The properties of the three-nucleon system with the dressed-bag model for nn interaction. I: New scalar three-body force

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Abstract. A multi-component formalism is developed to describe three-body systems with nonstatic pairwise interactions and non-nucleonic degrees of freedom. The dressed-bag model for $NN$ interaction based on the formation of an intermediate six-quark bag dressed by a $\sigma$-field is applied to the $3N$ system, where it results in a new three-body force between the six-quark bag and a third nucleon. Concise variational calculations of $3N$ bound states are carried out in the dressed-bag model including the new three-body force. It is shown that this three-body force gives at least half the $3N$ total binding energy, while the weight of non-nucleonic components in the $^3H$ and $^3He$ wavefunctions can exceed 10%. The new force model provides a very good description of $3N$ bound states with a reasonable magnitude of the $\sigma NN$ coupling constant. The model can serve as a natural bridge between dynamical description of few-nucleon systems and the very successful Walecka approach to heavy nuclei and nuclear matter.

1. Introduction

In recent years, few-nucleon theory based on traditional models for the $NN$ force had significant success [1,2]. In particular, a high level of precision for the agreement between $3N$ and $4N$ calculations and the corresponding experimental data has been reached in many cases. Such a degree of agreement obtained without any free parameter could promise a description of all properties of few-nucleon systems in the near future by solving the exact Faddeev-Yakubovsky equations with the inclusion of modern $NN$ potentials and $3N$ forces.
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However, rather significant disagreements between results of the most exact few-nucleon calculations and experimental data still remain. Among all such discrepancies, we mention here only the most important ones such as the A_y puzzle in $\overline{N} + d$ and $\overline{N} + ^3\text{He}$ scattering, disagreements at the minima of differential cross sections (Sagara puzzle) at $E \sim 150 \div 200$ MeV and polarization data for $N + \overline{d}$, $\overline{N} + d$, $\overline{N} + \overline{d}$, and $\overline{N} + ^3\text{He}$ scattering. The strongest discrepancy between current theories and experiments has been found in studies of the short-range $NN$ correlations in the $^3\text{He}(e,e'pp)$, $^4\text{He}(\gamma,pp)$, and $^3\text{He}(e,e'NN)$ processes. The strong short-range correlations found in these experiments are likely due to (at least partially) a three-body force acting in a space region where two nucleons are close to each other. Therefore, in order to quantitatively describe such correlations, some new types of three-body forces should be introduced.

In addition to these particular problems, there are more fundamental problems in the current theory of nuclear forces, e.g., strong discrepancies between the $\pi NN$, $\pi N \Delta$ and $\rho NN$ form factors used in OBE models for the description of elastic and inelastic scattering and in the parameterization of $2N$ and $3N$ forces. Many of these difficulties are attributed to a rather poor knowledge of the short-range behaviour of nuclear forces. This behaviour was traditionally associated with the vector $\omega$-meson exchange. However, the characteristic range of this $\omega$-exchange (for $m_\omega \simeq 780$ MeV) is equal to about $\lambda_\omega \simeq 0.2 \div 0.3$ fm, i.e., is deeply inside the internucleon overlap region. Therefore, the quark structure of nucleons should be important at this short range.

Within six-quark dynamics in its turn, it has long been known that the mixing of the completely symmetric $s^6[6]$ component with the mixed-symmetry $s^4p^2[42]$ component can determine the structure of the whole short-range interaction (in the $S$-wave). Assuming a reasonable $qq$ interaction model, many authors (see e.g. [18, 19, 20, 21]) have demonstrated that this mixture can result in both strong short-range repulsion (associated mainly with the $s^6$ component) and intermediate-range attraction (associated mainly with the above mixed-symmetry $s^4p^2$ component). However, recent studies for $NN$ scattering on the basis of the newly developed Goldstone-boson-exchange (GBE) $qq$ interaction have resulted in a purely repulsive $NN$ contributions from both $s^6[6]$ and $s^4p^2[42]$ six-quark components. There is no need to say that any quark-motivated model for the $NN$ force with $\pi$-exchange between quarks inevitably leads to the well-established Yukawa $\pi$-exchange interaction between nucleons at long distances.

Trying to solve the above problems (and to understand more deeply the mechanism for the short-range $NN$ interaction), we suggested to add to the conventional Yukawa meson-exchange ($t$-channel) mechanism (at intermediate and short ranges) contributions of $s$-channel graphs describing the formation of a dressed six-quark bag in an intermediate state such as $|s^6 + \sigma\rangle$ or $|s^6 + 2\pi\rangle$. It has been shown that, due to the change in the symmetry of the six-quark state in the transition from the $NN$ channel to the intermediate dressed-bag state, the strong scalar $\sigma$-field arises around the symmetric $6q$ bag. This intensive $\sigma$-field squeezes the bag and increases its density. The high quark...
density in the symmetric $6q$ bag enhances meson field fluctuations around the bag and thereby partially restores the chiral symmetry. Therefore, the masses of constituent quarks and $\sigma$ mesons decrease. As a result of this phase transition, the dressed bag mass decreases considerably (i.e., a large gain in energy arises), which manifests itself as a strong effective attraction in the $NN$ channel at intermediate distances. This attraction can be described in terms of the OBE model as the effective $t$-channel $\sigma$-exchange.

However, more accurate calculations of the intermediate-range $NN$ interaction within the $2\pi$-exchange model with the $\pi\pi$ $s$-wave interaction have revealed that this $t$-channel mechanism cannot give a strong intermediate-range attraction in the $NN$ sector, which is necessary for binding of a deuteron and fitting of $NN$ phase shifts. The fact that the conventional $t$-channel $2\pi$-exchange with the $\pi\pi$ $S$-wave interaction and reasonably soft $\Lambda_{NN}$ cut-off values cannot give the sufficiently strong intermediate-range $NN$ attraction has also been corroborated by recent independent calculations. Thus, the $t$-channel mechanism of the $\sigma$ exchange should be replaced by the corresponding $s$-channel mechanism. The contribution of the $s$-channel mechanism would generally be much larger due to resonance-like enhancement.

Based on this $s$-channel mechanism, we proposed a new model for the $NN$ interaction (referred to as the “dressed bag model” (DBM)) that provides a quite good description of both $NN$ phase shifts up to 1 GeV and the deuteron structure. The developed model includes both the conventional $t$-channel contributions (Yukawa $\pi$ and $2\pi$-exchanges) at long and intermediate distances and the $s$-channel contributions due to the formation of intermediate dressed-bag states at short distances. The most important distinction of such an approach from conventional models for nuclear forces is the explicit presence of a non-nucleonic component in the total wavefunction of the system, which necessarily implies the presence of new three-body forces (3BF) of several kinds in the $3N$ system. These new three-body forces differ from conventionally used models for three-body forces. One important aspect of the three-body force should be emphasized here. In conventional OBE models, the main contribution to $NN$ attraction is due to the $t$-channel $\sigma$ exchange. However, the three-body force models suggested until now (such as Urbana-Illinois or Tucson-Melbourne) are mainly based on the two-pion exchange with intermediate $\Delta$-isobar production, and the $\sigma$-exchange either is not taken into account or is of little importance in these models. In contrast, $\sigma$-exchange in our approach dominates in both $NN$ and $3N$ forces; i.e., the general pattern of the nuclear interaction appears to be more consistent.

The aim of this work is just to study both the new type of three-body forces and the properties of the $3N$ system with the dressed-bag model (DBM) proposed for $NN$ and $3N$ forces. In particular, we will show that the role of 3BF changes remarkably compared to the conventional OBE models. In the conventional models, the main contribution to the $3N$ binding energy comes from the pair $NN$ force, whereas 3BF gives a certain

\[ t\text{-channel mechanism can be associated with the direct nuclear reaction where only a few degrees of freedom are important, while the } s\text{-channel mechanism can be associated with resonance-like (or compound nucleus like) nuclear reactions with much larger cross sections at low energies.} \]
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(although significant) correction of about 15% to the total binding energy. In contrast, in our DBM approach the contribution of the new scalar three-body forces turns out to be larger by several times and is equal to about half the total 3N binding energy.

This paper is organized as follows. In Sect.II, we give a brief description of the DBM for the NN system. In Sect. III, we develop the multi-component formalism for the 3N system. In Sect. IV, the general formalism is specified to the one-pole approximation for the dressed-bag resolvent, which is used in our calculations. In Sect. V, three types of 3BF are considered: one-meson exchange (π and σ) between the dressed bag and third nucleon and the 2σ-exchange with breaking of the σ-loop in the two-nucleon interaction. The results of our variational calculation of the 3N system are given in Sect.VI and the results are discussed in Sect.VII. In the Conclusion we summarize the main results of the work.

