Demonstration of superluminal effects in an absorptionless, non-reflective system

D. R. Solli, C. F. McCormick, C. Ropers, J. J. Morehead, and R. Y. Chiao

Department of Physics, University of California, Berkeley, CA 94720-7300.

J. M. Hickmann

Department of Physics, University of California, Berkeley, CA 94720-7300. and Departamento de Física, Universidade Federal de Alagoas, Cidade Universitária, 57072-970, Maceió, AL, Brazil.

We present an experimental and theoretical study of a simple, passive system consisting of a birefringent, two-dimensional photonic crystal and a polarizer in series, and show that superluminal dispersive effects can arise even though no incident radiation is absorbed or reflected. We demonstrate that a vector formulation of the Kramers-Kronig dispersion relations facilitates an understanding of these counter-intuitive effects.

Superluminal group velocities have been observed in a number of different physical systems. These include passive absorptive [1], passive reflective [2, 3], and active transparent [4] media. There have also been numerous theoretical and experimental proposals to observe superluminality in the tunneling of electromagnetic wavepackets [5].

Here we report the first experimental observation of superluminal effects in a passive system with neither absorption nor reflection. The effects arise because of a transfer of energy or interference between two modes of the electromagnetic field, in this case two different polarizations of light. We are able to interpret these results using a new vector formulation of the Kramers-Kronig relations.

It is widely believed that the Kramers-Kronig (K-K) relations [6] require that passive systems be either absorptive or reflective in order to exhibit superluminal effects. In this paper, we demonstrate that this is not the case; while absorptive and reflective systems are required to have spectral regions of anomalous dispersion, the converse is not necessarily true. In fact, the exchange of energy between modes is a sufficient condition for superluminal propagation in any system. We show that these effects are consistent with causality.

Our experimental system consists of a slab of highly birefringent two-dimensional (2D) photonic crystal and a linear polarizer, placed in series. The photonic crystal has fundamental and second-order photonic band gaps in the regions of 10 and 20 GHz, and displays strong birefringence with very high transmission in the frequency range between the two gaps [7]. The crystal itself is an 18-layer hexagonal array of hollow acrylic rods (outer diameter 1/2") with an air-filling fraction (AFF) of 0.60. The crystal was constructed using a method which we have previously described [8].

We studied the transmission and dispersive properties of this system between the two band gaps, using an HP 8720A vector network analyzer (VNA). Microwaves were coupled to and from free space with polarization-sensitive horn antennae. The photonic crystal was placed in the far-field of the transmitter horn to ensure that planar wavefronts of a well-defined polarization were incident on it. The receiver horn was positioned immediately behind the photonic crystal on a direct line of sight with the transmitter horn. In addition, the crystal and receiver horn were placed inside a microwave-shielded box with an open square aperture, whose size was chosen to minimize diffraction effects while eliminating signal leakage around the crystal. This method has proven very effective for transmission measurements at centimeter wavelengths [8].

In order to control the polarizations of the incident and detected fields relative to the fast axis of the crystal, we mounted the transmitter and receiver horns on precision rotation stages. The angle of the incident polarization $\theta$ was held fixed at 45° relative to the fast axis of the crystal, while the angle of the receiver horn $\beta$ was allowed to vary. We define our coordinates such that the slab is oriented with its fast axis parallel to the vertical direction, and label the incident polarizations TM (transverse magnetic) and TE (transverse electric) for polarizations parallel and perpendicular to the fast axis, respectively. Our experimental setup is displayed in Fig. 1.

Our transmission amplitude and phase measurements with the receiver horn set at 40° and 50° are shown in Figs. 2(a) and 2(b), respectively. Since this spectral region is far from the band gaps, the transmission dip is not due to any band gap effect. It is caused by the fact that the photonic crystal rotates the polarization of the light by adding a different frequency-dependent phase to each polarization component [7]. For $\beta \leq 45^\circ$ there is clear anomalous dispersion in the vicinity of 16.5-17 GHz (the half-waveplate frequency for the photonic crystal) while at $\beta \geq 45^\circ$ the dispersion is normal.

A remarkable feature of our data is that while the transmission is identical for both $\beta = 40^\circ$ and $\beta = 50^\circ$, aside
FIG. 1: Experimental setup.

FIG. 2: Calculated (dotted lines) and measured (solid lines) transmission (left axis) and phase (right axis) for the detected polarization inclined at (a) 40° and (b) 50°.

from experimental errors, the phase properties are quite different. Motivated by these peculiar results, we would first like to derive a simple physical model capable of explaining these phenomena and illustrating their connection with causality.

