Discovery of geometry transition in nearby AGN jets

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ABSTRACT

Observational studies of collimation in jets in active galactic nuclei (AGN) are a key to understanding their formation and acceleration processes. We have performed an automated search for jet shape breaks in a sample of 367 AGN using VLBA data at 2 cm and 22 cm. This search has found ten nearby jets at redshifts \( z < 0.07 \) with a transition from a parabolic to conical shape, while the full sample is dominated by distant AGN with a typical \( z \approx 1 \). The ten AGN are UGC 00773, NGC 1052, 3C 111, 3C 120, TXS 0815−094, Mrk 180, PKS 1514+00, NGC 6251, 3C 371, and BL Lac. We conclude that the geometry transition may be a common effect in AGN jets. It can be observed only when sufficient linear resolution is obtained. Supplem enting these results with previously reported shape breaks in the nearby AGN 1H 0323+342 and M87, we estimate that the break typically occurs at a distance of \( 10^5 – 10^6 \) gravitational radii from the nucleus. We suggest that the jet shape transition happens when the bulk plasma kinetic energy flux becomes equal to the Poynting energy flux, while the ambient medium pressure is assumed to be governed by Bondi accretion. Our model predictions on the jet acceleration and properties of the break point are found to be supported observationally.

Key words: galaxies: jets – galaxies: active – radio continuum: galaxies – quasars: general – BL Lacertae objects: general

1 INTRODUCTION

Understanding the physical processes that determine the formation, acceleration and collimation of relativistic jets in active galactic nuclei (AGN) continues to be among the most challenging problems of modern astrophysics. There are a wide variety of analytical and numerical models for jet acceleration and its confinement (e.g., Vlahakis & Königl 2003; Beskin & Nokhrina 2006; McKinney 2006; Komissarov et al. 2007; Tchekhovskoy et al. 2011; McKinney et al. 2012; Potter & Cotter 2015) that consider different solutions for jet shapes, such as cylindrical, conical and parabolic. General relativistic magnetohydrodynamic simulations (e.g., McKinney et al. 2012) predict that a jet starting from its apex has a parabolic streamline within the magnetically dominated acceleration zone and transitions to a conical geometry at outer scales associated with equipartition between energy densities of the magnetic field and the radiating particle populations. It has been shown for cold jets that acceleration should not happen in a conical jet. This requires something akin to a parabolic jet shape closer to the jet base to allow differential expansion (Vlahakis & Königl 2004; Komissarov 2012).

In order to investigate these theories it is important to collect observational data on jet profile shapes for a large enough sample of AGN whose properties are well understood. The first observational evidence for a transition from parabolic to conical jet shape was detected in M87 (Asada & Nakamura 2012) at a distance of about 900 mas near the feature HST-1, about 70 pc in projection, corresponding to \( 10^7 \) Schwarzschild radii. A few more studies of nearby AGNs to probe their innermost jet regions were performed recently: Mkn 501 (Giroletti et al. 2008), Centaurus A (Müller et al. 2014), Cygnus A (Boccardi et al. 2016; Nakahara et al. 2019), NGC 6251 (Tseng et al. 2016), 0323+342 (Hada et al. 2018), 3C 273 (Akiyama et al. 2018),...
In a previous work (Pushkarev et al. 2017), we analyzed parsec-scale radio VLBI images of jets in 362 active galaxies. This sample is dominated by compact radio bright blazars with jet at a small angle to the line of sight and a typical redshift of one. However, some low luminosity nearby radio galaxies were also observed. Pushkarev et al. (2017) show that while the majority of resolved jets indeed have a shape close to conical, a significant fraction of the sample has observed deviations. A systematic change in jet width profile has been noted by Hervet et al. (2017), who explain it by using a stratified jet model with a fast spine and slow but relatively powerful outer layer. In this paper, we investigate if this outcome is partly affected by the typical finite resolution of VLBI observations. We probe a possible dependence of the jet shape on the distance \( r \) to the nucleus. Furthermore, we perform a systematic search for a possible transition from one jet shape to another on the basis of 2 cm and 22 cm VLBA images.

The observation of sources with the change in a jet shape from the parabolic to conical may provide an instrument to probe the MHD acceleration mechanism models as well as the ambient medium conditions. The change in jet shape in M87 (Asada & Nakamura 2012) coincides with the stationary bright feature HST-1, which can be associated with the change in ambient pressure profile and appearance of a recollimation shock due to pressure drop and jet abrupt expansion. This interpretation is supported by the measurements of external medium pressure by Russell et al. (2015) almost down to the Bondi radius, with an observed pressure profile \( P \propto r^{-3} \). The recently observed jet shape in 0323+342 (Hada et al. 2018) demonstrates a similar behavior. On the other hand, there are models predicting a jet shape transition for a single power law pressure profile. The analytical model by Lyubarsky (2009) predicts the transition from parabolic to conical form for certain regimes, as well as quasi-oscillations in jet shape in the conical domain. This solution has been applied to the reconstruction of the recollimation shock properties of M87 by Levinson (2017), with a predicted total jet power on the order of \( 10^{45} \text{erg/s} \). The recent semi-analytical results for the warm jet matching the ambient medium with a total electric current closed inside a jet by Beskin et al. (2017) predicts a change in a jet shape from parabolic to conical for the Bondi pressure profile \( P \propto r^{-2} \). In this work we follow the latter model and consider the results for a warm outflow in more detail.

The structure of the paper is the following: section 2 presents our results of a search for the jet profile change from parabolic to conical in a large sample of AGN jets, we interpret our findings in section 3. We summarize our work in section 4. Throughout this paper we will use the term “core” as the apparent origin of AGN jets that commonly appears as the brightest feature in VLBI images of blazars (e.g., Lobanov 1998; Marscher 2008). We adopt a cosmology with \( \Omega_m = 0.27 \), \( \Omega_b = 0.73 \) and \( H_0 = 71 \text{ km s}^{-1} \text{ Mpc}^{-1} \) (Komatsu et al. 2009).

2 A DISCOVERY OF SHAPE TRANSITION IN AGN JETS

2.1 Automated search of candidates with a change in jet geometry

For the purposes of our study, we made use of data at 15 GHz from the MOJAVE program, the 2-cm VLBA Survey, and the National Radio Astronomy Observatory (NRAO) data archive (Lister et al. 2018) for those sources that have at least five VLBA observing epochs at 15 GHz between 1994 August 31 and 2016 December 26 inclusive. We used the 2 cm VLBA total intensity MOJAVE stacked epoch images supplemented by single epoch 22 cm VLBA images to derive apparent jet widths \( (d) \) as a function of projected distance \( r \) from the jet core, and determined jet shapes similar to Pushkarev et al. (2017). In that work we fitted the \( d-r \) dependence with a single power law \( d \propto r^k \). The index is expected to be \( k \approx 0.5 \) for a quasi-parabolic shape and 1.0 for a conical jet. We note that even single-epoch observations at 1.4 GHz adequately reproduce source morphology, i.e., effectively fill jet cross-section due to a steep spectrum of synchrotron emission of the outflow, with a typical spectral index ~0.7 measured between 2 and 8 GHz (Pushkarev & Kovalev 2012) and ~1.0 between 8 and 15 GHz (Hovatta et al. 2014), making the low-frequency observations sensitive enough to probe jet morphology at larger scales.

We have carried out a similar analysis allowing for a change in the jet shape. Using all available data (15 GHz only or combined data set at 15 and 1.4 GHz) for each source, we performed a double power law fit of the jet width as a function of distance, dividing the jet path length in a logarithmic scale by two parts in proportion of 1:1, 1:2 and 1:3 to search for cases when the fitted \( k \)-index at inner scales was 0.5 ± 0.2, while at outer scales it was 1.0 ± 0.2. After such cases were identified automatically, we tuned the fits by setting the distance of the transition region by eye.

We ended up dropping 36 AGN jets as having unsatisfactory fits caused by either (i) non-optimal ridge line reconstruction for jets with strong bending, (ii) numerous large gaps in jet emission, (iii) too short a jet length (iv) low intensity regions not captured well by our jet width fitting. This resulted in a sample comprising 331 AGN jets.

