Quantum sealed-bid auction using a modified scheme for multiparty circular quantum key agreement

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Abstract A feasible, secure and collusion attack-free quantum sealed-bid auction protocol is proposed using a modified scheme for multiparty circular quantum key agreement. In the proposed protocol, the set of all \( n \) bidders is grouped into \( l \) subsets (sub-circles) in such a way that only the initiator (who prepares the quantum state to be distributed for a particular round of communication and acts as the receiver in that round) is a member of all the subsets (sub-circles) prepared for a particular round, while any other bidder is part of only a single subset. All \( n \) bidders and auctioneer initiate one round of communication, and each of them prepares \( l \) copies of a \((r - 1)\)-partite entangled state (one for each sub-circle), where \( r = \frac{n}{l} + 1 \). The efficiency and security of the proposed protocol are critically analyzed. It is shown that the proposed protocol is free from the collusion attacks that are possible on the existing schemes of quantum sealed-bid auction. Further, it is observed that the security against collusion attack increases with the increase in \( l \), but that reduces the complexity (number of entangled qubits in each entangled state) of the entangled states to be used and that makes the scheme scalable and implementable with the available technologies. The additional security and scalability are shown to arise due to the use of a circular structure in place of a complete-graph or tree-type structure used earlier.

Keywords Sealed-bid auction · Secure quantum communication · Quantum auction
1 Introduction

In our daily life, we often find it difficult to perform all the tasks ourselves. Consequently, we outsource some tasks. For outsourcing a task at the lowest possible price (or to sell a product at the highest possible price), we often use a process referred to as auction, which is extremely relevant for imperfect market. Specifically, in an imperfect market, it may be hard to find (a) an actual valuation of an item which you wish to sell or buy and (b) to identify potential buyers or sellers. Auction helps us in obtaining this information. This process (auction) is very important in today’s society and is frequently used in various forms. Interestingly, the notion of an auction is almost as old as the civilization [1]. In fact, this process links market and economics to cryptography and thus to mathematics and computer science. Interestingly, the recent developments in the field of quantum cryptography [2–4] further extend this link and establish a link between physics (especially, quantum mechanics) and market (see [5–8] and references therein). With time, various types of auction mechanism and the associated rules have been evolved. Based on those rules, auction schemes may be classified into a few classes which includes (but not restricted to), (i) English auction [9], (ii) Dutch auction [10], and (iii) Sealed-bid second-price (Vickrey) auction [11]. Here it would be apt to note that in the English auction, one of the bidders announces his bid publicly at the beginning of the auction process. The competitor bidders announce higher price bids, and the bid price rises until we find there is no further bid. The bidder, who announced the highest bid, wins the bid and pays his bid. In contrast, in a Dutch auction, the auctioneer announces a high bid and then gradually lowers the bid. The bidder who first accepts the bid wins, and he has to pay that price. These bidding mechanisms are traditionally performed publicly, where all bidders can listen the call of the other bidders. However, for various reasons, it may not be always possible (and desirable too) to arrange a physical presence of all the bidders at the same place. Further, it may not be desired that the bid of a party gets influenced by the bid of another as it happens in publicly organized English and Dutch actions, where the bids are public information. This led to the idea of another class of auction mechanism which is referred to as sealed-bid second-price auction. In this frequently used scheme for auction, bids are private information instead of public announcement. Further, the bids are made simultaneously instead of one after another, and computation of all the bids is done together, and the bidder who announces the highest bid wins the bid, and the winner bidder pays the price of the second-highest bid. These features distinguish it from English and Dutch auction schemes. As this type of auction demands security of the bids (to keep them private), it reduces to an important cryptographic problem. Several solutions using classical cryptographic methods have been proposed for this problem [11–15], but the fact that classical cryptography cannot provide an unconditional security, whereas quantum cryptography (cf. [16]) can led to a set of resent proposals for quantum sealed-bid auction—where quantum resources are used to perform sealed-bid auction in an unconditionally secure manner. The first quantum sealed-bid auction scheme was proposed by Naseri in 2008 [8]. In 2009, various attack and defense strategies were proposed in connection with the Naseri protocol [17–20]. In 2010, Zhao et al. [21] proposed a new sealed-bid auction scheme with post-confirmation. In the same year, Zhang [22] proposed another scheme for quan-
Quantum sealed-bid auction using EPR pairs. Subsequently, in 2014, Zhao [23] improved this post-confirmation-based protocol. The continual interest on this interesting and upcoming field led to a few more interesting schemes. For example, in 2015, Wang et al. [24] proposed a novel quantum sealed-bid auction protocol that utilizes a secret ordering in the post-confirmation, and very recently in 2016, Liu et al. [25] have proposed a scheme for multiparty quantum sealed-bid auction using single photons as message carrier. However, all these schemes of quantum sealed-bid auctions have some limitations, and most of them are vulnerable under specific eavesdropping strategy. This fact and the importance of the sealed-bid auction schemes in the modern economy motivated us to design a new protocol for sealed-bid auction that is free from the limitations of the previously proposed schemes.

