Nonperturbative Contributions
to the Hot Electroweak Potential
and the Crossover

Stephan J. Huber\textsuperscript{1,4}
Andreas Laser\textsuperscript{2}\textsuperscript{*}
Martin Reuter\textsuperscript{3}\textsuperscript{†}
Michael G. Schmidt\textsuperscript{4}\textsuperscript{§}

\textsuperscript{1,4} Institut für Theoretische Physik der Universität Heidelberg, Philosophenweg 16, D-69120 Heidelberg
\textsuperscript{2} Arthur Andersen GmbH, Mergenthalerallee 10-12, D-65760 Eschborn
\textsuperscript{3} Institut für Physik, Universität Mainz, Staudingerweg 7, D-55099 Mainz

Abstract

We discuss nonperturbative contributions to the 3-dimensional one-loop effective potential of the electroweak theory at high temperatures in the framework of the stochastic vacuum model. It assumes a gauge-field background with Gaussian correlations which leads to confinement. The instability of $\langle F^2 \rangle = 0$ in Yang-Mills theory appears for small Higgs expectation value $\langle \phi^2 \rangle$ in an IR regularized form. The gauge boson propagator obtains a positive momentum-dependent “diamagnetic” effective (mass)$^2$ due to confinement effects and a negative one due to “paramagnetic” spin-spin interactions which are related to the $\langle F^2 \rangle = 0$ instability. Numerical evaluation of an approximate effective potential containing these masses shows qualitatively the fading away of the first-order phase transition with increasing Higgs mass which was observed in lattice calculations. The crossover point can be roughly determined postulating that the effective $\phi^4$ and $\phi^2$ terms vanish there.

\textsuperscript{*}e-mail: s.huber@thphys.uni-heidelberg.de
\textsuperscript{†}e-mail: andreas.laser@arthurandersen.com
\textsuperscript{‡}e-mail: reuter@thep.physik.uni-mainz.de
\textsuperscript{§}e-mail: m.g. schmidt@thphys.uni-heidelberg.de
1 Introduction

The phase diagram of the electroweak standard model at high temperatures $T$ of about 100 GeV can be well described by the 3-dimensional effective Higgs field gauge theory which is obtained perturbatively by "integrating out" heavy degrees of freedom, i.e. by matching a set of amplitudes in 4 and 3 dimensions [1, 2]:

$$L_3 = \frac{1}{4} F^a_{\mu
u} F^a_{\mu\nu} + (D_{\mu}\phi)^+(D_{\mu}\phi) + m_3^2 \phi^+ \phi + \lambda_3 (\phi^+ \phi)^2. \quad (1.1)$$

Here higher derivative terms are neglected. This is adequate to order $g_3^3$, where $g_w$ is the weak coupling constant in $d = 4$. The SU(2) Yang-Mills action $\frac{1}{4} F^2$ and the covariant derivative $D_{\mu}$ contain a gauge coupling $g_3^2$ proportional to $g_3^2 T$. The U(1) part of the standard theory is neglected here since it is not important for our considerations, but it could be easily included. Furthermore, $m_3$ and $\lambda_3$ are the effective $T$-dependent mass and coupling constant of the dimensionally reduced theory, respectively.

The dimensionless ratio $g_3^2/m_{\text{IR}}$ with some typical physical infrared (IR) scale $m_{\text{IR}}$ is not small in general. Indeed, the gauge coupling in a Wilson-type effective action increases strongly with the IR cut-off going to zero [3]. Thus the further use of the perturbation theory based upon the Lagrangian (1.1) is very dangerous (like in QCD) and it is only consistent if the scale $<\phi^+ \phi>$ is not too small. Besides this, one also has to check if the quantity which one tries to calculate perturbatively is not dominated by nonperturbative contributions, in particular if the former turns out to be small, as in the case of the effective potential to be discussed.

Since the fermions are integrated out already, the action (1.1) can be easily put on a lattice [4, 5]. The critical temperature $T_c$ of the phase transition, $v(T_c)/T_c$ with $v(T_c)$ the Higgs field minimum, the latent heat, and the surface tension can be calculated ("measured") with good accuracy. The dimensionless quantities

$$y = \frac{m_3^2}{g_3^2}, \quad x = \frac{\lambda_3}{g_3^2} \quad \text{(1.2)}$$

span the phase diagram; $y$ is connected to $(T - T_c)$ and $x$ describes the strength of the phase transition and depends on the Higgs mass. For $x \lesssim 0.03 - 0.04$ the first order phase transition obtained in two-loop perturbation theory compares very well with lattice calculations [4] of the above-mentioned quantities.

For larger $x$ values there are deviations which become stronger for quantities of decreasing perturbative order in $g_w$. In the above list, the surface tension shows the worst failure of a perturbative calculation [4]. For $x \approx 0.1$ (corresponding to $m_H \sim m_W$) lattice calculations show [4] that the phase transition fades away, there is a crossover. On the other hand, a two-loop perturbative treatment [1, 4] gives a first-order phase transition. Perturbation theory can be formally applied since $g_3^2/g_w v(T)$ is still rather small in this $x$-range; but the effective potential becomes very shallow and $v(T_c)/T_c$ is quite small, so that the phase transition is very weakly first order. We conclude that in such a situation nonperturbative contributions to the effective potential are essential.

In this paper we develop a physical picture for such nonperturbative contributions. We will present analytic indications for an instability of the vacuum of the
3-dimensional theory (1.1) at \(< F^2 > = 0\) in case of small Higgs vevs. We then will develop a up to now qualitative model for the \(< F^2 > \neq 0\) vacuum contribution to the electroweak potential. This will lead to an understanding of the crossover.

Lattice studies reveal that in the high temperature phase of the effective theory (1.1), i.e. at values of \(m_2^2\) corresponding to \(T > T_c\), one has confinement phenomena like in QCD. There is a linearly rising part in the potential between static sources [3] and there exist \(W\)-ball states almost identical to those in pure SU(2) YM theory [8], which fits nicely together with the observation that the string tension is approximately the same in both theories. Also three-dimensional correlation masses of 0+, 1-, 2+ bound states of Higgses have been “measured” [8]. They can be described [9] in a simple relativistic bound state model analogous to the one of Simonov [10] in QCD whose main ingredient is an area law for the Wegner-Wilson loop.

This area law follows naturally from the stochastic vacuum model of Dosch and Simonov [11, 12] which is quite successful in QCD. It postulates the following gauge and Lorentz covariant correlation function between field strengths \(F_{\mu\nu} \equiv F^a_{\mu\nu} T^a\) at different positions \(x, x'\)

\[
\ll g^2 F^a_{\mu\nu}(x', x_0) \frac{F^b_{\lambda\sigma}(x, x_0)}{\langle g^2 F^2 \rangle} \gg_{\text{NP}} = \frac{\delta^{ab}}{N_c^2 - 1} \frac{\langle g^2 F^2 \rangle}{d(d-1)} \times [\ldots]
\]

(1.3)

to dominate the cumulant expansion (see eq. (3.4)). Here \(< g^2 F^2 \rangle\) is the usual \(x\)-independent condensate, and \(D\) and \(D_1\) are form factors, containing a correlation length \(a\) normalized such that \(D(0) = D_1(0) = 1\). The way it is used in this paper (1.3) only contains the nonperturbative correlation in the nontrivial QCD type vacuum (we skip the index NP in the following). Thus in the background field formalism with a decomposition \(A^a_\mu = A^a_\mu^{(\text{background})} + a^a_\mu^{(\text{quantum})}\) it describes a correlated gauge field background. To obtain the nonperturbative correlation in the continuum limit from lattice calculations the (diverging) perturbative part has to be subtracted (or suppressed by “cooling”). The quantum field \(a^a_\mu\) propagates and couples according to the usual perturbation theory. We expect that the propagation of the soft quanta should be suppressed by the background and this will indeed turn out to be the case.

Furthermore

\[
F^a_{\mu\nu}(x, x_0) = \left[ P \exp(i g \int_{x_0}^x d\bar{x} A^a_{\lambda}(\bar{x})) \right]^{ab} F^b_{\mu\nu}(x) \quad (1.4)
\]

is the field strength tensor parallel-transported to a fixed reference point \(x_0\) by a Schwinger string in the adjoint representation. The integration path is fixed to be a straight line. But even then the l.h.s. of (1.3) generically depends on \(x_0\) whereas the r.h.s. only depends on \(z = x' - x\). Neglect of the \(x_0\) dependence is an assumption [12], which is fulfilled in an appropriate range of \(x_0\) where the cumulant expansion converges rapidly: The Wegner-Wilson loop vev is \(x_0\) independent and if it is evaluated with stochastic correlations also these have to be \(x_0\) independent. With a linear path
in (1.4) and with \( x_0 \) on the line \( x - x' \) the ansatz (1.3) has been tested in 4-dimensional QCD lattice calculations [13] (and is being also probed in recent 3-dimensional lattice gauge theory investigations [14]).

In a coordinate (Fock-Schwinger) gauge with reference point at \( x_0 \) the parallel transport operator in (1.4), along a straight line, equals \( \delta^{ab} \). Using this gauge in the correlator (1.3) (or more general the nonabelian Stokes theorem) in order to perform a cumulant expansion of the vacuum expectation value of the Wegner-Wilson loop with the higher cumulants neglected, one easily obtains [11, 12] the area law. The string tension is related to the local condensate \( < g^2 F^2 > \) and to the correlation length \( a \) which enters the form factors \( D \) and \( D_1 \) (see eq. (3.6)).

Applying this picture to the three-dimensional Higgs-gauge theory (1.1), a stochastic gauge field background would not only be present in the “hot” phase, but more generally at small Higgs background fields \( \phi \). It leads to a modified effective potential containing nonperturbative effects of the gauge field background. This has been already proposed [13] some time ago for constant \( < F^2 > \). In the present paper the importance of nontrivial form factors describing a correlated gauge field background is demonstrated.

We propose that the nonperturbative dynamics of the theory (1.1) is dominated by a fluctuating IR-gauge field background like in pure YM theory but with the Higgs field background as a further parameter. Our most important starting point will be the instability of a vacuum with \( < F^2 >= 0 \) for small background Higgs fields. Such an instability we obtain due to paramagnetic interactions related to the spin of the “W-gluons”. It is similar to the Savvidy instability of QCD. But it is now stated for fluctuating fields, and there are no IR singularities.

Chapter 2 contains a discussion of the order \( F^2 \)-term in an effective potential \( V \) depending on the gauge and Higgs fields and of the instability of a vacuum with \( < F^2 >= 0 \). In chapter 3 we calculate the 1-loop effective potential in a certain approximation. We introduce a momentum dependent positive \((\text{mass})^2 m_{\text{conf}}^2 \) of the gauge boson due to confinement and a negative momentum dependent \((\text{mass})^2 - \Sigma \) related to paramagnetic spin-spin interactions. The evaluation of the potential in chapter 4 allows us to qualitatively discuss the crossover behavior of the hot electroweak theory. Appendix A contains a thorough discussion of the gluon propagator in the stochastic vacuum background. While the basic idea of how stochastic background fields can lead to the formation of condensates is very easy to understand, its implementation in a gauge theory is rather involved. Therefore, in Appendix B, we discuss this mechanism for the much more transparent case of a simple scalar theory. The reader might wish to turn to this Appendix before embarking on the detailed presentation in section 2 and 3.

