The time-reverse of any causal theory is eternal noise

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We consider a very general class of theories, process theories, which capture the underlying structure common to most theories of physics as we understand them today (be they established, toy or speculative theories). Amongst these theories, we focus on those which are causal, in the sense that they are intrinsically compatible with the causal structure of space-time as required by relativity. We demonstrate that there is a sharp contrast between these theories and the corresponding time-reversed theories, in which time is taken to flow backwards from the future to the past. While the former typically feature a rich gamut of allowed states, the latter only allow for a single state: eternal noise. We illustrate this result by considering the time-reverse of quantum theory. Moreover, we derive a strengthening of the result in PRL 108, 200403 on signalling in time-reversed theories.

The question of whether theories of physics should be formulated in a time-symmetric fashion has received a lot of attention in the past few years [1–6], because of its deep foundational implications and the pivotal role it plays in the relationship between quantum theory and general relativity. In this work, we show that if a theory obeys the requirements imposed by relativistic principles then the time-reverse of the theory is necessarily reduced to a trivial state of eternal noise. This paints a provocative picture, in which time-symmetry breaks down in the most extreme manner imaginable, but is the natural consequence of taking general relativity seriously and applying its constraints to the letter.

The assumption that nature is time-symmetric arises from taking reversible dynamics as the primary object of importance for a theory. In contrast, we choose to work within the framework of process theories [7, 8], which forces one to treat all processes – including states, effects and general transformations – on an equal footing with reversible dynamics. Our main result is then an immediate consequence of compatibility of processes with relativistic constraints.

PROCESS THEORIES

A process theory is comprised of a collection of systems, graphically represented by wires, and processes, graphically represented by boxes with wires as their inputs (at the bottom) and outputs (at the top). Special cases of processes include: states, which are processes with no inputs; effects, which are processes with no outputs; numbers, which are processes with neither input nor output. Processes can be wired together in order to form diagrams (a.k.a. circuits) such as in (1) below. Note that, further to assigning a meaning to wires and boxes, a process theory should explicitly specify what it means to form diagrams; in particular, every diagram must itself correspond to a process in the theory. In the case of diagram (1), this is a process with input $A$ and output $ABC$, i.e. a composite system. In algebraic terms, process theories are (strict) symmetric monoidal categories [9].

Recall that we adopted a convention whereby process inputs are drawn at the bottom, and process outputs are drawn at the top. When drawing diagrams, this means that the upward vertical direction is associated with time flowing forward:

The rather general formulation of process theories has helped to solve a number of concrete problems in quantum foundations and quantum computation. In quantum foundations, for example, it has provided a complete and consistent formulation of Spekkens’ toy theory, and pinpointed its key differences from quantum theory [10, 11]. In quantum computation, it has resulted in the derivation of exact correspondences between quantum computational models [12–14] and error-correction [15, 16].

One important thing to remark about process theories is that the diagrams capture all there is to know about the theory. This includes all relevant systems are represented explicitly in the diagrams e.g. also those carrying information about measurement outcomes and classical control in the case of quantum theory. This is in contrast to other ‘hybrid’ formalisms, such as [17–19], where classical systems are represented implicitly by means of indices and labels.
One example of a process theory is probability theory (a.k.a. classical theory), with probability distributions as states and stochastic maps as processes. Another interesting example is given by quantum theory: processes include classical processes between classical systems, quantum processes between quantum systems (i.e. CPTP maps), as well as mixed classical-quantum processes (e.g. preparations, measurements, instruments, controlled unitaries, etc.) [8]. Many other physical theories of interest can be formulated as process theories, even though they traditionally might not be. General relativity and quantum field theory, in particular, tend to take a “block-universe” approach (following Minkowski), as opposed to the “compositional” approach of process theories. However, recent work [20, 21] has provided initial evidence that those theories can be reformulated in a suitably compositional way, and be turned into process theories.

