Statistical Tools and Methodologies for URLLC - A Tutorial
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Abstract—Ultra-reliable low-latency communication (URLLC) constitutes a key service class of the fifth generation and beyond cellular networks. Notably, designing and supporting URLLC poses a herculean task due to the fundamental need of identifying and accurately characterizing the underlying statistical models in which the system operates, e.g., interference statistics, channel conditions, and the behavior of protocols. In general, multi-layer end-to-end approaches considering all the potential delay and error sources and proper statistical tools and methodologies are inevitably required for providing strong reliability and latency guarantees. This paper contributes to the body of knowledge in the latter aspect by providing a tutorial on several statistical tools and methodologies that are useful for designing and analyzing URLLC systems. Specifically, we overview the frameworks related to (i) reliability theory, (ii) short packet communications, (iii) inequalities, distribution bounds, tail approximations, and risk-assessment tools, (iv) rare events simulation, (v) large-scale tools such as stochastic geometry, clustering, compressed sensing, and mean-field games, (vi) queuing theory and information freshness, and (vii) machine learning. Throughout the paper, we briefly review the state-of-the-art works using the addressed tools and methodologies, and their link to URLLC systems. Moreover, we discuss novel application examples focused on physical and medium access control layers. Finally, key research challenges and directions are highlighted to elucidate how URLLC analysis/design research may evolve in the coming years.

Index Terms—age of information, clustering, compressed sensing, extreme value theory, finite blocklength, inequalities and distribution bounds, machine learning, meta distribution, queuing theory, rare event simulations, reliability theory, risk assessment, URLLC.

I. INTRODUCTION

Ultra-reliable low-latency communication (URLLC) aims to support wireless connectivity with wire-grade reliability and latency performance, and constitutes a key service class of the fifth generation (5G) and beyond cellular networks. The potential applications of URLLC, also referred to as critical machine-type communication (MTC), are indeed numerous and include smart grid, professional audio, intelligent transport systems, industrial and process automation, augmented reality, entertainment industry, e-health, Tactile Internet, as well as yet many unforeseen use cases [11–13].

In URLLC, reliability is often defined as the probability that a data unit is successfully transferred within a certain time period[1]. Noteworthy, although reliability and latency are inherently tied by definition, the technological enablers of URLLC can be often classified as delay-reducing or reliability-promoting [12]. For instance, latency figures can be mainly improved by leveraging short codes and transmission time intervals [11], [5], [13], [14], edge caching/computing/slicing [2], [6], [12], [15], limited-overhead protocols [2], [4], [16], [18], and non-orthogonal multiple-access (NOMA) [2], [5], [19], [20]. Meanwhile, high reliability levels can be attained by exploiting diversity techniques as in multi-connectivity [4], [5], [21–24], data replication [17], [25], automatic repeat request protocols [5], [26], and multiple-input multiple-output (MIMO) systems [4], [5], [18], [27], as well as efficient network coding and relaying [14], [28], [30] and space-time block codes [12], [13], [27]. A robust URLLC design may inevitably exploit several of the above technologies so that both latency and reliability metrics perform at the desired levels.

Remarkably, designing and supporting URLLC poses a herculean task due to the fundamental need of identifying and accurately characterizing the underlying statistical models in which the URLLC system operates, e.g., interference statistics, channel conditions, the behavior of protocols, etc [3], [4], as this is required for ultimately providing strong quality-of-service (QoS) guarantees. This inevitably requires multi-layer end-to-end approaches considering all the potential delay and error sources [3], [4], [27] and proper statistical tools and methodologies [12], [13], [31–33]. This paper contributes to the body of knowledge in the latter aspect by providing a tutorial on several statistical tools and methodologies that are useful for designing and analyzing URLLC applications.
URLLC systems. We specifically focus on physical and medium access control layer perspectives.

A. State-of-the-Art Tutorials

There are already some survey, tutorial, and overview papers on tools and methodologies for designing and analyzing URLLC systems in the literature. Table I collects those that cover at least two tools/methodologies and summarizes their main contributions. Next, we discuss the scope and contributions of such works in more detail.

In 2016, Durisi et al. [14] overviewed the main theoretical principles for the design and performance analysis of short packet communications, which are a key component in URLLC. Specifically, they delved into the structure of a data packet, finite blocklength (FBL) error characterization, and proper communication protocols. They discussed three exemplary scenarios, i.e., i) the two-way channel, ii) the downlink broadcast channel, and iii) the uplink random access channel, and illustrated how the transmission of control information can be optimized for short packets. The main conveyed message was that it is crucial to consider all the communication resources that are invested in the transmission of metadata when data messages are short.

In 2018, Benis et al. [12] discussed definitions of latency and reliability, examined various enablers of URLLC and their inherent tradeoffs, and overviewed a wide variety of techniques and methodologies for URLLC. The latter included risk analysis and optimization tools, extreme value theory (EVT), effective bandwidth, stochastic network calculus (SNC), the meta distribution, and mean-field (MF) game theory. Moreover, via four selected use cases related to i) ultra-reliable millimeter-wave communication, ii) virtual reality, iii) mobile edge computing, and iv) multi-connectivity for ultra dense networks, the authors showed how such tools provide a principled and clean-slate framework for modeling and optimizing URLLC-centric problems at the network level. The authors agreed that URLLC mandates a departure from expected utility-based approaches relying on average quantities to taming risk and distribution tails.

In 2019, Zhang et al. [31] reviewed the state-of-the-art of compressed sensing-based secure wireless communications. For this, they first studied the security aspects of compressed sensing according to different types of random measurement matrices such as Gaussian matrix, circulant matrix, and other special random matrices. Meanwhile, applications such as the wireless wiretap channel, wireless sensor network, Internet of Things, crowdsensing, smart grid, and wireless body area networks were thoroughly discussed immediately after.

In 2020, Yang et al. [32] investigated the challenges and solutions for enabling URLLC for vehicular networks. Specifically, the underlying application scenarios exploiting vehicle-to-vehicle/infrastructure/network/pedestrian communications were overviewed, together with the corresponding URLLC requirements and potential challenges. Moreover, the authors highlighted some frameworks for optimizing latency and/or reliability and discussed a few case studies. The optimization frameworks in question were related to FBL coding theory at the physical layer, and effective bandwidth/capacity, SNC, Lyapunov optimization, and Markov decision process at the medium access control layer.

In 2021, She et al. [33] illustrated how to integrate domain knowledge (models, analytical tools, and optimization frameworks) of communications and networking into deep learning (DL) algorithms for URLLCs. For this, they first overviewed URLLCs together with promising architectures and DL frameworks for the sixth-generation (6G) wireless networks. Then, they revisited model-based analytical tools and cross-layer optimization frameworks, including FBL coding theory, queuing theory, and stochastic geometry for delay analysis, and hinted at how to improve learning algorithms with such knowledge. Finally, they validated the effectiveness of several learning algorithms via simulation.

| Ref. | Main Contributions | Tools/Methodologies |
|------|--------------------|---------------------|
| [14] | Overview of FBL theoretical and protoclar principles | i) FBL Coding Theory, ii) Short Packet Protocols |
| [12] | Discussion about URLLC enablers and overview of techniques and methodologies for the design of URLLC systems | i) Risk Analysis/Optimization Tools, ii) EVT, iii) Effective Bandwidth, iv) SNC, v) Meta Distribution, vi) MF Game Theory |
| [31] | Revision of the security aspects of compressed sensing and applications to secure wireless communications | i) Compressed Sensing, ii) Security Tools |
| [32] | Discussion of requirements and challenges in URLLC vehicular networks, and overview of frameworks to enable them | i) FBL Coding Theory, ii) Effective Bandwidth, iii) SNC, iv) Lyapunov Optimization, v) Markov Decision Process |
| [33] | Review promising network architectures and DL frameworks for 6G URLLC | i) DL Frameworks, ii) FBL Coding Theory, iii) Queuing Theory Tools, iv) Stochastic Geometry for Delay Analysis |

Note that for a thorough understanding of a specific tool or methodology, the best approach might be to consult a book or a comprehensive tutorial paper on the matter.

TABLE I
Representative Survey/Overview/Tutorial Papers Related to Tools and Methodologies for Designing/Analyzing URLLC Wireless Systems
and experimental results.

B. Our Approach and Contributions

It is worth noting that the state-of-the-art tutorials covering tools and methodologies for the analysis and design of URLLC systems have mostly focused on a few approaches only. In some cases, the main contributions are not even around them, e.g., [31], [32]. Therefore, our approach here is more exhaustive and fully focused on tools and methodologies that seem appealing for the analysis and design of URLLC systems.

Last row of Table I collects the 15 specific tools and methodologies that we cover in this tutorial. Observe that some of them have been already discussed in prior tutorials, e.g., FBL coding theory in [14], [32], [33], short packet protocols in [13], EVT in [12], risk-assessment tools in [12], meta distribution in [12], [33], compressed sensing in [31], effective capacity/bandwidth and SNC in [12], [32], machine learning (ML) in [33], and MF game theory in [12]. For these, we provide fresh and intuitive discussions, in many cases accompanied by novel examples to propitiate a better understanding of the approaches and their, not always evident, link to URLLC design/analysis. Meanwhile, some other tools and methodologies that we discuss here have not been covered in the state-of-the-art tutorials. That is the case of reliability theory, inequalities and distribution bounds, rare events simulation tools, clustering, age of information (AoI), and time series prediction tools. Therefore, our contributions in this regard are even stronger and more evident, and comprise an overview of the tool/methodology, its potential for URLLC design/analysis, and novel examples. In some cases, we also identify convenient links among the different approaches, i.e., by highlighting how a certain tool/methodology can be leveraged by another one to (often more efficiently) meet the desired goals. Additionally, throughout the paper, we briefly review the state-of-the-art works using the addressed tools/methodologies, especially when they link them to URLLC systems, and identify key challenges. Finally, key research directions are highlighted, which might hint at how URLLC analysis/design research may evolve in the coming years.

C. Organization

In the following, Sections II-VIII overview the specific tools and methodologies for the design and/or analysis of URLLC systems as illustrated in Fig. 1 while providing examples and fresh views. Section IX concludes the paper and discusses interesting challenges and research directions. The acronyms and main notations used throughout this article are listed in Table II and Table III respectively.

| Table II | Acronyms and Corresponding Definitions |
|----------|---------------------------------------|
| Acronym  | Definition                            |
| #G       | #--th generation of cellular networks |
| 3GPP     | third generation partnership project  |
| AMP      | approximate message-passing           |
| AoI      | age of information                    |
| AWGN     | additive white Gaussian noise          |
| BLER     | block error rate                      |
| bpcu     | bits per channel use                  |
| CDF      | cumulative density function           |
| CSI      | channel state information             |
| CVaR     | conditional VaR                       |
| DL       | deep learning                         |
| EVT      | extreme value theory                  |
| FBL      | finite blocklength                    |
| FL       | federated learning                    |
| GPD      | generalized Pareto distribution       |
| LOS      | line of sight                         |
| LSTM     | long short-term memory                |
| MBB      | mobile broadband                      |
| MC       | Monte Carlo                            |
| MDO      | minimum duration outage               |
| MDT      | mean downtime                         |
| MF       | mean-field                            |
| MGF      | moment generating function            |
| MIMO     | multiple-input multiple-output        |
| ML       | machine learning                      |
| MTC      | machine-type communication             |
| MTTF     | mean time to first failure            |
| MUT      | mean uptime                           |
| NOMA     | non-orthogonal multiple access        |
| PDF      | probability density function          |
| QoS      | quality-of-service                    |
| QPSK     | quadrature phase shift keying         |
| RBD      | reliability block diagram             |
| RL       | reinforcement learning                |
| RV       | random variable                       |
| SINR     | signal-to-interference-plus-noise ratio|
| SNC      | stochastic network calculus           |
| SNR      | signal-to-noise ratio                 |
| SS       | subset simulation                     |
| UAV      | unmanned aerial vehicle               |
| URLLC    | ultra-reliable low-latency communication|
| VaR      | value-at-risk                         |
TABLE III
Notations and Corresponding Descriptions

| Notation | Description |
|----------|-------------|
| $f_X(x)$ | probability density function (PDF) of random variable (RV) $X$ |
| $f(x,y)$ | joint PDF of RVs $X$ and $Y$ |
| $F_X(x)$ | cumulative density function (CDF) of RV $X$ |
| $F_X(x)$ | complementary CDF of RV $X$ |
| $P(A)$ | probability of event $A$ occurrence |
| $A|B$ | RV $A$ conditioned on the realization of RV $B$ |
| $E(X|\cdot)$ | expected value of the argument w.r.t RV $X$ |
| $\sigma$ | binomial coefficient |
| $\Gamma(\cdot)$ | complete gamma function |
| $\Gamma(\cdot,\cdot)$ | upper incomplete gamma function |
| $\log(\cdot)$ | logarithmic integral function |
| $\gamma$ | Euler–Mascheroni constant $(0.577215664901532\ldots)$ |
| $Q(\cdot)$ | Gaussian Q function |
| $I_v(\cdot)$ | modified Bessel function of the first kind and order $v$ |
| $K_v(\cdot)$ | modified Bessel function of the second kind and order $v$ |
| $\Theta(\cdot)$ | big-O notation |
| $\text{sgn}(\cdot)$ | sign/signum function |
| $\odot$ | exclusive OR operator |
| $\sim$ | distributed as |
| $\exp(\lambda)$ | exponential distribution with mean $1/\lambda$ |
| $G(\alpha,\beta)$ | gamma distribution with shape $\alpha$ and scale $\beta$ |
| $\ll, \gg$ | much less/greater than |
| $T_{-,H}$ | transpose, Hermitian transpose operation |
| $\mathbb{C}^{N \times M}$ | domain of $N \times M$ complex matrices |
| $|\cdot|$ | absolute value |
| $\|\cdot\|_f$ | $f_\infty$-norm |
| $\inf$ | infimum operation |
| $a \in b$ | $a$ approaches (converges to) $b$, or domain-image mapping |
| $\mathbb{R}$, $\mathbb{R}^+$ | real, and non-negative real domain |
| $\mathbb{I}(\cdot)$ | indicator function: 1 (0) if the input is true (false) |

II. RELIABILITY THEORY

Reliability theory comprises a set of mathematical methods for analyzing the life cycles and failures of technical systems. Systems, subsystems, and components, hereinafter referred to as “items”, can be classified as repairable or non-repairable. In case of the latter, it does not matter what happens after the first failure occurs, i.e., the item might be discarded or even repaired, while all the operational and failed cycles matter in the case of repairable systems. Notice that a wireless communication system can be composed of several items, e.g., transmitters, receivers, and links/channels, all of which might constitute sources of failure. Next, we overview key aspects of the reliability theory framework and exemplify their potential application to URLLC analysis/design.

A. Key Dependability Quantities

Here, we overview key dependability quantities within the reliability theory framework. Such quantities are expressed by probabilities and time durations. A common condition is that the considered item is operational at time $t = 0$. By convention, “up” and “down” respectively refer to operational and failure (i.e., in repair if repairable) states.

1) Availability: The instantaneous availability, or point availability, of an item is defined as

$$A(t) = \Pr[\text{“item is up at time } t\text{”}] .$$

Meanwhile, the steady-state availability is defined as

$$A = \lim_{t \to \infty} A(t),$$

and allows characterizing the long-term probability that an item is available. Observe that (2) converges to the fraction of the mission time in which the item remains in the “up” state.

2) Reliability: Reliability refers to the probability of a failure-free operation of the item during the interval $[0, t]$, i.e.,

$$R(t) = \Pr[\text{“item is up throughout the interval } [0, t]\text{”}].$$

Here, $R(t)$ is also referred as survival function. Meanwhile, the amount of time elapsed before a failure occurs can be measured/characterized by the time to failure $R T$, and notice that

$$R(t) = 1 - F_T(t), \quad t \geq 0.$$  

Notice that reliability is usually the measure of interest for non-repairable systems, whereas for repairable systems, the probability that the item remains available is preferred. In general, $A(t) \geq R(t)$, while $\lim_{t \to \infty} R(t) = 0$ and $\lim_{t \to \infty} A(t) = A \geq 0$. In both cases, the equality holds only in the special case of no repairs.

3) Mean downtime/uptime: The mean downtime (MDT) and mean uptime (MUT) constitute the mean duration of a system failure and mean system operational time, respectively. Notice that $A = MUT/(MUT + MDT)$.

$$\text{MTTFF} = \int_0^\infty R(t) dt.$$  

Notice that this metric is equivalent to the mean time to failure and MUT, and also to the mean time between failures in memoryless settings. Nevertheless, the mean time to failure and mean time between failures are only conceptually applicable to non-repairable and repairable systems, respectively.

4) Mean time to first failure (MTTFF): MTTFF characterizes the average duration of an item in “up” state before the failure occurs, thus, it can be written as

$$\text{MTTFF} = \int_0^\infty R(t) dt.$$  

Moreover, as shown in [35], the probability that the item fails within the (short) time interval $\Delta t$ after being operational until time $t$ is approximately $\lambda(t)\Delta t$.

Failure rates of the form $K t^m$ with $m > -1$ are more common in practical systems [36, 37] and correspond...
to a Weibull-distributed time to failure. Interestingly, the exponential distribution (a Weibull distribution with $m = 0$) is the only continuous distribution with constant failure rate, i.e., $\lambda(t) = \lambda$, thus, memoryless. This implies that the distribution of the additional system lifetime does not depend on its age.

6) Maintainability: Maintainability refers to the ability of repairing a system within a given period of time. Notice that if the repair time follows an exponential distribution, it is possible to define the repair PDF and CDF based on a single parameter, the repair rate, $\mu$.

7) Importance measures: Importance measures are not dependability quantities per se, but rather they evaluate absolutely or relatively the contribution of the elements (e.g., failures modes, items, or events) to a dependability quantity of interest. Some examples are the Birnbaum importance, criticality importance measure, risk achievement worth, risk reduction worth, Fussell-Vesely, and differential importance measure. Since their potential benefits for URLLC design/optimization/analysis are unclear (have not yet been explored) and they seem more relevant in systems design and industrial engineering applications, we do not delve into them here. The interested reader can refer to [34] for more details.

B. Structure Functions

Assume a system is composed of $n$ items, and let the vector $x = [x_1, x_2, \ldots, x_n]^T$ indicate which of the items are functioning and which have failed. Mathematically,

$$x_i = \begin{cases} 
1, & \text{if the } i\text{-th item is "up"} \\
0, & \text{if the } i\text{-th item is "down"}
\end{cases} \quad (7)$$

The structure function of the system, denoted as $\phi(x)$, captures the operability of the whole system given a state vector $x$ as

$$\phi(x) = \begin{cases} 
1, & \text{if the system is "up" given } x \\
0, & \text{if the system is "down" given } x
\end{cases} \quad (8)$$

Meanwhile, the probability of the system being “up” or available can be written as $A = \Pr[\phi(x) = 1] = \mathbb{E}_x[\phi(x)]$. Notice that $A$ is solely a function of $p = [p_1, p_2, \ldots, p_n]^T$, where $p_i \triangleq \Pr[x_i = 1]$, when all the items’ states are independent. In such case, $A(p)$ is an increasing function of $p$, and $A(p^a) \geq A(p)^a$ with $a \in [0, 1]$ [39] Ch. 9).

A system, as a composition of several items, can be assessed via a fault tree analysis or a reliability block diagram (RBD). The former is a graphical tool used mainly for system-level risk assessment and mitigation purposes, while the latter illustrates, in a simple manner, how component reliability contributes to the success or failure of a system. Both can be used to model and analyze similar types of logical configurations required for system reliability and related analyzes, although RBDs are usually more mathematically abstracted and can be handled more straightforwardly. Here, we focus only on RBDs, while the interested reader can refer to [40], [41] for further details on fault tree analysis.

The way in which the items are interconnected determines the redundancy level of the whole system. Mathematically, a $k$-out-of-$n$ structure models a system that is functioning if and only if at least $k$ of the $n$ items are “up” [42], i.e.,

$$\phi_{k,n}(x) = \begin{cases} 
1, & \text{if } 1^T x \geq k \\
0, & \text{otherwise}
\end{cases} \quad (9)$$

There are two special systems: i) a parallel or full redundancy system, which corresponds to a 1-out-of-$n$ structure, and ii) a series or non-redundancy system, which corresponds to an $n$-out-of-$n$ structure. Their RBDs are illustrated in Fig. 2 while their structure functions are given by

$$\phi_{1,n}(x) = \max(x_1, x_2, \ldots, x_n) = 1 - \prod_{i=1}^{n}(1 - x_i), \quad (10)$$

$$\phi_{n,n}(x) = \min(x_1, x_2, \ldots, x_n) = \prod_{i=1}^{n} x_i. \quad (11)$$

Fig. 2. RBD for the two special systems, and Markov modeling for non-repairable and repairable systems with two items. Here, $\lambda_i$ and $\mu_i$ respectively denote the failure and repair rate of item $i \in \{1,2\}$. 
Moreover, $A_{1,n}(p) = 1 - \prod_{i=1}^{n} (1 - p_i)$ and $A_{n,1}(p) = \prod_{i=1}^{n} p_i$ in case of independent items’ states. Notice that any general structure (and corresponding structure function) can be scrutinized as a combination of series and parallel structures, thus, they may be seen as canonical structures.

