Persistent current in one-dimensional non-superconducting mesoscopic rings: effects of single hopping impurity, in-plane electric field and foreign atoms

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Abstract

Persistent current in one-dimensional non-superconducting mesoscopic rings threaded by a slowly varying magnetic flux $\phi$ is studied based on the tight-binding model. The behavior of the persistent current is discussed in three aspects: (a) single hopping impurity, (b) in-plane electric field and (c) in presence of some foreign atoms.

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1 Introduction

Advances in nanoscience and technology have made it possible to fabricate devices in sub-micrometer scale and the transport in such devices gives several novel and interesting new phenomena. For such mesoscopic systems, at sufficiently low temperatures, the semi classical theory of electronic transport breaks down. Here, two new important features occur. First, the system provides discrete electronic energy levels. Second, the motion of an electron is coherent in the sense that once the electron can propagate across the whole system without inelastic scattering, its wave function will maintain a definite phase. The electron will thus be able to exhibit variety of novel and interesting quantum interference phenomena. In what follows, we shall concentrate to one of the most striking evidences, called the persistent current in normal conducting loops. Starting from 1983, people are studying extensively the phenomenon of persistent current in mesoscopic one-channel rings, multi-channel cylinders and some twisted geometries both theoretically\(^{1,17}\) as well as experimentally\(^{18,22}\) Several aspects of persistent current those were observed experimentally can be explained by theoretical arguments, but till now there are lot of controversies between them.

One such promising discrepancy is the amplitude of the measured currents which is an order of magnitude larger than the theoretically estimated value. It was believed that the electron-electron correlation \((U)\) has a significant role to enhance the current amplitude in dirty systems. Some perturbative calculations have also been used to solve this problem and predicted some intuitive results, but no such clear explanations have yet been found. Most of the theoretical works those were performed to explain the combined effects of electron-electron correlation and impurity on persistent current are basically based on the tight-binding model with nearest-neighbor hopping (NNH) integral. This simple NNH model cannot explain the desired current amplitudes those are observed in the experiments. It is quite reasonable and also physical to take higher order hopping integrals in addition to the nearest-neighbor hopping, and in the theoretical papers\(^{21,25}\) we have shown that higher order hopping integrals have significant contribution to enhance the current amplitude in the presence of impurity.

Another one important controversy comes from the determination of the sign of low-field currents. In an experiment on \(10^7\) isolated mesoscopic Cu rings, Levy et al.\(^{20}\) have measured diamagnetic response of persistent currents at very low fields, while Chandrasekhar et al.\(^{18}\) have determined \(\phi_0\) periodic currents in Ag rings with paramagnetic response for these fields. Theoretically Cheung et al.\(^{3}\) have predicted that the direction of persistent current is random depending on the total number of electrons, \(N_e\), in the system and the specific realization of the randomness. Both diamagnetic and paramagnetic responses have been observed theoretically in mesoscopic Hubbard ring by Yu and Fowler.\(^ {26}\) They have shown that the rings with odd \(N_e\) exhibit paramagnetic response, while those with even \(N_e\) give diamagnetic response in the limit \(\phi \rightarrow 0\). In a recent experiment Jariwala et al.\(^{27}\) have got diamagnetic persistent currents with both integer and half-integer flux-quantum periodicities in an array of 30-diffusive mesoscopic gold rings. The diamagnetic sign of the currents in the vicinity of zero magnetic field were also found in an experiment\(^{21}\) on \(10^5\) disconnected Ag ring. The sign is a priori not consistent with the theoretical predictions for the average of persistent current. In the theoretical paper\(^{28}\) we have introduced in detail about the sign of the low-field \((\phi \rightarrow 0)\) currents both for one- and multi-channel mesoscopic loops to understand the controversies between these different predicted results.

Even though different theoretical models were used to explain several experimental results but a clear understanding of the experiments is still lacking. Here we study the behavior of persistent currents in one-dimensional mesoscopic rings and focus our attention to the effects of the single hopping impurity, the electric field in the plane of the ring and the existence of foreign atoms in different lattice sites on these currents. Depending on the values of the single hopping impurity we get three different regimes, ballistic, metallic and insulating phases respectively. The effect of the in-plane electric field on the current is also quite interesting. It is observed that the electric field shifts the electronic spectrum and damps the amplitude of persistent current. This behavior can be used to control the energy spectra and the amplitude of persistent currents externally. Now in the study of the effect of foreign atoms on persistent currents, we assume that only in these foreign atoms electron-electron correlation exists (here we neglect the electron correlation in parent atoms) and focus several interesting new results.

