The effect of the broad-line region with geometrical structures on gamma-ray absorption in blazars

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Abstract

The broad-line region (BLR) is an important component of blazars, especially for flat spectrum radio quasars. The soft photons arising from the BLR will substantially affect the transparency of the γ-ray photons produced in the relativistic jet. We study the effect of the geometrical structure of the BLR on the absorption of γ-rays. We find that the γ-ray optical depth strongly depends on the geometrical structure of the BLR. For a “flat” BLR geometry, γ-ray photons with specified energies could escape transparently even if their emission region is located inside the cavity of the BLR.

Key words: gamma-rays: general — quasars: emission lines — radiation mechanisms: non-thermal

1 Introduction

Blazars are generally considered as the most extreme class of radio-loud active galactic nuclei (AGNs), in which two-sided jets with magnetized plasmas are launched near the central engine at relativistic velocity aligned with the line of sight. Traditionally, blazars are classified as BL Lacertae objects (BL Lacs) and flat spectrum radio quasars (FSRQs) according to the equivalent width (EW) of their strong emission lines (Urry & Padovani 1995); objects with a rest frame EW < 5 Å are called BL Lacs, otherwise they are called FSRQs. A new classification has arisen, based on the location of the synchrotron peak frequency ($v_{\text{peak}}^5$; Abdo et al. 2010); blazars with values of $v_{\text{peak}}^5 \lesssim 10^{14}$ Hz are named low synchrotron peaked blazars (LSP); ones with values in the range $10^{14}$ Hz $\lesssim v_{\text{peak}}^5 \lesssim 10^{15}$ Hz are named intermediate synchrotron peaked blazars (ISP); others with values of $10^{15}$ Hz $\lesssim v_{\text{peak}}^5$ are named high synchrotron peaked blazars (HSP). Otherwise, from the ratio of luminosity of the broad-line region (BLR) in Edington units, i.e., $L_{\text{BLR}}/L_{\text{Edd}}$, blazars with $L_{\text{BLR}}/L_{\text{Edd}} > 10^{-3}$ mainly display the properties of FSRQs (Ghisellini et al. 2011). Actually, these classifications are potentially associated with the BLR.

The existence of the BLR around the central engine is an almost uncontroversial fact in blazars, especially for FSRQs. The BLR is known to emit a number of strong lines and diffuse continua in the optical to UV bands, but its structure, spherical or “flat” geometry, for example, is still under debate. Usually, a spherical BLR geometry is assumed, in which its photons follow a blackbody spectrum peaking at the frequency of $v_{\text{Ly}α} = 2.47 \times 10^{15}$ Hz (Celotti et al. 2007; Tavecchio & Ghisellini 2008; Ghisellini & Tavecchio 2009), and its radius $R_{\text{BLR}}$ is determined by the reverberation mapping technique (Blandford & McKee 1982) derived from its reprocessing emission of the accretion disk (Kaspi et al. 2000, 2005, 2007; Bentz et al. 2009; Ghisellini & Tavecchio 2008). If the γ-ray emission region of blazars is located within the cavity of the BLR, the photons from the BLR will significantly absorb the γ-rays with energies above a few tens of GeV via positron–electron pair production processes (Donea & Protheroe 2003;...
Tavecchio & Mazin 2009; Tavecchio & Ghisellini 2012). Liu and Bai (2006) used a spherical BLR with half-thickness $b (0.05 \rightarrow 0.3 \text{ pc})$ to investigate the absorption of $\gamma$-rays, and demonstrated that $\gamma$-rays with energies between 10 and 200 GeV cannot escape from the diffuse photon field of the BLR.

For the geometry of the BLR, Labita et al. (2006) found that the spherical BLR fails to explain the observed line width and shape in quasi-stellar objects. The “black hole mass deficit” observed in narrow line Seyfert Type-1 galaxies (NLS1s: Grupe & Mathur 2004; Hayashida 2000) disappears provided that NLS1s have a disc-like, rather than spherical, BLR (Decarli et al. 2008). Tavecchio and Mazin (2009) produced a reasonable hardening of the spectrum of quasar 3C 279 in the TeV bands assuming an anisotropic BLR structure. The stability of GeV breaks, as well as the $\gamma$-ray spectral breaks from a fraction of a GeV to tens of GeV band in blazars can be well produced by the absorption of He II Lyman recombination continuum and emission lines, indicating that $\gamma$-rays arise within the high ionization zone of the BLR (Poutanen & Stern 2010). Stern and Poutanen (2011) studied the spectral properties of 3C 454.3 in the GeV bands, implying that the $\gamma$-ray emission region is close to the inner boundary of the BLR, which is impossible for a spherical BLR. Recently, Decarli et al. (2011) studied the properties of the BLR for a set of blazars and reference quasars, and supported the conclusion that the BLR could have a flat geometry in blazars.

