Self-quenching of nuclear spin dynamics in the central spin problem

Arne Brataas$^1$ and Emmanuel I. Rashba$^2$

$^1$Department of Physics, Norwegian University of Science and Technology, NO-7491 Trondheim, Norway
$^2$Department of Physics, Harvard University, Cambridge, Massachusetts 02138, USA

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We consider, in the framework of the central spin $s = 1/2$ model, driven dynamics of two electrons in a double quantum dot subject to hyperfine interaction with nuclear spins and spin-orbit coupling. The nuclear subsystem dynamically evolves in response to Landau-Zener singlet-triplet transitions of the electronic subsystem controlled by external gate voltages. Without noise and spin-orbit coupling, subsequent Landau-Zener transitions die out after about $10^4$ sweeps, the system self-quenches, and nuclear spins reach one of the numerous glassy dark states. We present an analytical model that captures this phenomenon. We also account for the multi-nuclear-species content of the dots and numerically determine the evolution of around $10^7$ nuclear spins in up to $2 \times 10^5$ Landau-Zener transitions. Without spin-orbit coupling, self-quenching is robust and sets in for arbitrary ratios of the nuclear spin precession times and the waiting time between Landau-Zener sweeps as well as under moderate noise. In the presence of spin-orbit coupling of a moderate magnitude, and when the waiting time is in resonance with the precession time of one of the nuclear species, the dynamical evolution of nuclear polarization results in stroboscopic screening of spin-orbit coupling. However, small deviations from the resonance or strong spin-orbit coupling destroy this screening. We suggest that the success of the feedback loop technique for building nuclear gradients is based on the effect of spin-orbit coupling.

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I. INTRODUCTION

Electrical operation of electron spins in semiconductor double quantum dots (DQD) is one of the central avenues of semiconductor spintronics [1] and quantum computing [2–5]. There are three basic types of electronic spin qubits: (i) the Loss-DiVincenzo [2] qubits operating single-electron spins, (ii) singlet-triplet qubits operating a two-electron system, and (iii) three-electron qubits [6]. The second type is the center of our attention. Most widely explored singlet-triplet DQD qubits [7] are based on GaAs [8,9] and InAs [10–12]. Both in GaAs and InAs, there are three species of nuclei possessing nonvanishing angular momenta, and the coupling between electron and nuclear spins (mostly through contact interaction) strongly influences electron-spin dynamics. Primarily, this coupling has a destructive effect causing electron-spin relaxation, and many theoretical studies have focused on the challenging problem of determining the relaxation rate of an electron spin interacting with about $N \approx 10^6$ nuclear spins [13–20]. The problem of an electron spin interacting with a bath of nuclear spins is known as the central spin problem.

However, a controllable nuclear spin polarization, acting as an effective magnetic field, can also become a resource for manipulating electron spins [21,22]. In particular, the difference (gradient) in the nuclear polarization of the left and right dots can be used for $\sigma_z$ rotations of a $S$-$T_0$ singlet-triplet qubit on the Bloch sphere [8]. Efficient control of a vast ensemble of nuclear spins is very challenging, and many analytical and numerical works have been carried out on this subject [23–31]. The principal experimental tool for polarizing nuclear spins and building gradients is based on driving a two-electron DQD electrically through the avoided crossing ($S$-$T_+$ anticrossing) of its singlet level $S$ and the $T_+$ component of its electronic triplet $T = (T_+, T_0, T_-)$. $T_+$ is the lowest energy triplet component because the electron $g$ factor is negative, $g < 0$, both in GaAs and InAs. The width of the anticrossing is controlled by hyperfine and spin-orbit [32] interactions. When the electron state changes from $S$ to $T_+$ or vice versa by passing through the $S$-$T_+$ anticrossing, and there is no spin-orbit coupling, up to one quantum of the angular momentum is transferred to the nuclear subsystem, and such transfer facilitates polarizing the nuclear bath by performing multiple passages.

Unfortunately, experimental data show that the nuclear polarization saturates at a rather low level, typically of about 1% [33]. The origin of this low saturation level remains unclear and constitutes the critical obstacle for achieving higher levels of nuclear spin polarization. We recently uncovered a mechanism of dynamic self-quenching which, in absence of spin-orbit (SO) coupling, results in fast suppression of the transverse nuclear polarization under stationary pumping [30]. This is caused by screening the random field of the initial nuclear spin fluctuation by the nuclear polarization produced through pumping and closing the anticrossing. This conclusion is in a qualitative agreement with the data of Refs. [25,31] in the strong magnetic field limit, and the states with vanishing transverse magnetization are known as “dark states.” Meanwhile, by applying feedback loops, experimenters managed to achieve considerable and controllable nuclear spin polarizations [34]. This poses a challenging question in which way closing the $S$-$T_+$ intersection due to the self-quenching mechanism could be avoided. Our data of Ref. [30] indicate that SO coupling changes the patterns of self-quenching dramatically, which implies that it is the spin-orbit coupling that might resolve the problem. The main goal of the current paper is to shed more light on the mechanisms controlling the transfer of angular momentum from the electron qubit to the nuclear bath. For this purpose, we solve numerically the equations describing coupled electron and nuclear spin dynamics for DQDs of a realistic size of more than $N \sim 10^6$ nuclear spins and a shape...
of two overlapping Gaussian distributions. These simulations follow up to $2 \times 10^5$ sweeps and unveil intimate patterns of transferring spin polarization from the electronic to the nuclear subsystem.

To get quantitative insight onto the long time dynamics of spin pumping by multiple passages across the $S$-$T_\alpha$ anticrossing, we restrict ourselves to the strong magnetic field regime when the Zeeman split-off $T_0$ and $T_\alpha$ components of the electron-spin triplet are well separated from the $S$ and $T_+\!$ states; hence transitions to these states are disregarded. Therefore, with a semiclassical description of nuclear spins, electrons form a two-level system, and passages across the $S$-$T_\alpha$ anticrossing are described by the Landau-Zener type theory \cite{35,36}. The detailed patterns depend on the shape on the pulses on the gates and the instantaneous nuclear configuration. In turn, during each Landau-Zener (LZ) sweep the nuclear configuration changes due to the direct transfer of the angular momentum and shake-up processes \cite{29,30}.

Between the LZ sweeps, this configuration changes because of the difference in the Larmor precession rates of different nuclear species. To follow the long term evolution, we solve the problem of the coupled dynamics self-consistently. From the mathematical point of view, we arrive at a central spin problem with a driven dynamics of the electron spin. Hence, beyond the application to the spin pumping problem, our results are of general interest for coupled dynamics of many body systems.

