Research Article

Triple-Shear Unified Solution for Bearing Capacity of Unsaturated Soft Soil Foundations with Hard Crust

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The bearing capacity of unsaturated soft soil foundations is affected by natural or artificial hard crust, due to the stress diffusion and closure effect. Based on the triple-shear unified solution of shear strength for unsaturated soil, the analytical solutions of critical edge load and critical load of unsaturated soft soil with hard crust are derived with the consideration of the lateral pressure coefficient \( K_0 \neq 1 \). The formulas are degraded and compared. Meanwhile, the influences of stress diffusion, increased load, matric suction, intermediate principal, and lateral pressure coefficient are conducted on the bearing capacity of unsaturated soft soil foundations with hard crust. The results show that the critical edge load and critical load increased, and the stress diffusion coefficient decreased with the increase of the ratio of elastic modulus between hard crust and soft soil. The bearing capacity of soft soil foundation increased with the consideration of the stress diffusion and closure effect of hard crust. The effect of matric suction is remarkable, and the critical edge load and critical load linearly increase with the increase of low matric suction and variate with the dual influence of high matric suction. The bearing capacity of unsaturated soft soil foundation is obviously higher than that of saturated soil. The critical edge load and critical load increased with the increase of intermediate principal stress and lateral pressure coefficient. The triple-shear unified solution for bearing capacity of unsaturated soft soil foundation with hard crust contains different solutions under various working conditions, which has important theoretical significance and widely engineering application.

1. Introduction

Soft soil is not saturated due to evaporation and transpiration of water molecules, and this phenomenon is particularly prevalent in arid and semiarid regions [1–3]. With the long-term effect of dehydration, drying, weathering, ion exchange, and other factors or the influence of natural action, hard crust with different thickness is formed on the surface of soft soil [4]. The bearing capacity of soft soil foundation is effected by hard crust, due to the stress diffusion and closure effect [5]. Wen et al. [5, 6] deeply analyzed the influence of stress diffusion and closure effect of hard crust on soft soil foundation, the results show that the stress diffusion expands the load area on the top of soft soil foundation, and the maximum value decreases. The closure effect delays the load of soft soil into the plastic stage and improves the bearing capacity of the foundation. Chen et al. [7, 8] indicated that the in situ stress state of natural soil is usually inherently anisotropic, that is to say, the static lateral pressure coefficient \( K_0 \neq 1 \) in the actual stress state of soil. Fattah et al. [9–12] studied the shear strength properties for unsaturated soil with different matric suction values, and it is concluded that the matric suction is considered an important contributor to unsaturated shear strength properties. Wang [13] and Wen et al. [5] derived the critical edge load of soft soil foundation with hard crust based on Mohr-Coulomb criterion, the results take no account of intermediate principal stress, and the assumption \( K_0 = 1 \) is inconsistent with the actual stress state of soil. Zhu et al. [14] deduced the critical edge pressure and critical loading of soft soil foundation based on the unified strength theory from the static lateral pressure coefficient of soil under actual conditions, and the
results consider the stress diffusion effect and the influence of intermediate principal stress but do not consider the actual unsaturated state of soft soil. Zhang et al. [15] derived the critical edge load and critical load of unsaturated soft soil foundation based on the unified solution of plane shear strength of unsaturated soil, taking account of the stress dispersion and self-weight berm of hard crust. However, the results are derived based on the twin shear unified strength theory, and the problem of double slip angle exists under certain stress [16]. Gao et al. [17] obtained the calculation formula of plastic load and critical load of soft soil foundation with hard crust based on the triple-shear unified strength criterion, considering the influence of static lateral pressure coefficient, stress diffusion, and closure effect of hard crust but do not consider the actual unsaturated state of soft soil.

The above results show that considering the intermediate principal stress effect can better develop the strength of soil and improve the bearing capacity of soft soil foundation. The static lateral pressure coefficient $K_0$ has significant influence on the critical edge load and critical load of soft soil foundation, and $K_0 \neq 1$ is more consistent with the actual situation of soil. Matric suction of unsaturated soil has different effects on bearing capacity of soft soil foundation under high and low suction. Therefore, the effects of stress diffusion, closure effect, intermediate principal stress, static lateral pressure coefficient, and matric suction of unsaturated soil should be comprehensively considered.

