Exotic $1^{--}$ States in QCD Sum Rules

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Using the QCD sum rules we test if the charmonium-like structure $Y(4260)$, observed in the $J/\psi\pi\pi$ invariant mass spectrum, can be described as a exotic state, with a $J/\psi f_0(980)$ molecular current with $J^{PC}=1^{--}$. By exotic we mean a more complex structure than the simple quark-antiquark state and not exotic $J^{PC}$ quantum numbers. We consider the contributions of condensates up to dimension six and we work at leading order in $\alpha_s$. We keep terms which are linear in the strange quark mass $m_s$. The mass obtained for such state is $m_Y = (4.67 \pm 0.09)$ GeV, when the vector and scalar mesons are in color singlet configurations. We conclude that the proposed current can better describe the $Y(4660)$ state that could be interpreted as a $\Psi(2S)f_0(980)$ molecular state. We also use different $J^{PC}=1^{--}$ currents to study the recently observed $Y_6(10890)$ state. Our findings indicate that the $Y_6(10890)$ can be well described by a scalar-vector tetraquark current.

PACS numbers: 11.55.Hx, 12.38.Lg, 12.39.-x

I. INTRODUCTION

Most of the states recently observed at the B factories and the Tevatron, the $X$, $Y$ and $Z$ states, do not fit the quarkonia interpretation. Their production mechanism, masses, decay widths, spin-parity assignments and decay modes have been discussed in some reviews\cite{1-3}. Particularly interesting are the $1^{--}$ states, observed in $e^+e^-$ annihilation. The first state in the $1^{--}$ family discovered in the $e^+e^-$ annihilation through initial state radiation was the $Y(4260)$\cite{4}. Repeating the same kind of analysis leading to the observation of the $Y(4260)$ state, in the channel $e^+e^- \rightarrow \gamma_{ISR}\Psi(2S)\pi^+\pi^-$, BaBar\cite{5} has identified another broad peak at a mass around 4.32 GeV, which was confirmed by Belle\cite{6}. Belle found that the $\psi'\pi^+\pi^-$ enhancement observed by BaBar was, in fact, produced by two distinct peaks the $Y(4360)$ and the $Y(4660)$. In the bottom sector, the Belle's observation of an anomalously large $\Upsilon(nS)\pi^+\pi^-$, $n=1,2,3$ production around the $\Upsilon(9S)$ lead to the proposal of the existence of a new resonance. A Breit-Wigner resonance shape fit yields a peak mass of $(10888.4^{+2.7}_{-2.6} \pm 1.2)$ MeV/$c^2$, which is called $Y_6(10890)$\cite{7}.

There are many theoretical interpretations for these states\cite{1-3}. In the case of $Y(4260)$, although it seems not to fit the charmonium spectrum\cite{8}, a proposal to accommodate it as a $4S$ state has been made in\cite{9}. There are many other interpretations for this state: tetraquark state\cite{10}, hadronic molecule of $D_1 D, D_0 D^*$\cite{11,12}, $\chi_{c1}\omega$\cite{13}, $\chi_{c1}\rho$\cite{14}, $J/\psi f_0(980)$\cite{15}, a hybrid charmonium\cite{16}, a charm baryonium\cite{17}, a cusp\cite{18,20}, etc. Within the available experimental information, none of these suggestions can be completely ruled out. For the $Y_6(10890)$ it has been interpreted as a tetraquark state, in a $P$-wave scalar-diquark scalar-antidiquark configuration\cite{21,22}. An alternative scenario is that the anomalously large $\Upsilon(nS)\pi^+\pi^-$, $n=1,2,3$ production observed by the Belle Collaboration does not come from a new resonance but via sub-process $\Upsilon(5S) \rightarrow B^{(*)}B^{(*)} \rightarrow \Upsilon(1S,2S)\pi^+\pi^-$\cite{23,24}.

In this work we use the QCD sum rule approach (QCDSR)\cite{25-27} to check if the proposed $J/\psi f_0(980)$ assignment for the $Y(4260)$\cite{15} is supported by a direct QCDSR calculation. We also study if a similar $\Upsilon f_0(980)$ current and a tetraquark current (in a scalar-vector diquark configuration) could describe the $Y_6(10890)$ state.

II. QCD SUM RULES

The QCDSR approach is based on the two-point correlation function

$$\Pi_{\mu\nu}(q) = i \int d^4 x \, e^{i q \cdot x} \langle 0| T[j_\mu(x) j^*_\nu(0)] |0\rangle,$$  

(1)

where the current $j_\mu(x)$ contains all the information about the hadron of interest, like quantum numbers, quarks contents and so on.

