Modeling of the galvano mirror by lumped mass system and verification for the model through the experiments

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Abstract
The galvano mirror has been widely used in the field of laser processing. It has a variety of vibration modes which are due to the torsion of the motor and the elastic deformation of the mirror. To improve the mechanical structure and to identify the resonance causes, the model of the galvano mirror is constructed. Moreover, the lumped mass system model is parametrical considered to verify the relationship between each resonance frequency and the component of the galvano mirror. The galvano mirror is composed of the motor, the coupling, and the mirror. Then, the models of each part are separately constructed and finally connected to build the model of the galvano mirror. In detail, the motor model is constructed based on the torsion of the motor itself. The mirror model which is regarded as the elastic structure is constructed based on the reduced order model. The galvano mirror model includes the pitching vibration mode which is not able to be actually detected by encoder. Any parameters of the model which are able to be used for future improvements from the mechanical point of view are changed to verify the rate of change against each resonance. In addition, the coupling rigidity is changed to shift the resonances to high frequency region through various approaches and to verify the propriety of the model.

Keywords : Galvano mirror, Lumped mass system, Modeling, Reduced order model, Coupling rigidity, Simulation, Finite element method, Modal analysis

1. Introduction
High efficient and performance laser processing has been required for the mass production of small-size electronic equipment. A galvano mirror has been widely used as a positioning device of the laser beam. The galvano mirror is composed of a pair of devices which is comprised of a mirror, a motor, and an encoder. Then, the motor and the encoder are regarded as a rigid body. In addition, the motor and the mirror are connected by the rigid coupling. In the laser processing, the laser beam reflected with the mirror is irradiated in desired spots by rolling one. In its motion, the mirror is vibrated in resonance frequencies. Vibrations of mirror are unfavorable because these vibrations lead to misalignment of the laser beam irradiation position. It causes deterioration of manufacturing and printing accuracy. In order to suppress vibration at the resonance frequencies, a variety of control methods, e.g., feedback and feedforward compensation techniques were proposed (Hirose et al., 2009; Kato et al., 2012; Iwasaki et al., 2012; Maeda et al., 2013). On the other hand, the structure of the galvano mirror can be considered to be a cantilever whose supporting point is the coupling. For this reason, the galvano mirror is vertically vibrated against mirror reflective surface. Here in after, this vibration is called the pitching vibration. Information of the pitching vibration cannot be detected by the encoder. In order to cope with this problem, vibration suppression control using piezoelectric elements (Seki et al., 2010; Seki et al., 2012; Seki and Iwasaki, 2014), a structure to avoid the cantilever (Sugie and Takahashi, 2002), and vibration suppression of whirling of shaft by inserting rubbers were reported (Nakade et al., 2015). Although these researches have dealt with each vibration of the mirror, these have been unpractical in terms of improvements for the machine itself. In addition, the final-state control was reported (Hirata et al., 2009). In this method, the optimal control input was calculated in advance to the high-speed target value responses and inputted as feedforward signal for the
motor in order to introduce a specified final state. In this time, the mathematical model should be constructed from frequency response to suppress resonance. However, the resonance causes are not able to determine by the mathematical model. The detail of that reasons will be mentioned later in chapter 2.

In this paper, firstly, the lumped mass system of the galvano mirror which takes the component itself into consideration is constructed. Secondly, the propriety of the model is verified through various simulations. Thirdly, any parameters of the model are changed to verify the rate of change against each resonance. Finally, since it is found through the parameter study that the coupling rigidity should be adjusted, the rigidity is changed to shift the resonances to high frequency region by various approaches for the coupling. In addition, the propriety of the model are verified.

2. Previous modeling method

The typical modeling in conventional researches was to construct the mathematical model from the frequency response. Generally, the model, which was composed of the sum of the rigid body mode and the $n$th-degree oscillation mode, was expressed as follows:

$$P(s) = K_p \left( \frac{1}{s} + \sum_{i=1}^{n} \frac{k_i}{s^2 + 2\zeta_i \omega_i s + \omega_i^2} \right)$$  \hspace{1cm} (1)

where $K_p$ is the nominal gain, $n$ is the total of vibration mode, $\zeta_i$ is the damping coefficient of the $i$th vibration mode, $\omega_i$ is the natural angular frequency, and $k_i$ is the vibration mode gain. Equation (1) is able to be accurately matched to the measurement results and used for compensator design. However, since this model just expresses the resonances by fitting to the measurement result, these are unable to be perceived as a physical phenomenon. Therefore, it is impossible to determine the resonance causes. On the other hand, the lumped mass system which is expressed by the mass, the damping, and the spring can determine the causes, because it is physically interpreted from the corresponding relationship between each parameter and each resonance. Therefore, the vibration modes should be expressed by the lumped mass system, which is desirable representation, based on the vibration modes of the distributed parameter structure.

