Resolvent-based design and experimental testing of porous materials for passive turbulence control

Andrew Chavarin¹, Christoph Efthathiou¹, Shilpa Vijay¹, Mitul Luhar¹
Department of Aerospace and Mechanical Engineering, University of Southern California, Los Angeles, CA

Abstract
An extended version of the resolvent formulation is used to design anisotropic porous materials as passive flow control devices for turbulent channel flow. The effect of these porous substrates is introduced into the governing equations via a generalized version of Darcy’s law. Model predictions show that materials with high streamwise permeability and low wall-normal permeability (\(\phi_{xy} = k_{xx}/k_{yy} \gg 1\)) can suppress resolvent modes resembling the energetic near-wall cycle. Based on these predictions, two anisotropic porous substrates with \(\phi_{xy} \approx 8\) and \(\phi_{xy} \approx 1/8\) were designed and fabricated for experiments in a benchtop water channel experiment. Particle Image Velocimetry (PIV) measurements were used to compute mean turbulence statistics and to elucidate coherent structure via snapshot Proper Orthogonal Decomposition (POD). Friction velocity estimates based on the Reynolds shear stress profiles do not show evidence of discernible friction reduction (or increase) over the streamwise-preferential substrate with \(\phi_{xy} \approx 8\) relative to a smooth wall flow at identical bulk Reynolds number. A significant increase in friction is observed over the substrate with \(\phi_{xy} \approx 1/8\). Coherent structures extracted via POD analysis show qualitative agreement with model predictions.

Keywords: resolvent analysis, anisotropic permeable walls, proper orthogonal decomposition, passive flow control

1. Introduction
1.1. Motivation
Functional surfaces such as sharkskin-inspired riblets and denticles are some of the simplest and most effective control techniques tested thus far for turbulent friction reduction. Appropriately shaped and sized riblets have shown the ability to reduce drag up to 10% in laboratory experiments and up to 2% in real world conditions [1][4]. It is generally accepted that the drag-reducing ability of such surfaces arises from their anisotropy: they offer much less resistance to streamwise flows compared to spanwise flows [4]. The mean flow in the streamwise (x) direction is essentially unimpeded within the riblet grooves,
generating high interfacial slip. However, cross-flows in the wall-normal (y) and spanwise (z) directions arising from turbulence are blocked by the riblets and pushed further from the wall. This blocking effect weakens the quasi-streamwise vortices associated with the energetic near-wall (NW) cycle \[5, 6\], and reduces turbulent mixing and momentum transfer above the riblets \[7\]. Skin friction reduction initially increases with increasing riblet spacing and height. However, above a certain size threshold, performance deteriorates dramatically. Early studies attributed this deterioration of performance to the NW vortices lodging within the riblet grooves \[7, 8\]. More recently, Garcia-Mayoral and Jimenez \[4\] have shown that a Kelvin-Helmholtz (KH) instability may also contribute to the deterioration of performance.

Recent theoretical efforts and numerical simulations suggest that streamwise-preferential permeable substrates have the potential to reduce drag in wall-bounded turbulent flows through a similar mechanism as riblets \[9–11\]. Simulation results predict that as much as 25% drag reduction may be achievable through the use of anisotropic permeable substrates that have streamwise permeability \((k_{xx})\) that is higher than the wall-normal \((k_{yy})\) or spanwise \((k_{zz})\) permeabilities \[10, 11\]. Similar to flow over riblets, a KH instability is also predicted to arise for materials with high wall-normal permeability \[9, 11\]. However, these predictions remain to be tested in physical experiments. In this work, we seek to design and fabricate anisotropic porous materials that have the potential to reduce skin friction, and to test these materials in benchtop channel flow experiments.

1.2. Previous Theoretical Efforts and Simulations

As noted above, streamwise-preferential porous materials have the potential to reduce drag in turbulent flows through a similar mechanism as riblets. With anisotropic porous substrates, high porosity and streamwise permeability contribute to a substantial interfacial slip velocity for the mean flow, while low wall-normal and spanwise permeability limit turbulence penetration into the porous substrate. These effects can also be interpreted in terms of the slip length and virtual origin framework used by Luchini et al. \[1\] to characterize flows over riblets. Specifically, the interfacial slip velocity for the mean flow can be related to a streamwise slip length \(l_U^+\) that determines the virtual origin perceived by the mean flow below the porous interface. Following standard notation, a superscript + denotes normalization with respect to the friction velocity \(u_t\) and kinematic viscosity \(\nu\). Similarly, the distance to which the turbulent fluctuations penetrate into the porous substrate can be related to a transverse slip length \(l_t^+\) that determines the virtual origin for the turbulent cross-flows. Note that the virtual origin for the turbulent cross-flow can also be interpreted as the location at which the quasi-streamwise NW vortices perceive a non-slipping wall \[11, 12\]. The initial decrease in drag over riblets of increasing size has been shown to depend on the difference between the streamwise and transverse slip lengths, \(\Delta D \propto l_U^+ - l_t^+\). Physically, when there is a positive offset between the virtual origins for the mean flow and the transverse fluctuations \(l_U^+ > l_t^+\), the quasi-streamwise NW vortices are pushed into a region of lower
mean shear. This weakens the flow induced by the NW vortices and leads to a reduction in turbulent Reynolds stresses and skin friction. The virtual origin concept has recently been extended to the case of anisotropic porous substrates. Specifically, using the Brinkman equations, Abderrahaman-Elena and García-Mayoral [9] established the following relationships between the streamwise and transverse slip lengths and permeabilities: $l_U^+ \propto \sqrt{k_{xx}^+}$ and $l_t^+ \propto \sqrt{k_{zz}^+}$. These relationships indicate that the initial decrease in drag over anisotropic porous materials is expected to be proportional to the difference between the streamwise and spanwise permeability length scales, $\Delta D \propto l_U^+ - l_t^+ \propto \sqrt{k_{xx}^+} - \sqrt{k_{zz}^+}$.

Recent Direct Numerical Simulation (DNS) results obtained by Gómez-de-Segura and García-Mayoral [11] for turbulent flow over anisotropic permeable substrates show good agreement with the predictions made by Abderrahaman-Elena and García-Mayoral [9]. Specifically, DNS results show the presence of a linear regime in which drag reduction is initially proportional to $\sqrt{k_{xx}^+}$. Thus, drag reduction is expected for substrates with high streamwise permeability and low spanwise permeability. Further, DNS snapshots of the flow field over drag-reducing porous substrates indicate that the NW dynamics are similar to those observed over smooth walls (i.e., characterized by the presence of elongated streaky structures). However, there is a decrease in turbulent fluctuation intensity and Reynolds shear stress close to the wall [10, 13, 14]. These observations indicate that slip length-based models generate useful predictions for the linear drag reduction regime over anisotropic porous substrates. However, these models do not incorporate the effects of wall-normal permeability, which could also affect turbulence penetration into the substrate. Moreover, the maximum achievable drag reduction is thought to be limited by the onset of a KH-type instability. Linear instability analyses predict that the emergence of such instabilities is controlled by $k_{yy}^+ [9, 14]$. DNS results obtained by Gómez-de-Segura and García-Mayoral [11] confirm the emergence of energetic spanwise rollers as the wall-normal permeability increases beyond $\sqrt{k_{yy}^+} \approx 0.4$. Momentum balance arguments show that the additional Reynolds shear stress generated by these rollers is responsible for the deterioration of drag reduction performance and, eventually, an increase in drag over the porous substrates. For completeness, we note that spanwise rollers have also been documented in other simulations over permeable walls and isotropic porous substrates [10, 13, 15, 19].

The DNS results of Gómez-de-Segura and García-Mayoral [11] suggest that drag reduction over anisotropic permeable substrates depends on two key factors: the ability of the substrate to weaken the energetic NW cycle, which depends on $k_{zz}^+$ and $k_{xx}^+$, and the emergence of KH rollers, which is dictated by $k_{yy}^+$. However, these predictions remain to be tested in physical experiments.

