Large-$N$ nonlinear $\sigma$ models on $R^2 \times S^1$

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The large-$N$ nonlinear $O(N)$, $CP^{N-1}$ $\sigma$ models are studied on $R^2 \times S^1$. The $N$-components scalar fields of the models are supposed to acquire a phase $e^{i2\pi\delta}$ $(0 \leq \delta < 1)$, along the circulation of the circle, $S^1$. We evaluate the effective potentials to the leading order of the $1/N$ expansion. It is shown that, on $R^2 \times S^1$ the $O(N)$ model has rich phase structure while the phase of $CP^{N-1}$ model is just that of the model at finite temperature.

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I. INTRODUCTION

On 3-dimensional Euclidean spacetime, a wide class of quantum field theories which are not renormalizable in a weak-coupling expansion are renormalizable in the $1/N$ expansion [1-3]. These include the nonlinear $O(N)$, $CP^{N-1}$ models [1,2] which are of continuous interests since they describe two-dimensional antiferromagnets [4]. In the nonlinear $\sigma$ models there are critical coupling constants, $g_c$, which separate the ordered and disordered phase.

In this paper we will study the $O(N)$ model and the $CP^{N-1}$ model on $R^2 \times S^1$, where the circumference of the circle $S^1$ is $L$. We evaluate the effective potentials, $V_L$, to the leading order of $1/N$ expansion to find the phase structures of the models. The nonlinear $\sigma$ models share many properties with each other. And the phase structures of them are identical on $R^3$ [1,2] and at finite temperature [5].

For field theories on a spacetime of nontrivial topology the boundary condition of the fields should be specified. In this respect, we take the same point of view of Ref.[6] (see also our recent paper[7]): A field could take a phase $e^{i2\pi\delta}$ ($0 \leq \delta < 1$), along the circulation of the circle, $S^1$. In the $CP^{N-1}$ model, there is a Abelian gauge symmetry [8] while there is no such symmetry in the $O(N)$ model. One of the interesting properties of Abelian gauge theory coupled to matter field $R^n \times S^1$ is that a change of boundary condition of matter field is traded for a change of gauge field [6]. In the $CP^{N-1}$ model, the same phenomenon occurs and it leads to that the model be always in the disordered phase as in the model at finite temperature [5].

In the nonlinear $O(N)$ $\sigma$ model, however, the phase structure depends on the $\delta$ and the coupling constant $g$; For some cases there is critical circumference, $L_c$, which depends on $g$ and $\delta$ and separates the ordered and disordered phase, as will be found analytically. The $\delta \to 0$ limit is of particular interest, since $V_L(\delta = 0)$ is just the effective potential of the model at a temperature $T$ ($= 1/k_B L$). For $g < g_c$, $L_c$ approaches to $\infty$ in this limit, which corresponds to the phase transition at $T_c \to 0^+$ found in the finite-temperature analysis of the model [9].

In the next section we will analyze the $O(N)$ model and a similar analysis will be carried on $CP^{N-1}$ model in Sec. III. The final section is devoted to the discussions.

II. THE O(N) MODEL
A. Effective potential formalism for the model on $R^3$

The Lagrangian density of the $O(N)$ model is

$$\mathcal{L} = \frac{1}{2} \partial_\mu n \partial_\mu n + \frac{\sigma}{2} (n^2 - N/g_0^2)$$  \hspace{1cm} (1)$$

where $n$ is the $N$-component scalar field and $\sigma$ is a Lagrangian multiplier for the constraint

$$n^2 = \frac{N}{g_0^2}. \hspace{1cm} (2)$$

The effective potential in the leading order of the $1/N$ expansion is given for constant $\sigma$ by the tree and one-loop diagrams with external $\sigma$ lines [10,11] and on $R^3$ it is written as

$$\frac{V_0}{N} = -\frac{\sigma}{2g_0^2} + \frac{1}{2} \int_{|p_E|<\Lambda} \frac{d^3p_E}{(2\pi)^3} \ln[1 + \frac{\sigma}{p_E^2}], \hspace{1cm} (3)$$

where $\Lambda$ is the cutoff. The renormalization of $V_0$ can be done by demanding that

$$\frac{1}{N} \frac{\partial V_0}{\partial \sigma} \mid_{\sigma=M^2} = -\frac{1}{g^2}. \hspace{1cm} (4)$$

