Information Stickiness in General Equilibrium and Endogenous Cycles

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'Chaos represents a radical change of perspective on business cycles. Business cycles receive an endogenous explanation and are traced back to the strong nonlinear deterministic structure that can pervade the economic system. This is different from the (currently dominant) exogenous approach to economic fluctuations, based on the assumption that economic equilibria are determinate and intrinsically stable, so that in the absence of continuing exogenous shocks the economy tends towards a steady state, but because of stochastic shocks a stationary pattern of fluctuations is observed.'

Barnett, W.A.; A. Medio and A. Serletis (1997). *Nonlinear and Complex Dynamics in Economics*, EconWPA working paper number 9709001 (pages 36-37).
Nonlinear dynamics

- Linear models: *stability* (convergence towards a fixed-point) or *instability* (divergence away from the fixed-point) - all sources of fluctuations, in the long-run, are exogenous.

- Nonlinear models: other long-term outcomes are possible - cycles of any periodicity or complete a-periodicity / chaos (*bounded instability*).
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- Macroeconomics: many attempts to justify endogenous fluctuations - see Gomes (2006) for a survey.
## Endogenous fluctuations in the macro literature (some recent references: 2007-...)

| Author (year)      | Journal | Title                                             | Type of model            | Source of fluctuations                                                                 |
|--------------------|---------|---------------------------------------------------|--------------------------|----------------------------------------------------------------------------------------|
| Fanti and Manfredi (2007) | JEBO    | Neoclassical labour market dynamics, chaos and the real wage Phillips curve | Neoclassical labor market model | Consumption and leisure are modeled as weak substitutes                                  |
| Jaimovich (2007)   | JET     | Firm dynamic and markup variations: equilibria and endogenous economic fluctuations | Dynamic general equilibrium model | Interaction between firms’ entry-and-exit decisions and variations in competition (net business formation is endogenously pro-cyclical) |
| Yoshida and Asada (2007) | JEBO    | Dynamic analysis of policy lag in a Keynes-Goodwin model: stability, instability, cycles and chaos | Keynes-Goodwin model of the growth cycle | Lags in the implementation of stabilization policies                                      |
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| Author (year)       | Journal | Title                                                        | Type of model                        | Source of fluctuations           |
|---------------------|---------|--------------------------------------------------------------|--------------------------------------|----------------------------------|
| Chen, Li and Lin    | JEBO    | Chaotic dynamics in an overlapping generations model with myopic and adaptive expectations | Overlapping generations model with capital accumulation | Myopic and adaptive expectations |
| (2008)              |         |                                                              |                                      |                                  |
| Fujio               | JEBO    | Undiscounted optimal growth in a Leontief two-sector model with circulating capital: the case of a capital intensive consumption good | Two-sector optimal growth model with a Leontief technology | The shape of the production function |
| (2008)              |         |                                                              |                                      |                                  |
| Hallegatte, Ghil,   | JEBO    | Business cycles, bifurcations and chaos in a neo-classical model with investment dynamics into a Solow growth model | Non-equilibrium dynamic model that introduces instability | Investment-profit instability   |
| Dumas and Hourcade  |         |                                                              |                                      |                                  |
| (2008)              |         |                                                              |                                      |                                  |
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| Yokoo and Ishida      | JEDC    | Misperception-driven chaos: theory and policy implications | Economy with a continuum of firms that engage in innovation activities | Imperfect information        |
| (2008)                |         |                                                 |                                |                               |
| Dieci and Westerhoff  | JEDC    | Heterogeneous speculators, endogenous fluctuations and interacting markets: a model of stock prices and exchange rates | A model that integrates the stock markets of two countries via the foreign exchange market | Heterogeneous agents: technical traders and fundamentalists |
| (2009)                |         |                                                 |                                |                               |
| Kikuchi and Stachurski| JET     | Endogenous inequality and fluctuations in a two-country model | Two-country growth model      | Interaction between unequal countries through credit markets |
| (2009)                |         |                                                 |                                |                               |
| Stockmam              | JEDC    | Chaos and sector-specific externalities         | Two-sector growth model       | Sector-specific externalities |
| (2009)                |         |                                                 |                                |                               |
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| Author (year) | Journal | Title | Type of model | Source of fluctuations |
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| Gomes (2010)  | SNDE    | The sticky-information macro model: beyond perfect foresight | Sticky-information macroeconomic model | Formation of expectations under a learning rule |
| Lines and Westerhoff (2010) | JEDC | Inflation expectations and macroeconomic dynamics: the case of rational vs extrapolative expectations | Macro model composed by Okun's law, expectations-augmented Phillips curve and an aggregate demand relation | Heterogeneous expectations (trend-following and rational expectations) |
| Sushko, Gardini and Puu (2010) | JEBO | Regular and chaotic growth in a Hicksian floor/ceiling model | Hicksian trade-cycle model | Capital stock as a capacity limit (ceiling) for production |
Not yet explored ...