2. Dressed bag model for NN forces

Here, we give a brief description of the two-component dressed-bag model for the NN interaction. The detailed description has been presented in our previous papers [9, 22]. We consider a system that can exist in two different phase states: the NN phase and dressed six-quark bag phase. In the NN channel, the system can be described as two nucleons interacting via one-boson exchange; in the 6q+σ channel, the system is treated as a six-quark bag surrounded by the strong scalar-isoscalar σ-field (a ”dressed” bag).§ Accordingly, the wavefunction of the system consists of two components of different nature:

\[ \Psi = \left( \begin{array}{c} \Psi_{NN} \\ \Psi_{6q+\sigma} \end{array} \right). \]

The Hamiltonian of the system has the corresponding matrix form

\[ \hat{H} = \begin{pmatrix} H_{NN} & H_{NN,6q+\sigma} \\ H_{6q+\sigma,NN} & H_{6q+\sigma} \end{pmatrix}. \]

Here, \( H_{NN} \) includes the direct NN interaction induced by one- and two-pion exchanges with the \( \pi NN \) form factors taken with the correct “soft” cut-off parameters [9, 22]. Thus, \( H_{NN} \) describes the peripheral part (at \( r_{NN} > 1 \text{ fm} \)) of the NN interaction. \( H_{6q+\sigma} \) is the Hamiltonian of the dressed bag, while the operators \( H_{6q+\sigma,NN} \) and \( H_{NN,6q+\sigma} \) describe the transitions from \( NN \) to the \( (6q+\sigma) \) channel and vice versa. The Schrödinger equation|| reduced to two coupled equations and, by excluding the second component

§ Full description of the NN interaction at energies \( E \sim 1 \text{ GeV} \) still requires other fields in the bag such as 2π, ρ and ω but here we employ the version of DBM including only a leading σ-field [22] with parameters which are determined either phenomenologically from fit of NN phase shifts (variant (I)) or taken directly from the respective σ-loop diagram (the variant (II)).

|| In the force models described here the s-channel mesons are treated relativistically, while the heavy 6q bag and nucleons are taken in the nonrelativistic approximation. This is quite reasonable at the energy range \( E_{\text{Lab}} \leq 0.8 \text{ GeV} \).
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Ψ_{6q\sigma}, one obtains the following equation for the proper NN channel wavefunction with the effective Hamiltonian:

\[ H_{NN} \Psi_{NN} + H_{NN,6q\sigma}(E - H_{6q\sigma})^{-1}H_{6q\sigma,NN} \Psi_{NN} = E \Psi_{NN}. \]  

Having obtained the solution of this equation for the \( \Psi_{NN} \) component, one can also find straightforwardly the second component \( \Psi_{6q\sigma} \):

\[ \Psi_{6q\sigma} = (E - H_{6q\sigma})^{-1}H_{6q\sigma,NN} \Psi_{NN}. \]  

Using the following simple one-pole approximation for the dressed bag resolvent \((E - H_{6q\sigma})^{-1}\):

\[ (E - H_{6q\sigma})^{-1} = \sum_\alpha \int \frac{\langle \alpha, k | \langle \alpha, k |}{E - E_{6q\sigma}(\alpha, k)} \mathrm{d}k, \]

where \(|\alpha\rangle\) is the \(6q\) part of the wavefunction for the dressed bag and \(|k\rangle\) represents the plane wave of the \(\sigma\)-meson propagation, one obtains an effective interaction in the NN channel as a sum of separable terms.\footnote{The single-pole approximation for the dressed-bag resolvent should be quite valid at incident energies in NN-channel \(E_{\text{lab}} < 1 \text{ GeV}\) because the energy spacing \(\hbar \omega\) between different excited states of the dressed bag (with different parity and number of radial nodes) is expected around 300-350 MeV. So that the spacings between excited states in the fixed channel (with the same parity) should be around \(2\hbar \omega = 600 \div 700 \text{ MeV}\) in the c.m. system. Hence a few-quantum excitations of the bare \(6q\) bag should lie very high in the spectrum.}

We emphasize that the sign of the dressed bag wavefunction \(\Psi_{6q\sigma}\) at low and intermediate energies \(E < E_0\) (where \(E_0\) is the lowest eigenenergy of the dressed bag) is opposite to that of the NN wavefunction.

To solve equation (3) in such an approximation, the knowledge of total Hamiltonian \(H_{6q\sigma}\) of the dressed bag, as well as the total transition operator \(H_{NN,6q\sigma}\), is not necessary. Only the projections of the transition operator \(H_{NN,6q\sigma}|\alpha\rangle\) onto the bag states \(|\alpha\rangle\) are necessary to construct the interaction in the NN channel. These projections have been calculated in the microscopic quark-meson model \([9, 22]\). The effective interaction \(V_{NqN}\) induced in the NN channel by coupling with the intermediate dressed bag state is illustrated by the graph in Fig. 1.

\[ \]
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axis, respectively, and \( L_\sigma \) is the orbital angular momentum of the \( \sigma \) meson. However, in the present version of the DBM, only the s-wave state of the \( 6q \) bag with the \( s^6 \) configuration is taken into account, so that \( L_\theta = 0, J = S \), and the isospin of the bag is uniquely determined by its spin. The states of the dressed bag with \( L_\sigma \neq 0 \) should lie higher than those with \( L_\sigma = 0 \). For this reason, the former states are not included in the present version of the model. Therefore, the state index \( \alpha \) is specified here by the total angular momentum of the bag \( J \) and (if necessary) its \( z \) projection \( M \): \( \alpha \Rightarrow \{J(M)\} \).

Thus, the effective interaction in the \( NN \) channel \( V_{NqN} \equiv H_{NN,6q\sigma}(E - H_{6q\sigma})^{-1}H_{6q\sigma,NN} \), after partial-wave decomposition, can be written as a sum of separable terms in each partial wave:

\[
V_{NqN} = \sum_{J,L,L'} V_{LL'}^{J}(r,r'),
\]

with

\[
V_{LL'}^{J}(r,r') = \sum_{M} \varphi_{L}^{J,M}(r) \lambda_{LL'}^{J}(E) \varphi_{L'}^{J,M*}(r').
\]

The energy-dependent coupling constants \( \lambda_{LL'}^{J}(E) \) appearing in eq. (7) are calculated from the diagram shown in Fig. 1; i.e., they are expressed in terms of the loop integral of the product of two transition vertices \( B \) and the convolution of two propagators for the meson and quark bag with respect to the momentum \( k \):

\[
\lambda_{LL'}^{J}(E) = \int_{0}^{\infty} \frac{dk B_{L}^{J}(k,E) B_{L'}^{J*}(k,E)}{E - m_\alpha - \varepsilon_{\sigma}(k)},
\]

where \( \varepsilon_{\sigma}(k) \) is the kinetic energy of the dressed bag: \( \varepsilon_{\sigma}(k) = k^2/2m_\alpha + \omega_{\sigma}(k) \approx m_\sigma + k^2/2\tilde{m}_\sigma \), \( \tilde{m}_\sigma \) is the reduced mass in the \( 6q + \sigma \) channel \( \tilde{m}_\sigma = m_\sigma m_\alpha/(m_\sigma + m_\alpha) \), and \( \omega_{\sigma}(k) \) being relativistic energy of \( \sigma \)-meson: \( \omega_{\sigma}(k) = \sqrt{m_\sigma^2 + k^2} \). The vertex form factors \( B_{L}^{J}(k) \) and the potential form factors \( \varphi_{L}^{J,M}(r) \) have been calculated in the microscopic quark-meson model [9, 22].

When the \( NN \)-channel wavefunction \( \Psi_{NN} \) is obtained by solving the Schrödinger equation with the effective Hamiltonian \( H_{NN} + V_{NqN} \), the \( 6qN \)-component of the wavefunction can be found algebraically from eq. (10):

\[
|\Psi_{6q\sigma}^{J,M}(E)\rangle = \Psi_{\sigma}^{J}(k,E)|\alpha^{JM}\rangle,
\]

where the function

\[
\Psi_{\sigma}^{J}(k,E) = \sum_{L} C_{L}^{J}(E) \frac{B_{L}^{J}(k,E)}{E - m_\alpha - \varepsilon_{\sigma}(k)}
\]

can be interpreted as the mesonic part of the dressed-bag wavefunction. The coefficients \( C_{L}^{J}(E) \) in eq. (12) are overlap integrals of the nucleon-nucleon wavefunction and the form factors \( \varphi_{L}^{J,M}(r) \) of the separable potential (7):

\[
C_{L}^{J}(E) = \int \Psi_{NN}(r,E) \varphi_{L}^{J}(r) dr.
\]
The contribution of the dressed-bag component to the total wavefunction is proportional to the norm of $\Psi_{6q\sigma}$:

$$
\| \Psi_{6q\sigma} \|^2 = \| \alpha \|^2 \int \Psi^{JM}_\sigma(k, E) \Psi^{JM*}_\sigma(k, E) dk
$$

