Mixture of scalar bosons and two-color fermions in one dimension: Superfluid-insulator transitions

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The superfluid-insulator transitions in a one-dimensional mixture of two-color fermions and scalar bosons are studied in the framework of the Bose-Fermi-Hubbard model. Zero-temperature phase diagrams are built for repulsive intraspecies interactions and attractive or repulsive interspecies ones. In addition to the trivial Mott insulator phases, we report the emergence of new non-trivial insulator phases which depend on the sign of the boson-fermion interaction. These non-trivial insulator phases must satisfy the conditions \( \rho_B + \rho_F = n \) and \( \rho_B + \frac{1}{2} \rho_F = n \), with the plus (minus) sign for repulsive (attractive) interactions and \( n \) an integer. Far away from fermionic half-filling, the boson-fermion interaction drives a gapless-gapped transition in the spin sector. The model studied in the present work can be implemented in state-of-the-art cold-atom setups.

I. INTRODUCTION

The rapid advances in the cold-atom field have allowed the observation of several predicted physical phenomena, and have opened the possibility of experimenting with several dream scenarios. One of the latter corresponds to mixtures of particles that obey the Bose-Einstein or Fermi-Dirac statistics. Since the beginning of this century, experimentalists have mixed carriers with different statistics, using isotopes of different atoms or of the same type of atom. New phenomena, such as phase separation or Bose-Fermi superfluid mixtures, have been observed in clean and fully controllable setups, where the inter and intraspecies interactions can be tuned.

To fully comprehend the properties of such mixtures, those of the independent systems need to be well understood. This is indeed the case for several bosonic and fermionic gases. Namely, phase transitions between Mott insulator and gapless states in locally interacting systems have been widely studied for both statistics. It is well known that bosonic systems exhibit Mott insulator phases at integer densities, while for two-color fermions this phase emerges only at half-filling. When fermions and bosons are mixed a rich scenario is expected, and different levels of theoretical approach have been considered over the years.

The first approach to describe a mixture of bosons and fermions consists of freezing their internal degrees of freedom, a scenario that has been widely studied. Among the diverse phases revealed by these studies, we highlight the insulator phases at integer bosonic densities and the mixed Mott insulator determined by the relation \( \rho_B + \rho_F = 1 \), where \( \rho_B \) and \( \rho_F \) are the bosonic and fermionic densities respectively. An insulator that fulfills this commensurability relation has been verified in experiments.

To enrich the description it is necessary to consider internal degrees of freedom, which are relevant for both bosons and fermions. Inspired by the BCS theory, which is based on the interaction between electrons and phonons, several authors have studied a mixture of two-color fermions and scalar bosons at particular densities, using bosonization, renormalization group, mean-field theory, and dynamical cluster approaches for mixtures in one, two and three dimensions. In those studies diverse ground states were reported such as: superfluid, spin-density wave, charge-density wave (CDW), phase separation, Mott insulator, supersolid, antiferromagnetic order and evidence of various types of pairing, among other phenomena. In a recent paper, we explored numerically the above model in one dimension considering the hard-core limit and only repulsive interactions. Sweeping through a wide range of bosonic and fermionic densities, the superfluid-insulator transitions were studied, obtaining for a fixed fermionic density two non-trivial insulators phases that fulfill the relations \( \rho_B + \rho_F = 1 \) and \( \rho_B + \frac{1}{2} \rho_F = 1 \). Notice that considering the internal degrees of freedom of fermions leads to a new non-trivial insulator, but restricting the Hilbert space of the bosons prevents the emergence of bosonic Mott insulators.