In this paper, we study the effect of the geometrical structure of the BLR on the absorption of $\gamma$-rays in detail. In section 2, we characterize our model. In subsection 2.1 the geometry and emission of the BLR are presented, and the $\gamma$-ray optical depth is calculated in subsection 2.2. Our results are presented in section 3. Discussion and conclusions are given in section 4.

2 The model

2.1 Geometry and the BLR emission

In the following, we assume that the $\gamma$-ray emission region moves at relativistic velocity along the jet and locates at a position $R_o$ above the accretion disk at a certain time. The generalized geometry of the BLR is shown in figure 1. Two shaded areas with axis symmetry represent the BLR clouds, and the central strip corresponds to the accretion disk. The BLR is assumed to be flat and characterized by the aperture angle $\alpha$ measured from the disk plane. In calculations, $\alpha$ is set between 15° and 85°. We only consider the photons from the BLR that are not obscured by the accretion disk. The BLR is geometrically thick, with the scale from inner radius $r_{BLR,i}$ to outer radius $r_{BLR,o}$. Note that throughout this paper, $r_{BLR,i}$ is fixed at 5000$R_g$, while $r_{BLR,o}$ is fixed at $10^4R_g$ ($R_g$ is the gravitational radius, $R_g = GM/c^2$). This constraint is based on assumptions of canonical blazars as follows: (1) the accretion disk is a standard disk (Shakura & Sunyaev 1973). The inner and outer radius are $R_{\text{disk},i} = 6R_g$ and $R_{\text{disk},o} = 1000R_g$, respectively (Ghisellini & Tavecchio 2009); (2) the accretion disk is within the cavity of the BLR, while $r_{BLR,o}$ is fixed at $10^5R_g$ (Dermer et al. 2009). The blackbody temperature of the inner region of the accretion disk is assumed to be $T = 10^5 K$. The detailed mathematical treatments of the BLR are presented in the Appendix.

The accretion disk emits a total luminosity of $L_d$ uniformly in all directions; the BLR clouds intercept a fraction of the illuminating continuum and reprocess them to the emission lines and the diffuse continuum. Moreover, as stated by Poutanen and Stern (2010), the BLR also emits the recombination continua of hydrogen and He II, which could cause jumps in the $\gamma$-ray opacity at $\sim 19.2$ and 4.8 GeV. However, in this work, we do not consider the contributions from the recombination continua to the $\gamma$-ray absorption. For simplicity, we assume that the fractions $f_{\text{cov}}$ and $T_{BLR}$ of $L_d$ are reprocessed into the emission lines $\eta_{\text{line}}$ and the
diffuse continuum $J_{\text{cont}}$ respectively, $f_{\text{cont}}$ and $\tau_{\text{BLR}}$ are set to be 0.05, i.e., the total reprocessing fraction is fixed at 0.1. In order to evaluate the total emission line flux from the BLR, we consider the 35 components of the emission lines. The sum of line ratios is given by $N_\alpha = 555.77$ relative to the Ly$\alpha$ line ratio ($N_\alpha, \text{Ly}\alpha = 100$). Therefore, the flux contributed by a certain emission line equals

$$F_\nu = \frac{N_\nu}{555.77} F_{\text{BLR}}, \quad (1)$$

where $F_{\text{BLR}}$ is the total emission line flux contributing to the $\gamma$-ray emission region. Throughout this paper, the luminosity of the central disk is fixed at $L_d = 5 \times 10^{46} \text{erg s}^{-1}$, corresponding roughly to the intermediate value of the luminosity of FSRQs.