$$
= \sum_{LL'} C^J_L(E) C^{J*}_{L'}(E) \int_0^\infty \frac{B^J_L(k, E) B^{J*}_{L'}(k, E)}{(E - m_\alpha - \varepsilon_\sigma(k))^2} dk
$$

$$
= \sum_{LL'} C^J_L(E) C^{J*}_{L'}(E) P^J_{LL'}(E). \tag{12}
$$

If the vertex form factors $B^J_L$ are energy independent, the factors $P^J_{LL'}(E)$ are simply related to the coupling constants $\lambda^J_{LL'}(E)$ (see eq.(8)):

$$
P^J_{LL'}(E) = -\frac{d\lambda^J_{LL'}(E)}{dE}. \tag{13}
$$

The total wavefunction of the bound state $\Psi$ must be normalized. Assuming that the nucleonic part of the wavefunction $\Psi_{NN}$ found from the effective Schrödinger equation has the standard normalization $\| \Psi_{NN} \| = 1$, one obtains that the weight of the dressed bag component is equal to

$$
P_{6q} = \frac{\| \Psi_{6q\sigma} \|^2}{(1 + \| \Psi_{6q\sigma} \|^2)}. \tag{14}
$$

| Model          | $E_d$(MeV) | $P_D$(%) | $r_m$(fm) | $Q_d$(fm$^2$) | $\mu_d$($\mu_N$) | $A_S$(fm$^{-1/2}$) | $\eta(D/S)$ |
|----------------|------------|----------|-----------|---------------|------------------|-------------------|-------------|
| RSC            | 2.22461    | 6.47     | 1.957     | 0.2796        | 0.8429           | 0.8776            | 0.0262      |
| Moscow 99      | 2.22452    | 5.52     | 1.966     | 0.2722        | 0.8483           | 0.8844            | 0.0255      |
| Bonn 2001      | 2.224575   | 4.85     | 1.966     | 0.270         | 0.8521           | 0.8846            | 0.0256      |
| DBM (var. a)   | 2.22454    | 5.22     | 1.9715    | 0.2754        | 0.8548           | 0.8864            | 0.0259      |
| $P_{6q} = 3.66\%$ |           |          |           |               |                  |                   |             |
| DBM (var. b)   | 2.22459    | 5.31     | 1.970     | 0.2768        | 0.8538           | 0.8866            | 0.0263      |
| $P_{6q} = 2.5\%$ |           |          |           |               |                  |                   |             |
| experiment     | 2.224575   |          | 1.971     | 0.2859        | 0.8574           | 0.8846            | 0.0263*     |

* An average value of the asymptotic mixing parameter $\eta$ over a few most accurate results is presented here (see refs. [25, 26, 27, 28]).

The model constructed above gives a very good description for the coupled-channel $^3S_1 - ^3D_1$ phase shifts, mixing parameter $\varepsilon_1$ and singlet $^1S_0$ shifts in the energy region from zero up to 1 GeV [22]. The deuteron observables obtained in this model without any additional parameter are presented in Table 1 in comparison with some other $NN$ models and experimental values. The quality of agreement with experimental data for the deuteron static properties found with the presented force model, in general, is higher.
than those for the modern $NN$ potential model such as Bonn, Argonne, etc., especially for the asymptotic mixing parameter $\eta$ and the deuteron magnetic and quadrupole moments. The weight of the dressed bag component in the deuteron is calculated from the energy dependence of the coupling constants $\lambda^J_{LL'}$ (see eqs. (13) and (14) and is varied from 2.5% to 3.6% in different versions of the model [9, 22].

3. DBM for the three-nucleon system

When generalizing the above model to the $3N$ system, one meets two difficulties: the appearance of the energy dependent coupling constants $\lambda^J_{LL'}(E)$ and the presence of non-nucleonic degrees of freedom in an explicit form. Thus, we suggest that the $3N$ system can be found in the five different states (in three phases): $NNN$, $N_1 + (6q^{(23)}\sigma)$, $N_2 + (6q^{(31)}\sigma)$, $N_3 + (6q^{(12)}\sigma)$, and $9q + \sigma$. Therefore, the total wave function can be represented as the five-row Fock column

$$
\Psi = \begin{pmatrix}
\Psi_{3N} \\
\Psi_{(1)}^{(1)} \\
\Psi_{(2)}^{(2)} \\
\Psi_{(3)}^{(3)} \\
\Psi_{9q+\sigma}
\end{pmatrix}.
$$

The nucleonic component $\Psi_{3N}$ here describes the system as three nucleons interacting at large distances via (one- and two-) boson exchanges. In the $(6qN)^{(i)}$ channels, there are the dressed $6q$ bag and $i$th nucleon ($i = 1, 2, 3$). Hereinafter, the symbol $\sigma$ in the notation of these channels is omitted for brevity. The wavefunction component $\Psi_{(i)}^{(i)}$ depends on the dressed bag variables, including the momentum of the $\sigma$-meson, and the $6q-N$ relative motion variable (the Jacobi coordinate or momentum). The fifth component of the total wavefunction (15) describes a dressed nine-quark bag.

The admixture of the dressed $9q$-bag in $3N$ system and in "normal" nuclei should be rather low, at least one order of magnitude lower as compared to the weight of the $6q$ bag. This can be easily explained by the fact the $9q$-clusters in nuclei can be only generated from $6q$ and $3q$-clusters at their fusion which has rather low probability due to very high kinetic energy of $9q$ system. So the weight of $9q$ configuration must be a small fraction of that for $6q + 3q$ component. Thus one can expect only a very minor effects of this $9q$ configurations except the region of very high momentum transfer.

Therefore, we neglect this component in present study and consider below the 4-component system with the $(4x4)$-matrix Hamiltonian

$$
\hat{H} = \begin{pmatrix}
H_{3N} & H_{3N,1} & H_{3N,2} & H_{3N,3} \\
H_{1,3N} & H_1 & 0 & 0 \\
H_{2,3N} & 0 & H_2 & 0 \\
H_{3,3N} & 0 & 0 & H_3
\end{pmatrix}.
$$

Here, we use the brief notation for the following parts of the Hamiltonian: the operator $H_{3N,i} \equiv H_{3N,(6q^{(jk)}+\sigma)+N_i}$ describes the transition from the $\{(6q^{(jk)}+\sigma)+N_i\}$ channel to
the pure three-nucleon channel; the Hamiltonian $H \equiv H_{6q+iN}$ describes the interacting system of the dressed $6q$ bag and $i$th nucleon.

In our $3N$ dynamics, we neglect the direct transitions between different $6q - N$ channels. Such channels are coupled only via an intermediate $3N$ configurations. This assumption looks absolutely natural, while the incorporation of direct transitions between such channels requires some exotic new mechanisms for the coupling.

The Hamiltonian $H_{3N}$ of the nucleonic channel includes the kinetic energy of the relative motion of three nucleons and the direct $NN$ interactions (OPE and TPE with soft cut-offs) that enter into the two-body Hamiltonian:

$$h_{NN} = t + V_{NN} \Rightarrow H_{3N} = T + \sum_{i=1}^{3} V_{NN}^{(i)}.$$  \hfill (17)

Hereinafter, we use lowercase letters ($g$, $h$, etc.) for two-body operators and capital letters for three-body ones.

The $(6qN \rightarrow 3N)$-transition operators $H_{3N,i}$ do not affect the third nucleon and therefore take the form:

$$H_{3N,i} = h_{NN,6q+iN}^{(jk)} \otimes 1^{(i)} \text{, where } (ijk) = (123), (231), (312),$$  \hfill (18)

and $h_{NN,6q+iN}^{(jk)}$ are the two-body DBM transition operators for pair $(jk)$.

Since direct transitions between different $6qN$ channels are absent, the exclusion of the $6qN$ components is completely similar to the two-body case: hence one can rewrite the 4-component Hamiltonian in the two-component form

$$\hat{H} = \begin{pmatrix} H_{3N} & H_{3N,6qN} \\ H_{6qN,3N} & H_{6qN} \end{pmatrix},$$  \hfill (19)

where $H_{6qN}$ is the diagonal (3x3)-matrix operator with the elements $H_i$. The resolvent of $H_{6qN}$ is also a diagonal (3x3)-matrix.

