Numerical simulation of two-phase flow in screw-conveyor sedimentation centrifuge

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Abstract. The paper simulates the aerodynamics of a two-phase flow in the cylindrical part of the screw auger precipitation centrifuge in order to estimate the centrifugal forces in the central region of the centrifuge, due to problems of sedimentation of the solid fraction from the suspension. According to the obtained velocity field, the time and the trajectory of the deposition of solid particles of different diameters of a two-phase flow of different diameters are calculated. This is necessary to estimate the speed of rotation and the pitch of the screw for outputting solid sediment to the discharge windows. The resulting model will optimize the existing apparatus for centrifuging two-phase flows, as well as help in the design of new plants.

1. Introduction
Sedimentary centrifuges with screw unloading of the formed solid sludge are widely used in many industries for the separation of suspensions containing a solid phase with a particle size from 1 to 500 microns. The efficiency of centrifuges is characterized by the separation factor, which is the ratio of centrifugal acceleration to the acceleration of free fall. The value of this parameter for some of the centrifuges in the thousands. The advantages of these centrifuges include: a continuous process of suspension separation, stability of the work with fluctuations in the composition of the separated suspension and the content of particles of different sizes in it, greater flexibility in choosing the process parameters by changing the input parameters of the suspension flow. A certain disadvantage of these centrifuges in some cases is the increased humidity of the sludge. Given that the sediment of the centrifuge are often subjected to thermal drying, pressing is the improvement of the centrifuges with the aim of reducing the energy costs for drying and reducing the cost of the dehydration process in General [1]. Creating a mathematical model of the process of sedimentation of solid particles from a two-phase slurry stream in a screw precipitation centrifuge is relevant, because the mathematical model has the advantage of resource cost and optimization of operating parameters before the physical experiment. The model will allow evaluating the behavior of the ensemble of particles, the time of sedimentation and discharge of sediment; to optimize the operation of the auger precipitation centrifuge to obtain the optimal deposition regime and to get a visual representation of the process of sedimentation of the suspension.

2. Physical model of the screw-conveyor sedimentation centrifuge
Screw-conveyor sedimentation centrifuge, Fig. 1, is a set of cylindrical and conical sections, located on the axis of symmetry of the screw auger through which the slurry is fed; as well as the discharge window of the sludge and clarified liquid. The most important parameters are the centrifugal force,
which is set by the speed of rotation of the rotor; the speed of rotation of the screw to unload the solid sludge in the discharge window and the feed rate of the slurry for continuous operation of the centrifuge. These parameters are basic in the process of separation of suspensions into liquid and solid fractions.

![Figure 1](image_url)

**Figure 1.** General scheme of screw-conveyor sedimentation centrifuge

1 – housing of the centrifuge; 2 – auger blades; 3 – feeding pipe; 4 – feed opening; 5 – area of discharge of sludge; 6 – area of discharge of liquid.

The control scheme in the screw sedimentary centrifuges allows you to change the angular speed of rotation of the rotor and the screw and, thus, to change the mode of operation of the centrifuge in order to obtain the maximum possible degree of purification of the suspension for a particular type of raw material. Selection of optimal operating parameters of the separation process is mainly reduced to the selection of the rotor speed and the relative rotation of the screw. To achieve the best possible cleaning solution from solid impurities it is necessary to set a ratio of speeds of the rotor and the auger to the time of deposition of the particles and compaction of sludge on the rotor does not exceed the transport time of sediment to the Windows of the discharge.

