QUANTUM MEASUREMENTS AND INFORMATION
RESTRICTIONS IN ALGEBRAIC QM

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Abstract

It’s argued that Information-Theoretical restrictions for the systems
selfdescription are important for Quantum Measurement problem. As fol-
lows from Breuer theorem, for the quantum object S measurement by
information system O they described by O restricted states \( R_O \). \( R_O \)
ansatz can be introduced phenomenologically from the consistency with
Shrödinger dynamics and measurement statistics. The analogous restric-
tions obtained in Algebraic QM considering Segal algebra of \( S,O \) obser-
vables and the resulting \( O \) algebraic states \( \{ \varphi_j^O \} \) set defined as its dual
space. From Segal theorem for associative (sub)algebras it’s shown that
\( \varphi_j^O \) describes the random ‘pointer’ outcomes \( O_j \) observed by \( O \) in
the individual events.

Key Words: Quantum Measurements, Information, \( C^* \) algebras

1 Introduction

Despite that Quantum Mechanics (QM) is universally acknowledged physical theory,
there are still several unresolved problems concerned with its interpretation. Of them,
the State Collapse or Quantum Measurement Problem is the most widely and long
discussed (D’Espagnat, 1990; Busch, 1996). In this paper we regard the quantum mea-
surement process in the information-theoretical framework and demonstrate its impor-
tance for the state collapse consideration (Svozil, 1993). Really, both the quantum and
classical measurement is, eventually, the information acquisition by the information
system \( O \) (Observer) via the direct or indirect interaction with the studied system \( S \)
(Guilini, 1996, Duvenhage, 2002). Therefore, the possible restrictions on the informa-
tion pattern transferred from \( S \) to \( O \) can be important in the Measurement Theory
(Breuer, 1996). We concede in our study that QM description is applicable both for
a microscopic and macroscopic objects; in particular, \( O \) state described by Dirac vec-
tor \( |O\rangle \) or density matrix \( \rho \) relative to another observer \( O' \) (Rovelli, 1995; Bene, 2000).
considered as the information gaining and utilizing system (IGUS) which acquire and memorize the information as the result of S interactions with the measuring system (MS) which element is O (below S formally is also regarded as MS element). In principle, O can be either a human brain or some automatic device processing the information. In all cases it’s the system with some number of internal degrees of freedom (DF) which interacts during S information acquisition, so that O internal state changes after it.

S measurement by O described by MS state $|MS\rangle$ evolution relative to some $O'$, yet in this model the acquired S information memorized and processed by O, not by $O'$, which reflected by O internal state evolution. Therefore, the detailed description of S information recognition should be analyzed in the selfdescription framework (Svozil,1993). The information systems selfdescription was studied already in the context of the selfreference problem (Finkelstein,1988; Mittelstaedt,1998). It was shown that the arbitrary system selfdescription is always incomplete; this result often interpreted as the analog of Gödel Theorem for Information theory (Svozil,1993). In this framework Breuer developed the restricted states formalism for the selfdescription in the measurement process - the selfmeasurement which is applicable both in classical and quantum case (Breuer,1996). It follows that O internal state $R_O$, which is the partial (restricted) MS state, can differ principally from the standard QM ansatz for O state relative to $O'$ (Mittelstaedt, 1998). Basing on this results, we propose here the novel formalism which accounts O selfmeasurement effects and predicts the measured state collapse. Its main feature is the modification of quantum state ansatz which becomes the doublet $\Phi = \{\phi^D, \phi^I\}$, where $\phi^D = \rho$ is QM density matrix MS, $\phi^I(n)$ is O restricted state describing O subjective information in the given individual event $n$ (Mayburw,2001). $\phi^I$ can be independent of $\phi^D$ and ,in particular, demonstrates the stochastic behavior in S measurements. It will be shown that such formalism corresponds to the well-known generalization of standard QM - algebraic QM based on Jordan,Segal and $C^*$- algebras applications (Emch,1972). In its framework $\phi^D$ is MS state defined on MS observables algebra $U$, $\phi^I$ corresponds to the state defined on O observables subalgebra $U_0$.