- Endogenous fluctuations on the sticky-information general equilibrium macroeconomic model of Mankiw and Reis (2006, 2007) and Reis (2009).
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This model explains the gradual response (inertia) of aggregate variables to exogenous disturbances. It allows for a steady-state analysis, where policy shocks may temporarily deviate the economy from its fixed-point long-run locus.

How could endogenous cycles emerge within this setup? We just need to relax two benchmark assumptions and consider that:

1. Perfect foresight is not universal;
2. Information updating is counter-cyclical.
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- The monetary policy rule ignores real stabilization, and focuses on price stability;
- In order to emphasize the possible presence of endogenous fluctuations, stochastic disturbances (e.g., technological innovations) are overlooked;
- The rate at which the real interest rate converges to the steady-state is known at time $t$. 
Behavior of firms: profit maximization under monopolistic competition;

Optimization leads to desired price:

\[ p_t = p_t + mc_t; \]

Marginal costs:

\[ mc_t = \beta + \upsilon \left( 1 - \beta \right) w_t p_t + \frac{1}{\beta + \upsilon \left( 1 - \beta \right)} y_t; \]

\[ w_t: \text{nominal wage rate}; \]
\[ y_t: \text{output gap}; \]
\[ \upsilon > 0: \text{elasticity of substitution between different varieties of goods}; \]
\[ \gamma > 0: \text{elasticity of substitution between different varieties of labor}; \]
\[ \beta \in (0, 1): \text{output-labor elasticity}. \]
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  - $\pi_t = \frac{\lambda}{1 - \lambda} mc_t + \lambda \sum_{j=0}^{\infty} (1 - \lambda)^j E_{t-1-j} (\pi_t + \Delta mc_t) \Rightarrow$

    Sticky-information Phillips Curve.
Behavior of households - utility maximization for an individual consumer leads to: $c_{t,j} = -\theta E_{t-j}(R_t)$, with $R_t = E_t \left( \sum_{i=0}^{\infty} r_{t+i} \right)$ the long real interest rate and $\theta$ the intertemporal elasticity of substitution for consumption.
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- Applying first differences:
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y_{t+1} = y_t - \theta \lambda R_{t+1} - \theta \lambda \sum_{j=0}^{\infty} (1 - \lambda)^j E_{t-j}[(1 - \lambda) R_{t+1} - R_t].
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- Fisher equation: \( r_t = i_t - E_t(\pi_{t+1}) \); monetary policy rule: \( i_t = \phi[E_t(\pi_{t+1}) - \bar{\pi}], \phi > 1 \).
Sticky-Information Wage Curve

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The solution of the optimization problem for an individual worker is:

\[ w_{t,j} = E_{t-j} \left[ p_t + \frac{\gamma}{\gamma+\psi}(w_t - p_t) + \frac{1}{\beta(\gamma+\psi)} y_t - \frac{\psi}{\gamma+\psi} R_t \right], \text{ with } \psi > 0 \]