A linear system which is invariant under time translation and is described by a scalar response (Green’s) function $g(t)$ produces a time-dependent response $b(t)$ to an input $a(t)$ given by the convolution $b(t) = \int_{-\infty}^{\infty} g(t-\tau)a(\tau)d\tau$. This expression is the starting point for the usual derivation of the Kramers-Kronig relations [9]. Our system, however, accepts a time-dependent vector input and produces a vector output. We must therefore replace the scalar convolution with the expression
\[ b_i(t) = \int_{-\infty}^{\infty} g_{ij}(t-\tau)a_j(\tau)d\tau = \int_{-\infty}^{\infty} g_{ij}(t')a_j(t-t')dt' \]  

where \( b_i(t) \) and \( a_i(t) \) are the \( i^{th} \) components of the time-dependent output and input vectors \( b(t) \) and \( a(t) \), \( g_{ij}(t) \) is the \((i,j)^{th}\) component of the Green’s function matrix which describes the system. We have employed the standard summation convention over repeated indices. In the frequency domain, Eq. 1 takes the simple form

\[ \tilde{B}_i(\omega) = \tilde{G}_{ij}(\omega)\tilde{A}_j(\omega) \]  

where \( \tilde{B}_i(\omega) \), \( \tilde{A}_j(\omega) \), and \( \tilde{G}_{ij}(\omega) \) are the Fourier transforms of \( b_i(t) \), \( a_i(t) \), and \( g_{ij}(t) \), respectively (assuming these transforms exist).

Causality tells us that the output must vanish for times before the input has propagated through the system. Thus, we require \( \int_{-\infty}^{\infty} g_{ij}(\tau)a_j(t-\tau)d\tau = 0 \) where \( T \) must be greater than or equal to the relativistic propagation time. Since this relation must be satisfied for any choice of input, the components of the Green’s function matrix vanish individually for all time prior to \( T \). This directly implies that the components \( \tilde{G}_{ij}(\omega)e^{-i\omega T} \) have analytic continuities for complex frequencies \( \tilde{\omega} \) where \( \text{Im}(\tilde{\omega}) > 0 \) (i.e., the upper half-plane in complex frequency space). Therefore, it is possible to proceed in the usual way to show that the real and imaginary parts of each component \( \tilde{G}_{ij}(\omega) \) satisfy the Kramers-Kronig dispersion relations assuming the components \( \tilde{G}_{ij}(\omega) \) are square integrable:

\[ \text{Re} \tilde{G}_{ij}(\omega) = \frac{2}{\pi} P \int_0^{\infty} \frac{\Omega \text{Im} \tilde{G}_{ij}(\Omega)}{\Omega^2 - \omega^2} d\Omega \]  

(3)

\[ \text{Im} \tilde{G}_{ij}(\omega) = -\frac{2\omega}{\pi} P \int_0^{\infty} \frac{\text{Re} \tilde{G}_{ij}(\Omega)}{\Omega^2 - \omega^2} d\Omega \]  

(4)

where \( P \) denotes Cauchy’s principal value.

The signals in our experiment can be expressed as \( \tilde{A}(\omega) = \tilde{A}(\omega)e_A \) and \( \tilde{B}(\omega) = \tilde{B}(\omega)e_B \), where \( e_A \) and \( e_B \) are unit vectors in real space, and \( \tilde{A}(\omega) \) and \( \tilde{B}(\omega) \) are the complex amplitudes of the incident and detected electric fields. In this special case where \( e_A \) and \( e_B \) are frequency-independent, Eq. 2 implies that a scalar transfer function describes the relationship between the complex functions \( B(\omega) \) and \( A(\omega) \)

\[ \tilde{H}(\omega) = \frac{\tilde{B}(\omega)}{\tilde{A}(\omega)} = \tilde{G}_{ij}e_A^i e_B^j \]  

(5)

where \( e_A^i \) and \( e_B^j \) are the \( i^{th} \) components of the unit vectors \( e_A \) and \( e_B \), respectively. Since the elements \( \tilde{G}_{ij}(\omega) \) satisfy the Kramers-Kronig relations, it follows that the scalar function \( \tilde{H}(\omega) \) must also satisfy them. However, the components of \( \tilde{G}_{ij}(\omega) \) interfere with each other in \( \tilde{H}(\omega) \).

