A Note on Modeling Self-Suspending Time as Blocking Time in Real-Time Systems

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Abstract. This report presents a proof to support the correctness of the schedulability test for self-suspending real-time task systems proposed by Jane W. S. Liu in her book titled “Real-Time Systems” (Pages 164-165). The same concept was also implicitly used by Rajkumar, Sha, and Lehoczky in RTSS 1988 (Page 267) for analyzing self-suspending behaviour due to synchronization protocols in multiprocessor systems.

1 Introduction

This report presents a proof to support the correctness of the schedulability test for self-suspending real-time task systems proposed by Jane W. S. Liu in her book titled ”Real-Time Systems” [3, Pages 164-165]. The same concept was also implicitly used by Rajkumar, Sha, and Lehoczky [6, Page 267] for analyzing self-suspending behaviour due to synchronization protocols in multiprocessor systems.

The system model and terminologies are defined as follows: We assume a system composed of $n$ sporadic self-suspending tasks. A sporadic task $\tau_i$ is released repeatedly, with each such invocation called a job. The $j^{th}$ job of $\tau_i$, denoted $\tau_{i,j}$, is released at time $r_{i,j}$ and has an absolute deadline at time $d_{i,j}$. Each job of any task $\tau_i$ is assumed to have a worst-case execution time $C_i$. Each job of task $\tau_i$ suspends for at most $S_i$ time units (across all of its suspension phases). When a job suspends itself, the processor can execute another job. The response time of a job is defined as its finishing time minus its release time. Successive jobs of the same task are required to execute in sequence. Associated with each task $\tau_i$ are a period (or minimum inter-arrival time) $T_i$, which specifies the minimum time

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between two consecutive job releases of $\tau_i$, and a relative deadline $D_i$, which specifies the maximum amount of time a job can take to complete its execution after its release, i.e., $d_{i,j} = r_{i,j} + D_i$. The worst-case response time $R_i$ of a task $\tau_i$ is the maximum response time among all its jobs. The utilization of a task $\tau_i$ is defined as $U_i = C_i/T_i$.

In this report, we focus on constrained-deadline task systems, in which $D_i \leq T_i$ for every task $\tau_i$. We only consider preemptive fixed-priority scheduling on a single processor, in which each task is assigned with a unique priority level. We assume that the priority assignment is given.

We assume that the tasks are numbered in a decreasing priority order. That is, a task with a smaller index has higher priority than any task with a higher index, i.e., task $\tau_i$ has a higher-priority level than task $\tau_{i+1}$. When performing the schedulability analysis of a specific task $\tau_k$, we assume that $\tau_1, \tau_2, \ldots, \tau_{k-1}$ are already verified to meet their deadlines, i.e., that $R_i \leq D_i, \forall \tau_i | 1 \leq i \leq k-1$.

We also classify the $k-1$ higher-priority tasks into two sets: $T_1$ and $T_2$. A task $\tau_i$ is in $T_1$ if $C_i \geq S_i$; otherwise, it is in $T_2$.

2 Model Suspension Time and Blocking Time

To analyze the worst-case response time (or the schedulability) of task $\tau_k$, we usually need to quantify the worst-case interference caused by the higher-priority tasks on the execution of any job of task $\tau_k$. In the ordinary sequential sporadic real-time task model, i.e., when $S_i = 0$ for every task $\tau_i$, the so-called critical instant theorem by Liu and Layland [2] is commonly adopted. That is, the worst-case response time of task $\tau_k$ (if it is less than or equal to its period) happens for the first job of task $\tau_k$ when $\tau_k$ and all the higher-priority tasks release a job synchronously and the subsequent jobs are released as early as possible (i.e., with a rate equal to their period).

However, as proven in [3], this definition of the critical instant does not hold for self-suspending sporadic tasks. In [3 Pages 164-165], Jane W. S. Liu proposed a solution to study the schedulability of self-suspending tasks by modeling the extra delay suffered by a task $\tau_k$ due to the self-suspending behavior of the tasks as a blocking time denoted as $B_k$ and defined as follows:

- The blocking time contributed from task $\tau_k$ is $S_k$.
- A higher-priority task $\tau_i$ can only block the execution of task $\tau_k$ by at most $b_i = \min(C_i, S_i)$ time units.