The Hamiltonian $H_i$ describes the dressed bag $6q(jk)$ (formed by the fusion of the $j$th and $k$th nucleons) interacting as a whole with the third ($i$th) nucleon via exchanges by mesons ($\pi$, $\sigma$). Just this interaction results in the new three-body force arising in the two-phase model. The Hamiltonian

$$H_i = h_{6q+i\sigma}^{(jk)} \oplus h_{6q,N^{(i)}}^{(i)},$$  \hfill (20)

is the direct sum of the dressed-bag Hamiltonian $h_{6q+i\sigma}^{(jk)}$ and single-particle Hamiltonian of the $i$th nucleon $h_{6q,N^{(i)}}^{(i)}$, which includes its kinetic energy and the potential energy of its interaction with the bag:

$$h_{6q,N^{(i)}}^{(i)} = t_i + V_{6q,N^{(i)}}$$  \hfill (21)

It depends only on the single Jacobi coordinate (or momentum) of the third ($i$th) nucleon. Therefore, the resolvent of $H_i$ is equal to the convolution of two subresolvents: the first for the dressed-bag Hamiltonian $(E - h_{6q+i\sigma}^{(jk)})^{-1}$ and the second for the single-particle Hamiltonian (21):

$$(E - H_i)^{-1} = -\frac{1}{2\pi i} \int_{-\infty}^{\infty} (\varepsilon - h_{6q+i\sigma}^{(jk)})^{-1} g_{6q,N^{(i)}}(E - \varepsilon) d\varepsilon,$$  \hfill (22)
where
\[ G_{6q,N_i}(E) = (E - h_{6q,N_i})^{-1}. \]  (23)

Excluding the 6qN channels from the Schrödinger equation for the total four-component wave function, one arrives at the following effective Hamiltonian \( H_{3N} \) acting in the 3N channel:
\[ H_{3N} = H_{3N} + \sum_{i=1}^{3} H_{3N,i}(E - H_i)^{-1} H_{i,3N} \]  (24)
which has the form of the standard three-body Hamiltonian with pairwise energy-dependent interactions:
\[ H_{3N} = T + \sum_{i=1}^{3} (V_{NN}^{(i)} + V_{6qN,3N}^{(i)}); \]  (25)

This form of the effective Hamiltonian is quite suitable for the standard Faddeev reduction. Although the effective interaction is the sum of three "pairwise" interactions, each term \( V_{3N,6qN,3N}^{(i)} \) describes not a genuine two-body force but a three-body force including the interaction between the 6q bag and third nucleon. Moreover, even if this interaction is disregarded, the potential \( V_{3N,6qN,3N}^{(i)} \) still depends on the momentum of the third nucleon. This dependence on the momentum of the third particle reduces the strength of the effective interaction between two other particles due to a specific energy dependence of the coupling constants. Actually, the form of the effective Hamiltonian (24) implies that there are no pure two-body forces in the 3N system in our approach, except the peripheral part of the OBE interaction.

After solving the Faddeev equations (or the Schrödinger equation for bound states) with the effective Hamiltonian (24) for the 3N wave function, the "bag" components of the total wave function are "recovered" by means of the relationship:
\[ \Psi_{6q,N}^{(i)}(E - H_i)^{-1} H_{i,3N} \Psi_{3N}. \]  (26)

4. Single-pole approximation for the dressed bag resolvent

The momentum representation is more appropriate for description of the 3N system in the case of DBM. We will employ the same notation for functions both in the coordinate and momentum representations. In the single-pole approximation for the dressed-bag resolvent, the resolvent \((E - H_i)^{-1}\) (22) can be reduced to a sum of factorized terms:
\[ (E - H_i)^{-1} = \sum_\alpha \int |\alpha^{(i)}(k)| G_{6q,N_i}(E - m_\alpha - \varepsilon_\alpha(k)) \langle \alpha^{(i)}(k), \mathbf{k} | \mathbf{d} \mathbf{k} \]  (27)
and the effective 3N interaction \( V_{3N,6qN,3N}^{(i)} \) becomes the sum of integral operators with factorized kernels:
\[ V_{3N,6qN,3N}^{(i)}(p_i, p'_i, q_i, q'_i; E) = \sum_{J M, J'M', L, L'} \varphi_L^{JM}(p_i) W_{JJ'}^{LL'}(q_i, q'_i; E) \varphi_L^{J'M'}(p'_i). \]  (28)
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where \( W_{JL}^{\prime J_L}(q_i, q_i'; E) \) is expressed in terms of the integral of the product of the vertex functions and one-particle resolvent \( [23] \):

\[
W_{JL}^{\prime J_L}(q_i, q_i'; E) = \int dk B_L^J(k) G_{6q,N}(q_i, q_i'; E - m_\alpha - \varepsilon(k)) B_L^{J'}(k). \tag{29}
\]

If the interaction between the 6q bag and third nucleon is neglected, the resolvent \( G_{6q,N} \) becomes the simple propagator:

\[
G_{6q,N}(q_i, q_i'; E - m_\alpha - \varepsilon(k)) \Rightarrow \delta(q_i - q_i') \frac{1}{E - m_\alpha - \varepsilon(k) - q^2/2m}; m = \frac{m_N m_\alpha}{m_N + m_\alpha}. \tag{30}
\]

Then, the effective interaction reduces to the sum of two-body separable potentials with the coupling constants depending on the total three-body energy \( E \) and third-particle momentum \( q_i \):

\[
\tilde{V}_{3N,6qN,3N}^{(i)}(p_i, p_i', q_i, q_i'; E) = \delta(q_i - q_i') \sum_{J,M,L,L'} \varphi_J^{LM}(p_i) \lambda_L^{J}(E - \frac{q_i^2}{2m}) \varphi_M^{L'}(p_i'), \tag{31}
\]

where the coupling constants \( \lambda_L^{J} \) are determined by eq.\([8]\) with replacing \( E \) by \( E - \frac{q_i^2}{2m} \). In other words, the effective interaction in the three-body system reduces to a sum of three effective pairwise interactions \( V_{NqN} \) depending on the total three-body energy and momentum of the third (spectator) particle:

\[
V_{3N,6qN,3N} \Rightarrow \sum_{i=1}^{3} \delta(q_i - q_i') V_{NqN}^{(i)}(p_i, p_i'; E - \frac{q_i^2}{2m}). \tag{32}
\]

It should be noted that this result does not depend on specific choice of interaction model, but is quite general provided that there are not direct transition between different 6qN channels and the transition operators take the form \([18]\).

![Figure 2](image-url)

**Figure 2.** Different interactions in the 3N system for one of three possible combinations (1+23): the peripheral two-nucleon interaction is due to OPE + TPE, effective two-body interaction \( V_{NqN}^{(1)} \) is induced by the formation of dressed six-quark bag and meson-exchange three-body force (3BF).

When using the effective interaction \([32]\), one must also include an additional three-body force due to the meson-exchange interaction between the dressed bag and third
nucleon. The pattern of different interactions arising in the 3N system in such a way is illustrated in Fig. 2.

In the single-pole approximation, the components of the "bag" wave function can be expressed in terms of the nucleonic component as

$$\Psi_{6qN}^{(i)}(k, q_i; E) = \sum_{J_iM_iL_i} |\alpha^{J_iM_i}\rangle \frac{B_{L_i}^{J_i}(k) \chi_{L_i}^{J_iM_i}(q_i)}{E - m_\alpha - \frac{q_i^2}{2m} - \varepsilon(k)},$$

(33)

where $\chi_{L_i}^{J_iM_i}(q_i)$ are the overlap integrals of the three-nucleon component of the wavefunction and the potential form factors $\varphi_{L_i}^{J_iM_i}$:

$$\chi_{L_i}^{J_iM_i}(q_i) = \int \varphi_{L_i}^{J_iM_i}(p_i) \Psi_{3N}(p_i, q_i) \, dp_i.$$

(34)

These overlap functions depend on the momentum (or coordinate), spin, and isospin of the third nucleon. For brevity, the spin-isospin parts of the overlap functions and corresponding quantum numbers are omitted unless they are needed. In eqs. (33) and below, we keep the index $i$ in the quantum numbers $L$ and $J$ in order to distinguish the orbital and total angular momenta in the form factors from the respective angular momenta of the whole 3N system.

It should be noted that the angular part of the function $\chi_{L_i}^{J_i}(q_i)$ in eq. (34) is not equal to $Y_{L_iM_i}(\hat{q})$. This part includes other angular momenta due to coupling between the angular momenta and spins of the dressed bag and third nucleon. The total overlap function (without isospin part) can be written, for example, as (we use here the same letter for notation of both the total wavefunction and its radial part):

$$\chi_{L_i}^{J_iM_i}(q_i) = \sum_{l_i, J_{i/}, \vec{J}} \chi_{l_i, \vec{J}}^{J_i}(q_i) \langle J_{i/} J_{i/} \rangle \gamma_{l_{i1/2}}^{J_{i/} J} \chi_{l_{i1/2}}^{J_{i/} J}(q_i).$$

(35)

Here, $J$ and $M$ are the total angular momentum of the 3N system and its $z$ projection, respectively; $l_i$ and $\vec{J}$ are the orbital and total angular momenta of the third (ith) nucleon, respectively; $J_i$ is the total angular momentum of the bag ($J_i = S_i$, because in our case the bag has zero orbital momentum); and $L_i$ is the orbital angular momentum of a nucleon pair ($jk$) related to the transition vertex ($NN \rightarrow 6q + \sigma$). For the total angular momentum $J = 1/2$ (the ground state of $^3H$ and $^3He$), the orbital angular momentum of the third nucleon $l_i$ (in the c.m.s. of the entire system) can take the values $J_i \pm 1$ (we mean here the $6qN$ component only). Therefore, $l_i$ in our present model can be equal to 0 or 2.

