Spin-valley interplay in two-dimensional disordered electron liquid

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We report the detailed study of the influence of the spin and valley splittings on such physical observables of the two-dimensional disordered electron liquid as resistivity, spin and valley susceptibilities. We explain qualitatively the nonmonotonic dependence of the resistivity with temperature in the presence of a parallel magnetic field. In the presence of either the spin splitting or the valley splitting we predict novel, with two maximum points, temperature dependence of the resistivity.

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I. INTRODUCTION

Disordered two-dimensional (2D) electron systems have been in the focus of experimental and theoretical research for several decades. Recently, the interest to 2D electron systems has been renewed because of the experimental discovery of metal-insulator transition (MIT) in a high mobility silicon metal-oxide-semiconductor field-effect transistor (Si-MOSFET)\(^1\)\(^2\)\(^3\). Although, during last decade the behavior of resistivity similar to that of Ref.\(^1\)\(^2\)\(^3\) has been found experimentally in a wide variety of two-dimensional electron systems\(^4\)\(^5\)\(^6\) the MIT in 2D calls still for the theoretical explanation.

Very likely, the most promising framework is provided by the microscopic theory, initially developed by Finkelstein, that combines disorder and strong electron-electron interaction\(^7\)\(^8\) Punnoose and Finkelstein\(^7\)\(^8\) have shown possibility for the MIT existence in the special model of 2D electron system with the infinite number of the spin and valley degrees of freedom. The current theoretical results\(^9\)\(^10\) do not support the MIT existence for electrons without the spin and valley degrees of freedom. Therefore, it is natural to assume that the spin and valley degrees of freedom play a crucial role for the MIT in the 2D disordered electron systems.

Usually, in the MIT vicinity, from the metallic side, i.e., for an electron density higher than the critical one, and at low temperatures \(T \ll \tau^{-1}_r\) the initial increase of the resistivity \((\rho)\) with lowering temperature is replaced by the decrease of \(\rho\) as \(T\) becomes lower than some sample specific temperature\(^10\)\(^11\). Here, \(\tau_r\) denotes the elastic scattering time. This nonmonotonic behavior of the resistivity has been predicted from the renormalization group (RG) analysis of the interplay between disorder and electron-electron interaction in the 2D disordered electron systems\(^12\)\(^13\)\(^14\) As a weak magnetic field \(B\) is applied parallel to the 2D plane, decrease of the resistivity is stopped at some temperature \(T\) and \(\rho\) increases again\(^15\)\(^16\). Further increase of \(B\) leads to the monotonic growth of the resistivity as temperature is lowered, i.e., to an insulating behavior, in the whole \(T\)-range. These experimental results suggest the significance of the electron spin for the existence of the metallic phase in the 2D disordered electron systems.

As is well known, in both Si-MOSFET\(^1\)\(^2\)\(^3\) and n-AlAs quantum well\(^11\) 2D electrons can populate two valleys. Therefore, these systems offer the unique opportunity for an experimental investigation of an interplay between the spin and valley degrees of freedom. Recently, using a symmetry breaking strain to tune the valley occupation of the 2D electron system in the n-AlAs quantum well, as well as a parallel magnetic field to adjust the spin polarization, the spin - valley interplay has been experimentally studied\(^12\)\(^13\)\(^14\). However, the electron concentrations in the experiment were at least three times larger than the critical one\(^12\)\(^13\)\(^14\). Therefore, the spin - valley interplay has been studied in the region of a good metal very far from the metal-insulator transition.

In the present paper we report the detailed theoretical results on the \(T\)-behavior of the 2D electron system with two valleys in the MIT vicinity. In particular, we study the effect of a parallel magnetic field and/or a valley splitting \((\Delta_v)\) on the transport, and the spin and valley susceptibilities. We find that in the presence of either the magnetic field or the valley splitting the metallic behavior of the resistivity survives down to the zero temperature\(^12\)\(^13\)\(^14\). For example, this result implies that at \(B = 0\) the metallic \(\rho (T)\) dependence can be observed experimentally at temperatures \(T \ll \Delta_v\). Only if both the magnetic field and the valley splitting are present, then the metallic behavior of the resistivity crosses over to the insulating one. Next, we predict novel, with two maximum points, \(T\)-behavior of the resistivity in the presence of the magnetic field and/or the valley splitting. Finally, we find that as \(T\) vanishes the ratio of the valley susceptibility \((\chi_v)\) to the spin one \((\chi_s)\) becomes sensitive to the ratio of the valley splitting to the spin one. At high temperatures the ratio \(\chi_v / \chi_s\) is temperature independent and can be chosen equal unity. If the spin splitting is larger (smaller) than the valley splitting, then at low temperatures the ratio \(\chi_v / \chi_s < (>) 1\). If the spin and valley splittings are equal each other, then the ratio \(\chi_v / \chi_s = 1\) as temperature vanishes.

