A supersymmetric mean-field approach to the infinite-$U$ Hubbard model

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Abstract

A generalized supersymmetric representation of the Hubbard operator algebra is considered. This representation is applied to the infinite-$U$ Hubbard model. A mean-field theory which takes into account both on-site and inter-site virtual boson-fermion transitions is developed. Unlike previous approaches, the mean-field theory considered is free from divergences. A possible application of these results to the ferromagnet-paramagnet transition, as well as to other problems is discussed.

I. INTRODUCTION

The description of magnetism within the Hubbard model is the long-standing problem. At $U = \infty$ the ferromagnetic (but not antiferromagnetic) state can occur, and, as shown by Nagaoka, for small hole concentrations $\delta$ this is the ground state. The stability of saturated ferromagnetism for not too small $\delta$ was investigated within different approaches (see, e.g. Refs. [1–4]). The critical concentration of holes was estimated as $\delta_c \simeq 1/3$ for the transition into non-saturated ferromagnetic state, and as $\delta'_c \simeq 2/3$ for that into non-magnetic one. The same result for $\delta_c$ can be obtained by comparing the total energies in the slave-fermion approach (which describes correctly the excitations near half-filling) and slave-boson approach (which is suitable for the non-magnetic phase), see Ref. [5] and references therein.

Slave-fermion and slave-boson approaches have the advantage that they correspond to the
$N \to \infty$ limit of the generalized $U = \infty$ Hubbard model with $N$ fermion flavors (which will be referred to as $SU(N|1)$ model), and $1/N$ corrections can be found in a regular way. These two approaches treat different kind of excitations, and therefore have different applicability regions. While slave-fermion approach is convenient at small hole concentrations and enables one to describe naturally the magnetic state, the slave-boson approach works well at high $\delta$.

To obtain the physical picture at intermediate hole concentrations, a supersymmetric approach should be developed which interpolates between slave-fermion and slave-boson ones. The situation here is similar to that in $s-f$ model where boson representation of impurity spin operators describes correctly magnetic phases, while fermion representation is suitable for description of Kondo (i.e. nonmagnetic) state. To describe continuous transition between these phases, the supersymmetric approach of Ref. [8] can be used.

In this paper we consider the application of the supersymmetric approach to the $SU(N|1)$ model and obtain the effective action of the infinite-$U$ Hubbard model in supersymmetric representation. Then we consider the mean-field approach to this action and obtain the corresponding self-consistent equations.

**II. REPRESENTATIONS OF THE HUBBARD ALGEBRA**

The standard way of treating large-$U$ Hubbard model is introducing the Hubbard operators $X^{\alpha\beta}$ ($\alpha, \beta = 0, \pm$). These satisfy at a lattice site the commutation relations

$$[X^{\alpha\beta}, X^{\gamma\delta}]_{\pm} = \delta_{\beta\gamma} X^{\alpha\delta} \pm \delta_{\alpha\delta} X^{\gamma\beta}$$

and the constraint

$$\sum_{\alpha} X^{\alpha\alpha} = 1$$

The operators $X^{\alpha\beta}$ give the spin $S = 1/2$ realization of the $SU(2|1)$ superalgebra. To construct $1/N$ expansion we have to generalize this algebra to $SU(N|1)$ by introducing the operators $\chi^{\alpha\beta}$ ($\alpha, \beta = 0...N$) which satisfy the same commutation relations (1). We also use the generalized form of the constraint
\[ \sum_{\alpha} \chi^{\alpha\alpha} = Q_0 \] (3)

which reduces to standard one, \(Q_0\), for the physical case \(Q_0 = 1\). The operators \(\chi^{\alpha\alpha}\) yield a representation of the \(SU(N|1)\) superalgebra with the “superspin” \(Q_0/N\).

