Heat transfer in ice hockey halls: measurements, energy analysis and analytical ice pad temperature profile.

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Abstract

We consider heat transfer processes in an ice hockey hall, during operating conditions, with a bottom-up approach based upon on-site measurements. Detailed temperature data of both the ice pad and the air above the ice rink are used for a heat balance calculation in the steady-state regime, which quantifies the impact of each single heat source. We solve the heat equation in the ice slab in transient regime, and obtain a general analytical formula for the temperature profile. This solution is then applied to the resurfacing process by using our measurements as (time-dependent) boundary conditions (b.c.), and compared to an analogous numerical computation with good agreement. Our analytical formula is given with implicit initial condition and b.c., therefore it can be used not only in ice halls, but in a large variety of engineering applications.

Keywords: transient heat conduction, cooling, theoretical models, analytical solutions, ice rinks, energy efficiency, energy saving, eigenfunctions expansions

1. Introduction

Ice hockey arenas constitute a challenging setup for studies in building physics, due to the diverse physical processes, often concurring to each other, which take place in the environment. For instance, ventilation and air conditioning in such large and complex systems are an intriguing field of study.

Even more importantly, these structures consume a very large amount of energy, approaching ~ 1800MWh per year. In particular, refrigeration is responsible for nearly 43 percent of the total energy consumption of the ice hall [1, 2, 3, 4]. Process optimization and energy saving have therefore become key concepts in research in the latest years.

Heat and mass transfer processes take indeed a relevant part in the energy balance inside an ice hall, as they occur both above and inside the large ice/concrete slab forming the ice rink. A comprehensive literature about this
topic has been recently produced; thermal processes such as radiation from the
ceiling and heat transfer between the ice surface and the refrigeration system
have been thoroughly studied, see e.g. [5, 6, 7].

In particular, most studies focus on larger scales and rely importantly on
simulations and/or numerical methods [3, 8, 9, 10, 11], with a few exceptions
[12]. Here we address the topic on a smaller scale, i.e. an ice pad element, using
a bottom-up approach that comprises both accurate experimental data and the
derivation of an analytical temperature profile in the ice slab. Moreover, we
choose the resurfacing stages because they are the most complex and energy
consuming phases of the operational cycle of an ice hockey hall, due to the large
amount of heat transferred to the ice pad in a relatively short time. This affects
importantly the efficiency and energy consumption of the refrigerating system,
as discussed e.g. in [1].

The present article is aimed to finding applicable quantitative knowledge,
which can be used in the actual efforts towards energy costs reduction. In our
paper [1], we investigated the refrigerating system and studied energy consump-
tion and optimization possibilities. Through an analytical calculation, we found
the optimal configuration for a maximal coefficient of performance (COP). Here
we focus on a smaller scale, and consider only the heat transfer processes which
affect the ice pad temperature.

Though obtained for a specific case-study, our measurements can be applied
to most of ice hockey halls, for instance in the development of new control
methods for ice rink cooling systems [13, 14, 15, 16]. Our findings also help in
developing methods to collect a portion of the heat generated (see Eq. (2)) and
reuse it in the ice hockey hall. Moreover, the temperature profile formula (54)
is very general and can be applied to a whole class of heat conduction processes.

The present paper is organized as follows:

Section 2 addresses the experimental setup and measurements. We describe
the ice hall, the ice/concrete slab and the different phases of resurfacing. Our
indoor air data provide a stratification mapping at several heights from the
ice rink, from 4cm to 8m above the ice, together with temperature and heat
flux measurements at the ice/concrete interface. An according energy balance
analysis is also performed and validated with the experimental data, whose
interpolation curves are integrated and give a satisfactory agreement with the
theory.

In Section 3 we examine the heat flux on the ice pad, the relative humidity
(RH) close to the ice and the temperatures at surface and at the ice/concrete
interface, under the steady-state conditions preceding the resurfacing. Detailed
heat balance calculations pertain each contribution (such as the radiation from
the ceiling and the effect of the lighting system) separately.

We find the largest contribution to be thermal radiation from the ceiling
(74%), followed by lighting (14%), with excellent agreement between our calcu-
lations and the temperature and heat flux measurements.

This also constitutes a method for checking the data consistency.
In Section 4 we derive an analytical formula for the ice pad temperature in transient conditions, and verify its consistency in two limiting cases. In Appendix A we then apply it to the resurfacing process, using our measured data as initial condition and boundary conditions. Our result is compared with a numerical calculation, showing good agreement.

2. Ice hockey hall and temperature measurements

Good skating conditions occur when the ice surface is smooth, hard and slippery enough. Usually the ice temperature at the surface is between -3°C and -5°C. In any cases the temperature of the ice should be kept under -1°C to maintain ice hardness [17]. To keep the ice smooth and in optimal conditions after the wear due to skating, it is necessary to perform periodical maintenance, which is usually done by means of resurfacing machines. These devices first shave the ice, then brush it and eventually spread a thin layer of new water on the surface.

