The UV behavior of Gravity at Large N

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Abstract

A first step in the analysis of the renormalizability of gravity at Large N is carried on. Suitable resummations of planar diagrams give rise to a theory in which there is only a finite number of primitive superficially divergent Feynman diagrams. The mechanism is similar to the one which makes renormalizable the 3D Gross-Neveu model at large N. The connections with gravitational confinement and KLT relations are shortly analyzed. Some potential problems in fulfilling the Zinn-Justin equations are pointed out.

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The quantum theory of renormalizable interactions plays the main role in our present understanding of fundamental physical laws. It gave rise to the formulation of the Standard Model of elementary particles which is still today in a very good shape. Of course, there are many problems which have not been solved yet (such as confinement in QCD): some of these problems are likely to be "technical" problems in the sense that a better understanding of the actual standard theory should be enough to give the correct solutions. As far as other problems (such as the hierarchy problem, the cosmological constant, the quantum version of the gravitational interaction and so on) are concerned, the standard theory is likely to be inadequate. In particular, the lacking of a precise understanding of non perturbative phenomena occurring in the strongly coupled phase of gravity (which should clarify important and still poorly understood features of early cosmology) is unpleasant: being gravity perturbatively non renormalizable, it is not possible to make detailed predictions in such a phase. The two main candidates to be the final theory of quantum gravity, Superstring Theory and Loop Quantum Gravity (which are not necessarily to be thought as mutually exclusive), still are too complicated to be fully understood. Thus, it is worth to explore new ways in which "physical effects beyond the standard model" could be already manifest in the low energy (low with respect to the Planck scale) physics. The dominant point of view of string theorists is to see the Einstein-Hilbert action as an effective action in which the "heavy degrees of freedom" have been integrated out. Unfortunately, in order to make this idea predictive and to clarify how it is possible to improve in practice the UV behavior of gravity it would be important to understand in more details the dynamics of such degrees of freedom and, at the present stage, this is a rather difficult task.

It is not enough to say that "gravity is an effective field theory": even if this is the case, to perform meaningful computations in the strongly coupled regime with the technical tools at our disposal (perturbative expansions of various kind, renormalization group and so on), physical effects "beyond the standard model" (related, for instance, to string theory) which enable such a meaningful (and, hopefully, predictive) computations still have to be clarified.

To be more specific, string theory predicts various type of geometrical corrections to the "bare" Einstein-Hilbert action:

$$S_{corr} = S_{EH} + \sum_{n}(\alpha')^n \int_M f_n(R_{\mu\nu\rho\sigma})d\mu$$

where $S_{EH}$ is the Einstein-Hilbert action, $\alpha'$ is the string length and the $f_n$ are higher order curvature invariants. Even if a large but finite number of terms are added, the corrected action remains non renormalizable and, consequently, it is not yet clear how to perform meaningful quantum computations (unless one would be able to sum the whole series: a hopeless task indeed). In a sense, the analysis of the $\alpha'$ corrections as a tool to shed light on non perturbative phenomena in gravity is like the analysis of the standard perturbation expansion in QCD to understand confinement. Clearly, in QCD, there is a little hope to understand non perturbative phenomena with ordinary perturbation theory. In the same way, in gravity one should expect physics beyond the standard model to manifest itself in a different, perhaps more subtle, way. Renormalizability is not a mere aesthetic requirement; it is, in fact, the need to have a theory which is predictive in the strongly coupled phase: one should expect (it is better to say "hope") that physics beyond the standard model will improve the Einstein-Hilbert gravity precisely in this direction.

A sound theoretical idea which could give important indications on the way to follow is the Holographic Principle introduced in \[1,2\] (for two detailed reviews see \[3,4\]) which is at the basis of the string-theoretical AdS/CFT correspondence \[2\]. In a recent paper \[6\] a Large $N$ expansion for the gravitational interaction has been formulated which shed new light on the relations between higher spins, the holographic principle and non perturbative phenomena such as (a sort of) gravitational confinement. In SU($N$) Gauge Theory, the large $N$ expansion introduced by 't Hooft in \[7,5\] and refined by Veneziano \[8\] is indeed one of the most powerful non perturbative techniques available to investigate the strongly coupled phase (it provides the issues of confinement, chiral symmetry breaking and the relation with string theory with a rather detailed understanding; a clear analysis of the role of Baryons at large $N$ has been given in \[10\]: see, for two detailed pedagogical reviews, \[11\]).

One of the main properties of the large $N$ expansion is that many non trivial models (such as the $O(N)$ $\phi^4$ model in five space-time dimension and the Gross-Neveu model in three space-time dimensions which are not renormalizable in the standard perturbative expansion) become in fact renormalizable (see, for example, \[12\]): the Green functions are not anymore analytic in the small coupling constant region (see, for example, \[13,14,15\]). Thus, the large $N$ expansion can be seen as a strong coupling expansion which, besides to clarify non perturbative phenomena such as confinement and chiral symmetry breaking, greatly improve the UV behavior of theories which, at a first glance, would appear meaningless at high energies. Here, it will be argued that, under very reasonable assumptions, this could also happen in gravity. The large $N$ expansion suggests suitable resummations of planar diagrams which lead to an UV-softening of gravity: the relation between the Newton constant and the mass of higher spin field(s) seems to be quite similar to the relation between the Fermi coupling constant and the mass of the $W_\pm$ bosons of the electro-weak interactions.
The paper is organized as follows: in section 2 the diagrammatic formulation of General Relativity as a constrained BF theory is shortly described and the well known result about the perturbative non renormalizability of gravity is described in this formalism. In section 3, the large N resumptions which improve the UV behavior of gravity are introduced: for the sake of clarity, in section 3 the complications connected to ghosts will be neglected keeping manifest, however, the main physical features leading to the UV softening. In section 4, the ghosts effects which could prevent the UV softening are analyzed. In section 5, a possible physical interpretation of the large N "UV softening" is proposed and the connections with the KLT relations equation are pointed out. Eventually, the conclusions are drawn.

