A Fuzzy Best-Worst Multi-Criteria Group Decision-Making Method

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ABSTRACT This paper proposed a novel fuzzy best-worst multi-criteria group decision-making method to solve the group decision-making (GDM) problem with multi-granular linguistic approach, which is an effective and promising technique to tackle this issue. In the proposed method, the selectable multi-granularity linguistic term sets (LTS) are firstly provided for experts to express their individual assessment information. Then, the improved fuzzy BWM is employed to calculate the weights of criteria with the form of fuzzy numbers. In current several studies using the BWM for group decision-making, only two unified best and worst criteria are given, which cannot reflect the evaluation of the best and worst criteria by different experts, resulting in the omission of information. Moreover, the difference between the best and worst criteria initially given and the experts' ideas will cause the experts to be inaccurate in the comparison of each criterion. Therefore, in this paper, in order not to omit too much information, each expert will determine the best and the worst criteria. The evaluation information of each expert is integrated into two comparison vectors according to the transformation formula proposed in this paper. What's more, an improved input-based consistency measurement is proposed, which can provide the DMs with a clear guideline on the revision of the inconsistent judgement(s). Finally, two examples are given to illustrate the effectiveness and applicability of the proposed method.

INDEX TERMS Fuzzy best-worst method, group decision-making, linguistic assessment information, multi-granular fuzzy linguistic context, multi-criteria decision-making.

I. INTRODUCTION

People rank alternatives to express the importance, preference, or likelihood of a decision. As a very important branch of decision-making theory, the multi-criteria decision-making (MCDM) problems are gaining momentum in a wide range of real-world situations, such as engineering, economics, and management [1]–[5]. The essence of MCDM is the sorting of all the alternatives and then the selection of optimal one by employing certain approach and existing decision information with consideration of different criteria [6]. So, before alternatives are ranked, the decision criteria system and the importance (weight) of the criteria need to be given. After that, the performance of the alternatives with respect to the criteria can be constructed, which is called the performance matrix. Finally, different techniques can be applied to solve the problem.

To date, considerable MCDM techniques have been conducted to find efficient ranking decisions based on the preferences of the decision makers, such as TOPSIS (Technique for Order of Preference by Similarity to Ideal Solution) [7], [8], ANP (the Analytic Network Process), ELECTRE (Elimination and Choice Translation Reality) [9], [10], VIKOR (VlseKriterijumska Optimizacija I Kompromisno Resenje) [11], AHP (the Analytic Hierarchy Process) [12], and BWM (best-worst method) [13]. In particular, the AHP and the BWM are both pairwise comparison-based MCDM methods, but the BWM needs only 2n-3 pairwise comparisons and the AHP needs n(n-1)/2 pairwise comparisons, which makes the BWM a more data efficient method compared to AHP.

Despite these considerable techniques, due to the limited expertise, estimation inaccuracies and knowledge lack of decision makers, the application of fuzzy information to
reflect the decision information is an excellent way in many practical MCDM problems. Meanwhile, many fuzzy-based MCDM methods have been proposed and widely used in recent years, such as fuzzy TOPSIS [14], fuzzy ELECTRE [15], and fuzzy BWM [6], [16].

However, as the increasing complexity of the decision-making environment, many organizations have moved from a single decision maker to a group of experts to accomplish problems. In order to consider the evaluation preferences of different experts, different multi-granularity linguistic term sets are proposed. Multi-granular fuzzy linguistic modeling has been frequently used in group decision making field due to its capability of allowing each expert to express his/her preference using his/her own linguistic term set [17]–[20]. For example, Francisco et al. proposed the first fusion method to handle multi-granularity linguistic information with the use of fuzzy set theory [20]. Zhen Zhang and Chonghui Guo focused on dealing with multi-granularity uncertain linguistic group decision making problems with incomplete weight information. The uncertain linguistic evaluation information of each decision maker was transformed to trapezoidal fuzzy numbers, and then two optimization models were established to minimize the deviation between each decision maker’s evaluation and the group’s collective evaluation on each alternative [19]. Francisco and Luis expressed the linguistic information by means of 2-tuples, which are composed by a linguistic term and a numeric value assessed in [0.5,0.5] [21]. Moreover, Herrera and Martinez conducted linguistic hierarchies term sets, and applied 2-tuple linguistic representation model to unify multi-granularity information without loss of information to solve GDM problems with multi-granularity linguistic information.

About the BWM in GDM, there are some recent developments [18], [22]–[26]. Reference [16] obtained the final set of criteria through discussion when selecting the best and worst criteria, but this is the result of the compromise of different experts, which will cause information loss in the stage of selecting the best and worst criteria. Furthermore, since experts need to re-accept a new set of best and worst criteria, it is easier to cause inconsistencies in the values obtained from subsequent comparisons between criteria. In addition, many researches conduct the consistency test after the final result (such as criteria weight and criteria performance) has been calculated. For inconsistent results, this will increase the unnecessary workload of the calculation.

In this study, we propose a fuzzy best-worst multi-criteria decision-making method. The main contributions of this study include: (1) It introduces multiple experts into the fuzzy BWM method and allows them to use different Linguistic Term Sets (LTS) to express the different assessments of the linguistic variables; (2) In the selection of the best and worst criteria and the subsequent comparison and scoring, the method proposed in this paper allows the experts to score according to their own judgments of the best and worst criteria, and finally uses mathematical formulas to carry out the full information conversion without information loss; (3) Several methods to integrate the opinions of experts and the advantages and disadvantages of each method are proposed, and the geometric average method is used to integrate the fuzzy set of the experts’ language conversion, which presents the opinions of each expert completely and avoids the extreme value to influence the whole situation; and (4) a pre-emptive fuzzy consistency test method is constructed to test the consistency of expert opinions before mathematical model operation. This method can reduce the calculation amount compared with the traditional consistency test which needs to be modified and recalculated after the inconsistency occurs.

The remainder of this paper is organized as follows. In Section II, preliminaries about the multigranular fuzzy linguistic GDM problems and the consensus process are presented. The proposed model is described in detail in Section III. In section IV, the input-based consistency ratio is selected and extended so that the inconsistency of each expert can be corrected in time by pre-comparison. In Section V, two examples are given to illustrate the effectiveness and applicability of the proposed method, while in Section VI, we draw the conclusions.

II. PRELIMINARIES

A. TRIANGULAR FUZZY NUMBERS (TFN)

There are many researches on fuzzy set and fuzzy degree, such as triangular fuzzy number, trapezoidal fuzzy number and polygonal fuzzy number. This paper uses triangular fuzzy number and gives the sorting formula of triangular fuzzy number

Definition 1: Given two TFN $\tilde{a}_1 = (l_1, m_1, n_1)$ and $\tilde{a}_2 = (l_2, m_2, n_2)$ and any positive real number $\lambda$. Let $\oplus$ and $\otimes$ denote the extended addition and multiplication operation defined by the extension principle, and two main operations of $\tilde{a}_1$ and $\tilde{a}_2$ can be expressed as follows [27]:

\[
\begin{align*}
\tilde{a}_1 \oplus \tilde{a}_2 &= (l_1 + l_2, m_1 + m_2, n_1 + n_2) \\
\tilde{a}_1 \otimes \tilde{a}_2 &= (l_1 \ast l_2, m_1 \ast m_2, n_1 \ast n_2) \\
\lambda \otimes \tilde{a}_1 &= (\lambda \ast l_1, \lambda \ast m_1, \lambda \ast n_1) \\
\tilde{a}_1^{-1} &= \frac{1}{\tilde{a}_1} = \left(\frac{1}{m_1}, \frac{1}{n_1}, \frac{1}{l_1}\right)
\end{align*}
\]

Definition 2: Let the graded mean integration representation (GMIR) $R(\tilde{a})$ of a TFN $\tilde{a}$ represent the ranking of triangular fuzzy number [28]–[30]. Let $\tilde{a}_i = (l_i, m_i, n_i)$, and the GMIR $R(\tilde{a}_i)$ of TFN $\tilde{a}_i$ can be calculated by

\[
R(\tilde{a}_i) = \frac{l_i + 4m_i + n_i}{6}
\]

