Revisiting $B \rightarrow \phi \pi$ Decays in the Standard Model

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January 6, 2009

Abstract

In the standard model (SM), we re-investigate the rare decay $B \rightarrow \phi \pi$, which has been viewed as an ideal probe to detect the new physics signals beyond the SM. Contributions in the naive factorization method, the radiative corrections, the long-distance contributions, and the contributions due to the $\omega$-$\phi$ mixing are taken into account. We find that the tiny branching ratio in the naive factorization can be dramatically enhanced by the radiative corrections and the $\omega$-$\phi$ mixing effect, while the long-distance contributions are negligibly small. Assuming the Cabibbo-Kobayashi-Maskawa angle $\gamma = (58.6 \pm 10)^{\circ}$ and the mixing angle $\theta = -(3.0 \pm 1.0)^{\circ}$, we obtain the branching ratios of $B \rightarrow \phi \pi$ as $\text{Br}(B^{\pm} \rightarrow \phi \pi^{\pm}) = (3.2^{+0.8}_{-0.7}^{+1.8}) \times 10^{-8}$ and $\text{Br}(B^{0} \rightarrow \phi \pi^{0}) = (6.8^{+0.3}_{-0.3}^{+0.7}) \times 10^{-9}$. If the future experiment reports a branching ratio of order $10^{-7}$ for $B^{-} \rightarrow \phi \pi^{-}$ decay, it may not be a clear signal for any new physics scenario. In order to discriminate the large new physics contributions and those due to the $\omega$-$\phi$ mixing, we propose to measure the ratio of branching fractions of the charged and neutral B decay channel. We also study the direct $CP$ asymmetries of these two channels, and the results are about $(-8.0^{+0.9}_{-1.0}^{+1.5})\%$ and $(-6.3^{+0.5}_{-0.7}^{+2.5})\%$ for $B^{\pm} \rightarrow \phi \pi^{\pm}$ and $B^{0} \rightarrow \phi \pi^{0}$, respectively.

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$B$ meson decays provide valuable information on the flavor structure of the weak interactions so that they can be used to precisely test the standard model (SM) and to search for the possible signals of the new physics beyond the SM. Charmless two-body nonleptonic decay processes, such as $B \to \phi \pi$, are of great interests, since the branching fractions are very small. The experimentalists have reported the following measurements \cite{1}:

\[
BR(B^- \to \phi \pi^-) = (-0.04 \pm 0.17) \times 10^{-6}
\]
\[
BR(\bar{B}^0 \to \phi \pi^0) = (0.12 \pm 0.13) \times 10^{-6}
\]

at 68% probability, while the upper bounds at 90% probability are given as:

\[
BR(B^- \to \phi \pi^-) < 2.4 \times 10^{-7}
\]
\[
BR(\bar{B}^0 \to \phi \pi^0) < 2.8 \times 10^{-7}
\]

On the theoretical side, since these decay modes are absent from any annihilation diagram contribution, calculations of hadronic matrix elements are quite reliable, and these decays have been analyzed in the SM by different groups \cite{2,3}. In the SM, these channels are highly suppressed for several reasons. At the quark level, these decays proceed via $b \to d \bar{s}s$, which is a flavor changing neutral current (FCNC) process. The FCNC transition is induced by the loop effects and the relevant Wilson coefficients are very small. Secondly, the Cabibbo-Kobayashi-Maskawa (CKM) matrix element for this transition $V_{tb}V_{td}^*$ is tiny. Finally, in order to produce a $\phi$-meson from the vacuum, at least three gluons are required which suppresses these channels further. Feynmann diagrams for these decays are often referred to the hairpin diagram (the last reference in Ref.\cite{2}), which are shown in Fig.\cite{1}. Because of the tiny branching ratio in the SM, $B \to \phi \pi$ is usually considered as an ideal place to search for the possible new physics scenarios \cite{4}.