At each maintenance cycle, 300 to 800 liters of water are used, corresponding to a 0.25-0.5mm thick water layer. The water cooling and freezing processes generate a sudden increase in the ice temperature, which lowers the cooling efficiency of the refrigerating system, thus increasing the operational costs of the ice hall. Accurate analysis of this phenomenon is then important to optimize the refrigeration process and to achieve overall energy saving.

The field study discussed in this paper was carried out in August 2013 at the Reebok Arena in Leppävaara, Finland. The ice hall has two identical ice rinks of size 1624m² each. There are no major stands in the hall and the ventilation...
Figure 2: Measurements of temperature and heat flux at the ice/concrete interface.

The air flow rate varies between 3 – 6 m$^3$/s during the year. The refrigeration system is indirect, with ammonium as the refrigerant and ethylene glycol as the brine.

Consider now a modular component of the ice/concrete slab as in Fig. 1: ice 30 mm, concrete slab 150 mm with brine pipes at the middle, EPS-insulation 100 mm, sand 100 mm with frost pipes and a load bearing concrete slab 250 mm. The coefficient of performance for the refrigeration system is $\sim 1.6$ on the average [1].

We focus on the first resurfacing process in Fig. 2 corresponding to the peak near 12:45. The ice surface has an initial surface temperature $T_s \sim -4.5^\circ C$, the whole ice pad is at $-5^\circ C$ on the average. During resurfacing, $m_w = 450 \text{ kg}$ of water at temperature $T_w = 40^\circ C$ are spread on the ice surface at $t = 0$ s. The entire process consists of three phases:

1. cooling of the water layer by contact with the ice pad, convection and radiation with the surroundings,
2. complete freezing of the water by the same processes,
3. cooling of the new ice from $T_{freeze} = 0^\circ C$ to $T_3 \sim -4^\circ C$ (see Fig. 3). A thermal camera records the surface temperature point wise, taking pictures at intervals $\Delta t = 10$ s. Moreover, heat flux and temperature are measured with a heat flux plate and a pt-100 temperature sensor installed at the ice/concrete interface in the slab, as shown in Fig. 2. The according data are plotted in Fig. 3.

The physical properties of water are evaluated at the film temperature $T_f = 20^\circ C$. This implies the specific heat $c_{p,w} = 4.182 \text{ kJ/kg}$ and $c_{p,i} = 2.05 \text{ kJ/kgK}$ is the water latent heat of freezing. Since $m_w = 450 \text{ kg}$ of water are spread on the ice, the average thickness is only

$$x = \frac{m_w}{\rho A} = 0.28 \text{ mm,}$$

which is anyway just an indicative value, as the precision of the resurfacing machine is not high enough. The size of the ice rink is $A = 1624 \text{ m}^2$, $h_{fs} = 338 \text{ kJ/kg}$ is the water latent heat of freezing and $c_{p,i} = 2.05 \text{ kJ/kgK}$ is the
specific heat of ice at $T \sim 0^\circ C$. The water chilling and freezing loads are thus obtained as follows,

$$Q_w = [m_w h_{fs} + c_{p,w}(T_w - 0) + c_{p,i}(0 - T_{ice})]$$  \hspace{1cm} (2)

$$Q_1 = m_w c_{p,w}(T_w - 0) = 75.28 \text{ MJ} ; \quad q_1 = \frac{Q_1}{A} = 46.4 \frac{kJ}{m^2} , \hspace{1cm} (3)$$

$$Q_2 = h_{fs} m_w = 152.1 \text{ MJ} ; \quad q_2 = \frac{Q_2}{A} = 93.7 \frac{kJ}{m^2} , \hspace{1cm} (4)$$

$$Q_3 = m_w c_{p,i}(0 - T_{ice}) = 3.69 \text{ MJ} ; \quad q_3 = \frac{Q_3}{A} = 2.27 \frac{kJ}{m^2} , \hspace{1cm} (5)$$

where $T_{ice}$ is the temperature of the frozen resurfacing water under the new steady state conditions. The theoretical heat load Eq.(3) is consistent with the literature [18, 19].

Let us verify immediately that the theoretical heat flux in (3) is consistent with the measurements at the ice/concrete interface in Fig.3. The integral of the heat flux curve is the total heat transferred during the resurfacing, to be compared with Eq.(3). For better precision, we split the curve into three contributions: two polynomials plotted in Fig.4 and Fig.5 and a constant value $\dot{q}_{max} = 85.4839 \text{ W/m}^2$, corresponding to the narrow ($\Delta t \sim 30\text{ s}$) plateau at the top of the curve in Fig.3.

