Interaction-free measurements with superconducting qubits

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An interaction-free measurement protocol is described for a quantum circuit consisting of a superconducting qubit and a read-out Josephson junction. By measuring the state of the qubit one can ascertain the presence of a current pulse through the circuit at a previous time without any energy exchange between the qubit and the pulse.

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We know from classical optics that when an object blocks one arm of an interferometer the fringes disappear. In quantum physics, this effect acquires far more subtle features, and, counter-intuitively, it does not originate from the unavoidable disturbance on the photon trajectory or random phases due to scattering, but from the possibility of obtaining which-path information. Indeed, in the quantum treatment the interference would disappear if one can in principle obtain information about which path the light went through, even if this information is not actually extracted [1]. Quantum processes therefore result not only in changes in observables such as position and momentum (due to energy exchange), but, more importantly, in establishing new correlations between parts of the system. Recently, a lot of work has been put into harnessing the power of these correlations for performing computational tasks which are very difficult to implement on classical computers. Superconducting qubits based on the Josephson effect have been proposed [2] as the elements of future quantum computers, based on macroscopic quantum coherence effects in charge and flux devices [3]. Several species of superconducting qubits are currently under study, for example charge qubits [4], phase qubits [5], flux qubits [6], and a mixed charge-flux version called Quantronium [7]. This last type has a very large decoherence time (more than 500ns), and it will be the main focus of this paper. Besides quantum computing, fundamental research such as testing quantum mechanics at the macroscopic level is an important direction envisioned decades ago [8], with progress in this direction now enjoying a firm experimental basis.

In this paper we propose an experiment in which a Quantronium device (Fig. 1) could be used to ascertain the presence of a small pulse of electric current without any disturbance due to energy exchange with the continuum of states outside the washboard potential well in which the qubit is localized. An experiment of this type is feasible with the current Quantronium setup, and it would constitute a test, at the macroscopic level, of a strongly nonclassical prediction of quantum mechanics. The proposal is based on the interaction-free measurement scheme proposed by Elitzur and Vaidman for optical Mach-Zehnder interferometers [9]. This realization has found interesting applications in interaction-free imaging of objects [10], and more recently in quantum computing [11].

In the optical setup (upper schematic in Fig. 2), a balanced Mach-Zehnder interferometer is constructed from two 50% beam splitters, two perfectly reflecting mirrors, and two detectors D+ and D− (+ and − are the directions corresponding respectively to paths along the upper and lower arms of the interferometer). In the absence of an object in the lower arm this arrangement produces a destructive interference at the detector D+ (the state of the photon, which is initially |+⟩, becomes ⟨+ | +i|−⟩/√2 inside the interferometer and |i|−⟩ after the second beam splitter). If a quantum ultrasensitive object (i.e. triggered by the absorption of a single photon) is present, in 50% of the cases there will be no absorption event: the photon which has traveled in the upper arm of the interferometer must be then in the state |→⟩ (it did not "interact" with the object), and it will emerge from the second beam splitter in the state ⟨→ | +i|−⟩/√2, having now a 50% chance of being detected by D+ [9]. As a result, when D+ is triggered we can be certain about the presence of an object in a region in space without having to exchange energy (e.g. by photon absorption) with that object. The success rate (the fraction of photons detected by D+) is 25%.

![FIG. 1: Quantronium circuit with flux-current compensation.](image-url)

A Quantronium device (Fig. 1) consists of a split Cooper pair box of total capacitance $C_2$ and Josephson energy $E_J/2$ per junction, operated at a gate voltage $V_g$ and excited by microwave radiation coupled through the gate capacitance $C_g$. The box is connected in parallel...
with a read-out Josephson junction $E_{J0}$ of capacitance $C_0$. The Hamiltonian of the circuit is

$$H = \frac{1}{2C_0\Sigma}(q - C_g V_g)^2 - E_J \cos \frac{\gamma + \phi}{2} \cos \theta$$

$$+ \frac{Q^2}{2C_0} - E_{J0} \cos \gamma - I\phi_0 \gamma,$$

(1)

where $Q$ and $\gamma$ are the charge and the phase across the large junction, $I$ is the bias current, $\phi_0$ is the externally applied magnetic flux through the loop, $\phi_0 = h/2e$ is the reduced flux quanta, and $(q, \theta)$ is the pair of conjugate variables (commutation $[\phi_0\theta, q] = ih$) corresponding to the split Cooper pair box. The read-out junction is in the large-capacitance regime, $C_0 \gg e^2E_{J0}$, where $Q$ becomes a continuous operator $Q = -2e i \partial/\partial \gamma$. A two-level system (the qubit) is realized [2, 4] at the gate voltage $V_g = e/2g$, where the charging-energy degeneracy of the eigenvectors $|0\rangle$ and $|1\rangle$ of the operator $q$ ($q|0\rangle = 0$, $q|1\rangle = 2e|1\rangle$) is lifted by the tunneling term $-E_J \cos(\gamma/2 + \phi/2) \cos \theta$. The eigenvectors of this tunneling term (the qubit levels) are denoted by $|+\rangle$ (ground state) and $|\rangle$ (excited state), $|\pm\rangle = (|0\rangle \pm |1\rangle)/\sqrt{2}$, and in this basis the Hamiltonian Eq. (1) is of Stern-Gerlach type

