Constraining the density dependence of nuclear symmetry energy with heavy-ion reactions and its astrophysical impact

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Abstract. Recent analyses of several isospin effects in heavy-ion reactions have allowed us to constrain the density dependence of nuclear symmetry energy at sub-saturation densities within a narrow range. Combined with constraints on the Equation of State (EOS) of symmetric nuclear matter obtained previously from analyzing the elliptic flow in relativistic heavy-ion collisions, the EOS of neutron-rich nuclear matter is thus partially constrained. Here we report effects of the partially constrained EOS of neutron-rich nuclear matter on the mass-radius correlation, moment of inertia, elliptical deformation and gravitational radiation of (rapidly) rotating neutron stars.

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1. EOS of neutron-rich nuclear matter partially constrained by heavy-ion reactions

The EOS of isospin asymmetric nuclear matter can be written within the well-known parabolic approximation as

\[ E(\rho, \delta) = E(\rho, \delta = 0) + E_{\text{sym}}(\rho)\delta^2 + O(\delta^4), \]  

(1)
where \( \delta \equiv (\rho_n - \rho_p) / (\rho_p + \rho_n) \) is the isospin asymmetry with \( \rho_n \) and \( \rho_p \) denoting, respectively, the neutron and proton densities, \( E(\rho, \delta = 0) \) is the EOS of symmetric nuclear matter, and \( E_{\text{sym}}(\rho) \) is the density-dependent nuclear symmetry energy. The latter is very important for many interesting astrophysical problems \([1, 2]\), the structure of rare isotopes \([3]\) and heavy-ion reactions \([4, 5, 6, 7, 8]\). However, the density dependence of the nuclear symmetry energy has been the most uncertain part of the EOS of neutron-rich matter. Fortunately, comprehensive analyses of several isospin effects including the isospin diffusion \([9, 10]\) and isoscaling \([11]\) in heavy-ion reactions and the size of neutron skin in heavy nuclei \([12]\) have allowed us to constrain the density dependence of the symmetry energy at sub-saturation densities within approximately \(31.6 \rho^0.69 \) and \(31.6 \rho_1.05 \) as labelled by \( x = 0 \) and \( x = -1 \), respectively, in the lower panel of Fig.1 \([13, 14]\). While these constraints are only valid for sub-saturation densities and still suffer from some uncertainties, compared to the early situation they represent a significant progress in the field.

Further progress is expected from both the parity violating electron scattering experiments \([15]\) at the Jefferson lab that will help pin down the low density part of the symmetry energy and heavy-ion reactions with high energy radioactive beams at several facilities that will help constrain the high density behavior of the symmetry energy \([8]\).

For many astrophysical studies, the EOS is usually expressed in terms of the pressure as a function of density and isospin asymmetry. Shown in Fig. 1 are the pressures for two extreme cases: symmetric (upper panel) and pure neutron matter (lower panel). The green area in the density range of \(2 - 4.6 \rho_0\) is the experimental constraint on the pressure \(P_0\) of symmetric nuclear matter extracted by Danielewicz, Lacey and Lynch from analyzing the collective flow data from relativistic heavy-ion collisions \([6]\). It is seen that results from mean-field calculations using the phenomenological momentum-dependent (MDI) interaction \([16]\), the Dirac-Brueckner-Hartree-Fock approach with the Bonn B potential (DBHF) \([17]\), and the variational calculations by Akmal, Pandharipande, and Ravenhall (APR) \([18]\) are all consistent with this constraint. For pure neutron matter, its pressure is \(P_{\text{PNM}} = P_0 + \rho^2 dE_{\text{sym}} / d\rho\) and depends on the density dependence of nuclear symmetry energy. Since the constraints on the symmetry energy from terrestrial laboratory experiments are only available for densities less than about \(1.2 \rho_0\) as indicated by the green and red squares in the lower panel, which is in contrast to the constraint on the EOS of symmetric nuclear matter that is only available at much higher densities, the most reliable estimate of the EOS of neutron-rich matter can thus be obtained by extrapolating the underlying model EOS for symmetric matter and the symmetry energy in their respective density ranges to all densities. Shown by the shaded black area in the lower panel is the resulting best estimate of the pressure of high density pure neutron matter based on the predictions from the MDI interaction with \(x=0\) and \(x=-1\) as the lower and upper bounds on the symmetry energy and the flow-constrained EOS of symmetric nuclear matter. As one expects and consistent with the estimate in Ref. \([6]\), the estimated error bars of the high density pure neutron matter EOS is much wider than the uncertainty range of the
Fig. 1. (Color online) Pressure as a function of density for symmetric (upper panel) and pure neutron (lower panel) matter. The green area in the upper panel is the experimental constraint on symmetric matter. The corresponding constraint on the pressure of pure neutron matter obtained by combining the flow data and an extrapolation of the symmetry energy functionals constrained below $1.2\rho_0$ by the isospin diffusion data is the shaded black area in the lower panel. Results taken from Refs. [6,19].
EOS of symmetric nuclear matter. For the four interactions indicated in the figure, their predicted EOS's cannot be distinguished by the estimated constraint on the high density pure neutron matter. In the following, the astrophysical consequences of this partially constrained EOS of neutron-rich matter on the mass-radius correlation, moment of inertia, and the elliptical deformation and gravitational radiation of (rapidly) rotating neutron stars are briefly discussed. More details of our studies on these topics can be found in Refs. [19, 20, 21, 22, 23].

