Experimental Investigation of Spur Gear Tooth Crack Location and Depth Detection using Short-time Averaging Method and Statistical Indicators

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**Abstract**

Gear systems are one of the most functional power transmission systems in the industry. Crack is one of the common defects in gears which is caused by excessive loading, sudden impact and shortcomings in the gears construction. Initially, the crack will not result in structure collapse, but its growth can lead to irreparable damage. Therefore, detecting the crack and determining its location and depth are very important in this respect. In this paper, two encoders are used to obtain the spur gear pair transmission error speed. Moreover, the short-time averaging method (STAM) has been proposed thereby detecting the crack location and some statistical indicators have been used to estimate the crack depth in the spur gear tooth. For this purpose, a dynamical model in which mesh stiffness varying with time has been deployed to achieve the transmission error speed of the gear system. Additionally, a gear test rig including a single-stage gearbox, two encoders, and also an electronic board has been used. Encoders were installed on input and output shafts and the angular position of each shaft in time was saved in the computer using the electronic board. In addition, the transmission error speed was obtained by analyzing the received signals. Then, short-time averaging method was used to identify the crack location. Ultimately, some indicators such as ABS-max, FM0, Energy Ratio (ER) and Residual Signal Average were applied to the simulated results and experimental signals to fine the crack depth ratio. According to the results of this study, it seems safe to conclude that the STAM is a useful method in cracked tooth detection and the indicators have acceptable accuracy to find the crack depth ratio.

**1. INTRODUCTION**

Gear transmissions are widely utilized in various types of machines and also ranked as one of the significant mechanical components. The crack existing in the gear tooth causes an increase of vibration and generation of additional noise in the gear system. Initially, this defect does not result in the inefficiency of the system, but it can lead to irreparable damage as it grows. Therefore, the remaining life can be calculated and also the suitable program can be provided for the maintenance by detecting the faults in the early stages. The fault detection requires skill in defect detection using signals. The vibrations generated by gearboxes have a complex structure. In addition, these vibrations can be a sign of gear state, so they provide us with valuable information. According to An et al. [1], machinery fault diagnosis systems, which focus on the detection of health conditions after the occurrence of certain faults, have attracted considerable attention from practitioners and scholars. Gearbox vibration signals received from different situations have been used to detect gearbox fault. Then, the signals are processed and analyzed using certain math operations. Finally, a specific method is provided to find gearbox defects. Any tendency to deviate from healthy values indicates a system failure or defect. Currently, there are some methods such as statistical indicators, time synchronous averaging (TSA), amplitude and phase demodulation, residual signal method, autoregressive models (AR) utilized to detect some of the defects using gearbox signals [2]. Effective methods, like empirical mode decomposition [3], spectral kurtosis [4, 5], stochastic

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resonance [6], time–frequency analysis [7], [8] wavelet transform [9], sparse representation [10], and deep learning [11], have been applied and proposed for gearbox fault diagnosis. For monitoring vibration-based gearbox condition, Sait et al. [12] presented a review of several methods and indicators such as crest factor, Root Mean Square (RMS), kurtosis, wavelet transform, residual signal, difference signal, band-pass mesh signal, and Winger-Ville distribution. Braun et al. [13] simultaneously used the averaging method to identify gear defects. This method has a significant impact on the effecient extraction of various signal indicators and can be used to detect gear-related defects. Wan et al. [14] determined the dynamic response of a single-stage gearbox in which the driver's gear had a total crack in the tooth root. They utilized the Fourier transform and probed the effect of the crack on the dynamic response in the time and frequency domains. Qi et al. [15] proposed a method, which combines Fractional Fourier Transform (FrFT) and the Hilbert Transform (HT) to identify the sidebands of signal measured under the run-up process. The HT is utilized to construct the analytic representation of the measured signal, which has a better energy concentration in comparison with the measured signal in the fractional domain. Accordingly, the ability of extracting weak sidebands of FrFT is enhanced. Simulation case study and experimental case study are carried out to verify the effectiveness of the proposed method.

Mohammed and Rantatalo [16] calculated the mesh stiffness of spur gear pair and simulated the dynamics of a pair of the spur gear. They used the residual signal to signalize the crack effect on the simulated vibration signal and compared three different residual techniques. Rezaei et al. [17] proposed a new analytical approach for crack modeling in spur gears. This approach can model more than one crack in one tooth and in various locations of the gear. Rezaei et al. [18] investigated the detection possibility of two cracks in helical gear teeth in divergent situations of cracks placement using transmission error ratio method. They used the residual signal to signalize the crack effect in the simulated vibration signal.

