NEUTRINO EMISSION IN THE JET PROPAGATION PROCESS

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ABSTRACT

Relativistic jets are universal in long-duration gamma-ray burst (GRB) models. Before breaking out, they must propagate in the progenitor envelope along with a forward shock and a reverse shock forming at the jet head. Both electrons and protons will be accelerated by the shocks. High-energy neutrinos could be produced by these protons interacting with stellar materials and electron-radiating photons. The jet will probably be collimated, which may have a strong effect on the final neutrino flux. Under the assumption of a power-law stellar-envelope density profile $\rho \propto r^{-\alpha}$ with index $\alpha$, we calculate the neutrino emission flux by these shocks for low-luminosity GRBs (LL-GRBs) and ultra-long GRBs (UL-GRBs) in different collimation regimes, using the jet propagation framework developed by Bromberg et al. We find that LL-GRBs and UL-GRBs are capable of producing detectable high-energy neutrinos up to $\sim$PeV, from which the final neutrino spectrum can be obtained. Further, we conclude that a larger $\alpha$ corresponds to greater neutrino flux at the high-energy end ($\sim$PeV) and to higher maximum neutrino energy as well. However, such differences are so small that it is not promising for us to be able to distinguish these in observations, given the energy resolution we have now.

Key words: gamma-ray burst: general – neutrinos – relativistic processes

Online-only material: color figures

1. INTRODUCTION

The collapsar model of gamma-ray bursts (GRBs) suggests that during the core collapse of a progenitor massive star to a neutron star or a black hole, a relativistic jet punctures the stellar envelope and transports energy to electrons and protons through shock acceleration (Zhang & Mészáros 2004; Piran 2005; Mészáros 2006; Woosley 1993; MacFadyen & Woosley 1999). Accelerated electrons dominate radiation via synchrotron or inverse Compton mechanism while accelerated protons produce neutrinos by proton–proton (pp) collision and photopion process (Waxman & Bahcall 1997, 1999; Rachen & Mészáros 1998; Alvarez-Muniz et al. 2000; Bahcall & Waxman 2001; Guetta & Granot 2003; Murase et al. 2006; Becker 2008). Within this scenario, we can expect high-energy neutrinos originating from different stages in a GRB event. First, Waxman & Bahcall (2000) and Dai & Lu (2001) proposed $\epsilon_\nu > 10^3$ TeV neutrinos from an external reverse shock (RS). Similarly, neutrinos from an external forward shock (FS) have been discussed (Li et al. 2002; Dermer et al. 2003; Razzaque 2013). Second, the prompt photon emission from GRBs can be correlated to the production of neutrinos, since protons are also believed to be accelerated in relativistic internal shocks (Vietri 1995; Waxman & Bahcall 1997, 1999; Mészáros & Rees 2000; Guetta et al. 2004). Murase & Nagataki (2006) obtained a diffuse neutrino background spectrum from GRBs for specific parameter sets in the internal shock model. Alternatively, the neutrino emission might arise in a dissipative jet photosphere (Rees & Mészáros 2005; Murase 2008; Wang & Dai 2009; Gao et al. 2012). Third, neutrinos could also be produced while the jet is still propagating in the envelope. Generally, these neutrinos appear as a precursor burst. Mészáros & Waxman (2001) predicted a neutrino precursor of $\epsilon_\nu \geq 5$ TeV, which is produced by internal shocks at radius $r_{IS} \approx 10^{10}$–$10^{11}$ cm. Enberg et al. (2009) presented a new analysis of the neutrino flux of these internal shocks for two types of source environments: the slow-jet supernova model and the GRB model. Still further, Murase & Ioka (2013) studied high-energy neutrino production in collimated jets inside progenitors of GRBs and supernovae, considering both collimation and internal shocks. Pruet (2003) discussed the neutrinos produced by inelastic neutron–nucleon collisions of a relativistic jet propagating through a stellar envelope. Razzaque et al. (2003) dealt with the high-energy neutrino signature in a supernova remnant shell ejected prior to a GRB and then Ando & Beacom (2005) extended their model and significantly improved the detection prospects. Horiuchi & Ando (2008) proposed that an RS at the jet head probably will accelerate protons when crossing the jet material, and they calculated in detail the cumulative neutrino event number that may be observed by a km$^2$ scale detector like IceCube.

The jet propagation dynamics has been studied. Assuming a constant jet velocity, Begelman & Cioffi (1989) discussed the propagation of a galactic jet in the intergalactic medium. Mészáros & Waxman (2001) analyzed the propagation in the envelope of a red supergiant star, ignoring the surrounding cocoon. Matzner (2003) studied the jet-cocoon structure and jet head velocity that is constrained by ram pressure balance, but they did not consider the collimation by cocoon pressure. Lazzati & Begelman (2005) took into account the collimation effect and meanwhile they assumed that the jet expands adiabatically. Bromberg et al. (2011) considered a collimated shock that forms at the base of the jet and dissipates parts of the jet’s energy to counterbalance the cocoon’s pressure. They determined the geometry of a collimated shock, and thus further obtained the requirement for jet collimation. Moreover, it is worth mentioning that various numerical simulations of jet propagation in the envelope already have been performed (Zhang et al. 2003; Morsony et al. 2007; Mizuta & Aloy 2009; Mizuta & Ioka 2013).

