Diffractive jet production in a simple model
with applications to HERA

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Abstract

In diffractive jet production, two high energy hadrons $A$ and $B$ collide and produce high transverse momentum jets, while hadron $A$ is diffractively scattered. Ingelman and Schlein predicted this phenomenon. In their model, part of the longitudinal momentum transferred from hadron $A$ is delivered to the jet system, part is lost. Lossless diffractive jet production, in which all of this longitudinal momentum is delivered to the jet system, has been discussed by Collins, Frankfurt, and Strikman. We study the structure of lossless diffractive jet production in a simple model. The model suggests that the phenomenon can be probed experimentally at HERA, with $A$ being a proton and $B$ being a bremsstrahlung photon with virtuality $Q^2$. Lossless events should be present for small $Q^2$, but not for $Q^2$ larger than $1/R_P^2$, where $R_P$ is a characteristic size of the pomeron.
I. INTRODUCTION

In high energy $p\bar{p}$ collisions at hadron colliders, one sometimes produces high transverse momentum jets in the final state. Here high transverse momentum means, say, transverse momentum greater than 20 GeV. These jets are interpreted as the decay products of quarks and gluons (partons) produced by the hard scattering of two partons from the incoming hadrons. Cross sections for $A + B \rightarrow \text{jets} + X$ (1)
calculated in QCD according to this picture match the observations well [1,2]. Normally the two hadrons $A$ and $B$ are broken up in this process. It is clear that this should be so, since a 20 GeV momentum transfer is quite sufficient to disrupt a hadron. Nevertheless, in 1985 Ingelman and Schlein [3] predicted that events of the type $A + B \rightarrow A + \text{jets} + X$, (2)
where hadron $A$ is diffractively scattered, should occur with a small but not tiny probability.

Here by “diffractively scattered,” we mean that $A$ emerges with a fraction $(1 - z) > 0.9$ of its original longitudinal momentum and with a small transverse momentum $|P'_{A}| < \sim 1$ GeV. The transverse momentum transfer can also be characterized using the invariant momentum transfer $t$ from the hadron: $t = (P_{A} - P'_{A})^{2} = -(P'_{A}^{2} + z^{2}M_{A}^{2})/(1 - z) \approx -P_{A}^{2}$.

The picture for such diffractive hard scattering proposed by Ingelman and Schlein is that hadron $A$ exchanges a pomeron with the rest of the system, where “pomeron” means whatever is exchanged in elastic scattering at large $s$, small $t$. Thus the cross section is proportional to the pomeron coupling to hadron $A$ as measured in elastic scattering. The pomeron carries transverse momentum $-P'_{A}$ and a fraction $z$ of the hadron’s longitudinal momentum. Here we do not need to know what a pomeron is, only that its momentum is carried by quarks and gluons. One of these collides with a parton from hadron $B$ to produce the jets. Let the parton that participates in the hard scattering carry a fraction $x$ of the longitudinal momentum of the incoming hadron $A$, and thus a fraction $x/z$ of the longitudinal momentum transferred by the pomeron. Then the cross section in this model is proportional to a function $f_{a/P}(x/z; t; \mu)$, where $f_{a/P}(\xi; t; \mu) d\xi$ is interpreted as the probability to find a parton of kind $a$ in a pomeron, where the parton carries a fraction $\xi$ of the pomeron’s longitudinal momentum. In principle, $f_{a/P}$ can depend on the invariant momentum transfer $t$, and it should also depend on a scale parameter $\mu$ that characterizes the scale of virtualities or transverse momenta in the hard parton scattering.

Several guesses for the $\xi$ dependence of $f_{a/P}(\xi; t; \mu)$ were given in the literature [3–5], but ultimately it was left to experiment to measure these functions. An analysis of the theoretical expectations and of the approximate factorization hypothesis that is inherent in the model of Ingelman and Schlein was given in [5].

The reaction (2) anticipated by Ingelman and Schlein has been seen at the CERN collider by the UA8 experiment [6]. However, the experiment suggests a feature not anticipated in [3–5]. It was expected that the functions $f_{a/P}(x/z)$ would have support only for $x < z$. That is, some of the momentum fraction $x$ transferred from hadron $A$ would be lost, appearing in low $P_{T}$ particles instead of the jets. Instead, the experiment suggests that a fraction of
the events are lossless in the sense that \( x = z \). It is as if the formula for the cross section contained a term proportional to \( \delta(1 - x/z) \).

It will be important to confirm the existence of a term that is proportional to \( \delta(1 - x/z) \) or, more generally, that is singular as \( x/z \to 1 \). The UA8 experiment demonstrates that something is present in addition to terms that vanish as \( x/z \to 1 \). However, given the experiment’s rather limited resolution, it is not clear to us that the experimental findings could not be fit with a function that is merely finite as \( x/z \to 1 \). We shall present evidence in a forthcoming paper that such terms are to be expected [7].

Recently, the Zeus and H1 experiments at HERA have reported the first evidence for the analogous reaction in deeply inelastic electron scattering [8],

\[
e + A \to e + A + X.
\]

Here the diffractively scattered proton \( A \) has not been seen, but there is evidence for the “rapidity gap” expected in such events. Information on the \( x \) and \( z \) dependence of the cross section is not yet available. We will discuss HERA physics later in this paper, concentrating on \( e + A \to e + A + X \) events in which the virtual photon has low virtuality, rather than high virtuality.

The purpose of this paper is to consider some of the theoretical issues in light of the experimental results. In particular, we reexamine the factorization hypothesis of [3], and argue that this hypothesis is at best approximate. This conclusion is in agreement with that of Collins, Frankfurt, and Strikman [9], although our analysis of the source of the breakdown of Ingelman-Schlein factorization will be presented in more depth than that of Ref. [9]. We frame much of the analysis in terms of a simple model that allows a detailed examination of the theoretical issues.

Our analysis suggests three conclusions. First, the factorization model with a “distribution of partons in a pomeron” may ultimately not be a fruitful way of understanding the process. Second, the lossless events found in hadron-hadron collisions should be absent in deeply inelastic scattering.

Third, perhaps the most interesting conclusion suggested by the model is that if one examines the process \( e + A \to e + A + X \) as a function of the virtuality \( Q^2 \) of the virtual photon emitted by the electron, then the lossless events should be present for \( Q^2 = 0 \) but disappear gradually as \( Q^2 \) becomes larger than some value \( 1/R_P^2 \). The length scale \( R_P \) may be interpreted in the model as the transverse “size” of the pomeron.

Our discussion of the experimental possibilities at HERA bears some resemblance to that found in a recent paper of Brodsky, Frankfurt, Gunion, Mueller, and Strikman [10]. These authors study \( e + A \to e + A + V + X \), where \( A \) is a proton and \( V \) is a vector meson. They look at the region \( 1 \text{ GeV}^2 \ll Q^2 \ll s \). As in the present paper, large \( Q^2 \) implies a small size for the wave function of the quarks produced by the virtual photon. As in the present paper, these quarks couple to the proton via two gluons.

\section*{II. DIFFRACTIVE JET PRODUCTION}

Let us consider, as a definite example, the inclusive cross section for the production of two jets in a high energy collisions of two hadrons, \( A \) and \( B \). Let the initial hadron \( A \) have
momentum

\[ P_A^\mu = (P_A^+, P_A^-, P_A) = (P_A^+, \frac{M^2}{2P_A^+}, 0), \]  

(4)

where we denote \( P^\pm = (P^0 \pm P^3)/\sqrt{2} \) and where we denote transverse vectors by boldface letters. Similarly, hadron \( B \) enters the scattering with momentum

\[ P_B^\mu = (P_B^+, P_B^-, P_B) = (\frac{M^2}{2P_B^+}, P_B^-, 0). \]  

(5)

We specify the two jets by variables \( E_T, X_A, \) and \( X_B \), given in terms of the four momenta \( P_1^\mu \) and \( P_2^\mu \) of jets 1 and 2 by

\[ E_T = (|P_1| + |P_2|), \]

\[ X_A = (P_1^+ + P_2^+)/P_A^+, \]

\[ X_B = (P_1^- + P_2^-)/P_B^- \].  

