RADIAL FLOW OF DUST PARTICLES IN ACCRETION DISKS

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ABSTRACT

We study the radial migration of dust particles in accreting protostellar disks analogous to the primordial solar nebula. Our main objective is to determine the retention efficiency of dust particles, which are the building blocks of the much larger planetesimals. This study takes account of the two-dimensional (radial and normal) structure of the disk gas, including the effects of the variation in the gas velocity as a function of distance from the midplane. It is shown that the dust component of disks accretes slower than the gas component. At high altitude from the disk midplane (higher than a few disk scale heights), the gas rotates faster than the particles because of the inward pressure gradient force, and its drag force causes particles to move outward in the radial direction. Viscous torque induces the gas within a scale height from the disk midplane to flow outward, carrying small (size \( \lesssim 100 \ \mu m \) at 10 AU) particles with it. Only particles at intermediate altitude or with sufficiently large sizes (>1 mm at 10 AU) move inward. When the particles' radial velocities are averaged over the entire vertical direction, the particles have a net inward flux. The magnitude of their radial motion depends on their distance from the central star. At large distances, particles migrate inward with a velocity much faster than the gas accretion velocity. However, their inward velocity is reduced below that of the gas in the inner regions of the disk. The rate of velocity decrease is a function of the particles' size. While larger particles retain fast accretion velocity until they approach closer to the star, 10 \( \mu m \) particles have slower velocity than the gas in the majority of the disk (\( r \lesssim 100 \) AU). This differential migration of particles causes size fractionation. Dust disks composed mostly of small particles (size \( \lesssim 10 \mu m \)) accrete slower than gas disks, resulting in an increase in the dust-gas ratio during the gas accretion phase. If the gas disk has a steep radial density gradient or if dust particles sediment effectively to the disk midplane, the net vertically averaged flux of particles can be outward. In this case, the accretion of the dust component is prevented, leading to the formation of residual dust disks after their gas component is severely depleted.

Subject headings: accretion, accretion disks — planetary systems: formation — solar system: formation

1. INTRODUCTION

Planets form in circumstellar disks. In the standard scenario, the formation of Earth-size planets or planetary cores occurs through coagulation of small dust particles (e.g., Weidenschilling & Cuzzi 1993). Thus, the total amount, radial density distribution, and size distribution of dust particles in disks are the important initial conditions for planet formation. In many of the previous studies of planet formation, the dust-gas ratio of circumstellar disks is assumed to be similar to the solar value (\( \sim 10^{-2} \)), and dust particles are considered to be well mixed with gas, i.e., the dust-gas ratio is constant throughout the disk (Hayashi, Nakazawa, & Nakagawa 1985).

However, the motion of dust particles is different from that of gas. Dust particles have a radial motion that is induced by the gas drag force. Adachi, Hayashi, & Nakazawa (1976) and Weidenschilling (1977) studied the motion of particles in gas disks. Due to a radially outward pressure gradient force, the rotation velocity of the gas is generally slower than that of particles in nearly Keplerian circular orbits. Consequently, the gas drag force takes angular momentum from particles, resulting in their inward migration (Whipple 1972). At a few AU from the central star, the orbital decay time is estimated to be \( \sim 10^6 \) yr for 100 \( \mu m \) particles and \( 10^4 \) yr for 1 cm particles. These times are much shorter than the lifetime of gas disks (\( \sim 10^7 \) yr). Thus, the dust component may evolve much faster than the gas in protostellar disks, in which case the dust-gas ratio would decline. It appears that the initial conditions of planet formation need not be protostellar disks in which dust particles are well mixed with the gas and the dust-gas ratio is constant.

In this paper, we start a series of studies to determine the initial distribution of dust particles in the context of planet formation. Here, we study the radial migration of dust particles in gas disks. We focus our attention on particles smaller than \( \sim 1 \) cm in order to consider the evolution of dust disks before the initiation of planetesimal formation. The migration of these small particles establishes the initial density distribution of the dust component and sets the stage for planetesimal formation. In this phase of disk evolution, the gas component is considered to be turbulent and is accreting onto the star. The discussion in the above paragraph is based on the study of particle migration under the assumption that particles have completely sedimented on the midplane of laminar disks. However, in turbulent disks, the sedimentation of particles is prevented, because particles are stirred up to high altitude from the midplane by turbulent gas motion. In such disks, particles are distributed in the vertical direction, and the radial motion of particles depends on the distance from the midplane. Thus, there is a need for studying the radial migration of particles that reside above the midplane.

There are two important factors causing the vertical variation in the migration velocity of the particles. The first is the variation of the rotation velocity of the gas in the vertical direction. As mentioned above, the gas rotation differs from Keplerian rotation because of the gas pressure gradient. As the gas density decreases with the distance from the...
midplane, the radial pressure gradient varies, and under some circumstances even changes its sign. While at the midplane the pressure gradient force is outward, it is inward near the disk surface because the disk thickness increases with the radius. The gas drag force on the particles also varies with height, resulting in a variation of particle migration velocities. In § 3.1 we see that particles at high altitude, where the pressure gradient force is inward, flow outward.

The second factor is the variation in the radial gas flow. Small particles (≤ 100 μm) are well coupled to the gas, so that they migrate with the gas flow, as discussed in § 3.1 below. That is, if the gas is accreted onto the star, the small particles would also be accreted, while if the gas flows outward, particle flow would also be outward. In accreting protostellar disks, the gas does not always flow radially inward. The radial velocity of the gas varies with the distance from the midplane. Viscous stress can cause gas outflow near the midplane. Viscous stress can cause gas outflow near the midplane (Urry 1984; Kley & Lin 1992). Różycka, Bodenheimer, & Bell (1994) showed that the outflow occurs if the radial pressure gradient at the midplane is steep enough. The density (or pressure) gradient at the midplane is steeper than the average spatial (or surface) density gradient, e.g., if the surface density varies as \( \Sigma_{a} \propto r^{-1} \) and the disk thickness varies as \( h_{0} \propto r \), the midplane density has a steeper variation corresponding to \( \rho_{0} \propto r^{-2} \). Viscous diffusion at the midplane causes the outflow of the gas to reduce the surface density gradient. On the other hand, at high altitude the midplane, the density gradient is shallow, and the viscous torque causes the usual inward flow. Thus, in accretion disks, the outward flow at the midplane is sandwiched by the inflow at the disk surfaces (see § 2.2 for details). Because small particles are carried by the gas flow, they are expected to flow outward near the midplane and inward at higher altitude.