\section{Instability of \( F^2 = 0 \) for small Higgs field \( \varphi \)}

As a first step let us calculate the term

\[
\ll \Gamma_{FF} \gg = \int d^3 x V_{FF}(\varphi)
\]  

(2.1)
in the 3-dimensional effective action which is linear in the condensate
\[ <g_3^2 F^2 > \equiv < g_3^2 F^a \mu F^a \mu >. \]
\[
V_{FF}(\varphi) = < g_3^2 F^2 > P(\varphi^2)
\] (2.2)
where \( \varphi = \sqrt{2} \langle \phi^+ \phi \rangle \). The potential \( V_{FF} \) arises from the stochastic average of that term in the ordinary effective action \( \Gamma[A, \varphi] \) which contains two powers of \( F_{\mu \nu} \) and an arbitrary number of covariant derivatives acting on them:
\[
\ll \Gamma[A, \varphi] \gg = \ll -\frac{1}{4} g^2 \int d^d x \text{tr}_c F_{\mu \nu}(x) \Pi(-\partial^2) F_{\mu \nu}(x) + \ldots \gg \] (2.3)
\[
\ll \Gamma_{FF} \gg + \ldots
\]
Here \( \Pi \) is the \( \varphi \)-dependent polarization function (divided by \( g^2 \)) and \( \text{tr}_c \) denotes a color trace in the adjoint representation. In order to be slightly more general, we shall consider a \( SU(N_c) \) gauge theory in \( d \) dimensions and set \( N_c = 2, \quad d = 3, \quad g \equiv g_3 \) only at the very end. To proceed, we evaluate the gauge-invariant action (2.3) for gauge fields which satisfy the Fock-Schwinger gauge condition
\[
(x - x_0)_\mu A^a_\mu(x) = 0
\] (2.4)
for some arbitrary point \( x_0 \). Gauge fields satisfying (2.4) can be expressed in terms of the corresponding field strength according to
\[
A^a_\mu(x) = \int_0^1 d\eta (x - x_0)_\nu F^a_{\mu \nu}(x_0 + \eta(x - x_0))
\] (2.5)
As a consequence, there is a one-to-one correspondence between powers of \( A_\mu \) and powers of \( F_{\mu \nu} \), and being interested in \( F^2 \)-terms only we may ignore the \( A_\mu \)-terms in the covariant derivatives of (2.3)
\[
\Gamma[A, \varphi] = \frac{1}{4} g^2 N_c \int d^d x F^a_{\mu \nu}(x) \Pi(-\partial^2) F^a_{\mu \nu}(x) + \ldots
\] (2.6)
A priori the reference point \( x_0 \) in the stochastic correlator (1.3) is unrelated to the point \( x_0 \) above which characterizes a specific Fock-Schwinger gauge. In the following we shall identify these two points. This has the consequence that the parallel-transport operator in eq. (1.4) becomes equal to the unit matrix. Only in this case we are dealing with the correlator of two local field strength operators. In particular,
\[
\ll g^2 F^a_{\mu \nu}(x') F^a_{\mu \nu}(x) \gg = < g^2 F^2 > D_{\text{eff}} \left( \frac{z^2}{a^2} \right)
\] (2.7)
with the abbreviation
\[
D_{\text{eff}} \left( \frac{z^2}{a^2} \right) \equiv \kappa D \left( \frac{z^2}{a^2} \right) + (1 - \kappa) D_1 \left( \frac{z^2}{a^2} \right) + (1 - \kappa) \frac{2z^2}{da^2} D_1' \left( \frac{z^2}{a^2} \right)
\] (2.8)
Now it is straightforward to compute \( \ll \Gamma_{FF} \gg \) by applying (2.7) to (2.6):
\[
V_{FF}(\varphi) = \frac{1}{4} N_c < g^2 F^2 > \int \frac{d^d p}{(2\pi)^d} \Pi(p^2) \tilde{D}_{\text{eff}}(p^2)
\] (2.9)
Here $\tilde{D}_{\text{eff}}(p^2)$ is the Fourier transform of $D_{\text{eff}}(z^2/a^2)$. Denoting the Fourier transforms of $D(z^2/a^2)$ and $D_1(z^2/a^2)$ as $\tilde{D}(p^2)$ and $\tilde{D}_1(p^2)$, respectively, it reads

$$\tilde{D}_{\text{eff}}(p^2) = \kappa \tilde{D}(p^2) - \frac{2}{d}(1 - \kappa)p^2 \tilde{D}_1(p^2) \quad (2.10)$$

Later on in our numerical computations we shall consider the case $\kappa = 1$ only¹ and use the two form factors

$$D^{(1)} \left( \frac{z^2}{a^2} \right) = e^{-|z|/a} \quad (2.11)$$

$$D^{(2)} \left( \frac{z^2}{a^2} \right) = e^{-z^2/a^2} \quad (2.12)$$

Their Fourier-transforms for $d = 3$ are, respectively,

$$\tilde{D}^{(1)}(p^2) = \int d^3 z e^{ipz} e^{-|z|/a} = \frac{8\pi a^3}{(1 + a^2p^2)^2} \quad (2.13)$$

$$\tilde{D}^{(2)}(p^2) = \pi^{3/2} a^3 \exp \left( -\frac{a^2}{4p^2} \right) \quad (2.14)$$

It remains to compute the polarization function $\Pi$. We restrict ourselves to a one-loop calculation here. The dominant contributions to $\Pi$ come from gauge boson and ghost loops. (Higgs loops play a minor role and are considered later on.) Using the proper time method, their contribution to the effective action is given by

$$\Gamma = \frac{1}{2} \text{Tr} \ln[K] - \text{Tr} \ln[-D^2 + m^2]$$

$$= -\int_0^\infty \frac{dT}{T} \left\{ \frac{1}{2} \text{Tr}[e^{-TK}] - \text{Tr}[e^{-T(-D^2 + m^2)}] \right\} \quad (2.15)$$

Here

$$K^{ab}_{\mu\nu} = [-D^2 \delta_{\mu\nu} + 2igF_{\mu\nu} + m^2 \delta_{\mu\nu}]^{ab} \quad (2.16)$$

is the kinetic operator of the gauge bosons in the Feynman-’t Hooft gauge and $-D^2 + m^2$ the corresponding one for the ghosts. We identify

$$m^2 \equiv \frac{1}{4} g^2 \varphi^2, \quad \varphi = \sqrt{2} \langle \phi^+ \phi \rangle \quad (2.17)$$

Using the method of Barvinsky and Vilkovisky [10] or the corresponding world line technique [17] it is straightforward to identify the $F^2$-terms which are contained in the functional traces of (2.15):

$$\Pi(p^2) = -\frac{4}{(4\pi)^{d/2}} \int_0^\infty dTe^{-m^2TT^{-d/2 + 1}}$$

$$\cdot \left[ f(p^2T) + \frac{1}{4}(d - 2) \frac{f(p^2T) - 1}{p^2T} \right] \quad (2.18)$$¹The $D_1$ form factor does not contribute to the area law [11, 12]
with the familiar “second-order form factor”

\[ f(p^2T) \equiv \int_0^1 d\alpha \exp[-\alpha(1-\alpha)p^2T] \quad (2.19) \]

Performing the proper time integral yields \( \Pi = \Pi_G + \Pi_F \) with

\[ \Pi_G(p^2) = -\frac{(d-2)\Gamma(1-d/2)}{(4\pi)^{d/2}} \frac{1}{p^2} \int_0^1 d\alpha \left[ (m^2 + \alpha(1-\alpha)p^2)^{d/2-1} - m^{d-2} \right] \]

\[ \Pi_F(p^2) = -\frac{4\Gamma(2-d/2)}{(4\pi)^{d/2}} \int_0^1 d\alpha \left[ m^2 + \alpha(1-\alpha)p^2 \right]^{d/2-2} \quad (2.20) \]

The contribution \( \Pi_F \) originates from the nonminimal coupling of the gauge boson fluctuations to the background (the \( F_{\mu\nu} \)-term in \( K_{\mu\nu} \)), while \( \Pi_G \) arises from the \( D^2 \)-term in \( K_{\mu\nu} \) and from the ghosts. Using (2.20) in (2.9) for \( d = 3 \) and \( N_c = 2 \) we obtain the desired expression for the potential:

\[ V_{FF}(\varphi) = -\frac{1}{8\pi^3} < g_3^2 F^2 > \int_0^\infty dp \, p^2 \tilde{D}_{\text{eff}}(p^2) \]

\[ \cdot \int_0^1 d\alpha \left\{ \left[ m^2 + \alpha(1-\alpha)p^2 \right]^{-1/2} - \frac{1}{2p^2} \left[ (m^2 + \alpha(1-\alpha)p^2)^{1/2} - m \right] \right\} \quad (2.21) \]

The momentum integration in (2.21) converges both for \( p \to \infty \) and for \( p \to 0 \). In fact, \( V_{FF} \) is IR-finite even in the limit \( m \equiv g_3 \varphi/2 \to 0 \) of massless gauge bosons and ghosts. We emphasize that this IR finiteness has nothing to do with the dynamical generation of a mass for these particles (see below), but rather with the “inclusive” nature of the quantity at hand. For \( d = 3 \) and \( m = 0 \) the polarization function

\[ \Pi(p^2) = -\frac{1}{2} \left( 1 - \frac{1}{16} \right) \frac{1}{\sqrt{p^2}} \quad (2.22) \]

is singular at \( p = 0 \), of course, but the volume element \( d^3p = 4\pi p^2 dp \) in (2.9) has the effect of completely suppressing the contribution of \( p = 0 \) to \( V_{FF}(\varphi) \). (\( \tilde{D}_{\text{eff}}(0) \) is finite and nonvanishing.)

In eq. (2.22) the “1” comes from \( \Pi_F \) while the “\(-1/16\)” is due to \( \Pi_G \). We observe that the nonminimal \( F_{\mu\nu} \) interaction of the gauge boson fluctuations is the dominant effect. The contribution from the minimal couplings of gauge bosons and ghosts is smaller by more than one order of magnitude and has the opposite sign. This situation persists also for \( m > 0 \).

It is important to note that the one-loop contribution \( V_{FF}(\varphi) \) is negative. It has to be added to the stochastic average of the positive tree-level term,

\[ \ll \frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} \gg = < g_3^2 F^2 > / 4 g_3^2. \]

If it is large enough so that the sum is still negative, this indicates that the vacuum with a vanishing gauge field condensate is unstable and that the true ground state of the system is characterized by a nonvanishing value of \( < F^2 > \).\footnote{See ref. [24] for a related discussion in the exact renormalization group framework.}
At the linearized level, the dynamics of the gauge boson fluctuations $a_\mu(x)$ is governed by the Lagrangian

$$\mathcal{L} = \frac{1}{2} a_\mu \left[ -D^2 \delta_{\mu\nu} + 2igF_{\mu\nu} + m^2 \delta_{\mu\nu} \right] a_\nu + O(a_\mu^3)$$  \hspace{1cm} (2.23)$$

The coupling $a_\mu F_{\mu\nu} a_\nu$ is the spin-1 analogue of $\bar{\psi} \vec{\sigma} \cdot \vec{B} \psi$ for spin-$\frac{1}{2}$ particles. It gives rise to a “paramagnetic” interaction of the gauge bosons, while the terms resulting from the minimal substitution, $a_\mu D^2 a_\mu$ lead to the more conventional “diamagnetic” behavior [18]. Thus, using the terminology of solid state physics, we can say that the instability found above results from a dominance of paramagnetic over diamagnetic effects.

It is interesting to note that the possible existence of such an instability is closely related to the fact that the gauge bosons carry spin 1 and have a (tree level) Landé factor $g_L = 2$. This is most easily seen if we expose a “colored” particle of spin $S = \frac{1}{2}$ or $S = 1$ with $g_L = 2$ to a covariantly constant color magnetic field of strength $B$ along the z-axis. Its squared one-particle energies are given by [18, 19]

$$E^2 = m^2 + p_z^2 + gB(2n + 1) - 2gBS_z$$ \hspace{1cm} (2.24)$$

where $n = 0, 1, 2, \ldots$ enumerates the Landau levels and $S_z$ is the spin projection along the z-axis. In eq. (2.24) the terms $gB(2n + 1)$ and $-2gBS_z$ stem from the diamagnetic and the paramagnetic interactions, respectively. An instability is signalled by an imaginary part of $E$. In the case of fermions we have the two possibilities $S_z = -1/2$ and $S_z = +1/2$. In the former case, $E^2$ is strictly positive for all $n$, but in the latter the paramagnetic term precisely cancels the diamagnetic one at $n = 0$. For $m^2 = 0$ one finds at $p_z = 0$ a marginally stable state at $E^2 = 0$. For a spin-1 particle one has $S_z = 0, \pm 1$. Hence, for the spin properly aligned to the external magnetic field, the diamagnetic energy of the lowest Landau level, $+gB$, combines with a paramagnetic contribution $-2gB$ to yield a negative total magnetic energy $-gB$. Hence $E^2 < 0$ for $m^2 + p_z^2$ sufficiently small. (This effect is responsible for the instability discussed in the context of the Savvidy vacuum of QCD, for instance.)

The effective action which we have calculated above describes the interaction of gauge boson fluctuations with non-constant external fields of arbitrary momentum. Hence their spectrum is certainly not given by (2.24). However, what continues to be true is that for the spin-1 field the paramagnetic interaction overrides the diamagnetic one – which would not be possible for Dirac fermions.

When $\varphi^2$ is increased from $\varphi^2 = 0$ to larger values, the potential $V_{FF}(\varphi)$ approaches zero from below. Thus, starting from $< F^2 > \neq 0$ at $\varphi^2 = 0$, one will return to a situation with vanishing condensate for $\varphi^2$ larger than a certain critical value $\varphi^2_c$.

In Appendix B we discuss a scalar toy model and give a simple physical explanation of why stochastic background fluctuations tend to trigger the formation of condensates.

