Excitation of Vortices in Semiconductor Microcavities

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We predict that the transverse electric-magnetic polarization splitting of exciton-polaritons allows a simple technique for the generation of polariton vortices of winding number 2 in semiconductor microcavities. The vortices can be excited by circularly polarized light having no orbital angular momentum and observed as phase vortices in the opposite circular polarization or directly as linear polarization vortices. The prediction is explained by a simplified analytical linear model and shown fully with a numerical model based on the Gross-Pitaevskii equations, which includes the non-linear effects of polariton-polariton interactions.

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Introduction. The creation and evolution of vortices in atomic Bose-Einstein condensates (BECs) has attracted much attention [1, 2] since the experimental realization of a dilute atomic BEC [3] just over a decade ago. It is no coincidence that vortices are at the heart of our understanding of superfluidity, forming spontaneously in type II superconductors [4] and superfluids [5, 6]. In atomic BECs vortices can be optically created using light modes with orbital angular momentum (Laguerre-Gaussian modes) and exploiting the recent technology of ultraslow light pulses to couple light and matter fields [7]. Vortices written by light fields are stored in the atomic condensate and can later be re-written onto light fields.

In semiconductor systems, the high temperature BEC of the half-light half-matter quasiparticles called exciton-polaritons has been reported recently [8]. The condensation takes place in a system of several quantum wells sandwiched between two distributed Bragg reflectors, commonly known as a microcavity. Many unique properties of polaritons [9] arise from their non-parabolic in-plane dispersion (center left of Fig. 1). No vortices of polaritons have been observed so far, although superfluidity in polariton systems has been largely discussed theoretically [10, 11]. Here we show that a unique fine structure of polaritons in microcavities allows for a very simple mechanism of exciting vortices using conventional circularly polarized light beams having zero orbital angular momentum.

It is well known that transverse electric (TE) and magnetic (TM) normal modes of a cavity are non-degenerate for finite in-plane wavevectors. Polaritons also possess a polarization dependent TE-TM energy splitting [12], which was previously proposed to allow the optical spin Hall effect [13]. We will show that this splitting can cause an initially circularly polarized polariton distribution to form a phase vortex in the cross-circular polarization, characterized by a phase singularity around which the phase changes by $4\pi$. A vortex structure also appears in the linear polarization pattern.

This linear effect is a general property of all planar cavities, not only those with quantum wells, and could be useful for creating light modes with angular momentum. Such light modes can have interesting applications to quantum information [14]. Our next step is to take into account the many-body polariton-polariton interactions, which play a crucial role in a number of microcavity processes where large polariton populations are involved [15, 16].

Polariton-polariton interactions can be accounted for using the zero-range interaction and mean-field approximations that lead to the Gross-Pitaevskii equations. In Ref. [10] the polarization independent spatial dynamics of interacting polariton systems was derived by numerically solving the single component (scalar) Gross-Pitaevskii equations. For us, the inclusion of polarization is not only important due to effects caused by TE-TM splitting; polariton-polariton interactions themselves were previously shown to be spin-anisotropic, which led to effects such as self-induced Larmor precession of elliptical polarizations and inversion of linear polarizations [17].

Taking into account polarization, polaritons represent a two component Bose gas. Vortices in two-component bosonic systems are closely related to Skyrmions [18], which makes microcavities a potentially suitable solid state system for Skyrmion observation.

Linear Model. We will first outline a simple analytic model to show how vortices can be generated in microcavities. First we consider a lower branch polariton state system for Skyrmion observation.}

\begin{equation}
\begin{bmatrix}
\psi_{TMO} \\
\psi_{TE0}
\end{bmatrix} = \hat{M} \begin{bmatrix}
1 \\
0
\end{bmatrix}
\end{equation}

\begin{equation}
\hat{M} = \frac{1}{\sqrt{2}} \begin{pmatrix}
e^{i\phi} & e^{-i\phi} \\
e^{i\phi} & -ie^{-i\phi}
\end{pmatrix}
\end{equation}

The evolution of the initial state in the circular basis is:
\[
\begin{pmatrix}
\psi_+
\psi_-
\end{pmatrix}
= \mathbf{M}^{-1}
\begin{pmatrix}
\psi_{TMO}e^{i\omega_{TM}t'/\tau}
\psi_{TEO}e^{i\omega_{TE}t'/\tau}
\end{pmatrix}
= e^{i\omega_{TM}+\omega_{TE}t-t'/\tau}
\left(\cos\left(\frac{\omega_{TM}-\omega_{TE}}{2}t\right)
+ ie^{2i\phi} \sin\left(\frac{\omega_{TM}-\omega_{TE}}{2}t\right)\right),
\]