Remaining part of this paper is organized as follows. In Sect. 2, we discuss the limitations of the existing schemes and thus establish the motivation for designing a new protocol for quantum sealed-bid auction that would be free from the limitations of the existing protocols for the same task. In Sect. 3, we present a new protocol for quantum sealed-bid auction and illustrate the working of the same with an explicit example. In Sect. 4, we critically analyze the security and efficiency of the proposed scheme. Finally, the paper is concluded in Sect. 5.

2 Limitations of the existing schemes for quantum sealed-bid auction

The limitations of all the existing protocols of quantum sealed-bid auction may be summarized on the basis of structure of the arrangement of bidders and the auctioneer in the scheme. Naseri’s original protocol [8] was based on a tree-type structure. In a tree-type structure (see Fig. 1a), the auctioneer is like a root while the bidders work as nodes. Every node (bidder) is required to send his/her information to the root (auctioneer) directly, and none of the nodes would send his/her information (bid) to the remaining nodes (bidders). In this type of auction schemes, if the auctioneer colludes with one of the bidders, he can cheat the auction process by modifying the colluder’s bid, and therefore, announcing him the winner. In fact, all the remaining bidders have no choice but to trust the auctioneer.

To circumvent this problem, Zhao et al. [21] introduced a post-confirmation-based technique. All post-confirmation-technique-based quantum sealed-bid auction schemes convert the tree-type structure present in the initial schemes of sealed-bid auction to a complete-graph structure (see Fig. 1b). In the schemes that use a complete-graph-type structure, each node is directly connected to all the remaining nodes. Suppose there are \( n\) bidders (node) and an auctioneer then in the post-confirmation-technique-based quantum sealed-bid auction scheme a complete-graph of \( n + 1\) node is obtained. This complete-graph structure has its own limitations related to scalability, implementation, cost, more traffic, more memory requirement, etc. One can easily observe that these problems are present in all the post-confirmation-technique-based quantum sealed-bid auction schemes [21–25].

Further, in all the post-confirmation-technique-based quantum sealed-bid auction protocols, all the bidders send their bids to each other in an encoded form, which is used for post-confirmation after the winner of the auction is announced. They may
choose to send the bid values to each other either before they send it to the auctioneer or after that. However, both of these kinds of schemes can be attacked. Specifically, in the former case, the auctioneer may collude with one of the bidders to modify the bid initially proposed by the colluding bidder, and thus help him (the colluding bidder) to win the auction. While in the latter possibility, a single bidder (some of the colluding bidders) may extract the bid value of another bidder (all other bidders) using an optimized measurement on the accessible \((n-1)\) copies of the bids sent to all the competing bidders. As there exist some serious attack strategies against complete-graph-type schemes as well as tree-type schemes, we require a scheme of sealed-bid auction where all the parties (both auctioneer and bidders) get access to the information at the same time.

Here, we provide a solution of this problem by proposing a protocol using circular structure (see Fig. 1c) instead of tree or complete-graph structures. As there also exist a few collusion attacks on the circular quantum communication schemes (specifically, on multiparty circular quantum key agreement schemes), we have incorporated the solution provided in Refs. [26,27] against those attacks on the circular schemes for sending bids to auctioneer and competing bidders. We will discuss the possible attack strategies and the relevance of the solution adopted in the security section. In what follows, we first propose a protocol for sealed-bid auction that would be free from the limitations of the previous schemes mentioned in this section.

3 Proposed protocol: CMQKA-based scheme for sealed-bid auction

In this section, we propose a scheme using which the sealed-bid auction task can be accomplished without confronting the attacks possible in the schemes proposed in the recent past. Suppose there are \(n\) bidders \(B_1, B_2, B_3, \ldots, B_n\) and an auctioneer. Therefore, our task may be summed up as each bidder \(B_i\) wish to send his bid \(b_i\) to the auctioneer in a secure manner. Further, he also needs to share this information with the remaining bidders as well. At the same time, he must make sure that any single party (or a set of parties) does not get access to this information until he wishes them to, i.e., no one should be able to take advantage from this information.
In our scheme, to initiate the auction all the parties (the auctioneer and all the bidders) agree on a uniform arrangement in a circular manner. Additionally, they also decide to use disjoint subgroups (of a group of unitary operators which provide orthogonal states on the application) to encode their bid values as discussed in the past for MQKA scheme [28]. The disjoint groups are those groups which have only identity element in common, i.e., $A$ and $B$ are disjoint groups if $A \cap B = \{I\}$. Requirement of disjoint subgroups is justified as the receiver will lose the bijective mapping between each encoding operation and the initial and final state if two senders would apply the same operation.

