Description of He isotopes using the particle-particle random-phase approximation model

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The two-neutron RPA model has been used to describe helium isotopes with N (the neutron number) = 4, 6, 7, 8 in their ground and excited states. The properties of all these isotopes are given by a single system of equations so that their properties are interdependent. Since the ground states of $^9$He and $^{10}$He are still not well established, we have looked how the properties of $^9$He induce the properties of the other isotopes, energies and wave functions. Our results suggest an inversion of 2s–1p$_{1/2}$ shells in $^9$He. The corresponding ground states of $^8$He and $^{10}$He are slightly unbound and respectively 1/2$^+$ and 0$^+$ states while the 1/2$^-$ and 0$^+$ seen in experiments appear to be excited states. With this assumption on $^9$He, we get not only a nice picture of $^{10}$He but also a very good two-neutron separation energy in $^{8}$He.

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I. INTRODUCTION

Despite their small number of nucleons, the light nuclei of the nuclide chart provide a unique opportunity to investigate complex phenomena such as halo nuclei, cluster configurations, shell inversions, and give access to nuclear systems beyond the neutron dripline. This region thus challenges the theory in its capacity to reproduce the wide variety of these phenomena. In that sense, the unbound helium isotopes, $^3$He and $^{10}$He have been extensively studied both experimentally and theoretically. The first experiments on $^9$He showed a 1/2$^-$ state in the range of 1-2 MeV interpreted as its ground state and theoretically. This property of a 1/2$^-$ ground state was also the result of most of the calculations. A latter double proton knockout experiment shows the existence of a lower state but is still disputed in recent experiments. There is also an ambiguity concerning the 0$^+$ resonance seen experimentally in $^{10}$He and interpreted usually as the ground state while a suggestion that it is an excited state has been proposed by S. Aoyama but has not been confirmed by other calculations.

Such a situation of a 1/2$^+$ ground state with a 1/2$^-$ excited state reveals an inversion of the 2s and 1p$_{1/2}$ shells when $^9$He is described as a neutron added to a core of $^8$He. Such an inversion is well known in $^{11}$Be, $^{10}$Li, and $^{13}$Be. In $^{10}$Li and $^{13}$Be this inversion was first suggested theoretically in order to reproduce the two-neutron separation energy in $^{11}$Li and $^{13}$Be respectively, and then experimentally confirmed. These two-neutron separation energies were calculated in a two-neutron RPA model where $^{11}$Li and $^{14}$Be were considered as two neutrons added to an inert core of $^9$Li and $^{12}$Be respectively. In the present paper, the same two-neutron RPA model depicting $^{10}$He as two neutrons added to an inert core of $^8$He is used. This model gives one system of equations, describing both $^{10}$He and $^8$He; the latter being described as two-neutrons subtracted from a $^6$He core. Finally these calculations provide, among other properties, the two-neutron separation energies ($S_{2n}$) of $^8$He and $^{10}$He, the energies and wave functions of the 0$^+$ states in $^6$He and $^{10}$He as well as informations on the composition of the $^8$He core wave function.

In section II the two-neutron RPA model and the inputs used in the calculation are briefly presented. The results are given in section III and the conclusions are compiled in the final section.

II. THE MODEL

Only a brief summary of the used two-neutron RPA model is given. More details can be found in reference [29]. A core of $^8$He in its 0$^+$ ground state is assumed which, in the Hartree-Fock model, is a closed shell nucleus with the last neutrons filling the 1p$_{3/2}$ shell. Two neutrons are then added to, or subtracted from this core to describe $^{10}$He and $^6$He as eigenstates of a single system of equations. Eigenvalues and eigenvectors of the RPA matrix yield the energies of $^6$He and $^{10}$He referred to the core energy and the two-body amplitudes in $^6$He and $^{10}$He with the following definition for a solution of rank n:

$$X^{(n)}(a) = \langle \psi_n(^{10}He) | A_n^+ | \psi_0(^8He) \rangle,$$  \hspace{1cm} (1)

$$Y^{(n)}(a) = \langle \psi_n(^6He) | A_n | \psi_0(^8He) \rangle,$$  \hspace{1cm} (2)

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with the orthonormalization relations:

\[
\sum_{\text{unoccupied}} |X(a)|^2 - \sum_{\text{occupied}} |X(a)|^2 = 1, \\
\sum_{\text{occupied}} |Y(a)|^2 - \sum_{\text{unoccupied}} |Y(a)|^2 = 1,
\]