However, before we turn to the new physics scenario, it is logical to investigate all possible contributions in the SM: contributions in the naive factorization, radiative corrections (vertex corrections and the hard spectator diagram), long-distance contributions such as rescattering from $B \to KK^*$ decays and contributions due to the $\omega-\phi$ mixing. The motif of this letter is to investigate the possibility of the enhancement of $B \to \phi \pi$ decays in the SM.

The $\Delta B = 1$ effective weak Hamiltonian in SM is given by \cite{5}:

\[
H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left( C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3}^{10,7,8} C_i Q_i \right) + \text{h.c.},
\]

where $\lambda_p = V_{pb}V_{pd}^*$. $Q_{1,2}$ are the left-handed current–current operators arising from $W$-boson exchange, $Q_{3,...,6}$ and $Q_{7,...,10}$ are QCD and electroweak penguin operators, and $Q_{7\gamma}$ and $Q_{8g}$ are the electromagnetic and chromomagnetic dipole operators, respectively. Their explicit expressions can be found in, e.g., Ref. \cite{5}.

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Figure 1: Hairpin diagrams for $B \rightarrow \phi \pi$ decays

The physics above the scale $m_b$ in the $B$ meson weak decays have been incorporated into the Wilson coefficients of the effective Hamiltonian and the task left is to evaluate the matrix element of each four-quark operator. The simplest way is to decompose it into two simpler parts: one is the decay constant of the emitted meson; the other part is the $B$-to-light form factor which only contains one light meson in the final state. Both of these two parts can be directly extracted from the experimental data, or evaluated from some nonperturbative method such as the Lattice QCD and QCD Sum Rules. In this approach, often referred as the naive factorization[6], the decay amplitudes can be written as:

$$A_{B^+ \rightarrow \pi^- \phi} = \sqrt{2} A_{B^0 \rightarrow \pi^0 \phi} = A_{\pi \phi} \sum_{p=u,c} \lambda_p (a_3 + a_5 - \frac{1}{2} a_7 - \frac{1}{2} a_9),$$  \hspace{1cm} (5)$$

where

$$A_{\pi \phi} = -i \sqrt{2} G_F m_\phi f_\phi (e^*_\phi \cdot p_B) F_{\pi B}^B,$$  \hspace{1cm} (6)$$

and $a_i$ is the Wilson coefficient combination as defined in Ref.[3]. In the naive factorization, predictions for the branching ratios are given as:

$$\text{Br}(B^\pm \rightarrow \phi \pi^\pm) = 9.0 \times 10^{-10},$$
$$\text{Br}(B^0 \rightarrow \phi \pi^0) = 4.1 \times 10^{-10}. \hspace{1cm} (7)$$

In the calculation, we have used $f_\phi = 0.22$ GeV. The particle masses and lifetime of the $B$ meson are taken from [7]. The value of the form factor at zero recoil is taken as $F_{\pi B}^B = 0.25$. The value of the CKM matrix elements used are taken from [7]:

$$|V_{ub}| = 0.0039, \quad |V_{ud}| = 0.974, \quad |V_{cb}| = 0.0422, \quad |V_{cd}| = 0.226, \hspace{1cm} (8)$$

and the phase $\gamma$ associated with $V_{ub}$ as 58.6$^\circ$. Compared with Eq. (2) and (3), we can see the results in the naive factorization are far below the experimental upper bound. The tiny branching ratios are due to the cancelation of the Wilson coefficients $C_3, C_4, C_5, C_6$. This cancelation also reflects the fact that $\phi$ can only be produced by at least three gluons. The RG evolved Wilson coefficients at the scale $m_b$ have contains the multi-loop contributions above the scale $m_b$ which give small branching fractions to $B \rightarrow \phi \pi$ decays. Below this scale, the radiative corrections may
provide sizable contributions. In the QCD factorization (QCDF) approach \[8, 9\], the hadronic matrix elements of local operators \( Q_i \) can be written as

