Development of metacognitive skill in teaching mathematics

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Abstract

The study of metacognitive processes in the context of training and education is currently undoubtedly a priority in metacognitive psychology as a whole. The importance of this group of studies is due in many respects to the large representation of the class of processes in question in educational activities. At the same time, work in this area is not limited to a sample of primary school children, as this has been a significant amount of time. In recent years, the inclusion of the category of metacognition in education affects the educational activities of senior students, graduate students, and generally people involved in continuing education. The study described in this article aims at investigating the relation between the development of metacognition skills and success in learning algebraic identity formulas among middle school students. It uses a quantitative approach. Participants are about 50 students of 7th grade from Oral Bilim Innovation Liceum in Kazakhstan and are divided into two groups, namely experimental and control group. The data is collected by taking a pre-test and post-test from both classes. The diagnostic test is constructed by the researcher and confirmed by university professors. During the research students in the experimental group are familiarized with the concept of metacognition and its strategies such as thinking aloud, self-questioning, reflection and active learning strategies by applying them to solve problems with identity formulas. Finally, the data will be analyzed with t-test and much deeper conclusion will be drawn about the development of metacognitive strategies among students in teaching mathematics.

Keywords: metacognitive psychology, educational activities, pre-test, post-test.
The problem of formation of functional literacy among students at school is one of the promising research and methodological topics nowadays. All over the world, the concept of literacy has expanded significantly. Along with the traditional interpretation of literacy, which characterizes a person’s ability to read, write and perform arithmetic calculations, the concept of “functional literacy” has been actively used.

The development of functional literacy among students is defined as one of the goals with high priority in education. Consequently, one of the promising areas of educators’ innovative activity will be the introduction of forms, methods and technologies of instruction aimed at the formation of functional literacy among schoolchildren. In other words, we are talking about new literacy, which is becoming a factor in facilitating people's participation in social, cultural, political and economic activities, as well as lifelong learning. This feature of functional literacy is clearly visible in its definition as the ability to solve life problems in various fields of activity on the basis of applied knowledge necessary for everyone in the rapidly changing societies.

In Kazakhstan schools teach strong subject knowledge, but often it happens that they do not teach students how to apply that knowledge in real life situations. Functional literacy is an indicator of social well-being. A high level indicates certain socio-cultural achievements of society; while a low level is a warning of a possible social crisis. Since the requirements for applied knowledge grow as society develops (their range expand and new qualitative attributes are formed), we can say that the level of a person’s functional literacy is an indicator of his/her ability to adapt to microsocium conditions.

A modern lesson is an environment for the formation of vital competencies. In modern lessons, teachers and students are partners, the function of the teacher is to organize the lesson so that students have the opportunity to independently acquire knowledge. The purpose of the modern lesson provides an answer to the question: what will students be able to do by the end of the lesson, that is, the expected result. The content of the lesson is the activity of the teacher and students. First, students should dominate the lesson and focus on the efforts and attention. Secondly, to create an environment in which students would feel comfortable and could learn from each other. Such changes are important both for the teacher and students so that the teacher in this situation provides learning opportunities, materials, etc., but students must have the desire to act in order to develop a conscious understanding of the subject, that is, the development of metacognitive abilities. Why is it not enough to have a good memory, concentrated attention and
rational thinking skills? Flavell and others (1995) believed that if the learning process is brought to a conscious level, we will be able to help children better understand their own thought process and help them control or master the organization of their own learning. There are also three reasons why students need to develop metacognitive knowledge. First, the "information environment", as never before, has a strong, sometimes negative, effect on the youth, which is expressed in uncertainty, tension, and anxiety experienced by them. These factors, in turn, inhibit the processes of personality development, the growth of self-awareness, and the formation of individuality. However, in the modern socio-economic conditions, for the self-development of a person, the functioning of a number of abilities and skills is necessary. For example, critically thinking of incoming information; awareness of the possibility of many diverse views on the same phenomenon; substantiation of one’s own view of phenomena and processes. In other words, developed metacognitive processes do not allow a person to turn into a "vessel into which others fill information". Secondly, numerous studies have proven that the intellectual potential of students remains largely unfulfilled in the traditionally organized educational process, especially in the aspect of the development of its metacognition. Thirdly, the presence of metacognitive skills as elements of intellectual competence is a prerequisite for making the optimal decision, which is included in the functional literacy of students. The model of a competent specialist contains the ability to solve problems, choosing optimal solutions that can only be based on reflection, self-regulation and control. In other words, the metacognitive components of intelligence underlie the processes of personal self-development.

