Ideal fluid and acceleration of the universe

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The solution of the dark energy problem in models without scalars is presented. It is shown that a late-time accelerating cosmology may be generated by an ideal fluid with some implicit equation of state.

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According to recent astrophysical data the current universe is expanding with acceleration. Such accelerated behaviour of the universe is supposed to be due to the presence of mysterious dark energy which, at present, contributes about 70 percent of the total universe energy-mass. What this dark energy is and where it came from is one of the fundamental problems of modern theoretical cosmology (for a recent review, see [1], [2]). Assuming a constant equation of state (EOS), \( p = w\rho \), the dark energy may be associated with a (so far unobserved) strange ideal fluid with negative \( w \). Astrophysical data indicate that \( w \) lies in a very narrow strip close to \( w = -1 \). The case \( w = -1 \) corresponds to the cosmological constant. For \( w \) less than \(-1\) the phantom dark energy is observed, and for \( w \) more than \(-1\) (but less than \(-1/3\)), the dark energy is described by quintessence. It is interesting that the phantom phase is twice as probable than the quintessence phase. Moreover, there are indications that there occured a recent transition over cosmological constant barrier (over the phantom divide).

There are various approaches to describe the phantom dark energy. The simplest one is to work with negative kinetic energy scalar (phantom). The typical property of this phantom cosmology is a future singularity which occurs in a finite time. This is due to the growth of phantom energy which may lead to quite spectacular consequences. In particular, with growth of phantom energy, typical energies (as well as curvature invariants) increase in the expanding universe. As a result, in some scenarios the quantum gravity era may come back at the end of the phantom universe evolution. In this case, it was checked that quantum effects [3], [4], [5] may act against the Big Rip (or even stop it in case of quantum gravity [6]). The interesting approach to describe the dark energy universe is related to an ideal fluid with some (strange but explicit) EOS which may be sufficiently complicated [7], [8], [9]. Of course, this approach is phenomenological in some sense, because it does not describe the fundamental origin of dark energy. At the same time, it may lead to quite successful description of not only phantom phase but also of transition from decelation to acceleration or crossing of the phantom divide.

We now consider the FRW cosmology with an ideal fluid. The starting FRW universe metric is:

\[
ds^2 = -dt^2 + a(t)^2.
\]  

In the FRW universe, the energy conservation law can be expressed as

\[
0 = \dot{\rho} + 3H (p + \rho).
\]  

Here \( \rho \) is the energy density, and \( p \) is the pressure. The Hubble rate \( H \) is defined by \( H \equiv \dot{a}/a \) and the first FRW equation is

\[
\frac{3}{c^2} H^2 = \rho.
\]  

We often consider the case that \( \rho \) and \( p \) satisfy the simple EOS, \( p = w\rho \). Then if \( w \) is a constant, Eq. (2) can be easily integrated as \( \rho = \rho_0 a^{-3(1+w)} \). Using the first FRW equation (3), the well-known solution follows:

\[
a = a_0 (t - t_1) \frac{1}{\sqrt{w+1}} \quad \text{or} \quad a_0 (t_2 - t) \frac{1}{\sqrt{w+1}},
\]  

when \( w \neq -1 \), and

\[
a = a_0 e^{\kappa t \sqrt{w}}
\] 

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when \( w = -1 \).

The ideal fluid with a more general EOS may be considered \([10], [11]\):

\[
f(\rho, p) = 0 .
\]  

(6)

An interesting example is given by

\[
\rho \left( 1 + \frac{A}{2}(\rho + p) \right) + \frac{3}{\rho} \left( 2 + \frac{A}{2}(\rho + p) \right)^2 = 0 .
\]  

(7)

Solving Eqs. (2) and (3) one arrives at

\[
H = \frac{1}{t} \left( 1 - \frac{\kappa^2 t^2}{A} \right) , \quad \rho = \frac{3}{\kappa^2 t^2} \left( 1 - \frac{\kappa^2 t^2}{A} \right)^2 , \quad p = \frac{1}{\kappa^2} \left( \frac{1}{t^2} - \frac{8\kappa^2}{A} + \frac{3\kappa^4 t^2}{A^4} \right) .
\]  

(8)

Since

\[
\dot{H} = -\frac{1}{t^2} \frac{\kappa^2}{A} ,
\]  

(9)

it follows that

\[
\frac{\ddot{a}}{a} = \dot{H} + H^2 = \frac{\kappa^2}{A} \left( -3 + \frac{\kappa^2 t^2}{A} \right) .
\]  

(10)

Then if \( A > 0 \), the decelerating universe with \( \ddot{a} < 0 \) transits to an accelerating universe with \( \ddot{a} > 0 \) when \( t = \sqrt{\frac{3A}{\kappa^2}} \).

Eq. (9) also shows that if \( A < 0 \) - the non-phantom phase with \( \dot{H} < 0 \) changes to the phantom phase with \( \dot{H} > 0 \) and

\[
t = \sqrt{-\frac{A}{\kappa^2}}.
\]

This demonstrates that an ideal fluid with a complicated EOS may be the origin of dark energy and late-time acceleration.

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