THERMAL PROPERTIES OF OIL AND GAS RESERVOIRES ROCKS MODELING

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ABSTRACT

When, for a hydrocarbon deposit, the porosity, density and saturation of the rock, saturating fluid, the permeability, and the electrical formation factor are known, formulas for calculating the equivalent thermal conductivity can be used, by which this property is expressed, depending on the properties of deposit, more easily measurable. These computational expressions are the result of correlations between experimental determinations performed for the physical properties of a very large number of samples in the field. But these equations introduce measurement errors or interpretations by the authors of the research, which often lead to quite large differences from reality.

That is why we introduced an experimental model to determine the thermal conductivity and we determined this property on rocks taken from the productive layers of crude oil and gas from some geological research wells conducted in the Moesica Platform, Romania.

We also managed to introduce computational relationships between thermal conductivity and density and porosity of extracted rocks.

The role of these experiments is to find a new method for determining the thermal conductivity, a property necessary to simulate the flow of oil fluids and the design of oil recovery techniques.

Keywords: Oil and Gas Reservoir, Regression Modeling, Thermal Conductivity, Rocks Porosity, Rocks Density

1. INTRODUCTION

Thermal conductivity is another characteristic property of oil and gas deposits, analogous to electrical permittivity, magnetic permeability, electrical conductivity, filtration coefficient and molecular diffusion coefficient Cristescu (1998). Estimation of thermal conductivity by calculation is possible in two ways Cristescu (2009)

- considering an idealized model, consisting of a solid and a fluid: the solid medium is supposed to have a certain geometry, and the fluid environment is immobile; the thermal conductivity of the fluid and solid and the porosity are known.
- expression according to other properties of the porous medium saturated with fluids, more easily measurable (density, porosity, permeability).

It is useful to know the values of the thermal conductivity of the oil field when designing the exploitation by thermal methods or during the development of these processes.
The experimental law of thermal conduction or Fourier law is given by the expression:

\[ Q = -\lambda \cdot A \cdot \nabla T \]  \hspace{1cm} \text{Equation 1}

The possibility of heat passing through a body is highlighted by the thermal conductivity \( \lambda \) which is the heat that normally passes through the unit of surface, in the unit of time, at a temperature gradient of 1 k/m.

\[ \lambda = \frac{Q}{A \cdot \nabla T} \]  \hspace{1cm} \text{Equation 2}

Thermal conductivity is a characteristic property of everybody, which can be determined experimentally or can be calculated empirically based on other physical properties.

Conductive heat transport can take place through:

- Electronic transport-electrons move through high temperature areas to low temperature areas, transferring energy with them,
- Transport photon-ions of the body perform oscillating movements during which there are collisions with heat transfer, the transfer is performed from close to close, diffusive.

The contribution of ions to heat transfer is shaped by the introduction of an imaginary, weightless gas called photonic gas, which travels through bodies from high temperatures to low temperatures, transferring energy without changing the properties of the body.

- Radiant transport occurs when the emission and mutual absorption of radiation between neighboring elementary particles occurs.

In non-ionized gases, gases at \( t \leq 1800 \) °C, the conductive transport of heat takes place mainly under the effect of molecular oscillations (photonic gas) which have a small amplitude and as a result they are bad bodies conducting heat.

In the case of Newtonian liquids and non-metallic solids, heat transfer through conductivity takes place both through the oscillations of molecules, because the distance between them is relatively small, and through radiation \text{Sonney (2010)}. The coefficient of thermal conductivity varies with the nature of the body, with its state of aggregation, with the temperature and pressure, with the humidity of the body, with the porosity, with the nature and concentration of the impurities contained in the body, etc.

It is a scalar parameter, in the case of the isotropic medium (gases, liquids, metals, amorphous dielectric bodies) and vector, in the case of the anisotropic medium (crystals, materials with stratified arrangement).

One of the specific properties of crystalline bodies is the vectorial or tensor character that most physical-chemical properties of crystals represent, including thermal conductivity.

The vector properties can be continuous or discontinuous, which gives the crystals the fundamental property called anisotropy, which has its origin in the fact that the interatomic distances are different, depending on the crystallographic direction.
The most general hypothesis regarding thermal conductivity is given by Stokes’ matrix 
\[ \lambda_{jk} = \begin{bmatrix} \lambda_{11} & \lambda_{12} & \lambda_{13} \\ \lambda_{21} & \lambda_{22} & \lambda_{23} \\ \lambda_{31} & \lambda_{32} & \lambda_{33} \end{bmatrix} \]

Equation 3

Onsanger’s postulate shows that the matrix of thermal conductivity is symmetrical and therefore:
\[ \lambda_{jk} = \lambda_{kj}, j \neq k \]

Equation 4

Since the experiments did not confirm the existence of the rotary conduction \( (\lambda_{jk} = 0, \text{ for } j \neq k) \), the matrix Equation 3 is reduced to the equation:
\[ Q = -\lambda \cdot A \cdot \text{grad } T \]

Equation 5

So, in conclusion, thermal conductivity is a characteristic property of the environment, analogous to electrical conductivity, magnetic permeability, and electrical permittivity.