We must stress that the observer consciousness never referred directly and doesn’t play any role in our theory (London,1939). Rather, in our model observer O regarded as the quantum reference frame (RF) which interacts with studied object S (Aharonov,1981). S state description 'from the point of view' of the particular O referred by the terms 'S state in O RF' or simply 'S state for O'. The terms 'perceptions', 'impressions' used by us to characterize the IGUS O description of experimental results and defined below in strictly physical terms. In particular, the perception is the acquisition of some information by IGUS, i.e. the change of IGUS state; the different O impressions associated with the different, O physical states.

## 2 Measurements and Quantum States Restrictions

Our formalism exploits both the quantum states in the individual events - i.e. individual states and the statistical states describing the quantum ensembles properties (Mittelstaedt,1998). Remind that in QM the individual states are the pure states
which are isomorphic to Dirac vectors $|\Psi\rangle$ in $\mathcal{H}$; the statistical states described by the normalized, positive operators of trace 1 - density matrixes $\rho$ on $\mathcal{H}$. If the $\Psi_i$ composition is known for the given ensemble, its state can be described in more detail by the ensemble state (Gemenge) presented by the table $W^e = \{\Psi_i; P_i\}$ where $P_i$ are the corresponding probabilities (Busch,1996). Algebraic QM states will be considered the normalized, positive operators of trace 1 - density matrices $\rho$.

We'll regard the simple MS measurement model similar to von Neuman model (von Neuman,1932; Busch,1996). It includes the measured binary state $S$ which interacts with the observer $O$ storing the incoming $S$ information. In our model the detector $D$ omitted in MS chain, the role of $O$ decoherence effects will be discussed below. The regarded $O$ has one internal DF and in its Hilbert space $\mathcal{H}_O$ the basis consists of the three orthogonal states $|O_{0,1,2}\rangle$ which are the eigenstates of $Q_O$ 'internal pointer' observable with eigenvalues $q_1^O$. We'll consider the measurement of the binary $S$ observable $\bar{Q}$ on $S$ state $\psi_q$. Initial $O$ state is $|O_0\rangle$ and MS initial state is :

$$\Psi_{MS}^{in} = \psi_s |O_0\rangle = (a_1 |s_1\rangle + a_2 |s_2\rangle) |O_o\rangle \tag{1}$$

where $|s_1,2\rangle$ are $Q$ eigenstates with eigenvalues $q_{1,2}$. S-O measuring interaction starts at $t_0$ and finished effectively at some finite $t_1$, by the suitable choice of $S-O$ interaction Hamiltonian $\hat{H}_I$ Schrödinger equation (SE) results in MS final state $\rho_{MS}^\psi$ :

$$\Psi_{MS} = \sum_j \Psi_j^{MS} = \sum_i a_i |s_i\rangle \langle O_i| \tag{2}$$

As the result, for any $\psi_q$ one obtains $\bar{Q}_O = \bar{Q}$ which means that $O$ performs the unbiased $Q$ measurement. Meanwhile, for any $O$ observable $Q_O' \neq F(Q_O); \bar{Q}_O' = 0$ independently of $\psi_q$. Regarding $O$ as the information system, we'll assume that $|O_{1,2,0}\rangle$ corresponds to $O$ information pattern - an impressions notified by $q_{1,2,0}^O$ (Guilini,1996). Therefore, at $t > t_1$ for external O' MS is in the pure state $\Psi_{MS}$ of (2) which is the superposition of the states corresponding to the different measurement outcomes. Basing on our assumptions, from $O$ 'point of view' $\Psi_{MS}$ describes the simultaneous superposition (coexistence) of two contradictory impressions : $Q_O = q_1^O$ and $Q_O = q_2^O$ perceived by $O$ simultaneously. Yet it's well known that experimentally the macroscopic $O$ observes at random one of $Q_O$ values $q_{1,2}^O$. From that $S$ final state is $|s_1\rangle$ or $|s_2\rangle$ and $S$ state collapse occurs. In standard QM with Reduction Postulate $S$ final state described by the density matrix of mixed state:

$$\rho_s^m = \sum_i |a_i|^2 |s_i\rangle \langle s_i| \tag{3}$$

In accordance with it, in our model one can ascribe to MS the corresponding mixed state :

$$\rho_{MS}^m = \sum_i |a_i|^2 |s_i\rangle \langle s_i| |O_i\rangle \langle O_i| \tag{4}$$

which differs principally from $\rho_{MS}^\psi$ of (2). It's quite difficult to doubt both in the correctness of MS evolution description by SE and in the state collapse experimental observations. This obvious contradiction constitutes famous Wigner 'Friend Paradox' for $O, O'$ (Wigner,1961). We attempt here to unite this alternative systems descriptions 'from outside' by $O'$ and 'from inside' by $O$ in the same formalism.
Formally, both the classical and quantum measurement of the arbitrary system $S'$ is the mapping of $S'$ states set $N_S$ on the given IGUS $O^S$ states set $N_O$ (Mittelstaedt, 1998). If the final $O^S$ and $S'$ state can’t be factorized, then $O^S$ should be regarded as the subsystem of the large system $S_T = S' + O^S$ with the states set $N_T$.

In this situation - 'measurement from inside' $N_O$ is $N_T$ subset and $O^S$ state is $S_T$ state projection to $N_O$ - the restricted state $R_O$. From $N_T$ mapping properties the principal restrictions on $O^S$ restricted states obtained in Breuer theorem; if for two arbitrary $S_T$ states $\Phi_S, \Phi'_S$ their restricted states $R_O, R'_O$ coincide, then for $O^S$ this $S_T$ states are indistinguishable (Breuer, 1996). The origin of this results in classical case is easy to understand: $O^S$ has less number of DFs then $S_T$ and, therefore, can’t describe completely $S_T$ state (Svozil, 1993). In quantum case the observables noncommutativity and nonlocality introduce some new features regarded below. Despite that $R_O$ are incomplete $S_T$ states, they are the real physical states for $O^S$ observer - 'the states in their own right' as Breuer puts it.

The described $S', O^S, S_T$ relation corresponds to our MS model which can be regarded as 'the MS measurement from inside'. Breuer results doesn’t permit to derive the restricted states for an arbitrary system directly, and as the phenomenological $R_O$ ansatz it was proposed (Breuer, 1996) to use the partial trace which for MS final state (2) is equal to:

$$R_O = Tr_s \rho_{MS}^s = \sum |a_i|^2 |O_i\rangle\langle O_i|$$  \hspace{1cm} (5)

in particular, for the incoming $|s_j\rangle$ $R_O = |O_j\rangle\langle O_j|$. For MS state $\rho_{MS}^s$ of (4) appearing in the measurement of the incoming $S$ mixture, the corresponding restricted statistical state is the same $R_{O_{mix}}^s = R_O$. This equality doesn’t mean the collapse of MS pure state $\Psi_{MS}$ because the collapse appearance should be verified also for $O, S$ individual states. For the pure case MS individual state is always $\Psi_{MS}$, yet for the incoming $S$ statistical mixture (4) MS individual state differs from event to event:

$$\rho^o(n) = \rho^l = |O_i\rangle\langle O_i||s_i\rangle\langle s_i|$$  \hspace{1cm} (6)

where the random $l(n)$ described by the probabilistic distribution $P_l = |a_i|^2$. $\rho^o(n)$ differs from the state (2), correspondingly, its restricted state $\rho^O(n) = |O_i\rangle\langle O_i|$ also differs in any event from $R_O$ of (5). Due to it, the main condition of Breuer Theorem violated for the individual states and $O$ can differentiate pure/mixed states 'from inside' in the individual events (Breuer, 1996). Therefore, the proposed formalism doesn’t permit to obtain the state collapse for $O$ selfdescription in standard QM framework. Hence, $R_O$ is the consistent restriction of MS statistical state $\rho_{MS}^\prime$ to $O$ which coincides for the pure and mixed $S$ states with the same $|a_i|$. $R_O$ ansatz (5) regarded also as $O$ individual partial state relative to external classical observer in the standard Quantum Measurement Theory without selfdescription (Lahti, 1990).

