Effect of Temperature and moisture dependent material properties on the Bending behavior of Laminated Cylindrical Shell under Hygrothermal Loads

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Abstract. The effect of temperature and moisture dependent material properties on the bending behavior of simply supported angle-ply composite laminated shells has been investigated. A 13 term accurate higher order shear deformation theory with zigzag function is used in this analysis in which the effects of transverse shear deformation are taken into account. A number of examples are solved to illustrate the numerical results concerning bending response of simply supported angle-ply composite shell subjected to hygrothermal effects. It is suggested that temperature-dependent and/or moisture-dependent material properties ought to be used in the analysis of laminated composite shell subjected to hygrothermal loads.

1. Introduction

Laminated composite shell structures are widely used in civil, mechanical, aerospace, and other engineering applications. These materials are becoming popular because of their high specific strength and high specific stiffness. Structures used in the above fields are very often exposed to high temperatures as well as moisture during their service life. The varying environmental conditions due to moisture absorption and temperature seem to have an adverse effect on the stiffness and strength of the structural composites.

Considerable work has been done recently on composite laminated shell which can be found from a complete review of static analysis of shells by Qatu [1]. Very recently numerous works have been reported on the hygrothermal analysis of composite laminates. Salvatore [2] analyzed hygrothermal elastic analysis of multilayered composite and sandwich shells, by means of the Carrera Unified Formulation (CUF). Also Jafari and Sobhani [3] presented the thermo-elastic response of a 2-D functionally graded open cylindrical shell with temperature-dependent material properties based on third-order shear deformation plate theory of Reddy. Brischetto and Carrera [4] studied the static
analysis of multilayered smart shells subjected to mechanical, thermal and electrical loads using higher order models. Similar to these works, in this paper the higher order theory presented by Alsubari et al [5] and Ali et al[6,7] for analysis of thermal hygrothermal loads has been extended to include the temperature/moisture dependent material properties effects on shells. Thus, the aim of this study is to investigate the temperature/moisture dependent material properties behavior of simply supported, composite laminated cylindrical shell strip, using higher order shear deformation theory.

2. Governing Equation

Consider an infinitely long laminated cylindrical shell panel. The panel coordinate system is such that 0 \leq x \leq \infty, 0 \leq \theta \leq \theta_m$ and -h/2 \leq z \leq h/2. The radius of the shell strip is considered as R and the load is uniform along the x axis so that the shell undergoes cylindrical bending ($\varepsilon_x=0$). The displacement field used in this problem is [5-7]:

\[
\begin{align*}
(u,v)&=(u_0,v_0)+z(\phi_x, \phi_y)+z^2(\gamma_x, \gamma_y)+z^3(\psi_x, \psi_y)+\varphi(S_x, S_y) =w= w_0+2z\phi_z+z^2\gamma_z \\
\end{align*}
\]

where $u(x, \theta, z)$, $v(x, \theta, z)$ and $w(x, \theta, z)$ are the displacement at any point in the laminate. The parameters, $u_0, v_0$ are the in-plane displacements, and $w_0$ is the transverse displacement of a point $(x, \theta)$ on the middle plane respectively. The functions $\phi_x, \phi_y$ are the rotations of the normals to the middle plane about $x$ and $\theta$ respectively. The other parameters, $\gamma_x, \gamma_y, \psi_x$ and $\psi_y$ are unknown higher-order terms which are function of $x$ and $\theta$ only. $\varphi$ is zigzag function term and defined by:

\[
\varphi=2(-1)^k z_k/h_k
\]

where $z_k$ is a local transverse coordinate with its origin at the center of the $k$th layer and $h_k$ is the corresponding layer thickness.

The stress-strain law for the $k$th layer with respect to the $x, \theta$ coordinate is given by:

\[
\begin{pmatrix}
\sigma_x & \tau_{xz} & \tau_{xy} \\
\tau_{xz} & \sigma_z & 0 \\
\tau_{xy} & 0 & \sigma_y
\end{pmatrix}=egin{pmatrix}
C_{11} & C_{12} & C_{13} & 0 & 0 & C_{16} \\
C_{12} & C_{22} & C_{23} & 0 & 0 & C_{26} \\
C_{13} & C_{23} & C_{33} & 0 & 0 & C_{36}
\end{pmatrix}^k
\begin{pmatrix}
\varepsilon_{xx} - \Delta T \alpha_x - \Delta C \beta_x \\
\varepsilon_{yy} - \Delta T \alpha_y - \Delta C \beta_y \\
\varepsilon_{zz} - \Delta T \alpha_z - \Delta C \beta_z
\end{pmatrix}
\]

(3)

where the $C_{ij}^k$ are the transformed elastic coefficients and $\Delta T = T - T_0$, $\Delta C = C - C_0$ in which, $T(x,y,z)$ and $C(x,y,z)$, are the temperature and moisture distribution loads respectively, whereas $T_0$ and $C_0$ are the reference temperature and moisture concentration respectively. $\alpha_x$, $\alpha_y$, $\alpha_z$ and $\alpha_{xy}$ are the thermal expansion coefficients and $\beta_x$, $\beta_y$, $\beta_z$ and $\beta_{xy}$ are the moisture expansion coefficients.

