Hyperfine splitting in muonic hydrogen constrains new pseudoscalar interactions

W.-Y. Keung\textsuperscript{1,3} and D. Marfatia\textsuperscript{2,3}

\textsuperscript{1}Department of Physics, University of Hawaii at Chicago, Chicago, Illinois 60607, USA
\textsuperscript{2}Department of Physics and Astronomy, University of Hawaii, Honolulu, Hawaii 96822, USA and
\textsuperscript{3}Kavli Institute for Theoretical Physics, University of California, Santa Barbara, California 93106, USA

We constrain the possibility of a new pseudoscalar coupling between the muon and proton using a recent measurement of the 2S hyperfine splitting in muonic hydrogen.

Recent measurements of 2S – 2P transition frequencies in the exotic atom constituted by a proton orbited by a muon \textsuperscript{[1] 2} find the proton charge radius to be 7\(\sigma\) smaller than the 2010-CODATA \textsuperscript{[3]} value obtained using ordinary hydrogen and \(e−p\) scattering. The 2S hyperfine splitting deduced from the same measurements shows excellent agreement with predictions \textsuperscript{[1]}.

The discrepancy in the proton radius has generated a lot of interest, including the invocation of new fundamental interactions as an explanation. Here, we focus on the implications of the hyperfine splitting for new interactions between the muon and proton. Specifically, we consider the possibility of a new pseudoscalar particle that couples to the muon and proton. Such an interaction is spin and velocity dependent and has a negligible effect on the Lamb shift (which is used to extract the proton radius) in the nonrelativistic limit \textsuperscript{[4]}, but has a significant effect on the hyperfine splitting.

The measured value of the 2S hyperfine splitting (HFS) \textsuperscript{[1]}

\[
\Delta E_{\text{HFS}} = 22.8089 \pm 0.0051 \text{ meV},
\]

is to be compared with the theoretical prediction \textsuperscript{[5]}

\[
\Delta E_{\text{HFS}}^{\text{th}} = (22.9843\pm0.0030)-(0.1621\pm0.0010)r_Z + \delta E_a, \tag{2}
\]

in meV, where \textsuperscript{[6]}

\[
r_Z = 1.045 \pm 0.004 \text{ fm}, \tag{3}
\]

is obtained from \(e−p\) scattering\textsuperscript{[1]} \(\delta E_a\) is the contribution to HFS from the new pseudoscalar interaction. Taking the experimental and theoretical uncertainties in quadrature, the best-fit to the experimentally measured \(\Delta E_{\text{HFS}}\) and \(r_Z\) occurs for \(r_Z = 1.045\) fm and \(\delta E_a = -0.006\) meV, and

\[
-0.018 \text{ meV} \leq \delta E_a \leq 0.006 \text{ meV} \text{ at 2}\(\sigma\). \tag{4}
\]

We now compute \(\delta E_a\), and subject it to the above 2\(\sigma\) constraint. In the nonrelativistic (NR) limit, the pseudoscalar vertex becomes

\[
J_5 = \bar{u}(p')i\gamma_5 u(p) \rightarrow i\chi'^\dagger q\cdot\sigma \chi - \frac{1}{2}\chi'^\dagger q\cdot\sigma \chi',
\]

where \(\chi \) and \(\chi'\) are 2-component Pauli spinors. The \(\mu−p\) interaction in terms of the muon line (given by \(\chi_\mu, \sigma_\mu\)) and the proton line (given by \(\chi_p, \sigma_p\)) is then (see Fig. 1),

\[
J_{5,\mu}J_{5,p} = \int \chi_\mu^\dagger q\sigma_\mu \cdot (p - p')\chi_\mu \chi_p^\dagger q\sigma_p \cdot (P - P')\chi_p,
\]

and the NR scattering amplitude for \(p + P \rightarrow p' + P'\) is

\[
i\mathcal{M} = if_\mu J_{5,\mu} \int q e^{-m_\mu r} i\mathbf{f}_P J_{5,p}, \quad \text{with } q = p - p' = P' - p.
\]

The couplings of the light pseudoscalar \(a\) of mass \(m_a\) to the muon and to the proton are \(f_\mu\) and \(f_p\), respectively. Then,

\[
\mathcal{M} = -\frac{f_\mu f_p}{4m_\mu m_p} \chi_\mu^\dagger q\chi_\mu \chi_p^\dagger q\chi_p \frac{1}{q^2 + m_a^2}
\]

\[
= -\frac{f_\mu f_p}{4m_\mu m_p} \int \frac{1}{3} q^2 \chi_\mu^\dagger q\chi_\mu \chi_p^\dagger q\chi_p \frac{1}{q^2 + m_a^2},
\]

with the relative angle averaged for the \(s\) wave. The effective Hamiltonian is