The slave-fermion representation of the operators \(\chi^{\alpha\beta}\) through \(N\) bosons and one fermion has the form

\[ \chi^{\alpha\sigma'} = b^\dagger_\sigma b_{\sigma'}, \ \chi^{00} = f^\dagger f \] (4)

\((\sigma, \sigma' = 1...N)\), with \(b_{\sigma}\) and \(f\) being Bose and Fermi operators respectively. For \(Q_0 < N\) there exists also the slave-boson representation

\[ \chi^{\alpha\sigma'} = c^\dagger_\sigma c_{\sigma'}, \ \chi^{00} = a^\dagger a \] (5)

where \(c_{\sigma}\) and \(a\) are Fermi and Bose operators.

To obtain the supersymmetric representation which interpolates between slave-fermion and slave-boson ones, we introduce, following to Refs. [8–10], the operators

\[ \Psi_\sigma = \begin{pmatrix} c_{\sigma} \\ b_{\sigma} \end{pmatrix}, \ \Psi_0 = \begin{pmatrix} a \\ f \end{pmatrix} \] (6)

\((\sigma = 1...N)\) with

\[ \chi^{\alpha\beta} = \Psi_\alpha^\dagger \Psi_\beta, \]

\[ Q_0 = \Psi_\alpha^\dagger \Psi_\alpha \] (7)

To make a distinction between the representations with different symmetry, we consider the second-order Casimir operator of \(SU(N|1)\)

\[ C_2 = \chi^{\sigma\sigma'} \chi^{\sigma'\sigma} - \chi^{00} \chi^{0\sigma} + \chi^{0\sigma} \chi^{\sigma0} - (\chi^{00})^2 \] (8)

Expressing this in terms of \(\Psi\) we obtain

\[ C_2 = \Psi_\alpha^\dagger \Psi_\alpha [N - 1 - \Psi_\beta^\dagger \tau_3 \Psi_\beta] - [\Theta, \Theta^\dagger] \] (9)
where $\tau_3$ is the Pauli matrix,

$$\Theta = b_i^\dagger c_\sigma - f^i a$$

is the mixing fermion-boson operator with $\{\Theta, \Theta^\dagger\} = Q_0$, and the summation over repeated indices is assumed. Taking into account the constraint (8) we have finally

$$C_2 = Q_0(N - 1 - Y)$$

where

$$Y = \Psi_\beta^\dagger \tau_3 \Psi_\beta + \frac{1}{Q_0} [\Theta, \Theta^\dagger]$$

By fixing $Y = -Q_0 + 1 ... Q_0 - 1$, we obtain representations with different symmetry.

**III. SU($N\mid 1$) GENERALIZATION OF THE $U = \infty$ HUBBARD MODEL**

The Hamiltonian of the $SU(N\mid 1)$ model can be now rewritten as

$$\mathcal{H} = \sum_{ij} t_{ij} \chi_i^\sigma \chi_j^{0\sigma} = \sum_{ij} t_{ij} \Psi_i^\dagger \sigma \Psi_{i0} \Psi_j^\dagger \sigma \Psi_{j0}$$

For $N = 2$, $Q_0 = 1$ this coincides with that of the $U = \infty$ Hubbard model. The partition function reads

$$Z = \int D\Psi \exp \left\{-\int_0^\beta d\tau (L_0 + \mathcal{H})\right\}$$

where

$$L_0 = \sum_i \Psi_i^\dagger_{i\alpha} \left( \frac{\partial}{\partial \tau} + \lambda_i + \mu \delta_{i0} + \zeta_i \tau_3 \right) \Psi_i^\dagger_{\alpha} - \frac{2C}{Q_0} \sum_i \Theta_i^\dagger \Theta_i - \sum_i (\zeta_i Y + \lambda_i Q_0 + \mu \delta)$$

is the free Lagrangian, $\delta$ is the concentration of holes. Following to Ref. [8], we perform the replacement $\Psi \rightarrow g \Psi$ with

$$g = \begin{pmatrix} 1 + \eta \eta/2 & \eta \\ -\eta & 1 - \eta \eta/2 \end{pmatrix}$$
to obtain the gauge-invariant Lagrangian in the form

\[ \mathcal{L}'_0 = \sum_i \Psi^\dagger_{i\alpha} (\partial_\tau + \lambda_i + \mu \delta_{\alpha 0} + \zeta_i \tau_3) \Psi_{i\alpha} - \frac{1}{Q_0} \sum_i \Theta^\dagger_i (\partial_\tau + 2\zeta_i) \Theta_i - \sum_i (\zeta_i Y + \lambda_i Q_0 + \mu \delta) \]

