Modifying N=2 Supersymmetry via Partial Breaking

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Abstract

We study realization of $N = 2$ SUSY in $N = 2$ abelian gauge theory with electric and magnetic $FI$ terms within a manifestly supersymmetric formulation. We find that after dualization of even one $FI$ term $N = 2$ SUSY is realized in a partial breaking mode off shell. In the case of two $FI$ terms, this regime is preserved on shell. The $N = 2$ SUSY algebra is shown to be modified on gauge-variant objects.

1. A celebrated mechanism of spontaneous breakdown of rigid $N = 2$ SUSY consists in adding a Fayet-Iliopoulos ($FI$) term to the action of $N = 2$ gauge theory. Recently, Antoniadis, Partouche and Taylor (APT) \cite{1} have found that the dual formulation of $N = 2$ abelian gauge theory \cite{2} provides a more general framework for such a spontaneous breaking due to the possibility to define two kinds of the $FI$ terms (see also \cite{3}). One of them (‘electric’) is standard, while another (‘magnetic’) is related to a dual $U(1)$ gauge supermultiplet. APT show that a partial spontaneous breakdown of $N = 2$ SUSY to $N = 1$ becomes possible, if one starts with an effective $N = 2$ Maxwell action and simultaneously includes two such $FI$ terms.

Here we report on the results of studying the invariance properties of $N = 2$ Maxwell action with the two types of $FI$-terms in a manifestly supersymmetric $N = 2$ superfield approach. Our basic observation is that after duality transformation of a system even with one sort of the $FI$ term, off-shell $N = 2$ SUSY starts to be realized in a partial spontaneous breaking mode. When both $FI$ terms are included, this partial breaking is retained on shell by the APT mechanism. We study how these modified $N = 2$ SUSY transformations act on the gauge potentials and find that the $N = 2$ SUSY algebra also undergoes a modification.

2. Let us start from the following representation of the superfield action of $N = 2$ Maxwell theory with the $FI$ term \cite{4}

$$
S(W, L) = \frac{i}{4} \int d^4x d^4\theta \left[ \mathcal{F}(W) - WW_L + \frac{i}{2} E^{ik}(\theta_i \theta_k)W \right] + \text{c.c.}
$$

$$
\equiv S(W) + S_L + S_e, \quad W_L = (\bar{D})^4 D_{ik} L^{ik}.
$$

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Here, $\mathcal{F}(W)$ is an arbitrary holomorphic function, $W$ is a chiral $N = 2$ superfield, $L^i$ is a real unconstrained $N = 2$ superfield Lagrange multiplier, $E^{ik}$ is a real $SU(2)$ triplet of constants and $D^{ik} = D^i_\alpha D^k_\dot\alpha$. Varying $L^i$ yields the constraint

$$D^{ik}W - D^{ik}\bar{W} = 0 \quad (2)$$

which can be solved in terms of Mezincescu prepotential $V^{ik}$

$$W \equiv W_V = (\bar{D})^4 D^{ik} V^{ik} \quad (3)$$

Upon substituting this solution back into (1), the latter becomes

$$S(W, L) \Rightarrow S(V) = S(W_V) + S_e, \quad S_e = \int d^4x d^4\theta d^4\bar{\theta} E^{ik} V^{ik} \quad (4)$$

that is the standard ‘electric’ form of the action of $N = 2$ Maxwell theory with $FI$ term. On the other hand, one can first vary (1) with respect to $W$, which yields

$$\frac{\partial \mathcal{F}}{\partial W} = W_L - (i/2)(\theta_k \theta_l) E^{kl} \equiv \hat{W}_L \quad (5)$$

$$S(W, L) \Rightarrow i \int d^4x d^4\theta \hat{\mathcal{F}}(\hat{W}_L) + c.c., \quad \hat{\mathcal{F}}(\hat{W}_L) \equiv \mathcal{F}(W(\hat{W}_L)) - \hat{W}_L \cdot W(\hat{W}_L) \quad (6)$$

In this dual, ‘magnetic’ representation the $FI$ term-modified $N = 2$ Maxwell action is expressed through the dual (‘magnetic’) superfield strength and prepotential $W_L$ and $L^i$. Thus, (4) is a sort of ‘master’ action from which both the electric and magnetic forms of the $N = 2$ Maxwell action can be obtained (see [2, 8] for a similar discussion of the standard $E^{ik} = 0$ case).