The presence of the parallel magnetic field and the symmetry-breaking strain introduces new energy scales \(\Delta_v = g_L \mu_B B\) and \(\Delta_s\) in the problem. Here, \(g_L\) and \(\mu_B\) stand for the \(g\)-factor and the Bohr magneton, respectively. Let us assume that the following conditions
hold: $\Delta_{s} \ll \Delta_{\tau} \ll 1/\tau_{c}$. In addition, a magnetic field $B_{\perp} \gtrsim T/(D\varepsilon)$ is applied perpendicular to the 2D electron system in order to suppress the Cooper channel. Here, $e$ and $D$ denote the electron charge and diffusion coefficient, respectively. Due to the symmetry breaking, the spin and valley splittings set the cut-off for a pole in the diffusion modes (“diffusons”) with opposite spin and valley isospin projections. In the temperature range $\Delta_{\tau} \ll T \ll \tau_{c}^{-1}$ this cut-off is irrelevant and the 2D electron system behaves as if no symmetry breaking terms are applied. The temperature behavior of the resistivity is governed by one singlet and 15 triplet diffusive modes. At low temperatures $\Delta_{\tau} \ll T \ll \Delta_{s}$, eight diffusive modes with opposite spin projections do not contribute. Then, the $\rho(T)$ dependence is determined by the remaining one singlet and seven triplet modes. As we shall demonstrate below the behavior of the resistivity can be either metallic or insulating. Surprisingly, we found that the seven triplet diffusive modes are not equivalent. They have to split into two groups of six and one mode for the spin susceptibility to be $T$-independent. For temperatures $T \ll \Delta_{\tau}$, next four diffusive modes with opposite isospin projections become ineffective. In this case, the temperature dependence of the resistivity is determined by one singlet and three triplet diffusive modes. Although, the number of the remaining diffusive modes corresponds formally to single-valley electrons with spin, the $\rho(T)$ behavior is insulating.

The paper is organized as follows. In Section II we introduce the nonlinear sigma model that describes the disordered interacting electron system. Then, we consider the short length scales at which the system has $SU(4)$ symmetry in the combined spin and valley space (Sec. III). The behavior of the system at the intermediate and long length scales is studied in Sec. IV and Sec. V, respectively. We end the paper with discussions of our results and with conclusions (Sec. VI).

II. FORMALISM

A. Microscopic Hamiltonian

To start out, we consider 2D interacting electrons with two valleys in the presence of a quenched disorder and a parallel magnetic field at low temperatures $T \ll \tau_{c}^{-1}$. We assume that the perpendicular magnetic field $B_{\perp} \gtrsim T/(D\varepsilon)$ is applied in order to suppress the Cooper channel. Using one electron orbital functions, we write an electron annihilation operator as

$$\psi^{\sigma}(\mathbf{r}) = \sum_{\tau=\pm} \psi^{\sigma}_{\tau}(\mathbf{r}) \varphi(z) e^{i\tau z Q/2}, \quad (1)$$

where $z$ denotes the coordinate perpendicular to the 2D plane, $\mathbf{r}$ the in-plane coordinate vector, and $\mathbf{R} = \mathbf{r} + ze_{z}$. The subscript $\tau$ enumerates two valleys and $\psi^{\sigma}_{\tau}$ is the annihilation operator of an electron with the spin and isospin projections equal $\sigma/2$ and $\tau/2$, respectively. Let us assume that the wave functions $\varphi(z) \exp(\pm iQz/2)$ are normalized and orthogonal with negligible overlap $\int dz \varphi^{2}(z) \exp(iQz)$. The vector $\mathbf{Q} = (0, 0, Q)$ corresponds to the shortest distance between the valley minima in the reciprocal space: $Q \sim a_{\text{lat}}^{-1}$, with $a_{\text{lat}}$ being the lattice constant.

In the path-integral formulation 2D interacting electrons in the presence of the random potential $V(\mathbf{r})$ are described by the following grand partition function

$$Z = \int \mathcal{D}[\bar{\psi}, \psi] e^{S[\bar{\psi}, \psi]}, \quad (2)$$

with the imaginary time action

$$S = \int_{0}^{1/T} dt \left[ -\bar{\psi}^{\sigma}_{\tau}(\mathbf{r}, t) \partial_{t} \psi^{\sigma}_{\tau}(\mathbf{r}, t) - \mathcal{H}_{0} - \mathcal{H}_{\text{dis}} - \mathcal{H}_{\text{int}} \right]. \quad (3)$$