Taking $a \to \infty$ in eq. (2.21) with form factors (2.13) or (2.14) leads back to the perturbative result $\sim g_3^2 < F^2 > /m$, implying the 1-loop gauge $Z$-factor

$$Z_{\text{gauge}} = 1 - \frac{g_3^2}{8\pi} \left[ \frac{7}{m_W} + \frac{2}{3m_{gh}} - \frac{1}{8m_{Gb}} - \frac{1}{24m_H} \right]$$ \hspace{1cm} (2.25)$$

\text{3There is a misprint in formula (A.12) of ref. [20].}
Figure 1: The function $G(ma)$ appearing in $V_{FF}$ of eq. (2.26) for the two different form factors

$$m^2_W = m^2_{gh} = \frac{1}{4} g_3^2 \varphi^2; \quad m^2_{Gb} = \frac{1}{4} g_3^2 \varphi^2 + \lambda_3 \varphi^2 + m_3^2; \quad m^2_H = 3 \lambda_3 \varphi^2 + m_3^2$$

It has an IR singularity at $m^2 = \frac{1}{4} g_3^2 \varphi^2 = 0$ which is removed in our calculation with a gauge field correlator. Indeed for $a \to \infty$ we obtain the $W$-boson plus ghost part of $V_{FF}$ as $-(23/12\pi) g_3^2 \left(\frac{1}{4} F^2\right)$ as given in eq. (2.25).

The scalar Higgs contributions in the loop can be also taken into account in a Barvinsky-Vilkovisky type formula. Higgs fields in the loop are easily included in $V_{FF}$ by additional terms in $\Pi$ proportional to $(f(-T \partial^2) - 1)/(-T \partial^2)$, and by modified prefactors $\frac{1}{8N_c} e^{-m^2 \mu T}$ and $\frac{3}{8N_c} e^{-m^2 \epsilon T}$ for the Higgs boson and the Goldstone bosons, respectively, instead of $(d-2) e^{-m^2 T}$. This is in agreement with (2.25) for $a \to \infty$ ($N_c = 2, d = 3$). It is obvious that these parts are small compared to the gauge parts. Mixed gauge boson-Higgs field graphs do not contribute.

Evaluation of $V_{FF}$ in eq. (2.9) for $\kappa = 1$, i.e. with a pure $D$-form factor gives

$$V_{FF} = - <g_3^2 F^2> aG(ma)$$

with

$$G(ma) = \frac{(ma)^2}{16\pi^3} \int_0^\infty d\bar{p} \bar{D}(m^2 a^2 \bar{p}^2) \left[ \left(\frac{15}{4} \bar{p} - \frac{1}{\bar{p}}\right) \left(\arcsin \frac{\bar{p}}{\sqrt{\bar{p}^2 + 4}} - \frac{\bar{p}}{2}\right) + \frac{15}{8} \bar{p}^2 \right]$$

where $\bar{D}(p^2) \equiv a^3 \bar{D}(m^2 a^2 \bar{p}^2)$ with $\bar{p} \equiv p/m$. For $m^2 \to 0$ this simplifies to

$$G(0) = \frac{15}{128\pi^2} \int_0^\infty d\bar{p} \bar{D}(\bar{p}^2)$$

(2.28)
The first term inside the curly brackets of eq. (2.21) derives from the $F_{\mu\nu}$ part in $K_{\mu\nu}$ of eq. (2.16). It contributes $4\bar{p}\arcsin(\ldots)$ in the bracket in (2.27) and dominates $V_{FF}$. It is responsible for the negative sign of $V_{FF}$ which is capable of generating an instability at $F^2 = 0$.

The numerical evaluation of (2.27) for

1) $\bar{D}(1)(k^2) = \frac{8\pi}{1 + k^2}$ and

2) $\bar{D}(2)(k^2) = \frac{3\pi}{2}\exp(-k^2/4)$

corresponding to the form factors (2.11) and (2.12), respectively, is shown in fig. 1.

The massless limit is given by (2.28). $G(0)$ of eq. (2.27) is calculated as

\[ G(1)(0) \simeq 0.1492 \] (2.29)

and

\[ G(2)(0) \simeq 0.1322 \] (2.30)

respectively. Note that defining $\bar{a} = \int_0^\infty dz D(z^2)$ one has $\bar{a} = a$ in case (1) and $\bar{a} = 0.85a$ in case (2); thus $aG(0)$ has a very similar value in both cases if expressed in terms of $\bar{a}$.

There is a further contribution to $V_{FF}$ by a Higgs field tree term (fig. 2). Because of the correlator it is similar to a 1-loop term. The gauge-boson coupling to an external constant Higgs field is given by $(-ig\partial_\mu A_\mu^a)(\Phi^aT^a\varphi_{qu} - \varphi_{qu}^+T^a\Phi)$, where $\varphi_{qu}$ is the quantum fluctuation. The correlator

\[ \ll \partial_\mu A_\mu^a(x)\partial_\mu A_\mu^a(x') \gg \] (2.31)

can be conveniently evaluated in the Fock-Schwinger (coordinate) gauge (2.5) (like the Barvinsky-Vilkovisky form factors [16]) with $x_0 = \frac{x+x'}{2}$ (since we are discussing the contribution to an effective action with quasilocal terms). Note that the differentiation in (2.31) should not affect the $x, x'$ in $x_0$. Fourier transformation to momentum space gives

\[ V_{FF}^{\text{tree}} = \frac{2}{9} < g_3^2 F^2 > \Phi^+T^a\Phi \frac{1}{(2\pi)^2} \int_0^\infty dq d\eta d\bar{\eta} \frac{\eta\bar{\eta}}{(\eta + \bar{\eta})^3} \] (2.32)

\[
\lim_{p\to 0} \left( \frac{2}{p} \log \left( \frac{(p+q)^2 + m_3^2}{(p-q)^2 + m_3^2} \right) \right) \left( 3\bar{D}\left( \frac{4q^2}{(\eta + \bar{\eta})^2} \right) + \frac{8q^2}{(\eta + \bar{\eta})^2} \bar{D}'\left( \frac{4q^2}{(\eta + \bar{\eta})^2} \right) \right)
\]
The limit \( \lim_{p \to 0} (\ldots) \) equals \( 8q/(q^2 + m_3^2) \). After an integration by parts of the second term with respect to \( q \), the \( \eta - \bar{\eta} \) integration can be done analytically. The \( (\eta + \bar{\eta})/2 \) and \( q \) integrations were done numerically for the special form factors (2.13) and (2.14). This results in

\[
V_{\text{tree}}^{\text{FF}} = \frac{\varphi^2}{4} a^2 I_q(m_3a) \left< g_3^2 F^2 \right> > \frac{1}{6 \cdot 3} \tag{2.33}
\]

where

\[
I_q(m_3a) = \frac{6}{(2\pi)^3} \int_0^\infty dq (\tilde{q}^2 + a^2 m_3^2)^2 I_q(q\tilde{q})
\]

with some function \( I_\eta(qa) \) which can be extracted from (2.32). The function \( I_q(m_3a) \) is plotted in fig. 3. Note that \( I_q(m_3a) \) does not exist for \( m_3^2 < 0 \). \( I_q \) is a monotonically decreasing function of \( am_3 \) with \( I_q(0) \approx 0.005 - 0.01 \). \( V_{\text{tree}}^{\text{FF}} \) is positive and thus supports the vanishing of the gauge condensate for increasing \( \varphi^2 \).

The potential \( V_{\text{FF}}^{\text{tree}} \) of eq. (2.26) and the additional small positive pieces including the \( V_{\text{tree}}^{\text{FF}} \) discussed above have to be added to the tree potential \( < 1/4 F^2 > \). From eq. (2.26) and (2.33) we obtain an instability at \( F^2 = 0 \) if

\[
g_3^2 aG(ma) - m_3^2 a^2 I_q(m_3a)/18 > 0.25 \tag{2.34}
\]

An instability at \( F^2 = 0 \) requires a correlation mass \( 1/a < 4G(0)g_3^2 \approx 0.6g_3^2 \).

Lattice studies [14] indicate \( 1/a \sim 0.73g_3^2 \sim m_{\text{glueball}}/2 \), as suggested in ref. [3]. Strictly speaking, our calculation does not give an instability with such a value of \( 1/a \). However, keeping in mind the limitations of the present semi-quantitative approach, we have to be aware that it might be not very accurate. It is also gauge dependent. In perturbative calculations of the electroweak potential one observes that a strong gauge dependence of the 1-loop order result indicates that the next loop order contribution is needed and that one can arrive in 2-loop order at a quantitative reliable result without
much gauge dependence \[1, 7\]. Thus the gauge dependence of the 1-loop expression is a very useful hint. A remaining gauge dependence of the potential \[7\] together with a similar one of the Z-factor in the kinetic term should drop out in the calculation of the action of an extremal field configuration such as the critical bubble. But this has not really been possible to check up to now even in pure perturbative calculations. Here we employed the Feynman-’t Hooft gauge. In our present treatment the use of a specific model for the IR sector of the theory could weaken, but certainly not erase the gauge-fixing dependence. A 2-loop calculation in the perturbative part is very demanding, but would be very interesting.

A negative $F^2$ term then leads to a ground state with $< F^2 > \neq 0$ to be stabilized by confinement forces in an effective potential $V(\varphi^2, < g_3^2 F^2 >)$ as we will argue in the following chapters. The gauge boson condensate is determined as the minimum position of the “potential” in $F^2$. Given the minimum value of this potential, its negative linear part at small $F^2$ allows a rough estimate of the minimum position (e. g. allowing for an $F^4$ term). Thus the lattice determination of the nonperturbative finite part $a_4 g_3^2$ of the free energy in ref. \[31\] in pure Yang-Mills theory ($m^2 = 0$) may be used to connect the $F^2$ coefficient discussed in section 2 with the gauge condensate and with the string tension also known from the lattice. The small negative $a_4$ reported in \[31\] then points to a negative coefficient of $\frac{1}{4} F^2$ with an absolute value much smaller than 1. This strengthens our view that we are close to instability with the lattice value of $a$ and that only small further (2-loop) contributions are needed to get complete agreement. A careful analysis including lattice renormalization is required.

The instability of $< F^2 > = 0$ is strengthened with growing $g_3^2$. If we define the IR running of $g_{3, \text{eff}}^2$ via the prefactor of the $F^2$ term in the effective action, the negative sign of the 1-loop contribution tells us that $g_{3, \text{eff}}^2$ is indeed larger than $g_3^2$ and that it would diverge at the border of instability of $< F^2 > = 0$. In the Wilsonian renormalization-group approach of ref. \[4\] $g_{3, \text{eff}}^2(k)$ at a running IR scale $k$ is the central quantity. Its divergence at some $k = k_c$ signals confinement like in QCD. It is obtained from a differential renormalization-group procedure and certainly goes beyond a 1-loop order calculation. One way to look at the effective action (average action) of this approach is to supplement the usual background field formalism with a smooth IR cut-off $k$ for the momenta of inner propagators \[22\]. The stochastic vacuum with a correlation length $a$ (together with the condensate $< F^2 >$, both in principle to be determined dynamically) then constitute some “physical” value for the IR scale $k$.

### 3 The effective action with stochastic vacuum correlations

In the previous section, only those terms in $<< \Gamma[A, \varphi] >>$ were calculated which are linear in $< g_3^2 F^2 >$. Let us now turn to the much more complicated question of how $<< \Gamma >>$ depends on $< g_3^2 F^2 >$ to all orders.

The 1-loop effective action with a gauge boson circulating in the loop and with a Higgs field plus a correlated gauge field background can be easily written down as a
worldline path integral:

\[
\Gamma_{\text{gauge}}(m^2, <g^2F^2>) = -\frac{1}{2} \int_0^\infty \frac{dT}{T} \int_{x(0)=x(T)}^\infty [Dx] \exp \left\{ - \int_0^T d\tau \left( \frac{\dot{x}_\mu^2(\tau)}{4} + m^2 \right) \right\} \\
\ll \text{tr}L_P \exp \left\{ - \int_0^T d\tau (-ig) (A_\mu(x)\dot{x}_\mu + 2F(x)) \right\} \gg
\]

(3.1)

The subscripts “c” and “L” refer to the color and Lorentz matrix structure, respectively. We use the coordinate gauge based at the center of mass point \(x_0\) defined by

\[
x(\tau) = x_0 + y(\tau), \quad \int_0^T d\tau y(\tau) = 0, \quad Dx = d^4x_0Dy
\]

(3.2)

which amounts to

\[
A^a_\mu(x_0 + y) = \int_0^1 d\eta \eta y_\nu F^a_{\nu\mu}(x_0 + \eta y)
\]

(3.3)

The parallel transport implicit in the correlator on the l.h.s. of (1.3) is taken along a \(V\)-shaped path consisting of two straight lines from \(x\) to \(x_0\) and from \(x_0\) to \(x'\), respectively. The path is of course not the straight line connecting \(x\) to \(x'\) which is normally used in lattice calculations [13]. One has to make the assumption that the correlation length is independent of the position \(x_0\) somewhere inside the loop [12].