### 2.2 The $\gamma$-ray optical depth

According to the assumptions used by Liu and Bai (2006), the luminosity emitted by an accretion disk is reprocessed into the emission lines and the diffuse continuum. The emissivity of the emission lines is

$$j_{\text{line}}(r) = \frac{f_{\text{cont}} L_d r^{2\gamma - p - 2}}{16\pi^2} \int_{\text{BLR}}^r \gamma^2 - r^2 dr. \quad (2)$$

The emissivity of the diffuse continuum is

$$j_{\text{cont}}(r) = \frac{L_d \tau_{\text{BLR}} r^{s-2}}{16\pi^2} \int_{\text{BLR}}^r \gamma^2 - r^2 dr. \quad (3)$$

where the number densities of the clouds and electrons are assumed to follow power-law distributions, e.g., $n_c = n_{c0}(r/r_{\text{BLR},i})^{-p}$ and $n_e = n_{e0}(r/r_{\text{BLR},i})^{-q}$, where $n_{c0}$ and $n_{e0}$ are the number densities of clouds and electrons at $r_{\text{BLR},i}$, respectively. For a radial distribution of clouds, the same assumption is adopted, such as $R_{\text{cd}} = R_{\text{cd0}}(r/r_{\text{BLR},i})^q$, where $R_{\text{cd0}}$ is the radius of clouds at $r_{\text{BLR},i}$. In the present paper, we adopt the power-law exponents preferred by Kaspi and Netzer (1999): $p = 1.5$, $s = 1$, and $q = 1/3$.

The intensity of radiation at the angle $\theta$ with respect to the jet axis at position $R$ is

$$I(R, \theta) = \int_{\text{cont}}^r j(r) dl, \quad (4)$$

where $r = R^2 - 2 Rl \cos \theta + l^2$, and $j(r)$ is the total emissivity: $j(r) = j_{\text{line}}(r) + j_{\text{cont}}(r)$ and vanishes when $r > r_{\text{BLR},o}$ or $r < r_{\text{BLR},i}$.

The soft photon energy density of the BLR is given by

$$U(R) = \frac{2\pi}{c} \int_{\text{cont}}^r I(R, \theta) \sin \theta d\theta. \quad (5)$$

The photon number densities associated with the emission lines and the diffuse continuum are

$$n_{\text{line}}(R, \nu, \Omega) = \frac{I_{\text{line}}(R, \nu, \theta)}{b c},$$

$$n_{\text{cont}}(R, \nu, \Omega) = \frac{2v^2}{c^4[\exp(bv/kT) - 1]} n(R, \theta), \quad (6)$$

where $I_{\text{line}}(R, \nu, \theta)$ is the monochromatic intensity, and $n(R, \theta)$ is the normalization factor.

Finally, the $\gamma$-ray optical depth is calculated by

$$\tau_{\gamma\gamma}(\epsilon, \gamma) = \int_{R_0}^{R_{\max}} dR \int_{\epsilon_1}^{\epsilon_{\max}} d\epsilon \int_{\min}^{\max} d\theta \times \sigma(\epsilon, \epsilon, \theta)n(R, \epsilon, \Omega)(1 - \cos \theta), \quad (7)$$

where $\epsilon$ is the dimensionless energy of the soft photons, $n(R, \epsilon, \theta)$ is the soft photon number density at position $R$ with angle $\theta$ of integral path with respect to the jet axis. In the calculation of $\gamma$-ray attenuation optical depth, the soft photon frequencies are from $v_1 = 10^{14.6}$ Hz to $v_\gamma = 10^{16.5}$ Hz. The upper limit over $R$ is set to be 2 pc in all cases. $\sigma(\epsilon, \epsilon, \theta)$ is the pair-creation cross section, and the threshold of $\sigma(\epsilon, \epsilon, \theta)$ is determined by

$$\epsilon_\gamma = \frac{2}{(1 - \cos \theta)e}. \quad (8)$$

In order to increase the threshold of $\gamma$-ray photons in collisions, we can reduce the factor $(1 - \cos \theta)$. In a “flat” BLR geometry, decreasing $\alpha$ will reduce the probability of “head-on” collisions and increase the threshold of the $\gamma$-ray photons.

### 3 Results

To reduce the number of parameters, we keep the mass of the central supermassive black hole (BH) to be $M_{\bullet} = 5 \times 10^8 M_\odot$, which is a typical BH mass in AGNs. The total energy density distributions of the BLR are shown in figure 2. It is obvious that the aperture angle $\alpha$ has a dramatic impact on the energy density distributions. Actually, with increasing $\alpha$, the illuminating continuum intercepted by the BLR clouds increases, leading to an increase of the flux of the reprocessed emissions and the diffuse continuum. The energy density is almost constant within the cavity of the BLR, but at distance nearly close to $r_{\text{BLR},i}$, the energy density increases because the anisotropic effect of the BLR geometry becomes more apparent. Furthermore, the energy density declines outside $r_{\text{BLR},i}$, but when $\alpha$ is relatively low, the energy density declines even if the distance is less than $r_{\text{BLR},i}$. In the case of small $\alpha$, the BLR is disc-like, and the energy density is inversely proportional to the distance. In contrast, for larger $\alpha$ close to $90^\circ$, the energy density is
contributed by clouds within the BLR and will decrease along the radial distance due to the obscuring effect of the BLR clouds. Overall, the location of the energy density peaks shifts toward larger distance with increasing $\alpha$.

