Contraction based classification of supersymmetric extensions of kinematical Lie algebras

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We study supersymmetric extensions of classical kinematical algebras from the point of view of contraction theory. It is shown that contracting the supersymmetric extension of the anti-de Sitter algebra leads to a hierarchy similar in structure to the classical Bacry-Lévy-Leblond classification.

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I. INTRODUCTION

The contraction approach has been systematically applied in physics, among other problems, to classify the possible classical kinematical groups, basing on space isotropy and assuming that time-reversal and parity are automorphisms of the kinematical group, as well as non-compactness of one-parameter subgroups generated by boosts \cite{1}. Within this frame, all kinematical models arise as contractions of the de Sitter Lie algebras. It is therefore natural to ask whether for the supersymmetric extensions of the latter algebras, constructed in supersymmetric models, a similar procedure and classification holds, at least for those extensions proven to be of physical interest.

The main objective of this work is to extend the classical kinematical classification of Bacry and Lévy-Leblond (BBL classification) to the supersymmetric case, using generalized İnönü-Wigner contractions. By means of this procedure, we show that supersymmetric extensions of kinematical algebras considered in the literature \cite{2,3} fit into a contraction scheme. Contractions of supersymmetric extensions have usually been considered separately, for the most relevant cases \cite{4,5}. A BBL-classification however allows us to treat non-standard models like Carroll and Newton algebras in unified manner. This provides an alternative perspective to the numerous works developed in connection to nonrelativistic limits of supersymmetric theories \cite{5,6}.
We briefly recall the notion of contraction. Given a Lie algebra $\mathfrak{g}$ with structure tensor $C^k_{ij}$ over a fixed basis $\{X_i\}, i = 1, \ldots, n$, a linear redefinition of the generators via a matrix $A \in GL(n, \mathbb{R})$ gives the transformed structure tensor 

$$C''_{ij} = A_i^k A_j^\ell (A^{-1})_m^n C^m_{k\ell}. \quad (1)$$

Considering a family $\Phi_\epsilon \in GL(n, \mathbb{R})$ of non-singular linear maps of $\mathfrak{g}$, where $\epsilon \in (0, 1]$, for any $X, Y \in \mathfrak{g}$ we define

$$[X, Y]_{\Phi_\epsilon} := \Phi_\epsilon^{-1} \left[\Phi_\epsilon(X), \Phi_\epsilon(Y)\right], \quad (2)$$

which obviously reproduces the brackets of the Lie algebra over the transformed basis. Actually this is nothing but equation (1) for a special kind of transformations. Now suppose that the limit

$$[X, Y]_\infty := \lim_{\epsilon \to 0} \Phi_\epsilon^{-1} \left[\Phi_\epsilon(X), \Phi_\epsilon(Y)\right] \quad (3)$$

exists for any $X, Y \in \mathfrak{g}$. Then equation (3) defines a Lie algebra $\mathfrak{g}'$ which is a contraction of $\mathfrak{g}$, since it corresponds to a limiting point of the orbit. We say that the contraction is non-trivial if $\mathfrak{g}$ and $\mathfrak{g}'$ are non-isomorphic, and trivial otherwise. If there is some basis $\{Y_1, \ldots, Y_n\}$ such that the contraction matrix $A_\Phi$ adopts the form

$$(A_\Phi)_{ij} = \delta_{ij} \epsilon^{n_j}, \quad n_j \in \mathbb{Z},$$

the contractions is called a generalized Inönü-Wigner contraction (gen. IW). In this sense contractions were originally introduced in [7], to describe continuous transitions from relativistic to non-relativistic physics. It follows at once from these considerations that contractions are transitive, i.e., if $\mathfrak{g}$ contracts onto $\mathfrak{g}'$ and $\mathfrak{g}'$ onto $\mathfrak{g}''$, then $\mathfrak{g}$ contracts onto $\mathfrak{g}''$. This property will be useful in the following. The concept of contraction can be generalized without effort to other algebraic structures, in particular non-associative algebras [8].