3. Flow of modeling

The procedures of modeling are as follows under the assumption that the galvano mirror is separated to the motor and the mirror. Firstly, the lumped mass system of the motor is constructed in consideration of the torsion. Secondly, the coupling which connects the motor and the mirror is considered about incorporation into the model of the galvano mirror. Thirdly, the model of the mirror itself is constructed by converting the distributed parameter structure into the lumped mass system based on the reduced order model (Seto and Matsumoto, 1999). Finally, the lumped mass system model of the galvano mirror is built by connecting the motor model, the coupling element, and the mirror model. Figure 1 shows the frequency response of the galvano mirror from the motor torque $T$ [N·m] to the motor rotational angle $\theta_M$ [rad], i.e., the encoder’s signal. Multiple anti-resonance and resonance are generated in this figure. Therefore, the galvano mirror is composed of the multiple rotating mechanisms. The galvano mirror model which expresses these characteristics as the physical phenomenon is devised.

3.1 Modeling of the motor

Figure 2 shows the frequency response. In spite of without load, the anti-resonance and the resonance are generated. It means that the motor shaft is distorted. The model of the motor with torsion is constructed based on the gain diagram of Fig. 2. The anti-resonance and the resonance at specific frequencies are considered to be a function of the viscoelasticity which inherently presents in the shaft. In addition, the gain is flat at low frequency region of Fig. 2. Since it is the viscoelasticity of the motor inside, the motor viscoelasticity itself is also taken into consideration. From the above reasons, the model of Fig. 3 is composed. In this model, $T$ [N·m] be the motor torque and $J_m$ [kg·m²] be the motor inertia. In addition, $D_m$ [N·m/(rad/s)] be the damping coefficient and $K_m$ [N·m/rad] be the spring constant, respectively. $D_m$ [N·m/(rad/s)] and $K_m$ [N·m/rad] are regarded as a cause of torsion of the motor itself. The motor is
divided into two parts with the ratio \( \alpha (\alpha < 1) \) having the viscoelasticity. The rotational angle of the motor base side is \( \theta_1 \) [rad], which is equal to the motor rotational angle \( \theta_M \), and the motor tip side is \( \theta_2 \) [rad]. Moreover, the damping coefficient and the spring constant of inside of the motor are \( D_i [N \cdot m/(rad/s)] \) and \( K_i [N \cdot m/rad] \). From Fig. 3, the dynamical equations are expressed as follows:

\[
\begin{align*}
T &= (1 - \alpha)J_m\dot{\theta}_1 + D_m(\dot{\theta}_1 - \dot{\theta}_2) + K_m(\theta_1 - \theta_2) + D_1\dot{\theta}_1 + K_1\theta_1 \\
0 &= \alpha J_m\ddot{\theta}_2 + D_m(\ddot{\theta}_2 - \ddot{\theta}_1) + K_m(\theta_2 - \theta_1)
\end{align*}
\]

(2)

Then, the transfer function from \( T \) to \( \theta_1 \) is written as follows:

\[
\frac{\theta_1}{T} = \frac{\alpha J_m \omega^2 + D_m s + K_m}{\alpha(1 - \alpha)J_m s^4 + (J_mD_m + \alpha J_mD_1)s^3 + (J_mK_m + \alpha J_mK_i + D_mD_1)s^2 + (D_mK_i + D_1K_m)s + K_mK_i}
\]

(3)
Figure 4 shows Bode plots which are superimposed both the measurement and the simulation results. Each parameter used in the simulation is listed as Table 1. From Fig. 4, the simulation represents a resonance at 19 kHz corresponded with measurement result. Then, the ratio $\alpha$ is 0.27. From Fig. 3, it means that there is a node of torsion at the tip side of shaft. In addition, low frequency region in Fig. 4 shows that the flat gain is represented. Thus, it can be confirmed that the viscoelasticity is existed in the inside of the motor. Based on the above verifications, the block diagram of Eq. (2) are shown in Fig. 5.

