Evaluating Technical Efficiency of Stock Performance using Copula-Based Stochastic Frontier Analysis Approach

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Abstract. This study estimates the technical efficiency for 14 selected Malaysian trading and services companies for 2017 and 2018 using the Cobb-Douglas with the Stochastic Frontier production model. In standard stochastic frontier analysis (SSFA), the two-sided error, statistical noise, and one-sided error, inefficiency term are assumed to be independent. The assumption for statistical noise is normally distributed and the inefficiency term is half-normal distributed. In this paper, the copula model is applied to capture the joint distribution of these two error components. The copula model is an alternative method that is able to account for the joint multivariate distribution. Copula functions can be used to capture rank correlation and tail dependence between two error components, thus making the SFA more flexible. Seven copula models from the Archimedean copula family are considered in this study including the copulas with the trigonometric and hyperbolic generator. This study further compares the technical efficiency yielded by copulas with standard SFA, DEA-CCR and DEA-BCC models. The results raise the question of the reliability of SFA based on the standard model. Since the dependence error between statistical noise and inefficiency error cannot be ignored, the copula-based in SFA models can be considered as an alternative suitable tool for measuring efficiency performance.

1. Introduction
Researchers have been interested in exploring the gap between business and financial performance issues in financial studies for a long time. Recent research illustrated the company’s need to boost its efficiency and proposed using performance assessment as a method for continuous improvement. The general assessment of an organization’s performance and the director’s subsequent constructive criticism are essential to business transformation. The assessment helps companies to be measured on the basis of standardized data, thereby facilitating the recognition and broader implementation of best practices. The overall concept of efficiency refers to the gap between actual and optimal input, output, and input-output mixtures. Throughout literature, decision-maker units (DMUs) are defined as units of production such as companies, regions, and states that are supposed to generate output based common technology. A decision-maker unit (DMU) approaches the boundary when generating the maximum output for a given set of inputs that is if the analysis is output-oriented. When the orientation is reversed (input-oriented), the boundary is reached when the DMU retains an expected output by consuming the minimum available inputs. The quality evaluation has to do with many efficiency principles including...
profit, technical, productive and allocative. Market failure happens when it is difficult for companies to manage goods and services optimally. For this matter, the study for inefficiency is acceptable due to structural problems or market imperfections or even other factors, resulting in firms producing below their maximum attainable output.

Researcher’s persistence in calculating how effectively a company produces outputs with its available inputs resulted in the development of several efficiency measures such as scale efficiency, scope efficiency, economic efficiency, and technical efficiency. Technical efficiency needs only data on input and output while economic efficiency required cost information. Two specific paradigms for calculating efficiency had been reported by a substantial number of literatures, based on an essentially non-parametric, programming method for evaluating the effects observed; and another based on an econometric approach for estimating the theory-based models of production, cost or profit. For a company or DMU, technical efficiency represents the company’s ability to achieve maximum output from a given set of inputs. If the maximum output can be achieved from the minimum input number, a business is said to be technically successful. By comparing the technical efficiencies of firms, one can determine which company is most efficient in utilizing given inputs to produce maximum output. Hence, technical efficiency affects companies’ competitive positions.

Data Envelopment Analysis (DEA) and Stochastic Frontier Analysis (SFA) are the common methods used to measure the effectiveness of DMU. DEA is a frontier approach that is non-parametric and requires minimal assumptions. They do not impose restrictions on the inputs and outputs of functional types. In comparison, a suitable functional shape is assumed using SFA. DEA models conclude that all boundary deviations are inefficiencies while SFA models assign part of the deviations to inefficiency and part of the deviation to random noise. In other words, both inefficiency and random noise are taken into account by the SFA models. The specific distributional assumption is required to distinguish technical inefficiency from statistical. The types of distribution assumption for technical inefficiency used in past studies are exponential, half-normal, truncated-normal and gamma distribution and types for statistical noise is normal distribution.

The error term specification is given by \( \varepsilon_i = v_i - u_i \) for the two types of error, namely, noise, \( v_i \) and technical inefficiency error, \( u_i \). In estimating the efficiency based on SFA models, the error terms \( u_i \) and \( v_i \) are assumed independent. The non-negative error, technical inefficiency is the randomness resulting from factors under the control of the company. The error terms, noise represents the uncontrollable randomness. Nevertheless, the validity of the independent errors’ assumption has been questioned by many researchers, particularly in the context of inefficiency situation. Schmidt and Lovell [1] addressed the availability of dependency in the SFA model between technical efficiency and allocative efficiency. Although distinguishing between controllable sources of randomness and non-controllable sources, there is no statistical or economic reason to impose orthogonally between components of error, \( u_i \) and \( v_i \). In fact, only independent components would be rare for a productive system. In agriculture, for example, disruptive, seasonal variations often influence decisions about growth. Another example is the possible adverse effect of unregulated spillage of harmful by-product by heavy polluters on their own workforce output. If the assumption of dependence is ignored, the efficiency performance predictions will be inaccurate and biased.

The method used here generalizes the statistical model underpinning the SFA model by adding a framework of dependency that makes \( u_i \) and \( v_i \) is correlated with it. The parameterization of this structure allows data to be used to measure the extent of the association between \( u_i \) and \( v_i \). The stochastic frontier analysis of dependent error components is less restrictive than the standard SFA model, making it statistically more attractive. The analysis procedure used here was derived from the Sklar theorem where the joint distribution of random variables can be represented as a function of its univariate margins called the copula. The copula reflects the structure of dependence between random variables and defines their common behavior. Smith [2] was one of the early scholars who proposed the potential dependence between \( u_i \) and \( v_i \). He also proposed modeling this dependence using copulas and
then estimating the SFA models by using copulas. The use of copula-based SFA models can be found in studies by Carta and Steel [3], Prokhorov and Schmidt [4], Mehdi and Hafner [5]. Wiboonpongse et al.[6], Das [7] and Kınacı et al. [8].

