Research on Coordinate Conversion and Error Analysis of BLSS-PE

Lwen Yu1,2,*, Kai Zhan1,2 and Da Zhang1
1BGRIMM Technology Group, Beijing, China
2School of Mechanical Engineering, University of Science and Technology Beijing, Beijing, China

*Corresponding author email: yulewen@bgrimm.com

Abstract. In this paper, a 3D laser scanner for mining named BLSS-PE is proposed, based on a commercial rangefinder with two servo motors. The coordinate transformation model in different coordinate systems is put forward and the model solving strategy is given. By establishing the error model of BLSS-PE and comparing the measurement data, the results show that the accuracy of BLSS-PE data can be improved after the external factors are included in the system error model, which verifies the feasibility and correctness of the optimization model.

1. Introduction
The three dimensional laser measurement technology breaks through the traditional single point measurement method, and tens of thousands of high-precision point cloud data per second can be scanned [1-2]. With the wide application of this technology in scientific research, industrial production and other fields, engineering projects also put forward higher requirements for its measurement accuracy [3-6]. The system parameters of BGRIMM Laser scanning system-Portable and Embedded(BLSS-PE) are measured and given by the manufacturer under ideal conditions. Influenced by the uncertain factors of the instrument structure and production process, the actual indicators of the instrument are inconsistent with the nominal values, which brings errors to the measurement. This paper will introduce a 3D laser scanner for mining named BLSS-PE which is produced by Beijing General Research Institute of Mining and Metallurgy (BGRIMM) Group. The error sources of BLSS-PE are studied in this paper. The influences of laser ranging model, radial motor, axial motor, dip sensor and relative coordinate system on BLSS-PE are analyzed. The formula is deduced in detail, and the correctness and reliability of the optimization model are verified by experiments.

2. The Coordinates of BLSS-PE
BLSS-PE includes scanner, boom, controller, calibration system and cable. Also, there is an industrial laptop computer installed with the relative software. The mechatronic design comprises the mechanical system, the motion controller, and the ARM. These components and their relationships, which are described in the following sections, are described in Figure 1.
As shown in Figure 2, point P is the target point and point Z is the zero point of the ranging module. Point O₁ is the intersection of the central axis of the radial motor and the plane of the ranging module, point O₂ is the intersection of the central axis of the axial motor and the central axis of the radial motor, point O₃ is the intersection of the end face where the target laser logo is located and the central axis of the axial motor, and point O₄ is the central point of the target laser. The coordinate system 1-4 are successively established based on the right-hand spiral rule. L₁, L₂, L₃, L₄ and L₅ are all known fixed distances, \(d\) is the measured value of ranging module, \(\alpha\) is the rotation angle of radial motor, \(\beta\) is the rotation angle of axial motor. The coordinate solution process of BLSS-PE can be transformed into forward kinematics solution of two DOF series mechanism. It is known that the measurement distance of point P is \(d\), and the angles of two encoders are \(\alpha\) and \(\beta\). In coordinate system 1, the coordinates of point P are as follows.

\[
P^1 = \begin{bmatrix} d + L_1 & L_2 & 0 & 1 \end{bmatrix}^T
\]  

(1)

The coordinate system 1 is a rotation angle \(\alpha\) about the \(Z_2\) axis with respect to the coordinate system 2, and is translated \(-L_3\) along the \(Z_2\) axis. In coordinate system 2, the coordinate of point P is as follows.

\[
P^2 = \begin{bmatrix} (d + L_1) \cos \alpha - L_2 \sin \alpha \\ (d + L_1) \sin \alpha + L_2 \cos \alpha \\ -L_3 \\ 1 \end{bmatrix}
\]

(2)

The coordinate system 2 is a rotation angle \(\alpha\) about the \(X_3\) axis with respect to the coordinate system 3, and is translated \(-L_4\) along the \(X_3\) axis. In coordinate system 3, the coordinate of point P is as follows.

