Structure of S and T Parameters in Gauge-Higgs Unification

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Abstract

We investigate the divergence structure of one-loop corrections to S and T parameters in gauge-Higgs unification. We show that these parameters are finite in five dimensions, but divergent in more than six dimensions. Remarkably, a particular linear combination of S and T parameters becomes finite in six dimension case, which is indicated from the operator analysis in a model independent way.

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1 Introduction

Solving the gauge hierarchy problem motivates us to go to beyond the Standard Model (SM). Gauge-Higgs unification is one of the attractive approach to solve the gauge hierarchy problem without supersymmetry. In this scenario, Higgs is identified with zero mode of the extra component of the gauge field in higher dimensional gauge theories and the gauge symmetry breaking occurs dynamically through Wilson line phase dynamics. One of the remarkable features is that Higgs mass become finite thanks to the higher dimensional local gauge invariance. Furthermore, many applications of gauge-Higgs unification to the real world had been carried out in various aspects.

Here we would like to ask the following question; Is there any other finite (predictive) physical quantity such as Higgs mass? Noting that the gauge-Higgs sector is controlled by the higher dimensional local gauge invariance, S and T parameters

\[
S = -16\pi^2\Pi'_{3Y}(0),
\]

\[
T = \frac{4\pi^2}{M_W^2\sin^2\theta_W}(\Pi_{11}(0) - \Pi_{33}(0)),
\]

where \(\Pi_{ij}(p^2)\) is the \(g_{\mu\nu}\) part of the two-point function of currents and \(\Pi'_{ij} \equiv \frac{d^2}{dp^2}\Pi_{ij}(p^2)\) and \(\theta_W\) denotes the Weinberg angle, are one of the good candidates since these parameters are given as the coefficients of dimension six operators composed of the gauge fields and Higgs fields. In SM, S and T parameters are finite since SM is a renormalizable theory and these parameters are coefficients of dimension six higher dimensional operators. On the other hand, we consider here a nonrenormalizable theory, which implies that S and T parameters are in general divergent even if they are given by the coefficients of the nonrenormalizable operators. However, we know the fact that Higgs mass is finite, which is realized thanks to the higher dimensional gauge symmetry. Since S and T parameters can be also controlled by the higher dimensional gauge symmetry, we can expect that these parameters also become finite.

In this talk, we discuss the divergence structure of one-loop corrections to S and T parameters in the minimal \(SU(3)\) gauge-Higgs unification on an orbifold \(S^4/Z_2\) with a triplet fermion. We show that these parameters are finite in 5D case, but divergent in more than 6D case. The remarkable result is that in 6D case, one-loop corrections to S and T parameters themselves are certainly divergent, but a particular combination of them becomes finite. Its relative ratio agrees with that derived from operator analysis in a model independent way. This is the crucial difference from the universal extra dimension (UED) scenario.
2 Model

We introduce here a minimal model of 5D SU(3) gauge-Higgs unification on an orbifold $S^1/Z_2$, whose Lagrangian is given by

$$\mathcal{L} = -\frac{1}{4} F_{MN} F^{MN} + i \bar{\Psi} \slashed{D} \Psi$$

(2.1)

where $\Gamma^M = (\gamma^\mu, i\gamma^5)$,

$$F_{MN} = \partial_M A_N - \partial_N A_M - ig [A_M, A_N] \quad (M, N = 0, 1, 2, 3, 5),$$

(2.2)

$$\bar{\Psi} = \Gamma^M (\partial_M - ig A_M),$$

(2.3)

$$\Psi = (\psi_1, \psi_2, \psi_3)^T.$$  
(2.4)

The periodic boundary conditions for $S^1$ and $Z_2$ parities are imposed as follows,

$$A_\mu = \begin{pmatrix}
(+, +) & (+, +) & (-, -) \\
(+) & (+, +) & (-, -) \\
(-, -) & (-, -) & (+, +)
\end{pmatrix}, 
A_5 = \begin{pmatrix}
(-, -) & (-, -) & (+, +) \\
(-, -) & (-, -) & (+, +) \\
(+, +) & (+, +) & (-, -)
\end{pmatrix},$$

(2.5)

$$\Psi = \begin{pmatrix}
\psi_{1L}(+, +) & \psi_{1R}(-, -) \\
\psi_{2L}(+, +) & \psi_{2R}(-, -) \\
\psi_{3L}(-, -) & \psi_{3R}(+, +)
\end{pmatrix},$$

(2.6)

where $(+, +)$ means that $Z_2$ parities are even at $y = 0$ and $y = \pi R$, for instance. $y$ is the fifth coordinate and $R$ is the compactification radius. $\psi_{1L} \equiv \frac{1}{2} (1 - \gamma_5) \Psi$, etc. One can see that $SU(3)$ is broken to $SU(2) \times U(1)$ by these boundary conditions.

