Hiring Doctors in E-Healthcare With Zero Budget

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Abstract—The doctors (or expert consultants) are the critical resources on which the success of critical medical cases are heavily dependent. With the emerging technologies (such as video conferencing, smartphone, etc.) this is no longer a dream but a true fact, that for critical medical cases in a hospital, expert consultants from around the world could be hired, who may be present physically or virtually. Earlier, this interesting situation of taking the expert consultancies from outside the hospital had been studied, but under monetary perspective. In this paper, for the first time, to the best of our knowledge, we investigate the situation, where the Below Income Group (BIG) people of the society may be served efficiently through the expert consultancy by the renowned doctors from outside of the hospital under zero budget. This will help us saving many lives which will fulfill the present day need of biomedical research. We proposes two mechanisms: Random pick-assign mechanism (RanPAM) and Truthful optimal allocation mechanism (TOAM) to allocate the doctor to the patient. With theoretical analysis, we demonstrate that the TOAM is strategy-proof, and exhibits a unique core property. The mechanisms are also validated with exhaustive experiments.

Index Terms—E-Healthcare, hiring experts, core allocation.

I. INTRODUCTION

Considing the diverse potential application areas of crowdsourcing [1], [2] especially of PS [3]–[5], e-healthcare system, which is emerging as one of the most upcoming technologies to provide an efficient and automated healthcare infrastructure, can employ the crowdsourcing technology to enhance the services provided in that environment. Healthcare consultation (as for example consultation from physician, paediatricians, plastic and cosmetic surgeons, etc.) is said to be the crux of medical unit and operation suite. Earlier there had been some works to schedule the internal staffs of a medical unit (may be very large) for working in the operation theatre or controlling the outdoor units. Most of the literatures work on scheduling the internal staffs (inside the hospital) of the medical unit have been devoted to hospital nurses [6]–[9] and the physicians (or doctors) [10], [11]. The doctors are considered as the scarce resource in the medical unit. Scheduling doctors inside a hospital during a critical operations is a challenging task. In [10]–[13] the different methods of scheduling a physician in an emergency case (may be critical operation) are discussed and presented. Several companies have developed some physician scheduling softwares [14]–[17] that will schedule the physicians inside the hospital. However, it is observed that, with the unprecedented growth of the communication media (specially Internet), it may be a common phenomena to hire expert medical staffs (especially doctors) for a critical operation from outside of the medical unit where the operation is taking place. This event of hiring an external expert is a special case of crowdsourcing [1], [2] and participatory sensing [3], [18].

In literatures [19]–[21] it is explored, how to manage a limited resource (for example an Operation Theatre (OT)) to the competing tasks (surgeries) in a particular organization (a hospital or medical unit) and improves the efficiency of OT. In our future references, hospital, medical unit, organizations will be used interchangeably. In the past, OT planning and scheduling problem [19], [22] have been a major area of interest of researchers from various fields and is still an active area of research. However, one scenario that may be considered as a research area in healthcare is, say; during an operation, how to hire a well qualified personnel, including doctors, for a consultation. The challenges come from the following issues: (1) which doctors can be hired? (2) How to motivate the doctors to take part in the system as they may be very busy? (3) If the incentives are provided, how much can be offered? (4) If some renowned doctors are made themselves available for social work, how to grab the situation so poorest people may be served efficiently to save their valuable life? In our recent work [23], we have endeavoured solving the problem of hiring one or more expert from outside of the hospital for a critical operations answering some issues related to the challenges mentioned in points 1, 2, and 3. In [23] incentives were provided to motivate the doctors for their participation. However, it may be the case, that due to some social awareness, some doctors may impart some social services to the downtrodden community. This situation is mentioned in point 4. In this paper, to the best of our knowledge, first time we have tried to address the practical situation discussed in point 4 above in a game theoretic settings with the robust concept mechanism design without money or under zero budget environment.

In our paper, we have considered \( n \) hospitals as shown in Fig. 1. In each hospital several patients for a particular category (say for example gastric ulcer) are admitted who need expert consultation. The patients are of different income groups. Some of the patients may not be able to bear the cost of hiring expert consultation from outside the hospital. However, it may be the case that, due to some social awareness, several doctors throughout the world, may be available freely once in a while (e.g. once in a week). We have assumed that, at a particular time more than \( n \) number of doctors are available. They give their willingness to participate in the consultation process to some third party (platform). The third party selects \( n \) doctors out of all doctors available based on the quality of the doctors. Now the question is how to use the expert consultations that are available freely.
In this situation, each hospital selects one patient based on their income group (whoever has lowest income), who will be considered for free consultation shown on the left side in Fig. 1. Thus, we have $n$ patients available for $n$ doctors for a particular category (say for paediatric). In this paper, we have first time proposed an algorithm motivated by [24], [25] to allocate the doctors to the patients so that they will be happy. The main contributions of this paper are:

- First time the problem of hiring doctors is cast as a zero money problem or in zero budget environment.
- A truthful mechanism is proposed for allocating the doctors to the patients.
- A simulation is performed for comparing our scheme with a carefully designed benchmark scheme.

The remainder of the paper is organized as follows: In section II we describe the system model and formulate the problem as mechanism design without money (MDWM). In section III, we present two mechanisms: Random Pick-Assign Mechanism (RanPAM) and Truthful Optimal Allocation Mechanism (TOAM). A detailed analysis of the experimental results is carried out in section IV. Finally, we present a summary of our work and highlight some future directions in section V.

