The 2nd order corrections to the interaction of two reggeized gluons from the bootstrap

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Abstract. The 2nd order corrections are obtained to both forward and nonforward interaction of reggeized gluons in the octet colour channel using as a basis the bootstrap relation and a specific ansatz to solve it. The obtained forward kernel coincides with the logarithmic term plus two first non-logarithmic terms in the Pomeron 2nd order kernel. Both forward and nonforward kernels are found to be infrared finite.
1 Introduction.

Recently the 2nd order corrections were calculated for the forward hard (BFKL) pomeron equation [1,2]. They clearly divide into a correction for the running of the coupling and the 2nd order corrections in the running coupling proper. Generalization of this result for the other physically important case, that of the odderon, requires knowledge of the 2nd order corrections to the non-forward interaction of two reggeized gluons in the octet colour state. It is well-known that this interaction is severely restricted by the so-called bootstrap relation, which is, in fact, the unitarity requirement for the octet channel [3]. The form this relation takes in the 2nd order was recently discussed in [4].

In previous publications one of the authors (MAB) proposed to use the bootstrap relation and a specific ansatz to satisfy it for the introduction of the running constant, to all orders of the fixed one, into the gluon octet interaction [5]. Since in the lowest order this interaction is just one half of the interaction in the vacuum (pomeron) channel, this method also serves to find corrections for the running coupling in the pomeron equation both for forward and non-forward cases. Note that whereas for the forward case these corrections are trivial and can easily be reconstructed via the renormalization group, this is not so for the non-forward case, with more than one scale.

In this paper, using the bootstrap and the ansatz of [5], we calculate the full second order corrections for the non-forward gluon interaction in the octet channel necessary for the odderon equation. In this way we also automatically find the corrections for the running of the coupling for the non-forward pomeron equation.

2 Basic equations

Up to the 2nd order in the fixed coupling, two gluons are described by a Schrödinger equation in the transverse space

\[ H \phi(q_1, q_2) = E \phi(q_1, q_2). \]  

Here \( q_1 \) and \( q_2 \) are the momenta of the two gluons; \( E = 1 - j \) where \( j \) is the angular momentum. The Hamiltonian \( H \) is a sum of the kinetic energy, given by a sum of gluonic Regge trajectories with a minus sign \( -\omega(q_1) - \omega(q_2) \), and the interaction \( -V^{(R)} \), which depends on the colour state \( R \) of the two gluons. We shall be interested in either the vacuum state \((R = 1)\) or the octet state \((R = 8)\). As mentioned, to the lowest order in the running coupling, \( V^{(1)} = 2V^{(8)} \).

The bootstrap relation requires that the solution to Eq. (1) for the octet channel be the reggeized gluon itself. This can be fulfilled if the interaction \( V_8 \) is related to the trajectory as

\[ \int \left( \frac{d^2q_1'/(2\pi)^2}{(2\pi)^2} \right) V^{(8)}(q_1, q_1', q_2) = \omega(q) - \omega(q_1) - \omega(q_2), \quad q = q_1 + q_2. \]  

In [5] it was shown that this relation can be satisfied provided both \( V^{(8)} \) and \( \omega \) are expressed via a single function \( \eta(q) \) as follows

\[ \omega(q) = -\int \frac{d^2q_1 \eta(q)}{(2\pi)^2 \eta(q_1) \eta(q_2)}, \quad q = q_1 + q_2 \]  

and

\[ V^{(8)}(q, q_1, q_1') = \left( \frac{\eta(q_1)}{\eta(q_1')} + \frac{\eta(q_2)}{\eta(q_2')} \right) \frac{1}{\eta(q_1 - q_1')} - \frac{\eta(q)}{\eta(q_1) \eta(q_2')}. \]
In the lowest order of the fixed coupling one has (we take $N_c = 3$)

$$\eta^{(0)}(q) = \frac{q^2}{3\alpha_s}.$$  

(5)

The forms (3) and (4) guarantee that to the lowest order in the running coupling the full interaction in the vacuum channel is infrared stable [5]. A stronger result is that they also guarantee that the sum of single and pair terms in the odderon equation stays infrared stable in all orders in the running coupling. Indeed, this sum can be represented as a sum of three pair terms $-(1/2)(K_{12} + K_{23} + K_{31})$, where e.g.

$$K_{12} = \omega(q_1) + \omega(q_2) + 2V_{12}^{(8)},$$

(6)

and this combination is infrared stable provided $\eta(q)$ goes to zero as $q^2 \to 0$ modulo logarithms. Of course, this does not settle the question of the eventual infrared stability of the odderon equation in the higher orders, since already at the 2nd order triple interaction terms seem to appear. We shall not deal with this problem here.

