Closed trapped surfaces in higher dimensional self-similar Vaidya solution

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Abstract. Although an event horizon defines a black hole, it is difficult to explain a boundary of dynamical black holes. A trapped surface is a candidate of the boundary of dynamical black holes, and is studied in various spacetimes and settings. Usually, there is no trapped surface in a Minkowski region, however, Bengtsson and Senovilla showed an interesting result as follows: in a four-dimensional self-similar Vaidya spacetime, they considered non-spherical trapped surfaces and showed that trapped surfaces can extend into the Minkowski region, if and only if a mass function rises fast enough [1]. We extend this result in a higher-dimensional spacetime, because recently studying higher-dimensional black holes is significant in the context of large extra dimensions or TeV-scale gravity. In this paper we investigate a higher-dimensional Vaidya spacetime with a self-similar mass function. We match two kinds of \((n + 1)\)-surfaces and construct trapped surfaces extended into the Minkowski region by using Bengtsson and Senovilla’s way. Moreover, we demonstrate that there is no naked singularity, if the spacetime has trapped surfaces as above. These results might become a foothold to define the boundary of dynamical black holes in higher-dimensional spacetimes.

1. Introduction

A boundary of a region in a spacetime that cannot be observed from infinity is called event horizon. The event horizon defines a boundary between an inside and an outside of a black hole. Moreover, this horizon has a teleological property: an entire future history of the spacetime must be known before a position of the event horizon can be determined. Since the event horizon is defined at future timelike infinity, a shape of the event horizon does not change. However, some quantum effect might deform the boundary of dynamical black holes. It might be impossible to explain the boundary of dynamical black holes with the event horizon.

Eardley conjectured that the boundary of the region which contains marginally outer trapped surfaces coincide with the event horizon [2]. A surface called outer trapped surface is the closed spacelike two-surface (in four-dimensional case) and whose outer null expansion is negative. This conjecture is very interesting, because we can translate the event horizon with outer trapped surface, i.e., we can define the event horizon constructively. In a four-dimensional Vaidya spacetime, Ben-Dov showed that Eardley’s conjecture is true [3]. However, the outer trapped surface cannot be considered in the general spacetime, because it is defined only in asymptotically flat spacetimes [4]. Moreover, the outer trapped surface only consider an outer null ray. There remains a possibility of which we can observe an inner null ray. Since the black hole is an invisible region, to define this region we should consider a notion of which both null rays cannot be observed.

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A surfaces called trapped surface defines the boundary of an invisible region. The trapped surface is a closed spacelike two-surface (in four-dimensional case) and whose both null expansions are negative. Since the black hole cannot be observed even from infinity, its boundary might be defined with the trapped surface. Usually, there is no trapped surface in a Minkowski spacetime. However, in the four-dimensional Vaidya spacetime it was showed that trapped surfaces can extend into the Minkowski region. Numerical results of Schnetter and Krishnan showed that an outer boundary of trapped surfaces can extend into the Minkowski region [5]. Moreover, Bengtsson and Senovilla considered the self-similar Vaidya spacetime, and they analytically showed that trapped surfaces can extend into the Minkowski region, if and only if a mass function rises fast enough [1].

Recently, higher-dimensional scenarios with large [6] and warped [7] extra dimensions were proposed to resolve the hierarchy between the gravitational and electroweak interactions. One of the most striking predictions of such scenarios is productions of a large number of miniblack holes in high-energy particle collisions [8]. Therefore, studying higher-dimensional black holes is significant in the context of above scenarios.

In this paper, we extend Bengtsson and Senovilla’s way into a higher-dimensional Vaidya spacetime with a self-similar mass function. We match two kinds of \((n+1)\)-surfaces (where \(n = D - 3\)) and show that trapped surfaces can extend into the Minkowski region. Moreover, we demonstrate that there is no naked singularity, if the spacetime has trapped surfaces as above.

2. \(D\) dimensional Vaidya spacetime

We consider a \(D\)-dimensional Vaidya solution [9]

\[
ds^2 = - \left( 1 - \frac{2m}{nr^n} \right) dv^2 + 2dvdr + r^2 d\Omega_{n+1}^2 \quad (1)
\]

where \(n = D - 3\), \(D \geq 4\), and \(d\Omega_{n+2}^2\) is a line element of of a unit \((n+2)\)-sphere. We choose a mass function \(m(v)\) such as

\[
m = \begin{cases} 
0 & , \quad 0 \leq v \leq 0 \\
\mu v^n & , \quad 0 \leq v \leq M^{1/n}/\mu \\
M & , \quad v \geq M^{1/n}/\mu 
\end{cases} \quad (2)
\]

where \(\mu\) and \(M\) are constants, respectively. There is a radial influx of null fluid in an initially empty region of a \(D\)-dimensional Minkowski region. The region in \(0 \leq v \leq M^{1/n}/\mu\) is a \(D\)-dimensional Vaidya region with the self-similar mass function, and the region in \(v \geq M^{1/n}/\mu\) is a \(D\)-dimensional Schwarzschild region. We call a boundary between the Minkowski region and the Vaidya region \(MV\)-boundary, and also call the boundary between the Vaidya region and the Schwarzschild region \(VS\)-boundary. We know that a naked singularity will occur if and only if the mass function satisfies the following condition [10]:

\[
0 < \mu < \left[ \frac{n}{2(n+1)} \right]^{n+1}. \quad (3)
\]

