A photon mass on the brane

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Abstract

We discuss the impact of a bulk photon mass in a Dvali–Gabadadze–Porrati type brane model with Maxwell terms both on the brane and in the bulk, as proposed by Dvali, Gabadadze and Shifman.

The motivation to include the bulk photon mass is to suppress radiation loss into the bulk.

We point out that this modifies the photon propagator in such a way that it generates a small photon mass on the brane. Compatibility with present bounds on a photon mass imply that the transition to five-dimensional distance laws for the electromagnetic potentials would appear only at super-horizon length scales, thus excluding any direct detection possibility of a transition from four-dimensional to five-dimensional distance laws in electromagnetic interactions.

We also include results on fermion propagators with Dirac terms on the brane and in the bulk.

Key words: brane worlds, DGP brane model
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1 Introduction

In the present letter we would like to report results on the free propagators of fermions and gauge fields in a brane model where penetration and radiation into an infinitely large transverse dimension is suppressed by bulk mass terms.

After the pioneering work of Dvali, Gabadadze and Porrati on the emergence of an interpolating gravitational potential between four and five dimensions in a model with Einstein–Hilbert terms both on a brane and in the bulk [1], Dvali, Gabadadze and Shifman have shown that a similar interpolating static potential exists for gauge fields with both brane and bulk kinetic terms [2].

A generic feature of these models is the prediction that the four-dimensional potentials are realized at length scales well below a transition scale \( \ell \simeq M^{-1} \) beyond which the
five-dimensional distance laws become dominant. In Gaussian normal coordinates around a 3-brane \( x^\perp = 0 \) the relevant Lagrangians can be combined into

\[
S = \int dt \int d^3 x \int dx^\perp \sqrt{-g} \left( \frac{m_4^3}{2} R - \frac{1}{4q_3^2} F_{MN} a F^{MN}_a \right) + \int dt \int d^3 x \sqrt{-g} \left( \frac{m_3^2}{2} R^{(3)} - m_4^3 \overline{K} - \frac{1}{4q_3^2} F_{\mu\nu} a F^{\mu\nu}_a + \mathcal{L} \right) \bigg|_{x^\perp = 0},
\]

where \( \overline{K} \) is the mean extrinsic curvature scalar on the brane:

\[
\overline{K} = \frac{1}{2} \lim_{\epsilon \to +0} [K|_{x^\perp = -\epsilon} + K|_{x^\perp = \epsilon}].
\]

\( R^{(3)} \) is the intrinsic curvature scalar on the brane, and the Lagrangian \( \mathcal{L} \) accounts for scalar and fermionic matter degrees of freedom on the brane. If one starts from the bulk terms the model can be motivated by radiative generation of kinetic terms on the brane due to self energy contributions from brane modes [1,2,8,9,10].

Action principles like (1) will be denoted as dimensionally hybrid action principles in the sequel.

In the Newtonian limit (1) yields the static gravitational potential of a mass \( M \) on the brane as [1]

\[
U(r) = -\frac{M}{6\pi m_3^2 r} \left[ \cos \left( \frac{2m_4^3}{m_3^3} r \right) - \frac{2}{\pi} \cos \left( \frac{2m_3^3}{m_3^3} r \right) \text{Si} \left( \frac{2m_3^3}{m_3^3} r \right) \right] + \frac{2}{\pi} \sin \left( \frac{2m_3^3}{m_3^3} r \right) \text{ci} \left( \frac{2m_3^3}{m_3^3} r \right)
\]

with the sine and cosine integrals

\[
\text{Si}(x) = \int_0^x d\xi \frac{\sin \xi}{\xi}, \quad \text{ci}(x) = -\int_x^\infty d\xi \frac{\cos \xi}{\xi}.
\]

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1 Our conventions follow [3]. In particular, the perpendicular coordinate \( x^\perp \) is chosen such that \(|x^\perp|\) is the proper or geodesic distance from the 3-brane \( \mathcal{M}_3 \) of the DGP model, whereas the first 4 coordinates \( x^\mu = \{t, x\} \) cover patches of constant proper distance from \( \mathcal{M}_3 \):

\[
ds^2 = g_{\mu\nu} dx^\mu dx^\nu + (dx^\perp)^2.
\]

The reduced bulk and brane Planck masses are \( m_4 \) and \( m_3 \), and the bulk and brane gauge couplings are \( q_4 \) and \( q_3 \), respectively.