If $\dot{q}_A$ is the heat flux rate for the first contribution and $\dot{q}_B$ the flux for the second curve, we obtain the following expressions:

$$\dot{q}_A(t) = 3 \times 10^{-7} t^4 - 10^{-4} t^3 + 0.0106 t^2 + 0.0414 t + 45.191 , \hspace{1cm} (7)$$

$$\dot{q}_B(t) = 3 \times 10^{-12} t^4 - 2 \times 10^{-8} t^3 + 5 \times 10^{-5} t^2 - 0.0647 t + 85.873 . \hspace{1cm} (8)$$
Integrating the above over the respective time intervals gives

\[ q_A = \int_0^{140} \dot{q}_A(t) \, dt = 10.05 \, \frac{kJ}{m^2}, \]

\[ q_B = \int_0^{2300} \dot{q}_B(t) \, dt = 127.88 \, \frac{kJ}{m^2}, \]  

(9)  

(10)

to which we add the heat transferred at the peak, namely

\[ q_{\text{max}} = 85.4839 \, \frac{W}{m^2} \times \Delta t \, [s] = 2.57 \, \frac{kJ}{m^2}. \]

(11)

Thus the total heat transferred to the ice pad during the three phases of resurfacing in Eq.(3) is measured as

\[ q_{\text{exp}} = 140.49 \, \frac{kJ}{m^2}. \]

(12)

This is very close to the theoretical mean value in Eq.(3), namely

\[ q_w = 142.37 \, \frac{kJ}{m^2}, \]

(13)

we find indeed

\[ \Delta q \equiv q_w - q_{\text{exp}} = 1.88 \, \frac{kJ}{m^2}; \quad \% (\Delta q) = 1.32\%, \]

(14)

which is fairly satisfactory and verifies the consistency of the theoretical and measured heat load.\footnote{This is precise enough for our purposes, notice anyway that one has some freedom in setting the upper limit of integration in Eq.(10). The order of magnitude is anyway what matters.}
2.1. Indoor air measurements

Regarding the air temperature and RH, one set of data was obtained at 0.04m, 0.1m and 0.24m above the ice rink with accuracy ±2% for 0-90% relative humidity and ±3% for 90-100% RH. The accuracy of the temperature measurement is ±0.4°C. Additional measurements were made at 0.6m, 1.2m, 1.8m and 2.2m above the ice with PT-100 temperature sensors, with accuracy ±0.5°C. Moreover, the temperature was measured at 5.0m and 8.3m height with an accuracy of ±0.5°C. All measurements were done with Δt = 10s interval and lasted approximately four hours. The probes set at 0.04-2.2m height were set up in a metal holder standing above the ice; this was located at one corner of the ice rink, 1m from the edge of the ice pad.

A second set of measurements was made in April 2015 to verify and complement the earlier results. This measurement was done at 0.005m, 0.01m, 0.04m, 0.1m, 0.24m and 5m height over the ice. The accuracy of relative humidity of the probe was the same as in 2013, while for temperature measurements the accuracy was ±0.3°C. The measurements were done at one instance after the sensors had become steady. The probes at 0.005-0.24m height were again set up in a metal holder over the ice. Two locations of the holder were used, at the corner of the ice rink and close to the center of the ice pad, at approximately 8m from the edge.

The August measurements for the air temperature at 0.6m-8.3m over the ice are plotted in Fig.6 and those at 0.04m-0.24m in Fig.7. The air temperature, relative humidity and absolute humidity measured in April 2015 are shown in Table 1.

The plots show that, as expected, the air temperature above the ice rink is strongly stratified, see Fig.6, Fig.7 and Table 1. The temperature at 0.6m height is 2°C and 10°C at 5m height. Interestingly, by virtue of the mixing effect of the ventilation system shooting air at 25°C at 5m above the ice, the highest temperature is at 5m instead of 8.3m (Fig.6). Furthermore, the stratified air temperature at 0.6-2.2m turns into a more uniform temperature when the skaters enter the ice rink at 14:30 and start mixing the air layers.

The measured relative humidity (RH) and absolute humidity (AH), listed in Table 1 suggest that above the ice rinks the RH is uniform at heights 0.005m - 0.24m over the ice and distinctly lower farther from the ice pad. The relative humidity is higher above the ice than in the other parts of the space, yet it never reaches 95-100%.

The measurements did not imply any significant differences in temperature or relative humidity depending on the location over the ice pad (corner or central location).
Figure 6: Air temperature above the ice during resurfacing and skaters activity.

Figure 7: Air temperature above the ice rink at different heights, closer to the ice.

| Height over the ice [m] | T [°C] | RH [%] | AH [g/m³] |
|-------------------------|--------|--------|-----------|
| 5.0                     | 4.2    | 77.2   | 5.0       |
| 0.24                    | -0.8   | 90.2   | 4.1       |
| 0.1                     | -2.2   | 89.6   | 3.7       |
| 0.04                    | -2.7   | 89.3   | 3.5       |
| 0.01                    | -3.1   | 88.1   | 3.3       |
| 0.005                   | -3.5   | 88.2   | 3.2       |

Table 1: Air temperature, relative humidity (RH) and absolute humidity (AH) as measured at different heights from the ice pad surface.
3. Heat flux on the ice track and temperatures before resurfacing

In this section we use the field measurements to compute some estimates of the heat loads on the ice pad before resurfacing. We consider convection, condensation and radiation. In this case the heat transfer is steady state in good approximation, so this is easy to accomplish. Even though it is difficult to obtain very precise values, this shows well the various factors concurring to the overall energy balance on the ice hockey rink.