2. METHODS FOR DETERMINING THERMAL CONDUCTIVITY

Any systematic geothermal research presupposes the knowledge of the heat transfer mode in the researched environment Jugastreanu et al. (2022), Tabatabai et al. (2022)

Heat transfer takes place through three main processes, namely: conduction, convection, and radiation.

Heat transfer by conduction occurs only in solid media by molecular interaction Trochim (2001).

It is the main mechanism of heat transfer in the Earth's crust and the most important in geothermal probe research.

Convective heat transfer is associated with the free movement of fluids between two environments at different temperatures.

It becomes important in geothermal areas, in particular, in areas with volcanic activity and in areas with active groundwater circulation Tissot and Welte (1984).

The mechanism of heat transfer by convection must be taken into account in geothermal research conducted in boreholes, because it plays a significant role in changing the natural thermal regime of the geological formations crossed.

Radiative heat transfer occurs, on the one hand, on the Earth’s surface, where temperatures are conditioned by the exchange of heat between the Earth and the Sun, and on the other hand, in rocks at very high temperatures.

For temperatures encountered at usual probe depths, including deep probes, the radiative transfer is negligible.

The ability of media to transmit and absorb thermal energy depends on the thermal conductivity

Thermal conductivity is the property of media to transmit thermal energy to a greater or lesser degree.
Quantitatively, the thermal conductivity expresses the amount of heat $Q$ that flows in a time $\tau$ through a body with cross section and given length whose opposite faces are at temperatures $t_1$ and $t_2$:

$$\lambda = \frac{\Delta Q}{\Delta \tau} \cdot \frac{L}{A \Delta T}$$  \hspace{1cm} \text{Equation 6}$$

Where:
- $\lambda$ is the thermal conductivity,
- $\Delta Q$ is the amount of heat,
- $\Delta \tau$ represents the heat transfer time,
- $\Delta T$ is the thermal gradient,
- $A$ represents the contact area.

Thermal conductivity is the transfer mechanism that occurs at the molecular level as a result of elastic collisions between molecules or ions of the substance as a result of their oscillations or displacement \textit{Tissot et al. (1974), Waples (1985)}.

Molecules with higher energy by collision with molecules or ions with lower energy give them some of their kinetic energy, so that heat is transmitted from close to close throughout the body \textit{Tabatabai et al. (2022), Trochim (2001)}.

In liquids and gases, conductivity is the result of elastic collisions between molecules \textit{Cristescu (2009), Sonney (2010)}.

It is more intense in liquids than in gases because the distance between molecules is smaller in liquids than in gases.

Because this mechanism is realized at the molecular level, thermal conductivity is also known as heat transfer through the molecular mechanism.

Thermal conductivity is a characteristic property of the environment, analogous to electrical permittivity, magnetic permeability, electrical conductivity, filtration coefficient and molecular diffusion coefficient.

In permanent thermal conduction regime, a fluid-saturated porous medium can be assimilated with an equivalent continuous medium, for which a thermal conductivity tensor $\lambda$ is defined \textit{Trochim (2001), Tissot and Welte (1984)}.

### 2.1. CALCULATION METHODS FOR ESTIMATING THE EQUIVALENT THERMAL CONDUCTIVITY OF A FLUID-SATURATED POROUS MEDIUM

Thermal conductivity is a characteristic property of the environment, analogous to electrical permittivity, magnetic permeability, electrical conductivity, filtration coefficient and molecular diffusion coefficient \textit{Cristescu (2009), Sonney (2010)}.

In the permanent thermal conduction regime, a porous medium saturated with fluids can be assimilated with an equivalent continuous medium, for which a thermal conductivity tensor $\lambda$ is defined.

The values of the components of this tensor depend on the thermal conductivity and the distribution of each phase, the saturation in fluids, the porosity, the direction of the thermal flow, the thermodynamic parameters of state (pressure and temperature).
If it is assumed that the fluid-saturated porous medium is thermally isotropic, the thermal conductivity tensor is spherical and defined with the scalar $\lambda$ - equivalent thermal conductivity.

Estimation by calculation of equivalent thermal conductivity is possible in two ways:

- considering an idealized model, consisting of a solid and a fluid: the solid medium is supposed to have a certain geometry, and the fluid environment is immobile; the thermal conductivity of the fluid and the solid and the porosity are known Tissot and Welte (1984).

- expression according to other properties of the porous medium saturated with fluids, more easily measurable (density, porosity, permeability) Tissot et al. (1974).

The solids and fluids that make up a hydrocarbon deposit are a fluid-saturated porous medium.

When designing the exploitation by thermal methods or during the development of these processes, it is useful to know the values of the thermal conductivity of the oil field Tabatabai et al. (2022).

2.1.1. CALCULATION METHODS BASED ON IDEALIZED MODELS

1) The series model Cristescu (1998), Cristescu (2009)

The solid medium and the fluid medium consist of a succession of parallel layers, and the heat flux is perpendicular to the layers (Figure 1).

$$\frac{1}{\lambda_z} = \frac{\phi}{\lambda_f} + \frac{1-\phi}{\lambda_s}$$  \hspace{1cm} \text{Equation 7}

Where:

- $\lambda_z$ is the coefficient of thermal conductivity of the analyzed deposit, W/ (m K)?
- $\lambda_f$ is the coefficient of thermal conductivity of the fluids to oil deposit, W/ (m K)?
- $\lambda_s$ is the coefficient of thermal conductivity of the rocks to oil deposit, W/ (m K)?
- $\phi$ porosity.

