Duality in Non-Supersymmetric MQCD

Nick Evans

Department of Physics, Boston University, Boston, MA 02215

Abstract
The curves that describe the M-theoretic extension of type IIA string configurations with non-supersymmetric field theories on their surface exhibit a duality map. The map suggests a continued link between a \( SU(N) \) gauge theory with \( F \) flavours and an \( SU(F - N) \) gauge theory with \( F \) flavours (the duality of supersymmetric QCD) even when the gaugino mass is taken to infinity. Within the context of the field theory such a duality only continues to make sense if the scalar fields remain light. We discuss the difficulties of decoupling the scalars within this framework.
1 Introduction

The improved understanding of supersymmetric QCD over recent years has revealed many new aspects of gauge dynamics. Of particular interest from the study of N=1 SQCD have come examples of gauge theories that give rise to massless composites and exhibit a non-abelian electromagnetic duality [1]. Explicitly, N=1 $SU(N)$ SQCD with $F$ matter flavours, $Q, \bar{Q}$, is dual to an $SU(F-N)$ gauge theory with $F$ flavours $q, \bar{q}$, a meson superfield, $M$, transforming as $(F, \bar{F})$ under the flavour symmetry, and a superpotential $Mq\bar{q}$. Whether this sort of gauge dynamics is special to supersymmetric theories remains unclear. Soft supersymmetry breaking parameters may be introduced through the vevs of higher component fields of spurion superfields (the couplings and parameters of the theory) [2]-[6]. The symmetries of the model are not though sufficient to allow a map of soft breakings in the electric variables to those in the magnetic variables except in a few special cases [3]. Scalar masses result from Kahler terms that are non-holomorphic and so in general it is unclear whether the dual theory reacts to supersymmetry breaking in the alternative variables by a mass or higgs branch [4]. Whether the duality persists with massless states, the theory develops a mass gap, or whether the dual variables of SQCD are no longer the relevant degrees of freedom as supersymmetry is broken is therefore unclear. Recently [5] has suggested methods to overcome these issues for perturbing soft breakings.

A large literature has grown up on engineering supersymmetric field theories using D-brane constructions in type IIA string theory (a comprehensive review and list of references is to be found in [7]). The essential ingredient is that the massless string modes of strings ending on the surfaces of D-branes correspond to gauge fields living on the D-branes surface plus superpartners. Whilst a perturbative identification between the string states in the type IIA theory and the UV field theory may be made the IIA picture provides no information about the strongly coupled IR dynamics of the theory. Such information would correspond to short distance structure of the branes but due to the strongly coupled nature of the core of NS5 branes the precise structure of an NS5 D4 junction is unclear. To proceed one must move to M-theory [8] (type IIA string theory with coupling $g_s$ is the 11 dimensional M-theory compactified on a circle of radius, $R \sim g_s$). In M-theory NS5 branes and D4 branes become aspects of a single M5 brane wrapped in places on the compact dimension. The junctions between these objects may thus be smoothly described by a minimal area embedding of the M5 brane. Increasing the M-theory compactification radius from zero allows the study of the string theory with increased coupling at the string scale. It is therefore possible to make a strong coupling expansion to the field theory, that is, smoothly deforming the field theory of interest to a theory with the same global symmetries and parameters but which is fundamentally a theory of strongly interacting strings in the $R \to \infty$ limit. For intermediate $R$ the theory has Kaluza Klein states in addition to those of the field theory. We hope that
by making this transition between smoothly related theories there is no phase transition and that the two theories lie in the same universality class. This technique has been used to derive the existence of gaugino condensation in N=1 super Yang Mills theory [10] and duality [11][12] in the theories with flavour amongst other results (see [8]). For these supersymmetric theories the holomorphic and BPS properties of states in the theory contribute to making the motion to strong coupling smooth.

In [13]-[17] the construction of non-supersymmetric theories on D-brane surfaces has been discussed. The branes are rotated relative to their positions in the supersymmetric configurations. In the perturbative type IIA string theory the supersymmetry breaking of the branes can be mapped to supersymmetry breaking in the field theory using the strict constraints of N=2 supersymmetry on how spurions may enter the field theory [14][15]. M-theory curves for a subset of these configurations with and without matter fields have been proposed [10][15]. The low energy \(Z_N\) symmetries of the supersymmetric models are broken by the supersymmetry breaking terms in the curve and the degeneracy of the \(N\) supersymmetric vacua is lifted. The curves display the same phase structure with changing theta angle as softly broken field theories [16][15]. For the case with matter fields it has been shown that if parameters are chosen to decouple the gaugino then the low energy theory is described by two parameters, the quark mass and a parameter with the charges of a quark condensate [15].

