A Hypothesis on the Nature of Time

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Abstract

We present numerical evidence that fictitious diffusing particles in the causal dynamical triangulation (CDT) approach to quantum gravity exceed the speed of light on small distance scales. We argue this superluminal behaviour is responsible for the appearance of dimensional reduction in the spectral dimension. By axiomatically enforcing a scale invariant speed of light we show that time must dilate as a function of relative scale, just as it does as a function of relative velocity. By calculating the Hausdorff dimension of CDT diffusion paths we present a seemingly equivalent dual description in terms of a scale dependent Wick rotation of the metric. Such a modification to the nature of time may also have relevance for other approaches to quantum gravity.

1 Introduction

General relativity and quantum mechanics, in their respective domains of applicability, are extremely accurate theories. However, when strong gravitational fields interact over short distances, such as in the vicinity of the big bang singularity or near black holes, the description of such phenomena demand a unification of the two theories; a theory of quantum gravity.

The absence of experimental data at Planckian scales has produced an overabundance of observationally indistinguishable approaches to quantum gravity, yielding numerous often conflicting predictions about the nature of space, time and matter. Given the diverse number of different approaches to quantum gravity, perhaps it is prudent to look for common features shared by all such theories. One seemingly ubiquitous feature of quantum gravitational theories is the thermodynamic behaviour of black holes, which appears so consistently across such a diverse number of approaches it seems likely that it will feature in some form or other in the correct version of quantum gravity.

Another feature common to nearly all approaches to quantum gravity is dimensional reduction. A number of independent field-theoretic approaches to quantum gravity, using a variety of different techniques, have reported that the dimension of spacetime

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appears to be scale-dependent. Causal dynamical triangulations (CDT) \cite{1,2}, exact renormalisation group methods \cite{3}, Hořava-Lifshitz gravity \cite{4}, loop quantum gravity \cite{5} and string theory \cite{6,7} have all reported that the dimension of spacetime appears to reduce as one probes spacetime on ever decreasing distance scales, or conversely with ever increasing energies. Individually these results do not constitute substantial evidence in support of dimensional reduction; collectively, however, they form a compelling argument that demands further attention.\footnote{We refer the reader to Refs. \cite{8,9} for an excellent review of the accumulating evidence for the appearance of dimensional reduction, as well as an alternative proposal to explain the appearance of dimensional reduction on Planckian scales.}

\section*{1.1 Motivations for challenging the reality of dimensional reduction}

If the dimension of spacetime really does decrease as we probe spacetime on small distance scales then we must accept the seemingly unphysical consequences; that superluminal motion is possible \cite{10,11}, that relativistic symmetries are at the very least deformed \cite{12,13,14,15}, that it is possible to break Lorentz invariance \cite{16}, that gravitational waves cannot propagate \cite{6} and that Maxwell’s theory of electromagnetism completely breaks down \cite{17}. In light of these radical consequences, coupled with the absence of any solid theoretical explanation underpinning the mechanism of dimensional reduction, we are motivated to challenge the physical reality of this phenomenon and question what the appearance of dimensional reduction is really telling us. Although the central focus of this work is CDT quantum gravity, due to the almost universal observation of dimensional reduction in different approaches to quantum gravity it is possible that the arguments used and the conclusions reached in this work may be more widely applicable.

The spectral dimension $D_S$ is a measure of the effective dimension of a manifold over varying length scales (see e.g. \cite{18} for a thorough discussion), and is related to the probability $P_r$ that a random walk will return to the origin after $\sigma$ diffusion steps. The spectral dimension is defined by

$$D_S = -2 \frac{\text{d} \log P_r}{\text{d} \log \sigma}. \quad (1)$$

The majority of functions describing dimensional reduction using the spectral dimension can be derived from dispersion relations of the type \cite{11}\footnote{It should be noted that a number of exceptions exist, see e.g. \cite{19}, most notably within the ordinary asymptotic safety scenario.}