We can write the correlation function in Eq. (1)
in terms of two independent Lorentz structures:

\[ \Pi_{\mu\nu}(q) = -\Pi_1(q^2)(g_{\mu\nu} - \frac{q_{\mu}q_{\nu}}{q^2}) + \Pi_0(q^2)\frac{q_{\mu}q_{\nu}}{q^2}. \tag{2} \]

The two invariant functions, \( \Pi_1 \) and \( \Pi_0 \), appearing in Eq. (2), have respectively the quantum numbers of the spin 1 and 0 mesons. Therefore, we choose to work with the Lorentz structure \( g_{\mu\nu} \), since it gets contributions only from the vector state.

The QCD sum rule is obtained by evaluating the correlation function in Eq. (1) in two ways: in the OPE side, we calculate the correlation function at the quark level in terms of quark and gluon fields. We work at leading order in \( \alpha_s \) in the operators, we consider the contributions from condensates up to dimension six and we keep terms which are linear in the strange quark mass \( m_s \). In the phenomenological side, the correlation function is calculated by inserting intermediate states for the hadronic state, \( H \), and parametrizing the coupling of these states to the current \( j_\mu(x) \), in terms of a generic coupling parameter \( \lambda \), so that:

\[ \langle 0 | j_\mu | Y \rangle = \lambda \varepsilon_\mu, \tag{3} \]

where \( \varepsilon_\mu \) is the polarization vector.

The phenomenological side of Eq. (1), in the \( g_{\mu\nu} \) structure, can be written as

\[ \Pi_{phen}^{\mu\nu}(q^2) = \frac{\lambda^2}{M^2_H - q^2} + \int_0^{s_0} ds \frac{\rho^{cont}(s)}{s - q^2}, \tag{4} \]

where \( M^2_H \) is the hadron mass and the second term in the RHS of Eq. (4) denotes higher resonance contributions. The correlation function in the OPE side can be written as a dispersion relation:

\[ \Pi_{OPE}^{\mu\nu}(q^2) = \frac{\lambda^2}{4m_Q^2} \int \frac{ds}{s - q^2} \rho^{OPE}(s), \tag{5} \]

where \( m_Q \) is the heavy quark mass and \( \rho^{OPE}(s) \) is given by the imaginary part of the correlation function: \( \pi \rho^{OPE}(s) = \text{Im} \{ \Pi^{OPE}_{\mu\nu}(s) \} \).

As usual in the QCD sum rules method, it is assumed that the continuum contribution to the spectral density, \( \rho^{cont}(s) \) in Eq. (4), vanishes below a certain continuum threshold \( s_0 \). Above this threshold, it is given by the result obtained with the OPE. Therefore, one uses the ansatz

\[ \rho^{cont}(s) = \rho^{OPE}(s)\Theta(s - s_0). \tag{6} \]

In general, the continuum threshold \( s_0 \) is a parameter of the calculation which is connected to the mass of the studied state, \( H \), by the relation

\[ s_0 \sim (M^2_H + 0.5 \text{ GeV})^2. \]

To improve the matching between the two sides of the sum rule, we perform a Borel transformation, which introduces the Borel parameter \( \tau = 1/M^2 \), where \( M \) is the Borel mass. After transferring the continuum contribution to the OPE side the sum rule, in the \( g_{\mu\nu} \) structure, can be written as

\[ \lambda^2 e^{-M^2_H \tau} = \int \frac{ds}{4m_Q^2} e^{-s \tau} \rho^{OPE}(s). \tag{7} \]

To extract \( M^2_H \), we take the derivative of Eq. (7) with respect to Borel parameter \( \tau \) and divide the result by Eq. (7), so that:

\[ M^2_H = \frac{\int_0^{s_0} ds \int_0^{s_0} \rho^{OPE}(s)}{\int_0^{s_0} (\int_0^{s_0} e^{-s \tau} \rho^{OPE}(s))}. \tag{8} \]

### III. J/ψ f₀(980) MOLECULAR STATE

A possible current that couples with a J/ψ f₀(980) molecular state, with the quantum numbers \( J^{PC} = 1^{--} \), is given by:

\[ j_\mu = (\bar{c}_i \gamma_\mu c_i)(\bar{s}_j s_j). \tag{9} \]

where \( i, j \) are color indices and \( c_i, s_j \) are the charm and strange quark fields respectively. Although there are conjectures that the \( f_0(980) \) itself could be a tetraquark state \[^{29}\] in ref. \[^{30}\] it was shown that it is difficult to explain the light scalars as tetraquark states from a QCD calculation. Therefore, here we use a simple quark-antiquark current to describe the \( f_0(980) \).