\[
P^3 = \begin{bmatrix} (d + L_1) \cos \alpha - L_2 \sin \alpha - L_4 \\ (d + L_1) \sin \alpha \cos \beta + L_2 \cos \alpha \cos \beta + L_3 \sin \beta \\ (d + L_1) \sin \alpha \sin \beta + L_2 \cos \alpha \sin \beta - L_3 \cos \beta \\ 1 \end{bmatrix}
\]

(3)

The coordinate system 3 is a rotation angle 180° about the \(Z_4\) axis with respect to the coordinate system 4, and is translated \(-L_5\) along the \(Z_4\) axis. In coordinate system 4, the coordinate of point P is as follows.
3. Coordinate System Transformation

3.1. Establishment of Horizontal Coordinate System

The initial position of the BLSS-PE may not be horizontal. The angle between the X-axis, Y-axis and the horizontal plane is measured by the angle sensor. In coordinate system 4, the angle between axis X4 and the horizontal plane is $\theta$, and the angle between axis Y4 and the horizontal plane is $\rho$. Establishment of the horizontal coordinate system, X6 axis and Y6 axis are in the horizontal plane, X6-Z6 plane includes X4 axis, as shown in Figure 3. The auxiliary coordinate system 5 is established. The coordinate system 4 rotates around the X5 axis $\omega$ to get the coordinate system 5. The coordinate system 5 rotates around the Y6 axis $\theta$ to get the coordinate system 6. The rotation matrix needs to meet the right-hand rule, where $\angle AOC = \theta$, $\angle BOD = \rho$, $\angle BOE = \omega$. In coordinate system 5, the coordinate of point P is as follows.

$$
P^4 = \begin{bmatrix}
-(d + L_4) \cos \alpha + L_2 \sin \alpha + L_4 \\
-(d + L_4) \sin \alpha \cos \beta - L_2 \cos \alpha \cos \beta - L_3 \sin \beta \\
(d + L_4) \sin \alpha \sin \beta + L_2 \cos \alpha \sin \beta - L_3 \cos \beta - L_5 \\
1
\end{bmatrix}
$$

(4)

The coordinate system 5 is a rotation angle $\theta$ about the Y5 axis with respect to the coordinate system 6. In coordinate system 6, the coordinate of point P is as follows.

$$
P^5 = \begin{bmatrix}
P_x^4 \\
P_y^4 \cos \omega - P_z^4 \sin \omega \\
P_y^4 \sin \omega + P_z^4 \cos \omega \\
1
\end{bmatrix}
$$

(5)

3.2. Establishment of Relative Coordinate System

Coordinate system 8 is relative coordinate system, coordinate system 6 is horizontal coordinate system, and auxiliary coordinate system 7 is established, as shown in Figure 4. The H point is the target of the extension rod, and the vector $\overrightarrow{OH}$ is the light direction of the target laser. In Coordinate system 8, the coordinate of H point is $(H_x, H_y, H_z)$, the coordinate of O point is $(O_x, O_y, O_z)$. The coordinate system 6 is a rotation angle $\gamma$ about the Z8 axis with respect to the coordinate system 8. In coordinate system 8,
the coordinate of point P is as follows.

\[
P^8 = \begin{bmatrix}
P^x \cos \gamma - P^y \sin \gamma + O_x \\
P^x \sin \gamma + P^y \cos \gamma + O_y \\
P^x + O_z \\
1
\end{bmatrix}
\] (7)

4. Error Analysis

4.1. Error Analysis of Laser Ranging Module
The ranging accuracy of the laser ranging module is ±20mm, the sampling frequency is 200Hz, the resolution is 1mm, and the error of the laser ranging module is \(\Delta d\), then in the coordinate system 4, the coordinate of any point P in the space is expressed as follows.

\[
\begin{bmatrix}
P^x' \\
P_y' \\
P^z' \\
1
\end{bmatrix} = \begin{bmatrix}
-(d + \Delta d + L_1) \cos \alpha + L_2 \sin \alpha + L_4 \\
-(d + \Delta d + L_1) \sin \alpha \cos \beta - L_2 \cos \alpha \cos \beta - L_3 \sin \beta \\
(d + \Delta d + L_1) \sin \alpha \sin \beta + L_2 \cos \alpha \sin \beta - L_3 \cos \beta - L_5 \\
1
\end{bmatrix}
\] (8)