In this paper we show that the non-supersymmetric M-theory curves exhibit the same duality map as the supersymmetric curves. We discuss whether non-supersymmetric gauge theories could possess a non-abelian duality and conclude that it is only consistent if the scalar fields are remaining relevant to the low energy dynamics. A similar conclusion has been reached recently in [17]. With the loss of supersymmetry and its associated holomorphic and BPS properties we must be careful about treating the brane picture with its extra states as a proof of the gauge theory behaviour but the result is suggestive that the SQCD duality persists in the limit of the gaugino decoupling and fine tuned light scalars.

One might have hoped to be able to decouple the scalar fields and learn about the transition from SQCD to honest QCD without scalar fields from the brane set up. Unfortunately the available brane deformations do not seem to include those corresponding to positive masses for all scalars and hence this decoupling can not be performed at tree level. Nevertheless it is clear that QCD can not possess a description in terms of a flavour dependent dual gauge group, such as that suggested by the branes, since QCD has no higgs branch.

In section 2 we review the 4D field theories that may be engineered with type IIA string theory branes. In section 3 we review the derivation of duality in N=1 SQCD with branes following [11][12]. In section 4 we demonstrate the duality map holds for non-supersymmetric M-theory brane configurations and in section 5 discuss the implications for the field theories
on their surfaces.

2 Type IIA Configurations and Field Theories

The basic brane configuration from which we start corresponds to $SU(N)$ N=2 SQCD with $F$ quark matter flavours. It is given from left to right by the branes

\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
 & # & R^4 & x^4 & x^5 & x^6 & x^7 & x^8 & x^9 \\
\hline
NS5 & 1 & - & - & - & \bullet & \bullet & \bullet & \bullet \\
D4 & N & - & \bullet & \bullet & [-] & \bullet & \bullet & \bullet \\
NS5' & 1 & - & - & - & \bullet & \bullet & \bullet & \bullet \\
D4' & F & - & \bullet & \bullet & - & \bullet & \bullet & \bullet \\
\hline
\end{array}
\]

(1)

$R^4$ is the space $x^0 - x^3$. A dash $-$ represents a direction along a brane’s world volume while a dot $\bullet$ is transverse. For the special case of the D4-branes’ $x^6$ direction, where the world volume is a finite interval corresponding to their suspension between two NS5 branes at different values of $x^6$, we use the symbol $[-]$. The field theory exists on scales much greater than the $L_6$ distance between the NS5 branes with the fourth space like direction of the D4-branes generating the couplings of the gauge groups in the effective 4D theory. The semi-infinite D4$'$ brane is responsible for providing the quark flavours.

The $U(1)_R$ and $SU(2)_R$ symmetries of the N=2 field theory are manifest in the brane picture. They correspond to isometries of the configuration; an SO(2) in the $x^4$, $x^5$ directions and an SO(3) in the $x^7$, $x^8$, $x^9$ directions.

N=2 supersymmetry may be broken in the configuration by rotations of the branes away from this configuration of maximal symmetry. In the field theory these breakings correspond to soft breakings of supersymmetry introduced through the vevs of auxiliary components of spurion fields (couplings) of the theories $\Box$ $\Box$. The N=2 supersymmetry is sufficiently restrictive on how these spurions may enter the field theory that the correspondence between the brane motions and field theory parameters can be identified. The possible breakings are:

- The NS5 branes may be rotated in the $x^4$, $x^5$, $x^7$, $x^8$, $x^9$ space (rotations from the N=2 configuration into the $x^6$ direction cause the NS5 branes to cross changing the topology of the configuration in such a way that it can no longer be easily identified with a field theory). These rotations correspond to components of the spurion fields occurring as vector fields in the prepotential of the N=2 theory as $\mathcal{F} = (S_1 + i S_2) A^2$ $\Box$. The scalar spurion vevs generate the gauge coupling $\tau$. When we allow the auxiliary fields of the spurions to be non-zero we obtain the tree level masses
A number of consistency checks support the identification \[14\]. Switching on any one of the six independent real supersymmetry breakings in the field theory leaves the same massless spectrum in the field theory as in the brane picture when any one of the six independent rotations of the NS5 brane is performed. The field theory and brane configurations possess the same sub-manifold of N=1 supersymmetric configurations (we concentrate on these configurations below).

