REVENUE MANAGEMENT VIA MULTI-PRODUCT AVAILABLE TO PROMISE

SANDEEP DULLURI
JDA Software Group Inc., (formerly Manugistics Inc.,)
New Product Development - Demand Planning
Hyderabad, INDIA - 500 081

N. R. SRINIVASA RAGHAVAN
Associate Professor, Decision Sciences Laboratory
Department of Management Studies
Bangalore, INDIA - 560 012

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ABSTRACT. Today’s industries are highly complex in nature offering multiple customized quality products with shorter product life-cycles, volatile demand and tighter due-dates etc. to the customers. Manufacturers are focusing on Available-To-Promise (ATP) to their customers as a retention strategy. In other words manufacturers are forced to commit in advance to the customers the amount they can deliver by the specified due-date. In the current work we address a single manufacturer and multi-customer supply chain setting wherein there are multiple products, stochastic demands, varying profit rates, different learning rates etc. We restrict our focus to the multi-product ATP (MATP) strategies that maximize net profit of the manufacturer. We present optimization models in which there is a possibility of cancelling prior committed orders. We also model the dynamic pricing decision integrated with revenue management in MATP setting. We present the results of weak concavity of the MATP models and related structural insights. We support our thesis with rigorous numerical experimental results.

1. Introduction. Ability and capability to deliver product(s) in right quantity and right time forms the key for supply chain success. Manufacturers “promise” availability of capacity to their customers so as to sustain competition. This facilitates manufacturer to avoid the risk of loss of revenue due to perishable capacity.

Capacity commitment to downstream customers is one effective mechanism used by supply chain decision makers so as to smoothen the production and/or delivery schedule. Capacity management is key for effective revenue management (profit maximization). The basis for revenue management is an order acceptance/refusal process that integrates the marketing, financial, and operations functions to maximize profit from existing capacity. Revenue management in manufacturing is typically applicable in tactical decision making wherein the capacity is fixed and when there are high costs associated with capacity changes [23].

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Capacity commitment creates “win-win” situation both for the manufacturer (seller) and the customer (buyer). The manufacturer can streamline production due to deterministic orders from the customers. Customers usually have the advantage of minimizing cost due to committing for capacity from the manufacturer. Capacity commitments quoted in terms of inventory units are referred as “Available-To-Promise” (ATP) in industry. ATP is now common in supply chain vocabulary both in industry and academics. In particular ATP has received wide attention in the hi-tech supply chain networks. Hi-tech supply chains are quite complex and often characterized with (i) large product variety (ii) shorter product life-cycles (PLC) (iii) highly volatile demand etc.

Product life cycle (PLC) is an indicator for the manufacturer to assess whether the product is in the increasing growth stage vs stabilizing growth phase, based on the sales data from the market. PLC for any product at a given point of time is computed based on the ratio of the cumulative number of units of the product sold till that point of time to the total (expected) demand for the product. It is a noted fact that the (per unit) revenue for hi-tech products is increasing in the “initial growth stage” of PLC (product is relatively new to the market). The (per unit) revenue decreases as the product nears its end of life cycle, that is, product reaches “saturation” and there is no significant change in the market demand.

Note 1. We model the impact of PLC explicitly in our models by constructing monotonically increasing and monotonically non-increasing per unit revenue patterns to capture the increasing growth stage and saturated growth stage of the product respectively.

It should be noted that integrating ATP decision with product life-cycle (PLC) has received almost no attention in the literature. Another interesting issue which is often neglected in capacity planning is investigation of the manufacturing learning effects. In the current problem we investigate the effects of product life-cycle and manufacturing learning effects integrated with the MATP problem.

Note 2. Manufacturing learning refers to the activity by which the non-value adding (NVA) activities are removed/reduced to the maximal extent possible due to the time invested on “learning by doing”. For a detailed discussion on manufacturing learning, readers are referred to Macher and Mowery.

The rest of our paper is organized as follows: We survey recent literature on ATP and allied areas in Section 2. We present the problem, notation and optimization models for MATP in Section 3. We discuss analytical insights of the optimal MATP quantities based on the structure of optimization models in Section 4 following this in Section 5 we give numerical results strengthening the insights given in the earlier section. Finally we conclude with discussion and limitations of our current work. We also indicate some directions for future work in Section 6.

2. Relevant literature. We review some of the recent literature on ATP. The objective of our present work is to integrate two strands of literature viz., Available To Promise and Revenue Management.

Charnsirisakskul et al. propose a MILP formulation for order selection and scheduling. They also investigate the impact of lead-time flexibility on order scheduling. Our current problem has some commonalities with their work especially in modeling the due-date constraints. However their work is much on the scheduling
aspect whereas we are currently interested in revenue management. It is interesting to notice that our revenue management objective generates a “near optimal” schedule close to the objective of [5].

Feng and Xiao [7] propose a continuous time integrated dynamic pricing and capacity allocation model for perishable products. They also show that the optimal price comes from a sub-set of optimal prices which forms an increasing concave envelope of the “price-capacity (quantity) curves”. Current problem has some common things to share with this model such as “perishable” nature of capacity. However we do not use any threshold price for a class. In our model we assume the manufacturer is intelligent and there is no obligation on his part to accept the entire order placed by the buyer(s)/customer(s). She can choose a mix of orders placed by the buyers so as to increase the overall revenue.

Xu [12], gives a multi-period dynamic programming formulation for supply contracts with cancellation for single buyer and single seller model. He assumes that both buyer and seller have a common knowledge of the demand from the end customer(s). The author further presents optimal buyer’s ordering policy that minimizes his expected cost during the planning horizon and for supplier the optimal production policy characteristics are summarized. Harris and Pinder [23] discuss a revenue management problem in an assemble-to-order (ATO) manufacturing setting. The authors propose an order acceptance/refusal process coupled with differential pricing and capacity re-allocation strategies.

Tempelmeir [5] gives an overview of various existing supply chain solutions from i2, Manugistics, SAP-APO etc. The author highlights the issues of hierarchical supply chain planning capabilities of these softwares. He author also emphasizes on the missing of “stochastic” element in modeling of these software. Chakravarty and Balakrishnan [9] present a single period model that considers revision of order quantities in presence of capacity constraints. Our current model incorporates multiple revisions and is modeled in a multi-period setting.

