Optimization of rectangular mufflers inserted with baffles at high-order-modes using differential evolution method

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Abstract. Although rectangular mufflers have been widely used in industry, there has been scant research on both rectangular mufflers equipped with baffles under space-constrained conditions. Moreover, much muffler assessments have been restricted to lower frequencies using the plane wave theory. This has led to an underestimation of acoustical performances at higher frequencies. Beside, an analysis of three-dimensional waves propagating for mufflers using the finite element and boundary element methods is time-consuming in calculating the noise level. Also, the optimization of space-constrained mufflers is complicated. Therefore, to enhance the acoustical performance of mufflers, rectangular mufflers internally inserted with three kinds of baffles are presented. To illuminate the acoustical effect in high-order-modes and develop an optimally shaped muffler within a space-constrained situation, an eigen function in conjunction with a differential evolution is adopted. In this paper, a four-pole system matrix for evaluating the transmission loss of a muffler is derived using an eigen function.

1. Introduction
Davis [1] analysed mufflers using a plane wave effect in 1954. Munjal [2], in 1957, proposed an advanced four-pole transfer matrix using a fluid dynamic theory. Later, between 1958 and 1960, Igarashi, Toyama, Miwa, and Arai [3, 4, 5] also predicted the acoustical performance of a muffler using a four-pole transfer matrix. Sullivan and Crocker in 1978 and 1979 [6, 7] developed coupled equations for the outer and inner tubes of a perforated muffler. In 1981, Jayaraman and Yam [8] presented an analytic solution for the coupled equations. However, because the solutions mentioned above were based on the plane wave theory, the acoustical effect of a higher order wave was ignored. To overcome these drawbacks, Ih and Lee [9, 10], in 1985-1987, analysed the acoustical performance of an expansion and circular-sectioned muffler at a high-order-mode. Munjal [11], in 1987, simplified the calculation process using a numerical analysis method; however, the ratio of the expansion area to the inlet/outlet area should have been an integer value. Also, it proved to be difficult analysing a muffler using the analytic method when the angle between inlet and outlet was 90 degrees. Abom [12], in 1990, using higher order mode effects, developed a four-pole matrix for an extended muffler with a circular section. Ih [13], in 1992, proposed a numerical method for analysing the acoustical performance of an expansion muffler hybridized with a circular/rectangular section and an inlet/outlet duct. Although the acoustical effect for...
higher order modes had been considered, the shape optimization for maximizing acoustical performance for a space constrained muffler was overlooked. Because the high-order wave analysis of a muffler using the finite element method (FEM) [14] and the two-dimensional boundary element method (BEM) [15] was time-consuming, Chang and Chiu [16, 17, 18], in 2013 and 2014, proposed a simplified ANN (Artificial Neural Network) model in conjunction with the FEM & BEM and the genetic method to search optimally shaped rectangular mufflers equipped with simple baffles. However, the number of design parameters was limited to two due to the complicated model built by the ANN. In order to overcome this drawback, Chiu [19] analyzed the optimal shape of a rectangular expansion chamber using the eigen function, an analytic solution. With this, the number of design parameters increased. Also, the shape of a rectangular expansion chamber equipped with a simple baffle and a genetic algorithm with many control parameters (elitism, population, bit length of chromosome, crossover rate, mutation rate, and iteration) is complicated. So, to enhance acoustical performance, three kinds of rectangular mufflers (muffler A-muffler C) internally inserted with various baffles is proposed in this paper. Moreover, to simplify the optimization process (as compared to the genetic algorithm), a Differential Evolution (DE) method that has fewer control parameters is adopted as the optimizer in this paper.

2. Theoretical background

2.1. Mathematical model for a straight rectangular muffler

By utilizing the higher order wave propagating along a rectangular muffler shown in Figure 1 and assuming that a rigid rectangular tube is driven by a piston along the tube wall, a quiet medium with non-viscous and thermal-isolated properties is used to fill the chamber.

