Assessment of the Imperfections for Plate Buckling of Unstiffened Plates

Larissa Schönfeld, Bernd Naujoks, Christian Ludwig

1 Introduction

Thin-walled structures are often classified in class 4. In these cross-sections, stability failure in due to plate buckling will occur before the yield strength is reached. A typical example is shown in Figure 1. Another typical example of thin-walled components is a welded steel cross-section in bridge constructions, as shown in Figure 2.

Figure 1 Local plate buckling of a stiffened plate [1]

Figure 2 Stiffened steel box girder

Thin-walled structures can be verified according to EN 1993-1-5 [2]. This design standard provides three methods to consider plate buckling effects:

- Method of effective widths (MEW)
- Method of reduced stress (MRS)
- Finite element analysis (FEA)

Abstract

In this study different approaches and combinations of imperfections using GMNIA of unstiffened plates are examined. The imperfections have a crucial influence for the determination of the load bearing capacity of thin plates. According to EN 1993-1-5 Annex C, when using GMNIA, the user can choose between equivalent geometric imperfections and a combination of the geometrical and the structural imperfections. The exact influence of the individual imperfections and their combination on the load bearing behavior has not been conclusive clarified. Furthermore, EN 1993-1-5 Annex C does not recommend a suitable residual stress model. This study aims to compare the ultimate loads due to compression loading using different imperfection approaches. Ten different combinations are investigated by using GMNIA. The results show that for larger plate slenderness, many of the ultimate loads are above the Winter-Curve. The influence of the structural imperfections on the ultimate load is higher than the influence of the geometrical imperfections. An appropriate residual stress model is recommended. The geometrical imperfections should be given as a function of the plate slenderness.

Keywords

Geometrical and structural imperfections, residual stresses, unstiffened plates, plate buckling, Winter-Curve, FEA, GMNIA, ANSYS

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The MEW and MRS are generally suitable for a standard hand calculation. The bifurcation load factor \( \alpha_c \) can also be determined via numerical analysis for the MRS. Basic information about applying the FE-Analysis is presented in the Annex C of EN 1993-1-5 [2]. Emphasis is placed on the specification of the imperfections, which must be considered in a numerical determination of the ultimate load. These imperfections are already considered in the reducing factors of MRQ and MRS. Due to the different possibilities listed in EN 1993-1-5 Annex C [2], the influence of the imperfections is focused in this study.

2 Imperfections according to EN 1993-1-5

2.1 Overview

In Annex C of EN 1993-1-5 [2], imperfections are divided into three types:

- Geometrical imperfections
  (Deformation due to manufacturing and unintentional eccentric loads)
- Structural imperfections
  (Residual stresses and deviation of the material properties)
- Equivalent geometric imperfections
  (simplified combination of the geometrical and the structural imperfections)

The standard explicitly pronounces that Annex C’s application is reserved for engineers with appropriate experience in applying FE methods.

2.2 Geometrical imperfections

Typical geometrical imperfections are deviations caused by the manufacturing process of steel components. Geometrical imperfections can be measured optically (e.g. laser scan, photogrammetry) or mechanically (e.g. straight edge, displacement sensor). These measurements are usually non-destructive. The limit values for these manufacturing tolerances are presented in EN 1090-2 [3]. For the determination of limit values according to EN 1090-2 [3] the direction of stress is to be taken into account, Table 1.

| Table 1 Manufacturing tolerances (geometrical imperfections) for unstiffened plates, acc. to EN 1090-2 [3] |
|---|
| **Longitudinal direction** |
| \( \frac{a}{b} \leq 2 \):
| \( \pm \frac{a}{250} \) |
| \( \frac{a}{b} > 2 \):
| \( \pm \frac{b}{125} \) |
| **Transverse direction** |
| \( \frac{a}{b} < 0.5 \):
| \( \pm \frac{a}{125} \) |
| \( \frac{a}{b} \geq 0.5 \):
| \( \pm \frac{b}{250} \) |

Due to the column buckling behaviour it is necessary to distinguish between longitudinal and transverse loads (see Schmidt/Korth/Machura/Podleschny/Kammel/Volz [4]). For the first time the load directions were considered in DIN 18800-3 [5]. It has also to be noticed that DIN 18800-3 [5] is a design standard that refers to the execution standard DIN 1079 [6]. Both standards have been withdrawn. An analysis of the imperfections according to Table 1 is shown in Figure 3.

