A Numerical Simulation for Darcy-Forchheimer Flow of Nanofluid by a Rotating Disk With Partial Slip Effects

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This study examines Darcy-Forchheimer 3D nanoliquid flow caused by a rotating disk with heat generation/absorption. The impacts of Brownian motion and thermophoretic are considered. Velocity, concentration, and thermal slips at the surface of the rotating disk are considered. The change from the non-linear partial differential framework to the non-linear ordinary differential framework is accomplished by utilizing appropriate variables. A shooting technique is utilized to develop a numerical solution of the resulting framework. Graphs have been sketched to examine how the concentration and temperature fields are affected by several pertinent flow parameters. Skin friction and local Sherwood and Nusselt numbers are additionally plotted and analyzed. Furthermore, the concentration and temperature fields are enhanced for larger values of the thermophoresis parameter.

Keywords: rotating disk, Darcy-Forchheimer flow, nanoparticles, heat absorption/generation, slip conditions, numerical solution

1. INTRODUCTION

Flow due to a rotating disk plays an indispensable role in numerous modern items encompassing rotating machinery, apparatuses, rotors, and flywheels. As of late, rotating disks have become a significant component of many pieces of machinery, for example, thermal power-creation frameworks, rotor-stator turning circle reactors, electrical controls, stopping mechanisms, pivoting sawing machines, and rotational air cleaning systems. Close investigations of laminar boundary layer flow were carried out by Von Karman [1]. Turkyilmazoglu and Senel [2] examined the linked features of heat and mass exchange arising from the revolution of a hard and permeable disk. Entropy generation in slip flow by the turning of a permeable disk with MHD and variable properties was clarified by Rashidi et al. [3]. Nanofluid flow because of the revolution of a disk was explored by Turkyilmazoglu [4]. Hatami et al. [5] investigated the impacts of the contraction, turning, and heat of disks on the movement of nanofluids. They utilized a least-square strategy for solution development. Mustafa et al. [6] deciphered the three-dimensional rotating flow of nanofluids over a stationary disk. Sheikholeslami et al. [7] created numerical models of nanofluid splashing on a slanted turning disk. Transient thermophoretic molecule deposition through the constrained convective flow of micropolar liquid over a pivoting disk was examined by Doh and Muthtamilselvan [8]. Hayat et al. [9] discussed Darcy-Forchheimer flow of carbon nanotubes in
response to a turning disk. Aziz et al. [10] gave a numerical report on nanofluid flow from the pivoting of a disk, looking at the impacts of slip and heat absorption/generation. Synthetically responsive flow of third-grade nanofluid over a stretchable turning disk with heat generation was broken down by Hayat et al. [11]. The radiative flow of a suspension of nanoparticles and gyrotactic microorganisms by the variably thick surface of a turning disk was clarified by Qayyum et al. [12]. Hayat et al. [13] presented a numerical simulation of the radiative flow of carbon nanotubes due to the revolution of a disk with partial slip.

The low thermal productivity of working fluids is a guideline problem for several heat transport components in engineering applications. For this reason, some researchers are making efforts to develop an innovative course for the improvement of the thermal efficiency of working fluids. Various measures have been proposed by experts to improve the thermal efficiency of fluids. Accordingly, the incorporation of nanomaterial into the working fluid, making what is termed a nanofluid, is extremely promising. Recent assessments of nanofluids reveal that working fluid has totally different features with the addition of the nanomaterial. This is because the thermal efficiency of the working liquid is lower than that of the nanomaterial. Nanofluid is suspension of fluids containing standard fluid with the particles of nano-measure. Such nanomaterials are utilized in materials, MHD control generators, oil stores, cooling of nuclear reactors, vehicle transformers, and various others [14–18]. Choi and Eastman [19] coined the term nanofluid. They proposed that nanomaterials are a groundbreaking contender for the development of heat transport via the customary fluids. Buongiorno proposed a numerical model of convective transport by nanofluid [20]. Here, thermophoresis and Brownian motion are viewed as the most important slip mechanisms. Heat transfer increase by nanofluids in a two-sided top-driven heated square hole was considered by Tiwari and Das [21]. The significance of a CuO-water nanomaterial on the outside of heat exchangers was tentatively examined by Pantzali et al. [22]. Few ongoing studies on nanofluid flow can be found in the literature [23–45].

