Linearized Optomechanics Under Time-Dependent Phase Driving

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Abstract. We study the effect of steepness in smooth phase changes in the laser that drives an optomechanical system in the red-detuned, linearized regime. These phase changes take the semi-classical component out of its steady state. Steeper phase changes produce larger amplitudes for the fast oscillations in the mean fields. In contrast, sufficiently slow phase changes keep the system close to its steady state and allow the implementation of a phase driving scheme designed to minimize the variation of the quantum fluctuation mean excitation values.

1. Introduction

The quintessential optomechanical system is a driven cavity where one of the mirrors in the resonator is free to oscillate. In the frame rotating at the drive frequency, the standard optomechanical Hamiltonian [1,2],

$$\hat{H}_0 = (\omega_c - \omega_p) \hat{a}^\dagger \hat{a} + \omega_m \hat{b}^\dagger \hat{b} + g_0 \hat{a}^\dagger \hat{b} + i \frac{\varepsilon}{2} \left( e^{i \phi(t)} \hat{a}^\dagger - e^{-i \phi(t)} \hat{a} \right),$$

(1)
describes the interaction of cavity and mechanical modes with frequencies $\omega_c$ and $\omega_m$, in that order, and annihilation operators $\hat{a}$ and $\hat{b}$, with a bare optomechanical coupling $g_0$. The driving laser frequency is $\omega_p$, its strength $\varepsilon$ and relative time-dependent phase $\phi \equiv \phi(t)$. This Hamiltonian describes a trove of physical systems from ultracold atoms to photonic crystals [3–10]. The range of effects displayed in these systems [11–17] allow the control over the quantum states of electromagnetic and mechanical components and suggest their use for quantum information processing and communication [18–23].

Recently, we proposed a passive control technique to produce robust optomechanical state transfer in a standard driven optomechanical system in the presence of dissipation [24]. Our proposal relies on calculating an optimal phase that maximizes excitation exchange while minimizing the effect of random variations in the system parameters. Also, we presented a scheme to calculate these optimal sharp changes to produce smooth sequences that maintain the system close to its steady state. Here, we study the effect of steepness from smooth and
continuous changes in the phase of the driving laser. For this, we derive the semi-classical and quantum fluctuation dynamics for the system and realize that smooth phase changes produce an enhanced optomechanical coupling parameter and auxiliary phase that oscillate in time. We numerically study the effect of steepness in order to find adequate parameter values that bring and keep us closer to our initial and target enhanced coupling constant and auxiliary phase, respectively. Then, we use these parameters to show their effect on a control protocol designed to minimize the variation of the quantum fluctuation mean excitation values at the end of a tripartite composite driving sequence. We close with a brief discussion and our conclusions.

2. Model

In the strong driving regime of the standard optomechanical model, we use a mean field description [25, 26] for the creation operators, \( \hat{a} = \alpha e^{i\phi} + \hat{c} \) and \( \hat{b} = \beta + \hat{d} \). We introduce the driving phase into the semi-classical electromagnetic mean field in order to simplify the analysis. This allows us to split the dynamics [27, 28] into semi-classical components for the cavity and mechanical mode amplitudes,

\[
\dot{\hat{a}}(t) = -\left(\frac{\kappa}{2} + i\omega_m\right)\alpha - ig_0\alpha(\beta + \beta^*) - i\varepsilon/2 - i\dot{\phi}\alpha, \quad (2a)
\]

\[
\dot{\beta}(t) = -\left(\gamma/2 + i\omega_m\right)\beta - ig_0|\alpha|^2, \quad (2b)
\]

in that order, and their quantum fluctuations,

\[
\dot{\hat{c}}(t) = -(i\omega_m + \kappa/2)\hat{c}(t) - ig e^{i\phi}\hat{d}(t) - \sqrt{\kappa}\hat{\xi}_c(t), \quad (3a)
\]

\[
\dot{\hat{d}}(t) = -ie^{-i\phi}\hat{c}(t) - (i\omega_m + \gamma/2)\hat{d}(t) - \sqrt{\gamma}\hat{\xi}_m, \quad (3b)
\]

where the operators \( \hat{\xi}_c \) and \( \hat{\xi}_m \) describe quantum Gaussian noise in the electromagnetic and mechanical modes respectively with correlations,

\[
\left\langle \hat{\xi}^\dagger_{(c,m)}(t)\hat{\xi}_{(c,m)}(s) \right\rangle = n_{th}^{(c,m)}\delta(t-s) \quad \text{and} \quad \left\langle \hat{\xi}_{(c,m)}(t)\hat{\xi}^\dagger_{(c,m)}(s) \right\rangle = \left(n_{th}^{(c,m)} + 1\right)\delta(t-s). \quad (4)
\]