![Figure 3 Comparison of the geometrical imperfections acc. to [3] of unstiffened plates](image)

The imperfection due to transverse load is crucial in biaxial loading (longitudinal and transverse) and an aspect ratio of \( \alpha < 1.0 \).

2.3 Structural imperfections

The structural imperfections of a component are not visible from the outside. These so called "inner" inhomogeneities include e.g. the irregular distribution of strength properties and stresses within the cross-section. Residual stresses are the most important part of the structural imperfections. They emerge due to welding and also to hot-rolling. The residual stresses are determined from equilibrium. There are no resulting internal forces and moments. Figure 4 shows typical models of residual stresses for webs of welded I-sections. The residual stress model according to the current draft of EN 1993-1-14 [8] is identical to model ECCS 33 [7].

Different parameters are taken into account to determine the residual stresses, see Figure 4. In the model of ECCS 33 [7] and of Kubsch [9] the web height is considered, whereas BSK [10] is based on the web thickness. Furthermore, for the compressive stress in ECCS 33 [7] / EN 1993-1-14 [8] and Kubsch [9] fixed value is given. In BSK 94 [10], the compressive stress value has to be determined considering the equilibrium with the tensile residual stresses. Typical welding manufacturing parameters are not considered in these models, acc. to Figure 4 (e.g. welding method, weld seam thickness, welding sequence, flame cutting).
Recent investigations on the residual stress curves by Pasternak/Launert/Krausche [11], Schaper/Jörg/Winkler/Kuhlmann/Knobloch [12] and Trankova/Simoes/BalaKrishnam/Rodrigues/Launert/Tun [13] lead to different results and partly contradictory conclusions. For example, according to [11] the influence of the plate thickness can be omitted. In contrast an obvious influence is presented in [12]. Figure 5 shows the different residual stress models in a welded flange according to [11], ECCS 33 [7] and BSK 94 [10].

Compared to measurement of Pasternak/Launert/Krausche [11] the approach of BSK 94 [10] leads to the best results. As a result of the manufacturing process of the specimens (thermal cutting) tensile residual stresses occur at the unwelded outer edges of the flange in [12], s. Figure 6.

A comparison with other residual stress models is limited because of the tensile stresses at the flange edge. It is also not possible to compare the results of Pasternak/Launert/Krausche [11] to Schaper/Jörg/Winkler/Kuhlmann/Knobloch [12] due to different geometries. The influence of the cross-section geometry cannot be sufficiently determined.

The authors also apply different residual stress models for their further investigations. In [11], the residual stress model according to BSK 94 [10] is applied, but in [12], the model of ECCS 33 [7] is used. The choice of the residual stress model has a crucial influence on the load bearing behavior. Due to the tensile stresses at the edges of flanges a stabilizing effect occurs. This leads to a higher load bearing capacity. Slender cross-section parts are usually manufactured by welding. In this study, hot-rolled I-sections are not analysed. The geometrical and the structural imperfections are not completely independent of each other, see Figure 8.
The residual stresses that occur due to welding process can be relieved in the form of distortion in components with low stiffness.

2.4 Equivalent geometric imperfections

The equivalent geometric imperfections, according to EN 1993-1-5 [2] could be used as an alternative to the geometrical and the structural imperfections. The equivalent geometric imperfections according to Table 2 already take the geometrical and the structural imperfections into account.

Table 2 Equivalent geometric imperfection for unstiffened plates, acc. to [2]

| Type | Imperfection |
|------|--------------|
| A    | Equivalent geometric imperfection \( e_i \), acc. to Table 2 |
| B    | 80% Geometrical imperfection acc. to Table 1 |
| C    | Structural imperfection e.g. acc. to Figure 4 |

For the majority of numerical calculation, the equivalent geometric imperfections are used to determine the ultimate load, e.g. Braun [15], Zizza [16], Timmers [17] and Degée/Kuhlmann/Detzel/Maquoi [18].

