Distribution functions for continuous medium without probability hypotheses

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Abstract. A non-probabilistic interpretation of the distribution functions is obtained. The cases of instantaneous and retarded interactions are considered. The derivation is based on the use smoothing operator that does not change the symmetry of the equation of motion with respect to time inversion.

1. Introduction

The models underlying the continuum mechanics (MCM) are customarily justified within the framework of statistical physics \cite{1–3}. But the use of Hamiltonian mechanics in conjunction with probabilistic hypotheses, strictly speaking, is mathematically inconsistent approach. Even if we analyze the dynamics of the system within a probabilistic description, it is necessary to find and justify a method for calculating these probabilities. The reference to the limit theorems under such consideration is doubtful, since these theorems are related to independent identically distributed random variables, but the interaction between particles excludes their independence. Nevertheless, probabilistic hypotheses, ergodicity, and mixing formed the basis for the modern explaining the transition of a system of many particles to stochastization.

The basic method for studying non-equilibrium systems of many particles is the BBGKY hierarchy of kinetic equations for statistical distribution functions \cite{1–3}. At the same time, as was shown in \cite{4}, the kinetic equations written even for such a simple model as the system of elastic balls cannot contain the phenomenon of stochastization of the system. Moreover, formally the BBGKY hierarchy can be obtained without invoking probabilistic distribution functions. Vlasov \cite{5} noted that the BBGKY hierarchy has a solution that corresponds to the deterministic evolution of point particles. It was later shown \cite{6} that, in the general case, it is possible to construct a chain of equations for microscopic phase densities, similar to the BBGKY hierarchy, but completely equivalent to Newtonian deterministic description.

It is known \cite{7} that the Gibbs canonical distribution can be obtained by directly solving the Liouville equation. But since the Liouville equation is the basis for the BBGKY hierarchy, it can be argued that the Gibbs distributions are a simple consequence of this hierarchy. At the same time, the Liouville equation is deterministic and true for any function of canonical variables, and not only for a probabilistic function. This means that Gibbs distributions can be obtained without invoking probabilistic hypotheses and assumptions. Also the use of probabilities does not explain from the physical point of view the...
transition to thermodynamic equilibrium. Despite this, averaging involving probability measures is often used to derive kinetic equations. For example, the Klimontovich equation [8], written for phase microscopic density, being completely deterministic and reversible in time, after formal statistical averaging passes into the first equation of the BBGKY hierarchy for statistical distribution functions. But such averaging does not contain the phenomenon of stochastization as a physical process and does not explain the transition to equilibrium, but only describes it.

The mechanism of transition of the system to equilibrium can be explained by the influence of the thermostat [9]. Since a thermostat is an external environment, such a mechanism does not solve the problem of transition to stochastization of an isolated system. This means that the reason for the transition to equilibrium must be related to the interactions between the particles of the system itself. However, besides the particles themselves, it is also necessary to take into account the field through which the particles interact. The effect of the field leads to retarded potentials. Thus, one of the possible physical mechanisms causing irreversibility is not just the interaction of particles, but a retardation interaction. Accounting for the retardation effect allowed the authors of [10] to obtain the noninvariant with respect to time reversal equation of motion for the microscopic density of a system of point particles. Zakharov [11] obtained an irreversible equation for the microscopic phase density that generalizes the Klimontovich equation [8] to the case of retardation interactions.

This paper shows the possibility of formulating continuum physics without invoking probabilistic hypotheses and assumptions. A dynamic interpretation of the distribution functions that inherent in the kinetic equations of statistical physics is given.

2. The method of smoothing the microscopic dynamic functions and the equation of motion for the local density of the medium

To describe the physical properties of a continuous medium, we turn from microscopic values of the system of many particles to the macroscopic one in the following way [12]. Mentally divide the area occupied by the system into small volumes \( \Delta(r) \). Let \( N(r,t) \) be the number of particles in the volume \( \Delta(r) \). The value of the volumes \( \Delta(r) \) can be chosen rather arbitrarily depending on the research task. If the number of particles of radius \( r_0 \) at a “point” of a continuous medium is defined as \( N(r,t) \approx (n_0^o)^{-5/4} \) ( \( n \) is the average particle concentration), then using above mentioned division a unified description of both kinetic and gasdynamic processes is possible [8].