B. THE SIMPLIFICATION OF COMPARISON

Suppose there are $n$ criteria for a research object, and the fuzzy pairwise comparisons on these $n$ criteria can be performed based on the linguistic variables (terms) of decision makers [6]. Then, the fuzzy comparison matrix can be

\[
\begin{align*}
A_{ij} &= \min\left(\frac{\tilde{a}_{ij}}{\tilde{a}_{ji}}, \frac{\tilde{a}_{ji}}{\tilde{a}_{ij}}\right) \\
&= \frac{l_{ij} + m_{ij} + n_{ij}}{3}
\end{align*}
\]
obtained as follows:

\[
\tilde{A} = \begin{bmatrix}
    c_1 & c_2 & \cdots & c_n \\
    \tilde{a}_{11} & \tilde{a}_{12} & \cdots & \tilde{a}_{1n} \\
    \vdots & \vdots & \ddots & \vdots \\
    \tilde{a}_{n1} & \tilde{a}_{n2} & \cdots & \tilde{a}_{nn}
\end{bmatrix}
\]  

(7)

where \(\tilde{a}_{ij}\) represents the relative fuzzy reference of criterion \(i\) to criterion \(j\), which is a triangular fuzzy number; \(\tilde{a}_{ij} = (1, 1, 1)\) when \(i = j\). And as the same as the Best-Worst method (BWM), the FBWM has proved that the number of fuzzy reference comparisons are just \(2n - 3\).

To explain the above \(2n - 3\) comparisons, it can divide the pairwise comparisons into two main categories, namely reference comparisons and secondary comparisons [13].

**Definition 3:** Comparison \(\tilde{a}_{ij}\) is defined as a reference comparison if \(i\) is the best element and/or \(j\) is the worst element.

**Definition 4:** Comparison \(\tilde{a}_{ij}\) is defined as a secondary comparison if \(i\) or \(j\) are the best or the worst elements.

For FBWM, there are \(n - 2\) ‘Best-to-Others’ fuzzy comparisons, \(n - 2\) ‘Others-to-Worst’ fuzzy comparisons and one ‘Best-to-Worst’ fuzzy comparison. So, it can conclude that there are \(2n - 3\) reference comparisons, and the rest pairwise comparisons are secondary comparisons.

As discussed above, the secondary comparisons are executed based on the knowledge about the reference comparisons. Each secondary comparison \(\tilde{a}_{ij}\) appears in two relation chains, two members of which are reference comparisons: \(\tilde{a}_{Bi} \otimes \tilde{a}_{ij} = \tilde{a}_{Bj}, \tilde{a}_{ij} \otimes \tilde{a}_{W} = \tilde{a}_{W}\), which also means that \(\tilde{a}_{ij} = \tilde{a}_{Bj}/\tilde{a}_{Bi}, \tilde{a}_{ij} = \tilde{a}_{W}/\tilde{a}_{W}(i \neq j)\).

**C. MULTI-GRAINULARITY LINGUISTIC INFORMATION**

Let \(S^q = \{s^q_0, s^q_1, s^q_2, \ldots, s^q_T\}\) be the \(q\)th pre-established finite and totally ordered linguistic term set with odd cardinalities, \(q = 1, 2, \ldots, l\), where \(s^q_i\) denotes the \(i\)th linguistic term of \(S^q\), and \(T_q + 1\) is the cardinality of \(S^q\). For example, we can express a linguistic term set with five terms as \(S^q = \{s^q_0, s^q_1, s^q_2, s^q_3, s^q_4\}\), where \(s^q_0)\): Weakly Important(WI), \(s^q_1): Fairly Important(FI), \(s^q_2): Equally Important(EI), \(s^q_3): Very Important(VI), \(s^q_4): Absolutely Important(AI)\}. The linguistic term set should satisfy the following characteristics:

1. The set is ordered: \(s_i > s_j\), if \(i > j\);
2. There is a negation operator: \(\text{Neg}(s^q_i) = s^q_{\tilde{i}}\), such that \(j = T_q - i\);
3. Maximization operator: \(\max(s^q_i, s^q_j) = s^q_{\tilde{i}}\), if \(s^q_i \geq s^q_j\);
4. Minimization operator: \(\min(s^q_i, s^q_j) = s^q_{\tilde{i}}\), if \(s^q_i \leq s^q_j\).

In this study, we assume that different experts use the linguistic term set with different granularities to establish the parameters of the triangular membership functions.

**D. TRANSFORMING LINGUISTIC INFORMATION INTO FUZZY NUMBERS**

In order to apply fuzzy BWM and its related operations to GDM problems, the linguistic terms are usually transformed to fuzzy numbers. The linguistic term \(s^q_i\) in set \(S^q\), \(q = 1, 2, \ldots, h\) can be approximately expressed in the following triangular fuzzy number:

\[
\tilde{s}^q_i = (l_i^q, m_i^q, n_i^q), \quad i = 0, 1, 2, \ldots, T_q
\]

(8)

where \(l_i^q = 1 + 8 \times \max((i - 1)/T_q, 0), m_i^q = 1 + 8 \times i/T_q\), and \(n_i^q = 1 + 8 \times \min((i + 1)/T_q, 1)\).

The membership function of triangular fuzzy number is

\[
\mu_{\tilde{s}^q_i}(x) = \begin{cases} 
\frac{x - l_i^q}{m_i^q - l_i^q}, & m_i^q < x < n_i^q \\
\frac{n_i^q - x}{n_i^q - m_i^q}, & l_i < x < m_i^q \\
0, & \text{otherwise}
\end{cases}
\]

(9)

where \(\mu_{\tilde{s}^q_i}(x)\) is the degree to which the value \(x\) belongs to \(\tilde{s}^q_i\).

Some multi-granularity linguistic term sets and the rules of transformation are listed in Table 1, 2 and 3, respectively.

**TABLE 1. A set of five terms via triangular fuzzy numbers.**

| Term Set | Linguistic Terms | Fuzzy Number |
|----------|-----------------|--------------|
| \(s^q_0\): Poor | (1, 1, 3) |
| \(s^q_1\): Slightly Fair | (1, 3, 5) |
| \(s^q_2\): Fair | (3, 5, 7) |
| \(s^q_3\): Slightly High | (5, 7, 9) |
| \(s^q_4\): High | (7, 9, 9) |

**TABLE 2. A set of seven terms via triangular fuzzy numbers.**

| Term Set | Linguistic Terms | Fuzzy Number |
|----------|-----------------|--------------|
| \(s^q_0\): None | (1, 1, 2,33) |
| \(s^q_1\): Very Low | (1, 2,33, 3,66) |
| \(s^q_2\): Low | (2,33, 3,66, 5) |
| \(s^q_3\): Medium | (3,66, 5, 6,33) |
| \(s^q_4\): High | (5, 6,33, 7,66) |
| \(s^q_5\): Very High | (6,33, 7,66, 9) |
| \(s^q_6\): Total | (7,66, 9, 9) |

**III. THE PROPOSED METHOD**

Let \(E = \{E_1, E_2, \ldots, E_l\}\) be the set of decision makers, \(C = \{c_1, c_2, \ldots, c_n\}\) be a finite set of decision criteria, where \(E_i(i \in \{1, 2, \ldots, l\})\) and \(c_j(j \in \{1, 2, \ldots, n\})\) denote the \(i\)th expert and the \(j\)th criterion. We assume that the experts are identical in equal importance and use multi-granularity linguistic term sets \(S^1, S^2, \ldots, S^h\) to express their preference information, where \(S^q\) denote the \(ih\) preestablished finite and totally ordered linguistic term set with \(T_q + 1\) cardinalities, i.e. \(S^q = \{s^q_0, s^q_1, s^q_2, \ldots, s^q_T\}\).

