Does geometric coupling generate resonances?

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Abstract – Geometrical coupling in a co-dimensional one Randall-Sundrum scenario (RS) is used to study resonances of $p$-form fields. The resonances are calculated using the transfer matrix method. The studied model considers the standard RS with delta-like branes, and branes generated by kinks and domain walls as well. The parameters are changed to control the thickness of the smooth brane. With this a very interesting pattern is found for the transmission coefficient. The geometrical coupling does not generate resonances for the reduced $p$-form in all cases considered.

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Introduction. – Brane-world models consider the four-dimensional universe as a hyper-surface in a high-dimensional bulk. The four-dimensional world is obtained by integrating the action over the extra dimension. To ensure that this procedure is finite, Kaluza and Klein considered a compact extra dimension [1,2]. An alternative approach to compactification was proposed by Randall and Sundrum as a way to solve the hierarchy problem [3]. In brane scenarios with extended extra dimension the four-dimensional zero mode is not trivially obtained. Since the extra dimension is infinity, the field must be trapped in the membrane in order to recover the four-dimensional physics. The pathway is to solve an effective Schrödinger-like equation for the mass modes and verify its square integrability. It is well known that in anti-de Sitter bulk, as in Randall-Sundrum (RS) model, the gravity and the scalar fields are confined to the membrane [3]. However, due to its conformal invariance the vector field is not localized, which is a serious problem for a realistic model. Many solutions have been proposed to solve this problem. For example, most of these models introduce other fields or nonlinearities to the gauge field [4–6].

In the search for a model without introducing new degrees of freedom, a topological mass term in the bulk has been used. But it was not able to generate a massless photon in the brane [7]. However, it was shown by Ghoroku et al. that the addition of a mass term and a boundary mass term could solve the problem [8]. This result was generalized to include $p$-form fields [9], and more recently it was shown that the Ghoroku model can be generated by coupling the mass term with the Ricci scalar; this mechanism was called “geometrical localization” [10,11]. Interestingly, the same mechanism was proven to work for $p$-forms and Elko spinors [12–14]. A similar mechanism, with a geometrical coupling with the field strength, was shown to trap the zero mode of the Yang-Mills field [15]. It is important to mention that a different coupling between gravity and the gauge field in RS scenarios has first been considered in ref. [16] but localization was not fully obtained.

After the work of RS several recent results have been developed based on the idea of thick membranes and their implications for brane-world physics [17–25]. The advantage of these models is that the singularity generated by the brane is eliminated. In this scenario a transfer matrix method has been proposed to analyze resonances [26–28]. In this direction the geometrical localization mechanism has provided analytical solutions for any smooth version of the RS scenario and for any $p$-form [12] (for further analytic solutions see [28–30]).

As shown in ref. [12], the geometrical coupling traps the zero mode of all $p$-form fields in branes with asymptotic Randall-Sundrum behavior, while for reduced $(p−1)$-form only in special cases the zero mode is localized. It is also shown that the massive modes are non-localized. In this work we use the transfer matrix method to analyze the existence of unstable massive modes on the brane. We compute the transmission coefficient for some $p$-forms in co-dimension–one RS scenarios for delta-like branes, for branes generated by a domain wall and by a kink.
This detailed numerical study provides some hints about the structure of resonances in the geometrical localization model.

This paper is organized as follows: in the next section the geometrical localization mechanism is reviewed in order to obtain the effective potential of the Schrödinger equation for the p-form and the reduced (p - 1)-form in co-dimension-one bran-world. In the third section we use this potential to compute the transmission coefficient for some p-forms. In the fourth and fifth sections and smooth scenarios generated by topological defects are used to compute the transmission coefficient again considering different p-forms, dimensions of the spacetime and the thickness of the brane. In the last section the results are discussed.