Performing the Hubbard-Stratonovich transformation, we obtain

\[ \mathcal{L}'_0 = \sum_i \Psi^\dagger_{i\alpha} \left[ \partial_\tau + \lambda_i + \mu \delta_{\alpha 0} + \begin{pmatrix} \zeta_i & D_{0\alpha i} \\ D_{0\alpha i}^\dagger & -\zeta_i \end{pmatrix} \right] \Psi_{i\alpha} + Q_0 \sum_i \alpha^\dagger_i D_{0\alpha i} - \sum_i (\zeta_i Y + \lambda_i Q_0 + \mu \delta) \]

with \( D_0 = \partial_\tau + 2\zeta_i \). The Hamiltonian (13) is decoupled in the same way as in Ref. [11], and we derive

\[ H = \sum_{ij} t_{ij} \left[ A_{ij} c_{i\sigma}^\dagger c_{j\sigma} + c_{i\sigma} a_{i\sigma}^\dagger c_{j\sigma} + f_{ij} b_{i\sigma}^\dagger b_{j\sigma} + B_{ij} f_{ji}^\dagger f_i \\
+ \overline{F}_{ij} a_{i\sigma}^\dagger f_j + f_{i\sigma} a_{j\sigma}^\dagger P_{ji} + c_{i\sigma} b_{j\sigma} Q_{ji} + \overline{Q}_{ij} b_{i\sigma}^\dagger c_{j\sigma} \\
- A_{ij} C_{ji} + B_{ij} F_{ji} - \overline{P}_{ij} Q_{ji} - \overline{Q}_{ij} P_{ji} \right] \\
= \sum_{ij} t_{ij} \left[ \Psi^\dagger_{i\sigma} \nu_{ij} \Psi_{j\sigma} + \Psi^\dagger_{i0} Z_{ij} \Psi_{j0} - \text{STr}(\nu_{ij} Z_{ji}) \right] \]

where \( \text{STr}(...) \) is the supertrace; \( P, \overline{P}, Q, \overline{Q} \) are independent Grassmann variables,

\[ C_{ij} = -A_{ij}^\dagger, \quad B_{ij} = F_{ij}^\dagger \]

and

\[ \nu_{ij} = \begin{pmatrix} A_{ij} & Q_{ji} \\ \overline{Q}_{ij} & F_{ij} \end{pmatrix}, \quad Z_{ij} = \begin{pmatrix} C_{ij} & -P_{ji} \\ -\overline{P}_{ij} & B_{ij} \end{pmatrix} \]

The model (14) with (18) and (19) is invariant under the gauge transformation

\[ \Psi_i \rightarrow g_i \Psi_i \]
\[ \alpha \rightarrow \alpha + (\partial_\tau + 2\zeta_i) \eta_i \]
\[ \nu_{ij} \rightarrow g_i \nu_{ij} g_j^{-1}, \quad Z_{ij} \rightarrow g_i Z_{ij} g_j^{-1} \]
The action $S = \mathcal{L}_0' + \mathcal{H}$ can be rewritten in the form

$$
S = \sum_i \Psi_i^\dagger \left[ (\partial_\tau + \lambda_i + \zeta_i \tau_3) \delta_{ij} + \left( \begin{array}{c} A_{ij} t \\ Q_{ij} t \end{array} \right) \right] \Psi_j^\sigma 
+ \sum_i \Psi_i^\dagger \left[ (\partial_\tau + \lambda_i + \mu + \zeta_i \tau_3) \delta_{ij} + \left( \begin{array}{c} C_{ij} t \\ P_{ij} t \end{array} \right) \right] \Psi_j^0 
+ Q_0 \sum_i \alpha_i^\dagger D_0 \alpha_i - \text{STr}(\mathcal{V}_{ij} \mathcal{Z}_{ji}) - \sum_i (\zeta_i Y + \lambda_i Q_0 + \mu \delta)
$$

where

$$
Q_{ji} = D_0 \alpha_i \delta_{ij} + t \mathcal{Q}_{ji}, \quad \overline{Q}_{ij} = D_0 \alpha_i \delta_{ij} + t \mathcal{Q}_{ji} 
\overline{P}_{ij} = D_0 \alpha_i \delta_{ij} - t \mathcal{P}_{ij}
$$