Let us discuss peculiarities of realization of $N = 2$ SUSY in the magnetic representation. The electric action (4) is invariant under the standard $N = 2$ SUSY. The same is true for the dual actions (1), (6) in the absence of the $FI$ term. However, if $E^{ik} \neq 0$, the conventional $N = 2$ SUSY gets broken in (4), (6). The only way to restore it is to modify the transformation law of $W_L$

$$\delta \epsilon W_L = i(\epsilon_k \theta_l) E^{kl} + i(\epsilon Q + \bar{\epsilon} \bar{Q}) W_L \quad (7)$$

where $Q^i_\alpha, \bar{Q}^i_{\dot\alpha}$ are standard generators of $N = 2$ SUSY. It is easy to find the appropriate modified transformation of $L^i$.

An inhomogeneous term in the $N = 2$ SUSY transformation law means spontaneous breakdown of $N = 2$ SUSY. To see which kind of breaking occurs in the case at hand, let us firstly discuss the standard electric action (4).

Spontaneous breaking of $N = 2$ SUSY by the $FI$ term is related to the possibility of a non-zero vacuum solution for the auxiliary component $X^{ik} = -\frac{1}{4}D^{ik}W |_0$

$$< X^{ik} > \equiv x^{ik} \sim E^{ik} \quad (8)$$

Provided that such a solution of the equations of motion exists and corresponds to a stable classical vacuum, there appears an inhomogeneous term in the on-shell SUSY transformation law of the $N = 2$ gaugini doublet $\lambda^{i\alpha}$

$$\delta \lambda^{i\alpha} \sim \epsilon^i_k L^{ik} \quad (9)$$
\( \epsilon^k_\alpha \) being the transformation parameter. Thus there are Goldstone fermions in the theory, which is a standard signal of spontaneous breaking of \( N = 2 \) SUSY. Since the inhomogeneous terms in (3) appear as a result of solving equations of motion, it is natural to call \( \lambda^{i\alpha} \) on-shell Goldstone fermions. As the matrix \( E^{ik} \) is non-degenerate, both \( \lambda^{1\alpha}, \lambda^{2\alpha} \) are shifted by independent parameters, and so they both are Goldstone fermions. Thus, with the standard electric \( FI \) term, only total spontaneous breaking of \( N = 2 \) SUSY can occur (actually, for one vector multiplet this is possible only in the free case, with quadratic function \( F \)).

In the dual, magnetic representation of the same theory corresponding to the actions (1) or (6) the situation is radically different: the off-shell transformation law (7) contains an ‘inborn’ inhomogeneous piece. This leads us to interpret the magnetic gaugini as off-shell Goldstone fermions.

Both gaugini are shifted in (7), so at first sight we are facing the total off-shell spontaneous breaking of \( N = 2 \) SUSY in this case. However, by a proper shift

\[
W_L \rightarrow \tilde{W}_L = W_L + \frac{1}{2}(\theta_i\theta_k)C^{ik},
\]

one can restore a homogeneous transformation law with respect to one of two \( N = 1 \) supersymmetries contained in \( N = 2 \) SUSY (it is easy to find the appropriate redefinition of \( L^{ik} \)). The object \( \tilde{W}_L \) transforms as follows

\[
\delta_\epsilon \tilde{W}_L = (\epsilon_i\theta_i)(C^{kl} + iE^{kl}) + i(\epsilon Q + \bar{\epsilon}Q)\tilde{W}_L.
\]

One can always choose \( C^{ik} \) so that

\[
\det (C + iE) = 0 .
\]

This means that \( C^{ik} + iE^{ik} \) is a degenerate symmetric 2 \( \times \) 2 matrix, so it can be brought to the form with only one non-zero entry. As a result, \( \tilde{W}_L \) is actually shifted under the action of only one linear combination of the modified \( N = 2 \) SUSY generators \( \hat{Q}_\alpha^{1,2} \). The same is true for the physical fermionic components: only one their combination is the genuine off-shell Goldstone fermion.

Thus we arrive at the important conclusion: in the dual, magnetic representation of \( N = 2 \) Maxwell theory with \( FI \) term \( N = 2 \) SUSY is realized off shell in a partial spontaneous breaking mode, so that some \( N = 1 \) SUSY remains unbroken.

It is straightforward to show that the action (6) leads to the same vacuum structure as the original electric action (4). Thus on shell in the magnetic representation we again encounter the total spontaneous breaking of \( N = 2 \) SUSY.

