A Pilot Allocation Method of Jointing Cell Grouping and Alliance Game in Massive MIMO System

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Abstract. In order to improve the uplink channel estimation performance and reduce pilot contamination (PC), this paper proposes a pilot allocation method of jointing cell grouping and alliance game (JCG-AG) for massive multiple-input multiple-output (MIMO) cellular networks. It can be divided into two steps. First, by properly dividing all cells into different cell-groups, the users in different cell-groups are allocated different orthogonal pilot sets. Second, the users in the same cell-group are further divided into several mutually disjoint sub-alliances. Users belonging to different sub-alliances are allocated different pilots, and users belonging to the same sub-alliance reuse the same pilots. During the second step, the alliance formation algorithm is used to suppress PC. The JCG-AG method takes practical application as the starting point, focuses on the random distribution of users, and uses the idea of alliance game. All these make it more flexible and practical. As demonstrated in the simulation results, the JCG-AG method can greatly mitigate the PC and reduce the average mean square error (MSE) for all channel estimations in the uplink.

1. Introduction

As we all know, the new generation mobile communication systems, namely 5G, will come out in 2020. According to the objective laws of mobile communication development and the urgent needs of users, 5G will enhance greatly in frequency efficiency, energy efficiency, user experience and many other aspects on the basis of 4G. Moreover, compared with the traditional MIMO technology, the massive MIMO technology is one of the key technologies of 5G [1] and can obtain high channel capacity, more accurate spatial discrimination with low-cost and simple hardware facilities, so it causes great concern among professionals [2, 3, 4].

However, getting accurate channel state information (CSI) is very difficult for time division duplex (TDD) massive MIMO system. The number of orthogonal pilots is limited owing to the limitation of...
coherent time, but the number of users is large. Therefore, it is necessary for users in different cells to reuse the same pilot sequences, so that the base stations are unable to distinguish these pilot signals. This is the "pilot contamination".

Plenty of literatures propose methods to reduce PC. Literature [5] discusses the pilot power control method and uses the MSE of channel estimation in multi-cell one-dimensional network to alleviate PC. Literature [6] considers the characteristics of spatial domain and assigns the same pilot to the users of spatial orthogonal channels. In literature [7], an adaptive pilot allocation algorithm is provided and assigns pilots to users on the basis of the interference caused by inter-cell PC. Literature [8] utilizes the cooperation among base stations to decrease PC. Literature [9] attempts to combine power control and pilot time slot allocation to improve achievable rate in the downlink while reducing PC, but this method requires strict system synchronization. Literature [10] tries to apply game theory to pilot allocation, and takes the inter-cell game into account. In summary [5-10], these methods for suppressing PC are carried out for the system model that the same number of users per cell is assumed. However, in reality, the distribution of mobile users is disordered (i.e., random). Besides, these pilot allocation schemes lack flexibility and practicality.

In view of the above problems, this paper adopts the game between users and proposes a method based on JCG-AG to reduce PC. We first divide the cells into cell-groups reasonably. Then, users in each cell-group are divided into different sub-alliances, and corresponding pilots are allocated in combination with alliance game. After the end of alliance formation algorithm, the final inter-user pilot allocation is obtained.

2. System and Channel Model

As shown in Figure 1, a multi-user massive MIMO cellular network with \( L \) regular hexagonal cells is considered. Assuming that there are randomly and uniformly distributed single-antenna users per cell, each base station is equipped with \( M \) antennas. If the BS \( j \) in cell \( j \in \{1, 2, \ldots, L\} \) serves \( K_j \) users, there are \( N \) users in the area \( \mathbb{I} \). Here, user \((j, k)\) represents the \( k\)th user in the \( j\)th cell.

![Figure 1](image)

**Figure 1.** Multi-user massive MIMO cellular network with random user locations.

We give a general channel model in Figure 2. Assume the number of cells in the red slash area is \( T \), and all wireless channels are Rayleigh channels. The channel between user \((j, k)\) and the \( m\)th antenna in the \( l\)th cell is defined as \( C_{ljk} \), where

\[
m \in \{1, 2, \ldots, M\}, k \in \{1, 2, \ldots, K_j\}, l \in \{1, 2, \ldots, T\}, C_{ljk} \sim \mathcal{CN}(0, \gamma \rho_{ljk}) \text{ and } \rho_{ljk} = \frac{1}{\left(\frac{d_{jk}(\rho_{jk})}{\gamma}\right)^\mu}, \gamma \text{ represents the path fading factor. Here, the pilot transmission power of each user in the system uplink is } \rho. \text{ The channel matrix } C_{jk} \text{ at BS } j \text{ is } C_{jk} = [c_{j1}, L, c_{jM}]^T \in \mathbb{C}^{L \times 1}.
\]
3. Pilot Allocation Method of JCG-AG

3.1. Cell Grouping
The first step of JCG-AG is cell grouping, so we divide the cells in Fig. 1 into two cell-groups according to whether there are red slashes in the background. Two cell-groups are respectively denoted by $G_1$ and $G_2$. The total number of pilot sequences is $2P$, and all pilots are divided into two orthogonal pilot sets, and number of each pilot set is $P$. Then, correspondingly, the users in $G_1$ (i.e., red slashes area) and users in $G_2$ respectively allocate a pilot set. Thus there is no PC between users in different cell-groups.