The total norm of all three $6qN$ components for the 3N bound state is determined by the integral

$$\|\Psi_{6qN}\|^2 = 3 \sum_{J_i} \|\alpha^{J_i}\|^2 \sum_{L_iL_i'} \int \chi_{L_i}^{J_i}(q_i) \left\{ \int \frac{B_{L_i}^{J_i}(k)B_{L_i'}^{J_i}(k)}{(E - m_\alpha - \frac{q_i^2}{2m} - \varepsilon(k))^2} \, dk \right\} \chi_{L_i'}^{J_i}(q_i) \, dq_i,$$

(36)

If the vertex functions $B_{L_i}^{J_i}(k)$ are energy independent (it was just conjectured when deriving the effective Hamiltonian), the internal loop integral with respect to $k$ in
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(36) (in braces) can be replaced by the energy derivative of $\lambda_{J_{LL}'}$:

$$
\int \frac{B_{L_i}^J(k)B_{L_i'}^J(k)}{(E - m_\alpha - \frac{q_i^2}{2m} - \varepsilon_\sigma(k))^2} \, dk = - \frac{d}{dE} \int \frac{B_{L_i}^J(k)B_{L_i'}^J(k)}{E - m_\alpha - \frac{q_i^2}{2m} - \varepsilon_\sigma(k)} \, dk
$$

Thus, the weight of the $6qN$ component in the $3N$ system is determined by the same energy dependence of the coupling constants $\lambda_{J_{LL}'}(\varepsilon)$ as the contribution of the $6q$ component in the $NN$ system but at a shifted energy.

Using eq.(37), the norm of $6qN$ component can be rewritten eventually as

$$
\|\Psi_{6qN}\|^2 = \sum_{J_i} \|\alpha^{J_i}\| \sum_{L_iL_i'} \int \chi_{L_i}^{J_i}(q_i) \left( - \frac{d}{dE} \lambda_{L_iL_i'}(E - \frac{q_i^2}{2m}) \right) \chi_{L_i'}^{J_i}(q_i) \, dq_i,
$$

(38)

Due to the explicit presence of the meson variables in our approach, it is generally impossible to define the wavefunction of the third nucleon in the $6qN$ channel. However, by integrating $\Psi_{6qN}(k, q)$ with respect to the meson momentum $k$, one can obtain the momentum distribution of the third nucleon in the $6qN$ channel weighted with the $\sigma$-meson momentum distribution. Based on eq.(38), we can attribute the meaning of the wavefunction of the third nucleon in the $6qN$ channel to the quantity

$$
\tilde{\psi}_{L_i}^{J_iM_i}(q_i) = \sqrt{\left( - \frac{d}{dE} \lambda_{L_iL_i'}(E - \frac{q_i^2}{2m}) \right) \chi_{L_i}^{J_i}(q_i)},
$$

(39)

With this “wavefunction”, one can calculate the mean value of any operator depending on the momentum (or coordinate) of the third nucleon. We note that the derivative $-d\lambda/dE$ is always positive.

5. Three-body forces in the dressed bag model

In this work, we employ the effective interaction (31) and take into account the interaction between the dressed bag and third nucleon as an additional three-body force (3BF). We consider here three types of 3BF: one-meson exchange ($\pi$ and $\sigma$) between the dressed bag and third nucleon (see Figs.3a and 3b) and the exchange by two $\sigma$-mesons, where the third-nucleon propagator breaks the $\sigma$-loop of the two-body force (Fig.3c).

Figure 3. The graphs corresponding to three new types of three-body force
All these forces can be represented as some integral operators with factorized kernels similar to the effective interaction $V_{3N,6qN,3N}^{(i)}$.

$$3BF V^{(i)}(p_i, p'_i, q_i, q'_i; E) = \sum_{JL,J'L',L,L'} \varphi_{JL}^{J'M}(p_i) 3BF W_{LL'}^{JJ'}(q_i, q'_i; E) \varphi_{J'L'}^{J'M'}(p'_i),$$  \hspace{1cm} (40)

Therefore, matrix elements for 3BF include only the overlap functions, and thus the contribution of 3BF is proportional to the weight of the $6qN$ component in the total wavefunction. To our knowledge, the first calculation of the 3BF contribution induced by one-pion exchange between the $6q$ bag and third nucleon was done by Fasano and Lee [29] in the hybrid quark-compound bag model (QCB) using perturbation theory. They used the model where the weight of the $6q$ component in a deuteron is ca. 1.7%, and thus they obtained a very small value of -0.041 MeV for the 3BF OPE contribution to the $3N$ binding energy. Our results for the OPE 3BF agree with the results obtained by Fasano and Lee, because the OPE contribution to 3BF is proportional to the weight of the $6q$ component, and it should be in our case at least twice as large as in their calculation. However, we found that a much larger contribution comes from scalar-meson exchanges (OSE and TSE). We emphasize that, due to (proposed) restoration of chiral symmetry in our approach, the $\sigma$-meson mass is ca. 400 MeV, and thus the effective radius of the $\sigma$-exchange interaction is not so small as that in conventional OBE models. Therefore, we cannot use the perturbation theory anymore to estimate the 3BF contribution and have to do the full calculation including 3BF in the total three-body Hamiltonian.

5.1. One-meson exchange between the dressed bag and third nucleon

For the one-meson exchange (OME) term, the three-body interaction $3BF W_{LL'}^{JJ'}$ takes the form:

$$OME W_{LL'}^{JJ'}(q_i, q'_i; E) = \int \frac{B_{J'}^L(k)}{E - m_\alpha - q_i^2/2m - \varepsilon_\sigma(k)} V_{OME}(q_i, q'_i) \frac{B_J^L(k)}{E - m_\alpha - q_i'^2/2m - \varepsilon_\sigma(k)}.$$  \hspace{1cm} (41)

Therefore, the matrix element for one-meson exchange can be expressed in terms of the wave function of the "bag" components $\Psi_{6qN}^{(i)}$:

$$\langle \Psi_{3N}|OME|\Psi_{3N} \rangle = 3\langle \Psi_{6qN}^{i}|V_{OME}|\Psi_{6qN}^{i} \rangle.$$  \hspace{1cm} (42)

Including spin-isospin variables (it is important for evaluation of pion-exchange interaction), we can write down $\Psi_{6qN}^{(i)}$ as

$$\Psi_{6qN}^{(i)} = \sum_{J_i M_i, L_i} \frac{B_{J_i}^{L_i}(k)}{E - E_\alpha - q_i^2/2m - \varepsilon_\sigma(k)} \chi_{L_i}^{J_i M_i}(q) |\alpha^{J_i M_i}| T_d, \frac{1}{2} : TT_z,$$  \hspace{1cm} (43)

where $T_d$ is the bag isospin, $T$ and $T_z$ are the total isospin of the system ($T = 1/2$ for $^3$H and $^3$He) and its $z$-projection respectively and $\chi_{L_i}^{J_i M_i}(q)$ are the overlap functions [35].
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When calculating the matrix elements for OME, one can use a similar trick as in the calculation of the norm for the 6qN component. It enables us to exclude the vertex functions \( B_{L}^{J}(k) \) from the formulas for matrix elements. Replacing the product of propagators in the integral with respect to the meson momentum \( k \) (eq. (41)) by their difference, one obtains the following expression free of the vertex functions:

\[
\int \frac{B_{L}^{J}(k)B_{L'}^{J}(k)}{(E-\varepsilon(k)-q^2/2m)(E-\varepsilon(k)-q'^2/2m)} \, dk = \frac{\lambda_{LL'}^{J}(E-q^2/2m) - \lambda_{LL'}^{J}(E-q'^2/2m)}{q^2 - q'^2} \]

This quantity is the finite-difference analogue of the derivative of \( \lambda \) with respect to \( q^2 \).

In the present calculations, we employed a rational approximation for the energy dependence of \( \lambda_{LL'}^{J} \) [22]:

\[
\lambda_{LL'}^{J}(E) = \lambda_{LL'}^{J}(0) \frac{E_0 + aE}{E_0 - E},
\]

where the parameters \( E_0 \) and \( a \) are taken to be the same for all \( \lambda \)'s. We found that this simple rational form can reproduce quite accurately the exact energy dependence of the coupling constants \( \lambda_{LL'}^{J}(E) \) calculated from the loop diagram in Fig. 1. For an energy dependence such as in eq. (45), the energy derivative and \( \Delta \lambda(q,q') \) have the form

\[
-\frac{d}{dE} \lambda(E) = \lambda(0) \frac{E_0(1+a)}{(E_0 - E)^2};
\]

\[
\Delta \lambda(q,q') = \lambda(0) \frac{E_0(1+a)}{E - q^2/2m} \frac{1}{E - q'^2/2m}.
\]

Thus, the matrix elements for OME can be found without using the vertex functions \( B_{L}^{J}(k) \) explicitly.