Here, the parameters \( n_{th}^{(c,m)} \) are the average occupation numbers of the electromagnetic and mechanical modes in thermal equilibrium. Additionally, it is straightforward to note that the dynamics of the semi-classical modes are influenced just by the driving phase \( \phi(t) \) and are independent of the quantum fluctuations. The mean field components of the electromagnetic and mechanical modes influence the dynamics of the quantum fluctuations via the enhanced coupling constant \( g = g_0|\alpha(t)| \) and the auxiliary phase \( \varphi \equiv \varphi(t) = \phi + \arg\alpha(t) \). Additionally, the stationary values of the semi-classical modes do not depend on the actual value of the phase, as long as it remains constant. We need to define some extra parameters such as the semi-classical number of photons in the cavity, \( n_p = |\alpha(0)|^2 \), a dimensionless parameter related with the unbalanced losses in the cavity and mechanical modes, \( \Gamma = (\kappa - \gamma)/(4g) \), and a time scale \( \tau_0 = \pi/(2g\sqrt{1-\Gamma^2}) \) [24].

It is straightforward to assume that a change in the driving phase will induce an oscillation in the semi-classical mode amplitudes that will make an impact on the enhanced optomechanical coupling and the auxiliary phase, Fig. 1. Fast, or steep, phase changes produce fast and ample oscillations in the enhanced optomechanical coupling and auxiliary phase; first column in Fig. 1. In contrast, slower phase changes keep the system parameters close to their original value; second and third column in Fig. 1. We want to stress that sufficiently slow changes keep the enhanced coupling constant near its original value and, also, bring the auxiliary phase near to its target value. We use the experimental device in Ref. [29] and its parameters: \( g_0/\omega_m = 1.887 \times 10^{-5} \),

\[
1540 (2020) 012013 \quad \text{doi:10.1088/1742-6596/1540/1/012013}
\]
Figure 1. (a)-(c) Smooth driving phases $\phi(t)$ with varying degrees of steepness and their corresponding effects on (d)-(f) the normalized enhanced optomechanical coupling $g(t)/g(0)$ and (g)-(i) auxiliary phase $\varphi(t)$.

$\kappa/\omega_m = 1.119 \times 10^{-2}$, $\gamma/\omega_m = 9.434 \times 10^{-6}$ and $n_p = 180 \times 10^3$. The time-dependent phases in Fig. 1 have the form

$$\phi(t) = \frac{\pi}{2} \Theta_s(t - \tau_0/2, \sigma)$$

where $\Theta_s$ is a smooth step function,

$$\Theta_s(t, \sigma) = \frac{1}{2} \left( 1 + \frac{2}{\sqrt{\pi}} \int_0^{t/\sigma} e^{-y^2} \, dy \right).$$

where the parameter $\sigma$ controls the steepness; a smaller parameter value provides a steeper phase jump. The plots in the first column of Fig. 1 have a steepness to time scale ratio $\sigma/\tau_0 = 4.78 \times 10^{-3}$, those in the second column $\sigma/\tau_0 = 2.39 \times 10^{-2}$, and those in the third $\sigma/\tau_0 = 1.19 \times 10^{-1}$.

3. Results

A single change in the phase of the driving field can bring the enhanced optomechanical coupling constant and the auxiliary phase out of their target values. Our previous proposal for robust state transfer relies on composite phase sequences with an even number of jumps in the sequence [24]. For example, if we want to minimize the variation of the quantum fluctuation mean excitation values, the shortest possible sequence has three parts, each lasting a time interval $\tau_0$, with identical initial and final phases,

$$\phi(t) = \varphi_{\text{opt}} \Theta_s(t - \tau_0, \sigma) \Theta_s(2\tau_0 - t, \sigma),$$
the middle phase differs from the initial and final by the optimal phase value,

$$\varphi_{\text{opt}} = (3\Gamma^2 - 1)/2.$$  \hspace{1cm} (8)

Figure 2 displays the effects of steepness on the enhanced coupling constant and the auxiliary phase for our three-part composite sequence. Here, the steepness values are identical to those in Fig. 1. We can see that the composition of phase changes implies a strong instability on the phase, Fig. 2(a) and Fig. 2(b), unless a sufficiently slow change is chosen, Fig. 2(c).

