Acceleration and Deceleration Motion Control and Linear Interpolation based on Five-axis Linkage NC Machine

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Abstract. It is important to research on cradle-type five-axis vertical machining center. Based on the theory of five-axis linkage motion control algorithm, this paper studied the movement model of five-axis linkage control process, summarized the deceleration speed control strategy of area and deceleration control, five-axis linkage interpolation algorithm and so on. On the basis of these algorithms of five-axis linkage motion control, the mathematical model of the five-axis linkage is established. A typical impeller was processed in the five-axis linkage CNC system, and ultimately successful application was realized in the cooperative enterprise products.

1. Introduction

Five-axis vertical machining center is a high technology, high precision special machining center for machining complex curved surface, this kind of machining center system is important to the industries of a country, such as aviation, aerospace, military, scientific research, precision instruments, high precision medical equipment and so on[1]. At present, the five-axis CNC machining center system is the only means to solve the processing of impeller, blade, Marine propeller, heavy-duty generator rotor, turbine rotor, large diesel engine crankshaft and so on.

Equipment manufacturing industry is the cornerstone of a country's industry. It is an indispensable strategic industry for the development of new technologies, new products and modern industrial production. Even the developed industrialized countries attach great importance to it. With the rapid development of China's national economy and the need of national defense construction, there is an urgent need for high-grade CNC machining center. Machining center are a symbol of the level of a country's manufacturing industry. In a sense, it reflects the level of industrial development in a country [2-4].
2. METHODS

2.1. Five-axis linkage geometry model and coordinate system definition [5-8]

2.1.1. The geometry model of the cradle-type five-axis vertical machining center. There are many types of five-axis CNC structure. This paper developed the cradle-type structure, which divided into two categories: 1. using B axis to swing C axis in the rotary double turntable as Type I and using an axle to swing C axis in the rotary double turntable model as Type II. Their geometric model is as shown in the figure below:

![Type I](image1)

Figure 1. Type I: five-axis linkage CNC system of using B axis to swing C axis.

![Type II](image2)

Figure 2. Type II: five-axis linkage CNC system of using an axis to swing C axis.

2.1.2. Definition of Coordinate System. As shown in Fig.3, there are several coordinates defined as follows:

1) Absolute coordinate: the original frame whose origin is absolute zero of CNC machine;
2) Machine coordinate, the origin of programming coordinate.
3) Nominal coordinate: the coordinate system setting when process. The origin point generally is taken in the intersection point of the axis of A and C or B and C, if the two axis is not intersected, then it would be taken on the basic shaft axis, and the axis directions are in the same direction as the absolute coordinates of CNC machine.

![3D model and simulation model](image3)

Figure 3. 3D model and simulation model.
2.2. The vector rotation transformation of the two-way turntable model.

For the model of the Type I: When the vector $P'$ is shifted along $\{ L_x\ L_y\ L_z \}$, rotated $\beta$ angle around the B axis and $\gamma$ angle around the C axis, the vector matrix transformation calculation equation is shown as follows:

\[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
L_x & L_y & L_z & 1
\end{pmatrix}
\begin{pmatrix}
\cos \beta & 0 & -\sin \beta & 0 \\
0 & 1 & 0 & 0 \\
\sin \beta & 0 & \cos \beta & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
\cos \gamma & \sin \gamma & 0 & 0 \\
-\sin \gamma & \cos \gamma & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

(1)

For the model of the Type II: When the vector $P'$ is shifted along $\{ L_x\ L_y\ L_z \}$, rotated $\alpha$ angle around the A axis and $\gamma$ angle around the C axis, the vector matrix transformation calculation equation is shown as follows:

\[
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
L_x & L_y & L_z & 1
\end{pmatrix}
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \alpha & \sin \alpha & 0 \\
0 & -\sin \alpha & \cos \alpha & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
\cos \gamma & \sin \gamma & 0 & 0 \\
-\sin \gamma & \cos \gamma & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

(2)

The above composite transformation matrix is irreversible.