Ozdamar and Yazgac [21] propose a linear capacity planning model which addresses load-leveling (balancing) in bottleneck departments. They have linear programming with objective of minimizing total back-order costs and overtime costs over a finite planning horizon satisfying due-dates as well as capacity constraints.

Our current work is close to the following prior works [1, 2, 8, 9]. We majorly differ from these works in the following way:

- We address the ATP problem in a multi-product setting.
- We consider the impact of MATP across multiple customers who differ in their (rate of innovation and hence the) order size, especially the variance in order size.
- We further investigate the effect of delayed production on dynamic pricing and also on overall profit of the manufacturer.
- We explore the impact of manufacturing learning on MATP.
- We investigate the impact of product life cycle (PLC) on MATP.

3. The problem. We consider a hi-tech manufacturing firm. The firm manufactures a wide variety of products and in very high volumes. We term product portfolio as a typical customer demand comprising of product mix associated with volume requirements. The demand for the product portfolio is highly volatile and the manufacturer has finite capacity (usually fixed since we are focusing on tactical
planning). This is the internal force causing the manufacturer to commit capacity for a product type. Further the manufacturer is forced by the competition to promise/commit capacity for various product in each planning horizon.

Demand for product portfolio is highly volatile. Manufacturer would promise high amount of ATP to high profit (new) products. However, manufacturer fears the high “cancellation” of orders from customers in the initial growth stage/phase of new product. For old products the demand is quite stable and hence there are less cancellations. Manufacturer allows the accumulation of all orders for old and new product over a “week”. In the terminology used by Chen et al \[26\], our batching interval is one week. All orders accumulated for various products in this batching interval are now considered for MATP computation.

The problem faced by the manufacturer is two-fold. First, she has to decide on the orders to be accepted/rejected based on (i) profit per unit, (ii) demand (iii) due-date for accepted orders, (iv) available capacity. Secondly, the manufacturer has to decide on the amount of MATP across the product portfolio keeping in view cancellation(s) cost, inventory holding cost, spot market price.

In the current research, we investigate the following:

1. How should the manufacturer accept/reject customer orders so as to maximize the net profit without violating the due-date commitments for the accepted customer orders?
2. How should the manufacturer quote MATP to various customers?
3. How does the PLC affect MATP decision of the manufacturer?
4. What is the impact of manufacturing learning on the manufacturer’s net profit?
5. How does the delay in order fulfillment impact MATP decision of the manufacturer in the context of increasing risk of loss in revenue due to cancellation of customer orders?

**Indices**

- $t$: Time period (in weeks), $t = \{1, \ldots, T\}$
- $i$: Number of products, $i \in \{1, \ldots, I\}$ $I = 2$ in the current problem. We use $i = 1$ for representing “new product” and $i = 2$ for representing old product.
- $k$: Alias index created for time period - for indicating the cancellation of order.

**Parameters**

- $D_{i,t}$: Expected demand for product $i$ in week $t$. $D_{i,t} = \mu_{i,t} + z(1-(\alpha/2))\sigma_{i,t}$ Where $\alpha$ is the level of confidence (level of safety) usually taken as 95 % or 99 % and $z(1-(\alpha/2))$ is the Gaussian variate corresponding to mean $\mu_{i,t}$, variance $\sigma_{i,t}^2$ and confidence level $\alpha$.
- $\mu_{i,t}$: Mean order size for product $i$ placed in week $t$.
- $\sigma_{i,t}$: Allowed standard deviation of order size for product $i$ in week $t$.
- $D_{i,t}'$: Cancellation amount, in quantity of product $i$ in week $t$.
- $p_{i,t}$: Profit on processing unit of product $i$ in week $t$. 
$g_{i,t}$ : Cancellation cost for the manufacturer that is associated with the cancellation of a unit of product $i$ in week $t$.

$\text{spot}_{i,t}$ : Spot market price (salvage price) for a unit of product $i$ in week $t$.

$\langle \text{Start}_{i,t}, \text{Due}_{i,t} \rangle$ : MATP parameters $\text{Start}_{i,t}$ is the latest time by which the order for product $i$ placed in week $t$ is committed and $\text{Due}_{i,t}$ is the latest time by which the prior committed order in week $t$ for product $i$ needs to be delivered to the customer.

$h_{i,t}$ : Holding cost per week for a unit of product $i$ placed in week $t$.

$c_{i,t}$ : Manufacturing cost for a unit of product $i$ committed in week $t$.

$Q_t$ : Available production capacity in week $t$.

$\delta_t$ : Discount rate in week $t$; $\delta_t = \left(\frac{1}{1+r}\right)^t$; where “r” is the risk free interest rate.

| Functions | 
|-----------|
| $R_{i,t}(\cdot)$ : Revenue generated by processing unit order of product $i$ in week $t$ |
| $C_{i,t}(\cdot)$ : Operating cost associated with product $i$ in week $t$ |
| $H_{i,t}(\cdot)$ : Inventory holding cost associated with product $i$ in week $t$ |
| $G_{i,t}(\cdot)$ : Cancellation cost associated with product $i$ in week $t$ |

| Decision Variables | 
|--------------------|
| $x_{i,(t,t+n)}$ : MATP for product $i$, committed in week $t$ and due by date $[t+n]$ |
| $y_{i,t}$ : Binary variable indicating whether or not the order for product $i$ is delivered at the end of week $t$. $y_{i,t} = 1$, if the order for product $i$ is delivered at the end of week $t$, else $y_{i,t} = 0$. |

Table 1. Notation for MATP Models

3.1. MATP Model - I: Revenue Maximization Without Cancellations. We begin with a simple objective of maximizing revenue of the hi-tech manufacturer. We model a situation where the manufacturing delays are not allowed and the orders which could not be accepted by the manufacturer are lost to the competitors. Initially we develop a model wherein the manufacturer does not permit customers to cancel prior committed orders.

Assumption 1. We measure MATP quantity in terms of production capacity. In other words, the MATP quantity is converted to capacity units (for example, number of production hours). This assumption is common in literature, see [8].
Assumption 2. We consider the obsolescence cost of products to be very high and hence these products cannot be stocked to the next period in the case of an order being cancelled. This is a normal observation in high-tech manufacturing industries. Manufacturers are interested in “pushing” the finished products at the earliest possible time so as to meet customer demand.

