Stochastic Attribute-Value Grammars

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Abstract

Probabilistic analogues of regular and context-free grammars are well-known in computational linguistics, and currently the subject of intensive research. To date, however, no satisfactory probabilistic analogue of attribute-value grammars has been proposed: previous attempts have failed to define a correct parameter-estimation algorithm.

In the present paper, I define stochastic attribute-value grammars and give a correct algorithm for estimating their parameters. The estimation algorithm is adapted from Della Pietra, Della Pietra, and Lafferty [5]. To estimate model parameters, it is necessary to compute the expectations of certain functions under random fields. In the application discussed by Della Pietra, Della Pietra, and Lafferty (representing English orthographic constraints), Gibbs sampling can be used to estimate the needed expectations. The fact that attribute-value grammars generate constrained languages makes Gibbs sampling inapplicable, but I show how a variant of Gibbs sampling, the Metropolis-Hastings algorithm, can be used instead.

1 Introduction

Stochastic versions of regular grammars and context-free grammars have received a great deal of attention in computational linguistics for the last several years, and basic techniques of stochastic parsing and parameter estimation have been known for decades. However, regular and context-free grammars are widely deemed linguistically inadequate; standard grammars in computational linguistics are attribute-value grammars of some variety. Before the advent of statistical methods, regular and context-free grammars were considered too inexpressive for serious consideration, and even now the reliance on stochastic versions of the less-expressive grammars is often seen as an expedient necessitated by the lack of an adequate stochastic version of attribute-value grammars.
Attempts have been made to extend stochastic models developed for the regular and context-free cases to attribute-value grammars, but to date without success. 1 Brew 2 sketches a probabilistic version of HPSG, but admits that his way of dealing with re-entrances in feature structures is problematic. Eisele 3 attempts to translate stochastic context-free techniques to constraint-based grammar by assigning probabilities to SLD proof trees. Both Brew and Eisele propose associating weights with grammar-rule analogues (typed feature structures in Brew’s case; Horn clauses in Eisele’s case) and setting weights proportional to expected rule frequencies. For want of a standard term, I will call this the Expected Rule Frequency (ERF) method. Both propose using iterative reestimation of rule-frequency expectations when dealing with incomplete data (unannotated corpora), along the lines of the EM algorithm.

The attempt is ultimately unsuccessful. The ERF method is provably correct for the context-free case, but it fails in the presence of context dependencies, as will be discussed below. Both Brew and Eisele recognize that applying the ERF method has deficiencies. Eisele in particular identifies an important symptom that indicates that something has gone amiss: the grammar induced by the EM algorithm defines a probability distribution over trees that is not in accordance with their frequency in the training corpus. Moreover, Eisele recognizes that this problem arises only where there are context dependencies. That such dependencies lead to problems is not surprising, given the independence assumptions underlying Eisele’s model, but he is not able to explain why they manifest themselves in the way they do, nor what can be done to address the problem.

Now in fact solutions to the context-sensitivity problem have long been known, and are the subject of continuing study, in the image processing field and in related areas of statistics. The models of interest are known as random fields. Random fields can be seen as a generalization of Markov chains and stochastic branching processes. Markov chains can be seen as stochastic versions of regular grammars (Hidden Markov Models are in turn stochastic functions of Markov chains) and random branching processes are stochastic versions of context-free grammars. The evolution of a Markov chain describes a line, in which each stochastic choice depends only on the state at the immediately preceding time-point. The evolution of a random branching process describes a tree in which a finite-state process may spawn multiple child processes at the next time-step, but the number of processes and their states depend only on the state of the unique parent process at the preceding time-step. In particular,

1I confine my discussion here to Brew and Eisele because they aim to describe parametric models of probability distributions over the languages of constraint-based grammars, and to estimate the parameters of those models. Other authors have assigned weights or preferences to constraint-based grammars but not discussed parameter estimation. One approach of the latter sort that I find of particular interest is that of Stefan Riezler 4, who describes a weighted logic for constraint-based grammars that characterizes the languages of the grammars as fuzzy sets. This interpretation avoids the need for normalization that Brew and Eisele face, though parameter estimation still remains to be addressed.
stochastic choices are independent of other choices at the same time-step: each process evolves independently. If we permit re-entrancies, that is, if we permit processes to re-merge, we generally introduce context-sensitivity. In order to re-merge, processes must generally be “in synch,” which is to say, they cannot evolve in complete independence of one another. Random fields are a particular class of multi-dimensional random processes, that is, processes corresponding to probability distributions over an arbitrary graph. They were originally studied by Gibbs, nearly a hundred years ago, as a model for statistical mechanics, and the general family of probability distributions involved is still known by his name.

To my knowledge, the first application of random fields to natural language was by Mark et al. [3]. The problem of interest was how to combine a stochastic context-free grammar with n-gram language models. The resulting structures, e.g., (1), obviously involve re-entrancies and context-sensitivity.

\[
\begin{array}{c}
\text{S} \\
\text{NP} \\
\text{there was} \\
\text{NP} \\
\end{array}
\begin{array}{c}
\text{VP} \\
\text{no response} \\
\end{array}
\]

It was clear at that time that a similar approach ought to succeed for general attribute-value grammars, but the issue was not pursued.

Recent work by Della Pietra, Della Pietra, and Lafferty [5] (henceforth, DDL) also applies random fields to natural language processing. The application they consider is the induction of English orthographic constraints—inducing a grammar of possible English words. The authors describe an algorithm for selecting informative properties of words to construct a random field, and for setting the parameters of the field optimally for a given set of properties, to model an empirical word distribution.

The DDL algorithms require the computation of the expectations, under random fields, of certain characteristic functions. In general, computing these expectations involves summing over all configurations (all possible character sequences, in the orthography application), which is not possible when the configuration space is large. Instead, DDL use Gibbs sampling to estimate the needed expectations.

The orthography application cannot be immediately converted into a means of equipping attribute-value grammars with probabilities. Any labelling of a finite linear graph with ASCII characters yields a possible (though not necessarily probable) English word, and this unconstrainedness is essential for the use of Gibbs sampling. By contrast, the set of dags admitted by an attribute-value

\[\text{To be precise, DDL use closed linear graphs—i.e., polygons.}\]
grammar $G$ is highly constrained—most of the time, relabelling a dag admitted by $G$ does not yield a new dag admitted by $G$. Gibbs sampling is not applicable. However, I will show that a variant of Gibbs sampling, the Metropolis-Hastings algorithm, can be used. Indeed, we can use a random branching process much like Brew’s or Eisele’s to supply the so-called proposal matrix for the Metropolis-Hastings algorithm.

