Damping vibrations of an underground structure using a three-mass damper

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Abstract. The state of a rigid disk at elastic waves diffraction on it and the possibility of its oscillation suppression using a multi-mass damper was studied in the paper. The operation of a three-mass dynamic oscillation suppression was considered and its application was compared with the operation of a two-mass damper. The effectiveness of the use of multi-mass dampers was estimated by the relative amplitude of disk oscillations and by the width of the frequency zone. It was stated that to suppress disk oscillations in a state of low instability of external impact frequency, the use of single- or two-mass dampers was sufficient. To reduce the disk displacements at unstable frequency of external impact (to cover a wider frequency zone), it is necessary to use a three-mass damper; the width of the damper effective operation significantly increases (about 4-5 times) compared to the operation of a two-mass damper.

1. Introduction
In [1], the oscillation in the elastic space of an underground structure (hard disk) during diffraction of elastic harmonic waves on it is considered. In [2], the dynamic behavior of the soil was studied, and in [3, 4, 5], the stress state and dynamic characteristics of earth dams under various dynamic influences were investigated. Wave processes in determining the mechanical characteristics of the soil are considered in [6, 7]. Oscillation of an elastic half-space with a cylindrical cavity under the action of Rayleigh waves is considered in [8]. Vibrations of a rigid cylindrical disk in elastic space are studied in the article [9].

Since it is important to reduce the vibration amplitude of structures, in [10] the possibilities of reducing the vibration amplitude of a shallow underground structure from the action of Rayleigh waves are considered, in [11] - the possibility of reducing the vibration amplitude of a deeply buried underground structure from the action of elastic waves. The effectiveness of a shock absorber with bending vibrations of straight rods was studied in [12]. The effectiveness of the use of multi-mass dynamic vibration dampers under harmonic external influences was studied in [13, 14, 15, 16, 18]. The efficiency of using a two-mass dynamic damper under a periodic impulse disturbance was investigated in [17], and in [19, 20], the possibilities of damping the vibrations of chimneys and high-rise buildings using multi-mass dynamic absorbers were considered. In all these works, the effectiveness of the use of multi-mass vibration dampers is noted.
The use of multi-mass dynamic vibration dampers to reduce the amplitude of displacements of underground structures is interesting and relevant. In the practice of underground structures construction, various structural designs (transport, communication, etc.) are used, mostly of annular or circular cross section. Underground transport structures (tunnels), depending on the intensity of road traffic, have a diameter of 10 m to 18 m, the thickness of the bearing part - 0.4 - 1 m. Such structures can be located at different depths from the ground surface. There are underground structures of shallow laying located at a depth $H$ of 10-12 m, and of deep laying at $H> 12$ m.

Communication underground structures are mostly of annular cross section. In practice, the designs made of different materials (ceramic, metal, concrete, reinforced concrete, asbestos, plastic) and of different diameter are used. As a rule, the operating underground structures have a sufficiently rigid cross section, which allows their sections consider as undeformable ones in calculations.

Dynamic effects on structures buried in soil are caused by various sources, some of them are of industrial origin, others are the influence of transport operation. So, when trains pass through tunnels, vibrations occur and the wave effect affects nearby underground and ground buildings and structures. Or vice versa, dynamic operation of certain ground-based industrial equipment (hammers, crusher, etc.) causes the vibration of underground structure, and, when located in the vicinity, adversely affects its operating mode.

In order to avoid vibration propagation resulting from the cars motion in tunnels to the neighboring buildings and structures, underground structures must have an increased vibration absorption. Sometimes it is necessary to protect the structure itself from external influences. The article proposes one of the most effective and reliable measures to protect the structure from various kinds of dynamic effects – oscillation suppression using multi-mass dynamic vibration dampers. A comparison of single-mass, two-mass and three-mass dampers operation was carried out. The rigid disk oscillation suppression at elastic harmonic waves diffraction on it was considered in [1], where it was shown that by attaching a conventional single-mass vibration damper, a significant damping effect could be achieved. The use of multi-mass dampers to suppress oscillations of a system with one or two degrees of freedom significantly increases the damping efficiency and extends the frequency range in which the use of a damper is appropriate [2].