**Step 1:** Define the decision criteria system.

The decision criteria system consists of a set of decision criteria, which is very important to make a decision on alternatives. Suppose there are \(n\) decision criteria \(\{c_1, c_2, \ldots, c_n\}.

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TABLE 3. A set of nine terms via triangular fuzzy numbers.

| Term Set | Linguistic Terms | Fuzzy Number |
|----------|------------------|--------------|
| $s_1^q$  | None             | (1, 1, 2)    |
| $s_2^q$  | Very Low         | (1, 2, 3)    |
| $s_3^q$  | Low              | (2, 3, 4)    |
| $s_4^q$  | Almost Medium    | (3, 4, 5)    |
| $s_5^q$  | Medium           | (4, 5, 6)    |
| $s_6^q$  | Almost High      | (5, 6, 7)    |
| $s_7^q$  | High             | (6, 7, 8)    |
| $s_8^q$  | Very High        | (7, 8, 9)    |
| $s_9^q$  | Prefect          | (8, 9, 9)    |

Then we can denote that

$$\tilde{A}^q = \begin{bmatrix} c_1 & c_2 & \cdots & c_n \\ c_1 & \tilde{a}_{11}^q & \tilde{a}_{12}^q & \cdots & \tilde{a}_{1n}^q \\ c_2 & \tilde{a}_{21}^q & \tilde{a}_{22}^q & \cdots & \tilde{a}_{2n}^q \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ c_n & \tilde{a}_{n1}^q & \tilde{a}_{n2}^q & \cdots & \tilde{a}_{nn}^q \end{bmatrix}$$ (10)

where $\tilde{A}^q$ represent the $q$th decision maker’s pairwise comparison matrix, $\tilde{a}_{ij}^q$ show the relative of criterion $i$ to criterion $j$. And $\tilde{A}^q = (\tilde{a}_{11}^q, \tilde{a}_{12}^q, \ldots, \tilde{a}_{nm}^q)$ represents the vector of the $ith$ index compared to other indexes. Furthermore, the pairwise comparison matrix $\tilde{A}^q$ is considered to be perfectly consistent if:

$$\tilde{a}_{ik}^q \otimes \tilde{a}_{ij}^q = \tilde{a}_{lj}^q, \ \forall i, j$$ (11)

**Step 2:** Determine the best and the worst criterion.

Based on the built decision criteria system, decision makers are asked to determine the best and the worst criterion. The best criterion for $q$th decision maker is represented as $c_B^q$, and the worst criterion for $q$th decision maker is labeled as $c_W^q$. After which we can identify the two best and worst criteria that experts choose the most, named $c_{Bm}$ and $c_{Wm}$. Note that if all the experts choose different criteria for the best and worst, we can choose either of them as $c_{Bm}$ and $c_{Wm}$. However, it should be noted that the two indicators here are not necessarily the final best and worst criteria, and the final result should be the one calculated by the programming equation later. When making a decision, the expert selects the best and worst criteria just like a set of comparison benchmarks, and compares other criteria with these two criteria to obtain the ranking of each criterion. However, when there are many experts making decisions, the best and worst criteria in the eyes of each expert are often different, so that multiple groups of $\tilde{A}^q_B$ and $\tilde{A}^q_W$ are obtained. In order to finally integrate $\tilde{A}^q_B$ and $\tilde{A}^q_W$ of different experts, it is necessary to select a set of highly recognized best and worst criteria as the basis point in advance.

**Step 3:** Execute the fuzzy reference comparisons for the best criterion.

In this step, we define the preferences of the best criterion over the other criterion. The obtained fuzzy Best-to-Others vector for $q$th decision maker is $\tilde{A}^q_B = (\tilde{a}_{B1}^q, \tilde{a}_{B2}^q, \ldots, \tilde{a}_{Bn}^q)$, where $\tilde{a}_{Bj}^q$ is the fuzzy preference of the best criterion $B$ over the criterion $j$ ($j = 1, 2, \ldots, n$) and its value is a fuzzy number. Of which, $\tilde{a}_{BB}^q = (1, 1, 1)$.

Note that $\tilde{a}_{ij} = \tilde{a}_{ij}/\tilde{a}_{Bj},$ we can define the $q$th decision-maker’s $\tilde{a}_{Bm}^q = \tilde{a}_{B1}/\tilde{a}_{Bm}$ and $\tilde{a}_{Wm}^q = (\tilde{a}_{Bm1}^q, \tilde{a}_{Bm2}^q, \ldots, \tilde{a}_{Bmn}^q)$. With this transformation, we can ensure consistency in each expert’s decisions with minimal loss of information. When an expert selects a basis point and then compares a set of criteria based on that basis, the results tend to be consistent, and the comparison results can then be used to calculate the comparison values of the other two criteria. Because this set of numbers is based on $\tilde{A}^q_B$, it is the same as the expert’s original definition of the standard. It is worth mentioning that this transformation preserves the opinion of each expert and does not result in the loss of information.

**Step 4:** Execute the fuzzy reference comparisons for the worst criterion.

In this step, we define the preferences of all criteria $i$ over the worst criterion. The obtained fuzzy Others-to-Worst vector for $q$th decision maker is: $\tilde{A}^q_W = (\tilde{a}_{W1}^q, \tilde{a}_{W2}^q, \ldots, \tilde{a}_{Wn}^q)$, where $\tilde{a}_{Wj}^q$ is the fuzzy preference of the criterion $i$ ($i = 1, 2, \ldots, n$) over the worst criterion $W$ and its value is a fuzzy number. Note that $\tilde{a}_{WW}^q = (1, 1, 1)$.

In the same way, noted that $\tilde{a}_{ij} = \tilde{a}_{ij}/\tilde{a}_{Wj},$ we can define the $q$th decision-maker’s $\tilde{a}_{Wm}^q = \tilde{a}_{W1}/\tilde{a}_{Wm}$ and $\tilde{A}^q_W = (\tilde{a}_{Wm1}^q, \tilde{a}_{Wm2}^q, \ldots, \tilde{a}_{Wmn}^q)$.

**Step 5:** Incorporate all expert evaluations.

Since this paper uses the input-based consistency ratio, we need to check the consistency of each expert’s comparison vectors before we proceed to this step. The specific method will be given in the following section IV. After we get the evaluation of each expert, we can construct the pairwise comparison matrix of each expert. The $q$th decision maker’s pairwise comparison matrix is shown below:

$$\tilde{A}^q = \begin{bmatrix} c_1 & c_2 & \cdots & \cdots & \cdots & c_n \\ c_1 & \cdots & \cdots & \cdots & \cdots & \cdots \\ c_2 & \cdots & \cdots & \cdots & \cdots & \cdots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ c_{Bm} & \cdots & \cdots & \cdots & \tilde{a}_{Bm1}^q & \tilde{a}_{Bm2}^q & \cdots & \tilde{a}_{Bmn}^q \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ c_{Wm} & \cdots & \cdots & \cdots & \tilde{a}_{Wm1}^q & \tilde{a}_{Wm2}^q & \cdots & \tilde{a}_{Wmn}^q \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \cdots & \vdots \\ c_n & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \end{bmatrix}$$ (12)

The main purpose of aggregation is to produce appropriate results from the pairwise comparison matrix. There are many methods to synthesize the decisions of multiple experts. By learning from the integration method of AHP [31], the next part will give examples of several suitable methods and evaluate the merits and demerits of these methods.
1) MEAN METHOD

To the best of our knowledge, the mean method is a simple and effective method, which usually includes geometric mean and arithmetic mean, emphasizing the average of all judgments. As a result, the method can more respect the opinions of experts with the same weight. Arithmetic mean can merge all the \( A^q \) by the equation \( A = (1/q) \otimes (A^1 \oplus A^2 \oplus \cdots \oplus A^q) \). For the details, \( \tilde{a}_{BM} \) and \( \tilde{a}_{W} \) are calculated as follows:

\[
\tilde{a}_{BM} = (l_{BMj}, m_{BMj}, n_{BMj}) = \frac{1}{q} \otimes (\tilde{a}_{BMj} \oplus \tilde{a}_{BMj} \oplus \cdots \oplus \tilde{a}_{BMj})
\]

\[ j = 0, 1, 2, \ldots, n \tag{13} \]

\[
\tilde{a}_{W} = (l_{Wm}, m_{Wm}, n_{Wm}) = \frac{1}{q} \otimes (\tilde{a}_{Wm} \oplus \tilde{a}_{Wm} \oplus \cdots \oplus \tilde{a}_{Wm})
\]

\[ j = 0, 1, 2, \ldots, n \tag{14} \]

So that \( \tilde{A}_{B} = (\tilde{a}_{BM1}, \tilde{a}_{BM2}, \ldots, \tilde{a}_{BMn}) \) and \( \tilde{A}_{W} = (\tilde{a}_{W1}, \tilde{a}_{W2}, \ldots, \tilde{a}_{Wm}) \). Simple as it is, but there should be no extreme value due to its sensitivity.

