Processes that break baryon number by two units and the Majorana nature of the neutrino

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Abstract

We employ the simplest possible models of scalar-fermion interactions that are consistent with the gauge symmetries of the Standard Model and permit no proton decay to analyze the connections possible among processes that break baryon number by two units. In this context we show how the observation of $n$-$\bar{n}$ oscillations and of a pattern of particular nucleon-antinucleon conversion processes — all accessible through e-d scattering — namely, selecting from $e^- p \rightarrow e^+ \bar{p}$, $e^- p \rightarrow \bar{n}\nu$, $e^- n \rightarrow \bar{p}\bar{\nu}$, and $e^- n \rightarrow e^- \bar{n}$ would reveal that the decay $\pi^- \pi^- \rightarrow e^- e^-$ must occur also. This latter process is the leading contribution to neutrinoless double beta decay in nuclei mediated by new short-distance physics, in contrast to that mediated by light Majorana neutrino exchange. The inferred existence of $\pi^- \pi^- \rightarrow e^- e^-$ would also reveal the Majorana nature of the neutrino, though the absence of this inference would not preclude it.

1. Introduction

The quantity baryon number (B) - lepton number (L), B-L, is exactly conserved in the Standard Model (SM), so that the observation of B-L violation would reveal the existence of new physics. In this letter we consider the possibility of the discovery of B-L violation within the realm of the strong interactions and the quark sector — and its broader implications. We focus particularly on processes that break baryon number by two units because proton decay, or, more generally, processes with $|\Delta B| = 1$, are not only unobserved but also have exceptionally strong empirical limits on their non-existence [1]. Moreover, as long known, the new-physics origins of $|\Delta B| = 1$ and $|\Delta B| = 2$ processes can be completely distinct [2–5].

The prospect of B-L violation is often discussed in the context of the fundamental nature of the neutrino; its violation would both make the $|\Delta L| = 2$ process of neutrinoless double beta ($0\nu\beta\beta$) decay possible and give the neutrino a Majorana mass [6–8], revealing that the neutrino can be regarded as its
own antiparticle [9]. General parametrizations of the decay rate are associated with the long-range exchange of a light Majorana neutrino [10–12], or through a short-range process mediated by new B-L violating dynamics at roughly the TeV scale [13]. The nuclear matrix elements, which are needed to interpret 0νββ experiments, differ considerably in the two cases [10, 13–16]. Systematic analyses of the possible operators of 0νββ decay [17–20] and of the associated decay topologies [19], and of the decay rate within chiral effective theory [21–23] exist. The short-range mechanism is captured by $\pi^-\pi^-\to e^-e^-$ [24] at leading order in hadron chiral effective theory [25], and the size of the associated hadronic matrix element has recently been computed in lattice QCD [26]. We believe that insight on the mechanisms of 0νββ decay can be gleaned through the study of B-L violation in the quark sector, as it is the short-distance mechanism that can connect B-L violation with quarks to that with leptons.

The empirical study of $|\Delta B| = 2$ processes has traditionally been associated with the search for $n - \bar{n}$ oscillations with free or bound nucleons [2, 27–29] and dinucleon decay in nuclei [4, 5, 30–35]. Recently we have proposed the study of $n - \bar{n}$ conversion [36, 37], which, in contrast to $n - \bar{n}$ oscillation, would not be spontaneous but mediated by an external source. In this letter we discuss the connections between these possibilities in the context of simple models of B and B-L violation. Motivated by “minimal” models for connectors to new hidden sectors [38, 39], we introduce new scalar gauge bosons whose interactions are of mass dimension 3 and 4, so that the new interactions can be added to the SM in a theoretically consistent way. Scalar-fermion interactions in such models that respect the gauge symmetries of the SM have been studied in some detail [4, 40, 41]. In the current case our interest is in the models that permit $|\Delta B| = 2$ transitions without proton decay, and indeed in those that do not permit $|\Delta B| = 1$ transitions [4, 5]. Interestingly, we have discovered that a variant of the models of Arnold, Fornal, and Wise [4] can be used to generate a $|\Delta L| = 2$ transition, particularly, that of $\pi^-\pi^-\to e^-e^-$, whose existence drives the appearance of 0νββ decay if mediated by new short-distance physics [25]. Thus in what follows we consider not only how particular $n - \bar{n}$ oscillation and conversion processes can appear in these models, but we also show how such models can give rise to 0νββ decay in nuclei — and we consider the interconnections between them. Particularly, we discuss how possible patterns of discovery of $|\Delta B| = 2$ processes can reveal whether the short-distance dynamics that could give rise to $\pi^-\pi^-\to e^-e^-$ can be shown to exist. In contrast, Babu and Mohapatra have shown that in the case of the SO(10) grand unified theory — and independently from the expected existence of the SM sphaleron — that if $n - \bar{n}$ oscillations and a $|\Delta B| = 1$ process were observed to occur that one could also conclude the existence of a Majorana neutrino [42]. Here we show that such a connection can be demonstrated without requiring the observation of proton decay, or indeed of any $|\Delta B| = 1$ process. We emphasize that in this case, as in Ref. [42], the existence of such an inference does not imply that the short-distance mechanism ought saturate the experimental rate for 0νββ decay. Our approach, however, is different from that of Ref. [42], as it relies on the use of minimal scalar models.
Table 1: Scalar particle representations in the $SU(3)_c \times SU(2)_L \times U(1)_Y$ SM that carry nonzero $B$ and/or $L$ but permit no proton decay at tree level, after Ref. [4]. We indicate the possible interactions between the scalar $X$ and SM fermions schematically. Note that the indices $a,b$ run over three generations, that the symmetry of the associated coupling $g^{ab}_{ij}$ under $a \leftrightarrow b$ exchange is noted in brackets, and finally that our convention for $Y$ is $Q_{em} = T_3 + Y$. Please refer to the text for further discussion.

