Collective behavior in a granular jet: Emergence of a liquid with zero surface-tension

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(Dated: February 1, 2008)

We perform the analog to the “water bell” experiment using non-cohesive granular material. When a jet of granular material, many particles wide, rebounds from a fixed cylindrical target, it deforms into a sharply-defined sheet or cone with a shape that mimics a liquid with zero surface tension. The particulate nature of granular material becomes apparent when the number of particles in the cross-section of the jet is decreased and the emerging sheets and cones broaden and gradually disintegrate into a broad spray. This experiment has its counterpart in the behavior of the quark-gluon plasma generated by collisions of gold ions at the Relativistic Heavy Ion Collider. There a high density of inter-particle collisions gives rise to collective behavior that has also been described as a liquid.

PACS numbers: 45.70.-n, 45.70.Mg, 47.50.Tn

When one or two particles strike a smooth wall at normal incidence, they rebound in the direction whence they came. Yet, as we show here, a dense stream of non-cohesive particles hitting a target retains its integrity and deforms into a thin sheet with a shape resembling the structures created by an impinging water jet. Thus the collective behavior of many particles differs qualitatively from that of the individual components. When a jet of discrete particles be modeled as a liquid and how do the liquid patterns emerge out of individual particle scattering events from a target? Such questions, posed and investigated here with granular materials, have their counterpart in much more microscopic situations such as the quark-gluon plasma caused by relativistic high-energy collisions of gold ions, which also produces scattering patterns indicative of a liquid state. Our findings provide a macroscopic, purely classical example of how strong interactions, mediated by rapid collisions in a densely packed region, can give rise to liquid behavior.

When a water stream hits a flat target, it spreads symmetrically in the direction transverse to the impact and deforms into a thin sheet. For targets smaller than the stream diameter, the sheet forms a hollow bell-shaped structure that envelopes the target. Such “water bells” were reported in 1883 by Savart and have been systematically studied by Clanet. By contrast, with the exception of several simulations and experiments on granular streams passing by solid obstacles at relatively low particle density and speed, little is known about the structures emerging from equivalent experiments using granular material rather than a liquid. Here, we examine a non-cohesive granular jet colliding with a target. For jets many particles wide, the impact produces granular sheets and cones similar to those seen for water with structures that depend on the ratio of the jet to the target diameter. In this regime, we conclude that the granular medium behaves as if it were a liquid with infinite Weber number—a system appropriate for a system of non-cohesive particles. The particulate nature of the material becomes apparent when the sheets and cones broaden and gradually disintegrate as the number of particles in the beam is decreased.

We prepared our dense jets by packing granular material into a 40 cm section of a glass launching tube of inner diameter $D_{jet} = 0.73$ cm (Fig. 1a). The grains, which were compacted to a reproducible density by tapping, were mono-disperse spherical beads of glass ($\rho = 2.5 g/cm^3$) or copper ($\rho = 8.2 g/cm^3$) with diameters between $d = 50 \mu m$ and $d = 2.1 mm$. Prior to filling the tube, we baked the beads in vacuum to minimize any residual adhesion between beads. Pressurized gas accelerated this granular plug into a jet which hits a target 2.5 cm in front of, and collinear with, the tube. The jet velocity $U_0$ could be varied between 1 and 16 m/s. The impacts were filmed at 2000 frames/second with a Phantom V7.1 camera. Fig. 1b and c show the side and front views respectively of a colliding jet. Since there are very few collisions inside it, the jet maintains its cylindrical shape before hitting the target, consistent with jets produced when a sphere impacts loosely packed powders. After hitting the target, the jet deforms into an extraordinarily thin symmetric granular sheet clearly resembling a spreading liquid. To investigate the similarities between granular and ordinary liquid jets, we first keep the jet diameter, $D_{jet}$, fixed at values much larger than the particle diameter, $d$, and vary the target diameter, $D_{tar}$, as shown in Fig. 1c. When $D_{tar}$ is reduced, these planar sheets change into cones with opening angle, $\psi_0 \approx 90^\circ$. Fig. 2 shows that $\psi_0$ increases linearly with $D_{tar}/D_{jet}$ until $\psi_0$ saturates at $90^\circ$ above $D_{tar}/D_{jet} \sim 2$. Glass and copper beads produces essentially identical behavior.

