Evaluation of cosmological models in \( f(R, T) \) gravity in different dark energy scenario

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Abstract

In present paper, we search the existence of dark energy scalar field models within in \( f(R, T) \) gravity theory established by Harko et al. (Phys. Rev. D 84, 024020, 2011) in a flat FRW universe. The correspondence between scalar field models have been examined by employing new generalized dynamical cosmological term \( \Lambda(t) \). In this regards, the best fit observational values of parameters from three distinct sets data are applied. To decide the solution to field equations, a scale factor \( a = (\sinh(\beta t))^{1/n} \) has been considered, where \( \beta \) & \( n \) are constants. Here, we employ the recent ensues \( (H_0 = 69.2 \text{ and } q_0 = -0.52) \) from (OHD+JLA) observation (Yu et al., Astrophys. J. 856, 3, 2018). Through the numerical estimation and graphical assessing of various cosmological parameters, it has been experienced that findings are comparable with kinematics and physical properties of universe and compatible with recent cosmological ensues. The dynamics and potentials of scalar fields are clarified in FRW scenario in the present model. Potentials reconstruction is highly reasonable and shows a periodic establishment and in agreement with latest observations.

Keywords: \( F(R, T) \)-gravity; Tachyon field; k-essence; Quintessence

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1 Introduction

Various experimental observations like SN Ia \(^1\)-\(^2\), large scale structure (LSS) \(^3\) and Planks data \(^4\) etc. confirm accelerated expansion of present Universe. Apart from the baryon matter, a large amount of mysterious energy known as dark energy (DE) with large critical energy density and huge negative pressure is present in the universe, which is accelerating expansion of the universe. Recent observations supported the fact that nearly 70% matter/energy are in form of DE. The simplest effective component to describe the dynamics of the present expanding phase of universe is the Cosmology constant (\( \Lambda \)), as it possess a repulsive character. But the theoretical models with (\( \Lambda \)) faces problems of cosmic coincidence and fine tuning. Therefore, a dynamic cosmological constant (\( \Lambda = \Lambda(t) \)) with negative pressure has been considered a promising candidate of DE. The cosmological constant \( \Lambda \) is considered as a variable quantity by different authors for description of expanding universe. Chen \(^5\) took \( \Lambda \) as a function of \( \frac{1}{a^2} \), whereas Arbab \(^6\) describe \( \Lambda \) as a function of \( \frac{\dot{a}}{a} \). Waga \(^7\) and Carvalho et al. \(^8\) explored \( \Lambda \) in terms of \( (\frac{\dot{a}}{a})^2 \). Recently, several authors develop different cosmological models following
these particular phenomena, but most of these face certain theoretical and observational restrictions. Since observational data do not favour the growth of linear cosmic perturbations, in this way $\Lambda \propto H^2$ and $\Lambda \propto \dot{H}$ rules are strongly overrode [9]-[10]. In case of perfect fluid with equation of state $p = \omega \rho$, cosmological development appears with no DE scenario if $\Lambda \propto \rho$ is the only assumption taken into consideration. The introduction of $\Lambda \propto \rho$ effectively modifies the proportionality constant $\Lambda$ and the model becomes equivalent to one with merely a new fluid only. We tried to overcome this issue by introducing a new decay law which combines both matter and geometry into the dynamical cosmological term. Many theoretical models like tachyon field, k-essence, quintessence scalar field models, quintom, phantom field and chaplygin gas have been proposed recently [11]-[19].