Motivated by the above-mentioned articles, the objective here is to examine the impacts of heat absorption/generation in Darcy-Forchheimer 3D nanofluid flow caused by a rotating disk and the impacts of slip. Both Brownian diffusion and thermophoretic phenomena occur in view of the existence of nanoparticles. Velocity, concentration, and thermal slips are accounted for. The obtained framework is solved numerically by the shooting technique. Concentration, temperature, skin friction, and local Sherwood and Nusselt numbers are also analyzed through plots.

2. MATHEMATICAL MODELING

Let us examine steady Darcy-Forchheimer 3D nanoliquid flow caused by a rotating disk with slip and heat absorption/generation. A disk at \( z = 0 \) rotates with constant angular velocity \( \Omega \) (see Figure 1). The impacts of Brownian motion and thermophoresis are accounted for. The velocities are \((u, v, w)\) in the directions of increase in \((r, \phi, z)\), respectively. The resulting boundary layer expressions are [45, 46]:

\[
\begin{align*}
\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} &= 0, \\
\frac{\partial u}{\partial r} - \frac{v^2}{r} + \frac{\partial u}{\partial z} &= \nu \left( \frac{\partial^2 u}{\partial z^2} + \frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} \right) - \frac{\nu}{k^2} u - Fu^2, \\
\frac{\partial v}{\partial r} + uv + \frac{\partial v}{\partial z} &= \nu \left( \frac{\partial^2 v}{\partial z^2} + \frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} - \frac{v}{r^2} \right) - \frac{\nu}{k^2} v - Fv^2, \\
\frac{\partial w}{\partial r} + \frac{w}{r} + \frac{\partial w}{\partial z} &= \nu \left( \frac{\partial^2 w}{\partial z^2} + \frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} - \frac{w}{r^2} \right) - \frac{\nu}{k^2} w - Fw^2.
\end{align*}
\]

It is subject to the boundary conditions [10]:

\[
u = L_1 \frac{\partial u}{\partial z}, \quad \nu = r\Omega + L_1 \frac{\partial v}{\partial z}, \quad w = 0, \quad T = T_w + L_2 \frac{\partial T}{\partial z},
\]
\[ C = C_w + L_1 \frac{\partial C}{\partial z} \text{ at } z = 0, \quad (7) \]

\[ u \rightarrow 0, \; v \rightarrow 0, \; T \rightarrow T_\infty, \; C \rightarrow C_\infty \text{ as } z \rightarrow \infty. \quad (8) \]

Here \( u, v, \) and \( w \) represent the velocity components in the directions \( r, \phi, \) and \( z \) while \( \rho_j, \mu \) and, \( \nu \) (\( \equiv \mu/\rho_j \)) depict the fluid density, dynamic, and kinematic viscosities respectively, \( C_b \) the drag factor, \( L_1 \) the velocity slip factor, \( L_2 \) the thermal slip factor, \( L_3 \) the concentration slip factor, \( C_\infty \) the ambient concentration, \( D_T \) the thermophoretic factor, \( F = C_b/rk^{s/2} \) the non-uniform inertia factor, \( k^* = k/(\rho c)_f \) the thermal conductivity and thermal diffusivity, respectively, \( D_B \) the Brownian factor, \( Q \) the heat generation/absorption factor and \( T_\infty \) the ambient temperature. Selecting [10]:

\[
\begin{align*}
\phi(\zeta) &= \frac{C - C_\infty}{C_w - C_\infty}, \\
\zeta &= \left( \frac{2\Omega \nu}{r} \right)^{1/2} z, \\
\theta(\zeta) &= \frac{T - T_\infty}{T_w - T_\infty}.
\end{align*}
\]