Expanding in terms of Kaluza-Klein (K-K) modes and integrating out the fifth coordinate, we obtain a 4D effective Lagrangian for a fermion

$$\mathcal{L}_{\text{fermion}} = \sum_{n=1}^{\infty} \left\{ (\bar{\psi}_1^{(n)}, \bar{\psi}_2^{(n)}, \bar{\psi}_3^{(n)}) \right\} \begin{pmatrix}
i\gamma^\mu \partial_\mu - m_n & 0 & 0 \\
0 & i\gamma^\mu \partial_\mu - (m_n + m + gh) & 0 \\
0 & 0 & i\gamma^\mu \partial_\mu - (m_n - m - gh)
\end{pmatrix} \begin{pmatrix}
\psi_1^{(n)} \\
\psi_2^{(n)} \\
\psi_3^{(n)}
\end{pmatrix}$$

$$+ \frac{g}{2} (\bar{\psi}_1^{(n)} \bar{\psi}_2^{(n)} \bar{\psi}_3^{(n)}) \begin{pmatrix}
W_3^{\mu} + \frac{\sqrt{3}B^\mu}{3} & W^{+\mu} & W^{+\mu} \\
W_3^{\mu} & -\frac{W_3^{\mu}}{\sqrt{3}} - \frac{W_3^{\mu}}{6} & -\frac{W_3^{\mu}}{\sqrt{3}} + \frac{W_3^{\mu}}{6} \\
-W_3^{\mu} & \frac{W_3^{\mu}}{\sqrt{3}} + \frac{W_3^{\mu}}{6} & -\frac{W_3^{\mu}}{\sqrt{3}} - \frac{W_3^{\mu}}{6}
\end{pmatrix} \gamma_\mu \begin{pmatrix}
\psi_1^{(n)} \\
\psi_2^{(n)} \\
\psi_3^{(n)}
\end{pmatrix}$$

$$+ \bar{L}_L \gamma^\mu \partial_\mu L + \bar{b}(i\gamma^\mu \partial_\mu - m - gh) b$$

$$+ \frac{g}{\sqrt{2}} (\bar{\epsilon} \gamma_5 L b W^{+\mu} + \bar{b} \gamma_5 L t W^{-\mu}) + \frac{g}{2} (\bar{\epsilon} \gamma_5 L t - \bar{b} \gamma_5 L b) W_3^{\mu}$$

$$+ \frac{\sqrt{3}g}{6} (\bar{\epsilon} \gamma_5 L t + \bar{b} \gamma_5 L b - 2\bar{b} \gamma_5 R b) B^\mu$$

(2.7)
where the mass matrix for the non-zero K-K modes are diagonalized by use of the mass eigenstates $\tilde{\psi}_2^{(n)}$, $\tilde{\psi}_3^{(n)}$:

$$\begin{pmatrix} \psi_1^{(n)} \\ \tilde{\psi}_2^{(n)} \\ \tilde{\psi}_3^{(n)} \end{pmatrix} = U \begin{pmatrix} \psi_1^{(n)} \\ \psi_2^{(n)} \\ \psi_3^{(n)} \end{pmatrix}, \quad U = \frac{1}{\sqrt{2}} \begin{pmatrix} \sqrt{2} & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and $L \equiv \frac{1}{2}(1 - \gamma_5)$, $W^{1,2,3}_\mu$, $B_\mu$ are the $SU(2), U(1)$ gauge fields, respectively and $W^\pm_\mu \equiv (W^1_\mu \pm iW^2_\mu)/\sqrt{2}$. $m_n = \frac{m}{R}$ is the compactification scale. $m = g\langle A_5 \rangle$ is a bottom mass, where we consider $\Psi$ to be a third generation quark. Dirac particles are constructed as

$$\psi_{1,2,3}^{(n)} = \psi_{1,2,3R}^{(n)} + \psi_{1,2,3L}^{(n)} (n > 0)$$

$$b = \psi_{2L}^{(0)} + \psi_{3R}^{(0)}$$

and the remaining state is a Weyl spinor

$$t_L = \psi_{1L}^{(0)}.$$ 

We realized that zero mode part for $t$ and $b$ quarks are exactly the same as those in the SM with

$$m_t = 0, \quad m_b = m.$$ 

Thus, we can just use the result in the SM with (2.12). Note that the mass splitting occurs between the $SU(2)$ doublet component and singlet component. This pattern of mass splitting has a periodicity with respect to $m$, which is a remarkable feature of gauge-Higgs unification.