As stated earlier, many methods have been used to detect the crack. These methods do not have an optimal result in some cases. In the present study, short space time averaging of the main signal is proposed to extract the location and size of the local defects such as the tooth root crack. Based on this method, a rectangular movable window function with a width of a signal pulse (equivalent to a cyclic gear cycle) is used for processing.

2. EXPERIMENTAL TEST RIG
To identify the crack, a gear test device, including a motor, generator, inverter, single stage gearbox, encoders and electronic board, as shown in Figure 1, is used. The motor speed is controlled by the inverter and its power is converted through one-stage reduction gearbox with a ratio of 2. Moreover, the gearbox output is connected to the generator with a belt and a pulley. Power is transferred to the generator and eventually consumed in the thermal elements. Using a wire cutting machine, a crack with the dimensions, as illustrated in Figure 2, is created on the tooth 21 from the zero point of the encoder. The crack depth is 3.5 mm and its angle from the midline of the tooth is 70 degrees. The cracked gear and the utilized system characteristics are shown in Figure 2 and Table 1, respectively.

3. SHORT-TIME AVERAGING
In the process of fault diagnosis, the measured vibration signals of weak fault damage are often interfered by strong background noise, so that it is difficult to detect the localized damage at the early stage. According to the short-time averaging method, a window function with a constant width moves from the start point of the signal to the endpoint and in each point of its motion,

Figure 1. Spur gear pair experimental test rig (a) Real photo and (b) Schematic structure plan 1- Inverter 2- Electric motor 3- One stage gearbox 4- Encoder 5- Double-row pulley 6- Generator
the average of the data available in the window is
defined as the short-time average of the point. This
method can be used to diagnose and find the location of
the local faults in periodic signals. As shown in Figure
3, in a periodic signal such as transmission error with no
local faults, in various regions in which widths are
equal to signal period, such as (1), (2) and (3), the
average of the data in each window is equal. The mean
is normally taken from an equal number of
the data on
either side of a central value. This ensures that
variations in the mean are aligned with the variations in
the data rather than being shifted in time.

In the present study, the short-time averaging of the
original signal is proposed to extract the local defects
location and size such as the tooth gear crack. The
equation of the rectangular window function is defined
as Equation (1):

\[ f_w(x) = \begin{cases} 1 & -\frac{w}{2} < x < \frac{w}{2} \\ 0 & x \leq -\frac{w}{2}, x \geq \frac{w}{2} \end{cases} \] (1)

where \( w \) is the window width calculated as follows:

\[ w = \frac{360}{N_1} \] (2)

where \( N_1 \) is the number of pinion teeth.

Since the dynamic response of the gear system is
intermittent, if it is not defective, the mean values of the
stated short time will be constant. In the presence of
local defects such as tooth crack, the average value is
changed and the defect area is well represented. The
only problem of this method is its low precision in
displaying the desired defect range due to the window
width. If the average value of each window is assigned
to its midpoint, the defect area calculated by this method
is \( w/2 \) times faster and \( w/2 \) later than the actual value
of the defect area. In this regard, two different methods
of averaging are simultaneously used to accurately
calculate the exact area of the defect area: i.e. forward
averaging and backward averaging. The average value
of the data in each window in the forward and upward
averaging is allocated to the first and last points in the
window, respectively.

These two methods have been applied to a
transmission error signal having a crack of 270 degrees,
and the results are presented in Figure 4. In the
backward averaging, the first point of the climb is the
correct result, and its end point is the false result.
Conversely, in the forward averaging, the first point of
the climb is the wrong result and the final result is
correct. Therefore, the initial point of the backward
averaging and the endpoint of the forward averaging
can be considered the start and end points of the real
flaw area, respectively. The area between the two
vertical lines is magnified in Figure 5. Besides,
according to the theory and diagram of Figure 4, it can be easily observed that the only subtle difference between the two methods is concerned with a horizontal displacement of the window width. Therefore, as shown in Figure 5, only one of the two methods (here backward averaging) can be exerted to calculate the crack region.

In this case, the starting point of leaving $\alpha_1$ is correct and its ending point $\alpha_2$ must be corrected according to Equation (3) by decreasing the width of the window:

$$\alpha_{2c} = \alpha_2 - w$$

Additionally, using the value of $\alpha_1$, the number of the cracked teeth can be simply calculated as shown in Equation (4):

$$N_{\text{Crack}} = \left\lfloor \frac{N_1 \alpha_1}{360} \right\rfloor$$

where $\left\lfloor \frac{N_1 \alpha_1}{360} \right\rfloor$ represents the function of the nearest integer value.

