Dynamical dark energy with a constant vacuum energy density

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Abstract

We present a holographic dark-energy model in which the Newton constant $G_N$ scales in such a way as to render the vacuum energy density a true constant. Nevertheless, the model acts as a dynamical dark-energy model since the scaling of $G_N$ goes at the expense of deviation of concentration of dark-matter particles from its canonical form and/or of promotion of their mass to a time-dependent quantity, thereby making the effective equation of state (EOS) variable and different from $-1$ at the present epoch. Thus the model has a potential to naturally underpin Dirac’s suggestion for explaining the large-number hypothesis, which demands a dynamical $G_N$ along with the creation of matter in the universe. We show that with the aid of observational bounds on the variation of the gravitational coupling, the effective-field theory IR cutoff can be strongly restricted, being always closer to the future event horizon than to the Hubble distance. As for the observational side, the effective EOS restricted by observation can be made arbitrary close to $-1$, and therefore the present model can be considered as a “minimal” dynamical dark-energy scenario. In addition, for nonzero but small curvature ($|\Omega_k| \lesssim 0.003$), the model easily accommodates a transition across the phantom line for redshifts $z \lesssim 0.2$, as mildly favored by the data. A thermodynamic aspect of the scenario is also discussed.

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A symmetry principle of gravitational holography serves as a window to a complete theory of quantum gravity. According to that principle, the description of a physical system shows equivalence between a theory having the gravitational field quantized and a theory defined on the boundary encompassing a system whose dimension is lower by one. The most rigorous realization of holography is the AdS/CFT correspondence in all events. An important consequence of the holographic principle is that various entropy bounds should be manifest in quantum gravity in the semiclassical limit. All of them state that complete information stored in a physical system scales only with the area encompassing the system.

Motivated by a drastic reduction in effective degrees of freedom as predicted by gravitational holography, Cohen et al. showed that the application of the Bekenstein bound to the maximal possible entropy to effective field theories can even substantially improve the ‘old’ cosmological constant (CC) problem. By adapting the Bekenstein bound they showed that, if a certain relationship between the IR and the UV cutoff was obeyed, the information from quantum gravity could be consistently encoded in ordinary quantum field theory. Such a relationship prevents the formation of black holes within the effective field-theoretical treatment, leading to the bound which is far more restrictive than that in. With the notion that the size of the region (providing an IR cutoff) is varying in an expanding universe and that therefore the vacuum energy density \( \rho_\Lambda \) promotes to a dynamical quantity, a few years later their considerations became a core of a dynamical CC scenario generically dubbed ‘holographic dark energy’. Derived originally for zero-point energies, the bound predicted by Cohen et al. for \( \rho_\Lambda \) can be rewritten in the form

\[
\rho_\Lambda(\mu) = \kappa \mu^2 G_N^{-1}(\mu),
\]

where \( \mu \) denotes the IR cutoff and \( \kappa \) represents a degree of saturation of the bound. Note that \( \kappa \) should be of order of unity if \( \rho_\Lambda \) is to account for the present energy density in the universe. Since \( \rho_\Lambda \) is always dominated by UV modes, Eq. (1) is valid irrespective of whether the hierarchy between the UV cutoff and particle masses is normal or inverted.

With the appropriate choice for the IR cutoff, the setup described by (1) was intended to explain the present acceleration of the universe and possibly to shed some light on the still unresolved problem of ‘cosmic coincidence’. Two different sorts of generalization of the above setup can be found in the literature: the one also promotes the Newton constant \( G_N \) to a dynamical quantity [as already done in (1)], whereas the other relies
on some peculiar choices for $\mu$. {1}

The successfulness of the ‘holographic dark-energy’ scenario depends crucially on the choice for the IR cutoff and on the question whether $\rho_\Lambda$ describes perfect fluid or not. The generalization with the running $G_N$ but canonical matter dilution ($\sim a^{-3}$) has the advantage that the IR cutoff is univocally fixed by the continuity relation once the scaling law for $\rho_\Lambda$ (or $G_N$) is known \[8, 10\].

