Role of hexadecapole deformation of projectile $^{28}$Si in heavy-ion fusion reactions near the Coulomb barrier

Gurpreet Kaur$^1$, K. Hagino$^{2,3}$, and N. Rowley$^4$

$^1$Department of Physics, Panjab University, Chandigarh-160014, INDIA
$^2$Department of Physics, Tohoku University, Sendai 980-8578, Japan
$^3$Research Center for Electron Photon Science, Tohoku University, 1-2-1 Mikamine, Sendai 982-0826, Japan
$^4$Institut de Physique Nucléaire, UMR 8608, CNRS-IN2P3 and Université de Paris Sud, 91406 Orsay Cedex, France

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The vast knowledge of strong influence of quadrupole deformation $\beta_2$ of colliding nuclei on heavy-ion subbarrier fusion reactions inspires a desire to quest the sensitivity of fusion dynamics to higher order deformations, such as $\beta_4$ and $\beta_6$ deformations. However, such studies have rarely been carried out, especially for deformation of projectile nuclei. In this article, we investigated the role of $\beta_4$ of the projectile nucleus in fusion of the $^{28}$Si + $^{92}$Zr system. We demonstrated that the fusion barrier distribution is sensitive to the sign and the value of the $\beta_4$ parameter of the projectile, $^{28}$Si, and confirmed that the $^{28}$Si nucleus has a large positive $\beta_4$. This study opens an indirect way to estimate deformation parameters of radioactive nuclei using fusion reactions, which is otherwise difficult due to experimental constraints.

I. INTRODUCTION

Gaining insight into the role of nuclear intrinsic degrees of freedom in heavy-ion fusion reactions has been a motivation of many experimental and theoretical studies in the current nuclear research $^{1-7}$. During the fusion process, the nuclear intrinsic degrees of freedom, such as inelastic excitations, neutron transfers, static or dynamical deformation, are coupled to the relative motion of the interacting nuclei, and significantly affect the fusion dynamics. Experimental signatures of these couplings have been observed via a subbarrier fusion enhancement of fusion cross sections and a deviation of fusion barrier distributions from a simple one-peaked function $^{1-4}$. Comparisons of these experimental data with coupled-channels calculations have established the role of various couplings in heavy-ion fusion mechanism $^{2,3}$.

An important question to be addressed is what are the relevant degrees of freedom one has to consider in a description of the fusion dynamics. For deformed nuclei, the role of quadrupole deformation $\beta_2$ of the colliding nuclei in fusion is significant and has been well established $^{5}$. With the increasing experimental knowledge on the role of quadrupole collectivity in fusion, the sensitivity to the hexadecapole deformation $\beta_4$ is next to explore. In this connection, a measurement by Lemmon et al. $^{8}$ for $^{16}$O+$^{154}$Sm and $^{16}$O+$^{186}$W fusion reactions has clearly shown the sensitivity of fusion barrier distributions to the sign of $\beta_4$ of the target nuclei (see also Refs. $^{9,10}$). The effect of the $\beta_6$ (hexacontatetrapole) deformation, has also been investigated in Refs. $^{11,12}$.

A study of $\beta_4$ is significantly important also in connection to its association with the synthesis of superheavy elements (SHEs). That is, the hexadecapole deformation may significantly affect the height of fusion barrier, which in turn influences the fusion probability, thus the formation probability of SHEs $^{13}$. It has theoretically been argued that a $\beta_4$ deformation may help fusion (both hot and cold fusion reactions) leading to SHEs, depending on the choice of the reaction partners $^{14}$.