2. Methods

In this paper, the influence of a multi-mass damper is considered in the graphs of the frequency response characteristic (FRC) of the system. The maximum ordinate of the frequency response is the criterion for the suppression effectiveness. Attenuation in dampers is taken into account according to the Voigt theory (Figure 1).

\[ \text{Figure 1. Rigid disk oscillation at wave diffraction} \]

The system of differential equations of disk vibration equipped with a three-mass vibration damper has the form

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\[
\begin{align*}
    m_0 \frac{d^2 u}{dt^2} + m_1 \frac{d^2 u_1}{dt^2} + m_2 \frac{d^2 u_2}{dt^2} + m_3 \frac{d^2 u_3}{dt^2} = & \int_0^{2\pi} \left[ \sigma_r \cos \theta - \tau_{r\theta} \sin \theta \right] e^{iat} \cdot rd\theta, \\
    m_1 \frac{d^2 u_1}{dt^2} + m_2 \frac{d^2 u_2}{dt^2} + m_3 \frac{d^2 u_3}{dt^2} + \gamma_1 \left( \frac{du_2}{dt} \cdot \frac{du}{dt} \right) + k_1 (u_2 - u) = & 0, \\
    m_2 \frac{d^2 u_2}{dt^2} + \gamma_2 \left( \frac{du_1}{dt} \cdot \frac{du}{dt} \right) + k_2 (u_2 - u_1) = & 0, \\
    m_3 \frac{d^2 u_3}{dt^2} + \gamma_3 \left( \frac{du_3}{dt} \cdot \frac{du}{dt} \right) + k_3 (u_3 - u_1) = & 0,
\end{align*}
\]

where \( m_0, m_1, m_2, m_3 \) are the masses of the disk and the dampers, respectively, \( u, u_1, u_2, u_3 \) are the displacements of the disk and the dampers, respectively, \( \sigma_r, \tau_{r\theta} \) - are the normal and shear stresses of the medium at the side surface of a cylindrical structure, \( r_o \) is the radius of the disk cross section.

Omitting the intermediate calculations, represent the displacement of the disk with a damper in the form

\[
    u = \left[ \frac{4}{\pi} m H^{(2)}_2 (\alpha) \right] / \alpha [ H^{(2)}_1 (\alpha) H^{(2)}_1 (\alpha) A \left( \alpha H^{(2)}_1 \alpha \right) ] (1-c) + \alpha H^{(2)}_2 (\alpha) H^{(2)}_2 (\alpha) c, \quad (2)
\]

where \( H^{(2)}_a (z) \) is the Hankel function of the second kind;

\[
    m = \frac{c_1}{c_2}, \quad \alpha = \frac{\omega \rho}{c_1},
\]

\( c_1, c_2 \) are the velocities of longitudinal and transverse waves, \( b \) is the ratio of the disk linear mass to the reduced mass of soil, determined by the formula

\[
    b = \frac{m_0}{\rho r_o^2},
\]

\( c_1, c_2, c_3 \) are the relative masses of the primary and secondary dampers,

\[
    C = \frac{b}{\pi} \left[ 1 + v_1 (A_1 + A_2 \cdot v_2 + A_3 v_3) \right],
\]

\[
    v_1 = \frac{m_1}{m_0}, \quad v_2 = \frac{m_2}{m_1}, \quad v_3 = \frac{m_3}{m_1}, \quad f_1 = \frac{k_i}{m_i \omega_0^2}, \quad \mu_i = \frac{y_i}{m_i \omega_0^2},
\]

\[
    A_1 = (f_1 + i \mu_1 q) / \left[ \left( U_1 - v_2 U_2 + v_2 U_2 q^2 \right) + i \left( V_2 V_3 + v_3 V_3 \right) q^3 \right],
\]

\[
    A_2 = A_1 (U_2 - i V_2), \quad A_3 = A_1 (U_3 - i V_3) \quad U_1 = [(f_2^2 - q^2) + (\mu_2 q) q^3],
\]