In this paper we prove, both analytically and numerically, that self-quenching into dark states is a generic property of the pseudospin $s = 1/2$ model in absence of SO coupling, and that self-quenching sets in after only about $10^8$ sweeps. We also demonstrate that this result stands under a moderate noise. However, the main focus of the paper is on the effect of SO coupling. Because the SO field is static while the hyperfine Overhauser field oscillates in time with the Larmor frequencies of nuclei, self-quenching cannot set in. Nevertheless, if the waiting time between LZ sweeps coincides with the Larmor period of one of the species, self-quenching sets in stroboscopically (as was demonstrated in our previous paper for a single-specie model \cite{30}). More specifically, the Overhauser field of the resonant specie screens the SO field during the LZ sweeps (whose duration is small compared with Larmor periods). Therefore, during the sweeps the electron and nuclear subsystems become decoupled. As distinct from self-quenching in absence of SO coupling, the stroboscopic self-quenching is fragile. Even a small deviation from the resonance, about 1%, destroys the delicate compensation of the SO and hyperfine contributions during the LZ sweeps. Moreover, we were able to observe the stroboscopic self-quenching only for moderate values of the SO coupling that do not exceed considerably the random fluctuations of the hyperfine field.

We conclude that it is the SO coupling that endows the nuclear subsystem with a long term dynamics under the stationary LZ pumping. Therefore, we suggest that SO coupling is critical for efficient operating of the feedback loops that require accumulation of large polarization gradients at the scale of about $10^8$ sweeps \cite{34}. Effect of SO coupling at a shorter time scale has been recently unveiled by Neder et al. \cite{37} by comparing with experimental data of Ref. \cite{38}.
nuclear polarization as described by Eq. (5) below. In turn, semiclassical dynamics of nuclear spins is driven by the Knight fields $\Delta_{jk}(t)$ arising from electron dynamics
\[ \hbar \frac{dI_{jk}}{dt} = \Delta_{jk} \times I_{jk}, \tag{1} \]
where the subindex $j \lambda$ denotes a nuclear specie $\lambda$ at a lattice site $j$. Assuming the time scale $T_{\text{LZ}}$ of the LZ sweeps is much shorter than the nuclear precession times $t_\lambda$ in the external magnetic field, the total effect of the time-dependent fields $\Delta_{jk}(t)$ on each nuclear spin can be integrated over the LZ sweep. Then the change of an individual nuclear spin during a sweep is
\[ \Delta I_{jk} = \Gamma_{jk} \times I_{jk}, \tag{2} \]
where $\Gamma_{jk}$ accounts for the effective magnetic field induced by the hyperfine interaction during the LZ sweep and depends on the configuration of all the nuclear spins before the sweep.

Landau-Zener sweeps are repeated many times. Between consecutive LZ sweeps, electrons are in the singlet state and do not interact with nuclear spins. During this waiting time $T_\text{w}$ between consecutive sweeps, nuclear spins precess in an external magnetic field $B$ applied along the $z$ direction. The changes of the nuclear spins between LZ sweeps are
\[
\Delta I_{jk} = \cos \phi_{jk} I_{jk} - \sin \phi_{jk} I_{jk}^z, \tag{3a}
\Delta I_{jk} = \cos \phi_{jk} I_{jk} + \sin \phi_{jk} I_{jk}^z, \tag{3b}
\Delta I_{jk} = 0, \tag{3c}
\]
where the superscripts $x$, $y$, and $z$ denote Cartesian components of the nuclear spins, the transverse phase changes are $\phi_{jk} = -2\pi T_\text{w}/t_\lambda$ in terms of the spin precession times $t_\lambda = 2\pi h/g_\lambda \mu_\lambda B$, where $g_\lambda = \mu_\lambda/I_\lambda$ is the $g$ factor for a nuclear specie $\lambda$, $\mu_\lambda$ is its magnetic moment, and $\mu_\lambda = 3.15 \times 10^{-6} \text{ eV/T}$ is the nuclear magneton.

We also model the influence of noise by adding phenomenologically a random magnetic field along the $z$ direction for each nuclear spin so that the resulting effects are changeable in the phase-dependent factors in Eq. (3) as $\langle \cos \phi_{jk} \rangle_n = \cos \phi_{jk} \sin(2\pi T_\text{w}/t_\lambda)/2(\pi T_\text{w}/t_\lambda)$, and similarly for $\langle \sin \phi_{jk} \rangle_n$. Therefore, this model of transverse noise leads to a semiclassical dephasing of the transverse components of nuclear spins on the time scale $\tau$.

Let us next review how electronic Landau-Zener sweeps influence nuclear spins via $\Gamma_{jk}$. \cite{29}. The hyperfine electron-nuclear interaction is
\[ H_{\text{hm}} = V_i \sum_\lambda A_\lambda \sum_{j \in \lambda} \sum_{m=1,2} \delta(R_{jk}) (I_{jk} - m \sigma(m)) I_{jk}, \tag{4} \]
where $\sigma(m) = \sigma(m)/2$ are the electron-spin operators in terms of the vector of Pauli matrices $\sigma(m)$ for each electron $m = (1,2)$, $I_{jk}$ are the nuclear spin operators, $A_\lambda$ is the electron-nuclear coupling constant for a specie $\lambda$, $v_\lambda$ runs over all lattice sites populated by the specie $\lambda$, and $V_i$ is the volume per single nuclear spin. We consider GaAs or InAs quantum dots below; hyperfine coupling parameters for them can be found in Sec. VI.

Assuming that gate voltages keep the system close to the singlet–triplet $T_\text{s}$ transition, the effective Hamiltonian describing the electron qubit is
\[ H^{(S,T_\text{s})} = \left( \frac{\epsilon_S}{v^-} - \frac{\epsilon_T}{v^+} - \eta \right), \tag{5} \]
where $\epsilon_S$ is the singlet energy and $\epsilon_T$ is the triplet $T_\text{s}$ energy in the external magnetic field $B = B z$ when nuclear spins are unpolarized. By retaining only $S$ and $T_\text{s}$ states, the problem is reduced to a $1/2$ pseudospin problem, and we apply the term the central spin problem in this sense.

