Optimal designing of a ribbed cylindrical panel made of composite material

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Abstract. The shallow open-profile cylindrical shell strengthened in the middle with a stiffener rib and made of orthotropic composite material are considered under internal normal load. A problem of determination of optimal geometrical and physical parameters of the construction that maximize its rigidity and strength for given overall shell dimensions and fixed weight equal to the weight of a shell of constant thickness is investigated.

The stated optimization problem is reduced to a nonlinear programming problem, which is solved by the deformable polyhedron method in combination with the method of direct search and using the parallel computing package in the Wolfram Mathematic software application.

Numerical examples are given on the basis of which it is shown that the optimal choice of the shell and the rib parameters can substantially increase both the rigidity and strength of the ribbed construction in comparison with the shell of constant thickness of equal weight.

1. Introduction

The use of composite materials (CM) in the thin-walled structural elements makes it possible to take full advantage of the distinctive properties of these materials in optimal design of structures by criteria for maximum rigidity and load capacity. Designing of ribbed structures made of CM contributes to the same purpose.

Aspects of to the mechanics of thin-walled structures of composite materials were investigated in monographs [1–5] and others.

In the paper [6] of the authors, problem of optimal design of a shallow cylindrical shell of an open profile of a piecewise constant thickness made of CM under internal normal load was investigated, where it was shown that an improvement in the rigidity and strength characteristics of the shell occurs with a substantial increase of the thickness in its middle part, with a simultaneous decrease of its length. This result allowed to assume that the greatest effect should be expected in the designing of a shell, reinforced in its middle part with a stiffener rib.

In connection with this, the problem of optimal design of a cylindrical shell strengthened in the middle with a stiffener rib and made of CM is considered in this paper. Since the thin-walled structural elements of CM are mostly modeled as anisotropic, the theory of anisotropic shells of S. Ambartsumyan [1] is used here in the calculations. The mathematical model of the stress-strain state of the shell is described by differential equation with respect to the potential function for orthotropic shell, boundary conditions on its contour and conditions of elastic supporting on the rib. Determination of the optimal parameters of the shell is reduced to a nonlinear programming problem which is solved by the method of the deformable polyhedron [7] in conjunction with the
method of direct search [8] and using the parallel computing package in the Wolfram Mathematic software application [9].

At present, many investigations devoted to the problems of strength, stability, and vibrations of composite shell structures have been published.

The free vibrations of FGM composite panels were investigated in [10, 11]. The paper [12] is dedicated to the solution of the buckling problem for a uniaxially compressed composite cylindrical panel.

A review of recent researches on FGM cylindrical structures under coupled physical interactions, 2000–2015, is presented in [13].

Problems of calculation of the composite shells of variable thickness have been studied in the works [14, 15]. The paper [14] analyzes the effect of the variable thickness on nonlinear buckling of imperfect cylindrical panels made of sigmoid-functionally graded material (S-FGM) under combined axial compression and external pressure. Geometrically nonlinear mathematical models of the deformation of shells of variable thickness, taking into account transverse deformations, orthotropy, nonlinear elasticity, and creep are proposed in [15].

Aspects of optimization of the composite shells have been studied in [16, 17]. In [16] a free-form optimization method is proposed that maximizes the fundamental frequencies of the orthotropic shells to avoid vibration resonance. In [17] the structural optimization of a cantilever aircraft wing with stiffeners and curvilinear spars and ribs is described.

Papers [6, 18–20] are dedicated to the problems of the optimal design of cylindrical shells of piecewise constant thicknesses.

Anisotropic shells, supported by stiffening ribs, were considered in [21, 22]. In [21], the strength and stability of orthotropic ribbed shells were investigated. In [22], a mathematical model of the deformation of reinforced orthotropic shells of rotation was constructed.

