Application of Damage Mechanics for Prediction of Failure of Structural Materials and Elements

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Abstract. The article deals with the practical application of damage mechanics for modelling and predicting the failure of materials and structural elements. One of the recently developed and used models of Bai–Wierzbicki material is discussed in more detail. In this model the effect of hydrostatic pressure and the third invariant of the deviatoric stress on the process of the material plastic deformation is taken into account. It allows to model damage processes in the material and predict its total failure with high accuracy. Consequently, this enables the practical application in engineering calculations of structural elements. As an example, several such analyses were presented and discussed. They included the simulations of tensile elements made of C45E+N steel under proportional and non-proportional loading conditions in uniaxial and spatial stress state. Another analysis considered numerical simulations of the ductile fracture in Battelle Drop-Weight-Tear (BDWT) specimens used in pipeline sections tests. The problem concerned the phenomenon of block shear failure in bolted steel connections was the subject of the last presented simulations.

Introduction

The failures and disasters of building structures made of steel is caused by many factors, such as overloading, design errors, faulty performance, excessive slenderness leading to buckling, fatigue, defective joints, material defects, etc. Notwithstanding, the damage processes begin in the steel microstructure. As is well known, structural steel is highly imperfect in terms of its material structure. The basic factor which determines conditions for initiation of the material damage are particles that disturb the ordered system of the so-called material matrix. In the case of structural steels, they are all kinds of impurities in the form of precipitates and inclusions. They remain in the material as a result of technological processes related to their production.

The fracture process of the structural steels is complex due to their polycrystalline structure. Basically, three basic fracture patterns for this type of material have been identified: brittle, shear and ductile fracture. Structural steels used in construction, that interest us particularly, are damaged in accordance with the last mechanism, i.e. ductile fracture.

In the case of this type of fracture, the initiation of the process occurs due to the nucleation and evolution of so-called voids, that are formed during the material deformation process, between inclusions and precipitation and the material matrix. This impurities existing in the material are “neutral” as long as the intensity of the deformation process does not activate them. It is related to the disproportion between the stiffness of inclusions and precipitates, and the stiffness of the material matrix. In the situation of significant deformations, a less rigid matrix causes that the stiffer particles create considerable resistance and do not undergo such high deformations as the matrix. This phenomenon is observed primarily in the areas where the prevailing state of stress is spatial, triaxial. As a result, we observe separation of the material matrix from the walls of stiffer particles.

At a later stage, the voids grow (void growth), resulting in an increasingly softening of the material. The state of stress is reconfigured, which macroscopic effect is a decrease in nominal stress, being a measure of the effort of the material and its strength. The increasing neighbouring
voids reach such considerable sizes that they are joined (void coalescence). At the final stage, the size of the formed discontinuities is so significant that the material is separated, and its decohesion, fracture is observed. The idea of these processes is shown schematically in figure 1.

![Figure 1. Ductile fracture mechanism due to the voids evolution [1].](image)

The classic strength hypotheses do not allow to take into account the effect of described phenomena on the strength of the material due to the assumption of the material continuum. Therefore, their use to analyse and describe the materials that are in the post-elastic range, when the damage processes are initiated, is impossible. Thus, it is necessary to take into account the impact of damage parameters on the prevailing in the material stress state and deformation, and finally on failure of the material. Such models were developed by damage mechanics [2-14]. One of the latest approaches is this regard is the Bai–Wierzbicki (BW) plasticity model [15].

Application of the BW material model allows to describe more accurate the phenomena taking place in the phase of the material deformation, when intensive processes associated with its softening due to the growth of microdefects in the material, in comparison to many other models based on the damage mechanics. As a result, it is possibly to model and predict the time of a total loss of the material capacity, leading to its failure, which is particularly important from a practical point of view. As a consequence, application of the BW model enables to simulate the failure of structural elements, what is a theme of this study. The assumptions of the Bai–Wierzbicki material model are presented, together with examples of its practical application to model and predict the failure of structural materials and elements used in engineering and construction.