Then the renormalized effective potential is

$$\frac{V_0}{N} = -\frac{\sigma}{2g^2} - \frac{\sigma}{2} \int_{|p_E|<\Lambda} \frac{d^3p_E}{(2\pi)^3} \frac{1}{p_E^2 + M^2} + \frac{1}{2} \int_{|p_E|<\Lambda} \frac{d^3p_E}{(2\pi)^3} \ln[1 + \frac{\sigma}{p_E^2}] \hspace{1cm} (5)$$

$$= \frac{\sigma}{2} \left[ \frac{M}{4\pi} - \frac{1}{g^2} \right] - \frac{\sigma^{3/2}}{12\pi} + O(1/\Lambda). \hspace{1cm} (6)$$

For large $\sigma$, the behavior of $V_0$ is determined by the second term of the right-hand side of Eq.(6) which decreases as $\sigma$ increases. The necessary and sufficient condition for the existence of stationary point in $V_0$ is that $\frac{M}{8\pi} - \frac{1}{2g^2}$ (the first derivative of $V_0$ at $\sigma = 0$) is positive. This condition is satisfied for the coupling constant $g > g_c$ where the critical coupling constant $g_c$ is given as

$$g_c^2 = \frac{4\pi}{M}. \hspace{1cm} (7)$$

For the $g > g_c$, there is a global stationary point of $V_0$ at

$$\sigma = m_0^2 = (M - \frac{4\pi}{g^2})^2. \hspace{1cm} (8)$$
The presence of stationary point in effective potential which is absent for tree approximation denotes the dynamical mass generation [10]. \( m_0 \) is the dynamically generated mass of \( z \)-particles. On the other hand, for \( g < g_c \), there is no stationary point, which means that dynamical generation of mass does not occur in this phase.

The saddle point analysis through effective action has been carried out in Ref.[1] and the physical pictures of the model there agree with those in this paper. Though we use the manifestly \( O(N) \) invariant effective potential formalism, the results of Ref.[1] imply that for \( g < g_c \) (i.e. for the phase of no dynamical mass generation) the stationary condition can be satisfied with nonvanishing \( < n > \) while for \( g > g_c \) (i.e. for the phase of dynamical mass generation) \( < n > = 0 \); That is, the model would be in ordered phase for the case of no dynamical mass generation and the model is in disordered phase when dynamical mass generation takes place. This will be accepted [4] in this paper afterwards.

**B. Phase structure on \( R^2 \times S^1 \)**

Now we will evaluate the effective potential on \( R^2 \times S^1 \) to find the phase structure. As in Ref.[6,7], it will be assumed that the \( n \) fields are quasi-periodic under the circulation of the circle \( S^1 \) whose circumference is \( L \):

\[
n(x_1, x_2, x_3 + L) = e^{i2\pi \delta} n(x_1, x_2, x_3) \quad (0 \leq \delta < 1).
\]

(9)

In such boundary condition, the third component of the momentum of \( z \)-fields has the discrete values

\[
p_3 = \frac{2\pi}{L} (n + \delta) \quad (n = 0, \pm 1, \pm 2, \ldots),
\]

(10)

and the effective potential for \( \sigma \) in the leading order of the \( 1/N \) expansion is written as

\[
V_L(\sigma, \delta) = -\frac{\sigma}{2g_0^2} + \frac{1}{2L} \sum_{n=-\infty}^{\infty} \int \frac{d^2 p_E}{(2\pi)^2} \ln[1 + \frac{\sigma}{(\frac{2\pi}{L})^2(n + \delta)^2 + p_E^2}].
\]

(11)

Making use of the formula [6,7]

\[
\sum_{n=-\infty}^{\infty} \ln[1 + \frac{b^2}{(n + \delta)^2 + a^2}] = \int_{-\infty}^{\infty} \ln[1 + \frac{b^2}{\tau^2 + a^2}] d\tau
\]

