THE EFFECT OF A NONTHERMAL TAIL ON THE SUNYAEV-ZELDOVICH EFFECT IN CLUSTERS OF GALAXIES

P. Blasi, A. V. Olinto, and A. Stebbins

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ABSTRACT

We study the spectral distortions of the cosmic microwave background radiation induced by the effect in clusters of galaxies when the target electrons have a modified Maxwell-Boltzmann distribution with a high-energy nonthermal tail. Bremsstrahlung radiation from this type of electron distribution may explain the suprathermal X-ray emission observed in some clusters such as the Coma Cluster and A2199 and serve as an alternative to the classical but problematic inverse Compton scattering interpretation. We show that the Sunyaev-Zeldovich effect can be used as a powerful tool to probe the electron distribution in clusters of galaxies and discriminate among these different interpretations of the X-ray excess. The existence of a nonthermal tail can have important consequences for cluster-based estimators of cosmological parameters.

Subject headings: cosmology: theory — galaxies: clusters: general

1. INTRODUCTION

Clusters of galaxies are powerful laboratories for measuring cosmological parameters and for testing cosmological models of the formation of structure in the universe. These associations of large numbers of galaxies are confined by a much greater mass of dark matter, which also confines a somewhat smaller mass in very hot gas. The galaxies and the gas are in rough virial equilibrium with the dark matter potential well. While initially clusters were investigated through the observed dynamics of the galaxies they contain, in recent decades much information has been gathered from studies of the gas, primarily via X-ray observations. The bremsstrahlung emission but also through the Sunyaev-Zeldovich (SZ) effect (Zeldovich & Sunyaev 1969). Interpreting these observations requires a detailed understanding of the thermodynamic state of the gas. With increasingly more sensitive measurements, the gasdynamics should become clearer, which would allow for a better understanding of the structure and dynamics of clusters as well as their effectiveness as tests of cosmological models.

Both the X-ray emission and the SZ effect are sensitive to the energy distribution of the electrons. It is usually assumed that in the intracluster gas the electron energy distribution is described by a thermal (transrelativistic Maxwell-Boltzmann) distribution function. The typical equilibration time for the bulk of this hot and rarefied electron gas is of order \(10^3\) yr and is mainly determined by electron-electron Coulomb scattering (electron-proton collisions are much less efficient). This time rapidly increases when the electron energy becomes appreciably larger than the thermal average, so that thermalization takes longer for higher energy electrons. In the absence of processes other than Coulomb scatterings, the electron distribution rapidly converges to a Maxwell-Boltzmann distribution. However, the fact that the intracluster gas may be (and actually is often observed to be) magnetized can change this simple scenario: for instance, cluster mergers can modify the electron distribution either by producing shocks that diffusively accelerate part of the thermal gas or by inducing the propagation of MHD waves that stochastically accelerate part of the electrons and heat most of the gas (Blasi 2000). Although the bulk of the electron distribution is likely to maintain its thermal energy distribution, higher energy electrons, more weakly coupled to the thermal bath, may acquire a significantly nonthermal spectrum (Blasi 2000).

Until recently, X-ray observations could only probe energies below \(10\) keV, where the observed radiation is consistent with bremsstrahlung emission from the intracluster plasma with a thermal electron distribution with temperatures in the 1–20 keV range. The recent detection of a hard X-ray component in excess of the thermal spectrum of the Coma Cluster (Fusco-Femiano et al. 1999) may be the first indication that the particle distribution in (some) clusters of galaxies contains a significant nonthermal component. Observations of A2199 (Kaastra et al. 1999) show a similar excess, while no excess has been detected in A2319 (Molendi et al. 1999); thus, the source of this effect may not be universal.

As argued above, the presence of magnetic fields in the intracluster gas allows for acceleration processes that can modify the details of the heating processes, so that the electron energy distribution may differ from a Maxwell-Boltzmann distribution. In this case, the bremsstrahlung emission from a modified Maxwell-Boltzmann electron gas can account for the observed X-ray spectra up to the highest energies accessible to current X-ray observations (Ensslin, Lieu, & Biermann 1999; Blasi 2000). This model works as an alternative to the more traditional interpretation based on the inverse Compton scattering (ICS) emission from a population of shock-accelerated ultrarelativistic electrons (Volk & Atoyan 1998). The ICS model has many difficulties, such as the requirement that the cosmic-ray energy density be comparable to the thermal energy in the gas (Ensslin et al. 1999; Blasi & Colafrancesco 1999). This large cosmic-ray energy density might be hard to reconcile with the nature of cosmic-ray sources in clusters (Berezinsky, Blasi, & Ptuskin 1997) and with gamma-ray observations (Blasi 1999). Moreover, the combination of X-ray and radio observations within the ICS model strongly indicates a very low magnetic field, \(B \sim 0.1\) \(\mu\)G, much lower than the values derived from Faraday rotation measurements (Kim et al. 1990; Feretti et al. 1995), which by themselves represent only lower limits to the field.

