Discrete Time Leads to Quantum-Like Interference of Deterministic Particles

Andrei Khrennikov
International Center for Mathematical Modeling in Physics, Engineering and Cognitive science
MSI, Växjö University, S-35195, Sweden
email: Andrei.Khrennikov@msi.vxu.se

Yaroslav Volovich
Physics Department, Moscow State University
Vorobievi Gori, 119899 Moscow, Russia
email: yaroslav@aylabs.com

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Abstract

In this note we demonstrate that a quantum-like interference picture could appear as a statistical effect of interference of deterministic particles, i.e. particles that have trajectories and obey deterministic equations, if one introduces a discrete time. The nature of the resulting interference picture does not follow from the geometry of force field, but is strongly attached to the time discreetness parameter. As a demonstration of this concept we consider a scattering of charged particles on the charged screen with a single slit. The resulting interference picture has a nontrivial minimum-maximum distribution which vanishes as the time discreetness parameter goes to zero that could be interpreted as an analog of quantum decoherence.

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1 Introduction

It is well known that historically the results of experiments with elementary particles were interpreted as the evidence of the impossibility to provide deterministic, classical-like, description of motion of these objects. The main attitude in the development of quantum theory was deeper and deeper understanding of the fact that quantum randomness has fundamental, irreducible, character – in the opposite to classical randomness. Randomness in classical statistical mechanics can be reduced to uncertainty of initial conditions. The evolution of probabilistic density described by Liouville's equation can be reduced to deterministic evolution of an ensemble of particles described by Hamiltonian equations on the phase space. As we have just mentioned, such a picture was considered as totally inadequate to quantum situation. The formulation by N. Bohr the complementarity principle was the culmination of anti-deterministic development of views to experiments with elementary particles. The collection of these views (originated by Bohr, Heisenberg and many others, see e.g. [1], [2]) is now days known as the orthodox Copenhagen interpretation of quantum mechanics. By this interpretation it is (even in principle) impossible to provide deterministic description of motion of elementary particles. Thus we could not reproduce statistical results of quantum experiments by using classical statistical mechanical approach: deterministic equations for trajectories of e.g. electron that reproduce statistical behaviour described by a wave function. This idea appeared already in letter's exchange between Heisenberg and Bohr directly after the publication of the famous Heisenberg paper [3]. By discovering dynamical equations that describe physical observables, e.g. position and momentum observables, W. Heisenberg claimed that deeper, ontic, description of quantum systems is even in principle impossible. This viewpoint was strongly supported by the discovery of Heisenberg’s uncertainty principle. This principle was used by N. Bohr as the basis for starting great changes in philosophy of physics, resulted by the complementarity principle.

We would like to notice, see [4] for the details, that, in fact, Heisenberg-Bohr conclusions were not logically justified. By creating a mathematical formalism for physical observables, Heisenberg matrice-mechanics, we do not prove the impossibility of ontic deterministic description. Neither by referring to uncertainty principle. If this principle would be interpreted statistically, see e.g. [5], [6], then this would be simply a relation for dispersions
of two random variables. It seems very doubtful that such a relation should imply such strong restrictions on mathematical description of reality as e.g. impossibility to describe trajectories of e.g. electrons in configuration or phase space.

Now days we can definitely say that Heisenberg-Bohr conclusions were not totally justified, since we have e.g. Bohmian formalism [7], [8] or stochastic electrodynamics, see e.g. [9], [10] on results related to this paper. For example, the Bohmian formalism provides the deterministic description of trajectories of e.g. electron. Statistical results given by quantum formalism can be reproduced on the basis of this deterministic picture. However, by some reasons Bohmian theory is commonly considered as unacceptable. It seems that the appearance of this ontic model for quantum mechanics does not change essentially the orthodox Copenhagen orientation of quantum community. By ourself we do not have definite point of view to the validity of Bohmian model. In any case it could be used as an argument against Heisenberg-Bohr conclusions.

We think that there might be created other, non-Bohmian, ontic models reproducing probabilistic results given by quantum formalism. In particular, it might be that some of these models could be local. Of course, the reader may argue that there are Bell’s arguments. These arguments imply that local deterministic description is impossible, see e.g. [11], [12]. However, recent investigations, see e.g. [13] (papers of Accardy and Regoli, Ballentine, De Muynck, Gudder, Volovich), [14]–[16], demonstrated that Bell’s conclusions were not totally justified. It seems that experimental violations of Bell’s inequality need not be interpreted as arguments against local realism.

