Spin orientation of a two-dimensional electron gas by a high-frequency electric field

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I. INTRODUCTION

Spin dynamics of charge carriers and, particularly, possibilities of manipulating electron spins in semiconductor structures is attracting a great deal of attention [1]. Much effort in this research area is directed towards the development of efficient methods of injection and detection of spin-polarized carriers by electrical means. In particular, it has been shown that in gyrotropic semiconductor structures, i.e., structures of symmetry classes which allow spin-orbit splitting of the spectrum linear in the wave vector, spins of the free carriers can be oriented by an electric current flow [2, 3, 4, 5]. This current-induced spin polarization has been observed in bulk tellurium [6] and recently in strained bulk InGaAs [7] and GaAlAs quantum well (QW) structures [8, 9]. Microscopically, this effect represents a current-induced selective occupation of the spin subbands, which are split in $k$-space due to the spin-orbit interaction. Dynamics of the spin polarization is governed by the spin relaxation time that determines the rate of carrier redistribution between the spin subbands. The spin polarization induced by ac electric field oscillates at the field frequency, its amplitude being determined by the ratio between the field frequency and the spin relaxation rate. Under application of ac electric field of frequency higher than the spin relaxation rate, the spin polarization vanishes. On the other hand, as is known, perturbation of the electron gas with ac electric field causes an absorption of the field by the free carriers. In systems with spin-orbit interaction this process may also be spin-dependent [10].

In this paper we show that perturbation of a two-dimensional (2D) electron gas with a linearly polarized high-frequency electric field induces spin orientation of the carriers. The direction of the spin orientation is determined by the field polarization and the explicit form of the spin-orbit interaction. In particular, excitation with in-plane ac electric field in conventional (001)-grown QW structures leads to orientation of the electron spins along the QW normal, its sign and magnitude depending on the field polarization with respect to the crystallographic axes. From the phenomenological symmetry analysis, the effect under study is similar to the interband optical orientation of electron spins by linearly polarized light [11].

However, in contrast to the direct optical transitions from the valence to the conduction band, here only one type of carriers, namely, the conduction electrons, is excited by the electromagnetic field. Thus, the microscopic mechanism of the spin orientation of 2D electron gas proposed in the present paper differs from that based on the selection rules for interband optical transitions.

II. MICROSCOPIC MODEL

Absorption of a high-frequency electric field by free carriers, or Drude-like absorption, occurs in doped semiconductor structures and is always accompanied by electron scattering from acoustic or optical phonons, static defects, etc., because of the need for energy and momentum conservation. In systems with a spin-orbit interaction, processes involving change of the particle wave vector are spin-dependent. In particular, the matrix element of electron scattering by static defects or phonons $V_{k'k}$ in QW structures contains, in addition to the main contribution $V_0$, an asymmetric spin-dependent term [12, 13, 14]

$$V_{k'k} = V_0 + \sum_{\alpha\beta} V_{\alpha\beta} \sigma_\alpha(k_\beta + k_\beta'),$$

(1)

where $k$ and $k'$ are the initial and the scattered in-plane wave vectors, respectively, and $\sigma_\alpha$ ($\alpha = x, y, z$) are the Pauli matrices. Microscopically, this contribution originates from structural or bulk inversion asymmetry similar to $k$-linear Rashba and Dresselhaus spin splitting of the electron subbands in QWs grown from zinc-blende-type compounds. Due to the spin-dependent asymmetry of the scattering, electrons photoexcited from the subband bottom are scattered in preferred directions depending on their spin states [15, 16]. This concept is illustrated in Fig. 1(a), where the free-carrier absorption is shown as a combined two-stage process involving electron-photon interaction (vertical solid lines) and electron scattering (dashed horizontal lines). The scattering asymmetry is shown by thick and thin dashed lines: electrons with the spins $+1/2$ and $-1/2$ are scattered predominantly into the states with $k_x > 0$ and $k_x < 0$, respectively. Note, that such an electron distribution repre-
affected by the field with the frequency $\Omega_k$ opposite spin, $-\Omega_k$, with the Larmor frequency $\Omega_k$ an odd function of the wave vector, $k$. The effective magnetic field induced by spin-orbit coupling is $+1\hbar$ as shown in Fig. 1(b). Electrons with the initial spin directed according to the spin-dependent scattering, precession of the electron gas by a high-frequency electric field. (a) Asymmetry of scattering under photoexcitation followed by (b) spin precession in the effective magnetic field leads to appearance of the average electron spin.

