Slow stable hybrid stars: a new class of compact stars that fulfills all current observational constraints

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ABSTRACT
We study hybrid stars considering the effects on stellar stability of the hadron-quark conversion speed at the sharp interface. The equation of state is constructed by combining a model-agnostic hadronic description with a constant speed of sound model for quark matter. We show that current LIGO/Virgo, NICER, low-density nuclear and high-density perturbative QCD constraints can be satisfied in two scenarios, with low and high transition pressures. If the conversion speed at the interface is slow, a new class of dynamically stable hybrid objects is possible and very stiff hadronic equations of state cannot be discarded. Densities tens of times larger than the nuclear saturation density are possible at the centre of these objects. We discuss possible formation mechanisms for the new class of hybrid stars and smoking guns for their observational identification.

Key words: Stars: neutron – Dense matter – Gravitational waves

1 INTRODUCTION
In spite of several decades of observations and theoretical research, the nature of the deep interior of neutron stars (NSs) is still an unsolved issue. At present, a stringent constraint for the equation of state (EOS) comes from the observed masses of the pulsars PSR J1614-2230 (Demorest et al. 2010), PSR J0348+0432 (Antoniadis et al. 2013) and PSR J0740+6620 (Cromartie et al. 2019), which require that an acceptable EOS must be able to support a NS of at least $2 M_\odot$. New limits on the EOS were posed recently by the LIGO/Virgo detection of gravitational-waves (GWs) coming from the NS-NS merger event GW170817 (Abbott et al. 2017; Annala et al. 2018; Most et al. 2018; Raithel et al. 2018; Capano et al. 2020). Assuming that both NSs are described by the same EOS and have spins within the range observed in Galactic binary NSs, the dimensionless tidal deformability $\Lambda_{1.4}$ of a $1.4 M_\odot$ NS was found to be in the range 70 – 580 at the 90% level (Abbott et al. 2018). Also, the fact that the postmerger remnant of GW170817 did not suffer a prompt collapse was used to constrain the maximum gravitational mass of a non-rotating NS to be $2.17^{+0.17}_{-0.15} M_\odot$ (Rezzolla et al. 2018). The mass and radius of the merging objects in the GW190425 event were also inferred, but the possibility that one or both components are black holes cannot be ruled out (Abbott et al. 2020).

Additionally, the Neutron Star Interior Composition Explorer (NICER) has measured the mass and radius of the millisecond-pulsars PSR J0030+0451 (Riley et al. 2019; Miller et al. 2019) and PSR J0740+6620 (Riley et al. 2021; Miller et al. 2021) with great precision. Before the radius measurement of PSR J0740+6620, a comparison of observations with a large variety of EOSs led to the idea that both extremely stiff and soft matter would be ruled out (Abbott et al. 2018; Capano et al. 2020). However, the latest joint NICER and XMM-Newton observation showed that the radius of PSR J0740+6620 ($M \approx 2 M_\odot$) is very similar to that of PSR J0030+0451 ($M \approx 1.4 M_\odot$), even though they have very different masses (Riley et al. 2021; Miller et al. 2021). These results favour stiff EOSs (Raaijmakers et al. 2021) and create some tension with the masses and radii inferred for the objects in GW170817.

For decades, microscopic theories of matter have tried to reveal the EOS of NS interiors. At present, the EOS is well founded on nuclear theory and experiments for densities $\leq n_0$ (being $n_0 = 0.16$ fm$^{-3}$, the nuclear saturation density). Beyond $\sim n_0$, perturbative QCD (pQCD) can describe deconfined quark matter accurately (Kurkela et al. 2010; Gorda et al. 2018), but such large densities are not usually expected in NSs. Between these limits, a robust approach is to use a set of model-agnostic EOSs interpolating both regimes without violating causality, and requiring that the resulting NS configurations fulfil astrophysical constraints. However, it is still unclear at which density would a hadron-quark transition occur and whether hybrid stars (HSs) containing quark cores would exist in Nature.