According to Ih’s derivation [13], the inlet’s acoustical pressure in a normal direction is

\[
\bar{p}_{11} = (-1)^{1} j U_{1} Z_{0} \left\{ \frac{1}{\tan k L} \right. + \left( \frac{AH}{a_{1} b_{1}} \right)^{2} \left[ \sum_{m=0}^{1} \frac{1}{m \pi} \left( \frac{2}{m \pi} \frac{b_{1}}{H} \right)^{2} \psi_{111}^{2} + \sum_{m=0}^{1} \frac{1}{m \pi} \left( \frac{4}{m \pi} \right)^{2} \psi_{111}^{2} \psi_{211}^{2} \right] \frac{k}{k_{x} \tan k_{x} L} \right. \]

\[\]

\[\]  

where

\[\psi_{111} = \sin \frac{m \pi a_{1}}{2A} \cos \frac{m \pi a_{c1}}{A}, \quad \psi_{211} = \sin \frac{m \pi b_{1}}{2H} \cos \frac{m \pi b_{c1}}{H}\]

The outlet’s acoustical pressure in a normal direction is

\[
\bar{p}_{22} = (-1)^{1} j U_{2} Z_{0} \left\{ \frac{1}{\tan k L} \right. + \left( \frac{AH}{a_{2} b_{2}} \right)^{2} \left[ \sum_{m=0}^{1} \frac{1}{m \pi} \left( \frac{2}{m \pi} \frac{b_{2}}{H} \right)^{2} \psi_{122}^{2} + \sum_{m=0}^{1} \frac{1}{m \pi} \left( \frac{4}{m \pi} \right)^{2} \psi_{122}^{2} \psi_{222}^{2} \right] \frac{k}{k_{x} \tan k_{x} L} \right. \]

\[\]

\[\]  

\[\]
where

\[ \psi_{122} = \sin \frac{m \pi a_1}{2A} \cos \frac{m \pi a_2}{A}, \psi_{222} = \sin \frac{m \pi b_2}{2H} \cos \frac{m \pi b_2}{H} \]

If the inlet and outlet are in opposing directions, the inlet’s acoustical pressure in a tangential direction yields

\[ \tilde{p}_{12} = (-1)^2 j U_2 Z_0 \left\{ \frac{1}{\sin kL} \right\} \]

\[ + \left( \frac{AH}{a_1 b_1} \right) \left( \frac{AH}{a_2 b_2} \right) \left[ \sum_{n_0} \frac{1}{v_{mn}} \left( \frac{2}{m \pi} \right)^2 \left( \frac{a_1}{A} \right) \left( \frac{a_2}{A} \right) \psi_{121} \psi_{121}' \right] \frac{k}{k \sin kL} \]

\[ + \sum_{n_0} \sum_{m_2} \frac{1}{v_{mn}} \left( \frac{4}{m \pi n^2} \right)^2 \psi_{112} \psi_{122} \psi_{212}' \frac{k}{k \sin kL} \]

\[ = (-1)^2 j U_2 Z_0 E_{21} \]

(3a)

\[ \tilde{p}_{21} = (-1)^3 j U_1 Z_0 \left\{ \frac{1}{\sin kL} \right\} \]

\[ + \left( \frac{AH}{a_1 b_1} \right) \left( \frac{AH}{a_2 b_2} \right) \left[ \sum_{n_0} \frac{1}{v_{mn}} \left( \frac{2}{m \pi} \right)^2 \left( \frac{a_1}{A} \right) \left( \frac{a_2}{A} \right) \psi_{121} \psi_{121}' \right] \frac{k}{k \sin kL} \]

\[ + \sum_{n_0} \sum_{m_2} \frac{1}{v_{mn}} \left( \frac{4}{m \pi n^2} \right)^2 \psi_{112} \psi_{122} \psi_{222}' \frac{k}{k \sin kL} \]

\[ = (-1)^3 j U_1 Z_0 E_{21} \]