As a result of this analysis, we found a shape transition in ten jets (Table 1) out of 367 analyzed. We emphasize that all the AGNs with detected transition of the jet shape turned out to have low redshifts \( z < 0.07 \), i.e., have a high linear resolution of VLBI observations. This is highly unlikely to occur by chance and provides additional strong evidence that this result is not an observational artifact but a real effect. See discussion of the rest of analyzed low redshift AGN in the sample in subsection 2.4. Among the ten sources, there is one, the radio galaxy 0238−084 (NGC 1052), that shows a two-sided jet morphology. For this object, we analyzed the approaching, brighter outflow propagating to north-east direction, determining the position of a virtual VLBI core using a kinematic-based minimization method described in Vermeulen et al. (2003).
Figure 1. Jet profile with a sign of transition from parabolic to conical shape in ten well resolved nearby active galaxies. Dependence of the jet width on projected distance to apparent jet base is shown. The cyan and orange dots show measurements at 15 and 1.4 GHz, respectively. The red and black stripes represent Monte Carlo fits for jet regions before and beyond the jet shape transition region, respectively. The projected distance is shown in mas for targets with unknown redshift and in pc for known redshift cases. General properties of these AGN are presented in Table 1, parameters of the fits — in Table 2, parameters of the shape transition region — in Table 3.
Following our discovery of the shape transition preferentially occurring in nearby AGN, we supplemented our initial AGN sample of 362 targets from Pushkarev et al. (2017) with stacked images of five more low-z AGN which had five or more 15 GHz VLBA observing epochs after the Pushkarev et al. analysis was finished. These were: 0615−172, 1133+704, 1200+608, 1216+061, 1741+196. All the stacked images are available from the MOJAVE database.

### 2.2 Rigorous fitting of the jet shape

For each of the found 10 sources with a sign of jet geometry transition, we fit the data with the following dependencies: $d = a_1(r + r_0)^b$ and $d = a_2(r + r_1)^{b_2}$, describing a jet shape before and after the break. Here $r_0$ is understood as the separation of the 15 GHz apparent core from the true jet origin due to the synchrotron opacity (e.g., Lobanov 1998; Pushkarev et al. 2012a), while $r_1$ shows how much one underestimates the jet length if it is derived from the data only beyond the geometrical transition of the jet. We note that this approach is more accurate but more computationally intensive than that used by Pushkarev et al. (2017) and applied in the original selection of jet break candidates. It is needed in order to better fit for jet shape close to the apex.

We fit these dependencies with Bayesian modeling using the NUTS Markov Chain Monte Carlo sampler based on the gradient of the log posterior density. It was implemented in PYMC3 (Salvatier et al. 2016), which automatically accounts for uncertainties of all the parameters in further inferences. The best fit parameters are listed in Table 2, showing that initially the jet is quasi-parabolic with $k_1$ close to 0.5, while beyond the break point region the outflow manifests a streamline close to conical, with $k_2 \approx 1$. The location of the jet shape break given in Table 3 is estimated as the intersection point of these two $d = r$ dependencies. Note that Table 3 includes results on the jet shape transition region for two more sources, 0321+340 and 1228+126 (M87) taken from Hada et al. (2018) and Nokhrina et al. (2019), respectively.

For other sources without a detected shape break, we fit a single power-law $d = a(r + r_0)^b$ for consistency. Objects with unreliable ridge line detection or patchy structure in images (15 sources) and those with nonphysical $d = r$ dependence (24 sources) were excluded after visual inspection. They constitute only about one tenth of the dataset and thus the exclusion should not bias our estimates. To account for increased uncertainties of jet width measurements further from the core, the power law model is complemented as following:

$$d = \begin{cases} 
  a(r + r_0)^b + N(0, \sigma_1^2), & \text{if } r < R \\
  b + N(0, \sigma_2^2), & \text{if } r > R
\end{cases}$$

Here all of $a, r_0, k, R, \sigma_1, b, \sigma_2$ are treated as unknown parameters and inferred simultaneously using a Nested Sampling algorithm as implemented in PolyChord (Handley et al. 2015). As expected, $\sigma_2$ is typically significantly larger than $\sigma_1$. We find that this model generally captures the $d = r$ dependence and its uncertainty well. Fitting results are given

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**Table 1.** Source properties for 12 sources with the detected jet shape break. Ten of them are from this study (Figure 1), they are supplemented by 0321+340 (Hada et al. 2018) and M87 (Asada & Nakamura 2012). The full online only table adds properties of MOJAVE-1 sources for which redshift and Doppler factor estimates are available, the full table also contains references. Columns are as follows: (1) B1950 name; (2) alias; (3) optical class, where Q = quasar, B = BL Lac, G = radio galaxy, N = Narrow Line Seyfert 1 (NLSy1); (4) redshift; (5) maximum apparent radial speed from Lister et al. (2019); (6) variability Doppler factor from Hovatta et al. (2009); (7) viewing angle; (8) black hole mass estimated basing on assumption of virialized broad lines region (BLR) movement and correlation between the size of BLR and UV/optical luminosity; (9) black hole mass estimated by a stellar velocity dispersion method and associated fundamental plane method (for 2200+420).

| Source ID         | Alias     | Opt. ID | z     | $\beta_{app}$ (c) | $\delta$ (deg) | $M_{BH}$ (log($M_\odot$)) | $M_{BH}$ (log($M_\odot$)) |
|-------------------|-----------|---------|-------|-------------------|----------------|--------------------------|--------------------------|
| 0321+340          | 1H 0323+342 | N       | 0.005 | 0.42              | 0.3           | 49.0                     | 5.51                     |
| 0430+052          | 3C 120    | G       | 0.033 | 5.27              | 2.1           | 18.7                     | 7.52                     |
| 0851−094          | TXS 0851+094 | G       | 0.045 | 5.0              | 19.0           | 7.3                      | 8.13                     |
| 1133+704          | Mrk 180   | B       | 0.045 | 5.0              | 19.0           | 7.3                      | 8.21                     |
| 1228+126          | M87       | G       | 0.004 | 14.0              | 5.0           | 9.82                     | 8.78                     |
| 1514+004          | PKS 1514+00 | B       | 0.045 | 5.0              | 15.0           | 7.3                      | 8.51                     |
| 1637+826          | NGC 6251  | G       | 0.024 | 18.0              | 5.0           | 8.78                     | 8.23                     |
| 2200+420          | BL Lac    | B       | 0.060 | 7.2               | 7.1           | 8.78                     | 8.23                     |

Note. León Tavares et al. (2014) argue that using the single-epoch spectra scaling relations might lead to the significant black hole mass underestimation for 0321+340, they provide an independent estimate of $10^{8.6}M_\odot$.

a. Doppler factor is from Liodakis et al. (2017).

b. Assumed $\theta$ value as typical for BL Lacs.

c. Assumed $\theta$ value as typical for radio galaxies in the list which do not show a strong counter-jet.

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1. http://www.physics.purdue.edu/MOJAVE/
### Table 2. Derived best-fit parameters of the two fitted dependencies $d = a_1(r + r_0)^{k_1}$ and $d = a_2(r + r_0)^{k_2}$ before and after the jet break, respectively. The online only table presents properties of a single fit for 319 AGN without detected shape break, their $k$ values are presented in Figure 2.

| Source       | $a_1$            | $r_0$            | $r_0$        | $k_1$ | Band   | $r_{\text{min}}$ | $r_{\text{max}}$ |
|--------------|------------------|------------------|--------------|-------|--------|------------------|------------------|
| 0111+021     | 0.179 ± 0.010    | 0.143 ± 0.085    | 0.157 ± 0.093| 0.497 ± 0.077 | U      | 0.2              | 1.5              |
| 0238–084     | 0.078 ± 0.007    | 0.074 ± 0.038    | 0.740 ± 0.380| 0.391 ± 0.048 | U      | 0.3              | 2.5              |
| 0415+379     | 0.305 ± 0.011    | 0.042 ± 0.020    | 0.044 ± 0.021| 0.468 ± 0.026 | U      | 0.2              | 6.0              |
| 0430+052     | 0.202 ± 0.015    | 0.122 ± 0.071    | 0.188 ± 0.109| 0.556 ± 0.070 | U      | 0.2              | 2.0              |
| 0815–094     | 0.294 ± 0.015    | ...              | 0.163 ± 0.048| 0.527 ± 0.044 | U      | 0.2              | 1.0              |
| 1133+704     | 0.437 ± 0.013    | 0.061 ± 0.046    | 0.069 ± 0.052| 0.528 ± 0.040 | U      | 0.3              | 1.5              |
| 1514+004     | 0.171 ± 0.011    | 0.189 ± 0.088    | 0.189 ± 0.088| 0.564 ± 0.048 | U      | 0.2              | 3.5              |
| 1637+826     | 0.155 ± 0.005    | 0.098 ± 0.044    | 0.204 ± 0.092| 0.506 ± 0.041 | U      | 0.2              | 3.0              |
| 1807+698     | 0.207 ± 0.016    | 0.130 ± 0.089    | 0.133 ± 0.091| 0.388 ± 0.087 | U      | 0.2              | 1.4              |
| 2200+420     | 0.505 ± 0.029    | 0.087 ± 0.096    | 0.067 ± 0.074| 0.537 ± 0.057 | U      | 0.9              | 2.0              |

* Redshift unknown.