Thus, if most of the dust particles concentrate in the region where they migrate outward, i.e., near the disk surface or around the midplane, the dust component of the disk would accumulate in total mass and expand in size. In reality, the vertical spatial distribution of the particles is determined by the degree of sedimentation, and their size distribution is regulated by coagulation, condensation, and sublimation processes. In this paper, we calculate the vertical distribution of the particles in order to identify the mass flux of the dust particles as functions of their size and their distance from the midplane and their host stars. The results of the present calculation will be used to study the long-term dynamical evolution of the dust component of the disk. For the present task, we do not consider the effects of gas depletion, particles’ size evolution, or their feedback influence on the flow velocity of the gas. These effects will be considered in a future investigation.

There are several works on particle migration in turbulent disks. Stepiński & Valageas (1997) derived the vertically averaged velocity of migration for two limiting cases. One model is constructed for small particles that are well mixed with the gas, and the other is for large particles that are totally sedimented to the midplane. The velocity for intermediate-sized particles is calculated by the interpolation of these limiting cases. Supulver & Lin (2000) investigated the evolution of particle orbits in turbulent disks numerically. This paper gives an analytical expression for the particle velocity for any size of particle, which is estimated from interpolation by Stepiński & Valageas (1997). Several studies showed that vortices or turbulent eddies can trap particles within some size range (Barge & Sommeria 1995; Cuzzi, Dobrovolskis, & Hogan 1996; Tanga et al. 1996; Klahr & Henning 1997; Cuzzi et al. 2001; see, however, Hodgson & Brandenburg 1998). Particle trapping by eddies may be important for the local collection of size-sorted particles. In this paper, turbulent eddies are just treated as sources of particle diffusion and gas viscosity. The effect of particle trapping on their coagulation is not discussed here.

The plan of the paper is as follows. In § 2 the vertical variation of gas flow is derived. In § 3 we describe how the radial velocity of dust particles varies in the vertical direction and then calculate the net radial migration velocity. In § 4 we discuss the steady distribution of particles assuming no particle growth. We show that the size fractionation of particles occurs as a result of radial migration and that the distribution of particles differs from that of the gas.

2. VISCOUS FLOWS OF GAS DISKS

2.1. Rotation Law of the Gas

Gas disks rotate with a slightly different velocity from the Keplerian velocity. The rotation velocity is determined by the balance between the stellar gravity, the centrifugal force, and the gas pressure gradient. In cylindrical coordinates \((r, z)\), the balance of forces described by the momentum equation in the \(r\)-direction is given by

\[
\dot{r} \Omega_{g}^2 - \frac{GM_{*}}{(r^2 + z^2)^{3/2}} \frac{1}{\rho_{g}} \frac{\partial P_{g}}{\partial r} = 0 ,
\]

where \( \Omega_{g} \) is the angular rotation velocity of the gas, \( G \) is the gravitational constant, \( M_{*} \) is the stellar mass, \( \rho_{g} \) is the gas density, and \( P_{g} \) is the gas pressure. The density distribution in the vertical direction is determined by the balance between the \(z\)-components of the stellar gravity and the pressure gradient, which is written as

\[
\frac{GM_{*}}{(r^2 + z^2)^{3/2}} \frac{1}{\rho_{g}} \frac{\partial P_{g}}{\partial z} = 0 .
\]

We assume that the disk is isothermal in the vertical direction. Neglecting \((z/r)^2\) and higher order terms, the vertical density distribution is obtained as

\[
\rho_{g}(r, z) = \rho_{g}(r, 0) \exp \left( -\frac{z^2}{2h_{g}^2} \right) .
\]

The disk scale height \(h_{g}\) is given by

\[
h_{g}(r) = \frac{c}{\Omega_{K,\text{mid}}} ,
\]

where \( c \) is the isothermal sound speed and \( \Omega_{K,\text{mid}} = (GM_{*}/r_{\text{mid}}^3)^{1/2} \) is the Keplerian angular velocity at the midplane. The radial variations of physical quantities are assumed to have power-law forms, such that

\[
\rho_{g}(r, z) = \rho_{0} r_{\text{AU}}^p \exp \left( -\frac{z^2}{2h_{g}^2} \right) ,
\]

\[
c^2(r) = c_{0}^2 r_{\text{AU}}^q ,
\]

\[
h_{g}(r) = h_{0} r_{\text{AU}}^{(q+3)/2} ,
\]

where the subscript “ 0 ” means the value at 1 AU, \( r_{\text{AU}} \) is the radius in units of AU, and the power-law indices \( p \) and \( q \) are.
usually negative. The surface density is

$$\Sigma_g(r) = \int_{-\infty}^{+\infty} \rho_g \, dz = \sqrt{2 \pi \rho_0 h_0 r_h^2} ,$$

(6)

where \( p_+ = p + (q + 3)/2 \). To derive the rotation law of the gas, we retain \((z/r)^2\) and lower order terms in equation (1), then obtain

$$\Omega_g(r, z) = \Omega_{K, \text{mid}} \left[ 1 + \frac{1}{2} \left( \frac{h_g}{r} \right)^2 \left( p + q + \frac{q z^2}{2 h_g^2} \right) \right] .$$

(7)

The difference of the gas rotation from the Keplerian rotation is of the order of \((h_g/r)^2\), and the gas at high altitude rotates slower than the gas at the midplane.

2.2. Radial Velocity of the Gas Flow

In viscous disks, angular momentum is transferred from the inner part to the outer part of the disk under the action of viscous stress. The azimuthal component of the momentum equation implies

$$2 \pi \rho_g \left( v_{r,g} \frac{\partial}{\partial r} + v_{z,g} \frac{\partial}{\partial z} \right) (r^2 \Omega_g) = 2 \pi \left[ \frac{\partial}{\partial r} \left( r^2 \rho_0 \nu \frac{\partial \Omega_g}{\partial r} \right) + \frac{\partial}{\partial z} \left( r^2 \rho_0 \nu \frac{\partial \Omega_g}{\partial z} \right) \right] ,$$

(8)

where \( v_{r,g} \) and \( v_{z,g} \) are the \( r \)- and \( z \)-components of the gas velocity, respectively, and \( \nu \) is the kinetic viscosity. The right-hand side of equation (8) represents the torque exerted on an annulus with unit width in the \( r \)- and \( z \)-directions. The left-hand side is the variation of the angular momentum of the annulus accompanying the motion. We assume that the angular velocity is always adjusted to the equilibrium (eq. [7]) on a dynamical timescale. Since molecular viscosity is too ineffective, it is customary to assume that angular momentum is primarily transported by the Reynolds stress induced by the turbulent motion of the gas. Following conventional practice, we adopt an ad hoc but simple prescription to use \( \alpha \) for turbulent viscosity (Shakura & Sunyaev 1973), such that

$$\nu = \alpha c_h q = \alpha c_h h_0 r_h^{+3/2} .$$

(9)