A dominant contribution to absorption of a high-frequency electric field in QWs is introduced by processes with intermediate states in the same subband, $e1$. Taking into account the matrix element of electron scattering within the subband $e1$ in the form of Eq. (1), one derives two contributions to the matrix element of the indirect optical transitions

$$M^{(0)}_{k'k} = \frac{eA}{c\omega m^*} \mathbf{e} \cdot (\mathbf{k} - \mathbf{k}') V_0, \quad (4)$$

$$M^{(1a)}_{k'k} = \frac{eA}{c\omega m^*} \mathbf{e} \cdot (\mathbf{k} - \mathbf{k}') \sum_{\alpha\beta} V_{\alpha\beta} \sigma_\alpha (k_\beta + k'_\beta), \quad (5)$$

where $e$ is the elementary charge, $c$ is the light velocity, $m^*$ is the effective electron mass, $A = \mathbf{A} \mathbf{e}$ is the vector potential of the field, $\mathbf{e}$ is the (unit) polarization vector. We assume the electromagnetic field to be linearly polarized and, therefore, consider the polarization vector $\mathbf{e}$ to be real. The matrix element $\mathbf{M}$ determines the QW absorption coefficient, but does not contribute to spin phenomena alone, being independent of the electron spin state. On the contrary, the term given by Eq. (5) gives rise to spin-related effects under indirect optical transitions because of the spin-dependent scattering. In general, a dependence of the amplitude of electron scattering on the spin can be derived if one considers $\mathbf{k} \cdot \mathbf{p}$ admixture of spin-orbit interaction.

III. THEORY

Indirect optical transitions are treated in perturbation theory as second-order processes involving virtual intermediate states. The compound matrix element of this kind of the transition with the initial $|s\mathbf{k}\rangle$ and final $|s'\mathbf{k}'\rangle$ states in the electron subband $e1$ has the form (see, e.g., Ref. [4])

$$M^{(s)}_{s'k',sk} = \sum_j \left( \frac{V_{e1s'k',jk} R_{jk,ek1k} + R_{e1s'k',jk} V_{jk,ek1k}}{E_{e1k} - E_{jk} - \hbar \omega} \right). \quad (3)$$

Here $s$ and $s'$ are the spin indices, $j$ denotes the subband of an intermediate state; $E_{e1k}$, $E_{e1k'}$ and $E_{jk}$ are the electron energies in the initial, final and intermediate states, respectively; $V_{e1s'k',jk}$ is the matrix element of electron scattering, $R_{jk,ek1k}$ is the matrix element of electron interaction with the electromagnetic field, and $\omega$ is the field frequency. Spin-orbit splitting of the subbands $e1$ and $j$, odd in the wave vector, is neglected in Eq. (3), since it leads to no essential contribution to the pure spin current induced by free-carrier absorption.

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of the valence-band states to the conduction-band wave functions and takes into account the spin-orbit splitting of the valence band. To first order in the in-plane wave vector, the contribution to the matrix element of electron scattering by short-range static defects has the form

$$V_{s's',sk}^{(1)} = \frac{\hbar}{m_0} \sum_{j \neq 1} V_{c1s',j} \frac{k \cdot p_{j1s} + k' \cdot p_{c1s',j} V_{j1c}}{E_{c1} - E_j},$$

where $m_0$ is the free-electron mass, $E_{c1}$ and $E_j$ are the subband energies at $k = 0$, and $p_{j1s}$ is the matrix element of the momentum operator. This contribution leads to spin-dependent terms in the matrix element of electron scattering which may be rewritten in the form of Eq. (1), with the coefficients $V_{\alpha\beta}$ given by

$$V_{\alpha\beta} = \frac{\hbar}{2m_0} \sum_{s' = \pm 1/2} \sum_{j \neq 1} \frac{(\sigma_{\alpha})_{ss'} (p_{j\beta})_{c1s',j} V_{j1c}}{E_{c1} - E_j}.$$  