- Supersymmetry may also be broken by forcing the D4 and D4’ branes to lie at angles to each other. With the introduction of matter fields in the field theory a single extra spurion field is introduced associated with the quark mass. The only possibility is to promote the mass to an N=2 vector multiplet associated with $U(1)_B$. Switching on its auxiliary field vevs induce the tree level supersymmetry breaking operators

$$2Re(F_M q) + D_M \left( |q|^2 - |\tilde{q}|^2 \right)$$

(3)

Again a number of consistency checks support the identification of these field theory breakings with the angles between D4 branes \[15\]. There are three independent real parameters in both the field theory and the brane picture. The scalar masses in the field theory break $SU(2)_R$ but leave two $U(1)_R$ symmetries of the supersymmetric theory intact. The scalar masses may always be brought to diagonal form by an $SU(2)_R$ transformation that mixes $q$ and $\tilde{q}^*$. In the resulting basis there is an unbroken $U(1)$ subgroup of $SU(2)_R$. In the brane picture the D4’ branes lie at an angle in the $x^6 - x^9$ directions breaking the $SU(2)_R$ symmetry but leaving two $U(1)_R$ symmetries unbroken.

The NS5 branes may be rotated in such a way as to preserve $N = 1$ supersymmetry (corresponding to, for example, setting $F_2 = iF_1$ in the field theory) and the resulting configuration with the adjoint matter field decoupled is given by

$$-\frac{N_c}{8\pi^2} Im \left( (F_1^* + iF_2^*) \psi_{A}^{\alpha} \psi_{A}^{\alpha} + (F_1 + iF_2) \lambda^{\alpha} \lambda^{\alpha} + i\sqrt{2} (D_1 + iD_2) \psi_{A}^{\alpha} \lambda^{\alpha} \right)$$

$$-\frac{N_c}{4\pi^2 Im(s_1 + is_2)} \left( (|F_1|^2 + D_1^2/2) Im(\lambda^{\alpha})^2 + (|F_2|^2 + D_2^2/2) Re(\lambda^{\alpha})^2 \right)$$

(2)

$$+ (F_1 F_2^* + F_1^* F_2 + D_1 D_2) Im(\lambda^{\alpha}) Re(\lambda^{\alpha})$$

(4)
This configuration therefore describes an N=1 $SU(N)$ theory with $F$ matter fields. This is the theory that displays non-abelian electro-magnetic duality and will be our starting point below.

3 N=1 Duality

The brane configuration of (4) describes N=1 SQCD. The duality of the field theory can be deduced from the brane configuration as follows \[11\]. Assuming the theory has an IR fixed point then motion of the two NS5 branes relative to each other, which changes the effective gauge coupling, should be a modulus of the theory. In that case we can ask what happens when the two branes pass through each other. The result has been deduced from string theory charge conservation and winding number rules. The configuration is T dual to that discussed by Hanany and Witten \[3\]; they showed that by conservation of magnetic monopole number on the NS5 brane and equivalently by a linking number argument of the bulk fields that the number of D4s ending on the right of the NS5 brane minus those ending on the left must be conserved. Applying this rule to the brane configuration of (4) we obtain, after passing the two NS5 branes through each other, a configuration with $F - N$ D4 branes between the two NS5 branes and $F$ semi-infinite branes. The new configuration describes an $SU(F - N)$ gauge theory with $F$ flavours. This is Seiberg’s dual. A more quantitative approach is to move to M-theory on finite compactified radius and perform these motions \[12\] which we review next.

