Maximum Likelihood Identification of Cavitation Instabilities in Axial Inducers

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Abstract. The article illustrates the application of maximum likelihood estimation to the identification of cavitation instabilities in axial inducers from the unsteady pressure readings measured on the impeller casing. The typical triangular pressure distribution in the blade channels of the impeller is parametrized and modulated in time and space in order to theoretically reproduce the expected pressure generated by known forms of cavitation instabilities (cavitation auto-oscillations, n-lobed sub/super-synchronous rotating cavitation, higher-order surge/rotating cavitation modes). The Fourier spectra of the theoretical pressure so obtained in the rotating frame are transformed in the stationary frame and fitted by maximum likelihood estimation to the auto-correlation of the pressure measurements on the inducer casing. Each form of instability generates a characteristic spectral distribution of side bands in addition to its fundamental frequency. The identification makes use of this information for effective discrimination of simultaneous flow oscillations with significantly different intensities and partially overlapping frequencies. The method returns the estimates of the model parameters and their standard errors, allowing for both recognition of the forms of instabilities occurring in the inducer and assessment of the statistical significance of the results.

1. Introduction

Cavitation represents the main fluid dynamic phenomenon limiting the power density of hydraulic turbomachinery. For this reason, in the quest for optimum weight efficiency, liquid propellant rocket engine turbopumps are usually designed for operation under partial cavitation conditions and are therefore subject to the onset of cavitation-induced flow instabilities ([1] Natanzon et al., 1974; [13] Kamjio et al., 1977; [10] Hashimoto et al., 1997a; [11] Hashimoto et al., 1997b; [26] Shimura et al., 2002; [27] Shimura et al., 2003; [28] Tsujimoto et al., 1997; [4] Coutier-Delgosha et al., 2012; [2] Brennen, 2012; [5] d’Agostino, 2013; [20] Pace et al., 2015; [15] Lettieri et al. 2017). Historically, cavitation-auto-oscillations, partial cavitation instabilities, sub/super-synchronous rotating cavitation and, more recently, higher-order surge and rotating cavitation modes have been recognized as the origin of potentially catastrophic failures, ranging from the onset of fluid/structure POGO oscillations of the propulsion system ([25] Rubin, 1966; [19] Oppenheim and Rubin 1993, [14] Larsen, 2008; [12] Hori and Brennen, 2011), to the dynamic excitation of the impeller blades and the rotor shaft ([1] Bhattacharyya et al., 1997; [9] Goirand et al., 1992; [23] Pasini et al., 2011), and finally to the development of coupled blade flutter/cavitation oscillations ([16] NASA 2000a; [17] NASA, 2000b).
Effective identification, discrimination and characterization of unsteady cavitation phenomena is therefore a crucial aspect in the development of high-performance liquid propellant rocket propulsion systems for space applications and represents the main focus of the present work. Spectral analysis of the unsteady flow pressure measured in the stationary (laboratory) or, more recently, in the rotating (impeller) frames have traditionally been used to this purpose ([29] Tsujimoto, 2006; [21] Pace et al., 2017; [22] Pace et al., 2019). However, visual interpretation of the measured pressure spectra according to this approach is primarily based on consideration of what appear to be the fundamental frequencies of the flow oscillations. It becomes therefore increasingly difficult and uncertain when several instabilities are present with different intensities, significant side-bands and possibly overlapping fundamental frequencies, especially for measurements with low signal-to-noise ratios.

The present activity aims at demonstrating the possibility of effectively using maximum likelihood identification to reduce the impact of these limitations by improving the dynamic range and resolution of the spectral analysis of flow instabilities and eliminating its dependence on the subjective interpretation by the observer. Specifically, this article illustrates the results of the application of maximum likelihood estimation to the systematic discrimination of the various forms of cavitation instabilities occurring in inducers and turbopumps from the unsteady pressure readings measured on the impeller casing just downstream of the leading edges. The typical triangular pressure distribution in the blade channels is parametrized and modulated in time and space in order to model the expected pressure generated by known forms of cavitation instabilities (cavitation auto-oscillations, n-lobed sub/super-synchronous rotating cavitation, higher-order surge/rotating cavitation modes). The discrete Fourier spectrum so obtained in the rotating frame is transformed in the stationary frame and filtered to account for frequency broadening effects. Following the classical maximum likelihood approach in the presence of Gaussian errors, the predicted auto-correlation of the flow pressure is fitted to the measurements by minimizing its quadratic deviation from the experimental results. The identification makes use of the information provided by both the fundamental frequency of each instability as well as its side-band spectrum, allowing for increased sensitivity and resolution of unsteady flow phenomena even when their fundamental frequencies overlap. The method returns the estimates of the model parameters as well as their standard errors, which provide the necessary information for effective diagnostic identification and characterization of the instabilities and for quantitative assessment of the statistical significance and accuracy of the results.

