Joint TOA/DOA Estimation Using the SAGE Algorithm in OFDM Systems with Virtual Carriers

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Abstract. In this paper, we investigate the space-alternating generalized expectation-maximization (SAGE) algorithm with virtual carriers (VCs) in orthogonal frequency division multiplexing (OFDM) systems. The channel frequency response (CFR) at VCs cannot be estimated accurately due to edge effect after inverse discrete Fourier transform (IDFT). To solve the problem, an improved channel estimation method is introduced to minimize the errors of CFR via iterative technique. Then we apply the SAGE algorithm to estimate the direction of arrival (DOA) and time of arrival (TOA). The SAGE algorithm shows more excellent performance with higher resolution ability than subspace-based algorithms. Based on Monte Carlo trials, we test the performance of the improved method in terms of mean absolute error (MAE) at different signal-to-noise ratios (SNR). Simulation results indicate that the improved method behaves better at any SNR than the conventional DFT-based method.

1. Introduction

OFDM is a popular multi-carrier modulation technique for high-speed transmission of data. It is widely applied from theory to practice because of strong anti-interference ability in multipath fading environments.

Channel estimation is an important part for OFDM systems [1]. The three commonly used methods are least squares (LS), minimum mean square error (MMSE) and DFT-based method. The last one shows better performance than the first two methods. However, the OFDM systems normally add VCs for adjacent channel interference (ACI) suppression. It leads to edge effect, then large errors will occur near VCs when conventional DFT-based method is used [2]. Therefore, an improved channel estimation method is presented to reduce estimation errors by constructing artificial CFR values at VCs.

Channel parameters can be estimated by various algorithms. The first kind is subspace-based algorithm which estimates parameters by exploiting the orthogonality between signal subspace and noise subspace. Among the most popular methods are the estimation of signal parameter via rotational invariance techniques (ESPRIT) [3] and multiple signal classification (MUSIC) algorithm [4]. The other kind is an iterative technique based on maximum likelihood (ML) theory. It behaves better at low SNR than the first category when the number of snapshots is small or sources are coherent. Among the maximum likelihood estimation methods, several related algorithms have been proposed to extract channel parameters, such as alternating projection (AP), modified variable projection (MVP), expectation-maximization (EM) and SAGE algorithm [5-8]. The high accuracy and high resolution
ability of SAGE algorithm make it popular for multiple parameters estimation, though it has a higher computational cost than other methods.

In this paper, we present an improved channel estimation method to obtain complete CFR information at all carriers. In order to improve the accuracy of the estimation, the CFR values at VCs need to be initialized to satisfy the continuity condition. Then the values at VCs will get close to real values by iterative technique. The method can reconstruct relatively accurate values at VCs. Furthermore, it reduces the estimation errors caused by noise effect. It can converge quickly after a few iterations and improve the performance in channel estimation and channel parameter estimation. Then the method is applied to joint TOA/DOA estimation using the SAGE algorithm based on the CFR information obtained before.

2. System Model

The OFDM transmitter divides a complex sequence into a parallel data stream of length $N_d$ via $N$ point IDFT. The $N_d$ carriers are actively used to transmit data symbols and nothing is transmitted from $N_v = N - N_d$ carriers.

The transmitted signal can be expressed as

$$x(i) = \text{IDFT}[X_k] = \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} X_k \exp(j2\pi f_k T_i), \quad 0 \leq i \leq N - 1$$ (1)

Where $i$ represents the index of the $i$th sampled signal in time domain, $X_k$ is the modulated signal on the $k$th carrier frequency, $f_k$ is the $k$th carrier frequency, and $T_i$ is the sampling period. The channel impulse response (CIR) is characterized by

$$h(i) = \sum_{l=1}^{L} \alpha_l c(\theta_l) \delta(i - \frac{\tau_l}{T_s})$$ (2)

Where $\tau_l, \alpha_l, \theta_l$ represent TOA, complex amplitude and DOA of the $l$th path, respectively. $L$ and $M$ are the number of multipath and receiver antennas. The vector $c(\theta_l) = [c_1(\theta_l), ..., c_M(\theta_l)]^T$ is the steering vector of antenna array. The $m$th element of $c(\theta_l)$ is given by $c_m(\theta_l) = \exp(j2\pi \lambda^{-1} a^T r_m)$ with $\lambda, r_m, a$ denoting the wavelength, the location vector of the $m$th antenna, and the propagation direction of the $l$th incident wave. The propagation direction is given by

$$a = [\cos \phi \sin \theta, \sin \phi \sin \theta, \cos \theta]^T, \quad (\theta, \phi) \in [0, \pi] \times [0, 2\pi)$$ (3)

Where $[\cdot]^T$ denotes transpose operator. For simplicity, we only consider the elevation angle of the incident waves and the azimuth is fixed at $\phi = 90^\circ$.