In this respect, an interesting observation has appeared recently while investigating the experimental fusion barrier distribution for the $^{28}$Si + $^{154}$Sm system $^{15}$. In this experiment, the barrier distribution was extracted using quasi-elastic back-scattering $^{16,17}$. Despite the well-established rotational nature of $^{28}$Si (having both quadrupole and hexadecapole deformations), it was found that a coupled-channels calculation with a vibrational coupling to its first $2^+$ state reproduces the structure of the barrier distribution rather well. Subsequently, it was observed that the resolution of this anomaly lies in the large hexadecapole deformation parameter of $^{28}$Si, which has the opposite sign to the quadrupole deformation parameter. That is, the contribution to the reorientation coupling ($2_1^+ \rightarrow 2_1^+$) from the quadrupole deformation is largely canceled out by that from the hexadecapole deformation, making the rotational coupling scheme look like the vibrational coupling scheme for this system. This leads to almost identical results for the two coupling schemes. Since the quasi-elastic backward scattering is a process complementary to fusion, it thus shows a sensitivity of fusion mechanism to the hexadecapole deformation of $^{28}$Si.

In Ref. $^{17}$, Newton et al. studied the experimental fusion barrier distribution for the $^{28}$Si + $^{92}$Zr system, and reached the same conclusion as in Ref. $^{15}$ for the $^{28}$Si + $^{154}$Sm system. That is, the authors of Ref. $^{17}$ have reported that treating the $2^+$ state in $^{28}$Si as a phonon state rather than a rotational state with oblate deformation gives a somewhat better fit to the experimental fusion barrier distribution. Moreover, treating the $^{28}$Si nucleus as a prolate rotor leads to a poor representation of the data. They have argued that there is not strong evidence from the fusion data to distinguish between $^{28}$Si being a vibrational nucleus or an oblate deformed nucleus.

The aim of this paper is to reanalyse the fusion barrier
distribution for the $^{28}\text{Si} + ^{92}\text{Zr}$ system which Newton et al. have studied, and to clarify the role of hexadecapole deformation of the $^{28}\text{Si}$ nucleus. We shall show that a large positive value for $\beta_4$ leads to fusion barrier distributions calculated with the rotational coupling scheme which look similar to those with the vibrational scheme. This result cannot be regarded as a direct measurement of $\beta_4$, but it strongly suggests that $^{28}\text{Si}$ is a deformed nucleus with a large positive hexadecapole parameter, $\beta_4$.

II. COUPLED-CHANNELS CALCULATIONS FOR $^{28}\text{Si} + ^{92}\text{Zr}$ SYSTEM

To clarify the influence of hexadecapole deformation of $^{28}\text{Si}$ on the fusion of $^{28}\text{Si} + ^{92}\text{Zr}$ system, we have performed the coupled-channels calculations using the computer code CCFULL [18]. To this end, we have used a Woods-Saxon potential, whose diffuseness parameter was fixed to be $a_0 = 1.03$ fm. Notice that a large value of diffuseness parameter has been found to reproduce high-precision fusion cross sections in many systems [17]. The exact origin of this phenomenon has not been clarified, and the phenomenon has been referred to as the surface diffuseness anomaly. Here, we follow Ref. [17] and take $a_0 = 1.03$ fm. We have checked that the agreement of the calculation with the experimental data becomes worse if we use a smaller value of $a_0$, such as $a_0 = 0.7$ fm. Notice that results are almost independent of the precise values of $V_0$ and $R_0$ as long as the barrier height is reproduced. For excitations in the target nucleus, $^{92}\text{Zr}$, we have included a coupling to the one quadrupole phonon state at 0.934 MeV with the deformation parameter of 0.13.

The dashed line in Fig. 1 shows the fusion barrier distribution for the $^{28}\text{Si} + ^{92}\text{Zr}$ system when the coupling to the $2^+$ state in $^{28}\text{Si}$ is included assuming an oblate rotor with $\beta_2 = -0.407$ [20] and $\beta_4 = 0$. Here, the fusion barrier distribution is defined as [21],

