Optimization Settings in the Fuzzy Combined Mamdani PID Controller

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Abstract. In the present work the actual problem of determining the optimal settings of fuzzy parallel proportional-integral-derivative (PID) controller is considered to control nonlinear plants that is not always possible to perform with classical linear PID controllers. In contrast to the linear fuzzy PID controllers there are no analytical methods of settings calculation. In this paper, we develop a numerical optimization approach to determining the coefficients of a fuzzy PID controller.

Decomposition method of optimization is proposed, the essence of which was as follows. All homogeneous coefficients were distributed to the relevant groups, for example, three error coefficients, the three coefficients of the changes of errors and the three coefficients of the outputs P, I and D components. Consistently in each of such groups the search algorithm was selected that has determined the coefficients under which we receive the schedule of the transition process satisfying all the applicable constraints. Thus, with the help of Matlab and Simulink in a reasonable time were found the factors of a fuzzy PID controller, which meet the accepted limitations on the transition process.

1. Introduction

Currently, the use of fuzzy proportional-integral-derivative (PID) controllers is increasing, which have much greater flexibility compared to their linear counterparts. Methods for determination of optimal parameters of fuzzy PID controllers are quite difficult and time consuming, which greatly limits their application.

In the present work, a simple method is proposed for tuning the parameters of fuzzy PID controller using Matlab tools and certain provisions of the method of fuzzy linearization described in [1].

2. Structure of fuzzy PID Mamdani controller

Consider in figure 1a scheme of the most common discrete fuzzy controller which has two inputs: error of regulation e(k) and its change Δe(k) = e(k) - e(k - 1) or rate Δe(k) = Δe(k) / Δt and one output u(k) - is the control at time k Δt, k = 1, 2, ..., N, where Δt is the sampling time.
Valid values of inputs $e$, $\Delta e$ by using normalizing coefficients $K_e$, $K_{\Delta e}$ are converted to normalized $\hat{e}$, $\Delta \hat{e} \in [-1, 1]$. A central element of a fuzzy controller is the fuzzy operator $\tilde{f}_p$ (figure 1 b), which performs the operation of fuzzification (Fuz), fuzzy inference (FI) and defuzzyfication (Def) and rule base (RB).

![Figure 1](image1.png)

**Figure 1.** The scheme of fuzzy operator

Operation of Fuz converts the normalized inputs $\hat{e}$, fuzzy $\Delta \hat{e}$ in $E$, $\Delta E$. Fuzzy Inference FI, for example the Mamdani method, finds fuzzy output $U$ based on the fuzzy inputs $E$ and $\Delta E$ rule base RB of the form

$$R_p^\theta: \text{if } \hat{e} \text{ is } E^\theta, \Delta \hat{e} \text{ is } \Delta E^\theta, \text{ then } \hat{u} \text{ is } U^\theta, \theta = 1, 2, \ldots, q, \quad (1)$$

where $E^\theta$, $\Delta E^\theta$, $U^\theta$ - fuzzy sets having term sets $T_e$, $T_{\Delta e}$, $T_u$ with the elements characterizing the values of the corresponding variables $\hat{e}$, $\Delta \hat{e}$, $\hat{u}$ ($N$ - negative, $Z$ - zero, $P$ - positive) or ($NB$ - negative big, $NM$ - negative medium, $ZE$ is close to zero, $PM$ - positive medium, $PB$ - positive big).

Operation of Def converts the fuzzy output $U$, for example, the method of the median (Bisector) or center of gravity (Centroid), normalized value $\hat{u} \in [L, -L]$, $L = 1, 2$, which is multiplied by the coefficient $K_u$ becomes real $u$.

The proposed method of determining the parameters of fuzzy P, PI, PD and PID or PID controllers is linearization and optimization of fuzzy controllers.

To make the fuzzy operator (controller) $\hat{u} = \tilde{f}(\hat{e}, \Delta \hat{e})$ become the equivalent of a fuzzy linear $\hat{u} = \hat{e} + \Delta \hat{e}$, you must complete the following basic conditions set out in [1]:

1) use the premises in the triangular membership function (figure 2a);

2) intersecting if the value is 0.5, and at the end singletone (figure 2 b);

![Figure 2](image2.png)

**Figure 2.** Membership functions
3) combination of terms in the premises ∧ to construct a database of rules containing all possible rules.