Therefore,

$$B_k = S_k + \sum_{i=1}^{k-1} b_i. \quad (1)$$

In [3], the blocking time is then used to derive a utilization-based schedulability test for rate-monotonic scheduling. Namely, it is stated that if $\frac{C_k + B_k}{T_k} + \sum_{i=1}^{k-1} U_i \leq k(2^k - 1)$, then task $\tau_k$ can be feasibly scheduled by using rate-monotonic scheduling if $T_i = D_i$ for every task $\tau_i$ in the given task set. If the
above argument is correct, we can further prove that a constrained-deadline task \( \tau_k \) can be feasibly scheduled by the fixed-priority scheduling if

\[
\exists t \mid 0 < t \leq D_k, \quad C_k + B_k + \sum_{i=1}^{k-1} \left\lfloor \frac{t}{T_i} \right\rfloor C_i \leq t. \quad (2)
\]

The same concept was also implicitly used by Rajkumar, Sha, and Lehoczky \[6, \text{Page 267}\] for analyzing self-suspending behaviour due to synchronization protocols in multiprocessor systems. To account for the self-suspending behaviour, it reads as follows:\[3\]

For each higher priority job \( J_i \) on the processor that suspends on global semaphores or for other reasons, add the term \( \min(C_i, S_i) \) to \( B_k \), where \( S_i \) is the maximum duration that \( J_i \) can suspend itself. The sum ... yields \( B_k \), which in turn can be used in \( C_k + B_k + \sum_{i=1}^{k-1} U_i \leq k(2^k - 1) \) to determine whether the current task allocation to the processor is schedulable.

However, as there is no proof in \[3,6\] to support the correctness of the above tests, we present a proof in the next section of this report.

\section{Our Proof}

This section provides the proof to support the correctness of the test in Eq. (2). First, it should be easy to see that we can convert the suspension time of task \( \tau_k \) into computation. This has been proven in many previous works, e.g., Lemma 3 in \[1\] and Theorem 2 in \[4\]. Yet, it remains to formally prove that the additional interference due to the self-suspension of a higher-priority task \( \tau_i \) is upper-bounded by \( b_i = \min(C_i, S_i) \). The interference to be at most \( C_i \) has been provided in the literature as well, e.g., \[5,1\]. However, the argument about blocking task \( \tau_k \) due to a higher-priority task \( \tau_i \) by at most \( S_i \) amount of time is not straightforward.

From the above discussions, we can greedily convert the suspension time of task \( \tau_k \) to its computation time. For the sake of notational brevity, let \( C'_k \) be \( C_k + S_k \). We call this converted version of task \( \tau_k \) as task \( \tau'_k \). Our analysis is also based on very simple properties and lemmas enunciated as follows:

\begin{property}
In a preemptive fixed-priority schedule, the lower-priority jobs do not impact the schedule of the higher-priority jobs.
\end{property}

\begin{lemma}
In a preemptive fixed-priority schedule, if the worst-case response time of task \( \tau_i \) is no more than its period \( T_i \), preventing the release of a job of task \( \tau_i \) does not affect the schedule of any other job of task \( \tau_i \).
\end{lemma}

\begin{proof}
Since the worst-case response time of task \( \tau_i \) is no more than its period, any job \( \tau_{i,j} \) of task \( \tau_i \) completes its execution before the release of the next job
\[3\] We rephrased the wordings and notation to be consistent with this report.
τ_{i,j+1}. Hence, the execution of τ_{i,j} does not directly interfere with the execution of any other job of τ_i, which then depends only on the schedule of the higher priority jobs. Furthermore, as stated in Property 1, the removal of τ_{i,j} has no impact on the schedule of the higher-priority jobs, thereby implying that the other jobs of task τ_i are not affected by the removal of τ_{i,j}.

We can prove the correctness of Eq. (2) by using a similar proof than for the critical instant theorem of the ordinary sporadic task model. Let R'_k be the minimum t greater than 0 such that Eq. (2) holds, i.e., \( C'_k + B_k + \sum_{i=1}^{k-1} \left\lceil \frac{R(i)}{T} \right\rceil C_i \). We define R'_k and \( 0 < t < R'_k \) \( C'_k + B_k + \sum_{i=1}^{k-1} \left\lceil \frac{R(i)}{T} \right\rceil C_i > t \). The following theorem shows that R'_k is a safe upper bound if the worst-case response time of task τ'_k is no more than T_k.

**Theorem 1.** R'_k is a safe upper bound on the worst-case response time of task τ'_k if the worst-case response time of τ'_k is not larger than T_k.

**Proof.** Let us consider the task set τ' composed of \{τ_1, τ_2, …, τ_{k-1}, τ'_k, τ_{k+1}, …\} and let \( \Psi \) be a schedule of τ' that generates the worst-case response time of τ'_k.