The soft photons from the BLR are not only scattered by relativistic electrons in the jet to produce the high-energy component in the spectral energy distributions of FSRQs, but are also absorbers of $\gamma$-rays via the processes of pair creation. Liu and Bai (2006) found that $\gamma$-ray photons with energies from 10 to 200 GeV cannot escape from the diffuse field of a BLR with a spherical shell geometry. In this paper, we extend a “closed” geometry to a “flat” geometry for calculating $\gamma$-ray absorption. Because the emissivity of diffuse and line emissions is proportional to the disk luminosity $L_d$, the optical depth $\tau_{\gamma\gamma}$ is also proportional to $L_d$.

In figure 3, we plot the optical depths versus $\gamma$-ray energies. Absorption of $\gamma$-rays is shown at three different distances, $R_o = 0.01, 0.1, 0.3$ pc. It is shown that the aperture angle $\alpha$ affects the $\gamma$-ray optical depth $\tau$ greatly. From the top panel in figure 3, with the emitting region located at $R_o = 0.01$ pc, we can see that even if $\alpha$ is as low as 15°, the optical depth of the $\gamma$-ray photons with energies $\sim 35$ GeV approaches unity; photons with energies lower than this critical energy can escape the diffuse field. Increasing $\alpha$ to 25°, the critical energy is down to $\sim 50$ GeV. Similarly, if $\alpha$ grows to 35°, the critical energy is down to $\sim 30$ GeV.

When $R_o = 0.3$ pc within the BLR clouds and $\alpha$ is less than 35°, $\gamma$-ray photons with energies up to $\sim 85$ GeV can escape the diffuse field.

We calculate the optical depths of $\gamma$-ray photons in the framework of a “flat” BLR geometry, in which “flatness” is characterized by the aperture angle $\alpha$. For illustrating the dependence of $\tau$ on $\alpha$, we consider three positions as above, $R_o = 0.01, 0.1, 0.3$ pc, while the energies of $\gamma$-ray photons are considered from 50 to 100 GeV energy bands; the results are shown in figure 4. It shows that when the $\gamma$-ray emitting region is located at position $R_o = 0.01$ pc, the energy density of the diffuse field is large and opaque to all of the $\gamma$-ray photons with energies beyond 50 GeV, as shown in the upper panel in figure 4. In the middle panel, the $\gamma$-ray emitting region is close to the inner boundary of
the BLR, the $\gamma$-ray photons undergo a short path before they escape the BLR under a certain $\alpha$. We can observe $\gamma$-ray photons with energy 50 GeV in the case of $\alpha \sim 24^\circ$, 60 GeV in the case of $\alpha \sim 20^\circ$, 70 GeV in the case of $\alpha \sim 17^\circ$. When the $\gamma$-ray emitting region is located within the clouds of the BLR, as long as the aperture angle $\alpha \lesssim 30^\circ$, $\gamma$-ray photons from 50 GeV to 100 GeV will be observed.

Because the optical depth depends on the height of the emitting region and the aperture angle when $\tau_{\gamma\gamma} = 1$, we can find a critical height $R_c$ of $\gamma$-ray transparency for the aperture angle $\alpha$, as shown in figure 5, for $\gamma$-ray photons with energies from 50 GeV to 100 GeV. It can be seen that the critical aperture angle $\alpha_c$ moves toward smaller angles with an increase in $\gamma$-ray energies, because a decrease of $\alpha$ will reduce the probability of “head-on” collisions, leading to an increase of threshold energy. For $\gamma$-rays with $E_\gamma = 100$ GeV, $R_c = 0.1$ pc, the critical aperture angle $\alpha_c$ is nearly $15^\circ$, and the geometry of the BLR becomes disc-like.

