Study on Parametric Resonance of Support Parameters to Flow Pipeline

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Abstract. The influence of support parameters on the parametric resonance of fluid conveying pipeline under pulsating flow excitation is studied. Firstly, the mathematical model of fluid conveying pipeline and support system is established, then the Galerkin method is used for discretization, and the incremental harmonic balance method is used to solve the problem. The effects of support stiffness, support damping, support position, support number and support spacing on the boundary of parametric resonance region and the parametric resonance response are discussed. The results show that under the fixed pulsating amplitude, the pulsating frequency range of system instability becomes smaller with the increase of support stiffness, support damping and the number of support, while the support position and support spacing have less influence on it. The change of support stiffness and support position has little influence on the pulsation frequency that the parametric resonance lasts to the end, but the increase of brace damping makes its value decrease obviously. The research results can provide a theoretical basis for the design and installation of pipeline support system.

1. Introduction
Pipeline is a common device in mechanical hydraulic, chemical, petroleum and natural gas, and large water conveyance system. When the transport pipeline reaches a certain length, additional pipeline support is required to satisfy the normal operation of the whole system. The power source of fluid transmission is generally the pump. Due to the characteristics of the pump itself and the reasons of processing and manufacturing, the fluid must produce pulsation when flowing in the pipeline[1]. When the pulsation parameter satisfies a certain relationship, parameter resonance will occur, which will further affect the working performance of the entire pipeline system[2]. Due to the existence of pipeline support, the parameter resonance characteristics of the pipeline may change, which will affect the whole pipeline system. Therefore, it is of great engineering value and theoretical significance to study the influence of support parameters on parametric resonance.

Zhou Q Z[3] et al. used incremental harmonic balance method to study the influence of support parameters on Hopf bifurcation point of nonlinear braced cantilever transport pipeline system under the action of base excitation. Liang F et al.[1,4] used incremental harmonic balance method to study the parametric resonance of fixed transport tube with two ends under pulsating excitation. Jing S[5] using nonlinear theory, such as tilted support flow pipeline model is established, support Angle is studied for flow pipeline support parameters, such as the influence of dynamic response. Bai H H[6] studied the influence of bearing stiffness change on pipeline stress and other parameters by establishing a fluid-structure coupling dynamic model of straight pipe with elastic bearing with variable stiffness and finite
element simulation. Sheng S W[7] established a pipeline mathematical model by means of transfer matrix method and analysed the influence of support parameters on the natural frequency and vibration mode of the pipeline. Jin J D et al.[8,9] studied the parametric vibration characteristics of the hinge points transport pipeline under the action of pulsating flow and analysed the boundary of the unstable region under the conditions of different average velocity and mass ratio by establishing the equation of the two-end hinge points transport pipeline and using the average method. Ariaratnam S T et al.[10] studied and analysed the boundary range of the unstable region under different mean flow rates, boundary conditions and mass parameters by establishing a transport pipeline model supported by two ends and using the symbolism transformation method and the average method. Namchchivaya N S et al.[11] studied the subharmonic parameter resonance and combined parameter resonance of the pipeline under the action of pulsating fluid by establishing nonlinear pipeline model equation and using average method.

Although the above work has comprehensively studied the dynamic characteristics of pipeline conveying fluid and the influence of support parameters in it, there are few studies on the parametric resonance of pipeline conveying fluid with support parameters. Therefore, this paper establishes the dynamic model of pipeline and pipe support, obtains the numerical solution by Galerkin discrete method and incremental harmonic balance method, and uses numerical examples to study the influence of support parameters such as support position, support stiffness and support damping on the parametric resonance of flow conveying pipeline, so as to provide theoretical basis for the design and installation of pipeline support system.

2. Mathematical Model

2.1. Differential equations of motion for pipe conveying fluid

The vibration of a fixed pipe conveying fluid under the action of pulsating flow is analysed. The pipeline is placed vertically, and there is an elastic support at \( x = x_{m1}, x = x_{m2} \), and its bottom is fixed with a spacing of \( x_0 = x_{m2} - x_{m1} \).