2. Statement of the problem

The shallow cylindrical shell of a radius $R$, with planar dimension $2L \times b$, hinged at the sides $y = 0$ and $y = b$ rigidly fixed at the edges $x = \pm L$ and strengthened in the middle ($x = 0$) with a stiffener rib of the rectangular section $\alpha h_r \times h_r$ is examined. It is assumed that the shell is under internal normal pressure $q(y)$ and made of a CM by the successive piling of its monolayers at the angles $\pm \varphi$ at the $x$ axis of the shell (figure 1).

The goal is to find optimal values of parameters $\alpha$, $h$, $h_r$, $\varphi$, that maximize rigidity (the lowest value of the maximum deflection) and strength (the highest value of permissible load allowable under strength condition) of the ribbed shell at its constant weight equal to the weight of the shell of constant thickness $h_0$, and given overall dimensions $\xi = 2L/b$. 

![Figure 1. The design scheme of the shell](image-url)
3. Determination of the stress-strain state of the shell

Because of symmetry the stress-strain state of the shell is determined for the region \(x \geq 0\) with satisfaction of the boundary conditions on the sides \(y = 0, y = b, x = L\) and elastic supporting on the line \(x = 0\).

The adopted structure of the shell allows considering it orthotropic. According to the theory of extremely shallow orthotropic shells [1], the problem is reduced to the definition of potential function \(\Phi(x, y)\) of the shell satisfying to equation:

\[
P_1 \frac{\partial^2 \Phi}{\partial x^8} + P_3 \frac{\partial^2 \Phi}{\partial x^6 \partial y^2} + P_5 \frac{\partial^2 \Phi}{\partial x^4 \partial y^4} + P_4 \frac{\partial^8 \Phi}{\partial x^2 \partial y^6} + P_2 \frac{\partial^8 \Phi}{\partial y^8} + \frac{1}{R^2} \frac{\partial^4 \Phi}{\partial x^4} = q, \tag{1}
\]

where

\[
P_1 = D_{11} \frac{C_{11}}{\Omega}, \quad P_3 = D_{11} \left( \frac{1}{C_{66}} - 2 \frac{C_{12}}{\Omega} \right) + 2(D_{12} + 2D_{66}) \frac{C_{11}}{\Omega},
\]

\[
P_5 = D_{11} \frac{C_{22}}{\Omega} + 2(D_{12} + 2D_{66}) \left( \frac{1}{C_{66}} - 2 \frac{C_{12}}{\Omega} \right) + D_{22} \frac{C_{11}}{\Omega}, \tag{2}
\]

\[
P_4 = 2(D_{12} + 2D_{66}) \frac{C_{22}}{\Omega} + D_{22} \left( \frac{1}{C_{66}} - 2 \frac{C_{12}}{\Omega} \right), \quad P_2 = D_{22} \frac{C_{22}}{\Omega}, \quad \Omega = C_{11}C_{22} - C_{12}^2.
\]

\(C_{ik}\), \(D_{ik}\) are the rigidities of the shell, \(C_{ik} = B_{ik}h\), \(D_{ik} = B_{ik}h^3/12\), \(i, k = 1, 2, 6\); \(B_{ik}\) are elastic characteristics of the CM in the main geometric directions of the shell, determined through its characteristics on the main physical directions \(B_{ik}^0\) by known formulas of turning [1].

The expressions for the displacements of the shell through the potential function \(\Phi(x, y)\) as follows:

\[
u = -\frac{1}{R} \frac{C_{12}}{\Omega \frac{\partial^3 \Phi}{\partial x^3}} + \frac{1}{R} \frac{C_{22}}{\Omega \frac{\partial^3 \Phi}{\partial x^3}},
\]

\[
u = -\frac{1}{R} \left[ \frac{C_{22}}{\Omega} \frac{\partial^4 \Phi}{\partial y^4} + \left( \frac{1}{C_{66}} - \frac{C_{12}}{\Omega} \right) \frac{\partial^3 \Phi}{\partial x^2 \partial y} \right], \tag{3}
\]

\[w = \frac{C_{11}}{\Omega} \frac{\partial^4 \Phi}{\partial x^4} + \frac{C_{22}}{\Omega} \frac{\partial^4 \Phi}{\partial y^4} + \left( \frac{1}{C_{66}} - 2 \frac{C_{12}}{\Omega} \right) \frac{\partial^4 \Phi}{\partial x^2 \partial y^2}.
\]

