It Does Not Follow. Response to “Yes They Can! . . .”

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This a response to “Yes They Can! . . .” (a comment on [5]) by J.S. Shaari et al. [6]. We show that the claims in the comment do not hold up and that all the conclusions obtained in [5] are correct. In particular, the two considered kinds of two-way communication protocols (ping-pong and LM05) under a quantum-man-in-the-middle (QMM) attack have neither a critical disturbance $(D)$, nor a valid privacy amplification (PA) procedure, nor an unconditional security proof. However, we point out that there is another two-way protocol which does have a critical $D$ and a valid PA and which is resistant to a QMM attack.

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I. INTRODUCTION

In Ref. [5] we considered quantum-man-in-the-middle (QMM) attacks on two kinds of two-way quantum key distribution (QKD) protocols: ping-pong one with entangled photons [2] and LM05 one with single photons [4]. In the attacks, an undetectable eavesdropper (Eve) copies all messages in the message mode (MM) so as to keep sender’s (Bob) qubit in the quantum memory, sends her own qubits to the encoder (Alice), receives and decodes Alice’s messages, encodes it on kept Bob’s qubits, and sends them back to Bob. We showed that

(i) attacks leave no errors in the MM and hence the standard methods of establishing security from the literature [8] are not available;

(ii) standard privacy amplification (PA) procedure from the literature [8] cannot be executed due to the absence of a critical value of the disturbance $(D)$ in the MM;

(iii) the security proof for LM05 put forward in [5] does not cover the aforementioned QMM attack and therefore it cannot be considered to be a proof of unconditional security of LM05.

The authors of [9] claim that the above conclusions obtained in [5] are erroneous, though. In this paper we show that their claims do not hold up.

II. METHODS

We re-analyze the QMM attacks on two different two-way QKD protocols: entangled photon (ping-pong) and single photon (LM05) ones with reference to [2] and [4], in order to show that the conclusions obtained in [5] are correct and that the claims against them, put forward in [9], fail.

III. RESULTS AND DISCUSSION

The authors of [9] reformulated the above points (i-iii) and we denote those reformulations as (i)$'$, (ii)$'$, and (iii)$'$, respectively below. The points (i)$'$-(iii)$'$ do not faithfully correspond to (i)-(iii).

In what follows, we show that none of the claims of the authors of [9] against (i-iii) holds up. We analyze their points (i-iii)$'$ one by one and show that (i-iii)$'$ are all correct.

(i)$'$ attacks which leave no errors in the MM do not allow for security to be established by standard methods:

The security of a protocol, under individual, or collective, or general (coherent) attacks, alike, is standardly evaluated via the critical quantum bit error rate (QBER) by calculating the secret fraction

$$r = I_{AB} - I_E,$$

where $I_E = \min(I_{AE}, I_{BE})$, where $I_{AB}, I_{AE},$ and $I_{BE}$, are mutual information in the MM between Alice and Bob, Alice and Eve, and Bob and Eve, respectively [8]. The mutual information serves Alice and Bob to establish the quantum key distribution (QKD) key in the MM and Eve to snatch it from the MM.

Now, surprisingly, the authors of [9] claim: “In two-way QKD $I_E$ is obtained from the CM whereas $I_{AB}$ is estimated from the MM. Therefore, the CM makes it possible to properly execute the [privacy amplification] PA whereas the MM enables a proper execution of the [error correction] EC.”

In the control mode (CM) Alice and Bob can only estimate to which extent Eve was in the line and Eve can find out by which signals they did so (50%), but that cannot offer them any information on the key itself (via PA). On the other hand, there is no EC, since $I_{AB} = 1$.

Contradicting themselves, in the next paragraph the authors of [9] admit that $I_{AB} = 1$, that “there is no error in the MM,” and that the error rate in the CM is 50%, but claim that all that “is inconsequential” and write “Hence, if Eve attacks a fraction $f$ of the qubits, the resulting secret key rate will be given by $r = 1 - f \geq 0$, \[ \begin{align*}
I_{AB} & = 1, \\
I_E & = \min(I_{AE}, I_{BE}), \\
r & = I_{AB} - I_E.
\end{align*} \]

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with equality when $f = 1$, i.e., when Eve attacks all the qubits. Therefore, the protocol is always secure against such an attack, for any value of $f$.”

However, when Eve attacks all the qubits, she knows all the messages with certainty, i.e., the whole key. When $f < 1$, Alice and Bob do not have an available procedure to erase the parts of the key Eve snatched, as shown below. Such a protocol cannot be considered secure.

So, [9]’s claims against (i) do not hold up. Note that (i’) distorts the meaning of (i) which claims that “standard methods,” i.e., those from the literature, are not available, not that the security cannot be achieved at all as stated in (i)’. Let us look at the next point.

(ii’) privacy amplification (PA) cannot be executed due to the absence of a “critical value” similar to BB84’s famous 11%:

“PA [is a procedure] aimed at destroying Eve’s knowledge of the reference raw key... The fraction to be removed... is $I_E$. The procedure was established in and refined in. Eve’s knowledge comes exclusively from MM by the very definition of the mutual information and when $I_E$ exceeds $I_{AB}$ at the critical value of the $D$ (11% for BB84), Alice and Bob have to abort the transmission.

In contrast to that, the authors write: “The absence of errors in MM simply ensures the unity value for $I_{AB}$, to the legitimate parties’ benefit, and has no bearing in determining the PA rate, which is a function of $I_E$ estimated from the error rate in CM. Obviously, differing values for errors in CM would translate into differing PA rates, hence differing $r$, effectively dismissing the notion of ‘critical value’.”

