Long-distance contribution and $\chi_{c1}$ radiative decays to light vector meson

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The discrepancy between the PQCD calculation and the CLEO data for $\chi_{c1} \rightarrow \gamma V \ (V = \rho^0, \omega, \phi)$ stimulates our interest in exploring other mechanisms of $\chi_{c1}$ decay. In this work, we apply an important non-perturbative QCD effect, i.e., hadronic loop mechanism, to study $\chi_{c1} \rightarrow \gamma V$ radiative decay. Our numerical result shows that the theoretical results including the hadronic loop contribution and the PQCD calculation of $\chi_{c1} \rightarrow \gamma V$ are consistent with the corresponding CLEO data of $\chi_{c1} \rightarrow \gamma V$. We expect further experimental measurement of $\chi_{c1} \rightarrow \gamma V$, which will be helpful to test the hadronic loop effect on $\chi_{c1}$ decay.

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I. INTRODUCTION

In the past three decades, a series of the observations of S-wave, P-wave and D-wave charmonia make charmonium family abundant. Nowadays, charm physics is still an intriguing research field with challenges and opportunities [3]. Especially, the study of charmonium may provide valuable information on non-perturbative QCD effects.

As an important and effective approach to deeply learn the underlying properties of charmonium, charmonium decay is an extensively focused research topic. Among the observed charmonium states, $J/\psi$ and $\psi(2S)$ are of abundant experimental information of decay just listed in Particle Data Group (PDG) [2]. However, the experimental measurement relevant to the decay of P-wave charmonium is far less than that of $J/\psi$ and $\psi(2S)$. Thus, more experimental and theoretical explorations of P-wave charmonium decay are becoming active, especially with the running of BES-III.

Recently, the BES-III Collaboration announced its observations of $\chi_{cJ} (J = 0, 1, 2)$ decaying into two light vector mesons [3]. Among these decay modes of $\chi_{cJ}$ to two light vector mesons, the OZI suppressed processes $\chi_{c1} \rightarrow \omega \omega$, $\phi \phi$ and the double-OZI suppressed process $\chi_{c1} \rightarrow \omega \phi$ were firstly observed. In order to explain the evasion of the helicity selection rule in these processes, the hadronic loop effect, an important non-perturbative effect relevant to the decay of charmonia [4, 8] and molecular system [9–21], is introduced in Refs. [22–24], which also indicate that applying the hadronic loop mechanism to other $\chi_{cJ}$ decays will be helpful to further test the hadronic loop effect on $\chi_{cJ}$ decay.

In Ref. [25], the radiative decays of charmonia $J/\psi$ and $\chi_{cJ}$ into light meson are studied by the perturbative QCD (PQCD) approach, where a complete numerical calculation for the quark-gluon loop diagrams was performed. The obtained theoretical results for $J/\psi \rightarrow \gamma \eta, \gamma \eta'$ can well reproduce the experimental data. Furthermore, the branching ratios of $\chi_{cJ} \rightarrow \gamma \rho^0, \gamma \omega, \gamma \phi$ were predicted, which are $B(\chi_{c1} \rightarrow \gamma \rho^0) = 1.4 \times 10^{-5}$, $B(\chi_{c1} \rightarrow \gamma \omega) = 1.6 \times 10^{-6}$ and $B(\chi_{c1} \rightarrow \gamma \phi) = 3.6 \times 10^{-6}$.

In 2008, the radiative decays of $\chi_{cJ}$ were first measured by the CLEO Collaboration using a total of $2.74 \times 10^7$ decays of the $\psi(2S)$ collected with the CLEO-c detector [26]. The reported results are $B(\chi_{c1} \rightarrow \gamma \rho^0) = 243 \pm 19 \pm 22 \times 10^{-6}$, $B(\chi_{c1} \rightarrow \gamma \omega) = 85 \pm 15 \pm 12 \times 10^{-6}$ and $B(\chi_{c1} \rightarrow \gamma \phi) = 12.8 \pm 7.6 \pm 1.5 \times 10^{-6}$. One notices that the experimental results of the branching ratios of $\chi_{c1} \rightarrow \gamma \rho^0, \gamma \omega, \gamma \phi$ are an order magnitude larger than the corresponding theoretical predictions [25].

