Doppler Shift in Andreev Reflection from a Moving Superconducting Condensate in Nb/InAs Josephson Junctions

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We study narrow ballistic Josephson weak links in an InAs quantum wells contacted by Nb electrodes and find a dramatic magnetic-field suppression of the Andreev reflection amplitude, which occurs even for in-plane field orientation with essentially no magnetic flux through the junction. Our observations demonstrate the presence of a Doppler shift in the energy of the Andreev levels, which results from diamagnetic screening currents in the hybrid Nb/InAs-banks. The data for conductance, excess and critical currents can be consistently explained in terms of the sample geometry and the McMillan energy, characterizing the transparency of the Nb/InAs-interface.

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In recent years a detailed microscopic understanding of the proximity effect has emerged. There is now agreement that in high-transparent Josephson junctions formed by a metallic weak link in good contact with two superconducting (SC) banks the supercurrent is carried by Andreev bound states (ABS). These come in pairs corresponding to the opposite directions of Cooper-pair transfer mediated by multiple Andreev reflection (MAR) at the SC/metal interfaces provided that the acquired quasi-particle phase is a multiple of 2π. At currents exceeding the critical current \( I_c(T) \) MAR between the SC banks manifests itself in the current-voltage characteristics as subharmonic gap structures at voltages \( eV_n = 2\Delta/n \), where \( \Delta \) is the SC energy gap and \( n = 1, 2, ... \). At higher voltage \( eV \gg 2\Delta \), \( I(V) \) becomes linear with an excess current \( I_{exc} = I(V) - G_NV \) determined by a single Andreev reflection (AR) probability \( |a(\varepsilon)|^2 \) (\( G_N \) is the normal state conductance).

Weak links formed by a two-dimensional electron gas (2DEG) in semiconductor quantum wells are of particular interest because here the ballistic transport can be studied. In very high magnetic field perpendicular to the 2DEG, theory and experiments have demonstrated Andreev transport via edge states. Indirect evidence for a strong magnetic-field effect on AR was experimentally found in antidot billiards. The case of parallel field, with respect to the 2DEG, is equally intriguing: as ideally no magnetic flux threads the 2DEG, one may naively expect the Josephson current to survive up to the critical fields of the SC leads. This is not the case, but the underlying mechanism of the supercurrent suppression is still unclear. This question, also relevant for other 2D hybrid systems, is among the issues this study focuses on.

In this Communication, Nb/InAs Josephson junctions of different width are studied in a four-terminal lead configuration within the 2DEG. This allows us to separately determine the transparencies of the InAs weak link and Nb/InAs interfaces, and identify an additional energy scale in the electronic spectrum of the hybrid SC terminals. We observe a very strong suppression of both the AR probability and supercurrent in weak magnetic fields of 4 and 100 mT for perpendicular and parallel orientations, respectively. This unexpectedly rapid decay can be traced back to diamagnetic supercurrents in Nb, which break the time-reversal symmetry of Andreev reflection.

The samples were prepared from a heterostructure containing an InAs-quantum well with thickness \( d_N = 15 \) nm and a mean free path around \( 3.7 \) \( \mu \)m, confined between two AlGaSb layers. The electron density was \( 7.8 \cdot 10^{15} \) m\(^{-2} \), resulting in a Fermi wavelength \( \lambda_F = 2\pi/k_F = 28 \) nm. First, a constriction of width \( W \) with a four-terminal (4t) lead structure (shown in yellow in Fig. 1b) was patterned using electron beam lithography (EBL) and etched by reactive ion etching in a SiCl\(_4\)-plasma. In a second EBL step two Nb stripes were deposited onto the InAs (horizontal grey bars in Fig. 1b) after removal of the top AlGaSb layer and in-situ Ar-ion cleaning of the InAs surface. The Nb-strips have a width \( W_S = 1 \) \( \mu \)m and a thickness \( d_S = 35 \) nm; their distance \( L = 600 \) nm defines the length \( L \) of the junction. This results in a Thouless energy of \( \varepsilon_{Th} = h v_F/L = 0.8 \) meV \( \lesssim \Delta \) for all samples, \( v_F \) being the Fermi velocity in the normal metal. We have prepared four samples on two chips in the same batch. The chips contained junctions with \( W = 0.5 \) and \( 1 \) \( \mu \)m (denoted as \#1 and \#2a), and \( W = 1 \) and \( 2 \) \( \mu \)m (denoted \#2b and \#3), respectively.