### 4. STIMATION OF THE DEPTH OF THE CRACK

Statistical indicators are one of the tools used to detect defects in gears. Thus, the statistical indices and different diagnostic indicators are applied to the system dynamics response and finally the effects of crack length and depth can be closely observed and considered. Accordingly, the crack size in a gear is estimated based on its dynamic response. Here, the maximum statistical index and the FM-zero diagnosis indicator have been used.

#### 4.1. ABS Max

The maximum amount of dynamic response data can be an important indicator in determining an increase in dynamic response and decrease of the stiffness of the conflict due to the presence of the crack. This is due to the fact that an increase in the dynamic response at the moment of a faulty dentine contact directly affects the maximum value. The impact of this indicator is, of course, higher than the average. The ABS max is defined as the maximum value of the absolute values of the dynamic response data [19].

#### 4.2. FM0

This parameter has been developed as a strong display for large defects in gear cogs, although it is not very useful for small defects. By comparing the maximum distance between the peaks of the signal and the total frequency range of the gear 2 and its harmonics, large changes are detected in the conflict pattern. The FM0 index is calculated as follows [20]:

$$FM0 = \frac{PP_n}{\sum_{n=0}^{H} P_n}$$

where $PP_n$ is the maximum distance between the peaks, $P_n$ is the harmonic amplitude of the $n$-th signal and $H$ is the total number of harmonic signals in the frequency spectrum.

#### 4.3. Energy Ratio (ER)

The energy ratio is defined as the ratio of the square root mean squared of the difference signal to the root mean squared of the normal signal:

$$ER = \frac{RMS_d}{RMS_n}$$

In the event of a fault, the energy is transferred from the normal signal to the differential signal.

#### 4.4. Residual Signal Average

If the residual signal values in a full period are present as a vector over time, the mean value would be calculated as follows:

$$\text{mean}_r = \frac{1}{N} \sum_{i=1}^{N} x_i$$

where, $x_i$, $N$ and $\text{mean}_r$ are the residual signal data, the number of data and the mean of data, respectively. Due to the presence of cracks in a tooth, the mesh stiffness is reduced and the dynamic response of the system increases, as a result of which the average residual signal data will also increase.

### 5. RESULTS AND DISCUSSION

Due to the time interval availability of each pulse by an encoder, in calculating the transmission error, there is a
possibility of accumulated error. To prevent the accumulation of errors, the elimination of the integration and the derivation of the data, the transmission error speed of the system is measured. Initially, the output signals from the encoders are obtained and then, the transmission error speed is obtained for 10 rounds of pinion. The output signal from the encoder is so that the time difference between the pre-pulse and the current pulse is recorded as a number. As known, the transmission error can be calculated as follows [21]:

\[ TE = R_p \theta_p - R_g \theta_g \]  

By differentiating the equation concerning the time:

\[ \frac{dT_E}{dt} = R_p \frac{d\theta_p}{dt} - R_g \frac{d\theta_g}{dt} \]  

The encoders have 3600 pulses, so the angular distances between every two consecutive pulses are equal to 0.1 degrees as below:

\[ d\theta_p = d\theta_g = \Delta \theta = 0.1^\circ \]  

So, the transmission error speed can be obtained by:

\[ \frac{dT_E}{dt} = T_E' = \frac{\Delta \theta}{\Delta t_1 - \Delta t_2} \]  

To have a more precise and clear signal and reduce the noise, measured and computed errors and other unwanted components from the signal, it is crucial to get the signal for several rotations of the pinion and then use the time-synchronous averaging method (TSA). TSA can be calculated by dividing the raw signal of several rotations of the pinion into equal parts related to the number of rotations and averaged as follows [2]:

\[ y_i = \frac{1}{R} \sum_{j=0}^{n-1} x_{nj+i} \]  

where \( R \) is the number of rotations and \( n \) is the number of data points in each rotation.

The number of these data is 36,000 in the first encoder and 18,211 in the second encoder. The difference between these numbers is due to the gear ratio. The transmission error speed of the experimental gear pair system in 10 rotations of the pinion is presented in Figure 6. After using TSA, the transmission error speed of the system in one rotation of the pinion is obtained as shown in Figure 7. According to this diagram, the signal changes from the values of 0.026 to 0.028 second of the pinion rotation that will be 114 and 133 degrees of the pinion rotation with respect to the teeth number and rotational speed. To achieve exact results, the short-time averaging method is used to extract the crack effect in the signal and determine its location. In Figure 8, the results of this method are presented.