$$D_{\text{fus}}(E) = \frac{d^2(E\sigma_{\text{fus}})}{dE^2},$$

where $E$ is the incident energy in the center of mass frame and $\sigma_{\text{fus}}$ is a fusion cross section. The experimental fusion barrier distribution was extracted with a point difference formula with $\Delta E \sim 2$ MeV [17], and the same procedure was applied to the theoretical fusion barrier distribution as well. In the figure, one can find that this calculation captures the main structure of the barrier distribution, but the experimental data around $E_{\text{c.m.}} = 75$ MeV are not well accounted for. The calculation is somewhat improved by taking into account a finite value of $\beta_4$, e.g., $\beta_4 = +0.10$, the value which was employed in Ref. [22], as is shown by the dashed line with crosses. On the other hand, when the quadrupole deformation of $^{28}\text{Si}$ was taken to be positive, the shape of fusion barrier distribution becomes inconsistent with the experimental data (see the dashed line with triangles), supporting an oblate deformation of $^{28}\text{Si}$ [23]. The solid line in the figure shows the result with the vibrational excitation $^{28}\text{Si}$, in which the first $2^+$ state is treated as a one phonon state in the harmonic oscillator approximation. One can clearly see that this calculation better reproduces the experimental fusion barrier distribution, compared to the rotational coupling with $\beta_4 = 0.10$, as has been pointed out in Ref. [17].

In order to see the sensitivity of the results to $\beta_4$ in the rotational coupling scheme, the dashed line with triangles in Fig. 2 shows the barrier distribution obtained with a larger value of $\beta_4$, that is, $\beta_4 = 0.25$. This is the value obtained by Möller and Nix [24] by using the finite-range dropet model with spherical-harmonic expansions. This value is also consistent with the one obtained with proton scattering experiments, i.e., $+0.25 \pm 0.08$ [25]. Earlier experiments for electron scattering [26], neutron scattering [27, 28] and alpha particle scattering [29] indicate that the value of $\beta_4$ in $^{28}\text{Si}$ is $+0.10$, $+0.18 \pm 0.02/-0.20 \pm 0.05$ and $+0.08 \pm 0.01$, respectively. Although these values are somewhat different from each other, all of these values point to a large value of $\beta_4$. Interestingly, the rotational calculation with $\beta_4 = 0.25$ yields an almost identical result to the result of the vibrational coupling scheme shown by the solid line in the figure. This is in the same situation as in the $^{28}\text{Si} + ^{154}\text{Sm}$ system discussed in Ref. [15].

Within the space of the ground state ($0^+$) and the first $2^+$ state, the difference between the rotational and
the harmonic vibrational coupling schemes is found only in the re-orientation term. That is, there is no coupling from the 2\(^+\) state to the same state, 2\(^+\), in the vibrational coupling, while this coupling is finite in the rotational coupling (compare between Eqs. (3.41) and (3.49) in Ref. [4]). It is important to notice here that the 2\(^+\) similarity disappears when we take

\[ \beta_2 = -0.407, \beta_4 = +0.25 \]

with both the quadrupole and the hexadecapole terms. That is, there is no coupling of the 4\(^+\) state originated from the large value of \(\beta_4\), and the rotational excitation of the 28\(^{\text{Si}}\) projectile still plays an important role in the fusion of this nucleus.

A large hexadecapole deformation of 28\(^{\text{Si}}\) should accompany a strong direct coupling from the ground state to the 4\(^+\) state. The 4\(^+\) state also couples to the 2\(^+\) state with both the quadrupole and the hexadecapole terms (notice that there is no hexadecapole coupling between the 0\(^+\) state and the 2\(^+\) state). In order to check the influence of the 4\(^+\) state, the dashed line with stars in Fig. 3 shows the result obtained by including the ground state rotational band of 28\(^{\text{Si}}\) up to the 4\(^+\) state with the deformation parameters of \(\beta_2 = -0.407\) and \(\beta_4 = +0.25\). The inclusion of the 4\(^+\) state somewhat perturbs the shape of fusion barrier distribution, and the agreement with the experimental data is slightly worsened. However, the calculated fusion barrier distribution is still within the error bars of the experimental distribution and there remains a similarity to the barrier distribution for the vibrational coupling scheme. We have confirmed that the agreement is not significantly improved even with a larger value of \(\beta_4\), that is, \(\beta_4 = 0.30\). We have also checked the influence of the octupole excitation to the 3\(^-\) state at 6.878 MeV in 28\(^{\text{Si}}\) and have confirmed that the inclusion of this state simply shifts the barrier distribution in energy by \(\approx 1.5\) MeV without significantly changing its shape. As has been pointed out e.g., in Ref. [4], excitation to a state with large excitation energy, such as the 3\(^-\) state in 28\(^{\text{Si}}\), simply lead to a renormalization of the fusion barrier, thus do not significantly influence the fusion dynamics. We have also found that the results converge rapidly on adding the higher members in the rotational band, beyond 4\(^+\), of 28\(^{\text{Si}}\) due to the finite excitation energy. This latter fact is another necessary condition to have a simi-
larity between the rotational coupling and the vibrational coupling schemes. That is, when higher members in the ground state rotational band contribute significantly to the fusion dynamics, which is typically the case for fusion of medium-heavy nuclei such as $^{16}\text{O} + ^{154}\text{Sm}$, the resultant fusion barrier distribution differs considerably from fusion barrier distributions for vibrational nuclei $[3, 4, 9]$.

