Comment on the $W + 1$ jet to $W + 0$ jets ratio

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Abstract

To offer a possible resolution to the apparent discrepancy between the experimental and the theoretical values of the $W + 1$ jet to $W + 0$ jets ratio reported by the DØ group, we examine the effects of the multiple soft gluon radiation on the $W$ boson production at the Tevatron. Based on the calculation of the $W$ boson transverse momentum ($Q_T$) distribution in the Collins-Soper-Sterman resummation formalism, we conclude that the effect of the soft gluon radiation is important in the region of $Q_T < 50$ GeV, and it can be better tested by a more inclusive observable $R_{CSS}(Q_T^{\text{min}}) \equiv \frac{\sigma(Q_T > Q_T^{\text{min}})}{\sigma_{\text{Total}}}$. 

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Reports on the $W+1$ jet to $W+0$ jets ratio by the Fermilab DØ collaboration [1,2] have repeatedly shown a discrepancy between the experimental data and the next-to-leading-order (NLO) QCD predictions of the DYRAD Monte Carlo program [3]. In accordance with the experimental situation, a DYRAD event is considered to represent $W+0$ jets ($W+1$ jet) event if the transverse momentum ($E_T$) of the jet is smaller (larger) than a certain $E_T^{\text{min}}$ value. Using the above definition, the $W+1$ jet $E_T$ cross section is calculated in $\mathcal{O}(\alpha_S^2)$ while the $W+0$ jets $E_T$ distribution is calculated in $\mathcal{O}(\alpha_S)$ [4]. From these $W+$ jet cross sections, the following ratio is formed:

$$R_{10}^{\text{jet}}(E_T^{\text{min}}) = \frac{\int_{E_T^{\text{min}}}^{E_T^{\text{max}}} dE_T \frac{d\sigma}{dE_T}}{\int_{0}^{E_T^{\text{min}}} dE_T \frac{d\sigma}{dE_T}},$$

(1)

where $E_T^{\text{max}}$ is the maximal $E_T$ allowed by the phase space. This ratio is then compared with the experimental results. The DYRAD prediction of $R_{10}^{\text{jet}}$ is found to be consistently lower than the experimental central values by about 30%. Since, with the increase of $E_T^{\text{min}}$ the error bars of the experimental data increase, the confidence level of the statistical significance of the deviation is smaller in the $E_T^{\text{min}} > 50$ GeV region. Therefore, we focus our attention on the $E_T^{\text{min}} < 50$ GeV section, and ask the question: What physics can be responsible for this deviation?

The DØ analysis offers the uncertainty of the gluon parton distribution as the most likely reason for the discrepancy [2]. In this work we propose an additional possible explanation which originates from the perturbative QCD theory. We note that the situation is very similar to the one in the direct photon production, another process in which a vector boson produced together with a jet (or several jets). The NLO prediction of the transverse momentum ($p_T$) distribution of the photon is systematically lower than the measured cross sections [5] for the low $p_T$ region.

Different parts of the answer to the $W+$ jet puzzle might come from different sorts of physics. To illustrate this, we point out that the ratio $R_{10}^{\text{jet}}$ has several shortcomings from a theoretical point of view:

- The NLO calculation of $R_{10}^{\text{jet}}$ might not be sufficient to describe the data in the low transverse momentum region for it does not include the large effect of the multiple soft gluon emission.

- Calculating the numerator of $R_{10}^{\text{jet}}$ in $\mathcal{O}(\alpha_S^2)$ and the denominator in $\mathcal{O}(\alpha_S)$ implies a discontinuity in the $E_T$ distribution at $E_T^{\text{min}}$. It is therefore less natural than, say, a pure $\mathcal{O}(\alpha_S^2)$ calculation.
FIG. 1. The ratio $R_W^{10}$ calculated in the resummed formalism (solid), in $\mathcal{O}(\alpha_S^2)$ (long dash), in $\mathcal{O}(\alpha_S)$ (short dash), and with numerator in $\mathcal{O}(\alpha_S^2)$ and denominator in $\mathcal{O}(\alpha_S)$ (dots).

- The value of $R_{jet}^{10}$ depends on the detailed definition of the jet in both the theoretical calculation and the experimental measurement.

Each of these deficiencies may contribute to a different degree to the disagreement in various $E_T$ regions. Since no better calculation of the jet $E_T$ cross section is available than the one used by DØ, in order to analyze the situation, we turn to the calculation of the transverse momentum ($Q_T$) of the $W$ boson. In $\mathcal{O}(\alpha_S)$, the transverse momenta of the jet and the $W$ boson are the same: $E_T = Q_T$. In addition, the $Q_T$ distribution of the $W$ boson is theoretically well understood, and the contributions from the multiple soft gluon radiation can be resummed using the Collins-Soper-Sterman formalism [6–8].

We can form the ratio $R_W^{10}$ using the $W$ boson transverse momentum in the place of the jet $E_T$:

$$R_W^{10}(Q_T^{\min}) = \frac{\int_{Q_T^{\min}}^{Q_T^{\max}} dQ_T \frac{d\sigma}{dQ_T}}{\int_0^{Q_T^{\min}} dQ_T \frac{d\sigma}{dQ_T}}$$

where $Q_T^{\max}$ is the largest $Q_T$ allowed by the phase space. In Fig. 1, we plot the ratio $R_W^{10}$ for calculations done in different orders of the strong coupling constant $\alpha_S$. The difference of the $\mathcal{O}(\alpha_S)$ (short dashed) and $\mathcal{O}(\alpha_S^2)$ (long dashed) curves indicates that the K-factor is about

1We use the ResBos Monte Carlo code [7], $\sqrt{S} = 1.8$ TeV, and the CTEQ4M parton distribution.
1.4 in the region of interest, which suggests that higher order perturbative contributions might have to be considered. From comparing the resummed (solid) and the $O(\alpha_s^2)$ (long dashed) curves, we infer that the effects of the multiple soft gluon radiation increase the $Q_T$ cross section for $Q_T < 50$ GeV. This increase over the $O(\alpha_s^2)$ rate is about 30% around $Q_T = 20$ GeV, and remains sizable (more than 5%) even at $Q_T = 40$ GeV. The dotted curve, which is calculated with the mismatched numerator (to $O(\alpha_s^2)$) and denominator (to $O(\alpha_s)$), runs under the $O(\alpha_s^2)$ (long dashed) curve, but the difference coming from this mismatch is small in the $Q_T > 20$ GeV region.

Based on the results obtained for the $Q_T$ distribution of the $W$ boson, we conclude that soft gluon effects are important in the $W+$ jet production in the region of $Q_T < 50$ GeV. The ratio $R_{CSS} \equiv 1/(1 + R_{W}^{10})$ is more suitable to be compared to the experimental data for it is a more inclusive observable which does not involve any jet measurement but includes the large effect of multiple soft gluon contribution to all orders in $\alpha_s$.

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