Continuity Equation (1) is trivially verified, while Equations (2)–(8) yield

\[
\begin{align*}
2f''' + 2ff'' - f^2 + g^2 - \lambda f' - Frf' &= 0, \quad (10) \\
2g'' + 2fg' - 2f'g - \lambda g - Frg^2 &= 0, \quad (11) \\
\frac{1}{Pr} \theta'' + f\theta' + N_b \theta' \phi' + N_t \theta'^2 + \delta \theta &= 0, \quad (12)
\end{align*}
\]
\[
\frac{1}{Sc} \phi'' + f \phi' + \frac{1}{Sc N_l} \theta'' = 0, \quad (13)
\]

\[
f(0) = 0, \quad f'(0) = \alpha f''(0), \quad g(0) = 1 + \alpha g'(0), \quad (14)
\]

\[
\theta(0) = 1 + \beta \theta'(0), \quad \phi(0) = 1 + \gamma \phi'(0),
\]

\[
f'(\infty) \to 0, \quad g(\infty) \to 0, \quad \theta(\infty) \to 0, \quad \phi(\infty) \to 0. \quad (15)
\]

Here, \( Fr \) stands for the Forchheimer number, \( \alpha \) for the velocity slip parameter, \( \lambda \) for the porosity parameter, \( N_t \) for the thermophoresis parameter, \( \beta \) for the thermal slip parameter, \( Pr \) for the Prandtl number, \( N_b \) for Brownian motion, \( \delta \) for the heat absorption/generation parameter, \( \gamma \) for the concentration slip parameter, and \( Sc \) for the Schmidt number. Non-dimensional variables are defined by

\[
\begin{align*}
\lambda &= \frac{\nu}{k^2 \Omega}, & \alpha &= \frac{L_1}{k^2 \sqrt{2} \Omega}, & Fr &= \frac{C_b}{k^2 \nu}, & N_l &= \frac{(rcp)D_B(C_w-C_\infty)}{(rcp)v}, \\
\delta &= \frac{Q}{2 \Omega (rcp)}, & Pr &= \frac{\nu}{\alpha}, & N_b &= \frac{(rcp)D_T(T_w-T_\infty)}{(rcp)vT_\infty}, & \gamma &= L_3 \sqrt{2 \Omega} , \\
Sc &= \frac{\nu}{D_B}.
\end{align*}
\]
The coefficients of skin friction and the Nusselt and Sherwood numbers are

\[
Re_r^{1/2} C_f = f''(0), \quad Re_r^{1/2} C_g = g'(0), \quad Re_r^{-1/2} Nu = -\theta'(0), \quad Re_r^{-1/2} Sh = -\phi'(0),
\]

(17)

where \( Re_r = 2(\Omega r)r/\nu \) represents the local rotational Reynolds number.