3.2. Alliance Game among Users in the Same Cell-group
The second step of JCG-AG is the alliance game among users in the same cell-group. For each cell-group users, we use the idea of alliance game to allocate $P$ pilots reasonably. First of all, we divide the users in the same cell-group into $P$ sub-alliances. Because users belonging to different sub-alliances allocate orthogonal pilots, there is no PC. And there exists PC for users who reuse pilots in the same sub-alliance. So we need to consider how to reduce the PC in the same sub-alliance under the condition that each user (i.e., the participant) is profitable.

In this paper, only seven cells in the dashed circle of area $\Gamma$ (in figure 1) are considered here. Assume the number of users that fall in the red slashes area is $n$ and they are painted with different colors (black, yellow, green, purple, etc.). That is to say, users with the same color belong to the same sub-alliance.

**Definition 1** (Alliance structure) In the user set $\vartheta$, the alliance structure $R$ is defined as a set of disjoint sub-alliances $R_i (R = \{R_1, R_2, \ldots, R_L\})$, i.e., $\bigcup_{i=1}^{L} R_i = \vartheta$, $\bigcap_{i=1}^{L} R_i = \emptyset$.

Here, we define each user’s utility function (i.e., revenue function) as the MSE of channel estimation in the uplink. In order to theoretically analyze the effects of PC, we need to find the expression of channel estimation MSE.

For an arbitrary user $(j,k)$, the pilot $\psi_k \in \mathbb{C}^{P \times 1}$ satisfies $E(\psi_k \psi_k^H) = 1$ and $E(\psi_k \psi_m^H) = 1$ (when $(l,m)$ belongs to the sub-alliance $R_i(j,k)$). When it’s transmitted in the uplink, the pilot signal received at the BS $j$ is given by:

$$y_j^{\text{pilot}} = \sum_{l=1}^{T} \sum_{m=1}^{K} \sqrt{P_l} C_{jl} \psi_m^H + n_j^{\text{pilot}}$$

(1)

where $n_j^{\text{pilot}} \in \mathbb{C}^{M \times P}$ is additive complex white Gaussian noise, whose elements are all subject to
Thus, the MMSE estimation of channel matrix \( C_{j,k} \) at BS \( j \) can be written as:

\[
\hat{C}_{j,k} = \frac{\beta_{j,k}}{P_{j}} + \frac{\sigma_{j,k}^2}{P_{j}} \mathbf{P}_{j} \mathbf{W}_{j} = A_{j,k} \left( \sum_{l \in \mathcal{M}_{j,k}} \beta_{l,k} + \frac{\sigma_{l,k}^2}{P_{l}} \right) \mathbf{P}_{l} \mathbf{W}_{l} + \frac{1}{\sqrt{P_{j}}} \mathbf{n}_{j}^{\text{MSE}} \mathbf{W}_{j}
\]

In equation (2), \( A_{j,k} = \beta_{j,k} \left( \sum_{l \in \mathcal{M}_{j,k}} \beta_{l,k} + \frac{\sigma_{l,k}^2}{P_{l}} \right) \mathbf{P}_{l} \mathbf{W}_{l} \). \( r_{j}(j,k) \) is one of the sub-alliances in alliance structure \( R = \{ R_{1}, R_{2}, \ldots, R_{s} \} \), and user \((j,k)\) belongs to the sub-alliance \( r_{j}(j,k) \). Then, the user's channel estimation error is \( \bar{C}_{j,k} = \hat{C}_{j,k} - C_{j,k} \), which obeys \( \mathcal{N}(0, \sigma^{2}) \).