II. BF-GRAVITY AND FEYNMAN RULES

In this section the BF formulation of gravity and the corresponding Feynman rules will be shortly described. The topological BF theory in four dimensions is defined by the following action

\[ S[A,B] = \int_M B^{IJ} \wedge^* (F_{IJ}(A)) = \frac{1}{4} \int_M \varepsilon^{\alpha\beta\gamma\delta} B_{\alpha\beta} F_{\gamma\delta IJ} dx^\ast \]

\[ B^{IJ} = \frac{1}{2} B_{\alpha\beta} dx^\alpha \wedge dx^\beta, \quad F_{IJ} = \frac{1}{2} F_{\alpha\beta IJ} dx^\alpha \wedge dx^\beta \]

\[ F_{\alpha\beta IJ} = (\partial_\alpha A_\beta - \partial_\beta A_\alpha)_{IJ} + A^L_{\alpha I} A_{\beta LJ} - A^L_{\beta I} A_{\alpha LJ}, \]  

(2)

where \( M \) is the four-dimensional space-time, \( * \) is the Hodge dual, the greek letters denote space-times indices, \( \varepsilon^{\alpha\beta\gamma\delta} \) is the totally skew-symmetric Levi-Civita symbol in four-dimensional space-times, \( I, J \) and \( K \) are the internal Lorentz indices which are raised and lowered with the Minkowski metric \( \eta_{IJ} \): \( I, J = 1, \ldots, N \). Thus, the basic fields are a \( so(N-1,1) \)-valued differential 2-form \( B_{IJ} \) and a \( so(N-1,1) \) connection 1-form \( A_{\alpha LJ} \), the internal gauge group being \( SO(N-1,1) \). Also the Riemannian theory can be considered in which the internal gauge group is \( SO(N) \) and the internal indices are raised and lowered with the Euclidean metric \( \delta_{IJ} \); in any case, both \( B_{IJ} \) and \( A_{\alpha LJ} \) are in the adjoint representation of the (algebra of the) internal gauge group. The equations of motion are

\[ F = 0, \quad \nabla A B = 0, \]

\[ \nabla_A = \nabla = d + [A.] \]

(3)

where \( \nabla_A \) is the covariant derivative with respect to the connection \( A_{\alpha LJ} \). The above equations tell that \( A_{\alpha LJ} \) is, locally, a pure gauge and \( B^{IJ} \) is covariantly constant. When \( N = 4 \) and \( B^{IJ} \) has the form

\[ B^{IJ} = \frac{1}{2} \varepsilon_{KL}^I e^J K \wedge e^L. \]

(4)

The action (1) is nothing but the Palatini form of Einstein-Hilbert action. Eq. (4) can be enforced by adding to the action (1) a suitable constraint: the basic action in the BF formalism is

\[ GS_{GR} = S[A,B] - \int_M (\phi_{IJKL} B^{IJ} \wedge B^{KL} + \mu H(\phi)) \],  

(5)

where \( G \) is the gravitational coupling constant, \( \mu \) is fixed a differential 4-form and \( H(\phi) \) is a scalar which may have one of the following expressions:

\[ H_1 = \phi_{IJ}^I, \quad H_2 = \phi_{IJKL}^I e^{IJKL}, \quad H_3 = a_1 H_1 + a_2 H_2, \]

(6)

the \( a_i \) being real constants (see [16, 17, 18, 19, 20]). It is worth to note here that the Lagrange multiplier \( \phi \) has four internal indices. The form (4) of the Einstein-Hilbert action is a natural starting point to formulate the "gravitational" large N expansion since the connection formulation allows to adopt the double line notation (however, being the fundamental representation of \( so(N-1,1) \) real, the lines of internal indices carry no arrows).

The classical action in Eq. (5) is left invariant by \( so(N-1,1) \)–gauge transformations and by diffeomorphisms. The analysis of the BRST invariance of the gravitational action in the BF formalism can be found in [21]. As far as
the third terms on the right hand side of Eq. (8). The BF term is left in variant by Yang-Mills theory. It is worth to note that the symmetries δ
\_2 and δ
\_3 given by a shift (see below Eq. (10)). However, at this stage of the analysis, it seems to be unavoidable to add the Yang-Mills term (the second term on the right hand side of Eq. (7)) in order to obtain the gravitational action. This is possible since it has been shown in (22) (8).

The consequence is that, in order to produce the Yang-Mills kinetic term for η, it is convenient to add the second term on the right hand side of Eq. (7) to the gravitational action. This is lawful as is because the (classical) action of the 'enlarged' classical BF Yang-Mills theory is invariant under a further transformation

\[ B' = B - \nabla \eta, \]

whose role is to keep at the same time both the local degrees of freedom of Yang-Mills theory and the symmetries of the BF theory (physically, η represents the longitudinal components of B). It is worth to note here that, because of the Bianchi identities, one has

\[ S[A, B'] = S[A, B]. \]

In the Yang-Mills case, this procedure is lawful as is because the small e limit manifests no problem (the theory is "perturbative" in e). In other words, the second term on the right hand side of Eq. (7) can be regarded as a true vertex: therefore one can consider the gravitational case as the small e limit of the action (8).

Indeed, η has only a technical role since it just represents the longitudinal components of B and one of the η's transformation laws is given by a shift (see below Eq. (10)). However, at this stage of the analysis, it seems to be unavoidable to add the Yang-Mills term (the second term on the right hand side of Eq. (7)) in order to obtain the Feynman rules, vertices and propagators. On the other hand, due to the comparison with Yang-Mills theory, this scheme is mandatory if one wants to clearly identify the "guilty of the perturbative non renormalizability of gravity" in the BF scheme: but for this enlargement of the gravitational action, it would not be possible to "large N" improve the UV behavior of gravity.

The classical symmetries of the "enlarged" classical BF gravitational action

\[ GS_{cl} = S[A, B] - \int_M (\phi_{ab}(B')^a \wedge (B')^b + \mu H(\phi)) - e^2 \int_M tr (B')^a (B')_a \]