Hence, the received signal is

$$y(i) = x(i) \otimes h(i) + w(i)$$

$$= \frac{1}{\sqrt{N}} \sum_{k=0}^{N-1} \left[ \sum_{l=1}^{L} \alpha_l c(\theta_l) \exp(-j2\pi f_k \tau_l) \right] \cdot \exp(j2\pi f_k T_i) + w(i)$$ (4)

Where $\otimes$ represents the circular convolution, and $w(i)$ is the complex white Gaussian noise. The frequency domain signal $R(k)$ on the $k$th carrier can be written as equation (5).

$$R(k) = X_k \sum_{l=1}^{L} \alpha_l c(\theta_l) \exp(-j2\pi \frac{k \tau_l}{NT_s}) + W(k)$$ (5)

It follows from equation (2) that the CFR on the $k$th carrier is
\[
H(k) = \sum_{l=1}^{L} c_l \alpha_l e^{j 2\pi \frac{kl}{NT}}
\]

(6)

The channel frequency response can be easily obtained as follows
\[
\hat{H}(k) = \frac{X_k H(k) + W(k)}{X_k} = H(k) + \frac{W(k)}{X_k}
\]

(7)

Note that equation (7) can only be applied at data carriers and CFR values at VCs are set to zero. In OFDM systems, we adopt block-type pilots or comb-type pilots to estimate channel. If block-type pilots are used, CFR values can be obtained directly at all data carriers; otherwise, the values can be only obtained at pilots, and then interpolation methods are used for other data carriers.

3. An Improved Channel Estimation Method

Since the CFR values are lost at VCs, the continuity condition cannot be met and the energy leaks into all taps in time domain after IDFT. In order to solve this problem, we construct the artificial values via iterative technique at VCs to ensure the continuity.

Therefore, we need to initialize the values at VCs by linear interpolation method using the data on the left and right ends adjacent to virtual carriers. Mathematical expression is given by

\[
\hat{H}(k) = \begin{cases} 
\hat{H}(k), & \forall k \in \Gamma_{\text{data}} \\
\hat{H}(p_b - 1 + k_1) + k_i \cdot \frac{\hat{H}(p_z + 1) - \hat{H}(p_b - 1)}{N_v}, & \forall k \in \Gamma_v, k_1 = 1, 2, \ldots, N_v
\end{cases}
\]

(8)

where \( \Gamma_v, \Gamma_{\text{data}}, p_b, p_z \) represent virtual carriers area, data carriers area, the index of first and last virtual carrier, respectively. After IDFT, the CIR is obtained.

\[
\hat{h}(i) = \text{IDFT}\{\hat{H}(k)\}, \quad i = 0, 1, \ldots, N - 1
\]

(9)

The key step of the improved method is to find the position of \( L \) taps accurately. Since only \( L \) vital taps have energy in practical wireless environments, we need to determine the length of CIR and the position of \( L \) vital taps. In general, the position of the \( L \) significant taps is the index of time delay. Assuming the time delay and angle information are invariant on any receiver antenna, the time domain response information on all antennas is superimposed to find the \( L \) significant taps. The expression is shown in equation (10).

\[
b(k) = \sum_{m=1}^{M} \left| h_{m,k} \right|^2
\]

(10)

Therefore, the CIR values \( \hat{g}(i) \) are generated by retaining the energy of the \( L \) significant taps of \( \hat{h} \) and zeroing the energy of other taps. It is given by

\[
\hat{g}(i) = \begin{cases} 
\hat{h}(i), & \forall i \in \Lambda_L \\
0, & \forall i \not\in \Lambda_L
\end{cases}
\]

(11)

Note that \( h_{m,k} \) \( m = 1, \ldots, M, k = 1, \ldots, N \) is the \( (m,k) \)th element of \( \hat{h}(k) \), and \( \Lambda_L \) is the index of the \( L \) important taps of the vector \( b \). By transforming the time domain impulse response into the frequency domain, we have

\[
\hat{G}(k) = \text{DFT}\{\hat{g}(i)\}
\]

(12)

\[
\hat{H}(k) = \begin{cases} 
\hat{H}(k), & \forall k \in \Gamma_{\text{data}} \\
\hat{G}(k), & \forall k \in \Gamma_v
\end{cases}
\]

(13)

The values at VCs are replaced by equation (13) and other values are retained. Repeat the equation (8) to equation (13) until a certain condition is met. The last iteration is stopped at equation (12), and
then $\tilde{G}(k)$ is modified to $\hat{H}(k)$. The channel frequency response values can be estimated by the improved method. The CFR $\hat{H}(k)$ converges to a stable sequence after almost 6 iterations because the values are initialized at VCs.

4. Channel Parameter Estimation Using the SAGE algorithm

Channel parameters can be extracted if frequency response at each point is known. As an extension of the EM algorithm, the SAGE algorithm shows excellent performance in resolution ability and convergence rate [9].