For perfect fluids and for ad hoc chosen cutoffs, such as the Hubble distance or the particle horizon distance, one usually cannot obtain the equation of state $w$ (EOS) characterizing accelerated universes \[11\]. For interacting fluids, models with the Hubble distance show considerable improvement \[12\], and for a variable degree of saturation of the bound predicted by Cohen \textit{et al.} one can even obtain a transition from a decelerated to an accelerated era \[12\]. Another ad hoc chosen cutoff in the form of the future event horizon seems to work much better for both perfect (see second reference in \[11\]) and interacting fluids \[13\]. Still, most models with ad hoc chosen cutoffs suffer from the ‘cosmic coincidence problem’ or have an EOS too far from $-1$ to comply with the data. For related works, see \[14\].

In the present paper we study the implications of a holographic dark-energy model in which $G_N \sim \mu^2$ in Eq. (1) so as to make $\rho_\Lambda$ a true constant. The motivation for such a study is threefold. Firstly, the above model naturally accounts for recent data which converge rapidly towards an EOS $w = -1$, at the same time retaining its dynamic character. The dynamics of the model stems from the fact that the variation of $G_N$ goes at the expense of deviation of energy density of the cold dark matter (CDM) component $\rho_m$ from its canonical form $\sim a^{-3}$. Here three possibilities emerge \[15, 16\]: (i) the total number of CDM particles in a comoving volume changes while retaining its proper mass constant, or (ii) the proper mass promotes to a time-dependent quantity while retaining the total number of CDM particles constant, or (iii) both the total number of CDM particles and its proper mass change. Instead of dealing with ad hoc chosen IR cutoffs, we find that in our model the IR cutoff is univocally fixed by an amount of deviation of $\rho_m$ from its canonical shape. Secondly, our model can be considered a minimal model which can account for Dirac’s large-number hypothesis \[17\]. Namely, Dirac himself suggested a model with a time-varying gravitational

\[1\] The scenario \[9\] was primarily intended to unify the early-time inflation with the late-time acceleration of the universe.
constant $G_N$ supplemented with creation of matter in the universe. Thirdly, our model also accommodates an exciting possibility of having a transition from $w > -1$ to $w < -1$ at redshifts $z \lesssim 0.2$, of which there are already indications in recent data [18]. Finally, we explore thermodynamic consequences of the model, and find that the generalized second law (GSL) of gravitational thermodynamics demands creation of matter in the universe (as opposed to destruction), in accord with Dirac’s suggestion.

Our starting point for implementing a dynamical $G_N$ and $\rho_\Lambda$ in a cosmological model is the low-energy effective vacuum action which is just the Hilbert-Einstein action with a time-dependent CC and gravitational constants. Especially relevant are found those models in which the CC- and $G_N$-variation laws were inferred from some underlying physical theory, such as perturbative particle physics theory with the Hilbert-Einstein action treated semiclassically [19, 20], or the quantum-gravity approach where nonperturbative solutions were obtained within the “Hilbert-Einstein truncation” [21], or gravitational holography [8, 11]. A particularly interesting model which appeared recently in [22] also discussed Dirac’s cosmology and the large-number hypothesis, but in a slightly modified Hilbert-Einstein action containing an arbitrary function of the ratio of the CC over the 4-dimensional scalar curvature. Let us stress that at any rate the most popular modeling for the time-dependent gravitational coupling is through a time-varying scalar, especially the framework of the Brans-Dicke theory [23]. In the present paper we stick with the Hilbert-Einstein action and the $G_N$-variation law obtained from gravitation holography as given by (1), without introducing any geometrical- or quintessence-like scalar fields.