II. SYSTEM MODEL AND PROBLEM FORMULATION

We consider the scenario where $n$ distinct expert consultant providing their consultancy to $n$ different Below Income Group (BIG) patients. One hospital provides one BIG patient for the category under consideration. A platform acts as a third party and decides the patient – doctor allocation pair. Here, we assume that each hospital needs exactly one expert consultant and each expert consultant can provide their service to one hospital at a time. The case that each hospital need multiple expert consultants is left as our future work. In our model, expert consultation may be sought for several categories of diseases. The set of such categories is denoted by $x = \{x_1, x_2, \ldots, x_k\}$. The set of available expert consultants for a particular category $i \in \{1, \ldots, k\}$ is denoted by $P_i = \{p_{i1}^x, p_{i2}^x, \ldots, p_{in}^x\}$. The set of available BIG agents of particular category $i \in \{1, \ldots, k\}$ is denoted by $S_i = \{s_{i1}^x, s_{i2}^x, \ldots, s_{in}^x\}$. The strict preference ordering of $l^{th}$ category, $t$ prefers $s_{il}^x$ to $s_{ij}^x$. The set of preferences of all agents in $k$ different categories is denoted by $\succ = \{\succ_1, \succ_2, \ldots, \succ_k\}$. Where, $\succ_i$ is the preference of all the agents in $i^{th}$ category over all the doctors in $S_i$, represented as $\succ_i = \{\succ_i^1, \succ_i^2, \ldots, \succ_i^n\}$. The strict preference ordering of all the agents except agent $i$ is represented as $\succ_i^{-i}$. Given the preference of the agents, our proposed mechanisms allocates one doctor to one patient. Let us denote such an allocation vector by $A = \{A_1, A_2, \ldots, A_k\}$; where, each allocation vector $A_i \in A$ denotes the allocation vector of agents belongs to the $i^{th}$ category denoted as $A_i = \{a_{i1}^x, a_{i2}^x, \ldots, a_{in}^x\}$. Initially, one doctor is allocated to one patient randomly without loss of generality. Here, each $a_{il}^x \in A_i$ is a $(p_{il}^x, s_{il}^x)$ pair.

III. PROPOSED MECHANISMS

In this section, we have developed two algorithms motivated by [25] [26]. The first one i.e. RanPAM is given as a naive solution of our problem, that will help to understand better, the more robust DSIC mechanism i.e. TOAM presented next.

A. Random Pick-Assign Mechanism (RanPAM)

To better understand the model first we propose a randomized algorithm to assign doctors to the patients in the hospitals called RanPAM. The input to the RanPAM are: the set of $n$ available patients, the set of $n$ available doctors, the set of preferences of all the agents for the available doctors in a particular category. The output of the mechanism is the allocated patient-doctor pairs. From line 3, it is clear that the algorithm terminates, once the lists of patients showing preference becomes empty. Initially in line 4, using random() function, randomly selects a patient and the index of the selected patient is stored in a variable $i$. In line 5, the $p^*$ data structure holds the patient at the index returned in line 4. Line 6 checks the preference list of patient $i$. In line 7, the $p^*$ data structure holds the doctor present at index returned in line 4. Line 7 maintains the selected patient-doctor pairs in $R$ data structure. Line 10 and 11 removes the selected patients and selected doctors from their respective preference lists. Line 12 removes the selected doctor from the preference lists of the remaining patients. The RanPAM returns the final patient-doctor allocation pair set $R$.

**Algorithm 1 RanPAM ($S_i, P_i, \succ_i$)**

Output: $R \leftarrow \phi$.

1. begin
2. $i \leftarrow 0, j \leftarrow 0, p^* \leftarrow \phi, s^* \leftarrow \phi$
3. while $P_i \neq \phi$ do
4. $i \leftarrow$ random($P_i$).
5. $p^* \leftarrow \{p_{1i}^x, \ldots, p_{ni}^x\}$.
6. Check $>_i$, such that $>_{ij} \in >_i$.
7. $j \leftarrow$ random($>_i$).
8. $s^* \leftarrow s_{ij}^x$.
9. $R \leftarrow R \cup (p^* \cup s^*)$.
10. $P_i \leftarrow P_i \setminus p^*$.
11. $S_i \leftarrow S_i \setminus s^*$.
12. $>_{ij} \leftarrow >_{ij} \setminus s^*$, $\forall j \in P_i$.
13. end while
14. return $R$
15. end
1) Time complexity: The time taken by the Algorithm 1 is the sum of running times for each statement executed. The overall time complexity of Algorithm 1 is \( O(1) + O(n+1) + O(n) = O(n) \).

2) Essential properties: There are two essential properties that will help to develop the further mechanisms in our paper. The two properties are: Blocking coalition and Core. These two properties captures the fundamental idea: can we design a system where agents cannot gain by manipulating their preferences that are only known to them.

- **Blocking coalition.** For every \( T \subset P \) let \( A(T) = \{ u \in A : u_i^{p_i} \in T \} \) denote the set of allocations that can be achieved by the agents in \( T \) trading among themselves alone. Given an allocation \( a \in A \), a set \( T \subseteq P \) of agents is called a blocking coalition (for \( a \)), if there exists a \( u \in A(T) \) such that \( \forall i \in T \) either \( u_i > a_i \) or \( u_i = a_i \) and at least one agent is better off i.e. for at least one \( j \in T \) we have \( u_j > a_j \).