![Figure 1. Support model of pipeline](image)

Assuming that the pipe is an Euler beam model, the influence of shear deformation and gravity is ignored. The fluid is incompressible and has a steady flow, regardless of the influence of viscosity and gravity. Only radial vibration of the support is considered for pulsating fluid action. Then the differential equation of the transport pipeline motion can be written as:

\[
\begin{align*}
& aE T \frac{\partial^4 y}{\partial x^4} + \frac{E T}{x_0} \frac{\partial^2 y}{\partial x^2} + \left( m_f u^2 - \left( T - \bar{P} A (1 - 2\theta) \right) \right) + m_i \frac{\partial u}{\partial t} (L - x) - \frac{E A}{2L} \int_0^L \left( \frac{\partial^2 y}{\partial x^2} \right)^2 dx - a \frac{E A}{L} \int_0^L \frac{\partial y}{\partial x} \frac{\partial^2 y}{\partial x^2} dx \\ & + (m_i + m_f) \frac{\partial^2 y}{\partial t^2} + 2m_f \frac{\partial^2 y}{\partial x \partial t} + \left( k_1 + c_1 \frac{\partial y}{\partial t} \right) \delta(x - x_{m1}) + \left( k_2 + c_2 \frac{\partial y}{\partial t} \right) \delta(x - x_{m2}) = 0
\end{align*}
\]

(1)

Where \( a \) is the viscoelastic coefficient of the pipeline, \( E \) is the elastic modulus of the pipeline material, \( T \) is the moment of inertia of the pipeline section, \( m_f \) is the fluid mass per unit length, \( m_i \) is the mass of the pipeline per unit length, \( u \) is the fluid velocity, \( \bar{P} A \) is the...
static pressure in the pipe, $\bar{P}$ is the Poisson's ratio, $A$ is the effective cross-sectional area of the pipeline, L is the length of the pipe, $k$ is the stiffness of the support spring, $c$ is the damping coefficient of the support, $x_m$ is the support position, $\delta(x)$ is the $\delta$ Function, which is defined as a generalized function describing the distribution density of points.

2.2 Dimensionless
Introduction of dimensionless parameters and the periodic variation of fluid velocity with time is considered.

$$
\alpha = \frac{ET}{m_1 + m_2}, \quad \beta = \frac{ET}{m_1 + m_2}, \quad \gamma = \frac{ET(1 - 2\nu)}{E}, \quad \kappa = \frac{kL}{ET}.
$$

$$
Z_a = \frac{c_vL}{(ET(m_1 + m_2))^{\frac{1}{2}}}, \quad \xi_a = \frac{m_1}{L}, \quad \kappa_a = \frac{kL}{ET}, \quad Z_\omega = \frac{c_vL}{(ET(m_1 + m_2))^{\frac{1}{2}}}, \quad \xi_\omega = \frac{m_1}{L}, \quad \eta_a = \frac{m_2}{L}.
$$

$$
v = v_0 \left(1 + \varepsilon \cos(\omega t)\right), \quad v^2 = v_0^2 + 2\varepsilon v_0 \cos(\omega t), \quad \dot{v} = -\varepsilon \omega v_0 \sin(\omega t)
$$

(2)

Where $v_0$ is the dimensionless average velocity, $\omega$ is the dimensionless pulsation frequency, $\varepsilon$ is the dimensionless pulsation amplitude.

The dimensionless parameters and equation (2) are substituted into equation (1) to obtain the motion equation of the pipeline with pulsating flow:

$$
\alpha \frac{\partial^2 \eta}{\partial \eta^2} + \beta \frac{\partial^2 \eta}{\partial \xi^2} + (2\varepsilon \beta + 2\varepsilon \varepsilon \cos(\omega t)) \frac{\partial \eta}{\partial \eta^2} + \left(v_0^2 - \gamma \right) \frac{\partial \eta}{\partial \xi} d^2 - 2\omega \eta \left(\frac{\partial^2 \eta}{\partial \xi^2} \right) + 2\varepsilon \varepsilon \cos(\omega t) - \omega \varepsilon \omega \varepsilon \sin(\omega t) = 0
$$

(3)

2.3 Discretization of differential equations of motion
The mode function is assumed to be:[4]:

$$
\phi_n(\xi) = \cosh(\lambda_n \xi) - \cos(\lambda_n \xi), \quad \sinh(\lambda_n \xi) - \sin(\lambda_n \xi), \quad n = 1, 2, 3...N
$$

(4)

Where $\lambda_n$, when $n = 1, 2$, $\lambda_1$ and $\lambda_2$ are the first two order eigenvalues of the pipeline. According to the boundary conditions of the fixed support at both ends and referring to the calculation method in reference 12, it can be obtained that $\lambda_1 = 4.73$, $\lambda_2 = 7.8532$.