4
\[
\ln \frac{1 - 2 \cos(2\pi\delta)e^{-2\pi\sqrt{a^2 + b^2}} + e^{-4\pi\sqrt{a^2 + b^2}}}{1 - 2 \cos(2\pi\delta)e^{-2\pi|a|} + e^{-4\pi|a|}},
\]
(12)

the topology effect in the effective potential can be separated:
\[
V_L(\sigma, \delta) = V_0(\sigma) + NV_\Delta(\sigma, L, \delta),
\]
(13)

where
\[
V_\Delta = \frac{1}{4\pi L} \int_0^\infty dp \ln \frac{1 - 2 \cos(2\pi\delta)e^{-L\sqrt{p^2 + \sigma}} + e^{-2L\sqrt{p^2 + \sigma}}}{1 - 2 \cos(2\pi\delta)e^{-Lp} + e^{-2Lp}}
\]
\[
= \frac{1}{2\pi L^3} \sum_{n=1}^\infty \frac{\cos(2\pi\delta n)}{n^3} (1 - e^{-nL\sqrt{\sigma}})
\]
\[-\frac{1}{2\pi L^2} \sum_{n=1}^\infty \frac{\sqrt{\sigma}}{n^2} \cos(2\pi\delta n)e^{-nL\sqrt{\sigma}}.
\]
(14)

\(V_\Delta\), the effect of topology, is finite for any \(L, \delta,\) and \(\sigma,\) and reduced to zero as \(L\) approach to \(\infty\). Since \(V_\Delta\) is finite, the renormalization of \(V_L\) can be done by that of \(V_0\).

If \(V_L\) has stationary point at \(\sigma = m_L^2\), then \(m_L\) satisfies the following equation
\[
\frac{1}{N} \frac{\partial V_L}{\partial \sigma} \bigg|_{\sigma = m_L^2} = 0
\]
(15)
\[
= -\frac{1}{2g^2} + \frac{1}{8\pi} \left[M - m_L\right] + \frac{1}{4\pi L} \sum_{n=1}^\infty \frac{\cos(2\pi\delta n)}{n} e^{-nLm_L}
\]
\[-\frac{1}{8\pi L} \ln[1 - 2 \cos(2\pi\delta)e^{-Lm_L} + e^{-2Lm_L}].
\]

The (formal) solution of Eq.(15) is that
\[
m_L = \frac{1}{L} \cosh^{-1}\left[\frac{\exp\left\{(M - \frac{4\pi}{g^2})L\right\} + 2\cos(2\pi\delta)}{2}\right].
\]
(16)

It is instructive to expand \(V_L\) in terms of small \(\sigma:\)
\[
\frac{V_L}{N} = \left\{\frac{M}{4\pi} - \frac{1}{g^2} - \frac{1}{4\pi L} \ln(2 - 2 \cos(2\pi\delta))\right\}\sigma + O(\sigma^{3/2}) \quad (\delta \neq 0).
\]
(17)
Since $V_\Delta$ approaches to a fixed value as $\sigma$ goes to $\infty$, for large $\sigma$ the behavior of $V_L$ is determined by that of $V_0$ which decrease as $\sigma$ increase. A sufficient condition for the existence of stationary point is, therefore, that $V_L$ increases as $\sigma$ increases in the vicinity of $\sigma = 0$. The condition can be obtained from Eq. (17) as:

$$\frac{M}{4\pi g^2} - \frac{1}{4\pi L} \ln(2 - 2 \cos(2\pi \delta)) > 0,$$  \hspace{1cm} (18)

which is just the condition for the existence of $m_L$ in Eq. (16).

In the $\delta \rightarrow 0$ limit, the condition (18) is satisfied for any $g$ and finite $L$, which agrees with the result that the model is in disordered phase at any finite temperature [9]. For $\delta = 0$ it is easy to find a formula familiar through the finite-temperature analyses (for example, see Ref.[5]),

$$\frac{1}{N} \partial V_L(\delta = 0) \partial \sigma = -\frac{1}{2g^2} + \frac{1}{8\pi} [M - \sqrt{\sigma}] - \frac{1}{4\pi L} \ln[1 - e^{-L\sqrt{\sigma}}],$$

where the $V_L(\delta = 0)$ is the effective potential of the model at a temperature $T = 1/k_B L$.