Note that even in Breuer theory $O$ can’t observe the difference between MS states with different $D_{12} = a_1^*a_2 + a_1a_2^*$. Such difference revealed by MS interference term (IT) observable:

$$B = |O_1\rangle\langle O_2||s_1\rangle\langle s_2| + j.c.$$  \hspace{1cm} (7)

In standard QM, being measured by external $O'$ on $S, O$, it gives $B = 0$ for the mixed MS state (4), but $B \neq 0$ for the pure MS states (2); $B$ value principally can’t be measured by $O$ 'from inside'; note also that $B, Q_O$ doesn’t commute.

Formally, MS individual state for $O$ can be written in doublet form $\Phi^O(n) = |\phi^D, \phi'^I\rangle\rangle$, where $\phi^D = \rho_{MS}$ is the objective (dynamical) state component and the
information component $\phi^I$ describes $O$ subjective information in the given event $n$. In Breuer theory for the pure MS states $\phi^I$ is just $\phi^D$ projection but in the alternative formalism described below it will describe the novel $O$ state features. In this formalism the state collapse appears in MS 'measurement from inside' performed by $O$ and reflected in its information component $\phi^I$ (Mayburov, 2001). To agree with the quantum Schrödinger dynamics (SD), the particular formalism should satisfy to two operational conditions:

i) if an arbitrary system $S'$ doesn't interact with IGUS $O^S$, then for $O^S$ this system evolves according to Schrödinger-Liouville equation (SLE)

ii) If $S'$ interacts with $O^S$ and the entangled $S', O^S$ state produced i.e. measurement occurs, then SD can be violated for $O^S$ but for external, stand-by $O'$ the $S', O^S$ evolution should be described by SLE as follows from condition i).

Below it will be argued that this doublet state formalism (DSF) corresponds to the measurements description in Algebraic QM framework.

For the novel MS state: $\Phi = |\phi^O, \phi^I \rangle$ the dynamical component $\phi^D$ is also equal to QM density matrix $\phi^D = \rho$ and obeys to SLE:

$$\frac{\partial \phi^D}{\partial t} = [\phi^D, \hat{H}]$$

and the initial $\phi^D$ of (1) evolves at $t > t_1$ to $\phi^D(t) = \rho_{MS}^O$ of (2). $O$ information component $\phi^I$ differs principally from Breuer theory because it behaves stochastically in the individual events. Namely, for $t \leq t_0$ the initial $\phi^I = |O_i\rangle\langle O_i| - O$ has no information on $S$ at $t_0$. For the final $\phi^I(t)$ at $t \geq t_1$ after the measurement at $t > t_1 \phi^I$ is the stochastic state $\phi^I(n) = \phi^I_0$, where $\phi^I_0 = |O_i\rangle\langle O_i|$, with $i$ described by the probabilistic distribution with $P_i = |a_i|^2$. Therefore, such doublet, individual state $\Phi(n)$ can change from event to event and $\phi^I$ is partly independent of $\phi^D$ being correlated with it only statistically. DSF $O$ subjective states $\phi^I$ can’t differ the pure and mixed states with the same $|a_i|^2$. Therefore, Breuer theorem conditions fulfilled and the subjective state collapse observed by $O$. MS ensemble evolution described via the doublet statistical states $\Theta \gg = |\eta_D, \eta_I \rangle$, where $\eta_D = \phi^D$, $\eta_I(t)$ describes the probabilistic distribution $\{P_i(t)\}$ of $O$ $\phi^I_0$ observations at given $t$. Thereon, $\eta_I(t)$ defined by $\eta_D(t)$ which obeys to SLE. Due to it, $\Theta$ evolution is reversible and the acquired $O$ information can be erased completely. Naturally, the quantum states for external $O'$ (and other observers) also has the same doublet form $\Psi'$. In the regarded situation $O'$ doesn’t interact with MS and so $O'$ information doesn’t change after $S$ measurement by $O$, eventually, for $O'$ MS evolution described by SLE only.