2. Experimental Apparatus and Procedure

The three-bladed RAPDUD inducer (Figure 1) used in the present activity, whose main characteristics are illustrated in Table 1 and more extensively in [6] d’Agostino, 2017, has been designed as proposed by some of the authors ([7] d’Agostino et al., 2008a; [8] d’Agostino et al., 2008b) and tested in a series of cavitation experiments ([22] Pace et al., 2019) in the Cavitating Pump Rotodynamic Test Facility (CPRTF) ([3] Cervone et al., 2007). Its pumping and suction performance are shown in Figure 2. The unsteady pressure in the blade channels has been measured by PCB S112A22 piezoelectric transducers (0 to 345 kPa pressure range, 7 Pa resolution, 14.5 mV/kPa sensitivity, –73 to +135 °C temperature range) flush-mounted on the inducer casing as shown in Figure 1.

Table 1. Characteristics of the RAPDUD inducer.

| Parameter                | Value          |
|--------------------------|----------------|
| Design flow coefficient, $\Phi_D$ | 0.070          |
| No. of blades, $N$       | 3              |
| Specific speed, $\Omega_S$ | 1.70           |
| Rotational speed, $\Omega$ | 1500 rpm       |
| Tip solidity, $\sigma_t$ | 2.28           |
| Diffusion factor, $D_F$  | 0.25           |
| Parameter                                      | Value   |
|-----------------------------------------------|---------|
| Tip radius, \( r_T \)                        | 81 mm   |
| Inlet hub radius, \( r_{HI} \)               | 35 mm   |
| Inlet hub radius (fully developed blade), \( r_{Hf} \) | 45 mm   |
| Inlet tip blade angle, \( \gamma_{Hf} \)     | 82.8°   |
| Flow incidence angle at blade leading edge tip, \( \alpha_{He} \) | 1.42°   |
| Flow incidence-to-blade angle ratio at blade leading edge tip, \( \alpha_{He}/\beta_{He} \) | 0.197   |
| Axial length (fully developed blade), \( L_a \) | 69 mm   |
| Axial length, \( L \)                        | 90 mm   |
| Outlet hub radius, \( r_{He} \)             | 58.5 mm |
| Outlet tip blade angle, \( \gamma_{He} \)    | 79.65°  |

**Figure 1.** The RAPDUD inducer (left) mounted in the CPRTF test section (right) with the positions of the statoric pressure taps ([22] Pace et al., 2019). The measurements from the second series of transducers mounted just downstream of the full-blade cross-section have been used in the present identification.

**Figure 2.** Pumping (left) and suction characteristics (right) of the RAPDUD inducer for operation in water at \( T = 20 \, ^\circ C \) and \( \Omega = 3000 \, \text{rpm} \) ([22] Pace et al., 2019).

The pressures \( p(\vartheta', t) \) and \( p(\vartheta, t) \) in the rotating and stationary frames are represented in \( \vartheta' \) and \( \vartheta \) by Fourier series with discrete wavenumber spectra \( \tilde{p}'_m(t) \) and \( \tilde{p}_m(t) \):

\[
p(\vartheta', t) = \frac{1}{2\pi} \sum_{m=-\infty}^{\infty} \tilde{p}'_m(t) e^{im\vartheta'} \quad \Leftrightarrow \quad \tilde{p}'_m(t) = \int_0^\pi p(\vartheta', t) e^{-im\vartheta'} \, d\vartheta'
\]