We can compare our results with those from Clanet for water jets, who found that the opening angle, $\psi$, of the “water bell” depends on the Weber number $W_e = \rho U_0^2 D_{jet}/\sigma$, where $U_0$ is the jet velocity and $\rho$ the density and $\sigma$ the surface tension of water. In the
large-We limit (large $U_0$ or small $\sigma$), $\psi$ approaches a constant, $\psi_0$, that depends only on $D_{Tar}/D_{Jet}$. Fig 2 shows excellent overlap of our values for $\psi_0$ with those of Clanet. As shown in the inset to Fig 2, $\psi$ for granular material remains constant as $U_0$ is varied over our entire experimental range (an order of magnitude) whereas $\psi$ for water decreases at small velocities. This suggests a granular jet impacting a the target behaves like a liquid with negligible surface-tension: regardless of the value of $U_0$, the Weber number is pinned at infinity since the absence of significant cohesive forces effectively drives the surface tension to zero.

The linear dependence of $\psi_0$ on $D_{Tar}/D_{Jet}$ in the large-We limit can be understood from momentum conservation. The magnitude of the momentum of the incoming granular jet reaching the target in time $\tau$ is $P_{in} = (\pi/4)D_{Jet}^2\rho U_0^2\tau$. The collisions of the jet with the target are inelastic so the outgoing momentum magnitude $P_{out} = CP_{in}$ where $C$ is the average coefficient of restitution. Along the axis, the momentum balance is given by: $P_{in} - P_{out} \cos \psi_0 = P_{in}(1 - C \cos \psi_0) = F_{Tar}\tau$, where $F_{Tar}$ is the average force along the axis exerted by the target on the jet. Transverse to the axis, the momentum (averaged over the entire sheet) remains zero. When $D_{Tar}/D_{Jet} < 1$, $F_{Tar} = (A\pi/4)D_{Jet}^2\rho U_0^2$. Here $A$ is a constant describing the average glancing collision angle for a particle. In this region, the average restitution coefficient, $C$, also depends on the fraction of particles hitting the target: $C = [1 - B(D_{Tar}/D_{Jet})^3]$ where $B$ depends on the coefficient of restitution for single collisions. When $D_{Tar}/D_{Jet} \gg 1$, the entire momentum of the jet is reflected by the target. Finally we obtain:

$$\psi_0 = \left\{ \begin{array}{ll} \cos(1 - (A - B)(\frac{D_{Tar}}{D_{Jet}})^2); & \frac{D_{Tar}}{D_{Jet}} \ll 1 \\ 90^\circ; & \frac{D_{Tar}}{D_{Jet}} \gg 1 \end{array} \right. \quad (1)$$

Clanet reached the same result by considering the momentum transfer during the impact of a water jet on a target using hydrodynamic equations [2]. By fitting the experimental data in Fig 2, we find $(A - B) = 0.30 \pm 0.02$ which is close to the value $\approx 0.352$ for water [2].

The impact of dense particle streams clearly generates similar patterns as do liquids. How does the particulate nature of granular material becomes manifest as the number of particles within the jet decreases? To vary this number we change the ratio, $D_{Jet}/D$, of jet to particle diameter. In the limit $D_{Tar}/D_{Jet} \gg 1$, Figs 3a–c show
major qualitative changes in the particle trajectories as $D_{jet}/d$ is reduced. The images were created by superimposition of many different consecutive still images; each pixel shows the maximum intensity at that location over all images in the time period. For $D_{jet}/d = 73$, almost all particles emerge in a sheet normal to the jet axis. For $D_{jet}/d = 14.6$, the sheet becomes more diffuse as some particles leave the plane in the bright lines. For $D_{jet}/d = 3.5$, a firework-like pattern results after impact and sheet structure is no longer apparent as particles rebound from the target in a broad angular distribution.

To quantify this trend, we plot in Fig.3 the angular scattering distribution, obtained by averaging images as in Fig.3a-c along a circle of radius $r = 8.7D_{jet}$ centered on the jet axis (shown partially in Fig.3a) as the dashed line). The axial position of the center is chosen to be in the middle of the ejected particles, which gradually moves away from the target in upstream direction as the particle diameter $d$ increases. To avoid the target holder, only the scattering profile along the upper half of the circle is plotted. Each profile is normalized to its peak value. The scattering angle $\theta$ is zero along the axis of the jet and increases clockwise. As $D_{jet}/d$ decreases, these profiles become broader.