Concerning HSs, it is under debate whether quarks and hadrons are separated by a sharp discontinuity or by a mixed phase where they coexist along a wide density region forming globally charge-neutral geometrical structures. Mixed phases are energetically preferred if the quark matter surface tension $\sigma$ is smaller than a critical value $\sigma_{\text{crit}}$ of the order of tens of MeV/fm$^2$; but if $\sigma > \sigma_{\text{crit}}$, the mixed phase is unstable and a sharp interface is favoured (Wu & Shen 2019; Maslov et al. 2019). Unfortunately, theoretical values of $\sigma$ span a wide range depending on the EOS and on the calculation method: some authors obtain $\sigma < \sigma_{\text{crit}}$ (Garcia & Pinto 2013; Lugones & Grunfeld 2017, 2019; Fraga et al. 2019; Gao & Liu 2016) but very large $\sigma$ favouring
a sharp interface is obtained using the multiple reflection expansion method with the Nambu-Jona-Lasinio EOS (Lugones et al. 2013) and the MIT bag EOS with vector interactions (Lugones & Grunfeld 2021). In this work, we adopt as a working hypothesis that the interface is sharp.

On the other hand, it has been shown that the conversion speed between quarks and hadrons at a sharp interface in a HS is deeply related to the dynamic stability of the object (Vasquez Flores, Lenz & Lugones 2012; Pereira, Flores & Lugones 2018). In fact, if conversions have a sufficiently long timescale compared to the typical oscillation timescale of the star ($\sim 1$ ms), a new branch of stable hybrid stars is possible. In this work, we will explore systematically this possibility and will analyse several physical and astrophysical consequences of their hypothetical existence.

The paper is organised as follows. In Section 2, we summarise the role of interface reactions on the dynamic stability of hybrid stars emphasising that a new class of compact stars is possible if the reaction timescale is slower that the typical oscillation frequency. In Section 3, we discuss the microphysics of the hadron-quark conversion in degenerate matter and present arguments supporting the idea that it may be slow. In Section 4, we describe the EOS that will be used in our calculations. In Section 5, we present our results for the mass-radius relationship and the tidal deformability of the new class of hybrid objects, showing that they are in agreement with current observations. In Section 6, we explore some physical and astrophysical consequences of our results and give some prospects for the detection of the new class of objects.

2 THE ROLE OF INTERFACE REACTIONS ON STELLAR STABILITY

As shown by Chandrasekhar (1964), stellar stability can be assessed by inspecting the response of equilibrium configurations to small radial perturbations. In a dynamically unstable star, small perturbations grow without limit, leading to the collapse or disruption of the object. In a stable star, fluid elements along the stellar interior oscillate around their equilibrium positions, compressing and expanding periodically. The formalism of small radial perturbations of spherically symmetric stars shows that stellar configurations are stable if the frequency $\omega_0$ of the fundamental mode verifies $\omega_0^2 \geq 0$ and unstable if $\omega_0^2 < 0$.

However, in the case of HSs, Chandrasekhar’s analysis is not straightforward because radial perturbations may induce phase conversions in the neighbourhood of the interface. Let us first assume that phase conversions are slow, i.e. that the conversion timescale $\tau_{\text{conv}}$ turns out to be much larger than the oscillation period $\tau_{\text{osc}} = 2\pi/\omega_0^{-1}$ of perturbed fluid elements (notice that for typical NSs, $\tau_{\text{osc}} \sim 1$ ms). When matter close to the quark-hadron interface is disturbed and radially displaced from its equilibrium position, it will maintain its composition even if its pressure fluctuates above and below the transition pressure, $p_f$. Thus, the interface oscillates around its unperturbed position and fluid elements on either side cannot traverse it. Since the interface oscillates with the same period as the disturbances, its movement can be encoded in junction conditions for the relative radial displacement, $\xi$, and the Lagrangian perturbation of the pressure, $\Delta p$, which must be continuous across the phase splitting surface (Pereira, Flores & Lugones 2018):

$$[\xi]^+ \equiv \xi^+ - \xi^- = 0, \quad [\Delta p]^+ \equiv \Delta p^+ - \Delta p^- = 0,$$

where $^+$ and $^-$ indicate the function values on each side of the interface. Otherwise, if conversions are rapid ($\tau_{\text{conv}} \ll \tau_{\text{osc}}$), the interface stays at rest with respect to the stellar centre and matter moves across it changing instantly its composition. In this case $[\xi \Delta p/(r_1 p_0')]$ and $\Delta p$ are continuous across the interface,

$$[\xi]^+ = \Delta p \left[ \frac{1}{r_1 p_0'} \right]^+, \quad [\Delta p]^+ = 0,$$

being $r_1$ its radial position and $p_0'$ the pressure’s radial derivative in the unperturbed configuration (Pereira, Flores & Lugones 2018).