2) The parallel models Cristescu (1998), Cristescu (2009)

The solid medium and the fluid medium have the same arrangement as in the case of the series model, but the heat flux is parallel to the layers (Figure 1).

$$\lambda_z = \Phi \lambda_f + (1 - \Phi) \lambda_s$$  \hspace{1cm} \text{Equation 8}

Where:

- $\lambda_z$ is the coefficient of thermal conductivity of the analyzed deposit, W/ (m K)?
- $\lambda_f$ is the coefficient of thermal conductivity of the fluids to oil deposit, W/ (m K)?
- $\lambda_s$ is the coefficient of thermal conductivity of the rocks to oil deposit, \( \text{W/ (m K)} \)?
- $\Phi$ porosity.

3) *Weighted geometric mean* Cristescu (1998), Cristescu (2009)

It does not have a physical basis, but it is easy to apply, and an intermediate value is obtained compared to the two variants set out above.

$$\lambda_z = \lambda_f^\Phi \lambda_s^{1-\Phi}$$

Equation 9

Where:
- $\lambda_z$ is the coefficient of thermal conductivity of the analyzed deposit, \( \text{W/ (m K)} \)?
- $\lambda_f$ is the coefficient of thermal conductivity of the fluids to oil deposit, \( \text{W/ (m K)} \)?
- $\lambda_s$ is the coefficient of thermal conductivity of the rocks to oil deposit, \( \text{W/ (m K)} \)?
- $\Phi$ porosity.

4) *Maxwell’s equation* Cristescu (1998), Cristescu (2009)

It has been proposed for the calculation of electrical conductivity in the case of some distribution of solid spheres in a continuous fluid environment.

![Figure 1 Thermal conductivity calculation models (series and parallel)](image)

Applied by Eucken for the calculation of equivalent thermal conductivity, the equation has the form:

$$\lambda_z = \frac{2\Phi \lambda_f + (3-2\Phi)\lambda_s}{(3-\Phi)\lambda_f + \Phi\lambda_s}$$

Equation 10
Where:
- $\lambda_z$ is the coefficient of thermal conductivity of the analyzed deposit, W/(m K)?
- $\lambda_f$ is the coefficient of thermal conductivity of the fluids to oil deposit, W/(m K)?
- $\lambda_s$ is the coefficient of thermal conductivity of the rocks to oil deposit, W/(m K)?
- $\Phi$ porosity.

Equation 10 is only applicable if the porosity has a high value, which implies that the solid spheres are sufficiently far apart and do not interact with each other.

5) The Beck Model Cristescu (1998), Cristescu (2009)
It is a modified Maxwell model which, in the opinion of the one who proposed it, leads to good results, if the porous medium has the following characteristics $\Phi < 0.5$ și $\frac{\lambda_s}{\lambda_f} \in (1 ÷ 300)$.

$$\lambda_z = \lambda_s \left[ \left( \frac{2\lambda_s^2 + 1}{2\lambda_f^2} \right)^{-2\Phi} \left( \frac{2\lambda_f^2 - 1}{\lambda_f^2} \right)^{-2\Phi} \right]$$

Equation 11

- $\lambda_z$ is the coefficient of thermal conductivity of the analyzed deposit, W/(m K)?
- $\lambda_f$ is the coefficient of thermal conductivity of the fluids to oil deposit, W/(m K)?
- $\lambda_s$ is the coefficient of thermal conductivity of the rocks to oil deposit, W/(m K)?
- $\Phi$ porosity.

Equation 11 is based on the real physical situation, in which fluid spheres, of thermal conductivity $\lambda_f$ are dispersed in a solid environment, of thermal conductivity $\lambda_s$.

6) Model de Vries Cristescu (1998), Cristescu (2009)
The model is a generalization of Maxwell’s equation for a medium consisting of a continuous fluid phase and a dispersed solid phase consisting of ellipsoidal particles.

$$\lambda_z = \frac{\Phi \lambda_f + (1-\Phi) G \lambda_s}{\Phi + (1-\Phi) G}$$

Equation 12

- $\lambda_z$ is the coefficient of thermal conductivity of the analyzed deposit, W/(m K)?
- $\lambda_f$ is the coefficient of thermal conductivity of the fluids to oil deposit, W/(m K)?
• \( \lambda_s \) is the coefficient of thermal conductivity of the rocks to oil deposit, W/ (m K)?

• \( \Phi \) porosity.

and:

\[
G = \frac{1}{3} \sum_{j=1}^{3} \left[ 1 + \left( \frac{\lambda_s}{\lambda_f} - 1 \right) \xi_j \right]^{-1} \quad \text{si} \quad \sum_{j=1}^{3} \xi_j = 1 \tag{13}
\]

\( \xi \) refers to the shape of the particles.

When \( \xi_1 = \xi_2 = \xi_3 \) (spherical particles), the relationship Equation 11 se reduce la forma Equation 12.

