Linear Programming Model for Investment Problem in Maximizing the Total Return

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Abstract. In this paper, we concentrate on the investment problem of fixed deposit (FD). Our problem is to allocate the amount invested to a suitable tenure and obtain the optimal investment. There are two types of maturity that need to be considered which are short-term and long-term. Our objective is to maximize the total return of the total amount invested with a different percentage of annual return. A linear programming (LP) model is proposed to solve this investment problem using scheduling methodology. We conduct a computational experiment of a real case study for one company located in Kuala Lumpur with RM 20.6 million of investment to see the performance of the model by using the Excel Solver Parameter package. The results show that a significant improvement obtains by our model compared to the original investment practice by the company.

1. Introduction

Nowadays, there are wide ranges of investment alternatives. For instance, deposits, equity shares, mutual fund scheme, insurance products and real estate [1]. Fixed deposit (FD) is a financial tool offered by the banks which provides more profit than a saving account. In some country, FD known as term deposit or time deposit. Investment in a FD is the most common ways of receiving passive income. FD accounts are a very liquid investment and less risky compared to other investments. FD has maturity date and the interest rate are varies based on tenure. Therefore, financial planning is noteworthy in turn to demand highest of yearly return from the total amount invested.

One of the challenges in FD investment is the ability to make a right decision when there are many possible alternatives and options. The issues that always rise are: (1) the optimal duration for the investor to gain the profit and (2) the allocation amount to a particular period. In a traditional way, a company will decide on the FD investment through manual trial-and-error calculations to obtain the highest possible return. There will be a lot of possibilities that need to be considered in term of duration and amount before obtain a right decision. Therefore, in this paper, we consider the the existence of different length of tenure in FD investment problem. The different tenure relates with type of maturity where it will give impact to the total annual return received. Our model will give an automated optimal value for
the amount that should be invested based on the preferences by the company. Our objective is to schedule the fixed deposit to the tenure that maximize the total annual return.

To address this issue, a new methodology is applied. A scheduling techniques in parallel processing system has been considered. Many parallel processor scheduling problems are NP-hard. Scheduling is a method of assigning a number of tasks to process for processing. A scheduling system can be considered as consisting of a set of consumers, a set of resources and a scheduler. In this investment problem, the ‘consumers’ are the amount of the investment and the ‘resources’ are the different tenure. The strategy is to have a scheduler that generates a feasible allocation for the consumer (amount) to the resources (tenure).

A number of different types of scheduling in parallel processors has been studied including identical parallel processors ($P$), unrelated parallel processors ($R$) and uniform parallel processors ($Q$). Each types have different characteristics and specific environment need to be applied. In the application of FD investment problem, unrelated parallel processor system has the most similar features that can be used. In the case the processor are unrelated, the consumer (amount) can be differ for each resources (tenure). There is a representation scheme introduced by Graham et al. [2] to describe the problem in three-field notation $\alpha|\beta|\gamma$ where $\alpha$ represent the processor environment, $\beta$ indicated the task characteristics and $\gamma$ shows the performance criteria. The three field notations for the FD investment problem can be denoted as $R|A_{\text{max}}$, i.e the investment problem in maximizing the total annual return, $A_{\text{max}}$, on unrelated parallel processors characteristics, $R$. We develop a linear programming assignment model to solve the problem $R|A_{\text{max}}$.

This paper is organized as follows. Section 2 presents the literature review of the works in the investment problem. Then, the detail of the investment problem is discussed with the mathematical formulation. A case study of the model will be conducted and the results will be discussed.

2. Literature Review
Managing income is more effectively through planning. Investment is one technique to provide financial security. Therefore, the knowledge of optimization problem in investment financial market was studied many years ago [3],[4],[5],[6] until present [7],[8],[9]. There are many approached in the optimization for the investment problem. For example, Drexl and Kimms [10], used lagrangean relaxation and column generation techniques in minimizing the cost in the resource’s investment problem. They considered an issue of providing resources to a project that deal with the deadline to be met. Grigorij [11] suggested investment portfolio rebalancing decision making method that developed two main portfolio characteristics which are expected return and risk. Jay et al.[12] used linear programming in the selection of subset projects that optimizing the profit goal in the exploration and production of oil and gas industry. Another approach that has been developed is stochastic programming by Anton et al.[13] after considering the uncertainty of the future returns. The authors aimed to minimize the maximum downside semi deviation of the risk selection model.