As for geometric mean, it is less affected by extreme value and more suitable to average values, \( \tilde{a}_{BM} \) and \( \tilde{a}_{W} \) are calculated as follows:

\[
\tilde{a}_{BM} = (l_{BMj}, m_{BMj}, n_{BMj})
\]

\[ = (\tilde{a}_{BMj} \otimes \tilde{a}_{BMj} \otimes \cdots \otimes \tilde{a}_{BMj})^{1/q} = (\prod_{j=1}^{q} \tilde{a}_{BMj})^{1/q}
\]

\[ j = 0, 1, 2, \ldots, n \tag{15} \]

\[
\tilde{a}_{W} = (l_{Wm}, m_{Wm}, n_{Wm})
\]

\[ = (\tilde{a}_{Wm} \otimes \tilde{a}_{Wm} \otimes \cdots \otimes \tilde{a}_{Wm})^{1/q} = (\prod_{j=1}^{q} \tilde{a}_{Wm})^{1/q}
\]

\[ j = 0, 1, 2, \ldots, n \tag{16} \]

2) MAX-MIN METHOD

Compared to the mean methods using an average solution, the max–min method extends the aggregated value range by including the ‘worst’ and the ‘best’ judgements. Max and min, as two aggregation operators, choose the largest and smallest values respectively [30]. They decide the upper and lower bounds of the aggregated TFN.

The aggregated TFN \( \tilde{a}_{BM} = (l_{BMj}, m_{BMj}, n_{BMj}) \) by max–min with geometric mean is:

\[
l_{BMj} = \min_{t=1,2,\ldots,q} (l_{BMj}^t)
\]

\[ m_{BMj} = (\prod_{t=1}^{q} m_{BMj}^t)^{1/q}, \quad j = 1, 2, \ldots, n \tag{17} \]

\[
n_{BMj} = \min_{t=1,2,\ldots,q} (n_{BMj}^t)
\]

And the aggregated TFN \( \tilde{a}_{BM} = (l_{BMj}, m_{BMj}, n_{BMj}) \) by max–min with arithmetic mean is:

\[
l_{BMj} = \min_{t=1,2,\ldots,q} (l_{BMj}^t)
\]

\[ m_{BMj} = \frac{1}{q} \sum_{q} m_{BMj}^t, \quad j = 1, 2, \ldots, n \tag{18} \]

\[ n_{BMj} = \min_{t=1,2,\ldots,q} (n_{BMj}^t)
\]

3) METHOD BASED ON CONSENSUS DEGREE

The aggregation principle of this method borrows the idea of weighted arithmetic mean, but there are great differences in the derivation of weight coefficient. The key part of this approach is how to derive reasonable weights so that expert opinion can be integrated according to the weights. This method introduces a ‘consensus coefficient’ variable which is a compromise between the weight of expert and the difference of its opinion from the opinions of all the others, and multiplies it by a separate judgment instead of the weight of the expert in the weighted arithmetic average. The process is as follows:

**Step a:** Calculate the degree of agreement \( S(\tilde{a}_{BMj}^f, \tilde{a}_{BMj}^g) \) of the opinions between each pair of experts DM\( f \) and DM\( g \), where \( S(\tilde{a}_{BMj}^f, \tilde{a}_{BMj}^g) \in [0, 1] \) and \( f \neq g \). The degree is calculated as follows:

\[
S(\tilde{a}_{BMj}^f, \tilde{a}_{BMj}^g) = \frac{1}{3} \left[ |l_{BMj}^f - l_{BMj}^g| + |m_{BMj}^f - m_{BMj}^g| + |n_{BMj}^f - n_{BMj}^g| \right]
\]

\[ \tag{19} \]

**Step b:** Calculate the average degree of agreement \( A(DM_g) \) of expert DM\( g \) (\( g = 1, 2, \ldots, n \)) with all the others.

\[
A(DM_g) = \frac{1}{q-1} \sum_{f=1, f \neq g}^{q} S(\tilde{a}_{BMj}^f, \tilde{a}_{BMj}^g), \quad j = 1, 2, \ldots, n
\]

\[ \tag{20} \]

**Step c:** Calculate the relative degree of agreement \( RA(DM_g) \) of expert DM\( g \) (\( g = 1, 2, \ldots, n \))

\[
RA(DM_g) = \frac{A(DM_g)}{\sum_{g=1}^{q} A(DM_g)} \tag{21}
\]

**Step d:** Calculate the consensus degree coefficient \( C(DM_g) \) of expert DM\( g \) (\( g = 1, 2, \ldots, n \))

\[
C(DM_g) = \frac{y_1}{y_1 + y_2} \ast \omega_{DM_g} + \frac{y_2}{y_1 + y_2} \ast RA(DM_g) \tag{22}
\]

\( \omega_{DM_g} \) is the weight of expert DM\( g \). In this paper, each expert has the same weight; \( y_1 \) and \( y_2 \) are the weight of the importance of experts and the weight of the relative degree of agreement of experts.

**Step e:** Aggregate the fuzzy judgements.
where, for each fuzzy pair \( \tilde{o}_j \) of decision maker. Therefore, in order to satisfy these conditions for all \( j \), the absolute gaps \( |\tilde{\omega}_B/\tilde{\omega}_j - \tilde{\omega}_B/\tilde{\omega}_W | \) and \( |\tilde{\omega}_j/\tilde{\omega}_W - \tilde{\omega}_j/\tilde{\omega}_j | \) for all \( j \) can be calculated. So, we can obtain the constrained optimization problem for determining the optimal fuzzy weights \( (\tilde{\omega}_1, \tilde{\omega}_2, \ldots, \tilde{\omega}_n) \) as follows:

\[
\begin{align*}
\min \max & \left\{ \left| \frac{\tilde{\omega}_R}{\tilde{\omega}_j} - \tilde{\omega}_B \right|, \left| \frac{\tilde{\omega}_B}{\tilde{\omega}_j} - \tilde{\omega}_B \right| \right\} \\
\text{s.t.} & \sum_{j=1}^{n} R(\tilde{\omega}_j) = 1 \\
& 1_j \leq \tilde{\omega}_j \leq n_j \\
& 1_j \geq 0 \\
& j = 1, 2, \ldots, n
\end{align*}
\]

(24)

where \( \tilde{\omega}_R = (1_j, m_j, n_j), \tilde{\omega}_B = (1_j, m_j, n_j), \tilde{\omega}_W = (1_j, m_j, n_j), \tilde{\omega}_B = (1_j, m_j, n_j), \tilde{\omega}_W = (1_j, m_j, n_j), \tilde{\omega}_B = (1_j, m_j, n_j), \tilde{\omega}_W = (1_j, m_j, n_j).