The average over the gauge field background \(\ll .. \gg\) in (3.1) can be evaluated using the lowest (quadratic) term in the cumulant expansion. This is the “stochastic vacuum” model assumption

\[
\ll e^{\int A} \gg \approx \exp(-1/2 \int \ll AA \gg)
\]

(3.4)

The correlator in the exponential follows from eq. (1.3) with (3.3). After rescaling \(\tau = T\bar{\tau}, \quad y = T^{1/2}\bar{y}\) and taking only the form factor \(D\) in (1.3) (\(\kappa = 1\)), we obtain for the correlated part in (3.1)

\[
\Gamma_{\text{gauge}}(m^2, <g^2F^2>) = -\frac{1}{2} \int_0^\infty \frac{dT}{T} T^{-d/2} \int d^4x_0 \int [D\bar{y}] \exp \left\{ - \int_0^1 d\bar{\tau} \frac{\dot{y}_\mu^2}{4} - m^2 T \right\} \\
\times \text{tr} \exp \left[ - \frac{g_3^2 F^a A^a}{d(d-1)} \frac{1}{(N_c^2 - 1)} T^2 \int_0^1 d\bar{\tau} d\tilde{\tau}' \left\{ \int_0^1 d\eta d\eta'((\eta\eta'')(-\dot{\eta} \cdot \dot{\eta}' + \dot{\eta} \cdot \dot{\eta}' \cdot \dot{\eta}'')1l_L D \left( \frac{(\eta\dot{\eta} - \eta'\dot{\eta}')^2 T}{a^2} \right) \\
+ 4(1 - d) 1l_L D \left( \frac{(\bar{y} - \bar{y}')^2 T}{a^2} \right) \\
+ \int_0^1 d\eta (\bar{y} \times \dot{\eta} - \dot{\bar{y}} \times \bar{y}') D \left( \frac{(\eta\dot{\eta} - \eta'\dot{\eta}')^2 T}{a^2} \right) + (\text{primed} \leftrightarrow \text{unprimed}) \right\} \right]\]

(3.5)

Expansion to order \(F^2\) would reproduce the contribution from gauge bosons in the loop calculated in chapter 2.

\(^4\)Some indication that the correlation length is path independent can be found in ref. [20].
The ghost contribution $\Gamma_{\text{ghost}}$ has the opposite sign and an additional factor of 2 compared to (3.1). The trace is only over adjoint color indices, and only the first term in the curly brackets in (3.5) appears.

The first term of (3.5) originates from the correlation of two Wegner-Wilson loop integrals $\exp(-\oint A dx)$. Using the nonabelian Stokes theorem one obtains area integrals and only the first term in the curly brackets in (3.5) appears.

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The first term in (3.5) is just this area integral. For large areas it reduces to
\[
\bar{\sigma} = \pi < g_3^2 F^a F^a > \frac{1}{d(d-1)} \frac{N_c}{N_c^2 - 1} T \int_0^\infty dz z D \left( \frac{z^2}{a^2} \right) = \sigma_{\text{adj}} T = \frac{2N_c^2}{N_c^2 - 1} \sigma_{\text{fund}} T
\] (3.6)

\[
\bar{A} \equiv A/T \quad \text{where} \quad A \text{ is the area of the (minimal) surface whose boundary is given by the closed path } y(\tau). \]

In eq. (3.6) we wrote $\sigma_{\text{adj}}$ and $\sigma_{\text{fund}}$ for the confining string tension in the adjoint and fundamental representation, respectively.

The second term in the curly brackets in (3.5) arises from the correlation of two explicit factors of $F_{\mu\nu}$ in the exponential of eq. (3.1). It is related to a gauge boson spin interaction which is short-ranged. Its range is of the order of the correlation length $a$. This interaction owes its existence to the nonminimal “paramagnetic” coupling of the gauge boson fluctuations $a_\mu(x)$ to the background field, i.e. to the $F_{\mu\nu}$-term in the kinetic operator $K_{\mu\nu}$. The third term in (3.5) is due to the interference of the confinement and spin effects.

To start, let us first disregard the “diamagnetic” interactions coming from the $D^2$-term in $K_{\mu\nu}$ and let us focus on the spin interaction. For large loops ($d \gg a$) the $\tau, \tau'$-correlations are well approximated by nearest neighbour interactions along the closed path. Instead of doing the above world-line path integral one can change to the ordinary QFT language and calculate the gluon self-energy $\Sigma$ in a correlated gauge background. This calculation is described in Appendix A in some detail. There we also include the covariant derivative induced parts of the interaction. Of course, the use of the correlator (1.3) again requires the specification of a reference point for the parallel transport operator and for the coordinate gauge. In the computation of $\Sigma(x,x')$ we identify both of those points with $x_0 \equiv (x + x')/2$.

For the spin-induced part of $\Sigma$ we obtain
\[
\Sigma(x,x')_{\mu\nu}^{ab} = S_F(z^2 = (x - x')^2)\delta_{\mu\nu}\delta^{ab}
\] (3.7)

with
\[
S_F(z^2) = < g^2 F^2 > \frac{4N_c}{d(N_c^2 - 1)} H \left( \frac{z^2}{2} \right) D \left( \frac{z^2}{a^2} \right)
\] (3.8)

where $H(z^2)$ is a massive ($m^2 = g_3^2 \varphi^2 / 4$) scalar propagator function. Fourier transformation leads to
\[
\tilde{S}_F(p^2) = + < g^2 F^2 > \frac{1}{9\pi^2} \int_0^\infty dq \frac{q^2}{\ln \left( \frac{(p + q)^2 + m^2}{(p - q)^2 + m^2} \right) D(q^2)}
\] (3.9)
where we have inserted the physical values $d = 3$ and $N_c = 2$ now. The contribution $S_F(z^2)$ is given by the last term of the expression (A.31) for the complete function $S(z^2)$.

Eq. (3.8) shows that the range of the spin interaction is determined by the fall-off behavior of the form factor $D$. Furthermore it is important to observe that $S_F(z^2)$ is a positive function. The self-energy $\Sigma$ is defined in such a way that the inverse dressed propagator is $G(0)^{-1} - \Sigma$ where $G(0)$ is the free one (see eq. (A.38)). Hence $S_F > 0$ implies a negative (“tachyonic”) contribution to the effective squared mass of the gauge boson. This means that the interaction with the stochastic background destabilizes the “empty” vacuum where the gauge boson fluctuations have a vanishing expectation value: They have the tendency to condense which then gives rise to a nonzero vacuum expectation value of $F^2_{\mu\nu}$.

This conclusion does not change if we retain the interactions coming from the $D^2$-term in $K_{\mu\nu}$, i.e. the $\dot{x}_\mu A_\mu(x)$-term in the world line path integral (3.1) in a coordinate gauge with $x_0 = (x_1 + x_2)/2$. The complete result is given in eq. (A.35). This delivers part of the gauge interaction in $D^2$ into $\Sigma$ whereas the main part constitutes the confining force summarized in the area law term discussed below. In eq. (A.38) we give the complete result for the mass shift $\Delta m^2$ of the gauge field fluctuations. The spin interaction contributes the “1” in the curly brackets of eq. (A.38), whereas the $D^2$-interactions produce the terms proportional to $\theta_1$ and $\theta_2$. In view of (A.39) the latter are seen to be negligible at our present level of accuracy.

Inserting the self-energy (3.9) in a gauge boson loop leads to an effective potential

$$V_{S_F} \sim 2 \frac{9}{2} \int \frac{d^3p}{(2\pi)^3} \ln[(p^2 + m^2 - \tilde{S}_F(p^2))]$$

(3.10)

Here we have multiplied by 2 because of the two different ways to get a chain of neighbours. This has to be corrected for the zeroth and first order terms in $\tilde{S}_F$. $S_F(p^2)$ is positive and contributes negatively in the propagator. Of course (3.10) has to be UV-renormalized but the counter terms are identical to those of the case $S_F = 0$. Thus (3.10) reads more correctly

$$V_{S_F} = \frac{9}{2} \int \frac{d^3p}{(2\pi)^3} \left\{ 2 \ln(p^2 + m^2 - \tilde{S}_F(p^2, m^2)) - \ln(p^2 + m^2) \\
+ \frac{\tilde{S}_F(p^2, m^2)}{p^2 + m^2} - \frac{m^2}{p^2} - \ln p^2 \right\}$$

(3.11)

The ghost loop-induced part has to be added - it does not contain $\tilde{S}_F$.

Let us now focus on the other extreme and consider the approximation of a pure area law in the IR for the first term in (3.5). Then it behaves as $\sim \exp(-\sigma \bar{A} T)$ for large $\bar{A} T$. Furthermore, we assume that when $\bar{A} T$ becomes smaller this term is damped as $\sim \exp(-c \bar{A}^n <g^2_T>^n) \sim \exp(-c \bar{A}^n <g^2_T>^n) \sim \exp(-c \bar{A}^n (a^2)^{n-2})$ with an exponent $n > 2$. This damping is supposed to set in for $\bar{A} T < \tilde{c} a^2$ where $\tilde{c} = O(1)$ is treated as a free parameter. In the following

5Such a behavior with $n = 2$ was argued for in the context of the stochastic vacuum model in ref. [34]. It would lead to a $m^2_{\text{conf}}(p^2)$ in (3.13) below with a rather weak fall-off in $p^2$ (like $\tilde{S}_F(p^2)$). The additional piece in $\tilde{S}$ in appendix A due to $n$-integrals has also such a behaviour. Thus a contribution with $n = 2$ would be better associated with the $\Sigma(x, x')$ evaluated before.
we will use the interpolating ansatz with \( n = 3 \)

\[
\sim \exp \left( -\sigma \bar{A} \frac{T^3}{(\tilde{c} a^2 / \bar{A})^2 + T^2} \right) \tag{3.12}
\]

If we include also an area law for the ghost loop and renormalize at \( m^2 = F^2 = 0 \) we obtain

\[
V_{\text{area}} = -\frac{(9 - 6)}{2} \int_0^\infty \frac{dT}{T} T^{-3/2} \int D\bar{y} \exp \left\{ -\int_0^1 d\bar{\tau} \frac{\bar{y}^2}{4} \right\} \times \exp \left( \frac{-\sigma \bar{A} T^3}{(\tilde{c} a^2 / \bar{A})^2 + T^2} - m^2 T \right) - 1 + m^2 T \tag{3.13}
\]

where \( \bar{A} \) is a complicated functional of the interaction variable \( \bar{y}(\bar{\tau}) \): it is the area of the minimal surface spanned by the loop \( \bar{y} \).

Substituting \((4\pi T)^{-3/2} = \int \frac{d^3p}{(2\pi)^3} e^{-p^2 T^2} \) in \((3.13)\) as a procedure to trade \( T \) for a momentum integration, the \( T \) integration can be performed (numerically if \( \tilde{c} \neq 0 \)) and we arrive at

\[
V_{\text{area}} = -\frac{(9 - 6)}{2} \int \frac{d^3p}{(2\pi)^3} (4\pi)^{3/2} \int D\bar{y} \exp \left\{ -\int_0^1 d\bar{\tau} \frac{\bar{y}^2}{4} \right\} \times \left[ \ln[p^2 + m^2 + m_{\text{conf}}^2(p^2, <\bar{A}>, m^2)] - \ln p^2 - \frac{m^2}{p^2} \right] \tag{3.14}
\]

where \( m_{\text{conf}}^2 \) is a complicated function defined by the requirement of producing the \( \ln[p^2 + (\text{mass})^2] \) form of the integrand. It is easy to convince oneself that this function always exists and is positive. It would be \( p^2 \)-independent and equal to \( \sigma \bar{A} \) if \( \tilde{c} = 0 \). Thus we have an area-dependent, momentum-dependent positive \( m_{\text{conf}}^2 \) related to confinement. It acts as an IR regulator in the same way as the Higgs mass but it is momentum-dependent.

The functional integral over \( \bar{y}(\bar{\tau}) \) cannot be performed in closed form. It could be performed numerically. Here we make the rather crude approximation that it is dominated by paths with areas peaked near a mean value \(<\bar{A}>\). The numerical value of \(<\bar{A}>\) will be treated as a free parameter. For \( \tilde{c} = 2 \) and an average value \(<\bar{A}> = 2.5 \) the function \( m_{\text{conf}}^2(p^2, <\bar{A}>, m^2 = 0) \) is plotted in fig. 5.