Figure 1b shows the two-terminal (2t) resistance of samples \#1, \#2a, and \#3 measured through the Nb stripes vs. temperature \( T \). The resistance drops at the transition temperature of the Nb stripes around 8.3 K and again between 2 and 3 K, where the constriction becomes superconducting. In Fig. 1b: we plot the 2t-resistance measured across the Nb wires and the 4t-resistance measured within the InAs layer at \( T = 10 \) K as a function of \( W \). The 4t-resistance scales as 1/\( W \), obeying the ballistic Landauer–Büttiker formula:

\[
G_N = \frac{2e^2}{h} \sum_n T_n = \frac{2e^2}{h} \frac{e_F W}{\pi} \langle \mathcal{T} \rangle ,
\]

where \( \langle \mathcal{T} \rangle \) is the average channel transparency. From the
three samples with width grey stripes are made from a 35 nm thick Nb. (b) Regions with InAs two-dimensional electron gas are shown in sample #3 with width vs. $1/W$ function of temperature for different widths $W$. (c) Resistance vs. $1/W$ at 10 K for the 4t-configuration with InAs leads (■) and the 2t-configuration with Nb leads (□). (d) Normalized supercurrent $I_C$ vs. perpendicular magnetic field $B_\perp$ for different widths $W$. The black curve is the standard Fraunhofer pattern fitting the data for sample #3 ($W = 2 \mu m$).

slope of $G_N(W)$ we extract $\langle T \rangle = 0.8$. The 2t-resistance at 10 K is also proportional to $1/W$, however, with an offset caused by the normal state resistance of the Nb-stripes and the Nb/InAs-interface resistance. Finally, we show in Fig. 1 the interference pattern of the junctions in a transversal magnetic field. For the widest junction the critical current $I_C(B)$ exhibits the expected Fraunhofer pattern. For the narrower ones the higher order maxima are suppressed, and in the 0.5 µm-wide device $I_C(B)$ resembles more a Gaussian rather than a Fraunhofer pattern. This will be addressed below.

Information about the spectral distribution of ABS is contained in the $T$-dependence of the supercurrent. In Fig. 2 we present the critical current $I_C$ at $B = 0$ as a function of temperature for different widths $W = 0.5$, 1 and 2 µm of the InAs constriction. With decreasing temperature $I_C(T)$ initially increases and then saturates at $T \lesssim 1 K$. The solid lines are fits based on the approach of Grajcar et al. who adapted the scattering theory for the Josephson current to 2D ballistic proximity structures. Two parameters were used in these fits: the average normal-state transmission probability $\langle T \rangle$ and the McMillan energy $\Gamma_{McM} = \tau_{SN} \cdot \hbar v_F / 2 d_S$ which is the Thouless energy of the InAs quantum well multiplied by the Nb/InAs interface transparency $\tau_{SN}$. $\Gamma_{McM}$ determines the strength of the proximity-induced superconducting correlations in the InAs regions underneath the Nb film, which act as effective superconducting terminals described by the Green’s functions of the McMil-}

lan model. Clearly, the smaller $\tau_{SN}$, the weaker the proximity effect. Because $\langle T \rangle$ influences mainly the saturation value of $I_C(T \to 0)$, while $\Gamma_{McM}$ determines the decay of $I_C$ with $T$ these two parameters can be extracted independently from the measured $I_C(T)$.

The fit parameters are given in the inset of Fig. 2. The values of $\langle T \rangle$ scatter by ±5% around 0.9, which is slightly higher, but still close to $\langle T \rangle = 0.8$ estimated independently from 4t-resistance (■). The values of $\Gamma_{McM}$ scatter by ±15% around 0.9 meV. From this, we find $\tau_{SN}(T \approx 0.6$ which is much smaller than $\langle T \rangle$. The high mobility of the InAs-quantum well provides the high transparency of the constriction, while the comparatively low Nb/InAs interface transparency is sufficient to transform the Nb/InAs bilayer into an effective superconductor with the additional energy scale $\Gamma_{McM}$ (see also inset in Fig. 6).