The existence of junction conditions embodying the conversion speed has strong consequences for the dynamic stability of hybrid stars. In particular, it can be shown that in the case of slow conversions, the maximum mass star is no longer the one that separates stable from unstable configurations, as we explain below. For cold catalysed stars, it is known that changes of stability ($\omega_0 = 0$) occur at maxima or minima in the $M - e_\epsilon$ diagram, being $e_\epsilon$ the energy density at the NS centre. Thus, the following static stability criterion holds (Harrison et al. 1965):

$$\frac{\partial M}{\partial e_\epsilon} < 0 \quad \Rightarrow \quad \omega_0^2 < 0 \quad \text{(unstable star)}, \quad (3)$$

$$\frac{\partial M}{\partial e_\epsilon} > 0 \quad \Leftrightarrow \quad \omega_0^2 \geq 0 \quad \text{(stable star)}. \quad (4)$$

However, when chemical reactions are possible in some part of the star, matter is not necessarily in thermodynamic equilibrium everywhere and Eqs. (3) and (4) may fail 1. More specifically, if interface conversions are slow, matter in the neighbourhood of the phase splitting surface is non-catalysed (there is not enough time to attain equilibrium) and the prescriptions that lead to Eqs. (3) and (4) are not fulfilled. In fact, numerical calculations have shown that in the case of HSs with sharp density discontinuities and slow interface conversions, $\omega_0$ can be a real number (indicating stability) even

\footnote{1 Some examples of the failure of Eqs. (3) and (4) are already known in the literature in different contexts; for example, in purely hadronic stars with frozen oscillations (Gourgoulhon et al. 1995), in electrically charged strange quark stars (Arbain & Malheiro 2015), and in compact objects with dark matter (see e.g. Fig. 2 of Kain 2020).}
if $\partial M/\partial \epsilon_c < 0$ (Vasquez Flores et al. 2012; Pereira et al. 2018; Mariani et al. 2019; Tonetto & Lugones 2020; Parisi et al. 2020; Di Clemente et al. 2020; Pereira et al. 2021; Rodriguez et al. 2021; Mariani et al. 2022; Ranea-Sandoval et al. 2022). HSs that are stable even if $\partial M/\partial \epsilon_c < 0$ will be called slow-stable (SS) configurations or SS hybrid stars (SSHSs). A schematic representation of SSHSs in the mass-radius diagram is shown in Figs. 1 and 2. When the transition density is high enough, they usually arise to the left of the maximum mass object, i.e. their central densities are larger than the central density of the star with $M_{\text{max}}$ (see Fig. 1b). If the hadron–quark transition occurs at a low enough density, configurations like the one shown in Fig. 2(b) are possible. Although other possible forms of the HS mass-radius relation may arise when a single first-order phase transition is taken into account (Alford et al. 2013), in this work we will focus mainly on the case represented in Fig. 1(b), due to its important physical and astrophysical consequences.