The main contribution to \( m_{\text{conf}}^2(p^2, <\bar{A}>, m^2) \) comes from \( T<\bar{A}> \)-values smaller than \( \sim 10 - 20g_3^{-4} \) which is not very large. With increasing values of \( p^2 \) even smaller \( T<\bar{A}> \)-values dominate. At very large areas we would expect gluon loop holes in the area corresponding to the splitting of the confining string and virtual production of glueballs. In the case of static valence fields this would lead to a screening and a flattening of the linear rise of the 2-dimensional potential. As recently measured on the lattice this only happens in the case of very large distances \( d \sim 5 - 10g_3^{-2} \) much bigger than the inverse glueball mass \( \sim a/2 \sim (2 \cdot 0.73g_3^2)^{-1} \) \([4]\). The relation between the 2-dimensional static potential and our effective action is not clear but we would expect that screening is important only at very large areas \( T\bar{A} \). The screening effect corresponds to higher correlations and is not described by the stochastic vacuum model with Gaussian correlations.
The spin-induced $\Sigma$ and the confinement effects have opposite impact on the “magnetic mass”. In (3.5) there is also the last term which arises from the interference of the two effects. It changes sign under the substitution $\bar{y}(\bar{\tau}) \Rightarrow \bar{y}(1 - \bar{\tau})$ and thus the term would cancel if it is small. Leaving it aside one can combine the first (confinement) and second (spin interaction) term in the exponential in (3.5) and add a corresponding expression without spin interaction for the ghosts. Thus one arrives at

$$V = \int_0^\infty \frac{dT}{T} T^{-3/2} e^{-m^2 T} \int D\bar{y} \exp \left\{ - \int_0^1 d\bar{\tau} \frac{\bar{y}^2}{4} \right\} \times \left[ -\frac{9}{2} \exp \left\{ -\sigma \bar{A}^3 T^3 \left( \bar{c}^2 + T^2 \right)^3 + \frac{4}{9} < g_3^2 F^2 > T^2 \right\} \times \left\{ -\sigma \bar{A}^3 \left( \frac{\bar{c}^2}{T^2} + T^2 \right) \right\} \right]$$

(3.15)

This is an interesting structure but it seems to be impossible to treat the functional integral analytically. For our numerical estimates we replace the $\bar{y}$-integral by the integrand at $\bar{A} = \bar{A}$. We will come back to this point in the discussion in section 4.

If the spin-spin interaction has a shorter range than the confinement effects one could work with the integrand of (3.11) as a rough estimate for momenta $p > a$. At even shorter ranges (large $p \gg 1/a$) one might suspect that (3.11) is not appropriate any longer: The correlation via the form factor $D$ in (1.3) can be neglected and we have the case of a constant uncorrelated background. If we assume this background to be pseudoabelian, it can be accounted for by an Euler-Heisenberg type formula. In the case of $d = 3$ we evaluate [21, 17] the gauge field and ghost contributions as

$$V_{EH} = -\frac{g_3 |B|}{8\pi^{3/2}} \int_0^\infty dT T^{-3/2} e^{-m^2 T} \left\{ \frac{1}{\sinh(g_3 |B|T)} - \frac{1}{g_3 |B|T} + \frac{9 - 6}{2} \frac{1}{8\pi^{3/2}} \int_0^\infty \frac{dT}{T} T^{-3/2} e^{-m^2 T} \right\}$$

(3.16)

After renormalization, the last term is just the well-known cubic term $-\frac{1}{12}(m^2)^{3/2} \sim \varphi^3$. The first integral in (3.16) is UV and IR convergent. For $m^2 < g_3 |B|$ we have an imaginary contribution due to a cut in $(m^2 - g_3 |B|)^{1/2}$ signalizing instability of the background field. This seems to be similar to the case $\hat{S}_F(p^2, m^2) > m^2$ in eq. (3.10). As is clear from the above, (3.16) is only justified below some IR cut-off $T_0 \sim a^2/\nu^2$, i. e. for momenta $p > p_0 = \nu/a$, where $\nu = \mathcal{O}(1)$. One can again replace the $T$ integration by a $p$ integration using the $T^{-3/2}$ substitution trick. (3.16) can be brought to the form (3.11) with a new polarization function $\hat{S}_B(p^2, m^2)$ instead of $\hat{S}_F$. $\hat{S}_B$ is compared to $\hat{S}_F$ in fig. [4]. For $p^2 \gg 2/a$ indeed both quantitites converge to each other. This is as expected since for these momenta only the lowest order in $\hat{S}_F \sim F^2$ contributes effectively and the nonlocality of the condensate does not play a role any more.

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6 Renormalization in the first of refs. [21] after eliminating the unstable mode is not correct.
Figure 4: The Euler-Heisenberg $\tilde{S}_B(p^2, m^2)$ compared to $\tilde{S}_F(p^2, m^2)$ for $m^2 = 0$ (all in $g_3^2$ units)

4 Evaluation and Discussion

As outlined in the previous chapter, even the 1-loop effective action generated by gauge bosons and ghosts propagating in the nonperturbative vacuum which is modelled by a correlated gauge field and a constant Higgs field background is a complicated expression which cannot be evaluated analytically without drastic approximations. From our experience with perturbative calculations of the thermal electroweak effective potential we expect that also (perturbative) 2-loop contributions are necessary in order to arrive at reliable (gauge-fixing and renormalization point insensitive) results. It also might turn out necessary to include higher cumulants in a description of the long-wave length modes by means of the stochastic vacuum model.

If we had such an improved effective potential at our disposal, we then could actually determine the “local” gauge condensate $< g^2 F^2 >$ as a function of $\varphi$ by minimizing the effective potential in $F^2$ and in $\varphi^2 \propto m^2$. Without a detailed evaluation of the pertinent world-line path integrals (using lattice methods, for instance) we only can advocate a qualitative picture: Due to spin forces the state with $< F^2 > = 0$ is unstable while the true vacuum is stabilized at some value $< F^2 > \neq 0$ due to confinement forces.

In the hot electroweak phase ($\varphi^2 \sim 0$), we learned from lattice studies that the confining string tension is approximately that of pure $SU(2)$-YM theory. In the stochastic vacuum model there is the relation (3.6) between $< g_3^2 F^2 >$ and the string tension $\sigma$. Thus we may deduce the value of the condensate from the lattice value of $\sigma$ rather than by minimizing the effective potential.

For $m^2 \neq 0$ the destabilizing effect of the 1-loop $F^2$ term diminishes (see eq. (2.24)) and at some $m_0^2$ the $< F^2 > = 0$ vacuum stabilizes. Clearly $< F^2 >$ depends on $m^2$. Again $m_0^2$ could in principle be read off from a balance between tree and loop $F^2$ terms, but as we said before, in praxi a 1-loop calculation is not very reliable. We thus will
parametrize the \(m^2\)-dependence of \(< F^2 >\) by

\[
<F^2>(m^2) = <F^2>(0) h(m^2)
\]  \hspace{1cm} (4.1)

where the function \(h\) satisfies \(h(0) = 1\), \(h(m_0^2) = 0\), \(h'(m_0^2) = 0\). We choose it in the form

\[
h(m^2) = \cos^2\left(\frac{\pi m}{2 m_0}\right)
\]  \hspace{1cm} (4.2)

for \(m \leq m_0\), and fix\(^7\) \(< g_3^2 F^2 > (0) = \frac{24}{\pi} a^{-2} \sigma_{\text{fund}}(m^2 = 0)\). The numerical values \(\sigma_{\text{fund}}(m^2 = 0) \sim 0.13 g_3^4\), \(1/a \sim 0.73 g_3^2\) are known from lattice studies \([14]\). For the new parameter \(m_0\) in \(\text{(4.2)}\) we choose \(m_0 = 4a^{-1}\) so that \(F^2(m^2)\) has a mean “decay width” \(2/a\).

In general the correlation length \(a\) also will depend on \(\varphi^2 \propto m^2\). For large \(m^2\) the perturbative part of the form factor \(D\) (with scale \(1/m\)) which is not included in our analysis (because it is represented by the usual Feynman graphs) dominates. A dependence \(a(m^2)\) should come out from a self-consistency equation for the stochastic vacuum \([33]\), but this is beyond the scope of this paper. In the practical evaluation of our formulas \(a\) is always accompanied by a factor \(< g_3^2 F^2 > (m^2)\) which we had to parametrize anyway in lack of a fully dynamical treatment. Thus we keep the correlation length constant.\(^8\)

As outlined in section 3 and appendix A the gauge boson vacuum polarization \(\Sigma\) contains a destabilizing spin force effect \(S_F\). In the case of a nearest neighbour interaction in a sufficiently extended loop we obtain a negative IR magnetic (mass)\(^2\).

\(^7\)In all the numerical evaluations we present in section 4 and appendix A we used the form factor \(D^{(1)}\) of eq. \(\text{(2.11)}\).

\(^8\)A simple substitution \(\frac{1}{a^2(m^2)} = \frac{1}{a^2(0)} + m^2\) together with \(\text{(1.1)}/\text{(1.2)}\) turned out to give a very rapid fall-off of \(m^2_{\text{conf}}(p^2, m^2)\) and \(S_F(p^2, m^2)\) in \(m^2\).
the 1-loop effective potential. We also add the ghost part which contains the \( m_{\text{conf}}^2(p^2) \). Such a positive \( (mass)^2 \) in the IR region acts as an IR cutoff. Indeed, the soft gauge quanta are already taken into account by the stochastic vacuum background. They should not circulate in the gauge boson loop of the effective action.

There seems to be a paradoxon: On one hand, in order to destabilize \( < F^2 > = 0 \), the spin forces have to dominate the \( F^2 \) radiative correction calculated in section 2. On the other hand confinement forces should stabilize \( F^2 \) at some value \( < F^2 > \neq 0 \) and these forces should dominate, i.e. the singularity in \( p^2 \) shifted towards positive \( p^2 \) by the \( \tilde{S}_F \) term in (3.10), (3.11) should not be reached because now the positive \( m_{\text{conf}}^2 \) dominates. However, this is not a contradiction if \( \tilde{S}_F(p^2) \) and \( m_{\text{conf}}^2(p^2) \) have different \( p^2 \) dependences and dominate at intermediate and small \( (p \lesssim \frac{1}{a}) \) loop momenta, respectively. For suitable values of \( < A > \), this can be indeed the case as we see from fig. 5. Here we have chosen \( < A > = 2.5, \bar{c} = 2 \) in (3.14) and \( m^2 = 0 \).

It is important to note that also \( \tilde{S}_F \) is changed (diminished!) by the effective IR cutoff through \( m_{\text{NJL}}^2 \) of refs. [25] who give a pole mass \( m_{\text{NJL}} = 0.28g_3^2 \) for “the magnetic mass” in a 1-loop Nambu-Jona-Lasinio-type equation for the gluon propagator. It is also in the range of the Landau gauge gluon propagator mass, \( m_L = 0.35g_3^2 \) in the lattice evaluation of [30] and of the magnetic mass obtained from gap equations for pure YM theory [32,33]. Extrapolating our \( m_{\text{conf}}^2(p^2) \) to the “pole-mass” (at negative \( p^2 = -m_{\text{conf}}^2(p^2) \)) would increase its value according to fig. 5.