### 3 Calculation of S and T parameters in 5D case

In this section, we calculate one-loop corrections to T-parameter, which is obtained from the mass difference between the neutral W-boson and the charged W-bosons $\Delta M^2 \equiv \delta \Pi_{33}(0) - \delta \Pi_{11}(0)$. The result is given by

$$\Delta M^2 = i \frac{3g^2 2^{D/2}}{16 \sqrt{D}} \int_{n=-\infty}^{\infty} \frac{d^D k}{(2\pi)^D} \left[ \frac{(2 - D)k^2 + D(m_n^2 - m^2)}{[k^2 - (m_n - m)^2][k^2 - (m_n + m)^2]} - 4 \frac{(2 - D)k^2 + D(m_n^2 + m^2)}{[k^2 - m_n^2][k^2 - (m_n + m)^2]} \right].$$

Let us evaluate T-parameter in 5D by carrying out the dimensional regularization for 4D momentum integral before taking the mode sum in order to keep 4D gauge invariance and expanding the non-zero mode part of (3.1) in $m/m_n$, that is, we consider the case
where the compactification scale is larger than the bottom mass. It is straightforward to check that the pole terms in $D \rightarrow 4$ limit are exactly cancelled and the finite value can be calculated from the log terms.

$$\Delta M^2_{(n\neq 0)} = -\frac{3g^2}{40\pi^2} \sum_{n=1}^{\infty} \frac{m^4_n}{m^2_n} = -\frac{g^2}{80}(mR)^2 m^2,$$

(3.2)

where $\sum_{n=1}^{\infty} n^{-2} = \zeta(2) = \pi^2/6$ is used. The fact that the leading order term is proportional to $m^4$ corresponds to four Higgs vacuum expectation values (VEV) insertions in dimension six operator contributing to T-parameter ($\phi^\dagger D_{\mu} \phi)(\phi^\dagger D^\mu \phi)(\phi$ : Higgs doublet).

This finite value can be also obtained by taking the mode sum before 4D momentum integration. If we take the mode sum explicitly in (3.1), we find

$$\Delta M^2 \simeq -\frac{3g^2}{(8\pi)^2} \left( m^2 + \frac{4\pi^2}{15}(mR)^2 m^2 \right),$$

(3.4)

The $m^2$ is known to be coincide with zero mode contribution. The remaining $m^4$ term also agrees with the finite result of non-zero K-K mode contributions (3.2), which was calculated by performing the dimensional regularization for 4D momentum before taking the mode sum.

Similarly, one-loop corrections to S-parameter, which is obtained from the kinetic mixing term for $U(1)$ gauge bosons, can be calculated as

$$\Pi_{3V}(0) = i\frac{\sqrt{3}g^2}{48} 2^{D/2} \int \frac{d^D k}{(2\pi)^D} \sum_{n=-\infty}^{\infty} \left[ \frac{2}{(k^2 - m_n^2)^2} + \frac{1}{[k^2 - (m_n + m)^2]^2} \right].$$
\[-18 \int_0^1 dt (1-t) \left\{ \frac{1}{[k^2 - (m_n + (2t-1)m)^2 - 4t(1-t)m^2]^2} + \frac{2t-1}{[k^2 - (m_n + (2t-1)m)^2 - 4t(1-t)m^2]^3} \right\} \].

(3.5)

The finite value is found in a similar way.

\[\Pi'_{3Y}(0) = -\frac{23\sqrt{3}g^2}{120} \frac{1}{(2\pi)^2} \sum_{n=1}^{\infty} \left( \frac{m}{m_n} \right)^2 = -\frac{23\sqrt{3}g^2}{2880} (mR)^2.\] 

(3.6)

\(m^2\) dependence is consistent with the dimension six operator representing S-parameter \((\phi^\dagger W_\mu \bar{z}_a \phi) B^\mu (\phi^\dagger \phi)\). \(m_n^{-2}\) dependence is also consistent with the decoupling nature of K-K particles.