Review of localization of p-form field with geometric coupling. – In this section the geometrical localization mechanism of p-form fields in a co-dimension–one bran-world is reviewed based on [12]. The action used is given by

\[ A = \int d^D x \sqrt{-\gamma} \left[ -\frac{1}{2(p+1)!} \gamma_{\mu_1...\mu_{p+1}} Y_{\mu_1...\mu_{p+1}} Y^{\mu_1...\mu_{p+1}} - \frac{1}{2p!} \gamma_{\mu} R X_{\mu_2...\mu_{p+1}} X^{\mu_2...\mu_{p+1}} \right], \]

(1)

where \( Y_{\mu_1...\mu_{p+1}} \equiv \partial_{\mu_1} X_{\mu_2...\mu_{p+1}} \). The components of the equation of motion are given by

\[ e^{\alpha_p A} \partial_{\mu} Y^{\mu_1...\mu_{p+1}} - \gamma_p R e^{\alpha_p - 1 A} X^{\mu_1...\mu_{p+1}} = 0, \]

(2)

\[ \partial_{\mu} Y^{\mu_1...\mu_{p+1}} = 0, \]

(3)

where \( \alpha_p \equiv D - 2(p + 1) \), the prime means a z derivative and from now on all (D - 1)-dimensional indices will be contracted with \( \eta^{\mu_p} \). Due the geometric coupling term the divergence of the equation of motion is not trivial and provides the condition

\[ (Re^{\alpha_p - 1 A} X^{\mu_1...\mu_{p-1}})' + Re^{\alpha_p - 1 A} \partial_{\mu_p} X^{\mu_1...\mu_{p-1}} = 0. \]

(4)

Also the effective gauge invariance for the (p - 1)-form is provided, \( \partial_{\mu_p} X^{\mu_1...\mu_{p-1}} = 0. \)

The gauge symmetry has been broken in the D-dimensional action because of the coupling term. To avoid this difficulty we must divide the p-form into transversal and longitudinal parts as

\[ X^{\mu_1...\mu_p}_T \equiv X^{\mu_1...\mu_p} + \left( -\frac{1}{p} \right) \partial_{\mu_1} \partial_{\mu_p} X^{\mu_2...\mu_{p-1}}; \]

\[ X^{\mu_1...\mu_p}_L \equiv \left( -\frac{1}{p-1} \right) \partial_{\mu_1} \partial_{\mu_p} X^{\mu_2...\mu_{p-1}}. \]

(5)

As shown in ref. [12] the decoupling can be obtained using the definitions (5) and the divergence equation (4) leading to

\[ e^{\alpha_p A} \Box X^{\mu_1...\mu_p}_T + e^{\alpha_p A} (X^{\mu_1...\mu_p}_T)' = 0, \]

(6)

\[ \Box X^{\mu_1...\mu_p}_L + \left[ Re^{\alpha_p - 1 A} (Re^{\alpha_p - 1 A} X^{\mu_1...\mu_p}_L)' \right]' = 0. \]

(7)

To determine the longitudinal part of the p-form we can use the divergence equation

\[ (Re^{\alpha_p - 1 A} X^{\mu_1...\mu_{p-1}})' + Re^{\alpha_p - 1 A} \partial_{\mu_p} X^{\mu_1...\mu_{p-1}}_L = 0. \]

(8)

Starting with the transversal part of the p-form field, we impose the separation of variables in the form \( X^{\mu_1...\mu_p}_T (z, x) = f(z) \tilde{X}^{\mu_1...\mu_p}_T (x) \) in (6) to obtain the set of equations

\[ \Box \tilde{X}^{\mu_1...\mu_p}_T - m^2 \tilde{X}^{\mu_1...\mu_p}_T = 0, \]

(9)

\[ (e^{\alpha_p A} f'(z))' - \gamma_p Re^{\alpha_p - 1 A} f(z) = -m_X^2 e^{\alpha_p} f(z). \]

(10)

To write eq. (10) in a Schrödinger form we must consider \( f(z) = e^{-\alpha_p A/2} \psi. \) The potential obtained after this transformation is

\[ U(z) = \left[ \frac{\alpha_p^2}{4} - (D - 1)(D - 2)\gamma_p \right] A'(z)^2 + \frac{\alpha_p}{2} - 2(D - 1)\gamma_p A''(z), \]

(11)

where we have used that

\[ R = -(D - 1) \left[ 2A'' + (D - 2)A^2 \right] e^{-2A}. \]

(12)

Imposing a solution for zero mode in the form \( \psi \propto e^{\phi A} \) we obtain that

\[ \gamma_p = -\frac{(D - 2) - 2\alpha_p}{4(D - 1)}, \]

(13)

and \( b = p \). Fixing the coupling constant \( \gamma_p \) by (13) the zero mode of the transversal part of the p-form field is localized in a brane with asymptotic Randall-Sundrum behavior. To compute the massive modes we must specify the brane scenario.