The energies $\epsilon_S$ and $\epsilon_T$ are controlled by the gate voltages. The off-diagonal components $v^\pm = v^+_\text{s} + v^+_{\text{SO}}$, coupling the singlet $S$ and triplet $T_\text{s}$ states, contain contributions from nuclear spins
\[ v^\pm_n = V_i \sum_\lambda A_\lambda \sum_{j \in \lambda} \rho_{jk} I_{jk}^\pm, \tag{6} \]
and SO coupling $v^\pm_{\text{SO}}$ \cite{26,39}. When nuclear spins are polarized, the energy of the triplet state is affected by the Overhauser shift
\[ \eta = -V_i \sum_\lambda A_\lambda \sum_{j \in \lambda} \xi_{jk} I_{jk}. \tag{7} \]
The singlet-triplet electron-nuclear couplings are
\[ \rho_{jk} = \int dl \psi_S^*(r,R_{jk}) \psi_T(r,R_{jk}), \tag{8} \]
and the electron-nuclear couplings in the $T_\text{s}$ state are
\[ \xi_{jk} = \int dl \left| \psi_T(r,R_{jk}) \right|^2, \tag{9} \]
where $\psi_S$ ($\psi_T$) is the orbital part of the singlet (triplet) wave function. Beyond the $2 \times 2$ $S$-$T_\text{s}$ model, it is possible to define a hyperfine term that determines the singlet $S$–triplet $T_\text{0}$ coupling (as in Eq. 4 in Ref. \cite{29}), but it is not at the center of our attention and will not be discussed here. The effect of the electron spin $T_\text{0}$ and $T_\text{s}$ components is critical for the development of nuclear polarization gradients and has been investigated in Refs. \cite{25,31}.

In terms of these parameters, the changes of the nuclear spins $\Delta I_{jk} = \Gamma_{jk} \times I_{jk}$ during a Landau-Zener sweep are determined by coefficients
\[
\Gamma_{jk}^{(s)} = -V_i A_\lambda \rho_{jk} (P v^- + Q v^+)/(2v^2), \tag{10a}
\Gamma_{jk}^{(T_\text{s})} = V_i A_\lambda \rho_{jk} (P v^- - Q v^+)/(2v^2), \tag{10b}
\Gamma_{jk}^{(\text{SO})} = V_i A_\lambda \xi_{jk} R/(2v), \tag{10c}
\]
with $v^2 = |v_\text{s}|^2 = (v^2_\text{s} + v^2_{\text{SO}})/2$. In these expressions, $0 \leq P \leq 1$ is the $S$-$T_\text{s}$ transition probability, a real number $Q$ is the shake-up parameter defined via \cite{29}
\[ P + i Q = -i 2v \int_{-T_{\lambda z}}^{T_{\lambda z}} \frac{dt}{\hbar} c^*_S(t) e^{-i T_{\lambda z} t}, \tag{11} \]
in terms of the singlet (triplet) amplitude \( c_S(t) \) [\( c_T(t) \)], and
\[
R = 2v \int_{-T_{1Z}}^{T_{1Z}} dt |c_{T}(t)|^2 / \hbar
\]
(12)
accounts for the Overhauser shift due to the triplet \( T_{1+} \)
component of the electron state during the interval \((-T_{1Z}, T_{1Z})\).
In the absence of SO coupling, \( v_{SO} = 0 \), the change \( \Delta I^z \) of
the total angular momentum of nuclei \( I = \sum_j I_j^z \) during a
single sweep equals \( \Delta I^z = -P \), as follows from the angular
momentum conservation [29].

Using the amplitudes [\( c_S(t), c_T(t) \)] found from solving
the time-dependent Schrödinger equation with the Hamiltonian
\( H^{STz} \) of Eq. (5) in combination with the dynamical equations
for nuclear spins of Eq. (2) makes our approach completely
self-consistent.

We assume that electrons are loaded into the singlet (0,0)
state with energies far away from the S-\( T_{1+} \) anticrossing.
After loading electrons, gate voltages are changed to bring
the system closer to the level anticrossing, and this change is
performed fast enough to keep the system in the singlet state
[40]. From there on, an LZ sweep brings the system through
the anticrossing. After the slow sweep, the system is moved
back to the recharging point where it exchanges electrons
with the reservoir. This back motion is fast at the scale of
the narrow anticrossing and therefore does not influence
the nuclear spin subsystem, but slow at the scale of the electron
Zeeman splitting to keep the system inside the S-\( T_{1+} \) subspace.

IV. SIMPLE MODEL WITH ANALYTICAL SOLUTION

Let us first present a simplified model that can be solved
analytically and that manifests basic features of self-quenching
[30] in the absence of SO coupling, \( v_{SO} = 0 \). Different versions
of this “box” or “giant spin” model were applied to various
problems; see Refs. [25,26,41,42]. Subsequently, we will in
Sec. V outline a more complex and extensive numerical
procedure and discuss numerical results for realistic models
in Sec. VI. Restricting ourselves to a single nuclear specie,
we simplify the system by modeling it as a box inside which
the electron wave functions are independent on position, all
nuclei possess the same Larmor frequency, and all hyperfine
coupling constants \( A_\alpha \) and electron-nuclei couplings \( \rho_{\alpha j} \)
are equal, \( A_\alpha = \tilde{A} \) and \( \rho_{\alpha j} = \tilde{\rho} \); typically, \( \tilde{A} \approx 10^{-4} \text{ eV} \). Then
Eqs. (6) and (8) simplify to \( v^0 = A_0 I^0, \alpha = (+, -, \cdot) \)
with \( A_0 = V_i \tilde{A} \tilde{\rho} \sim \tilde{A} / N \). Here \( N \) is the number of nuclei in the
box, and \( I^0 = \sum_j I_j^0 \) are components of the “giant” collective
angular momentum of nuclei. In the framework of this model,
nuclear spin precession in the Zeeman and Overhauser fields
does not influence the coupled electron-nuclear spin dynamics,
\( \Delta^z = 0 \). With these assumptions, Eq. (1) becomes
\[
\begin{align*}
\frac{dI^z}{dt} &= -\frac{i}{\hbar} \Delta^+ I^z, \\
\frac{dI^z}{dt} &= \frac{i}{\hbar} \Delta^- I^z, \\
\end{align*}
\]
(13)
For LZ pulses, the energy level difference changes linearly
with \( t \), \( \epsilon_S(t) - \epsilon_T(t) = \beta^2 t / \hbar \), for \(-T_{1Z} \leq t \leq T_{1Z} \), and
the dynamics of the qubit is controlled by a dimensionless
parameter \( \gamma = v^+ v^- / \beta^2 \). The equation of motion for \( \gamma 
following from (13) is
\[
\frac{d\gamma}{dt} = -\frac{A_0^2}{2\beta^2} \frac{d}{dt}(I^z)^2.
\]
(14)

During each sweep, \( I^z \) changes by \( \Delta I^z = -P \) and \( \gamma 
changes by \( \Delta \gamma = (A_0^2/\beta^2)P \). Precession of the collective
nuclear spin \( I \) in the external magnetic field during the
waiting times between sweeps changes neither \( I^z \) nor \( \gamma \) and
is disregarded. The discrete number of sweeps \( n \) can be
considered as a continuous variable since the changes \( \Delta \gamma 
and \( \Delta I^z \) during a single sweep are small as compared to \( \gamma 
and \( I^z \). We then arrive at the differential equations that determine
the evolution of the \( \gamma(n) \) and \( I^z(n) \):
\[
\begin{align*}
\frac{d\gamma}{dn} &= \frac{A_0^2 P}{\beta^2} I^z, \\
\frac{dI^z}{dn} &= -P.
\end{align*}
\]
(15a)
(15b)