Internal forces of the shell are determined by the following formulas:

\[
T_1 = \frac{1}{R} \frac{\partial^2 \Phi}{\partial x^2 \partial y^2}, \quad T_2 = \frac{1}{\Omega} \frac{\partial^4 \Phi}{\partial x^4}, \quad S = -\frac{1}{R} \frac{\partial^4 \Phi}{\partial x^2 \partial y^2}, \quad H = -2D_{66} \frac{\partial^2 \Phi}{\partial x^2 \partial y}, \tag{4}
\]

\[
M_1 = -\left( D_{11} \frac{\partial^2 \Phi}{\partial x^2} + D_{12} \frac{\partial^2 \Phi}{\partial y^2} \right), \quad M_2 = -\left( D_{12} \frac{\partial^2 \Phi}{\partial x^2} + D_{22} \frac{\partial^2 \Phi}{\partial y^2} \right),
\]

\[
N_1 = -\left[ D_{11} \frac{\partial^3 \Phi}{\partial x^3} + (D_{12} + 2D_{66}) \frac{\partial^3 \Phi}{\partial x \partial y^2} \right], \quad N_2 = -\left[ D_{22} \frac{\partial^3 \Phi}{\partial y^3} + (D_{12} + 2D_{66}) \frac{\partial^3 \Phi}{\partial x^2 \partial y} \right].
\]

The boundary conditions are written as follows:

- hinged support on the sides \(y = 0\) and \(y = b\):
  \[
  w = 0, \quad u = 0, \quad T_2 = 0, \quad M_2 = 0 \quad \text{at} \quad y = 0, \quad y = b, \tag{5}
  \]

- rigidly fixation on the side \(x = L\)
  \[
  u = 0, \quad v = 0, \quad w = 0, \quad \frac{\partial w}{\partial x} = 0 \quad \text{at} \quad x = L, \tag{6}
  \]

\[\text{doi:10.1088/1742-6596/1474/1/012011}\]
• elastic supporting on the line \( x = 0 \)

\[
\begin{align*}
\frac{\partial w}{\partial x} &= 0, \quad \frac{\partial}{\partial y} \left( EA \frac{\partial v}{\partial y} \right) = -2S, \quad -2D_{11} \frac{\partial^3 w}{\partial x^3} = B \frac{\partial^4 w}{\partial y^4} \quad \text{at} \quad x = 0, \\
(7)
\end{align*}
\]

where \( A = \alpha h^2_r \), \( B = EJ = E\alpha h^2_r/12 \) are the cross-sectional area and the moment of inertia of the stiffening rib, \( E \) is the modulus of elasticity of the CM in the direction of reinforcement.

In the fourth of the conditions (7), due to the flatness of the shell, the curvature of the rib is not taken into account.

Expanding the load function in a series

\[
q(y) = \sum_{m=1}^{\infty} q_m \sin(\lambda_m y), \quad q_m = \frac{2}{b} \int_0^b q(y) \sin(\lambda_m y) \, dy, \quad \lambda_m = \frac{m\pi}{b},
\]

the solution of equation (1), satisfying the conditions (5), is presented as follow:

\[
\Phi = \sum_{m=1}^{\infty} \frac{q_m}{P_2 \lambda_m^8} \sin(\lambda_m y) + \sum_{m=1}^{\infty} \Phi_m \sin(\lambda_m y),
\]

where

\[
\Phi_m = \sum_{j=1}^{8} C_{jm} e^{\alpha_j \lambda_m x} = C_{1m} e^{\alpha_1 \lambda_m x} + C_{2m} e^{\alpha_2 \lambda_m x} + C_{3m} e^{\alpha_3 \lambda_m x} \\
+ C_{4m} e^{\alpha_4 \lambda_m x} + \ldots + C_{5m} e^{\alpha_5 \lambda_m x} + C_{6m} e^{\alpha_6 \lambda_m x} + C_{7m} e^{\alpha_7 \lambda_m x} + C_{8m} e^{\alpha_8 \lambda_m x}.
\]