| Scalar | SM Representation | B | L | Operator(s) | $g^{ab}_{ij}$ |
|--------|-------------------|---|---|-------------|--------------|
| $X_1$  | $(1, 1, 2)$       | 0 | -2| $X e^a e^b$ | $S$          |
| $X_2$  | $(1, 1, 1)$       | 0 | -2| $XL^a L^b$  | $[A]$        |
| $X_3$  | $(1, 3, 1)$       | 0 | -2| $XL^a L^b$  | $S$          |
| $X_4$  | $(6, 3, -1/3)$    | -2/3| 0 | $XQ^a Q^b$  | $S$          |
| $X_5$  | $(6, 1, -1/3)$    | -2/3| 0 | $XQ^a Q^b, Xu^a d^b$ | $[A, -]$ |
| $X_6$  | $(3, 1, 2/3)$     | -2/3| 0 | $Xd^a d^b$  | $[A]$        |
| $X_7$  | $(6, 1, 2/3)$     | -2/3| 0 | $Xd^a d^b$  | $S$          |
| $X_8$  | $(6, 1, -4/3)$    | -2/3| 0 | $Xu^a u^b$  | $S$          |
| $X_9$  | $(3, 2, 7/6)$     | 1/3| -1| $XQ^a e^b, XL^a u^b$ | $[-, -]$ |

2. Minimal scalar models with baryon number violation but no proton decay

The minimal scalar models that give rise to $|\Delta B| = 2$ and not $|\Delta B| = 1$ processes while respecting SM gauge symmetries contain either three or four scalar interactions. Following Refs. [4, 40, 41, 43] we consider all the interactions permitted by Lorentz and $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge symmetry. Models for processes with both $|\Delta B| = 1, 2$ have been constructed [4, 41, 43, 44], though in this paper we follow Ref. [4]. The particular scalars that allow $B$ or $L$ violation to appear but do not admit $|\Delta B| = 1$ processes at tree level are enumerated in Table 1. We have also noted the schematic interactions of the scalars $X_i$ to right-handed leptons and quarks of generation $a$ as $e^a$ and $u^a$, $d^a$ and to left-handed leptons and quarks as $L^a$ and $Q^a$, respectively. The symmetries of the scalar representations under color $SU(3)$ and/or weak isospin $SU(2)$ can fix the symmetry of the associated coupling constant under $a,b$ interchange, and we have noted that as well in Table 1 — the relation $g^{ab}_{ij} = \pm g_{ij}^{ab}$ indicates $S(+)$ or $A(-)$, respectively, and “–” denotes no interchange symmetry. We note that $X_9$ cannot generate a $B$ and/or $L$ violating interaction of mass dimension four or less, so that we do not consider it further, and that interactions denoted by “$A$” cannot involve only first-generation fermions.