This central result for the simple model clarifies the
different modes of self-quenching. The evolution of the LZ
parameter \( \gamma \) that controls the LZ probability \( P \) differs in
two scenarios that manifest themselves for opposite signs of
\( I^z \). (i) When \( I^z \) is initially negative, it continues to decrease
(becoming more negative) and magnitudes of both \( \gamma \) and
the LZ probability \( P \) decrease; hence the process slows down.
Finally, self-quenching sets in exponentially; see Eq. (19)
below. (ii) When \( I^z \) is initially positive, \( \gamma \) first increases so
that the LZ probability \( P \) becomes larger. However, since
\( I^z \) only can be reduced, it eventually becomes negative and
self-quenching of scenario (i) sets in. So, self-quenching
ultimately sets in generically independent on the original
sign of \( I^z \).

We can get a more detailed insight into the self-quenching
dynamics by using the first integral of Eq. (14) [43],
\[
\gamma = -(A_0^2 / 2\beta^2)(I^z)^2 + \gamma_0,
\]
where \( \gamma_0 \) is an integration constant that depends on the initial
values of \( \gamma \) and \( I^z \); \( \gamma_0 \) and \( I^z \) are defined so that \( \gamma \leq \gamma_0 \), and \( \gamma_0 \geq 0 \)
because \( \gamma \geq 0 \) by definition. Therefore,
\[
I^z = \pm \sqrt{2(\beta / A_0)\sqrt{\gamma_0 - \gamma}}
\]
(17)
and
\[
\frac{d\gamma}{dn} = \pm \sqrt{2A_0(\beta / A_0)\gamma_0 - \gamma} P(\gamma).
\]
(18)

In scenario (i), when \( I^z < 0 \), the minus sign should be chosen
in Eqs. (17) and (18), and \( \gamma(n) \) decreases monotonically.
In scenario (ii), when \( I^z > 0 \), the dynamics first follows the
plus branches of Eqs. (17) and (18), and \( \gamma(n) \) increases until
it reaches its maximum value \( \gamma = \gamma_0 \). At this point, \( I^z(n) \)
vanishes, changes sign, and continues to decrease as follows
from Eq. (16) and Eq. (15b) [because \( P(\gamma_0) > 0 \)]. At the same
point, the signs in Eqs. (17) and (18) switch from plus to minus,
and afterwards \( \gamma(n) \) decreases monotonically as follows from
Eq. (15a).
self-quenching sets in for arbitrary initial conditions. Of SO coupling, the large-
universal exponent. The rate of decay increases with decreasing
original fluctuation includes $N_{\text{LZ}}$ in units of $\beta/\sqrt{A_0}$. (a) Initial nuclear polarization is negative, $I_i < 0$. (b) Initial nuclear polarization is positive, $I_i > 0$. In the plots, the lower bounds of $\gamma_i$ and $\gamma_f$ were chosen to be 0.01 to cut off logarithmic singularities in $n(\gamma)$ developing because of the $P_{\text{LZ}}(\gamma)$ factor in Eq. (18). Curves $n_\infty(\gamma_i)$ in front panels show the number of sweeps before the self-quenching sets in. See text for details.

Equation (19) describes an exponential decay with a nonuni-
versal exponent. The rate of decay increases with decreasing $\beta$, when sweeps become more adiabatic. Therefore, in absence of SO coupling, the large-$n$ behavior of $\gamma(n)$ is exponential and self-quenching sets in for arbitrary initial conditions.

Let us make a rough estimate of the number of sweeps $n_\infty$ before self-quenching sets in based on Eq. (19). A typical original fluctuation includes $N^{1/2}$ spins; hence $v \sim A_0 \sqrt{N}$. For LZ pulses with an amplitude of about $v$ and duration of about $h/v$, $\beta$ is about $\sim v$. Therefore, $n_\infty \sim \beta/A_0 \sim \sqrt{N}$, i.e., about the number of nuclear spins in a typical fluctuation. The dependence of $n_\infty$ on $\beta$ demonstrates the effect of the sweep duration $T_{\text{LZ}}$; $n_\infty$ is smaller for longer sweeps. A similar estimate for the length $\Delta n$ of the exponential tail in Eq. (19), with $2\pi \sqrt{2}/v \approx 10$, results in $\Delta n \sim n_\infty/10$, i.e., it is shorter than $n_\infty$ by a numerical factor.

More detailed estimates for both regimes require specific assumptions about the shape and duration of sweeps. For sufficiently long sweeps, $P_{\text{LZ}}(\gamma)$ can be used for $P(\gamma)$ and Eq. (18) can be integrated. The number of sweeps $n = n(\gamma_i, \gamma_f)$ in units of $\beta/\sqrt{2A_0}$, between the initial $\gamma_i$ and final $\gamma_f$ values of $\gamma$ is plotted in Fig. 1 for two modes; the value of $\gamma_0$ has been chosen equal to $\gamma_0 = 2$. Figure 1(a) is plotted for $I_i < 0$ and Fig. 1(b) for $I_i > 0$. Front sections of $n(\gamma_i, \gamma_f)$ surfaces by $\gamma_f = 0$ planes demonstrate $n_\infty(\gamma_i)$, the number of sweeps before self-quenching. For $I_i < 0$, the curve increases fast with $\gamma_i$ and reaches its maximum value at $\gamma = \gamma_0$. It is achieved at a ridge at the $n(\gamma, \gamma_f)$ surface that originates from the square-root singularity in the $dn/d\gamma$ dependence and is well seen in Fig. 1(a). For $I_i > 0$, the $n_\infty(\gamma_i)$ dependence is much slower and becomes fast only near $\gamma \approx \gamma_0$. In both cases, $n_\infty \sim \beta/A_0$, in agreement with the previous estimate.

Therefore, the model not only provides analytical justifi-
cation of the self-quenching phenomenon found numerically in Ref. [30] for systems without SO coupling but relates, for single-specie systems, two modes of behavior (monotonic and nonmonotonic) to the difference in initial conditions. It is the first analytical solution of the central spin problem (i) describing dynamical evolution of a pumped system into a “dark state” [25,44] and (ii) establishing a connection between the initial and final states of the system.

