Coulomb Blockade in the Fractional Quantum Hall Effect Regime

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We use chiral Luttinger liquid theory to study transport through a quantum dot in the fractional quantum Hall effect regime and find rich non-Fermi-liquid tunneling characteristics. In particular, we predict a remarkable Coulomb-blockade-type energy gap that is quantized in units of the non-interacting level spacing, new power-law tunneling exponents for voltages beyond threshold, and a line shape as a function of gate voltage that is dramatically different than that for a Fermi liquid. We propose experiments to use these unique spectral properties as a new probe of the fractional quantum Hall effect.

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Despite enormous theoretical and experimental effort during the past decade, the nature of transport in the fractional quantum Hall effect (FQHE) regime of the two-dimensional electron gas remains uncertain. Although chiral Luttinger liquid (CLL) theory has successfully predicted transport and spectral properties of sharply confined FQHE systems near the center of the $\nu = 1/3$ plateau, the situation at other filling factors and in smooth-edged geometries is poorly understood. This has motivated us to consider a new, alternative probe of FQHE edge states.

In a certain sense, tunneling spectra of single-branch edge states are ultimately measurements of $g$, the dimensionless parameter characterizing a CLL that measures the degree to which it deviates from a Fermi liquid, for which $g = 1$. In particular, the zero-temperature density-of-states (DOS) of a macroscopic CLL varies as $\epsilon^{-2}$, which is responsible for its well-known power-law tunneling characteristics. It is not surprising (and will be established below) that transport through a large quantum dot in the FQHE regime is primarily governed by the DOS of a mesoscopic CLL. We shall show here that this finite-size DOS has a remarkable low-energy structure that depends on $g$ in an intricate manner. We therefore propose tunneling through a quantum dot in the FQHE regime as a new probe of edge-state dynamics.

It has been appreciated for some time that transport through a strongly correlated FQHE droplet would be interesting in its own right, and this motivated Kinaret et al. to do their work on the subject. Their work, which mostly focused on the linear response regime and on small system sizes, led to a number of proposed experiments, which have not been carried out yet. We would like to emphasize, however, that the experiments proposed by Kinaret et al., and by us in the present work, although far from routine, should be possible using current nanostructure fabrication techniques.

The main difference between our work and previous work is that we are the first to directly calculate the retarded electron propagator for a mesoscopic CLL, which has required the development of finite-size bosonization methods appropriate for the CLL. As mentioned, this Green’s function has a fascinating low-energy structure, which will be described below. This result has enabled us to map out a considerable portion of the low-temperature phase diagram for transport through a large quantum dot: In the $\nu = 1/q$ state with $q$ an odd integer we predict a remarkable Coulomb-blockade-like energy gap of size $(q - 1)\Delta \epsilon$, where $\Delta \epsilon$ is the noninteracting level spacing. Unlike a conventional Coulomb blockade, however, the energy gap here is precisely quantized. Furthermore, the low-temperature tunneling current scales nonlinearly with voltage as $V^q$ at a Coulomb blockade tunneling peak, as one might expect, but as the voltage is increased between these peaks to overcome the Coulomb blockade the current at the threshold varies as $V^{q+1}$. The finite-bias line shape as a function of gate voltage depends nontrivially on $q$ and is also dramatically different than that for a Fermi liquid.

The model we adopt here for the quantum dot system is as follows: Two macroscopic $g = 1$ edge states, L and R, are weakly coupled to a mesoscopic FQHE edge state, D, in the quantum dot, by a tunneling perturbation

$$\delta H = \sum_{I=L,R} \gamma_1 \psi_I(x_1) \psi_D^\dagger(x_1) + \gamma_2 \psi_D(x_1) \psi_I^\dagger(x_1).$$

The edges of the two-dimensional electron gas are assumed to be sharply confined, and the interaction short-ranged (screened by a nearby gate), so that the low lying excitations consist of a single branch of edge-magnetoplasmons with linear dispersion $\omega = v|k|$. Although we will be working at zero-temperature, it is assumed that there is a small temperature present to help suppress coherence and resonant tunneling. Finally, the dot should be large enough (a few $\mu m$ in circumference) so that the charging energy is smaller than the bulk FQHE energy gap.