(6)

The production of high transverse momentum jets is a hard process, to which QCD factorization applies. Thus the total cross section for inclusive two jet production can be written as

\[ \frac{d\sigma(A + B \to \text{jets} + X)}{dE_T \, dX_A \, dX_B} \sim \sum_{a,b} \int dx_a f_{a/A}(x_a; \mu) \int dx_b f_{b/B}(x_b; \mu) \frac{d\hat{\sigma}(a + b \to \text{jets} + X)}{dE_T \, dX_A \, dX_B}. \]  

(7)

Corrections to this asymptotic equality are suppressed by a power of \( m/E_T \), where \( m \) represents a typical hadronic momentum scale, say 300 MeV. Here \( d\hat{\sigma} \) is the parton-level cross section, computed in perturbation theory with a certain prescription for removing infrared divergences. The functions \( f_{a/A}(x_a; \mu) \) are the parton distribution functions. Notice that the Born-level cross section \( d\hat{\sigma} \) is proportional to \( \delta(x_a - X_A) \delta(x_b - X_B) \) since, at the Born level, all of the momentum carried by the colliding partons is transferred to the observed jets. In higher orders of perturbation theory, one will have \( X_A \leq x_a \) and \( X_B \leq x_b \).

The topic of this paper is inclusive jet production in which we make a very restrictive demand on the final state. We demand that hadron \( A \) appear in the final state with momentum \( P_A' \) given by

\[ P_A'^\mu = ((1 - z)P_A^+, \frac{M^2 + P_A'^2}{2(1 - z)P_A^+}, P_A'). \]  

(8)

The invariant momentum transfer to hadron \( A \) is

\[ t = (P_A - P_A')^2 = -\frac{1}{1 - z} \left\{ P_A'^2 + z^2 M^2 \right\}. \]  

(9)

For diffractive scattering, we require that \( t \) be small compared to \( s = (P_A + P_B)^2 \) and that \( z \) be small, say \( z < 0.1 \). Since \( X_A \leq x_a \leq z \), one must look for diffractive hard scattering in the region \( X_A < 0.1 \). We thus consider the cross section

\[ \frac{d\sigma_{\text{diff}}(A + B \to A + \text{jets} + X)}{dE_T \, dX_A \, dX_B \, dz \, dt} \]  

(10)
in the region described.
III. MODEL OF INGELMAN AND SCHLEIN

We begin our discussion of diffractive jet production by reviewing the model of Ingelman and Schlein [3]. We follow much of the notation of Ref. [5], but break the presentation into two stages. In the first stage, we hypothesize that the cross section for diffractive jet production can be written in terms of a diffractive parton distribution:

\[
\frac{d\sigma_{\text{diff}}(A + B \rightarrow A + \text{jets} + X)}{dE_T dX_A dX_B dz dt} \sim \sum_{a,b} \int dx_a \frac{d f_{a/A}^{\text{diff}}(x_a, \mu)}{dz dt} \int dx_b f_{b/B}(x_b; \mu) \frac{d\hat{\sigma}(a + b \rightarrow \text{jets} + X)}{dE_T dX_A dX_B}.
\]

(11)

Here

\[
\frac{d f_{a/A}^{\text{diff}}(x_a, \mu)}{dz dt} dx_a
\]

(12)

represents the probability to find in hadron A a parton of type \(a\) carrying momentum fraction \(x_a\), while leaving hadron A intact except for the momentum transfer \((z, t)\). In the second stage, we hypothesize that \(d f_{a/A}^{\text{diff}}(x_a, \mu)/dz dt\) has a particular form:

\[
\frac{d f_{a/A}^{\text{diff}}(x_a, \mu)}{dz dt} = \frac{1}{16\pi} |\beta_A(t)|^2 z^{-2\alpha(t)} f_{a/P}(x_a/z, t, \mu).
\]

(13)

Here \(\beta(t)\) is the pomeron coupling to hadron A and \(\alpha(t)\) is the pomeron trajectory. The function \(f_{a/P}(x_a/z, t, \mu)\) is then the “distribution of partons in the pomeron.”

In writing Eq. (13), one thinks of the pomeron as a continuation in the angular momentum plane of a set of hadron states. Since hadrons contain partons, the pomeron should also. Thus one has in Eq. (13) the standard factors describing the coupling of the pomeron to hadron A, together with a distribution of partons in the pomeron [3,5]. Eq. (11), on the other hand, is more basic. It says only that factorization still applies when hadron A is diffractively scattered. It is Eq. (11) that will be of concern in this paper. We shall argue that, in fact, factorization does not generally apply when hadron A is diffractively scattered.

IV. FACTORIZING AND NON-FACTORIZING GRAPHS

In this section, we discuss the Ingelman-Schlein model in terms of the contributing Feynman graphs. We first give a graph-based argument that the Ingelman-Schlein picture is plausible. Fig. 1 shows a typical graph that contributes to diffractive jet production. The shaded circles represent Bethe-Salpeter wave functions for the two hadrons, A and B. In the center of the diagram, there is a tree level contribution to the hard scattering of two gluons with momenta approximately collinear to the momenta of their respective parent hadrons. The hard scattering produces the observed high \(E_T\) jets. The jets carry plus-momentum \(X_A P_A^+\). The gluon that enters the hard scattering from hadron A carries plus-momentum \(x_a P_A^+\), which equals \(X_A P_A^+\) for Born-level hard scattering. We may view this gluon as a
normal part of the gluon cloud that accompanies any hadron. Since only a small fraction $X_A < 0.1$ of hadron $A$’s plus-momentum has been lost to the hard scattering, it seems plausible that the gluon cloud and the constituent quarks can reassemble themselves into the hadron bound state. It is only necessary to dispose of one more gluon, as indicated in the figure. Then the net color transfer from hadron $A$ can be zero. If this gluon carries a small momentum fraction $x'$ into the final state, then the net momentum fraction lost from hadron $A$, $z = X_A + x'$, will also be small ($z < 0.1$, say). Similarly, the net transverse momentum $-P_A^\perp$ transferred from the hadron can be small ($|P_A^\perp| \lesssim 1$ GeV).

Let us now compare Fig. 1 with a typical graph that contributes to pomeron exchange, $i.e.$ hadron-hadron elastic scattering at large $s$ and small $t$, as depicted in Fig. 2. Of course, the pomeron is much more complicated than this, so the reader is invited to imagine some much more complicated graphs [11]. Nevertheless, it seems an attractive proposition that whatever graphs contribute to the bottom half of the pomeron in Fig. 2 can contribute equally to the bottom half of Fig. 1.

This graphical argument makes the Ingelman-Schlein model seem plausible. Since the pomeron-proton coupling is large, the argument also suggests that the cross section for diffractive jet production at a given $E_T$ is not an infinitesimal fraction of the total cross section for jet production with the same jet $E_T$. In this paper, however, we will focus on two other features of Fig. 1: factorization and longitudinal momentum loss.

