Finite Temperature Quark Matter and Supernova Explosion

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We study the equation of state of quark matter at finite temperature, using a confinement model in which chiral symmetry remains broken in the deconfined phase. Implications for type II supernova explosion and for the structure and evolution of the proto-neutron star are discussed.

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The Equation of State (EOS) of matter at high density and temperature is a very interesting but controversial topic. Among the possible applications of such an EOS are the structure of neutron stars (very high density and low temperature), supernova explosions (high density and moderate temperature) and relativistic heavy ions collisions (high temperature and not very high density).

It is possible, at least in principle, to obtain information about the behavior of matter under extreme conditions through lattice QCD calculations. Recently it was shown that the deconfinement transition seen at zero baryon density becomes a smooth crossover at very small density and that at low enough temperature chiral symmetry remains broken at all densities. Although these results need to be confirmed by other calculations, it is anyway interesting to explore a model where these features are implemented. In particular, if a constituent quark model is used, the deconfinement transition need not to be as discontinuous as in MIT-like models, where the transition to quark matter implies also chiral symmetry restoration.

In this letter we compute a finite temperature EOS based on the transition to quark matter, and study in particular its connection to the problem of supernova explosion. The most recent calculations show that using traditional EOS a successful explosion is not achieved via the prompt shock mechanism and late neutrino transport still have to be more exhaustively investigated. The prompt explosion mechanism needs a very soft EOS, what seems apparently incompatible with the observed masses of neutron stars. The last conclusion is based essentially on the results coming from the phenomenological BCK EOS. This difficulty can be circumvented. In order to obtain a successful explosion one needs a softening of the EOS at densities slightly larger than the nuclear matter saturation density $\rho_0$. On the other hand, the maximum mass of a neutron star depends on the very large density behavior of the EOS and requires a not too soft EOS in that density region.

The idea to correlate the softening of the EOS to the presence of a phase transition and in particular to the formation of quark matter is rather old. This possibility, in connection with the problem of supernova explosion, has been considered recently by Gentile et al. in a phenomenological way, without any attempt to relate the parameters governing the transition to other quark model calculations. Moreover the dependence of the EOS on the temperature and on the electron fraction was not investigated. Since during the first seconds of the life a proto-neutron star deleptonizes, it is particularly important to study how the transition to quark matter is affected by the electron fraction.

We present here a calculation of the transition to quark matter at finite temperature. In order to satisfy Gibbs conditions when more than one charge is conserved, one has to use the technique developed by Glendenning. Until now, the only calculation incorporating this technique at finite temperature was the study of the liquid-gas phase transition in nuclear matter made by Müller and Serot.

We will use the Color Dielectric Model (CDM) to describe quark matter. The CDM is a confinement model which has been used with success to study properties of single nucleons, such as structure functions and form factors, or to investigate zero temperature quark matter and the structure of neutron stars. An important feature of the CDM is that effective quark masses are of the order of 100 MeV even at very large densities, hence chiral symmetry is broken and the Goldstone bosons are relevant degrees of freedom. This is to be contrasted with models like the MIT bag, where quarks have masses of a few MeV.

The Lagrangian of the model reads:

$$\mathcal{L} = i\bar{\psi}\gamma^\mu\partial_\mu\psi + \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 - U(\sigma, \vec{\pi}) + \sum_{f=u,d} \frac{g_f}{f\pi}\bar{\psi}_f(\sigma + i\gamma_5\vec{\tau} \cdot \vec{\pi})\psi_f + \frac{g_s}{\chi}\bar{\psi}_s\psi_s + \frac{1}{2}(\partial_\mu\chi)^2 - \frac{1}{2}M^2\chi^2$$

(1)

where $U(\sigma, \vec{\pi})$ is the “mexican-hat” potential, as in Ref. The Lagrangian describes a system of interacting $u$, $d$ and $s$ quarks, pions, singaus and a scalar–isoscalar chiral singlet field $\chi$. The scalar field $\chi$ characterizing the CDM is related to the fluctuation of the gluon condensate around its vacuum expectation value. In the model this fluctuation is rather small indicating a smooth transition between the exterior and the interior of the nucleon and allowing a soft deconfinement.
The coupling constants are given by \( g_{u,d} = g(f_π ± ξ_π) \) and \( g_s = g(2f_K - f_π) \), where \( f_π = 93 \text{ MeV} \) and \( f_K = 113 \text{ MeV} \) are the pion and the kaon decay constants, respectively, and \( ξ_π = f_K - f_K^0 = -0.75 \text{ MeV} \). These coupling constants depend on a single parameter \( g \).