3. **Mathematical model of the screw-conveyor sedimentation centrifuge**

Simulation of viscous swirling gas in the working area of the centrifuge is based on the Navier – Stokes equations in a cylindrical coordinate system. In the separation process, as a rule, deals with the suspension moving with relatively small velocities, so as a model carrier environment, you can use the model of incompressible liquid. Thus, we obtain an axisymmetric system of dimensionless differential equations of momentum transfer and the continuity equation in the form of [2]:

\[
\frac{\partial u_r}{\partial \tau} + \frac{\partial u_r}{\partial r} + \frac{\partial u_z}{\partial z} + \frac{u_r^2}{r} - \frac{u_z^2}{r} = -\frac{\partial p}{\partial \tau} + \frac{1}{Re} \left( \frac{\partial^2 u_r}{\partial r^2} + \frac{\partial^2 u_r}{\partial z^2} + \frac{1}{r} \frac{\partial u_r}{\partial r} - \frac{u_r}{r} \right); \\
\frac{\partial u_\phi}{\partial \tau} + \frac{\partial u_\phi}{\partial r} + \frac{\partial u_\phi}{\partial z} + 2 \frac{u_r u_\phi}{r} = \frac{1}{Re} \left( \frac{\partial^2 u_\phi}{\partial r^2} + \frac{\partial^2 u_\phi}{\partial z^2} + \frac{1}{r} \frac{\partial u_\phi}{\partial r} - \frac{u_\phi}{r^2} \right); \\
\frac{\partial u_z}{\partial \tau} + \frac{\partial u_z}{\partial r} + \frac{\partial u_z}{\partial z} + \frac{u_z u_r}{r} = -\frac{\partial p}{\partial \tau} + \frac{1}{Re} \left( \frac{\partial^2 u_z}{\partial r^2} + \frac{\partial^2 u_z}{\partial z^2} + \frac{1}{r} \frac{\partial u_z}{\partial r} \right); \\
\frac{\partial u_r}{\partial \tau} + \frac{\partial u_z}{\partial z} + \frac{u_r}{r} = 0.
\]
The dimensionless form of the equations is obtained using the scale of the length is radius \( R_0 \) of the cylindrical part of the centrifuge, the velocity - the average flow rate of the gas \( U_0 \) through the cylindrical part of the apparatus, the constant density - \( \rho \), the dimensionless pressure of the form \( p = P/(\rho U_0^2) \) and the Reynolds criterion - \( Re = \rho U_0 R_0/\mu \).

At the first stage of modeling of the auger sedimentary centrifuge, the cylindrical area of the working zone of the apparatus without the auger is considered. The following boundary conditions were used. At the entrance to the considered zone, the value of the axial velocity was taken \( u_z = 1 \), for the radial component the Neumann conditions \( \partial u_r / \partial z = 0 \) were used, for the circumferential dimensionless velocity the Neumann condition \( \partial u_\phi / \partial z = 0 \) was also accepted. Conditions \( u_r = 0; \partial u_r / \partial r = 0; u_\phi = 0 \) were used on the axis of symmetry. Neumann conditions \( \partial / \partial z = 0 \) for all required functions were used at the exit from the computational domain. On the surface of the rotor for the radial and axial components of the velocity vector due to the condition of adhesion of the viscous gas, the condition of equality to zero was used, for the circumferential velocity the boundary condition takes the form: \( u_\phi = R_0 \), where the criterion \( R_0 = \omega_0 R_0/U_0 \).

![Figure 2](image.png)

**Figure 2.** Mathematical model of work area of the screw-conveyor sedimentation centrifuge. Where \( r \) is the radius of the centrifuge; \( r_0 \) is the radius of the screw-conveyor; \( L \) is the length of the centrifuge; \( L_0 \) is the length of the feed opening.

### 4. Solution method of the system of governing equations

The main problem of solving the equations of motion of an incompressible viscous fluid is the presence of an unknown hydrodynamic pressure function in these equations. Unlike problems about the compressible flow, the pressure can't be expressed through any physical variables. Since in the considered system of equations there is not partial derivative from the pressure over time, it is impossible to formulate a problem with the initial conditions. Currently, the most promising approach is the method of solution equations of momentum transfer in physical variables "velocity – pressure".

One of the effective ways to solve the equations of motion of an incompressible medium in the variable "velocity – pressure" is a method of physical splitting over time velocity and pressure fields [3].