1.3. Previous Experiments

Previous experimental studies of turbulent flow over porous materials have focused primarily on granular media such as packed beds of spheres [e.g., 20, 23] or commercially-available materials such as reticulated foams [24, 25], providing significant insight into how porous substrates modify the near-wall turbulent
mean flow and statistics. For instance, they have characterized the effect of substrate permeability on the interfacial slip velocity and the logarithmic region of the mean flow reasonably well, and documented the emergence of spanwise rollers. Recent work by Kim et al. [26] also attempts to systematically delineate the effect of porosity and interfacial roughness.

However, sphere beds and reticulated foams are approximately isotropic. Few studies have explicitly considered the effect of anisotropic porous materials on turbulent flows. Even fewer studies have considered the effect of streamwise-preferential materials that have the potential to reduce drag. The early experiments of Kong and Schetz [27] over mesh and perforated sheets considered materials with significantly higher wall-normal permeability compared to streamwise permeability, i.e., materials with anisotropy ratio $\phi_{xy} = k_{xx}/k_{yy} < 1$. Similarly, recent experiments by Suga et al. [28] have also focused on materials with $\phi_{xy} < 1$. These experiments considered channel flow at bulk Reynolds numbers $Re_b = 900 - 13600$, where one wall was lined with a porous material. The porous substrates consisted of layers of co-polymer nets and the resulting anisotropy ratio for these substrates was $\phi_{xy} = 1/190 - 1/1.5$. In all the cases considered by Suga et al. [28], an increase in friction velocity was observed at the porous wall, and the total friction drag increased by 13-73%.

To the best of our knowledge, the only prior experimental evidence of turbulent drag reduction over porous materials comes from the seal fur tests pursued by Itoh et al. [29]. Given the streamwise-preferential nature of sea fur, these observations provide limited support for the theoretical predictions and simulation results discussed in the previous subsection. However, further verification of these prior results requires a more complete characterization of how streamwise-preferential materials with known permeability affect turbulent flows.

Finally, note that canopies of terrestrial or aquatic vegetation and corals reefs can also be considered anisotropic porous materials. However, since such substrates do not exhibit high streamwise permeability, the extensive literature on flows over vegetation canopies and coral reefs is not reviewed here for brevity.

1.4. Contribution and Outline

In this paper we seek to design and fabricate anisotropic porous materials that have the potential to reduce skin friction, and to test these materials in laboratory experiments. In particular, we leverage advances in additive manufacturing (3D-printing) to fabricate cellular porous materials that have desirable anisotropy ratios $\phi_{xy} > 1$, and test the effect of these materials in benchtop channel flow experiments. In order to predict the viability of these substrates for passive drag reduction, we extend the resolvent analysis framework the resolvent framework of McKeon and Sharma [30] to account for porous substrates. Recent work by Chavarin and Luhar [31] shows that the resolvent framework can serve as a useful assessment and design tool for riblets. In particular, Chavarin and Luhar [31] show that the resolvent framework can: (i) predict whether riblets of specified geometry are likely to suppress or amplify the energetic NW cycle, and (ii) be used to test for the emergence of spanwise rollers resembling KH vortices. Here, we use resolvent analysis as a preliminary design tool: we use
it to identify streamwise-preferential porous geometries with \( \phi_{xy} > 1 \) that are likely to suppress the NW cycle. These geometries are then 3D printed for the laboratory experiments. Measurements made over the material with \( \phi_{xy} > 1 \) are compared against measurements made over a geometrically-similar porous material with \( \phi_{xy} < 1 \) as well as a solid smooth wall. In addition to serving as a test for whether streamwise-preferential materials can reduce friction, these experiments also provide preliminary insights into how anisotropic materials with \( \phi_{xy} < 1 \) and \( \phi_{xy} > 1 \) modify the near-wall flow physics.

The remainder of this paper is structured as follows. The resolvent-based modeling framework is described further in §2. The experimental methods are presented in §3. Model predictions and experimental results are discussed together in §4. Specifically, the resolvent-based predictions used to design the porous materials tested in the experiments are shown in §4.1. Experimental measurements for the mean profile and turbulence statistics are discussed in §4.2 and the flow features identified via snapshot proper orthogonal decomposition (POD) are shown in §4.3. One of the inputs required for resolvent analysis is an estimate of the turbulent mean profile. The model predictions shown in §4.1 are obtained using a synthetic mean profile computed using an eddy viscosity formulation. These predictions are compared against those made using the mean profiles measured in the experiments in §4.4. Brief concluding remarks are presented in §5.

2. Modeling

In this section, we describe the extension to the resolvent framework to account for porous substrates and provide details on numerical implementation.

2.1. Extended Resolvent Formulation

We utilize a modified version of the resolvent formulation proposed by McKeeon and Sharma [30] to predict the drag performance of anisotropic permeable materials for passive turbulence control. For wall-bounded turbulent flows, the resolvent formulation interprets the Navier-Stokes equations, Fourier-transformed in the (approximately) homogeneous streamwise and spanwise directions and in time, as a forcing-response system. For this system the nonlinear convective terms are treated as internal forcing (input) to the system composed from the remaining linear terms of the Navier-Stokes equations. At every wavenumber-frequency combination \( \kappa = (\kappa_x, \kappa_z, \omega) \) this internal forcing generates a turbulent velocity and pressure response. A gain-based singular value decomposition of the forcing-response transfer function—the resolvent operator—yields a set of highly amplified velocity and pressure response modes (left singular vectors) and the corresponding forcing-response gains (singular values). The response modes—termed resolvent modes—are flow structures with streamwise and spanwise wavelength \( \lambda_x = 2\pi/\kappa_x \) and \( \lambda_z = 2\pi/\kappa_z \), respectively, traveling at speed \( c = \omega/\kappa_x \). The forcing-response gain is a measure of energy amplification in the system, and serves as a metric of control performance.
Previous work shows that specific high-gain response modes can serve as useful surrogates for energetic structures such as the NW cycle \[32\]. These resolvent modes can therefore serve as building blocks for the design and optimization control strategies \[31, 33–36\]. Specifically, these prior efforts show that suppression of the NW resolvent mode is a useful indicator of drag reduction performance for both active and passive control of wall turbulence. In other words, if a control technique is unable to suppress the surrogate NW resolvent mode, then it is unlikely to yield drag reduction for the full turbulent flow field. In addition, recent work by Chavarin and Luhar \[31\] shows that the resolvent framework is also able to predict the emergence of energetic spanwise rollers over riblets that contribute to the deterioration of drag reduction performance \[4\]. Building on these prior studies, here we use the resolvent framework to test whether a given porous material can (i) suppress the gain for the resolvent mode that serves as a surrogate for the NW cycle, and (ii) limit the emergence of energetic spanwise rollers resembling KH vortices.

For this analysis we formulate the resolvent framework using the volume-averaged Navier-Stokes (VANS) equations in which the effect of anisotropic porous substrates is included via a generalized version of Darcys law\[16\]:

\[
\begin{align*} 
\frac{\partial \langle u \rangle}{\partial t} + \frac{1}{\varepsilon} \nabla \cdot (\varepsilon \langle u \rangle \langle u \rangle + \varepsilon \tau) &= -\frac{1}{\varepsilon} \nabla \langle p \rangle + \frac{1}{\varepsilon Re_\tau} \nabla^2 (\varepsilon \langle u \rangle) - \frac{\varepsilon}{Re_\tau} K^{-1} \langle u \rangle, \\
\nabla \cdot (\varepsilon \langle u \rangle) &= 0. 
\end{align*}
\]

(1a) (1b)

Here, \(\langle u \rangle\) and \(\langle p \rangle\) represent the dimensionless volume averaged velocity and pressure respectively, \(\varepsilon\) represents the porosity, and \(K\) is the dimensionless permeability tensor. The equations above have been normalized by the channel height \(h\) (see Fig.1) and the friction velocity \(u_\tau\) and the friction Reynolds number is given by \(Re_\tau = h^+ = u_\tau h/\nu\). The sub-filter scale stresses which arise from volume-averaging the Navier Stokes equations are defined as \(\tau = \langle uu \rangle - \langle u \rangle \langle u \rangle\). The unobstructed fluid domain is characterized by porosity \(\varepsilon = 1\) and infinite permeability. For this region, the Darcy term drops out of the governing equations, and Eq. 1 reduces to the standard Navier-Stokes equations.

