Shape-Invariant Single and Double-well Potentials under Spectral Invariance

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Abstract: We formulate the structure of spectral invariance in shape invariant single and double well potentials using derivative invariance principle.

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Supersymmetry [1] has many applications on quantum mechanical problems involving $V_\pm$ reflecting unbroken spectra[2,3,4].

Spectral nature becomes interesting, when analytical results are possible.

Mathematically, susy becomes shape-invariance (SI) in nature[5].

Here, we extend the SI using derivative invariance principle.

$$V_\pm = W^2 - \frac{dW}{dx}$$ (1)

Here, $W$ is the superpotential. Since, we stress on different models, we change the notion as The model susy partner potentials are

$$V_\pm^k = W_k^2 \mp \frac{dW_k}{dx}$$ (2)
Selection of $k$ is such that

$$\frac{dW_1}{dx} = \frac{dW_2}{dx} = \ldots = \frac{dW_k}{dx}$$ \hfill (3)

However, wave functions nature are different i.e

$$\psi_0^{(k)} \sim e^{-\int W_k dx}$$ \hfill (4)

Mathematically,

$$\psi_0^{(1)} \neq \psi_0^{(2)} \neq \psi_0^{(3)} \ldots.$$ \hfill (5)

Here all forms of super potentials $W_1, W_2 \ldots W_k$ come independently under shapeinvariant in nature.

Let us consider a model example to highlight the above development.

$$W_1 = A \tanh(x)$$ \hfill (6)

The corresponding SUSY potentials are

$$V_{\pm}^{(1)} = A^2(\tanh(x))^2 \mp A(1/\cosh(x))^2$$ \hfill (7)

$$\int W_1 dx = \log_e \cosh(x)$$ \hfill (8)

$$W_2 = A \tanh(x + \sinh\left(\frac{x}{|x|}\right))$$ \hfill (9)

$$V_{\pm}^{(2)} = A^2(\tanh(x + \tanh(x/|x|))^2 \mp A(1/\cosh(x + \sinh(x/|x|)))^2$$ \hfill (10)

$$\int W_2 dx = A \log (\cosh(x + \sinh\left(\frac{1}{\text{sgn}(x)}\right)))$$ \hfill (11)

$$W_3 = A \tanh(x - \tanh(x/|x|))$$ \hfill (12)
Now we focus on shape-invariance principle[1-5] i.e

\[ V_+^{(3)} = A^2(\tanh(x - \tanh\left(\frac{x}{|x|}\right)))^2 \mp A\left(\frac{1}{\cosh(x - \tanh\left(\frac{x}{|x|}\right))}\right)^2 \] (13)

\[ \int W_3dx = A \log \left(\cosh(\tanh\left(\frac{1}{\text{sgn}(x)} - x\right))\right) \] (14)

\[ W_4 = A \tanh(x - \sin\left(\frac{x}{|x|}\right)) \] (15)

\[ V_+^{(4)} = A^2(\tanh(x + \sin\left(\frac{x}{|x|}\right)))^2 \mp A\left(\frac{1}{\cosh(x + \sin\left(\frac{x}{|x|}\right))}\right)^2 \] (16)

\[ \int W_4dx = A \log \left(\cosh(\sin\left(\frac{1}{\text{sgn}(x)} - x\right))\right) \] (17)

Now we focus on shape-invariance principle[1-5] i.e

\[ V_+ = V_- + R(A) \] (18)

\[ E_n^- = \sum R(A) \] (19)

More explicitly, we have

\[ E_n^- = A^2 - (A - n)^2 \] (20)

Using computational analysis[6], we find for all the cases, energy levels remain invariant. In conclusion, suitable choice of superpotential, susy can reflect shape-invariance principle in single well as well as in double well potentials.
Figure 1: Single well

Figure 2: Single well

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Figure 3: Double well

Figure 4: Double well

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