Based on the triple-shear unified solution of shear strength for unsaturated soil, with the consideration of stress diffusion and closure effect of hard crust, the analytical solution of the critical edge load and critical load of unsaturated soft soil with hard crust are deduced under the lateral pressure coefficient $K_0 \neq 1$. The formulas are degraded and compared. Meanwhile, the influences of stress diffusion $\eta$, increased load $P_{GW}$, matric suction $(u_a - u_w)$, strength criterion parameter $b$, and lateral pressure coefficient $K_0$ are conducted on the bearing capacity of unsaturated soft soil foundation with hard crust. The results can provide theoretical reference for the design and construction of soft soil foundation with hard crust and have important engineering application value.

2. Model Establishment and Verification

2.1. The Triple-Shear Unified Solution of Shear Strength for Unsaturated Soil. The triple-shear unified strength criterion [18] overcomes the problem of double slip angle under certain stress in twin-shear unified strength criterion, which considers the influence of intermediate principal stress effect. Based on the triple-shear unified strength criterion and shear strength of unsaturated soil in terms of two state stress variables, a triple-shear unified solution of shear strength for unsaturated soil is obtained with comprehensively considering the effects of intermediate principal stress, matric suction, strength disparity of material, and strength criterion [19]. The formula is as follows:

$$
\tau_f = c_t' + (\sigma - u_a) \tan \varphi_t' + (u_a - u_w) \tan \varphi_t'' = c_t + (\sigma - u_a) \tan \varphi_t',
$$

where

$$
c_t = c_t' + (u_a - u_w) \tan \varphi_t''\sin \varphi_t',
$$

$$
\varphi_t' = \tan^{-1}\left(\frac{2(1 + b) \sin \varphi'}{2 + b(1 + \mu_c^2)}\right),
$$

$$
\varphi_t'' = \tan^{-1}\left(\frac{2(1 + b) \sin \varphi''}{2 + b(1 + \mu_c^2)}\right).
$$

In the formula, $\tau_f$ is the shear strength of unsaturated soil, $c_t$ is the unified total cohesion of unsaturated soil, $c_t'$ is the triple-shear unified effective cohesion, $\varphi_t'$ is the triple-shear unified effective angle of internal friction, $\varphi_t''$ is the triple-shear unified angle related to matric suction, $\sigma$ is the total normal stress, $u_a$ is the pore gas pressure, $u_w$ is the pore water pressure, $(\sigma - u_a)$ is the net normal stress, $(u_a - u_w)$ is the matric suction, $c'$ is the effective cohesion intercept, $\varphi'$ is the effective angle of internal friction, $\varphi''$ is the angle of matric suction contribution to shear strength; $\mu_c$ is the lode parameter, $b$ is the parameter of different strength criteria, which reflect
the influence of intermediate principal shear stress and normal stress on material failure, and the value of $b$ is related to the mechanical properties of the material itself, which is determined by the test, $b = [(1 + \alpha)\tau_x - \sigma_y]/\sigma_y$, where $\alpha = \sigma_y/\sigma_y$ is the tensile yield limit, $\sigma_y$ is the compression yield limit, and $\tau_x$ is the shear yield strength.

Equation (1) take intermediate principal stress effect into account by strength criterion parameter $b$ and lode parameter to reflect the influence of strength disparity of material and strength criterion difference on its shear strength. Meanwhile, effect of matric suction on shear strength of unsaturated soil also takes into account by $(u_x - u_y)$. When $b = 0$, equation (1) degenerates into shear strength of saturated soil based on Mohr-Coulomb strength criterion [20], without considering the influence of intermediate principal stress. When $(u_x - u_y) = 0$, it degenerates into shear strength of saturated soil based on the triple-shear strength criterion [21].

2.2. Stress Analysis. Ignoring the compression and shear deformation of hard crust, the stress of any point $M$ in the foundation is shown in Figure 1. In the figure, $p$ is the uniform strip load on foundation, $B$ is the load width, $h$ is the thickness of hard crust, $\gamma_h$, $E_h$, and $v_h$ are gravity, elastic modulus, and Poisson ratio of the hard crust, respectively, $\gamma_s$, $E_s$, and $v_s$ are gravity, elastic modulus, and Poisson ratio of the soft layer, respectively, $c'$ and $\phi'$ are effective cohesion intercept and effective angle of internal friction of the soft soil, and $\phi''$ is the suction angle related to matric suction of soft soil.

The hard crust has stress diffusion and closure effect on the underlying soft soil layer. Stress diffusion is generalized by stress diffusion coefficient $\eta$, so, the uniform strip load $p$ transmits to the top of soft soil and becomes $\eta p$ [22]. Closure effect can be simplified by calculating that the increments of horizontal stress and vertical stress are the same, and both are the load $p_0$ imposed on the top of hard crust in the first stage [5]. Therefore, the stress at any point in the foundation consists of additional stress due to stress diffusion, self-weight stress, and stress generated by considering closure effect.