- **Core.** An allocation \( a \) will be said to be a core allocation, if there is no allocation \( \hat{a} \) that could have done better for any \( p_i^p \in P \). The core of a \( M(\succ) \), is the collection of all \( a \in A \) in which no allocation is blocked by any subset of agents. It can be shown that every allocation \( a \) lying in the core of \( M(\succ) \) is stable in the sense that no player be benefited by unilaterally deviating from the given allocation in core.

3) Drawback: From the perspective of blocking coalition, it can be concluded that the RanPAM is suffering from the blocking coalition. This leads to the violation of one of the major properties in the area of Mechanism design without money (MDWM) named as core.

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**B. Truthful Optimal Allocation Mechanism (TOAM)**

The proposed truthful mechanism needs to overcome several non-trivial challenges: firstly, the patients preferences are unknown and need to be reported in a truthful manner; secondly, the allocation of doctors made to the patients must satisfy the core. The previously discussed RanPAM mechanism failed to handle such challenges. To overcome these challenges, in this paper a truthful mechanism is proposed which is termed as TOAM. Along with, truthfulness, TOAM satisfies Pareto Optimal (defined later) property. The main idea of the TOAM is to develop a mechanism where the agent can’t gain by manipulation. If there is no manipulation we can reach to the equilibrium of the system very quickly and the market become stable. The TOAM satisfies two useful properties mentioned in the previous subsection. The two properties are:

- **Truthfulness or Incentive compatibility.** Let \( A_i = M(\succ^i_1, \succ^i_2, ..., \succ^i_k) \) and \( \hat{A}_i = M(\succ^i_1, \succ^i_2, ..., \hat{\succ}^i_k) \). TOAM is truthful if \( a(i)^{\hat{a}} \succeq a(i)^{\hat{a}} \), for all \( p_i^p \in P_i \).

- **Pareto optimal.** An allocation \( A_i \) is pareto optimal if there exists no allocation \( b^i \in A_i \) such that for all \( i \in P_i \), \( b(i)^{\hat{a}} > a(i)^{\hat{a}} \) or \( b(i)^{\hat{a}} = a(i)^{\hat{a}} \), and for some \( j \in P_i \), \( b(j)^{\hat{a}} > a(j)^{\hat{a}} \).

1) **Sketch of the TOAM:** More formally, the proposed TOAM, unlike the previous mechanism, can be thought of as a four stage allocation mechanism: Main routine, Graph initialization, Graph creation and Cycle detection.

   **a) Main routine:** The idea lies behind the construction of main routine is to handle the system, consisting of patients and doctors partitioned on the basis of category \( x = \{x_1, x_2, ..., x_k \} \). The input to the main routine is the sets of vertices representing the patients in each category at a time i.e. \( C \), the set of vertices representing expert consultants (doctors) in each category at a time i.e. \( Q \), and the set \( x \) containing the information about the type of expertise present in the system. The output of the main routine is the allocation set \( A \) containing the sets of all feasible allocation \( A_i \), for all the available categories \( i \in 1...k \). In line 4, the select() returns the index of the set of vertices representing the patients belonging to each \( x_i \).

**Algorithm 2 Main routine (C, Q, x)**

```
Output: A ← \{A_1, A_2, ..., A_k\}
1: begin
2:    A ← φ, Q* ← φ, C* ← φ
3:    for each \( x_i \in x \) do
4:        i ← select(C)
5:        C* ← C*
6:        i ← select(Q)
7:        Q* ← Q*
8:    A_i = Graph initialization (C*, Q*)
9:    A ← A ∪ A_i
10: end for
11: return A
12: end
```

In line 5, the \( C^* \) data structure holds the set of vertices present at the index returned by line 4. Similarly, the select() in line 6, returns the index of the set of vertices representing the doctors belonging to \( x_i \) category. The \( Q^* \) data structure in line 7 holds the set of vertices present at index returned by line 6. For each category \( x_i \in x \), for loop calls the graph initialization \( (C^*, Q^*) \) and the resultant allocation vector is returned for each category \( i \in 1...k \). Line 9, maintains the set of all feasible allocations \( A_i \) for all available categories \( i \in 1...k \) in \( A \).

**b) Graph initialization:** The input to the graph initialization phase is the selected set of patients and the doctors in each iteration of for loop of main routine. The output of the graph initialization is the incomplete graph \( G \) in the form of adjacency matrix \( \mathcal{F} \). Line 3, initializes the adjacency matrix \( \mathcal{F} \) of size \( |\mathcal{V}| \times |\mathcal{V}| \) to null matrix. Line 4, keeps track of the strict preference ordering of the agents of different categories.

**Algorithm 3 Graph initialization (C*, Q*)**

```
1: begin
2:    i ← 0
3:    \( \mathcal{F} = \{0\}_{|\mathcal{V}||\mathcal{V}|} \)
4:    \( \succ = \{\succ^{\hat{a}}, \succ^1, \succ^2, ..., \succ^k\} \)
5:    for each vertex \( c_i \in C^* \) do
6:        i ← Random_assign(Q*)
7:        c_i* ← \{q_i\}
8:        \( \mathcal{F}_{c_i, c_i} = 1 \)
9:    Q^* ← Q^* \setminus q^*
10: end for
11: return \( \mathcal{F} \)
12: Graph creation (C*, Q*, \( \mathcal{F}, \succ \))
13: end
```