In what follows we extend the models of Ref. [4] to include the possibility of $|\Delta L| = 2$ processes as well. That earlier work focused on the possibility of $|\Delta B| = 2$ processes without proton decay as mediated by interactions of the form $X^2_a X_b$ or $X^3_a X_b$, where $X_a$ and $X_b$ are simply two distinct scalars that yield the SM gauge invariant interactions indicated, because it turns out not to be possible to add just one scalar and achieve that end. Here we enumerate all the possible $B$ and/or $L$ violating interactions that appear in mass dimension of four or less without regard to the number of different scalars that can appear. With three different scalars we can produce $|\Delta L| = 2$ processes that also couple to
Table 2: Minimal interactions that break $B$ and/or $L$ from scalars $X_i$ that do not permit $|\Delta B| = 1$ interactions at tree level, indicated schematically, with the Hermitian conjugate implied. Interactions labelled M1-M9 appear in models 1-9 of Ref. [4]. Interactions A-G possess $|\Delta L| = 2$, $|\Delta B| = 0$. M19, M20, and M21 follow from M8, M17, and M18 under $X_7 \rightarrow X_6$, respectively, but they do not involve first-generation fermions only.

| Model | Model | Model |
|-------|-------|-------|
| M1    | $X_2X_3X_7$ | A | $X_1X_8X_7$ | M10 | $X_7X_6X_8X_1$ |
| M2    | $X_4X_3X_7$ | B | $X_3X_4X_1$ | M11 | $X_5X_3X_4X_3$ |
| M3    | $X_7X_7X_8$ | C | $X_3X_8X_1$ | M12 | $X_5X_5X_8X_1$ |
| M4    | $X_5X_6X_8$ | D | $X_5X_2X_7$ | M13 | $X_4X_4X_5X_2$ |
| M5    | $X_5X_5X_5X_2$ | E | $X_5X_2X_5$ | M14 | $X_4X_4X_5X_3$ |
| M6    | $X_4X_4X_4X_2$ | F | $X_2X_2X_7$ | M15 | $X_4X_4X_8X_1$ |
| M7    | $X_4X_4X_4X_3$ | G | $X_3X_3X_1$ | M16 | $X_4X_7X_8X_3$ |
| M8    | $X_7X_7X_7X_7$ | | M17 | $X_5X_7X_7X_7$ |
| M9    | $X_5X_5X_5X_3$ | | M18 | $X_4X_7X_7X_3$ |

quarks, and we study the connections between $|\Delta B| = 2$ and $|\Delta L| = 2$ processes explicitly.

We begin by fleshing out the precise interactions indicated in Table 1. Specifically, the possible scalar-fermion interactions mediated by each $X_i$ are

\[
\begin{align*}
-g_1^{ab}X_1(\epsilon^ae^b), & \quad -g_2^{ab}X_2(L^a\varepsilon L^b), & \quad -g_3^{ab}X_3^\alpha(L^a\xi^\alpha L^b), \\
-g_4^{ab}X_4^{\alpha\beta}(Q_\alpha^a\xi^\beta), & \quad -g_5^{ab}X_5^{\alpha\beta}(Q_\alpha^a\varepsilon Q_\beta), & \quad -g_6^{ab}X_6^{\alpha\beta}(u_a^b d_b^\dagger), \\
-g_7^{ab}X_6^{\alpha\beta}(d_\beta^a u_a^b), & \quad -g_8^{ab}X_7^{\alpha\beta}(u_a^b d_\beta^a), & \quad -g_9^{ab}X_8^{\alpha\beta}(u_a^b u_b^a),
\end{align*}
\]  

where $\varepsilon = i\tau^2$ is a totally antisymmetric tensor, $\xi^\alpha = ((1 + \tau^3)/2, \tau^1/\sqrt{2}, (1 - \tau^3)/2)$, and $\tau^A$ are Pauli matrices with $A = 1, 2, 3$. We note $\varepsilon\tau^A$ was used in place of $\xi^A$ in Ref. [4], but that choice couples a single component of the scalar weak triplet to fermion states of differing total electric charge, incurring couplings that break electric charge conservation. The Greek indices are color labels, and we employ the SU(3) notation of Ref. [45] for fundamental and complex conjugate representations. We adopt 2-spinors such that the fermion products in parentheses are Lorentz invariant, and we map to 4-spinors via $(u_L, R_\alpha d_{L,R\beta}) \rightarrow (u_L^\dagger C P_{L,R} d_{L,R\beta})$ where $C = i\gamma^0\gamma^2$ and $P_{L,R} = (1 \mp \gamma_5)/2$ in Weyl representation [46].

