A Dark Energy Model with Generalized Uncertainty Principle in the Emergent, Intermediate and Logamediate Scenarios of the Universe

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This work is motivated by the work of Kim et al (2008), which considered the equation of state parameter for the new agegraphic dark energy based on generalized uncertainty principle coexisting with dark matter without interaction. In this work, we have considered the same dark energy interacting with dark matter in emergent, intermediate and logamediate scenarios of the universe. Also, we have investigated the statefinder, kerk and lerk parameters in all three scenarios under this interaction. The energy density and pressure for the new agegraphic dark energy based on generalized uncertainty principle have been calculated and their behaviors have been investigated. The evolution of the equation of state parameter has been analyzed in the interacting and non-interacting situations in all the three scenarios. The graphical analysis shows that the dark energy behaves like quintessence era for logamediate expansion and phantom era for emergent and intermediate expansions of the universe.

I. INTRODUCTION

Cosmological observations suggest that our universe is currently undergoing a phase of accelerated expansion driven by some unknown energy component characterized by negative pressure (Perlmutter, et al., 1999; Bachall et al, 1999; Copeland et al, 2006). Recently, the combination of WMAP3 and Supernova Legacy Survey data shows a significant constraint on the equation of state (EOS) for the dark energy, \( w_{ob} = -0.97^{+0.07}_{-0.09} \) in a flat universe (Seljak et al, 2006). This unknown energy component is dubbed as “dark energy” and a great variety of models have been proposed so far to describe this dark energy. Observations show that the energy density of DE occupies about 70% of today’s universe (Cai et al, 2010). However, at early cosmological epochs DE could not have dominated since it would have destroyed the formation of the observed large scale structure. These features have significantly challenged our thoughts about Nature. People begin to ask questions like (Cai et al, 2010): What is the constitution of DE? Why it dominates the evolution of our universe today? What is the relation among DE, dark matter and particle physics, which is successfully constructed? Some recent reviews on dark energy are Copeland et al (2006), Padmanabhan (2005, 2006), Sahni and Starobinsky (2006) and Sahani (2005). The simplest candidate of dark energy is a tiny positive cosmological constant. However, as is well known, it is plagued by the so-called “cosmological constant problem” and “coincidence problem” (Copeland et al, 2006). Other dark energy models include quintessence (Ratra and Peebles, 1988), phantom (Nojiri et al, 2005), quintom (Guo et al, 2005; Elizalde et al, 2004), Chaplygin gas (Gorini et al, 2003), tachyon (Calcagni and Liddle, 2006), hessence (Wei et al, 2005), Ricci dark energy (Feng, 2008), and electro magnetic dark energy (Beck et al, 2008). There are two other candidates of dark energy based on holographic principle (Bousso, 2002). They are holographic dark energy (Li, 2004) and agegraphic dark energy model (Wei and Cai, 2008). The first is based on the Bekenstein-Hawking energy bound \( E_A \leq E_{BH} \) with the energy \( E_{BH} \) of a universe-sized black hole which produces \( L^3 \rho_A \leq m_p^2 L \) with the length scale \( L \) (IR cutoff) of the universe and the Planck mass \( m_p \). The largest \( L \) allowed is the one saturating this inequality, thus the holographic dark energy density is \( \rho_A = 3c^2 m_p^2 L^{-2} \) (Li, 2004; Pavon and Zimdahl, 2005), where \( c^2 \) is a constant. The later is based on the Karolyhazy relation of \( \delta t \) and the time-energy uncertainty of \( \Delta E \sim t^{-1} \) in the Minkowski spacetime with a given time scale \( t \), which gives \( \rho_q \sim \frac{\Delta E}{(\delta t)^2} \sim \frac{m_p^2}{L} \) (Maziashvili, 2007). Nojiri and Odintsov (2006) suggested generalized holographic dark energy where infrared cutoff is identified with combination of
FRW parameters: Hubble constant, particle and future horizons, cosmological constant and universe life-time (if finite). This study of Nojiri and Odintsov (2006) also reviewed other known holographic dark energy models.