The for loop in line 5 iterates over all the patients in the \(i^{th}\) category. In line 6, the `Random_assign()` function takes the set of vertices \(Q^*\) (analogous to the doctors with \(x_i\) expertise area) as the input and returns the index of the randomly selected vertex \(q_i \in Q^*\). In line 6, the \(q^*\) data structure holds the vertex \(q_i\) present at randomly selected index. The mechanism generates an independent directed graph for each category. Line 8, places a directed edge from each \(q^* \in Q^*\) to each \(c_i \in C^*\). Line 9, removes the randomly allocated vertex \(q_i \in Q^*\) in the current iteration. Line 11, returns the incomplete directed graph \(G\) in the form of adjacency matrix \(F\). In line 12, a call to `Graph creation` phase is done.

c) Graph creation: The input to the `graph creation` are: the set of vertices \(C^*\), the set of vertices \(Q^*\), the incomplete graph in the form of adjacency matrix \(F\), and the strict preference ordering of the agents of different categories \(\succ\).

The output of the `graph creation` is the adjacency matrix \(F\). In line 2, a variable is initialized to 0. Line 3 iterates for all the vertices \(c_i \in C^*\). In line 4, the strict preference ordering of \(t^{th}\) agent is checked in the preference list set \(\succ\). In line 5, the `Select_best()` function takes the strict preference ordering list of \(t^{th}\) agent as input and returns the index of the best vertex \(q_i \in Q^*\). In line 6, the \(q^*\) data structure holds the vertex \(q_i\) present at the best selected index in line 5. Line 7 places a directed edge from each \(c_i \in C^*\) to \(q^* \in Q^*\) in current iteration. Line 10 returns the graph \(G\) in the form of adjacency matrix \(F\). In line 11, a call to `Optimal allocation` phase is done.

**Algorithm 4** Graph creation (\(C^*, Q^*, F, \succ\))

```java
1: begin
2: \(j \leftarrow 0\)
3: for each vertex \(c_i \in C^*\) do
4: if \(\succ_j \in \succ^*\) then
5: \(j \leftarrow \text{Select_best}(\succ_j)\)
6: \(q^* \leftarrow \{q_i\}\)
7: \(F_{c_i, q^*} = 1\)
8: end if
9: end for
10: return \(F\)
11: Optimal allocation (\(F, S\))
12: end
```

d) Optimal allocation: The next challenge is to determine a finite cycle in a directed graph \(G\). The input to the `Optimal allocation mechanism` is the adjacency matrix \(F\) returned from the previous stage and an empty stack \(S\). Initially, all \(v_i \in V\) are marked unvisited. Random vertex \(v_i \in V\) is selected and after marking that \(v_i\) as visited is pushed into the stack. Line 10–19, computes a finite directed cycle in the graph \(G = (V, E)\) by following the outgoing arcs, until a vertex \(v_i \in V\) gets repeated. Line 20, reallocates as suggested by directed cycle. Each patient on a directed cycle gets the expert consultant better than the expert consultant it initially points to or the initially pointed expert consultant. Each cycle gets the graph creation phase to generate the updated graph from the available number of patients and the expert consultants until the patients set and doctor sets are not empty.

**Algorithm 5** Optimal allocation (\(F, S\))

```java
1: begin
2: \(\pi \leftarrow \phi, C^* \leftarrow \phi, Q^* \leftarrow \phi\)
3: for each \(v_i \in V\) do
4: Mark \(v_i\) as unvisited
5: end for
6: \(\pi \leftarrow \text{random}(v_i \in V)\)
7: Mark \(\pi\) as visited
8: push(S, \(\pi\)
9: while \(S\) is non-empty do
10: \(\pi \leftarrow \text{pop}(S)\)
11: for each \(\pi^*\) adjacent to \(\pi\) do
12: if \(\pi^*\) is unvisited then
13: Mark \(\pi^*\) as visited
14: push(S, \(\pi^*\)
15: else if \(\pi^*\) is visited then
16: Exists a finite cycle.
17: end if
18: end for
19: end while
20: Allocate each \(v_i \in C^*\) in cycle the doctors it points in \(Q^*\)
21: \(C^* \leftarrow C^* \cup v_i\)
22: \(C^* \leftarrow C^* \setminus C^*\)
23: \(Q^* \leftarrow Q^* \setminus v_i\)
24: \(Q^* \leftarrow Q^* \setminus Q^*\)
25: \(S = Q^* \cup C^*\)
26: if \(C^* \neq \phi\) and \(Q^* \neq \phi\) then
27: for all \(c_i \in C^*\) do
28: \(\succ^* \leftarrow \phi\)
29: \(\succ^* \leftarrow \succ^* \cup C^*\)
30: \(\succ^* \leftarrow \succ^* \cup \succ^*\)
31: for all \(q_k \in Q^*\) do
32: if \(F_{c_i, q_k} = 1\) then
33: \(F_{c_i, q_k} = 0\)
34: end if
35: end for
36: end for
37: Graph creation (\(C^*, Q^*, F, \succ\)
38: end if
39: end
```

2) Several properties of TOAM: The proposed TOAM has several compelling properties. These properties are discussed next.

a) Running time: Total running time of TOAM: \(O(n) + O(n^2) = O(n^2)\). The detailing of the running time calculation is excluded because of space contraint.

b) DSIC: The second property, that distinguishes the proposed TOAM from any direct revelation allocation mechanism is its DSIC property. In TOAM, the strict preference ordering revealed by the agents in any category \(x_i \in x\) over the set of doctors \(S_i\) are unknown or private to the agents. As the strict preference ordering is private, any agent \(i\) belonging to category \(x_j \in x\) can misreport their private information to make themselves better off, TOAM, an obvious direct revelation mechanism claims that agents in any category \(i \in 1 \ldots k\) cannot make themselves better off by misreporting their private valuation, i.e. TOAM is DSIC.

