Thermal explosion characteristics of a combustible gas containing fuel droplets

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Abstract
This paper investigated the critical ignition conditions of combustible gas containing liquid fuel droplets. The analysis is done based on the criteria of the thermal explosion theory. It includes analytical and numerical solutions of modeling equations of fuel droplets heating and evaporation by convection and radiation from the surrounding reactive hot gas. The exothermic reaction is usually modeled as a single-step reaction obeying an Arrhenius temperature dependence. The thermal conductivity of the fuel droplet is dependent on temperature. The analytical solution produced relations between the main critical characteristic parameters in all planes of the solution. The results obtained from investigating the effect of the characteristic parameters on the explosion behavior of gas and fuel droplets and the thermal radiation proved that both of them are significant. The study proved that the criticality definitions of the thermal explosion of a single-phase system can be used effectively and efficiently to determine the critical conditions of a multi-phase system. Finally, the application of the numerical solutions of the modeling equations was used to analyze the explosion characteristics of a diesel fuel spray system.

Keywords
Thermal explosion, combustible gas, fuel droplets, critical conditions for ignition, numerical analysis

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Introduction
The thermal explosion of a reactive gas containing fuel droplets is a significant problem because it has many industrial applications such as in furnaces, boilers, gas turbines, internal combustion engines, and rocket engines where it is considered a critical state in these systems’ operations. Combustion of liquid fuel droplets spray in a reactive gaseous mixture (two-phase reaction) can be described as a different form of an identical pure reactive gaseous mixture (single-phase combustion). The previous experimental work showed that the addition of fuel droplets increased the flammable pre-gaseous mixture burning velocity, which in turn increased the combustion efficiency, but on the other hand, the combustion hazards also increased. The use of liquid fuel droplets, rather than pre-vaporized ones, produced an explicit oscillation during the combustion at a frequency of about 1–5 MHz. The thermal explosion of stationary exothermic chemical reaction with heat loss (Newton’s cooling law) has first been established and solved analytically. In reality, the assumption of uniform temperature, neglecting reactant consumption, measuring ignition temperature, simple thermal reaction, and others raise criticisms and limitations on applications of the theory. After that, different treatments and applications of the theory were handled in and so many others. Analytical and numerical solutions to the critical ignition and transition conditions of a dust cloud of carbon particles were obtained. The critical conditions of a gas-solid mixture in an adiabatic confined vessel was investigated. A mathematical model for the thermal explosion in a combustible dusty gas containing fuel droplets with general Arrhenius reaction-rate laws and convective and radiative heat losses considering the interphase heat exchange between the gas and inert solid particles was investigated. The effects of liquid droplets on reactive gas combustion behavior have less concern in their work. The effect of a liquid fuel spray on the thermal explosion of a combustible droplet-gas cloud

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is treated mathematically by using the integral manifold method. Their results showed that there are three major regimes: non-explosive slow, conventional fast explosive, and thermal explosion with delay. The geometrical asymptotic of the integral manifold method was also used to model the thermal explosion problem of a hot gas mixture with fuel droplets. This method was applied to the ignition process problem in diesel engines. They tried to show the role of the reduced chemistry described by the shell model on the solution of the energy equation that used the Arrhenius form of the reaction rate with the time-dependent frequency factor. The asymptotic method of the integral manifolds was also used to study the explosion characteristics of a flammable gas mixture containing volatile fuel droplets. In this study, the radiative heating of droplets model takes into account the droplets’ semi-transparency, and simplified droplets heat-up were used. The heating and evaporation of semi-transparent diesel fuel droplets in the presence of thermal radiation were investigated in ref. Their model was based on the geometrical optics approximation, which is valid for typical diesel-fuel droplets with radii of more than 3 μm. The geometrical asymptotic method of the integral manifold has been applied to the thermal explosion of combustible gas containing fuel droplets in ref. Simplified models of droplet transient heating were investigated. They studied the effect of the temperature gradient inside droplets and recirculation within them on droplets’ evaporation, ignition, and break-up. The influence of the Frank-Kamenetski critical parameter on the thermal explosion of flammable gas containing fuel droplets was investigated in. They showed that the ignition time decreases as increases. The thermal explosion of a two-phase poly-disperse combustible mixture was examined. The effect of fuel droplets’ size distribution and radiation heat transfer on the ignition conditions have been shown and discussed. The study of the combustion behavior of gel fuel droplets in the next-generation propulsion systems is essential for enhancing burn rates, lowering ignition delay, and improving the operational performance of these systems, as shown in. A mathematical model of fuel droplet combustion in a hot gas with a uniform free stream motion has been solved and discussed. The conservation equations of heat, mass, and momentum of both fuels and gas phases have been solved numerically. A simplified one-dimensional model has been studied to explore the physics of two-mode spray combustion. Also, a simplified droplet ignition model under forced convection heat transfer by using Frank-Kamenetski’s approximation model for handling the highly nonlinear reaction term has been introduced. Validation of the droplet ignition model against experimental studies of droplet ignition has been also done and discussed. The theory of integral manifolds of singularly perturbed systems has been applied to investigate the ignition behavior of combustible gas containing liquid fuel droplets. This method allowed them to define and clarify different chemical regimes, including the critical mode. An analytical method to estimate the ignition time of the fuel droplet injected into the combustion chamber of an I.C.E. was examined in ref. A correlation between the characteristic parameters of the fuel droplet and the ignition time of the droplet has been established. Combustion and ignition of a single droplet represent the appearance of a flame surrounding the droplet or in its wake region with a dimension of the order of droplet diameter was examined. A mathematical model of single fuel droplet evaporation and combustion based on the numerical simulation method of VOF (Volume of Fluid) using FLUENT was established. In this paper, the combustion behavior and flame propagation characteristics of a small droplet in the convection medium under different pressures were investigated. Recent progress in the development of a combined analytical, asymptotic, and numerical approach to model heating and evaporation of fuel droplets and ignition of a fuel vapor-air mixture was presented and discussed. Detailed modelling of the spray ignition process in the diesel engines was presented in ref. The authors used a CFD package in their analysis by considering heat and mass transfer as well as combustion processes in the mixture of gas and fuel droplets. The dynamics of thermal explosion in a fuel droplets/hot air mixture were investigated using the geometrical version of the method of integral manifolds. The results were applied to the modelling of the ignition process in diesel engines. The authors used a CFD package in their analysis by considering heat and mass transfer as well as combustion processes in the mixture of gas and fuel droplets. The dynamics of thermal explosion in a fuel droplets/hot air mixture were investigated using the geometrical version of the method of integral manifolds. The results were applied to the modelling of the ignition process in diesel engines.