Another possibility for the current is considering the vector and scalar parts in a color octet configuration:

\[ j_\mu = (\bar{c}_i \lambda^A_{ij} \gamma_\mu c_j)(\bar{s}_j \lambda^A_{ik} s_k). \tag{10} \]

where \( \lambda^A \) are the Gell-Mann matrices. The two currents can be related by the change: \( \lambda^A_{ij} \rightarrow \delta_{ij} \) with \( \lambda^A_{ij} \rightarrow \delta_{ij} \). Although the current in Eq. (10) can not be interpreted as a meson-meson current, since the vector and scalar parts carry colour, for simplicity we still call it a molecular current. Since the currents in Eqs. (9) and (10) have the lowest dimension for a four-quark current with the \( 1^{--} \) quantum numbers, from the theory of composite-operator renormalization \[^{31}\] we expect these currents to be multiplicatively renormalizable.

The spectral density \( \rho^{OPE}(s) \), for the \( J^{PC} = 1^{--} \) exotic state described by a \( J/ψ f_0(980) \) molecular current, up to dimension-six condensates, can be written as:
TABLE I: QCD input parameters.

| Parameters | Values                  |
|-----------|-------------------------|
| $m_b(m_b)$ | (4.24 ± 0.05) GeV       |
| $m_c(m_c)$ | (1.23 ± 0.05) GeV       |
| $m_s$     | (0.13 ± 0.03) GeV       |
| $\langle q\bar{q} \rangle$ | $-0.23 ± 0.01$ GeV$^3$ |
| $\langle g_s^2 G^2 \rangle$ | 0.88 GeV$^4$          |
| $\kappa \equiv \langle s\bar{s} \rangle / \langle q\bar{q} \rangle$ | (0.74 ± 0.03)         |
| $m_0^2 \equiv \langle sG_s \rangle / \langle s\bar{s} \rangle$ | 0.8 GeV$^2$          |

\[
\rho^{\text{OPE}}(s) = \rho^{\text{pert}}(s) + \rho^{\langle q\bar{q} \rangle}(s) + \rho^{\langle G^2 \rangle}(s) + \rho^{\langle qGq \rangle}(s) + \rho^{\langle q\bar{q} \rangle^2}(s).
\]

The expressions for $\rho^{\text{OPE}}(s)$ for the currents in Eqs. (9) and (10), using factorization hypothesis, are given in Appendix A.

To extract reliable results form the sum rule is necessary to stabilize the Borel window. A valid sum rule exist when one can find a Borel window where there is an OPE convergence, a $\tau$-stability and where there is a dominance of the ground state contribution. The maximum value of $\tau$ parameter is determined by imposing that the contribution of the higher dimension condensate is smaller than 20% of the total contribution: $\tau_{\text{max}}$ is such that

\[
\left| \frac{\text{OPE summed up dim n-1 (\tau_{\text{max}})}}{\text{total contribution (\tau_{\text{max}})}} \right| = 0.8.
\]

Since the continuum contribution decreases with $\tau$, due to the dominance of the perturbative contribution, the minimum value of $\tau$ is determined by imposing that the ground state contribution is equal to the continuum contribution. To guarantee a reliable result extracted from sum rules it is important that there is a $\tau$ stability inside the Borel window.

For a consistent comparison with the results obtained for the other molecular states using the QCDSR approach, we have considered here the same values used for the quark masses and condensates as in refs. [12, 32–38], listed in Table I.

We start with the current in Eq. (10). As mentioned above, the continuum threshold is a physical parameter that should be determined from the spectrum of the mesons. The value of the continuum threshold in the QCDSR approach is, in general, given as the value of the mass of the first excited state squared. In some known cases, like the $\rho$ and $J/\psi$, the first excited state has a mass approximately 0.5 GeV above the ground state mass. In the cases that one does not know the spectrum, one expects the continuum threshold to be approximately the square of the mass of the state plus 0.5 GeV: $s_0 = (M_{H} + 0.5 \text{ GeV})^2$. Therefore, to fix the continuum threshold range we extract the mass from the sum rule, for a given $s_0$, and accept such value of $s_0$ if the obtained mass is in the range 0.4 GeV to 0.6 GeV smaller than $\sqrt{s_0}$. Using this criterion, we obtain $s_0$ in the range $5.0 \leq \sqrt{s_0} \leq 5.2 \text{ GeV}$. In Fig. (1), we show the relative contribution of the terms in the OPE side of the sum rule, for $\sqrt{s_0} = 5.10 \text{ GeV}$. From this figure we see that
the contribution of the dimension-6 condensate is smaller than 20% of the total contribution for values of \( \tau \leq 0.33 \text{GeV}^{-2} \), which indicates a good OPE convergence. From Fig. 1(b), we also see that the pole contribution is bigger than the continuum contribution only for \( \tau \geq 0.28 \text{GeV}^{-2} \). Therefore, we fix the Borel window as: \((0.28 \leq \tau \leq 0.33)\text{GeV}^{-2}\). From Eq. (9), we can calculate the ground state mass, which is shown, as a function of \( \tau \), in the Fig. 1(b). From this figure we see that there is a very good \( \tau \) stability in the determined Borel window, which is shown, through the crosses, in Fig. 1(b).