where \(\omega_{TM}\) and \(\omega_{TE}\) are the frequencies of the two eigenstates and \(\tau\) accounts for the life-time of polaritons. The \(k\)-dependent energy splitting between \(\omega_{TM}\) and \(\omega_{TE}\) is shown in the center right plot of Fig. 1. From Eq. \(3\) we see that the phase of the cross-circularly polarized component \(\psi_-\) is angle dependent. So far we have only considered a single state with wavevector \(\vec{k}\). To obtain the field distribution from multiple wavevector states we should take the convolution integral with a pump distribution, \(f(k, \phi, t)\):

\[
\begin{pmatrix}
\Psi_+(\vec{r}, t)
\Psi_-(\vec{r}, t)
\end{pmatrix}
= 2\pi Ak_p e^{i\omega_{TM}+\omega_{TE}t-t'/\tau} \times
\frac{J_0(k_p r) \cos\left(\frac{\omega_{TM}-\omega_{TE}}{2}t\right)}{-ie^{2i\theta} J_2(k_p r) \sin\left(\frac{\omega_{TM}-\omega_{TE}}{2}t\right)},
\]

where \(J_n\) denotes the \(n\)th order Bessel function of the first kind. The amplitude and phase of \(\Psi_+\) and \(\Psi_-\) are plotted in Fig. 1. It shows that the phase of the cross-circular component changes by \(4\pi\) around the origin. In other words we have found a vortex of winding number \(2\). Note also the role of oscillatory terms in Eqs. \(3\) and \(5\): they describe the beats between the two spin states of polaritons. Initially, all polaritons lie in the spin-up state and there is no vortex. After a quarter of the period \(T = \frac{2\pi}{\omega_{TM}+\omega_{TE}}\) all polaritons lie in the spin-down state and the vortex appears. Another quarter period later polaritons return to the spin-up state and the vortex disappears. At the intermediate time \(t = \frac{T}{4}\) the polaritons are linearly polarized, forming a polarization vortex as sketched in the bottom left plot of Fig. 1.

**Non-Linear Model.** In this section we use a numerical model to include the effects of polariton-polariton interactions and the finite extent of any real pulse in space and time. We will use the Gross-Pitaevskii equations, which are a familiar tool for the treatment of dilute Bose-gases \([19, 20]\). For a coherent ensemble of polaritons the set of coupled Gross-Pitaevskii equations is written:

\[
\begin{align*}
&i\hbar \frac{\partial \chi_i}{\partial \tau} = \hat{T}_{ij} \chi_j + V \phi_i + (\alpha_1 - \alpha_2)|\chi_i|^2 \chi_i + \\
&\quad + \alpha_2 \chi_i^* \chi_j \chi_i - \frac{i\hbar}{2\tau_\chi} \chi_i,
\end{align*}
\]

\[
\begin{align*}
&i\hbar \frac{\partial \phi_i}{\partial \tau} = \hat{T}_{ij} \phi_j + V \chi_i + f_i(\vec{x}, t) - \frac{i\hbar}{2\tau_\phi} \phi_i
\end{align*}
\]

**FIG. 1:** (color) (Top:) Plots of the absolute values of \(\Psi_+\) (left) and \(\Psi_-\) (right) for \(k_p = 100\text{mm}^{-1}\). Note that the gray-scales are not the same; the cross-circular component is much weaker (depending strongly on the value of the TE-TM splitting at \(k_p\)). (Center Left:) The dispersion of photons and excitons (gray) and polaritons (black) for an exciton-photon coupling energy (vacuum field Rabi splitting) of 5.1meV. The TE-TM splitting of the photon is given by the different effective masses of TE and TM branches. We chose \(m_t = 10^{-3}m_e\) and \(m_l = 0.95m_t\) where \(m_e\) is the free electron mass. (Center Right:) The parabolic TE-TM splitting of the photon causes an TE-TM splitting of the lower (solid) and upper branch (dashed) polaritons. (Bottom Left:) The linear polarization vortex. Arrows indicate the linear polarization along the circle \(k = k_p\) after a time \(t = \frac{T}{4}\). (Bottom Right:) The phase of \(\Psi_-\) changes by \(4\pi\) around the origin.
and densities where the thermal and quantum depletion by the mean field treatment, valid at low temperatures, are accounted for by the two circular polarizations. The operators $\hat{T}_x$ and $\hat{T}_\phi$ are the kinetic energy operators of uncoupled excitons and photons, which have parabolic dispersion with effective masses $m_x$ and $m_\phi$:

$$\hat{T}_x = -\frac{\hbar^2}{2m_x} \begin{pmatrix} \partial_{xx} + \partial_{yy} & 0 \\ 0 & \partial_{xx} + \partial_{yy} \end{pmatrix}$$

$$\hat{T}_\phi = -\frac{\hbar^2}{2m_\phi} \begin{pmatrix} \partial_{xx} + \partial_{yy} & \zeta (\partial_x - i\partial_y)^2 \\ \zeta (\partial_x + i\partial_y)^2 & \partial_{xx} + \partial_{yy} \end{pmatrix}$$

We have chosen an energy scale such that the energy of the photonic component is zero at zero in-plane wavevector and consider the case where there is no detuning between the exciton and photon modes. The parameter $\zeta$ allows for a (parabolic) TE-TM splitting of the photonic component in the microcavity (we neglect the much smaller splitting of the excitonic component).