Note the the expression above omits the nonlinear Forchheimer term. For the remainder of the paper, we focus on porous substrates for which the permeability tensor is diagonal and has the following form \(K = \text{diag}(k_{xx}, k_{yy}, k_{zz})\). Further, since we are primarily considering structures (i.e., NW cycle and KH rollers) that are much larger than the pore scale, we assume that the sub-filter scale stresses are negligible. These assumptions and modeling simplifications are consistent with those made in recent numerical simulations of flow over anisotropic porous substrates \[10, 11\]. To further simplify the expressions in Eq. 1 we assume that the porous substrate is spatially homogeneous and has constant porosity. This
yields:

\[ \frac{\partial u}{\partial t} + \nabla \cdot (uu) = -\nabla p + \frac{1}{Re_T} \nabla^2 u - \frac{1}{Re_T} \varepsilon K^{-1} u, \]  

(2a)

\[ \nabla \cdot u = 0, \]  

(2b)

where the \( \langle \cdot \rangle \) notation has been omitted for simplicity. Resolvent analysis proceeds as follows. First, we employ a standard Reynolds-averaging procedure such that velocity is decomposed into a mean component \( (U) \) and a fluctuation about this mean \( (u') \). Next, the governing equations for the fluctuations are Fourier-transformed and expressed as

\[
\begin{bmatrix}
 u_\kappa \\
p_\kappa
\end{bmatrix} = H_\kappa \mathbf{f}_\kappa.
\]  

(3)

Here, \( u_\kappa \) and \( p_\kappa \) represents the Fourier-transformed velocity and pressure fluctuations, \( \mathbf{f}_\kappa \) represents the nonlinear forcing terms, and \( H_\kappa \) is the resolvent operator representing the linear forcing-response dynamics. At every wavenumber-frequency combination \( \kappa \), an SVD of the discretized resolvent operator, i.e.,

\[ H_\kappa = \sum_m \psi_{\kappa,m} \sigma_{\kappa,m} \phi_{\kappa,m}^* \]  

(4)

yields forcing modes (right-singular vectors, \( \phi_{\kappa,m} \)) and velocity/pressure response modes (left-singular vectors, \( \psi_{\kappa,m} \)) that are ordered based on their forcing-response gain (singular values, \( \sigma_{\kappa,m} \)). For our analysis, the resolvent operator is scaled to enforce an \( L_2 \) energy norm and so the change in singular value relative to the smooth wall case can be interpreted as a measure of energy amplification or suppression. Importantly, previous work shows that the resolvent operator tends to be low-rank at wavenumber-frequency combinations that are energetic in turbulent flows [30, 32]. Consequently the resolvent operator can be well approximated using a rank-1 truncation after the SVD, i.e., by only considering the first singular values, \( \sigma_{\kappa,1} \), and response modes, \( \psi_{\kappa,1} \). Chavarin and Luhar [31] show that resolvent analysis with this rank-1 approximation provides useful insight into the effect of riblets on wall turbulence. We retain the rank-1 approximation here as well, and drop the additional subscript 1. For further discussion pertaining to resolvent analysis for wall-bounded turbulent flows and the rank-1 approximation, the reader is referred to several recent studies in this area [30, 37, 38].

For the remainder of this work, we focus on modes serving as a surrogate model for the dynamically important NW cycle (i.e., with \( \kappa \) corresponding to \( \lambda_z^+ = 10^3, \lambda_z^+ = 10^2, \) and \( c^+ = 10 \)) and consider the highest singular value as a measure of performance. If the singular value—or gain—is reduced over the porous material \( (\sigma_{\kappa,p}) \) relative to the smooth wall value \( (\sigma_{\kappa,s}) \), the porous material is likely to suppress the corresponding flow structure. We also test for the emergence of high-gain spanwise-constant rollers resembling Kelvin-Helmholtz vortices (i.e., with \( \kappa_z = 0 \)) at the porous interface.
2.2. Numerical Implementation

Previous theoretical and numerical efforts have primarily considered a symmetric channel geometry, with porous materials at both the upper and lower walls \[10, 11\]. However, this geometry would have limited optical access for Particle Image Velocimetry (PIV) in the laboratory experiments discussed below. Instead, we consider an asymmetric channel geometry corresponding to the experimental setup shown in Fig. 1(a). The unobstructed region of the channel spans \( y \in [0, h] \) and the porous material occupies the region corresponding to \( y \in (h, H + h) \), with \( H = h \). We generate model predictions for \( Re = u \tau h / \nu = 120 \), which corresponds roughly to the conditions tested in the experiments, and for \( Re = 360 \), which corresponds to prior numerical simulations \[10\]. As noted earlier, the spatially-homogeneous porous substrate is defined by its principal permeability components \( K = \text{diag}(k_{xx}, k_{yy}, k_{zz}) \). For simplicity, the porosity is set to \( \varepsilon = 1 \), though the materials tested in the experiment have a porosity \( \varepsilon = 0.87 \).

No-slip boundary conditions are applied at the true walls, which are located at \( y = 0 \) and \( y = H + h \). The interface between the porous substrate and the unobstructed domain is located at \( y = h \). The resolvent operator is discretized in the wall-normal direction using the Chebyshev collocation method detailed by Aurentz and Trefethen \[39\]. This approach allows us to discretize the unobstructed and porous domains independently and couple these two domains through the jump boundary conditions proposed by Ochoa-Tapia and Whitaker \[40\]. The following boundary conditions are applied at the interface:

\[
\begin{align*}
    u_{\mid y=h^-} &= u_{\mid y=h^+},
\end{align*}
\] (5a)
where $h^-$ and $h^+$ refer to $y$-locations on either side of the interface.

Construction of the resolvent operator also requires a mean velocity profile, \( \mathbf{U} = [U(y), 0, 0] \). The mean velocity is predicted from the Reynolds-averaged mean flow equation which contains a linear Darcy drag term, with the Reynolds stress term modeled using an eddy viscosity. For the unobstructed region, the eddy viscosity profiles for our analysis are generated using the analytical model developed by Reynolds and Tiederman [41]. For the porous layer, the eddy viscosity is set to zero.

Since resolvent-based predictions are sensitive to the exact form of the mean profile, in Section 4.4, we compare model predictions obtained using the synthetic mean profile discussed above against predictions obtained using a profile fitted to the experimental measurements described below. This fitted profile is synthesized from the experimental measurements as follows. First, the mean velocity ($U(y)$) and Reynolds shear stress ($= -u'v'$) profiles are obtained from the PIV measurements described below by averaging in time and in the streamwise direction. These profiles are then used to estimate the eddy viscosity profile, $\nu_T = -\frac{u'v'}{dU/dy}$, where the mean shear is $d \bar{U} / dy$ is approximated using a finite difference scheme. The points near the maximum in the mean profile, corresponding to $d \bar{U} / dy \approx 0$, are removed and a smooth cubic spline is fitted to the resulting profile. Finally, the eddy viscosity profile is allowed to smoothly transition to zero in the porous medium and values for which $\nu_T(y) < 0$ are removed and set to 0. This fitted eddy viscosity profile is then used to generate predictions for the mean profile (see Fig. 5) used in the resolvent operator.

For the model predictions presented in this paper, a total of 226 Chebyshev nodes are used for discretization in the wall-normal direction. The nodes are divided evenly between the porous and the unobstructed domain. Further grid refinement beyond this point led to changes in singular values smaller than $O(10^{-4})$.