The complexity of studying the combustion, ignition behavior, and characteristics of the two-phase systems makes its mathematical analysis to some extent rather rough. In a real combustion system, the researchers are worried about simultaneous heat, mass, and momentum transfer of a multi-component, and multi-phase mixture that happened in these systems. In order to separate the different physical and chemical processes of the two fuels and try to characterize their domain of links and importance, it is reasonable and trusted to choose the thermal explosion of a single-phase system and its definition of criticality to handle this complex two-phase explosion. So, in this paper, well-known definitions of the criticality of a single-phase system’s thermal explosion are used to determine the thermal explosion characteristics and critical conditions for this two-phase mixture in various planes of the solution using the exact form of Arrhenius reaction rate and gas thermal conductivity dependent on temperature. Also, the mathematical modelling equations were integrated numerically by using the Runge-Kutta method with high precision accuracy. The application of the solution of the model equations was used to analyze the explosion characteristics of a diesel fuel spray.
Mathematical equations

Assuming that the spray ignition is considered as an explosion problem (the droplets are considered as a source of endothermicity), the explosion regimes depend on physical and chemical parameters of the system. The environment is modelled as an optically thin, spatially homogenous mixture, and a combustible gas with a mono-dispersed spray of fuel droplets is considered. The deformation of the incident radiation by surrounding droplets, the droplet movement, and the temperature gradient inside the droplets have not been considered. The incident radiation has a black body spectrum, the system is adiabatic, both convective and radiative droplet heating are considered, gas pressure is constant (as in a diesel engine), and the thermal conductivity of the liquid fuel droplet is assumed to be infinitely large compared to the gaseous phase. The reaction on the right-hand side of equation (1) represents the heat transferred from the gas to the fuel droplets, respectively.

The energy balance equation for the gas phase is

\[ c_{pg} \rho_g \frac{dT_g}{dt} = C_f C_{ao} A_o e^{-E/R \theta_g} M_f Q_f / \Theta_g - 4 \pi R_g^2 \rho_g \frac{dT_d}{dt} \]  

where \( C_f C_{ao} A_o e^{-E/R \theta_g} \) represents the chemical reaction rate of highly exothermic one-step first-order reaction with gaseous fuel as a deficient reactant, and the second term on the right-hand side of equation (1) represents the heat transferred from the gas to the fuel droplets, respectively.

The concentration of fuel vapor equation is

\[ \frac{dC_{f}}{dt} = -\nu_f C_f C_{ao} A_o e^{-E/R \theta_g} + 4 \pi r_d^2 h_c(T_g - T_d) + \alpha A_d (\theta_g^4 - \theta_d^4)(1 - e^{-2k_r d}) \]  

where \( T_g \) is the temperature of the fuel vapor source and combined heat transfer mechanism, respectively. Assuming: \( q_f = h_c(T_g - T_d) \), \( h_c = \text{Nu} k_g / 2 r_d \), and \( \text{Nu} = 2.0 \) in the presence of \( \text{Re} \ll 1.0 \). The combustible gas components’ content is controlled by both oxidation and droplet evaporation.

The oxygen concentration is

\[ \frac{dC_{ox}}{dt} = -\nu_{ox} C_f C_{ao} A_o e^{-E/R \theta_g} \]  

and their further evaporation (first and second terms on the right-hand side of equation (4)) of fuel. In addition, the droplet mass \( m_d \) becomes zero when the liquid-fuel temperature reaches its boiling temperature \( T_b \). The droplet’s average temperature \( T_d \) is

\[ C_f m_d \frac{dT_d}{dt} = 4 \pi r_d^2 \left( h_c(T_g - T_d) + \alpha A_d (\theta_g^4 - \theta_d^4)(1 - e^{-2k_r d}) \right) \]  

(4)

The rate of fuel evaporation is

\[ \frac{d}{dt} \left( \frac{4 \pi r_d^3 \rho_f}{3} \right) = -4 \pi r_d^2 h_c(T_g - T_d) + \alpha A_d (\theta_g^4 - \theta_d^4)(1 - e^{-2k_r d}) \]  

(5)

The right-hand side of equation (4) becomes zero when the liquid-fuel temperature reaches the boiling temperature \( T_b \).

Note that \( 1 - (T_b - T_d) / (T_b - T_{do}) = T_d - T_{do} / T_b - T_{do} \) and \( (T_d - T_{do}] \) is the fraction of heat supplied from the gas to heat the droplet. \( (T_d - T_{do}] \) decreases from 1.0 to 0 when \( T_d \) increases from \( T_{do} \) to \( T_b \). This reflects the decrease in heat spent on droplet heating and evaporation when the droplet temperature increases, and the range of \( T_d \) under consideration is the interval \( [T_{do}, T_b] \). Assuming that gas pressure is constant where: \( \rho_{ga} T_{ga} = \rho_g T_g \) and taking into consideration that the thermal conductivity of the gas is temperature-dependent, that takes the nonlinear form as

\[ k_g = k_{ga} \left( \frac{T_g}{T_{ga}} \right)^a \]  

(6)

Regarding radiation heat transfer, as was demonstrated in21 satisfactory results for droplets of diesel fuel are produced by the following simple formula:

\[ Q_r = 1 - e^{-2 \sigma \tau_{op}} \]  

(7)

Therefore, the radiation heat flux can be obtained in the following form:

\[ q_r = \alpha \sigma (T_g^4 - T_d^4) \]  

(8)

The results in21 for diesel fuel have demonstrated that the absorptivity \( (\alpha) \) increases with increasing droplet radius \( r_d \) and decreases with increasing gas temperature \( T_g \).

All of the above governing equations are subjected to these initial conditions: \( T_d = T_{do} \), \( T_g = T_{ga} \), \( T_d = T_{do} \), \( C_f = C_{fo} \), \( C_{ao} = C_{ao} \) at \( t = 0 \).

The behaviour of the system is driven by two main processes: heat transfer from the gas phase to the liquid phase due to the evaporation process, and heat release, which is associated with
an exothermic reaction in the gas phase and is described by the Arrhenius equation. Competition between these two processes determines the main dynamic peculiarities of the system.

The dimensionless forms of equations (1)–(5) are as follows:

\[
\frac{d\theta_g}{d\tau} = \eta \beta e^{-\theta_g} - \xi \delta \left( \frac{\theta_g}{\theta_{g,\alpha}} \right)^2 (\theta_g - \theta_d) \tag{9}
\]

\[
\frac{d\eta}{d\tau} = -\eta \beta e^{-\theta_g} + \xi \psi \delta \xi \mu \left( \frac{\theta_g}{\theta_{g,\alpha}} \right)^a (\theta_g - \theta_d) + \xi^2 \left(1 - \xi \delta \epsilon \right) \tag{10}
\]

\[
\frac{d\theta_d}{d\tau} = \frac{\delta \xi \Phi \left( \frac{\theta_g}{\theta_{g,\alpha}} \right)^a (\theta_g - \theta_d) + \xi^2 \delta \epsilon (1 - \xi) (\theta_g^4 - \theta_d^4)(1 - e^{-2\xi})}{1 - e^{-2\xi}} \tag{12}
\]

\[
\frac{d\theta_d}{d\tau} = \frac{\delta \xi \Phi \left( \frac{\theta_g}{\theta_{g,\alpha}} \right)^a (\theta_g - \theta_d) + \xi^2 \delta \epsilon (1 - \xi) (\theta_g^4 - \theta_d^4)(1 - e^{-2\xi})}{1 - e^{-2\xi}} \tag{13}
\]

where characteristic values of the parameters \(\beta\) and \(\gamma\) are small compared with unity for most gaseous mixtures due to the high exothermicity of the chemical reaction and high activation energy. In addition, \(\psi\) represents the characteristics of the fuel, whereas for all realistic fuels, \(\psi \gg 1\).

**Critical conditions in \(\theta_g - \tau\) plane**

The determination of critical conditions to foresee if any system undergoes a thermal explosion or not is very important in many different applications. It is very helpful in estimating the risk or hazard accompanied by highly exothermic chemical reactions and activation energies or in thermal stability of chemical compounds or in avoiding ignition during the storage and transportation of hazardous materials or in chemical reactors or in chemical batteries, and so on.