2.3. Quintic polynomial S curve acceleration and deceleration motion control algorithm [9-13]

![Figure 4. Quintic polynomial S curve acceleration and deceleration motion.](image)

The acceleration curve of the quantic polynomial S curve acceleration and deceleration motion control algorithm is shown in Fig.4. It is devieded into 3 stages, namely acceleration stage $t \in [T_0, T_1]$, constant speed stage $t \in (T_1, T_2]$ and deceleration stage $t \in (T_2, T_3]$. The acceleration of acceleration calculation formula is as follows:
16

\begin{equation}
\begin{cases}
    j(t) = -A_{mp} \frac{64}{T_1^3} \left( t - \frac{T_2}{2} \right)^3 & t \in (T_0, T_1) \\
    j(t) = 0 & t \in (T_1, T_2) \\
    j(t) = A_{mm} \frac{64}{(T_2 - T_1)^3} \left( t - \frac{T_3 + T_2}{2} \right)^3 & t \in (T_2, T_3)
\end{cases}
\end{equation}

Where $A_{mp}$ is the max acceleration, $A_{mm}$ is the max deceleration; $V_S$ is the initial velocity; $V_T$ is the terminal velocity; $V_T$ is the constant velocity.

The acceleration calculation formula is as follows:

\begin{equation}
\begin{cases}
    a(t) = -A_{mp} \frac{16}{T_1^2} \left( t - \frac{T_2}{2} \right)^2 + A_{mp} & t \in [T_0, T_1] \\
    a(t) = 0 & t \in (T_1, T_2) \\
    a(t) = A_{mm} \frac{16}{(T_2 - T_1)^2} \left( t - \frac{T_3 + T_2}{2} \right)^2 - A_{mm} & t \in (T_2, T_3)
\end{cases}
\end{equation}

Where when $t = \frac{T_1}{2}, \frac{T_1 + T_2}{2}$, there are the max acceleration $A_{mp}$ and $A_{mm}$; when $t = 0, T_1, T_2, T_3$, the acceleration is 0.

The speed calculation formula is as follows:

\begin{equation}
\begin{cases}
    v(t) = -A_{mp} \frac{16}{5T_1^2} \left( t - \frac{T_2}{2} \right)^2 + A_{mp} t - A_{mp} \frac{T_2^3}{10} + V_S & t \in [T_0, T_1] \\
    v(t) = V_M & t \in (T_1, T_2) \\
    v(t) = A_{mm} \frac{16}{5(T_2 - T_1)^2} \left( t - \frac{T_3 + T_2}{2} \right)^2 - A_{mm} (t-T_2) + A_{mm} \frac{T_2 - T_3}{10} + V_M & t \in (T_2, T_3)
\end{cases}
\end{equation}

Where when $t=0$, the initial velocity is $V_S$; when $t=T_1$, the velocity is $V_M$; when $t=T_2$, the velocity is $V_M$; when $t=T_3$, the velocity is $V_T$.

The displacement calculation formula is as follows:

\begin{equation}
\begin{cases}
    s(t) = -A_{mp} \frac{8}{15T_1^3} \left( t - \frac{T_1}{2} \right)^3 + A_{mp} \frac{t^2}{2} - A_{mp} \frac{T_2 t}{10} + A_{mp} \frac{T_1^2}{120} + V_S t & t \in [T_0, T_1] \\
    s(t) = V_M \cdot (t - T_1) + S_1 & t \in (T_1, T_2) \\
    s(t) = A_{mm} \frac{8}{15(T_2 - T_1)^3} \left( t - \frac{T_3 + T_2}{2} \right)^3 - A_{mm} \frac{(t - T_2)^3}{2} + A_{mm} \frac{T_2 - T_3}{20} (t-T_2) & t \in (T_2, T_3) \\
    & - A_{mm} \frac{(T_2 - T_1)^2}{120} + V_M (t-T_3) + S_2 & t \in (T_2, T_3)
\end{cases}
\end{equation}

Where when $t=0$ the initial displacement is 0; when $t=T_1$, the displacement is $S_1$; when $t=T_2$, the displacement is $S_2$; when $t=T_3$, the displacement is $S_3$. Taking $t = T_1$ into the equation (5), it will get the following equation (7):
Simplifying the equation (7), it could get acceleration time $T_1$ from $V_S$ to $V_M$ when the max acceleration is $A_{mp}$, shown as equation (8).

$$T_1 = \frac{5(V_M - V_S)}{4A_{mp}}$$

(8)

By the same token, taking $t = T_3$ into equation (5), it could get the following equation (9).

$$V_E = \frac{16A_{mm}}{5(T_2 - T_3)^5} (\frac{T_3 - T_2}{2})^5 - A_{mm}(T_3 - T_2) - A_{mm} \frac{1}{10} (T_2 - T_3) + V_M$$

(9)