### Table 3. Derived parameters of the jet shape break for 10 AGN with addition of 0321+340 adopted from Hada et al. (2018) and M87 from Nokhrina et al. (2019). Columns are as follows: (1) source name (B1950); (2) jet width at the break in mas; (3) jet width at the break in pc; (4) projected distance of a break from the apparent core along the jet in mas; (5) projected distance of a break from the BH along the jet in mas; (6) projected distance of a break from a BH along the jet in pc; (7) deprojected distance of a break from a BH along the jet in pc, the parameter uses value of a viewing angle; (8) presence of a bright low pattern speed feature (Lister et al. 2019).

| Source       | $d_{\text{break}}$ | $r_{\text{break}}$ | $r_{\text{break}}^{\text{proj}}$ | $r_{\text{break}}^{\text{proj}}$ | $r_{\text{break}}^{\text{proj}}$ | $r_{\text{break}}^{\text{proj}}$ | Stationary |
|--------------|---------------------|---------------------|-----------------------------------|-----------------------------------|-----------------------------------|-----------------------------------| jet feature |
|              | (mas)               | (pc)                | (mas)                             | (mas)                             | (mas)                             | (pc)                             |            |
| 0111+021     | 0.30 ± 0.03         | 0.28 ± 0.03         | 2.46 ± 0.27                       | 2.62 ± 0.29                       | 2.38 ± 0.26                       | 27.31 Y            |
| 0238–084     | 0.53 ± 0.05         | 0.50 ± 0.05         | 2.93 ± 0.57                       | 3.73 ± 0.65                       | 0.37 ± 0.06                       | 0.49 ...           |
| 0321+340     | 1                   | 1.16                | 10                                | 10.04                            | 11.64                            | 106.07 Y           |
| 0415+379     | 0.78 ± 0.03         | 0.74 ± 0.03         | 7.03 ± 0.50                       | 7.07 ± 0.50                       | 6.72 ± 0.47                       | 29.00 ...          |
| 0430+052     | 0.45 ± 0.06         | 0.29 ± 0.04         | 2.67 ± 0.40                       | 2.85 ± 0.41                       | 1.85 ± 0.27                       | 5.77 ...           |
| 0815–094     | 0.37 ± 0.05         | ...                 | 1.37 ± 0.30                       | 1.54 ± 0.30                       | ...                              | ...                |
| 1133+704     | 0.57 ± 0.02         | 0.50 ± 0.02         | 1.39 ± 0.09                       | 1.46 ± 0.10                       | 1.29 ± 0.09                       | 14.80 ...           |
| 1228+126     | 15.73 ± 0.39        | 2.42 ± 0.06         | ...                               | 273 ± 12                         | 21.0 ± 0.9                       | 87.0 Y ...          |
| 1514+004     | 0.34 ± 0.02         | 0.34 ± 0.02         | 3.10 ± 0.22                       | 3.39 ± 0.30                       | 3.39 ± 0.30                       | 13.10 Y            |
| 1637+826     | 0.32 ± 0.02         | 0.16 ± 0.01         | 1.92 ± 0.26                       | 2.13 ± 0.28                       | 1.02 ± 0.13                       | 3.30 ...           |
| 1807+698     | 0.26 ± 0.04         | 0.25 ± 0.04         | 1.53 ± 0.33                       | 1.67 ± 0.34                       | 1.63 ± 0.33                       | 12.83 Y            |
| 2200+420     | 0.74 ± 0.03         | 0.95 ± 0.04         | 2.45 ± 0.10                       | 2.52 ± 0.13                       | 3.25 ± 0.16                       | 24.57 ...           |

* This refers to the well-known HST-1 feature (Chang et al. 2010) which is normally not sampled by MOJAVE images.

in Table 2 and the source distribution of exponents $k$ is shown in Figure 2. Even though the estimates for individual sources have a large spread, the median exponent is very close to 1. This indicates a conical average outflow shape, and agrees with previous results using slightly different estimation method (Pushkarev et al. 2012b). We note the peak in the histogram bin at $k = 0.5$ which corresponds to the parabolic jet shape; the number of objects with $k \approx 0.5$ is not high enough in the sample to make it significant (Table 2).
Table 4. Angular size of the VLBA core at 15 GHz, $d_{\text{MC}}$, and its offset from the true jet origin, $r_0$, derived by our Monte Carlo modeling of the jet width compared with independent MOJAVE core size measurements in the visibility plane, $d_{\text{MC}}$, and the core offset, $r_0$, estimated from the multi-frequency core shift measurements (Pushkarev et al. 2012a). The shown $d_{\text{MC}}$ value is a median over all epochs from Lister et al. (2019).

| Source     | $d_{\text{MC}}^c$ (mas) | $d_{\text{MC}}^w$ (mas) | $r_0$ (mas) | $r_0^c$ (mas) |
|------------|--------------------------|--------------------------|-------------|---------------|
| 0111+021   | 0.075 ± 0.025            | 0.079 ± 0.030            | 0.157 ± 0.093 | 0.159 ± 0.050 |
| 0238–084   | 0.282 ± 0.067            | 0.284 ± 0.042            | 0.740 ± 0.380 | ...           |
| 0415+379   | 0.075 ± 0.017            | 0.075 ± 0.008            | 0.044 ± 0.021 | 0.275 ± 0.050 |
| 0430+052   | 0.096 ± 0.034            | 0.182 ± 0.014            | 0.188 ± 0.109 | 0.051 ± 0.050 |
| 0815–094   | 0.113 ± 0.020            | 0.062 ± 0.041            | 0.163 ± 0.048 | ...           |
| 1133+704   | 0.116 ± 0.049            | 0.089 ± 0.048            | 0.072 ± 0.056 | ...           |
| 1514+004   | 0.067 ± 0.018            | 0.043 ± 0.027            | 0.189 ± 0.088 | ...           |
| 1637+826   | 0.100 ± 0.025            | 0.069 ± 0.004            | 0.204 ± 0.092 | 0.198 ± 0.050 |
| 1807+698   | 0.096 ± 0.031            | 0.067 ± 0.007            | 0.133 ± 0.091 | 0.240 ± 0.050 |
| 2200+420   | 0.105 ± 0.064            | 0.044 ± 0.003            | 0.067 ± 0.074 | 0.090 ± 0.050 |

Figure 2. A histogram of the best fit exponents $k$ assuming a single power law $d \sim (r + r_0)^k$ for all spatial scales. Shown here are 319 sources from Table 2, which exclude those listed in this table with detected shape break.

23 Checking consistency of the fits and analyzing for possible biases

We complemented our analysis using 43 GHz data from the Boston University (BU) AGN group$^2$ for two sources, BL Lac and 3C120, which are present in the MOJAVE and BU samples. For each of these sources we (i) produced stacked total intensity maps, aligning single epoch-images by the position of the VLBA core derived from structure modeling of the visibility data, (ii) determined the reconstructed jet ridge line, and (iii) fitted the transverse jet width as a function of distance from the core. Both sources showed a jet shape transition at core separations comparable to those found from the 15 GHz data.