The one-particle Hamiltonian

$$\mathcal{H}_{0} = \int d\mathbf{r} \bar{\psi}^{\sigma}_{\tau}(\mathbf{r}) \left[ -\frac{\nabla^{2}}{2m_{\varepsilon}} - \mu + \frac{\Delta_{s}}{2} \sigma + \frac{\Delta_{\tau}}{2} \tau \right] \psi^{\sigma}_{\tau}(\mathbf{r}), \quad (4)$$

describes a 2D quasiparticle with mass $m_{\varepsilon}$ in the presence of the parallel magnetic field and the valley splitting. Here, $\mu$ denotes the chemical potential. Next,

$$\mathcal{H}_{\text{dis}} = \int d\mathbf{r} \bar{\psi}^{\sigma}_{\tau_{1}}(\mathbf{r}) V_{\tau_{1}\tau_{2}}(\mathbf{r}) \psi^{\sigma}_{\tau_{2}}(\mathbf{r}) \quad (5)$$

involves matrix elements of the random potential:

$$V_{\tau_{1}\tau_{2}}(\mathbf{r}) = \int dz \, V(\mathbf{r}) \varphi^{2}(z) e^{i(\tau_{2} - \tau_{1})Qz/2}. \quad (6)$$

In general, the matrix elements $V_{\tau_{1}\tau_{2}}(\mathbf{r})$ induce both the intravalley and intervalley scattering. We suppose that $V(\mathbf{r})$ has the Gaussian distribution, and

$$\langle V(\mathbf{r}) \rangle = 0, \quad \langle V(\mathbf{r}_{1}) V(\mathbf{r}_{2}) \rangle = W(|\mathbf{r}_{1} - \mathbf{r}_{2}|, |z_{1} - z_{2}|), \quad (7)$$

where the function $W$ decays at a typical distance $d$. If $d$ is larger than the effective width of the 2D electron system, i.e., $d \gg \langle dz \varphi^{4}(z) \rangle^{-1}$, then one can neglect the $z$-dependence of $V(\mathbf{r})$ under the integral sign in Eq. (6). In this case, the intravalley scattering survives only:

$$\langle V_{\tau_{1}\tau_{2}}(\mathbf{r}_{1}) V_{\tau_{1}\tau_{4}}(\mathbf{r}_{2}) \rangle = W(|\mathbf{r}_{1} - \mathbf{r}_{2}|, 0), \quad (8)$$

In the opposite case, $d \ll \langle dz \varphi^{4}(z) \rangle^{-1}$, one finds\textsuperscript{16}

$$\langle V_{\tau_{1}\tau_{2}}(\mathbf{r}_{1}) V_{\tau_{1}\tau_{4}}(\mathbf{r}_{2}) \rangle = \left[ \delta_{\tau_{1}\tau_{2}} \delta_{\tau_{1}\tau_{4}} W(|\mathbf{r}_{1} - \mathbf{r}_{2}|, 0) + \delta_{\tau_{1}\tau_{2}} \delta_{\tau_{1}\tau_{4}} W(|\mathbf{r}_{1} - \mathbf{r}_{2}|, 2Q) \right] \int dz \varphi^{4}(z). \quad (9)$$

where $\bar{W}(r, Q) = \int dz \, W(r, z) \exp(iQz)$. The other correlation functions, e.g., with $\tau_{1} = -\tau_{2}$ and $\tau_{1} = \tau_{4}$, vanish due to integration over $(z_{1} + z_{2})/2$ coordinate. It is the last term in Eq. (9) that contributes to the intervalley scattering rate $1/\tau_{c}$. Assuming $Q^{-1} \ll d$, one can neglect the intervalley scattering rate in comparison with
the intravalley scattering rate $1/\tau_i \sim \tilde{W}(r,0)$. At last, allowing for a low electron concentration $n_e$ in 2D electron systems, we consider the case when the following inequality holds, $n_e d^2 \ll 1$. Then, both Eqs. 8 and 9 read

$$\langle V_{\tau_1 \tau_2}(r_1)V_{\tau_3 \tau_4}(r_2) \rangle = \frac{1}{2\pi i \tau_1} \delta_{\tau_1 \tau_2} \delta_{\tau_3 \tau_4} \delta(r_1 - r_2),$$

where $S_\alpha$ represents the free electron part of the Coulomb interaction $H$.

Here, $\alpha = 1, 2, \ldots, 15$ are the non-trivial generators of the $SU(4)$ group.

**B. Nonlinear sigma model**

At low temperatures, $T \tau_e \ll 1$, the effective quantum theory of 2D disordered interacting electrons described by the Hamiltonian (4) is given in terms of the non-linear sigma-model. This theory involves unitary matrix field variables $Q_{\alpha \beta \gamma \delta}^{\tau_1 \tau_2}(r)$ which obey the nonlinear constraint $Q^2(r) = 1$. The integers $q_1 = 1, 2, \ldots, N_q$ denote the replica indices. The integers $m, n$ correspond to the discrete set of the Matsubara frequencies $\omega_n = \pi T (2n + 1)$. The integers $\alpha, \beta, \gamma, \delta$ are spin and valley indices, respectively. The effective action is

$$S = S_\sigma + S_F + S_{ab} + S_{eb} + S_0,$$

where $S_F$ involves the electron-electron interaction amplitudes which describe the scattering on small $(\Gamma)$ and large $(\Gamma_2)$ angles and the quantity $z$ originally introduced by Finkelstein which is responsible for the specific heat renormalization. The interaction amplitudes are related with the standard Fermi liquid parameters as $\Gamma_2 = -z F^{\alpha \beta}_0/(1 + F^{\alpha \beta}_0)$, $4\Gamma_2 = -z F^{\alpha \beta}_0/\Gamma^2_0$ and $\Gamma_2 = 2 \pi T z$ where $\Gamma^2_0$ is $\Gamma_0^2$. The effective mass $\gamma$ is $\gamma^2 = m'/(2\pi)$ with $m'$ being the effective mass.

