble average operator, \( * \) is the symbol for complex conjugate and \( A_0 = L^2 \) is the illuminated area.

Substituting (3) and (4) into (9) we have

\[
\Gamma_{qp}(\vec{k}_i, \vec{k}_i'; \vec{k}_i, \vec{k}_i') = \Gamma_{qp}^h + \Gamma_{qp}^v + \Gamma_{qp}^{h'} + \Gamma_{qp}^{v'}
\]

where

\[
\Gamma_{qp}^h(\vec{k}_i, \vec{k}_i'; \vec{k}_i, \vec{k}_i') = \frac{4\pi R^2}{A_0 E_0^2} \left(\langle E_{s,qp}^h E_{s,qp}^{v*}\rangle - \langle E_{s,qp}^h \rangle \langle E_{s,qp}^{v*} \rangle\right)
\]

\[
\Gamma_{qp}^v(\vec{k}_i, \vec{k}_i'; \vec{k}_i, \vec{k}_i') = \frac{4\pi R^2}{A_0 E_0^2} \left(\langle E_{s,qp}^v E_{s,qp}^{v*}\rangle - \langle E_{s,qp}^v \rangle \langle E_{s,qp}^{v*} \rangle\right)
\]

\[
\Gamma_{qp}^{h'}(\vec{k}_i, \vec{k}_i'; \vec{k}_i, \vec{k}_i') = \frac{4\pi R^2}{A_0 E_0^2} \left(\langle E_{s,qp}^{h'} E_{s,qp}^{v*}\rangle - \langle E_{s,qp}^{h'} \rangle \langle E_{s,qp}^{v*} \rangle\right)
\]

\[
\Gamma_{qp}^{v'}(\vec{k}_i, \vec{k}_i'; \vec{k}_i, \vec{k}_i') = \frac{4\pi R^2}{A_0 E_0^2} \left(\langle E_{s,qp}^{v'} E_{s,qp}^{v*}\rangle - \langle E_{s,qp}^{v'} \rangle \langle E_{s,qp}^{v*} \rangle\right)
\]
To obtain the more explicit expression of each term, let us substitute (5)-(8) into (11)-(14). Note that the field coefficients \( f_{q-p} \) and \( f_{q-p}^{\prime} \) are related to the reflection coefficient, which is in general a function of the local incident angle and therefore depends on the spatial variables. However, for surfaces with small scale roughness the local incident angle can be approximated by the incident angle and for surfaces with moderate or large-scale roughness it can be calculated by the stationary-phase approximation. In both cases the Fresnel reflection coefficient becomes independent of the spatial coordinates, therefore \( f_{q-p} \) and \( f_{q-p}^{\prime} \) become constants. In addition to carry out the ensemble average operation we must make an assumption about the type of rough surface height distribution, and in this paper the Gaussian distribution is assumed. If we let \( \bar{v} = \bar{k}_s - \bar{k}_i \), \( \bar{v}' = \bar{k}'_s - \bar{k}'_i \), \( \bar{w} = \bar{k}_s + \bar{k}_i \), and \( \bar{w}' = \bar{k}'_s + \bar{k}'_i \), then the expression of each term of the two-frequency MCF in (10) can be given by detailed derivations as follows

\[
\begin{align*}
\Gamma^i_{q-p} & = \frac{4\pi R^2}{A_{k'}} f_{q-p} f_{q-p}^{\prime} \Phi_{k'} \sum_{n=1}^{\infty} \left( \frac{\sigma v k v k'}{2} \right) \left( \frac{v + v'}{2} \right) \\
\Gamma^{iv}_{q-p} & = \frac{R^2 \sum_{n=1}^{\infty} \left( \frac{\sigma v k v k'}{2} \right) \left( \frac{v + v'}{2} \right)}{A_{k'}} f_{q-p} f_{q-p}^{\prime} \Phi_{k'} \\
\Gamma^{iiv}_{q-p} & = \frac{R^2 \sum_{n=1}^{\infty} \left( \frac{\sigma v k v k'}{2} \right) \left( \frac{v + v'}{2} \right)}{A_{k'}} f_{q-p} f_{q-p}^{\prime} \Phi_{k'} \\
\Gamma^{ivv}_{q-p} & = \frac{R^2 \sum_{n=1}^{\infty} \left( \frac{\sigma v k v k'}{2} \right) \left( \frac{v + v'}{2} \right)}{A_{k'}} f_{q-p} f_{q-p}^{\prime} \Phi_{k'}
\end{align*}
\]
where the expressions of $\Phi$, $\Phi_k$, $\Phi_{\omega}$, $\Phi_{\omega_k}$, and $\Phi_{\omega_k}$ can be found in reference [8]. $\sigma$ is the root mean square (rms) height of rough surfaces, and $W^{(\omega)}(k_x,k_y)$ is the roughness spectrum of rough surfaces related to the $n$th power of the surface correlation function $\rho(x,y)$ by the Fourier transformation. For rough surfaces generated by a Gaussian process, $W^{(\omega)}(k_x,k_y)$ can be solved analytically as follows

$$W^{(\omega)}(k_x,k_y) = \frac{\pi l_x l_y}{n} \exp\left(\frac{k_x^2 l_x^2 + k_y^2 l_y^2}{2n}\right)$$

(19)

where $l_x$ and $l_y$ are the correlation length along $x$-axis and $y$-axis, respectively.

