Finite element analysis of the reinforced concrete special-shaped columns under axial force and torsion

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Abstract. The finite element analysis software ANSYS is used to analyze 9 reinforced concrete special-shaped columns under the combined action of axial force and torque. The parameters of the numerical model are given through repeated trial calculation. The influence of different values of axial compression ratio on the torsional capacity of test specimens is studied. The results show that the prediction error of the finite element model is small, which means the model is capable of accurate simulation. But the whole process analysis of the torque-torsion angle curve needs to be improved. The cracking and ultimate torque of special-shaped columns with different sizes and section shapes increase linearly with the growth of axial compression ratio. However when the ratio exceeds 0.6, the torsional bearing capacity of the columns will decrease. The increasing of axial compression ratio will also shorten the process from cracking to ultimate torque.

1. Introduction
As one of the basic stress forms of reinforced concrete members with special shape, the research of pure torsion performance has made some progress [1-3]. Due to the connection effect, the pure torsional bearing capacity of the special-shaped section is larger than the sum of the bearing capacity calculated by dividing them into several rectangles. Therefore, the calculation method of dividing the special-shaped section into several rectangles is safe and feasible[4].

Under the earthquake action, the special-shaped columns in the frame structure will be in the composite stress state of compression, bending, shear and torsion due to the asymmetry of the structure, accidental eccentric load and other factors. The current national standard GB50010-2010 "Code for Design of Concrete Structures" gives the design formula of torsional capacity of rectangular columns under compression and torsion, but the corresponding design formula applicable to special-shaped columns is not given in the code [5]. The profession standard JGJ 149-2017 "Technical Specification for Concrete Special-shaped Column Structure" does not provide any relevant design provisions on the torsional bearing capacity of special-shaped columns [6]. Compared with rectangular columns, due to the existence of multiple limbs, the shear center and section centroid of special-shaped columns are often not coincident, and its torsional effect is more complicated than that of rectangular columns. When bearing the load, it depends on the coordinated deformation of the core concrete at the intersection of limbs to bear the load, so that there are considerable warping normal stress and shear stress in each column limb. This will make the ductility of the columns worse, and a sudden fracture is more likely to happen [7]. Xinzheng Lu [8] used a finite element software to carry
on the comparative analysis under the earthquake action to the special-shaped column frame structure and the rectangular column frame structure, indicated that the torsional bearing capacity of the special-shaped column frame structure is far lower than that of the rectangular ones with the same moment of inertia.

In a word, with the increasing application of special-shaped columns in engineering structures, it is urgent and important to study on the torsional behavior of special-shaped columns under the action of compression and torsion. For this reason, research work was carried out on the torsional performance test of nine reinforced concrete special-shaped columns under the combined action of pressure and torque. According to the test results, the influence rule of axial compression ratio, section shape and reinforcement configuration on the torsional performance of columns has been analyzed [9]. Based on that, this paper uses the finite element numerical analysis model to further analyze the influence of axial compression ratio on the cracking and ultimate torque with different section shapes and sizes.

2. Finite element model and its experimental verification

2.1. Type of units
Solid65 unit in ANSYS unit library is used for concrete and Link8 unit is used for reinforcement. The element of Mass21 is used to establish the main node at the center of the component's enlarged top surface, and it is connected with other nodes on the top surface through MPC184 element. Each node of MPC184 element has 6 degrees of freedom, and the key word of the element is 1. At this time, the element is equivalent to a rigid beam, which can be used to transfer forces or moments, suitable for the case of applying twist angle.

2.2. Material properties
The standard value of concrete axial compressive strength and axial tensile strength should be calculated according to the formula in GB50010-2010 [5], but the correction coefficient of concrete strength 0.88 was not considered. The reason was that the coefficient is to consider the difference between the concrete strength in the structure and in the test. In actual project, there may be some fluctuations in the strength, while in the test, the specimen were relatively accurate, which could guarantee the concrete strength. The calculation formula is written as follows:

\[ f_{ck} = \alpha_{c1} \alpha_{c2} f_{cu,k} \]  

(1)

\[ f_{tk} = 0.395 f_{cu,k}^{0.55} (1 - 1.645 \delta)^{0.45} \times \alpha_{c2} \]  

(2)

In formula 1 and 2, \( f_{ck} \), \( f_{tk} \), \( f_{cu,k} \) are the standard values of axial compressive strength, axial tensile strength and cube compressive strength of concrete, respectively. \( \alpha_{c1} \), \( \alpha_{c2} \) and \( \delta \) are the ratio of prism strength to cube strength, brittleness reduction coefficient and variation coefficient, respectively, according to literature [5].