### 3.2 Consideration of the coupling element

As mentioned the above, the mirror and the motor are connected by the rigid coupling. It tends to be ignored as a factor in modeling in terms of the strength of the stiffness. However, the rigid coupling should also be incorporated as one of the elements that have the finite stiffness. It is confirmed that the coupling elements also influences the response of the mirror. The detail of the confirmation will be mentioned later in chapter 4. Therefore, the coupling element is considered for the constructed model.

### 3.3 Modeling of the mirror

The lumped mass system of the mirror which is regarded as the elastic structure is constructed. The procedures of modeling which is carried out by the reduced order model proposed by Kazuto Seto’s book (Seto and Matsumoto, 1999) are as follows. Firstly, to search the natural frequencies of the mirror itself, the mirror is analyzed by the finite element method (FEM) under the condition of fixed end. In this paper, since modeling target is limited up to fourth mode of the mirror, fifth mode which is higher order than the target is analyzed. Secondly, the masses are set on the

Table 1 Parameters of each model

| Parameter [unit] | Description | Value |
|------------------|-------------|-------|
| $D_i$ [N·m/(rad/s)] | Damping coefficient of the motor | $7.00 \times 10^{-5}$ | $1.70 \times 10^{-4}$ |
| $K_i$ [N·m/rad] | Spring constant of the motor | $8.00 \times 10^{-2}$ | $1.40 \times 10^{-1}$ |
| $J_m$ [kg·m²] | Motor inertia | $0.82 \times 10^{-7}$ | $0.82 \times 10^{-7}$ |
| $D_m$ [N·m/(rad/s)] | Damping coefficient of the torsion | $1.00 \times 10^{-4}$ | $3.50 \times 10^{-5}$ |
| $K_m$ [N·m/rad] | Spring constant of the torsion | $1.60 \times 10^{-2}$ | $3.40 \times 10^{-2}$ |
| $\alpha$ | Motor partition rate | 0.27 | 0.54 |
| $J_c$ [kg·m²] | Coupling inertia | — | $0.14 \times 10^{-7}$ |
| $D_c$ [N·m/(rad/s)] | Damping coefficient of the coupling | — | $1.00 \times 10^{-7}$ |
| $K_c$ [N·m/rad] | Spring constant of the coupling | — | $2.30 \times 10^{6}$ |
| $M_1 = M_2$ [kg] | Mass of point 1 and point 2 | — | $0.39 \times 10^{-3}$ |
| $M_3 = M_4$ [kg] | Mass of point 3 and point 4 | — | $0.71 \times 10^{-3}$ |
| $k_{11} = k_{22}$ [N/m] | Spring constant between fixed end and point 1 and point 2 | — | $2.03 \times 10^{6}$ |
| $k_{33} = k_{44}$ [N/m] | Spring constant between fixed end and point 3 and point 4 | — | $5.27 \times 10^{6}$ |
| $k_{13} = k_{24}$ [N/m] | Spring constant between point 1 and point 3 and point 2 and point 4 | — | $0.46 \times 10^{6}$ |
| $k_{14} = k_{23}$ [N/m] | Spring constant between point 1 and point 4 and point 2 and point 3 | — | $-1.89 \times 10^{6}$ |
| $k_{12}$ [N/m] | Spring constant between point 1 and point 2 | — | $0.30 \times 10^{6}$ |
| $k_{34}$ [N/m] | Spring constant between point 3 and point 4 | — | $7.35 \times 10^{6}$ |
| $r_1 = r_2$ [m/rad] | Transmission constant from motor to point 1 and point 2 | — | $2.80 \times 10^{5}$ |
| $r_3 = r_4$ [m/rad] | Transmission constant from motor to point 3 and point 4 | — | $5.60 \times 10^{5}$ |
| $r_1' = r_2' = r_3' = r_4'$ [m] | Transmission constant from each position to motor | — | $2.50 \times 10^{5}$ |
node of the vibration mode in the highest order of analysis results. The number of masses is equal to the modeling order. Thirdly, the mode matrix is constructed from the first to forth modes of each mass. Finally, the mass and the spring matrixes which are the physical system are derived from the above mentioned information.