Consider the linearization of a fuzzy PID controller and find relationships that determine their coefficients.

Fuzzy PID controller that has a fuzzy PD component and a linear I component, and two inputs $e, \dot{\Delta}e$ is shown in Figure 3.

In this scheme the integral error $e_i$ is calculated as the sum of the incremental controls, obtained by adder $C_1$ and the delay element for 1 cycle $z^{-1}$.

$$e_i(k) = \sum_{i=1}^{k} e(i)\Delta t.$$  

Then the output of the controller formed by the addition of fuzzy PD and clear integral (I) $u_i$ components of the adder $C_2$ are defined as

$$u(k) = K_u \left[ \tilde{f}_{pd}(K_e e(k), K_{se} \dot{\Delta}e(k)) + K_e e_i(k)\Delta t \right].$$  \hfill (2)

Linearization of the expression (2) has the form

$$u(k) = K_u \left[ K_e e(k) + K_e e_i(k)\Delta t + K_{se} \dot{\Delta}e(k) \right] = K_e K_u \left[ e(k) + \frac{K_i}{K_e} \sum_{i=1}^{k} e(i)\Delta t + \frac{K_{se}}{K_e} \dot{\Delta}e(k) \right].$$  \hfill (3)

Comparing (3) with the expression of the linear discrete PID controller

$$u(k) = K_p e(k) + \frac{1}{T_i} \sum_{i=0}^{k} e(i)\Delta t + T_d \dot{\Delta}e(k),$$

get the following rather simple ratio

$$K_e K_u = K_p,$$  \hfill (4)

$$\frac{K_i}{K_e} = \frac{1}{T_i};$$  \hfill (5)

$$\frac{K_{se}}{K_e} = T_d.$$  \hfill (6)

linking the parameters of the linear and fuzzy PID controllers.

On the basis of ratios (2), (3), (4) – (6) in table 1 can be set the correspondence between the coefficients of linear traditional and fuzzy PID controllers.
By determining and tuning linear PID controllers by the method of Ziegler – Nichols \((K_p = 4.8, \frac{1}{T_i} = 1.9, T_d = 3.7)\) and putting them in a table 1, it is possible to obtain the relations (4) - (6) to calculate \(K_e, K_u, K_{\Delta e}, K_i\) and \(K_u\) equivalent linear fuzzy PID controllers. We now proceed to a description of procedures of constructing, modelling and optimization in MATLAB - Simulink of a closed-loop digital control system (figure 4) containing the fuzzy PID controller and the control object with transfer function

\[
W(s) = \frac{1}{s^3 + 3s^2 + 3s + 1}.
\]  

\textbf{Table 1.} The coefficients of fuzzy PID controller

| Fuzzy controller | \(K_p\) | \(1/T_i\) | \(T_d\) |
|------------------|---------|----------|---------|
| PID              | \(K_e, K_u = 4.8\) | \(K_i/K_e = 0.51\) | \(K_{\Delta e}/K_e = 3.7\) |

Let us create a model of a digital control system with fuzzy PID controller the dynamic simulation package Simulink in the MATLAB command window, File, New, Model, open the model window called “Untitled”, then click on Library Browser - open the items window “Simulink Library Browser”. From the “Simulink Library Browser” drag the desired blocks into the model window called “Untitled” and create a digital model of control system presented on figure 4 and having a combined fuzzy PID controller and the control object with transfer function (7).