### 5.1.1. One-pion exchange

For one-pion exchange, we take the interaction operator in the standard form

\[
V_{\text{OPE}}^{(i)} = -\frac{g_{\pi NN}^2}{(2m_N)^2} (\sigma^{(i)} \sigma) \frac{1}{p^2 + m_{\pi}^2} (S_d \sigma)(\sigma^{(i)} \tau_d), \ p = q - q', \]

where \( \sigma^{(i)} \) and \( \tau^{(i)} \) are the spin and isospin variables of the third \( (i) \)th nucleon, whereas \( S_d \) and \( T_d \) are the operators of the total spin and isospin of the 6q bag, respectively. We found that the contribution of OPE is so small that it is sufficient to include here only \( S \) waves. In this case, only the central part of the OPE interaction remains:

\[
V_{\text{OPE}}^{c} = g_{\pi NN}^2 \frac{m_{\pi}^2}{4m_N} \frac{1}{3} (\sigma^{(i)} S_d)(\sigma^{(i)} \tau_d) \frac{1}{p^2 + m_{\pi}^2}. \]

The spin-isospin matrix element is nonzero only for a singlet-triplet transition:

\[
\langle S_d = 0, T_d = 1 | \frac{1}{3} (\sigma^{(i)} S_d)(\sigma^{(i)} \tau_d) | S_d = 1, T_d = 0 \rangle = \frac{4}{9}, \]

(50)
Then, the matrix element of the OPE contribution for $s$ waves takes the form
\[
\langle\text{OPE}\rangle_c = \frac{8}{3} f_{NN}^2 \sqrt{\lambda_{00}^0(0)\lambda_{10}^1(0)} E_0(1 + a) \times \int \frac{\chi_0^0(q)}{E - E_0 - \frac{q^2}{2m}} \frac{1}{(q - q')^2 + m^2} \frac{\chi_0^1(q')}{E - E_0 - \frac{q'^2}{2m}} dq dq'.
\]

(51)

Here, we take the vertex functions $B_0^0$ and $B_1^1$ differing from each other only by a constant. Therefore, using eq. (44), one can exclude these functions from the formula for the matrix elements.

We did not introduce here any cut-off factor for OPE, because the overlap functions truncate the OPE interaction at large $q$ values. It should be noted that the overlap functions $\chi_0^0$ and $\chi_1^0$ for singlet and triplet bags, respectively, are very similar in shape and magnitude but have opposite signs (see Fig.5). Therefore, the OPE contribution (51) is always negative. In our case, it is equal to only $-0.1$ MeV and thus generally agrees with the result obtained in ref. [29] when the latter is rescaled to the larger weight of the 6$q$ component in our case.

5.1.2. One-sigma exchange (OSE)

The scalar meson exchange operator does not include any spin-isospin variables. Therefore, eq. (41) for $W_{JJ'}_{LL'}$ can be simplified and, in view of the energy dependence given in eq. (45), reduces to the form
\[
\text{OSE} W_{JJ'}_{LL'}(q, q'; E) = \delta_{JJ'} \lambda_{LL'}^J(0) E_0(1 + a) \times \frac{1}{E - E_0 - \frac{q^2}{2m}} \frac{-g_{\sigma NN}^2}{(q - q')^2 + m^2} \frac{1}{E - E_0 - \frac{q'^2}{2m}} \chi_{JM,L,L'}^J(q) \chi_{JM,L,L'}^{J'}(q') dq dq'.
\]

(52)

Thus, the matrix element of OSE 3BF is determined by the formula
\[
\langle\text{OSE}\rangle = -3g_{\sigma NN}^2 \sum_{J,M,L,L'} \lambda_{LL'}^J(0) E_0(1 + a) \times \int \frac{\chi_{JM}^J(q)}{E - E_0 - \frac{q^2}{2m}} \frac{1}{(q - q')^2 + m^2} \frac{\chi_{JM'}^{J'}(q')}{E - E_0 - \frac{q'^2}{2m}} dq dq'.
\]

(53)

In the actual calculations, we used the “light” $\sigma$-meson mass $m_\sigma = 400$ MeV.

The physical meaning of the OSE three-body force is easily understood: it represents the interaction of the third nucleon with the $\sigma$-meson cloud of the bag. In view of the enhancement of the $\sigma$-field around the dense 6$q$ bag, this contribution is very important for our understanding of the properties of 3$N$ and heavier nuclei.

5.2. Two-sigma exchange (TSE)

The two-sigma process shown in Fig.4 also contributes significantly to 3BF. This 3$N$ interaction seems less important than the OSE force, because this interaction imposes a specific kinematic restriction on the 3$N$ configuration.\(^+\)

\(^+\) It follows from the intuitive picture of this interaction that this force can be large only if the momentum of the third nucleon is almost opposite to the momentum of the emitted $\sigma$-meson. Thus,
The operator of this interaction includes the vertex functions for the \((NN \rightarrow 6q+\sigma)\) and reverse transitions so that these vertices cannot be excluded similarly to the case of OME. Therefore, we have to choose some form of these functions. It is naturally to suppose that these vertices are the same as those assumed in two-body DBM; i.e., they can be normalized by means of the coupling constants \(\lambda(E)\), which, in turn, are chosen in the two-nucleon sector to accurately describe \(NN\) phase shifts and deuteron properties (see eq.(56) for vertex normalization). We use the Gaussian form factor for these vertices:

\[
B^J_L(k) = B^J_{0L} \frac{e^{-b^2k^2}}{\sqrt{2\omega_\sigma(k)}},
\]

where \(k\) is the meson momentum and the parameter \(b\) is taken from the microscopical quark model [22]:

\[
b^2 = \frac{5}{24} b_0^2; \quad b_0 = 0.5 \text{ fm}.
\]

Then, the vertex constants \(B_0\) should be found from the equation:

\[
\frac{1}{(2\pi)^3} \int d\mathbf{k} \frac{B^J_{0L} B^{J'L'}_0 e^{-2b^2k^2}}{(E-m_\alpha-\varepsilon_\sigma(k))2\omega_\sigma(k)} = \lambda^J_{LL'}(E),
\]

where \(\lambda^J_{LL'}(E)\) are the coupling constants employed in the construction of the DBM in the \(NN\) sector and are fixed by \(NN\) phase shifts.

For the \(\sigma NN\) vertices, we also take the Gaussian form factor:

\[
g_{\sigma NN} e^{-\alpha^2k^2}, \quad \text{with} \quad \alpha^2 = \frac{1}{6} b_0^2.
\]

Then, the box diagram in Fig. 4 can be expressed in terms of the integral with respect to the momentum \(\mathbf{q}_0\) of the third nucleon in the intermediate state:

\[
T^{3SE}_{W_L L'}(\mathbf{q}, \mathbf{q}'; E) = \delta_{JJ'} g_{\sigma NN}^2 B^J_0 B^{J'}_0
\]

\[
\times \frac{1}{(2\pi)^3} \int d\mathbf{q}_0 \frac{e^{-(\alpha^2+b^2)(\mathbf{q}_0-\mathbf{q})^2}}{m_\sigma^2 + (\mathbf{q}_0-\mathbf{q})^2} \frac{1}{E-m_\alpha-\frac{q_0^2}{2m}} \frac{e^{-(\alpha^2+b^2)(\mathbf{q}_0-\mathbf{q}')^2}}{m_\sigma^2 + (\mathbf{q}_0-\mathbf{q}')^2},
\]

a specific 3N kinematic configuration is required when two nucleons approach close to each other to form a bag, while the third nucleon has a specific space localization and momentum.
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Thus, the matrix element for the total contribution of TSE takes the form

\[ \langle \text{TSE} \rangle = 3 \sum_{J M, L, L'} \int \chi_L^{JM}(q) TSE W_{LL'}^{JJ}(q, q'; E) \chi_{L'}^{JM}(q') \, dq \, dq'. \] (59)

After the partial wave decomposition, these six-dimensional integrals can be reduced to two-dimensional integrals, which are computed numerically by means of the Gaussian quadratures.