**PROCESS TERMINALITY**

Our starting point is the following principle, underpinning general scientific practice: the description of reality here and now cannot depend on things happening too far away in space, or in the future. In general relativity, this principle manifests itself as the causal structure of space-time. From the point of view of process theories, this means that there should be a way of discarding those systems which don’t affect our local description of reality, compatibly with the constraints imposed by relativity. Implementing this prescription can be achieved by providing a designated discarding effect for each system in the theory:

\[
\begin{array}{c}
\text{Discarding effect} \\
\hline
\end{array}
\]

A special case of the discarding effect appears when there is nothing to actually discard e.g. at the output of an effect. In that case, the discarding effect is simply denoted by an empty diagram:

\[
\begin{array}{c}
\hline
\end{array}
\]

In order for discarding effects to accurately model the idea that a system has been discarded, the processes in the theory have to obey the following constraint: discarding all outputs after a process \( f \) has occurred is equivalent to discarding all inputs beforehand; either way the system ends up discarded. Formally, the constraint is expressed by the following definition.

**Definition 1.** A process \( f \) is terminal if it satisfies:

\[
\begin{array}{c}
\begin{array}{c}
\hline
\end{array}
\end{array} = \begin{array}{c}
\text{Discarding effect} \\
\hline
\end{array}
\]

and a process theory is terminal if all its processes are.

Note that from this formal definition it clearly follows that we mean ‘truly discarding’, i.e. the process did not “stash away” any of its input, as there are no outputs, and diagrams are all there is to know about the theory. Within the more restricted context of Operational Probabilistic Theories [19], Definition 1 corresponds to the principle of ‘no signalling from the future’, which was introduced as a postulate in [19] and shown to be equivalent to uniqueness of deterministic effects.

The special of equation (2) where \( f \) is an effect immediately implies that the only effects are the discarding ones themselves:

\[
\begin{array}{c}
\begin{array}{c}
\hline
\end{array} = \begin{array}{c}
\hline
\end{array}
\end{array}
\]

In the case of probability theory, discarding effects are row vectors with all entries equal to 1, and in particular the empty diagram is the number 1 itself. Discarding a state in classical probability theory (a column vector with non-negative real components) amounts to producing a number equal to the sum of its components: hence a terminal state is one with components summing to 1, i.e. a probability distribution. More generally, discarding a process (a matrix with non-negative real components) amounts to producing a row vector equal to the sum of its rows: hence a terminal process is a matrix with entries of each column summing up to 1, i.e. a stochastic matrix. In the case of quantum theory, the discarding effect for a quantum system is the process of taking the (partial) trace of the system: hence terminal states are unit-trace density matrices, and terminal processes are trace-preserving CP maps.

It has been shown [19, 22] that equation (2) allows one to derive the “no-signalling” principle, showcasing a close connection between the notion of terminality and relativity theory. A much stronger result was obtained in [23], where it was shown that the very existence of relativistic causal structure is equivalent to terminality of a process theory.

**Theorem 2** [23]. A process theory is causal—in the sense of general relativity—if and only if it is terminal.

The importance of this result to the understanding of physics stems from the different mathematical standing of causal structure and process terminality. Traditionally, causal structure is defined by a partial ordering between events [24, 25], with the interpretation that
event $a$ can affect event $b$ if and only if $a \leq b$. Under this definition, causal structure is not intrinsic to a process theory, instead having to be imposed on top of it. By passing to process terminality, relativistic constraints are directly incorporated into the process theory, which becomes a full-fledged, stand-alone description of physical reality.

**MAIN RESULT**

A major advantage of the process-theoretic format is the canonical manner in which time-reversal arises, even in absence of a time parameter $t$: it is simply obtained by inverting the temporal reading order of the diagrams, turning our bottom-to-top convention into a top-to-bottom one without altering the diagrams themselves:

$$
\begin{array}{c}
\text{A} & \text{B} & \text{C} \\
\text{g} \\
\text{A} & \text{D} \\
\text{f} \\
\hline
\end{array} \\
\begin{array}{c}
\text{A} & \text{B} & \text{C} \\
\text{g} \\
\text{A} & \text{D} \\
\text{f} \\
\hline
\end{array}
\quad \begin{array}{c}
\text{A} & \text{B} & \text{C} \\
\text{g} \\
\text{A} & \text{D} \\
\text{f} \\
\hline
\end{array}
$$

Alternatively, we can retain our bottom-to-top reading convention, and flip the diagrams themselves upside-down, obtaining the corresponding time-reversed diagrams:

$$
\begin{array}{c}
\text{A} & \text{B} & \text{C} \\
\text{g} \\
\text{A} & \text{D} \\
\text{f} \\
\hline
\end{array} \\
\begin{array}{c}
\text{A} & \text{B} & \text{C} \\
\text{g} \\
\text{A} & \text{D} \\
\text{f} \\
\hline
\end{array}
$$

Notably, our definition of time-reversal does not include a specification of what time-reversing a box actually means: the reason for this is that such specifics are of no importance to our main result, which applies to any notion of time-reversal which respects the structure of diagrams. In the special case where the original process theory is causal, we refer to its time-reverse as a retro-causal process theory.