In this way, we can assign probabilities to the classes of dags admitted by attribute-value grammars. We can use these probabilities to disambiguate sentences (by selecting the most-probable parse), and we can give a parameter-estimation algorithm that is correct, in the sense that, if we generate a training corpus of size $n$ from a model $M$, and then estimate parameters from the training corpus to yield a model-estimate $\hat{M}_n$, then $\hat{M}_n$ converges to $M$ as $n \rightarrow \infty$.

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2 Stochastic Context-Free Grammars

Let us begin by examining stochastic context-free grammars and asking why the “obvious” generalization to attribute-value grammars fails. A point of terminology: I will use the term grammar to refer to an unweighted grammar, be it a context-free grammar or attribute-value grammar. The combination of a grammar and weights (later, also properties) I will refer to as a model. (Occasionally I will use model to refer to the weights themselves, or the probability distribution they define.)

Throughout we will use the following stochastic context-free grammar for illustrative purposes. Let us call the underlying grammar $G_1$ and the grammar equipped with weights as shown, $M_1$: 
The probability of a given tree is computed as the product of probabilities of rules used in it. For example:

Let $x$ be tree (3) and let $q_1$ be the probability distribution over trees defined by model $M_1$. Then:

$$q_1(x) = \beta_1 \cdot \beta_3 \cdot \beta_3 = \frac{1}{2} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{2}{9}$$

In parsing, we use the probability distribution $q_1(x)$ defined by model $M_1$ to disambiguate: the grammar assigns some set of trees $\{x_1, \ldots, x_n\}$ to a sentence $\sigma$, and we choose that tree $x_i$ that has greatest probability $q_1(x_i)$. For example, $G_1$ assigns two parses to the sentence $aa$: tree (3) above and tree (5):

The probability of tree (3) is $2/9$, as we have seen. The probability of tree (5) is $\beta_2 \beta_6 = 1/2 \cdot 1/2 = 1/4$. Since $1/4 > 2/9$, a stochastic parser for $M_1$ should return tree (5) on input $aa$.

The issue of efficiently computing the most-probable parse for a given sentence has been thoroughly addressed in the literature. The standard parsing techniques can be applied as is to the random-field models to be discussed below, so I simply refer the reader to the literature. Instead, I concentrate on
parameter estimation, which for attribute-value grammars cannot be accomplished by standard techniques.

By parameter estimation we mean determining values for the weights $\beta$. In order for a stochastic grammar to be useful, we must be able to compute the correct weights, where by correct weights we mean the weights that best account for a training corpus. The degree to which a given set of weights account for a training corpus is measured by the similarity between the distribution $q_\beta(x)$ determined by the weights $\beta$ and the distribution of trees $x$ in the training corpus.

2.1 The Goodness of a Model

The distribution determined by the training corpus is known as the empirical distribution. For example, suppose we have a training corpus containing twelve trees of the following four types from $L(G_1)$:

\[
\begin{array}{cccc}
  & x_1 & x_2 & x_3 & x_4 \\
  & S & S & S & S \\
 & A & A & A & B \\
 & a & b & a & b \\
\end{array}
\]

If $c_i$ is the count of how often the $i$-th tree (type) appears in the corpus, then

\[
\tilde{p}(x_i) = \frac{c_i}{\sum_j c_j}
\]

In comparing a distribution $q$ to the empirical distribution $\tilde{p}$, we shall actually measure dissimilarity rather than similarity. Our measure for dissimilarity of distributions is the Kullback-Leibler distance, defined as:

\[
D(\tilde{p}||q) = \sum_x \tilde{p}(x) \ln \frac{\tilde{p}(x)}{q(x)}
\]

The distance between $\tilde{p}$ and $q$ at point $x$ is the log of the ratio of $\tilde{p}(x)$ to $q(x)$. The overall distance between $\tilde{p}$ and $q$ is the average distance, where the averaging is over tree (tokens) in the corpus; i.e., point distances $\ln \tilde{p}(x)/q(x)$ are weighted by $\tilde{p}(x)$ and summed.

For example, let $q_1$ be, as before, the distribution determined by model $M_1$. The following table shows $q_1$, $\tilde{p}$, the ratio $q_1(x)/\tilde{p}(x)$, and the weighted point
distance \( \tilde{p}(x) \ln(\tilde{p}(x)/q_1(x)) \). The sum of the fourth column is the Kullback-Leibler distance \( D(\tilde{p}||q) \) between \( \tilde{p} \) and \( q_1 \). The third column contains \( q_1(x)/\tilde{p}(x) \) rather than \( \tilde{p}(x)/q_1(x) \) so that one can see at a glance whether \( q_1(x) \) is too large \((q_1(x)/\tilde{p}(x) > 1)\) or too small \((< 1)\).

One set of weights is better than another if its distance from the empirical distribution is less. For example, let us consider a different set of weights for grammar \( G_1 \). Let \( M' \) be \( G_1 \) with weights \((1/2, 1/2, 1/2, 1/2, 1/2)\), and let \( q' \) be the probability distribution determined by \( M' \). Then the computation of the Kullback-Leibler distance is as follows:

\[
\begin{array}{cccc}
\text{x} & \tilde{p} & q_1/\tilde{p} & \tilde{p} \ln(\tilde{p}/q_1) \\
1 & 2/9 & 1/3 & 0.67 & 0.14 \\
2 & 1/18 & 1/6 & 0.33 & 0.18 \\
3 & 1/4 & 1/4 & 1.00 & 0.00 \\
4 & 1/4 & 1/4 & 1.00 & 0.00 \\
\end{array}
\]

The total distance \( D(\tilde{p}||q_1) = 0.32 \).

Another reason for adopting the definition of goodness in terms of Kullback-Leibler distance is the following. Suppose Nature secretly chooses some set of weights \( M \) for \( G_1 \). These are the true weights; they define the true distribution \( q \). Nature then generates trees at random from \( M \) in accordance with \( q \). Let \( \tilde{p}_n \) be the empirical distribution determined by the first \( n \) trees that Nature generates. A parameter-setting method must choose a model \( \hat{M}_n \) given \( \tilde{p}_n \), for each \( n \). A parameter-setting method is correct if it converges to \( M \), the true model. The sequence of hypotheses \( \hat{M}_1, \hat{M}_2, \ldots \) defining distributions \( \hat{q}_1, \hat{q}_2, \ldots \) is said to converge to \( M \) (defining distribution \( q \)) just in case, for all tolerances \( \epsilon \), there is some point \( n \) such that \( D(q||\hat{q}_n) < \epsilon \) for all \( n' > n \). It can be shown that \( D(q||\hat{p}_n) \) converges to 0; that is, \( \lim_{n \to \infty} \hat{p}_n = q \). If a parameter-setting method returns the model \( \hat{M}_n \) that minimizes \( D(\tilde{p}_n||\hat{q}_n) \), then \( \lim_{n \to \infty} \hat{q}_n = \lim_{n \to \infty} \tilde{p}_n \), if the limiting distribution for \( \tilde{p}_n \) is generable.