The proof is built upon the two following steps:

1. **Step 1: Reducing the schedule \( \Psi \).** During this step, we iteratively build an artificial schedule \( \Psi^j \) from \( \Psi^{j+1} \) (with 1 ≤ j < k) so that the response time of τ'_k remains identical. At each iteration, we define t_j for task τ_j in the schedule \( \Psi^{j+1} \) (with j = k - 1, k - 2, …, 1) and build \( \Psi^j \) by removing all the jobs released by τ_j before t_j.

2. **Basic step (definition of \( \Psi^k \) and t_k):** Suppose that the job J_k of task τ'_k with the largest response time in \( \Psi \) arrives at time τ_k and finishes at time f_k. We know by Property 1 that the lower priority tasks τ_{k+1}, τ_{k+2}, …, τ_n do not impact the response time of J_k. Moreover, since we assume that the worst-case response time of task τ'_k is no more than T_k, Lemma 1 proves that removing all the jobs of task τ'_k but J_k has no impact on the schedule of J_k. Therefore, let \( \Psi^k \) be a schedule identical to \( \Psi \) but removing all the jobs released by the lower priority tasks τ_{k+1}, …, τ_n as well as all the
jobs released by $\tau_k^j$ at the exception of $J_k$. The response time of $J_k$ in $\Psi$ is thus identical to the response time of $J_k$ in $\Psi$. We define $t_k$ as the release time of $J_k$ (i.e., $t_k = r_k$).

**Induction step (definition of $\Psi^j$ and $t_j$ with $1 \leq j < k$):**

Let $t_j$ be the arrival time of the last job released by $\tau_j$ before $t_j+1$ in $\Psi^{j+1}$ and let $J_j$ denote that job. Removing all the jobs of task $\tau_j$ arrived before $t_j$ has no impact on the schedule of any other job released by $\tau_j$ (Lemma 1) or any higher priority job released by $\tau_1, \ldots, \tau_{j-1}$ (Property 1). Moreover, because by the construction of $\Psi^{j+1}$, no task with a priority lower than $\tau_j$ executes jobs before $t_j+1$ in $\Psi^{j+1}$, removing the jobs released by $\tau_j$ before $t_j+1$ does not impact the schedule of the jobs of $\tau_{j+1}, \ldots, \tau_k$. Therefore, we can safely remove all the jobs of task $\tau_j$ arrived before $t_j$ without impacting the response time of $J_k$. Two cases must then be considered:

(a) $\tau_j \in T_1$, i.e., $S_j < C_j$. In this case, we analyze two different subcases:

- $J_j$ completed its execution before or at $t_j+1$. By Lemma 1 and Property 1, removing all the jobs of task $\tau_j$ arrived before $t_j+1$ has no impact on the schedule of the higher-priority jobs (jobs released by $\tau_1, \ldots, \tau_{j-1}$) and the jobs of $\tau_j$ released after or at $t_j+1$. Moreover, because no task with lower priority than $\tau_j$ executes jobs before $t_j+1$ in $\Psi^{j+1}$, removing the jobs released by $\tau_j$ before $t_j+1$ does not impact the schedule of the jobs of $\tau_{j+1}, \ldots, \tau_k$. Therefore, $t_j$ is set to $t_j+1$ and $\Psi^j$ is generated by removing all the jobs of task $\tau_j$ arrived before $t_j+1$. The response time of $J_k$ in $\Psi^j$ thus remains unchanged in comparison to its response time in $\Psi^{j+1}$.

- $J_j$ did not complete its execution by $t_j+1$. For such a case, $t_j$ is set to $r_j$ and hence $\Psi^j$ is built from $\Psi^{j+1}$ by removing all the jobs released by $\tau_j$ before $r_j$.

Note that because by the construction of $\Psi^{j+1}$ and hence $\Psi^j$ there is no job with priority lower than $\tau_j$ available to be executed before $t_j+1$, the maximum amount of time during which the processor remains idle within $[t_j, t_j+1)$ is at most $S_j$ time units.

(b) $\tau_j \in T_2$, i.e., $S_j \geq C_j$. For such a case, we set $t_j$ to $t_j+1$. Let $c_j(t_j)$ be the remaining execution time for the job of task $\tau_j$ at time $t_j$. We know that $c_j(t_j)$ is at most $C_j$. Since by the construction of $\Psi^j$, all the jobs of $\tau_j$ released before $t_j$ are removed, the job of task $\tau_j$ arrived at time $r_j (< t_j)$ is replaced by a new job released at time $t_j$ with execution time $c_j(t_j)$ and the same priority than $\tau_j$. Clearly, this has no impact on the execution of any job executed after $t_j$ and thus on the response time of $J_k$. The remaining execution time $c_j(t_j)$ of $\tau_j$ at time $t_j$ is called the residual workload of task $\tau_j$ for the rest of the proof.

