Multi-bit quantum digital signature based on quantum temporal ghost imaging

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Digital signature scheme is commonly employed in modern electronic commerce and quantum digital signature (QDS) offers the information-theoretical security by the laws of quantum mechanics against attackers with unreasonable computation resources. The focus of previous QDS was on the signature of 1-bit message. In contrast, based on quantum ghost imaging with security test, we propose a scheme of QDS in which a multi-bit message can be signed at a time. Our protocol is no more demanding than the traditional cases for the requirements of quantum resources and classical communications. The multi-bit protocol could simplify the procedure of QDS for long messages in practical applications.

I. INTRODUCTION

Digital signatures aim to guarantee the authenticity and transferability of signed messages in modern digital communications. The classical protocols of digital signatures commonly employ the public-key encryption and provide the computational- assumption security for legitimate users. For example, the security of the famous Rivest-Shamir-Adleman protocol [1] is based on the reasonable capabilities in the computation of factoring large integers, and such security is susceptible to algorithmic breakthroughs, large-scale computational resources and the emerging technologies of quantum computation [2-4]. In contrast, quantum digital signature (QDS), which was firstly proposed in 2001 [5], is robust against attackers with unrestricted capabilities in the computation, since QDS offers the information-theoretical security by fundamental principles of quantum mechanics. The early versions of QDS [5,6] required the quantum memory for the interval between the distribution stage of quantum signature and the messaging stage, which is unfeasible in practical applications because of the immature technologies of quantum memory [7,8]. Later, this demanding requirement was removed by using unambiguous state elimination for quantum states and only store classical outcomes for the messaging stage [9,10]. Wallden et al. furthermore presented QDS protocols [11] requiring only the same technical components as quantum key distribution (QKD), which has been greatly developed over the past two decades [12,13]. Such breakthroughs motivated several notable advances in experimental demonstrations of QDS, where the transmission distance has been remarkably extended by utilizing phase-encoded states [14,15] and polarization-encoded states [16,17] in recent years. Additionally, several proposals have been implemented for improving the security of QDS, such as measurement-device-independent (MDI) QDS [18,19], which is immune to detector side-channel attacks, and passive decoy-state QDS for circumventing the leakage of the signal and decoy information to attackers [20]. However, those QDS protocols were dealing with the case of one-bit signature and the iterations of the procedure would be considerable for longer messages, limiting the feasibility in practical applications.

In this work, we propose and experimentally demonstrate a scheme of QDS in which the multi-bit message can be signed at a time. The multi-bit QDS protocol could markedly simplify the signature procedure for long messages in comparison with previous one-bit QDS. This work is inspired by ghost imaging in time domain [21-24] developed over the last several years. Ghost imaging is an intriguing technique of indirect imaging by the correlation of two beams. In a typical scheme of ghost imaging, one beam (so-called signal beam) passing through the object is collected by the bucket detector without spatial resolution, while the multi-pixel detector which can image the object is placed in another spatially separated beam (test beam). The first ghost imaging experiment was realized by the spatial correlation of photon pairs generated by spontaneous parametric down conversion (SPDC) in nonlinear crystals [25]. Thereafter, ghost imaging has been rapidly developed from quantum to classical to computational [26-28] with schemes of thermal-source ghost imaging [26], computational ghost imaging [27] and compressive ghost imaging [28]. Recently, Ryczkowski et al. demonstrated a thermal ghost imaging scheme in time domain with an all-fiber setup [21]. Meanwhile, Zhang’s group also realized quantum temporal ghost imaging and quantum secure ghost imaging over optical fibers of 50 km by utilizing time-frequency entanglement of photon pairs [23,24], since this entanglement can be well maintained during the distribution of photon pairs over optical fibers. Quantum ghost imaging process can be treated as the transfer of multi-bit...
information between legal parties and suitable security test over the quantum channel can limit the attacker’s knowledge on the imaging information, which can be applied in scenarios of quantum communications [24].

In this paper, we will firstly introduce the principle of quantum temporal ghost imaging based on time correlation of photon pairs, and then present the protocol and experimental demonstration of multi-bit QDS, in which the multi-bit message is signed by the way of ghost imaging. The multi-bit scheme would promote QDS toward practical applications by reducing the iteration of the signature procedure.