D.A. de Vries considered \( \xi_1 = \xi_2 = \frac{1}{8} \) si \( \xi_3 = \frac{3}{4} \), which corresponds to particles in the shape of an ellipsoid of revolution, with the major axis 6 times the minor axis.

7) Model Woodside and Messmer Cristescu (1998), Cristescu (2009)

Theoretical and experimental research has led to the adoption of an equivalent resistor model, in a modified form.

The three-element resistor model belongs to Wyllie and Southwick and has been proposed to calculate the equivalent electrical conductivity of a fluid-saturated porous medium.

The model comprises a conductive particle aggregate, saturated with a conductive electrolyte. They are arranged as three components in parallel.

Element 1 is a series group of particles and the electrolyte, element 2 is formed by the particles, and the 3\textsuperscript{rd} is the electrolyte (Figure 2).

The equivalent conductivity of this aggregate is calculated by the relation:

\[
\lambda_z = \frac{\xi_1 \lambda_s \lambda_f}{\lambda_s (1-\xi_4) + \xi_4 \lambda_f} + \xi_2 \lambda_s + \xi_3 \lambda_f \tag{14}
\]

Factors \( \xi_1, \xi_2, \xi_3, \xi_4 \) have certain forms for the calculation of equivalent electrical conductivity.

It is considered that the use of equation Equation 14, for the calculation of the equivalent thermal conductivity of an unconsolidated environment, leads to results close to those obtained experimentally, if the following relations are adopted:

\[
\xi_2 = 0, \xi_3 = \Phi - 0.03, \xi_1 = 1 - \xi_3, \xi_4 = \frac{(1-\Phi)}{\xi_1} \tag{15}
\]

Where:

• \( \lambda_z \) is the coefficient of thermal conductivity of the analyzed deposit, W/ (m K)?

• \( \lambda_f \) is the coefficient of thermal conductivity of the fluids to oil deposit, W/ (m K)?

• \( \lambda_s \) is the coefficient of thermal conductivity of the rocks to oil deposit, W/ (m K)?

• \( \Phi \) porosity.
The calculation formula proposed by this model is:

$$\lambda_z = \lambda_f \left( \frac{\lambda_s}{\lambda_f} \right)^{A' + B \log \frac{\lambda_s}{\lambda_f}}$$

Equation 16

where:

$$A' = 0.280 - 0.757 \log \Phi$$ și $$B = -0.057$$

Equation 17

The porosity is between 0.215 și 0.476.

Where:

- $$\lambda_z$$ is the coefficient of thermal conductivity of the analyzed deposit, W/ (m K)?
- $$\lambda_f$$ is the coefficient of thermal conductivity of the fluids to oil deposit, W/ (m K)?
- $$\lambda_s$$ is the coefficient of thermal conductivity of the rocks to oil deposit, W/ (m K)?
- $$\Phi$$ porosity.

Testing 165 data from specialized publications Tabatabai et al. (2022), Trochim (2001) it was found that, in 76% of them, the difference between the values obtained experimentally and those calculated with the relation Equation 16 is ± 30% Tabatabai et al. (2022).

3. MATERIALS AND METHODS

The oil field is a porous medium saturated with fluids.
The extremely complicated and varied composition of a hydrocarbon deposit, as well as the conditions in which it is located, are the reasons why the physical properties have specific values for each case.

The existence on Earth of large accumulations of heavy and / or viscous crude oil, shale and bituminous sands, as well as the fact that, after the application of the classic methods of exploitation, 60-70% of the geological reserve remain in the field, on the one hand, and on the other hand, maintaining the predominant place of hydrocarbons as a resource in world energy are the factors that have captivated the interest in the application of thermal methods of oil exploitation.

These are hot fluid injection, underground combustion, and combinations thereof.

It is useful to know the values of the thermal conductivity of the oil field when designing the exploitation by thermal methods.

The equivalent thermal conductivity of an oil field can be estimated by calculation, either by applying idealized models or by expressing this heat transfer property as a function of other properties of the field (density, porosity, permeability).

Using data appropriate to the oil fields, equivalent thermal conductivity values can be obtained Tabatabai et al. (2022), Trochim (2001)

Experimental research has been undertaken to measure the equivalent thermal conductivity for rock and reservoir fluids.

Thus, the effect of factors such as the mass composition of the rock and the nature of the fluid that saturates the rock pores can be highlighted and calculation models can be proposed that lead to results consistent with the measurements.

The measurement of the thermal conductivity of non-metallic solid bodies, in stationary regime, is determined from the expression of Fourier's law, aiming that, after reaching the stationary regime, to ensure the constancy of the thermal flux transmitted through the test material and the temperatures at its surfaces. (Through which the heat exchange takes place).

Electric heaters are generally used as heat sources, and the temperatures on the outer surfaces of the samples are measured with small thermocouples evenly distributed on them.

Various methods and devices for determining the thermal conductivity of solid materials are described in specialized publications Cristescu (1998), Cristescu (2009), Sonney (2010), Jugastreanu et al. (2022), Tabatabai et al. (2022)

Of these, the method of the plate, the cylindrical tube and the spherical one is distinguished by the shape of the test specimens.

