Ediction of railway settlement deformation based on improved GM-AR model

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Abstract: In view of the randomness and fluctuation characteristics of railway settlement deformation, an unbiased grey model based on Markov optimization was established to predict railway settlement deformation. On the basis of unbiased optimization of the grey model, Markov correction is carried out to improve the prediction accuracy. Through the measured data of the settlement deformation of a high-speed railway, the prediction results of the model presented in this paper are compared with those of the grey model and the regression model, and the model presented in this paper has higher prediction accuracy and stability.

1. Introduction
With the rapid development of western China and the accelerating process of railway construction, the deformation analysis and prediction of railway settlement are particularly important. Many scholars have done a lot of research on railway deformation analysis and prediction. Wei Liangjian1] used the method of support vector machine to predict the deformation and settlement of the surface around the railway, and achieved a good prediction effect. Li Qiuquan[2] predicted the railway deformation by combining the models and combining the advantages of different models, which improved the stability and accuracy of the prediction of railway sidings. Zhang Xianzhou3] et al. made the IGM and FM model in series to predict the settlement of high-speed railway foundation, and improved the prediction accuracy of the settlement of high-speed railway foundation by correcting the residual. Jin Pengwei obtained a better prediction result by improving the grey model to predict the deformation of high-speed railway. Although the above research on high-speed railway settlement prediction has achieved certain results, only one aspect of the model improvement method is considered, which has certain limitations.

Therefore, an improved GM-AR model is proposed in this paper based on the settlement monitoring data of high-speed railway and the relationship between the settlement deformation law of high-speed railway and the grey theory. On the one hand, the background value of the grey GM (1,1) model was improved and unbiased optimized to solve the background value construction error and the inherent error of the grey model. On the other hand, AR model is used for residual correction to solve the randomness and non-linearity of residual and improve the accuracy and adaptability of model prediction.
2. Model principle

2.1 Construction and solution of GM (1,1) model in prediction of high speed railway settlement

Taking the settlement monitoring data of high-speed railway as the original time series of the grey model, the grey model of high-speed railway settlement deformation prediction is constructed.

Set the settlement deformation data corresponding to a monitoring point of high-speed railway as:

\[ x^0 = (x^0_1, x^0_2, ..., x^0_n) \]  

The original settlement deformation sequence of high-speed railway was accumulated once to generate the following generation formula:

\[ x^1 = (x^1_1, x^1_2, ..., x^1_n) \]  

The differential equation of the grey model for the cumulative generation sequence is as follows\(^{[4-6]}\):

\[ \frac{dx^1}{dt} + ax^1 = u \]  

In the equation, A is the development coefficient, reflecting the variation trend of high-speed railway settlement, and U is the gray action, representing the external influence degree of high-speed railway settlement.

Solve the grey development coefficient and grey action in Equation (3) according to the least square principle:

\[ \begin{bmatrix} a \\ u \end{bmatrix} = (B^T B)^{-1} B^T Y_n \]  

\[ B = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} & \cdots & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & \cdots & \frac{1}{2} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{1}{2} & \frac{1}{2} & \cdots & \frac{1}{2} \end{bmatrix}, \quad Y_n = \begin{bmatrix} x^0_1 \\ x^0_2 \\ \vdots \\ x^0_n \end{bmatrix} \]  

Where: B is the background value of the gray model

Substitute the calculation results a and u into Equation (1) to obtain the time response sequence by solving the model:

\[ \hat{x}^1(k + 1) = \left( x^0(1) - \frac{u}{a} \right) e^{-ak} + \frac{u}{a} \]  

Degradation and reduction of time response sequence were carried out to obtain the fitting and predicted values of the settlement deformation of high-speed railway:

\[ \hat{x}^0(k + 1) = \hat{x}^1(k + 1) - \hat{x}^1(k) \]  

2.2 Background value construction optimization

According to the construction process of the traditional grey model, the fitting and prediction accuracy of the grey GM (1,1) model is mainly affected by the accuracy of the grey parameters, and the accuracy of the grey parameters depends on the rationality of the construction of grey background values and the calculation of the original settlement deformation sequence. In the traditional grey model, the trapezoidal formula is used to calculate the geometric area of \( x^1(k) \) on the interval \([k,k-1]\). The error of the trapezoidal formula to calculate the geometric area of the curve affects the accuracy of the grey parameter solution.

Integrate both sides of the grey differential equation

\[ \int_{k-1}^{k} \frac{dx^1}{dt} + \int_{k-1}^{k} x^1 dt = b \]  

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The integral $\int_{k-1}^{k} x^4 dt$ is used as the background value [4] to replace the background value of the traditional grey model to construct the formula $0.5 \times (x^4(k) + x^4(k-1))$ to improve the accuracy of grey parameter solute.