Simplifying the above equation, it could get the deceleration time $T_3 - T_2$ from $V_E$ to $V_M$, when the max deceleration is $A_{mm}$, as shown in equation (10)

$$T_3 - T_2 = \frac{5(V_M - V_E)}{4A_{mm}}$$

(10)

Taking $t = T_1$ into equation (6), it could get the displacement in the acceleration time.

$$S_1 = \frac{5}{8}, \frac{(V_M^2 - V_S^2)}{A_{mp}}$$

(11)

By the same token, it could get the displacement in the deceleration time.

$$S_3 = \frac{5}{8}, \frac{(V_M^2 - V_E^2)}{A_{mm}}$$

(12)

Setting the total displacement $S_0$, it could get the displacement in the constant speed time.

$$S_2 - S_1 = S_0 - S_1 - S_3 = S_0 - \frac{5}{8} \frac{(V_M^2 - V_S^2)}{A_{mp}} - \frac{5}{8} \frac{(V_M^2 - V_E^2)}{A_{mm}}$$

(13)

The constant speed time $T_2 - T_1$ is as follows:

$$T_2 - T_1 = \frac{S_2 - S_1}{V_M} = \frac{S_0}{V_m} - \frac{5}{8V_M} \frac{(V_M^2 - V_S^2)}{A_{mp}} - \frac{5}{8V_M} \frac{(V_M^2 - V_E^2)}{A_{mm}}$$

(14)
With the quantic S-shape curve deceleration motion control algorithm, the displacement, velocity, acceleration, and acceleration of acceleration are flexible. It not only can effectively decrease the vibration when the machine speeding up or down, or frequent switching back and forth, but also improve the positioning accuracy, improve the running speed, shorten the operation time, and finish the processing fast and efficiently.

3. Multi-axis linear interpolation method [14]

A linear interpolation control algorithm for multi-axis linkage data is set up in pulse control mode: Firstly, according to the trajectory description information (NC code data), using the data sampling interpolation processing method to process the multidimensional space curve interpolation by coordinate projection, it could get the digital incremental instructions \( \Delta x, \Delta y, \Delta z, \Delta a, \Delta c \) (or \( \Delta b, \Delta c \)) of all coordinate axis. Secondly, according to the interpolation cycle, the digital incremental instructions such as \( \Delta x \) are divided into the equal sections with the linear proportion, and then to calculate the increment value of each interpolation cycle coordinate. Thirdly, to calculate the digital-pulse conversion and line interpolation calculation.

Setting the interpolation cycle \( T_0 \), and after the acceleration and deceleration process, it could get the instantaneous feed speed \( F_i \) (mm/min), the current displacement \( \Delta L_i \) (mm) of the current interpolation cycle could be calculated:

\[
\Delta L_i = \frac{F_i T_0}{60000}
\]  

The geometrical length \( L \) of the interpolation line is calculated in Cartesian coordinates.

\[
L = \sqrt{\Delta x^2 + \Delta y^2 + \Delta z^2 + L'}
\]  

Where \( L' \) is the interpolation length correction which is determined by the influence of the rotation coordinates on linear motion.

The feed proportionality coefficients could be calculated as follows:

\[
\begin{align*}
K_x &= \Delta x / L \\
K_y &= \Delta y / L \\
K_z &= \Delta z / L \\
K_a &= \Delta a / L \\
K_b &= \Delta b / L \\
K_c &= \Delta c / L
\end{align*}
\]  

Finally, according to feed proportionality coefficients, the values of each coordinate component of the current interpolation points in the five-dimension space are:
For multi-coordinate NC machining, such as five axes, it usually uses the G93 instruction. In the numerical control program, the feed rate \( F \) is given, which means the time of a program segment completion is equal to \( 1/F \).

We assumed that because every period of NC program takes short displacement and the program segments are large, it would take the fixed feed rate \( F \) during interpolation, namely the time required for a program segment completion of a NC program is equal to \( 1/F \) which is constant. It would only adjust the feed rate \( F \) when it changes greatly.

Based on this assumption, a five-axis NC interpolation algorithm is developed:

The interpolation period is \( T_0 \)(ms), the feed rate is \( F \), and the completion time of a segment program is \( t = 1/F \) (min).

The number \( m \) of cycles required to complete a certain NC program segment is:

\[
m = \frac{60000}{T_0} = \frac{60000}{FT_0}
\]

Supposed that the increment of each axis of the NC program is \( \Delta X \), \( \Delta Y \), \( \Delta Z \), \( \Delta A \), \( \Delta B \), \( \Delta C \) respectively, and \( K = \frac{1}{m} = \frac{FT_0}{60000} \).