Therefore, the ranging error of the laser ranging module \(\Delta d\) is expressed as follows.

\[
\begin{bmatrix}
\Delta P^x \\
\Delta P_y \\
\Delta P^z \\
1
\end{bmatrix} = \begin{bmatrix}
-\Delta d \cos \alpha \\
-\Delta d \sin \alpha \cos \beta \\
\Delta d \sin \alpha \sin \beta \\
1
\end{bmatrix}
\] (9)

4.2. Error Analysis of Axial Motor
The axial motor error includes encoder error and electromagnetic zero return switch error, which affects the axial rotation angle \(\beta\). Assuming that the axial rotation angle error is \(\Delta \beta\), the coordinates of any point P in coordinate system 4 are expressed as follows.

\[
\begin{bmatrix}
P^x' \\
P_y' \\
P^z' \\
1
\end{bmatrix} = \begin{bmatrix}
-(d + L_1) \cos \alpha + L_2 \sin \alpha + L_4 \\
-(d + L_1) \sin \alpha \cos(\beta + \Delta \beta) - L_2 \cos \alpha \cos(\beta + \Delta \beta) - L_3 \sin(\beta + \Delta \beta) \\
(d + L_1) \sin \alpha \sin(\beta + \Delta \beta) + L_2 \cos \alpha \sin(\beta + \Delta \beta) - L_3 \cos(\beta + \Delta \beta) - L_5 \\
1
\end{bmatrix}
\] (10)

The angle error of the axial motor \(\Delta \beta\) is expressed as follows.

\[
\begin{bmatrix}
\Delta P^x \\
\Delta P_y \\
\Delta P^z \\
1
\end{bmatrix} = \begin{bmatrix}
0 \\
-(d + L_1) \sin \alpha - L_2 \cos \alpha \sin(\beta + \Delta \beta) - L_3 \sin(\beta + \Delta \beta) - \sin \beta \\
(d + L_1) \sin \alpha + L_2 \cos \alpha \sin(\beta + \Delta \beta) - \sin \beta - L_3 \cos(\beta + \Delta \beta) - \cos \beta \\
1
\end{bmatrix}
\] (11)

4.3. Error Analysis of Radial Motor
The radial motor error includes encoder error and electromagnetic zero return switch error, which affects
the radial rotation angle $\alpha$. Assuming that the axial rotation angle error is $\Delta \alpha$, the coordinates of any point P in coordinate system 4 are expressed as follows.

$$
\begin{bmatrix}
    P_x^4 \\
    P_y^4 \\
    P_z^4 \\
    1
\end{bmatrix} =
\begin{bmatrix}
    -(d + L_r) \cos(\alpha + \Delta \alpha) + L_z \sin(\alpha + \Delta \alpha) + L_4 \\
    -(d + L_r) \sin(\alpha + \Delta \alpha) \cos \beta - L_z \cos(\alpha + \Delta \alpha) \cos \beta - L_5 \sin \beta \\
    (d + L_r) \sin(\alpha + \Delta \alpha) \sin \beta + L_z \cos(\alpha + \Delta \alpha) \sin \beta - L_5 \cos \beta - L_5 \\
    1
\end{bmatrix}
$$

(12)

The angle error of the radial motor $\Delta \alpha$ is expressed as follows.

$$
\begin{bmatrix}
    \Delta P_x^4 \\
    \Delta P_y^4 \\
    \Delta P_z^4 \\
    1
\end{bmatrix} =
\begin{bmatrix}
    -(d + L_r)[\cos(\alpha + \Delta \alpha) - \cos \alpha] + L_z[\sin(\alpha + \Delta \alpha) - \sin \alpha] \\
    \{-(d + L_r)[\sin(\alpha + \Delta \alpha) - \sin \alpha] - L_z[\cos(\alpha + \Delta \alpha) - \cos \alpha]\} \cos \beta \\
    \{(d + L_r)[\sin(\alpha + \Delta \alpha) - \sin \alpha] + L_z[\cos(\alpha + \Delta \alpha) - \cos \alpha]\} \sin \beta \\
    1
\end{bmatrix}
$$