**Case of convection heat transfer**

If we consider \(\eta = \beta = \xi = 1.0\) and \(\alpha = 0\), equation (9) becomes the classical Semenov problem with neglecting reactive consumption of zero-order reactions. Instead of that, for \(\beta = \xi = 1.0\) and \(\alpha = 0\), equation (9) represents the classical Semenov model, taking into account the convection heat loss and reactive consumption \(\eta\) via equation (10). For this study case, neglecting the last terms in the right-hand side of equations (10), (12), (13) (neglecting the radiation heat transfer). Criticality was defined as the attainment of inflection in the temperature-time trajectories, before the maximum temperature was reached (Rice et al. 1935).\(^{40}\) This definition was further used by Todes and Melentjew,\(^{41}\) Shouman and Gill,\(^{42}\) Shouman and El-Sayed\(^{9}\) and many others. In the temperature–time trajectories, the value of the induction period at which the curve begins to rise rapidly is largely determined by its behaviour in the neighbourhood of the inflection point. At this critical point, the system will ignite if its temperature is higher than the temperature at this point (supercritical) and it will not ignite if its temperature is less than the temperature at this point (subcritical). So, the critical can be defined as the point at which the locus of inflection \((d^2\Phi/d\tau^2 = 0)\) touches the integral curve of the governing equations at that point, where \((d\Phi/d\tau)^* = (d\Phi/d\tau)\). Differentiating equation (9) for \(\tau\) and equating the result to zero (inflection point) produces

\[
\theta_d^g = \frac{(2A\theta_g^* + B) \pm \sqrt{(2A\theta_g^* + B)^2 - 4A(A\theta_g^* + B\theta_g^* + C)}}{2A}
\]

where:

\[
A = \frac{\gamma \delta \xi \Phi \left( \frac{\theta_g^*}{\theta_{g,\alpha}} \right)^a (\theta_g^* - \theta_d) + \xi^2 \delta \epsilon (1 - \xi) (\theta_g^4 - \theta_d^4)}{1 - e^{-2\xi}}
\]

\[
B = -\xi \delta \psi \delta \xi \mu \left( \frac{\theta_g^*}{\theta_{g,\alpha}} \right)^a e^{-\left(1/\theta_g^*\right)} + \xi^2 \delta \epsilon \left( \frac{\theta_g^*}{\theta_{g,\alpha}} \right)^{2a}
\]

\[
C = \eta \beta \delta \epsilon e^{-\left(2/\theta_g^*\right)} - \eta^2 \beta \delta \epsilon e^{-\left(2/\theta_g^*\right)} - \eta \beta \theta_g^* \left( \frac{\theta_g^*}{\theta_{g,\alpha}} \right)^a e^{-\left(1/\theta_g^*\right)}
\]

**Convection and radiation heat transfer case**

The critical conditions for ignition can be obtained from differentiating equation (9) with respect to \(\tau\) and taking into consideration that, \(\eta\), \(\beta\), \(\theta_d\) and \(\xi^2\) are also functions of time and equating the result to zero (inflection point), that produces

\[
A\theta_d^5 - (A + B + C)\theta_d^4 + (D + E)\theta_d^2 - [A\theta_d^2 + 2B\theta_d (D + E) - S\theta_d^2]
\]

\[
+ [A\theta_d^5 + (B + C + E)\theta_d^4 + D\theta_d^2 + S\theta_d^2 + Q]
\]

\[
= 0
\]

where:

\[
A = \gamma \delta \xi \Phi \left( \frac{\theta_d^*}{\theta_{g,\alpha}} \right)^a, \quad B = \xi^2 \xi \psi \beta \delta \epsilon e^{-\left(1/\theta_g^*\right)},
\]

\[
C = \delta \xi \lambda \left( \frac{\theta_d^*}{\theta_{g,\alpha}} \right)^a (1 - e^{-2\xi})
\]
\[ D = \frac{\gamma}{\xi} \Phi(\theta^*_g/\theta_{g_*})^{2\alpha}, E = (\alpha \gamma^2 \xi^2 / \theta^*_g)(\theta^*_g/\theta_{g_*})^{2\alpha}, \]
\[ F = -\eta \beta \xi \Phi e^{-10\eta} (\theta^*_g/\theta_{g_*})^a, \]
\[ G = \xi \eta \beta \xi \mu e^{-10\eta} (\theta^*_g/\theta_{g_*})^a, \]
\[ L = \frac{\gamma}{\xi} \phi \xi \mu e^{-10\eta} (\theta^*_g/\theta_{g_*})^a, \]
\[ P = (\eta \beta \xi \eta \beta \xi e^{-10\eta}(\theta^*_g/\theta_{g_*})^a, \]
\[ Q = \eta^2 \beta (e^{-10\eta} / \theta^*_g)^2 - \eta^2 \beta (e^{-10\eta} / \theta^*_g)^2 - \eta^2 \beta (e^{-20\eta} / \theta^*_g)^2 - \eta^2 \beta (e^{-20\eta} / \theta^*_g)^2 \]

\[ , S = F + G + H + L + P \]

The positive real root of the fifth order degree equation (15) gives the critical conditions at different characteristic parameters.

Figure 1 shows the effect of the initial gas temperature \( \theta_{g_*} \) on the critical temperatures \( \theta^*_g \) and \( \theta^*_d \). At low \( \theta_{g_*} \), high \( \theta^*_g \) is accompanied by low \( \theta^*_d \) and vice versa. Because high \( \theta^*_g \) means more heat is transferred to the particle heating and evaporation, which delays gas ignition and advanced vapor particle ignition. As \( \theta_{g_*} \) increases both critical temperatures increase, because the chemical reaction sustains gas ignition. As the increase in \( \theta_{g_*} \) continue, two boundaries (flammability) limits of gas ignition have appeared. Results also showed that the behavior of \( \omega, \Phi \), and \( \psi \) like \( \theta_{g_*} \) behavior with \( \theta^*_g \) and \( \theta^*_d \) but with different values of these parameters. Figure 2 shows the effect of radiation added on the critical temperatures. At certain \( \tau_x \), the high critical gas temperature is accompanied by low critical droplets' temperature. As \( \tau_x \) increases in value, \( \theta^*_d \) increases where more radiation heat is transferred from the gas to particles. The results also revealed that the behavior of \( \lambda \) resembles \( \tau_x \) behavior for both \( \theta^*_g \) and \( \theta^*_d \).

Figure 3 shows that for \( \mu = 2.0 \), almost the S-shape for explosion limits was obtained where each \( \theta^*_g \) value could be accompanied by three values of \( \theta_{g_*} \). As \( \mu > 2.0 \), a U-shape appeared and gives the two explosion limits of gas critical temperature where the bounded area represents ignition. Only a single value of gas critical temperature is accompanied with a minimum \( \theta^*_d \) below which this value, no matter how high or low the \( \theta^*_g \) value is. As \( \mu \) increases, the ignition area collapses, and the critical temperatures of fuel droplets become higher. Figure 4 shows that, at a certain value of \( \chi \), almost S-shape for explosion limits is obtained where each \( \theta^*_g \) value could be accompanied by three values of \( \theta_{g_*} \). The high critical gas temperature is accompanied by low critical droplet temperatures. After that, the high critical gas temperature is accompanied by high critical droplet temperatures until reaching a maximum \( \theta^*_d \), above which, no matter how high or low the \( \theta^*_g \) value is high or low, then, the behavior returns to the initial state. This may be due to the inadequate heat transferred from the gas to particles, and the slow initial reaction of gas, or the opposite is true. Both the mechanisms of heat transfer and gas reaction that lead to explosion depend on the initial hot gas temperature, speed of reaction, rate of heating and evaporation of fuel particles depending on their sizes, movements of particles inside the gas, etc. As \( \chi \) increases, \( \theta^*_d \) values become higher at the same value of \( \theta^*_g \). Because, low \( \chi \) means that the heat transfer from gas related to the molar concentration of
Figure 2. \( \theta_g^* - \theta_d^* \) for \( \eta = 0.2, \beta = 0.2, \xi = 0.2, \delta = 10, \epsilon = 200, \zeta = 50, \alpha = 0.5, \lambda = 25, \mu = 0.5, \gamma = 0.005, \psi = 1500, \chi = 0.5, \omega = 0.5, \Phi = 50, \tau = 0.2, 0.4, \text{ to } 1.0, \theta_{go} = 0.05. \)