The effective action is invariant under the global rotations $\delta_{\tau_1 \tau_2} \delta_{\tau_3 \tau_4}$.

**C. $J$-algebra**

The action (16) involves the matrices which are formally defined in the infinite Matsubara frequency space.
In order to operate with them we have to introduce a cut-off for the Matsubara frequencies. Then, the set of rules which is called $\mathcal{F}$-algebra can be established. At the end of all calculations one should tend the cut-off to infinity.

The global rotations of $Q$ with the matrix $\exp(i\hat{\chi})$ where $\hat{\chi} = \sum_{\alpha,n} \chi_n^\alpha I_n^\alpha$ play the important role. For example, $\mathcal{F}$-algebra allows us to establish the following relations

$$sp I_n^\alpha e^{i\hat{\chi}} Q e^{-i\hat{\chi}} = sp I_n^\alpha Q + 2im\chi_{\alpha,n}^t,
\text{tr} \eta e^{i\hat{\chi}} Q e^{-i\hat{\chi}} = \text{tr} Q + \sum_{\alpha,n} \text{in}(\chi_n^\alpha r_{t1}^\alpha) \text{sp} P_n^\alpha Q s,s_v^\alpha r_{t1}^\alpha$$

where $sp$ stands for the trace over replica and the Matsubara frequencies.

D. Physical observables

The most significant physical quantities in the theory containing information on its low-energy dynamics are physical observables $\sigma_{xx}$, $\chi_{xx}$, and $z_{xx}$, $z_{xx}$, $z_{xx}$, and $z_{xx}$ of the action (16). The observable $\sigma_{xx}$ is the DC conductivity as one can obtain from the linear response to an electromagnetic field. The observable $\chi_{xx}$ is associated with the static spin ($\chi_{xx}^t$) and valley ($\chi_{xx}^v$) susceptibilities of the 2D electron system as $\chi_{xx}^s = 2\chi_{xx}^t/\pi$. Extremely important to remind that the observable parameters $\sigma_{xx}$, $\chi_{xx}$, and $\chi_{xx}$ are precisely the same as those determined by the background field procedure.

The conductivity $\sigma_{xx}$ is obtained from

$$\sigma_{xx}(i\omega_n) = -\frac{i\omega_n}{16\pi} \text{tr}[I_n^\alpha Q][I_n^\alpha Q]$$

$$+\frac{i\omega_n}{64\pi n} \int dr \langle\{\text{tr} I_n^\alpha Q(r)\nabla Q(r) \text{tr} I_n^\alpha Q(r')\nabla Q(r')\rangle$$

after the analytic continuation to the real frequencies, $i\omega_n \rightarrow \omega + i0^+$ at $\omega \rightarrow 0$. Here, $\mathbb{D} = 2$ stands for the space dimension, and the expectations are defined with respect to the theory (16).

A natural definition of $z'$ is obtained through the derivative of the thermodynamic potential $\Omega$ per the unit volume with respect to $T$,

$$z' = \frac{1}{2\pi \text{tr} \eta \sigma} \frac{\partial}{\partial T} \Omega$$

The observables $z_{xx}$ are given as

$$z_{xx} = \frac{\pi}{2N^r} \frac{\partial^2 \Omega}{\partial \Delta^2_{xx}}$$

III. SU(4) SYMMETRIC CASE

A. $\mathcal{F}$-invariance

At short length scales $L \ll L_s, L_v$ where $L_{s,v} = \sqrt{\sigma_{xx}/(16\pi x_{s,v}\Delta_{s,v})}$, the symmetry breaking terms $S_{sb}$ and $S_{sb}$ can be omitted and the effective theory becomes $SU(4)$ invariant in the combined spin-valley space. Then, Eqs. (17) and (18) should be supplemented by the important constraint that the combination $z + \Gamma_2 - 4\Gamma$ remains constant in the course of the RG flow. Physically, it corresponds to the conservation of the particle number in the system. In the special case of the Coulomb or other long-ranged interactions which are of the main interest for us in the paper the relation

$$z + \Gamma_2 - 4\Gamma = 0$$

holds. With the help of Eqs. (22), one can check that Eq. (20) guarantees the so-called $\mathcal{F}$-invariance of the action $S_{r} + S_{F}$ under the global rotation of the matrix $Q$:

$$Q(r) \rightarrow e^{i\chi} Q(r) e^{-i\chi}, \quad \hat{\chi} = \sum_{\alpha,n} \chi_n^\alpha I_n^\alpha.$$

Here, $\chi_n^\alpha$ is the unit matrix in the spin-valley space. In virtue of Eq. (21), it is convenient to introduce the triplet interaction parameter $\gamma = \Gamma_2/\gamma$ such that $\Gamma = (1+\gamma)/4$. We notice that the triplet interaction parameter is related with $F_0^\alpha$ as $\gamma = -F_0^\alpha/(1 + F_0^\alpha)$.