The above results are based on the assumption that the outer radius $R_{\text{disk}, o}$ of the accretion disk extends to the inner radius $r_{\text{BLR}, i}$ of the BLR, i.e., $R_{\text{disk}, o} = r_{\text{BLR}, i}$. In the following, we try to consider the effect of disk size on soft photon fields and $\gamma$-ray absorption. The results are shown in figure 6 for $U_{\text{BLR}}$ versus $R$, in which $\alpha$ is set to $15^\circ$, and in figure 7 for $R_c$ versus $\alpha$, in which the outer radius of the disk varies as $R_{\text{disk}, o}/r_{\text{BLR}, i} = 0.6, 0.4, 0.2, 0$. The disk size mainly influences the energy density of the diffuse photon field inside the BLR. When $R_{\text{disk}, o}/r_{\text{BLR}, i} = 0$, the central engine is a point source of radiation that isotropically emits and radially locates behind the jet, and the energy density inside the BLR becomes a constant. It can be seen that the disk size does not affect the relationship between $R_c$ and $\alpha$.

It is evident that BLR clouds do not exist near the jet axis under the disk when $\alpha$ is small, and $R_{\text{disk}, o}$ does not change the energy density of the diffuse photon field. Furthermore, when $\alpha$ is large, the critical position $R_c$ will be within the BLR clouds, and the energy density remains constant due to the obscuring effect of BLR clouds.
4 Discussion and conclusions

The geometry of the BLR is usually assumed to be an isotropic shell; the absorption of $\gamma$-ray photons by the diffuse field is serious when the $\gamma$-ray emitting region is located within the cavity of the BLR (Liu & Bai 2006; Tavecchio & Ghisellini 2012). The present work extends an isotropic BLR to a “flat” BLR described by an aperture angle $\alpha$. In the previous work (Tavecchio & Ghisellini 2012), the BLR clouds were assumed to intercept a fraction $\Omega_{\text{BLR}}/2\pi = 0.1$ of the illuminating continuum, where $\Omega_{\text{BLR}}$ is the solid angle surrounded by the BLR clouds, i.e., $\Omega_{\text{BLR}} = 2\pi \sin \alpha_{\text{min}}$. The minimum aperture angle is then given by $\alpha_{\text{min}} = \arcsin C$; we get $\alpha_{\text{min}} \approx 5:7$. Given this limit, we can determine the upper limit $\alpha_{\text{max}}$ that increases with the decline of $\gamma$-ray energy shown in figure 5. Alternatively, the geometry of the BLR could be reflected by the deprojection factor $f$. In the commonly assumed isotropic shell, $< f > = \sqrt{3}/2$. When the geometry is disc-like (McLure & Dunlop 2001),

$$f = 0.5 \left[ \left( \frac{H}{R} \right)^2 + \sin^2 \theta \right]^{-1/2}, \quad (9)$$

where $H/R$ is the scale ratio of the BLR, $H$ is the thickness, $R$ is the radius, and $\theta$ is the angle between the normal to the disk and the line of sight. We then find that the relationship between $\alpha$ and $f$ is given by

$$\tan \alpha = \left( \frac{1}{16 f^2} - \frac{1}{4} \sin^2 \theta \right)^{1/2}. \quad (10)$$

When we take $f = 2.0$, according to the result given by Decarli, Dotti, and Treves (2011), and $\theta \simeq 1/\delta_\theta$ for blazars, in which $\delta_\theta$ is the Doppler factor from 10 to 50 randomly, we obtain $\alpha \approx 7^\circ$, indicating that the geometry of the BLR is rather flat. In this case, almost all the $\gamma$-ray photons can escape the diffuse photon field.

In summary, we extend the spherical geometry of the BLR to the flat geometry characterized by the aperture angle $\alpha$. In the calculations, we take the central BH mass of $5 \times 10^8 M_\odot$, the inner and the outer radius of the BLR to be $5 \times 10^3 R_g$ and $10^3 R_g$, and the luminosity of the central disk to be $L_\delta = 5 \times 10^{46}$ erg s$^{-1}$. The diffuse photon energy density decreases with decreasing $\alpha$, but it remains constant inside $r_{\text{BLR},1}$ and declines outside $r_{\text{BLR},1}$ for a given $\alpha$. The optical depth $\tau$ of $\gamma$-rays depends not only on $R_\delta$ and the $\gamma$-ray energy, but also strongly on $\alpha$. Setting $\tau = 1$, we can obtain a relation between the critical distance $R_\delta$ and the aperture angle $\alpha$ for different energies. We find that when the geometry of the BLR is flat, $\gamma$-rays with specified energies can escape the cavity of the BLR.