In our paper, we try to calculate the flux of high-energy neutrinos from the shocks formed at the jet head for low-luminosity GRBs (LL-GRBs) and ultra-long GRBs (UL-GRBs), when the jet is still propagating inside the envelope. Most
importantly, we take into account the jet collimation which could largely affect the final neutrino flux but was ignored in the previous studies. For simplicity, we only account for jet propagation in the helium core because collimation mainly happens there. We assume a power law envelope density profile \( \rho(r) = Ar^{-\alpha} \), where \( A = (3 - \alpha)M_{\text{He}}/(4\pi r_{\text{He}}^3) \) and \( 2 < \alpha < 3 \) with \( M_{\text{He}} \) and \( r_{\text{He}} \) being the mass and radius of the helium envelope. We employ the analytical solutions described in Bromberg et al. (2011) to further calculate the neutrino flux in different collimation regimes, and finally, we discuss its dependence on the index \( \alpha \), providing an alternative way to probe the GRB progenitor through the neutrino precursor signal in the future.

This paper is organized as follows. In Section 2 we discuss the jet propagation dynamics. In Section 3 we perform detailed calculations of neutrino flux in each regime and a comparison between different regimes. Dependence on \( \alpha \) is considered in Section 4. We make a comparison with previous works in Section 5 and finally we present discussions and conclusions in Section 6.

2. JET PROPAGATION DYNAMICS

According to Matzner (2003), the head velocity is constrained by ram pressure balance,

\[
\rho_j h_j \Gamma_j^2 (\beta_j - \beta_h)^2 + P_j = \rho_a h_a \Gamma_a^2 \beta_h^2 + P_a, \tag{1}
\]

where \( \rho, P, \beta, \Gamma, \) and \( h \equiv 1 + 4P/\rho c^2 \) are the density, pressure, Lorentz factor, and dimensionless specific enthalpy, and the subscripts \( j \) and \( a \) refer to the jet and ambient material, respectively. Then the jet head velocity is

\[
\beta_h = \frac{\beta_j}{1 + \bar{L}^{-1/2}}, \tag{2}
\]

where \( \bar{L} \equiv \frac{\rho_j h_j \Gamma_j^2}{\rho_a} \simeq \frac{L_j}{\Sigma_j \rho_a c^3} \)

\[
\tag{3}
\]

with \( L_j \) is the jet luminosity and \( \Sigma_j = \pi r_j^2 \) is the jet’s cross section.

The parameter \( \bar{L} \) is crucial for determining collimation regimes (see Bromberg et al. 2011, Table 1). While \( \bar{L} \lesssim \theta_{0,1}^{-3/4} \) the jet is strongly collimated by the cocoon pressure, where \( \theta_0 \) is the jet opening angle. Bromberg et al. (2011) further divide it into two situations: \( \bar{L} \ll 1 \) and \( 1 \ll \bar{L} \lesssim \theta_{0,1}^{-4/3} \). The uncollimated regime corresponds to \( \bar{L} > \theta_{0,1}^{-4/3} \) accordingly.

The internal energy and particle number density of the shocked and unshocked regions are correlated by (Blandford & McKee 1976; Sari & Piran 1995)

\[
\frac{e_f}{n_f m_p c^2} = \Gamma_h - 1, \quad \frac{n_f}{n_a} = 4\Gamma_h + 3, \tag{4}
\]

\[
\frac{e_r}{n_r m_p c^2} = \bar{\Gamma}_h - 1, \quad \frac{n_r}{n_j} = 4\bar{\Gamma}_h + 3, \tag{5}
\]

where the subscripts \( f \) and \( r \) represent regions that have been crossed by the FS and RS. \( \Gamma_h \) is the Lorentz factor of the head and \( \bar{\Gamma}_h \) is the Lorentz factor of the unshocked jet measured in the jet head frame,

\[
\bar{\Gamma}_h = \bar{\Gamma}_j \Gamma_h(1 - \beta_j \beta_h). \tag{6}
\]

Moreover, the density of the unshocked jet materials \( \rho_j \) is determined by

\[
L_j = \Gamma_j^2 \pi r_j^2 \rho_j c^3. \tag{7}
\]

Based on the above equations, we can carry on with our calculation.

3. THEORETICAL CALCULATION OF NEUTRINO FLUX

In this section, we take \( \alpha = 2 \) as our premise, and discuss the dependence on \( \alpha \) later in Section 4.

The most promising acceleration process in a GRB is the Fermi acceleration mechanism. However, now there are noteworthy arguments that once the jet bulk kinetic energy is dissipated at the collimation shock, the collimated jet would become radiation-dominated and then the RS occurring at the interface of the jet head and collimated jet would also be radiation-mediated. At such shocks, photons produced in the downstream diffuse into the upstream and interact with electrons or pairs. There would no longer be a strong shock jump and Fermi acceleration will no longer work (Levinson & Bromberg 2008; Katz et al. 2010; Murase & Ioka 2013). However, this case is changed for LL-GRBs (Soderberg et al. 2006; Toma et al. 2007; Liang et al. 2007; Murase & Ioka 2013) and UL-GRBs (Levan et al. 2013; Gendre et al. 2013; Murase & Ioka 2013). Because of low power or large radii, the Thomson optical depth is low even inside a star (Murase & Ioka 2013) so that efficient Fermi acceleration would be expected. We consider LL-GRBs and UL-GRBs separately and they fall into different collimation regimes, which we will discuss later.