(3)

where \(A^+_\alpha\) and \(A_\alpha\) are respectively the creation and annihilation operators of two neutrons in state \(a\) formed of two occupied or unoccupied states in the Hartree-Fock core. \(\psi_0(\text{He})\) is the correlated wave function of the \(\text{He}\) core. Then, for example, a non zero amplitude \(X\) for \(a\), representing two occupied states, means that in the core wave function there is a configuration with two holes in that state. Similarly, if \(Y\) is non zero for a state \(a\) formed of two unoccupied states, then the \(\text{He}\) wave function contains a configuration with two particles in that state \(a\). The energies of the \(\text{He}\) and \(\text{He}\) ground states give directly the two-neutron separation energies in \(\text{He}\) and \(\text{He}\) respectively.

To solve the RPA equations, we need the single neutron energies in the field of the core and the neutron-neutron interaction which we take as the Gogny DIS effective interaction \(30, 31\). For the neutron states in the field of \(\text{He}\), we take a Saxon-Woods potential with a surface term which simulates the contribution to the one body potential of neutron-phonon couplings \(24, 32\). It is written as:

\[
V_{nc}(r) = -V_{NZ} \left( f(r) - 0.44r_0^2 l_0 \frac{1}{r} \frac{df(r)}{dr} \right)
+ 16a^2 \alpha_n \left( \frac{1}{r} \frac{df(r)}{dr} \right)^2,
\]

with \(f(r) = \left( 1 + \exp \left[ \frac{r - R_0}{r_0} \right] \right)^{-1}.\) (4)

The following parameters suggested by Bohr and Mottelson \(33\), are used:

\[
V_{NZ} = \left( 51 - 33N-Z \right) MeV, \quad (6)
R_0 = r_0A_\alpha^{1/3}, \quad r_0 = 1.27 \text{ fm}, \quad (7)
\]

where \(N, Z, A_\alpha\) refer to the core nucleus. The diffusivity is set to a larger value, \(a = 0.75 \text{ fm}\), to account for the diffuse surface of such a light nucleus. It has been shown that neutron-phonon couplings are responsible of the inversion of \(\text{He}\) and \(2s\) shells in \(11\text{Be}\) \(32, 33\). Most of the contribution of such couplings comes from the low energy \(2^+\) excited state in the core of \(10\text{Be}\). This inversion is also present in \(10\text{Li}\) and \(11\text{Be}\) where the cores of \(9\text{Li}\) and \(12\text{Be}\) have also low energy \(2^+\) states with large B(E2). Such couplings are also responsible for an inversion of \(2s\) and \(1d_{5/2}\) shells in the light carbon isotopes and also due to \(2^+\) phonons \(34\). Since \(8\text{He}\) has a \(2^+\) state at \(3.54 \text{ MeV}\) \(34\), we expect that neutron-\(2^+\) phonon coupling will induce large modification of the neutron energies close to the Fermi surface and could be responsible

for an inversion of \(1/2^-\) and \(1/2^+\) states in \(9\text{He}\). In our calculation we take the strength \(\alpha_n\) of the surface term as a parameter fitted on known or assumed energy for states close to the Fermi surface as \(1\text{p}_{3/2}, 1\text{p}_{1/2}\) and \(2s\) states. For higher states, the coupling effects are small and we take \(\alpha_n = 0\). For \(1\text{p}_{3/2}, \alpha_n\) is fitted to give \(\epsilon(1\text{p}_{3/2}) = -S_{1n}(\text{He}) = -2.57 \text{ MeV}\). To solve the one-body equation we use a discretisation of unbound states with a box of 16 fm. This discretisation of continuum states has been tested in \(9\text{He}\) \(34\) and shown to bring an under-estimation of pairing energy of few percent only.

Working with discrete states means that we get the position of the resonances but do not get any width.