The fit for \( x_2 \) improves, but that is more than offset by a poorer fit for \( x_1 \). The distribution \( q_1 \) is a better distribution than \( q' \), in the sense that \( q_1 \) is more similar (less dissimilar) to the empirical distribution than \( q' \) is.
by any model with underlying grammar $G_1$. Since $q$ is generable by such a grammar, and $q$ is the limit distribution for $\tilde{p}_n$, it follows that $q$ is also the limit distribution for $\hat{q}_n$, and the method is correct.

Note that the model $\tilde{M}$ that minimizes the distance $D(q\|\tilde{q})$ is $M$ itself, and $D(q\|q) = 0$. This does not mean, however, that $D(\tilde{p}_n\|\hat{q}_n) = 0$ for the model minimizing $D(\tilde{p}_n\|\hat{q}_n)$. The empirical distributions $\tilde{p}_n$ converge to $q$, but do not necessarily equal $q$. Intuitively, the relative frequency of any given tree converges to its true probability, but need not be precisely its true probability, even in very large corpora.

### 2.2 The ERF Method

For stochastic context-free grammars, it can be shown that the Expected Rule Frequency (ERF) method mentioned in the introduction always yields the best model for a given training corpus. To define the ERF method, we require a bit of terminology and notation. With each rule $i$ in a stochastic context-free grammar is associated a weight $\beta_i$ and a function $f_i(x)$ that returns the number of times rule $i$ is used in the derivation of tree $x$. For example, consider tree (3), repeated here as (10):

![Tree Diagram]

Rule 1 is used once and rule 3 is used twice; accordingly $f_1(x) = 1$, $f_3(x) = 2$, and $f_i(x) = 0$ for $i \in \{2, 4, 5, 6\}$.

The expectation of a function over a probability space (for each $i$, $f_i$ is such a function) simply the average value of the function. We use the notation $p[f]$ to represent the expectation of $f$ under probability distribution $p$. It is defined as:

$$p[f] = \sum_x p(x)f(x)$$

The ERF method instructs us to choose the weight for rule $i$ proportional to the average frequency of rule $i$ in the corpus. That is:

$$\beta_i \propto \hat{p}[f_i]$$

Algorithmically, we compute the expectation of each rule’s frequency, and normalize among rules with the same lefthand side. For example, consider corpus...
The expectation of each rule frequency \( f_i \) is a sum of terms \( \tilde{p}(x)f_i(x) \). These terms are shown for each tree, in the following table.

|       | A A   | S   | A a | B a a | A b | B a a | B b b |
|-------|-------|-----|-----|-------|-----|-------|-------|
| \( \tilde{p} \) | \( \tilde{p}_{f_1} \) | \( \tilde{p}_{f_2} \) | \( \tilde{p}_{f_3} \) | \( \tilde{p}_{f_4} \) | \( \tilde{p}_{f_5} \) | \( \tilde{p}_{f_6} \) |
| \( x_1 \) | \( S [A a] [A a] \) | 1/3 | 1/3 | 2/3 |
| \( x_2 \) | \( S [B a a] \) | 1/6 | 1/6 | 2/6 |
| \( x_3 \) | \( S [A b] [A b] \) | 1/4 | 1/4 | 1/4 |
| \( x_4 \) | \( S [B b b] \) | 1/4 | 1/4 | 1/4 |

For example, in tree \( x_1 \), rule 1 is used once and rule 3 is used twice. The empirical probability of \( x_1 \) is 1/3, so \( x_1 \)'s contribution to \( \tilde{p}_{f_1} \) is 1/3 · 1, and its contribution to \( \tilde{p}_{f_3} \) is 1/3 · 2. The weight \( \beta \) is obtained from \( \tilde{p}_{f_i} \) by normalizing among rules with the same lefthand side. For example, the expected rule frequencies \( \tilde{p}_{f_1} \) and \( \tilde{p}_{f_2} \) of rules with lefthand side \( S \) already sum to 1, so they are adopted without change as \( \beta_1 \) and \( \beta_2 \). On the other hand, the expected rule frequencies \( \tilde{p}_{f_5} \) and \( \tilde{p}_{f_6} \) for rules with lefthand side \( B \) sum to 1/2, not 1, so they are doubled to yield weights \( \beta_5 \) and \( \beta_6 \). It should be observed that the resulting weights are precisely the weights of model \( M_1 \).

It can be proven that the ERF weights are the best weights for a given grammar, in the sense that they define the distribution that is most similar to the empirical distribution. That is, if \( \beta \) are the ERF weights (for a given grammar), then \( D(\tilde{p}|\tilde{q}) < D(\tilde{p}||q') \) for all sets of weights \( \beta' \neq \beta \).

As noted earlier, one might expect the best weights to yield \( D(\tilde{p}||q) = 0 \), but such is not the case. We have just seen, for example, that the best weights for grammar \( G_1 \) yield distribution \( q_1 \), yet \( D(\tilde{p}||q_1) = 0.32 > 0 \). A close inspection of the distance calculation (8) reveals that \( q_1 \) is sometimes less than \( \tilde{p} \), but never greater than \( \tilde{p} \). Could we improve the fit by increasing \( q_1 \)? For that matter, how can it be that \( q_1 \) is never greater than \( \tilde{p} \)? As probability distributions, \( q_1 \) and \( \tilde{p} \) should have the same total mass, namely, 1. Where is the missing mass for \( q_1 \)?

The answer is of course that \( q_1 \) and \( \tilde{p} \) are probability distributions over \( L(G) \), but not all of \( L(G) \) appears in the corpus. Two trees are missing, and they account for the missing mass. These two trees are:

(11)  
```
     S
   /   \\  \
A   A  A
  / \ / \  \
 a  b b  a
```
Each of these trees have probability 0 according to $\tilde{p}$ (hence they can be ignored in the distance calculation), but probability $1/9$ according to $q_1$.