Modified theories of gravitation have been largely focused regarding their cosmological consequences. Several researchers have been reviewing DE theories in distinct context of modified speculations of gravitation [20]-[21]. Despite the sturdy sign of the late-time acceleration of universe and also the existence of DE and substance, a great interest are developed in the modified theories of gravity. Amongst all such theories, $f(R)$ theory is a suitable theory which probed Einstein’s (GR) theory by using the arbitrary function $f(R)$ of the gravitational action $R$. $f(R)$ theory depicts more developed conditions as compared with GR and provides broad measures. Starobinsky [22] proposed a singularity free isotropic cosmological model and Sotiriou [23] gives scalar-tensor theory in framework of $f(R)$ gravity. Nojiri and Odintsov[24] reconstruct the modified theories to get cosmological behaviour of universe and analyzed Big Rip along with the future singularities. The inflation model of $f(R)$ have been explored by Huang [25]. In the similar manner, numerous cosmologists have also explored $f(T)$ gravity models in distinct context. Other modified theory models are $f(R)$ gravity, $f(T)$ gravity, scalar-tensor speculations, Galileon gravity, Braveworld models, Gauss-Bonnet gravity etc. [26]-[32]. Houndjo [33] has reconstructed $f(R,T)$ cosmological model showing the transition from a decelerated to an accelerated era. Ahmed et al., [34] conferred the Bianchi type-V accelerating model with $\Lambda(T)$. Some recent researches in $f(R,T)$ gravity have been discussed in [35]-[44] in different contexts.

Singh et al. [45] have described signature changed model from early decelerated to present accelerated of the universe through the bulk viscosity effects in $f(R,T)$ theory of gravity. Sahoo et al. [46] established the Kaluza Klein cosmological model in $f(R,T)$ theory. Bamba et al. [47] explored expansion of universe in presence of DE in f(R) gravity using inhomogeneous equation of the state. Sebastiani et al. [48] has develop k-essence cosmology in $f(T)$ theory. Guendelman et al. [49] described a quintessence DE model showing the dynamic effect of vacuum energy density and dust-like matter. Aktas [50] has described various DE models in $f(R,T)$ theory like the k-essence, tachyon field, and quintessence in FRW universe with variable $\Lambda$ and $G$ by considering a linearly varying deceleration parameter (DP). The role of $G$ and $\Lambda$ become important in explaining accelerating expansion of the universe. Various authors have consider the cosmological constant and $G$ as time function in numerous frameworks. Norman Cruz [51] assume a relationship between the DE densities of k-essence and holographic, in this case the HDE is defined in form of k-essence scalar field for $c > 1$. Granda et al. [52] formulate a relationship between energy density of k-essence, dilation, quintessence, and tachyon with HDE in the flat FRW universe. This correlation permits the models of the scalar fields to reconstruct the potentials and dynamics to characterize accelerated expansion. A general Lagrangian density $p(\phi, X)$ scalar-field energy model was reconstructed by Tsujikawa [53]. Korumur [54] and Sharif and Jawad [55] developed DE models using scalar field in the flat Kaluza-Klein universe. The
most attractive and exciting state in cosmology is the detection of the accelerated expansion of the universe [56]-[59]. So many ways are there to address this problem, including: models of k-essence, the quintessence field, the cosmological constant, a brane and cosmology scenarios [60]-[67].

We have revisited in the recent work [68] by considering a distinct value of the cosmological term $\Lambda$ as a simple linear combination of energy density $\rho$ and scalar factor $a$ and obtained a new and better scenario for different dark energy. The scalar field DE models can effectively clarify the basic DE theory, the application of scalar field DE models become very exciting to describe the energy density as influential theory. In the present work, we discuss the evaluation of the scalar field with the tachyon, k-essence, and the quintessence models in the framework of the $f(R, T)$ gravity. We consider a new generalized dynamic form of $\Lambda$. Sect. 2 consists of field equations. In Sect. 3, we propose the solution of the field equations. Sect. 4 contains three sub-sections expressing the tachyon, k-essence, and quintessences scalar fields. Sect. 5 consists concluding summary and results of derived model.

2 Field Equations in $f(R, T)$ gravity

We consider the modified $f(R, T)$ theory first researched by Harko et al.[69]:

$$S = \int \left( \sqrt{-g} \frac{f(R, T)}{2k} + \sqrt{-g} L_m \right) dx^4,$$

(1)

The Einstein’s field equations with variable $\Lambda$ and $G$ in the framework of $f(R, T)$ theory are assumed [70]:

$$G_{\alpha\beta} = [8\pi G(t) + 2\lambda'_a(T)] T_{\alpha\beta} + [2p\lambda'_a(T) + \lambda_a(T) + \Lambda(t)] g_{\alpha\beta},$$

(2)

Here ‘dash’ represent the differentiation.