3. Experimental Methods

3.1. 3D Printed Porous Materials

For this study, two custom anisotropic porous materials were fabricated using a stereo-lithographic 3D printer (formlabs Form 2) based on input from the resolvent-based predictions described below. The porous material microstructure consisted of a cubic lattice of rectangular rods with constant cross-section
and varying spacing in the $x$, $y$, and $z$ directions. Fabrication constraints (printing resolution, allowable unsupported lengths, resin drainage) limited the maximum anisotropy that could be achieved. The anisotropy was varied by controlling the size of the pores normal to the spanwise and streamwise directions. The minimum pore size was dictated by the printer resolution as the rods fused and the surface became solid if the separation between two rods fell below the minimum resolution. Moreover, the maximum pore size was limited by the maximum overhang between rods because with excessive overhang, the horizontal rod sagged and deviated from the design geometry. Considering these limitations, the following two geometries represented a good compromise between reliable fabrication and anisotropy. The first case with spacings $s_x = 0.8$ mm and $s_y = s_z = 3.0$ mm contained larger pores facing the streamwise direction and small pores facing the wall-normal and spanwise directions (see Fig. 1(b)). The second geometrically similar, but rotated, case with $s_x = s_z = 3.0$ mm and $s_y = 0.8$ mm contained larger pores facing the wall-normal direction (see Fig. 1(c)). These materials are referred to as $x$-permeable and $y$-permeable, respectively, for the remainder of this paper. For both geometries, the rod cross-section was a square of size $d \times d$, with $d = 0.4$ mm. The porosity was $\varepsilon \approx 0.87$. A 3D-printed sample of the $x$-permeable material is shown in Fig. 1(d).

Following the approach of Zampogna and Bottaro [42], the permeability tensor ($K$) for these anisotropic porous materials was determined by solving independent forced Stokes flow problems for a unit cell of the cubic lattice in the ANSYS Fluent software package. Due to the symmetric nature of the microstructures tested, only two Stokes flow problems were required to determine the permeability components $k_{xx}$ and $k_{yy}$; $k_{zz}$ is equal to either $k_{xx}$ or $k_{yy}$ depending on configuration (i.e., $x$-permeable or $y$-permeable). For these two Stokes flow problems, a uniform body forcing of unit amplitude was applied in the direction of the permeability component being evaluated. The permeability was determined from the resulting volume-averaged velocity using Darcy’s law. Periodic boundary conditions were applied to the boundaries of the unit cell and a no-slip condition was applied at the solid boundaries. A time-marching scheme was used for each of these simulations. The solutions were determined to be at steady state when the residual in the permeability was less than $10^{-6}$. A mesh independence study confirmed that our results were grid converged. The resulting permeability estimates are shown in Table I. The anisotropic ratio is $\phi_{xy} = k_{xx}/k_{yy} \approx 8$ for the $x$-permeable case and $\phi_{xy} \approx 1/8$ for the $y$-permeable case.

|                | $\varepsilon$ | $k_{xx}/H^2$ | $k_{yy}/H^2$ | $k_{zz}/H^2$ |
|----------------|---------------|---------------|---------------|---------------|
| $x$-permeable  | 0.87          | 4.3e-3        | 5.5e-4        | 5.5e-4        |
| $y$-permeable  | 0.87          | 5.5e-4        | 4.3e-3        | 5.5e-4        |

Table 1: Dimensionless permeability estimates for the 3D-printed porous materials. $H = 6.34$ mm is the height of the porous substrates tested in the channel flow experiments.
3.2. Channel Flow Experiment

The anisotropic porous substrates described above were tested in a turbulent channel flow experiment, albeit at very low Reynolds number. A schematic of the experimental setup is shown in Fig. 1. A custom test section was machined from acrylic with a cutout of length $L = 320$ mm designed to hold the porous substrates. The width of the test section was $W = 50$ mm, and the height of the unobstructed region was $h = 6.34$ mm. The cutout was located approximately 150 mm from the inflow, and allowed for 3D-printed tiles of thickness $H = h = 6.34$ mm to be mounted flush with the smooth wall upstream of the cutout. Note that the number of pores accommodated over the height of the tiles was limited to 2 for the $x$-permeable case and 8 for the $y$-permeable case, indicating limited separation between the pore-scale and outer-scale flow. For a baseline comparison, experiments were also carried out with a solid smooth walled insert placed in the cutout.

We recognize that there is insufficient scale separation between the pore size and the height of the porous medium and, as such, the volume-averaged representation shown in Eq. 1 is not truly valid. The large pore sizes are driven by the minimum pore size and the desire to generate a maximum anisotropy ($\phi_{xy} = k_{xx}/k_{yy}$). Nevertheless, this represents the first set of experiments over custom-designed, anisotropic porous media.

Flow in the channel was generated using a submersible pump placed in a large water tank. The flow rate was controlled using an electronic proportioning valve. The volumetric flow rate was $Q = 92$ cm$^3$/s for the smooth wall and $x$-permeable cases, and $Q = 82$ cm$^3$/s for the $y$-permeable case. Thus, the bulk Reynolds number was $Re_b = Q/(W \nu) = 1840$ for the smooth wall and $x$-permeable case, and $Re_b = 1640$ for the $y$-permeable case. This corresponds to the lower end of the $Re_b$ ranges considered by Chang in recent experiments over anisotropic porous materials.

A 5W continuous wave laser with integrated optics was used to generate a laser sheet in the streamwise-wall normal direction at mid-span. A high-speed camera (Phantom VEO-410L) was used to capture images near the downstream end of the porous section. Recent turbulent boundary layer experiments over isotropic porous foams show that the flow adjusts to the new substrate over a streamwise distance of $\approx 30H - 40H$, where $H$ is the porous layer thickness [25]. To provide an adequate development length therefore, the PIV field of view began 195 mm ($\approx 31H$) from the leading edge of the porous section and extended 22 mm ($\approx 3.5H$) downstream. Images were acquired at 2 kHz for 10 seconds for a total of 20,000 images. The total duration of the measurements is approximately 100 turnover times, where the turnover time is estimated as $(H + h)/U_b$, and the bulk-averaged velocity is defined as $U_b = Q/[W(H + h)]$. The images were processed in PIVlab [43] using the Fast-Fourier transform routine with a minimum box size of 16 pixels and 50% overlap, which yielded 36 (vertical) x 125 (horizontal) data points in the unobstructed section. Based on friction velocities computed from the PIV measurements, the vertical resolution was $\Delta y^+ = \Delta x^+ = 3.3 - 5$ in inner units.
4. Results and Discussion

We first present resolvent-based predictions for the mode that serves as a surrogate for the NW cycle and for spanwise constant KH-rollers, focusing on the x-permeable and y-permeable cases tested in experiment (Section 4.1). We then present experimental measurements for the mean flow, turbulence statistics and flow structure (Sections 4.2-4.3). A comparison of model predictions made using the synthetic mean profiles and those obtained from fits to experimental data is presented at the end (Section 4.4).

4.1. Model Predictions

![Graphs showing predicted changes in singular values for resolvent modes resembling the NW cycle.](image)

Figures 2(a,b) show the predicted change in singular values for resolvent modes resembling the NW cycle as a function of their streamwise and wall-normal permeability length scales, $\sqrt{k_{xx}}$ and $\sqrt{k_{yy}}$. Color contours show predictions for $Re_\tau = 120$; solid black lines correspond to $Re_\tau = 360$. Predictions in panel (a) are for substrates which have a similar configuration to x-permeable material $K = \text{diag}(k_{xx}, k_{yy}, k_{zz} = k_{yy})$. Predictions in panel (b) are for substrates with a similar configuration to the y-permeable substrate, $K = \text{diag}(k_{xx}, k_{yy}, k_{zz} = k_{xx})$. The (◦) symbols in (a) and (b) correspond roughly to the permeabilities for the x-permeable and y-permeable materials tested in the experiments. (c,d) Amplification of spanwise-constant modes at $Re_\tau = 120$ relative to the smooth wall case as a function of streamwise wavelength and mode speed for (c) the x-permeable substrate and (d) for the y-permeable substrate, i.e., for permeability values labeled with (◦) symbols in (a) and (b).