(13)

4.4. Error Analysis of Inclination Sensor

The error of inclination sensor includes inclination error and rotation error. The inclination error is the angle error between the Y axis of X axis and the horizontal plane, and the rotation error is the angle deviation between the Y axis of X axis of sensor and the Y axis of X axis of coordinate system 4. If the angle error between the Y axis of the X axis of the inclination sensor and the horizontal plane is $\Delta \theta$ and $\Delta \rho$, The axis rotation angle error can be expressed as follows.

$$
\Delta \omega = \arccos\left(\frac{\sin(\rho + \Delta \rho)}{\cos(\theta + \Delta \theta)}\right) - \arccos\left(\frac{\sin \rho}{\cos \theta}\right)
$$

(14)

The coordinate of point P is expressed as follows.

$$
\begin{bmatrix}
    P_x^6 \\
    P_y^6 \\
    P_z^6 \\
    1
\end{bmatrix} =
\begin{bmatrix}
    P_x^4 \cos(\theta + \Delta \theta) - P_y^4 \sin(\theta + \Delta \theta) \sin \omega' - P_z^4 \cos(\theta + \Delta \theta) \cos \omega' \\
    P_y^4 \cos \omega' - P_z^4 \sin \omega' \\
    P_x^4 \sin(\theta + \Delta \theta) + P_y^4 \sin \omega' \cos(\theta + \Delta \theta) + P_z^4 \cos(\theta + \Delta \theta) \\
    1
\end{bmatrix}
$$

(15)

4.5. Conversion Error of Relative Coordinate System

Mapping the point cloud data under the horizontal coordinate system to the relative coordinate system, the error includes the rotation angle error $\Delta \gamma$, and the origin coordinate error of coordinate system $\Delta O_x$, $\Delta O_y$, $\Delta O_z$. The coordinate of point P is expressed as follows.

$$
\begin{bmatrix}
    P_x^8 \\
    P_y^8 \\
    P_z^8 \\
    1
\end{bmatrix} =
\begin{bmatrix}
    P_x^6 \cos(\gamma + \Delta \gamma) - P_y^6 \sin(\gamma + \Delta \gamma) + O_x + \Delta O_x \\
    P_y^6 \cos(\gamma + \Delta \gamma) + P_z^6 \cos(\gamma + \Delta \gamma) + O_y + \Delta O_y \\
    P_z^6 + O_z + \Delta O_z \\
    1
\end{bmatrix}
$$

(16)

5. Experimental Analysis

In order to verify the reliability and correctness of the optimized error model, the system error of BLSS-PE is studied and analyzed. Ten targets are evenly distributed in the experimental site and numbered. Firstly, the target area is precisely scanned by BLSS-PE to obtain the point cloud data in this area.
According to the intensity information of the point cloud data, the target area is obtained by threshold segmentation method. Then the point cloud data in the target area is weighted to obtain the coordinates of the geometric center of the target. Then, Leica TS60 total station is used to obtain the geometric center coordinates of the target with the same name point, which is used as the absolute coordinate to check the systematic error of BLSS-PE. The ranging accuracy and angle measurement accuracy of total station is significantly higher than that of BLSS-PE. Table 1 shows the 3D coordinates obtained by BLSS-PE and total station.

Table 1. Coordinate comparison between BLSS-PE and TS60.