Having all sample parameters fixed, we now turn to the main topic of our study, i.e. the magnetic field dependence of Andreev reflection. First, we examine $IV$- and $dI/dV$-characteristics. Figure 3 shows the differential conductance $dI/dV$ of sample #2a ($W = 1 \mu m$) for different values of perpendicular field $B_\perp$. At $B_\perp = 0$ subharmonic gap structures (see arrows) appear at integer fractions $V_n = 2\Delta_{N_{b}} / n e$ of $2\Delta_{N_{b}}$. The location of the steps vs. $1/n$, plotted in the inset of Fig. 3, agrees very well with the value of $\Delta_{N_{b}} = 1.35$ meV. The $n = 1$ step appears at lower voltage as predicted in Ref. [16]. A tiny perpendicular field of 4 mT washes out these features, vanishing completely at 10 mT. Also, the enhancement of $dI/dV$ at low bias is suppressed. It is known that the subharmonic gap structure and low bias enhancement of $dI/dV$ originate from multiple AR at finite $V$. The large number of steps independently confirms the high AR probability $|a|^2$ expected from the high values of $\langle T \rangle$. The rapid smearing of these steps thus indicates a surprisingly strong magnetic field dependence of $|a|^2$.

This observation is further substantiated by the behavior of the excess current $I_{exc}$ obtained by integrating $dI/dV - G_N$ (see Fig. 3) over voltage. Since $I_{exc}$ results
from a single AR at $V > 2\Delta_{Nb}$, it is more robust than MAR whose amplitude is $\propto |a|^{2n}$. Figure 1 shows $I_{exc}$ on a logarithmic scale for different field orientations. It is suppressed to $\sim 50\%$ at $B_{\parallel} = 30$ mT and $B_{\perp} = 300$ mT. The field $B_{\parallel}$ exceeds $B_{\perp}$ by an order of magnitude, but is still a factor of 5 smaller than needed for an appreciable reduction of the gap $\Delta_{Nb}(B)$ as follows from the pair-breaking theory (dashed line in Fig. 3).

We attribute the behavior of $I_{exc}(B)$ to the magnetic field suppression of the Andreev reflection in the InAs proximity regions underneath the Nb films. In the presence of the external field $B = \nabla \times A(r)$, the SC order parameter acquires an inhomogeneous phase, $\gamma(r) = -2\pi\Phi_{0}^{-1}\int_{r}^{r'} A(r')dr'$, whose gradient $k_S = \nabla \gamma$ induces diamagnetic currents in the entire Nb/InAs structure, including the InAs proximity layer (see, Fig. 4b and c). If $k_i$ and $k_f$ denote the wave vectors of initial and final quasi-particle states in the AR-process, momentum conservation requires $k_i + k_f = k_S$, since the Cooper-pairs absorbed by the moving SC condensate have momentum $\hbar k_S$. This leads to a Doppler shift $\varepsilon_D = -h^2 k_S^2 / 2m^* \varepsilon_{\text{transverse wave number}}$. The inset in Fig. 3 illustrates the energy dependence of $|a(\varepsilon)|^2$. The fact that each of the Nb/InAs-banks constitutes a single hybrid superconducting terminal is illustrated the two BTK-like peaks in the spectrum, one at $\Delta_{Nb}$ and another at a smaller energy determined by $\Gamma_{McM}$. Similar hybrid excitation spectra have recently been observed by scanning tunneling spectroscopy in diffusive Al/Cu-structures. The suppression of $|a(\varepsilon)|^2$ occurs in the range $|\varepsilon| \lesssim \Gamma_{McM}$, when $\varepsilon_D$ becomes comparable to the McMillan energy $\Gamma_{McM}$.

The solid lines in Fig. 4 show $I_{exc}(B)$ obtained from the BTK-model, in which we express the AR amplitude $a(\varepsilon)$ in terms of the McMillan’s Green’s functions, accounting for the Doppler shift $\varepsilon_D$ in the InAs proximity layer. Using the parameters extracted from Fig. 2, the model reproduces the measured curves quite well (except for the shoulder in $I_{exc}(B_{\perp})$, which at present is not understood). The model accounts for the ratio of the $B_{\parallel}$ and $B_{\perp}$ scales. This has a simple interpretation if we notice that the relevant value of $k_S$ is related, via the circulation theorem, to the magnetic flux $\Phi$ threading the Nb/InAs bilayer for given field orientation (Fig. 3a and c): $k_S = -\pi / W \times \Phi / \Phi_0$. Neglecting the field in-homogeneity in Nb, we find $\Phi = B_{\parallel}W(d_S + d_N)$ and $\Phi = B_{\perp}W_S$ for the parallel and perpendicular cases, respectively. Consequently, the expected field ratio is $B_{\parallel}/B_{\perp} = W_S/(d_S + d_N) = 20$, which is within the margins of the experimental uncertainty.