3. Possible baryon-number and/or lepton-number violating processes

We now turn to the possible minimal scalar interactions that mediate either baryon and/or lepton number violation but conserve SM gauge symmetries. The possible interactions, including as many as four distinct scalars, are enumerated in Table 2. The models labelled M1-M9 are those of Models 1-9, respectively, in Ref. [4]. A particular model contains terms that couple the scalars to fermions and terms that couple the scalars to each other. We find we must modify the
scalar self-couplings of M2 and M7 in order to maintain electric charge conservation for each term of the scalar self-interaction. Rather than recapitulate M1-M9 we simply summarize the detailed versions of the scalar forms enumerated in Table 2:

\[
\begin{align*}
\lambda_1 X_5^{\alpha} X_5^{\beta} \epsilon_{\alpha\beta} X_7^\gamma & \epsilon_{\alpha\beta\gamma'} \epsilon_{\alpha\beta} & \lambda_2 [X_3^{\alpha} X_3^{\beta} X_3^\beta]_0 X_7^\gamma & \epsilon_{\alpha\beta} \epsilon_{\alpha\beta'} \gamma' , \\
\lambda_3 X_7^{\alpha} X_7^{\beta} \epsilon_{\alpha\beta} X_8^\gamma & \epsilon_{\alpha\beta} X_8 & \lambda_4 X_6\alpha X_6\beta X_8^\gamma , \\
\lambda_5 X_5^{\alpha} X_5^{\beta} \epsilon_{\alpha\beta} X_5^\gamma X_2^\alpha & \epsilon_{\alpha\beta} & \lambda_6 X_4^{\alpha} X_4^{\beta} X_3^\gamma X_2^\epsilon & \epsilon_{\alpha\beta} \epsilon_{\alpha'} \gamma' , \\
\lambda_7 [X_4^{\alpha} X_4^{\beta} X_4^\beta X_3^\gamma X_3^\delta] & 0 \epsilon_{\alpha\beta} \epsilon_{\alpha'} \gamma' , & \lambda_8 X_7^{\alpha} X_7^{\beta} X_7^\gamma X_5^\gamma & \epsilon_{\alpha\beta} \epsilon_{\alpha'} \gamma' , \\
\lambda_9 X_6\alpha X_6\beta X_6\gamma & \epsilon_{\alpha\beta} ,
\end{align*}
\]

where Hermitian conjugation is implied. The noted weak singlets follow from SU(2) Clebsch-Gordon coefficients [1], so that

\[
[X_4^{\alpha} X_4^{\beta} X_4^\beta]_0 \equiv \frac{1}{\sqrt{3}} [X_4^{\alpha} X_4^{\beta} + X_4^{\alpha} X_4^{\beta} - X_4^{\alpha} X_4^{\beta}] (3)
\]

and

\[
[X_4^{\alpha} X_4^{\beta} X_4^\beta X_4^\gamma X_4^\delta]_0 \equiv \frac{1}{\sqrt{3}} \left\{ \left[ \frac{3}{5} \lambda_4^{\alpha} X_4^{\beta} X_4^{\gamma} X_4^{\delta} X_4^{\beta} \right] - \left[ \frac{3}{20} - \frac{1}{2} \right] \lambda_4^{\alpha} X_4^{\beta} X_4^{\gamma} X_4^{\delta} X_4^{\beta} \right\} (4)
\]

where "\(1" \leftrightarrow "3"" denotes the expression found by exchanging 1 and 3 superscripts. Turning to the \(|\Delta L| = 2\) models in Table 2, we find

\[
\begin{align*}
\lambda_A X_8^{\alpha} (X_7^{\alpha})^\dagger X_1 , & \lambda_B [X_3^{\alpha} X_3^{\beta}]_0 (X_7^{\alpha})^\dagger , & \lambda_C [X_3^{\alpha} (X_3^{\beta})^\dagger]_0 X_8^{\alpha} , \\
\lambda_D X_8^{\alpha} (X_5^{\alpha})^\dagger X_2 , & \lambda_E X_8^{\alpha} (X_5^{\alpha})^\dagger X_2 , & \lambda_F X_2 X_2 X_1^\dagger , \\
\lambda_G [X_3^{\alpha} X_3^{\beta}]_0 X_1^\dagger ,
\end{align*}
\]
whereas for the remaining baryon-number-violating models, we have