$$H = -\frac{2e^2}{C_0 \Sigma^2} - E_{J0} \cos \gamma - I\phi_0 \gamma - \frac{E_J}{2} \cos \frac{\gamma + \phi}{2} \sigma_z.$$

(2)

The bias current and the flux are externally controlled parameters $(\phi(t), I(t))$ which are manipulated adiabatically compared to the time-scale of the qubit frequency. The eigenvalue-eigenfunction problem $H\psi_{\pm}(\gamma) \pm > = \epsilon_{\pm} \psi_{\pm}(\gamma) \pm >$ can be thus solved at every moment $t$, where, to keep the notation simple, we will specify the corresponding values of $(\phi, I)$ each time we refer to $\epsilon_{\pm}$. The phase $\gamma$ is almost classical-like ($\gamma \approx 0$), and the qubit energy is $\epsilon = \epsilon_+ - \epsilon_- \approx E_J$ [2]. Adiabatic excitations at nonzero values of $I$ help differentiate between the states $\pm$: a typical measurement protocol for this circuit proceeds by raising the bias current to a value close to the critical current $E_{J0} \phi_0^{-1}$ of the large read-out junction for a longer read-out time $\tau_p$. The junction then can tunnel in the running-wave state $|\pm\rangle$ with a switching rate $\Gamma_{\pm}(\gamma)$ [2] which depends on the state $|\pm\rangle$ of the qubit.

The idea of the proposed experiment is to create the time-equivalent of a Mach-Zehnder interaction-free experiment by inserting a bias pulse (referred to in the following as “interaction pulse”) inside a Ramsey sequence of $\pi/2$ microwave gate pulses. Ramsey techniques are common to many fields of physics; they can be interpreted as two-paths interferometry, the first $\pi/2$ pulse separating the paths and the second rejoining them to test what has happened between. The experiment proceeds as follows (Fig. 2): at all times, the system is kept at the charge degeneracy point $C_g V_g = e$ where the decoherence due to gate voltage fluctuations is zero in the first order. A first $\pi/2$ pulse initializes the qubit in an equal-weight superposition of the states $|+\rangle$ and $|\rangle$. After that the bias current is increased adiabatically to a value $I^{(p)}$ that allows tunneling during a time interval $\tau_p$, then turned off back to zero. Finally, another $\pi/2$ pulse is applied and after that a standard switching current measurement sequence $[7, 12]$ follows. Therefore, in this proposal the role of the ultrasensitive quantum object is played by the quasi-continuum of modes outside the well of the washboard potential of the large junction, into which the phase $\gamma$ is allowed to tunnel during the interaction pulse.

![FIG. 2: Mach-Zehnder interferometric setup (upper schematic) and the Quantumtron pulse sequence (graphs below) with the axes of the flux graph slightly displaced for clarity.]

The duration of the whole procedure must be less than the decoherence time $T_2 \approx 500$ns, and the value of $I^{(p)}$ has to be such that the dominant switching process is macroscopic quantum tunneling (and not thermal activation). It is simple to maintain the qubit at the optimal charge parameter point $C_g V_g = e$, but changes in the bias current will displace the system along the phase direction [7]. We propose to compensate this displacement by adding a magnetic field pulse such that $\phi(t) = -\gamma_m(t)$, where $\gamma_m(t)$ is the phase corresponding to the minimum of the washboard well potential of the large junction at a time $t$ during the current pulse, defined by $\sin \gamma_m(t) = I(t)/E_{J0}$, $0 < \gamma_m(t) < \pi/2$. As a result, there will be no linear longitudinal noise component due to fluctuations of the flux in the loop (e.g. vortices moving in and out, fluctuations of the externally applied magnetic field) and the decoherence time $T_2$ is expected to stay of the order of $0.5\mu$s. Technically, it is easier to apply a magnetic field pulse and then model the current bias pulse according to the relation above $(I(t) = -E_{J0}\phi_0^{-1} \sin \phi(t))$, as shown in Fig. 2. This simultaneous manipulation of the bias current and flux results in a different physics than that of the usual read-out pulse. In the last case, the two states of the qubit are distinguished by the appearance of small electrical currents.
that substract from or add to the externally imposed bias current. In our procedure, during the excursion toward the switching point, these currents are maintained to zero to reduce decoherence; however, the two states are still distinguishable by different values of the plasma oscillation frequency and of the tunneling barrier height, which lead to different macroscopic quantum tunneling probabilities. Finally, one can notice that the qubit Larmor frequency $\epsilon$ remains unchanged during the compensated interaction pulses, therefore no extra phase differences are introduced in the Ramsey interference pattern.