We can bring together the negative \( -\tilde{S}_F(p^2) \) and the positive \( m_{\text{conf}}^2(p^2) \) in one expression in order to obtain a rough approximation to the gauge particle contribution to the 1-loop effective potential. We also add the ghost part which contains the \( m_{\text{conf}}^2(p^2) \)-modification only:

\[
V_1^a = \frac{9}{2} \int_0^\infty \frac{dp^2}{2\pi^2} \left\{ 2 \ln[p^2 + m^2 + m_{\text{conf}}^2(p^2, m^2)] - \tilde{S}_F(p^2, m^2) \right\}
- \ln[p^2 + m^2 + m_{\text{conf}}^2(p^2, m^2)] + \frac{\tilde{S}_F(p^2, m^2)}{p^2 + m^2 + m_{\text{conf}}^2(p^2, m^2)} - \frac{m^2}{p^2} - \ln p^2 \right\}
- \frac{6}{2} \int_0^\infty \frac{dp^2}{2\pi^2} \left\{ \ln[p^2 + m^2 + m_{\text{conf}}^2(p^2, m^2)] - \frac{m^2}{p^2} - \ln p^2 \right\}
\]

with the modified \( \tilde{S}_F \) of (3.9)

\[
\tilde{S}_F(p^2, m^2) = \frac{< g_3^2 FF > (m^2)}{9\pi^2} \int_0^\infty \frac{dq^2}{pq} \ln \left( \frac{(p + q)^2 + m^2 + m_{\text{conf}}^2((p + q)^2, m^2)}{(p - q)^2 + m^2 + m_{\text{conf}}^2((p - q)^2, m^2)} \right) \tilde{D}(q^2)
\]

As we argued before (fig. 4) also the Euler-Heisenberg region \( p \gtrsim \frac{a}{\bar{c}} \) is properly included.
A similar 1-loop potential due to scalars (Higgs plus Goldstone bosons) in the loop has the form

\[
V_1^H = \frac{1}{2} \int \frac{dpp^2}{2\pi^2} \left\{ \ln(p^2 + m_3^2 + 3\lambda\varphi^2 + m_{\text{conf(fund)}}^2(p^2, m^2)) - \ln p^2 - \frac{m_3^2 + 3\lambda\varphi^2}{p^2} \right\} + 3 \left\{ \ln(p^2 + m_3^2 + \lambda\varphi^2 + m^2 + m_{\text{conf(fund)}}^2(p^2, m^2)) - \ln p^2 - \frac{m_3^2 + \lambda\varphi^2 + m^2}{p^2} \right\}
\]

(4.5)

\(m_{\text{conf(fund)}}^2\) is related to \(m_{\text{conf}}^2\) via the relation between the string tension in the adjoint and fundamental representation (3.6) as

\[
m_{\text{conf(fund)}}^2(p^2, m^2) = \frac{3}{8} m_{\text{conf}}^2(p^2, m^2) \quad (4.6)
\]

In the evaluation of (4.3) we vary \(m_3^2\) starting from positive values (high temperature) and avoid the unstable part with negative \(m_3^2\) where a treatment in the framework of coarse grained actions seems to be indispensible.

The 1-loop potentials (4.3) and (4.5) are the main tool for our investigation of the phase structure of the hot electroweak theory. They are plotted in fig. 6 for \(\lambda = 0.11g_3^2\) and \(m_3^2 = -0.0041g_3^4\) together with the perturbative \(\varphi^3\)-term. They have to be added to the tree Higgs potential

\[
V(\varphi_{DL}) = \frac{1}{2} y\varphi_{DL}^2 + \frac{1}{4} x\varphi_{DL}^4 + V_{FF}^{\text{tree}} + (V_1^g + V_1^H)/(g_3^2)^3
\]

(4.7)

written with the dimensionless quantities (1.2) and the dimensionless field \(\varphi_{DL}^2 = \varphi^2/g_3^2\) \((m^2 = g_3^2\varphi^2/4 = g_3^4\varphi_{DL}^2/4)\). \(V_{FF}^{\text{tree}}\) of eq. (2.33) turns out to be small numerically.

As we can see in fig. 7, starting at small \(x\) the potential (4.7) signals the usual first-order phase transition, i.e. two degenerate minima at the appropriate \(y\) (temperature
Figure 7: Fading away of the first order phase transition with increasing $x = \frac{\lambda}{g_3}$, where $x_1 = 0.06$, $x_2 = 0.08$ and $x_3 = 0.11$

$T_c$) and in between a bump with a maximum. Increasing $x$ the bump between the minima and the “size of the broken phase” $\varphi_{\text{min}}$ becomes smaller. In the case $A = 2.5$, $\tilde{c} = 2$ and $m_0 = 4a^{-1}$ the bump vanishes at $x \sim 0.11$. This is in agreement with lattice calculations: we have a crossover. For this result the $\tilde{S}_F$ spin part is very important.

In the work of Buchmüller and Philipsen [25] the crossover point was determined by comparing the “magnetic mass” $m_{\text{NJL}}^2$ of the gauge bosons in the hot phase with the (perturbative) gauge boson mass in the broken phase in the unitary gauge. Since our “magnetic mass” is momentum dependent this is not a clear prescription in our case. Trying a similar reasoning we could compare $m_{\text{conf}}^2(p^2 = 0, m^2 = 0) - \Sigma(p^2 = 0, m^2 = 0)$ with the perturbative gauge boson mass in the “broken” phase.

At the endpoint of the first-order phase transition line in the $x - y$ plane ("crossover point") we expect a second-order phase transition and thus conformal symmetry. The latter leads to the requirement that the dimensionful parameters $m_{\text{conf}}^2$ and $\lambda_{\text{eff}}$ multiplying the $\phi^2$- and $\phi^4$- terms of the effective 3-dimensional scalar action should be zero at the crossover point. At the present level of accuracy we can be gentle about anomalous dimensions which may indeed be rather small [26]. Taylor expanding the potential $V$ of eq. (4.7) allows us to read off the coupling $\lambda_{\text{eff}}$ and with the parameters chosen in fig. 7 we obtain

$$x_{\text{cro}} = 0.11$$

(4.8)

It is an interesting question whether the second-order phase transition is in the universality class of scalar $\phi^4$ theory and the Ising model [26, 27]. Of course our approach is not quite adequate to give a satisfactory answer to this.

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9For the parameter set used in fig. $m_{\text{conf}}^2 - \tilde{S}_F$ is of the order of $m_{\text{NJL}}^2$ obtained in ref. [23]. The Higgs part of the potential just contains $\frac{1}{4} m_{\text{conf}}^2$ which is also of that order whereas the ghost part has the full $m_{\text{conf}}^2$.

10This was reported recently as a lattice result [24].
The table below contains the contribution of $V^{g}_1$ and $V^{H}_1$ to the 1-loop $\phi^4$ term and the effective coupling at the $x$ values of fig. 7. We also give the surface tension in the thin wall approximation $\sigma_s = \int_{\phi_s}^{\phi_A} \sqrt{2V(\phi)}d\phi$ for the full and the perturbative potential. We get the qualitative features of the comparison of perturbative and lattice results for $\sigma_s$.

| $x$  | $\delta x^g$ | $\delta x^H$ | $x_{\text{eff}}$ | $\sigma_s$ | $\sigma_{\text{per}}$ |
|------|--------------|---------------|-----------------|------------|------------------|
| 0.06 | -0.063       | -0.039        | -0.041          | 0.016      | 0.013            |
| 0.08 | -0.063       | -0.043        | -0.026          | 0.004      | 0.007            |
| 0.11 | -0.063       | -0.051        | 0               | 0          | 0.004            |

Our most important calculation of the $F^2$ term in chapter 2 only contained the correlation length $\alpha$ which we took from lattice data; there is also some uncertainty in this term because of its gauge dependence which should be reduced by a 2-loop calculation. The construction of an effective potential in 1-loop order contained many more parameters: $< F^2 > (0)$ and the function $h(m^2)$ containing $m^2_0$ in (4.1) could in principle be obtained by studying the nontrivial minimum of the effective potential in $F^2$; $\tilde{c}$ and $n = 3$ in (3.13) are related to the modification of the area law at small areas. $< \bar{A} >$ could be determined in a numerical study of the path integral (3.15).

Given our ignorance about the last three parameters $\tilde{c}$, $n$ and $< \bar{A} >$ we just fitted a potential in order to reproduce the crossover at the $x$-value given in lattice calculations. We then did not consistently determine $< F^2 > (0)$ and $h(m^2)$ from the potential but fixed the former by lattice data on the string tension and used a plausible value for $m^2_0$. This point requires more exhaustive numerical studies, not to speak about a 2-loop calculation again.

We have described the IR effects of the gauge interactions by use of the stochastic vacuum correlation (1.3). This is quite different from a renormalization group treatment like in ref. [3] where differential renormalization group equations (at least in principle) lead to an effective IR action starting from the usual tree action in the UV. The stochastic vacuum prescription is a pure IR concept and - in a sense - complementary to the renormalization group approach. The latter is applied most easily if the relevant momentum scale is not too far below the UV cutoff where the initial condition (classical action) is specified. In this situation simple truncations of the space of actions are quite successful since the effective (average) action does not deviate very strongly from the classical one. Once the variable IR cutoff enters the deep IR-regime a more exhaustive parametrization of the space of actions becomes indispensable. For pure Yang-Mills theory, a first step in this direction has been done in ref. [24], but a complete ab initio treatment of the IR physics is necessarily quite involved and not available yet.

The stochastic vacuum model, on the other hand, makes a phenomenological ansatz for the low momentum modes of the gauge field. In principle this effective IR dynamics should follow from the renormalization group analysis but clearly the actual proof is a formidable task. Accepting the stochastic vacuum as a starting point one is in the opposite situation as in the renormalization group framework where one is given an
effective average action at a certain scale \( k \). In the former case, only the modes with very small momenta are described by the correlators used and all other modes with \( \text{larger} \) momenta, have to be dealt with ("integrated out") explicitly, e. g. by perturbation theory in the background of the low-momentum modes. This is what we did in sec. 2, for instance, when we stochastically averaged the one-loop effective action. Conversely, in the renormalization group case the modes with momenta \( \text{smaller} \) than \( k \) are the ones which still have to be integrated out.

It would also be very interesting to see if the stochastic vacuum model can be derived from a self-consistent set of Dyson-Schwinger equations as proposed in ref. [33].

Our approach is also quite different from the Nambu-Jona-Lasinio-type equations for the "magnetic mass" which are proposed in [25]. In the latter approach, this magnetic mass seems to have a similar meaning as the confinement mass \( m_{\text{conf}}^2 \) introduced in our discussion. We argued that this is an effective (mass)\(^2\), which is only important for very small momenta and we have an additional negative (mass)\(^2\), \(-S_F\), related to the gauge boson spin-spin interaction which is in a delicate balance with the confinement mass. This avoids the appearance of a tachyonic pole in the gluon propagator related to the well known IR instability of YM theory. A higher (two) loop calculation in the nonperturbative gauge field background would be desirable (though very demanding) both in order to reduce the gauge dependence and in order to treat the IR instability properly. Indeed this also seems to be an important point in the discussion of gauge invariant gap equations [32, 25] in a recent paper of Cornwall [35]. It would be interesting to include a gauge background in the gap equations of ref. [25].

In our approach the nonperturbative dynamics mainly enters through the correlated gauge field background like in pure gauge theory but with a further Higgs field parameter \( m^2 \propto \phi^2 \). We have been using lattice results for the values of \( a \) and \( \sigma_{\text{fund}}(m = 0) \). It might be possible to explore the \( m^2 \) dependence of these quantities on the lattice.

In finite temperature QCD (and in ordinary QCD) we have no Higgs expectation value \( \phi^2 \) at our disposal which could act as an IR regulator and the 4-dimensional gauge coupling is big. Thus the status of naive perturbation theory is even much worse [28]. Still an IR-improved perturbative picture using a correlated gauge field background might be possible [29] and the methods introduced in the present paper may be still relevant. This deserves further investigation.

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Appendix A

In this appendix we calculate the stochastic average of the gauge boson propagator \( G = K^{-1} \) with the kinetic operator \( K_{ab}^{\mu\nu} \) given in eq. (2.16). We shall determine \( G \) to first order in the condensate \( \langle g^2 F^2 \rangle \).

To start with, we decompose \( K_{\mu\nu} \) according to

\[
K_{\mu\nu} = K_{\mu\nu}^{(0)} + K_{\mu\nu}^{(1)} + K_{\mu\nu}^{(2)}
\]

(A.1)

where the superscripts denote the powers of \( F_{\mu\nu} \) which are contained in the respective terms once the gauge field

\[
A_\mu(x) = \int_0^1 d\eta \eta x_\nu F_{\mu\nu}(\eta x)
\]

(A.2)
is inserted. Thus we employ the Fock-Schwinger gauge centered at \( x_\mu = 0 \)

\[
K_{\mu\nu}^{(0)} = (\hat{p}^2 + m^2)\delta_{\mu\nu}
\]

\[
K_{\mu\nu}^{(1)} = -g(\hat{p}_\alpha A_\alpha + A_\alpha \hat{p}_\alpha)\delta_{\mu\nu} + 2ig F_{\mu\nu}
\]

\[
K_{\mu\nu}^{(2)} = g^2 A_\alpha A_\alpha \delta_{\mu\nu}
\]

(A.3)

with \( \hat{p}_\mu \equiv -i\partial_\mu \) and with \( A_\mu \) and \( F_{\mu\nu} \) considered as matrices in the adjoint representation of the gauge group.

To order \( F^2 \), the propagator then reads

\[
G = G^{(0)} - G^{(0)} K^{(1)} G^{(0)} - G^{(0)} K^{(2)} G^{(0)} + G^{(0)} K^{(1)} G^{(0)} K^{(1)} G^{(0)} + 0(F^3)
\]

(A.4)

where \( G^{(0)} = [K^{(0)}]^{-1} \) is the free propagator.