\[
p(\vartheta, t) = \frac{1}{2\pi} \sum_{m=-\infty}^{\infty} \tilde{p}_m(t) e^{im\vartheta} \quad \Leftrightarrow \quad \tilde{p}_m(t) = \int_0^\pi p(\vartheta, t) e^{-im\vartheta} \, d\vartheta
\]
and in time $t$ by continuous frequency spectra $\hat{p}(i\varphi', \omega)$ and $\hat{p}(i\varphi, \omega)$:

$$p(\varphi', t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \hat{p}(\varphi', \omega) e^{i\omega t} d\omega \Leftrightarrow \hat{p}(\varphi', \omega) = \int_{-\infty}^{+\infty} p(\varphi', t) e^{-i\omega t} dt$$

$$p(\varphi, t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \hat{p}(\varphi, \omega) e^{i\omega t} d\omega \Leftrightarrow \hat{p}(\varphi, \omega) = \int_{-\infty}^{+\infty} p(\varphi, t) e^{-i\omega t} dt$$

Hence, from the above expressions of $p(\varphi', t)$ and $p(\varphi, t)$, interchanging summation and integration:

$$\hat{p}(\varphi', \omega) = \frac{1}{2\pi} \sum_{m=-\infty}^{\infty} e^{im\varphi'} \int_{-\infty}^{+\infty} \hat{p}_m(t) e^{-i\omega t} dt$$

$$\hat{p}(\varphi, \omega) = \frac{1}{2\pi} \sum_{m=-\infty}^{\infty} e^{im\varphi} \int_{-\infty}^{+\infty} \hat{p}_m(t) e^{-i\omega t} dt$$

Besides, the azimuthal coordinates in the rotating and stationary frames are related by the transformation:

$$\varphi' = \varphi - \Omega t \Rightarrow p(\varphi', t) = p(\varphi - \Omega t, t) = p(\varphi, t)$$

where $\Omega$ is the rotational speed of the impeller. Hence, from earlier representations of $p(\varphi', t)$ and $p(\varphi, t)$:

$$\hat{p}_m(t) = \hat{p}_m(t) e^{-im\Omega t}$$

Consideration of the unsteady pressure measured by the PCB transducers flush-mounted on the inducer casing 10.7 mm downstream of the blade leading edge tips of the RAPDUD inducer (Figure 3, on the left) confirms that the expected azimuthal profile of the pressure $p_j(\varphi', t)$ generated by the $j$-th blade channel of azimuthal spacing $\theta = 2\pi/N$ can be modeled as a sequence of $N$ triangular pressure waves, whose shape is illustrated on the right in Figure 3.

**Figure 3.** Typical pressure history (left) measured in the blade channels of the RAPDUD inducer (blue line) and its low-pass filtered value (red line). Triangular pressure profile assumed in the blade channels for the development of the parametric model (right).

Axial surge cavitation with frequency $\omega_{SC}$ and rotating cavitation with $N_{RC}$ lobes and angular speed $\omega'_{RC}$ in the relative frame are then modeled by respectively modulating the peak-to-peak amplitude $P_j(t)$ of the pressure signal as:

$$P_j(t) = \Delta P_{SC} \cos(\omega_{SC} t + \varphi_{SC})$$

and:

$$P_j(t) = \Delta P_{RC} \cos[N_{RC}(i\varphi' - \omega_{RC} t) + \varphi_{RC}]$$

where $\varphi_{SC}$ and $\varphi_{RC}$ are the relevant phase angles. With these positions the resulting surge cavitation pressure spectrum in the absolute frame is:

$$\hat{p}_{SC}(\varphi, \omega) = \sum_{m=-\infty}^{\infty} a_{mSC} \delta(\omega + m\Omega - \omega_{SC}) + \sum_{m=-\infty}^{\infty} b_{mSC} \delta(\omega + m\Omega + \omega_{SC})$$

with:
$a_{mRC}, b_{mRC} = \begin{cases} 
\Delta P_{SC} \frac{\pi}{2} e^{i\varphi_{SC}} & \text{for } m = 0 \\
\Delta P_{SC} \frac{1}{2} e^{i\varphi_{SC}} \sum_{j=1}^{N} e^{i(\omega - \vartheta_j)} \left( e^{i\omega_{m}} - 1 \right) \theta + \left( 1 - e^{i\omega_{m}} \right) \theta \frac{\Delta \sigma}{\sigma^2} \left( \theta - \varphi_j \right) & \text{for } m \neq 0 
\end{cases}$

and, correspondingly, for rotating cavitation:

$\hat{p}_{RC}(\vartheta, \omega) = \sum_{m=-\infty}^{\infty} a_{mRC} \delta(\omega + m\Omega + N_{RC} a_{mRC}') + \sum_{m=-\infty}^{\infty} b_{mRC} \delta(\omega + m\Omega - N_{RC} a_{mRC}')$

with:

$a_{mRC}, b_{mRC} = \begin{cases} 
\Delta P_{RC} \frac{\pi}{2} e^{i (N_{RC} \vartheta_{RC}' + \varphi_{RC})} & \text{for } m = 0 \\
\Delta P_{RC} \frac{1}{2} e^{i (N_{RC} \vartheta_{RC}' + \varphi_{RC})} \sum_{j=1}^{N} e^{i(\omega - \vartheta_j)} \left( e^{i\omega_{m}} - 1 \right) \theta + \left( 1 - e^{i\omega_{m}} \right) \theta \frac{\Delta \sigma}{\sigma^2} \left( \theta - \varphi_j \right) & \text{for } m \neq 0 
\end{cases}$

where $m$ is integer and $\delta(\omega - \omega_0)$ is the Dirac function centered at $\omega = \omega_0$. Each form of instability is characterized by a distinctive pattern of side-bands above the fundamental frequency, which suggests the possibility of effectively identifying multiple, possibly overlapping, flow oscillation modes and discriminating their most intense side-bands from other forms of flow oscillations at higher frequencies.

For efficient fitting to the experimental data the delta functions of the theoretical spectra must be transformed in peaks of finite amplitude and bandwidth $\Delta \omega$. Frequency broadening of the experimental spectra generally occurs as a consequence of:

- the intrinsically noisy nature of the phenomena observed, here assumed to be Gaussian-distributed around the relevant means with unknown standard error $\sigma$;
- the evaluation of the experimental spectra as the averages of the spectra obtained by partitioning the sampling time in four equal time intervals;
- windowing of the sampled data ($\Delta \omega \sim 1/\Delta t_{\text{sampling}}$);
- measurement errors (repeatibility and A/D discretization, $\Delta \omega \sim 0.1$ rad/s);
- fluctuations $\Delta \Omega$ of the impeller rotational speed $\Omega$ and/or, equivalently, manufacturing imperfections of the impeller blading ($\Delta \Omega/\Omega \sim 0.1\%$ to 1%);

the first two effects being the dominant ones in present experiments. In order to predictively account for frequency broadening, the arguments of the delta functions in the theoretical pressure spectra are therefore considered as the sum $y = x_1 + x_2 + \cdots + x_n = n\bar{x}$ of $n$ terms Gaussian-distributed with equal mean $\mu$ and standard deviation $\sigma$, so that the variable:

$t = \frac{\bar{x} - \mu}{s/\sqrt{n}} = \frac{n\bar{x} - n\mu}{s/\sqrt{n}} \text{ where } s^2 = \frac{n}{n-1} \sigma^2$

follows the Student's $t$-distribution with $n-1$ degrees of freedom. Consequently, the theoretical delta functions are transformed in finite bandwidth peaks by convolution with a Gaussian of standard deviation $\sigma$ when $n=1$ and with a Student's $t$-distribution with $n-1$ degrees of freedom when $n>1$.

The theoretical pressure spectra are therefore expressed as explicit functions of a finite number of free parameters, namely: $\vartheta_j$, $\Delta P_{SC}$, $\omega_{SC}$, $\Delta P_{RC}$, $\omega_{RC}$, $N_{RC}$ and $\sigma$. Finally, the set of parameters describing a specific combination of cavitation-induced flow instabilities is determined by best fitting the theoretical pressure power density spectra $S_{th}(\omega, \mathbf{a})$ to the experimental ones $S_{exp}(\omega)$. Gaussian maximum likelihood estimation is used to this purpose, by numerically minimizing in the multidimensional space of the model parameters $\mathbf{a}$ the sum of the quadratic deviations of the predicted and measured spectra for all of the $k_s$ values of the discretized frequency:
Experience shows that the convergence of the iterative minimization of $\chi^2(a)$ to the instabilities of interest is critically dependent on the initialization of the unstable frequency. In order to address this problem, the identification of a specific form of instability of fundamental frequency $\omega_{in}$ is carried out in two steps:

- **first the correlation coefficient:**
  
  $$r(\omega_{in}) = \frac{\sum_{k=1}^{k_s} S_{\text{exp}}(\omega_k, \omega_{in}) \left[ \sum_{k=1}^{k_s} S_{\text{exp}}^2(\omega_k) \right]^{-1/2}}{\left[ \sum_{k=1}^{k_s} S_{\text{th}}^2(\omega_k, \omega_{in}) \right]^{-1/2}}$$

  between the unconvolved theoretical power spectrum of that instability $S_{\text{th}}(\omega, \omega_{in})$ and the experimental one $S_{\text{exp}}(\omega)$ is computed;

- **then simultaneous parametric identification of steady cavitation, axial surge cavitation and rotating cavitation is carried out using the frequency-broadened theoretical spectrum at the statistically significant value of the rotating cavitation frequency $\omega_{in}$ corresponding to the statistically significant largest values of $r(\omega_{in})$.

The accuracy of the results is finally assessed from the standard errors of the estimated parameters returned by the maximum likelihood method.

### 3. Results and Discussion

The experiments carried out in the CPRTF generally indicate that the main forms of cavitation instabilities in the RAPDUD inducer are ([6] d’Agostino, 2017; [22] Pace et al., 2019; [24] Pasini et al., 2019):

- **rotating cavitation modes** (indicated as RC# in Figure 4) with one or more co-rotating and/or counter-rotating lobes, arising when the suction performance curve starts declining;

- **low frequency** (subsynchronous) cavitation auto-oscillation surge modes (indicated as A# in Figure 4), also arising when approaching breakdown conditions;

- **higher-order cavitation surge modes**, occurring at super-synchronous frequencies over a wider range of cavitation numbers.

![Figure 4. Reduced frequency $\omega/\Omega$ of the pressure power spectrum $S_{\text{exp}}(\omega)$ measured in the RAPDUD inducer for operation at design flow $\Phi = 0.0703$ and $\Omega = 3000$ rpm in water at $T = 20 ^\circ \text{C}$, plotted as a function of the cavitation number $\sigma$ (left, [22] Pace et al., 2019). Different colors indicate the intensity in Pa (left) and phase in deg (right) of the pressure spectra.](image)
design flow, when flow instabilities are relatively less intense. Experimental evidence confirmed the performance of the estimation method also at off-design conditions.

Figure 5. Schematic of the axial stations \( f_a, f_b \) and \( f_c \) and casing taps of the pressure transducers mounted on the test inducer (left). Suction performance of the RAPDUD inducer (right) for operation at design flow \( \Phi = 0.0703 \) and \( \Omega = 3000 \) rpm in water at \( T = 20 \) °C, plotted as a function of the cavitation number \( \sigma \) ([22] Pace et al., 2019) and illustrating the effect of the onset of rotating cavitation for \( \sigma < 0.70 \).

Figure 4 illustrates the amplitude (left) in Pa of the power spectrum \( S_{\text{exp}}(\omega) \) and the phase (right) in deg of the cross-power spectrum of the pressure measured by transducers 1 and 2 mounted on station \( f_a \) upstream in the impeller eye (on the left in Figure 5). They are plotted as functions of the cavitation number \( \sigma \) and reduced frequency \( \omega / \Omega \) for inducer operation in water at \( T = 20 \) °C, rotational speed \( \Omega = 3000 \) rpm and design flow coefficient \( \Phi = 0.0703 \). Consideration of the cross-correlation phase and azimuthal separation of the sensors indicated the occurrence of the axial flow oscillation and the single-lobe rotating cavitation instabilities denoted in the figure as A1, R2, R3 and R4, respectively, whose effects on the suction performance of the inducer when the cavitation number drops below \( \sigma = 0.70 \) are illustrated in Figure 5.