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SIGE model: SIPC-SIISC-SIWIC.
Steady-state

Steady-state: \((p^*, y^*, w^*)\) with \(p^* := p_t = E_{t-j}(p_t)\), \(y^* := y_t = E_{t-j}(y_t)\), \(w^* := w_t = E_{t-j}(w_t)\), \(\forall t, j = 0, 1, 2, \ldots\)
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- Applying the definition: \(y^* = 0; \ p^* = w^*; \ R^* = r^* = 0; \ \pi^* = i^* = \frac{\phi}{\phi - 1} \bar{\pi}.\)
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- Assumption - \(r_t\) converges to the steady-state at rate \(a \in (0, 1):\)
  \(R_t = E_t \left( \sum_{i=0}^{\infty} r_{t+i} \right) = \sum_{i=0}^{\infty} (1-a)^i r_t = \frac{1}{a} r_t.\)
For the newly considered variables, the SIGE dynamic system can be further rearranged:
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\[ \pi_{t+1} = \frac{1}{1-\lambda} \pi_t + \frac{\lambda}{1-\lambda} (\Delta mc_{t+1} + \Delta mc_t) \]

\[ + \lambda \sum_{j=0}^{\infty} (1 - \lambda)^j E_{t-j} \left[ \pi_{t+1} + \Delta mc_{t+1} - \frac{1}{1-\lambda} (\pi_t + \Delta mc_t) \right] \]

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- \( g_{t+1} = -\theta \lambda \frac{\phi-1}{\alpha} E_t (\pi_{t+1}) \)
  \[ -\theta \lambda \sum_{j=0}^{\infty} (1 - \lambda)^j E_{t-j} \left[ (1 - \lambda) \frac{\phi-1}{\alpha} \pi_{t+2} - \frac{\phi-1}{\alpha} \pi_{t+1} \right] \]
Two central assumptions - 1) Expectations

- Benchmark approach: perfect foresight
  \[ E_{t-j}(\pi_t) = \pi_t, E_{t-j}(\mu_t) = \mu_t, E_{t-j}(g_t) = g_t, \forall j; \]

- More sensible approach: agents lose capacity to predict future values with accuracy, as we go further back in time.
  \[ E_{t-j}(\pi_t) = \alpha_j \pi_t + (1-\alpha_j) \pi_t, E_{t-j}(\mu_t) = \alpha_j \mu_t + (1-\alpha_j) \mu_t, E_{t-j}(g_t) = \alpha_j g_t + (1-\alpha_j) g_t, \]

\[ \text{with } 1-\alpha_j \in (0,1), \]
Two central assumptions - 1) Expectations

- Benchmark approach: perfect foresight
  \[ E_{t-j}(\pi_t) = \pi_t, \ E_{t-j}(\mu_t) = \mu_t, \ E_{t-j}(g_t) = g_t, \ \forall j; \]

- More sensible approach: agents lose capacity to predict future values with accuracy, as we go further back in time.
  \[ E_{t-j}(\pi_t) = \alpha^j \pi_t + (1 - \alpha^j)\pi^*, \ E_{t-j}(\mu_t) = \alpha^j \mu_t + (1 - \alpha^j)\mu^*, \ E_{t-j}(g_t) = \alpha^j g_t + (1 - \alpha^j)g^*; \] with \( 1 - \alpha \in (0, 1) \) the probability of interpreting \( t \) as the steady-state, when formulating the expectation at \( t - 1 \).
'We (...) find evidence supporting that consumers update their expectations about the economy much more frequently during periods of high news coverage than in periods of low news coverage; high news coverage of the economy is concentrated during recessions and immediately after recessions, implying that 'stickiness' in expectations is countercyclical.'

Doms, M. and N. Morin (2004). Consumer Sentiment, the Economy, and the News Media, FRBSF 2004-09 (abstract).