Consider first Fig.2 and Fig.3 before resurfacing, namely before \( \sim 12:43 \). The measured heat flux through the ice pad at the ice/concrete interface is on the average \( \dot{q} = 41.85 \text{ W/m}^2 \), and the temperature is \( T_I = -5.2^\circ \text{C} \). The ice pad thickness is \( L = 30 \text{mm} \). We can derive the ice temperature at surface very easily, if \( k_{\text{ice}} = 2.25 \text{ W/mK} \),

\[
T_S = T_I + \frac{L}{k_{\text{ice}}} \dot{q} = -4.64^\circ \text{C} , \quad (15)
\]

which is consistent with the ice surface temperature given in Fig.3.

The heat flux on the ice track surface is the sum of several contributions,

\[
\dot{q} = \dot{q}_{\text{rad}} + \dot{q}_{\text{conv}} + \dot{q}_{\text{wvcond}} + \dot{q}_{\text{lamp}} = h_{\text{rad}}(T_{\text{ceiling}} - T_S) + (h_{\text{conv}} + h_{\text{wvcond}})(T_{\text{in}} - T_S) + \dot{q}_{\text{lamp}} , \quad (16)
\]

namely thermal radiation from the ceiling, convection and water vapor condensation at the surface, and heat load from the lighting system. For simplicity, we neglect the thermal radiation from the vertical walls and from the audience stands (they do not give a relevant contribution anyway).

For convection we use \( T_{\text{in}} = T(0.04m) = -3.5^\circ \text{C} \), see Table 1. Accordingly, the heat transfer coefficient takes into account both natural convection and a correction given by forced convection [20]

\[
h_{\text{conv}} = 3.41 + 3.55 V = 3.94 \frac{W}{m^2 \text{K}} , \quad (17)
\]

corresponding to \( V = 0.15 \text{ m/s} \) for the air flow right on top of the ice, specifically at the height 4cm, as stated above. The convection heat flow is therefore

\[
\dot{q}_{\text{conv}} = h_{\text{conv}}(T_{\text{in}} - T_S) = 4.49 \frac{W}{m^2} . \quad (18)
\]

The radiation heat transfer coefficient is written instead as

\[
h_{\text{rad}} = \varepsilon_{12} \sigma (T_{\text{ceiling}}^2 + T_{\text{ice.sur}}^2)(T_{\text{ceiling}} + T_S) = 1.39 \frac{W}{m^2 \text{K}} , \quad (19)
\]

where \( T_{\text{ceiling}} = 18^\circ \text{C} \) and the resulting emissivity is [21]

\[
\varepsilon_{12} = \left[ \frac{1}{F_{ci}} + \left( \frac{1}{\varepsilon_{\text{ceiling}}} - 1 \right) + \frac{A_{ci}}{A_i} \left( \frac{1}{\varepsilon_{\text{ice.sur}}} - 1 \right) \right]^{-1} . \quad (20)
\]
The view factor is $F_{ci} = 0.68$, and emissivities $\varepsilon_{ice\_surf} = 0.98$ and $\varepsilon_{ceiling} = 0.28$ for the ice surface and the ceiling (a load bearing sheet of galvanized steel) are respectively used [2]. These give $\varepsilon_{12} = 0.28$.

One must also take into account the radiative heat transferred to the ice pad by the lighting system. The lamps in Leppävaara are metal halide, which implies a contribution of 400 W per lamp. The portion of this power that is turned into heat is nearly 62% [22]. Using the upper limit for the heat generation for 40 lamps gives the following contribution:

$$\dot{Q}_{lamp} = 9.92 \text{ kW},$$

(21)
corresponding to the following heat flux,

$$\dot{q}_{lamp} = \frac{\dot{Q}_{lamp}}{A} = 6.11 \frac{W}{m^2}.$$  

(22)
The water vapor condensation heat load is computed via the formula [23, 4]

$$\dot{q}_{wvcond} = h_d (T_{in} - T_S) \left[ \frac{W}{m^2} \right],$$

(23)
where the heat transfer coefficient for condensation $h_d$ is calculated from

$$h_d = 1750 h_{conv} \frac{\Delta p}{\Delta T}, \quad [\Delta] = [atm]$$

$$\Delta p = \varphi_{in} p_{in} - p_s.$$  

(24)
Here $\Delta T = T_{in} - T_S$, and $\varphi_{in} = 0.88$ is the relative humidity at 4cm from the ice surface, as in Table 1. The saturation pressures are calculated from (here $[T]=[K]$)

$$p_{in} = 10^5 \exp \left( 17.391 - \frac{6142.83}{273.15 + T_{in}} \right),$$

(25)
$$p_s = 10^5 \exp \left( 17.391 - \frac{6142.83}{273.15 + T_S} \right).$$

(26)
We thus obtain $h_d = 0.9 W/m^2K$ and $\dot{q}_{wvcond} = 1.03 W/m^2$. By substituting this result into (16), together with Eqs. (18), (19) and (20), and adding also (22), we get

$$\dot{q} = \dot{q}_{rad} + \dot{q}_{conv} + \dot{q}_{wvcond} + \dot{q}_{lamp} = 31.25 + 4.49 + 1.03 + 6.11 = 42.87 \frac{W}{m^2},$$