Using \( (1.3) \) together with \( \ll 1 \gg = 1 \) and \( \ll F_{\mu\nu} \gg = 0 \) the stochastic average \( \bar{G} \equiv \ll G \gg \) is seen to be

\[
\bar{G} = G^{(0)} + G^{(0)} \Sigma G^{(0)} + 0(F^3)
\]

(A.5)
i.e.,

\[
\bar{G}^{-1} = G^{(0)-1} - \Sigma + 0(F^3),
\]

(A.6)

with the “mass operator”

\[
\Sigma \equiv \ll K^{(1)} G^{(0)} K^{(1)} \gg - \ll K^{(2)} \gg
\]

(A.7)
The quadratic averages in (A.7) have to be performed using the correlator (1.3). For simplicity we restrict ourselves to the case \( \kappa = 1 \). Furthermore, we choose the reference point \( x_0 = 0 \), which coincides with the point at which the Fock-Schwinger gauge is centered. As a consequence, the path-ordered exponential in eq. (1.4) equals unity and it follows that

\[
\ll g^2 F^a_{\mu\nu}(x) F^b_{\rho\sigma}(y) \gg = \omega \delta^{ab} \times [\delta_{\mu\rho} \delta_{\nu\sigma} - \delta_{\mu\sigma} \delta_{\nu\rho}] D((x-y)^2/a^2)
\]

(A.8)
with
\[ \omega \equiv \frac{<g^2 F^2>}{d(d-1)(N_c^2-1)} \]  
(A.9)

It is convenient to write
\[ \Sigma = \Sigma_1 + \Sigma_2 \]  
(A.10)

\[ \Sigma_1 \equiv \ll Q \gg, \quad Q \equiv K^{(1)}G^{(0)}K^{(1)} \]  
(A.11)

\[ \Sigma_2 \equiv - \ll K^{(2)} \gg \]  
(A.12)

and to further decompose
\[ Q = Q_{AA} + Q_{FF} + Q_{AF} \]  
(A.13)

with
\[ [Q_{AA}]_{\mu\nu} = g^2(\hat{p}_\alpha A_\alpha + A_\alpha \hat{p}_\alpha)G^{(0)}(\hat{p})(\hat{p}_\beta A_\beta + A_\beta \hat{p}_\beta) \]  
(A.14)

\[ [Q_{FF}]_{\mu\nu} = (2i g)^2 F_{\mu\alpha} G^{(0)}(\hat{p}) F_{\beta\nu} \]  
(A.15)

\[ [Q_{AF}]_{\mu\nu} = -2i g^2 \left\{ (\hat{p}_\alpha A_\alpha + A_\alpha \hat{p}_\alpha)G^{(0)}(\hat{p}) F_{\beta\nu} + F_{\mu\alpha} G^{(0)}(\hat{p}) (\hat{p}_\beta A_\beta + A_\beta \hat{p}_\beta) \right\} \]  
(A.16)

Working in a position-space representation, quantities such as \( \Sigma_1 \) or \( Q \) are kernels with respect to the space-time coordinates and matrices with respect to Lorentz and adjoint group indices. Before we can evaluate the average
\[ \Sigma_1(x_2, x_1)^{ab}_{\mu\nu} = \ll Q(x_2, x_1)^{ab}_{\mu\nu} \gg \]  
(A.17)

we have to determine the position-space matrix elements
\[ Q(x_2, x_1)^{ab}_{\mu\nu} = <x_2|Q^{ab}_{\mu\nu}(\hat{x}, \hat{p})|x_1> \]  
(A.18)

for a fixed background field. For the contribution coming from \( Q_{AA} \), say, one finds
\[ Q_{AA}(x_2, x_1)^{ab}_{\mu\nu} = -g^2(T^c T^d)^{ab}_{\mu\nu} \times \left[ 4A^c_\rho(x_2)A^d_\sigma(x_1)\partial_\rho \partial_\sigma G^{(0)}(x_2 - x_1) - 2A^c_\rho(x_2)\partial A^d_\rho(x_1)\partial_\sigma G^{(0)}(x_2 - x_1) + 2\partial A^c(x_2)A^d_\sigma(x_1)\partial_\sigma G^{(0)}(x_2 - x_1) - \partial A^c(x_2)\partial A^d_\rho(x_1)G^{(0)}(x_2 - x_1) \right] \]  
(A.19)

with \( \partial A^a \equiv \partial_\mu A^a_\mu \) and \( <x_2|G^{(0)}_{\mu\nu}|x_1> \equiv \delta_{\mu\nu}G^{(0)}(x_2 - x_1) \). The next step is to express all \( A_\mu \)’s in terms of \( F_{\mu\nu} \)’s by using (A.2), and to apply (A.8) to the resulting \( \ll FF \gg \) correlators. In particular one needs
\[ \ll g^2 A^c_\mu(x_2)A^d_\nu(x_1) \gg = \omega \delta^{cd}[(x_1 \cdot x_2)\delta_{\mu\nu} - x_{1\mu}x_{2\nu}] \times \int_0^1 d\eta_1 \int_0^1 d\eta_2 \eta_1 \eta_2 D(\delta^2/a^2) \]  
\[ \ll g^2 A^c_\mu(x_2)\partial A^d_\mu(x_1) \gg = -2\omega \delta^{cd}a^{-2}[(x_1 \cdot x_2)x_{2\mu} - (x_2)^2 x_{1\mu}] \]

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\[ \times \int_0^1 d\eta_1 \int_0^1 d\eta_2 (\eta_1 \eta_2)^2 D'(\delta^2/a^2) \]
\[ \ll g^2 \partial A^d(x_2) \partial A^d(x_1) \gg = -2\omega \delta^{\alpha\beta} a^{-2} \]
\[ \times \left\{ (d-1)(x_1 \cdot x_2) \int_0^1 d\eta_1 \int_0^1 d\eta_2 (\eta_1 \eta_2)^2 D'(\delta^2/a^2) \right. \]
\[ + 2a^{-2} v_{12} \int_0^1 d\eta_1 \int_0^1 d\eta_2 (\eta_1 \eta_2)^3 D''(\delta^2/a^2) \right\} \]  
(A.20)

with the abbreviations
\[ \delta_{\mu} \equiv \eta_2(x_2)_\mu - \eta_1(x_1)_\mu \]  
(A.21)

and
\[ v_{12} \equiv (x_1)^2(x_2)^2 - (x_1 \cdot x_2)^2 \]  
(A.22)

The derivatives acting on \(G^{(0)}\) in eq. (A.19) can be easily removed by noting that the free propagator has the structure
\[ G^{(0)}(x_2 - x_1) = H \left( \frac{1}{2}(x_2 - x_1)^2 \right) \]  
(A.23)

for some function \(H\). In the massless case, for instance,
\[ H(\nu) = \frac{1}{2}(2\pi)^{-d/2} \Gamma(d/2 - 1) \nu^{-d/2+1}, \quad d > 2 \]  
(A.24)

After some calculation this leads to
\[ \ll Q_{AA}(x_2, x_1)_{\mu\nu}^{ab} \gg = -2\omega N_c \delta^{ab} \delta_{\mu\nu} \]
\[ \times \left\{ 2[(d-1)(x_1 \cdot x_2)H' + v_{12} H''] \int_0^1 d\eta_1 d\eta_2 (\eta_1 \eta_2)^2 D(\delta^2/a^2) \right. \]
\[ + 4a^{-2} v_{12} H' \int_0^1 d\eta_1 d\eta_2 (\eta_1 \eta_2)^2 D'(\delta^2/a^2) \]
\[ + (d-1)a^{-2}(x_1 \cdot x_2)H \int_0^1 d\eta_1 d\eta_2 (\eta_1 \eta_2)^2 D'(\delta^2/a^2) \]
\[ + 2a^{-4} v_{12} H \int_0^1 d\eta_1 d\eta_2 (\eta_1 \eta_2)^3 D''(\delta^2/a^2) \right\} \]  
(A.25)

Here a prime denotes the derivative with respect to the argument, and the function \(H\) and its derivatives are always evaluated at the point \((x_2 - x_1)^2/2\). Likewise one finds for the other contributions to \(\Sigma_1\):
\[ \ll Q_{FF}(x_2, x_1)_{\mu\nu}^{ab} \gg = 4(d-1)\omega N_c \delta^{ab} \delta_{\mu\nu} H((x_2 - x_1)^2/2) D((x_2 - x_1)^2/a^2) \]  
(A.26)
\[ \ll Q_{AF}(x_2, x_1)_{\mu\nu}^{ab} \gg = -2g^2 \omega N_c \delta^{ab}(x_1 x_2 - x_1 x_2) \]
\[ \cdot \left\{ H'((x_2 - x_1)^2/2) \int_0^1 d\eta_1 D((x_2 - x_1)^2/a^2) \right. \]
\[ + 2a^{-2} H((x_2 - x_1)^2/2) \int_0^1 d\eta_1^2 D'(x_2 - x_1)^2/a^2) \]
\[ + (x_1 \leftrightarrow x_2) \right\} \]  
(A.27)
The contribution $\Sigma_2$ is a pure contact term which vanishes for $x_1 \neq x_2$:

$$
\Sigma_2(x_2, x_1)_{\mu\nu}^{ab} = -(d-1)\omega N_c \delta^{ab} \delta_{\mu\nu} x_1^2 \delta(x_2 - x_1) 
\cdot \int_0^1 d\eta_1 d\eta_2 \eta_1 \eta_2 D((\eta_1 - \eta_2)^2 x_1^2 / a^2) \quad (A.28)
$$

The complete “mass operator” $\Sigma$ is given by the sum of the terms in eqs. (A.25), (A.26), (A.27) and (A.28). We observe that for the gauge chosen, $\Sigma$ is not translational invariant; it depends on $x_1$ and $x_2$ separately and not only on the difference $x_1 - x_2$.

Generally speaking $\Sigma$ is a function of the two translational invariant combinations of $x_1, x_2$ and the point $x_0$ at which the Fock-Schwinger gauge is based (cf. eq. (3.3)) and which was chosen to act also as the reference point in the $\ll FF \gg$-correlator, see eq. (1.3). Without loss of generality, one of these three points can be fixed at will. We have exploited this freedom in order to set $x_0 = 0$. The general case is recovered by substituting $x_{1,2} \rightarrow x_{1,2} - x_0$. We see that $\Sigma$ is proportional to the unit matrix in color space. It is not, however, a unit matrix with respect to the Lorentz indices since $Q_{AF}$ has a nontrivial tensor structure involving $x_1$ and $x_2$.

Next we consider $\Sigma$ with a different choice of the point $x_0$ which, again, serves both as the reference point in the correlator and as the base point of the Fock-Schwinger gauge. We define $x_0$ to be in the middle of the straight line connecting $x_1$ to $x_2$:

$$
x_0 = \frac{1}{2}(x_1 + x_2) \quad (A.29)
$$

For this choice of $x_0$, $\Sigma$ depends on $x_1$ and $x_2$ only via $(x_1 - x_2)^2$. One finds

$$
\Sigma(x_2, x_1)_{\mu\nu}^{ab} = \delta_{\mu\nu} \delta^{ab} S((x_1 - x_2)^2) \quad (A.30)
$$

The function $S$ can be determined from our first calculation by setting

$$
\begin{align*}
   x_1 &= \frac{1}{2} z, \\
   x_2 &= -\frac{1}{2} z
\end{align*}
$$

for some vector $z$. Then $x_0 = \frac{1}{2}(x_1 + x_2) = 0$, and the reference points agree. In this manner one is led to the comparatively simple result $(z \equiv x_1 - x_2)$

$$
S(z^2) = \frac{N_c}{d(N_c^2 - 1)} < g^2 F^2 > 
\cdot \left\{ z^2 H \left( \frac{z^2}{z} \right) \int_0^1 d\eta_1 d\eta_2 \eta_1 \eta_2 D \left( (\eta_1 + \eta_2)^2 \frac{z^2}{4a^2} \right) 
+ \frac{z^2}{2a^2} H \left( \frac{z^2}{2} \right) \int_0^1 d\eta_1 d\eta_2 (\eta_1 \eta_2)^2 D' \left( (\eta_1 + \eta_2)^2 \frac{z^2}{4a^2} \right) 
+ 4H \left( \frac{z^2}{2} \right) D \left( \frac{z^2}{a^2} \right) \right\} \quad (A.31)
$$

The $Q_{AF}$ contribution vanishes.