Theorem 1. The TOAM is DSIC.

**Proof.** The truthfulness of the TOAM is based on the fact that each agent \(i\) gets the best possible choice from the reported strict preference, irrespective of the category \(i \in 1 \ldots k\) of the agent \(i\). It is to be noted that, the third party (or the platform) partition the available patients and doctors into different sets based on their category. The partitioning of doctors set \(S = \{S_1, S_2, \ldots, S_k\}\) is independent of the partitioning of
the available patients into the set \( P = \{ P_1, P_2, \ldots, P_k \} \). So, if we select the patient set \( P_i \in P \) and the doctor set \( S_i \in S \) randomly from category \( x_i \in x \) and show that for any agent \( p_1^{x_i} \in P_i \), misreporting the private information (in this case strict preference over \( S_i \)) will not make the agent \( p_1^{x_i} \) better-off, then its done. Our claim is that, if any agent belonging to \( x_i \) category, cannot be better off by misreporting their strict preference, then no agent from any category can be better off by misreporting the strict preference.

Fix category \( x_i \). Let us assume that, if all the agents in \( x_i \) are reporting truthfully, then all the agents gets a doctor till the end of \( m \)th iteration. From the construction of the mechanism in each iteration of the TOAM, at least a cycle \( \Omega_i \in \Omega \) is selected. The set of cycles chosen by the TOAM in \( m \) iterations are: \( \Omega = (\Omega_1, \Omega_2, \ldots, \Omega_m) \), where \( \Omega_i \) is the cycle chosen by the TOAM in the \( i \)th iteration, when agents reporting truthfully. Each agent in \( \Omega_i \), gets its first choice and hence no strategic agent can be benefited by misreporting. From the construction of the mechanism, no agent in \( \Omega_i \) will ever be pointed by any agents in \( \Omega_1, \ldots, \Omega_{i-1} \); if this is not the case, then agent \( i \) could have been belong to one of the previously selected cycle.

Once the doctor is allocated to the agent, the mechanism remove the agent along with the allocated doctor, and the strict preference list of the remaining agents are updated. Since, agent \( i \) gets its first choice outside of the doctors allocated in \( \Omega_1, \ldots, \Omega_{i-1} \), it has no incentive to misreport. Thus, whatever agent \( i \) reports, agent \( i \) will not receive a doctor owned by an agent in \( \Omega_1, \ldots, \Omega_{i-1} \). Since, the TOAM gives agent \( i \) its favourite doctor outside the selected cycle till now. Hence, agent \( i \) did not gain by misreporting the strict preference ordering. From our claim it must be true for any agents in any category \( i \in 1 \ldots k \). Hence, TOAM is DSIC.

c) Core: The third property exhibited by the proposed TOAM is related to the uniqueness of the resultant allocation or in some sense optimality. The term used to determine the optimal allocation of TOAM is termed as unique core allocation. The claim is that, the allocation computed by the proposed TOAM is the unique core allocation.

Theorem 2. The allocation computed by TOAM is the unique core allocation.

Proof. The proof of unique core allocation for any category \( x_i \in x \) can be thought of as divided into two parts. First, it is proved that the allocation vector \( A_i \) computed by TOAM for any category \( x_i \in x \) is a core allocation. Once the allocation vector in \( x_i \) category is proved to be The core, the uniqueness of the core allocation for \( x_i \) is taken into consideration. Our claim is that, if the allocation \( A_i \) computed by TOAM for any arbitrary \( x_i \in x \) is a unique core allocation, then the allocation computed by TOAM for all \( x_i \in x \) will be a unique core allocation.

Fix category \( x_i \). In order to prove the allocation computed by TOAM is a core allocation, consider an arbitrary sets of agents \( S^* \), such that \( S^* \subseteq P \). Let \( \Omega_i \) is the cycle chosen by TOAM in the \( i \)th iteration and \( \delta(\Omega_i) \) is the set of agents allocated a doctor, when reporting truthfully. When TOAM will allocate the agents, at some cycle \( \Omega_k, i \in S^* \) will be included for the first time. In that case \( \delta(\Omega_k) \cap S^* \neq \phi \). As any agent \( i \in S^* \) is being included for the first time, it can be said that no other agent in \( S^* \) is included in the cycles \( \Omega_1, \ldots, \Omega_{k-1} \). As the TOAM allocates the favourite doctor to any arbitrary agent \( i \in \delta(\Omega_k) \) outside the doctors allocated to \( \delta(\Omega_1), \ldots, \delta(\Omega_{k-1}) \), it can be concluded that \( i \in \delta(\Omega_k) \) and \( i \in S^* \) such that \( \delta(\Omega_k) \cap S^* \neq \phi \) gets his favourite doctor at the \( k \)th iteration. Hence no internal reallocation can provide a better doctor to any agent \( i \in S^* \). Inductively, the same is true for any agent \( j \in S^* \) that will satisfy \( \delta(\Omega_k) \cap S^* \neq \phi \). Now, we prove uniqueness. In TOAM, each agent in \( \Omega_1 \) receives the best possible doctor from his preference list. Any core allocation must also do the same thing, otherwise the agents who didn’t get the first choice could be better off with internal reallocation. So the core allocation agrees with the TOAM allocation for the agents in \( \delta(\Omega_1) \). Now in TOAM, as all the agents in \( \delta(\Omega_2) \) get their favourite doctors outside the set of doctors allocated to the agents \( \delta(\Omega_1) \), any core allocation must be doing the same allocation, otherwise the agents in \( \delta(\Omega_2) \) who didn’t get their favourite choice can internally reallocate themselves in a better way. In this way we can inductively conclude that the core allocation must follow the TOAM allocation. This proves the uniqueness of TOAM.