**Theorem 1** For a given alliance structure \( R \), the MSE of the channel estimation \( \bar{C}_{j,k} \) is

\[
MSE_{j,k} = (1 - 2A_{j,k}) \cdot M \beta_{j,k} + MA_{j,k} \sum_{l \in \mathcal{M}_{j,k}} \beta_{l,k} + \frac{1}{\sqrt{P_{j}}} \mathbf{n}_{j}^{\text{MSE}} \mathbf{W}_{j}
\]

Proof:

\[
\begin{align*}
MSE_{j,k} &= E\left[ (\hat{C}_{j,k} - C_{j,k})^2 \right] \\
&= E\left[ (\hat{C}_{j,k} - C_{j,k})^2 \right] \\
&= E\left[ (A_{j,k} - 1) C_{j,k}^2 + A_{j,k} \sum_{l \in \mathcal{M}_{j,k}} C_{l,k} + \frac{1}{\sqrt{P_{j}}} \mathbf{n}_{j}^{\text{MSE}} \mathbf{W}_{j} \right] \\
&= (A_{j,k} - 1)^2 E\left[ (C_{j,k})^2 \mathbf{C}_{j,k} \right] + A_{j,k} \sum_{l \in \mathcal{M}_{j,k}} E\left[ (C_{l,k})^2 \mathbf{C}_{l,k} \right] + \frac{1}{P_{j}} E\left[ \mathbf{n}_{j}^{\text{MSE}} \mathbf{C}_{j,k} \mathbf{W}_{j} \right]
\end{align*}
\]

Here, \( \mathbf{C}_{j,k} \) (when \( (l,m) \neq (j,k) \)) are independent of each other, \( \mathbf{C}_{j,k} \) and \( \mathbf{n}_{j}^{\text{MSE}} \) are also independent of each other, so

\[
MSE_{j,k} = (A_{j,k} - 1)^2 E\left[ (C_{j,k})^2 \mathbf{C}_{j,k} \right] + A_{j,k} \sum_{l \in \mathcal{M}_{j,k}} E\left[ (C_{l,k})^2 \mathbf{C}_{l,k} \right] + \frac{1}{P_{j}} E\left[ \mathbf{n}_{j}^{\text{MSE}} \mathbf{C}_{j,k} \mathbf{W}_{j} \right]
\]

Here, we know that \( E[\mathbf{C}_{j,k}^T \mathbf{C}_{j,k}] = M \beta_{j,k} \), \( E[\mathbf{C}_{j,k}^T \mathbf{C}_{l,k}] = M \beta_{l,k} \), \( E[\mathbf{W}_{j}^T \mathbf{W}_{j}] = 1 \), and the elements of \( \mathbf{n}_{j}^{\text{MSE}} \) are subject to \( \mathcal{N}(0, \sigma^{2}) \). Hence, expression (5) can be simplified as expression (3). The end of the proof of Theorem 1.

**Definition 2** (Alliance adjustment rules) According to the definition of alliance game, we give the adjustment rules. If \( R \xrightarrow{\text{R \to R'}} \) (i.e., users withdraw from its sub-alliance and join another sub-alliances, the alliance structure changes from \( R \) to \( R' \)) is permissible, two prerequisites are required, 1) \( MSE_{j,k}(R) > MSE_{j,k}(R') \), 2) \( \frac{1}{n} \sum_{(l,m) \neq (j,k)} MSE_{l,m}(R) \geq \frac{1}{n} \sum_{(l,m) \neq (j,k)} MSE_{l,m}(R') \).

According to the theory of final stability in alliance game [11], we give the end stable conditions of
the algorithm.

**Definition 3** (Alliance stable conditions) When there is no permission of adjustment $R_{(j,k)} \rightarrow R'$ for all users $(j,k) \in \emptyset$ and all sub-alliances $R_i, R_j, R_p \ L$, the alliance forms a stable structure.

The alliance formation algorithm is shown as follows in algorithm 1.

**Algorithm 1:**

Initialization: randomly initial alliance structure $R = \{R_i, R_j, R_p \}$, and $\omega = 0$, $\omega$ is the current search times.

Loop:
1) Cyclic search all users $(j,k)$
2) Search all $R_i \in \{R_i, R_j, R_p \}$, $R_p \ (j,k)$

According to the alliance adjustment rules in definition 2, conditions 1) and 2) are judged. If $R_{(j,k)} \rightarrow R'$ is allowed, the user $(j,k)$ leaves its sub-alliance, joins sub-alliance $R_i$, and the alliance structure is updated ($R = R'$); otherwise, alliance structure remains unchanged ($R = R$).

End (2))
$\omega = \omega + 1$
End (1))

Until all the conditions of $R_{(j,k)} \rightarrow R'$ are not permitted or $\omega > \partial$, end the loop. (here $\partial$ is the user's maximum number of search limits.)

4. Simulation Results

In this part, we will carry out simulations about three pilot allocation methods, joint cell grouping and fixed alliance (JCG-FA), joint cell grouping and random alliance (JCG-RA) and JCG-AG proposed in section 3. These methods are compared according to the simulation results. We choose the fixed area of the $5 \times 5$ and set $N = 20$, $n = 8$, $\sigma^2 = 1$, $\gamma = 2$, $M = 500$, $p_o = 10$, $\partial = 800$, $L = 10$.