Maximize
\[
\sum_{i=1}^{I} \sum_{t=1}^{T} \sum_{n=\text{Start}_{i,t}}^{\text{Due}_{i,t}} R_{i,t}(D_{i,t}, x_{i,(t,n)}, \delta_{i,t}, y_{i,t})
\]
\[
- \sum_{i=1}^{I} \sum_{t=1}^{T} \sum_{n=\text{Start}_{i,t}}^{\text{Due}_{i,t}} C_{i,t}(D_{i,t}, x_{i,(t,n)}, \delta_{i,t}, y_{i,t})
\]
\[
- \sum_{i=1}^{I} \sum_{t=1}^{T} \sum_{n=\text{Start}_{i,t}}^{\text{Due}_{i,t}} H_{i,t}(D_{i,t}, x_{i,(t,n)}, \delta_{i,t}, y_{i,t})
\] (1)

Subject to the following constraints:
\[
\sum_{t=1}^{n} x_{i,(t,n)} \leq \sum_{t=1}^{n} D_{i,t} y_{i,t}
\] ∀i = \{1, 2, \ldots, I\} n ∈ [\text{Start}_{i,t}, \text{Due}_{i,t}] (2)
\[
\sum_{i=1}^{I} \sum_{n=\max(\text{Start}_{i,t}, t)}^{\text{Due}_{i,t}} x_{i,(t,n)} y_{i,t} \leq Q_t
\] ∀t = \{1, 2, \ldots, T\} (3)
\[
x_{i,(t,n)} \geq 0
\] ∀t = \{1, 2, \ldots, T\} n ∈ [\text{Start}_{i,t}, \text{Due}_{i,t}] ∀i = \{1, 2, \ldots, I\} (4)
\[
y_{i,t} \in [0, 1]
\] ∀t = \{1, 2, \ldots, T\} ∀i = \{1, 2, \ldots, I\} (5)

Thus we have formulated the manufacturer’s problem of revenue management (profit maximization) as a “Non-Linear Mixed Integer Programming” problem.

Note 3. We give the expressions for the revenue function (8), operating cost function (9), inventory holding cost function (10) and cancellation cost function (11) later in this paper in section 3.3.

The first term in the objective, equation (1) corresponds to the “Revenue function”. It indicates the total revenue generated in the planning horizon taking into account the MATP across various products.

Second term in the objective function corresponds to the “manufacturing cost function”. It indicates the cumulative manufacturing cost incurred due to ATP processing in the planning horizon.

Third term in the objective function corresponds to the “inventory holding cost function”. It is the cumulative cost incurred due to holding inventories in the planning horizon.
Constraint (2) takes care that the ATP amount for any product does not exceed the demand for that product in that week.

Constraint (3) is capacity constraint i.e., the ATP amount across the entire product portfolio cannot exceed maximum available capacity.

Constraint (5) imposes “non-negativity” restriction on ATP across entire product portfolio.

Constraint (6) indicates the “binary” decision variable. It takes value “1” if the order is accepted and delivered by the committed due-date, else it takes a value “0”.

Note 4. We can extend the above model closer to reality by considering lead-time in formulation i.e., the revenue on orders committed in week \( t \) is realized at the end of order fulfillment i.e., after \( \lceil t + l_i \rceil \). Where \( l_i \) is the expected lead-time for order processing.

Note 5. We can relax constraint (6) from “integrality” restrictions and allow it to take any value on \( \mathbb{R}_{[0, 1]} \). Thus modeling the partial ATP.

3.2. MATP Model-II: Revenue maximization With Cancellations.

Note 6. Cancellation of an order by a buyer causes loss to the manufacturer. In order to recover this loss the manufacturer will ideally want to transfer the risk of revenue loss due to cancellations to the buyer. However it is observed that transferring the risk of revenue loss in its entirety would result in under commitment of demand from the buyers. Especially in hi-tech supply chains with heavy competition, it is often seen that manufacturer bears a significant portion of risk of revenue loss due to cancellations of prior committed orders [38, 39].

We use a piece-wise linearly increasing cost structure for cancellation cost. We consider the difference of cancellation cost and the spot market price of the product for which the order is cancelled. Intuitively it can be seen that supplier would incur ‘high” cancellation cost as the cancellation instant for a prior committed order nears the delivery date/due-date of that order.

\[
\begin{align*}
\text{Maximize} & \quad \sum_{i=1}^{I} \sum_{t=1}^{T} \sum_{n=\text{Start}_{i,t}}^{\text{Due}_{i,t}} R_{i,t}(D_{i,t}, x_{i,(t,n)}, \delta_t, y_{i,t}) \\
& - \sum_{i=1}^{I} \sum_{t=1}^{T} \sum_{n=\text{Start}_{i,t}}^{\text{Due}_{i,t}} C_{i,t}(D_{i,t}, x_{i,(t,n)}, \delta_t, y_{i,t}) \\
& - \sum_{i=1}^{I} \sum_{t=1}^{T} \sum_{n=\text{Start}_{i,t}}^{\text{Due}_{i,t}} H_{i,t}(D_{i,t}, x_{i,(t,n)}, \delta_t, y_{i,t}) \\
& - \sum_{i=1}^{I} \sum_{t=1}^{T} \sum_{n=\text{Start}_{i,t}}^{\text{Due}_{i,t}} G_{i,t}(D'_{i,t}, \delta_t, y_{i,t}, \text{spot}_{i,t}) \\
\end{align*}
\] (7)

Subject to the constraints, (2) – (6).

Above objective is different from the earlier objective (See equation (1)) in the last term. The last term in the objective (See equation (7)) gives the whole of loss to the manufacturer due to cancellation of a prior committed order minus the salvage value of the cancelled order realizable from the spot market.
3.3. General form of revenue, manufacturing, inventory holding cost and cancellation cost functions. We present the general form for the revenue, manufacturing, inventory holding cost and cancellation cost functions used in the objective functions refer equations (1) and (7).

\[ R_{i,t}(D_{i,t}, x_{i,(t,n)}, \delta_{t}, y_{i,t}) = \delta_{t}p_{i,t}x_{i,(t,n)}y_{i,t} \quad \forall i = \{1, 2, \ldots, I\} \quad n \in [Start_{i,t}, Due_{i,t}] \quad \forall t = \{1, 2, \ldots, T\} \]  