By assuming $\lambda_a(T) = \mu T$, Eq. (2) can be rewrite as [70],

$$G_{\alpha\beta} = [8\pi G(t) + 2\mu] T_{\alpha\beta} + [\mu \rho - \mu p + \Lambda(t)] g_{\alpha\beta},$$

(3)

where ($\mu \rightarrow$ constant).

Now, we will report about the distinct DE models in the formalism of $f(R, T)$ theory taking variable $G$ & $\Lambda$ in FRW space-time. By applying $\mu = 0$ in Eq. (3), we obtained GRT scenario.

The spatially flat FRW universe is given by

$$ds^2 = -dt^2 + a^2(t) \left(dx^2 + dy^2 + dz^2\right),$$

(4)

the Ricci scalar is given as:

$$R = -6 \left( \frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2} \right),$$

(5)

Here, the energy-momentum tensor can be determined as:

$$T_{\alpha\beta} = -pg_{\alpha\beta} + (\rho + p)u_\alpha u_\beta,$$

(6)
where $\rho$ and $p$ stand for energy density and cosmic pressure respectively. $u^i = (0, 0, 0, 1)$ indicates the four-velocity components. The trace of energy-momentum tensor is given by $T^{DF} = \rho - 3p$.

For the flat FRW universe, the field equations of DE model in framework of $f(R, T)$ theory are described as:

\[ 3H^2 + 2\dot{H} = -8\pi G\rho + \mu \rho - 3\mu p + \Lambda \]  
\[ 3H^2 - 8\pi G\rho = 3\mu \rho + \Lambda - \mu p \]

Here, $H = \frac{\dot{a}}{a} \to$ Hubble parameter.

3 Solution of the Field Equations

We have two independent field equations (7) and (8) in five unknowns $H, \rho, p, G$, and $\Lambda$. Thus, to determine explicit solution of field equations, we need three more assumptions consider the some more assumptions as:

(i) We assume the dynamic cosmological term $\Lambda(t)$ as a simple linear combination of $\rho$ and $a$ [71]:

\[ \Lambda = l \frac{\ddot{a}}{a} + \lambda \left(\frac{\dot{a}}{a}\right)^2 + 4\pi G\delta \rho \]

where $l$, $\lambda$ and $\delta$ are constants.

(ii) The EoS parameter is consider as $\omega = \frac{p}{\rho}$.

(iii) A deceleration parameter is supposed as [72, 73, 74, 75]

\[ /q = -\frac{a\ddot{a}}{a^2} = n \left[ -\tanh^2(\beta t) + 1 \right] - 1 \] (9)

Using above assumption in Eqs. (7), Eq. (8), the expressions for physical quantities $\rho, p, G$, and $\Lambda$ are derived as:

\[ \rho = \frac{\beta^2 \text{csch}^2(\beta t) [(\omega + 1) \{(\lambda - 3) - l(2n - 1) + (\lambda + l - 3) \cosh(2\beta t)\} + 2n(\delta + 2)]}{2 \mu n^2(\omega + 1)(\delta + \omega - 1)} \] (10)

\[ p = \frac{\omega \beta^2 \text{csch}^2(\beta t) [(\omega + 1) \{(\lambda - 3) - l(2n - 1) + (\lambda + l - 3) \cosh(2\beta t)\} + 2n(\delta + 2)]}{2 \mu n^2(\omega + 1)(\delta + \omega - 1)} \] (11)

\[ G = -\frac{\mu [(\omega + 1) \{(\lambda - 3) - l(2n - 1) + (\lambda + l - 3) \cosh(2\beta t)\} - 2n(\omega - 3)]}{4\pi [(\omega + 1) \{(\lambda - 3) - l(2n - 1) + (\lambda + l - 3) \cosh(2\beta t)\} + 2n(\delta + 2)]} \] (12)