The best way to resolve the question of whether the observed hard X-rays are due to ICS or are the first evidence for a

1 NASA/Fermilab Astrophysics Group, Fermi National Accelerator Laboratory, Batavia, IL 60510-0500.
2 Department of Astronomy and Astrophysics and Enrico Fermi Institute, University of Chicago, Chicago, IL 60637.
modified thermal electron distribution in clusters is to probe directly such a distribution. We propose that this probe can be achieved by detailed observations of the SZ effect, which is the change in brightness temperature of the cosmic microwave background (CMB) photons when they traverse a hot electron gas such as the gas in clusters. We discuss the SZ effect in detail in the next section, where the main results are also discussed. Additional implications of the scenario proposed here are presented in § 3.

2. THE SZ EFFECT AS A PROBE OF NONTHERMAL PROCESSES

In this section, we calculate the SZ effect for a modified electron distribution, including a high-energy tail. We follow the procedure outlined by Birkinshaw (1999).

Photons of the CMB propagating in a gas of electrons are Compton scattered, and their energy spectrum is modified. As long as the center-of-mass energy of the collision is less than \( m_e c^2 \), the scattering is accurately described by the Thomson differential cross section. For CMB photons at low redshift, this only requires that the electron energy in the cosmic rest frame be less than \( \sim 1 \) TeV. For scattering of a photon with initial frequency \( \nu_0 \), off an isotropic distribution of electrons each with speed \( v \), the probability distribution of the scattered photon having frequency \( \nu(1 + \Delta) \) is (Stebbins 1997)

\[
P(\Delta, \beta) d\Delta = \frac{F(\Delta, \beta \text{ sgn}(\Delta))}{(1 + \Delta)^3} d\Delta,
\]

\[
\Delta \in \left[ -\frac{2\beta}{1 + \beta}, \frac{2\beta}{1 - \beta} \right],
\]

where \( \beta = v/c \) and

\[
F(\Delta, b) = \left| \frac{3(1 - b^2)^2(3 - b^2)(2 + \Delta)}{16b^6} \right| \times \ln \left( \frac{1 - b(1 + \Delta)}{1 + b} \right) + \frac{3(1 - b^2)(b - 1)(1 + \Delta)}{32b^6(1 + \Delta)} \times \left[ 4(3 - 3b^2 + b^4) + 2(6 + b - 6b^2 - b^3 + 2b^4)\Delta \right. \\
\left. + (1 - b^2)(1 + b)\Delta^2 \right|.
\]

If instead of a fixed speed we consider the scattering off electrons with a distribution of speeds \( p(\beta)d\beta \), the distribution of \( \Delta \) after one scattering is

\[
P_1(\Delta) = \int_{[\Delta/(1+\Delta)]}^{1} d\beta p(\beta) P(\Delta, \beta).
\]

This expression can be easily applied to determine the change in the spectrum of the CMB as seen through the hot gas in a cluster of galaxies. Since clusters have a small optical depth to Compton scattering (\( \sim 10^{-2} \)), the fraction of photons that are scattered is given by the optical depth \( \tau_{\gamma} \), where \( \gamma \) is the projected surface density of free electrons. The change in brightness of the CMB at frequency \( \nu \) due to the SZ effect is then given by

\[
\Delta I(\nu) = \frac{2\nu^3}{c^2} \tau_{\gamma} \int_{-\infty}^{\infty} d\Delta P_1(\Delta) \left[ \left( 1 + \Delta \right)^2 - 1 - \frac{1}{e^{\nu\tau_{\gamma}} - 1} \right],
\]

where \( \nu = h\nu/k_B T_{\text{CMB}} \). \( T_{\text{CMB}} \) is the CMB temperature at the present epoch, and \( k_B \) is Boltzmann’s constant. It is conventional in CMB studies to use the change in the thermodynamic brightness temperature rather than the change in brightness, the former being given by

\[
\frac{\Delta T}{T_{\text{CMB}}} = \left( \frac{e^{\nu \tau_{\gamma}} - 1}{x^2 e^{\nu \tau_{\gamma}}} \right) I_0,
\]

where \( I_0 = 2(k_B T_{\text{CMB}})^3/\hbar c^3 \).