Then we can define smoothing density of the mass of particles located in \( \Delta(r) \)

\[
\rho(r,t) = \frac{1}{\Delta} \sum_{i=1}^{N(r,t)} m_i = \frac{1}{\Delta} \int_{\Delta} m_i \delta(\mathbf{r} + \xi - \mathbf{r}_i(t)) d^3 \xi.
\] (1)

In general case [12] each microscopic dynamic additive quantity \( \chi_i \) corresponding to the microscopic density \( \chi(r,t) = \sum_{i=1}^{N(r,t)} \chi_i \delta(\mathbf{r} - \mathbf{r}_i(t)) \) can be associated with the corresponding field function \( \psi(r,t) \) using the smoothing operator \( \hat{S} \):

\[
\psi(r,t) = \hat{S}[\chi(r,t)] = \frac{1}{\Delta} \sum_{i=1}^{N(r,t)} \chi_i = \frac{1}{\Delta} \int_{\Delta} \chi(\mathbf{r} + \xi, t) d^3 \xi.
\] (2)

It should be emphasized that the operation (2) is a transition to the description of the dynamics in new (field) variables. Under the action of the operator \( \hat{S} \), of course, most of the information about the motion of the particles of the system is lost, which, however, does not lead to its irreversible behavior. In statistical physics, it is averaging (over any Gibbs ensemble) that is considered the cause of irreversibility. Here the situation is different. Just as the transition to the equation of motion of the center
of inertia of a particle system in classical mechanics does not lead to irreversible behavior, so in our case operation (2) does not change the symmetry of the equation of motion with respect to time inversion.

The vector field of the mass flux density associated with the motion of particles of the medium with velocities $v_i(t)$ is defined as

$$j(r,t) = \frac{1}{\Delta} \sum_{i=1}^{\mathcal{N}} m_i v_i(t) \delta(r + \xi - r_i(t)) d^3 \xi = \frac{1}{\Delta} \sum_{i=1}^{\mathcal{N}(r,t)} m_i v_i(t) = v(r,t) \rho(r,t).$$  \hspace{1cm} (3)

Here we introduced a vector field $v(r,t)$, i.e. the velocity of a center of volume $\Delta(r)$:

$$v(r,t) = \sum_{i=1}^{\mathcal{N}(r,t)} m_i v_i(t) / \sum_{i=1}^{\mathcal{N}(r,t)} m_i.$$  \hspace{1cm} (4)

Further we shall consider the motion of particles of the same mass. In this case

$$j(r,t) = m v(r,t) n(r,t).$$  \hspace{1cm} (5)

Here $n(r,t)$ is a density of the number of particles. From (3), (1) and (5) the equation of continuity follows:

$$\frac{\partial n(r,t)}{\partial t} = \frac{\partial}{\partial t} \int_{\Delta)}^{\mathcal{N}(r,t)} \delta(r + \xi - r_i(t)) d^3 \xi = -\frac{\partial}{\partial r} \int_{\Delta)}^{\mathcal{N}(r,t)} \sum_{i=1}^{\mathcal{N}(r,t)} v_i \delta(r + \xi - r_i(t)) d^3 \xi = -\frac{1}{m} \text{div} j(r,t).$$  \hspace{1cm} (6)

Differentiating the flux density with respect to time and taking into account the equation (6), we obtain the equation for the density of number of particles:

$$\frac{\partial^2 n(r,t)}{\partial t^2} = -\frac{1}{m} \frac{\partial \Phi_a(r,t)}{\partial x_a} + \frac{1}{m} \frac{\partial^2 \Pi_{a\beta}(r,t)}{\partial x_a \partial x_\beta}.$$  \hspace{1cm} (7)