Clearly, mixtures consisting of scalar bosons and two-color fermions hide much more phenomena to be discovered. This motivates the present work, in which we determine the phase diagrams that emerge when allowing more than one boson per site, i.e. when considering the soft-core approach. This has only been analyzed in a very recent report where the authors study the FFLO physics in a spin-imbalanced mixture. Taking into account that in cold-atom setups the amplitude and sign of interspecies interspecies interactions can be tuned, we consider both repulsive and attractive interactions. Exploring the superfluid-insulator transitions in soft-core mixtures, we have found that regardless of the sign of the boson-fermion interaction and for a fixed fermionic density \( \rho_F \), there are always two non-trivial insulator...
FIG. 1. Illustration of schematic ground states of a mixture of scalar bosons and two-color fermions in one-dimension. Here, we consider a lattice with eight ($L = 8$) sites, and draw different possible distribution of particles. Blue (golden) circles represents bosons (fermions). (a) Coexistence of Mott insulators for fermions ($\rho_F = 1$) and bosons ($\rho_B = 2$); here $U_{BF} \leq 0$. (b) Mixed Mott insulator state with $\rho_F = 1/2$ and $\rho_B = 1/2$ for repulsive interparticle coupling. (c) Noncommensurate insulator state with a fermionic density $\rho_F = 1$ and bosonic density $\rho_B = 1/2$, for $U_{BF} \leq 0$. (d) Phase separation state for repulsive interactions, for $\rho_F = 1$ and $\rho_B = 1$.

In the following we describe the model and the main approximations considered in the current work to study a degenerate mixture of bosons and fermions.

A system of scalar bosons in one dimension can be modeled by the Hamiltonian
\begin{equation}
\hat{H}_B = -t_B \sum_{\langle i,j \rangle} (\hat{b}_i^{\dagger} \hat{b}_j + \text{h.c.}) + \frac{U_{BB}}{2} \sum_i \hat{n}_i^B (\hat{n}_i^B - 1),
\end{equation}
which takes into account the kinetic energy (first term) and the local repulsive interaction between bosons (second term). In Hamiltonian $\hat{H}_B$, $\hat{b}_i^{\dagger}$ ($\hat{b}_i$) creates (annihilates) a scalar boson at size $i$. The local boson number operator is $\hat{n}_i^B = \hat{b}_i^{\dagger} \hat{b}_i$. The parameter $U_{BB}$ quantifies the local interaction and $t_B$ is the hopping amplitude between neighboring sites ($(i,j)$).

A system composed of two-color fermions that interact locally is described by the Hamiltonian
\begin{equation}
\hat{H}_F = -t_F \sum_{\langle i,j \rangle \sigma} (\hat{\tilde{f}}_{i,\sigma}^{\dagger} \hat{\tilde{f}}_{j,\sigma} + \text{h.c.}) + \frac{U_{FF}}{2} \sum_{i,\sigma \neq \sigma'} \hat{n}_{i,\sigma} \hat{n}_{i,\sigma'},
\end{equation}
being $\hat{\tilde{f}}_{i,\sigma}^{\dagger}$ ($\hat{\tilde{f}}_{i,\sigma}$) an operator that creates (annihilates) a fermion with internal degree of freedom $\sigma = \uparrow, \downarrow$ at site $i$. The local operator $\hat{n}_{i,\sigma} = \hat{\tilde{f}}_{i,\sigma}^{\dagger} \hat{\tilde{f}}_{i,\sigma}$ corresponds to the density operator for $\sigma$-fermions. The nearest-neighbor fermionic hopping parameter is $t_F$, and $U_{FF}$ quantifies the fermion-fermion interaction. The fermionic density for systems with two-color fermions varies in the interval $[0, 2]$, such that $\rho_F = 1$ corresponds to half-filling.

When two-color fermions and scalar bosons are mixed in a one-dimensional optical lattice and interact with each other, they are described by the Hamiltonian
\begin{equation}
\hat{H}_{BF} = \hat{H}_B + \hat{H}_F + U_{BF} \sum_{i,\sigma} \hat{n}_{i,\sigma} \hat{n}_{i,\sigma'},
\end{equation}
where the boson-fermion interaction $U_{BF}$ can be repulsive or attractive ($U_{BF} \leq 0$). In the following, we measure energies and gaps in units of the fermionic hopping parameter $t_F$ i.e., we set $t_F = 1$.