3. Let us show that the phenomenon of partial breaking of \( N = 2 \) SUSY becomes valid both off and on shell upon adding to the ‘master’ action (4) the new sort of \( FI \) term, the ‘magnetic’ one:

\[
S(W,L) \Rightarrow S(W,L)' = S(W,L) + S_m , \quad S_m = \frac{1}{8} \int d^4xd^4\theta \; M^{kl}(\theta_k\theta_l)W_L + c.c. ,
\]

\( M^{ik} \) being another triplet of real constants. It is easy to show that \( S_m \) is invariant under the Goldstone-type transformation (7).
When one descends to the electric representation of (13) (by varying \(L\)), the only effect of the magnetic FI term \(S_\text{m}\) is the modification of the constraint (2):

\[D^{ik}W - \bar{D}^{ik}\bar{W} = 4iM^{ik}.\]  

(14)

It suggests the redefinition

\[W = W_\nu - \frac{i}{2}(\theta_i\theta_k)M^{ik},\]  

(15)

with \(W_\nu\) satisfying eq. (2) and hence given by eq. (3). This shift amounts to the appearance of the constant imaginary part \(-\frac{i}{2}M^{ik}\) in the auxiliary field of \(W\),

\[\hat{X}^{ik} \equiv -\frac{1}{4}D^{ik}W|_0 = X^{ik} - \frac{i}{2}M^{ik} .\]

The inclusion of magnetic FI term cannot change the standard transformation properties of \(W\) under \(N = 2\) SUSY. Then the relation (15) requires to modify the transformation law of \(W_\nu\), and, respectively, of \(V^{ik}\) on the pattern of eq. (7)

\[\delta_\epsilon W_\nu = i(\epsilon_k\bar{\theta}_l)M^{kl} + i(\epsilon Q + \bar{\epsilon}\bar{Q})W_\nu .\]  

(16)

(it is easy to find the appropriate transformation law of \(V^{ik}\)). In other words, \(N = 2\) SUSY is now realized in a Goldstone-type fashion in the electric representation as well, but with \(M^{ik}\) instead of \(E^{ik}\) as the ‘structure’ constants. So, when both FI terms are present, there is no way to restore the standard \(N = 2\) SUSY off shell. The same arguments as in the previous Section show that \(N = 2\) SUSY in both representations is realized off shell in the partial breaking mode.

A general electric effective action of the abelian gauge model with the \((E, M)\)- mechanism of the spontaneous breaking can be obtained by substituting the expression for \(W\), eq.(15), into the action (13):

\[S_{(E,M)} = \left[\frac{i}{4} \int d^4x d^4\theta \mathcal{F}(W) + \text{c.c.}\right] + \int d^4x d^4\theta d^4\bar{\theta} E^{ik} V^{ik}.\]  

(17)

Taking the standard vacuum ansatz

\[< W_\nu >_0 = a + (\theta_i\theta_k) x^{ik},\]  

(18)

it is easy to show that the superfield equation of motion following from (17) implies the following equations for moduli \(a, x^{ik}\)

\[(i) \quad x^{kl} = \frac{1}{2\tau_2(a)}(\tau_1(a)M^{kl} - E^{kl}), \quad (ii) \quad \tau' \hat{x}^{ik}\hat{x}_{ik} = 0 ,\]  

(19)

where

\[\tau = \mathcal{F}'' = \tau_1 + i\tau_2 , \quad \hat{x}^{ik} = < \hat{X}^{ik} > = x^{ik} - (i/2)M^{ik} .\]

A crucial new point compared to the case of \(M^{ik} = 0\) is that the vector \(\hat{x}^{ik} = x^{ik} - (i/2)M^{ik}\) is complex, so the vanishing of its square does not imply it to vanish. As a result, besides the trivial solution \(\tau' = 0\) [1], eq.\((ii)\) in (19) possesses the non-trivial one

\[\tau' \neq 0 , \quad \hat{x}^{ik}\hat{x}_{ik} = 0 .\]  

(20)
This solution amounts to the following relations

\[ \tau_1(a) = \vec{E} \vec{M} \quad , \quad |\tau_2(a)| = \sqrt{\frac{\vec{E}^2 \vec{M}^2 - (\vec{E} \vec{M})^2}{\vec{M}^2}} = \frac{|\vec{E} \times \vec{M}|}{\vec{M}}. \]  \hspace{1cm} (21)

This is just the vacuum solution found in [1] based on the component version of the action (17). It triggers a partial spontaneous breaking of \( N = 2 \) SUSY down to \( N = 1 \). Only one combination of gaugini is the Goldstone fermion in this case.

