Research on surface quality of Si$_3$N$_4$ ceramics by internal grinding

H Yan$^{1,2,*}$, Y Wu$^{1,2}$ and H Wang$^{1,2}$

$^1$School of Mechanical Engineering, Shenyang Jianzhu University, Shenyang 110168, China.
$^2$National-Local Joint Engineering Laboratory of NC Machining Equipment and Technology of High-Grade Stone, Shenyang Jianzhu University, Shenyang 110168, China.

$^*$E-mail: sjzuwuyh@163.com

Abstract. The machined surface quality of hard and brittle materials was studied based on the analysis of the material removal mechanism of ceramics. According to the geometrical relationship of the internal grinding, the equivalent diameter of grinding wheel was deduced. In the internal grinding, the models of undeformed chip thickness, peak-valley surface roughness and arithmetic mean surface roughness were established with the assistance of planar grinding. The proposed models were verified utilizing the orthogonal test of internal grinding Si$_3$N$_4$ ceramics. The trend of experimental results was consistent with the theoretical model. The scientific validity of the proposed models was supported by the experimental results, and the investigation provides a theoretical reference and experimental basis for further study on the grinding of ceramic materials.

1. Introduction

Ceramic materials such as Si$_3$N$_4$ ceramic have advantages of high strength, high hardness, oxidation resistance, corrosion resistance, wear resistance and ablation resistance. Ceramic products have been used in military engineering, aerospace engineering and other high-tech fields [1]. With the development of Si$_3$N$_4$ ceramics, the requirements of machining quality, precision, shape, dimension precision and surface complexity are improved gradually. Especially in recent years, with the advent and development of ceramic motorized spindle, the machining accuracy requirement of ceramic bearing rings is extremely demanding. However, ceramic materials are difficult to process because of their hard and brittle properties [2, 3]. The efficient and precise machining has become a focus recently. Nowadays, the common method of precision machining for ceramic materials is grinding with diamond wheel [4-6].

Grinding is generally applied in the final stage of processing mechanical parts to ensure the shape accuracy and diminish surface roughness [7]. The formation of grinding surface is a constant interference between abrasive grains and workpiece surface. Due to the complex process of grinding, numerous factors can interfere in the surface roughness, thus the relationship between them is difficult to calculate accurately [8]. Generally, the proposed theoretical models about grinding surface roughness were based on some assumptions to better simulate the mechanism of workpiece surface formation. The theoretical formula showed high calculation accuracy, whereas the calculation results...
were often smaller than the actual values of surface roughness [9, 10]. As the empirical formula is suitable for specific grinding conditions, it has a narrow scope for analyzing and forecasting workpiece surface roughness values, which limits its application [4]. When the grinding condition changes greatly, the calculation accuracy decreases. To choose a proper grinding wheel is also crucial [11]. Reducing the grain size, which can improve the machining surface morphology, determines a reduction of the material removal rate and prolongs the processing cycle. In addition, Liu et al. [12] investigated the influence of planar grinding on Si$_3$N$_4$ ceramics, which proved that the grinding parameters regulated the surface roughness. Jing et al. [13] established a model between the surface morphology and process parameters based on BP neural network, providing the support for surface morphology optimization. Mochida et al. [14] pointed out that small grinding depth was necessary for low damage grinding by increasing the wheel speed at an even high material removal rate. Rodriguez et al. [15] studied the influence of grinding depth and workpiece velocity on Si$_3$N$_4$ ceramics surface damages. The experimental results showed that the surface damage decreased under "hard" grinding conditions.

In sum, the existing literature focused on the prediction for surface roughness of the hard and brittle materials after grinding. As the mechanism model of grinding surface roughness is limited to planar grinding, the mechanism of internal grinding is reported rarely. In this paper, taking Si$_3$N$_4$ ceramics as an example, the mechanism of internal grinding ceramic materials was investigated by means of the orthogonal test. Peak-valley surface roughness, arithmetic mean surface roughness and surface morphology were utilized to characterize the machined surface quality. Theoretical models of undeformed chip thickness and surface roughness for internal grinding were established. Furthermore, the reliability and accuracy of models were verified by means of the orthogonal test to provide a theoretical basis for surface roughness prediction in actual internal grinding.

2. Removal mechanism of ceramic materials in internal grinding

2.1. Equivalent diameter of grinding wheel

As shown in figure 1, similar to surface grinding, internal grinding can be considered a material removal process, in which the abrasive grains were utilized as cutting tools. Under the same grinding condition, the contact arc length of internal grinding is larger than that of planar grinding, thus the chip is not easy to eliminate. Due to the poor heat dissipation of internal grinding, the workpieces are prone to be burnt by high temperature [16].

