A minimax p-robust optimization approach for planning under uncertainty

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Abstract
Planning under uncertainty is one of the important issues in production planning. The development of mathematical models under uncertainty has long been studied in order to avoid the impact of uncertain factors and to maintain stable and excellent performance of manufacturing system. In this paper, we propose a minimax p-robust production planning problem in the presence of parameter uncertainty. Particularly, we apply p-robust measure to a multi-period production planning and inventory control problem considering a set of demand scenarios. A scenario based robust optimization problem is extended to a minimax p-robust optimization problem by combining a p-robustness measure and a minimax objective function. The proposed model is compared with a deterministic model and a minimax model using simulation experiments. The results show that the minimax p-robust solution improves the average cost compared to other approaches while maintaining similar level of worst-case cost from the minimax model.

Key words : Production planning, Demand uncertainty, Robust optimization, p-robustness

1. Introduction

Production planning is basically concerned with determining the production and inventory level over a finite planning horizon to satisfy customer demand in the most efficient way. Such a decision has long been an important issue in manufacturing since it leads higher utilization of resources and customer satisfaction, and eventually, more profit for a company. So far, several mathematical optimization models have been proposed, including linear programming, mixed integer programming, and goal programming (Nam and Logendran, 1992; Pochet and Wolsey, 2006). When researchers and practitioners develop mathematical models, dealing with uncertainty is recognized as a critical concern, since models without consideration of uncertainty can be expected to perform worse than the models accounting for the uncertainty (Mula et al., 2006). Ho (1989) classified several types of uncertainty affecting production processes into two groups: environmental uncertainty beyond the production processes, and system uncertainty within the production processes. Similarly, Ravindran and Warsing (2012) divided the sources of uncertainty into internal and external sources. Examples of system uncertainty are production lead time, yield, and quality. Environmental uncertainty includes demand uncertainty and supply uncertainty. Typically, environmental uncertainty is less manageable than system uncertainty, since it arises outside of a manufacturing system. Therefore, determining a robust production planning and inventory management policy that is less sensitive to environmental uncertainty may be one of the most effective ways to mitigate loss for a company.

There have been several approaches to optimization under uncertainty, such as stochastic optimization (Birge and Louveaux, 1997; Shimizu et al, 2011) and robust optimization (RO) (Ben-Tal et al., 2009; Seo and Chung, 2014). In traditional stochastic optimization, it is assumed that an uncertain parameter follows a certain probability distribution, and an optimal solution minimizes the average objective value. In contrast, discrete scenarios or a continuous range is assumed in RO to find an optimal solution that minimizes the worst-case objective value. The use of discrete scenarios...
to represent uncertainty is referred to as scenario-based robust optimization. RO with continuous range uncertainty is referred to as set-based robust optimization.

The basic idea of RO is originated from decision theory for worst case analysis based on minimax or maximin model. Then, the idea of set based RO was first introduced by Soyster (1973). A linear programming model was proposed to find an uncertainty-immunized solution when the coefficients of constraints belong to prescribed convex sets. The model was based on the column-wise uncertainty and, unfortunately, produced extremely conservative solutions. Ensuring robustness required sacrificing too much optimality, and thus, the issue of robustness did not draw attention. Significant steps towards RO approaches were made by Mulvey et al. (1995), Ben-Tal and Nemirovski (1998, 1999), and Bertsimas and Sim (2004). Mulvey et al. (1995) developed a RO model in which uncertainty is characterized by a set of scenarios by integrating goal programming formulation and scenario description. Moreover, Ben-Tal and Nemirovski (1998, 1999) and Bertsimas and Sim (2004) proposed reformulation techniques for convex optimization problems with set-based uncertainty.