The dramatic reduction of the AR amplitude $a(B)$, inferred from the behavior of $I_{exc}$, can also explain the suppression of the higher lobes of the critical current in the Fraunhofer pattern in weak perpendicular magnetic field (see Fig. 1), since the same $a(B)$ determines the energies of the ABS and hence $I_C(B)$. We also note that the smaller the channel width $W$ is, the higher are the values of $B_{\perp}$ required to reach $\Phi = \Phi_0$ in the junction and the stronger is the suppression of $I_C(B)$ at this point.

Finally, we examined the critical current $I_C$ in a parallel magnetic field for samples #1 and #2a. Again, even for the perfectly parallel field there is the phase gradient due to the vector potential (Fig. 4b) that induces a Doppler energy shift of the ABS. When it becomes comparable to $\Gamma_{McM}$, counter-propagating ABS overlap in energy, and their contributions to $I_C$ gradually cancel. Similar to $I_{exc}$, the suppression of $I_C$ occurs at $\sim 100$ mT. In Fig. 4 we plot $I_C(B)$ for sample #1 for several almost parallel field orientations. The small perpendicular component of B still leads to a Fraunhofer-like minimum in $I_C$, which allows a precise determination of the angle $\alpha$ between B and the junction plane. This can be described by $I_C(B) = I_C(B_{\parallel}) |\sin(\pi AB_{\parallel}/\Phi_0)/|\pi AB_{\parallel}/\Phi_0|$, where $\Phi_0$ is the magnetic flux quantum.
FIG. 5: Critical current vs. $B$ for sample #1 at small angles $\alpha$ between $B$ and 2DEG. The lines are theoretical fits (see text). The inset shows the double peak structure of $\alpha$ characteristic of hybrid SC and its suppression by $B_{\parallel}$.

where $B_{\perp} = B \sin \alpha$, $B_{\parallel} = B \cos \alpha$ and $A$ is the junction area. Unlike the standard Fraunhofer pattern, we take into account the dependence of the critical current $I_c(B_{\parallel})$ on the parallel component of the field. This is done by straightforwardly generalizing the ballistic formula for the Josephson current in which we express the AR amplitude $a$ in terms of the McMillan’s Green’s functions with the Doppler shift in the InAs proximity layers. The dependence $I_c(B_{\parallel})$ makes the Fraunhofer-like oscillations decay much faster than the usual $1/B_{\perp}$ law, which is clearly seen in Fig. 5. For $\alpha \leq 1.3^\circ$ no minimum is detected, while it is still observed for a wider sample #2a on the same chip (not shown). The curve for $\alpha \approx 0$ resulted from a careful maximization of $I_c$ at fixed field, since in this case, evidently, no Fraunhofer minimum can be observed anymore. The fits (solid lines in Fig. 5) are obtained by varying the effective thickness of the Nb/InAs bilayer, $d_{\text{eff}}$. We find $d_{\text{eff}} = d_N + d_S = 50 \text{ nm}$ for sample #2a (not shown) in agreement with the geometry, while for sample #1 $d_{\text{eff}} \approx 25 \text{ nm}$. The reason for this discrepancy is unclear, since the excess current data in Fig. 4 agree very well. The deviation of the data for $\alpha \approx 0$ from the theoretical $B_{\perp} = 0$ curve is probably due to a weak perpendicular stray field of the Nb electrodes.

In conclusion, we have studied the IV-characteristics and critical current of ballistic Nb/InAs Josephson junctions in parallel and perpendicular magnetic fields. The observed field dependence is much stronger than anticipated from standard models and can be traced back to the Doppler shift of Andreev levels due to diamagnetic supercurrents in hybrid Nb/InAs contacts. Several different aspects of the proximity effect are consistently and nearly quantitatively described by our theoretical model.

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while, for the perpendicular field, the Pearl penetration depth $\lambda_P = 2\lambda_L^2/d_S \simeq 700$ nm is larger than half of the Nb stripe width $W_S/2 = 500$ mm.