In this work we focus primarily on generalized Inönü-Wigner (IW) contractions. As the generators of kinematical Lie algebras are identified with physical operators, contractions obtained by re-scaling certain of its generators still preserve this physical meaning, up to some phase transitions for the rescaled elements. Even if successive composition of IW-contractions is not necessarily equivalent to a general IW-contraction, i.e., the transitivity does not necessarily preserve diagonalization properties, in each step we deal with diagonal transformations, which enables us to interpret how the symmetry changes when modifying the main parameters.
II. SUPERSYMMETRIC EXTENSIONS OF THE DE SITTER ALGEBRAS

According to the BLL classification, our starting point must be the supersymmetric extension of the de Sitter algebras. The difference between these is just the signature of the metric tensor. We give the algebraic structure for the $\mathfrak{so}(2,3)$ algebra. Consider the usual basis $\langle L_{MN} = -L_{NM}, 0 \leq M < N \leq 4 \rangle$ with commutation relations

$$[L_{MN}, L_{PQ}] = \eta_{NP} L_{MQ} - \eta_{MP} L_{NQ} + \eta_{QN} L_{PM} - \eta_{QM} L_{PN},$$

(4)

where $\eta_{MN} = \text{diag}(1,-1,-1,-1,1)$. The generators $L_{MN}$ with $M, N \neq 4$ span the Lorentz algebra $\mathfrak{so}(1,3)$. In the basis $\langle L_{\mu\nu}, P_{\rho} = L_{\mu4}, \mu, \nu = 0, \ldots, 3 \rangle$ the non-vanishing commutation relations are rewritten as

$$[L_{\mu\nu}, L_{\rho\sigma}] = \eta_{\nu\rho} L_{\mu\sigma} - \eta_{\mu\rho} L_{\nu\sigma} + \eta_{\sigma\nu} L_{\rho\mu} - \eta_{\sigma\mu} L_{\rho\nu},$$

$$[L_{\mu\nu}, P_{\rho}] = \eta_{\nu\rho} P_{\mu} - \eta_{\mu\rho} P_{\nu},$$

$$[P_{\mu}, P_{\nu}] = L_{\nu\mu}.$$  

(5)

In terms of the BLL basis, with $K_i = L_{0i}$, $P_i = L_{i4}$, $H = L_{04}$ and $L_i = L_{jk}$, where $i, j, k$ are taken in cyclic order, the brackets are expressed as:

$$[L_i, L_j] = L_k, \quad [L_i, K_j] = K_k, \quad [L_i, P_j] = P_k, \quad [K_i, K_j] = -L_k,$$

$$[P_i, P_j] = -L_k, \quad [K_i, P_j] = -\delta_{ij} H, \quad [K_i, H] = -P_i, \quad [P_i, H] = K_i.$$

The remaining classical kinematical algebras and their commutators are given in Table 1.

Since Poincaré and Para-Poincaré ($I\mathfrak{so}(1,3)'$ in Table 1) algebras are isomorphic as Lie algebras, though physically different, as happens with the two Galilei algebras, we will only consider the standard Poincaré and Galilei algebras in the following.

A. The $\mathfrak{osp}(1|4)$ algebra

We construct a supersymmetric extension of the anti-de Sitter algebra starting from the real forms of $\mathfrak{osp}(1|4,\mathbb{C}) = \mathfrak{sp}(4,\mathbb{C}) \oplus \mathbb{C}^4$. Since only $\mathfrak{so}(2,3)$ admits a four-dimensional real spinor representation, only $\mathfrak{so}(2,3)$ will have a supersymmetric extension.
Table I: Non-vanishing brackets of classical kinematical algebras in the standard basis. The common brackets to all Lie algebras below are those corresponding to space isotropy: \([L, L] = L, [L, K] = K\) and \([L, P] = P\).

| \(\mathfrak{so}(2,3)\) | \(\mathfrak{so}(1,4)\) | \(\mathfrak{iso}(1,3)\) | \(\mathfrak{iso}(4)\) | \(\mathfrak{iso}(1,3)'\) | Carroll \(Ne^{\mathfrak{osp}}\) | \(Ne^{\mathfrak{osc}}\) | \(G(2)\) | \(G(2)\)' |
|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|-------------------------|
| \([K, K]\)             | \(-L\)                  | \(-L\)                  | \(-L\)                  | 0                       | 0                       | 0                       | 0                       | 0                       |
| \([K, P]\)             | \(-H\)                  | \(-H\)                  | \(-H\)                  | \(-H\)                  | 0                       | 0                       | 0                       | 0                       |
| \([P, P]\)             | \(-L\)                  | \(-L\)                  | \(-L\)                  | 0                       | 0                       | 0                       | 0                       | 0                       |
| \([K, H]\)             | \(-P\)                  | \(-P\)                  | \(-P\)                  | 0                       | 0                       | -P                      | -P                      | 0                       |
| \([P, H]\)             | \(K\)                   | \(-K\)                  | 0                       | \(-K\)                  | 0                       | \(-K\)                  | 0                       | \(-K\)                  |