Varying the value of the continuum threshold in the range \( \sqrt{s_0} = 5.10 \pm 0.10 \) GeV, and the other parameters as indicated in Table I we get:

\[
M_Y = (4.67 \pm 0.09) \text{ GeV}.
\]  

This mass is not compatible with the proposition in [13], which describes the \( Y(4260) \) state as the \( J/\psi f_0(980) \) molecular state. By the other hand, this result is in an excellent agreement with the mass of the \( Y(4660) \) state. The obtained mass is largely above the \( J/\psi f_0(980) \) threshold and, therefore, such molecular state would not be bound. One has to remember, however, that the current in Eq (9) is written in terms of the currents that couples with the \( J/\psi \) and \( f_0(980) \) mesons, but it also couples with all excited states with the \( J/\psi \) and \( f_0(980) \) quantum numbers. From the QCDSR analysis presented here we can only warranty that the mass in Eq. (13) is the mass of the ground state of all states described by the current in Eq (9), but not that its constituents, described by the \( \bar{c}\gamma_{\mu}c \) and \( \bar{s}\gamma_5s \) currents, are the ground states of these currents: the \( J/\psi \) and \( f_0(980) \) mesons. Therefore, it is possible that the mass obtained in Eq (13) describes a \( \psi' f_0(980) \) molecular state, since the \( \psi' f_0(980) \) threshold is at 4.66 GeV, compatible with a loosely bound state. The interpretation of the \( Y(4660) \) as a \( \psi' f_0(980) \) molecular state was first proposed in ref. [39] and is also in agreement with the \( Y(4660) \) main decay channel: \( Y(4660) \rightarrow \Psi(2S) \pi^+\pi^- \). It is also important to mention that our result indicates that, from a QCDSR point of view, there is no \( J/\psi f_0(980) \) bound state.

In the case of the current in Eq. (10) we obtain \( s_0 \) in the range \( 5.4 \leq \sqrt{s_0} \leq 5.6 \) GeV. The results for this current are shown in Fig. 2 from where we can see that, for \( \sqrt{s_0} = 5.50 \) GeV, the Borel window is fixed as: \((0.23 \leq \tau \leq 0.33)\text{GeV}^{-2}\). Varying the continuum threshold in the range \( \sqrt{s_0} = 5.50 \pm 0.10 \) GeV, and the other parameters as indicated in Table I we get:

\[
M_{\chi_Y} = (5.00 \pm 0.10) \text{ GeV}.
\]

![Fig. 2](image)

**FIG. 2:** \( J/\psi f_0(980) \) molecule in a color octet configuration. a) OPE convergence in the region \( 0.13 \leq \tau \leq 0.37 \text{GeV}^{-2} \) for \( \sqrt{s_0} = 5.50 \text{GeV} \). We plot the relative contributions starting with the perturbative contribution (dot-dashed line), and each other line represents the relative contribution after adding of one extra condensate in the expansion: \(+\langle\bar{s}s\rangle\) (solid line), \(+\langle G^2 \rangle\) (long-dashed line), \(+\langle\bar{s}Gs\rangle\) (dotted line) and \(+\langle\bar{s}s\rangle^2\) (dashed line). b) The pole (solid line) and continuum (dotted line) contributions for \( \sqrt{s_0} = 5.50 \text{GeV} \). c) The mass as a function of the sum rule parameter \( \tau \) for \( \sqrt{s_0} = 5.40 \text{GeV} \) (dotted line), \( \sqrt{s_0} = 5.50 \text{GeV} \) (solid line) and \( \sqrt{s_0} = 5.60 \text{GeV} \) (dot-dashed line). The crosses indicate the valid Borel window.