The experiment with 7th grades started with a diagnostic test (pre-test) which has a number of open ended and multiple-choice test questions and duration was one lesson hour. The same exam questions were taken as a post-test. The questions are as follows:

**Exam**

1. Expand and simplify:
   a) $(2x + 3)^2 =$
   b) $(x + 2)^3 =$
   c) $(a+b)(a-b)=

2. Factor the expression: $4x^2 - 25 =$

3. Rewrite by completing the square:
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II. Equations

a. \( x^2 - 2x = \)
b. \( x^2 + x + 1 = \)

IV. True (T) or False (F)

a. \((p + q)^2 = p^2 + q^2\) T/F
b. \((x - y)^3 = x^3 + 3x^2y - 3xy^2 - y^3\) T/F
c. \(x^2 + 4 = (x + 2)^2\) T/F

V. Find the coefficient of \(x^3\) in the expansion of \((x - 2)^3\).

VI. Questions 6-10:

Choose the correct answer:

6. One factor of \(27x^3 - 8y^3\) is:
   A. \(9x+2y\)
   B. \(9x-2y\)
   C. \(3x+2y\)
   D. \(3x-2y\)
   E. None of the above

7. \((x-4)(x+4) =\)
   A. \(x^2 - 16\)
   B. \(x^2 + 16\)
   C. \(x^2 - 8x + 16\)
   D. \(x^2 + 8x + 16\)
   E. None of the above

8. Factor: \(25x^2 - 16y^2\)
   A. \((5x - 4y)^3\)
   B. \(5(5x - 4y)\)
   C. \((5x+4y)(5x-4y)\)
   D. \((5x+2y)(5x-8y)\)
   E. None of the above

9. Multiply \(3(2x - 5)^2\)
   A. \((6x - 15)^2\)
   B. \(12x^2 - 75\)
   C. \(12x^2 - 60x+75\)
   D. \(36x^2 - 90x+225\)
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10. Find a possible middle term to make this polynomial a complete square: \((x^2 + \underline{\quad} + 49)\)
   A. 12x
   B. 13x
   C. 7x
   D. 3x
   E. 14x

Development of functional literacy results in the mastery of a system of key competencies that form school graduates’ basis of the personality. They are informational, communicative, social, personal, civil, and technological competencies. How can these competencies be developed in mathematics lessons by teaching metacognitive skills? For example, to develop the ability to perceive information, the ability to highlight the main idea, the ability to establish similarities and differences between the square formula and cubic formula in 7th grade, I used “My Reliance” technique, giving information on a poster from where students in groups extract information on their own. When designing lessons with the goal “being able to learn,” teacher creates conditions for students to learn teaching techniques, including techniques for working with a text as a source of information, techniques for presenting information and its critical comprehension in the form of diagrams, models, story-telling. Students learn to work in collaboration, listen to each other, and at the same time gain experience of self-learning and mutual learning. In training new self-regulation, teamwork is the most manifested on the basis of a social model. It is a form of work in which students combined into one team, jointly solve a problem or carry out some task. The principle of teamwork is that some functions of the teacher are transferred to the students for a period of time: informational (the students themselves obtain and process information), organizational (independent support of the roles in the group), control (students exercise control over each group member’s activity), evaluative (students evaluate each group member’s participation in the discussion) - this is a number of functional literacy competencies. This means that collaborative learning environment is conducive to the social and emotional development of students, because it provides opportunities for getting acquainted with various points of view, expressing and substantiating one's ideas. At the same time, students begin to form their own unique conceptual foundations, and do not rely on existing ones.
In the course of group work, from my side and from the side of more successful students support was provided, that is called metacognitive “forests”. This kind of support helps students bridge the gap between what they can do on their own and what they can do under the guidance of others. In this case, it is extremely important to correctly evaluate the help that students really need. If the intellectual help of an adult is excessive, the child’s consciousness does not need to grow, and the child becomes dependent on it. Optimal support involves the development of a metacognitive skill that makes the student more independent, able to draw on past experience to model possible solutions by the example of an algorithm, solution model, or leading questions. I very often used modeling in teaching. For example, to say out loud and write on the board a geometrical interpretation of the identity formulas, and thereby acting as a “reflective agent” for students.