**Example 1 (RBD Analysis of a Relaying System).**

Consider the wireless link between a source and a destination node is obstructed, thus, a potential direct communication between these nodes is discarded. Instead, the source sends a data packet to the destination via a wireless network composed of two relays as illustrated in Fig. 3a. There are five active wireless links in total, and each link $i \in \{1,2,3,4,5\}$ is in state $x_i$ at a given time instant. Notice that the operation of a wireless link, thus, the realizations of $|x_i|$ are commonly affected in practice by, e.g., the fading, shadowing, path-loss, and interference phenomena.

Based on Fig. 3a, one can straightforwardly draw the structure representation of the system, followed by its series-parallel equivalent representation, as shown respectively in Fig. 3b and Fig. 3c. Based on the latter, one can write the structure function of such a hybrid system as shown in (12) (at the top of the next page), where the last step follows from simple algebraic transformations, including $x_5^3 = x_1$. Meanwhile, assuming independent items, the availability of the system can be expressed as

$$A_{\text{hyb}}(p) = E_x[\Phi_{\text{hyb}}(x)]$$

$$= p_1 p_2 p_4 p_5 + p_1 p_3 p_5 + p_2 p_3 p_4 + p_2 p_4 p_5$$

$$- p_1 p_4 p_5 - p_1 p_2 p_5 - p_1 p_3 p_4 p_5 - p_1 p_2 p_4 p_5$$

$$- p_2 p_3 p_4 p_5 - p_1 p_2 p_3 p_4 + 2 p_1 p_2 p_3 p_4 p_5.$$  \hspace{1cm} (13)

Notice that by exploiting the methodological approach discussed earlier in this section, one has rigorously and straightforwardly derived the structure and availability functions of the example scenario.

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**C. Markov Models**

RBDs and fault tree analysis are essentially static and Boolean (there are only two possible states: “up” and “down”) tools. For more complex systems, one may resort to Markovian analysis, which is essentially dynamic and can include several component states, thus allowing to capture a variety of important aspects of the temporal and sequential dependence among events [43]. The interested reader can refer to [44] for a detailed analysis and mathematical characterization of Markov models.

To formulate a Markov model, the system behavior is abstracted into a set of mutually exclusive system states, and a set of equations, describing the probabilistic transitions among states and initial state probability distributions, is set. Moreover, the transition from one state to another depends only on the current state, thus, the way in which the entire past history affects the future of the process is completely summarized in the current state of the process. A Markov model can be defined over discrete (either finite or countable infinite) or continuous state spaces. The former case is commonly known as Markov chain, whereas the latter is defined as a continuous-space Markov model. Regarding the time granularity of the state transitions, Markov models are classified as continuous-time or discrete-time if they allow transitions at any time or at fixed intervals, respectively. Next, we discuss and exemplify key principles and methodological aspects of Markov modeling within the reliability theory framework.

In Markov models for repairable systems, the repairs are represented by cycles depicting the loss and the restoration of the functionality of an item. Meanwhile, the Markov models for non-repairable systems are acyclic. As an example, the last two blocks in Fig. 2 illustrates the Markov models for the series and parallel repairable and non-repairable systems with two items.

Table [IV] summarizes the main advantages and disadvantages of Markov modeling within the reliability theory framework [45]. Notice that Markov modeling should not be used when: i) the system can be well-modeled with simpler combinatorial methods, e.g., RBDs and fault tree analysis,
\( \phi_{\text{hyb}}(x) \)

\[ = \phi_{4,4} \left( \left[ \phi_{1,2}(\{x_1, x_4\}^T), \phi_{1,3}(\{x_1, x_3, x_5\}^T), \phi_{1,2}(\{x_2, x_3\}^T), \phi_{1,3}(\{x_2, x_3, x_4\}^T) \right]^T \right) \]

\[ = \min \{ \max \{x_1, x_4\}, \max \{x_1, x_3, x_5\}, \max \{x_2, x_3\}, \max \{x_2, x_3, x_4\} \} \]

\[ = \min \{ 1 - \frac{1}{1 - x_1} (1 - x_4), 1 - \frac{1}{1 - x_1} (1 - x_3), 1 - \frac{1}{1 - x_2} (1 - x_3), 1 - \frac{1}{1 - x_2} (1 - x_3) (1 - x_4) \} \]

\[ = \min \left\{ x_1 + x_4 - x_1 x_4, x_1 + x_3 + x_3 - x_1 x_3 - x_3 x_5 - x_1 x_3 - x_1 x_3 x_5, x_2 + x_5 - x_2 x_5, x_2 + x_3 + x_3 - x_2 x_3 - x_3 x_4 - x_2 x_3 + x_3 x_4 \right\} \]

\[ = x_1 x_2 x_3 + x_3 + x_3 x_4 + x_2 x_4 x_5 - x_1 x_3 x_5 - x_1 x_2 x_5 - x_3 x_4 - x_1 x_3 x_4 + x_2 x_3 x_4 + x_3 x_4 + 2 x_1 x_2 x_3 x_4 x_5 \] (12)

ii) modeling the system requires a very large number of states; iii) system behavior is too detailed or complex to be expressed in a Markov/semi-Markov model, or iv) a detailed performance estimate is required. In the case of the latter three, simulation tools are usually preferred.

Example 2 (Markov-Enabled Reliability Analysis).
Consider that a Rayleigh-faded receive signal can be successfully decoded if the instantaneous signal-to-noise ratio (SNR) at time \( t \), \( \gamma(t) \), is above a certain threshold, \( \gamma_{th} \). Thus, there are two states, i.e.,

\[ x(t) = \begin{cases} 0, & \text{if } \gamma(t) < \gamma_{th} \rightarrow \text{"down"} \\ 1, & \text{if } \gamma(t) \geq \gamma_{th} \rightarrow \text{"up"} \end{cases} . \] (14)

According to the Gilbert-Elliott model [46, 47], one can interpret the wireless channel as a repairable item and model it as a continuous-time Markov chain of two states. The corresponding failure rate and repair rate are respectively given by [48]

\[ \lambda = \sqrt{\frac{2 \pi \gamma_{th} f_D}{\bar{\gamma}}} \quad \mu = \frac{\lambda}{\exp(\gamma_{th} / \bar{\gamma}) - 1} , \] (15)

and are obtained based on a level crossing analysis [49]. In [15], \( \bar{\gamma} \) is the average SNR over time, and \( f_D \) is the maximum Doppler frequency. Notice that the rates \( \lambda \) and \( \mu \) are constants, which is consistent with the exponential distribution of \( \gamma(t) \) since the channel is subject to Rayleigh fading.

Meanwhile, the steady state probabilities are given by

\[ P_0 = \Pr[x(t) = 0] = \frac{\lambda}{\lambda + \mu} , \quad P_1 = \Pr[x(t) = 1] = \frac{\mu}{\lambda + \mu} . \] (16)

They are obtained by solving \( \{P_0, P_1\} M = 0, P_1 + P_2 = 1 \), where

\[ M = \begin{bmatrix} -\mu / \lambda & \mu / \lambda \\ \lambda & -\lambda \end{bmatrix} \] (17)

is the system transition matrix. With all this at hand, one obtains

\[ A(t) = A = P_1 = \frac{\mu}{\lambda + \mu} , \] (18a)

\[ R(t) = \exp(-\lambda t) , \] (18b)

\[ \text{MTTF} = \text{MUT} = \frac{1}{\lambda} , \] (18c)

\[ \text{MDT} = \frac{\mu}{\lambda} . \] (18d)

Notice that \( 1 - A = \frac{\lambda}{\lambda + \mu} = 1 - \exp(-\gamma_{th} / \bar{\gamma}) \), which, as expected, matches the so-called outage probability or packet loss rate in wireless communications.

Some comments are in order regarding the previous example:

- In URLLC systems with short packet transmissions, there is no such a threshold \( \gamma_{th} \) that fully guarantees a failure-free system operation. The reasons are discussed in Section VII. Instead, an appealing approach that is leveraged in [23] relies on defining the system “up” state as the state in which the failure-free operation is guaranteed with a certain (potentially high) confidence level. By setting a target confidence level, the value of \( \gamma_{th} \) can be computed using [53] in Section VII.

- There is no system redundancy. The interested reader can refer to [50], [51], where the authors assume there are \( n \geq 1 \) links and the system is operational if at least one of the links is “up”, i.e., redundancy via selection combining. In the case of [50], this leads to a birth-death continuous-time Markov chain, while the Markov chain is more evolved in [51] since the authors assume Rician fading and incorporate another cause of failure, interference. The analysis in [51] is facilitated by a hybrid series-parallel structure representation of the system (see Section VII).

- One may wonder how the dependability metrics in [18d] are really relevant in a URLLC context. In the case of the steady-state availability, \( A \), there is not much more to say as this quantity (or its complement, the outage probability \( 1 - A \)) has been the most commonly adopted for assessing the performance of URLLC systems in the literature. In the case of the instantaneous availability and the reliability quanti-
ties, the main attraction lies in their time-dependency feature, specially in systems with non-constant failure rate. Finally, although MTTFF, MUT and MDT are quantities comprising averaging over time, which is not per se appealing in a URLLC context, they provide an overarching view of the system in terms of average times in “up”, “down” and transition states. Maybe more importantly, they can facilitate more rigorous reliability analysis as we illustrate in the following example.

**Example 3 (Minimum Duration Outage).**

Consider the same system setup as in Example 2. Moreover, assume that such wireless system can tolerate error bursts of $u$ symbols by implementing an appropriate channel coding mechanism. The symbol duration is denoted by $T_s$. Herein, we are interested in determining the SNR margin, defined as $F = \gamma / \gamma_{dB}$, such that the packet loss rate does not exceed the value of $\xi < 1$.

Obviously, $uT_s$ must be greater than MDT, but by how much? The notion of minimum duration outage (MDO) introduced in [52] addresses this kind of problems. Specifically, a MDO occurs only when the signal level remains below a pre-specified threshold for a certain minimum duration. Mathematically, the probability of MDOs is given by

$$P_{MDO} = \Pr[\tau > uT_s | x(t) = 0] \Pr[x(t) = 0],$$

where $\tau$ is a RV defined as the duration of a fade. Observe that this formulation neglects the possibility that a given packet experiences more than one deep fade event. Nevertheless, this would extremely rarely occur for $\xi \ll 1$, thus, we can safely adopt it. Then, one must configure $F$ such that $P_{MDO} \leq \xi$.

Since $\Pr[x(t) = 0]$ can be easily computed as $1 - A = \lambda / (\lambda + \mu)$, the only inconvenience for evaluating (19) lies in finding the distribution of $\tau$ [53]. Herein, we adopt a much simpler approach that exploits the already available MDT quantity. Specifically, based on the simplest form of Markov’s inequality (see first row of Table VII in Section III), one obtains

$$\Pr[\tau > uT_s | x(t) = 0] \leq \frac{\mathbb{E}[\tau] = \frac{\text{MDT}}{uT_s}}{uT_s}.$$

By substituting (20) into (19), while exploiting (16d), we have that

$$P_{MDO} \leq \frac{\text{MDT}}{uT_s} (1 - A) = \frac{\lambda}{uT_s \mu (\lambda + \mu)} = \frac{2F \cosh(1/F) - 1}{uT_s f_D}. \quad (21)$$

As expected, the upper bound of $P_{MDO}$ is a decreasing function of $F$, and $\lim_{F \to \infty} P_{MDO}(F) = 0$. This means that there is a unique solution $F^*$ for the equation

$$\sqrt{\frac{2F \cosh(1/F) - 1}{uT_s f_D}} = \xi,$$  

which, in fact, constitutes our solution for the original problem. Finally, we can obtain a closed-form solution for $F^*$ for the high-SNR regime. Specifically, by using $\cosh(1/F) \approx 1 + 1/(2F^2)$, which comes from the Taylor series expansion, one obtains

$$F^* \approx \frac{1}{\sqrt{2\pi \xi^2 u^2 T_s^2 f_D^2}}. \quad (23)$$

Fig. 4 shows $F^*$ as a function of $\xi$ for a certain system configuration, while confirming the high accuracy of $F^*$ driven by the relatively large values of $F^*$. Notice that the adopted upper-bound in (21) is only relevant when $\text{MDT} / (uT_s) < 1$ since $P_{MDO} < 1 - A$ must hold. Therefore, as shown in the figure, it can be only used when $\xi < 2 \times 10^{-3}$.

According to the figure, $F = 35.5$ dB and $F = 55.5$ dB can respectively guarantee a MDO probability below $10^{-4}$ and $10^{-7}$. Meanwhile, in terms of outage probability, which is agnostic of the channel coding correcting errors potentialities, the required SNR margin values would be $F = 40$ dB and $F = 70$ dB, respectively.

### III. Taming Distribution Tails

URLLC is about limiting the occurrence of extreme and rare events. Thus, a proper URLLC design relies ultimately on taming the tail distribution of latency and/or reliability system performance. Next, we overview key bounds, approximations, and risk-assessment tools for taming distributions’ tails.

#### A. Bounds & Limiting Forms

Assume $X$ is a RV and $X \leq Y$, where $Y = g(X)$ and $g(\cdot)$ is a real increasing function, hence, $F_X(x) \geq F_Y(g(x))$. Therefore, one may tractably bound the distribution of $X$ by properly designing $g(\cdot)$. Alternatively, bounds to the
distribution of $X$, if known, can be directly established, e.g., by applying fundamental algebra.

In practice, bounding the entire distribution of $X$ is not often necessary, but only one of its tails according to the specific URLLC problem at hand. Therefore, one may resort to limiting forms, which are often obtained by taking only the asymptotically-dominant terms of a series (e.g., Taylor, Laurent, and Puiseux) expansion or similar (e.g., Poincaré expansion and Padé approximants). Table V lists simple limiting forms of popular functions, which can be used to construct the limiting forms of more evolved functions.

Notice that
- if the distribution of $X$ is known, the limiting forms may help to simplify it in the tail region (see Example 4 below); otherwise,
- if RV $X$ is, or can be represented, as a transformation of other RV(s), one may leverage their limiting forms to bound/approximate the tail distribution of $X$. For instance, assume $X = \exp(Y - 1) + \cosh(1/Z) - 2$, where $Y,Z \geq 1$ are some arbitrary RVs, and one is interested in the left tail of the distribution of $X$, i.e., $F_X(x)$ as $x \to 0$. In this case, finding the whole distribution of $X$ may be cumbersome even in the case that the distributions of $Y$ and $Z$ are simple. Instead one may realize that the small realizations of $X$ are obtained when $Y$ and $Z$ approach 1 and $\infty$, respectively, and consequently may leverage the limiting forms in the rows one and four of Table V. As a result, one obtains that the distribution of $Y + 1/(2Z^2) - 1$ converges to the distribution of $X$ in the left tail. Notice obtaining the distribution of such expression seems much more analytically friendly than directly obtaining the distribution of $X$.

### Example 4 (Reliable Interference Networks [54, 55])
Consider a point-to-point wireless communication link subject to the interference of $K$ neighboring nodes. Assume single-antenna nodes and that all channels are subject to Rayleigh fading. Then, the receive SINR can be written as

$$\text{SINR} = \frac{\bar{\gamma}_0 h_0}{\sum_{i=1}^{K} \bar{\gamma}_i h_i + 1},$$

where $\bar{\gamma}_0$ and $h_0 \sim \text{Exp}(1)$ denote the average SNR of the received signal from the intended transmitter and the experienced normalized channel power gain, respectively. Similarly, $\bar{\gamma}_k$ and $h_k \sim \text{Exp}(1)$ denote the average SNR of the receive interfering signal from node $k$ and the corresponding normalized channel power gain, respectively.

The CDF of the SINR, i.e., outage probability for a certain SINR threshold $\gamma_{th}$, can be computed as follows

$$F_{\text{SINR}}(\gamma_{th}) = 1 - \Pr[\text{SINR} > \gamma_{th}],$$

where (a) comes from using (24), (b) from using the complementary CDF of $h_0$, (c) from exploiting the property $\exp(\sum a_i) = \prod \exp(a_i)$, and (d) after taking the expectation over every $h_k$. Observe that although (25) is in closed-form, it is of little, if any, relevance for designing practical resource allocation mechanisms. For instance, let us focus on a power control mechanism and try to shed some light to the following problem: what is the average SNR $\bar{\gamma}_0$ required for satisfying $F_{\text{SINR}}(\gamma_{th}) \leq \xi$, where $\xi \ll 1$? It is important to note that neither the specific values of the average SNR of the interfering signals $\{\bar{\gamma}_k\}$ nor the number of interferers are generally known in practice. Nevertheless, observe that $\gamma_{th}/\bar{\gamma}_0$ must be small to guarantee $\xi \ll 1$, thus, one may
leverage the limiting forms in the sixth and first row of Table I to state
\[
\prod_{k=1}^{K} \left( 1 + \frac{\tilde{y}_k}{\gamma_0} \right)^{\gamma_m/\gamma_0 - 0} = \left[ 1 + \frac{\gamma_m}{\gamma_0} \right]^{K} = \exp \left( \gamma_m \tilde{y}_0 / \gamma_0 \right),
\]
where \( \tilde{y}_0 = \sum_{k=1}^{K} \tilde{y}_k \). Substituting this into (25) and combining with (26) yields
\[
F_{\text{SNR}}(\gamma_m) = 1 - \exp \left( - \gamma_m / S N R R \right),
\]
where \( S N R R = \tilde{y}_0 / (1 + \tilde{y}) \) is the average-signal-to-average-interference-plus-noise ratio. Note that the complexity can be further reduced, e.g., for diversity order analysis, by exploiting again the limiting form in the first row of Table IV such that (27) converges to \( \gamma_m / S N R R \) as \( \gamma_m / S N R R \to 0 \). Fig. 5 illustrates the incredible tightness of (27) with respect to left tail of (25). This evinces that the outage probability in the URCL regime for the considered scenario depends merely on the \( S N R R \), whose value may be acquired in practice. Back to the original power control problem, \( F_{\text{SNR}}(\gamma_m) \leq \xi \) with \( \xi \ll 1 \) is satisfied for \( \tilde{y}_0 \geq -(1 + \tilde{y}) \gamma_m / \ln(1 - \xi) \) or simply \( \tilde{y}_0 \geq (1 + \tilde{y}) \gamma_m / \xi \).

When threshold, one may exploit the following results.

Theorem 1 (Markov’s inequality [56]). Let \( X \in \mathbb{R}^+ \) be a RV and \( g : \mathbb{R}^+ \to \mathbb{R}^+ \) a non-decreasing function such that \( E[g(X)] < \infty \) exists, then
\[
\bar{F}_X(x) \leq \frac{E[g(X)]}{g(x)}. \tag{28}
\]
Observe that (28) includes an infinite variety of bounds since there are infinite choices for \( g \). The most popular bounds derived from (28) are illustrated in Table VI.

Example 5 (Channel History-Based Precoding [57]). Assume that a critical data message arriving to the data queue of a multi-antenna node \( A \) demands an urgent URCL transmission to a single-antenna node \( B \). Because of the stringent latency constraints, the transmission needs to be carried out immediately in one shot, thus instantaneous channel state information (CSI) cannot be acquired at \( A \). However, in the past, \( A \) had kept a record of \( L \) CSI entries of the communication channel with \( B \), which it may now exploit for the current URCL transmission. The set of CSI entries is given by \( L = \{ b_{1}, b_{2}, \ldots, b_{L} \} \), where \( b_{i} \in \mathbb{C}^{M \times 1} \) is the channel vector coefficient corresponding to the \( i \)-th CSI entry, and \( M \) is the number of transmit antennas.

Assume an interference-free scenario, and denote by \( h \in \mathbb{C}^{M \times 1} \) the unknown channel coefficient for the URCL transmission. Then, the corresponding SNR is given by
\[
\text{SNR} = \frac{|w^H h|^2}{N}, \tag{29}
\]
where \( N \) is the receive noise power, and \( w \in \mathbb{C}^{M \times 1} \) is the transmit precoder. Here, we aim to attain the lowest-power precoder to guarantee a data reception’s SNR of at least \( \gamma_m \) with probability \( 1 - \xi \), where \( \xi \ll 1 \), thus \( \min_{w} \|w\|^2 \leq \text{Pr}[\gamma_m < \gamma_m] \leq \xi \). Observe that for a sufficiently large record of CSI entries, i.e., \( L \to \infty \), the distribution of \( h \) (thus, the distribution of the SNR for a fixed \( w \)) can be acquired, making the design of \( w \) relatively easy to optimize. However, in a practical setup, the number of channel samples \( L \) may be rather limited due to limited training, memory and/or channel coherence time. Herein, we employ the simple form of Markov’s inequality to state
\[
\text{Pr}[\gamma_m < \gamma_m] = \text{Pr}[\gamma_m < \gamma_m] \leq \frac{E[|w^H h|^2]}{(N \gamma_m)} \leq \xi. \tag{30}
\]
Now, the best estimator for \( E[|w^H h|^2] \) is the sample mean \( \frac{1}{L} \sum_{i=1}^{L} |w_{i}^H h_{i}|^2 \). Then, the problem translates to minimizing \( \|w\|^2 : \frac{1}{L} \sum_{i=1}^{L} |w_{i}^H h_{i}|^2 \leq \xi \), which is herein solved numerically to illustrate some performance results in the following.

