Entanglement driven self-organization via a quantum seesaw mechanism

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Atom-field entanglement is shown to play a crucial role for the onset of spatial self-organization of ultracold atoms in an optical lattice within a high-Q cavity. Like particles on a seesaw, the atoms feel a different potential depending on their spatial distribution. The system possesses two stable configurations, where all atoms occupy either only even or only odd lattice sites. While for a classical cavity field description a distribution balanced between even and odd sites is a stationary equilibrium state at zero temperature, the possibility of atom-field entanglement in a quantum field description yields an instant simultaneous decay of the homogeneous atomic cloud into an entangled superposition of the two stable atomic patterns correlated with different cavity fields. This effect could be generic for a wide class of quantum phase transitions, whenever the quantum state can act back on the control parameter.

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A spatially modulated laser field far red detuned from an atomic resonance creates a designable optical potential to trap and manipulate ultra-cold atoms. For a periodic lattice potential this enables tailored implementations of the Bose-Hubbard Hamiltonian to study quantum phase transitions or ideas of quantum information processing. While a free-space laser field acts as a prescribed classical potential, a field enclosed in a cavity is significantly influenced by the atoms and takes part in the coupled atom-field dynamics. Nowadays in such setups the regime of strong light-matter interaction (cavity QED) is experimentally accessible using cold atoms in high finesse Fabry-Perot resonators. Here even a single cavity photon exerts significant forces and the quantum properties of the field can no longer be ignored.

One striking consequence of the coupled atom-field dynamics is spatial self-organization of a laser-illuminated atoms. Above a threshold pump intensity the atoms spontaneously break the continuous translational symmetry of the cloud and form one of two regular patterns in a phase transition. These patterns maximize light scattering from the pump into the cavity with two possible phases of the field, as observed experimentally. The atoms find their stable configurations by a feedback mechanism, i.e: the optical potential is modified by the cavity field, which is generated by phase coherent scattering of pump light in the mode by the atoms. Accumulating around every other antinode the scattering into the cavity mode is enhanced by constructive interference (superradiance), and the potential depth of those lattice sites where atoms sit is maximally increased.

This self-organizing process gets even more intriguing, when the atoms have a kinetic energy less than the recoil energy, i.e. their wave function is flat on the wavelength scale, e.g. using a BEC as an initial state. For a “classical”, or mean-field description of the light field no self-organization can occur, because of the destructive interference of the field amplitudes scattered into the cavity by different parts of the atomic wave function. However, this conclusion is invalid since the cavity field realizes a quantum feedback for the atomic motion, in which entanglement is a crucial element. Scattered field amplitudes with opposite phases do not cancel but entangle to different atomic wave functions. The quantum average of the field amplitude is still zero, but the photon number is not, which is clearly incompatible with the mean-field description. Field components of the superposition create different forces, which pull the atomic wave functions towards the corresponding self-organized configurations. Hence self-organization is started immediately even at $T = 0$ and without measurement induced projections (no spontaneous symmetry breaking).

This system is an experimentally accessible implementation of a “quantum seesaw”. The seesaw is a generic example of a system where the particle is subject to a dynamically varying potential (feedback). In its unusual quantum version, the seesaw undergoes an entanglement-assisted decay from the unstable equilibrium towards the left- and right-tilted positions. In this Letter we present a two-site optical lattice model for the atom-cavity system producing analogous decay. The decay from an initially symmetric wavefunction is the quantum limit of the self-organization. Entangled states, inherent to the effect, are influenced by cavity losses, thus the lattice model will be demonstrated by fully quantum Monte Carlo simulations.

As first toy model we consider a particle moving in 1D along $x$ on a seesaw potential parameterized by the tilt angle $\varphi$ (Fig. 1):

$$V(x, \varphi) = \omega_x^2 x^2 + \omega_\varphi^2 \varphi^2 - 2J \sin(\varphi) x ,$$

where harmonic retaining forces have been added both to the particle and the seesaw (all quantities are dimensionless). Classically $x = \varphi = 0$ is a stationary point, which is unstable for $J > \omega_x \omega_\varphi$. To model a semiclassical seesaw we describe $\varphi$ by a classical variable but the $x$-motion quantum mechanically. Any symmetric wave packet sitting on the balanced seesaw $\varphi = 0$ will exert...
no net force and is stationary, whereas placing the center of mass of the wave packet on either side of the seesaw induces tilting of the potential following by growing acceleration of the wave packet.