Specifically, they obtain $n$ disjoint subgroups of order two, where one element is identity, to encode each bit of their secret bids. The bidder $B_i$ is assigned a disjoint subgroup $\{U_I, U_i\}$, where $U_I$ is the identity operator, and the set of all these unitary operators $\{U_I, U_1, \ldots, U_i, \ldots, U_n\}$ should be a part of a larger group of at least order $n$. It should be noted that an operational definition of the group (or a modified group as mentioned in [28–30]) is used hereafter neglecting the global phase.

Before we proceed with the protocol, we would also like to mention that we have already modified the MQKA scheme [28] to circumvent the participant attack mentioned beforehand and in the security section. Due to this modification, the structural arrangement of participants in our scheme (which is a circular structure) may be viewed as an intermediate structure between the tree-type and complete-graph structures. In what follows, we will establish that tree-type structure-based scheme and complete-graph structure-based scheme for quantum sealed-bid auction can be obtained as special cases of the circular structure-based scheme proposed here. The quantum sealed-bid auction scheme works as follows, where we assume the bidders and the auctioneer arranged in a circular manner (cf. Fig. 1c).

**Step 1** The auctioneer divides all the bidders (arranged in a large circle) in $l$ number of sub-circles in such a way that each sub-circle contains equal number of bidders, i.e., $r = \frac{n}{l} + 1$. It should be noted here that only the auctioneer is part of all the sub-circles. Thus, each sub-circle contains $(r - 1)$ distinct bidders, who are part of only a single sub-circle.

Similarly, each bidder also prepares $l$ sub-circles of the remaining bidders and the auctioneer from the larger circle. The division of circle in sub-circles by each bidder is similar, but not the same to each other or auctioneer. In fact, it is important in this arrangement that the bidder (initiator) himself should remain part of all the sub-circles he prepares.

**Step 2** To start the auction, the auctioneer prepares $l$ number of $(r - 1)$-qubit entangled states $|\psi\rangle^1, |\psi\rangle^2, \ldots, |\psi\rangle^l$, one for each sub-circle formed in Step 1. He may/may not prepare different entangled states for each sub-circle. Subsequently, he divides each $(r - 1)$-qubit entangled state into two sequences. The first sequence containing $p$ number of qubits is called “travel qubit sequence ($t_i$),” and the other one with $(r - p - 1)$ qubits is known as “home qubit sequence ($h_i$).”

**Step 3** All the bidders also prepare the same number of $(r - 1)$-qubit entangled states and divide them into two sequences, namely, home and travel qubit sequences, similar to what the auctioneer has performed in Step 2.
Step 4 Auctioneer can begin with the bidding process by sending his $l$ different travel sequences ($t_i$s) to the first bidders in each sub-circle. Prior sending these qubits, he has to insert an equal number of decoy qubits randomly in each $t_i$ to ensure security against an adversary [31,32]. At the same time, all bidders $B_i$s also send their $l$ number of $t_i$s (only after inserting an equal number of decoy qubits) to the first members of their respective $l$ sub-circles. At the end of this step, all the legitimate parties have received travel qubits from one of their adjacent parties (either auctioneer or a bidder). Subsequently, they perform a security check using the decoy qubits inserted in $t_i$s among the sender and the receiver of each $t_i$. If they find traces of an eavesdropping attempt inferred from the errors more than a threshold value, then they abort the protocol; otherwise, proceed to the next step.

Step 5 Once it is ensured that all the parties have received the $t_i$s in a secure manner, they can encode their messages on it. Specifically, each bidder $B_i$ encodes his respective bid $b_i$, and the auctioneer may choose to encode some random bits or information regarding the auction on the traveling qubits, which they have received in Step 4. For encoding each bit of the secret they use the prior decided unitary operations $U_i$s to obtain the encoded sequence $t'_i$. Subsequently, all the parties prepare the same number of decoy qubits as in the travel sequence and insert them randomly in $t'_i$ to obtain an enlarged sequence. In fact, at the end of this step, each bidder and the auctioneer have encoded his information only on the quantum state prepared by his adjacent party (bidder/auctioneer).

Step 6 The auctioneer and all the bidders send the enlarged sequences to their next party in their respective sub-circles, i.e., $B_i \rightarrow B_{i+1}$. On the receipt of this sequence, each receiving party performs a security check using the decoy qubits with the sending party following the same strategy as in Step 4. In case of fewer than threshold errors, each party encodes the bid/information on the received travel sequence $t'_i$ using the assigned unitary operations to obtain $t''_i$. Finally, he sends this encoded sequence to the next party only after inserting adequate decoy qubits in it. The same process is repeated for the next $(r-3)$ steps until all the $(r-1)$ parties in each sub-circle have encoded their information. In the end, the last party in each sub-circle sends this encoded sequence to the first party who had prepared the quantum state in a secure manner.