Fig. 6 shows the truly attainable information outage as a function of the number of channel history samples for \( \xi = 10^{-3} \) and quasi-static Rician fading channels with line-of-sight (LOS) factor of -10 dB. Observe that for a relatively small number of samples, i.e., \( L < 16 \), the outage constraint may not be met. This is because the sample mean converges to the population mean only as \( L \to \infty \). Thus, substituting the sample mean into (30) does not fully guarantee that the bound still holds, but only for large \( L \). The situation may worsen if tighter bounds such as Chernoff’s or moment bounds are used. In such cases, one might need to limit the possible range of values for parameter \( \theta \) (see Table VII) to prevent excessive skewness of the distribution of \( g(\text{SNR}) \) that may significantly slow down the convergence of the sample mean to the population mean. Alternatively, and/or in addition, one might leverage a probabilistic upper bound of the sample mean to ensure (30) is met with a certain confidence level. Such bound can be designed exploiting the central limit theorem, as in [57]. Meanwhile, Fig. 7 leverages \( 10^4 \) Monte Carlo runs to illustrate the empirical distribution of the outage probability (Fig. 27) and transmit

| Corollary | Transformation of (25) | Resulting Bound |
|-----------|------------------------|-----------------|
| Simple form of Markov’s inequality | \( g(t) = t \) | \( \bar{F}_X(x) \leq \frac{E[X]}{x} \) |
| Chebychev’s bound | \( g(t) = t & X \to X - E[X] \) | \( \bar{F}_X(X) \leq \frac{||X - E[X]||}{\theta} \) |
| Chernoff’s bound | \( g(t) = e^{t^2}, \theta \geq 0 \) | \( \bar{F}_X(X) \leq e^{-\theta E[X]} \) |
| moment bound | \( g(t) = e^{t^2}, \theta \geq 0 \) | \( \bar{F}_X(X) \leq e^{-\theta E[X]} \) |
sumed, thus intrinsically discarding an optimization over systems, e.g., [15], [17], [30], [64]–[66], including URLLC. For the analysis and design of wireless communication requirements tend to decrease as LOS increases. Moreover, the transmit power features, although less tightly as LOS increases since channel noise equal to 10 dB.

In practice, the main challenge lies in having accurate estimates of $E[g(X)]$ since these statistics converge at a different pace for different functions $g$ given limited data samples. Finally, a selection of other less popular, but valuable, distribution bounds is compiled in Table VII.

| Assumptions                  | Bound                                                                 |
|------------------------------|-----------------------------------------------------------------------|
| Paley-Zygmund [58]          | $F_X(x) \geq (1-\theta)^2 + \frac{2(1-\theta)^2}{\theta^2|x|^2}$     |
| Hoeffding [59]              | $F_{X^n}(x) = \exp \left( \frac{2(x+\sum_{i=1}^n E[X])}{\sum_{i=1}^n (\theta_i-a_i)^2} \right)$ |
| Vysochanskij–Petunin [60]   | $F_X(x) = \frac{V_x(x)}{V(X)}$, $x > E(X)$                          |
| Cantelli [61]               | $F_X(x) = \frac{V_x(x)}{|x|^r}$, $x > E(X)$                          |
| (tightened) Gauss (Dharmadhikari & Joag-Dev [62]) | $F_X(x) = \max \left\{ \left( \frac{r}{(r+1)} \right)^r E[|X|^r], \frac{E(|X|^r)}{r+1} - \frac{1}{r+1} \right\}$ |
| Dvoretzky–Kiefer–Wolfowitz [63] | $F_{\sup{\left\{ |x_E-F_X(y)| \right\}}}(x) \leq 2e^{-2nx^2}$, $F_{\sup{\left\{ |x_E-F_X(y)| \right\}}}(x) \leq e^{-2nx^2}$, $x \geq \sqrt{\frac{2}{n\theta}}$ |

Table VII | OTHER USEFUL DISTRIBUTION BOUNDS |

Fig. 6. Outage probability as a function of the number of history channel samples. Channel is subject to quasi-static Rician fading with LOS factor of $-10$ dB. We set $\gamma_{th} = 1$, $\xi = 10^{-3}$, and average channel path-loss over noise equal to $10$ dB.

power (Fig. 2b) for $L = 32$ and quasi-static Rician fading with different LOS factors. Results evince that the target outage probability $\zeta = 10^{-3}$ is met regardless of the channel features, although less tightly as LOS increases since channel becomes more deterministic. Moreover, the transmit power requirements tend to decrease as LOS increases.

Theorem 1 has been extensively utilized in the literature for the analysis and design of wireless communication systems, e.g., [15], [17], [20], [64]–[66], including URLLC. However, a fixed transformation function $g$ is always assumed, thus intrinsically discarding an optimization over $g$ for potentially tightening the bounds. For instance, let’s say we have a function space of positive non-decreasing real functions $\mathcal{G}$, then [20] can be tightened as

$$F_X(x) = \inf_{g \in \mathcal{G}} \frac{E[g(X)]}{g(x)}.$$ (31)

Fig. 7. Empirical distribution of the a) outage probability (left) and b) transmit power (right). Channel is subject to quasi-static Rician fading. We set $M = 4$, $L = 32$, $\gamma_{th} = 1$, $\xi = 10^{-3}$, and average channel path-loss over noise to $10$ dB. We consider $10^4$ Monte Carlo realizations.
B. Extreme Value Theory

Estimating the extreme values of an underlying process, which are scarce by definition, inevitably requires extrapolation from available observations. The EVT provides a class of models to enable such extrapolation \[\text{[67]}.\] The cornerstone of EVT revolves around the statistical characterization of the following RV

\[M_n = \max\{X_1, X_2, \ldots, X_n\} \text{ as } n \to \infty, \tag{32}\]

where \(X_1, X_2, \ldots, X_n\) are independent RVs sharing a common, but unknown, CDF \(F_X(x)\).

**Theorem 2 (Extreme Value Distribution \[\text{[64, Th.3.1.1]}\]).** If there exists a sequence of constants \(\{a_n > 0\}\) and \(\{b_n\}\) such that

\[\Pr\left(\frac{M_n - b_n}{a_n} \leq z\right) \to F_Z(z) \text{ as } n \to \infty \tag{33}\]

for a non-degenerate distribution function \(F_Z(z)\), then

\[F_Z(z) = \exp\left(-\left[1 + \kappa \left(\frac{z - \mu}{\sigma}\right)\right]^{-1/\kappa}\right), \tag{34}\]

defined on \(\{z : 1 + \kappa(z - \mu)/\sigma > 0\}\), where \(-\infty < \mu < \infty\) is the location parameter, \(\sigma > 0\) is the scale parameter, and \(-\infty < \kappa < \infty\) is the shape parameter.

Theorem 2 states that if suitably normalized maxima of RVs converge in distribution, then the limit distribution must be an extreme value distribution as in \(\text{[34]}\) for some \(\mu, \sigma,\) and \(\kappa\). The apparent difficulty that \(\{a_n\}\) and \(\{b_n\}\) are unknown in practice is easily resolved as follows. Assume that \(\Pr\left(\left.M_n - b_n\right)/a_n \leq z\right) \approx F_Z(z)\) for large enough \(n\). Then, \(\Pr\left[M_n \leq z\right] \approx F_Z\left(z - b_n\right)/a_n = F^*_Z(z)\), and note that \(F_Z^*(z)\) also has the form \(\text{[34]}\) but with potentially different parameters \(\mu\) and \(\sigma\). Since the parameters of the distribution must be estimated anyway, it is irrelevant in practice that the parameters of the distribution \(F_Z(z)\) are different from those of \(F^*_Z(z)\). Theorem 2 can be easily transformed to characterize the asymptotic distribution of the minima of RVs \(\text{[67, Th.3.3]},\) and the asymptotic distribution of the \(k\)-th ordered statistics \(\text{[67, Th.3.5]}\).

A key EVT result describing the behaviour of large observations exceeding high thresholds is presented next.

**Theorem 3 (Limiting distribution of excesses \[\text{[62, Th.4.1]}\]).** For large enough \(u\), the distribution function of \(X-u\), conditioned on \(X > u\), and where \(X \in \{X_1, X_2, \ldots, X_n\}\) with \(\Pr[\max_{i=1,\ldots,n} X_i < z] \approx F_z(z)\) given in \(\text{[34]}\), follows approximately a generalized Pareto distribution (GPD) such as

\[F_{X-u|X>u}(y) \approx 1 - \left[1 + \frac{\kappa y}{\sigma} \right]^{-1/\kappa}, \tag{35}\]

where \(\sigma = \sigma + \kappa(u - \mu)\). Such distribution is defined on \(\{y : y > 0, (1 + \kappa y/\sigma) > 0\}\).

Although Theorem 2 and Theorem 3 assume an underlying process consisting of a sequence of independent RVs, their claims are still usually valid when such an assumption does not hold, as in case of time-correlated data \(\text{[67]}\).

**Example 6 (Channel History-Based Precoding (II)).** Let’s exploit the EVT framework to solve the problem described in Example 5, i.e.,

\[\min_{w} \|w\|_2^2 \Pr[\text{SNR} < \gamma_{th}] \leq \xi, \tag{36}\]

where \(\xi \ll 1\). Here, the difficulty lies in addressing the URLLC constraint, thus, we proceed as follows

\[\Pr[\text{SNR} < \gamma_{th}] = \Pr\left[N |w^H h|^2 > 1/\gamma_{th}\right] \overset{(a)}{=} \Pr[\Theta > f(1/\gamma_{th})]\]

\[\overset{(b)}{=} \frac{1}{L} \left(1 - F_{\Theta - u > u}(f(1/\gamma_{th}) - u)\right) \sum_{l=1}^L \|\Theta_l > u\]\n
\[\overset{(c)}{=} \frac{1}{L} \left(1 + \frac{\kappa(w, \mathcal{L})}{\sigma(w, \mathcal{L})} f\left(rac{1}{\gamma_{th}} - u(w, \mathcal{L})\right)\right) \frac{1}{\gamma_{th}} \times \sum_{l=1}^L \|\Theta_l > u(w, \mathcal{L})\], \tag{37}\]

where \((a)\) comes from introducing an increasing function \(f\) and defining \(\Theta \overset{\text{df}}{=} f(N |w^H h|^2)\). Note that by adopting a concave \(f\) one can mitigate the impact of potentially disperse samples \(\Theta_l \overset{\text{df}}{=} f(N |w^H h|^2)\) as a more compact sample set \(f(\Theta_l))\) is obtained, thus, facilitating a better GPD fitting in the next steps. Thereafter, \((b)\) comes from exploiting the conditional probability framework, specifically, the probability conditioned on exceeding a threshold \(u\), which is introduced here and designed to be relatively large but smaller than \(1/\gamma_{th}\). Meanwhile, the empirical distribution of \(\Theta\) (obtained from \(\Theta_l\)) is used in \((c)\) to estimate \(\Pr[\Theta > u]\). Note that for this estimation to be somewhat accurate it is required that \(u < \max \Theta_l\). Finally, Theorem 3 result is leveraged to obtain \((d)\), and therein we have highlighted the dependency of \(\kappa, \sigma, \) and \(\mu\) on the adopted beamformer \(w\) and channel sample set \(\mathcal{L}\).

Then, the following procedure can be followed to determine if a certain beamformer \(w\) satisfies the URLLC constraint:

1. Set \(u(w, \mathcal{L})\) as the quantile of \(\{\Theta_l\}\) for the cumulative probability \(p \in (0.5, 1)\). This leads to \(\frac{1}{L} \sum_{l=1}^L \left\{\Theta_l > u(w, \mathcal{L})\right\} = p\)
2. Fit the GPD \(\text{[55]}\) to the samples \(\{\Theta | \Theta > u\}\) and obtain a \(100 \times \alpha\%\) confidence intervals \(\left[\kappa_{\bar{w}}, \kappa_{\bar{u}}\right], \left[\sigma_{\bar{w}}, \sigma_{\bar{u}}\right]\) for \(\kappa(w, \mathcal{L}), \sigma(w, \mathcal{L})\) respectively for \(\kappa(w, \mathcal{L}), \sigma(w, \mathcal{L})\)
3. by exploiting \(\text{[67]}\), \(w\) surely satisfies the URLLC constraint if \(\left(1 + \frac{\kappa(w, \mathcal{L})}{\sigma(w, \mathcal{L})} \left(\frac{1}{\gamma_{th}} - u(w, \mathcal{L})\right)\right) \leq \xi / p, \forall \kappa(w, \mathcal{L}) \in \left[\kappa_{\bar{w}}, \kappa_{\bar{u}}\right], \sigma(w, \mathcal{L}) \in \left[\sigma_{\bar{w}}, \sigma_{\bar{u}}\right]\).

Observe that \(p\) and \(\alpha\) are heuristic parameters that must be pre-defined. The closer \(p\) is to 1, the more extreme, right-tailed, data samples \(\Theta | \Theta > u(w, \mathcal{L})\) become, thus, the better the suitability of a GPD fitting. However, few samples are employed for the fitting, which ultimately reduces the accuracy. Meanwhile, the closer \(\alpha\) is to one, the more flexible the GPD fitting becomes, but the chances of misdetecting an appropriate beamformer increase.

We resort to a brute force numerical optimization where the beamformer, out of \(10^2\) randomly generated beamformers, with the lowest power but satisfying the URLLC...
averages. Therefore, Markov-inequality-based solutions, which are based on some tail data, thus, they usually require more data than reliability constraints. Moreover, EVT-based solutions exploit \( x \) by reducing GPD estimator confidence intervals (smaller EVT-based solutions may be more energy efficient, specially based solving framework used in Example 5. Observe that in Fig. 8 and compared to the that of the Markov inequality resulting from employing the EVT framework is illustrated constraint as described above, is selected. The performance in Fig. [9] and compared to the that of the Markov inequality-based solving framework used in Example 5. Observe that EVT-based solutions may be more energy efficient, specially by reducing GPD estimator confidence intervals (smaller \( a \)), although this leads to greater chances of violating the reliability constraints. Moreover, EVT-based solutions exploit some tail data, thus, they usually require more data than Markov-inequality-based solutions, which are based on averages. Therefore, Markov-inequality-based solutions are preferable when a limited data set is available, while EVT-based solutions become more appealing as more data can be exploited.

Several works have exploited the EVT framework for designing and assessing the performance of URLLC systems, e.g., [3, 68, 69]. For example, the authors in [68] leverage EVT to approximate the extreme probability of exceeding a queue length threshold, and develop a federated learning (FL) approach for resource allocation mechanisms upon it. Meanwhile, an EVT-based methodology, which includes determining the optimum threshold \( a \) and ascertaining the required number of channel samples, is introduced in [69] to statistically model the behavior of extreme events in a URLLC wireless channel. Moreover, the Pickands-Balkema-de Haan theorem [70, Th. 2.1.1] in EVT, which states that

\[
F_X(x) \rightarrow ax^\beta \quad \text{as} \quad x \to 0, \quad \text{for some} \ a, \beta > 0
\]

(or alternatively, \( F_X(x) \rightarrow 1-x^{-\beta} \) as \( x \to \infty \) in case the interest is in the right tail) holds for a large class of distributions, has been exploited in [3] for estimating channel tails followed by transmit rate adaptation with ultra-reliability guarantees. Note that estimators of the tail index, i.e., \( 1/\beta \), include Pickands estimator [71], the moment estimator [72], and Hill’s estimator [73]. Note that the asymptotic convergence of [39] tends to be slower compared to the previously discussed Pareto approximation, thus although simpler, its accuracy is more limited, specially when exploiting limited data sets.

C. Risk-assessment tools

Risk management theory [74], widely used in the field of finance, represents a fundamental tool for the design and analysis of URLLC systems [75]. Two widely used metrics to characterize risks are:

- value-at-risk (VaR), which is defined as the worst loss over a target horizon within a given level of confidence, thus, it is a quantile of the loss distribution;
- conditional VaR (CVaR) or expected shortfall, which characterizes the expected loss in the right tail of the distribution given a particular threshold has been crossed, thus, measuring the risky realizations.

In terms of a RV \( X \), and given a confidence of \( 100\times (1-a)\% \), these metrics are given by

\[
\text{VaR}_{1-a}(X) = F_X^{-1}(1-a),
\]

\[
\text{CVaR}_{1-a}(X) = \mathbb{E}[X | X > \text{VaR}_{1-a}(X)]
\]

\[
= \frac{1}{a} \int_0^a \text{VaR}_{1-t}(X) \, dt,
\]

and are illustrated in Fig. [9].

Relying on distribution bounds such as those discussed in Section III-A one can obtain bounds for VaR and CVaR as well. For instance, by exploiting \( (20) \), one attains

\[
\text{VaR}_{1-a}(X) \leq g^{-1}(\mathbb{E}[g(X)]/a),
\]

which holds for every non-negative non-decreasing function \( g \). Similarly, Theorem [3] can be leveraged to attain VaR and CVaR estimates, which are less conservative than those based on bounds, thus, generally more efficient for URLLC-related resource allocation.

Recently, there has been an increasing interest in applying the risk-management framework to design and analyze URLLC systems. Specifically, the authors in [15] use a risk-sensitive approach exploiting CVaR to schedule URLLC systems.
traffic in coexistence with punctured mobile broadband (MBB) services. Meanwhile, CVaR rate is a metric introduced and thoroughly investigated in [76] to characterize the expected rate of a worst-case fraction of clients in broadcast/multicast URLLC systems. Finally, in [72], the authors realize that the FBL error probability (see related discussions) is in fact a RV in fading scenarios, for which they provide accurate PDF analytical approximations. Based on this, they leveraged VaR and CVaR measures to illustrate that statistically different risks may be experienced even on services with similar performance in terms of average reliability.

Note that VaR can be directly exploited for modeling URLLC given its design constraints. Meanwhile, CVaR quantifies the performance impact of violating the URLLC operation, thus can be indirectly used for URLLC system design as well. In addition, CVaR can be leveraged to design resource-efficient resilience mechanisms. For instance, the number of wireless re-transmissions of a data message after failure can be designed based on the CVaR level. The higher the CVaR, the more re-transmissions should be scheduled after a data message fails.

IV. RARE EVENT SIMULATION

In wireless communications, we are often confronted with noisy observations from which we would like to estimate some unknown distribution and/or its parameters, or the unknown parameters. Among the many solutions, constructing posterior distributions is normally adopted as it facilitates incorporation of the prior information into the observed data. However, closed-form solutions for such problems are often infeasible, leading to the development of approximate inference techniques, such as Monte Carlo (MC) methods [78], [79]. MC methods comprise a large class of simulation-based algorithms optimized to perform random drawing from a target probability distribution. These methods can be used i) to generate independent random samples from a probabilistic model, or ii) to perform numerical integration, or iii) in optimization.

Therefore, MC methods became an essential tool in URLLC systems because reliability can be posed as rare-event estimation/simulation. In the reminder of this section, we overview classical MC methods and assess their limitations in terms of computation time and accuracy. We then overview alternative solutions dedicated to rare-event sampling and provide examples of their efficiency in terms of computation time, complexity and accuracy.

A. Conventional MC

To introduce the MC method, let us define the following high-dimensional integral, that estimates the probability, \( p \), of a rare-event

\[
p = \Pr \{ S(x_i) \geq x_{th} \} = \int f_X(x) \{ S(x_i) \geq x_{th} \} \, dx = E_X \{ S(x_i) \geq x_{th} \},
\]

(42)

where \( X \) is a \( d \times N \) random matrix, where \( d \) denotes the dimensions and \( N \) the number of samples, its PDF is \( f_X \), \( S(x) : R^d \rightarrow R \) is a real-valued function, and \( x_{th} \) is a threshold parameter. The \( x_{th} \) parameter determines the rarity of the event, i.e., \( p = \Pr \{ S(x_i) \geq x_{th} \} \rightarrow 0 \) as \( x_{th} \rightarrow \infty \). In other words, this parameter indicates how far into the tail of the distribution we are attempting to estimate the probability \( p \).