2.5 Application of imperfections according to EN 1993-1-5

In EN 1993-1-5 Annex C [2], three possibilities are available for applying imperfections to determine the ultimate load in numerical analyses. In Table 1 these approaches are summarized as types A, B and C.

Table 3 Approach options for imperfections acc. to EN 1993-1-5 Annex C [2]

| Type | Imperfection |
|------|--------------|
| A    | Equivalent geometric imperfection \( e_i \), acc. to Table 2 |
| B    | 80% Geometrical imperfection acc. to Table 1 |
| C    | Structural imperfection e.g. acc. to Figure 4 |

Applying type C the measurement of the imperfections is only possible after manufacturing the components. Usually, the components have to be cut (sectioning method, e.g. in [12]) to determine the residual stresses (structural imperfections).

3 Existing studies

In recent years it has been published more researches using FE method (FEA) due to increasing high-performing computational technology. Analysing these researches, it is decisive to distinguish which boundary conditions have been considered for numerical calculations by the authors. In particular the imperfections are focused. An overview is summarised in Table 4.

Table 4 Overview of applied approaches of the imperfections in selected scientific researches according to Table 3

| Type | Reference | Design |
|------|-----------|--------|
| A    | [15] [16] [17] [18] | Plate buckling |
|      | [11] [18] [19] [20] | Plate buckling |
|      | [21]      | Plate buckling |
| B    | [12] [22] | Lateral torsional buckling |
| C    | [12]      | Lateral torsional buckling |

Table 4 shows that type B according to Table 3 (geometrical imperfection according to EN 1090-2 [3]) was only analysed by Pasternak/Launert/Krautsche [11]. Furthermore, it should be mentioned that in most numerical analysis, the geometrical imperfections or the equivalent geometric imperfections are assumed to be sinusoidal. In the following numerical analysis type B is used, considering the eigenmodes of the analysed plate.

4 Numerical simulations

4.1 Numerical model

The ultimate loads are determined numerically (GMNIA), considering geometrical and physical non-linear conditions. The FE software Ansys [23] is used for the calculations. For the modeling are applied:

- Steel grade S355 · bilinear material behaviour (linear elastic – plastic with E / 10 000)
- Shell element 181
- Geometrical and equivalent geometric imperfections according to the scaled eigenmode
- Structural imperfections due impressed stresses
- Four-sided hinged with an aspect ratio \( \alpha = 1 \)
- Constant compression stress \( \nu = 1 \)
- Calculation method Newton-Raphson + stabilization energy [24]

(extended comparative calculations using arc-length method show no significant deviations)

The Newton-Raphson method for analysing stability behaviour is also used, for example by Ruff/Schul [19]. The analysed structural elements are hinged and non-sway in vertical direction to the plane (Navier’s boundary condition). The elements at the transverse edges are linked in plane. Thus, loaded by constant compression a uniform
deformation in plane is ensured. The longitudinal edges are not linked (s. Figure 9).

![Boundary conditions and loading of the plane](image)

**Figure 9** Boundary conditions and loading of the plane

The boundary conditions according to Figure 9, are also assumed by Braun [15], Zizza [16] and Rusch/Lindner [20]. These conditions result in the best correspondence of the ultimate loads with the Winter-Curve.

### 4.2 Analysed approaches

An overview of the numerical calculations is summarised in Table 5.

| Combination | Deviations from Winter-Curve [%] | Deviations | Winter-Curve |
|-------------|---------------------------------|------------|-------------|
| I 100 %    | 1 %                             | -          | -           |
| II 100 %   | 1 %                             | -          | -           |
| III 80 %   | 1 %                             | -          | -           |
| IV 80 %    | 1 %                             | -          | -           |
| V 80 %     | 1 %                             | -          | -           |
| VI 80 %    | 1 %                             | -          | -           |

Combinations I, IV, V and VI correspond to the specifications in EN 1993-1-5 [2]. Compared to Table 3, the combination I can be assigned to type A and IV-VI to type B. This scope of parameters was chosen to determine the influence of the geometrical and the equivalent geometric imperfections (type I-III) and the residual stresses (type IV-VI). The calculated ultimate loads are compared to the Winter-Curve, whose accuracy has been confirmed by many experimental studies. An overview of these studies is presented by Scheer/Peil/Fuchs [25], Braun [15], Zizza [16] and Rusch/Lindner [20] also use the Winter-Curve to verify their results.