Importantly, the number of bosons per site is unbounded, making the local Hilbert space exactly intractable. To treat the model numerically, it is necessary to perform a cutoff, i.e., we consider the soft-core approximation and restrict the number of bosons per site to a maximum of $\hat{n}_{\text{max}} = 3$. This results in a large yet treatable Hilbert local space of dimension $d = 16$. Note that it has been argued in several reports that the qualitative physical properties obtained for $\hat{n}_{\text{max}} = 3$ do not change when $\hat{n}_{\text{max}}$ is increased [65, 66].

The ground state energy $E(N_\uparrow, N_\downarrow, N_B)$ for $N_B$ bosons, and $N_\uparrow, N_\downarrow$ fermions of a Bose-Fermi mixture described by Hamiltonian [63] is obtained using the density matrix renormalization group algorithm (DMRG) with open boundary conditions. We perform several finite-system sweeps until the ground-state energy is converged to an absolute error of $10^{-3}$, keeping a discarded weight of $\sim 10^{-7}$ in the dynamical block selection state (DBSS) protocol [67].
In Fig. 1 we sketch some possible distributions of carriers along the lattice, which will emerge depending on the sign of the boson-fermion interaction. For instance the coexistence of fermionic and bosonic Mott insulator states is depicted in (a), insulator states with commensurate or not commensurate total number of carriers are shown in (b) and (c), respectively, and an immiscible phase separation state is sketched in (d). Other carriers distributions can be obtained by varying the densities and interaction parameters, as discussed below.

III. REPULSIVE BOSON-FERMIOM INTERACTION ($U_{BF} > 0$)

In a previous work we showed that a mixture of two-color fermions and scalar bosons in the hard-core limit possesses two different insulator phases located at the bosonic densities $\rho_B = 1 - \rho_F$ and $\rho_B = 1 - \frac{1}{2}\rho_F$, being $\rho_F$ the fixed fermionic density [63]. Motivated by these findings and intrigued to unearth new properties and characteristics of these mixtures, we consider a more general situation, namely the soft-core limit. Fixing the fermionic density to $\rho_F = 1/2$, we increase the number of bosons from zero up to a global density $\rho_B \leq 3$, considering a boson-boson interaction $U_{BB} = 16$ and fermion-fermion repulsion $U_{FF} = 6$ [see Fig. 2 (a)]. For a weak

Finally, we note that the system studied in the present work can be implemented in the laboratory. In particular, several mixtures of bosonic and fermionic atoms in a degenerate regime have been achieved in cold-atom setups, even though their stability is severely limited by 3-body recombinations. A promising candidate to emulate the Hamiltonian [63] is a mixture containing $^{174}$Yb and $^{171}$Yb atoms, given that the later has a nuclear spin $I = 1/2$ whereas for the former the nuclear spin is zero [68].

FIG. 2. Physical properties in the thermodynamic limit of a mixture of scalar bosons and two-color fermions with repulsive boson-fermion coupling. Here the boson-boson and fermion-fermion interactions are $U_{BB} = 16$ and $U_{FF} = 6$, respectively. (a) Bosonic density ($\rho_B$) versus the bosonic chemical potential ($\mu_B$) for a fixed fermionic density of $\rho_F = 1/2$ and two values of boson-fermion coupling. In the inset, we show the evolution of the width of the plateaus as a function of the inverse of the lattice size, indicating that they are finite when $1/L \rightarrow 0$ (extrapolated diamond points). (b) Phase diagram in the bosonic chemical potential ($\mu_B$)- interparticle interaction ($U_{BF}$) plane for a fixed fermionic density $\rho_F = 1/2$. The white areas are superfluid regions, while the colored lobes correspond insulator phases. (c) Fermionic density ($\rho_F$) versus the fermionic chemical potential ($\mu_F$) for a fixed bosonic density of $\rho_B = 1/4$ and two values of boson-fermion coupling. Again, the width of the plateaus is finite when $1/L \rightarrow 0$. (d) Phase diagram in the $\mu_F$ vs $U_{BF}$ plane for a fixed bosonic density of $\rho_B = 1/4$. Again, white (colored) areas represent superfluid (insulator) phases. In all the figures the points correspond to DMRG results and the lines are visual guides. The values in the thermodynamic limit were obtained by using a second-order polynomial extrapolation.
the commensurability condition and satisfy the relation at the bosonic densities namely \( \rho_n = \frac{N_f}{N_B} + \frac{N_B}{N_f} \), given by the relation \( \rho_B + \rho_F = n \), where \( n \) is an integer, namely \( n = 1 \) and 2 for the plateaus at \( \rho_B = 1/2 \) and 3/2, respectively. On the other hand, the non-trivial plateaus at the bosonic densities \( \rho_B = 3/4 \) and 7/4 do not fulfill the commensurability condition and satisfy the relation \( \rho_B + \frac{1}{2} \rho_F = n \) instead, recovering the particular bosonic densities with \( n = 1 \) and 2. The above discussion and calculations for other fermionic densities (not shown) allow us to conclude that a mixture of two-color fermions and scalar bosons can have two insulator states (one of them commensurate) between trivial (integer density) bosonic insulators, generalizing the hard-core results [63].

