NUCLEAR PAIRING: SURFACE OR BULK?

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Abstract

We analyse how the spatial localisation properties of pairing correlations are changing in a major neutron shell of heavy nuclei. It is shown that the radial distribution of the pairing density depends strongly on whether the chemical potential is close to a low or a high angular momentum level and has very little sensitivity to whether the pairing force acts in the surface or in the bulk. The averaged pairing density over one major shell is however rather flat, practically independent of the pairing force. Hartree-Fock-Bogoliubov calculations for the isotopic chain $^{100-132}$Sn are presented for demonstration purposes.
I. INTRODUCTION

There is an ongoing debate whether pairing in nuclei is concentrated preferentially in the bulk or in the surface. Quite a few authors plead for a surface enhancement of nuclear pairing \[1, 2\]. This belief is based mostly on two arguments. First, pairing is concentrated around the Fermi energy and in heavy nuclei levels at the Fermi energy are dominated by high \(l\)-values with corresponding wave functions peaked at the surface. Second, the idea of surface dominance of pairing was probably mostly fostered from the Local Density Approximation (LDA) picture. Indeed, from Figure 3 of Ref. \[3\], where the neutron gap at the Fermi surface is shown in symmetric nuclear matter as a function of \(k_F\), calculated once with a realistic bare force and once with the Gogny D1S-force, one can see that, adopting the LDA, the gap in the interior of a nucleus should be very small, while in the surface it should pass through a huge peak over 2.0 MeV high. On the other hand, using LDA one obtains on average a reasonable agreement for the correlation energy in comparison with quantal calculations \[4\]. However, to conclude from this fact that the LDA provides also a good approximation for the \(r\)-dependence of \(\Delta\) in nuclei is a little premature since one knows that Thomas-Fermi theory, on which LDA is based, yields local quantities which must be interpreted as distributions, useful under integration but locally quite erroneous \[5\].

Actually, the two arguments presented above are related in fact to two different aspects of pairing in finite nuclei. Thus, the first argument refers to the radial distribution of pairing correlations, i.e., of Cooper pairs formed upon various single-particle states. Commonly, the spatial localisation of pairing correlations is described by the pairing density. In finite nuclei the pairing density is ascribed to the radial form factor of pair transfer reactions \[6, 7, 8\]. These reactions probe the strength of pairing correlations as manifested in pair-vibration and pair-rotation modes \[9\]. On the other hand, the pairing field \(\Delta(r)\), on which the second argument is based, provides not only informations about the localisation of pairing correlations but also on the properties of the effective pairing force. This is evident from the fact that, by definition, \(\Delta(r)\) is given by the convolution between the pairing force and the pairing density. As it will be shown in this paper, in finite nuclei the radial dependence of the pairing density is rather insensitive to the type of the pairing force. Consequently, in finite nuclei \(\Delta(r)\) and the associated global quantities (e.g., pairing gaps) carry in fact informations related essentially to the effective pairing force and much less on Cooper pairs.
An indication that pairing correlations may not be tremendously surface peaked came already from the study of pairing density in half infinite nuclear matter with the Gogny force \[10\]. There, the peaking of the pairing density at the surface was only very moderate. Looking at Figure 1 of Ref.\[10\] it is even not evident whether the peaking is not a pure consequence of Friedel-oscillations \[11\]. For finite nuclei only very few calculations of the pairing density as a function of the radius are available \[12\]. These calculations were focused mainly on how the pairing density is changing passing from one major shell to the other, up to the neutron dripline. The scope of the present paper is to give a more systematic investigation of the localisation properties of the pairing density in one major shell. Thus, it will be shown that the bulk versus surface localisation of pairing correlations can change very much already in one major shell, depending on whether the chemical potential is close to a low or a high angular momentum level. This behaviour is illustrated here for the chain of Sn-isotopes with the neutron number between N=50 and N=82. The study is performed in the Hartree-Fock-Bogoliubov approach and using surface versus bulk dominated pairing interactions. The general framework of the calculations is described in Section II. Then, in Section III, we discuss the radial distribution of pairing correlations as provided by the HFB calculations. Finally in Section IV we present our conclusions.