(8)

\[ C_{i,t}(D_{i,t}, x_{i,(t,n)}, \delta_{t}, y_{i,t}) = \delta_{t}c_{i,t}x_{i,(t,n)}y_{i,t} \quad \forall i = \{1, 2, \ldots, I\} \quad n \in [Start_{i,t}, Due_{i,t}] \quad \forall t = \{1, 2, \ldots, T\} \]  

(9)

\[ H_{i,t}(D_{i,t}, x_{i,(t,n)}, \delta_{t}, y_{i,t}) = \delta_{t}h_{i,t}x_{i,(t,n)}y_{i,t} \quad \forall i = \{1, 2, \ldots, I\} \quad n \in [Start_{i,t}, Due_{i,t}] \quad \forall t = \{1, 2, \ldots, T\} \]  

(10)

\[ G_{i,t}(D'_{i,t}, \delta_{t}, y_{i,t}) = \delta_{t}g_{i,t}D'_{i,t}y_{i,t} \left[ 1 + \left( \frac{n - Start_{i,t}}{Due_{i,t}} \right) \right] - \text{spot}_{i,t}D'_{i,t}\delta_{t}y_{i,t} \quad \forall i = \{1, 2, \ldots, I\} \quad n \in [Start_{i,t}, Due_{i,t}] \quad \forall t = \{1, 2, \ldots, T\} \]  

(11)

3.4. MATP Model- III: Revenue Maximization with Dynamic Pricing - With Cancellations Vs Without cancellations. It should be noted that revenue management and pricing decisions are strongly correlated. We can extend our models to incorporate the pricing decisions by a known “mapping” between the price i.e., knowing the demand-price trajectories [15].

Some examples of integrating pricing decisions with capacity commitment can be seen in telecommunication networks. These can be adopted to manufacturing networks with some care. Keon and Anandalingam [6] present a simple pricing scheme in monopoly setting that is applicable for real-time telecommunication services. They use a linear demand model and exponential demand model to couple demand with price. However in the case of supplier - manufacturing networks, exponential demand model would not be appropriate.

In the current research, we consider a “linear inverse relation model between price and demand” as shown in equation (12). In addition to the constraints we link demand and price constraints in the model and thus obtain the “dynamic prices”. In the current case we consider profit per unit is positively correlated with the demand for capacity. We can relate “price” (\(P_{i,t}\)) and “profit” (\(p_{i,t}\)) by a known mapping (\(\Phi()\)) \(P_{i,t} = \Phi(p_{i,t}, x_{i,<t,t+n>}\)).

**Linear Demand Model**

\[ D_{i,t} = \beta^{0}_{i,t} - \beta^{1}_{i,t} p_{i,t} \]  

(12)

Where, \(\beta^{0}_{i,t}\) and \(\beta^{1}_{i,t}\) are constants indicating the map between demand and profit.
\[ \text{Maximize} \quad \sum_{i=1}^{I} \sum_{t=1}^{T} \sum_{n=\text{Start}_{i,t}}^{\text{Due}_{i,t}} R_{i,t}(D_{i,t}, x_{i,(t,n)}, \delta_{t}, y_{i,t}) - \sum_{i=1}^{I} \sum_{t=1}^{T} \sum_{n=\text{Start}_{i,t}}^{\text{Due}_{i,t}} C_{i,t}(D_{i,t}, x_{i,(t,n)}, \delta_{t}, y_{i,t}) - \sum_{i=1}^{I} \sum_{t=1}^{T} \sum_{n=\text{Start}_{i,t}}^{\text{Due}_{i,t}} H_{i,t}(D_{i,t}, x_{i,(t,n)}, \delta_{t}, y_{i,t}) - \sum_{i=1}^{I} \sum_{t=1}^{T} \sum_{n=\text{Start}_{i,t}}^{\text{Due}_{i,t}} G_{i,t}(D'_{i,t}, \delta_{t}, y_{i,t}, \text{spot}_{i,t}) \] (13)

Subject to the constraints, (2)–(6). In addition to this we have a demand - profit mapping given by equation (12). We add one more constraint on profit i.e., profit ought to be strictly positive.

\[ p_{i,t} > 0 \quad i = \{1, 2, \ldots, I\}, \ t = \{1, 2, \ldots, T\} \] (14)

Note 7. We use the term “period” to indicate the number of integer time slots in \(\langle \text{Start}_{i,t}, \text{Due}_{i,t} \rangle\).

3.5. An Estimate of Problem Size. To give an overview of the problem size; consider an example with 2 products (new, old), 5 periods difference between a typical start date and due date. The models are considered for a period of 50 weeks.

**Number of constraints = 1150.**

2 Products * 5 Periods * 50 Weeks + 2 Products * 5 Periods * 50 Weeks + 2 Products * 50 Weeks

First term corresponds to the number of equations corresponding to constraint (2), likewise second to fourth term represents number of equations corresponding to constraints (3), (5) and (6) respectively.

**Number of decision variables = 1000.**

2 Products * 5 Periods * 50 Weeks = 500 General Integer decision variables.

2 Products * 5 Periods * 50 Weeks = 500 Binary decision variables \{0, 1\}.

4. Structural results. We can implement the above formulations in commercial optimization solvers such as GAMS, LINGO etc. We are not interested in that for a moment. Our primary objective is to consider the impact of product life-cycle (PLC) on the MATP. Secondly, we are interested in investigating the impact of learning on MATP and overall profit of the manufacturer. We begin with proof of “weak concavity” of the models described above.

4.1. Weak Concavity Result. The MATP models I, II and III, are special cases of resource allocation models. There are two decision variables in each stage (i) accept/reject the order(s) and to deliver the order latest by the due-date of the committed order \(y_{i,t}\) - binary variable and (ii) MATP for product \(i\), \((x_{i,(t,t+n)})\) - Integer variable.

**Theorem 1.** MATP models I, II and III belong to the category of “weakly concave” optimization problems. [18].
**Definition:** A function \( f(.) \) is said to be “weakly concave” if it satisfies following two properties.

1. If \( y \geq x, f(x) \geq f(x + e_i) \), then \( f(y) \geq f(y + e_i) \), \( i \in E \).
2. If \( y \geq x, x_i = y_i, \text{and } f(x + e_i) \geq f(x + e'_i) \), then \( f(y + e_i) \geq f(y + e'_i) \), \( i, j \in E \).

Where \( E \) is a convex region and \( e_i \) is a small positive increment.