4 \(D > 5\) case

In this section, we would like to clarify whether these parameters are finite or not in the case higher than five dimensions. S and T parameters are given by the coefficients of dimension six operators such as \((\phi^\dagger W_\mu \phi) B^\mu (\phi^\dagger \phi)\) for S-parameter and \((\phi^\dagger D_\mu \phi)(\phi^\dagger D^\mu \phi)\) for T-parameter. Naively, the corresponding operators in the gauge-Higgs unification can be regarded as the operators where Higgs doublet \(\phi\) is replaced with \(A_i\) (\(i\): extra space component index). Since \(A_i\) transform as \(A_i \to A_i + \text{const}\) by the higher dimensional local gauge symmetry, it seems that the local operators for S and T parameters are forbidden as in the case of Higgs mass. Therefore, we are tend to conclude that S and T parameters in gauge-Higgs unification become finite, but this argument is too naive, and not correct.

The point is that the gauge invariant local operators for S and T parameters are allowed by a single gauge invariant operator \(\text{Tr}[(D_L F_{MN})(D^L F^{MN})]\). Therefore, there is no physical reason for S and T parameters to be finite.

\[\text{Tr}[(D_L F_{MN})(D^L F^{MN})] \geq \frac{1}{2} (8m^4)(W_\mu^3)^2 + (2m^4)W_\mu^+ W^- + 2\sqrt{3}m^2 p^2 g_{\mu\nu} W^{3\mu} B^\nu + 2\sqrt{3}m^2 (p^2 g_{\mu\nu} - p_\mu p_\nu) W^{3\mu} B^\nu.\] 

(4.1)

What a remarkable thing is that we can predict some combination of S and T parameters although these parameters themselves are divergent. We can read off the ratio of them as

\[C \text{Tr}[(D_L F_{MN})(D^L F^{MN})] \to \begin{cases} \frac{\Delta M^2}{6C m^4} = 6C m^4 \\ \Pi'_{3Y} = 4\sqrt{3}C m^2, \end{cases}\]

(4.2)

where \(C\) is an undetermined overall constant. Thus, we can expect the combination \(\Pi'_{3Y} - \frac{2\sqrt{3}}{3m^2} \Delta M^2\) to be finite even in more than five dimensions.
In fact, we can show that $\Pi'_{3Y} - \frac{2\sqrt{3}}{3m} \Delta M^2$ is finite in 6D case because

\[
\Delta M^2 = \frac{3g^2}{40\sqrt{2}\pi^2} \sum_{n=1}^{\infty} \left[ -\frac{m^4}{m_n} + \frac{1}{12} \frac{m^6}{m_n^3} \right],
\]

(4.3)

\[
\Pi'_{3Y}(0) = \frac{\sqrt{3}g^2}{20\sqrt{2}\pi^2} \sum_{n=1}^{\infty} \left[ -\frac{m^2}{m_n} + \frac{3}{14} \frac{m^4}{m_n^3} \right],
\]

(4.4)

where the first term indicates logarithmic divergence. Combining these results (4.3) and (4.4), we obtain the finite result

\[
\Pi'_{3Y} - \frac{2\sqrt{3}}{3m^2} \Delta M^2 = \frac{11\sqrt{6}g^2}{3360\pi^2} m^4 R^3 \zeta(3).
\]

(4.5)

5 Conclusions

In this talk, we have discussed the divergence structure of one-loop corrections to S and T parameters in gauge-Higgs unification. Taking a minimal $SU(3)$ gauge-Higgs model with a triplet fermion, we have calculated S and T parameters at one-loop order. In five dimensions, we have shown that one-loop corrections to S and T parameters are finite and evaluated their finite values explicitly. In more than six dimensions, S and T parameters are divergent as in the UED scenario. However, a particular combination of S and T parameters is shown to be finite in six dimension case, whose relative ratio was found to agree with that derived from the operator analysis in a model independent way. This is the crucial difference from the UED scenario.

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References

[1] C.S. Lim and Nobuhito Maru, “Calculable One-loop Contributions to S and T Parameters in the Gauge-Higgs Unification”, hep-ph/0703017 and related references therein.