Witnessing Interpretation proposed by Kochen (Kochen,1985) is quite close to DSF but doesn’t exploits the selfdescription effects. It phenomenologically supposed that for apparatus $A$ ($O$ in our notations) some $S$ measured value $Q$ in pure state always has random definite value $q_i$ relative to $A$, yet no new mathematical formalism different from standard QM wasn’t constructed for its proof (Lahti,1990).

Plainly, in DSF $|O_i\rangle$ constitutes the preferred basis (PB) in $\mathcal{H}_O$ and its appearance should be explained in the consistent theory, this problem is well-known in standard QM with the Reduction Postulate (Busch,1995). In DSF PB problem acquires the additional aspects related to the information recognition by $O$. The plausible explanation prompts $O$ decoherence - i.e. $O$ interaction with environment $E$ (Zurek,1982; Guilini,1996). In this case the produced, entangled $S, O, E$ state admits the unique,
3 Selfmeasurement in Algebraic QM

Now the quantum measurements and O selfdescription will be regarded in Algebraic QM framework (Bratelli,1981). Besides the standard quantum effects, Algebraic QM describes successfully the phase transitions and other nonperturbative phenomena which standard QM fails to incorporate (Emch,1972). Consequently, there are the serious premises to regard Algebraic QM as the consistent generalization of standard QM. Algebraic QM was applied extensively to the superselection model of quantum measurements when the detector D or environment E regarded as the infinite systems (Pimas,1990; Guilini,1996). The algebraic formalism of nonperturbative QFT was applied also to the study of measurement dynamics in some realistic systems (Mayburov,1998). In standard QM the fundamental structure is the fixed states set - Hilbert space $\mathcal{H}$ on which an observables - Hermitian operators defined. Yet for some systems the states set structure principally differs from the arbitrary $\mathcal{H}$ and the standard QM axiomatics becomes preposterous. In distinction, in Algebraic QM the fundamental structure is the Segal algebra $\mathcal{U}$ of observables $A,B,...$ which incorporate the main properties of the studied system $S_f$ and eventually, defines $S_f$ state set $\Omega$ (Emch,1972). Technically, it’s more convenient to consider $C^*$-algebra $\mathcal{C}$ for which $\mathcal{U}$ is the subset and calculate $\mathcal{U}$ properties afterward. For our problems $\mathcal{C}$, $\mathcal{U}$ are in the unambiguous correspondence $\mathcal{C} \leftrightarrow \mathcal{U}$ and below their use is equivalent in this sense. $S_f$ states set $\Omega$ defined by $S_f \mathcal{U}$ via the notorious GNS construction; it demonstrates that $\Omega$ is the vector space dual to the corresponding $S_f \mathcal{C}$ (Bratelli,1981). Such states called here the algebraic states $\phi \in \Omega$ and are defined as the normalized, positive, linear functionals on $\mathcal{U}$: $\forall A \in \mathcal{U}; \forall \phi \in \Omega$ it gives $\bar{A} = \langle \phi; A \rangle$.

Here only unitarily equivalent $S_f$ will be regarded; for them $S_f \phi$ formally corresponds to QM density matrices $\rho$ (Segal,1947). The algebraic pure states are $\Omega$ extremal points and they regarded as the algebraic individual states (AIS) $\xi$; their set denoted $\Omega^p$ (Emch,1972; Primas,1990). The arbitrary $\phi$ doesn’t admit the unambiguous decomposition into AIS $\xi$ ensemble, except the situation when $\phi$ is pure; in this case $\xi = \phi$. The algebraic mixed states $\phi_{mix}$ can be constructed as $\xi_i$ ensembles; the ensemble states $W_{A}$ defined analogously to the described QM ansatz.