$\int_{k-1}^{k} x^4 dt$ is used as the background value for grey model modeling, and the time response function of grey differential equation $\frac{dx^4}{dt} + ax^1 = u$ is:

$$x^0(t) = \frac{b}{a} + (x^0(1) - \frac{b}{a})e^{-a(t-1)}$$

(9)

The time response of grey differential equation $\frac{dx^4}{dt} + ax^1 = u$ is as follows:

$$x^0(k) = \frac{b}{a} + (x^0(1) - \frac{b}{a})e^{-a(k-1)}, k=1,2,\ldots,n$$

(10)

Finally, the reduction value formula is obtained as follows:

$$\hat{x}^4(k+1) = a^1 \left( \hat{x}^1(k+1) - \frac{b}{a} \right) = \hat{x}^4(k+1) - \hat{x}^4(k)$$

(11)

2.3 Unbiased optimization improvement

The grey model has a good prediction effect on the settlement deformation series with small fluctuation and exponential growth. The characteristics of weak regularity and complex variation of the settlement deformation data of high-speed railway lead to the deviation of the prediction results of the model. When building the model, unbiased optimization is carried out on the gray model to eliminate prediction bias:

According to the grey model parameters $a^1$ and $u^1$, the parameters of the unbiased optimization model are calculated:

$$L = \ln \left( \frac{2-a^1}{2+a^1} \right) S = \frac{2u^1}{2+a^1}$$

(12)

Unbiased optimization model results calculation:

$$y(k) = \begin{cases} y^0(1), k = 1 \\ Se^{L(k-1)}, k = 2,3,\ldots,n \end{cases}$$

(13)

2.4 AR model

AR model can achieve a better prediction effect on the data with randomness, and the prediction error of grey model for high-speed railway settlement deformation has the characteristics of volatility and randomness. AR model can achieve a better prediction effect on predicting residual error and improve the prediction accuracy of grey model.

Residual sequence is listed as a random sequence, which can be described as:

$$y_k = \varphi_1 y_{k-1} + \varphi_2 y_{k-2} + \cdots + \varphi_p y_{p+1}$$

The $p$-order autoregressive model equation of the residual sequence of this equation. From $\varphi_1, \varphi_2, \ldots, \varphi_p$ is the autoregressive parameter, and $p$ represents the model order.

When the model conforms to the formula: $\Omega = E\{ (y_k - \hat{y}_k)^2 \} = \min$ the partial autocorrelation function has ended in step $p$, that is, the sequence is AP $(p)$ model.

3. An engineering example

3.1 Data Sources

The example in this paper adopts the deformation monitoring data of the settlement monitoring point of DK0511+610 section of a high-speed railway base. The settlement deformation monitoring strictly follows the requirements of the second-class leveling specification with the electronic leveling instrument of Tianbo DINI03. The first 15 periods of samples are selected to predict the settlement
deformation of high-speed railway, and the settlement deformation of the next 5 periods is predicted. The prediction results of this model are compared with the grey model and the regression model, The data are shown in Table 1:

| Nper(d) | Cumulative settlement (mm)  |
|--------|-----------------------------|
| 1      | 34.07                       |
| 2      | 34.74                       |
| 3      | 34.65                       |
| 4      | 35.04                       |
| 5      | 35.22                       |
| 6      | 35.85                       |
| 7      | 35.66                       |
| 8      | 36.83                       |
| 9      | 36.83                       |
| 10     | 37.58                       |
|        | 38.02                       |
|        | 38.41                       |
|        | 41.36                       |
|        | 40.49                       |
|        | 40.47                       |
|        | 42.83                       |
|        | 41.89                       |
|        | 39.93                       |
|        | 41.77                       |
|        | 42.67                       |

3.2 Comparison results of model prediction
The data in Table 1 were analyzed and predicted by the model in this paper, the grey model and the regression model, and the relative error comparison results were shown in Table 2:

| Nper(d) | The regression model(%) | Grey model(%)  | The article model(%) |
|--------|--------------------------|----------------|---------------------|
| 16     | 5.202%                   | 6.106%         | 2.594%              |
| 17     | 1.745%                   | 1.965%         | 0.060%              |
| 18     | 4.490%                   | 7.471%         | 0.503%              |
| 19     | 1.257%                   | 5.116%         | 1.223%              |
| 20     | 0.480%                   | 5.362%         | 0.007%              |

Through the comparative analysis of the three models for the prediction of high-speed railway settlement, it can be seen that the relative error of the model in this paper is obviously better than that of the regression model and the grey model, and the prediction results of the model in this paper are more stable than that of the grey model and the regression model. Based on the background value and unbiased optimization of the improved grey model, the AR model is used to correct the residual errors to improve the prediction accuracy.

4. Conclusion
1. In this paper, the background value construction formula is improved to improve the calculation accuracy of gray parameters, and the model prediction accuracy and stability are improved by eliminating the inherent errors of the model through unbiased optimization of the grey model.
2. Based on the improved grey model, the AR model residuals were modified to improve the prediction accuracy of the model.

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