### 3.3.1 Calculation of equivalent mass

Figure 6 shows the vibration modes and movement of the masses in each mode. These are verified through FEM. In this figure, only actual analysis result of the first mode is indicated. Then, the second, the third, and the fourth modes are omitted. From Fig. 6, the first and the third modes are the pitching vibration against the mirror surface. In addition, the second and the fourth modes are the rolling vibration. Figure 7 shows the lumped mass system of the mirror which is traced the position relationship of the masses. In this figure, each mass is mutually connected and grounded via the springs. The equivalent mass of each mass is calculated by the rotational inertia, the vertical inertia and the dimension of the mirror. Figure 8 shows the calculation models of the rotational and the vertical inertias. The equations to calculate the equivalent masses \( M' \) and \( M'' \) [kg], which are shown in Fig. 8, are expressed as follows:

\[
\begin{align*}
M' + 4M'' &= 2.883 \times 10^{-3} \\
1.342M' + 0.138M'' &= 0.534 \times 10^{-3}
\end{align*}
\]  

(4)

The parameters of Fig. 8 and the calculated equivalent masses are listed as Table 2. From this table, the total value of \( M' \) and \( M'' \) are \( 1.94 \times 10^{-3} \) kg. That value is equal to the mass of the mirror.

### 3.3.2 Calculation of the mass and the stiffness matrixes

The springs which connect with each mass are calculated. From the amplitude of each vibration mode which is introduced by FEM, the mode matrix \( \Phi \) is expressed as follows:

\[
\Phi = \begin{bmatrix}
1.000 & -1.000 & 1.000 & -0.598 \\
1.000 & 1.000 & 1.000 & 0.598 \\
0.179 & -0.876 & -0.887 & 1.000 \\
0.179 & 0.876 & 0.887 & -1.000
\end{bmatrix}
\]  

(5)

Then, from the natural frequencies of each vibration mode, the natural angular frequency matrix \( \Omega \) [rad/s] is expressed
as follows:

\[
\Omega = \begin{bmatrix}
1.26 & 0 & \times 10^4 \\
4.77 & 8.25 & 0 \\
0 & 16.40 & 0
\end{bmatrix}
\]  

(6)

In addition, the normalized mode matrix \( \Phi' \), which is calculated from the mode matrix \( \Phi \) and the equivalent masses \( M' \) and \( M'' \), is expressed as follows:

\[
\Phi' = \begin{bmatrix}
1.000 & -1.000 & 1.000 & -0.598 \\
-1.000 & 1.000 & 1.000 & 0.598 \\
0.179 & -0.876 & -0.887 & 1.000 \\
0.179 & 0.876 & 0.887 & -1.000
\end{bmatrix}
\]  

(7)

![Fig. 7 Lumped mass system model of the mirror](image)

![Fig. 8 Calculation model of each inertia](image)

Table 2 Calculated equivalent masses

| Parameter [unit] | Description                  | Value    |
|------------------|------------------------------|----------|
| \( J \, [\text{kg} \cdot \text{m}^2] \) | Mirror rotational inertia    | \( 0.452\times10^7 \) |
| \( I \, [\text{kg} \cdot \text{m}^2] \) | Mirror vertical inertia      | \( 0.534\times10^7 \) |
| \( l \, [\text{m}] \) | Distance from central axis to \( M' \) | \( 2.800\times10^3 \) |
| \( L' \, [\text{m}] \) | Distance from the shaft root to \( M' \) | \( 25.900\times10^4 \) |
| \( L'' \, [\text{m}] \) | Distance from the shaft root to \( M'' \) | \( 8.300\times10^3 \) |
| \( M' \, [\text{kg}] \) | Masses of \( M_1 \) and \( M_2 \) | \( 0.332\times10^4 \) |
| \( M'' \, [\text{kg}] \) | Masses of \( M_3 \) and \( M_4 \) | \( 0.637\times10^4 \) |

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By using $\Phi'$, the mass matrix $M$ [kg] and the stiffness matrix $K$ [N·m/rad] are expressed as follows:

$$M = \left(\Phi\Phi^T\right)^{-1} \quad (8)$$
$$K = \left(\Phi^T\right)^{-1}Q\Phi^{-1} \quad (9)$$