IV. MEAN-FIELD THEORY

Our purpose now is to consider the mean-field approximation for the action (23), which does not violate the gauge invariance (22). To this end, we will take into account the fluctuations of $Q, P$ and $\alpha$, while other fields will be considered in the mean-field approximation. The motivation for this approximation is as follows.

(i) The fields $Q, P$ and $\alpha$ are equally important: while the field $\alpha$ describes the on-site virtual transitions of bosons into fermions and vice versa, the fields $Q$ and $P$ describe the same transitions with simultaneous intersite hopping. As can be seen from (23), $Q$ and $P$ play the role of “spatial components” of the gauge field, while $\alpha$ is only its time component.

(ii) All the fields, besides $Q, P$ and $\alpha$, being taken into account at the mean-field level, are shifted properly by the gauge transformation. At the same time, the fields $Q, P$ and $\alpha$ have zero mean-field value due to their fermionic nature and therefore can not be transformed at the mean-field level.

Thus, $Q, P$ and $\alpha$ make the minimal set of fields, fluctuations of which should be taken into account to keep gauge invariance. The gauge transformation of resulting theory is
considered in Appendix. Note that taking into account only fluctuations of the field $\alpha$ leads to divergences due to violation of gauge invariance \[10\].

Unfortunately, unlike treating of single-impurity problem of Ref. \[8\], the fields $Q$ and $P$ can not be completely removed by gauge transformation. Thus it will be more convenient for us to work in the gauge where $\alpha_i = 0$.

Integrating over $\Psi_{i\alpha}$, expanding the action to second order in $P_{ij}, Q_{ij}$ at $\alpha_i = 0$ we obtain the effective action in the form

$$S_{\text{eff}} = -\text{NSTr} \ln[G^{-1}_b(q, i\omega_n)G^{-1}_c(q, i\omega_n)] - \text{STr} \ln[G^{-1}_a(q, i\omega_n)G^{-1}_f(q, i\omega_n)]$$

$$+ \sum_{q, i\omega_n} \sum_{\delta, \delta'} \mathcal{R}_\delta(q, i\omega_n) \left( \begin{array}{cc} \Pi^{\delta \delta'}_{cb}(q, i\omega_n) & \delta_{\delta \delta'} \\ \delta_{\delta' \delta} & -\Pi^{\delta \delta'}_{af}(q, i\omega_n) \end{array} \right) \mathcal{R}_{\delta'}(q, i\omega_n)$$

$$- \sum_i (\zeta_i Y + \lambda_i Q_0 + \mu \delta)$$

(25)

where $\mathcal{R}_\delta = (Q_\delta, \mathcal{P}_\delta)$, and the polarization operators are given by

$$\Pi^{\delta \delta'}_{cb}(q, i\omega_n) = N \sum_{k, i\nu_n} G_b(k, i\nu_n)G_c(k + q, i\nu_n + i\omega_n) e^{ik(\delta - \delta')}$$

$$= N \sum_k \frac{n^b_{k} + n^c_{k+q}}{i\omega_n - \varepsilon^c_{k+q} + \varepsilon^b_k} e^{ik(\delta - \delta')}$$

$$\Pi^{\delta \delta'}_{af}(q, i\omega_n) = \sum_{k, i\nu_n} G_f(k, i\nu_n)G_a(k + q, i\nu_n + i\omega_n) e^{ik(\delta - \delta')}$$

$$= \sum_k \frac{n^a_{k-q} + n^f_{k}}{i\omega_n - \varepsilon^a_{k-q} + \varepsilon^f_k} e^{ik(\delta - \delta')}$$

with $G_{b,c,a,f}(k, i\nu_n)$ are standard Bose (Fermi) Green functions of corresponding fields with the spectra

$$\varepsilon^c_k = At_k + \lambda + \zeta$$

$$\varepsilon^a_k = Ct_k + \lambda + \zeta + \mu$$

$$\varepsilon^f_k = Bt_k + \lambda - \zeta + \mu$$

$$\varepsilon^b_k = Ft_k + \lambda - \zeta$$

(26)

