NUMERICAL STUDY OF WATER CIRCULATION IN SHALLOW BASIN USING LATTICE BOLTZMANN METHOD

K. MACHROUHI¹, M. M. CHARAFI²*, A. HASNAOUI¹

¹LS3M polydisciplinary faculty of Khouribga, Sultan Mly Slimane University 25000, Morocco.
²Corresponding author: Charafi.moulaymustapha@usms.ac.ma

Abstract: In the present work, a code based on the Lattice Boltzmann method (LBM) has been developed to study the hydrodynamic field in a shallow basin with complicated geometry. It is a non-regular border basin with dikes built inside. The LBM method was therefore used to solve the Navier Stokes equations with boundary conditions which express that the components of the horizontal two-dimensional velocity field are zero at closed boundaries (sealing and non-slip conditions), while the flow is imposed at the entrance of the basin, and a zero speed gradient is imposed at the free exit of the basin (condition of continuity of the flow). The validation of the developed code was verified by comparing the results obtained with those of Charafi et al. [5], where the finite difference method with a MacCormack scheme was used. A good agreement of the results was noted. A series of simulations was made for different values of the Reynolds number, and showed good performance of the code. The main advantage of the LBM method seen in this work lies in the simplicity of its implementation. However, the model suffers from some limitations such as its validity for low Reynolds numbers (Re = 400)

Keywords: Shallow water flow, Numerical methods, Lattice Boltzmann Method (LBM), Finite difference method (FD), dikes.

1 Introduction

The Lattice Boltzmann method is increasingly used to describe the behavior of flowing fluids, whether they are incompressible or compressible. By a Chapman-Enskog expansion, it leads to the Euler equation at order 1 and to the Navier-Stokes equation at order 2. Its main advantage is that it allows to describe fairly easily free surface flows, flows in complex media (porous media), flows with heat transfers (conduction, convection and phase changes), as well as flows of multi-component mixtures. The LBM method therefore differs from other simulation methods by its simplicity of use [1] - [3]. In the present work, it was a question of making investigations on the capacity of the Lattice Boltzmann method to simulate the problems of shallow water in a complex geometry. Hence the idea of developing a numerical code to simulate a flow in a shallow basin with an irregular border and equipped with two dikes. The said basin was the subject of a study carried out by Charafi et al [5], where the MacCormack scheme was used to solve the Saint-Venant equations discretized using the finite difference method. We will therefore use the results of this study to validate our code. After a description of the Navier-Stokes equations in section 2, section 3 will be devoted to the formulation of the problem using the LBM method, and to the presentation of the boundary conditions. In section 4, the validation of the developed code will be confirmed by comparing the results with those of Charafi et al [5].
2. Navier–Stokes Equation

In fluid mechanics, flows are governed by the Navier-Stokes (NS) equations (equations of continuity and momentum conservation). For a two-dimensional incompressible flow, the conservative form of the N-S equation can be written in Cartesian coordinates, without bodily force, as follows [4]:

The x – momentum equation:

\[
\frac{\partial (\rho u)}{\partial t} + \frac{\partial (\rho uu)}{\partial x} + \frac{\partial (\rho uv)}{\partial y} = -\frac{\partial p}{\partial x} + \frac{\partial}{\partial x}\left(\mu \frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) 
\]

(1)

The y – momentum equation:

\[
\frac{\partial (\rho v)}{\partial t} + \frac{\partial (\rho uv)}{\partial x} + \frac{\partial (\rho vv)}{\partial y} = -\frac{\partial p}{\partial y} + \frac{\partial}{\partial x}\left(\mu \frac{\partial v}{\partial x}\right) + \frac{\partial}{\partial y}\left(\mu \frac{\partial v}{\partial y}\right)
\]

(2)

The continuity equation:

\[
\frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} = 0
\]

(3)

3. Formulation by the LBM method

The LBM method comes from the kinetic theory of gases of Boltzmann. The basic idea of kinetic theory is that fluids are made up of a large number of particles which move in random motion [7]. The exchange of momentum and energy is obtained by elastic collisions between the particles and the propagation of these particles. This process is modeled by the Boltzmann transport equation [4]:

\[
\frac{\partial f}{\partial t} + \mathbf{c} \cdot \nabla f + \Omega f = 0
\]

(4)

Where the distribution function \((x, c, t)\) of a simple particle represents the probability of finding at time \(t\) a particle at position \(x\) and having a speed \(c\). The collision operator \(\Omega\) characterizes the rate of evolution of the distribution function \(f\) due to collisions between particles. The second term on the left represents the modification of the function of distribution due to the movement of the particles (in the case where they are not subjected to any force). [8]

\[
f_h(x + c_h \Delta t, t + \Delta t) - f_h(x, t) = \frac{\Delta t}{\tau} \left( f_h(x, t) - f_h(x, t) \right)
\]

(5)

where \(f_h\) is the distribution function of particles; \(\Delta x\) is the lattice size; \(\Delta t\) is the time step; \(\tau\) is the single relaxation time which controls the rate of approach to the local equilibrium; \(c_h\) is the velocity vector of a particle in the \(h\) link, which is defined by [4]:

\[
c_0, c_1, c_2, c_3, c_4, c_5, c_6, c_7, c_8
\]