The extrinsic curvature tensor in these coordinates is

\[
K_{\mu\nu} = -\frac{1}{2} \partial_\perp g_{\mu\nu}.
\]

2 See [4] for the first realization that brane models should include a Gibbons–Hawking term, and [5,6] for a discussion of alternatives to a Gibbons–Hawking term. See also [7] for a further discussion.
\[ U(\mathbf{r}) = U(\mathbf{r}, 0) \] is the gravitational potential at a point \( \mathbf{r} \) on the brane. The off brane gravitational potential \( U(\mathbf{r}, x^\perp) \) can be expressed in terms of special functions using

\[ U(\mathbf{p}, p^\perp) = \frac{4 M}{3 (\mathbf{p}^2 + p^\perp_2)(2m_3^4 + m_3^2|\mathbf{p}|)}. \]

(3) yields four-dimensional gravity at short distances and five-dimensional gravity at large distances

\[ r \ll \ell_{\text{DGP}} : \]

\[ U(\mathbf{r}) = -\frac{M}{6\pi m_3^2 r} \left[ 1 + \left( \gamma - \frac{2}{\pi} \right) \frac{r}{\ell_{\text{DGP}}} + \frac{r}{\ell_{\text{DGP}}} \ln\left( \frac{r}{\ell_{\text{DGP}}} \right) + \mathcal{O}\left( \frac{r^2}{\ell_{\text{DGP}}^2} \right) \right], \]

\[ r \gg \ell_{\text{DGP}} : \]

\[ U(\mathbf{r}) = -\frac{M}{6\pi^2 m_3^4 r^2} \left[ 1 - 2 \frac{\ell_{\text{DGP}}^2}{r^2} + \mathcal{O}\left( \frac{\ell_{\text{DGP}}^4}{r^4} \right) \right], \]

with the transition scale

\[ \ell_{\text{DGP}} = \frac{m_3^2}{2m_3^4}. \]

The presence of the higher-dimensional terms in the weak field limit of the Einstein equation for \( U = -h_{00}/2 \) causes the modification in the numerical factor between the reduced four-dimensional Planck mass \( m_3 \) and Newton’s constant to \( m_3 = (6\pi G_N)^{-1/2} \) [3].

The Coulomb potential on the brane is

\[ A^0(\mathbf{r}) = \frac{q_3}{4\pi r} \left[ \cos\left( \frac{2q_3^2}{q_4^2} r \right) - 2 \frac{\gamma}{\pi} \cos\left( \frac{2q_3^2}{q_4^2} r \right) \text{Si} \left( \frac{2q_3^2}{q_4^2} r \right) + 2 \frac{\sin\left( \frac{2q_3^2}{q_4^2} r \right)}{\pi} \text{Ci} \left( \frac{2q_3^2}{q_4^2} r \right) \right], \]

which corresponds to a transition scale

\[ \ell_{\text{DGS}} = \frac{q_4^2}{2q_3^2} \]

between four-dimensional behavior at short distances and five-dimensional behavior at large distances [2].

A further attractive feature of the DGP model is that it allows for an implementation of standard Friedmann cosmology on the brane [3,11,12].

The motivation for the present work was twofold: On the one hand we were seeking a corresponding model for fermions with both brane and bulk terms, and on the other hand we wanted to understand the impact of a bulk mass term for the gauge fields in this class

\[ 3 \text{ See [13,14,15,16,17,18] for discussions of } \mathbb{Z}_2 \text{ symmetric cosmology in the DGP brane model, where Friedmann cosmology can be realized approximately.} \]
of large extra dimension models. Massless bulk gravitons can be acceptable due to their extremely weak coupling and the great difficulties of observing gravitational waves, but for photons the time-dependent propagator following from (1) would imply unacceptable radiation loss into the transverse dimension from dynamical sources on the brane.