Here $\Pi_{a\beta}(r,t)$ is a tensor field of flux density

$$\Pi_{a\beta}(r,t) = \frac{1}{\Delta} \sum_{i=1}^{\mathcal{N}(r,t)} [mv_{i\alpha}^\beta(t) v_i(t) \delta(r + \xi - r_i(t))] d^3 \xi,$$  \hspace{1cm} (8)

and $\Phi_a(r,t)$ is volume density of the force

$$\Phi_a(r,t) = \frac{1}{\Delta} \sum_{i=1}^{\mathcal{N}(r,t)} F_{ai}(r_i(t), v_i(t), t) \delta(r + \xi - r_i(t)) d^3 \xi.$$  \hspace{1cm} (9)

The dynamic of the continuum is completely determined by the value of volume density of the force (9).

3. Smoothing microscopic distribution functions and continuum dynamics

The Klimontovich microscopic phase density for a system comprised of $N$ particles is given by the following delta-function expression

$$f\text{micro}(r,p,t) = \sum_{i=1}^{\mathcal{N}} \delta(r - r_i(t)) \delta(p - p_i(t)).$$  \hspace{1cm} (10)

After smoothing $f\text{micro}(r,p,t)$ turns into macroscopic phase density

$$f\text{macro}(r,p,t) = \frac{1}{\Delta} \sum_{i=1}^{\mathcal{N}} \delta(p - p_i(t)) \delta(r + \xi - r_i(t)) d^3 \xi.$$  \hspace{1cm} (11)

To obtain the one-particle distribution function we integrate eqn. (11) with respect to $p$. This yields

$$F_i(r,t) = \int f\text{macro}(r,p,t) d^3 p = \frac{1}{\Delta} \sum_{i=1}^{N} \delta(r + \xi - r_i(t)) d^3 \xi.$$  \hspace{1cm} (12)

The resulting expression coincides with the definition of the density of the number of particles $n(r,t)$.

Next we define the one-particle distribution function
\[ F_2(\mathbf{r}_i, \mathbf{r}_j, t) = \frac{1}{\Delta_v} \sum_{i=1}^{N} \sum_{k=1, k \neq i}^{N} \delta(\mathbf{r}_i + \xi - \mathbf{r}_j) \delta(\mathbf{r}_i + \xi - \mathbf{r}_j) d^3 \xi. \]  

(13)

It is significant that smoothing is carried out only on the configuration subspace. Doing additional smoothing over the momentum subspace, we would cut off a set of possible values of the particle momentum in the volume \( \Delta(\mathbf{r}) \). This would lead, firstly, to inevitable errors, and secondly, could lead to an artificial noninvariance of the equations of motion of a continuous medium with respect to the time reversal.

A smoothing density of the mass (1) defined in terms of the macroscopic phase density takes the form

\[ \rho(\mathbf{r}, t) = m \int f_{\text{macro}}(\mathbf{r}, \mathbf{p}, t) d^3 \mathbf{p}. \]  

(14)

The scalar density field of kinetic energy may be written as

\[ E_k(\mathbf{r}, t) = \frac{1}{2} \sum_{i=1}^{N} \left[ \frac{mv_i^2}{2} \delta(\mathbf{r} + \xi - \mathbf{r}_i(t)) d^3 \xi \right] = \int \frac{p_i^2}{2m} f_{\text{macro}}(\mathbf{r}, \mathbf{p}, t) d^3 \mathbf{p}. \]  

(15)

If we introduced a local temperature \( T(\mathbf{r}, t) \) as a smoothed kinetic energy of the particles in their relative movement

\[ \frac{1}{3} \sum_{i=1}^{N} m \left( v_i(t) - \mathbf{v}(\mathbf{r}, t) \right)^2 \delta(\mathbf{r} + \xi - \mathbf{r}_i(t)) d^3 \xi = n(\mathbf{r}, t)T(\mathbf{r}, t), \]  

(16)

then (15) may be written as

\[ E_k(\mathbf{r}, t) = \frac{1}{2} \mathbf{v}^2(\mathbf{r}, t) \rho(\mathbf{r}, t) + \frac{3}{2} n(\mathbf{r}, t)T(\mathbf{r}, t). \]  