Figures 2(a,b) show the predicted change in singular values for resolvent modes resembling the NW cycle (i.e., resolvent modes with $\lambda_x^+ = 10^3$, $\lambda_z^+ = 10^2$, and $c^+ = 10$) over anisotropic porous substrates as a function of their streamwise and wall-normal permeability length scales, $\sqrt{k_{xx}^+}$ and $\sqrt{k_{yy}^+}$. Note that the contours show the forcing-response gain for the porous material normalized.
by the smooth wall value at the same \( Re_\tau \); \( \sigma_{\kappa,p}/\sigma_{\kappa,s} < 1 \) indicates mode suppression and \( \sigma_{\kappa,p}/\sigma_{\kappa,s} > 1 \) indicates mode amplification. For all the predictions shown in Fig. 2 the mean flow was computed using the synthetic eddy viscosity profile.

Consistent with prior simulation results [10, 11], porous substrates with high streamwise permeability and low wall-normal permeability are found to suppress the NW mode, which is known to be a useful predictor of drag reduction performance [31]. In general, mode suppression increases as the permeability ratio increases, \( \phi_{xy} \gg 1 \), though there are some subtle differences between the results presented in Fig. 2(a) for substrates with \( k_{zz}^+ = k_{yy}^+ \) and in Fig. 2(b) for substrates with \( k_{zz}^+ = k_{xx}^+ \). The substrates shown in panel (a) produce greater mode suppression than those shown in panel (b) for \( \phi_{xy} \gg 1 \). This is consistent with the virtual origin model proposed in previous studies [9, 11], which suggests that turbulence penetration into the porous medium is dictated by the spanwise permeability, and that the initial decrease in drag depends on the difference between the streamwise and spanwise permeability length scales \( \Delta D \propto \sqrt{k_{xx}^+} - \sqrt{k_{zz}^+} \).

The predictions shown in Figs. 2(a,b) do not change substantially from \( Re_\tau = 120 \) (colored shading) to \( Re_\tau = 360 \) (solid black lines). In particular, the location of the neutral curve corresponding to \( \sigma_{\kappa,p}/\sigma_{\kappa,s} = 1 \) (i.e., no change in gain) is very similar for both Reynolds numbers. In addition, the trends in the suppression and amplification of the NW-mode remain the same between \( Re_\tau = 120 \) and \( Re_\tau = 360 \).

For the specific porous materials tested in our experiments, the permeability length scales correspond to \( (\sqrt{k_{xx}^+}, \sqrt{k_{yy}^+}) = (7.9, 2.8) \) for the x-permeable case and \( (\sqrt{k_{xx}^+}, \sqrt{k_{yy}^+}) = (2.8, 7.9) \) for the y-permeable case at \( Re_\tau = 120 \). These values were computed from the dimensionless permeability listed in Table 1 assuming \( H^+ = Re_\tau = 120 \). These specific permeability ratios are labeled using \( \circ \) markers in Figs. 2(a,b). Model predictions indicate that the x-permeable substrate suppresses resolvent modes resembling the NW cycle by approximately \( 25 - 30\% \) (see \( \circ \) marker in Fig. 2(a)). In contrast, the y-permeable substrate leads to significant mode amplification, with \( \sigma_{\kappa,p}/\sigma_{\kappa,s} > 2 \) (see \( \circ \) marker in Fig. 2(b)). This is broadly consistent with previous simulations, which indicate that drag reduction is only expected over streamwise-preferential materials.

In addition the the suppression or amplification of the NW cycle, the other factor that controls the drag-reduction performance of anisotropic porous substrates is the emergence of KH rollers [9, 11, 16, 44]. Linear stability analysis and simulations suggest that the appearance of such rollers is linked to a relaxation of the wall-normal permeability. Specifically, the recent simulations of Gómez-de-Segura and Garca-Mayoral [11] indicate that the spanwise rollers emerge as the wall normal permeability increases beyond \( \sqrt{k_{yy}^+} \approx 0.4 \). Unfortunately, due to fabrication constraints, both the x-permeable and y-permeable substrates tested here are expected to have wall-normal permeabilities larger than this threshold value. Figures 2(c,d) show the normalized gain for spanwise-constant (\( \kappa_z = 0 \)) resolvent modes over the x-permeable and y-permeable material, respectively.
For the y-permeable case a region of high amplification is visible in Fig. 2(d) for structures with streamwise wavelength \( \lambda^+ \approx 700 - 1200 \) and mode speed \( c^+ \approx 5 - 10 \). The wave speed of these structures indicates that these structures are localized close to the fluid-porous interface. The most amplified structure in this region has \( \lambda^+ \approx 1000 \) and \( c^+ \approx 8.5 \). The gain for this structure increases by a factor of approximately 100 relative to the smooth wall case. In contrast, Fig. 2(c) shows that there is no localized maximum in relative amplification for spanwise-constant structures for the x-permeable case. Model predictions indicate that spanwise rollers with \( \lambda^+ \geq 600 \) and \( c^+ \approx 4 - 15 \) are amplified relative to the smooth wall case and amplification generally increases with increasing wavelength. The gain for the most amplified structure in Fig. 2(c) increases by a factor of approximately 6 relative to the smooth wall case. Thus, even though the x-permeable material is susceptible to the emergence of spanwise-constant structures, the degree of energy amplification relative to the smooth wall case is more limited compared to that for the y-permeable material.

Together, the predictions presented in section indicate that the x-permeable material is likely to suppress the energetic NW cycle but could give rise spanwise rollers resembling KH vortices. The y-permeable material is likely to further amplify the NW cycle and give rise to spanwise rollers that are amplified significantly relative to the smooth wall case. These predictions are compared against the measurements made in the benchtop channel flow experiments in the following sections.

4.2. Mean Flow and Turbulence Statistics

Figure 3 shows the measured mean statistics for the channel flow experiments. These statistics were computed by averaging both in time and in the streamwise direction. Results in the region \( y/h \geq 0.95 \) were affected by reflections at the smooth/porous tiles and should be treated with caution.

As seen from Fig. 3(a), the mean profile remains relatively symmetric across the unobstructed region for the x-permeable material. However, for the y-permeable case, the bulk of the flow in the unobstructed region deviates from the parabolic profile and is shifted towards the smooth wall. The location of the maximum mean velocity is \( y/h = 0.45 \) for the x-permeable case as compared to \( y/h = 0.38 \) for the y-permeable case. Interestingly, the slip velocity at the porous interface appears to be higher for the y-permeable case despite the substantially lower streamwise permeability. However, this observation could be attributed to the specific porous geometry tested here. For the y-permeable material, the porous interface is characterized by much higher local porosity compared to the x-permeable material. The visibly lower bulk-normalized mean profile for the x-permeable material in the unobstructed region is indicative of greater flow through the porous medium itself.

The Reynolds shear stress profiles in Fig. 3(b) show the presence of an (almost) linear region in the middle of the unobstructed domain. Friction velocities at the porous and smooth walls, \( u^+_f \) and \( u^+_s \), were estimated by extrapolating the total stress (i.e., Reynolds shear stress plus viscous stress) from this linear region to \( y = h \) and \( y = 0 \), respectively [see e.g., 16] and are listed in Table 2.
Figure 3: PIV results for the channel flow experiment. The smooth wall is located at \( y = 0 \), while the interchangeable wall is located at \( y = h \). Panel (a) shows the measured mean velocity profiles normalized by the bulk-averaged velocity (calculated as \( U_b = Q / (W(H + h)) \) in all cases). Panel (b) shows Reynolds shear stress profiles normalized by the smooth wall friction velocity \( u_* \). Friction velocities at the smooth and porous walls were estimated by extrapolating a linear fit to the total stress profile to the wall locations. Panels (c) and (d) show profiles of the root-mean-square streamwise fluctuations normalized by the friction velocities at the smooth wall and porous interface, \( u^*_{s} \) and \( u^*_{p} \), respectively.