Our aim will be to study the 2nd order corrections to the kernel $K$ which follow from the 2nd order corrections in $\eta(q)$. The latter can be found from the known form of the trajectory $\omega(q)$ to the 2nd order in the fixed coupling [6]:

$$\omega(q) = \omega^{(1)}(q) + \omega^{(2)}(q),$$

(7)

where the superscripts refer to the order in the coupling, and, in the dimensional regularization in the MS scheme,

$$\omega^{(1)}(q) = -\bar{g}^2 \left( \frac{2}{\epsilon} + 2t + \epsilon(t^2 - 2\psi'(1)) \right),$$

$$\omega^{(2)}(q) = -\bar{g}^4 \left( A\left(\frac{1}{\epsilon^2} - t^2\right) + B\left(\frac{1}{\epsilon} + 2t\right) + C \right),$$

(8)

(9)

Here and in the following $t = \ln(q^2/\mu^2)$; $\mu$ is the normalization point; $\epsilon \to 0$:

$$\bar{g}^2 = \frac{3\alpha_s \Gamma(1 - \epsilon)}{(4\pi)^{1+\epsilon}},$$

(10)

and $\bar{g}^4$ is just the square of (10). The coefficients $A$, $B$ and $C$ are

$$A = \frac{11}{3} - \frac{2}{9}N_f; \quad B = \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27}N_f; \quad C = -\frac{404}{27} + \frac{1}{27} \zeta(3) + \frac{56}{81}N_f.$$  

(11)

We have retained a term proportional to $\epsilon$ in (8) for future reference.

### 3 Function $\eta(q)$ to the 2nd order

We present function $\eta(q)$ in the form

$$\eta(q) = \eta^{(0)}(q)(1 + \xi(q) + ...),$$

(12)

where $\xi$ represents the terms of the 2nd order in $\alpha_s$. The substitution of the running coupling instead of the fixed one into (5) evidently gives a contribution to $\xi$

$$-\frac{3\alpha_s A}{4\pi} t.$$
This term is what we call a correction for the running of the coupling. However there may be other terms in $\xi$ which represent corrections of the 2nd order in the already running coupling constant. We are going to find all these by matching the form of $\omega^{(2)}$ found via function $\eta(q)$ given by (12) with the expression (9) found from the perturbative QCD.

Putting (12) into Eq. (3) we obtain in the 2nd order

$$\omega^{(2)}(q) = \omega^{(1)}(q)\xi(q) + \frac{6\alpha_s}{4\pi^2} \int \frac{d^2 q_1 q_2^2}{q_1^2 q_2^2} \xi(q_1).$$  \hspace{1cm} (13)

With $\omega^{(2)}(q)$ known, this is an integral equation for $\xi$. We shall solve it by choosing an appropriate functional form of $\xi(q)$:

$$\xi(q) = c + dt + \epsilon ft^2,$$ \hspace{1cm} (14)

where the coefficients $c$, $d$ and $f$ in principle may depend on $\epsilon$ but are regular at $\epsilon = 0$. We introduce an extra factor $\epsilon$ into the third term of (14) to avoid poles of the third order in $\epsilon$ absent both in $\omega^{(2)}$ and the first term on the r.h.s. of (13).

We denote the integral term in (13) as $X$. In the dimensional regularization we have

$$X = \frac{6\alpha_s q^2}{(2\pi)^D} (cX_0 + dX_1 + f\epsilon X_2),$$ \hspace{1cm} (15)

where

$$X_n = \int \frac{d^D q_1 t_1^n}{q_1^2 q_2^2},$$ \hspace{1cm} (16)

$D = 2 + 2\epsilon$ and $t_1 = \ln(q_1^2/\mu^2)$. $X_n$ can be calculated as the $n$-th derivative in $\alpha$, taken at $\alpha = 0$ of a general integral

$$I = \int \frac{d^D q_1 q_2^{2\alpha}}{q_1^2 q_2^2}.$$ \hspace{1cm} (17)

