Loop Corrections to Primordial Non-Gaussianity

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We discuss quantum gravitational loop effects to observable quantities such as curvature power spectrum and primordial non-gaussianity of Cosmic Microwave Background (CMB) radiation. We first review the previously shown case where one gets a time dependence for zeta-zeta correlator due to loop corrections. Then we investigate the effect of loop corrections to primordial non-gaussianity of CMB. We conclude that, even with a single scalar inflaton, one might get a huge value for non-gaussianity which would exceed the observed value by at least 30 orders of magnitude. Finally we discuss the consequences of this result for scalar driven inflationary models.

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I. INTRODUCTION

Most probably, the founders of quantum gravity did not have high hopes that what they were doing would someday be tested or even have observational consequences. The CMB power spectrum opened that window to us and now we can name cosmological perturbations as first quantum gravitational observables that were predicted by Mukhanov and Chibisov [1] for scalar part and by Starobinsky [2] for tensor part.

Since we already measured the lowest order effect in perturbation theory, the next logical step in any quantum field theory calculation is to go beyond this level, so-called the tree-level. This is mostly done in order to make precision tests of a particular model. Although one would expect only small corrections to already known physics(precision predictions) by calculating these higher order terms in perturbation theory, i.e. loops, new phenomena beyond our expectations can arise as was the case in the famous 1-loop beta function calculation in quantum chromodynamics.

For the last fifteen years, there have been many efforts towards understanding loops in cosmology. Among these relatively large literature, the most influential works were that of Weinberg’s [3, 4]. In one these works, Weinberg asserted a theorem [4] related to quantum loop effects in cosmology:

in N-th order perturbation theory, quantum corrections can at most be of order \((\alpha \ln a(t))^N\), where \(\alpha\) is the loop counting parameter and \(a(t)\) is the scale factor.

There were plenty of discussions [5–9] not about the existence but the type of “infrared logarithm” that arise from the quantum contributions in cosmological correlations. Two types of infrared logarithm factors that appeared are: time-dependent and time-independent. The obvious enticing aspect of time-dependent infrared logarithm is that it grows with time. Having this case, the smallness of the loop counting parameter in quantum gravity gets counterbalanced by time-dependent infrared logarithms. If one assumes the we observe 50 e-folds of inflationary era, this time-dependent enhancement would only bring a factor of 50. Therefore there is not much hope of observing this effect any time soon.

Most of the discussions were around the quantum corrections to 2-point correlation functions, i.e. power spectrum; since it is a quantity which is measured more accurately. The debate between time-dependence and time-independence camp went on and both parties published explicit calculations to support their claims [5–9]. The author of this work also contributed to this discussion claiming the time-dependence. In this work, we do not want to discuss the strengths and weaknesses of each point of view, but rather want to point out that the “small” time-dependent loop effects are not really that small; if one looks at higher orders of correlation functions such as 3-point functions and even higher.

The loop effects to 3-point function was discussed in two separate works. The first one is work of Giddings and Sloth where they assumed the semi-classical approximation holds [10]. The second work is the work of Cogollo et al. [11] where there is an extra scalar field which induces time-dependent effect. In this we will show that the 1-loop correction might dominate the tree-level term by 30 orders of magnitude. This would immediately lead to ruling out all single field inflationary models due to observational constraints on non-Gaussianity of the primordial curvature perturbation [12]. This claim might appear very odd, if one naively looks at the above theorem of Weinberg. But it turns out that
the constancy of the tree-level mode function and time-dependency of 1-loop correction determines the faith of their contribution to the 3-point function, bispectrum.

This paper is organized as follows. In Section II, we review the case of time-dependent zeta-zeta correlator arising from self interaction of zeta at 1-loop order. In Section III, we use this 1-loop corrected time-dependent mode function to calculate 1-loop corrected primordial bispectrum. We give our conclusions in Section IV.