There are numerous stock market studies using stochastic frontier models to estimate technical efficiency using the function of Cobb-Douglas. However, past stock market studies, along with copula-based SFA studies, did not use the DuPont model. In the early 1900s, the DuPont model or DuPont analysis was developed, which is still considered a reliable method to be used for profitability evaluation. DuPont’s research uses Return on Equity (ROE) to calculate the number of profits available to stockholders based on their total investment in equity. In other words, DuPont research can be used by a client to find factors that cause the company to have low ROE. Because of its ability to generate a high return on investment by stockholders, a company with the highest value of ROE can be regarded as a high-performing company. DuPont Model is a useful tool to evaluate the financial statement, which is supported by the results to predict future profitability. DuPont analysis highlights a company’s performance in three areas: profitability (net profit margin), total assets turnover and equity multiplier (leverage). Hasan et al. [9] analyzed the technical efficiency of selected companies in the Bangladesh stock market by selecting the return of individual companies from the Dhaka Stock Exchange as a performance-dependent variable. The return on the prepared individual company depending on the regular closing price of the individual company. Janang et al. [10] examines the technical efficiency of government-linked companies (GLCs) and benchmarks the result with top foreign-owned firms listed in Bursa Malaysia and the output selected is the firms’ turnover (net revenue). Research by Arsad and Isa [11] applied DuPont analysis to select as an output factor to assess company performance, but they use the assumption that technical inefficiency and noise error are independent.

The application of SFA models with dependent error in stock market studies are still limited because of the complex and challenging computations of copula models. A study by Tibprasorn et al. [12] found that the SFA model was being used to calculate stock efficiencies of the top 50 with the highest market capitalization in the stock exchange of Thailand. They decomposed the real variance from the expected return into stochastic noise and inefficiency term and used the copula method for the components of the joint error. This analysis used four copula forms, namely, the copula of Gaussian, Frank, Clayton, and Independence. Tibprasorn et al. [13] used the idea of a stochastic frontier in production to analyze the problem of pricing in the Stock Exchange of Thailand. By using copulas, to adjust the classical stochastic frontier model to accommodate error dependence, they have shown that their extended stochastic frontier model is more suitable for financial analysis. Differently from previous studies, this study estimates the technical efficiency of stock companies in terms of return to shareholders equity by using DuPont analysis. This article also offers an overview of stochastic frontier analysis (SFA) and their benefits in estimating SFA parameters through modeling error terms. Following a study by Najjari et al. [14], we focus on seven Archimedean families. Three of the Archimedean copulas have trigonometric and hyperbolic generators and they are more flexible in modeling dependence structure. The Cat-copula family has a trigonometric generator and proposed by Pirmoradian and Hamzah [15], Csch-copula and Coth-copula have hyperbolic generators and were proposed by Hasan and Najjari [16] and Najjari et al. [17]. After estimating the technical efficiency of 14 selected companies using SFA with copula models, we compare the technical efficiencies (TEs) with those generated from the Charnes, Cooper & Rhodes (CCR), Banker, Charnes & Cooper (BCC) and standard SFA (SSFA) models.

The remainder of the paper is organized as follows. Section 2 provides a brief description of the SFA model, copula and its characteristics and the copula SFA. Section 3 presents the procedure on the chosen dataset for estimating the models and Section 4 presents the findings and discussion. Finally, in the final part of the paper, the conclusion and the future work are observed.

2. Stochastic Frontier Analysis

Stochastic frontier analysis (SFA) is a method of economic modeling, which was firstly introduced by Aigner et al. [18] and Meeusen and van Den Broeck [19]. Standard parametric stochastic frontier models assume that there is a production function, \( f \) that converts \( X \in \mathbb{R}_+^p \), a vector of inputs of dimension \( p \)
into a scalar output $Y \in \mathbb{R}$. If one has $n$ observations of $(X_i, Y_i)$, the model can be written for the $i^{th}$ DMU as:

$$y_i = f(x_i, \beta) + \epsilon_i, \quad i = 1, \ldots, n$$

(1)

where $y_i = \log(Y_i)$, $x_j = \log(X_j)$, $j = 1, 2, \ldots, p$, $i = 1, 2, \ldots, n$ for $n$ is the number of DMUs, $\beta$ is a vector of parameters to be estimated, and $\epsilon_i$ is a composite error term. The function $y_i = f(x_i, \beta)$ represents the production frontier. The stochastic error term $\epsilon_i$ contains information about the noise and the inefficiency given by:

$$\epsilon_i = v_i - u_i$$

(2)

where $v_i$ is the normal error term, $v_i \sim N(0, \sigma^2_v)$ and $u_i$ is the technical inefficiency error term with non-negative value, $u_i \geq 0$. The types of distribution assumption for technical inefficiency $u_i$ used in past studies are exponential, half-normal, truncated-normal and gamma distribution and types for statistical noise is normal distribution. The types of distributional choices are guided by the theoretical consideration and computational convenience. Researchers tend to use half-normal and truncated-normal as inefficiency error distribution due to the ease of estimation and interpretation. Although there is no consensus on the type of distribution, one should choose to arrive at the inefficiency measure which most of the works that are available in the literature suggest different distributional assumptions that tend to yield similar efficiency. Based on a study conducted by Aigner et al. [18], distribution of the inefficiency term takes a half-normal and exponential distribution, and the result shows a little difference in inefficiency scores and the efficiency rank are roughly the same order when different assumptions were used for the inefficiency term. The random noise error component, $v_i$ affecting the production process is not directly attributable to the producers or the underlying technology. The noise may come from economic adversities [20]. The technical inefficiency $u_i$ measures the shortfall of output $y_i$ from its maximal possible value given by the stochastic frontier, $y_i = f(x_i, \beta) + v_i$. The one-sided error term $u_i \geq 0$ allows the distinction between DMUs. For example, DMUs that are on the frontier will have $u_i = 0$ and others that are below the frontier will have $u_i > 0$.

The stochastic model also permits to estimate $\beta$ and its standard errors. The parameters of the model described in equation (1) and equation (2) can be estimated using the maximum likelihood method and $\epsilon_i$ can be predicted by $\hat{\epsilon}_i = y_i - f\left(x_i, \hat{\beta}\right)$, which contains information on $u_i$. Since $TE_i$ denotes technical efficiency for the $i^{th}$ DMU, therefore it can be written as $TE_i = \exp(-u_i)$ where $u_i \geq 0$. In this study, the function for $f(x_i, \beta)$ is log-linear Cobb-Douglas form and can be written as:

$$\ln y_i = \beta_0 + \sum \beta_j \ln x_{ij} + v_i - u_i$$

(3)

where $v_i \sim N(0, \sigma^2_v)$ and $u_i \sim N^+(0, \sigma^2_u)$. The Cobb-Douglas major strengths are its ease of use and its seemingly good empirical fit across many data sets.