Dynamical binding mechanisms of matter and gravity to submanifolds as a feature of solitonic solutions or as a consequence of couplings to solitonic backgrounds [19,20,21] has been discussed well before the recent formulation of brane models in terms of dimensionally hybrid action principles. One possibility is to think of the models discussed here as effective descriptions of underlying binding mechanisms e.g. in string inspired brane models. Of course, this may not be the only possibility in which actions like (1) or (7,16) below may arise.

We consider the impact of a bulk photon mass in Sec. 2 and bulk fermion masses in Sec. 3.

2 A photon mass in the bulk

With brane sources and a bulk mass term the action for photons on and off the brane becomes

$$S = \int d^4x \left( j^\mu A_\mu - \frac{1}{4} F_{\mu\nu} F^{\mu\nu} \right) \bigg|_{x^+ = 0} + \mathcal{M} \int d^4x \int dx^\perp \left( -\frac{1}{4} F_{MN} F^{MN} - \frac{1}{2} M^2 A^M A_M \right).$$

The source terms $j^\mu$ on the brane are assumed to satisfy a conservation law

$$\partial_\mu j^\mu = 0.$$  (8)

There are two mass scales in the problem: $\mathcal{M}$ determines the relative weight of the bulk and brane contributions, and from the results of [2] we know that small $\mathcal{M}$ corresponds to a large transition length $\ell_{DGS}$ to five-dimensional behavior of the static Coulomb potential. However, for $\mathcal{M} = 0$ and time-dependent brane sources we would find radiation leaking into the bulk also at length scales much smaller than $\ell_{DGS}$, and therefore the bulk photon mass term has been included to suppress missing energy from radiation loss.

The equations of motion

$$\mathcal{M} \left( \partial_M F^{MN} - M^2 A^N \right) + \delta(x^\perp) \eta^N_\nu \partial_\mu F^{\mu\nu} = -\delta(x^\perp) \eta^N_\nu j^\nu$$

split into

$$\partial_M A^M = 0$$  (9)

and

$$\mathcal{M} \left( \partial_M \partial^M A^N - M^2 A^N \right) + \delta(x^\perp) \eta^N_\nu \left( \partial_\mu \partial^\mu A^\nu + \partial^\nu \partial^\perp A^\perp \right) = -\delta(x^\perp) \eta^N_\nu j^\nu.$$  (10)

\footnote{We assume that the vacuum solutions of the underlying brane model allow for expansion around a flat background, as is the case e.g. in the DGP model [1,3,22,23].}
$A^\perp$ should be symmetric across the brane and decouples from the sources. We set it to zero in the sequel. To determine the potentials

$$A_\mu(t, x, x^\perp) = \int dt' \int d^3x' G(t-t', x-x', x^\perp) j_\mu(t', x')$$

for the electromagnetic fields generated by the currents on the brane we have to solve

$$\mathcal{M} \left( \partial_M \partial^M - M^2 \right) G(x, x^\perp) + \delta(x^\perp) \partial_\mu \partial^\mu G(x, 0) = -\delta^4(x) \delta(x^\perp).$$

(11)

The Fourier ansatz

$$G(x, x^\perp) = \frac{1}{(2\pi)^5} \int d^4p \exp[i(p \cdot x + p_\perp x^\perp)].$$

yields

$$\mathcal{M} \left( p^2 + p_\perp^2 + M^2 \right) G(p, p_\perp) + \frac{p_\perp^2}{2\pi} \int_{-\infty}^{\infty} dp_\perp G(p, p_\perp') = 1,$$

which implies the following form for the Green’s function

$$G(p, p_\perp) = \frac{f(p)}{p^2 + p_\perp^2 + M^2}.$$

With

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} dp_\perp \frac{1}{p^2 + p_\perp^2 + M^2} = \frac{\Theta(p^2 + M^2)}{2\sqrt{p^2 + M^2}}$$

we then find for $p^2 + M^2 = p^2 + M^2 - E^2 > 0$, i.e. below the threshold set by the bulk photon mass:

$$G(p, p_\perp) = \frac{2\sqrt{p^2 + M^2}}{(2\mathcal{M}\sqrt{p^2 + M^2 + p^2})(p^2 + p_\perp^2 + M^2)},$$