The generalized equation of continuity in the framework consisting of constant $\rho_\Lambda$ but variable $G_N$ [i.e. where $G_N$ scales as $\mu^2$ in Eq. (1)] is given by

$$\dot{G}_N(\rho_\Lambda + \rho_m) + G_N(\dot{\rho}_m + 3H\rho_m) = 0.$$  \hspace{1cm} (2)

Eq. (2) understands that $G_N T^\mu_\nu_{\text{total}}$ and $T^\mu_\nu_{\Lambda}$ are conserved separately \[^2\]. We find the scaling $G_N(a)$ from Eq. (2), and the function $a(t)$ from the Friedman equation for flat space

$$H^2 = \frac{8\pi G_N}{3}(\rho_\Lambda + \rho_m).$$  \hspace{1cm} (3)

\[^2\] A similar framework for net creation of energy was studied in the transplanckian approach to inflation in [24], and in gravitational holography in [25].
by making a specific \textit{ansatz} for the matter energy density

\[ \rho_m = \rho_{m0} a^{-3+\epsilon}, \]

where \( \epsilon \) is a constant. Although the parametrization (4) is not the most general one, it certainly covers a large number of interesting cases, including small deviations of \( \rho_m \) from the canonical form.\(^3\) As mentioned above, \( \rho_{m0} a^{-3+\epsilon} \) may understand: \( m(a) = m_0; n_m = n_m a^{-3+\epsilon} \) or \( m(a) = m_0 a^{\epsilon}; n_m = a^{-3} \) or \( m(a) = m_0 a^{\delta}; n_m = n_m a^{-3+\epsilon-\delta} \), where \( m \) is the mass of the CDM particles, \( n_m \) is their concentration, and \( \delta \) is another constant. In the following, we always consider the overall change of \( \rho_m \) without paying attention to the particular cases listed above, although mass-varying particles deserve attention of their own as they may even shed a new light on the nature of dark energy [16, 28].

Using (2), (3), and (4), one obtains explicit expression for \( G_N(a) \) and a differential equation that determines \( a(t) \);

\[ G_N(a) = G_{N0} \left( \frac{r_0^{-1} + a^{-3+\epsilon}}{r_0^{-1} + 1} \right)^{\epsilon/(3-\epsilon)}, \]

\[ \dot{a} = H_0 a \left( \frac{r_0^{-1} + a^{-3+\epsilon}}{r_0^{-1} + 1} \right)^{3/(2(3-\epsilon))}, \]

where \( r_0 \) is the present-day ratio \( \rho_m/\rho_\Lambda \).

Strong restriction on the \( \epsilon \)-parameter can be obtained by considering the time variation of \( G_N \). Writing the time variation of \( G_N \) as \( \dot{G}_N/G_N = \gamma H \), where \( \gamma = -\epsilon/\rho_\Lambda/\rho_m + 1 \), with the aid of the present observational upper bound \( |\gamma| < 0.1 \) [29] (for astrophysical and cosmological constraints, see e.g. [30]), we obtain that \( |\epsilon| \lesssim 0.1 \). As another important observational constraint we may take the redshift \( z_{tr} \), at which the deceleration parameter vanishes, and therefore denotes a transition from deceleration to acceleration. We obtain \( a_{tr} = (r_0/2)^{1/(3-\epsilon)} \). For \( r_0 = 3/7 \), this gives \( z_{tr} = 0.64 \) for \( \epsilon = -0.1 \) and \( z_{tr} = 0.70 \) for \( \epsilon = 0.1 \), to be compared with the 2\( \sigma \) constraint, \( 0.2 \lesssim z_{tr} \lesssim 0.72 \) [6]. Although we see that both signs of the \( \epsilon \)-parameter almost equally fit in observationally, we show below that only \( \epsilon > 0 \) survives considerations of gravitational thermodynamics.