Considering the decomposition \(\mathfrak{osp}(1|4) = \mathfrak{so}(2,3) \oplus \mathbb{R}^4 = \langle L_{\mu \nu}, P_{\mu} \rangle \oplus \langle S_\alpha, \bar{S}^{\dot{\alpha}} \rangle\), where \(\alpha, \dot{\alpha} = 1, 2\), \((S_\alpha, \bar{S}^{\dot{\alpha}})\) is a four dimensional Majorana spinor\(^1\) of \(\mathfrak{so}(1, 3)\). Consider the Dirac \(\Gamma\)-matrices \(\Gamma_{\mu} = \begin{pmatrix} 0 & \sigma_{\mu} \\ \bar{\sigma}_{\mu} & 0 \end{pmatrix}\), \(\Gamma_4 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\), where \(\sigma_{\mu} = (1, \sigma_i)\), \(\bar{\sigma}_{\mu} = (1, -\sigma_i)\) and \(\sigma_i, (i = 1, 2, 3)\) denote the Pauli spin matrices \((\sigma_{\mu} \rightarrow \sigma_{\mu \alpha \dot{\alpha}}, \bar{\sigma}_{\mu} \rightarrow \bar{\sigma}_{\mu \dot{\alpha} \alpha})\). Consider now \(S_\alpha = \varepsilon_{\alpha \beta} S^\beta, \ S^{\dot{\alpha}} = \varepsilon^{\alpha \dot{\beta}} S_{\dot{\beta}}, \ S_{\dot{\alpha}} = \bar{\varepsilon}^{\dot{\alpha} \dot{\beta}} \bar{S}_{\dot{\beta}}\) and \(\bar{S}_\dot{\alpha} = \bar{\varepsilon}^{\dot{\alpha} \dot{\beta}} \bar{S}_{\dot{\beta}}\) with \(\varepsilon, \bar{\varepsilon}\) antisymmetric matrices given by \(\varepsilon_{12} = \bar{\varepsilon}_{12} = -1\) and \(\varepsilon^{12} = \bar{\varepsilon}^{12} = 1\), respectively. Defining the Majorana spinor \(S_A = \begin{pmatrix} S_\alpha \\ \bar{S}_{\dot{\alpha}} \end{pmatrix}\) and introducing the anti de Sitter generators into the spinor representation, we obtain the matrices

\[
\Gamma_{\mu \nu} = \frac{1}{4} \begin{pmatrix} \sigma_{\mu} \sigma_{\nu} - \sigma_{\nu} \sigma_{\mu} & 0 \\ 0 & \bar{\sigma}_{\mu} \sigma_{\nu} - \bar{\sigma}_{\nu} \sigma_{\mu} \end{pmatrix}, \quad \Gamma_{\mu 4} = \frac{1}{2} \begin{pmatrix} 0 & -\bar{\sigma}_{\mu} \\ \bar{\sigma}_{\mu} & 0 \end{pmatrix}.
\]

In order to express the orthosymplectic algebra with real structure constants, we need to consider generators in the fermionic sector which respect to the following convention: we use rescaled \(S_A\)'s such that \(S_\alpha^* = i \bar{S}_{\dot{\alpha}}\) and \(\bar{S}_{\dot{\alpha}}^* = i S_\alpha\). Then \(\mathfrak{osp}(1|4)\) reads

\[
\{S_A, S_B\} = b \, \Gamma_{M N A}^D C_{5 B D L}^{M N}, \quad b \neq 0,
\]