The most important issue in RO is the tradeoff between robustness and solution conservativeness. The minimax objective function usually used in RO finds the best solution in the worst case. Under some conditions, a RO model with uncertain parameters can be reformulated as a deterministic model with extreme values of the parameters. For example, Yao et al. (2009) showed that a RO model for an evacuation planning problem is equivalent to a deterministic problem with the maximum demand case. RO models in Pishvae et al. (2011) and Chung et al. (2011) also found a solution for extreme values of uncertain parameters. Extreme values may improve the robustness of a solution but the solution (e.g. Just consider a maximum demand case and prepare products as many as possible) may not be applicable in practice. Also, there is no guarantee that a RO solution is better than other approaches in other cases except the worst case. For example, even though RO approach is good in terms of worst case performance and standard deviation, average performance can be worse than other models including a deterministic model with nominal values (Yin and Lawphongpanich (2007)). To handle the conservativeness issue, Ben-Tal and Nemirovski (1999) proposed RO with ellipsoidal uncertainty set and showed that it is less conservative than RO with box uncertainty even though constraints may be violated with small probability. Bertsimas and Sim (2004) used the parameter $\Gamma_i$ to adjust the trade-off between violation probability and the performance of the solution, which is so called the price of robustness. In this paper, we propose a scenario-based robust optimization model constrained by $p$-robustness measure to overcome the conservativeness issue. The aim of this paper is to develop a model with better average performance than classical minimax RO model maintaining similar worst-case performance.

The remainder of this paper is organized as follows. In Section 2, we introduce a $p$-robustness measure and then develop a minimax $p$-robust model. In Section 3, a deterministic production planning problem is formulated as a linear programming problem, and then minimax production planning problem and $p$-robust minimax problem are developed to consider demand uncertainty. Using simulation experiments, the performance of deterministic, minimax, and $p$-robust minimax solutions are compared in Section 4. Finally, in Section 5, we summarize our findings and conclude with future research.

2. Minimax $p$-robust optimization

In a recent work on stochastic programming considering solution robustness, Snyder and Daskin (2006) proposed a robustness measure called $p$-robustness based on a set of scenarios, and applied it to a stochastic $p$-robust facility location problem. The novel approach has also been successfully applied to supply chain network design problems (Liu et al., 2010; Peng et al., 2011).

Let us consider a deterministic mathematical model, Problem (1), with an objective function $f(x, \xi)$ and a set of constraints $g_i(x, \xi) \leq 0$ for all $i \in I$. The problem finds an optimal solution of a set of decision variables $x$ that minimizes the objective function when parameter values for $\xi$ are known in advance.

$$\min_x f(x, \xi)$$

s.t. $g_i(x, \xi) \leq 0 \ \forall i \in I$
Now, we are interested in a problem where the parameter values are unknown. In this case, the uncertainty could be described via a set of scenarios. For each scenario \( s \in S \), we can solve the deterministic problem by setting \( \alpha = \alpha_s \). The minimum objective value and optimal solution of an individual scenario are denoted as \( z^*_s \) and \( x^*_s \), respectively. To define \( p \)-robustness, let \( p \) be a nonnegative constant value. Then, a feasible solution \( x \) is \( p \)-robust if:

\[
\frac{f(x, \alpha_s) - z^*_s}{z^*_s} \leq p \quad \forall s \in S
\]  

(2)

The left-hand side of Eq. (2) represents the relative deviation of the optimal objective value when an optimal solution \( x^*_s \) under scenario \( s \) is replaced by a feasible solution \( x \). Note that when \( z^*_s > 0 \) for all \( s \in S \), Eq. (2) can be rewritten as:

\[
f(x, \alpha_s) \leq (1 + p)z^*_s \quad \forall s \in S
\]  

(3)

Next, we propose new model integrating the \( p \)-robust measure and minimax objective function, which is commonly used in robust optimization models. This problem finds a solution that minimizes the maximum value of \( f_s(x) \) such that Eq. (3) and a set of constraints given in the deterministic problem hold. We call this model a minimax \( p \)-robust model.

\[
\min_{x} \max_{s} f(x, \alpha_s) \\
\text{s.t.} f(x, \alpha_s) \leq (1 + p)z^*_s \quad \forall s \in S
\]  

(4)

3. Mathematical formulation of a production planning problem

In this section, we consider a production planning problem under scenario-based demand uncertainty. It is assumed that there are two ways to fulfill customer demand. Multiple items can be produced from resources of a manufacturer, or purchased from vendors at higher cost. These are called in-house production and vendor supply, respectively. It is assumed that there is a capacity limit of in-house production, while vendor supply is not restricted by capacity constraints. The manufacturer can purchase any products from vendors when its inventory level or capacity is insufficient. This assumption can be easily modified by adding additional constraints related to vendor capacity. The notations used for the production planning problem are listed in Table 1 for convenience.