\[
\delta H_a = \frac{1}{3} \int \frac{f_\mu f_p}{4m_\mu m_p} \left[ \delta^3(r) - \frac{m_a^2 e^{-m_a r}}{4\pi r} \right] \sigma_\mu \cdot \sigma_p,
\]

so that

\[
\delta E_a = \frac{f_\mu f_p}{3m_\mu m_p} \left[ |\psi(0)|^2 - m_a^2 \int |\psi(r)|^2 \frac{e^{-m_a r}}{4\pi r} d^3r \right],
\]
where $\psi$ is the wave function of the $2S$ state:

$$\psi(r) = \frac{1}{2\sqrt{2\pi a_B}} (1 - \frac{r}{2a_B}) e^{-\frac{r}{2a_B}}.$$  

Here, $a_B = \frac{1}{\alpha m_e}$ is the Bohr radius for muonic hydrogen with $m_r = m_\mu m_p/(m_\mu + m_p)$, the reduced mass of the system. On convolving, we obtain

$$\delta E_a = \frac{f_\mu f_\mu \alpha^3 m_r^3}{3m_\mu m_p} \frac{1}{8\pi} F\left(\frac{m_\mu}{m_r}\right),$$  

where

$$F(x) = 1 - x^2 \frac{\alpha^2 + 2x^2}{2(\alpha + x)^4}.$$  

It is important to distinguish between $m_r$ and $m_\mu$ in the equations above. The $m_r$ dependence comes from the Bohr radius $a_B$, and $m_\mu$ from the NR reduction. The function $F(x)$ interpolates between 1 and 0 for $x = 0$ and $x \to \infty$ which is consistent with decoupling behavior.

In Fig. 2 we show the 2\(\sigma\) allowed values of $f_\mu f_\mu$ as a function of $m_\alpha$. The region between the solid curves is allowed. We restrict $m_\mu \leq 100$ MeV so that $f_\mu f_\mu$ remains comfortably in the perturbative regime.

Note that in the potential model of the proton with nonrelativistic quarks, the proton pseudoscalar coupling $f_\mu$ arises from the pseudoscalar couplings $f_u, f_d$ of the up and down quarks, which are of the same order of magnitude. In this simplified picture, we have $f_\mu = \frac{1}{2} f_d - \frac{1}{2} f_u$ (as for the magnetic moments). If $f_u = f_d$, we have the simple result, $f_\mu = f_u = f_d$.

In principle, the the anomalous magnetic moment of the muon places a stringent independent constraint on $f_\mu$ since pseudoscalar couplings yield a negative contribution to $a_\mu$ [\textsuperscript{2}][\textsuperscript{7}][\textsuperscript{9}][\textsuperscript{10}] while the measured value is higher than the standard model expectation: $\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{th}} = (29 \pm 9) \times 10^{-10}$ [\textsuperscript{5}]. However, the scalar sector may be more trinitate than envisioned here, and may offer a fine-tuned (and perhaps unnatural) cancellation of the pseudoscalar contribution.

For the sake of comparison, the QED contribution at leading order is

$$\delta H_{\text{QED}} = \frac{e^2}{6} \frac{g_\mu g_p}{4m_\mu m_p} \delta^3(\mathbf{r}) \mathbf{\sigma}_\mu \cdot \mathbf{\sigma}_p .$$

Here, $g_\mu (\approx 2)$, and $g_p (\approx 5.5857)$ are the gyromagnetic ratios for the muon and proton. Correspondingly,

$$\delta E_{\text{QED}} = \frac{\alpha^4 m_\mu^3}{12m_\mu m_p} g_\mu g_p .$$

The above QED result, though simple, represents the first three significant digits of the dedicated theoretical calculation, and is consistent with the recent measurement of Ref. [\textsuperscript{1}].

The ratio of the pseudoscalar contribution to the leading QED contribution is

$$\frac{\delta E_a}{\delta E_{\text{QED}}} = \frac{2}{4\pi \alpha} \frac{f_\mu f_\mu}{g_\mu g_p} F\left(\frac{m_\mu}{m_r}\right)\backslash \bigg[ 1 - \frac{m_\mu^2}{m_r^2} \frac{\alpha^2 + 2(m_\mu/m_r)^2}{2(\alpha + m_\mu/m_r)^4} \bigg] .$$

In sum, the 2\(\sigma\) hyperfine splitting in muonic hydrogen constrains the product of the pseudoscalar couplings of the muon and proton $f_\mu f_\mu$ to lie in the 2\(\sigma\) ranges $[-0.00040, 0.00013]$, $[-0.00173, 0.00058]$ and $[-0.015, 0.005]$ for $m_\mu = 0$, 10 MeV and 100 MeV, respectively. As the pseudoscalar mass is further increased, the constraint is weakened. The couplings have no impact on the discrepant measurements of the proton radius.

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