\[
\langle \pi(p) \phi(q) | Q_i | B(p) \rangle = F_{\pi} T^I(v) \Phi_{\phi}(v) + \int_0^1 d\xi dudv T^{II}(\xi, u, v) \Phi_B(\xi) \Phi_{\pi}(u) \Phi_{\phi}(v) \tag{9}
\]

where \( \phi_M (M = \phi, \pi, B) \) are light-cone distribution amplitudes of the meson \( M \), \( T^I \) and \( T^{II} \) are hard scattering kernels. To be more specific, the Wilson coefficient combination called final state interaction. In the long-distance contribution to the \( p \rightarrow K^{(*)} \pi \) decays also receive some nonperturbative corrections: \( B \rightarrow K^{(*)} \pi \rightarrow K^{(*)} \pi \rightarrow \phi \pi \) through exchanging a \( K^{(*)} \) meson, which is also called final state interaction. In the \( m_b \rightarrow \infty \) limit, the FSI is power suppressed and believed to vanish. Since the \( b \) quark mass is finite, the FSI is not zero and the \( \pi^0 \) decay is given as:

\[
\text{Br}(B^0 \rightarrow \phi \pi^0) = 5.2 \times 10^{-5} \tag{10}
\]

Compared with results in the naive factorization approach, we find that the branching ratios are enhanced by a factor of 12.3. We also find that our results are larger than those evaluated in the QCDF approach in Ref.\[4, 9\]. The reason is that we have chosen the factorization scale for the hard-spectator diagram as \( \mu = 1 \text{ GeV} \). If we chose the factorization scale as \( \mu = 2.1 \text{ GeV} \), the branching ratio will be reduced by a factor of 2. The difference caused by the factorization scale characterizes the size of the subleading corrections for the hard scattering diagrams.

Despite of the perturbative contributions, \( B \rightarrow \phi \pi \) decays also receive some nonperturbative corrections: \( B \rightarrow K^{(*)} K^{(*)} \rightarrow K^{(*)} \pi \rightarrow K^{(*)} \pi \rightarrow \phi \pi \) through exchanging a \( K^{(*)} \) meson, which is also called final state interaction. In the \( m_b \rightarrow \infty \) limit, the FSI is power suppressed and believed to vanish. Since the \( b \) quark mass is finite, the FSI is not zero and the \( t \)-channel FSI has been modeled as the one-particle-exchange picture \[10\]. As an example, we will study the FSI effect from the \( B^- \rightarrow K^{*-} \pi^- \) decays. The short distance contribution to the \( B^- \rightarrow K^{*-} K^0 \) decays is given as:

\[
A(B^- \rightarrow K^{*-} K^0) = -i \frac{G_F}{\sqrt{2}} f_{KB} A_0 (2 m_K \epsilon_K \cdot p_B) \sum_\rho \lambda_\rho \alpha_4^\rho - \frac{1}{2} \alpha_{4,EW}^\rho \tag{11}
\]

The long-distance contribution to \( B^- \rightarrow \phi \pi^- \) is given as:

\[
A_{abs} = -i \frac{G_F}{\sqrt{2}} f_{KB} A_0 \sum_\rho \lambda_\rho \alpha_4^\rho - \frac{1}{2} \alpha_{4,EW}^\rho \int_{-1}^1 \frac{d \cos \theta}{16 \pi m_B} 4 m_K g_{K^* \pi} g_{\phi KK} \times \left( -p_B \cdot p_3 + \frac{p_B \cdot p_1 \cdot p_3}{m_{K^*}^2} \right) \times \frac{E_2 |\tilde{p}_4| - E_4 |\tilde{p}_2| \cos \theta}{m_\phi} \times \frac{F(t, m^2)}{t - m^2}, \tag{12}
\]

where \( p_1, p_2, p_3, p_4 \) denotes the momentum of the \( K^{*-}, K^0, \pi^-, \phi \) mesons, respectively. \( \theta \) is the angle between the momenta \( \tilde{p}_1 \) and \( \tilde{p}_3 \). The coupling constants \( g_{\phi KK} \) and \( g_{K^* \pi} \) can be determined.
through the experimental data on $\phi \to KK$ and $K^* \to K\pi$ decays [7], and we get $g_{\phi KK} = 4.51$ and $g_{K^*K\pi} = 4.86$. Because of the small branching ratios (of order $10^{-7}$) of $B \to KK^*$ decays [11], the long-distance contributions to $B \to \pi\phi$ decays are not expected to give sizable corrections. The numerical results also show that these contributions are negligibly small.