However, the mass and the stiffness matrices constructed by Eqs. (8) and (9) are the linear transformation from the physical coordinate system to mode coordinate one. Therefore, each matrix becomes the diagonal matrix which is the condition to establish the lumped mass system. However, when the reduced order is performed from the distributed parameter system to the lumped mass system, the mass matrix $M$ is not generally diagonalized. Therefore, the normalized mode matrix $\Phi'$ has to be corrected by using the sensitivity of the error function (Seto and Matsumoto, 1999). The corrected mode matrix $\Phi^\dagger$ is expressed as follows:

$$\Phi^\dagger = \begin{bmatrix}
31.86 & 35.24 & 16.44 & -6.61 \\
31.86 & -35.24 & 16.44 & 6.61 \\
-12.16 & 4.89 & 23.56 & 26.06 \\
-12.16 & -4.89 & 23.56 & -26.06
\end{bmatrix} \quad (10)$$

Then, the mass matrix $M$ and the stiffness matrix $K$ calculated by Eqs. (8), (9), and (10) are expressed as follows:

$$M = \begin{bmatrix}
0.39 & 0 & 0 \\
0.39 & 0.71 & 0 \\
0 & 0.71 & 0
\end{bmatrix} \times 10^{-3} \quad (11)$$
$$K = \begin{bmatrix}
0.91 & -0.30 & -0.46 & 1.89 \\
-0.30 & 0.91 & 1.89 & -0.46 \\
-0.46 & 1.89 & 11.19 & -7.35 \\
1.89 & -0.16 & -7.35 & 11.19
\end{bmatrix} \times 10^{6} \quad (12)$$

Each parameter in Fig. 7 which is calculated from Eqs. (11) and (12) is listed as Table 1. The sum of masses is $2.2 \times 10^{-3}$ kg which is almost equal to the value calculated by Eq. (4). The dynamical equations of each mass in Fig. 7 are expressed as follows:

$$\begin{align*}
M_1 \ddot{x}_1 + K_1 x_1 - k_{12} x_2 - k_{13} x_3 - k_{14} x_4 &= F \\
M_2 \ddot{x}_2 - k_{12} x_1 + K_2 x_2 - k_{23} x_3 - k_{24} x_4 &= 0 \\
M_3 \ddot{x}_3 - k_{13} x_1 - k_{23} x_2 + K_3 x_3 - k_{34} x_4 &= 0 \\
M_4 \ddot{x}_4 - k_{14} x_1 - k_{24} x_2 - k_{34} x_3 + K_4 x_4 &= 0
\end{align*} \quad (13)$$

These equations are the case of that the mass 1 is vibrated by the excited force $F$ [N]. Moreover, the displacement of each mass is from $x_1$ to $x_4$ [m]. Validity of the model by the simulation is verified from the above equations. The simulation result is shown in Fig. 9. In this figure, the vertical dashed lines indicate each resonance frequency. Since all resonances are expressed, the model is reasonable. Based on the above verifications, the block diagram of Eq. (13) is shown in Fig. 10.

### 3.4 Modeling of the galvano mirror

By using Figs. 5 and 10, the model of the galvano mirror is constructed by connecting the motor with the mirror
through the coupling. On the other hand, the element of the coupling is not included in each model of Figs. 5 and 10. Therefore, the element of the coupling is added against Fig. 5 in order to connect with Fig. 10. Then, the output in Fig. 5 is the rotational angle $\theta_2$. To transfer the force from the motor to the mirror, the rotation of the motor has to be converted. Therefore, distance from the center of the motor to each mass is set as the transmission constants from $r_1$ to $r_4$ [m/rad]. In addition, in Fig. 10, the excited force $F$ is inputted to a particular mass. However, it is actually impossible that $F$ is inputted to a particular point on the mirror. Therefore, it is considered that the motor torque is added to all the masses. Then, the reaction force to the motor that comes back from each mass should be considered by the transmission constant from $r'_1$ to $r'_4$ [m]. From the above, the block diagram of the modeling of the galvano mirror is shown in Fig. 11. Then, the parameters of this figure are listed as Table 1. From this, the ratio $\alpha$ which expresses the position of torsion is 0.54. It is indicated that the torsion of motor is shifted to almost center by attaching the load.