\[ \Lambda = \frac{\beta^2}{n} \left[ \frac{\beta(\omega - 3)}{(\omega + 1)(\delta + \omega - 1)} - l \right] \cosh^2(\beta t) + \frac{\beta^2(\lambda + 1)}{n^2} \coth^2(\beta t) \]

\[ -\frac{\beta^2 \delta}{2n^2} \left[ \frac{(\lambda - 3) - l(2n - 1) + (\lambda + l - 3) \cosh(2\beta t)}{(\delta + \omega - 1)} \right] \cosh^2(\beta t). \] (13)
From Eq. (5), Eq. (6), the trace and Ricci scalar of DE matter for $f(R, T) = R + 2\mu T$ model are determined as:

$$R = 6 \left( \frac{n\beta^2 - 2\beta^2 \cosh^2 (\beta t)}{n^2 \sinh^2 (\beta t)} \right)$$  \hspace{1cm} (14)$$

$$T^{DF} = (1 - 3\omega) \left( \frac{(\mu - 3)(1 + \omega)\beta^2 \cosh^2 (\beta t) + \beta^2 n}{n^2 \lambda (\omega^2 - 1) \sinh^2 (\beta t)} \right)$$ \hspace{1cm} (15)$$

Using Eq. (14) and Eq. (15), we get $f(R, T) = R + 2\mu T$ as

$$f(R, T) = 6 \left( \frac{n\beta^2 - 2\beta^2 \cosh^2 (\beta t)}{n^2 \sinh^2 (\beta t)} \right) + 2(1 - 3\omega) \left( \frac{(\mu - 3)(1 + \omega)\beta^2 \cosh^2 (\beta t) + \beta^2 n}{n^2 (\omega^2 - 1) \sinh^2 (\beta t)} \right)$$ \hspace{1cm} (16)$$

From Eq. (9), it has been found that $q > 0$ for $t < \frac{1}{\beta} \tanh^{-1} \left( 1 - \frac{1}{n} \right)^{1/2}$ and $q < 0$ for $t > \frac{1}{\beta} \tanh^{-1} \left( 1 - \frac{1}{n} \right)^{1/2}$. Our model is also shown to be in an accelerating phase for $0 < n \leq 1$, but the universe transitions from a decelerating to an accelerating phase for $n \geq 1$. Values of $n$ and $\beta$ can be used in the resulting model to ensure that the decelerating parameter behaves properly. The current universe is accelerating, according to recent SNe Ia observations, and the value of DP is in the range $-1 \leq q < 0$.

The DP in terms of red shift $z$ is given by

$$q(z) = -1 + n + \frac{n(1 - n + q_0)}{n + 1 + q_0[1 + (1 + z)2^n]}$$ \hspace{1cm} (17)$$

where $z = -1 + \frac{a_0}{a}$ is red shift parameter, $a_0$ is the present value of scale factor, $q_0$ is present value of DP at $z = 0$.

The metric potential for the model form Eq. (9) is obtained as,

$$a = [\sinh (\beta t)]^{1/n}$$ \hspace{1cm} (18)$$

where $n$ and $\beta$ are constants.

From Eq. (9), a relationship between $n$ and $\beta$ can be considered for the present universe ($t_0 = 12.36$ Gyr with $q_0 = -0.52$ Yu et al. [76]) as

$$\beta = \frac{1}{12.36} \tanh^{-1} \left[ 1 - \frac{0.48}{n} \right]^{1/2}$$ \hspace{1cm} (19)$$

which states that the model is valid for $0.48 < n \leq 1$ and shows accelerated expansion of present universe.

From Eq. (19), we get a set of values ($n = 2$, $\beta = 0.1084504$), these experimental values of $n$ and $\beta$ can be used for plotting and validation of the given model.