\(^1\) \(S^{\alpha*} = \bar{S}_{\dot{\alpha}}, \ \ast\) denoting complex conjugation.
where \( C^A_B = \begin{pmatrix} \epsilon^{\alpha\beta} & 0 \\ 0 & \bar{\epsilon}_{\dot{\alpha}\dot{\beta}} \end{pmatrix} \). \(^2\) Choosing the normalization \( b = 4 \) we finally obtain the brackets
\[
[L_{\mu\nu}, S_\alpha] = (\sigma_{\mu\nu})_\alpha^\beta S_\beta, \quad [L_{\mu\nu}, \bar{S}_\dot{\alpha}] = (\bar{\sigma}_{\mu\nu})^{\dot{\alpha}}_{\dot{\beta}} \bar{S}_{\dot{\beta}}, \quad [L_{\mu4}, \bar{S}_\dot{\alpha}] = \frac{1}{2} \bar{\sigma}^{\dot{\alpha}}_\mu S_\alpha,
\]
\[
\{S_\alpha, S_\beta\} = 4(\sigma^{\mu\nu})_{\alpha\beta} L_{\mu\nu}, \quad \{S_\alpha, \bar{S}_\dot{\beta}\} = -4(\bar{\sigma}^{\mu\nu})^{\dot{\alpha}}_{\dot{\beta}} L_{\mu\nu}, \quad \{S_\alpha, \bar{S}_\dot{\beta}\} = 2(\sigma^{\mu}_{\alpha\dot{\beta}}) P_\mu.
\]

The brackets of the bosonic sector \(^5\) must be added to the former. We observe that the convention adopted in this work is different from the standard one \([3, 9]\).

## III. SUPERSYMMETRIC POINCARÉ ALGEBRAS

The Poincaré algebra is classically derived from the anti-de Sitter algebra by means of the contraction defined by the transformations \( L'_{\mu\nu} = L_{\mu\nu}, \ P'_\mu = \epsilon P_\mu \) and taking the limit \( \epsilon \to 0 \). The non-vanishing brackets are
\[
[L'_{\mu\nu}, L'_{\rho\sigma}] = \eta_{\nu\rho} L'_{\mu\sigma} - \eta_{\mu\rho} L'_{\nu\sigma} + \eta_{\sigma\nu} L'_{\rho\mu} - \eta_{\eta\mu} L'_{\rho\nu},
\]
\[
[L'_{\mu\nu}, P'_\rho] = \eta_{\nu\rho} P'_\mu - \eta_{\mu\rho} P'_\nu.
\]

We remark that the isomorphism of the anti-de Sitter algebra with \( \mathfrak{sp}(4, \mathbb{R}) \) implies that the choice for the contraction in the supersymmetric case must be the orthosymplectic algebra \( \mathfrak{osp}(1|4) \). \(^3\) To the previous generators, we add transformed generators in the Fermi sector of \( \mathfrak{osp}(1|4) \) defined by \( Q_\alpha = \epsilon^a S_\alpha, \ \bar{Q}_{\dot{\alpha}} = \epsilon^{\dot{a}} \bar{S}_{\dot{\alpha}} \). In these conditions, the limit for \( \epsilon \to 0 \) exists and defines a superalgebra only if \( 2a \geq 1 \) is satisfied. For \( a = \frac{1}{2} \) we recover the algebra with non-trivial brackets
\[
[L'_{\mu\nu} Q_\alpha] = (\sigma_{\mu\nu})_\alpha^\beta Q_\beta, \quad [L'_{\mu\nu} \bar{Q}_{\dot{\alpha}}] = (\bar{\sigma}_{\mu\nu})^{\dot{\alpha}}_{\dot{\beta}} \bar{Q}_{\dot{\beta}}, \quad \{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = 2(\sigma^{\mu}_{\alpha\dot{\beta}}) P'_\mu,
\]
which is the well-known supersymmetric algebra. This supersymmetric extension of the Poincaré algebra was first proposed in 1971, although its formal introduction in physics is

\(^2\) Since in our algebra we have \( S^*_\alpha = i \bar{S}_{\dot{\alpha}} \) there is a \( e^{i\pi/4} \) factor for our supercharge with respect to the usual conventions. As a consequence, there is no \( i \) factor in the bracket \( \{S_\alpha, \bar{S}_{\dot{\alpha}}\} \).