As discussed in [13], Eq.(24) may result in multiple optimal solutions. Then we can minimize the maximum among the set of \( \left\{ |\tilde{\omega}_R - \tilde{\omega}_B|, |\tilde{\omega}_j - \tilde{\omega}_W| \right\} \). The constraint equation can be formulated as follows:

\[
\begin{align*}
\min \max & \left\{ \left| \tilde{\omega}_R - \tilde{\omega}_B \right|, \left| \tilde{\omega}_j - \tilde{\omega}_W \right| \right\} \\
\text{s.t.} & \sum_{j=1}^{n} R(\tilde{\omega}_j) = 1 \\
& 1_j \leq \tilde{\omega}_j \leq n_j \\
& 1_j \geq 0 \\
& j = 1, 2, \ldots, n
\end{align*}
\]

(25)

What’s more, Eq.(25) can be transferred to the following nonlinear programming problem:

\[
\begin{align*}
\min & \xi \\
\text{s.t.} & \sum_{j=1}^{n} R(\tilde{\omega}_j) = 1 \\
& 1_j \leq \tilde{\omega}_j \leq n_j \\
& 1_j \geq 0 \\
& j = 1, 2, \ldots, n
\end{align*}
\]

(26)

where \( \xi = (1^k, m^k, n^k) \). Considering \( l^k \leq m^k \leq n^k \), we suppose \( \xi^* = (k^*, k^*, k^*) \), \( k^* \leq l^k \). By solving Eq.(26), the optimal fuzzy weights \( (\tilde{\omega}_1^*, \tilde{\omega}_2^*, \ldots, \tilde{\omega}_n^*) \) can be obtained.

So far, the main steps of the proposed method in this paper have been introduced, and the relevant process diagram is shown in Figure 1. With the aim of determining the fuzzy weight of criteria, we can execute the fuzzy comparison on relative criteria. Moreover, with the aim of determining the fuzzy weights of alternatives with respect to different criteria, the related alternatives should be fuzzily compared against each criterion. Finally, the fuzzy ranking scores of alternatives can be derived from the fuzzy weights of alternatives with respect to different criteria multiplied by the fuzzy weights of the corresponding criteria [31].

**FIGURE 1.** The process diagram of fuzzy BWM in GDM in this paper.

Compared with the best and worst criteria determined by a single leader in [16], this method reflects the fairness of group decision making. Compared with the multi-round discussion in [20], this method simplifies the number of iterations and introduces prior consistency comparison to achieve the same collective decision effect as the multi-round discussion.

**IV. CONSISTENCY RATIO FOR FUZZY BWM IN GDM**

When a decision-maker provides the pairwise comparisons in fuzzy BWM, it is important to check the acceptable inconsistency and ensure the rationality of the assessment.

To check how inconsistent a full set of pairwise comparisons may be, several consistency indices have been proposed. Liang et al. [32] proposed an input-based consistency measurement, which is simple to use and has several desirable properties. By using the simple calculation of the input-based consistency measurement, it is easy to determine a DM with immediate feedback. The consistency ratio can be obtained after the entire elicitation process has finished, which means that it provides a DM with a clear and immediate idea of his/her consistency level. Based on their research, this paper extends this method to the fuzzy environment.

**Definition 5:** The Input-based Consistency Ratio \( CR_l^j \) is formulated as follows:

\[
CR_l^j = \max \left\{ \left| \tilde{\omega}_j^* \right| \right\}
\]

(27)
where

\[
CR_j = \begin{cases} 
\frac{|R(\tilde{a}_{BW} \otimes \tilde{a}_{IW} - \tilde{a}_{BW})|}{R(\tilde{a}_{BW} \otimes \tilde{a}_{BW} - \tilde{a}_{BW})} & \tilde{a}_{BW} \neq (1, 1, 1) \\
0 & \tilde{a}_{BW} = (1, 1, 1) 
\end{cases}
\] (28)

CR_j is the global input-based consistency ratio for all criteria, CR_j represents the local consistency level associated with criterion c_j. Besides we use CR_{IE} and CR_{IE} to represent the global input-based consistency ratio and the local consistency level of the nth expert respectively. In this paper, we treat the triangular fuzzy numbers obtained by pairwise comparison accurately and convert them into crisp values. Defuzzification converts the fuzzy results produced by aggregation methods into crisp values. At the same time, according to Table 4 of consistency ratio threshold in [32], consistency evaluation of the results is given.

**TABLE 4. Thresholds for different combinations using input-based consistency measurement.**

| Scales | 3   | 4   | 5   | 6   | 7   | 8   | 9   |
|--------|-----|-----|-----|-----|-----|-----|-----|
|        | 0.1667 | 0.1667 | 0.1667 | 0.1667 | 0.1667 | 0.1667 | 0.1667 |
|        | 0.1121 | 0.1529 | 0.1898 | 0.2206 | 0.2527 | 0.2577 | 0.2683 |
|        | 0.1354 | 0.1994 | 0.2356 | 0.2546 | 0.2716 | 0.2844 | 0.2960 |
|        | 0.1330 | 0.1990 | 0.2643 | 0.3044 | 0.3144 | 0.3221 | 0.3262 |
|        | 0.1294 | 0.2457 | 0.2819 | 0.3029 | 0.3144 | 0.3251 | 0.3403 |
|        | 0.1309 | 0.2521 | 0.2958 | 0.3154 | 0.3408 | 0.3620 | 0.3657 |
|        | 0.1339 | 0.2681 | 0.3062 | 0.3337 | 0.3517 | 0.3620 | 0.3662 |

In Table 4, let the Scales of the row dimension represents the estimated size of R(\tilde{a}_{BW}). Because the size of R(\tilde{a}_{BW}) may not be integer, and the row dimension data in the table are all integer, so it can approximate the integer value to obtain R(\tilde{a}_{BW}) for convenience. It is worth noting that even though we can easily identify inconsistent judgments by using consistency measures, it is unrealistic to expect DM to achieve perfect consistency. However, this kind of prior consistency comparison can avoid the workload of recalculation caused by the possible inconsistencies in the last consistency test in [6], [17], [16], which is also one advantage of the method proposed in this paper.

**V. CASE STUDIES**

In this section, we give two examples for the different group decision-making problems to illustrate the method proposed in this paper. Note that, the mathematical models of the examples are solved by Lingo Version 17.0 software to obtain the optimal weights.

**A. CASE 1**

Imagine a situation where you want to buy a high cost-performance car, but have no idea which car to choose, so ask three car experts to give you some ideas. At this point, the method mentioned in the paper can be applied.

First of all, five criteria, namely price (c_1), quality (c_2), comfort (c_3), safety (c_4) and style (c_5), are considered for making this car-buying decision. Then, three experts need to choose multi-granularity linguistic term sets. In this case, each expert picks according to his or her own preference, where Expert 1(E_1) picks S^1, Expert 2(E_2) picks S^2, and expert 3(E_3) picks S^3. They give their preference information on criteria weights using the following different linguistic term sets:

\[
E_1 : S^1 = \{s_0^1, s_1^1, s_2^1, s_3^1, s_4^1\} \\
E_2 : S^2 = \{s_0^2, s_1^2, s_2^2, s_3^2, s_4^2, s_5^2\} \\
E_3 : S^3 = \{s_0^3, s_1^3, s_2^3, s_3^3, s_4^3, s_5^3, s_6^3\}
\]

The selection results of the best and worst criteria by each expert are shown in Table 5. Quality (c_2) and Style (c_5) are defined as c_B and c_W, respectively. Furthermore, these three experts compare and evaluate the five criteria in pairs using their linguistic term set in Table 6-8. Each expert gets a set of two vectors based on his or her best and worst criteria. The results are shown below:

\[
\tilde{A}_B = (\tilde{a}_{B1}, \tilde{a}_{B2}, \tilde{a}_{B3}, \tilde{a}_{B4}, \tilde{a}_{B5}) = (\tilde{a}_{21}, \tilde{a}_{22}, \tilde{a}_{23}, \tilde{a}_{24}, \tilde{a}_{25}) \\
(3, 5, 7, (1, 1, 1), (5, 7, 9), (1, 3, 5), (7, 9, 9))
\]

\[
\tilde{A}_W = (\tilde{a}_{W1}, \tilde{a}_{W2}, \tilde{a}_{W3}, \tilde{a}_{W4}, \tilde{a}_{W5}) = (\tilde{a}_{15}, \tilde{a}_{25}, \tilde{a}_{35}, \tilde{a}_{45}, \tilde{a}_{55}) \\
((3, 5, 7), (7, 9, 9), (1, 3, 5), (5, 7, 9), (1, 1, 1))
\]