**Theorem 3 (Eternal noise).** In a retro-causal process theory every system has a unique state, namely the time-reversal of the discarding effect:

$$
\begin{array}{c}
\& \\
\hline
\end{array}
\quad \begin{array}{c}
\& \\
\hline
\end{array}
$$

*Proof.* By flipping the process terminality condition of equation (2) upside-down we obtain the following equation, holding for every process $\chi$ in the time-reverse of the theory:

$$
\begin{array}{c}
\& \\
\hline
\end{array} = \begin{array}{c}
\& \\
\hline
\end{array}
$$

In the special case where the process $\chi$ is chosen to be a state, we have that the process $f$ in the original theory must necessarily be an effect. But $f$ has to be terminal, i.e. it must be the discarding effect itself, and hence $\chi$ is always necessarily equal to the time-reverse of the discarding effect (we have used the fact that the time-reversal of the empty diagram is again the empty diagram). This concludes our proof.

A system with a single state cannot possibly carry any information, so we can naturally interpret this state as (white) noise. Taking the standard notion of time reversal within quantum and classical theory (as provided by the Hermitian adjoint and transpose respectively [8]), we find that this is explicitly the case. Hence the result of Theorem 3 can be restated as follows: any retro-causal process theory is necessarily in a state of eternal (white) noise.

**EXAMPLE: TIME-REVERSING QUANTUM THEORY**

To illustrate the extremely general result from the previous section, we will now consider the concrete example of quantum theory. Quantum theory is captured by a process theory in which systems correspond to finite dimensional Hilbert spaces, and processes to CPTP maps between them. Typically time-reversal is described by the Hermitian Adjoint, which respects the structure of diagram as required by our assumptions [7, 8].

If we consider reversible dynamics for a quantum system, i.e. a unitary $U$, then the corresponding retro-causal process is $\Omega = U^\dagger$. Thise is clearly a suitable choice of time-reversed process, since it ‘undoes’ the evolution of the quantum system as expected:

$$
\begin{array}{c}
\& \\
\hline
\end{array} = \begin{array}{c}
\& \\
\hline
\end{array}
$$

Note that any unitary will belong to both the causal and retro-causal theory: this follows from the fact that the time-reverse of the discarding effect is proportional to the maximally mixed state, and it means that unitaries are unital CPTP maps. Because unitaries belong to both, we must go beyond reversible dynamics to observe the distinction between the causal and retro-causal theories. Specifically, we will consider measurements, i.e. physical interactions between quantum and classical systems. In
order to readily distinguish between classical and quantum systems, we will adopt the graphical convention of \[8\], by which quantum systems are denoted by thick wires and classical systems are denoted by thin wires. For example, a non-demolition measurement associated with a quantum observable—a complete family \((P_x)_{x \in X}\) of orthogonal projectors on a quantum system—is a process with quantum input and quantum-classical output: it takes a quantum state, non-deterministically applies a projector \(P_x\) form the observable to it, and returns the projected state in output, together with the classical label \(x \in X\) identifying the projector that has been applied. In the diagrammatic style of \[8\] and \[27\], one such non-demolition measurement takes the following form:

\[
\overline{x} = \sum_{x \in X} P_x \overline{x}
\]  

(5)

The corresponding demolition measurement is the process which is obtained by discarding the quantum output:

\[
\overline{x} := \overline{x} = \sum_{x \in X} P_x \overline{x}
\]  

(6)

The demolition measurement is a quantum-to-classical process. Applied to a density matrix \(\rho\) it results in a probability distribution \(P[x \in X]\) on the set \(X\) indexing the projectors:

\[
P[x \in X] = \text{Tr}(P_x \rho)
\]

The time-reverse of the non-demolition measurement from equation (5) is the following process:

\[
\overline{x} = \sum_{x \in X} P_x \overline{x}
\]  