These results clearly indicate that both the fermionic and bosonic densities, as well as their coupling, determine the existence and properties of the insulating phases. To present a more complete picture, we show a phase diagram of the bosonic chemical potential versus the boson-fermion interaction, keeping constant the fermionic density \( \rho_F = 1/2 \), the boson-boson interaction \( U_{BB} = 16 \) and the fermion-fermion repulsion \( U_{FF} = 6 \) (see Fig. 2 (b)). The colored regions are insulating phases, while the white ones correspond to gapless phases, i.e. superfluid states. The trivial bosonic Mott lobes (green areas) shrink as the repulsive boson-fermion coupling increases, with critical points indicating their suppression at \( U_{BF} \approx 11.7, 11.3 \) for \( \rho_B = 1, 2 \) respectively. In our case, the fermion-fermion interaction makes the Mott insulator lobes disappear more quickly than the prediction for a mixture of scalar bosons and polarized fermions, namely \( U_{BF} \approx 2U_{BB} \) [54]. In the latter study and in our previous hard-core approach [63], it was shown that the non-trivial lobes emerge from a finite value of the boson-fermion repulsion, a scenario that is seen here in the most general case. Contrary to what is observed in the hard-core limit [63], the mixed Mott lobes (cyan areas) appear earlier than the non commensurate lobes due to the lower repulsion between bosons. Specifically, the non-trivial lobes for densities \( \rho_B = 1/2, 3/4, 3/2, \) and 7/4 emerge at the critical points \( U_{BF}^{\ast} \approx 1.9, 2.5, 2.9, 3.9 \), respectively. Notice that the width of the mixed Mott lobes tends to saturate for large values of the boson-fermion interaction, showing that this feature does not depend on the boson repulsion. Furthermore, the evolution of non commensurate lobes (yellow areas) differs from the hard-core result, where the width always increases. Now we see that for \( \rho_B = 3/4 \), the width saturates for larger values of \( U_{BF} \), and vanishes for \( \rho_B = 7/4 \), determining a closed lobe in the phase diagram.

When studying mixtures of bosons and fermions, it is a common practice to fix the fermionic density and vary the number of bosons; however, in experiments both can be controlled. To provide more evidence of the unveiled density conditions, we explore the superfluid-insulator transitions fixing the bosonic density \( \rho_B = 1/4 \) and varying the number of fermions. We define the fermionic chemical potential as \( \mu_F = E(N_f + 1, N_B + 1) - E(N_f, N_B) \), determining a closed lobe in the phase diagram.

FIG. 3. Spin gap \( \Delta_S \) as a function of the boson-fermion interaction \( U_{BF} \) for a fixed \( \rho_F = 1/2 \). For all the considered bosonic densities a gapless and a gapped regions are clearly seen. The lines are visual guides.
lattice size is shown in the inset of Fig. 2 (c). Using a second-order polynomial extrapolation, we obtained that the charge gap is always finite, therefore these insulating phases survive in the thermodynamic limit.