The average friction velocity was estimated as \( u^*_f = \sqrt{((u^*_s)^2 + (u^*_p)^2)/2} \). Note that the profiles shown in Fig. (3b) are normalized by \( u^*_s \). With this normalization, it is clear that \( u^*_p \) is higher than \( u^*_s \) for the y-permeable material, indicative of greater friction generated at the porous interface than at the smooth wall. However, for the x-permeable material, the estimated friction velocities at the smooth wall and porous interface are comparable, \( u^*_s \approx u^*_p \). In other words, there is no clear increase or decrease in friction at the porous interface relative for the x-permeable material. Note that friction velocities at both interfaces are approximately equal for the smooth wall case, as expected. Thus, the friction velocity estimates are broadly consistent with the mean profiles shown in Fig. (3a); only the y-permeable material shows a significant difference in behavior at the porous interface.

Moreover, relative to the smooth wall case, x-permeable material does not
Table 2: Friction velocity estimates at the smooth wall ($u^*_u$) and porous interface ($u^*_p$). The average friction velocity is $u^*_t$. For the smooth wall case $u^*_p$ corresponds to the solid tile placed in the cutout.

| Case          | $u^*_u$ [m/s] | $u^*_p$ [m/s] | $u^*_t$ [m/s] |
|---------------|---------------|---------------|---------------|
| smooth wall   | 0.0194        | 0.0198*       | 0.0196        |
| x-permeable   | 0.0201        | 0.0205        | 0.0203        |
| y-permeable   | 0.0235        | 0.0326        | 0.0284        |

lead to a significant change in friction velocities; $u^*_u$ values differ by 3% and at the porous wall $u^*_p$ is higher by approximately 4% (note that $u^*_p$ for the smooth wall corresponds to the value at the smooth tile at $y/h = 1$). In contrast, the y-permeable material leads to a significant increase in friction velocities relative to the smooth wall case; $u^*_u$ increases by 20% and $u^*_p$ by 70%. Thus, in contrast to the resolvent-based predictions for the NW cycle, no reduction in friction is observed at the porous interface for the x-permeable material. However, the y-permeable material leads to a substantial increase in friction at both the smooth wall and the porous interface. This is qualitatively consistent with model predictions, which show a substantial increase in NW cycle gain as well as the emergence of high-gain spanwise rollers over the y-permeable material.

Profiles for the root-mean-square streamwise velocity fluctuations normalized by $u^*_u$ and $u^*_p$, are plotted in Figs. 3(c) and 3(d) respectively. When normalized by $u^*_u$, the near-wall peaks on the smooth wall side collapse together for all cases. Further, the normalized peak value for the fluctuations is $\sqrt{\overline{u'^2_s}} \approx 3$, which is close to that expected in a canonical turbulent channel flow configuration. However, as per Fig. 3(d), only the peaks for the smooth wall and x-permeable case collapse reasonably well near the porous interface when normalized by $u^*_p$, with a peak value of $\sqrt{\overline{u'^2_p}} \approx 3$. This confirms that the flow physics are qualitatively similar for the smooth wall and x-permeable substrate. The profile for the y-permeable case has no discernible peak near the porous interface and the maximum value for the normalized fluctuations $\sqrt{\overline{u'^2_p}} \approx 2.5$, is lower than the other two cases. These observations are consistent with previous results for flow over porous materials [16, 28] and are indicative of a change in flow structure.

4.3. Flow Structure

To provide further insight into the changes in mean statistics discussed above, snapshot proper orthogonal decomposition (POD) was performed on the fluctuating velocity fields obtained from PIV. The streamwise velocity fields associated with the first 2 spatial modes for the smooth wall, x-permeable, and y-permeable cases are shown in Fig. 4.

As expected, the most energetic modes for the smooth wall case shown in Figs. 4(a,b) resemble long streaky structures that are symmetric across the
Figure 4: The first two POD modes for the smooth wall case (a,b), x-permeable case (c,d), and the y-permeable case (e,f). The modes are computed using 20,000 PIV frames. The shading represents normalized levels for the streamwise velocity component. The solid and dashed black lines represent positive and negative contours for the wall-normal velocity component. The porous interface is at the top wall.

channel. The first POD mode for the x-permeable case shown in Fig. 4(c) does not have a clear physical interpretation. It bears some resemblance to the second POD mode for the smooth wall shown in Fig. 4(b) but also some to the asymmetric first POD mode over the y-permeable case shown in Fig. 4(e). However, the second POD mode for the x-permeable substrate closely resembles the first mode for the smooth wall case. The streamwise extent of the plots shown in Fig. 4 corresponds to the PIV field of view, which is roughly 3.5h or 22 mm. With this in mind, the first smooth wall mode and the second x-permeable mode appear to have a streamwise wavelength that is more than twice the PIV field of view, $\lambda_x > 44$ mm (or $\lambda_x > 7h$). For the friction velocity estimates shown in Table 2, this translates into $\lambda_x^+ = (\lambda_x u'_t / \nu) > 800$, which is consistent with the scale of NW streaks [5].

Unlike the smooth wall and x-permeable cases, POD modes for the y-permeable material have a visibly asymmetric structure in the wall-normal direction (see Fig. 4(c,f)). For both modes, the streamwise velocity field is much more intense near the porous interface. This is in contrast to the symmetric streaky structures observed over both the smooth wall and x-permeable material. Moreover, when the streamwise and wall-normal velocity contours are considered together, the full velocity field for these POD modes is indicative of a counter-rotating structure. Such rollers have been observed in previous numerical simulations [11,16], and are typically associated with a Kelvin-Helmholtz instability mechanism. This observation provides qualitative support for the model predictions shown in Fig. 2(d) which shows that the y-permeable material is susceptible to the emergence of large high-gain spanwise constant structures.
More quantitatively, the first POD mode over the y-permeable material appears to have a streamwise wavelength more than twice the PIV field of view, $\lambda_x > 7h \approx 44 \text{ mm}$. Using the $u^+_t$ estimate for the y-permeable material shown in Table 2, this translates into $\lambda^+_x > 1200$. Thus, the size of this structure is larger than the region of peak amplification around $\lambda^+_x \approx 1000$ predicted in Fig. 2(d). However, assuming that the spanwise rollers scale in outer units, $\lambda^+_x \approx 1000$ for the predictions shown in Fig. 2(d) at $Re_\tau \approx 120$, corresponds to $\lambda_x/h = \lambda^+_x/Re_\tau \approx 8$. This is more consistent with the streamwise wavelength of the first POD mode for the y-permeable substrate shown in Fig. 4(e). Put another way, if the resolvent-predictions had been carried out at the measured friction Reynolds number for the y-permeable substrate, $Re_\tau = u^+_t h/\nu \approx 180$ for the $u^+_t$ estimate shown in Table 2, they might show a region of higher amplification at higher streamwise wavelengths. This issue is explored further in Section 4.4.

The streamwise extent of the second POD mode over the y-permeable substrate (see Fig. 4(f)) is slightly larger than the PIV window, $\lambda_x \approx 4h$. This corresponds to a streamwise wavelength of $\lambda^+_x \approx 700$, which is at the lower end of the predicted region of highly-amplified spanwise rollers in Fig. 2(d).

4.4. Model Sensitivity to Mean Profile

![Figure 5: Comparison of the mean velocity profiles used in the construction of the resolvent operator for the smooth wall (a), x-permeable material (b), and y-permeable material (c).](image)

Figure 5: Comparison of the mean velocity profiles used in the construction of the resolvent operator for the smooth wall (a), x-permeable material (b), and y-permeable material (c). Experimental profiles are shown using the circular makers (●). Synthetic profiles generated using the eddy viscosity model of Reynolds and Tiederman [41] are plotted as dashed lines (—). Fitted profiles computed using eddy viscosity profile determined from experimental data are plotted as solid lines (—).  