\(^3\) The same parametrization has been employed in several recent papers ([16, 26] and the first reference in [33]) considering the variable \( \rho_\Lambda \) but constant \( G_N \) scenario. In addition, a strong bound on the \( \epsilon \)-parameter was derived in [27].
Using the holographic dark-energy relation (1), Friedman equation (3) and the ansatz (4), one finds a compact formula for the IR cutoff

\[ \mu = \sqrt{\frac{3}{8\pi\kappa}} \frac{H}{\sqrt{1 + r_0 a^{-3+\epsilon}}} \]  

(6)

Note the explicit dependence of \( \mu \) on \( \epsilon \). In Figs. 1 and 2 we depict the dependence of \( \mu^{-1} \) on \( a \) along with the dependence on \( a \) of the other two most-popular ad hoc chosen cutoffs, namely, the future event horizon and the Hubble distance, for both signs of \( \epsilon \). In both cases we find that our IR cutoff as given by (6) is always closer to the inverse future horizon than to the Hubble scale \( H \).

![Graph 1](image1.png)

**FIG. 1:** The evolution of various cosmological scales \( d \) in units of \( H_0^{-1} \) as functions of \( a \), for \( r_0 = 3/7 \) and \( \epsilon = 0.1 \). The future event horizon is represented by the dotted curve, the scale \( \mu^{-1}\sqrt{3/8\pi\kappa} \) by the solid curve and the Hubble distance by the dashed curve.

![Graph 2](image2.png)

**FIG. 2:** The same as in Fig. 1, but for \( \epsilon = -0.1 \).

One may wonder why in this study of dark energy we insist on the holographic point of view, when obviously the scaling law \( G_N(a) \) and the evolution law \( a(t) \) as given by Eqs. (5)
(from which all phenomenological implications considered below emerge) are the same in any setup consisting of constant $\rho_\Lambda$ and variable $G_N$. In addition, Eq. (6) for the IR cutoff appears as redundant here, since a dependence of $G_N$ on the scale factor is given directly by (5), with no obvious reference to holography. The reason lies in the fact that it is nontrivially to provide a theoretical background for the above setup sticking only with the Hilbert-Einstein action and variable cosmological parameters. In this context, scalar-tensor theories appear more natural for accommodating the above setup because there generally $G_N \sim 1/\phi$, with no reference to $\rho_\Lambda$, which can always be put in as a constant term in the action. In the present situation, however, one is forced to rely on RG approaches [19, 20, 21] which employed the Hilbert-Einstein action, and where both $G_N$ and $\rho_\Lambda$ varied through the chosen evolving RG scale. From the point of view of our scenario the problematic point in these approaches is that the same mechanism is responsible for the RG-running of both quantities. Therefore the RG approach generally cannot support a scenario where only one quantity is varying. On the other hand, one can show (see the first reference in [8]) that the RG approach in quantum gravity [21] is manifestly underpinned by the generalized holographic dark-energy relation (1), while the RG approach in a conventional field-theoretical model in the classical curved background [19, 20] can also be supported by holography for certain choices of the RG scale (see the second reference in [8]). Obviously, the generalized holographic principle (1) is able to accommodate a larger class of models with running cosmological quantities, and for this reason we find the model-independent holographic relation (1) a suitable background for our setup. Although originally derived by Cohen et. al. [3] for the opposite limit, where $\rho_\Lambda$ is variable and $G_N$ is static, we make use of the flexibility of the generalized relation (1) to explore the opposite limit, where $\rho_\Lambda$ is static and $G_N$ is variable. Therefore, the scale $\mu$ in (6) should not be confused with any of the RG scales, and although with no practical meaning here, it serves as an internal check for the holographic approach of Cohen et. al.. In contrast to other approaches where the IR cutoff is ad hoc chosen, it is here unequivocally determined by dynamics, at the same time retaining its intuitive interpretation in measuring an 'extension' of the system (see Figs. 1 and 2).

Although $G_N \sim \mu^2$ always in our model, we can notice from Figs. 1 and 2 a different behavior of $\mu$ for small $a$ for each sign of $\epsilon$. Specifically, it can be easily seen that for $a \to 0$, $G_N \to 0$ for $\epsilon < 0$. Such a scale dependence implies that the coupling $G_N$ is asymptotically free; a feature exhibited, for instance, by higher-derivative quantum gravity models at the
1-loop level \[31\]. Although for positive sign of $\epsilon$ there is no such feature, in both cases the gravitational coupling soon tends to a constant value, implying the absence of large quantum gravity effects on cosmological scales.