The plate method determines the thermal conductivity of a material in the form of perfectly flat and parallel plates, of surface $A$ and thickness $h$, crossed by a heat flux $\dot{Q}$, and a temperature difference $\Delta t = t_1 - t_2$.

$$\lambda = \frac{\dot{Q}h}{A(t_1-t_2)} \quad \text{Equation 18}$$

Figure 3 shows a device for determining the thermal conductivity by the plate method, with a single specimen.

The heat flux, represented by the electrical power consumed by the resistors 4, is read at a wattmeter inserted in their supply circuit.
The thermal flow, transmitted through the test tube 1, is taken over by the water circuit 7 whose thermostat ensures the uniformity of the temperatures on the lateral surfaces of the sample, temperatures measured by the thermocouples 2.

In addition to the thermal insulation 6 of the entire device, the side faces of the test plates are protected by guard rings 3, made of the test material, in the form of annular bodies; by heating with electrical resistances 5, the guard rings are kept inwards at a temperature approximately equal to the average temperature of the side faces of the protected parts Jugastreanu et al. (2022), Tabatabai et al. (2022).

Figure 3 Device for determining the thermal conductivity by the plate method, with a single test tube Jugastreanu et al. (2022), Tabatabai et al. (2022)

1 - test plate; 2 - thermocouples; 3 - guard ring; 4 - electrical resistance; 5 - electrical resistance of the guard ring; 6 - thermal insulation; 7 - water cooler; 8 - compensation plate; 9 - electrical resistance of the compensation plate.

To measure the thermal conductivity of these preparations, capsules were made in which the mixtures of solids and liquids were introduced.

The capsules were text Olite cylinders, 32.4 / 42 mm in diameter and 24.5 mm high, with metal caps, screwed.

The lids are 42/44 mm in diameter and 1.5 mm thick.

The thermal conductivity of the cover materials is 40 W/mK and as a result, their thermal resistance to heat transfer through conduction is very low.

Also, the side walls of the text Olite were further thermally insulated to reduce the heat flux dissipated through the side walls to the outside environment.

In order to make such determinations, the possibilities existing in a series of laboratories were analyzed, but the work with crude oil, which when the temperature increases flows or even starts to burn, as well as a certain geometry of the specimens, imposed by the respective equipment, have limited approaches.

The text Olite capsules, described above, have been designed and made so that their shape and dimensions correspond to those required by the conductivity measuring apparatus.

Also, the materials were in the form of pastes, for which the thermal conductivity could not be measured, if they were not placed in the supports described.

In fact, during the measurement of the thermal conductivity, as the temperature increased, the oil and water began to flow from the capsules, which created difficulties.
Another set of samples was the carrots. The cores come from various areas of the oil fields I studied.

The thermal conductivity of some samples containing crude oil could not be measured because, under the thermal effect, they started to smoke.

In the case of other samples, the thermal conductivity was determined for the solid matrix, after the oil traces had vaporized.

When measuring the thermal conductivity, following the study and calculations regarding the thermal resistances that appear in the process of heat transfer through conduction and the comparison of the data obtained with the specific ones from the works it is estimated that the error is 10%.

In the case of samples prepared in the laboratory, the error is mainly caused by the thermal contact resistances between the test tube and the plates of the apparatus, by the heat dissipated to the external environment and the thermal resistances of the metal caps of the capsules.

4. RESULTS AND DISCUSSIONS

Knowing the thermal conductivity of rocks is important due to the implications of this property in the exploitation of oil fluid deposits.

Thermal conductivity conditions the distribution of heat in the earth's crust through the phenomenon of conductive transmission.

Also, the study of the crust-mantle discontinuities and the explanation of the distribution of temperatures inside the earth and of the geothermal flow, is another property of conductivity.

Fluid saturation and thermal conductivity influence the equivalent thermal conductivity of the oil field.

It is observed that samples with identical solid composition have different values of thermal conductivity, depending on the nature of the fluid they contain.

Also, calculations were performed to determine the equivalent thermal conductivity, applying the relation:

\[ \lambda_z = 1.272 - 2.25\Phi + 0.39\lambda_s \cdot \sqrt{S_a} \]  \hspace{1cm} \text{Equation 19}

This ratio is recommended for unconsolidated rocks, with porosity 28-37%. It follows that, for unconsolidated rocks, the relation Equation 18 can be used to estimate the equivalent thermal conductivity.

**Transitional measurement methods (non-stationary)**

Transitional measurement methods, \( \frac{\partial t}{\partial t} \neq 0 \), can be performed in practice in multiple variants.

In some variants, a cylindrical sample with a uniform initial temperature, is constantly heated by a source-line and the temperature increases over time.

The temperature rise at a point of a heated sample (via a line source) can be written as follows:

\[ t = \frac{Q}{2nk} I \left( \frac{r}{2\sqrt{at}} \right) \]  \hspace{1cm} \text{Equation 20}
Where:

- $Q$, is the amount of heat per unit length of the source,
- $r$ is the radial distance of the measuring point from the source-line,
- $\tau$ it is the time that has passed since the beginning of the warm-up,

\[
I(x) = e - \ln x + \frac{x^2}{2} - \frac{x^4}{8} \ldots
\]

Equation 21

Where:

- $e$ is Euler number = 0.5772.