We note that, in accordance with the time inversion symmetry, the coefficients of the terms $\sigma_{\alpha}k_{\beta}$ and $\sigma_{\alpha}k'_{\beta}$ in the matrix element of electron scattering appear to be real in the appropriate basis and coincide, as presented in Eq. (1). Thus this follows from Eq. (1), taking into account that all electron and hole states $j$ and $j'$ are two-fold degenerate at $k = 0$. The states $j$ and $j'$ corresponding to the same energy $E_j$ are interconnected by the time inversion operator. Contributions to $V_{\alpha\beta}$ from the states $j$ and $j'$ are complex conjugate fulfilling the requirements imposed on the matrix element of electron scattering by the time inversion symmetry.

Together with the spin-dependent contribution to the indirect optical transitions $M_{kk'}^{(1a)}$, a comparable term comes from the processes with virtual intermediate states in other, e.g. valence, subbands. This term is given by

$$M_{s's',sk}^{(1b)} = \frac{eA}{cm_0} \sum_{j \neq 1} V_{c1s',j} \frac{e \cdot p_{j1s} + e \cdot p_{c1s',j} V_{j1c}}{E_{c1} - E_j}.$$  

As is shown, it is governed by the matrix elements of interband coupling responsible for the spin-dependent scattering, Eq. (9). Therefore, its spin-dependent part may be expressed in terms of the coefficients $V_{\alpha\beta}$ as follows

$$M_{kk'}^{(1b)} = \frac{2eA}{\hbar c} \sum_{\alpha\beta} V_{\alpha\beta} \sigma_{\alpha} e_{\beta}.$$  

To summarize, to first order in spin-orbit interaction the matrix element of the indirect optical transitions accompanied by electron scattering from short-range potentials may be presented as a sum of three contributions

$$M_{kk'} = M_{kk'}^{(0)} + M_{kk'}^{(1a)} + M_{kk'}^{(1b)},$$  

with the terms given by Eqs. (10-12), respectively.

We assume the frequency of the ac electric field to exceed the reciprocal relaxation times of the carriers. Then, the interaction of the electrons with the electromagnetic field may be treated quantum-mechanically, assuming that the field induces indirect optical transitions. Neglecting the spin-orbit splitting of the electron subband, the spin matrix of the carrier photogeneration has the form (see, e.g., Ref. [14]).

$$G = \frac{2\pi}{\hbar} \sum_{k'} (f_{k'}^{(0)} - f_k^{(0)}) [M_{kk'}^{0} M_{k'k}^{\dagger}(\varepsilon_{k'} - \varepsilon_k + \hbar \omega) + M_{k'k}^{0} M_{kk'}^{\dagger}(\varepsilon_{k'} - \varepsilon_k - \hbar \omega)],$$

where $f_{k'}^{(0)}$ is the equilibrium carrier distribution function and $\varepsilon_k = \hbar^2 k^2/2m^*$ is the kinetic electron energy for the in-plane motion.

As is shown above, the absorption of the high-frequency electric field by free carriers results in an asymmetrical spin-dependent distribution where particles with the opposite spins flow in the opposite directions. Such a photoexcitation asymmetry induced by the spin-dependent scattering does not generally correspond to the eigenstate of the spin-orbit coupling in the subband. The subsequent rotation of the electron spins in the spin-orbit interaction-induced effective magnetic field leads to appearance of a net spin polarization of the electron gas. We assume the relaxation time of the pure spin current $\tau_0$ to be shorter than the Larmor precession period, $\Omega_k \tau_0 \ll 1$. Then, in the steady-state regime, the spin generation rate is given by

$$\dot{S} = \sum_k \tau_0 \left[ \Omega_k \times \mathbf{g}_k \right],$$

where the Larmor frequency corresponding to effective magnetic field in QWs, $\Omega_k$, is a linear function of the electron wave vector

$$\Omega_{k,\alpha} = \frac{2}{\hbar} \sum_{\beta} \gamma_{\alpha\beta} k_{\beta},$$

and $\mathbf{g}_k$ is the spin part of the photogeneration matrix [14]. $g_k = \text{Tr}(\sigma \mathbf{G})/2$.