![Figure 1. Sketch of planar grinding and internal grinding.](image-url)
in form of tiny debris, forming certain scratches on the machined surface and even visible microcracks.

Figure 2. Material removal of ceramics by internal grinding.

Figure 3. Undeformed cutting thickness and surface roughness of planar grinding.

Figure 4. Undeformed cutting thickness and surface roughness of internal grinding.
In the process of high speed finish grinding, the contact arc length was also tiny due to the minute grinding depth. In this paper, plunge grinding was applied as an example to study the mechanism of internal grinding of ceramic materials.

As shown in figure 3, in the planar grinding, the contact arc length \( l \) is considered the chord length \( AB \) because the grinding depth is quite tiny. So,

\[
l \approx AB = \left( a_p \cdot d_s \right)^{\frac{1}{2}}
\]

where \( a_p \) is the grinding depth, \( d_s \) is the external diameter of the grinding wheel.

As shown in figure 4, similar to planar grinding, the contact arc length \( l_c \) of internal grinding \[18\] can be expressed as:

\[
l_c = \left( AE \cdot d_e \right)^{\frac{1}{2}}
\]

where \( AE \) is equivalent to grinding depth of planar grinding. In the process of internal grinding chip formation, \( AE \) is solved by geometric relation as equation (3):

\[
AE = \frac{a_p}{1 - d_e/d_w}
\]

where \( d_e \) is the internal diameter of workpiece.

The contact arc length \( l_c \) of internal grinding is finally denoted as:

\[
l_c = \left( a_p \cdot d_e \right)^{\frac{1}{2}}
\]

where \( d_e \) is defined as equivalent diameter of wheel.

The \( d_e \) of internal grinding is calculated by:

\[
d_e = \frac{d_s}{1 - d_e/d_w}
\]

2.2. Undeformed chip thickness of internal grinding

Undeformed chip thickness is the maximum cutting depth of a cutting edge, which is often referred to as “abrasive cutting depth”. In planar surface grinding, the workpiece is stationary when the cutting edge is working, and a distance of \( s \), i.e., the feed per cutting edge, is moved instantaneously before the next cutting edge enters the cutting process. As shown in figure 3, the undeformed cutting thickness of the planar surface grinding \[18\] can be described as:

\[
h_w = 2s \left( \frac{a_p}{d_s} \right)^{\frac{1}{2}} - \frac{s^2}{d_s}
\]

where \( s \) is:

\[
s = \frac{Lv_w}{v_s}
\]

where: \( L \) is grain spacing, which indicates the distance between two adjacent grains, \( v_s \) is grinding wheel speed, \( v_w \) is workpiece velocity.
As shown in figure 4, in internal grinding, $s_c$ is the moved arc length of the center of the grinding wheel, which correspondsto the feed per cutting edge of the planar surface grinding. $s_c$ is defined as follows:

$$s_c = s(1-d_e/d_w)$$

(8)

The undeformed chip thickness of internal grinding for ceramic materials can be obtained using $s_c$ in equation (8) instead of $s$ in equation (6). This is actually the same as the result that $d_e$ in equation (5) substitutes for $d_s$ of equation (6).

$H_m$ can be represented as:

$$H_m = 2s\left(\frac{a_p}{d_e}\right)^{\frac{1}{2}} - \frac{s^2}{d_e}$$

(9)

The equations (5) and (7) are plugged into equation (9) to obtain the final expression of $H_m$, shown as equation (10).

$$H_m = 2L\left(\frac{v_w}{v_s}\right)^{\frac{1}{4}} a_p^\frac{1}{2} \left(\frac{1-d_e/d_w}{d_s}ight)^{\frac{1}{2}}$$

(10)

2.3. Surface roughness model of internal grinding

As shown in figure 3, the theoretical formulas of peak-valley surface roughness $R_t$ and arithmetic mean surface roughness $R_a$ in the planar surface grinding are given [18], respectively, by:

$$R_t = \frac{1}{4} \left(\frac{v_w L}{v_s d_e^\frac{5}{2}}\right)^2$$

(11)

$$R_a = \frac{1}{9\sqrt{3}} \left(\frac{v_w L}{v_s d_e^\frac{5}{2}}\right)^2$$

(12)

When the $d_e$ in equation (11) is regarded as the equivalent diameter of $d_o$, which is replaced by equation (5), the peak-valley surface roughness in internal grinding ceramic materials can be rewritten as:

$$R_t = \frac{1}{4} \left(\frac{v_w L}{v_s}\right)^2 \frac{1-d_s/d_w}{d_s}$$

(13)

According to the equation (13) and figure 4, when $d_e$ tends to infinity, i.e. $d_s/d_w = 0$, here equation (13) is the same as equation (11), which is equivalent to the peak-valley surface roughness of the planar surface grinding.