(3b)

where

\[ \psi_{121} = \sin \frac{m \pi a_1}{2A} \cos \frac{m \pi a_2}{A}, \psi_{112} = \sin \frac{m \pi a_2}{2A} \cos \frac{m \pi a_2}{A}, \]

\[ \psi_{122} = \sin \frac{m \pi b_2}{2H} \cos \frac{m \pi b_2}{H}, \psi_{112}' = \sin \frac{m \pi a_2}{2A} \cos \frac{m \pi a_2}{A}, \]

\[ \psi_{222} = \sin \frac{m \pi b_2}{2H} \cos \frac{m \pi b_2}{H}, \psi_{211}' = \sin \frac{m \pi b_2}{2H} \cos \frac{m \pi b_2}{H}, \]

\[ \psi_{222}' = \sin \frac{m \pi b_2}{2H} \cos \frac{m \pi b_2}{H}, \psi_{121}' = \sin \frac{m \pi a_2}{2A} \cos \frac{m \pi a_2}{A}, \]

\[ \psi_{112}' = \sin \frac{m \pi a_2}{2A} \cos \frac{m \pi a_2}{A}, \psi_{122}' = \sin \frac{m \pi b_2}{2H} \cos \frac{m \pi b_2}{H}, \]

Combining Eqs.(1)~(4) yields

\[ \bar{p}_1 = \bar{p}_{11} + \bar{p}_{12} = -j Z_0 (U_1 E_{11} - U_2 E_{12}); \bar{p}_2 = \bar{p}_{21} + \bar{p}_{22} = -j Z_0 (U_1 E_{21} - U_2 E_{22}) \]

(4)

where \( P \) is the total acoustical pressure in the inlet and \( P \) is the total acoustical pressure in the outlet.

Rearranging Eq. (4) into a matrix yields

\[ \begin{bmatrix} P_1 \\ U_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} P_2 \\ U_2 \end{bmatrix} \]

(5)

where

\[ T_{11} = (\bar{P}_1 / \bar{P}_2)|_{p_2=0} = E_{11}/E_{12}, T_{12} = (\bar{P}_1 / \bar{U}_2)|_{p_2=0} = j Z_0 (E_{12} E_{22}/E_{12}); \]

\[ T_{21} = (\bar{U}_1 / \bar{P}_2)|_{U_2=0} = j Z_0 (E_{12} E_{22}/E_{12})^{-1}, T_{22} = (\bar{U}_1 / \bar{U}_2)|_{p_2=0} = E_{22}/E_{12} \]

The transmission loss (TL) between the inlet (node 1) and the outlet (node 2) is

\[ TL = 20 \log \left[ \left| T_{11} + \frac{T_{12}}{Z_2} + \frac{T_{22}}{Z_1} + T_{21} (S_2/S_1) \right|/2 \right] \]

(6)

where

\[ Z_1 = \rho_0 c/S_1, Z_2 = \rho_0 c/S_2, S_1 = b_1 h_1, S_2 = b_2 h_2 \]

2.2. System matrices and transmission loss for three kinds of mufflers
In order to enhance the acoustical performance of the expansion chamber muffler, six kinds of mufflers (muffler A: a rectangular muffler internally inserted with a rectangular-shaped baffle; muffler B: a rectangular muffler internally inserted with a U-shaped baffle; muffler C: a rectangular muffler internally inserted with two parallel rectangular-shaped baffles) shown in Figures 2~4 are proposed.

