INFLUENCES OF CARBON EMISSION ABATEMENT ON FIRMS' PRODUCTION POLICY BASED ON NEWSBOY MODEL

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ABSTRACT. Carbon emission allowance (CEA) has been becoming an important factor for firms to make production policies. Cap-and-trade system is fulfilling in many countries and regions as a market scheme promoted by many politicians and economists for its efficiency in resources assignment and promotion to abatement of carbon emission. More and more firms take CEA into their production plan which makes them confronted with influences from two markets, product market and CEA trade market in the meanwhile. Based on the Newsboy model for simplicity, and with assumption that demand of product is a stochastic variable, this paper establishes optimization models to get the optimal production policy under administrative scheme (command-and-control) and market scheme (cap-and-trade) respectively. By comparing the firms' production policy and expected net income (ENI) with or without the existence of CEA trade market, it is found that CEA trade market can reduce the optimal amount of production and carbon emission on the one hand, and it does not decrease firms' ENI on the another hand because the CEA trade market provides more options for firms to make production policy. Hence, in the proposed complete and perfect market, we concluded cautiously that market-based carbon emission abatement scheme is effective to reduce carbon emission and to accomplish regulatory carbon emission abatement goal.

1. Introduction. The Intergovernmental Panel on Climate Change (IPCC, 2007) noted that worldwide annual carbon emissions need to be cut approximately to half by 2050. Different policies have been conducted all over the world to abate carbon emissions, such as direct control, cap-and-penalty, carbon tax, cap-and-trade, subsidiary and a mix of these approaches.

Carbon emission allowance (CEA) trading has been recommended as one of the flexible mechanisms in the Kyoto Protocol to reduce greenhouse gas emissions in a cost-effective way. European Union has introduced a carbon-trading system in the beginning of 2005 as a means for achieving its carbon reduction target [4].

The most widely used scheme may be the cap-and-trade scheme. In this system, regulator assigns carbon emission allowance, may be not free such as auctioned
CEA, to every emission-dependent firm in a given period. Any firm can buy CEA from or sell CEA to others in the CEA trade market. This scheme is appreciated by many governments and researchers because they believe that cap-and-trade is based on the market scheme and it can distribute CEA more efficiently. As the scheme has been established, many researches have studied the macro influences on macro-economics of world- or region-wide or industry level, such as [7, 2, 17, 9, 11] and others, which we find that it is impossibly hard to list them all. In industrial level, [4] analyzed the impacts of the EU CO\textsubscript{2} emission trading on power plant operators, energy-intensive industries, and other consumer groups in Finland. Their results found that large windfall profits were estimated to incur to electricity producers in the Nordic electricity market, while the metal industry and private consumers were estimated to be most affected by the electricity market price increases. Among various energy-intensive industries, the pulp and paper industry may actually be a net beneficiary of the EU emission trading. [9] analyzed the impacts of combining a carbon tax and emission trading on different industry sectors. Their results indicate that the grandfathering rule is the more feasible approach in allocating the emission permit to each industry sector. And they also find that the accumulated GDP loss of the petrochemical industry by the carbon tax during the period 2011-2020 is 5.7%. However, the accumulated value of GDP will drop by only 4.7% if carbon taxation is implemented together with emission trading. Besides, among petrochemical-related industry sectors, up-stream sectors earn profit from emission trading, while down-stream sectors have to purchase additional emission permits due to failure to achieve their emission targets. [12] investigated the cost impacts of the European Emission Trading System (ETS) on energy-intensive manufacturing industries. Their results indicate that the ETS affects the industry sectors quite differently. The cost impacts of the steel and cement industries are 3-4 fold compared with the least-affected pulp and paper and oil refining. They therefore suggest that some correcting mechanisms may be worth considering in securing the operation of some industry sectors. [8] presented a global simulation model to quantitatively analyze the impacts of three carbon emission trading schemes on the cement sector. In a multi-regional “bottom-up” energy-system optimization MARKAL model, [1] discussed the impacts of emission constraints on a given region. They concluded that Incorporating endogenous technology learning and allowing for spillovers across regions appears as an important mechanism for capturing the possibility of induced technological change due to environmental constraints in “bottom-up” models.