Similarly, the arithmetic mean surface roughness of internal grinding ceramic materials is determined as:
According to the model established above, the undeformed cutting thickness and surface roughness are not only related to the geometrical size of grinding wheel and workpiece, but also to the cutting parameters and the grain spacing. Average spacing of grains $L_{\text{avg}}$ can be indicated as equation (15) [19].

$$L_{\text{avg}} = 137.9M^{-1.43} \sqrt[3]{\frac{\pi}{32 - S}}$$

where: $M$ is the grain size number of abrasives, $S$ is the structure number of the grinding wheel. When the grain size of the wheel is settled, the average grain spacing is determined.

3. Test procedures

3.1. Materials and equipment

As shown in figure 5a, the test was carried out in high precision MK2710 NC internal and cylindrical modular grinding machine tool manufactured by Wuxi Machine Tool Group of China. The four axes of the machine tool are composed of a servo motor and a ball screw through a flexible coupling. It can realize no gap and high sensitivity movement, and the motion resolution of each axis can both reach 1 μm.

![Experimental machine tools and grinding test](image)

**Figure 5.** Experimental machine tools and grinding test. a) MK2710 grinding machine and b) Internal grinding.

**Table 1.** Performance of grinding wheel.

| Wheel attribute       | Values |
|-----------------------|--------|
| External diameter (mm)| 55     |
| Thickness (mm)        | 23.5   |
| Aperture (mm)         | 10     |
| Ring width (mm)       | 3      |
| Ring height (mm)      | 5      |
| Abrasive              | RVD    |
| Concentration (%)     | 100    |
| Average grain spacing (μm) | 108    |
Table 2. Properties of Si₃N₄ ceramic.

| Material attribute                        | Values |
|-------------------------------------------|--------|
| Density (g·cm⁻³)                          | 3.25   |
| Coefficient of thermal expansion (10⁻⁶·K⁻¹) | 3.2    |
| Elastic modulus (GPa)                     | 310    |
| Poisson ratio                             | 0.26   |
| Fracture toughness (MPa·m¹/²)             | 6.0    |
| Bending strength (MPa)                    | 900    |
| HRC (GPa)                                 | 80     |
| Inner diameter of material ring (mm)      | 65     |

As shown in figure 5b, the grinding test of Si₃N₄ ceramic inner ring was carried out by using a diamond grinding wheel with metal bonded and grain size of 120#. The grinding wheel parameters and material properties are shown in tables 1 and 2, respectively. The grinding fluid was water-based emulsified oil (1 : 20).

3.2. Test scheme
To investigate the parameters which affect the surface formation mechanism of Si₃N₄ ceramic via internal grinding, an orthogonal test was carried out, which included wheel speed, workpiece velocity and grinding depth. The orthogonal test design is shown in table 3.

Table 3. Orthogonal test design.

| Experiment number | vₛ (m/s) | vₘ (mm/min) | aₚ (μm) |
|-------------------|----------|-------------|---------|
| 1                 | 34       | 1200        | 4       |
| 2                 | 34       | 6000        | 12      |
| 3                 | 34       | 12000       | 20      |
| 4                 | 51       | 1200        | 20      |
| 5                 | 51       | 6000        | 4       |
| 6                 | 51       | 12000       | 12      |
| 7                 | 17       | 1200        | 12      |
| 8                 | 17       | 6000        | 20      |
| 9                 | 17       | 12000       | 4       |
4. Results and discussion

4.1. Surface morphology

As shown in figure 6, the surface morphology of workpiece after grinding was observed under the scanning electron microscope (S-4800). In the internal grinding, the influence of grinding parameters on the surface quality of Si$_3$N$_4$ ceramic was analysed.

Figure 6. Scanning electron microscope.