3.1 Deterministic problem

Before developing a scenario-based mathematical model, we first present a deterministic production planning model:

| Sets | Notations |
|------|-----------|
| \( M \) | A set of machines |
| \( J \) | A set of products |
| \( T \) | A set of planning periods |
| \( S \) | A set of demand scenarios |
### Decision Variables

- $x_j$: The amount of product $j$ produced in-house at time $t$
- $y_j$: The amount of product $j$ produced at a vendor at time $t$
- $I_j$: Inventory of product $j$ at time $t$

### Parameters

- $h_j$: Inventory holding cost per unit of product $j$ at time $t$
- $c^1_j$: Production cost per unit product $j$ at time $t$
- $c^2_j$: Supply cost per unit product $j$ from vendors at time $t$
- $a_{mj}$: The amount of capacity of machine $m$ required for a unit of product $j$
- $b_{mt}$: The available capacity of machine $m$ at time $t$
- $I_{j0}$: Initial inventory level of product $j$

### Demand Data

- $d_j$: Deterministic demand for product $j$ at time $t$
- $d'_j$: Demand for product $j$ at time $t$ under demand scenario $s$

\[
\begin{align*}
\min_{x,y, I} & \sum_{j \in J} c^1_j x_j + \sum_{j \in J} h_j I_j + \sum_{j \in J} c^2_j y_j \\
\text{s.t.} & \quad I_{j,t-1} - I_{j,t} + x_{j,t} + y_{j,t} = d_{j,t} \quad \forall j \in J, t \in T \\
& \quad \sum_{j \in J} a_{mj} x_{j,t} \leq b_{mt} \quad \forall m \in M, t \in T \\
& \quad x_{j,t}, I_{j,t}, y_{j,t} \geq 0 \quad \forall j \in J, t \in T \\
& \quad I_{j0} = k_j \quad \forall j \in J
\end{align*}
\]  

(5)  

(6)  

(7)  

(8)  

(9)

This model is a modification of a production planning problem from Escudero et al. (1993). Our objective is to determine the best production plan at minimum cost. The cost function consists of production cost, inventory holding cost, and purchase cost from vendors in Eq. (5). The dynamics of the system is related to the change of inventory level determined by production quantity and customer demand. The constraints of the problem are inventory conservation constraints, the manufacturer’s capacity constraints, and nonnegative constraints Eq. (6) to Eq. (8). Also, the initial inventory level of product $j$ at the beginning of the planning period is given as $k_j$ in Eq. (9).

### 3.2 Minimax and Minimax p-robust problems

Minimax problem is the problem that modifies objective function of deterministic problem as minimax objective function after introducing a set of uncertain demand scenarios $s$. In this model, production quantity $x_j$ is determined before the realization of uncertainty and therefore it is not dependent on $s$. However, inventory level and
purchase quantity are dependent on \( s \) and therefore we use \( y'_{jl} \) and \( I'_{jl} \) to represent \( y_{jl} \) and \( I_{jl} \) under scenario \( s \). Objective function of the minimax problem can be formulated as Eq. (10)

\[
\min_{x,y,I} \max_{s} \sum_{s} \sum_{t} c^s_{jl} x_{jl} + \sum_{s} \sum_{t} h_{jl} I^s_{jl} + \sum_{s} \sum_{t} c^2_{jl} y^s_{jl} \tag{10}
\]

The problem is nonlinear due to the nested max operator. However, the minimax objective function can be reformulated as the following mathematical model

\[
\min_{x,y,I,z} \sum_{s} \sum_{t} c^s_{jl} x_{jl} + \sum_{s} \sum_{t} h_{jl} I^s_{jl} + \sum_{s} \sum_{t} c^2_{jl} y^s_{jl} \leq z \quad \forall s \in S \tag{11}
\]

We use \( z \) to denote the maximum value of total costs. The mathematical model equivalent to the minimax model becomes the following linear programming model, Problem (12), together with other constraints.