‘Local realism’ is the standard terminology used in Bell-discussions. Of course, the use of such a terminology was a consequence of EPR-discussions. We would not like to tell about realism. Of course, we do not deny the existence of independent physical reality. But we understood well that all our models are simply approximations of this reality. We would never create the model that would be totally adequate to physical reality. We prefer to speak about deterministic models that provide some mathematical picture of reality. The crucial point of this consideration is that, in principle, a mathematical model of space-time need not be identified with the ‘continuous’ model given by real numbers. When we say deterministic description, we do not mean that this is some model based on differential equations in the continuous real space. For instance, it can be some discrete deterministic
model or a p-adic model. The latter models were intensively studied in the connection to string theory, see, for instance, [17]-[21]. At the moment we do not say the p-adic program of reconstruction of theoretical physics was totally successful. In any case it is an interesting (deeply developed) alternative approach to mathematical modelling of physical phenomena. However, it is not the subject of our present consideration.

Regarding to space-time models, we, finally, remark that locality in the most of ‘local realism’ discussions also is considered in the old fashioned, real continuous, form. It would be natural to extend this 18th-20th century approach and consider not only real locality, but e.g. locality on a discrete space or p-adic locality. As having large experience of the work with p-adic numbers, we can tell that there is crucial difference between e.g. real and p-adic locality.

As the reader understood, we are looking for local deterministic models in above mentioned extended meaning that could reproduce probabilistic behaviour described by quantum theory.

We recall that one of the most distinguishing features of ‘quantum’ probabilistic behaviour is the appearance of interference structures created by ensembles of elementary particles. In fact, the impossibility to provide deterministic description of such a phenomenon was the main reason to create the orthodox Copenhagen interpretation. Thus deterministic models reproducing interference pictures are of the large interest as at least simulating quantum behaviour.

We remark that according to the orthodox Copenhagen interpretation it is (even in principle) impossible to create such models. In particular, we can not describe ‘self-interference’ of e.g. electron in the two slit experiment without to use wavelike arguments. In particular, there is a rather common opinion that there is crucial difference between classical and quantum probabilistic rules for addition of probabilities of alternatives:

\[ P = P_1 + P_2 \]  

\[ P = P_1 + P_2 + 2\sqrt{P_1 P_2} \cos \theta. \]  

However, recently it was demonstrated in series [22]-[24] of papers that the difference between ‘classical’ and ‘quantum’ probabilities can be explained in classical probabilistic terms by taking into account contextual dependence of
probabilities involved into quantum interference. Moreover, in [22]–[24] there was presented the idea that quantum formalism was merely the pure mathematical discovery of such calculus of probabilities depending on contexts, complexes of experimental physical conditions. Of course, everybody knows that contextualism was originally incorporated into quantum theory by N. Bohr. Thus, in fact, contextual probabilistic approach is nothing else than mathematical (classical probabilistic) formalization of Bohr’s contextualism. We also remark that ‘classical probabilistic’ should not identified with some concrete probabilistic model; in particular, conventional measure-theoretical model, Kolmogorov, 1933 [25]. For us, ‘classical probabilistic’ is a frequency description. There can be various mathematical models for such a description. Of course, quantum probabilistic calculus, Hilbert space calculus, also gives a frequency description. However, here we start with Hilbert space that appears without direct relation to frequencies; such a Hilbert space description (with corresponding interpretation of superposition principle) looks merely as the mathematical model for wave phenomena. In [22]–[24] we start with frequencies and reproduce complex wave amplitudes and the Hilbert space structure as a consequence of contextual dependence of probabilities.

Understanding of the contextual structure of quantum probabilities implies that, on one hand, we need not follow to orthodox Copenhagen approach; on the other hand, quantum-like probabilistic behaviour need not be related only to experiments with elementary particles. In particular, we can try to obtain interference-like effects in experiments with macroscopic systems by taking into account context dependence. In our paper [26] we presented numerical experiment for macroscopic charged balls that can be considered as the direct analogue of the two slit experiment. We found the interference effect and introduced corresponding complex waves (of course, of probabilities).