Intuitively, the problem is this. The distribution $q_1$ assigns too little weight to trees $x_1$ and $x_2$, and too much weight to the trees of (11); call them $x_5$ and $x_6$. Yet exactly the same rules are used in $x_5$ and $x_6$ as are used in $x_1$ and $x_2$. Hence there is no way to increase the weight for trees $x_1$ and $x_2$, improving their fit to $\tilde{p}$, without simultaneously increasing the weight for $x_5$ and $x_6$, making their fit to $\tilde{p}$ worse. The distribution $q_1$ is the best compromise possible.

To say it another way, our assumption that the corpus was generated by a context-free grammar means that any context dependencies in the corpus must be accidental, the result of sampling noise. There is indeed a dependency in corpus (11): in the trees where there are two A’s, the A’s always rewrite the same way. If corpus (11) was generated by a stochastic context-free grammar, then this dependency is accidental.

This does not mean that the context-free assumption is wrong. If we generate twelve trees at random from $q_1$, it would not be too surprising if we got corpus (11). More extremely, if we generate a random corpus of size 1 from $q_1$, it is quite impossible for the resulting empirical distribution to match the distribution $q_1$. But as the corpus size increases, the fit between $\tilde{p}$ and $q_1$ becomes ever better.

3 Attribute-Value Grammars

But what if the dependency in corpus (11) is not accidental? What if we wish to adopt a grammar that imposes the constraint that both A’s rewrite the same way? We can impose such a constraint by using an attribute-value grammar. Consider the following grammar, in which rewrite rules are now represented as feature structures. Let us call this grammar $G_2$:

\[
\begin{align*}
1. & \quad S \\
2. & \quad S \quad [B] \\
3. & \quad A \quad a \\
4. & \quad A \quad b \\
5. & \quad B \\
6. & \quad B \\
\end{align*}
\]

The language $L(G_2)$ is a set of dags, namely:

\[
\begin{align*}
1. & \quad S \\
2. & \quad S \\
3. & \quad S \\
4. & \quad S \\
\end{align*}
\]

\[
\begin{align*}
A & \quad A \\
A & \quad A \\
B & \quad B \\
B & \quad B \\
\end{align*}
\]

\[
\begin{align*}
a & \quad b \\
a & \quad b \\
\end{align*}
\]
3.1 AV Grammars and The ERF Method

Now we face the question of how to attach probabilities to grammar $G_2$. The approach followed by Brew and Eisele is basically as follows. Associate a weight with each of the six "rules" of grammar $G_2$. For example, let $M_2$ be the model consisting of $G_2$ plus weights $(\beta_1, \ldots, \beta_6) = (1/2, 1/2, 2/3, 1/3, 1/2, 1/2)$. The weight assigned to a tree $x$ is then (as before) the product of the weights of the rules used in $x$. For example, the weight $\tilde{q}_2(x_1)$ assigned to tree $x_1$ of (13) is $2/9$, computed as follows:

$$(14)$$

Rule 1 is used once and rule 3 is used twice; hence $\tilde{q}_2(x_1) = \beta_1 \beta_3 \beta_3 = 1/2 \cdot 2/3 \cdot 2/3 = 2/9$.

Observe that $\tilde{q}_2(x_1) = \beta_1 \beta_3^2$, which is to say, $\beta_1^{f_1(x_1)} \beta_3^{f_3(x_1)}$. Moreover, since $\beta^0 = 1$, it does not hurt to include additional factors $\beta_i^{f_i(x_1)}$ for those $i$ where $f_i(x_1) = 0$. That is, we can define $\tilde{q}_\beta$ corresponding to weights $\beta = (\beta_1, \ldots, \beta_n)$ generally as:

$$\tilde{q}_\beta(x) = \prod_{i=1}^{n} \beta_i^{f_i(x)}$$

Now let us consider how to estimate weights. Brew and Eisele propose using the ERF method, as in the context-free case. To be sure, Brew and Eisele are more concerned about the case in which the training corpus consists of sentences alone, rather than parses (dags), and they concentrate on the application of the EM algorithm to estimate rule-frequency expectations in the absence of complete information. But their basic method is the ERF method: rule weights $\beta_i$ are set in accordance with the formula $\beta_i \propto \tilde{p}[f_i]$, under the constraint that the weights for rules with the same lefthand side sum to 1. The EM algorithm enters the picture only as a means of estimating $\tilde{p}[f_i]$ when it cannot be determined by simple counting.

\[3\]To be precise, neither Brew nor Eisele adopt the attribute-value framework discussed here, but the approaches they take in the related frameworks they do adopt are clearly analogous to the one I describe here.

\[4\]The reason for the '˘' will be made clear shortly.
To illustrate, let us assume a corpus distribution for the dags (13) analogous to the distribution in (6):

\[
\tilde{p} = \begin{bmatrix}
    x_1 & x_2 & x_3 & x_4 \\
    1/3 & 1/6 & 1/4 & 1/4
\end{bmatrix}
\]

Using the ERF method, we estimate rule weights as follows:

\[
\begin{array}{cccccc}
    \tilde{p} & \tilde{p}_{f_1} & \tilde{p}_{f_2} & \tilde{p}_{f_3} & \tilde{p}_{f_4} & \tilde{p}_{f_5} \\
    x_1 & 1/3 & 1/3 & 2/3 & \text{} & \text{} \\
    x_2 & 1/6 & 1/6 & 2/6 & 1/4 & 1/4 \\
    x_3 & 1/4 & 1/4 & \text{} & \text{} & 1/4 \\
    x_4 & 1/4 & 1/4 & \text{} & \text{} & \text{}
\end{array}
\]

This table is identical to the one given earlier in the context-free case. We arrive at the same weights we considered above for the AV grammar \(G_2\), yielding the distribution \(\tilde{q}_2\).