II. QUANTUM TEMPORAL GHOST IMAGING

Fig. 1 illustrates the sketch of quantum temporal ghost imaging. Energy-time entangled photon pairs are generated by the quantum light source placed at Alice. She keeps signal photons and sends idler photons to Bob by the quantum channel. The timing clocks of their detectors are synchronized and the measurement time is equally divided into many frames (Fig. 1(b)). Alice and Bob record the photons’ arrival times at detectors and the frames of the detected photons. For a photon pair, the signal photon and idler photon are in the same frame after the clock synchronization. The imaging object at Alice is a repeated binary pattern to select the single photon events of SPD1. The period of the pattern is equal to the frame size. After the selection by the object, Alice sends the frame numbers, denoted as $A_{frame}$, of the selected photon records to Bob through the classical channel. Then, Bob sifts his photon records $B_{record}$ by keeping the records of photons in the same frames as $A_{frame}$ and discarding others. Finally, Bob could retrieve the temporal pattern by making the statistics of photons’ relative positions in the corresponding frames. The quantum temporal ghost imaging is based on the time correlation of photon pairs. Frame numbers from Alice $A_{frame}$ actually inform Bob how the temporal pattern selects the photon pairs. From the view of ghost imaging, $A_{frame}$ are the outputs of the bucket detector without temporal resolution in the test beam, while Bob’s precise timing information $B_{record}$ are from the multi-pixel detector in the reference beam. Neither $A_{frame}$ nor $B_{record}$ can singly retrieve the temporal pattern.

For reducing the background noise of temporal ghost imaging, we adopt the approach of large-alphabet QKD encoding [29] in which the photons’ arrival times are encoded by three layers. A frame contains several time slots and a slot consists of a few bins. For the photon record in Fig. 2, its frame number is 1, slot number is 1 and bin number is 4. In ghost imaging, the period of the binary pattern is as large as the frame size and the bit size of the pattern is the same as the slot size. The pattern displayed in Fig. 2 is 1001 and the four-bit message can be transferred from Alice to Bob by ghost imaging. To be specific, Alice and Bob firstly make the frame and bin sifting to reduce the effect of noise photons and detector jitter. The two parties publicly announce the photons’ frame numbers and bin numbers, only keeping the records of those photons with same frame and bin numbers and discarding other records. Then, Alice utilizes the temporal pattern to select her single photon records and sends the corresponding frame numbers to Bob. The multi-bit message can be retrieved at Bob’s side by the correlation of Alice’s frame numbers and Bob’s precise records.

In the three-layer encoding mechanism, Alice and Bob don’t announce the slot numbers in the classical communication for ghost imaging, and the third party cannot eavesdrop the multi-bit message over the classical channel since the slot layer conveys the information of the temporal pattern. Furthermore, the security of quantum channel (Fig. 1(b)) can be checked by the measurement of time-energy entanglement quality [29,30] or the protocol of nonlocal dispersion cancellation [31,32]. The upper bound of the eavesdropping fraction indicates the leakage of Bob’s slot information and determines the security level of QDS protocol based on ghost imaging, which will be discussed in the following sections.

III. PROTOCOL OF MULTI-BIT QUANTUM DIGITAL SIGNATURE

There are generally three parties in the model of QDS wherein Alice signed the message and Bob (Charlie) are recipients. Fig. 3 illustrates the basic scheme of multi-bit QDS. Alice holds the entanglement source and keeps the signal photons of entangled pairs. Idler photons pass through a beam splitter and are distributed to Bob
and Charlie over quantum channels. There are pairwise authenticated classical channels between Alice and Bob (Charlie), which can be realized with short preshared keys [33]. Additionally, there is a secure classical channel between Bob and Charlie, which can be guaranteed by the state-of-the-art technology of QKD. In QDS, Alice sends the message with her signature to Bob and Bob forwards the message to Charlie if he accepts it. QDS should be immune to Alice’s repudiation and Bob’s forging. A successful repudiation by Alice means Bob accepts the message but Charlie rejects it.

Similar to previous QDS, the multi-bit QDS has two stages: distribution stage and messaging stage. In the former stage, the quantum states are distributed to the parties and measured by them, while the latter stage corresponds to the transmission of the signed message by the classical communication.

Basically, there are five steps in the distribution stage of the proposed protocol.

1. Alice, Bob and Charlie implement the clock synchronization, and announce the sizes of time frame, slot and bin.

2. The three parties measure the single photons, recording the arrival times of photons and the corresponding frame, slot and bin numbers.

3. Alice and Bob (Charlie) publish part of records to estimate the error rate of slot encoding and the upper bound of the eavesdropping fraction $\chi_{AB}$ ($\chi_{AC}$) over the quantum channel $X$ ($Y$). A successful forging of the signed message is closely related to the considerable eavesdropping on photons toward the recipients of the signature, i.e., Bob and Charlie. Suitable security check can be utilized to monitor the quantum channel [29-32] and the success probability of the malicious forging exponentially increases as the eavesdropping fraction increases [14,15].