The coupling between excitons and photons is described by the Rabi splitting of the cavity, $2V$, which is related to the quality factor and the exciton decay rate $\Gamma$. The non-linear terms in Eq. (6) describe Coulomb interactions between the excitonic components where $\alpha_1$ describes the parallel spin configuration and $\alpha_2$ describes the anti-parallel configuration. In Eq. (7) the term $f_i(\vec{x}, t)$ represents the optical pumping of the microcavity.

For a coherent Gaussian pump pulse exciting the lower polariton branch at the wavevector $k_0$ and energy $E_p$ at time $t_p$, the pump in reciprocal space is:

$$f_i(\vec{k}, t) = A_i e^{-iE_p t_p / \hbar} \frac{\Gamma e^{-(\vec{K} - \vec{k}_0)^2 / 2\Delta_k^2} e^{-(t-t_p)^2 / 2\Delta t^2}}{(E_{LP}(\vec{k}) - E_p - i\Gamma)}$$

$A_i$ represent the amplitudes of the two circular polarizations of the pump pulse. $\Delta k$ and $\Delta t$ define the width of the pump in reciprocal space and time. The fraction $\Gamma / \left( E_{LP}(\vec{k}) - E_p - i\Gamma \right)$ is the linear susceptibility of a system of Lorentz oscillators where $\Gamma$ is the homogeneous oscillator (HWHM) linewidth. In Eq. (7) the pumping term $F(\vec{x}, t)$ is the inverse Fourier transform of $f_i(\vec{k}, t)$. $E_{LP}(\vec{k})$ is the bare dispersion of the lower polariton branch, which is calculated by finding the eigenvalues of the linear Hamiltonian (without interaction terms) in the exciton-photon basis. Finally, $\tau_\chi$ and $\tau_\phi$ are the lifetimes of the excitonic and photonic components, which account for the inelastic scattering and radiative decay of polaritons respectively.

Equations (6) and (7) completely determine the dynamics of interacting polaritons once initial wavefunctions and pumping terms are defined. We acknowledge that the polariton dynamics from a linear pulse pump was recently studied using similar equations (and our model was able to reproduce these earlier results). In this paper we focus on the case of excitation with a circularly polarized pump pulse ($A_+ = 1, A_- = 0$) with broad spread in wavevector tuned slightly ($0.35\text{meV}$) above the bottom of the lower polariton branch. Although the...
The pump is centered at \( \vec{r}_0 = 0 \) (normal incidence) it is resonant with the lower polariton branch along a ring \((k = 435\text{mm}^{-1})\) in reciprocal space with a radius set by the pump energy. We numerically solved Eqs. (4) and (7) to find the results plotted in Figs. 2 and 4. We show the behaviour of the photonic wavefunctions, which are directly accessible experimentally. The excitonic wavefunctions demonstrate an identical behaviour.

The appearance of expanding rings in the distributions should be expected since we are exciting a ring in reciprocal space. In the cross-circular polarization there is again a phase singularity and vortex of winding number 2 (in agreement with the analytical linear model).

The expanding polariton rings are influenced significantly by the polariton-polariton interactions as shown in Fig. 1 where we plot cross sections of the wavefunctions along the x-axis for different values of the scattering parameter, \( \alpha_1 \). In the absence of polariton-polariton interactions \( \alpha_1 = 0 \) a ring is observed in both circular polarizations. As \( \alpha_2 \) increases the intensity of the wavefunctions increases slightly (formally an increase in the interaction constants is identical to an increase in the pump amplitude) but as the interaction constants increase further the intensity decreases, and the wavefunctions spread due to the repulsion of polaritons in real space. Also note that as \( \alpha_1 \) increases we observe a central peak in the co-circular distribution instead of a ring. This is due to the larger (time-dependent) blueshift of the system when \( \alpha_1 \) is increased, which results in the resonant excitation of a smaller ring in reciprocal space. In real space we also observe the appearance of darker rings within the brighter ring in cross-circular polarisation as \( \alpha_1 \) is increased from zero. Such structures seem similar to dark ring solitons, which are known to appear in the equilibrium scalar Gross-Pitaevskii equations and similar non-linear Schrödinger equations. (24, 25). The in-coherent polariton scattering, introduced via dissipative terms in Eqs. (7) and (8), does not affect the appearance of the vortices, whilst it does influence the intensity of polarized light emitted by the cavity.

**Conclusion.** We calculated the dynamics of an interacting polariton condensate, which was excited by a circularly polarized Gaussian pump pulse tuned above the bottom of the lower polariton branch. The presence of TE-TM splitting in the microcavity allows coupling into the cross-circular polarization in which the phase shows the profile of a vortex of winding number 2. The appearance of a linear polarization vortex and spreading ring-like structures is also derived.

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