To discuss the phase structure, it is convenient to consider the cases $g > g_c$ (that is, $\frac{M}{4\pi} - \frac{1}{g^2} > 0$), $g < g_c$, separately.

a. $g > g_c$

On $R^3$, for this coupling constant the $z$-fields have the dynamically generated mass $m_0$ of Eq. (8) and the model is in disordered phase. On $R^2 \times S^1$, there are two cases:

(i) For $\frac{1}{6} < \delta < \frac{5}{6}$, there is a critical circumference of the $S^1$, $L_c$, which is given as

$$L_c = \frac{1}{M - \frac{4\pi}{g^2}} \ln\{2 - 2 \cos(2\pi \delta)\}. \hspace{1cm} (19)$$

If $L$ is smaller than $L_c$, there is no dynamical mass generation and the model is in ordered phase, which is absent for $g > g_c$ on $R^3$. If $L$ is larger than $L_c$, the dynamical mass generation takes place and the model is in disordered phase as in the model on $R^3$. $m_L$ in Eq. (16) is the mass of $z$-particles.

(ii) For $\delta$ not mentioned in (i), the dynamical mass generation occurs for all $L$ and the model is in disordered phase as in the model on $R^3$. Again, the mass of $z$-particles is given as $m_L$ in Eq. (16).
b. \( g < g_c \)

(i) For \( \frac{1}{6} < \delta < \frac{5}{6} \), there is no dynamical mass generation for all \( L \) and the model is in ordered phase, as in the model on \( R^3 \).

(ii) For \( \delta \) not mentioned in (i), the critical circumference \( L_c \) in Eq.(19) is positive and separate the ordered and disordered phases. For \( L < L_c \) the model is in disordered phase and the mass of \( z \)-particle is given as \( m_L \) in Eq.(16), which is absent for the model of \( g < g_c \) on \( R^3 \). When \( L > L_c \), there is no dynamical mass generation. In the \( \delta \to 0 \) limit, \( L_c \) approaches to \( \infty \) which denote the phase transition at \( T_c \to 0^+ \) of Ref.[9].

In every case of a. and b., when \( L \) approaches to \( \infty \) the results of the model on \( R^2 \times S^1 \) reproduce those on \( R^3 \).

III. THE \( CP^{N-1} \) MODEL

In this section, to denote this model we use the same notations used for the \( O(N) \) model. This will go on throughout the paper unless there is confusion.

A. Effective potential formalism of the model on \( R^3 \)

The model is described by the Lagrangian density

\[
\mathcal{L} = (\partial_{\mu} - iA_{\mu}) z^\dagger (\partial^\mu + iA_{\mu}) z + \sigma (z^\dagger z - N/g_0^2),
\]  

(20)

where \( z \) is an \( N \)-component complex scalar field. At the classical level, as in the 2-dimensional model [8], \( A_{\mu} \) is an auxiliary field which can be replaced by

\[
\frac{ig_0^2}{2N} [z^\dagger \partial_{\mu} z - (\partial^\dagger_{\mu}) z],
\]

and \( \sigma \) is a Lagrangian multiplier for the constraint

\[
z^\dagger z = \frac{N}{g_0^2}.
\]  

(21)