Figure 7 shows the surface morphology of Si$_3$N$_4$ ceramics by internal grinding. When the grinding wheel speed was low, the gully regions were wide and the bulge parts were high on the surface (figure 7a and b). Along with the increase of the wheel speed, the gully was obviously narrowed (figure 7c). As the wheel speed continues to increase, the bulge on the surface may be removed (figure 7d). From all the pictures in figure 7, we can find that the surface scratch caused by the plastic removal and fracture pit by brittle removal were obvious. The increase of the wheel speed reduced the maximum undeformed chip thickness, which resulted in a shallow action depth and a narrow action width of the abrasive grain. As a result, the gully became shallow and narrow. Furthermore, the increase of the wheel speed led to an increase of the number of abrasive grains participating grinding per unit time, so that the bulges were removed in time or squeezed into the gully, so the increase of the wheel speed was beneficial to improve the machined surface. Therefore, in the internal grinding, the wheel speed is an important factor which affects the surface morphology of Si$_3$N$_4$ ceramics.

Figure 7. Surface morphology of Si$_3$N$_4$ ceramics by internal grinding. a) Experiment number 1, b) Experiment number 3, c) Experiment number 6 and d) Experiment number 8.
In addition, according to figure 7a and b, when the grinding wheel speed was the same, the increase of grinding depth would widen the gully and slightly heighten the bulge in the machined surface. From figure 7b and d, a better surface quality could be obtained using a smaller workpiece velocity. With the increase of grinding depth, the removal of material is mainly in a brittle manner [20]. The increase of the workpiece velocity reduced the time of abrasive grain cutting in the same area, thus the gully and bulge of the machined surface will be increased. Consequently, it is not conducive to achieving a better surface quality through increasing the grinding depth and the workpiece velocity.

4.2. Surface roughness
The roughness of machined surface was measured by Taylor Hobson roughness measuring instrument, and the results are shown in table 4. According to the results of orthogonal test, the relation curve of the surface roughness with the cutting parameters is drawn as shown in figure 8.

| Experiment number | $R_{\text{e}}$ (μm) | $R_{\text{a}}$ (μm) |
|------------------|---------------------|---------------------|
| 1                | 1.4763              | 0.3957              |
| 2                | 1.5861              | 0.4134              |
| 3                | 1.7903              | 0.4762              |
| 4                | 1.5160              | 0.3635              |
| 5                | 1.4358              | 0.3793              |
| 6                | 1.6842              | 0.4216              |
| 7                | 1.7907              | 0.4598              |
| 8                | 1.8723              | 0.4874              |
| 9                | 1.9212              | 0.5424              |

As shown in figure 8, the surface roughness decreased and the surface quality was improved by the gradual increase of the grinding wheel speed. Increasing the workpiece velocity led to an increase of surface roughness, which resulted in the deterioration of the surface. The increase of the grinding depth also made the surface roughness an increase trend, but the change was small. In this experiment, when the wheel speed increased from 17 m/s to 51 m/s, the surface roughness value $R_{\text{e}}$ decreased from 1.8614 μm to 1.5453 μm, down by 16.98% and the surface roughness value $R_{\text{a}}$ decreased from 0.4965 μm to 0.3881 μm, reduced by 21.83%. When the workpiece velocity increased from 1200 mm/min to 12000 mm/min, the $R_{\text{e}}$ increased from 1.5943 μm to 1.7986 μm, an 12.81% upwards and the $R_{\text{a}}$ increases from 0.4063 μm to 0.4801 μm, an increase of 18.16%. The effect of the grinding depth on the surface roughness was that the grinding depth increased from 4 μm to 20 μm, the $R_{\text{e}}$ increased from 1.6111 μm to 1.7262 μm, which increased by up to 7.15%. The $R_{\text{a}}$ decreased slightly from 0.4391 μm to 0.4316 μm when the grinding depth increased from 4 μm to 12 μm, while increasing the grinding depth to 20 μm, the $R_{\text{a}}$ increased to 0.4424 μm. In the process, the increase of the grinding depth led to an increase of 0.75% only for the $R_{\text{a}}$ value.
Figure 8. Effect of grinding parameters on surface roughness in the internal grinding process of Si₃N₄ ceramics. a) Wheel speed, b) Workpiece velocity and c) Grinding depth.

By increasing the grinding wheel speed, the number of effective abrasive grain raises in per unit time, which can improve the machined surface quality. Reducing the workpiece velocity can diminish the height of the residual area, which leads to the decrease of the surface roughness value. For the grinding depth, the increase of the grinding depth can cause the increase of the surface roughness; but the change is not obvious. In the actual grinding process, an increase in cutting depth will increase the number of grains that participate in cutting simultaneously, bringing a tremendous friction extrusion in the contact area. In the meantime, the heat produced by the process is not easy to dissipate in internal grinding, leading to a rise of cutting temperature, which may also burn the workpiece surface and reduce the process quality.