2.2.1. Additional Stress considering Stress Diffusion. The stress diffusion of hard crust to the underlying soft soil is related to the thickness of hard crust, the width of the load, and the compression modulus of the hard crust and soft soil. Xu [23] obtained the stress diffusion coefficient by fitting the finite element calculation, and its expression is

$$\eta = \left( d \sqrt{\frac{h}{B}} + c \right) \left( \frac{E_s'}{E_h'} \right)^{1/4},$$

where

$$E_h' = \frac{E_h}{1 - v_h^2}, E_s' = \frac{E_s}{1 - v_s^2},$$

$$d = 0.1461 \ln \left( \frac{E_h'}{E_s'} \right) + 0.0018,$$

$$e = -0.1334 \ln \left( \frac{E_h'}{E_s'} \right) + 1.0284.$$
2.2.4. The Total Stress at Point M of Foundation. According to formulas ((5)–(11)), the total stress at any point M in the soft soil foundation is

\[ \sigma_z = \frac{\eta p}{\pi} [\theta + \sin \theta \cos (\theta + 2\delta)] + (\gamma_h + \gamma_z)z + p_0, \quad (12) \]

\[ \sigma_x = \frac{\eta p}{\pi} [\theta - \sin \theta \cos (\theta + 2\delta)] + K_0(\gamma_h + \gamma_z) + p_0. \quad (13) \]

\[
\frac{\sigma_1}{\sigma_3} = \frac{\sigma_z + \sigma_x}{2} = \frac{\sigma_z - \sigma_x}{2} \quad (15)
\]

\[
\pm \sqrt{\left(\frac{\eta p}{\pi} \sin \theta \cos (\theta + 2\delta) + \frac{1}{2}K_0(\gamma_h + \gamma_z) + p_0 \right)^2 + \left[\frac{\eta p}{\pi} \sin \theta \sin (\theta + 2\delta)\right]^2}
\]

The stress \( (\sigma_z - \sigma_x)/2 > \tau_{xz} > 0 \) in the plastic zone is established within the depth range of \( B/4 \) under the foundation edge [25], which meets the application conditions of the mathematical approximation formula \( \sqrt{s^2 + \pi^2} \approx 1.0s + 0.38t \) [26]. So, equation (15) can be simplified as

\[ \sigma_1 = \frac{\eta p}{\pi} [\theta + \sin \theta \cos (\theta + 2\delta) + 0.38 \sin \theta \sin (\theta + 2\delta)] + (\gamma_h + \gamma_z)z + p_0, \quad (16) \]

\[ \sigma_3 = \frac{\eta p}{\pi} [\theta - \sin \theta \cos (\theta + 2\delta) - 0.38 \sin \theta \sin (\theta + 2\delta)] + K_0(\gamma_h + \gamma_z) + p_0. \quad (17) \]

2.3. Triple-Shear Unified Solution of Foundation Bearing Capacity. According to the triple-shear unified strength criterion, when point M is in the limit equilibrium state, the major and minor principal stresses satisfy the following equilibrium conditions:

\[ \frac{\sigma_1 - \sigma_3}{2} = \frac{\sigma_1 + \sigma_3}{2} \sin \phi_t' + c_t \cos \phi_t'. \quad (18) \]

Substituting equation ((16) and (17)) into equation (18) can be obtained

\[ \frac{1}{2} \left[ (1 - K_0) - (1 + K_0) \sin \phi_t' \right] y_z \]

\[ = c_t \cos \phi_t' + \frac{\eta p}{\pi} \theta \sin \phi_t' + p_0 \sin \phi_t' \]

\[ - \frac{1}{2} \left[ (1 - K_0) - (1 + K_0) \sin \phi_t' \right] y_h \]

\[ - \frac{\eta p}{\pi} \sin \theta \cos (\theta + 2\delta) + 0.38 \sin \theta \sin (\theta + 2\delta). \]

(19)

According to the approximate formula \( \sqrt{s^2 + \pi^2} = 1.0s + 0.38t \), the upper formula

\[ [\sin \theta \cos (\theta + 2\delta) + 0.38 \sin \theta \sin (\theta + 2\delta)] \]

\[ = \sin \theta \cos (\theta + 2\delta) + 0.38 \sin \theta \sin (\theta + 2\delta) \quad (20) \]

\[ = \sin \theta \sqrt{\cos^2(\theta + 2\delta) + \sin^2(\theta + 2\delta)} = \sin \theta. \]