$$
\sqrt{2}A_{B^+ \to \pi^-}^\phi = A_{\phi\pi} \sum_{p=u,c} \lambda_p \left[ \delta_{pu}(\alpha_2 + \beta_2) + 2\alpha_3^p + \alpha_4^p + \frac{1}{2}\alpha_{3,EW}^p - \frac{1}{2}\alpha_{4,EW}^p + \beta_3^p + \beta_3^p \right] + A_{\phi\pi} \sum_{p=u,c} \lambda_p \left[ \delta_{pu}(\alpha_1 + \beta_2) + \alpha_4^p + \alpha_{4,EW}^p + \beta_3^p + \beta_3^p \right], \quad (14)
$$

$$
-2A_{B^0 \to \pi^0}^\phi = A_{\phi\pi} \sum_{p=u,c} \lambda_p \left[ \delta_{pu}(\alpha_2 - \beta_1) + 2\alpha_3^p + \alpha_4^p + \frac{1}{2}\alpha_{3,EW}^p - \frac{1}{2}\alpha_{4,EW}^p + \beta_3^p - \frac{1}{2}\beta_{3,EW}^p - \frac{3}{2}\beta_{4,EW}^p \right] + A_{\phi\pi} \sum_{p=u,c} \lambda_p \left[ \delta_{pu}(-\alpha_2 - \beta_1) + \alpha_4^p - \frac{3}{2}\alpha_{3,EW}^p - \frac{1}{2}\alpha_{4,EW}^p + \beta_3^p - \frac{1}{2}\beta_{3,EW}^p - \frac{3}{2}\beta_{4,EW}^p \right], \quad (15)
$$

![Figure 2: Feynman diagrams of the final state interactions](image)

All the above investigations are based on the hypothesis that $\omega - \phi$ are ideally mixing: $\omega = \frac{u\bar{d} + d\bar{u}}{\sqrt{2}}$ and $\phi = s\bar{s}$. But generally, the $\omega$ and $\phi$ can mix with each other via the SU(3) symmetry breaking effect. With the aid of a mixing angle $\theta$, one can parameterize the $\omega - \phi$ mixing, so that the physical $\omega$ and $\phi$ are related to the two states $n\bar{n} = \frac{u\bar{d} + d\bar{u}}{\sqrt{2}}$ and $s\bar{s}$

$$
\begin{pmatrix}
\omega \\
\phi
\end{pmatrix} =
\begin{pmatrix}
\cos \theta & \sin \theta \\
-\sin \theta & \cos \theta
\end{pmatrix}
\begin{pmatrix}
n\bar{n} \\
\bar{s}\bar{s}
\end{pmatrix}, \quad (13)
$$

Recent studies within the chiral perturbative theory imply a mixing angle of $\theta = -(3.4 \pm 0.3)^\circ$ [12], while the most recent treatment implies an energy-dependent mixing which varies from $-0.45^\circ$ at the $\omega$ mass to $-4.64^\circ$ at the $\phi$ mass [13]. Although the $n\bar{n}$ component in the $\phi$ meson is tiny, it may sizably contribute to some observables such as branching ratio and direct $CP$ violation parameters of the rare decays such as $B \to \phi\pi$ [14].