First, we note that the top half of the graph is connected to the bottom half only at the hard scattering. The absolute square of the graph breaks up into a convolution of a factor that contributes to the distribution function of gluons in hadron $B$, times a contribution to the hard scattering cross section, times a contribution to the distribution function of gluons in hadron $A$, with the additional requirement that hadron $A$ is diffractively scattered. This is just the structure of the factorized cross section formula (11). Thus the usual factorization program (as described in [12]) is relatively straightforward for this graph (although there are some subtleties related to the gluon polarizations).

Second, we note that, in order to get the color right, it was necessary to emit a gluon (or more than one gluon) into the final state. Inevitably, then, some of the net plus-momentum $zP^+$ transferred by the pomeron is not contributed to the hard scattering, but is lost into the final state in the form of low transverse momentum particles. Thus the contribution to the cross section from graphs like Fig. 1 will be nonzero only for $X_A/z < 1$.

Consider, however, the graph of Fig. 3. Here the gluon that had to be emitted in order to make the color right is absorbed by a parton from hadron $B$, which is moving in the opposite direction. As we will see in the following sections, the plus-momentum transferred by the extra gluon in this process is negligible compared to $zP^+$. Thus the contributions from graphs like Fig. 3 will be effectively proportional to $\delta(1 - X_A/z)$. Furthermore, graphs
like this break the Ingelman-Schlein factorization represented by Eq. (11). As argued in [12], such factorization breaking effects cancel when one calculates an inclusive cross section to produce jets with no restriction on the final state. However, here there is an important restriction on the final state: we allow only final states consisting of an elastically scattered proton. Thus factorization breaking effects may survive. As noted in [9], a signature of these effects is the appearance of $\delta(1 - X_A/z)$ terms in the cross section.

**diffjetmod.epsf here: just uncomment the macro.**

**FIG. 3.** Diffractive jet production with color transferred to spectator parton.

In the following sections, we explain a simple model in which this effect can be examined in some detail. Within this model, we find that the factorization breaking effect has some simple features. We abstract the simple features from the model and propose that one can test for these features in electron-proton scattering at HERA.

**V. A SIMPLE MODEL**

We will examine diffractive jet production within the context of scalar-quark QCD. In this model, there is an SU(3) gauge field $A_\mu^a(x)$ as in normal QCD. There is also a color triplet quark field $q_i(x)$ with quark mass $m$, but we take $q_i(x)$ to be a scalar field instead of a Dirac field. Since the theory includes a scalar quark field, a 4-quark coupling is necessary, but we set the renormalized coupling constant $g_4$ to a negligibly small value. The lagrangian is

$$L = (D_\mu q)^{(\dagger)}(D^\mu q) - m^2 q^{(\dagger)}q - \frac{1}{4} G^{\alpha\nu}_{\mu\nu} G^{\alpha\nu}_{\mu\nu} - \frac{1}{2\xi} (\partial_\mu A^\mu_a)(\partial_\nu A^{\nu}_a) + \text{Faddeev-Popov terms} - \frac{g_4}{4} (q^{(\dagger)}q)^2.$$  (14)

This theory has the same behavior as spinor QCD for collinear and soft gluon emission from quarks. Its chief advantage is that it allows a perturbative model for a quark-antiquark bound state. We introduce a scalar, color singlet meson field $\phi(x)$ and couple it to the quarks using

$$L_\phi = G \phi(x) q_i^{(\dagger)}(x) q_i(x).$$  (15)

We work to lowest nontrivial order in the $\phi q^{(\dagger)}q$ coupling $G$, letting the $\phi q^{(\dagger)}q$ vertex play the role that is played by the (amputated) Bethe-Salpeter wave function of a meson in spinor QCD. We denote the mass of the meson by $M$ and take $M$ to be smaller than $2m$, so that the meson cannot decay into a quark and an antiquark. This model has been used for similar purposes in Refs. [13] and [14].

In the following sections, we examine diffractive jet production within the context of low order perturbative graphs in this model. The idea is to understand some of the basic physics by means of examples. We will be working with perturbation theory for the cross section at order $\alpha_s^4 G^6$ and $\alpha_s^5 G^6$. Of course, the real strong coupling $\alpha_s(\mu)$ is not small except at large $\mu$, and some parts of our graphs contain soft momentum flows, for which small $\mu$ would
be relevant. We hope (but do not prove) that the basic lessons we learn here will be valid at order $\alpha_s^N G^6$ for any $N$ in the model, and at order $\alpha_s^N$ in real QCD. However, such an analysis at arbitrary order $N$ remains a challenge for the future.

VI. WAVE FUNCTIONS

The analysis that follows will make use of the null-plane wave function of the meson state in our model. For a meson moving in the minus direction, the wave function $\psi(x, k_{ij})$ is the amplitude to find that the meson with momentum $P^\mu = (M^2/(2P^-), P^-, 0)$ consists of a quark and an antiquark of colors $i$ and anti-$j$ respectively, with the quark having minus-momentum $k^- = xP^-$ and transverse momentum $k$. The wave function is measured by operators defined on the null-plane $y^- = 0$. The precise definition, following the formalism of [15–17], is

$$\psi(x, k_{ij}) = 2x(1-x)P^- \int d^4y \ e^{ik \cdot y} \delta(y^-) \langle 0 | q_i(y) q_j^\dagger(0) | P \rangle.$$  

Here we have chosen the normalization

$$(2\pi)^{-3} \int_0^1 \frac{dx}{2x(1-x)} \int dk \sum_{ij} |\psi(x, k_{ij})|^2 = P_2,$$  

where $P_2$ is the probability, which is of order $G^2$, that the meson state consists of a $(q, q^\dagger)$ pair. In terms of the covariant $\phi q q^\dagger$ Green function amputated on the $\phi$-leg, the definition (16) can be written as

$$\psi(x, k_{ij}) = 2x(1-x)P^- \int \frac{dk^+}{2\pi} G(k^+, P)_{ij},$$  

At lowest order in $\alpha_s$ and $G$ one has,

$$G(k^\alpha, P^\beta)_{ij} = iG \delta_{ij} \frac{i}{k^2 - m^2 + i\epsilon} \frac{i}{(P-k)^2 - m^2 + i\epsilon}.$$  

By integrating according to Eq. (18), we find

$$\psi(x, k_{ij}) = \frac{Gx(1-x)}{k^2 + m^2 - x(1-x)M^2} \delta_{ij}.$$  

Notice that $|\psi|^2 \propto 1/k^4$ for large $k^2$. This good behavior in the ultraviolet, which arises from the fact that $G$ has dimensions of mass, is the essential reason for the usefulness of this model.

The wave function can be used to calculate, to order zero in $\alpha_s$, the probability for finding a quark in a meson i.e. the parton distribution function:

$$f_{q/\phi}(x) = (2\pi)^{-3} \frac{1}{2x(1-x)} \int dk \sum_{ij} |\psi(x, k_{ij})|^2$$
$$= \frac{3G^2}{16\pi^2 m^2 - x(1-x)M^2}.$$  

8
We pause here to consider a more realistic example, chosen in anticipation of the application to HERA physics in the final section of this paper. We let the meson be a photon with polarization vector \( \epsilon \) and we let the quarks be genuine spin-\( \frac{1}{2} \) quarks with negligible mass. The perturbative Green function for the photon to turn virtually into the quark-antiquark system is

\[
G(k^\alpha; P^\beta) = i e Q \frac{k \cdot \gamma}{k^2 + i \epsilon} \epsilon \cdot \gamma \frac{(k - P) \cdot \gamma}{(k - P)^2 + i \epsilon}.
\] (22)

Here \( e \) is the proton charge and \( Q \) is the quark charge in units of the proton charge. The Dirac indices are left implicit and there is an implicit unit matrix in the 3 \( \times \) 3 color space.