II. SLOW-ROLL INFLATION AND TIME DEPENDENT ZETA-ZETA CORRELATOR

The action of the model that we would like to consider consists of a scalar field \( \phi \), an Einstein-Hilbert part and a standard kinetic term

\[
S = \int d^4x \sqrt{-g} \left[ \frac{R}{16\pi G} - \frac{1}{2} g_{\mu\nu} \phi_{,\mu} \phi_{,\nu} - V(\phi) \right].
\]  

(1)

The metric is that of the FRW one which our universe seems to prefer,

\[
d\tilde{s}^2 = -dt^2 + a^2(t)d\vec{x} \cdot d\vec{x} \Rightarrow H \equiv \frac{\dot{a}}{a}.
\]  

(2)

We choose the background inflaton field to be constant at equal-time hypersurfaces as Maldacena [13] and Weinberg [3],

\[
\phi(t, \vec{x}) - \phi_0(t) = 0.
\]  

(3)

The other \((D - 1)\) conditions come from defining the uni-modular part of the metric \( \tilde{g}_{ij} \),

\[
g_{ij} = a^2(t)e^{2\zeta(t, \vec{x})} \tilde{g}_{ij}(t, \vec{x}) \Rightarrow \sqrt{g} = a^{D-1}e^{(D-1)\zeta}.
\]  

(4)

By choosing the gauge in the above manner we switched from inflaton field, which is the dynamical variable \( \phi \) of our theory, to \( \zeta \) now parametrizing scalar fluctuations. Using Einstein’s equations for the background scalar field \( \phi \), one can express its time-derivative in terms of the Hubble parameter as

\[
\dot{\phi}^2 = -\frac{\dot{H}}{4\pi G}.
\]  

(5)

Another important quantity, called the first slow-roll parameter \( \epsilon \), is defined as

\[
\epsilon \equiv -\frac{\dot{H}}{H^2} \ll 1.
\]  

(6)

The next step is using perturbation theory for small fluctuations of the scalar and tensor fields. It became customary to use ADM formalism to get the quadratic, cubic and even higher order parts of the action. The quadratic part of the action for \( \zeta \) is [13]:

\[
S^{(2)}_\zeta = \frac{1}{8\pi G} \int d^4x \epsilon \left[ a^3 \dot{\zeta}^2 - a(\partial \zeta)^2 \right]
\]  

(7)

and the cubic part of zeta is:

\[
S^{(3)}_\zeta = \frac{1}{2\pi G} \int d^4x \epsilon^2 a^5 H \dot{\zeta}^2 \partial^{-2} \zeta \equiv \int dt L_3(t).
\]  

(8)

One can vary the quadratic part of the action and equate to zero, to get the equation of motion for \( \zeta \) as

\[-\partial_t(a^3 \epsilon \dot{\zeta}) + a \epsilon \partial^2 \zeta = 0.\]  

(9)

The standard way of solving this equation for quantum fields is going into momentum space and expressing \( \zeta \) as a mode sum

\[
\zeta_k(t) = u_k(t)a_k + u_k^*(t)a^*_k.
\]  

(10)
where $a_{-k}$ and $a_k$ are creation and annihilation operators that obey canonical quantization conditions.

It is best to go to conformal time to write the expression for the mode function,

$$d\eta \equiv -adt \Rightarrow ds^2 = -dt^2 + a^2(t) \, d\vec{x} \cdot d\vec{x} = a^2(t)(-d\eta^2 + d\vec{x} \cdot d\vec{x}),$$

(11)

so that the geometry is conformally flat. Another motivation for choosing conformal time is related to the fact that there is not a unique choice of vacuum in curved space. One takes the expression (10) and uses that to solve equation (9) for $u_k(\eta)$

$$u_k(\eta) = \frac{H}{\sqrt{2\epsilon}} \frac{1}{\sqrt{2k^3}} (1 + ik\eta)e^{-ik\eta},$$

(12)

which corresponds to the positive frequency modes. By choosing conformal time for coordinate system, it is easy to see that this solution for the mode function behaves like Minkowski in the early time limit. This solution is called Bunch-Davies vacuum solution [14].

Let us define the curvature power spectrum:

$$\Delta_R^2(k,t) \equiv k^3 \int d^3x \, e^{-i\vec{k} \cdot \vec{x}} \langle \Omega | R(t,\vec{0})R(t,\vec{x}) | \Omega \rangle$$

(13)

where $R$ is related to the 3-curvature and is equal to $\zeta$ at the linearized order [15]:

$$R(t,\vec{x}) \equiv -\frac{a^2(t)}{4\nabla^2} R = \zeta(t,\vec{x}) + O(\zeta^2, \zeta h, h^2).$$

(14)

Therefore curvature power spectrum also goes by the name “zeta-zeta correlator” as well. The latest value of the curvature power spectrum constructed from measurements is [16]

$$\Delta_R^2(k) = \left( 2.198^{+0.076}_{-0.085} \right) \times 10^{-9} \left( \frac{k}{0.002 \text{ Mpc}^{-1}} \right)^{-0.0345 \pm 0.0062}.$$  

(15)

And the theoretical prediction at tree-level gives us

$$\left[ \Delta_R^2(k) \right]_{\text{tree}} \approx \frac{4Gk^3}{\pi} \times |u(t,k)|^2 \approx \frac{GH^2(t_k)}{\pi\epsilon}.$$  

(16)

Although one would expect that tree-order quantum gravity calculations to capture the full effect, it is natural to wonder what happens beyond that. Therefore we want to know if loop corrections to this measurable quantity make any difference. There have been many efforts to answer this questions in the last ten years [17–28].