The Poisson's ratio of concrete is 0.2, the cracking option is turned on, the crushing option is turned off, the strengthening criterion adopts the multi-linear isotropic reinforcement model (MISO), and the failure criterion adopts the Willam-Warnke yield criterion. By means of repeated calculation, compared with the test results, it was found that when the shear transfer coefficient of the concrete open crack is 0.4 and that of the closed crack is 0.95, the numerical analysis results of the cracking and ultimate torque are in good agreement with test results. The constitutive relation of concrete adopts the Hongnestad expression [10], as follows:

\[ \sigma_c = \sigma_0 [2 (\frac{\varepsilon_c}{\varepsilon_0}) - (\frac{\varepsilon_c}{\varepsilon_0})^2] \quad \varepsilon_c \leq \varepsilon_0 \]  

(3)
In formula 3 and 4, \( \sigma_c \) is the stress of concrete, \( \varepsilon_c \) is the strain of concrete. \( \sigma_0 \) is the standard value of axial compressive strength. \( \varepsilon_0 \) is the peak strain of concrete. \( \varepsilon_u \) is the ultimate strain of concrete. In the process of numerical analysis, 0.0025 is taken for \( \varepsilon_0 \) and 0.0038 for \( \varepsilon_u \) in this paper [10].

Poisson’s ratio of reinforcement is taken as 0.3, the strengthening criteria is simulated by multi-linear kinematic strengthening model (MKIN), and the constitutive relation is taken as tri-linear model. The constitutive relation expression is written as follows:

\[
\sigma_s = E_s \varepsilon_s \quad \varepsilon_s \leq \varepsilon_y \tag{5}
\]

\[
\sigma_s = f_y \quad \varepsilon_y < \varepsilon_s \leq \varepsilon_{s,h} \tag{6}
\]

\[
\sigma_s = \left(\frac{f_{s,u} - f_y}{\varepsilon_{s,u} - \varepsilon_{s,h}}\right)(\varepsilon_s - \varepsilon_{s,h}) + f_y \quad \varepsilon_{s,h} < \varepsilon_s \leq \varepsilon_{s,u} \tag{7}
\]

In formula 5~7, \( \sigma_s \) is the stress of reinforcement. \( E_s \) is the modulus of elasticity of reinforcement. \( \varepsilon_s \) is the strain of reinforcement. \( \varepsilon_y \) is the maximum strain corresponding to the elastic stage. \( \varepsilon_{s,h} \) is the maximum strain corresponding to the yield platform. \( \varepsilon_{s,u} \) is the ultimate tensile strain of the reinforcement. \( f_y \) is the yield strength of the reinforcement. \( f_{s,u} \) is the ultimate strength of the reinforcement. \( \varepsilon_{s,h} \) is taken as 5\( \varepsilon_y \), \( \varepsilon_{s,u} \) is taken as 0.02.

2.3. Entity modeling and unit division

The finite element model of the test specimen is shown in Figure 1~3. Firstly, the finite element model of the reinforcement is established to form the reinforcement cage. The longitudinal reinforcement extends to the top of the enlarged head, and then the geometric model of the concrete is established. The Boolean operation is used to bond several entities into a whole, and the geometric entities are divided by the position of the reinforcement to ensure that the concrete and the reinforcement share the same node in the division process. The NUMMRG and NUMCMP commands are used to merge the nodes with the same coordinate, regardless of the bond and slip between the reinforcement and the concrete. At the centroid of the top surface of the enlarged head, the main node is established by Mass21 element, and other nodes on top are connected with the main node by Mpc184 element to form a rigid surface, and the torque angle is applied on the main node. Size of the unit element grid is about 50 mm.

![Figure 1. L-shaped column.](image1)

![Figure 2. T-shaped column.](image2)

![Figure 3. Cross-shaped column.](image3)

2.4. Boundary conditions and loading methods

Three translational and rotational degrees of freedom of all nodes at the bottom of the column are all constrained as fixed ends. The load is applied in two steps.