Due to the first order Fermi acceleration, the accelerated proton spectrum is (Achterberg et al. 2001; Keshet & Waxman 2005; Horiuchi & Ando 2008):

\[
\frac{dn_p}{d\epsilon_p} \propto \epsilon_p^{-p}, \tag{8}
\]

where \( \epsilon_p \) and \( n_p \) are the proton energy and number density. We optimistically take \( p = 2 \) in our calculation. The minimum proton energy \( \epsilon_{p,\text{min}} \sim \Gamma_0^3 m_p c^2 \) and maximum energy \( \epsilon_{p,\text{max}} \) is determined by the balance between proton acceleration and the cooling process.

3.1. \( \bar{L} \ll 1 \)

In this regime, the jet head moves forward with a non-relativistic velocity, \( \Gamma_h \simeq 1 \). The unshocked jet’s Lorentz factor in the head frame is \( \Gamma_h = \bar{\Gamma}_j \). We can easily see that the RS is strong while the FS is so weak that we need not consider it. The internal energy of the shocked jet is \( e_r \approx (\Gamma_j - 1)(4\Gamma_j + 3)\rho_j c^2 \).

In addition, we deduce some crucial terms below analytically (see Bromberg et al. 2011, Appendix B).

\[
\bar{L} = \left( \frac{16}{\pi} \right)^{2/3} \Gamma_j^{2/3} A_{-2/3} \theta_0^{8/3} c^{-2}, \tag{9}
\]

\[
\theta_j = 2^{-4/3} \pi^{-1/6} L_j^{1/6} \theta_0^{8/15} c^{-1/2}, \tag{10}
\]

\[
\rho_j = 2^{8/3} \pi^{-2/3} \Gamma_j^{2/3} A_{-2/3} \theta_0^{16/15} c^{-2}. \tag{11}
\]

where \( \theta_0 \) is the initial jet opening angle and \( \theta_j \) is the opening angle after collimation. Fortunately, \( \bar{L}, \theta_j \) do not vary with radius when and only when \( \alpha = 2 \).

The internal energy

\[
e_r = 2^{8/3} \pi^{-2/3} (4\Gamma_j + 3)(\Gamma_j - 1) L_j^{2/3} A_{-2/3} \theta_0^{-16/15}. \tag{12}
\]
We assume the energy equipartition factor \( \epsilon_r = \epsilon_B = 0.1 \) and thus we can obtain the comoving magnetic field from

\[
\frac{B^2}{8\pi} = \epsilon_B \epsilon_T,
\]

\[
B = 2^{17/6} \pi^{1/6} \left( \frac{4(e^j + 3)(e_j - 1)}{\Gamma_j^2} \right)^{1/15} \tilde{L}^{1/3} A_1^{1/3} r_p^{-1} \theta_0^{-8/15}.
\]

(10)

Because of the large opacity of the envelope, the radiation of relativistic electrons will be thermalized, with a typical black body temperature

\[
\approx -3 \frac{8n_T^5(kT)^4}{15(hc)^3} = \epsilon_e \epsilon_T.
\]

\[
kT = 2^{2/3} \pi^{1/6} \left( \frac{15 \epsilon_e (hc)^3}{8 \pi^5} \left( \frac{4(e^j + 3)(e_j - 1)}{\Gamma_j^2} \right) \right)^{1/4} \times L_j^{1/6} A^{1/12} r_p^{-1} \theta_0^{-4/15}.
\]

(11)

The average number density of thermal photons is

\[
n_\gamma = 19.232\pi \times \frac{1}{(hc)^3} \times (kT)^3.
\]

(12)

Furthermore, in order to obtain the numerical values, we adopt the typical values of LL-GRBs, \( L_{iso} = 10^{46} \text{ erg s}^{-1}, \theta_0 = 0.02, \) which suggests \( L_j = 10^{42} \text{ erg s}^{-1} \). Thus, the Lorentz factor of collimated jet is \( \Gamma_j \sim 1/\theta_0 \sim 50 \) (Mizuta & Ioka 2013). According to Heger et al. (2000) and Woosley & Heger (2006), we assume a typical progenitor with a helium core of mass \( \sim 2M_\odot \) and radius \( r_{10} = 4 \times 10^{11} \text{ cm} \) so that the ambient envelope density can be expressed as \( \rho_{j}(\Gamma_j) = 7.96 \times 10^{29} r^{-2} \text{ g cm}^{-3} \). From Equation (8) we get \( \bar{L} \approx 0.013, \rho_j = 4.95 \times 10^{-11} \text{ g cm}^{-3} \) so that this LL-GRB meets the requirement \( \bar{L} \ll 1 \). Moreover, Fermi acceleration is efficient because the Thomson optical depth is \( \tau_T = \sigma_T l = (\rho_j / m_p) \sigma_T (r_{10} / \Gamma_j) \zapprox 0.158 L_j / 10^{42} \text{ erg s}^{-1} \) \( \times (\theta_0 / 0.02)^{20/15} (M_{10}/2 M_\odot)^{1/5} (r_{10}/4 \times 10^{11} \text{ cm})^{-4/5} \sim 0.1 \). \( \Gamma_\tilde{N} < 1 \), where \( C = 1 + 2 \ln \Gamma_\tilde{N} \) is the possible effect from pair production (Murase & Ioka 2013; Budnik et al. 2010; Nakar & Sari 2012). Now we can get \( \bar{B} = 3.33 \times 10^7 \text{ G}, kT = 0.76 \text{ keV}, n_\gamma = 1.36 \times 10^{22} \text{ cm}^{-3}. \)