The second-order Galerkin expansion is used to discretize equation (4):

$$
\eta(\xi, \tau) = \phi_1(\xi) q_1(\tau) + \phi_2(\xi) q_2(\tau)
$$

(5)

Write the above formula in matrix form, $\mathbf{I} \mathbf{q}^T = \left[ \phi_1, \phi_2 \right]^T, \quad \mathbf{q}^T = [q_1, q_2]^T$, then it can be written as:

$$
\mathbf{I} \mathbf{q}^T = \phi^T \mathbf{q}
$$

(6)

By substituting equation (6) into equation (3), both sides are the same left multiplication, and integral on $[0, 1]$, the following results are obtained:

$$
\mathbf{I} \dot{\mathbf{q}} + \left[ \dot{\mathbf{C}} + 2\varepsilon \varepsilon \beta \cos(\omega \tau) \right] \ddot{\mathbf{q}} + \mathbf{C} \ddot{\mathbf{q}} + \left[ \mathbf{K} - 2\varepsilon \varepsilon ^2 \beta \sin(\omega \tau) \right] \mathbf{q} + \mathbf{K} \mathbf{q} = 0
$$

(7)

where
3. Incremental Harmonic Balance Method

First, a new time variable is introduced \( t = \omega \tau \). By substituting equation (7),

\[ \frac{d^2 \tilde{q}}{dt^2} + \omega^2 \tilde{q} = \bar{G}(\tilde{q}) + \bar{F}(\tilde{q}) \]

(8)

Where \( \dot{\tau} \) is the partial derivative of \( t \). The first step is the incremental process. Let sum be the solution of the vibration equation, then its adjacent states are expressed in incremental form as follows:

\[ q = q_0 + \Delta q, \quad \epsilon = \epsilon_0 + \Delta \epsilon, \quad \omega = \omega_0 + \Delta \omega \]

(9)

By substituting equation (9) into equation (8) and omitting high-order trace terms,

\[ \frac{d^2 \tilde{q}}{dt^2} + \omega^2 \tilde{q} = \bar{G}(\tilde{q}) + \bar{F}(\tilde{q}) + \left( \bar{G}(\tilde{q}) + \bar{F}(\tilde{q}) \right) \Delta \omega - \left( 2 \omega \epsilon_0 + \omega^3 \tilde{q} \right) \Delta \omega + \left( 2 \omega \epsilon_0 + \omega^3 \tilde{q} \right) \Delta \epsilon \]

(10)

where

\[ R = \left( \omega_0 \Delta \omega \right) \]

(11)

The second step is harmonic balance process.

\[ q = S \Delta; \quad \Delta q = S \Delta \Delta \]

(12)

where

\[ S = \begin{pmatrix} C_0 & 0 \\ 0 & C_1 \end{pmatrix}, \quad A = \begin{pmatrix} A_0 & \Delta A_0 \\ \Delta A_0^T & \Delta A_1 \end{pmatrix}, \quad C_1 = \begin{pmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{pmatrix}, \]

(13)

By substituting equation (12) into equation (10) and (11) and applying Galerkin method, we can obtain the system of equations with \( \Delta A_1, \Delta \omega, \Delta \epsilon \) as unknown quantity.

(14)

The number of unknowns in equation (13) is two more than that in equation (13). Therefore, one increment must be selected as the active increment, and the other increment as the reference increment, and selected as the fixed value. In this way, the unique solution of equation (13) can be solved. In this paper, the unstable region of parametric resonance and the relationship between vibration amplitude and pulsation frequency are studied. Therefore, \( \Delta \omega \) is selected as the active increment and \( \Delta \epsilon \) is the reference increment, and the initial value \( A_0, \omega_0, \epsilon_0 \) is given. By calculation and error correction, the solution of equation (13) can be obtained.

4. Influence of Support Parameters on Parametric Resonance Region

When the fluid in the pipe is pulsating, the average velocity, amplitude and frequency of pulsating fluid will affect the stability of the system [13]. This section analyzes the influence of different support parameters on the system stability. Referring to reference 4, the parameters of pipeline and fluid are selected as follows: when the fluid in the pipe has pulsating property, the average velocity, amplitude...
and frequency of pulsating fluid will affect the stability of the system. The stability of the support system is affected by different parameters in this section. According to reference 4, the parameters of pipeline and fluid are selected as follows: \( v_0 = 4, \beta = 0.447, \gamma = 5000, \Gamma = 0 \).