In order to compare our model with observations, we need to adapt the concept of effective EOS (for dark energy), as put forward by Linder and Jenkins \[32\]. It is defined by the second term in the Hubble parameter squared that encapsulates the entire modification of the standard Friedmann equation:

$$H^2 = \frac{8\pi G N_0}{3}(\rho_m a^{-3} + \rho_\Lambda^{eff}),$$

(7)

as

$$w_{eff}(a) = -1 - \frac{1}{3} \frac{a}{\rho_\Lambda^{eff}} \frac{d\rho_\Lambda^{eff}}{da}.$$  

(8)

For our model, $\rho_\Lambda^{eff}$ is given by

$$\rho_\Lambda^{eff} = \rho_\Lambda r_0[-a^{-3} + A(a)],$$

(9)

so that

$$w_{eff}(a) = -1 + \frac{-a^{-3} + A(a)B(a)}{-a^{-3} + A(a)},$$

(10)

where

$$A(a) \equiv \frac{(r_0^{-1} + a^{-3+\epsilon})^{3/(3-\epsilon)}}{(r_0^{-1} + 1)^{\epsilon/(3-\epsilon)}},$$

$$B(a) \equiv \frac{3 - \epsilon}{3} \frac{a^{-3+\epsilon}}{r_0^{-1} + a^{-3+\epsilon}}.$$  

(11)

It is interesting to calculate the present-day value for $w_{eff}(a)$. We obtain $w_{eff}(a = 1) = -1 - (r_0/3)\epsilon$, giving −0.986 for $\epsilon = -0.1$ and $r_0 = 3/7$. Also, $dw_{eff}(a)/da|_{a=1} = \epsilon[(6 - \epsilon)r_0^{-1} + 3]/[2(1 + r_0)]$, giving −0.40 for $\epsilon = -0.1$. One sees that our $w_{eff}$ is maximally elastic, in the sense that it can be made arbitrary close to −1, thus easily complying with the recent data which give the EOS converging rapidly towards −1. Hence, the ‘minimal’ character of our dynamical dark-energy model is evident. It is also interesting to examine the limit of the vanishing CC, i.e. when $r_0^{-1} = 0$. Although the $\epsilon$-dependence is still present in the scaling of $G_N$ and $\rho_m$ in this limit, their product becomes $\epsilon$-independent, giving $\rho_\Lambda^{eff} = 0$, thus reducing cosmology to the standard CDM case. In addition, one can be convinced that, asymptotically, our model always gives a de Sitter universe for both signs of $\epsilon$, meaning that arguments leading to the Big Rip \[33\] no longer apply here. The reason for having
$w_{\text{eff}} < -1$ for some time in the future lies in the modified expansion rate for matter caused by the variable $G_N$, and not in the exotic nature of dark energy.

However, the analysis of the recent data indicates a slightly better fit for the time-varying EOS than for a CC [18]. Specifically, the EOS evolution from $> -1$ to $< -1$ in the recent past is mildly favored for redshifts $z \sim 0.1 - 0.2$. However, to have this property implemented in our scenario, we need to switch to curved spaces. Namely, it is evident for the above flat-space case that crossing of the phantom line, $w_{\text{eff}} = -1$, occurs always in the (near) future (for both signs of $\epsilon$). For instance, for $\epsilon = -0.1$, the phantom line is crossed at $z \simeq -0.33$ ($a \simeq 1.5$). To switch the crossing of the phantom line from the near future to the recent past, we introduce another modification of the standard Hubble parameter in the form of curvature, i.e. a term $-k/a^2$ in the Friedmann equation that modifies (9) in (7).