\[
\begin{align*}
\lambda_{10} x_7^{\alpha_0} x_8^{\beta_0} x_8^{\gamma_0} x_1 \epsilon_{\alpha\beta\gamma} \epsilon_{\alpha^\prime\beta^\prime\gamma^\prime}, & \quad \lambda_{11} x_5^{\alpha_0} x_4^{\beta_0} x_4^{\gamma_0} x_3^{\alpha} x_4^{\beta} x_4^{\gamma} [0 \epsilon_{\alpha\beta\gamma} \epsilon_{\alpha^\prime\beta^\prime\gamma^\prime}], \\
\lambda_{12} x_5^{\alpha_0} x_5^{\beta_0} x_5^{\gamma_0} x_1 \epsilon_{\alpha\beta\gamma} \epsilon_{\alpha^\prime\beta^\prime\gamma^\prime}, & \quad \lambda_{13} [x_4^{\alpha_0} x_4^{\beta_0} x_4^{\gamma_0} x_1 x_4^{\beta_0} x_4^{\gamma_0} [0 \epsilon_{\alpha\beta\gamma} \epsilon_{\alpha^\prime\beta^\prime\gamma^\prime}], \\
\lambda_{14} x_4^{\alpha_0} x_5^{\beta_0} x_5^{\gamma_0} x_4^{\alpha} x_4^{\beta} x_4^{\gamma}, & \quad \lambda_{15} [x_4^{\alpha_0} x_4^{\beta_0} x_4^{\gamma_0} x_4^{\alpha} x_4^{\beta} x_4^{\gamma} [0 \epsilon_{\alpha\beta\gamma} \epsilon_{\alpha^\prime\beta^\prime\gamma^\prime}], \\
\lambda_{16} x_4^{\alpha_0} x_3^{\beta_0} x_3^{\gamma_0} x_4^{\alpha} x_4^{\beta} x_4^{\gamma}, & \quad \lambda_{17} x_5^{\alpha_0} x_7^{\beta_0} x_7^{\gamma_0} x_1^{\alpha} x_7^{\beta} x_7^{\gamma} [0 \epsilon_{\alpha\beta\gamma} \epsilon_{\alpha^\prime\beta^\prime\gamma^\prime}], \\
\lambda_{18} x_4^{\alpha_0} x_3^{\beta_0} x_3^{\gamma_0} x_4^{\alpha} x_3^{\beta} x_3^{\gamma} & \quad \epsilon_{\alpha\beta\gamma} \epsilon_{\alpha^\prime\beta^\prime\gamma^\prime},
\end{align*}
\]

(6)

and Hermitian conjugation is implied throughout. Models with \(X_2\) and \(X_6\) couple to leptons and quarks of different generations. Only models M1, M2, and M3 can produce \(n - \bar{n}\) oscillations, though these models do not generate all the low-energy effective operators expected if SM gauge symmetry holds \([37, 47, 48]\).

In particular, we find that M1 yields the operator \((O_2)_{RRR}\), M2 yields \((O_1)_{LLR}\) and \((O_2)_{LLR}\) \([47]\), though an operator relation combines these to \((O_3)_{LLR}\) \([48]\) and M3 yields \((O_1)_{RRR}\). An operator of form \((O_3)_{LLR}\) can also appear \([47, 48]\), but it is not generated in the minimal scalar-fermion models we consider.

Only models A, B, and C can produce \(\pi^-\pi^- \rightarrow e^-e^-\) decay, though B and C can also yield a weak isospin triplet of \(|\Delta L| = 2\) processes. These models all correspond to the second case of decay topology “T-II-3” in Ref. \([19]\), as that decomposition considers the scalars’ electric and color charge only. At energies below the \(X_1\) mass scale, model A generates the operator combination \(O_{3+}^{+} - O_{3-}^{-}\), whereas models B and C generate linear combinations of \(O_{2\pm}^{\pm}\) \([25]\).

4. Phenomenology

The models of Table 2 possess a rich array of possible \(|\Delta B| = 2\) and \(|\Delta L| = 2\) processes. They also reveal the possibility of scattering-mediated \(|\Delta B| = 2\) processes, which we term “conversion” modes \([36, 37]\), and we show some of the more experimentally accessible ones in Table 3. As they are mediated by mass dimension 12 operators, they do not break B-L \([49]\). Other models show additional features. Models D and E support \(\pi^-\pi^0 \rightarrow e^-\nu_{\mu,\tau}\) and \(\pi^-\pi^0 \rightarrow \mu^-\nu_e\), whereas F supports \(\mu^- \rightarrow e^-e^+e^-\nu_e\nu_e\) and G supports \(e^+e^- \rightarrow e^+e^-\nu_e\nu_e\). Models M8 and M18 can mediate \(nn \rightarrow \pi^+\pi^+e^-\bar{e}^-\) decay, and finally M17 and M18 can yield \(e^+n \rightarrow \Delta^+\nu_{\mu,\tau}\) and \(e^-n \rightarrow \Delta^+\nu_e\) processes, respectively. We review the existing experimental constraints on the scalars we have considered in Sec. 6.