Let \( Q = 1/2[ (1 - K_0) - (1 + K_0) \sin \phi_t' ] \), and substituting equation (20) into equation (19) can be obtained development depth of plastic zone \( z \)

\[ \frac{z}{y_z} = \frac{1}{Qy_z} \left( c_t \cos \phi_t' + p_0 \sin \phi_t' \right) - \frac{\gamma_h}{y_z} + \frac{1}{Qy_z} \left( \theta \sin \phi_t' \sin \theta \right). \]

(21)

When the properties of hard crust and soft soil are certain, \( z \) is just a function of \( \theta \); so, the maximum development depth of plastic zone \( z_{\text{max}} \) can be obtained by

\[ \tau_{xz} = \frac{\eta p}{\pi} \sin \theta \sin (\theta + 2\delta). \quad (14) \]
derivation of \( \theta \) by equation (21), namely,

\[
\frac{dz}{d\theta} = \eta p \frac{1}{Q\gamma'} \left( \sin \phi' \cdot \cos \theta \right) = 0. \tag{22}
\]

So, \( \theta = (\pi/2)\cdot\phi'_t \), and substitute into equation (21) to obtain

\[
z_{\text{max}} = \frac{1}{Q\gamma'} \left( c_{tt} \cos \phi'_t + p_0 \sin \phi'_t \right) - \frac{\gamma_h h + \eta p_0 \left( \frac{\pi}{2} \cdot \phi'_t \right) \sin \phi'_t \cdot \cos \phi'_t}{Q\gamma'} \tag{23}
\]

When the maximum development depth of the plastic zone \( z_{\text{max}} = 0 \), the critical edge load \( p_{\text{cr}} \) of unsaturated soft soil foundation with hard crust can be obtained by equation (23).

\[
p_{\text{cr}} = \frac{\pi Q}{\eta} \left[ \left( \frac{\pi}{2} \cdot \phi'_t \right) \sin \phi'_t \cdot \cos \phi'_t \right] \gamma_h h + \frac{1}{\eta} \left[ 1 - \left( \frac{\pi}{2} \cdot \phi'_t \right) \tan \phi'_t \right] c_{tt} + \frac{1}{\eta} \left[ \cot \phi'_t - \left( \frac{\pi}{2} \cdot \phi'_t \right) \right] p_0 = M_h \gamma_h h + M_c \left[ c_{tt} + (u_a - u_w) \tan \phi'_t \right] + M_p p_0. \tag{24}
\]

When the maximum development depth of the plastic zone \( z_{\text{max}} = B/4 \), the critical load \( p_{1/4} \) of unsaturated soft soil foundation with hard crust can be obtained by equation (23).

\[
p_{1/4} = M_h \gamma_h B + M_c \left[ c_{tt} + (u_a - u_w) \tan \phi'_t \right] + M_p p_0. \tag{25}
\]

In the formula, \( M_h, M_c, \) and \( M_p \) are the coefficients of foundation bearing capacity, and their expressions are, respectively,

\[
M_h = \frac{\pi}{2\eta} \left[ \frac{1 - K_0}{(\pi/2)\cdot\phi'_t} \sin \phi'_t \cdot \cos \phi'_t \right];
\]

\[
M_c = \frac{1}{4} M_h;
\]

\[
M_p = \frac{\pi}{\eta} \left[ \frac{1}{\cot \phi'_t - (\pi/2)\cdot\phi'_t} \right]. \tag{26}
\]

From formula (24) and formula (25), it can be seen that the critical edge load of unsaturated soft soil founda-

tion with hard crust is composed of three components: one is related to the weight of hard crust \( \gamma_h h \), one is related to the unified total cohesion of soft soil \( c_{tt} \), and the other is related to the increasing load \( p_0 \) with the consideration of closure effect of hard crust; the critical load increases one more related to the weight of hard crust \( \gamma_s \).

B. From equation (26), it can be seen that the weight coefficients of critical edge load and critical load are related to the stress diffusion coefficient \( \eta \), the static lateral pressure coefficient \( K_0 \), and the triple-shear unified effective angle of internal friction \( \phi'_t \), which are the functions of strength criterion parameters \( b \) and lode parameter.