This integral can be easily calculated by standard methods using the generalized Feynman representation. We find:

$$I = \pi^{1+\epsilon}(q^2)^{\alpha+\epsilon}\frac{\Gamma(1-\alpha-\epsilon)\Gamma(\alpha+\epsilon)\Gamma(\epsilon)}{\Gamma(1-\alpha)\Gamma(\alpha+2\epsilon)}.\hspace{1cm} (18)$$

From this we obtain

$$X_0 = \pi^{1+\epsilon}(q^2)^{-\epsilon}\frac{\Gamma(1-\epsilon)\Gamma^2(\epsilon)}{\Gamma(2\epsilon)},$$ \hspace{1cm} (19)

$$X_1 = X_0 Y, \quad Y = (t - \psi(1-\epsilon) + \psi(\epsilon) + \psi(1) - \psi(2\epsilon)),$$ \hspace{1cm} (20)

$$X_2 = X_0 (Y^2 + \psi'(1-\epsilon) + \psi'(\epsilon) - \psi'(1) - \psi'(2\epsilon)).$$ \hspace{1cm} (21)

One has to take into account that the 1st order trajectory (8) is related to $X_0$ as

$$\omega^{(1)}(q) = -\frac{3\alpha_s q^2}{2(2\pi)^{2+2\epsilon}} X_0.$$ \hspace{1cm} (22)

Combining the two terms on the r.h.s (13) we get

$$\omega^{(2)}(q) = \frac{3\alpha_s q^2}{(2\pi)^{2+2\epsilon}} X_0 \left(-\frac{u}{\epsilon} + c + ht + f\epsilon t^2 \right)$$ \hspace{1cm} (23)

with $u = d - 2f$. Recalling the form of $\omega^{(1)}$, Eq. (8), separating $g^2$ and neglecting terms of the order $\epsilon$ or higher, we finally get

$$\omega^{(2)}(q) = 2\bar{g}^2 \left(-\frac{u}{\epsilon^2} + \frac{c}{\epsilon} + ct + \frac{1}{2} dt^2 + u\psi'(1) \right).$$ \hspace{1cm} (24)
Comparing this expression with the one found from the perturbative QCD (9), we note at once that to match them we have to assume that both u and c contain terms of the higher order in $\epsilon$:

$$u = u_0 + u_1 \epsilon + u_2 \epsilon^2, \quad c = c_0 + c_1 \epsilon.$$  \hspace{1cm} (25)

With this form the 2nd order trajectory becomes

$$\omega^{(2)}(q) = 2\bar{g}^2 \left( -\frac{u_0}{\epsilon^2} + \frac{c_0 - u_1}{\epsilon} + c_0 t + \frac{1}{2} dt^2 + c_1 - u_2 + u_0 \psi'(1) \right)$$  \hspace{1cm} (26)

So we have just 5 parameters $c_0, c_1, d, u_0, u_1, u_2$ to match this expression with 5 terms in (9) containing $1/\epsilon^2, 1/\epsilon, t, t^2$ and a constant. We get for our parameters

$$u_0 = (1/2)\bar{g}^2 A, \quad c_0 - u_1 = -(1/2)\bar{g}^2 B, \quad c_0 = -\bar{g}^2 A, \quad d = \bar{g}^2 A, \quad u_0 \psi'(1) + c_1 - u_2 = -(1/2)\bar{g}^2 C.$$  \hspace{1cm} (27)

We observe that to match the perturbative trajectory it is not sufficient to take into account only the corrections to $\eta(q)$ due to the running of the coupling, which corresponds to all coefficients equal to zero, except for equal $d$ and $u_0$. Quite a few proper 2nd order corrections have to be also included.

4 Forward interaction in the octet channel

Having determined the 2nd order corrections for $\eta(q)$ we are now in a position to find these corrections for the interaction of gluons in the octet channel. As mentioned, to the lowest order in the running coupling, the vacuum channel interaction is just twice the octet one. So we also find the corrections due to the running of the coupling for the vacuum channel. In this section we calculate the 2nd order interaction for the forward case $q = 0$, which is a much simpler task due to the fact that the eigenfunctions of the 1st order kernel are simple.