2.1. Copula

A copula connects a given number of one-dimensional marginal distribution to form a joint multivariate distribution. Copula can be defined as informally as follows:
Let $U$ and $V$ be continuous random variables with distribution function, $G_1(u) = P(U \leq u)$ and $G_2(v) = P(V \leq v)$. The joint density function is given below:

$$
  h(u, v) = P(U \leq u, V \leq v)
$$

(4)

For every $(u, v)$ in $[-\infty, \infty]^2$, consider the point in $I^1$ ($l = [0,1]$) with coordinates $(G_1(u), G_2(v), h(u, v))$. The mapping from $I^2$ to $I$ is a copula. Copula is also known as a dependence function or uniform representation. A function $C : [0,1]^2 \rightarrow [0,1]$ is a copula if it satisfies the following properties:

C1: For every $u$ and $v$ in $l$, $C(u, 0) = 0 = C(0, v)$ for all $u$ and $v$ and $C(u, 1) = u$ and $C(1, v) = v$ for all $u$ and $v$;

C2: For every $u_1, u_2, v_1, v_2$ in $l$, such that $u_1 \leq u_2$ and $v_1 \leq v_2$, $C(u_2, v_2) - C(u_2, v_1) - C(u_1, v_2) + C(u_1, v_1) \geq 0$ (5)

For this study, the bivariate copulas are applied, Sklar’s theorem (1959) explains the role that copulas in showing the relationship between a multivariate distribution function and their univariate marginals. Let $h$ be a two-dimensional distribution function with marginal distribution functions, $G_1$ and $G_2$. Then, there exists a copula $C$ such that:

$$
  h(u, v) = C(G_1(u), G_2(v))
$$

(6)

where $G_1(\cdot)$ and $G_2(\cdot)$ are the marginal cumulative distribution functions (CDFs) of $u$ and $v$, respectively, $C$ is a bivariate function called copula. If $u$ and $v$ are independent, then $C$ is the product copula. Conversely, for any distribution functions, $G_1$ and $G_2$, for any copula $C$, the function $h$ defined above is a two-dimensional distribution function with marginal, $G_1$ and $G_2$. Furthermore, if $G_1$ and $G_2$ are continuous, then $C$ is unique. The function $\Pi(u, v) = u \cdot v$ is called the product copula and has an important statistical interpretation. Let $u = G_1(u)$ and $v = G_2(v)$; the product copula is important as a benchmark because it corresponds to independence. It is easy to see that function $\Pi(u, v) = u \cdot v$ satisfies conditions (C1) and (C2) and hence is a copula. Other details of families of copula are provided in the appendix section.

The most common measure of dependence between random variables is Pearson’s correlation coefficient. However, it only measures linear dependence and is not very informative for asymmetric distributions. To measure non-linear dependence, rank correlation coefficients such as Kendall’s tau and Spearman’s rho are more suitable measures. The copula families used in this study are summarized in the appendix section. Copula families can be selected based on their ability to capture positive and negative dependence, as well as tail dependence. For instance, Clayton copula can measure lower tail dependence, while Gumbel can measure upper tail dependence. To summarize, different copulas have different characteristics, such as upper tail dependence, lower dependence, positive and negative.

2.2. Copula Stochastic Frontier Analysis

Referring to equation (3), the SFA model assumes that $v$ and $u$ are dependent errors. In this study, we assume the normal distribution for $v$ case, $v \sim N(0, \sigma_v^2)$ and $u$ has a half-normal distribution with $u \sim N^+(0, \sigma_u^2)$. With mean, $E(u) = \sigma_u \sqrt{2/\pi}$, variance, $Var(u) = \sigma_u^2(\pi - 2/\pi)$ and composite error $\varepsilon = v - u$, the variance for $\varepsilon$ is given as below:
Var(ε) = Var(u) + Var(v) - 2Cov(u, v)  \quad (7)

The equation of variance for composite error, ε in equation (7) shows that a positive correlation between u and v will reduce the variance of ε and a negative correlation between u and v will increase the variance of ε. For all u ≥ 0; u and v are assumed dependent, the joint density of u and v can expressed as follows:

\[ h(u, v) = g_1(u)g_2(v) \theta(G_1(u), G_2(v)) \quad (8) \]

The joint density of u and ε is obtained by replacing v in h(u, v) by v = ε + u. To obtain the density of ε, the joint density of u and ε is then integrated by the variable u as follows:

\[ h(\varepsilon) = \int_{-\infty}^{\infty} h(u, \varepsilon) du = \int_{-\infty}^{\infty} g_1(u)g_2(\varepsilon + u) \theta(G_1(u), G_2(\varepsilon + u)) du \quad (9) \]

Replacing \( \varepsilon = y - f(x, \beta) \) in the expression \( h(\varepsilon) \) gives the density of \( y \). Based on the maximum likelihood estimator (MLE), one can address the issue of estimating technical efficiencies. Clearly, copulas allow to model marginal distributions separately from their dependence structure, so we have a flexible joint distribution function whose marginal are specified by the researchers. Through observed input \( x \) and output \( y \), the technical efficiency, \( TE \) for each DMU can be estimated by the conditional expectations given \( y \) is \( TE = E(\exp(-u) | \varepsilon) \). Based on equation (8) and equation (9), the \( TE \) can expressed as follows:

\[ TE = \frac{1}{h(\varepsilon)} \int_{-\infty}^{\infty} \exp(-u) h(u, \varepsilon) du \quad (10) \]

3. Methodology

This study estimates the technical efficiencies for 14 trading and services companies listed at Bursa Malaysia. The sample dataset was collected from theDataStream database which includes asset turnover (AT), market capitalization (MC), debt to equity ratio (D/E) and returns on equity (ROE) for the years 2017 and 2018. The selected companies are Tenaga Nasional Bhd (DMU1), IHH Healthcare (DMU2), Maxis Bhd (DMU3), MISC Bhd (DMU4), Genting Berhad (DMU5), Hap Seng Consolidate (DMU6), Dialog Group Berhad (DMU7), Malaysia Airlines (DMU8), Sime Darby Bhd (DMU9), AirAsia Group (DMU10), Sunway Bhd (DMU11), Analabs (DMU12), AWC Berhad (DMU13) and Atlan Holdings Bhd (DMU14). There are three inputs and one output for computing the efficiency score, which is listed as follows:

- **Input 1:** The value of the sales and revenue of a company in relation to the value of its assets was measured by AT. It is used as a measure of the efficiency in which the assets used to generate revenue;
- **Input 2:** MC shows the size of a company which is known as the fundamental determinant of different characteristics, including risk which investors are interested in;
- **Input 3:** D/E ratio is used in this study to evaluate a company’s leverage;
- **Output:** ROE indicates how much profit a company gained compared to the total shareholders’ equity listed on balance sheets.

