Inflation method based on confidence intervals for data assimilation in soil hydrology using the ensemble Kalman filter

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Abstract
The ensemble Kalman filter (EnKF) is a widely used data assimilation method in soil hydrology. However, underestimation of the modeling errors and of the sampling errors may cause systematic reduction of state variances and rejection of the observations. Inflation methods are used to alleviate this phenomenon. Here, we suggest a novel inflation method based on confidence intervals constructed using the collected ensemble of the measurements. The proposed method is illustrated via two synthetic examples of a three-layer soil with (i) precipitation and evaporation boundary condition and (ii) irrigation boundary condition. We present a comparison of two existing inflation methods and discuss the advantages and limitations of the proposed method. Basically, the suggested method behavior is superior to the behavior of the existing methods.

1 | INTRODUCTION

Data assimilation combines information from both models and measurements to obtain optimal estimates of variables of interest (Reichle, 2008). The advantage of data assimilation over other methods (such as inverse modeling) is that it intrinsically considers the model uncertainty.

Data assimilation has been applied in a wide range of applications, among them dynamic modeling of crop–soil–atmosphere interactions. In recent years, most of these studies involved measurements obtained via remote sensing techniques (de Wit & van Diepen, 2007; Dente, Satalino, Mattia, & Rinaldi, 2008; Ines, Das, Hansen, & Njoku, 2013). Despite the popularity of remote sensing, this technique has some limitations in terms of temporal and spatial resolution. In addition, remote sensing does not provide the moisture profile in the whole root zone, which is of particular interest for managing crop irrigation. Furthermore, remote sensing cannot penetrate through dense vegetation (Ines et al., 2013). Accordingly, direct measurements collected from in situ, pointwise sensors, such as soil moisture sensors, are still very much relevant in many practical applications. From the point of view of data assimilation, such measurements present unique challenges due to (i) the low number of sensors (due to cost and operational constraints), and (ii) the natural heterogeneity existing in the field leading to large differences between measurements provided by similar sensors at different locations. The present work focuses on assimilation of such measurements.

One of the most important methods for data assimilation is the Kalman filter (Kalman, 1960) which in its basic formulation can be applied only to linear models, but for which extensions to nonlinear systems have been developed. One extension is the extended Kalman filter (EKF), which linearizes the nonlinear model using first order approximation of Taylor series (Chirico, Medina, & Romano, 2014). In particular, the ensemble Kalman filter (EnKF) (Burgers, van Leeuwen, & Evensen, 1998; Evensen, 1994) is a popular extension of the Kalman filter to nonlinear systems due to its
ease of implementation and simple conceptual formulation (Evans, 2003). The EnKF uses the Monte Carlo approach to approximate the conditional second-order moments using a finite number of randomly generated model “replicates” (i.e., ensemble members) and then estimates model forecast and error covariance (Evans, 2003; Houtekamer & Mitchell, 1998). Many studies have successfully applied EnKF to assimilate measured data in meteorological and hydrological models (Bauser, Berg, Klein, & Roth, 2018; Brandhorst, Erdal, & Neuweiler, 2017; Das & Mohanty, 2006; Das, Mohanty, Cosh, & Jackson, 2008). In addition, various studies have shown that EnKF is more efficient in state retrieval than other methods such as EKF (Chirico et al., 2014).

One of the factors that strongly influence the performance of data assimilation algorithms are model errors, which can be due to various causes: limited ensemble size in Monte Carlo methods, incorrect model parameters, low-resolution models, and lack of consideration of subgrid interaction in large-scale cases (Berardi, Andrisani, Lopez, & Vurro, 2016; Grooms, Lee, & Majda, 2015). In addition, when considering water flow in the layered soil, a numerical solution might result in drifted values of the state in the interface between two layers, inducing large model errors (Berardi et al., 2016).