Conventional (also known as naive or crude) MC method is a convenient way to estimate \( p \), i.e., \( \hat{p} = E_X \{ S(x_i) \geq x_{th} \} \). The idea behind conventional MC to estimate the probability \( \hat{p} \) using many independently generated samples of \( X = [x_1, \ldots, x_N]^T \) per dimension. In this case, the unbiased estimator of choice is the sample mean, i.e., \( \hat{p} = \frac{1}{N} \sum_{i=1}^{N} I[S(x_i) \geq x_{th}] \); in other words, the ratio between the samples that belong to the event \( \{ S(x_i) \geq x_{th} \} \) and the total number of samples. Therefore, conventional MC relies on the law of large numbers and the central limit theorem to increase the accuracy of the estimation, as the number of samples grows to infinity, the sample mean converges to the expectation, i.e.,

\[
E[\hat{p}] = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{i=1}^{N} I[S(x_i) \geq x_{th}]
= \frac{1}{N} \sum_{i=1}^{N} E_X[I[S(x_i) \geq x_{th}]] = p.
\]

(43)

We resort to the coefficient of variation, \( \delta(\hat{p}) \) as a measure of the accuracy of the conventional MC, and therefore as \( \delta(\hat{p}) \rightarrow 0 \) the more accurate is the estimation. We define this measure as \( \delta(\hat{p}) = \frac{\sqrt{\text{Var}[\hat{p}]} \cdot \hat{p}}{\hat{p}} \), which is the ratio of the standard deviation and expected value of the estimate \( \hat{p} \). Following the same steps as in [43] we calculate the estimate’s variance

\[
\text{Var}[\hat{p}] = \text{Var}_X \left[ \frac{1}{N} \sum_{i=1}^{N} I[S(x_i) \geq x_{th}] \right]
\]

\[
(\text{a}) = \frac{1}{N^2} \sum_{i=1}^{N} E_X[I[S(x_i) \geq x_{th}]^2] - p^2
\]

\[
(\text{b}) = \frac{1}{N^2} \sum_{i=1}^{N} (p - p)^2 = \frac{p(1 - p)}{N},
\]

(44)
where (a) expands the definition of the variance, and (b) uses the identity $l[\cdot] = l[\cdot]$ and (43). Now, we can calculate the coefficient of variance as

$$
\delta(p) = \sqrt{\frac{1-p}{Np}} \approx \frac{1}{\sqrt{Np}}.
$$

(45)

where last step holds accurate for $p \ll 1$. The latter corresponds to the probability of a rare event, which is of particular interest in URLLC setups.

One advantage of conventional MC is the independence on the dimension of the input state for the accuracy measure. Note that the accuracy varies with the number of samples and the target probability; therefore, the lower the desired estimate, the higher the number of samples required. Let us exemplify this point with the 3GPP target probability for URLLC of $p = 10^{-5}$. We expect a low variability, therefore, a coefficient of variation of 1% is appropriate. Then, replacing these values into (45), the required number of samples for such estimate is $N = 10^9$, which is costly with conventional simulation tools. In addition, for every sample there are additional computations required, e.g., $l[\cdot]$ and $S[\cdot]$, which may increase the computational cost and render the URLLC analysis infeasible with conventional MC.

Notably, conventional MC is particularly useful when the target probability is around the mean of the distribution. However, conventional MC becomes inefficient when the target probability is at the distribution’s tail. This is the case in URLLC scenarios as discussed in Section III. Therefore, robust MC methods are needed to evaluate URLLC with limited costs (e.g., computation and time). Indeed, we are interested in computationally affordable methods that allow evaluating reliability levels in the order of five or more nines, as in (extreme) URLLC use-cases [75], [82], by effectively sampling the region of the distribution’s domain where such rare events occur. Fortunately, several methods can help in this task.

B. Importance sampling

Importance sampling (IS) is perhaps one of the most common MC methods for rare-event simulation, which is a variance reduction technique in the estimation of (42). In general, for a fixed degree of relative error, IS achieves sufficient variance reduction to decrease total computational effort by several orders of magnitude compared to that of conventional MC. This is achieved because IS draws samples from another distribution where the original rare events happen more often. Precisely, assume a PDF $g_Y(x)$ such that if $g_Y(x) = 0$ then $\mathbb{I}[S(x_i) \geq x_{th}] f(x) = 0 \forall x$; thus, we can re-write the expectation in (42) as

$$
\mathbb{E}_X [\mathbb{I}[S(x_i) \geq x_{th}]] = \int \mathbb{I}[S(x_i) \geq x_{th}] \frac{f_X(x)}{g_Y(x)} g_Y(x) dx = \mathbb{E}_Y \left[ \mathbb{I}[S(x) \geq x_{th}] \frac{f_X(x)}{g_Y(X)} \right].
$$

(46)

Notice that one must carefully select the PDF such that the variance of $X$ is reduced. For this, the PDF must have $i)$ finite variance, i.e.,

$$
\mathbb{E}_X \left[ \mathbb{I}[S(x_i) \geq x_{th}] \frac{f_X^2(X)}{g_Y^2(X)} \right] = \mathbb{E}_Y \left[ \mathbb{I}[S(x) \geq x_{th}] \frac{f_X^2(X)}{g_Y^2(X)} \right] < \infty,
$$

and $ii)$ heavier tails than $f_X$ and the bounded likelihood ratio $\frac{f_X}{g_Y}$. Notably, a poor selection of the PDF compromises the quality of the estimate [83].

Observe that the optimum PDF is the one that minimizes the variance of the estimate, i.e.,

$$
\arg\min_{g_Y} \mathbb{E}_Y \left[ \mathbb{I}[S(x) \geq x_{th}] \frac{f(X)}{g(X)} \right],
$$

(47)

whose solution is the zero-variance IS density [83]

$$
\pi(x) = \frac{f(x)}{g(x)}
$$

(48)

However, (48) depends on the unknown estimate $l$, therefore it cannot be directly used. Instead, we approximate the $\pi$ and the IS density $g_Y$, e.g., by resorting to differential entropy and by allowing $g_Y$ to be in product form $g_Y(x) = \prod_{i=1}^d \pi_i(x_i)$, which usually simplifies the analysis.

Thought relevant, IS method is particularly useful for light-tailed distributions, e.g., where the rare-event probability decays exponentially. Moreover, the selection of the desired density can be optimized for instance with the cross-entropy method [83], which minimizes a density $g_Y$ for a parametric family of probability densities instead of a single function. By doing so, the cross-entropy method allows to iteratively update the target density, helping with effective sampling.

C. Markov-chain MC Algorithms

Current MC methods are based on the initial works of Stanislaw Ulam, John von Neumann, among others, during the Manhattan Project in the 1940s. One of the key contributions was the Metropolis algorithm, which relies on a Markov chain for the generation of a desired distribution based on symmetrized proposal densities. After improvements to accept asymmetric proposal densities by Hastings, the algorithm is now known as Metropolis-Hastings algorithm [78]. The Markov-chain MC algorithms rely on a basic idea: run a sufficiently long Markov chain such that its stationary distribution approximates an arbitrary distribution, known as target distribution, from which we can generate samples. In other words, suppose our target distribution is $f_X(x)$, as in (42), then construct a Markov chain with states $\{X_t, t = 0, 1, 2, \cdots\}$, where $X_t$ is a given state at time $t$ and stationary probability distribution is $f_X(x)$. The Metropolis-Hastings algorithm core idea relies on calculating the transition from a state to the next. To do so, we first draw a proposal state, e.g., $Y$ from a transition density $q_Y(\cdot | X_t)$. Then, the proposal state is accepted based on the acceptance probability

$$
\alpha(x, y) = \min \left\{ \frac{f_X(y) q_Y(x | y)}{f_X(x) q_Y(y | x)} \right\}.
$$

(49)

Please, see [83] for detailed discussion on alternative algorithms.
Algorithm 1: Metropolis-Hastings Algorithm

| Input: $X_0$, $N$, $f_X(x)$, $q_Y(y|x)$ |
|---|
| Initialization: $t = 0$ |
| Draw $Y \sim q_Y(y|X_t)$ |
| Calculate $\alpha(X_t, Y)$ in (49) Draw $U \sim \mathcal{U}(0,1)$ (uniform distribution) |
| $X_{t+1} = \begin{cases} Y, & \text{if } U \leq \alpha(X_t, Y) \\ X_t, & \text{otherwise} \end{cases}$ |
| Increase $t$ |
| until $N$ samples were generated, i.e., $t = N$; |
| Output: $X_1, \cdots, X_N$ |

Otherwise, the proposed state is rejected and the the chain remains in current state. This can be observed in Algorithm 1 lines 4-5 when the proposal is accepted or rejected compared to a draw form a uniform distribution. Notice that the transition density satisfies the balanced detailed equations, i.e., $f_X(x)q_Y(y|x) = f_Y(y)q_Y(y|x)$. Therefore, $f_X(x)$ is the stationary distribution of the chain. Notably, if the event $\{X_{t+1} = X_t\}$ has positive probability, i.e., $\Pr(\{X_{t+1} = X_t\}) > 0$ and $q_Y(y|x) > 0 \forall x, y$ in the domain, then the Markov chain is ergodic and its limiting distribution is $f_X(x)$.

Note that computing the acceptance probability $\alpha(x,y)$ requires knowing the distribution $f_X(x)$ only up to a normalizing constant, i.e., $f_X(x) = c \tilde{f}(x)$ where $\tilde{f}(x)$ is a known function while $c > 0$ is an unknown constant. Moreover, the efficiency of the Metropolis-Hastings algorithm heavily depends on the choice of the transition density, which ideally should be close to the target function $f_X(x)$ regardless of $x$. In such circumstances, the proposal function can be independent of $x$, thus $q_Y(y|x) = g_Y(y)$ for some distribution $g_Y(y)$. Consequently, (49) reduces to $\alpha(x,y) = \min \left\{ \frac{f_Y(y)g_Y(y)}{f_X(x)g_Y(y)} \right\}$, and this method is known as an independence sampler. However, notice that the generated samples are dependent and the proposal function should be close to the target function. Moreover, if $f_X(x) \leq c g_Y(x) \forall x$, where $c > 0$ is a normalizing constant, then, the acceptance rate in Algorithm 1 Step 5 is at least $1/c$. A simple alternative is to consider a symmetric proposal function, i.e., $q_Y(x|y) = q_Y(y|x)$, hence $\alpha(x,y) = \min \left\{ \frac{f_X(x)q_Y(y|x)}{f_Y(y)q_Y(x|y)} \right\}$.

A particular case of the Metropolis-Hastings algorithm is the Gibbs sampler, commonly used for n-dimensional random vectors. In this case, the conditional distribution governs the construction of the Markov chain. Notably, Gibbs sampler is helpful when sampling the conditional distribution is less computationally expensive than the joint distribution. In addition, the Gibbs sampler distribution, $f_X$, is the stationary distribution of the Markov chain and has a geometrically fast convergence.

D. Subset simulation

With origins in structural reliability theory, subset simulation (SS) is a powerful and efficient method for simulation of rare events and estimation of tail probabilities. Moreover, SS decomposes the rare event into a sequence of nested events such that the most frequent events are supersets of the less frequent ones. In other words, let the rare event $F$ be decomposed into a sequence of $L$ progressively rarer events from $F_1$ to $F_L$, i.e., $F = F_L \subset F_{L-1} \subset \cdots \subset F_1$, where $F_1$ is a frequent event. Therefore, the probability of the rare event $Pr[F]$ follows the chain rule as

$$Pr[F] = Pr[F_1] = Pr[F_1|F_L]Pr[F_2|F_L]\cdots Pr[F_L|F_{L-1}].$$

However, in many applications, it is cumbersome to decompose the events and obtain closed-form or tractable expressions for the conditional probabilities. Thus, to overcome this issue, it relies on Markov-chain MC methods to adaptively obtain the conditional probabilities, and consequently the intermediate events.

In a nutshell, the SS algorithm generates a small number of samples to explore the input space, thus, generating a rough approximation of the failure domain $F$. This process is repeated iteratively such that the domain approximations converge to the domain $F$, and sufficient samples are generated in $F$ to estimate the rare event, as illustrated in Fig. 10. Notice that some samples serve as seeds for the Markov-chain MC algorithm at the upper level $L$, thus, drawing from $\pi(\cdot|F_L)$. These steps are summarized in Algorithm 2. Notice that any Markov-chain MC algorithm can be used to sample from the conditional distribution. The particular choice depends on the scenario, e.g., difficulty on sampling from the conditional distribution.

Based on (34), we describe the key points of the SS algorithm. For the first domain, SS generates the initial sample set comprising the realizations $x_0 = [x_0^1, \cdots, x_0^n]^T \sim \pi(x)$, $n < N$, $N = n^L$, as shown in line 2 of Algorithm 2. Then, the SS algorithm computes the response $y_0 = S(x_0)$ in line 3. As the sample size is small and we want to estimate a rare event, it is likely that the samples $x_0$ do not belong to $F$. However, $x_0$ contains relevant information about $F$ since by ordering $y_0$ we can identify which are the samples that are closest to the target set. As in (22), we can define the first intermediate failure domain, i.e., $F_1 = x : S(x_i) > y_i^*$, which is the set of samples that lead toward the rare event and are ordered in line 4. By construction, $x_0^{n+1}, \cdots, x_0^n$ belong to $F_1$ and $x_0^{n+1}, \cdots, x_0^n$ do not, which is obtained by the
intermediate failure domain \( y^*_q = (y^*_0 + y^*_n q^{n+1})/2 \) (line 6), where \( q \in (0, 1) \) is an arbitrary input parameter such that \( n q \) is a natural number. Consequently, the probability estimate of \( F_1 = \Pr[F_1] = q \), which can be estimated via conventional MC since \( F_1 \) is designed not rare and the number of samples, \( n \), is small. Moreover, we can calculate the rare event probability as in (50), which for the first domain reduces to \( q \Pr[F_1|F_0] \). Therefore, \( \Pr[F_1|F_0] \) using a Markov-chain MC method (line 7), e.g., a modified Metropolis-Hastings or Gibbs sampling [83, 84]. Once we generate samples based on \( \pi(x|F_1) \), we apply the same steps and determine the next domain. This process is repeated iteratively until the stopping criteria are met (see [84] for details on the stopping criteria and an alternative implementation of the algorithm). By design, the SS algorithm controls the probability of the intermediate steps, therefore, (50) approximates to \( q^L \Pr[F|F_1] \) where \( L \leq L \) indicates the number of domains, \( \Pr[F|F_1] = n(L)/n \), and \( n(L) \) is the number of samples in the last domain. Thus, the output of Algorithm 2 line 8 is the rare event probability, which is given as

\[
\Pr[F] \approx q^L \frac{n(L)}{n} \tag{51}
\]

A key advantage of the SS algorithm is that by design one can control input variable \( q \), which influences the number of chains, number of samples in each chain, and the stopping criteria. Therefore, with proper selection of \( q \), the SS algorithm efficiently samples the conditional distributions to (approximately) match this probability ensuring that each factor in (50) is larger than \( q \). Such an iterative process yields an accurate estimate of the rare event even for a small sample size.

**Example 7.** Multi-connectivity emerged as a promising solution for URLLC as it allows the devices to connect to two or more base stations. See for instance [85] for a comprehensive overview and [4, 22, 23] for related works and applications. Herein, we assume a simple system model where a device connects to \( d \in \{2, 4, 8, 16\} \) base stations. The channel coefficients follow a Rayleigh distribution and we are interested in coherently combining these signals via maximal ratio combining. Therefore, the SNR of each link \( X \) follows an exponential distribution \( \gamma \), i.e., \( X \sim \exp(1) \). Under this setting, the SNR of the combined received signal can be described as \( Y = \sum_{i=1}^d X_i \). For this particular case the distribution of the sum is known in closed-form, i.e., \( Y \sim \chi^2(d, 1) \), which helps the comparison between conventional MC and SS algorithms here. However, in many cases, we are unable to express the target distribution in closed-form due to the complexity of the model, or intractability of the functions that lead to intricate integrals that require numerical analysis. This is one more example of the usefulness of MC methods in practice.

We are interested in the outage probability, i.e.,

\[
P_{\text{out}} = \Pr[\log_2(1 + \gamma Y) < r] = \Pr[Y < (2^r - 1)/\bar{\gamma}],
\]

where \( r \) represents a target rate and \( \bar{\gamma} \) the average SNR. In Fig. 11, we compare the analytical expressions to the conventional MC and SS algorithms as a function of \( \bar{\gamma} \) for different number of base stations (dimensions) and \( r = 1 \) bits per channel use (bpcu). For the conventional MC, we draw \( N = 10^7 \) samples for each dimension \( d \), while for the SS algorithm we draw \( n = 10^4 \) samples and we assume \( q = 0.1 \). We implement Algorithm 2 using the modified Metropolis-Hastings algorithm, as in Algorithm 1.

As discussed above, conventional MC is inefficient as the \( \bar{\gamma} \) grows, thus, as the events become more rare, and therefore, it can only reliably estimate the probabilities as \( P_{\text{out}} > 10^{-6} \). Increasing the sample size, \( N \gg 10^7 \) becomes computationally expensive. On the other hand, SS algorithm, uses a small sample size for each domain yielding a faster simulation compared to conventional MC. The number of samples grows with the increase in \( \bar{\gamma} \). For instance, for \( d = 4 \), \( \bar{\gamma} = 20 \text{ dB} \), SS requires approximately 91000 samples to estimate an event with probability around \( 410^{-10} \). Moreover, it is able to estimate the probabilities for region below \( 10^{-10} \). This is particularly relevant for the analysis of performance metrics in the extreme URLLC cases. Notice that the SS algorithm can be optimized to reduce the variance for extreme events, e.g., by selecting a different target distribution or different input parameters such as the number of samples or \( q \).

**V. Finite Blocklength Coding**

In any communication systems, the received message is not always the same as the transmitted message. An error is said to occur when this happens. This can happen due to a number of reasons in a wireless system, for example, deep fades due to random channel fluctuations or outages due to strong interference. One way to recover such errors is to use use error correction codes, where redundant information are added to the transmitted message to facilitate error detection and/or correction. However, this requires additional decoding algorithms at the receiver end to recover the original message from the transmitted

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9The feasibility of such solution is discussed in, e.g., [1], [22].

10For simplicity of presentation, we assume all links have the same mean, which in practice presumes that the nodes are equidistant from the base station, or the devices employ some path-loss compensation.
encoded information [85]. The first error correcting code was the Hamming \((7, 4)\) code invented in 1950 [87].

Error-correcting codes are usually classified into convolutional codes and block codes. Convolutional codes work on a bit-by-bit basis as a sliding window of the bits. A \((k_c, k_d)\) block code acts on a block of \(k_d\) information bits of input data to produce \(k_c\) \((k_c > k_d)\) bits of coded output data, also known as a codeword symbol. The ratio \(r_c \triangleq k_d / k_c\) is known as the coding rate. The price to pay for this error resilience is that the introduction of redundant bits decreases the data rate in proportion to the coding rate \(r_c\).

### A. Finite Blocklength Theory

The capacity of a wireless channel dictates the maximum data rate that can be transmitted with asymptotically small error rates in the absence of any complexity or delay constraints in the coding and encoding process. For example, the well known Shannon capacity of additive white Gaussian noise (AWGN) channels for SNR \(\gamma\) given by \(C_{\text{AWGN}} = \log_2(1 + \gamma)\) in bpcu assumes that the channel’s mutual information is maximized over all possible input distributions, i.e., an infinite blocklength. Encoding over a large number of bits increases the diversity order of the code and hence provides better error resilience. Notice that such increased reliability adds complexity and delay to the encoding and decoding processes.

The payload in the novel URLLC and massive MTC service classes introduced in 5G are usually small in size. Moreover, the low-latency constraint in URLLC implies that messages have to be transmitted as soon as they arrive and cannot afford to be buffered in a queue. Thus, the amount of data available for encoding is usually small. Alongside, the massive contention to access the channel and/or strict energy limitations that are common in massive MTC imposes further constraints. Altogether, many URLLC and massive MTC use cases do not allow coding over a large block length and hence there is a need to evaluate the performance of a wireless system transmitting FBL data. We next present Polyanskiy, Poor and Verdú’s FBL theory, which is a unified approach to obtain tight bounds on the maximum transmission rate \(r(N, \epsilon)\) as a function of the blocklength \(N\) and the error probability \(\epsilon\).

**Theorem 4 (Maximum FBL Transmission Rate [88]).**

For a given channel with capacity \(C\), the difference between the channel capacity and the maximum transmission rate in the FBL regime is a function of the blocklength \(N\), the error probability \(\epsilon\), and the channel dispersion \(V\). The latter is a measure of the stochastic variability of the channel relative to a deterministic channel with the same capacity. In concrete terms,

\[
r(N, \epsilon) = C - \sqrt{N}Q^{-1}(\epsilon) + O\left(\frac{\log N}{N}\right),
\]

where \(Q^{-1}(\cdot)\) is the inverse of the Gaussian Q function.

Notice that for an AWGN channel with SNR \(\gamma\), the capacity matches the well known Shannon rate \(C_{\text{AWGN}}\), and the dispersion is given by

\[
V(\gamma) = \frac{2 + \gamma}{(1 + \gamma)^2} \log^2 e.
\]

Fig. 12 leverages (52) and (53) to illustrate the maximum transmission rate achievable in AWGN channels as a function of the error probability \(\epsilon\) for SNR values \(\gamma \in [10, 20]\) dB, and blocklengths \(N \in [100, 1000]\). The transmission rate is normalized by the corresponding AWGN capacity in the infinite blocklength regime, \(C_{\text{AWGN}}\). We can observe that the penalty incurred with FBL transmissions is higher for shorter blocklengths, lower SNRs, and tighter error probability targets, as typical in the case of URLLC transmissions. The interested reader is referred to [14] and [89] for expositions on the derivation of the maximum FBL transmission rate under fading channels and in a random wireless network, respectively.