The problem of discriminating different dark energy models is now emergent. In order to solve this problem, a sensitive and robust diagnostic for dark energy is a must. The statefinder parameter pair \( r, s \) introduced by Sahni et al (2003) and Alam et al (2003) is proven to be useful tools for this purpose. The statefinder pair is a ‘geometrical’ diagnostic in the sense that it is constructed from a space-time metric directly, and it is more universal than ‘physical’ variables which depends upon properties of physical fields describing dark energy, because physical variables are, of course, model-dependent (Feng, 2008). Details of the statefinder parameters would be discussed in the subsequent section. The spatially flat \( \Lambda\text{CDM} \) scenario corresponds to a fixed point \( \{r, s\} = \{1, 0\} \) in the \( r-s \) plane. The statefinder can successfully differentiate between a wide variety of dark energy models including the cosmological constant, quintessence, the Chaplygin gas, braneworld models and interacting dark energy models. The statefinder diagnostics have been investigated for tachyonic field (Chattopadhyay et al, 2008), holographic dark energy (Zhang, 2005), Ricci dark energy (Feng, 2008), quintessence (Zhang, 2005), Yang-Mills dark energy (Feng, 2008), quintom dark energy (Wu and Yu, 2005), dilaton dark energy (Huang et al, 2008). In a study on interacting new agegraphic dark energy by Zhang et al (2010) it was found that the \( r-s \) trajectory is confined in the first quadrant of the \( r-s \) plane for various forms of interaction. Statefinders generalize such well known observational characteristics of the expansion as the Hubble (first-order) and the deceleration (second-order) parameters. The expansion factor or scale factor \( a \) of the universe can be Taylor expanded around the present epoch \( t_0 \) as \( a(t) = a_0 \left[ 1 + \sum_{n=1}^{\infty} \frac{A_n(t)}{n!} (t - t_0)^n \right] \); where \( A_n = \frac{a^{(n)}}{H} \), \( n \in \mathbb{N} \) (Arabsalmania and Sahni, 2011). Here, \( a^{(n)} \) is the \( n \)-th derivative of the scale factor with respect to time. For various values of \( n \), we get different parameters like jerk ‘\( j \)’, snap ‘\( s \)’, jerk ‘\( l \)’, etc (Visser, 2005; Arabsalmania and Sahni, 2011; Dabrowski, 2005). It should be mentioned that for \( n = 3 \) we get statefinder parameter ‘\( r \)’, which is also known as ‘jerk’ parameter (Arabsalmania and Sahni, 2011).

In this work, we consider the new agegraphic dark energy model with the generalized uncertainty principle (GUP). This work is motivated by the work of Kim et al (2008), who were first consider the new agegraphic dark energy models with the GUP. The GUP and its consequences has been discussed in the papers like Garay (1995), Scardigli (1999) and Rama (2001). Although the GUP has its origins in the string theory, may play a role of evolution of the universe Kim et al (2008). In this paper we consider the interacting new agegraphic dark energy model with the generalized uncertainty principle in emergent, intermediate and logamediate scenarios of the universe. We investigate the behavior of the equation of state parameter, statefinder, kerk and lerk parameters under this interaction and also the fate of the universe through statefinder diagnostics.

## II. NEW AGEGRAPHIC DARK ENERGY MODEL WITH GENERALIZED UNCERTAINTY PRINCIPLE (GUP)

The metric of a spatially flat homogeneous and isotropic universe in FRW model is given by

\[
ds^2 = dt^2 - a^2(t) \left[ dr^2 + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right]
\]

where \( a(t) \) is the scale factor. The Einstein field equations are given by

\[
H^2 = \frac{1}{3} \rho \tag{2}
\]

and

\[
\dot{H} = -\frac{1}{2} (\rho + p) \tag{3}
\]

where \( \rho \) and \( p \) are energy density and isotropic pressure respectively (choosing \( 8\pi G = c = 1 \)).
The conservation equation is given by

\[ \dot{\rho} + 3H(\rho + p) = 0 \]  \hspace{1cm} (4)

Next, we consider the interaction between the new agegraphic dark energy using GUP and dark matter. According to the GUP, the energy density is defined by (Kim et al, 2008)

\[ \rho_G = \frac{\Delta E_G}{(\delta t)^3} \]  \hspace{1cm} (5)

where, where \( \delta t \) is given by the Károlyházy relation of time fluctuations as \( \delta t = t_p^{2/3} t^{1/3} \). Solving the saturation of the GUP leads to

\[ \Delta E_G = \frac{1}{t} + \frac{\zeta}{t^3} \]  \hspace{1cm} (6)

labelling of \( \zeta \) and \( t_p \) are

\[ \zeta = \left( \frac{\xi}{n} \right)^2, \quad t_p^2 = \frac{1}{3n^2m_p^2} \]  \hspace{1cm} (7)

Consequently, the dark energy density is described with two parameters \( (n, \xi) \) as (Kim et al, 2008)

\[ \rho_G = \frac{3n^2m_p^2}{t^2} + \frac{3\xi^2}{t^4} \]  \hspace{1cm} (8)