\[
\min_{x,y,I,z} \sum_{s} \sum_{t} c^s_{jl} x_{jl} + \sum_{s} \sum_{t} h_{jl} I^s_{jl} + \sum_{s} \sum_{t} c^2_{jl} y^s_{jl} \leq z \quad \forall s \in S
\]

\[
I^s_{jl} - I^s_{jl} + x_{jl} + y^s_{jl} = d^s_{jl} \quad \forall j \in J, t \in T, s \in S
\]

\[
\sum_{m \in M} a_{m} x_{jl} \leq b_{mo} \quad \forall j \in J, t \in T
\]

\[
x_{jl} \geq 0 \quad \forall j \in J, t \in T
\]

\[
I^s_{jl} = k_j \quad \forall j \in J
\]

Finally, the minimax p-robust model is formulated as Problem (13) after adding p-robust measure.

\[
\min_{x,y,I,z} \sum_{s} \sum_{t} c^s_{jl} x_{jl} + \sum_{s} \sum_{t} h_{jl} I^s_{jl} + \sum_{s} \sum_{t} c^2_{jl} y^s_{jl} \leq z \quad \forall s \in S
\]

\[
\sum_{s} \sum_{t} c^s_{jl} x_{jl} + \sum_{s} \sum_{t} h_{jl} I^s_{jl} + \sum_{s} \sum_{t} c^2_{jl} y^s_{jl} \leq (1+p)z^* \quad \forall s \in S
\]

\[
I^s_{jl} - I^s_{jl} + x_{jl} + y^s_{jl} = d^s_{jl} \quad \forall j \in J, t \in T, s \in S
\]

\[
\sum_{m \in M} a_{m} x_{jl} \leq b_{mo} \quad \forall j \in J, t \in T
\]

\[
x_{jl} \geq 0 \quad \forall j \in J, t \in T
\]

\[
I^s_{jl} = k_j \quad \forall j \in J
\]

**Proposition 1.** The upper bound of the optimal objective value obtained by the minimax p-robust problem is the maximum value of \((1+p)z^*\) for all \( s \).

Proof. Let us consider the first two constraints of the minimax p-robust model. From the definition of \( z \) as well as
the first constraint of Problem (12), the following equation holds:

\[ z = \max_{j} \sum_{j=t}^{j=t} c_{j}^{c} x_{j} + \sum_{j=t}^{j=t} h_{j} I_{j} + \sum_{j=t}^{j=t} c_{j}^{p} y_{j}^{p} \]

The maximum value of \( \sum_{j=t}^{j=t} c_{j}^{c} x_{j} + \sum_{j=t}^{j=t} h_{j} I_{j} + \sum_{j=t}^{j=t} c_{j}^{p} y_{j}^{p} \) is \((1+p)z_{j}^{*}\) due to the second constraint of Problem (12). Therefore,

\[ z \leq \max_{j} (1+p)z_{j}^{*}. \]

4. Numerical Experiments

This section considers two numerical examples to compare the performance of deterministic and robust solutions, and to illustrate the robustness of the solutions. The input data consist of the number of machines, the number of products, the planning horizon, the number of demand scenarios, and the parameter values for cost and capacity. In the numerical experiments, a problem with 1 machine, 3 products, and 12 time intervals is constructed. The production cost \( c_{j}^{c} \) and the supply cost \( c_{j}^{p} \) are 1 and 1.5, respectively. The inventory holding cost \( h_{j} \) is equal to 1, and the initial inventory level is 11 for all products. The data related to cost and capacity are taken from Leung (2007) as shown in Table 3.

| Product | Scenario | Time Period |
|---------|----------|-------------|
| 1       | Boom     | 1500 2000 1600 1700 2000 1800 1700 1900 2000 2300 2400 2500 |
|         | Good     | 1100 1600 1200 1300 1400 1300 1500 |
|         | Fair     | 700 1200 800 900 1200 1000 900 1100 |
|         | Poor     | 500 800 400 500 800 600 500 700 |

| 2       | Boom     | 1400 1900 1600 1500 1900 1700 1600 1700 1900 2300 2300 1300 |
|         | Good     | 1100 1600 1300 1200 1400 1300 1400 |
|         | Fair     | 800 1300 1000 900 1300 1100 1000 1100 |
|         | Poor     | 500 1000 700 600 1000 800 700 800 |

| 3       | Boom     | 1300 1800 1600 1300 1800 1600 1500 1500 1800 2300 2200 2100 |
|         | Good     | 1100 1600 1400 1100 1600 1400 1300 1300 |
|         | Fair     | 900 1400 1200 900 1400 1200 1100 1100 |
|         | Poor     | 700 1200 1000 700 1200 1000 900 900 |