For constant speed interpolation operations in every interpolation cycles, the coordinate values of each coordinate in the \( j \)-th interpolation cycle could be calculated by the following formula:

\[
\begin{align*}
\Delta x_j &= jK\Delta X \\
\Delta y_j &= jK\Delta Y \\
\Delta z_j &= jK\Delta Z \\
\Delta a_j &= jK\Delta A \\
\Delta b_j &= jK\Delta B \\
\Delta c_j &= jK\Delta C
\end{align*}
\quad j \in N \{1 \sim m\}
\]

\[
\begin{align*}
\delta x &= L_1 K_x \\
\delta y &= L_1 K_y \\
\delta z &= L_1 K_z
\end{align*}
\]

\[
\begin{align*}
\delta b &= L_1 K_b \\
\delta c &= L_1 K_c
\end{align*}
\]

\[
\begin{align*}
\delta a &= L_1 K_a \\
\delta c &= L_1 K_c
\end{align*}
\]
4. Experiment and Results
To test quintic S-shape curve deceleration motion control algorithm and multi-axis linkage data sampling control algorithm, the algorithms were transplanted into K8000-2 CNC system as shown in Fig. 6(a) which was developed by the cooperation company, and the typical impeller was processed to verify the algorithm as shown in Fig. 6 (b)

![Diagram of interpolation cycle](image)

**Figure 5.** Interpolation Cycle of a certain NC program segment.

**Figure 6.** The processing of a typical impeller.
### Table 1. Comparison of the Processing demand of old and new algorithm

| F (mm/s) Setting speed | Vm (mm/s) Max speed | a (mm/s.s) setting acceleration | Old algorithm Processing Demand of Mold | Processing Demand of Products | New algorithm (this paper) Processing Demand of Mold | Processing Demand of Products |
|------------------------|---------------------|--------------------------------|----------------------------------------|-----------------------------|----------------------------------------|-----------------------------|
|                        |                     |                                | Processing Demand of Mold             | Processing Demand of Products | Processing Demand of Mold             | Processing Demand of Products |
| 100                    | 100                 | 2000                           | Satisfied                              | Satisfied                   | Satisfied                              | Satisfied                   |
| 200                    | 200                 | 2000                           | basically satisfied                    | Satisfied                   | Satisfied                              | Satisfied                   |
| 300                    | 300                 | 4000                           | Unsatisfied                           | basically satisfied         | Satisfied                              | basically satisfied         |
| 400                    | 400                 | 5000                           | Unsatisfied                           | basically satisfied         | basically satisfied                    | basically satisfied         |
| 500                    | 500                 | 5000                           | Unsatisfied                           | Unsatisfied                 | basically satisfied                    | basically satisfied         |
| 600                    | 600                 | 6000                           | Unsatisfied                           | Unsatisfied                 | basically satisfied                    | basically satisfied         |
| 700                    | 700                 | 8000                           | Unsatisfied                           | Unsatisfied                 | Unsatisfied                           | Unsatisfied                 |

From the table 1, we could see that:

For mold processing: F=100mm/s is the appropriate maximum speed; F=200mm/s is the highest feed rate when it meets basic satisfaction; When the new motion control algorithm is adopted, F=200mm/s is the appropriate maximum speed; While F=300mm/s is the highest feed rate when it meets basic satisfaction;

For product processing: F=200mm/s is the appropriate maximum speed; F=400mm/s is the highest feed rate when it meets basic satisfaction; When the new motion control algorithm is adopted, F=300mm/s is the appropriate maximum speed; F=600mm/s is the highest feed rate when it meets basic satisfaction;

We could conclude that: the operation stability and reliability of the five-degree polynomial acceleration and deceleration motion control algorithm can improve the machining efficiency and improve the working efficiency.

### 5. Conclusion

Motion control algorithm is the core of the five-axis vertical machining center control system, it could determine the CNC machine performances and its precision.

This paper proposed a quintic curve S acceleration and deceleration motion control algorithm, and its displacement, velocity, acceleration, and acceleration is flexible changes. At the same time by using linear interpolation control algorithm of the multi-axis linkage data sampling interpolation, the calculations of displacement, speed, acceleration and acceleration of acceleration were completed. Finally by using five-axis vertical machining center control system, this paper verified and optimized the algorithm, and improved the quality and efficiency of NC machining.

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