In the cap-and-trade scheme, CEA becomes an important factor for emission-dependent firms who must optimize their production plan in consideration of influences from CEA trade market. Some researchers focused on the influences of emission trading based on the micro-level. There are several papers that have proposed mathematical models to deal with different aspects of environmental management and control. [13] developed a non-linear dynamic model to illustrate that changing public environmental policies (taxes and standards) may lead to a more erratic production profile. [5] established a basic conceptual model of environmental management within operations. The model proposes that the general orientation of operations managers on environmental issues ranges from proactive to reactive, and this is intrinsically related to the investment pattern in environmental technologies. [14] utilized nonlinear programming to study whether the profit maximizing output of ammonia changes when the production planning considers emissions of substances such as nitrogen oxides and carbon dioxide.
In addition, there are some models for special applications that may give deeper insights into the relevance of environmental constraints for production management. [15] analyzed how to achieve optimal allocation of emission reduction measures in the energy production sector in Germany by integration of emission reduction technologies into the model. [3] discussed the growing importance of environmentally responsible manufacturing. He then uses an empirical approach to develop constructs and measures that are critical to the development and growth and research in Environmentally Responsible Manufacturing (ERM) area. [10] presented two mathematical models that can be used by firms to determine their product mix and production quantities in the presence of several different types of environmental constraints, in addition to typical production constraints. [6] offered a general approach to evaluating investments in end-of-pipe-technologies with special regard to an emissions trading scheme. The model considered joint production, activity-level-dependent and -independent payments and specifically includes the indivisibility of the investment. [16] and [19] established an optimal model where the firm can obtain emission permits in three different ways, government quota, market trade and purifying, to determine the firm’s production policy by making a trade-off between them. [13] a modified multi-factor market model has been used to investigate how the development of EU emission allowance (EUA) prices has influenced corporate value.

It can be found that it is of great importance to study the influences of environmental constraints on firms’ production decision. To achieve this end, assumptions on product and/or CEA trade market are very crucial to get reasonable results, which leads to a trend of utilizing more and more complicated models. In this paper, we will try to get meaningful results in a simple way. From the micro-level stand, the impacts from CEA trade market on a single emission-dependent firm are discussed by a simple model originated from Newsboy model. The considered firm is confronted with two markets: the product market characterized by the exogenous price and stochastic demand, and the CEA trading market characterized by the exogenous price. Based on the Newsboy model’s framework, two models are developed by whether there is the existence of CEA trading market. And based on the results of comparative analysis, it is found that the existence of CEA trading market presents more options for the firm to optimize its expected net income. These options make the firm more profitable even the firm reduce its production. Therefore, market-based carbon emission abatement scheme can be considered as an effective way to reduce carbon emission.

2. Production optimization without CEA trading. According to the framework of Newsboy model, our model is designed to include two times. At time 0, the regulator distributes CEA to every emission-dependent firm. At time 1, firms make their own production plan under the carbon emission constraints set by CEA. The product market is assumed as a perfect competition one, and the demand on product is stochastic in the given period, that is, the price of product is assumed to be exogenous and firms are price-takers. There is a CEA trade market where firms can sell or buy CEA if they have surplus or insufficient CEA, and the price of CEA is assumed to be exogenous too.

The notations are listed below.

A: the amount of CEA assigned to a given firm by the regulator at time 0.
Q: the amount of product. It is viewed as a production policy and should be optimized by the firm at time 1 with constraint from CEA, other parameters of product and CEA market.

\( P_m \): the price of product and it is assumed to be exogenous.

\( C_1 \): the production cost for per product and for a given time period. Generally speaking, \( P_m \) is assumed to be greater than \( C_1 \).

\( C_2 \): the storage cost for per product for a given time period when oversupply occurs.

\( C_3 \): the stock loss for per product for a given time period when demand exceeds supply.

\( r (r \geq 0) \): the stochastic demand on product at time 1. And its probability density function is set to be \( \varphi (r) \) and the probability distribution function is set to be \( \Phi (r) = \int_0^r \varphi (r) \, dt \). Its expected value and variance are denoted as \( \mu \) and \( \sigma^2 \) individually.

\( E_m \): the amount of carbon emission for producing one product. It usually can be normalized to one for convenience. Hence, the total carbon emission is \( QE_m \) when the production is assumed to be \( Q \).