The mass conservation of the gas is

$$\frac{\partial \rho_g}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} \left( r \rho_g v_{r,g} \right) + \frac{\partial}{\partial z} \left( \rho_g v_{z,g} \right) = 0 .$$

(10)

In the accretion disks considered here, the second term is of the same order as the third term. Then, we see that \( v_{z,g} \sim (h_g/r) v_{r,g} \). In addition, from equation (7) we see \( \partial(r^2 \Omega_g)/\partial z \sim (h_g/r) \partial(r^2 \Omega_g)/\partial r \). Thus, \( v_{z,g} \partial(r^2 \Omega_g)/\partial z \) is a factor \((h_g/r)^2\) smaller than \( v_{r,g} \partial(r^2 \Omega_g)/\partial r \) and can be neglected in equation (8). Using the power-law expressions in equations (5) and (9) and retaining terms of the order of \((h_g/r)^3\), the radial velocity is reduced to

$$v_{r,g} = -2 \pi \alpha \left( \frac{h_0}{\text{AU}} \right)^2 \times \left( 3p + 2q + 6 + \frac{5q + 9 z^2}{2 h_g^2} \right) r_h^{+3/2} \text{AU yr}^{-1} .$$

(11)

In the following discussions, we adopt these values as being representative of a typical model: \( M = 1 \ M_\odot \), \( \rho_0 = 2.83 \times 10^{-10} \text{ g cm}^{-3} \), \( h_0 = 3.33 \times 10^{-2} \text{ AU} \), \( p = -2.25 \), and \( q = -0.5 \). These values correspond to a minimum-mass solar nebula in which the surface density distribution has a power-law form with index \( p_+ = p + (q + 3)/2 = -1.0 \) and the gas mass inside 100 AU is \( 2.5 \times 10^{-2} \ M_\odot \).

Figure 1 shows the radial velocity of the gas. The radial velocity is outward (positive) near the midplane, while it is inward (negative) above the height \( 0.73 h_g \). In the standard model \((q = -0.5)\), the radial velocity is independent of the distance from the star, as seen from equation (11). At the midplane, the radial gradient of the gas density is so steep \((p = -2.25)\) that the gas receives more torque from the inner disk than the torque it exerts on the outer disk. This radial dependence means that viscous stress acts to move the gas outward in such a steep density gradient. The condition for the outward gas flow at the midplane is

$$p + \frac{3}{2} q < -2 ,$$

(12)

which is satisfied in many accretion disk models. The rotation of the gas is slower at higher altitude, as seen in equation (7). Thus, the gas at the midplane loses its angular momentum through the action of the viscous stress that it has with the gas at higher altitude. However, in the standard model, this angular momentum loss is smaller than the angular momentum gain through the viscous stress in the \( r \)-direction. The outflow zone around the midplane shrinks as the radial density gradient is reduced \((p \to 0)\), and when the condition in equation (12) is violated, the gas at all altitudes flows inward. The radial density gradient is shallower at higher altitude, because the disk scale height increases with the radius. Thus, at high altitude, the gas loses more angular momentum through exerting torque on the outer disk than it receives from the inner disk. For such gas, the presence of viscous torque induces it to flow toward the star.
The net radial velocity of the gas is given by averaging in the vertical direction:

\[
(v_{r,g}) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} v_{r,g}\rho_g \, dz
\]

\[= -6\pi\alpha \left( \frac{h_0}{\text{AU}} \right)^2 \left( p_s + q + 2 \right) \rho_{g,AU}^{q+1/2} \text{AU yr}^{-1}. \tag{13}\]

The net accretion of the gas toward the star occurs if

\[p_s + q = p + \frac{1}{2}(q + 1) > -2. \tag{14}\]

We can consider two models in which a steady state is achieved. One model is the case in which the net radial velocity is 0, i.e., \(p_s + q = -2\). For example, if \(p_s = -1.5\) and \(q = -0.5\), there is no net accretion of the disk. A more realistic model is the case in which the mass flux of the gas is constant with radius, i.e., \(\Sigma_g(v_{r,g}) \propto \rho_p(r)^{q+3/2}\) is constant. We take this as our standard model, i.e., \(p_s = -1.0\) and \(q = -0.5\). This surface density profile is expected to arise after the disk has undergone a period of initial viscous evolution.

3. RADIAL FLOW OF DUST PARTICLES

3.1. Vertical Dependence of Radial Velocity

The azimuthal velocities of dust particles are different from that of the gas. The resulting gas drag force transfers angular momentum between the particles and the gas and moves particles in the radial direction. If there were no gas drag force, the particles would orbit with the Keplerian angular velocity, which is approximated as

\[\Omega_K(r, z) \approx \Omega_{K,\text{mid}} \left( 1 - \frac{3z^2}{4r^2} \right). \tag{15}\]

We express the deviation of the angular velocity of the gas from the Keplerian angular velocity as

\[\Omega_g = \Omega_K(1 - \eta)^{1/2}. \tag{16}\]

From equation (1) it is seen that \(\eta = -(r\Omega_K^2\rho_g)^{-1}\partial P_g/\partial r\) is the ratio of the gas pressure gradient to the stellar gravity in the radial direction. From equations (7), (15), and (16), \(\eta\) is written within the order of \((h_0/r)^2\) as

\[\eta = -\left( \frac{h_0}{r} \right)^2 \left( p + q + \frac{3z^2}{2h_0^2} \right). \tag{17}\]

Note that the pressure gradient force is outward (\(\eta\) is positive) around the midplane, while it is inward (\(\eta\) is negative) where \(|z| > (2p + q)/(q + 3)\). In the standard model, \(\eta\) changes sign at \(z \approx 1.5h_0\). Dust particles near the midplane rotate faster than the gas, and particles at high altitude rotate slower than the gas.

The equations of motion of a particle are

\[
\frac{dv_{r,d}}{dt} = \frac{v_{\theta,d}}{r} - \frac{\Omega_K^2}{r} - \frac{\Omega_{K,\text{mid}}}{T_s} (v_{r,d} - v_{r,g}), \tag{18}\]

\[
\frac{d}{dt} (rv_{\theta,d}) = \frac{v_{\theta,K,\text{mid}}}{T_s} (v_{\theta,d} - v_{\theta,g}), \tag{19}\]

where \(v_r\) and \(v_{\theta}\) are the \(r\)- and \(\theta\)-components of the velocity, respectively, with the subscripts “\(g\)” and “\(d\)” distinguishing gas and dust, and \(v_{K,\text{mid}} = r\Omega_{K,\text{mid}}\) is the Keplerian velocity at the midplane. The gas drag force is expressed through the nondimensional stopping time \(T_s\) (normalized by the Kepler time at the midplane). We do not solve the equation of motion in the \(z\)-direction. Instead, we simply assume \(v_z = 0\). When an equilibrium in the vertical dust distribution is achieved, the dust sedimentation to the midplane is balanced by the diffusion of particles, as discussed below in §3.2. The vertical velocity of a particle is 0 when time averaged.