Our scheme of this paper is as follow. In Section 2, we study persistent current in one-dimensional non-interacting mesoscopic rings in the presence of single hopping impurity. Section 3 provides the effect
of in-plane electric field on persistent current in one-dimensional non-interacting rings. In Section 4, we discuss the effect of e-e correlation, exists only in the foreign atoms, on persistent current. Finally, we summarize our results in Section 5.

2 One-dimensional mesoscopic ring with single hopping impurity

The system under consideration is a one-dimensional non-interacting mesoscopic ring (Fig. 1) with single hopping impurity. Such a ring with \( N \) atomic sites is modeled by a single-band tight-binding Hamiltonian within a non-interacting picture, which can be written as,

\[
H = \sum_{i=1}^{N} \epsilon_i c_i^\dagger c_i + v \sum_{i=1}^{N-1} \left( e^{i\theta} c_i^\dagger c_{i+1} + e^{-i\theta} c_{i+1}^\dagger c_i \right) \\
+ v(1-\rho) \left( e^{i\theta} c_N^\dagger c_1 + e^{-i\theta} c_1^\dagger c_N \right)
\]  

(1)

where \( \epsilon_i \)'s are the on-site energies, \( c_i^\dagger (c_i) \) is the creation (annihilation) operator of an electron at site \( i \), \( v \) gives the hopping strength between two nearest-neighbor sites and \( \theta = 2\pi \phi/N \), the phase factor due to the flux \( \phi \) threaded by the ring. The single hopping impurity in this tight-binding Hamiltonian is inserted between the sites 1 and \( N \), and the hopping strength can be controlled by the parameter \( \rho \). Depending on the value of \( \rho \), three possible cases appear. Case I : \( \rho = 0 \), the system is free from any impurity. This system shows purely ballistic nature. Case II : \( \rho = 1 \), here electrons cannot hop between the sites 1 and \( N \) and accordingly, system goes to the insulating phase. Thus for such a case no current will appear. Case III : \( 0 < \rho < 1 \), here ring is treated as a single hopping impurity system or metallic system. For all these non-interacting cases, the spin of the electrons does not give any new qualitative behavior in the persistent currents and accordingly, in this section we ignore the spin dependent term. Throughout this article we take \( v = -1 \) and use the units where \( c = e = h = 1 \).

At absolute zero temperature, persistent current in the ring threaded by a magnetic flux \( \phi \) is determined from the expression,

\[
I(\phi) = -\frac{\partial E_0(\phi)}{\partial \phi}
\]  

(2)

where \( E_0(\phi) \) is the ground state energy.

In Fig. 2 we plot the current-flux characteristics for some typical one-dimensional non-interacting mesoscopic rings taking the ring size \( N = 200 \). Figures 2(a) and (b) correspond to the rings with \( N_e = 75 \) (odd \( N_e \)) and \( N_e = 80 \) (even \( N_e \)) respectively. The solid curves represent the persistent currents for the rings with \( \rho = 0 \), while the dashed curves correspond to the currents for the rings with \( \rho = 0.5 \). In the absence of any impurity, the current shows saw-tooth like nature (see solid curves) as a function of the flux \( \phi \). It is observed from the solid curves that, at half-integer or integer multiples of the flux-quantum \( \phi_0 \), the current has a sharp transition. This is due of the existence of the degenerate energy eigenstates at these respective field points. But as long as the impurities are introduced, all these degeneracies move
Figure 2: Current-flux characteristics for some one-dimensional non-interacting mesoscopic rings, where (a) $N_e = 75$ and (b) $N_e = 80$. Here we take the ring size $N = 200$. The solid and the dashed curves are respectively for the rings with $\rho = 0$ and 0.5.

out and the current varies continuously with $\phi$ and achieves much reduced value (see dashed lines). This reduction of the current amplitude is due to the localization effect\textsuperscript{29,30} of the energy eigenstates caused by the presence of impurity in the rings. Both for the perfect and dirty rings, the currents exhibit $\phi_0$ periodicity.