It is this interference which leads to the superluminal effects in our experiment. The Green’s function matrix for our (birefringent) system is

\[ \tilde{G}(\omega) \leftrightarrow \begin{pmatrix} e^{i\phi_{TE}(\omega)} & 0 \\ 0 & e^{i\phi_{TM}(\omega)} \end{pmatrix} \]  

(6)

where \( \phi_{TE}(\omega) = n_{TE}(\omega)d/c \) and \( \phi_{TM}(\omega) = n_{TM}(\omega)d/c \) are the frequency-dependent phases imparted to TE and TM polarizations, \( n_{TE}(\omega) \) and \( n_{TM}(\omega) \) represent the frequency-dependent indices of refraction for the two polarizations, \( d \) is the slab thickness, and \( c \) is the vacuum speed of light. Substituting these matrix elements into Eq. 6 we find

\[ \tilde{H}(\omega) = \sin(\beta)\sin(\theta)e^{i\phi_{TE}(\omega)} + \cos(\beta)\cos(\theta)e^{i\phi_{TM}(\omega)}. \]  

(7)

For incident linear polarization at an angle of \( \theta = \pi/4 \) from the vertical, the magnitude and phase of the transfer function are, respectively,
\[
\left| \hat{H}(\omega) \right| = \frac{1}{\sqrt{2}} \{1 + \sin(2\beta) \cos [\Delta \phi(\omega)]\}^{1/2}
\]

(8)

\[
\arg[\hat{H}(\omega)] = \arg \left[e^{i\phi_{TM}(\omega)}\right] + \arctan \left\{ \frac{\sin[\Delta \phi(\omega)]}{\cos[\Delta \phi(\omega)] + \cot(\beta)} \right\}
\]

(9)

where \(\Delta \phi(\omega) = \phi_{TE}(\omega) - \phi_{TM}(\omega)\).

The transmission \(\left| \hat{H}(\omega) \right|\) has a minimum for \(\Delta \phi(\omega_m) = (2m + 1)\pi\) where \(\omega_m\) is a half-waveplate frequency of the photonic crystal and \(m\) is an integer. For \(\beta = \pi/4\), \(\left| \hat{H}(\omega_m) \right| = 0\) since the polarization which emerges from the slab has zero projection along the unit vector \(e_B\). At this point, the phase \(\arg[\hat{H}(\omega)]\) is undefined since the transmission vanishes. For \(\beta = \pi/4 \pm \epsilon\) where \(0 < \epsilon \ll 1\), we find

\[
\left| \hat{H}(\omega_m) \right| = \epsilon + \theta(\epsilon^3);
\]

(10)

\[
\left. \frac{\partial \arg[\hat{H}(\omega)]}{\partial \omega} \right|_{\omega_m} = \pm \frac{1}{2\epsilon} \left. \frac{\partial \Delta \phi(\omega)}{\partial \omega} \right|_{\omega_m} + \theta(1),
\]

(11)

where \(\tau_g = \partial \arg[\hat{H}(\omega)] / \partial \omega\) is the group delay of the transmitted wave. As \(\epsilon \to 0\) (i.e., as we approach the singularity in the transfer function), the transmission goes to zero at \(\omega_m\) and the group delay is unbounded. For example, if \(\beta = \pi/4 - \epsilon\) and \(\partial \Delta \phi(\omega)/\partial \omega\rvert_{\omega_m} > 0\), it is clear that the group delay can become superluminal, zero, or even negative depending on the value of \(\epsilon\). Thus, if one were to measure the time of flight of an analytic pulse based on the arrival of its peak, superluminal results can be obtained. However, there is no violation of causality here: the group velocity of the pulse has nothing to do with the signal velocity (i.e., the velocity of “information”), the quantity restricted by relativity. As we will demonstrate empirically, these superluminal group velocities are in fact required by causality through the Kramers-Kronig relations.

In Figs. 2a and 2b, we have plotted the results of this simple model described by Eqs. 8 and 9 for comparison with the experimental results. The frequency-dependent phases \(\phi_{TE}(\omega)\) and \(\phi_{TM}(\omega)\) were calculated using an independent measurement of the indices of refraction of the photonic crystal, and include phase delays associated with free-propagation in the air-spaces between the horns and the crystal. It is clear that the model correctly predicts the behavior of this system. Some discrepancies between the model and the experiment are observed at the edges of the plotted regions due to the effects of the band structure of the photonic crystal, which were not included in the model.

In order to transform between the real and the imaginary parts of \(\hat{H}(\omega)\), slightly modified versions of the Kramers-Kronig integrals must be used since the function \(\left| \hat{H}(\omega) \right|\) is not square integrable over all real frequencies. In practice, however, one only has actual data for \(\hat{H}(\omega)\) over a finite range of positive frequencies between values we label as \(\omega_1\) and \(\omega_2\). In this case, it is possible to approximate transforms by truncating the integrals of Eqs. 8 and 9 at \(\omega_1\) and \(\omega_2\). These transformations are approximately correct for \(\hat{H}(\omega)\) given in Eq. 7 if \(\omega_1 \ll \omega \ll \omega_2\).