The mean free path of gas molecules is larger than 1 cm for \(r \gtrsim 1\) AU in our models (Nakagawa, Sekiya, & Hayashi 1986). In this paper, we consider particles smaller than 1 cm and use Epstein’s gas drag law. Then, the nondimensional stopping time \(T_s\), as given by Takeuchi & Artymowicz (2001), is

\[
T_s = \frac{\rho_p s \Omega_{K,\text{mid}}}{\rho_g F_V}, \tag{20}\]

where \(\rho_p\) is the particle internal density, \(s\) is the particle radius, and the mean thermal velocity is \(v_t = (8/\pi)^{1/2} c^1\). We take the particle internal density as \(\rho_p = 1.25 \text{ g cm}^{-3}\).

For particles smaller than 1 cm, the nondimensional stopping time is much smaller than unity through most of the disk. Figure 2 shows the radii of particles whose stopping time is unity. At the midplane (solid line), the nondimensional stopping time is smaller than unity for particles of size \(s \leq 1\) cm at \(r \leq 100\) AU. At \(z = 2h_0\) (dashed line), although the gas density is lower than at the midplane and the stopping time is longer, particles of size \(s \leq 1\) mm still have a nondimensional stopping time smaller than unity. These particles are well coupled to the gas, and their angular velocity is similar to the gas angular velocity in equation (7). Only particles with \(s \gtrsim 1\) cm that are located at high altitude and \(r \gtrsim 100\) AU become decoupled from the gas.

\[1\text{ In Takeuchi & Artymowicz (2001), } v_t \text{ is defined to be } 4/3\text{ times the mean thermal velocity. This definition causes a factor of } 4/3\text{ difference between eq. (20) and their eq. (10).}\]
We assume that motions of both the gas and the particles are close to Keplerian, i.e., \( v_{g, d} \approx v_{g, z} \approx v_{K, mid} \), and that \( d(rv_{g, d})/dt \approx v_{r, d} d(rv_{K, mid})/dr = v_{r, d} v_{K, mid}/2 \). Then, from equation (19), we have
\[
v_{g, d} - v_{g, z} = -\frac{1}{2} T_s v_{r, d}.
\]
Using equation (16) and neglecting terms of the order of \((h_g/r)^4\) and higher, equation (18) is reduced to
\[
\frac{dv_{r, d}}{dt} = -\frac{v_{K, mid}^2}{r} + \frac{2v_{K, mid}}{r} (v_{g, d} - v_{g, z}) - \frac{\Omega_{K, mid}}{T_s} (v_{r, d} - v_{g, z}).
\]

The left-hand side is of the order of \(v_{r, d}/r\) and is neglected if \(v_{r, d} \ll c\). Substituting equation (21) into equation (22), we find the radial velocity of the particle to be
\[
v_{r, d} = \frac{T_s^{-1} v_{r, g} - \eta^2 v_{K, mid}}{T_s + T_s^{-1}}.
\]

In the above derivation of the particle’s radial velocity, we assume that in the \(z\)-direction, the particle sedimentation is balanced by turbulent diffusion, but that in the \(r\)-direction, turbulent effects are neglected.

For particles well coupled with the gas (\(T_s \ll 1\)), the radial velocity reduces to
\[
v_{r, d} = v_{r, g} + v_{r, drift},
\]
where \(v_{r, drift} = -\eta T_s v_{K, mid}\) is the relative velocity from the gas. Using equations (5), (17), and (20),
\[
v_{r, drift} = 2\pi \left(\frac{h_0}{\Lambda U}\right)^2 \left(p + q + \frac{q}{2} \frac{z^2}{h_0}\right)
\times \frac{r^{r+1/2}}{T_{s, mid}} \left(\frac{z^2}{2h_0}\right) A U \text{ yr}^{-1},
\]
where \(T_{s, mid}\) is the nondimensional stopping time at the midplane. The radial drift velocity increases exponentially with the height. Because the particles are strongly coupled with the gas, the gas drag force suppresses their drift velocity. At the midplane where the gas density is highest, the suppression of the drift is most effective. The drift velocity at the midplane is
\[
v_{r, drift, mid} = \sqrt{\frac{\pi^2}{2} h_0 \rho_p \rho_0 (p + q) r_{AU}^{r+q/2-1}} A U \text{ yr}^{-1},
\]
which agrees with the derivations of Adachi et al. (1976) and Weidenschilling (1977).

For large particles decoupled from the gas (\(T_s \gg 1\)), the radial motion of the gas does not affect the particles’ velocity, so that \(v_{r, d} = -\eta T_s^{-1} v_{K, mid}\). In this paper, we do not consider such large particles.

Figure 3 shows the radial velocity \(v_{r, d}\) of particles of \(s = 10 \mu m, 100 \mu m\), and 1 mm at 10 AU in the standard model disk. Near the midplane, the drift velocity \(v_{drift}\) (relative to the gas velocity shown by the dotted line) of 10 \(\mu m\) particles is small. The particles move outward almost together with the gas. As the altitude increases and the gas velocity decreases, the radial velocity decreases, then becomes negative. At \(z \approx 1.5 h_g\), where \(\eta\) changes its sign, the radial pressure gradient vanishes and the gas rotates with the Keplerian velocity. The particles corotate with the gas at that location and have the same radial velocity as the gas. The drift velocity changes its sign to become positive. At high altitude (\(z \gtrsim 2 h_g\)), where the gas drag is weak, the particles’ drift velocity begins to increase rapidly, and then the radial velocity becomes positive again at \(z \gtrsim 3h_g\). The behavior of the radial velocity of 100 \(\mu m\) particles is qualitatively similar to that of 10 \(\mu m\) particles. i.e., it is outward at the midplane, inward at intermediate altitude, and outward again at high altitude. The coupling of 100 \(\mu m\) particles to the gas is weaker, so their radial velocity is very different from the gas velocity, even at the midplane. However, the drift velocity at the midplane is still slightly smaller than the gas outflow velocity, and the particles move outward. Particles of 1 mm move inward even at the midplane, because the inward drift velocity of such large particles is larger than the outflow velocity of the gas. These large particles also move outward at high altitude where the gas rotates faster than the particles.