Step 7 The auctioneer and all the bidders now possess all the qubits of $l$ entangled states they have prepared. Therefore, they can perform a measurement on the entangled particles from the travel and home qubit sequences in the basis set it was initially prepared. Due to the presence of a bijective mapping between the set of encoding operations of all the bidders and auctioneer and the initial-final entangled state pair, one can easily deduce all the bid values $b_i$s. The auctioneer declares the bidder with the highest bid as the winner of the auction, which can be easily verified by each bidder.

In what follows, we will show an explicit example of the proposed scheme for six bidders and an auctioneer. Before that, it is customary to mention that the proposed
scheme has one unexplored advantage that it can also be performed without an auctioneer. Apart from this, breaking a larger circle into smaller sub-circles not only provides security against participants’ collusion attack, but also reduces the requirement of the number of entangled qubits required. As the preparation and maintenance of a higher-dimensional entangled state are difficult, it becomes significant from the point of view of an experimental realization of the proposed scheme.

Further, as mentioned beforehand that the proposed scheme can be modified to obtain both tree-type and complete-graph-type auction schemes (cf. Fig. 1) as its limiting cases. Specifically, in our scheme, if \( l = n \), i.e., all the bidders are sending their bid information to the auctioneer and all the remaining bidders, it reduces to a complete-graph-type of auction scheme. Additionally, if only the auctioneer prepares the state and receives the bid values from all the bidders (i.e., the bidders do not send bid values among themselves), then a tree-type auction scheme can be deduced.

### 3.1 An example

Here, we show an explicit example in which we suppose there are six bidders \( B_1, B_2, B_3, B_4, B_5, B_6 \) and one auctioneer \( A \). The bidders (\( B_i \)s) want to encode their bid values \( b_i \)s, respectively. To initiate the auction, all the parties agree on a uniform arrangement in a circular manner. They further decide their unitary operations \( U_i \)s to be used for encoding their bids. In this case, without loss of generality, we may assume that the bidders \( B_2, B_4, \) and \( B_6 \) use the unitary operation \( \{ I, iY \} \) for encoding their bids, while \( B_1, B_3, \) and \( B_5 \) use \( \{ I, X \} \). The auctioneer encodes using the unitary operations \( \{ I, Z \} \). It is also predecided that all the parties will perform \( I \) to encode 0, and the other unitary to encode 1.

The working of our example scheme is summarized in the following steps.

**Example-Step1:** In this example, we wish to ensure that fewer than four participants cannot successfully collude to cheat. For the same, the auctioneer (and all bidders) divides all the participants in \( l = 3 \) number of sub-circles in such a way that each sub-circle contains an equal number of bidders, i.e., \( r = \frac{n}{l} + 1 = \frac{6}{3} + 1 = 3 \). One such arrangement is shown in Fig. 2. Note that only \( A \) is present in all the sub-circles; otherwise, all the bidder are a part of only single sub-circle. Namely, the sub-circle \( s_1^A \) contains \( A, B_1, B_2 \); sub-circle \( s_2^A \) contains \( A, B_3, B_4 \); and sub-circle \( s_3^A \) contains \( A, B_5, B_6 \). Using a similar approach, all the bidders make their three sub-circles which are mentioned in detail in Table 1.

**Example-Step2:** The auctioneer prepares three distinct 2-qubit entangled states (Bell states) to receive each bit value of the secret bids from each bidder. For example, he may prepare three copies of \( |\psi^+\rangle = \frac{1}{\sqrt{2}} (|00\rangle + |11\rangle) \). He may also to use different entangled state for each sub-circle, like \( |\psi^+\rangle \) for \( s_1^A \), \( |\psi^-\rangle \) for \( s_2^A \), and \( |\phi^+\rangle \) for \( s_3^A \). In general, he prepares \( 3N \) Bells states independently to receive bids of \( N \)-bits from all the bidders. Then he divides all three sets of \( N \) Bell states (prepared for each sub-circle) to obtain two sequences of all the first qubits \( h_i \) (home qubit sequence) and the second qubits \( t_i \) (travel qubit sequence).

**Example-Step3:** All the bidders also prepare the same number of Bell states and obtain two sequences of home and travel qubits as the auctioneer in previous step.
Fig. 2 (Color online) A circular-type of sealed-bid auction scheme. The auctioneer \((A)\) and all the bidders \((B_i)\) are arranged in a circle shown in smooth (blue) line. The sub-circle \(s_1^A\) is presented by gray (dotted) arrows; sub-circle \(s_2^A\) by green (solid dense) arrows; and sub-circle \(s_3^A\) by yellow (solid semi-transparent) arrows.

