Tunneling through a barrier in Tomonaga-Luttinger liquid connected to reservoirs

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Abstract

The transport through a barrier is studied for a Tomonaga-Luttinger liquid of finite length connected to reservoirs. An effective action for the phase variable at the barrier is derived for spatially varying electric field. In the d.c. limit only the total voltage drop between the left and right reservoirs appears in the action. We discuss crossovers of the renormalization group flow and hence of the temperature and wire length dependence of the conductance, taking into account the location of the barrier.

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Recently the quantum transport in Tomonaga-Luttinger (TL) liquids has been studied intensively both experimentally and theoretically. One of the subtle issues in this problem is the effects of the leads and contacts which are often ignored in the treatment of infinitely long TL wires. Several authors reached the conclusion that the renormalization of the conductance $G$ due to the electron-electron interaction is absent in the clean TL liquids when the reservoirs, i.e., leads, are properly taken into account. The same conclusion has been obtained even for the infinite TL wire by considering carefully the definition of the voltage drop, i.e., the chemical potential difference between the right and left leads.

Inspired by these recent works, we examine in this paper the transport through a barrier in a finite-length quantum wire connected to the leads. We find nontrivial dependence of the conductance on the temperature, wire length, and also the location of the barrier.

Our model is similar to those of Ref. and its Lagrangian in the imaginary time is given as

$$L = \int dx \left[ \frac{1}{2K(x)} \left( \frac{1}{v(x)} \left( \frac{\partial \phi}{\partial \tau} \right)^2 + v(x) \left( \frac{\partial \phi}{\partial x} \right)^2 \right) \right] + \frac{\lambda_B}{\pi \alpha} \cos[2k_F a + 2\sqrt{\pi} \phi(a, \tau)] + \int dx E(x, \tau) \frac{1}{\sqrt{\pi}} \phi(x, \tau), \quad (1)$$

where $K(x)$ and $v(x)$ is the spatially varying exponent and velocity of the TL liquid, and $\alpha$ is a short-distance cutoff. Following Ref., we assume that the wire is confined in $0 < x < L$ and the leads extend for $x < 0$ and $x > L$. Correspondingly $K(x) = K_W$, $v(x) = v_W$ for the wire and $K(x) = K_L = 1$, $v(x) = v_L$ for the leads. The second term on the rhs of Eq. (1) describes the backward scattering by the barrier potential at $x = a$ ($0 < a < L$) whose strength is $\lambda_B$. The third term is the coupling with the electric field $E(x)$. Now we integrate over the continuum degrees of freedom $\phi(x, \tau)$ in the functional integral with the fixed value of $\phi(x = a, \tau) = \phi_0(\tau)$ and $E(x, \tau)$. This procedure can be done if one knows the Green’s function $G_{\omega_n}(x, x')$ for the unperturbed TL liquid described by the first term in Eq. (1). Note that it is no longer a function of $x - x'$ because the translational symmetry is broken in our system. In fact $G_{\omega_n}(x, x')$ has been obtained in Ref., but here we derive the effective action first without referring to its explicit form:
\[ S = \frac{1}{2\beta} \sum_{\omega_n} \frac{1}{G_{\omega_n}(a,a)} \tilde{\phi}_0(-\omega_n) \tilde{\phi}_0(\omega_n) + \frac{\lambda_B}{\pi \alpha} \int_0^\beta d\tau \cos[2k_Fa + 2\sqrt{\pi} \phi_0(\tau)] \\
+ \sum_{\omega_n} \int dx \frac{G_{\omega_n}(x,a)}{\sqrt{\pi} G_{\omega_n}(a,a)} \tilde{E}(x,\omega_n) \tilde{\phi}_0(-\omega_n) \\
- \frac{\beta}{2\pi} \sum_{\omega_n} \int dx \int dx' \left[ G_{\omega_n}(x,x') - \frac{G_{\omega_n}(x,a)G_{\omega_n}(x',a)}{G_{\omega_n}(a,a)} \right] \tilde{E}(x,\omega_n) \tilde{E}(x',-\omega_n), \] (2)

where \( \tilde{X}(\omega_n) = \int d\tau e^{i\omega_n \tau} X(\tau) \) (\( X = \phi_0 \) and \( E \)). In this calculation we used the relation \( G_{\omega}(x,x') = G_{|\omega|}(x,x') = G_{|\omega|}(x',x) \). Only the following information is needed for our purpose:

\[ \lim_{\omega \to 0} G_{\omega}(x,x') = \frac{K_L}{2|\omega|}, \] (3)

\[ G_{\omega}(a,a) = \frac{K_W}{2|\omega|} + \frac{K_W}{|\omega|} \frac{(K_L - K_W)^2 e^{-L/L_\omega} + (K_L^2 - K_W^2) \cosh((L - 2a)/L_\omega)}{(K_L + K_W)^2 e^{L/L_\omega} - (K_L - K_W)^2 e^{-L/L_\omega}}, \] (4)

where \( L_\omega = v_W/|\omega| \).