3.2. Vertical Distribution of Particles

At the beginning stage of star formation from molecular cloud cores, dust particles in circumstellar disks may be well mixed with the gas, and the dust-gas ratio may be uniform throughout the disk. After the termination of gas infall onto the disks, the disks become hydrostatic, and particles begin to sediment toward the midplane because of the \(z\)-component of the stellar gravity. The particles reach terminal velocity (Nakagawa, Nakazawa, & Hayashi 1981), at which the gravity and the gas drag balance with each other,
\[
v_{z, d} = -\Omega_{K, mid} T_s z.
\]

The timescale of the sedimentation is \(t_{sed} \sim z/v_{z, d} \sim T_s^{-1} \Omega_K^{-1}\). If this timescale is much smaller than the radial migration timescale \(t_r \sim r/v_{r, d} \sim (\alpha + T_s)^{-1}(h_g/r)^{-1} \Omega_K^{-1}\), i.e., if \(T_s/\alpha \gg (h_g/r)^2\), then the particles sediment before any large migration in the \(r\)-direction. At high altitudes or around the midplane, particles may drift in the opposite direction as the gas, as seen in Fig. 3. Some particles with \(T_s \approx \alpha\) may have a radial migration timescale \(t_r\), much larger than the above estimate. Even for
such particles, the condition $T_s/\alpha \gg (h_g/r)^2$ for the fast sedimentation is still valid. For example, at 10 AU in a disk with $\alpha = 10^{-3}$ and $(h_g/r)^2 \sim 10^{-3}$, particles larger than 0.2 $\mu$m sediment to the midplane without large radial movement.

We consider disks at later stages, such as the T Tauri stage. The disks are still turbulent, and the turbulent motion of the gas stirs dust particles up to high altitude to prevent dust sedimentation. An equilibrium distribution of particles in the vertical direction is achieved by the balance between the sedimentation and the diffusion due to the turbulent gas.

The turbulent diffusion is modeled in an analogy of molecular diffusion, provided that the particles are considered to be passive tracers of fluid, i.e., the particles have no influence on the gas motion and have a velocity similar to the surrounding gas (see, e.g., Monin & Yaglom 1971, p. 579; Morfill 1985, § 3.5.1). The equation of continuity is written as

$$\frac{\partial}{\partial t}\rho_d + \mathbf{V} \cdot (\rho_d \mathbf{v}_d + \mathbf{j}) = 0,$$

where $\rho_d$ is the particle density and $\mathbf{v}_d$ is the particle velocity. The diffusive mass flux $\mathbf{j}$ is estimated by

$$\mathbf{j} = -\frac{\rho_d \nu}{\text{Sc}} \left( \frac{\partial \rho_d}{\partial z} \right),$$

where the Schmidt number $\text{Sc}$ represents the strength of coupling between the particles and the gas. For small particles, $\text{Sc}$ approaches unity, while it becomes infinite for large particles. For intermediate particle sizes, $\text{Sc}$ can be as small as 0.1, which means that particle diffusion occurs effectively (see, e.g., Fig. 1 in Cuzzi, Dobrovolskis, & Champney 1993). In our standard model, we use $\text{Sc} = 1$. If $\text{Sc}$ is not unity, the particles are not the passive tracers that completely follow the fluid motion. In this case, the formulation according to the analogy of molecular diffusion may be inappropriate. In addition, if the velocity of sedimentation $v_z$ is comparable to or larger than the turbulent velocity, the particles could not be passive tracers. The estimate of the diffusive mass flux by equation (29) should be considered as just a "gradient diffusion hypothesis."

In a steady state of axisymmetrical disks, $\partial \rho_d / \partial t$ and $\partial \mathbf{j} / \partial z$ in equation (28) are 0. In addition, for particles satisfying the condition $T_s/\alpha \gg (h_g/r)^2$ (i.e., for particles settling fast without large radial migration), we see that $\partial (\rho_d v_z) / \partial z \gg \partial (\rho_d v_{z,d}) / \partial r$. Therefore, the mass flux in the $z$-direction must be 0, i.e.,

$$\rho_d v_z - \rho_g \nu \frac{\partial}{\partial z} \left( \frac{\rho_d}{\rho_g} \right) = 0.$$

This equation is the same as the one derived by Dubrulle, Morfill, & Sterzik (1995). Solving equation (30) with equations (3), (4), (9), (20), and (27) gives the particle density:

$$\rho_d(r, z) = \rho_d(r, 0) \exp \left[ -\frac{z^2}{2h_g^2} \frac{\text{Sc} T_{s,\text{mid}}}{\alpha} \left( \exp \frac{z^2}{2h_g^2} - 1 \right) \right].$$

The first term in the exponential comes from the gas distribution, and the second term represents the sedimentation. In the limit of tight coupling of the particles and the gas

$$\Sigma_d(r) = \int_{-\infty}^{+\infty} \rho_d(r, z) \, dz.$$

Figure 4a shows variation of the dust-gas ratio in the vertical direction at 10 AU in the standard model disk. We see the sedimentation of particles. Large particles ($\gtrsim 1$ mm) sediment around the midplane, while small particles ($\lesssim 10$ $\mu$m) spread over a few scale heights of the gas disk. These small particles are stirred up to high altitude by the turbulence of the gas. The gas drag force becomes weaker at higher altitude, where the gas density decreases, so the gas cannot sustain the dust particles there. Thus, the particle

$$T_{s,\text{mid}} = 0,$$

the particle distribution is the same as that of the gas. The surface mass density of dust particles is

$2$ We obtained a slightly thinner distribution of particles than that by Dubrulle et al. (1995), which is shown in their Fig. 3. They assumed the stopping time of particles, which is inversely proportional to the gas density, to be constant and used the value at the midplane. This assumption causes the dust disk to puff out.
density drops rapidly at some altitude, and the dust disk obtains a relatively sharp surface (e.g., at \( z \approx 2.5h_g \) for 10 \( \mu m \) particles). The sedimentation of particles is more effective in the outer part of the disk, farther away from the star (see Fig. 4b). For example, 100 \( \mu m \) particles at 1 AU spread over the gas disk, while the particles at 100 AU concentrate around the midplane. At the outer part of the disk, the gas density is lower, so the gas drag is weaker and turbulence cannot loft the particles to high altitudes.

Dust particles are well mixed with the gas if

\[
\frac{ScT_{z,mid}}{\alpha} \left( \exp \left( \frac{z^2}{2h_g^2} - 1 \right) \right) \ll 1 ,
\]

i.e., if they are below the height

\[
\frac{z}{h_g} = \left[ 2 \ln \left( \frac{\alpha}{ScT_{z,mid}} + 1 \right) \right]^{1/2}.
\]

In the standard model, particles smaller than

\[
s \ll 20r_{AU} \mu m
\]
distribute uniformly with regard to the gas under the height \( z = 3h_g \).

### 3.3. Net Radial Velocity

As discussed in § 3.1, the radial velocity of particles varies with the altitude. In this subsection, we discuss the net radial velocity averaged in the vertical direction.