Therefore, the triple-shear unified solution formula (24) and formula (25) of the bearing capacity of unsaturated soft soil foundation with hard crust takes into account the combined effects of the stress diffusion (stress diffusion coefficient \( \eta \), the closure effect (increasing load \( p_0 \)), the characteristics of unsaturated soil (matric suction \( (u_a - u_w) \)), the intermediate principal stress effect (strength criterion parameters \( b \) and lode parameter), and the actual stress state (static lateral pressure coefficient \( K_0 \)). It has important theoretical significance and broad engineering application prospects.

3. Analysis

3.1. Formula Degradation and Comparative Analysis

3.1.1. Formula Degradation Analysis. When \( \eta = 1 \) and \( p_0 = 0 \), equation (24) and equation (25) degenerate into ignoring the stress diffusion and closure effect, and the expression is

\[
p_{\text{cr}} = \frac{\pi}{2} \left[ \frac{(1 + k_0) \tan \phi'_t \cdot (1 - k_0) / \cos \phi'_t}{(\pi/2)\cdot\phi'_t} \right] \gamma_h h + \frac{\pi}{2} \left[ \frac{(1 + k_0) \tan \phi'_t \cdot (1 - k_0) / \cos \phi'_t}{(\pi/2)\cdot\phi'_t} \right] \left[ c_{tt} + (u_a - u_w) \tan \phi'_t \right] + \gamma_h B + p_{\text{cr}}. \tag{27}
\]

\[
p_{1/4} = \frac{\pi}{8} \left[ \frac{(1 + k_0) \tan \phi'_t \cdot (1 - k_0) / \cos \phi'_t}{(\pi/2)\cdot\phi'_t} \right] \gamma_h B + \gamma_h h \frac{\pi}{8} \left[ \frac{(1 + k_0) \tan \phi'_t \cdot (1 - k_0) / \cos \phi'_t}{(\pi/2)\cdot\phi'_t} \right] \gamma_h B + p_{\text{cr}}. \tag{28}
\]

Formula ((27), (28)) is basically the same as the expression in literature [27], and the main difference is that the load on the soft soil in literature [27] is \( p \cdot \gamma_s \). Therefore, \( \gamma_s \) is added to the expressions of critical edge load and critical load, and the linear distribution of matric suction is also added, while other terms are the same.

When \( \eta = 1, p_0 = 0, b = 0, \) and \( (u_a - u_w) = 0 \), it degenerates into the Mohr-Coulomb solution of saturated soft soil foundation without considering the stress diffusion and
closure effect, and the expression is

$$P_{cr} = \frac{\gamma_h h [\sin \varphi (1 + k_0) - (1 - k_0)]}{2 \cos \varphi \cdot \sin \varphi} + \frac{c \pi \cos \varphi}{\cos \varphi \cdot \sin \varphi}$$

(29)

$$P_{u/d} = \frac{\pi (\gamma_h h + \gamma_s B/4) [\sin \varphi (1 + k_0) - (1 - k_0)]}{2 \cos \varphi \cdot \sin \varphi} + \frac{c \pi \cos \varphi}{\cos \varphi \cdot \sin \varphi}$$

(30)

Formula ((29), (30)) is basically the same as the expression in literature [28], and the main difference is that the load on the soil in literature [28] is \( p - \gamma_h h \). So, \( \gamma_h h \) is added to the expressions of critical edge load and critical load. Meanwhile, the literature is single-layer soil, while this paper is double-layer soil (hard crust and soft soil), and other items are the same.

When \( b = 0, (u_a - u_w) = 0 \), and \( k_0 = 1 \), it degenerates into the Mohr-Coulomb solution of saturated soft soil foundation with hard crust in the hydrostatic self-weight stress field and is expressed as

$$p_{cr} = \frac{\gamma_h h + p_0}{\eta} \cdot \frac{\pi}{\cot \varphi + \varphi - \pi/2} + \frac{c}{\eta} \cdot \frac{\pi \cot \varphi}{\cot \varphi + \varphi - \pi/2},$$

(31)

$$P_{u/d} = \frac{\gamma_h h + 0.25 \gamma_s B + p_0}{\eta} \cdot \frac{\pi}{\cot \varphi + \varphi - \pi/2} + \frac{c}{\eta} \cdot \frac{\pi \cot \varphi}{\cot \varphi + \varphi - \pi/2}.$$  

(32)

Formula (31) is the same as the expression in literature [5], which considering the stress diffusion and closure effect, but not considering the matric suction of unsaturated soil, intermediate principal stress effect, and \( k_0 \neq 1 \).