\( P_c \): the price of CEA in the CEA trade market. It is assumed to be exogenous as well. But in order to specify the influences of CEA trading on the firm’s production policy, its variation will be discussed too.

The firm needs to make its production plan under the carbon emission constraint when there is no CEA trade market. According to Newsboy model, the profit maximization model for the firm is established as

\[
\max \Pi (Q) = P_m \min \{Q, r\} - C_1 Q - BC_2 (Q - r) - (1 - B) C_3 (r - Q) \tag{1}
\]

\[
s.t. \quad 0 \leq QE_m \leq A \tag{2}
\]

where \( \Pi (Q) \) is the net income. \( B \) is binomial variable if \( Q \geq r \) then \( B = 1 \); otherwise, \( B = 0 \). The first expression on the right-hand side of (1) is gross income, the second one is the production cost, the third one is storage cost and the fourth one is the stock cost.

The net income is a stochastic variable because of randomness of product demand \( r \). Therefore, the expected net income (ENI) should be calculated, and the result (see Appendix) is

\[
E[\Pi (Q)] = k_1 Q - C_3 \mu - k_2 \mathcal{M} (Q) \tag{3}
\]

where \( k_1 = P_m - C_1 + C_3, \quad k_2 = P_m + C_2 + C_3 \) and \( \mathcal{M} (Q) = \int_0^Q \Phi (r) \, dr \).

It is easy to find that \( E[\Pi (Q)] \) is a continuous function with respect to \( Q \). According to the first order condition, the ENI-maximizing decision is

\[
Q_1^* = \min \left\{ \frac{A}{E_m}, Q_1 \right\} \tag{4}
\]

where

\[
Q_1 = \Phi^{-1} \left( \frac{k_1}{k_2} \right) \tag{5}
\]

Therefore, the optimal amount of product is \( A/E_m \) when the carbon emission is constrained by CEA and the optimal amount is \( Q_1 \) when it is not. To be clear, the optimal decision is depicted in Figure [ ]

From Figure [ ] the firm’s optimal decision on amount of product under the constraint of CEA can be classified into two cases. In case I, the ENI-maximizing amount of product is less than CEA limited amount \( A/E_m \), and the optimal decision is
$Q_1^* = Q_1$. In this case the CEA does not limit firm’s production, so the market and firm producing parameters determinate firm’s optimal amount of product. Therefore, CEA is enough for the firm to maximize its ENI. In case II, the ENI-maximizing amount of product is larger than the CEA limit. However, because of limit of CEA the optimal decision can only be $Q_1^* = A/E_m$, and the CEA limit is functional for the firm. In this case, the firm’s ENI does not meet its optimal level $Q_1$. And its ENI can be leveled up if the CEA is larger or there is a CEA trade market where the firm can buy the needed CEA to increase its profit.

From another side, the firms can be classified into two types according to (4). Firms in Type I have surplus CEA and their optimal amount of production is smaller than CEA limited amount. Firms in Type II are short of CEA and their optimal amount of production are limited by the CEA constraint. The two types of firms can be looked as potential suppliers and buyers of CEA, which provides the foundation of market-based scheme.

3. **Production optimization with CEA trading.** When there is a CEA trade market, the firm can sell or buy the surplus or insufficient CEA to maximize its ENI. Hence, the existence of CEA trade market makes the CEA become a source of cost or profit for the firm, and the firm must take its influences into the consideration of production policy. Under this situation, the firm’s profit maximizing problem is

$$\max \Pi(Q) = \begin{cases} \varpi_1(Q), & Q > 0 \\ \varpi_2(Q), & Q = 0 \end{cases}$$

where

$$\varpi_1(Q) = P_m \min\{Q, r\} - C_1Q - BC_2(Q - r) - (1 - B)C_3(r - Q) + P_e(A - QE_m)$$

$$\varpi_2(Q) = AP_e - C_3r$$

![Diagram](image_url)

**Figure 1.** Optimal decision making without CEA trading
Similarly as ongoing the firm’s ENI can be calculated and the result is

$$E[\Pi(Q)] = \begin{cases} 
E[\pi_1(Q)] = k_1 Q - \mu C_3 \\
- k_2 M(Q) + P_c (A - Q E_m), & Q > 0 \\
E[\pi_2(Q)] = A P_c - C_3 \mu, & Q = 0
\end{cases}$$