For $\rho = 1$, the system goes to the insulating phase and no current will be available.

3 One-dimensional mesoscopic ring with in-plane electric field

In this section we describe the effect of in-plane electric field on persistent current of a one-dimensional non-interacting mesoscopic ring (Fig. 3) within the tight-binding framework. For a $N$-site ring, the
single-band tight-binding Hamiltonian is written as,

\[
H = \sum_{i=1}^{N} \epsilon_i c_i^\dagger c_i + v \sum_{i=1}^{N} \left( e^{i\theta} c_i^\dagger c_{i+1} + e^{-i\theta} c_{i+1}^\dagger c_i \right)
\]  

(3)

where the symbols carry their usual meaning as in Eq. 1. Due to the in-plane electric field, site energy gets modified and is expressed through the relation,

\[
\epsilon_i = \left( eEaN/2\pi \right) \cos \left[ 2\pi(i - 1)/N \right] = \left( ev \right) \left( E^*N/2\pi \right) \cos \left[ 2\pi(i - 1)/N \right]
\]  

(4)

where \( E \) is the electric field and \( a \) is the lattice spacing. Here we define the dimensionless electric field \( E^* = Ea/v \).

Figure 4 shows the current-flux characteristics for some one-dimensional non-interacting rings \( (N = 60) \) in the presence of in-plane electric field. The behavior of the persistent currents for the rings with odd number of electrons \( (N_e = 27) \) are shown in Fig. 4(a), while for the rings with even \( N_e \) \( (N_e = 32) \) the results are shown in Fig. 4(b). In the absence of any electric field, the currents exhibit saw-tooth like behavior (see solid curves) with sharp transitions at half-integer (for odd \( N_e \)) and integer (for even \( N_e \)) multiples of the elementary flux quantum \( \phi_0 \), similar to that as given by the solid curves in Fig. 2. On the other hand, in the presence of in-plane electric field, the saw-tooth like behavior of the currents disappears and the currents vary continuously with \( \phi \) as shown by the dotted \( (E^* = 0.18) \) and the dashed \( (E^* = 0.2) \) curves. Our numerical results predict that, the current amplitude gets reduced with
the increase of the electric field which is really an interesting one. It is also examined that the current
amplitude decays exponentially with this electric field (not shown here in the figure). Thus we can
emphasize that, the behavior of the persistent current in one-dimensional non-interacting rings with an
in-plane electric field is quite similar to that of one-dimensional non-interacting rings in the presence of
impurity, but the significant feature is that in the previous systems i.e., the rings with in-plane electric
field, one can control the current amplitude externally by tuning the electric field which provides a key
idea for fabrication of efficient nano-scale devices.

4 One-dimensional mesoscopic ring with foreign atoms in
various lattice sites

This section demonstrates persistent currents in one-dimensional mesoscopic ring with foreign atoms in
different lattice sites. The speciality of the foreign atoms is that, only in these atoms the electron-electron
correlation exists, while the parent atoms are free from any such interaction. Actually such systems can
be observed where one dopes some foreign atoms in the parent system and we also notice such systems
in reality. In the previous two sections, we have studied the persistent currents in normal conducting
rings within the framework of one-electron picture, but in the presence of e-e correlation, we have to
consider the many-body Hamiltonian to describe our model. The tight-binding model Hamiltonian for
an interacting ring with \( N \) atomic sites is expressed in this form,

\[
H = \sum_{i=1}^{N} \varepsilon_{i,\sigma} c_{i,\sigma}^\dagger c_{i,\sigma} + v \sum_{i=1}^{N} \left( e^{i\theta} c_{i,\sigma}^\dagger c_{i+1,\sigma} + e^{-i\theta} c_{i+1,\sigma}^\dagger c_{i,\sigma} \right) + U \sum_{d=1}^{N_d} c_{d,\uparrow}^\dagger c_{d,\downarrow}^\dagger c_{d,\downarrow} c_{d,\uparrow}
\]  