(7)

The classical outcome is now classical control, and picks out a choice of projector to be applied to the quantum system. Similarly, the time-reverse of the demolition measurement from equation (6) is now a classically controlled preparation of quantum states:

\[
\overline{x} := \overline{x} = \sum_{x \in X} P_x \overline{x}
\]  

(8)

In a retro-causal theory, however, the only classical state available to control these projector-selection and state-preparation processes is the uniform probability distribution over controlling states. As a consequence, all we can achieve by classical control in time-reversed quantum theory is decoherence in an observable and preparation of the totally mixed state:

\[
\overline{x} = \sum_{x \in X} P_x
\]  

(9)

This observation resolves an apparent contradiction between this work and the result in \[2\] stating that the time-reverse of classical probability theory is a signalling theory: in principle, the retro-causal classical theory does admit signalling processes, but there is no information to actually signal, only eternal noise. One might therefore be tempted to ask: what happens if we de-couple classical agents from the theory that we reverse, and retain their normal abilities?

**NO-GO: RETAINING AN AGENT’S CONTROL**

A process theoretic perspective highlights how de-coupling classical agents from the physical theory is a somewhat odd thing to do: if everything else is modelled explicitly, there is no real reason to believe that agents should not. In other approaches this may appear less odd, but it is only because agents held an external position in the first place. Thus said, de-coupling agents from the theory does not solve our problems: as it turns out, the agent would still be able to superluminally signal. Indeed, all they have to do is apply the time-reverse of a non-demolition Bell measurement to deterministically realize the projector on the Bell state (where we have exploited the Choi isomorphism to represent all Bell-basis projectors in terms of the Bell state):

\[
\overline{x} = \sum_{x \in X} P_x
\]

This in turn result in the possibility of performing deterministic quantum teleportation \[28\]:

Because the agent is now in possession of states other than noise, deterministic quantum teleportation immediately leads to a violation of no-signalling. For example, they could use the time-reverse of the demolition Pauli Z measurement to selectively prepare computation basis states, which they would then deterministically teleport:

\[
\overline{x} = \overline{x}
\]  

for \(x \in \{0, 1\}\)
We conclude that retaining an agent’s full classical control does nothing to help restore time-symmetry: the original causal theory is non-signalling, while the retro-causal theory is signalling, so the symmetry is broken.

DISCUSSION AND CONCLUSION

The physical interpretation of our main result depends on a choice of ontological stance on states and processes. In modern physics, the standard assumption is that the state of a physical system describes its actual “state of affairs”, evolved and studied by solving appropriate sets of equations: for example, the ontic state of a physical system in classical Hamiltonian mechanics is fully described by a considering a point in its phase space, and can be evolved in time by solving Hamilton’s Equations. In this sense, a retro-causal process theory is truly eternal noise, and hence utterly trivial.

One could consider the alternative, long-standing viewpoint that it is processes, rather than states, that fully characterize the “state of affairs” of physical systems: it is becoming that matters, rather than being [29, 30]. In this case, the retro-causal theory is no longer trivial, as it ends up having as many processes as the causal theory originally had. It would therefore be tempting to conclude that a causal process theory and its time-reverse have the same “informational content”, with the only distinction that inputs and outputs have exchanged roles [31]. However, these two process theories are not at all equivalent: the processes that can be applied to a particular system will be different for the two theories, and in particular the states never coincide!

The results we present here seem to be in conflict with recent time-symmetric approaches to quantum theory, such as those by [1, 2, 3]. The resolution of this apparent conflict lies in the observation that those time-symmetric theories are not terminal in their most general formulation: they are not a priori compatible with relativity, at least not in the compositional, process-theoretic sense laid out by [21]. Instead, asymmetric boundary conditions must be imposed to ensure compatibility with causal structure (e.g. to forbid superluminal signalling). It was shown in [4] that suitable boundary conditions and dynamics can be chosen so that process terminality is recovered: in that case, the results of the time-symmetric framework coincide with those presented in this work. To us, however, this entire setup seems unnecessarily complicated: after all, relativistic principles do not make any obvious statements about the boundary conditions of the universe. In contrast, the terminality condition that we impose is directly related to the causal structure of relativity, and as such we deem it to be a far more natural constraint.

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