\(^3\) The “natural” choice \( \mathfrak{osp}(5|N) \) violates the principle requiring that the elements of the Fermi sector transform as Lorentz spinors.
considered \[10\]. Since it arises as contraction of \( \mathfrak{osp}(1|4) \), we get a consistent contraction pattern for supersymmetric extensions.

We remark the existence of another Poincaré-compatible supersymmetric extension arising as contraction of \( \mathfrak{osp}(1|4) \) \[11\]. However, no hermitian representations for the generators in the Fermi part exist, thus the model is of no use in supersymmetric considerations.

**IV. SUPERSYMMETRIC GALILEI ALGEBRA**

Contracting the Poincaré algebra \( \mathfrak{iso}(1, 3) \) using the transformations

\[
L'_i = L_i, \quad K'_i = \epsilon K_i, \quad P'_i = \epsilon P_i, \quad H' = H.
\]

(see Table 1). A supersymmetric extension is obtained by adding the following odd generators to the previous:

\[
Q'_\alpha = \epsilon^a Q_\alpha, \quad \bar{Q}'_\alpha = \epsilon^b \bar{Q}_\alpha.
\]

Taking into account the embedding \( \mathfrak{so}(3) \subset \mathfrak{so}(1, 3) \), it turns out that both representations \( \langle Q_\alpha \rangle \) and \( \langle \bar{Q}_\alpha \rangle \) are equivalent, although complex conjugate. We denote \( \bar{Q}_\alpha \rightarrow \bar{Q}_\alpha \) and \( \sigma_{i\alpha\dot{\beta}} \rightarrow \sigma_{i\alpha\beta} \). The contraction of the supersymmetric Poincaré algebra with respect to these transformations lead to a superalgebra whenever the condition \( a + b - 1 \geq 0 \) is satisfied. Among these supersymmetric extensions, that given by \( a + b - 1 = 0 \) is the usual \( N = 2 \) supersymmetric extension (without central charge) of the Galilei algebra:

\[
\begin{align*}
[L'_k, Q'_\alpha] &= -i_2 (\sigma_k)_\alpha^{\beta} Q'_\beta, \quad [K'_k, Q'_\alpha] = 0, \\
[L'_k, \bar{Q}'_\alpha] &= -i_2 (\sigma_k)_\alpha^{\beta} \bar{Q}'_\beta, \quad [K'_k, \bar{Q}'_\alpha] = 0,
\end{align*}
\]

\[
\{Q'_\alpha, \bar{Q}'_\beta\} = 2\sigma_{i\alpha\beta} P'_i,
\]

Using the transitivity of contractions, it can be easily seen that the extended Galilei algebra can be obtained contracting the supersymmetric extension of the anti de Sitter algebra. We remark that other extension considered in the literature is obtained using a special \( \mathbb{Z}_4 \)-grading of the Poincaré superalgebra \[4\]. It seems however that no supersymmetric field models based on this Galilei superalgebra have been developed.

**V. SUPERSYMMETRIC CARROLL ALGEBRA**

The Carroll algebra, a quite strange object, appears as an alternative non-relativistic limit of the Poincaré group, and was first described in the work \[12, 13\]. It is obtained from
the Poincaré algebra through the contraction determined by rescaling the boosts and time translations:

\[ J'_i = J_i, \quad K'_i = \epsilon K_i, \quad P'_i = P_i, \quad H' = \epsilon H. \]  

(12)

Although the Carroll algebra has played no distinguished role in kinematics, it appears in the study of tachyon condensates in string theory [14]. In this context a possible interpretation of this limit and supersymmetric extensions regain some interest. To construct a supersymmetric extension of the Carroll algebra, we add to the generators specified in (12) the additional generators of the symmetric part

\[ Q'_{\alpha} = \epsilon^a Q_{\alpha}, \quad \bar{Q}'_{\alpha} = \epsilon^b \bar{Q}_{\alpha}. \]  

(13)