This just does not make any operational sense. All messages Eve snatched are perfectly correct and there is nothing to be erased in the PA procedure. On the other hand, due to exponential attenuation of signals in fibers, Alice and Bob can hardly guess which messages Eve snatched even when the CM and MM signals are equally numerous and when $f$ is low. When $f$ is high or close or equal to 1, this is absolutely impossible because then they erase the whole key. There is simply no reliable method of identifying Eve’s messages, especially not with the present PA procedure. So, the considered protocols under the QMM attack is equivalent to sending a plain text under an unspecified “protection” of the CM.

It helps to dig into the literature. For instance, “some considered ... quantum secure direct communication [two-way protocols] and generated some interest. However, it was soon recognized that the idea suffers from a major default with respect to standard QKD. It allows no analogue of privacy amplification: if an eavesdropper obtains information, it is information on the message itself and of course cannot be erased.” [8]

The authors of [9] further claim that “a similar argument applies even in the BB84 protocol, where the amount of PA executed depends on the actual error rate that has been detected, not on a predetermined critical value like the 11% quoted in [8].”

As stressed above and shown in Fig. 5(a) of [8], $D = 0.11$ is not “predetermined;” it corresponds to $I_{AB} = I_{AE}$. For $D > 0.11$ we have $I_{AB} < I_{AE}$, i.e., Eve has more information than Alice and Bob who therefore cannot possibly carry out PA beyond $D = 0.11$ at which point they are left with no bits for the key.

Hence, [9]’s claims against (ii) fail, too. Note that (ii’) distorts the meaning of (ii) which claims that “standard privacy amplification,” i.e., the one from the literature, cannot be executed, not that the privacy amplification cannot be executed at all as stated in (ii)’. As for the last point

(iii’) the existing security proofs [8] are flawed as they do not consider this specific class of attacks, first of all, nowhere in [8] did we write that the proofs from [8] are “flawed.” We just pointed out that they do not cover the QMM attack, i.e., that they are not general enough to provide an unconditional security proof for the LM05 protocol.

In [8] we read: “This is clearly untrue as QMM attack can be described as a specific case of such proofs. It suffices to consider Eve’s ancilla as a qubit and the unitary transformation as the well known SWAP gate.”

But, when we look into [8] this does not make sense. Eq. (1), III.B. Eve’s attack in Bob-Alice channel of [8] reads:

$$U_{BE}|0\rangle_B|E\rangle = c_{00}|0\rangle_B|E_{00}\rangle + c_{01}|1\rangle_B|E_{01}\rangle,$$

where $|0\rangle_B, |1\rangle_B$ are Bob’s qubits and $|E\rangle, |E_{00}\rangle, |E_{01}\rangle$ Eve’s ancillas. How can we now consider $U$ as a SWAP gate for her ancillas and what does it swap for what? Our Eve does not make use of ancillas at all and does not resend Bob’s qubits. So, the security analysis is for another type of attack, not for the QMM one.

The authors even admit that “an attack in the backward path is not made explicit as an extremely pessimistic stand is taken where Eve is allowed to extract all possible information from the entire Bob-Eve system without specifying the actual mechanism.”

What does an “extremely pessimistic stand” mean in the context of a rigorous proof? Equally so, within a rigorous proof of a theorem, Eve cannot be “allowed” to “extract all possible information from the entire Bob-Eve system without specifying the actual mechanism.” And most importantly, an analysis of a QMM attack cannot be carried out without the “the backward path” being considered. If one does not analyse the backward path then one cannot detect Eve at all because in the QMM Eve just lets all the messages in both CM and MM through to Alice.

This shows that the proof in [8] does not cover a QMM attack and, hence, that it is not an unconditional security proof. Ergo, claims against (iii) from [8] fail, as well.
IV. CONCLUSIONS

In this paper we analyze the claims put forward by the authors of [9] that the conclusions obtained in [5] and cited in Sec. I as (i), (ii), and (iii), are erroneous.

In Sec. III we show that none of the claims from [9] holds up and that the conclusions (i), (ii), and (iii) about the QMM attacks on the two-way communication protocols from [5] are correct.

The most important point of [5] is the following. It is obvious that when Eve is in the line all the time (caring out a QMM attack) she has all the messages (without inducing any error in the MM) and therefore the PA cannot be done. When Eve is in the line only up to a certain percentage of the transmission time, then PA might be possible based on the disturbance ($D$) from the CM but we do not know up to which value of $D$ (usually called critical $D$) the PA would be valid under a QMM attack.

So, we need an elaborated procedure and algorithm which would ensure that Eve would possess no significant number of messages after the PA procedure. A good example of how to estimate when and how we should design a PA procedure is given in [10].

Having said that, we would like to stress that the two-way direct communication protocols covered in [5] include just two best-known ones of their kind: the ping-pong and LM05 protocols (see Sec. I). These two kinds of two-way protocols have neither a critical $D$, nor a valid PA for a QMM attack since no one has come forward with them in the literature as of yet. The security proof from [5] does not contain or elaborate on them and therefore cannot be considered “unconditional” until one closes this open question.

Still, a two way communication protocol with a critical $D$, valid PA, and resistant to a QMM attack, is possible, as shown in [6]. So, “quite complex two-way QKD security proofs” which the authors of [9] refer to in their Conclusions might indeed help us all to develop security analysis of an implementable high capacity (four bits) two-way protocol.

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