What property of the electron gas is probed in a mea-
measurement of tunneling through the dot? It is known that the conductance of a simple resistive barrier measures the transmission probability of that barrier, a one-particle property, along with the single-particle Green’s function of the leads, even when there is strong electron-electron interaction \([10]\). In contrast, transport through a quantum dot containing other electrons generally probes two-particle (and higher order) properties of the dot, even at the grand-canonical ensemble with chemical potential \(\mu\). However, it is clear that for a large, weakly coupled dot, and small enough currents, an electron can tunnel onto the dot, dissipate energy, and then tunnel incoherently through the second barrier, and in this so-called sequential tunneling limit the resistance will probe the one-particle Green’s function of the dot.

To establish this relationship we write the current from \(L\) to \(R\) as

\[
I = \sum_N P(N) [w_{LD}(N) - w_{DL}(N)],
\]

where \(w_{LD}\) and \(w_{DL}\) are transition rates to go from \(L\) to \(D\) and from \(D\) to \(L\), given that there are \(N\) electrons in the quantum dot, and \(P(N)\) is the probability that the dot has \(N\) electrons. The zero-temperature rate from an initial ground state \(|\Psi_0^{NL}\rangle_{L} \otimes |\Psi_0^{N}\rangle_{D}\) to final excited states of the form \(|\Psi_{\alpha_1}^{N-1}\rangle_{L} \otimes |\Psi_{\alpha_2}^{N+1}\rangle_{D}\) is given by

\[
w_{LD}(N) = 2\pi |\gamma_L|^2 \sum_{\alpha L} |\langle \Psi_{\alpha L}^{N-1} | \psi_0 \rangle_{L} |^2 \sum_{\alpha D} |\langle \Psi_{\alpha D}^{N+1} | \psi_0 \rangle_{D} |^2 \delta(E_{\alpha L}^{N-1} - E_0^{NL} + E_{\alpha D}^{N+1} - E_0^{N} - V_L + V_D),
\]

where \(V_L\) and \(V_D\) are potential energies produced by gates above \(L\) and \(D\). This can be written as

\[
w_{LD}(N) = 2\pi |\gamma_L|^2 \Theta(V) \int_0^V d\epsilon \, A_+^{\prime}(\epsilon) \, A_L^+(V - \epsilon),
\]

where \(V\) is the electrochemical potential difference between \(L\) and \(D\), \(\Theta\) is the unit step function, and

\[
A_+(\omega) = \sum_{\alpha} \langle \Psi_{\alpha}^{N+1} | \psi_0 \rangle_{D}^2 \delta(\omega + \mu^N - E_{\alpha}^{N+1} + E_0^{N})
\]

\[
A_-(\omega) = \sum_{\alpha} \langle \Psi_{\alpha}^{N-1} | \psi_0 \rangle_{D}^2 \delta(\omega - \mu^N - E_{\alpha}^{N-1} + E_0^{N}),
\]

where \(\mu^N = E_0^{N+1} - E_0^{N}\). The chemical potential in the quantum dot is \(N\) dependent. With noninteracting leads,

\[
w_{LD}(N) = 2\pi |\gamma_L|^2 N_0(0) \Theta(V) \int_0^V d\epsilon \, A_+^{\prime}(\epsilon),
\]

where \(N_0(0)\) is the density of states at the Fermi energy in the left lead. If we define the interacting DOS as \(N(\epsilon) \equiv \frac{1}{2} \text{Im} G(0,\epsilon)\), where \(G(x,t) \equiv \langle \psi_\alpha(x,t), \psi_\alpha^\dagger(0) \rangle\) is the retarded electron propagator calculated in the grand-canonical ensemble with chemical potential \(\mu\), then for \(\epsilon > 0\) it follows that \(A_+(\epsilon) = N(\epsilon)|_{\mu=\mu^N}\).