In the first load step, the axial force was applied and the setting value was maintained. The method of loading is to apply the additional face load on the expanded top, which can ensure that the axial
force applied is always perpendicular to the loading surface, and prevent the premature cracking caused by the stress abnormality.

In the second load step, the torsion angle was applied to the main node to form the torque. In order to accelerate convergence, linear search and predictor are opened. L2 norm is used to control the convergence in the process of axial force loading. Infinite norm is used to control the convergence in the process of torsional angle loading, and the maximum number of equilibrium iterations is 30.

2.5. Calculation results and analysis

The cracking and ultimate torque calculated by ANSYS are compared with the test results, as shown in Table 1. n is the axial compression ratio, \( n=N/(f_{ck}A), \) A is the cross-sectional area of the column. \( T_{cr,t} \) and \( T_{cr,s} \) are the test and simulation values of cracking torque respectively. \( T_{u,t} \) and \( T_{u,s} \) are the test and simulation values of ultimate torque respectively. \( \eta_{cr} \) and \( \eta_{u} \) are the error rates of the cracking and limited torque respectively.

| Specimen number | Section shape | \( T_{cr,t} \) /kN · m | \( T_{cr,s} \) /kN · m | \( \eta_{cr} \) /% | \( T_{u,t} \) /kN · m | \( T_{u,s} \) /kN · m | \( \eta_{u} \) /% |
|-----------------|---------------|-------------------------|-------------------------|-----------------|-------------------------|-------------------------|-----------------|
| C1              | L-shaped      | 6.58                    | 5.96                    | 9.42            | 10.58                   | 10.32                   | 2.46            |
| C2              | L-shaped      | 8.14                    | 8.23                    | 1.11            | 12.08                   | 12.79                   | 5.88            |
| C3              | L-shaped      | 11.45                   | 10.25                   | 10.48           | 14.67                   | 15.26                   | 4.02            |
| C4              | L-shaped      | 5.78                    | 6.13                    | 6.06            | 5.98                    | 5.44                    | 9.03            |
| C5              | T-shaped      | 6.49                    | 6.69                    | 3.08            | 12.53                   | 11.83                   | 5.59            |
| C6              | T-shaped      | 11.10                   | 11.79                   | 6.22            | 14.54                   | 15.04                   | 3.44            |
| C7              | T-shaped      | 5.46                    | 6.01                    | 10.07           | 6.08                    | 6.65                    | 9.38            |
| C8              | Cross-shaped  | 6.36                    | 6.89                    | 8.33            | 9.39                    | 8.98                    | 4.37            |
| C9              | Cross-shaped  | 5.04                    | 4.76                    | 5.56            | 6.08                    | 6.55                    | 7.73            |

The comparison between the torque and torsional angle (T-\( \phi \)) curve calculated by ANSYS and the test results is shown in Figure 4–6. The specimen numbers in Table 1 and Figure 4–6 correspond to those in reference [9].

Figure 4. L-shaped column.

Figure 5. T-shaped column.
Figure 6. Cross-shaped column.

It can be seen from Table 1 that the average error between simulation and test value of ultimate torque is within 10.5%. By comparing the simulation and test value of cracking torque, the error is less than 10%, except that the error of individual specimens is slightly more than 10%. Compared with the error of ultimate torque, the error of cracking torque is relatively large, which may be due to the fact that in order to ensure the safety of the test personnel during the test, the cracks are not observed manually until the completion of a certain level of load, and the cracks may appear in the loading process of the load level.

It can be seen from Figure 4~6 that the torque and torsional angle curve of numerical simulation is consistent with the rising section of the test curve. However, when the test specimen reaches the ultimate torque, the torsional angles of numerical simulation are smaller than those of test. The reason may be that the test process uses reinforced concrete beam to apply torque, and the beam is installed on the enlarged head of the test specimens. With the increasing of the torque, the gap between the beam and the enlarged head is tighter, resulting in the void torque angle. In addition, the deformation of the beam itself is inevitable during the loading process, so the torsional angle measured when the test reaches the ultimate load will be larger than the simulation ones.