Finally, assuming that the high-frequency electric field is polarized in the QW plane, one derives the electron spin generation rate

$$\dot{S}_\alpha = \sum_{\beta\gamma\mu} \epsilon_{\alpha\beta\gamma}(\gamma_{\beta\mu} - 2e_\nu e_\mu \gamma_{\beta\mu}) \frac{V_{0\nu}}{V_0} \frac{\tau_0 m^*}{\hbar^3} \eta.$$  

Here $\epsilon_{\alpha\beta\gamma}$ is the third-rank antisymmetric (Levy-Civita) tensor, $I$ is the light intensity related to the vector potential of the electromagnetic field by $I = A^2 \omega^2 n_0/(2\pi c)$, $n_0$ is the refractive index of the medium, and $\eta$ is the QW light absorbance in this spectral range. The latter is given by

$$\eta = \frac{2\pi \tilde{a} V_0^2 N_d}{n_0 (h\omega)^2} N_e \kappa,$$

where $\tilde{a} = e^2/\hbar c$ is the fine-structure constant, $N_e$ is the electron concentration, $N_d$ is the sheet density of defects,
and $\kappa$ is a dimensionless parameter that depends on the carrier distribution,

$$\kappa = \int (1 + 2\varepsilon/\hbar\omega)(f^{(0)}_x - f^{(0)}_{x+\hbar\omega})d\varepsilon / \int f^{(0)}_x d\varepsilon,$$

and is equal to 1 and 2 for the limiting cases $\hbar\omega \gg \varepsilon$ and $\hbar\omega \ll \varepsilon$, respectively, with $\varepsilon$ being the mean electron kinetic energy.

In the case of elastic scattering by static defects, the relaxation time $\tau_c$ coincides with the conventional momentum relaxation time and is governed by the same matrix element of scattering $V_0$ that determines the QW absorbance,

$$\tau_c^{-1} = V_0^2 N_d m^*/\hbar^3.$$

Assuming this, the equation for the spin generation rate \([18]\) can be simplified to the following expression

$$\dot{S}_\alpha = \frac{2\pi \tilde{\alpha}}{\hbar \omega} \sum_{\beta\lambda\mu} \epsilon_{\alpha\beta\lambda\mu}(\gamma_{\beta\mu} - 2\epsilon_{\mu\nu}\gamma_{\beta\mu}) \frac{V_{\lambda\nu}}{V_0} \frac{N_c}{(\hbar\omega)^2} I. \quad (16)$$

It is independent of the structure mobility and determined by the ratio of the spin-dependent to the spin-independent parts of the scattering amplitude, $V_{\lambda\nu}/V_0$, as well as by the constants of the subband splitting, $\gamma_{\beta\mu}$. However, it should be noted, that electron-electron collisions, which do not affect the mobility, can control the spin dynamics and spin transport of the conduction electrons and decrease the time $\tau_c$ \([18], [19]\).

**IV. (001)-GROWN QUANTUM WELLS**

The direction of the spin orientation induced by ac electric field depends on the field polarization and the explicit form of the spin-dependent scattering and the spectrum spin splitting. The latter is governed by the QW symmetry and can be varied. In (001)-grown QWs based on zinc-blende-lattice semiconductors, there are two types of $\mathbf{k}$-linear contributions to the spin-orbit splitting of the conduction subbands. First, a contribution can originate from the lack of an inversion center in the bulk compositional semiconductors and/or from the anisotropy of chemical bonds at the interfaces (so-called Dresselhaus term). Second, $\mathbf{k}$-linear spin-orbit splitting can be induced by the heterostructure asymmetry unrelated to the crystal lattice (Rashba term). Similarly to the spectrum splitting, Dresselhaus-type and Rashba-type contributions to the spin-dependent part of the electron scattering amplitude can be distinguished. Correspondingly, non-zero components of the tensors $\gamma_{\alpha\beta}$ and $V_{\alpha\beta}$ are expressed in terms of the Dresselhaus and the Rashba contributions as follows

$$\gamma_{x'y'} = \gamma_D + \gamma_R, \quad \gamma_{y'x'} = \gamma_D - \gamma_R; \quad (17)$$

where $x' \parallel [1\overline{1}0]$ and $y' \parallel [110]$ are the axes in the QW plane, and $z \parallel [001]$ is the structure normal. Experimental data indicate that the Dresselhaus and Rashba contributions to the $\mathbf{k}$-linear subband splitting in real 2D structures are comparable (see references in Ref. \([11]\)). Furthermore, the ratio between these terms may be tuned, varying the heteropotential asymmetry with an external gate voltage.

