Reply to Cohen’s comment on the rotation–vibration coupling in chiral soliton models

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In this short note we summarize the main results of our paper [hep-ph/0510055] and reply to a recent comment [hep-ph/0511174] on that paper.

In a recent comment [1] Cohen criticized our conclusion in ref. [2] that the rigid rotator approach (RRA) to generate baryon states with non–zero strangeness in chiral soliton models is suitable to estimate excitation energies and decay properties of exotic baryons such as the Θ+ pentaquark. Starting point for this criticism is the so–called bound state approach (BSA) to chiral soliton models. The BSA describes baryons with non–zero strangeness as compound objects of the soliton and kaon modes that are treated as harmonic vibrations about the soliton. It is well established that the BSA becomes exact in the limit that the number of colors, \( N_C \), approaches infinity. Cohen’s criticism is based on the (correct) observation that the excitation energy of the mode needed to build the Θ+ pentaquark does not vanish even in the combined limit of large \( N_C \) and \( m_K \rightarrow m_\pi \). Hence rotational and vibrational modes do not decouple for pentaquark baryons. Cohen then argues that this prevents the introduction of collective coordinates to describe these modes as rigid rotations and that the RRA would be inadequate to compute physical properties of exotic baryons. For this and other reasons we concluded that the Θ+ symmetry breaking can straightforwardly be included and the exact eigenstates of the Hamiltonian (4.15) may well be considered as a collective excitation of the soliton. Here we will back up this conclusion by briefly recapitulating the central results of ref. [2].

In the RRA the SU(3) Euler angles \( \vec{\alpha} \) that parameterize the orientation of the soliton in flavor space are introduced as collective coordinates and quantized canonically. In the flavor symmetric case the RRA then predicts the excitation energy and wave–function of the exotic Θ+ to be

\[
\omega_\Theta = E_\Theta - E_N = \frac{N_C + 3}{4 \Theta_K}, \quad \langle \vec{\alpha}|\Theta^+ \rangle \propto D^{10}_{(2,0,0), (1, \frac{1}{2}, J_3)}(\vec{\alpha}).
\]

Form and numerical value of the kaonic moment of inertia, \( \Theta_K \), depend on the considered model. The baryon wave–functions are Wigner D–functions of the Euler angles, characterized by the left and right quantum numbers \((Y, T, T_3)\) and \((Y_R, J, -J_3)\), respectively, and the SU(3) representation "10" with \((p, q) = (0, \frac{N_c-4}{2})\) for arbitrary \( N_C \). Flavor symmetry breaking can straightforwardly be included and the exact eigenstates of the Hamiltonian (4.15)2 for the collective coordinates are obtained as linear combinations of states from different SU(3) representations. To study the rotation–vibration coupling small amplitude fluctuations must be introduced in addition to the collective rotations. In ref. [2] we have utilized Dirac’s quantization procedure under constraints to quantize these additional fluctuations in the subspace that is orthogonal to the rigid rotations parameterized by the collective coordinates. This then defines the rotation–vibration approach (RVA). An important feature of the RVA is that it generates a contribution in the Hamiltonian, \( H_{\text{int}} \) that is linear in these fluctuations. Since \( H_{\text{int}} \) also contains collective coordinate operators it gives rise to a Yukawa coupling between the nucleon and its collective excitations. Actually, rotation–vibration coupling has been frequently considered in former soliton calculations, both in SU(2) and in SU(3) (see [2] for references).

Since the RVA contains collective rotations and orthogonal fluctuations but the BSA contains fluctuations only, the large \( N_C \) correspondence is such that the fluctuations in the two approaches are equal in the subspace orthogonal to the rotations. In the rotational subspace the BSA fluctuations must thus correspond to the collective rotations of the RVA. In section III and IV of ref. [2] we therefore have carefully compared the BSA and RVA in the rotational subspace. Projecting the BSA equation (3.5) onto its rotational subspace immediately leads to the criticized eqs. (3.10) and (3.11) for the mass differences \( \omega_\Lambda = E_\Lambda - E_N \) and \( \omega_\Theta = E_\Theta - E_N \). In Fig. 3 of ref. [2] we compared these mass differences to the excitation energies predicted by the RSA for arbitrary \( N_C \) and \( m_K = 495 \text{MeV} \). Their equality

1 This (wrong) argument would also invalidate the RRA for non–exotic \( 8 \) and \( 10 \) baryons in the full calculation, where a sizable symmetry breaking must be included. Note e.g. that the excitation energy of the Ω(1670) is also order \( N_C^0 \) but even larger than that of the \( \Theta^+ \).