Meanwhile, one can obtain the achievable error probability given a fixed transmission rate \(r = k/N\) from [52] as

\[
\epsilon(\gamma) \approx Q\left(\frac{\log N}{\sqrt{V(\gamma)/N}}\right),
\]

for which we have ignored the residual terms of order \(\log N/N\) in [52] for simplicity. Consequently, the average error probability for block fading channels is given by \(\bar{\epsilon} = E_r[\epsilon(\gamma)]\). Interestingly, it has been shown in [14], [90], [91] that the effect of the fading on \(\bar{\epsilon}\) vanishes the FBL impact in two cases, when \(i)\) \(k\) is not extremely small, and \(ii)\) there is not a strong LOS component (e.g., as in Rayleigh fading). In such cases, the asymptotic outage probability is a good approximation for \(\bar{\epsilon}\), i.e., \(\bar{\epsilon} = \Pr[\gamma < 2^{k/N} - 1] = F_r(2^{k/N} - 1)\). Unfortunately, this is not yet completely understood by the scientific community, which often adopts \(E_r[\epsilon(\gamma)]\) in situations where \(F_r(2^{k/N} - 1)\) is very accurate and much easier to evaluate.

Next, we present an application example of the above complete framework and some follow-up discussions.
Example 8 (Error-Constrained FBL Power Control).

Assume a node A wants to transmit a data message of \( k \) bits through \( N \) channel uses to a node B. For this, A has already acquired the CSI of the communication link. Specifically, A perfectly knows the value of \( \gamma' \), which is defined as the receive SNR at B given a normalized transmit power. What is the minimum transmission power that A must set to ensure a decoding error probability \( \epsilon \) at B?

In this case, both the transmission rate \( r = k/N \) and the error probability \( \epsilon \) are fixed. Also, given a transmit power \( p \), the receive SNR at B becomes \( \gamma = \gamma'p \). Therefore, the problem reduces to finding the required SNR \( \gamma \) such that (54) (or alternatively (52)) holds. It has been shown in (52) that the required \( \gamma \) is a fixed point solution of

\[
\gamma^{(t+1)} = 2^{N+V(\gamma')Q^{-1}(\epsilon)/\sqrt{N}} - 1,
\]

where \( t \) is the iteration index. Here, one can initially set \( \gamma^{(0)} \to \infty \) for which \( V = \log_2 e \) according to (53). The corresponding iterative procedure was shown in (52) to converge fast, i.e., in no more than five iterations for \( N \leq 1000 \) with an accuracy superior to 99.9%. Finally, after solving (55), the required transmit power is given by \( p = \gamma/\gamma' \).

In the previous example, the solution of (55) becomes \( \gamma = 2^k/N - 1 \) when \( N \to \infty \) (and also \( k \to \infty \) such that \( k/N = r \) is fixed). This agrees with Shannon’s framework in which the error probability becomes arbitrarily small when the received SNR exceeds \( 2^k/N - 1 \). Here, an interesting question is: how much greater must the SNR at a FBL N be in comparison with the asymptotic result? The answer comes by defining and simplifying the quotient between the required SNR at FBL and infinite blocklength as (52)

\[
\delta \triangleq \frac{2^N + \sqrt{V(\gamma')Q^{-1}(\epsilon)/N} - 1}{2^N - 1} = \frac{2V(\gamma')Q^{-1}(\epsilon)/\sqrt{N} + 2^NQ^{-1}(\epsilon)/N - 1}{2^N - 1}.
\]

Interestingly \( \delta \) is a decreasing function of \( r \) (52), thus

\[
\delta \geq \delta_0 = \lim_{r \to \infty} \delta = e^{Q^{-1}(\epsilon)/\sqrt{N}},
\]

which exploits the fact that \( r \to \infty \) implies \( \gamma \to \infty \), for which \( V(\gamma) \to \log_2 e \). Fig. 13 corroborates this behavior, and already evinces the convergence of \( \delta \) to \( \delta_0 \) for \( r = 4 \) bpcu. Notably, the main remark from (57) and Fig. 13 is that the SNR has to be at least \( e^{Q^{-1}(\epsilon)/\sqrt{N}} \) times greater than the asymptotic threshold of \( 2^r - 1 \) to reach an error probability no lower than \( \epsilon \) while transmitting the information through \( N \) channel uses. Obviously, this criterion is also applied to the transmit power.

B. Encoding of data and metadata in URLLC transmissions

Most wireless systems group the information bits to be transmitted into transmission blocks whose duration is sufficiently short compared to the channel coherence time. Thus, all bits in the block experience the same channel condition. The bits in a transmission block are of two types, metadata, i.e., the control information needed to decode the packet, and the intended data message.

In conventional MBB transmissions, the modulation and coding scheme of the data message within each transmitted block is selected to meet a target block error rate (BLER). It has been shown that for a large range of system parameters, a 10% BLER leads to near-optimal performance (26). In contrast, the metadata is transmitted with a much lower BLER target, which means that the contribution of the metadata in the final error probability calculations is negligible.

This is no longer the case for URLLC, where both metadata and the data have to be transmitted with ultra-low BLER targets. Instead, the final success probability is the product of the probabilities of successfully decoding both metadata and the data, i.e.,

\[
P_s = (1 - P_{e,m})(1 - P_{e,d}),
\]
where $P_{e,m}$ and $P_{e,d}$ are the error probabilities for the metadata and the data, respectively. The success probability can be further improved via retransmissions in the event of a failure \[53\]. In this case too, the error probability of the feedback link should not be ignored. The success probability $P_{s,n}$ after the $n$-th ($n > 1$) retransmission is given by

$$P_{s,n} = P_{s,n-1} + \left(1 - P_{e,m}\right)P_{e,d} \left(1 - P_{e,f}\right) \frac{\text{metadata decoded, data failed}}{\text{success after retransmission}} \right) ,$$

where $P_{e,f}$ is the error probability of the feedback link. Eq. \[53\] indicates that a feedback is only possible if the metadata is successfully decoded, allowing the receiver to establish the transmitter and the packet identity. It is worth mentioning here that this only holds under the assumption of independence between subsequent transmission slots. Such assumptions may not hold under very stringent latency constraints where the retransmission occurs within the channel coherence time \[53\].

Let us illustrate the importance of considering the metadata error probability in the success probability calculation through an example.

**Example 9 (Impact of the Metadata Error Probability).** Consider two services: i) MBB transmissions characterized by $\{P_{e,m} = P_{e,f} = 10^{-3}, P_{e,d} = 0.1\}$, and ii) URLLC transmissions characterized by $\{P_{e,m} = P_{e,f} = P_{e,d} = 10^{-3}\}$. The success probabilities up to the first retransmission ($n = 1$) is shown in Table VIII for the case when the error probability of the metadata is considered (the middle column) and compared against the case when the metadata is ignored (the right column). We can observe that the difference between the two cases (considering vs. neglecting the metadata error probability) is almost negligible for conventional MBB transmissions, whereas it is several orders of magnitude for URLLC transmissions. This highlights the importance of considering the metadata error probability when analyzing URLLC transmissions.

| Transmission type | metadata considered | metadata neglected |
|-------------------|---------------------|--------------------|
|                   | $n = 0$             | $n = 1$            |
|                   | $n = 0$             | $n = 1$            |
| MBB               | 0.9                 | 0.899              |
| URLLC             | 0.999               | 0.998              |

**C. Polar codes for FBL transmissions**

We have seen in Section \[\ref{PolarCodes}A\] that using FBL in URLLC transmissions reduces the maximum achievable rate. One of the reason for this is the reduced coding gain due to the reduced blocklength. Hence there is a need to study channel codes that perform well in the FBL regime. The channel code selected by 3GPP for MBB data transmissions, namely low-density parity check codes, are not particularly suitable for FBL scenarios \[13\]. Recently, 3GPP has adopted Polar codes for short blocks of metadata in MBB transmissions. Current investigations also demonstrated that Polar codes outperform low-density parity check codes in short block lengths and low code rates, therefore they might be appealing for URLLC use cases \[13\].

Polar codes, invented by Professor Erdal Arikan in 2009 \[54\], are the first codes with an explicit construction to achieve the channel capacity using low-complexity encoding and decoding. Specifically, low complexity successive cancellation and belief propagation decoding are used at the receiver side. A polar code $\mathcal{P}(k_c, k_d)$ is a code sequence containing a codeword symbol of $k_c$ bits carrying $k_d$ information bits. Channel polarization is proposed in \[54\] as a way to construct code sequences in order to achieve the symmetric channel capacity of the binary-input discrete memoryless channel.

Channel polarization refers to the fact that it is possible to synthesize the channel into a set of $k_c$ binary-input channels such that, as $k_c$ becomes large, a fraction of the channels becomes reliable (i.e., the symmetric capacity approaches 1) while the remaining fraction becomes unreliable (i.e., the symmetric capacity approaches 0). Thus, the information bits can be transmitted at rate 1 through the $k_d$ most reliable channels, while “dummy” frozen bits (usually zero) are transmitted through the unreliable channels. Reliable transmission can be achieved if $k_d$ is less than equal to the number of reliable polarized channels.

1) **Encoding of Polar codes:** The transformation matrix for a polar code of length $k_c$ is constructed through a log$_2 k_c$–Kronecker product of the polarization kernel $\mathcal{G} = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$.

As $k_c \to \infty$, this construction creates channels that are either perfectly noiseless or completely noisy. For smaller values of $k_c$, the synthetic channels polarization may be incomplete, generating intermediary channels that are only partially noisy.

2) **Decoding of Polar codes:** The successive cancellation algorithm used to decode Polar codes can be represented as a depth-first binary tree search with priority to the left branch, which has a decoding complexity of $\mathcal{O}(k_c \log_2 k_c)$ \[54, 55\]. The leaf nodes are the $k_c$ bits to be estimated. The soft information on the received code bits are input at the root node. Fig. \[14\] shows the decoding tree of a $(8,4)$ Polar code where bits numbered 1,2,3,5 are frozen. The black leaf nodes represent information bits and the white ones are the frozen bits.

Each node at stage $t_p$ uses the $2^{p-n}$-length soft input vector $\alpha_{t_p}$, received from its parent node, to calculate the soft output $\alpha_{t_p-1}$ and forward it to its left child. The $i$-th element of $\alpha_{t_p-1}$ is given by $\alpha_{t_p-1}^i = f(\alpha_{t_p-i}, \alpha_{t_p,i+2^{p-1}})$, with $f(a, b) = \text{sgn}(a)\text{sgn}(b) \min(|a|, |b|)$. The soft output $\alpha_{t_p-1}$ to the right child is calculated by combining $\alpha_{t_p}$ with the
2^{t_p-1}$-length hard decision $\hat{\beta}_i^{t_p-1}$ received from its left child, where $a_i^{t_p-1} = g(\alpha_i^{t_p-1}, \beta_i^{t_p-1}, \gamma_i^{t_p-1}, \delta_i^{t_p-1})$ with $g(a, b, c) = b + (1 - 2c)a$. The final hard decision vector is then obtained as $\beta_i^{t_p} = [\hat{\beta}_i^{t_p-1} \oplus \hat{\beta}_i^{t_p-1}, \hat{\beta}_i^{t_p-1}]$. Lastly, the hard decision at the $i$-th leaf node is given by $\hat{u}_i = \hat{\beta}_i^{t_p} = (1 - \text{sgn}(\alpha_i^{t_p})) / 2$; while frozen bits are always decoded as zero.

The BLER of Polar codes for different codeword sizes $(k_c)$, data size $(k_d)$, and different coding rates $(k_d/k_c)$ are illustrated in Fig. 15. The codeword and the data sizes are chosen to cover a range of coding rates, from $\sim 0.24$ to a value close to one. The downlink transmission and quadrature phase shift keying (QPSK) modulation are considered. The BLER performance results indicate that polar codes can deliver very low BLERs in the FBL regime with a blocklengths in the order of 100–200 bits. The performance is compared against the ideal FBL error rates obtained by Eq. (54). We observe that the performance gap is small with robust coding whereas a larger gap is seen for coding rates close to one. It is worth highlighting that even lower BLERs can be achieved through retransmissions (cf. Section V-B), albeit at the cost of higher latency.

The recursive nature of the successive cancellation decoding algorithm may impose a large latency and low error-correction performance in the FBL regime [95]. Different approaches have therefore been proposed to enhance the coding efficiency and reduce the latency for URLLC applications. For example, the binary-tree can be pruned to decrease the number of calculations, thereby reducing the decoding latency and complexity [96]. Altering the polar code construction is another way to further reduce the latency of polar codes, though this induces a trade-off with the error-correction performance [97]. This can be addressed by taking advantage of the high degree of parallelism to provide a favorable tradeoff between throughput and energy efficiency at short to medium block length [98], or through a more efficient memory utilization [99].

### D. GRAND: a generic maximum likelihood decoder

In this last subsection, we briefly introduce a novel low-latency decoding algorithm called guessing random additive noise decoding, GRAND for short. Traditional channel codes are co-designed with specific decoders that use the codebook structure to decode the received encoded signal, which may limit their applicability. Recently, GRAND has been proposed as a new algorithm for realizing maximum likelihood decoding in discrete channels with random codebooks [100]. The algorithm operates by subtracting ordered noise sequence from the received signal and estimating the first instance that results in a member of the codebook to be the transmitted message.

The basic principle of the GRAND algorithm is as follows. Let $X^N, Y^N$ and $Z^N$ be RVs representing the coded transmitted message, received message and random noise of blocklength $N$ over a discrete channel. The arbitrary codebook $\mathcal{C}_N$ used to generate the encoded message $X^N$ is commonly known to the transmitter and the receiver. Assume that the input output relation of the channel is reversible such that noise can be subtracted from the received message to recover the transmitted message, i.e.,

$$Y^N = X^N + Z^N \quad \Rightarrow \quad X^N = Y^N - Z^N.$$  

For a given received message $y^N$, the receiver orders the possible noise sequences from the most likely to the least likely, and then queries one by one whether the sequence $y^N - z_p^N$ is an element of the codebook $\mathcal{C}_N$. Here $z_p^N$ is the $p$-th ordered noise sequence. For the channel structure described above, irrespective of how the codebook is constructed, the first instance such that $x^N = y^N - z_p^N \in \mathcal{C}_N$ corresponds to the maximum likelihood decoding. GRAND operates by attempting to identify the random noise that has corrupted the transmitted message, and hence is independent of the code structure. It is inherently highly parallelizable, resulting in the low latency desired in URLLC and other similar applications [101].

GRAND’s accuracy depends on the ability to query the noise sequences, whose probability distribution depend on the channel statistics. For instance, in a binary symmetric channel, the relationship among the binary output symbol $y^N$, the binary input codeword $x^N$, and the independent binary additive channel noise $z^N$ is given in [60]. Considering the likelihood ordering determined by such a channel, the most likely noise sequence is the all zeros, followed by
VI. Queuing Theory & Information Freshness

URLLC is about taming the tail distributions of reliability and latency (see Section XIII), while considering the inherent trade-offs associated to energy consumption, throughput, and data freshness. All this makes the design/analysis of URLLC systems extremely challenging in practice, and motivates the exploitation of queuing theory metrics/tools such as effective capacity, Aol, and SNC, which can natively capture several of these trade-offs and constitute the scope of our discussions in this section.

A. Effective capacity

Metrics such as effective capacity and effective bandwidth capture tail statistical delay requirements in parallel with transmission throughput. Specifically, the effective capacity is defined as the highest arrival rate that can be served by the network under a particular latency constraint. Conversely, its dual twin, the effective bandwidth, characterizes the minimum service rate required to support the arrival of data in a certain network subject to a QoS constraint. Here, we focus on the foundations of effective capacity, which can be easily extended to effective bandwidth as well.

To begin with, the effective capacity, denoted as $C_e$, is a metric that captures the physical and link layers characteristics in terms of specific latency QoS guarantees, thus, allowing a further investigation of the latency-reliability trade-off. For low latency communication, $C_e$ is a powerful metric that characterizes the relation between the communication rate and the tail distribution of the packet delay violation probability $P_{out}$. A statistical delay violation model implies that for relatively large delay, an outage occurs when a packet delay exceeds a maximum delay bound $D_{max}$, and its probability is defined as:

$$P_{out} = \Pr\{\text{delay} \geq D_{max}\} \approx e^{-\theta C_e D_{max}}. \quad (61)$$

Meanwhile, system’s availability (or reliability in case of no repairs) is given by $A = 1 - P_{out}$. Conventionally, a system’s tolerance to long delay is measured by the delay exponent $\theta$. The system tolerates large delays for small values of $\theta$ (i.e., $\theta \to 0$) while for large values of $\theta$, it becomes more delay-sensitive.

The following examples aim to facilitate a practical understanding of the effective capacity and the corresponding trade-off between latency and availability in a real communication scenario.

**Example 10.** Consider 5G New Radio numerology 1 transmission with $C_e = 1$ bpcu, for which the symbol period is 35.7 $\mu$s. For a delay outage probability of $P_{out} = 10^{-5}$ (i.e., 99.999% availability), the network can tolerate, on average, a maximum delay of $\delta = 1151$ symbol periods ($\approx 41$ ms) for $\theta = 0.01$, and $\delta = 115$ symbol periods ($\approx 4.1$ ms) when $\theta = 0.1$.

**Example 11.** Consider a Rayleigh block fading channel with blocklength $T_f$. For a transmission rate $r$ and channel fading coefficient $|h|^2$, then

$$C_e(r, \theta) = -\frac{1}{\theta T_f} \ln \left( E_{|h|^2} \left[ e^{-\theta T_f r} \right] \right) \quad (62)$$

in bpcu. In case of fixed rate transmission with no CSI availability at the transmitter, a transmission outage may occur if the transmission rate is higher than the channel capacity. For Rayleigh fading channel, the mean outage probability for a transmission SNR of $\rho$ is given by

$$\epsilon = \Pr\{\log_2 (1 + \rho z) < r\} = 1 - e^{-\frac{r}{\rho}}. \quad (63)$$

Due to the fast channel variation, the transmission might be successful with probability of $1 - \epsilon$. In case of transmission failure, the transmission rate is effectively zero; then, the average effective capacity is given by

$$C_e(r, \theta) = -\frac{1}{\theta T_f} \ln \left( e + (1 - \epsilon) e^{-\theta T_f r} \right). \quad (64)$$

**Fig. 16.** Trade-off between availability, latency, and power consumption for 5G New Radio in Rayleigh fading.

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Each of those with one bit flip in any order, followed by those with two bit flips in any order, and so on [102].

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Fig. 16 depicts the trade-off between availability, latency, and power consumption from the effective capacity perspective for 5G New Radio numerology 1 with symbol period of 35.7 $\mu$s, and transmission rate of 0.5 bpcu. The maximum arrival rate is set to $C_e = 0.2$ bpcu. The figure shows a contour plot for the achievable availability-latency regions for different transmit power ranges. This reveals the contradicting achievability of URLLC transmission where achieving both high availability and low latency together requires high power consumption. We also observe that there is a dead zone of extremely low latency (below 1.5 ms) and high availability (>99%) that can not be achieved at the same time for this setup except at extremely high transmit power. For more details about how Fig. 16 was generated, one can refer to Algorithm [5].
B. Stochastic Network Calculus

SNC is a framework that enables the end-to-end analysis of networks by means of non-asymptotic performance bounds. SNC builds upon the theory of network calculus, which analyzes performance guarantees of queuing systems via convolutional forms based on (min,+) dioid algebra. Network calculus enables the derivation of work-consumption bounds such as backlog and delay. In the deterministic case, network calculus models the arrival and service process as envelope functions, and thus, it does not capture the stochasticity of the arrival and service processes. SNC addresses this issue by relaxing the deterministic assumptions introducing envelope violation probability [104]–[106]. Compared to conventional queuing theory, SNC is broader and includes several stochastic processes such as long-range dependent, self-similar, and heavy-tailed traffic. In addition, it can incorporate the variability of wireless fading channels.

Fidler and Rizk provide a comprehensive guide to SNC in [104]. Commonly, SNC resorts to envelope functions of the MGF of the arrival and service processes. Nonetheless, the framework is robust and supports general envelope models and models that provide strong guarantees. For instance, statistical delay analysis in fading channels is introduced [105] using a new representation based on the Mellin transform, and a (min,+) calculus, leading to tractable closed-form results that incorporate the channel variability to the model, yielding non-asymptotic performance bounds of the fading and arrival processes [105]. Due to its properties, SNC, and its simplified variants, effective bandwidth and effective capacity, are suitable for modeling end-to-end performance in URLLC (e.g., see [108]–[111]).

Next, we focus on the fundamental components of SNC, namely the arrival process (cumulative number of bits arriving) between times \( t \) and \( t + \tau \geq 0 \), \( A^\tau(t, t) \), the service process (e.g., dynamic server), \( S^\tau(t, t) \), and the departure process, \( D^\tau(t, t) \). We can relate these quantities as

\[
D^\tau(t) \geq \min_{r \in [0, t]} \left\{ A^\tau(r) + S^\tau(r, t) \right\},
\]

where \( A^\tau(0) = A^\tau(t, t) = 0 \) and \( S^\tau(0) = S^\tau(t, t) = 0 \), \( \forall \tau \geq 0 \), and both arrival and service functions are non-negative. In addition, assuming a first-come first-served approach, the delay bound at time \( t \geq 0 \) is

\[
W(t) \leq \min \left\{ w \geq 0 : \max_{r \in [0, t]} \left\{ A^\tau(r, t) - S^\tau(r, t + w) \right\} \leq 0 \right\}.
\]

For other arrival-service approaches see [104].