This value for the mass is not compatible with any observed charmonia state. Besides, comparing the results in Eqs. (13) and (14), we conclude that a molecular state with \( \bar{c}\gamma_{\mu}c \) and \( \bar{s}s \) in color octet configurations has a bigger mass than the similar state when \( \bar{c}\gamma_{\mu}c \) and \( \bar{s}s \) are in a color singlet configuration. This result is the opposite than the result obtained in ref. [40] for a \( J/\psi\pi \) current. However, in ref. [40] the same range of the continuum threshold was used for both currents. In the present case...
we see that if we use $\sqrt{s_0} = 5.10 \pm 0.10$ GeV for the current in Eq. (10), we do not find a Borel window. This is the reason why we had to work with bigger values of $s_0$ for the current in Eq. (10).

IV. $J/\psi \sigma(600)$ MOLECULAR STATE

It is straightforward to extend the study presented in the above section for the non-strange case. To do that one only has to use $\langle \bar{s}s \rangle = \langle \bar{q}q \rangle$ and $m_s = 0$ in the spectral density expressions given in appendix A. In this case, to obtain a Borel window we need to release the condition in Eq. (12) and allow $\tau_{\text{max}}$ to be determined by imposing that the dimension-6 condensate could be 25% of the total contribution. This indicates that the OPE convergence is worse in this case as compared with the $J/\psi f_0(980)$ case. This is due to the fact that the dimension-3 and dimension-5 condensates do not contribute in this case. In Table II we present the result obtained for the mass with the two currents, together with the used continuum threshold range.

| TABLE II: Results for the $J/\psi \sigma$ currents. |
|---------------------------------------------------|
| Current in Eq. | $M_H$ (GeV) | $\sqrt{s_0}$ (GeV) |
|----------------|-------------|-------------------|
| (9)            | 4.63 ± 0.10 | 5.1 ± 0.1         |
| (10)           | 4.97 ± 0.08 | 5.5 ± 0.1         |

As one can see from Table III, the results obtained with the $J/\psi \sigma(600)$ molecular current are in agreement with the results obtained with the $J/\psi f_0(980)$ current, which is not what one would naively expect. However, this kind of findings is not uncommon in QCDSR calculations for multi-quark states [32]. Again, since the masses obtained are largely above the $J/\psi \sigma(600)$ threshold, we conclude that there is no $J/\psi \sigma(600)$ bound state. In this case, since the mass obtained is also above the $\psi' \sigma(600)$ threshold, we cannot interpret the $Y(4660)$ as a $\psi' \sigma(600)$ molecular state, despite the fact that the obtained mass is in agreement with the $Y(4660)$ mass.

V. $Y f_0(980)$ AND $Y \sigma(600)$ MOLECULAR STATES

It is also straightforward to extend the previous study to the b-sector. To do that one only has to make the change $m_c \rightarrow m_b$ and for the non-strange case use $m_s = 0$ in the spectral density expressions given in the appendix A. This allows us to study the molecular currents: $Y f_0(980)$ and $Y \sigma(600)$, in both, color singlet and color octet configuration, as given in Eqs. (9) and (10). We obtain similar OPE convergence and pole dominance as in the charm sector.

| TABLE III: Results for the $Y f_0$ and $Y \sigma$ currents. |
|-------------------------------------------------------|
| States | $M_H$ (GeV) | Borel Window $\sqrt{s_0}$ (GeV$^{-2}$) |
|---------|--------------|------------------------------------------|
| Color Singlet |
| $Y f_0(980)$ | 10.75 ± 0.12 | 0.11 $\leq \tau \leq 0.15$ | 11.3 ± 0.1 |
| $Y \sigma(600)$ | 10.74 ± 0.09 | 0.11 $\leq \tau \leq 0.13$ | 11.3 ± 0.1 |
| Color Octet |
| $Y f_0(980)$ | 11.09 ± 0.10 | 0.10 $\leq \tau \leq 0.13$ | 11.7 ± 0.1 |
| $Y \sigma(600)$ | 11.09 ± 0.10 | 0.10 $\leq \tau \leq 0.13$ | 11.7 ± 0.1 |

In Table III we present the results obtained for the masses of the states described by $Y f_0(980)$ and $Y \sigma(600)$ currents, together with their respective continuum threshold range and valid Borel window. From this Table we see that, as in the charm sector, the masses obtained with the $Y f_0$ and $Y \sigma$ currents are very similar and that the relative differences with the singlet and octet currents are smaller than in the charm sector. This is also consistent with the findings in ref. [41], where different $1^{--}$ tetraquark currents were used, in a QCDSR calculation, with similar results for the different currents and also for non-strange and strange sectors. Considering the errors, all the masses obtained with these currents are compatible with the mass of the recently observed $Y_b(10890)$ state. However, since the $Y(1S) f_0$ and $Y(2S) f_0$ thresholds are at 10.44 GeV and 11.00 GeV respectively, and that the thresholds with $\sigma$ are around 380 MeV below these numbers, the only possible molecular interpretation for the $Y_b(10890)$ is that it could be a $Y(2S) f_0$ molecular state.