Start to build the fuzzy PID controller, type 1 editor the fuzzy inference system FIS Editor. Replacing \(\hat{e}\) to \(e_1\), \(\hat{a}e\) to \(d_1\), \(u\) to \(u_1\), \(K_e\) to \(K_e\), \(K_{\Delta e}\) to \(K_{de}\), \(K_i\) to \(K_i\), \(K_u\) to \(K_u\) we write rules for fuzzy linear PD controller in the following form [2]:

Figure 4. The scheme of the fuzzy control system
\[ R^1_{pd} : \text{if } e_1 \text{ is } N, \ de_1 \text{ is } N, \text{ then } u_1 \text{ is } NB, \]
\[ R^2_{pd} : \text{if } e_1 \text{ is } N, \ de_1 \text{ is } Z, \text{ then } u_1 \text{ is } NM, \]
\[ R^3_{pd} : \text{if } e_1 \text{ is } N, \ de_1 \text{ is } P, \text{ then } u_1 \text{ is } ZE, \]
\[ R^4_{pd} : \text{if } e_1 \text{ is } Z, \ de_1 \text{ is } N, \text{ then } u_1 \text{ is } NM, \]
\[ R^5_{pd} : \text{if } e_1 \text{ is } Z, \ de_1 \text{ is } Z, \text{ then } u_1 \text{ is } ZE, \]
\[ R^6_{pd} : \text{if } e_1 \text{ is } Z, \ de_1 \text{ is } P, \text{ then } u_1 \text{ is } PM, \]
\[ R^7_{pd} : \text{if } e_1 \text{ is } P, \ de_1 \text{ is } N, \text{ then } u_1 \text{ is } ZE, \]
\[ R^8_{pd} : \text{if } e_1 \text{ is } P, \ de_1 \text{ is } Z, \text{ then } u_1 \text{ is } PM, \]
\[ R^9_{pd} : \text{if } e_1 \text{ is } P, \ de_1 \text{ is } P, \text{ then } u_1 \text{ is } PB. \]

In the window “Untitled FIS Editor” by command File, New FIS, we choose the Mamdani type fuzzy inference Mamdani and ask for the variables \( e_1 \) and \( de_1 \) triangle, and for the variable \( u_1 \) single-tone membership functions. In the field choose Defuzzification method bisector. Start the modeling process of control system by pressing the Start simulation. The units Display will reflect the current values of the coefficients \( Ke \) and \( Ku \). At the command prompt MATLAB input different values of \( Ke \) and \( Ku \). The best reaction to a single jump was obtained with \( Ke = 4; Ku = 1.2 \), but this was a violation of all constraints.

3. Optimization of fuzzy PID coefficients

Double-clicking the block Check Step Response Characteristics will open the window of the task block parameter “Sink Block Parameters: Check Step Response Characteristics”, and ask the edge of the output signal: rise time (Rise time) – not more than 6 and the duration of the transition process (Setting time) – not more than 15 s (figure 5).

![Figure 5. Setting limits](image)

put a tick next to Show plot on block open or select additional optimization settings press the button Response Optimization, which opens the window “Design Optimization”. Drag the mouse to box the Model Workspace, the coefficients \( Ke, Kde \) and \( Kdu \) to be optimized. Then we define the variables, due to the optimization which will be improved as the transition process. On the Response tab field Optimization in the Design Variables Set, select New. A window will open the “Create Design Variables Set” on the right side which will choose custom variables \( Ke, Kde, Ku \) and \( Ki \), and using the arrow move them to the left side of the window, as shown in figure 6.
On the Optimization method tab, choose the optimization method as the Simplex Search, and then start the process of optimizing the parameters of the controller by pressing the button Optimize. Will get a graph of the transient process (figure 7) that satisfies all the constraints specified in the window in figure 5 and displayed in figure 7.

![Figure 6. Enter initial values of parameters](image)

![Figure 7. Optimal transient process](image)

The resulting mean modular error is 2.7%, not exceeding the maximum allowable (5%) at the following values of coefficients: $K_e = Ke = 2.82 \times 10^{-4}; \ K_{de} = Kde = 5.57; \ Ki = K_i = 0.053; \ Ku = Ku = 2.69$.

4. Conclusion

Implemented linearization of the combined fuzzy PID controller Mamdani. By the method of Ziegler and Nichols determined the initial values of the tuning coefficients of the linearized fuzzy PID controller. In Simulink is composed of digital circuit modeling of the control system with fuzzy PID controller. With this scheme, carried out parametric optimization, which are factors of a fuzzy PID controller, which provide the required quality of the transition process.

5. References

[1] Jantzen J. 2007 Foundations of Fuzzy Control (Chichester. John Wiley & Sons).
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