The results presented in the previous sections show that there is no clear evidence of NW-cycle suppression and friction reduction over the x-permeable material. For the y-permeable material, a substantial increase in friction is observed at the porous interface relative to the smooth wall and x-permeable material. In addition, the POD modes shown in Section 4.3 suggest that large-scale spanwise rollers are energetic over the y-permeable substrate. These results are in partial agreement with the model predictions shown in Section 4.1. Here, we revisit resolvent-based predictions for the NW mode and spanwise rollers, but using the mean profile estimated from the experiments based on the procedure described in Section 2.2.
Figure 6: Normalized gain ($\sigma_{\kappa,p}/\sigma_{\kappa,s}$) for spanwise-constant resolvent modes as a function of streamwise wavelength and mode speed. Panels (a,b) show predictions for the x-permeable case computed using the synthetic eddy viscosity profile (a) and the fitted eddy viscosity profile (b). Panels (b,c) show predictions for the y-permeable case computed using the synthetic eddy viscosity profile (c) and the fitted eddy viscosity profile (d).

Figure 5 compares mean velocity profiles measured in the experiments (●) with those generated using the synthetic eddy viscosity profile [41] (---) and the fitted eddy viscosity profile (--). In general, the synthetic and fitted profiles are in close agreement with the measurements for the smooth wall and x-permeable material, as shown in Figs. 5(a,b). However, Fig. 5(c) shows that the synthetic mean profile for the y-permeable case does not reproduce the asymmetry observed in the experiments. This is because the synthetic eddy viscosity profile from Reynolds and Tiederman [41] was developed for smooth wall flows. Therefore, it assumes that the turbulence is symmetric across the unobstructed channel. This is reasonable for the smooth wall case and the x-permeable material but not for the y-permeable materials (see $u_p^2$ and $u_{\tau}^2$ estimates in Table 2). The fitted eddy viscosity profile accounts for the asymmetry in turbulence across the unobstructed region over the y-permeable substrate. As a result, it is able to better reproduce the shape of the measured mean profile. Note that all the experimental measurements are under-predicted by the eddy viscosity models to some extent. This could be attributed to spanwise variation in the mean flow in the finite-width channel (recall that the channel has an aspect ratio of $W/h \approx 8$). The mean profile measured at the channel centerline may be larger than the true spanwise average.

Table 3 shows how the normalized gain for the NW resolvent mode changes with the mean profile for the x-permeable and y-permeable substrates. These
Figure 7: High-gain spanwise-constant resolvent modes identified from model predictions for the y-permeable substrate. (a) Structure corresponding to the highest-gain mode from the synthetic mean profile predictions in Fig. 6(c) with \((\lambda^+, c^+) \approx (2000, 9.5)\). (b) Structure corresponding to the highest-gain mode from the fitted mean profile predictions in Fig. 6(d) with \((\lambda^+, c^+) \approx (530, 9)\). The shading represents normalized levels for the streamwise velocity component. Solid and dashed black lines respectively represent positive and negative contours for the wall-normal velocity component. The porous interface is at the top wall \((y/h = 1)\).

|                     | synthetic | fitted |
|---------------------|-----------|--------|
| x-permeable, \(\sigma_{\kappa,p}/\sigma_{\kappa,s}\) | 0.71      | 0.80   |
| y-permeable, \(\sigma_{\kappa,p}/\sigma_{\kappa,s}\) | 2.55      | 1.46   |

Table 3: Comparison of the normalized gain for NW resolvent modes computed using the synthetic and fitted mean velocity profiles.

normalized singular values are computed for friction Reynolds numbers estimated from the experiments: \(Re_\tau = u'^*_t h/\nu \approx 124\) for the smooth wall case, \(Re_\tau \approx 129\) for the x-permeable substrate, and \(Re_\tau \approx 180\) for the y-permeable substrate. In other words, the singular values over the porous substrates \((\sigma_{\kappa,p})\) are normalized by the singular values for a smooth wall \((\sigma_{\kappa,s})\) at the \(Re_\tau\) estimated from the experiments. For the x-permeable material, the singular value ratio increases from \(\sigma_{\kappa,p}/\sigma_{\kappa,s} = 0.71\) for the synthetic mean profile to \(\sigma_{\kappa,p}/\sigma_{\kappa,s} = 0.80\). In other words, the predicted suppression for the NW resolvent mode decreases from 29\% with the synthetic profile to 20\% for the fitted profile. Despite this slight deterioration, the model still predicts suppression for the NW mode which we consider a necessary, but not sufficient, condition for friction reduction. For the y-permeable substrate, the singular value ratio decreases from \(\sigma_{\kappa,p}/\sigma_{\kappa,s} = 2.55\) for the synthetic profile to \(\sigma_{\kappa,p}/\sigma_{\kappa,s} = 1.46\) for the fitted profile. Thus, the fitted profile yields more limited NW mode amplification (46\%) compared to the synthetic profile (155\%). Interestingly, the measured increase in the average friction velocity for the y-permeable material relative to the smooth-wall case is approximately 45\% (see \(u'^*_t\) values in Table 2), which is close to the predicted increase in NW mode amplification. However, one must be cautious in using NW mode amplification as a direct measure of drag reduction performance. The x-permeable material yields a 4\% increase in \(u'^*_t\) relative to the smooth wall case, even though the resolvent model predicts suppression for the NW mode.

Together, the predictions shown in Table 3 indicate that: (i) the exact shape of the mean profile can have a significant effect on resolvent-based predictions, and (ii) the eddy viscosity model from Reynolds and Tiederman [41] may not
be the most appropriate choice for generating mean profile predictions over porous substrates. However, the overarching design guidelines do not change: only materials with high streamwise permeability and low wall-normal/spanwise permeabilities are likely to reduce drag.

Next, we evaluate how resolvent-based predictions for the spanwise-constant modes change with the mean profile. For the x-permeable case, there is no localized region of very high amplification for either the synthetic or fitted mean profiles. This is broadly similar to the results shown in Fig. 2(c). However, for the y-permeable case, these new predictions show significant changes relative to the model predictions in Fig. 2(d). With the synthetic profile, the localized region of high amplification shifts to larger wavelengths, from $\lambda^+_x \approx 1000$ in Fig. 2(d) to $\lambda^+_y > 1800$ in Fig. 6(c). This could be partially due to the increase in Reynolds number from the a priori estimate, $Re_\tau = 120$, used to generate the model predictions in Fig. 2(d) to the measured value for the y-permeable substrate, $Re_\tau = 180$, in Fig. 6(c). In other words, this shift in the peak to larger $\lambda^+_y$ for higher $Re_\tau$ may indicate that the energetic spanwise rollers scale in outer units.

Interestingly, when the fitted mean profile is used to generate predictions for the y-permeable substrate, the amplification map for the spanwise rollers changes substantially (see Fig. 6(d)). There is no region of high amplification for $\lambda^+_y > 1800$ and $c^+ \approx 9$ as observed with the synthetic profile (or perhaps this gets pushed beyond the field of view, $\lambda^+_y > 2000$). Instead a region of high gain is observed from $(\lambda^+_y, c^+) \approx (300, 11)$ to $(\lambda^+_y, c^+) \approx (1000, 6)$. It could be argued that the second POD mode for the y-permeable substrate shown in Fig. 4(f) belongs in this region. However, there is no clear evidence of a longer structure like resembling the first POD mode shown in Fig. 4(e).

The predicted structure for the modes with the highest normalized gain in Fig. 6(c) and Fig. 6(d) is shown in Fig. 7(a) and Fig. 7(b), respectively. The streamwise extent of these structures is broadly consistent with the first and second POD modes shown in Figs. 4(e,f). However, there are significant differences in flow structure between the predictions and the measurements. This discrepancy is the source of ongoing research.

