1. INTRODUCTION

Gravitational waves emitted during gravitational collapse carry away linear momentum, causing the center of mass of the collapsing object to recoil (Peres 1962; Bekenstein 1973). In the case of binary black holes, coalescence can be accompanied by kicks as large as \( \sim 200 \, \text{km s}^{-1} \) if the holes are non-spinning prior to coalescence, increasing to \( \sim 4000 \, \text{km s}^{-1} \) in the case of maximally spinning, equal-mass holes with the optimum orientations (e.g., Campanelli et al. 2007; González et al. 2007; Herrmann et al. 2007; Koppitz et al. 2007; Baker et al. 2008; Schnittman et al. 2008).

Recently, the first compelling candidate for a recoiling SMBH was discovered (Komossa et al. 2008). The quasar SDSS J092712.65+294344.0 at \( z = 0.7 \) exhibits two separate sets of unusual emission lines: a set of broad emission lines, presumably associated with gas bound to a SMBH, and a second set of atypically narrow emission lines. The broad lines are shifted by \( \sim 2650 \, \text{km s}^{-1} \) relative to the narrow lines. This is one of the key predicted signatures of a recoiling SMBH, which would carry with it the gas from the broad-line region while leaving the bulk of the narrow-line gas behind (Merritt et al. 2006; Bonning et al. 2007).

Gas near to the SMBH is expected to respond to the kicks in a number of other, potentially observable ways. Accretion activity may be temporarily interrupted during the final phases of binary coalescence but would restart thereafter with a certain time delay (Liu et al. 2003; Milosavljević & Pinney 2005). The recoiling SMBH would then appear, temporarily, as a “quasar” that is spatially offset from the core of its host galaxy (Madau & Quataert 2004; Loeb 2007). UV (Lippai et al. 2008), soft X-ray (Shields & Bonning 2008), and IR flaring (Schnittman 

2. DISRUPTION RATE OF BOUND STARS

A recoiling SMBH carries with it a cloud of stars on bound orbits. We first compute the mass of this cloud (expressed as a fraction \( f_b \) of the SMBH’s mass); then we compute the rate at which the stars would be scattered into the SMBH’s tidal disruption sphere.

Figure 1a shows the steady-state distribution of bound stars following an instantaneous kick of magnitude \( V_k \), assuming an initial, power-law density profile \( \rho = \rho_0 (r/r_b)^{-\gamma} \) around the SMBH. Stars initially at distances \( r \geq r_h \equiv G M_{\text{BH}}/V_k^2 \) from the SMBH will be unbound after the kick. For \( V_k \equiv 0.4V_{\text{esc}}, i.e., \), large enough to remove the SMBH from the galaxy core (Gualandris & Merritt 2008; \( V_{\text{esc}} \) is the escape velocity), \( r_h \) is small compared to the SMBH influence radius and stars that follow the SMBH will be moving on essentially Keplerian orbits both before and after the kick. The final distribution can be computed uniquely from the initial distribution by randomizing the orbital phases of the stars that remain bound. Beyond \( r_h \) the cloud consists of an elongated, \( \rho \sim r^{-4} \) envelope containing stars that were pushed into nearly unbound orbits by the kick (Fig. 1b).

The bound mass must scale as \( M_b \sim \rho(r_b) r_b^3 \approx \rho_0 r_b^3 (G M_{\text{BH}}/r_b V_k^2)^{3-\gamma} \). Choosing for \( r_b \) the (prekick) influence radius \( r_b \) of the SMBH, defined as the radius containing a mass in stars equal to twice \( M_{\text{BH}} \), this becomes