6. Results

The present calculations of 3N bound states have been made with the two variants of the DBM, versions I and II, one of which being used in our previous two-nucleon studies [22]. For the other variant, we use a weaker energy dependence \( \lambda(E) \), which is taken directly from the loop integral (eq. (56)) corresponding to the diagram in Fig. 1. The effective three-nucleon Hamiltonian in the 3N channel includes both two-body and three-body forces (although, as was noted above, such a division is justified only for the Yukawa \((t\text{-channel})\) exchange contribution):

\[ H_{3N} = T + \sum_{i=1}^{3} V_{i}^{NN} + \sum_{i=1}^{3} V_{i}^{3BF}. \] (60)

The parameters of two-body potentials \( V_{i}^{NN} \) (for the two versions) provide a quite good description of the deuteron observables \( NN \) and phase shifts in the wide energy interval \( 0 - 1 \) GeV. The parameters of the three-body force \( V_{i}^{3BF} \) were discussed in the previous section. We took into account both the OSE and TSE contributions to 3BF. The contribution of OPE interaction between the dressed bag and third nucleon to the 3N binding energy was found to be only about -0.1 MeV. Therefore, we did not include this term in the complete variational calculation but estimated it with the perturbation theory (PT). The present 3N calculations were made using a highly accurate variational method and the antisymmetrized Gaussian basis [30, 31] with non-linear scale parameters. This basis is extremely flexible and has an enormous advantage for such tedious calculations. It enables us to obtain completely analytical formulas for the matrix elements of effective two- and three-body forces, except two-sigma exchange (these formulas will be presented in another paper). In the present calculations, we used the variational basis including even partial waves with the total orbital angular momenta \( L = 0, 2 \) and took into account the \( NN \) force components for the \( ^1S_0 \) and \( ^3S_1 - ^3D_1 \) channels. The dimension of the basis was increased until the converged results were reached. The results presented in Table 2 are obtained with the basis that includes six three-body partial components and has the total dimension \( N = 688 \).

Table 2 presents our results for the following static properties of \( ^3\text{H} \): binding energy \( E \), \( D\)-wave percentage \( P_D \), and weight of the dressed “bag” \( 6qN \) component \( P_{6qN} \). In addition, this table shows the contributions of the following individual parts of the Hamiltonian to the total three-body expectation value: three-nucleon kinetic energy \( T \), two-body effective force \( V_{NN}^{NN} \), and three-body force due to one-sigma \( (V_{OSE}^{3BF}) \) and
Table 2. Results of $3N$ calculations with two- and three-body forces for two versions (I and II) of the dressed-bag model.

|          | $E$ (MeV) | $P_D$ (%) | $P_{6qN}$ (%) | Individual contributions to $H$, MeV |
|----------|-----------|-----------|---------------|--------------------------------------|
|          |           |           |               | $V_{NN}$ | $V_{3BF}^{OSE}$ | $V_{3BF}^{TSE}$ | $V_{3BF}^{OPE(PT)}$ |
| Version I, $g_{\sigma NN} = 9.5$ |
| $^3$H$^{(1)}$ | -5.83 | 6.44 | 7.87 | 75.0 | -64.45 | -1.54 (PT)* | -0.43 (PT)* |
| $^3$H$^{(2)}$ | -4.14 | 5.81 | 6.24 | 58.1 | -49.88 | -0.83 (PT)* | -0.22 (PT)* |
| $^3$H$^{(3)}$ | -8.326 | 6.84 | 10.84 | 110.7 | -92.9 | -4.72 | -2.03 | -0.1 |
| $^3$He$^{(3)}$ | -7.588 | 6.80 | 10.66 | 108.2 | -91.0 | -4.55 | -1.99 | -0.1 |
| Version II, $g_{\sigma NN} = 8.6$ |
| $^3$H$^{(1)}$ | -6.12 | 6.67 | 5.45 | 79.0 | -67.82 | -1.30 (PT)* | -0.56 (PT)* |
| $^3$H$^{(2)}$ | -5.01 | 6.23 | 4.67 | 66.9 | -57.3 | -0.87 (PT)* | -0.36 (PT)* |
| $^3$H$^{(3)}$ | -8.358 | 7.06 | 7.31 | 110.8 | -94.2 | -2.79 | -1.66 | -0.1 |
| $^3$He$^{(3)}$ | -7.565 | 7.00 | 7.14 | 107.8 | -91.8 | -2.66 | -1.58 | -0.1 |

(1) – without 3BF, without $q^2$-dependence,
(2) – without 3BF but with $q^2$-dependence,
(3) – with 3BF and with $q^2$-dependence,

* These values have been found with the $3N$ wavefunctions in those calculation the 3BF have been omitted, i.e. within PT. The comparison of these entries with those found in the complete calculation (see rows denoted $^3$H$^{(3)}$) shows with evidence inapplicability of the PT.

two-sigma exchanges ($V_{TSE}^{3BF}$). The contribution of 3BF due to OPE calculated with perturbation theory (PT) is also given in Table 2. The other details of the results presented in the Table, especially related to the $^3$He properties, are discussed in the forthcoming paper.

For both variants of the model, we present also the result calculated disregarding both 3BF and the $q^2$ dependence of the effective two-body force $V_{NqN}$ on the momentum of the third nucleon (see the first and fifth rows of Table 2). The results in the second and sixth rows of Table 2 are obtained including the $q^2$ dependence of $V_{NqN}$, but disregarding 3BF. The percentages of the $D$-wave $P_D$ and $6qN$ component $P_{6qN}$ given in Table 2 were obtained with incorporation of the three $6qN$ components; i.e., these values correspond to the normalization of the total (four-component) wavefunction of the system to unity.

7. Discussion

The $3N$ results presented in the previous section differ significantly both from the $3N$ results found with the conventional models for $NN$ and $3N$ forces (based on Yukawa’s meson exchange mechanism) and from the results obtained in the framework of hybrid models [32], which include the two-component representation of the $NN$ wavefunction $\Psi = \Psi_{NN} + \Psi_{6q}$. It is convenient to discuss these differences in the following order.

(i) We found that the $q^2$ dependence of pair $NN$ forces on the momentum of the third
particle in the $3N$ system is more pronounced in our case than in other hybrid models [32, 33, 34, 35]: the $3N$ binding energy decreases by more than 1.7 MeV, from 5.85 to 4.14 MeV (cf. the first and second rows in Table 2). From the more general point of view, it means that, in our approach, pair $NN$ interactions (except Yukawa OPE and TPE terms), being “embedded” into a many-body system, loose their two-particle character and become substantially many-body forces (depending on the momenta of other particles of the system).

(ii) Due to such a strong $q^2$ dependence (of “repulsive” character), the $3N$ system calculated including only pair forces turns out to be strongly underbound ($E = −4.14$ MeV). In other words, the “pairwise” $NN$ forces (including their $q^2$ dependence on the momenta of the third nucleon) give only about half the total $3N$ binding energy, leaving the second half for the 3BF contribution. Therefore, the following question is decisively important: can the three-body force (inevitably arising in our approach) give the large missing contribution to the $3N$ binding energy? Usefulness of the developed model for the description of nuclear systems depends directly on the answer to this important question. It is appropriate here to remind that in the conventional 3BF models such as Urbana-Illinois or Tucson-Melbourne, the contribution of 3BF to the total $3N$ binding energy does not exceed 1 MeV; i.e., this contribution can be considered as some correction ($\sim 15\%$), although it is significant for the precise description of the $3N$ system.

(iii) Fortunately, the contribution of 3BF induced by $\sigma$ and $2\sigma$ exchanges enables one to fill this 4.4 MeV gap between the two-body force contribution and experimental value. In fact, including both one- and two-sigma exchange contributions to 3BF, taken with the same parameters as in the initial $NN$-force model and a quite reasonable coupling constant $g_{\sigma NN} = 8 \div 9.5$, we obtain the $3N$ binding energy that is very close to the experimental value (see rows 3, 4 and 7, 8 in Table 2). Thus, the presented force model leads to a very reasonable binding energy for the three-nucleon system but with the strongly enhanced (as compared to the traditional $3N$ force model) contribution of three-body forces.

(iv) The contributions of pair and different three-body forces to the total $3N$ binding energy for $^3H$ are given in the third and fourth rows of Table 2. From the results presented in these rows, one can conclude that the total 3BF contribution to the $3N$ binding energy dominates and, in fact, determines the structure of the $^3H$ ground state*. Moreover, comparing the third and fourth rows of the table, one can see a “nonlinear” effect of self-strengthening for the 3BF contribution. The comparison of the results presented in the third and fourth rows of the table (see the second, fourth, and fifth columns) shows clearly that the binding energy is almost proportional to the weight $P_{6qN}$ of the $6qN$ component in the total $3N$ wavefunction. Thus, when the weight of the $6qN$ component increases, the 3BF contribution, which is related

* It should be noted that the relative contribution of the pair effective force $V_{NN}qN$ to the binding energy decreases noticeably when including 3BF (due to strengthening the $q^2$ dependence of pair forces).
directly only to this component of the total wavefunction, increases accordingly. However, the enhancement of the pure attractive 3BF contribution squeezes the \(3N\) system and thus reduces its rms radius, i.e., the mean distance between nucleons, which, in turn, again increases the weight of the \(6qN\) component. In other words, a some chain process strengthening the attraction in the system arises. This process is balanced both by the weakening of the effective pair interaction due to the \(q^2\) dependence and by the repulsive effect of the orthogonalizing pseudopotentials included in each pair interaction.