Substituting Eq. \((17)\) into \((14)\), one obtains that excitation with the in-plane linearly polarized ac electric field in (001)-grown QWs leads to orientation of the electron spins along the QW normal, with the spin generation rate being

$$\dot{S}_z = 4e_{x'x'}e_{y'}(\gamma_D V_R - \gamma_R V_D) \frac{m^*}{V_0 \hbar^2} I \eta. \quad (18)$$

The spin orientation \([18]\) depends on the polarization of the ac electric field, that provides additional experimental means to observe the effect under study. Indeed, the spin generation $\dot{S}_z$ has opposite sign for the field polarized along the [100] and [010] crystallographic axes and vanishes for the field polarized along the [110] or [100] axes. In general, the dependence of the spin orientation on the field polarization is given by $e_{x'x'}e_{y'} \propto \sin 2\varphi$, where $\varphi$ is the angle between the polarization vector $\mathbf{e}$ and the [110] axis.

The spin generation rate given by Eq. \((18)\) requires both the Rashba and the Dresselhaus contributions to the spin-dependent scattering and the subband splitting. In particular, the spin orientation vanishes, if the spin effects are determined only by the Dresselhaus term, as happens in the symmetrical (001)-grown QWs, or if they are related only to the Rashba contribution, as can be the case of asymmetrical structures grown of centrosymmetrical compounds.

Finally, we present an estimation for the efficiency of the electron spin orientation by the high-frequency electric field. The spin orientation efficiency may be considered as a ratio of the generated spins to the total energy absorbed in the structure, $\dot{S}/I\eta$. Following Eq. \((18)\) one estimates the efficiency as $\dot{S}/I\eta \sim 1 \text{ eV}^{-1}$ for the structure parameters: $\gamma_{\alpha\beta}/\hbar \sim 10^5 \text{ cm/s}$, $V_{\alpha\beta}/V_0 \sim 10^{-8} \text{ cm}$, and $\tau_s \sim 10^{-11} \text{ s}$. This value corresponds with the efficiency of the conventional interband optical orientation by circularly polarized light, where photons of the energy comparable to the band gap, $E_g \sim 1 \text{ eV}$, excite the spin-polarized carriers. The spin orientation by ac electric field has an advantage that only one type of carriers, here electrons, are involved in excitation, allowing to consider it as a kind of spin injection. Recent progress in optical spectroscopy of spin polarization by means of the magneto-optical Faraday and Kerr rotation \([17], [18], [20]\) allows one to expect that the effect discussed in this paper is observable at present.

In conclusion, it is shown that the absorption of linearly polarized high-frequency electric field by two-dimensional electron gas leads to spin orientation of the carriers. The direction and sign of the spin orientation
are determined by the polarization of an electric field as well as by the structure symmetry.

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[1] I. Žutić, J. Fabian, and S. Das Sarma, Spintronics: Fundamentals and applications, Rev. Mod. Phys. 76, 323 (2004).

[2] E.L. Ivchenko and G.E. Pikus, New photogalvanic effect in gyrotropic crystals, Pis'ma Zh. Eksp. Teor. Fiz. 27, 640 (1978) [JETP Lett. 27, 604 (1978)].

[3] F.T. Vas’ko and N.A. Prima, Spin splitting of the spectrum of two-dimensional electrons, Fiz. Tverd. Tela 21, 1734 (1979) [Sov. Phys. Solid State 21, 994 (1979)].

[4] (a) A.G. Aronov and Yu.B. Lyanda-Geller, Nuclear electric resonance and orientation of carrier spins by an electric field, Pis'ma Zh. Eksp. Teor. Fiz. 50, 398 (1989) [JETP Lett. 50, 431 (1989)]; (b) A.G. Aronov, Yu.B. Lyanda-Geller, and G.E. Pikus, Spin polarization of electrons by an electric current, Zh. Eksp. Teor. Fiz. 100, 973 (1991) [Sov. Phys. JETP 73, 537 (1991)].