B. Perturbative expansions

To define the theory for the perturbative expansions we use the “square-root” parameterization

$$Q = W + \lambda \sqrt{1 - W^2}, \quad W = \begin{pmatrix} 0 & w \\ 0 & 0 \end{pmatrix}$$

The action (10) can be written as the infinite series in the independent fields $w_{n1}^\alpha a^\dagger_1 a_1^\sigma_1$ and $w_{n2}^\alpha a^\dagger_2 a_2^\sigma_2$. We use the convention that the Matsubara frequency indices with odd subscripts $n_1, n_3, \ldots$ run over non-negative integers whereas those with even subscripts $n_2, n_4, \ldots$ run over negative integers. The propagators can be written in the following form...
\[ \langle w^{\alpha_1 \alpha_2} \sigma_1 \sigma_2 (p) w^{\alpha_3 \alpha_4} \sigma_3 \sigma_4 (-p) \rangle = \frac{16}{\sigma_{xx}} \delta^{\alpha_1 \alpha_3} \delta^{\alpha_2 \alpha_4} \delta_{n_1 n_3} \delta_{n_2 n_4} \left\{ \delta^{\sigma_1 \sigma_3} \delta^{\sigma_2 \sigma_4} \delta_{\tau_1 \tau_3} \delta_{\tau_2 \tau_4} \left[ \delta_{n_1 n_3} D_p(\omega_{12}) - \frac{32 \pi T \gamma}{\sigma_{xx}} \delta^{\alpha_1 \alpha_2} \right] \right\}, \]  

(29)

where \( \omega_{12} = \omega_{n_1} - \omega_{n_2} \) and

\[ D_p^{-1}(\omega_n) = p^2 + \frac{16 z \omega_n}{\sigma_{xx}}, \quad [D_p^*(\omega_n)]^{-1} = p^2, \]  

(30)

\[ [D_p^*(\omega_n)]^{-1} = p^2 + \frac{16 (z + \Gamma_2) \omega_n}{\sigma_{xx}}. \]

C. Relation of \( z_s, \gamma \) with \( z, \gamma \)

The dynamical spin susceptibility \( \chi_s(\omega, p) \) can be obtained from

\[ \chi_s(i\omega_n, p) = \chi_s^0 - T \gamma^2 \left( \text{tr} I_{\alpha \alpha} \sigma_3 Q(p) \text{tr} I_{\beta \beta} \sigma_3 Q(-p) \right) \]  

(31)

by the analytic continuation to the real frequencies, \( i\omega_n \to \omega + i0^+ \). Similar expression is valid for the valley susceptibility. Evaluating Eq. (31) in the tree level approximation with the help of Eqs. (29), we obtain

\[ \chi_s(i\omega_n, p) = \frac{2z}{\pi} \left( 1 - \frac{16 z \omega_n}{\sigma_{xx}} D_p^*(\omega_n) \right). \]  

(32)

In the case \( \Delta_s = \Delta_v = 0 \) the total spin conserves, i.e., \( \chi(\omega, p = 0) = 0 \). In order to be consistent with this physical requirement, the relation

\[ z_s = z + \Gamma_2 \equiv z(1 + \gamma) \]  

(33)

should hold. Similarly, the total valley isospin conservation guarantees that

\[ z_v = z + \Gamma_2 \equiv z(1 + \gamma). \]  

(34)

Being related with the conservation laws, Eqs. (33) and (34) are valid also for the observables:

\[ z'_s = z'_v = z'(1 + \gamma'). \]  

(35)

Therefore, three physical observables \( z_s, z_v \) and \( \gamma \) completely determines the renormalization of the theory at short length scales \( L \ll L_s, L_v \).

D. One loop renormalization group equations

As is shown in Ref. [3], the standard one-loop analysis for the action \( S_{\pi} + S_F \) yields the following renormalization group functions that determine the zero-temperature behavior of the observable parameters with changing of the length scale \( L \)

\[ \frac{d\sigma_{xx}}{d\xi} = \beta_\sigma = -\frac{2}{\pi} \left[ 1 + 15 f(\gamma) \right], \]  

(36)

\[ \frac{d\gamma}{d\xi} = \beta_\gamma = (1 + \gamma)^2, \]  

(37)

\[ \frac{d\ln z}{d\xi} = \gamma_z = 15 \gamma - 1. \]  

(38)

Here, \( f(\gamma) = 1 - (1 + \gamma^{-1}) \ln(1 + \gamma) \), \( \xi = \ln L/l \) and we omit prime signs for a brevity. Physically, the microscopic length \( l \) is the mean-free path length. It is the length at which the bare parameters of the action (16) are defined. Renormalization group Eqs. (36)–(38) are valid at short length scales \( L \ll L_s, L_v \).