Strictly speaking, the SSHS branch does not extend exactly up to the star with $\omega_Q^2 = 0$ ($\tau_{\text{osc}} = \infty$), but up to an object with a smaller central density that has $\omega_Q^2 \sim 2 \tau_{\text{osc}}^{-1}$ (i.e. $\tau_{\text{osc}} \sim \tau_{\text{conv}}$). However, in practice $\omega_Q$ is of the order of a few kilohertz for almost all HSs in the SS branch, and tends steeply to zero very close to the mass of the zero-frequency object. This behaviour can be verified in Fig. 11 of Pereira, Flores & Lugones (2018), where it is seen that $\omega_Q$ has a significantly large value (some fraction of 1 kHz) even for HSs that are extremely close to the zero-frequency object. Therefore, if $\tau_{\text{conv}}$ is larger than some milliseconds, the difference between locating the stable-unstable boundary at the zero-frequency point ($\tau_{\text{osc}} = \infty$) or at the point with $\tau_{\text{osc}} \approx \tau_{\text{conv}}$ is negligible. If $\tau_{\text{conv}} \sim 1$ ms, the stable-unstable border would be shifted to the right in Fig. 1(b), resulting in a shorter SSHS branch. As $\tau_{\text{conv}}$ decreases below ~1 ms, the border gets closer and closer to the maximum of $M \sim R$ curve. In the limiting case of $\tau_{\text{conv}} = 0$ (infinitely rapid conversions) the border coincides with the maximum, Eqs. (3)-(4) become valid again, and the SSHS branch would not exist. This is easy to understand because for $\tau_{\text{conv}} = 0$ perturbed matter is in thermodynamic equilibrium at all times, i.e. the assumption of catalysed matter that led to Eqs. (3)-(4) (Harrison et al. 1965) is always fulfilled.

**Figure 2.** Same as in the previous figure but in a scenario where the transition density is low. The green branch (fully stable HSs) is stable for any conversion speed (rapid or slow). As in the previous figure, the blue branches (SSHSs) are stable only for slow conversions.

**Figure 3.** Schematic representation of the Gibbs free energy per baryon as a function of pressure for hadron and quark matter in chemical equilibrium ($H^\text{eq}$ and $Q^\text{eq}$) and for the non-catalysed transition states ($H^*$ and $Q^*$). The quark phase $Q^*$ is out of chemical equilibrium and is defined as the one that has the same flavour composition than $H^\text{eq}$ at the same pressure (see Olesen & Madsen 1994; Iida & Sato 1998; Lugones & Benvenuto 1998; Bombaci et al. 2004; Bombaci et al. 2007; Lugones 2016) for more details). Analogously, $H^*$ has the same flavour composition than $Q^\text{eq}$ at the same $p$. The HS interface is located at the point $1^\text{eq}$ with a pressure $p_1$. A hadronic fluid element with a pressure slightly below $p_1$ will deconfine if a perturbation is able to take it to the point $2^*$ at which $H^\text{eq}$ and $Q^*$ have the same free energy. Similarly, a quark fluid element with a pressure slightly above $p_1$ will hadronize only if it reaches the point $3^*$ after a perturbation. Quantum and thermal nucleation timescales for these transitions are extremely long at low temperatures (see text).

### 3 THE SPEED OF INTERFACE CONVERSIONS

According to the previous discussion, interface conversions in a HS are slow if $\tau_{\text{conv}} \gg 1$ ms, and fast if $\tau_{\text{conv}} \ll 1$ ms. Unfortunately, the actual conversion timescale in the hypothetic sharp interface of a HS is uncertain. Nonetheless, the transition timescale is not expected to be simply the result of particles that interact independently, because phase transitions are highly collective and nonlinear phenomena. In other words, although typical interaction timescales are $\sim 10^{-23}$ s for the strong force and $\sim 10^{-9}$ s for the weak one, the conversion timescale in a HS is not necessarily that fast. In principle, one expects that the conversion would be driven by nucleation or spinodal decomposition, since these are the two major ways by which first-order phase transitions proceed in a wide variety of systems (see e.g., Slezov 2009). However, other mechanisms based on strangeness diffusion have been proposed (Olinto 1987; Alford et al. 2015).