Remember that this is measured in the rest frame of the jet head.

A high energy proton loses its energy through radiative and hadronic processes. Radiative cooling includes synchrotron and inverse Compton scattering, with typical cooling timescales (Horiuchi & Ando 2008):

\[
t_{\text{sync}} = \frac{6\pi m_p^4 c^3}{\sigma_T m_e^2 B^2 e_p},
\]

\[
t_{\text{Th}} = \frac{3m_p^4 c^3}{4\sigma_T m_e^2 \epsilon_e n_p e_p},
\]

\[
t_{\text{KN}} = \frac{3\epsilon_p e_p}{4\sigma_T m_e^2 c^2 n_p},
\]

(13)

where the subscripts \( \text{Th}, \text{KN} \) represent the Thomson limit and the Klein–Nishina limit of inverse Compton scattering, \( \sigma_T \) is the Thomson cross section, and the average photon energy \( \bar{\epsilon}_p = 2.7kT \). If the proton energy \( \epsilon_p \) is in units of GeV, the numerical values of the timescales given above are

\[
t_{\text{sync}} = t_{\text{Th}} = 4.07 \times 10^7 \frac{1}{\epsilon_p} \text{ s}, \quad t_{\text{KN}} = 2.16 \times 10^{-8} \epsilon_p \text{ s}.
\]

Hadronic cooling mechanisms mainly contain pp collision, the Bethe–Heitler interaction, and photopion production in the following ways, respectively,

\[
p + p \rightarrow \pi^\pm, K^\pm \rightarrow \mu^\pm + \nu_\mu(\bar{\nu}_\mu),
\]

\[
p + p \rightarrow p + e^\pm.
\]

\[
p + p \rightarrow p + \pi^0 \quad \text{or} \quad p + p \rightarrow n + \pi^+.
\]

We can expect muon neutrinos to be produced in pp collision and photopion process, via charged pion or kaon decay. Here we do not consider the secondary electron neutrino production.

The cooling timescale of pp collision is

\[
t_{pp} = \frac{\epsilon_p}{c\sigma_{pp} n_p \Delta \epsilon_p}.
\]

(14)

We estimate the proton number density in the shocked jet as \( n_p = (4e^j + 3)\rho_j / m_p \sim 6.02 \times 10^{15} \text{ cm}^{-3} \). Assuming in each collision a fraction \( 20\% \) of the proton energy is lost and \( \sigma_{pp} = 5 \times 10^{-26} \text{ cm}^2 \) (Eidelman et al. 2004), we can get \( t_{pp} = 0.55 \text{ s}. \)

At relative higher energy, the protons start to cool through the Bethe–Heitler interaction, for which the energy loss every times is \( \Delta \epsilon_p = 2m_ec^2 \gamma_{c.m.} \), where \( \gamma_{c.m.} \) is the Lorentz factor of the center of inertia in the comoving frame and can be expressed as

\[
\gamma_{c.m.} = (\epsilon_p + \epsilon_\gamma) / (m^2_p c^4 + 2m_p \epsilon_\gamma + \epsilon_\gamma^2)^{1/2}. \quad \text{BH cross section is given by} \quad \sigma_{BH} = (28/9) a r^2_c \text{ ln}(2(\epsilon_p \epsilon_\gamma) / (m_p m_c^2)) - 106/9, \quad \text{so the Bethe–Heitler (BH) cooling time is} \quad (Razzaque et al. 2004; Horiuchi & Ando 2008)
\]

\[
t_{BH} = \frac{\epsilon_p}{2n_p \sigma_{BH} m_c^2 \gamma_{c.m.}}.
\]

(15)

The photopion production dominates the cooling even at higher energy. We adopt the photopion cross section described in Stecker (1968) and Asano (2005) as a broken power law: \( \sigma(\chi) = 5 \times 10^{-28} (\chi / 590)^{-3} \text{ cm}^2 \) for \( 290 < \chi < 590 \) and \( \sigma(\chi) = 5 \times 10^{-28} (\chi / 590)^{-0.7} \text{ cm}^2 \) for \( 590 < \chi < 9800 \), where \( \chi m_c^2 \) is the photon energy in the proton rest frame. Thus the cooling timescale is

\[
t_{\gamma} = \frac{\epsilon_p}{c\sigma_{\gamma} n_p \Delta \epsilon_p},
\]

(16)

where the conventional inelasticity \( K = \Delta \epsilon_p / \epsilon_p = [1 - (m^2_p - m^2_c) / s] / 2 \) and \( s \) is the invariant mass of the system.

