Numerical analysis of thermal deformation for constructive variants of mirror segments in a parabolic antenna

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Abstract. This work is devoted to the research of thermal deformation of segments in a parabolic reflector mirror in connection with the limitations determined by the radio frequency range of antenna operation. The thermal deformation analysis is performed based on the results of a numerical solution of the coupled problem of stationary thermal conductivity under conditions of radiation-convective heat transfer and thermoelasticity. A comparative analysis of five form factors of mirror segments was performed, which allowed choosing its preferred configuration.

1. Introduction
The efficiency of mirror reflectors in terrestrial satellite communication systems is directly determined by their geometric stability - the shape constancy of the reflecting surface of the mirror under the action of complex operational loads and impacts. As the aperture of the reflector increases, it is natural that the absolute deformations of the mirror increase with constant loads and impacts. However, as the frequency of the radio bands increases, the requirements to geometric stability sharply increase. This is based upon the following considerations.

As is well known, the relation between the wavelength $\lambda$ and its propagation velocity $v$ is expressed by the formula $\lambda = v/f$, where $f$ is the oscillation frequency. For electromagnetic waves in a vacuum, their propagation velocity is equal to the speed of light, and the wavelength expressed in meters is

$$\lambda = \frac{299792458}{f},$$  \hspace{1cm} (1)

where the frequency $f$ is measured in hertz.

In modern communication systems, radio bands with frequencies reaching hundreds of gigahertz are used [1, 2]. As the frequencies increase, the wavelength obviously decreases. For example, for the Y-band frequencies are from 325 to 500 GHz [3], and the wavelength in accordance with (1) is in the range from 0.60 to 0.92 mm. It is known that the maximum linear deviation of the actual shape of the mirror from the theoretical should usually not exceed [4] (1/10 ... 1/16) $\cdot \lambda$. According to other sources [5], this value is 1/8 $\cdot \lambda$. Otherwise, there is a significant decrease in the efficiency of the antenna. Then, for the Y-band, the tolerance should not exceed 0.06 mm. This value can be considered as the limiting value of absolute deformation.
Thus, the reflectors of parabolic antennas operating in the high-frequency bands belong to the class of precision structures with increased requirements for geometric constancy. To achieve levels of geometric constancy, characterized by limiting deformations that are in millimeters and tenths of a millimeter, with an increase in the aperture of the reflector it turns out to be a nontrivial task. An obligatory condition for its solution is a comprehensive calculation analysis of the nature of the deformation of the reflector in the range of operational loads and impacts. The present paper takes stock of the experience of the calculation analysis of the thermodynamic state and deformation of the reflector mirror in a wide range of operating temperatures.

2. Problem definition
This study is dedicated to reflector antennas for terrestrial satellite communication systems working in conditions of $Q/K_a$ frequency band. The reflector diameter is in the range 9 to 12 m and the ratio of the focal length to the diameter is in the range 0.3 to 0.5. At the stage of research and development, several alternative constructive variants of the mirror using segments of the reflecting surface are considered. These options are determined by the configuration of the power frame and the limitations of the technological process of manufacturing segments of polymer composite materials.

The common for the alternative constructive solutions of the mirror segments considered below is their cross-section structure: the external layers of 0.6 mm thickness carbon fiber composite, the internal layer of 40 mm thickness plastic foam. Two essentially different constructive variants of the mirror structure are considered (Figure 1). In one case (configuration I), the mirror consists of six parts, each of which includes one segment of form factor 1 and two segments of form factors 2 and 3 (Figure 1, a). In the other case (configuration II), the mirror is formed by fifteen parts consisting of one segment of form factor 4 and two segments of form factor 5 (Figure 1, b). Another difference is that the reflecting segments of configuration II (form factors 4 and 5) are reinforced by radially extended 100 mm high stiffening plates made of 1.4 mm thickness carbon fiber composite.