II. FORMALISM: HARTREE-FOCK-BOGOLIUBOV APPROACH

In superfluid Fermi liquids, pairing correlations are usually characterized by the "condensate" wave function \[13\]. For the case of \(^1S_0\) pairing the condensate wave function, referred below as to the pairing density, is given by:

\[
\kappa(r) = \langle \psi(r, s = 1/2) \psi(r, s = -1/2) \rangle,
\]

where the operator \(\psi(r, s = 1/2)\) annihilates a nucleon in the point \(r\) and having the spin projection \(s = 1/2\). The local pairing density defined above describes the center of mass distribution of the Cooper pairs inside the superfluid. For translationally invariant superfluid systems at zero temperature, \(\kappa(r)\) is a constant field with the modulus equal to \(\sqrt{n_0}\), where \(n_0\) is the number of the condensed (Cooper) pairs \[13\]. In large superfluid systems, in which the condensate is macroscopically occupied, \(\kappa(r)\) can still be regarded as a classical coherent...
field with negligible variations over distances greater than the coherence length. However, when the range of pairing correlations (associated to the mean square radius of Cooper pairs) is comparable with the size of the system, as happens in finite nuclei, the local variations of the pairing density are important and should be calculated from a quantum equation of motion of the Bogoliubov-de-Gennes type \[14\]. In finite nuclei for such a task one usually employs the Hartree-Fock-Bogoliubov equations \[5\], in which both the mean field and the pairing field are calculated self-consistently.

In this study the HFB equations are solved by imposing spherical symmetry and employing zero-range forces in both the particle-hole and particle-particle channels. Under these conditions the radial HFB equations have the following form:

\[
\begin{pmatrix}
  h(r) - \lambda & \Delta (r) \\
  \Delta (r) & - h(r) + \lambda
\end{pmatrix}
\begin{pmatrix}
  U_i(r) \\
  V_i(r)
\end{pmatrix}
= E_i
\begin{pmatrix}
  U_i(r) \\
  V_i(r)
\end{pmatrix},
\]

(2)

where \( h(r) \) and \( \Delta (r) \) are, respectively, the mean field hamiltonian and pairing field, while \( \lambda \) is the chemical potential. All basic quantities such as particle density \( \rho(r) \) and pairing density \( \kappa(r) \) are expressed in terms of the upper and lower components of the HFB wave function. Thus:

\[
\rho(r) = \frac{1}{4\pi} \sum_i (2j_i + 1)V_i^*(r)V_i(r),
\]

(3)

\[
\kappa(r) = \frac{1}{4\pi} \sum_i (2j_i + 1)U_i^*(r)V_i(r).
\]

(4)

In the present HFB calculations we use in the particle-hole channel a Skyrme-type force, i.e., SLy4 \[15\]. Hence the mean field has the standard expression in terms of single-particle densities \[16, 17\]. For the pairing interaction we use two forces, i.e., a pure delta-force and a density-dependent delta (DDD) -force. For the latter one we take the form \[18\]:

\[
V(r - r') = V_0[1 - \eta(\frac{\rho}{\rho_0})^\alpha]\delta(r - r') \equiv V_{eff}(\rho(r))\delta(r - r').
\]

(5)

The pairing interaction acts upon the pairing density through the pairing field, which for the DDD-force is given by:

\[
\Delta(r) = V_{eff}(\rho(r))\kappa(r).
\]

(6)

Due to the divergencies associated to a zero range pairing force, the HFB calculations should be performed with an energy cut-off or by using more sophisticated regularisation.
procedures [19]. In the present calculations the energy cut-off and the strength of the DDD-force are related to each other through the scattering length of the di-neutron system [18]. The density dependence of the DDD-force is fitted so as to reproduce approximately the pairing gap in nuclear matter provided by the Gogny D1S-force [3]. Guided by these prescriptions we have taken for the DDD-force the following parameters: $V_0 = -430$ MeV fm$^{-3}$, $\eta=0.45$ and $\alpha=0.47$. For the cut-off energy (in the quasiparticle spectrum) we use a value equal to 60 MeV. In the HFB calculations with the delta-force we have taken a strength $V_0 = -220$ MeV fm$^{-3}$ and we have used the same cut-off energy. With this value of the strength we get for Sn isotopes approximately the same pairing energies as for the DDD-force.