Confinement is obtained via the effective quark masses \( m_{u,d} = g_{u,d}Ω/(Ωf_π), m_s = g_s/Ω \), which diverge outside the nucleon. Indeed, the classical fields \( Ω \) and \( Ω \) are solutions of the Euler–Lagrange equations and \( Ω \) goes asymptotically to zero at large distances.

In the following we will use for the model parameters the values:

\[
g = 0.023 \text{ GeV}, \quad M = 1.7 \text{ GeV},
\]

giving a nucleon isoscalar radius of 0.80 fm (exp.val. = 0.79 fm) and an average delta–nucleon mass of 1.129 GeV (exp.val. = 1.085 GeV). A similar set of parameters has been used to compute structure functions \( f_H \) and form factors \( f_H \) and to study neutron stars [14].

We describe the hadronic phase with a relativistic field theoretic model of the Walecka type [15], including protons and neutrons only. The parameters used to define the Lagrangian of the hadronic part are the ones labeled HS81 in the work by Knorren et al. [19].

The transition to quark matter has been studied using the technique developed by Glendenning [9], since in the transition two quantities are conserved, the baryon (B) and the electric (C) charge:

\[
ρ_B = (1 - χ)ρ_B^0 + χρ_B^0 + χρ_B \tag{3}
\]
\[
ρ_C = (1 - χ)ρ_C^0 + χρ_C^0 + ρ_e + ρ_µ = 0.
\]

Here \( χ \) is the fraction of matter in the quark phase. The superscripts \( h \) and \( q \) label the density in the hadronic and in the quark phase, respectively. The electron \( (ρ_e) \) and the muon \( (ρ_µ) \) charge densities contribute to make the total electric charge equal to zero. Due to the presence of strange quarks electron and muon densities are suppressed.

Since matter has to be in chemical equilibrium under \( β \)-decay and deconfinement, the following equations have to be satisfied:

\[
μ_n - μ_p = μ_e - μ_{ν_e}, \quad μ_n - μ_p = μ_µ - μ_{ν_µ},
\]
\[
2μ_d + μ_u = μ_n, \quad μ_u - μ_d = μ_p - μ_n
\]
\[
μ_s = μ_d, \tag{4}
\]

together with the usual condition for mechanical equilibrium, \( i.e. \) the equality of the pressure in the two phases:

\[
P^h = P^q. \tag{5}
\]

Other conditions depend on the specific problem under discussion. For instance, in order to study the structure of the star after deleptonization, one assumes that neutrinos can escape freely, so their chemical potential is set to zero in previous equations. This assumption is incorrect in the first seconds of the life of the proto-neutron star. Due to neutrino opacity lepton numbers are conserved, neutrino chemical potentials are different from zero and the EOS is computed for fixed values of the lepton fractions:

\[
Y_ν = (ρ_ν + ρ_ν)/ρ_B, \quad Y_µ = (ρ_µ + ρ_µ)/ρ_B. \tag{6}
\]

Since before the collapse \( Y_ν = 0 \), this quantity has been kept fixed and the EOS has been computed for various values of \( Y_ν \).

In the computation of the EOS all previous equations have been solved together with the mean-field equations of the Walecka model and of the CDM. Finite temperature has been taken into account using the standard technique developed \( e.g. \) in Ref. [20].

![FIG. 1. Boundaries separating hadronic matter, mixed phase and quark matter in the density-temperature plane. The labels indicate various values of \( Y_ν \), \( s \) is for symmetric matter.](image)
large densities and the mixed phase extends on a broad density range. Using the MIT model, which assumes the interior of the nucleon to be in a perturbative regime, it is almost unavoidable to have a first order transition. In the CDM it is conceivable, at least in principle, to obtain a smoother transition by taking into account quark correlations beyond mean field approximation.