The mean-field values $F, A, C$ and $B$ have to be determined self-consistently.
\[ F = -\langle f_i^\dagger f_j \rangle, \quad B = -\langle b_i^\dagger b_{j\sigma} \rangle \]
\[ C = \langle c_i^\dagger c_{j\sigma} \rangle, \quad A = \langle a_i^\dagger a_j \rangle \]  
(27)

For \( \lambda, \zeta \) and \( \mu \) we have the constraint equations

\[ \langle f_i^\dagger f_i \rangle + \langle a_i^\dagger a_i \rangle = \delta \]
\[ \langle b_i^\dagger b_{i\sigma} \rangle + \langle c_i^\dagger c_{i\sigma} \rangle = 1 - \delta \]
\[ \langle b_i^\dagger b_{i\sigma} \rangle + \langle f_i^\dagger f_i \rangle + \frac{1}{Q_0} \langle \theta_i^\dagger \theta_i \rangle = 2S \]  
(28)

Further we consider the most interesting case of a “mixed” phase where both the bosons \( a, b \) are condensed. Physically, this corresponds to a non-saturated ferromagnetic state. The then concentrations \( \delta_c, \delta'_c \) where the condensates of \( b \) and \( a \) bosons vanish will determine the transitions into saturated ferromagnetic and nonmagnetic states respectively.

First, we consider the standard (noninteracting) mean-field theory which neglects the fluctuations of the \( Q \) and \( P \) fields, i.e. the second line of (25). We obtain in this case at \( T = 0 \)

\[ n_a = A_0 = \delta - \sum_k n_{k}^f \]
\[ n_b = -B_0 = 1 - \delta - \sum_k n_{k}^c \]  
(29)

where \( n_{k}^{c,f} = N_F(\varepsilon_{k}^{c,f}), \) \( N_F(\varepsilon) \) is the Fermi distribution function, the spectra \( \varepsilon_{k}^{c,f} \) are determined by (26) with

\[ \lambda_0 = \zeta - F_0 t_0 \]
\[ \lambda_0 + \mu_0 = -\zeta - C_0 t_0 \]  
(30)

and \( n_{a,b} \) are densities of condensates, index 0 at the parameters stands for their noninteracting mean-field values. It can be checked numerically that the equations (29) do not have the solutions with positive \( n_a, n_b \) for any \( \delta, \zeta \).

To improve the behavior of solutions of equations (29), we consider the corrections to above noninteracting mean-field theory owing to fluctuations of \( Q, P \) (remember that the fluctuations of \( \alpha \) were excluded by gauge transformation). Introducing Fourier components
\[ Q_{q,\delta} = \sum_i Q_{i, i+\delta} e^{i\mathbf{q} \cdot \mathbf{R}_i} \]
\[ P_{q,\delta} = \sum_i P_{i, i+\delta} e^{i\mathbf{q} \cdot \mathbf{R}_i} \]

and taking the functional derivatives of action (25), we obtain for the following expressions

\[
\langle b_{k\sigma}^\dagger b_{k\sigma} \rangle = n_k^b + \sum_{q,i\omega_n} \left[ \frac{1}{T} \frac{n_k^b (1 + n_k^b)}{i\omega_n - \varepsilon_k^b + \varepsilon_q^b} + \frac{n_k^b + n_{k+q}^b}{(i\omega_n - \varepsilon_k^b + \varepsilon_{k+q}^b)^2} \right] G^{\delta\delta'}_{Q}(\mathbf{q}, i\omega_n) e^{i\mathbf{k}(\delta - \delta')} - \frac{\delta F_{k\ell} + \delta \lambda}{T} n_k^b (1 + n_k^b) \]