Now, we consider the Hubble data sets $H(z)$ [77]-[82], 580 data sets of apparent magnitude $m(z)$ from union 2.1 compilation of SNe Ia data sets [83] and 51 data sets of $m(z)$ from Joint Light Curve Analysis (JLA) data sets [84] and using the $R^2$-test formula, we obtain the best fit values of various model parameters $l$, $\lambda$ and $\delta$ for the best fit curve of Hubble function $H(z)$ and apparent magnitude $m(z)$. The best fit values of $l$, $\lambda$ and $\delta$ for the various data sets at 95% level
Table 1: Table of values of parameters

| Parameter | H(z) | SNe Ia | JLA |
|-----------|------|--------|-----|
| \( l \)   | 1.764| 0.1881 | 0.1137 |
| \( \lambda \) | 0.7351 | 0.8632 | 0.9196 |
| \( \delta \) | 0.4916 | 0.7292 | 0.9696 |

Figure 1: Plot of Energy density \( \rho \) against \( z \) with \( n = 2, \beta = 0.1084504, \mu = -2, \omega = -2/3 \).

Figure 2: Variation of pressure \( p \) versus \( z \) with \( n = 2, \beta = 0.1084504, \mu = -2, \omega = -2/3 \).

of confidence are mentioned in the table-1. These values of parameters are used for plotting and describing the dynamics of model.

Figure 1, depict the variation of \( \rho \) against red shift \( z \) for sets of observational data. Figure shows that \( \rho \) is a positive decreasing function that approaches to zero in the current era. It’s worth noting that the energy density \( \rho \) decrease sharply for \( H(z) \) data as compared to SNe Ia and JLA observational data.

Figure 2 shows the variability of fluid pressure \( p \) against the redshift \( z \) for three observations data sets. Figure confirms that the isotropic pressure is negative all over the expansion. It has been observed from the figure that pressure is a increasing function and converge to zero in late time. The high negative pressure corresponding to JLA observational data in comparison with
set of observation data explains the accelerated expansion of the universe.

In the late time the gravitational term $G(t)$ rises slowly and leads to infinity as noticed from the Figure 3. It is clearly observed from the figure that $G$ increases correspond to all set of observations but for SNe Ia data, it increase very sharply.

The cosmological term $\Lambda$ explicate the behaviour of the universe in the present cosmological model. Figure 4 clarify the nature of $\Lambda$ against redshift $z$. One can easily observed from the figure that $\Lambda$ is a decreasing function and approach to zero in present epoch. Our universe is in accelerated expansion era as specified by various recent studies [85, 86]. In this regards, behaviour of $\Lambda$ in the derived model is in good agreement with observations and trends. However, as compared to $H(z)$ and SNe Ia observational data sets, it decreases for JLA universe.
4 The f(R, T) DE Models with correspondence of scalar fields

The scalar field DE models are efficient to explain the occurrence of DE. There are various such model already mentioned in the literature like quintessence, condensate, k-essence, ghost dilaton phantom and tachyon, etc.

Reconstruction of the efficient DE models necessarily explain the nature of quantum gravity by the means of scalar fields. These models demonstrate quintessence nature of the universe and generate an efficient explanation of the existence of DE in the universe. In this section, we search the DE models in correspondence with tachyon, k-essence and, quintessence scalar fields taking variable $G$ and $\Lambda$ in the frame work of $f(R, T)$ theory. In derived model, we assume the values of EoS parameter as $\omega = -1/3$ and $-2/3$. Now, the behaviour of kinetic energy and scalar potential against $z$ are demonstrated through the plots for three sets of observational data by employing $a = a_0(1 + z)^{-1}$, here we assume $a_0 = 1$ as considered by distinct authors [68-87].

4.1 DE model with Tachyon field

The tachyon field (TF) is one of the dark energy components used to characterize the universe’s rapid expansion [88, 89]. Some tachyon condensate with an effective Lagrangian density [90, 91] suggests a tachyon field.