In the case of nucleation, there are quantum and thermal nucleation models describing the case where there is no pre-existing quark matter, and quark drops must nucleate somewhere in the high-density region of the NS (see Bombaci et al. 2016, and references therein). The same analysis can be applied to the present case where a pre-existing quark-hadron interface moves in response to a density fluctuation. The mechanism can be understood by analysing the Gibbs free energy per baryon $G/n_B$ of the different phases involved in the transition (see Fig. 3). Catalysed hadronic matter in the neighbourhood of the interface (point $1^\text{eq}$) will deconfine only if it reaches the point $2^*$, while quark matter close to the point $1^\text{eq}$ will convert into hadrons only if it reaches the point $3^*$. A direct conversion $H^\text{eq} \leftrightarrow Q^\text{eq}$ is strongly suppressed because both phases have in general a very different flavor composition and a high-order weak interaction process.
would be needed (Lugones 2016; Bombaci et al. 2007; Lugones & Benvenuto 1998; Iida & Sato 1998; Olesen & Madsen 1994). Quantum and thermal nucleation timescales for the $1^{\text{eq}} \rightarrow 2^{\text{eq}}$ transition are typically orders of magnitude larger than the age of the Universe for temperatures below a few MeV (Bombaci et al. 2016). We are not aware of calculations of the $1^{\text{eq}} \rightarrow 3^{\text{eq}}$ transition, but similar timescales can be reasonably expected.

Finally, a conversion mechanism based on strangeness diffusion has been considered (Alford et al. 2015) and seems powerful enough to saturate $r$-modes at very low amplitude, of order $10^{-10}$. A similar saturation can be expected for radial oscillations.

The above discussion shows that although the conversion mechanism is not fully understood, slow conversions represent a feasible scenario. Moreover, since in the limit of $\tau_{\text{conv}} \ll 1$ ms and $\tau_{\text{conv}} \gg 1$ ms the stellar response is independent of the kinetic details of the conversion (they can be encoded in junction conditions), we can obtain robust conclusions about stellar stability in spite of the microphysical uncertainties of the phase transformation, whose understanding is not the objective of this work.

4 HYBRID EOS MODEL

For the hadronic crust we use a generalised piecewise polytropic (GPP) fit (O’Boyle et al. 2020) to the SLy(4) EOS found in Douchin & Haensel (2001), which accurately reproduces the EOS and the adiabatic index. For the hadronic part of the core with baryon number density above $0.3n_0$, we construct several different model-agnostic GPP EOSs using the prescription of O’Boyle et al. (2020) to ensure continuity in the pressure, the energy density, and the sound speed $c_s$. The EOS parameters are chosen arbitrarily to obtain ultrastiff EOSs that match at $1.1n_0$ to the upper limit of a calculated EOS range based on chiral effective field theory (cEFT) interactions including theoretical uncertainties (see Hebeler et al. (2013) for details). In all cases, causality and consistency with $2M_\odot$ pulsars are satisfied. Yet, these stiff EOSs are intentionally built to produce hadronic $M$-$R$ curves that do not satisfy the constraints from GW170817 (Annala et al. 2018; Annala et al. 2020).

We assume that at a given pressure $p_t$ a first-order phase transition between hadronic and quark matter occurs, with both phases separated by a sharp interface. To keep our analysis as general as possible, quark matter is described by the constant speed of sound model (Alford et al. 2013), which is parametrized in terms of $p_t$, the energy density jump $\Delta \epsilon$ between both phases, and $c_s$ (assumed to be constant). The phase transition is described by the Maxwell construction, which guarantees that the pressure and the Gibbs free energy per baryon are the same at both sides of the interface. In this way, we constructed over 3000 hybrid EOSs using parameters in the following ranges:

$$10 \text{ MeV/fm}^3 \leq p_t \leq 300 \text{ MeV/fm}^3,$$

$$100 \text{ MeV/fm}^3 \leq \Delta \epsilon \leq 3000 \text{ MeV/fm}^3,$$

$$0.2 \leq c_s^2 \leq 1.$$

checking consistency with pQCD calculations for $n \geq 40n_0$ (Kurkela et al. 2010; Gorda et al. 2018) and picking only EOSs whose stellar models verify $2M_\odot < M_{\max} < 2.3M_\odot$ (Rezzolla et al. 2018). From these results we selected eight representative hybrid EOS (see Fig. 4 and Table 1) which satisfy all recent astrophysical constraints and exemplify qualitatively the broad range of results we obtained.

5 RESULTS

In the following, we present our results for the $M$-$R$ relationship (Fig. 5) and the tidal deformability (Figs. 6 and 7) using the EOSs of Fig. 4. In all figures, we show simultaneously the results for slow and rapid conversions. When slow conversions are assumed, all points of the thick curves in Fig. 5 and all points of the curves in Figs. 6 and 7 represent stable configurations. In the rapid case, all HSSs with $\partial M/\partial \epsilon_c < 0$ are unstable (i.e. HSSs of models 1-4, 7 and 8).