Wei and Cai (2008) proposed the new agegraphic dark energy model characterized by the energy density

\[ \rho_A = \frac{3n^2m_p^2}{\eta} \]  \hspace{1cm} (9)

where the conformal time \( \eta \) is given by

\[ \eta = \int \frac{dt}{a} \]  \hspace{1cm} (10)

Using (10) in (8) the dark energy density based on GUP takes the form (Kim et al, 2008)

\[ \rho_G = \frac{3n^2m_p^2}{\eta^2} + \frac{3\xi^2}{\eta^4} \]  \hspace{1cm} (11)

Here, now we are considering interaction between dark matter and the dark energy. That is why, in equations (2), (3) and (4) we replace \( \rho \) and \( p \) by \( \rho_{\text{total}} \) and \( p_{\text{total}} \) respectively with

\[ \rho_{\text{total}} = \rho_G + \rho_m, \quad p_{\text{total}} = p_G + p_m \]  \hspace{1cm} (12)

where \( p_G, p_m \) and \( \rho_m \) denote the pressure of the GUP based dark energy, pressure of dark matter and the density of the dark matter respectively. Consequently, the conservation equation (4) becomes

\[ \dot{\rho}_{\text{total}} + 3H(\rho_{\text{total}} + p_{\text{total}}) = 0 \]  \hspace{1cm} (13)

As in the case of interaction the components do not satisfy the conservation equation separately, we need to reconstruct the conservation equation by introducing an interaction term \( Q \). It is important to note that the
conservation equations imply that the interaction term should be a function of a quantity with units of inverse of time (a first and natural choice can be the Hubble factor $H$) multiplied with the energy density. Therefore, the interaction term could be in any of the forms (Sheykhi, 2010; Wei and Cai, 2009): $Q \propto H \rho_G$, $Q \propto H \rho_m$, and $Q \propto H \rho_{total}$.

Considering the interaction term $Q$ as $Q = 3H \delta \rho_m$, where $\delta$ is the interaction parameter, the conservation equation (13) takes the form

$$\dot{\rho}_G + 3H(\rho_G + p_G) = Q \quad (14)$$

and

$$\dot{\rho}_m + 3H \rho_m(1 + w_m) = -Q \quad (15)$$

where, $w_m = \frac{p_m}{\rho_m}$ is the equation of state parameter for dark matter. It may be noted that similar choice of the interaction term has been made in Wang et al (2005), Sheykhi (2009). If $Q > 0$, there is a flow of energy from dark matter to dark energy (Cataldo et al, 2008). We are going to discuss the said interactions in three scenarios:

1. **Emergent scenario** (Mukherjee et al, 2006), where the scale factor has the form $a(t) = a_0(B + e^{At})^m$ with $a_0 > 0$, $A > 0$, $B > 0$, $m > 1$.

2. **Intermediate scenario** (Barrow and Nunes, 2007; Barrow and Liddle, 1993), where $a(t) = \exp(\lambda t^\beta)$ with $\lambda > 0$; $0 < \beta < 1$.

3. **Logamediate scenario** (Barrow and Nunes, 2007), where $a(t) = \exp(\mu(\ln t)^\alpha)$ with $\mu \alpha > 0$, $\alpha > 1$.

At this juncture it should be stated that some authors first choose the scale factor in power law, exponential or in other forms and then find out other variables with some conditions under these solutions. This ‘reverse way of investigations had earlier been used extensively by Ellis and Madsen (1991) who chose various forms of scale factor and then found out the other variables from the field equations. Subsequently, this approach has been adopted by Banerjee and Das (2005) who clearly stated “This is not the ideal way to find out the dynamics of the universe, as here the dynamics is assumed and then the fields are found out without any reference to the origin of the field. But in the absence of more rigorous ways, this kind of investigations collectively might finally indicate towards the path where one really has to search”. In another study, Feinstein (2002) assumed scale factor in the power law form to model the potential by an inverse square law in terms of the tachyon field. Campuzano et al (2010) studied the curvaton reheating assuming assuming the scale factor in the logamediate scenario i.e. in the form given under item 3 of the above list. So in particular, we have chosen the scale factor in the forms enlisted above. Mukherjee et al (2006) obtained the general solution of the scale factor for the emergent universe without referring to the actual source of the energy density. So we use the form under item 1 of the above list as the choice of scale factor for the emergent universe. Such choice of scale factor has been used in the references like Mukherjee et al (2005), Debnath (2008) and Paul and Ghose (2010). In the particular scenario of ‘intermediate’ inflation the expansion scale factor of the Friedmann universe evolves as in item 2; the expansion of the Universe is slower for standard de Sitter, which arises when $\beta = 1$, but faster than in power-law inflation, $a = t^\beta$, with $\beta > 1$ constant (Barrow and Nunes, 2007). This form of scale factor has been used in Khatua and Debnath (2010). Barrow and Nunes (2007) considered ‘logamediate’ inflation where the cosmological scale factor expands in the form expressed in the item 3 of the above list. This form of scale factor has been used in Khatua and Debnath (2010). All of the above three scenarios have been discussed at length in the references cited in the above list. In this paper we are not going into the descriptions of the said scenarios. For all of the said scenarios, we have considered the characteristics of the universe expansion as:

**The deceleration parameter:** The acceleration of the universe can be quantified through a dimensionless cosmological function known as the ‘deceleration parameter’ $q$ given by (Dabrowski, 2005)

$$q = -\frac{1}{H^2} \frac{\ddot{a}}{a} \quad (16)$$

where $q < 0$ describes an accelerating universe, whereas $q \geq 0$ for a universe which is either decelerating or expanding at the ‘coasting’ $a \propto t$ (Alam et al, 2003).
The “jerk” or “statefinder” parameter: It is known that “jerk” parameter is another name of the “statefinder” parameter. The statefinder parameter is given by (Alam et al, 2003; Arabsalmania and Sahni, 2011)

\[
r = \frac{1}{H^3} \dot{a}
\]  

(17)

It is easy to see that \( r \) is a natural next step beyond \( H \) and \( q \). We can easily see that this diagnostic is constructed from the \( a(t) \) and its derivatives up to the third order. So, the statefinder probe is the expansion dynamics of the universe through higher derivatives of the scale factor (Huang et al, 2008).

The “second statefinder” parameter: The second statefinder parameter is given by (Alam et al, 2003)

\[
s = r - \frac{1}{3(q - \frac{1}{2})}
\]  

(18)

By far, many models have been differentiated by this geometrical diagnostic method. Its important property is that \( \{r, s\} = \{1, 0\} \) is a fixed point for the flat \( \Lambda \)CDM FRW cosmological model (Huang et al, 2008). Departure of a given DE model from this fixed point is a good way of establishing the “distance” of this model from flat \( \Lambda \)CDM.

The “kerk” or “snap” parameter: Snap, which involves the fourth time derivative of scale factor, is also sometimes called “kerk”. This parameter is given by

\[
k = -\frac{1}{H^3} \frac{a^{(4)}}{a}
\]  

(19)

This parameter has been discussed in the references like Dabrowski (2005), Dunajski and Gibbons (2008) and Arabsalmania and Sahni (2011).

The “lerk” parameter: This parameter involves the fifth time derivative of scale factor. This parameter is given by (Dabrowski, 2005)

\[
l = \frac{1}{H^5} \frac{a^{(5)}}{a}
\]  

(20)

In this work we have investigated all of the said parameters for the new agegraphic dark energy based on generalized uncertainty principle under the three different scenarios mentioned earlier.

III. INTERACTION IN THE EMERGENT SCENARIO

In this section we consider the interaction between dark energy and dark matter under emergent scenario and discuss the \( r-s \) trajectories along with other parameters involving various orders of derivative of the scale factor. For emergent scenario using \( a(t) = a_0 \left( B + e^{At} \right)^m \) in (10) we get

\[
\eta = -\frac{(1 + Be^{-At})^m (B + e^{At})^{-m} 2F_1 [m, m, 1 + m, -Be^{-At}]}{Aa_0 m}
\]  

(21)

Consequently, using (11) and (16) we get the dark energy density as

\[
\rho_G = \frac{3A^4a_0^4 (1 + Be^{-At})^{-4m} (B + e^{At})^{4m} m^2 \xi^2}{2F_1 [m, m, 1 + m, -Be^{-At}]^4} + \frac{3A^2a_0^2 (1 + Be^{-At})^{-2m} (B + e^{At})^{2m} a^2m^2m^2}{2F_1 [m, m, 1 + m, -Be^{-At}]^2}
\]  

(22)

From the conservation equation (15) we get under emergent scenario

\[
\rho_m = \rho_{m0} \left[ a_0 \left( B + e^{At} \right)^m \right]^{-3(1 + \omega_m + \delta)}
\]  

(23)
Fig. 1 shows the evolution of the equation of state parameter $w_{\text{total}}$ in the interacting (thick line) and non-interacting (dashed line) situations in the emergent universe scenario.

Fig. 2 shows the total energy density $\rho_{\text{total}}$ and pressure $p_{\text{total}}$ in the interacting (thick line) and non-interacting (dashed line) situations in the emergent universe scenario.

Fig. 3a shows the evolution of the deceleration (red), jerk or statefinder (green), kerk (brown) and lerk (blue) parameters. The continuous lines represent the interacting and the dotted lines represent the non-interacting situations in the emergent scenario.