(17)

3.1. The case of instantaneous interactions

In this case we may introduce the potential energy of particles \( W_a(\mathbf{r}_i, \mathbf{r}_j) \) and, as a consequence, the volume density of this energy:

\[ U(\mathbf{r}, t) = -\frac{1}{2} \sum_{i=1}^{N} \sum_{k=1, k \neq i}^{N} W_a(\mathbf{r}_i, \mathbf{r}_j) \delta(\mathbf{r} + \xi - \mathbf{r}_i(t)) d^3 \xi = \frac{1}{2} \int d^3 R W(R) F_2(\mathbf{r} - \mathbf{R}, \mathbf{r}, t). \]  

(18)

The volume density of the force can be represented as the divergence of the stress tensor \( \sigma_{\text{aff}} \):

\[ \Phi_a(\mathbf{r}, t) = -\frac{1}{2} \sum_{i=1}^{N} \sum_{k=1, k \neq i}^{N} \frac{\partial W(\mathbf{r}_i, \mathbf{r}_j)}{\partial x_{i\alpha}} \left[ \delta(\mathbf{r} + \xi - \mathbf{r}_i(t)) - \delta(\mathbf{r} + \xi - \mathbf{r}_j(t)) \right] d^3 \xi = -\frac{\partial \sigma_{\text{aff}}}{\partial x_{\beta}}. \]  

(19)

The stress tensor, in turn, is determined by the two-particle distribution function:

\[ \sigma_{\text{aff}}(\mathbf{r}, t) = -\frac{1}{2} \int d^3 \mathbf{R} \frac{\partial W(\mathbf{R}) R_{\mathbf{R}_\beta}}{\partial R} F_2(\mathbf{r} - s \mathbf{R}, \mathbf{r} - s \mathbf{R}, t) = \]  

\[ = -\frac{1}{2} \sum_{n=1}^{\infty} \int d^3 \mathbf{R} \frac{\partial W(\mathbf{R}) R_{\mathbf{R}_\beta}}{\partial R} \left( -\frac{\partial}{\partial \mathbf{R}} \right)^{n-1} F_2(\mathbf{r} + \mathbf{R}, \mathbf{r} + \mathbf{R}, t). \]  

(20)

We see that equations (14)–(20) are the complete analogue of the classical statistical theory [1, 2]. But in our case, the distribution functions (11)–(13) have no any probabilistic sense, because no statistical averaging was performed. All that has been done is smoothing (11) and (13), as a result of which information on the motion of individual particles have switched to dynamic distribution functions (11)–(13).

We now rewrite (16) in the following form:

\[ U(\mathbf{r}, t) = \frac{1}{2\Delta_v^2} \int d^3 \mathbf{r}_1 \int d^3 \mathbf{r}_2 \int d^3 \xi \sum_{i=1}^{N} \sum_{k=1, k \neq i}^{N} W \left[ (\mathbf{r} + \xi - \mathbf{r}_1) \delta(\mathbf{r} + \xi - \mathbf{r}_1(t)) \delta(\mathbf{r} + \xi - \mathbf{r}_1(t)) \right]. \]  

(21)
Here we clearly presented $U(r, t)$ as the energy density of the interaction of particles from a volume $\Delta(r)$ with each other and with all particles of all other volumes $\Delta(r')$. We expand the potential energy in a series in powers of $\xi^a - \xi'^a$ and introduce the moments

$$S^{(\alpha_1, \ldots, \alpha_N)}(r, r', t) =$$

$$= \frac{1}{\Delta^3} \sum_{i=1}^{\infty} \sum_{\alpha} \int d^3\xi \int d^3\xi' (\xi^a - \xi'^a) (\xi'^b - \xi'^b') \cdots \cdots (\xi'^m - \xi'^m) \delta(r + \xi - r_i(t)) \delta(r + \xi' - r_i(t)). \tag{22}$$