Because the locations of the base stations are generally fixed in practice, we consider the fixed location of base stations in table 1. Moreover, we first consider the situation where the users’ position are fixed in table 1, and then consider the situation where the users are randomly distributed. In simulation results, the solid line represents the former, and the dash line represents the latter. Table 1 shows the positions of eight users and four base stations respectively.

| Table 1. Position of base stations and users in simulations |
|------------------------------------------------------------|
| Locations of base stations | (1.5, 0.8660), (-1.5, 0.8660), (-1.5, -0.8660), (1.5, -0.8660) |
| Locations of users | (1, 1), (2, 1), (-1, 1), (-2, 1), (-1, -1), (-2, -1), (1, -1), (2, -1) |

The relationship between the average MSE of all channel estimations and the pilot transmitting power $p_o$ in the system uplink is shown in Figure 3. Here $M = 500$. As can be seen from figure 3, regardless of whether the user is fixed or randomly distributed, the JCG-AG is superior to the JCG-RA, and the JCG-RA is better than the JCG-FA. In addition, the starting point value of each line in the graph is relatively large, which is related to the formula of the third term in expression (3). When the pilot transmitting power $p_o$ tends to infinitesimal, the value of the third term is large, and the MSE of channel estimation for each user depends mainly on this term. When the pilot transmitting power $p_o$ tends to infinity, once the alliance structure reaches final stability, the MSE of the channel estimation for per user is a constant, which coincides with the trend of the red curve in the figure.
above.

![Figure 3. MSE versus $p_t$ with $M=500$.](image)

The relationship between the average MSE of all channel estimations and the number of antennas $M$ is shown in Figure 4. Here $p_t=10$ dB. As can be seen from figure 4, increasing the number of antennas leads to increasing the average MSE of all channel estimations. The reason is that the greater the number of antennas, the greater the number of channels transmitted in parallel between the users and the base stations, and so the greater the number of channel estimation errors. For a fixed $M$, the JCG-AG is superior to the JCG-RA and the JCG-FA whether the user is fixed or randomly distributed.

![Figure 4. MSE versus $M$ with $p_t=10$ dB.](image)

The relationship between the average MSE of all channel estimations and the pilot transmitting power $p_t$ for JCG-AG is shown in Figure 5. Here $M=500$, the number of pilot is 2, 4, 6 and 8, and we compare the MSE of different number of pilots for JCG-AG. As can be seen from figure 5, the average MSE of all channel estimations is decreasing when $P$ is increasing in the uplink for JCG-AG, which indicates that the rich orthogonal pilots can improve the channel estimation performance of uplink, and alleviate PC.
The relationship between the average MSE of all channel estimations and the number of antennas M for JCG-AG is shown in Figure 6. Here \( p_t = 10 \) dB, the number of pilot is 2, 4, 6 and 8. Similarly, figure 6 also demonstrates that plenty of orthogonal pilots contribute to improving the channel estimation performance of uplink and reducing PC.

**Figure 5.** MSE versus \( p_t \) for JCG-AG with \( M = 500 \), \( p = 2, 4, 6, 8 \).

**Figure 6.** MSE versus \( M \) for JCG-AG with \( p_t = 10 \) dB, \( p = 2, 4, 6, 8 \).

**Figure 7.** MSE versus \( \sigma^2 \) with \( p_t = 10 \) dB, \( M = 500 \).
The relationship between the average MSE of all channel estimations and the variance $\sigma^2$ is shown in Figure 7. Here $p_t = 10$ dB, $M = 500$. We can see that the average MSE of all channel estimations increases when the variance $\sigma^2$ increases. On the part of the average MSE of channel estimation, the JCG-AG is much smaller than JCG-RA for a fixed $\sigma^2$. This indicates that the JCG-AG can obtain more accurate CSI than the JCG-RA. Here, we omitted the results of JCG-FA for convenience.

5. Conclusion
In this paper, in order to obtain accurate CSI and minimize the average MSE of channel estimations in the uplink, JCG-AG pilot allocation method is proposed. This method can be applied to the case of random user distribution, and the idea of alliance game is applied to reduce PC. In order to make every user profitable, we give the adjustment principles of the alliance structure, the final stable conditions, and the alliance formation algorithm. The expression of MSE of each user’s channel estimation is given for JCG-AG. And the influences of the pilot transmission power, the number of antennas and the variance of additive complex white Gaussian noise on the average MSE are simulated. Simulation results are implemented and indicate that the proposed JCG-AG scheme can obtain smaller average MSE than JCG-RA and JCG-FA. It is true that we can also consider the irregular shape of the cells, which means the random distribution of base stations. This will be carried out in our follow-up work.

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