![Figure 2. A rectangular muffler internally inserted with a rectangular-shaped baffle (muffler A).](image)

![Figure 3. A rectangular muffler internally inserted with a U-shaped baffle (muffler B).](image)

![Figure 4. A rectangular muffler internally inserted with two parallel rectangular-shaped baffles (muffler C).](image)

According to the mathematical form of the straight rectangular muffler shown in Eq. (6), the four-pole transfer matrix for muffler A between nodes 1 and 3 yields

\[
\begin{bmatrix}
    P_1 \\
    U_1
\end{bmatrix}
= \prod_{n=1}^m \begin{bmatrix}
    T_n
\end{bmatrix}
\begin{bmatrix}
    P_3 \\
    U_3
\end{bmatrix}
= \begin{bmatrix}
    T_{F11} & T_{F12} \\
    T_{F21} & T_{F22}
\end{bmatrix}
\]

(7a)

\[
TL_1(f, X_1) = 20 \log \left[ \frac{T_{F11} + T_{F12} + T_{F21}Z_2 + T_{F22}(S_2/V)}{2} \right]
\]

(7b)

where \( X_1 = (L_1, P_1) \)

Similarly, for muffler B, the system’s four-pole matrix and \( TL \) between nodes 1 and 3 yields

\[
\begin{bmatrix}
    P_1 \\
    U_1
\end{bmatrix}
= \prod_{n=1}^2 \begin{bmatrix}
    T_n
\end{bmatrix}
\begin{bmatrix}
    P_3 \\
    U_3
\end{bmatrix}
= \begin{bmatrix}
    T_{F11} & T_{F12} \\
    T_{F21} & T_{F22}
\end{bmatrix}
\]

(8a)

\[
TL_2(f, X_2) = 20 \log \left[ \frac{T_{F11} + T_{F12} + T_{F21}Z_2 + T_{F22}(S_2/V)}{2} \right]
\]

(8b)

where \( X_2 = (L_1, P_1) \)

Also, for muffler C, the system’s four-pole matrix and \( TL \) between nodes 1 and 3 yields

\[
\begin{bmatrix}
    P_1 \\
    U_1
\end{bmatrix}
= \prod_{n=1}^2 \begin{bmatrix}
    T_n
\end{bmatrix}
\begin{bmatrix}
    P_3 \\
    U_3
\end{bmatrix}
= \begin{bmatrix}
    T_{F11} & T_{F12} \\
    T_{F21} & T_{F22}
\end{bmatrix}
\]

(9a)
5

$$TL_i(f, \bar{X}_i) = 20 \log \left[ \frac{T_{\text{in}_1} + T_{\text{in}_2} + T_{\text{out}_1} Z_i + T_{\text{out}_2} \left( S_i / S_1 \right)}{2} \right] \quad (9b)$$

where $\bar{X}_i = (L_i, P_i)$

2.3. Objective function

By using Eqs. (7)-(9), the objective functions used in the DE optimization with respect to each type of muffler are established as below:

$$OBJ_i = TL_i(f, \bar{X}_i) \quad (10)$$

where $i = 1 \sim 6$

3. Theoretical background

Before performing the DE optimal simulation on the mufflers, a comparison of the simulated results of a rectangular one-chamber straight muffler with Ih’s analytic solution is performed and shown in Figure 5. As depicted in Figure 5 they are in an agreement. Therefore, the proposed mathematical model for a straight rectangular muffler is acceptable. Consequently, the model linked with the numerical method is applied to the shape optimization of muffler A ~ muffler C in the following section.

4. Analysis of location for the openings of the inlets and outlets

In order to find a better location for the opening of a muffler’s inlet and outlet, the transmission loss analysis with respect to various locations of the inlet and the outlet is required and discussed below.

4.1. Muffler A

Three kinds of locations (position A: with the opening a little higher at the bottom; position B: with the opening in the middle of the vertical wall; position C: with the opening on the bottom) for muffler A’s inlet and outlet are depicted in Figure 6. The TL with respect to position A ~ position C is shown in Figure 7. As indicated in Figure 7, the design of position C (with the opening on the bottom) is superior to the others. Therefore, position C is chosen as the best location for the inlet and outlet of muffler A.
4.2. Muffler B

Similarly, position A has an opening that is a little higher at the bottom. Position B’s opening is in the middle of the vertical wall. In addition, position C’s opening is on the bottom. The related inlet/outlet position for muffler B is depicted in Figure 8. The related TL with respect to position A~position C is simulated and shown in Figure 9. As indicated in Figure 9, the location of the opening at position C is also superior to the others. Therefore, position C is selected as the best location of the inlet and the outlet for muffler B.