Then the expression for the volume density of the force can be expressed in the form

$$U(r, t) = \frac{1}{2} \int d^3r \sum_{\alpha} \frac{1}{n^1} \frac{\partial^2 W(r-r')}{\partial x_\alpha} S^{(\alpha_1, \ldots, \alpha_N)}(r, r', t), \tag{23}$$

If the internal fields vary slightly within volumes $\Delta(r)$ then in a series in powers $\xi^a - \xi'^a$ we can leave the first two terms of the expansion (23). In this approximation we get

$$U(r, t) = \frac{1}{2} \int d^3r' W(r-r') \left[ n(r, t) n(r', t) + \nabla_r \left[ n(r', t) I(r, t) - n(r, t) I(r', t) \right] \right]. \tag{24}$$

Here $l^a(r, t)$ is the vector field of the displacements of particles relative to the center of a volume $\Delta(r)$:

$$l^a(r, t) = \frac{1}{\Delta^a} \sum_{i=1}^{\infty} \xi^a \delta(r + \xi - r_i(t)) d^3\xi. \tag{25}$$

Comparing (16) and (24), we conclude that

$$F_i(r, r', t) = n(r, t) n(r', t) + \nabla_r \left[ n(r', t) I(r, t) - n(r, t) I(r', t) \right]. \tag{26}$$

Thus, the two-particle distribution function in this approximation is determined by the smoothed density $n(r, t)$ and the divergence of the combination of the smoothed density and the vector field of particle displacements $l(r, t)$. The first term in (26) corresponds to the so-called mean-field approximation, which does not take into account correlations in the location of particles. The second term in (26) takes into account the inhomogeneity of the distribution of particles in volumes $\Delta(r)$. The reason for the heterogeneity of the distribution of particles is associated with the inconstancy of the gradient of the inter particle fields inside $\Delta(r)$. Approximation of the mean field corresponds to the constancy of the gradient of internal fields within the volumes $\Delta$. We can continue to consistently take into account the degree of heterogeneity of the gradients of the internal fields, obtaining a more accurate expression for the two-particle distribution function. Thus, the origin of the distribution functions has a completely mechanical (deterministic) explanation, not connected with probabilistic representations. Thus, the origin of the distribution functions has a completely mechanical explanation, associated with probabilistic representations.

The equation for the local density of the medium (7) in the approximation (24) takes the form

$$\frac{\partial^2 n(r, t)}{\partial t^2} = \nabla_r \cdot \nabla_r \left( n(r, t) v(r, t) \otimes v(r, t) \right) +$$

$$+ \frac{1}{3m} \Delta \left( S p P(r, t) \right) + \frac{1}{m} \nabla_r \cdot \nabla_r \cdot d v \left[ n(r, t) n(r', t) + \nabla_r \left[ n(r', t) I(r, t) - n(r, t) I(r', t) \right] \right], \tag{27}$$

where $P_{\alpha\beta}(r, t)$ is the tensor field of kinetic pressure

$$P_{\alpha\beta}(r, t) = \frac{1}{\Delta} \sum_{i=1}^{\infty} m \left( v_i(t) - v(r, t) \right)_{alpha} \left( v_i(t) - v(r, t) \right)_{beta} \delta(r + \xi - r_i(t)) d^3\xi.$$
and \( S_P(\mathbf{r}, t) = 3n(\mathbf{r}, t)T(\mathbf{r}, t) \) is the trace of the tensor \( \mathbf{P}(\mathbf{r}, t) \), and \( \text{dev} \mathbf{P}(\mathbf{r}, t) \) denotes its deviator.