Scheduling is one of the methodology that has been implemented in investment problem. Several studies have been applied this approach [14],[15],[16]. Leyman & Vanhoucke [14] considered a resources investment problem with objective function of project net present value maximization with discounted cash flow. They consider the cash inflow and outflow at the completion time for resources-constraint project scheduling problem. Fatemeh et al. [15] proposed a water pipe replacement scheduling plan of annual investment time-series to obtain efficient budget limit. Their objective functions are to minimize the life cycle cost and have annual investment smoothing. Mathias & Martin [16] presented a few market model for transportation sector especially in airline industry. They provide a new model framework on optimal long run investment of aircraft scheduling. They elaborate on optimal fleet investment that lead to future profits of airlines revenue for flight tickets. Based on the literature review, we address scheduling technique in investment problem specifically in fixed deposit fund. An optimal mathematical model will be developed in solving both tenure in the FD which are short and long term. Under the short term tenure, the maturity of the investment is between 3 to 6 months while the long term
will be up to 12 months. The tenure consideration is important for optimizing the investment by maximizing the annual profit. Then, we will describe the details of our problem involving tenure feature in the investment problem using scheduling in parallel processor approach.

3. Mathematical Formulation

In general, the scheduling problem consists of a set of consumers that get processes by a set of parallel resources. Therefore, the scheduling approach that have been considered in this investment problem is the application of the scheduling model that involve amount of investment as tasks. The tenure with respective return are assumed as unrelated parallel processor with processing time.

In this section, we will develop a model for investment problem that we have adopted and converted to $R|A_{\text{max}}$ problem. We modified from the general formulation of task scheduling on unrelated parallel processor in minimizing the makespan that denoted as $R|C_{\text{max}}$ [16]. In order to strengthen the quality of the model, a few assumptions and requirements from the department should be taken as priority. Therefore, the following are the assumptions that has been made:

a. All RM20.6 million must be deposit from the first month.
b. Only one cycle will be considered i.e. 12 months duration.
c. The allocation of the money is compounding for one cycle.
d. No premature withdrawal is allowed.
e. Withdrawal of the total return is considered for each mature tenure.

3.1. Linear Programming (LP) model

The following notation is used for the problem under consideration.

Set:
- $N$ – set of FD
- $S$ – set of short term FD
- $L$ – set of long term FD
- $s_j$ – set of selected short term FD $j \in S, s_j \subset N$
- $l_k$ – set of selected long term FD $k \in L, l_k \subset N$
- $P$ – set of allocation percentage

Parameters:
- $i$ – type of FD for $i = 1, 2, \ldots, n$ where $i \in N$
- $p_m$ – fixed allocation percentage for $m = 1, 2, \ldots, M$
- $A$ – total amount invested
- $Q$ – total amount required
- $C$ – total annual cost
- $r_i$ – percentage of annual return for $i = 1, 2, \ldots, n$ where $i \in N$

Variables:
- Decision variables, $x_i$ – amount of money to invest in FD$_i$
- Objective function, $f(x_i)$ – maximize the total annual return

The formulation for the $R|A_{\text{max}}$ problem can be written as the LP model with the following objective function and subject to constraint (1) – (5):

$$\text{maximize } f(x_i) = \sum_{i=1}^{n} r_i x_i$$
The objective function of the model is to maximize the percentage of annual return with the invested amount of money.

$$\sum_{i=1}^{n} x_i = A, \forall i \in N$$  \hspace{1cm} (1)

Constraint (1) is to ensure that all money will be invested in the FD.

$$\sum_{j=1}^{n} s_j x_j \geq p_m A, \forall j \in S, s_j \subset N, p_m \in P$$ \hspace{1cm} (2a)

or

$$\sum_{j=1}^{n} s_j x_j \leq p_m A, \forall j \in S, s_j \subset N, p_m \in P$$ \hspace{1cm} (2b)

Constraint (2a) and (2b) are to ensure that the investment is allocated for short-term issues. Constraint (2a) is to ensure that the allocation percentage for short term is at least $p_m$ and constraint (2b) is to ensure that the allocation percentage for short term is not more than $p_m$.

$$\sum_{k=1}^{n} l_k x_k \geq p_m A, \forall k \in L, l_k \subset N, p_m \in P$$ \hspace{1cm} (3a)

or

$$\sum_{k=1}^{n} l_k x_k \leq p_m A, \forall k \in L, l_k \subset N, p_m \in P$$ \hspace{1cm} (3b)

