Dynamic Modulation Transfer Function Analysis of Images Blurred by Sinusoidal Vibration

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The dynamic modulation transfer function (MTF) for image degradation caused by sinusoidal vibration is formulated based on a Bessel function of the first kind. The presented method makes it possible to obtain an analytical MTF expression derived for arbitrary frequency sinusoidal vibration. The error obtained by the use of finite order sum approximations instead of infinite sums is investigated in detail. Dynamic MTF exhibits a stronger random behavior for low frequency vibration than high frequency vibration. The calculated MTFs agree well with the measured MTFs with the slant edge method in imaging experiments. With the proposed formula, allowable amplitudes of any frequency vibration are easily calculated. This is practical for the analysis and design of the line-of-sight stabilization system in the remote sensing camera.

Keywords: Imaging system, Modulation transfer function, Sinusoidal vibration, Slant edge method

OCIS codes: (110.0110) Imaging systems; (110.4100) Modulation transfer function; (110.3000) Image quality assessment; (120.0280) Remote sensing and sensors

I. INTRODUCTION

In the remote sensing imaging system, image resolution is generally limited resulting from motion degradation caused by mechanical vibration [1]. Sinusoidal vibration is a basic form of mechanical vibration, such as low frequency, rigid body motion due to platform control instabilities and medium- to high frequency disturbances due to structural elasticity [2]. A convenient form with which to characterize motion degradation quantitatively is dynamic MTF [3, 4].

Earlier MTF calculation methods were based on spatial frequency analysis of sine-wave response or on spatial domain spread function analysis. Analytical MTF expressions were established only for simple kinds of motion such as linear motion and high frequency vibration whose frequency is integral times the detector exposure frequency [5-7]. A MTF expression for low frequency vibration was approximated by a sinc function, regarding square root of the average central second moment during exposure time as constant velocity motion [8]. The calculation method presented by A. Stern and N. S. Kopeika for any kind of motion was expressed in the form of a power series in terms of statistical moments of the motion during the exposure [9]. MTF for 2-D relative motion is obtained with the method of numerical integration [10]. However, the previous calculation methods are not very suitable for arbitrary frequency sinusoidal vibration.

In this paper a formula is established to calculate dynamic MTF for sinusoidal vibration at arbitrary frequency. The MTF expression is in the form of an infinite sum of integral order Bessel functions of the first kind. Whereas, only finite order of Bessel functions can be used for practical implementation. The truncation error caused by replacing the infinite sum with a finite sum approximation is described in detail. In the simulation analysis, there exists a strong random characteristic especially for low frequency vibration when the ratio of the exposure period to the vibration period is not an integer. Admissible vibration amplitudes...
at arbitrary frequency are calculated with the MTFs in the worst case. The actual MTF due to vibration is measured with the knife edge chart. Using a fast steering mirror (FSM) actuated by the piezoelectric transducer (PZT), sinusoidal vibrations are generated. The measured MTFs agree well with the calculated results, indicating the validity of the proposed MTF calculation formula.

II. MTF CALCULATION FORMULA

2.1. Analytical MTF Expression

According to the transfer function theory of the linear optical system, the line spread function (LSF) in one direction is expressed as $\delta(X)$ in ideal conditions. The relative motion between the object and detector is assumed to be $x(t)$. The LSF becomes $\delta[X-x(t)]$ when the motion component is in the direction which is perpendicular to the optical axis direction. The LSF of the imaging system during the exposure period is given by normalization processing.

$$LSF(X) = \frac{1}{t_e} \int_{-\infty}^{\infty} \delta[X-x(t)]\,dt$$

(1)

where $t_0$ is the beginning time of exposure, $t_e$ is the exposure period. The optical transfer function (OTF) is given in terms of the Fourier Transform of the LSF.

$$OTF(f) = \int_{-\infty}^{\infty} LSF(X) \exp(-2\pi j f X)\,dX$$

(2)

Hence OTF can be deduced from Eqs. (1) and (2):

$$OTF(f) = \frac{1}{t_e} \int_{-\infty}^{\infty} \exp[-2\pi j f x(t)]\,dt$$

(3)

Dynamic MTF is derived as the modulus of the OTF:

$$MTF(f) = \frac{1}{t_e} \int_{-\infty}^{\infty} \exp[-2\pi j f x(t)]\,dt$$

(4)