(12)

$$G(p, x^\perp) = \frac{\exp\left(-\sqrt{p^2 + M^2}|x^\perp|\right)}{2\mathcal{M}\sqrt{p^2 + M^2 + p^2}},$$

(13)

and above the threshold

$$G(p, p_\perp) = \frac{1}{\mathcal{M}(p^2 + p_\perp^2 + M^2)},$$

(14)

$$G(p, x^\perp) = -\frac{1}{2\mathcal{M}\sqrt{-p^2 - M^2}} \sin\left(\sqrt{-p^2 - M^2}|x^\perp|\right).$$

(15)

For $\mathcal{M} \to 0$ we find the ordinary 4-dimensional potential $G(p) = G(p, x^\perp)|_{x^\perp=0}$ below the bulk photon threshold. For $\mathcal{M} \to \infty$ we find $G \to 0$. This is correct, since $G$ describes
the potentials generated from brane sources, and in the limit $\mathcal{M} \to \infty$ the electromagnetic fields decouple from the brane sources. If we want to calculate the fields from bulk sources residing on the brane we would have to rescale the source terms on the right hand side of (11) by $\mathcal{M}$, which of course amounts to a rescaling of $G$ by the same factor. The limit $\lim_{\mathcal{M} \to \infty} \mathcal{M}G$ yields exactly the usual 5-dimensional potentials, as expected.

### 3 Fermion masses in the bulk

To formulate a dimensionally hybrid action principle for fermions we can take e.g. $\gamma^\perp = i\gamma_5 = \gamma_0\gamma_1\gamma_2\gamma_3$. The formulation of the action principle introduces again a bulk fermion mass and an extra mass scale $\mathcal{M}$:

$$S = \int d^4x \frac{\overline{\psi}(i\gamma^\mu \partial_\mu - m)\psi}{x^\perp=0} + \mathcal{M} \int d^4x \int d^4x' \frac{\overline{\psi}(i\gamma^\mu \partial_\mu + i\gamma^\perp \partial_\perp - M)\psi}{x^\perp=x'^\perp = 0}. \quad (16)$$

The corresponding equation for the free fermion propagator for sources on the brane is

$$\mathcal{M}\left(i\gamma^\mu \partial_\mu + i\gamma^\perp \partial_\perp - M\right) S(x, x^\perp) + \delta(x^\perp) (i\gamma^\mu \partial_\mu - m) S(x, 0) = -\delta^4(x)\delta(x^\perp), \quad (17)$$

and with

$$S(x, x^\perp) = \frac{1}{(2\pi)^5} \int d^4p d^4p_\perp S(p, p_\perp) \exp[i(p \cdot x + p_\perp x^\perp)]$$

we find

$$\mathcal{M}\left(\gamma^\mu p_\mu + \gamma^\perp p_\perp + M\right) S(p, p_\perp) + \frac{\gamma^\mu p_\mu + m}{2\pi} \int_{-\infty}^{\infty} dp_\perp S(p, p'_\perp) = 1. \quad (18)$$

This determines the $p_\perp$-dependence of the propagator

$$S(p, p_\perp) = \frac{M - \gamma^\mu p_\mu - \gamma^\perp p_\perp}{M^2 + p^2 + p_\perp^2} s(p),$$

and $s(p)$ is then determined from (18) by taking into account

$$\frac{1}{2\pi} \int_{-\infty}^{\infty} dp_\perp \frac{M - \gamma^\mu p_\mu - \gamma^\perp p_\perp}{M^2 + p^2 + p_\perp^2} = \frac{M - \gamma^\mu p_\mu}{2\sqrt{p^2 + M^2}} \Theta(p^2 + M^2).$$