The wave function, defined analogously to Eq. (16), is

\[
\psi(x, k) = \frac{1}{2P^-} \int \frac{dk^+}{2\pi} \bar{U} \gamma^- G(k^+, xP^-, k; P^\beta) \gamma^- V,
\] (23)

where \( U \) and \( V \) are the Dirac spinors for the quark and antiquark, respectively.

We consider an off-shell photon with \( P^2 \equiv -Q^2 < 0 \) and zero transverse momentum, so that

\[
P^\mu = (-\frac{Q^2}{2P^-}, P^-, 0).
\] (24)

To define the possible polarization states, we use light-like axial gauge with gauge-fixing vector \( v^\mu = (1, 0, 0) \). In this gauge the photon propagator is \( i N^\mu\nu/(P^2 + i \epsilon) \) where

\[
N^\mu\nu = -g^\mu\nu + \frac{v^\mu P^\nu + P^\mu v^\nu}{P \cdot v} = \sum_{i=1}^{2} \epsilon(P, 1)^{\mu} \epsilon(P, i)^{\nu} - \epsilon(P, L)^{\mu} \epsilon(P, L)^{\nu}.
\] (25)

Here \( \epsilon(P, 1)^{\mu} \) and \( \epsilon(P, 2)^{\mu} \) are the two transverse polarization vectors, with \( v \cdot \epsilon = P \cdot \epsilon = 0 \) and \( \epsilon \cdot \epsilon = -1 \). The longitudinal polarization vector is

\[
\epsilon(P, L)^{\mu} = \frac{Q}{P \cdot v} v^{\mu}.
\] (26)

As \( Q^2 \to 0 \), only the transverse polarization contributes. The longitudinal mode becomes important for \( Q^2 \neq 0 \). It does not propagate, but is responsible for the “instantaneous scalar photon exchange” force \([15,16]\).

We now insert polarization vectors into \( G \) and perform the \( k^+ \)-integration in Eq. (23). For transverse polarization, we find

\[
\psi(x, k) = -\frac{eQ}{2P^-} \frac{U}{k^2 + x(1-x)Q^2} [x \epsilon \cdot \gamma k_T \cdot \gamma - (1-x) k_T \cdot \gamma \epsilon \cdot \gamma] \gamma^- V,
\] (27)

where \( k_T^{\mu} = (0, 0, k) \). This is quite similar to the scalar wave function, Eq. (20). The chief difference is the factor of \( k \) in the numerator, which leads to a logarithmic divergence in the normalization integral for \( \psi \). (The spinors \( U \) and \( V \) depend on \( k \), but this dependence is eliminated when the spinors stand next to a \( \gamma^- \).) For longitudinal polarization, we obtain

\[
\psi(x, k) = \frac{eQ}{P^-} \frac{x(1-x)Q}{k^2 + x(1-x)Q^2} \overline{U} \gamma^- V.
\] (28)

Again, \( \overline{U} \gamma^- V \) is independent of \( k \). This wave function is small compared to that for transverse polarization when \( Q^2 \ll k^2 \) but becomes comparable when \( Q^2 \sim k^2 \).
VII. LOWEST ORDER CONTRIBUTION

We consider the cross section

\[
\frac{d\sigma^{\text{diff}}(A + B \to A + \text{jets} + X)}{dE_T dX_A dX_B dz dt}
\]

for diffractive jet production in meson-meson collisions within the model described above. In Fig. 4 we show a lowest order graph for this process. The jets are produced in a quark-gluon collision. The active quark in this collision comes from meson B. The gluon comes from meson A, where it has been emitted from one of the quark lines. Another gluon is emitted into the final state and meson A is reconstituted. There are eight ways of attaching the gluons to the scalar quark and antiquark lines; only one of them is shown. Similarly, there are four lowest order graphs for quark-gluon scattering. Thus one needs to sum over the 32 Feynman graphs contributing to the amplitude. There is also an analogous set of graphs in which the hard collision is between an antiquark and a gluon.

FIG. 4. Factorizable graph for diffractive jet production.

The contribution to the cross section calculated from graphs like Fig. 4 takes the factorized form of Eq. (11) in the high energy limit. This is a simple application of the general analysis of [12]. We do not give this analysis in any detail, but we do mention three key points.

First, we discuss the polarization of the gluon carrying momentum \( k_A \) in Fig. 4. We consider all graphs to be in Feynman gauge in this paper, since to use a physical gauge introduces non-physical singularities into the graphs. In Feynman gauge, the relevant part of the graph has the form

\[
H^\alpha - ig_{\alpha\beta} \frac{J^\beta}{k_A^2 + i\epsilon},
\]

where \( H \) represents the hard scattering part of the graph and \( J \) represents the meson wave function part at the bottom of the graph. Now we can replace \(-g^{\alpha\beta}\) by

\[
\sum_{\lambda=1}^{2} e^\alpha(k_A^+, k_A, \lambda) e^\beta(k_A^+, k_A, \lambda) - \frac{u^\alpha k_A^\beta + k_A^\alpha u^\beta}{u \cdot k_A} + \frac{k_A^2}{(u \cdot k_A)^2} u^\alpha u^\beta,
\]

where \( u^\alpha = (0, 1, 0) \) is a lightlike vector in the minus-direction and \( e^\alpha \) is the polarization vector for a gluon with momentum \( (k_A^+, k_A) \) and polarization \( \lambda \) in null-plane gauge, \( u \cdot A = A^+ = 0 \). (See ref. [13] for definitions.) This much is an identity. Now using the power counting of ref. [12], one finds that the terms proportional to \( u^\alpha k_A^\beta \) and \( u^\alpha u^\beta \) are negligible compared to the terms proportional to \( e^\alpha e^\beta \) in the leading integration region, in which \( k_A \) is approximately collinear to \( P_A \). Thus these terms can be dropped. On the other hand, for each Feynman graph, the term proportional to \( k_A^\alpha k_A^\beta \) is larger than the terms proportional to \( e^\alpha e^\beta \). This is an artifact of Feynman gauge. When one sums \( H^\alpha k_A^\beta \) over the graphs contributing to \( H \), there is an almost complete cancellation and the remaining terms are
negligibly small. The result of this is that \(-g^{\alpha \beta}\) can be replaced by \(\sum \epsilon^\alpha \epsilon^\beta\). That is, only physical polarizations contribute.

The second point concerns the gluon carrying momentum \(q\) in Fig. 4. The leading integration region for \(q\) is the collinear region, in which \(q\) is the same order of magnitude as the transverse momenta inside the meson: \(q^2 \sim m^2\). In an abelian theory, the graphs without this gluon would constitute the lowest order graphs for diffractive jet production. Fig. 4 would represent another contribution, a contribution that is formally of higher order in \(\alpha_s\) but is still a leading contribution in the sense of not being suppressed by a power of \(m/E_T\). However, in the non-abelian theory, Fig. 4 gives the lowest order contribution because the emission of a second gluon is necessary to restore the meson to a color singlet state.

The third point concerns the factor representing meson \(B\):

\[
iG\delta_{ij} \frac{i}{k_B^2 - m^2 + i\epsilon}.
\]

(32)

Taking into account that \(P_B^\mu = (M^2/(2P_B^-), P_B^-, 0)\) and that \((P_B - k_B)^2 = m^2\), this factor is

\[
G\delta_{ij} \frac{1 - x_B}{k_B^2 + m^2 - x_B(1 - x_B)M^2},
\]

(33)

where \(x_B = k_B^-/P_B^-\) and \(k_B\) is the transverse part of \(k_B\). Comparing to the definition (20) of the meson wave function \(\psi\), this is

\[
\frac{1}{x_B} \psi(x_B, k_B^2)_{ij}.
\]

(34)

Thus when we square the amplitude and integrate over \(k_B\) we obtain the parton distribution function for meson \(B\), Eq. (21).