Our results show that astrophysical constraints are satisfied in two different situations (see Fig. 5):

(a) Scenario with high-pressure transition (models 1-4, 7 and 8). A long SS hybrid branch is found to the left of the maximum mass point in the $M$-$R$ diagram and astrophysical constraints are fulfilled for a wide range of values of the quark EOS parameters. Twin stars with

| # | Hadronic EOS | Quark EOS |
|---|---|---|
| log$10 \rho_1$ | log$10 \rho_2$ | $\Gamma_2$ | $\Gamma_3$ | $p_t$ [MeV/fm$^3$] | $\Delta \epsilon$ [MeV/fm$^3$] | $c_s^2$ |
| 1 | 14.43 | 14.58 | 5.9 | 2.0 | 150 | 1800 | 0.33 |
| 2 | 14.43 | 14.58 | 6.2 | 2.3 | 150 | 3000 | 0.33 |
| 3 | 14.45 | 14.58 | 6.5 | 2.6 | 150 | 2000 | 0.30 |
| 4 | 14.45 | 14.68 | 6.5 | 2.6 | 70 | 1000 | 0.33 |
| 5 | 14.45 | 14.58 | 8.5 | 2.0 | 20 | 100 | 0.50 |
| 6 | 14.45 | 14.58 | 10.0 | 2.0 | 10 | 100 | 0.50 |
| 7 | 14.45 | 14.58 | 10.0 | 2.0 | 60 | 600 | 0.20 |
| 8 | 14.45 | 14.58 | 10.0 | 2.0 | 60 | 900 | 0.33 |

Table 1. Parameters of the selected hybrid EOSs. In all cases, we adopted log$10 \rho_1 = -27.22$ and $\Gamma_1 = 2.764$ for the first piece of the hadronic core EOS to match the upper limit predicted by cEFT EOSs (see Fig. 4).
In this work we explored new prospects for HS models emerging when the speed of quark-hadron interface conversions is taken into account, and confronted them with current observational constraints. When rapid interface conversions are assumed, we find that stable HSs compatible with astrophysical observations are possible only if the discontinuity occurs at low enough pressures and the quark EOS parameters are fine-tuned. This occurs because we concentrated intentionally on HSs with very stiff hadronic EOSs and on quark EOSs with only one linear piece. The use of hadronic EOSs of intermediate stiffness and different quark EOSs, may allow other fits to present observational data, albeit less easily than before the NICER measurement of PSR J0740+6620 (Annala et al. 2020).

On the other hand, our results for HSs with slow interface conversions bring a novel view of the structure of compact stars, which is completely consistent with current astrophysical observations and nuclear/pQCD restrictions. In this new scenario, purely hadronic NSs made of extremely stiff nuclear matter would have large radii and masses exceeding that of observed $2 M_\odot$ pulsars. To its left in the $M-R$ diagram, there would be a long branch of HSs whose density at the centre of SSHSs can be as high as some tens of $n_0$ (e.g., model 2 reaches $\sim 66 n_0$ at the centre of the last stable object). In this scenario, the stiff hadronic branch fulfils NICER and $2 M_\odot$ pulsar constraints, and SSHSs explain GW170817 as well as $2 M_\odot$ pulsars and PSR J0740+6620 observations. For a fixed hadronic EOS and a given $p_t$, the length of the SS hybrid branch depends mostly on $\Delta \varepsilon$ and on the stiffness of the quark EOS. Thus, a sufficiently large $\Delta \varepsilon$ is needed at the interface for explaining GW170817.

(b) Scenario with low-pressure transition (models 5 and 6). The main feature of this case is the existence of a long hybrid stellar branch that is stable for both rapid and slow conversions. For this totally stable hybrid branch the condition $\partial M/\partial \varepsilon_c > 0$ is verified. Observations are not easily fulfilled in this scenario when the PSR J0740+6620 radius is taken into account and fine tuning is needed, with low values of $p_t$ and $\Delta \varepsilon$.