MTF for any kind of motion can be calculated by numerical integration without obtaining the probability density function (PDF) of the motion in previous calculation methods. Sinusoidal vibration can be expressed as:

$$x(t) = D \sin\left(\frac{2\pi}{T} t\right)$$

(5)

where $D$ stands for the amplitude and $T$ stands for the period of the sinusoidal motion. MTF for sinusoidal vibration becomes:

$$MTF(f, t_e) = \left| \frac{1}{t_e} \int_{-\infty}^{\infty} \exp\left[-2\pi j f D \sin\left(\frac{2\pi}{T} t\right)\right] dt \right|$$

(6)

The exponential function can be expressed as an infinite sum of integral order Bessel functions of the first kind:

$$\exp\left[-2\pi j f D \sin\left(\frac{2\pi}{T} t\right)\right] = J_0\left(2\pi j f D \sin\left(\frac{2\pi}{T} t\right)\right)$$

$$+ 2 \sum_{k=1}^{\infty} J_{2k}\left(2\pi j f D \sin\left(\frac{2\pi}{T} t\right)\right) \cos\left(\frac{4k\pi}{T} t\right)$$

$$- 2 \sum_{k=1}^{\infty} J_{2k-1}\left(2\pi j f D \sin\left(\frac{2\pi}{T} t\right)\right) \sin\left(\frac{(4k-2)\pi}{T} t\right)$$

(7)

Because the sum of infinite series is coincident convergence, the operation order of integrals and sums can be exchanged. The MTF expression becomes:

$$MTF(f, t_e) = \left| J_0\left(2\pi j f D \sin\left(\frac{2\pi}{T} t\right)\right) + 2 \sum_{k=1}^{\infty} J_{2k}\left(2\pi j f D \sin\left(\frac{2\pi}{T} t\right)\right) \cos\left(\frac{4k\pi}{T} t\right) \right.$$

$$- 2 \sum_{k=1}^{\infty} J_{2k-1}\left(2\pi j f D \sin\left(\frac{2\pi}{T} t\right)\right) \sin\left(\frac{(4k-2)\pi}{T} t\right) dt$$

(8)

where

$$\frac{1}{t_e} \int_{-\infty}^{\infty} \cos\left(\frac{4k\pi}{T} t\right) dt = \sin\left(\frac{2\pi}{T} t\right) \cos\left(\frac{2\pi}{T} t\right)$$

(9)

$$\frac{1}{t_e} \int_{-\infty}^{\infty} \sin\left(\frac{(4k-2)\pi}{T} t\right) dt$$

$$= \sin\left(\frac{2\pi}{T} t\right) \sin\left(\frac{(4k-2)\pi}{T} t\right)$$

(10)

Substituting Eqs. (9) and (10) into Eq. (8), we have the MTF expression:

$$MTF(f, t_e) = \left| J_0\left(2\pi j f D \sin\left(\frac{2\pi}{T} t\right)\right) + 2 \sum_{k=1}^{\infty} J_{2k}\left(2\pi j f D \sin\left(\frac{2\pi}{T} t\right)\right) \cos\left(\frac{2\pi}{T} t\right) \right.$$}

$$- 2 \sum_{k=1}^{\infty} J_{2k-1}\left(2\pi j f D \sin\left(\frac{2\pi}{T} t\right)\right) \sin\left(\frac{2\pi}{T} t\right)$$

(11)

Thus, Eq. (11) defines a direct relation between dynamic MTF and the vibration amplitude $D$, the ratio $t_e/T$ between the exposure period and vibration period and the initial exposure time $t_0$. With different initial exposure time MTF may change even if the vibration amplitude and the ratio are certain. However, MTF is the function of the vibration amplitude $D$ when the ratio is an integer. The expression becomes:
FIG. 1. The upper bound error for the 2Nth order approximation at spatial frequency $f = 1/D$.