The timescale of the first order Fermi acceleration is \( t_{acc} = \theta_F \epsilon_p / (\epsilon B c) \). If the diffusion coefficient is assumed to be proportional to the Bohm diffusion coefficient and \( \theta_F = 0. \) then \( t_{acc} \sim 3.32 \times 10^{-11} \epsilon_p \text{ s}. \)

We now can plot the inverse of all these timescales as functions of proton energy in Figure 1. The maximum proton energy can be obtained by \( t_{sync} = t_{acc} \), so \( \epsilon_{p,\text{max}} \approx 1.11 \times 10^7 \text{ GeV}. \)
Hence we expect the maximum neutrino energy produced by this LL-GRB as $\epsilon_{\nu, max} \simeq (1/10) \epsilon_{p, max} \simeq 1.11 \text{ PeV}$.

We can define two threshold proton energies $\epsilon_{p, \text{th}}^{(BH)}$ and $\epsilon_{p, \text{th}}^{(pp)}$, corresponding to $t_{pp} = t_{BH}$ and $t_{pp} = t_{p, \gamma}$ respectively (Horiuchi & Ando 2008). We know that the pp collision and photopion process will produce muon neutrinos while the Bethe–Heitler interaction does not. Hence, if the proton energy $\epsilon_p$ falls into the range $\epsilon_{p, \text{th}}^{(BH)} < \epsilon_p < \epsilon_{p, \text{th}}^{(pp)}$, the BH interaction dominates and has a strong suppression on the final neutrino spectrum. This suppression factor can be written as $\xi_{BH}$,

$$\xi_{BH} = \begin{cases} \frac{\epsilon_{p,\text{th}}^{(BH)}}{\epsilon_p} & \text{if } \epsilon_p < \epsilon_{p,\text{th}}^{(pp)} \\ \frac{\epsilon_{p,\text{th}}^{(pp)}}{\epsilon_p} & \text{if } \epsilon_p > \epsilon_{p,\text{th}}^{(pp)} \end{cases},$$

(17)

Further on, the cooling of mesons also needs to be considered. It is similar to the radiative and the hadronic cooling times of protons:

$$t_{\text{rad}} = \frac{3m_p^4c^3}{4\sigma_f m_e^2 \epsilon (U_{\nu} + U_{\bar{\nu}})},$$

$$t_{\text{had}} = \epsilon/(c \sigma_f \nu \Delta \epsilon),$$

(18)

where $\epsilon$ is in units of GeV, the numerical values are $t_{\text{rad}} = 1.0(1/\epsilon_{KK})$, $t_{\text{rad}} = 1.56 \times 10^3(1/\epsilon_{KK})$, and $t_{\text{prod}} = 1.04 \times 10^3$. As in Horiuchi & Ando (2008), for our jet parameters, the meson goes from being decay-dominated to radiation-cooling-dominated. We can define the break energy for neutrinos, $\epsilon_{\nu,\text{brk}}$, which satisfies $\gamma\tau \sim t_{\text{rad}}$, and thus the suppression factor due to meson cooling is expressed as

$$\zeta(\nu) = \begin{cases} 1 & \text{if } \epsilon_\nu < \epsilon_{\nu,\text{brk}} \\ \epsilon_{\nu,\text{brk}}/\epsilon_\nu & \text{if } \epsilon_\nu > \epsilon_{\nu,\text{brk}}. \end{cases}$$

(19)

With the cooling suppression effect of both protons and mesons, we obtain the final neutrino flux (Totani 2003; Horiuchi & Ando 2008)

$$F_\nu = \frac{(n)B_\nu}{\kappa} \frac{L_{\text{iso}}}{4\pi D_L^2 \ln(\epsilon_{p,\text{max}}/\epsilon_{p,\text{min}})} \frac{\xi_{BH}(\nu)\zeta(\nu)}{\epsilon_\nu^2},$$

(20)

where $(n)$ is the meson multiplicity (1 for pions and 0.1 for kaons), $B_\nu$ is the branching ratio of meson decay into neutrinos (1 for pions and 0.6 for kaons), and $\kappa^{-1}$ is the fraction of the primary proton energy carried by neutrinos, regardless of energy loss (1/8 for pions and 1/4 for kaons). The factor $\ln(\epsilon_{p,\text{max}}/\epsilon_{p,\text{min}})$ normalizes the proton spectrum to the jet power. In this paper, we just assume that highly efficient acceleration occurs and take the acceleration efficiency $\xi_{\text{acc}} \simeq 1$ so that $\epsilon_{\text{acc}} = \epsilon_{\text{acc}} = \xi_{\text{acc}}(1 - \epsilon_{\nu} - \epsilon_{\nu}) = 0.8 \sim 1$. This is consistent with the fiducial value of the baryon loading parameter $\xi_{\text{acc}} = \xi_{\text{acc}} = 10$ according to Murase (2007).