For very nonrelativistic electrons, \( P(\Delta) \) is narrowly peaked and can be accurately estimated via a first-order Fokker-Planck approximation. This gives the classical formula (Zeldovich & Sunyaev 1969) \( \Delta T/T_{\text{CMB}} \approx y \left[ x(e^{\nu \tau_{\gamma}} - 1) - 4 \right] \), where \( y = \frac{1}{4} \tau_{\gamma}(\beta^2) \). In this limit, the shape of the spectral distortion yields no useful information (only the amplitude \( y \) is interesting), but it depends only on the second moment of \( p(\beta) \). Fortunately the gas in rich clusters is hot enough for relativistic corrections to become important, leading to deviations from this classical formula at the \( \sim 10\% \) level (Birkinshaw 1999; Rephaeli 1995; Stebbins 1997; Challinor & Lasenby 1998; Itoh, Kohyama, & Nozawa 1998). Through these relativistic corrections, changes in the electron energy distribution can be measured by the modified shape of the SZ spectrum, hence the shape of the SZ effect can be used to differentiate between thermal and nonthermal models. Even without spectral information, nonthermality can be inferred by the comparison of the X-ray flux and temperature with the amplitude of \( \Delta T_{\text{SZ}} \); however, this requires a detailed model of the density structure of the cluster since the effect of synchrotron and bremsstrahlung emission scale differently with density.

The SZ effect is usually computed assuming a thermal \( p(\beta) \), but here we include the effect of a nonthermal tail. We adopt the model for the distribution function used by Ennslin et al. (1999) which fits both the nonthermal hard X-ray data and the thermal soft X-ray data. In particular, a thermal distribution for momenta smaller than \( \rho^* \) (\( \equiv m_e c \beta^* \gamma \)) is matched to a power-law distribution in momentum above \( \rho^* \), and cut off at momentum \( \rho_{\text{max}} \) (\( \equiv m_e c \beta_{\text{max}} \gamma_{\text{max}} \)), i.e.,

\[
p(\beta) = \frac{C \gamma^\gamma\beta^2}{\Theta K_\gamma(1/\Theta)} \begin{cases} \exp \left( -\frac{\beta}{\Theta} \right), & \beta \in [0, \beta^*], \\ \exp \left( -\frac{\beta^*}{\Theta} \right) \left( \frac{\beta}{\beta^*} \right)^{\gamma^\gamma+2}, & \beta \in [\beta^*, \beta_{\text{max}}], \\ 0, & \beta \in [\beta_{\text{max}}, 1]. \end{cases}
\]

Here \( \gamma = 1/(1 - \beta^3)^{1/2} \), \( \gamma^\gamma = 1/(1 - \beta^2)^{1/2} \), \( \Theta = kTm_e c^2 \) gives the temperature of the low-energy thermal distribution, and \( C \) \( (\approx 1) \) normalizes the function to unit total probability. For instance, in the model proposed by Blasi (2000), a cutoff at \( \beta_{\text{max}} \gamma_{\text{max}} \sim 1000 \) arises naturally and ensures that the electrons
in the tail do not affect the synchrotron radio emission. For \( \gamma_e \gg 1 \) one finds \( C = 0.982 \), indicating that only 1.8% of the electrons are in the nonthermal tail; however, the electron kinetic energy is increased by 73% and the electron pressure by 48%, so the hydrodynamical properties of the gas can be greatly influenced by the nonthermal component.

The bremsstrahlung emissivity is given by \( q_{\text{brem}}(p_e) = n_{\text{gas}} \int dp v(p) \sigma_{\text{brem}}(p, k_e) \), where \( n_{\text{gas}} \) is the gas density in the cluster, \( v(p) \) is the velocity of an electron with momentum \( p \), and \( k_e \) is the photon momentum. The bremsstrahlung cross section \( \sigma_{\text{brem}} \) is taken from Haug (1997). We assumed for simplicity that the cluster has constant density and temperature, but our results can be easily generalized to the more realistic spatially varying case.