#### 3.3.1. Particles Well Mixed with the Gas

At the beginning stage of the formation of circumstellar disks, small particles should be distributed uniformly in the disk gas. Before the particles have undergone enough sedimentation, their net radial velocity averaged in the vertical direction is

\[
\langle v_{r,d} \rangle = \frac{1}{\Sigma_d} \int_{-\infty}^{+\infty} \rho_d v_{r,d} \, dz.
\]

Figure 5 shows the net radial velocities of small particles. The velocity of particles approaches the gas velocity as the distance from the star becomes smaller. Near the star, the gas density is high enough to induce almost all particles to move together with the gas. As the distance from the star increases, the gas density decreases, and particles at high altitude begin to drift outward from the gas. At sufficiently large distance from the star, the net radial velocity can be positive, i.e., particles can move outward. However, before they move over a large distance in the radial direction, they also sediment to the midplane. The timescale of the outward radial motion is \( t_{out} \sim r/(v_{r,d}) \sim 1/(\eta T_s \Omega_K) \) for outflowing particles, \( |v_{r,d}| \) is smaller than \( |v_{r,drift}| \) and is neglected in the estimate of \( t_{out} \), while the sedimentation timescale is \( t_{sed} \sim z/v_{z,d} \sim 1/(T_s \Omega_K) \). Because \( \eta < 1 \), \( t_{out} \gg t_{sed} \). The outward motion of those particles that are well mixed with the gas may proceed shortly after the formation of the circumstellar disk, but it does not significantly modify the radial distribution of dust particles. The evolution of radial distribution occurs primarily through the motion of sedimented particles.

#### 3.3.2. Removal of Outflow at High Altitude by Sedimentation

The net radial velocity of sedimented particles is

\[
\langle v_{r,d} \rangle = \frac{1}{\Sigma_d} \int_{-\infty}^{+\infty} \rho_d v_{r,d} \, dz.
\]

The function \( \rho_d v_{r,d}/\Sigma_d \), which is proportional to the mass flux, is shown in Figure 6. Because of the sedimentation to the midplane, which is efficient for larger particles, the mass flux is dominated by the particles near the midplane. For example, the mass flux of 10 \( \mu m \) particles is mainly carried by particles moving outward around the midplane (\( |z| \leq 0.7h_g \)) and by those moving inward at intermediate altitude (0.7\( h_g \leq |z| \leq 2.9h_g \)). Although particles at high altitude (\( |z| \geq 2.9h_g \)) move outward, their contribution to the mass flux is negligibly small. Very large particles, for example, 1 mm particles, move inward even at the midplane. Thus, sedimentation causes the vast majority of particles to move inward.

Fig. 5.—Net radial velocity of particles well mixed with the gas, \( \langle v_{r,d} \rangle \). The three solid lines correspond to \( s = 10, 1, \) and 0.1 \( \mu m \) particles from the top line. The dashed line shows the gas accretion velocity.

![Fig. 5.](image)

Fig. 6.—Normalized mass flux \( \rho_d v_{r,d}/\Sigma_d \) at 10 AU. The solid, dashed, and dot-dashed lines are for \( s = 10 \mu m, 100 \mu m, \) and 1 mm particles, respectively. For 1 mm particles (dot-dashed line), 1/10 of the value is plotted (or refer to the ordinate at the right-hand side). The distance from the midplane \( z \) is normalized by the disk scale height \( h_g = 0.59 \) AU. The dotted line shows the zero velocity for reference.
Sedimentation effectively removes outflowing particles at high altitude. This flow pattern is seen as follows. The particles flowing outward have a drift velocity larger than the gas inflow velocity, i.e., \( |v_{r,\text{drift}}| > |v_{r,g}| \). From equations (11) and (25), it is seen that \( \alpha^{-1} T_{s,\text{mid}} \exp\left( z^2/2h_g^2 \right) \geq 1 \) for such particles. However, from equation (31), we also see that the density of these particles is as small as \( \rho_d(z)/\rho_d(0) \leq e^{-1} \). Therefore, very few particles remain in the outflow region at high altitude.

If the sedimentation is so effective that most particles concentrate around the midplane \( |z| \leq h_g \), e.g., for particles larger than 1 mm at 10 AU (see Fig. 4a), the inward drift velocity at the midplane will be larger than the gas outflow velocity. Thus, such particles flow inward. Again, from equation (31), we see that \( T_{s,\text{mid}}/\alpha \geq 1 \) for such particles. Then, it is seen from equations (11) and (25) that \( |v_{r,\text{drift}}| \geq |v_{r,g}| \) at the midplane. This inequality implies that when particles grow to sizes sufficiently large to sediment toward and concentrate around the midplane, they become decoupled from the outflow motion of the gas. If the sedimentation is so efficient that all particles concentrate in a thin layer at the midplane \( T_{s,\text{mid}}/\alpha \gg 1 \), the radial motion of the particles will not be affected by the radial motion of the gas. In this limit, the particles’ motion is similar to that deduced by Adachi et al. (1976) and Weidenschilling (1977).

3.3.3. Net Velocity of Sedimented Particles

Figure 7a shows the net radial velocities of sedimented particles. These net radial velocities are negative for particles of all sizes. Particles move rapidly inward when they are at large distances from the star. In the outer regions of the disk, particles sediment and concentrate at the midplane, and their net radial velocity, which is much faster than the gas outflow velocity \( v_{r,g} \), is approximated by the drift velocity at the midplane (eq. [26]). Their inward velocity decreases as they approach the star, because the suppression of inward velocity by the gas drag becomes stronger. The timescale of their orbital decay is \( r/(v_{r,d}) \propto r^{2+\rho-\omega/2} \). When they approach the location where the gas density is dense enough to make \( T_{s,\text{mid}}/\alpha \leq 1 \), the inward drift velocity becomes smaller than the gas outflow velocity at the midplane. The particles begin to move with the gas flow. At the same time, particles are spread over more than the disk scale height. The particles’ net radial velocity approaches the net gas velocity (Fig. 7a, dashed line). However, because the particle distribution is slightly more concentrated at the

![Fig. 7](https://example.com/f7.png)

Fig. 7.—Net radial velocity \( (v_{r,d}) \) of sedimented particles. The three solid lines correspond to \( s = 1 \) mm, 100 \( \mu \)m, and 10 \( \mu \)m particles from the left line. The accretion velocity of the gas is shown by the dashed line, and the dotted line shows the zero velocity for reference. (a) Standard model \( (p_v = -1.0, q = -0.5, \text{and } Sc = 1.0) \) (b) Model with \( p_v = -0.5 \). (c) Model with \( p_v = -1.3 \). (d) Model with \( Sc = 10 \).
midplane than the gas distribution, the number of particles riding on the outflowing gas around the midplane is larger than in the case in which the particle distribution is same as the gas. The enhancement of outflowing particles retards their net inflow velocity to values below that of the gas. At the inner part of the disk, the net inward velocity of the particles is always slower than the gas velocity. Near the star, the particles mix more with the gas, and their motion converges with that of the gas. The particles' inward velocity has a minimum magnitude which is about half of the gas velocity. Particles smaller than 10 $\mu$m have a slower inward velocity than the gas throughout nearly the entire disk ($r \leq 1$ AU), while particles larger than 1 mm have a slower velocity only in the innermost part of the disk ($r \lesssim 1$ AU). The difference in the inward velocity induces the size fractionation of particles, as discussed in § 4.1 below. The dust-gas ratio may increase during the viscous evolution of disks, because the accretion velocity of small particles is slower than that of the gas.