By teaching metacognitive strategies, I helped children become self-sufficient. My teaching goal is to make students independent of the teacher. To do this, I used the technique “Undeclared topic”. This is a universal technique aimed at creating external motivation for studying the topic of the lesson. This technique allows you to attract students' interest in studying a new topic without blocking the perception of obscure terms. Reflective questions and motivation are important tools for teachers in the teaching process, starting from focusing students on the details and ending with the development of critical thinking and making an action plan. There is some difference between question and motivation. Questions are understood as more general, for example: “And what is this?”, “And how is it?”, “What's next?”, Prompting the student to think about what he/she has done, what he/she is doing and what he/she will do next. Motivation is understood as a more specific question, for example: “Can your goal be changed?” (drawing students to the specifics and examples.) Another way to motivate is to rephrase and summarize what the student said in a situation where he/she is asking for help. Third, motivation can redirect the question to the student. It can also stimulate students to express their thoughts for better understanding of themselves and the material. The only difficulty with this technique is that it is sometimes difficult to choose the right moment to interrupt the student and draw his attention to what he is doing or why he is doing it. Good teachers, however, know when to intervene and ask a question or voice a motivation. In a lesson, I offered one of the students a technique called “Thinking out loud and explaining to yourself” - this is when the student says out loud everything that he/she thinks and what he/she feels in the course of solving the problem. This
method is used when working in pairs, groups, and also when students are working independently. This allowed me to notice errors and incorrect courses of thought, as well as to see the specific psychological difficulties of students. “Explaining to yourself” is a technique that allows the students to clarify to themselves the elements of solving the problem. Moreover, self-explanations are effective because they actively use students’ existing knowledge.

The methods “Before-After”, “Basket of ideas, concepts, names” from the technology for the development of critical thinking, were used in updating the knowledge of students, and also at the stage of reflection in order to determine what students know or think about the topic under discussion. Even students with a low level of knowledge, discuss with a neighbor at the desk, systematize the knowledge gained in the lesson and actively participate. This confirms the constructivist theory: “the development of student's thinking occurs in the context of the interaction of existing knowledge with new knowledge obtained in the classroom from various sources, that can be teachers, peers, textbooks, and information sites. Graphs and drawings can help to understand the text or to solve various problems. Text, graphics can help to identify the structure, to see the relationship between concepts (“conceptual map”), etc. And they can be very different: tree or network diagrams, cyclic, etc. charts, underscores, comparative matrices, chronological chains, etc.

Designing level learning tasks based on Bloom's taxonomy helped me, as a teacher, to bring the organization of independent learning activities for students throughout the lesson. Differentiation of teaching tasks based on Bloom's taxonomy enables students to maximize their personal potential, on the one hand, and on the other, allows the teacher to realize the principles of personality-oriented learning, since in such a lesson everyone moves from level to level at their own pace, gets knowledge within his/her own needs and capabilities. A multiple return to the information component of the lesson, growing from reproductive tasks to cognitive and behavioral, not only enables each student to learn the teaching material, but also allows the formation of logical and creative thinking skills as a vital competency.