For the (p - 1)-form we will separate the variables imposing that \( X^{\mu_2...\mu_p}_T (z, x) = u(z) \tilde{X}^{\mu_2...\mu_p}_T(x) \) in (7) to obtain the set of equations

\[ \Box \tilde{X}^{\mu_2...\mu_p}_T - m^2 \tilde{X}^{\mu_2...\mu_p}_T = 0, \]

(14)

\[ u''(z) + \left[ R^{-1} e^{-\alpha_p - 1 A} (Re^{\alpha_p - 1 A})' u(z) \right]' - \gamma_p Re^{2A} u(z) = -m^2 u(z), \]

(15)

and as we have made for the p-form, we will make \( u(z) = (Re^{(D - 2p)A}/2)\psi \) to transform eq. (15) into a Schrödinger-like equation with the potential

\[ U(z) = \left[ \frac{1}{4} (\alpha_p + 2) A' + (\ln R)' \right]^2 - \frac{1}{2} ([\alpha_p + 2] A'' + (\ln R)')^2 + \gamma_p Re^{2A}. \]

(16)
The above potential is not simple to be analyzed in its complete form. Therefore, the study of localization must be made by the asymptotic behavior, which is

\[
U(z) = \left[ \frac{\alpha_p}{D-1} + \frac{1}{2} - 2(D-1) \right] A''(z) + \frac{1}{2} A'(z)^2.
\]

Since we have fixed the coupling constant \( \gamma_p \) to localize the zero mode of the \( p \)-form field transversal part, the same cannot be done for the \((p-1)\)-form. An exception occurs when \( \alpha_p = -1 \) and in five dimensions this condition is satisfied only by the Kalb-Ramond field.

Finally, for the longitudinal part of the \( p \)-form field, making the separation of variables in the form \( X^{p-1\ldots p}\tau(x,z) = F(z)X^{p-1\ldots p}(x) \) we obtain (4),

\[
C_0 \left( R_0^{(D-2p)} A \right)' = R_0^{(D-2p)} A F(z),
\]

\[
\tilde{X}^{p-1\ldots p} + C_0 \partial_{\nu} \tilde{X}^{p-1\ldots p} = 0,
\]

where \( C_0 \) is a constant. To solve the massive modes we must specify the brane scenario explicitly. In the following sections we will study this for some brane-world scenarios.

**The delta-like brane case.** — The first brane scenario discussed here is the one with a delta-like brane. Despite the singularity, this scenario has an historical importance and serves as an important paradigm in physics of extra dimensions and field localization. The warp factor for this scenario in a conformal form is given by

\[
A(z) = -\ln |kz| + 1.
\]

The Schrödinger equation potential, eq. (11), for the transversal part of the \( p \)-form field is given by

\[
U(z) = \frac{p(p+1)k^2}{(kz)^2} - 2\pi k\delta(z),
\]

and is illustrated in fig. 1 for some \( p \)-forms. An interesting result is that the potential of the transversal part of the \( p \)-form does not depend on the dimensionality of space-time. It depends only on the degree of the form. As imposed in the previous section the solution of zero mode is \( \psi \propto (kz+1)^{p} \). For the massive case eq. (10) provides the solution

\[
\psi(z) = (kz+1)^{1/2} \left[ C_1 J_\nu(m_Xz+m_X/k) + C_2 Y_\nu(m_Xz+m_X/k) \right],
\]

where \( \nu = (2p+1)/2 \) and \( C_1 \) and \( C_2 \) are constants related by the boundary conditions. Then the massive modes are non-localized. To obtain more information about massive modes we can evaluate the transmission coefficient. As made in previous cases, the transmission coefficient can be written as

\[
T = \frac{m_X^2}{|F_\nu(0)F_\mu'(0)+\alpha_p kF_\nu'(0)|^2},
\]

where \( F_\nu(z) = \sqrt{2} (m_Xz+m_X/k)^{1/2} H^{(1)}_\nu(m_Xz+m_X/k) \) and \( H^{(1)}_\nu(z) \) is the Hankel function of the first kind. The transmission coefficient is plotted in fig. 2 as a function of energy for some \( p \)-forms. The figure does not show peaks, indicating that there is no unstable massive mode. For the \((p-1)\)-form in the Randall-Sundrum scenario the potential of the Schrödinger equation, (16), can be written as

\[
U(z) = \frac{p(p+1)k^2}{(kz)^2} - 2[p-(\alpha_p+1)]k\delta(z).
\]