\[
f_b \equiv \frac{M_b}{M_{\text{BH}}} = F(\gamma) \left( \frac{G M_{\text{BH}}}{r_b V_k^2} \right)^{3-\gamma};
\]
we find $F(\gamma) \approx 11.6 \gamma^{-1.75}$, $0.5 \leq \gamma \leq 2.5$. An alternative form is $f_{\text{kick}} \equiv G(\gamma)M^2_{\text{BH}}/V_{\text{esc}}^3 (V/k_{\text{esc}})^{2(\gamma-3)}$, where $f_{\text{kick}} \equiv f_{\text{kick}}/10^{-3}$, $M_{\text{BH}} \equiv M_{\text{BH}}/10^7 M_\odot$, $r_{\text{kick}} \equiv r_{\text{kick}}/10$ pc, $V_{\text{esc}} \equiv V_{\text{esc}}/10^3$ km s$^{-1}$, and $G(\gamma) = (0.048, 0.22, 1.6, 15)$ for $\gamma = (0.5, 1, 1.5, 2)$. For kick velocities in the range of interest, equation (1) predicts bound masses of order of the SMBH mass, 1% (Milosavljević & Merritt 2003) and coalesce. In lower mass (e.g., Merritt 2006); however it is not clear whether binary galaxies, $\approx$, binary evolution can continue to $10^{-5}$ in a large, low-density galaxy produces a flat core, $\approx$, galaxies (e.g., Drinkwater et al. 2003).

The appropriate value for $\gamma$ is the density slope just prior to the kick, and after the binary SMBH has completed its inspiral; $\gamma$ is likewise defined at this time. Slow inspiral of the SMBHs in a large, low-density galaxy produces a flat core, $\gamma \approx 0.5$ (e.g., Merritt 2006); however it is not clear whether binary SMBHs in such nuclei can overcome the “final parsec problem” (Milosavljević & Merritt 2003) and coalesce. In lower mass galaxies, $M_{\text{gal}} \approx 10^{10} M_\odot$, binary evolution can continue to coalescence in $\leq 10$ Gyr via collisional loss-cone repopulation; in this case the prekick density profile would have $\gamma \approx 1.75$ (Merritt et al. 2007). Rapid, gas-driven inspiral would tend to preserve the initial density profile and might even steepen it via star formation. Henceforth we take $\gamma = 1$ as a fiducial value.

In standard loss-cone theory, stars are scattered into the tidal disruption sphere $r \leq r_{\text{kick}}$ at a rate $\sim N(\gamma)/(t_{\text{kick}} \ln (r_{\text{kick}}))$ where $N(\gamma)$ is the number of stars within $r$ and $t_{\text{kick}}$ is the local (non-resonant) relaxation time. The total disruption rate from the cloud of bound stars would be

$$N_{\text{diss}} \approx C_{\text{diss}}(\gamma) \frac{\ln \Lambda}{\ln (r_{\text{kick}}/r_{L})} \frac{V_{\text{esc}}}{r_{\text{kick}}} f_{\text{kick}}^2,$$

with $\ln \Lambda \approx \ln (M_{\text{BH}}/m_\star)$ the Coulomb logarithm; the ratio of logarithmic terms is of order unity. Equation (2) assumes that gravitational encounters are uncorrelated; however, near the SMBH, orbits are slowly precessing Keplerian ellipses and “encounters” are highly correlated (Rauch & Tremaine 1996), shortening the effective, angular momentum relaxation time by a factor $\sim m_\star N(\gamma)/M_{\text{BH}}$ (Hopman & Alexander 2006). The contribution of this “resonant relaxation” to tidal disruption rates of SMBHs embedded in galaxies is small since most of the disrupted stars come from orbits with $r \approx r_{\text{kick}}$. However in our case, stars beyond $\sim r_{\text{kick}} \ll r_{\text{kick}}$ were removed by the kick and loss-cone repopulation is dominated by resonant scattering, yielding

$$N_{\text{RR}} \approx C_{\text{RR}}(\gamma) \frac{\ln \Lambda}{\ln (r_{\text{kick}}/r_{L})} \frac{V_{\text{esc}}}{r_{\text{kick}}} f_{\text{kick}}^2,$$

i.e., $N_{\text{RR}} \gg N_{\text{diss}}$ for $f_{\text{kick}} \approx 1$.