Students use self-assessment by a descriptor only in order to understand and test their level of knowledge after independent work. Monitoring knowledge is a person’s ability to know what he/she knows and does not know. Monitoring of prior learning is a fundamental or prerequisite for metacognitive process” (Tobias and Everson, 2002). Therefore, one who can clearly distinguish between what he/she has already learned from what he/she still needs to learn. It is an
important metacognitive skill and can more quickly and successfully advance the learning process, using more time to study new material. A couple forms of worksheets for monitoring knowledge used in the experiment are as follows:

Figure 1. “Thinking about my thinking” student worksheet 1. Monitoring

| Name: __________________ | Date: _____________ | Topic: __________________ |
|--------------------------|--------------------|--------------------------|
| What I know              | What I want to know| What I have learned      | What I still need to know|

Figure 2. “Thinking about my thinking” student worksheet 2. Planning

| Name: __________________ | Date: _____________ | Topic: __________________ |
|--------------------------|--------------------|--------------------------|
| What I know              | What I want to know| How will I find out?      | What I have learned      |

Figure 3. “Thinking about my thinking” student worksheet 3. Evaluating

| What I know NOW | What I STILL need to Know | How DID I find out | How WILL I do it next time |
|-----------------|---------------------------|-------------------|---------------------------|

It is also important that the ability to teach metacognitive skills allows the teacher to reconsider the view on the content component of the lesson. Considering the fact that functional literacy is based on the "atomic" level of knowledge, we can conclude: in preparation for the lesson, the teacher must determine the minimum necessary and sufficient amount of information that allows to realize the objectives of the lesson from the position of competent education. The ability to accurately measure the presence of metacognitive skills allows the teacher to select the necessary content according to the following algorithm:

1) vital information;
2) information encountered in the study of other subjects;
3) information necessary for further study of the subject.
The results of control and experimental groups for pre-test and post-test is shown below:

| ID | Pre_test | Post_test |
|----|----------|-----------|
| 1  | 10,00    | 13,00     |
| 2  | 5,00     | 6,00      |
| 3  | 8,00     | 13,00     |
| 4  | 11,00    | 15,00     |
| 5  | 6,00     | 12,00     |
| 6  | 7,00     | 11,00     |
| 7  | 11,00    | 11,00     |
| 8  | 9,00     | 12,00     |
| 9  | 3,00     | 12,00     |
| 10 | 10,00    | 14,00     |
| 11 | 8,00     | 11,00     |
| 12 | 5,00     | 8,00      |
| 13 | 6,00     | 12,00     |
| 14 | 2,00     | 6,00      |
| 15 | 10,00    | 11,00     |
| 16 | 7,00     | 11,00     |
| 17 | 8,00     | 13,00     |
| 18 | 6,00     | 10,00     |
| 19 | 4,00     | 7,00      |
| 20 | 6,00     | 9,00      |
| 21 | 8,00     | 12,00     |
| 22 | 6,00     | 9,00      |
| 23 | 8,00     | 10,00     |
| 24 | 3,00     | 8,00      |
| 25 | 6,00     | 12,00     |

Figure 4. Results for experimental group
Data analysis was done through SPSS Statistics viewer using “paired samples t-test” and “independent samples t-test”. Paired samples t-test was used to check if there was difference between pre-test and post-test results in each group (Figure 6, 7). According to the results, calculated t-value was compared with the critical t-value and the difference between pre and post-test was confirmed in both groups.

| ID | Pre_test | Post_test |
|----|----------|-----------|
| 1  | 5.00     | 8.00      |
| 2  | 9.00     | 9.00      |
| 3  | 7.00     | 9.00      |
| 4  | 6.00     | 9.00      |
| 5  | 8.00     | 11.00     |
| 6  | 4.00     | 8.00      |
| 7  | 11.00    | 11.00     |
| 8  | 9.00     | 12.00     |
| 9  | 5.00     | 8.00      |
| 10 | 10.00    | 13.00     |
| 11 | 4.00     | 10.00     |
| 12 | 12.00    | 12.00     |
| 13 | 11.00    | 13.00     |
| 14 | 10.00    | 8.00      |
| 15 | 7.00     | 11.00     |
| 16 | 11.00    | 14.00     |
| 17 | 9.00     | 12.00     |
| 18 | 9.00     | 13.00     |
| 19 | 8.00     | 10.00     |
| 20 | 7.00     | 14.00     |
| 21 | 12.00    | 14.00     |
| 22 | 7.00     | 10.00     |
| 23 | 4.00     | 10.00     |
| 24 | 12.00    | 13.00     |