5. Conclusion

Consistent with previous theoretical efforts and numerical simulations, the present experiments show a very different flow response over the x-permeable and y-permeable materials. The x-permeable material leads to a marginal increase in friction velocity at the porous interface (see Fig. 3(b) and Table 2). This is counter to resolvent-based predictions, which suggest that the x-permeable material should lead to a reduction in gain for the NW cycle (Fig. 2(a)). Possible explanations for this discrepancy include: the emergence of energetic spanwise rollers, as predicted by the resolvent formulation and previous numerical simulations for materials with $\sqrt{k_{yy}} > 0.4$; roughness effects at the porous interface that are neglected in the VANS equations; nonlinear (Forchheimer) effects becoming important in the porous medium; and perhaps
most importantly, insufficient scale separation between the pore-scale and outer flow. The y-permeable material leads to a significant increase in friction velocities at both walls relative to the smooth wall and x-permeable cases. This is consistent with resolvent-based predictions, which indicate a substantial increase in NW cycle gain as well as the emergence of large, high-gain spanwise rollers over the y-permeable material. POD confirms the presence of such spanwise rollers over the y-permeable material (Fig. 4). Resolvent-based predictions are able to predict the streamwise length scale of the POD modes. However, there are important differences in structure between the highest-gain resolvent modes and the computed POD modes.

The model predictions and experimental results shown here confirm that materials with high streamwise permeability and low spanwise and wall-normal permeability are good candidates for drag reduction. Ongoing work seeks to alleviate some of the weaknesses associated with the current experimental setup (limited development length, insufficient scale separation, low Reynolds number) and to identify streamwise-permeable porous materials that could be more effective.

6. Acknowledgements

This material is based on work supported by the Air Force Office of Scientific Research under awards FA9550-17-1-0142 (program manager Dr. Gregg Abate) and FA9550-19-1-7027 (program manager Dr. Douglas Smith).

References

[1] P. Luchini, F. Manzo, A. Pozzi, Journal of fluid mechanics 228 (1991) 87–109.

[2] J. Robert, Drag reduction: an industrial challenge, Technical Report, AIRBUS INDUSTRIE BLAGNAC (FRANCE), 1992.

[3] M. Walsh, A. Lindemann, in: 22nd Aerospace Sciences Meeting, p. 347.

[4] R. Garcia-Mayoral, J. Jimenez, Journal of Fluid Mechanics 678 (2011) 317–347.

[5] S. K. Robinson, Annual Review of Fluid Mechanics 23 (1991) 601–639.

[6] A. J. Smits, B. J. McKeon, I. Marusic, Annual Review of Fluid Mechanics 43 (2011) 353–375.

[7] H. Choi, P. Moin, J. Kim, Journal of fluid mechanics 255 (1993) 503–539.

[8] S.-J. Lee, S.-H. Lee, Experiments in fluids 30 (2001) 153–166.

[9] N. Abderrahaman-Elena, R. García-Mayoral, Phys. Rev. Fluids 2 (2017) 114609. doi:10.1103/PhysRevFluids.2.114609
[10] M. E. Rosti, L. Brandt, A. Pinelli, Journal of Fluid Mechanics 842 (2018) 381–394.
[11] G. Gómez-de-Segura, R. Garca-Mayoral, Journal of Fluid Mechanics 875 (2019) 124172. doi:10.1017/jfm.2019.482
[12] R. Garca-Mayoral, G. Gmez-de Segura, C. T. Fairhall, Fluid Dynamics Research (2019) 011410.
[13] A. Busse, N. D. Sandham, Physics of Fluids 24 (2012) 055111. doi:10.1063/1.4719780
[14] G. Gómez-de Segura, A. Sharma, R. García-Mayoral, Flow, Turbulence and Combustion 100 (2018) 995–1014.
[15] J. Jiménez, A. Pinelli, Journal of Fluid Mechanics 389 (1999) 335–359. doi:10.1017/S0022112099005066
[16] W. Breugem, B. Boersma, R. Uittenbogaard, Journal of Fluid Mechanics 562 (2006) 35–72.
[17] M. E. Rosti, L. Cortelezzì, M. Quadrio, Journal of Fluid Mechanics 784 (2015) 396–442.
[18] Y. Kuwata, K. Suga, International Journal of Heat and Fluid Flow 61 (2016) 145 – 157. doi:https://doi.org/10.1016/j.ijheatfluidflow.2016.03.006
[19] Y. Kuwata, K. Suga, Journal of Fluid Mechanics 831 (2017) 41–71.
[20] A. F. Zagni, K. V. Smith, Journal of the Hydraulics Division 102 (1976) 207–222.
[21] D. Pokrajac, C. Manes, Transport in porous media 78 (2009) 367.
[22] N. Horton, D. Pokrajac, Physics of fluids 21 (2009) 045104.
[23] T. Kim, G. Blois, J. Best, K. T. Christensen, in: River Flow 2016, CRC Press, 2016, pp. 950–955.
[24] C. Manes, D. Poggi, L. Ridolfi, Journal of Fluid Mechanics 687 (2011) 141–170.
[25] C. Efstathiou, M. Luhar, Journal of Fluid Mechanics 841 (2018) 351–379.
[26] T. Kim, G. Blois, J. L. Best, K. T. Christensen, Journal of Fluid Mechanics 887 (2020).
[27] F. Kong, J. Schetz, in: 20th Aerospace Sciences Meeting, p. 30.
[28] K. Suga, Y. Okazaki, U. Ho, Y. Kuwata, Journal of Fluid Mechanics 855 (2018) 983–1016.
[29] M. Itoh, S. Tamano, R. Iguchi, K. Yokota, N. Akino, R. Hino, S. Kubo, Physics of Fluids 18 (2006) 065102.

[30] B. J. McKeon, A. S. Sharma, Journal of Fluid Mechanics 658 (2010) 336–382. doi:10.1017/S002211201000176X.

[31] A. Chavarin, M. Luhar, AIAA Journal 58 (2020) 589–599.

[32] R. Moarref, A. S. Sharma, J. A. Tropp, B. J. McKeon, Journal of Fluid Mechanics 734 (2013) 275–316. doi:10.1017/jfm.2013.457.

[33] M. Luhar, A. S. Sharma, B. J. McKeon, Journal of Fluid Mechanics 749 (2014) 597–626.

[34] M. Luhar, A. S. Sharma, B. McKeon, Journal of Fluid Mechanics 768 (2015) 415–441.

[35] S. Nakashima, K. Fukagata, M. Luhar, Journal of Fluid Mechanics 828 (2017) 496–526.

[36] S. S. Toedtli, M. Luhar, B. J. McKeon, Physical Review Fluids 4 (2019) 073905.

[37] M. Luhar, A. Sharma, B. McKeon, Journal of fluid mechanics 751 (2014) 38–70.

[38] B. McKeon, Journal of Fluid Mechanics 817 (2017).

[39] J. Aurentz, L. Trefethen, SIAM Review 59 (2017) 423–446. doi:10.1137/16M1065975.

[40] J. A. Ochoa-Tapia, S. Whitaker, International Journal of Heat and Mass Transfer 38 (1995) 2635–2646. doi:10.1016/0017-9310(94)00346-W.

[41] W. C. Reynolds, W. G. Tiederman, Journal of Fluid Mechanics 27 (1967) 293–272. doi:10.1017/S002211206700308.

[42] G. A. Zampogna, A. Bottaro, Journal of Fluid Mechanics 792 (2016) 535. doi:10.1017/jfm.2016.66.

[43] W. Thielicke, E. J. Stamhuis, Journal of Open Research Software 2 (2014). URL: http://openresearchsoftware.metajnl.com/articles/10.5334/jors.bl doi:10.5334/jors.bl.

[44] M. Chandesris, A. d'Hueppe, B. Mathieu, D. Jamet, B. Goyeau, Physics of Fluids 25 (2013) 125110.