In Fig. 2 (d), we show the corresponding phase diagram in terms of the fermionic chemical potential versus the boson-fermion repulsion for a fixed bosonic density $\rho_B = 1/4$, and boson-boson (fermion-fermion) interaction $U_{BB} = 16$ ($U_{FF} = 6$). As in Fig. 2 (b) the white regions are superfluid, whereas the colored ones correspond to insulator phases. In the absence of boson-fermion interaction only the trivial Mott insulator phase (green area) emerges, which shrinks as the interaction between fermions and bosons increases, disappearing at $U_{BF} \approx 7.8$. Also, as the boson-fermion coupling increases from zero, the non-trivial commensurate lobes (cyan areas) emerge at the critical points $U_{BF} \approx 1.4$ and 0.7 for $\rho_F = 3/4$ and 7/4, respectively. However, the evolution of these lobes is different; whereas the charge gap for $\rho_F = 3/4$ saturates for larger values of $U_{BF}$ the charge gap for $\rho_F = 7/4$ varies, determined by the increase in chemical potentials for adding or decreasing the number of fermions. The non-commensurate insulator lobe (yellow area) arises from $U_{BF}^* \approx 2.1$, and grows monotonously. We expect that these critical points take different values as the densities and the other interaction parameters vary.

The fermionic particles of our mixture have an internal degree of freedom; therefore a natural question is whether gapped excitations related to it take place. To explore this issue, we calculate the spin gap $\Delta_S = E(N_{\uparrow} + 1, N_{\downarrow} - 1, N_B) - E(N_{\uparrow}, N_{\downarrow}, N_B)$ at each insulator phase. In Fig. 3 we show the spin gap in the thermodynamic limit as a function of the boson-fermion repulsion for a system with fixed fermionic density $\rho_F = 1/2$, $U_{FF} = 6$ and $U_{BB} = 16$. Note that for this fermionic density, repulsive coupling between fermions and without boson-fermion interaction, we expected a metallic ground state with dominant spin density fluctuations, i.e. both charge and spin gaps vanish. Turning on the boson-fermion repulsion and for all the insulating regions, we obtain spin gapless states for a range of values of $U_{BF}$; however a finite spin gap arises from a critical value, which depends on the bosonic density. We highlight that similar results were obtained for attractive boson-fermion coupling and that this unexpected result, which suggest a quantum phase transition in the spin sector corresponds to an unveiled phenomenon that has not been discussed before in Bose-Fermi mixtures.

IV. ATTRACTIVE BOSON-FERMION INTERACTION ($U_{BF} < 0$)

Attractive interactions between bosons and fermions have been considered by several authors, and interesting effects have been predicted and observed \cite{46, 53}. Now, we wish to establish wheter the conditions for the emergence of the insulator phases of the mixture change with the nature of the boson-fermion interaction. For this, we maintain the same values of the boson-boson and fermion-fermion couplings considered in Fig. 2 and explore the superfluid-insulator transitions with $U_{BF} < 0$; our results are shown in Fig. 4. First, for a constant global density of fermions $\rho_F = 1/2$, we increase the number of bosons from zero. The corresponding chemical potential is shown in Fig. 4 (a), where attractive boson-fermion interactions of $U_{BF} = -1$ and $U_{BF} = -8$ were considered. Figures 2 (a) and 4 (a) have the same parameters except that the former is for the repulsive case and the latter for the attractive one; this allows us to clearly see the influence of the nature of the boson-fermion interaction. Again, for weak boson-fermion couplings only
trivial plateaus at integer densities appear, and their widths are independent of the sign of the boson-fermion coupling. The most interesting situation takes place for larger strengths with the emergence of four more plateaus in the bosonic density versus chemical potential curve, as we illustrate for $U_{BF} = -8$ (see Fig. 4(a)). This figure confirms that between trivial plateaus two insulating states arise regardless of the sign of the boson-fermion interaction, being this fact a main conclusion of the present study. For two of the new non-trivial plateaus, namely those at bosonic densities $\rho_B = 1/4$ and $\rho_B = 5/4$, the total number of particles is not commensurate with the lattice size. Since the fermionic density is $\rho_F = 1/2$, these new insulator states fulfill the relation $\rho_B - \frac{1}{2}\rho_F = n$, where the integer $n$ take the values 0 and 1 for the plateaus at $\rho_B = 1/4$ and $\rho_B = 5/4$, respectively. For the other two plateaus, taking place at bosonic densities $\rho_B = 1/2$ and $3/2$, the total number of carriers is commensurate with the lattice size, and the condition $\rho_B - \rho_F = n$ (n integer) is satisfied. For $\rho_F = 1/2$, we recover the remaining plateaus at $\rho_B = 1/2$ and $3/2$, with $n = 0$ and 1, respectively. However, we note that for other fermionic densities, the plateaus that satisfy the latter condition are such that the total number of carriers is incommensurate with the lattice size. For example, we observed that for $\rho_F = 1/3$, the four non-trivial insulating plateaus emerge at the bosonic densities $\rho_B = 1/6, 1/3, 7/6, 4/3$, none of which satisfy the commensurability relation with the lattice size. Therefore we are in front of a new scenario where there is no mixed Mott state.