At the plateau $I^{(p)}$ of the interaction pulse, by expanding the potential energy around $\gamma_m^{(p)} = \arcsin(I^{(p)} \phi_0 / E_{J0})$, we find the Josephson plasma frequency

and the corresponding washboard potential height

$$\Delta U^{(p)} = (2/3) E_{J0} \left[ \sqrt{1 - (I^{(p)} \phi_0 / E_{J0})^2} \pm E_J / 8E_{J0} \right]^3.$$  

The switching rate can be calculated in the WKB approximation \[3\] \[12\] and will be different for the two qubit states, $\Gamma^{(p)}_\pm = 52 \sqrt{\Delta U^{(p)} / \hbar \omega^{(p)}_\pm} (\omega^{(p)}_\pm / 2\pi) \exp \left( -7.2 \Delta U^{(p)} / \hbar \omega^{(p)}_\pm \right)$.

We start with the qubit in the ground state $\psi_+ (\gamma)|+> > 0$; after the first $\pi/2$ Ramsey pulse this state (all the states from now on will be written in the interaction picture) changes into the superposition $|+\rangle + i|\psi_-(\gamma)|- > \sqrt{2}$. In the absence of an interaction pulse, one can check that after the second Ramsey $\pi/2$ pulse the state of the system is $i|\psi_-(\gamma)|- > \sqrt{2}$ (destructive interference toward the $D+$ detector, in the Mach-Zehnder setup). Therefore, during the read-out pulse of length $\tau_r$, the non-switching probability is $\exp(-\Gamma^{(r)}_- \tau_r)$.

In the case in which we have an interaction pulse, the status of the quantum circuit immediately at the end of the interaction pulse depends on whether the large junction has switched or not. Immediately after the interaction pulse, if the large junction did not switch, which happens with probability $N = \exp(-\Gamma^{(p)}_+ \tau_p) + \exp(-\Gamma^{(p)}_- \tau_p) / 2$, the state of the circuit is $\sqrt{2}N^{-1/2} \exp(-\Gamma^{(p)}_+ \tau_p/2) |\psi_+ (\gamma)| + i \sqrt{2} N^{-1/2} \exp(-\Gamma^{(p)}_- \tau_p/2) |\psi_-(\gamma)| - >$. The second Ramsey pulse transforms this state into

$$(2\sqrt{N})^{-1} \left( e^{-\Gamma^{(p)}_+ \tau_p/2} - e^{-\Gamma^{(p)}_- \tau_p/2} \right) |\psi_+ (\gamma)| + > + (2\sqrt{N})^{-1} \left( e^{-\Gamma^{(p)}_+ \tau_p/2} + e^{-\Gamma^{(p)}_- \tau_p/2} \right) |\psi_-(\gamma)| - >.$$  

Finally, after the read-out pulse, the probability that the system did not switch during the entire sequence of pulses is a conditional probability obtained by multiplying the probability $N$ that the system did not switch during the first pulse with the nonswitch probabilities during the read-out pulse for the state Eq. \[3\], with the result

$$\left( e^{-\Gamma^{(p)}_+ \tau_p/2} - e^{-\Gamma^{(p)}_- \tau_p/2} \right) e^{-\Gamma^{(r)}_- \tau_r/4} + \left( e^{-\Gamma^{(p)}_+ \tau_p/2} + e^{-\Gamma^{(p)}_- \tau_p/2} \right) e^{-\Gamma^{(r)}_- \tau_r/4}.$$  

We analyze now the read-out pulse sequence. To avoid the case in which non-switching events occur in the absence of the interaction bias pulse, we have to set the corresponding probability $\exp(-\Gamma^{(r)}_- \tau_r)$ as small as possible: suppose we consider this spurious non-switching rate as satisfactory when it is of the order of $10^{-3}$. To determine the read-out bias pulse parameters, we will use directly the experimental data reported so far. Suppose we use a bias read-out current of 1.11 $\mu$A (94.9% of the critical current $E_{J0} \phi_0$); then at 14 mK the switching can be estimated as $\Gamma^{(r)}_+ = 15$ MHz, and $\Gamma^{(r)}_- = 2$ MHz. For a read-out time $\tau_r = 400$ ns we obtain $\exp(-\Gamma^{(r)}_- \tau_r) = 10^{-3}$ and $\exp(-\Gamma^{(r)}_- \tau_r) = 0.4$.