By setting $r = 0$ we can estimate the apparent core size $d_{\text{MC}}$ at 15 GHz from the Monte-Carlo fit of the jet width as $a_1 r_0^c$ and compare it with a median value of the core size $d_{\text{MC}}^c$, derived from structure modelfit in visibility plane taken from Lister et al. (2019), see Table 4. Two sources, 0111+021 and 0415+379, show a good agreement between $d_{\text{MC}}^c$ and $d_{\text{MC}}^w$, while for the other seven objects $d_{\text{MC}}^w$ is somewhat larger than $d_{\text{MC}}^c$. This is likely due to a non-ideal determination of the core position throughout the epochs, which is used to align single-epoch maps to produce stacked images. The radio galaxy 0430+052 is the only source having $d_{\text{MC}}^c < d_{\text{MC}}^w$ for reasons that are unclear.

A bias related to this effect might affect the results. Statistically analyzing jet shapes for the whole sample of 331 sources with stacked VLBA images by introducing different ridge line path length limits we have found the following. A near-parabolic streamline for quasars and BL Lacs can be derived if the innermost jet, only up to $\sim 1$ mas from the apparent core, is considered. This is not a real effect. The bias is found to be the most pronounced for curved jets or jets with features emerging at different position angles over time (Lister et al. 2013). This is confirmed by the found artificial correlation of median jet width with the number of epochs in a stacked image for such AGN. Uncertainties in the core position also contribute to this effect due to the non-ideal alignment of images while performing the stacking. Variability of opacity conditions and apparent position of the core (Plavin et al. 2019a) affect this partially even though the alignment of the stacked single epoch images is done on the core position. Together, it causes an additional artificial widening of the jet near the core region up to distances $r \approx 0.3$ mas. The effect quickly vanishes at larger scales. Thus, if we exclude jet width measurements at distances $\leq 0.4$ mas, the effect becomes much weaker and disappears completely if we rule out the measurements within 0.5 mas from the core. We also note that radio galaxies, being at low redshift and thus having apparently wider outflows, are much less subject to this effect. The same is true for the sources with a jet shape break shown in Figure 1, as these are low-redshift objects. Only for BL Lac, as the most remote source among them and also having a bright quasi-stationary component near the core (Cohen et al. 2014), we put a conservative limit of 0.9 mas, while for the other sources we used the non-cut intervals listed in Table 2, because dropping measurements at $r < 0.5$ mas did not significantly change the fit parameters. For the rest of the sources, we

https://www.bu.edu/blazars/VLBAproject.html
have dropped all measurement for $r < 0.5$ mas while analyzing the data.

Another possible problem might be related to cases where the jet width is completely unresolved. Indeed, this was found for some AGN targets at some epochs from the visibility model fitting of the core (e.g., Kovalev et al. 2005; Lister et al. 2019). This issue is taken care of by the requirement to drop all measurement for $r < 0.5$ mas. Interestingly, the rest of the measured deconvolved jet width values are always positive. If we assume that this is some sort of a positive bias overestimating the width, it should not depend on $r$ for unresolved jets and will result in $k$ values close to zero. This behavior was not seen in our fitting results.

We have also compared the fitted parameter $r_0$ with the core offset from the jet base estimated from the core shift measured between 15 and 8 GHz (Pushkarev et al. 2012a) assuming an inverse frequency dependence $r \propto \nu^{-1}$. These quantities, also listed in Table 4, agree well within the errors in four out of six sources having measured core shifts. The large discrepancy for two sources can be explained by the recently recently established phenomenon of significant core shift variability (Plavin et al. 2019a) or the difference between the true jet shape derived by us and the assumed conical jet shape in (Pushkarev et al. 2012a). We note that this result opens a new way to estimate the distance to the true jet origin which does not require an assumption regarding the jet geometry.

We warn readers about deriving jet shapes from structure model fitting of single-epoch data, as the jet may appear quasi-parabolic ($k < 1$) up to a certain (typically short) distance from the core and then change its shape to conical ($k \approx 1$). This effect takes place in the sources that show variations in their inner jet position angle. Lister et al. (2013) established this as a common, decade-timescale phenomenon for the most heavily monitored AGNs in the MOJAVE sample. Thus, single-epoch VLBI maps may not reveal the whole jet cross-section, but rather a portion of it, especially in the inner jet regions where images are dynamic range limited. Therefore, the conclusions on jet geometry based strictly on a model fit approach should be treated with caution.

### 2.4 Jet shape transition: a common effect in AGN jets, its consequences and prospects

We have found evidence for geometry transition in many jets for which sufficient linear resolution was achieved. This means that a change in jet shape is a common phenomenon which has significant consequences for many high angular resolution astrophysical and astrometric studies. It is difficult to conclude if the geometry transition with measured properties is specific to only nearby radio galaxies and BL Lacs, or can be extended to the AGN class in general. The radio luminosities of the nearby ($z < 0.07$) jets are much lower than the rest of the sample and this might affect the geometry and transition zone. We note that Figure 3 presents a consistent picture of the power index dependence on the downstream distance for nearby and distant jets.

In total, indications of the transition from parabolic to conical shape are found in 10 out of 29 nearby ($z < 0.07$) sources observed as part of the MOJAVE program or by other investigators (VLBA archival data from the latter were processed by the MOJAVE team). The reasons for non-detection of the geometry transition in nearby AGN jets are varied. Some jets, e.g., 0007+106 and 1959+650, have too compact structure to study the shape. Some others, e.g., 0241+622, 0316+413, 1216+061, show purely parabolic streamlines (Table 2), and their transition regions are expected at larger angular scales than those probed by our observations. MERLIN or low-frequency VLBA observations are needed. For example, the nearby radio galaxy 1216+061 ($z = 0.0075$, scale factor 0.15 pc mas$^{-1}$) not shown in Figure 3 has a parabolic streamline with $k = 0.64 \pm 0.05$ up to 7 mas at 15 GHz, corresponding to a deprojected distance of about 1 pc only. We are studying the remaining 12 low-redshift jets that show no sign of a profile break in a followup program that has been approved by the VLBA.

The other jets in the sample (Table 2), namely 97%, do not show a clear significant change in jet geometry. We explain this by (i) a large scale factor of the order of 8 pc mas$^{-1}$ for a typical source in the sample at a redshift of $z \sim 1$ and (ii) a small viewing angle typically about several degrees (Pushkarev et al. 2017). Jet power may also play a role, since the MOJAVE sample is flux-density limited and the AGN with $z > 0.1$ typically have jet luminosities $\sim 2$ orders of magnitude higher than the lower-redshift ones. The jets with a detected shape change have an average scaling factor of 0.7 pc mas$^{-1}$ and, on average, larger viewing angle since 6 out of 12 are radio galaxies. Thus, if a transition region is located at a distance of a few tens of pc, it corresponds to a projected angular separation of $\lesssim 1$ mas from the apparent jet base at 15 GHz, which is comparable to the typical interferometric restoring beam size. VLBI observations at higher frequencies may be more effective in registering the jet shape transition, since they provide a better angular resolution and are less subject to opacity effects. This would...
probe scales closer to the jet apex and possible dependencies between acceleration zone extension and the maximum bulk Lorentz factor or jet power, as predicted by Potter & Cotter (2015). On the other hand, the steep spectrum of the optically thin jet emission hinders the tracing of the jet for long distances. The small viewing angles of the bright AGN jets set another limit on any jet shape investigation in the innermost parts. The streamline of an outflow can be studied down to distances at which the jet half-opening angle is still smaller than viewing angle. As shown by Pushkarev et al. (2017), the intrinsic jet opening angle reaches values of a few degrees at scales of the order of 10 pc. This suggests that the jet shape transition phenomenon might be more effectively studied for nearby AGNs that are oriented at larger angles to the line of sight. After considering all the points discussed above, we have started a dedicated VLBA program in 2019 to search for geometry transition in 61 AGN jets with $z < 0.07$ from observations at 15 and 1.4 GHz.

It is a challenging problem to estimate the consequences of this result on astrometry and astrophysics of AGN. VLBI astrometry delivers the position of the true jet apex only if the opacity driven core shift $r$ is proportional to the frequency $\nu$ as $r \propto \nu^{-1}$ (Porcas 2009). However, this is expected only for conical jets and synchrotron opacity (Lobanov 1998). A non-conical jet base results in an extension of the true jet length between the apex and the observed opaque core. This also produces somewhat larger VLBI-Gaia offsets for AGN positions (Kovalev et al. 2017; Plavin et al. 2019b) than predicted by Kovalev et al. (2008).