V. NUMERICAL PROCEDURE

During a sweep, the difference in the singlet and triplet energies $\epsilon_S = \epsilon_{T_0}$ varies linearly in time within the sweeping interval $-T_{LZ} \leq t \leq T_{LZ}$. We impose no restrictions on the relative magnitude of the sweep duration $2T_{LZ}$ and the inverse $S-T_+$ coupling $\beta/v$, but, as stated above, $T_{LZ}$ is long as compared to the inverse electron Zeeman energy. Furthermore, it is assumed that the variation of the energies of both the upper and lower spectrum branches is symmetric with respect to the $S-T_+$ anticrossing for the first transition when the initial position of the $T_+$ level is $\eta = \eta_i$. We denote the amplitude of the change in the energy difference between the singlet and triplet energies as $\epsilon_{\text{max}}$. In other words, we use

$$\epsilon_S(t) = \epsilon_{\text{max}}t/2T_{LZ},$$  \hspace{1cm} (20a)

$$\epsilon_{T_0}(t) = \epsilon_{\text{max}}(t/2T_{LZ} - (\eta - \eta_i))$$  \hspace{1cm} (20b)

in Eq. (5) [45]. Note that, as a result of the dynamical nuclear polarization, LZ sweeps become asymmetric with respect to the anticrossing point because of the changing Overhauser shift $\eta$. There is no longer any traditional LZ passage whenever $|\eta - \eta_i| > \epsilon_{\text{max}}$, i.e., after the anticrossing point passes across one of the ends of the sweeping interval. This naturally implies a slowdown in accumulating dynamical nuclear polarization. Maintaining the LZ passages requires additional feedback mechanisms by changing the energy level difference, which we introduce below by shifting the edges of the integration interval.

Assuming $|\eta - \eta_i| \leq \epsilon_{\text{max}}$, the $S$ and $T_+$ states are degenerate at $t^* = -T_{LZ}(\eta - \eta_i)/2\epsilon_{\text{max}}$. To avoid trivial quenching due to the shift in $\eta$ caused by the accumulating polarization far away from the degeneracy point, the electronic energies were renormalized after every 100 sweeps keeping $\eta - \eta_i \approx 0$ and ensuring the $S-T_+$ anticrossing be passed during all LZ sweeps, $-T_{LZ} < t^* < T_{LZ}$. As a result, the center of the sweep was permanently kept close to the anticrossing point. Such a regime can be achieved experimentally by applying appropriate feedback loops.

In order to relate the properties of the sweeps to the conventional notations of the LZ transition probabilities in the limit $T_{LZ} \to \infty$, it is helpful to introduce the dimensionless initial $\tau_i = -T_{LZ}[1 + t^*/T_{LZ}]\beta/h$ and final $\tau_f = T_{LZ}[1 - t^*/T_{LZ}]\beta/h$ times, where $\beta = (\epsilon_{\text{max}}h/T_{LZ})^{1/2}$. The Landau-Zener parameter is $\gamma = (v/\beta)^2$. When $-\tau_i \gg \sqrt{\gamma}$
\[ \psi_S(1,2) = \cos \nu \psi_R(1) \psi_R(2) + \sin \nu [\psi_L(1) \psi_R(2) + \psi_L(2) \psi_R(1)]/\sqrt{2} \] (21)

and the triplet part is

\[ \psi_T(1,2) = [\psi_L(1) \psi_R(2) - \psi_L(2) \psi_R(1)]/\sqrt{2}, \] (22)

where \( \psi_L \) (\( \psi_R \)) denotes the wave function in the left (right) dot. The angle \( \nu \) depends on the electron Zeeman energy. We assume the electrons are in the lowest orbital harmonic oscillator state so that the wave functions are

\[ \psi(x,y,z) = \frac{\exp[-(x^2 + y^2)/l^2 - z^2/w^2]}{\sqrt{wl^2/\pi/2}w^{3/2}}, \] (23)

where \( l \) is the lateral size of each dot and \( w \) is its height. For two dots that are separated by a distance \( d \) we form an orthonormal basis set based on the functions \( \psi(x-d/2,y,z) \) and \( \psi(x + d/2,y,z) \), that defines the above \( \psi_L \) and \( \psi_R \). While both dots are chosen in the same size, hyperfine couplings in them differ due to the dependence of \( \rho_{j\lambda} \) of Eq. (8) on the mixing angle \( \nu \).

We solve the nuclear dynamics numerically by using Mathematica 9. First, we include all nuclear spins that are in the vicinity of the double quantum dot and satisfy the condition that the electron-nuclear coupling constants \( \zeta_{j\lambda} \geq \kappa \max(|\zeta_{j\lambda}|) \), where \( \kappa \) is a small parameter. We checked, by changing \( \kappa \), that our results converged and have found that reducing \( \kappa \) below \( \kappa = 0.01 \) does not produce any visible changes in the plots we present. Initial configurations of the nuclear spin directions are chosen by a pseudorandom number generator. The initial nuclear spin configuration determines the 2 × 2 electron S–T+ Hamiltonian. We solve the time-dependent 2 × 2 differential equation numerically for linear LZ sweeps and compute the probability \( P \), the shake-up parameter \( Q \), and the time-integrated effect of the Overhauser shift of the triplet state \( T_+ \) described by the parameter \( R \). We then let the nuclear spins precess in the external magnetic field and a random noise field before the next LZ sweep takes place. We record all electron singlet-triplet coupling parameters as a function of the sweep number, as well as \( P \), \( Q \), and the change in the total magnetization.

We choose realistic parameters for a double quantum dot of a height \( w = 3 \) Å, size \( l = 50 \) Å, and distance \( d = 100 \) Å. We consider an external magnetic field of \( B = 10 \) mT. Using a cutoff \( \kappa = 0.01 \) implies that we explicitly include in our calculations around ten million spins. A single such calculation takes about one week on our state-of-the-art workstation. We have studied the evolution of the nuclear spin dynamics during up to 2 × 10⁵ LZ sweeps for 10² spins and used various pseudorandom initial configurations of nuclear spins. While the detailed pattern of the dynamics depends on the initial conditions, all basic regularities were exactly the same in all simulations. Hence our results are representative for the generic behavior of a pumped electron-nuclear system.
A. Two-specie systems: InAs

Let us first consider InAs which effectively consists of two species because the parameters of $^{113}$In and $^{115}$In practically coincide. Therefore, species $^{113}$In and $^{115}$In behave as a single specie and $^{75}$As as a second specie. We first demonstrate that, in absence of SO coupling, the dynamical evolution of nuclear spins in InAs is similar to the dynamics in GaAs reported earlier [30]. In all our InAs simulations, we start in the same initial (pseudorandom) configuration of the nuclear spins. We have checked that similar results are obtained when we start in several other configurations. In all our simulations in this section, the waiting time between LZ sweeps equals the precession time of specie $^{75}$As in the external magnetic field, $T_w = \tau_{75\text{As}}$.