Fig. 3b shows the $r$-$s$ trajectory under interaction and non-interaction. The trajectories have coincided. We have taken the cosmic time $t \in [0, 4]$. The $r(s)$ trajectory has passed through the fixed point $\{r = 1, s = 0\}$ of the $\Lambda$CDM. We have taken $n = 1.1$, $m = 2.1$, $a_0 = 0.02$, $\delta = 0.03$, $B = 3.2$, $m^2_p = 1$ and $A = 2.2$.

Using $\rho_m$ and $\rho_G$ in the first field equation (1) it is easy to express $H$ as a function $t$ under interaction. Using this $H$ in the second field equation (2) we get $p_G$ under interaction as
\[ p_G = \left[ -\left( a_0 \left( B + e^{A t}\right) \right)^{m} \right]^{-3(1+w_m+\delta)} \rho_m (1+w_m) - \]
\[
\frac{3A^4 a_0^3(B + e^{At})^{-4m}(B + e^{At})^{4m}m^{4k^2}}{2F_1[m, m, 1 + m, -B e^{-At}]} - \frac{3A^4 a_0^3(B + e^{At})^{-2m}(B + e^{At})^{2m}n^2 m^2 m^2}{2F_1[m, m, 1 + m, -B e^{-At}]^2} \times 
\]
\[
\left( B + e^{At}\right)^{-1-3m} \left( a_0 \left( B + e^{At}\right) \right)^m \left( -4A^4 a_0^7 (B + e^{At})^1 + 7m \left( a_0 \left( B + e^{At}\right) \right)^m \right)^{3(w_m+\delta)} \times 
\]
\[
m^{4k^2} + e^{At} (1 + Be^{-At})^5 m (1 + w_m + \delta) \rho m 2F_1[m, m, 1 + m, -Be^{-At}]^{5} - 
\]
\[
2A^2 a_0^5(1 + Be^{-At})^{2m}(B + e^{At})^{1+5m} \left( a_0(B + e^{At})^m \right)^{3(w_m+\delta)} n^2 m^2 \times 2F_1[m, m, 1 + m, -Be^{-At}]^{2m_p^2} \right) \times 
\]
\[
\left( 2a_0^3 2F_1[m, m, 1 + m, -Be^{-At}]^{5} \right)^{-1} \times 
\]
\[
\left( a_0 \left( B + e^{At}\right) \right)^m \left( -3(1+w_m+\delta) \right) + \frac{3A^4 a_0^3(B + e^{At})^{-4m}(B + e^{At})^{4m}m^{4k^2}}{2F_1[m, m, 1 + m, -B e^{-At}]} + \frac{3A^4 a_0^3(B + e^{At})^{-2m}(B + e^{At})^{2m}n^2 m^2 m^2}{2F_1[m, m, 1 + m, -B e^{-At}]^2} \right)^{-1/2}
\]

(24)

Using the above forms of \( \rho_G, p_G, \rho_m \) and \( p_m = w_m \rho_m \) we calculate \( w_{total} = \frac{\rho_{total}}{\rho_{total} + p_{total}} \) and plot against cosmic time \( t \) in figure 1. In figure 1 we have considered \( \delta \neq 0 \) as well as \( \delta = 0 \). Non-zero \( \delta \) implies interaction between dark energy and dark matter, whereas \( \delta = 0 \) implies the co-existence of dark energy and matter without interaction. In both of the interacting and non-interacting situations, it is observed that the equation of state parameter \( w_{total} < -1 \), which indicates phantom like behavior. In figure 2 we have plotted \( \rho_{total} \) and \( p_{total} \) against \( t \). Here also we have considered both interacting and non-interacting situations. It is observed that \( p_{total} \) is increasing in the negative direction and \( \rho_{total} \) is increasing with \( t \). This indicates that the energy density is increasing and pressure is decreasing under both interacting and non-interacting situations.

In figures 3a we have plotted the various parameters characterizing the accelerating universe against time \( t \). We find that “lerk” and “statefinder” parameters are having similar patterns and are staying at positive level. We also find in this figure that the deceleration parameter is staying at the negative level throughout the evolution of the universe. This indicates that under this interaction in emergent scenario we are getting an ever accelerating universe. The “lerk” parameter is also behaving like the deceleration parameter. The deceleration moving upwards with evolution of the universe. It may be interpreted from this pattern that the acceleration is decreasing at late time.