TF is considered as one of the DE constituent which explain the accelerated expansion of the universe. The EoS parameter of the tachyon DE matter distribution falls between $-1$ and $0$ Gibbon [92]. For tachyon matter distribution, the energy density $\rho$ and pressure $p$ associated to SF ($\Phi$) and scalar potential $V(\Phi)$ in flat FRW context are as follows:

$$p_{TF} = -T_i^i = V(\Phi) \left(1 - \dot{\Phi}^2\right)^{1/2}$$  \hspace{1cm} (20)

$$\rho_{TF} = T_4^4 = \frac{V(\Phi)}{\sqrt{1 - \dot{\Phi}^2}}$$  \hspace{1cm} (21)

Here, $\dot{\Phi}^2$ → kinetic energy (KE), $V(\Phi)$ → scalar potential. $\Phi$ and $V(\Phi)$ in terms of redshift are obtained as:

$$\Phi_{TF} = \frac{(\omega + 1)^{1/2} \sinh^{-1} \left((z + 1)^{-n}\right)}{\beta} + c_1$$  \hspace{1cm} (22)

$$V(z)_{TF} = \frac{\beta^2 \sqrt{-\omega} \left(\frac{1}{z+1}\right)^{-2n} \left(l \left((z + 1)^{-2n} + 1 - n\right) + \frac{(\delta+2)n}{\omega+1} + (\lambda - 3) \left((z + 1)^{-2n} + 1\right)\right)}{\mu n^2 (\delta + \omega - 1)}.$$  \hspace{1cm} (23)

Figure 5, represents the variation of the kinetic tachyon energy with respect to redshift $z$ for observational values [76] by assuming the $\omega = -1/3$ & $-2/3$. Plots shows that kinetic energy is increasing along the redshift $z$. The SF ($\Phi$) is very low in early universe while it is very high.
Figure 5: Plot of SF for tachyon against $z$ with $n = 2.0, \beta = 0.1084504$ and $c_1 = 0$.

Figure 6: Plot of $V(\Phi)$ for tachyon field against $z$ with $n = 2.0, \beta = 0.1084504, \mu = -2$.

Similarly, Figures 6(a,b, & c) depict the effect scalar potential against $(z)$. $V(\Phi)$ against $z$ decreases for all the three set of observations. From the above discussion for the tachyon potential, we observe that the tachyon field decreases as the universe expands [76-83].
Figure 7: Plot of SF for k-essence against $z$ with $n = 2.0, \beta = 0.1084504$ and $c_2 = 0$.

### 4.2 DE model with k-essence field

To explain the universe’s late-time acceleration, the k-essence scalar field (SF) model was proposed which is motivated by the Born-Infeld action string theory [93, 94].

K-essence models produced a very attractive dynamical scenario of the universe, which shows the nature in agreement with current cosmological observations. Thus, the fine tuning of the initial scalar field conditions can be ignored in these models [95]. In a flat FRW background, $p$ and $\rho$ in terms of $\Phi$ and $V(\Phi)$ for k-essence field are established as [96]:

$$p_{ke} = -T^i_i = \frac{V(\Phi)\left(\dot{\Phi}^4 - 2\dot{\Phi}^2\right)}{4}$$  \hspace{1cm} (24)$$

$$\rho_{ke} = T^i_4 = \frac{V(\Phi)\left(3\dot{\Phi}^4 - 2\dot{\Phi}^2\right)}{4}$$  \hspace{1cm} (25)$$

The $\Phi$ and $V(\Phi)$ for the k-essence field are establish as:

$$\Phi_{ke} = \sqrt{\frac{2\omega - 2}{3\omega - 1}} \sinh^{-1}\left(\left(\frac{1}{z+1}\right)^n\right) \beta + c_2$$  \hspace{1cm} (26)$$

$$V(z)_{ke} = \frac{\beta^2(3\omega - 1)^2\left(\frac{1}{z+1}\right)^{-2n}\left(l\left((z + 1)^{-2n} + 1 - n\right) + \frac{(\delta + 2)m}{\omega + 1} + (\lambda - 3)\left((z + 1)^{-2n} + 1\right)\right)}{2\mu n^2(1 - \omega)(\delta + \omega - 1)}$$  \hspace{1cm} (27)$$

Plot 7, demonstrate the k-essence KE with $z$ for recent observations [76] by choosing $\omega = -1/3 \& -2/3$. The graph convey that the KE is very high in late universe and express a behaviour similar to tachyon. Figure 8(a,b,c), draw the scalar potential effect against $(z)$, it decreases with redshift and approach to zero in late universe for three sets of observations. For $\omega = -1$, derived model shows a singularity [76]-[83].