4. Verification of the model through the simulations

The proposed model shown in Fig. 11 is verified by the simulations. Firstly, since the motor rotational angle $\theta_1$ is actually detected and controlled by the encoder, the response from the motor torque $T$ to $\theta_1$ is simulated. Figure 12 shows the simulation result. In addition, Fig. 13 shows Bode plots which are superimposed both Fig. 1 and the simulation result. In Fig. 12, the vertical dashed lines mean each resonance frequency. In these figures, from the first to fourth resonances are expressed. Moreover, the first and the second resonances are matched for the measurement result.
Secondly, the displacement $x_1$, $x_2$, $x_3$, and $x_4$ against $T$ are simulated. Figure 14 shows the gain diagrams of the simulations. In this figure, the resonance frequencies are matched to the dashed line of Fig. 12. Therefore, four resonances which are generated by the rotation of both the motor and the mirror are verified. Finally, the responses of the displacement which are vibrated by the excited force $F$, not inputted the torque $T$, are verified. Figure 15 shows the
Fig. 13 Simulation and measurement results of the galvano mirror

Fig. 14 Gain diagrams from $T$ to $x_i$

Fig. 15 Gain diagrams from $F$ to $x_i$

gain diagrams of the simulations. In this figure, two resonances which are not appeared in Fig. 14 are emerged. It is indicated that the pitching vibrations are not excited in the case of inputted the torque. Moreover, the pitching vibrations which are related to quality problems in industrial applications become the hidden mode. Figure 16 shows the phase diagrams of Fig. 14. In these diagrams, the red vertical dashed lines mean each frequency of the mirror in Fig. 6. From the phase relationship at each frequency, it is verified that the vibration modes of the mirror are expressed for the model in rolling motion. Firstly, the first mode is about to be verified. The phases of all masses are not changed at the vertical dashed line which indicates the first mode. Therefore, the first mode of the mirror which is the pitching vibration is not generated under the rolling. Secondly, the second mode is going to be verified. In this mode, the masses 1 and 3 are a pair of phase change. The former and the latter pairs have different 180 degrees. Therefore, the second mode is emerged under the rolling. Thirdly, the third mode is about to be verified. The phases of all masses are not changed at the vertical dashed line which means the third mode. Therefore, the third mode is not excited under the rolling motion. Finally, the fourth mode is going to be verified. In this mode, the masses 1 and 4 are a pair of phase change. In addition, the masses 2 and 4 are also a pair of phase change. The former and the latter pairs have different 180 degrees. Therefore, the fourth mode is appeared under the rolling. From the above verifications, the model of Fig. 11 is the rotating body model including the pitching vibration.
5. Parameter study

Any parameters of Table 1 which are able to be used for future betterment from the mechanical point of view, i.e., $J_c$, $K_c$, $J_m$, $K_m$, and $M$ are changed to verify the rate of change for the first, the second, and the third resonance frequencies. The transitions of each resonance frequency against each parameter are listed as Table 3. From this table, to shift the resonance frequencies to high frequency region, it is thought that $K_c$ and $K_m$ are increased or $J_c$, $J_m$, and $M$ are decreased. By using these characteristics, the resonances of the galvano mirror are sequentially shifted from the low frequency region. It is thought that a person, who is given the galvano mirror as a machine, can approach against the coupling to improve the positioning characteristics. Therefore, in this paper, the propriety of the model is verified by changing the coupling rigidity i.e., $K_c$, with the purpose which is to shift the resonance frequencies to high frequency region.

5.1 Change the coupling rigidity by the damping coating material

By adding the stiffness for the contact site of the motor and the coupling, the coupling rigidity $K_c$ is increased. Therefore, the damping coating material is applied into the coupling and the influences for each resonance frequency are verified. Figure 17 shows the measurement results with and without the damping coating material. From this figure, the resonance frequencies are shifted to high frequency region in the case with the damping coating material. It is found that the result is similar to the simulations (see Table 3 *).