(where a, b, c and so on are indices in the adjoint representation of so(N - 1, 1)) are

\[ \delta_1 A_\mu = \nabla \theta^{(1)}_\mu; \quad \delta_1 B_{\mu\nu} = [B_{\mu\nu}, \theta^{(1)}]; \quad \delta_1 \eta_\mu = [\eta_\mu, \theta^{(1)}]; \]

\[ \delta_2 A_\mu = 0; \quad \delta_2 B_{\mu\nu} = \nabla_{\mu\nu} \omega_{\nu}; \quad \delta_2 \eta_\mu = \omega_\mu; \]

\[ \delta_3 A_\mu = 0; \quad \delta_3 B_{\mu\nu} = [F_{\mu\nu}, \theta^{(3)}]; \quad \delta_3 \eta_\mu = \nabla_\mu \theta^{(3)}; \]

where θ
\_i are so(N - 1, 1) valued gauge scalars. δ_1 is a simple gauge transformation so that the action is invariant. As far as δ_2 and δ_3 are concerned, the transformations of B cancel out the transformations of η in the second and in the third terms on the right hand side of Eq. (8). The BF term is left invariant by δ_1 because of the Bianchi identities and by δ_2 because it reduces to a trivial gauge transformation on the usual F² term of (the standard formulation of) Yang-Mills theory. It is worth to note that the symmetries δ_2 and δ_3 are reducible, as it is clear if one considers in Eqs. (10) and (11)

\[ (\nabla \theta^{(3)})_\mu = \omega_\mu. \]
To obtain the Feynman rules, the gauge fixing and ghost terms related to the above symmetries have to be included. A convenient gauge fixing is

$$\partial_\mu A^\mu = 0, \quad \partial_\mu \eta^\mu = 0, \quad \partial_\mu B^{\mu\nu} = 0$$  \hspace{1cm} (12)

so that the corresponding gauge-fixing term of the action is

$$S_{gf} = \int_M \{ \bar{\psi} (-\partial_\mu \nabla^\mu) c + h_A (\partial_\mu A^\mu) +$$

$$+\bar{\psi}^a \partial_\mu \left\{ -[B^{\mu\nu}, c] + \nabla_{[\mu} \psi_{\nu]} + [F^{\mu\nu}, \rho] \right\} +$$

$$+h_B (\partial_\mu B^{\mu\nu}) + \bar{\rho} \partial_\mu \left\{ -[\eta_{\mu}, c] + \psi_\mu + \nabla_\mu \rho \right\} +$$

$$+h_\eta (\partial_\mu \eta^\mu) + u (\partial^\mu h_B) + h_\psi \left( \partial_\mu \bar{\psi} \right) +$$

$$+\bar{\xi} \delta^\mu \left\{ [\psi_\mu, c] + \nabla_\mu \xi \right\} + h_\psi (\partial_\mu \psi^\mu) \} ,$$  \hspace{1cm} (13)

where \((c, \bar{\psi}, h_A), (\psi, \bar{\psi}, h_B), (\rho, \bar{\rho}, h_\eta)\) are respectively the ghost, the antighost and the Lagrange multiplier for \(\delta_1\) and \(\delta_2\); \((\xi, \bar{\xi}, h_\psi)\) the ghost, the antighost and the Lagrange multiplier for the zero modes of the topological symmetry \(\delta_3\) and \((u, h_\psi)\) a pair of fields which takes into account a further degeneracy associated with \(\bar{\psi}\). It is worth to stress here that all the fields appearing in the gauge fixing term \(13\) are in the adjoint representation of the gauge group: this will be important when the role of ghosts loops effects in not preventing the UV softening will be discussed. The following tables summarize the ghost numbers and dimensions of the fields:

| Fields \( A \) \( B \) \( \eta \) \( c \) \( \bar{\tau} \) \( \psi \) \( \bar{\psi} \) \( h_A \) \( h_B \) | dimension \( 1 \) \( 2 \) \( 1 \) \( 0 \) \( 2 \) \( 1 \) \( 2 \) | ghost number \( 0 \) \( 0 \) \( 1 \) \( -1 \) \( 1 \) \( -1 \) |
|-----|-----|-----|-----|-----|-----|-----|

The natural choice is to consider, as the Gaussian part of the fields \( A \) and \( B \), the off-diagonal kinetic term

$$S_0 = \frac{1}{\kappa} \sqrt{\varepsilon} B^{a\beta} \partial_\alpha \partial_\beta A_\delta a$$  \hspace{1cm} (14)

plus the quadratic terms for ghosts in the gauge fixing term \(13\). The \( A \to B \) propagator (which propagates \( A_\mu \) into \( B_{\nu\gamma} \)) has the following structure (a simple method to find them can be found in \(24\) \(25\)):

$$\Delta^{(a,b)}_{(A,B)\mu\nu\gamma} = -\delta^{ab} \frac{1}{2} \varepsilon^{\mu\nu\gamma\alpha} \frac{p^\alpha}{p^2}.$$  \hspace{1cm} (15)

The internal index structures of the propagators tells that, as one should expect, the internal index is conserved along the gravitational internal color lines. The \( A \to A \), \( B \to B \) and the \( \eta \to \eta \) propagators are

$$\Delta^{(a,b)}_{(A,A)\mu\nu} = \delta^{ab} \frac{1}{p^2} (\delta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}) , \quad \Delta^{(a,b)}_{(B,B)\mu\nu\gamma\rho} = -\delta^{ab} \varepsilon_{\mu\nu\alpha\lambda} \varepsilon_{\gamma\rho\beta\lambda} \frac{p^\alpha p^\beta}{p^2}$$  \hspace{1cm} (16)

$$\Delta^{(a,b)}_{(\eta,\eta)} = (\delta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}) \delta^{ab}.$$  \hspace{1cm} (17)

The ghosts propagators can be deduced from the gauge-fixing term in Eq. \(13\):

$$\Delta^{(a,b)}_{(\psi,\bar{\psi})\mu\nu} = -i \delta^{ab} \frac{1}{p^2} (\delta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}) , \quad \Delta^{(a,b)}_{(c,\bar{\tau})} = -i \delta^{ab} \frac{1}{p^2}$$

$$\Delta^{(a,b)}_{(\xi,\bar{\xi})} = -i \delta^{ab} \frac{1}{p^2} , \quad \Delta^{(a,b)}_{(\rho,\bar{\rho})} = -i \delta^{ab} \frac{1}{p^2} .$$

Graphically, all the propagators in the double line notation are represented by two parallel internal "gravitational" color lines (along which the internal \(\text{so}(N-1,1)\) index is conserved) with no arrows \(\text{[5]} \text{[6]}\).
The coupling with matter fields (as discussed in \cite{6}) shows that the number of internal "gravitational color" lines associated with each matter field is connected with its spin: the higher the spin, the more internal lines are needed (that is, the higher the representation of the internal gauge group associated with each matter field is connected with its spin: the higher the spin, the more internal lines are needed). For the sake of simplicity, in this paper the purely gravitational case will be considered, however the inclusion of material fields should not destroy the main conclusions since, usually, matter fields do not worsen the UV behavior. This point will be discussed in slightly more details later on.