In many practical situations only some restricted linear subspace $\mathcal{M}_R$ or subalgebra $\mathcal{U}_R$ of $S_f$ observables algebra $\mathcal{U}$ is available for the observation. For such subsystems the restricted algebraic states $\phi_R$ can be defined consistently via $A_R \in \mathcal{U}_R$ expectation values:

$$\bar{A}_R = \langle \phi; A_R \rangle = \langle \phi_R; A_R \rangle$$

defining $\phi \rightarrow \phi_R$ restrictions; their set denoted $\Omega^p_R$. $\phi_R$ doesn’t depend on any $A' \notin \mathcal{U}_R$, therefore, $\forall \phi_R, \langle \phi_R; A' \rangle = 0$ (Emch,1972). For our MS only $O$ observables supposedly are available for the observation (perception) and that makes the
subalgebras studies important for us. Remind that any classical system $S^c$ can be described by some associative Segal algebra $U^c$ of $S^c$ observables $\{A\}$ (Emch,1972); in algebraic QM $U$ associativity corresponds to QM observables commutativity. The theorem by Segal proves that any associative Segal (sub)algebra $U'$ is isomorphic to some algebra $U^c$ of classical observables (Segal,1947); thereon, its $\varphi^a$ states set $\Omega^a$ is isomorphic to the set $\Omega^c$ of the classical statistical states $\varphi^c$. The corresponding AIS - i.e. the pure states corresponds to the classical, individual states $\xi^c_i$ - points in $S^c$ parameters space. For us the most important is the case when $U'$ includes only $I$ and single $A \neq I$; there $\xi_i^c = \delta(q^a_i - q_i)$, corresponding to $A$ eigenvalues $q_i^a$ spectra. Consequently, even if quantum $S_T$ described by nonassociative $U$, it contains the subalgebra $U' \in U$ (and may be not unique) for which the restricted AIS $\xi_i^c$ are classical with the objective properties $q_i^a$.

For the classical observing system $S^c_T$ described by some $U^c$ its selfmeasurement $O$ restrictions are easy to find - $O$ state depends only on $S^c_T$ coordinates $\{x^c_i\}$ which are $O$ internal coordinates (Breuer,1996). They constitute $U^c_R$ subalgebra of $U^c$ but the realistic $O$ effective subalgebra $U^c_R \in U^c_F$ can be even smaller because some $x^c_i$ can be uninvolved directly into the measurement process. QM Correspondence principle prompts that for the transition to the quantum case $S^c_T \rightarrow S_T$ $O$ restricted subalgebra $U^c_R$ also includes only $O$ internal observables. In quantum case any effective subalgebra $U^c_O \in U^c_R$ stipulating the restricted states sets $\Omega_R, \Omega_O$, correspondingly. Our main hypothesis is that in any individual event to the arbitrary $S_T$ AIS $\xi$ responds some restricted $O$ AIS $\xi^c_i$. It advocated below for $U_O$, for $U_R$ it accepted ad hoc.

MS described by $U$ Segal algebra for MS observables which defines $\varphi^{MS} \in \Omega$ properties. $O$ subalgebra is $U^c_R$ which includes all $O$ internal observables. Then, $\varphi^R \in \Omega_R$ is equivalent to $O$ QM statistical states $\rho$ set. Consequently, $O$ AIS set $\Omega_R^O$ is equivalent to $H_O$ and any $O$ AIS $\xi^R$ corresponds to some $O$ state vector $|O_i^c\rangle \in H_O$. We don’t study here $\Omega_R$ states further, note only that Breuer $O$ state $\langle 5 \rangle$ $R_0 \notin \Omega_R^O$ and can’t be AIS on $U_R$ for $a_{1,2} \neq 0$. To define $U_O$, let’s consider $\varphi^{MS} \rightarrow \varphi^O$ restriction properties. Remind that for the regarded MS dynamics of $\langle 2 \rangle$ $O$ can measure only the observable $Q_O$, for any other $Q_O \neq F(Q_O)$ the final $Q_O = 0$: it means $\forall \varphi^O; \langle Q_O^c; \varphi^O \rangle = 0$ for $O$ restricted, algebraic states. From that follows that $U_O \in U^c_R$ effective $O$ subalgebra includes only $Q_O$ and $I$. Really, only in this case $\forall \varphi^O \in \Omega_O; \langle Q_O^c; \varphi^O \rangle = 0$: each $\varphi^O$ corresponds to $\varphi^c$ with the same $Q_O$ and vice versa. Therefore, $\varphi^O$ set $\Delta_O$ is isomorphic to $\Omega_O$. There is no other $U_R$ subalgebras with such properties and that settles $U_O$ finally. Therefore, obtained $\varphi^O$ are equivalent to $R_O$, in agreement with MS the statistical states $\rho_{MS}$ restriction to $O$ which are equal to $R_O$ of $\langle 5 \rangle$ as was shown above. From Segal theorem for $U^c_R$ the restricted algebraic $O$ states $\varphi^O \in \Omega_O$ are isomorphic to classical, probabilistic $q_i^O$ distributions, $O$ AIS $\xi^O$ are isomorphic to the classical, pointlike states:

$$\xi^O_i = \delta(q^O_i - q_i^O)$$

for $Q_O$ eigenvalues. For the incoming $S$ state $\psi_s = |s_i\rangle$ results in $\Psi^{MS}_s = |s_i\rangle|O_i\rangle$ which are $\Omega$ extremal points, $O$ restricted states $\varphi^O_i = |O_i\rangle\langle O_i|$ are $\Omega_O$ extremal point and AIS $\xi^O_i = \varphi^O_i$. In any $Q$ eigenstate $|s_i\rangle$ measurement the final MS restricted state from $O$ 'point of view' describes the definite $Q_O$ value $q_i^O$ which establishes operationally $\xi^O_{i,j}$ distinction in the individual events.

For the incoming $S$ mixture with the $|s_i\rangle$ probabilities $|a_i|^2$ MS algebraic final state is $\varphi_{mix} = \rho_{MS}^m$ of $\langle 4 \rangle$; the corresponding $O$ restricted state $\varphi^O_{mix}$ defined from
the relation for \( Q_O \):

\[
Q_O = \langle \phi_{mix}^O; Q_O \rangle = \langle \phi_{mix}; Q_O \rangle = \sum |a_i|^2 q_i^O
\]

which results in the solution \( \phi_{mix}^O = \sum |a_i|^2 \phi_i^O \). From the regarded correspondence of MS \( \xi^{MS} \) and O AIS \( \phi_{mix}^O \) represents the stochastic mixture of AIS \( \xi_i^O \) described by O ensemble state \( W_{mix}^O = \{ \xi_i^O; P_i = |a_i|^2; i = 1, 2 \} \). If the incoming S state is Q eigenstates superposition \( \Psi_{MS}^O \) of (1), MS final algebraic state \( \phi^{MS} \) with the same \( |a_i|^2 \) results in the same \( Q_O \) value. Therefore, its restricted algebraic state coincides with the mixed one \( \phi^O = \phi_{mix}^O \). From Segal theorem in \( \Omega^O \) all O individual states are AIS \( \xi_i^O \), possessing the definite properties \( q_i^O \). There is no other individual states \( \xi_a^O \neq \xi_i^O \), consequently, MS restricted AIS in each event can be only one of \( \xi_i^O \). Eventually, if MS state is \( \Psi^{MS} \) superposition of (2), to dispatch the correct \( Q_O \) for \( \xi_i^O \) ensemble, \( \xi_i^O \) should appear at random with the probabilistic distribution \( P_i^O \) defined by \( Q^O \). Yet for such \( W^O \) content the only solution which results in the necessary \( Q_O \) value is \( P_i^O = |a_i|^2 \) and so \( W^O = W_{mix}^O \). It demonstrates that \( \xi^{MS} \to \xi_i^O \) restriction map is stochastic.