Expression for the principle of virtual work is

\[
\iint (\sigma_x \delta \varepsilon_x + \sigma_y \delta \varepsilon_y + \sigma_z \delta \varepsilon_z + \tau_{xz} \delta \gamma_{xz} + \tau_{xy} \delta \gamma_{xy} + \tau_{xz} \delta \gamma_{xz} + \tau_{xy} \delta \gamma_{xy}) dzdA - \iint q \delta w dA = 0
\]

(4)

where $q$ is the load per unit area on the mid-surface of the shell.

Integrating the displacement gradient given in Eq. 4 by parts and by setting the coefficient of $\delta u_0, \delta v_0, \delta w_0, \delta \phi_x, \delta \phi_y, \delta \phi_z, \delta \psi_x, \delta \psi_y, \delta \psi_z, \delta \phi_z, \delta \psi_z, \delta S_x$ and $\delta S_y$ to zero separately, we obtain the following equilibrium equations with associated boundary conditions.

\[
\begin{align*}
N_{x0/0} + N_{y0/0} &= 0 \\
N_{x0/0} + N_{y0/0} - N_{zz} &= 0 \\
M_{x0/0} - M_{yz} - 3K_{x0/0} &= 0 \\
J_{x0/0} - J_{y0/0} - 3 \alpha_{xy} &= 0 \\
H_{0,0}^{0} - H_{0,0} &= 0 \\
M_{0,0} - M_{0,0} - RN_{x0/0} &= 0 \\
K_{0,0} - T_{0} &= 0 \\
H_{0,0}^{0} - H_{0,0} &= 0 \\
M_{0,0} - M_{0,0} - RN_{x0/0} &= 0 \\
K_{0,0} - T_{0} &= 0 \\
H_{0,0}^{0} - H_{0,0} &= 0 \\
M_{0,0} - M_{0,0} - RN_{x0/0} &= 0 \\
K_{0,0} - T_{0} &= 0 \\
H_{0,0}^{0} - H_{0,0} &= 0
\end{align*}
\]

(5)
Boundary conditions are given as
At θ=constant: one from each of the following bracketed quantities should be specified:

\[ (N_{\theta\theta}, u_0), (N_0, v_0), (M_{x\theta}, \phi_x), (M_\theta, \phi_\theta), (K_{\phi\phi}, \gamma_x), (K_\theta, \gamma_\theta), (J_{x\theta}, \kappa_x), (J_\theta, \kappa_\theta), (H_{x\theta}, S_x), (H_\theta, S_\theta), \]

\[ (N_{0\theta}, w_0), (M_{0\theta}, \phi_z) \quad \text{and} \quad (K_{0\theta}, \gamma_z) \]

The assumed solution for Eq. 5 which satisfies the above boundary conditions is

\[ (u_0, \phi_x, \gamma_x, \kappa_x, S_x) = (p_1, p_3, p_5, p_7, p_9) \cos(\pi \theta/\theta_m) \]

\[ (v_0, \phi_\theta, \gamma_\theta, \kappa_\theta, S_\theta) = (p_2, p_4, p_6, p_8, p_{10}) \cos(\pi \theta/\theta_m) \]

\[ (w_0, \phi_z, \gamma_z) = (p_{11}, p_{12}, p_{13}) \sin(\pi \theta/\theta_m) \]

From Eq. 1, Eq. 3, Eq. 5, and Eq. 6, the stress resultants can be expressed in terms of displacements, and finally a system of linear algebraic equations will be formed as:

\[ [L]\{\delta\} = \{f\} \]

where \([L]\) is a matrix of differential operators with respect to \(x\) and \(y\), \(\{\delta\}\) represent the displacement vector \([u_0 \ v_0 \ w_0 \ \phi_x \ \phi_\theta \ \gamma_x \ \gamma_\theta \ \kappa_x \ \kappa_\theta \ S_x \ S_\theta \ S_\theta]\), and \(\{f\}\) is the load vector that depends on the mechanical and hygrothermal load. Using Eq. (7) unknown coefficient \(\{\delta\}\) can be obtained readily, and subsequently in-plane stresses and transverse stresses can be obtained. Although the transverse shear stresses can be calculated from the constitutive relations, these stresses may not satisfy the continuity conditions at the interface between layers, hence transverse shear stresses are obtained by integrating three dimensional equilibrium equations of elasticity.