In this paper we study the deterministic model for scattering of charged particles on the charged screen with a single slit. The resulting interference picture has a nontrivial minimum-maximum distribution. This interference picture has no relation to ‘self-interference’ of particles, no wave-structure is involved into considerations. The basic source of interference is the discrete time scale used in our mathematical model: instead of Newton’s differential equations, continuous time evolution, we consider difference equations, discrete time evolution. Interference effect disappears as the time discreetness

\footnote{In particular, there can be created local deterministic models for quantum statistics.}
parameter goes to zero that could be interpreted as an analog of quantum decoherence.

The common viewpoint might be that we study just a discrete approximation to continuous Newton model. It is supposed that the latter model gives the right picture of 'classical physical reality.' However, we think that \textit{continuous Newton’s model is just an approximation of physical reality}. The right picture is given by discrete difference equations. Hence the contradiction between statistical description provided by quantum formalism and Newtonian mechanics could not be considered as a contradiction between quantum and classical (deterministic) physics. Such a contradiction, that typically discussed in quantum literature, should be interpreted as simply a consequence of the use of an approximation, namely continuous Newtonian mechanics, instead of the use of the adequate model, namely discrete model with some level of discretization depending on an experimental context.

2 Discrete Time in Newton’s Equations

Classical particles are believed to obey the well known Newton’s equation

\begin{equation}
F = m\ddot{r}
\end{equation}

Here we modify this equation to produce an interference picture similar to quantum interference. We introduce a parameter of time discreetness \( \tau \) described below.

Let us rewrite the second order differential equation (3) as a system of first order differential equations, we have

\begin{align*}
F &= m\dot{v} \\
v &= \dot{r}
\end{align*}

In the system (4) the derivatives assume the continuousness of time. Let us now introduce a discreetness parameter \( \tau \).

\begin{align*}
F &= m\frac{v(t + \tau) - v(t)}{\tau} \\
v(t + \tau) &= \frac{r(t + \tau) - r(t)}{\tau}
\end{align*}

(5)
In the limit of $\tau \to 0$ (5) is equivalent to (3) and (4).

In the model described below we consider particles which move obeying the system (5) where the force is produced by a charged screen.

Please note that in our model the coordinate space is left continuous, although it would be interesting to consider it on the discrete coordinate space, i.e. on the lattice.

3 The Model

![Figure 1: Single slit experiment. Charged particles are emitted at point e pass through a slit in the screen $S_1$ and gather on the screen $S_2$.](image)

We consider a scattering on the single slit (Fig.1). Uniformly charged round particles are emitted at point $e$ (emitter) with fixed velocity with angles evenly distributed in the range $(-\pi/2, \pi/2)$. Each particle interacts with the uniformly charged flat screen $S_1$. The charge distribution on the particle and the screen stays unchanged even if the particle comes close to the screen. Physically this is a good approximation when the particle and the screen are both made of dielectric. There is a rectangular slit in the screen (on the Fig.1 the slit is perpendicular to the plane of the picture). Particles pass through the slit in screen $S_1$ and gather on screen $S_2$. We are interested in the particle distribution on the second screen.
Now let us write the laws of motion for the particles. The force affecting the particle is given by the Coulomb’s law

\[ \mathbf{F}_i = \int_{D_i} \frac{q\sigma}{|\mathbf{r}'|^2} \cdot \frac{\mathbf{r}'}{|\mathbf{r}'|} \, ds \]  

(6)

where \( r' \) is a vector from an element on the screen to the particle, \( q \) is charge of the particle, \( \sigma \) is charge density on the screen, i.e. charge of a unit square. We integrate over the surface of the screen, the integration region \( D_i \) is plane of the screen except the split.