\[
    U_2 = [(f_2^2 - q^2) f_2 + (\mu_2 q) q^3], \quad V_2 = [\mu_2 q] / [(f_2^2 - q^2) + (\mu_2 q) q^3],
\]

\[
    U_3 = [\mu_2 q] / [(f_3^2 - q^2) + (\mu_2 q) q^3], \quad V_3 = [\mu_2 q] / [(f_3^2 - q^2) + (\mu_2 q) q^3],
\]

\[
    f_1, f_2, f_3 - \text{the oscillation frequencies of masses } m_1, m_2, m_3, \text{ respectively.}
\]

When optimizing the parameters of vibration dampers, the structure parameters are considered constant; the damper parameters \( f, \mu \) will be optimized. The width of the resonance zone is taken within \( \alpha = 0.05 - 0.40 \), which simplifies the optimization of the dampers parameters.

3. Results and Discussion

In the case of a single-mass damper (\( \theta_1 = 0 \)) with broadband optimization, the resulting effect is negligible. So, at \( b = 10, \theta_1 = 0.01 - 0.10 \), the efficiency coefficient is \( K_e = 1.03 \), and at \( b = 20, \theta_1 = 0.01 - 0.10, K_e = 1.02 - 1.06 \), respectively. To achieve a greater damping effect, the frequency range should be reduced, i.e. to conduct optimization not over the entire frequency domain, but only over some part of it, near the resonance zone (Table1).
| b  | θ₁ | Optimal parameters | Damping coefficient, Kₑ |
|----|----|--------------------|--------------------------|
| 10 | 0.05 | 0.998           | 0.192                    | 1.04          |
| 0.10 | 0.997 | 0.192 | 1.08 |
| 20 | 0.01 | 0.99          | 0.190                    | 1.02          |
| 0.05 | 0.984 | 0.160 | 1.08 |
| 0.10 | 0.989 | 0.144 | 1.18 |

The table shows that the damping coefficient Kₑ is 3-4 times greater than in the broadband version, in which the selected parameters of the damper (f₁, µ₁) depending on b and θ₁ have a fairly wide range of variation (f₁ = 0.98 – 0.998, µ₁ = 0.14 – 0.19). The so-called narrow-band version makes it possible to damp the disk oscillations in the resonance zone, i.e. at a small deviation of the impact frequency from the frequency of natural oscillations. This approach is usually called optimization with low instability in frequency. Instability can be different - 2.5%, 5%, 10%. In this case, the partial frequency of the damper varies approximately in the same range (0.96 - 0.995). Figure 2 shows the dependence of the damping coefficient on the relative mass of a damper. As seen from the graph, for small values of b, between Kₑ and θ₁ there is an almost linear dependence.

From the optimization results it follows that for the systems where there is high energy dissipation, the partial frequency should be 15-25% lower than the frequency of the structure, and the attenuation in the damper should be quite high - 0.1 - 0.2. At ± 5% instability, the damping effect reaches 25%. It was stated that even at small instability in frequency and small b, the damping coefficient is small, and in soils of less density (b=20), the use of dampers gives a good effect. As noted in [2], the use of a two-mass damper allows expanding the range of effective vibration damping. When using a two-mass damper, the main damper is m₁, connected by elastic links to the protected system. An additional (constructional) damper m₂, having a sufficiently small mass, is connected consecutively with the main damper.

The damped frequency range is greater for the case of a two-mass damper (θ₂(0) (Figure 4) than for a single-mass damper (θ₂ =0) (Figure 3). With increasing θ₂ the damping effect increases. The coefficient Kₑ increases to θ₂ =0.05, and at θ₂ > 0.05 the coefficient of efficiency Kₑ begins to decrease.