As the $n\bar{n}$ component concerned, both the emission and annihilation topologies contribute to these decays. Therefore, not only penguin operators but also tree operators should be taken into account. For the $n\bar{n}$ part, the decay amplitudes are given:
where $A_{\phi \pi}$ and $A_{\phi \pi}$ are defined by:

$$A_{\phi \pi} = -i\sqrt{2} G_F m_\phi f_{+}^{\pi} (\epsilon_\phi \cdot p_B);$$

$$A_{\phi \pi} = -i\sqrt{2} G_F m_\phi A_0^{B \rightarrow \phi} f_{\pi} (\epsilon_\phi \cdot p_B),$$

(16)

with the $B \rightarrow \phi$ form factor $A_0^{B \rightarrow \phi} = 0.28$ for the $\bar{n}n$ component. The Wilson coefficients $\alpha_i$ come from vertex corrections and hard spectator corrections, and $\beta_i$ represent contribution of annihilation diagrams, which can be found in Ref. [9].

The total amplitudes are given as:

$$A_{B^- \rightarrow \pi^- \phi} = \left[ A_{\pi^- \phi}^{QCDF} + i Abs(B^- \rightarrow \pi^- \phi) \right] \cos \theta + A_{B^- \rightarrow \pi^- \phi}^{n} \sin \theta,$$

(17)

$$A_{B^0 \rightarrow \pi^0 \phi} = \left[ A_{\pi^0 \phi}^{QCDF} + i Abs(B^0 \rightarrow \pi^0 \phi) \right] \cos \theta + A_{B^0 \rightarrow \pi^0 \phi}^{n} \sin \theta.$$  

(18)

If one adopts the mixing angle $\theta = -3^\circ$, the branching ratios of $B \rightarrow \phi \pi$ are:

$$\text{Br}(B^- \rightarrow \phi \pi^-) = 3.2 \times 10^{-8},$$

$$\text{Br}(B^0 \rightarrow \phi \pi^0) = 6.8 \times 10^{-9}.$$  

(19)

Comparing with results in Eq.(11), we found that the branching ratio of charged channel $B^- \rightarrow \phi \pi^-$ is enhanced remarkably. However, the $\omega-\phi$ mixing does not change $B^0 \rightarrow \phi \pi^0$ so much. We plot the relations between branching ratios and the mixing angle $\theta$, and the CKM angle $\gamma$ in Fig.3. In the left diagram of Fig.3 we set $\gamma = 58.6^\circ$ and let $\theta$ change from $-5^\circ$ to zero; in the right part, set $\theta = -3^\circ$ and $\gamma \in (50^\circ, 90^\circ)$. As indicated in this diagram, the branching ratio of $B^- \rightarrow \phi \pi^-$ is sensitive to both $\theta$ and $\gamma$, whereas the $B^0 \rightarrow \phi \pi^0$ does not have this character. Physically, $B^0 \rightarrow \pi^0 \phi(n\bar{n})$ is a color-suppressed process associated with angle $\gamma$, whose decay amplitude is much smaller than color favored mode $B^\pm \rightarrow \pi^\pm \phi(n\bar{n})$. Hence, for the charged channel, the mixing contribution becomes larger with the mixing angle $\theta$ decreasing. For the $B^0 \rightarrow \pi^0 \phi$, its branching ratio is not sensitive to these two angles because of the small amplitude of $B^0 \rightarrow \pi^0 \phi(n\bar{n})$.

Although our results are still below the experimental bound, the branching ratio of $B^- \rightarrow \phi \pi^-$ is dramatically enhanced: to roughly $0.6 \times 10^{-7}$ if the mixing angle is $-4.64^\circ$. This value is smaller than the upper bound only by a factor of 4. If the future experiment reports a branching ratio of order $10^{-7}$, it may not be the signal for any new physics at all. Since both of the new physics scenarios and the mixing effect can give the branching ratio of order $10^{-7}$, it is necessary to find a way to discriminate them. We propose a ratio $R$ of branching fractions which is defined as:

$$R = \frac{\text{Br}(B^- \rightarrow \phi \pi^-)}{\text{Br}(B^0 \rightarrow \phi \pi^0)} \frac{\tau_{B^-}}{\tau_{B^0}} \left| \frac{A_{B^- \rightarrow \phi \pi^-}}{A_{B^0 \rightarrow \phi \pi^0}} \right|^2.$$  