Hence, it is proved that the allocation by TOAM for category \( x_i \) is a unique core allocation. From our claim it must be true for any agents in categories \( i \ldots k \). Hence, the allocation computed by TOAM for any category \( x_i \in x \) is the unique core allocation.

3) Correctness of the TOAM: The correctness of the TOAM mechanism is proved with the loop invariant technique [27][23]. The loop invariant: At the start of \( j \)th iteration, the number of patient-doctor pairs to be explored are \( n - \sum_{i=1}^{j-1} k_i \) in a category, where \( k_j \) is the number of patient-doctor pairs processed at the \( i \)th iteration. Precisely, it is to be noted that \( n - \sum_{i=1}^{j-1} k_i \leq n \). From definition of \( k_i \), it is clear that the term \( k_i \) is non-negative. The number of patient-doctor pairs could be at least 0. Hence, satisfying the inequality \( n - \sum_{i=1}^{j-1} k_i \leq n \). We must show three things for this loop invariant to be true.

Initialization: It is true prior to the first iteration of the loop. Just before the first iteration of the while loop, in optimal allocation mechanism \( n - \sum_{i=1}^{j-1} k_i \leq n \) \( \Rightarrow n - 0 \leq n \) i.e. no patient-doctor pair is explored a priori in, say \( i \)th category. This confirms that \( A_i \) contains no patient-doctor pair.

Maintenance: For the loop invariant to be true, if it is true before each iteration of while loop, it remains true before the next iteration. The body of while loop allocates doctor(s) to the patient(s) with each doctor is allocated to one patient present in the detected cycle; i.e. each time \( A_i \) is incremented or each time \( n \) is decremented by \( k_i \). Just before the \( j \)th iteration the number of patient-doctor pairs allocated are \( \sum_{i=1}^{j-1} k_i \), implies that the number of patient-doctor pairs left are: \( n - \sum_{i=1}^{j-1} k_i \leq n \). After the \( j \)th iteration, two cases may arise:

Case 1: If \( k_j = n - \sum_{i=1}^{j-1} k_i \)

In this case, all the \( k_j \) patient-doctor pairs will be exhausted
in the \( j^{th} \) iteration and no patient-doctor pair is left for the next iteration. The inequality \( n - (\sum_{i=1}^{j-1} k_i + k_j) = (n - \sum_{i=1}^{j-1} k_i) - (n - \sum_{i=1}^{j-1} k_i) = 0 \leq n \).

**Case 2:** If \( k_j < n - \sum_{i=1}^{j-1} k_i \)

In this case, \( j^{th} \) iteration allocates few patient-doctor pairs from the remaining patient-doctor pairs; leaving behind some of the pairs for further iterations. So, the inequality \( n - (\sum_{i=1}^{j-1} k_i + k_j) \leq n - n - \sum_{i=1}^{j} k_i \leq n \) is satisfied.

From Case 1 and Case 2, at the end of \( j^{th} \) iteration the loop invariant is satisfied.

**Termination:** In each iteration at least one patient-doctor pair is formed. This indicates that at some \((j + 1)^{th}\) iteration the loop terminates and in line no. 9, \( S \) is exhausted, otherwise the loop would have continued. As the loop terminates and \( S \) is exhausted in \((j + 1)^{th}\) iteration. We can say \( n - \sum_{i=1}^{j} k_i = 0 \leq n \). Thus indicates that all the \( n \) agents are processed and each one has a doctor assigned when the loop terminates.

4) **Illustrative example:** The number of patients is \( n = 5 \) i.e. \( P_2 = \{p_1^{z_2}, p_2^{z_2}, p_3^{z_2}, p_4^{z_2}, p_5^{z_2}\} \) and the number of expert consultant (or doctors) is \( n = 5 \) \( S_2 = \{s_1^{z_2}, s_2^{z_2}, s_3^{z_2}, s_4^{z_2}, s_5^{z_2}\} \). The strict preference ordering given by the patient set \( P_2 \) is: \( p_1^{z_2}: (s_2^{z_2}, s_3^{z_2}, s_4^{z_2}, s_1^{z_2}, s_5^{z_2}); p_2^{z_2}: (s_3^{z_2}, s_1^{z_2}, s_2^{z_2}, s_4^{z_2}, s_5^{z_2}); p_3^{z_2}: (s_4^{z_2}, s_2^{z_2}, s_3^{z_2}, s_5^{z_2}, s_1^{z_2}); p_4^{z_2}: (s_5^{z_2}, s_1^{z_2}, s_3^{z_2}, s_4^{z_2}, s_2^{z_2}); p_5^{z_2}: (s_1^{z_2}, s_4^{z_2}, s_3^{z_2}, s_2^{z_2}, s_5^{z_2}) \). Following the graph initialization phase a directed edge is placed between the following pairs: \( \{(s_1^{z_2}, p_1^{z_2}), (s_2^{z_2}, p_2^{z_2}), (s_3^{z_2}, p_3^{z_2}), (s_4^{z_2}, p_4^{z_2}), (s_5^{z_2}, p_5^{z_2})\} \). Now, Following the graph creation phase, say, a patient \( p_3^{z_2} \) is selected from \( P_2 \). As, \( s_3^{z_2} \) is the most preferred doctor in the preference list of \( p_3^{z_2} \). So, a directed edge is placed from \( p_3^{z_2} \) to \( s_3^{z_2} \). The for loop of the graph creation phase places a directed edge between the remaining patients in \( P_2 \) and the most preferred doctors in \( S_2 \), resulting in a directed graph.