III. RESULTS

All experiments agree on the presence in the \(9\text{He}\) spectrum of a \(1/2^-\) state in the vicinity of \(1.27 \text{ MeV}\) energy range. This state was, up to L. Chen and collaborators’ experiment \(3\), considered as the ground state of \(9\text{He}\). This was attested by several shell model calculations. However L. Chen et al. experiment and most of the recent ones agree that the \(1/2^-\) state is an excited state and that the ground state is an unbound virtual \(1/2^+\) state, with an energy very close to the \(n^+\text{He}\) threshold. On the other end, there is an ambiguity in the interpretation of experiments on \(10\text{He}\). Indeed until recently the resonance found at about 1 MeV above the \(2n^+\text{He}\) threshold was considered to be the ground state of \(10\text{He}\). However there is a recent suggestion by \(19\) that this resonance corresponds to a \(0^+\) excited state and that the ground state is close to the threshold. The use of the two-neutron RPA model assuming a core of \(8\text{He}\) gives the opportunity to make a simultaneous study of \(9\text{He}, 9\text{He}, 9\text{He}\) and \(10\text{He}\) where the choice of parameters defining \(9\text{He}\) states determines the states of the other three nuclei without any further fitting possibility. The calculations proceed as follows: the \(1\text{p}_{1/2}\) energy is set to the experimental energy of \(-2.57 \text{ MeV}\). We put the \(1\text{p}_{1/2}\) state close to the measured energy of the \(11/2^-\) state seen in all experiments and vary the energy of the \(2s\) state. The resolution of the RPA equations gives us the corresponding properties of the other nucleus. The results are presented in table 4 for \(\epsilon(1\text{p}_{1/2}) = 1.25 \text{ MeV}\), which is the value given in most of the experiments’ analysis and in table 11 for \(\epsilon(1\text{p}_{1/2}) = 0.92 \text{ MeV}\), which is not excluded by experiments. In these tables are given the results for \(S_{2n}(\text{He}), S_{2n}(\text{He})\) and \(E(0^+).\) The latter corresponds to the energy of the first excited \(0^+\) state in \(10\text{He}\) relative to the \(n + n^+\text{He}\) threshold. For \(\epsilon(1\text{p}_{1/2}) = 1.25 \text{ MeV}\) and \(\epsilon(2s) \approx 0.3 - 0.4 \text{ MeV},\) \(10\text{He}\) is unbound with a small value of \(S_{2n}\) in agreement with the assumption that the resonance at about 1 MeV is not the ground state but a \(0^+\) excited state. Yet the energy of this excited state is too high. Furthermore \(S_{2n}(\text{He})\) is slightly too large compared to the experimental measured value of \(2.14 \text{ MeV}\) \(35\). If we assume the \(2s\) shell to
be higher than the $1p_{1/2}$ one, for any value of $\epsilon(p_{1/2})$, the results are in complete disagreement with measurements.

On the other hand, in table for $\epsilon(p_{1/2}) = 0.92$ MeV a general agreement is observed for all calculated quantities though the most accurate are obtained for $\epsilon(2s) \approx 0.3$ MeV. This value of 0.92 MeV is slightly smaller than the values derived from experiments. However if one looks, for example, at the figure 1 in ref. 39, a $p_{1/2}$ resonance at about 0.9 MeV would give the same or even better agreement than 1.27 MeV. With these energies of 2$s$ and 1$p_{1/2}$ states, the two-neutron separation energy in $^8$He is very close to the measured value of 2.14 MeV while the energy of the excited $0^+$ state in $^{10}$He is close to the energy of 1.2 MeV deduced from experiments. The advantage of this model is that, once the single neutron energies are fixed, the properties of all nuclei are given, without any possibility to introduce further parameters to modify the results for one or the other of the nuclei.

The RPA equations also yield the wave functions of $^6$He and $^{10}$He through the two-neutron amplitudes defined in Eqs. 13. In the case where $\epsilon(p_{1/2}) = 0.92$ MeV, $\epsilon(2s) = 0.29$ MeV, the following RPA amplitudes of Eqs. 13 for the $^{10}$He ground state are obtained as:

$$X(2s^2) = 0.98,$$
$$X(p_{1/2}^2) = -0.28,$$
$$X(p_{3/2}^2) = -0.28.$$

The main contribution comes from the $(2s)^2$ configuration for the two last neutrons but the amplitude for the two neutrons added to the $(1p_{3/2})$ shell is quite large and means that in the core of $^8$He there is a non negligible contribution of a 2 holes-2 particles configuration with the holes in the $p_{3/2}$-shell.