![Figure 1. Variants of mirror construction: a – configuration I; b – configuration II.](image-url)

In accordance with regulatory requirements, the antennas are operated in the ambient air temperature range from minus 50 to plus 55 °C. The thermal state of the reflector is formed under the influence of the integrated flux density of solar radiation $Q_{sol}$ under conditions of radiation-convective heat exchange with the surrounding medium.

The research task is to determine the fields of temperature, strain and stress for the mirror segments of five form factors, and to perform a comparative analysis of the results and an estimate of the operability of the antenna mirror for the two configurations.
3. Mathematical formulation of the problem
To determine the thermodynamic and stress-strain states of mirror segments, it is necessary to solve the conjugate problems of stationary thermal conductivity and thermoelasticity. At the first stage, the temperature distribution over the volume of the segments is calculated under the boundary conditions of radiation-convective heat transfer (Figure 2).

Under the action of solar radiation on the mirror surface, most of the radiation is reflected (radiation heat exchange). The effective value of the integrated flux density of solar radiation, expended on the heated surface of the segments, is

\[ q^+ = Q_{sol} \mu, \]

where \( \mu \) is the reflection coefficient of the surface (albedo).

\[ q^- = \int_S q^- dS, \quad (2) \]

where \( S \) is the outer surface of the segments; \( q^- \) is the amount of heat removed from a unit surface area due to convective heat transfer.

Under the Newton-Richman law

\[ q^- = h_f (T_S - T_B), \quad (3) \]

where \( h_f \) is the coefficient of convective heat transfer (heat transfer); \( T_S, T_B \) – respectively the surface temperatures of the segments and ambient air temperature.

The temperature distribution over the volume of segments in the Cartesian coordinate system \( xyz \) (the temperature field \( T(x,y,z) \)) is determined by the equation of stationary thermal conductivity

\[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = 0, \quad (4) \]

and the Fourier thermal conductivity law, which establishes a unique relationship between the temperature gradient along the normal to the isothermal surface \( n \) (Figure 2) and the heat flux vector

\[ \bar{q} = -k \frac{\partial T}{\partial n}, \quad (5) \]

where \( k \) is the coefficient of thermal conductivity.

The result of solving equations (2) - (5) is the calculated temperature field \( T(x,y,z) \) in the volume of the mirror segments, which makes it possible to determine the temperature difference \( \Delta T(x,y,z) \) on the faces of the elementary volume at the point with the coordinates \( x, y, z \). Then for the reflector mirror, which configuration is described by the domain \( \Omega \) in three-dimensional space, the absolute deformations (displacements) along each axis of the Cartesian coordinate system are determined by summing the deformations along elementary volumes in the region \( \Omega \).
\[ \{\Delta x, \Delta y, \Delta z\} = \alpha \int_{\Omega} \Delta T(x, y, z) d\Omega, \]  

where \( \alpha \) is the coefficient of linear temperature expansion.

Further, the relative deformations and stresses are determined in accordance with the classical equations of the three-dimensional theory of elasticity.

4. Numerical modeling and results

Equations (2) - (6) were solved for discrete (finite-element) models of mirror segments I and II configurations in a three-dimensional formulation. A regular mesh of finite elements was constructed under the control of the step size of the mesh, which was varied in order to ensure convergence of the results along the finite element mesh. Finally, for all five mirror segments, the same degree of sampling is adopted. The kinematic boundary conditions consist of assigning zero displacements along all translational degrees of freedom at the points of segments in which they are mounted on the reflector skeleton: five points for segments 1, 2, 3, seven points for segment 4 and eight points for segment 5.

Figures 3 and 4 show the typical distribution fields of the calculated temperature values and absolute deformations (displacements) for segment 4 at an ambient air temperature of minus 20 °C.