**The Bai–Wierzbicki Material Model**

Although many damage mechanics based material models have been developed, in many cases they do not fully describe the phenomena that occur in the material during its plastic deformation, especially in terms of significant softening due to the microdefects growth. During the ductile fracture, the initiation of the process is local and is usually preceded by significant plastic deformations. Classical plasticity theories ignore the impact of the third invariant of the deviatoric stress on the plastic deformation process of the material. With regard to ductile fracture, this type of assumption may be incorrect. Determination of stress and strain gradients that occur in this case requires considerable accuracy of calculations. This entails taking into account the impact of the third invariant of deviatoric stress, which will increase the likelihood to obtain the correct solutions.
This is the subject of extensive research conducted for many years. One of the basic problems was to examine the relationship between the type of material failure, and the parameters describing the state of stress and deformation at the time of material damage leading to rupture. Bao and Wierzbicki noticed in the study [16] that the third invariant of the deviatoric stress has an impact on the local strain function extremes, and the critical strain of the ductile fracture is described by monotonic function of the stress triaxiality. This phenomenon is presented in figure 2.

![Figure 2. The equivalent strain to fracture versus the average stress triaxiality relation [16].](image)

The stress triaxiality \( \eta \) contains the link between hydrostatic and equivalent stress, according to the expression:

\[
\eta = \frac{-p}{q} = \frac{\sigma_m}{\sigma}
\]

(1)

where:

\[
p = -\sigma_m = -\frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3),
\]

\[
q = \sigma = \sqrt{\frac{1}{2}[((\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]} ,
\]

\( p \) – equivalent stress,  
\( \sigma_1, \sigma_2, \sigma_3 \) – principal stress, 
\( \sigma_m \) – hydrostatic stress (mean stress).

In many research and numerical simulations it has been shown that due to the assumption of the material continuum, the classic Huber–Mises hypothesis does not allow predict the correct force-displacement relationship, especially in the range of the material damage. For this reason Bai and Wierzbicki presented a new concept of the material model [15], taking into consideration the effect of hydrostatic pressure and the third invariant of the deviatoric stress on the process of the material plastic deformation. The impact of the third deviatoric stress invariant was taken into account by the so-called Lode angle parameter, described later in this chapter in detail. It was assumed that the elastic-plastic material, with isotropic hardening, is homogeneous and isotropic. The yield function in the Bai–Wierzbicki model takes the form [15]:


\[ \sigma_{yld} = \overline{\sigma} \left[ 1 - c_\eta (\eta - \eta_0) \left( c_\theta^s + (c_\theta^{ax} - c_\theta^s) \left( \gamma - \frac{\gamma_m + 1}{m + 1} \right) \right) \right] \]

where:

\[ \gamma = \frac{\cos \left( \frac{\pi}{6} \right)}{1 - \cos \left( \frac{\pi}{6} \right)} \left[ \frac{1}{\cos \left( \theta - \frac{\pi}{6} \right)} - 1 \right] \]

\[ c_\theta^{ax} = \begin{cases} c_\theta^t & \text{for } \theta \geq 0 \\ c_\theta^c & \text{for } \theta < 0 \end{cases} \]

\( \overline{\sigma}(\overline{\varepsilon}_p) \) – strain hardening function,

\( c_\eta, c_\theta^s, c_\theta^t, c_\theta^c, m \) – material constants,

\( \eta \) – stress triaxiality,

\( \eta_0 \) – reference stress triaxiality.

The graphic representation of the Bai–Wierzbicki model in the deviatoric stress plane is revealed by the envelope shown in Figure 3 [15].

![Figure 3. The representation of the Bai–Wierzbicki model in the deviatoric stress plane [15].](image-url)

For a certain, specific values of the \( c \) parameters, the Bai–Wierzbicki model is reduced to the classic strength hypothesis of Huber-Mises or Treska, which results from the relationship (2), and is clearly visible in the three yield loci in the deviatoric stress plane shown in figure 3.

The basic problem and requirement of the Bai–Wierzbicki model is knowledge of the specific material constants necessary to describe its behaviour in the entire range of deformation. Current research is also devoted to this. Determining all Bai–Wierzbicki material parameters is of great importance in many cases. For instance, in [15] it was shown that for aluminum 2024-T351 the omission of the effect of stress triaxiality and Lode angle parameter leads to significant errors in modelling the tensile tests of flat grooved plates in comparison to the results of experiments. On the other hand, in the case of notched round samples subjected to tension it does not lead to significant inaccuracy of calculations. One should also mention the theoretical research conducted on the Bai and Wierzbicki model itself. For example, in [17] a corrected version of the original yield function (2) is presented.