At low energies $p^2 + M^2 > 0$ this yields again exponential damping in the transverse direction:

$$S(p, p_\perp) = 2\sqrt{p^2 + M^2} \frac{M - \gamma^\mu p_\mu - \gamma^\perp p_\perp}{M^2 + p^2 + p_\perp^2} \times \frac{2\mathcal{M}\sqrt{p^2 + M^2} + mM + p^2 + (m - M)\gamma^\mu p_\mu}{4\mathcal{M}^2(p^2 + M^2) + 4\mathcal{M}\sqrt{p^2 + M^2}(mM + p^2) + p^4 + (m^2 + M^2)p^2 + m^2M^2}, \quad (19)$$
\[ S(p, x^\perp) = \left( M - \gamma^\mu p_\mu - i\gamma^\perp \sqrt{p^2 + M^2 \text{sign}(x^\perp)} \right) \exp \left( -\sqrt{p^2 + M^2 |x^\perp|} \right) \]

\[ \times \frac{2\mathcal{M} \sqrt{p^2 + M^2} + mM + p^2 + (m - M)\gamma^\mu p_\mu}{4\mathcal{M}^2 (p^2 + M^2) + 4\mathcal{M} \sqrt{p^2 + M^2} (mM + p^2) + p^4 + (m^2 + M^2) p^2 + m^2 M^2} \]

while above the threshold for bulk fermions we have

\[ S(p, p^\perp) = \frac{1}{\mathcal{M}} \frac{M - \gamma^\mu p_\mu - \gamma^\perp p^\perp}{M^2 + p^2 + p^\perp^2} \]

\[ S(p, x^\perp) = -\frac{1}{2\mathcal{M}} \left[ \frac{M - \gamma^\mu p_\mu}{\sqrt{-p^2 - M^2}} \sin \left( \sqrt{-p^2 - M^2} |x^\perp| \right) \right. 

\left. + i\gamma^\perp \text{sign}(x^\perp) \cos \left( \sqrt{-p^2 - M^2} |x^\perp| \right) \right]. \]

The fermion propagators have the usual hermiticity properties

\[ \gamma^0 S^+(p, p^\perp) \gamma^0 = S(p, p^\perp) \]

\[ \gamma^0 S^+(p, x^\perp) \gamma^0 = S(p, -x^\perp) \]

and the expected 4-dimensional and 5-dimensional limiting behavior: \( \lim_{\mathcal{M} \to \infty} \mathcal{M} S \) is the usual 5-dimensional fermion propagator, and below the bulk fermion threshold we have

\[ \lim_{\mathcal{M} \to 0} S(p) = \lim_{\mathcal{M} \to 0} S(p, x^\perp) \bigg|_{x^\perp = 0} = \frac{m - \gamma^\mu p_\mu}{m^2 + p^2}. \]

4 Conclusion

The dimensionally hybrid action principles (7,16) yield propagators which interpolate between genuine four-dimensional propagators at small distances \( \ll \mathcal{M}^{-1} \) and five-dimensional propagators at large distances \( \gg \mathcal{M}^{-1} \). However, suppression of radiation into the bulk requires bulk masses \( M \gg \mathcal{M}, \) and as a consequence of that the four-dimensional behavior actually persists only in an energy range \( \mathcal{M} \ll E \ll M. \) Furthermore, the bulk mass shifts the pole of the photon propagator \( G(p) = G(p, x^\perp)|_{x^\perp = 0} \) also in the four-dimensional regime to

\[ m_\gamma^2 = 2\mathcal{M} \left( \sqrt{\mathcal{M}^2 + M^2} - M \right) \approx 2\mathcal{M}M. \]

That this corresponds to the generation of a photon mass in four dimensions may not be as obvious as in a genuine four-dimensional propagator, since the pole appears as a simple algebraic pole only in an expansion of the denominator in \( G(p) \) around \( m_\gamma^2, \) and also the integration to \( G(x) \) is not straightforward.