The dynamics of the mesoscopic CLL is governed by the action \((g = 1/q\) with \(q\) an odd integer\)

\[
S = \frac{1}{4\pi} \int_0^L dx \int_0^\beta d\tau \left[ \pm i (\partial_x \phi_\pm(\tau)) (\partial_x \phi_\pm(\tau)) + v (\partial_x \phi_\pm(\tau))^2 \right],
\]

where \(\rho_\pm = \pm \partial_x \phi_\pm / 2\pi\) is the charge density fluctuation for right (+) or left (−) moving electrons [3]. Momentum space quantization is achieved by decomposing the chiral scalar field \(\phi_\pm\) into a nonzero-mode contribution \(\phi_{\pm}^0\) satisfying periodic boundary conditions, and a zero-mode part \(\phi_\pm^0\). The bosonized electron field is \(\psi_\pm(x) \equiv (2\pi a)^{-1} e^{iq\phi_\pm(x)/x} e^{iq\pi x/L}\), where \(a\) is a microscopic cutoff length. In the presence of an Aharonov-Bohm flux \(\Phi = \varphi \Phi_0\) (with \(\Phi_0 \equiv hc/e\)) the grand-canonical Hamiltonian corresponding to [8] is

\[
H = \frac{1}{2g}(N \pm g\varphi)^2 \Delta \epsilon + \sum_k \Theta(\pm(\epsilon)) v |k| a_k^\dagger a_k - \mu N,
\]

where \(\Delta \epsilon \equiv 2\pi v/L\) is the noninteracting level spacing and \(N \equiv \int_0^L dx \, \rho_\pm\). At zero temperature,

\[
G(x,t) = \frac{1}{2\pi a} \Theta(t) e^{\pm i q x \varphi (x \mp vt)/L} \left( e^{iq\phi_\pm(x,t) - \phi_\pm^0(0))} \right) \left( e^{iq\phi_\pm(x,t) - \phi_\pm^0(0))} \right) e^{i q x f_\pm(x,t)} + e^{\mp i q x f_\pm(x,t)} \left( e^{iq\phi_\pm(x,t) - \phi_\pm^0(0))} \right) e^{i q x f_\pm(-x,-t)},
\]

where \(f_\pm(x,t) \equiv \langle \phi_\pm^0(x,t) - \phi_\pm^0(0)) ( - \phi_\pm^0(0)) \rangle.\) The time-evolution of the zero-mode field under the action of \(\Psi(x,t)\) is found to be \(\Phi_\pm^0(x,t) = \mp 2\pi N(x \mp vt)/L - g\chi + g(\mu \mp \varphi \Delta \epsilon)t\), where \([\chi, N] = i\). Then
$G(x,t) = \pm \Theta(t) (i/L)^q (\pi a)^{q-1} e^{\pm i q(x \mp vt)/L} e^{i(\mu \mp \varphi) \xi t} e^{\pm 2 \pi i q(N)(x \mp vt)/L} \text{Im} \sin^{-q}[\pi(x \mp vt \pm \mu)/L], \hspace{1cm} (10)$

where $\langle N \rangle = q^{-1} \text{int}(\frac{q}{2\pi} \mp \varphi)$. Here $\text{int}(x)$ denotes the integer closest to $x$. The transform may be written as

$$G(x,\omega) = \frac{\pi i (i/2)^{q-1}}{(q-1)!} (1 - \Omega^2)(3^2 - \Omega^2) \times \cdots \times [(q - 2)^2 - \Omega^2] \times G_0(x,\omega), \hspace{1cm} (13)$$

where $G_0(x,\omega)$ is the retarded propagator for the noninteracting chiral electron gas, given below. The $q$ dependence of $\Omega$ is extracted by writing $\Omega = 2z - q\langle N \rangle$, where $z = \frac{2\pi}{a} + \text{frac}(\frac{2\pi}{a} \mp \varphi)$ and $\text{frac}(x) = x - \text{int}(x)$. Finally, after using the identity (proved by induction) for $q$ an odd integer greater than one,

$$(1 - \Omega^2)(3^2 - \Omega^2) \times \cdots \times [(q - 2)^2 - \Omega^2] = [1 - (2z - q^2)](3^2 - (2z - q^2)) \times \cdots \times [(q - 2)^2 - (2z - q)^2]$$