The theory has the following matter vertices:

\begin{align}
V_1(A^a_{\mu}, A^b_{\nu}, B^c_{\alpha\beta}) &= Gf^{abc}\varepsilon_{\mu
u\alpha\beta}, \\
V_2(B^a_{\mu\nu}, B^b_{\alpha\beta}, \phi^{cd}) &= G\delta_{\alpha\beta}\delta_{bd}\varepsilon^{\mu
u\alpha\beta},
\end{align}

where $f^{abc}$ are the structure constants (the Newton constant $G$, as usual, has been absorbed in the fields in such a way that it appears only in the vertices which are depicted in Fig. 1). The first vertex is also present in the BF-Yang-Mills theory, while the second one only pertains to General Relativity and is likely to be the main responsible for the quantum realization of the holographic principle \cite{6}. The ghosts vertices in the gauge fixing term are:

\begin{align}
V_3(A^a_{\mu}, \psi^b_{\nu}, \gamma^d_{\sigma}) &= -Gf^{abd}\gamma^{\mu}_{\nu\sigma}, \\
V_4(\gamma^d_{\nu\sigma}, B^b_{\gamma\delta}, c^d) &= -Gf^{abd}\gamma^{\nu\delta}_{\sigma}, \\
V_5(A^a_{\mu}, \psi^b_{\nu}, \psi_{\sigma}) &= -f^{abd}\gamma_{\nu\sigma}, \\
V_6(A^a_{\mu}, \psi^b_{\nu}, \psi_{\sigma}) &= f^{abd}\gamma_{\nu\sigma},
\end{align}

It is worth to note that the internal index structures of the ghosts vertices are standard, that is, they have a connected structure (see, Fig. 2): this fact will play an important role in the following.

The aim of this paper is to show that, in the large $N$ expansion, there is only a finite number of superficially divergent diagrams. However, large $N$ resummations (even if they improve the UV behavior of the theory) could give rise to some problems in fulfilling the Zinn-Justin equation which is needed to ensure that the infinities can be absorbed in counterterms not violating the symmetries of the original action. This point will be discussed in slightly more details later on.

\subsection*{A. The "non renormalizable" vertex}

The perturbative non renormalizability of (super)gravity \cite{26} was an important result: even if at a first glance this could be rather obvious by power counting, there are many examples (such as gravity in three dimensions \cite{27}) of theories which are trivially renormalizable (being BRS-exact) and, in fact, would not appear in this way by naive power
FIG. 2: In this picture the ghosts vertices have been drawn: they have only two types of connected internal index structures.

counting arguments. Moreover, the results in [28] about the one-loop finiteness of the Einstein-Hilbert action lead to the expectations that the powerful symmetries of gravity could give rise to some "miracle" at least in supergravity: indeed, in the standard perturbative formulation, such a miracle does not occur. It is important to understand this result in the present formalism: the question is: which is the "wrong" vertex responsible for the perturbative non renormalizability of the theory? The answer is as follows: if one would drop the vertex in Eq. (19), one would obtain the topological BF Yang-Mills theory which, obviously, is renormalizable. Thus, the vertex responsible for the perturbative non renormalizability of the theory is the one in Eq. (19). In gravity, the problem is mainly connected to the 4-uple $B$ vertex. Indeed, $B$ has a bad asymptotic behavior (see Eq. (16)): such a propagator is also present in the BF Yang-Mills theory [25]. However, in the Yang-Mills case the $B$ field is always attached to the better-behaved $A$ field through the good $AAB$ vertex. Consequently, loops with only $B$ fields cannot arise. Such loops in which only the $B$ fields appear are at the origin of the perturbative non renormalizability of gravity. The 4-uple vertex for $B$ (see the left column of Fig. 5 in which there is a typical non-renormalizable diagram which cannot arise in the BF formulation of Yang-Mills theory) gives rise to bad UV-behaved loops in which only $B$-fields appear: at any order in perturbation theory, new diagrams diverging in the UV appear in the expansion.

III. LARGE N RESUMMATIONS AND EFFECTIVE PROPAGATORS

Here, it will be shown that the large $N$ expansion suggests a useful resummation of a certain class of planar diagrams which greatly improves the UV behavior of gravity. For the sake of clarity, it will be firstly presented a simplified version of the argument [17] (in which, however, the main physical ideas are manifest) leading to the UV improvement. In the next section, the effects of ghosts in the loops will be discussed.

It is worth to briefly recall what happens in the large $N$ expansion of the 5D $O(N)$ $\phi^4$ model [12]. The Lagrangian is

$$L_{\phi} = \frac{1}{2} \left( \partial_{\mu} \phi_j \partial^{\mu} \phi_j + m^2 \phi_j \phi_j \right) + \frac{\lambda}{8} \left( \phi_j \phi_j \right)^2, \quad j = 1, \ldots, N$$

(25)

$\phi_j$ being an $N$-component scalar field, $\lambda$ the coupling constant and $m$ the mass. It is convenient to introduce a Lagrange multiplier field $\sigma$ as follows

$$L_{\phi, \sigma} = \frac{1}{2} \left( \partial_{\mu} \phi_j \partial^{\mu} \phi_j + \left( m^2 + i\sigma \sqrt{\lambda} \right) \phi_j \phi_j \right) + \frac{1}{2} \sigma^2,$$

(26)

of course, due to the equation of motion of $\sigma$, this Lagrangian is equivalent to the previous one. In the momentum space, the bare propagator $D_{\sigma}(k)$ of $\sigma$ is constant:

$$D_{\sigma}(k) = 1.$$
The Lagrangian in Eq. (26) in five space-time dimensions gives rise to a non renormalizable theory. On the other hand, by the taking into account that at large $N$ the dominating diagrams are the well known bubble diagrams, it is possible to obtain an improved propagator $D_\sigma(k)$ at the leading order in $1/N$ (see Fig. 3):

$$D_\sigma(k) = \frac{1}{1 + g \Pi(k)}, \quad \Pi(k) \to k, \quad g = \lambda N.$$  

Such an effective propagator improves the UV behavior of the theory in such a way that there are only three superficially divergent diagrams and the theory becomes renormalizable in the $1/N$ expansion. This "miracle" happens in the following way: many diverging diagrams in the standard perturbative expansion (which are of different order in the standard coupling constant $\lambda$) are, in fact, of the same order in $1/N$: this fact tells that such diagrams should be summed together (such a resummation is easy being a geometric series). The difference with respect the standard perturbative renormalizability is that the propagator is not anymore analytic in the effective coupling constant $g$ in the region of small $g$ (see, for instance, [13, 14, 15]).