Here we are interested in the d.c. conductance, i.e., \( \tilde{E}(x,\omega_n) = E(x) \delta_{\omega_n,0} \) and from Eq. (3) the action becomes

\[ S = \frac{1}{2\beta} \sum_{\omega_n} \frac{1}{G_{\omega_n}(a,a)} \tilde{\phi}_0(-\omega_n) \tilde{\phi}_0(\omega_n) + \frac{\lambda_B}{\pi \alpha} \int_0^\beta d\tau \cos[2k_Fa + 2\sqrt{\pi} \phi_0(\tau)] \\
+ \frac{1}{\sqrt{\pi}} \Delta V \tilde{\phi}_0(\omega_n = 0). \] (5)

Note that only the total voltage drop \( \Delta V = \int_{-\infty}^\infty dx E(x) \) appears in the action, which couples linearly with the phase variable at the barrier. Thus we can calculate the d.c. conductance without the detailed knowledge of the spatial dependence of the electric field \( E(x) \), which should be determined by solving the Maxwell equations self-consistently.

We study the action Eq. (5) in the limit of weak and strong barrier potential by employing the renormalization group (RG) method. Since the analysis is now standard, we skip the detailed derivation and give only the final results and their physical picture. Assuming \( 0 < a < L/2 \) without loss of generality, we can approximate Eq. (4) as

\[ G_{\omega}(a,a) \approx \frac{K}{2|\omega|}, \] (6)
where the effective exponent \( K \) is given by

\[
K = \begin{cases} 
K_W & |\omega_n| \gg v_W/a, \\
\frac{2K_LK_W}{K_W+K_L} & v_W/L \ll |\omega_n| \ll v_W/a, \\
K_L & |\omega_n| \ll v_W/L,
\end{cases}
\]

(7)

where we have assumed \( a \ll L \). When \( a \sim L/2 \) the second region \( v_W/L \ll |\omega_n| \ll v_W/a \) collapses. As the frequency \( |\omega_n| \) decreases, the longer range properties become relevant.

In the low-frequency limit the presence of the Fermi-liquid leads is essential, while in the high-frequency regime the exponent of the TL wire controls the renormalization. In the intermediate frequency regime, both \( K_W \) and \( K_L \) contribute because \( \phi_0 \) sees both the Fermi-liquid lead \((x < 0)\) and the TL wire \((a < x < L)\).

First we consider the weak-potential limit. The RG equation for \( \lambda_B \) reads

\[
\frac{d\lambda_B}{dl} = (1 - K)\lambda_B,
\]

(8)

where \( l = -\ln \Lambda \) (\( \Lambda \): frequency cutoff). Up to second order in \( \lambda_B \) the conductance \( G(T) \) at temperature \( T \) is then given by

\[
G(T) = \begin{cases} 
\frac{e^2}{2\pi} - c_1 e^2 \left( \frac{\lambda_B}{v_W} \right)^2 \left( \frac{T}{D} \right)^{2K_W-2} & T \gg v_W/a, \\
\frac{e^2}{2\pi} - c_2 e^2 \left( \frac{\lambda_B}{v_W} \right)^2 \left( \frac{v_W}{\alpha D} \right)^{2K_W-2} \left( \frac{T}{v_W} \right)^{2(K_W-1)/(K_W+1)} & v_W/L \ll T \ll v_W/a, \\
\frac{e^2}{2\pi} - c_3 e^2 \left( \frac{\lambda_B}{v_W} \right)^2 \left( \frac{v_W}{\alpha T} \right)^{2K_W-2} \left( \frac{\alpha T}{v_W} \right)^{2(K_W-1)/(K_W+1)} & T \ll v_W/L,
\end{cases}
\]

(9)

where \( c \)'s are constants of order unity and \( D \) is a high-energy cutoff (band width, \( \alpha D \sim v_W \)).