The situation is different, however, if $\varphi$ is also a quantum variable, so that we have a two-component coupled quantum system. Even for a perfectly symmetric initial condition of the wave packet and the $\varphi$-oscillator, $\langle \varphi \rangle = 0$, part of the wave packet will immediately start moving to the right with growing amplitude $\varphi(t)$, while the second half will move to the left with opposite tilt $-\varphi(t)$. Thus by forming an entangled state the wave packet can leave its unstable equilibrium and escape towards right and left simultaneously. Although the expectation values of $x$ and $\varphi$ remain zero again, their variances rapidly grow in time. A simple model Hamiltonian is

$$H = \frac{1}{2}(P_x^2 + P_\varphi^2) + V(x, \varphi)$$

$$\approx \hbar \omega_x a_x^\dagger a_x + \hbar \omega_\varphi a_\varphi^\dagger a_\varphi - \frac{J}{4}(a_\varphi^\dagger + a_\varphi)(a_x^\dagger + a_x),$$

where the $\sin$-function was expanded to first order, valid for small $\varphi$. This corresponds to two position-coupled oscillators with $a_i$ ($i \in \{x, \varphi\}$) denoting the annihilation operators. Starting from the product of the ground states of the uncoupled oscillators, the system immediately evolves into a strongly entangled state. The growth of variances and entanglement is shown in Fig. 2, where this latter is measured by the negativity of the partial transpose [11].

Let us now turn to ultracold atoms in a 1D optical lattice created by a standing wave laser field perpendicular to the axis of a cavity (Fig. 1). The atoms scatter photons between the cavity field and the lattice laser, which is associated with momentum transfer and a modification of the lattice potential. In the limit of large detuning (small atomic saturation) the single-atom Hamiltonian reads [7]

$$H = \frac{P_x^2}{2m} + V_0 \sin^2(kx) - \hbar (\Delta_c - U_0) a^\dagger a$$

$$+ \sqrt{\hbar V_0 \omega_0 \sin(kx)} (a + a^\dagger),$$

where $a$ and $a^\dagger$ are the cavity photon annihilation and creation operators. The lattice potential depth is given by $V_0$, the detuning between the lattice field and the cavity resonance by $\Delta_c$, and $U_0$ describes the shift of the cavity resonance frequency per atom. This enables us to construct an $N$-atom Hamiltonian in second quantized form. On expanding the atomic operators in a localized Wannier basis of the lattice potential and keeping only the lowest vibrational state, the following Bose-Hubbard type Hamiltonian [8] is obtained:

$$H = \sum_{m,n} J_{m,n} b_m^\dagger b_n + \hbar \left( \Delta_c - U_0 \sum_n b_n^\dagger b_n \right) a^\dagger a$$

$$+ (a + a^\dagger) \sum_{m,n} \hbar \tilde{J}_{m,n} b_m^\dagger b_n. \quad (4)$$

The nonlinear onsite interaction term is omitted because the atom-atom collision negligibly contributes to the dynamics. The operator $b_n$ annihilates an atom at the site $kx = (n - 1/2)\pi$. The coupling matrix elements for the kinetic and potential energy $\hbar^2/2m + V_0 \sin^2(kx)$ between sites $m$ and $n$ are denoted by $J_{m,n}$, whereas $\tilde{J}_{m,n}$ gives the matrix elements of $\sqrt{U_0 V_0 / \hbar \sin(kx)}$. The model holds as long as the scattering induced potential change is smaller than the depth of the lattice potential, $U_0 \ll V_0$.

It is enough to consider only two sites, left and right, centered on $kx = \pm \pi/2$. The matrix elements $J_{l,r} = J$, $\tilde{J}_{l,l} = -\tilde{J}_{r,r} = \tilde{J}$ are important, the others either vanish or amount to additive constants.

The “classical” mean field approach consists in replacing the field operators $a$ and $a^\dagger$ by their expectation values $\langle \alpha(t) \rangle$ and $\langle \alpha^*(t) \rangle$. The atomic motion is governed by

$$H = J \left( b_l^\dagger b_r + b_r^\dagger b_l \right) + \hbar \tilde{J} \left( b_l^\dagger b_l - b_r^\dagger b_r \right) 2 \text{Re} \{ \alpha(t) \}, \quad (5)$$

where $\alpha(t)$ fulfills a $c$-number equation containing expectation values of atomic operators and a damping term with decay rate $\kappa$:

$$\dot{\alpha}(t) = [i (\Delta_c - U_0 N) - \kappa] \alpha(t) - i \tilde{J} \left<b_l^\dagger b_l - b_r^\dagger b_r \right> . \quad (6)$$
By construction, entanglement is absent in this model. A perfectly symmetric atomic distribution with no average field implies $\dot{\alpha}(t) = \alpha(t) = 0$ and is stationary. As shown in Fig. 3, starting with a tiny initial population asymmetry reveals the bistability of the system and will dynamically confine the atoms to one of the two wells correlated with a nonzero field amplitude.