As is well-known, a solution of the RG Eqs. (36)–(37) yields the dependence of the resistivity \( \rho = \pi \sigma_{xx} \) on \( \xi \) which has the maximum point and \( \gamma(\xi) \) dependence that monotonically increases with \( \xi \).

IV. \( SU(2) \times SU(2) \) SYMMETRY CASE

A. Effective action

In this and next sections we assume that the spin splitting is much larger than the valley splitting, \( \Delta_s \gg \Delta_v \). Then, at intermediate length scales \( L_s \ll L \ll L_v \) the symmetry breaking term \( S_{sb} \) becomes important. In the quadratic approximation it reads

\[ S_{sb} = \int \frac{d\xi}{2} \sum_{n_1 \tau_1, n_2 \tau_2} \left( \sigma_2 - \sigma_1 \right) \left[ w^{\alpha_1 \alpha_2} \sigma_1 \sigma_2 \right] \left[ w^{\alpha_3 \alpha_4} \sigma_3 \sigma_4 \right]. \]  

(39)

Hence, the modes in \( Q_{n_1 n_2 \tau_1 \tau_2}^{\alpha_3 \alpha_4} \sigma_3 \sigma_4 \) with \( \sigma_1 \neq \sigma_2 \) acquire a finite mass of the order of \( \Delta_s \), and, therefore, are negligible at length scales \( L \gg L_s \). As a result, \( Q \) becomes a diagonal matrix in the spin space. Then, the spin susceptibility has no renormalization on these length scales, i.e.,

\[ \frac{d\Delta_s}{d\xi} = 0, \quad L_s \ll L \ll L_v. \]  

(40)

Let us denote \( Q_{n_1 n_2 \tau_1 \tau_2}^{\alpha_3 \alpha_4} \sigma_3 \sigma_4 \). Then, the ac-
tion (10) becomes $S = S_\sigma + S_F + S_{sb}$ where
\[ S_\sigma = -\frac{\sigma_{xx}}{32} \sum_{\sigma=\pm} \int d^2r \text{tr}(\nabla Q_\sigma)^2 \quad (41) \]
and
\[ S_F = 4\pi T z \sum_{\sigma} \int d^2r \text{tr}\eta(Q_\sigma - \Lambda) \quad (42) \]
\[ + \pi T \int d^2r \sum_{\alpha \sigma_1,\sigma_2=\pm} \Gamma_{\sigma_1\sigma_2} \text{tr} I_\sigma^n Q_{\sigma_1} \text{tr} I_\sigma^- n Q_{\sigma_2} \]
\[ - \pi T \Gamma_2 \int d^2r \sum_{\alpha \sigma=\pm} (\text{tr} I_\sigma^n Q_\sigma) \otimes (\text{tr} I_\sigma^- n Q_\sigma). \]

Now, the symbol $\text{tr}$ stands for the trace over replica, the Matsubara frequencies, and the valley indices whereas $Tr = \int d^2r \text{tr}$. The action (42) corresponds to the following low energy part of the Hamiltonian describing electron-electron interactions:
\[ \mathcal{H}_{int} = \frac{1}{2} \int d\sigma \left[ \rho^\sigma \Gamma_\sigma \rho^\sigma + m^a \Gamma_1 m^a \right], \quad (43) \]
\[ \rho^\sigma = \sum_{\sigma\tau} \bar{\psi}_{\tau}^\sigma \psi_{\tau}^\sigma, \quad m^a = \sum_{\sigma\tau\sigma'} \bar{\psi}_{\tau}^{\sigma'} (t^a)_{\sigma\sigma'} \psi_{\tau}^\sigma. \]

It is worthwhile mentioning that Eq. (43) is in agreement with the ideas of Ref. 27,28.

The symmetry breaking part reads
\[ S_{sb} = iz_\alpha \Delta_{\alpha} \sum_{\sigma=\pm} \int d^2r \text{tr}\tau_\sigma Q_\sigma. \quad (44) \]

At length scales $L \sim L_s$, the couplings $\Gamma_{\sigma_1\sigma_2}$ are all equal to each other, $\Gamma_{\sigma_1\sigma_2}(L \sim L_s) = \Gamma = (z + \Gamma_2)/4$. However, the symmetry allows the following matrix structure of $\hat{\Gamma}$:
\[ \hat{\Gamma} = \begin{pmatrix} \Gamma_{++} & \Gamma_{+-} \\ \Gamma_{-+} & \Gamma_{++} \end{pmatrix}. \quad (45) \]

As we shall see below, this matrix structure is consistent with the renormalization group. Physically, $\Gamma_{++}$ and $\Gamma_{+-}$ describe interactions between electrons with the same and opposite spins, respectively.

