Dispersion Quark Model of Heavy Meson Decays

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We discuss applications of the dispersion quark model to exclusive semileptonic decays of heavy mesons. The transition form factors in this relativistic formulation of the quark model are given by double spectral representations through the wave functions of the initial and final mesons both in the scattering and the decay regions. An important feature of the model is that the form factors have the correct behavior in accordance with QCD in the case of meson decays induced by heavy parent quark transition both to heavy and light daughter quarks. We demonstrate that the dispersion quark model gives results compatible with the lattice calculations at large $q^2$ and thus provides reliable form factors in the whole decay region.

Theoretical description of hadronic amplitudes of the quark currents is one of the key problems of particle physics as such amplitudes provide a bridge between QCD formulated in the language of quarks and gluons and observable phenomena which deal with hadrons. In particular, the(105,629),(893,826)
mesons, $G_1(s_1)$ and $G_2(s_2)$, respectively

$$f(q^2) = \int \frac{ds_1}{s_1 - M_1^2} \frac{G_1(s_1)}{s_1 - M_1^2} \frac{ds_2}{s_2 - M_2^2} \frac{G_2(s_2)}{s_2 - M_2^2} \times \left[ \tilde{f}_D(s_1, s_2, q^2) + \tilde{f}_{\text{sub}}(s_1, M_1^2, s_2, M_2^2, q^2) \right], \tag{1}$$

where $s_1$ ($s_2$) is the invariant mass of the initial (final) $\bar{q}q$ pair. The double spectral density of the representation corresponding to the initial and final mesons and to the quark transition current $\frac{ds_1 G_1(s_1)}{s_1 - M_1^2}$ consists of the two parts: the unsubtracted part $\tilde{f}_D$ and the subtraction part $\tilde{f}_{\text{sub}}$.

The unsubtracted part $\tilde{f}_D(s_1, s_2, q^2)$ is calculated unambiguously from the double cut triangle Feynman graph with a relevant spinorial structure of the vertices corresponding to the initial and final mesons and to the quark transition current $\frac{ds_1 G_1(s_1)}{s_1 - M_1^2}$.

The subtraction part $\tilde{f}_{\text{sub}}(s_1, M_1^2, s_2, M_2^2, q^2)$ contains the factors $s_1 - M_1^2$ or $s_2 - M_2^2$ and accounts for an ambiguity connected with possible subtractions in the dispersion representations inherent to any dispersion approach. To decide on the necessity and the structure of the subtraction terms we need some additional arguments.

Notice that once we are working within an approach not directly deduced from QCD it is important to keep essential features of the underlying fundamental theory in the model. Thus matching the results obtained within the quark model to rigorous QCD results might be helpful for bringing more realistic features to the model.

QCD gives rigorous predictions for the structure of the form factors in the case of the meson transition induced by the heavy quark transition. And it turns out possible to use matching the form factors of the dispersion quark model to form factors in QCD in the heavy quark limit for determining the subtraction terms in the spectral representations.

Namely, we consider the case of a pseudoscalar heavy meson decay into final pseudoscalar and vector mesons induced by the heavy-to-heavy and heavy-to-light quark transitions through vector, axial-vector and tensor currents. We perform the expansion of the quark model form factors in inverse powers of the heavy quark mass $m_Q$ and match this expansion to the heavy quark expansion in QCD in leading and next-to-leading orders. This matching provides constrains on the structure of the subtraction terms. Additional constraints allowing us to fix the subtraction terms are obtained by requiring the form factors of vector, axial-vector and tensor currents to obey the general relations found in the case of heavy-to-light transitions.

It is important that the only property of the soft wave function of a heavy meson necessary for obtaining the correct $1/m_Q$ expansion of the transition form factors is a strong peaking of this soft wave function in the relative momenta of the constituent quarks in the region of order $\Lambda_{\text{QCD}}$. No other constraints on the soft wave function are required.

Finally, we arrive at the double spectral representations of the form factors with fixed subtraction terms. To obtain numerical predictions for the form factors we must specify the parameters of the quark model, i.e. constituent quark masses and the meson wave functions. To this end we take as an example the parameters of the ISGW2 model. The form factors of the semileptonic decay of $B$ and $D$ mesons calculated within dispersion quark model adopting the ISGW2 quark masses and wave functions provide reasonable description of all experimental data. Fig.1 illustrates the form factor $f_{B\to\pi}^{B\to\pi}$ vs the lattice results, and Fig. 2 presents form factors for the transition $B \to K^*$.
$B \to K^*\gamma$ transition we find

$$T_2(0) = 0.14.$$ 

which agrees well with a recent light-cone sum rule estimate $[7]$. 

It should be understood that the reported form factors are obtained with a very simple exponential ansatz for soft wave functions, which can be expected to describe only the effects of the confinement scale. Figs. 1 and 2 show that even this rather crude approximation works well and suggest that the bulk of the form factor behavior can be explained by the chiral symmetry breaking in the soft region and a strong peaking of the meson wave functions with a width of order of the confinement scale $[4]$. 

For obtaining more reliable predictions for the form factors one needs realistic meson wave functions and constituent quark masses relevant for the meson decays. These can be found by applying a combined fit based on the representations $[1]$ to all available lattice points on meson transition form factors at large $q^2$ considering the wave functions and constituent quark masses as variational parameters. In this case the spectral representations of the dispersion quark model may be treated as some generalized parametrizations of the transition form factors based on the constituent quark picture and obeying the rigorous QCD results in the limit of heavy-to-heavy and heavy-to-light meson transitions. 

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