In this research, output selection is based primarily on the relationship between efficiency and profitability known as the DuPont analysis or the theory of DuPont. DuPont theory highlights a company’s performance in three areas: profitability (net profit margin), total assets turnover and equity.
multiplier (leverage). The net profit margin shows how effective a company is to control costs. The higher the company’s net profit margin, the more effectively the company in turn sales into actual profit. A company with a high ROE is more likely to be able to generate cash internally. DuPont also provides management with a roadmap in assessing their effectiveness in managing the company’s resources to maximize the return earned on owners’ investments. Based on DuPont, the value of ROE is based on the following model:

$$\text{ROE} = \frac{\text{Net Profit Margin} \times \text{Total Assets Turnover}}{1 - \text{Debt Ratio}}$$ (11)

Assets turnover has been identified in this analysis as one of the inputs. Assets turnover was measured is determined by the income and profit value of a business compared to its assets. It is used as a measure of how well the resources were used to generate revenue. The reason of selecting asset turnover as one of the input variables is that the changes in the asset turnover ratio provide information on future profitability [21]. Market capitalization is very essential to estimate stock return and risk. Dias [22] studied the roles of market capitalization by estimating the value at risk (VaR) in order to diversify the portfolio with different market capitalizations. The study by Reinganum [23] investigated the relationship between stock return and market capitalization and found that market capitalization was an excellent indicator for a long-run rate of return. Besides that, the average portfolio return was found systematically related to market capitalization. In a Bhandari [24] study, the expected returns on common stocks are found positively linked to the beta (risk) and firm size controlling debt per equity ratio. In another report, Mokhtar et al. [25] identified the debt-equity ratio as one of the most significant financial ratios used by the Fuzzy Delphi Method (FDM) to measure stock performance. This study considers the Cobb-Douglas production model, which is expressed as:

$$\ln(\text{ROE}) = \beta_0 + \beta_1 \ln(\text{AT}) + \beta_2 \ln(\text{MC}) + \beta_3 \ln(\text{DE}) + \nu_i - u_i$$ (12)

The estimation of TE s based on standard SFA and copula-based on SFA models are conducted on MATLAB software. The selected copulas for this study and their details are provided in Table 1 [28].

| Copula      | Generator, $\ell_{\theta}(t)$ | Kendall’s Tau, $\tau$ | $\lambda_L$ | $\lambda_U$ | Interval $\theta$ |
|-------------|--------------------------------|------------------------|--------------|--------------|-------------------|
| Clayton     | $\frac{1}{\theta} \left( \frac{1}{t^\theta} - 1 \right)$ | $\frac{\theta}{\theta + 2}$ | $2^{\frac{1}{\theta}}$ | 0 | $(0, \infty)$ |
| A12         | $\left( \frac{1}{t} - 1 \right)^\theta$ | $1 - \frac{2}{3\theta}$ | $2^{\frac{1}{\theta}}$ | $2 \cdot 2^{\frac{1}{\theta}}$ | $[1, \infty)$ |
| Gumbel      | $(- \ln t)^\theta$ | $\frac{\theta - 1}{\theta}$ | 0 | $2 \cdot 2^{\frac{1}{\theta}}$ | $[1, \infty)$ |
| Csch-copula | $\csc h(t^\theta) - \csc h(1)$ | $\frac{\theta}{\theta + 2}$ | $2^{\frac{1}{\theta}}$ | 0 | $(0, \infty)$ |
| Coth-copula | $\coth(\theta t) - \coth(\theta)$ | $1 + \frac{2}{\theta^2} - \frac{2}{\theta} \coth(\theta)$ | $\frac{1}{2}$ | 0 | $[1, \infty)$ |
| Cot-copula  | $\cot \left( \frac{\pi}{2} \right)$ | $1 - \frac{8}{\pi^2 \theta}$ | $2^{\frac{1}{\theta}}$ | $2 \cdot 2^{\frac{1}{\theta}}$ | $[1, \infty)$ |
The fundamental concept of frontier estimation methods is the comparison between the DMUs to understand how inputs generate outputs. The comparison is based on the technical performance ranking of each unit. The efficiency score (ES) is the capability of the company to achieve maximum output from the set of inputs obtained. The maximum likelihood estimator (MLE) is used to estimate the parameters of standard SFA and copula-based SFA models. The parameter estimates and TEs are displayed in the next section. A TE value can either be 1 or less than 1.

- The producers or DMUs at the frontier with TE of 1 can be called fully efficient.
- When they fail to reach the frontier, they are called production inefficiency and the TE score will be less than one.

The estimation of the technical efficiency of the company’s stocks based on Data Envelopment Analysis (DEA) (CCR and BCC models) was computed in R software using the ‘benchmarking’ package. The original DEA model was presented by Charnes et al. [26] and Banker et al. [27].

DEA is a non-parametric frontier technique used to measure the relative efficiency of a set of DMUs with multiple inputs and outputs without specifying a priori of a production function. Consider a set of \( n \) DMUs. For DMU \( k \), let \( y_{rk} (r=1,\ldots,s) \) denotes the level of the \( r^\text{th} \) output, and \( x_{ik} (i=1,2,\ldots,m) \) denotes the level of the \( i^\text{th} \) input. To measure the efficiency of DMU \( k \), the weights \( u_r \) and \( v_i \) will be found to maximize output. The value \( E_k \) is between zero and one, with higher values indicating greater efficiency. The optimal \( \theta \), denoted by \( \theta^* \), should satisfy \( 0 < \theta^* < 1 \). If \( \theta^* \) equals to one, then the DMU understudy is said to be technically efficient and lies on the efficiency frontier that composed of the set of efficient units. To measure the efficiency of DMU \( k \), the weights \( u_r \) and \( v_i \) will be found to maximize the following ratio \( E_k \) subject to a set of constraints:

\[
\text{Max } E_k = \sum_{r=1}^{s} u_r y_{rk} \\
\text{Subject to } \sum_{r=1}^{s} u_r y_{rk} - \sum_{i=1}^{m} v_i x_{ik} \leq 0, \quad j = 1,2,\ldots,n \\
\sum_{i=1}^{m} v_i x_{ik} = 1 \\
u_r ; v_i \geq 0, \quad r = 1,2,\ldots,s \quad i = 1,2,\ldots,m
\] (13)