### 4.3 DE model with Quintessence field

Quintessence is a theoretical form of DE characterized by homogeneous variable SF as well as the scalar potential, which is responsible for universe’s expansion [94]. For quintessence model, EoS parameter specifies that the accelerated expansion of universe exists for $-1 \leq \omega \leq -\frac{1}{3}$ [94].
Figure 8: Plot of $V(\Phi)$ for k-essence against $z$ with $n = 2.0, \beta = 0.1084504, \mu = -2$. (a) $l = 1.764, \lambda = 0.7351, \delta = 0.4916$, (H(z) data)  
(b) $l = 0.1881, \lambda = 0.8632, \delta = 0.7292$ (SNe Ia data)  
(c) $l = 0.1137, \lambda = 0.9196, \delta = 0.9696$ (JLA data)

Figure 9: Plot of $\Phi$ for Quintessence against $z$ with $n = 2.0, \beta = 0.1084504, \mu = -2$ and $c_3 = 0$.  

The relations of $\rho$ and $p$ in form of $\Phi$ and $V(\Phi)$ for quintessence model in flat FRW universe are established as [61]:

\[ p_Q = -T_i^i = \frac{1}{2} \left( \dot{\phi}^2 - 2V(\Phi) \right) \]  
\[ \rho_Q = T_4^4 = \frac{1}{2} \left( \dot{\phi}^2 + 2V(\Phi) \right) \]
The Φ and \(V(\Phi)\) for the quintessence model are determined as:

\[
\Phi_Q = \frac{a_2}{a_4} \tanh^{-1} \left( \sqrt{\frac{a_2}{a_1}} \cosh \left( \sinh^{-1} \left( \frac{1}{z+1} \right)^n \right) \right)
\]

\[
-\frac{a_2}{a_4} \log \left( \sqrt{2} a_3 \cosh \left( 2 \sinh^{-1} \left( \frac{1}{z+1} \right)^n \right) + a_1 \right) + c_3
\]

where,

\[
a_1 = \sqrt{(\omega + 1) \left\{ (\lambda - 3) - l(2n - 1) + (\lambda + l - 3) \cosh \left( 2 \sinh^{-1} \left( \frac{1}{z+1} \right)^n \right) \right\}} + 2n(\delta + 2)
\]

\[
a_2 = \sqrt{-l(n - 1)(\omega + 1) + (\delta + 2)n + (\lambda - 3)(\omega + 1)}
\]

\[
a_3 = \sqrt{(\omega + 1)(\lambda + l - 3)}
\]

\[
a_4 = \sqrt{\mu n^2(\delta + \omega - 1)}
\]

\[
V(z)_Q = \frac{\beta^2(1 - \omega) \left( \frac{1}{z+1} \right)^{-2n} \left( l \left( \frac{1}{z+1} \right)^{2n} - n + 1 \right) + \frac{\delta + 2}{\mu} n + (\lambda - 3) \left( \frac{1}{z+1} \right)^{2n} + 1}{2\mu n^2(\delta + \omega - 1)}
\]

Figure 10: Plot of \(V(\Phi)\) for quintessence against \(z\) with \(n = 2.0, \beta = 0.1084504, \mu = -2\).
Plot 9, convey the variation of quintessence KE versus $z$ for set of observations 76–83 by choosing $\omega = -1/3$ & $-2/3$. Similar to k-essence and tachyon field, the behaviour of the quintessence field is increasing and in late time it approaches to zero. Figure 10(a,b,c) depicts the effect of Scalar potential along $(z)$, it decreases and approaches to zero in late universe for all three sets of observations. For $\omega = -1$ model shows a kind of singularity.