The effect of hydrostatic pressure and the third invariant of the deviatoric stress on the process of the material plastic deformation is the essential element of the Bai–Wierzbicki model. It should be noted that the third invariant of deviatoric stress is widely used in constitutive models of brittle
materials due to their specificity. In various forms it was also introduced into the constitutive equations of ductile materials by modifying the shape of the yield surface, e.g. [18, 19]. The problem of the impact of the third invariant of the deviatoric stress on the process of the plastic deformation of the materials was investigated in [20, 21]. However, the proposed solutions did not allow for obtaining accurate results for plain strain.

In the Bai–Wierzbicki model, the impact of the third invariant of the deviatoric stress is taken into account by so-called Lode parameter \( \xi \) or Lode angle \( \theta \), which are defined as follows [15]:

\[
\xi = \left( \frac{r}{q} \right)^3 = \cos(3\theta)
\]

where:

\[
r = \frac{27}{2} \frac{(\sigma_1 - \sigma_m)(\sigma_2 - \sigma_m)(\sigma_3 - \sigma_m)}{} \\
\xi - \text{Lode parameter (normalized third deviatoric stress invariant)}, \\
\theta - \text{Lode angle}.
\]

The geometrical interpretation of the Lode angle in the space of principal stresses is shown in figure 4 [15].

The Lode angle range is \( 0 \leq \theta \leq \pi/3 \), which corresponds to the Lode parameter range \(-1 \leq \xi \leq 1\). The normalized Lode angle is defined as follows:

\[
\bar{\theta} = 1 - \frac{6\theta}{\pi} = 1 - \frac{2}{\pi} \arccos \xi
\]

Its values ranges from \(-1\) to \(+1\). The normalized Lode angle allows for a clear representation of typical stress states. This is shown in figure 5 [15], where limit values for the stress triaxiality and normalized Lode angle are presented for typical strength cases.
Finally, it should be noted that the description of the stress state by the Lode angle parameter has been used in recent years to describe ductile materials, often using classic material concepts, such as the Coulomb-Mohr model, for example.

**Prediction of Failure of Structural Materials and Elements Based on the Bai–Wierzbicki Material Model**

Application of the models based on damage mechanics is of fundamental importance for modelling the failure processes of materials used in technique. From a practical point of view, the most important utility is the possibility to model the material failure that allows predicting the failure of the element from which it is made. In this regard, it is necessary to emphasize the high utility of the BW model, which has been used in failure analysis of various structural materials and components for many years. Below are some examples of application of the BW material model to predict the failure of various materials and structural elements.

Problems of stress state prevailing in the material and loading history was the subject of the research done by Wu et al. [22]. In this study, the cases of complex loading conditions were considered. Authors proposed a new concept of damage initiation and failure indicators and corresponding evolution laws, which enabled to predict ductile damage of C45E+N steel by using the modified Bai–Wierzbicki model. The stress triaxiality and the Lode angle parameter were taken into account in damage initiation and propagation model. Authors introduced to the modified Bai–Wierzbicki model several additional parameters in order to model damage phenomena. In particular, the parameter $G_f$ which controls the rate of damage accumulation as well the softening rate of material, and the critical damage variable to fracture $D_{cr}$, were used in order to describe the damage initiation and subsequent propagation processes. The mechanical tests and numerical simulations were applied in order to calibrate model parameters for C45E+N steel. The presented method and model were validated in several various mechanical tests under proportional and non-proportional loading conditions. Finally, it allowed to predict the ductile damage behaviour in considered loading cases and tests. Exemplary of these tests are shown in figures 6 a, b and c. The results of simulations for the smooth and notched tensile round bar specimens with notch radii of 3 mm and plane strain conditions for quasi-proportional loading are presented, respectively.
In presented cases good agreement between the results of simulations and experiments is observed. For smooth round bar and notched round bar tensile tests the force-displacement curves determined numerically by using Bai–Wierzbicki model are fully consistent with the experimental results. Slight differences are observed in shear test during the hardening phase. Generally, for all considered in the study cases high compliance of numerical and experimental results was found. The modified Bai–Wierzbicki model was positively verified as useful to simulate the damage behaviour of considered material subjected to quasi-proportional loadings under uniaxial as well as spatial stress states.