The Abelian gauge symmetry of the model is that the substitutions of \( z \) and \( A_{\mu} \) fields by \( e^{i\alpha(x)} z \) and \( A_{\mu} - \partial_{\mu} \alpha(x) \) respectively do not cause change for the Lagrangian density of Eq.(20). Because of the gauge fields, we use a
different method for evaluating effective potential. The Lagrangian density can be written as
\[
\mathcal{L} = z^\dagger D z - N\sigma/g_0^2,
\]
where
\[
D = -(\partial_\mu + iA_\mu)(\partial^\mu + iA^\mu) + \sigma.
\]
The Gaussian functional integral for the effective potential gives the radiative contribution, \((\text{Tr } \ln D + C)\) divided by the spacetime volume \([12]\), or
\[
\frac{V_0}{N} = -\frac{\sigma}{g_0^2} + \text{Tr } \ln D + C. \quad (22)
\]
\(C\) is constant which may arise from functional integral and it will be fixed by demanding that \(V_0 |_{\sigma=0} = 0\). In \(1/N\) expansion, the diagrams which have \(\sigma\) or \(A_\mu\) propagator as internal lines give contribution of next to the leading order and can be ignored in the large-\(N\) analysis. This means that the \(\sigma\) and \(A_\mu\) look like spacetime constants in the large-\(N\) approximation. These considerations give the effective potential
\[
\frac{V_0}{N} = -\frac{\sigma}{g_0^2} + \int \frac{d^3p_E}{(2\pi)^3} \ln[1 + \frac{\sigma}{(p_\mu + A_\mu)(p_\mu + A_\mu)}]. \quad (23)
\]
From this potential, it is clear that the constant \(A_\mu\), which can be gauged away, does not give rise to any physical result for the renormalizable theory of the \(CP^{N-1}\) model on \(R^3\), and we will set \(A_\mu = 0\). The effective potential of the \(CP^{N-1}\) model can be treated almost identically with that of the \(O(N)\) model. The renormalization can be done by demanding that
\[
\frac{1}{N} \frac{\partial V_0}{\partial \sigma} |_{\sigma=M^2} = \frac{-1}{g^2}, \quad (24)
\]
and then with the cutoff \(\Lambda\) the effective potential is written as
\[
\frac{V_0}{N} = -\frac{\sigma}{g^2} - \sigma \int_{|p_E|<\Lambda} \frac{d^3p_E}{(2\pi)^3} \frac{1}{p_E^2 + M^2} + \int_{|p_E|<\Lambda} \frac{d^3p_E}{(2\pi)^3} \ln[1 + \frac{\sigma}{p_E^2}] \quad (25)
\]
\[
= \sigma \left[ \frac{M}{4\pi} - \frac{1}{g^2} \right] - \frac{\sigma^{3/2}}{6\pi} + O(1/\Lambda).
\]
The phase structure from the effective potential is identical with that of $O(N)$ model on $R^3$. The critical coupling constant is

$$g_c^2 = \frac{4\pi}{M}.$$  \hfill (26)

For $g > g_c$ the mass $m_0$ of $z$-particles are dynamically generated as

$$m_0 = M - \frac{4\pi}{g^2},$$  \hfill (27)

and from the saddle point analysis of effective action [2,5] one can find that $\langle z \rangle$ is zero which means that the model is in disordered phase. Recently it has been suggested that the hedgehog-like instantons which exist in this model may yield some interesting consequences for the properties of antiferromagnet in disordered phase while there is no significant effect of instanton in ordered phase [13]. For $g < g_c$, there is no dynamical mass generation and stationary point exists with nonvanishing $\langle z \rangle$ which means that the model is in ordered phase.

**B. The model on $R^2 \times S^1$**

The boundary condition of $z$-fields along the circulation of the circle $S^1$ again will be assumed as

$$z(x_1, x_2, x_3 + L) = e^{i2\pi \delta} z(x_1, x_2, x_3) \quad (0 \leq \delta < 1).$$  \hfill (28)

We can use the periodic $z$-field through the gauge transformation $z' = e^{-i2\pi \delta \frac{2\pi}{L}} z$, however, with the changed gauge field $A'_\mu = A_\mu + \frac{2\pi \delta}{L} A_3$. Instead of doing this, we will assume a $\delta$ and gauge field $A_\mu = A_3 \delta_{\mu,3}$. The effective potential is then

$$\frac{V_L(\sigma, \delta)}{N} = -\frac{\sigma}{g_0^2} + \frac{1}{L} \sum_{n=-\infty}^{\infty} \int \frac{d^2 p_E}{(2\pi)^2} \ln[1 + \frac{\sigma}{(\frac{2\pi}{L})^2(n + \delta + \frac{L}{2\pi} A_3)^2 + p^2_E}],$$  \hfill (29)