The M-theory description of the N=1 SQCD configuration has an M5 brane that is flat in $R^4$ and described in the remaining 7 dimensions (we define $v = x^4 + ix^5$, $w = x^7 + ix^8$ and $t = \exp(x^6 + ix^{10}/R)$ by the curve

$$v = z, \quad w = \frac{\xi}{z}, \quad t = z^N m^{F-N}/(z - m)^F$$

The curve has two $U(1)$ symmetries associated with rotations in the $v$ and $w$ planes. The parameters of the curve break the symmetries but may be made to transform spuriously

|       | $v$ | $w$ | $z$ | $m$ | $\xi$ |
|-------|-----|-----|-----|-----|-------|
| $U(1)_v$ | 2   | 0   | 2   | 2   | 2     |
| $U(1)_w$ | 0   | 2   | 0   | 0   | 2     |

These symmetries correspond to the field theory symmetries

|       | $W$ | $Q$ | $\bar{Q}$ | $m$ | $\Lambda^{b_0}$ |
|-------|-----|-----|-----------|-----|----------------|
| $U(1)_R$ | 1   | 0   | 0         | 2   | $2(N - F)$ |
| $U(1)_{R'}$ | 1   | 1   | 1         | 0   | $2N$       |
where $\Lambda = \exp(2\pi i \tau/b_0)$ and $b_0 = 3N - F$. We may make the identifications $\xi = \Lambda^{b_0/N} m^{F/N}$ and $m$ is the matter field mass. The UV field theory displays $Z_N$ and $Z_{N-F}$ discrete subgroups of these symmetries. Viewing the curve asymptotically and as $\Lambda \to 0$

$$
\begin{align*}
  z &\to \infty \quad w = 0 \quad t = v^{N-F} m^{F-N} \\
  z &\to 0 \quad v = 0 \quad t = \left( \frac{1}{w} \right)^N \Lambda^{b_0} m^{F-N}
\end{align*}
$$

(8)

The $U(1)_v$ and $U(1)_w$ symmetries (allowing $m$ to transform spuriously but not $\Lambda$) are indeed asymptotically broken to $Z_N$ and $Z_{N-F}$ discrete subgroups. Other combinations of the two $U(1)_R$ symmetries may also be identified in the asymptotic curve. For example $U(1)_A$ symmetry is given by the rotations

$$
\begin{array}{cccc}
  U(1)_A & v & w & t & z \\
  -2 & 2 & 0 & -2 & 2F & -2
\end{array}
$$

(9)

The $N=1$ theory behaves like supersymmetric Yang Mills theory below the matter field mass scale and has $N$ degenerate vacua associated with the spontaneous breaking of the low energy $Z_N$ symmetry. In the curve this corresponds to the $N$ curves in which $\xi_n = \xi_0 \exp(2\pi in/N)$ (equivalently $\Lambda^{b_0}_n = \Lambda^{b_0}_0 \exp(2\pi in)$). In the UV these curves can be made equivalent by a $Z_N$ transformation.

For $F > N$ the curve may be recast in such a way that the dual picture of the strong dynamics as $\Lambda \to \infty$ appears. In particular we make the transformations

$$
\begin{align*}
  m &\equiv M^{N/F-N} \Lambda^{-b_0/F-N} \\
  \Lambda^{b_0} &\equiv \tilde{\Lambda}^{-\tilde{b}_0} \\
  z &\equiv \frac{M^{F/F-N} \Lambda^{-b_0/F-N}}{z'}
\end{align*}
$$

(10)

and the curve is transformed to

$$
\begin{align*}
  v = \frac{M^{F/N} \tilde{\Lambda}^{b_0}/\tilde{N}}{z'}, \quad w = z', \quad t = \frac{M^{F-N} z' \tilde{N}}{(M - z')F}
\end{align*}
$$

(11)

where $\tilde{N} = F - N$ and $\tilde{b}_0 = (3\tilde{N} - F)$.

In the limit $\tilde{\Lambda} \to 0$ at fixed $M$ this curve degenerates to the IIA configuration describing a $SU(F-N)$ gauge theory with $F$ quark flavours of mass $M$. This theory is Seiberg’s dual SQCD theory. Note that in terms of the electric curve this configuration is obtained when $\Lambda \to \infty$ with $m$ scaled to zero appropriately to keep $M$ fixed. The duality of the field theory is a strong-weak duality.