To apply the EnKF optimally, all sources of errors should be sampled. Inevitably, some sources will be undersampled, resulting in systematic reduction of state variances, rejection of the observations, and the estimate state vector drifting away from the true state vector. With time, this problem progressively worsens and results in a condition known as “filter divergence,” in which the state vector variance becomes very small and the observations are totally ignored (Whitaker & Hamill, 2012).

Covariance inflation (i.e., artificially increasing the state covariances) can be applied to alleviate this phenomenon. Different methods of inflation have been proposed in the literature. One of the methods is the additive inflation, which applies an additive model error after propagating the state vector in time (Whitaker, Hamill, Wei, Song, & Toth, 2008). This method is useful when good prior knowledge of the model error is available. For instance, Whitaker et al. (2008) applied successfully additive inflation in atmospheric sciences. Another covariance inflation method is the relaxation method, which relaxes the analyzed state vector (the updated state vector) to the prior state vector’s spread (Whitaker & Hamill, 2012; Ying & Zhang, 2015). This can be performed by reducing the extent of the update, which results in a smaller decrease in the state variance. However, the implementation of the above methods for models that deal with soil hydrology is limited due to the spatiotemporally varying model errors, meaning that it is complex to know the true values of the errors and determine the correct inflation magnitude.

**Core ideas**
- The ensemble Kalman filter (EnKF) was used to estimate soil water content.
- A covariance inflation method based on confidence interval was suggested.
- The EnKF method showed superior performance to other existing methods.

One more type of covariance inflation is multiplicative covariance inflation, according to which the complete state vector is inflated (i.e., multiplied by some factor). This factor can be set a priori (Anderson & Anderson, 1999; Ines et al., 2013) or can be estimated based on the measurements (Anderson, 2007; Li, Kalnay, & Miyoshi, 2009). Ines et al. (2013) applied constant inflation factors for soil moisture (factor = 1.05) and crop canopy cover (factor = 1.15) estimations. Their suggestion was for future works to consider variable inflation factors to further improve the performance. Recently, Bauser et al. (2018) suggested a variable inflation method that estimates the inflation factor via the measurements by considering the inflation factor as a virtual state in the general Kalman filter framework. However, the use of Kalman filter equations can result in poor performance because of the difficulty of satisfying the underlying Kalman filter assumptions. In addition, the suggested method includes parameters that affect the performance of the method but for which no general tuning rules were provided.

Here, we present a new adaptive covariance inflation method, in which the inflation factors are updated at each time step according to the model predictions and the collected measurements. This method was tested for soil moisture estimation, which is important for various applications in agriculture, such as irrigation scheduling. As detailed below, the method estimates the inflation factors as measurements become available, based on confidence intervals constructed from the ensemble of measurements collected within a predetermined confidence level. Compared with other inflation methods, the proposed method has the advantage that, although it includes a few parameters that must be defined by the user, these do not have a drastic influence on the overall performance. In addition, this method combines the advantage of using data assimilation together with the advantage of using the statistical properties of the measurements. The performance of the suggested method is compared with the performance of two methods: (i) constant inflation factor of 1.05, as suggested by Ines et al. (2013), and (ii) considering the inflation factor as state, as suggested by Bauser et al. (2018). By comparing the suggested method with these two methods, we covered two types of methods: (i) a...
1.1 Study scope and assumptions

The present study focuses on assimilation of soil moisture measurements in a one-dimensional flow model under the following general assumptions:

- Only the soil dynamics are considered (i.e., no crop).
- The soil profile consists of several sublayers, whose thickness and mechanical and hydraulic properties are roughly known (e.g., soil mixture, saturation water content).
- The top-layer boundary conditions are known or measured (soil evaporation, rain or irrigation).
- Measurements of water content are performed continuously at the center of each sublayer.
- Assimilation of the measurements is performed at the end of each day.
- The “soil” covers a relatively large area (e.g., field) that exhibits some inherent spatial variability, and measurements are performed at a (small) number of random locations using identical sensors.