We plot in Figure 2 the flux of neutrinos for the LL-GRB jet mentioned above, with luminosity $L_j = 10^{45}$ erg s$^{-1}$ and initial opening angle $\theta_0 = 0.02$. We assume that this LL-GRB is at a rather close distance $D_L = 10$ Mpc. We can see that at the low energy end, $\nu^2F_\nu \sim const$ suggests a power law neutrino spectrum. The neutrino number from kaon decay is one or two orders of magnitude more than that from pions at the high energy end. It is mainly because kaons are heavier and experience less energy loss (Horiuchi & Ando 2008). A sharp jump is obvious in the spectrum due to the transition of the dominant process from BH interaction to photopion process in proton cooling mechanisms, and thus prominently more neutrinos are produced, that is to say, it is caused by $\xi_{BH}(\nu)$.

We now can simply estimate the neutrino events for this LL-GRB in IceCube. We use the following fitting formula of the probability of detecting muon neutrinos (Murase & Nagataki 2006; Abbasi et al. 2011).

$$P(E_\nu) = 7 \times 10^5 \frac{E_\nu}{10^{4.5} \text{ GeV}}^\beta,$$

where $\beta = 1.35$ for $E_\nu < 10^{4.5}$ GeV, while $\beta = 0.55$ for $E_\nu > 10^{4.5}$ GeV. The number of muon neutrinos from a burst are given by

$$N(> E_{\nu,3}) = A_{\text{det}} \int_{E_{\nu,3}}^{\nu_{\text{max}}} dE_\nu P(E_\nu) \frac{dN_{\nu}(E_\nu)}{dE_\nu} dA.$$

Using a geometrical detector area of $A_{\text{det}} = 1$ km$^2$, the expected neutrino number is $N \simeq 4.2 \times 10^{-3}$ for the LL-GRB above with a neutrino emission duration of $T_{\text{det}} \simeq 117s$. Thus, it is currently not easy to detect.
3.2. \( I \ll \dot{L} \lesssim \theta_0^{-4/3} \)

In this case, the jet is still collimated but the jet head velocity will become subrelativistic. Similar to the above discussions, we deduce some crucial terms below:

\[
\dot{\beta}_h \simeq 1,
\]

\[
\dot{L} = 4(2\pi)^{-2/5} L_j^{2/5} \dot{A}^{-2/5} \theta_0^{-8/5} c^{-6/5},
\]

\[
\Gamma_h = \sqrt{\frac{2}{5}} L_{\dot{L}}^{4/5} = (2\pi)^{-1/10} L_j^{1/10} \dot{A}^{-1/10} \theta_0^{-2/5} c^{-3/10},
\]

\[
\tilde{\Gamma}_h = \Gamma_j \sqrt{\frac{2}{5}} L_{\dot{L}}^{1-4/5} = 2^{-9/10} \pi^{1/10} \Gamma_j L_j^{-1/10} A^{1/10} \theta_0^{-2/5} c^{-3/10},
\]

\[
\theta_j = 2^{-4/5} \pi^{-3/10} L_j^{3/10} \dot{A}^{-3/10} \theta_0^{-4/5} c^{-9/10},
\]

\[
\rho_j = 2^{8/5} \pi^{-2/5} L_j^{5/10} \Gamma_j^{-2} A^{3/5} \theta_0^{-8/5} c^{-6/5}.
\]

The internal energy of the shocked ambient medium by FS and the shocked jet by RS are

\[
\epsilon_f = (4\Gamma_h + 3)(\Gamma_h - 1) \rho_a c^2, \quad \epsilon_r = (4\tilde{\Gamma}_h + 3)(\tilde{\Gamma}_h - 1) \rho_j c^2.
\]

The protons will be accelerated simultaneously by FS and RS, but the FS’s contribution is negligible because the FS would be radiation-mediated, and the shock acceleration would be inefficient. In this case, we choose a UL-GRB with \( L_{iso} = 10^{50} \text{ erg s}^{-1} \) and \( \theta_0 = 0.01 \), which suggests \( L_j = 2.5 \times 10^{44} \text{ erg s}^{-1} \). The extremely long duration \( \sim 10^8 \) s suggests a progenitor like a blue supergiant (BSG) of radii up to \( \sim 10^{13} \) cm. We assume this BSG to have a helium core of mass \( \sim 2 M_\odot \) and radius \( r_{He} = 5 \times 10^{13} \) cm so that the envelope density can be expressed as \( \rho_a(r) = 6.37 \times 10^{18} r^{-2} \text{ g cm}^{-3} \). The assumed radius may be relatively larger than that of typical BSGs, but we choose this value in order to realize efficient Fermi acceleration here. These parameters are possible according to Woosley & Heger (2012) and we then focus on the neutrino emission of this single UL-GRB. From Equation (21) we get \( \dot{L} \simeq 14.06 \), \( \rho_j = 3.58 \times 10^{-12} \text{ g cm}^{-3}, \Gamma_h \simeq 1.37, \tilde{\Gamma}_h \simeq 36.5 \) thus this UL-GRB satisfies \( I \ll \dot{L} \lesssim \theta_0^{-4/3} \). The Thomson optical depth is \( \tau_T = \sigma_T l = (\rho_j c/m_p) \sigma_T (r_{He}/\Gamma_j) \simeq 0.71 (L_j/2.5 \times 10^{44} \text{ erg s}^{-1})^{2/5} \theta_0^{-0.01})^{7/5} (M_{He}/2 M_\odot)^{3/5} (r_{He}/5 \times 10^{13} \text{ cm})^{-8/5} \sim 0.1 C^{-1} \tilde{\Gamma}_h \sim 1 \) so Fermi acceleration for the RS is efficient. In line with current observation (see Gendre et al. 2013), we assume this UL-GRB to be at a relatively close distance \( D_L = 500 \) Mpc. We calculate the neutrino flux for RS and then we plot it in Figure 3. The maximum proton energy \( \epsilon_{p,\text{max}} \) can be obtained from \( t_{\text{sync}} = t_{\text{occ}}, \) and the maximum neutrino energy produced by this UL-GRB is \( \epsilon_{\nu,\text{max}} \simeq \Gamma_{\delta}(1/10) \epsilon_{p,\text{max}} \simeq 3.42 \) PeV. Also, greater total neutrino fluence is exhibited in this UL-GRB case. As before, the expected neutrino number in IceCube for this UL-GRB is \( N \simeq 8.3 \times 10^{-2}, \) with a neutrino emission duration of \( T_{\text{dur}} \simeq r_{He}/\dot{\beta} c \simeq 2438 \) s.