2 The equations in this note are labeled (R1), (R2) and (R3), all other numbers refer to formulas in ref. [2].
FIG. 1: Full phase shifts as calculated directly from the RVA equations (5.10) and (7.4) for various values of $N_C$.

for large $N_C$ and arbitrary $m_K \neq m_\pi$ unambiguously confirms the above described scenario for the correspondence between the BSA fluctuations and the collective excitations.

The central equations of the RVA are the integro–differential eqs. (5.10) for $m_K = m_\pi$ and (7.4) for $m_K \neq m_\pi$. As a matter of fact, these equations are fundamental to the RVA and everything else directly follows thereof. We have solved these two equations numerically in order to obtain the phase shifts. For completeness we show these phase shifts here in an extra figure although they may be easily extracted from Figs. 2, 5 and 6 of ref. [2]. For $N_C = 3$ we notice a sharp and pronounced resonance with almost a full $\pi$ jump in the phase shifts. In the RVA the transition matrix element $\langle N|H_{\text{int}}|\Theta^+ \rangle$ between the nucleon and the $\Theta^+$ is essential. This matrix element can be expressed as a sum of terms that are products of two factors, (i) a spatial integral over the wave–functions of the fluctuations and the soliton profile and (ii) a collective coordinate matrix element involving the Wigner $D$–functions of the nucleon and the $\Theta^+$, cf. eq. [4]. Of course, we have taken configuration mixing into account in the physical case of $m_K \neq m_\pi$.

Although our results do not rely on separating background and resonance phase shifts it is instructive to do so. For simplicity we consider the SU(3) symmetric case (5.10) and switch off the $\Lambda$ pole contribution. In the $\Theta^+$ resonance region that contribution is unimportant and in large $N_C$ it vanishes anyhow if $m_K = m_\pi$ [2]. Using standard scattering theory techniques we then find the exact and unambiguous relation

$$\delta(k) = \overline{\delta}(k) + \arctan \frac{\Gamma_\Theta(\omega_k)/2}{\omega_\Theta - \omega_k + \Delta_\Theta(\omega_k)}.$$  \hspace{1cm} \text{(R2)}

The $N_C$ independent background phase shift $\overline{\delta}(k)$ is obtained from (5.10) for vanishing Yukawa coupling. Eq. [4] corroborates that the RRA excitation energy $\omega_\Theta$ is absolutely essential to reproduce the correct phase shift within the RVA. The width, $\Gamma_\Theta(\omega_k)$ is proportional to the square of the transition matrix element $\langle N|H_{\text{int}}|\Theta^+ \rangle$ between the nucleon and the $\Theta^+$. The unique resonance contribution arises solely due to the Yukawa coupling. It emerges in the standard shape parameterized by the width $\Gamma_\Theta$ and the pole shift $\Delta_\Theta$ that are listed in eqs. (6.5) and (6.6). The collective RRA quantities, eq. [4], inevitably enter the computation of $\omega_\Theta$, $\Gamma_\Theta$ and $\Delta_\Theta$, therewith emphasizing the collective nature of the $\Theta^+$. Furthermore these collective coordinate matrix elements induce a strong $N_C$ dependence in the resonance contribution. In the flavor symmetric case $\langle N|H_{\text{int}}|\Theta^+ \rangle$ contains only a single SU(3) structure. This is in sharp contrast to the approaches of refs. [2] that attempt to describe the (potentially) small width of the $\Theta^+$ from cancellations between contributions from different SU(3) structures. Moreover, the SU(3) structure in $H_{\text{int}}$ is not related to the transition operator for the decay $\Delta \to \pi N$. For $N_C = 3$ we have calculated a small pole shift $\Delta_\Theta = -14$MeV. This small number has to be contrasted with the RRA excitation energy $\omega_\Theta = 792$MeV. Obviously, the coupling to the continuum yields a negligible correction to the RRA prediction for the excitation energy of the $\Theta^+$. This additionally indicates its collective nature.