Figure 6. Typical autocorrelation pressure spectrum predicted by the model for rotating cavitation with reduced frequency \( \omega_{\text{RC}} / \Omega = 0.686 \) (left). Correlation coefficient \( r(\omega / \Omega) \) of the experimental power spectrum \( S_{\text{exp}}(\omega) \) measured in the test inducer operating at \( \sigma = 0.0505 \) with single-lobe rotating cavitation as a function of the reduced frequency \( \omega / \Omega \) in the relative frame (right).

Figure 6 on the left shows as an example the theoretical spectrum generated by rotating cavitation (index RC) for \( \omega_{\text{RC}} / \Omega = 0.686 \). Similar plots are predicted for steady cavitation (index C) and surge cavitation (index SC). The correlation coefficient \( r(\omega / \Omega) \) of the theoretical and experimental spectra for single-lobe rotating cavitation at \( \sigma = 0.0505 \) and angular speed \( \omega' = \omega'_{\text{RC}} \) in the relative frame is shown on the right in the same figure as a function of the reduced frequency \( \omega' / \Omega \). It confirms the occurrence of a single-lobed rotating cavitation mode (RC2) with a statistically significant value of \( r > 0.7 \) at a reduced frequency \( \omega'_{\text{RC}} / \Omega = -0.21 \).
Figure 7. Comparison between the experimental power spectrum \( S_{\exp}(\omega) \) (black line) of the pressure fluctuations, measured in the RAPDUD inducer for operation at design flow \( \Phi = 0.0703 \) in water at \( T = 20 \, ^\circ \text{C} \), \( \Omega = 3000 \, \text{rpm} \), \( \sigma = 0.505 \), and the theoretical power spectrum \( S_{\text{th}}(\omega) \) (green line) predicted by the model for single-lobe rotating cavitation with frequency \( \omega_{\text{RC}}/\Omega = -0.21 \) in the relative frame. Statistically significant spectral peaks not identified by the assumed mode of flow instability are circled in red.

Parametric identification of the simultaneous occurrence of steady cavitation, surge cavitation and single-lobe rotating cavitation at \( \omega_{\text{RC}}/\Omega = -0.21 \), corresponding to the highest value of \( r(\omega) \), yielded the comparison illustrated in Figure 7 between the experimental and theoretical spectra. Visual examination qualitatively indicates that close agreement is obtained between the experimental and predicted power spectra of the pressure fluctuations in the test inducer. Similar results, not shown here for conciseness, are also obtained at off-design conditions corresponding to 80% of the nominal flow rate.

Table 2. Relative standard errors of the identified parameters of steady cavitation, surge cavitation and single-lobe rotating cavitation with reduced frequency \( \omega_{\text{RC}}/\Omega \) in the RAPDUD inducer operating in water at \( T = 20 \, ^\circ \text{C} \), \( \sigma = 0.505 \), \( \Omega = 3000 \, \text{rpm} \) and \( \Phi = 80\% \) and 100% of the design value \( \Phi_D = 0.0703 \).

| Parameter | \( \omega_{\text{RC}}/\Omega = -0.21 \) | \( \omega_{\text{RC}}/\Omega = -0.19 \) | \( \omega_{\text{RC}}/\Omega = 1.21 \) |
|-----------|----------------|----------------|----------------|
| \( P_C \) | 3.4% \( \Phi/\Phi_D = 1.0 \) | 2.2% \( \Phi/\Phi_D = 0.8 \) | 5.4% \( \Phi/\Phi_D = 1.0 \) |
| \( \Delta P_{\text{SC}} \) | 9.2% | 11.2% | 13.8% |
| \( \omega_{\text{SC}} \) | 0.50% | 0.48% | 0.89% |
| \( \Delta P_{\text{RC}} \) | 1.0% | 1.1% | 2.0% |
| \( \omega_{\text{RC}} \) | 0.05% | 0.06% | 0.02% |
| \( \sigma \) | 1.4% | 1.5% | 2.8% |