Tables 3 and 4 show that the minimum value of $R_{\text{E}}$ was obtained (1.4358 μm) under the following conditions: wheel speed $v_1$ at 51 m/s, workpiece velocity $v_w$ at 6000 mm/min and grinding depth $a_p$ at 4 μm. Whereas the surface roughness value $R_{aE}$, 0.3793 μm, under the same conditions, was not its minimum value. The best condition for $R_{aE}$, 0.3635 μm, was when the wheel speed $v_1$ was 51 m/s, the workpiece velocity $v_w$ was 1200 mm/min and the grinding depth $a_p$ was 20 μm. Correspondingly, the value $R_{eE}$ was 1.5160 μm, also not the minimum.

From the analysis presented above, it results that, in this experiment, the wheel speed has the main influence on the surface roughness, followed by workpiece velocity, with the minimum effect of grinding depth. Under the test parameters of the No. 5, that was $v_1 = 51$ m/s, $v_w = 6000$ mm/min and $a_p = 4$ μm, the optimal surface quality was obtained by comprehensive analysis of $R_{eE}$ and $R_{aE}$.
5. Comparison between theoretical and experimental results

5.1. Theoretical values

The measured values of surface roughness after processing were larger than the theoretical values, due to the machine vibration, size error, wheel dressing or the measurement accuracy, which would affect the experimental results. In this paper, all these factors can be attributed to the size effect $M_r$, so the formulas (13) and (14) respectively are corrected as:

$$R_t = \frac{M_r}{4} \left( \frac{v_w L}{v_s} \right)^2 \frac{1 - d_s/d_w}{d_s}$$  \hspace{1cm} (16)

$$R_w = \frac{M_r}{9\sqrt{3}} \left( \frac{v_w L}{v_s} \right)^2 \frac{1 - d_s/d_w}{d_s}$$  \hspace{1cm} (17)

In order to compare the change of undeformed chip thickness and grinding depth conveniently, an amplification factor $K_H$ is added into formula (10), as shown in equation (18).

$$H_m = 2K_H L \left( \frac{v_w}{v_s} \right) a_p^\frac{1}{2} \left( \frac{1 - d_s/d_w}{d_s} \right)^{\frac{1}{2}}$$  \hspace{1cm} (18)

$$-K_H L^2 \left( \frac{v_w}{v_s} \right)^2 \left( \frac{1 - d_s/d_w}{d_s} \right)$$

The size effect $M_r$ and the coefficient $K_H$ are calculated by the latest square method. The values of $M_r$ and $K_H$ are $4 \times 10^5$ and 700, respectively. Table 5 presents the theoretical and experimental values of the modified model.

| Experiment number | $a_p$ (μm) | $H_m$ (μm) | $R_{te}$ (μm) | $R_{te}$ (μm) | $R_{ea}$ (μm) | $R_{ea}$ (μm) |
|-------------------|------------|------------|---------------|---------------|---------------|---------------|
| 1                 | 4          | 0.7658     | 1.4763        | 0.0075        | 0.3957        | 0.0019        |
| 2                 | 12         | 6.6321     | 1.5861        | 0.1870        | 0.4134        | 0.0480        |
| 3                 | 20         | 17.1238    | 1.7903        | 0.7480        | 0.4762        | 0.1920        |
| 4                 | 20         | 1.1416     | 1.5160        | 0.0033        | 0.3635        | 0.0008        |
| 5                 | 4          | 2.5526     | 1.4358        | 0.0831        | 0.3793        | 0.0213        |
| 6                 | 12         | 8.8427     | 1.6842        | 0.3324        | 0.4216        | 0.0853        |
| 7                 | 12         | 2.6528     | 1.7907        | 0.0299        | 0.4598        | 0.0077        |
| 8                 | 20         | 17.1238    | 1.8723        | 0.7480        | 0.4874        | 0.1920        |
| 9                 | 4          | 15.3160    | 1.9212        | 2.9921        | 0.5424        | 0.7678        |
5.2. Undeformed chip thickness and grinding depth

Figure 9 shows the calculated undeformed chip thickness and the experimental grinding depth. From the data of view, the overall variation trend of undeformed chip thickness and grinding depth was similar. It proved that the theoretical model of undeformed chip thickness established in this paper can reflect the effect of cutting parameters on the undeformed chip thickness. Thus, a brief description of the material removal and surface formation by internal grinding on the hard brittle materials was obtained.