It is also of interest to note that the UV dual theory breaks supersymmetry as observed in [12]. Including a mass term $mQ\tilde{Q}$ in the electric theory maps in the magnetic variables to
a superpotential term $mM$ or equivalently to a shift in the vev of the auxiliary component of $M$. The dual gauge dynamics restores supersymmetry in the IR. In the brane picture this effect shows itself as follows: in the electric variables the mass term is a relative displacement in the $w$ direction of the end point of the semi-infinite D4 brane and the $w$ coordinates of the $v$ plane NS5 brane. After the duality motion the semi-infinite D4s connect directly to that NS5 brane and hence lie at an angle in the $w$ direction. This is precisely the configuration we identified in section 2 with corresponding to a shift in the vev of the auxiliary field of the adjoint chiral multiplet of the N=2 flavour gauge group, in other words, $M$. The duality maps interchanges an electric theory mass with a magnetic theory $F_M$ term.

4  N=0 Duality

In section 2 we have reviewed how non-supersymmetric gauge theories may be constructed in type IIA string theory. Adjoint matter fields may be given masses by rotations of the NS5 branes and scalars by placing the two sets of D4 branes at angles to each other. For the purposes of this discussion we shall consider deformations from N=1 SQCD that restrict rotations of the NS5 branes to those corresponding to a gaugino mass with the adjoint chiral multiplet always decoupled. At the level of the string theory these deformations of the brane configuration do not seem to preclude the brane motions required to derive duality for the supersymmetric theory. The relative angles of the branes do not disrupt the linking number argument of Hanany and Witten. Naively we may move the branes through each other as before and obtain an $SU(F - N)$ gauge theory with $F$ matter flavours. Any rotations of the NS5 branes will presumably remain and so a gaugino mass in one set of variables maps to a gaugino mass in the other variables. Similarly an angle between the sets of D4s in the $x^9$ direction will remain after the motion and thus a $D_M$ mass term in the electric variables maps to the same in electric variables.

To place these observations on more solid and quantitative grounds we will move to the M-theory curve describing such a brane configuration. A curve describing a subset of possible configurations has been proposed in [15]. The non-holomorphic curve

$$v = z + \frac{\bar{\epsilon}}{z} \quad w = \frac{\Lambda^{k_0/N} m^{F/N}}{z}, \quad t = z^N m^{F-N}/(z - m)^F, \quad x^9 = 4e^{1/2} Re \ln z$$

(12)

is a minimal area embedding ($\epsilon^{1/2}$ must be real for the configuration to be single valued in $x^9$).

In the $R \rightarrow 0$ limit the D4 branes lie in the $x^6$ and $x^9$ direction and the NS5 brane lying in the $w$ direction has been rotated in a non-supersymmetric fashion into the $v$ direction. We generically expect the field theory to have the supersymmetry breaking terms

$$D \left( |q|^2 - |\bar{q}|^2 \right) + m_\lambda \lambda \bar{\lambda}$$

(13)
We may identify the parameter $\epsilon$ with field theory parameters from its symmetry charges. Since it is chargeless under both $U(1)$s we may only identify it as a function of $m^{N}N^{b_{0}}m^{F}$

$$m^{N}N^{b_{0}}m^{F}$$ (14)

To complete the identification we note that the field theory retains a $Z_{F}$ subgroup of $U(1)$ even after the inclusion of the soft breaking terms. Requiring this property of the curve asymptotically forces

$$\epsilon = \left(m^{N}N^{b_{0}}m^{F}\right)^{1/N}$$ (15)

Asymptotically the curve is then

$$z \to \infty \quad w = 0 \quad t = v^{N-F}m^{F-N}$$

$$z \to 0 \quad v = \bar{m}_{\lambda}\bar{w} \quad t = \left(\frac{1}{v}\right)\bar{m}_{\lambda}N^{b_{0}}m^{F-N}$$

which possesses a $Z_{F}$ subgroup of the U(1)

$$
\begin{array}{cccccccc}
\text{U(1)} & v & w & t & z & \Lambda^{b_{0}} & m & m_{\lambda} & D \\
\text{A} & -2 & 2 & 0 & -2 & 2F & -2 & 0 & 0
\end{array}
$$

It is important to understand to what extent this identification of the parameters between the brane configuration and the field theory is valid. The identification has been made in the $R \to 0$ limit where the field theory on the surface is perturbative. As $R$ is increased or $\Lambda$ increased from zero we have no holomorphy properties and corrections between the brane parameters and field theory which are arbitrary functions of the symmetry neutral quantities $|m|, |\Lambda|$ and $|m_{\lambda}|$ may enter the relationship. The perturbative identification is a helpful labelling of the parameters because it displays their symmetry charges more readily.