3. Analysis of influencing factors

In order to study the influence trend of axial compression ratio on the cracking and ultimate torque of equal length special-shaped columns with different section shapes and thickness, relevant curves are shown in Figure 7~9. In Figure 7~9, the size of the section is represented by h×b, h and b are the length and the thickness of the limb respectively, and the height of the column is 2000mm. Three shapes of special-shaped columns with different cross-section sizes were selected, which were 500mm×200mm, 500mm×250mm, 600mm×200mm. The concrete strength class is C30. The thickness of concrete cover is 30mm. The tensile strength of longitudinal reinforcement is HRB400, with a diameter of 18mm and a total of 12. HPB300 was taken as the hoop reinforcement, with a diameter of 8mm and a spacing of 150mm. The yield strength of longitudinal and the hoop reinforcement is 400Mpa and 300Mpa respectively, and the ultimate strength is 540Mpa and 420Mpa respectively.

The variation trend of cracking torque (T_c) and ultimate torque (T_u) of L-shaped, T-shaped and Cross-shaped columns with different section sizes and axial compression ratios is shown in the Figure 7~9. Following results can be get from the figures:
1. Basically, the cracking and ultimate torque with different cross-section sizes increases with the growth of axial compression ratio, but when the ratio is greater than 0.6, the cracking and ultimate torque begin to decrease. This shows that the increase of the axial compression ratio is limited, and when it exceeds a certain limit, on the contrary, it will reduce the bearing capacity of the columns. This is because when the axial compression ratio exceeds 0.6, part of the concrete column may have been crushed and lose the ability to continue to bear the torque.

2. With the increasing of axial compression ratio, the process from cracking torque to ultimate torque is shortened. This may be due to the higher cracking torque of columns which under high axial compression ratio. When the column cracks, the compression zone is already in a high stress state, which makes the column easier to reach the ultimate state of concrete compression.
4. Conclusion
Based on the finite element software ANSYS, a numerical analysis model of the torsional behavior of special-shaped columns combined axial force and torsion is established in this paper, and 9 test specimens are analyzed by using this model, which verifies the established finite element model is feasible through error analysis. At the same time, the effects of axial compression ratio on the cracking and ultimate torque with different section shapes and sizes is analyzed by using the established model. Following main conclusions can be get from the research:

1. ANSYS can simulate the cracking torque, ultimate torque and rising stage of torque and torsion angle curve of concrete special-shaped columns combined axial force and torsion, but the simulation of the whole process curve, especially the decline phase, needs to be further improved.

2. The favorable influence of axial compression ratio on the torsional capacity is limited. With the growth of the axial compression ratio, the cracking and ultimate torque of the columns with different cross-section shapes and sizes increase approximately linearly. However, the torsional capacity will decrease when the ratio exceeds 0.6. So it is not feasible to increase the axial compression ratio endlessly.

3. With the increasing of axial compression ratio, the process from cracking to ultimate torque is shortened, which means that the column will damage rapidly under a high level of axial force.

References
[1] Xiaoming L, Hai C, Yaoqing G 2019 Torsional center of beams of arbitrary complex cross-section Mechanics in Engineering 04 453-457
[2] Bo D, Qian W, Yang S 2007 Experiment on additional torsion of L-shaped column frame and bearing capacity analysis of shear and torsion Journal of Architecture and Civil Engineering 04 21-28
[3] Qifeng L 2012 The research of the L-shaped column on the performance of special-shaped column frame Southwest Jiaotong University
[4] Xiaming W, Pusan H Chunyang L 1996 Discussion on calculation of reinforced concrete L-beams and T-beams in torsion Journal of Guangxi University Natural Science Edition 21 70-73
[5] Ministry of housing and urban rural development of the people's Republic of China 2011 Code for Design of Concrete Structures
[6] Ministry of housing and urban rural development of the people's Republic of China 2017 Technical Specification for Concrete Special-shaped Column Structure
[7] Tianjia Zhao 2018 Study on seismic behavior of reinforced concrete frame with special-shaped columns Northeast Petroleum University
[8] Xinzheng L, Jianjing J 2002 Studies on improving the torsion resistance ability of special shape column frames using inclined supports Industrial Construction 32 39-41
[9] Zhenghe Q 2011 Study on the torsional behavior of reinforced concrete special-shaped columns under the combined action of pressure and torque Huaqiao University
[10] Jianjing J, Xinzheng L, Lieping Y 2005 Finite element analysis of concrete structure Tsinghua University Press