We therefore obtain a contraction of \( \mathfrak{osp}(1|4) \) if the constraint \( a + b - 1 \geq 0 \) is satisfied. Again, for the special case \( a + b - 1 = 0 \), we get a \( N = 2 \) supersymmetric extension of the Carroll algebra given by

\[
\begin{align*}
[L'_k, Q'_{\alpha}] &= -\frac{i}{2}(\sigma_k)_{\alpha}^{\beta} Q'_{\beta}, \quad [K'_k, Q'_{\alpha}] = 0, \\
[L'_k, \bar{Q}'_{\alpha}] &= -\frac{i}{2}(\sigma_k)_{\alpha}^{\beta} \bar{Q}'_{\beta}, \quad [K'_k, \bar{Q}'_{\alpha}] = 0, \\
\{Q'_{\alpha}, \bar{Q}'_{\beta}\} &= 2\delta_{\alpha\beta} H.
\end{align*}
\]  

(14)

For this supersymmetric extension we find an interesting feature, namely, that in the fermionic sector the symmetric product reproduces a structure of Clifford algebra [15]. It is not difficult to verify that this algebra corresponds to the \( N = 4 \) supersymmetric quantum mechanics extension of the Galilei algebra. The main difference between this and the Galilean supersymmetric model reside in the fact that the supercharges are in the spinor representation of the rotation group.

VI. SUPERSYMMETRIC NEWTON ALGEBRA

Newton-Hooke kinematical algebras arise as contractions of the de Sitter algebras, and there is no relation, by means of continuous contractions, i.e., a limiting process, between these and the Poincaré algebra. However, in analogy with the latter, they have the Galilei algebra as non-relativistic limit. The contraction is determined by the transformations

\[ P'_i = \epsilon P_i, \quad K'_i = \epsilon K_i. \]  

(15)

The limit \( \epsilon \to 0 \) corresponds to the oscillating Newton Lie algebra. As observed above, properties of these models are close, in some sense, to those of the de Sitter algebras, like
curvature properties of space-time. In a different context, they have been studied to some extent in non-relativistic brane theory [16]. A supersymmetric extension of the Newton algebra is obtained by joining to the generators of (15) the additional elements

$$Q_{\alpha} = \epsilon^{\alpha} S_{\alpha}, \quad \bar{Q}_{\alpha} = \epsilon^{\dot{\alpha}} \bar{S}_{\alpha}. \quad (16)$$

If we compute the corresponding brackets, it follows that the action of $P'$ on $Q$ and $\bar{Q}$ is trivial, although the anti-commutators lead to the constraint $a + b - 1 \geq 0$. Here there is only one possibility to obtain a contraction with non-trivial symmetric product, given by the condition $a + b - 1 = 0$. Therefore the $N = 2$ supersymmetric extension has brackets

$$[L'_{k}, Q'_{\alpha}] = -i \frac{1}{2} (\sigma_{k})_{\alpha}^{\beta} Q'_{\beta}, \quad [K'_{k}, Q'_{\alpha}] = 0, \quad \{Q'_{\alpha}, \bar{Q}'_{\beta}\} = 2 \sigma_{i\alpha\beta} P'_{i}, \quad (17)$$

VII. SUPERSYMMETRIC STATIC ALGEBRA

A supersymmetric extension of the static Lie algebra appears as contraction of any superextension, by simply considering the pure inequalities in all the constraints obtained previously. Actually this algebra is nothing but the sum of the classical static algebra and a four dimensional space with trivial symmetric product. In this sense, this extension can be seen as the final superalgebra that still preserves the condition of space isotropy. The general contraction pattern can be resumed in the following diagram:

VIII. FINAL REMARKS

Applying the contraction ansatz of Bacry and Lévy-Leblond to classify the possible kinematics, we have extended the method, to obtain a similar classification of "kinematical" supersymmetric extensions of these Lie algebras. Various possibilities arise, although only those leading to physically interesting superalgebras, due to the non-apparent relation with field theoretic realizations for the remaining solutions. We however remark that this general
approach, leading to all possible supersymmetric extensions of kinematical algebras, including exotic models like the alternative Poincaré or Galilean superalgebras and their respective contractions, can be done in analogy with the general analysis of [17]. Work in this direction is in progress.

We finally remark that the classical approach of [1] is based on the assumption that parity and time reversal (PT) are automorphisms of the kinematical group. This is well known to fail for weak interactions, thus a relaxation of the hypothesis arising from this general analysis could lead to physically interesting models. In this sense, a kinematical classification of cubic extensions has been worked out [18], allowing to recover the known extensions studied in the literature [6, 19, 20]. In this third step, the possibility of discrete symmetries in the form of graded contractions must be taken into account.

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