2. Nuclear constraints on the mass-radius correlation, moment of inertia, elliptical deformation and gravitational radiation of rapidly rotating neutron stars

Fig. 2. (Color online) Gravitational mass versus equatorial radius for neutron stars rotating at $\nu = 716$ Hz and $\nu = 1122$ Hz. Taken from Ref. [22].

The partially constrained EOS of neutron-rich nuclear matter has important ramifications on properties of neutron stars. As a first example, in Fig. 2 we show the mass-radius correlations for the two fastest rotating neutron stars known as of today. These pulsars spin at 716 [24] and 1122 Hz [25], respectively. However, based only on the observational data available so far, their properties have not yet been fully understood. The analysis of their properties based on the EOS and symmetry energy constrained by the terrestrial laboratory data is thus especially interesting. Setting the observed frequency of the pulsar as the Kepler frequency, corresponding to the highest possible frequency for a star before its starts to shed mass at the equator, one can obtain an estimate of its maximum radius as a function of mass $M$,

$$R_{\text{max}}(M) = \chi \left( \frac{M}{1.4M_\odot} \right)^{1/3} \text{km},$$

(2)
with $\chi = 20.94$ for rotational frequency $\nu = 716$ Hz and $\chi = 15.52$ for $\nu = 1122$ Hz. The maximum radii are shown with the dotted lines in Fig. 2. It is seen that the range of allowed masses supported by a given EOS for rapidly rotating neutron stars becomes narrower than the one of static configurations. This effect becomes stronger with increasing frequency and depends upon the EOS. Since predictions from the $x = 0$ and $x = -1$ EOSs represent the limits of the neutron star models consistent with the experimental data from terrestrial nuclear laboratories, one can predict that the mass of the neutron star rotating at 1122 Hz is between 1.7 and 2.1 solar mass [22].

Another interesting example is the gravitational radiation expected from elliptically deformed pulsars. Gravitational waves (GWs) are tiny disturbances in space-time and are a fundamental, although not yet directly confirmed, prediction of General Relativity. Gravitational wave astrophysics would open an entirely new non-electromagnetic window to the Cosmos, making it possible to probe physics that is hidden or dark to current electromagnetic observations [26]. Elliptically deformed pulsars are among the primary possible sources of the GWs. Very recently the LIGO and GEO collaborations have set upper limits on the GWs expected from 78 radio pulsars [27]. Gravitational waves are characterized by a strain amplitude $h_0$ which can be written as

$$h_0 = \chi \frac{\Phi_{22} \nu^2}{r},$$

with $\chi = \sqrt{2048\pi^4/15G/c^4}$. In the above equation, $r$ is the distance between the pulsar and the detector, and the $\Phi_{22}$ is the quadrupole moment of the mass.
distribution. For slowly rotating neutron stars, one has [28]

$$\Phi_{22,\text{max}} = 2.4 \times 10^{38} \text{ g cm}^2 \left(\frac{\sigma}{10^{-2}}\right) \left(\frac{R}{10 \text{ km}}\right)^{6.26} \left(\frac{1.4 M_\odot}{M}\right)^{1.2}.$$  \hspace{1cm} (4)