The coefficients \( \alpha_j \) are the roots of the characteristic equation:

\[
P_1 \alpha^8 - P_3 \alpha^6 + \left( P_5 + \frac{1}{R^2 \lambda_m^4} \right) \alpha^4 - P_4 \alpha^2 + P_2 = 0.
\]

(10)

Considering expressions (3), (4), and (8), the boundary conditions (6)–(7) will be reduced to the form:

• rigidly fixation on the side \( x = L \)

\[
C_{12} \frac{d^3 \Phi_m}{dx^3} + C_{22} \lambda_m^2 \frac{d\Phi_m}{dx} = 0, \\
q_m \frac{C_{22}}{\Omega \lambda_m^5 P_2} + \frac{C_{22}}{\Omega} \lambda_m^3 \Phi_m - \left( \frac{1}{C_{66}} - \frac{C_{12}}{\Omega} \right) \frac{d^2 \Phi_m}{dx^2} = 0, \\
w_m = 0, \quad \frac{dw_m}{dx} = 0 \quad \text{at} \quad x = L;
\]

(11)

• elastic supporting on the line \( x = 0 \)

\[
C_{12} \frac{d^3 \Phi_m}{dx^3} + C_{22} \lambda_m^2 \frac{d\Phi_m}{dx} = 0, \quad \frac{dw_m}{dx} = 0,
\]

\[
EA \left[ \frac{C_{22}}{\Omega} \left( \frac{q_m}{\lambda_m^4 P_2} + \lambda_m^4 \Phi_m \right) - \left( \frac{1}{C_{66}} - \frac{C_{12}}{\Omega} \right) \lambda_m^2 \frac{d^2 \Phi_m}{dx^2} \right] = -\frac{d^3 \Phi_m}{dx^3} \quad \text{at} \quad x = 0,
\]

\[
-2D_{11} \frac{\partial^3 w_m}{\partial x^3} = B \lambda_m^4 w_m.
\]

(12)
5. Optimization of the shell by the strength criterion

The problem of optimal design of a shell of maximum load-carrying capacity is to determine permissible under strength conditions at its constant weight and overall dimensions.

4. Optimization of the shell by the rigidity criterion

Determination of the optimal parameters of the shell \( h, h, \varphi \), where the largest deflections at constant weight and overall dimensions \( \xi = 2L/h \) of the construction reach to the lowest value is reduced to the following nonlinear programming problem.

\[
\min_{\varphi, h, b} e
\]

subject to the constraints:

\[
f(h, b, h) = 0, \quad 0.005 \leq h \leq 0.26.
\]

Accordingly, the stresses in the directions of setting of the composite will be:

\[
e_1 = B_1 \xi_1 + B_2 \xi_2, \quad e_2 = B_3 \xi_1 + B_4 \xi_2, \quad e_6 = B_5 \xi_1 + B_6 \xi_2.
\]

(14)

Substituting (9) into conditions (11)-(12), the values of the coefficients \( C_m \) are determined and then the calculated values are defined according to the formulas (3) and (10). The values of the coefficients \( C_m \) will be determined by the following formulas:

\[
C_m = \left( \frac{C_2}{C_1} \right)^{1/2} \sin \frac{\varphi}{2}.
\]

(13)
Solution of this problem reduces to the following nonlinear programming problem. Find:

\[ q_0 = \max_{\bar{x}} q, \quad \bar{x} = \{\alpha, h, h_r, \varphi\}, \]  

(18)

with the constraints (16) and (17).