We have already noted that the BSA is exact for $N_C \to \infty$. Indeed we have verified that in this limit $\delta(k)$ is identical to the BSA phase shift. Nevertheless for $N_C \to \infty$ the separation in eq. [4] still holds and we observe a broad resonance hidden by repulsive background phase shifts (cf. Fig. 2 in ref. [2] for the individual contributions). Eq. [4] also applies to the $\Delta$ decay in the SU(2) version of the model, where nobody doubts the validity of the RRA. Apart from the different transition operator, the collective $\Theta^+$ quantities, eq. [4], are simply replaced by those of the $\Delta$ in the two flavor model

$$\omega_\Delta = E_\Delta - E_N = \frac{3}{2 \Theta_\pi}, \quad \langle \tilde{a}|\Delta \rangle \propto D_{T_3,J_3=\frac{3}{2}}^{T=\frac{1}{2}}(\tilde{a}),$$ \hspace{1cm} \text{(R3)}

where $\Theta_\pi$ is the pionic moment of inertia. A small pole shift $\Delta_\Delta$ due to the coupling to the continuum appears also
In the large $N_C$ limit width and pole shift become sizable for the $\Theta^+$ (cf. Fig. 1) but vanish for the $\Delta$ in SU(2). For the $\Delta$ this reflects the above mentioned decoupling of rotational and vibrational modes and the fact that the $\Delta$ excitation becomes purely collective in that limit. In the real world, $N_C = 3$, the situation is just reversed, namely width and pole shift for the $\Theta^+$ are smaller than the corresponding quantities for the $\Delta$, implying that the collective portion in the total wave function is even higher for the $\Theta^+$ than the $\Delta$. In any case, we may safely conclude that both excitations, the $\Delta$ and the $\Theta^+$ can reliably be described as collective excitations of the soliton.

Finally we briefly comment on the $1/N_C$ expansion. Admittedly there is an inconsistency which we frankly discussed in chapter V of [2]. Namely, we have selected the leading $N_C$ Yukawa couplings only, but treated them to all orders in $N_C$ while we omitted subleading terms. This is completely sufficient to investigate the relation between the BSA (which does not have subleading terms to begin with) and the RVA. Because the leading terms taken into account introduce already an extreme (for $N_C = 3$ diverging) $1/N_C$ dependence (cf. section VI.B of ref. [2]) serious doubts concerning the applicability of $1/N_C$ expansion methods in the context of exotic baryons are in place. There is no reason to expect the subleading rotation–vibration couplings to be small. Eventually all possible terms would have to be taken into account. This would lead to a tremendous computational effort including many Yukawa coupling terms into a complicated coupled channel calculation [5] (the inclusion of processes like $KN \rightarrow K\pi N$ would stand at the very end of our wish list). Improvements in that direction probably have to wait for a clarification of the experimental situation concerning the status of exotic states.

To summarize, we fully reject the criticism raised in the comment, ref. [1], in all points. Moreover, from the presented argumentation it is obvious that the exotic $\Theta^+$, alike the non-exotic $\Delta$, is predominantly a collective soliton excitation. Thus we have to reiterate the conclusion drawn in ref. [2] that the rigid rotator approach is indeed appropriate in predicting pentaquark masses and properties in chiral soliton models, in sharp disagreement to the statements made in ref. [1] (and refs. [2-7] therein) put forward to discredit this approach to chiral soliton models in flavor SU(3).

[1] T. D. Cohen, arXiv:hep-ph/0511174 "Comment on the Walliser-Weigel approach to exotic baryons in chiral soliton models".
[2] H. Walliser and H. Weigel, arXiv:hep-ph/0510055 "Bound state versus collective coordinate approaches in chiral soliton models and the width of the $\Theta^+$ pentaquark".
[3] D. Diakonov, V. Petrov and M. V. Polyakov, Z. Phys. A359, 305 (1997), M. Praszalowicz, Phys. Lett. B583, 96 (2004).
[4] B. Schwesinger, H. Weigel, G. Holzwarth and A. Hayashi, Phys. Rep. 173, 173 (1989).
[5] B. Schwesinger, Nucl. Phys. A537, 253 (1992).