As for the pomeron, at $q = 0$ the eigenvalue equation for the octet channel interaction

$$-K\phi = E\phi,$$  \hspace{1cm} (28)

with $K$ given by (6), can be simplified by substituting

$$\phi(q) \rightarrow \eta(q)\phi(q).$$

Then (28) takes the form

$$2\omega(q)\psi(q) + \frac{4}{4\pi^2} \int \frac{d^2q'\phi(q')}{\eta(q - q')} = -E\psi(q).$$  \hspace{1cm} (29)

In the 1st order this is just the BFKL equation. In the 2nd order we get for the left-hand side

$$K^{(2)}\psi \equiv 2\omega^{(2)}\psi(q) - \frac{12\alpha_s}{4\pi^2} \int \frac{d^2q'\xi(q - q')\psi(q')}{(q - q')^2},$$  \hspace{1cm} (30)

where $\xi$ is defined by (12) and determined by (14) and (27).

We shall calculate the action of the kernel $K^{(2)}$ on the 1st order BFKL eigenfunctions

$$\psi_{\lambda}(q) = q^{2\lambda}.$$  \hspace{1cm} (31)

In fact, for the normalizability one should take $\lambda = -1/2 + i\nu$ and multiply (31) by the appropriate normalization factor, but this is irrelevant for the properties of the kernel $K^{(2)}$. 
We again introduce a notation $X$ for the integral part of the left-hand side (without the minus sign). Putting (14) into the integrand we find the same form (15) for $X$, with an extra factor two and $X_n$, which are now given by

$$X_n = \int \frac{dD q T \alpha}{q_1 - q_2}, \quad q_2 = q - q_1, \quad t_2 = \ln(q_2^2/\mu^2).$$ (32)

These integrals can again be obtained by differentiating in $\alpha$ at $\alpha = 0$ the generic integral

$$I = \int \frac{dD q T \alpha}{q_1 - q_2} = \pi^{1+\epsilon}(g^2)^{\epsilon} \frac{1}{\Gamma(1-\epsilon)} \frac{\Gamma(-\lambda - \epsilon)}{\Gamma(-\lambda)\Gamma(1 + \lambda + \epsilon)} \frac{\Gamma(1 + \epsilon)}{\Gamma(1 + \alpha + \epsilon)}.$$ (33)

>From this we find

$$X_0 = \pi^{1+\epsilon}(g^2)^{\epsilon} \frac{1}{\Gamma(1-\epsilon)} \frac{\Gamma(-\lambda - \epsilon)}{\Gamma(-\lambda)\Gamma(1 + \lambda + \epsilon)} X_0 Y, \quad Y = t - \psi(-\lambda - \epsilon) + \psi(1) - \psi(1 + \lambda + 2\epsilon),$$ (34)

$$X_1 = X_0 Y, \quad Y = t - \psi(-\lambda - \epsilon) + \psi(1) - \psi(1 + \lambda + 2\epsilon),$$ (35)

$$X_2 = X_0 Y^2 + \psi'(-\lambda + \epsilon) + \psi'(1 - \psi'(1 + \lambda + 2\epsilon)).$$ (36)

Combining the terms we get for $X$

$$X = \frac{12\alpha_s}{(2\pi)^2 + 2\epsilon} X_0 \left[ - \frac{u}{\epsilon} + c + u(t + \Delta) + \epsilon \left( d(\psi'(-\lambda) + \psi'(1 + \lambda)) + f((t + \Delta)^2 - \psi'(1 + \lambda)) + u \psi'(1) \right) \right],$$ (37)

where we denoted

$$\Delta = 2\psi(1) - \psi(-\lambda) - \psi(1 + \lambda)$$ (38)

the eigenvalue of the 1st order kernel (up to a constant factor). To obtain our final result we separate a factor to be included into $\bar{g}^2$ and develop the rest, including $X_0$, in $\epsilon$ up to constant terms. Then we get

$$X = 4\bar{g}^2 q^2 \chi \left( - \frac{u}{\epsilon^2} + \frac{c}{\epsilon} + c(t + \Delta) + \frac{1}{2} d((t + \Delta)^2 + \psi'(1 + \lambda)) + u \psi'(1) \right).$$ (39)

Now we have only to subtract it from the 1st term on the righthand side of (30). Taking $\omega(2)$ in the form (24) we find that the terms singular in $\epsilon$ cancel, so that the kernel turns out to be infrared stable, as expected. Our final result for it is

$$K^{(2)} \psi = 4\bar{g}^2 \left( - d\Delta t - c \Delta + \frac{1}{2} d(-\Delta^2 + \psi'(1 + \lambda) - \psi'(-\lambda)) \right) \psi(q).$$ (40)