The above difference between the experimental measurement from CLEO and the theoretical result calculated in PQCD shows that there should exist the extra effect on the $\chi_{c1}$ radiative decays into a light vector meson, which stimulates our interest in exploring the underlying mechanism to resolve this large discrepancy between the theoretical prediction by PQCD [25] and the experimental measurement by CLEO [26].

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As indicated in Refs. 22, 24, the hadronic loop effect can explain the experimental observation of the OZI suppressed processes $\chi_{c1} \rightarrow \omega \gamma$, $\phi \gamma$ and the double-OZI suppressed process $\chi_{c1} \rightarrow \omega \phi$ well. In this work, we extend the hadronic loop effect to $\chi_{c1} \rightarrow \gamma V$ ($V = \rho^0, \omega, \phi$) process to answer whether large discrepancy between the theoretical calculation and the experimental measurement of $\chi_{c1} \rightarrow \gamma V$ can be alleviated. $\chi_{c1} \rightarrow \gamma V$ is similar to $\chi_{c1} \rightarrow VV$ since both processes occur via the intermediate charmed mesons under considering the hadronic loop effect. What is more important of this work is to offer an effective approach to test the proposed hadronic loop effect applied to explain $\chi_{c1} \rightarrow VV$ decay processes.

The paper is organized as following. After the introduction, we present the formula of the hadronic loop contributions to $\chi_{c1}$ radiative decays to a light vector meson, which includes the effective Lagrangian employed in this work and the decay amplitudes. In Sec. III the numerical results of $\chi_{c1} \rightarrow \gamma \rho^0$, $\gamma \omega$, $\gamma \phi$ are given. The last section is the discussion and conclusion.

II. HADRONIC LOOP EFFECT ON $\chi_{c1} \rightarrow \gamma V$

As indicated in Ref. 23, the nonperturbative QCD mechanism, i.e., hadronic loop effect, may play a crucial role in understanding $\chi_{c1}$ decay. In Table. I the typical diagram depicting the hadronic loop effect on $\chi_{c1} \rightarrow \gamma V$ at quark level is given, which is different from the quark-gluon loop diagrams depicting $\chi_{c1} \rightarrow \gamma V$ at PQCD approach in Ref. 25. The transition element of $\chi_{c1} \rightarrow \gamma V$ can be expressed as

$$\mathcal{M}[\chi_{c1} \rightarrow \gamma V] = \sum \langle \gamma V | \mathcal{H}^{(2)} | i \rangle \langle i | \mathcal{H}^{(1)} | \chi_{c1} \rangle,$$

which reflects the intermediate state contribution to $\chi_{c1} \rightarrow \gamma V$. Here, $\mathcal{H}^{(1)}$ represents the interaction of $\chi_{c1}$ and $D D^* + h.c.$ and $\mathcal{H}^{(2)}$ describes interaction $D D^* + h.c. \rightarrow \gamma V$ by exchanging an appropriate charmed meson.

![Diagram](image)

FIG. 1: The quark level typical diagrams (the first column) describing the hadronic loop effect on $\chi_{c1} \rightarrow \gamma V$ and the hadron level schematic diagram corresponding to $\chi_{c1} \rightarrow \gamma \rho^0$ (the second column). The red and black lines denote the charm quark and light quark. The photon emits from charm quark line or light quark line. By the charge conjugate transformation $D^{(*)^+} = D^{(*)0}$ and $D^{(*)-} = D^{(*)0}$, the rest two diagrams of $\chi_{c1} \rightarrow DD^* + h.c. \rightarrow \gamma \rho^0$ can be obtained by diagrams (a) and (d).

With $\chi_{c1} \rightarrow \gamma \rho^0$ as an example, we list the schematic diagrams at hadron level in Table II where $\chi_{c1}$ first dissolves into two virtual charmed mesons which is originated from the coupled channel effect. Then these two virtual charmed mesons $D D^* + c.c.$ turn into a photon and $\rho^0$ meson by exchanging the charmed meson. Due to the mass of $\chi_{c1}$ being lower than the threshold of $D D^*$, the charmed mesons in the loop are off-shell.

In the following, we still use $\chi_{c1} \rightarrow \gamma \rho^0$ as an example to illustrate the relevant calculation of hadronic loop diagrams listed in Fig. 1 where the effective Lagrangian approach is applied to write out the decay amplitude throughout this work.