(20)

Without the $\omega-\phi$ mixing, $R = 2$, while if the $\omega-\phi$ mixing is present, the ratio is a function of the mixing angle $\theta$. Here we plot the relation of $R$ and $\theta$ in Fig.4 as $\gamma = (58.6 \pm 10)^\circ$, through
Figure 3: Dependence of the CP averaged branching ratios on the mixing angle $\theta$ (left panel) and the CKM phase angle $\gamma$ (right panel), where the dot-dashed and solid lines correspond to charged channel and neutral channel, respectively.

Figure 4: Dependence of the ratio $R$ on the mixing angle $\theta$ with $\gamma = (58.6 \pm 10)^\circ$. The solid line is the central value of $\gamma$, while the short-dashed line and the long dashed line correspond to the upper limit and the lower limit, respectively.
which we can determine the mixing angle $\theta$ with the observable $R$. From this diagram, we can obtain $R = 4.3$ when $\theta = -3^\circ$ and $\gamma = 58.6^\circ$. As stated above, the neutral channel is not changed so much, whereas the charged decay $B^\pm \rightarrow \pi^\pm \phi$ is enhanced by the mixing contribution, so the ratio $R$ is controlled by the channel $B^\pm \rightarrow \pi^\pm \phi$. If the mixing angle was tiny and a large branching ratio of the order $10^{-7}$ is observed in the future, the large branching ratio must receive dominant contributions from the new physics scenario, either enhance the Wilson coefficients of the operators in the SM or introduce new effective operators beyond. They will contribute to both contributions from the new physics scenario, either enhance the Wilson coefficients of the operators and $\bar{B}^0 \rightarrow \phi \pi^0$ decays. In this case, the ratio R is identically 2 which is dramatically different with the one caused by the $\omega-\phi$ mixing effect.

Another observable in $B$ decays is the direct $CP$ violation parameter, which is defined as:

$$A_{CP} = \frac{\Gamma(B^- \rightarrow \phi \pi^-) - \Gamma(B^+ \rightarrow \phi \pi^+)}{\Gamma(B^- \rightarrow \phi \pi^-) + \Gamma(B^+ \rightarrow \phi \pi^+)}. \quad (21)$$

In order to have non-zero direct $CP$ asymmetry, the decay amplitude needs to contain at least two interfering contributions with different strong and weak phases. Since only penguin operators does contribute to this decay mode in the absence of $\omega-\phi$ mixing, the direct $CP$ asymmetry turns out to be identically zero. In the mixing scenario, there is a small portion of the $u\bar{u}$ component in $\phi$ meson, and tree operators contribute so that the direct $CP$ asymmetries are non-zero. If $\theta < -3^\circ$, the contribution from $n\bar{n}$ part dominate the $\phi \pi^-$ progressively, and the direct $CP$ violation becomes stable as $\theta$ grows down. Because $B^0 \rightarrow \pi^0 \phi(n\bar{n})$ has a small amplitude, the direct $CP$ of this decay mode comes from interference between tree contribution of $n\bar{n}$ and penguin from both $n\bar{n}$ and $s\bar{s}$, which leads to the $CP$ violation are sensitive to mixing angle $\theta$. With the definition in Eq. (21) and the mixing angle $\theta = -3^\circ$, the direct $CP$ violation parameters are given as:

$$A_{CP}(B^- \rightarrow \phi \pi^-) = -8.0\%, \quad A_{CP}(\bar{B}^0 \rightarrow \phi \pi^0) = -6.3\%. \quad (22)$$

In the left part of Fig.5, we illustrate the dependence of $A_{CP}$ on the mixing angle $\theta$. In the right part of the Fig.5 we set $\theta = -3^\circ$, and draw the relation between $A_{CP}$ and the CKM angle $\gamma$.