To investigate our EOS in connection with the problem of supernova explosion, we compare with BCK EOS (parameters as in model 38). The latter is a totally phenomenological EOS which is soft enough to allow for supernova explosion, but gives a maximum mass smaller than the mass of PSR 1913+16 (1.44 $M_{\odot}$). In our model the maximum value of the gravitational mass for a non-rotating cold star is 1.59 $M_{\odot}$.

In Fig. 2 we present results for $Y_e = 0.4$ and entropy per baryon number $S/R = 1$. Due to the presence of strange quarks, the electron fraction $Y_e$ is rather small in the quark phase, $Y_e \sim 0.3$, and the muon fraction $Y_\mu$ is always very small. In the upper box we compare the pressure in the Walecka model, in our model and in BCK EOS. Due to the phase transition, our EOS is rather soft from $\rho = 0.17 \text{ fm}^{-3}$ to $\rho = 0.34 \text{ fm}^{-3}$. On the other hand, after $\rho = 0.34 \text{ fm}^{-3}$ it is considerably stiffer than BCK, allowing higher masses for the proto-neutron star. These conclusions are strengthened by the computation of the adiabatic index, shown in the lower box of Fig. 2. Clearly in the mixed phase matter offers little resistance to collapse, but when pure quark matter phase is reached the collapse is halted. In the mixed phase region our adiabatic index is even smaller than in BCK.

Another way of investigating the possibility of a prompt explosion using our EOS is to estimate the

|    | $V_E$ | $V_9$ | $E_i$ |
|----|-------|-------|-------|
| Walecka | 24    | -73   | 97    |
extra energy at disposal for the shock wave in our model respect to pure Walecka model. To this purpose we follow the analysis of Gentile et al. [8]. Their way of estimating the extra energy is the following: they compute the binding energy of the central part of the proto-neutron star, immediately before bounce, integrating on a mass of $0.5M_\odot$. The binding energy is the sum of gravitational and internal potential energy: $BE = V_g + E_i = \text{gravitational mass} – \text{baryonic mass}$. Integrating only on the inner region, a positive value for the $BE$ is obtained, indicating that this region tries to expand, pushing the exterior envelope. The larger the (anti-)binding energy, the stronger the push. We can therefore compare these numbers in Walecka and in our model, using the EOS with $S/R = 1$ and $Y_e = 0.4$. The result is shown in the Table. An extra energy of about 8 foe is at disposal for the explosion.

![Figure 3](image1.png)

**FIG. 3.** Composition of a star having $S/R = 1$ and $Y_e = 0.4$. A different length’s scale has been used where the mixed phase is formed.

![Figure 4](image2.png)

**FIG. 4.** Same as Fig.3 for a star having $S = 0$ and $\beta$-stable.

We come now to the second problem we like to study, namely the structure and evolution of the proto-neutron star. In Fig.3 we show the composition of a star with $Y_e = 0.4$ ($Y_e \sim 0.3$) and entropy per particle $S/R = 1$. The central temperature is of the order of 10 MeV. These conditions should be realized at the bounce. Immediately after the bounce the entropy increases. After a time of the order of 10 seconds the proto-neutron star cools down and deleptonizes. After that the composition of the star does not change any more. We show this later stage in Fig.4, where we assume entropy $S = 0$ and $\beta$-stability. In both figures the baryonic mass is the same, 1.54$M_\odot$, which corresponds to a gravitational mass of 1.4$M_\odot$ for the final star of Fig.4.

![Figure 5](image3.png)

**FIG. 5.** Mass fractions in hadronic, mixed and quark phase (upper box) and binding energy (lower box) as function of $Y_e$ for entropy $S/R = 1$ (solid) and $S/R = 3$ (dashed), respectively.