The coefficient $C_{\text{RR}}$ in equation (3) is poorly determined (Rauch & Ingalls 1998; Hopman & Alexander 2006). Because the number $M_{\text{BH}}/m_\star$ of stars remaining bound to a recoiling SMBH is relatively small, tidal disruption rates in this regime can be computed directly via sufficiently accurate $N$-body integrations. Figure 1c shows the results of a series of such experiments starting from initial conditions generated from the steady-state (nonspherical and anisotropic) distribution of Figure 1a, realized with $N = 1.5 \times 10^4$ particles and various values of $m_\star$. We used the hybrid $N$-body code $\phi$GRAPEch (Harfst et al. 2008) running on the RT GRAPE cluster (Harfst et al. 2007). Stars were initially removed from the model if their periapsis fell below $10^{-4}r_{\text{kick}}$, the assumed radius of the disruption sphere; the model was then integrated forward and stars that approached the SMBH particle at distances $\leq r_{\text{kick}}$ were recorded and removed from the integration. The $N$-body integrations confirmed the $N \propto f_{\text{kick}}$ dependence predicted by equation (3). In the adopted units ($G = M_{\text{BH}} = V_{\text{esc}} = 1$), the disruption rate is $\sim 0.15f_{\text{kick}}^2$, assuming that $\Lambda \approx r_{\text{kick}}/(2Gm_\star/V_{\text{esc}}^2) \approx M_{\text{BH}}/m_\star$ in the simulations then implies $C_{\text{RR}}(\gamma = 1) \approx 0.14$.

1 We note that the various other conditions required for resonant relaxation to be present in these simulations were satisfied; i.e., the integration times were long compared with orbital precession times and most of the stars were in the diffusive, as opposed to pinhole, loss-cone regime.

1 The logarithmic factor is associated with diffusive loss-cone repopulation which is appropriate in the “empty loss cone” regime near a SMBH (e.g., Lightman & Shapiro 1977).
Using the derived value of $C_{\text{BH}}$, equation (3) predicts for the disruption rate of bound stars ($\gamma = 1$)

$$\dot{N}_b \approx 6.5 \times 10^{-6} \text{yr}^{-1} M_\star^3 V_{\text{clump}}^3 f_{b,3},$$

(4)

here and below, In $\Delta / \ln (\gamma / \sigma)$ has been set to 2. Combining equation (4) with equation (1) for the bound mass gives

$$\dot{N}_b \approx 1.5 \times 10^{-6} \text{yr}^{-1} M_\star^3 \rho_\star^3 V_{\text{clump}}^3.$$ 

(5)

Figure 2 (solid lines) plots $\dot{N}_b$ for SMBHs ejected from the centers of two representative galaxies, with masses $4.5 \times 10^{10}$ and $1.5 \times 10^8 M_\odot$; the galaxies were modeled as Prugniel-Simien (1997) (i.e., deprojected Sérsic) spheroids with Sérsic indices (4, 2.5). How do the rates we estimate compare with flare rates of nonrecoiling SMBHs? The steady-state stellar disruption rates of SMBHs embedded in galactic nuclei are predicted to be in the range $10^{-5} \leq N \leq 10^{-4} \text{yr}^{-1}$ for $M_{\text{BH}} \approx 10^7 M_\odot$ (e.g., Wang & Merritt 2004, Fig. 5b). For $V_{\epsilon} \approx 10^4 \text{km s}^{-1}$, rates derived here are roughly an order of magnitude lower. Were resonant relaxation not effective, this ratio would be $\sim 10^4$ rather than $\sim 10$.

Ignoring changes in the shape of the density profile of the bound population with time, the disruption rate is predicted to drop off as $\dot{N} = \dot{N}(0) \exp (-t/\tau)$, $\tau \approx 3.6 G M_{\text{BH}}^2 / V_{\epsilon}^3 M_\star$. This decay is included in the curves of Figure 2. In fact, after $\sim 1$ (nonresonant) relaxation time, the density would evolve to the Bahcall-Wolf $\rho \propto r^{-7/4}$ slope and maintain this profile as its amplitude decayed; this complication was ignored in the curves of Figure 2.