2.2 Shadowing effect

For large incident or scattering angle, the shadowing effect due to the roughness of surfaces should be included. Here a shadowing function is utilized to describing the shadowing effect, which is a function of the rms slope of surfaces and the observation angle [9], that is

$$S(\theta, \sigma) = \left[1 - \frac{1}{2} \text{erfc}\left(\frac{\cot \theta}{\sqrt{2} \sigma}ight)\right][1 + f(\theta, \sigma)]^{-1}$$

(20)

where

$$f(\theta, \sigma) = \frac{1}{2} \left[\frac{2}{\pi} \frac{\sigma}{\cot \theta} \exp\left(-\cot^2 \theta \frac{2 \sigma^2}{\sqrt{2} \sigma y}\right) - \text{erfc}\left(\frac{\cot \theta}{\sqrt{2} \sigma}ight)\right]$$

(21)

and $\text{erfc}(\cdot)$ is the error function complement.

From the above we know that the two-frequency MCF consists of four terms and the detailed expression of each term has been obtained, where the single sum terms are single scattering terms and the double sum terms represent multiple scattering. For single scattering only the incident and scattering shadowing should be considered, hence we only need to multiply the single scattering terms by $S(\theta_i)S(\theta_s)$, where $S(\theta_i)$ and $S(\theta_s)$ are the incident shadowing function and scattering shadowing function, respectively. While for multiple scattering the propagation shadowing should be taken into consideration besides the incident and scattering shadowing, therefore we need to multiply the multiple scattering terms by $S(\theta_i)S(\theta_s)$ outside the integral and $S(\theta_{\omega_s})$ inside the integral, and the cotangent function in $S(\theta_{\omega_s})$ is defined as [8]

$$\cot \theta_{\omega_s} = \sqrt{\frac{k^2 - u^2 - v^2}{u^2 + v^2}}$$

(22)

3. Numerical simulation and analysis

Without loss of generality it is assumed that the incident wave and scattered wave lie in the same plane and the rough surface is a perfect conductor. The incident angle is $\theta_i = 30^\circ$ and the top frequency is $f = 14$ GHz. For comparison the variations of the two-frequency MCF from two-dimensional rough surfaces with frequency difference and scattering angle are calculated by KA and IEM, respectively, and the results are displayed in Figure 1 and Figure 2. It can be shown that KA is only applicable to the case of like polarization since the multiple scattering need to be included for cross polarization, in addition due to the assumption of perfect conductor the results with KA are the same for HH polarization and VV polarization. IEM not only applies for the case of cross polarization, but can give different results for HH polarization and VV polarization, respectively, which is because the multiple scattering is included in IEM. By investigating the variation of the two-frequency MCF with the frequency difference, it is concluded that as the frequency difference increases, the two-frequency MCF drops from the maximum value to zero, which means the coherence between the two waves with different frequency declines. In addition, the two-frequency MCF decreases more quickly in cross
polarization than in like polarization, which means the coherence bandwidth in cross polarization is less than that in like polarization.

Figure 1. Variation of two-frequency MCF with frequency difference and scattering angle by KA for HH polarization and VV polarization \((k\sigma = 0.8, \ kl = 3.14, \ \theta_i = 30^\circ)\)

Moreover, in the case of cross polarization, the two-frequency MCF reaches a peak in the backscattering direction, which is called backscattering enhancement. This phenomenon is different from that in like polarization where the backscattering enhancement occurs only for surfaces with

Figure 2. Variation of two-frequency MCF with frequency difference and scattering angle by IEM \((k\sigma = 0.8, \ kl = 3.14, \ \theta_i = 30^\circ)\)
large-scale roughness, this is because the backscattering enhancement is caused by the constructive interference of multiple scattering waves. In like polarization the contribution of the multiple scattering is negligible compared with that of the single scattering for surface with small scale roughness, hence the backscattering enhancement does not occur. While in the case of cross polarization when the incident wave and scattered wave lie in the same plane the single scattering contribution is negligible and only multiple scattering contributes, therefore even for surfaces with small scale roughness the backscattering enhancement still occurs.

4. Conclusion
In this paper the analytical expression of two-frequency MCF of the scattered wave from two-dimensional rough surfaces is derived with the integral equation method. By numerical simulation it is concluded that as the frequency difference increases, the two-frequency MCF drops from the maximum value to zero and it decreases more quickly in cross polarization than in like polarization, which means the coherence bandwidth in cross polarization is less than that in like polarization. In addition in the case of cross polarization the backscattering enhancement occurs.

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