Constraint (3a) and (3b) are to ensure that the investment is allocated for long-term issues. Constraint (3a) is to ensure that the allocation percentage for long term is at least $p_m$ and constraint (2b) is to ensure that the allocation percentage for long term is not more than $p_m$.

$$\sum_{i=1}^{n} r_i x_i \geq C, \forall i \in N$$ \hspace{1cm} (4)

Constraint (4) is to guarantee that the total annual return must exceed the total annual cost.

$$x_i, x_j, x_k \geq 0, \forall i \in N, \forall j \in S, \forall k \in L$$ \hspace{1cm} (5)

Constraint (5) is to ensure the value obtained is positive and at least equal to 0.

4. Case Study

This section carries out a computational testing of our preliminary LP model to see how well the model performs. We implement a case study to validate the performance of the model. Then, we compare the results obtained and disclose the best solution.
The case study is intended to demonstrate the whole process for selection in different type of periods. The case study is taken from a company in Kuala Lumpur. The case study consists of four types of FD that we reflect as tasks. The tasks need to be consigned to four different tenure. These four durations represent the four unrelated parallel processors. It has been expressed by two type of maturity. The job data for the investment is indicated in Table 1. There are four type of the FD. The annual return $r_i$ displayed are considers as the value for the processing time for each task. Noted that the percentage of annual return given will have varying levels of required minimum deposits depends on the bank. We classified the 3 to 6 months tenure by short term and 6 to 12 month as long term based on the company description.

| FD  | Tenure (months) | Annual Return (%) | Maturity |
|-----|-----------------|-------------------|----------|
| FD_A | 3               | 3.85              | Short    |
| FD_B | 6               | 3.90              | Short    |
| FD_C | 9               | 3.95              | Long     |
| FD_D | 12              | 4.00              | Long     |

4.1. Computational Result
We now demonstrate our result of the LP model. The LP model has been implemented and compiled using the Simplex LP model that run with Excel Solver Parameter package. The LP offers the optimum result for the instance problem that has RM 20.6 million of investment. This model capable to give the automated optimal results for the allocation amount to a particular duration and hence, maximize the total return. There are five pairs of weightages for selecting the short-term $S$ and long-term $L$ maturity of the investment: \{1,0\}, \{≥0.5, <0.5\}, \{<0.5, ≥0.5\}, \{≥0.25, <0.25\}, \{<0.25, ≥0.25\}.

4.1.1. Case 1. In case 1, the preference is 100% of the investment is in short-term maturity. Thus, only the condition of short term FD $s_j$ for $j \in S$ will be selected i.e $s_1 = 1$ and $s_2 = 1$. The values of $l_k$ for $k \in L$ are all 0. $p_m$ is notation used in constraint (2) and (3) that need to be satisfied for allocation percentage condition for $m = \{1, 2\}$ where $M = 2$. The inputs of allocation percentage for $p_m$ are $p_1 = 100$ and $p_2 = 0$ where $p_1, p_2 \in P$.

From the computational results, the total amount for each FD is $x_1 = 0$, $x_2 = 2,060,000$, $x_3 = 0$, $x_4 = 0$. The value $x_2 = 2,060,000$ means that, RM20.6 million is suggested to be compounded from the first month until six month of tenure. Here, the objective value of $A_{max}$ is 803,400. For easy understanding, the optimal solutions of the required value of the investment for every FD are given in Table 2. The result of the total amount invested is 2,060,000 on FD_B.

Table 2. The result of the amount invested to the tenure with ratio of \{1,0\}
Case 2. In case 2, the condition for the investment is at least 50% of the investment must be made in short-term basis and not more than 50% is in long-term issues. The short-term \( s_j \) \( \forall j \in S \) and long-term \( l_k \) \( \forall k \in L \) are given as \( s_1 = 1, s_2 = 1, l_1 = 1 \) and \( l_2 = 1 \). Allocation percentage condition \( p_m \) are now \( p_j \) and \( p_2 \) are both equal to 50 \( \forall p_j, p_2 \in P \).