In the above expression, $\sigma$ is the breaking strain of the neutron star crust which is rather uncertain at present time and lies in the wide range $\sigma = [10^{-5} - 10^{-2}]$ [29]. In our estimate, we use the maximum breaking strength, i.e. $\sigma = 10^{-2}$. In Fig. 3 we display the GW strain amplitude, $h_0$, as a function of stellar mass for three selected millisecond pulsars which are relatively close to Earth ($r < 0.4$ kpc) and have rotational frequencies below 300 Hz. It is interesting to note that the predicted $h_0$ is above the design sensitivity of LIGO detector. The error bars in Fig. 3 between the $x = 0$ and $x = -1$ EOSs provide a constraint on the maximal strain amplitude of the gravitational waves emitted by the millisecond pulsars considered here. The specific case shown in the figure is for neutron star models of $1.4 M_\odot$. Depending on the exact rotational frequency, distance to detector, and details of the EOS, the maximal $h_0$ is in the range $\sim [0.4 - 1.5] \times 10^{-24}$. These estimates do not take into account the uncertainties in the distance measurements. They also should be regarded as upper limits since the quadrupole moment (Eq. (4)) has been calculated with $\sigma = 10^{-2}$ (where $\sigma$ can go as low as $10^{-5}$).

To emit GWs a pulsar must have a quadrupole deformation. The latter is normally characterized by the ellipticity which is related to the neutron star maximum quadrupole moment $\Phi_{22}$ and the moment of inertia via [28]

$$\epsilon = \sqrt{\frac{8\pi}{15} \Phi_{22}},$$ \hspace{1cm} (5)

For slowly rotating neutron stars, one can use the following empirical relation [30]

$$I_{zz} \approx (0.237 \pm 0.008) M R^2 \left[ 1 + 4.2 \frac{M \text{ km}}{M_\odot R} + 90 \left(\frac{M \text{ km}}{M_\odot R}\right)^4 \right]$$ \hspace{1cm} (6)

This expression is shown to hold for a wide class of equations of state which do not exhibit considerable softening and for neutron star models with masses above $1 M_\odot$ [30].

Fig. 4 displays the neutron star moment of inertia (left panel) and ellipticity (right panel). It is interesting to mention that a fiducial value of $I_{zz} = 10^{45}$ g cm$^2$ is normally assumed in the literature. Our calculations indicate that the $I_{zz}$ is strongly mass dependent. This observation is consistent with previous calculations. Moreover, the ellipticity decreases with increasing mass. The magnitude is above the lowest upper limit of $4 \times 10^{-7}$ estimated for the PSR J2124-3358 [27]. Interestingly, essentially all observables depend strongly on the EOS of neutron-rich matter. In particular, the MDI EOSs, adopting the same symmetric matter EOS but different density dependence of the symmetry energy, sets useful nuclear boundaries for these gravitational wave observables.
In summary, the heavy-ion physics community has made significant progress in constraining the EOS of neutron-rich nuclear matter in recent years. In particular, comprehensive analyses of several isospin effects including the isospin diffusion and isoscaling in heavy-ion reactions and the size of neutron skins in heavy nuclei have allowed us to constrain the symmetry energy at sub-saturation densities within approximately $3.6\left(\rho/\rho_0\right)^{0.69}$ and $3.6\left(\rho/\rho_0\right)^{1.05}$. While the currently existing data only allowed us to constrain the symmetry energy and thus the EOS of neutron-rich matter in a narrow range, it can already help to put some useful constraints on several interesting observables in astrophysics, such as the mass-radius correlation, moment of inertia, and the elliptical deformation and gravitational radiation of (rapidly) rotating neutron stars. With the parity violating electron scattering experiments and heavy-ion reactions with high energy radioactive beams, it will be possible in the future to map out accurately the entire density dependence of the symmetry energy.

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