\[
\langle c_{k\sigma}^\dagger c_{k\sigma} \rangle = n_k^c + \sum_{q,i\omega_n} \left[ \frac{1}{T} \frac{n_k^c (1 - n_k^c)}{i\omega_n - \varepsilon_k^c + \varepsilon_q^c} - \frac{n_{k-q}^c + n_k^c}{(i\omega_n - \varepsilon_k^c + \varepsilon_{k-q}^c)^2} \right] G^{\delta\delta'}_{Q}(\mathbf{q}, i\omega_n) e^{i(\mathbf{k} - \mathbf{q})(\delta - \delta')} + \frac{\delta \mathcal{A}_{k\ell} + \delta \lambda}{T} n_k^c (1 - n_k^c) \]

\[
\langle a_{k\sigma}^\dagger a_{k\sigma} \rangle = n_k^a + \sum_{q,i\omega_n} \left[ \frac{1}{T} \frac{n_k^a (1 + n_k^a)}{i\omega_n - \varepsilon_k^a + \varepsilon_q^a} + \frac{n_k^a + n_{k-q}^a}{(i\omega_n - \varepsilon_k^a + \varepsilon_{k-q}^a)^2} \right] G^{\delta\delta'}_{P}(\mathbf{q}, i\omega_n) e^{i\mathbf{k}(\delta - \delta')} - \frac{\delta C_{k\ell} + \delta \lambda + \delta \mu}{T} n_k^a (1 + n_k^a) \]

\[
\langle f_{k\sigma}^\dagger f_{k\sigma} \rangle = n_k^f + \sum_{q,i\omega_n} \left[ \frac{1}{T} \frac{n_k^f (1 - n_k^f)}{i\omega_n - \varepsilon_k^f + \varepsilon_q^f} - \frac{n_{k+q}^f + n_k^f}{(i\omega_n - \varepsilon_k^f + \varepsilon_{k+q}^f)^2} \right] G^{\delta\delta'}_{P}(\mathbf{q}, i\omega_n) e^{i(\mathbf{k} + \mathbf{q})(\delta - \delta')} + \frac{\delta B_{k\ell} + \delta \lambda + \delta \mu}{T} n_k^f (1 - n_k^f) \]

and

\[
\langle \theta_i^\dagger \theta_i \rangle = - \sum_{q,i\omega_n} \left[ \Pi^{cb}_{\delta\delta'}(\mathbf{q}, i\omega_n) - \Pi^{af}_{\delta\delta'}(\mathbf{q}, i\omega_n) \right] \]

where \( \delta \mathcal{F}, \delta \mathcal{A}, \delta \mathcal{C}, \delta \mathcal{B}, \delta \lambda \) and \( \delta \mu \) denote the corrections to corresponding quantities of noninteracting mean-field theory owing to interaction with the fields \( Q \) and \( P \); the Green functions \( G^{\delta\delta'}_{Q,P} \) are defined by

\[
G^{\delta\delta'}_{Q}(\mathbf{q}, \tau) = \langle T[Q_{q,\delta}(\tau) \overline{Q}_{q,\delta'}(0)] \rangle
\]
\[
G^{\delta\delta'}_{P}(\mathbf{q}, \tau) = \langle T[P_{q,\delta}(\tau) \overline{P}_{q,\delta'}(0)] \rangle
\]

and can be found by inverting corresponding matrix in (23). To keep the spectrum of the bosons \( a, b \) gapless we choose

\[
\delta \lambda = \sum_{q,i\omega_n} \frac{1}{i\omega_n - \varepsilon_q} G^{\delta\delta'}_{Q}(\mathbf{q}, i\omega_n) - \delta \mathcal{F}_0
\]
\[
\delta \lambda + \delta \mu = \sum_{q,i\omega_n} \frac{1}{i\omega_n - \varepsilon_q} G^{\delta\delta'}_{P}(\mathbf{q}, i\omega_n) - \delta \mathcal{C}_0
\]
The expressions (31) and (32) together with the self-consistent equations (27), (28) give the parameters of supersymmetric mean-field theory.