Consider the unconstrained objective function II; refer equation (7). We can see that objective functions for MATP-I and MATP-III can be customized the objective function of MATP-II. Hence we discuss the proof of weak concavity for MATP-II and infer the results to MATP-I and MATP-III.

**Proof by induction.**

**Step 1.** For week 1 and single period ATP model, the conditions of weak concavity holds for single period problem is quite evident. That is assuming inventory holding cost, cancellation cost and manufacturing cost to be invariant with time. The revenue maximization would depend on the per unit profit generated by MATP computation.

Let \( x_{i,1,2}^1 \) and \( x_{i,1,2}^2 \) be two feasible solution for the single period problem. Assume \( x_{i,1,2}^1 \geq x_{i,1,2}^2 \)

We have, \( R_{i,1}^1(D_{i,1}, x_{i,1,2}^1, \delta_1, 0) \geq R_{i,1}^2(D_{i,1}, x_{i,1,2}^1, \delta_1, 0) \)

Now consider a small increments in MATP amount say \( \varepsilon^1 \) and \( \varepsilon^2 \); also \( \varepsilon^1, \varepsilon^2 > 0 \). And let \( \varepsilon^1 \geq \varepsilon^2 \). We consider a set of feasible MATP amount \( (x_{i,1,2}) \) and an alias \((y_{i,1,2})\).

Mathematically \( x_{i,1,2}^1 = y_{i,1,2}^1 \).

If, \( R_{i,1}^1(D_{i,1}, y_{i,1,2}^1 + \varepsilon^1, \delta_1, 0) \geq R_{i,1}^2(D_{i,1}, y_{i,1,2}^1 + \varepsilon^2, \delta_1, 0) \) holds then this implies \( R_{i,1}^1(D_{i,1}, y_{i,1,2}^1 + \varepsilon^1, \delta_1, 0) \geq R_{i,1}^2(D_{i,1}, y_{i,1,2}^1 + \varepsilon^2, \delta_1, 0) \)

Therefore for \( t = 1 \), weak concavity holds.

**Step 2.** We assume that the weak concavity holds for “k”-period MATP problem. We consider the scenario wherein the manufacturer receives orders in periods prior to \( k \) but she promises the order fulfillment only in period \( k \). Note that we ignore cancellations in the current proof. We can model the order cancellations by a linear term in MATP to be deducted from the overall profit without cancellations for the manufacturer. Since the cancellation cost term happens to be linear, it will preserve the “weak concavity” structure [20]. For two feasible MATP sets, \( x_{i,<t,k>} \geq x_{i,<t,k>} \) \( \forall i = \{1, 2, \ldots, I\} \) and \( \forall t = \{1, 2, \ldots, k\} \). We have the net profit equation satisfying the first condition of weak concavity as:

\[
\sum_{t=1}^{k} R_{i,t}^1(D_{i,t}, x_{i,<t,k>}^{1}, \delta_t, 0) \geq \sum_{t=1}^{k} R_{i,t}^2(D_{i,t}, x_{i,<t,k>}^{1}, \delta_t, 0) \quad (15)
\]

For second order condition of weak concavity, we have following inequality:

Let \( x_{i,<t,k>} = y_{i,<t,k>} \) \( \forall i = \{1, 2, \ldots, I\} \) and \( \forall t = \{1, 2, \ldots, k\} \).

\[
\sum_{t=1}^{k} R_{i,t}^1(D_{i,t}, x_{i,<t,k>}^{1} + \varepsilon^1, \delta_t, 0) \geq \sum_{t=1}^{k} R_{i,t}^2(D_{i,t}, x_{i,<t,k>}^{2} + \varepsilon^2, \delta_t, 0) \quad \Rightarrow
\]

\[
\sum_{t=1}^{k} R_{i,t}^1(D_{i,t}, y_{i,<t,k>}^{1} + \varepsilon^1, \delta_t, 0) \geq \sum_{t=1}^{k} R_{i,t}^2(D_{i,t}, y_{i,<t,k>}^{2} + \varepsilon^2, \delta_t, 0) \quad (16)
\]
Step 3. We need to prove that the “weak concavity” conditions hold for \( k + 1 \) period problem. We start with net revenue generated in \( k + 1 \) period MATP problem given by equation (17).

\[
\sum_{t=1}^{k+1} R_{i,t}(D_{i,t}, x_{i,<t,k+1>, \delta_{t}, 0}).
\] (17)

Above equation (17) can be expressed as summation of two terms viz., net revenue in \( k \) periods starting from 0 plus the net revenue generated in period \([k, k + 1]\).

\[
\left[ \sum_{t=1}^{k} R_{i,t}(D_{i,t}, x_{i,<t,k>, \delta_{t}, 0}) \right] + \left[ R_{i,k+1}(D_{i,k+1}, x_{i,<k,k+1>, \delta_{k+1}, 0}) \right]
\] (18)

Consider weak concavity conditions for \( k \) period MATP problem given by equation (15) and equation (16). Also for \( k + 1 \) period we have, \( x_{i,<k,k+1>}^1 \geq x_{i,<k,k+1>}^2 \)

\[
R_{i,k+1}(D_{i,k+1}, x_{i,<k,k+1>, \delta_{k+1}, 0}) \geq R_{i,k+1}(D_{i,k+1}, x_{i,<k,k+1>, \delta_{k+1}, 0})
\] (19)

Also, for \( x_{i,<k,k+1>} = y_{i,k,k+1}^1 \),

\[
R_{i,k+1}(D_{i,k+1}, y_{i,k,k+1}^1, \delta_{k+1}, 0) \geq R_{i,k+1}(D_{i,k+1}, y_{i,k,k+1}^2, \delta_{k+1}, 0)
\] (20)

Thus we show weak concavity conditions 1 and 2 hold for the \( k + 1 \) period MATP problem.

Step 4. From steps 1 through 4, we proved that weak concavity holds for MATP problem with one period. Assuming that the weak concavity holds for a \( k \)-period MATP problem (step 2), we proved the weak concavity holds in the case of \( k + 1 \) period MATP problem.

Hence by mathematical induction we can conclude that the general \( n \)-period MATP problem is “weakly concave”.

4.2. Generalization of Weak Concavity Result. Without loss of generality assume that these two decisions (variables) are evaluated in a sequential manner. Consider a single period problem say week \( t \), there are two possible cases for \( y_{i,t} \).