3.2 Why the ERF Method Fails

But at this point a problem arises: \(\tilde{q}_2\) is not a probability distribution. Unlike in the context-free case, the four trees in (13) constitute the entirety of \(L(G)\). This time, there are no missing trees to account for the missing probability mass. There is an obvious “fix” for this problem, as Brew and Eisele observe: we can simply normalize \(\tilde{q}_2\). (This, by the way, is the reason for the ‘˘’ in ‘\(\tilde{q}_2\)’—it is meant to indicate that \(\tilde{q}_2\) is an “unnormalized” probability distribution.) That is, for the AV-grammar case, we must define the distribution \(q_\beta\) corresponding to the weights \(\beta\) as:

\[
q_\beta(x) = \frac{1}{Z} \tilde{q}_\beta(x)
\]

where \(Z\) is a normalizing constant defined as:

\[
Z = \sum_{y \in L(G)} \tilde{q}_\beta(y)
\]

In particular, for the ERF weights given in (16), we have \(Z = 2/9 + 1/18 + 1/4 + 1/4 = 7/9\). Dividing \(\tilde{q}_2\) by \(7/9\) yields the ERF distribution:

\[
q_2(x) = \begin{bmatrix}
    x_1 & x_2 & x_3 & x_4 \\
    2/7 & 1/14 & 9/28 & 9/28
\end{bmatrix}
\]
On the face of it, then, we can transplant the methods we used in the context-free case to the AV case and the only problem that arises ($\tilde{q}_2$ not summing to 1) has an obvious fix (normalization). However, something has actually gone very wrong. The theorem according to which the ERF method yields the best weights makes certain assumptions that we inadvertently violated by changing $L(G)$ and re-apportioning probability via normalization. In point of fact, we can easily see that the ERF weights are not the best weights for our example grammar. Consider the alternative model $M^*$ given in (18), defining probability distribution $q^*$:

$$q^*(x) = \begin{align*}
\beta_1 &= \frac{3+2\sqrt{2}}{6+2\sqrt{2}} \\
\beta_2 &= \frac{3}{6+2\sqrt{2}} \\
\beta_3 &= \frac{1}{1+\sqrt{2}} \\
\beta_4 &= \frac{1}{2} \\
\beta_5 &= \frac{1}{6} \\
\beta_6 &= \frac{1}{4}
\end{align*}$$

These weights are proper, in the sense that weights for rules with the same lefthand side sum to one. The reader can verify that $\tilde{q}^*$ sums to $Z = \frac{4+\sqrt{2}}{3}$ and that $q^*$ is:

$$q^*(x) = \begin{align*}
x_1 &= \frac{1}{3} \\
x_2 &= \frac{1}{6} \\
x_3 &= \frac{1}{4} \\
x_4 &= 1/4
\end{align*}$$

That is, $q^* = \tilde{p}$. Comparing $q_2$ (the ERF distribution) and $q^*$ to $\tilde{p}$, we observe that $D(\tilde{p}||q_2) = 0.07$ but $D(\tilde{p}||q^*) = 0$.

In short, in the AV case, the ERF weights do not yield the best weights. This means that the ERF method does not converge to the correct weights as the corpus size increases. If there are genuine dependencies in the grammar, the ERF method converges systematically to the wrong weights. Fortunately, there are methods that do converge to the right weights. These are methods that have been developed for random fields.

4 Random Fields

A random field defines a probability distribution over a set of labelled graphs $\Omega$ called configurations. In our case, the configurations are the dags generated by the grammar, i.e., $\Omega = L(G)$. The weight assigned to a configuration is the product of the weights assigned to configuration properties. Those familiar with random fields will recognize that identifying configurations with the dags of $L(G)$ is not entirely unproblematic. For one thing, configurations are standardly taken to be labelings over a fixed graph, not graphs with varying topologies. For another thing, the configuration space is standardly taken to be finite, not countably infinite, as $L(G)$ may be. These issues will be dealt with in the course of discussion.

The standard term in the random-fields literature is feature; I use the term property to avoid confusion with feature in the sense of an attribute plus value.

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\[ q(x) = \prod_i \beta_i^f_i(x) \]

where \( \beta_i \) is the weight for property \( i \) and \( f_i(x) \) is the frequency of occurrence of property \( i \) in configuration \( x \). The probability of a configuration is proportional to its weight, and is obtained by normalizing the weight distribution. That is:

\[
q(x) = \frac{1}{Z} \bar{q}(x) \\
Z = \sum_{y \in \Omega} \bar{q}(y)
\]

If we identify properties of a configuration with the rules used in it, the random field model is almost identical to the model we considered in the previous section. There are two important differences. First, we no longer require weights to sum to one for rules with the same left-hand side. Second, we no longer require properties to be identical to the rules of the grammar. We use the grammar to define the set of configurations \( \Omega = L(G) \), but give ourselves more flexibility in choosing the properties of dags we would like to use to define the probability distribution over \( L(G) \).

Let us consider an example. Let us continue to assume grammar \( G_2 \) generating language (13), and let us continue to assume the empirical distribution (15). But now rather than taking rule applications—local trees—to be properties, let us adopt the following two properties:

\[
1. \quad A \quad \text{a} \\
2. \quad B \\
\]

For purpose of illustration, take property 1 to have weight \( \beta_1 = \sqrt{2} \) and property 2 to have weight \( \beta_2 = 3/2 \). The functions \( f_1 \) and \( f_2 \) represent the frequencies of properties 1 and 2, respectively:

\[
\begin{align*}
&f_1 = \begin{pmatrix} & S & A & A & S & S & S \\ A & a & b & a & b & a & b \\ \end{pmatrix} \\
&f_2 = \begin{pmatrix}
2 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 \\
\end{pmatrix} \\
&\bar{q} = \sqrt{2} \cdot \sqrt{2} \\
&q = \begin{pmatrix}
2/6 & 1/6 & (3/2)/6 & (3/2)/6 \\
2/6 & 1/6 & 1/4 & 1/4 \\
\end{pmatrix}
\end{align*}
\]

In short, we are able to exactly recreate the empirical distribution using fewer properties than before. Intuitively, we need only use as many properties as are necessary to distinguish among trees that have different empirical probabilities.
This added flexibility is welcome, but it does make parameter estimation more involved. Now we must not only choose values for weights, we must also choose the properties that weights are to be associated with. We would like to do both in a way that permits us to find the best model, in the sense of the model that minimizes the Kullback-Leibler distance with respect to the empirical distribution. Methods for doing both are given in a recent paper by Della Pietra, Della Pietra, and Lafferty [5].

5 Field Induction

In outline, the DDL algorithm is as follows:

1. Start \((t = 0)\) with the null field (no properties).
2. **Property Selection.** Consider every property that might be added to the field \(q_t\) and choose the best one.
3. **Weight Adjustment.** Readjust weights for all properties. The result is a new field \(q_{t+1}\).
4. Iterate until the field cannot be improved.

One has a great deal of flexibility in defining the space of properties. For the sake of concreteness, let us take properties to be labelled subdags. In step 2 of the algorithm we do not consider every conceivable labelled subdag (there are simply too many of them), but only the atomic (i.e., single-node) subdags and those complex subdags that can be constructed by combining properties already in the field or by combining a property in the field with some atomic property.