In sharp contrast to the mean-field description, the quantum field model of Eq. (4) predicts that the symmetrical population triggers instantly an increase of the photon number associated with the buildup of atom-field entanglement. This is shown in the insert of Fig. 3 and also in Fig. 4 for two atoms. This latter plot displays the self-organization process also in terms of the decaying probability of the two atoms sitting in different wells $(\langle n_l n_r \rangle)$. Note that the entanglement decays on a longer time scale only after that the self-organized state with finite intensity was occupied.

We can keep track of the quantum behavior analytically in the bad cavity limit, where the field is slaved dynamically to the atomic motion and can formally be expressed in terms of the atomic operators:

$$a = -i\frac{\hat{J}}{\kappa - i(\Delta_c - U_0 N)} \left( b_l^\dagger b_l + b_r^\dagger b_r \right).$$

As the atom number $\hat{N} = b_l^\dagger b_l + b_r^\dagger b_r$ is a constant of motion, the dynamics can be consistently restricted to irreducible subspaces. Let us first consider the single-atom subspace, where $b_l^\dagger b_l$ acts as a projector on the $\alpha$-th well. Depending on the atom's position, $(a)$ changes sign, and it vanishes for a symmetric atomic state. However, the photon number operator $a^\dagger a \propto \left( b_l^\dagger b_l - b_r^\dagger b_r \right)^2$, which is proportional to the unit operator in position space. That is, the photon number is nonzero and independent of the atomic state. For more than one atom the dynamics depends also on the quantum statistics. This is displayed in Fig. 4 via the example of two different quantum states producing the initially symmetric population distribution. The superfluid state, when both atoms are in a symmetric superposition of the two wells, $(1/2 (b_l^\dagger + b_r^\dagger) 2 | 0 \rangle)$, self-organizes faster than the Mott insulator state $(b_l^\dagger b_r^\dagger | 0 \rangle)$, indicated by the lines with circles in the figure. The Mott state is a perfectly balanced initial state with exactly one atom in each well. It is an eigenstate of the operator $a$, given in Eq. (7), with zero eigenvalue, so there is indeed a destructive interference of the quantum mechanical amplitudes of all the excited photon states. Nevertheless, entanglement drives the decay towards the self-organized state via the coherence between the left and right sites induced by tunneling.

Let us now check the key features of the lattice model by the Monte Carlo Wave Function (MCWF) method restricted to one wavelength with periodic boundary conditions. The method allows to solve the coupled atom-field dynamics given by Eq. (3) including cavity decay and atomic spontaneous emission. Individual trajectories are simulated and the full atom-cavity density operator can be approximated by an ensemble average. This calculation is exempt from constraining the atomic dynamics on the lowest-energy Wannier-basis states.

In Fig. 5 the evolution of the mean photon number and the negativity characterizing the atom-field entanglement is plotted. Light scattering immediately creates photons entangled to the atomic state in very good agreement with the quantum lattice model. Oscillations are related to that the distribution of the number of spontaneously lost photons is dominated by the 0, 1, 2, etc. numbers. The photon number oscillates around a constant and does not vanish for long times. On the other hand, dissipation destroys entanglement so that the negativity oscillates around a decaying mean. The individual trajectories reveal that the system evolves into the “stochastic” state

$$|\psi_\pm \rangle = 1/\sqrt{2} (|\alpha \rangle \pm |\text{right} \rangle | -\alpha \rangle),$$

FIG. 3: (color online). (a) Time evolution of the field intensity (dashed line) and site-occupation difference (solid line) for mean-field approximation (c.f. Eq. (3)) and taking four atoms. The initial distribution deviates slightly from a symmetric one, $U_0 = -0.25\kappa$, $\Delta_c = -2/3\kappa$, $V_0 = -4\hbar\kappa$ and $\kappa = 1.4\hbar^2$. The insert shows the much faster growth of the field intensity (solid line) and entanglement of the corresponding quantum model (note the different time range).