**TABLE 5. Experts on the selection of the best and worst criteria.**

| Criterion | Expert 1 (E_1) | Expert 2 (E_2) | Expert 3 (E_3) |
|-----------|---------------|---------------|---------------|
| Quality (c_2) | Price (c_2) | Quality (c_2) | (c_5) |
| Style (c_5) | Style (c_5) | (c_5) | (c_5) |

The selected best criterion (c_{Bw}) Quality (c_2)

The selected worst criterion (c_{Ww}) Style (c_5)
TABLE 6. Evaluation information of decision maker E₁.

| Criteria  | Price (c₁) | Quality (c₂) | Comfort (c₃) | Safety (c₄) | Style (c₅) |
|-----------|------------|--------------|--------------|-------------|------------|
| Best criterion c₁ (Quality) | Fair        | Equal        | Slightly High | Slightly Fair | High       |
| Worst criterion c₅ (Style)    | Fair        | High         | Slightly Fair | Slightly High | Equal      |

TABLE 7. Evaluation information of decision maker E₂.

| Criteria  | Price (c₁) | Quality (c₂) | Comfort (c₃) | Safety (c₄) | Style (c₅) |
|-----------|------------|--------------|--------------|-------------|------------|
| Best criterion c₂ (Price)    | Equal      | None         | Medium       | Low         | Very High  |
| Worst criterion c₅ (Style)    | Very High  | High         | Low          | Medium      | Equal      |

TABLE 8. Evaluation information of decision maker E₃.

| Criteria  | Price (c₁) | Quality (c₂) | Comfort (c₃) | Safety (c₄) | Style (c₅) |
|-----------|------------|--------------|--------------|-------------|------------|
| Best criterion c₃ (Quality)    | Medium     | Equal        | Almost Medium | None        | Very High  |
| Worst criterion c₅ (Style)     | Low        | Very High    | Almost High  | Very High   | Equal      |

Given by each expert. If the experts’ opinions are inconsistent, we can calculate the results and adjust accordingly. The input-based consistency ratio of the three experts are obtained by using Equation (27) and Equation (28), and the results are shown in the Table 9. By comparing with the values in Table 4, it is found that each expert satisfies the consistency. After that geometric mean is used to integrate the experts’ opinions because it’s less affected by extreme value. By using Equation (15) and Equation (16), the final two integrated vectors can be obtained as:

\[
\tilde{A}_{B'} = (\tilde{a}_{B^1}, \tilde{a}_{B^2}, \tilde{a}_{B^3}, \tilde{a}_{B^4}, \tilde{a}_{B^5}) = ((1.727, 2.924, 3.476), (1, 1, 1), (2.866, 5.192, 6.580), (1, 2.223, 3.684), (5.102, 8.201, 9))
\]

\[
\tilde{A}_{W'} = (\tilde{a}_{W^1}, \tilde{a}_{W^2}, \tilde{a}_{W^3}, \tilde{a}_{W^4}, \tilde{a}_{W^5}) = ((3.361, 4.862, 6.316), (6.257, 7.696, 8.529), (2.267, 4.039, 5.593), (5.041, 6.542, 8.004), (1, 1, 1))
\]

Based on the above analysis, for getting the optimal fuzzy weights of all the criteria, the following optimization problem can be built according to Equation (26).

\[
\min \xi \sum_{j=1}^{n} R(\tilde{a}_j) = 1
\]

\[
l_j^o \leq m_j^o \leq n_j^o
\]

\[
l_j^o \geq 0
\]

\[
j = 1, 2, 3, 4, 5
\]

(29)
Then, we can obtain the nonlinear constrained optimization problem which is further calculated as follows.

\[
\min \tilde{\xi} = \left\{ \begin{array}{l}
(l_1^{o}, m_1^{o}, n_1^{o}) \oplus (-n_{21}, -m_{21}, -l_{21}) \otimes (l_1^{o}, m_1^{o}, n_1^{o}) \\
\leq (k^*, k^*, k^*) \\
(l_2^{o}, m_2^{o}, n_2^{o}) \oplus (-n_{22}, -m_{22}, -l_{22}) \otimes (l_2^{o}, m_2^{o}, n_2^{o}) \\
\leq (k^*, k^*, k^*) \\
(l_3^{o}, m_3^{o}, n_3^{o}) \oplus (-n_{23}, -m_{23}, -l_{23}) \otimes (l_3^{o}, m_3^{o}, n_3^{o}) \\
\leq (k^*, k^*, k^*) \\
(l_4^{o}, m_4^{o}, n_4^{o}) \oplus (-n_{24}, -m_{24}, -l_{24}) \otimes (l_4^{o}, m_4^{o}, n_4^{o}) \\
\leq (k^*, k^*, k^*) \\
(l_5^{o}, m_5^{o}, n_5^{o}) \oplus (-n_{25}, -m_{25}, -l_{25}) \otimes (l_5^{o}, m_5^{o}, n_5^{o}) \\
\leq (k^*, k^*, k^*) \\
(l_6^{o}, m_6^{o}, n_6^{o}) \oplus (-n_{26}, -m_{26}, -l_{26}) \otimes (l_6^{o}, m_6^{o}, n_6^{o}) \\
\leq (k^*, k^*, k^*) \\
(l_1, m_1, n_1) \oplus (-n_{15}, -m_{15}, -l_{15}) \otimes (l_1, m_1, n_1) \\
\leq (k^*, k^*, k^*) \\
(l_2, m_2, n_2) \oplus (-n_{16}, -m_{16}, -l_{16}) \otimes (l_2, m_2, n_2) \\
\leq (k^*, k^*, k^*) \\
(l_3, m_3, n_3) \oplus (-n_{17}, -m_{17}, -l_{17}) \otimes (l_3, m_3, n_3) \\
\leq (k^*, k^*, k^*) \\
(l_4, m_4, n_4) \oplus (-n_{18}, -m_{18}, -l_{18}) \otimes (l_4, m_4, n_4) \\
\leq (k^*, k^*, k^*) \\
(l_5, m_5, n_5) \oplus (-n_{19}, -m_{19}, -l_{19}) \otimes (l_5, m_5, n_5) \\
\leq (k^*, k^*, k^*) \\
(l_6, m_6, n_6) \oplus (-n_{20}, -m_{20}, -l_{20}) \otimes (l_6, m_6, n_6) \\
\leq (k^*, k^*, k^*) \\
\left[ \sum_{j=1}^{n} R(\tilde{a}_j) = 1 \\
\right]_{m_j} = m_j \leq n_j \\
\left[ l_j \geq 0 \\
j = 1, 2, 3, 4, 5 \right]
\end{array} \right. 
\]  

(30)

By solving the problem, it can be obtained that

\[ \tilde{a}_1 = (0.087, 0.149, 0.442), \quad \tilde{a}_2 = (0.233, 0.364, 0.693), \quad \tilde{a}_3 = (0.046, 0.084, 0.266), \quad \tilde{a}_4 = (0.083, 0.196, 0.686), \quad \tilde{a}_5 = (0.019, 0.038, 0.122), \quad \tilde{\xi} = (0.07, 0.07, 0.07) \]

The optimal fuzzy weights of six criteria in this case is also shown in Figure 2.

Then, it can be seen that Quality \((c_2)\) is the most important criterion in terms of supplier’s willingness for supplier performance evaluation, the next important criteria are Safety \((c_4)\), Price \((c_1)\), and Comfort \((c_3)\), and Style \((c_5)\) is ranked as the least important criterion. When we defuzzify the obtained fuzzy numbers and turn them into \(R(\tilde{a}_j)\) for sorting, through the result that \(R(\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4, \tilde{a}_5) = (0.187, 0.397, 0.108, 0.259, 0.049)\), the same sorting results can be obtained. The dispersion degree of the fuzzy numbers also reflects the degree of inconsistency of experts’ opinions to some extent.