In our running example, the atomic properties are:

\[(22) \quad S \quad A \quad B \quad a \quad b\]

Properties can be combined by adding connecting arcs. For example:

\[(23) \quad \begin{array}{c}
\text{(22)} \quad A + a = \quad S + A = \\
\text{(23)} \quad S + A = \quad \begin{array}{c}
\text{S} \\
A \\
A \\
\end{array}
\end{array}\]

5.1 The Null Field

Field induction begins with the null field. With the corpus we have been assuming, the null field takes the following form.
No dag $x$ has any features, so $\tilde{q}(x) = \prod_i \beta_i f_i(x)$ is a product of zero terms, and hence has value 1. As a result, $q$ is the uniform distribution. The Kullback-Leibler distance $D(\tilde{p}||q)$ is 0.03. The aim of property selection is to choose a property that reduces this distance as much as possible.

The astute reader will note that there is a problem with the null field if $L(G)$ is infinite. Namely, it is not possible to have a uniform distribution over an infinite set. If each dag in an infinite set of dags is assigned a constant nonzero probability $\epsilon$, then the total probability is infinite, no matter how small $\epsilon$ is. There are a couple of ways of dealing with the problem. The approach that DDL adopt is to assume a consistent prior distribution $p(k)$ over graph sizes $k$, and a family of random fields $q_k$ representing the conditional probability $q(x|k)$; the probability of a tree is then $p(k)q(x|k)$. All the random fields have the same properties and weights, differing only in their normalizing constants.

I will take a slightly different approach here. Let us adopt an initial distribution like that proposed by Brew and Eisele. There is a natural correspondence between AV grammars and CFG’s, a correspondence that we implicitly adopted in earlier discussion. We assume that the rules of an AV grammar are typed feature structures in which all types (of toplevel feature structures) are disjoint. Types correspond to categories in a CFG, and the righthand side of the CF analogue of rule $r$ is the list of types of immediate constituents of $r$, viewed as a feature structure. For example, the AV grammar $G_2$ has corresponding CF grammar $G_1$.

In this framework, a model consists of: (1) An AV grammar $G$ whose purpose is to define a set of dags $L(G)$. (2) An SCFG $H$ derived from $G$, with weights $\theta$, defining a distribution $\tilde{p}(d)$ over derivations $d$. There is a unique derivation corresponding to each dag in $L(G)$, but some derivations correspond to no well-formed dag—intuitively, some derivations lead to unification failures. Discarding the bad derivations and renormalizing yields the initial distribution $p(x)$ over dags $L(G)$. (3) A set of properties $f$ with weights $\beta$, to define the final distribution $q(x) = \frac{1}{Z} \prod_i \beta_i f_i(x) p(x)$.

There are a couple possible choices of weights $\theta$ for the initial distribution. The easiest approach would be to adopt the ERF weights. Field induction would then be a way of adding context-sensitivities to the ERF distribution. An alternative would be to adopt maximum-entropy weights. The intuitive reason for adopting the uniform distribution (in the finite case) is that it distinguishes dags in $L(G)$ from dags not in $L(G)$, but otherwise makes no assumptions about the distribution. The uniform distribution maximizes entropy over a
finite set. Maximizing entropy is more generally applicable, however, and can
be applied to infinite sets as well. Maximum entropy distributions for context-
free languages are discussed in a paper by Miller and O’Sullivan [4], though a
number of technical questions arise that I do not wish to pursue here.

5.2 Property Selection

At each iteration, we select a new property \( f \) by considering all atomic properties
and all complex properties that can be constructed from properties already in
the field. Holding the weights constant for all old properties in the field, we
choose the best weight \( \beta \) for \( f \) (how \( \beta \) is chosen will be discussed shortly),
yielding a new distribution \( q_f = q_f, \beta \). The score for property \( f \) is the reduction
it permits in \( D(\hat{p}||q_{old}) \), where \( q_{old} \) is the old field. That is, the score for \( f \)
is \( D(\hat{p}||q_{old}) - D(\hat{p}||q_f) \). We compute the score for each candidate property and
add to the field that property with the highest score.

To illustrate, consider the two atomic properties ‘a’ and ‘B’. Given the null
field as old field, the best weight for ‘a’ is \( \beta = \frac{7}{5} \), and the best weight for ‘B’
is \( \beta = 1 \). This yields \( q_f \) and \( D(\hat{p}||q_f) \) as follows:

\begin{align*}
\hat{p} & \begin{bmatrix} 1/3 & 1/6 & 1/4 & 1/4 \end{bmatrix} \\
\hat{q}_a & \begin{bmatrix} 7/5 & 1 & 7/5 & 1 \end{bmatrix} \\
q_a & \begin{bmatrix} 7/24 & 5/24 & 7/24 & 5/24 \end{bmatrix} \\
\hat{p} \ln \hat{q}_a & \begin{bmatrix} 0.04 & -0.04 & -0.04 & 0.05 \end{bmatrix} \\
\hat{p} \ln \hat{q}_a & \begin{bmatrix} Z = 24/5 \end{bmatrix} \\
\hat{q}_B & \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix} \\
q_B & \begin{bmatrix} 1/4 & 1/4 & 1/4 & 1/4 \end{bmatrix} \\
\hat{p} \ln \hat{q}_B & \begin{bmatrix} 0.10 & -0.07 & 0 & 0 \end{bmatrix} \\
\hat{p} \ln \hat{q}_B & \begin{bmatrix} Z = 4 \end{bmatrix} \\
D & \begin{bmatrix} 0.01 & 0.03 \end{bmatrix}
\end{align*}

The better property is ‘a’, and ‘a’ would be added to the field if these were the
only two choices.

Intuitively, ‘a’ is better than ‘B’ because ‘a’ permits us to distinguish the
set \( \{x_1, x_3\} \) from the set \( \{x_2, x_4\} \); the empirical probability of the former is
\( 1/3 + 1/4 = 7/12 \) whereas the empirical probability of the latter is \( 5/12 \). Distingui-
ishing these sets permits us to model the empirical distribution better (since
the old field assigns them equal probability, counter to the empirical distri-
bution). By contrast, the property ‘B’ distinguishes the set \( \{x_1, x_2\} \) from \( \{x_3, x_4\} \).
The empirical probability of the former is \( 1/3 + 1/6 = 1/2 \) and the empirical
probability of the latter is also \( 1/2 \). The old field models these probabilities
exactly correctly, so making the distinction does not permit us to improve on
the old field. As a result, the best weight we can choose for ‘B’ is \( 1 \), which is
equivalent to not having the property ‘B’ at all.