Under these general assumptions, the goal of the assimilation procedure is to improve the daily estimations of the water content of the whole profile.

2 MATERIALS AND METHODS

2.1 The HYDRUS-1D model

The HYDRUS-1D model, developed by Šimůnek, Šejna, and van Genuchten (2005), is widely used for hydrological research and other uses. It is a physically based model that solves Richards’ equation for saturated–unsaturated flow numerically. It can be written as

\[ \frac{\partial \theta}{\partial t} = \frac{\partial}{\partial z} \left[ K (h_p) \left( \frac{\partial h_p}{\partial z} + 1 \right) \right] - S \]  

where \( \theta \) is the volumetric water content (cm\(^3\) cm\(^{-3}\)), \( t \) is the time (s), \( z \) is the Euclidian coordinate orthogonal to the soil surface (cm), \( h_p \) is the pressure head (cm), and \( S \) is a sink term (cm\(^3\) cm\(^{-3}\) s\(^{-1}\)). \( K(h_p) \) is the unsaturated hydraulic conductivity function. The van Genuchten–Mualem parametrization (van Genuchten, 1980) was used to parametrize the hydraulic conductivity and retention function. Thus, the retention curve is defined as

\[ \theta = \begin{cases} \theta_0 + \frac{\theta_s - \theta_0}{(1 + |\alpha h_p|^n)^m} & \text{for } n \leq 1 \\ \theta_s & \text{otherwise} \end{cases} \]  

where \( \theta_0 \) and \( \theta_s \) are the residual and saturated water content (cm\(^3\) cm\(^{-3}\)), respectively. The parameter \( \alpha \) (cm\(^{-1}\)) can be related to the air entrance value, and \( n \) is a shape parameter. The parameter \( m \) can be determined as \( m = 1 - 1/n \) when \( n > 1 \). The hydraulic conductivity curve is defined by

\[ K (h_p) = K_s \frac{\left( 1 - (\alpha h_p)^m \right)^{1-n} + (\alpha h_p)^n}{\left( 1 + (\alpha h_p)^n \right)^{m/n}} \]  

where \( K_s \) is the saturated hydraulic conductivity (cm h\(^{-1}\)), and \( l \) is the tortuosity (–).

2.2 Ensemble Kalman filter

The EnKF (Burgers et al., 1998; Evensen, 1994) is the Monte Carlo extension of the Kalman filter (Kalman, 1960) for nonlinear models. As a filter, the EnKF works in two steps: forecast and analysis. The forecast step is the propagation of the uncertain states (i.e., ensemble of states) in time, and the analysis step is the combination of the uncertain measurements and the predicted states to obtain an updated ensemble of states. Below, these two steps are described for completeness.

The forecast step is the propagation of each of the state ensemble members, \( \varphi^n \in \mathbb{R}^n \) (where \( n \) is the size of state vector), forward in time from \( k - 1 \) to \( k \) according to a model \( f \),

\[ \varphi^{f,n}_k = f (\varphi^{a,n}_{k-1}) + w_k \]  

where the superscripts \( f \) and \( a \) denote forecast and analysis, respectively, the superscript \( n \) denotes the state ensemble members with \( n = 1, \ldots, N \) and \( w_k \in \mathbb{R}^n \) is the process (model) noise at time \( k \). Generally, the model noise is unknown and must be chosen somewhat arbitrarily. In the following, the model noise is chosen as zero, and the state uncertainties are directly represented through the spread of the new propagated ensemble.