The above construction of $\Psi^{k-1}, \Psi^{k-2}, \ldots, \Psi^1$ is repeated until producing $\Psi^1$. The procedures are well-defined. Therefore, it is guaranteed that $\Psi^1$ can be constructed. Note that after each iteration, the number of jobs considered in the schedule have been reduced, yet without affecting the response time of $J_k$. 

Step 2: Analyzing the final reduced schedule $\Psi^1$

We now analyze the properties of the final schedule $\Psi^1$ in which all the unnecessary jobs have been removed. The proof is based on the fact that for any interval $[t_1, t)$, there is

$$\text{idle}(t_1, t) + \text{exec}(t_1, t) = (t - t_1) \quad (3)$$

where $\text{exec}(t_1, t)$ is the amount of time during which the processor executed tasks within $[t_1, t)$, and $\text{idle}(t_1, t)$ is the amount of time during which the processor remained idle within the interval $[t_1, t)$.

Because there is no job released by lower priority tasks than $\tau_k$ in $\Psi^1$, the workload released by $\tau_1, \ldots, \tau_k$ within any interval $[t_1, t)$ is upper bounded by $\sum_{\tau_i \in T_2} C_i$. Therefore, considering the fact that no job of $\tau_j$ is released before $t_j$ in $\Psi^1$ ($j = 1, 2, \ldots, k$), the workload released by the tasks (by treating the residual workload in $T_2$ as released workload as well) within any time interval $[t_1, t)$ in schedule $\Psi^1$ such that $t_1 < t \leq f_k$ is upper bounded by

$$\sum_{i=1}^{k} \left( c_i(t_i) + \max\{0, \left\lceil \frac{t-t_i}{T_i} \right\rceil C_i \} \right) \leq \sum_{\tau_i \in T_2} C_i + \sum_{i=1}^{k} \max\{0, \left\lceil \frac{t-t_i}{T_i} \right\rceil C_i \},$$

leading to

$$\forall t \mid t_1 \leq t < f_k, \quad \text{exec}(t_1, t) \leq \sum_{\tau_i \in T_2} C_i + \sum_{i=1}^{k} \max\{0, \left\lceil \frac{t-t_i}{T_i} \right\rceil C_i \} \quad (4)$$

Furthermore, from case (a) of Step 1, we know that the maximum amount of time during which the processor is idle in $\Psi^1$ within any time interval $[t_1, t)$ such that $t_1 < t \leq f_k$, is upper bounded by $\sum_{\tau_i \in T_1} S_i$. That is,

$$\forall t \mid t_1 \leq t < t_k, \quad \text{idle}(t_1, t) \leq \sum_{\tau_i \in T_1} S_i \quad (5)$$

Hence, injecting Eq. (4) and Eq. (5) into Eq. (3), we get

$$\forall t \mid t_1 \leq t < t_k, \quad \sum_{\tau_i \in T_1} S_i + \sum_{\tau_i \in T_2} C_i + \sum_{i=1}^{k} \max\{0, \left\lceil \frac{t-t_i}{T_i} \right\rceil C_i \} \geq t - t_1.$$

Since $C'_k > 0$ and $\max\{0, \left\lceil \frac{t-t_k}{T_k} \right\rceil C'_k \} = 0$ for any $t$ smaller than $t_k$, it holds that

$$\forall t \mid t_1 \leq t < t_k, \quad \sum_{\tau_i \in T_1} S_i + \sum_{\tau_i \in T_2} C_i + \sum_{i=1}^{k-1} \max\{0, \left\lceil \frac{t-t_i}{T_i} \right\rceil C_i \} > t - t_1,$$
and using the definition of $b_i$

$$\forall t \mid t_1 \leq t < t_k, \quad C'_k + \sum_{i=1}^{k-1} b_i + \sum_{i=1}^{k-1} \max\left\{0, \left\lfloor \frac{t-t_i}{T_i} \right\rfloor C_i \right\} > t - t_1. \quad (6)$$

Furthermore, because $J_k$ is released at time $t_k$ and does not complete its execution before $f_k$, it must hold that

$$\forall t \mid t_k \leq t < f_k, \quad C'_k + \sum_{i=1}^{k-1} b_i + \sum_{i=1}^{k-1} \max\left\{0, \left\lfloor \frac{t-t_i}{T_i} \right\rfloor C_i \right\} > t - t_1. \quad (7)$$