Note 8. The solution set(s) corresponding to the binary variable \( y_{i,t} \) for the MATP model(s) happens to be the union of two distinct sets that are mutually exclusive and collectively exhaustive.

4.2.1. Case 1: \( y_{i,t} = 0 \). In this case the proof is trivial. Since \( y_{i,t} \) is the deciding variable whether to accept an order or not. If \( y_{i,t} = 0 \) it indicates that the order is rejected and hence it does not figure in the objective function. Hence our interest will be limited to the orders that are accepted i.e., \( y_{i,t} = 1 \).

4.2.2. Case 2: \( y_{i,t} = 1 \). This case is interesting in the sense, we deal with orders that are given prior commitment. We assume a sequential decision making approach that is, to decide on order acceptance first and then compute ATP amount. We focus on a (sub)optimization problem where we consider the orders that are accepted. In this case we replace all \( y_{i,t} \) terms with “1”. It is clear that the resulting problem now
happens to be a Mixed Integer Linear Programming (MILP) i.e., linear in objective and constraints too. There by ensuring the concavity in the current case due to the following two important theorems. (1) Any linear function is by nature convex and also concave. (2) Linear combination of concave (convex) functions is concave (convex) [20].

We use above theorems in conjunction with induction approach to generalize that objective function of models I; please see equation (1) is concave. However for model II the objective function refer (7) has an additional “cancellation function” whose structure is assumed to be “piecewise - linear”. This makes the function non-concave in cases when the marginal increase in the cancellation cost is “non-increasing”. The objective function retains the structure of “weak concavity”.

**Note 9.** The models happen to be “weakly concave”. Hence any greedy algorithm can solve the problem to optimality. For a detailed maths on employing greedy algorithms for weakly concave problems, we direct interested readers to a seminal work of [19]. We make use of the fact that our models happen to be weakly concave in setting the solver options to use “myopic/greedy search” to locate optima in a polynomial time.

### 4.3. Impact of product life cycle.

**Theorem 2.** \( x^*_i(t,n) \) be the optimal ATP amount for product \( i \), committed in week \( t \) and due by date \([t+n]\). Let “\( h_i \)”, “\( c_i \)” and “\( g_i \)” represent the unit inventory holding cost, manufacturing cost and cancellation cost associated with product \( i \). These costs are same across all time periods in the planning horizon.

**Part A: Product in Increasing Growth Stage:** Consider the prices for product \( i \), varies in the non-decreasing sequence with the time \( t \). The optimal MATP \( x^*_i(t,n) \) will be in the descending order of \( p_{i,t} \).

**Part B: Product in Saturated Growth Stage:** Consider the prices for product \( i \), varies in the non-increasing sequence with the time \( t \). The optimal MATP \( x^*_i(t,n) \) will be in the ascending order of \( p_{i,t} \).

**Proof.**

**Part A: Increasing Growth Stage of Product Life Cycle:**

Construct two sequences of feasible MATP solutions and also two sequences of profit per unit to the objective (7) as prescribed by equations (21), (22), (23) and (24) respectively.

\[
X^1 = \left\{ x^*_i(t,n) : i = \{1, 2, \ldots, I\}, \{t = 1, 2, \ldots, T\}, n \in [Start_{i,t}, Due_{i,t}] \right\} 
\]

\[
X^2 = \left\{ \begin{array}{l}
 x^*_i(t,n) : i = \{1, 2, \ldots, I\}, \{t = 1, 2, \ldots, T\}, n \in [Start_{i,t}, Due_{i,t}] \\
 x^*_i(t,n) \leq x^*_i(t+k,n), \forall k = \{1, 2, \ldots, T-t\}
\end{array} \right\} 
\]

\[
\Phi^1 = \left\{ p_{i,t} : i = \{1, 2, \ldots, I\}, \{t = 1, 2, \ldots, T\} \right\}
\]

\[
\Phi^2 = \left\{ \begin{array}{l}
 p_{i,t} : i = \{1, 2, \ldots, I\}, \{t = 1, 2, \ldots, T\} \\
 p_{i,t} \leq p_{i,t+k}, \forall k = \{1, 2, \ldots, T-t\}
\end{array} \right\}
\]

**Claim:** For profit sequence \( \Phi^2 \) MATP \( X^2 \) fetches more revenue when compared to MATP \( X^1 \).
**Interchange Argument:** We make use of “interchange argument” and prove by contradiction that our claim holds. Let us for a moment assume our claim does not hold. In other words, the profit sequence $\Phi^2$ fetches more revenue with MATP $X^1$ than $X^2$. We assume all other costs remain constant.

We take two consecutive stages say $k$ and $k + 1$ and compare the relative profits in both cases, See equations (25) and (26).

$$ \text{profit}^1 = \left\{ p_{i,k}x^*_{i,(k,n)} + p_{i,k+1}x^*_{i,(k+1,n)} - W : p_{i,k} \in P^2, x^*_{i,(k,n)} \in X^1 \right\} \quad (25) $$

$$ \text{profit}^2 = \left\{ p_{i,k}x^*_{i,(k,n)} + p_{i,k+1}x^*_{i,(k+1,n)} - W' : p_{i,k} \in P^2, x^*_{i,(k,n)} \in X^2 \right\} \quad (26) $$

Where $W$ and $W'$ indicate the cumulative inventory holding cost, manufacturing processing cost till $k + 1$.

**Assumption 3.** Without loss of generality let us consider $W \approx W'$. This holds true for many high-tech manufacturing firms. The inventory holding cost and manufacturing costs are “almost” similar for “minor” changes in MATP.

It is easy to see by construction of sequences of profits and ATPs’ we could get $\text{profit}^2 \geq \text{profit}^1$. This contradicts our earlier claim and makes our claim is true.

**Part B: Declining (Saturated) Growth Stage of Product Life Cycle:** We can extend the above proof for the declining growth stage of PLC. We need to construct sequences with descending order of prices and MATP, and employ interchange arguments as presented above with appropriate changes.

This concludes the proof.

4.4. **Impact of Manufacturing Learning.**

**Proposition 1.** If the manufacturing price per unit follow a learning (experience) curve we see that the unit manufacturing costs are in decreasing sequence with increase in time $t$. With other things remaining constant we see an increase in overall revenue and profit with the increase in learning.