### 4.3 Influence of the geometrical imperfections

Figure 10 shows the reduction factors depending on plate slenderness $\lambda_p$ resulting from combinations I-III according to Table 5.

![Ultimate loads as a function of the geometrical and the equivalent geometric imperfections](image)

**Figure 10** Ultimate loads as a function of the geometrical and the equivalent geometric imperfections

In general, the differences in ultimate load of the combination II compared to the combination III are low. Thus, the results of the numerical researches of Ruft/Schulz [19] and Rusch/Lindner [20] can still be used for comparison, although these calculations consider imperfections according to DIN 18800-3 [5], s. Table 4.

### 4.4 Influence of the residual stresses

Figure 11 shows the ultimate loads of the combinations IV-VI.
In summary, in this research, the residual stress model leads to the best approximation of the ultimate loads compared to the Winter-Curve.

4.5 Various combinations of the imperfections

In order to improve the results (closer to Winter-Curve), the geometrical and the structural imperfections are modified by Rusch/Lindner [20]. A combination of 50% geometrical and 50% structural imperfection is proposed. Analogous to this method, further combinations are analysed in this research. In the following numerical calculations, the residual stress approach according to ECCS 33 [7] is used. The reason for this is that the residual stress model ECCS 33 [7] and EN 1993-1-14 [8] are identical. Thus, this residual stress approach is going to form the basis of FE-analysis. An overview of further combinations is summarised in Table 8.

Analogous to Figure 10, Figure 11 shows different ultimate load effects above and below the plate slenderness $\lambda_p = 2.0$. Applying the combinations V and VI with $\lambda_p < 2.0$ results in similar conservative ultimate loads. In contrast, using the combination IV, the results are farther below the Winter-Curve. With increasing plate slenderness, the deviations are reduced until $\lambda_p = 2.0$. At $\lambda_p > 2.0$ the ultimate loads of the combinations IV and VI are almost similar and conservative. The ultimate loads of the combination V lay above the Winter-Curve in this plate slenderness range. Table 7.

Table 7 Deviations of ultimate loads of the combinations IV-VI according to Table 5

| Combination | Deviations from Winter-Curve [%] | $\lambda_p = 0.673 \cdot 1.75$ | $\lambda_p > 2$ |
|-------------|----------------------------------|-----------------------------|-----------------|
| IV $\diamond$ 80 % EN 1090-2 [3] + ECCS 33 [7] | 86.2 – 91.1 | 92.8 – 95.4 |
| V $\Delta$ 80 % EN 1090-2 [3] + BSK 94 [10] | 93.1 – 94.9 | 95.5 – 103.2 |
| VI $\times$ 80 % EN 1090-2 [3] + Kubsch [9] | 91.7 – 94.4 | 94.7 – 97.3 |

The reason for this combination is that hot-rolled plates exhibit compressive stresses at their outer edges, s. Hänsch [26]. Due to the welding process tensile stresses occur at the outer edges. It cannot be guaranteed whether a complete inversion of the algebraic signs occur. The determined ultimate loads of the combinations VII-X are shown in Figure 13.

The combination X in Table 8 corresponds to the proposal by Rusch/Lindner [20]. Another special research is the combination IX. The algebraic signs of the residual stresses are inverted, Figure 12a).