In turn in [23] Kei et al. have taken up the problems of prediction of ductile fracture in pipelines due to increasing requirements on properties of materials used for the modern systems. In order to make possible the use of modern high toughness linepipe steels, new assessment methods were necessary to apply to fully characterize the ductile fracture behaviour of these materials. Thus, the modified Bai–Wierzbicki material model was used to simulate numerically the ductile fracture in Battelle Drop-Weight-Tear (BDWT) specimens used in pipeline sections tests. During the analysis, another damage mechanics based models, i.e. the Gurson-Tvergaard-Needleman (GTN) and Cohesive Zone (CZ) models, were applied. One of the crucial simulations are presented in figure 7, where results of modelling of the BDWT test is presented.

As can be seen, the modified Bai–Wierzbicki (MBW) predictions are very consistent with the results of experiments in terms of dynamic ductile fracture prediction. Other models, such as the GTN and the CZ models give acceptable but less accurate results. According to the authors conclusions, it is related to taking into consideration strain rate hardening, temperature softening and stress state effects in the MBW model. The differences are negligible, e.g. the peak load determined numerically is underestimated by 3 kN (1%) only in comparison to the experimental results.
Very interesting research with an extremely practical application was done by Wen and Mahmoud [24]. Authors attempted to model numerically steel joints in building structures. The problem concerned phenomenon of block shear failure in bolted steel connections. Due to high complexity, i.e. the varying stress-state conditions along the failure path, the applied damage model required to take into account all components of stress-state prevailing in the failure region. For this type of joints the ductile fracture is observed, especially on the shear plane. This failure mechanism is dependent on stress triaxiality and on Lode angle parameters. Basing on this assumptions, the numerical simulations of block shear in gusset plate and coped beam connections were conducted. By using newly developed ductile fracture criterion, taking into consideration the stress triaxiality and Lode angle parameter it was possible to model and analyse total failure material and elements.

The numerical simulations were validated and compared with experiments done by Huns et al. [25] and Franchuk et al. [26]. The effect of various levels of beam end rotations was considered on connection behaviour. The mechanisms of block shear in gusset plate and coped beam connections were analyzed numerically. Simulations were compared with results of experimental tests, including force-displacement curves, fracture sequences, and fracture profiles. One of the analyzed connections are presented in figure 8.

![Figure 8. Views of analyzed connections (a) T1 and (b) T2 [24, 25].](image)

As can be seen, the simulated failure mechanisms were consistent with their experimental equivalents. The damage model applied in the calculations allow to simulate the fracture sequences observed during the experiments. They included the horizontal tensile necking, horizontal tensile fracture and shear yielding, and vertical shear fracture.

The damage processes are very clear visible in load-displacement curves, determined experimentally and numerically. They are presented in figures 9 a and b, where comparison of the results of simulations and experiments of elements shown in figures 8 a and b is included.
Figure 9. Load-displacement curves determined during numerical simulations and experiments for specimen (a) T1 and (b) T2 [24].

Similarly as during failure mechanism simulations (Figs. 8 a and b), high agreement of curves determined numerically and experimentally is observed. It is due to the high level of accuracy in simulating the fracture sequence. The residual strength observed at the final fracture propagation stage was due to the processes taking place in the shear plane. It was connected with the strength of the tension plane until complete fracture of the connection. Modelling of the damage phenomena was possible by introducing the fracture to the applied models. When damage parameters reached critical value, element was deleted.

Conclusions

Although the basic relationships of solid mechanics were established long ago, this branch of continuum mechanics is still intensively developed. It is enough to mention fracture mechanics and damage mechanics, which are the subject of intense interest of many researchers. This is related to, among others, with the possibilities offered by numerical methods that allow verification of theoretical models. The examples of practical application of damage mechanics methods based on the Bai-Wierzbicki model presented in the article show that this direction of research is correct. As seen in discussed cases, the results of numerical simulations are very similar to experiments, which allows modeling and predicting the failure of materials and structural elements.

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