5. Connecting \(|\Delta B| = 2\) to \(|\Delta L| = 2\) processes with new physics

The scalar-fermion models that yield \(n - \bar{n}\) oscillations can differ in just one scalar from models that generate \(|\Delta L| = 2\) processes and indeed \(0\nu\beta\beta\) decay. We now discuss how an observed pattern of baryon-number-violating conversion modes, all accessible through e-d scattering, can determine both the \(n - \bar{n}\) model and whether such an additional scalar exists. To distinguish the
possibilities, detecting both the appearance of an antinucleon and the electric charge of a final-state charged lepton is necessary. For context, we note that M3 has scalar content $X_7 X_7 X_8$ but A has $X_1 X_8 X_7^\dagger$, that M2 has $X_4 X_4 X_7$ but B has $X_3 X_3 X_7^\dagger$, that M1 has $X_3 X_3 X_7$ but D has $X_5 X_7^\dagger X_2$ — and finally that C has $X_3 X_8 X_1^\dagger$, where the Hermitian conjugate is implied here and henceforth. If $n - \bar{n}$ oscillation occurs, then $e^- n \rightarrow e^- \bar{n}$ can appear also, if the mediating operator is not $(O_1)_R H_R^R [37]$. Thus the latter process acts as a diagnostic of the possible $n - \bar{n}$ model.

Possible patterns of $|\Delta B| = 2$ discovery are shown for the different $n - \bar{n}$ models in Table 4. Model M3 can connect to model A through models M8, containing $X_7 X_7 X_7 X_7^\dagger$, and M10, containing $X_7 X_3 X_3 X_3^\dagger$, though only the latter can be probed through $e - p$ scattering, as shown in Table 3. Consequently, observing a $n - \bar{n}$ oscillation and the process $e^- p \rightarrow e^+ \bar{p}$ in the absence of $e^- n \rightarrow e^- \bar{n}$ and $e^- p \rightarrow \nu X \bar{n}$ would point to model M3 and the existence of $X_1$. With these observations, we then would have experimental evidence for all the new degrees of freedom in model A. Thus model A, with its minimal scalar interaction, should also exist because there would be no reason that it should not. This thinking was promoted by Gell-Mann in the early days of the quark model: that what is not forbidden is compulsory [50]. We can also make a connection to model A by drawing a Feynman diagram for $\pi^- \pi^- \rightarrow e^- e^-$ utilizing the interactions of models M3 and M10; this is illustrated in Fig. 1. However, this suggests that the rate for $\pi^- \pi^- \rightarrow e^- e^-$, although nonzero, would also be vanishingly small. We do not think this latter conclusion is necessary because model A itself is minimal.

Other connections are possible and can be distinguished by the pattern of observables shown in Table 4. Observing a $n - \bar{n}$ oscillation and $e^- n \rightarrow e^- n$ would reveal that either M2 or M1 operate, though the pattern of $|\Delta B| = 2$ $e - p$ processes shown can also discriminate between the three $n - \bar{n}$ models. Model M2 is associated with the interaction $X_4 X_4 X_7$, and model B is associated with $X_3 X_4 X_7^\dagger$. Minimal models with a four-scalar interaction that connect them are M7, with $X_4 X_4 X_4 X_3$, or M18, with $X_4 X_7 X_7 X_1^\dagger$, though only M7 can generate a process with an $e^- p$ or $e^- n$ initial state. Model M7 can give rise to $e^- p \rightarrow e^+ \bar{p}$ and $e^- p \rightarrow \nu \bar{X} \bar{n}$. Note that a Feynman diagram utilizing M2 and M7 can generate model B and $\pi^- \pi^- \rightarrow e^- e^-$. In contrast, model M1 is
associated with $X_5X_5X_7$, and model D, that yields lepton number and flavor violation, is associated with $X_5X_2X_7^\dagger$. Here the minimal four-scalar models are M5, with $X_5X_5X_5X_2$, or M17, with $X_5X_7X_7X_2^\dagger$, though only M5 can give rise to $e^-p \rightarrow \bar{\nu}_{\mu,\tau}\bar{n}$. Here a Feynman diagram utilizing M1 and M5 generates model D and, e.g., $\pi^-\pi^0 \rightarrow e^-\bar{\nu}_{\mu,\tau}$. The two sets of possibilities can be distinguished as follows. If $e^-p \rightarrow \bar{\nu}_X\bar{n}$ and $e^-p \rightarrow e^+\bar{p}$ are both observed, in addition to a $n-\bar{n}$ oscillation, then this would point to the existence of $X_3$ and thus models M2 and B. However, if $e^-p \rightarrow e^+\bar{p}$ is instead absent, this would point to the existence of $X_2$ and thus models M3 and D. Note that the various model possibilities cannot combine to show that only $X_8$ exists, even if the noted $|\Delta B| = 2$ processes are observed, so that we cannot show that model C operates. The observed patterns would establish the existence of $|\Delta L| = 2$ processes from new short-distance physics, but the connections we argue would not exclude the latter possibility if no $|\Delta B| = 2$ processes were observed.