Column 2 of Table 2 lists three Bell states; each party prepares to obtain information regarding \(i\)th bit of the secret, like \((|\psi_{s_1}^J\rangle \otimes |\psi_{s_2}^J\rangle \otimes |\psi_{s_3}^J\rangle)\) is the \(i\)th set of Bell states prepared by \(J\)th party.

Example-Step4: The auctioneer sends all three travel qubit sequences \(t_i\)s to the first bidders (i.e., \(B_1, B_3, B_5\)) of all three sub-circles \(s_1^A, s_2^A, s_3^A\), respectively. He also inserts an equal number of decoy qubits randomly in all the traveling sequences. At the same time, all the bidders \(B_i\) also send their \(t_i\)s (after inserting decoy qubits) to the next bidder \(B_{i+1}\) in their respective sub-circles \(s_j^B\). All the recipients now perform the security check with corresponding senders using decoy qubits; and if they find an eavesdropper, they abort the protocol; otherwise, continue the process.

Example-Step5: Each bidder encodes his respective bid \(b_i\), and the auctioneer encodes random bits on \(t_i\), which they had received in the previous step, using a unitary operation \(U\) mentioned in Column 2 of Table 1. In fact, Column 3 of Table 2 lists the operations performed in this round. Though we have not mentioned the transformed state in the table, it is a trivial task to obtain that the auctioneer’s (and similarly other’s) transformed state becomes \((U_{B_1}|\psi_{s_1}^J\rangle \otimes U_{B_3}|\psi_{s_2}^J\rangle \otimes U_{B_5}|\psi_{s_3}^J\rangle)\).

Example-Step6: Subsequently, all the parties send the encoded qubits to the next party in a secure manner, who also performs the unitary operations corresponding to his bit value before sending the travel qubits finally to the party who has
Table 1  The unitary operations assigned to each party along with the arrangement of sub-circles are mentioned explicitly across the auctioneer $A$ and each bidder $B_i$ in an example scheme with one auctioneer and six bidders in a circular sealed-bid auction scheme

| Parties in auction | Unitary operation ($U$) | 1st sub-circle | 2nd sub-circle | 3rd sub-circle |
|--------------------|-------------------------|----------------|----------------|----------------|
| $A$                | $\{I, Z\}$             | $s_1^A = A, B_1, B_2$ | $s_2^A = A, B_3, B_4$ | $s_3^A = A, B_5, B_6$ |
| $B_1$              | $\{I, X\}$             | $s_1^{B_1} = B_1, B_2, B_3$ | $s_2^{B_1} = B_1, B_4, B_5$ | $s_3^{B_1} = B_1, B_6, A$ |
| $B_2$              | $\{I, iY\}$            | $s_1^{B_2} = B_2, B_3, B_4$ | $s_2^{B_2} = B_2, B_5, B_6$ | $s_3^{B_2} = B_2, A, B_1$ |
| $B_3$              | $\{I, X\}$             | $s_1^{B_3} = B_3, B_4, B_5$ | $s_2^{B_3} = B_3, B_6, A$ | $s_3^{B_3} = B_3, B_1, B_2$ |
| $B_4$              | $\{I, iY\}$            | $s_1^{B_4} = B_4, B_5, B_6$ | $s_2^{B_4} = B_4, A, B_1$ | $s_3^{B_4} = B_4, B_2, B_3$ |
| $B_5$              | $\{I, X\}$             | $s_1^{B_5} = B_5, B_6, A$ | $s_2^{B_5} = B_5, B_1, B_2$ | $s_3^{B_5} = B_5, B_3, B_4$ |
| $B_6$              | $\{I, iY\}$            | $s_1^{B_6} = B_6, A, B_1$ | $s_2^{B_6} = B_6, B_2, B_3$ | $s_3^{B_6} = B_6, B_4, B_5$ |

prepared it. The unitary operations performed on the quantum state are mentioned in Column 4 of Table 2. The auctioneer’s quantum state may be written as \( U_{B_3} U_{B_1} |\psi_1^A\rangle \otimes U_{B_4} U_{B_3} |\psi_2^A\rangle \otimes U_{B_6} U_{B_5} |\psi_3^A\rangle \).  

It is interesting to see here that the combined state contains the encoding from all the bidders. Similarly, all the parties hold the message encoded travel qubits and home qubits they have prepared.

Example-Step7: Finally, the auctioneer and all the bidder perform the Bell state measurement on all the three states and extract the information regarding each $U_i$s. Using which a winner may be chosen.

In Table 2, it is important to note that the operation in Round 2 (Column 4) can be obtained from the operation in Round 1 (Column 3), just by performing a cyclic permutation of rows. This can be attributed to the cyclic property of the scheme proposed here.