Linear programming for DEA-CCR model is:

\[
\text{Min } \theta \\
\text{Subject to } \theta_{ik} - \sum_{j=1}^{n} \lambda_{ij} x_{ij} \geq 0, \quad i = 1,2,\ldots,m \\
\sum_{j=1}^{n} \lambda_{ij} y_{ij} \geq y_{rk}, \quad r = 1,2,\ldots,s \\
\lambda_{ij} \geq 0, \quad j = 1,2,\ldots,n, \quad \theta \text{ unrestricted in sign}
\] (14)

The DEA-BCC model equation as follows:

\[
\text{Min } \theta \\
\text{Subject to } \theta_{ik} - \sum_{j=1}^{n} \lambda_{ij} x_{ij} \geq 0, \quad i = 1,2,\ldots,m
\]
\[ \sum_{j=1}^{n} \lambda_j = 1 \]
\[ \sum_{j=1}^{n} \lambda_j y_j \geq y_u, \quad r = 1, 2, \ldots, s \]
\[ \lambda_j \geq 0, \quad j = 1, 2, \ldots, n, \quad \theta \text{ unrestricted in sign} \quad (15) \]

4. Results and Discussion

This research was carried out on 14 trading and services companies in Malaysia for the years, 2017 and 2018. The MLE method was used to estimate the parameters. The estimated parameters for SSFA, copula-based SFA models and Kendall’s \( \tau \) are given in Table 2. Kendall’s \( \tau \) is the probability of concordance minus the probability of discordance and is thus standardized to the interval \([-1,1]\].

Kendall’s \( \tau \) has a direct relationship with the copula parameter, \( \theta \) as shown in Table 1. Therefore, we can interpret the degree of association between technical inefficiency error, \( u \) and random noise error, \( v \) based on Kendall’s \( \tau \) parameter value.

For the Clayton copula, the relationship between the copula parameter and Kendall’s \( \tau \) is given by \( \tau = \theta / \theta + 2 \). Hence, the estimated value Kendall’s \( \tau \) for the year 2017 is \( \tau = 0.7542 \), which corresponds to the parameter \( \theta = 6.1357 \). These results indicate that there is a strong relationship between \( u \) and \( v \). For 2018, the estimated Kendall’s tau is \( \tau = 0.3383 \) thus signifying the weak relationship between \( u \) and \( v \). Similarly to Clayton copula, Gumbel copula has a parameter \( \theta = 6.0129 \) with Kendall’s \( \tau \) value of 0.8337, which indicates a strong relationship between \( u \) and \( v \) for the year 2017. The strongest association is shown by the A12 family with \( \tau = 0.8913 \). Referring to the last column of Table 2, it can be concluded that the relationship between \( u \) and \( v \) in 2017 is strong while in 2018, the relationship appears weak as evident by the low Kendall’s \( \tau \) value. However, A12 copula proves that the relationship between \( u \) and \( v \) are strongest for both years, 2017 and 2018 with 0.8913 and 0.8448 respectively.

| Copula        | Year | \( \sigma_u \) | \( \sigma_v \) | \( \theta \) | \( \beta_0 \) | \( \beta_1 \) | \( \beta_2 \) | \( \beta_3 \) | \( \tau \) |
|--------------|------|----------------|----------------|----------|-------------|-------------|-------------|-------------|-----|
| Clayton      | 2017 | 0.2724         | 0.3158         | 6.1357   | -5.2358     | 1.4153      | 0.0822      | 0.3900      | 0.7542 |
|              | 2018 | 0.2947         | 0.1620         | 1.0224   | -3.0509     | 1.4571      | -0.001      | 0.1513      | 0.3383 |
| A12          | 2017 | 0.2704         | 0.3149         | 6.1324   | -5.2428     | 1.4170      | 0.0826      | 0.3886      | 0.8913 |
|              | 2018 | 0.1686         | 0.1328         | 4.2944   | -3.5253     | 1.4959      | -0.001      | 0.2331      | 0.8448 |
| Gumbel       | 2017 | 0.2812         | 0.3188         | 6.0129   | -5.2336     | 1.4151      | 0.0821      | 0.3901      | 0.8337 |
|              | 2018 | 0.3264         | 0.1884         | 1.1388   | -3.8684     | 1.5956      | -0.001      | 0.2498      | 0.1219 |
| Csch         | 2017 | 0.2724         | 0.3158         | 6.1357   | -5.2358     | 1.4153      | 0.0822      | 0.3900      | 0.7542 |
|              | 2018 | 0.3309         | 0.1841         | 1.1554   | -3.9249     | 1.6019      | -0.001      | 0.2468      | 0.3662 |
| Coth         | 2017 | 0.2839         | 0.3198         | 5.9973   | -5.2311     | 1.4149      | 0.0820      | 0.3902      | 0.7221 |
|              | 2018 | 0.2862         | 0.2468         | 1.0031   | -4.1458     | 1.6055      | -0.001      | 0.2967      | 0.3742 |
| Cot          | 2017 | 0.2804         | 0.3186         | 6.0191   | -5.2328     | 1.4145      | 0.0823      | 0.3897      | 0.8653 |
|              | 2018 | 0.3264         | 0.1884         | 1.1388   | -3.9184     | 1.5956      | -0.001      | 0.2498      | 0.2882 |
| Product      | 2017 | 0.2823         | 0.3191         | -       | -5.2332     | 1.4152      | 0.0821      | 0.3900      | -    |
|              | 2018 | 0.2431         | 0.2208         | -       | -3.0411     | 1.4076      | -0.0009     | 0.1899      | -    |
| SSFA         | 2017 | 0.3152         | 0.2544         | -       | -5.2479     | 1.4161      | 0.0830      | 0.3891      | -    |
|              | 2018 | 0.9704         | 0.1116         | -       | -4.2381     | 1.6235      | -0.0007     | 0.3035      | -    |
As mentioned in equation (7), a positive association between $u$ and $v$ decreases the composite error variance, while a negative association between $u$ and $v$ increases the variance of $e$. Based on Table 2, the proportion of standard deviations lambda $\lambda$ can be estimated by $\lambda = \sigma_u / \sigma_v$. Its measures the amount of composite error distribution. For example, for 2017 and 2018, the Clayton copula will have $\lambda$ of 0.8626 and 1.8191 respectively. The coefficient of the parameter gamma, $\gamma$ which can be estimated by $\gamma = \sigma_u^2 / \sigma_v^2 = \lambda^2 / \lambda^2 + 1$, provides information regarding the presence of efficiency in the production. The parameter $\gamma$ lies between 0 and 1 where $\gamma = 0$ specifies that all frontier deviations are due to random error (noise), while $\gamma = 1$ means all deviations are triggered from the technical inefficiency. For the SSFA model, the values of gamma for 2018 are 0.9869. Since this value is close to one, the deviation comes mainly from technical inefficiency.