17
5.3 Selecting the Initial Weight

DDL show that there is a unique weight that maximizes the score for a new property $f$ (provided that the score for $f$ is not constant for all weights), and that the maximizing weight is the solution to the equation

\begin{equation}
q_{f,\beta}[f] = \tilde{p}[f]
\end{equation}

in the single unknown $\beta$. Intuitively, we choose the weight such that the expectation of $f$ under the resulting new field is equal to its empirical expectation.

Solving equation (26) for $\beta$ is easy if $L(G)$ is small enough to enumerate. Then the sum over $L(G)$ that is implicit in $q_{f,\beta}[f]$ can be expanded out, and solving for $\beta$ is simply a matter of arithmetic. Things are a bit trickier if $L(G)$ is too large to enumerate. DDL show that we can solve equation (26) if we can estimate $q_{\text{old}}[f = k]$ for $k$ from 0 to the maximum possible value for $f$.

We can estimate $q_{\text{old}}[f = k]$ by means of random sampling. The idea is actually rather simple: to estimate how often the property appears in “the average dag”, we generate a representative mini-corpus from the distribution $q_{\text{old}}$ and count. That is, we generate dags at random in such a way that the relative frequency of dag $x$ is $q_{\text{old}}(x)$ (in the limit), and we count how often the property of interest appears in dags in our generated mini-corpus.

The application that DDL consider is the induction of English orthographic constraints—inducing a field that assigns high probability to “English-sounding” words and low probability to non-English-sounding words. For this application, Gibbs sampling is appropriate. Gibbs sampling does not work for the application to AV grammars, however. Fortunately, there is an alternative random sampling method we can use: Metropolis-Hastings sampling. We will discuss the issue in some detail shortly.

5.4 Readjusting Weights

When a new property is added to the field, the best value for its initial weight is chosen, but the weights for the old properties are held constant. In general, however, adding the new property may make it necessary to readjust weights for all properties. The second half of the DDL algorithm involves finding the best set of weights for a given set of properties.

The method is very similar to the method for selecting the initial weight for a new property. Let $(\gamma_1, \ldots, \gamma_n)$ be the old weights for the properties. Consider the equation

\begin{equation}
q_{\gamma}[\beta^f f_i] = \tilde{p}[f_i]
\end{equation}

where $f^#(x) = \sum_i f_i(x)$ is the total number of properties of dag $x$. Without going into exactly why $f_i$ is weighted as it is on the lefthand side, the idea is the same as before: we want to adjust $\beta_i$ so that the average number of instances
of property $f_i$ according to the model matches the average number of instances of property $f_i$ in dags in the corpus.

If the weights $\gamma_1, \ldots, \gamma_n$ are not already as good as they can be, solving equation (27) for $\beta_i$ (for each $i$) is guaranteed to improve the weights, but it does not necessarily immediately yield the globally best weights. We can obtain the globally best weights by iterating. Set $\gamma_i \leftarrow \beta_i$, for all $i$, and solve equation (27) again. Repeat until the weights no longer change.

As with equation (26), solving equation (27) is straightforward if $L(G)$ is small enough to enumerate, but not if $L(G)$ is large. In that case, we must use random sampling. We generate a representative mini-corpus and estimate expectations by counting in the mini-corpus.

### 5.5 Random Sampling

We have seen that random sampling is necessary both to set the initial weight for properties under consideration and to adjust all weights after a new property is adopted. Random sampling involves creating a corpus that is representative of a given model distribution $q(x)$. To take a very simple example, a fair coin can be seen as a method for sampling from the distribution $q$ such that $q(H) = 1/2$, $q(T) = 1/2$. Saying that a corpus is representative is actually not a comment about the corpus itself but the method by which it was generated: a corpus representative of distribution $q$ is one generated by a process that samples from $q$. Saying that a process $M$ samples from $q$ is to say that the empirical distributions of corpora generated by $M$ converge to $q$ in the limit. For example, if we flip a fair coin once, the resulting empirical distribution over $(H, T)$ is either $(1, 0)$ or $(0, 1)$, not the fair-coin distribution $(1/2, 1/2)$. But as we take larger and larger corpora, the resulting empirical distributions converge to $(1/2, 1/2)$.

One of the advantages of SCFGs, that is lost when we go to random fields, is that there is a transparent relationship between an SCFG defining a distribution $q$ and a sampler for $q$. We can sample from the distribution defined by an SCFG as follows. Consider the grammar (3), repeated here as (28):

\[
\begin{align*}
1. \quad S &\rightarrow A A \quad \beta_1 = 1/2 \\
2. \quad S &\rightarrow B \quad \beta_2 = 1/2 \\
3. \quad A &\rightarrow a \quad \beta_3 = 2/3 \\
4. \quad A &\rightarrow b \quad \beta_4 = 1/3 \\
5. \quad B &\rightarrow a a \quad \beta_5 = 1/2 \\
6. \quad B &\rightarrow b b \quad \beta_6 = 1/2 
\end{align*}
\]

The language of (28) consists of the six trees \{$x_1 = [S [A a] [A a]], x_2 = [S [B a a]], x_3 = [S [A b] [A b]], x_4 = [S [B b b]], x_5 = [S [A a] [A b]], x_6 = [S [A b] [A a]]$\} with probability distribution $q : x_1 \mapsto 2/9, x_2 \mapsto 1/4, x_3 \mapsto 1/18, x_4 \mapsto 1/4, x_5 \mapsto 1/9, x_6 \mapsto 1/9$. 19
We sample from \( q \) via stochastic derivations. In a stochastic derivation, we start with the start symbol, \( S \). There are two rules expanding \( S \): \( S \rightarrow AA \) and \( S \rightarrow B \). We flip a coin to choose between them, heads for \( AA \), tails for \( B \). Suppose the coin comes up heads. We expand \( S \) to \( AA \), and then expand each of the \( A \)'s in turn. To expand the first \( A \), we consider the two rules \( A \rightarrow a \) and \( A \rightarrow b \). To decide between them, we flip a loaded coin that comes up heads (\( A \rightarrow a \)) \( 2/3 \) of the time and tails (\( A \rightarrow b \)) \( 1/3 \) of the time. Suppose this coin also comes up heads. We rewrite the first \( A \) as \( a \) and go to the second \( A \). We flip the loaded coin again; suppose it comes up heads again. We rewrite the second \( A \) as \( a \), and the result is \( x_1 \). The chances of throwing three heads in this manner are \( 1/2 \cdot 2/3 \cdot 2/3 = 2/9 = q(x_1) \). If we sample repeatedly in this manner, the proportion of \( x_1 \) in the resulting corpus will converge to \( 2/9 \). This is the sense in which stochastic derivations of this sort sample from the distribution defined by the given SCFG.