$$E^2 = p^2 \left(1 + (\lambda p)^{2\gamma}\right). \quad (2)$$

From this modified dispersion relation we obtain a modified speed of light $c_m$ given by

$$c_m = \frac{E}{p} = \sqrt{1 + (\lambda p)^{2\gamma}}. \quad (3)$$

Nearly all approaches that report dimensional reduction find a large distance value for the spectral dimension of four, and a small distance value of two. This small distance dimensionality translates to a $\gamma$ value of 2 \cite{11}, thus giving a speed of light $c_m$ that is
dependent on the momentum scale $\lambda p$. In fact, as is evident from Eq. (3), any non-zero value of $\gamma$ results in a speed of light that is dependent on the momentum scale $\lambda p$.

In a spacetime with $(d_H + t_H)$ Hausdorff dimensions one finds the general form of the spectral dimension [11, 4]

$$D_S = t_H + \frac{d_H}{1+\gamma}. \quad (4)$$

In Ref. [10] an expression for the ratio of the phase and group velocity $v_{\text{phase}}/v_{\text{group}}$ in terms of the spectral dimension is derived in $(d + 1)$ topological dimensions independently of any particular approach to quantum gravity [7]

$$D_S = 1 + d \frac{v_{\text{phase}}}{v_{\text{group}}} + ... \quad (5)$$

For electromagnetic waves in a vacuum one should obtain $v_{\text{group}}/v_{\text{phase}} = 1$, which we take to define a dimensionless speed of light parameter $c_m$. However, for any degree of dimensional reduction $D_S < 4$, Eq. (5) clearly indicates that the dimensionless value for the speed of light $c_m = v_{\text{group}}/v_{\text{phase}}$ must exceed unity [4].

From these two examples we see that dimensional reduction strongly suggests a speed of light that is scale dependent. Additionally, any theory with a running dimensionality must at the very least deform the relativistic symmetries [12, 13, 14, 15], and may not preserve Lorentz invariance [16], although in certain cases it may still preserve the relativity principle [12].

The more radical stance would be to interpret dimensional reduction as support for variable speed of light theories (VSL) [20, 21], or theories that explicitly break Lorentz invariance such as Hořava–Lifshitz gravity [22]. However, the much more conservative stance is to explicitly preserve the constancy of the speed of light from the outset, and compute the consequences.

2 Using CDT to derive how time dilates as a function of distance

The canonical point in the physical phase of CDT, which has an established macroscopic 4-dimensional de Sitter geometry [23], has been shown to have a scale dependent spectral dimension given by the functional form

$$D_S = a - \frac{b}{c + \sigma}. \quad (6)$$

Where $a$, $b$ and $c$ are free fit parameters, and $\sigma$ is the diffusion time. CDT simulations yield a fit to the data with $a = 4.02$, $b = 119$ and $c = 54$ [1]. A more recent study at the same point in the CDT parameter space also gives similar results, namely $a = 4.06$, $b = 135$ and $c = 67$ [2]. This particular functional form of the spectral dimension was also arrived at using purely analytic considerations in Ref. [24, 25]. Integration of Eq. [4] is based on a saddlepoint approximation (see Ref. [10] for details).

Obviously the phase velocity of light can in certain circumstances exceed $c$, for example when travelling through certain dispersive media, but the signal velocity is strictly forbidden from doing so. In a vacuum, electromagnetic waves must obey the relation $v_{\text{group}}/v_{\text{phase}} = 1$, a relation that is violated by dimensional reduction.
\( (6) \) gives an expression for the probability \( P_r \) that the diffusion process will return to the origin after \( \sigma \) steps

\[
P_r = \frac{1}{\sigma^{a/2} (1 + \frac{c}{\sigma})^{\frac{b}{c}}}.
\]

As found in Refs. \([1, 2]\) \( a \approx 4 \) and the ratio \( b/2c \approx 1 \), hence to a good approximation one finds\(^5\)

\[
P(\sigma) \approx \frac{1}{\sigma^2 + c\sigma}.
\]