As shown by Ensslin et al. (1999), there is a wide region in the \( \rho^*-\alpha \) parameter space that matches the observations. We choose the values \( \beta^* \gamma = 0.5 \) and \( \alpha = 2.5 \) that provide a good fit to the overall X-ray data, as shown in Figure 1, where the thermal component has a temperature \( T = 8.21 \) keV. The data points are from BeppoSAX (R. Fusco-Femiano 1999, private communication) observations, while the thick curve is the result of our calculations for a suitable choice of the emission volume.

The basic question that we want to answer is whether the nonthermal tail in the electron distribution produces distortions in the CMB radiation that can be distinguished from the thermal SZ effect. To answer this question, we calculate the SZ spectrum using equation (4), plotting the results in Figure 2 for a thermal model and two nonthermal models, each based on Coma. There is an appreciable difference between the curves, as large as \( \sim 60\% \) at high frequencies (\( x > 5 \)). At low frequencies (\( x < 1.7 \)), the region currently probed by most SZ observations, the relative difference is at the level of \( \sim 10\%-20\% \).

To establish the existence of a nonthermal contribution to the SZ effect, say in Coma, one should measure \( \Delta T \) at three or more frequencies. While \( T_e \) is well constrained by X-ray measurements, \( \tau_e \) is not, and in addition the SZ distortion is contaminated by a frequency-independent constant, \( \Delta T_{\text{CMBR}} + \Delta T_{\text{SZ}} \), i.e., the sum of the background primordial CMB radiation anisotropy, and the kinematic SZ effect caused by a line-of-sight velocity \( v_0 \) in the CMB radiation frame. Two measurements are required to determine these unknowns before one is able to detect nonthermality. In Figure 3 we estimate the difference in \( \Delta T \) for a thermal and nonthermal spectrum after allowing for these unknowns. The residual spectral differences remain both at low and high frequencies and might be accessible by ground observation. From space a nonthermal signature should be detectable by the Planck Surveyor, but not by MAP, mainly owing to sensitivity and beam dilution rather than frequency coverage.

Of particular interest observationally is the frequency of the zero SZ spectral distortion \( x_0 \), defined by \( \Delta T(x_0) = \Delta T_{\text{thermal}}(x_0) = 0 \). Measuring the difference in the CMB flux on and off the cluster near the zero allows the measurement of small deviations...
tions from the classical behavior with only moderate requirements on the calibration of the detector and is very sensitive to \(v_e\). For a thermal plasma (Birkinshaw 1999)

\[
x_0 = 3.830 \left(1 + 1.13\Theta + 1.14 \frac{\beta_e}{\Theta} \right) + O(\Theta^2, \beta_e^2),
\]

where \(\Theta = kT/m_e c^2\), \(\beta_e = v_e/c\). This equation is no longer valid for a nonthermal electron distribution. For our canonical parameters, no cutoff, and \(v_e = 0\), we find that \(x_0\) is shifted to 3.988, the same as would be obtained for a thermal distribution with an unreasonably large temperature of 18.62 keV and \(v_e = 0\), or with the “correct” temperature (8.21 keV) and \(v_e = 111 \text{ km s}^{-1}\). Even with our nonthermal tail, it is the velocity that mostly determines the value of \(x_0\), although the nonthermal electrons can bias the \(v_e\) determinations by \(\pm 100 \text{ km s}^{-1}\) (i.e., away from the observer).

3. OTHER IMPLICATIONS

In this section, we mention other important consequences of the existence of a nonthermal electron distribution. As noted above, the nonthermal component might correspond to only a few percent in additional electrons that do not contribute significantly to the nearly thermal 1–10 keV X-ray emission, while at the same time the electron pressure may be increased by nearly a factor of 2 (we have no evidence whether there is similar increase in the ion pressure). Many cluster mass estimates which are based on X-ray observations use the hydrostatic relation \(M \propto \nabla \rho / \rho\), and if the pressure has been significantly underestimated due to nonthermal electrons, the cluster mass would also be underestimated. Cluster masses play an important role in normalizing the amplitude of inhomogeneities in cosmological models, and the nonthermal electron populations may lead to an underestimate in this cluster normalization. The baryon fraction in clusters have also been used as an indicator of the universal baryon-to-mass ratio \(\Omega_b / \Omega_m\). If a cluster mass is underestimated due to nonthermal electrons, then the cluster baryon fraction will be overestimated. Note that the Coma Cluster, which does have a nonthermal X-ray excess, has played a particularly important role in cluster \(\Omega_b / \Omega_m\) estimates (White et al. 1993), although optical mass estimates are also used here. These cosmological consequences would be true even if the excess pressure was provided by a population of relativistic cosmic rays, as discussed by Ensslin et al. (1997).