Many uncertainties in two body non-leptonic $B$ decays, such as the decay constants, the amplitude distributions, are constrained by many well measured decay channels as $B \rightarrow \pi \pi, K \pi$ [11]. In the decay mode $B \rightarrow \phi \pi$, the major uncertainties are from the mixing angle $\theta$ and the CKM phase $\gamma$. Then we set the CKM angle $\gamma = (58.6 \pm 10)^\circ$ and the mixing angle $\theta = -(3.0 \pm 1.0)^\circ$, and get the results with errors,

$$\text{Br}(B^\pm \rightarrow \phi \pi^\pm) = (3.2^{+0.8}_{-0.7} \pm 1.2^{+0.7}_{-1.0}) \times 10^{-8}, \quad (23)$$

$$\text{Br}(B^0 \rightarrow \phi \pi^0) = (6.8^{+0.3}_{-0.3} \pm 0.7^{+1.0}_{-0.7}) \times 10^{-9};$$

$$A_{CP}(B^\pm \rightarrow \phi \pi^\pm) = (-8.0^{+0.9}_{-1.0} \pm 1.5)\%, \quad (24)$$

$$A_{CP}(B^0 \rightarrow \phi \pi^0) = (-6.3^{+0.5}_{-0.7} \pm 2.5)%;$$

$$R = 4.3^{+1.0}_{-0.9} \pm 1.4. \quad (25)$$
Figure 5: Dependence of the direct CP asymmetries (in units of %) on the mixing angle $\theta$ (left panel) and the CKM phase angle $\gamma$ (right panel), where dot-dashed lines and the solid lines correspond to charged channel and neutral channel respectively.

Our result can be directly generalized to other similar channels such as $B \to \phi \rho$ decays. There are several differences between $B \to \phi \rho$ and $B \to \phi \pi$ decays. The contributions from the mixing mechanism are larger, since the branching ratio of $B^- \to \omega \rho^-$ is larger than that of $B^- \to \omega \pi^-$ (in unit of $10^{-6}$): $\text{Br}(B^- \to \omega \rho^-) = (10.6^{+2.6}_{-2.3}) > \text{Br}(B^- \to \omega \pi^-) = (6.9 \pm 0.5) [11]$. The transverse polarization of $B \to \phi \rho$ also receives sizable contributions from the dipole operator $O_{7 \gamma} [15]$.

Because of tiny branching ratios in the SM, the authors in Refs. [4] argued that the decay mode $B \to \phi \pi$ is a good place for probing the new physics effect. In the present paper, we have studied several contributions to $B \to \pi \phi$ decays in the SM. We find that the small branching fraction, expected in the naive factorization approach, can be remarkably enhanced by the radiative corrections and the $\omega \phi$ mixing mechanism. The final results for the branching ratio of $B^- \to \pi^- \phi$ is smaller than the present upper limit by a factor of $4 - 20$. We conclude that, the observation of this channel with the branching ratio of the order $10^{-7}$ may not be a clear signal for the new physics scenario. On the contrary, that may be induced by the $\omega \phi$ mixing effect. In order to discriminate the two different contributions, we propose to measure the ratio $R$ of the branching fractions in the future. The contributions from the $\omega \phi$ also provide nontrivial strong phases, which result in large direct CP asymmetries. These results can be tested on the future experiments.

Acknowledgements

This work is partly supported by the National Science Foundation of China under Grant Nos. 10747156, 10625525, 10735080 and 10805037.
References

[1] B. Aubert et al. [BABAR Collaboration], Phys. Rev. D 74, 011102 (2006) [arXiv:hep-ex/0605037].