When \( \eta = 1, p_0 = 0, b = 0, (u_a - u_w) = 0, k_0 = 1, \) and \( h = 0, \) it degraded to

$$p_{cr} = \frac{c \pi \cot \varphi}{\cot \varphi + \varphi - \pi/2},$$

(33)

$$P_{u/d} = \frac{\pi (c \cot \varphi + 0.25 \gamma_s B)}{\cot \varphi + \varphi - \pi/2}.$$  

(34)

Equation ((33), (34)) is the same as the formula of critical edge load and critical load of homogeneous foundation with zero burial depth in soil mechanics; so, the critical edge load and critical load of homogeneous foundation in classical soil mechanics are special case of this paper.

From the above analysis, it can be seen that the analytical solutions equation (24) and equation (25) obtained in this paper contain different solutions under various working conditions, which can be reasonably selected according to the specific conditions in practical engineering and have wide applicability.

3.1.2. Comparative Analysis. Based on the unified solution of plane strain shear strength of unsaturated soil, Zhang et al. [15] established the critical edge load and critical load of unsaturated soft soil foundation with hard crust, taking account of the stress dispersion and self-weight berm of hard crust. The analysis idea and derivation process of this paper are similar, and the obtained expression is basically the same. Comparing literature [15] with equation (24) and equation (25) in this paper, it can be seen that there are three differences in the expressions of critical edge load and critical load: first, this paper believes that the increment value of horizontal stress and vertical stress by closure effect is the same, and both are the loads imposed on the top surface of hard crust in the first stage; so, a term related to the increase load of the closure effect is added to the expression. Second, when calculated the total stress of \( M \) in literature [15], the normal stress and shear stress of self-weight stress are obtained in the direction of additional stresses \( \sigma_1 \) and \( \sigma_3 \) and then superimposed, but the direction of total stress is not the direction of principal stress. In this paper, the stress in \( X \) and \( Z \) directions are calculated, respectively, and then superimposed. At this time, the stress direction is consistent, and the principal stress should be sought after superposition. Third, the meaning of \( c' \) and \( \varphi' \) is different. In literature [15], unified effective cohesion and effective angle of internal friction are analogy obtained based on the twin shear unified strength theory, while in this paper, triple-shear unified effective cohesion and effective angle of internal friction are obtained based on the triple-shear unified strength criterion. The twin shear unified strength criterion exist double slip angle under some specific stress, the analytical solution of this paper adopts the triple-shear unified strength criterion, which overcomes the problem of double slip angle.

3.2. Analysis of Influencing Factors. In this paper, the critical edge load and critical load of unsaturated soft soil foundation with hard crust are established based on the triple-shear unified solution of shear strength for unsaturated soil, taking into account the effects of the stress diffusion, the closure effect, the characteristics of unsaturated soil, the intermediate principal stress effect, and the actual stress state. Therefore, the influence of the above five factors is analyzed in this section.

The basic parameters of unsaturated soft soil foundation with hard crust are [7, 14] as follows: load width \( B = 12 \) m, thickness of hard crust \( h = 3 \) m, gravity \( \gamma_h = 18.4 \) kN/m\(^3\), elastic modulus \( E_h = 16 \) MPa, and Poisson ratio \( \nu_h = 0.25 \). Here are the following parameters: the gravity of soft soil \( \gamma_s = 17.4 \) kN/m\(^3\), elastic modulus \( E_s = 3 \) MPa, Poisson ratio \( \nu_s = 0.42 \), effective cohesion intercept \( c' = 10 \) kPa, effective angle of internal friction \( \varphi' = 8^\circ \), and under plane strain state \( \mu_o = 0 \) [19].

3.2.1. The Stress Diffusion (Stress Diffusion Coefficient \( \eta \)). It can be seen from equation (3) that the ratio of elastic modulus between hard crust and soft soil \( E_s/E_h \) directly affects the stress diffusion, namely, the stress diffusion coefficient \( \eta \), and then has a certain influence on the critical edge load.
and critical load $p_{1/4}$, Figures 2 and Figures 3, respectively, show the variation rules of $\eta$, $p_{cr}$, and $p_{1/4}$ with $E_h/E_s$ and $p_{cr}$, $p_{1/4}$ with $\eta$ when $b = 0$, $(u_a - u_w) = 0$, $k_0 = 1$, and $p_0 = 0$.

It can be seen from Figures 2 and 3 that with the increase of the ratio of elastic modulus between hard crust and soft soil, the stress diffusion coefficient $\eta$ gradually decreases, and the critical edge load $p_{cr}$ and critical load $p_{1/4}$ gradually increase. When $E_h/E_s$ increases from 3.0 to 7.0, $\eta$ decreases by 9.9%, and $p_{cr}$ and $p_{1/4}$ increase by 11.0%.