In figure 3b, we have presented the \( r(s) \) curve. The trajectory passes through \( \{ r = 1, s = 0 \} \) corresponding to \( \Lambda CDM \). Also, we find that the \( r-s \) trajectory is confined within the first and fourth quadrant of the \( r(s) \) plane and \( r \) increases with decrease in \( s \) with the evolution of the universe. The section of the plot with positive \( r \) and \( s \) gives the radiation phase of the universe. After passing through the \( \Lambda CDM \) we get the end point of the \( r(s) \) curve at \( r = 1.55, s = -0.2 \). The above calculation has been done under the interaction (\( \delta = 0.05 \)). If we take \( \delta = 0 \), then the trajectory coincides with that for interacting model. This indicates that the interaction does not affect the fate of the universe. At this point this model differs from the interacting new agegraphic dark energy model proposed by Zhang et al (2010) where the \( r(s) \) curve got the endpoint at \( \Lambda CDM \). When we are considering new agegraphic dark energy based on generalized uncertainty principle in the emergent universe we can go beyond \( \{ r = 1, s = 0 \} \) i.e. the \( \Lambda CDM \).

IV. INTERACTION IN THE INTERMEDIATE SCENARIO

Interacting dark energy in the intermediate scenario is presented in this section. For this purpose, using \( a(t) = \exp(\lambda t^\beta) \) in (10) we get
Fig. 4 shows the evolution of the equation of state parameter \( w_{\text{total}} \) in the interacting (thick line) and non-interacting (dashed line) situations in the intermediate scenario.

Fig. 5 shows the total energy density \( \rho_{\text{total}} \) and pressure \( p_{\text{total}} \) in the interacting (thick line) and non-interacting (dashed line) situations in the intermediate universe scenario.

\[
\eta = \frac{\lambda}{\beta} \Gamma \left[ \frac{1}{\beta}, \lambda t^\beta \right]
\]

where \( \Gamma[x, z] \) is the incomplete gamma function defined by \( \Gamma[x, z] = \int_z^\infty u^{x-1} e^{-u} du \). Using the above form of conformal time \( \eta \) and using (11), we get

\[
\rho_G = -\frac{3\xi^2(\lambda t^\beta)^{4/\beta} \beta^4}{t^4 \Gamma \left[ \frac{1}{\beta}, \lambda t^\beta \right]^4} + \frac{3n^2(\lambda t^\beta)^{2/\beta} \beta^2 m_p^2}{t^2 \Gamma \left[ \frac{1}{\beta}, \lambda t^\beta \right]^2}
\]

Using the same technique as in emergent scenario we get the form of pressure \( p_G \) under interaction as

\[
p_G = \left[-e^{-3\lambda t^\beta}(e^{\lambda t^\beta})^{-3w_m-3\delta} \rho_m(1+w_m) - \frac{3\xi^2(\lambda t^\beta)^{4/\beta} \beta^4}{t^4 \Gamma \left[ \frac{1}{\beta}, \lambda t^\beta \right]^4} - \frac{3n^2(\lambda t^\beta)^{2/\beta} \beta^2 m_p^2}{t^2 \Gamma \left[ \frac{1}{\beta}, \lambda t^\beta \right]^2}
\]

\[
- \left(\frac{2}{\sqrt{5}}e^{-3\lambda t^\beta}(e^{\lambda t^\beta})^{-3w_m-3\delta} \left(4e^{3\lambda t^\beta}(e^{\lambda t^\beta})^{3w_m+3\delta} \xi^2(\lambda t^\beta)^{5/\beta} \beta^4 - \lambda t^\beta \beta^4 \rho_m \Gamma \left[ \frac{1}{\beta}, \lambda t^\beta \right]^5 (1+w_m+\delta) \right)
\]

\[
+2e^{\lambda t^\beta(2+3w_m+3\delta)} t^2 (\lambda t^\beta)^{3/\beta} \Gamma \left[ \frac{1}{\beta}, \lambda t^\beta \right]^2 m_p^2 \right) \right] \times
\]

\[
\left(2t^5 \Gamma \left[ \frac{1}{\beta}, \lambda t^\beta \right]^5 \sqrt{(e^{\lambda t^\beta})^{-3(1+w_m+\delta)} \rho_m} + \frac{3\xi^2(\lambda t^\beta)^{4/\beta} \beta^4}{t^4 \Gamma \left[ \frac{1}{\beta}, \lambda t^\beta \right]^4} + \frac{3n^2(\lambda t^\beta)^{2/\beta} \beta^2 m_p^2}{t^2 \Gamma \left[ \frac{1}{\beta}, \lambda t^\beta \right]^2} \right)^{-1}
\]