The dimensionless tidal deformability $\Lambda$ is shown in Fig. 6 as a function of $M$. When rapid conversions are assumed, $\Lambda$ for stable configurations has the standard behaviour (Chatziioannou et al. 2018), i.e. the larger the mass the smaller the $\Lambda$. However, for slow conversions, $\Lambda$ can decrease or increase with $M$ meaning that, for a given hybrid EOS, not necessarily the most massive component of a binary NS merger (BNSM) will have the smallest $\Lambda$. As for the $M-R$ relationship, the GW170817 constraint is satisfied in the low and in the high pressure interface scenario.

In Fig. 7, we show the $\Lambda_1-\Lambda_2$ relationship spanning all possible combinations for the masses of the BNSM compatible with the GW170817 event. All BNSMs involving two hadronic stars are outside the 90% confidence contour of GW170817 (label I in Fig. 7), which is an expected result due to our choice of extremely stiff hadronic EOSs. In the case of slow conversions many combinations are in agreement with GW170817: binaries with two SSHSs are inside the 50% region (II), binaries involving a hadronic star and a SSHS are mostly inside the 90% contour (IIA and IIb), and binaries with two totally stable HSs are inside the the 90% region (IV). If rapid conversions are assumed, only binaries involving two totally stable HSs fall inside the 90% confidence contour (IV).

6 DISCUSSION

the same $M$ but different $R$ are possible for a wide range of masses. The density at the centre of SSHSs can be as high as some tens of $n_0$ (e.g., model 2 reaches $\sim 66 n_0$ at the centre of the last stable object). In this scenario, the stiff hadronic branch fulfils NICER and $2 M_\odot$ pulsar constraints, and SSHSs explain GW170817 as well as $2 M_\odot$ pulsars and PSR J0740+6620 observations. For a fixed hadronic EOS and a given $p_t$, the length of the SS hybrid branch depends mostly on $\Delta \varepsilon$ and on the stiffness of the quark EOS. Thus, a sufficiently large $\Delta \varepsilon$ is needed at the interface for explaining GW170817.

Figure 5. Mass-radius relationships for the EOSs of Fig. 4 (same colour-coding and symbols are used). We also show astrophysical constraints from the $\sim 2 M_\odot$ pulsars, GW170817 (Raithel et al. 2018; Annala et al. 2018), GW190425 (Abbott et al. 2020), and NICER (Riley et al. 2019; Miller et al. 2019) observations. In grey and black, we show radius constraints from the analysis of Dietrich et al. (2020) and Capano et al. (2020), respectively. In brown, the region excluded due to the maximum mass constraint for NSs, $M_{\text{max}} \leq 2.17^{+0.17}_{-0.15} M_\odot$, by Rezzolla et al. (2018). When slow conversions are assumed, all points of the thick curves represent stable stars and if conversions are rapid some hybrid segments are unstable (see text). Thin lines represent unstable stars in both scenarios.
in this work has not been to perform an exhaustive analysis of all possible hybrid models that agree with current constraints, but to show that this explanation is feasible and does not require a fine tuning of EOS parameters. Our results also draw attention to the great relevance of microphysical properties such as the surface tension, the curvature energy and reaction timescales, which can completely change our understanding of NS structure but cannot be encoded in \( p(\varepsilon) \) relationships and derived quantities, no matter how general or comprehensive they may be.

The probable existence of SSHSs opens new interesting scenarios in NS physics and astrophysics. For the hadronic matter EOS, it remarks that stiff and ultra stiff hadronic EOSs are still viable and compatible with current observations. For the quark matter EOS, the possibility of reaching densities of times greater than those normally expected in NSs, reinforces the astrophysical significance of studies that explore perturbative QCD in the low-temperature and high-density regime (Kurkela et al. 2010, 2014; Gorda et al. 2018). It also shows that there is not necessarily a tension between astrophysical observations and the theoretically expected conformal limit of the speed of sound (Reed & Horowitz 2020; Bedaque & Steiner 2015): large observed masses and radii would be explained by hadronic matter with very repulsive contributions and a large sound speed, while the small deformability of GW170817 is naturally explained by a high-pressure first-order phase transition to weakly interacting dense quark matter with \( c_s^2 \rightarrow 1/3 \). For astrophysics, a new scenario for the existence of two families of NSs is available, together with the standard hybrid star one (Alford et al. 2013; Benic et al. 2015; Christian & Schaffner-Bielich 2020; Shahrbaf et al. 2020) and the proposal of joint existence of hadronic and self-bound strange quark stars (Bombaci et al. 2004; Drago et al. 2016). Compared to twin stars already studied in the literature, twins involving SSHSs may span a wider mass range, approximately between \( \sim 1 M_\odot \) to more than \( 2 M_\odot \).