The connections we consider exist regardless of whether the neutrino also has a Dirac mass. Note that if $\nu_R$ fields existed in the low-energy theory, not only could the neutrino have a Dirac mass, but the $X_6$ scalar could also induce
Table 4: Possible patterns of $\Delta B = 2$ discovery and their interpretation in minimal scalar-fermion models. Note that only $n-\bar{n}$ oscillations and $e^-n \rightarrow e^-\bar{n}$ break B-L symmetry and that the pertinent conversion processes can be probed through electron-deuteron scattering. The latter are distinguished by the electric charge of the final-state lepton accompanying nucleon-antineucleon annihilation. Note that the $0\nu\beta\beta$ query refers specifically to the existence of $\pi^-\pi^- \rightarrow e^- e^-$ from new, short-distance physics. Note that we can possibly establish model D and $|\Delta L| = 2$ violation, but that model does not give rise to $\pi^-\pi^- \rightarrow e^- e^-$. In contrast we cannot establish $X_8$ alone and thus cannot establish model C.

| Model | $n\bar{n}$? | $e^- n \rightarrow e^- \bar{n}$? | $e^- p \rightarrow \nu_X \bar{n}$? | $e^- p \rightarrow e^+ \bar{p}$? | $0\nu\beta\beta$? |
|-------|-------------|---------------------------------|---------------------------------|---------------------------------|------------------|
| M3    | Y           | N                               | Y                               | Y [A]                           |
| M2    | Y           | Y                               | Y                               | Y [B]                           |
| M1    | Y           | Y                               | Y                               | N [D]                           |
| -     | N           | N                               | Y                               | Y [C?]                          |

proton decay. Thus this possibility would rule out models M4, M9, M19-M21, but they are not pertinent to our arguments. We also note that independent constraints on $X_7$ and $X_8$ can be had from studies of $K\bar{K}$ and $D\bar{D}$ mixing, respectively. Thus the discovery of new physics in $D\bar{D}$ mixing could also help anchor evidence for Model C and $0\nu\beta\beta$ decay from new short-distance physics.

