Sensitivity plots for WIMP modulation searches

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Prospects of WIMP searches using the annual modulation signature are discussed on statistical grounds, introducing sensitivity plots for the WIMP–nucleon scalar cross section.

1. INTRODUCTION

The annual modulation effect[1] provides a distinctive signature for the identification of a Dark Matter signal in the direct searches of WIMPS through their elastic scattering off the nuclei of a detector. Due to this effect the relative velocity between the detector and the WIMP Maxwellian distribution (assumed at rest in the Galactic rest frame) is given by:

\[ v_{\text{earth}} = v_{\text{sun}} + v_{\text{orb}} \sin \delta \cos \omega (t - t_0) \] (1)

where \( v_{\text{sun}} \) is the Sun’s velocity in the galactic rest frame, \( v_{\text{orb}} \approx 30 \text{ km sec}^{-1} \), \( \sin \delta \approx 0.51 \) (\( \delta \) is the angle between the Ecliptic and the Galactic plane), \( \omega = 2\pi/T \), \( T=1 \text{ year} \) and \( t_0 \approx 2^{nd} \text{ june} \).

2. EXTRACTING THE MODULATION SIGNAL

Given a set of experimental count rates \( N_{ik} \) representing the number of events collected in the i-th day and k-th energy bin, the mean value of \( N_{ik} \) (expressed in number of counts per unit of detector mass, time and interval of recoil energy) is:

\[ \langle N_{ik} \rangle = \mu_{ik} = [b_k + S_{0,k} + S_{m,k} \cos \omega (t - t_0)] \cdot W_{ik} \] (3)

where the \( b_k \) represent the average background while \( S_{0,k} \) and \( S_{m,k} \) are the constant and the modulated amplitude of the WIMP signal respectively. The various parameters of the WIMP model are contained in \( S_{0,k} \) and \( S_{m,k} \). In particular they depend on the WIMP-nucleus elastic cross sections \( \sigma \) and the WIMP mass \( m_W \). The \( W_{ik} = M \Delta T_i \Delta E_k \) are the corresponding exposures, where \( M \) is the mass of the detector, \( \Delta E_k \) is the amplitude of the k-th energy–bin, while \( \Delta T_i \) represents the i-th time bin (in the following we will assume all \( \Delta T_i = 1 \text{ day} \)). For simplicity \( t_0 \) will be omitted in the following equations.

The general procedure to compare theory with experiment is by making use of the maximum-likelihood method. The combined-probability function of all the collected \( N_{ik} \), assuming that they have a poissonian distribution with mean values \( \mu_{ik} \), is given by:

\[ L = \prod_{ik} e^{-\mu_{ik}} \frac{\mu_{ik}^{N_{ik}}}{N_{ik}!}. \] (4)

The most probable values of \( m_W \) and \( \sigma \) maximize \( L \) or, equivalently, minimize the function:

\[ y(m_W, \sigma) \equiv -2 \log L - \text{const} \]

\[ = 2\mu - 2 \sum_{ik} N_{ik} \log [b_k + S_{0,k} + S_{m,k} \cos \omega t_i] \]

where \( \mu \equiv \sum_{ik} \mu_{ik} \) and all the parts not depending on \( m_W \) and \( \sigma \) may be absorbed in the constant because are irrelevant for the minimization.

3. STATISTICAL SIGNIFICANCE OF THE SIGNAL

Once a minimum of the likelihood function has been found, a positive result excludes the absence of modulation at some confidence level probabil-
ity. This can be checked by evaluating the quantity $\delta^2 = y(\sigma = 0) - y(m_W, \sigma)_{\text{min}}$ to test the goodness of the null hypothesis. In order to study the distribution of $\delta^2$ we make use of the asymptotic behaviour:

$$\delta^2 \simeq \chi^2(\sigma = 0) - \chi^2_{\text{min}}$$

$$\chi^2(\sigma, m_W) \equiv \sum_k \frac{(S_{m,k}(m_W, \sigma) - X_k)^2}{\text{Var}(X_k)}$$

$$X_k \equiv \frac{\sum_i N_{ik} \cos \omega t_i - N_k \beta_k}{W_k(\alpha_k - \beta_k^2)}.$$  (7)

where $\beta_k \equiv \sum_i W_{ik} \cos \omega t_i$, $\alpha_k \equiv \sum_i W_{ik} \cos^2 \omega t_i$, and $N_k \equiv \sum_i N_{ik}$. In the case of absence of a modulation effect numerical simulations show that the quantity $\delta^2$ belongs asymptotically to a $\chi^2$ distribution with two degrees of freedom. We explain this by the fact that once the cross section $\sigma$ is set to zero the likelihood function $L$ no longer depends on $m_W$ (all the $S_0$ and $S_n$ functions vanish) and this is equivalent to fixing both the parameters of the fit at the same time. In the case of presence of a modulation, $\delta^2$ has the asymptotic distribution of a non central $\chi^2$ with one degree of freedom and with a mean value given by

$$< \delta^2 > = \frac{1}{2} \sum_k \frac{S_{m,k}(\sigma, m_W)^2}{b_k + S_{0,k}} \Delta E_k (MT\alpha + 2)$$  (8)

where the same days of data taking have been assumed for all the energy bins, and the approximations $\sum_i N_{ik} \cos^2 \omega t_i \simeq < N_{ik} > \sum \cos^2 \omega t_i$, $< N_{ik} > \simeq \sum_i W_k(b_k + S_0)$ have been made. In Eq. (8) we have also defined the factor of merit $\alpha = \frac{1}{7} \sum \cos^2 \omega t_i \ (\alpha = 1$ in case of a full period of data taking) and the terms depending on the $\beta_k$ have been neglected.