4.1 Example 1 – simple fluctuation

In the previous section, we present the demand uncertainty using four scenarios to develop a production plan. However, in reality, we do not know which economic condition would occur. Also, demand would not be realized according to the scenarios. Therefore, a production plan developed based on a specific scenario may not be an optimal solution when the realized demand does not match the scenarios.

For the comparison of the three production plans obtained from deterministic, minimax, and minimax p-robust
problems with the scenarios, we first solved the three types of problems. For the deterministic problem, average demand for products at each period is used since demand scenario has no meaning for the deterministic problem. Next, in order to see the performance of three plans, we conducted simulation experiments by matching the production plans and randomly generated demand.

In the example 1, it is assumed that each economic condition has equal probability, and the future demand is generated in a certain range of variation from the demand scenarios. Demand variation can be interpreted as uncertainty level. When we assume that the demand variation is 20%, the realized demand is a value between 80% and 120% of the scenarios. For example, as shown in Fig. 1, the solid line is the demand scenario of product 1 when future economic condition is boom. The dotted lines are upper and lower bounds of demand used for random number generation in the simulation experiments.

For the simulation study, demand variations of 10%, 20% and 30% are considered. After 1000 iterations of simulation, the average value, maximum value, and standard deviation of total costs are calculated. Table 4 shows the performance of three approaches to the production planning based on the simulation results when p-robustness value is 0.2, which is determined experimentally.

![Fig. 1 Demand scenario and realized demand (economic condition: boom, demand variation: 20%).](image)

| Demand Variation | Deterministic | Minimax | Minimax p-robust |
|------------------|---------------|---------|------------------|
| **Average**      |               |         |                  |
| 10%              | 813.076       | 783.184 | 599.305          |
| 20%              | 822.674       | 790.711 | 610.179          |
| 30%              | 825.811       | 797.544 | 616.745          |
| **Maximum** (Worst Case) |         |         |                  |
| 10%              | 1060.651      | 811.558 | 820.148          |
| 20%              | 1085.543      | 843.963 | 842.169          |
| 30%              | 1111.949      | 874.638 | 863.044          |
| **Standard Deviation** |         |         |                  |
| 10%              | 150.172       | 15.681  | 128.631          |
| 20%              | 154.307       | 23.445  | 131.796          |
| 30%              | 153.152       | 32.087  | 128.412          |

According to the results in table 4, we can see that the minimax p-robust solutions and the minimax solutions outperform the deterministic solutions in terms of total cost of production and inventory. That is, the average and maximum values of the deterministic approach are highest in all cases. It is clear that the deterministic model is the worst among the three models since it does not consider uncertain factors. Among the minimax and minimax p-robust models considering uncertainty, the average costs of the minimax p-robust solution is better than the minimax solution regardless the value of demand variation. It is because the minimax approach only focuses on the worst case scenario. However, interestingly, it is observed that the maximum costs of the minimax solutions and the minimax p-robust solutions are similar. As we mentioned, the weakness of the minimax approach is solution conservativeness and, thus, the purpose of the p-robust minimax model is to overcome the solution conservativeness. This example demonstrated the proposed minimax p-robust model works well on average while maintaining the worst case performance. Also, we note that a minimax solution does not guarantee the best performance in terms of maximum cost since the realized demand is deviated from the scenario. For example, when demand variation is 20%, the maximum cost with minimax
A p-robust solution is 842.169 and the maximum cost with minimax solution is 843.963.

One important measure to show the performance of a solution is standard deviation of objective values. Standard deviation stands for how the model is insensitive for the uncertainty. The results showed that the minimax model is much better than the others. However, it is not meaningful in this case since average cost is close to the worst case solution. In other words, the variation of cost is small but it is always close to the worst case cost. The results also show the solution conservativeness of the minimax solution.