One notes that out of five coefficients entering $\xi$ only two remain in the final expression for the action of the kernel. Using (27) we find

$$K^{(2)} \psi = 4\left( \frac{3\alpha_s}{4\pi} \right)^2 \left( -\Delta A t + B \Delta + \frac{1}{2} A(-\Delta^2 + \psi'(1 + \lambda) - \psi'(-\lambda)) \right) \psi(q).$$ (41)

The first term, proportional to $\ln(q^2/\mu^2)$ gives the correct contribution from the running of the coupling. As mentioned this term is the same for the octet and vacuum channels. So the bootstrap indeed gives the correct description of the running of the coupling, at least, in the 2nd order. Other terms are corrections to the interaction in the 2nd order in the running coupling constant. They refer only to the octet interaction since at this order the vacuum channel interaction is different. It is remarkable however that they exactly coincide with the first two terms found for corresponding correction in the vacuum channel. As a whole the found expression is much simpler than the one for the vacuum channel, which includes many more terms with a complicated dependence on $\lambda$. 

5 The non-forward pair kernel in the octet colour state

The bootstrap gives a relation between the gluon trajectory and its pair interaction in the octet colour state. As mentioned this interaction is important for the the odderon, where it enters in the combination (6). In this section we shall obtain the 2nd order correction to the kernel (6) using the found form of the function $\eta$ and Eqs (3) and (4).

It is convenient to present the kernel (6) in the form which explicitly shows its infrared stability. We use an identity

$$
\frac{1}{\eta(q_1)\eta(q_2)} = \frac{1}{\eta(q_1)(\eta(q_1) + \eta(q_2))} + \frac{1}{\eta(q_2)(\eta(q_1) + \eta(q_2))}
$$

(42)

to present the trajectory (3) in an equivalent form

$$
\omega(q) = -2 \int \frac{d^2q_1\eta(q)}{(2\pi)^2\eta(q)(\eta(q_1) + \eta(q_2))}, \; q = q_1 + q_2.
$$

(43)

Then denoting $q_{11'} = q_1 - q_1'$ for brevity and combining the terms with $\eta(q_1 - q_1')$ in the denominator in the trajectories and in the interaction, we obtain for the action of the kernel (6) on an arbitrary function $\psi(q_1)$

$$
K\psi = 2 \int \frac{d^2q_1'\eta(q_1)}{(2\pi)^2\eta(q_1 - q_1')} \left( \frac{\psi(q_1')}{\eta(q_1') - \eta(q_1) - \eta(q_1')} \right) - \frac{\psi(q_1)}{\eta(q_1)} (\xi(q_1) - \xi(q_1') - \xi(q_{11'}))
$$

$$
+ (1 \leftrightarrow 2) - 2 \int \frac{d^2q_1'd^2q_1'q_2'\psi(q_1')}{(2\pi)^2\eta(q_1')\eta(q_2')} \xi(q_1') - \xi(q_2')).
$$

(44)

As in the case of the standard BFKL equation, this form shows explicitly that all singularities at small momenta $q_1', q_2'$ and $q_{11'}$ cancel and the kernel is infrared stable if function $\eta(q)$ goes to zero not essentially faster than $q^2$.

Now, preserving the infrared stability, we present in (44) function $\eta$ in the form (12) keeping terms up to the 2nd order in $\alpha_s$. For the 2nd order contribution we obtain

$$
K^{(2)}\psi = 6\alpha_s \int \frac{d^2q_1'd^2q_1}{(2\pi)^2 q_{11'}^2} \left( \frac{\psi(q_1')}{q_{11'}^2 + q_1^2} (\xi(q_1) - \xi(q_1') - \xi(q_{11'})) - \frac{\psi(q_1)}{q_1^2 + q_{11'}^2} (\xi(q_1) - \xi(q_1') - \xi(q_{11'})) \right)
$$

$$
+ (1 \leftrightarrow 2) - 6\alpha_s \int \frac{d^2q_1d^2q_1'q_2\psi(q_1)}{(2\pi)^2 q_{11'}^2 q_2'} (\xi(q) - \xi(q_1') - \xi(q_2')).
$$

(45)

Presenting

$$
\frac{q_1^{'2}}{q_1^2 + q_{11'}^2} = 1 - \frac{q_{11'}^2}{q_1^2 + q_{11'}^2},
$$

(46)

we separate from the first term a contribution

$$
6\alpha_s \psi(q_1) \int \frac{d^2q_1d^2q_1'}{(2\pi)^2(q_1^2 + q_{11'}^2)^2} (\xi(q_1') - \xi(q_{11'})),
$$

(47)

which is equal to zero, since it changes sign under the substitution $q_1' \rightarrow q_1 - q_1'$.