3. NUMERICAL RESULTS AND DISCUSSION

This section depicts the contributions of various physical variables like thermophoresis parameter \( N_t \), Forchheimer number \( Fr \), thermal slip parameter \( \beta \), heat generation/absorption parameter \( \delta \), Brownian number \( N_b \), and concentration slip number \( \gamma \) on The concentration \( \phi(\zeta) \) and temperature \( \theta(\zeta) \) distributions. The effect of Forchheimer variable \( Fr \) on \( \theta(\zeta) \) is portrayed in Figure 2. A larger value for \( Fr \) shows expanding behavior of \( \theta(\zeta) \) and the related thermal layer. Figure 3 shows the impact of thermal slip \( \beta \) on temperature \( \theta(\zeta) \). Temperature is reduced by increasing thermal slip \( \beta \). Figure 4 demonstrates the effect of \( N_t \) on the temperature field \( \theta(\zeta) \). A larger thermophoresis parameter \( N_t \) value leads to a higher temperature field and thicker dynamically warm layer. The reason for this conflict is that growth in \( N_t \) yields high grounded thermophoresis control, which further allows movement of the nanoparticles in the fluid zone. A long way from the surface, a more grounded temperature scattering \( \theta(\zeta) \) and continuously warm layer is thus created. The impact of \( N_b \) on the temperature profile \( \theta(\zeta) \) is portrayed in Figure 5. Physically, the irregularity of nanoparticle movement increases by enhancing Brownian motion, due to which collision of particles occurs. Thus, kinetic energy is converted into heat energy, which produces an increase in the temperature field. Figure 6 shows how heat generation/absorption \( \delta \) influences
temperature dispersion $\theta(\zeta)$. Here, $\delta > 0$ portrays heat generation and $\delta < 0$ for heat absorption. Both temperature $\theta(\zeta)$ and the warm layer are upgraded with increasing $\delta$. Figure 7 shows that concentration $\phi(\zeta)$ is higher for larger values of the Forchheimer variable Fr. Figure 8 shows how concentration $\phi(\zeta)$ is influenced by concentration slip $\gamma$. Concentration is reduced at higher estimations of $\gamma$. Figure 9 demonstrates how the thermophoresis parameter $N_t$ influences the concentration $\phi(\zeta)$. By improving thermophoresis parameter $N_t$, the concentration $\phi(\zeta)$ is increased. Figure 10 depicts the impact of Brownian motion $N_b$ on concentration $\phi(\zeta)$. It has been noted that a stronger concentration $\phi(\zeta)$ is developed by utilizing greater $N_b$. Figures S1, S2 display the impacts of Fr on $C_{f}Re_{r}^{1/2}$ and $C_{g}Re_{r}^{1/2}$, respectively. It is noted that $C_{f}Re_{r}^{1/2}$ is a decaying function of Fr, while the reverse situation is observed for $C_{g}Re_{r}^{1/2}$. The effects of $N_t$ and $N_b$ on $Nu(Re_{r})^{-1/2}$ are highlighted in Figures 11, 12, respectively. Here, $Nu(Re_{r})^{-1/2} =-\lambda$ for $N_t$ and $N_b$. The effects of $N_t$ and $N_b$ on $Sh(Re_{r})^{-1/2}$ are portrayed in Figures 13, 14, respectively. Here, $Sh(Re_{r})^{-1/2}$ is an increasing factor of $N_t$, while the opposite trend is seen for $N_b$. The figures in Table 1 were computed to validate the present results with previously published results in a limiting sense. Here, we see that the present numerical solution is in good agreement with the previous solution by Naqvi et al. [45] in a limiting sense.

4. CONCLUSIONS

In this paper, Darcy-Forchheimer 3D nanofluid flow caused by a rotating disk with heat generation/absorption is studied. Brownian motion and thermophoretic phenomena occur with the existence of nanoparticles. Velocity, concentration, and thermal slips are accounted for. A higher Forchheimer number $Fr$ depicts similar behavior for concentration and temperature. A larger $\beta$ corresponds to a lower temperature field. Higher $\gamma$ depicts decreasing behavior for the concentration field. A stronger temperature field is observed for $N_b$ and $N_t$. Concentration $\phi(\zeta)$ displays the reverse behavior for $N_b$ and $N_t$.

DATA AVAILABILITY STATEMENT

The datasets generated for this study are available on request to the corresponding author.

AUTHOR CONTRIBUTIONS

All authors listed have made a substantial, direct and intellectual contribution to the work, and approved it for publication.

ACKNOWLEDGMENTS

This project was funded by the Deanship of Scientific Research (DSR), King Abdulaziz University, Jeddah, Saudi Arabia under grant no. KEP-16-130-40. The authors, therefore, acknowledge with thanks DSR technical and financial support.

SUPPLEMENTARY MATERIAL

The Supplementary Material for this article can be found online at: https://www.frontiersin.org/articles/10.3389/fphy.2019.00219/full#supplementary-material

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Conflict of Interest: The authors declare that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

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