To calculate the actual delay bounds, we resort to i) affine traffic envelope functions, introduced in deterministic network calculus, and denoted as \( \varphi(t - \tau) + b \), where \( \varphi \) is a rate parameter related to the traffic and \( b \) is the burst parameter; and ii) MGFs, from effective bandwidth theory, since it determines the distribution of a random process, and the sum of two or more random processes can be calculated by the product of their MGFs. We write the MGF of the arrival process as

\[
M_A(\theta, t - \tau) = \mathbb{E}\left[ \exp\left( \theta A^\tau(t, t) \right) \right] \leq \exp\left( \theta (\varphi(t - \theta) + b) \right), \quad (67)
\]

where \( \theta \geq 0 \) is a free parameter. Similarly, we can write the MGF of the service process as

\[
M_S(\theta, t - \tau) = \mathbb{E}\left[ \exp\left( -\theta S^\tau(t, t) \right) \right] \leq \exp\left( -\theta (\varphi(t - \theta) + b) \right). \quad (68)
\]

Then, we calculate the delay bound as in [66] and upper bound the delay violation probability, i.e., \( P_{out} = \text{Pr}[W(t) > w] \) as in [76]. Note that when we introduce fading to the model, the rate expressions include logarithmic terms, which makes it cumbersome to find closed-form expressions for MGFs and the metrics. To alleviate this issue, [105] introduces a transformation facilities analysis under fading conditions.

\textbf{Example 12. The log normalized MGFs of the arrival and service process are known as effective bandwidth [104], and effective capacity [103], respective as}

\[
B_v(\theta) = \frac{1}{\theta} \ln M_A(\theta, t),
\]

\[
C_v(\theta) = -\frac{1}{\theta} \ln M_S(-\theta, t). \quad (70)
\]

Then, we can derive \( C_v \) as in [69] in Example 11.

Let us consider a simple arrival model known as two-state (ON-OFF) Markovian model. In this model, the data arrival process is described as a two-state discrete-time Markov chain, \( r \) bits arrive during ON state with an arrival rate of \( r \) bits/block, while no arrivals occur during the OFF state. Such system has a transition probability matrix \( \mathbf{P} = \left( \begin{array}{cc} p_{11} & p_{12} \\ p_{21} & p_{22} \end{array} \right) \), where \( p_{11} \in [0, 1] \) denotes the probability of staying in the off state, while \( p_{22} \in [0, 1] \) denotes the probability of staying on the ON state, while the transition probabilities are \( p_{21} = 1 - p_{22} \) and \( p_{12} = 1 - p_{11} \). At the steady state, the probability of ON state is \( p_{\text{ON}} = \frac{p_{11} p_{22}}{p_{11} p_{22} + p_{12} p_{21}} \) [112], [113]. Then, working out the MGF expressions for the arrival process, we obtain a closed-form expression for the effective bandwidth as

\[
B_v(\theta) = \frac{1}{\theta} \log \left( \frac{1}{2} \left( p_{11} + p_{22} e^{\theta} \right) \right.
\]

\[
+ \frac{1}{2} \sqrt{\left( p_{11} + p_{22} e^{\theta} \right)^2 - 4(p_{11} + p_{22} - 1)e^{2\theta}} \bigg) ^{(a)} - 1 - s + se^{\theta}, \quad (71)
\]

where \( (a) \) comes from a simplified version of the source with \( p_{11} = 1 - s \) and \( p_{22} = s \), hence \( p_{\text{ON}} = s \). Thus, the parameter \( s \) can be interpreted as a measure of the burstiness, which is relevant to model different traffic generated by a device. Then, the maximum average arrival rate is \( \bar{r}_{\text{max}} = r p_{\text{ON}} \).
For the particular case when $B_s(\theta) = C_s(\theta)$, we can calculate the maximum average arrival rate of the discrete-time Markov source supported by the wireless channel as

$$\tau_{\text{max}} = \frac{s}{\theta} \log \left( \frac{e^{\theta E_c(\theta)} - (1 - s)}{s} \right).$$

(72)

Note that the SNC is general and accommodates the cases when $B_s(\theta) \neq C_s(\theta)$ given queuing stability constraints [104], [105]. Moreover, this framework can be extend to other types of sources for the effective bandwidth, or in general for the SNC. The maximum average arrival rate enables us to characterize the impact of the variability of the arrival process as function of the service process. Therefore, when the effective capacity is optimized, as in Example 11, we assess the impact of the arrivals so that fluctuations are accommodated, reducing delay violation and latency. All in all, SNC constitutes a valuable framework for the statistical QoS delay and end-to-end performance analysis of URLLC networks, whose traffic diverges from conventional light tailed models as discussed in [4], [12].

C. Age of Information

AoI was introduced in [114] as a tool to measure the degree of freshness of the received data corresponding to a certain process. It is defined as the time elapsed since the latest received packet about a process under observation was generated. Hence, the AoI at time $t$ can be written as

$$\Delta(t) = t - t_g,$$

(73)

where $t_g < t$ is the generation time (i.e, timing anchor) of the most recently received packet. Herein, in order to maintain fresh information, AoI should be minimized.

Note that, the minimization of AoI is a non-trivial problem of great interest. In fact, system utilization can be maximized by making the source send more and faster updates which would lead to packets congestion and higher delay. In this case, delay can be reduced by decreasing the rate of updates. Alternatively, decreasing the update rate can lead to the destination having outdated information due to lack of updates. Hence, timely updating a destination about a remote process is neither the same as maximizing the system utilization nor reducing the delay.

The design of URLLC focuses more on peak and tail age metrics rather than the average AoI. In what follows, we discuss two important metrics of information freshness in time-sensitive applications, e.g., remote factory automation, where decisions should be based on fresh information.

1) Peak AoI: measure of the maximum AoI that could occur within a network. Therefore, the peak AoI measures the worst case scenario with respect to reliability. Many works have studied the peak AoI for different network models. According to [116], the average peak AoI for the M/M/1 model is given by

$$\tilde{\Delta}_{\text{M/M/1}} = \frac{1}{\mu} \left( 1 + \frac{1}{\rho_o} + \frac{\rho_o}{1 - \rho_o} \right),$$

(74)

where $\lambda$ is the arrival rate, $\mu$ is the service rate, $\rho_o = \frac{\lambda}{\mu}$ is the server utilization. Moreover, for the just-in-time transmission model, a packet is generated at time $t_i$ and received at $t_{i+1} = t_i + d_{i+1}$, where $d_{i+1}$ is the service time of packet $i$. Notice that a new packet is instantaneously generated by the source node and starts its service time right after the current update packet in service is received at the destination node. The average peak AoI of this model is given by [117]

$$\tilde{\Delta}_{\text{JIT}} = \frac{1}{N-1} \sum_{i=1}^{N-1} d_i + d_{i+1},$$

(75)

which is dependent only on the average service time.

2) Age survival function: complimentary CDF of the AoI, i.e., $\tilde{F}_{\text{AoI}}(t)$. It allows computing the probability that the AoI exceeds a certain threshold $\Delta_{th}$, i.e., $\tilde{F}_{\text{AoI}}(\Delta_{th})$, which is referred to as age violation probability. This is relevant for time-sensitive networking, where the age violation probability should not exceed a small value $\epsilon$, i.e.,

$$\tilde{F}_{\text{AoI}}(\Delta_{th}) = \text{Pr}[\Delta(t) \geq \Delta_{th}] \leq \epsilon.$$  

(76)

**Example 13.** Consider an M/M/1 last-come first-serve transmission with preemption. According to [118], the age survival function for this case is given by

$$\tilde{F}_{\text{ AoI}}(t) = \begin{cases} \frac{\lambda}{\lambda - \mu} e^{-\mu t} - \frac{\mu}{\lambda - \mu} e^{-\lambda t}, & \lambda \neq \mu \\ \frac{\lambda}{(\lambda + 1)} e^{-\lambda t}, & \lambda = \mu \end{cases},$$

(77)

which is depicted in Fig. 17 for different values of $\mu$ and $\lambda$. Obviously, the age survival function decays with the age, and more rapidly for higher values of $\lambda$ and/or $\mu$. For example, $\lambda = 3$ and $\mu = 3$, the age survival function is about $10^{-2}$ for $\Delta(t) > 2$ s. This means that the AoI exceeds 2 seconds for only 1% of the time. This is because the queue updates arrive and are served more rapidly resulting in fresher information at the receiver side. For the same setup, the age survival percentage becomes even lower ($\approx 0.5\%$) when increasing the service rate $\mu$ to 5. This is because the queue is able to deliver data faster which improves information freshness. Meanwhile, for $\lambda = 1$ and $\mu = 1$, the AoI exceeds 2 s for...
more than 50% of the time, since the amount of updates and service time are low, which results in less fresh data at the receiver.

VII. ANALYSIS & DESIGN OF LARGE-SCALE URLLC

Here, we overview some key mathematical and algorithmic tools for designing and/or assessing the performance of large-scale URLLC systems.

A. Meta Distribution

In general, the performance of wireless networks is highly influenced by their spatial configuration because phenomena like large and small scale fading, shadowing, and co-channel interference, are location dependent. Hence, it is not surprising that the stochastic geometry, which allows modeling the network as a point process, constitutes a fundamental theoretical tool for the analysis and characterization of large-scale wireless systems such as cellular, ad-hoc, vehicular, RF wireless power transfer, and satellite networks [19, 68, 119–131].

Within the stochastic geometry framework, the most popular performance metric is the success probability of the transmission over the typical link of a point process $\Phi$, which is defined as the probability that the corresponding SINR is greater than a given threshold $\gamma_{th}$, i.e., $p_s(x_{th}) \triangleq \Pr[\text{SINR} \geq \gamma_{th}] = 1 - F_{\text{SINR}}(\gamma_{th})$. However, large-scale URLLC systems cannot directly benefit from $p_s(\gamma_{th})$, neither for system design nor analysis, since the computation of such metric involves over space (and/or time) [11]. Instead, a more refined metric has been proposed and leveraged in the last years: the meta distribution of the SINR, which is defined as

$$p_m(\gamma_{th}, \xi) \triangleq \mathbb{P}[\Pr[\text{SINR} \geq \gamma_{th} | \Phi] \geq 1 - \xi].$$  \hspace{1cm} (78)

Here, $\mathbb{P}$ denotes the reduced Palm measure of $\Phi$, while $1 - \xi$, with $\xi \in [0, 1]$, denotes a target per-node success probability, where $\xi \ll 1$ in the context of URLLC. Note that $p_m(\gamma_{th}, \xi)$ denotes the fraction of users that attain an SINR of at least $\gamma_{th}$ in $(1 - \xi)$ fraction of the time, whereas $p_s(\gamma_{th})$ just characterizes the fraction of users that can communicate successfully. Observe that $p_s(\gamma_{th})$ can be obtained from $p_m(\gamma_{th}, \xi)$ as $p_s(\gamma_{th}) = \int_{0}^{1} (1 - \xi) p_m(\gamma_{th}, \xi) \, d\xi$, thus, $p_m(\gamma_{th}, \xi)$ is indeed a broader performance metric.

Example 14 (Meta-Analysis of a Large-Scale Network). Assume the setup described in Example 3 but where the communicating (typical) nodes are at a distance $r_0$ between each other while the interfering nodes are distributed according to a bi-dimensional homogeneous Poisson point process $\Phi$ with density $\lambda$. Thus, the total number of interfering nodes is infinite, i.e., $K \to \infty$, but the number of interfering nodes in a given area $A \subset \mathbb{R}^2$ is finite and follows a Poisson distribution with mean $\lambda A$. Ignore the noise power for simplicity, and assume no power control mechanism such that the average SNR (with respect to the Rayleigh small-scale fading) of the receive and interfering signal is given by $\gamma_k = r_k^{-\alpha}$, for $k \in \Phi \cup 0$, where $\alpha > 2$, thus $\mathbb{E}[\text{SNR}] = \frac{r_{th}^{-\alpha}}{\sum_{k \in \Phi} r_k^{-\alpha}}$. Then, by substituting $\gamma_k$ into (78), one obtains

$$p_m(\gamma_{th}, \xi) \leq \mathbb{P}\{\exp(-\gamma_{th}r_{th}^{-\alpha}) \geq 1 - \xi\} = F_V\left(-\frac{\ln(1 - \xi)}{\gamma_{th}r_{th}^{-\alpha}}\right),$$  \hspace{1cm} (79)

where $v \triangleq \sum_{k \in \Phi} r_k^{-\alpha}$ is the average interference with respect to small-scale fading realizations (or alternatively, interference without fading [119]). Now, since $V$ is obtained via transformations to RV’s $\{r_k\}$, one may exploit lattices’ distributions, which were provided in [120] for Poisson networks, and/or a procedure exploiting the probability generating functional of a Poisson point process, to compute [79]. Unfortunately, this has been proved to be a cumbersome task for general path-loss exponents, i.e., $\alpha > 2$. Fortunately, for some special cases, e.g., $\alpha \in \{3, 4, 5, 6, 8, 10, 12\}$, $F_V(v)$ has been derived in closed- or semi-closed form [119, 121]. For instance, for $\alpha = 4$, the PDF of $V$ is given by [119]

$$f_V(v) = \frac{\pi \lambda}{2v^{3/2}} \exp\left(-\frac{\pi^2 v^2}{4v}\right).$$  \hspace{1cm} (80)

By leveraging (80), one can get back to (79) to bound $p_m(\gamma_{th}, \xi)$ as

$$p_m(\gamma_{th}, \xi) \leq \frac{1}{\sqrt{\pi}} \int_{0}^{\gamma_{th}r_{th}^{-\alpha}} f_V(v) \, dv = \frac{1}{\sqrt{2\pi}} \int_{0}^{\gamma_{th}r_{th}^{-\alpha}} \frac{\pi \lambda}{2v^{3/2}} \exp\left(-\frac{\pi^2 v^2}{4v}\right) \, dv \leq \frac{1}{\sqrt{2\pi}} \Gamma\left(1, \frac{\pi^2 - \ln(1 - \xi)}{\gamma_{th}r_{th}^{-\alpha}}\right),$$  \hspace{1cm} (81)

where (a) follows from solving the indefinite integral by exploiting [119, eq. (2.325.6)], while (b) comes from evaluating the extremes.

The above bound is expected to be tight because of the same reasons given in Example 4. This is corroborated in Fig. 15 where $p_m(\gamma_{th}, \xi)$ is plotted against $1 - \xi$. In Fig. 15, we do not only plot $p_m(\gamma_{th}, \xi)$ but also the success probability $p_s(\gamma_{th})$. The latter metric reveals that the average success probability in the network is 90% and 99.85% for a density of $10^{-3}$ and $10^{-7}$ nodes/m$^2$, respectively, but says nothing about the reliability performance per node. Instead, $p_m(\gamma_{th}, \xi)$ reveals directly the percentage of nodes achieving a reliability of $1 - \xi$. Based on the results in Fig. 15, we can observe that only 44% of the nodes are operating with a success probability not inferior to 99% when $\lambda = 10^{-5}$ nodes/m$^2$, while that percentage of nodes increases up to 99.4% when $\lambda = 10^{-7}$ nodes/m$^2$.

Note that $p_m(\gamma_{th}, \xi)$ can be leveraged to properly adjust the per-link reliability via rate control, which does not affect the SINR realizations, thus potentially guaranteeing
network-wide reliability guarantees \[129\]. Moreover, similar work, the so-called guarantee. In fact, outside the stochastic geometry framework, transmission attempts to accomplish a success or reliability guarantee. While the secrecy rate in wireless-powered networks \[127\], the meta distribution of other relevant figures, e.g., the SINR or SIR meta distribution, one can formulate an expression for the meta-probability is leveraged in \[3, 4\], at least implicitly, to impose stricter and more firm guarantees on URLLC design/performance.

Unfortunately, MC methods for estimating \(p_m(\gamma_{\text{th}}, \xi)\) are in general computationally-expensive as they require separate averaging over fading and point realizations. Meanwhile, analytical closed-form characterizations are not often attainable, as in case of Example 14 for any \(\alpha > 2\). Therefore, efficient numerical and approximate analytical methods for obtaining \(p_m(\gamma_{\text{th}}, \xi)\) take significant relevance here. The most prominent methods include moment-matching, the Gil-Pelaez theorem, and bounding approaches such as Markov inequalities and Paley-Zygmund bound (refer to Section III \[66\], \[123\]). Interestingly, the beta distribution (by default defined in \([0, 1]\)) and the Fourier-Jacobi expansion framework fit typical meta distributions with high accuracy in most cases \[123\].

**B. Clustering**

Clustering relies on building groups or hierarchies among points/observations based on their (dis-)similarities. The main clustering methods and example algorithms are summarized in Table IX and described briefly in the following.

1) **Partitioning methods**: The most popular partitioning method, K-Means (K-Modes for categorical data), iteratively relocates the cluster centers by computing the points’ mean (mode). K-Means scales well on huge data sets, even though, it is very sensitive to outliers and fails to perform well on arbitrary shapes of data. These issues are mitigated by K-Medoids, which chooses actual data points as centers (medoids or exemplars), and can be used with arbitrary dissimilarity measures.

2) **Hierarchical methods**: There are two main types of hierarchical methods \[133\]: i) agglomerative, where each point starts in its own cluster, and pairs of clusters are merged as one moves up the hierarchy; and ii) divisive, where all points start in one cluster, and splits are performed recursively as one moves down the hierarchy. For clusters’ merging/splitting, not only a distance metric, but also a linkage criterion specifying the dissimilarity of clusters as a function of the pairwise distances of points in the clusters, must be specified. Merges/splits are often determined in a greedy manner, and the results are presented in a dendrogram.

3) **Other methods**: In density-based clustering, the points that are highly dense are grouped together, which allows identifying low-density/extreme points. Meanwhile, model-based approaches involve applying a model to find the best cluster structures.

In the context of large-scale URLLC, clustering algorithms can be mainly leveraged for:

- Configuration of NOMA groups.
- Pilot allocation. Assigning orthogonal pilots to all users becomes inefficient, or even unaffordable, as the network grows. This issue can be addressed by resorting to clustering based on i) channel covariance matrices such that the pilots are reused among the users having sufficiently orthogonal channel subspaces \[135, 136\], and/or ii) traffic profiles such that devices with smaller activation probabilities (more stringent reliability requirements) can be grouped in clusters with smaller (greater) pilot pools \[16\].
- Data aggregation.

In large-scale systems, devices may be clustered in...
Moreover, assume that there is a set \( \mathcal{N} \) of devices. A collision occurs if at least two devices attempt to access the medium by exploiting the same orthogonal resource, thus, the collision probability within the \( l \)-th spectrum resource is given by

\[
P_{\text{col}}^{(l)} = \sum_{i \in \mathcal{N}_l} A_{i,l} \prod_{j \in \mathcal{N}_j, j \neq l} (1 - A_{i,j}),
\]

while

\[
P_{\text{col}} = \frac{1}{L} \sum_{l \in \mathcal{L}} P_{\text{col}}^{(l)}
\]

denotes the average collision probability in the network.

The optimum resource allocation comes from solving \( \arg\min_{\mathcal{N}_l \times \mathcal{L} \times \mathcal{L}} P_{\text{col}} \), which is an NP-hard problem due to its combinatorial structure. Interestingly, a sub-optimal but also scalable and simple approach relying on K-Medoids clustering can be designed by adopting \( d_{i,j} = \|i \neq j\| A_{i,j} \) as dissimilarity measure between points (devices) \( i, j \in \mathcal{N} \). In this way, two devices with small joint activation probability are likely to be clustered together. Fig. 19 illustrates the attainable performance gains of such an approach with respect to a traditional pure random allocation. Here, we would like to emphasize that the adopted K-Medoids approach is by no means optimized, and better results may be attainable by, e.g., more tightly coupling the dissimilarity measure with the problem’s objective.

C. Compressed Sensing

Compressed sensing comprises a set of mathematical tools for either fully or partially recovering sparse signals from a small set of measurements. The sparsity \( f(\mathbf{s}) \) of a given signal \( \mathbf{s} \) is defined as the number of non-zero entries, i.e., \( f(\mathbf{s}) = ||\mathbf{s}||_0 \). Moreover, \( \mathbf{s} \) is considered to be sparse if \( f(\mathbf{s}) \) is sufficiently small compared with the dimension of \( \mathbf{s} \).