| Target number | BLSS-PE coordinates | Total station coordinates | Coordinate difference |
|---------------|---------------------|--------------------------|-----------------------|
|               | X/m Y/m Z/m         | X/m Y/m Z/m              | ∆X/m ∆Y/m ∆Z/m        |
| 1             | 12.3634 16.1433 13.249 | 12.3597 16.1421 13.2431 | 0.0037 0.0012 0.0059 |
| 2             | 12.3631 16.1101 13.242 | 12.3576 16.1126 13.2436 | 0.0055 -0.0025 -0.0016 |
| 3             | 11.4564 16.9504 13.5734 | 11.4543 16.9468 13.5689 | 0.0021 0.0036 0.0045 |
| 4             | 11.4231 16.944 13.5752 | 11.4191 16.9426 13.5685 | 0.004 0.0014 0.0067 |
| 5             | 11.7643 16.3616 13.2415 | 11.7713 16.3567 13.2458 | -0.007 0.0049 -0.0043 |
| 6             | 12.7622 16.3522 13.2398 | 12.7548 16.356 13.245 | 0.0074 -0.0038 -0.0052 |
| 7             | 12.1258 16.5365 13.7683 | 12.129 16.5324 13.7651 | -0.0032 0.0041 0.0032 |
| 8             | 12.9832 16.5264 13.764 | 12.981 16.5329 13.7654 | 0.0022 -0.0065 -0.0014 |
| 9             | 10.8914 16.6696 13.8451 | 10.8946 16.679 13.8543 | -0.0032 -0.0094 -0.0092 |
| 10            | 12.3783 16.6866 13.8625 | 12.38 16.6783 13.8541 | -0.0017 0.0083 0.0084 |

After inputting the error model into the BLSS-PE, measure the target points again, and compare with the target coordinate measured by TS60, as shown in Table 2.

The experimental results show that the average error in X, Y and Z directions is 0.004m, 0.0046m and 0.005m before the establishment of BLSS-PE error model and 0.0016m, 0.0021m and 0.0021m after the establishment of the error model without considering the coordinate measurement error of TS60. It can be seen that the accuracy of the point position is improved by about 53% after the establishment of the system error model, which proves that the optimized system error model can improve the accuracy of the measurement data.

Table 2. Coordinate comparison between BLSS-PE which corrected by error model and TS60.

| Target number | BLSS-PE coordinates | Total station coordinates | Coordinate difference |
|---------------|---------------------|--------------------------|-----------------------|
|               | X/m Y/m Z/m         | X/m Y/m Z/m              | ∆X/m ∆Y/m ∆Z/m        |
| 1             | 12.3609 16.1429 13.243 | 12.3597 16.1421 13.2431 | 0.0012 0.0008 0.0012 |
| 2             | 12.3599 16.1114 13.243 | 12.3576 16.1126 13.2436 | 0.0023 -0.0012 -0.0006 |
| 3             | 11.4544 16.9489 13.571 | 11.4543 16.9468 13.5689 | 0.0001 0.0021 0.0021 |
| 4             | 11.4204 16.9435 13.5717 | 11.4191 16.9426 13.5685 | 0.0013 0.0009 0.0032 |
| 5             | 11.7693 16.3596 13.244 | 11.7713 16.3567 13.2458 | -0.002 0.0029 -0.0018 |
| 6             | 12.7584 16.3547 13.2432 | 12.7548 16.356 13.245 | 0.0036 -0.0013 -0.0018 |
| 7             | 12.1278 16.5345 13.766 | 12.129 16.5324 13.7651 | -0.0012 0.0021 0.0009 |
| 8             | 12.9824 16.5295 13.765 | 12.981 16.5329 13.7654 | 0.0014 -0.0034 -0.0004 |
| 9             | 10.8925 16.6747 13.849 | 10.8946 16.679 13.8543 | -0.0021 -0.0043 -0.0053 |
| 10            | 12.3791 16.68 13.8577 | 12.38 16.6783 13.8541 | -0.0009 0.0017 0.0036 |

6. Conclusions
This paper introduces a new 3D laser scanner named BLSS-PE. This mechatronic system is based on a commercial rangefinder with two servo motors. The 3D coordinate solution method of BLSS-PE in scanner coordinate system, horizontal coordinate system and mining coordinate system is introduced in detail. The error sources of BLSS-PE are studied and the influences of laser ranging model, radial motor, axial motor, dip sensor and relative coordinate system on BLSS-PE are analyzed. The formula is deduced in detail, and the correctness and reliability of the optimization model are verified by experiments. It can be seen from the experimental results that after the system error correction, the data quality of BLSS-PE has been greatly improved, which verifies the feasibility and correctness of the system error model, and provides a reference for the development and improvement of the system error calibration of 3D laser scanner.
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