we arrive at the remarkable relation

$$G(x,\omega) = G_0(x,\omega) \times \frac{1}{(q-1)! \epsilon_F^{q-1}} \prod_{j=1}^{q-1} (\omega - \omega_j), \hspace{1cm} (15)$$

where $\epsilon_F \equiv v/a$ is an effective Fermi energy and where $\omega_j \equiv [j - \text{frac}(\frac{2\pi}{a} \mp \varphi)] \Delta \epsilon$ are the noninteracting energy levels. Whereas in the $q = 1$ case the propagator has poles at each of the $\omega_j$, in the interacting case the first $q - 1$ poles above $\mu$ are removed. This effect, which leads to a Coulomb-blockade-type energy gap, is a consequence of the first term in the Hamiltonian (3). At higher frequencies or in the large $L$ limit where $\omega \gg \Delta \epsilon$, the additional factor becomes $\omega^{q-1}/(q-1)! \epsilon_F^{q-1}$. The polynomial factor in Eqn. (13) is plotted in Fig. (4).

In (3) we have taken the single-particle dispersion to be $\epsilon_{\pm}(k) = \pm v(k + 2\pi \varphi/L)$. The noninteracting chiral propagator is therefore $G_0(\omega) = (1/2v) \cot[\theta(\omega)/2]$, where $\theta(\epsilon) = 2\pi(\epsilon/\Delta \epsilon \mp \varphi)$ is the phase subject to an electron of energy $\epsilon$ after going around the edge state. Having obtained the transition rate (7) we turn to a calculation of the probability $P(N)$, which satisfies

$$\partial_t P(N) = \sum_{l=L,R} [w_{\text{ID}}(N-1) P(N-1) + w_{\text{DI}}(N+1) P(N+1) - w_{\text{ID}}(N) P(N) - w_{\text{DI}}(N) P(N)]. \hspace{1cm} (16)$$

The steady-state solution of (16) yields the final result for the tunneling current. For the case $q = 3$,

$$I = 2\pi |\gamma|^2 [N_0(0)]^2 \left\{ \frac{V^2(V - 4U^2[N_G - (n + \frac{1}{2})]^2)^3}{V^2 + 12U^2[N_G - (n + \frac{1}{2})]^2} \right\} \text{ when } V > 2U|N_G - (n + \frac{1}{2})|, \hspace{1cm} (17)$$

and is zero otherwise. Eqn. (17) is valid for $n < N_G < n+1$, where the gate charge $N_G$ is the number of positive charges induced by the gate, and for symmetric leads. $U \equiv \Delta \epsilon$ is the quantized charging energy plus the single-particle level spacing.

The $q = 3$ result (13) clearly exhibits the novel transport properties present at all $q \neq 1$. The Coulomb blockade boundary, shown as a solid line in Fig. (3) has the familiar diamond shape, but the scale $U$ is now quantized in units of $\Delta \epsilon$. It can be shown that the current in a Fermi liquid would be proportional to a term of the
form $V - \frac{4U^2}{N} [N_G - (n + \frac{1}{2})]^2$ alone. The additional structure present in Eqn. (17) describes how the quantum dot becomes a non-Fermi-liquid conductor when threshold is exceeded. Examples of this non-Fermi-liquid behavior are shown in Fig. 2. Along path (i) the current varies as $V^q$, as one might naively expect, but on (ii) it varies as $(V - U)^q + 1$. The line shape along (iii) depends nontrivially on $q$; for $q = 3$ it varies as $(1 - 4x^2)^3/(1 + 12x^2)$, which, surprisingly, is in excellent agreement with the finite-bias numerical results for just 8 electrons [7]. The transport properties at other values of $q$ can be determined from Eqn. (15).

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FIG. 1. Polynomial factor for the cases $q = 3$ and 5, plotted as a function of $\omega/\Delta \epsilon$. 
FIG. 2. Phase diagram for the tunneling current as a function of bias voltage $V$ and gate charge $N_G$. 

(i) non-Fermi-liquid conductor

(ii) insulating (Coulomb blockade)