[5] V.M. Edelstein, Spin polarization of conduction electrons induced by electric current in two-dimensional asymmetric electron systems, Solid State Commun. 73, 233 (1990).

[6] L.E. Vorob’ev, E.L. Ivchenko, G.E. Pikus, I.I. Farbstein, V.A. Shalygin, and A.V. Sturbin, Optical activity in tellurium induced by a current, Pis'ma Zh. Eksp. Teor. Fiz. 29, 485 (1979) [JETP Lett. 29, 441 (1979)].

[7] Y.K. Kato, R.C. Myers, A.C. Gossard, and D.D. Awschalom, Current-induced spin polarization in strained semiconductors, Phys. Rev. Lett. 93, 176601 (2004).

[8] A.Yu. Silov, P.A. Blajnov, J.H. Wolter, R. Hey, K.H. Ploog, and N.S. Averkiev, Current-induced spin polarization at a single heterojunction, Appl. Phys. Lett. 85, 5929 (2004).

[9] S.D. Ganichev, S.N. Danilov, Petra Schneider, V.V. Bel’kov, L.E. Golub, W. Wegscheider, D. Weiss, and W. Prettl, Can an electric current orient spins in quantum wells?, cond-mat/0403641.

[10] E.L. Rashba and A.L. Efros, Efficient electron spin manipulation in a quantum well by an in-plane electric field, Appl. Phys. Lett. 83, 5295 (2003).

[11] S.A. Tarasenko, Optical orientation of electron spins by linearly polarized light, Phys. Rev. B 72, 113302 (2005).

[12] V.I. Belinicher, Asymmetry of the scattering of spin-polarized electrons and mechanisms of the photogalvanic effects, Fiz. Tverd. Tela 24, 15 (1982) [Sov. Phys. Solid State 24, 7 (1982)].

[13] N.S. Averkiev, L.E. Golub, and M. Willander, Spin relaxation anisotropy in two-dimensional semiconductor systems, J. Phys.: Condens. Matter 14, R271 (2002).

[14] E.L. Ivchenko and S.A. Tarasenko, Monopolar optical orientation of electron spins in bulk semiconductors and heterostructures, Zh. Eksp. Teor. Fiz. 126, 476 (2004) [JETP 99, 379 (2004)].

[15] S.A. Tarasenko and E.L. Ivchenko, Pure spin photocurrents in low-dimensional structures, Pis’ma Zh. Eksp. Teor. Fiz. 81, 292 (2005) [JETP Lett. 81, 231 (2005)].

[16] V.V. Bel’kov, S.D. Ganichev, E.L. Ivchenko, S.A. Tarasenko, W. Weber, S. Gigberger, M. Olteanu, H.-P. Tranitz, S.N. Danilov, Petra Schneider, W. Wegscheider, D. Weiss, and W. Prettl, Magnetogyrotropic photogalvanic effects in semiconductor quantum wells, J. Phys.: Condens. Matter 17, 3405 (2005).

[17] (a) M.I. D’yakonov and V.I. Perel’, Spin relaxation of conduction electrons in noncentrosymmetric semiconductors, Fiz. Tverd. Tela 13, 3581 (1971) [Sov. Phys. Solid State 13, 3023 (1972)]; (b) M.I. D’yakonov and V.Yu. Kachorovskii, Spin relaxation of two-dimensional electrons in noncentrosymmetric semiconductors, Fiz. Tekh. Poluprovodn. 20, 178 (1986) [Sov. Phys. Semicond. 20, 110 (1986)].

[18] M.A. Brand, A. Malinowski, O.Z. Karimov, P.A. Marden, R.T. Harley, A.J. Shields, D. Sanvitto, D.A. Ritchie, and M.Y. Simmons, Precession and motional slowing of spin evolution in a high mobility two-dimensional electron gas, Phys. Rev. Lett. 89, 236601 (2002).

[19] C.P. Weber, N. Gedik, J.E. Moore, J. Orenstein, J. Stephens, and D.D. Awschalom, Observation of spin Coulomb drag in a two-dimensional electron gas, Nature 437, 1330 (2005).

[20] S.A. Crooker and D.L. Smith, Imaging spin flows in semiconductors subject to electric, magnetic, and strain fields, Phys. Rev. Lett. 94, 236601 (2005).