3. Results And Discussion

In previous studies [5-7], the material properties of the composite laminate are assumed to be independent of temperature and moisture concentration. However the elastic moduli of laminate in general degenerate with the elevation of temperature and moisture concentration. Thus, in this section, we will illustrate the effect of temperature and moisture on the material properties, and hence its effect on deflections and stresses.

3.1 Effect of temperature variation

In this section, we will investigate the static response of composite cylindrical shell laminate by taking into account the change of material properties due to temperature variation. This example will examine the effect of temperature variation on the deflections and stresses for four layered angle-ply composite laminate \((45/15)\). All laminae are assumed to have same thickness and made of the same orthotropic materials. The material properties used are as given in Table 1 [8]. The load composite laminated cylindrical shells considered to be subjected to a sinusoidal temperature load that is varying through thickness [9]

\[ \Delta T = (T - t) z (1 - 4/3 z^2) \] \( \sin(\pi \theta/\theta_m) \] (8)

The results obtained expressed in dimensionless terms given as \(\bar{\sigma} = \bar{w}/(h \alpha_t \bar{T}_0 S^2), \bar{v} = v/(h \alpha_t \bar{T}_0 S)\) and \(\bar{\sigma}_{ij} = \sigma_{ij}/(E_t \alpha_t \bar{T}_0)\) with \(S = R/h\).

The results of deflections and stresses are presented for two different cases. The case of dependent material properties (DMP) on temperature variation and independent material properties of temperature (IMP) are given in Table 2 and Figure 1. It is to be noted that the IMP means that material properties are considered at \(T = 300 \text{ K}\).

Table 2 shows that, when the effects of temperature on material properties are considered, the deflections and displacement decreased, while in-plane and shear stresses increased. Also Figure 1 shows that, as temperature rise, the difference between IMP and DMP results increased. It is to be noted that, transverse shear stresses results for the case of DMP are less affected by temperature rise than the corresponding one for the case of IMP. In general one can conclude that, as the temperature rises, the difference between DMP and IMP results increased.
3.1 Effect of moisture variation

In this section, we will investigate the effect of moisture variation on the material properties and the static hygrothermal bending response of the simply-supported \((45/15)_2\) cylindrical shells. The composite laminate cylindrical shell assumed to be subjected to sinusoidal moisture and thermal distribution across the thickness of the laminate as follows [9]

\[
\Delta T = (T - t) z(1 - 4/3 z^2/h) \sin(\pi \theta / \theta_m) \\
\Delta C = C z(1 - 4/3 z^2/h) \sin(\pi \theta / \theta_m)
\]

(9a)  
(9b)

The relationship between the moisture concentration and material properties are as given in Table 3 [8]. In this section, “T” and “t” given in Eq. 9 are assumed to be 300 K and 375 K respectively. Different values of moisture concentration are considered. In this example displacements and the stresses of a four-layer \((45/15)_2\) cylindrical shell strip subjected to hygrothermal effect are reported in Table 4 with \(R/h=4\). Similarly displacements of the four-layer \((30/-30)_2\) laminate for \(R/h=4\) are plotted in Figure 2. Once again, Table 4 shows that all displacements and stresses results are increasing with the increase of the moisture concentration. In addition, Figure 2 shows displacements for the shell strip for different moisture concentration. It can be noted that the transverse displacement \(\bar{w}\) increases as moisture concentration increase.

**Table 1. Elastic properties at different temperature \((\alpha_1 = -0.3, \alpha_2 = 28.1)\)**

| Temperature T [K] | 300 | 325 | 350 | 375 | 400 | 425 |
|-------------------|-----|-----|-----|-----|-----|-----|
| \(E_2\)           | 9.5 | 8.5 | 8.0 | 7.5 | 7.0 | 6.75|
| \(G_{12}\)        | 6.0 | 6.0 | 5.5 | 5.0 | 4.75| 4.5 |

*Note: \(E_1 = 130\) GPa, \(E_3 = E_2\), \(G_{13} = G_{12}\), \(G_{23} = 0.5G_{12}\), \(v_{12} = v_{23} = v_{13} = 0.3\)*