VI. TETRAQUARK CURRENT FOR THE $Y_b(10890)$

In ref. [21] the $Y(10890)$ was interpreted as a bound tetraquark state $[bq][\bar{b}q] = Q\bar{Q}$ with the spin and angular momentum quantum numbers: $S_Q = 0$, $S_{\bar{Q}} = 0$, $S_{Q\bar{Q}} = 0$, $L_{Q\bar{Q}} = 1$. This same configuration was used in a QCDSR calculation [22] and the obtained mass was $10.88 \pm 0.13$ GeV, in a very good agreement with the $Y(10890)$ mass. In this section we want to check if the tetraquark current
constructed with scalar and vector diquarks:

\[
J_\mu^Y = \frac{\epsilon_{ijk}\epsilon_{lmn}}{\sqrt{2}} \left((q_i^T C \gamma_5 b_j)(\bar{q}_l \gamma_\mu \gamma_5 C \bar{r}_m^T)
+ (q_i^T C \gamma_5 \gamma_\mu b_j)(\bar{q}_l \gamma_5 C \bar{r}_m^T)\right)
\]

(15)

can also be used to describe the \(Y(10890)\). In Eq. (15) \(i, j, k, \ldots\) are color indices, \(C\) is the charge conjugation matrix, \(q = u, d, s\) is the light quark field and \(b\) is the quark bottom field. Notice that the main decay channel: \(Y_b \to Y(1S)\pi^+ \pi^-\), does not necessary indicate that \(Y_b\) has only light-quarks in its composition, if it is interpreted as a four-quark state. In fact, it is very interesting to investigate any possibility to the quark content for this tetraquark state. The expressions for \(\rho^{\text{OPE}}(s)\) for the current in Eq. (15) are given in appendix A.

We consider first the \([bq][\bar{b}q]\) tetraquark state and the results are shown in Fig. 3. As explained above, we extract the mass from the sum rule, for a given \(s_0\), and accept such value if the obtained mass is around \(\sqrt{s_0} - 0.5\) GeV. Using this criteria.
we got $11.4 \text{ GeV} \leq \sqrt{s_0} \leq 11.6 \text{ GeV}$.

Varying the value of the continuum threshold in the range: $\sqrt{s_0} = 11.50 \pm 0.10 \text{ GeV}$, and taking into account the uncertainties as indicated in Table I, we get:

$$M_{Y_b} = (10.91 \pm 0.07) \text{ GeV} , \quad (16)$$

which is in an excellent agreement with the observed mass for $Y_b(10890)$ Therefore, we conclude that this state could also be described as a $[bq][\bar{b}q]$ tetraquark state, in a scalar-vector configuration.

Considering the strange quark mass in the current $[15]$ and doing the same analysis as before, we obtain the results shown in Fig. 4 for the $[bs][\bar{b}s]$ tetraquark state. The valid Borel window in this case is: $(0.10 \leq \tau \leq 0.15) \text{ GeV}^{-2}$. Varying the continuum threshold in the range: $\sqrt{s_0} = 11.60 \pm 0.10 \text{ GeV}$, and the parameters as indicated in Table I, we get:

$$M_{Y_{ss}} = (10.97 \pm 0.10) \text{ GeV} . \quad (17)$$

In this case the differences in the masses obtained for the non-strange and strange exotic tetraquark states are larger than is the case of the $\Upsilon(1S)$ and $\Upsilon(2S)$ molecular currents. Therefore, comparing the results presented in Table III and in Eqs. (16) and (17), we conclude that the $Y_b(10890)$ is better described by a $[bq][\bar{b}q]$ tetraquark current, as suggested in ref. [21].

VII. CONCLUSIONS

We have studied the mass of the $1^{-+}$ exotic states using QCD sum rules. We find that the molecular currents $J/\psi f_0(980)$ and $J/\psi \sigma(600)$ lead to (almost) the same mass predictions, the difference being only around 30 MeV. The mass obtained for the molecular state is smaller when the two constituents mesons are in the color singlet configuration.