Recommend non-segmenting, when the elastic modulus of hard crust and soft soil are quite different, the stress diffusion effect of hard crust on soft soil increases and then leads to the increase of the bearing capacity of soft soil foundation. $p_{cr}$ increased by 8.52 kPa and $p_{1/4}$ increased by 12.03 kPa when $\eta$ decreased from 0.90 to 0.81, both of which increased by 11%.

In other words, the diffusion effect of hard crust on soft soil foundation can improve the bearing capacity of soft soil foundation, but the effect is not significant. Therefore, reasonable natural or artificial hard crust should be selected to treat soft soil foundation in practical engineering, and the stress diffusion effect of hard crust on soft soil should be fully utilized to increase the bearing capacity of soft soil foundation.

3.2.2. The Closure Effect (Increasing Load $p_0$). According to equations (24) and (25), it can be seen that the influence of closure effect on the critical edge load and critical load is reflected in $M_4p_0$. Figure 4 shows the variation rules of $p_{cr}$ and $p_{1/4}$ with $p_0$ when $b = 0$, $(u_a - u_w) = 0$, $k_0 = 1$, and $\eta = 0.84$.

It can be seen from Figure 4 that the critical edge load and critical load increase linearly with the increase of $p_0$ (load applied on the top of hard crust in the first stage). When $p_0$ increases by 1 kPa, the critical edge load and critical load increase 0.66 kPa. When $p_0$ increases from 0 kPa to 10 kPa, the critical edge load and critical load increase 7.91% and 5.60%, respectively. Therefore, considering the closure effect of hard crust, the bearing capacity of soft soil foundation increases, and the increase depends on the load applied on the top of hard crust in the first stage.

3.2.3. The Characteristics of Unsaturated Soil (Matric Suction $(u_a - u_w)$). From equation (24) and (25), it can be seen that the critical edge load and critical load are related to the unified total cohesion $c_{tt}$ of soft soil, which is the function of
effective cohesion and matric suction, and matric suction directly affects the bearing capacity of soft soil foundation. Figure 5 shows the variation rules of critical edge load $p_{ct}$ and critical load $p_{1/4}$ with matric suction $(u_a - u_w)$ when $b = 0$, $k_0 = 1$, $\eta = 0.84$, $p_0 = 0$, and the suction angle of matric suction adopts the hyperbolic model in literature [15]. It can be seen from Figure 5 that the variation of critical edge load $p_{ct}$ and critical load $p_{1/4}$ with matric suction $(u_a - u_w)$ is related to $m$, which is the intercept of the hyperbolic model of suction angle $\phi$ under high suction, reflecting the change rate of suction angle. When $(u_a - u_w) \leq 15 \text{kPa}$ is in the range of low suction, $p_{ct}$ and $p_{1/4}$ are independent of the intercept $m$ and increase linearly with the increase of $(u_a - u_w)$. When $(u_a - u_w) = 15 \text{kPa}$ is the air-entry value of unsaturated soft soil, $p_{ct}$ and $p_{1/4}$ reach the peak point ($m = 0$) or the turning point ($m > 0$), and the larger the intercept $m$, the less obvious the turning trend. When $(u_a - u_w) > 15 \text{kPa}$ is in the range of high suction, $p_{ct}$ and $p_{1/4}$ are closely related to the intercept $m$. When $m = 0$, $p_{ct}$ and $p_{1/4}$ increase linearly with the increase of $(u_a - u_w)$. With the increase of $m$, it gradually turns to nonlinear increase. This curve reflects the nonlinearity of unsaturated soil strength in the range of high matric suction, namely, the double influence of matric suction on unsaturated soil strength. When $m = 0 \text{kPa}$, the decrease of suction angle makes the effect of matric suction on strength reduction more obvious than that of matric suction increase, and the transition is obvious. With the increase of $m$, the improvement effect of matric suction gradually increases, which exceeds the influence of the decrease of suction angle, occupies a dominant position, and the turning trend is less obvious. When $(u_a - u_w) = 0$, unsaturated soft soil becomes saturated soft soil, $p_{ct}$ and $p_{1/4}$ are less than the bearing capacity of unsaturated soft soil foundation. Therefore, the double influence of matric suction should be considered in the design and construction of soft soil foundation and fully utilize the bearing capacity of unsaturated soft soil significantly higher than that of saturated soil.