In this paper, a numerical model is a system of differential equations in partial derivatives of the second order was approximated by the finite difference method. The resulting system of algebraic equations was solved numerically on the spaced difference grid under given initial and boundary conditions. The implicit scheme of equations was written in the "Delta" form was solved by the numerical method of longitudinal-transverse run [4, 5].

We write the equations in the "Delta" form to obtain the second order of accuracy in time, for this we use a layer \( \left( n + \frac{1}{2} \right) \).
The time derivative of the function F will have the form:

\[ \frac{\Delta F}{\Delta \tau} = \frac{F^{n+1} - F^n}{\Delta \tau}; \]

\[ F^{n+1} = F^n + \Delta F; \]

\[ \frac{\Delta F}{\Delta \tau} = \left( \frac{1}{\text{Re}} \left( \frac{\partial^2 F}{\partial r^2} + \frac{\partial^2 F}{\partial z^2} \right) - \left( U \frac{\partial F}{\partial r} + V \frac{\partial F}{\partial z} \right) \right)^{n+1/2}; \]

Let us rewrite the convective and diffusion components of equation in the form of:

\[ D = \frac{1}{\text{Re}} \left( \frac{\partial^2 F}{\partial r^2} + \frac{\partial^2 F}{\partial z^2} \right); \]

\[ K = U \frac{\partial F}{\partial r} + V \frac{\partial F}{\partial z}, \]

Let's express the right part of the equation through the previous layer "n":

\[ \left( D - K \right)^{n+1/2} = \left( D - K \right)^n + \frac{1}{2} \Delta (D - K). \]

Therefore,

\[ \frac{\Delta F}{\Delta \tau} = \left( D - K \right)^n + \frac{1}{2} \Delta (D - K). \]

We find the solution of the system of equations in the variables "velocity – pressure" by splitting the equations by coordinates:

\[ \frac{\Delta F^*}{\Delta \tau} - \frac{1}{2} \Delta (D - K)^* = \left( D - K \right)_{r,z}; \]

\[ \frac{\Delta F^{**}}{\Delta \tau} - \frac{1}{2} \Delta (D - K)^{**} = \frac{\Delta F^*}{\Delta \tau}, \]

Where,

\[ \Delta (D - K) = \frac{\partial^2 \Delta F}{\partial r^2} - U \frac{\partial \Delta F}{\partial r}; \]

\[ \Delta (D - K) = \frac{\partial^2 \Delta F}{\partial z^2} - V \frac{\partial \Delta F}{\partial z}. \]

The numerical solution of the presented system of equations was carried out in the physical variables "velocity – pressure" by physical splitting of the velocity and pressure fields. According to this method, the solution of the Navier – Stokes equations written in vector form includes two stages:

\[ \frac{\Delta F^*}{\Delta \tau} = \left( D - K \right)^{n+1/2} - \text{grad} \left( P \right)^n; \]

\[ \frac{\Delta F^{**}}{\Delta \tau} = -\text{grad} \left( \delta P \right). \]

The solution of the stationary problem is carried out by the method of establishing the time, so the Poisson equation is written in the form of a non-stationary differential equation:

\[ \frac{\partial (\delta P)}{\partial \tau} - \nabla^2 (\delta P) = - \frac{\text{div}(F^*)}{\partial \tau}. \]
After determining the values of the intermediate velocity vector and pressure correction, you can proceed to the calculation of velocities and pressure on the n+1 time layer.

5. Results of numerical simulation
The mathematical model was solved numerically by the finite difference method. For this, the difference analogs of the system of equations of motion were separated by coordinates on the difference grid and calculated by the method of alternating directions.

To assess the adequacy of the created numerical model, test calculations were performed laminar flow in a cylindrical tube. A good agreement between numerical and analytical solutions allows us to conclude about the possibility of using this numerical model in the future for numerical studies of the modes of operation of the screw-conveyor sedimentation centrifuge.