As we see in Eq. (9), in the presence of a single barrier near the edge of the wire \((a \ll a \ll L)\), the conductance shows the power-law temperature dependence characteristic of the TL liquid in the high and intermediate temperature regime. In the low-temperature limit, the conductance does not depend on \( T \) because the renormalization is cut off by the finite length of the wire. If the barrier is in the middle of the wire \((a \approx L/2)\), the intermediate temperature regime does not exist, and furthermore in the low-temperature regime the correction term proportional to \( \lambda_B^2 \) depends on the wire length as \( L^{2-2K_W} \). When many weak impurities (barriers) are distributed in the wire, we should average over the location \( a \) of the barriers. For a uniform distribution, there remains only a single crossover temperature \( v_W/L \) because
the dominant contribution comes from the case $a \sim L/2$. We thus reproduce the result of Maslov, Eq. (14) in Ref. We note that Eq. (9) is valid only when the second term on the rhs is much smaller than the first term, $e^2/2\pi$. This condition is the most severe at low temperatures, and is always satisfied if $\left(\frac{\lambda_B}{v_W}\right)^2 \left(\frac{v_W}{aD}\right)^{2K_W-2} \left(\frac{a}{T}\right)^{(K_W-1)/(K_W+1)} \ll 1$. Otherwise we can use Eq. (9) only above some temperature, below which the barrier potential should be regarded as strong.

We now turn to the limit of strong barrier potential, which is not discussed in Ref. The condition for this limit to be realized is $D(\lambda_B/v_W)^{1/(1-K_W)} \gg v_W/a$. Duality mapping using the instantons is useful in this case, and the effective action for the dual field $\theta_0(\tau)$ is given by

$$S = 2 \sum_{\omega_n} \omega_n^2 G_{\omega_n}(a,a) \tilde{\theta}_0(-\omega_n) \tilde{\theta}_0(\omega_n) + 2t \int_0^\beta d\tau \cos[2\sqrt{\pi} \theta_0(\tau)], \quad (10)$$

where $t$ is the fugacity of the instanton, i.e., the tunneling matrix element through the barrier. The RG equation for $t$ is readily obtained as

$$\frac{dt}{dl} = \left(1 - \frac{1}{K}\right) t, \quad (11)$$

where $K$ is given in Eq. (7). Then it is straightforward to derive the temperature dependence of the conductance $G(T)$ in this limit:

$$G(T) = \begin{cases} \tilde{c}_1 e^2 \left(\frac{T}{T}\right)^2 \left(\frac{v_W}{aD}\right)^{2/2K_W-2} & T \gg v_W/a, \\ \tilde{c}_2 e^2 \left(\frac{T}{T}\right)^2 \left(\frac{v_W T}{aD^2}\right)^{1/(K_W-1)} & v_W/L \ll T \ll v_W/a, \\ \tilde{c}_3 e^2 \left(\frac{T}{T}\right)^2 \left(\frac{v_W^2}{aLD^2}\right)^{1/2K_W-1} & T \ll v_W/L. \end{cases} \quad (12)$$

Equation (12) is valid for $\alpha \ll a \ll L/2$. If $a \approx L/2$, the intermediate temperature regime does not exist, and $G(T) \propto L^{2-2/K_W}$ in the low-temperature limit. Since the strongest barrier determines the transport of the whole system, at low temperatures where the impurity potentials become strong due to the renormalization, the $T$- and/or $L$-dependence of the conductance shown in Eq. (12) might be observed in experiments of long quantum wires.

So far we have discussed the transport of spinless TL liquids. To compare the theory with experiments in quantum wires, we only need to replace the RG equations, Eqs. (8)
and (11), by $\frac{d\lambda_B}{dt} = \frac{1}{2}(1 - K)\lambda_B$ and $\frac{dt}{dl} = \frac{1}{2}(1 - \frac{1}{K})t$. Accordingly, the exponents of the renormalization factors in Eqs. (9) and (12) should be divided by 2.

Finally we briefly comment on the effect of the Coulomb interaction. With this interaction the effective exponent $K$ is a function of the energy even for the infinite wire and goes to zero in the low-energy limit. For the finite-length wires the RG is cut off by the energy scale $v_W/L$ or $v_W/a$. For example, when a single strong barrier is present in the middle of the wire, the conductance should show the anomalous temperature dependence discussed in Ref. 2 at $T \gg v_W/L$, $G(T) \propto T^{-2} \exp \left[ -A \left( \ln \frac{v_W}{W T} \right)^{3/2} \right]$ ($A$ is a constant and $W$ is the width of the wire). At lower temperatures $T \ll v_W/L$, $T$ should be replaced by $v_W/L$ and the conductance becomes independent of $T$. We note, however, that the above discussion is possibly oversimplified, and the screening of the Coulomb interaction due to the electrons in the Fermi-liquid leads and the gates near the wire should be properly taken into account.

In summary we have studied the tunneling through a barrier in a finite-length TL liquid connected to Fermi-liquid leads. The conductance shows a peculiar dependence on the temperature, the wire length, and the location of the barrier. We propose that systematic studies of these dependence will give a firm evidence for the TL liquid in the quantum wire.

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