Since \( R \) in the RS scenario is a constant, it does not contribute to the potential. Unlike the \( p \)-form case, the potential of the \((p-1)\)-form depends on the dimensionality of the space-time. As the finite part of the potential is the same as in (21), the regular part of the potential and the solution are the same of the \( p \)-form field. Now the boundary condition at \( z = 0 \) imposes

\[
\psi(z) = f_0 \left[ (kz+1)^{-p} - \frac{\alpha_p+1}{D-2} (kz+1)^{p-1} \right].
\]

As discussed in the previous section, only when \( \alpha_p = -1 \) it is possible to obtain a convergent solution. The potential for the \((p-1)\)-form is the same of the \( p \)-form with a modified boundary condition. The behavior of the massive modes are the same, i.e., non-localized. The behavior of the transmission coefficient is shown in fig. 3(a) for some forms in \( D = 5 \). In fig. 3(b) we consider the 1-form case...
in some bulk dimensions. The figure shows the same behavior as in the $p$-form case.

The longitudinal part of the $p$-form can be found replacing the solution (25) in (18). This procedure provides

$$F(z) = F_1 \sgn(z) \left[15 (k|z| + 1)^{-3/2 + \alpha_p} + (2\alpha_p - 3)(\alpha_p + 1)(k|z| + 1)^{3/2}\right],$$

where $F_1$ is a constant. As in the $(p - 1)$-form case only when $\alpha_p = -1$ the above expression provides a localized solution.

The smooth domain-wall brane case. – In this section we investigate the localization of a $p$-form field in a smooth warp factor scenario. Since the metric can be written in a conformal form we can use all results obtained in the previous section which did not use the explicit form of the warp factor. The smooth warp factor produced by a domain wall is given by [19,31],

$$A(z) = -\frac{1}{2n} \ln \left[(kz)^{2n} + 1\right],$$

which recovers the Randall-Sundrum metric for large $z$ and $n \in N^*$. Using this metric in eq. (11) we obtain the Schrödinger potential for the transversal part of the $p$-form

$$U(z) = p(p + 2n) \frac{k^2(kz)^{4n-2}}{[(kz)^{2n} + 1]^2} - \frac{p(2n - 1)k^2(kz)^{2n-2}}{(kz)^{2n} + 1}.$$  

The plots of this potential are shown in fig. 4(b) for some values of $n$ with $p = 1$, and in fig. 4(a) for some values of $p$ with $n = 1$. As in the delta-like case, the potential of the $p$-form does not depend on the dimensionality of the spacetime.

For the massless mode of the transversal $p$-form, the Schrödinger equation with the above potential provides the following convergent solution:

$$\psi \propto \left[(kz)^{2n} + \beta\right]^{-p/2n},$$

as computed in the second section. The solution of massive modes for the transversal part of the $p$-form cannot be found analytically. To obtain information about these states we use the transfer matrix method to evaluate the transmission coefficient. The behavior is illustrated in fig. 5(b) for $p = 1$ and some values of the parameter $n$. In fig. 5(a) the transmission coefficient is shown for different $p$-forms with $n = 1$. Both figures do not exhibit peaks, indicating the absence of unstable modes.

Considering now the massless mode of the reduced $(p - 1)$-form, eq. (16) gives a complicated potential. One must be careful since it involves $\ln R$ and a vanishing $R$ generates a divergent contribution to the potential. To deal with this one must consider cases with a regular Ricci scalar. Its explicit form is given by

$$R = (D - 1)z^2(2n-1)\frac{2(1 - 2n) + Dz^{2n}}{(1 + z^{2n})^{2-1/n}}.$$  

Fig. 3: Transmission coefficient for the $(p - 1)$-form in the Randall-Sundrum scenario as a function of adimensional energy, $E = m_{D-1}^2/k^2$. (a) For some $p$-forms in $D = 5$, (b) for $1$-form for some $D$.