The probability of return for infinitely flat 4-dimensional Euclidean space with no dimensional reduction is given by \( P(\sigma) = \sigma^{-2} \). The path length traced out by a diffusing particle is just proportional to the number of diffusion steps \( \sigma \). We ask what function \( \Gamma \) rescales this path length \( \sigma \) such that we obtain the probability of return found in CDT, namely that of Eq. \((8)\). Hence we form the equation

\[
\frac{1}{\Gamma^2(\sigma)\sigma^2} = \frac{1}{\sigma^2 + c\sigma},
\]

which gives a \( \Gamma \) function of

\[
\Gamma(\sigma) = \sqrt{1 + \frac{c}{\sigma}}.
\]

Thus, by rescaling \( \sigma \) in the expression for infinitely flat 4-dimensional Euclidean space by the \( \Gamma \) function of Eq. \((10)\) we obtain the probability of return found in CDT, and hence upon taking the logarithmic derivative we obtain the functional form of dimensional reduction reported in CDT \([1, 2]\). Since \( \sigma \) is proportional to the square of the distance scale \( \Delta x \) with which one probes the manifold, we can write this \( \Gamma \) function as

\[
\Gamma(\Delta x) = \sqrt{1 + \frac{c}{\Delta x^2}}.
\]

By substituting the fit to the functional form \( D_S = a - b/(c + \sigma) \) found using the CDT approach to quantum gravity into Eq. \((5)\) we obtain a modified speed of light \( c_m \), as implied by dimensional reduction in CDT,

\[
c_m = \frac{v_{\text{group}}}{v_{\text{phase}}} = \frac{d}{a - \frac{b}{c + \sigma} - 1}.
\]

Figure \((1a)\) shows this modified speed of light \( c_m \) as a function of \( \sigma \), with the fit parameters \( a = 4.06, b = 135 \) and \( c = 67 \) determined from CDT calculations \([2]\)\(^6\). Figure \((1b)\) shows the ratio \( c_m/\Gamma \), where \( \Gamma \) is defined by Eq. \((10)\).

\(^5\) This expression is also equivalent to the continuum return probability guessed at in Ref. \([1]\) upon equating \( c = A l_p^2 \), where \( l_p \) is the Planck length and \( A \) is a numerical constant.

\(^6\) Ref. \([26]\) provides an independent derivation of superluminality in CDT that is almost identical to the one presented in this work.
Figure 1: (1a) The scale dependent speed of light as predicted by dimensional reduction. (1b) The ratio $c_m/\Gamma$ as a function of $\sigma$.

As can be seen in Fig. (1a) the modified speed of light $c_m$ increases above unity as one probes the manifold on ever decreasing distance scales. This kind of variable speed of light is a generic prediction of theories containing dimensional reduction. We attribute this to the distance the diffusing particle travels dilating as we probe the manifold on smaller scales, as is common to all fractal curves. To get an intuitive understanding for why we expect to see an apparently decreasing dimension, and hence a variable speed of light, if length dilates on smaller distance scales consider the following example. Imagine a diffusion process on a one-dimensional line, at each point on the line the diffusing particle has just two options; it can either go left or right (assuming the particle must move). For a two-dimensional diffusion process on a grid, at each point the particle has four options. In three-dimensions the particle will have six options, etc. Since having more options decreases the probability of returning to the origin, it is clear that dimension is inversely proportional to the probability of return. Hence, in order to explain why the dimension appears to reduce on small scales, and by extension why the speed of light increases, one only has to explain a relative increase in the probability of return on smaller scales compared with larger ones. Now, if the path length of the diffusing particle increases on smaller distances the probability that the diffusing particle will return to the origin will be comparatively increased, because for a given number of diffusion steps the density of points sampled will be greater. Therefore the dimension will appear to decrease on smaller distance scales.