To illustrate the general behavior of the insulating phases for attractive boson-fermion couplings, a phase diagram in the $\mu_B$ vs $U_{BF}$ plane is shown in Fig. 4(b), keeping constant a fermionic density of $\rho_F = 1/2$, and a boson-boson (fermion-fermion) interaction of $U_{BB} = 16$ ($U_{FF} = 6$). This phase diagram was obtained by replicating Fig. 4(a) for several negative values of $U_{BF}$. The white areas correspond to superfluid regions, which surround the insulator (colored) ones. As in the repulsive case the trivial lobes shrink and vanish at $U_{BF} \approx -11.7$ and $-12.6$ for $\rho_B = 1$ and 2, respectively. A finite value of the boson-fermion coupling is required for non-trivial lobes to arise, determining the critical point located at $U_{BF}^{*} \approx -3.5, -1.7, -3.4, -3.0$ for the bosonic densities $\rho_B = 1/4, 1/2, 5/4, 3/2$, respectively.

In Fig. 4(c), we display the evolution of the fermionic chemical potential as the number of fermions increases, for a mixture with a bosonic density of $\rho_B = 1/4$ and an attractive boson-fermion interaction. This figure corresponds to the attractive version of Fig. 2(c) and, like before, only the antiferromagnetic Mott insulator emerges for small strengths. However, as the boson-fermion coupling increases the width of this trivial plateau decreases and vanishes, while other non-trivial ones arise at the fermionic densities $\rho_F = 1/4, 1/2$, and $5/4$ (the forth one for $n = -1$ being unphysical, with $\rho_F = 5/2 > 2$). The above fact reinforces our result that the attractive boson-fermion interaction generates different insulator regions to the repulsive one. Note that the positions of these non-trivial insulator regions fulfill the relations discussed before.

V. HALF FILLING

A case that deserves especial attention is the half-filling one, because it is well-known that this configuration leads to interesting physical phenomena in fermionic systems. At fermionic half-filling ($\rho_F = 1$) without bosons only one insulator phase is expected, which corresponds to the well-known Mott insulator state, where each site is occupied by one fermion and antiferromagnetic order is established along the lattice. Adding bosons to the system, but without coupling them to the fermions, we trivially expect a superfluid to Mott insulator transition under a fermionic Mott background, which takes place when the bosonic density reaches integer values; a coexistence of fermionic and bosonic Mott insulators is thus established, as seen in Fig. 5 (a) for $U_{BF} = 0$. Turning on the boson-fermion coupling, we observe that the trivial boson plateaus shrink as the interparticle interaction grows, and both will disappear at some large $U_{BF}$. Therefore, the Mott insulator for bosons and fermions coexist in the system for finite values of the interparticle coupling. Regardless of the sign of the boson-fermion interaction, only one non-trivial plateau emerges between the trivial bosonic plateaus, namely at densities $\rho_B = 1/2$ and $3/2$, which agrees with the relations found above for the repulsive and attractive cases.