But how could the SSHS branch be populated in a realistic astrophysical scenario? Many cold hadronic stars of models 1-4, 7 and 8 are in metastable states because it is energetically convenient for them to convert into more compact SSHSs with same baryonic mass. However, these hadronic stars attain \( p_\mathrm{f} \) only at the centre of the maximum mass object. Therefore, mass accretion onto them would not be able to produce SSHSs because the object would collapse to a black hole when \( p_\mathrm{f} \) is attained at the stellar centre. However, in hot hadronic objects such as a protoneutron stars (Fischer et al. 2018) or NSs created after a compact star merger (Baiotti et al. 2008; Weih et al. 2020) the conversion to quark matter can be triggered at the core at a pressure significantly smaller than \( p_\mathrm{f} \). The reasons are the following. On one hand, the QCD phase diagram suggests that the density of hadron-quark phase equilibrium gets smaller as the temperature, \( T \), increases. Indeed, although the state-of-the-art understanding of the phase structure of QCD matter allows robust conclusions only at finite temperature with a small density and at an asymptotically high density, there are several investigations of the whole phase diagram using effective models. Many of these models find that the phase transition from a deconfined quark phase to confined hadronic matter is of the first order at large chemical potential with a critical point at intermediate non-zero chemical potential at the end of the first order phase transition line. This generic behaviour suggests that the hadron-quark transition gets easier as the temperature gets larger. On the other hand, if the conversion is triggered by quantum or thermal nucleation, it has been shown that the critical stellar mass above which a metastable hadronic star could undergo a phase transition is significantly reduced when the object is hot. As shown in Figs. 14 and 15 of Bombaci et al. (2016), when the entropy per baryon of the protoneutron star is \( \sim 2 k_B \), the critical stellar mass decreases by \( \sim 10 - 20\% \) with respect to the cold star. Thus, a significant portion of the upper part of the hadronic branch of our models 1-4, 7 and 8 would be prone to a transition to the SSHS branch in the protoneutron star phase. Hot post-merger hadronic stars would be even more propitious environments for conversion due to the high
To conclude, we mention some features of SSHSs that can help in their observational identification. Certainly, precise mass and radius measurements for a sufficiently large population of sources will significantly reduce the degeneracy of theoretical models and may open up the possibility of identifying SSHS branches if they exist and are long enough. Additionally, future GW detector networks will be able to measure the masses and tidal deformabilities to high accuracy, as well as some quasinormal mode frequencies to within tens of Hz (Pratten et al. 2020). The tidal deformability of SSHSs is significantly smaller and the \( f \)-mode frequency, \( v_f \), considerably larger (Tonetto & Lugones 2020; Mariani et al. 2022) than the corresponding values of a hadronic star of the same mass. This characteristic is in agreement with claims that hyper-excited dynamical tides, i.e., anomalously small \( v_f \), are disfavoured by GW170817 (Pratten et al. 2020). Moreover, discontinuity \( g \)-modes can be excited in SSHSs but don’t exist in the case of rapid conversions due to the absence of a buoyancy force (Tonetto & Lugones 2020). Contrary to \( g \)-modes of totally stable HSs which have \( v_g \) \( \lesssim \) 1 kHz and very long damping times, \( g \)-modes of SSHSs have \( v_g \) \( \approx \) \( 1 \) kHz and much shorter damping times that facilitate their detection for a given excitation amplitude (Tonetto & Lugones 2020; Mariani et al. 2022). This property makes SSHSs falsifiable by GW asteroseismology.

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Data Availability

All data are incorporated into the article.

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