After all the parameters substituted in the model, we can realize that the results obtained are \( x_1 = 0, x_2 = 1,030,000, x_3 = 0, x_4 = 10,299,917.60 \). Consequently, the optimal objective function \( f(x_i) \) has a value of 813,696.70. We can conclude that, for this preferences, the company should invests RM1.03 million from the first month up to six months to gain the return and another RM10,299,917.60 need to compounded until 12 month i.e. the model proposed the selection of FDB and FDC respectively. For FDB and FDC the company received RM401,700 and RM411,996.70 respectively. Hence, the total return obtained is RM813,696.70 for cycle one. The results are illustrated clearly in Table 3.

Table 3. The result of the amount invested to the tenure with ratio of \( \{ \geq 0.5, < 0.5 \} \)

Case 3. In case 3, the requirement is not more than 50% of the seed amount need to invest in short-term and at least 50% in long-term issues. Now, the value of \( s_j \) \( \forall j \in S \) are all 1 for \( j = \{1, 2\} \) and the value for \( l_k \) \( \forall k \in L \) also equivalent to 1 for \( k = \{1, 2\} \). The allocation percentage \( p_m \in P \) for \( m = \{1, 2\} \) where \( M = 2 \) have the input parameters which are \( p_j = 50 \) and \( p_2 = 50 \).

The results obtained are 0 for \( x_1, x_2 \) and \( x_3 \) as displayed in Table 4. In this case, we achieved \( x_4 = 20,600,000 \). Hence, the total annual return \( A_{max} \) is 824,000 on FDD. For the case 3 preference, the optimal decision making is to compounding the RM 20.6 million until 12 months and received the total return of RM824,000. The results obtained from the LP model displayed in Table 4.

Table 4. The result of the amount invested to the tenure with ratio of \( \{ < 0.5, \geq 0.5 \} \)

Case 4. In case 4, the investment constraint is need to be at least 25% in both short-term and not more than 25% in both long-term issues. So, value of short-term \( s_j = 1 \) for \( \{ j | j = 1, 2, 3, 4 \in S \} \)
and long-term \( l_k = 1 \) for \( \{ k \mid k = 1, 2, 3, 4 \} \in L \). The input percentages for \( p_m \in P \) are \( p_1 = 25 \), \( p_2 = 25 \), \( p_3 = 25 \), \( p_4 = 25 \).

The model attained \( x_i = 5,150,000 \forall \{ i \mid i = 1, 2, 3, 4 \} \in N \) as the optimal solution for the preferences. Accordingly, \( f(x_i) = 808,550 \). Each FD has equivalent amount of investment. The constraint lead to a decision where each RM5.15 mil is compounding from the first month until the third, sixth, ninth and twelfth month respectively. For FDA, the return received is RM198,275, followed by FDB that obtained RM200,850. The following FD_C and FD_D achieved RM203,425 and RM206,000. The results for these constraints are shown in Table 5.

**Table 5.** The result of the amount invested to the tenure with ratio of \( \{ \geq 0.25, < 0.25 \} \)

| Bond | Tenure (months) | Amount Invested | Return | Maturity | Invest\(\text{7} \) (1-year, 0-no) | Invest\(\text{7} \) (1-year, 0-no) | Invest\(\text{7} \) (1-year, 0-no) | Invest\(\text{7} \) (1-year, 0-no) |
|------|----------------|-----------------|--------|----------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| A    | 3              | RM5,150,000.00  | 3.85%  | Short    | 1                             | 0                             | 0                             | 0                             |
| B    | 6              | RM5,150,000.00  | 3.90%  | Short    | 0                             | 1                             | 0                             | 0                             |
| C    | 9              | RM5,150,000.00  | 3.95%  | Long     | 0                             | 0                             | 1                             | 0                             |
| D    | 12             | RM5,150,000.00  | 4%     | Long     | 0                             | 0                             | 0                             | 1                             |
| Total Invested: | RM20,600,000.00 | Total Required: | RM5,150,000.00 | RM5,150,000.00 | RM5,150,000.00 | RM5,150,000.00 | RM5,150,000.00 | RM5,150,000.00 |
| Total Available: | RM20,600,000.00 | Total Required: | RM5,150,000.00 | RM5,150,000.00 | RM5,150,000.00 | RM5,150,000.00 | RM5,150,000.00 | RM5,150,000.00 |
| Total Return: | RM808,550.00 | (at least) | (at least) | (not more) | (not more) | (at least) | (at least) |

4.1.5. **Case 5.** In case 5, the limitation for the investment is not more than 25% can be invested in both short-term and at least 25% in both long-term. The short-term \( s_j \forall j \in S \) and long-term \( l_k \forall k \in L \) are given as \( s_1 = s_2 = s_3 = s_4 = 1 \) and \( l_1 = l_2 = l_3 = l_4 = 1 \) respectively. The allocation of \( p_m \in P \) are given by \( p_1 = p_2 = p_3 = p_4 = 25 \).