This modifies the effective EOS to

$$w_{\text{eff}}(a) = -1 + \frac{-a^{-3} + A(a)B(a) + 2ba^{-2}/3}{-a^{-3} + A(a) + ba^{-2}},$$

(12)

where

$$b \equiv \frac{8\pi r_0 + 1}{3r_0} \frac{\Omega_{k0}}{1 - \Omega_{k0}},$$

(13)

and $\Omega_{k0} \equiv -k/H_0^2$. In particular, for $r_0 = 3/7$, $\epsilon = -0.1$, and $\Omega_{k0} = -0.0025$, we get $w_{\text{eff}} = -1$ for $z = 0.124$, with $w_{\text{eff}}$ decreasing with $a$, as it should. In addition, as today $w_{\text{eff}}$ behaves as

$$w_{\text{eff}}(1) = -1 + \frac{2f - \epsilon}{3(r_0^{-1} + f)},$$

$$\left. \frac{dw_{\text{eff}}}{da} \right|_{a=1} = \frac{\epsilon(r_0^{-1} + b)[(6 - \epsilon)r_0^{-1} + 3](r_0^{-1} + 1) + 2b(2b - \epsilon)}{3(r_0^{-1} + b)^2},$$

(14)

we see that the ‘minimal’ character of the scenario is still preserved for curved universes. Finally, in the limits $\rho_{\Lambda} \to 0$ or $a \to \infty$, our curved-space model behaves identically as the flat-space one above, with no Big Rip occurrence in the future. Let us stress that crossing of the phantom line has recently been shown [36] to be a general feature of models with variable cosmological parameters.

To this end, we investigate some thermodynamic features of the present scenario and show that the $\epsilon$-parameter is restricted by the GSL of gravitational thermodynamics to assume

\footnote{We mention here that there is a hint from more recent analyses [34, 35] that this is not so obvious.}
only positive values. The positivity of $\epsilon$ entails creation of matter. We start with the fact that in an ever accelerating universe there always exists a future event horizon. Thus, analogously to the black-hole horizon, it can be attributed some thermodynamical quantities, like entropy and temperature. Although, in a strict sense, this has been proved for a de Sitter horizon only, where the temperature is proportional to the inverse apparent horizon, $\sim H$, many authors use to apply the same concept when exploring the thermodynamical behavior of accelerated universes driven by other sorts of dark energy. In these cases, when the degeneracy between the apparent and the event horizon is broken, the horizon entropy always refers to the entropy of the future event horizon, while the not-well-defined temperature of the dark-energy fluid as well as the Hawking temperature are usually assumed to be the same as the de Sitter temperature $\sim H$ (see, however, [39]).

We seek for information on the $\epsilon$-parameter by assuming that the GSL of gravitational thermodynamics is satisfied. The GSL states that the entropy of the event horizon plus the entropy of matter and radiation in the volume within the horizon cannot decrease in time. The easiest way to gain information on the $\epsilon$-parameter is by considering the change of entropy in the asymptotic regime, $a \gg 1$. In this case, one should also add the entropy of Hawking particles since it is conceivable that the CMB temperature will drop below the Hawking temperature after some time in the (distant) future (see the first reference in [38]). Hence, we have

\[
\frac{dS_{tot}}{da} \geq 0, \\
S_{tot} = S_e + S_m + S_r + S_{Haw},
\]

where

\[
S_e = \frac{\pi d_e^2}{G_N}, \\
S_m = \frac{\rho_m 4\pi}{m} d_e^3, \\
S_r = \alpha T_{CMB}^3 \frac{4\pi}{3} d_e^3, \\
S_{Haw} = \beta T_{Haw}^3 \frac{4\pi}{3} d_e^3,
\]