In a particular curious case [6] it was shown that one can get an enhanced time-dependent correction to $\zeta-\zeta$ correlator at 1-loop order coming from the Feynman diagrams in Figure 1. The 1-loop corrected curvature power spectrum gives

$$\left[ \Delta_R^2(k,t) \right]_{\zeta \text{ loops}} \approx \frac{GH^2}{\pi\epsilon} \left( 1 + \frac{27GH^2}{4\pi\epsilon} \ln(a) + O(G^2H^4) \right).$$

(17)

This corresponds to a correction to the tree-level scalar mode function as:

$$\Rightarrow u(t,k) \approx \frac{H}{\sqrt{2\epsilon}} \frac{1}{\sqrt{2k^3}} \left( 1 + \frac{27GH^2}{8\pi\epsilon} \ln(a) \right),$$

(18)

if one uses Hartree approximation.
The $\zeta\zeta$ correlator becomes time dependent if there is least one undifferentiated field in the action at the relevant order. These so called infrared logarithms, as well as $1/\epsilon$ term, enhance this 1-loop effect by 3 orders of magnitude. But the smallness of $GH^2 \approx 10^{-10}$ overshadows this enhancement and make the total 1-loop correction to be at most at the order of $10^{-6}$. The degree of the precision of the current experiments are well below the necessary level to untangle this 1-loop effect. But still the effect is not hopelessly small.

For the last five years there has been some discussion about the time-dependence of the $\zeta\zeta$ correlator. It has even been claimed [9] that this quantity is constant at all loops, which we find to be highly dubious since even at tree-level it only asymptotes to a constant. The point of this work is not to argue the time-dependence of $\zeta\zeta$ correlator, but rather go towards another direction; which is loops corrections to 3-point function and to see what the consequences of time dependence of $\zeta$ are. We believe that the real enhancement of time-dependent $\zeta\zeta$ correlator arises if one calculates 3-point function for $\zeta$. It turns out the 1-loop correction to this quantity might totally dominate the tree-level result. Therefore when searching for enhanced quantum gravity corrections, the more interesting quantity to calculate is the 3-point function, which is the subject of the next section.

### III. BISPECTRUM AT TREE-LEVEL

One can write the primordial bispectrum in terms of the Fourier transformed 3-point function as

$$\langle \zeta_{k_1} \zeta_{k_2} \zeta_{k_3} \rangle = (2\pi)^3 \delta^3(k_1 + k_2 + k_3) B_\zeta(k_1, k_2, k_3).$$

(19)

Assuming a local form for the bispectrum where the non-Gaussian $\zeta$ field is produced from the Gaussian background $\zeta_g$ field as

$$\zeta(x) = \zeta_g(x) + (3/5) f_{NL} \zeta_g^3(x) + \mathcal{O}(\zeta_g^3).$$

(20)

One can show that the bispectrum peaks at the so called “squeezed” triangle, for which one takes one wave number much smaller than the other two, i.e. $k_1 \approx k_2 \gg k_3$. For the case of squeezed limit bispectrum can be expressed in terms of power spectrum as

$$B^{\text{local}}_\zeta(k_1, k_2, k_3)_{k_1 \approx k_2 \gg k_3} \approx \frac{12}{5} f_{NL} P(k_1) P(k_3),$$

(21)

where the late time limit of the power spectrum is

$$P(k) = \left| u_k \right|^2_{\eta \rightarrow 0} = \frac{H^2}{2\epsilon} \frac{1}{2k^3}.$$}

(22)

If Creminelli-Zaldarriaga consistency [29] condition for single field inflation models hold, bispectrum in the local limit (or squeezed-limit) can be written as

$$B_\zeta(k_3 \ll k_1) = (1 - n_s) P(k_1) P(k_3),$$

(23)

where $n_s(k)$ is called the spectral tilt index and is defined as

$$n_s - 1 \equiv \frac{dln(P(k))}{dln(k)}.$$}

(24)