The action (41) and (42) is invariant under the global rotations $[Q_{nm;\tau_1\tau_2}^\sigma(r) \rightarrow u_{\sigma_{\tau_1\tau_2}}(r)]Q_{nm;\tau_1\tau_2}(r) [u_{\sigma_{\tau_1\tau_2}}^{-1}(r)]$ in the valley space for $u_{\sigma} \in SU(2)$. In order to preserve the invariance under the global rotations:
\[ Q_\pm(r) \rightarrow e^{i\chi} Q_\pm(r)e^{-i\chi}, \quad \chi = \sum_{\alpha n} \chi_\alpha^n I_\alpha^n, \quad (46) \]
where $\chi_\alpha^n$ is the unit matrix in the valley space, the following relation has to be fulfilled:
\[ z + \Gamma_2 - 2\Gamma_{++} = 2\Gamma_{+-}. \quad (47) \]

Physically, this equation corresponds to the particle number conservation and is completely analogous to Eq. (20).

### B. Perturbative expansions

In order to resolve the constraint $Q_\pm^2 = 1$ we use the “square-root” parameterization:
\[ Q_\pm = W_\pm + \Lambda\sqrt{1 - W_\pm^2}. \quad (48) \]

Then, the action (41) and (42) determines the propagators as follows:
\[ (w_{n_1n_2}^{\alpha_1\alpha_2;\tau_1\tau_2}(q)_{\sigma}[w_{n_4n_3}^{\alpha_3\alpha_4;\tau_3\tau_4}(-q)]_{\sigma'}) = \frac{32}{\sigma_{xx}} \hat{D}_{\sigma\sigma'}. \quad (49) \]

where
\[ \hat{D} = \delta^{\alpha_1\alpha_2} \delta^{\alpha_3\alpha_4} \delta_{n_1,n_2} \left[ \delta_{n_1,n_3} \delta_{\tau_1\tau_3} \delta_{\tau_2\tau_4} D_{q}(\omega_{12},\tau_1,\tau_2) \right] \]
\[ - \frac{32\pi T}{\sigma_{xx}} \Gamma_2 \delta^{\alpha_1\alpha_2} \delta^{\alpha_3\alpha_4} \delta_{\tau_1\tau_3} \delta_{\tau_2\tau_4} D_{q}(\omega_{12},\tau_1,\tau_2) D_{q}^{\dagger}(\omega_{12},\tau_1,\tau_2) \]
\[ + \frac{32\pi T}{\sigma_{xx}} \delta^{\alpha_1\alpha_2} \delta^{\alpha_3\alpha_4} \delta_{\tau_1\tau_3} \delta_{\tau_2\tau_4} \delta_{\tau_3\tau_4} D_{q}(\omega_{12}) D_{q}^{\dagger}(\omega_{12}) \quad (50) \]

with
\[ [\hat{D}_{q}^{\dagger}(\omega_{n})]^{-1} = q^2 + \frac{16}{\sigma_{xx}}(z + \Gamma_2 - 2\Gamma)\omega_{n}. \quad (51) \]

\[ D_{q}^{-1}(\omega_{n},\tau_1,\tau_2) = D_{q}^{-1}(\omega_{n}) + i \frac{8z_{\alpha} \Delta_{\alpha}}{\sigma_{xx}}(\tau_1 - \tau_2). \quad (52) \]

\[ [D_{q}^{\dagger}(\omega_{n},\tau_1,\tau_2)]^{-1} = [D_{q}^{\dagger}(\omega_{n})]^{-1} + i \frac{8z_{\alpha} \Delta_{\alpha}}{\sigma_{xx}}(\tau_1 - \tau_2). \quad (53) \]

In the same way as in Sec. IIIC the conservation of the total valley isospin guarantees the relation $z_{\alpha} = z + \Gamma_2$. The conservation of the $z$-component of the total spin, $\rho^+ - \rho^-$, implies that $z_{\alpha} = 4\Gamma_{++}$ (see Eq. (31)). Therefore,
\[ \frac{d\ln \Gamma_{++}}{d\xi} = 0 \quad (54) \]

for the length scales $L_s \ll L \ll L_{\alpha}$.

### C. One-loop approximation

Evaluation of the conductivity according to Eq. (23) in the one-loop approximation yields
\[ \sigma'_{xx}(i\omega_n) = \sigma_{xx} + \frac{2\pi}{\sigma_{xx}} \int p^2 T \sum_{\omega_{n}>0} \min\left\{ \frac{\omega_m}{\omega_{n}}, 1 \right\} D_{p}^{\dagger}(\omega_{m}) \]
\[ \times D_{p}(\omega_{m} + \omega_{n}) \left[ \sum_{\sigma=\pm}(\hat{\Gamma} \delta_{p}^{\dagger}(\omega_{m}))_{\sigma\sigma'} - 4\Gamma_2 D_{p}(\omega_{m}) \right]. \quad (55) \]