Here, \( q \) is the objective function which is defined from the condition:

\[ q = \min(q_1, q_2), \]  

(19)

where \( q_1 \) and \( q_2 \) are the permissible load values determined from the strength conditions written in the form of equalities for the most dangerous points of the shell

\[ \max_{x,y,z} \left[ \left( \frac{\sigma_{11}}{R_1} \right)^2 + \left( \frac{\sigma_{22}}{R_2} \right)^2 + \left( \frac{\sigma_{12}}{T_0} \right)^2 - \frac{\sigma_{11}\sigma_{22}}{R_1^2} \right] = 1, \]  

(20)

and of the stiffener rib

\[ \max_{y,z} \sigma_y^* = R_1, \]  

(21)

where: \( \sigma_y^* = -Ez\partial^2 w/\partial y^2 \) at \( x = 0 \), \( \sigma_{11}, \sigma_{22}, \) and \( \sigma_{12} \) are determined from (14), \( R_1, R_2, T_0 \) are the strength characteristics of the CM.

The problem (18) is also solved by the MDP in conjunction with the method of direct search and using the package of parallel computing in the Wolfram Mathematic software application.

6. Numerical results and discussion

Numerical calculations are made for the values \( h_0 = h_0/b = 0.01, 0.02, \bar{b} = b/R = 0.05\pi \) at \( \xi = 1, 2, 3, 5 \). CM with the following characteristics \( B_{22} = B_{22}/B_{11} = 0.6164, B_{12} = B_{12}/B_{11} = 0.12, B_{66} = B_{66}/B_{11} = 0.1572, R_2 = 0.403R_1, T_{20} = 0.264R_1 \) is considered.

As the objective function in the considered optimization problems is multi-extremely, the search by MDP was held with a number of starting vectors, using parallel computing. A suitable calculation program was compiled in Wolfram Mathematic using a computer with four core processor. From each of the considered optimization problems, four independent tasks were modeled, which were simultaneously launched for solution on different processor cores. This parallel processing yielded the final selection of the lowest (highest) value of the objective function and the corresponding optimal parameters.

The optimum values of the parameters \( \alpha, h_r, \bar{h}, \bar{h} = h/b, \varphi \) ensuring the highest rigidity of the construction, and the corresponding greatest values of the maximal reduced deflections of the shell \( \bar{w} = wB_{11}^0 h_0^3/(12q_0^b^4) \) are given in table 1. In the same table, the corresponding greatest values of the maximal reduced deflections \( \bar{w}_0 \) are given for the equilibrium shell of a constant thickness \( h_0 \) for comparison, as well as the values of coefficient \( k = \bar{w}/\bar{w}_0 \) characterizing the increase in the rigidity of the optimal ribbed shell.

The optimum values of parameters \( \alpha, h_r, \bar{h}, \varphi \) ensuring the highest strength of the construction, and also the corresponding values of the reduced load \( \bar{q} = qR/(R_1b) \) and maximal deflection \( \bar{w} \) are given in table 2. Also, the corresponding values of the reduced load \( \bar{q}_0 \) are given for the shell of a constant thickness \( h_0 \) for comparison, as well as the values of coefficient \( k_1 = \bar{q}/\bar{q}_0 \) characterizing the increase in the strength of the optimal ribbed shell.

As follows from the results of the calculation, by an optimal choice of parameters of the ribbed shell it is possible to achieve a significant increase in the stiffness (reduction in the maximum deflection) and strength (increase in the bearing capacity) characteristics in comparison with a shell of constant thickness with its constant weight and overall dimensions. For example, when \( h_0 = 0.01, \xi = 1.0 \) the greatest reduced deflection \( \bar{w} \) of the finned optimal shell is 3.43
It should be noted that, according to the calculation results, the values of the optimal parameters and corresponding values of maximum deflections of the shell.

| $h_0$ | $\xi$ | $\alpha$ | $h_r$ | $\bar{h}$ | $\varphi$ | $10^2\bar{w}$ | $10^2\tilde{w}$ | $k_0$ |
|-------|-------|----------|-------|-----------|-----------|---------------|---------------|------|
| 0.01  | 1     | 0.2      | 0.100 | 0.0081    | 90°       | 0.0487        | 0.1669        | 3.43 |
| 2     | 0.2   | 0.095    | 0.0092| 90°       | 0.2242    | 0.6171        | 2.75          |
| 3     | 0.2   | 0.090    | 0.0095| 90°       | 0.4662    | 0.9856        | 2.11          |
| 5     | 0.2   | 0.01     | 0.01  | 90°       | 1.2603    | 1.2603        | 1             |