Next, we investigated the performance of the minimax p-robust solutions by adjusting the value of p-robustness. There is no theoretically proven way to determine the value of p-robustness (Snyder et al. 2006). Therefore it must be determined by a decision maker by considering the trade-offs between regret allowance and solution feasibility. When p-robustness value is large enough, it is easily expected that Eq. (3) becomes dummy constraint and a minimax p-robust problem becomes a minimax problem. Table 5 shows that the average cost increase and standard deviation decrease along the increase of p-robustness value.

Table 5  Comparison of simulation results by adjusting the value of p-robustness

|          | Demand Fluctuation | Minimax | Minimax p-robust (p=0.3) | Minimax p-robust (p=0.2) |
|----------|-------------------|---------|--------------------------|--------------------------|
| **Average** |                  |         |                          |                          |
| 10%      | 783.184           | 613.194 | 599.305                  |                          |
| 20%      | 790.711           | 617.756 | 610.179                  |                          |
| 30%      | 797.544           | 623.674 | 616.745                  |                          |
| **Maximum** (Worst Case) |         |         |                          |                          |
| 10%      | 811.558           | 818.084 | 820.148                  |                          |
| 20%      | 843.963           | 837.816 | 842.169                  |                          |
| 30%      | 874.638           | 857.946 | 863.044                  |                          |
| **Standard Deviation** |         |         |                          |                          |
| 10%      | 15.681            | 115.119 | 128.631                  |                          |
| 20%      | 23.445            | 112.102 | 131.796                  |                          |
| 30%      | 32.087            | 109.593 | 128.412                  |                          |

4.2 Example 2 - growing fluctuation

In the previous example, it was assumed that demand variation is fixed during the planning horizon. However, the accuracy of forecasting may decrease as time period increases, especially in a long term planning horizon. To reflect this feature, simulation is carried out under the assumption that the realized demand at each time period is generated within increasing variation range. As shown in Fig. 2, the upper and lower bonds for the random demand are increasing as the planning period covered in the problem increases. In this example, it is assumed that demand variation is increased 2% and 3% at each time period. Simulation is conducted similarly as example 1 to compare the performance of solutions except for the demand variation.

Fig. 2 Increasing demand fluctuation

The simulation results are summarized in table 6. Similar to the results of example 1, the minimax p-robust model outperforms the deterministic model in all three aspects; average cost, standard deviation, and cost of the worst case.
The minimax p-robust model has better average performance than minimax model maintaining similar worst-case cost.

| Increasing Fluctuation | Deterministic | Minimax | Minimax p-robust |
|------------------------|---------------|---------|------------------|
| Average                |               |         |                  |
| 2%                     | 812.353       | 782.866 | 600.187          |
| 3%                     | 812.412       | 784.403 | 603.406          |
| Maximum                |               |         |                  |
| 2%                     | 1067.153      | 829.188 | 827.194          |
| 3%                     | 1084.364      | 843.168 | 840.582          |
| Standard Deviation     |               |         |                  |
| 2%                     | 150.170       | 20.056  | 127.597          |
| 3%                     | 150.628       | 26.450  | 125.276          |

5. Conclusion

In this paper, we proposed a minimax p-robust model with scenario based uncertainty. The model has been applied to a production planning problem under environment uncertainty. In particular, a minimax p-robust model was formulated for determining an optimal production plan and inventory level in the consideration of scenario-based demand uncertainty. The problem becomes a linear programing problem, and an optimal production plan can be easily obtained using a commercial solver. The numerical experiments with two examples showed that the minimax p-robust solution outperforms deterministic and minimax solutions in terms of average objective value. Also, it was shown that the minimax p-robust model provides a similar worst-case solution with the minimax solution and better worst-case solution than deterministic model which is less sensitive to the perturbation of uncertain data. Therefore, the minimax p-robust solution is more favorable in regard to robust optimization. However, a decision maker can select any of the p-robust methods depending on the output measure of interest. An issue has been raised in this investigation. A small p-value can lead to infeasibility. A possible direction for further studies is to investigate the minimum p-value that provides a feasible solution.

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