**Figure 2. Damping coefficient versus damper mass**
Figure 3. FRC for a disk with a damper. \( b = 10; \) 1 - \( \vartheta_1 = 0, \) 2 - \( \vartheta_1 = 0.01, \) 3 - \( \vartheta_1 = 0.03, \) 4 - \( \vartheta_1 = 0.05, \) 5 - \( \vartheta_1 = 0.10 \)

Figure 4. FRC for a disk with a damper. \( b = 10, \) \( \vartheta_2 = 0.05; \) 1 - \( \vartheta_1 = 0, \) 2 - \( \vartheta_1 = 0.01, \) 3 - \( \vartheta_1 = 0.05, \) 4 - \( \vartheta_1 = 0.10 \)

Figure 5. FRC for a disk with a damper. \( b = 20. \) 1 - \( v_1 = 0, v_2 = v_3 = 0; \) 2 - \( v_1 = 0.05, v_2 = v_3 = 0; \) 3 - \( v_1 = 0.05, v_2 = v_3 = 0.01 \)
Present the results of a three-mass damper to suppress the amplitude of disk displacement oscillations. Here, the total mass of the damper system for all compared options is assumed to be the same and \( f_2 = f_3 \). The structure of a three-mass damper is formed by attaching two additional masses \( (m_2 \text{ and } m_3) \) to the main damper \( m_1 \) (Figure 1b).

Calculations show that the difference in the effects of damping the amplitude of the disk oscillations with three-mass damper and single-mass damper is small. However, the three-mass damper significantly expands the frequency zone of effective damping (Figure 5).

The masses \( m_2 \text{ and } m_3 \) of the dampers work effectively in the entire frequency domain in the case \( f_2 = 0.95 \lt f_1 \) (figure 6). But if it is possible to reduce the frequency interval, then the damper is more efficient when \( f_2 \gt f_1 \). In this case, the damping result is improved by 6-15%.

![Figure 6. FRC for a disk with a damper.](image)

In the case of similar setting of all masses, i.e. when \( f_1 = f_2 = f_3 \), the whole system works as a single-mass damper. The same conclusion can be drawn when \( f_1 \) changes at fixed, \( f_2, f_3 \) (Figure 7).

![Figure 7. FRC for a disk with a damper.](image)

A comparison of the performance of different vibration dampers is presented in Table 2.
**Table 2. Comparison of damper systems**

| Type of damper system | $b$ | $v_1$ | Parameters of the main damper | Damping effect | By the amplitude of displacements | By the width of damped frequencies *, % |
|-----------------------|-----|-------|-------------------------------|---------------|-----------------------------------|---------------------------------------|
| Single mass damper    | 20  | 0.05  | 0.984 0.144                  | 1.08          | -                                 |                                       |
| Two-mass damper with consecutive mass connection | 20  | 0.05  | 0.905 0.124                  | 1.13          | 6                                 |                                       |
| Three-mass damper     | 20  | 0.05  | 1.00 0.24                    | 1.16          | 40                                |                                       |

* Compared to a single-mass damper.

4. Conclusions
Calculations show that the difference in the effects of damping the amplitude of disk oscillations by three-mass and single-mass absorbers is small. A three-mass absorber significantly expands the frequency zone of effective damping (Figure 5). Table 2 shows that the three-mass damper almost seven times expands the frequency attenuation zone compared to the two-mass one. Since the influence of the masses $m_2$ and $m_3$, was studied, their parameters changed in calculations, and for $m_3$, the quantities $v_1$, $f_1$, $\mu_1$ were taken constant. With increasing mass $m_1$, the damping effect of the oscillation amplitude improves. Therefore, if it is necessary to reduce the amplitude of disk oscillations, then it is necessary to optimize the mass parameter $m_1$, and if to expand the range of frequencies of the damper effective operation — it is necessary to optimize the mass parameters $m_2$ and $m_3$.

The masses $m_2$ and $m_3$ effectively work in the entire frequency domain in the case $f_2 < f_1$ (Figure 6). But if it is possible to reduce the frequency interval, then the damper is more effective provided $f_2 > f_1$. In this case, the quenching result is improved by 6-15%. In the case of the same setting of all masses, the entire system works as a single-mass absorber.

It should only be noted that the damper will operate effectively in the case with a stable impact frequency close to the eigenfrequency of the disk, which is almost similar for both cases.

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