(27)
that overestimates only slightly the measured value 41.85 $W/m^2$. The percentage of each contribution is listed in Fig[8]
Now we focus on the ice/concrete pad and consider only conduction. The steady-state conditions provide for the heat flow inside the slab

$$\dot{q} = \frac{T_S - T_p}{R_{tot}} = \frac{T_S - T_p}{R_{ice} + R_{conc}}, \quad (28)$$

where $T_p$ is the temperature at the top of the pipes and the thermal resistances of the ice and concrete slabs are written as

$$R_{ice} = \frac{L}{k_{ice}} = 1.33 \times 10^{-2} \frac{Km^2}{W}, \quad (29)$$
$$R_{conc} = \frac{d}{k_{conc}} = 1.67 \times 10^{-2} \frac{Km^2}{W}, \quad (30)$$

where $k_{ice} = 2.25 \text{ W/mK}$ and $k_{conc} = 1.8 \text{ W/mK}$. The temperature at the top of the pipes $T_p$ is then estimated from

$$\dot{q} = 41.85 \text{ W/m}^2 = \frac{T_S - T_p}{R_{tot}}, \quad (31)$$

which gives $T_p = -5.9^\circ C$ and can be cross-checked with heat balance inside the concrete slab only,

$$\dot{q} = \frac{T_I - T_{p}}{R_{conc}}. \quad (32)$$

This in fact returns $\dot{q} = 41.92 \text{ W/m}^2 \approx 41.85 \text{ W/m}^2$.

Let us finally check the agreement between the thermal camera measurement and the result [(15)] for the ice surface temperature. The heat transfer via conduction inside the ice pad is written as

$$\dot{q}_I = \frac{T_S - T_I}{R_{ice}}. \quad (33)$$
Since this also must be equal to \( \dot{q} \), namely \( \dot{q}_I = \dot{q} = 41.85 \, W/m^2 \) because of the steady state conditions, substituting and solving with respect to the temperature at the ice/concrete interface we get

\[
T_I = T_S - R_{\text{ice}} \dot{q} = -5.2^\circ C,
\]

that is consistent with the measured value indeed.

4. Analytical temperature profile for the ice pad

In this section we derive an analytical formula for the temperature profile of the ice pad \( T_{\text{ice}}(t, x) \), which can be used under any specific situation occurring in the ice hockey hall.

The problem consists of solving the heat equation

\[
\frac{\partial u}{\partial t} = \alpha_I \frac{\partial^2 u}{\partial x^2},
\]

where \( 0 < x < L = 30 \, mm \), and \( \alpha_I \) is the thermal diffusivity of ice, with the time-dependent boundary conditions

\[
u(0, t) = T_S(t),
\]
\[
u(L, t) = T_I(t),
\]

and the initial condition

\[
u(x, t = 0) = f(x) = 18.67(0.03 - x) - 5.2,
\]

which is easily retrieved from the temperature data in the steady state regime. It gives indeed \( T_S(0) = -4.64^\circ C \) at the ice surface and \( T_I(0) = -5.2^\circ C \) at the ice/concrete interface, as in Fig.11.

The boundary conditions are in general given by the measurements. In this specific case, \( T_S(t) \) is computed at the ice surface (at the water/ice interface) and \( T_I(t) \) at the ice/concrete interface. They both are illustrated in Fig.3.

To obtain the analytical form of \( T_S(t) \), we interpolate the temperature of the ice pad at the surface. The overall trend, including the entire curve from the beginning of resurfacing to \( t = 620 \, s \), is clearly logarithmic. It gives the equation

\[
T_S(t) = -0.641 \ln(t) + 0.4016 \quad t[s] \leq 620.
\]

If instead we consider e.g. only the first minute, the surface temperature becomes

\[
T_S(t) = 2 \times 10^{-5} t^3 - 0.0015 t^2 - 0.0065 t - 1.0633, \quad t[s] \leq 60.
\]

These interpolations are given in Figs.9 and 10. The temperature raise in the last 10s is due to the third order polynomial (this is non physical, still it remains within the experimental error and does not affect the result sensibly).
The formula for the ice-concrete interface temperature, in the first approximation, is also a simple third-order polynomial obtained from the field measurements (Fig.3),

\[ T_I(t) = 6 \times 10^{-6}t^3 - 3 \times 10^{-4}t^2 + 4.3 \times 10^{-3}t - 5.2071 , \]  

(41)

which is plotted in Fig.12. We will apply these in the Appendix.