In summary, the graph of Fig. 4 factors in a simple way. In the next section, we examine graphs that do not factor.

**VIII. COLOR EXCHANGE WITH SPECTATOR PARTON**

The graph in Fig. 5 is similar to that in Fig. 4, but here the second gluon is absorbed on the spectator antiquark line from meson \(B\) rather than emitted into the final state. As in Fig. 4, there are eight ways of attaching the gluons to the scalar quark and antiquark lines associated with meson \(A\); and there are four lowest order graphs for the hard quark-gluon scattering. Thus one needs to sum over the 32 Feynman graphs contributing to the amplitude.

We will find two important differences between the cross section associated with Fig. 4 and that associated with Fig. 5. First, there is an important observable signature associated with Fig. 5. Essentially all of the plus-momentum \(P_A^+ - P_A'^+ \equiv zP_A^+\) transferred from meson \(A\) appears in the system of high \(E_T\) jets. A negligible amount is transferred to the spectator antiquark. Second, the cross section corresponding to graphs does not take the factored form (11). The cross section is not simply proportional to the probability to find the active partons from mesons \(A\) and \(B\), but also contains information on the spectator interaction.
Consider the integration over $q^\mu$ in Fig. 5. Using power counting arguments similar to those in Ref. [12], one finds that the dominant integration region for the transverse components of $q^\mu$ is $q^2 \sim m^2$. For $q^2$ much larger than this, the propagators in the loop for meson A are thrown far off shell, while for $q^2$ much smaller than this there is a cancellation between attachments to the quark and antiquark lines in meson A, since meson A is a color singlet. For $q^-$, the dominant integration region is $q^- \sim m^2/P_A^+$, since, for $q^-$ much larger than this the propagators in the loop for meson A are again thrown far off shell. Similarly, for $q^+$, the dominant integration region is $q^+ \sim m^2/P_B^-$ since, for $q^+$ much larger than this the propagators for the quark and antiquark in meson B are thrown far off shell. This integration region is denoted the Glauber region in Ref. [12].

We now examine more carefully the integration over $q^+$. In the Glauber region, the only two propagators that have a significant dependence on $q^+$ are the spectator-antiquark propagator and the active-quark propagator. These have the form

$$\frac{i}{(P_B - k_B - q)^2 - m^2 + i\epsilon} \approx \frac{i}{-2(1 - x_B)P_B^- q^+ - \Lambda_1^2 + i\epsilon} \quad (35)$$

and

$$\frac{i}{(k_B + q)^2 - m^2 + i\epsilon} \approx \frac{i}{2x_B P_B^- q^+ - \Lambda_2^2 + i\epsilon}, \quad (36)$$

where $x_B = k_B^-/P_B^-$ and

$$\Lambda_1^2 = (k_B + q)^2 - k_B^2,$$

$$\Lambda_2^2 = \frac{x_B}{1 - x_B} (k_B^2 + m^2) + (k_B + q)^2 + m^2 - x_B M^2. \quad (37)$$

Notice that $\Lambda_1^2$ and $\Lambda_2^2$ are each of order $m^2$ in the dominant integration region, in which $x_B$ is of order 1 and all $k_B^2$ and $q^2$ are of order $m^2$.

The crucial fact about these propagators is that the poles in $q^+$ are on opposite sides of the integration contour along the real $q^+$ axis, creating a pinch singularity as discussed in [12]. We can put the integral in a useful form by deforming the contour into the upper half $q^+$ plane, past the pole from the spectator propagator. The deformation can be continued until $q^+$ is large. Then the propagators are far off shell, so that the contribution from the deformed contour is negligible. The contribution that survives comes from the residue at the spectator pole. Thus we make the replacement $q^+ \rightarrow -\Lambda_2^2/[2(1 - x_B)P_B^-]$ in the active quark propagator. This value of $q^+$ is very small, and all other factors in the diagram depend only weakly on $q^+$, so we may replace $q^+$ by 0 everywhere else in the diagram. Thus we make the replacements

$$\frac{i}{(k_B + q)^2 - m^2 + i\epsilon} \rightarrow -i \left[ \frac{x_B}{1 - x_B} \Lambda_1^2 + \Lambda_2^2 \right]^{-1}$$

$$= \frac{-i (1 - x_B)}{(k_B + q)^2 + m^2 - x_B (1 - x_B) M^2} \quad (38)$$
followed by
\[
\frac{i}{(P_B - k_B - q)^2 - m_b^2 + i\epsilon} \rightarrow \frac{1}{2(1 - x_B)P_B^+} 2\pi \delta(q^+) .
\]

(39)

Finally, the interaction of the Glauber gluon with the spectator quark is
\[
- ig (2P_B - 2k_B - q)\mu a ,
\]
where \(\mu\) is the polarization index of the gluon and \(a\) is its color index. This can be approximated by
\[
- ig 2 (1 - x_B) P_B^- u^\mu t_a .
\]

(41)

Here \(u^\mu = (0, 1, 0)\) is a vector in the minus direction, so \(u \cdot A = A^+\).

What is the net result? The part of the diagram referring to meson B and the interaction with the Glauber gluon begins as
\[
F^\mu_a (k_B, q) = \frac{i}{(k_B + q)^2 - m_b^2 + i\epsilon} G \frac{i}{(P_B - k_B - q)^2 - m_b^2 + i\epsilon} (-ig)(2P_B - 2k_B - q)^\mu t_a .
\]

(42)

With the replacements described above, this factor becomes
\[
F^\mu_a (k_B, q) = - \frac{G (1 - x_B)}{(k_B + q)^2 + m_b^2 - x_B (1 - x_B) M^2} 2\pi \delta(q^+) i g u^\mu t_a .
\]

(43)

We can write \(F\) as
\[
F^\mu_a (k_B, q) = - \frac{1}{x_B} \psi(x_B, k_B + q) J^\mu_a (q) ,
\]
where
\[
J^\mu_a (q) = 2\pi \delta(q^+) i g u^\mu t_a .
\]

(45)

and \(\psi\) is the meson wave function, Eq. (20).

We can understand this as follows. Let \(A^\mu_a (q)\) represent the lower half of the graph. (\(A\) depends on other variables, but we suppress this dependence here and think of \(A\) as a classical gluon field.) Then the factor
\[
\int \frac{d^4 q}{(2\pi)^4} F^\mu_a (k_B, q) A^\mu_a (q)
\]

is approximated by
\[
\int \frac{d^4 q}{(2\pi)^4} \frac{-1}{x_B} \psi(x_B, k_B + q) 2\pi \delta(q^+) i g u^\mu t_a A^\mu_a (q) .
\]

(47)

Define Fourier transforms by
\[ A_\mu^a(q) = \int d^4x \ e^{iq\cdot x} \tilde{A}_\mu^a(x), \]
\[ \psi(x_B, k_B + q) = \int d^2r \ e^{-i(k_B + q)\cdot r} \tilde{\psi}(x_B, r). \]  

Then the factor in Eq. (48) can be written as
\[ -\frac{1}{x_B} \int d^2r \ e^{-ik_B\cdot r} \tilde{\psi}(x_B, r) \int_{-\infty}^{\infty} dx^- i\gamma_\mu \tilde{A}_a^+(0, x^-, -r). \]  

One can interpret Eq. (49) as follows. The amplitude to find a quark at the annihilation point (say \( x^\nu = 0 \)) with momentum fraction \( x_B \) while the antiquark carries transverse momentum \(-k_B\) is \( \int d^2r \ \exp(-ik_B\cdot r) \tilde{\psi}(x_B, r)\). For the present graph, this amplitude is modified by multiplication by a line integral of the color potential along a line in the minus-direction at the transverse coordinate of the antiquark, \(-r\) and the plus-coordinate of the antiquark \(x^+ \approx 0\). The antiquark is not appreciably deflected, but provides a color current that can absorb the gluon that was needed to balance the color and maintain meson \( A \) as a color singlet. The overall minus sign in Eq. (49) arises because the spectator parton is an antiquark instead of a quark.

spectatorsq.epsf here: just uncomment the macro.