Using the above forms of \( p_G \) and \( \rho_G \) we calculate \( w_{\text{total}} = \frac{p_G + p_m}{\rho_G + \rho_m} \) and plot against cosmic time \( t \) in figure 4. We find that the equation of state parameter \( w_{\text{total}} \) is staying below \(-1\) in the case of interaction (\( \delta \neq 0 \)) as well as non-interaction (\( \delta = 0 \)). This indicates the phantom-like behavior of the equation of state parameter. It is tending towards \(-1\), but is not crossing the barrier of \(-1\). Hence, it can be said that the behavior of \( w_{\text{total}} \) is more or less similar to that in the case of emergent scenario. In figure 5 we have plotted \( \rho_{\text{total}} = \rho_G + \rho_m \) and \( p_{\text{total}} = p_G + p_m \). It is observed that \( \rho_{\text{total}} \) is increasing and \( p_{\text{total}} \) is increasing in negative direction with time \( t \). Here also we get almost similar behavior to that of the emergent scenario.
Fig. 6a shows the evolution of the deceleration (red), jerk or statefinder (green), kerk (blue) and lerk (brown) parameters. The thick lines represent the interacting and the dotted lines represent the non-interacting situations in the intermediate scenario.

Fig. 6b shows the r-s trajectory under interaction and non-interaction. The trajectories have coincided. We have taken the cosmic time $t \in [0, 4]$. The $r(s)$ trajectory has passed through the fixed point $\{r = 1, s = 0\}$ of the ΛCDM. We have taken $n = 2$, $\lambda = 1.9$, $\beta = 0.5$, $\delta = 0.05$ and $m_p^2 = 1$.

In figure 6a we have plotted the various parameters characterizing the accelerating universe. In this case also we are getting an ever accelerating universe as suggested by the negative deceleration parameter. However, there is a difference between its behavior in the case of emergent and in the intermediate scenario. In the emergent scenario we have seen the deceleration parameter to increase throughout the evolution of the universe. However, in the intermediate scenario we find it to decrease in the early epoch and then to increase in the later stages. This suggests that in this scenario, the acceleration is increasing fast and then decreasing gradually. This behavior has been observed in the interacting ($\delta = 0.05$) as well as non-interacting ($\delta = 0$) situations. The statefinder or jerk and the lerk parameters are continuously decreasing throughout the evolution of the universe and are always staying at the positive level. The kerk parameter transiting from positive to negative level and then showing asymptotic behavior.

The statefinder diagnostics has been presented through $r(s)$ trajectory in figure 6b. Like emergent scenario we are getting the radiation and the ΛCDM in this trajectory. However, in the fourth quadrant of the universe we are seeing that $r$ is finite with $s \to -\infty$. This indicates the dust phase of the universe. This could not be derived in the emergent scenario. In this way, the fate of the universe in the intermediate scenario differs from emergent scenario when we are considering the new agegraphic dark energy based on generalized uncertainty principle interacting with dark matter. The $r(s)$ curve for the non-interacting case has coincided with the trajectory in the interacting case. Here also, the dark energy model proposed in the present paper differs from the interacting new agegraphic dark energy model proposed by Zhang et al (2010) where the $r(s)$ curve got the endpoint at ΛCDM. When we are considering new agegraphic dark energy based on generalized uncertainty principle in the intermediate scenario we can get the transition from radiation to ΛCDM stage through dust stage.

\section*{V. INTERACTION IN THE LOGAMEDIATE SCENARIO}

In this section we have discussed the interacting dark energy under consideration in the logamediate scenario. For this purpose, using $a(t) = \exp(\mu(\ln t)^\alpha)$ in (10) we get the conformal time as

$$
\eta = \int \frac{dt}{\exp(\mu(\ln t)^\alpha)}
$$

(28)
Fig. 7 shows the evolution of the equation of state parameter $w_{\text{total}}$ in the interacting (thick line) and non-interacting (dashed line) situations in the logamediate scenario.

Fig. 8 shows the total energy density $\rho_{\text{total}}$ and pressure $p_{\text{total}}$ in the interacting (thick line) and non-interacting (dashed line) situations in the logamediate universe scenario.

In figure 9a we find that the deceleration parameter is staying at negative level throughout the evolution of the universe. This means accelerated expansion of the universe. Moreover the acceleration is increasing with cosmic time $t$. Similar behavior has been observed in interacting ($\delta = 0.05$) and non-interacting ($\delta = 0$) situations.
The jerk or statefinder parameter is transiting from negative to positive sign. The jerk parameter is staying at negative level. The jerk parameter is behaving similarly to that of the jerk or statefinder parameter. This parameter is transiting from negative to positive sign with the evolution of the universe. Behaviors of the parameters are similar in the cases of interaction \((\delta = 0.05)\) and non-interaction \((\delta = 0)\).