| $h_0$ | $\xi$ | $\alpha$ | $h_r$ | $\bar{h}$ | $\varphi$ | $10^2\bar{q}$ | $10^2w$ | $10^3\tilde{q}_0$ | $k_1$ |
|-------|-------|----------|-------|-----------|-----------|---------------|--------|------------------|------|
| 0.01  | 1     | 0.2      | 0.0081| 0.0092    | 90°       | 0.2392        | 0.0487 | 0.0796           | 3.0  |
| 2     | 0.2   | 0.093    | 0.0092| 90°       | 0.0603    | 0.2243        | 0.0303 | 1.99             |
| 3     | 0.2   | 0.086    | 0.0095| 90°       | 0.0354    | 0.4667        | 0.0238 | 1.49             |
| 5     | 0.2   | 0.01     | 0.01  | 90°       | 0.0225    | 1.2603        | 0.0225 | 1               |

| $h_0$ | $\xi$ | $\alpha$ | $h_r$ | $\bar{h}$ | $\varphi$ | $10^2\bar{w}$ | $10^2\tilde{w}$ | $k_0$ |
|-------|-------|----------|-------|-----------|-----------|---------------|---------------|------|
| 0.02  | 1     | 0.2      | 0.153 | 0.0158    | 90°       | 0.0719        | 0.2582        | 3.59 |
| 2     | 0.2   | 0.135    | 0.0184| 90°       | 0.3690    | 0.9050        | 2.45          |
| 3     | 0.2   | 0.121    | 0.0192| 90°       | 0.7535    | 1.1889        | 1.58          |
| 5     | 0.2   | 0.02     | 0.02  | 90°       | 1.2894    | 1.2894        | 1             |

Table 2. Optimal parameters and corresponding values permissible load and maximum deflections of the shell.

| $h_0$ | $\xi$ | $\alpha$ | $h_r$ | $\bar{h}$ | $\varphi$ | $10^2\bar{q}$ | $10^2w$ | $10^3\tilde{q}_0$ | $k_1$ |
|-------|-------|----------|-------|-----------|-----------|---------------|--------|------------------|------|
| 0.01  | 1     | 0.2      | 0.151 | 0.0159    | 90°       | 1.3738        | 0.0720 | 0.4339           | 3.16 |
| 2     | 0.2   | 0.130    | 0.0185| 90°       | 0.2997    | 0.3696        | 0.1615 | 1.86             |
| 3     | 0.2   | 0.112    | 0.0193| 90°       | 0.1725    | 0.7560        | 0.1424 | 1.21             |
| 5     | 0.2   | 0.02     | 0.02  | 90°       | 0.140    | 1.2894        | 0.140  | 1               |

The requirements of the greatest strength and rigidity.

Table 1. Optimal parameters and corresponding values of maximum deflections of the shell.

| $h_0$ | $\xi$ | $\alpha$ | $h_r$ | $\bar{h}$ | $\varphi$ | $10^2\bar{w}$ | $10^2\tilde{w}$ | $k_0$ |
|-------|-------|----------|-------|-----------|-----------|---------------|---------------|------|
| 0.01  | 1     | 0.2      | 0.100 | 0.0081    | 90°       | 0.0487        | 0.1669        | 3.43 |
| 2     | 0.2   | 0.095    | 0.0092| 90°       | 0.2242    | 0.6171        | 2.75          |
| 3     | 0.2   | 0.090    | 0.0095| 90°       | 0.4662    | 0.9856        | 2.11          |
| 5     | 0.2   | 0.01     | 0.01  | 90°       | 1.2603    | 1.2603        | 1             |

It should be noted that, according to the calculation results, the values of the optimal parameters of the ribbed shell, obtained by the criteria of rigidity and strength differ insignificantly, which makes it possible to manufacture shells that simultaneously satisfy the requirements of the greatest strength and rigidity.

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