Since the operator $\Sigma$ calculated according to the second prescription is translational invariant, it is meaningful to Fourier-transform it with respect to $z$, and to interpret
Figure 8: Defining $\tilde{S}(p^2, m^2) = \langle g_3^2 F^2 > a^2 \cdot s(p^2, m^2)$ the full line is $s(p^2 = 0, m^2)$, the dashed and the dot-dashed lines are the contributions of the spin and the minimal interactions to $s(p^2 = 0, m^2)$, respectively. We used the form factor $D(1)$.

$\tilde{\Sigma}(p^2 = 0)$ as a kind of mass shift originating from the interaction of the gauge bosons with the stochastic background field. If we define

$$\tilde{S}(p^2) \equiv \int d^dz e^{-ipz} S(z^2)$$

(A.32)

and use the Fourier transforms

$$H \left( \frac{z^2}{2} \right) = \int \frac{d^d q}{(2\pi)^d} e^{iqz}(q^2 + m^2)^{-1}$$

$$D \left( \frac{z^2}{a^2} \right) = \int \frac{d^d q}{(2\pi)^d} e^{iqz} \tilde{D}(q^2)$$

(A.33)

it is straightforward to arrive at

$$\tilde{S}(p^2) = \frac{N_c}{d(N_c^2 - 1)} \langle g^2 F^2 > \int \frac{d^d q}{(2\pi)^d} \left\{ \frac{4 \tilde{D}(q^2)}{(p-q)^2 + m^2} \right. \right.$$

$$-2^d \int_0^1 d\eta_1 d\eta_2 \frac{\eta_1 \eta_2}{(\eta_1 + \eta_2)^d} \left[ \frac{d-2}{(p-q)^2 + m^2} + \frac{2m^2}{[(p-q)^2 + m^2]^2} \right] \tilde{D} \left( \frac{4q^2}{(\eta_1 + \eta_2)^2} \right)$$

$$-2^d \int_0^1 d\eta_1 d\eta_2 \frac{(\eta_1 \eta_2)^2}{(\eta_1 + \eta_2)^{2+d}} \left[ d\bar{D} \left( \frac{4q^2}{(\eta_1 + \eta_2)^2} \right) \right. \right.$$

$$\left. + \frac{8q^2}{(\eta_1 + \eta_2)^2} \bar{D}' \left( \frac{4q^2}{(\eta_1 + \eta_2)^2} \right) \right] \frac{1}{(p-q)^2 + m^2}$$

(A.34)

For the remaining analysis we restrict ourselves to the physically most interesting case in this paper, $d = 3$ and $N_c = 2$. The angular integration in (A.34) can be evaluated
analytically. One obtains
\[
\hat{S}(p^2) = \frac{2}{g^2} < g_2 F^2 > \int_0^\infty dq^2 \\
\left\{ \frac{1}{2 \bar{p} q} \bar{D}(q^2) \ln \frac{(p + q)^2 + m^2}{(p - q)^2 + m^2} \\
- \int_0^1 d\eta_1d\eta_2 \frac{\eta_1 \eta_2}{(\eta_1 + \eta_2)^3} \left[ \frac{1}{\bar{p} q} \ln \frac{(p + q)^2 + m^2}{(p - q)^2 + m^2} + \frac{8m^2}{(p^2 + q^2 + m^2)^2 - 4p^2 q^2} \right] \\
\right. \\
\left. \bar{D} \left( \frac{4q^2}{(\eta_1 + \eta_2)^2} \right) \\
- \int_0^1 d\eta_1d\eta_2 \frac{(\eta_1 \eta_2)^2}{(\eta_1 + \eta_2)^3} \frac{1}{\bar{p} q} \left[ 3\bar{D} \left( \frac{4q^2}{(\eta_1 + \eta_2)^2} \right) + \frac{8q^2}{(\eta_1 + \eta_2)^2} \bar{D}' \left( \frac{4q^2}{(\eta_1 + \eta_2)^2} \right) \right] \\
\right. \\
\left. \ln \frac{(p + q)^2 + m^2}{(p - q)^2 + m^2} \right\} \\
\tag{A.35}
\]

The integral representation (A.35) is the main result of this appendix. The remaining \(q\)-integration has to be performed numerically. Results are given in fig. 8. As we can see the first term due to the spin interaction dominates. The other terms related to minimal interaction have opposite sign and go into the same direction as the area law-induced mass square discussed in section 3.

Indeed, instead of the choice (A.29) for the base point \(x_0\) one could choose \(x_0\) to be the center of mass introduced in eqs. (3.2). The \(\eta\)-integrals together with the \(\tau\)-integrals in the first part of this appendix then belong to area integrals filling the loop in the sense of the nonabelian Stokes theorem. This is similar in spirit to ref. [33]. One could imagine a split of these area integrals into segments related to the \(\eta\)-contributions to (A.35) and a main area integral approximated as in section 3 by a (modified) area law.

We mentioned already that \(\Delta m^2 = -\hat{S}(p^2 = 0)\) should be interpreted as the mass shift due to the interaction with a stochastic background. This quantity turns out to be IR-finite even in the case when the gauge bosons are massless a priori, i.e., setting \(m = 0\) on the RHS of eq. (A.35) yields a finite function \(\hat{S}(p^2)\) and in particular a finite value for \(\hat{S}(0)\). It is even possible to calculate this value analytically. Returning to the representation (A.31), one obtains for arbitrary \(m^2, d\) and \(N_c\):

\[
-\Delta m^2 = \int d^d z S(z^2) = \frac{\pi^{d/2} N_c}{d \Gamma(d/2)(N_c^2 - 1)} < g^2 F^2 > \\
\cdot \left\{ \int_0^1 d\eta_1d\eta_2d\eta_1 \eta_2 \int_0^\infty dw \; w^{d/2-1} w H' \left( \frac{w}{2} \right) D \left( \eta_1 + \eta_2 \right) w 4a^2 \right\} \\
+ \int_0^1 d\eta_1d\eta_2 \left( \eta_1 \eta_2 \right) \int_0^\infty dw \; w^{d/2-1} (w/2a^2) H \left( \frac{w}{2} \right) D' \left( \eta_1 + \eta_2 \right) w 4a^2 \right\} \\
+ 4 \int_0^\infty dw \; w^{d/2-1} H \left( \frac{w}{2} \right) D \left( \frac{w}{a^2} \right) \right\} \\
\tag{A.36}
\]

\(^{11}\)For the form factor \(D^{(2)}, s(0, p^2)\) differs by roughly a factor of 2 but the relation between \(< g_2^2 F^2 >\) and \(\sigma_{\text{fund}}\) also changes and the result for \(\hat{S}\) is close to that for \(D^{(3)}\).

\(^{12}\)The minus sign is due to the fact that the inverse dressed propagator is \(G^{(0)-1} - \Sigma\).
Assuming now the propagator function $H$ to be of the massless form (A.24), the integrals in (A.36) can be simplified considerably. In this case one may use the identities

\[ wH'(\frac{w}{2}) = -(d-2)H(\frac{w}{2}) \]
\[ w^{d/2-1}H(\frac{w}{2}) = \frac{1}{4\pi^{d/2}}\Gamma(d/2 - 1) \]

in order to bring (A.36) to the following explicit form:

\[ -\Delta m^2 = 2N_c \frac{d}{d(d-2)(N_c^2 - 1)}a^2 \left< g^2 F^2 \right> \cdot \left\{ 1 - (d-2)\theta_1 - 2\theta_2 \right\} \int_0^\infty dw D(w) \]

with the constants

\[ \theta_1 \equiv \int_0^1 d\eta_1 \int_0^1 d\eta_2 \frac{\eta_1 \eta_2}{(\eta_1 + \eta_2)^2} = \frac{2}{3}(1 - \ln 2) = 0.2046... \]
\[ \theta_2 \equiv \int_0^1 d\eta_1 \int_0^1 d\eta_2 \frac{(\eta_1 \eta_2)^2}{(\eta_1 + \eta_2)^4} = \frac{1}{24} = 0.0417... \]  

(A.39)

The remaining integral in (A.38) slightly depends on the precise shape of the model function $D$ we have chosen. For the exponential $D(w) = e^{-w}$, corresponding to eq. (2.12) the integral $\int_0^\infty dw D(w)$ equals unity, for instance. In any case it is a positive number of order unity.

Since in the physically interesting situation $d = 3$ the expression inside the curly brackets of (A.38) is positive, we are led to the important conclusion that $\Delta m^2$ is negative (tachyonic). This shows that the interaction with the stochastic background destabilizes the perturbative gauge field vacuum and drives the system towards the formation of a condensate.

Appendix B

In this appendix we explain the basic mechanism of why stochastic background fields lead to the formation of a condensate within a simple scalar toy model.

Let $J(x,\mu)$ be a real scalar Gaussian random variable, i.e. the average of an arbitrary functional $F[J]$ is given by

\[ \left< F[J] \right> = \int D J \exp \left( -\frac{1}{2} \int J \Omega J \right) F[J] \]

where

\[ \int J \Omega J \equiv \int d^d x d^d y J(x) < x | \Omega | y > J(y) \]

with some positive definite operator $\Omega$. Eq. (B.1) implies that for any function $\phi(x)$

\[ \left< \exp \left( \pm \int d^d x J(x) \phi(x) \right) \right> = \exp \left( \pm \frac{1}{2} \int \phi D \phi \right) \]

(B.3)
where the operator $D \equiv \Omega^{-1}$ is positive definite, too. We assume that $D$ (and $\Omega$) is translational invariant with real matrix elements $< x | D | y > = D(x - y)$. Then the only nonzero connected correlation function reads

$$< < J(x)J(y) >> = D(x - y) \quad (B.4)$$

This equation should be compared for the correlator (1.3) of the stochastic vacuum model in QCD.

Let us consider the partition function $Z[J]$ of a scalar field theory governed by the action $S[\phi]$ with an additional linear coupling of the field $\phi(x)$ to a stochastic background $J(x)$:

$$Z[J] = \int D\phi \exp \left\{ -S[\phi] - \int d^d x J(x)\phi(x) \right\} \quad (B.5)$$

Its average is given by

$$< < Z[J] >> = \int D\phi e^{-S[\phi]} < < e^{-\int J\phi} >> = \int D\phi \exp \left\{ -S[\phi] + \frac{1}{2} \int \phi D\phi \right\} \quad (B.6)$$

We see that a linear coupling of $\phi$ to a Gaussian random field has the effect of changing the classical action according to

$$S[\phi] \to S[\phi] - \frac{1}{2} \int d^d x d^d y \phi(x) D(x - y) \phi(y) \quad (B.7)$$

It is important to observe that the new nonlocal term in (B.7) is negative for any $\phi(x)$. This means in particular that it supplies a negative contribution to the mass square of $\phi$. If $S$ is of the $Z_2$-symmetric form $S[\phi] = \int d^d x \left\{ \frac{1}{2} (\partial_\mu \phi)^2 + \frac{1}{4} m^2 \phi^2 + \frac{1}{4} \lambda \phi^4 \right\}$, say, then the stochastic background induces a mass shift

$$m^2 \to m^2 + \Delta m^2 \quad (B.8)$$

with

$$\Delta m^2 = - \int d^d x D(x) = - \tilde{D}(p = 0) < 0 \quad (B.9)$$

where $\tilde{D}$ is the Fourier transform of $D$. If $m^2 > 0$ in absence of the $J\phi$ coupling, the theory has a $Z_2$-symmetric vacuum with the minimum of the potential at $\phi = 0$. If, however, the coupling to the stochastic background is switched on and if $|\Delta m^2| > m^2$, then $m^2 + \Delta m^2$ becomes negative. Hence the modified classical potential develops two minima at $\phi \neq 0$ and the $Z_2$-symmetry is spontaneously broken.

In Yang-Mills theory a similar but technically less transparent mechanism is at work. The Lagrangian for the gauge field fluctuations $a_\mu(x)$ reads $\frac{1}{4} a_\mu K_{\mu\nu}[A]a_\nu + O(a_\mu^3)$, and clearly $a_\mu$ and $A_\mu$ are analogous to $\phi$ and $J$, respectively. The coupling of $a_\mu$ to the background $A_\mu$ is more complicated than the linear $J\phi$-term, however. In particular, the analogue of the correlator (B.4) is formulated in terms of $F_{\mu\nu}[A]$ rather than $A_\mu$ itself. This problem can be overcome by employing the Fock-Schwinger gauge which
provides a simple formula for $A_\mu$ in terms of $F_{\mu\nu}$, see eq. (2.3). Both the computation of the effective action in the main body of the paper and the calculation of the mass operator in Appendix A show that at least for $d = 3$ the $\Delta m^2$ term for $a_\mu$ which is induced by the stochastic background is negative, see eq. (A.38), for instance. For large values of $d$ we find $\Delta m^2 > 0$, however.

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