Through the same methods for the $O(N)$ model, we obtain the renormalized effective potential

$$V_L(\sigma, \delta) = V_0(\sigma) + NV_\Delta(\sigma, L, \delta),$$  \hfill (30)
where

\[ V_\Delta = \frac{1}{\pi L^2} \sum_{n=1}^{\infty} \frac{\cos(2\pi \delta' n)}{n^3} (1 - e^{-nL\sqrt{\sigma}}) \]

\[ -\frac{1}{\pi L^2} \sum_{n=1}^{\infty} \frac{\sqrt{\sigma}}{n^2} \cos(2\pi \delta' n) e^{-nL\sqrt{\sigma}}. \] (31)

The $\delta'$ denotes $\delta + LA_3/2\pi$ which may be justly termed effective boundary condition parameter. $V_L$ depends only on $\delta'$ not separately on $\delta$ or $A_3$, which is a reflection of gauge invariance on $R^2 \times S^1$.

For the stationary point, $V_L$ must satisfy

\[ \frac{1}{N} \frac{\partial V_L}{\partial \sigma} \bigg|_{\sigma=m_L^2} = 0 \] (32)

\[ = -\frac{1}{g^2} + \frac{1}{4\pi} [M - m_L] - \frac{1}{4\pi L} \ln[1 - 2 \cos(2\pi \delta') e^{-Lm_L} + e^{-2Lm_L}] \]

which is similar to the Eq.(15) of the $O(N)$ model. However, there is another condition for the stationary point in $CP^{N-1}$ model:

\[ \frac{\partial V_L}{\partial A_3} = 0 \] (33)

\[ = \frac{\partial \cos(2\pi \delta')}{\partial A_3} \frac{\partial V_L}{\partial \cos(2\pi \delta')} = -\frac{L}{2\pi} \frac{\sin(2\pi \delta')}{\cos(2\pi \delta')} \frac{\partial V_L}{\partial \cos(2\pi \delta')} \] (34)

which is absent in the $O(N)$ model. The two conditions for stationary point can be satisfied with

\[ m_L = \frac{1}{L} \cosh^{-1}[1 + \frac{\exp\{(M - \frac{4\pi}{g^2})L\}}{2}] \] (35)

and

\[ \delta' = 0 \pmod{1}. \] (36)

At the stationary point of the model on $R^2 \times S^1$, $A_3$ is to be arranged so that the effective boundary parameter is 0. Therefore, the dynamical mass generation takes place for any $\delta$ and $L$, which implies the model is in disordered phase.
IV. CONCLUSION

We evaluate the large-$N$ effective potentials of the $O(N)$ model and the $CP^{N-1}$ model. It is shown that the $O(N)$ model has rich phase structure on $R^2 \times S^1$ while the $CP^{N-1}$ model is always in disordered phase.

Recently we studied four-fermion interaction model on $R^2 \times S^1$[7], where we also obtained the rich phase structure similar to that of the $O(N)$ model. The phase structure of the $CP^{N-1}$ model is relatively very simple. This is because the $CP^{N-1}$ model has Abelian gauge symmetry, and the presence of Abelian gauge symmetry enforces the effective boundary condition parameter to be 0 as in the Abelian gauge theory coupled to charged scalar field. It suggests that on a nontrivial topology phase structure of a model which has Abelian gauge field would be relatively simple. Through the similar analysis of this paper it is easy to find that the $CP^{N-1}$ model on $R^1 \times S^1$ is in disordered phase.

The limit $\delta = 0$ coincides with a case of finite temperature. In both cases of the $O(N)$ model and the $CP^{N-1}$ model, we recover the well-known result that, at any finite temperature, dynamical mass generations take place and the models are in disordered phases [5,9] (see also Ref.[11]). For the $O(N)$ model, the $L_c$ ($T_c$) of $g < g_c$ approaches to $\infty$ ($0$) in this limit. In the previous analysis of the finite temperature $CP^{N-1}$ model [5] it was assumed that $A_3 = 0$, while our results show that why it must be so. The $m_L$ in Eq.(35) is equal to the finite temperature mass of Ref.[5] in the large-$N$ limit when $L = \beta = 1/k_B T$.

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