In general, any two physically different states operationally discriminated by the particular observation procedure which reveals this states difference via the difference of some observables values distributions. For the statistical states it demonstrated by their probabilistic distributions parameters, for the individual states \( \xi_a,b \) such difference can be extracted from some observables \( A, B \) eigenvalues \( q_{a,b} \) for which these states are the eigenstates. In that case this values can be obtained and compared in the single event per each state (Mittelstaedt, 1998). In our case the only \( U_O \) observable is \( Q_O = \sum q_i^O P_i \), and \( \xi_i^O \) difference reflected by \( q_i^O \) difference. If to assume that some other \( \xi_a^O \neq \xi_i^O \) exists, it needs also some other observable \( Q^O \neq F(Q^O) \); \( Q^O \) should belong to \( U_O \) to differ it from \( \xi_i^O \), but it’s inconsistent with the obtained \( U_O \) structure. In particular, Breuer restricted state \( R_O \) of (5) analog \( \xi_R^O = \sum |a_i|^2 \xi_i^O \) for \( a_i \neq 0 \) can’t be O individual state on \( U_O \) because it isn’t \( \Omega_O \) extremal point. Moreover, the arbitrary \( \phi^O \) admits the unique decomposition into \( \xi_i^O \) set and can be interpreted as \( \xi_i^O \) ensemble with the given probabilities. Since \( \xi_i^O \in \Omega_R \), it can be taken also as the possible ansatz for MS states restriction on \( U_R \). DSF doublet state \( \Phi \) components \( \phi^{O}, \phi^O \) are equivalent to \( \xi^{MS}, \xi^O \) correspondingly.

If to analyze this results from the Information-Theoretical premises, note that the difference between the pure and mixed MS states reflected by B IT of (7) expectation values. Therefore, O possible observation of S pure/mixed \( W^O \) states difference means that O can acquire the information on B expectation value. But \( B \notin U_R \) and isn’t correlated with \( Q_O \) via \( S,O \) interaction alike \( Q \) of S; so this assumption is preposterous. Note that MS individual states \( \xi^{MS} \) symmetry is larger than the symmetry of the restricted O states. In Algebraic QM such symmetry reduction results in the phenomena of Spontaneous Symmetry Breaking, by the analogy the discussed randomness appearance can be called Information Symmetry Breaking. In practice it’s possible that O effective subalgebra is larger then \( U_O \) but this case will demand more complicated calculations which we plan to present in the forthcoming papers. In Algebraic QM the only important condition for the classicality appearance is \( U_O \) observables commutativity and it’s reasonable to expect it to be feasible also for complex IGUS structures.

Despite of the acknowledged Algebraic QM achievements, its foundations are still
discussed and aren’t finally established. In particular, it’s still unclear whether all the algebraic states corresponds to the physical states (Primas, 1983). This questions are important by themself and are essential for our formalism feasibility. By our choice of the initial MS states we avoid it in the regarded model. In particular, we admitted without proof that for MS AIS $\xi^M S$ some $O$ restricted AIS responds in any event. It agrees with the restricted states consideration as the real physical states, but on the whole, this assumption needs further clarification.

For the conclusion, the information-theoretical restrictions on the quantum measurements were studied on the simple selfdescription model of IGUS $O$. Breuer selmeasurement study shows that by itself the $O$ inclusion as the quantum object into the measurement scheme doesn’t result in the state collapse appearance (Breuer, 1996). Our considerations indicates that to describe the state collapse and in the same time to conserve Schrödinger linear evolution, it’s necessary to extend the quantum states set over standard QM Hilbert space. Such modification proposed in DSF involving the doublet states $\Phi$, where one of its components $\phi^i$ corresponds to $O$ subjective information - i.e. $O$ selfdescription. Algebraic QM presents the additional arguments in favour of this approach, in its formalism $O$ structure described by $O$ observables algebra $\mathcal{U}_O$ which defines the multiplet states set analogous to $\Phi$. In Algebraic formalism the stochastic events appearance stipulated by MS individual states restriction to $O$. In our opinion the obtained results evidence that it’s impossible to solve the Measurement Problem without accounting of the information system $O$ interactions at quantum level and its information acquisition restrictions (Zurek, 1998).

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