2.5 Deprojected position of the jet break

We chose the MOJAVE-1 sample (Lister et al. 2009) to perform a direct comparison with the 12 jets showing the breaks. Our reasoning is as follows. Most of MOJAVE-1 targets were observed by VLBA not only at multiple 15 GHz epochs but also in a single epoch at 1.4 GHz, which increases the jet distance being probed by our analysis. In addition, VLBI measurements of the apparent kinematics $\beta_{\text{app}}$ (Lister et al. 2019) and variability Doppler factor estimates $\delta$ (Hovatta et al. 2009; Liodakis et al. 2017) are available for a large fraction of the sample. We need this information to derive deprojected distance values. These requirements result in a sample of 65 sources (Table 1) described in Pushkarev et al. (2017).

We derived viewing angle estimates through the relation

$$\theta = \arctan \frac{2 \beta_{\text{app}}}{\beta_{\text{app}}^2 + \delta^2 - 1}$$

to convert the jet distance from angular projected to linear deprojected. Note that this assumes the same beaming parameters for the flux density variability and jet kinematics. For $\beta_{\text{app}}$ we used the fastest non-accelerating apparent jet speeds from the MOJAVE kinematic analysis. For 0321+340 we use $\theta = 6.3^\circ$, based on the observed superluminal motion (Lister et al. 2016) assuming $\theta = \gamma^{-1} = (1 + \beta_{\text{app}})^{-0.5}$. The other possible viewing angle value for this target $\theta = 4^\circ$ based on variability time scale is discussed by Hada et al. (2018). For the BL Lac objects 0111+021 and 1133+704 we assumed a viewing angle of $5^\circ$, typical for this class of AGN (Hovatta et al. 2009; Savolainen et al. 2010; Pushkarev et al. 2017; Liodakis et al. 2017). For the radio galaxy 1514+004 we assumed a viewing angle of $15^\circ$.

In Figure 3, we plot the corresponding single power-law $k$-index values derived from the 15 and 1.4 GHz VLBA data (Pushkarev et al. 2017) versus deprojected distance from the 15 GHz VLBA core for 62 sources. There are eleven sources among them with known deprojected linear jet distance that have a jet shape transition (Figure 1, Table 1). They are all shown by a pair of points each from the double power-law fits. The BL Lac object 0815−094 is not shown in Fig. 3, as it does not have a measured spectroscopic redshift. Our results on jet shape transition in nine targets (Table 2, Table 3, Figure 3) are supplemented by multi-frequency data for M87 from Nakamura et al. (2018), with $k_1 = 0.57$, $k_2 = 0.90$, and break point position obtained by Nakirina et al. (2019). For M87 we adopt $\theta = 14^\circ$ (Wang & Zhou 2009), consistent with more recent results by Mertens et al. (2016). For NLSy1 0321+430 we use 1.4–2.3 GHz measurements from VLBA observations (Hada et al. 2018), with $k_1 = 0.6$ and $k_2 = 1.41$, for which the jet shape break point position is estimated.

Horizontal lines represent the scales at which $k$-indices were derived, starting from several tens of mas distance from the 15 GHz VLBA core (see subsection 2.3) and up to jet distances limited by the sensitivity of our observations. The nearby jets, for which we are probing closer to the central engine, have low $k$ values and show a transition from quasi-parabolic values at small scales to quasi-conical at larger scales (Figure 3). It is possible that at larger scales of a few hundred kiloparsec, where jets become diffuse and disruptive, their geometry further changes from conical to hyperbolic, characterized by more rapid expansion (Owen et al. 2000).

In order to plot the observed $k$-index values as a function of the deprojected distance along jets in gravitational radius $r_g = 2GM/c^2$ units, we use the
black hole masses estimated assuming virialized broad lines region (BLR) motion and correlation between BLR size and UV/optical luminosity (Torrealba et al. 2012; McLure & Jarvis 2002; Vestergaard & Peterson 2006; Landt et al. 2017; Palma et al. 2011; Shaw et al. 2012; Liu et al. 2006). We also have masses inferred by stellar or gas kinematics methods (e.g., Woo & Urry 2002) for the closest sources. The mass values and references can be found in Table 1. Results are presented in Figure 4. It turns out, that the sources with BH masses obtained by stellar velocity dispersion method or stellar/gas kinematics measurements, are the subset of the sources with the detected jet shape break, being the closest ones.

Since estimating the black hole mass is a complicated and strongly model-dependent method, some of the values might be significantly in error. By dropping the highest and lowest values as possible outliers of the derived jet break position \( r_{\text{break}} \) measured in \( r_g \) we are able to bound its values in the narrower range \( r_{\text{break}} \in (10^6, 10^8) r_g \). This is an important result, especially when taken together with our finding that the jet shape transition may be a common phenomenon in nearby or even most of the AGN.

We note the following. The black hole mass of 0321+340 is suspected to be underestimated (León Tavares et al. 2014; Hada et al. 2018). If we use for this source the mass \( M = 10^8 M_\odot \), obtained using the relation between black hole mass and bulge luminosity (León Tavares et al. 2014), 0321+340 yields \( r_{\text{break}} = 2.7 \times 10^9 r_g \), falling within the discussed above range of \( r_{\text{break}} / r_g \) distances. This may provide an additional argument favoring a higher black hole mass for this source.

3 MODELING RELATIVISTIC JET WITH A SHAPE BREAK

3.1 Qualitative consideration

Both analytical (see below) and phenomenological (Potter & Cotter 2013, 2015) considerations as well as numerical simulations (Komissarov et al. 2009; Tchekhovskoy et al. 2009; Porth et al. 2011) suggest that for moderate initial magnetization of a jet \( \sigma_M \sim 10^{-10} \), where

\[
\sigma_M = \frac{\Omega^2 \gamma_0}{8 \pi^2 \mu_0 c^2}
\]

is the Michel magnetization parameter, the flow transits from a magnetically dominated regime at small distances \( r \) from the origin to a particle dominated regime at larger distances. Here \( \gamma_0 \) and \( \Omega \) are the total magnetic flux and characteristic angular velocity of the “central engine” respectively. Accordingly, \( \mu = m_e c^2 + m_e u \) is the relativistic enthalpy, where \( u \) is the nonrelativistic enthalphy, and \( m_e \) is a particle mass. Here we assume a lepton jet, so \( m_e \) is the electron mass. Below for simplicity we consider not so large temperatures, so that \( u \ll c^2 \). Finally, \( \eta \) is the particle-to-magnetic flux ratio.

Indeed, the physical meaning of the Michel magnetization parameter is the maximum Lorentz factor \( \gamma \) of the hydrodynamical flow when all the electromagnetic energy flux is transferred to particles. On the other hand, for quasi-cylindrical jets the following asymptotic solution for magnetically dominated flow exists (see e.g., Beskin 2009)

\[
\gamma(r_{\perp}) = \frac{r_{\perp}}{R_{\text{te}},}
\]

where \( R_t = c/\Omega_0 \) is the light cylinder radius, and \( r_{\perp} \) is the distance from the jet axis. For the black hole spin parameter \( a_* = 0.5, R_t \approx 7.4 \times 10^5 (M_{\text{BH}}/10^9 M_\odot) \) cm \( \approx 6.8 \times 10^{-4} (M_{\text{BH}}/10^9 M_\odot) \) pc. For observed pc scale jets, the jet width \( d \) reaches 1 pc, so that \( d/2R_t > \sigma_M \). This means that the flow cannot be still magnetically dominated. As was shown by Nokhrina et al. (2015) who have analysed about 100 AGN jets, \( \sigma_M \approx 10 \) is a reasonable value constrained by the observations.

The observed median value of 1.02 for k-index clearly points to ballistic plasma motion, suggesting the jet is dominated by the plasma bulk motion kinetic energy rather than by the Poynting flux, as expected close to the launching region.

For this reason we aim to explain the break in the \( d(r) \) dependence as the consequence of a transition from the magnetically dominated to the particle dominated regime. Below we present the main results of our semi-analytical consideration. Our goal is in evaluating the dependence of the jet width \( d \) on an ambient pressure profile \( P_{\text{ext}}(r) \). The results for the cold jet are presented in Beskin et al. (2017), while here we consider the semi-analytical results for a warm outflow.