Let us now focus on the interaction pulses. For an interaction bias pulse of height $I^{(p)} \phi_0 / E_{J0} = 95.2\%$, we obtain $\Gamma^{(p)}_+ = 2.22$ MHz and $\Gamma^{(p)}_- = 6.58$ MHz. Since $\exp(-\Gamma^{(r)}_- \tau_r)$ is negligible, we find from Eq. \[4\] that the maximum success rate is obtained when the difference between the nonswitching probabilities of the two states, $\exp(-\Gamma^{(p)}_+ \tau_p/2) - \exp(-\Gamma^{(p)}_- \tau_p/2)$ is maximal. For $\tau_p \approx 250$ ns – a value of the same order as that used routinely in switching current experiments – we reach $\exp(-\Gamma^{(p)}_+ \tau_p/2) - \exp(-\Gamma^{(p)}_- \tau_p/2) = 2.5\%$. Note that the pulse duration is below 500 ns; we have checked also that the thermal activation rate is negligible. Now, taking into account the $\exp(-\Gamma^{(p)}_- \tau_r) = 0.4$ read-out discrimination factor estimated above, we obtain as the final result a success rate of $1\%$, much larger than the error due to spurious nonswitching events and easily detectable experimentally. In a typical experiment recording about $10^4$ events per second, this corresponds to 100 successful events every second. This success rate is smaller than the theoretical maximum of 25%, resulting from the imperfect discrimination between the states $|+> > 0$ and $|->$. Due to this, our estimates show that the success rate cannot be improved by using the quantum Zeno effect \[8\]. In a Mach-Zehnder setup, this would correspond to the quantum ultrasensitive object interacting with different probabilities with photons propagating in both arms of the interferometer.

It is instructive to consider the case in which a phase error $\Delta = \hbar^{-1} \int \delta \epsilon(t) dt$ (the change of the qubit energy at each instant $t$) is introduced due to an imperfect compensation $\phi (t) \neq -\gamma_m (t)$. This will produce a dephasing of the qubit with respect to the microwave radiation at the second $\pi/2$ Ramsey pulse. Immediately after the interaction pulse, the circuit is in the state $\sqrt{2}N^{-1/2} \exp(i\Delta/2) \exp(-\Gamma^{(p)}_+ \tau_p/2) |\psi_+ (\gamma)| + >$.
\[+i(2N)^{-1/2}\exp(-i\Delta/2)\exp(-\Gamma^{(p)}\tau_p/2)\psi_-(\gamma)|+\rangle + i(2N)^{-1}\left(e^{i\Delta/2-\Gamma^{(p)}\tau_p/2} - e^{-i\Delta/2-\Gamma^{(p)}\tau_p/2}\right)\psi_+(\gamma)|+\rangle,\]

and after the second Ramsey pulse the state assumes the form

\[\left(2\sqrt{N}\right)^{-1}\left(e^{i\Delta/2-\Gamma^{(p)}\tau_p/2} - e^{-i\Delta/2-\Gamma^{(p)}\tau_p/2}\right)\psi_+(\gamma)|+\rangle + \left(e^{i\Delta/2-\Gamma^{(p)}\tau_p/2} + e^{-i\Delta/2-\Gamma^{(p)}\tau_p/2}\right)\psi_-(\gamma)|-\rangle.\]

The probability of not observing a switch during the entire sequence (with the read-out strategy \(\exp(-\Gamma^{(r)}\tau^{(r)})\approx 0\) adopted before), is obtained as

\[\left(e^{\Gamma^{(p)}\tau_p/2} - e^{-\Gamma^{(p)}\tau_p/2}\right)^2 e^{-\Gamma^{(r)}\tau^{(r)}}/4 + e^{-\Gamma^{(p)}\tau_p/2}\sin^2(\Delta/2)e^{-\Gamma^{(r)}\tau^{(r)}}. \tag{5}\]

One can now clearly distinguish between classical interference (oscillations of probability as a sine function, the first term in the sum above) and the pure interaction-free effect (the second term in the expression in square brackets above)

\[\text{and much less at the plateau - where the flux and the current being constant it is easier to ensure precisely the compensation.}\]

Finally, one can realize interaction-free experiments with even less control over the accumulated phase difference \(\Delta\), using interaction pulses with slightly higher \(I^{(p)}\), which tend to nullify predominantly the interferometric term. The price to pay for this is a reduction in the success rate. As an example, if \(I^{(p)}\Phi_0/E_{J0} = 96\%\) and for a maximum error \(\Delta = \pm \pi\), the interference term becomes 7 times smaller than the interaction-free term (now also reduced to 0.27%).

In conclusion, two fundamental physical processes, interferometry and tunneling, can be combined to demonstrate the equivalent of the non-classical interaction-free detection scheme for a superconducting quantum circuit. The crossover between standard interference effects and the interaction-free phenomenon is also discussed.

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