(5)

where \( c_{d,\sigma}^\dagger \) \( (c_{d,\sigma}) \) is the creation (annihilation) operator of an electron with spin \( \sigma \) \( (\uparrow \text{ or } \downarrow) \) at site \( d \) where
the foreign atom situates. The parameter \( U \) corresponds to the strength of the Hubbard correlation,
exists only in \( N_d \) foreign atoms where \( N_d \leq N \). In these interacting systems, since the dimension of the
many-body Hamiltonian matrices increase very sharply with \( N \) for higher number of electrons \( N_e \), and
also the computational operations are so time consuming, we restrict our study only on the systems of
smaller \( N \) and \( N_e \). In what follows, we shall describe our results for the few cases those are respectively
given as: (I) ring with two opposite \( (\uparrow, \downarrow) \) spin electrons, (II) ring with three \( (\uparrow, \uparrow, \downarrow) \) spin electrons, (III)
ingoing with four \( (\uparrow, \uparrow, \downarrow, \downarrow) \) spin electrons and (IV) ring with five \( (\uparrow, \uparrow, \downarrow, \downarrow, \downarrow) \) spin electrons.

At absolute zero temperature \((T = 0)\), the persistent current in such interacting rings is determined
by taking the first order derivative of the many-body ground state energy \( E_0(\phi) \), which is obtained from
the exact numerical diagonalization of the tight-binding Hamiltonian Eq. \( 5 \)

4.1 Ring with two \( (\uparrow, \downarrow) \) spin electrons

To understand the precise dependence of the electron-electron correlation, exists in foreign atoms, on
persistent current let us first take the simplest possible system which is the case of a ring with two
opposite spin electrons. Figure 4 shows the current-flux characteristics of some one-dimensional rings
\((N = 20)\) with two opposite spin electrons.

Figure 5(a) gives the results for the rings in the absence of any impurity \((W = 0, \text{ where } W \text{ is the})
strength of the randomness) with Hubbard correlation strength \( U = 2 \). The dashed, dotted, small
dotted and solid curves correspond to the rings with \( N_d = 4, 8, 12 \) and 16 respectively. It is observed
that, suddenly the direction and the magnitude of the current change across \( \phi = \pm \phi_0 / 2 \) and a kink-like
structure appears in the current. With the increase of \( N_d \), the length of the kink increases, which is clearly
visible from the curves plotted in this figure. Though the effective Hubbard correlation strength increases
with the increment of \( N_d \), but yet the kinks are situated at the same region and they overlap with each
other. This feature clearly manifests that the kinks are independent of the correlation strength \( U \). The
appearance of the kink-like structures and their independence on the e-e correlation can be explained
as follows. For the two opposite spin electron system, total spin $S$ has two values which are 0 and 1. The Hamiltonian of such a system, for any $\phi$, can be block diagonalized by proper choices of the basis states. This can be achieved by choosing the basis states from the two different sub-spaces with $S = 0$ and $S = 1$. The sub-space spanned by the basis set with $S = 1$, the block Hamiltonian is free from $U$, and hence the corresponding energy eigenvalues and eigenstates are $U$-independent. On the other hand, for the other sub-space with $S = 0$, all the energy eigenstates are $U$-dependent. In the absence of any electron correlation, the $U$-independent energy eigenstates situate always above the ground state for any value of $\phi$. But for non-zero values of $U$, one of these $U$-independent energy levels achieves the ground state energy of the system in certain domains of $\phi$ and the length of these domains increases with $N_d$ which produce larger kinks. In these regions, we observe the kinks in the $I-\phi$ curves, and it is obvious that the persistent currents inside these kinks are independent of the Hubbard correlation $U$.

The behavior of the persistent current in perfect rings with two opposite spin electrons is quite similar to that of Fig. 5(a) if we fix $N_d$ instead of the e-e correlation $U$. As illustrative example, in Fig. 5(b) we plot the current-flux characteristics for some perfect rings considering $N_d = 10$. The dashed, dotted, small dotted and solid lines correspond to the rings with $U = 2, 4, 6$ and 8 respectively, where all these curves almost overlap with each other.