Compared to other intelligent fault diagnosis methods, this method has advantages such as simplicity of learning for the researchers, no need for additional knowledge in the field of intelligent methods, much easier coding and execution, and much less computing volume due to a much lower number of simulation runs. In the short-time averaging results, half of the window width should be used, that is, half denture involvement cycle (about four degrees). The angular values obtained for the beginning and the end of the change closed are considered as a result; the values
obtained in this method will be 155 and 177 degrees, respectively. According to this figure, the 21st tooth of the pinion enters into contact at 155 degrees and exits at 177 degrees of the pinion rotation angle, so the crack is certainly in the 21st tooth. It should be noted that the 20th and 22nd teeth cannot cover the whole region of the crack effect. Moreover, according to the angles that obtained for the crack zone and the contact zone of the 21st tooth, it can be figured out that the crack is a full face-width crack. Completely monotonic effects (completely ascending or descending) of the depth of the crack on the applied indicators on the transmission error speed make it possible to find the fault by such a simple method instead of complex and intelligent methods of fault diagnosis or optimization. In the following, it is assumed that the cracks on the total pinion tooth and its depth vary linearly, the stiffness of the gear mesh for the different crack depths is calculated and the dynamic response is obtained. Then, the values obtained from some indicators applied to the dynamic response are achieved for different crack depths. On the other hand, the values of these indices applied to the experimental test signal are also available. The graph of the variations of each indicator and the method of estimating the crack depth using its experimental values are shown in Figure 9. Also, the experimental values of the indices and the depth of the crack ratio estimated by each indicator and their response error percentage are presented in Table 2. Respecting the results of this Table, crack depth has been estimated by the indicators with plausible accuracy. As can be seen, the FM0, ER and mean_r estimate the crack depth size with good agreement (fewer than 5% error) and positive error; the indicator ABSmax also estimates it with the negative error about 3.4 percent with respect to the previous factors. The average of the estimated values of the crack depth ratio is calculated as 3.585 mm, which has a 2.42% error to its real value of 3.5 mm.

### TABLE 2. The estimated crack depth ratio by applying the indicators to experimental signal

| Index   | Experimental value | Crack depth | Error (%) |
|---------|--------------------|-------------|-----------|
| ABS Max | 5.9e-3             | 3.38 mm     | 3.4       |
| FM0     | 3.1104             | 3.67 mm     | 4.8       |
| ER      | 0.2922             | 3.63 mm     | 3.7       |
| Mean_r  | -2.1414e-5         | 3.66 mm     | 4.6       |

6. CONCLUSION

The tooth root cracks can increase the generated vibration and noise in the gear systems. The crack does not initially result in the collapse of the structure. However, its growth can lead to irreparable damage. Therefore, detecting crack and determining its location and depth are very important for this purpose. Fault
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چکیده
چرخدنده‌ها یکی از کارآمدترین سیستم‌های انتقال نیرو در صنعت هستند. ترک یکی از نقص‌های مطرح در چرخدنده‌ها می‌باشد. این اتفاق اغلب اغلب به دلیل افزایش فشار در داخل دندان می‌باشد. در ابتدا، ترک منجر به شکست دندان نمی‌شود و سپس خطا به دلیل تغییرات زمانی و تغییرات ناشی از تغییرات ترک در دندان، ادامه یافته‌است. در این مطالعه، از دو انکودر برای محاسبه سرعت خطای انتقال استفاده شده است. همچنین، روش میانگین‌گیری زمان کوتاه‌سازی در محل و عمق ترک استفاده گردیده است. برای این منظور، از یک مدل الکترینیک با ساختار متنی از دانه‌های چرخدنده، برای بدست‌آوردن سرعت خطای انتقال سیستم دندان استفاده گردید. سپس از یک مدل دینامیکی جهت آزمایش عملی استفاده گردید. اینکوره‌ها را به‌دست‌آورده و بر اساس شایعه و نسبت به زمان ترک در دندان، میزان ترک را بدست‌آورده‌اند. سپس از این نشانه‌ها، سرعت نشتی خطای انتقال سیستم به تجزیه و تحلیل سیگنال‌های دریافت شده بدست‌آورده‌اند. نتیجه‌گیری‌ها از این آزمون‌ها نشان می‌دهد که این روش به‌عنوان یکی از روش‌های جدید برای شناسایی محل و عمق ترک را می‌تواند به‌عنوان یکی از روش‌های جدید برای شناسایی محل و عمق ترک را انتخاب کننده‌اند.