Fig. 4: Behavior of the Schrödinger potential in a smooth scenario generated by domain walls. (a) For some $p$-forms with $n = 1$, (b) for $1$-form and for some values of the parameter $n$. 

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The Ricci scalar vanishes at \( z = \pm (2(2n - 1)/D)^{1/2n} \), making the potential (16) divergent at these points. Because of this, the transmission coefficient cannot be calculated by the transfer matrix method. The longitudinal part of the \( p \)-form is determined by eq. (4). As no analytic solution for the massless mode of the \((p-1)\)-form is discussed here, it is not possible to find an analytic solution for the massless mode of the longitudinal part of the \( p \)-form.

**The smooth kink brane case.** – In this section we must consider branes generated by a kink. As shown in previous work, this scenario produces resonances for some free fields [26]. The warp factor is given by

\[
A(y) = -4 \ln \cosh y - \tanh^2 y, \tag{31}
\]

where the variable \( y \) is related to the conformal coordinate \( z \) by \( dz = e^{-A(y)} dy \). The behavior of the potential of the transversal part of the \( p \)-form field, eq. (11), with this warp factor is illustrated in fig. 6(a). For the massive modes, like in previous sections, the transfer matrix method is used to compute the transmission coefficient. The results are plotted in fig. 6(b), and, unlike the previous cases, it exhibits the most probable state when \( p = 3 \). A careful analysis indicates that the peak for the 3-form is not a resonance, as shown in detail in fig. 6(b).

For the \((p-1)\)-form field the Schrödinger potential of eq. (16) with the warp factor of eq. (31) diverges at finite points due to the fact that the Ricci scalar vanishes at these same points. Because of this, the transmission coefficient cannot be calculated by the transfer matrix method.

**Conclusion.** – In this manuscript we have calculated the transmission coefficient for massive modes of \( p \)-form fields with geometrical coupling in some brane scenarios. The important result found is that for all scenarios covered in this paper no resonances appear for the massive modes of \( p \)-form fields. To draw this conclusion we studied a thin and two thick brane scenarios: one generated by domain walls and the others generated by a kink. Since the potential of these cases does not depend on the dimension \( D \), the number of parameters to be considered is lower than for the reduced \((p-1)\)-form. The case with a thin or delta-like brane has also been considered previously in [9] and no resonance has been found. However, in [9], the Ghoroku model [8] was used. This model contains two free parameters and is very similar to the geometrical localization mechanism if the RS warp factor is considered. Here we have considered a different range in the parameter. For the cases \( p = 1, 2, 3 \), fig. 2 shows that no resonance were found. The indication that there is no resonance is given by the plots of the potential 1, which have only one barrier. Next, the domain wall case was considered. For this case, the effective potential has a volcano profile, fig. 4(b), and at first sight resonances should be expected. However, as shown in figs. 5(a) and (b), for several possible parameters, no resonance appears. Finally we consider the kink case. The effective potential was plotted in fig. 6(a) and
again a volcano-like potential was obtained. At this time a possible resonance appears for the $\mathcal{J}$-form field. However, when the numerical calculation is improved, we see in fig. 6(b) that there is no resonance at that mass. For all other cases no resonance was found. Therefore, this result indicates that the geometric coupling does not produce resonances for $p$-form fields. Since all the results presented here are from numerical calculations, more cases must be studied in order to reinforce that conclusion. Therefore, by the study made in this manuscript, it is not possible to obtain a definitive conclusion about the possibility that this result is a universal property of the model. However, it is surprising that no resonance appears in all brane scenarios used, both in the thin models as in the thick ones. Maybe this points to some hidden intrinsic property of this kind of coupling. This absence of resonances affects the phenomenological properties of the model, since there would be no imprints of extra dimensions through massive unstable excitation. However, in a recent paper [32] some of the present authors has shown that a specific residual photon mass is to be expected, being the first phenomenological test of the model.

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