As far as gravity is concerned, one has to cure mainly the "non renormalizable" vertex in Eq. (19). In the scalar $O(N)$ case, the large $N$ expansion tells that a suitable class of diverging diagrams (the "bubble diagrams") should be summed together: of course, from the technical point of view, the gravitational case is more difficult. However, similarities in the two cases are indeed present. It is easy to see that the dominant contributions to the 4-uple $B$ vertex come from bubble-like diagrams which, form the internal index point of view, are "tree diagrams" (see Fig. 4): that is, they are planar diagrams with no closed color loops so that they contribute to the "large $N$-bare" vertex which is the relevant quantity as far as power counting arguments are concerned [48]. Thus, one is simply computing the tree-like term of the topological expansion at genus zero. These large-$N$-tree-diagrams can be formally summed as geometric series as it will be shown in a moment.

This is very much in the spirit of the large $N$ expansion [7, 9]: at any fixed genus, the contributions are weighted by the effective coupling constant

$$g_{eff} = GN$$

(kept fixed at large $N$) to the power of the number of closed color loops $L$ and, consequently, the tree diagrams should be summed together as it happens in standard Feynman diagrams calculations. To be more precise, at any given genus in the topological expansion, the large $N$ diagrams are weighted by the effective coupling constant in Eq. (27). To any closed color loop in the topological expansion it corresponds a factor $g_{eff}$ in the same way as the powers of the Planck constant weight the usual loops in the standard Feynman diagrams. Any interesting physical observable $\langle O \rangle$ can be expanded at large $N$ as follows:

$$\langle O \rangle = \sum_{g,b} N^{2-2g-b} \sum_L (g_{eff})^L O_{g,b,L}$$

FIG. 3: In this picture the large $N$ improvement of the scalar $\phi^4 - O(N)$ model is depicted: the constant propagator of the Lagrange multiplier is UV-softened since the leading large $N$ contribution is given by the geometric sum of bubble diagrams.
where \( g \) is the genus, \( b \) is the number closed matter loops, \( L \) is the number of closed color loops and \( O_{g,b,L} \) is the sum of fat diagrams contributing at genus \( g \), with \( b \) matter loops and \( L \) closed color loops. In the same way, in the standard Feynman expansion for physical quantities such as amplitudes one has

\[
\langle O \rangle_{(F)} = \sum_{L_F} (\hbar)^{L_F} O_{L_F}^{(F)}
\]

where \( \hbar \) is the Planck constant, \( L_F \) is the number of loops and \( O_{L_F}^{(F)} \) is the sum of the Feynman diagrams contributing at \( L_F \) loops: the bare propagators and vertices have, by definition, \( L_F = 0 \) (it is apparent the similarity between the topological expansion with \( g \) and \( b \) fixed and the standard Feynman expansion). Thus, in very much the same way, in the large \( N \) expansion the bare propagators and vertices must not contain any closed color loops [49]. The key point is that to count the number of primitive superficially divergent diagrams, one only needs tree-like propagators and vertices (see, for instance, [29]).

1. **UV-softening of the 4-uple B-vertex**

The "guilty of perturbative non-renormalizability" 4-uple B vertex is UV softened by large \( N \) effects: the factor which dresses \( V_2 \) is related to the geometric "bubble-like" series in Fig. 4. It is worth to stress here that the above "bubble-like" series is made of "tree-like large \( N \)" diagrams: that is, it is the sum of diagrams with no closed internal color lines \( (L = 0) \). It is a peculiar feature of gravity the possibility to construct a "tree-like large \( N \)" quantity (which, therefore, enters at the leading order in the large \( N \) expansion) which, in fact, contains an infinite number of standard Feynman loops. In the large \( N \) expansion of Yang-Mills theory the "tree-like large \( N \)" vertices and propagators coincide with the standard tree-like Feynman vertices and propagators since the vertices have a connected structures.

In the standard Feynman expansion the basic building blocks are tree-like vertices and propagators (that is, vertices and propagators without Feynman loops): starting from them, well known arguments (and, in particular, the BPHZ theorem) tell whether or not the perturbation expansion has a finite number of primitive superficially divergent diagrams. In very much the same way, the analogous question in the large \( N \) expansion have to be answered by looking at the "tree-like large \( N \)" vertices and propagators [50] (as is clear from well studied \( O(N) \)-vectorial examples: see, for instance, [12]). Thus, to understand whether or not gravity in the large \( N \) expansion has a finite number of primitive superficially divergent diagrams one has to use "tree-like large \( N \)" vertices and propagators: thus the 4-uple B vertex (which is a "tree-like quantity in the Feynman expansion) has to be replaced by the "bubble-like" series in Fig. 4 which correctly accounts for all the tree-like large \( N \) contributions.

Any potentially dangerous 4-uple B vertex is dressed by a factor vanishing in the UV as \( 1/q^2 \). In Fig. 5 is depicted a diagrams which diverges in the standard perturbative expansion and is finite in the large \( N \) improved expansion.
Divergent in the usual perturbation theory

Convergent at large $N$

This UV improvement appears to be quite consistent with the Weinberg asymptotic safety scenario \[30\]. The large $N$ dressed vertex is

$$V_{4B}(q) = V_2 \frac{1}{1 - \Pi(q^2)}, \quad (29)$$

where $\Pi(q^2)$ is the $B$ self-energy giving rise to the geometric series:

$$\Pi(q^2) = G^2 \int_A d^4p \Delta_{(B,B)}(p) \Delta_{(B,B)}(p - q) = \quad (30)$$

$$\rightarrow \left( I_1 G^2 \Lambda^2 \right) q^2 + \left( G^2 I_2 \Lambda^4 \right) + ... \quad (31)$$

where $\Delta_{(B,B)}$ is the $B$ propagator, in the momentum integrals it has been introduced a cut off $\Lambda$ (to be removed in a suitable way), terms which are subleading for $\Lambda \to \infty$ and $N \to \infty$ have been neglected and $I_1$ and $I_2$ are two non vanishing real constants (whose precise values are not important as far as the present discussion is concerned so that, from now on, they will be set equal to one). It is worth to note that, in the geometric series giving rise to $\Pi(q^2)$, no factor $N$ appears since there are no closed color loops ($\Pi(q^2)$ is a sum of the large $N$ tree diagrams). It is useful to rewrite Eq. \[29\]

$$V_{4B}(q) = V_2 \frac{M^2}{M_0^2 - q^2 + ...}, \quad (32)$$

$$M_0^2 = M^2 + \delta M_0^2 = M^2 - b\Lambda^2, \quad M^2 = \frac{1}{(G\Lambda)^2},$$

where in the denominator of Eq. \[32\] subleading terms when $\Lambda \to \infty$ have been neglected. The most convenient way to remove the cutoff is

$$\frac{\Lambda}{N} \xrightarrow{\Lambda,N \to \infty} \text{finite value} \Rightarrow M^2 = \frac{1}{(G\Lambda)^2} \to \text{fixed}, \quad (33)$$

where $M$ could be interpreted as a sort of renormalized Planck mass. The divergent term $\delta M_0^2 \sim -b\Lambda^2$ has the typical form which can be removed by a tadpole contribution.