Finally we have studied the scalar-vector tetraquark current in the strange and no-strange sectors. We find that the newly observed $Y_b(10890)$ state can be well described by a $[bq][\bar{b}q]$ tetraquark current, as suggested in ref. [21]. However, considering the uncertainties, the $f_0$ and $\sigma$ molecular currents cannot be discarded. More experimental data on the $Y_b(10890)$ decay channels could be used to discriminate between different assignments.

Acknowledgements

This work has been partly supported by FAPESP and CNPq-Brazil. We would like to thank Prof. S. Narison for bringing our attention to the work in ref. [21] and for discussions at the beginning of this calculation.

Appendix A: Spectral Densities

The spectral densities expressions for $J/\psi f_0(980)$, $J/\psi \sigma(600)$, $\Upsilon f_0(980)$ and $\Upsilon \sigma(600)$ molecular currents, were calculated up to dimension-6 condensates, in the $g_{\mu\nu}$ structure, at leading order in $\alpha_s$. We have kept terms which are linear in the strange quark mass $m_s$. To keep the heavy quark mass finite, we use the momentum-space expression for the heavy quark propagator. We calculate the light quark part of the correlation function in the coordinate-space, and we use the Schwinger parameters to evaluate the heavy quark part of the correlator. To evaluate the $d^4x$ integration in Eq. (1), we use again the Schwinger parameters, after a Wick rotation. Finally we get integrals in the Schwinger parameters that can be performed after scaling these parameters and introducing a delta function in the scale parameter $\mu$. The result of these integrals are given in terms of logarithmic functions, from where we extract the spectral densities and the limits of the integration. The same technique can be used to evaluate the condensate contributions. For that we have only to use the OPE expansion for the propagators. For the light quark propagator we use [2]:

$$S_{ab}(x) = \langle 0T[q_a(x)\bar{q}_b(0)]0 \rangle = \frac{i\delta_{ab}}{2\pi^2 x^4} \delta^4(x^2) - \frac{\alpha_s}{32\pi^2} \frac{G_{\mu\nu}^A}{x^2} \frac{(i\sigma^{\mu\nu} + \sigma^{\mu\nu} x^\mu x^\nu)}{x^2} - \frac{\delta_{ab}}{12} \langle \bar{q}q \rangle + \frac{i\delta_{ab}}{48} m_q \langle \bar{q}q \rangle x^2 \frac{\delta(x^2)}{2^6} \frac{G_{\mu\nu}}{x^2}$$
where we have used the fixed-point gauge. For heavy quarks, as explained above, we work in the momentum space and the OPE expansion for the heavy quark propagator is given by:

\[
S_{ab}(p) = i \frac{p^\mu + m}{p^2 - m^2} \delta_{ab} + \frac{i \lambda^\mu_{ab} g G_\mu^\lambda}{4 (p^2 - m^2)^2} [\sigma^{\mu\nu}(p + m) + (p + m)\sigma_{\mu\nu}] + i \frac{\delta_{ab} m (g_s^2 G^2)}{12 (p^2 - m^2)^2}.
\]  

(A2)

We have considered the compact notation for both color singlet and octet configurations, since the difference between them is only proportional to a color factor. For this we define:

| Color Factor | Mesons in | Mesons in |
|--------------|-----------|-----------|
|               | Color Singlet | Color Octet |
| \(N\)         | 9/25      | 1         |
| \(N^*\)       | -9/2²     | 1         |

where we have normalized the factor to the color octet configuration. Therefore, we obtain the following expressions:

\[
\rho_M^{(G)}(s) = \frac{N}{3^2 \cdot 2^2 \pi^2} \int \frac{\alpha}{\alpha} \int \frac{\beta}{\beta} \int \frac{\gamma}{\gamma} (1 - \alpha - \beta) F_Q^2(\alpha, \beta)
\]

\[
\times \left[(1 + \alpha + \beta) F_Q(\alpha, \beta) - 4 m_Q^2 (1 - \alpha - \beta)\right],
\]

\[
\rho_M^{(\bar{q}q)}(s) = \frac{N m_Q(s) \bar{s}}{4 \pi^4} \left\{ \int \frac{\alpha}{\alpha} \int \frac{\beta}{\beta} \int \frac{\gamma}{\gamma} \left[ F_Q(\alpha, \beta) - 2 m_Q^2 \right] \right\},
\]