Finally, we discuss the spatial distribution of particles across the lattice for different states. A homogeneous profile of carriers is obtained in the nontrivial plateaus for weak values of the boson-fermion interaction, which is characterized by one fermion per site, and one or three bosons extended across two sites at the densities $\rho_B = 1/2$ and $3/2$, respectively. For larger values of $U_{BF}$, interwoven CDW orderings for bosons and fermions emerge, with some small peculiarities depending on the repulsive or attractive character of the interaction.

A different scenario emerges for weak interparticle interaction and $\rho_F = \rho_B = 1$, where the Mott insulator for bosons and fermions coexist, and in average there is one boson and one fermion per site. However this picture can change depending on the magnitude and sign of $U_{BF}$. For larger and positive values, fermions and bosons occupy different domains along the lattice, establishing a phase separation state as depicted in Fig. 6(d), and clearly seen in the density profiles show in Fig. 6(b) for $U_{BF} = 20$. Note that the bosons and fermions miscibility, which emerged in our previous results is a very interesting question that arises naturally, and which has become a subject of great interest in mixtures [10, 36, 42]. For weak interparticle coupling, the ground-state is a coexistence of Mott insulators for bosons and fermions. Therefore a quantum phase transition between insulating states of
different nature takes place. A similar effect occurs for attractive interparticle interactions; however, in this case and for large magnitudes of $|U_{BF}|$, bosons and fermions share the same domains (see Fig. 5(c)), leaving regions of the lattice without particles. In other words, the ground state is a miscible phase separation characterized by domains with or without carriers. It is important to highlight that recently a phase separation state was observed in a mixture of $^{41}$K and $^6$Li atoms [29] using an interspecies Feshbach resonance for controlling the repulsive interaction. They clearly seen that bosons and fermions occupied different domains in the lattice as we show in Fig. 5(b).

VI. CONCLUSIONS

We have studied the ground state of a one-dimensional mixture of scalar bosons and two-color fermions in the soft-core regime, using the density matrix renormalization group technique. Relaxing the hard-core restriction, the local Hilbert space can be huge, and we decided to consider up to three bosons per site, making this approximation numerically treatable leading us to unveil new phenomena. For repulsive intraspecies interactions, we swept through a wide range of bosonic and fermionic densities, for positive and negative interspecies couplings, and obtained rich zero temperature phase diagrams.

Choosing repulsive boson-fermion interactions and fixing the fermionic density $\rho_F$, two non-trivial plateaus arise between the trivial Mott insulators as the number of bosons increase from zero, which satisfy the relations $\rho_B + \rho_F = n$ and $\rho_B + \frac{1}{2}\rho_F = n$, being $n$ an integer. As the boson-fermion coupling increases the non-trivial insulator phases emerge; for stronger couplings the Mott insulator phases disappear, establishing critical points for each phase. This generalizes the previous results for polarized atoms and mixture of two-color fermion and scalar bosons in the hard-core limit [54, 58]. To reinforce our conclusions, we also fixed the bosonic density and varied the fermionic density. Here we found the trivial antiferromagnetic Mott insulator and three non-trivial insulators that fulfilled the relations given before.

For attractive boson-fermion interactions we observed insulator phases for integer and fractional bosonic densities, where the latter can be commensurate or not with the lattice size; this establishes a fundamental difference to the repulsive case. Therefore, the relations that determine the non-trivial insulator states for attractive interspecies interactions differ to those reported before. Namely, the corresponding relations are give by $\rho_B - \rho_F = n$ and $\rho_B - \frac{1}{2}\rho_F = n$, being $n$ an integer. This constitutes one of the main results reported in our paper. We also showed that for a finite interparticle coupling, the trivial Mott insulator states for bosons and fermions coexist in the system.

The energy cost to generate a spin flip in the system was calculated to each insulator phase, finding that the spin gap is zero for a range of values of the boson-fermion coupling and there is a different critical point for each insulator phase, which suggest a diverse magnetic behavior of the system.

Considering that the mixed Mott state (commensurate), phase separation among other interesting phenomena have been observed in experiments with bosonic and fermionic isotopes in cold-atoms setups, we expect that our results will stimulate experimentalists to implement the insulator states reported in our work.

![Fig. 5](image-url)
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