This brings us to a simple expression

$$
K^{(2)}\psi = 6\alpha_s \int \frac{d^2q_1d^2q_1'}{(2\pi)^2 q_{11'}^2} \left( \frac{\psi(q_1')}{q_1^2} - \frac{\psi(q_1)}{q_1^2 + q_{11'}^2} \right) (\xi(q_1) - \xi(q_1') - \xi(q_{11'})).
$$
\[ + (1 \leftrightarrow 2) - 6\alpha_s \int \frac{d^2 q_1' q^2 q_2' \psi(q_1')}{(2\pi)^2 q_1^2 q_2^2} (\xi(q) - \xi(q_1') - \xi(q_2')). \quad (48) \]

Now we have only to put our expression (14) for \( \xi(q) \) with the coefficients given by (25) and (27). Since the kernel \( K^{(2)} \) is infrared finite, all terms in (14) proportional to \( \epsilon \) or \( \epsilon^2 \) give no contribution, and, as in the forward case, the result only depends on \( c_0 = -\bar{g}^2 B \) and \( d = \bar{g}^2 A \). It is also clear that the constant term \( c_0 \) in \( \xi(q) \) just rescales the first order kernel \( K^{(1)} \). So in the end we finally obtain an explicit form for the action of \( K^{(2)} \) as

\[ K^{(2)} \psi = \frac{3\alpha_s}{4\pi} \left[ BK^{(1)} \psi + 6A \int \frac{d^2 q_1' q^2 q_2' \psi(q_1')}{(2\pi)^2 q_1^2 q_2^2} \left( \frac{\psi(q_1')}{q_1^2} - \frac{\psi(q_1)}{q_1^2 + q_1^2} \right) \ln \frac{q_1^2 \mu^2}{q_1^2 q_1^2} \right] \]

\[ + (1 \leftrightarrow 2) - 6A \alpha_s \int \frac{d^2 q_1' q^2 q_2' \psi(q_1')}{(2\pi)^2 q_1^2 q_2^2} \ln \frac{q_1^2 \mu^2}{q_1^2 q_1^2}, \quad (49) \]

As in the forward case, one expects that acting on the eigenfunctions of the lowest order kernel, this 2nd order kernel will give a shift for the eigenvalue plus some terms which come from the running of the coupling. As mentioned, the latter should be the same as for the non-forward kernel in the vacuum colour state. Unfortunately, although the found kernel does not look too complicated, we have not able to find its action on the first order eigenfunctions up to now. Work in this direction is in progress.

6 Conclusions

We have found the 2nd order corrections to the interaction of two reggeized gluons in the octet channel, using the bootstrap relation and a specific ansatz to solve it proposed in [5]. This interaction is important for the odderon. The relevant kernel for a pair of gluons is found to be infrared finite and have a rather simple form. For the forward case, apart from an expected term which describes the running of the coupling, it contains a correction which coincides with the first two terms found for the vacuum channel in [1,2].

The infrared stability of the found octet pair kernel, if correct, implies that the three-body interaction in the odderon has to be infrared finite by itself. In our approach this stability, in fact, automatically follows from the applied ansatz to satisfy the bootstrap relation. We have no proof that this ansatz is unique. Calculation of the three-body interaction for the odderon and study of its infrared properties is thus essential to support our approach.