If $x = \frac{r}{2 \sqrt{\alpha \tau}}$ is small, that is, when $\tau$ is large and $r$ smal, the terms in $x^2$ and higher powers can be neglected so that equation Equation 20 can be written:

\[
t = \frac{Q}{2\pi K} \left[ e - \ln \left( \frac{r}{2 \sqrt{\alpha \tau}} \right) \right]
\]

Equation 22

For two measurement times $\tau_1$ și $\tau_2$, the temperature rise will be:

\[
\Delta t = t_2 - t_1 = \frac{Q}{4\pi K} \ln \frac{\tau_2}{\tau_1}
\]

Equation 23

So, the thermal conductivity $K$ can be evaluated from the slope of the line $\frac{Q}{4\pi K}$.

\[
\Delta t = f(\ln \tau)
\]

Equation 24

Transient working methods are characterized by rapid measurements, good accuracy, good accuracy and in the conditions of rocks saturated with hydrocarbons and water, there is no fluid displacement in the porous space, respectively uneven distributions of saturation fluid.

These methods are used to determine the thermal conductivity of unconsolidated rocks (ocean floor sediments).

**Methods of measurement in stationary thermal regime**

In this case $\frac{\partial t}{\partial r} = 0$ and the principle of the method is to measure the temperature of a split bar consisting of two polycarbonate disks as the reference material with known thermal conductivity and the analyzed sample (Figure 4).

After a sufficient time to reach thermal equilibrium, assuming that the heat flux is axial and there are no significant radial losses between the disks:

\[
Q_2 \approx Q_1
\]

Equation 25

\[
Q_2 \approx Q_3
\]

Equation 26
Gathering the above relationships:

\[ 2Q_2 \equiv Q_1 + Q_3 \]  
Equation 27

So, the heat flux through the rock sample can be taken as the sum of the fluxes \(Q_1\) and \(Q_3\).

\[ Q_2 \equiv \frac{Q_1 + Q_3}{2} \]  
Equation 28

\[ k_r \frac{\Delta t_2}{z_2} S_2 = \frac{k_t}{z_1} \frac{\Delta t_1 S_1}{z_1} + k_t \frac{\Delta t_3 S_3}{z_3} \]  
Equation 29

Where:
- \( k_r \) is the thermal conductivity of the rock sample with cross section \( S_2 \) and thickness \( z_2 \) (mcal/ºC cms)?
- \( k_t \) is the thermal conductivity of textolite with cross section \( S_1, S_3 \) and thickness \( z_1, z_3 \) (mcal/ºC cms)?

So, the thermal conductivity is:

\[ k_r = \frac{k_t}{z_1} \frac{\Delta t_1 S_1}{z_1} + \frac{k_t}{z_3} \frac{\Delta t_3 S_3}{z_3} \]  
Equation 30

Measurement data

In the first part of the practical work, we determined the thermal conductivity of polycarbonate.

The equation of variation of the thermal conductivity of polycarbonate with respect to temperature is (Figure 5):

\[ y = 0.001x + 0.1295 \]  
Equation 31
where:
- $y$ represents the value of thermal conductivity, mcal/$^\circ$C cms,
- $x$ is the temperature of determination, $^\circ$C.

The analysis of the thermal conductivity of the cores taken from the geological structures analyzed in the chapter starts from the determination of the conductivity of the productive layers.

![Figure 5 Thermal conductivity of polycarbonate](image)

Table 1. Analysis of the productive states of the studied deposits (oil and gas)

| Drilling position | Drilling depth | Geologic layer | rocks | $k_r$ mcal/$^\circ$C cms |
|-------------------|----------------|----------------|-------|-------------------------|
| Belciugatele      | 3350           | Malm           | Chalk | 18,126                  |
| Slobozia          | 1603           | Cretacic lower | Chalk | 27,273                  |
| Slobozia          | 1603           | Cretacic lower | Fissure chalk | 19,808          |
| Smeeni A          | 4343           | Sarmatian      | Marl  | 0,6794                  |
| Smeeni A          | 4343           | Badenian       | Compact clay | 0,6363         |
| Smeeni A          | 4343           | Badenian       | Chalk | 11,353                  |
| Suraia A          | 4956           | Sarmatian      | Compact clay | 12,735          |
| Suraia B          | 4330           | Sarmatian      | Compact floor tiles | 15,299       |
| Smirna            | 4050           | Carbonifer     | Floor tiles | 20,923         |
| Smirna            | 4050           | Devonian higher | Compact clay | 17,226          |
| Smirna            | 4050           | Devonian lower | Compact clay | 19,403          |
| Ianca Berlescu    | 3550           | Sarmatian      | Conglomerate | 10,831          |
| Ianca Berlescu    | 3550           | Devonian       | Chalk /Fissure dolomite | 16,704       |
| Ianca Berlescu    | 3550           | Devonian       | Chalk /dolomite compact | 20,696       |
| Cireșu A          | 3146           | Sarmatian      | Fissure marl | 0,2890          |
| Zăvoia            | 3500           | Sarmatian      | Marne whith inclusion | 0,9095       |
| Zăvoia            | 3500           | Albian         | Chalk | 15,827                  |
| Zăvoia            | 3500           | Malm           | Limestone tiles | 15,127         |

It is found that the values of this thermal transport property depend on the composition of the porous medium, respectively the deposit area where they come from.
The cores were consolidated rocks, and the results obtained refer to the solid matrix, because either this is how they were initially presented, or during the measurements the oil was expelled from the rock pores.