The relative errors associated to the estimation of the relevant model parameters (\( P_C \), \( \Delta P_{\text{SC}} \), \( \omega_{\text{SC}} \), \( \Delta P_{\text{RC}} \), \( \omega_{\text{RC}} \) and \( \sigma \)) so obtained at 100% and 80% of the design flow are listed in the first two columns of Table 2. Their moderate values confirm also quantitatively the accuracy of the identifications.
Figure 7 also displays a number of relatively intense spectral lines (highlighted in red along the frequency axis) not captured by the model predictions for rotating cavitation at $\omega'_{RC}/\Omega = -0.21$, thus suggesting the presence of other flow instabilities in the test inducer. The second largest statistically significant maximum ($r = 0.65$) of the correlation coefficient plot indicates that an additional single-lobe rotating cavitation mode may actually occur at $\omega'_{RC}/\Omega = 1.21$, corresponding to the instability indicated as RC4 in Figure 4. Consideration of this instability in the model leads to the comparison of the experimental and theoretical spectra illustrated in Figure 8. The relative errors in the estimation of the relevant parameters of the model are reported in the last column of Table 2. Although slightly less accurate that for the first mode of rotating cavitation at $\omega'_{RC}/\Omega = -0.21$ owing to the lower intensity of this second instability, also in this case the model predictions captures a number of additional frequency bands of the experimental spectrum, demonstrating the capability of the method of accurately identifying multiple overlapping instability modes.

![Figure 8](image_url)

**Figure 8.** Comparison between the experimental power spectrum $S_{exp}(\omega)$ (black line) of the pressure fluctuations, measured in the RAPDUD inducer for operation at design flow $\Phi = 0.0703$ in water at $T = 20$ °C, $\Omega = 3000$ rpm, $\sigma = 0.505$, and the theoretical power spectrum $S_{th}(\omega)$ (green line) predicted by the model including single-lobe rotating cavitation modes with reduced frequency $\omega'_{RC}/\Omega = 1.21$ in the relative frame.

4. Conclusions

The proposed maximum likelihood approach proved capable of using statoric unsteady pressure measurements for efficiently identifying and characterizing multiple cavitation-induced axial/azimuthal flow instabilities simultaneously occurring at design and off-design operation and subsynchronous or supersynchronous frequencies in a typical high-head inducer for space applications. The theoretical spectra obtained from the parametrization of the relevant flow phenomena display characteristic patterns, with the presence of significant side-band contributions in addition to the fundamental frequency. The prediction of these spectra yields the following distinct advantages:

- provides the information needed for improving the visual interpretation of the pressure spectra from characterization experiments of cavitating inducers and, in particular, for eliminating the possibility of misinterpreting relatively intense side-bands of already identified phenomena as additional distinct instabilities;
- provides a very powerful means for effectively detecting and discriminating different flow oscillation modes even in measurements with relatively low signal-to-noise ratio;
- allows for the characterization of the nature, spatial structure and rotational speed of multi-lobed azimuthal flow instabilities using just one single sensor, eliminating the need for cross-correlating...
the signals from multiple transducers and dealing with aliasing limitations, thus significantly simplifying the acquisition and reduction of the experimental data.

Finally, if successfully extended and validated to the analysis of complex Fourier spectra (capable of retaining the phase information of the pressure signals), the present estimation method would open the possibility of providing diagnostic information on possible geometric imperfections of the impeller blades even before cavitation occurs in the machine.

In the authors’ opinion the proposed approach represents therefore a promising tool in high-performance cavitating turbopump research.

5. References

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6. Nomenclature

\( \rho, \rho_t \) static and total pressures

\( \rho_i, \rho_v \) inlet and vapor pressures

\( r_t \) tip radius

\( t \) time

\( T \) temperature

\( \dot{V} \) volumetric flow rate

\( \vartheta, \vartheta' \) absolute and relative azimuthal coordinates

\( \rho \) flow density

\( \sigma \) cavitation no., \( \sigma = (\rho_1 - \rho_v)/\rho \Omega^2 r_t^2 \), standard deviation
Φ flow coefficient, $\Phi = \dot{V}/\pi \Omega r_T^3$

Ψ head coefficient, $\Psi = \Delta p_T/\rho \Omega^2 r_T^2$

ω angular frequency

Ω rotational speed

$\Omega_S$ specific speed, $\Omega_S = \Omega \lambda^{1/2} (\Delta p_T/\rho)^{-3/4}$

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