FIG. 6. Cut graphs for inclusive jet production. Graph a) contributes to diffractive jet production, while graph b) does not.

We have seen that the contribution to the cross section calculated from the graph of Fig. 5 does not take the factorized form of Eq. (11). To see why the usual factorization theorem \[12\] does not apply here, consider the square of Fig. 4, which is represented by the cut Feynman diagram of Fig. 6(a). If we were considering an inclusive jet production cross section, then other graphs would also be present and would cancel Fig. 6(a) to leading order in \( m/E_T \). These other graphs do not contribute to the cross section for diffractive jet production. For instance, one graph that contributes to inclusive jet production but not to diffractive jet production is shown in Fig. 6(b). In this graph, the constituents of hadron \( A \), rather than a diffractively scattered version of hadron \( A \), appear in the final state.

IX. COLOR EXCHANGE WITH ACTIVE PARTON

We now consider graphs in which the gluon from meson \( A \) is absorbed on the active quark from meson \( B \) before it enters the hard interaction. A typical graph of this sort is shown in Fig. 7. For convenience, we shall refer to the gluon from meson \( A \) as the Glauber gluon, although the important integration region for its momentum \( q^\mu \) is, strictly speaking, not confined to the Glauber region as described in the previous section. We will concentrate on the factor associated with the absorption of the Glauber gluon on the active quark line from hadron \( B \),
\[ G_I \equiv \frac{i}{(k_B + q)^2 - m^2 + i\epsilon} i\gamma_\mu (2k_B + q)^\mu t_a. \]
As in the case of Fig. 5, the dominant integration region for the transverse components of $q^\mu$ is $q^2 \sim m^2$ and the dominant integration region for $q^-$ is $q^- \sim m^2/P_A^+$. The factor $ig (2k_B + q)^ \mu t_a$ in (50) multiplies a vector associated with meson $A$ that is predominantly in the plus direction. Thus we can replace this factor by $ig 2x_B P^{-}_B u^\mu t_a$ with $x_B = k^{-}_B/P^{-}_B$ and $u^\mu = (0, 1, 0)$. We now examine the integration over $q^+$. The only propagator that has a significant dependence on $q^+$ for small $q^+$ is the active quark propagator in (50),

$$i \left( \frac{k_B + q}{(k_B + q)^2 - m^2 + i\epsilon} \right) \approx i \frac{2x_B P^+_B q^+ - \Lambda_2^2 + i\epsilon}{2x_B P^+_B q^+ - \Lambda_2^2 + i\epsilon},$$

(51)

where $\Lambda_2^2$ is given in Eq. (37). Recall that $\Lambda_2$ is of order $m^2$ in the dominant integration region. Thus there is a pole in the lower half $q^+$ plane near $q^+ = 0$. However, as discussed in Ref. [12], the integration contour is not pinched between two nearby poles. Thus the contour can be deformed away from the pole. We then see that it is a good approximation to replace $\Lambda_2^2$ by 0 in (51). Then

$$G_I \approx ig u^\mu t_a \frac{i}{q^+ + i\epsilon}.$$  

(52)

Before completing work on the interaction with the active quark entering the hard interaction, we need to consider the two graphs, as shown in Fig. 8, in which the gluon attaches to one of the partons emerging from the hard scattering. Here the factor associated with the absorption of the gluon is approximately

$$G_F \approx ig (2k_F - q)^ \mu t_a \left( \frac{i}{(k_F - q)^2 - m_F^2 + i\epsilon} \right),$$

(53)

where $t_a$ is the appropriate color matrix, depending on whether the final state parton in question is a quark or a gluon, and where $m_F = m$ for a quark or 0 for a gluon. This is exactly the vertex factor for the graph in which the Glauber gluon is absorbed on the outgoing quark line. It is approximate in the case that the gluon with small momentum $q^\mu$ is absorbed on the outgoing gluon line that carries large momentum $k_F$ into the final state. As before, the factor $ig (2k_F - q)^ \mu t_a$ can be approximated by $ig 2k_F u^\mu t_a$. The propagator can be approximated by

$$\frac{i}{(k_F - q)^2 - m_F^2 + i\epsilon} = \frac{i}{-2k_F \cdot q + q^2 + i\epsilon} \approx \frac{i}{-2k_F \cdot q + i\epsilon} = \frac{i}{-2k_F q^+ - 2k_F q^- + 2k_F \cdot q + i\epsilon}.$$  

(54)
This propagator has a pole in the upper half \( q^+ \) plane. The poles in the \( q^+ \) plane from the other propagators are all at large \( q^+ \), \( q^+ \sim E_T \). Thus we can deform the contour into the lower half \( q^+ \) plane until \( |q^+| \gg m \). Now the components of \( k_F \) are all of the same order of magnitude (about \( E_T \)), while the minus and transverse components of \( q \) are both small.

Thus on the deformed contour, the approximation

\[
\frac{i}{(k_F - q)^2 - m_F^2 + i\epsilon} \approx \frac{i}{-2k_F^{-}q^+ + i\epsilon}.
\]  

(55)

is valid. Of course, the integral is independent of the contour deformation, so we can move the contour back to the real \( q^+ \) axis. (Similar manipulations may be found in Ref. \[12\].)

With these approximations, \( G_F \) is

\[ G_F \approx -igu^\mu t_a \frac{i}{q^+ - i\epsilon}. \]  

(56)

Notice that there is a certain similarity among the effective factor \( G_I \) giving the coupling of the Glauber gluon to the incoming parton from hadron \( B \) and the two factors \( G_F \) giving the coupling to the outgoing partons. For the moment, let us consider \( q^+ \neq 0 \), so that we can ignore the \( i\epsilon \). Then the only difference is how the color matrix \( t_a \) for the Glauber gluon is connected to the color matrix \( M_{cd}^{ij} \) for the hard scattering. Here the color indices are \( i \) for the final quark, \( j \) is the initial quark, \( c \) for the final gluon, and \( d \) for the initial gluon.

Attaching to the final quark, we have \((t^{(3)}_a)_{ii'} M_{cd}^{ij}\), where the superscript (3) denotes the triplet representation of SU(3). Attaching to the final gluon, we have \((t^{(8)}_a)^{cc'} M_{cd}^{ij}\). Attaching to the initial quark, we have \(-M_{cd}^{ij} (t^{(3)}_a)^{jj'} \), where we have noted the opposite signs between Eq. (52) and Eq. (56). We can sum the three contributions using the group theoretic identity

\[
(t^{(3)}_a)_{ii'} M_{cd}^{ij} + (t^{(8)}_a)^{cc'} M_{cd}^{ij} - M_{cd}^{ij} (t^{(3)}_a)^{jj'} = M_{cd}^{ij} (t^{(8)}_a)^{dd'},
\]  

(57)

which merely says that \( M \) is invariant under SU(3) rotations. In the color matrix on the right hand side of (57), the Glauber gluon from hadron \( A \) first couples to the active gluon from hadron \( A \) to make a color octet object, then this color octet couples to the hard scattering color matrix. However, these two gluons are in a color singlet state, so coupling the two to make a color octet gives zero. (That is, the lower part of the graph is proportional to \( \delta_{ad} \), while \((t^{(8)}_a)^{dd'}\) is antisymmetric in \( \{a,d\} \).)