Table 3. Efficiency score and ranking using Clayton, A12, Gumbel, and Csch copulas

| Year | 2017    | 2018    | 2017    | 2018    | 2017    | 2018    |
|------|---------|---------|---------|---------|---------|---------|
| Year | ES      | Rank    | ES      | Rank    | ES      | Rank    |
| 1    | 0.7974  | 4       | 0.3993  | 9       | 0.7975  | 4       | 0.3861  | 9       |
| 2    | 0.3510  | 13      | 0.2016  | 12      | 0.3508  | 13      | 0.2041  | 13      |
| 3    | 0.9999  | 3       | 1       | 1       | 0.9422  | 3       |
| 4    | 1       | 1       | 0.8011  | 5       | 1       | 2       | 8.558   | 4       |
| 5    | 0.4383  | 10      | 0.4850  | 8       | 0.4388  | 10      | 0.4792  | 8       |
| 6    | 0.7875  | 5       | 0.8242  | 4       | 0.7875  | 5       | 0.8021  | 5       |
| 7    | 0.4170  | 11      | 0.6213  | 7       | 0.4166  | 11      | 0.6258  | 7       |
| 8    | 0.2334  | 14      | 0.9882  | 2       | 0.2337  | 14      | 0.9865  | 2       |
| 9    | 0.4620  | 9       | 0.1337  | 14      | 0.4607  | 9       | 0.1384  | 14      |
| 10   | 0.6867  | 6       | 0.9128  | 3       | 0.6872  | 6       | 1       |
| 11   | 0.6241  | 8       | 0.7257  | 6       | 0.6252  | 8       | 0.6951  | 6       |
| 12   | 0.6595  | 7       | 0.2469  | 10      | 0.6584  | 7       | 0.2857  | 10      |
| 13   | 0.4035  | 12      | 0.1983  | 13      | 0.4025  | 12      | 0.2169  | 13      |
| 14   | 1       | 2       | 0.2122  | 11      | 0.9956  | 3       | 0.2392  | 11      |

Table 3 and Tables 4 provide the efficiency score (ES) and ranking performance for trading and services companies or DMUs. Specifically, Table 3 provides the ES and ranks of each DMU for Clayton
Table 4 reveals the ES and ranking based on hyperbolic and trigonometric copula which is Coth-copula and Cot-copula. Malaysia Airports (DMU₈) has a top performance with maximum scores for Coth-copula and Cot-copula while Malaysia Airports (DMU₈) has the bottom rank in 2017. For 2018, the best performance is the AirAsia Group (DMU₁₀) for Coth-copula and Malaysia Airports (DMU₈) for Cot-copula. The lowest performance in 2018 for both Coth and Cot is Sime Darby Bhd (DMU₃). Another

| Year | 2017 | 2018 | 2017 | 2018 |
|------|------|------|------|------|
| Coth-copula | | | | |
| DMU | ES | Rank | ES | Rank | ES | Rank | ES | Rank |
| 1 | 0.7976 | 4 | 0.3677 | 9 | 0.7972 | 4 | 0.3747 | 9 |
| 2 | 0.3511 | 13 | 0.2053 | 13 | 0.3508 | 13 | 0.2044 | 2 |
| 3 | 1 | 1 | 0.8562 | 4 | 1 | 1 | 0.8822 | 4 |
| 4 | 1 | 2 | 0.9279 | 3 | 0.9991 | 3 | 0.9006 | 3 |
| 5 | 0.4382 | 10 | 0.4792 | 8 | 0.4380 | 10 | 0.4848 | 8 |
| 6 | 0.7875 | 5 | 0.7559 | 5 | 0.7875 | 5 | 0.7658 | 5 |
| 7 | 0.4170 | 11 | 0.5959 | 7 | 0.4170 | 11 | 0.5905 | 7 |
| 8 | 0.2333 | 14 | 0.9942 | 2 | 0.2332 | 14 | 1 | 1 |
| 9 | 0.4624 | 9 | 0.1240 | 14 | 0.4618 | 9 | 0.1194 | 14 |
| 10 | 0.6865 | 6 | 1 | 1 | 0.6869 | 6 | 0.9421 | 2 |
| 11 | 0.6238 | 8 | 0.6694 | 6 | 0.6240 | 8 | 0.6881 | 6 |
| 12 | 0.6590 | 7 | 0.3053 | 10 | 0.6594 | 7 | 0.2799 | 10 |
| 13 | 0.4035 | 12 | 0.2104 | 12 | 0.4036 | 12 | 0.1976 | 13 |
| 14 | 1 | 3 | 0.2327 | 11 | 1 | 2 | 0.2141 | 11 |

| Cot-copula | | | | |
| DMU | ES | Rank | ES | Rank | ES | Rank | ES | Rank |
| 1 | 0.7976 | 4 | 0.3677 | 9 | 0.7972 | 4 | 0.3666 | 9 |
| 2 | 0.3511 | 13 | 0.2006 | 13 | 0.3509 | 13 | 0.2062 | 13 |
| 3 | 1 | 1 | 1 | 1 | 1 | 1 | 0.8478 | 4 |
| 4 | 1 | 2 | 0.7944 | 5 | 1 | 2 | 0.9421 | 3 |
| 5 | 0.4383 | 10 | 0.4718 | 8 | 0.4386 | 10 | 0.4815 | 8 |
| 6 | 0.7875 | 5 | 0.8290 | 4 | 0.7878 | 5 | 0.752 | 5 |
| 7 | 0.4169 | 11 | 0.6438 | 7 | 0.4171 | 11 | 0.5928 | 7 |
| 8 | 0.2333 | 14 | 0.9658 | 3 | 0.2337 | 14 | 1 | 1 |
| 9 | 0.4621 | 9 | 0.1506 | 14 | 0.4612 | 9 | 0.1217 | 14 |
| 10 | 0.6866 | 6 | 0.9977 | 2 | 0.6875 | 6 | 0.9988 | 2 |
| 11 | 0.6240 | 8 | 0.7035 | 6 | 0.6253 | 8 | 0.6694 | 6 |
| 12 | 0.6590 | 7 | 0.2710 | 10 | 0.6608 | 7 | 0.3078 | 10 |
| 13 | 0.4034 | 12 | 0.2219 | 12 | 0.4036 | 12 | 0.2090 | 12 |
| 14 | 0.9994 | 3 | 0.245 | 11 | 1 | 3 | 0.2309 | 11 |
two models, product copula and standard SFA (SSFA) also show that Maxis Bhd (DMU₁) has top performance and Malaysia Airports (DMU₈) are at the bottom rank in 2017. However, for the year 2018, Malaysia Airports (DMU₈) tops the ranking list for SSFA and Maxis Bhd (DMU₁) for product copula. Meanwhile, Sime Darby Bhd (DMU₄) places last in the ranking for product copula and SSFA in 2018.