The approximation error is given:

$$\text{Err} (N) \leq 2 \sum_{k=2N+1}^\infty |I_j(2\pi fD)|$$

(19)

The integral order Bessel function of the first kind can be expressed in the form of a power series.

$$I_j(2\pi fD)| = \frac{1}{\pi} \sum_{m=0}^\infty \frac{(\pi fD)^{2m+j}}{m!}$$

(20)

Substituting Eq. (20) into Eq. (19)

$$\text{Err} (N) \leq 2 \sum_{k=2N+1}^\infty \left[ \frac{(\pi fD)^2}{k!} \right] \left[ \frac{(\pi fD)^{2m+j}}{m!} \right]$$

(21)

The 2Nth order approximation error is calculated by the remainder of the sum in Lagrange form.

$$\text{Err} (N) \leq 2 \exp\left[ (\pi fD)^2 + \xi \frac{(\pi fD)^{2m+j}}{(2N+1)!} \right]$$

$$\xi \in (0, \pi fD)$$

(22)

Finally, the upper bound to the absolute error is expressed as follows:

$$\text{Err} (N) \leq \frac{2 \exp\left[ (\pi fD)^2 + \xi \right]}{(2N+1)!}$$

(23)

With Eq. (23), the upper bound error can be calculated. However it is necessary to emphasize that fewer order Bessel functions are required generally. A plot of the upper limit
error for the 2Nth order approximation at the spatial frequency \( f = 1/D \) is shown in Fig. 1. As can be seen, up to 18th order Bessel functions are needed for the approximation error less than 0.02.

III. SIMULATION ANALYSIS OF MTF FOR SINUSOIDAL VIBRATION

3.1. Simulation of MTF for Low and High Frequency Sinusoidal Vibration

Because of turbines and motors or structural resonance, vibration is a critical factor that degrades the image quality when the imaging system is located on aircraft and other vehicles [11]. There is still residual image motion even though sinusoidal vibration is mostly attenuated by passive isolation and gyro stabilization techniques. Sinusoid oscillations are often classified as low frequency \( (t_e/T < 1) \) and high frequency \( (t_e/T > 1) \) based on the ratio of the exposure period to the vibration period as shown in Fig. 2. For low frequency vibration, there is a great change in the relative motion during the exposure period. Whereas the relative motion for high frequency vibration is nearly the same.

In accordance with the calculation formula, MTF for

![FIG. 2. Sinusoidal vibration: (a) low frequency vibration; (b) high frequency vibration.](image)

![FIG. 3. The MTFs for sinusoidal vibration different ratios: (a) \( t_e/T = 0.25 \); (b) \( t_e/T = 0.5 \); (c) \( t_e/T = 1.5 \); (d) \( t_e/T = 5.5 \).](image)
sinusoidal vibration depends on the parameters (the amplitude \(D\) and the ratio \(t_e/T\)). It is also a random process, depending on the beginning time of the exposure when the ratio \(t_e/T\) is not an integer. The MTFs for sinusoidal vibration (\(D = 10\ \mu m\) and \(t_e/T = 0.25, 0.5, 1.5, 5.5\)) are calculated with different initial time of the exposure as shown in Fig. 3.

As can be seen, the MTFs exhibit a strong stochastic characteristic for low frequency. Image blur due to low frequency vibration is variable because the beginning time of the exposure is usually random. However, the random behavior is not evident for high frequency vibration and the exposure period can be approximated to an integral number of the vibration period when the ratio \(t_e/T\) is more than 5. MTF can be calculated simply by using the zero order Bessel function of the first kind.

3.2. Tolerance of Residual Sinusoidal Vibration Amplitude

Dynamic MTF in the worst case should be analyzed to make sure that the image resolution can be high enough. With the initial exposure time \(t_e = (T - t_e)/2\), image degradation is the most severe. The analytical MTF expression becomes:

\[
MTF_{\text{worst}}(f) = J_0(2\pi D) + 2 \sum_{k=1}^\infty J_0(2\pi D) \operatorname{sinc}\left(\frac{2\pi k T}{T}\right)
\]  

(24)

\(MTF_{\text{worst}}\) is a function of vibration amplitude \(D\) and the ratio \(t_e/T\). The MTFs with different vibration amplitudes and ratios are calculated as shown in Fig. 4. As can be seen, for low frequency vibration, image quality can be improved by reducing vibration amplitude and the exposure period of the detector. However, the only effective way of enhancing image resolution for high frequency vibration is limiting the vibration amplitude.

As previously discussed, the MTF due to vibration at the Nyquist frequency should be more than 0.9 to ensure high resolution of images [11]. The Nyquist frequency is defined as

\[
f_{\text{Nyquist}} = \frac{1}{2p}
\]  

(25)

where \(p\) is the pixel size of the detector. The upper bound of vibration amplitudes with arbitrary ratio is obtained with Eq. (26).

\[
MTF_{\text{worst}}(f_{\text{Nyquist}}) \geq 0.9
\]  

(26)

As illustrated in Fig. 5, the upper bound of low frequency vibration amplitudes declines as the ratios rise. And the allowable amplitudes of high frequency vibration are almost consistent with the value 0.2 pixel size.