**Table 2. Normalized deflections and stresses for the \((45/15)_2\) cylindrical shell for \(R/h=4\)**

| Results | IMP | DMP | IMP | DMP | IMP | DMP |
|---------|-----|-----|-----|-----|-----|-----|
| \(\bar{w}(-1/2)\) | -0.93 | -0.91 | -1.39 | -1.37 | -1.86 | -1.81 | -2.32 | -2.27 |
| \(\bar{v}(-1/2)\) | -0.59 | -0.58 | -0.89 | -0.87 | -1.19 | -1.15 | -1.483 | -1.44 |
| \(\tilde{\sigma}_x(1/2)\) | 238.79 | 265.71 | 231.75 | 268.62 | 224.71 | 272.19 | 217.67 | 269.57 |
| \(\tilde{\sigma}_d(1/2)\) | 234.33 | 260.96 | 225.01 | 261.72 | 215.79 | 262.88 | 206.51 | 258.12 |
| \(\tilde{\tau}_{xx}(0)\) | 122.21 | 149.29 | 116.64 | 153.79 | 111.06 | 158.76 | 105.49 | 157.67 |
| \(\tilde{\tau}_{yy}(0)\) | 7.8329 | 9.73 | 7.36 | 9.85 | 6.89 | 10.06 | 6.43 | 9.84 |
| \(\tilde{\tau}_{xy}(0)\) | 24.28 | 24.69 | 23.22 | 23.65 | 22.14 | 22.61 | 21.09 | 21.47 |

**Table 3. Elastic properties at different moisture concentration \((E_1 = 130\) GPa and \(G_{12} = 6\) GPa)**

| Moisture concentration C % | 0.00 | 0.25 | 0.50 | 0.75 | 1.00 | 1.25 | 1.5 |
|---------------------------|-----|-----|-----|-----|-----|-----|-----|
| \(E_2\)                   | 9.5 | 9.25 | 9.0 | 8.75 | 8.5 | 8.5 | 8.5 |

*Note: \(E_3 = E_2\), \(G_{13} = G_{12}\), \(G_{23} = 0.5G_{12}\), \(v_{12} = v_{23} = v_{13} = 0.3\)*

\(\text{and } \beta_1 = 0, \beta_2 = 0.44, \alpha_1 = -0.3 \cdot 10^{-6}, \beta_2 = 28.1 \cdot 10^{-6}\)*
Table 4. Normalized deflections and stresses for the 
\((45/15)_2\) cylindrical shell for \((R/h=4)\)

| Results | 0.25 | 0.5  | 0.75 | 1.0  | 1.25 | 1.5  |
|----------|------|------|------|------|------|------|
| \(\bar{w}(-1/2)\) | -2.13 | -2.85 | -3.57 | -4.29 | -5.02 | -5.76 |
| \(\bar{v}(-1/2)\) | -1.37 | -1.83 | -2.30 | -2.76 | -3.23 | -3.70 |
| \(\bar{\sigma}_x(1/2)\) | 246.53 | 284.03 | 310.58 | 339.55 | 362.32 | 385.09 |
| \(\bar{\sigma}_\theta(1/2)\) | 256.95 | 269.70 | 292.25 | 316.96 | 335.92 | 354.87 |
| \(\bar{\tau}_{x\theta}(1/2)\) | 129.84 | 144.82 | 159.20 | 175.85 | 186.52 | 197.19 |
| \(\bar{\tau}_{xz}(0)\) | 8.14  | 9.06  | 9.93  | 10.98 | 11.57 | 12.17 |
| \(\bar{\tau}_{\theta z}(0)\) | 25.05 | 26.90 | 28.74 | 30.59 | 32.36 | 34.13 |

Fig 1. Normalized deflection \(\bar{w}\) for \((45/15)_2\) cylindrical shell with and without the effect of temperature on material properties \((R/h=4)\)

Fig 2. Normalized deflection \(\bar{w}\) for \((30/-30)_2\) cylindrical shell strip at different moisture concentration

4. Conclusion

A higher order shear deformation theory is proposed to study the hygrothermal analysis of composite laminated shells including the effect of temperature and moisture dependent material properties. The present results show that the transverse displacements and shear stresses are sensitive to the variation due to change on material properties caused by the rise of temperature and moisture concentration. Moreover, numerical results shows that, the hygrothermal responses taking into account the hygrothermal effect into material properties is quite different from those ignoring hygrothermal effect on material properties, thus in the accurate stress analysis the change in material properties with temperature and moisture variation should be considered when such a laminate is subjected to hygrothermal loading.
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