Now, running the optimal allocation phase on the directed graph a cycle \( (p_2^{z_2}, s_2^{z_2}, p_2^{z_2}, s_2^{z_2}, p_2^{z_2}) \) is determined. Similarly, the remaining patients \( P_2 = \{p_1^{z_2}, p_4^{z_2}, p_5^{z_2}\} \) will be allocated a doctor. The final allocation of patient – doctor pair are: \( \{(p_1^{z_2}, s_4^{z_2}), (p_2^{z_2}, s_2^{z_2}), (p_3^{z_2}, s_2^{z_2}), (p_4^{z_2}, s_5^{z_2}), (p_5^{z_2}, s_1^{z_2})\} \).

**IV. Experimental findings**

In this section, we compare the efficacy of the proposed mechanisms via simulations. The experiments are carried out in this section to provide a simulation based on the data (the strict preference ordering of the patients) generated randomly. Our proposed naive mechanism i.e. RanPAM is considered as a benchmark scheme. It is to be noted that RanPAM is not truthful and is vulnerable to manipulation. In our simulation results, this manipulative nature of RanPAM can be seen evidently. The experiments are done using Python. Random library is used in python to randomly generate the strict preference ordering for the patients.

A. **Simulation setup**

For the simulation purpose, the number of agents and the number of available doctors i.e. \( n \) is bounded above by \( n = 50 \). In simulation, \( n \) value is taken as: \( n = 10, n = 20, n = 30, n = 40, \) and \( n = 50 \). The strict preference ordering of each patient is generated randomly. It is to be noted that the simulation is done for a particular category.

**B. Performance metrics**

The performance of the proposed mechanisms is measured under the banner of two important parameters:

- **a) Efficiency loss (EL):** It is the sum of the difference between the index of the doctor allocated from the agent preference list to the index of the most preferred doctor by the agent from his preference list. Mathematically, the \( EL \) is defined as:

\[
EL = \sum_{i=1}^{n} (T_{iA} - T_{iMP})
\]

Where, \( T_{iA} \) is the index of the doctor allocated from the initially provided preference list of the patient \( i \), \( T_{iMP} \) is the index of the most preferred doctor in the initially provided preference list of patient \( i \). More formally, the \( EL \) measures the gap between the most preferred doctor by the patient and the allocated doctor by our proposed mechanisms to the patient.

- **b) Number of best allocation (NBA):** It measures the number of patients (say \( k \)) gets their best choice (most preferred doctor) from their provided preference list over the available number of doctors.

**C. Simulation directions**

As the benchmark scheme is vulnerable to manipulation, the two direction are seen for measuring the performance of RanPAM and the TOAM. The two directions are: (1) When all the agents (patients) are reporting their true preference list. (2)When the agents (patients) are misreporting their true preference list.

**D. Result analysis**

Our simulation results are analysed by considering the agenda of *truthful* and *non-truthful* revelation of the preference ordering by the patients. First, the two parameters: \( EL \) and \( NBA \) are discussed under the banner of "*truthful revelation of the preference ordering" i.e. no patient vary his preference list (\( \delta = 0 \), \( \delta \) is defined later). Next, these stated parameters (\( EL \) and \( NBA \)) are studied under the manipulative environment of the market i.e. patients are varying their preference ordering (\( \delta > 0 \)). In this section, the result is simulated for the two cases and discussed.

- **When \( \delta = 0 \):**

  In this case, it is assumed that, all the patients are declaring their true preference ordering to the proposed mechanisms. Under this assumption, the RanPAM and TOAM are simulated with \( EL \) and \( NBA \) parameters with consideration of two issues. The two issues are: (1) Effect on the performance of RanPAM and TOAM when the patient is selected randomly. (2) Effect on the performance of RanPAM and TOAM when the initial selection of patient is made fixed.

- **Random selection (RS):** The patient is randomly selected
for allocating a doctor to the selected patient from his preference list in case of RanPAM. In case of TOAM, it means that the source node (i.e., patient or doctor) will be selected randomly for detecting the directed cycle in a directed graph.

- **Fixed selection (FS):** The initial selection of patient is fixed (say $p_{n}^{2}$ is selected) for allocating a doctor from the preference list of $p_{n}^{2}$ in RanPAM. In TOAM, it means that the source node will be $p_{n}^{2}$, for the determination of the directed cycle in the first iteration of the mechanism.

**EL and NBA with RS and FS:** It is seen in Fig. 2 and Fig. 4 that EL of the agents using TOAM is lower than the EL calculated using RanPAM independent of type of initial selection (i.e., fixed or random). This is natural as in TOAM, each of the agents gets a best possible doctor (i.e., lower indexed doctor in preference list) from their reported preference ordering from among available doctors. The value generated by equation 1 in case of TOAM is smaller as compared to RanPAM. In Fig. 3 another interesting observation is made in the simulation experiments in terms of NBA (no. of best allocation). It is observed that the NBA in case of TOAM is larger than in RanPAM independent of initial selection of agents. It is natural from the construction of the TOAM scheme. It is to be noted that, the FS scheme is applied in the first iteration of both the mechanisms. From the next iteration, the general RS scheme is followed.