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Appendix. Mathematical treatments of the BLR

As shown in figure 1, we first perform integration for the shaded region surrounded by the two blue dashed lines which connect with the emitting region; the black solid line is the integral path to $l$ that has an angle of $\theta$ with respect to the jet axis, and $l_{\text{min}}, l_{\text{max}}$ are the lower and upper limits, respectively. The integral limits are given by

$$\theta_{\text{min}} = 0,$$

$$\theta_{\text{max}} = \arccos \left( \frac{R^2 + x_1^2 - r_{\text{BLR},1}^2}{2 Rx_1} \right), \quad (A1)$$

and

$$\theta_{\text{min}}^{\text{cr}} = \arccos \left( \frac{R}{\sqrt{R^2 + r_{\text{BLR},1}^2}} \right),$$

$$\theta_{\text{apex}}^{\text{cr}} = \arccos \left( \frac{R^2 + \xi_0^2 - r_{\text{BLR},1}^2}{2 R \xi_0} \right). \quad (A2)$$

Here, $\theta_{\text{min}}^{\text{cr}}$ is the angle that the jet axis makes with the line linking the emitting region and outer boundary of the disk, while $\theta_{\text{apex}}^{\text{cr}}$ is the angle that the jet axis makes with the line linking the emitting region and apex N. $l_{\text{max}}$ is determined by if $\theta > \theta_{\text{apex}}^{\text{cr}}$:

$$l_{\text{max}} = R \cos \theta + R \left[ \cos^2 \theta + \left( \frac{r_{\text{BLR},1}}{R} \right)^2 \right]^{1/2}, \quad (A3)$$

if $\theta_{\text{apex}}^{\text{cr}} > \theta > \theta_{\text{min}}^{\text{cr}}$:

$$l_{\text{max}} = R \cos \theta + R \sin \theta \tan(\alpha + \theta), \quad (A4)$$

if $\theta_{\text{apex}}^{\text{cr}} > \theta_{\text{min}}^{\text{cr}} > \theta$:

$$l_{\text{max}} = R / \cos \theta, \quad (A5)$$

if $\theta > \theta_{\text{min}}^{\text{cr}} > \theta_{\text{apex}}^{\text{cr}}$:

$$l_{\text{max}} = R \cos \theta + R \left[ \cos^2 \theta + \left( \frac{r_{\text{BLR},1}}{R} \right)^2 \right]^{1/2}. \quad (A6)$$
if $\theta_{\text{cr}}^{\text{min}} > \theta_{\text{cr}}^{\text{apex}} > \theta$:

$$l_{\text{max}} = \frac{R}{\cos \theta}, \quad \text{(A7)}$$

where

$$x_1 = \left[ (r_{\text{BLR},0} \cos \alpha)^2 + (R - r_{\text{BLR},0} \sin \alpha)^2 \right]^{1/2},$$

$$\theta_4 = \angle_1 - \angle_2,$$

$$\xi = \left[ R^2 + r_{\text{BLR},0}^2 - 2Rr_{\text{BLR},0} \cos(\pi/2 + \alpha) \right]^{1/2}. \quad \text{(A8)}$$

Note that when we integrate the shaded region surrounded by the two blue dashed lines, the red region will not be integrated, because with the increase of $\alpha$, the line linking the $\gamma$-ray emitting region and apex $M$ becomes steeper, leading to the loss of the red region. To compensate in the above calculation, we consider a critical position that satisfies the condition $\alpha + \eta_c = \pi/2$; if $\alpha + \eta_c < \pi/2$, then the contributions from the red region to the soft seed photon field can be neglected. Inversely, we have to integrate the red region. The integral limits of the red region are given by

$$\theta_{\text{min}} = \arccos \left( \frac{R^2 + x_1^2 - r_{\text{BLR},0}^2}{2Rx_1} \right),$$

$$\theta_{\text{max}} = \arccos \left( \frac{\sqrt{R^2 - r_{\text{BLR},0}^2}}{R} \right), \quad \text{(A9)}$$

$$l_{\text{min}} = R \cos \theta - R \left[ \cos^2 \theta + \left( \frac{r_{\text{BLR},0}}{R} \right)^2 - 1 \right]^{1/2}, \quad \text{(A10)}$$

while $l_{\text{max}}$ is taken as equations (A3)–(A7).

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