There are two another important stabilizing factors weakening the strong three-body attraction in the \(3N\) system. First, the generation of the short-range repulsive vector \(\omega\)-field, where all three nucleons are close to each other \(36\). Since the \(\omega\)-meson is heavy, this field is located in the deep overlap region of all three nucleons. In this study, we omitted the three-body contribution of this repulsive \(\omega\)-field. This repulsive contribution will keep the whole system from the further collapse due to the strong attractive \(3N\) force induced by the scalar field.

The second factor slightly weakening the effective \(3N\) attraction is associated with the conservation of the number of scalar mesons generated in the \(2N\) and \(3N\) interaction process. The problem is that \(2\sigma\)-exchange giving the 3BF contribution (see Fig. 4) arises due to the break of the \(\sigma\)-meson loop, which induces the main \(2N\) force. In other words, the \(\sigma\)-meson generated in the transition of pair nucleons from the \(NN\) phase state to the \(6q\) state is absorbed either in the \(6q\) bag with closing the loop or by the third nucleon resulting in the 3BF contribution. Thus, the appearance of such a 3BF should weaken attraction between nucleons in the pair. We carefully estimated the effect of the meson-number conservation for the TSE contribution on the total \(3N\) binding energy. Its magnitude occurred to be rather moderate in the absolute energy scale (ca. 0.3 – 0.4 MeV) but quite noticeable within the whole TSE contribution. However, when the total nucleon density increases (and the relative TSE contribution also increases), the effect is enhanced.

(v) Dependence of the two-body coupling constants \(\lambda(\varepsilon)\) upon the average momentum of other nucleon in three-nucleon system (see e.g. eq. \(31\)) can be interpreted generally as a density dependence of the two-body force in many-nucleon system. Thus, one fixes (e.g. at energy \(\varepsilon = \varepsilon_d\)) the value of energy dependent coupling constant \(\lambda(E-q^2/2m)\) of our two-body force, i.e., if one disregards its \(q^2\)-dependence (this \(q^2\)-dependence leads to a weakening of the two body force in a many-nuclear system when \(q^2\) is rising), than the neglected \(q^2\)-dependence must be compensated by an additional repulsive density-dependent effective three-body force.

On the other hand, it is well known from the Skirme-model calculations of nuclei that just similar repulsive phenomenological density-dependent three-body force should be added to conventional \(2N\)- and \(3N\)-forces to guarantee the saturation properties of heavy nuclei. Thus, in this respect the present force model is also in a qualitative agreement with phenomenological picture of nuclear
Figure 5. $6qN$ component of the $^3H$ wavefunction: the triplet and singlet s-wave overlap functions $\chi^T_0(\rho)$ (solid line) and $\chi^S_0(\rho)$ (dashed line) and the corresponding "quasi-wavefunctions" $\tilde{\psi}^T_0(\rho)$ (dash-dotted line) and $\tilde{\psi}^S_0(\rho)$ (dot-dot-dashed line). The triplet functions are multiplied by a factor of (-1).

(vi) Figure 5 shows two types of the $^3H$ s-wave wavefunctions in the $6qN$ channel: the triplet $\chi^T_0(\rho)$ (solid line) and singlet $\chi^S_0(\rho)$ (dashed line) overlap functions given by eqs. (32, 33) and the corresponding “quasi-wavefunctions” $\tilde{\psi}^T_0(\rho)$ and $\tilde{\psi}^S_0(\rho)$ normalized according to eq. (39). The triplet functions are multiplied by (-1) for convenience. It is evident that there is an inner node (at $r_n \approx 0.56$ fm) in all s-wave components of the quasi-wavefunctions and the overlap functions. These inner nodes are stationary when the total energy increases and the $Nd$ scattering problem is considered. As is seen in Fig. 5, the overlap function $\chi^T_0(\rho)$ for the triplet $6qN$ channel has a more extended tail than the singlet component $\chi^S_0(\rho)$. This is due to the fact that the $3N$ system in the spin-triplet channel (coinciding with deuteron channel) is less bound, by about 2 MeV, than in the spin-singlet channel.
8. Conclusion

In this paper, we have developed a formalism for a multi-component description of the three-nucleon system within the new approach to the 2N and 3N interactions based on the dressed dibaryon and \( \sigma \)-field generation. It has been shown that the DBM applied to the 3N system results automatically in a new three-body scalar force due to the interaction between the dressed dibaryon and third nucleon. This force plays a crucial role in the structure of few-nucleon systems. Our accurate variational calculations have demonstrated that new 3BF gives half the 3N binding energy, whereas the 3BF contribution in the traditional \( NN \)-force approaches gives about 15\% of the total binding energy. Thus, the suggested approach to the \( NN \) interaction can lead to significant revision of relative contributions of two- and many-body forces in nuclear systems.

It is crucially important that the DBM gives an 8 - 11\% non-nucleonic component in the 3N wavefunction, while this component in the deuteron is equal to only about 3\%, leading to a reformulation of many effects in few-nucleon systems and other nuclei as well. It is probable that the weight of such non-nucleonic components in heavy nuclei can be even higher with an increase in the mass number and nuclear density.

Generalization to 4N (and more) systems can be done rather similarly to the case of conventional force model. Having 2N- and 3N-forces fixed from 2N- and 3N-calculations one can straightforwardly consider systems with larger number of nucleons with these forces to be incorporated, i.e. ignoring the contribution of 4N etc. forces. However there is still another possibility to proceed for many-body systems. If the contribution of such many-body forces will occur to be significant one can reformulate the whole scheme in terms of collective \( \sigma \)- (and other) fields interacting with relativistic nucleons and dibaryons, i.e. in the spirit of Walecka-Serot model of hadrodynamics. The only essential difference in this point from the Serot-Walecka model is adding dibaryon components to the nucleonic ones. Moreover, our approach, contrary to the conventional OBE-approach, leads inevitably to a strong enhancement of collective \( \sigma \)-field in nuclei and thus to validity of Walecka-type model.

Numerous modern experiments could corroborate these results. In particular, according to the recent experiments \(^3\text{He}(e,e'p)p\)\[^6\] and their theoretical interpretation on the basis of fully realistic 3N calculations, the cross sections for the \(^3\text{He}(e,e'p)p\) process cannot be explained within a fully realistic 3N model incorporating the process where a proton knocked out by a virtual \( \gamma \)-ray photon is sequentially rescattered by the second nucleon. These calculations show that the leading contribution comes from the one-step process where a virtual photon is absorbed by one nucleon in the target, while the remaining two nucleons (which are spectators in this process) are emitted (in coincidence) at the second step. This important conclusion has been further corroborated in recent experiments at the Jefferson Laboratory when the incident electron beam energy has been increased up to \( E_e = 2.2 \text{ GeV} \) and 4.4 GeV \[^8\]. The data of the two different experiments give a clear evidence of very strong short-range \( NN \)
correlation in the $^3$He ground state. This correlation still cannot be explained within the traditional pattern for the 3N system. In addition, our approach has recently been partially corroborated from the other side by considering a model for $2\pi$ production in $pp$ collisions at $E_p = 750$ and 900 MeV. The authors have found that almost all particle energy- and angular correlations (e.g. $\pi^+\pi^-$, $pp$, $\pi pp$ and etc.) can be explained quantitatively by assuming that $\pi^+\pi^-$ production occurs through the generation of an intermediate light $\sigma$-meson with the mass $m_\sigma \simeq 380$ MeV and rather narrow width. These values generally agree with the parameters adopted in our $NN$ model and drastically disagree with the values assumed in OBE and other potential models.

Very interesting general implication of the results presented here is their evident interrelation to the famous Walecka hadrodynamical model for nuclei [38]. It is well known that the Walecka model describes nuclei and nuclear matter in terms of the scalar $\sigma$ and vector $\omega$-fields, where the $\sigma$-field gives the attractive contribution, while the vector $\omega$-field balances this attraction by short-range repulsion. It is very important that both basic fields exist (in the model) as the explicit degrees of freedom (together with relativistic nucleons) in contrast to conventional meson-exchange models for nuclear forces, where mesons appear as the carriers of forces rather than as the explicit field degrees of freedom. Our approach does include the $\sigma$-meson (and potentially the $\omega$-meson) degrees of freedom in an explicit form similarly to the Walecka model. Moreover, since the average kinetic energy of the $3N$ system is high in our model (it is higher than that in the conventional OBE approach by a factor of more than 2), nucleon motion is closer to the relativistic case, and thus the similarity with the Walecka model is even closer.

These general arguments give an additional strong support for the $2N$- and $3N$-force model presented here. Quite independent numerous arguments in favour of this approach in nuclear physics are presented in the subsequent paper [39].

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