Small particles that satisfy the condition in equation (35) are mixed well with the gas. The net radial velocities of such small particles can be calculated by equation (36), even after larger particles have undergone sedimentation. It is seen from comparison between Figures 5 and 7a that, e.g., for 10 $\mu$m particles, the radial velocities are about the same in both figures for $r \leq 1$ AU, while they deviate significantly between these figures for $r \gtrsim 10$ AU. As shown in Figure 5, the radial velocity of 10 $\mu$m particles at $r \gtrsim 10$ AU is outward when the particles are mixed with the gas, i.e., in the very first stage of disk formation. However, after particle sedimentation, the radial velocity becomes inward even at $r \gtrsim 10$ AU, as shown in Figure 7a. It is concluded that particle sedimentation suppresses the outward flow of particles in the standard model.

3.3.4. Various Models

In Figure 7b we show the case with $p_s = -0.5$ in which the surface density profile of the gas is flatter than in the standard model. The velocity profiles are qualitatively similar to those in the standard model. Particles of 10 $\mu$m move slower than the gas throughout nearly the entire disk, while 1 mm particles rapidly migrate inward.

In Figure 7c the case with a steeper surface density gradient is shown. In this model, we adopt $p_s = -1.3$, which satisfies the condition in equation (14) for inward gas accretion flow. In this case, we see that the net radial velocity of particles becomes positive at some locations. In these regions, the accretion of particles is prevented. The particles flowing inward from large distances terminate their migration at the location where the net radial velocity becomes 0 and then accumulate at that particular orbital radius. Particles located just inside this critical radius flow outward and accumulate there also. Particles at the innermost part of the disk flow inward onto the star. Size fractionation occurs because the location of the accumulation depends on the particle size. The accumulation continues as particles drift radially inward from large radii until particles in the outer disk are significantly depleted or the number density of accumulated particles becomes so high that the removal of particles through coagulation or collisional destruction becomes efficient. Although accretion of dust particles terminates at some locations, the gas disk continues to accrete onto the star. The dust-gas ratio increases as the accretion of the gas proceeds.

3.3.5. Self-Similarity of the Velocity Profile

The functions of net radial velocities shown in Figure 7 for various sizes have self-similar forms. From equations (11), (24), (25), (31), (32), and (37), the net radial velocity can be written as

$$\langle v_r, d \rangle = 2\pi\alpha \left( \frac{h_0}{\text{AU}} \right)^2 \frac{\text{AU}}{\text{yr}^{-1}} F \left( p, q, \text{Sc}; \frac{T_{s, \text{mid}}}{\alpha} \right),$$

(38)

where $F$ is a function of $p$, $q$, Sc, and $T_{s, \text{mid}}/\alpha$,

$$F \left( p, q, \text{Sc}; \frac{T_{s, \text{mid}}}{\alpha} \right) = \int_{-\infty}^{+\infty} \left\{ -3p - 2q - 6 - (5q + 9)z^2 + \left[ p + q + (q + 3)z^2 \right] \frac{T_{s, \text{mid}}}{\alpha} \exp z^2 \right\} \exp \left[ z^2 \right] \exp \left[ \frac{\text{Sc} T_{s, \text{mid}}}{\alpha} (\exp z^2 - 1) \right] dz \times \exp \left[ z^2 \right] \left[ \int_{-\infty}^{+\infty} \exp \left[ z^2 \right] \exp \left[ \frac{\text{Sc} T_{s, \text{mid}}}{\alpha} (\exp z^2 - 1) \right] dz \right]^{-1}.$$  

(39)

and $z' = z/\sqrt{2h_0}$. If we vary the properties of particles, keeping $T_{s, \text{mid}}/\alpha$ to be constant, the function $F$ yields the same value. For example, when we vary the particle size $s \rightarrow s'$, then transform the orbital radius as $r \rightarrow r' = (s'/s)^{1/[p+(q+3)/2]} r$, we have the same value of $T_{s, \text{mid}}/\alpha$ and the same functional form of $F$. In our models, $q = -\frac{3}{2}$, so $(v_r, d)$ depends on $r$ only through the function $F$. In this case, the profiles of the net radial velocities for different sizes are obtained by transformation in the $r$-direction from one profile. In Figure 8 the function $F$ is plotted for various values of the Schmidt number Sc.

![Figure 8](image)

**Fig. 8.**—Function $F$ for various values of the Schmidt number: Sc = 10, 1, and 0.1 from the top line. The dotted line shows the zero for reference.
3.3.6. Various Schmidt Numbers

In the standard model, the net radial velocity of the particles is inward everywhere in the disk. This flow pattern is partially due to the removal of particles through sedimentation from high altitudes where they would flow outward. For particles that are sufficiently large to concentrate near the midplane, the magnitude of their inward drift velocity is larger than that of the outflowing gas. Thus, the strength of sedimentation is always moderate enough to remove outflowing particles. The sedimentation is controlled by the coupling of particles to the turbulent motion of the gas. If the coupling were relatively strong (Sc were smaller than unity), the sedimentation would be less effective, and particles would mix more completely with the gas, while if it were relatively weak (Sc were larger), particles would concentrate more at the midplane. Usually, Sc is assumed to be unity, but the actual value may be different from unity. Some experimental data are plotted in Figure 1 in Cuzzi et al. (1993), which show Sc as small as 0.1 when the Stokes number St is about 0.01 (the Stokes number is considered to be of the same order as the nondimensional stopping time $T_s$ in our models). For large particles ($St \sim T_s \geq 1$), Sc increases with the particle size.