V. CONCLUSION

In this paper we have obtained the supersymmetric action of $t - J$ model, which interpolates between slave-fermion and slave-boson ones. We have also developed a mean-field approximation for this action, which takes into account the fluctuations owing to the interaction with gauge fields. Physically, these fluctuations correspond to fluctuations of the total boson (fermion) number in the system, since only total number of bosons and fermions is conserved in the supersymmetric representation [8]. In other words, these interactions describe virtual boson-fermion transitions within the supersymmetric fermion-boson representation. The account of these virtual transitions can give in principle a possibility to investigate the crossover from the saturated ferromagnetic state into nonmagnetic one and. After a generalization to finite $U$, the crossovers from antiferromagnetic state to the states which are described well by slave-boson representation, e.g. pseudo-gap and superconducting one, could be described. The application of the results of the present paper to solving these problems is the aim of future work.

VI. APPENDIX. GAUGE TRANSFORMATION OF MEAN-FIELD HAMILTONIAN PARAMETERS AND GREEN FUNCTIONS

In this Appendix we consider the transformation laws for different quantities under the gauge transformation (22). We treat $\eta_i(\tau)$ as a sort of fluctuations (see, e.g. [12]) with zero average and the pair correlation function

$$f(q, \omega) = \langle \eta(q, \omega) \eta(q, \omega) \rangle$$

Calculating the contributions up to quadratic terms in $\eta$, we obtain following transformation laws of the Hamiltonian parameters:
\[ \lambda \rightarrow \lambda - \sum_{\mathbf{q}, i\omega_n} (2\zeta - i\omega_n) f(\mathbf{q}, i\omega_n) \]
\[ \mathcal{F} \rightarrow \mathcal{F} - \sum_{\mathbf{q}, i\omega_n} (\mathcal{F} - \mathcal{A}t_\mathbf{q}/t_0) f(\mathbf{q}, i\omega_n) \]
\[ \mathcal{A} \rightarrow \mathcal{A} + \sum_{\mathbf{q}, i\omega_n} (\mathcal{A} - \mathcal{F}t_\mathbf{q}/t_0) f(\mathbf{q}, i\omega_n) \]
\[ \mathcal{B} \rightarrow \mathcal{B} - \sum_{\mathbf{q}, i\omega_n} (\mathcal{B} - \mathcal{C}t_\mathbf{q}/t_0) f(\mathbf{q}, i\omega_n) \]
\[ \mathcal{C} \rightarrow \mathcal{C} + \sum_{\mathbf{q}, i\omega_n} (\mathcal{C} - \mathcal{B}t_\mathbf{q}/t_0) f(\mathbf{q}, i\omega_n) \]  

(35)

The gauge fields are transformed as

\[ \alpha_i \rightarrow \alpha_i + (\partial_\tau + 2\zeta_i) \eta_i \]
\[ Q_{ij} \rightarrow Q_{ij} + \mathcal{A} \eta_i - \mathcal{F} \eta_j \]
\[ \mathcal{P}_{ij} \rightarrow \mathcal{P}_{ij} + \mathcal{C} \eta_i - \mathcal{B} \eta_j \]  

(36)

Using (36) one can obtain the transformation laws for the gauge fields Green functions, e.g.,

\[ G_\alpha(\mathbf{q}, i\omega_n) \rightarrow G_\alpha(\mathbf{q}, i\omega_n) + (2\zeta - i\omega_n)^2 f(\mathbf{q}, i\omega_n) \]  

(37)

\[ G_\delta' Q(\mathbf{q}, i\omega_n) \rightarrow G_\delta' Q(\mathbf{q}, i\omega_n) + (\mathcal{A} - \mathcal{F} e^{i\phi})(\mathcal{A} - \mathcal{F} e^{i\phi'}) f(\mathbf{q}, i\omega_n) \]
\[ G_\delta' P(\mathbf{q}, i\omega_n) \rightarrow G_\delta' P(\mathbf{q}, i\omega_n) + (\mathcal{C} - \mathcal{B} e^{i\phi})(\mathcal{C} - \mathcal{B} e^{i\phi'}) f(\mathbf{q}, i\omega_n) \]

The transformation laws for other quantities can be obtained by combining the results (35), (37).
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