Instead of expressing the 3-point function in terms of 2-point functions, one can directly compute the in-in expectation value of $\zeta$ and use [33] for the interaction Hamiltonian and can get the following expression for the bispectrum

$$B_\zeta(k_1, k_2, k_3) = 8i \frac{\epsilon^2}{H^2} \sum_{k_i} \left( \frac{1}{k_i^2} \right) u_{k_1}(\bar{\eta}) u_{k_2}(\bar{\eta}) u_{k_3}(\bar{\eta}) \int_{\eta_0}^{\bar{\eta}} \frac{d\eta}{\eta^3} u_{k_1}^* u_{k_2}^* u_{k_3}^* + c.c. \right.$$}

(25)

The main point of this calculation is the integral that we have in the above expression for the bispectrum

$$\int_{\eta_0}^{\bar{\eta}} \frac{d\eta}{\eta^3} u_{k_1}^* u_{k_2}^* u_{k_3}^*.$$

(26)
If we take the tree-order mode function for the above expression it is obvious that we will get a small non-gaussianity. This is due to the fact that the 3-point function, therefore non-gaussianity, is proportional to the change of the mode function for each wave number $k_1, k_2, k_3$. Since for each mode the mode function itself goes to a constant after the horizon crossing, change of those tree-level mode functions will be very small. On the other hand, the 2-point function, therefore power spectrum, is proportional to the magnitude of the mode function. Let us highlight this point by giving equations:

\[
\text{Power Spectrum} \sim \langle \zeta_{k_1}\zeta_{k_2}\rangle \sim \delta^3(k_1 + k_2)|u_k|^2 \\
\text{Non-Gaussianity} \sim \langle \zeta_{k_1}\zeta_{k_2}\zeta_{k_3}\rangle \sim \delta^3(k_1 + k_2 + k_3)u_{k_1}(\bar{\eta})u_{k_2}(\bar{\eta})u_{k_3}(\bar{\eta}) \int_{\eta_0}^{\bar{\eta}} d\eta \frac{1}{\eta^2} u_{k_1}^* u_{k_2}^* u_{k_3}^* + \text{c.c.} \quad (27)
\]

\[u_{\text{tree}} = \frac{H}{\sqrt{2\epsilon}} \frac{1}{\sqrt{2k^3}} (1 + i k \eta)e^{-i k \eta} \implies \frac{H}{\sqrt{2}} \frac{1}{\sqrt{2k^3}} \left\{ 1 + \frac{k^2 \eta^2}{2} + \ldots \right\}, \quad (28)\]

\[u_{1-\text{loop}} \implies \frac{H}{\sqrt{2\epsilon}} \frac{1}{\sqrt{2k^3}} \left\{ 1 + \mathcal{O}(1) \right\} = u_{\text{tree}} \left\{ 1 + \mathcal{O}(1) \right\}, \quad (29)\]

\[u'_{\text{tree}} \implies \frac{H}{\sqrt{2\epsilon}} \frac{1}{\sqrt{2k^3}} \left\{ k^2 \eta + \ldots \right\}, \quad (30)\]

\[u'_{1-\text{loop}} \implies \frac{H}{\sqrt{2\epsilon}} \frac{1}{\sqrt{2k^3}} k^2 \eta \left\{ 1 + \mathcal{O}(1) \right\} = u'_{\text{tree}} \left\{ 1 + \mathcal{O}(1) \right\}, \quad (31)\]

The difference between the one-loop corrected mode function’s and tree-level mode function’s time derivative is of the order of $GH^2 \approx 10^{-10}$ as expected, but also multiplied with an additional factor of $1/k^2 \eta^2$. For the super-horizon

\[
\text{FIG. 2: Bispectrum: Tree-level and corrections to external legs.}
\]
modes ($k\eta \ll 1$) with the relevant 50 e-folds this brings an extra factor of $10^{20}$ which makes the 1-loop correction to
dwarf the tree-level part of the mode function.

The mathematical reasons for this huge effect is the following. The derivative with respect to conformal time brings
an extra factor of $H a(t)$ when it acts on a power of $a(t)$. Since the tree-level mode function is constant after horizon
crossing, this time derivative does no good to boost the leading term, it simply annihilates it. But for the case 1-loop
corrected mode function, the time derivative acts on the $\ln a$ and gives a chance to the constant leading term of the

The integrals that appear in (27) can be evaluated [32] analytically in a general vacuum choice, called non-Bunch-
Davies initial state which would lead to an increase of the non-Gaussianity parameter $f_{NL}$ even at the tree-level
[33, 34]. For the case of non-Bunch-Davies initial state the one-loop term again dominates the tree-level one and be
ruled out as well.