III. RESULTS: PAIRING LOCALISATION IN SN ISOTOPES

Before we start analysing the local properties of pairing correlations, we first present shortly the global quantities which characterize the HFB solution. Thus, in Table I is shown how the chemical potential $\lambda$, the averaged gaps and the occupation probabilities of single-particle states are evolving by filling the major shell N=50-82. The changes of particle density with the neutron number are shown in Figure 1a. These changes can be easily traced back to the occupation probabilities of single-particle wave functions shown in Figure 1b. Thus, one notices the progressive increase of the particle density at small distances, produced by the filling of the state $3s_{1/2}$.

Next we discuss how the localisation properties of the pairing density are changing with the filling of the major shell N=50-82. The results for $\kappa(r)$ obtained by using the two zero-range pairing forces introduced in the previous section are shown in Figure 2.

First, we notice that for both pairing forces $\kappa(r)$ is changing quite strongly passing from A = 104 to A = 128. Thus, we see that at the beginning of the shell, when the contribution of the states $2d_{5/2}$ and $1g_{7/2}$ is dominant, the pairing density is slightly larger inside the nucleus than in the surface. Then, at midshell, the pairing density becomes in average almost flat for an extended region from about 1 fm to 5 fm. We also notice a large bump close to the center of the nucleus produced by the neutrons distributed in the state $3s_{1/2}$.

In the second half of the major shell the chemical potential becomes closer to the intruder state $1h_{11/2}$, which starts progressively to dominate the structure of $\kappa(r)$. Since the wave
TABLE I: Results of HFB calculations for Sn isotopes obtained with the DDD-force. $A$ is the atomic mass, $\lambda$ is the chemical potential (in MeV) and $\langle \Delta \rangle$ is the averaged pairing gap (in MeV). The latter is calculated by convoluting the pairing field with the pairing density. In the next 5 rows are given the occupations probabilities for the single-particle states of the valence shell. For each of these states is shown in the bracket the Hartree-Fock energy corresponding to the midshell isotope $^{116}\text{Sn}$.

FIG. 1: The particle density (a) for various Sn isotopes and the Hartree-Fock wave functions (b) corresponding to the valence shell of $^{116}\text{Sn}$.

function of $1h_{11/2}$ is localized in the surface region (see Figure 1b), the pairing density is getting also a bump around 5 fm. This bump becomes more pronounced towards the end of the shell. On the other hand, one can see that towards the end of the shell the pairing density decreases rather strongly in the center of the nucleus. This behaviour reflects the
A somewhat surprising but important finding, shown in Figure 2, is that the radial structure of $\kappa(r)$ is changing very little with the density dependence of the $\delta$-force. Through this dependence the strength of the pairing force is reduced inside the nucleus compared to the surface region. Consequently, the relative contribution of the state $3s_{1/2}$ to pairing correlations is suppressed. However, because of its small degeneracy compared to the other states, this suppression has not important consequences on $\kappa(r)$, as can be seen in Figure 2.

As we have seen above, the radial dependence of the pairing density in Sn-isotopes is dominated by the individual shells structure. In order to reveal a generic behaviour one should follow the old idea of Strutinsky [20] and average the pairing density over one major shell. Here we have taken the crude arithmetic average over all even Sn isotopes, leaving the study of more refined averaging methods for future work. The result is shown in Figure 3. We remark that on average the pairing density $\bar{\kappa}(r)$ is quite flat over the bulk. Since we are considering an even parity shell, we have at the origin a quite pronounced bump from the s-wave. Had we considered an odd parity shell, we certainly would see a corresponding hole at the origin [21]. In Figure 3 we also see that in spite of the fact that the density dependent force yields, with respect to the pure $\delta$-force, slight surface enhancement and volume depression, the difference induced upon $\bar{\kappa}(r)$ by the two pairing forces is practically insignificant.
In order to further understand what happens, let us take the midshell situation and approximate the $uv$ factors in the BCS expression of kappa with a unique constant equal to one, i.e., \[ \kappa(r) = \frac{1}{4\pi} \sum_i u_i v_i |\phi_i(r)|^2 \approx \frac{1}{4\pi} \sum_i' |\phi_i(r)|^2, \] where the prime indicates that the sum runs over the major shell only and $\phi_i(r)$ are the single-particle wave functions plotted in Figure 1b. The result of this approximation is shown in Figure 4. By summing in $\kappa$ all single-particle states with the same $uv$ factor one overestimates the contribution of those single-particle states which are far from the chemical potential, mainly of the states $2d_5/2$ and $h_{11/2}$. This fact produces the extended tail seen in Figure 4. Apart from that, we can see that this rough approximation of $\kappa$ follows rather well the flat radial structure of the HFB curve.