Let us consider a continuous medium at a temperature \( T \) placed in an external field \( \varphi(\mathbf{r}) \) in which there are no macroscopic flows of any type. Choosing \( \Delta \) as a physically infinitesimally small volume. In this case the field function (2) can be considered sufficiently smooth, and we can accept the condition of local equilibrium, at which \( \nabla \cdot \nabla, \text{dev} \mathbf{P}(\mathbf{r}, t) = 0 \), \( \partial n(\mathbf{r}, t)/\partial t = 0 \) and \( \partial I(\mathbf{r}, t)/\partial t = 0 \). Then equation (27) take the form

\[
T \Delta n(\mathbf{r}) = -\nabla_{\mathbf{r}} \left( n(\mathbf{r}) \nabla_{\mathbf{r}} \varphi(\mathbf{r}) \right) -
-\nabla_{\mathbf{r}} \left[ d^3r' \left( n(\mathbf{r}) n(\mathbf{r}') + \nabla_{\mathbf{r}} \left[ I(\mathbf{r}) n(\mathbf{r}') - n(\mathbf{r}) I(\mathbf{r}') \right] \right) \nabla_{\mathbf{r}} W(\mathbf{r} - \mathbf{r}').
\]

Introducing the two-particle correlation distribution function

\[
g_2(\mathbf{r}, \mathbf{r}') = \frac{g_z(\mathbf{r}, \mathbf{r}')}{n(\mathbf{r}) n(\mathbf{r}')} = 1 + \frac{\nabla_{\mathbf{r}} \left[ I(\mathbf{r}) n(\mathbf{r}') - n(\mathbf{r}) I(\mathbf{r}') \right]}{n(\mathbf{r}) n(\mathbf{r}')}.
\]

we get the final equation for smoothed density

\[
-T \ln n(\mathbf{r}) = \int d^3r' W(\mathbf{r} - \mathbf{r}') n(\mathbf{r}') g_2(\mathbf{r}, \mathbf{r}') + \varphi(\mathbf{r}).
\]

which corresponds to the well-known equation of the classical density functional theory [13, 14].

### 3.2. The case of retarded interactions

The force acting on \( i \)-th particle taking into account the retarded interactions, has the form

\[
\mathbf{F}_i = -\frac{\partial}{\partial \mathbf{r}_i} \sum_{k=1}^N \int d^3r' \left( W(\mathbf{r}_i(t) - \mathbf{r}_i(t')) \delta \left( t - t' - \frac{\mathbf{r}_i - \mathbf{r}_i}{c} \right) \right) d\mathbf{r}'.
\]

where \( \frac{\mathbf{r}_i - \mathbf{r}_i}{c} = \tau_{i,i} \) is the interactions retardation between \( i \)-th and \( k \)-th particles, \( c \) is the light speed. It should be specially noted that the function \( W(\mathbf{r}) \) is defined for resting particles, while the function (31) depends not only on \( \mathbf{r} \) and \( t \), but also on the interactions retardation \( \tau_{i,k} \) between all the particles.

Taking into account (31), the expression for the volume density of force (9) can be represented in the form [15]:

\[
\Phi(\mathbf{r}, t) = -\int d^3r' \int d^3p' \sum_{k=1}^N \frac{1}{n!} \left( F_{(a_1, \ldots, a_n)}^{(1)} + F_{(a_1, \ldots, a_n)}^{(2)} \frac{p'}{m c} \frac{\partial}{\partial \mathbf{r}'} \right) S_{(a_1, \ldots, a_n)}(\mathbf{r}, \mathbf{r}', \mathbf{p}', t, t - \bar{\tau}).
\]

Here \( S_{(a_1, \ldots, a_n)}(\mathbf{r}, \mathbf{r}', \mathbf{p}', t, t') \) are the generalized moments:

\[
S_{(a_1, \ldots, a_n)}(\mathbf{r}, \mathbf{r}', \mathbf{p}', t, t') =
\frac{1}{\Delta^2} \sum_{i=1}^N \sum_{k=1}^N \int d^3\xi \int d^3\xi' \left( \xi_{a_i} - \xi'_{a_i} \right) \left( \xi_{a_1} - \xi'_{a_1} \right) \ldots \left( \xi_{a_n} - \xi'_{a_n} \right) \times \delta \left( \mathbf{r}' + \xi' - \mathbf{r}_i(t') \right) \delta \left( \mathbf{r} + \xi - \mathbf{r}_i(t) \right) \delta \left( \mathbf{p}' - \mathbf{p}_i(t') \right),
\]