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V. DISCUSSION

The success of inflationary cosmology is appalling. This simple idea solves homogeneity, flatness, horizon, isotropy
and primordial monopole problems of standard cosmology with a single shot [35–37]. With inflation, linking quantum
physics with cosmology, we can understand the origin of all matter from primordial quantum fluctuations. Getting
first quantum gravitational data, such as curvature power spectrum with small error bars is a major success in itself.

It is therefore time to go beyond this tree-level effect and investigate possible consequences, which we can name as
precision inflationary cosmology. Towards this direction, one possible thing to do is calculating loop corrections to
cosmological correlations. At first look, one would naturally think that this is a futile effort due to the smallness of
the loop counting parameter $GH^2$. There was a lot of attention to loop corrections to power spectrum despite the
smallness of them.

Cosmological loop corrections bring a typical infrared logarithm and are divided into three categories according
to the form of the logarithmic factors: $\log(Ha)$, $\log(kL)$ and $\log(a(t))$ [8]. The first case is claimed to be due to
making an error in the implementing diffeomorphism invariant regularization and the second being a projection effect
that will be removed if one computes observable quantities. The final case is also dismissed in the mentioned work on
the grounds of symmetry arguments as well as extrapolating this time-dependent effect to reheating and baryogenesis
and claiming that predictivity of inflation will be lost. One can certainly reply to the above criticisms and perhaps
one should. But in this work, we would like to bring a different viewpoint to this discussion that is more dramatic.

First of all, time-dependent zeta do occur even without loop corrections, such as multi-field inflationary models and
entropy perturbations. The time-dependence that we are interested in, that has the form of $\log(a(t))$, are originated
from loop corrections. We investigated the minimal case where the only scalar field is the inflaton. We first reviewed
time-dependent loop corrections to 2-point functions, i.e. power spectrum, which arises due to $\zeta$-$\zeta$ self interactions
at 1-loop order. They in principle are important; on the other hand from an observational perspective are irrelevant
at the moment. We took the 1-loop corrected scalar mode function and used that to correct 3-point functions and
concluded that they might grow with the square of the scale factor. That would result into an immediate ruling out
all single-scalar driven models of inflation.

Therefore, non gaussianity is a better place, compared to power spectrum, to look for quantum gravitational

corrections. The main reason of this is:
1. Power spectrum is related to the magnitude of mode function. 
   Therefore it goes like: constant(tree-level) + a small correction (loops)

2. Non-Gaussianity is related to the time derivative of the mode function. 
   Therefore it goes like: almost zero(tree-level) + a not so small correction(loops) compared to zero

Therefore the real treasure is hidden in the higher order correlation functions not in the power spectrum. We also showed that this would imply a huge (10^{20} times bigger than tree-level prediction) non-Gaussianity $f_{NL}$ parameter, leading to an immediately contradiction with the constraints on observed value of $f_{NL}$ parameter.

At this point let us discuss the possible ways of avoiding huge $f_{NL}$ parameter. It might turn out that the diagram that we did not include at 1-loop order might cancel the total contribution coming from Figure 2. In the work of

![FIG. 3: The remaining diagram at 1-loop order](image)

Cogolo et al. [11] it was shown that this kind of diagram dominates the whole series of diagrams. They consider and extra scalar field and the effect arises due to that, but still a similar thing might happen for the single scalar field situation.

If the above do not happen at least one of our assumptions should not be true. The main ones are:

- Hartree approximation
- Single scalar field(inflaton), adiabatic perturbations
- Time-dependent $\zeta$-$\zeta$ correlator from loops
- Almost constant slow-roll parameter $\epsilon$.

It might be that using Hartree approximation is the source of the mentioned effect. For that, one should do the full calculation and see if the effect is not there. One can imagine scenarios where a spectator field causing a similar effect which might cancel the $\zeta$ loops, which only happens for particular situations [11]. Another possibility is summing the whole perturbative series which was done by Starobinsky & Yokoyoma [38] for self interacting scalar fields and Tsamis & Woodard for SQED [39]. Then investigate the consequences of the non-perturbative result. Or one can simply say that it is the third assumption that is wrong and maybe it is so. But no matter what the solution is, quantum loop corrections that result into time-dependent scalar mode functions have consequences that are so important and will be of such magnitude that they can not be swept under the rug.