Figure 3. \( \theta_g^* - \theta_d^* \) for \( \eta = 0.2, \beta = 0.2, \xi = 0.2, \delta = 10, \epsilon = 50, \zeta = 50, \alpha = 0.5, \lambda = 5, \mu = 2.0 \text{ to } 6.0, \psi = 500, \chi = 0.5, \omega = 0.5, \Phi = 20, \tau = 0.2, \theta_{go} = 0.05. \)
Figure 4. \( \theta_{d}^* - \theta_{g}^* \) for \( \eta = 0.2, \beta = 0.2, \xi = 0.2, \delta = 10, \epsilon = 50, \zeta = 50, \alpha = 0.5, \mu = 2.0, \lambda = 5, \gamma = 0.005, \psi = 500, \chi = 0.5 \) to 2.5, \( \omega = 0.5, \Phi = 20, \tau_s = 0.2, \theta_{go} = 0.05 \).

Figure 5. \( \theta_{d}^* - \theta_{g}^* \) for \( \eta = 0.2, \beta = 0.2, \tau_s = 0.2, \xi = 0.2, \delta = 10, \epsilon = 50, \zeta = 50, \alpha = 0.5, \mu = 2.0, \gamma = 0.0025 \) to 0.0125, \( \lambda = 5, \psi = 500, \chi = 1.0, \omega = 0.5, \Phi = 20, \theta_{go} = 0.05 \).
combustible components in the gas is low, the ignition of fuel droplet vapor takes a longer time, and the opposite is true for high $\chi$. In addition, results showed that the same behavior is noticed for $\delta$, $\epsilon$, and $\zeta$ parameters with different values of critical temperatures, $\theta_g^*$ and $\theta_d^*$.

Figure 5 shows that for $\gamma = 0.0025$, the left-hand side of the curve shows that high $\theta_g^*$ is accompanied by low $\theta_d^*$ reaching a minimum of $\theta_d^*$. The right-hand side shows that high $\theta_g^*$ is accompanied by low $\theta_g^*$. Because this depends on the rate of combined heat transfer mechanisms from gas and the gas reaction. This U-shape gives the two explosion limits where the bounded area represents the possible ignition. At $\gamma = 0.005$, almost the S-shape for explosion limits is obtained where each $\theta_g^*$ value can be accompanied by three values of $\theta_d^*$. Under this S-shape, no explosion occurs. As $\gamma$ increases more, a single value of $\theta_g^*$ is accompanied by a single value of $\theta_d^*$. Above the curves, an explosion is possible. At low $\theta_d^*$, a very high critical gas temperature is needed to accomplish ignition and vice versa. Results also showed that the same behavior is noticed for the $\alpha$ parameter with different values of critical temperatures, $\theta_g^*$ and $\theta_d^*$. Figure 6 shows that for each value of $\beta$, the left-hand side of the curve shows that high $\theta_g^*$ is accompanied by low $\theta_d^*$ reaching a minimum value of $\theta_d^*$. Also, the right-hand side shows that high $\theta_g^*$ is accompanied by low $\theta_g^*$. This U-shape gives the two explosion limits where the bounded area represents the possible ignition. As $\beta$ increases, the bounded ignition zone (U-shape) shrinks as the temperature rises. The minimum $\theta_d^*$ below which ignition is impossible increases with increasing $\beta$. As $\beta$ increases, more oxygen is consumed in a gas reaction where more heat is transferred to fuel particles and the gas reaction sustains gas explosion at a lower temperature.

In Figure 7, at low $\xi$, the high $\theta_g^*$ is accompanied by $\theta_d^*$. As the particle diameter reduces more ($\xi$ increases), $\theta_g^*$ increases at the same explosion gas temperature. When the particle diameter diminishes to half its original diameter, explosion limits (U-shape) of gas start to appear and particles ignite at high temperatures because this depends on the heat transfer mechanisms, gas reaction, and particle diameter itself.

**Critical conditions in $\xi^3 - \tau$ plane**

The critical conditions for ignition can be obtained from differentiating equation (13) with respect to ($\tau$) and equating

![Figure 6](image.png)
the result to zero (inflection point) that it produces

\[
\begin{align*}
\left[ y + \frac{\alpha \gamma}{\theta_g} (\theta_g^* - \theta_d^*)^2 \right] e^{3} \\
- \left[ \eta \beta \left( \frac{\theta_g^*}{\theta_{g_0}} \right)^2 e^{-3/3} + \frac{\alpha \beta \eta}{\theta_g} \left( \frac{\theta_g^*}{\theta_{g_0}} \right)^{-3} e^{-3/3} (\theta_g^* - \theta_d^*) \right]
\end{align*}
\]

\[\xi^2 + \xi \theta_g \left( \frac{\theta_g^*}{\theta_{g_0}} \right)^{-3} (\theta_g^* - \theta_d^*) = 0 \tag{16}\]

where the real root of this equation gives \( \xi \) at a critical point for other different values of characteristic parameters.

**Critical conditions in \( \eta - \tau \) plane**

The critical conditions for ignition can be obtained from differentiating equation (10) with respect to (\( \tau \)) and equating the result to zero (inflection point), which produces

\[ \eta^* = \frac{a + b}{c} \tag{17} \]

where:

\[
\begin{align*}
a &= \frac{\delta}{\chi^2} \left( \frac{\theta_g^*}{\theta_{g_0}} \right)^{\alpha} (\theta_g^* - \theta_d^*) + \frac{\delta}{\chi} \left( \frac{\theta_g^*}{\theta_{g_0}} \right)^{\lambda} (1 - e^{-2\tau}) (\theta_g^{4d} - \theta_d^{4d}) \\
+ 4 \frac{\delta^2}{\chi^2} \chi \theta_d^{3d} \left( \frac{\theta_g^*}{\theta_{g_0}} \right)^{\lambda} (\theta_g^* - \theta_d^*) \\
+ 4 \delta^2 \lambda^2 \theta_d^{3d} (1 - e^{-2\tau}) (\theta_g^{4d} - \theta_d^{4d}) \\
b &= \delta \xi \theta_g \left( \frac{\theta_g^*}{\theta_{g_0}} \right)^{2\alpha} (\theta_g^* - \theta_d^*) + \frac{\alpha \delta \xi \theta_g}{\theta_{g_0}} \left( \frac{\theta_g^*}{\theta_{g_0}} \right)^{\alpha} (\theta_g^* - \theta_d^*) \\
+ 4 \delta^2 \delta \lambda \theta_g \left( \frac{\theta_g^*}{\theta_{g_0}} \right)^{2\alpha} (1 - e^{-2\tau}) (\theta_g^* - \theta_d^*) \\
c &= \beta \delta \xi \theta_g e^{-10\tau} \left( \frac{\theta_g^*}{\theta_{g_0}} \right)^{3\alpha} + \delta \lambda \theta_g e^{-10\tau} \left( \frac{\theta_g^*}{\theta_{g_0}} \right)^{3\alpha} (\theta_g^* - \theta_d^*) \\
+ 4 \beta \delta \xi \lambda \theta_g e^{-10\tau} (1 - e^{-2\tau}) \theta_g^3
\end{align*}
\]

where at critical point, \( \eta^* \) can be determined for other different characteristic parameters. In addition, you can see that \( \theta_g^* \) or \( \theta_d^* \) can be determined in this plane at different characteristic parameters.