3.2 Semi-analytical model

Basic equations describing the internal structure of relativistic and nonrelativistic jets within the Grad-Shafranov approach are now well-established (Heyvaerts & Norman 1989; Pelletier & Pudritz 1992; Lery et al. 1998; Beskin & Malyshekin 2000; Beskin 2009; Lyubarsky 2009). This approach allows us to formulate the problem of finding a stationary axisymmetric magnetohydrodynamic outflow structure, a jet solution, as a set of two differential equations on a magnetic flux function \( \Psi \) and an Alfvénic Mach number \( M \). These equations are Bernoulli equation and Grad–Shafranov equation of a force balance perpendicular to magnetic surfaces. The approach allows us to determine the internal structure of axisymmetric stationary jets knowing in general case five “integrals of motion”, i.e., energy \( E(\Psi) \) and angular momentum \( L(\Psi) \) flux, electric potential which connects with angular velocity \( \Omega(\Psi) \), entropy \( s(\Psi) \), and the particle-to-magnetic flux ratio \( \eta(\Psi) \). All these values are to be constant along magnetic surfaces \( \Psi = \text{const} \). Once the Grad–Shafranov and Bernoulli equations solved for the given integrals, all the other flow properties, such as particle number density, four-velocity, electric current, and Lorentz factor, can be determined from algebraic equations (see, e.g., Beskin (2009)). In particular, it was shown that a jet with total zero electric current can exist only in the presence of an external medium with finite pressure \( P_{\text{ext}} \). Thus, it is the ambient pressure \( P_{\text{ext}} \) that is expected to determine the transverse dimension of astrophysical jets. In general, it is a complicated problem to solve the set of Bernoulli and Grad–Shafranov equations. Additional complication is connected with the change of a system type from elliptical to hyperbolic. So, to tackle the problem different simplifications are introduced. In this
paper we simplify the problem, assuming the flow is highly collimated and can be described within the cylindrical geometry, in which case it can be solved numerically (Beskin & Malyshkin 2000).

On the other hand, careful matching of a solution inside the jet with the external medium has not been produced up to now. The difficulty arises with having a very low energy density of the external medium in comparison with the energy density inside the relativistic jet. For this reason, in most cases an infinitely thin current sheet was introduced. Moreover, an ambient pressure was often modelled by homogeneous magnetic field $B_0^2/8\pi = P_{\text{ext}}$. Neither an external magnetic field nor infinitely thin current sheet are assumed. The main new aspect involves the boundary conditions at the jet boundary $r_+ = d/2$. In what follows we propose to construct a solution so that both magnetic field and flow velocity vanish at the jet boundary $r_+ = d/2$. On the other hand, we suppose that the flow remains supersonic, so that Alfvénic Mach number $M(d/2) > 1$.

As was shown by Beskin & Malyshkin (2000), for a cylindrical flow it is convenient to reduce the Grad-Shafranov and Bernoulli equations to two first-order ordinary differential equations for magnetic flux $\Psi(r_\perp)$ and poloidal Alfvénic Mach number $M(r_\perp)$:

$$M^2 = \frac{4\pi \mu n^2}{\eta}.$$  \hfill (3)

Here $n$ is an electron number density in the co-moving reference frame. The first of this set of equations is the relativistic Bernoulli equation $u_n^2 = \gamma^2 - u_0^2 - 1$ which has the form

$$\frac{M^2}{64\pi^2 r_\perp^4} \frac{d\Psi}{dr_\perp}^2 = \frac{K}{r_\perp^2 M - \mu^2 \eta^2}.$$  \hfill (4)

Here

$$A = 1 - \frac{\Omega_\psi^2 r_\perp^2}{c^2} - M^2$$

is the Alfénic factor,

$$K = r_\perp^2 \left( e^2 A - M^2 \right) + M^4 r_\perp^2 E^2 - M^4 L^2 c^2,$$

and $e' = E - \Omega_\psi L$. The second equation on the Mach number is as follows:

$$\left[ \frac{(e')^2}{\mu^2 \eta^2} - 1 + \frac{\Omega_\psi r_\perp}{c^2} - A c^2 \right] \frac{dM^2}{d\Omega_\psi} + \frac{M^6 L^2 c^2}{Ar^2 \mu^2 \eta^2} - \frac{M^6 r_\perp^2}{2c^2} \frac{d\Psi}{d\Omega_\psi} \frac{d^2\Omega_\psi}{d\Psi^2} - M^2 \left[ \frac{2 - \frac{(e')^2}{\mu^2 \eta^2}}{\mu^2 \eta^2 \frac{d\Omega_\psi}{d\Psi}} + \frac{M^2 e'}{\mu^2 \eta^2 \frac{d\Omega_\psi}{d\Psi}} \frac{d\Psi}{d\Omega_\psi} \frac{d\Psi}{d\Omega_\psi} \right]$$

$$- \frac{A}{n} \left( \frac{dP}{d\Omega_\psi} \right)_n + \left( 1 - \frac{\Omega_\psi^2 \mu^2}{c^2} \right) T \frac{M^2}{\mu} \frac{d\Psi}{d\Omega_\psi} \frac{d\Omega_\psi}{d\Psi}.$$  \hfill (7)

Here $T$ is the temperature, $c_0$ is the sound velocity defined as $c_0^2 = (dP/\rho_n)_{\mu, \rho_0}$, $\rho_0$ is a particle mass, and $P$ is a gas pressure. As a result, Bernoulli Equation 4 and Equation 7 form the system of two ordinary differential equations for the Mach number $M^2(r_\perp)$ and the magnetic flux $\Psi(r_\perp)$ describing cylindrical relativistic jets.

As it was already stressed, the solution results from our choice of five integrals of motion. It is important that by determining the functions $M^2(r_\perp)$ and $\Psi(r_\perp)$ one can find the jet radius $d/2$ as well as the profile of the current $I(r_\perp)$, the particle energy, and the toroidal component of the four-velocity from a solution of a problem under consideration. In particular, as

$$\frac{I}{2\pi} = \frac{L - \Omega_\psi r_\perp^2 E/c^2}{1 - \Omega_\psi^2 r_\perp^2 /c^2 - M^2}.$$  \hfill (8)

the condition of the closing of the electric current within the jet $I(\Psi_0) = 0$ can be rewritten as $L(\Psi_0) = 0$ and $\Omega_\psi(\Psi_0) = 0$, where $\Psi_0$ is the given total magnetic flux in a jet.

For this reason in what follows we use the following expressions for these integrals

$$L(\Psi) = \frac{\Omega_\psi \Psi}{4\pi^2} \left( 1 - \frac{\Psi}{\Psi_0} \right),$$

$$\Omega_\psi(\Psi) = \frac{\Omega_\psi}{\Psi_0} \left( 1 - \frac{\Psi}{\Psi_0} \right).$$  \hfill (9, 10)

In the vicinity of a rotation axis these integrals correspond to the well-known analytical force-free solution for a homogeneous poloidal magnetic field. On the other hand, they both vanish at the jet boundary, which guarantees the fulfilment of the condition $I(\Psi_0) = 0$. For a linearly decaying magnetic field $B(\delta r_\perp) \propto (d/2 - \delta r_\perp)$ this root dependence on $\Psi$ ensures the linear decay of the integrals $L(\Psi(r_\perp))$ and $\Omega_\psi(\Psi(r_\perp))$ with respect to a transverse distance.

The analytical analysis of Equation 4 and Equation 7 provides that both magnetic field $B$ and flow four-velocity $u$ vanish at the jet boundary $r_\perp = d/2$. As one can see from Equation 8, our choice of $L(\Psi)$ and $\Omega_\psi(\Psi)$ guarantees that toroidal components $B_\theta$ vanish at the jet boundary. One can find that the same holds for the toroidal velocity $u_\theta$. On the other hand, using relation $n u_\eta = \eta B_\theta$, one can conclude that the conditions $u_\eta = 0$ and $B_\theta = 0$ can be realised for finite $n$ and $\eta$. For simplicity we consider here the case $\eta(\Psi) = \eta = \text{const}$.