3.2.4. The Intermediate Principal Stress Effect (Strength Criterion Parameters $b$). Under plane strain state $\mu_0 = 0$, parameter $b$ reflects the effect of intermediate principal stress, which is the selection parameter of different strength criteria. Figure 6 shows the variation rules of critical edge load $p_{ct}$ and critical load $p_{1/4}$ with strength criterion parameters $b$ when $(u_a - u_w) = 0$, $k_0 = 1$, $\eta = 0.84$, and $p_0 = 0$.

It can be seen from Figure 6 that with the increase of $b$, $p_{ct}$ and $p_{1/4}$ increase nonlinearly. When $b$ increases from 0 to 1, $p_{ct}$ increases from 83.05 kPa to 120.69 kPa, with an increase of 50.77%, and $p_{1/4}$ increases from 117.36 kPa to 170.53 kPa, with an increase of 45.31%. In other words, considering the intermediate principal stress effect can give full play to the strength potential of soft soil and improve its bearing capacity. Therefore, the effect of intermediate principal stress should be reasonably considered in the design of soft soil foundation, which can give full play to the self-bearing capacity of materials, save materials, and reduce costs.

3.2.5. The Stress State of Foundation Soil (Static Lateral Pressure Coefficient $K_0$). The static lateral pressure coefficient $K_0$ reflects the initial stress state of foundation soil. The general soil mechanics textbook assumes that $K_0 = 1$; that is, the self-weight stress of the soil is a hydrostatic stress state, which artificially increases the lateral stress of the foundation soil and overestimates the bearing capacity of the foundation. But in practice, most of the soil is $K_0 \neq 1$. Figure 7 shows the variation rules of critical edge load $p_{ct}$ and critical load $p_{1/4}$ with static lateral pressure coefficient $K_0$ when $b = 0$, $(u_a - u_w) = 0$, $\eta = 0.84$, and $p_0 = 0$.

It can be seen from Figure 7 that with the increase of $K_0$, $p_{ct}$ and $p_{1/4}$ increase significantly. When $K_0 = 0.76$ ($K_0$ reaches its minimum when $M_0 = 0$ while $b = 0$), the difference between critical edge load and critical load is small, but the difference gradually becomes obvious with the increase of $K_0$. When $K_0$ increases from 0.76 to 1.0, the critical edge load increases 75.18%, and the critical load
increases 144%. That is to say, the artificial setting of $K_0 = 1$ significantly improves the bearing capacity of the foundation, which is seriously inconsistent with the actual. Therefore, the real static lateral pressure coefficient of the foundation should be reasonably measured in the design of soft soil foundation.

4. Conclusions

(1) The triple-shear unified solution of the foundation bearing capacity of unsaturated soft soil with hard crust is established in this paper, considering the comprehensive effects of the stress diffusion (stress diffusion coefficient $\eta$), the closure effect (increasing load $p_0$), the characteristics of unsaturated soil (matric suction $(u_a - u_w)$), the intermediate principal stress effect (strength criterion parameters $b$), and the actual stress state (static lateral pressure coefficient $K_0$). It contains different solutions under various working conditions, which is widely applicable to more engineering situations and has important theoretical significance and broad engineering application prospects.

(2) The critical edge load $p_{cr}$ and critical load $p_{1/4}$ increase with the increase of the ratio of elastic modulus between hard crust and soft soil and increase with the decrease of the stress diffusion coefficient, while linearly increase with the increase of $p_0$ (the load applied on the top of the first stage of the hard crust). Therefore, the foundation bearing capacity of unsaturated soft soil is increased by considering the stress diffusion and closure effect of hard crust.

(3) The influence of matric suction on critical edge load $p_{cr}$ and critical load $p_{1/4}$ is related to the values of matric suction. $p_{cr}$ and $p_{1/4}$ increase linearly with the increase of matric suction in the range of low suction and reach the peak point or the turning point when the matric suction equal to the air-entry value of unsaturated soft soil, while changes with the relative magnitude of the double influence due to the matric suction in range of the high suction.

(4) The critical edge load $p_{cr}$ and critical load $p_{1/4}$ increase nonlinearly with the increase of strength theoretical parameter $b$, and increase with the increase of static lateral pressure coefficient $K_0$. The calculation result of the foundation bearing capacity of unsaturated soft soil by Mohr-Coulomb criterion is too conservative, and the calculation by $K_0 = 1$ is seriously inconsistent with the actual. Therefore, the intermediate principal stress effect should be reasonably considered, and the real static lateral pressure coefficient of foundation soil should be measured.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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