Assume \( \mathbf{s} \in \mathbb{C}^n \) is sparse, i.e., \( f(\mathbf{s}) \ll n \), and needs to be estimated from \( m < n \) samples that are collected via a sensing matrix \( \mathbf{H} \in \mathbb{C}^{m \times n} \) as

\[
\mathbf{y} = \mathbf{Hs} + \mathbf{w},
\]

where \( \mathbf{y} \in \mathbb{C}^m \) and \( \mathbf{w} \in \mathbb{C}^m \) denote respectively the receive and noise signals. Then, compressed sensing problems are commonly formulated to find the sparsest signal such that the estimated measurement noise power is upper bounded by a pre-configured noise level \( \eta \) as

\[
\mathbf{s}^* = \arg\min_{\mathbf{s}} f(\mathbf{s}), \text{ s.t. } ||\mathbf{y} - \mathbf{Hs}||_2^2 \leq \eta.
\]

Observe that the number of possible vectors \( \mathbf{s} \) that satisfy \( ||\mathbf{y} - \mathbf{Hs}||_2^2 \leq \eta \) is potentially infinite since \( \mathbf{y} = \mathbf{Hs} \) is an underdetermined system of equations. However, the sparsity-reduction objective function forces the solution to be unique, the sparsest one. Unfortunately, since \( f : \mathbb{C}^n \to \mathbb{C}^{m \times n} \), there is no way but to rely on a combinatorial search to get the optimum solution of (85). The complexity of this approach increases exponentially in \( n \), thus, the \( \ell_0 \)-norm minimization approach is infeasible for most real-world applications.
Several techniques and heuristics have been suggested in the last few years to solve \cite{85}, including regularization, greedy, and message-passing approaches. Specifically,

- regulation techniques, e.g., the $\ell_1$-norm approximation of the $\ell_p$-norm \cite{148}, \cite{149} and the alternative direction method of multipliers \cite{150}, rely on transforming \cite{85} into a convex problem via regularized, often iterative, procedures;
- greedy algorithms, e.g., orthogonal matching pursuit \cite{151}, make a local optimal selection at each time with either the hope of ultimately finding the global optimum solution, or just a sufficiently good local solution with low complexity and/or in a reasonable amount of time; and
- message-passing algorithms exploit factor graphs \cite{152}, thus, the a posteriori distribution of the signal to be reconstructed.

In wireless communications, compressed sensing has been exploited for solving problems related to sparse estimation/detection and support identification \cite{147}, \cite{153}–\cite{155}. The latter relates to a partial recovering of $s$, specifically, of the indexes of the non-zero elements of $s$, and it is particularly relevant here as it can mimic the multi-user detection problem in massive MTC systems with grant-free and sporadic random access. We next discuss an example linking both the multi-user detection and support identification problems. A simple approximate message-passing (AMP) solving algorithm relying on iterative thresholding is also provided. Readers are encouraged to refer to \cite{147} for additional examples on how the compressed sensing framework can be applied to wireless communications-related problems.

**Example 16 (Sparse MTC Activity Detection \cite{153}).**

Consider the uplink communication between a coordinator with $M$ antennas and $N$ single antenna devices, out of which only a few are simultaneously active. Quasi-static fading is assumed, and the small-scale channel between the $n$-th device and the coordinator is denoted as $h_n$. The coherence interval comprises $\tau$ samples, out of which $\tau_p$ are used for user identification (and potentially channel estimation). The coordinator is aware of the pilot sequence associated to each device, whereas $h_n$ coefficients, which change independently between consecutive coherence intervals, are unknown.

The signal $Y \in \mathbb{C}^{\tau \times M}$ received at the coordinator in a given user identification phase can be written as

$$Y = \sum_{n=1}^{N} \sqrt{\tau_p} \gamma_n \alpha_n \varphi_n h_n^T + W = \sqrt{\tau_p} \Phi \Psi^T + W$$  \hspace{1cm} (86)

where $\alpha_n$ denotes the device activity indicator for device $n$, $\gamma$ is the average SNR of the signal received at the coordinator from each active device, $\sqrt{\tau_p} \varphi_n \in \mathbb{C}^{\tau}$ with $||\varphi||_2^2 = 1$ is the pilot sequence of the $n$-th device, and $W$ is the power-normalized AWGN samples. Observe that the signals arrive at the coordinator with the same average SNR, which can

**Algorithm 4: AMP for MTC activity detection**

_\hspace{1cm} Input: $Y, \psi$

1. Initialization: $\hat{\alpha}^0 = 0$, $R^0 = Y$, $t = 0$

2. repeat

3. $\hat{S}^{t+1} = \partial((R^t)^H \varphi_n + S_n^t)$, $\forall n$

4. $R^{t+1} = Y - \Phi (\hat{S}^{t+1})^T + \frac{1}{\tau_p} R^t \sum_{n} \partial((R^t)^H \varphi_n + S_n^t)$

5. $t = t + 1$

6. until convergence or maximum number of iterations;

7. $\forall n: \hat{\alpha}_n = 1$ if $||\hat{S}_n^t||_2 \geq \psi$, $\hat{\alpha}_n = 0$ otherwise

_Output: $\{\hat{\alpha}_n\}$

be motivated by the use of a statistical inverse power control at the devices. Moreover, $\Phi = [\varphi_1, \varphi_2, \ldots, \varphi_N] \in \mathbb{C}^{\tau \times N}$ constitutes the pilot matrix, while $S = [s_1, s_2, \ldots, s_N] \in \mathbb{C}^{M \times N}$, where $s_n \triangleq \alpha_n h_n$, is an effective channel matrix. Here, $S$ has a sparse structure as the rows corresponding to inactive users are zero. Therefore, the activity detection problem reduces to finding the non-zero rows of $S$.

Algorithm 4 iteratively solves the above problem. Note that $\theta(\cdot)$ is a denoising function, which for the multi-user detection problem can be defined as in \cite{153}, eq. (11). Meanwhile, $\theta(\cdot)'$ is the first order derivative of $\theta(\cdot)$ and $R^t$ is the signal residual at iteration $t$. In step 3, all the devices are assumed inactive, while the estimate $\hat{S}' = [\hat{s}_1', \hat{s}_2', \ldots, \hat{s}_N']$ and the residual signals are then iteratively updated in steps 4 and 5 respectively. In the case of the latter, a crucial term containing $\theta(\cdot)'$, called the Onsager term, is included as it has been shown to substantially improve the performance of the algorithm. Observe that the estimated activity indicators are returned as output of the algorithm after hard-thresholding. Specifically, the $\ell_2$-norm of each $\hat{s}_n'$ is compared with a given threshold to decide whether the $n$-th device is active.\footnote{The channel estimates could be easily obtained as well as they match the entries of $S$ associated to the non-zero activity indicators. Nevertheless, they could be still refined in a last step by leveraging traditional MMSE channel estimators but considering only the group of detected devices.}

Unfortunately, AMP algorithms such as that illustrated in Algorithm 4 face several inconveniences. On the one hand, prior distributions of the device activity patterns and channels are usually unknown. On the other hand, the design of the denoising function $\theta(\cdot)$ becomes tedious, if not impossible, in most practical cases of known prior distributions. Indeed, typical prior assumptions that are difficult to handle include correlated activation patterns and/or other than zero-mean Gaussian channels. To alleviate these issues, expectation propagation algorithms \cite{156}, \cite{157} have emerged as a more flexible (less model-dependent) approach. Moreover, it should be noted that although the basic expectation propagation algorithms are in general more computationally complex than the AMP several state-of-the-art complexity-reduction techniques (e.g., see \cite{158}–\cite{161}) may be implemented to make them even more appealing in practice.

Notice that configuring the hyperparameters of the algorithm might also be quite challenging, specially for URLLC
systems with stringent performance requirements. In case of Algorithm 4, the main issue is related to properly configuring the decision threshold $\psi$. Notice that a relatively large (small) $\psi$ reduces the probability of misdetection (false-alarm), but at the cost of triggering many false-alarm (misdetection) events. In fact, this is not only an issue of AMP implementations, but of most state-of-the-art procedures, as those based on regulation techniques, greedy and other message-passing algorithms.

In general, prior procedures aiming at estimating certain raw features of $s$ can potentially mitigate the previous issue and assist the posterior estimation based on compressed sensing. For instance, the authors in [162] proposed coordinated pilot transmission mechanisms that can be run prior to the multi-user detection phase, as that described in Example 16 to detect the signal sparsity level (number of active devices), i.e., $\kappa = 1 + \epsilon$ and $Pr(\kappa = 0) = 1 - \epsilon$, and Bernoulli pilots as in [153]. Moreover, we set $t_p = 48$, $M = 64$, $N = 200$, and $y = 20$ dB.

D. Mean Field Game Theory

Mean-field (MF) game theory is an attractive tool for describing and optimizing networks with an asymptotically large number of inter-device interactions [163].

Consider a very large set $\mathcal{N}$ of nodes, and let $s \in \mathcal{N}$ to represent a rational node/agent with associated set of states $\mathcal{X}_s \subset \mathbb{R}^n$ and set of actions $\mathcal{A}_s$. Then, one can formulate a strategic game $\mathcal{G} = (\mathcal{N}, \mathcal{A}, \mathcal{X}, \{u_s\})$, where $\mathcal{A} = \mathcal{A}_1 \times \cdots \times \mathcal{A}_|\mathcal{N}|$ and $\mathcal{X} = \mathcal{X}_1 \times \cdots \times \mathcal{X}_|\mathcal{N}|$ denote the overall action and state profiles, respectively, while $u_s$ is the average utility function of $s$. While traditional game theory approaches consider the individual interactions among agents, MF game theory takes advantage of the massiveness of the network assuming that $|\mathcal{N}| \rightarrow \infty$ to approximate the solution.

The common underlying assumptions are:

- at any given time instance $t \in [0, T]$, individual agents’ states $x_s(t) \in \mathcal{X}_s$ and actions $a_s(t) \in \mathcal{A}_s$ have a negligible impact on the game;
- the optimization criterion is invariant to the permutation of agents’ indices;
- agents are indistinguishable, i.e., they share a common state and action space. This allows reducing the game to a generic agent with states $\bar{x}(t) \in \mathcal{X}$ and action $\bar{a}(t) \in \mathcal{A}$ playing against the MF distribution.

Let the state $\bar{x}(t)$ to evolve according to $\dot{\bar{x}}(t) = (1 + \Lambda)dx(t)dt + \Delta dw(t)$, where $\mu : [0, T] \rightarrow \mathbb{R}^n$ is a deterministic function, $w : [0, T] \rightarrow \mathbb{R}^n$ is a random process, and $\Lambda \in \mathbb{R}^{n \times n}$ is a constant matrix. Given a state transition from time $t \in [0, T]$ to $T$: $\bar{x}(t) \rightarrow \bar{x}(T)$, the utility function for the generic agent is given by

$$u(t, \bar{x}(t), \bar{a}(t)) = \mathbb{E}_x \left[ \int_t^T f(r, \bar{x}(t), \bar{a}(t))dr + c(T, \bar{x}(T)) \right],$$

where $f : [0, T] \times \mathcal{X} \times \mathcal{A} \rightarrow \mathbb{R}$ is the instantaneous payoff, and $c : T \times \mathcal{X} \rightarrow \mathbb{R}$ is a terminal cost associated with reaching state $\bar{x}(T)$. The MF distribution, i.e.,

$$\rho(t, \bar{x}(t)) = \lim_{|\mathcal{N}| \rightarrow \infty} \frac{1}{|\mathcal{N}|} \sum_{i=1}^{|\mathcal{N}|} \mathbb{1}[x_s(t) = \bar{x}(t)],$$

which approximates the collective behaviour of the whole population, is an implicit argument of $f$.

To solve the game, agents depart from a local estimate of $\rho$ using other agents’ initial states, and then run an iterative procedure. The latter consists in alternating between solving the optimal action from the Hamilton-Jacobi-Bellman equation, i.e.,

$$\frac{\partial u(t, \bar{x}(t), \bar{a}(t))}{\partial t} + \max_{\bar{a}(t)} \left[ \mu^T(t) \frac{\partial u(t, \bar{x}(t), \bar{a}(t))}{\partial \bar{x}} \right] + f(t, \bar{x}(t), \bar{a}(t)) + \frac{1}{2} \text{tr} \left( \Lambda^2 \frac{\partial^2 u(t, \bar{x}(t), \bar{a}(t))}{\partial \bar{x}^2} \right) = 0$$

(89)

for a given MF distribution, and an improved estimate of the MF distribution from the Fokker-Planck-Kolmogorov equation, i.e.,

$$\frac{\partial \rho(t, \bar{x}(t))}{\partial t} + \frac{\partial}{\partial \bar{x}} \left[ \frac{\partial u(t, \bar{x}(t))}{\partial \bar{x}} \rho(t, \bar{x}(t)) \right] + \text{tr} \left( \Lambda^2 \frac{\partial^2 \rho(t, \bar{x}(t))}{\partial \bar{x}^2} \right) = 0$$

(90)

given the action obtained from (89). Notice that the solution of a MF game, known as MF equilibrium, is guaranteed to

Fig. 20. Average error rate as a function of the detection threshold. We have assumed Rayleigh fading channels, i.i.d activation of the devices with $Pr(\alpha_i = 1) = 1 - \epsilon$ and $Pr(\alpha_i = 0) = 1 - \epsilon$, and Bernoulli pilots as in [153].
converge to the Nash equilibrium in the asymptotic case $|\mathcal{S}| \to \infty$. However, since practical network deployments have a large but finite number of nodes, a MF game converges to an $\epsilon$-approximate Nash equilibrium in most of these settings. That is, some players may benefit when deviating from the equilibrium, thus violating the Nash equilibrium conditions, but the expected payoff gain will not be superior to $\epsilon$.

In MF games, no information is exchanged among agents; thus, this approach is effective in reducing the signaling overhead of the network. This is appealing for meeting the ultra-low latency requirements in massive/dense URLLC systems. Take as an example a ultra-dense network, where the cells may experience high levels of interference due to their proximity. This demands effective interference management techniques, which may induce a severe signalling overhead if the cells treat the interfering links individually. Instead, the problem can be recast as a MF game, which allows modeling the mutual interference experienced by all cells as an average interference over the state distribution. Therefore, each cell can locally determine a transmission policy without coordinating with its interfering cells.

With respect to the ultra-high reliability requirements in massive/scalable URLLC systems, one may need to incorporate the notion of risk or other tail statistics (refer to Section III in the instantaneous payoff function). Another interesting applications are those related to studying neural networks and multi-agent RL, and simplifying rare events simulations (refer to Section IV). Regarding the latter one, note that modeling how each agent interacts with its peers may require significant computational effort if traditional Monte Carlo methods or agent-based models are used. Fortunately, with MF game theory we can significantly reduce the complexity by abstracting the massive interactions in the evolution of one agent.

Among the main limitations related to MF games, there are the three main underlying assumptions from the framework enumerated above, and the issue that solving becomes computationally expensive for high-dimensional states due the curse of dimensionality. This limits the applications of classical MF game approaches for real-time applications and networks where devices have limited computational capabilities. In view of this, there is a growing interest in using DL or RL to solve MF games. Indeed, neural networks are used to approximate the optimal solutions of equations at the cost of violating the conditions for convergence of the MF game framework. In another approach, a FL-based MF game approach was proposed to allow the devices periodically exchange their model parameters so that convergence is guaranteed.

VIII. MACHINE LEARNING FOR URLLC

ML constitutes a popular set of tools, which are formally classified as supervised, unsupervised, and reinforcement learning (RL), for addressing data-driven problems. In the field of wireless communications, ML has been applied, for instance, in traffic and interference prediction, physical layer design, heterogeneous network traffic control, and multi-access edge communications. More recently, URLLC systems have also been designed/optimized using ML, and some of the proposed ML approaches are collected in Table X together with popular challenges, practical use cases that could benefit from solving them, and technology enablers.

It should be noted that ML tools may fall short in handling strict URLLC requirements if not wisely used. This is because:

- prior contextual information might be difficult to exploit;
- it can be hard to obtain large training data sets in case of URLLC applications with sporadic (rare) error events (cf. Section IV);
- the accuracy of most ML algorithms may not match the stringent reliability requirements; and the required time for (re)training can violate the strict latency constraints.

Fig. 21 illustrates the suitability of ML vs. conventional mathematics in solving different problems. For instance, RL adapts well to dynamic behaviours through a reward function, and leverages gradient methods to find efficient policies online, thus, it may be appealing for solving dynamic problems. Meanwhile, deep Q-learning is able to deal with the curse of dimensionality by sampling the state and action spaces and reducing the problem complexity. On the other hand, typical convex and near-convex problems may be easily solved using well-known methods. For URLLC applications, the recent advances in ML algorithms can be harnessed by adopting a model based learning framework which promotes integrating domain knowledge with learning. Here, certain aspects of the system can be assumed to operate under a known model, of which some parameters may be learned by applying ML algorithms.

A. Large-Scale Optimization

Conventional convex/non-convex optimization approaches do not scale well with the number of non-linear parameters, making them computationally prohibitive for solving large-scale/high-dimensional problems. Given this, the recent few years have witnessed a growing trend towards applying ML tools to solve such problems, including those related to wireless communications.
Decentralized ML approaches such as FL and multi-agent RL are attractive candidates for solving large-scale problems with high dimensional action and state spaces. For instance, Choi in [191] proposed a distributed stochastic gradient approach for large-scale MTC scenarios without CSI, which seems favourable for distributed URLLC applications. Moreover, such a distributed approach does not require user specific information because only the gradients are transmitted over the air and hence, data privacy is preserved.

Deep RL can be applied to reduce the state space in extremely high dimensional problems. A typical example is the large-scale UAV trajectory planning problem for serving massive MTC deployment with URLLC requirements, which has been recently addressed in works such as [192], where the authors applied combinatorial neural networks in order to optimize the user scheduling and AoI, and [193], where deep RL and device clustering are used to facilitate trajectory planning for multiple UAVs with the target of reducing the AoI and energy consumption of MTC networks. These works imposed maximum thresholds for AoI to guarantee minimum level of reliability.

### B. Learning with Limited Data Samples

In many URLLC scenarios, it is not feasible to obtain large data sets due to the transmission overhead involved, while a large number of trials are required to witness rare events. Unfortunately, conventional supervised learning schemes are not adequate in such cases as they require large number of trials. Instead, meta-learning seems to be a more appealing tool as illustrated in [186]. Meta-learning refers to “learning to learn” and aims at solving many tasks that are different but related to each other through acquiring a common parameter \( \theta \) called the “prior” among the learning model. This would assist learning new tasks faster than learning them from scratch.

**Example 17 (ML-assisted CSI acquisition [194])**. Consider a channel estimation scenario, where URLLC devices sporadically transmit their uplink data to the base station using short packets with a few pilot symbols. In the context of channel estimation, meta learning could be applied to estimate a channel using only few pilot symbols. As shown in Fig. 22, the target is to estimate the channel and find an efficient demodulator for the meta-test device. Meta-learning would utilize the channel estimation of other meta-training devices to facilitate the channel learning of the meta-test device. Herein, the base station can make use of the channel estimation of meta-training devices in order to facilitate the channel learning of a meta-test device transmitting a few number of pilot symbols. A demodulator consisting of multi-layer neural network is constructed. Then the shared learning parameter \( \theta \) and the task specific parameter \( \phi \) are jointly and iteratively updated using a model-agnostic meta learning algorithm [194], which depends mainly on the following two gradient update equations for each device \( k \)

\[
\phi_k = \theta - \alpha \nabla L_{\text{test}}(\theta),
\]

\[
\theta = \theta - \beta \sum_{k=1}^K \nabla L_{\text{test}}(\phi_k),
\]

where \( \nabla(\cdot) \) is the gradient operator, and \( \alpha \) and \( \beta \) are the step sizes. For an input vector \( s \) and output vector \( y \), \( L \) is the cross entropy loss-function given by

\[
L(x) = - \sum_{s \in x, y \in y} \log p_x(s | y).
\]

The algorithm details and the results can be consulted in [186]. Therein, it is shown that the meta-learning assisted
The goal is then to estimate the parameters of the model from the observed data. A complete probabilistic time series model for the sequence of RVs corresponding to the observed samples would specify all of the joint distributions of the underlying RVs. In general, such a specification will contain far too many parameters to be estimated. Hence, usually only the first- and second-order moments of the joint distributions are specified. The general approach to time series modeling involves the following steps:

- Processing the data to get a stationary series. The processed stationary series is known as the residual.
- Choose a model to fit the residual, making use of various (first and second-order) sample statistics including the sample autocorrelation function.

Commonly used models in the literature include the autoregressive moving average (and its variants) [196], Markov chains [197], [198], Bayesian network [199], and Kalman filter [200].

Next, we present an example of a simple model-based interference prediction scheme enabling efficient allocation of resources for URLLC applications [197]. Specifically, the interference variation is modeled as a discrete-time Markov chain, while the state transition probability matrix, obtained by observing the history of state transitions, is used to predict future interference states and allocate resources accordingly.

EXAMPLE 18 (MODEL-DRIVEN RESOURCE ALLOCATION [197]). Consider the downlink of a wireless network where the URLLC link of interest operates in the presence of $N$ interferers. Single shot [201] transmissions are considered for the sake of low latency transmissions, i.e., there are no retransmissions.