- We extrapolate this logic to price-setting firms and wage-setting labor suppliers.
Two central assumptions - 2) Information updating

- Information stickiness is counter-cyclical! - how to model this?

\[ \lambda(g_t) = 1 + \frac{\lambda_2}{\pi \arctan(g_t + \tan(\pi/2) + \lambda_0)} \]

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Two central assumptions - 2) Information updating

- Information stickiness is counter-cyclical! - how to model this?
- Let $\lambda_0 \in (0, 1)$ be the attentiveness rate for $g_t = 0$, and $\Lambda \in (0, \lambda_0)$ a benchmark minimal level of attention that asymptotically holds for large growth rates. Attentiveness increases as the growth rate becomes smaller. Full attentiveness ($\lambda = 1$) is a virtual outcome for extremely negative growth rates.

Function that captures the mentioned properties:

$$\lambda(g_t) = 1 + \frac{\lambda_2}{\lambda_0} \arctan(g_t + \pi) + \frac{\lambda_2}{\lambda_0}$$
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- Function that captures the mentioned properties:

$$\lambda(g_t) = \frac{1 + \lambda}{2} - \frac{1 - \lambda}{\pi} \arctan \left[ g_t + \tan \left( \frac{\pi}{2} \frac{1 + \lambda}{1 - \lambda} - 2\lambda_0 \right) \right]$$
\[ \lambda_0 = 0.25; \lambda = 0.1: \]

- Note that in the vicinity of \( \lambda_0 \), \( \lambda(g_t) \) is an increasing and slightly convex function.
System under new assumptions

- Let \( \mu^R_t := \mu_t - \pi_t \) - growth rate of the real wage.
System under new assumptions

- Let $\mu^R_t := \mu_t - \pi_t$ - growth rate of the real wage.
- System:

$$
\begin{align*}
  g_{t+1} &= \left[ \alpha (1 - \lambda) + \frac{\Omega_1}{(\phi - 1)\Omega_2\Omega_6} \right] g_t + \frac{(1 - \alpha)(1 - \lambda)}{(1 + \Omega_3)\Omega_6} \mu^R_t \\
  \mu^R_{t+1} &= -\frac{\Omega_1}{(\phi - 1)\Omega_2\Omega_6} \left[ \Omega_6 + \frac{1 - \beta}{\beta} \right] g_t \\
  &\quad + \frac{1 - \lambda}{1 + \Omega_3} \left[ 1 + \alpha \Omega_3 - \frac{1 - \alpha}{\Omega_6} \left( \Omega_6 + \frac{1 - \beta}{\beta} \right) \right] \mu^R_t
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\end{aligned}
\end{align*}
$$

$\Omega_1 := \frac{1 - \alpha(1 - \lambda)}{\beta \alpha^2 \lambda (1 - \lambda)}$; $\Omega_2 := \frac{\lambda}{(1 - \alpha)(1 - \lambda)} \frac{\beta}{\beta + \nu (1 - \beta)}$;

$\Omega_3 := \frac{\lambda}{1 - \alpha (1 - \lambda)} \left[ \frac{\beta}{\beta + \nu (1 - \beta)} - \frac{\gamma}{\gamma + \psi} \right]$;

$\Omega_4 := \frac{\lambda}{1 - \alpha (1 - \lambda)} \left[ \frac{1 - \beta}{\beta + \nu (1 - \beta)} - \frac{1}{\beta (\gamma + \psi)} \right]$; $\Omega_5 := \frac{\lambda}{1 - \alpha (1 - \lambda)} \frac{\psi}{\gamma + \psi} \frac{\alpha}{a}$;

$\Omega_6 := \frac{\Omega_4 - \Omega_1 \Omega_5}{1 + \Omega_3} - \frac{1 - \beta}{\beta}$, with $\lambda = \lambda(g_t)$. 
Perfect foresight