To solve the Cauchy problem given by Eqs.(35), (36), (37) and (38), we adopt the method of Eigenfunctions Expansions [24, 25, 26] as follows. First impose the following Ansatz

\[ w(x,t) = T_S(t) + x \left( \frac{T_I(t) - T_S(t)}{L} \right) , \]  

(42)

that implies

\[ w(0,t) = T_S(t) , \]  

(43)

\[ w(L,t) = T_I(t) . \]  

(44)

Now define the temperature profile \( u(x,t) \) as the sum

\[ u(x,t) = w(x,t) + v(x,t) , \]  

(45)

and substitute this expression in the Cauchy problem. We obtain

\[ v_t = \alpha_I v_{xx} - w_t , \]  

(46)

where we have simplified the derivatives notation, with the Dirichlet boundary conditions \( v(0,t) = v(L,t) = 0 \) and the initial condition \( v(x,0) = f(x) - w(x,0) \).

Now use the Eigenfunction expansion

\[ v(x,t) = \sum_{n=1}^{\infty} \hat{v}_n(t) \sin (\lambda_n x) , \]  

(47)
to separate space and time dependence. The eigenvalues and eigenfunctions associated to the Dirichlet b.c. are
\[ \lambda_n = \left( \frac{n\pi}{L} \right), \quad n \in \mathbb{N}; \quad X_n(x) = \sin (\lambda_n x) \]  
(48)

\[ S(x,t) = -w_I = -\left( \hat{T}_I(t) - \hat{T}_S(t) \right) \left( \frac{x}{L} \right) - \hat{T}_S(t). \]  
(49)

Therefore we obtain a first order linear ODE
\[ \frac{d\hat{v}_n}{dt} + \alpha_I \lambda_n^2 \hat{v}_n = \hat{S}_n(t), \]  
(50)

where
\[ \hat{S}_n(t) = \frac{2}{L} \int_0^L \left[ -\hat{T}_S(t) - \frac{x}{L} \left( \hat{T}_I(t) - \hat{T}_S(t) \right) \right] \sin \left( \frac{n\pi x}{L} \right) dx \]

\[ = \frac{2}{n\pi} \left[ \left( \frac{\sin n\pi}{n\pi} - 1 \right) \hat{T}_S(t) - \left( \frac{\sin n\pi}{n\pi} \cos n\pi \right) \hat{T}_I(t) \right], \]  
(51)

with an integrating factor \( F(t) = e^{\alpha_I \lambda_n^2 t} \). Eq.(50) can then be integrated to give the following solution,
\[ v(x,t) = \sum_{n=1}^{\infty} \left\{ \int_0^t d\tau e^{-\alpha_I \lambda_n^2 (t-\tau)} \hat{S}_n(\tau) + e^{-\alpha_I \lambda_n^2 \tau} c_n \right\} \sin (\lambda_n x), \]  
(52)

where the coefficients
\[ c_n = \frac{2}{L} \int_0^L \left\{ f(x) - \left[ (T_I(0) - T_S(0)) \left( \frac{x}{L} \right) + T_S(0) \right] \right\} \sin \left( \frac{n\pi x}{L} \right) dx, \]  
(53)

are given by the initial condition \( u(0,x) = f(x) \).

Putting everything together, we can now write the analytical temperature profile for the ice pad Eq.(45) as
\[ u(x,t) = T_{ice}(x,t) = [T_I(t) - T_S(t)] \left( \frac{x}{L} \right) + T_S(t) \]
\[ + \sum_{n=1}^{\infty} \left\{ \int_0^t d\tau e^{-\alpha_I \lambda_n^2 (t-\tau)} \hat{S}_n(\tau) + e^{-\alpha_I \lambda_n^2 \tau} c_n \right\} \sin (\lambda_n x), \]  
(54)
that constitutes a novel result of this paper. This is a completely general for-
mula, with implicit initial condition \( f(x) \) and b.c. \( T_I(t) \) and \( T_S(t) \).

The above clearly reduces to the ice temperature \( T_S(t) \) at the surface for
\( x = 0 \), while it is slightly less immediate to verify that (54) is consistent with
the temperature at the bottom of the ice pad \( T_I(t) \) for \( x = L \). In this case we get

\[
\begin{align*}
u(L, t) &= T_{\text{ice}}(L, t) \equiv T_I(t) = [T_I(t) - T_S(t)] + T_S(t) \\
+ &\sum_{n=1}^{\infty} \left\{ \int_0^t d\tau e^{-\alpha_I \lambda_n^2 (t-\tau)} \hat{S}_n(\tau) + e^{-\alpha_I \lambda_n^2 t} c_n \right\} \sin (\lambda_n L) . \quad (55)\end{align*}
\]

Recall now that

\[
\lambda_n = \frac{n\pi}{L} , \quad (56)
\]

which implies

\[
-2T_S(t) \sum_{l=1}^{\infty} \frac{\sin (\lambda_n L)}{n\pi} = -2T_S(t) \sum_{l=1}^{\infty} \frac{\sin n\pi}{n\pi} = 0 , \quad (57)
\]

since each term in the summation is identically zero. Regarding the term con-
taining the integral,

\[
2 \sum_{l=1}^{\infty} \frac{\alpha_I \lambda_n^2}{n\pi} \sin (\lambda_n L) e^{-\alpha_I \lambda_n^2 t} \int_0^t d\tau e^{\alpha_I \lambda_n^2 \tau} T_S(\tau) \propto \sum_{l=1}^{\infty} n \sin n\pi e^{-\alpha_I \lambda_n^2 t} \equiv 0 , \quad (58)
\]

again because \( \sin n\pi = 0, \forall n \in \mathbb{N} \). So we get an identity \( 0 = 0 \), as required.