IV. EXPERIMENTS AND RESULTS

4.1. Experimental Setup

The optical system for measuring dynamic MTF under sinusoidal vibration is illustrated in Fig. 6. The uniform illumination for the knife-edge target is provided by an integrating sphere and the edge target is at the focal plane of the collimation lens. The light is incident to the CMOS camera after being reflected by a FSM actuated by PZT.
The sinusoidal motion at the focal plane is generated when the mirror actuator is supplied with a sinusoidal signal by the function generator. The image motion is expressed as:

\[ x(t) = 2F\theta \sin \left( \frac{2\pi t}{T} \right) \]  

(27)

where \( F \) is the focal length of the camera, \( \theta \) is the angular amplitude of the mirror rotation, \( T \) is the period of the sinusoidal signal.

In this paper the slanted edge method is used to provide a fast and effective MTF measurement [12]. The slanted edge method has been used to measure MTF on orbit and modified methods have been proposed to improve the measuring accuracy [13-15]. For the sake of increasing the sampling frequency and eliminating the aliasing effect, the knife-edge is slightly slanted compared with the knife-edge method. The procedure of the slanted edge method is illustrated in Fig. 7. Firstly, the edge angle is estimated by performing the linear regression on the collected line-to-line estimated edge positions. In Fig. 7(a), the pixels in the rectangular region of interest (ROI) are projected along the gradient direction into the bins whose width is a quarter of the sampling frequency on the horizontal axis [16]. The one-dimensional edge spread function (ESF) is generated by averaging the values of the pixels in each bin in Fig. 7(b). The line spread function (LSF) is yielded by the derivative of the ESF in Fig. 7(c) and the MTF in Fig. 7(d) is measured after performing a discrete Fourier transform and normalizing.

The total MTF for the imaging system denoted by \( MTF_{\text{total}} \) is obtained from Eq. (28):

\[ MTF_{\text{total}} = MTF_{\text{static}} \times MTF_{\text{vibration}} \]  

(28)

where \( MTF_{\text{static}} \) is the MTF of the camera without vibration and \( MTF_{\text{vibration}} \) is the dynamic MTF of image degradation caused by vibration.

4.2. Experimental Results

As Fig. 8 shows, the COMS camera we used is with 1K×1K pixels and the size of each pixel is 5.5 μm × 5.5 μm. The exposure period is set as 20 ms and the focal length of the camera is 50 mm. The frequency range of the vibration generated by the FSM is from 10 Hz to 300 Hz and the amplitude provided by the FSM is 0.5 mrad. Figure 9 shows the static image and blurry images caused by the sinusoidal vibration whose frequency is 25 Hz. Figure 9(a) shows the static image without vibration and as shown in Fig. 9(b) ~ Fig. 9(f), the blurred images are dissimilar due to the different beginning time of the exposure. The dynamic MTFs calculated by Eq. (28) though the static MTF and the total MTF measured with the slant edge method are shown in Fig. 10(a). The MTFs measured from the degraded images agree well with the calculated MTFs shown in Fig. 10(b). For the sinusoidal vibration whose frequency is 50, 100, 200 Hz, the blurry images are mostly uniform as in Fig. 9(g) ~ Fig. 9(i) and the MTFs measured from images and the MTFs in theory are basically consistent as shown in Fig. 11.
V. CONCLUSION

In this paper a new formula to calculate dynamic MTF for sinusoid oscillation has been described. The great advantage of the proposed method is that analytical MTF expressions for any frequency sinusoidal vibration are obtained. Dynamic MTF is expressed as an infinite sum of integral order Bessel functions of the first kind. For practical application, low order sum approximations give satisfactory results. And the validity of the MTF calculation formula is verified with imaging experiments.

For low frequency, MTF exhibits a strongly random characteristic. And the random behavior is not obvious for high frequency vibration especially when the ratio $t_e/T$ is more than 5. With the same vibration amplitude, a comparison shows that image degradation due to high frequency vibration is more severe than for low frequency vibration.

This new method can be implemented in image motion degradation analysis. Allowable vibration amplitudes can be predicted simply with the presented method which has an effective guidance for the line-of-sight stabilization system design.

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