5 Concluding Remarks:

In the present work, cosmological consequences are developed using the scalar fields in the framework of $f(R, T)$ theory. We look at a specific dynamical form of cosmological term $\Lambda(t) = \Lambda = l\ddot{a} + \lambda (\dot{a})^2 + 4\pi G\delta \rho$, that has been generated using modification of Einstein-Hilbert action and is used to describe cosmological inflation, late-time acceleration of the cosmos, or dark matter in the sense that a set of specific scalar field models. The best fit values of $l, \lambda$ and $\delta$ for the various data sets at 95% level of confidence for different sets of observational data and are used for plotting the dynamics of model. In this work, we have searched various scalar field DE models in flat FRW universe for with variable $G$ and $\Lambda$ in formalism of $f(R, T)$ theory by considering different set of cosmological observational values.

The following are the primary outputs of the generated model:

- In the present model, Fig. 1 shows that the energy density $\rho$ is a positive decreasing function and approaches to zero in present epoch, which is consistent with the Big bag theory. It is important to note that energy density $\rho$ decrease sharply for $H(z)$ data as compared to SNe Ia and JLA observational data.

- Figure 2 demonstrate that the pressure $p$ is negative throughout the cosmic era and converge to zero in late time. It has been observed from the figure that the high negative pressure exists corresponding to JLA observational data in comparison with other sets of observations. The rapid expansion of the cosmos is caused by this negative pressure.

- $G(t)$ increases slowly and tends to infinity in the late universe as noticed from the Figure 3. It is clearly observed from the figure that the gravitational term increases correspond to all set of observations but for SNe Ia data, it increase very sharply. The cosmological constant $\Lambda$ is a positive constant and has a magnitude of $\Lambda(Gh/c^3) = 10^{-23}$ as shown by Schmidt et al. [96]. In the derived model, the Figure 4 shows that $\Lambda$ is a decreasing function and approach to zero in present epoch. Through the findings of the magnitude and redshift of cosmological $\Lambda$-term by Ia supernova observations, it get impression that expansion of universe can be accelerated with induced cosmological density.

- Scalar field DE models like tachyon, k-essence and quintessence are explored for constant values of EoS parameter $\omega = -1/3, -2/3$ as shown in Figs.5-10. The kinetic energy $\Phi$ and corresponding potential are derived for respective models. We observe that kinetic energy $\Phi$ is very high at present epoch while the respective potentials are very low (approach to zero). The derived model shows a kind of singularity for k-essence and tachyon scalar fields at $\omega = -1$. 

• In the end, we can conclude that the present model initiate with Big-Bang scenario while terminate with Big Rip. These kinds of results are the consequences of the substantial swing in the evolutionary growth of matter and DE dominated eras. The potentials \( V(\Phi) \) remains positive for negatively varying \( \mu \). We may derive a GRT solution for FRW world with the tachyon, k-essence, and quintessence scalar field distributions in this way since \( G = 0 \) for \( \mu = 0 \). The current model’s results describe that the current state of universe, which is consistent with the scalar field and its related potential. The formulation dynamics of scalar field models are used to explain the tachyon, k-essence, and quintessence cosmologies in the framework of gravity \( f(R, T) \).

• In previous work the authors [68] have taken the ratio between \( H^2 \) and \( \Lambda \), i.e. \( \Lambda = \xi H^2 \), where \( \xi \) is a constant. But in the present paper we have considered the dynamic cosmological term \( \Lambda(t) \) as a simple linear combination of \( \rho \) and \( a \)

\[
\Lambda = \frac{l}{a} + \lambda \left( \frac{\dot{a}}{a} \right)^2 + 4\pi G \delta \rho
\]

• In previous work the authors [68] assumed the deceleration parameter as: 
\[
q = -\left( \frac{\dot{H} + H^2}{H^2} \right) = \frac{m}{(m + \kappa t)^2} - 1,
\]

whereas in the present article we consider
\[
q = -\frac{a\ddot{a}}{a^2} = n \left[ -\tanh^2 (\beta t) + 1 \right] - 1
\]

Hence, the results obtained in the present paper is more general than the previous one [68]. The results in the given work may be useful to understand the evolution of universe through the tachyon, k-essence, and quintessence field models with FRW space-time in the formalism of \( f(R, T) \) gravity.

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