Thus the sum of the three diagrams cancels, except for the fact that the denominators in Eqs. (52) and (56) have the opposite \( i\epsilon \)’s. In order to take the \( i\epsilon \)’s into account we now make the replacement

\[
\frac{i}{q^+ + i\epsilon} \to \frac{i}{q^+ - i\epsilon} + 2\pi\delta(q^+).
\]  

(58)

in \( G_I \). The first term has the right \( i\epsilon \) to participate in the cancellation, and we are left with an uncancellation term

\[ G'_I \equiv igu^\mu t_a 2\pi\delta(q^+). \]  

(59)
We are grateful to John Collins for suggesting this to us.

What is the net result? The part of the diagram referring to meson B and the interaction with the Glauber gluon begins as

\[ F_{\mu}^{\prime}(k_B, q) = \frac{i}{(k_B + q)^2 - m^2 + i\epsilon} \left( \frac{iq}{k_B} \right) \frac{iG}{k_B - m^2 + i\epsilon}. \tag{60} \]

With the replacements described above, this factor becomes

\[ F_{\mu}^{\prime}(k_B, q) = \frac{G(1 - x_B)}{(k_B + m^2) - x_B(1 - x_B)M^2} \frac{2\pi \delta(q^+)}{\psi(x_B, k_B)J_{\mu}(q)}. \tag{61} \]

We can write \( F' \) as

\[ F_{\mu}^{\prime}(k_B, q) = \frac{1}{x_B} \psi(x_B, k_B) J_{\mu}(q), \tag{62} \]

where \( J \) and \( \psi \) are defined as in Eq. \((44)\).

It is instructive to write this in transverse coordinate space, as we did for the attachment to the spectator antiquark. We let \( A_{\mu}(q) \) represent the lower half of the graph. Then the factor

\[ \int \frac{d^4q}{(2\pi)^4} F_{\mu}^{\prime}(k_B, q) A_{\mu}(q) \tag{63} \]

is approximated by

\[ \int \frac{d^4q}{(2\pi)^4} \frac{1}{x_B} \psi(x_B, k_B) 2\pi \delta(q^+) \frac{igu_{\mu}t_a}{A_{\mu}(q)}. \tag{64} \]

Define Fourier transforms as before, we find that the factor in Eq. \((63)\) becomes

\[ \frac{1}{x_B} \int d^2r \ e^{-ik_B \cdot r} \tilde{\psi}(x_B, r) \int_{-\infty}^{\infty} dx^+ \frac{igt_a}{A_{\mu}^+(0, x^+, 0)}. \tag{65} \]

This is the same as the factor \((49)\), except for the overall sign and except that the Glauber gluon is absorbed at transverse position \( 0 \) instead of transverse position \(-r\).

X. RESULTS

We have investigated diffractive jet production within the context of a certain simple model. We have seen that the cross section has contributions that are lossless in the sense that the plus-momentum \( zP_A^+ \) transferred from hadron \( A \) is transferred without loss to the jet system. Thus these terms contain a factor \( \delta(1 - X_A/z) \). We now assemble the results and rewrite them in a suggestive form.

We can build up the result using four ingredients. The first ingredient is the line integral of the color potential along a lightlike line in the minus direction (the direction of hadron \( B \)) at a transverse position \( b \), where the hard interaction is at transverse position \( 0 \):
\[ A(b) = \int_{-\infty}^{+\infty} dy^- \sum_c i g t_c A_c^+(0, y^-, b). \]  

(66)

Notice that, in addition to being an operator, \( A \) is a matrix that acts on vectors in the 3-representation of color SU(3). The next ingredient is the color field operator \( F_{a}^{ij}(0, 0, 0) \), which destroys a gluon of transverse polarization \( j \) and color \( a \) at the origin of space-time. Combining these operators, we form the amplitude to annihilate the gluon at the origin along with one more Glauber gluon at transverse position \( b \), while scattering the meson \( A \) with momentum transfer \((z, t)\):

\[ G_a^b(b; t, z) = \frac{1}{4\pi X_A P_A^+} \langle P_A^T \{ A(b) F_{a}^{++j}(0, 0, 0) \} | P_A \rangle. \]  

(67)

(We have indicated a time-ordered product here, but within the context of the model the operators commute, so the time-ordering is not relevant.) The next ingredient is the wave function to find the quark in meson \( B \) carrying a momentum fraction \( X_B \) and separated from the antiquark by a transverse separation \( r \), \( \psi(X_B, r) \).

Finally, we have a hard scattering function \( H_{ab}^{jk}(\hat{s}, E_T)_{IJ} \), where \( \hat{s} = X_A X_B s \). Up to a normalization, this function is the product of the Born-level scattering amplitude \( M \) for the gluon of plus-momentum \( x_A P_A^+ \) to scatter from a (scalar) quark of minus-momentum \( x_B P_B^- \) times the complex conjugate amplitude \( M^\dagger \) with different color and spin labels. The gluon has of polarization \( k \) and color \( b \), while the quark has color \( J \). In the complex conjugate amplitude, the corresponding indices are \( j, a, \) and \( I \). We consider \( H \) as a matrix in the quark color space and do not write the indices \( I J \) explicitly. The normalization is such that the usual hard scattering cross section \( d\hat{\sigma} \) for inclusive jet production, as used in Eq. (7), is, at the Born level, the color and spin average of \( H \), with the momentum-conserving delta functions removed:

\[
\left[ \frac{d\hat{\sigma}(a + b \to \text{jets} + X)}{dE_T dX_A dX_B} \right]_{\text{Born}} = \delta(x_A - X_A) \delta(x_B - X_B) \frac{1}{2} \sum_j \frac{1}{8} \sum_a \frac{1}{3} \text{Tr} \{ H_{aa}^{jj} \}. \]  

(68)

With these ingredients, the lossless part of the cross section for diffractive jet production within the model described in the preceding sections is

\[
\left[ \frac{d\sigma^{\text{diff}}(A + B \to A + \text{jets} + X)}{dE_T dX_A dX_B dz dt} \right]_0 \sim \delta(1 - X_A/z) \int d\hat{r} \frac{||\psi(X_B, \hat{r})||^2}{2X_B(1 - X_B)} \times \frac{2}{\sum_{j,k=1}^8} \sum_{a,b=1}^8 \text{Tr} \left\{ \left[ G_a^j(-\hat{r}; t, z) - G_a^j(0; t, z) \right]^\dagger H_{ab}^{jk}(\hat{s}, E_T) \times \left[ G_b^k(-\hat{r}; t, z) - G_b^k(0; t, z) \right] \right\}. \]  

(69)

There are several features of this result that are notable. First is the \( \delta(1 - X_A/z) \), which provides the experimental signature for the lossless contribution to the cross section. In contrast, the normal contributions yield values of the variable \( X_A \) that are spread over the region \( X_A < z \). The second feature is that the factorization that applies to inclusive jet production does not apply here. The factor corresponding to hadron \( A \) is tied to the factor
representing hadron $B$ by a convolution over a transverse position $\mathbf{r}$. There is also a more complicated color and spin structure than in the normal contributions.

We emphasize that the fact that factorization does not apply here does not represent a failure of the factorization theorem for the total cross section for $A + B \rightarrow \text{jets + X}$. The factorization theorem requires an unrestricted sum over the final state $X$ [12]. When we demand that $X$ contain the a diffractively scattered hadron $A$, the conditions for the theorem are violated.