The first-order parametric resonance regions with different support stiffness are plotted on the \( \omega - \varepsilon \) parameter plane, as shown in figure 2. It can be seen from figure 2 that increasing the support stiffness will make the instability region move along the direction of increasing the transverse axis. This is because the increase of stiffness increases the natural frequency, so the parametric resonance region will move in the direction of increasing frequency. At the same time, with the increase of support stiffness, the width of resonance region becomes smaller, that is to say, for the same amplitude, the increase of support stiffness makes the fluctuation frequency range of instability smaller.

It can be seen from figure 3 that the increase of support damping makes the parametric resonance region move upward along the direction of vertical axis increase, which is due to the increase of support damping which increases the minimum amplitude of instability. When the amplitude of pulsation is 0.1, the system will lose stability near the pulsation frequency under condition 1 or condition 2 or condition 3. However, under the condition of condition 4, the system will not lose stability no matter how large the pulsation frequency is. For the same pulsation amplitude, the increase of support damping reduces the frequency range of instability.

It can be seen from figure 4 that changing the value of the support position can make the resonance region move to the right along the pulsating frequency direction, but it does not reduce the width of the resonance region, that is, for the same pulsation amplitude, the change of the support position will not change the fluctuation frequency regions with instability. This is due to the change of the support position which changes the natural frequency of the system.

It can be seen from figure 5 that in the process of changing the number of supports from 0 to 2, the resonance region moves to the right along the direction of increasing the pulsation frequency. At the same time, the opening width of the parametric resonance boundary becomes smaller, that is, for the same amplitude, the increase of the number of supports makes the fluctuation frequency range of instability smaller. This is due to the increase of the number of supports, which changes the natural frequency of the system and increases the rigidity and stability of the system.
Figure 4. The first order parametric resonance region with different
(1. $\xi_{m1}=0.1$, $Z_i=0$, $\kappa_i=100$, $\kappa_j=0$, $\theta=10$, $m_0=0$, $\phi=0$),
(2. $\xi_{m1}=0.2$, $Z_i=0$, $\kappa_i=100$, $\kappa_j=0$, $\theta=10$, $m_0=0$, $\phi=0$),
(3. $\xi_{m1}=0.3$, $Z_i=0$, $\kappa_i=100$, $\kappa_j=0$, $\theta=10$, $m_0=0$, $\phi=0$),
(4. $\xi_{m1}=0.5$, $Z_i=0$, $\kappa_i=100$, $\kappa_j=0$, $\theta=10$, $m_0=0$, $\phi=0$)

Figure 5. The first order parametric resonance region with different support numbers
(1. Without support, 2. one support, 3. two supports)

It can be seen from figure 6 that with the increase of the distance between the two supports, the resonance region first moves to the right and then to the left along the pulsating frequency direction. However, for parametric resonance, the V-shaped width of the unstable region does not change, that is, for the same amplitude, the change of the spacing between the two supports will not change the unstable pulsation frequency range. This is because the natural frequency of the system first increases and then decreases with the increase of the distance between the two supports.

Figure 6. The first order parametric resonance region with different spacing between two supports
(1. $\xi_{m1}=0.2$, $Z_i=Z_j=0.1$, $\kappa_i=\kappa_j=100$, $\theta=0.1$, $\phi=0$, $\omega=0.7$, $\Gamma=0$, $m_0=0$, $\phi=0$)

5. Influence of support parameters on parametric resonance response of pipeline
The parameters of pipeline and fluid are: $v_0=4$, $\beta=0.447$, $\gamma=5000$, $\Gamma=0$. When the support parameters are $\xi_{m1}=0.5$, $Z_i=0$, $\kappa_i=0$, $\xi_{m2}=0.7$, $Z_j=0$, $\kappa_j=0$, the first-order natural frequency and the second-order natural frequency of the pipeline corresponding to this parameter value are $\omega_{n1}=16.9938$, $\omega_{n2}=56.7521$. The actual amplitude fluctuation frequency response curve in figure 7(b) is the pipeline amplitude at $\xi=0.65$. According to figure 7(a), when $\varepsilon=0.25$, the pulsating frequency values of points a and B corresponding to the first-order parametric resonance boundary points are $\omega_a=30.3$, $\omega_b=37.4$. These two points are at both ends of $\omega=2\omega_{n1}=33.9876$. When the pulsating frequency $\omega$ increases gradually and reaches $\omega=2\omega_{n1}$, the pipeline begins to lose stability and parametric resonance occurs. With the
increase of the pulsating frequency $\omega$, the actual amplitude $\omega$ of the pipeline gradually increases, and this vibration continues to $\omega = 122$.