**Critical conditions in \( \theta_d - \tau \) plane**

The critical conditions for ignition can be obtained from differentiating equation (12) with respect to (\( \tau \)) and equating the result to zero (inflection point), which produces

\[ A \theta_d^{4d} + B \theta_d^{3d} + C \theta_d^3 + D = 0 \tag{18} \]

where:

\[
\begin{align*}
A &= -4 \lambda \xi (1 - e^{-2\tau}) \left( \frac{\theta_g^*}{\theta_{g_0}} \right)^{\alpha} + \frac{2\delta \xi \lambda \theta_g}{\theta_{g_0}} (1 - e^{-2\tau}) \left( \frac{\theta_g^*}{\theta_{g_0}} \right)^{\alpha} \\
+ \frac{\delta \lambda \theta_g}{\theta_{g_0}} (1 - e^{-2\tau}) \left( \frac{\theta_g^*}{\theta_{g_0}} \right)^{3\alpha} + \frac{4 \lambda \xi \delta}{\theta_{g_0}} (1 - e^{-2\tau}) \theta_g^3
\end{align*}
\]

Figure 7. \( \theta_g^* - \theta_d^* \) for \( \eta = 0.2, \beta = 0.2, \xi = 0.05, 0.15,\ldots,0.45, \delta = 10, \varepsilon = 50, \zeta = 50, \alpha = 0.2, \lambda = 5, \gamma = 0.005, \mu = 2.0, \psi = 500, \chi = 1.0, \omega = 0.5, \phi = 0.2, \tau = 0.2, \theta_{g_0} = 0.05. \)
$B = -4\eta \beta (1 - e^{-2\tau}) e^{-10\tau} + 4\lambda \xi \gamma (1 - e^{-2\tau}) \left( \frac{\theta^v_g}{\theta^g_0} \right)^\alpha \theta^v_g$

$C = \xi \gamma \left( \frac{\theta^v_g}{\theta^g_0} + \frac{a_1 \xi \gamma^2}{\xi} \left( \frac{\theta^v_g}{\theta^g_0} \right)^{2\alpha} + \frac{a_2 \xi \gamma^2}{\xi} \left( \frac{\theta^v_g}{\theta^g_0} \right)^{2\alpha} \right) + 4\delta \lambda \xi \gamma \left( \frac{\theta^v_g}{\theta^g_0} \right)^{(1 - e^{-2\tau})} \theta^v_g$

$D = -\eta \beta (1 - e^{-10\tau})$

$+ \xi \gamma \left( \frac{\theta^v_g}{\theta^g_0} \right)^\alpha \frac{\delta \xi \gamma^2}{\xi} \left( \frac{\theta^v_g}{\theta^g_0} \right)^{2\alpha} \theta^v_g$

$- \frac{2\delta \xi \gamma^2}{\xi} \left( \frac{\theta^v_g}{\theta^g_0} \right)^{2\alpha} \theta^v_g$

$- \frac{\delta \xi \gamma^2}{\xi} \frac{\theta^v_g}{\theta^g_0} \left( \frac{\theta^v_g}{\theta^g_0} \right)^{2\alpha} \theta^v_g$

$- \frac{4\delta \lambda \xi \gamma}{\xi^2} \left( \frac{\theta^v_g}{\theta^g_0} \right)^{(1 - e^{-2\tau})} \theta^v_g$

$- \frac{\delta \xi \gamma}{\xi} \left( \frac{\theta^v_g}{\theta^g_0} \right)^{(1 - e^{-2\tau})} \theta^v_g$

$- \frac{4\xi^2 \delta \xi \gamma}{\xi} \left( \frac{\theta^v_g}{\theta^g_0} \right)^{(1 - e^{-2\tau})} \theta^v_g$

**Numerical solution**

Equations (9)–(13) subjected to the initial conditions can be solved numerically using the fourth-order high precision Runge-Kutta technique. Solution trajectories of different characteristic parameters as a function of time in various planes have been obtained as shown below. The study concentrates on analyzing regimes characterized by a delay period before the occurrence of the blow-up (thermal runaway), which are particularly important for practical applications.

**Convection and radiation heat transfer case**

Figure 8 shows the trajectories of $\theta_f$ and $\theta_d$ with $\tau$ for different values of $\psi$ at certain values of other characteristic parameters. The moment that the fuel droplet is suddenly exposed to a hot environment, its surface temperature increases due to the heat transfer from the environment to it until vaporization starts. The produced fuel vapor mixes with the oxidizer, forming a locally combustible mixture. As the chemical reactivity intensifies in the gas, heat-releasing reactions are initiated, and the gas temperature in the droplet vicinity starts rising, reaching ignition. The droplet ignition delay is determined by calculating the time from the moment of the droplets introduced into the hot gas to the moment of the ignition (inflection point in the temperature-time trajectories also). This ignition delay (chemical delay) represents the time required for chemical reactions of the gas mixture to reach a thermal runaway condition. As shown in Figure 8, at a modified Semenov number ($\gamma \leq 0.002$), gas explodes at a smaller ignition time compared to vapor fuel droplets in a combustible gas mixture. Because the fuel droplets are cooler and it takes more time for heating, evaporation, and ignition in the mixture depending on the gas chemical reaction and the rate of heat transferred from the gas to droplets. The figure also shows that the ignition temperature of the gas is higher than that of fuel droplets. Once the fuel droplet is suddenly exposed to a hot environment, gas is cooled with accompanying heating and evaporation of droplets. The fuel vapor is allowed to diffuse through the gas sustaining-ignition of the fuel vapor-air mixture. It can be seen, that for $\gamma > 0.002$, the gas mixture did not explode because the rate of heat transfer from the gas to the fuel droplets is higher and the gas heat reaction is not enough to sustain ignition. It can be seen that the gas temperature decreases with increasing time and the fuel droplet temperature increases until a certain time, then it remains fixed as time increases (self-heating or non-ignition case). Therefore, the effect of changing $\gamma$ values can be strictly noticeable in ignition or non-ignition cases, as shown in Figure 8. Figure 9 shows the trajectories of $\theta_f$, $\theta_d$, $\gamma$, $\eta$, and $\beta$ with $\tau$ for different values of $\psi$ at certain values of other characteristic parameters. The figure shows that as time proceeds, the gas and fuel particles’ temperature increase. Then, at a certain time, a thermal runaway occurs, reaching the maximum temperature where both trajectories become coincident (combustible gas mixture), then decaying as time proceeds, reaching its almost initial state. The ignition time of the gas is lower than that of the fuel droplets at the same $\psi$. As $\psi$ increases, the ignition time decreases for both gas and fuel droplets because the specific combustion energy increases due to chemical reaction, which lets more heat be transferred from the gas to fuel particles that enhance the heating and evaporation of the droplets, so ignition occurs at a shorter time. Thermal radiation leads to a decrease in the gas temperature during the period of intensive evaporation of fuel. As a result, on the contrary, it leads to some delay in the thermal explosion at low $\psi$. It is also noticed that, as $\psi$ increases, the reduction in oxygen concentration ($\beta$) increases and vanishes in a shorter time.