The model found that \( x_1 = 0 \), \( x_2 = 0 \), \( x_3 = 5,150,000 \), \( x_4 = 15,450,000 \) and gained 821,425 for total annual return after invested in FD_C and FD_D. At the end of 9-months term, the company can withdraw RM203,425. For another RM15.45mil that has been compounded up to 12 months, the generated return after the mature term is RM618,000. The computational result for case 5 is displayed in Table 6.

**Table 6.** The result of the amount invested to the tenure with ratio of \( \{ < 0.25, \geq 0.25 \} \)

| Bond | Tenure (months) | Amount Invested | Return | Maturity | Invest\(\text{7} \) (1-year, 0-no) | Invest\(\text{7} \) (1-year, 0-no) | Invest\(\text{7} \) (1-year, 0-no) | Invest\(\text{7} \) (1-year, 0-no) |
|------|----------------|-----------------|--------|----------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| A    | 3              | RM0.00          | 3.85%  | Short    | 1                             | 0                             | 0                             | 0                             |
| B    | 6              | RM0.00          | 3.90%  | Short    | 0                             | 1                             | 0                             | 0                             |
| C    | 9              | RM5,150,000.00  | 3.95%  | Long     | 0                             | 0                             | 1                             | 0                             |
| D    | 12             | RM5,150,000.00  | 4%     | Long     | 0                             | 0                             | 0                             | 1                             |
| Total Invested: | RM20,600,000.00 | Total Required: | RM0.00 | RM0.00 | RM5,150,000.00 | RM5,150,000.00 | RM5,150,000.00 | RM5,150,000.00 |
| Total Available: | RM20,600,000.00 | Total Required: | RM0.00 | RM0.00 | RM5,150,000.00 | RM5,150,000.00 | RM5,150,000.00 | RM5,150,000.00 |
| Total Return: | RM16,275.00 | (not more) | (not more) | (at least) | (at least) | (at least) | (at least) |

4.1.6. **Conclusion.** Case 1 – 5 gained different annual return for different choice of FD tenure based on the developed mathematical formulation as shown in Table 7. Figure 1 illustrated the Gantt Chart for \( R\|A_{\text{max}} \) problem. The current investment practiced by the company contributed RM803,100 per annum. In the experiment, case 3 indicated the highest value of total return with RM824,000 per annum which implied 2.6% greater than the current. Secondly, case 5 offered RM821,425 each year and revealed the different of 2.28% better than the practiced. Case 1, case 2 and case 4 provided the percentage of 0.04%, 1.32% and 0.68% higher than the current practice respectively.
Table 7. The selection of FD in the case study

| Cases | FDA | FBD | FDC | FDD |
|-------|-----|-----|-----|-----|
| 1     | ✓   |     |     |     |
| 2     | ✓   | ✓   |     | ✓   |
| 3     | ✓   |     | ✓   | ✓   |
| 4     | ✓   | ✓   | ✓   | ✓   |
| 5     |     | ✓   | ✓   | ✓   |

Figure 1. Gantt Chart for $R\|A_{\text{max}}$ problem

5. Summary

In this paper, a new methodology using scheduling in parallel processors system approach has been implemented in solving FD investment problem. The objective function of the problem is the maximization of the total annual return with different type of maturity and denoted as $R\|A_{\text{max}}$ problem. To solve this problem, a linear programming model is developed and implemented in a case study to allocate the investment of RM 20.6 million of a company. From the experiment, all our five cases obtained higher return than the investment practiced by the company. Case 3 obtained the best solution with the amount invested to the tenure with ratio $\{S, L\} = \{< 0.5, \geq 0.5\}$. We can view that there is a significance value of the return between the selection of tenure. Therefore, the ratio for the short-term and long-term can affect the assignment of the FD to the tenure. In future study, the scheduling in parallel processor system approach can be extend using other characteristics for example additional or withdrawal investment for various maturity date with dynamic transactions and compounding interest.

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