When we went from SCFGs to random fields, we lost the transparent connection between the probability distribution defined by the field and a method for sampling from it. Since weights do not sum to one for rules with the same lefthand side—indeed, since the properties with which weights are associated are not even necessarily rule applications—we cannot sample in the same way as we sample from an SCFG.

There is, however, a method that can be adapted for sampling from the random field defining a probability distribution over the language of an AV grammar. This method is the Metropolis-Hastings algorithm. Specifically, in the case of sets of dags with probability distribution \( q \), we proceed as follows.

Recall that we have a grammar \( G \) consisting of feature structures. We also have a context-free analogue \( H \) of \( G \) with weights \( \theta \), which we use to define the initial distribution \( p(x) \). In addition, we have a field consisting of a set of properties \( f_i \) with weights \( \beta_i \). The grammar defines a set of \( \Omega = L(G) \) and the field plus initial distribution define a probability distribution \( q(x) = \frac{1}{Z} \prod_i \beta_i f_i(x) p(x) \) over \( \Omega \).

We can sample from the initial distribution \( p(x) \) by performing stochastic derivations using grammar \( H \). The derivations map to dags in \( L(G) \) according to the correspondence between context-free rules and the AV rules of \( G \). It is possible that some of the derivations will fail—that they will map to inconsistent dags. Those derivations are simply discarded. That is, the probability that \( H \) assigns to a derivation is actually \( \tilde{p}(x) \); when we throw away derivations that map to inconsistent dags, the result is to restrict \( \tilde{p}(x) \) to consistent dags and normalize it, so that we end up sampling from \( p(x) \).

In this way, we can sample from \( L(G) \), but not in accordance with the field probability \( q(x) \). The essence of the Metropolis-Hastings algorithm is a means of converting the sampler for \( p(x) \) into a sampler for \( q(x) \). Suppose we are generating a corpus, and have generated dags \( x_1, \ldots, x_n \). Now we wish to add another dag, \( x_{n+1} \), to the corpus. We generate a dag \( y \) at random using the
sampler for $p(\cdot)$. Now, instead of simply adding $y$ to the corpus, we flip a loaded coin, that comes up heads with probability

$$A(y|x) = \min\{1, \frac{q(y)p(x_n)}{q(x_n)p(y)}\}$$

If the coin comes up heads, we do include $y$ in the corpus, that is, $x_{n+1} = y$. But if the coin comes up tails, we throw $y$ away and make a copy of $x_n$ instead, that is, $x_{n+1} = x_n$.

The acceptance probability $A(y|x)$ reduces in our case to a particularly simple form. If

$$q(y)p(x_n) \geq q(x_n)p(y),$$

then obviously $A(y|x) = 1$. Otherwise, writing $F(x)$ for the “field weight” $\prod_i \beta_f(x_i)$, we have:

$$A(y|x) = \frac{Z^{-1}F(y)p(y)p(x_n)}{Z^{-1}F(x_n)p(x_n)p(y)}$$

$$= \frac{F(y)}{F(x_n)}$$

It can be shown that the result of generating a new dag with probability $p(\cdot)$ and accepting it with probability $A(\cdot|x_n)$ yields a sampler for $q(\cdot)$ (see e.g. Winkler [7]). The final “acceptance” step intuitively serves the role of “punishing” dags that the $p$-sampler proposes more often than a $q$-sampler would, and shifting their probability to dags that the $p$-sampler would propose less often than a $q$-sampler would.

In somewhat more detail, if we think of the corpus $x_1, x_2, \ldots$ as a random walk through the space $L(G)$, the Metropolis-Hastings algorithm works because it forces the random walk to spend time in a region $R$ proportional to the probability of $R$. This is accomplished, intuitively, by preservation of what is known as detailed balance. Detailed balance requires that the probability of making a transition from dag $x$ to dag $y$ in the course of the random walk should balance the probability of making a transition from dag $y$ to dag $x$.

Let $q(x)$ be, as always, the model probability that we wish to sample from and let $q(y|x)$ be the transition probability—the probability of the next dag in the corpus being $y$ if the previous dag is $x$. In our case, $q(y|x)$ (for $y \neq x$) is the probability that we generate $y$ at random, and then also accept it: $q(y|x) = p(y)A(y|x)$. Define $q(x,y)$ (for $x \neq y$) to be the joint probability that $x$ is the previous dag and $y$ is the next dag; that is, $q(x,y) = q(x)q(y|x)$. Detailed balance requires that $q(x,y) = q(y,x)$. If detailed balance is preserved, it can be shown that the empirical distribution of the corpus generated by the random walk converges to $q(\cdot)$, and that the expectation of a function $f$ taken with respect to the empirical distributions converges to $q[f]$.

We can see that the transition probability we have assumed does indeed preserve detailed balance, as follows. Let $x$ be the last-generated tree and $y$ the new tree, and suppose that $q(y)p(x) > q(x)p(y)$. Then:
\[ q(y|x) = p(y) \quad q(x|y) = p(x) \frac{q(x)p(y)}{q(y)p(x)} = \frac{q(x)}{q(y)}p(y) \]
\[ q(x,y) = q(x)p(y) \quad q(y,x) = q(y) \frac{q(x)}{q(y)}p(y) = q(x)p(y) \]

That is, \( q(x,y) = q(y,x) \) and detailed balance is confirmed. The remaining cases \( q(y)p(x) < q(x)p(y) \) and \( q(y)p(x) = q(x)p(y) \) are similar and are left as an exercise for the reader.

6 Final Remarks

In summary, we cannot simply transplant CF methods to the AV grammar case. In particular, the ERF method yields correct weights only for SCFGs, not for AV grammars. We can define a probabilistic version of AV grammars with a correct weight-selection method by going to random fields. Property selection and weight adjustment can be accomplished using the DDL algorithms. In property selection, we need to use random sampling to find the initial weight for a candidate property, and in weight adjustment we need to use random sampling to solve the weight equation. The random sampling method that DDL used is not appropriate for sets of dags, but we can use the Metropolis-Hastings method.

As a closing note, it should be pointed out explicitly that the random field techniques described here can also be profitably applied to context-free grammars. As Stanley Peters nicely put it, there is a distinction between possibilistic and probabilistic context-sensitivity. Even if the language described by the grammar of interest—that is, the set of possible trees—is context-free, there may well be context-sensitive statistical dependencies. Random fields can be readily applied to capture such statistical dependencies whether or not \( L(G) \) is context-sensitive.

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