However, that \( m_\gamma \) is indeed an effective photon mass on the brane can also be inferred from the free equation

\[ \mathcal{M} \left( \partial_\mu \partial^\mu - M^2 \right) A_\nu(x, x^\perp) + \delta(x^\perp) \partial_\mu \partial^\mu A_\nu(x, 0) = 0, \]
which yields

\[ A_\mu(p, p_\perp) \propto \frac{1}{p_\perp^2 + \left(\sqrt{M^2 + M^2 - M}\right)^2} \delta \left(p^2 + 2M\sqrt{M^2 + M^2 - 2M^2}\right). \]

The corresponding mass shift for massive fields (20) is of much less significance, since generically it will provide only a small shift of \( m \): In leading order in \( M \) the shift of a mass \( m < M \) is

\[ \delta m^2 = 4mM\sqrt{\frac{M - m}{M + m}}. \]

The current upper bound on a photon mass is [24,25]

\[ m_\gamma < 2 \times 10^{-16}\text{ eV}. \]

No significant missing energy events have been found in accelerator searches up to \( \sqrt{s} = 1.8\text{ TeV} \). Therefore bulk photon masses in the class of brane models discussed here must certainly exceed \( M > 1\text{ TeV} \). This translates into a bound on the transition scale to five-dimensional behavior

\[ (2M)^{-1} = \frac{M}{m_\gamma^2} > 1.7 \times 10^{11}\text{ Gpc}, \]

which exceeds the size of our Hubble horizon by a factor \( \approx 2 \times 10^{10} \). In those brane models where photon localization is dynamical and can effectively be described by a bulk mass term, laboratory constraints on brane and bulk photon masses are more restrictive than constraints from large scale observations of four-dimensional distance laws.

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**References**

[1] G. Dvali, G. Gabadadze, M. Porrati, Phys. Lett. B 485 (2000) 208.
[2] G. Dvali, G. Gabadadze, M. Shifman, Phys. Lett. B 497 (2001) 271.
[3] R. Dick, Class. Quantum Grav. 18 (2001) R1.
[4] A. Chamblin, H.S. Reall, Nucl. Phys. B 562 (1999) 133.
[5] R. Dick, D. Mikulović, Phys. Lett. B 476 (2000) 363.
[6] R. Dick, Phys. Lett. B 491 (2000) 333.
[7] Z. Lalak, R. Matyszkiewicz, JHEP 0111 (2001) 027.
[8] G. Dvali, G. Gabadadze, Phys. Rev. D 63 (2001) 065007.
[9] C. Deffayet, G. Dvali, G. Gabadadze, A. Vainshtein, Phys. Rev. D 65 (2002) 044026.
[10] G. Dvali, G. Gabadadze, M. Kolanovic, F. Nitti, Phys. Rev. D 65 (2002) 024031.
[11] R. Cordero, A. Vilenkin, hep-th/0107175.
[12] R. Dick, Acta Phys. Pol. B 32 (2001) 3669.
[13] C. Deffayet, Phys. Lett. B 502 (2001) 199.
[14] C. Deffayet, G. Dvali, G. Gabadadze, Phys. Rev. D 65 (2002) 044023.
[15] P.P. Avelino, C.J.A.P. Martins, Astrophys. J. 565 (2002) 661.
[16] C. Deffayet, G. Dvali, G. Gabadadze, astro-ph/0106449.
[17] C. Deffayet, S.J. Landau, J. Raux, M. Zaldarriaga, P. Astier, astro-ph/0201164.
[18] G. Dvali, G. Gabadadze, M. Shifman, hep-th/0202174.
[19] K. Akama, in: Gauge Theory and Gravitation (Springer–Verlag, Berlin, 1983) pp. 267–271; V.A. Rubakov, M.E. Shaposhnikov, Phys. Lett. B 125 (1983) 136.
[20] M. Cvetič, S. Griffies, S.-J. Rey, Nucl. Phys. B 381 (1992) 301.
[21] M. Cvetič, H.H. Soleng, Phys. Rep. 282 (1997) 159.
[22] I. Giannakis, H.-c. Ren, Phys. Lett. B 528 (2002) 133.
[23] M. Kolanovic, hep-th/0203136.
[24] R. Lakes, Phys. Rev. Lett. 80 (1998) 1826.
[25] D.E. Groom et al. (Particle Data Group), Eur. Phys. J. C 15 (2000) 1.