The same phenomenon also occurs in simpler models in which the large $N$ technique works (see, for instance, \[31\]). In such models the quadratically divergent contribution to the mass is already included in the so called gap equation.
A subleading contribution to the 4-uple B-vertex

![Diagram](image)

FIG. 6: In this picture it has been drawn a typical ghosts contribution to the 4-uple B vertex. It is clear that, in the large $N$ expansion, the ghosts contributions are always subleading since, due to their connected structures, they contain at least one closed color loop (consequently, they do not contribute to the tree level large $N$ 4-uple B vertex).

The gap equation accounts for the tadpole embodying it in the effective coupling constant(s). However, to do this, it is necessary to write down and solve the saddle point equations at large $N$ for the full Lagrangian. In the gravitational case this seems to be rather difficult so that one is enforced to remove the divergent contribution to the mass "by hand" with counterterms. The typical tadpole diagram contributing to the quadratically divergent contribution to the mass is in Fig. 8: one can easily see that it is of order $\Lambda^2$ due to the UV behavior of the $B$ propagator.

Once the quadratically divergent term in the denominator is removed, one gets

$$V_{4B}(q) = V_2 \frac{M^2}{M^2 - q^2 + ...}$$

(34)

where in the denominator terms which are subleading in the large $N$ limit have been neglected.

It is now possible to compute the superficial degree of divergence of the large $N$ diagrams in which one simply has to use the 4-uple $B$ vertex in Eq. (34) instead of $V_2$. Since any 4-uple $B$ vertex now decreases by 2 the superficially degree of divergence $\Omega$ of a loop integral, $3$ vertex in Eq. (34) are enough to make a loop integral convergent: thus $\Omega \leq 4 - E$ (where $E$ is the number of external legs in the diagrams) as in the BF-Yang-Mills case (in Fig. 5 there is meaningful example). The large $N$ prescription gives rise to a gravitational theory with only a finite number of primitive superficially divergent diagrams (in good agreement with the Weinberg asymptotic safety scenario): indeed, this is a rather non-trivial step since it opens the unexpected possibility to control the UV behavior gravity exploring, for instance, the inflationary phase of cosmology.

However, in theories with local symmetries, one has also to show that the infinities of the theory have the suitable symmetries which allow to cancel such infinities by adding counter-terms not violating the original symmetries of the action (see, for instance, 27). It is not clear at this stage of the analysis if such "large $N$ reorganization" prevents the fulfillment of the Zinn-Justin equation: for instance, the large $N$ expansion could not commute with the BRS and Slavnov operators. This is a rather involved technical question: I hope to return on this issue in a future publication.

It would be very interesting to compare more closely the above results with the ones obtained in a series of papers in which the authors began the analysis of the Einstein-Hilbert in the standard metric variables using the method of the non perturbative renormalization group. The authors found sound evidences supporting the existence of a non trivial UV fixed point (which, obviously, would confirm the Weinberg asymptotic safety scenario). The present results, although derived in a completely different way, seem to be consistent with such a scheme.

The above UV-improvement is conceptually very similar to what happens in the change from the Fermi effective model of weak interaction to the Glashow-Weinberg-Salam model of electro-weak interactions: in this case the non renormalizable Fermi coupling constant $G_F$ is replaced by the $W^\pm$ propagators (see, for a detailed pedagogical exposition, 24)

$$G_F \rightarrow \frac{g_W^2}{M_W^2 - q^2},$$

where $g_W$ is the (dimensionless) electro-weak coupling constant and $M_W$ is the mass of the $W$-bosons. In the gravitational case, the large $N$ expansion suggests that, in the strongly coupled phase of gravity, a similar phenomenon
Examples of planar boxes with loops

"Loopless" Planar Boxes

PB(1)  PB(2)

Examples of planar boxes with loops

PB(3)  C  C

PB(4)  B  C  C

FIG. 7: In this picture the double line structures of the possible contributions to the 4-uple B vertex have been drawn. Only two structures could contribute at the tree level large $N$ 4-uple B vertex: PB(1) (which, however, is not present in the theory) and PB(2). All others planar boxes with four external $B$ lines have at least one closed color loop inside: therefore, they are not relevant as far as UV-power counting is concerned.

should occur:

$$G \rightarrow G \frac{M^2}{M^2 - \Pi(q^2)}.$$

Thus, the sentence "gravity is an effective field theory" would acquire a rather precise meaning. Such a non renormalizability should be removed by non perturbative effects, displayed by the large $N$ expansion, which correctly take into account the heavy degrees of freedom in a way very similar to what happens in the electro-weak model.

IV. POSSIBLE COMPLICATIONS

The main effects which have been neglected in the previous discussion are related to the ghosts. In particular, ghosts loops could cancel out, asymptotically, the $B$ loops vanishing the previous UV softening. In fact, the large $N$ expansion itself provides one with a natural recipe to deal with this problem.