In the EnKF, the state covariances, \( \mathbf{P}^f \in \mathbb{R}^{n \times n} \), are estimated from the propagated ensemble as follows

\[ \mathbf{P}^f_k = \frac{1}{N-1} \left[ \varphi^{f,n}_k - \bar{\varphi}^f_k \right] \left[ \varphi^{f,n}_k - \bar{\varphi}^f_k \right]^T \]  

where \( \bar{\varphi}^f_k \) is the ensemble mean, which is considered as the best estimate of the true state. \( T \) is the transpose of the matrix.
When measurements become available, the analysis step is conducted. This step combines the information from the model ensemble and the measurements. It is assumed that the measurements are related to the state linearly as follows:

$$d_k = H_k \phi_{f,k} + v_k, v_k \sim N(0, R_k)$$

where \(d_k \in \mathbb{R}^{n_m}\) (where \(n_m\) is the number of measurements) denotes the measurements at time \(k\), \(\phi_{f,k} \in \mathbb{R}^{n_t}\) is the true state, \(H_k \in \mathbb{R}^{n_m \times n_t}\) is the measurement operator that maps from the state space to the measurement space at time \(k\), \(v_k \in \mathbb{R}^{n_m}\) is the measurements noise, and \(R_k\) is the covariance matrix of the measurements, which is determined according to the sensor accuracy with no correlation between the measurements. The measurement vector is perturbed to produce a measurement ensemble with \(N\) members,

$$d_k^n = d_k + v_k^n$$

where \(v_k^n \in \mathbb{R}^{n_m}\) is the \(n\)th member sampled from the Gaussian distribution in Equation 6. Then, at time \(k\), the covariance of the measurements, \(R_k\), is estimated from the perturbed ensemble. As \(N\) increases, \(R_k\) converges to \(\tilde{R}_k\).

Each member of the state ensemble is updated to \(\phi_{k,n}^{a,n}\) according to

$$\phi_{k,n}^{a,n} = \phi_{f,k}^{f,n} + K_k [d_k^n - H_k \phi_{f,k}^{f,n}]$$

where the Kalman gain is calculated as follows:

$$K_k = P_k^{f} H_k^T [H_k P_k^{f} H_k^T + R_k]^{-1}$$

The Kalman gain, \(K_k \in \mathbb{R}^{n_t \times n_m}\), weights the forecasted state covariances and the measurements covariances: the more reliable the measurements, the higher the value of Kalman gain and the largest the state update. Basically, in the analysis step, the state covariance is increasingly reduced as state update increases, especially through spurious correlation between states. In particular, using limited ensemble size results in spuriously large covariances, and as a result, states uncorrelated to predictions are updated unreasonably. This causes variances to become too small, which, as mentioned above, causes rejection of observations and filter divergence (Whitaker & Hamill, 2012). State inflation can alleviate this problem.

### 2.3 The proposed inflation method

We assume that some of the states are measured directly (i.e., the matrix \(H\) has 1 on some of its entries and 0 elsewhere), which is indeed the case when soil moisture measurements are performed at several depths along the soil profile. We adopt the multiplicative inflation approach (Anderson & Anderson, 1999) to enlarge the state covariances:

$$\Phi_{inf}^f, n = \lambda (\Phi_{f,n}^f - \bar{\Phi}^f) + \bar{\Phi}^f$$

where \(\lambda \in \mathbb{R}^{n_t}\) is the inflation factor vector, \(\Phi_{f,n}^f\) is the \(n\)th ensemble member, \(\bar{\Phi}^f\) is the mean of the ensemble, and \(\Phi_{inf}^f, n\) is the \(n\)th ensemble member after the inflation. The obvious question is how to determine the values of \(\lambda\). In the present work, we suggest determining \(\lambda\) using the uncertainty of the corresponding measurements. As mentioned in the introduction, our focus is on systems that are inherently heterogeneous but in which the number of locations at which measurements are performed is limited, so that, loosely speaking, the reliability associated with the measurements is low. Mathematically this can be expressed by considering the system (i.e., the field) as a population being sampled, so that the true mean of the state \(\phi_{f,k}\) can be estimated using the mean of the collected measurements and a confidence interval corresponding to a predetermined confidence level:

$$H \Phi_{f,k} = [d_k \pm t_{m-1, \alpha} \frac{s}{\sqrt{m}}]$$

where \(t_{m-1, \alpha}\) is the \(t\) score from the \(t\) distribution with \(m - 1\) degrees of freedom and \(1 - \alpha\) confidence level, \(m\) is the number of measurements collected (arbitrarily chosen as 10 in this study), and \(\alpha\) is the significance level (typically equal to .05). \(s\) is the estimation of the standard deviation of the population, which is calculated from the collected measurements:

$$s^2 = \frac{1}{m - 1} [d_k^n - d_k][d_k^n - d_k]^T$$

The \(t\) distribution is used instead of the more common Gaussian distribution, since it is more appropriate for situations where the sample size is small (Whitley & Ball, 2002). The confidence interval (Equation 11) defines the range within which the true measured state is believed to be. In other words, the estimations of the EnKF should be inside the confidence interval, which means that the confidence interval constitutes a uniform distribution of the estimated state. On the one hand, the state update should be such that the updated state approaches the mean of the confidence interval (i.e., the average measurement). On the other hand, inflation must be performed only if the updated state is not inside the confidence interval, since our belief is that the true state could be anywhere within the confidence interval. Accordingly, we propose estimating the inflation factors as follows: first, all the inflation factors for all the states are initialized as 1. Then, the predictions that are outside their respective confidence interval are identified as “governing states” that should be updated to the mean of the confidence interval, and hence determine the extent of required inflation. The inflation factors of the governing states are increased incrementally by \(\Delta \lambda\),
ideally until the inflated states reach their target values. However, since increasing the inflation for the governing states might cause other predictions to leave their respective confidence interval, the procedure has to be applied in a stepwise and iterative fashion, and inflation factors for predicted measurements that leave the confidence interval are increased by $\Delta \lambda$ as well. Since a layered soil is assumed, states of a given layer are multiplied by the same inflation factor, since these states are expected to have similar covariance behavior. Algorithm 1 (in the supplemental material) details the determination of the inflation factors. Since the increase in inflation factors is incremental and discrete, the inflated states do not reach their target values exactly, but rather the inflation is stopped when the updated inflated states are within some predefined distance, $\Delta$, from the measurements averages. Clearly, performing inflation of a specific state has repercussions on other (possibly all) states and may cause other states to grow or decrease in an unreasonable manner. To avoid such extreme values, inflation is also stopped whenever some predefined number of states, $\theta_{\text{max}}$, reach their upper or lower bounds. As illustrated in the examples below, this means that in some situations, inflation is stopped even though some of the predicted states are not inside the desired confidence interval.

### 2.4 Example

The proposed inflation method was tested by simulating a one-dimensional flow in a soil profile consisting of three 20-cm layers (broadly speaking, sand-loamy, sand, and sand-loamy). The simulations were performed using HYDRUS-1D, which implements Richard’s equation to describe the change of the volumetric water content over time, together with the parametrization of Mualem–van Genuchten (Mualem, 1976; van Genuchten, 1980). The soil column consisted of 60 layers, each of 1-cm thickness, and the parameterization of each layer was described by a set of soil hydraulic parameters: $\theta_s$ (cm$^3$ cm$^{-3}$), $\theta_r$ (cm$^3$ cm$^{-3}$), $\alpha$ (cm$^{-1}$), $n$ (–), and $K_s$ (cm d$^{-1}$).

Two models were considered: the “True model” and “Biased model,” which differed in terms of hydraulic parameters, as detailed in Table 1. These parameters values were such that there were obvious differences in the soil moisture profiles predicted by the two models. In both models, the lower boundary condition was set as free drainage.

Synthetic measurements were produced at three depths: 10, 30, and 50 cm. For each depth, 10 measurements were generated daily from the True model, assuming that the measurements followed a Gaussian distribution with the True model state value as a mean and a standard deviation equal to .05 [–]. Such synthetic measurements mimic the situation of a heterogeneous field (in terms of soil) equipped with 10 sets of sensors placed at random locations, and which do not provide the same readings. The average of the 10 measurements was considered as the measured value and perturbed to generate a measurement ensemble as described in Equation 7. The standard deviation of the 10 measurements was used for calculating the confidence interval in Equation 11.