It is easy to see that with increase in time $t$, manufacturing learning increases thereby “non-value adding” (NVA) activities are compressed to a great extent. Assume that the profit sequence remains same. We see a decrease in the manufacturing costs. Hence a net increase in overall revenue. Thus we can conclude as the manufacturing learning increases the overall revenue increases.

**Proposition 2.** In the increasing growth stage of product life cycle for a product. The overall revenue also profit increases with increase in the MATP quantity to that product. Another interesting thing to note is this increase occurs at a “decreasing” rate. Correspondingly in the decreasing growth stage (after maturity of product), we notice decrease in overall revenue and profit with increase in the MATP quantity to that product; this decrease is at an “increasing” rate.

5. **Numerical results.** We begin with description of generating the scenarios (datasets) for optimization via simulation. We implemented MATP models I, II and III in Lingo [17] extended non-linear solver on a windows machine with 1GB
RAM. The computation time ranged from 60 seconds to 3000 seconds. Median computation time was observed to be around 500 seconds.

5.1. Scenario generation. In order to justify our claims made earlier, we generated various scenarios depicting the MATP problem. We construct three different scenarios for the problem, based on the manufacturer's capacity namely, \{High = 120 units, Medium = 110 units, Low = 100 units\}. We generated scenario datasets for optimization problem with uniform distribution random variates with values given in Table (2).

$$
T = 50 \text{ weeks} \\
\mu_{1,t} = 90 - -110 \, (\text{Pdt/week}) \\
\sigma_{1,t} = 1 - -10 \, (\text{Pdt/week}) \\
\mu_{2,t} = 50 \, (\text{Pdt/week}) \\
\sigma_{2,t} = 1 - -10 \, (\text{Pdt/week}) \\
p_{1,t} = 500 - -600 \, (\$/\text{Pdt}) \\
p_{2,t} = 300 - -310 \, (\$/\text{Pdt}) \\
c_{1,t} = 20 - -70 \, (\$/\text{Pdt}) \\
c_{2,t} = 10 - -60 \, (\$/\text{Pdt}) \\
h_{1,t} = 5 - -10 \, (\$/\text{Pdt}) \\
h_{2,t} = 2 - -5 \, (\$/\text{Pdt}) \\
g_{1,t} = 1 - -10 \, (\$/\text{Pdt}) \\
g_{2,t} = 1 - -10 \, (\$/\text{Pdt}) \\
\text{spot}_{1,t} = 250 - -380 \, (\$/\text{Pdt}) \\
\text{spot}_{2,t} = 50 - -200 \, (\$/\text{Pdt}) \\
\text{Start}_{1,t} = 1 - -50 \, (\text{week}) \\
\text{Due}_{1,t} = 1 - -60 \, (\text{week}) \\
\text{Start}_{1,t} \leq \text{Due}_{1,t} \\
\text{Start}_{2,t} = 1 - -50 \, (\text{week}) \\
\text{Due}_{2,t} = 1 - -65 \, (\text{week}) \\
\text{Start}_{2,t} \leq \text{Due}_{2,t} \\
r = 0.90 - -0.98
$$

Table 2. Parameters for scenario generation

5.2. Impact of Product Life-Cycle.

5.2.1. Without cancellation.

Result 1. We notice increase in the MATP for new product and correspondingly decrease in MATP for old product. Also we notice increase in overall revenue and profit of the manufacturer with increase in MATP for new product. This result strengthens Theorem [1]. The amount of MATP for new product is with high variability but with an increasing pattern. For old product, MATP is less variable and constantly declining.

Note 10. Datasets 1 to 5 contains data for two product types, product 1 is a “new product” and is in its increasing growth stage. Datasets 6 to 10 contains data for two products and new product has reached “matured growth stage”.

1Dataset1 for capacity 110 units with cancellations took 35 minutes, with a solver generator memory of 120KB
Justification: In the increasing growth phase for new product we see increasing per unit profit rates. This in turn motivates manufacturer to promise more capacity to the new products. A revenue or profit maximizing manufacturer will target at allocating MATP equal to her maximum capacity of allocation for new products in this phase.

5.2.2. With Cancellation.

Result 2. There is a difference in MATP for both new and old products when cancellations are allowed vs when the cancellations are not allowed. We notice that MATP with cancellation for new products is less than or equal to the MATP for new products without cancellation. With the increase in growth of new product and reduction in the variability of cancellation we see MATP for new products with cancellation converges with MATP without cancellation. For old products we notice increase in MATP with increase in variability of cancellation of new products. However the pattern of decrease in MATP amount with cancellation holds for old products also, see Figures 1 and 2.

Justification: The manufacturer is interested in maximizing resource utilization and thereby increasing her revenue. With the high variability of cancellation of orders for new products, manufacturer will shift the MATP from new products to MATP for old products, making sure of minimum revenue. The amount of transfer of MATP from new to old products is highly dependent on the spot market price. For new products we can observe spot price positively correlated with the per unit profit.

![ATP for New Product vs Old Product with Cancellation](image)

**Figure 1.** ATP New vs Old with cancellations.

5.3. Impact of Manufacturing Learning.

Result 3. Manufacturing learning has a significant impact on the net revenue and profit of the manufacturer. With the increase in learning rate we notice that the
manufacturer’s net revenue and profit is increasing. We notice the same effect both when cancellation of orders are not allowed and allowed. An interesting point to be noted is with the increase in learning, the net revenue gap and profit gap between cancellation(s) vs no-cancellation(s) is non-increasing. Please see Figure 4. The details of learning rates along with profit are given in Tables 3 and 4.
Figure 4. Manufacturing Learning Effects - cancellations vs No cancellations.

| Data | Old Product | New Product | Manufacturer's Net Profit (in $) |
|------|-------------|-------------|----------------------------------|
|      | (µ₁,t)      | (σ₁,t)      | (µ₂,t) | (σ₂,t) | c₂,t= 8.5-0.094t | c₂,t,= 7.1-0.079t |
| Set1 | 45.90       | 17.49       | 15.27  | 4.42   | 2934928        | 3113457        |
| Set2 | 42.84       | 16.33       | 14.25  | 4.13   | 2975342        | 3007771        |
| Set3 | 36.72       | 14.00       | 12.22  | 3.54   | 2990156        | 3025385        |
| Set4 | 39.78       | 15.16       | 13.23  | 3.83   | 2503790        | 2581229        |
| Set5 | 33.66       | 12.82       | 11.20  | 3.60   | 2478430        | 2547908        |
| Set6 | 30.60       | 11.66       | 10.19  | 2.91   | 2469519        | 2507822        |
| Set7 | 27.54       | 10.50       | 9.16   | 2.63   | 2447259        | 2478430        |
| Set8 | 24.48       | 9.33        | 8.14   | 2.36   | 2425039        | 2453450        |
| Set9 | 21.42       | 8.16        | 7.13   | 2.06   | 2402829        | 2425039        |
| Set10| 18.36       | 7.90        | 6.11   | 1.77   | 2380619        | 2402829        |
| Set11| 15.30       | 5.93        | 5.09   | 1.47   | 2358399        | 2380619        |