4. Alice and Bob announce the frame and bin numbers of the remaining photon records. They keep the records of photons in the same frames and bins, discarding other records. Then, Alice’s records are labelled as $X^A$ and Bob’s records as $X^B$ since their records correspond to the quantum channel $X$. The records here only contain the frame numbers and slot numbers of photons. Similarly, Alice and Charlie also make the frame and bin sifting.

After the sifting, Alice’s records are labelled as $Y^A$ and Charlie’s records as $Y^C$ by quantum channel $Y$.

5. Bob and Charlie secretly and randomly exchange half of photon records with each other to make the records symmetric from Alice’s view. The secret exchange can be guaranteed by the classical secure channel between Bob and Charlie based on QKD technology. Similar to previous QDS, this step is to prevent Alice’s repudiation. After the distribution stage, Bob has the records $S^B = (X^B_{\text{keep}}, Y^C_{\text{forward}})$, wherein $X^B_{\text{keep}}$ are the half of records Bob keeps in the secret exchange and $Y^C_{\text{forward}}$ are the records forwarded from Charlie. Also, Charlie’s records are denoted as $S^C = (X^C_{\text{forward}}, Y^C_{\text{keep}})$. Alice’s records are $S^A = (X^A, Y^A)$.

The messaging stage is the classical communication, which could occur much later. There are three steps in this stage.

6. Alice randomly keeps half the records $S^A = (X^A, Y^A)$ and discards others. Thereafter, Alice chooses the records with specific slot numbers according to the message to be signed, and then sends the frame numbers of those records she chooses to Bob through the classical channel. Here the frame numbers are the signature elements, denoted as $S_{\text{frame}}$. If a frame consists of $M$ slots, M-bit signed message has been transmitted.

7. When Bob receives the frame numbers $S_{\text{frame}}$, he can retrieve the M-bit signed message by the temporal ghost imaging, as discussed in Section I. Bob has two record blocks $X^B_{\text{keep}}$ and $Y^C_{\text{forward}}$, and hence he can perform ghost imaging twice. The first ghost imaging is realized by $X^B_{\text{keep}}$ and $S_{\text{frame}}$, and the second one is by $Y^C_{\text{forward}}$ and $S_{\text{frame}}$. The noise of ghost imaging is due to system error rate, which includes setup imperfection and the eavesdropping perturbation over the quantum channel, and the forging of signature numbers $S_{\text{frame}}$. The low leakage of slot information would lead to a high noise of ghost imaging in case of forging the frame numbers and vice versa. If both of the noise factors in the two ghost images, defined as the inverse of signal-to-noise ratio, are lower than the acceptance threshold $Th_{\text{accept}}$, Bob will accept the message and forwards it to Charlie through the classical channel. The noise factor here is...
Timing jitters of \( \sim 100 \) Hz. The detection efficiency is high due to the nonlocal dispersion cancellation \([31,32]\).

In the experiment, the photons’ timing information are encoded by the third-layer mechanism as shown in Fig. 3. Considering the filtering bandwidth of photon pairs is 50 GHz, the size of bin is set as 20 ps, similar to Ref. \([31]\). In detail, a time slot contains 15 bins and a frame consists of 10 slots. Therefore, the size of the frame is 3 ns. Taking into account of the single count rate of Alice’s detector SNSPD1 as 1.5 MHz, there is basically no more than one photon in each frame and multi-photon records are discarded. In the step (5) of distribution stage, Bob and Charlie secretly and randomly exchange half of the records, i.e., frame and slot numbers of photons, with each other through the classical secure channel. After this stage, Alice holds the photon records \( S^A = (X^A, Y^A) \), where \( X^A (Y^A) \) corresponds to quantum channel \( X (Y) \) between Alice and Bob (Charlie). Bob has the records \( S^B = (X^B_{\text{keep}}, Y^B_{\text{forward}}) \), where \( X^B_{\text{keep}} \) are the records he keeps in the exchange and \( Y^B_{\text{forward}} \) are forwarded from Charlie. Similarly, Charlie’s records are \( S^C = (X^C_{\text{forward}}, Y^C_{\text{keep}}) \), where the former are from Bob and the latter are kept by Charlie himself.