Despite its rather complicated structure, the interpretation of Eq. (19) is straightforward. In the model, meson $B$ consists of a quark and an antiquark. With probability $\propto |\psi(X_B, \mathbf{r})|^2$, they are separated by a transverse distance $\mathbf{r}$. In order to restore the color of hadron $A$, we must absorb a gluon on either the antiquark (at position $-\mathbf{r}$) or the quark (at position $\mathbf{0}$). Since the quark and antiquark have opposite color charges, the absorption amplitude is proportional to the difference $G_a^j(-\mathbf{r}; t, z) - G_a^j(0; t, z)$. Here $G_a^j(\mathbf{b}; t, z)$ is the amplitude to absorb a color field quantum at transverse position $\mathbf{b}$ when the “active” gluon is annihilated at the origin of space-time and hadron $A$ is diffractively scattered. Thus $G$ describes the color field associated with the pomeron when one gluon from the pomeron has been annihilated at the origin.

Here we meet an interesting experimental possibility. The $\mathbf{b}$ dependence of $G_a^j(\mathbf{b}; t, z)$ reflects the transverse structure of the pomeron. It has significant structure on some distance scale $R_P$ characteristic of the pomeron. In the present model, $1/R_P$ is of order of the quark mass $m$. Thus $G_a^j(-\mathbf{r}; t, z) - G_a^j(0; t, z)$ is small when $|\mathbf{r}| \ll R_P$. On the other hand, $|\psi(X_B, \mathbf{r})|^2$ is small when $|\mathbf{r}| \gg R_B$, where $R_B$ is a characteristic size of hadron $B$. This size is also of order $1/m$ in the model. However, suppose that we generalize the model so that $R_B$ can be separately adjusted. Then when $R_B \approx R_P$, there will be a substantial contribution to the cross section proportional to $\delta(1 - X_A/z)$. But when $R_B \ll R_P$, this contribution will vanish.

So far, we have worked only with a simple model. But the model suggests a plausible conjecture. First, there can be a sizable contribution to diffractive jet production proportional to $\delta(1 - X_A/z)$, arising from using one gluon from the pomeron to make the jets and absorbing on the partons of hadron $B$ the rest of the color field needed to make hadron $A$ back into a color singlet. Second, when the size $R_B$ of hadron $B$ is small compared to the transverse size $R_P$ associated with the color field in pomeron exchange, then hadron $B$ should act as a color singlet and this contribution should disappear.

In order to test this conjecture, and probe the transverse structure of the pomeron, one needs to use hadrons of adjustable size. This is simple (for theorists). At HERA, one manufactures bremsstrahlung photons from the electron beam. The virtuality $Q = \left[-P_B^\mu P_B^\mu\right]^{1/2}$ of the photon is measured by the deflection of the electron, and can be anything from nearly zero to many GeV. The photon can collide with a proton (hadron $A$) to make jets with $E_T \gg Q$. The cross section for this process can be (roughly) divided into two parts. In one part, the photon acts as a parton and scatters directly with a parton from hadron $A$ to make the jets. In the other part, the photon acts as a hadron, made of constituent partons. For $Q \approx 0$, this hadron is essentially a $\rho$-meson, with a size $R_B \approx 1$ fm. For $Q \gg 1$ fm$^{-1}$, the “hadron” consists of a quark-antiquark pair, with wave functions given in Eq. (27) for transverse polarization and Eq. (28) for longitudinal polarization. These wave functions are characterized by a size $R_B \approx [X_B(1 - X_B)Q^2]^{-1/2}$. Since $Q^2$ and $X_B$ are measurable, this
Thus the experimental possibility is to select photon-proton scattering events with $Q$ in the range from zero up to one or two GeV and look for jets accompanied by a diffractively scattered proton. These can be divided into events with a partonic photon ($X_B \approx 1$) and those with a hadronic photon ($X_B < 1$). Some of the events should be of the lossless ($X_A \approx z$) type, while some most should be of the normal ($X_A < z$) type. For the partonic-photon events, there should be diffractive jet production, but it should have $X_A < z$, since the photon has no color structure. For the hadronic-photon events, both $X_A < z$ and $X_A \approx z$ events should occur, but the fraction of $X_A \approx z$ events should approach zero as $Q$ becomes large. The scale of $X_B(1 - X_B)Q^2$ at which this happens reflects the transverse size of the pomeron.

In the model we have used, the “lossless” contributions to the cross section are proportional to $\delta(1 - X_A/z)$ while the “normal” contributions are non-singular functions of $X_A/z$. We expect that this clean division is not so clean when higher-order contributions are taken into account. Presumably there are contributions that are singular as $X_A/z \to 1$ but are not as singular as a delta-function. Thus one may anticipate that the lowest order $\delta(1 - X_A/z)$ appears as events with $X_A \approx z$ rather than $X_A = z$.

We must emphasize that the proposal given above is a conjecture based on a simple model, not a proven consequence of QCD. It should be a challenge to investigate the structure of diffractive hard scattering further and to discover what features of the model survive a higher order analysis.

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REFERENCES

[1] S. D. Ellis, Z. Kunszt, and D. E. Soper, Phys. Rev. Lett. 69, 3615 (1992); 69, 1496 (1992); 64, 2121 (1990); 62, 726 (1989).
[2] CDF Collaboration (F. Abe, et al.), Phys. Rev. Lett. 68, 1104 (1992)
[3] G. Ingelman and P. Schlein, Phys. Lett. B152, 256 (1985).
[4] H. Fritzsch and K. H. Streng, Phys. Lett. 164B, 391 (1985).
[5] E. L. Berger, J. C. Collins, D. E. Soper, and G. Sterman, Nucl. Phys. B286, 704 (1987).
[6] A. Brandt, et. al., Phys. Lett. B297, 417 (1992).
[7] A. Berera and D. E. Soper, Arizona preprint AZPH-TH/94-05, in preparation.
[8] ZEUS Collaboration (M. Derrick, et al.), Phys. Lett. B 315, 481 (1993); K. Desch, ZEUS Collaboration, talk at XXIV International Symposium on Multiparticle Dynamics, Aspen, Colorado, September, 1993 (to be published); J. F. LaPorte, H1 Collaboration, ibid.
[9] J. C. Collins, L. Frankfurt, and M. Strikman, Phys. Lett. 307, 161 (1993).
[10] S. J. Brodsky, L. Frankfurt, J. F. Gunion, A. H. Miller and M. Strikman, SLAC preprint SLAC-PUB-6412.
[11] See, for example, E. Gotsman, E. M. Levin, U. Maor, Z. Phys. C 57, 677 (1993); J. Bartels and E. Levin, Nucl. Phys. B387, 617 (1992); J. Bartels, Nucl. Phys. A546, 107c (1992).
[12] J. C. Collins, D. E. Soper, and G. Sterman, Nucl. Phys. B261, 104 (1985); B308, 833 (1988); G. Bodwin, Phys. Rev. D 31, 2616 (1985); 34, 3932 (1986).
[13] W. W. Lindsay, D. A. Ross, and C. T. Sacrajda, Nucl. Phys. B214, 61 (1983).
[14] J. C. Collins, D. E. Soper, and G. Sterman, Nucl. Phys. B223, 281 (1983).
[15] J. B. Kogut and D. E. Soper, Phys. Rev. D 1, 2901 (1970).
[16] J. D. Bjorken, J. B. Kogut and D. E. Soper, Phys. Rev. D 3, 1382 (1971).
[17] S. J. Brodsky and G. P. Lepage, Phys. Rev. D 22, 2157 (1980).