Thus, the phenomenon of the off-shell partial breaking of \( N = 2 \) SUSY is preserved on shell, provided that both electric and magnetic \( FI \) terms are included.

4. The modified \( N = 2 \) SUSY, when realized on the superfield strengths (eqs. (7), (16)), still closes on space-time translations. One can wonder how it is realized on the gauge-variant objects: gauge potentials and prepotentials. The \( N = 2 \) SUSY algebra itself proves to be modified in this case. We will demonstrate this in the \( N = 1 \) superfield formalism, on the example of the model considered in Sect. 3.

We define

\[ W = W_V + \hat{x}^{ik}(\theta_i \theta_k) , \quad < W_V >_0 = 0 . \]  \hspace{1cm} (22)

Decomposing \( W_V \) in powers of \( \theta_2, \bar{\theta}^2 \)

\[ W_V = \phi(x, \theta_1) + i\theta_2^a W_\alpha(x, \theta_1) + (\theta_2)^2 (1/4)(\bar{D}_1)^2 \bar{\phi} , \]  \hspace{1cm} (23)

it is easy to find how the \( N = 1 \) superfield components \( \phi(x, \theta_1), W^\alpha(x, \theta_1) = (\bar{D}_1)^2 D^\alpha V \) behave under the modified \( N = 2 \) SUSY transformations (16). We will be interested in how the latter are realized on the \( N = 1 \) gauge prepotential \( V(x, \theta_1, \bar{\theta}^1) \). Modulo gauge transformations, the \( N = 2 \) SUSY acts on \( V \) as (we choose the \( SU(2) \) frame so that \( M_{12} = 0, M^{11} = M^{22} = m \) )

\[ \delta V = m(\bar{\theta}^1)^2 \theta_1^a \epsilon_{a_2} + (i/2)\theta_2^a \epsilon_{a_2} \bar{\phi} + c.c + i(\epsilon_1 Q^1 + \bar{\epsilon} \bar{\bar{Q}}^1) V . \]  \hspace{1cm} (24)

Defining \( Q_\alpha \equiv Q_1^\alpha, S_\alpha \equiv Q_2^\alpha \), one finds that the (anti)commutation relations of the \( N = 2 \) SUSY algebra are modified as follows:

\[ \{ Q_\alpha, S_\beta \} = \epsilon_{a\beta} G , \quad \{ Q_\alpha, \bar{S}_\beta \} = G_{a\beta} , \quad ( \text{and c.c.} ) . \]  \hspace{1cm} (25)

The newly introduced generators possess the following action on \( V \)

\[ GV = (i/2)m(\bar{\theta}^1)^2 , \quad \bar{G}V = (i/2)m(\theta^1)^2 , \quad G_{a\beta} V = im\theta_{1a} \bar{\theta}_\beta^1 , \]  \hspace{1cm} (26)

and are easily seen to be particular \( N = 1 \) gauge transformations. Thus, off-shell \( N = 2 \) SUSY algebra turns out to be non-trivially unified with the gauge group algebra. Such a unification does not contradict the famous Coleman-Mandula theorem. Indeed, in gauge theories it is impossible to simultaneously satisfy two important conditions of this theorem: manifest Lorentz covariance and positive definiteness of the metric in the space of states. Note that the \( N = 2 \) transformation (24) is defined up to pure gauge terms which are capable to change the above algebra. However, in the closure of spinor generators one will always find some gauge generators in parallel with 4-translations. An interesting problem is to construct a non-linear realization of this modified \( N = 2 \) algebra along the lines of ref. [8].
and to compare it with that constructed in [9] based on the standard $N=2$ algebra. It is also tempting to elaborate on a possible stringy origin of the modified algebra.

Finally, we would like to point out that the difficulty with the self-consistent incorporation of charged matter hypermultiplets into the $N=2$ Maxwell theory with the two sorts of $FI$ terms recently discussed in [10] is directly related to the modification of $N=2$ SUSY algebra in the presence of these terms. The standard minimal coupling of the $q^+$ hypermultiplets to the harmonic superspace $N=2$ Maxwell potential $V^{++}$ [11] is not invariant under the modified $N=2$ SUSY.

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