In fact, one can actually map the path a fictitious diffusing particle traces out in a given ensemble of triangulations defined by the CDT approach to quantum gravity. In CDT one approximates a continuous spacetime manifold by connecting adjacent 4-dimensional simplices via their mutual tetrahedra, forming a discretised simplicial geometry. The resulting ensemble of triangulations can be used to study different geometric properties of the simplicial manifold, one of which is the spectral dimension, which can be calculated by studying how a test particle diffuses within the geometry. The test particle starts in a randomly chosen simplex and diffuses throughout the ensemble by jumping to adjacent simplices, making $\sigma$ diffusion steps in total. Individual diffusion processes on a triangulation are typically averaged over in order to determine the spectral dimension of the simplicial manifold. However, such diffusion processes might also be used to give information about how the length of such trajectories vary with distance scale, and hence allow one to define an effective velocity of the diffusing test particle. For combinatorially unique triangulations, each simplex the test particle
visits during its random walk has a unique label, allowing one to track the diffusion process along each step of its trajectory.

Figure 2: A schematic representation of a trajectory that a fictitious particle diffusing between points A and B in \((1 + 1)\)-dimensional CDT might take.

A defining feature of CDT is that it distinguishes between space-like and time-like links on the lattice so that an explicit foliation of the lattice into space-like hypersurfaces of fixed topology can be introduced. The space-like hypersurfaces are separated by time intervals \(t_N\), as shown schematically in Fig. 2, thereby explicitly introducing a time coordinate. In CDT the time-like and space-like simplicial edge lengths, \(a_t\) and \(a_s\), respectively, do not have to be equal. The path length a diffusing particle traces out within the ensemble of triangulations is just equal to the number of diffusion steps taken, \(\sigma\), multiplied by the average distance between adjacent simplices, which we encode by the constant of proportionality \(C\), which is a function of \(a_t\) and \(a_s\). A CDT triangulation defines a time coordinate that exists within the triangulation at times \(t = 0, t = 1, \ldots, t = N\), as shown schematically in Fig. 2 defining a causal slice of spacetime of duration \(t = N\). The elapsed time observed by a particle diffusing between points A and B is then given by the number of times the test particle crosses a space-like hypersurface, thus incrementing its time coordinate \(t\). The number of times the particle’s trajectory intersects a space-like hypersurface we denote by \(t_d\). Hence, we can now define an effective velocity \(v_d\) for the massless diffusing test particle within the triangulation via

\[
v_d = \frac{C\sigma}{t_d}.
\]
Figure 3 shows $v_d$ averaged over 1000 different diffusion processes for the canonical point in the de Sitter phase of CDT, and for a constant of proportionality $C = 0.18$, chosen such that $v_d = 1$ in the large distance limit, as one would expect of a massless diffusing particle. The important feature of Fig. 3 is that the measured velocity of the diffusing particle $v_d$ in a typical ensemble of triangulations in CDT closely matches the scale dependent speed of light of Eq. (12). Figure 3 therefore provides numerical evidence for superluminality on small distance scales within the CDT approach to quantum gravity. Based on this numerical evidence it is tempting to conclude that Lorentz invariance in CDT is broken on small distance scales, effectively yielding a scenario along the lines of Hořava-Lifshitz gravity in which space and time scale differently. Indeed, many similarities between CDT and Hořava-Lifshitz gravity have been reported in the literature [27, 28, 29]. If this correspondence between the two theories holds, it suggests the effective action of CDT may contain higher spatial derivatives, yielding a description of dimensional reduction in CDT similar to Eq. (4), which was originally derived within the context of Hořava-Lifshitz gravity.