(6)
Figure 1 Model D2Q9

\( f_{\mathbf{c}}^{eq} \) is the local equilibrium distribution function defined as:

\[
f_{\mathbf{c}}^{eq} = \rho \omega_N \left( 1 + 3 \left( \mathbf{c}_{\mathbf{c}} \cdot \mathbf{V} \right) + \frac{3}{2} \left( \mathbf{c}_{\mathbf{c}} \cdot \mathbf{V} \right)^2 - \frac{\mathbf{c}_{\mathbf{c}} \cdot \mathbf{V}}{2} \right) \tag{7}
\]

The fluid viscosity is related to the relaxation frequency by and the kinematic viscosity is:

\[
\nu = \frac{\Delta t}{2 \sigma_t} \left( \tau_f - 0.5 \right) \tag{8}
\]

The weight coefficients are [4]

\[
\omega_0 = \frac{4}{9}, \quad \omega_{-1} = \frac{1}{9} \quad \text{and} \quad \omega_2 = \frac{1}{9}
\]

The density \( \rho \) is the momentum density:

\[
\rho = \sum_{\mathbf{c}=-2}^{2} f_{\mathbf{c}} \rho \mathbf{V} = \sum_{\mathbf{c}=-2}^{2} f_{\mathbf{c}} \mathbf{c}_\mathbf{c} \tag{9}
\]

On the boundaries (see Figure 2), the fluid speed being zero, the dynamic boundary conditions are defined as follows:

\[
\rho \mathbf{u} = f_1 + f_5 + f_0 - f_0 - f_0 - f_5 = 0 \tag{10}
\]

\[
\mu \mathbf{v} = f_4 + f_2 + f_6 - f_6 - f_6 - f_4 = 0 \tag{11}
\]

Solid boundary conditions: We may apply no-slip, slip or semi-slip boundary conditions. For no-slip conditions, the standard bounce-back scheme can be used.

Figure 2 Configuration of boundary conditions
Inflow boundary conditions: If the velocity are known, the unknown distribution function $f_i$ at the boundary can be decided with the method described by Zou and He [6]. For example in Figure 2, given the velocity at the inflow boundary, after streaming, the unknown $f_1$, $f_2$, and $f_5$ can be decided as:

\[
\begin{align*}
    f_1 &= f_4 + \frac{3}{2} \rho u \\
    f_2 &= f_7 - \frac{1}{2} (f_1 - f_2) + \frac{1}{2} \rho u + \frac{1}{2} v \\
    f_5 &= f_6 - \frac{1}{2} (f_3 - f_5) + \frac{1}{2} \rho u + \frac{1}{2} v
\end{align*}
\]

Outflow boundary conditions: In some problems, the outlet velocity is unknown. The normal practice is to use extrapolation for the unknown distribution functions. For instance, if the east boundary conditions (right-hand-side) of Figure 2 represents the outlet condition, then $f_3$, $f_4$, and $f_5$ need to be calculated at the east boundary, $i = n$. A second-order polynomial can be used as

\[
\begin{align*}
    f_3 &= 2f_{3,n-1} - f_{3,n-2} \\
    f_4 &= 2f_{4,n-1} - f_{4,n-2} \\
    f_5 &= 2f_{5,n-1} - f_{5,n-2}
\end{align*}
\]

4. Validation
To validate the developed code, several simulations were carried out on the case of a shallow basin with irregular borders. The performance of the LBM method to solve the Naviers stokes equations in the case of a shallow flow has therefore been examined, by comparing the results obtained with those found by Charafi et al [5].

4.1 Brief description of the studied case

Figure 3 geometry of the basin studied

Figure 3 shows the geometry of the basin with an inlet, an outlet and an irregular border. A set of meshes was used in our code in order to describe the same field by defining the nodes at the borders.

4.2 Results and discussions
As illustrated in FIG. 4, an almost perfect agreement was found by comparing the results of this work and those of Charafi et al [5], and this for different Reynolds numbers. In these simulations, we can notice the influence of the borders which therefore generate a circulation area which begins to mobilize downstream. Figure 4 therefore presents a flow model up to 1000 s of simulation which illustrates the stability of the LBM method and its efficiency in the field of flows and more particularly in the case of complex geometries for different values of the number of Reynolds: Re = 90, 150, and 240. The structure of the flow determined numerically using the two methods LBM and FDM therefore shows figures which are almost identical, meaning that the two methods give the same results.

**Figure 4.** Hydrodynamic field obtained using the two methods for Re = 90, 150, and Re = 240.

### 4.3 Effect of dikes construction
Figure 5. Effect of dikes in model of the basin: (a) circulation pattern with a dike using McCormack-FDM scheme; (b) circulation with a dike using LBM.

After modifying the structure of the basin by adding dikes, the speed field illustrated still shows the perfect agreement of the results obtained using the LBM method with those obtained using the McCormack-FDM scheme method (Charafi et al. [5]). The purpose of adding the dikes was to assess their impact on increasing the length of stay in the flow. The shape of the hydrodynamic field indeed confirms this increase since the length of the advective path of the flow increases.

5. Conclusion

In this work, a code based on the LBM method was used to study the hydrodynamic field in a shallow basin, with irregular border and dikes built inside it. The validation of the code was verified by comparing the results obtained with those of Charafi et al. [5], where the finite difference method - MacCormack scheme was used. Several simulations for different Reynolds numbers were carried out and a good agreement of the results was noted, as well in the case of a basin without dikes as a basin with dikes. The simulation with the addition of dikes made it possible to obtain a shape of the hydrodynamic field which shows an increase in the length of the advective path of the flow, and consequently the increase in its residence time.

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