- Under perfect foresight, $\alpha = 1$; dynamics are reduced to

$$
\begin{align*}
  g_{t+1} &= [1 - \lambda(g_t)] g_t \\
  \mu_{t+1}^R &= [1 - \lambda(g_t)] \mu_t^R
\end{align*}
$$

This system implies convergence to $h$, $\mu^R$ independently of parameter values and initial state.
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- This system implies convergence to $[g^*, (\mu^R)^*] = (0, 0)$, independently of parameter values and initial state.
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  \]

- This system implies convergence to $[g^*, (\mu^R)^*] = (0, 0)$, independently of parameter values and initial state.

**Fact (Proposition)**

Under perfect foresight, there is stability in the SIGE model. This result holds for constant information updating and for counter-cyclical information updating.
Partial perfect foresight ($\alpha < 1$)

- Parameter values in Mankiw and Reis (2006): $\psi = 4$, $\beta = 2/3$, $\theta = 1$, $\gamma = 10$, $\nu = 20$. 

Local dynamics:

$$
\begin{align*}
\Delta g_{t+1} &= f_{11}(\lambda(g_t)) g_t + f_{12}(\lambda(g_t)) \mu_R \mu_{R+1} \\
\Delta \mu_{R+1} &= f_{21}(\lambda(g_t)) g_t + f_{22}(\lambda(g_t)) \mu_R
\end{align*}
$$

Linearization in the vicinity of the steady-state $(g_t, \mu_R) = (0, 0)$:

$$
\begin{align*}
\Delta g_{t+1} &= f_{11}(\lambda_0) g_t + f_{12}(\lambda_0) \mu_R \\
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Partial perfect foresight ($\alpha < 1$)

- Parameter values in Mankiw and Reis (2006): $\psi = 4$, $\beta = 2/3$, $\theta = 1$, $\gamma = 10$, $\nu = 20$.
- Take, as well, $\alpha = 0.75$; $a = 0.01$. Let $\phi$ be the bifurcation parameter.
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- Take, as well, \(\alpha = 0.75\); \(a = 0.01\). Let \(\phi\) be the bifurcation parameter.
- **Local dynamics:**
  - Our system is
    \[
    \begin{align*}
    g_{t+1} &= f_{11}[\lambda(g_t)]g_t + f_{12}[\lambda(g_t)]\mu_t^R \\
    \mu_t^R &= f_{21}[\lambda(g_t)]g_t + f_{22}[\lambda(g_t)]\mu_t^R
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    \end{cases}
    \]
  - Linearization in the vicinity of the steady-state $(g^*, \mu_{R}^*) = (0, 0)$:
    \[
    \begin{bmatrix}
    g_{t+1} \\
    \mu_{t+1}^R
    \end{bmatrix} =
    \begin{bmatrix}
    f_{11}(\lambda_0) & f_{12}(\lambda_0) \\
    f_{21}(\lambda_0) & f_{22}(\lambda_0)
    \end{bmatrix}
    \cdot
    \begin{bmatrix}
    g_t \\
    \mu_t^R
    \end{bmatrix}
    \]
Partial perfect foresight ($\alpha < 1$)

- Local dynamics are identical for constant information updating and counter-cyclical information updating.
Partial perfect foresight (\(\alpha < 1\))

- Local dynamics are identical for constant information updating and counter-cyclical information updating.