Figure 10 shows the trajectories of $\theta_f$, $\theta_d$, $\gamma$, $\eta$, and $\beta$ with $\tau$ for different values of $\eta$ and constant values of other characteristic parameters. At certain fuel reactant depletions ($\eta$), one can see that gas explodes at a lower ignition time compared to the ignition time of fuel droplets vapor. As reactant fuel depletion increases ($\eta$), the ignition temperature of gas and fuel droplets vaporizes decreases. Because gas heat reaction increases, more heat is transferred from the gas phase to droplets due to its thermal conductivity.
and radiation heat transfer. As time increases, more reactants are consumed until they are exhausted at the end of the reaction. As time proceeds, the oxygen concentration β is almost not consumed until the reaction strongly leads to ignition, then it decreases vastly until the end of the reaction. As η increases, the oxygen concentration is consumed in a shorter time since the ignition time of the gas mixture decreases with increasing η.

Figure 11 shows the trajectories of θg, θd, η, and β with τ for different values of β and c values of other characteristic parameters. c must be ‘constant’. The figure shows that at 10% and 25% reduction in oxygen concentration (β), the temperature-time trajectories did not show ignition (gas and droplets fail to ignite) at pre-determined conditions. Because the chemical energy to be released, while burning the fuel vapor in the initial gaseous mixture (per unit volume) must be much greater than the energy needed to evaporate all the droplets (in the same unit volume). As trajectories show explosion behavior (an inflection point appears in the trajectories), the reduction in (β) increases due to the progress of the chemical reaction. As β value increases (more oxygen has been consumed), the ignition time decreases. This is due to the high heat of the reaction being released, which leads to a low ignition temperature. As time proceeds, the reduction in β increases and vanishes towards the end of the reaction. Also, as β increases, more oxygen depletion is noticed, and the β trajectories remain unchanged early on, then they increase with time, showing a peak (maximum depletion), after which they decrease rapidly until they vanish at the end of the reaction. The increase in β means that more reactions are taking place, which promotes an increase in heat removal from gas to droplets as well as thermal radiation, which causes thermal explosion to form. Regarding η, the figure shows that for low, β, and the change in reactant depletion cannot be noticed early, but as β increases, the reduction in η is remarkably noticed at an early time, and this reduction proceeds fast with time when more oxygen is consumed due to the progress of chemical reactions.

Figure 12 shows the trajectories of θg, θd, η, and β with τ for different values of χ at certain values of other characteristic parameters. As shown in the figure, the ignition time of the fuel droplet is higher than that of gas at χ = 0.05 and the opposite is true for χ > 0.05. Because for low χ, the heat transfer from the gas is low, so the ignition of a combustible fuel vapor takes more time, and the opposite is true for high χ. Regarding, η, as time increases η increases due to the added fuel droplets vapor reaches a maximum, then
Figure 9. \( \theta_g, \theta_d, \eta, \beta \) vs \( \tau \) for \( \eta = 0.1, \beta = 0.1, \xi = 0.5, \delta = 0.05, \varepsilon = 0.5, \zeta = 0.05, \alpha = 0.1, \gamma = 0.0002, \lambda = 0.5, \mu = 0.2, \psi = 1.0, 3.0, 5.0, \chi = 0.05, \Phi = 0.000005, \tau_s = 0.15, \omega = 0.5, \theta_{go} = 0.15, \theta_{do} = 0.005 \).

Figure 10. \( \theta_g, \theta_d, \eta, \beta \) vs \( \tau \) for \( \eta = 0.40, 0.55, 0.70, 0.85, \beta = 0.5, \xi = 0.5, \delta = 0.01, \varepsilon = 0.5, \zeta = 0.05, \alpha = 0.1, \gamma = 0.0002, \lambda = 0.5, \mu = 0.2, \psi = 1, \chi = 1.0, \omega = 0.5, \Phi = 0.000005, \tau_s = 0.15, \theta_{go} = 0.15, \theta_{do} = 0.005 \).
decreases as the concentration of combustible components in the gas decreases until it is exhausted at the end of the reaction. As time proceeds, there is almost no oxygen consumed ($\beta$) until the reaction strongly leads to ignition, then the oxygen concentration decreases vastly until the end of the reaction. As $\chi$ increases, the oxygen concentration is consumed in a shorter time since the ignition time of gas decreases as $\chi$ increases. Figure 13 shows the trajectories of $\theta_g$, $\theta_d$, $\eta$, and $\beta$ with $\tau$ for different values of $\delta$ at certain values of other characteristic parameters. At a
Figure 13. $\Theta_g, \Theta_d, \eta, \beta$ vs $\tau$ for $\eta = 0.1, \beta = 0.1, \xi = 0.5, \delta = 0.05, 0.5, 0.25, \epsilon = 0.5, \zeta = 0.05, \alpha = 0.1, \gamma = 0.0002, \lambda = 0.5, \mu = 0.2, \psi = 1.0, \chi = 0.05, \omega = 0.5, \tau_s = 0.15, \phi = 0.000005, \theta_g = 0.15, \theta_d = 0.005$.

Figure 14. $\Theta_g, \Theta_d, \eta, \beta, \xi$ vs $\tau$ for $\eta = 0.1, \beta = 0.2, \xi = 0.5, \delta = 0.05, \epsilon = 0.5, \zeta = 0.05, \alpha = 0.1, \gamma = 0.0002, \lambda = 0.5, \mu = 0.2, 0.4, 0.6, \psi = 5.0, \chi = 0.05, \omega = 0.5, \phi = 0.000005, \theta_g = 0.15, \theta_d = 0.005, \tau_s = 0.15$. 
Figure 15. $\theta_g, \theta_d, \eta, \beta,$ and $\xi$ vs $\tau$ for $\eta = 0.1$, $\beta = 0.5$, $\xi = 0.5$, $\delta = 0.05$, $\epsilon = 0.5$, $\zeta = 0.05$, $\alpha = 0.1$, $\gamma = 0.0005$, $\lambda = 0.5$, $\mu = 0.2$, $\psi = 5.0$, $\chi = 0.05$, $\omega = 0.5$, $\Phi = 0.000005$, $\tau_s = 0.15$, $\theta_{gs} = 0.15$, 0.3 and $\theta_{ds} = 0.005$.