The Lagrangian for $\chi_{c1} D D^*$ coupling reads 27

$$\mathcal{L}_{\chi_{c1} D D^*} = i g_{\chi_{c1}} D D^* \chi_{c1} \cdot D_i^{*} D^i + h.c.$$

(2)
The effective Lagrangians responsible for $D^{(*)} D^{(*)} V$ interactions are
\[
\mathcal{L}_{D^{(*)} D^{(*)} V} = -ig_D D V D_i^+ \partial_\mu D_i^j (\mathcal{V})^{\mu j} - 2f_{D^{(*)} D^{(*)}} \epsilon_{\mu \nu \alpha \beta} (\partial^\mu \mathcal{V})^j (D_i^j \partial^\alpha D_i^\beta - D_i^\beta \partial^\alpha D_i^j) + ig_{D^{(*)} D^{(*)}} \epsilon_{\mu \nu \alpha \beta} \partial_\mu D_\nu^j (\mathcal{V})^j + 4f_{D^{(*)} D^{(*)}} \epsilon_{\mu \nu \alpha \beta} (\partial^\mu \mathcal{V})^j (D_i^j - \partial^\nu \mathcal{V})_j D_i^j,
\]
where $D^{(*)} = (D^{(*)0}, D^{(*)+}, D_s^{(*)+})$ and $A \partial_\mu B = (\partial_\mu A - (\partial_\mu A) B$. The matrix of the nonet vector mesons $\mathcal{V}$ is defined as
\[
\mathcal{V} = \begin{pmatrix}
\frac{1}{\sqrt{2}} (\rho^0 + \omega) & \rho^+ & K^{*+} \\
\rho^- & \frac{1}{\sqrt{2}} (\rho^0 + \omega) & K^{*0} \\
K^{*-} & K^{*0} & \phi
\end{pmatrix}.
\]

We need to specify that the Lagrangians in Eq. (4) is just the first term in an infinite series of terms that represents the hadronic representation of the QCD Lagrangian, which is restored from the Lagrangian constructed in the chiral and heavy quark limits [28–32]. Thus, we adopt usual $i/(k^2 - m^2)$ and $i(-g_{\mu \nu} + k^\mu k^\nu/(k^2 - m^2)$ propagators when writing out the decay amplitude of $\chi_{c1} \rightarrow D D^* + h.c. \rightarrow \gamma \rho^0$. The relevant coupling constant will be presented in the next section.

The Lagrangians for $\gamma D D$ and $\gamma D^* D^*$ interactions can be obtained from the Lagrangian for free scalar and massive vector fields by the minimal substitution $\partial^\mu \rightarrow \partial^\mu + i e A^\mu$, which are [33]
\[
\mathcal{L}_{D D} = i e A^\mu D^- \partial^\mu D^+ + i e A_\mu D^- \partial^\mu D^+_s,
\]
\[
\mathcal{L}_{D^* D^*} = i e A^\mu \left\{ g^{\beta \alpha} D^\alpha D^\beta \partial^\mu D^+ + g^{\nu \mu} D^\nu D^\mu \partial^\alpha D^\beta - g^{\nu \nu} \partial^\alpha D^\nu D^\nu \right\} + i e A_\mu \left\{ g^{\beta \alpha} D^\alpha D^\beta \partial^\mu D^+ + g^{\nu \mu} D^\nu D^\mu \partial^\alpha D^\beta - g^{\nu \nu} \partial^\alpha D^\nu D^\nu \right\},
\]
respectively. Here, the electromagnetic interactions of $D^0 D^0 \gamma$ and $D^{*0} D^{*0} \gamma$ does not exist.