As

$$\lim_{Q \to 0^+} E[\pi_1(Q)] = A P_c - C_3 \mu = E[\pi_2(Q)]$$

so the firm’s ENI can be unified as

$$E[\Pi(Q)] = k_1 Q - \mu C_3 - k_2 M(Q) + P_c (A - Q E_m)$$

(6)

To maximize ENI, it can be found that $E[\Pi(Q)]$ may have two shapes, as shown in Figure 2 based on the first-order and second-order condition.

**Figure 2.** Production decision-making with CEA trading

In Case I, $E[\Pi(Q)]$ is a monotonous decreasing function when $Q \geq 0$. Hence, the optimal decision for the firm is not to produce any products to achieve the expected income as $A P_c - C_3 \mu$ by selling its CEA in the corresponding market. To clarify, there exists

$$\frac{dE[\Pi(Q)]}{dQ} \bigg|_{Q=0} \leq 0$$

so

$$k_1 - k_2 \Phi(Q) - P_c E_m \leq 0 \implies P_c E_m \geq k_1$$

The above equation states that if $P_c E_m \geq k_1$ holds then the firm will not produce any products. Hence, we call it as the condition for the firm not to produce any products.

In Case II, $E[\Pi(Q)]$ is a concave function when $Q \geq 0$. Therefore, there is a single point $Q_2$ to maximize the firm’s ENI. According to the first order condition, the optimal value is

$$Q_2 = \Phi^{-1} \left( \frac{k_1 - P_c E_m}{k_2} \right)$$

(7)

In this case, it is more profitable for the firm to produce products. The firm will increase its profit in product market at the cost of consuming some own CEA or
buying some CEA if necessary. So the condition for the firm to produce products is
\[
\frac{dE[\Pi(Q)\mid Q]}{dQ}\bigg|_{Q=0} > 0
\]
that is
\[P_cE_m < k_1\]
So, if \(P_cE_m < k_1\) holds the firm will produce products. Hence, we call it as condition for the firm to produce products.

To sum up the above two cases, the optimal decision for the firm with CEA trading is
\[
Q^*_2 = \begin{cases} 
0, & P_cE_m \geq k_1 \\
Q_2, & P_cE_m < k_1
\end{cases}
\]  
(8)

In the above equation, \(P_cE_m\) is the loss in CEA trade market for the firm to produce one product, which can be considered as the opportunity cost of producing one product in CEA trade market. \(P_m - C_1\) is the net profit for the firm to produce one product and \(C_3\) is the reduced loss for the firm to produce one product. Hence, \(k_1 = P_m - C_1 + C_3\) can be looked as opportunity income in product market for the firm to produce one product. When the opportunity loss in the CEA trade market is larger than the opportunity income in the product market (Case I, \(P_cE_m \geq k_1\)), it is not profitable for the firm to produce products (so \(Q^*_2 = 0\)). On the other side, when the opportunity loss in the CEA trade market is less than the opportunity income in the product market (Case II, \(P_cE_m < k_1\)), it is profitable so the firm will produce products (so \(Q^*_2 = Q_2\))

4. Influences of CEA trading on production optimization. In this section, the influences of CEA trading on the firm’s production decision will be analyzed from the views of amount of production and ENI respectively, which are stated as theorems as below.

**Theorem 4.1.** If the firm is not limited by the CEA constraint, then \(Q^*_1 \geq Q^*_2\) exists; If the firm is limited by the CEA constraint, and if \(P_cE_m \leq k_1 - k_2\Phi(A/E_m)\), then \(Q^*_1 \leq Q^*_2\), and if \(k_1 \geq P_cE_m \geq k_1 - k_2\Phi(A/E_m)\), then \(Q^*_1 \geq Q^*_2\).

**Proof.** As \(\Phi(r)\) is a probability distribution function, so it is a increasing function and \(\Phi^{-1}(r)\) is a increasing function too. Therefore, it is easy to find that \(Q_2 < Q_1\) exists.

