A Stochastic Process for Music: The Example of K-pop Music

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Abstract. This study analyzes music using a stochastic process, particularly, the Vasicek model. This approach interprets note progression in a song as a mean-reverting process, allowing the estimation of three parameters such as the speed of the revision to the mean, long-term level of the mean, and volatility. In addition, the entropy is evaluated for each song to identify the randomness of rise–fall patterns for each music genre. Our results characterize certain types of music and could be used to create new indicators for music classification.

1. Introduction

Mathematics and computation contribute in numerous ways to music research, including music technology, clarification of music theoretical concepts, and creation of educational music tools. Yet, prior studies have concentrated on scale theories, and current challenges are predominantly focused on modeling musical behavior rather than music products [1].

Accordingly, this study aims to uncover the behavior of notes in music by applying the mean-reverting process, which is widely used in mathematics and physics. In particular, the Vasicek model is considered to model the evolution of notes, and entropy is employed to quantify its randomness.

Prior studies have employed numerous approaches to explain music mathematically. Some of these works have used the Gabor analysis, a type of time-frequency analysis, to investigate music signals [2]. Other works have attempted musical segmentation by finding patterns using musical parallelism [3]. However, to the best of the authors’ knowledge, there has not been a single study utilizing the Vasicek model and symbolic time-series analysis (STSA) to examine the mean-revision process and time-varying patterns of note progression in music.

2. Theoretical framework

The notes in a song are analyzed through a mean-reverting process, in particular using the Vasicek model [4–8], which is a one-factor model widely adopted to describe the interest rate movements in finance. According to this model, the progression of a note at time $t$ is defined as

$$dN_t = k(\mu - N_t)dt + \sigma dW_t,$$

where $k$, $\mu$, $\sigma$, and $W_t$ are the speed of adjustment, mean reversion level, volatility, and standard Wiener process, respectively.

To calibrate the parameters in equation (1), we rewrite it in the following regression form [9]:

$$y_t = \alpha + \beta x_t + \epsilon_t,$$
where \( y_t = dN_t \), \( \alpha = k\mu dt \), \( \beta = -kd t \), \( x_t = N_t \), and \( \epsilon_t = \sigma dW_t \). We estimate the parameters of equation (2), i.e., \( \alpha \), \( \beta \), and \( \sigma \), by means of the ordinary least-squares method, where volatility is proportional to the standard error of the residuals according to

\[
\sigma = \sqrt{\frac{\text{VAR}(\epsilon_t)}{dt}}.
\]

(3)

3. Method and data

3.1. Entropy

We apply the STSA and detect the dispersion of the probability allocation onto the rise–fall patterns of consecutive note series. The symbolization of consecutive notes is encoded as 1 for the positive change and 0 for the other changes. Subsequently, the size of the rolling window is determined to quantify the subsequence bundles consisting of \( S \) binary numbers. Each subsequence bundle is converted from a binary sequence to a new decimal number, \( X^S \) [10–15]. Then, the entropy of the random variable \( X^S \) is derived as

\[
H(X^S) = -\sum_{i=1}^{S=(S-1)} p(x_i^S) \log_2 p(x_i^S),
\]

(4)

where \( M \) is the number of changes across all notes. Finally, the normalized Shannon entropy is given by

\[
h(X^S) = \frac{1}{S} H(X^S).
\]

(5)

Hereafter, “Shannon entropy” or simply “entropy” will be used to refer to a normalized entropy, that is, \( h(X^S) \).

3.2. Data

We reference the music chart data from Melon [16], and the top 30 songs from the chart of May 2021 are retrieved to analyze the latest trending K-pop music. The first verses of these songs are converted into numerical data according to the following rules: (i) the sixteenth note length is taken as the unit length 1, and (ii) the C4 note is taken as the base note 0.

Table 1 summarizes the descriptive statistics. The mean note is between A4 and C5. The minimum note is F3, whereas the maximum note is E6. The skewness and excess kurtosis values suggest that the distribution of notes in K-pop music does not deviate significantly from the normal distribution. In particular, R&B/soul music has the highest variability, whereas rock/metal music exhibits the lowest variability.

|       | Obs. | Mean | Standard deviation | Maximum | Minimum | Skewness | Excess kurtosis |
|-------|------|------|--------------------|---------|---------|----------|----------------|
| K-pop | 350.20 | 5.83 | 3.41 | 16.00 | -4.00 | 0.15 | -0.50 |
| Dance | 355.75 | 5.40 | 3.61 | 16.00 | -1.50 | 0.55 | -0.12 |
| Ballad | 381.75 | 6.52 | 2.84 | 12.50 | -1.00 | -0.37 | -0.44 |
| R&B/Soul | 466.00 | 5.70 | 3.94 | 14.00 | -4.00 | 0.01 | -0.91 |
| Rap/Hip-Hop | 349.25 | 5.20 | 3.22 | 14.50 | 0.00 | 0.52 | -0.92 |
| Rock/Metal | 175.33 | 6.84 | 2.76 | 14.00 | -4.00 | -0.45 | 0.94 |

4 Melon has the largest market share in the music streaming industry of South Korea.
"Obs." indicates the average number of data points in each song.

4. Results and discussion

Figure 1 displays the histograms of the Vasicek parameters $k$, $\mu$, and $\sigma$. Blue bar represents rock/metal music, and yellow represents the others. The dataset consists of 12 dance, 8 ballad, 3 R&B/soul, 4 rap/hip–hop, and 3 rock/metal music songs. In particular, songs belonging to the rock/metal category exhibit consistent values for all three parameters. Rock/metal music is more volatile than other music types and exhibits a higher mean reversion level and a lower speed of adjustment than other genres.

Figure 1. Histogram of the Vasicek parameters for Rock/Metal (see appendix for details).

Table 2 shows the entropy of each genre. R&B/soul and dance music show a higher entropy level than other music types, indicating that their randomness of note progression is higher than that of other genres. In addition, rap/hip–hop and rock/metal music have a lower randomness value, indicating that they have a more consistent rise–fall note pattern. Finally, the results show that the entropy values are robust regardless of the window size.

Table 2. Entropy.

| Window size | Dance     | Ballad    | R&B/Soul | Rap/Hip–Hop | Rock/Metal |
|-------------|-----------|-----------|----------|-------------|------------|
| 3           | 0.602 (2) | 0.595 (3) | 0.608 (1)| 0.549 (5)   | 0.574 (4)  |
| 4           | 0.595 (2) | 0.590 (3) | 0.603 (1)| 0.581 (4)   | 0.567 (5)  |
| 5           | 0.587 (2) | 0.584 (3) | 0.597 (1)| 0.567 (4)   | 0.559 (5)  |

5. Conclusion

This study analyzes note progression in music by applying the Vasicek model. Furthermore, the STSA is employed to quantify the unpredictability of the rise–fall patterns in the note progression. Our results characterize certain music genres with three parameters of the Vasicek model and evaluate the randomness of note progression through entropy. Accordingly, our findings suggest the possibility of using a mathematical approach to explain the evolution of music history and propose new indicators for music classification.

Appendix
Figure 2. Histogram of the Vasicek parameters for all the details.
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