We start by presenting results for a system without spin-orbit coupling, $v_{SO} = 0$, to prove that self-quenching occurs and investigate its stability with respect to nuclear noise. Figure 2(a) shows the evolution of the magnitude of the singlet-triplet coupling $|v_n^\pm|$ with increasing number of sweeps $n$. For a sweep duration of $T_{LZ} = 40$ ns, the initial LZ probability for the first few sweeps is $P \sim 0.5$, see Fig. 3, and the singlet-triplet coupling is self-quenched already after about 20000 sweeps. The number of sweeps $n$ required to reach self-quenching is about the same as for GaAs [30]. The appearance of several peaks of $v_n^\pm$ in the range of 5000–20000 sweeps before the self-quenching sets in is typical of multispecie systems. In contrast to single-specie systems, and especially the model of Sec. IV, in multispecie systems final self-quenching is usually preceded by partial self-quenches followed by revivals. We attribute this behavior to competition between subsystems with the different Zeeman precession times. As seen in Fig. 3, in each peak of $|v_n^\pm|$ the $S$-$T$ transition probability $P$ increases strongly, near it $I^2$ shows a steplike behavior (not shown), and accompanying peaks of $Q$ indicate massive shake-ups which flop many nuclear spins per LZ sweep. The model of Sec. IV that only deals with the total magnetization $I^2$ does not describe such events and provides a smoothened picture of the nuclear spin evolution.

Transverse noise transforms the dynamical evolution into a dissipative one. Figure 2 shows the effect of the increase of the level of noise from (a) through (b) to (c). In Fig. 2(a), the noise correlation time is of the order of the self-quenching time $\tau/t_{75\text{As}} = 10^4$. In this case, transverse noise only modestly perturbs the nuclear spin evolution as compared to the nondissipative regime (simulated and analyzed, but not shown). Note the presence of a long slightly visible tail with irregular oscillations along it. In contrast, Figs. 2(b) and 2(c) demonstrate that when the transverse noise correlation time $\tau$ is shorter than the typical self-quenching set-in time in un-noisy systems, self-quenching is suppressed and eventually does not happen at all; in particular, Fig. 2(b) demonstrates a possibility of revivals. We conclude that for high noise levels the chaotic evolution of the nuclear spins persists, but the magnitudes of the peaks of $|v_n^\pm|$ seem to gradually decrease in time.

Figure 2(a) suggests a glassy behavior of the nuclear system with an extensive manifold of dark states separated by low barriers. In the absence of noise, repeated LZ sweeps cause the system to end in one of the dark states (usually after passing through several peaks of $|v_n^\pm|$). Weak noise produces slow diffusion between adjacent dark states across low saddle points. During this diffusion, the magnetization $I^2$ changes only slightly. With increasing noise, the system experiences revivals as seen in Fig. 2(b) as a sharp peak in $|v_n^\pm|$. During such events $P$ increases strongly, $I^2$ shows steplike behavior, and peaks in $Q$ (not shown) indicate massive shake-ups, similar to the patterns discussed as applied to Fig. 2(a) above.

We demonstrated earlier that SO coupling is screened stroboscopically in a single-specie system [30]. Next, we will demonstrate that SO coupling can be screened stroboscopically also in multispecie systems, and investigate this phenomenon in more detail. Figure 4 shows simulations of the singlet-triplet coupling $v_n^\pm$ for $v_{SO} = 62$ neV and three
FIG. 3. (Color online) (a) Landau-Zener transition probability $P$ as function of the sweep number $n$ for a double quantum dot with the nuclear parameters of InAs in absence of SO coupling. (b) Shake-up parameter $Q$ as a function of sweep number $n$. The parameters are as in Fig. 2(a).

LZ sweep durations $T_{LZ}$. In comparison, the straight black line indicates the value of the spin-orbit coupling $v_{SO} = 62$ neV (which is independent of the sweep number $n$). We see that, in all these simulations, the spin-orbit coupling eventually becomes screened so that all the colored lines approach the black line which implies that $|v_{n}^{\pm}| = |v_{SO}|$. For longer LZ sweep durations, oscillations of $v_{n}^{\pm}$ are more rapid, but screening eventually occurs faster because nuclear spins are more strongly affected during each sweep.

Screening of the SO coupling even in multispecie systems seems counterintuitive at first glance. Indeed, the spin-orbit coupling $v_{SO}$ is static while the transverse components of the nuclear spins contributing to $v_{n}^{\pm}$ precess in time. In InAs, two nuclear species $^{113}$In and $^{115}$In precess at the same frequency and behave effectively as a single spin specie, whereas the third spin specie, $^{75}$As, precesses at a different frequency. So, while screening indicates that the magnitude of the singlet-triplet coupling $v_{n}^{\pm}$ remains finite, it must inevitably precess in time. Therefore, it cannot compensate the spin-orbit coupling $v_{SO}$ at all instants of time. The screening we observe is only possible because the waiting time $T_{w}$ is exactly equal to the precession time of the $^{75}$As specie, $T_{w} = t_{\nu_{As}}$. The data used in our plots of $|v_{n}^{\pm}|$ were taken at exact multiples of the waiting time, which was equal to the precession time of the $^{75}$As specie. Therefore, the self-quenching that manifests itself in Fig. 4 is a stroboscopic self-quenching.

Stroboscopic self-quenching can be understood in the following way. The dynamical evolution of nuclear spins causes self-quenching of the sum of the contributions from the transverse components of species $^{113}$In and $^{115}$In (that are out of resonance with the pumping period $T_{w}$, hence their contribution to $v_{n}^{\pm}$ vanishes). In contrast, the contribution from the specie $^{75}$As to $v_{n}^{\pm}$ exactly compensates the spin-orbit coupling $v_{SO}^{\pm}$ at every time instant when a LZ sweep happens. In other words, the matrix elements $v_{n}^{\pm}(t)$ change in time harmonically with the amplitude $v_{SO}^{\pm}$ and a period $t_{\nu_{As}}$:

$$v_{n}^{\pm}(t) = v_{SO}^{\pm} \cos \left(2\pi t / t_{\nu_{As}} \right).$$

This generalizes our previous findings of the screening of SO coupling in single-specie systems [30]. For a single-specie imitation of GaAs, we found that the SO coupling was screened in such a way that the matrix element changed harmonically with the amplitude $v_{SO}$ and a period $t_{\nu_{Ga}}$:

$$v_{n}^{\pm}(t) = v_{SO}^{\pm} \cos \left(2\pi t / t_{\nu_{Ga}} \right).$$

Let us now demonstrate explicitly that, when self-quenching sets in, the sum of the contributions from the transverse components of $^{113}$In and $^{115}$In to $v_{n}^{\pm}$ vanishes while the contribution from $^{75}$As equals $v_{SO}$. We show in Fig. 5(a) the contribution from $^{113}$In and $^{114}$In to $|v_{n}^{\pm}|$ as a function of the number of sweeps $n$. Clearly, it vanishes for large $n$. On the other hand, $^{75}$As whose nuclear precession time equals the waiting time $T_{w}$, makes a contribution to $|v_{n}^{\pm}|$ that exactly compensates $v_{SO}$ at all integers of $T_{w}$; see Fig. 5(b). Hence Eq. (24) is satisfied.