Table 8 Further combinations of unstiffened plates

| Geom. imperfection $e_g$ | Struc. imperfection $e_s$ |
|--------------------------|--------------------------|
| VII 20 % EN 1090-2 [3] $e_g = \frac{1}{250}$ | ECCS33 [7] |
| VII 80 % EN 1090-2 [3] $e_g = \frac{1}{250}$ | 40 % ECCS 33 [7] |
| IX 80 % EN 1090-2 [3] $e_g = \frac{1}{250}$ | ECCS 33 [7] inverted |
| X 50 % EN 1090-2 [3] $e_g = \frac{1}{250}$ | 50 % Kubsch [9] |

In order to improve the results (closer to Winter-Curve), the geometrical and the structural imperfections are modified by Rusch/Lindner [20]. A combination of 50% geometrical and 50% structural imperfection is proposed. Analogous to this method, further combinations are analysed in this research. In the following numerical calculations, the residual stress approach according to ECCS 33 [7] is used. The reason for this is that the residual stress model ECCS 33 [7] and EN 1993-1-14 [8] are identical. Thus, this residual stress approach is going to form the basis of FE-analysis. An overview of further combinations is summarised in Table 8.
Figure 13 shows similar effects as Figure 10 and Figure 11. The ultimate loads of the various combinations deviate larger in plate slenderness range $\bar{\lambda}_p \leq 2.0$. As expected, the combination IX leads to the highest ultimate loads, which are partly located above the Winter-Curve. The reason for this is the stabilizing influence of the tensile stresses in the plate centre, according to Figure 12 a). Within a larger plate slenderness range, the approach of Rusch/Lindner [20] (combination X) also leads to ultimate loads above the Winter-Curve. Comparing the combinations VII and VIII, residual stresses influence is more significant than the geometrical imperfection. The reason for this is that the combination VII leads to lower ultimate loads than the combination VIII. Comparable results are also published by Kubsch [9] and Brune [21]. In Table 9 the deviations of the combinations of Table 8 are summarised. Only the combination VII always leads to conservative ultimate loads. However, these results are located significant far below the Winter-Curve at lower plate slenderness range.

| Combination | Deviations from Winter-Curve [%] | $\bar{\lambda}_p$ = 0.673 - 1.75 | $\bar{\lambda}_p \geq 2$ |
|-------------|---------------------------------|-------------------------------|-----------------|
| VII | $\times$ 20% EN 1090-2 [3] + ECCS 33 [7] | 86.6 - 99.1 | 92.4 - 99.1 |
| VIII | $\square$ 80% EN 1090-2 [3] + 40% ECCS 33 [7] | 95.1 - 98.0 | 99.5 - 104.1 |
| IX | $\triangle$ 80% EN 1090-2 [3] + ECCS 33 inv. | 97.5 - 100.0 | 102.0 - 111.6 |
| X | $\bigcirc$ 50% EN 1090-2[3] + 50% Kubsch [9] | 96.7 - 98.9 | 100.3 - 103.7 |

### 5 Conclusions

Basic notes for the analysis of plate buckling according to FE method are presented in EN 1993-1-5 Annex C [2]. For numerical calculations considering the geometrical and the physical non-linearities (GMNIA), imperfections must be taken into account. Applying Annex C of EN 1993-1-5 [2], the equivalent geometric imperfections or a combination of the geometrical (according to EN 1090-2[3]) and the structural imperfections (essentially residual stresses) can be used. In this study, different approaches of imperfections and combinations of the imperfections were analysed and compared to the Winter-Curve. Different residual stress approaches were taken into account. Furthermore, the influence of individual imperfections and 10 combinations of imperfections were analysed. The following conclusions can be drawn from this study:

- The equivalent geometric imperfections approach leads to higher ultimate loads compared to the combination of geometrical and structural imperfections (s. Table 6 vs. Table 7).
- The deviations of the individual combinations are decreased with increasing plate slenderness (s. Figure 10, Figure 11, Figure 13).
- The ultimate loads due to the equivalent geometric imperfections are located significantly above the Winter-Curve in the range of higher plate slenderness ratios (s. Table 6).
- The combination of geometrical and structural imperfections, according to Kubsch [9] leads to the best approximation compared to the Winter-Curve (s. Table 7).
- The influence of the residual stresses is significantly larger than the influence of geometrical imperfections (s. Table 6 vs. Table 7).

For this reason, the value of the geometrical and the equivalent geometric imperfections should be applied depending on plate slenderness $\bar{\lambda}_p$. A comparable approach presented by Beier-Tertel [27] is found in EN 1993-1-1/NA [28] for analysis of lateral torsional buckling of beams.

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