We note that any postmerger galaxy would likely be highly inhomogeneous. Tidal disruption rates could be temporarily increased during close encounters of the SMBH to a massive perturber, e.g., a giant molecular cloud, in much the same way that “comet showers” are triggered by near passage of the solar system to a star (Hills 1981). The same is true for the rate of disruption of unbound stars ($\S$ 3) if the SMBH passes through a dense clump.

3. CONTRIBUTION OF UNBOUND STARS

Even a naked SMBH encounters stars as it traverses a galaxy (Kapoor 1976). Unbound stars are deflected into the tidal disruption sphere at an instantaneous rate

$$\dot{N}_{\text{unb}} \approx 2 \pi G M_{\text{BH}}(\rho/r_\star) V_{\epsilon}^3,$$

$$\approx 1.7 \times 10^{-9} \text{yr}^{-1} \rho(r/R_\star)_{\text{core}} M_\star^{7/3} V_{\epsilon}^{-1} r_{\text{eff},11},$$

(6)

where $\rho$ is the local (stellar) density, $\bar{\rho} \equiv \rho(r/R_\star)$, $R_\star$ is the galaxy effective (projected half-light) radius, $\rho_{\text{core}} \equiv \rho(r/R_\star) / M_\odot \text{pc}^{-3}$, and $r_{\text{eff},11} \equiv R_\star / 10^{11} \text{cm}; V_\epsilon \equiv V_\epsilon(r)$ is the instantaneous SMBH velocity. Figure 2 (dashed lines) shows $N_{\text{unb}}(r)$ in two representative galaxy models. (The curves in Fig. 2 include the correction factor of Danby & Camm [1957] that accounts for the decrease in the capture rate when $V_\epsilon \approx \sigma$, with $\sigma$ the stellar velocity dispersion.) In the larger of the two galaxy models considered, of order 10 flaring events are predicted from both bound and unbound stars for $V_\epsilon = 10^4 \text{km s}^{-1}$ during the time required for the SMBH to travel from the core to the half-mass radius; in the smaller galaxy, $N_{\text{unb}} \ll N_b$.

4. OBSERVABILITY

We have shown that rates of stellar disruption by recoiling SMBHs are interestingly high; only moderately lower than those of SMBHs in the cores of galaxies. For a typical galaxy, we predict $\sim 20$ flares as the SMBH travels through the central parts of the galaxy (50% of these from bound stars, 50% from unbound stars), and $\sim 10^2$--$10^3$ more after the SMBH has left the galaxy. Now, we discuss observational consequences.

4.1. Powerful Off-Nuclear X-Ray Flares and Feedback Trails

Stellar tidal disruptions appear as luminous X-ray flares which reach quasar-like luminosities and then decline on month–year timescales (e.g., Komossa & Bade 1999). Tidal flares from recoiling SMBHs would look similar but would occur off-nucleus, and would be easily identified since no other known mechanism produces quasar-like luminosities far from the nucleus.

Future X-ray all-sky surveys like eROSITA (Predehl et al. 2006) and EXIST (Grindlay 2005) will be sensitive to these types of events. While sky surveys are most suited to identifying the candidates, follow-up observations with high spatial resolution ($\sim 0.5$; Chandra) will then allow the measurement of spatial offsets from the galaxy core. If the flare is associated with jet emission, radio observations would greatly improve the positioning accuracy. With future spectroscopic instrumentation, we will be able to determine the line-of-sight recoil velocity directly, if emission lines form in the temporary accretion disk of the stellar debris. Such instrumentation will be available aboard XEUS, its NFI reaching 3 eV resolution at 6 keV (Turner et al. 2007).