![Fig. 16 Phase diagrams from $T$ to $x_i$](image)

Table 3 Summary of parameter study

| Parameter [unit] | Rate of change | First resonance (7600 Hz) | Second resonance (18500 Hz) | Third resonance (24000 Hz) |
|-----------------|----------------|---------------------------|-----------------------------|---------------------------|
| $J_c$ [kg·m²]   | +20%           | 7600 Hz (0%)              | 18100 Hz (-2.16%)           | 23750 Hz (-1.04%)         |
|                 | -20%           | 7600 Hz (0%)              | 19200 Hz (3.78%)           | 24250 Hz (1.04%)         |
| $K_c$ [N·m/rad] | +20%*          | 7800 Hz (2.63%)           | 18800 Hz (1.62%)           | 24600 Hz (2.50%)         |
|                 | -20%           | 7450 Hz (-1.97%)          | 18400 Hz (-0.54%)          | 23300 Hz (-2.92%)        |
| $J_m$ [kg·m²]   | +20%           | 7450 Hz (-1.97%)          | 17700 Hz (-4.32%)          | 23100 Hz (-3.75%)        |
|                 | -20%           | 7950 Hz (4.61%)           | 19600 Hz (5.95%)           | 25350 Hz (5.63%)         |
| $K_m$ [N·m/rad] | +20%           | 7700 Hz (1.32%)           | 19600 Hz (5.95%)           | 24800 Hz (3.33%)         |
|                 | -20%           | 7600 Hz (0%)              | 17350 Hz (-6.23%)          | 23400 Hz (-2.50%)        |
| $M$ [kg]        | +20%           | 7200 Hz (-5.26%)          | 18300 Hz (-1.08%)          | 23000 Hz (-4.17%)        |
|                 | -20%           | 8300 Hz (9.21%)           | 19000 Hz (2.70%)           | 25100 Hz (4.58%)         |
5.2 Change the coupling rigidity by the visco-elastic materials

The coupling is connected to the motor by tightening the screws. There are the clearance between the screw and the coupling. Therefore, the visco-elastic materials (Shore hardness: 45) are inserted in the clearance to increase the coupling rigidity $K_c$. Figure 18 shows the measurement results with and without the visco-elastic materials. In the former case, the resonance frequencies are slightly shifted to high frequency region. Similar to the previous section, the result indicates same responses of the simulations (see Table 3 *) by inserting one.

5.3 Change the coupling rigidity by altering the tightening torque

The mirror is connected to the motor by narrowing the inner diameter of the coupling. It is thought that the coupling rigidity $K_c$ is increased by strengthening the tightening torque of the screws. The torque of the above mentioned results is defined as 100% (0.2 N·m) torque. Then, 80% and 120% torques are prepared to measure three kind results which are different of the tightening torque. In this measurement, the frequency responses are measured in the vicinity of each resonance frequency to verify the effect in detail. In addition, the galvano mirror which is different from the above sections is measured. Figure 19 shows the results which are changed the torque. In this figure, the first and the third resonances are shifted to high frequency region by strengthening the tightening torque. In addition, the second resonance is not significantly shifted. The coupling rigidity $K_c$ is increased by strengthening the torque and the resonance frequencies are shifted similar to the simulations (see Table 3 *). $K_c$ which has been regarded as part of the mirror is changed by altering the tightening torque. From this fact, it is obvious that transitions of the resonance frequencies are contributed to the coupling rigidity. Moreover, the coupling, which is seemingly the rigid coupling, is the flexible coupling concerning the galvano mirror.
5.4 Change the coupling rigidity by altering the connection method

By altering the way of connection, the coupling rigidity $K_c$ is changed. Figure 20(a) shows the conventional connection method. While Fig. 20(b) shows the new connection method. In this method, the screws are directly tightened to the motor. The contact area for the motor is decreased and then $K_c$ is changed. The measurement results in before and after the change of the connection method is shown in Fig. 21. In this figure, the resonance frequencies are shifted to low frequency region after the change of the method. It is found that $K_c$ is decreased by tightening the screws directly for the motor. Therefore, the resonance frequencies are obviously influenced on $K_c$.

6. Conclusion

The galvano mirror model which is composed of the motor, the coupling, and the mirror was constructed. In the model, four resonances and the six vibration modes of the mirror were expressed. Moreover, the two pitching modes of the mirror which were not detected in rolling were included. In order to shift the resonances to high frequency region...
for improvement of both the positioning time and the accuracy, it was found through the parameter study that the coupling rigidity should be increased. Its rigidity was increased by various methods to shift the resonances and to verify the propriety of the model.

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