Figure 16. $\theta_g, \theta_d, \eta, \beta,$ and $\xi$ vs $\tau$ for $\eta = 0.1$, $\beta = 0.5$, $\xi = 0.5$, $\delta = 0.05$, $\epsilon = 0.5$, $\zeta = 0.05$, $\alpha = 0.1$, $\gamma = 0.0005$, $\lambda = 0.5$, $\mu = 0.2$, $\psi = 5.0$, $\chi = 0.05$, $\omega = 0.5$, $\Phi = 0.000005$, $\tau_s = 0.25$, 0.5, 0.75, $\theta_{gs} = 0.15$, 0.30 and $\theta_{ds} = 0.005$. 
certain value of \( \delta \), one can see that hot gas explodes to some extent at a smaller ignition time (inflection point) comparable to fuel droplets vapor. As time proceeds, the trajectories of gas and fuel droplets coincide as one gas combustible mixture trajectory. As \( \delta \) increases, the ignition time for both gas and fuel droplets becomes lower because the reaction and heat transfer process proceed faster. The \( \eta \) value increases as time increases due to the added fuel droplets vapor reaching a maximum, then decreases as the concentration of combustible components in the gas decreases until it is exhausted at the end of the reaction. As time proceeds, there is no almost oxygen consumed (\( \beta \)) until the reaction strongly leads to ignition, then, \( \beta \) decreases vastly until the end of the reaction. As \( \mu \) increases, the particle diameter (\( \xi \)) vanishes in a shorter time since the ignition of fuel particles decreases with increasing \( \mu \). As time passes, the rate of depletion increases, reaching a maximum at the start of ignition, then decreases and remains to some extent fixed up to a certain time (rate of depletion is so slow), and then drops to almost zero after the combustible gas reaction disappears. Regards \( \beta \), as time proceeds, there is no oxygen almost consumed until the reaction strongly leads to ignition, then the oxygen concentration decreases vastly until the end of the reaction. As \( \mu \) increases, the oxygen concentration is consumed in a shorter time since the ignition time of the gas decreases as \( \mu \) increases.

![Figure 17. \( \Theta_g, \theta_d, \eta, \beta, \) and \( \xi \) vs \( \tau \) for \( \eta = 0.1, \beta = 0.5, \xi = 1.0, \delta = 0.05, \epsilon = 0.5, \zeta = 0.05, \alpha = 0.1, \gamma = 0.00002, \lambda = 0.5, \psi = 5.0, \chi = 0.05, \omega = 0.5, \Phi = 0.0005, \theta_{g_i} = 0.15, \theta_{d_i} = 0.005, \tau_i = 0.15, \mu = 0.2. \) ](image)

Figure 15 shows the effect of \( \theta_{g_{0i}} \) on \( \theta_g, \theta_d, \eta, \beta, \) and \( \xi \) at certain values of other characteristic parameters. The behavior of the main parameters with time does not change, but one can see that increasing the initial gas temperature decreases the ignition times. Because more heat is transferred from the gas to fuel droplets to sustain its ignition in a shorter
time, where the gas reaction sustained its ignition. Generally, it can be noticed that the time of the thermal explosion of gas almost coincides with the time of droplet heating and evaporation. As $\theta_{go}$ increases, the droplet diameter reduces rapidly and vanishes in a shorter time. Also, as the initial gas temperature increases, the oxygen concentration ($\beta$) reduces rapidly and vanishes in a shorter time. Regards, $\eta$, one can see that as $\theta_{go}$ increases, the maximum reactant depletion appears at such an early time, and it decreases rapidly and vanishes in a short time due to the high speed of the reaction that leads to the explosion.

Figure 16 shows that for $\tau_s < 0.5$, the ignition delay for both gas and the fuel droplets is lower and as $\tau_s (=0.75)$ increases, the ignition delay becomes so long, but at $\tau_s = 1.0$, the gas temperature decreases as time increases (no ignition) due to the enhanced radiation heat loss from the gas to the fuel droplets. On the other side, fuel droplets heat up with time until it reaches a certain time, and then the temperature remains fixed up to the end of the reaction. As $\tau_s$ increases, more heat is transferred from the gas that delays the speed of the reaction and cools the gas so that no ignition is occurring. With regards to, $\eta$, one can see that as $\eta$ decreases, the maximum reactant depletion appears so early and decreases rapidly and vanishes in a short time due to the high speed of the reaction.20 As $\zeta$ increases, the reduction in droplet diameter takes a longer time. The reduction in oxygen concentration $\beta$ dropped early and the radiation has little effect.

**Convection against radiation heat transfer cases**

Figures 17 and 18 show the trajectories of $\theta_g$, $\theta_d$, $\eta$, $\xi$, and $\beta$ with $\tau$ at certain values of characteristic parameters for convection and convection with radiation heat transfer processes. One can see that the addition of radiation heat loss early the ignition times of both gas and fuel droplets vapor. Also, adding radiation heat loss enhances the heat transferred to the particles, which aids them to be vaporized and ignited early. So, the reduction in particle diameter occurred early and dropped suddenly in a shorter time where most of the fuel is consumed. Then, the rate of particle diameter reduction is so slow that it drops to zero at the end of the reaction. Also, the thermal radiation would increase at the initial stage of evaporation, especially, for larger droplets. The increase of the combined heat-transfer coefficient due to the contribution of thermal radiation would decrease the total ignition delay. For the heat transferred by convection only, the reduction in the particle diameter needs a long time until the gas reaction heat is so high enough to enhance the heat transferred to particles.

![Figure 18](image.png)

**Figure 18.** $\theta_g$, $\theta_d$, $\eta$, $\xi$, and $\beta$ vs $\tau$ for $\eta = 0.1$, $\beta = 0.5$, $\xi = 0.5$, $\delta = 0.05$, $\epsilon = 0.5$, $\zeta = 0.05$, $\alpha = 0.1$, $\gamma = 0.00002$, $\lambda = 0.5$, $\mu = 0.2$, $\psi = 5.0$, $\chi = 0.05$, $\omega = 0.5$, $\phi = 0.0005$, $\tau_s = 0.15$, $\theta_{go} = 0.15$, $\theta_{do} = 0.005$. 
and the gas mixture starts the ignition. Regarding the effect of radiation on the reactant consumption ($\eta$), one can see that $\eta$ increases with time until reaching a maximum after the gas explodes, then decreases gradually until reaching zero at the end of the reaction. The effect of radiation on oxygen concentration ($\beta$) is not noticeable.

**Application to diesel fuel droplets**

The values of main and characteristic parameters in a diesel-engine combustion chamber vary widely, so the values of $r_{do} = 5 \mu$m and $50 \mu$m have been chosen for the calculations as in literature, and other main parameters are taken from\textsuperscript{18-20,23,43} as $E = 7.6 \times 10^7$ (J/kmol), $k_p = 0.061$ (W/m K), $\rho_g = 23.8$ (kg/m$^3$), $c_{ps} = c_{pv} = 1120$ (J/kg K), $T_g = 900$ (K), $Q_f = 4.3 \times 10^7$ (J/kg), $\sigma = 5.67 \times 10^{-8}$ (W/m$^2$ K$^4$), $T_p = 600$ (K), $T_d = 300$ (K), $C_{do} = 2830$ (J/kg K), $L = 3.6 \times 10^5$ (J/kg), $\rho_{do} = 600$ (kg/m$^3$), $M_f = 170$ (kg/kmol), $\omega = 38.54$, $\phi = 0.0011$, $\tau = 1.0$, $\theta_{do} = 0.0328$.

For 5-micron fuel particle diameter. The trajectories of time histories of gas temperature, liquid-fuel temperature, droplet radius, oxygen concentration, and fuel-vapor concentration are shown in Figure 19.
obtained from solving equations (9)–(13) numerically using the above data of diesel engine fuel for 5-μm fuel particle diameter are presented in Figure 19(a)–(c). The ignition starts at about 1.57 to 1.96 ms and the sharp increase in fuel-vapor concentration occurs at times ranging from 1.6 to 2.3 ms. During this period, the time of heating and evaporation of the particles coincides nearly with the gas explosion time. On the other hand, the time of droplet heating until evaporation occurs is small before ignition. The figure also shows that the decrease in the ignition time with increasing the initial gas temperature predicted from this figure is consistent with predictions of.20,23

Figure 20. (a–c) \( \eta = 0.012, \beta = 1.0, \zeta = 1.0, \delta = 15.416, \varepsilon = 1337, \zeta = 0.66, \alpha = 0.5, \gamma = 0.0001005, \lambda = 0.551, \mu = 4.3, \psi = 0.7, \chi = 0.001528, \omega = 3854, \Phi = 0.000011, \tau_i = 1.0, \theta_{d} = 0.0328.