![Figure 3](image1.png)

**Figure 3.** The distribution component of the velocity $u_z$ at the expiration of the suspension from the feed pipe.

![Figure 4](image2.png)

**Figure 4.** The distribution component of the velocity $u_\phi$.

Where $r$ and $z$ are dimensionless coordinate axes:

$$
\frac{r}{R - r_0}; \quad \frac{z}{L} = \frac{L}{R - r_0}.
$$

Where $R$ is the radius of the centrifuge; $r_0$ is the radius of the screw-conveyor; $L$ is the length of the centrifuge.

The created mathematical model, which allows calculating the hydrodynamics in the working area of the centrifugal precipitation centrifuge, can be used as a component of the General model of the process of centrifugal separation of solid sludge from the suspension.

The model is extended by equations that allow calculating the behaviour of the ensemble of solid particles in the field of centrifugal force:
\[
\frac{\partial w_r}{\partial \tau} = \frac{w_\varphi^2}{r} + \frac{u_r - w_r}{Stk} \xi; \\
\frac{\partial w_\varphi}{\partial \tau} = -\frac{w_\varphi w_r}{r} + \frac{u_\varphi - w_\varphi}{Stk} \xi, \\
\frac{\partial w_z}{\partial \tau} = \frac{u_z - w_z}{Stk} \xi - \frac{1}{Fr}; \\
d\tau = \frac{dr}{w_r} = \frac{rd\varphi}{w_\varphi} = \frac{dz}{w_z}.
\]

Where \( \xi \) – coefficient showing the deviation of the coefficient of aerodynamic drag of the particle from its value found from the Stokes law; \( Fr, Stk, \text{Re}_p \) – the criteria of Froude, Stokes and Reynolds particles, respectively:

\[
\xi = 1 + \frac{Re_p^{2/3}}{6}; \quad Fr = \frac{U_0^2}{gR_0}; \quad Stk = \frac{\rho_p \delta^2 U_0}{18\nu R_0}; \quad \text{Re}_p = \frac{|U - \bar{W}|}{\nu},
\]

Where \( \rho_p, \rho \) – the density of solid particles and the carrier medium, respectively, \( \delta \) – the diameter of the solid particle, \( g \) – the acceleration of gravity.

Since the volume concentration of solid particles in the suspension is relatively small, therefore, in the formulation of the problem, the interaction of particles with each other and their reverse effect on the carrier phase is neglected. Also, given the particle size of the order of 100 microns, we can assume that at the initial time the particles have a flow rate. In the model, the motion of continuous medium and discrete particles is considered separately [6, 7].

Figure 5. The path of deposition of particles with a diameter of 100 microns.
**Figure 6.** The path of deposition of particles with a diameter of 50 microns.

**Figure 7.** The path of deposition of particles with a diameter of 30 microns.

**Figure 8.** The dependence of the particle deposition length on the diameter of particles.
Dark areas at the beginning and end of the deposition process in Fig. 5 – 7 indicate the high intensity of the rotation of the particles near the shaft of the screw and the outer wall of the drum centrifuge due to close to zero the longitudinal component of the velocity of the flow. The obtained numerical results allow us to estimate the behavior of the ensemble of solid particles in the separation process, to obtain the trajectory, deposition time and the distance traveled by the solid particle of the two-phase flow in the apparatus of the sedimentary centrifuge under the action of centrifugal force.

6. Conclusion
Thus, the results of theoretical studies, algorithms, calculated ratios and a developed set of programs can be used to quickly, qualitatively and visually simulate the process of centrifugal separation of low-concentrated suspensions and isolate particles of the solid phase. Also this mathematical model can be used to predict changes in performance centrifuges in the period of its work. Practical application of the results will reduce the number of field experiments in the design of industrial centrifuges. The created mathematical model of a two-phase flow in an auger precipitation centrifuge allows a detailed study of the characteristic laws of a two-phase swirling flow in the process of separation and classification of particles; can be used when designing new original devices of this type and optimizing the operating modes of existing centrifugal devices.

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