[2] D. Du, H. Gong, J. Sun, D. Yang and G. Zhu, Phys. Rev. D 65, 094025 (2002); R. Aleksan, P. -F. Giraud, V. Morenas, O. Pene and A. S. Safir, Phys. Rev. D 67, 094019 (2003); B. Melic, Phys. Rev. D 59, 074005 (1999); R. Fleischer, Phys. Lett. B 321, 259 (1994); D. S. Du and Z. Z. Xing, Phys. Lett. B 312, 199 (1993).

[3] A. Ali, G. Kramer and C. D. Lu, Phys. Rev. D 58, 094009 (1998) [arXiv:hep-ph/9804363]; A. Ali, G. Kramer and C. D. Lu, Phys. Rev. D 59, 014005 (1999) [arXiv:hep-ph/9805403].

[4] S. Bar-Shalom, G. Eilam and Y. D. Yang, Phys. Rev. D 67, 014007 (2003); J. F. Cheng and C. S. Huang, Phys. Lett. B 554, 155 (2003); A. K. Giri and R. Mohanta, Phys. Lett. B 594, 196 (2004); ibid. 660, 376 (2008); J. F. Cheng, Y. N. Gao, C. S. Huang and X. H. Wu, Phys. Lett. B 647, 413 (2007); B. Mawlong, R. Mohanta and A. K. Giri, Phys. Lett. B 668, 116 (2008).

[5] G. Buchalla, A. J. Buras and M. E. Lautenbacher, Rev. Mod. Phys. 68, 1125 (1996) [arXiv:hep-ph/9512380].

[6] M. Bauer and B. Stech, Phys. Lett. B 152, 380 (1985); M. Bauer, B. Stech and M. Wirbel, Z. Phys. C 34, 103 (1987).

[7] C. Amsler et al. [Particle Data Group], Phys. Lett. B 667, 1 (2008).

[8] M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Phys. Rev. Lett. 83, 1914 (1999) [arXiv:hep-ph/9905312]; M. Beneke, G. Buchalla, M. Neubert and C. T. Sachrajda, Nucl. Phys. B 591, 313 (2000) [arXiv:hep-ph/0006124].

[9] M. Beneke and M. Neubert, Nucl. Phys. B 675, 333 (2003) [arXiv:hep-ph/0308039].

[10] H. Y. Cheng, C. K. Chua and A. Soni, Phys. Rev. D 71, 014030 (2005) [arXiv:hep-ph/0409317].

[11] E. Barberio et al. [Heavy Flavor Averaging Group], arXiv:0808.1297 [hep-ex]; and online update at http://www.slac.stanford.edu/xorg/hfag.

[12] M. Benayoun, L. DelBuono, S. Eidelman, V. N. Ivanchenko and H. B. O’Connell, Phys. Rev. D 59, 114027(1999) [arXiv:hep-ph/9902326]; A. Kucukarslan and U. G. Meißner, Mod. Phys. Lett. A 21, 1423(2006) [arXiv:hep-ph/0603061], M. Benayoun, L. DelBuono
and H. B. O’Connell, Eur. Phys. J. C 17, 593(2000) [arXiv:hep-ph/9905350]; M. Benayoun, L. DelBuono, P. Leruste and H. B. O’Connell, Eur. Phys. J. C 17, 303(2000) [arXiv:nucl-th/0004005]; M. Benayoun and H. B. O’Connell, Eur. Phys. J. C 22, 503(2001) [arXiv:nucl-th/0107047].

[13] M. Benayoun, P. David, L. DelBuono, O. Leitner and H. B. O’Connell, Eur. Phys. J. C 55, 199(2008) [arXiv:0711.4482 [hep-ph]].

[14] M. Gronau and J. L. Rosner, Phys. Lett. B 666, 185 (2008) [arXiv:0806.3584 [hep-ph]].

[15] M. Beneke, J. Rohrer and D. Yang, Phys. Rev. Lett. 96, 141801 (2006) [arXiv:hep-ph/0512258]; C. D. Lu, Y. L. Shen and W. Wang, Chin. Phys. Lett. 23, 2684 (2006) [arXiv:hep-ph/0606092].