\[
3.3. \dot{L} > \theta_0^{-4/3}
\]

This case corresponds to the uncollimated regime. That is, the collimation effect is weak and to a good approximation the jet remains conical. The head will move forward with relativistic velocity, \( \dot{\beta}_h = 1 \). In this case, we have

\[
\dot{L} = \frac{L_j}{\pi A \theta_0^2 c^3},
\]

\[
\Gamma_h = 2^{-1/2} \pi^{-1/4} L_j^{1/4} A^{-1/4} \theta_0^{-1/2} c^{-3/4},
\]

\[
\tilde{\Gamma}_h = \Gamma_j \sqrt{\frac{2}{5}} L_{\dot{L}}^{1-4/5},
\]

\[
\theta_j = \theta_0, \quad \rho_j = \pi^{-1} L_j^{-2} \Gamma_j^{-2} \theta_0^{-2} c^{-3}.
\]

Unfortunately, this situation is not suitable for high energy neutrino production. The reason is that, to meet the requirement \( \dot{L} > \theta_0^{-4/3} \), we need \( L_j = 10^{53} \text{ erg s}^{-1} \) for \( \theta_0 = 0.1 \) in the helium core we previously assumed. Leaving aside the existence of such a powerful jet, the Fermi acceleration is no longer efficient and meson cooling in this jet is so severe that there will hardly be any high energy neutrinos. Hence we do not need to carry on.

4. DEPENDENCE ON \( \alpha \)

It is reasonable to argue that the final neutrino flux depends on the density profile of the progenitor envelope. Apparently, with the same assumed envelope mass and radius, different values of the power law index lead to different ambient envelope densities, which directly influence the cocoon pressure and the collimation of the jet (Bromberg et al. 2011). Moreover, changing the power law index does affect the jet dynamics and result in a different dependency on radius (as we will see later). These two reasons cause the collimation effect on the final neutrino spectrum to be different. For this sake, we would like to study the influence of the different values of the power law index of the density profile. Here we display one situation (UL-GRB) for simplicity and clearness. We still use the progenitor envelope properties of a helium core of mass \( \sim 2 M_\odot \).
Figure 4. Total neutrino flux multiplied by the square of neutrino energy vs. neutrino energy for a UL-GRB with different envelope density profile. Green solid lines, blue dashed lines, and red dotted lines represent $\alpha = 2$, $\alpha = 2.5$, $\alpha = 2.7$, respectively. Relevant parameters: $L_j = 2.5 \times 10^{54}$ erg s$^{-1}$, $\theta_0 = 0.01$, $r_{\text{He}} = 5 \times 10^{13}$ cm, $\epsilon_\nu = \epsilon_B = 0.1$, at a distance $D_L = 500$ Mpc. (A color version of this figure is available in the online journal.)

and radius $r_{\text{He}} = 5 \times 10^{13}$ cm, but with $\alpha = 2.5, 2.7$. Respectively, we can write them as $\rho_0(r) = 2.25 \times 10^{25} r^{-2.5}$ g cm$^{-3}$ and $\rho_0(r) = 7.42 \times 10^{27} r^{-2.7}$ g cm$^{-3}$. We could repeat all those calculations above for $\alpha = 2.5, 2.7$.