![Figure 8](image1)

**Figure 8.** Adjustment of the location of the inlet and the outlet (muffler B).

![Figure 9](image2)

**Figure 9.** Transmission loss for various locations of the inlet and the outlet (muffler B).

4.3. Muffler C

The related inlet/outlet position for muffler C is depicted in Figure 10. The related TL with respect to position A~position C is simulated and shown in Figure 11. As illustrated in Figure 11, the location of opening at position C is superior to the others. There is no doubt that position C will be selected for muffler C.

![Figure 10](image3)

**Figure 10.** Adjustment of the location of the inlet and the outlet (muffler C).

![Figure 11](image4)

**Figure 11.** Transmission loss for various locations of the inlet and the outlet (muffler C).

5. Differential evolution method

Differential Evolution (DE), a population-based search algorithm, was first presented by Price and Storn in 1995 [20, 21]. The design variable is set as a real-valued vector in DE. The optimal design variable will be optimally search by using mutation, crossover, and selection. The mutation operator of DE is performed by multiplying a scaling factor to a vector difference. A new offspring is then generated by using crossover and selection. Because of fewer control parameters in the DE, DE has become increasingly popular worldwide.

For the Differential Evolution procedures, the initial population of DE will be first generated randomly between the lower and upper bounds for each design variable. For a specified population (NP), the initial i-th design vector (Xi) will be generated by the random value (ρi) as

\[ X_i = X_{\text{min}} + \rho_i \cdot (X_{\text{max}} - X_{\text{min}}), i = 1,2,\ldots, NP \]  

Here, is within 0 and 1. Xmin and Xmax are the minimum and maximum values of the parameters.

As indicated in Figure 12, in order to increase the diversity of the perturbed parameter vectors which can expand the searching space during the DE operation, a crossover mechanism is used. The trial vector (Uept) is created by using the crossover mechanism between the mutant vector (Vept) and the target vector (Utarg).
which is chosen from the population. For an m-dimensional search space, \( X_{j,i,g} \) is the target vector at the j-th parameter and the i-th population for the g-th generation. The initial target vector was

\[
X_{j,g} = (X_{1,j,g}, X_{2,j,g}, X_{3,j,g}, \ldots, X_{m,j,g}), \quad j \in [1,m], i \in [1, NP]
\]

(12)

The initial mutant vector yields

\[
V_{j,g+1} = (V_{1,j,g+1}, V_{2,j,g+1}, V_{3,j,g+1}, \ldots, V_{m,j,g+1}), \quad j \in [1,m], i \in [1, NP]
\]

(13)

For a j-th searching space, using the crossover on mutant vector \( V_{j,g+1} \) and the target vector \( X_{j,g} \), the trial vector \( U_{j,g+1} \) will be deduced as

\[
U_{j,g+1} = \begin{cases} 
X_{j,g} & \text{rand}(j) \leq CR \\
V_{j,g+1} & \text{else} \end{cases}, \quad j \in [1,m]; i \in [1, NP]
\]

(14)

Here, rand (j) and CR (the crossover constant) are within 0 and 1.

In order to escape from a local optimum during the optimization procedure, a mutation mechanism is also used. In a m-dimensional search space, for each target vector \( X_{j,g} \), a mutant vector generated by three vectors \( X_{r1,g}, X_{r2,g}, \) and \( X_{r3,g} \) will be randomly selected with the mutant vector \( V_{j,g+1} \) generated by using the mutation factor (F) as

\[
V_{j,g+1} = X_{r1,g} + F \cdot (X_{r2,g} - X_{r3,g})
\]

where \( r1, r2, r3 \) \{1, 2, ..., NP\} are randomly chosen as integers.