The Lagrangian describing the electromagnetic $D^* D^* \gamma$ vertex is [33]
\[
\mathcal{L}_{D^* D^* \gamma} = \left\{ \frac{g_{D^* D^* \gamma}}{4} \epsilon^{\mu \nu \alpha \beta} F_{\mu \nu} D^*_{\alpha \beta} D^0 - \frac{g_{D^* D^* \gamma}}{4} \epsilon^{\mu \nu \alpha \beta} F_{\mu \nu} D^{*0}_{\alpha \beta} D^0 \right\} + h.c.
\]
where $F_{\mu \nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and $D^{\mu0,+} = \partial_\mu D^{\mu0,+} - \partial_\mu D^{\mu0,+}$ is the stress tensor of the vector charmed meson. To some extent, the Lorentz structure just shown in Eq. (7) is same as that in Ref. [34, 35]. In Eq. (7), parameters $g_{D^* D^* \gamma}$ and $g_{D^{*0} D^{*0} \gamma}$ are introduced to get consistent results with experimental measurements of $D^{*+} \rightarrow D^+ \gamma$ and $D^{*0} \rightarrow D^0 \gamma$. The theoretical decay widths of $D^{*+} \rightarrow D^+ \gamma$ and $D^{*0} \rightarrow D^0 \gamma$ are
\[
\Gamma(D^{*+} \rightarrow D^+ \gamma) = \frac{\alpha}{24} g_{D^{*+} D^+ \gamma} m_{D^{*+}} (1 - \frac{m_{D^+}^2}{m_{D^{*+}}^2}),
\]
\[
\Gamma(D^{*0} \rightarrow D^0 \gamma) = \frac{\alpha}{24} g_{D^{*0} D^0 \gamma} m_{D^{*0}} (1 - \frac{m_{D^0}^2}{m_{D^{*0}}^2}).
\]
According to the experimental widths $\Gamma(D^{*+} \rightarrow D^+ \gamma) = 1.54 \text{ keV}$ [2] and $\Gamma(D^{*0} \rightarrow D^0 \gamma) = 26.04 \text{ keV}$ [2, 18], the coupling constant $g_{D^* D^* \gamma}$ is fixed as
\[
|g_{D^{*+} D^+ \gamma}| = 0.5 \text{ GeV}^{-1}, \quad |g_{D^{*0} D^0 \gamma}| = 2.0 \text{ GeV}^{-1}.
\]
Both calculations based on Lattice QCD [40] and QCD sum rules (QSR) [33] predict that the coupling constant for the radiative decay of the neutral charmed meson has a positive sign while the coupling constant for the charged charmed meson radiative decay is negative. In present work, we follow such a convention and take $g_{D^{*+} D^+ \gamma} = -0.5$ GeV$^{-1}$ and $g_{D^{*0} D^0 \gamma} = 2.0$ GeV$^{-1}$. For the coupling constant of $D_s^* D_s \gamma$ interaction, the calculation from QSR gives $g_{D_s^* D_s \gamma} = -0.3 \pm 0.1 \text{ GeV}^{-1}$ [33]. In present work, the central value is adopted.
According to the Lagrangian just listed above, we obtain the decay amplitudes of $\chi_{c1} \to \gamma \rho^0$ corresponding to the diagrams in Fig. 1.

$$\mathcal{M}^{(a)}_C = (i)^3 \int \frac{d^4q}{(2\pi)^4} [ig_{\chi_1\rho D^*} \epsilon^{\mu}_{\chi_1}] [-ig_{\tau D^* V} \epsilon^{\nu}_V (iq_\nu + i\gamma_\nu)] \times \left[ \frac{e}{4} g^{D_+ D^+ + + \epsilon \theta \phi \lambda \epsilon^{\mu}_{\chi_1} [(ip_4g^\rho - ip_4^\rho g^\phi)(-ip_3^\lambda g^{\tau \kappa} + ip_3^\kappa g^{\tau \lambda})] \times \frac{i}{p_1^2 - m_{D^+}^2} \frac{i}{p_2^2 - m_{D^*}^2} \frac{i}{q^2 - m_{D^*}^2} \mathcal{F}^2(q^2), \right]$$

$$\mathcal{M}^{(b)}_C = (i)^3 \int \frac{d^4q}{(2\pi)^4} [ig_{\chi_1\rho D^*} \epsilon^{\mu}_{\chi_1}] [-2f_{D^* D V} \epsilon_\theta \epsilon_\rho \epsilon_\tau \epsilon_\nu (ip_3^\theta)(iq_\nu + ip_1^\nu)] \times \left[ \frac{e^2}{4} (g_{\rho \phi}(p_2^\rho - g_{\rho \phi}q_\tau + g_{\rho \tau}p_2^\rho)) \frac{i}{p_1^2 - m_{D^+}^2} \frac{i}{p_2^2 - m_{D^*}^2} \frac{i}{q^2 - m_{D^*}^2} \mathcal{F}^2(q^2), \right]$$