Physically, the terms in brackets in Eq.(54) give the transient state correction
to temperature, that depends on the history of the process (via the \( t \)-integral)
and on the initial and boundary conditions by virtue of Eqs.(51) and (53). This
is plotted in Fig.A.18 for our specific example calculated in the Appendix, where
the initial condition (38) and the boundary conditions (40) and (41) pertain the
resurfacing process.

The according analytical temperature profile (54) is given in Figure 13. In
Fig.15 this is compared to a numerical solution by a Finite Element Model
(FEM), which is plotted in Figure 14. Details of the analytical computation are
discussed in the Appendix.

In conclusion, Eq.(54) provides a general formula for the temperature profile
\( T(x, t) \) at any point \( x \) of the ice pad, at any generic time \( t \) corresponding to any
situation in the ice hockey hall (closing hours, resurfacing or skaters’ activity).
The practical problem of studying the complex physical processes over and
under the ice pad is here avoided by virtue of the boundary conditions (i.e.
experimental data), which encode all the involved phenomenology in a very
simple analytical form.
Figure 13: Temperature profile in the ice pad, analytical solution.

Figure 14: FEM solution for $t = 0...60s$.

Figure 15: Temperature profile after $t = 30s$, analytical (solid) versus FEM (dashed).
5. Conclusions

In this paper we have considered thermophysical processes in an ice hockey hall during standard operation hours. Detailed heat flux, air and ice temperature and relative humidity data are provided and discussed in a quantitative analysis with specific focus on the maintenance (resurfacing) phase. An energy balance calculation shows the different contributions to the heat load on the ice rink, and we find an analytical formula for the temperature inside a medium which is subject to conduction with time-dependent boundary conditions. We apply this formula to the case at hand, and obtain a good agreement with an analogous numerical computation.

Outside the occupation hours, we find a strong temperature stratification in the air above the ice rink, which is compromised once the skaters start their activity. The effect of the ventilation system, delivering air at 25°C at about 5m above the ice, is instead independent of the skaters, yet it is critically affecting the air layers temperature. As it is shown in Figure 6, our data read warmer air at 5m than at 8m. Therefore one should control the ventilation system to avoid energy dissipation, and/or use waste energy techniques [15, 16].

The energy balance calculations in Section 3 give the thermal radiation from the ceiling as the largest contribution (74%), followed by lighting (14%). Together with the data measurements discussed in Section 2 such quantitative knowledge can aid energy saving efforts in a wide range of energy saving studies, since these processes occur in an average-sized ice hall under standard operating conditions.

Furthermore, the analytical formula Eq. (54) holds for any problem pertaining one-dimensional heat conduction inside a generic sample with thermal diffusivity α, under time-dependent boundary conditions at the two surfaces

In the bottom-up approach adopted here, all the physics is encoded in the boundary conditions, circumventing an otherwise involved phenomenological analysis. This article contains therefore a methodology which is firmly grounded on experimental data, using induction to obtain theoretical (predictive) results, and deduction to apply these formulas and to check the data consistency.

The accurate measurements, energy balance analysis and the analytical temperature profile in the ice pad presented in this work can constitute useful tools for a proper scientific assessment, in view of today’s pressing energetic efficiency demands.

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3This work is indeed part of a wider project aiming to energy efficiency in this class of buildings, which was started in [1].
Appendix A. Analytical temperature profile during resurfacing.

The temperature profile (54) was obtained in Section 4 in implicit form. In this section we put it into context, by using the initial and boundary conditions Eqs. (38), (40) and (41) to obtain the temperature profile inside the ice pad during the resurfacing process. First, we expand Eq. (51),

\[
\hat{S}_n(t) = \frac{2}{n\pi} \left[ \left( \frac{\sin n\pi}{n\pi} - 1 \right) \dot{T}_S(t) - \left( \frac{\sin n\pi}{n\pi} - \cos n\pi \right) \dot{T}_I(t) \right]
\]

\[
= \frac{1}{n\pi} \left[ \left( 8.4 \frac{\sin n\pi}{n\pi} + 3.6 \cos n\pi - 12 \right) \times 10^{-5}t^2 
+ \left( -0.0048 \frac{\sin n\pi}{n\pi} - 0.0012 \cos n\pi + 0.006 \right) t 
+ \left( -0.0261 \frac{\sin n\pi}{n\pi} + 0.0086 \cos n\pi + 0.013 \right) \right], \quad (A.1)
\]