Other implications are instead peculiar to the nonthermal tail scenario: using a combination of X-ray and SZ measurements, clusters have been used to estimate Hubble’s constant, \(H_0 \propto L_5 / (\Delta T_{SZ})^2\) (Birkinshaw 1999). We have shown that a nonthermal electron distribution generally increases \(\Delta T_{SZ}\) for fixed \(\tau\) and \(\Theta\), and therefore one should use a larger proportionality constant when nonthermal electrons are present. Therefore, cluster estimates of \(H_0\) without taking into account a nonthermal electron distribution would underestimate \(H_0\).

If our model of the nonthermal tail held universally, then naive estimates of \(M_h, \Omega_b / \Omega_m\), and \(H_0\) should be respectively adjusted upward, downward, and upward by tens of percentage points. However, estimates of cosmological parameters using clusters generally make use of measurements of an ensemble of clusters. Suprathermal X-ray emission does appear in a two of three clusters, but the statistics are not good enough for an accurate prediction of how frequently a nonthermal electron distribution might be present in a sample of clusters. Therefore, the overall bias introduced in parameter estimates is necessarily uncertain. In any individual cluster the bias in a parameter estimator will depend on the spatial distribution of the nonthermal electrons, which is also uncertain and not well-constrained by present hard X-ray measurements. The important constraint is that the magnitude of cosmological parameter misestimation might be quite large.

Confirmation or refutation of the hypothesis that the X-ray excess is due to a nonthermal tail will have important consequences not only for the understanding of cluster structure but for cosmology as well. We argue that SZ measurements are the best way to test this hypothesis and that this is within the capabilities of present technology.

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REFERENCES

Berezinsky, V. S., Blasi, P., & Ptuskin, V. S. 1997, ApJ, 487, 529
Birkinshaw, M. 1999, Phys. Rep., 310, 97
Blasi, P. 1999, ApJ, 525, 603
———. 2000, ApJ, 532, L9
Blasi, P., & Colafrancesco, A. 1999, Astropart. Phys. 12, 169
Challinor, A., & Lasenby, A. 1998, ApJ, 499, 1
Ensslin, T. A., Biermann, P. L., Kronberg, P. P., & Wu, X.-P. 1997, ApJ, 477, 560
Ensslin, T. A., Itoh, R., & Biermann, P. L. 1999, A&A, 344, 409
Feretti, L., Dallacasa, D., Giovannini, G., & Tagliani, A. 1995, A&A, 302, 680
Fusco-Femiano, R., Dal Fiume, D., Feretti, L., Giovannini, G., Grandi, P., Matt, G., Molendi, S., & Santangelo, A. 1999, ApJ, 513, L21
Haug, E. 1997, A&A, 326, 417
Herbing, T., Lawrence, C. R., Readhead, A. C. S., & Gulkis, S. 1995, ApJ, 449, L5
Itoh, N., Kohyama, Y., & Nozawa, S. 1998, ApJ, 502, 7
Kaastra, J. S., Lieu, R., Mittaz, J. P. D., Bleeker, J. A. M., Mewe, R., Colafrancesco, S., & Lockman, F. J. 1999, preprint (astro-ph/9905209)
Kim, K.-T, Kronberg, P. P., Dewdney P. E., & Landecker, T. L. 1990, ApJ, 355, 29
Molendi, S., Grandi, S., Fusco-Femiano, R., Colafrancesco, S., Fiore, F., Neschi, R., & Tamurelli, F. 1999, ApJ, 525, L73
Rephaeli, Y. 1995, ApJ, 445, 33
Stebbins, A. 1997, in The Cosmic Microwave Background, ed. C. H. Line weaver, J. G. Bartlett, A. Blanchard, M. Signore, & J. Silk (Dordrecht: Kluwer), 241
Volk, H. J., & Atoyan, A. M. 1998, preprint (astro-ph/9812458)
White, S. D. M., Navarro, J. F., Evrard, A. E., & Frenk, C. S. 1993, Nature, 366, 429
Zeldovich, Y. B., & Sunyaev, R. A. 1969, Ap&SS, 4, 301