This particular composite driving sequence is designed to minimize the variation of the mean photon and phonon numbers of the quantum fluctuations near the end of the sequence, $$\langle\hat{n}(c)(3\tau_0)\rangle$$ and $$\langle\hat{n}(m)(3\tau_0)\rangle$$ in that order. It is optimized to minimize the first derivative of these quantities under thermal fluctuations. Unbalanced cavity and mechanical resonator losses induce anharmonic oscillations of the quantum fluctuation mean excitation numbers [30, 31], Fig. 3(a), but the introduction of our tripartite phase sequence minimizes the variation in the excitation number at the end of the sequence, Fig. 3(b).

4. Conclusion
We presented the dynamics for both the semi-classical and quantum fluctuations of the electromagnetic and mechanical modes under a time-dependent driving. In particular, we are interested in smooth approximations to ideal step changes. The steepness in these smooth and continuous phase sequences introduce high frequency oscillations in the semi-classical mode amplitudes that inform the enhanced coupling constant and the auxiliary phase parameter.
Figure 3. Mean excitation values of the electromagnetic (blue solid curve) and mechanical (red dashed curve) quantum fluctuation modes as a function of time (a) with no phase sequence and (b) with a smoothed phase sequence, Fig. 3(c), designed to minimize the variation of these mean values at the end of the sequence, $t = 3\tau_0$. The parameters in this figure are the same as those in Fig. 1, 2, and the evolution in (b) has the same driving phase as the third column in Fig. 2.

determining the dynamics of the quantum fluctuations. We show that it is imperative to find phase driving sequences that change sufficiently slow to keep these parameters stable and near their target values. As an example, we presented a tripartite phase sequence designed to minimize the variation of the quantum fluctuation mean excitation values at the end of the sequence.