4 Security and efficiency analysis

A sealed-bid auction process may be viewed as a multiparty computational task, where each party sends inputs to the auctioneer, who computes the value of the function (i.e., maximum/minimum of all the inputs/bids). The security of a multiparty scheme should be ensured both against an outsider and insider attacker. In fact, an insider attacker is more powerful than an outsider attacker. Here, we will show the security of our sealed-bid auction scheme against both types of attacks.

4.1 Outsider’s attacks

As the outsider’s attacks can be checked by using the standard eavesdropping checking techniques (subroutines) discussed in Ref. [32], we just note that it is straightforward to observe that the use of BB84 subroutine or GV subroutine can protect our
scheme at least from the following types of outsider’s attacks: (i) Intercept-resend attack, (ii) Entanglement measure attack, (iii) Disturbance or modification attack, (iv) Impersonation or man-in-the-middle attack, and (v) Trojan-horse attack.

Specifically, in intercept-resend attack Eve may attempt to gain a bidder’s encoding by sending him freshly prepared qubits (equal number of qubits as that of auctioneer has sent him) after intercepting the original qubits from the auctioneer. This will allow her to intercept those encoded qubits when the bidder sends them to the next bidder and by measuring them she can decode the secret bid. Suppose, the legitimate parties incorporate BB84 subroutine to foil this particular attack by using extra qubits prepared randomly prepared in either computational \( |0\rangle \) or diagonal \( \{+\rangle, \{-\rangle \} \) basis. In that case, Eve will introduce detectable errors with probability \( \left(1 - \frac{1}{4^p} \right) \) for \( n \) decoy qubits inserted with message qubits. Due to the decoy qubits inserted with travel qubits, Eve cannot succeed to replace the qubits intended to be sent to a bidder. Therefore, this attack can be circumvented.

Independently, Eve can also attempt entanglement measure attack by entangling her ancilla qubits with the travel qubits instead of replacing them. Suppose Eve prepares ancilla qubits in \( |E\rangle = (\alpha |0\rangle_E + \beta |1\rangle_E \) applies a CNOT with control on ancilla and target on the travel qubits. Security against this attack can be obtained using decoy qubits. As the sender and receiver observe the presence of an eavesdropper by analyzing decoy qubits, which can be obtained as

\[
U_{E_{\text{Eve}}} |E\rangle |t\rangle = \text{CNOT}_E (\alpha |0\rangle_E + \beta |1\rangle_E) |t\rangle |1\rangle_E
\]

which becomes

\[
U_{E_{\text{Eve}}} |E\rangle |t\rangle = \cos \theta (\alpha |00\rangle_E + \beta |11\rangle_E) + \sin \theta (\alpha |10\rangle_E + \beta |01\rangle_E)
\]
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for $|t\rangle = \cos \theta |0\rangle_t + \sin \theta |1\rangle_t$. Without loss of generality, one can assume that all decoy states \{\{0\}, \{1\}, \{+\}, \{-\}\} are equally probable $\frac{1}{4}$. For $\theta = 0 \left( \frac{\pi}{2} \right)$, we can obtain that for $|t\rangle = |0\rangle (|1\rangle)$ a measurement by the receiver in the correct basis would result in $|1\rangle (|0\rangle)$ with probability $|\beta|^2$. However, no such error can be detected in the diagonal basis. Therefore, the average detection probability for Eve for this particular attack can be calculated as $\frac{|\beta|^2}{2}$.

The disturbance or modification attack can be viewed as a denial of service attack; as Eve has no intention to gain the secret information, she rather intends to disturb its content. Incorporation of decoy qubits restricts Eve to interrupt the message qubits selectively, and this attack on decoy qubits will be detected at the receiving end.

Further, to check the impersonation attack, a suitable authentication protocol is used and all the parties may inform regarding sending and receiving of the qubits via an authenticated classical channel. It is also known that there exist suitable technical measures to circumvent trojan-horse attack. Keeping this in mind, we now proceed to discuss insider’s attacks in detail.

4.2 Insider’s attacks

1. Participant attack A single bidder may also try to attack the auction scheme. For example, a bidder $B_i$ may choose to send different bid values to the auctioneer and each voter. As a specific case, consider that the bid value sent to the auctioneer is higher than that sent to the remaining bidders, and the auctioneer chooses him as the winner of the auction and announces his bid value. However, the tally of the remaining bidders will show different value of the bid by that bidder. This may be ascribed as a case of cheating, and the auction will be called off. However, if $B_i$ wishes to disrupt the auction process, he may do so by using this strategy. In summary, a participant will not gain any benefit from this attack as he will always be detected. Therefore, this attack can be viewed as a denial of service attack.