Figure 7. (a) First order parametric resonance region, (b) Actual amplitude fluctuation frequency response curve

From the above analysis, given the amplitude of fluid pulsation, when the pulsation frequency reaches a certain value, the pipeline will lose stability and produce parametric resonance. Therefore, this section will study the influence of support parameters such as support stiffness, support damping, support position and support number on parametric resonance response when the fluctuation amplitude is $\varepsilon = 0.25$.

The influence of support stiffness on the amplitude $A$ of $q_1$ and $q_2$ modes at first-order parametric resonance is analyzed by IHB method. Figure 8 shows the variation of modal amplitudes $A_1$ and $A_2$ when $\xi = 0.5$, $Z = 0$, $\xi_1 = 0$, $\kappa_1 = 100$, $\kappa_1 = 150$, $\kappa_1 = 200$. It can be seen from Figure 8 that with the increase of support stiffness, the pulsating frequency corresponding to the parametric resonance boundary point of the first-order 1/2 harmonic resonance due to the instability of the system increases, which is due to the change of the linear natural frequency of the system due to the existence of the support stiffness. At the same time, the value of the pulsation frequency which lasts until the end of the resonance increases correspondingly. Combined with Figure 8 and Figure 9, since the modal amplitude $A_1$ is much larger than $A_2$, $q_1$ is the main component in the actual amplitude (the amplitude at $\xi = 0.65$). Therefore, in a certain range, with the increase of stiffness, the change of actual amplitude is similar to that of mode amplitude $A_1$.

Figure 8. Amplitude frequency response curve of different support stiffness
1. $\xi = 0.5$, $Z = 0$, $\xi_1 = 0$, $\kappa_1 = 0$, $\kappa_1 = 0$

Figure 9. Actual amplitude of pipeline with different support stiffness
1. $\xi = 0.5$, $Z = 0$, $\xi_1 = 0$
It can be seen from figure 10 and figure 11 that with the increase of support damping, the fluctuation frequency value corresponding to the boundary point of parametric resonance for the first-order 1/2 harmonic resonance due to system instability changes very little, but the pulsating frequency value of the parametric resonance lasting to the end decreases obviously, from 126 to 85. Combined with figures 10 and figure 11, it can be seen that the variation of the actual amplitude (the amplitude at $\xi=0.65$) is similar to that of the modal amplitude $A_1$.

Combined with figure 12 and figure 13, it can be seen that the change of support position makes the system unstable and the first-order 1/2 harmonic resonance occurs. The fluctuation frequency corresponding to the boundary point of parametric resonance also changes correspondingly, but the change of support position has little effect on the fluctuation frequency value of parametric resonance lasting to the end.

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6. Conclusion
In this paper, by establishing the mathematical model of pipeline system and support system, the influence of support parameters on parametric resonance of pipeline system under pulsating flow is studied by Galerkin method and incremental harmonic balance method. The influence of parameters such as support stiffness, support damping, support position, support number and support spacing on the boundary of parametric resonance region is analysed, and the influence of support stiffness, support damping and support position on parametric resonance response is analysed. The results show that: (1) For a fixed amplitude, the increase of support stiffness and the number of supports makes the fluctuation frequency range of system instability smaller, while the change of support position and support spacing does not change the fluctuation frequency range of system instability; the increase of support damping makes the system parameter resonance boundary move upward along the fluctuation amplitude. (2) With the increase of support stiffness, the fluctuating frequency corresponding to the boundary point of parametric resonance will increase. However, the increase of support damping has little effect on the pulsation frequency corresponding to the boundary point of parametric resonance, and the change of support position will also increase the critical pulsation frequency of subharmonic resonance of the system, but the influence is not as great as the change of support stiffness. The results show that the change of support stiffness has obvious effect on the pulsation frequency of parametric resonance, while the change of support stiffness and support position has little effect on it. (3) Through the above analysis, it is necessary to comprehensively consider the influence of excitation form and support parameters in the design and installation of pipeline support system to meet the working performance of pipeline system.

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