Combining Eq. (6) and Eq. (7), we get

$$\forall t \mid t_1 \leq t < f_k, \quad C'_k + \sum_{i=1}^{k-1} b_i + \sum_{i=1}^{k-1} \max\left\{0, \left\lfloor \frac{t-t_i}{T_i} \right\rfloor C_i \right\} > t - t_1. \quad (8)$$

Since $t_i \geq t_1$ for $i = 1, 2, \ldots, k$, there is

$$\left\lfloor \frac{t-t_i}{T_i} \right\rfloor \leq \left\lfloor \frac{t-t_1}{T_1} \right\rfloor,$$

thereby leading to

$$\forall t \mid t_1 \leq t < f_k, \quad C'_k + \sum_{i=1}^{k-1} b_i + \sum_{i=1}^{k-1} \max\left\{0, \left\lfloor \frac{t-t_i}{T_i} \right\rfloor C_i \right\} > t - t_1. \quad (9)$$

By replacing $t - t_1$ with $\theta$, Eq. (9) becomes

$$\forall \theta \mid 0 < \theta < f_k - t_1, \quad C'_k + \sum_{i=1}^{k-1} b_i + \sum_{i=1}^{k-1} \left\lfloor \frac{\theta}{T_i} \right\rfloor C_i > \theta.$$  

The above inequation implies that the minimum $t$ such that $C'_k + \sum_{i=1}^{k-1} b_i + \sum_{i=1}^{k-1} \left\lfloor \frac{\theta}{T_i} \right\rfloor C_i \leq t$ is larger than or equal to $f_k - t_1$. And because by assumption the worst-case response time of $\tau'_k$ is equal to $f_k - t_k \leq f_k - t_1$ which is obviously smaller than or equal to $R'_k$, it holds that $R'_k$ is a safe upper bound on the worst-case response time of $\tau'_k$.  

For the simplicity of presentation, we assume that $\Psi$ is a schedule of $\tau'_k$ that generates the worst-case response time of $\tau'_k$ in the proof of Theorem 1. This can be relaxed to start from an arbitrary job $J_k$ in any fixed-priority schedule by using the same proof flow with similar arguments.

**Corollary 1** $R'_k$ is a safe upper bound on the worst-case response time of task $\tau'_k$ if $R'_k$ is not larger than $T_k$.  

*We take $0 < \theta$ instead of $0 \leq \theta$ since $C'_k$ is assumed to be positive.*
Proof. Directly follows from Theorem 1.

Corollary 2 $R'_k$ is a safe upper bound on the worst-case response time of task $\tau_k$ if $R'_k$ is not larger than $T_k$.

Proof. Since, as proven in [5], the worst-case response time of $\tau'_k$ is always larger than or equal to the worst-case response time of $\tau_k$, this corollary directly follows from Corollary 1.

Note that the proof of Theorem 1 does not require to start from the schedule with the worst-case response time for $\tau'_k$. The analysis still works well by considering any job with any arbitrary fixed-priority schedule. To illustrate Step 1 in the above proof, we also provide one concrete example. Consider a task system with the following 4 tasks:

- $T_1 = 6, C_1 = 1, S_1 = 1,
- T_2 = 10, C_2 = 1, S_2 = 6,
- T_3 = 18, C_3 = 4, S_3 = 1,
- T_4 = 20, C_4 = 5, S_4 = 0.$

Figure 1 demonstrates a schedule for the jobs of the above 4 tasks. We assume that the first job of task $\tau_1$ arrives at time $4 + \epsilon$ with a very small $\epsilon > 0$. The first job of task $\tau_2$ suspends itself from time 0 to time $5 + \epsilon$, and is blocked by task $\tau_1$ from time $5 + \epsilon$ to time $6 + \epsilon$. After some very short computation with $\epsilon$ amount of time, the first job of task $\tau_2$ suspends itself again from time $6 + 2\epsilon$ to 7. In this schedule, $f_k$ is set to $20 - \epsilon$.

We define $t_4$ as 7. Then, we set $t_3$ to 6. When considering task $\tau_2$, since it belongs to $T_2$, we greedily set $t_2$ to $t_3 = 6$ and the residual workload $C'_2$ is 1. Then, $t_1$ is set to $4 + \epsilon$. In the above schedule, the idle time from $4 + \epsilon$ to $20 - \epsilon$ is at most $2 = S_1 + S_3$. We have to further consider one job of task $\tau_2$ arrived before time $t_1$ with execution time $C_2$. 

Fig. 1. An illustrative example of the proof.
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