The EnKF was implemented using the Biased model to estimate the daily soil water content in the whole soil profile. Fifty ensemble members were used, which were generated from a Gaussian distribution with mean equals the initial state values and .005 standard deviation. The following parameters might be considered as nongeneric (case dependent) tuning parameters, which were set by trial and error as follows: $O_{\text{max}} = 5$, $P_{\text{max}} = O_{s}$, $P_{\text{min}} = O_{r}$, and $\Delta = .1$ (see the supplemental material for the definition of these parameters).

Two cases were tested. In the first case, the initial conditions were set as equilibrium (field capacity), and the boundary condition at the upper layer consisted of climatic data (precipitation and evaporation) obtained from the Technion meteorological station and Ein HaHoresh station, Israel Meteorological Service, in November 2017 through February 2018. This period was chosen to test the suggested method over a rainy period that caused high dynamics of the system. Interpolations were conducted to fill in missing records. Figure 1 describes the precipitations and soil evaporation during the simulation period. In the second scenario, the upper-layer boundary condition consisted of “irrigation events” on Days 5–7 (20, 10, and 15 mm, respectively). Precipitation and evaporation were set to 0 in this scenario. Furthermore, in this second test case, the initial conditions of the True and Biased

![Table 1](image-url)
systems differed: the initial water content (along the whole profile) was set to .2 in the True model and .18 in the Biased model. The goal was to investigate the performance of the proposed method in response to external step-wise excitation and to determine whether the estimated states converged toward the true states after the excitation died out.

3 | RESULTS

3.1 | Case study 1: Precipitation and evaporation boundary condition

One hundred-day simulations were run with the assimilation procedure described above (named “ByConfidenceInterval”) and two methods from the literature for comparison: the method suggested by Bauser et al. (2018), which considers the inflation factor as states (named “AsStates”), and the method suggested by Ines et al. (2013), which suggests using the inflation factor as constant factor of 1.05 (named “ConstantFactor”). The performances of the assimilation procedures were compared using the sum of square errors (SSE) between the true (calculated by the True model) and estimated states:

$$\text{SSE} = \sum_{i=1}^{T} \left[ \Phi_{t,i} - \Phi_{f,i} \right]^T \left[ \Phi_{t,i} - \Phi_{f,i} \right]$$

(13)

The results are summarized in Table 2. The estimation error in each layer and the whole soil profile is the lowest for the ByConfidenceInterval method. When considering the whole soil profile, the AsStates method improves the predictions compared with “no assimilation,” so that the SSE is lower than the SSE of “no assimilation” by ~12%. However, the ByConstantFactor method does not improve the predictions compared with the “no assimilation” case. These behaviors can be explained by the low inflation, low covariances, and consequently low state updates. For the ByConfidenceInterval and AsStates methods, significant improvements are achieved in the 21- to 40-cm and 41- to 60-cm layers. Contrarily, the predictions are actually worse in the 1- to 20-cm layer for the AsStates method, and the improvement is the lowest for the ByConfidenceInterval method, which has the fastest dynamics. This occurred because of the high correlation with other layers, which causes large updates based on measurements in other layers.

The results of the estimated state (i.e., the ensemble mean and spread after the update) for the ByConfidenceInterval method at the measured depths, together with the confidence interval lower and upper bounds, are shown in Figure 2. The bold (black) line corresponds to the true model, and the red and blue dots define the confidence interval constructed from the 10 simulated “measurements.” The poor reliability of the
TABLE 2  The sum of square errors for all the inflation methods tested in this study in Case Study 1

| Layer cm   | ByConfidenceInterval | AsStates | ByConstantFactor | Without assimilation |
|------------|----------------------|----------|-----------------|---------------------|
| 1–20       | 1.42                 | 2.88     | 3.19            | 2.25                |
| 21–40      | .60                  | 1.24     | 1.42            | 2.20                |
| 41–60      | .63                  | 2.01     | 2.48            | 2.50                |
| Whole soil profile | 2.65     | 6.13     | 7.09            | 6.95                |