The situation becomes much more interesting when we add impurity in these rings. Here we assume that the impurities, taken randomly from a “Box” distribution function of width $W$, are given only in $N_d$ atomic sites where the e-e correlation exists. The characteristic behavior of the persistent currents for some disordered rings ($W = 1$) is shown in Fig. 5(c), where we set $U = 2$. The dashed, dotted, small dotted and solid curves correspond to the same values of $N_d$ as in Fig. 5(a). It is known that the repulsive Coulomb interaction doesn’t favor occupancy of the two electrons in a same site and also it opposes confinement of the electrons due to localization of energy eigenstates. Hence the mobility of the electrons increases as we introduce the Hubbard interaction and the current amplitude gets enhanced. But this enhancement ceases to occur after certain value of $U$ due to the ring geometry, and the persistent current then decreases as we increase $U$ further. This is the basic feature of persistent current in any

Figure 5: Current-flux characteristics for some interacting rings ($N = 20$) with two opposite ($\uparrow, \downarrow$) spin electrons. The dashed, dotted, small dotted and solid lines in different figures are respectively for: (a) $N_d = 4, 8, 12, 16$ with $U = 2$; (b) $U = 2, 4, 6, 8$ with $N_d = 10$; (c) $N_d = 4, 8, 12, 16$ with $U = 2$ and $W = 1$; (d) $U = 2, 4, 6, 8$ with $N_d = 10$ and $W = 1$. 

$N_d$ atomic sites where the e-e correlation exists. The characteristic behavior of the persistent currents for some disordered rings ($W = 1$) is shown in Fig. 5(c), where we set $U = 2$. The dashed, dotted, small dotted and solid curves correspond to the same values of $N_d$ as in Fig. 5(a). It is known that the repulsive Coulomb interaction doesn’t favor occupancy of the two electrons in a same site and also it opposes confinement of the electrons due to localization of energy eigenstates. Hence the mobility of the electrons increases as we introduce the Hubbard interaction and the current amplitude gets enhanced. But this enhancement ceases to occur after certain value of $U$ due to the ring geometry, and the persistent current then decreases as we increase $U$ further. This is the basic feature of persistent current in any
dirty rings in the presence of the electron correlation. The curves in Fig. 5(c) show that the currents vary continuously with flux $\phi$ without giving any kink and their amplitudes decrease gradually with the increase of $N_d$. This is due to the fact that with the increase of $N_d$, both the effective electron correlation and randomness increase but since the later effect dominates over the previous one, the current amplitude decreases gradually. For these cases, the $U$-independent energy eigenstates do not contribute to the lowest energy in any energy domain and accordingly, no kink appears in the current.

The dependence of the randomness, the total number of foreign atoms and the electron-electron correlation on the appearance of kinks in the persistent current is much more clearly observed from Fig. 5(d), where we plot the current-flux characteristics for some disordered rings with fixed $N_d$. The different curves in this figure correspond to the same values of $U$ as in Fig. 5(b). Comparing the results of Figs. 5(b) and (d), we clearly observe that for the dirty rings the kinks disappear for $U = 2$, 4 and 6, while the kink resides only for $U = 10$ (solid curve of Fig. 5(d)). Thus we can emphasize that the electron-electron correlation, randomness and $N_d$ have important significance on the behavior of persistent current in such small rings. For all these cases the persistent currents exhibit $\phi_0$ flux-quantum periodicity.

4.2 Ring with three ($\uparrow, \uparrow, \downarrow$) spin electrons

With the above background we now study the behavior of persistent current in mesoscopic interacting rings with higher number of electrons $N_e$. Here we consider rings with two up and one down spin electrons as illustrative example of three spin electron systems. In Fig. 6, we plot the current-flux characteristics for some of such interacting rings taking the ring size $N = 10$.

Figure 6(a) gives the variation of the persistent currents for the rings in the absence of any impurity ($W = 0$). Here we set the correlation strength $U = 16$. The dashed, small dotted and solid lines are respectively for the rings with $N_d = 4, 6, 10$ with $U = 16$; (b) $U = 4, 6, 8$ with $N_d = 6$; (c) $N_d = 4, 6, 10$ with $U = 16$ and $W = 1$; (d) $U = 4, 6, 8$ with $N_d = 6$ and $W = 1$.

as illustrative example of three spin electron systems. In Fig. 6 we plot the current-flux characteristics for some of such interacting rings taking the ring size $N = 10$.