It is natural to ask what has become of the dual description of the N=1 theory? Again performing the redefinitions of the curve parameters using (10) we obtain a curve for the dual variables

$$v = \frac{M^{F}/\bar{N}^{b_{0}/\bar{N}}}{z'} + \bar{m}_{\lambda}\bar{z}', \quad w = z', \quad t = \frac{M^{F-S}z'/\bar{S}}{(M-z')F}$$

$$x^{9} = -4(m_{\lambda}M^{F}/\bar{N}^{b_{0}/\bar{N}})^{1/2}\text{Re} \ln z'$$

The dual description (taking the $R \to 0$ limit) also has one NS5 brane rotated and an angle between the semi-infinite and finite D4 branes in the $x^{6}x^{9}$ plane. In the field theory the gaugino is massive and the dual squarks are also massive through a non-zero D.

In fact we may take the decoupling limit for the gaugino mass, that is take $\epsilon \to \infty$. $m$ is the only R-charged parameter remaining and there is therefore nothing that can play the role of either a gaugino mass or condensate fitting the assumption that the gaugino
has been decoupled. We must define a new strong scale parameter below the gaugino mass
\( \Sigma = m_\lambda^{N/F} \Lambda_b^{-F} \) which has the R-charges of a quark condensate and was identified as such in [13]. In fact we will see below that in the light of the duality this precise identification must be re-evaluated though the parameter is still plausibly related to a quark condensate.

The decoupled curve is
\[
\begin{align*}
v &= v = z + \left( \frac{\Sigma}{z} \right)^{F/N}, \\
t &= t = \frac{z^N m^{F-N}}{(z - m)^F}, \\
x^0 &= x^0 = 4(m\Sigma)^{F/2N} Re \ln z \\
\end{align*}
\]
(19) Again a duality map exists
\[
\begin{align*}
m &\equiv M^{N/F-N}\Sigma^{-F/F-N} \\
\Sigma &\equiv \tilde{\Sigma}^{-1} \\
z^' &\equiv m^{F-F-N}\Sigma^{-F/F-N} \\
\end{align*}
\]
(20) The dual curve is
\[
\begin{align*}
v &= v' = z' + \left( \frac{M\Sigma}{z'} \right)^{F/N} , \\
t &= t' = z^{F-N} M^{F-N} / (M - z')^F, \\
x^0 &= x^0 = -4(M\tilde{\Sigma})^{F/2N} Re \ln z' \\
\end{align*}
\]
(21) Again the dual picture emerges from the M-theory curve in the limit where \( \Sigma \to \infty \) and \( m \to 0 \). Surprisingly the string/M theory seems to be telling us that for massless quarks there is a duality symmetry between an \( SU(N) \) gauge theory with \( F \) quarks and an \( SU(F - N) \) gauge theory with \( F \) quarks. Presumably some components of the dual meson of SQCD also survive though we have been cavalier in the above discussion as to the boundary conditions at infinity of the semi-infinite D4s. In the next section we address the issue of whether there is any reasonable field theoretic interpretation of this duality that the curve has led us too.

## 5 Duality In Non-Supersymmetric Field Theory

Is the possibility that duality survives the breaking of supersymmetry and the complete decoupling of the gaugino reasonable within the context of the field theory? The first worry is that the massless fermions will no longer satisfy the 'tHooft anomaly matching conditions in the absence of the gaugino. In fact they do; the gaugino’s mass breaks the \( U(1)_R \) symmetries of the supersymmetric model and the only remaining symmetries are the flavour symmetries of the matter fields. Since the gaugino does not transform under the flavour symmetries the other fermions’ anomaly matching for the flavour symmetries is maintained [14].

Naively we would expect that with the decoupling of the gaugino there would no longer be supersymmetry to stabilize the scalar fields masses against radiative effects of order \( m_\lambda \).
If this were the case then the low energy behaviour of the theory on the branes surface
would be that of non-supersymmetric QCD. Could the duality be one of QCD? We must conclude that it could not. For the duality to link an $SU(N)$ theory to a flavour dependent $SU(F - N)$ theory requires that for the mass branch (giving a mass to the quarks) of each of the two descriptions there is a higgs branch in the other. QCD has no higgs branch and so there is no way to flow to an $SU(N - 1)$ gauge theory as one dual quark is given a mass. One might be tempted to seek a description with four-Fermi interactions triggering a higgs branch by a colour superconductivity mechanism (for example the dynamical generation of a $\langle \psi_q \psi_M \rangle$ condensate). Such a mechanism is though limited by the strong belief from NJL type models that a four Fermi or other higher dimension operator has a none zero critical coupling for the triggering of chiral symmetry breakdown. For the duality the higgs branch would have to switch on for an infinitessimal breaking of the flavour symmetry so the theory behaved correctly below the scale of the quark mass. 