The statefinder diagnostics has been presented through \(r-s\) trajectory in figure 9b. Like emergent and intermediate scenarios we are getting the radiation \((r \text{ and } s \text{ positive})\) and the ΛCDM i.e. \(\{r = 1, s = 0\}\) in this trajectory. Moreover, Like emergent scenario, in the second quadrant of the \(s - r\) plane we are seeing that \(s\) is finite (negative) with \(r \to \infty\). There is one characteristic of the \(r(s)\) curve that differs from both emergent and intermediate scenarios. At the point \(\{s = 0.65, r = 0\}\) there is a loop that makes \(r \to -\infty\) with finite \(s\) (positive). Here also, the dark energy model proposed in the present paper differs from the interacting new agegraphic dark energy model proposed by Zhang et al (2010).

VI. DISCUSSIONS

In the present work, we have considered the interaction between the dark energy based on generalized uncertainty principle and dark matter. We have considered three scenarios, namely, emergent, intermediate and logamediate scenarios. In figure 1 we have plotted the equation of state parameter for the interacting dark energy against cosmic time \(t\). This figure shows that the equation of state parameter \(w_{\text{total}}\) in both interacting \((\delta \neq 0)\) and non-interacting \((\delta = 0)\) situations stays below \(-1\) and then after a certain cosmic time it tends to \(-1\) (never crosses \(-1\) barrier), which shows phantom like behavior. Figure 2 shows that the total energy density \(\rho_{\text{total}}\) is increasing with time. From the choice of the interaction term it may be stated that dark matter is getting transferred to dark energy. Under this situation, the total energy density \(\rho_{\text{total}}\) is increasing with evolution of the universe under emergent scenario. It also shows that the total pressure \(p_{\text{total}}\) is increasing in negative direction with cosmic time \(t\) under this scenario. It is apparent from the figure that the evolution of the \(\rho_{\text{total}}\) and \(p_{\text{total}}\) with cosmic time \(t\) in the case of non-interaction are quite similar to that in the case of interaction.

In figure 3a we have plotted the deceleration parameter \(q\) against cosmic time \(t\) under this interaction. We find that it stays at negative level in interacting as well as non-interacting situations. This is consistent with the ever accelerating nature of the emergent universe. Also we find that \(q\) is increasing with time. This indicates the acceleration of the universe is decreasing as we move towards late stage of the universe.

In figure 4, where we have considered the universe under intermediate scenario, the equation of state parameter \((\delta \neq 0)\) \(w_{\text{total}}\) is below \(-1\) that indicates phantom-like behavior. However, like emergent scenario it is tending to \(-1\) with evolution of the universe. However, it never crosses the boundary of \(-1\). Similar pattern is available in the case of non-interaction \((\delta = 0)\) situation. From figure 5, we find the similar behavior of \(\rho_{\text{total}}\) and \(p_{\text{total}}\) to that of emergent scenario. In figure 6a we find that the deceleration parameter \(q\) is negative throughout the evolution of the universe characterized by intermediate scenario. However, we observe that after a decay up to a certain time, it starts moving upwards with the evolution of the universe. This indicates that although the universe is ever accelerating under intermediate scenario, the acceleration itself decreases as we reach late stage of the universe. However, in the early stage of universe the acceleration increased with time.

Considering the logamediate scenario, we found from figure 7 that the equation of parameter \(w_{\text{total}} \geq -1\) that indicates quintessence like behavior. From figure 8 we understood that the \(\rho_{\text{total}}\) is decaying and \(p_{\text{total}}\) is decreasing in the negative direction. Similar patterns are found in interacting \((\delta = 0.05)\) and non-interacting \((\delta = 0)\) situations. Like the emergent and intermediate scenarios the deceleration parameter \(q\) stays at negative level and the acceleration decreases with time (figure 9a).

We have also investigated the statefinder diagnostics in figures 3b, 6b and 9b. We found that the two-component dark energy model consisting of new agegraphic dark energy based on the generalized uncertainty principle and dark matter generated \(r(s)\) trajectory passing through \(\{r = 1, s = 0\}\) i.e. ΛCDM irrespective of the scenario of the universe and interaction. In all of the cases the radiation phase is achieved. However, only in the case of intermediate scenario the dust phase was obtained. In all the cases we could go beyond ΛCDM.
contrary to what obtained by Zhang et al (2010) for interacting new agegraphic dark energy model. Finally, it may be conclude that the new agegraphic dark energy based on generalized uncertainty principle coexisting with dark matter behaves like quintessence era for logamediate expansion and phantom era for emergent and intermediate expansions of the universe.

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