The simulation in Fig. 4 and Fig. 5 is performed to show that TOAM is not affected by the originating point for the determination of the directed cycle but RanPAM gets affected by the initial selection of patient for the allocation process.

- **When $\delta > 0$:**

In this case, it is assumed that, the patients are varying their true preference list in the proposed mechanisms. Now, the next question that comes is that, how many of the patients can vary their true preference list (i.e., what fraction of the total available patients can vary their true preference list?) To answer this question, the calculation is done using indicator random variable.

**Expected amount of variation:** The following analysis mathematically justifies the idea of choosing the parameters of variation.

### Table I. Experimental parameters

| Available patients | Type of variation | Fraction of agents varying ($\delta$) | Resultant |
|--------------------|-------------------|---------------------------------------|-----------|
| $n$                | small             | $1/n$                                 | $n/8$     |
| $n$                | medium            | $1/4$                                 | $n/4$     |
| $n$                | large             | $1/2$                                 | $n/2$     |

The analysis is motivated by [27]. Let $N_i$ be the random variable associated with the event in which $i^{th}$ patient varies his true preference ordering. Thus, $N_i = \{i^{th}$ patient varies preference ordering$.\} We have, from the definition of expectation that $E[N_i] = \Pr\{i^{th}$ patient varies preference ordering$.\}$. Let $N$ be the random variable denoting the total number of patients vary their preference ordering. By using the properties of random variable, it can be written that $N = \sum_{i=1}^{n} N_i$. We wish to compute the expected number of variations, and so we take the expectation both sides and by linearity of expectation we can write $E[N] = \sum_{i=1}^{n} E[N_i] = \sum_{i=1}^{n} \Pr\{i^{th}$ patient varies preference ordering$.\} = \sum_{i=1}^{n} 1/8 = n/8$. Here, $E[\{i^{th}$ patient varies preference ordering$.\}]$ is the probability that given a patient whether he will vary his true preference ordering. The probability of that is taken as $1/8$ (for small variation). If the number of agents varies from $1/4$ and $1/2$, then the expected number of patient that may vary their preference ordering can be $n/4$ and $n/2$ respectively. These values of $\delta$ are shown in Table I.

In this section the simulation results shows the effect of variation on the two proposed mechanisms using EL and NBA. The simulation result with the patients varying their preference ordering are studied under three different cases.

**Case 1 ($\delta = n/8$):** It is to be noted that, when $n/8$ of the patients are varying their preference ordering among $n$ available patients, then the EL of the agents in case of RanPAM with small variation (RanPAM S-var) is more than the EL of the agents in case of RanPAM without variation. If we consider the case of TOAM, the EL of the agents in case of TOAM is less than the EL of agents in RanPAM and RanPAM S-var. The EL of agents in TOAM with small variation (TOAM S-var) is more than the EL in TOAM without small variation. (as shown in Fig. 6). It is intuitive, as TOAM is not vulnerable to manipulation, if the patients are varying their preference ordering, then the patients must not be allocated doctors better than the doctors allocated to the patients when reporting their true preference ordering. As the patients are not getting their best doctor (i.e., low indexed doctors) in case of non truthful reporting of preference list, so the value of the EL is increased by small amount. In Fig. 7 considering the second parameter of simulation i.e. NBA, it can be seen that the NBA of the agents in case of RanPAM S-var is more than the NBA of the agents in case of RanPAM without variation. It means that, in case of
RanPAM, if the system (here system means patients, doctors, and patients preference lists) consists of strategic patients then the patients can be allocated better doctor by misreporting their preference ordering. If we consider the case of TOAM, the NBA of TOAM without variation is more than the NBA of TOAM S-var. The NBA of TOAM is more than the NBA of RanPAM and RanPAM S-var as shown in Fig. 7 So, it is natural, from the construction of TOAM is to report their true preference ordering. The entail case with medium variation ($\delta = n/4$) is excluded because of space constraint.

**Case 2 ($\delta = n/2$):** Similarly, When $n/2$ of the agents are varying their preference ordering among $n$ available doctors, then the EL of the patients in case of RanPAM with large variation (RanPAM L-var) is less than the EL of the agents in case of RanPAM without variation, RanPAM S-var, and RanPAM M-var.

In TOAM, the EL in TOAM with large variation (TOAM L-var) is more than the EL in TOAM, TOAM S-var, and TOAM M-var as shown in Fig. 8 In Fig. 9 the effect of large variation by the patients on the parameter NBA is shown. It can be seen that the NBA of the agents in case of RanPAM L-var is more than the NBA of the agents in case of RanPAM without variation, RanPAM S-var, and RanPAM M-var. Consider the case of TOAM, the NBA of TOAM L-var is less than the TOAM without deviation, TOAM S-var, and TOAM M-var.

**V. Conclusions and Future Works**

In this paper, we studied the problem of hiring renowned expert consultants (doctors) from around the world, to serve BIG patients of our society under zero budget environment. Designing a more general mechanism for the set-up consisting of $n$ patients and $m$ doctors ($m \neq n$) is the immediate future work. The work can be further extended to many different settings, e.g., multiple doctors are allocated to a BIG patient in different hospitals around the globe. The other interesting direction is studying the discussed set-ups under budget constraints.

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