If the firm is not limited by the CEA constraint, then \(Q^*_1 = Q_1\) and \(Q^*_2 = 0\) or \(Q^*_2 = Q_2\), so it can be concluded that \(Q^*_1 \geq Q^*_2\) stands in this case.

If the firm is limited by the CEA constraint, then \(Q^*_1 = A/E_m\). And if \(P_cE_m \leq k_1 - k_2\Phi(A/E_m)\), then it can be deduced that
\[
\Phi\left(\frac{A}{E_m}\right) \leq \frac{k_1 - P_cE_m}{k_2}
\]
that is
\[
\frac{A}{E_m} \leq \Phi^{-1}\left(\frac{k_1 - P_cE_m}{k_2}\right) = Q^*_2
\]
Hence, in this case \(Q^*_1 \leq Q^*_2\) holds.

If \(k_1 \geq P_cE_m \geq k_1 - k_2\Phi(A/E_m)\), then
\[
P_cE_m \geq k_1 - k_2\Phi\left(\frac{A}{E_m}\right) \Rightarrow \frac{A}{E_m} \geq \Phi^{-1}\left(\frac{k_1 - P_cE_m}{k_2}\right) = Q^*_2
\]
As we know that $Q_1^* = A/E_m$ in this case, so we can come to the conclusion that $Q_1^* > Q_2^*$ exists.

To be clear, Theorem 4.1 is depicted in Figure 3.

\[\begin{align*}
Q_2^* & \geq A/E_m \\
Q_2^* & < A/E_m \\
Q_2^* & = 0
\end{align*}\]

\[\begin{align*}
A/E_m & \\
Q_1^* & \leq Q_2^* \\
Q_1^* & > Q_2^*
\end{align*}\]

\[\begin{align*}
0 & \leq k_1 - k_2 \Phi(A/E_m) \\
k_1 & \leq P_c E_m
\end{align*}\]

\textbf{Figure 3.} Influence of CEA trading on firm’s decision on production plan

From Theorem 4.1 it can be concluded that

1) The optimal decision of amount of production is decreasing with respect to $P_c E_m$ if there is a CEA trade market. When $P_c E_m$ is increasing which means that the opportunity loss in the CEA trade market is increasing, the firm tends to decrease its production (so the carbon emission is decreasing too) to increase profit or reduce the loss from CEA trade market.

2) The CEA trade market can promote abatement of carbon emission more efficiently. When the firm is limited by the CEA constraint, Theorem 4.1 shows that $Q_1^* > Q_2^*$ which means the optimal production is less if there is a CEA trade market. In this case, the emission is reduced too. When the firm is limited by the CEA constraint, the firm has two options. If the product market is more profitable than CEA trade market ($P_c E_m \leq k_1 - k_2 \Phi(A/E_m)$), the firm will buy some CEA to increase production to pursue more profits. It seems that the carbon emission is increased in this case, however the optimal production is still less than $Q_1^*$ actually. Hence, it can be thought that as CEA market has a potential suppression function to carbon emission. On the other side, if CEA market is more profitable than product market ($k_1 \geq P_c E_m \geq k_1 - k_2 \Phi(A/E_m)$), the firm will reduce its production (emission) to save some CEA to sell them in CEA market to make more profits.

Theorem 4.1 clarifies the influence of the existence of CEA trade market on firm’s optimal production plan. The following theorem will show its influence on firm’s ENI.

Let $\Delta \Pi$ be the amount of discrepancy of ENI if there is a CEA trade market or not. Hence,

\[
\Delta \Pi = E[\Pi(Q_2^*)] - E[\Pi(Q_1^*)] = k_1 \Delta Q - k_2 \Delta M + (A - Q_2^* E_m)P_c \tag{9}
\]

where $\Delta Q = Q_2^* - Q_1^*$ and $\Delta M = M(Q_2^*) - M(Q_1^*)$. 