We note that the contributions from $^{113}$In and $^{115}$In to $v_{n}^{\pm}$ vanish not only stroboscopically but identically, at each instant of time (not shown). We have also checked that, in systems without spin-orbit coupling, self-quenching sets in for all species and for an arbitrary ratio between $T_{w}$ and the precession times of the species (not shown). For three-specie systems the last statement is proven below; see Sec. VI.B.

Now we will illustrate that stroboscopic screening of SO coupling can be practically achieved only for small and moderate magnitudes of $v_{SO}$. Since stroboscopic screening implies that the contribution from $^{75}$As to $v_{n}^{\pm}$ compensates $v_{SO}$ while the combined contribution from species $^{113}$In and $^{115}$In...
vanishes, we show in Fig. 6 the evolution of the contribution of
75As to $v_n^\pm$ as a function of sweep number for three values of
spin-orbit coupling $v_{SO} = 31, 62,$ and 91 neV [50]. While all
the results in Fig. 6 were found for the same value of $T_{LZ}$ and
the same initial conditions, screening sets in at $n \approx 75\,000$ for
spin-orbit coupling $v_{SO} = 31$ neV, is delayed to $n \approx 125\,000$ for $v_{SO} = 62$ neV,
and is far from complete even at $n = 200\,000$ for $v_{SO} = 93$ neV. These data suggest that stroboscopic self-quenching sets in
when $v_{SO} \lesssim v_n^0$, where $v_n^0 \approx A/\sqrt{N}$ is a typical
fluctuation of the Overhauser field, and cannot be practically achieved for $v_{SO} \gtrsim v_n^0$; see estimates of the magnitude of the
spin-orbit coupling in the Appendix. This criterion resembles
the criterion of the phase transition of Ref. [26].

One should keep in mind that, with the interval between
LZ pulses of about 1 $\mu$s, a set of $n \sim 10^6$ pulses takes about
1 s which is a typical scale of nuclear spin diffusion [51], which is not taken into account in the above considerations. We expect, but have not checked, that inhomogeneity of
magnetic field should have a detrimental effect on stroboscopic
self-quenching. Therefore, we conclude that stroboscopic quenching of SO coupling is less generic and more fragile
than self-quenching in systems without spin-orbit coupling.

We estimate in the Appendix that SO coupling is weaker or
comparable to (stronger than) the typical nuclear polarization
induced singlet-triplet coupling in GaAs (InAs). As a conse-
quence, we expect the SO coupling might be stroboscopically
screened in GaAs systems, but that stroboscopic screening is improbable in InAs systems.

B. Three-specie systems: GaAs

In this section, we present results for GaAs that complete the
picture of the generic nature of self-quenching in multispecie systems. Furthermore, we show that screening of the SO
coupling requires that the waiting time $T_w$ is in resonance
with the precession time of one of the nuclear species. When
the resonance condition is not satisfied, screening of the SO
coupling is partial and irregular.

In Sec. VI A and in Ref. [30], self-quenching in multispecie
systems in absence of SO coupling was demonstrated only
under the conditions when the waiting time $T_w$ was in resonance with the precession time $T_{75As}$ of the 75As specie,
$T_w = T_{75As}$. We demonstrate here that, while self-quenching
is generic and independent of the waiting time, the evolution
towards the self-quenched states depends on the waiting time.

To this end, we plot in Fig. 7 the evolution of the singlet-
triplet coupling $v_n^\pm$ for two different values of the waiting time
$T_w$. Figure 7(a) displays results of simulations for the resonant
case when $T_w = T_{75As}$, in which pronounced oscillations are
distinctly seen. For $n \gtrsim 3000$, the plot consists of five branches
that reflect coupled dynamics of three species. In contrast, in
the absence of the resonance, Fig. 7(b), the evolution is
chaotic. Nevertheless, self-quenching sets in in both cases and,
what is most remarkable, at the same time scale of $n \approx 10^4$.
Remarkably, the processes of Figs. 7(a) and 7(b) ended in
states with the same $I^z$ (not shown). While the set of dark
states is vast (as follows from our discussion in Sec. IV),
this observation indicates that the number of strong attraction
centers in which self-quenching ends is more scant.

We conclude that self-quenching in systems without spin-
orbit coupling is generic and robust, at least in the framework
of $S-T+$ scheme.

We checked that not only does the total matrix element
$v_n^\pm$ vanish, but also the matrix elements for all of three species
contributiong to it. Because between the LZ sweeps the electron

FIG. 6. (Color online) Contribution of the specie 75As to the
singlet-triplet coupling $v_n^\pm$ as a function of sweep number $n$ for
different values of $v_{SO}$: 31 neV (red curve), 62 neV (black curve),
and 91 neV (blue curve). Duration of LZ pulses $T_{LZ} = 40$ ns. Waiting
time between consecutive LZ pulses equals the precession time of
75As, $T_w = T_{75As}$. Nuclear parameters of InAs.
The critical sensitivity of stroboscopic self-quenching to small deviations from resonance looks indicative of a chaotic behavior of the system [52]. This is not surprising because the system of integro-differential equations of Eq. (1) is highly nonlinear because the coefficients \( \Delta_{j\lambda} \) depend through Eqs. (10) on the electronic amplitudes \( c_{\lambda}, c_{\rho} \) that, in turn, depend on all nuclear angular momenta \( I_{j\lambda} \). In this context, we speculate that a strong revival of all black curves in Fig. 9 near \( n \approx 15000 \) where all red curves saturate, and the return of black curves close to their initial values near \( n \approx 28000 \), is reminiscent of the strange attractor pattern [52]. These signatures of chaotic nuclear dynamic in SO coupled systems require a more detailed study.

We conclude that stroboscopic screening of the SO coupling is not a robust phenomenon.

While the above simulations are focused on the large \( n \) region, we mention that commensurability oscillations in the polarization accumulation per sweep were observed experimentally [38] and described theoretically [37] in the small \( n \) region, \( n \lesssim 10^3 \).

In addition to the regular investigation of the transverse magnetization, we also followed the time dependence of the longitudinal magnetization \( v_n^z = V_{j} \sum_{\lambda} \rho_{j\lambda} \sum_{j'\lambda'} I_{j'\lambda'} \). In presence of SO coupling, it shows an oscillating sign-alternating behavior, and we were unable to detect any signatures of its accumulation.