\[
\rho_M^{(G^2)}(s) = \frac{- (g_s^2 G^2)}{3^2 \cdot 2^2 11 \pi^6} \left\{ \int \frac{\alpha}{\alpha} \int \frac{\beta}{\beta} \int \frac{\gamma}{\gamma} (1 - \alpha - \beta) \right\}
\]

\[
\times \left\{ 6 N^* \int \frac{\alpha}{\alpha} \int \frac{\beta}{\beta} \left[ H_Q^2(\alpha) \right] + 32 N m_Q \right\}
\]

\[
\times \left\{ (3 F_Q(\alpha, \beta) + m_Q^2 \beta) - \beta (1 + \alpha + \beta) F_Q(\alpha, \beta) \right\}
\]

\[
\times \left\{ (3 - \alpha - \beta) + 6 m_Q^2 (2 + \alpha + \beta) \alpha \beta - 6 \alpha^2 \beta^2 \right\}.
\]

The spectral densities expressions for \([bq][\bar{b}q]\) and \([bs][\bar{b}s]\) tetraquark states with the current defined in Eq. (13) are given by:

\[
\rho_T^{(G)}(s) = \frac{1}{3 \cdot 2^{10} \pi^6} \int \frac{\alpha}{\alpha} \int \frac{\beta}{\beta} \int \frac{\gamma}{\gamma} (1 - \alpha - \beta) F_Q^2(\alpha, \beta)
\]

\[
\times \left\{ 2 m_Q^2 (1 - \alpha - \beta) - 3 (1 + \alpha + \beta) F_Q(\alpha, \beta) \right\}
\]

\[
\times \left\{ \int \frac{\alpha}{\alpha} \int \frac{\beta}{\beta} \int \frac{\gamma}{\gamma} \left[ m_Q^2 (5 - \alpha - \beta) + 2 F_Q(\alpha, \beta) \right] \right\}
\]

\[
\rho_T^{(G^2)}(s) = \frac{- (g_s^2 G^2)}{3^2 \cdot 2^2 11 \pi^6} \int \frac{\alpha}{\alpha} \int \frac{\beta}{\beta} \int \frac{\gamma}{\gamma} (1 - \alpha - \beta)
\]

\[
\times \left\{ 2 m_Q^2 (1 - \alpha - \beta) - 2 (1 + \alpha + \beta) \right\}
\]

\[
\times \left\{ \int \frac{\alpha}{\alpha} \int \frac{\beta}{\beta} \int \frac{\gamma}{\gamma} \left[ (1 - \alpha - \beta) + 2 \alpha (\alpha + \beta) \right] \right\}
\]

\[
\rho_T^{(\bar{q}q)}(s) = \frac{m_Q(s) \bar{s}}{2^{9} \pi^4} \left\{ \int \frac{\alpha}{\alpha} \int \frac{\beta}{\beta} \int \frac{\gamma}{\gamma} \left[ 8 m_Q^2 \alpha - (\alpha - 1) s \right] \right\}
\]

\[
\times \left\{ \int \frac{\alpha}{\alpha} \int \frac{\beta}{\beta} \int \frac{\gamma}{\gamma} \left[ 8 m_Q^2 \alpha - (\alpha - 1) s \right] \right\}
\]

\[
\times \left\{ \int \frac{\alpha}{\alpha} \int \frac{\beta}{\beta} \int \frac{\gamma}{\gamma} \left[ 8 m_Q^2 \alpha - (\alpha - 1) s \right] \right\}
\]

\[
\times \left\{ \int \frac{\alpha}{\alpha} \int \frac{\beta}{\beta} \int \frac{\gamma}{\gamma} \left[ 8 m_Q^2 \alpha - (\alpha - 1) s \right] \right\}
\]

In all expressions we have used the following definitions:

\[
F_Q(\alpha, \beta) = m_Q^2 (\alpha + \beta) - \alpha \beta s,
\]

\[
H_Q(\alpha) = m_Q^2 - \alpha (1 - \alpha) s
\]
and the integration limits are given by:

\[ \alpha_{\text{min}} = (1 - \sqrt{1 - 4m_Q^2/s})/2, \quad (A5) \]

\[ \alpha_{\text{max}} = (1 + \sqrt{1 - 4m_Q^2/s})/2, \quad (A6) \]

\[ \beta_{\text{min}} = \alpha m_Q^2/(\alpha s - m_Q^2). \quad (A7) \]

The index \( Q = c \) or \( b \) indicates the heavy quark content in the current. We have neglected the contribution of the dimension-six condensate \( \langle g^2 G^3 \rangle \), since it is assumed to be suppressed by the loop factor \( 1/16\pi^2 \).

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