\[\begin{align*}
p_c & = \frac{k_1}{E_m} \\
\Phi & = \frac{1}{A/E_m}
\end{align*}\]
As for $\Phi(r)$ is a probability function and it is continuous, and according to mean value theorem of integrals,

$$\Delta \mathcal{M} = \mathcal{M}(Q_2^*) - \mathcal{M}(Q_1^*) = \int_{Q_1^*}^{Q_2^*} \Phi(r) \, dr = \Phi(r_0) \Delta Q$$

where $Q_1^* \leq r_0 \leq Q_2^*$ can be looked as a real value of product market demand. Hence, (9) can be rewritten as

$$\Delta \Pi = [k_1 - k_2 \Phi(r_0)] \Delta Q + (A - Q_2^* E_m) P_c$$

From above equation, it can be found that the amount of discrepancy of ENI is composed of two parts. The first part, $[k_1 - k_2 \Phi(r_0)] \Delta Q$, is the difference from product market. And the second part, $(A - Q_2^* E_m) P_c$, is the difference from CEA trade market. Therefore, the total discrepancy of ENI is the result of trade-off between the influence from two markets.

**Theorem 4.2.** The firm's ENI will not be decreased by CEA constraint if there is a CEA market, i.e., $\Delta \Pi \geq 0$.

**Proof.** According to Theorem 4.1, there are three kinds relationship between $Q_1^*$ and $Q_2^*$, so we will calculate the discrepancy of ENI for different kinds of relationship.

1) If $Q_2^* \geq Q_1^*$, then $\Delta Q \geq 0$ and

$$Q_1^* \leq r_0 \leq Q_2^* \Rightarrow \Phi(Q_1^*) \leq \Phi(r_0) \leq \Phi(Q_2^*)$$

Therefore,

$$\Delta \Pi = [k_1 - k_2 \Phi(r_0)] \Delta Q + (A - Q_2^* E_m) P_c$$

$$\geq [k_1 - k_2 \Phi(Q_2^*)] \Delta Q + (A - Q_2^* E_m) P_c$$

$$= P_c E_m \Delta Q + \left( \frac{A}{E_m} - Q_2^* \right) P_c E_m$$

$$= P_c E_m \left( \frac{A}{E_m} - Q_1^* \right)$$

$$\geq 0$$

2) If $0 < Q_2^* \leq Q_1^*$, then $\Delta Q \leq 0$ and

$$Q_2^* \leq r_0 \leq Q_1^* \Rightarrow \Phi(Q_2^*) \leq \Phi(r_0) \leq \Phi(Q_1^*)$$

Therefore,

$$\Delta \Pi = [k_1 - k_2 \Phi(r_0)] \Delta Q + (A - Q_2^* E_m) P_c$$

$$\geq [k_1 - k_2 \Phi(Q_2^*)] \Delta Q + (A - Q_2^* E_m) P_c$$

$$= P_c E_m \Delta Q + \left( \frac{A}{E_m} - Q_2^* \right) P_c E_m$$

$$= P_c E_m \left( \frac{A}{E_m} - Q_1^* \right)$$

$$\geq 0$$
3) If $Q'_2 = 0$, then $P_c E_m \geq k_1$ and

$$
\Delta \Pi = -k_1 Q'_1 + k_2 M(Q'_1) + AP_c \\
\geq -k_1 Q'_1 + k_2 M(Q'_1) + k_1 \frac{A}{E_m} \\
= k_1 \left( \frac{A}{E_m} - Q'_1 \right) + k_2 M(Q'_1) \\
\geq 0
$$

To sum up, $\Delta \Pi \geq 0$ holds. \hfill \square

Theorem 4.2 shows that CEA trading scheme is an efficient instrument to promote abatement of carbon emission in our framework. It has been thought that abatement of carbon emission is not beneficial to firms because CEA will put a limit on the firm’s profit-maximizing production. However, according to Theorem 4.2, if the regulator establishes a CEA trade market at the same time when he puts an emission limit on firms, the firm’s profit will not be decreased compared with the case where there is only emission constraint. In fact, CEA trade market provides a option for a firm to make production decision, which can be viewed as the reason of increase of ENI. Therefore, CEA trading scheme can be beneficial to the firm and the regulator at the same time. However, we have to clarify that the result is based on our limited framework, and it may not true when the market is incomplete and imperfect. More general considerations can be found in [6].

5. **Numerical examples.** In this section, some numeric examples are provided to verify conclusions achieved before in this paper. The demand of product is set to be a random variable with a normal distribution as $r \sim N(250, 25^2)$. And other parameters are set to be $C_1 = 10, C_2 = 5, C_3 = 15, E_m = 1$.