2. Collusion attack Another interesting insider’s attack on a multiparty scheme is a collusion attack. Our scheme being a circular-type scheme of multiparty quantum communication increases the number of parties need to collude to obtain any useful information. To elaborate this point, we may consider that the bidders $B_i$ and $B_{i+m}$ collude to learn the encoding of all the $m$ bidders lying between them in the circle. Best strategy for them would be that $B_i$ would generate an entangled state having an adequate number of qubits, and would send the same number of qubits that $B_i$ has received from $B_{i-1}$ to $B_{i+1}$. He would send the remaining qubits of the entangled states directly to $B_{i+m}$. At a later time, $B_{i+m}$ will receive the set of travel qubits form $B_{i+m-1}$, which would have encoded bidding information of all the $(m - 1)$ bidders’ lying between them. As $B_i$ has already sent the home qubits of the entangled state he had prepared to $B_{i+m}$, he may now use both set of qubits and the knowledge of initial state prepared by $B_i$ to obtain information of all the $(m - 1)$ bidders.

However, this attack will not be as effective as they wish it to be. Specifically, an effective attack that can deterministically affect the outcome of the bidding process would require that $B_i$ and $B_{i+\frac{n}{2}}$ collude, as in that case, the colluding parties would
obtain the bid values of all the other bidders. Precisely, both $B_i$ and $B_i + \frac{n}{2}$ prepare entangled states and send their travel qubits to $B_{i+1}$ and $B_{i+\frac{n}{2}+1}$ and home qubits to $B_{i+\frac{n}{2}}$ and $B_i$, respectively. After $n/2$ rounds, when both $B_i$ and $B_i + \frac{n}{2}$ receive the encoding of all the remaining party, they can choose their appropriate bid values. Further, it is important to note that at least one of the colluding parties will always have access to the travel qubits after they have learnt other’s secret. Therefore, they will leave no traces in this kind of an attack.

Here, the security against this attack is achieved here by breaking the circle into $l$ sub-circles. In this case, if less than $l$ attackers collude, then they cannot cheat the remaining bidders and the auctioneer. Thus, increase in the number $l$ would decrease the effectivity of this attack.

Last but not least, possible collusion attack between a participating bidder and the auctioneer should also be considered. In the proposed scheme, we put all the bidders and the auctioneer on the same footing, i.e., both the bidders and auctioneer obtain information regarding bids at the same time. Therefore, this kind of a collusion attack reduces to that discussed previously for collusion by participants and security against such an attack is already discussed.

### 4.3 Efficiency analysis

There is a quantitative measure to analyze the efficiency of a quantum communication scheme known as the qubit efficiency, proposed in Ref. [33], given by $\eta = \frac{c}{q + b}$. Here, $c$ corresponds to the number of classical bits transmitted with the help of $q$ number of qubits and $b$-bits of classical communication. We will show the efficiency of the example protocol to give an idea about the performance of the proposed scheme. However, qubit efficiency of the proposed scheme can also be calculated using the same approach.

In the example of the proposed scheme, qubit efficiency for sending each bit of bids from all the bidders can be calculated as follows. In sub-circle $s_1^A$, $c_A = 2$ bits of the message regarding bids of $B_1$ and $B_2$ were encoded using a Bell state ($q_1^A = 2$). Additionally, one decoy qubit was inserted by each party in the sub-circle (i.e., $A$, $B_1$, and $B_2$) resulting in total $d_A = 3$. Therefore, total number of qubits used is $q_A = q_1^A + d_A = 2 + 3 = 5$. Further, no classical communication is involved so $b_A = 0$. The same scenario is repeated in sub-circles $s_2^A$ and $s_3^A$. Hence, for all the sub-circles initiated by $A$, the total contribution in $c$ and $q$ is thrice of that calculated for one sub-circle, i.e., 6 and 15, respectively.

In one of the sub-circles initiated by the bidders $B_i$, the auctioneer $A$ does not encode a useful message so it is not counted as classical bit. Therefore, in six sub-circles $\left( s_3^B, s_3^A, s_2^B, s_2^A, s_1^B, s_1^A \right)$, where $A$ is present contribution in $c$ becomes 1 bit, while the number of qubits used remain the same. The remaining 12 sub-circles initiated by the bidders will have the same contribution in the calculations of $c$ and $q$ as that of the sub-circle initiated by $A$. Therefore, the total amount of classical information transmitted is $c = 2 \times (3 + 12) + 1 \times 6 = 36$ bits and the same is done using $q = 5 \times 21 = 105$ qubits, and involving no classical communication $b = 0$ (except for eavesdropping checking). Finally, the auctioneer announces the winner out.
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Table 3  Qubit efficiency of various types of structures of sealed-bid auction schemes

| Task | Structure type                                      | Reference          | Efficiency ($\eta$) (%) | Remarks          |
|------|-----------------------------------------------------|--------------------|-------------------------|------------------|
| Task 1 | Tree structure                                     | Yang et al. [17]   | 30.00                   | $n \gg 2.585$    |
|       | Circular structure with auctioneer                 | Present work       | 34.28                   |                  |
| Task 2 | Complete-graph structure using single photon       | Liu et al. [25]    | 6.74                    |                  |
|       | Complete-graph structure using 7-qubit GHZ state   | Luo et al. [34]    | 6.12                    |                  |
|       | Circular structure with auctioneer                 | Present work       | 5.71                    |                  |
|       | Circular structure without auctioneer              | Present work       | 6.67                    |                  |