Summarizing the results of Secs. VI A and VI B, we conclude that SO coupling eliminates self-quenching and causes the nuclei of a pumped system to exhibit a persistent irregular dynamics. We speculate that this phenomenon is closely related to the feedback loop technique for building controllable nuclear gradients which is inherently based on employing such a dynamics [34]. Indeed, any long-term control of the nuclear ensemble by alternating \( S \to T_+ \) and \( T_+ \to S \) sweeps is impossible after the self-quenching set-in time that is of a millisecond scale in absence of SO coupling.

Our data, especially Fig. 8, suggest that near the resonance nonresonant case, and the contributions from \( ^{75}\text{As} \) do not screen the SO coupling.

FIG. 8. (Color online) Transverse nuclear polarization as a function of sweep number \( n \) for a GaAs double quantum dot with \( v_{\text{SO}} = 8 \text{ neV} \). The duration of LZ pulses \( T_{\text{LZ}} = 80 \text{ ns} \). The red curve shows results for the waiting time in resonance with the precession time of \( ^{75}\text{As} \), \( T_w = T_{3\text{Ar}} \), and the black curve for a 1% deviation from the resonance.
between the waiting time of LZ pulses and the Larmor frequency of one of the nuclear species the quasiperiods of nuclear fluctuations become longer and are controlled by the deviations from the exact resonance. We also expect that under these conditions the nuclear gradients should be dominated by the resonant specie.

VII. CONCLUSIONS

An analytical solution of a simplified model, and extensive numerical simulations for a realistic geometry, prove that self-quenching is a generic property of the central spin-1/2 problem in absence of spin-orbit coupling. As applied to a double quantum dot of a GaAs type, where electron and nuclear spins are coupled via hyperfine interaction, pumping nuclear magnetization across a $S$-$T_+$ avoided crossing through successive Landau-Zener sweeps ceases after about $10^4$ sweeps. This is a result of the screening of the initial fluctuation of the nuclear magnetization by the injected magnetization and vanishing of the $S$-$T_+$ anticrossing width, and this sort of self-quenching is robust. Under the influence of moderate noise, the system wanders through a set of dark states belonging to a vast landscape of the system including about $10^6$ nuclear spins coupled through inhomogeneous electron spin density. With time intervals depending on the level of the noise, the system experiences revivals when additional magnetization is injected, and afterwards it wanders through a new set of dark states.

Due to the violation of the angular momentum conservation, spin-orbit coupling changes the situation drastically. Self-quenching sets in only stroboscopically under the condition that the waiting time between consecutive Landau-Zener sweeps is in resonance with the Larmor precession time of one of the nuclear species. Then the precessing Overhauser field of the resonant specie compensates the spin-orbit field $v_{SO}$ during the sweep, while contributions of other species vanish. This sort of self-quenching is fragile and sensitive even to minor deviation from the resonance. Generically, injection of nuclear polarization oscillates in time and changes sign. Therefore, spin-orbit coupling causes the nuclear magnetization of a pumped $S$-$T_+$ double quantum dot to exhibit a persistent dynamics.

We suggest that the feedback loop technique for building controllable nuclear field gradients [34] is based on the oscillatory behavior of the nuclear spin magnetization caused by spin-orbit coupling. Spin-orbit coupling is a natural mechanism of overcoming self-quenching. The technique employs persistent oscillations and selects the sign of the pumping response to the changing magnetization gradient.

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APPENDIX: SPIN-ORBIT COUPLING

The Rashba spin-orbit Hamiltonian is

$$H_{SO} = \alpha \sum_{n=1,2} \left[ \sigma_z(n) k_x(n) - \sigma_y(n) k_y(n) \right], \quad (A1)$$

where $\alpha$ is the strength of SO interaction, and $k_x(n)$ and $k_y(n)$ are the in-plane momenta for the electron $n$. The singlet and triplet states are $\Psi_S(1,2) = \psi_S(1,2) \chi_S(1,2)$ and $\Psi_T(1,2) = \psi_T(1,2) \chi_T(1,2)$, where the orbital components of the singlet and triplet wave functions have been defined in Sec. V and the spin parts of the wave functions are $\chi_S(1,2) = \cdots$
\((|\uparrow_1\rangle|\downarrow_2\rangle - |\downarrow_1\rangle|\uparrow_2\rangle)/\sqrt{2}\) and \(\chi_{T_+}(1,2) = |\uparrow_1\rangle|\uparrow_2\rangle\). In terms of the orbital wave functions, the SO induced \(S\cdot T_+\) coupling is then \(v^+_{SO} = i\alpha\cos v (\psi_L|k_x + ik_y|\psi_L)\). This matrix element can be estimated similarly to Refs. [26] and [39]. It depends exponentially on the overlap between the wave functions of the dots, \(\psi_L\) and \(\psi_R\). Therefore, the SO coupling \(v^+_{SO}\) can be tuned and strongly decreases with the interdot distance \(d\). In InAs quantum wires the SO coupling parameter is around \(\alpha \sim 10^{-11}\) eV m [53] corresponding to a spin precession length of \(l_{SO} = h^2/(2m^*\alpha)\) of around 100 nanometers. With typical parameters of \(d = 100\) nm, \(l = 50\) nm, and \(\cos v = 1/\sqrt{2}\), we find that \(v_{SO}\) is around \(4 \times 10^{-5}\) eV. This is about two orders of magnitude larger than the typical \(S\cdot T_+\) coupling \(10^{-7}\) eV induced by the hyperfine interaction, but decreases with \(d\) exponentially. Theoretical estimates of \(v_{SO}\) hold only with exponential accuracy. Preexponential factors are model dependent, and a somewhat different estimate was proposed in Ref. [54]. In GaAs quantum dots [55], the SO coupling constant \(\alpha\) is two orders of magnitude smaller than in InAs with \(v_{SO} \approx 30\) \(\mu\)eV, so that the SO coupling may be comparable to the hyperfine-induced coupling, and is usually considered as weaker than it. These estimates should be treated with caution since the SO coupling is not only a function of the material but is sample specific.

For GaAs, an estimate of a typical fluctuation as \(v_n^0 \approx A/\sqrt{N}\) with \(A\) from Table II and \(N \approx 10^6\) results in \(v_n^0 \approx 100\) neV. Our estimates of \(v_{SO}\) of Ref. [30] gave \(v_{SO} \approx 50\) neV, while the estimate of Ref. [37] is \(v_{SO} \approx 15\) neV [56]. Recently, Shafiee et al. [57] managed to resolve the SO and hyperfine components of electric dipole spin resonance (EDSR) [58] in the same system, a GaAs double quantum dot. Their data suggest that both components were of a comparable magnitude, with the hyperfine component maybe somewhat stronger.

Keeping in mind the uncertainty in the values of \(v_{SO}\), we carry out simulations both with SO coupling and without it.

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