Below we assume that the flow remains supersonic up to the very boundary: $M(d/2) > 1$. This assumption allows us to simplify our consideration. Indeed, in this case Equation 7 has no additional singularity at the Alfvénic surface $A = 0$ in the vicinity of the jet boundary. We check this assumption numerically, and we find that it holds for our solution.

Further, as is shown in Beskin et al. (2017), energy integral $E(\Psi)$ can be written down as

$$E(\Psi) = \Omega_\psi - \mu \eta \gamma(\Psi).$$  \hfill (12)

Here $\gamma(0) = \gamma_{\text{in}}$ is the injection Lorentz-factor along the jet axis, and $\gamma(\Psi_0) = 1$. The last relation just implies that the flow velocity vanishes at the jet boundary. As to relativistic enthalpy $\mu_{\text{jet}} = m_p c^2 + m_e \gamma_{\text{jet}}$, we consider it here as a constant with $\gamma_{\text{jet}} = c^2_{\text{jet}}/(\Gamma - 1)$, where the constant $c^2_{\text{jet}}$ corresponds to sonic velocity of a flow at the very boundary. For simplicity in this paper we use polytropic equation of states with polytropic index $\Gamma$. The relativistic enthalpy $\mu$ in Equation 4 and Equation 7 is determined via local value $w = c^2_\perp/(\Gamma - 1)$ (see below). Finally, in what follows we assume

$$s(\Psi) = s = \text{const}.$$  \hfill (13)

Here the entropy $s$ enters the polytropic equation of state.
as \( P(n, s) = k(s)n^r \) with the function \( k(s) \) which particular form is not used explicitly in the following equations. It is this nonzero value that allows us to match magnetically dominated flow to the external media with finite pressure.

In addition to five integrals of motion, the system of \( \text{Equation 4} \) and \( \text{Equation 7} \) requires two boundary conditions. The first one is the obvious condition at the symmetry axis
\[
\Psi(0) = 0. \tag{14}
\]
As to the second one, it can be found from the pressure balance \( P_{\text{ext}} = \Gamma^{-1}c_{\text{jet}}^2n_{\text{jet}} \), where \( n_{\text{jet}} \) is the number density of a flow at the jet boundary. Thus, we can determine the jet radius \( d/2 \) as a function of \( P_{\text{ext}} \) self-consistently as the condition \( \psi(d/2) = 0 \) and \( \mathbf{b}(d/2) = 0 \) are automatically fulfilled at \( \Psi = \Psi_0 \). Using now the definition (\( \text{Equation 3} \)) one can write down the second boundary condition \( M_{\text{jet}}^2 = M^2(d) \) as
\[
M_{\text{jet}}^2 = \frac{4\pi \Gamma^{-1}c_{\text{jet}}^2 n_{\text{jet}}}{P_{\text{ext}}}. \tag{15}
\]
The local non-relativistic enthalpy \( w \) can be written as
\[
w = \frac{c_{\text{jet}}^2}{(1 - 1)} \left( \frac{n}{n_{\text{jet}}} \right)^{1 - 1}, \tag{16}
\]
where the local particle number density \( n \) is obtained from the equation
\[
M^2n = 4\pi \eta^2 m_pc^2 \left[ 1 + \frac{1}{1 - 1} \left( \frac{n}{n_{\text{jet}}} \right)^{1 - 1} \right]. \tag{17}
\]
Thanks to these definitions the system of \( \text{Equation 4} \), \( \text{Equation 7} \) becomes now fully determined.

We solve the system of \( \text{Equation 4} \) and \( \text{Equation 7} \) for the boundary conditions \( \text{Equation 14} \) and
\[
P|_{r+ = d/2 - 0} = P_{\text{ext}}. \tag{18}
\]
We should note that due to the root dependence of the integrals \( L(\Psi) \) and \( \Omega_t(\Psi) \) the thickness of the final current closure domain tends to zero and is not resolved numerically. However, as it has been shown in (Beskin et al. 2017), the total pressure in this region is strictly conserved.

We find that for the chosen sound velocity at the boundary \( c_{\text{0}}^2 = 0.001c^2 \) the thermal effects may be neglected in the outflow volume, playing an important role only at the outflow boundary. It turns out that the resultant dependence of pressure at the jet boundary as a function of jet radius for the cold (Beskin et al. 2017) and the warm outflow starts to differ considerably for large \( M_{\text{jet}}^2 \) (this value is of an order of 10, but depends on the initial magnetization), affecting the flow boundary shape downstream of the equipartition transition.

The proposed jet model with an electric current enclosed inside the jet has a natural sheath structure, observed, for example, in the M87 jet (Mertens et al. 2016). Due to choice of integrals, the outer parts of a jet have slower velocities, tending to non-relativistic with \( \gamma(d/2) = 1 \). Such a sheath may be produced by different mechanisms: it may be a slower disk wind or an outer jet disturbed and slowed down by the pinch instability (Chatterjee et al. 2019). In our model it appears naturally as a consequence of a jet transiting into the ambient medium with the hydrodynamical discontinuity only (Beskin et al. 2017).

### 3.3 Transition from magnetically dominated to particle dominated flow

It is necessary to stress that this system of equations can describe both magnetically and particle dominated flow, the physical answer (including the jet boundary radius \( d/2 \)) depending on one external parameter only, namely, on the amplitude of the magnetic field. In our interpretation, which one may neglect by the derivatives over \( r \) in comparison with the derivatives over \( r_+ \) in the two-dimensional Grad-Shafranov and Bernoulli equations. This can be done for the highly collimated, at least as a parabola, outflows (Nokhrina et al. 2015) and the flows with small opening angles (Tchekhovskoy et al. 2009).

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bient pressure $P_{\text{ext}}$. In Figure 5 we show the dependence of the ambient pressure $P_{\text{ext}}$ on a jet transverse dimension $d/2$ obtained by numerical solving the system of Equation 4 and Equation 7 for central mass $M = 10^9 M_\odot$ (gravitational radius $r_g \sim 3 \cdot 10^{14}$ cm) and black hole spin $a_s = 0.1$. We also set the total magnetic flux in an outflow $\Psi_0 = 10^{32}$ G cm$^2$, which gives the value $B(r_b) = 350$ G, and $B_L = 3.5$ G. We observe (see Figure 5) that the pressure has different power law dependence on the jet radius for small and large $d/2$. For each magnetization $\sigma_M$, this behavior holds, with the change between two profiles occurring at different jet widths. For fiducial $\sigma_M = 10$ the pressure changes its dependence on $d$ from $P \propto d^{-3.7}$ to $P \propto d^{-2.3}$.

In order to model a jet shape break position along the jet we need to introduce the pressure dependence on $r$, which we choose in the power law form

$$P_{\text{ext}} = P_0 \left( \frac{r}{r_0} \right)^{-b}.$$  

(20)

Such a pressure profile is consistent with Bondi flow (Quataert & Narayan 2000; Shcherbakov 2008; Narayan & Fabian 2011) having $b \in [1; 2.5]$ for different models. This power law with $b \approx 2.0$ allows to reproduce well both the parabolic jet form upstream the break and conical downstream. We obtain for small distances $r$ (magnetically dominated regime)

$$d \propto r^{0.54}.$$  

(21)

Accordingly, for large distances (saturation regime)

$$d \propto r^{0.87}.$$  

(22)

As we see, qualitatively, the power indices are in good agreement with observational data.