B. CASE 2

In this case, the selection of the mobile phone is discussed, which was applied when the BWM approach was first proposed. The choice of mobile phone refers to six criteria, namely price \((c_1)\), dimensions \((c_2)\), weight \((c_3)\), display \((c_4)\), data inputs \((c_5)\), and memory \((c_6)\). In terms of personnel choices, five students were asked to rank six criteria to consider when buying a phone.

In this case, each student picks according to his or her own preference, where Expert 1\((E_1)\) picks \(S_1\), Expert 2\((E_2)\) picks \(S_2\), expert 3\((E_3)\) picks \(S_3\), expert 4\((E_4)\) picks \(S_3\), and expert 5\((E_5)\) picks \(S_2\).

They give their preference information on criteria weights using the following different linguistic term sets:

\[ E_1 : S_1 = \{ s_1^1, s_1^2, s_1^3, s_1^4 \} \]
\[ E_2 , E_5 : S_2 = \{ s_2^1, s_2^2, s_2^3, s_2^4, s_2^5 \} \]
\[ E_3 , E_4 : S_3 = \{ s_3^1, s_3^2, s_3^3, s_3^4, s_3^5, s_3^6, s_3^7 \} \]

The selection results of the best and worst criteria by each expert are shown in Table 10. Data inputs \((c_5)\) and Weight \((c_3)\) are defined as \(c_{bm}\) and \(c_{w}\) respectively. Each expert gets a set of two vectors based on his or her best and worst criteria. The results are shown below:

\[ A_{B}^1 = (a_{B1}^1, a_{B2}^1, a_{B3}^1, a_{B4}^1, a_{B5}^1, a_{B6}^1) \]
\[ A_{W}^1 = (a_{W1}^1, a_{W2}^1, a_{W3}^1, a_{W4}^1, a_{W5}^1, a_{W6}^1) \]
\[ A_{B}^2 = (a_{B1}^2, a_{B2}^2, a_{B3}^2, a_{B4}^2, a_{B5}^2, a_{B6}^2) \]
\[ A_{W}^2 = (a_{W1}^2, a_{W2}^2, a_{W3}^2, a_{W4}^2, a_{W5}^2, a_{W6}^2) \]
\[ A_{B}^3 = (a_{B1}^3, a_{B2}^3, a_{B3}^3, a_{B4}^3, a_{B5}^3, a_{B6}^3) \]
\[ A_{W}^3 = (a_{W1}^3, a_{W2}^3, a_{W3}^3, a_{W4}^3, a_{W5}^3, a_{W6}^3) \]

\[ E_1 : \]

\[ A_{B}^1 = (a_{B1}^1, a_{B2}^1, a_{B3}^1, a_{B4}^1, a_{B5}^1, a_{B6}^1) \]
\[ A_{W}^1 = (a_{W1}^1, a_{W2}^1, a_{W3}^1, a_{W4}^1, a_{W5}^1, a_{W6}^1) \]
\[ A_{B}^2 = (a_{B1}^2, a_{B2}^2, a_{B3}^2, a_{B4}^2, a_{B5}^2, a_{B6}^2) \]
\[ A_{W}^2 = (a_{W1}^2, a_{W2}^2, a_{W3}^2, a_{W4}^2, a_{W5}^2, a_{W6}^2) \]
\[ A_{B}^3 = (a_{B1}^3, a_{B2}^3, a_{B3}^3, a_{B4}^3, a_{B5}^3, a_{B6}^3) \]
\[ A_{W}^3 = (a_{W1}^3, a_{W2}^3, a_{W3}^3, a_{W4}^3, a_{W5}^3, a_{W6}^3) \]
TABLE 10. Experts on the selection of the best and worst criteria.

| Criterion | Expert | E1 | E2 | E3 | E4 | E5 |
|-----------|--------|----|----|----|----|----|
| Each expert’s Best criterion | Data inputs (cₐ) | Memory (cₐ) | Display (cₐ) | Data inputs (cₐ) | Display (cₐ) |
| (cₐₙ) | (36, 712) = 4 | | | (6.33, 76.6) | | |
| Each expert’s Worst criterion | Weight (cₐ) | | | | Data inputs (cₐ) |
| (cₐₙ) | (36, 712) = 4 | | | (6.33, 76.6) | | |
| The selected best criterion | | | | | |
| (cₐₙ) | (36, 712) = 4 | | | (6.33, 76.6) | | |
| The selected worst criterion | | | | | |
| (cₐₙ) | (36, 712) = 4 | | | (6.33, 76.6) | | |

According to formula \( \bar{a}_{ij} = a_{Bi}/\bar{a}_{Wi} \) and \( \bar{a}_{ij} = a_{Wi}/\bar{a}_{Wj} \), the converted vectors are shown below:

**E₂:**

\[
\bar{A}_B^2 = (\bar{a}_{B1}^2, \bar{a}_{B2}^2, \bar{a}_{B3}^2, \bar{a}_{B4}^2, \bar{a}_{B5}^2, \bar{a}_{B6}^2)
\]

\[
= (\bar{a}_{B1}^2, \bar{a}_{B2}^2, \bar{a}_{B3}^2, \bar{a}_{B4}^2, \bar{a}_{B5}^2, \bar{a}_{B6}^2)
\]

\[
= ((3, 4, 5), (6, 7, 9), (6, 7, 9), (6, 7, 9), (6, 7, 9))
\]

**E₃:**

\[
\bar{A}_W^3 = (\bar{a}_{W1}^3, \bar{a}_{W2}^3, \bar{a}_{W3}^3, \bar{a}_{W4}^3, \bar{a}_{W5}^3, \bar{a}_{W6}^3)
\]

\[
= (\bar{a}_{W1}^3, \bar{a}_{W2}^3, \bar{a}_{W3}^3, \bar{a}_{W4}^3, \bar{a}_{W5}^3, \bar{a}_{W6}^3)
\]

\[
= ((3, 4, 5), (6, 7, 9), (6, 7, 9), (6, 7, 9), (6, 7, 9))
\]

**E₅:**

\[
\bar{A}_B^5 = (\bar{a}_{B1}^5, \bar{a}_{B2}^5, \bar{a}_{B3}^5, \bar{a}_{B4}^5, \bar{a}_{B5}^5, \bar{a}_{B6}^5)
\]

\[
= (\bar{a}_{B1}^5, \bar{a}_{B2}^5, \bar{a}_{B3}^5, \bar{a}_{B4}^5, \bar{a}_{B5}^5, \bar{a}_{B6}^5)
\]

\[
= ((3, 4, 5), (6, 7, 9), (6, 7, 9), (6, 7, 9), (6, 7, 9))
\]

Before integrating the comparison vectors of experts, it is necessary to check the consistency of the comparison vectors.
given by each expert. If the experts’ opinions are inconsistent, we can calculate the results and adjust accordingly. The input-based consistency ratio of the five experts are obtained by using Equation (27) and Equation (28), and the results are shown in Table 11 below. By comparing with the values in Table 4, it is found that each expert satisfies the consistency. After that geometric mean is used to integrate the experts’ opinions because it’s less affected by extreme value. By using Equation (15) and Equation (16), this paper obtains the final two integrated vectors.

\[
\begin{align*}
\tilde{A}_{BW} &= (\tilde{a}_{BW1}, \tilde{a}_{BW2}, \tilde{a}_{BW3}, \tilde{a}_{BW4}, \tilde{a}_{BW5}, \tilde{a}_{BW6}) \\
&= ((1.600, 3.645, 5), (4.153, 6.995, 8.094), \\
&\quad (4.794, 7.838, 8.790), (2.332, 4.076, 4.791), \\
&\quad (1, 1, 1), (0.739, 1.149, 2.111)) \\
\tilde{A}_{Ww} &= (\tilde{a}_{Ww1}, \tilde{a}_{Ww2}, \tilde{a}_{Ww3}, \tilde{a}_{Ww4}, \tilde{a}_{Ww5}, \tilde{a}_{Ww6}) \\
&= ((2.419, 4.782, 6.095), (0.735, 1.149, 2.111), \\
&\quad (1, 1, 1), (1.522, 3.173, 4.076), \\
&\quad (4.879, 7.977, 8.512), (4.346, 7.257, 8.586))
\end{align*}
\]