FIG. 4: (color online). Entanglement (solid lines), photon number (dot-dashed lines), and two site atom-atom correlation function (dotted lines) for two atoms in two wells. Lines with extra circles show the case of exactly one atom in each well, while the others correspond to a symmetric superposition state for each atom at start. The parameters are $U_0 = -2\kappa$, $\Delta_c = -6\kappa$, $J = 1/100\kappa$ and $\bar{J} = 1.6\kappa$. 

FIG. 5: (color online). The evolution of the mean photon number (dot-dashed lines) and the negativity characterizing the atom-field entanglement (solid line). Oscillations are related to the distribution of the number of spontaneously lost photons.
of Eq. (8), having no mean field but non-vanishing photon fluctuations it eventually escapes and evolves into the state. The atom is initially well localized in the right well ($|\langle x \rangle = -\pi/2 \rangle$ and generates approximately a coherent field of intensity $|\langle a^\dagger a \rangle| \approx |\alpha|^2$. Due to fluctuations it eventually escapes and evolves into the state of Eq. (8), having no mean field but non-vanishing photon: $\langle a \rangle = 0$, $|\langle a^\dagger a \rangle| \approx |\alpha|^2$. Parameters: $U_0 = -\kappa/2$, $\Delta_\kappa = -1.2\kappa$, $V_0 = -2\hbar\kappa$, $\kappa \approx 200\hbar^2/\bar{m}$. The results of the MCWF simulations agree well with the lattice model.

FIG. 5: (color online) Fast dynamic growth of entanglement and photon number of a single atom starting from a flat wavefunction coupled to a vacuum field. Parameters: $U_0 = -1.7\kappa$, $\Delta_\kappa = -12\kappa$, $V_0 = -6.7\hbar\kappa$, $\kappa \approx 200\hbar^2/\bar{m}$. The results of the MCWF simulations agree well with the lattice model.

FIG. 6: (color online). Dynamic formation of a symmetrically self-organized state. The atom is initially well localized in the right well ($|\langle x \rangle = -\pi/2 \rangle$) and generates approximately a coherent field of intensity $|\langle a^\dagger a \rangle| \approx |\alpha|^2$. Due to fluctuations it eventually escapes and evolves into the state of Eq. (8), having no mean field but non-vanishing photon: $\langle a \rangle = 0$, $|\langle a^\dagger a \rangle| \approx |\alpha|^2$. Parameters: $U_0 = -\kappa/2$, $\Delta_\kappa = -1.2\kappa$, $V_0 = -2\hbar\kappa$, $\kappa \approx 200\hbar^2/\bar{m}$.

where $|\text{left}\rangle$ (right$\rangle$) means an atomic wave packet centered on the left (right) site in the lattice potential, and the radiated field states $|\alpha\rangle$ and $|-\alpha\rangle$ are coherent ones.

The state of Eq. (8) is stochastic in the sense that each photon loss event flips the sign. After a few jumps the density matrix describes a left-right mixed state as from the classical model with random initial conditions.

Let us start the system from the initial condition $|\langle x \rangle \rangle$ where it starts to radiate the coherent field state $|\langle \alpha \rangle \rangle$. As we see in Fig. (8) this state is remarkably stable for a long time, but eventually collapses due to fluctuations, and the system ends up in the state of Eq. (8), which is made obvious by the fact that $\langle x \rangle = \langle \alpha \rangle = 0$, while the photon number is not affected: $\langle a^\dagger a \rangle \approx |\alpha|^2$. Hence in the atom-cavity implementation of the quantum seesaw, even if the seesaw is tilted to one direction with the atomic wave packet completely on that side, fluctuations eventually enable the system to escape from this state to a symmetric final state.

In summary, at the example of self-organization of ultracold atoms in an optical lattice we found that the possibility of entanglement is an essential ingredient for dynamical decay of a quantum system from an unstable equilibrium point. The classical (factorized atom-field state) description of the optical potential predicts a stationary homogeneous distribution, while a quantum description implies immediate atomic ordering via atom-field entanglement formation. The entanglement involves states, not describable as small quantum fluctuations, around a large mean-field, even if starting the system from coherent states with large photon and atom number. Entanglement driven decay can be a generic feature in the dynamics of quantum phase transitions induced by a classical control parameter, whenever the quantum system acts even minimally back on its control. Recent experimental progress in cavity QED should allow to study such models with current technology.

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