### 3.4 Magnetization

In this subsection we check whether the break in a jet shape corresponds to the transition from magnetically-dominated into equipartition regime. The jet magnetization is defined as the ratio of Poynting flux

$$S = \frac{c}{4\pi} E \times B$$  

(23)

to particle kinetic energy flux

$$K = \gamma mc^2 n u_n,$$  

(24)

where $n$ is particle number density in the jet proper frame. Using the standard expressions for ideal MHD velocities and electric and magnetic fields, one can obtain the following expression for the magnetization:

$$\sigma = \frac{|S|}{K} = \frac{\Omega r I}{2\pi c^2 \gamma m_{\text{jet}}},$$  

(25)

Using the definitions of bulk Lorentz factor $\gamma$ and total current $I$, we rewrite it as

$$\sigma = \frac{\Omega r L - \Omega r \gamma^2 E/c^2}{E - \Omega r L - M^2 E}.$$  

(26)

In order to check $\sigma$ along the jet, we calculate the maximal magnetization across the jet for each given distance $r$ along the jet. The magnetization is always much less than the unity at the jet axis and at the jet boundary. The first holds always, since the Poynting flux behaves at the jet axis as

$$|S| \propto I = \pi j r_\perp^2 + o(r_\perp^2)$$  

(27)

if the current density $j$ has no singular behavior at $r_\perp = 0$. Thus, $\sigma \to 0$ at the axis. The same holds for the boundary in a case of a full current closure. Due to specific choice of integrals $E(\Psi)$, $L(\Psi)$, and $\Omega \Psi(\Psi)$ (Equations 9, 10, 12), the Poynting flux together with the magnetization reach their maximum values at $\Psi = \Psi_0/2$. It is at this magnetic field line the flow attains its highest Lorentz factor across the jet for the given distance from the central source. Thus, we choose the maximal magnetization reaching approximately the unity as a criteria of a flow attaining the ideal MHD equipartition regime.

In Figure 6 we present the maximal magnetization and the break in a jet form. We see that the break in a jet shape occurs roughly at the distance from the BH $r$, where the flow magnetization becomes equal to unity. For the higher initial magnetization it takes the larger transverse jet dimension in $R_L$ to accelerate the flow up to equipartition, according to Equation 2.

The first change in jet shape has been reported by Asada & Nakamura (2012), and this break is associated with the standing bright feature HST-1 in M87. The change in jet form may be explained by the change in ambient pressure at the break radius (Asada & Nakamura 2012). Another explanation has been proposed by Levinson (2017), who considered a disk wind. The shock between the jet and a disk wind followed by ambient pressure decrease accounts for both jet boundary shape change and the presence of HST-1. This point is supported by the presence of a standing bright feature, approximately coincident with a jet shape break location, in 6 of 12 sources (see details in Table 3). Another model for a jet boundary shape is an analytical asymptotic solution by Lyubarsky (2000), which predicts both the change in a jet form without the change in an ambient pressure profile and the oscillations of a jet form under the conditions of a jet being not in equilibrium state with

Figure 6. An example of a jet boundary shape (blue solid line) for $\sigma_M = 50$. The jet magnetization at a given distance from its base is plotted by a red solid line. The transition from one power law to the other (green dashed lines) for the jet boundary roughly coincides with the point where the outflow transits from the magnetically dominated to particle dominated (equipartition) regime, designated by a star.
3.5 Additional observational evidence of the break point and predicted evolution of plasma acceleration

We find in six out of twelve sources (Table 3) that the jet shape transition region is associated with a low pattern speed feature from the MOJAVE analysis by Lister et al. (2019). They define such features as having angular speed smaller than 20 μas yr−1 and being at least 10 times slower than the fastest component in the same jet. This is a factor of 2.5 larger compared to overall statistics of jet kinematics analysis performed at 15 and 43 GHz that reveals a fraction of quasi-stationary jet features to be about 20% (Lister et al. 2019; Jorstad et al. 2017). We note that the MOJAVE team uses conservative criteria in cross-identifying components between epochs and selecting robust ones (Lister et al. 2019). This means that the 50% fraction of sources which show a standing feature in the break point region should be considered as a lower limit.

We plot in Figure 7 the maximum Lorentz factor of a bulk plasma motion along a jet, which we obtain within our semi-analytical model. The predicted pattern of a bulk Lorentz factor acceleration in magnetically dominated domain is \( \gamma \propto r^{-1.5} \), which provides for a parabolic jet \( \gamma \propto r^{0.5} \). After the flow reaches equipartition, the acceleration continues slower than any power-law (logarithmically slow) (e.g., Beskin & Nokhrina 2006). There is also a transitional zone between the two regimes. Thus, we would expect for the sources with the detected jet shape break and superluminal motion the following kinematics pattern: efficient Lorentz factor growth before the break point, and cessation of it in the conical region.

This prediction is consistent with observations by the MOJAVE program that acceleration is a common property of jet features (e.g., Homan et al. 2015; Lister et al. 2019), reflecting a tendency for increasing Lorentz factors near the base of the jet with decreasing or constant speeds being more common at distances \( \gtrsim 10 - 20 \) parsecs projected (Homan et al. 2015). While decreasing speeds are not a prediction of this model for a change in jet shape, they could naturally occur if the reduction in positive acceleration is also accompanied by entrainment of external material into the jet.

4 SUMMARY

Pushkarev et al. (2017) studied the jet shapes by measuring the power low index \( k \) assuming a \( \gamma \propto r^k \) dependence of the observed deconvolved jet width on the apparent distance from its core \( r \). Most of the jets exhibited \( k \) values in the range from 0.5 to 1.5. As it was clearly shown by Pushkarev et al. (2017), high-quality, high-dynamic-range stacked images are needed for an analysis of this kind in order to trace the full jet channel. In view of a few recent exciting reports on jet shape transitions from parabolic to conical (e.g., Asada & Nakamura 2012; Giroletti et al. 2008; Boccardi et al. 2016; Tseng et al. 2016; Hada et al. 2018; Akiyama et al. 2018; Nakahara et al. 2018), we have performed a systematic search of such transition using MOJAVE 15 GHz stacked images supplementing some of them with available single epoch 1.4 GHz VLBA images to trace larger scales.

Using an automated analysis approach, we have found 10 jets with such transition out of 331 analyzed: 0111+021, 0235−084, 0415+379, 0430+052, 0815−094, 1133+704, 1514+004, 1637+826, 1807+698, 2200+420. Their redshift lies in the range \( z < 0.07 \) except for 0815−094, whose redshift is unknown. For the full analyzed sample the typical redshift is about 1. This low-z coincidence is unlikely to have occurred by chance. Taken together with an analysis of possible biases, we conclude that a genuine effect is found. We could predict that the BL Lac object 0815−094 should also be a nearby AGN.

This finding leads to the following important conclusion. The transition of jets from the parabolic to the conical shape might be a general property of AGN. At the same time, we note that AGN observed at higher redshifts typically have higher luminosities and kinetic power which can affect the collimation properties. This conclusion has important implications for jet models, astrophysics and astrometry of AGN. Measuring this phenomenon requires a search within nearby AGN (the subject of our current followup study), or increasing the resolution by using Space VLBI (e.g., Giovannini et al. 2018) or high dynamic range high frequency VLBI imaging.

The deprojected distance \( r_{\text{break}} \) from the nucleus to the break zone is found to be typically 10 pc. Even more interesting due to its relation to jet formation and acceleration models is this value measured in Schwarzschild radius units. We find typical range of \( r_{\text{break}} \in (10^5, 10^6) r_g \).
We suggest the following model to explain the observed jet shape break. The accurate matching of a jet outflow with an ambient medium (Beskin et al. 2017) predicts a change in jet shape from parabolic to conical if the ambient medium pressure is assumed to be governed by Bondi accretion. Within this model, a smaller external pressure is needed to support a jet. The transition of predicted jet shape from parabolic to conical occurs in the domain where the bulk plasma kinetic energy flux becomes equal to the Poynting energy flux, i.e., where the bulk flow acceleration reaches saturation (Beskin & Nakhrina 2006). From studying the break properties it allows us to estimate black hole spin and/or mass, jet total magnetic flux, and ambient medium properties as discussed by Nakhrina et al. (in prep.).

The following two model predictions are supported observationally. The break point, where jets start to be plasma dominated energetically, might be a preferable domain for shocks. We detect standing jet features in this region from the MOJAVE analysis (Lister et al. 2019) in at least a half of the AGN targets. The plasma acceleration is predicted to decrease significantly at the transition region which is consistent with MOJAVE acceleration results (Homan et al. 2015; Lister et al. 2019).

Our finding also implies the following. The well-known effect of the apparent shift of the core position with frequency due to synchrotron self-absorption does not follow the $r_{\text{core}} \propto \nu^{-1}$ law all the way up to the true jet base, since a $–1$ power low index is expected only for a conical jet (Blandford & Königl 1979; Lobanov 1998). Geometrical and physical estimates made on the basis of core shift measurements will need to take this into account while VLBI and VLBI-Gaia astrometry applications will need to correct for it (Porcas 2009) in cases where very high accuracy is required.

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