For the $^6$He ground state, the obtained amplitudes are:

$$Y(p_{3/2}^2) = 1.08,$$
$$Y(p_{1/2}^2) = 0.21,$$
$$Y(2s^{-2}) = -0.20.$$

These values of the Y amplitudes indicate again a contribution of a 2 holes-2 particles configuration in the $^6$He core with the two particles in the $1p_{1/2}$ or $2s$ shells. Then from these two results it can be understood that $^8$He in its ground state is not a pure closed shell nucleus but has a contribution of $(1p_{3/2})^{-2} - (1p_{1/2})^2$ or $(1p_{3/2})^{-2} - (2s)^2$ configurations. The $0^+$ excited state of $^{10}$He appears to have mainly a $(1p_{1/2})^2$ configuration with some mixture of $(2s)^2$, with the respective RPA amplitudes of 0.96 and 0.25.

### IV. CONCLUSIONS

The difference between our work and previous theoretical works relies on two facts. First, we use a two-neutron RPA model which gives simultaneously properties of the $^6$He, $^5$He, $^9$He and $^{10}$He isotopes while usually calculations concern one of these nuclei only. Second, the only parameters of the model are the neutron energies for 2$s$ and 1$p_{1/2}$ states in $^{10}$He considered as a $^8$He + n system. The energy of the $1p_{1/2}$ neutron state is varied in the vicinity of the measured energy of the $1/2^+$ state seen in experiments. For the 2$s$ state we vary its energy starting from very low positive energy. Then for each couple we look at the results of the RPA system of equations which gives interrelated informations on the other nuclei. We then look for the best overall results, namely which is the best case for which all known quantities are well reproduced, then see what are the implications for other properties. The results can be summarized as follows:

- in $^9$He the ground state should be an unbound $1/2^+$ state very close to the $n+^8$He threshold with an energy of 0.2-0.3 MeV, while the $1/2^-$ found experimentally close to 1-1.2MeV is an excited state. Therefore, we see in $^9$He the same inversion of $2s$-$1p_{3/2}$ shells as known in $^{11}$Be, $^{10}$Li and $^{13}$Be. We guess from our previous works that this inversion is due to the coupling of the neutron with the $2^+$ phonon known at 3.59 MeV in $^8$He.

- the $^{10}$He has an unbound ground state very close to the $n+n+^8$He threshold and the resonance seen in experiments at 1.2 MeV and identified as the ground state in many theoretical or experimental works, is the first $0^+$ excited state.

Therefore our results give support to the suggestions of L. Chen for $^9$He and S. Aoyama for $^{10}$He which are still considered as doubtful in recent theoretical as well as experimental studies.

Furthermore the RPA amplitudes obtained for $^6$He and $^{10}$He show that the core of $^8$He is not a pure closed $(1p_{3/2})$ shell nucleus but has a qualitatively important mixture of $(1p_{3/2})^{-2}(1p_{1/2})^2$ and $(1p_{3/2})^{-2}(2s)^2$ configurations. The wave functions of $^6$He and $^{10}$He show also some mixtures of various configurations with still a strong component on two neutron holes in the $p_{3/2}$-shell for $^6$He and

| TABLE I. Results in terms of $\epsilon(2s)$ when $\epsilon(p_{1/2}) = 1.25$ MeV |
|-----------------|---------|---------|---------|---------|
| $\epsilon(2s)$ | 0.78    | 0.45    | 0.29    | 0.19    |
| $S_{2n}({}^9He)$ | 2.52    | 2.48    | 2.48    | 2.43    |
| $S_{2n}({}^{10}He)$ | -1.33   | -0.53   | -0.11   | 0.12    |
| $E(0_{2}^{+})$ | 2.3     | 2.3     | 2.3     | 2.3     |

| TABLE II. Results in terms of $\epsilon(2s)$ when $\epsilon(p_{1/2}) = 0.92$ MeV |
|-----------------|---------|---------|---------|---------|
| $\epsilon(2s)$ | 0.785   | 0.45    | 0.29    | 0.19    |
| $S_{2n}({}^9He)$ | 2.36    | 2.31    | 2.29    | 2.24    |
| $S_{2n}({}^{10}He)$ | -1.11   | -0.44   | -0.07   | 0.22    |
| $E(0_{2}^{+})$ | 1.6     | 1.4     | 1.4     | 1.4     |
on two neutrons in the 2s-shell and $p_{1/2}$-shell for respect-
tively the ground state and the $0^+_2$ excited state in $^{10}$He.

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