Since the degree of overlapping between the distributions of $\delta^2$ in the two cases of absence and presence of modulation depends on $< \delta^2 >$, equation (8) allows to estimate the needed exposure $MT\alpha$ in order to observe a modulation effect with a given probability: for instance, $< \delta^2 > = 14.9 \ (5.6)$ corresponds to a 90% (50%) probability to see an effect at least at the 95% (90%) C.L. Once a required $< \delta^2 >$ is chosen, a sensitivity plot may be obtained by showing the curves of constant $MT\alpha$ in the plane $m_W - \sigma$.

4. SENSITIVITY PLOTS AND QUANTITATIVE DISCUSSION

In Figures 1–2 we discuss the example of a Germanium detector with background b=0.01 cpd/kg/keV (assumed constant with energy) and energy thresholds $E_{th}=2$ keV, values not unrealistic, taken into account the recent performances of some Ge detectors. Parameter values used in the plots are the local halo mass density $\rho=0.3$ GeV/cm$^3$, $v_{\text{loc}}=220$ km sec$^{-1}$ ( $v_{\text{loc}}$ is the measured rotational velocity of the Local System at the Earth’s position), the WIMP r.m.s. velocity $v_{\text{rms}}^2 = \frac{3}{4} \bar{v}_{\text{loc}}^2$ and $v_{\text{sun}} \simeq (v_{\text{loc}} + 12)$ km sec$^{-1}$.

In Figure 1 the sensitivity plots for $< \delta^2 > = 5.6$ is shown in the plane $m_W - \sigma^{(n)}$, where $\sigma^{(n)}$ is the WIMP cross section $\sigma$ rescaled to the nucleon by adopting a scalar–type interaction. The different curves correspond to values of $MT\alpha$ from 10 kg·year to 100 kg·year in steps of 10 (from top to bottom). The closed contour and the cross indicate respectively the $2\sigma$ C.L. region singled out by the DAMA modulation search experiment and the minimum of the likelihood function found by the same authors[3]. Note that an exposure of 10 kg·year of a Ge detector of the above–quoted performances would explore almost totally the DAMA region.

In Figure 2 we show, as a function of $m_W$, the minimal exposures required for the same ge-
Table 1
Summary of minimal exposures, all in kg · year. Values off (in) parenthesis refer to $v_{\text{loc}}=220(170) \text{ km sec}^{-1}$. $E_{th}$ indicates the energy thresholds expressed in keV, $b$ the background (assumed not dependent on energy) in cpd/kg/keV. Exposures are estimated for the WIMP mass range $10^{10} \lesssim m_W \lesssim 1000$ unless specified otherwise.

| Material | $E_{th}$ | $b$ | Exploration of not DAMA region | DAMA region $\delta^2 = 5.6$ | DAMA region $\delta^2 = 15$ |
|----------|---------|-----|--------------------------------|-----------------------------|-----------------------------|
| Ge, $E_{th} = 2$, $b=0.1$ | 80(50) | | | 50(25) | 175(90) |
| Ge, $E_{th} = 12$, $b=0.01$ | 25(19)* | | | 50(25) | 190(95) |
| TeO$_2$, $E_{th} = 5$, $b=0.01$ | 40(25) | | | 40(20) | 150(80) |
| NaI, $E_{th} = 2$, $b=0.1$ | 50† | | | 180(100) | 660(355) |

* $45\text{ GeV} \lesssim m_W \lesssim 110\text{ GeV}$; † $m_W \lesssim 70(125)\text{ GeV}$.

nium set–up in order for its sensitivity contour to lie below the upper limit on $\sigma^{(n)}$ implied by the exclusion plot obtained in Ref.[3] with pulse shape discriminated spectra. Since the present uncertainty in $v_{\text{loc}}$ can affect the results in a significant way[2], the different values $v_{\text{loc}} = 170, 220$ and 270 km sec$^{-1}$ are shown by the dotted, solid and dashed curves respectively. In each case the values $b=0,0.01$ and 0.1 are given from bottom to top.

Some examples of minimal exposures for other target materials are given in Table 1 for $v_{\text{loc}} = 220$ and 170 km sec$^{-1}$. The second column of table 1 shows the exposures necessary to explore the regions of the $m_W$–$\sigma^{(n)}$ plane below the exclusion plot or Ref.[3]. The third and fourth columns show the lowest values of MT$\alpha$ that give a sensitivity plot encompassing all the $2\sigma$ DAMA contour for $< \delta^2 > = 5.6$ and 15 respectively. The experimental thresholds and resolutions assumed in Table 1 are close to those already obtained (or foreseeable) in Ge, NaI and TeO$_2$ detectors.

A systematic study of sensitivity plots (not shown here for lack of space[4]) concludes that prospects of modulation searches seem promising provided that the WIMP signal is not far below present sensitivities, the lowest values of explorable $\sigma^{(n)}$ falling in most cases in the typical range of few $\times 10^{-10}$ barn. An important feature of all the plots is that the sensitivity to modulation is generally a decreasing function of the WIMP mass, the highest sensitivities corresponding roughly to the interval $10\text{ GeV} \lesssim m_W \lesssim 130\text{ GeV}$, and depending in a sensitive way on the value of the parameter $v_{\text{loc}}$.

Figure 2. Minimal exposures MT$\alpha$ for the $< \delta^2 > = 5.6$ calculated for a germanium detector with threshold energy $E_{th}=2$ keV.

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