**Figure 3.** Typical temperature distribution field (°C).

**Figure 4.** Typical displacements distribution field (nm).
Analysing the obtained results we found that in the range of ambient air temperatures from minus 50 to plus 55 °C with an effective integral flux density of solar radiation \( q = 112 \text{ W/m}^2 \) and convective heat transfer coefficient \( h_f = 1.5 \cdot 10^3 \text{ W/(m}^2\text{°C)} \), the temperature difference across the segment volume for different configurations is in the range 7.03...7.62 °C. This difference is equal to the excess of the reflecting surface temperature of the mirror segment due to solar heating above the ambient temperature.

These results practically coincide with the following values obtained by other authors. The temperature regime of the radio telescope RT-70 is characterized by an excess of the temperature of the reflecting surface above the ambient temperature by an average of 7.1...9.8 °C [6]. Studies [7] have shown that the maximum temperature changes in the reflector constructions do not exceed 5...8 °C, respectively, reflecting the possibilities of paint and varnish protection. Similar estimates are known for building structures: it is reported that the maximum temperature changes in the reflecting surface above the ambient temperature is only 8.0...8.5 °C [8].

Interest in analyzing the deformed state was the maximum displacement of the outer layer of carbon fiber - the reflecting surface proper (Table 1). These are the movements that are compared with the maximum permissible deviations of the surface from the original shape in accordance with the wavelength by (1). For band antenna reflectors in accordance with (1), the wavelength of the electromagnetic wave is in the range of 6.0 to 11.3 mm. We apply the rule of multiplication of intervals [9]

\[
[a, b] \times [c, d] = [\min(ac, ad, bc, bd), \max(ac, ad, bc, bd)],
\]

where \( a = 1/16 \), \( b = 1/8 \), \( c = 6.0 \), \( d = 11.3 \). We obtain the range of the maximum permissible deviations 0.375...1.413 mm. Comparing these values with the data of Table 1, we arrive at the following conclusions.

For segments of form factors 1, 2, 3 of mirror configuration I, the maximum displacements do not exceed the upper limit of the range of permissible deviations (1.413 mm) in the range from minus 35 to plus 55 °C. At a temperature of minus 50 °C, the calculated displacements exceed the permissible values.

For segments of form factors 4, 5 of mirror configuration II, the maximum displacements do not exceed the upper limit of the range of permissible deviations in the full range of operating temperatures.

| Form factor | Ambient air temperature (°C) |   |   |   |   |   |   |
|-------------|-----------------------------|---|---|---|---|---|---|
|             | −50 | −35 | −20 | −5 | 10 | 25 | 40 | 55 |
| 1           | 1.60 | 1.23 | 0.86 | 0.49 | 0.16 | 0.26 | 0.63 | 1.00 |
| 2           | 1.09 | 0.85 | 0.60 | 0.35 | 0.14 | 0.15 | 0.39 | 0.64 |
| 3           | 1.82 | 1.40 | 0.98 | 0.56 | 0.23 | 0.29 | 0.71 | 1.13 |
| 4           | 0.70 | 0.53 | 0.36 | 0.23 | 0.11 | 0.17 | 0.34 | 0.51 |
| 5           | 0.98 | 0.76 | 0.53 | 0.31 | 0.11 | 0.14 | 0.36 | 0.58 |

### 5. Conclusion

As a result of the formulation and numerical solution of the problem of analyzing the thermodynamic and deformed states of the five form factors of reflector segments, the regions (intervals) of the characteristic values of temperature differences across the volume of segments and absolute deformations of the reflecting surface of the mirror are established. Temperature differences are
characterized by high stability and correlate with similar results of other authors. This makes it possible to use the temperature range of 7...8 °C as the temperature boundary conditions for the preliminary thermoelastic analysis of alternative constructive variants of the mirror without first solving the problem of stationary thermal conductivity under conditions of radiation-convective heat transfer.

The results of the analysis of the deformed state demonstrate a high sensitivity to structural inhomogeneities that ensure the stiffness of the structure. The introduction of radial stiffeners results in both a significant reduction in the overall level of deformations and a decrease in their spread within the operating temperature range.

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