FIGURE 2  The ensemble mean and spread after the update for the ByConfidenceInterval method. WC is the water content, and CI is the measurements confidence interval.

measurements can be appreciated from the large confidence intervals. Ideally, the ensemble prediction (ensemble mean, thin blue line) at each depth should fall within its respective confidence interval. However, since inflation in one layer influences the predictions in other layers, it is not always possible to satisfy the confidence interval constraint simultaneously in all layers. This can be seen, for instance, on Day 77, where the top-layer ensemble does not fall within the confidence interval.

Figure 3 describes the inflation factors of the three layers. The inflation frequency increases after Day 40 when the precipitations increase (high dynamics). The high inflation values occurred when the maximum number of iterations was reached. Before Day 40, the inflation was implemented in low frequencies because of the low precipitation amounts. Despite the low inflation in this period, the state spread was not decreasing drastically (Figure 2), because no large updates occurred. In addition, it is likely that the sampling errors were described properly via the large size of the ensemble (50 members). Comparing the inflation factor in the ByConfidenceInterval method with the AsStates method (shown in Figure 4), the inflation in the AsStates method was more frequent (inflation was implemented almost continuously), but in low magnitudes. In addition, after Day 40, the inflation factors decreased to ~1 and remained around there, indicating that the method stopped working. Note that the inflation factors in the AsStates method do not differ very much along the profile, even though the method estimates the inflation factor for each state. This means that using the same inflation factor for a group of states (as done in the
FIGURE 3 The inflation factors of the three layers in the ByConfidenceInterval method, together with the precipitation, in Case Study 1

Figure 5 shows the differences between the true and obtained state values, as well the variances of the states, which are an indication of state spread. The low inflation in the AsStates method causes low variances compared with the ByConfidenceInterval method. For the AsStates and ByConfidenceInterval methods, the differences between the estimated state and the true state are larger in the top layer than in the lower layers. It can be seen that the lowest differences of the AsStates method were before Day 40, when the inflation factors were larger than those after Day 40.

As mentioned above, for the ByConfidenceInterval method, the search of the inflation factors for each state (water content at each depth) was preformed iteratively and implemented for the layers with similar materials as described in the section above. Figure 6b describes the inflation factor for each layer as a function of the number of iterations, together with the states at each measured depth before and after the inflation for a specific day (Figure 6a). At depths of 10 and 30 cm, the estimated water content was initially outside of the confidence interval, whereas at a depth of 50 cm, it was inside the confidence interval. Hence, the inflation factors for the 10- and 30-cm depths were incrementally increased, whereas the inflation factor for the third depth remained unchanged ($\gamma = 1$, no inflation). After $\sim$7000 iterations, the inflation factor at the 30-cm depth stopped increasing, indicating that the updated state reached the measurement value. The inflation factor for the proposed method) may indeed work properly. However, note that the inflation factors estimated for the 10- and 50-cm depths (at which measurements were performed) were not close to the inflation factors for the nearby layers, despite the proximity and high correlation, which show that this method failed to evaluate the true correlations between the states.

FIGURE 4 The inflation factors in the AsStates method in Case Study 1
FIGURE 5  The differences between the true and obtained state values and the variances of the states for all the inflation methods in Case Study 1.