The optimization diagram is indicated in Figure 13. As indicated in Figure 13, to simplify the numerical assessment of the DE method, maximal evolution iteration (iter\text{max}) is predetermined in advance. The process is continually repeated until the predetermined number (iter\text{max}) of the outer loop is achieved.

![Figure 12. The mechanism of crossover for a target vector and a mutant vector in the DE method.](image1)

![Figure 13. The flow diagram of the DE optimization.](image2)
6. Results and discussion

6.1. Results

Four types of DE control parameters including NP (population number), CR (crossover rate), F (mutation factor), and iter\textsubscript{max} (maximum iteration) are adopted in the DE optimization. According to Rainer and Kenneth [22], an appropriate number for population (NP) is predetermined as 5\*m where m is the number of parameters. To achieve a good optimization, the following parameters are varied step by step:
CR (0.1, 0.3, 0.5, 0.7, 0.9); F (0.1, 0.3, 0.5, 0.7, 0.9); iter\textsubscript{max} (50, 100, 500).

Results reveal that the optimal design data can be obtained from the last set of DE parameters at (CR, F, iter\textsubscript{max}) = (0.7, 0.1, 500).

The optimization of straight rectangular mufflers internally inserted with three kinds of baffles is carried out and described below.

6.1.1. Muffler A. By using Eqs. (7) and (10), the maximization of the TL with respect to muffler A at the specified pure tone (1500 Hz) is performed first. The range of design parameters in muffler A is shown in Table 1. The optimal design parameters are illustrated in Table 2. The corresponding DE control parameters at (CR, F, iter\textsubscript{max}) are (0.7, 0.1, 500). Putting the optimal design parameters of LL1 and PP1 through a theoretical calculation, the resulting TL is plotted in Figure 14. As indicated in Figure 14, the transmission loss at 1500 Hz can be tremendously improved from 2 dB to 47 dB. Doing an optimization at the target tone of 2500 Hz, the optimal design parameters and acoustical performance are also shown in Table 2 and Figure 15. As indicated in Figure 15, the transmission loss at 2500 Hz can be remarkably improved from 3 dB to 30 dB.

| Min. | Max. |
|------|------|
| LL1  | 0.05  |
| PP1  | 0.05  |
|      | 0.20  |
|      | 0.15  |

Table 1. The range of design parameters for a rectangular muffler internally inserted with a rectangular-shaped baffle (muffler A).

Table 2. The design parameters before and after optimization is performed (muffler A).

|                  | Min. | Max. |
|------------------|------|------|
| original         | 0.150| 0.100|
| Optimization at 1500 Hz | 0.104 | 0.127 |
| Optimization at 2500 Hz | 0.083 | 0.090 |

Figure 14. Transmission loss before and after the optimization is performed at 1500 Hz (muffler A).

Figure 15. Transmission loss before and after the optimization is performed at 2500 Hz (muffler A).

6.1.2. Muffler B. The range of design parameters in muffler B is shown in Table 3. By using Eqs. (8) and (10) and the same DE control parameters of (CR, F, iter\textsubscript{max}) = (0.7, 0.1, 500), the maximization of the TL with respect to muffler B at the specified pure tone of 1500 Hz is carried out. The optimal design parameters are illustrated in Table 4. Putting the optimal design parameters of LL1 and PP1 through a theoretical calculation, the resulting TL with respect to the target tone of 1500 Hz is plotted in Figure 16. As indicated in Figure 16, the transmission loss at 1500 Hz can be improved from 0 dB to 53 dB.

|                  | Min. | Max. |
|------------------|------|------|
|                  | LL1  | PP1  |
| original         | 0.150| 0.100|
| Optimization at 1500 Hz | 0.104 | 0.127 |
| Optimization at 2500 Hz | 0.083 | 0.090 |

Figure 16. Transmission loss before and after the optimization is performed at 1500 Hz (muffler B).
Table 3. The range of design parameters for a rectangular muffler internally inserted with a U-shaped baffle (muffler B).

|       | Min. | Max. |
|-------|------|------|
| LL1   | 0.05 | 0.20 |
| PP1   | 0.05 | 0.15 |

Table 4. The design parameters before and after optimization is performed (muffler B).

|       | LL1 | PP1 |
|-------|-----|-----|
| original | 0.150 | 0.100 |
| Optimization at 1500 Hz | 0.103 | 0.078 |

Figure 16. Transmission loss before and after the optimization is performed at 1500 Hz (muffler B).