On the other hand, it seems counter-intuitive, from a Kramers-Kronig point of view, that the function \(\left| \hat{H}(\omega) \right|\) is symmetric around \(\beta = \pi/4\), while \(\arg \hat{H}(\omega)\) lacks symmetry about this point. In particular, it is usually possible to obtain the phase of a causal physical response function given its amplitude using the equation

\[
\arg \hat{H}(\omega) = d_0 \omega - \frac{2\omega}{\pi} \int_0^\infty \ln \left| \hat{H}(\Omega) \right| \frac{d\Omega}{\Omega^2 - \omega^2}
\]

(12)

where \(d_0\) is a fixed, undetermined constant which characterizes the system in spectral regions of constant transmission. Clearly, this transformation cannot produce two different phase functions given a single magnitude. However, it is known that this expression must be modified if the function \(\hat{H}(\omega)\) has zeros in the upper half-plane. Thus, the phase of the response function is not uniquely determined from a measurement of the amplitude alone; if the response
function has zeros in the upper-half plane, these zeros increase the group delay in a nontrivial way over what is expected simply from \( |\vec{H}(\omega)| \). For the transfer function given in Eq. 7, we find that the zeros satisfy

\[
\Delta \phi(\tilde{\omega}_n) = -i \ln |\cot(\beta)| + 2\pi (n + \frac{1}{2})
\]  

(13)

where \( n \) is an integer and \( 0 < \beta < \pi/2 \). Given our form of \( \Delta \phi(\omega) \), it follows from Eq. 13 that all the relevant zeros lie in the lower-half plane for \( 0 < \beta < \pi/4 \), while all are in the upper-half plane for \( \pi/4 < \beta < \pi/2 \). Under these conditions, we see that it should be possible to apply a relatively simple Kramers-Kronig transformation to obtain the phase of the response function given its magnitude if \( 0 < \beta < \pi/4 \), but not if \( \pi/4 < \beta < \pi/2 \).

In Fig. 3, we display the Kramers-Kronig transformation of Eq. 12 applied to the amplitude data shown in Fig. 2(a) with \( d_0 = 1.34d/c \) (\( d_0 \) was determined by comparing the result of the integral transformation with the actual phase data far from the half-waveplate frequency). The actual phase data are also shown in this plot for comparison. Clearly, the transformation works quite well at \( \beta \approx \pi/4 - 0.087 \), confirming that the transfer function does not have any complicating zeros in the upper half-plane at this angle. However, this simple transformation cannot produce the radically different phase data obtained for \( \beta \approx \pi/4 + 0.087 \) (shown in Fig. 2(b)).

The interference between terms in \( \vec{H}(\omega) \) arises because of the two (polarization) modes available to light in this system. The birefringent photonic crystal allows a coupling between these modes, and energy can flow from one to the other. This feature is common to all systems that display superluminal effects. In absorptive media energy is scattered from the incident field into other directions by fluorescence, while in transparent active media two frequencies are present and the medium provides a coupling between them. In passive, reflective systems energy is transferred between the incoming and reflected waves. Since the vector formulation of the K-K relations presented above can apply to any pair of modes, any medium that displays this coupling between an incident mode and another mode should also display superluminal effects. This result extends the conclusion of Bolda et al., that the (scalar) K-K relations imply superluminal group velocities must exist in systems that are absorptive over some range of frequencies [11].

In conclusion, we have shown that superluminal and even negative group velocities can exist as a result of interference, instead of absorption or reflection. Moreover, it is evident from this discussion that causal superluminal propagation can potentially be observed in any system in which input energy can “escape” into other modes. Our results also demonstrate an example of an unusual application of the amplitude-phase Kramers-Kronig relations; for the system we have described, an infinitesimal adjustment of parameters radically affects the validity of a simple transformation between the amplitude and phase.

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[1] S. Chu and S. Wong, Phys. Rev. Lett. 48, 738 (1982).
[2] A. M. Steinberg, P. G. Kwiat and R. Y. Chiao, Phys. Rev. Lett. 71, 708 (1993).
[3] D. R. Solli, C. F. McCormick, R. Y. Chiao and J. M. Hickmann, IEEE J. of Sel. Topics in Quant. Elec. 9, (2003), to appear.
[4] L. J. Wang, A. Kuzmich and A. Dogariu, Nature 406, 277 (2000).
[5] G. Nimtz and W. Heitmann, Prog. Quant. Electr. 21, 81 (1997).
[6] R. Kronig, J. Opt. Soc. Am. 12, 547 (1926).
[7] D. R. Solli, C. F. McCormick, R. Y. Chiao, and J. M. Hickmann, Optics Express 11, 125 (2003).
[8] J. M. Hickmann, D. Solli, C. F. McCormick, R. Weakley, and R. Y. Chiao, J. Appl. Phys. 92, 6918 (2002).
[9] H. M. Nussenzveig, Causality and Dispersion Relations, (Academic Press, New York, 1972), Chap. 1.
[10] J. S. Toll, Phys. Rev. 104, 1760 (1956).
[11] E. L. Bolda, R. Y. Chiao, and J. C. Garrison, Phys. Rev. A 48, 3890 (1993).