In the gravitational case, due to the "higher spin" Lagrange multiplier $\phi_{ab}$, it is possible to construct planar diagrams without closed color loops (thus contributing to the large $N$ bare propagators) containing an infinite number of loops of the standard Feynman expansion\[54\]. The question is: do the ghosts contributions to the 4-uple $B$-vertex survive at the tree-level in the large $N$ expansion in such a way to prevent the above analyzed UV softening? In other words, is it possible to construct a contribution to the 4-uple $B$-vertex by using the ghosts vertices in Eqs. (20), (21), (22), (23) and (24) without closed internal lines? The key point is that the only way in which ghosts effects could vanish the previously considered UV softening is by changing the tree-like large $N$ vertices and, in particular, the 4-uple $B$ vertex (since the large $N$ power counting only depends on tree-like large $N$ vertices and propagators): ghosts effects have to come into play at tree-level otherwise the power-counting does not change. It is easy to see that all the possible ghosts-mediated corrections to the 4-uple $B$ vertex contain at least one closed color loop (since the vertices in Eqs. (20), (21), (22), (23) and (24) have connected structures) and, therefore, do not contribute to the bare large $N$ propagators and vertices. Let us consider the ghost contribution to the $B$ vertex in Fig. 6; indeed, at genus zero, it contains at least one closed color loops. More in general, in the presence of fields living in the adjoint representation of the gauge group and vertices with the ghosts vertices of the BF formulation of gravity it is not possible to construct "tree-like large $N$" diagrams without closed color loops contributing to the 4-uple $B$ vertex (see Fig. 7). Therefore, ghosts effects do not affect the UV softening of the previously considered "bubble-like" series.
FIG. 8: In this picture the typical tadpole diagram has been drawn. In simpler models (such as the scalar $\phi^4$ -- $O(N)$ model) one can take care of the tadpoles by using the so called gap equation. In gravity, it seems difficult to write and solve the gap equation so that one has to remove by hand the typical tadpole quadratic divergences.

A. The role of $\eta$

Up to now, the role of $\eta$ has not been discussed yet. Its introduction (as in the Yang-Mills case) is dictated by technical reasons. To deduce the large $N$ effects on the interactions of $\eta$ and $B$ one can consider the large $N$ improvements for the 4-uple vertex without separating $B'$ in $B$ and $\nabla \eta$. Obviously, since $B$ and $\nabla \eta$ can be "assembled" back in a unique field, the vertices between $\nabla \eta$ and $B$ experience at large $N$ similar UV improvements as in Eq. (34). In other words, $\eta$ represents the longitudinal components of $B'$ so that the separation of $B'$ in $B$ and $\nabla \eta$ is an artifact, the physical field is $B + \nabla \eta$. This implies that one can trivially deduce the large $N$ effects on the interactions between $\nabla \eta$ and $B$ by analyzing the large $N$ improvements of the effective 4-uple vertex for the physical field $B + \nabla \eta$ and then separating it into $B$ and $\nabla \eta$. On the other hand, since $\eta$ can be gauged to zero (being one of its transformation laws given by a shift) its properties do not affect the UV behavior.

As it has been already mentioned, at this stage of the analysis, it is not clear if and how one could avoid the introduction of the auxiliary $\eta$ field from the very beginning. Indeed, the introduction of $\eta$ and of the "Yang-Mills" term is not a mere avoidable technical complication: it is in fact the more transparent way to identify the "guilty" vertex and to cure it by the means of the large $N$ expansion.

V. A POSSIBLE PHYSICAL INTERPRETATION

It will be now discussed a possible interpretation of the previous results in terms of a sort of gravitational confinement (in which the gravitational color, that is the spin, is confined) which should be dual, in the sense of the Gauge/Gravity correspondence (see, for two detailed reviews, [37, 38]) to the standard gauge theoretical confinement.

In many other cases, such as the $O(N)$ scalar (in five space-time dimensions) and Fermionic (in three space-time dimensions) models with quartic interactions, the seemingly "magic" properties of the large $N$ resummations are, in fact, related in a very simple way to physical properties of the models. In the above mentioned cases, the large $N$ expansion is able to explore the strongly coupled phase of the theory in which phenomena such as the appearance of "color-less" bound states in the spectrum, spontaneous breaking of symmetries and so on occur. Thus, a natural question is: which is the phenomenon behind the improvement of the UV behavior of gravity at large $N$? In the present formulation of the gravitational action matter fields should be described as scalar fields living in suitable representation of the internal gauge group according to their spin [6]. At large $N$, in the strongly coupled phase of gravity (as it is also suggested by the Gauge/Gravity correspondence), the physical spectrum should be dominated by colorless bound states. In this context colorless particles simply means scalar particles which, therefore, should dominate the spectrum in the UV region [57]. Gravitational confinement would dramatically improve the renormalizability of the theory. The reason is that, if the gravitational interaction confines at the Planck scale, one is left with only scalar fields at high energy: this would soften the UV behavior of the theory. This can be seen forgetting for a moment the present formalism (in which the spin of particles is represented by an internal index in a suitable representation of the internal gauge group) and returning to the standard "space-time" interpretation of spinful particles. The contribution to the scattering amplitude $A_J(s,t)$ in the $t$-channel (where $t$ and $s$ are Mandelstam variables) at high energies of a particles with spin equal to $J$ is roughly

$$A_J(s,t) \approx -e^2 \frac{s^J}{t - m_J^2}.$$

$m_J$ being the mass of the particle and $e$ being a suitable coupling constant. When $J < 1$, loops integrals in which such a spin $J$ particle appears are convergent; when $J = 1$ there are logarithmic divergences (which, in case, can be
KLT relations\cite{42}. The closed string amplitudes in terms of open string amplitudes. Schematically, the factorization of the vertex operators \( \phi \) expansion such as the Gross-Neveu model in 3 dimensions or the Lagrangian\cite{59}. the theory is wrong or the UV degrees of freedom are non trivial non local combinations of the ones appearing in the theory of this paper). As far as the present scheme is concerned, it is enough to say that an UV renormalizable theory is manifest in the Wilson point of view (to discuss the Wilson view in a gravitational context is far beyond the scope of this paper). As far as the present scheme is concerned, it is enough to say that an UV renormalizable theory is a theory which allows meaningful and predictive computations (in the UV) because the fields appearing in the Lagrangian are suitable to describe the UV phase\cite{53}. When this does not happen, there are two possibilities: either the theory is wrong or the UV degrees of freedom are non trivial non local combinations of the ones appearing in the Lagrangian\cite{53}.

Thus, a pragmatic answer to the question on the physical meaning of renormalizability at large \( N \) could be the following: a theory which is UV renormalizable at large \( N \) (and is not UV renormalizable in the standard perturbative expansion such as the Gross-Neveu model in 3 dimensions or the \( \phi^4 \) model in 5 dimensions\cite{12}) is a theory which is formulated in terms of fields not suitable to describe the UV phase. However, it is not wrong: the large \( N \) recipe tells how to sum class of diagrams of the standard expansion in order to obtain a meaningful expansion (with a finite number of primitive superficially divergent diagrams). In the already mentioned examples, this has a very precise meaning: in the UV phase new degrees of freedom appear which are bound states of the original fields (see, for instance, \cite{31}). This is exactly the picture which seems to emerge from the present scheme: scalar bound states which soften the UV behavior of amplitudes of particles carrying spin 2 or greater. However, as it will be discussed in a moment, in gravity there is the further complication of local symmetries: one should also prove that local symmetries are preserved by the large \( N \) resummations.