$$\mathcal{M}^{(c)}_C = (i)^3 \int \frac{d^4q}{(2\pi)^4} [ig_{\chi_1\rho D^*} \epsilon^{\mu}_{\chi_1}] [ie\epsilon^{\nu}_V (-ip_1^\nu - iq_\nu)] [-2f_{D^* D V} \epsilon_\theta \epsilon_\rho \epsilon_\tau \epsilon_\nu (ip_3^\theta)(-iq_\nu + ip_2^\nu)] \times \left[ \frac{e^2}{4} (g_{\rho \phi}(p_2^\rho - g_{\rho \phi}q_\tau + g_{\rho \tau}p_2^\rho)) \frac{i}{p_1^2 - m_{D^+}^2} \frac{i}{p_2^2 - m_{D^*}^2} \frac{i}{q^2 - m_{D^*}^2} \mathcal{F}^2(q^2), \right]$$

$$\mathcal{M}^{(d)}_C = (i)^3 \int \frac{d^4q}{(2\pi)^4} [ig_{\chi_1\rho D^*} \epsilon^{\mu}_{\chi_1}] \left[ \frac{e}{4} g^{D^* D^* + + \epsilon \theta \phi \lambda \epsilon^{\mu}_{\chi_1} [(ip_4g^\rho - ip_4^\rho g^\phi)(-ip_3^\lambda g^{\tau \kappa} + ip_3^\kappa g^{\tau \lambda})] \times \frac{i}{p_1^2 - m_{D^+}^2} \frac{i}{p_2^2 - m_{D^*}^2} \frac{i}{q^2 - m_{D^*}^2} \mathcal{F}^2(q^2), \right]$$

which are resulted from the charge intermediate charmed mesons. Thus, the total decay amplitude of $\chi_{c1} \to D\bar{D}^* + h.c. \to \gamma \rho^0$ is

$$\mathcal{M}(\chi_{c1} \to D\bar{D}^* + h.c. \to \gamma \rho^0) = [\mathcal{M}^{(a)}_C + \mathcal{M}^{(b)}_C + \mathcal{M}^{(c)}_C + \mathcal{M}^{(d)}_C] + [\mathcal{M}^{(a)}_N + \mathcal{M}^{(d)}_N],$$

where subscripts $C$ and $N$ denote the corresponding amplitudes being from charge charmed meson loop and neutral charmed meson loop, respectively. $\mathcal{M}^{(a)}_N$ and $\mathcal{M}^{(d)}_N$ is obtained by amplitudes $\mathcal{M}^{(a)}_C$ and $\mathcal{M}^{(d)}_C$ with the replacements of the mass and coupling constants, i.e., $g_{D^* D^* \gamma} \to g_{D^* D^* \rho \rho} \gamma$, $m_{D^*}^{(0)\rho} \to m_{D^*}^{(0)\rho}$ and $m_{D^*}^{(\pm)\rho} \to m_{D^*}^{(\pm)\rho}$. In Eqs. $11-14$, the form factor $\mathcal{F}(q^2)$ is introduced to depict the inner structure of the interaction vertex of the exchanged charmed meson and the intermediate state. As what we have done in Ref. $23$, a dipole form of the form factor is employed

$$\mathcal{F}(q^2) = \left( \frac{\Lambda^2 - m_{D^*}^2}{\Lambda^2 - q^2} \right)^2.$$
Before performing the numerical calculations, we need to introduce the coupling constant relevant to the effective Lagrangian listed in the previous section. The coupling constants for $\chi_{c1}DD^*$ and $DDV$ interactions are listed in Table. I.

| Coupling constant | Expression | Value |
|-------------------|------------|-------|
| $g_{\chi_{c1}DD^*}$ | $2\sqrt{2}g_1\sqrt{m_Dm_{D^*}m_{\chi_{c1}}}$ | $-21.44 \text{ GeV}$ |
| $g_{DDV}$ | $\beta g_{\gamma}/\sqrt{2}$ | $3.71$ |
| $f_{D^*D^*V}$ | $\lambda g_{\gamma}/\sqrt{2}$ | $2.31 \text{ GeV}^{-1}$ |
| $f_{D^*D^*V}$ | $\lambda m_D g_{\gamma}/\sqrt{2}$ | $4.88$ |

TABLE I: The coupling constants relevant to the calculation of $\chi_{c1} \to \gamma V$. Here, $g_1$ is related to the $\chi_{c0}$ decay constant $f_{\chi_{c0}}$ via relation $g_1 = -\sqrt{\frac{m_{\chi_{c0}}}{f_{\chi_{c0}}}}$ with $f_{\chi_{c0}} \approx 0.51 \text{ GeV}$ [37]. Other parameters include $g_{\gamma} = m_{\rho}/f_{\pi}$, $m_{\rho} = 0.77 \text{ MeV}$, $\beta = 0.9$, $\lambda = 0.56 \text{ GeV}^{-1}$, $g = 0.59$ and $f_{\pi} = 132 \text{ MeV}$ [32, 38, 39].