For 50-micron fuel particle diameter. Figure 20(a)–(c) shows the trajectories of time histories of gas temperature, liquid-fuel temperature, droplet radius, oxygen concentration, and fuel-vapor concentration obtained from the above data for diesel engine fuel with a particle diameter of 50 m. The ignition starts at about 2.8 ms and the sharp increase in fuel-vapor concentration occurs at times of 5 to 5.5 ms. During
this period, the time of heating and evaporation of the particles coincides to some extent with the gas explosion time. Because large droplets need more heat, which is taken from gas to break-up and evaporate. This means that a higher initial gas temperature is needed to ignite the gas and evaporate particles in a shorter time, as shown in Figure 20(a) and (d). On the other hand, the time of droplet heating until evaporation occurs is higher compared with the small particles. The figure also shows that the decrease of ignition time with the increasing initial gas temperature predicted from this figure is consistent with predictions of.\textsuperscript{20,23}

**Conclusions**

Analytical and numerical solutions of droplets heating and evaporation by convection and radiation from the surrounding reactive hot gas obeying the exact form of Arrhenius reaction rate with thermal conductivity dependent on temperature as well as ignition were examined. The critical conditions and solution trajectories of the main characteristic parameters in different planes produced the following conclusions as

1. Thermal radiation leads to a decrease in the gas temperature during the period of intensive evaporation of fuel and, as a result, on the contrary, to some delay in thermal explosions.
2. During the droplet heat up and evaporation, the solution indicated synchronization between the temperature of the droplets and that of the hot gas, with only higher changes in the other dependent variables.
3. The increase of the combined heat-transfer coefficient due to the contribution of thermal radiation would decrease the total ignition delay time.
4. The maximum reactant depletion ($\eta$) appeared at an early time, then decreased rapidly and vanished in a short time due to the high reaction speed.
5. As the initial gas temperature increases, the droplet diameter reduces rapidly and vanishes in a shorter time.
6. Thermal radiation leads to a decrease in the gas temperature during the period of intensive evaporation of fuel and, as a result, on the contrary, to some delay of the thermal explosion at low $\psi$.
7. As $\beta$ value increases (more oxygen consumed), the ignition time decreases due to the high heat of reaction released that leads to a low ignition temperature.
8. At modified Semenov number ($\gamma$$<$$0.002$), the gas explodes at a smaller ignition time, but for $\gamma$ > 0.002, the gas mixture does not ignite because the rate of heat transfer regarding the gas heat reaction is not enough to sustain ignition.
9. It is important to note that the ignition delay before the onset of the explosion is very important from a practical point of view. Its dependence on the physical and chemical characteristics of the liquid fuel can be important in estimating the safety advantages of a given system.

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**Appendix**

**Notation**

- $A_o$: frequency factor, m$^3$/kmol s,
- $c$: specific heat capacity, J/kg K,
- $C$: molar concentration of the combustible component in the gas mixture, kmol/m$^3$,
- $E$: activation energy, J/mol,
- $K_a$: spectral absorptivity of fuel and assumed constant,
- $L$: latent heat of vaporization, J/kg,
- $m$: fuel droplet mass, kg,
- $M_f$: molar mass of fuel vapor, kg/kmol,
- $n_d$: number of fuel droplets per unit volume, 1/m$^3$,
- $q$: heat flux, J/m$^2$ s,
- $Q$: specific combustion energy, J/kg,
Dimensionless parameters

- $\beta$: concentration of oxygen ($\beta = \frac{C_{ox}}{C_{p,o}}$).
- $\gamma$: modified Semenov number ($\gamma = \frac{4\pi \nu \sigma k_{g} \rho_{f} E}{R \nu \sigma C_{f} \rho_{f} E \phi}$).
- $\delta$: represents the heat transfer process of gas and heat of combustion ($\delta = \frac{\tau_{s} \phi_{g}}{\rho_{f} \sigma \rho_{f} \nu \sigma C_{f} \rho_{f} E \phi}$).
- $\epsilon$: relates the activation energy and energy needed to evaporate all fuel droplets ($\epsilon = \frac{3k_{g} \rho_{f} \phi \rho_{f} \nu \sigma C_{f} \rho_{f} E \phi}{\rho_{f} \sigma \rho_{f} \nu \sigma C_{f} \rho_{f} E \phi}$).
- $\zeta$: represents the fraction of heat supplied from gas to heat the droplet ($\zeta(\theta_d) = \frac{\theta_{d} - \theta}{\theta_{bo} - \theta_{d}}$), and
- $\eta$: concentration of fuel ($\eta = \frac{C_{f}}{C_{p,f}}$).
- $\theta$: temperature ($\theta = \frac{E}{R}$).
- $\lambda$: relates between radiation heat transfer and the molar concentration of combustible components in the gas ($\lambda = \frac{3k_{g} \rho_{f} \phi \rho_{f} \nu \sigma C_{f} \rho_{f} E \phi}{\rho_{f} \sigma \rho_{f} \nu \sigma C_{f} \rho_{f} E \phi}$).
- $\mu$: relates the gas conductivity and the total fuel heat ($\mu = \frac{4\pi \nu \sigma k_{g} \rho_{f} E}{R \nu \sigma C_{f} \rho_{f} E \phi}$).
- $\xi$: dimensionless radius ($\xi = \frac{r_{d}}{r_{do}}$).
- $\tau_{s}$: dimensionless optical thickness of particle ($\tau_{s} = K_{a} \xi$).
- $\Phi$: represents the heat transfer process of gas and the latent heat of vaporization of fuel droplets ($\Phi = \frac{3k_{g} \rho_{f} \phi}{\rho_{f} \sigma \rho_{f} \nu \sigma C_{f} \rho_{f} E \phi}$).
- $\phi$: volumetric phase content (where $\phi_g = 1 - \phi_{fd}$ and $\phi_{fd} = \frac{1}{3} \pi n_{d} r_{d}^{3}$ of average droplet sizes).
- $\chi$: relates the heat transfer of gas to the molar concentration of combustible components in the gas ($\chi = \frac{3k_{g} \rho_{f} \phi \rho_{f} \nu \sigma C_{f} \rho_{f} E \phi}{\rho_{f} \sigma \rho_{f} \nu \sigma C_{f} \rho_{f} E \phi}$).
- $\psi$: ratio of specific combustion energy and latent heat of vaporization ($\psi = \frac{\bar{Q}}{\overline{Q_{f}}}$).
- $\omega$: relates between radiation heat transfer in gas and the molar heat released by fuel during combustion ($\omega = \frac{4\pi \nu \sigma k_{g} \rho_{f} E \phi}{\rho_{f} \sigma \rho_{f} \nu \sigma C_{f} \rho_{f} E \phi}$).

Subscripts

- $b$ refer to boiling,
- $c$ refer to convection,
- $d$ refer to droplet,
- $f$ refer to fuel,
- $g$ refer to gas mixture,
- $o$ refer to initial condition,
- $ox$ refer to oxidizer,

Superscript

- * refer to critical