then the coefficients (53),

\[
c_n = \frac{2}{L} \int_0^L \left\{ f(x) - \left[ (T_I(0) - T_S(0)) \left( \frac{x}{L} \right) + T_S(0) \right] \sin \left( \frac{n\pi x}{L} \right) \right\} dx 
\sim \frac{1}{n\pi} \left[ 7.167 \left( \frac{\sin n\pi}{n\pi} - 1 \right) - 0.0142 \cos n\pi \right], \quad (A.2)
\]

because 0.998 \sim 1. Already at this stage we notice the suppression factor 1/n. The integral in Eq. (54) is now computed as

\[
I_n(t) = \int_0^t d\tau e^{-\alpha_1 \lambda_n^2 (t-\tau)} \hat{S}_n(\tau) = 
\]

\[
= \frac{1}{n^3} \left\{ e^{-0.01327n^2t} - 1 \right\} \left[ \left( 0.1649 - \frac{2.762}{n^2} - \frac{7.284}{n^4} \right) \frac{\sin n\pi}{n} 
- \left( 0.206 + \frac{2.169}{n^2} + \frac{9.808}{n^4} \right) \cos n\pi - 0.312 + \frac{10.846}{n^2} + \frac{32.693}{n^4} \right] 
+ \left( 0.00064 \frac{\sin n\pi}{n} + 0.00086 \cos n\pi - 0.0029 \right) t^2 
- \left[ \left( 0.037 + \frac{0.097}{n^2} \right) \frac{\sin n\pi}{n} + \left( 0.029 + \frac{0.13}{n^2} \right) \cos n\pi - 0.144 - \frac{0.434}{n^2} \right] t \right\} 
\]

(A.3)

since \( e^{\alpha_1 \lambda_n^2 \tau} = e^{0.01327n^2\tau} \).
Figure A.16: Contribution of the first 30 terms in the summation (A.4), at \( t = 30 \) s.

Figure A.17: Largest \((n = 1)\) contribution, where \( 0 < t < 60 \) s.

We can thus recast the overall summation as

\[
\sum_{n=1}^{\infty} \left\{ \int_0^t d\tau e^{-\alpha_1 \lambda_n^2 (t-\tau)} \hat{S}_n(\tau) + e^{-\alpha_1 \lambda_n^2 t} c_n \right\} \sin (\lambda_n x)
\]

\[= \sum_{n=1}^{\infty} \left\{ I_n(t) + \frac{e^{-0.01327 \pi^2 t}}{n} \left( 0.73 \sin \frac{n\pi}{n} - 0.01 \cos n\pi - 2.28 \right) \right\} \sin \frac{n\pi}{L} x
\]

(A.4)

which is clearly convergent, since everything is proportional to \( \propto 1/n^a \).

The first 30 terms in the summation at \( t = 30 \) s are plotted separately in Fig A.16; we see that only the first six or seven matter significantly. The largest contribution, for \( n = 1 \), is shown in Fig A.17, where the correction to the temperature (in absolute value) is maximal at \( t = 0 \) s and minimal at \( t \approx 50 \) s.

The temperature profile inside the ice pad Eq.(54) is accordingly rewritten as follows,

\[
u(x, t) = T_{ice}(x, t) = [T_I(t) - T_S(t)] \left( \frac{x}{L} \right) + T_S(t)
\]

\[+ \sum_{n=1}^{\infty} \left\{ \int_0^t d\tau e^{-\alpha_1 \lambda_n^2 (t-\tau)} \hat{S}_n(\tau) + e^{-\alpha_1 \lambda_n^2 t} c_n \right\} \sin (\lambda_n x)
\]

\[= -(1.4 \times 10^{-5} t^3 - 0.0012 t^2 - 0.0108 t + 4.1438) \frac{x}{L}
\]

\[+ 2 \times 10^{-5} t^3 - 0.0015 t^2 - 0.0065 t - 1.0633
\]

\[+ \sum_{n=1}^{\infty} \left\{ \int_0^t d\tau e^{-\alpha_1 \lambda_n^2 (t-\tau)} \hat{S}_n(\tau) + e^{-\alpha_1 \lambda_n^2 t} c_n \right\} \sin (\lambda_n x), \quad \text{(A.5)}
\]

where the temperature correction generated by the summation is computed with \( \text{[A.3]} \) and \( \text{[A.4]} \); it is shown in Figure A.18 after 30s, where the first 100 terms are summed.

Figure 15 compares the analytical solution \( \text{[A.5]} \) to an FEM calculation for \( t = 30 \) s, with \( n = 1...100 \). We notice a good agreement between the two curves.

We remark that this result is specifically valid for any \( t \leq 60 \) s, since the boundary conditions \( T_S(t) \) and \( T_I(t) \) are interpolations corresponding only to
such time interval. Choosing different times will change the explicit form of both $T_S(t)$ and $T_I(t)$. On the contrary, the temperature profile Eq. (54) found in Section 4 is given with implicit initial and boundary conditions, which makes it general and applicable to a range of diverse engineering problems.

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