References

[1] Pace A F, Collett M J and Walls D F 1993 Phys. Rev. A 47 3173 URL http://dx.doi.org/10.1103/PhysRevA.47.3173
[2] Law C K 1995 Phys. Rev. A 51(3) 2537 URL http://dx.doi.org/10.1103/PhysRevA.51.2537
[3] Anetsberger G, Rivière R, Schliesser A, Arcizet O and Kippenberg T J 2008 Nat. Photonics 2 627 URL http://dx.doi.org/10.1038/nphot.2008.199
[4] Regal C A, Teufel J D and Lehnert K W 2008 Nat. Phys. 4 555 article URL http://dx.doi.org/10.1038/nphys974
[5] Kippenberg T J and Vahala K J 2008 Science 321 1172 URL http://dx.doi.org/10.1126/science.1156032
[6] Stamper-Kurn D M 2014 Cavity Optomechanics ed Aspelmeyer M, Kippenberg T and Marquardt F (Berlin, Heidelberg: Springer) chap 13, p 283
[7] Metcalfe M 2014 Appl. Phys. Rev. 1(3) 031105 URL http://dx.doi.org/10.1063/1.4896029
[8] Zhang Y, Zhao Y and Lv R 2015 Sens. Actuator A Phys. 233 374 URL https://doi.org/10.1016/j.sna.2015.07.025
[9] Bernier N R, Tóth L D, Feofanov A K and Kippenberg T J 2018 IEEE Antennas Wirel. Propag. Lett. 17 1983 URL http://dx.doi.org/10.1109/LAWP.2018.2856622
[10] Grinwal P and Rodríguez-Lara B M 2019 OSA Continuum 2 175 (Preprint arXiv:1808.03725 [physics.optics]) URL http://dx.doi.org/10.1364/OSAC.2.000175
[11] Dorsel A, McCullen J D, Meystre P, Vignes E and Walther H 1983 Phys. Rev. Lett. 51 1550 URL http://dx.doi.org/10.1103/PhysRevLett.51.1550
[12] Aldana S, Bruder C and Nunnenkamp A 2013 Phys. Rev. A 88 043826 (Preprint arXiv:1306.0415 [quant-ph]) URL http://dx.doi.org/10.1103/PhysRevA.88.043826
[13] Weis S, Rivière R, Deleglise S, Gavartin E, Arcizet O, Schliesser A and Kippenberg T J 2010 Science 330 1520 (Preprint arXiv:1007.0565 [quant-ph]) URL http://dx.doi.org/10.1126/science.1195596
[14] Karuza M, Biancofiore C, Bawaj M, Molinelli C, Galassi M, Natali R, Tombesi P, Di Giuseppe G and Vitali D 2013 Phys. Rev. A 88 013804
[15] Mancini S, Vitali D and Tombesi P 1998 Phys. Rev. Lett. 80 688 (Preprint arXiv:quant-ph/9802034) URL http://dx.doi.org/10.1103/PhysRevLett.80.688
[16] Marquardt F, Chen J P, Clerk A A and Girvin S M 2007 Phys. Rev. Lett. 99 093902 (Preprint arXiv:cond-mat/0701416) URL http://dx.doi.org/10.1103/PhysRevLett.99.093902
[17] Marquardt F, Clerk A A and Girvin S 2008 J Mod Opt. 55 3329 (Preprint arXiv:0803.1164 [quant-ph]) URL http://dx.doi.org/10.1080/09500340802454971
[18] Stannigel K, Rabl P, Sørensen A S, Zoller P and Lukin M D 2010 Phys. Rev. Lett. 105 220501 (Preprint arXiv:1006.4361 [quant-ph]) URL http://dx.doi.org/10.1103/PhysRevLett.105.220501
[19] Stannigel K, Rabl P, Sørensen A S, Lukin M D and Zoller P 2011 Phys. Rev. A 84 042341 (Preprint arXiv:1106.5394 [quant-ph]) URL http://dx.doi.org/10.1103/PhysRevA.84.042341
[20] Safavi-Naeini A H and Painter O 2011 New J. Phys. 13 013017 (Preprint arXiv:1009.3529v1 [physics.optics]) URL http://dx.doi.org/10.1088/1367-2630/13/1/013017
[21] Stannigel K, Komar P, Habraken S J M, Bennett S D, Lukin M D, Zoller P and Rabl P 2012 Phys. Rev. Lett. 109 013603 (Preprint arXiv:1202.3273 [quant-ph]) URL http://dx.doi.org/10.1103/PhysRevLett.109.013603
[22] Tian L 2015 Ann. Phys 527 1 (Preprint arXiv:1407.3035 [quant-ph]) URL http://dx.doi.org/10.1002/andp.201400116
[23] Vostrosablin N, Rakhubovsky A A and Filip R 2017 Opt. Express 25 18974 (Preprint arXiv:1704.01784v2 [quant-ph]) URL http://dx.doi.org/10.1364/OE.25.018974
[24] Ventura-Velázquez C, Jaramillo Ávila B, Kyoseva E and Rodríguez-Lara B M 2019 Sci. Rep. 9 4382 (Preprint arXiv:1806.04266 [quant-ph]) URL http://dx.doi.org/10.1038/s41598-019-40492-y
[25] Mancini S and Tombesi P 1994 Phys. Rev. A 49 4055 URL http://dx.doi.org/10.1103/PhysRevA.49.4055
[26] Paternostro M, Gigan S, Kim M S, Blaser F, Böhm H R and Aspelmeyer M 2006 New J. Phys. 8 107 URL https://dx.doi.org/10.1088/1367-2630/8/6/107
[27] Gardiner C W and Collett M J 1985 Phys. Rev. A 31 3761 URL http://dx.doi.org/10.1103/PhysRevA.31.3761
[28] Walls D and Milburn G J 2008 Quantum Optics 2nd ed (Springer-Verlag Berlin Heidelberg) URL http://dx.doi.org/10.1007/978-3-540-28574-8
[29] Leocq F, Teufel J D, Aumentado J and Simmons R W 2015 Nat. Phys. 11 635 (Preprint arXiv:1409.0872 [quant-ph]) URL http://dx.doi.org/10.1038/nphys3365
[30] Jaramillo Ávila B, Ventura-Velázquez C, de J León-Montiel R, Joglekar Y N and Rodríguez-Lara B M 2020 Sci. Rep. 10 1761 (Preprint arXiv:1908.03240 [quant-ph]) URL http://dx.doi.org/10.1038/s41598-020-58582-7
[31] Quiroz-Júaiz M A, Perez-Leija A, Tschernig K, Rodríguez-Lara B M, Magaña-Loaiza O S, Busch K, Joglekar Y N and de J León-Montiel R 2019 Photon. Res. 7 862 (Preprint arXiv:1905.06993 [quant-ph]) URL http://dx.doi.org/10.1364/PRJ.7.000862