\[
F_{(a_1, \ldots, a_n)}^{(1)}(\mathbf{r} - \mathbf{r}') = \frac{\partial^n}{\partial \mathbf{r}^{a_i} \partial \mathbf{r}^{a_i} \ldots \partial \mathbf{r}^{a_i}} W(\mathbf{r} + \xi - \mathbf{r}' - \xi'),
\]

\[
F_{(a_1, \ldots, a_n)}^{(2)}(\mathbf{r} - \mathbf{r}') = \frac{\partial^n}{\partial \mathbf{r}^{a_i} \partial \mathbf{r}^{a_i} \ldots \partial \mathbf{r}^{a_i}} \left( W(\mathbf{r} + \xi - \mathbf{r}' - \xi') \left( \frac{1}{c} \frac{\mathbf{r} + \xi - \mathbf{r}' - \xi'_{a_i}}{\mathbf{r} + \xi - \mathbf{r}' - \xi'} \right) \right).
\]

\( \bar{\tau} \) is the interactions retardation between points with coordinates \( \mathbf{r} + \xi \) and \( \mathbf{r}' + \xi' \), \( \bar{\tau} = \left| \mathbf{r} + \xi - \mathbf{r}' - \xi' \right| / c \).

Then we can, after carrying out the expansion with respect to the interactions retardation, limit ourselves by the terms of the first order in \( \bar{\tau} \). Taking account the first two first moments \( S_{(0)}^{(1)} \) and \( S_{(a)}^{(2)} \), the volume density of the force takes the form
\[ \Phi_a (r, t) = - \int d^3 r [ n(r, t) n(r', t) + \nabla_n \left[ n(r', t) I(r, t) - n(r, t) I(r', t) \right] ] \frac{\partial W(r-r')}{\partial x^a} - \\
- \int d^3 r [ \frac{\partial n(r', t)}{\partial t} I(r, t) - n(r, t) \frac{\partial I(r', t)}{\partial t} ] \frac{r-r'}{c} \frac{\partial W(r-r')}{\partial x^a} + \\
+ \int d^3 r [ n(r, t) \frac{\partial n(r', t)}{\partial t} + \nabla_n \left[ I(r, t) \frac{\partial n(r', t)}{\partial t} - n(r, t) \frac{\partial I(r', t)}{\partial t} \right] ] \frac{\partial W_{\text{eff}}(r-r')}{\partial x^a}. \]

Here \( W_{\text{eff}}(r-r') = W(r-r') |r-r'|/c \).

The first term in (34) corresponds to the absence of interactions retardation and may be written in the form (19). The last two terms on the right-hand side of the resulting equation are a consequence of the finiteness of the rate of transfer of interaction between particles. In this case, the potential energy of type (18) no longer exists, and it only makes sense to talk about the kinetic energy of the particles and the energy of the field through which the interaction takes place. These terms correspond to the correlations due to the influence of the field on the evolution of the particle system.

The equation of motion (7), taking into account (34), becomes non-invariant with respect to the time reversal and, as a consequence, describes the irreversible evolution of a system of particles. This explains the transition of the system to equilibrium. With time, the kinetic energy of the particles transforms into the energy of the field, which plays the role of a kind of thermostat. A similar effect is observed even for the dynamics of a two-particle harmonic oscillator [16].

Correlations due to the presence of a thermostat-field, as is easily seen from (34), are determined by the rates of change of field variables \( n(r, t) \) and \( I(r, t) \), as well as the divergences of their combinations. The latter arise only when the inconstancy of the interparticle and potential gradient is taken into account.

4. Conclusions

a) Distribution functions that determine the macroscopic characteristics of a particle system have dynamic origin and do not require the introduction of probabilistic hypotheses to their foundation.

b) When taking into account of the retardation of interactions, additional correlations arise due to the rate of change of smoothing local density and other field functions characterizing the inhomogeneity of the distribution of particles. The presence of these terms indicates the influence of the field on the evolution of a system of particles.

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