From all these results it appears that in a particular nucleus the radial distribution of the pairing correlations, given by the pairing density, is determined essentially by the localisation of single-particle states which are closer to the chemical potential and much less by the type of the pairing force. The dependence of pairing density profile on the filling of the major shells and its rather small sensitivity to the assumption made on the pairing force can be eventually checked in pair transfer reactions.

The quantity which, by definition, depends explicitly on the assumption made on the type of pairing force is the pairing field. This dependence is clearly seen in Figure 5, where the averaged pairing fields corresponding to the $\text{delta-force}$ and the DDD-force are plotted.
FIG. 4: Pairing density in $^{116}\text{Sn}$. The solid line is the HFB result. The dotted line corresponds to the summation of the square of single-particle wave functions (see text).

Hence, indications about which type of pairing force might be more appropriate in finite nuclei can eventually be extracted from quantities related to the pairing field, e.g., the pairing gaps. Thus, according to Ref. [22] a DDD-force with $\eta = 1/2$ and $\alpha = 1$ would give better results for the odd-even mass staggering than a $\delta$-force or a DD-force with $\eta = 1$ and $\alpha = 1$. However, a pure $\delta$-force apparently gives for the one- and the two-neutron separation energies almost the same results as a DDD-force [23]. Therefore, from these calculations one cannot draw yet a definite conclusions on how much stronger the pairing force should be on the surface compared to the bulk. On the other hand, the fact that the pairing force should have some surface peaking is suggested by all bare interactions, which are more attractive for small momenta than for large ones (see also the $V_{\text{low-}k}$ force based on realistic bare nucleon-nucleon interactions [24]).

IV. SUMMARY AND CONCLUSIONS

In this work we have investigated the radial distribution of pairing correlations in the major neutron shell $N=50-82$ of Sn isotopes. The localisation of pairing correlations, given by the pairing density, is calculated in the framework of HFB approach and using two effective pairing interactions, i.e., a pure $\delta$-force and a $\delta$-force with a density dependent
factor which enhances the strength of the interaction at low density, i.e., in the surface region. It was found that the pairing density changes strongly going from one end to the other of the major shell $N=50-82$, depending on whether the chemical potential is close to a level with low or high $l$-value. However, the differences with respect to the two pairing forces stayed insignificant.

Then, in order to obtain some generic behaviour of the pairing density $\kappa$ in nuclei, we performed an arithmetic average of $\kappa$’s over one major shell. The resulting averaged pairing density, $\overline{\kappa}$, is, apart from some oscillations, practically constant over the whole volume besides a quite pronounced peak at the origin, produced by the $3s_{1/2}$-state contained in the open major shell of Sn-isotopes. The results for $\overline{\kappa}$ show again very little sensivity to the type of the pairing force used in the calculations.

We have also calculated the corresponding pairing fields $\Delta(r)$. By definition, the pairing field depends explicitly on the pairing force. Thus, even if for the DDD-force the pairing density is almost constant over the nuclear volume, the corresponding pairing field would get a similar surface enhancement as the DDD-force. Therefore in finite nuclei quantities relating to the radial distribution of the pairing field, e.g. odd-even mass differences, probes the
density dependence of the pairing forces rather than the localisation of pairing correlations, i.e., of Cooper pairs.

Summarizing, the radial distribution of pairing correlations in finite nuclei depends strongly on the localisation of single-particle states which are closer to the chemical potential and much less on the surface or bulk enhancement of the pairing force.

The conclusions of this study are based on zero range pairing forces. However, the results with a finite-range force of Gogny type, which will be published in a future paper together with a semiclassical study of nuclear pairing localisation, show the same trends.

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