It appears therefore that the brane picture above is not decoupling the scalar fields and they remain light or at least relevant to the low energy dynamics. This is presumably an artifact of moving to strong coupling. The duality is observed in the brane picture when $\Lambda$ is taken to infinity, that is the strong coupling scale is taken to the UV cut off. Tree level massless scalar fields are not in this limit given the opportunity to radiatively acquire masses above the strong coupling scale and decouple. Alternatively we must bare in mind that the curve may not be describing the global minimum of the theory. There may have been a first order phase transition for some value of the supersymmetry breaking and the global minimum is described by a curve other than (12). We would then be studying a local minimum of the theory where the scalars might have vevs at the expense of a large cost in energy. Nevertheless the continued existence of duality at this local minimum is interesting. The correct question therefore is whether non-supersymmetric field theories with fine tuned light scalars can continue to exhibit duality. Once we include the scalar fields the correct flow in the two theories with the addition of mass terms (which become $F_m$ terms in the dual) is correctly satisfied following the SQCD pattern and the duality is possible. 

We must also ask whether the duality continues to make sense in the presence of a $D_M$ term from (3). Including such a term in the electric variables triggers a higgs branch through the potential term 

$$ (D_M + |Q|^2 - |\tilde{Q}|^2)^2 $$

In the brane picture this can be seen because when the semi-infinite D4 branes are placed at an angle into the $x^9$ direction the D4 branes may decrease their lengths and hence tension by detaching from the central NS5 brane. They are then free to move along the single NS5 brane to which they are attached. The result is that the gauge group is higgsed of one colour and one flavour of matter is lost. In the dual construction the equivalent motion of the semi-infinite D4s along that NS5 brane now does not involve the $F - N$ D4s associated with
the gauge group. The motion corresponds to a meson vev and hence a mass branch in the dual. In the field theory this is provided by the potential

\[(D_M + |q|^2 - |\tilde{q}|^2 - [M^I, M])^2\]

which has a mass branch. The duality again is at least potentially consistent with the addition of these soft breaking terms. It was the assumption that this was the case in the supersymmetric limit that was used in [11] to derive the field theory duality by moving the NS5 branes past each other in the \(x^9\) direction.

As the scalars remain light in the theory the precise identification of the parameter \(\Sigma\) is harder. Perturbatively softly breaking the supersymmetric field theory with a gaugino mass is known to generate a quark condensate since

\[\langle \psi_Q \psi_{\tilde{Q}} \rangle \simeq F_M \simeq m_\lambda m^{N/F - N} \Lambda^{b_0/N}\]

In [15] where it was assumed that the scalars radiatively decoupled \(\Sigma = m_\lambda m^{N/F - N} \Lambda^{b_0/F}\) was identified with the quark condensate in the decoupling limit when \(m_\lambda \gg m\). If the scalars remain light and the theory possesses a strong weak duality then presumably at the origin of moduli space of the massless theory the theory is conformal. This can be seen since by suitable choice of large \(N\) and \(F\) we may place the massless theory (with the gaugino decoupled) arbitrarily close to where asymptotic freedom is lost and where the theory is believed to have a weakly coupled IR Banks-Zaks fixed point. At the origin of moduli space, if the duality is correct in the field theory, then there is a strongly interacting theory at a conformal fixed point that flows to the same low energy physics. These theories can not dynamically generate a quark condensate since they’re conformal and so we conclude that a condensate only develops with the introduction of a mass term.

In the decoupling limit to bring the M-theory curve to a standard form we rescaled the curve parameter \(z\) by \(m_\lambda\). In the field theory this corresponds to rescaling the dual fields by \(m_\lambda\) and this is presumably the correct basis for the decoupled field theory. In this basis the vev of the scalar, \(a_M\), has the same symmetry charges as the quark condensate. This is why the condensate appears as the mass parameter in the curve (24). In the field theory we expect the scalar vev and quark condensate to grow together.