Next, we take the data block \( X^B_{\text{keep}} \) as an example to explain the messaging stage of 10-bit QDS by ghost imaging. 10-bit message can be signed at a time since a frame contains 10 slots in our setting. Fig. 5(a) presents the distribution of Bob’s records \( X^B_{\text{keep}} \) in each slot with the single-photon measurement time of 2 s. The average count of \( \langle X^B_{\text{keep}} \rangle \) is 554.9 and the error rate of slot number between Alice and Bob is 3.59%. For signing the message 1000000000, Alice firstly randomly chooses half of photons, with each other through the classical secure channel. After this stage, Alice holds the photon records with \( X^A = (X^A_{\text{keep}}, Y^A_{\text{forward}}) \), where \( X^A_{\text{keep}} \) are the records she keeps in the exchange and \( Y^A_{\text{forward}} \) are forwarded from Charlie. Similarly, Charlie’s records are \( S^C = (X^C_{\text{forward}}, Y^C_{\text{keep}}) \) along with the quantum channel. In the classical channel.

IV. PROOF-OF-PRINCIPLE DEMONSTRATION

The experimental setup of multi-bit QDS is shown as Fig. 4. A continuous-wave laser with center-wavelength of 1552.52 nm pumps a piece of dispersion shifted fiber (DSF) to generate time-energy entangled photon pairs by the spontaneous four-wave mixing process. The length of DSF is about 200 meters and it is cooled at about 40 K in the Gifford-McMahon cryocooler for suppressing Raman noise photons. The generated photon pairs are filtered by a dense wavelength division multiplexer (DWDM) with center-wavelength of 1549.32 nm and bandwidth of 50 GHz. The signal photons are collected by Alice, while the reflected photons pass through the 50:50 coupler and are distributed to Bob and Charlie. At Bob side, he uses another DWDM with center-wavelength of 1552.72 nm and bandwidth of 50 GHz to collect the idler photons. Charlie’s setup is identical to Bob’s and therefore is not shown in detail. For the three parties, they all directly detect half the daughter photons to estimate the channel error rate and perform the digital signature based on ghost imaging, while the other half of photons at each side pass through the positive (negative) dispersion module to estimate the security of quantum channel by the nonlocal dispersion cancellation \([31,32]\).

The positive (negative) dispersion module (DCM DB, Proximion Corp.) is based on the fiber Bragg grating with group velocity dispersion of 1981 (-1980) ps/nm at 1545 nm. The detection efficiencies of superconducting nanowire single photon detectors (SNSPD) are \( \sim 50\% \) with dark count rates of \( \sim 100 \) Hz, timing jitters of \( \sim 80 \) ps and maximum count rates of \( \sim 2 \) MHz \([35]\). The coincidence measurements of the single photon events are realized by a time-to-digital converter (TDC) modules (Hydra Harp 400, Pico Quant). The full width at half maximum (FWHM) of the coincidence peak between SNSPD1 and SNSPD3 is 128 ps, while coincidence between SNSPD2 and SNSPD3 manifests the large dispersion effect with the FWHM of 896 ps. The peak of nonlocal dispersion is 160 ps by the coincidence measurement of SNSPD2 and SNSPD4. The eavesdropping of the collective-attack level on photons’ timing information would be indicated by the nonlocal dispersion cancellation \([31,32]\).

It’s worth noting that in our protocol two special cases should be avoided: all bits of the signed message are 1 or 0, since Bob could easily forge any message to be 1111 or 0000 according to the frame setting of Alice and Charlie in the classical channel.
FIG. 4. Experimental setup. The time-energy entangled photon pairs are generated in the cooling dispersion shifted fiber (DSF). Alice holds the quantum source and keeps the signal photons of pairs, while idler photons are collected by Bob and Charlie. Charlie’s setup is identical to Bob’s. Positive and negative dispersion modules are used for the measurement of nonlocal dispersion cancellation to estimate the eavesdropping fraction of the quantum channel between Alice and Bob (Charlie). EDFA: erbium doped fiber amplifier; DWDM, dense wavelength division multiplexer; SNSPD, superconducting nanowire single photon detector; TDC, time-to-digital converter.

is attributed to the perturbation by the eavesdropping in the distribution stage and the malicious forging in the messaging stage. Actually, the noise factor can be treated as the mismatch of signature elements in the traditional QDS [14-20]. On the other hand, Bob has an acceptance threshold \( T_{\text{accept}}^B \). After receiving the signature elements \( \text{Sig}_{\text{frame}} \), if all the noise factors in each slot of the two ghost images (\( \text{Sig}_{\text{frame}}, X_{\text{keep}}^B, \text{Sig}_{\text{frame}} \) and \( Y_{\text{forward}}^C \)) are less than the threshold \( T_{\text{accept}} \), Bob will accept the message. The value of \( T_{\text{accept}}^B \) should be slightly larger than the system error rate. Furthermore, Bob forwards \( \text{Sig}_{\text{frame}} \) to Charlie and Charlie will also accept the message if the noise factors are less than his threshold \( T_{\text{verify}}^C \). By this step, a 10-bit QDS is realized.