Overall, for the year 2017, the rankings of companies based on copulas and SSFA are similar to one another. However, there are some differences in the rankings for the year 2018. We further proceed with the investigation of the differences by determining the correlation for each copula family and SSFA. Table 5 provides ES produced by non-parametric frontier technique, DEA with CCR and BCC models. The CCR model evaluates both technical and scales efficiency and therefore the general technical efficiency can be calculated by combining both measures in a single efficiency score. However, the pure TE of DMUs is evaluated for the BCC model which is called local TE. For the years 2017 and 2018, the top-performing company is Maxis Bhd (DMU₁) for the CCR model. The bottom rank company in 2017 is Malaysia Airports (DMU₈) with ES of 0.1647. For 2018, the bottom rank company is IHH Healthcare (DMU₂) with ES of 0.1431.

There are three efficient companies with ES equals to one for the CCR model for both years. For the BCC model, nine companies are efficient in 2017 and seven companies are efficient in 2018. The number of efficient companies using the BCC model is greater than using the CCR model. This is because of the different assumptions of technology. These results are lined with results by Chen [29]. DEA-CCR model assumes that all the DMUs are constant return to scale (CRS). Under the BCC model, all DMUs are considered to be a variable return to scale (VRS). There is no functional relationships between production output and inputs or any particular statistical distribution of error terms is assumed in the DEA. Therefore, DEA does not provide statistical information on the effects of goodness and reliability. The ES values for DEA and SFA (with and without copulas) models produce several perfect ES values (ES = 1) for several companies. Therefore, it is difficult to distinguish which company is the best company.

**Table 5: Efficiency score and rank performance using DEA models**

| Model | CCR | BCC |
|-------|-----|-----|
|       | 2017 | 2018 | 2017 | 2018 |
| DMU  | Year | Rank | ES | Rank | ES | Rank | ES | Rank |
| 1    | 6.598 | 9 | 0.3345 | 12 | 1 | 1 | 0.5923 | 13 |
| 2    | 0.2968 | 12 | 0.1431 | 14 | 1 | 8 | 0.6538 | 12 |
| 3    | 1 | 1 | 1 | 1 | 0.8788 | 11 | 1 | 1 |
| 4    | 0.5801 | 11 | 0.4403 | 10 | 1 | 7 | 1 | 2 |
| 5    | 0.2765 | 13 | 0.3345 | 13 | 0.8428 | 13 | 0.8979 | 9 |
| 6    | 0.9212 | 4 | 0.8114 | 4 | 1 | 2 | 0.9036 | 8 |
| 7    | 0.7384 | 5 | 0.6817 | 9 | 0.9524 | 10 | 0.8128 | 10 |
| 8    | 0.1647 | 14 | 0.7328 | 7 | 1 | 9 | 1 | 3 |
| 9    | 0.6829 | 8 | 0.3900 | 11 | 1 | 4 | 0.5164 | 14 |
| 10   | 1 | 2 | 1 | 2 | 0.7611 | 14 | 1 | 4 |
| 11   | 0.6564 | 10 | 0.7070 | 8 | 1 | 6 | 1 | 5 |
| 12   | 0.7171 | 7 | 0.7643 | 5 | 0.8669 | 12 | 1 | 6 |
| 13   | 0.7348 | 5 | 0.7516 | 6 | 1 | 3 | 0.8088 | 11 |
| 14   | 1 | 3 | 1 | 3 | 1 | 1 | 1 | 7 |

Table 6 and Table 7 displays the correlation between the copula-based SFA, standard SFA, and DEA models for the years 2017 and 2018. Clayton, A12, Gumbel, Csch, Coth, Cot, Product copulas and SSFA have very high correlation values ranging from 0.9 to 1 for the year 2017. These values indicate a very strong relationship between the models. For the year 2018, the correlations between copula families and SSFA are high where the values range between 0.7 to 1. Besides that, the correlation between the BCC model and copulas (Clayton copula, A12 copula, Gumbel copula, Csch-copula, Coth-copula) as well as SSFA are above 0.8 thus showing a strong relationship between the models. In 2017, the relationship
between all models with BCC is very low. In addition, the correlation between the CCR model and copulas (Clayton copula, A12 copula, Gumbel copula, Csch-copula, Cot-copula) as well as SSFA are mostly with smaller correlation values. The minimum correlation value is 0.4840 which depicts the moderate level relationship between the CCR model and Clayton copula in the year 2017. For the year 2018, the lowest correlation value is 0.0726 which measures the relationship between the CCR model and Cot-copula and product copula.

Overall, most of Kendall’s $\tau$ estimates for the models reveal the presence of a strong relationship between $u$ and $v$. This study focuses on two years period, 2017 and 2018. However, in the year 2018, based on Kendall’s $\tau$, only A12 copula shows a strong relationship between $u$ and $v$, while the other copulas that shows a weak relationship between $u$ and $v$. The alternative approach for determining the best model to estimate efficiency scores is by checking the information criterion (AIC or BIC) [30],[12]. Therefore, the AIC or BIC approach is the best method for determining the best model to estimate TEs for DMUs so that the predictions of performance are more accurate and not biased. The negligence of the error dependence assumption in an SFA model will result in inaccurate parameters’ estimations and efficiency scores. Future study can further support the findings of the present study by using information criterion (Akaike Information Criterion) as the basis to determine the best SFA models for the dataset. If the relationship between $u$ and $v$ are strong the copula approach should be considered to estimate reliable TEs.