Task 1 (2) corresponds to the sealed-bid auction schemes without post-confirmation (with post-confirmation)

The qubit efficiency of a tree-type sealed-bid auction (without post-confirmation) scheme [17] that would do a task similar to what is done in the proposed scheme (however, at the end of the bidding process none of the bidders would obtain the bid of others as it happens in our scheme) is calculated as $\frac{6n}{105n+\log_2 6} \approx 30\%$, which appears to be lower than the qubit efficiency of the proposed scheme. On the contrary, a tree-type scheme would appear more efficient in comparison with the proposed scheme, if we consider that the task (auction process) is restricted to a situation where bidding information is only communicated to the auctioneer. To be specific, we may consider that the information communicated by each bidder to all other bidders is not meaningful information (as far as the task is concerned), and it does not contribute to $c$, which corresponds to meaningful classical information. In that case, the efficiency of the proposed scheme would be $\frac{6n}{89n+\log_2 6} \approx 5.71\%$. In the above discussed situation, qubit efficiency of a sealed-bid auction scheme based on complete-graph structure and proposed to be realized using single photons [25] is obtained as $\frac{6n}{89n+\log_2 6} \approx 7.23\%$, which should be modified to $\frac{6n}{89n+\log_2 6} \approx 6.74\%$ after concatenating some decoy photons for security as mentioned in [35]. Similarly, the efficiency of the sealed-bid auction scheme using 7-qubit GHZ states [34] is calculated as $\frac{6n}{98n+\log_2 6} \approx 6.52\%$, which falls to $\frac{6n}{98n+\log_2 6} \approx 6.12\%$ due to the addition of some decoy photons in accordance with the scheme proposed in Ref. [35].

There is an interesting feature of the proposed scheme—it can be performed without the auctioneer. This is so because each party obtains bidding information of the others in the post-confirmation stage. If bidding is performed without an auctioneer in that case the qubit efficiency of the scheme would have been $\frac{6n}{90n} = 6.67\%$, which is
comparable with the qubit efficiency of the schemes that use complete-graph structure. In brief, our scheme is only slightly less efficient compared to the schemes based on complete-graph structure (cf. Table 3), but it provides higher security against collusion attacks and requires quantum resources that are easier to prepare.

5 Conclusion

We proposed a scheme for quantum sealed-bid auction, which is free from the limitations of the existing schemes for the same purpose proposed by other groups, and have shown that the proposed scheme is not only secure it is also efficient. The advantages of the present scheme are actually obtained by transforming a complete-graph structure to a circular structure and subsequently transforming that to sub-circles. This unique strategy and the benefits obtained by adopting this strategy are expected to open up a new window for more research related to quantum auction for a couple of reasons. Firstly, sealed-bid auction is an extremely important process in our daily life; secondly, with an increase in $l$ (the number of sub-circles) the size of the entangled state required reduces, whereas the security against collusion attack increases. This trade-off has a fantastic effect as at present preparation and maintenance of $s$-qubit entangled state is very difficult when $s$ is large. However, in our case smaller $s$ leads to better security in one hand and scalability on the other. It seems feasible that the proposed scheme can be realized experimentally using available technologies, but this was not the case with most of the other proposals as $s$ was very high for them. Further, auction process demands security and quantum version of the same can provide unconditional security which is not possible by any of its classical counterparts. This unconditionally secure protocol for sealed-bid auction is essentially obtained by modifying a scheme for multiparty circular quantum key agreement. This observation leads to a possibility of future investigation on designing similar schemes by modifying other schemes of group communication like Xu et al.’s scheme of multiparty quantum key management [36]. Here, it would be apt to note that in the practical realization of the scheme, the noise would affect the result. However, the effect of various types of noise on the scheme proposed here can be studied easily using the methods adopted in Refs. [37–40] and references therein. Further, tree-based structures are often used in classical schemes for multiparty secure communication [41,42], and an effort can be made to obtain their quantum counterparts and to check whether the change is the graph structure (as is done in the present study) improves security in other cases, too. Specially, it would be interesting and relevant to explore the possibility of designing quantum counterparts for the classical schemes designed in the context of encrypted cloud data [41–43]. The relevance of such future investigations in the line of the present work has recently been amplified with the introduction of IBM quantum experience [44], a 5-qubit superconductivity-based quantum computer that is placed in the cloud. From the above-mentioned facts and possibilities in mind, we conclude this paper with an expectation that the works reported here will be realized experimentally and will find applications in daily life.
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