Before the eROSITA mission will start its all-sky survey, a deep survey of the nearest clusters of galaxies will be performed (G. Hasinger 2008, private communication). This will provide an excellent opportunity to detect the nearest flare events. Disruption
rates of unbound stars are highest in the galaxy core (Fig. 2); in order to identify these as off-nuclear events, high spatial resolution is required (at the distance of the Virgo cluster, 1” corresponds to \( \sim 100 \) pc). The long timescales for damping of orbital oscillations (Gualandris & Merritt 2008) when \( V_e \lesssim V_{esc} \) will increase the chances of detecting an offset. In addition, the abundance of “fossil” intracluster recoiling SMBHs from past mergers (Volonteri 2007) would be especially high in clusters.

While the brightest phases of a tidal disruption flare might only last a few years or decades, interaction with the environment would leave a feedback trail along the path of the SMBH. If radio jets are temporarily formed (Wong et al. 2007), they would create local cavities in the ISM, similar to but smaller than the X-ray cavities that have been observed in nearby ellipticals (e.g., Biller et al. 2004; Wang & Hu 2005). The bright flare will also excite emission lines in any surrounding gas. Even though these lines would be faint given the short time span of the energy input, low-density gas would retain memory of the flare for a long time (e.g., for a particle density \( n = 10 \) cm\(^{-3}\), the hydrogen recombination timescale is on the order of 10,000 yr). This way, quasar-like emission lines could be produced far off-nucleus.

In addition to stellar disruption and related observational signatures, mass loss from evolving bound stars (e.g., Kudritzki & Puls 2000) will provide episodic low-level fuelling of the SMBH, causing repeated episodes of X-ray emission and variability. Assuming a mass-loss rate of a massive star of \( 10^{-4} M_\odot \) yr\(^{-1}\) of which 1% will be accreted in the most efficient mode with a radiative efficiency of \( \eta = 0.37 \) and radiating mostly in X-rays would result in an X-ray luminosity of \( L_x \approx 10^{40} \) erg s\(^{-1}\).

4.2. Intergalactic Flares and Other Signatures

Tidal disruptions continue long after the recoiling SMBH (with \( V_e > V_{esc} \)) has left its host galaxy and wanders in intergalactic space (Fig. 2). Such events will appear as luminous flares without host galaxies. Furthermore, the bound stars will undergo stellar evolution and will therefore ultimately produce intergalactic planetary nebulae, and intergalactic supernovae (SNe) of Type I.\(^3\) While the most massive recoiling SMBHs (\( M > 10^7 M_\odot \)) would generally not disrupt solar-type stars, they would still partially disrupt giant stars, and they would also show repeated phases of activity from fuelling due to stellar mass loss. As such they might hide among the “blank field sources” with bright X-ray emission but no optical counterpart (e.g., Cagnoni et al. 2002) if the absence of optical hosts persists in deeper optical imaging.

After several Gyr, stars will start turning into white dwarfs, and SMBHs which left their galaxies long ago will have a cloud of white dwarfs, neutron stars, and stellar mass BHs bound to them. While the white dwarf tidal radius is inside the Schwarzschild radius except for low (\(< 10^{-6} M_\odot \)) BH masses, Dearborn et al. (2005) have shown that relativistic compression causes white dwarfs to ignite and explode as SN at distances up to \( 100 R_s \) from a SMBH. Part of the disrupted white dwarf will escape while the rest will eventually be accreted; both the SN signal and the accretion phase would allow us to detect such fossil intergalactic SMBHs.

In summary, we have discussed observational signatures of recoiling black holes related to the bound (and unbound) stellar population. All these signals would generically be associated with recoiling SMBHs, whether or not an accretion disk is present initially, and they would continue episodically for a time of \( \sim 10 \) Gyr.



3 We note that high-mass stars that produce Type II SNe evolve quickly; within \( t < 100 \) Myr, a SMBH moving at \( 1000 \) km s\(^{-1}\) has reached 100 kpc. Therefore, most of the Type II SNe would be produced when the SMBH has not yet escaped into intergalactic space. These events might be identified from their large velocities with respect to their host galaxy.

This work was supported by grants AST 04-20920 and AST 04-37519 from the NSF and grant NNG04GJ48G from NASA.