### 6. Observability

The non-observation of $n-\bar{n}$ oscillations [51, 52] can be interpreted as a limit on the neutron’s Majorana mass of $2 \times 10^{-33}$ GeV at 90% CL [52], with greatly improved sensitivity anticipated at a new experiment proposed for the European Spallation Source [53]. Such limits do not preclude the observation of processes associated with the dimension-12 operators we have considered, because different scalars can have different masses. The scalar self-interactions we consider do not select a particular mass scale; rather, the allowed masses and couplings should be determined from experiment, as in hidden-sector searches [54]. We find that the various $e - p$ processes we have considered should be appreciable if the scalars possess masses of $\mathcal{O}(1 - 10 \text{ GeV})$. Existing collider constraints on color-sextet scalars (of $\mathcal{O}(500 \text{ GeV})$ with $\mathcal{O}(1)$ couplings) come from studies of $t$-quark final states [55–58], and flavor-physics constraints, while more severe, also involve second- and third-generation quark-scalar couplings [4, 59–63]. Thus these constraints are not really pertinent to our case. However, there are also limits specific to scalars that couple to first-generation fermions; here we summarize findings that we plan to report on detail elsewhere [64]. Severe limits on $pp \rightarrow e^+e^+$ in $^{16}$O have recently been reported by the Super-Kamiokande collaboration [65]. Such limits must be interpreted carefully because conventional physics can act to make the spontaneous process impossible, regardless of whether new physics is present. It has been claimed that earlier studies already limit the scalar mass scale to no less than 1.6 TeV [66], though that analysis neglects the role of Coulomb repulsion in the $pp$ initial state. Its inclusion should weaken that bound by orders of magnitude. In addition, $e^- p \rightarrow e^+ \bar{p}$ from K-shell capture in $^{16}$O would not occur spontaneously because only the initial
lepton can be in an atomic bound state. There are also astrophysical limits on hydrogen-antihydrogen \((H - \bar{H})\) oscillation from attributing a measured excess of gamma radiation to the annihilation of \(H\) atoms from \(H - \bar{H}\) oscillations \[67\]. That analysis neglects Galactic magnetic fields, which act to make the energy of \(H\) and \(\bar{H}\) unequal, quenching the oscillation probability. Magnetic fields of about 1 nT have been established in cold, HI clouds \[68\], and magnetic fields of no less than 0.1 nT exist in the warm interstellar medium \[69\]. Thus we believe that cold, HI regions continue to drive the assessed \(H - \bar{H}\) oscillation limit as estimated in Ref. \[67\]. Computing the \(H - \bar{H}\) energy splitting, we estimate the oscillation limit to be weakened by a factor of \(10^8\). Collider searches for events with same-sign dileptons and multiple jets at the center-mass energies of \(\sqrt{s} = 7, 8,\) and 13 TeV have been performed by the CMS collaboration \[70–72\]. Due to backgrounds from \(b\)-hadron decays, they reject same-sign dilepton events with an invariant mass of less than 8 GeV \[70\]. Thus, these collider constraints do not exclude possibility of models with scalars that couple to dileptons with masses that are less than 8 GeV. With these various refinements in place we believe that scalars with masses of \(O(1 - 10 GeV)\) are a viable possibility.

Models that support \(e^- p \rightarrow e^+ \bar{p}\) have low-energy operators whose quark parts correspond to those found in \(n - \bar{n}\) oscillations under \(u \leftrightarrow d\) exchange. Exploiting this and a MIT bag model \[73, 74\] computation of \(\langle \bar{n} | (\mathcal{O}_{1,2})_{LLL} | n \rangle\) \[47, 75\] yields

\[
\sigma \sim 1.5 \times 10^{-4} |g_4^{11}|^6 |\lambda_7|^2 |g_5^{11}|^2 \left( \frac{5 \text{ GeV}}{M_{X_4}} \right)^{12} \left( \frac{1 \text{ GeV}}{M_{X_3}} \right)^4 \text{ab} \tag{7}
\]

in model M7 for an electron beam energy of 155 MeV with a fixed target \[76\]. Model M7 contains scalars distinct from those that generate \(n - \bar{n}\) oscillations, and existing phenomenological analyses allow scalars in the \(O(1 - 10 \text{ GeV})\) mass range to appear. The experimental searches we propose, given Eq. (7) and the established accelerator and target capacities we have collected in Ref. \[37\], can discover or constrain them.

7. Summary

We have considered different physical processes that could reveal \(|\Delta B| = 2\) violation, both \(n - \bar{n}\) oscillation and conversion, and we have considered their interrelationships within minimal scalar-fermion models that support \(|\Delta B| = 2\) processes without proton decay. To realize this we have extended the models of Ref. \[4\] to include all possible minimal-scalar models that satisfy SM gauge invariance. Three distinct scalars are required to realize neutrinoless double \(\beta\) decay in these models, and Ref. \[4\] considered no more than two distinct scalars. Moreover, we have shown how the patterns of observation of particular \(|\Delta B| = 2\) processes would speak to the existence of particular new scalars within these models, and we have employed Gell-Mann’s totalitarian principle \[50\] to invoke the new combination of these scalars needed to predict the existence of \(\pi^- \pi^- \rightarrow e^- e^-\) and thus of neutrinoless double \(\beta\) decay, though the latter
connection also follows from a Feynman diagram approach once the particular $|\Delta B| = 2$ processes are observed. Thus, finally, we conclude that the observation of particular $|\Delta B| = 2$ processes could be used to infer the existence of a $|\Delta L| = 2$ process, $0\nu\beta\beta$ decay in nuclei, speaking to the Majorana nature of the neutrino and to new dynamics at accessible energy scales.

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