Figure 7d shows the net radial velocities in the case in which the sedimentation is efficient (Sc = 10). The net radial velocity becomes outward at its peak. Thus, an accumulation of particles and an increase in the dust-gas ratio would occur. Because of the strong sedimentation, the dust disk is much thinner than in the standard model (Sc = 1) and concentrates more at the midplane where the gas flows outward. In the outer part of the disk (for example, $r > 10$ AU for 100 $\mu$m particles), the inward drift velocity of particles (relative to the gas) at the midplane is faster than the outward gas velocity. As the particles drift in, the inward velocity decreases, and at some distances from the star ($r \sim 10$ AU for 100 $\mu$m particles), the inward drift velocity at the midplane becomes smaller than the outward gas velocity, and the particles around the midplane flow outward. At such distances the dust disk is still concentrated around the midplane, and the net radial velocity is dominated by the particles around the midplane and is outward. As the distance from the star decreases, the particles mix more with the gas, and the concentration at the midplane becomes weaker. The net radial velocity approaches the gas velocity, which is inward, at the innermost part of the disk.

If the Schmidt number Sc is less than unity, the sedimentation of particles is less efficient. Therefore, the particles are always well mixed with the gas at where the inward drift velocity of particles is smaller than the gas outflow velocity (i.e., at where the particle outflow at the midplane occurs). Thus, the net radial velocity of particles is inward everywhere.

Figure 8 shows the function $F$ for various values of the Schmidt number: Sc = 0.1, 1, and 10. The net radial velocity has the same sign as $F$ (see eq. [38]). It is seen that the outflow of dust particles occurs if the sedimentation is efficient (Sc ≥ 10).

4. DISCUSSION AND SUMMARY

4.1. Steady Density Distribution of Dust Particles

Because the inward velocity of particles depends on their size, particles of different sizes accumulate at different localizations. In this subsection, we discuss the density distribution of dust particles as a consequence of their radial flow and show how their size fractionation may occur. We adopt the models in which the net velocity of particles is always inward (Sc = 1 and $p_s \geq -1.0$). We assume that the distribution of particles approaches a steady state after a brief stage of initial evolution. When a steady state is achieved, the mass flux of dust particles becomes constant in the radial direction. We calculate the mass flux of particles as

$$
\frac{dM_d}{ds} = 2\pi r(v_{rd}) d\Sigma_d,
$$

where $M_d$ and $\Sigma_d$ are the mass flux and the surface density, respectively, of particles smaller than size $s$. For simplicity, we neglect three physical processes. First, the evolution of the particle size through coagulation, collisional destruction, condensation, and sublimation is neglected. Second, in turbulent disks, the particles' mass flux comes not only from the mean flow with an average velocity $\langle v_{rd} \rangle$ but also from the turbulent diffusion of particles, which appears as $j$ in equation (28), for example. We neglect the mass flux from turbulent diffusion. Third, we assume that the structure of the gas component of the disk is entirely determined by the gas itself. We neglect the feedback drag induced by the particles on the gas. In the limit that the spatial density of the dust component becomes comparable to that of the gas near the midplane, this effect would speed up the azimuthal velocity of the gas to the Keplerian value and quench the radial migration of the dust (Cuzzi et al. 1993). Such a dust concentration requires substantial sedimentation of relatively large particles. Contributions from all of these factors will be investigated in future grain-evolution calculations. Here, we adopt the simplest assumptions to focus on showing the size fractionation of the particles. The mass flux of particles in all size ranges is

$$
M_{d,\text{all}} = \int_{s_{\text{min}}}^{s_{\text{max}}} \frac{dM_d}{ds} ds,
$$

where $s_{\text{min}}$ and $s_{\text{max}}$ are the minimum and maximum sizes of particles, respectively. The surface density in the steady state is calculated from equation (40) with a given $M_d$. Figure 9a shows the surface densities of dust components composed of single-size particles. Particles of different sizes have different density profiles. If the dust component is composed of relatively large particles, it would be concentrated at the inner region of the disk. Thus, as particles grow in size, their surface density distribution becomes more centrally concentrated. Figure 9b shows the surface density, assuming that the size distribution of the particles is a power law with index $-3.5$, $s_{\text{min}} = 0.1 \mu$m, and $s_{\text{max}} = 10$ cm. The surface density distribution of the dust particles is different from that of the gas. The power-law index of the dust distribution $\Sigma_d$ is about $-1.5$ for a gas disk with index $p_s = -1.0$ ($-1.2$ for a gas disk with $p_s = -0.5$). The implied power-law index, $-1.5$, is similar to the value anticipated from the present mass distribution of planets in the solar system (Hayashi et al. 1985). However, note that the density distributions in Figure 9 are derived assuming no size evolution of particles and no turbulent diffusion in the radial direction. The growth of particles during the radial flow adds a source term in the equation of continuity. The evolution of...
cause the dust component to become unstable and promote planetesimal formation through gravitational instability (Goldreich & Ward 1973; Sekiya 1998).

If the particle growth proceeds to make 1 mm–1 cm particles during the gas accretion phase, such large particles will migrate inward rapidly and accumulate in the inner part of the disk (see Fig. 9a). This size fractionation will cause an increase in the dust-gas ratio at the inner disk, while it might also decrease the dust-gas ratio at the outer disk, resulting in a dust disk concentrated in the inner part of the gas disk.

### 4.3. Summary

The radial migration of dust particles in accretion disks is studied. Our results are as follows:

1. Dust particles move radially both inward and outward by the gas drag force. Particles at high altitude \((|z| > 2h_g)\) move outward because they rotate slower than the gas whose pressure gradient force is inward. Small particles \((s \lesssim 100 \mu m \text{ at } 10 \text{ AU})\) near the midplane \((|z| \lesssim h_g)\) are advected by the gas outflow. On the other hand, particles at intermediate altitude and large particles \((s \gtrsim 1 \text{ mm at } 10 \text{ AU})\) move inward.

2. The net radial velocity, averaged in the vertical direction, is usually inward, provided that the radial gradient of the gas surface density is not too steep \((\rho_r \gtrsim -1.3)\). Sedimentation removes outflowing particles from high altitudes. Small particles, which can be advected by the outflowing gas around the midplane, do not concentrate at the midplane. In the inner part of the gas disk \((r \lesssim 100 \text{ AU for } 10 \mu m \text{ particles})\), the inflow velocity of particles is smaller than the gas accretion velocity, resulting in an increase in the dust-gas ratio.

3. The particle sedimentation would be efficient if dust-gas coupling were relatively weak \((Sc > 1.0)\). In the special case in which the sedimentation is very efficient \((Sc > 10)\), the number of outflowing particles around the midplane would be large, and the direction of the net radial velocity of particles would change to outward at some distance from the star. The accumulation of particles at such locations serves to increase the local dust-gas ratio.

4. The inflow velocity of particles depends on the particle size. Therefore, inflow causes size fractionation of the particles. Larger particles accumulate at distances closer to the star.

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