In Ref. [30] Abbott and Wise examine the quantum mechanical trajectory of a particle by measuring its position with an increasingly refined resolution $\Delta x$, finding that the length of the quantum mechanical path increases as one increases the resolution with which we probe spacetime. Note this property is also a general feature of fractal curves [30]. We argue the same thing is happening for the diffusing particle that defines the spectral dimension. As we resolve spacetime on ever decreasing scales the path length of the diffusing particle increases in length according to $\Gamma$, hence we obtain a modified speed of light $c_m$, as shown in Fig. 1a and supported by the numerical evidence presented in Fig. 3 given by

$$c_m = \frac{\Gamma \Delta l}{\Delta t}. \quad (14)$$

The fictitious particles diffusing within the CDT ensemble of triangulations are
massless and so should propagate at the speed of light. Now, if the massless diffusing particle is a photon bouncing between two parallel mirrors, the photon’s path length will appear to get longer as an observer probes the trajectory with ever finer measurements. If we set up a light-clock such that each time the photon traverses the distance between the two mirrors we define the tick of a clock, and if we are to assume the constancy of the speed of light then the observer must see the light-clock ticking slower than it would if probed on larger distance scales. We argue the same thing is true of diffusion processes that define the spectral dimension. Since our postulate assumes the speed of light is scale invariant, and since the path length increases by a factor Γ as a function of σ, time must also dilate on smaller distances by the same Γ factor, such that we obtain a scale invariant speed of light c, as shown in Fig. (1b), hence

\[ c_m \rightarrow c = \frac{\Gamma \Delta l}{\Gamma \Delta t}. \]  

If we define Δt as the time it takes a photon to traverse the distance between the mirrors along the shortest possible path, and Δt' as the time it takes the photon to traverse the distance between the two mirrors when an observer probes the trajectory with a resolution of Δx, we obtain a relation reminiscent of time dilation in special relativity, namely

\[ \Delta t' = \Gamma \Delta t. \]  

We conclude that dimensional reduction as reported in CDT, and possibly other approaches to quantum gravity exhibiting dimensional reduction, is in fact telling us that in order to retain a scale invariant speed of light time must dilate as a function of distance scale.

3 A dual description?

Following Hausdorff, we can introduce a new definition of length \( \langle L \rangle \) that is independent of the measurement resolution Δx via a rescaling by the number of spatial Hausdorff dimensions \( d_H \) [30].

\[ \langle L \rangle = \langle l \rangle (\Delta x)^{d_H - 1}. \]  

By defining the diffusion paths in terms of the Hausdorff length \( \langle L \rangle \) they are by construction scale invariant, with the scale dependence now encoded in the Hausdorff dimension. The ratio of the invariant Hausdorff length \( \langle L \rangle \) and the variable length \( \langle l \rangle \) is by definition \( 1/\Gamma \), where \( \Gamma \) is defined by Eq. (11), so that

\[ \frac{\langle L \rangle}{\langle l \rangle} = \frac{1}{\Gamma} = (\Delta x)^{d_H - 1}. \]  

Which gives,

\[ d_H = \frac{\ln (1/\Gamma)}{\ln (\Delta x)} + 1. \]  

Equation (19) describes how the spatial Hausdorff dimension \( d_H \) of particles diffusing in a typical CDT geometry changes as a function of the distance scale with which they
are probed $\Delta x$. Plotting Eq. (19) we see that in the infra-red limit we have $d_H = 1$, whereas in the ultraviolet limit we have $d_H \to 0$, as shown in Fig. 4.

We now return to Eq. (3), namely

$$c_m = \frac{E}{p} = \sqrt{1 + (\lambda p)^{2\gamma}},$$

we see that in order to get a speed of light $c_m$ that is independent of the scale $\lambda p$ we must have $\gamma = 0$. For simplicity we can normalise Eq. (3) so that $c_m = 1$ when $\gamma = 0$, giving

$$c_m = \frac{E}{p} = \sqrt{\frac{1}{2} + (\lambda p)^{2\gamma}}.$$  

We now plug $\gamma = 0$ into Eq. (4), and upon rearranging we obtain the condition

$$d_H = D_S - t_H.$$

Combining Eqs. (19) and (22), and fixing $D_S = 2$, tells us that the number of temporal Hausdorff dimensions $t_H$ transforms as a function of scale according to