### A. Connections with the KLT relations

It is worth to mention here an interesting relation (which is worth to be further investigated) with the so called KLT relations\cite{12}. The KLT relations were first obtained in a string theoretical framework: they allow to express closed string amplitudes in terms open string amplitudes. Schematically, the factorization of the vertex operators

\[
V_{\text{closed}} = V_{\text{left}}^{\text{open}} \times V_{\text{right}}^{\text{open}}
\]

(where the \( V \)'s are vertex operators of the closed string, open string left modes and right modes) is related to the results in \cite{43} stating that correlations of vertex operators factorize at the level of integrands (that is, before the world sheet integrations are performed). Kawai, Lewellen and Tye were able to prove a stronger result: the complete closed string amplitudes factorize into products of open string amplitudes even after the world sheet integrations. Indeed, the KLT relations go well beyond string theory itself: they imply, for instance, in the low energy limit highly non trivial relations among tree amplitudes of gravity in four dimensions which factorize (in suitable non linear gauges) into gauge theoretical tree amplitudes in four dimensions. These results have been generalized to include loops by using unitarity relations (for a pedagogical review see \cite{44}), however it is not available yet a complete proof of these very useful factorizations results. Usually, one deals with the KLT relations in the standard metric formalism in which, having in mind small deviations from a flat metric

\[
g_{\mu\nu} \sim \eta_{\mu\nu} + h_{\mu\nu},
\]

the basic variable is the metric fluctuation \( h_{\mu\nu} \). In the standard metric formulation of gravity the structure of the many vertices present in the theory is very complicated. To fully exploit the KLT relations one needs to choose a
FIG. 9: In this picture a typical gravitational large $N$ diagram which could be responsible for the KLT relations has been drawn. It is manifest the important role which could have the 4-uple $B$ vertex (which is disconnected from the "internal lines" point of view) in explaining such important relations between gravity and gauge theory (which, in the BF formalism, are distinguished precisely by the above mentioned 4-uple $B$ vertex).

gauge in which, roughly speaking, the right index and the left index of $h_{\mu\nu}$ are never contracted with each others (otherwise no "left-right" factorization

$$h_{\mu\nu} \sim \epsilon_{\mu} \otimes \epsilon_{\nu}^*$$

would be apparent\cite{1}. Such (non linear) gauge choices are highly non trivial and the many vertices of the theory in the metric formalism sometimes obscure the physical origin behind the KLT relations. In the present framework such features are rather manifest: in particular, the presence of a Lagrange multiplier field $\phi_{ab}$ which, in the double line notation, is represented by four internal lines leads directly to amplitudes fulfilling the generalized KLT relations. The reason is that in the BF formulation of gravity the propagators and vertices can be chosen to be equal to the propagators and vertices appearing in the BF formulation of Yang-Mills theory: the only, crucial, difference is the higher spin Lagrange multiplier field. Such a field gives rise, quite generically, to gravitational amplitudes which manifestly are factorized into pieces in which only "YM fields" (that is, fields which are also present in the BF Yang-Mills Lagrangian) appear (see, for instance, Fig.9). In other words, $\phi_{ab}$ "allows" to attach amplitudes of the BF Yang-Mills theory to obtain amplitudes of the BF formulation of gravity: for this reason, the present scheme seems to be suitable to fully exploit and, hopefully, to establish in general the KLT relations.

VI. CONCLUSIONS AND PERSPECTIVES

In this paper an analysis of renormalizability of gravity at large $N$ expansion for General Relativity has been carried on. It is based on the BF formulation of General Relativity in which the Einstein-Hilbert action is splitted into a topological term plus a constraint. It has been shown that the large $N$ expansion dictates resummations of a suitable class of planar diagrams which lead to a great improvement of the UV behavior of gravity: only a finite number of superficially divergent diagrams are present at large $N$. This is an important step in proving renormalizability. The next steps are the analysis of the infinities and their fulfilment of the symmetry constraints such as the Slavnov-Taylor identities and the Zinn-Justin equation. The analysis of the KLT relations in this scheme is also worth to be further investigated.

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If one would not remove such a term (which appears in the denominator of the improved vertex), the improved 4-uple \( B \) vertex would actually vanish when removing the cut-off. Therefore, one would obtain that the gravitational action would be equal to the large \( N \) Yang-Mills action in the limit of small coupling constant. Even if this would produce a renormalizable theory in the large \( N \) expansion, it seems more natural to remove the quadratically divergent term in the denominator as it is usually done in similar situations [31].

As it has been already remarked, this feature tells far apart gravity from gauge theories: in gravity the bare large \( N \) propagators and vertices do not coincide with the standard bare propagators and vertices in the Feynman expansion.

Which is given by the already discussed bubble-like geometric series "mediated" by the "higher spin" Lagrange multiplier \( \phi_{ab} \) which allows "bubble-like" series with the same internal-line structure in Fig 4. Thus, the 4-uple vertex of physical field \( B + \nabla \eta \) is dressed by the same factor which has been discussed in the previous section.

The introduction of the Yang-Mills term has not played any explicit role in the whole discussion. Its only role is, by the way, to arrive at a theory which has the same propagators of the BF Yang-Mills theory.

Gravitational confinement would be of great cosmological importance since it would provide the standard inflationary scenario with sound basis: such a mechanism could lead to the expected decrease of the number of degrees of freedom expected in a holographic theory.

That is, the fields in the Lagrangian represents small fluctuations around the UV vacuum so that the perturbation expansion works.

For instance, in QCD quarks and gluons are good UV degrees of freedom but they are not good IR degrees of freedom. This, of course, does not imply that QCD is wrong: it is an indication that the IR degrees of freedom are non trivial combinations of the UV degrees of freedom.

Large \( N \) renormalizability (in the cases in which there are not local symmetries) can be "detected" as standard renormalizability using power counting arguments by looking at the UV behavior of large \( N \) propagators and vertices (that is, vertices and propagators which have been corrected in order to encode the leading corrections in the expansion).

\( \epsilon^{\pm} \) have to be thought as the "gluons" of the gauge theory responsible of the factorization of the gravitational amplitudes (the plus and minus signs refer to the elicitices).