Figure 5. Results for control group

| ID | Pre_test | Post_test |
|----|----------|-----------|
| 1  | 5.00     | 8.00      |
| 2  | 9.00     | 9.00      |
| 3  | 7.00     | 9.00      |
| 4  | 6.00     | 9.00      |
| 5  | 8.00     | 11.00     |
| 6  | 4.00     | 8.00      |
| 7  | 11.00    | 11.00     |
| 8  | 9.00     | 12.00     |
| 9  | 5.00     | 8.00      |
| 10 | 10.00    | 13.00     |
| 11 | 4.00     | 10.00     |
| 12 | 12.00    | 12.00     |
| 13 | 11.00    | 13.00     |
| 14 | 10.00    | 8.00      |
| 15 | 7.00     | 11.00     |
| 16 | 11.00    | 14.00     |
| 17 | 9.00     | 12.00     |
| 18 | 9.00     | 13.00     |
| 19 | 8.00     | 10.00     |
| 20 | 7.00     | 14.00     |
| 21 | 12.00    | 14.00     |
| 22 | 7.00     | 10.00     |
| 23 | 4.00     | 10.00     |
| 24 | 12.00    | 13.00     |

Figure 6. Paired samples t-test for experimental group

| Mean | Std. Deviation | Std. Error Mean | 95% Confidence Interval of the Difference | t | df | Sig. (2-tailed) |
|------|----------------|-----------------|------------------------------------------|---|----|----------------|
| Pair 1 | Pre_test-Post_test | -3.00006 | 1.97083 | .37417 | -6.57224 | -3.63776 | -8.156 | 24 | .003 |

Figure 7. Paired samples t-test for control group
To meet the research aim, the results for the independent samples t-test is found to be more important:

![Figure 8. Independent samples t-test for control and experimental groups.](image)

When the calculated t-values were compared with the corresponding critical values, the results failed to reject the null hypothesis, i.e. the means for experimental group and the control group in post-test didn’t vary very much. However, this conclusion should not imply that metacognitive strategies were not helpful in teaching mathematics. There are some factors which might have affected the final result. First of all, the experiment duration was only one month and in this period of time students had to be acquainted with the concept of metacognition and main mathematical concept. This leads us to the second difficulty occurred in implementing metacognition since students lacked background about metacognition. Thirdly, instructional strategies might not have been effective and needed more adjustments. Fourth, metacognition is not easy to assess because it is not directly observable. To solve this problem verbal reporting can be considered as a recommendation; and to record nonverbal behaviors observational techniques can be applied.

|                  | Pre_Tests | Post_Tests |
|------------------|-----------|------------|
| **Independent Samples Test** |           |            |
| **Levene's Test for Equality of Variances** | F | Sig | t | df | Sig (2-tailed) | 95% Confidence Interval of the Difference | Lower | Upper |
| **Equal variances assumed** | .007 | .422 | -1.425 | 47 | .161 | -1.2157 | .7870 | -2.7806 | .6523 |
| **Equal variances not assumed** | -1.419 | .4448 | .163 | .161 | -1.1717 | .7565 | -2.7143 | .4703 |
| **Equal variances assumed** | .140 | .710 | .077 | 47 | .939 | .9333 | .5938 | -1.2459 | 1.4024 |
| **Equal variances not assumed** | .077 | 46.660 | .939 | .0535 | .6652 | 1.3476 | 1.4542 |
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