**Statistical relationships between rock properties**

The cores harvested from the wells were subjected to other determinations, namely:

- Density, g/cm³,
- Porosity, %,
- Permeability, mD.

For some analyzed geological layers we performed statistical analyzes to see what the correlation equations between thermal conductivity are and the above-mentioned properties.

The equations are of the type:

\[ y = ax + b \]

Equation 32

Where:

- \( x \) is the property determined above (density, porosity, permeability) and \( y \) is the thermal conductivity?
- \( a \) and \( b \) are numerical coefficients.

### Table 2 Statistical equations for determining the density (g/cm³) as a function of thermal conductivity (mcal/ºC cms)

| Drilling   | Rocks                      | \( k_r \) | Density, g/cm³ | Ecuation                      | \( R^2 \) |
|------------|----------------------------|-----------|----------------|-------------------------------|----------|
| Slobozia   | Chalk                      | 19,808    | 2,76           | \( y = 0.2858x + 2.1612 \)   | 1        |
| Slobozia   | Fisure chalk               | 27,273    | 2,95           |                               |          |
| Smeeni A   | Chalk                      | 0.6363    | 2,69           | \( y = 0.1367x + 2.605 \)    | 1.00     |
| Smeeni A   | Compact clay               | 0.6794    | 2,7            |                               |          |
| Smeeni A   | Compact floor tiles         | 11,353    | 2,76           |                               |          |
| Smirma     | Floor tiles                | 10,831    | 2,56           | \( y = 0.2045x + 2.3353 \)   | 0.9952   |
| Smirma     | Compact clay               | 17,226    | 2,68           |                               |          |
| Smima      | Compact clay               | 19,403    | 2,73           |                               |          |
| Smima      | Conglomerate               | 20,923    | 2,77           |                               |          |
| Ianca Berlescu | Compact Chalk             | 16,704    | 2,3            | \( y = 1.4071x - 0.0341 \)   | 0.9623   |
| Ianca Berlescu | Chalk/Fissure dolomite    | 19,686    | 2,8            |                               |          |
| Ianca Berlescu | Chalk/dolomite compact     | 20,696    | 2,83           |                               |          |
| Zăvoia     | Marne white inclusion      | 0,9095    | 2,5            | \( y = 0.3778x + 2.1556 \)   | 0.9976   |
| Zăvoia     | Chalk                      | 15,127    | 2,72           |                               |          |
| Zăvoia     | Limestone tiles            | 15,827    | 2,76           |                               |          |
### Table 3 Statistical equations for determining the porosity as a function of thermal conductivity (mcal/ºC cms)

| Drilling  | Roks       | kr   | porosity, % | equations                        | $R^2$ |
|-----------|------------|------|-------------|----------------------------------|-------|
| Slobozia  | Chalk      | 19,808 | 0,5         | $y = 0,3561x + 2,022$            | 1     |
| Slobozia  | Fisure chalk | 27,273 | 2,2         |                                  |       |
| Smeeni A  | Chalk      | 0,6363 | 0,8         | $y = 0,9509x + 0,2231$           | 0,9875|
| Smeeni A  | Compact clay | 0,6794 | 0,9         |                                  |       |
| Smeeni A  | Compact floor tiles | 11,353 | 1,3         |                                  |       |
| Smirna    | Floor tiles | 10,831 | 1,2         | $y = 0,7897x + 0,2975$           | 0,9164|
| Smirna    | Compact clay | 17,226 | 1,50        |                                  |       |
| Smirna    | Compact clay | 19,403 | 1,9         |                                  |       |
| Smirna    | Conglomerate | 20,923 | 1,99        |                                  |       |
| Ianca Berlescu | Compact Chalk | 16,704 | 1           | $y = 11,588x - 18,651$         | 0,8409|
| Ianca Berlescu | Chalk /Fisure dolomite | 19,686 | 3           |                                  |       |
| Ianca Berlescu | Chalk /dolomite compact | 20,696 | 6,2        |                                  |       |
| Zăvoia    | Marne whith inclusion | 0,9095 | 5,2         | $y = 9,4431x - 3,4729$           | 0,9534|
| Zăvoia    | Chalk      | 15,127 | 10          |                                  |       |
| Zăvoia    | Limestone tiles | 15,827 | 12,2        |                                  |       |