2.1. Two types of trapped surfaces

In order to construct trapped surfaces extended into the Minkowski region, we consider two kinds of \((n+1)\)-surfaces as in Table. 1. We call the surface in which \(r\) and \(v\) are the function of \(\rho\), and in which \(\theta_i = \pi/2\) constant inclination surface (CIS). Similarly, we call the surface in which \(\theta_i\) and \(v\) are the function of \(\rho\), and in which \(r = r_0\) constant radius surface (CRS).
Table 1. The type of \((n+1)\)-surfaces (where \(i, j = 1, \ldots, n\))

| Type | \(v\)  | \(r\)  | \(\theta_i\) | \(\theta_{n+1}\) | \(\theta_{j \neq i}\) |
|------|--------|--------|---------------|-----------------|------------------|
| CIS\(_i\) | \(V(\rho)\) | \(R(\rho)\) | \(\pi/2\) | \(0 \leq \theta_{n+1} \leq 2\pi\) | \(0 \leq \theta_j \leq \pi\) |
| CRS\(_i\) | \(V(\rho)\) | \(r_0\) | \(\Theta_i(\rho)\) | \(0 \leq \theta_{n+1} \leq 2\pi\) | \(0 \leq \theta_j \leq \pi\) |

2.2. Closed trapped surfaces

We consider the following types of \((n+1)\)-surfaces in each region, respectively:

- The Minkowski region: We consider CISs which are a topological disk given by the hyperboloid \(v = t_0 + r - \sqrt{r^2 + k^2}\), where \(t_0\) and \(k\) are constants. We have chosen \(t = v - r\) and \(R = \rho\). These surfaces have negative both null expansions automatically.

- The Vaidya region: We consider CISs in which \(V\) and \(R\) satisfy the relation \(dV/dR = a = (b - X)\), where \(X = V/R\), \(a\) and \(b\) are constants, and we have chosen \(R = \rho\). If \(a\) satisfies \(a > b^2/4\) and \(a > (n/\mu)^{1/n}\), both null expansions of these surfaces are negative.

- The Schwarzschild region: We consider CRSs which is a capping disk defined by \(\Theta_i^2 + (\frac{V(n/(\gamma M))^{1/n} - \sigma_0}{\gamma})^2 = \pi^2/4\), where \(\gamma = nR^n/M\) and \(\sigma_0\) is a constant. Both null expansions of this surface are negative, if \(\gamma\) satisfies the following inequalities which are different in each CRSs: In the case of \(i = 1\) the inequality is given by

\[
\frac{2}{\sqrt{\gamma}} n - 1 \left(\frac{1}{\gamma} - 1\right) > \frac{2}{\pi}.
\]

We solve numerically this inequality with respect to \(\gamma\), and plot an upper bound of \(\gamma\) in Figure 1. The upper bound of \(\gamma\) approaches one for infinitely large dimension. Therefore,

Figure 1. The upper bound of \(\gamma\), where ”+” is the upper bound of \(\gamma\) for each dimensions. We have connected each ”+” by a dashed line. The dashed line approaches one for infinitely large dimension.

\(\gamma\) is less than one in any dimensions. On the other hand, in the case of \(i \neq 1\) the inequalities
are given by
\[ n \sqrt{\frac{2}{\gamma} - 1} \left( \frac{1}{\gamma} - 1 \right) > \frac{2}{\pi} \prod_{k=1}^{l-1} \sin \theta_k, \]  
(5)

where \( l = a, \cdots, n \). Since \( \theta_k \) exists in the range of \( 0 \leq \theta_k \leq \pi \), the right hand side in Eq. (5) exists in the range of zero to \( 2/\pi \). In this case \( \gamma \) is also less than one in any dimensions.

In both cases both null expansions are negative, if \( \gamma \) is less than one.

Matching these surfaces we construct trapped surfaces extended into the Minkowski region. To do this we impose the following matching conditions into parameters: On the MV-boundary if the mass function satisfies
\[ \mu > \frac{1}{4 \left( \frac{n}{\gamma} \right)^{1/n}}, \]  
(6)
surfaces are matched. On the other hand, On the VS-boundary if \( \gamma \) is less than one, surfaces are matched. Because conditions for getting negative both null expansions are not inconsistent to matching conditions, we are able to construct trapped surface extended into the Minkowski region.

2.3. Naked singularity

We shall consider the naked singularity in this spacetime. Combining Eqs. (6) and (3) we get the condition with respect to \( \gamma \) such as \( \gamma > n \left[ (2(n+1)/n)^{n+1} / 4 \right]^{1/n} \). If \( \gamma \) satisfies this inequality, the spacetime has the naked singularity. However, this inequality is inconsistent to \( \gamma < 1 \). Therefore, if trapped surfaces given by above discussions exist in the spacetime, there is no naked singularity.

3. Conclusion

We have considered the higher-dimensional self-similar Vaidya spacetime. By using Bengtsson and Senovilla’s way we have constructed trapped surfaces extended into the Minkowski region. Moreover, we have shown that there is no naked singularity, if the spacetime has trapped surfaces constructed above. Although in the higher-dimensional case there have been many CISs and CRSs compared to four-dimensional case, a feature of trapped surfaces constructed in this study and the possibility of the naked singularity have been same to four-dimensional case.

The event horizon is the surface which satisfies \( \gamma = 2 \), on the other hand, trapped surfaces constructed in this study satisfies \( \gamma < 1 \). Therefore, trapped surfaces are far from the event horizon. In higher-dimensional spacetime it is possible to consider many kinds of \((n+1)\)-surfaces without CISs and CRSs. By using these \((n+1)\)-surfaces, we could construct trapped surfaces which exist near the event horizon. Moreover, similar to this study, if we extend the Ben-Dov’s way into the higher-dimensional Vaidya spacetime, we may show that Eardley’s conjecture is true in this spacetime.

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