$$t_H = 1 - \frac{\ln (1/\Gamma)}{\ln (\Delta x)}.$$  

Equation (23) is plotted in Fig. 5 showing that the number of temporal Hausdorff dimensions increases from $t_H = 1$ on macroscopic scales to at least $t_H = 2$ on microscopic scales. Taking into account the processes occurring in both Figs 4 and 5 the CDT diffusion paths suggest that a spatial dimension is transforming into a temporal dimension as we decrease the distance scale, which can be interpreted as an inverse Wick rotation. The recent work of Ref. [31] reports a possible scale dependent signature change of the metric in CDT, which if confirmed would provide numerical evidence for this claim. A Wick rotation has also been reported in loop quantum gravity [32]. A dual description of dimensional reduction of the spectral dimension in CDT then appears to exist: the initially linear diffusion paths on macroscopic scales dissolve into a series of
zero-dimensional points on microscopic scales, while $t_H$ simultaneously increases from $t_H = 1$ to at least $t_H = 2$. Such an explanation is similar to results reported in [33] and is somewhat reminiscent of asymptotic silence.

![Figure 5: The temporal Hausdorff dimension of CDT diffusion paths as a function of distance scale $\sigma$ using the $\Gamma$ function of Eq. (10).](image)

Clearly, if we are to restore $d_H = 1$ and $t_H = 1$ on all distance scales then we must reverse this process, transforming a dimension of time back into a dimension of space as a function of distance scale, i.e. perform a scale dependent Wick rotation $t \rightarrow -i\tau$ in order to once again recover a scale invariant speed of light.

4 Discussion and conclusions

Combining key elements of general relativity and quantum mechanics predicts a particular property of spacetime known as quantum foam. The energy-time uncertainty relation coupled with mass-energy equivalence implies that as one probes spacetime on Planckian scales one encounters an extremely turbulent geometry, with the possibility of creating infinite energy fluctuations as the distance scale is taken to zero. Such fluctuations of spacetime on small distance scales can lead to large scale observable effects, such as the energy dependence of the speed of light in a vacuum. Likewise, many attempts to combine general relativity and quantum mechanics in a more rigorous mathematical framework predict a similar dispersion of light in a vacuum. However, experiments performed by the Fermi space telescope and others have already measured the difference in arrival times for photons with different energies emitted from the same cosmic source [34]. The idea being that shorter wavelength photons resolve the turbulent geometry to a greater extent than longer wavelength photons, thus comparatively delaying their time of flight. The Fermi GBM/LAT collaboration using the Fermi Gamma-ray space telescope have put such severe constraints on Lorentz violating effects, even at energies exceeding the Planck scale for dispersion relations linear in the energy scale, as to effectively rule out such phenomena [35].

How then do we reconcile the experimental observations that tells us Lorentz invariance holds even beyond the Planck scale, with approaches to quantum gravity that either include Lorentz invariance violations as foundational principles or theories in which such effects emerge as a prediction? Clearly, it does not make sense to attempt
to modify the first class of theories by making them conform with Lorentz invariance since they’re axiomatically Lorentz violating, however we argue that wherever possible the second class of theories should be made to conform with Lorentz invariance. We propose that CDT quantum gravity, and possibly other approaches to quantum gravity, might be made to conform with Lorentz invariance by the inclusion of a scale dependent time coordinate. The inclusion of such a modification to the nature of time may have important implications for the perturbative renormalizability of gravity, as the zero distance limit may no longer correspond to the infinite energy limit.

This work provides numerical evidence for superluminality on small distance scales within CDT quantum gravity. We argue that it is this scale dependent speed of light that is responsible for the appearance of dimensional reduction. By using the Hausdorff definition of invariant length we determine the spatial and temporal Hausdorff dimensions of diffusing particles within CDT geometries, finding an effective scale dependent signature change of the metric on small distance scales, thus providing a possible dual description of dimensional reduction in models of quantum gravity. We propose that if we are to maintain a scale invariant speed of light in approaches to quantum gravity that predict superluminality then we must admit a modification to the nature of time; time must dilate as a function of relative scale, just as it does as a function of relative velocity.

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