III. SUMMARY AND DISCUSSIONS

In summary, we have carried out the coupled-channels calculations for the $^{28}\text{Si} + ^{92}\text{Zr}$ fusion reaction and have shown that the fusion process is sensitive to the hexadecapole deformation of the $^{28}\text{Si}$ nucleus. We have demonstrated that the reorientation term for the $2^+$ state is largely canceled out, leading to similar results between the rotational and the vibrational coupling schemes, even though in reality the $^{28}\text{Si}$ nucleus is not a spherical nucleus. This nicely follows the earlier conclusion obtained for the $^{28}\text{Si} + ^{154}\text{Sm}$ reaction $[15]$, making a strong evidence for that $^{28}\text{Si}$ possesses a large positive hexadecapole moment.

In order to have such similarity between results with the rotational coupling scheme and those with the vibrational coupling scheme, the following two conditions are necessary. The first condition is that the quadrupole and the hexadecapole deformation parameters have opposite sign to each other and the ratio is close to $\beta_4/\beta_2 = -\sqrt{5}/3 = -0.745$. The second condition, which is usually satisfied for light deformed nuclei, is that the excitation energy of the first $2^+$ state is large so that higher members of the ground state rotational band do not significantly contribute. The $^{28}\text{Si}$ nucleus satisfies both conditions. In addition to other Si isotopes, another candidate which shows the same kind of similarity might be $^{38}\text{Ne}$.

Even though several aspects related to the weakly-bound nature of this neutron-rich nucleus would have also to be taken into account, this nucleus satisfies the two conditions, as the deformation parameters for this nucleus are predicted to be $\beta_2 = -0.302$ and $\beta_4 = +0.163$ with the FRDM(2012) mass model $[30]$ and the energy of the $2^+$ state is predicted to be around 1.45 MeV with a shell model calculation $[31]$.

The coupled-channels calculations for the $^{28}\text{Si} + ^{92}\text{Zr}$ system presented in this paper suggest that the fusion mechanism is sensitive to projectile excitations. This is also relevant to the synthesis of superheavy elements. Very recently, barrier distributions were extracted using quasi-elastic scattering for reactions to form superheavy elements $[32]$. Quasi-elastic barrier distributions are complimentary to fusion barrier distributions and they have smaller error bars on the high energy side. It will be an interesting future work to study how the projectile excitations influence evaporation residue cross sections for fusion reactions of the $^{28}\text{Si}$ projectile to form superheavy elements. Note also that the method based on a quasi-elastic barrier distribution will be useful to discuss the shape of radioactive nuclei, for which the beam intensity is low $[16]$.

In the past, $\alpha$-particle scattering $[33, 34]$, electron scattering $[35]$, and muonic $x$-rays methods $[36]$ have been used in order to determine experimentally the shape of a deformed nucleus. However, $\beta_4$, especially its sign, is difficult to extract. All the available results for $\beta_4$ are model-dependent and quite different from each other with large uncertainties. As we have discussed in this paper, fusion is sensitive not only to the target excitations but also to the projectile excitations, and the barrier distribution analysis will offer an alternative powerful method to extract the magnitude and sign of $\beta_4$ for deformed nuclei.

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