5.1. **Influence of CEA’s price when the firm is limited by CEA constraint.**

Let $A = 150$ and $P_m = 60$, and price of $P_c$ is set to be 0, 30, 50, 70, 90 respectively. All calculations are running in Maple 14 software. Based on the above parameters, the optimal decision on production are plotted in Figure 4 and listed in Table 1.

![Figure 4. Influence of CEA’s price when production limited by CEA constraint](image-url)
Table 1. Influence of CEA’s price when production limited by CEA

| $P_c$ | $Q^*_1$ | $E[\Pi(Q^*_1)]$ | Limited by CEA | $Q^*_2$ | $E[\Pi(Q^*_2)]$ | Producing |
|-------|---------|-----------------|---------------|---------|-----------------|-----------|
| 0     | 150.00  | 5999.99         | Y             | -       | -               | -         |
| 30    | 150.00  | 5999.99         | Y             | 246.07  | 8711.93         | Y         |
| 50    | 150.00  | 5999.99         | Y             | 227.82  | 6961.68         | Y         |
| 70    | 150.00  | 5999.99         | Y             | 0.00    | 6570.00         | N         |
| 90    | 150.00  | 5999.99         | Y             | 0.00    | 6570.00         | N         |

From the above results, it can be found that

1) When there is no CEA trade market ($P_c = 0$), the firm’s optimal decision is equivalent to the CEA limited production $A/E_m = 150$. And if there is no CEA constraint, the optimal production should be 272.18 and corresponding ENI should be 11961.68. Therefore, CEA constraint prevents the firm from achieving its optimal ENI.

2) When there is existence of CEA trade market ($P_c > 0$), and if price of CEA is lower (when $P_c = 30$ and $P_c = 50$), the firm starts to produce products ($Q^*_2 = 246.07$ and $Q^*_2 = 227.82$). It should be noticed that the production is still less than the production without CEA constraint ($Q = 272.18$) as Theorem 4.1 stated. And it also expresses that CEA constraint has suppression function on the firm’s production. When price of CEA increases (when $P_c = 70$ and $P_c = 90$), the firm will stop produce real products instead of making profits by selling CEA in CEA trade market.

3) From Table 1, the firm’s ENI is always higher when there is existence of CEA trade market, which proves that Theorem 4.2 is correct.

5.2. Influence of CEA’s price when the firm is not limited by CEA constraint. Based on the current parameters, if the regulator loosens up CEA constraint, for instance let $A$ be 300, then the firm will not be limited by CEA constraint. Figure 5 and Table 2 are the results in this situation.

Figure 5. Influence of CEA’s price when the firm is not limited by CEA constraint

From the above results, it can be found that when there is no CEA trade market ($P_c = 0$), the firm’s optimal production is 272.18 and the corresponding ENI is
Table 2. Influence of CEA’s price when the firm is not limited by CEA constraint

| $P_c$ | $Q_1^*$ | $E[\Pi(Q_1^*)]$ | Limited by CEA | $Q_2^*$ | $E[\Pi(Q_2^*)]$ | Producing |
|-------|---------|-----------------|----------------|---------|-----------------|-----------|
| 0     | 272.18  | 11961.68        | N              | -       | -               | -         |
| 30    | 272.18  | 11961.68        | N              | 246.07  | 13211.93        | Y         |
| 50    | 272.18  | 11961.68        | N              | 227.82  | 14461.68        | Y         |
| 70    | 272.18  | 11961.68        | N              | 0.00    | 17250.00        | N         |
| 90    | 272.18  | 11961.68        | N              | 0.00    | 23250.00        | N         |

When there is existence of CEA trade market and if the price of CEA is relatively lower (when $P_c = 30$ and $P_c = 50$), the firm’s optimal production is cut down to 246.07 and 227.82. However its ENI is raised up to 13211.93 and 14461.68 correspondingly. And when the price of CEA goes more higher ($P_c = 70$ and $P_c = 90$), the condition for the firm not to produce any products is satisfied and the firm will stop produce any products and make profits by selling CEA. But its ENI is still higher.

To sum up, in this case the existence of CEA trade market decreases firm’s production and carbon emission without decreasing its ENI.