Based on the above analysis, for getting the optimal fuzzy weights of all the criteria, the following optimization problem can be built according to Equation (26).

\[
\begin{align*}
\min \xi & \quad \left\{(l^o_1, m^o_1, n^o_1) \oplus ((-n_51, -m_51, -l_51) \otimes (l^o_1, m^o_1, n^o_1)) \right\} \\
&\quad \leq (k^*, k^*, k^*) \\
&\quad \left\{(l^o_2, m^o_2, n^o_2) \oplus ((-n_52, -m_52, -l_52) \otimes (l^o_2, m^o_2, n^o_2)) \right\} \\
&\quad \leq (k^*, k^*, k^*) \\
&\quad \left\{(l^o_3, m^o_3, n^o_3) \oplus ((-n_53, -m_53, -l_53) \otimes (l^o_3, m^o_3, n^o_3)) \right\} \\
&\quad \leq (k^*, k^*, k^*) \\
&\quad \left\{(l^o_4, m^o_4, n^o_4) \oplus ((-n_54, -m_54, -l_54) \otimes (l^o_4, m^o_4, n^o_4)) \right\} \\
&\quad \leq (k^*, k^*, k^*) \\
&\quad \left\{(l^o_5, m^o_5, n^o_5) \oplus ((-n_55, -m_55, -l_55) \otimes (l^o_5, m^o_5, n^o_5)) \right\} \\
&\quad \leq (k^*, k^*, k^*) \\
&\quad \left\{(l^o_6, m^o_6, n^o_6) \oplus ((-n_56, -m_56, -l_56) \otimes (l^o_6, m^o_6, n^o_6)) \right\} \\
&\quad \leq (k^*, k^*, k^*) \\
&\quad \left\{(l^o_1, m^o_1, n^o_1) \oplus ((-n_13, -m_13, -l_13) \otimes (l^o_3, m^o_3, n^o_3)) \right\} \\
&\quad \leq (k^*, k^*, k^*) \\
&\quad \left\{(l^o_2, m^o_2, n^o_2) \oplus ((-n_23, -m_23, -l_23) \otimes (l^o_3, m^o_3, n^o_3)) \right\} \\
&\quad \leq (k^*, k^*, k^*) \\
&\quad \left\{(l^o_3, m^o_3, n^o_3) \oplus ((-n_33, -m_33, -l_33) \otimes (l^o_3, m^o_3, n^o_3)) \right\} \\
&\quad \leq (k^*, k^*, k^*) \\
&\quad \left\{(l^o_4, m^o_4, n^o_4) \oplus ((-n_43, -m_43, -l_43) \otimes (l^o_3, m^o_3, n^o_3)) \right\} \\
&\quad \leq (k^*, k^*, k^*) \\
&\quad \left\{(l^o_5, m^o_5, n^o_5) \oplus ((-n_53, -m_53, -l_53) \otimes (l^o_3, m^o_3, n^o_3)) \right\} \\
&\quad \leq (k^*, k^*, k^*) \\
&\quad \left\{(l^o_6, m^o_6, n^o_6) \oplus ((-n_63, -m_63, -l_63) \otimes (l^o_3, m^o_3, n^o_3)) \right\} \\
&\quad \leq (k^*, k^*, k^*) \\
\sum_{j=1}^{n} R(\tilde{a}_j) &= 1 \\
\sum_{j=1}^{n} l^o_j &= \sum_{j=1}^{n} m^o_j \leq \sum_{j=1}^{n} n^o_j \geq 0 \\
\end{align*}
\]

where with solving the problem, we have \(\tilde{\omega}_1 = (0.057, 0.110, 0.300)\), \(\tilde{\omega}_2 = (0.035, 0.057, 0.100)\), \(\tilde{\omega}_3 = (0.020, 0.036, 0.100)\), \(\tilde{\omega}_4 = (0.059, 0.098, 0.208)\), \(\tilde{\omega}_5 = (0.224, 0.339, 0.424)\), \(\tilde{\omega}_6 = (0.135, 0.318, 0.491)\), \(\xi = (0.06, 0.06, 0.06)\).

The optimal fuzzy weights of six criteria in this case is also shown in Figure 3.

Therefore, through the above calculation, the optimal fuzzy weights of six criteria are obtained in this paper. Then, we can de-fuzzy the obtained fuzzy numbers and turn them into \(R(\tilde{a}_j)\) for sorting, namely \(R(\tilde{a}_1, \tilde{a}_2, \tilde{a}_3, \tilde{a}_4, \tilde{a}_5, \tilde{a}_6) = (0.133, 0.060, 0.044, 0.110, 0.334, 0.316)\). Therefore, it can be concluded that the data inputs criterion is the most important, followed by memory, price, display, dimensions, and weight criteria.

\[
\text{VI. CONCLUSION AND FURTHER RESEARCH}
\]

In this paper, a new model based on the fuzzy best-worst method in GDM environment is proposed to integrate DM opinions, which solves the consistency problem in fuzzy BWM and helps to provide immediate feedback on the consistency of pair comparison. At the same time, a number of experts are introduced to evaluate the opinions, and the transformation of expert opinions by using mathematical formulas is convenient for the integration of experts’ opinions. Finally, the geometric average method is used for integration. The results show that the method is effective and suitable for group decision making problems.

In the examination of the degree of consistency of expert opinions, we refer to the input-based consistency ratio proposed in [32] and extend the input-based consistency ratio to the fuzzy environment. What’s more, with the thresholds.

\[
\begin{array}{|c|c|c|c|c|c|c|c|}
\hline
\text{Expert} & CR^I & CR^I & CR^I & CR^I & CR^I & CR^I & CR^I \cr
\hline
E1 & 0.114 & 0.114 & 0.020 & 0 & 0.114 & 0.035 & 0.03 \cr
E2 & 0.216 & 0.202 & 0.002 & 0.012 & 0.202 & 0.026 & 0.01 \cr
E3 & 0.218 & 0.201 & 0.003 & 0 & 0.01 & 0.018 & 0.1 \cr
E4 & 0.146 & 0.136 & 0.011 & 0.016 & 0.136 & 0.01 & 0 \cr
E5 & 0.105 & 0.095 & 0.044 & 0.026 & 0.01 & 0.01 & 0.01 \cr
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|c|c|c|c|c|}
\hline
\text{Criteria} & \text{Memory} & \text{Data inputs} & \text{Price} & \text{Display} & \text{Dimensions} & \text{Weight} \cr
\hline
\end{array}
\]

\[
\begin{array}{|c|c|c|c|c|}
\hline
\text{Criteria} & \text{Memory} & \text{Data inputs} & \text{Price} & \text{Display} \cr
\hline
\end{array}
\]
proposed in [32], the DM can decide whether to modify his/her previous assessment. This not only shows the ratio of DM violation ordinal degree of consistency, but also provides a convenient way to identify and correct the conflict.

There are several aspects of improvement for the proposed method in this paper, which are also the future research directions. Firstly, one of the defuzzification methods is used for improved input-based consistency ratio and constrained optimization equations. However, there are many other defuzzification methods that can be applied to the model, which can be a direction for future research. Secondly, the main purpose of aggregating the opinions of different experts is to produce appropriate results from pairwise comparison matrices. Each approach has its own strengths and weaknesses, and subsequent research can focus on the strengths and weaknesses of the different aggregation approaches. Thirdly, the proposed fuzzy best-worst multi-criteria group decision-making method in this paper can be employed in some real-world problems to further verify the effectiveness of the proposed method.

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