Table 3. Manufacturing Learning Effect - Without Cancellations.

| Data | Old Product | New Product | Manufacturer's Net Profit (in $) |
|------|-------------|-------------|----------------------------------|
|      | (µ₁,t)      | (σ₁,t)      | (µ₂,t) | (σ₂,t) | c₂,t= 8.5-0.094t | c₂,t,= 7.1-0.079t |
| Set1 | 45.90       | 17.49       | 15.27  | 4.42   | 2934928        | 3113457        |
| Set2 | 42.84       | 16.33       | 14.25  | 4.13   | 2975342        | 3007771        |
| Set3 | 36.72       | 14.00       | 12.22  | 3.54   | 2990156        | 3025385        |
| Set4 | 39.78       | 15.16       | 13.23  | 3.83   | 2503790        | 2581229        |
| Set5 | 33.66       | 12.82       | 11.20  | 3.60   | 2478430        | 2547908        |
| Set6 | 30.60       | 11.66       | 10.19  | 2.91   | 2469519        | 2507822        |
| Set7 | 27.54       | 10.50       | 9.16   | 2.63   | 2447259        | 2478430        |
| Set8 | 24.48       | 9.33        | 8.14   | 2.36   | 2425039        | 2453450        |
| Set9 | 21.42       | 8.16        | 7.13   | 2.06   | 2402829        | 2425039        |
| Set10| 18.36       | 7.90        | 6.11   | 1.77   | 2380619        | 2402829        |
| Set11| 15.30       | 5.93        | 5.09   | 1.47   | 2358399        | 2380619        |

Table 4. Manufacturing Learning Effect - With Cancellations.

Result 4. With increase in learning rate, the total revenue and profit of the manufacturer increases. This effect is clearly evident in the case of new product in increasing growth.
5.4. Impact of Available Capacity on Manufacturer’s Net Revenue.

Result 5. With the increase in the manufacturer’s available capacity, there is a corresponding increase in MATP for new products. We notice a corresponding increase in the manufacturer’s net profit. Figure 6 and Tables 5 and 6 are in support of this result. We also notice that in the case of new products, MATP with cancellation is less than MATP without cancellation.

Justification: It is evident that revenue is dependent on the manufacturer’s capacity. As the manufacturer’s capacity increases, her “commitment capacity”
increases thereby fetching him higher profits. Now considering the impact of PLC, as the cancellation rate increases for a particular product, manufacturer is forced to commit lesser amount and hence shift the excess amount to other product(s) which can generate more profit.

![Manufacturer's Available Capacity vs Net Profit](image)

**Figure 5.** Manufacturer’s Available Capacity Against Net Profit Without Cancellation vs With Cancellation.

5.5. **Dynamic Pricing with MATP.**

**Result 6.** We notice that the dynamic prices with cancellation are more variable compared to without cancellation. For new products, the MATP price with cancellation always exceeds the price without cancellation. In fact the price corresponding to the MATP without cancellation for both old and new products appear to be uniform, see Figure 6. Hence a uniform pricing strategy might be better for MATP without cancellation. We notice a similar result in our earlier work [15]. We also observe that for old products the price corresponding to MATP with cancellations is less than price for MATP without cancellations, this is quite counter-intuitive. As discussed earlier, the MATP with cancellations is higher than MATP without cancellation for new products and the converse hold for old products, as illustrated in Figure 7.

**Justification:** With cancellations the orders received by manufacturer are highly variable. This “variability” in-turn results in variation of profits and thereby the dynamic prices. The price corresponding to MATP with cancellation exceeds the price for MATP without cancellation is justified in the following manner. When the manufacturer allows for cancellations, she prices the capacity high to account for loss of revenue due to cancellations. We notice that in spite of dynamic pricing mechanism manufacturer’s profit with cancellation is always less than (or atmost equal to) profit without cancellations.
We observe a counter-intuitive result in this regard. The price corresponding to MATP for old products with cancellation is less than that of price without cancellation. This can be justified with following explanation. When the manufacturer allows for cancellation of orders placed, it is seen that the variability in new product orders cancelled is higher than variability in cancellation orders for old products. This is attributed to the manufacturing learning effects for old product. Manufacturer displays intelligent pricing mechanism wherein she attracts orders for old products pricing lower than the original price. The orders for old products are retained for ensuring minimum revenue.

**Figure 6.** MATP Quantities and Dynamic Pricing For a Linear Demand Model - With Cancellations vs Without Cancellations.

**Figure 7.** Dynamic Prices and MATP For a Linear Demand Model - With Cancellations vs Without Cancellations.
Note 11. Validation of results: We use a simple validation for our models. It is easy to infer that our model is a more general form of the traditional "capacitated lot sizing problem". We obtain same results as that of multi-item single level capacitated lot sizing problem with no explicit backlogs as discussed in Bylka and Rempala [22].

6. Limitations and conclusions. We discussed the problem of MATP. We presented models for revenue management of a hi-tech manufacturer via MATP (profit maximization via MATP models I, II and III). We addressed the impact of order cancellation(s) on net profit of the manufacturer. Further, we investigated the impact of PLC on MATP and net profit. We also studied the impact of manufacturing learning on MATP. Another interesting dimension that we added to the MATP problem, was to integrate dynamic pricing problem with revenue management in MATP setting. We supported the claims based on structural insights - such as the proof of weak concavity of MATP models I, II and III with numerical experiments.

Future work. Our models do not address two coupled problems in revenue management (i) Distributed Order Scheduling and (ii) Gaming between multiple manufacturers and customers. Integrating these problems in revenue management with MATP framework can be taken as a direction for future research.

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E-mail address: Sandeep.Dulluri@JDA.com
E-mail address: raghavan@mgmt.iisc.ernet.in