There have been many speculations about the possibility of a late neutrino emission, based on the SN1987 data [21]. In particular the idea of a late transition to strange matter with a new emission of energetic neutrinos has been invoked. We investigate this possibility in Fig.5, where we show the composition of the star and its binding energy as a function of the lepton fraction $Y_e$ and for two values of the entropy per particle. To study this problem one should really solve the dynamics’ equations giving lepton fraction and entropy as a function of time. What can be learnt from Fig.5 is the absence of a sudden jump in the composition of the star and in its binding energy as a function of $Y_e$. The conclusion we can draw is that an emission of energetic neutrinos during the first seconds is indeed possible, since the binding energy increases steadily as $Y_e$ is diminishing. Anyway a peak in the neutrino luminosity can be due only to the dynamics of the explosion, not to a discontinuity in the EOS of the proto-neutron star.

Recently there has been a speculation about the possibility of having a proto-neutron star collapsing later to a black hole, due to the softening of the EOS after deleptonization [22]. In our calculation we do not have this effect, because the maximum baryonic mass for the proto-neutron star is essentially independent of the value of $Y_e$ and $S$ and remains always near 1.82$M_\odot$. 


In this letter we have shown the possibility to have a phase transition at densities slightly larger than $\rho_0$ in pre-supernova matter. This transition softens the EOS which in this range of densities is comparable with the phenomenological BCK EOS, but gives acceptable values for the maximum gravitational mass of the final neutron star. Our EOS indicates also the possibility of an energetic neutrino emission in the first seconds of the life of the proto-neutron star. All these conclusions will soon be tested in a dynamical simulation of the explosion.

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[1] T. Blum, J.E. Hetrick and D. Toussaint, Phys. Rev. Lett. 76, 1019 (1996).
[2] D.Arnett, *Supernovae and Nucleosynthesis*, Princeton University Press, Princeton 1996.
[3] E.A.Baron, J.Cooperstein and S.Kahana, Phys. Rev. Lett. 55, 126 (1985).
[4] G.E. Brown, Phys. Rep. 163, 167 (1988).
[5] F.D. Swesty, J.M. Lattimer and E.S. Myra, Astrophys. J. 425, 195 (1994).
[6] A.B. Migdal, A.I. Cherenoutsan and I.N. Mishustin, Phys. Lett. B83, 158 (1979).
[7] M. Takahara and K. Sato, Phys. Lett. B156, 17 (1985).
[8] N. A. Gentile, M. B. Aufderheide, G. J. Mathews, F. D. Swesty and G. M. Fuller, Astrophys. J. 414, 701 (1993).
[9] N. K. Glendenning, Phys. Rev. D46, 1274 (1992).
[10] H. Müller and B. D. Serot, Phys. Rev. C52, 2072 (1995).
[11] H. J. Pirner, Prog. Part. Nucl. Phys. 29, 33, (1992).
[12] M. C. Birse, Prog. Part. Nucl. Phys. 25, 1 (1990).
[13] J. A. McGovern, Nucl. Phys. A533, 553 (1991).
[14] V. Barone and A. Drago, Nucl. Phys. A552, 479 (1993); A560, 1076 (1993); V. Barone, A. Drago and M. Fiolhais, Phys. Lett. B338, 433 (1994).
[15] A. Drago, M. Fiolhais, U. Tambini, Nucl. Phys. A609, 488 (1996).
[16] W. Broniowski, M. Čibeg, M. Kutschera and M. Rosina, Phys. Rev. D41 (1990) 285; A. Drago, A. Fiolhais and U. Tambini, Nucl. Phys. A588, 801 (1995).
[17] A. Drago, U. Tambini, M. Hjorth-Jensen, Phys. Lett. B380, 13 (1996).
[18] B. D. Serot and J. D. Walecka, Adv. Nucl. Phys. 16 (1986) 1.
[19] R. Knorren, M. Prakash and P. J. Ellis, Phys. Rev. C52, 3470 (1995).
[20] J. I. Kapusta, *Finite-temperature field theory*, Cambridge University Press, Cambridge 1989.
[21] S. H. Kahana, Ann. Rev. Nucl. Part. Sci. 39, 231 (1989).
[22] M. Prakash, I. Bombaci, M. Prakash, P. J. Ellis, J. M. Lattimer and R. Knorren, [nucl-th/9603042](http://arxiv.org/abs/nucl-th/9603042), to appear in Phys.Rep.