The possibility then is that non-supersymmetric \(SU(N)\) gauge theories with \(F\) quark and scalar fields (fine tuned to masslessness) in the fundamental representation is dual to an \(SU(F - N)\) gauge theory with \(F\) dual quarks and scalars plus \(F^2\) singlet meson fermions and scalars. In the massless limit the dual theories do not generate a fermion condensate but as a mass is introduced a fermion condensate and scalar vev develop together and the theory has a mass gap. Of course we emphasise again that the brane construction strong coupling expansion does not prove this for these field theories though they are amusingly suggestive.
The failure of the scalar fields to decouple even in the absence of supersymmetry is awkward for the discussion of true QCD dynamics. Ideally we would like to decouple the scalars at tree level and not rely on radiative effects. Unfortunately the deformations of the brane configuration do not appear to correspond to a suitable scalar mass term. The best we could hope to achieve is to switch on masses of the form of (3) but these masses are always unbounded and trigger a higgs branch of the theory. We do not know of anyway to give all scalars a positive tree level mass and so their decoupling remains frustratingly elusive. It is though clear that a flavour dependent dual gauge theory such as the branes suggest is not possible for QCD since it possesses no higgs branch.

Acknowledgements: This work was in part supported by the Department of Energy under contract #DE-FG02-91ER40676.

References

[1] N. Seiberg, Nucl. Phys. B435 (1995) 129, hep-th/9411149.

[2] A. Masiero and G. Veneziano, Nucl. Phys. B249 (1985) 593; N. Evans, S.D.H. Hsu and M. Schwetz, Phys. Lett. B355 (1995) 475, hep-th/9503186; N. Evans, S.D.H. Hsu, M. Schwetz and S. Selipsky, Nucl. Phys. B456 (1995) 205, hep-th/9508002.

[3] N. Evans, S.D.H. Hsu and M. Schwetz, Phys. Lett. B404 (1997) 77, hep-th/9703197.

[4] O. Aharony, J. Sonnenschein, M. E. Peskin, S. Yankielowicz, Phys. Rev. D52 (1995) 6157, hep-th/9507013; E. D’Hoker, Y. Mimura and N. Sakai Phys. rev. D54 (1996) 7724, hep-th/9603200; M. Chaichian, W. Chen and T Kobayashi, hep-th/9803146.

[5] H. Cheng and Y. Shadmi, hep-th/9801146; N. Arkani-Hamed and R. Rattazzi, hep-th/9804068.

[6] L. Álvarez-Gaumé, J. Distler, C. Kounnas and M. Mariño, Int. J. Mod. Phys. A11 (1996) 4745, hep-th/9604003; L. Álvarez-Gaumé and M. Mariño, Int. J. Mod. Phys. A12 (1997) 975, hep-th/9606191; L. Álvarez-Gaumé, M. Mariño and F. Zamora, hep-th/9703072; M. Mariño.
and F. Zamora, hep-th/9804038; N. Evans, S.D.H. Hsu and M. Schwetz, Nucl. Phys. B484 (1997) 124, hep-th/9608135.

[7] A. Giveon and D. Kutasov, hep-th/9802067.

[8] A. Hanany and E. Witten, Nucl. Phys. B492 (1997) 152, hep-th/9611230.

[9] E. Witten, Nucl. Phys. B500 (1997) 3, hep-th/9703166.

[10] E. Witten, Nucl. Phys. B507 (1997) 658, hep-th/9706109.

[11] S. Elitzur, A. Giveon and D. Kutasov, Phys. Lett. B400(1997) 269, hep-th/9702014.

[12] M. Schmaltz and R. Sundrum, hep-th/9708015.

[13] A. Brandhuber, J. Sonnenschein, S. Theisen and S. Yankielowicz, Nucl. Phys. B502 (1997) 125, hep-th/9704044. A. Hanany, M.J. Strassler and A. Zaffaroni, hep-th/9707244.

[14] N. Evans and M. Schwetz, hep-th/9708122

[15] N. Evans, hep-th/9801159

[16] J.L.F Barbon and A. Pasquinucci, hep-th/9711030

[17] J.L.F. Barbon and A. Pasquinucci, hep-th/9804029