### Table 4 Statistical equations for determining the permeability (mD) function of thermal conductivity (mcal/ºC cms)

| Drilling  | Roks       | kr   | permeability mD | Equations                        | $R^2$ |
|-----------|------------|------|-----------------|----------------------------------|-------|
| Slobozia  | Chalk      | 19,808 | 0,01            | $y = 0,775x + 1,1923$            | 1     |
| Slobozia  | Fisure chalk | 27,273 | 1,2             |                                  |       |
| Smeeni A  | Chalk      | 0,6363 | 0,005           | $y = 3,6512x - 2,3481$           | 0,999 |
| Smeeni A  | Compact clay | 0,6794 | 0,1             |                                  |       |
| Smeeni A  | Compact floor tiles | 11,353 | 1,8             |                                  |       |
| Smirna    | Floor tiles | 10,831 | 0,016           | $y = 0,0048x + 0,0103$           | 0,806 |
| Smirna    | Compact clay | 17,226 | 0,017           |                                  |       |
| Smirna    | Compact clay | 19,403 | 0,02            |                                  |       |
| Smirna    | Conglomerate | 20,923 | 0,021           |                                  |       |
| Ianca Berlescu | Compact Chalk | 16,704 | 0,03            | $y = 0,0048x + 0,0103$         | 0,91  |
| Ianca Berlescu | Chalk /Fisure dolomite | 19,686 | 0,397          |                                  |       |
| Ianca Berlescu | Chalk /dolomite compact | 20,696 | 0,8             |                                  |       |
| Zăvoia    | Marne whith inclusion | 0,9095 | 0,01            | $y = 1,3172x - 1,1884$         | 1     |
| Zăvoia    | Chalk      | 15,127 | 0,8             |                                  |       |
| Zăvoia    | Limestone tiles | 15,827 | 0,9             |                                  |       |
Qualitative analysis of the thermal conductivity of the geological layers analyzed according to the data from the literature

The literature has given values for the thermal conductivity of geological strata and for density. The accuracy of our determinations can be expressed by the absolute deviation ratio (AAD%) calculated with the equation:

$$AAD = \frac{100}{N} \sum_{i=1}^{N} \left| \frac{v_{exp,i} - v_{cal,i}}{v_{exp,i}} \right|$$

Equation 33

where:

- $v_{exp,i}$, experimental values.
- $v_{cal,i}$, calculated values.

| Geologic layer | Thermal conductivity (calculated), $\lambda$, (mcal/ºC cms) | Thermal conductivity (analysis), $k$, (mcal/ºC cms) | Absolute deviation, thermal conductivity (%) | Density calculated, $\rho$, (kg/m$^3$) | Density determinations, $\rho$, (kg/m$^3$) | Absolute deviation, thermal conductivity (%) |
|----------------|----------------------------------------------------------|-------------------------------------------------|---------------------------------------------|----------------------------------|----------------------------------|---------------------------------------------|
| Cretacic       | 1,85                                                     | 19,809                                          | 6,56                                        | 2620                             | 2950                             | 11,18                                       |
| Carbonifer     | 1,73                                                     | 20,923                                          | 17,22                                       | 2780                             | 2770                             | 6,85                                        |

The error is a maximum of 17% for carboniferous.

| Rocks          | Thermal conductivity, (calculate d), $\lambda$, (W/m°C) | Density, value of literatures, $\rho$, (kg/m$^3$) | Thermal conductivity, measurement, $k$, (W/m°C) | Absolute deviation, thermal conductivity (%) | Density determinations, $\rho$, (kg/m$^3$) | Absolute deviation, density (%) |
|----------------|----------------------------------------------------------|-------------------------------------------------|---------------------------------------------|---------------------------------------------|----------------------------------|---------------------------------------------|
| Marne with inclusion | 0,65                                                     | 2,77                                            | 0,64                                        | 1,56                                        | 2,77                             | 24,47                                       |
| Shale clay     | 0,48                                                     | 2,57                                            | 0,53                                        | 9,43                                        | 2,68                             | 35,72                                       |
| Chalk          | 0,6                                                      | 11,455                                          | 0,66                                        | 9,09                                        | 2,80                             | 29,69                                       |
| Chalk with inclusion | 0,9                                                      | 2,62                                            | 0,82                                        | 9,76                                        | 2,83                             | 26,87                                       |

The error is a maximum of 10% in conductivity, in the case of shale clay, due to the fact that it was not pure.

The density errors are large because the chosen cores were not pure, being impure with other materials.
5. CONCLUSIONS AND RECOMMENDATIONS

The purpose of this analysis was to compare the experimental results with those obtained by calculation, so as to recommend the most suitable theoretical models for estimating the equivalent thermal conductivity of an oil field.

In the initial state, before applying a process of thermal recovery or in areas of deposit downstream of the thermal front, far from it, of the idealized models, the Krupiczka model is recommended, for rock saturated with crude oil or crude oil and water.

The series model is suitable for water-saturated rock.

It should be noted that it is necessary to know the composition and thermal conductivity of the phases.

The calculation ratio, based on the correlation with more easily measurable properties (porosity, saturation), is also appropriate in the case of solid environment composed of unconsolidated rocks.

Experimental research has shown changes in thermal conductivity depending on the composition of the fluid-saturated porous medium.

The equivalent thermal conductivity of the fluid-saturated porous medium increases as the thermal conductivity of the solid medium and that of the fluid medium increase.

The solid environment of the oil field is the bedrock, and the fluid one is made up of crude oil, water, gas.

Their thermal conductivity depends on the composition and thermal conductivity of the components, where they come from.

The analyzed cores were consolidated rocks, and the results obtained refer to the solid matrix, because either this is how they were initially presented, or during the measurements the oil was expelled from the rock pores.

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