Equation (7) can be rewritten as follows:

$$\hat{H}(k) = \sum_{l=1}^{L} S(k; \theta_l) + W(k), \quad k = 0, ..., N - 1$$

$$S(k; \theta_l) = \alpha_c(\theta_l) \exp(-j2\pi \frac{k\tau_l}{NT})$$

The SAGE algorithm divides the matrix $\hat{H}$ into several subsets and performs the expectation (E) step and the maximization (M) step while the EM algorithm updates parameters simultaneously. Complete data and incomplete data are two key concepts involved in the EM algorithm. The choice of the complete data should ensure that the parameter estimation process can converge as quickly as possible. A reasonable choice is given by

$$X_l(k) = S(k; \theta_l) + W(k), \quad l = 1, ..., L$$

$$\hat{H}(k) = \sum_{l=1}^{L} X_l(k)$$

Since $X_l(k)$ is unknown in practice, it is generally obtained by the E step from the known data $\hat{H}$. This means that

$$\hat{X}_l(k; \hat{\theta}_l) = \mathbb{E}[X_l(k)|\hat{H}(k), \hat{\theta}_l] = \hat{H}(k) - \sum_{l \neq j \neq l} S(k; \hat{\theta}_j)$$

Where $\hat{\theta}_l$ represents the estimated direction of arrival of the $l$th path. The parameters of multipath components can be estimated by maximizing the cost function which is given by

$$z = c^H(\theta_l) \sum_{k=0}^{N-1} \exp(j2\pi \frac{k\tau_l}{NT}) \hat{X}_l(k; \hat{\theta}_l)$$

$$\tau_l^* = \arg\max_{\tau} \left| z(\tau; \hat{\theta}_l; \hat{X}_l(k; \hat{\theta}_l)) \right|$$

$$\hat{\theta}_l^* = \arg\max_{\theta} \left| z(\tau_l^*; \hat{\theta}_l; \hat{X}_l(k; \hat{\theta}_l)) \right|$$

$$\alpha_l^* = \frac{1}{MN} z(\tau_l^*; \hat{\theta}_l; \hat{X}_l(k; \hat{\theta}_l))$$

Where $[.]^H$ denotes Hermitian operation.

A series of values are generated until the cost function converges to a stable point. The initial values of each path have a great impact on the convergence rate of the iteration. All received signals are sorted in descending order according to the power by successive interference cancellation initialization (SICI) method. It can avoid inaccurate estimation caused by the interference of strong multipath components to weak multipath components.
5. Simulation
A QPSK-OFDM system is considered with 1024 carriers and 600 carriers of them are used for data transmission. The system bandwidth is 10MHz and the length of CP is 128. Sampling frequency is 15.36 MHz, and sampling period is 65.104 ns. The SNR is 10dB. Simulation parameter settings are shown in Table 1.

| Path | Complex Amplitude | DOA / degree | TOA / ns |
|------|-------------------|--------------|----------|
| 1    | 0.17 + 0.9854i    | 75           | 6380.192(98T_s) |
| 2    | -0.5 + 0.866i     | 75           | 6510.4(100T_s)   |
| 3    | -0.6 - 0.8i       | 80           | 6510.4(100T_s)   |

We use the improved channel estimation method to estimate the CFR values at all frequency point on each antenna. An 8-element uniform linear array with half-wavelength spaced sensors is adopted. We take the amplitude of CFR on the first antenna to show the improved method behaves better than the conventional DFT-based method. The results are given in Figure 1. Obviously, the CFR values constructed by the improved method are close to real values in Figure 1(b) while the conventional method has poor performance shown in Figure 1(a). We also test the performance of the improved method comparing with the conventional method in terms of MAE at different SNR in Figure 2. Define the MAE as

\[ MAE(\theta) = \frac{1}{D} \sum_{i=1}^{D} |\hat{\theta}_i - \theta_i| \]  

(21)

Where \( \hat{\theta}_i, \theta_i \) represent the estimated and real values of parameters, respectively. It can be seen from Figure 2 that the improved channel estimation method has lower MAE than conventional method at any SNR.

% Figure 1. Conventional (left) and improved (right) channel estimation methods.

Then the improved channel estimation method is applied to joint TOA/DOA estimation for 500 trials with similar parameter settings except SNR. As is shown in Figure 2-5, the improved method has lower estimation error than the conventional DFT-based method. Also, we can see that although the TOA of the second and the third signal are same, all parameters can still be accurately estimated based on the improved method.

6. Conclusion
An improved channel estimation method for OFDM systems with VCs has been proposed in this paper. It can not only reduce the estimation errors caused by noise, but also avoid the estimation errors emerging from discontinuous points at VCs. Combining with the SAGE algorithm, this method can
also estimate the channel parameters more correctly than the conventional DFT-based method. It can be seen that the waves can be separated even if the DOA or TOA of the incident waves are same. Simulation results show that the improved method has superior performance in channel estimation and channel parameter estimation.

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{fig2.png}
\caption{MAE of CFR.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{fig3.png}
\caption{MAE of complex amplitude.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{fig4.png}
\caption{MAE of TOA.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{fig5.png}
\caption{MAE of DOA.}
\end{figure}

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