For a successful jet meeting the condition $1 \ll \tilde{L} \leq \theta_0^{-4/3}$, we show the crucial, analytical quantities for $\alpha = 2.5$,

$$
\tilde{L} \simeq 2.85 L_j^{2/5} A^{-2/5} r^{1/5} \theta_0^{-8/5} c^{-6/5},
$$

$$
\theta_j \simeq 0.334 L_j^{3/10} A^{-3/10} r^{3/20} \theta_0^{4/5} c^{-9/10},
$$

$$
\rho_j \simeq 2.85 L_j^{2/5} \Gamma_j^{-2} A^{3/5} r^{-23/10} \theta_0^{-8/5} c^{-6/5},
$$

and for $\alpha = 2.7$,

$$
\tilde{L} \simeq 3.66 L_j^{2/5} A^{-2/5} r^{7/25} \theta_0^{-8/5} c^{-6/5},
$$

$$
\theta_j \simeq 0.295 L_j^{3/10} A^{-3/10} r^{21/100} \theta_0^{4/5} c^{-9/10},
$$

$$
\rho_j \simeq 3.66 L_j^{2/5} \Gamma_j^{-2} A^{3/5} r^{-121/50} \theta_0^{-8/5} c^{-6/5}.
$$

In all three cases $\alpha = 2, 2.5, 2.7$, we only consider the RS’s contribution to the final high energy neutrino flux and we plot it in Figure 4. As before, we obtain the maximum neutrino energies, and these are $3.42$ PeV, $4.42$ PeV, $5.31$ PeV, respectively. Once we can correlate an observed high energy ($\sim$ PeV) neutrino precursor with a GRB in the future, we can constrain the properties of the progenitor envelope by the maximum neutrino energy.

We can see in this figure that $\alpha$ has a visible influence on the total neutrino flux, though it is not very prominent. Differences occur mostly at the high energy end. Also, the energy at which the neutrino spectrum peaks and the maximum neutrino energy in these three cases are a bit different. For a steeper envelope density profile, the density of the jet material and the outer envelope at which neutrino production begins is generally smaller. Hence, the steepest $\alpha = 2.7$ case encounters the least cooling impact at the high-energy end, thus leading to the highest maximum neutrino energy and greatest neutrino flux. In any case, we can make this dependency more striking, for example, by choosing $\alpha = 2.9$. This could result in a difference of several times compared to the $\alpha = 2$ case at the high energy end. However, given the energy resolution of IceCube now, these differences are so small that we can hardly distinguish them. Here we just provide an alternative way to probe the stellar structure and wish to do this if we could realize better energy resolution in the future.

5. COMPARISON WITH PREVIOUS WORKS

The main calculation of our paper is based on the jet propagation dynamics developed by Bromberg et al. (2011), and we further consider the neutrino emission during the jet propagation process. This neutrino emission serves as a precursor signal prior to GRB prompt emission. We differ from Mészáros & Waxman (2001), Enberg et al. (2009), and Murase & Ioka (2013) in that we deal with the high-energy neutrino emission produced by shocks formed at the jet head, while they focused on internal shocks. We have a similar handle on the calculation with Horiiuchi & Ando (2008), but our result may be very different from theirs because they adopted a progenitor model from Heger et al. (2000) with a simplified dynamics in which the jet opening angle remains constant and thus just ignored the collimation effect, which should play an important role. Collimation depends on the jet luminosity $L_j$, initial opening angle $\theta_0$ and the progenitor density profile.

We calculate in detail the high energy neutrino flux in each collimation regime, and choose the promising LL-GRBs and UL-GRBs as high energy neutrino sources, leading to a more likely result. What is more, we discuss the dependence of the maximum neutrino energy and high energy neutrino flux on the progenitor density profile.

6. DISCUSSIONS AND CONCLUSIONS

High-energy neutrinos can be produced while the jet is still propagating in the envelope. These neutrinos appear as a precursor signal, with energy ranges from GeV to PeV. Analytically, we calculate this neutrino flux. To handle this, we first need to determine whether the jet is in the collimation regime. Collimation has a crucial effect on jet propagation dynamics. We adopt the previous propagation framework developed by Bromberg et al. (2011). We assume separate cases (LL-GRBs and UL-GRBs) in which Fermi first order acceleration works and they fall into different collimation regimes. With a power law spectrum of accelerated protons with $p = 2$, we then calculate the neutrino flux in various situations.

At the low-energy end, we always obtain $\epsilon_\nu^2 F_\nu \sim \text{const}$, and this suggests a power law neutrino spectrum. Neutrinos are mainly produced by pp collision. As the neutrino energy increases, a sharp jump is obvious in the spectrum because of the photopion process starting to dominate, and thus more neutrinos are expected. Moreover, the neutrino flux from kaon decay is almost two orders of magnitude more than that from pions at the high energy end. It is mainly because kaons are heavier and experience less energy loss.

From our expectations, the final neutrino flux will depend on the density profile parameter $\alpha$. We take $\alpha = 2, 2.5, 2.7$ to verify this dependence. We get a good result in Figure 4, which shows that the dependence exists. Further, the maximum neutrino energy for the three cases is $3.42$ PeV, $4.42$ PeV, $5.31$ PeV, respectively. At a given radius, the density of the jet material is lower for a steeper envelope density profile. There is less cooling impact so that higher
maximum neutrino energy and greater high-energy neutrino flux is expected.

In this paper, we only calculated the high-energy neutrino flux of one GRB for the given parameters, but it is not easy to detect this flux with the current instruments. We wish to obtain a diffuse GRB neutrino background that can be correlated with current observations in our future work.

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