**Table 6.** Correlation between selected models for the year 2017

| *F | C       | A12 | Gumbel | Csch | Cot | Cot | P     | S     | CCR | BCC         |
|----|---------|-----|--------|------|-----|-----|-------|-------|-----|-------------|
| ^aC | 1       | 0.9868 | 0.9868 | 0.9868 | 0.9824 | 0.9868 | 0.9868 | 0.4840 | 0.2442 |
| A12 | 0.9868 | 1    | 1    | 1    | 0.9956 | 1    | 1    | 0.5369 | 0.1849 |
| ^cG | 0.9868 | 1    | 1    | 1    | 0.9956 | 1    | 1    | 0.5369 | 0.1849 |
| Csch | 0.9868 | 1    | 1    | 1    | 0.9956 | 1    | 1    | 0.5369 | 0.1849 |
| Cot | 0.9868 | 1    | 1    | 1    | 0.9956 | 1    | 1    | 0.5369 | 0.1849 |
| ^aP | 0.9868 | 1    | 1    | 1    | 0.9956 | 1    | 1    | 0.5369 | 0.1848 |
| ^aS | 0.9868 | 1    | 1    | 1    | 0.9956 | 1    | 1    | 0.5369 | 0.1848 |
| CCR | 0.4840 | 0.5369 | 0.5369 | 0.5369 | 0.5369 | 0.5369 | 0.5369 | 1     | 0.0463 |
| BCC | 0.2442 | 0.1848 | 0.1848 | 0.1848 | 0.1848 | 0.2090 | 0.1848 | 0.0463 | 1     |

*Family; ^aClayton; ^cGumbel; ^aProduct; ^aSSFA

**Table 7.** Correlation between selected models for the year 2018

| *F | C       | A12 | Gumbel | Csch | Cot | Cot | P     | S     | CCR | BCC         |
|----|---------|-----|--------|------|-----|-----|-------|-------|-----|-------------|
| ^aC | 1       | 0.9769 | 0.9648 | 0.9648 | 0.9560 | 0.7393 | 0.7393 | 0.9604 | 0.4637 | 0.8022 |
| A12 | 0.9769 | 1    | 0.9901 | 0.9901 | 0.9945 | 0.7478 | 0.7478 | 0.9901 | 0.4466 | 0.8053 |
| ^cG | 0.9648 | 0.9901 | 1    | 1    | 0.9912 | 0.7811 | 0.7811 | 0.9956 | 0.3670 | 0.8022 |
| Csch | 0.9648 | 0.9901 | 1    | 1    | 0.9912 | 0.7811 | 0.7811 | 0.9956 | 0.3670 | 0.8022 |
| Cot | 0.9560 | 0.9945 | 0.9912 | 0.9912 | 1    | 0.7283 | 0.7283 | 0.9956 | 0.4242 | 0.8022 |
| ^dP | 0.7393 | 0.7478 | 0.7811 | 0.7811 | 0.7283 | 1    | 0.7349 | 0.0726 | 0.5787 |
| ^eS | 0.9604 | 0.9901 | 0.9956 | 0.9956 | 0.9956 | 0.7349 | 0.7349 | 1     | 0.4022 | 0.8066 |
| CCR | 0.4637 | 0.4466 | 0.3670 | 0.3670 | 0.4242 | 0.0726 | 0.0726 | 0.4022 | 1     | 0.6000 |
| BCC | 0.8022 | 0.8053 | 0.8022 | 0.8022 | 0.8022 | 0.5787 | 0.5787 | 0.8066 | 1     | 1     |

*Family; ^aClayton; ^cGumbel; ^dProduct; ^eSSFA

5. Conclusion
The stochastic frontier analysis (SFA) model is a linear regression model that has two component errors where the errors are often assumed to be independent. This study uses the Cobb-Douglas function in the
standard SFA to calculate the TEs of 14 trading and services companies. Due to the issue of error dependence assumption in the standard SFA model, this study evaluates TEs using the copula-based on SFA models where the copula component captures the joint the distribution between technical inefficiency and noise.

In addition, TEs were also produced from DEA models, a non-parametric frontier method, namely, CCR and BCC. For comparison purposes, TEs produced by all models used in this study were displayed together with the ranks of the companies (DMUs). This study found slight differences in TEs produced by copula-based SFA models. However, in terms of the ranking, the positions of DMUs showed were almost the same for each model. The accuracy of TEs results is very important in making decisions based on stock performance predictions for investment decision making. If a company has a very low TE value, the company is deemed inefficient at maximizing output to shareholders. Thus, this affects an investor’s decision to invest in the company’s shares.

In conclusion, the copula-based SFA approach is a better model compared to the standard model because it captures the dependence between technical inefficiency and noise. These two types of errors depend on each other in the real situation. Every decision made in operating of a company is influenced by other elements such as economic problems that are out of control. If the dependence of the assumption of these two errors is ignored, the decision making, and performance predictions will be imprecise and biased. This study did not consider the selection of the best copula model. Further study in this area should investigate the proper selection of copula functions using the AIC method.

**Appendix**

In this appendix, there is a description generally given in [28] that can be used to model the dependence between the random variable:

**Clayton copula:** Copula of the form 
\[ c(u, v) = \ell^{-1}(\ell(u) + \ell(v)) \]

is said to be Archimedean and function \( \ell \) is said to be its generator. The Clayton family can reflect lower tail dependence \( \theta > 0 \). It is characterized by the following formula:
\[ c_{\theta}(u, v) = \left[ \max\{u^{-\theta} + v^{-\theta} - 1, 0\} \right]^{\frac{1}{\theta}}. \]

This copula can only capture a strong lower tail and positive dependence, but it can be rotated and used to capture negative dependence or reflect strong upper tail dependence. The lower tail dependence coefficient is \( \lambda_u = 2 - \frac{1}{\theta} \).

**Gumbel copula:** The bivariate Gumbel copula is given by:
\[ c_{\theta}(u, v) = \exp\left\{ -\left[ \ln u + \ln v \right]^{\frac{1}{\theta}} \right\}. \]

It is an asymmetric Archimedian copula that allows for strong upper tail dependence. The upper tail dependence coefficient is \( \lambda_u = 2 - 2^{\frac{1}{\theta}} \).

**A12 copula:** The bivariate A12 copula is given by:
\[ c_{\theta}(u, v) = \left( 1 + \left[ u^{\theta} - 1 \right]^{\frac{1}{\theta}} + \left[ v^{\theta} - 1 \right]^{\frac{1}{\theta}} \right)^{-1}. \]

The upper tail dependence coefficient \( \lambda_u = 2 - 2^{\frac{1}{\theta}} \) and lower tail dependence coefficient is \( \lambda_u = 2 - 2^{\frac{1}{\theta}} \).

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