5.3. Influence of product’s price when there is existence of CEA trade market. The price of product plays an important role in firm’s production policy too. In this subsection, we will analyse influences of price of product on firm’s production decision when there is existence of CEA trade market. Let $A = 150$ and $P_c = 30$, and the price of product is increasing from 35 to 75 by 10. The results are plotted in Figure 6 and listed Table 3.

![Figure 6. Influence of product price when production limited by CEA constraint](image)

It can be found that from the above results that when there is no CEA trade market, the firm’s optimal production is limited to 150 though the price of product is increasing. Hence, it can be thought that the market failure occurs. However, when there is existence of CEA trade market and the price of CEA is set as a constant, the firm’s optimal production increases with the price of product. So in this case, the market scheme is still working.
Table 3. Influence of product price when production limited by CEA constraint

| $P_m$ | $Q_1^*$ | $E[Π(Q_1^*)]$ | Limited by CEA | $Q_2^*$ | $E[Π(Q_2^*)]$ | Producing |
|-------|---------|----------------|---------------|---------|----------------|-----------|
| 35    | 150.00  | 2249.99        | Y             | 227.29  | 2886.92        | Y         |
| 45    | 150.00  | 2249.99        | Y             | 237.44  | 5178.58        | Y         |
| 55    | 150.00  | 2249.99        | Y             | 243.67  | 7525.61        | Y         |
| 65    | 150.00  | 2249.99        | Y             | 248.16  | 9904.55        | Y         |
| 75    | 150.00  | 2249.99        | Y             | 251.65  | 12304.57       | Y         |

6. Conclusions. CEA has been becoming a very important resource to the firms and it has great influences on the firms’ production policy and operations.

In this paper, based on the Newsboy model framework we introduced CEA constraint into emission-dependent firms’ production decision with random demand in product market. By comparing the optimal production decision and corresponding ENI whether there is existence of CEA trade market, it was found that CEA trading scheme decreases the amount of product and carbon emission without decreasing firms’ ENI, which indicates that CEA trading scheme is an efficient instrument to control abatement of carbon emission. It is necessary to clarify that our model is very simple and the results are limited in the proposed framework, which cannot be assured to be true in more generalized situations on incomplete and imperfect markets.

Appendix. Let $M(Q) = \int_Q^\infty \Phi(r)dr$ and $[x]^+ = \max\{x, 0\}$, then

\[
E[P \min\{Q, r\}] = P \int_0^Q r\varphi(r)dr + P \int_Q^{+\infty} Q\varphi(r)dr \\
= P \int_0^Q r\Phi(r) + P\Phi[1 - \Phi(Q)] \\
= P[r\Phi(r)]_0^Q - P \int_0^Q \Phi(r)dr + P\Phi[1 - \Phi(Q)] \\
= QP - PM(Q)
\]

\[
E[-C_2[Q - r]^+] = -C_2 \int_0^Q (Q - r)\varphi(r)dr \\
= -C_2Q \int_0^Q \varphi(r)dr + C_2 \int_0^Q r\varphi(r)dr \\
= -C_2Q\Phi(Q) + C_2 \int_0^Q r\Phi(r) \\
= -C_2Q\Phi(Q) + C_2[r\Phi(r)]_0^Q - C_2 \int_0^Q \Phi(r)dr \\
= -C_2M(Q)
\]
\[ E[-C_3[r - Q]^+] = -C_3 \int_Q^{+\infty} (r - Q) \varphi(r) dr \]
\[ = -C_3 \int_Q^{+\infty} r \varphi(r) dr + QC_3 \int_Q^{+\infty} \varphi(r) dr \]
\[ = -C_3 \mu + C_3 \int_0^Q r \varphi(r) dr + QC_3 [1 - \Phi(Q)] \]
\[ = -C_3 \mu + C_3 M(Q) \]

Hence,
\[ E[\Pi(Q)] = Q(P - C_1) - (\mu - Q)C_3 - (P + C_2 + C_3)M(Q) \]

Let \( k_1 = P - C_1 + C_3, k_2 = P + C_2 + C_3 \), then it comes to
\[ E[\Pi(Q)] = k_1 Q - C_3 \mu - k_2 M(Q) \]

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