Since we are dealing here with the ‘true’ CC, the entropy of the dark-energy fluid inside the cosmological event horizon is equal to zero.
$d_e$ is the future event horizon, $T_{\text{Hawk}} = H/2\pi$ and $\alpha$ and $\beta$ are order-of-one constants. In the asymptotic regime, $dS/da$ for the particular entropies in (16) reads

$$\frac{dS_e}{da} \approx \pi H_0^{-2} G_{N_0}^{-1} (1 + r_0)^{(3+\epsilon)/(3-\epsilon)} r_0 2(6 - \epsilon) a^{-4+\epsilon},$$

$$\frac{dS_m}{da} \approx -\rho m_0 H_0^{-3} (1 + r_0)^{9/(2(3-\epsilon))} \frac{4\pi(3 - \epsilon)}{3} a^{-4+\epsilon},$$

$$\frac{dS_r}{da} \approx -\alpha 4\pi T_{CMB0}^3 H_0^{-3} (1 + r_0)^{9/(2(3-\epsilon))} a^{-4},$$

$$\frac{dS_{\text{Hawk}}}{da} \approx \beta 3r_0 (3 - \epsilon) 4 a^{-4+\epsilon}.$$  \hspace{1cm} (17)

We see that for $\epsilon < 0$ the left-hand side of (15) is dominated by $dS_r/da$. Since the entropy of radiation inside the horizon, $S_r$, always decreases with time, the GSL as given by (15) cannot be satisfied for $\epsilon < 0$. On the other hand, for $\epsilon > 0$, the GSL can be satisfied provided the following constraint is obeyed:

$$(1 + r_0)^{(3+\epsilon)/(3-\epsilon)} 2(6 - \epsilon) - \frac{H_0}{m} (1 + r_0)^{(3+\epsilon)/(2(3-\epsilon))} (3 - \epsilon) = \frac{\beta H_0^2 G_{N_0} 3(3 - \epsilon)}{4\pi} \geq 0. \hspace{1cm} (18)$$

Notice that (18) entails a trivial bound on the mass of the CDM particles, $m \gtrsim H_0$.

We would like to conclude with a few additional remarks and comments. One notices that the positivity of the $\epsilon$-parameter, as predicted by the assumed validity of the GSL, may, in a lesser extent, pose a drawback on our model. Namely, for $\epsilon > 0$, the crossing of the phantom line in the recent past (for curved space) is always such that $w_{\text{eff}}$ increases with $a$, a trend not confirmed by the data. Still, one should note that there is only a marginal ($2\sigma$) evidence for the phantom-line crossing, being, in addition, strongly dependent on the subsampling of the SNe dataset. On the other hand, although there appear strong arguments for believing in the validity of the GSL, one argues that in some cases, comprising both phantom (see the third reference in [38]) and nonphantom [39] fluids, the GSL might not be fulfilled. In addition, $\epsilon > 0$ corroborates Dirac’s hypothesis. Hence, we see from the above discussion that arguments towards either $\epsilon > 0$ or $\epsilon < 0$ can by no means be deemed as decisive.

Finally, we should stress that a more pressing issue is the ‘cosmic coincidence problem’, which, beyond anthropic considerations, has no solution in the present scenario. In models with the running CC and static $G_N$, in which there is a continuous energy transfer between the CC and matter (see the first reference in [33]), the constant term in $\rho_\Lambda$ is crucial since otherwise a transition between deceleration to acceleration cannot be obtained. On the other hand, holography cannot underpin such models as, by Eq. (1), the constant term in $\rho_\Lambda$ is
always set to zero. Hence, although such models can ameliorate the ‘cosmic coincidence problem’ to some extent, they do not comply with observation. It would be interesting to explore the generalized equation of continuity within the holographic dark-energy model (1),
\[
\dot{G}_N(\rho_\Lambda + \rho_m) + G_N\dot{\rho}_\Lambda + G_N(\dot{\rho}_m + 3H\rho_m) = 0,
\]
in which both \(\rho_\Lambda\) and \(G_N\) are variable and the scaling of \(\rho_m\) is different from \(a^{-3}\). In this way it would be possible to see if such a scenario can keep nice features of the present minimal model regarding observation, at the same time offering a solution to the ‘cosmic coincidence problem’. Any other improvement of the minimal scenario, for instance, that with additional degrees of freedom in the form of scalar fields, would certainly have much less predictive power.

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