To prevent Alice’s repudiation, \( T_{\text{accept}}^B \) is smaller than \( T_{\text{verify}}^C \), i.e., Charlie will certainly accept the message if Bob accepts it. Fig. 5(d) is the case of the signed message 1010101010.

Finally, the security level can be calculated in the 10-bit QDS. As the noise factor can be treated as the mismatch in previous QDS schemes, the security-level equations in Ref. [15] can be adopted in the multi-bit model as:

\[
\text{Prob(Honest Abort)} \leq 2 \exp[-(T_{\text{accept}}^B - \epsilon)^2 L]; \quad (1a)
\]

\[
\text{Prob(Repudiation)} \leq 2 \exp[-(T_{\text{verify}}^C - T_{\text{accept}}^B)^2 L]; \quad (1b)
\]

\[
\text{Prob(Forge)} \leq \exp[-(P_\epsilon - T_{\text{verify}}^C)^2 L]. \quad (1c)
\]

In the experiment, the system error rate \( \epsilon = 3.59\% \) and the average count in ghost imaging is \( L = \langle X_{\text{keep}}^B \rangle / 2 + \langle Y_{\text{forward}}^C \rangle / 2 \) with the count rate of 273 per second. \( P_\epsilon = 1 - \chi_{\text{AC}} \) is Bob’s probability of incorrectly guessing the slot numbers of Charlie’s records \( Y_{\text{keep}}^C \), when Bob wants to forge the message. Here we have \( P_\epsilon = 0.447 \) for the eavesdropping at the collective-attack level [32]. In general, the security level \( \varepsilon \), which means the failure possibility of the protocol, is the maximum value of the three
FIG. 5. (a) The distribution of Bob’s records $X^B_{\text{keep}}$ in each slot after the distribution stage; Bob’s ghost images after Alice signs (b) 1000000000, (c) 0111111111 and (d) 1010101010 in the messaging stage.

FIG. 6. (a) The security level $\varepsilon$ versus Bob’s and Charlie’s thresholds ($Th^B_{\text{accept}}$ and $Th^C_{\text{verify}}$); (b) the optimized value of $\varepsilon$ versus the count $L$.

possibilities described above. Fig. 6(a) displays $\varepsilon$ versus the Bob’s and Charlie’s thresholds when $L = 546$, corresponding to the measurement time of 2 s in the distribution stage. We can also obtain the dependence of the optimized value $\varepsilon$ on the count $L$ in the ghost imaging (Fig. 6(b)). For the QDS with typical security level of $10^{-4}$, $L$ is taken as 928. Bob’s threshold $Th^B_{\text{accept}}$ is set as 0.1396 and Charlie’s threshold $Th^C_{\text{verify}}$ is 0.3470. Hence, we have the security level $\varepsilon = 0.93 \times 10^{-4}$. Therefore, the three parties need to make the single photon measurement for 4 s to control the failure probability below $10^{-4}$ in the 10-bit QDS.

V. CONCLUSION

We have presented a protocol of multi-bit QDS with a proof-of-principle demonstration in which the 10-bit message can be signed at a time. The message transmission is actually based on the quantum temporal ghost imaging, which is compatible with the security mechanism of nonlocal dispersion cancellation in the large alphabet-QKD scheme. In the distribution stage, the parties implement the measurements of the entangled photon pairs and store the classical measurement outcomes after the security check of pairwise quantum channels and necessary sifting on single photon events. In the messaging stage, the recipients, Bob and Charlie, retrieve the message by ghost imaging between their photon records and the forwarded signature elements from Alice. Bob (Charlie) accepts (verifies) the message by checking whether the noise factors in ghost images are below the acceptance (verification) thresholds. In real applications, the noise may due to the perturbation of the attacker’s eavesdropping in the distribution stage and the forging of the forwarded signature elements in the messaging stage. The noise factor in our scheme is similar to the mismatch of raw-key sequence in traditional QDS and hence the security level of multi-bit protocol can be calculated by the approaches discussed in previous cases. In the experiment, 10-bit QDS is implemented with the distribution stage of 4 s and the security level of $10^{-4}$. The volume of 10 bit could be furthermore increased by suitably setting the parameters in three-layer encoding mechanism, simplifying the procedure of signing long messages in practical applications.

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