6.1.3. Muffler C. For muffler C, the range of design parameters is shown in Table 5. By using Eqs. (9) and (10) and adopting the same DE control parameters of (CR, F, itermax) = (0.7, 0.1, 500), the optimization of the TL at the specified pure tone of 2500 Hz is performed. The optimal design parameters are illustrated in Table 6. Putting the optimal design parameters of LL1 and PP1 through a theoretical calculation, the resulting TLs with respect to the target tone of 2500 Hz are plotted in Figure 17. As indicated in Figure 17, the transmission loss at 2500 Hz can be improved from 0 dB to 37 dB.

Table 5. The range of design parameters for a rectangular muffler internally inserted with two parallel rectangular-shaped baffles (muffler C).

|       | Min. | Max. |
|-------|------|------|
| LL1   | 0.05 | 0.20 |
| PP1   | 0.02 | 0.05 |

Table 6. The design parameters before and after optimization is performed (muffler C).

|       | LL1 | PP1 |
|-------|-----|-----|
| original | 0.150 | 0.050 |
| Optimization at 2500 Hz | 0.174 | 0.035 |

Figure 17. Transmission loss before and after the optimization is performed at 2500 Hz (muffler C).

To achieve a sufficient optimization, the selection of the appropriate DE parameter set is essential. Because of the influence of the acoustical performance on the opening’s location, a serious investigation of acoustical inlet/outlet openings for three kinds of mufflers has been investigated. As illustrated in Figure 7, the acoustical performance of a muffler that has a bottom position for its inlet/outlet is superior to that of one having a low positioned or a center positioned inlet/outlet. As indicated in Figures 2~4, a rectangular chamber is partitioned with a baffle. As illustrated in Figures. 14 and 16, the optimum
transmission loss of muffler A and muffler B at the target tone of 1500 Hz reaches 47 dB and 53 dB. Results reveal that the U-shaped baffle used in muffler B has a better acoustical performance than that of the rectangular-shaped baffle used in muffler A. Also, as indicated in Figures 15 and 17, the optimum transmission loss for muffler A and muffler C at the target tone of 2500 Hz reaches 30 dB and 37 dB. Therefore, muffler B with two parallel rectangular-shaped baffles is superior to muffler A with a rectangular-shaped baffle.

7. Conclusion
It has been shown that the acoustical performance of a straight rectangular muffler in high order modes can be calculated by using the eigen function. Three kinds of straight rectangular mufflers internally equipped with various baffles are used and optimally shaped within a space-constrained situation. The optimization of muffler shapes within a limited space can be easily and efficiently carried out by using a four-pole transfer matrix as well as a DE optimizer. Three kinds of DE control parameters (CR, F, and iter\text{max}) are adopted in the DE optimization. As indicated in Figures 14–17, the predicted maximal value of the TL is roughly located at the desired frequency. Hence, the tuning ability established by adjusting the design parameters of mufflers is reliable. As indicated Figure 7, the acoustical performance of a muffler that has a bottom position for inlet/outlet is superior to that of one having a low positioned or a centre positioned inlet/outlet. In addition, as indicated Figures 14-16 and 15-17, the U-shaped baffle’s noise reduction is superior to the rectangular-shaped baffle. The acoustical performance of a two parallel rectangular-shaped baffle is also superior to that of a rectangular-shaped baffle. Consequently, considering acoustical effect of high-order-mode waves, the approach used for optimal noise elimination in the space-constrained straight rectangular mufflers internally inserted with various shapes of baffles proposed in this study is quite important and can be efficiently assessed.

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