- E.g., \(\lambda = 0.25\),

\[
\begin{bmatrix}
  g_{t+1} \\
  \mu^R_{t+1}
\end{bmatrix}
= 
\begin{bmatrix}
-0.2547/(\phi - 1) + 0.5625 & -0.2167 \\
-0.2149/(\phi - 1) & 0.6709
\end{bmatrix}
\cdot
\begin{bmatrix}
g_t \\
\mu^R_t
\end{bmatrix}
\]

Stability conditions:

- \(\text{Det} > 0\);
- \(\text{Tr} + \text{Det} > 0\);
- \(\text{Tr} + \text{Det} > 0\)

Stability: \(\phi > 1.1808\).
Partial perfect foresight (alfa < 1)

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- Stability conditions:
  - \(1 - Det = 0.6226 + 0.2174/(\phi - 1) > 0\);
Partial perfect foresight (alfa<1)

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- E.g., \( \lambda = 0.25 \),

\[
\begin{bmatrix}
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\end{bmatrix}
\]

- Stability conditions:
  - \( 1 - \text{Det} = 0.6226 + 0.2174/ (\phi - 1) > 0 \);
  - \( 1 - \text{Tr} + \text{Det} = 0.1436 + 0.0373/ (\phi - 1) > 0 \);
Partial perfect foresight ($\alpha < 1$)

- Local dynamics are identical for constant information updating and counter-cyclical information updating.
- E.g., $\lambda = 0.25$,

$$
\begin{bmatrix}
  g_{t+1} \\
  \mu_t^R \\
  \mu_t^{R+1}
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- Stability conditions:
  - $1 - \text{Det} = 0.6226 + 0.2174/ (\phi - 1) > 0$;
  - $1 - \text{Tr} + \text{Det} = 0.1436 + 0.0373/ (\phi - 1) > 0$;
  - $1 + \text{Tr} + \text{Det} = 2.6107 - 0.4721/ (\phi - 1) > 0 \Rightarrow \text{Stability:} \ 
    \phi > 1.1808.$
Fig. 2 - Trace-determinant diagram in the partial perfect foresight case

- Flip bifurcation (line $1 + Tr + Det = 0$ is crossed).
Sensitivity analysis

For other values of $\lambda_0$,

| $\lambda$ | Stability Condition | $\lambda$ | Stability Condition |
|-----------|---------------------|-----------|---------------------|
| 0.1       | $\phi > 1.9787$    | 0.6       | $\phi > 1.0311$    |
| 0.2       | $\phi > 1.2720$    | 0.7       | $\phi > 1.0200$    |
| 0.3       | $\phi > 1.1293$    | 0.8       | $\phi > 1.0118$    |
| 0.4       | $\phi > 1.0750$    | 0.9       | $\phi > 1.0053$    |
| 0.5       | $\phi > 1.0475$    | 1         | $\phi > 1$         |
Fact (Proposition)

Under partial perfect foresight, the stronger the level of inattentiveness, the more aggressive monetary policy is required to be, in order for stability to hold.
Constant information updating $\Rightarrow$ the model is linear: local and global dynamics coincide (there are no endogenous fluctuations).
Global dynamics

- Constant information updating $\Rightarrow$ the model is linear: local and global dynamics coincide (there are no endogenous fluctuations).

- Counter-cyclical information updating: endogenous cycles are found for reasonable parameter values.
Global dynamics

- Constant information updating $\Rightarrow$ the model is linear: local and global dynamics coincide (there are no endogenous fluctuations).

- Counter-cyclical information updating: endogenous cycles are found for reasonable parameter values.
  - E.g., $\lambda_0 = 0.25; \lambda = 0.1$ [stability: $\phi > 1.1808$].
Fig. 4 - Bifurcation diagram

\( (g_t; \phi) \)
Fig. 5 - Attractor

\[(g_t, \mu^R_t); \phi = 1.1\]
Conclusion

- **Setup**: Macroeconomic general equilibrium model with information stickiness;

- Departures from perfect foresight, that depend on the timing of the expectations;
- Counter-cyclical information updating.

- Nonlinearities emerge on an otherwise linear model;
- Local dynamics are coincident between linear / nonlinear cases;
- Global dynamics: endogenous cycles - the initially mentioned new perspective on business cycles can be associated with a sticky-information environment;

Orlando Gomes (Business Research Unit)
Setup: Macroeconomic general equilibrium model with information stickiness;

Assumptions:
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