After this work had been completed and published as a preprint [7] a paper by V.Fadin et al. [8] appeared in which the quark contribution to the interaction in the octet colour channel was directly calculated from the asymptotics of the relevant Feynman diagrams. It was claimed in [8] that their results disagreed with ours and that therefore our bootstrap condition and the ansatz to solve it were wrong. In fact this claim is absolutely unfounded: the quark contribution to the interaction found by our method identically coincides with the results of [8] (see Appendix), although the amount of labour necessary to obtain it is incomparably smaller.
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Appendix

The authors of [8], in all probability, have not properly taken into account that the unsymmetric kernel \( V \) calculated in this paper is a different quantity as compared to their symmetric kernel \( K_r \). It is however straightforward to relate them. First one goes over from \( V \) to a symmetric kernel \( W \) in the trivial metric (that is with the integration measure \( d^2 q_1 \)) by expressing

\[
V(q, q_1, q'_1) = \sqrt{\frac{\eta(q_1)\eta(q_2)}{\eta(q_1)\eta(q_2)}} W(q, q_1, q'_1)
\]  

(50)

On the other hand, the kernel \( K_r \) of [8] is defined in the metric with an extra factor \( q_1^2 q_2^2 \) in the denominator. Taking this into account we obtain the desired relation

\[
K_r(q, q_1, q'_1) = \sqrt{\frac{q_1^2 q_2^2 q_1^2 q_2^2}{\eta(q_1)\eta(q_2)}} V(q, q_1, q'_1)
\]  

(51)

Now we are going to show that, first, the quark contribution to the gluon Regge trajectory has the form implied by our anzatz (3). We shall determine the part of \( \eta(q) \) coming from this contribution. Then using (4) and (51) we shall find the corresponding part of the irreducible kernel \( K_r \) and compare it with the results of [3]. To simplify the comparison with [8] we shall use the relevant expressions in the unrenormalized form and for an arbitrary number of colours \( N \).

The part of the gluon trajectory which comes from quark is given by the expression [8]

\[
\omega^{(2)}_Q = 2b q^2 \int \frac{d^{D-2} q_1}{q_1^2 q_2^2} (q^{2\epsilon} - q_1^{2\epsilon} - q_2^{2\epsilon}),
\]  

(52)

where

\[
b = \frac{g^4 N_F \Gamma(1-\epsilon)\Gamma(2+\epsilon)}{(2\pi)^{D-1}(4\pi)^{2+\epsilon}\Gamma(4+2\epsilon)}.
\]  

(53)

On the other hand using (3) and (12)

\[
\omega^{(2)}(q) = -aq^2 \int \frac{d^{D-2} q_1}{q_1^2 q_2^2} (\xi(q) - \xi(q_1) - \xi(q_2))
\]  

(54)

where

\[
a = \frac{g^2 N}{2(2\pi)^{3+2\epsilon}}
\]  

(55)

As we observe, the form of (52) follows this pattern. Comparing (52) and (54) we identify

\[
\xi_Q(q) = -\frac{2b}{a} q^{2\epsilon}
\]  

(56)

Now we pass to the irreducible kernel \( K_r \). From (4) and (51) we express it via \( \xi(q) \) as

\[
K_r^{(2)}(q_1, q'_1) = \frac{1}{2} \sqrt{\frac{q_1^2 q_2^2}{q_1^2 q_2^2}} \frac{1}{K^2} \frac{\eta(q_1)\eta(q_2)}{\eta(q_1)\eta(q_2)} (\xi(q_1) + (q_1^2 q_2^2 q_1^2 q_2^2) (\xi(q_1) + \xi(q_2) - 2 \xi(q_1) + \xi(q_2) - 2 \xi(q_1) q_2^2))
\]
\[\frac{1}{k^2} \sqrt{\frac{q_2^2 q_1^2}{q_2^2 q_1^2} (\xi(q_2) + \xi(q_1') - \xi(q_1) - \xi(q_2') - 2\xi(q_1')) - \frac{q^2}{\sqrt{q_1^2 q_2^2 q_1' q_2'}} (2\xi(q) - \xi(q_1) - \xi(q_2) - \xi(q_1') - \xi(q_2'))}\]  

Putting here the found \(\xi(q)\), Eq. (56), we obtain the quark contribution

\[K_{1Q}(q_1, q_1') = -b \left[ \frac{q_1^2 q_2^2}{k^2} (q_1^2 + q_2^2 - q_1^2 - q_2^2 - 2k^2) + \frac{q_1^2 q_2^2}{k^2} (q_2^2 - q_1^2 - q_2^2 - 2k^2) - q^2 (2q_2^2 - q_1^2 - q_2^2 - q_1^2 - q_2^2 - q_2^2) \right]\]  

Comparing this expression with the one found in [8] (Eq. (47) of that paper) we observe that they are identical. This means that our bootstrap condition and the ansatz to solve it are valid at least for the quark contribution in the next-to-leading order.

9 References.

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