To investigate this crossover to diffuse scattering we examined the roles of air and inelasticity during inter-particle collisions. To exclude air as the primary cause of the sheet formation, three experiments were performed. (i) Doing the entire experiment in a helium atmosphere and using helium as the accelerating gas produced no change in the angular distribution, as shown in Fig.3a for $D_{jet}/d = 73$. (ii) Performing our experiment at reduced ambient pressure of 31 Pa also resulted in a thin, planar sheet structure. (31 Pa corresponds to a mean free path of air molecules three times the 100 $\mu$m grain diameter used in Fig.3. Although air accelerates the granular column from behind, we estimate that the velocity of air penetrating into a 40 cm granular pack comprised of 100 $\mu$m beads with packing fraction around 0.6 is two orders of magnitude smaller than the velocity of the jet. Thus, by the time the front of the jet hits the target, a negligible amount of air has entered the chamber.) (iii) We decreased the tube diameter $D_{jet}$, but fixed the particle-air interaction by keeping $d = 100 \mu$m. In this case the planar structure shown in Fig.3 disappeared gradually, and for $D_{jet} \sim d$ we regained the firework pattern shown in Fig.3. In addition, we altered the inter-particle collision dynamics by changing the particle material or the surface roughness. Fig.3 shows that for copper particles that are less elastic than glass or for rough sand particles the angular scattering distribution remains unchanged. However, distributions for the same $D_{jet}/d$ do not overlap quantitatively for different $d$, and the liquid-like sheets tend to be more sharply delineated with smaller particles.

While we cannot rule out that inelastic collisions or effects of air affect the detailed shape of the scattering profiles, our results imply that the creation of the sheets arises fundamentally from the rapid collisions occurring in an interaction region right in front of the target. We found similar behavior when two granular jets collide head-on, implying that the target serves primarily to reverse the direction of particles incident upon it. For this situation, a simple geometric model can capture the essence of the crossover from fluid- to granular behavior. As shown in Fig.3, inset, we divide the system into three zones: two external regions in which the jets are traveling towards one another but have not yet collided, and an interaction region with the same diameter as the jets, $D_{jet}$, and axial length, $l$. Inside the interaction region particles undergo rapid collisions and are confined by pressure from the incoming jets on both sides. The only way in which particles can escape (which they must since new particles are entering the region continuously) is to emerge perpendicularly to the jet axis. A measure for deviations from this transverse axis and thus for the half-width of the angular distributions in Fig.3 is the an-
Transverse to the beam (azimuthal direction) we expect azimuthally anisotropic patterns to be created. For example, when $D_{jet}/d = 7$, the cross-section of jet are shown in the inset; $\phi$ is taken to be zero along the long axis of the jet cross-section.

FIG. 4: Azimuthal scattering profiles for granular jets with rectangular cross-section (aspect ratio 2). Shown are views along the axial direction, looking at the target from behind. (a) Anisotropic profile resulting for $D_{jet}/d = 73$, where $D_{jet}$ is the long axis of the jet cross-section. The target holder blocks the view of the scattered portion shooting downward. Scale bar is 3.0cm. (b) Isotropic profile resulting for $D_{jet}/d = 7$. (c) Azimuthal profiles along a circle with $r = 8.7D_{jet}$ for $D_{jet}/d = 73$, $D_{jet}/d = 29.2$, $D_{jet}/d = 14.6$, and $D_{jet}/d = 3.5$. The direction and the cross section of jet are shown in the inset; $\phi$ is taken to be zero along the long axis of the jet cross-section.

The crossover between diffuse and sharp scattering profiles appears to be controlled primarily by the number of collisions in the jet. These findings may have analogous in other, disparate parts of physics, where a high density of collisions dominates the behavior.

We thank R. Bellwied, E. Corwin, S. Gavin, T. Pöschel, J. Royer, T. Witten and L. Xu. This work was supported by the NSF through its MRSEC program under DMR-0213745 and through its Inter-American Materials Collaboration under DMR-0303072.

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