Projecting equation (6) to \( xy \)-plane, where \( x \) and \( y \) denotes horizontal and vertical coordinates of the particle respectively we get

\[ F_x = q \sigma \int_{\Gamma} dy' \int_{\mathbb{R}} dz' \frac{x}{(x^2 + (y - y')^2 + z'^2)^{3/2}} \]

\[ F_y = q \sigma \int_{\Gamma} dy' \int_{\mathbb{R}} dz' \frac{y - y'}{(x^2 + (y - y')^2 + z'^2)^{3/2}} \]  

(7)

where \( 2R \) is the height of the slit, \( F_x \) and \( F_y \) denote the projections of the force \( \mathbf{F} \) to \( x \) and \( y \) axes, and the integration region

\[ \Gamma = (-\infty, -R) \cup (R, +\infty) \]  

(8)

Integrating the rhs of (7) we get

\[ F_x = 2q \sigma \left( \pi + \arctan \frac{y - R}{x} - \arctan \frac{y + R}{x} \right) \]

\[ F_y = q \sigma \ln \frac{x^2 + (R - y)^2}{x^2 + (R + y)^2} \]  

(9)

We take the following initial values

\[ x(0) = -D \quad \dot{x}(0) = v_0 \cos \alpha \]

\[ y(0) = 0 \quad \dot{y}(0) = v_0 \sin \alpha \]  

(10)

where angle \( \alpha \) is a random variable uniformly distributed in \([0, 2\pi]\). The constant parameters \( v_0 \) and \( D \) are initial velocity and distance between emitter and the screen.
Particles are emitted at point \( e \) (see Fig.1), move obeying affected by force \( \mathbf{F} \) pass through slit in the screen \( S_1 \) and gather on the screen \( S_2 \). Having points where particles hit the screen \( S_2 \) we compute frequencies with which particles appear on screen \( S_2 \) as a function of coordinates on the screen, we call this function a particle distribution. We are interested in computing the particle distribution over a vertical line on screen \( S_2 \) with \( z = 0 \). That is why we consider a motion only in the \( xy \)-plane and initial values (10) do not contain \( z \)-coordinate.

The second screen was separated with cells of equal size, the diameter of a particle. The number of particles which hit into each cell was calculated and interpreted as a particle distribution. The details of numeric computations are given in the appendix.

4 Conclusion

In this note we have shown that a quantum-like interference picture could appear as a statistical effect of deterministic particles, i.e. having trajectories and obeying deterministic equations, if one introduces a discrete time. The nature of the resulting interference picture (particle distribution, see Fig. 2-5 in appendix) does not follow from the geometry of force field, but is strongly attached to the discreetness parameter \( \tau \).

The described behavior stays without contradiction with a contextual approach to quantum probabilities. It would be interesting to investigate the scattering on the two slit screen.

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Appendix

Below we present a sample trajectories plots for two different discretness parameters $\tau$ (Fig.4,5). And the corresponding particle distributions (Fig.2,3).

Parameters of the model where the following $D = 5$, $d = 25$, $R = 5$, $v_0 = 12$ the particle and the screen $S_1$ (Fig.1) had the opposite charges, i.e. $q\sigma = -1$ and the radius of a particle $r = 0.2$.

To produce a particle distribution pictures with a trustable precision about $10^7$-10$^8$ trajectories where computed. To produce such a large amount of computations even for modern stations we used a parallel Sun-UltraSPARC 4-processor station located at Växjö University. The program was implemented using GNU C++ (g++).

Since the computation time is proportional to $1/\tau$ and the computations where performed simultaneously the total number of computed trajectories for $\tau = 0.05$ (Fig.4) and $\tau = 0.01$ (Fig.3) differs approximately five times.

Figure 2: Particle distribution on the second screen. See also the corresponding (Fig.4). Parameters: $\tau = 0.05$, total = 138582362 particle trajectories where computed.
Figure 3: Particle distribution on the second screen. See also the corresponding (Fig.5). Parameters: $\tau = 0.01$, total = 28200885 particle trajectories where computed.
Figure 4: Single slit experiment. Particles are emitted with even distribution, although it is seen that the distribution on the second screen is nontrivial. See the corresponding (Fig. 2) where several millions trajectories were computed to plot a distribution on the second screen. Parameters: discretness: $\tau = 0.05$, trajectories: 250, angles: $-45.5 \leq \alpha \leq 45.5$
Figure 5: Single slit experiment. The corresponding distribution on the second screen is given on (Fig.3). Parameters: discretness: $\tau = 0.01$, trajectories: 250, angles $-45.5 \leq \alpha \leq 45.5$