Figure 6(a) gives the variation of the persistent currents for the rings in the absence of any impurity ($W = 0$). Here we set the correlation strength $U = 16$. The dashed, small dotted and solid lines are respectively for the rings with $N_d = 4, 6, 10$. The current shows a kink-like structure across $\phi = 0$ only for the ring with $N_d = 10$ (see solid curve). It is due the $U$-independent eigenstates like as the two electron systems, as explained earlier, and the current inside the kink is independent of the strength of
the Hubbard correlation $U$. It is also noticed that the kinks in the current for such three spin electron systems appear comparatively at quite high values than the two spin electron systems.

For the perfect rings described with fixed $N_d$ instead of $U$, no kink appears in the currents as shown by the curves in Fig. 6(b). Here we put $N_d = 6$. The dashed, small dotted and solid lines respectively correspond to the current for the rings with $U = 4, 6$ and $8$, where all these curves almost coincide with each other. It is observed that for all such values of $U$, the $U$-independent energy eigenstates do not contribute to the lowest energy in any energy domain and the kinks will appear for more higher values of $U$ than those are considered here.

Now we describe the behavior of the current-flux characteristics for these three spin electron systems in the presence of impurity. Figure 6(c) shows the results for some disordered rings ($W = 1$) with same parameters as taken in Fig. 6(a). The dashed, small dotted and solid curves correspond to the similar meaning as in Fig. 6(a). From this figure we see that initially the current amplitude decreases, but it again increases for higher value of $N_d$ ($N_d = 10$, solid curve). This is due to the fact that for $N_d = 10$, the effective Hubbard interaction dominates over the randomness. Here the kink also exists only for $N_d = 10$, similar to that as observed in Fig. 6(a).

The behavior of the persistent current in the presence of impurity for the rings specified by the same parameters as in Fig. 6(b) is plotted in Fig. 6(d) and it shows almost similar behavior to that as drawn in Fig. 6(b). Due to the randomness, the current amplitudes get reduced slightly and there is no possibility for the appearance of any kink-like structure for these parameter values. For all such rings with three spin electrons the current exhibits $\phi_0$ flux-quantum periodicity.

### 4.3 Ring with four ($\uparrow, \uparrow, \downarrow, \downarrow$) spin electrons

For a systematic approach, next we focus the behavior of persistent current in four electron systems and as illustrative example, we consider rings with two up and two down spin electrons. In Fig. 7 we draw the current-flux characteristics for some interacting rings ($N = 8$) with four spin electrons. The dotted and solid lines in different figures are respectively for: (a) $N_d = 6, 8$ with $U = 10$; (b) $U = 6, 8$ with $N_d = 8$; (c) $N_d = 6, 8$ with $U = 10$ and $W = 1$; (d) $U = 6, 8$ with $N_d = 8$ and $W = 1$.

Figure 7: Current-flux characteristics for some interacting rings ($N = 8$) with four ($\uparrow, \uparrow, \downarrow, \downarrow$) electrons. The dotted and solid lines in different figures are respectively for: (a) $N_d = 6, 8$ with $U = 10$; (b) $U = 6, 8$ with $N_d = 8$; (c) $N_d = 6, 8$ with $U = 10$ and $W = 1$; (d) $U = 6, 8$ with $N_d = 8$ and $W = 1$. 

For a systematic approach, next we focus the behavior of persistent current in four electron systems and as illustrative example, we consider rings with two up and two down spin electrons. In Fig. 7 we draw
Hubbard correlation. Here we set $U = 10$ which is quite high for these rings. The dotted and the solid lines in this figure are respectively for $N_d = 6$ and 8. The current shows kink-like structure across $\phi = 0$ and the appearance of the kinks is due to the additional crossing of the ground state energy level with the other energy levels as we vary the flux $\phi$. Here it is noted that, in the present case the kinks arise due to the $U$-dependent eigenstates, not from the $U$-independent eigenstates as in the earlier cases. Quite similar feature is also observed for the perfect rings specified with fixed $N_d$ ($N_d = 8$), instead of $U$. The results are plotted in Fig. 7(b), where the dotted and the solid curves correspond to the currents for the rings with $U = 6$ and 8 respectively.