The sketch of the interference prediction algorithm is pictorially outlined in Fig. 22. First, we discretize the interference space $\mathcal{I}$ into the state space $\mathcal{L} = \{\mathcal{I}_1, \ldots, \mathcal{I}_L\}$, where the state $\mathcal{I}_i$ corresponds to the interference values in the range $[I_{i-1}, I_i)$. Besides, we define $\hat{\mathcal{I}} \triangleq \{\mathcal{I}_0, \mathcal{I}_1, \ldots\}$ as the set of observed interference states at time index $t = 0, 1, \ldots$. For example, $\mathcal{I}_t = S_i$ implies that the interference state at time $t$ is $\hat{\mathcal{I}}$. Next, we compute the entries of the transition matrix $\hat{\mathcal{P}}$ which describes the transition probabilities of the states in $\mathcal{L}$ as

$$p_{ij} = \frac{\sum_{t=1}^T \mathbb{1}\{\mathcal{I}_{t+1} = \mathcal{I}_j \text{ \& } \mathcal{I}_t = \mathcal{I}_i\}}{\sum_{t=1}^T \mathbb{1}\{\mathcal{I}_t = \mathcal{I}_i\}} \quad \forall i, j \in \{1, 2, \ldots, L\},$$

where $p_{i,j}$ is the probability of transition from state $\mathcal{I}_i$ to $\mathcal{I}_j$.

Now, we introduce the confidence level parameter $\eta < 1$ to utilize the tail statistics of the interference distribution. We aim to ensure that the predicted interference at time $t+1$, $\hat{I}_{t+1}$, is greater than the actual interference, $I_{t+1}$, with probability greater than or equal to $\eta$, i.e., $\Pr[I_{t+1} \leq \hat{I}_{t+1}] \geq \eta$.

For this, we first predict the next state $\hat{\mathcal{I}}_{t+1}$ to be $\mathcal{I}_j$, where

A time series is said to be (weakly) stationary if its mean and the variance are independent of the time observation, i.e., the joint probability distribution does not change when shifted in time.
$j$ is the smallest integer such that $\sum_{l=1}^{j} p_{il} \geq \eta$. The predicted interference level at time index $t+1$ is then

$$I_{t+1} = \begin{cases} I_j & \text{if } j \neq L \\ 2I_{L-1} - I_{L-2} & \text{if } j = L \end{cases}$$

(95)

where $I_j$ is the right-endpoint of state $S_j$. Lastly, once a transition is made from state $S_i$ to state $S_j$, the transition matrix $P$ is updated as

1. $p_{ij} \leftarrow p_{ij} + \omega_{ij}$ for each $i, j$,

2. normalize $i$-th row such that $\sum_{j=1}^{L} p_{ij} = 1$,

where $0 \leq \omega_{ij} \ll 1$ is the learning rate.

Let $\hat{\gamma} = \frac{\text{SNR}}{\hat{I}}$ be the predicted SINR, where the SNR of the desired transmission is assumed to be known using CSI estimates. Using the FBL theory results detailed in Section V-A, one can obtain the channel uses that need to be allocated for a given number of information bits and a target outage probability. After the transmission takes place, the achieved outage can be calculated using Eq. (54), where $\hat{\gamma}$ is replaced by the actual SINR.

The outage probability performance attained with the outlined interference prediction based resource management algorithm vs. the target outage for $\eta = 0.95$ is illustrated in Fig. 24. The conventional weighted average based interference estimator adopted as an estimator in link adaptation for conventional MBB services [202] is considered as the baseline scheme. The proposed interference estimate based resource management scheme is found to always fulfill the target outage. On the other hand, the baseline scheme is only able to meet a BLER target of about 10%, which is the usual BLER target in MBB applications. Conversely, the number of information bits that can be transmitted with a desired decoding error probability for a given FBL size can be obtained using the FBL theory outlined in Section V, where $\gamma$ is set to be the predicted SINR value.

On the other hand, a data-based approach advocates learning the dependence structure from past data without the need to consider the underlying structure. Data-driven methods employ ML tools to predict future instances of the observed data. These include applying DL schemes like deep belief network [203], convolutional neural network [204], recurrent neural network [205], LSTM networks [206], [207], and the novel "transformer" architecture [208]. The latter may provide unprecedented support to tackle sophisticated and high dimensional time series problems, but much research is still needed for a comprehensive assessment of its potentialities.

Observe that model-based approaches have several advantages over data-based approaches, namely, i) they allow easily taking advantage of prior information; ii) they can be easily adapted to different environments even in the absence of training data; and iii) it can be gradually improved using training information. However, they are built on some
underlying assumptions, such as the stationarity of the residual and the validity of the assumed model. In contrast, a data-driven approach can capture the nonlinearity and randomness of actual time series, albeit usually at the cost of large training overhead and complexity. Nonetheless, the advantages of both approaches can be harnessed by integrating domain knowledge in data-based approaches.

IX. Conclusions & Future directions

URLLC is about taming the tail of reliability and latency under the uncertainty arising from the stochastic nature of wireless communications. Moreover, most URLLC use cases are very different from conventional human-type communications as characterized by traffic with smaller payloads and sporadic arrivals, large device density, and heterogeneous service demands. Thus, designing and/or analyzing URLLC systems mandates adopting a vastly different set of statistical tools and methodologies. Aiming to fulfill the corresponding gap in existing surveys and overview papers on URLLC, this article presented a unified framework for designing and/or analyzing URLLC. Specifically, several relevant statistical tools and methodologies were introduced, and their applications to URLLC were highlighted together with thorough discussions on how these tools can be used interdependently towards designing/analyzing URLLC systems. As a unique feature of the tutorial-like nature of this article, each of the introduced tools and methodologies was further elaborated through concrete numerical examples and selected applications. This article is targeted toward graduate students, early-stage researchers, and professionals interested in getting a comprehensive overview of the statistical tools and methodologies relevant to URLLC, and will help foster more research in URLLC and its evolution towards next-generation wireless systems. In what follows, we present some potential future research directions for each of the discussed tools and methodologies.

1) From URLLC to dependable communications: Current 5G approaches for meeting URLLC requirements based on tweaking the system design is not scalable nor efficient. This is specially evident for use cases related to emerging sophisticated cyber-physical systems, which rely on embedded, decentralized, real-time computations and interactions, where physical and software components are deeply intertwined. The design and analysis of such systems mandate a departure from link-specific URLLC to holistic dependability. For this, there is a series of associated challenges, which are mainly related to i) the scalable modeling of the different kinds of adverse events, e.g., errors, faults, failures (refer to), and their cross-correlation, ii) the extrapolation of dependability concepts and tools as those overviewed in Section III which are originally conceived for systems engineering, to wireless communications engineering in different domains/layers, and iii) the development of data-driven dependable mechanisms. Moreover, notice that a thorough joint optimization of the communication, control, computing, and sensing processes is fundamental for resource-efficient truly-dependable systems. Therefore, communication and control co-design, quantum computing and communication, and joint communication and sensing, are instrumental for this and may continue receiving dedicated research in the coming years. All in all, the notion of dependability is expected to play a more dominant role in beyond 5G wireless systems to provide a richer performance measure of critical systems.

2) Statistical bounds, inequalities, approximations, and risk measures on small sample sizes: The bounds, approximations, and risk-assessment tools discussed in Section III inherently rely on a statistical pre-processing of data samples. For instance, applying Markov’s inequality mandates computing the expected value of a certain random process, the GPD fitting of the conditional CDF is data-driven, and the evaluation of the VaR and CVaR risk metrics leverages the distribution and conditional expectation of an underlying random process, respectively. Therefore, the confidence in such approaches increases with the sample size. However, the number of samples corresponding to a certain relevant random process may be strictly limited in many URLLC use cases, which calls for novel probabilistic bounds/approximations/risk-metrics with accompanied confidence guarantees. As commented in Example 5 they may be designed using the same tools and approaches outlined in Section III but enriched with additional procedures considering the data finitude, e.g., exploiting the central limit theorem. Finally, the risk metrics discussed in Section III-C, i.e., VaR and CVaR, are by no means the only ones within the risk management theory, and further research is needed to identify and link more risk-related metrics to the design/analysis of URLLC systems.

3) Ultra-efficient sampling for rare event simulations: The methods discussed in Section IV rely on statistical models, including some underlying prior knowledge, for proper sampling. Therefore, efficient approximations/upper bounds and optimization of target distributions are needed to reduce computational complexity and/or increase accuracy. In addition, many URLLC use cases require efficient sampling to assess novel probabilistic metrics with a significant number of samples from the probability space where the URLLC event exists. Therefore, further research is needed to properly link efficient sampling algorithms and URLLC research. One direction may be using Hamiltonian Monte Carlo methods, particularly for use cases with large dimensions, as it is a non-Markovian-based method able to explore the probability space efficiently by avoiding the dependencies of the Markov chain. Similarly, variational Bayes methods, poorly explored in the URLLC literature, deserve more attention.

4) Fixed to variable length coding: The FBL channel codes considered in Section V are fixed-length codes where each codeword has the same number of bits. Variable-length codes, on the other hand, encodes message symbols to codewords with a variable number of bits. This has the advantage that the coding rate can be adjusted to the channel conditions leading to enhanced resource efficiency, albeit requiring a feedback channel.
of the feedback loop incurs extra latency and an additional error-source (as illustrated through the example in Section V-B). A recent study has shown that noise in the feedback link can cause a significant increase in the minimum average latency when applying variable-length codes to URLLC use cases [214]. This raises the question whether variable-length coding schemes are suitable for URLLC, or alternately how to design variable-length codes for URLLC applications with FBL transmissions?

5) FBL queueing and dependable AoI: Notably, SNC may help identify and define new delay-bounded QoS metrics incorporating the channel's and the arrival and service processes' stochasticity. However, the relationship between SNC and FBL theory/coding still requires further investigation, especially the characterization of the tail distribution and the delay bounds in extreme URLLC [111]. Meanwhile, open challenges in the context of information freshness include the characterization of AoI tail distributions for different network models, part of which were characterized in [118]. Moreover, future research should focus on critical decision making in inter-disciplinary problems such as communication control co-design, which demands timely and reliable fresh information. In such cases, the link between AoI and dependability metrics, such as mean time to first failure, deserves particular attention [118].

6) Strengthening the large-scale URLLC design/analysis: In Section VII-B, we only referred to hard-clustering, where a point can only belong to exactly one cluster. However, soft (or fuzzy) clustering [214, 215], where a point can potentially belong to multiple clusters, may be more appealing for assessing the inherent complexity/heterogeneity of wireless networks, specially URLLC. Meanwhile, tools and methodologies related to the meta distribution, compressed sensing, and MF games rely heavily on underlying models and assumptions. Since ultimately “all models are wrong” (quote from George Box), say imperfect, they must be investigated considering modeling imperfections/uncertainties. Finally, regarding Example 16 in Section VII-C and the discussions that followed it, we would like to emphasize that multi-user detection in grant-free URLLC systems can be assisted by a preliminary sparsity level estimation phase as discussed in [164]. However, there are numerous associated research directions yet to explore, including i) sparsity level estimators exploiting prior traffic history knowledge and/or optimized for MIMO operation, and ii) a joint optimization of the sparsity level estimation and multi-user detection phases since the total number of symbols for these tasks might be very limited and accuracy of the sparsity level estimation is intrinsically imperfect.

7) Towards Intelligent URLLC: The learning process in multi-agent RL depends only on the states and actions observed by each agent without or with partial information about the states and actions of the whole environment. Therefore, its application in large-scale systems may significantly reduce the action-state space and complexity, thus, the incurred latency, which is beneficial for URLLC. However, how to reduce the state and action spaces through either partial or no information sharing between the learning agents remains an open question. Further research is still needed to characterize the performance degradation resulting from the limited information of states and actions, the accuracy of distributed approaches, and the factors affecting their performance. Meanwhile, in another direction, accurate and intelligent prediction techniques that integrate model-driven approaches with ML/AI tools via deep unfolding principles must be developed, together with efficient resource management schemes that utilize the prediction information to enable URLLC without over-provisioning the system redundancy.

REFERENCES

[1] M. A. Lema, A. Laya, T. Mahmoodi, M. Cuevas, I. Sachs, I. Markendahl, and M. Dohler, “Business case and technology analysis for 5G low latency applications,” IEEE Access, vol. 5, pp. 5917–5935, 2017.

[2] H. Chen, R. Abbas, P. Cheng, M. Shirvanimoghaddam, W. Hardjawana, W. Bao, Y. Li, and B. Vucetic, “Ultra-reliable low latency cellular networks: Use cases, challenges and approaches,” IEEE Communications Magazine, vol. 56, no. 12, pp. 119–125, 2018.

[3] M. Angjelichinoski, K. F. Trillingsgaard, and P. Popovski, “A statistical learning approach to ultra-reliable low latency communication,” IEEE Transactions on Communications, vol. 67, no. 7, pp. 5153–5166, 2019.

[4] P. Popovski, C. Stefanović, J. J. Nielsen, E. De Carvalho, M. Angjelichinoski, K. F. Trillingsgaard, and A.-S. Bana, “Wireless access in ultra-reliable low-latency communication (URLLC),” IEEE Transactions on Communications, vol. 67, no. 8, pp. 5783–5801, Aug. 2019.

[5] G. J. Sutton, J. Zeng, R. P. Liu, W. Ni, D. N. Nguyen, B. A. Jayawickrama, X. Huang, M. Abolhasan, Z. Zhang, E. Dutkiewicz, and T. Lv, “Enabling technologies for ultra-reliable and low latency communications: From PHY and MAC layer perspectives,” IEEE Communications Surveys & Tutorials, vol. 21, no. 3, pp. 2488–2524, 2019.

[6] N. H. Mahmood, H. Alves, O. L. A. López, M. Shehab, D. P. M. Osorio, and M. Latva-Aho, “Six key features of machine type communication in 6G,” in 2nd 6G Wireless Summit (6G SUMMIT), 2020, pp. 1–5.

[7] N. Mahmood, O. López, O. Park, I. Moerman, K. Mikhaylov, E. Mercier, A. Munari, F. Clazzer, S. Böcker, and H. Bartz (Eds.), “White paper on critical and massive machine type communication towards 6G [white paper],” 6G Research Visions, vol. 11, 2020. [Online]. Available: http://urn.fi/URN:ISBN:9789526226781

[8] N. H. Mahmood, S. Boecker, I. Moerman, O. A. López, A. Munari, K. Mikhaylov, F. Clazzer, H. Bartz, O.-S. Park, E. Mercier et al., “Machine type communications: key drivers and enablers towards the 6G era,” EURASIP Journal on Wireless Communications and Networking, vol. 2021, no. 1, pp. 1–25, 2021.

[9] 3GPP TS 22.261 V16.0.0, Service Requirements for the 5G System, [Online]. Available: https://portal.3gpp.org/desktopmodules/Specifications/SpecificationDetails.aspx?specificationId=3107

[10] H. Ji, S. Park, J. Yeo, Y. Kim, J. Lee, and B. Shim, “Ultra-reliable and low-latency communications in 5G downlink: Physical layer aspects,” IEEE Wireless Communications, vol. 25, no. 3, pp. 124–130, 2018.

[11] R. E. Barlow and F. Proschan, Mathematical theory of reliability. Wiley, 1965.

[12] M. Bennis, M. Debbah, and H. V. Poor, “Ultrareliable and low-latency wireless communications: Tail, risk, and scale,” Proceedings of the IEEE, vol. 106, no. 10, pp. 1834–1853, 2018.

[13] M. Shirvanimoghaddam, M. S. Mohammadi, R. Abbas, A. Minja, C. Yue, R. Matuz, G. Han, Z. Lin, W. Liu, Y. Li, S. Johnson, and B. Vucetic, “Short block-length codes for ultra-reliable low latency communications,” IEEE Communications Magazine, vol. 57, no. 2, pp. 130–137, Feb. 2019.

[14] G. Durisi, T. Koch, and P. Popovski, “Toward massive, ultrareliable, and low-latency wireless communication with short packets,” Proceedings of the IEEE, vol. 104, no. 9, pp. 1711–1726, Sep. 2016.

[15] M. Alsenwi, N. H. Tran, M. Bennis, A. Kumar Bairagi, and C. S. Hong, “eMBB-URLLC: Resource slicing: A risk-sensitive approach,” IEEE Communications Letters, vol. 23, no. 4, pp. 740–743, 2019.
of Machine Learning Research, D. Precup and Y. W. Teh, Eds., vol. 70. PMLR, 06–11 Aug 2017, pp. 1126–1135. [Online]. Available: https://proceedings.mlr.press/v70/finn17a.html

[195] P. J. Brockwell and R. A. Davis, *Introduction to time series and forecasting*, 3rd ed. Switzerland: Springer, 2016.

[196] X. Luo, L. Niu, and S. Zhang, "An algorithm for traffic flow prediction based on improved SARIMA and GA," *KSCE Journal of Civil Engineering*, vol. 22, no. 10, pp. 4107–4115, Oct. 2018.

[197] N. H. Mahmood, O. L. A. López, H. Alves, and M. Latva-Aho, "A predictive interference management algorithm for URLLC in Beyond 5G networks," *IEEE Communications Letters*, vol. 25, no. 3, pp. 995–999, Mar. 2021.

[198] S. Zhang, Z. Kang, Z. Zhang, C. Lin, C. Wang, and J. Li, "A hybrid model for forecasting traffic flow: Using layerwise structure and markov transition matrix," *IEEE Access*, vol. 7, pp. 26,002–26,012, Feb. 2019.

[199] Z. Li, S. Jiang, L. Li, and Y. Li, "Building sparse models for traffic flow prediction: an empirical comparison between statistical heuristics and geometric heuristics for Bayesian network approaches," *Transportmetrica B: Transport Dynamics*, vol. 7, no. 1, pp. 107–123, 2019.

[200] J. F. Schmidt, U. Schilcher, M. K. Atiq, and C. Bettstetter, "Interference prediction in wireless networks: Stochastic geometry meets recursive filtering," *IEEE Transactions on Vehicular Technology*, vol. 70, no. 3, pp. 2783–2793, Mar. 2021.

[201] N. H. Mahmood, N. Pratas, T. H. Jacobsen, and P. E. Mogensen, "On the performance of one stage massive random access protocols in 5G systems," in *Proc. 9th International Symposium on Turbo Codes and Iterative Information Processing, (ISTC)*, Brest, France, Sep. 2016, pp. 340–344.

[202] G. Pocovi, B. Soret, K. I. Pedersen, and P. Mogensen, "MAC layer enhancements for ultra-reliable low-latency communications in cellular networks," in *IEEE International Conference on Communications Workshops (ICC Workshops)*, Paris, France, May 2017, pp. 1005–1010.

[203] W. Huang, G. Song, H. Hong, and K. Xie, "Deep architecture for traffic flow prediction: Deep belief networks with multitask learning," *IEEE Transactions on Intelligent Transportation Systems*, vol. 15, no. 5, pp. 2191–2201, May 2014.

[204] S. Sakib, T. Tazrin, M. M. Fouda, Z. M. Fadlullah, and N. Nasser, "A deep learning method for predictive channel assignment in beyond 5G networks," *IEEE Network*, vol. 35, no. 1, pp. 266–272, 2021.

[205] W. Jiang and H. D. Schotten, "Neural network-based fading channel prediction: A comprehensive overview," *IEEE Access*, vol. 7, pp. 118,112–118,124, 2019.

[206] L. Hu, Y. Miao, J. Yang, A. Ghoneim, M. S. Hossain, and M. Alrashoud, "IF-RANs: Intelligent traffic prediction and cognitive caching toward fog-computing-based radio access networks," *IEEE Wireless Communications*, vol. 27, no. 2, pp. 29–35, Apr. 2020.

[207] W. Zhai, H. Han, L. Liu, and J. Zhao, "An LSTM-aided hybrid random access scheme for 6G machine type communication networks," 2020, arXiv:2012.13537v8.

[208] H. Jiang, M. Cui, D. W. K. Ng, and L. Dai, "Accurate channel prediction based on transformer: Making mobility negligible," *IEEE Journal on Selected Areas in Communications*, vol. 40, no. 9, pp. 2717–2732, 2022.

[209] Y. Wang, M. C. Vuran, and S. Goddard, "Cyber-physical systems in industrial process control," *ACM Sigbed Review*, vol. 5, no. 1, pp. 1–2, 2008.

[210] W. Elghazel, J. Bahi, C. Guyeux, M. Hakem, K. Medjaher, and N. Zerhouni, "Dependability of wireless sensor networks for industrial prognostics and health management," *Computers in Industry*, vol. 68, pp. 1–15, 2015.

[211] A. Mousavi, R. Monsefi, and V. Elvira, "Hamiltonian adaptive importance sampling," *IEEE Signal Processing Letters*, vol. 28, pp. 713–717, 2021.

[212] Z. Wang, M. Broccardo, and J. Song, "Hamiltonian Monte Carlo methods for Subset Simulation in reliability analysis," *Structural Safety*, vol. 76, pp. 51–67, Jan. 2019.

[213] T.-Y. Chen, N. Seshadri, and B.-Z. Shen, "Is feedback a performance equalizer of classic and modern codes?" in *2010 Information Theory and Applications Workshop (ITA)*, Texas, USA, Jun. 2010, pp. 1–5.

[214] J. Östman, R. Devassy, G. Durisi, and E. G. Ström, "Short-packet transmission via variable-length codes in the presence of noisy stop feedback," *IEEE Transactions on Wireless Communications*, vol. 20, no. 1, pp. 214–227, 2021.

[215] J. C. Dunn, "A fuzzy relative of the ISODATA process and its use in detecting compact well-separated clusters," 1973.