With the above preparation, the radiative decays of $\chi_{c1}$ to a light vector meson are estimated. As a free parameter, $\alpha$ is introduced in the cutoff $\Lambda$ of the form factors, which is usually dependent on the particular process and taken to be of the order of unity. In Fig. 2, we present the branching ratios of $\chi_{c1} \to \gamma \rho^0$, $\gamma \omega$, $\gamma \phi$ dependent on the parameter $\alpha$. For comparing with the experimental data [26], the theoretical result includes the hadronic loop contribution obtained in this work and the PQCD estimation in Ref. [25].

FIG. 2: (color online) The branching ratios of $\chi_{c1} \to \gamma \rho^0$, $\gamma \omega$, $\gamma \phi$ dependent on the parameter $\alpha$. The red dashed-lines with the blue bands are the experimental measurement [26]. The blue solid lines correspond to the theoretical calculations including the hadronic loop effect and the PQCD calculation [25]. The vertical yellow bands in the sub-figure denote the overlap of our results with the corresponding experimental measurement. With the same $\alpha$ range sandwiched between two green vertical solid lines, the obtained branching ratios of $\chi_{c1} \to \gamma \rho^0$, $\gamma \omega$, $\gamma \phi$ in this work are consistent with the experimental data.
As shown in Fig. 2 there exists overlap between the numerical result obtained in this work and the experimental data announced by CLEO. The corresponding $\alpha$ ranges for $\chi c_1 \to \gamma \rho^0$, $\gamma \omega$, $\gamma \phi$ are $2.18 < \alpha < 2.35$, $2.06 < \alpha < 2.28$ and $1.16 < \alpha < 2.77$, respectively, which are in the reasonable parameter space. Especially, we need to emphasize that there exists a common $\alpha$ range $2.18 < \alpha < 2.28$ for $\chi c_1 \to \gamma \rho^0$, $\gamma \omega$, $\gamma \phi$ radiative decays.

**IV. DISCUSSION AND CONCLUSION**

As an important non-perturbative QCD effect, the hadronic loop mechanism was proposed in studying $J/\psi$ and $\psi(3770)$ decays \cite{1,2,3}. By using this mechanism, the OZI suppressed processes $\chi c_1 \to VV$ with $VV = \omega \omega$, $\phi \phi$ and the double-OZI process $\chi c_1 \to \omega \phi$ announced by BES-III were explained well in the recent work \cite{21,22}.

The CLEO Collaboration announced the experimental results of $\chi c_1 \to \gamma V$ in 2008 \cite{26}, which are an order magnitude larger than the corresponding theoretical estimations calculated by PQCD \cite{25}. To search for the source of the discrepancy between the PQCD calculation $\chi c_1 \to \gamma V$ and the CLEO data \cite{26} for $\chi c_1 \to \gamma V$, in this work we propose the hadronic loop contribution to $\chi c_1 \to \gamma V$. Under the hadronic loop mechanism, $\chi c_1 \to \gamma V$ is similar to $\chi c_1 \to VV$ process, both of which occur via the intermediate $D^*$ just shown in Fig. 1. To some extent, the study of this work can be as a test for the hadronic loop effect, which was applied to explain $\chi c_1 \to VV$ decays \cite{23,24}.

Our numerical result of $\chi c_1 \to \gamma V$ indicates that the theoretical result including the hadronic loop contribution and the result in PQCD calculation for $\chi c_1 \to \gamma V$ can reach up to the experimental data of $\chi c_1 \to \gamma V$. Thus, non-perturbative QCD effect, i.e., hadronic loop mechanism, can be as the underlying source to alleviate the difference between the PQCD calculation and the CLEO data of $\chi c_1 \to \gamma V$. As indicated in Refs. \cite{23,24}, the hadronic loop effect also plays an important role to $\chi c_1 \to VV$. Thus, the success of explaining $\chi c_1 \to VV$ and $\chi c_1 \to \gamma V$ under the hadronic loop mechanism not only tests the model itself, but also shows that the non-perturbative effect on $\chi c_1$ is important. Further experimental and theoretical studies of $\chi c_1$ decay are encouraged.

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