The current-flux characteristics for some dirty ($W = 1$) rings with four spin electrons are shown in Figs. 7(c) and (d). For both of these two figures we take the same values of the different parameters those are considered respectively in Fig. 7(a) and (b). Since the values of $N_d$ and $U$ are quite large for these rings compared to the system size $N$, the current shows almost similar variation even in the presence of impurity. The persistent current amplitude decreases slightly due to the effect of the impurity in the rings and for all these rings the current shows only $\phi_0$ flux-quantum periodicity.

We find striking similarity in the behavior of the persistent currents with rings containing two opposite spin electrons with the rings containing four spin electrons. Hence it becomes apparent that mesoscopic Hubbard rings with even number of electrons exhibit similar characteristic features in the persistent current.

4.4 Ring with five ($\uparrow, \uparrow, \uparrow, \downarrow, \downarrow$) spin electrons

Lastly, we describe the characteristic behavior of persistent current in five electron systems and as representative example we take rings with three up and two down spin electrons. Figure 8 shows the variation of the persistent currents for some specific rings ($N = 7$) with five electrons.

Figure 8: Current-flux characteristics for some interacting rings ($N = 7$) with five ($\uparrow, \uparrow, \uparrow, \downarrow, \downarrow$) electrons. The dotted and solid lines in different figures are respectively for: (a) $N_d = 5, 7$ with $U = 12$; (b) $U = 8, 12$ with $N_d = 6$; (c) $N_d = 5, 7$ with $U = 12$ and $W = 1$; (d) $U = 8, 12$ with $N_d = 6$ and $W = 1$.
yet for this value of $U$ no kink-like structure appears in the currents and it is noted that the kink will appear for some higher values of $U$ (not shown here in this figure). Now the variation of the persistent currents for some perfect rings described with fixed $N_d$ is given in Fig. 8(b), where the dotted and the solid lines are respectively for $U = 8$ and $12$ and they almost overlap with each other. It is observed that the current amplitude gets reduced, even in these perfect rings, to nearly half of the value given in Fig. 8(a). This is due to the strong repulsive effect caused by the higher value of the effective $U$.

In the presence of impurity, the variation of the persistent currents are plotted in Figs. 8(c) and (d), where all the parameters have the same values as considered respectively in Fig. 8(a) and (b). The feature of these currents are almost similar to that of the perfect ring results. Similar to the previous cases, here also the persistent currents exhibit $\phi_0$ flux-quantum periodicity.

It is clear from these results that the behavior of the persistent current in rings with three spin electrons is quite similar to that of rings with five spin electrons. Hence it becomes apparent that mesoscopic Hubbard rings with odd number of electrons exhibit similar characteristic features in the persistent current.

5 Concluding remarks

In conclusion, we have investigated the characteristic features of persistent currents in one-dimensional mesoscopic rings based on the tight-binding model. Here we have focused our attention to the effects of the single hopping impurity, the in-plane electric field and the Hubbard correlation in foreign atoms on persistent currents.

The first part of this article has described the dependence of the persistent current on the single hopping impurity, and we have seen that depending on the strength of $\rho$, three possible regimes (ballistic ($\rho = 0$), metallic ($0 < \rho < 1$) and insulating ($\rho = 1$)) appear. In the presence of impurity, the current amplitude gets reduced due to the localization effect.

Next we have studied the effect of the in-plane electric field on the persistent current, and the main motivation for that calculation was to observe how one can control the current amplitude externally, which provides a key idea for the fabrication of efficient nano-scale devices.

In the last part, we have described the combined effects of the electron-electron correlation (exist only in the foreign atoms) and the randomness on the persistent current. An important finding is the appearance of kink-like structures in the current-flux characteristics. Quite interestingly we have observed that, in some cases, persistent currents inside the kinks are independent of the strength of the interaction $U$. These kinks give rise to anomalous Aharonov-Bohm oscillations in the persistent current, and recently Keyser et al. experimentally observed similar anomalous Aharonov-Bohm oscillations in the transport measurements on small rings.

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