FIGURE 6  The states at each measured depth before and after the inflation, and the inflation factor for each layer as a function of the number of iterations for a specific day in the ByConfidenceInterval method in Case Study 1. CI is the measurements confidence interval.
**Table 3** The sum of square errors for all the inflation methods tested in this study in Case Study 2

| Layer cm | ByConfidenceInterval | ByConfidenceInterval | ByConfidenceInterval | ByConfidenceInterval |
|----------|-----------------------|-----------------------|-----------------------|-----------------------|
| 1–20     | .18                   | .46                   | .49                   | .49                   |
| 21–40    | .05                   | .18                   | .20                   | .21                   |
| 41–60    | .11                   | .28                   | .33                   | .33                   |
| Whole soil profile | .34 | .92 | 1.02 | 1.03 |

**Figure 7** The soil profile for Days 4–9 in the ByConfidenceInterval method in Case Study 2. CI is the measurements confidence interval.

10-cm depth continued to increase, since the corresponding updated state was not yet within the prescribed interval. Due to the cross-coupling between the states, in the present case, increasing the inflation factor of the 10-cm depth ultimately caused the prediction at 30 cm to leave its respective confidence interval. This occurred at iteration 13,000 (when the 10-cm-depth inflation factor reached a value of ~10). In response, the inflation factor at the 30-cm depth had to be further increased. After ~23,000 iterations the desired result was obtained—namely, all predicted states were within the respective confidence intervals (Figure 6a). Notice that throughout the process, the inflation factor at the 50-cm depth remained constant because of the low correlation between the measurement at that depth and at two other depths.

### 3.2 Case Study 2: Irrigation boundary condition

Top-layer boundary conditions mimicking irrigation events were introduced to determine the performance of the different assimilation methods in case of rapid and significant changes in soil water content. Table 3 presents the sum of squared error for each method in each layer and over the entire soil profile. The ByConfidenceInterval method yielded the best estimations in all layers and in the soil profile as a whole. Despite the high dynamics, both methods still improve the estimations. In contrast with what observed in Case Study 1, an improvement of the estimations (compared with the “without assimilation” case) in the top layer was observed in the AsStates.
method. The ByConstantFactor method results were similar to the results without assimilation because of the low variances and consequently small state updates.

Figure 7 shows the soil profile for Days 4–9 to further appreciate the performance of the various methods. As mentioned, irrigation was implemented on Days 5–7. On Day 4, after days without irrigation, the water content in the profile is close to equilibrium. The ByConstantFactor method yielded estimations that were very close to the “no assimilation” case, due to the low variances and low updates occurred. The estimations of the AsStates and ByConfidenceInterval methods on this day are better than for the “no assimilation” case. After Day 4, the ByConfidenceInterval method improves the estimations compared with the “no assimilation” case in the three layers despite the high dynamics. By comparison, the AsStates method estimations became close to the “no assimilation” case because of the underestimation of the inflation factors during the period with high dynamics. It can also be seen that the estimations of the ByConfidenceInterval method were almost always inside the confidence interval (except on Day 6). This indicates that there were low covariances between the predicted measurements (measured states), and each “governing state” was updated almost solely according to its corresponding measurement.

4 | CONCLUSION

In this work, we proposed an adaptive inflation method within the EnKF framework. The method estimates inflation factors based on the assumption that the actual states lie within confidence intervals that can be estimated from the dispersion of the measurements.

The proposed assimilation method was tested over two test cases that were generated by changing soil hydraulic parameters. These test cases mimic the situation expected to occur in real applications where the actual soil hydraulics are known only approximately. The proposed method outperformed the other inflation methods tested and demonstrated stable behavior by keeping the estimations inside, or close to, confidence intervals, which were determined by taking into account the reliability of the measurements. The performance of the suggested method is superior to that of the other methods in the two case studies, both for the whole soil profile and for each layer individually.

The suggested method estimates the inflation factors based on the difference between the predictions and the measurements, similar to the method of Bauser et al. (2018) (AsStates), which is based on Kalman filter. However, using strict Kalman filter is somewhat questionable, since the assumptions on which the Kalman filter is based are not satisfied for the inflation factors. This may explain the poorer results obtained using this approach. In future work, we plan to evaluate the performance of the method for estimating the model states and model parameters simultaneously.

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