![Figure 9](image)

**Figure 9.** Comparison between undeformed chip thickness and grinding depth.

5.3. Measured values and theoretical values of surface roughness

Figure 10 shows the relationship between peak-valley surface roughness and arithmetic mean surface roughness, in which the fitting curve is obtained by the least square method. The coefficient of the fitted curve ($k_T = 3.8971$) is very close to that of the theoretical curve ($k_T = 3.8971$), and the error is only 2.17%. The relationship between peak-valley surface roughness and arithmetic mean surface roughness obtained by the experiment is consistent with the theoretical prediction and validates the correctness of the theoretical model.

![Figure 10](image)

**Figure 10.** Relationship between peak-valley surface roughness and arithmetic mean surface roughness.

The surface roughness values calculated by the theoretical model and obtained by the experimental measurement are presented in Figure 11. In Figures 11a and b, both the calculated values of test No. 9 are greater than the measured values. The others show an opposite trend, where the actual measured
values of surface roughness are greater than the theoretical values. According to the variation trend of peak-valley surface roughness, the experimental measurement results had a parallel trend with the theoretical values. The same results were observed in arithmetic mean surface roughness.

The theoretical model was built based on pure cutting state, which did not consider the effects of axial feed on the undeformed chip thickness and the surface roughness, assuming that the grinding wheel abrasive cutting edge was evenly distributed and had the same radial bump height. Furthermore, several factors that affect the undeformed chip thickness and surface roughness were not included in the modelling process, such as wheel wear, scratch overlap, cutting temperature and vibration of machine tool and grinding wheel. Moreover, the surface morphology obtained completely according to the theoretical cutting path is unsatisfactory in actual grinding. Comprehensively analyzing the above factors, it is not difficult to understand that the actual measured surface roughness values are much larger than the theoretical values, while in a similar trend. Therefore, the theoretical model for the surface roughness of the internal grinding is identified by the experimental results. The model can provide a theoretical reference for the investigation of surface formation mechanism of hard and brittle materials by the internal grinding, and also a valuable application for grinding internal surfaces of ceramic bearings.

Figure 11. Comparison between theoretical and experimental values of surface roughness. a) Peak-valley surface roughness, $R_t$ and b) Arithmetic mean surface roughness, $R_a$.

In addition, the influence of grinding depth on the surface roughness is not considered in the theoretical model. As the experimental results showed, the grinding depth had little effect on the surface roughness. From this point of view, the theoretical model is reliable.

6. Conclusions

This paper investigated the Si$_3$N$_4$ ceramics internal grinding material removal mechanism. By analysing the internal grinding process, the wheel equivalent diameter of internal grinding was deduced. Furthermore, a theoretical model for the internal grinding undeformed chip thickness and surface roughness of Si$_3$N$_4$ ceramics was developed based on the planar grinding model. Additionally, the size effect was utilized to correct the theoretical model to better simulate the machined surface quality. Internal grinding test of Si$_3$N$_4$ ceramics was carried out using the orthogonal test scheme. Then, the influence law of wheel speed, workpiece velocity and grinding depth on the Si$_3$N$_4$ ceramics grinding surface roughness were investigated and discussed according to the experimental results. And through the analysis of the theoretical model and the experimental results, the conclusions are described as follows:

1. Surface morphology of Si$_3$N$_4$ ceramics showed that the grinding wheel speed was an important factor which affected the surface formation in internal grinding. By increasing the grinding wheel
speed and reducing the workpiece velocity, the quality of the machined surface, on which the gully and bulge tend to shrink can be improved.

2. According to the experimental results, the surface roughness will be alleviated by increasing grinding wheel speed and reducing workpiece velocity. Analysed comprehensively, the increase of the grinding depth accelerated the surface roughness a little. The experimental results were consistent with the trend of the theory model. It can be also obtained from the test results, according to which the grinding wheel speed has the greatest influence on the surface roughness, followed by the workpiece velocity, and the influence of the grinding depth is last.

3. Through the comparison of undeformed chip thickness and actual grinding depth, experimental and theoretical values of peak-valley surface roughness and arithmetic mean surface roughness, the experimental results have a similar trend with the theoretical values. The results showed that the theoretical model can reflect the influence of cutting parameters on surface formation and surface roughness. It provides a theoretical and experimental basis for the study of the formation mechanism, which can predict the surface roughness of the hard and brittle materials such as ceramics.

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