Hence, we find
\[ \sigma'_{xx}(i\omega_n) = \sigma_{xx} + \frac{2\pi}{\sigma_{xx}} \int p^2 T \sum_{\omega_{n}>0} \min\left\{ \frac{\omega_m}{\omega_{n}}, 1 \right\} D_{p}(\omega_{m}) \]
\[ \times D_{p}(\omega_{m} + \omega_{n}) \left[ (\hat{\Gamma} \delta_{p}^{\dagger}(\omega_{m}))_{\sigma\sigma'} - 6\Gamma_2 D_{p}(\omega_{m}) \right] \quad (56) \]
where
\[ [\hat{D}_n^\dagger(n)]^{-1} = q^2 + \frac{64}{\sigma_{xx}} \omega_n \Gamma_{+-}. \] (57)

Performing the analytic continuation to the real frequencies, \(i\omega_n \to \omega + i0^+\) in Eq. (54), one obtains the DC conductivity in the one-loop approximation:
\[ \sigma_{xx}^\prime = \sigma_{xx} - \frac{2^8 \pi}{D_0 \sigma_{xx}} \int_{p}^{p_2} d\omega D_p^2(\omega) \left[ z D_p^\dagger(\omega) \right. \]
\[ - 6\Gamma_2 D_p^0(\omega) \left. - (z + 2\Gamma_2 - 4\Gamma_{+-}) \hat{D}_p^\dagger(\omega) \right] \] (58)

In order to compute \(z'\) and \(z''\) we have to evaluate the thermodynamic potential \(\Omega\) in the presence of the finite valley splitting \(\Delta_v\). In the one-loop approximation we find
\[ T^2 \frac{\partial \Omega}{\partial T} = 8N_\tau T \sum_{\omega_n > 0} \omega_n \left[ z + \frac{4}{\sigma_{xx}} \int_{p}^{p_2} 2\Gamma_0 \hat{\chi}_p(\omega_n) \right. \]
\[ \left. (z + \Gamma_2) D_p^0(\omega_n) - (z + 2\Gamma_2) \sum_{\tau_1, \tau_2} D_p^\dagger(\omega_n, \tau_1, \tau_2) \] (59)

Following definitions (24) and (25) of the physical observables, we obtain from Eq. (59)
\[ z' = z + \frac{8}{\sigma_{xx}} (2\Gamma_2 - \Gamma_{++}) \int_{p} D_p(0) \] (60)
and
\[ z'' = z \left[ 1 + 4\pi \left( \frac{16}{\sigma_{xx}} \right)^3 (z + \Gamma_2) T \sum_{\omega_n > 0} \omega_n \int_{p} z D_p^3(\omega_n) \right. \]
\[ \left. -(z + \Gamma_2) D_p^0(\omega_n) \right]. \] (61)

We mention that the results (58), (60), (61) can be obtained with the help of the background field procedure applied to the action (11)-(12).

**D. One loop RG equations**

Using the standard method, we derive from Eqs. (58), (60), and (61) one-loop results for the RG equations which determine the \(T = 0\) behavior of the physical observables with changing the length scale \(L\). It is convenient to define \(\gamma_v = \Gamma_2 / z\) and \(\gamma_s = -1 + 4\Gamma_{+-} / z\). Then, for \(D = 2\) we obtain
\[ \frac{d\sigma_{xx}}{d\xi} = -\frac{2}{\pi} \left[ 1 + 6f(\gamma_v) + f(\gamma_s) \right] \] (62)
\[ \frac{d\gamma_v}{d\xi} = \frac{1 + \gamma_v}{\pi \sigma_{xx}} (1 + 2\gamma_v - \gamma_s) \] (63)
\[ \frac{d\gamma_s}{d\xi} = \frac{1 + \gamma_s}{\pi \sigma_{xx}} (1 - 6\gamma_v - \gamma_s) \] (64)
\[ \frac{d\ln z}{d\xi} = -\frac{1}{\pi \sigma_{xx}} [1 - 6\gamma_v - \gamma_s]. \] (65)

**FIG. 1:** The projection of the RG flow in the three dimensional parameter space \((\sigma_{xx}, \gamma_v, \gamma_s)\) onto \((\gamma_v, \gamma_s)\) plane for the \(SU(2) \times SU(2)\) symmetry case (Eqs. (62)-(65)). Dots denote the line at which \(1 + 6f(\gamma_v) + f(\gamma_s) = 0\). The dashed line indicates the line \(\gamma_v = \gamma_s\) (see text).

**FIG. 2:** Schematic dependence of the resistance \(\rho = 1/(\pi \sigma_{xx})\) on \(\xi\). Curves a, b, and c corresponds to the flow lines a, b, and c in Fig. 1 (see text).

Eqs. (62)-(65) constitute one of the main results of the present paper and describe the system at the intermediate length scales \(L_s \ll L \ll L_v\). We mention that the length scale \(l\) involved in \(\xi = \ln L / l\) is now of the order of \(L_s\).

In Figure 1 we present the projection of the RG flow in the three dimensional parameter space \((\sigma_{xx}, \gamma_v, \gamma_s)\) onto \((\gamma_v, \gamma_s)\) plane. There is the unstable fixed point at \(\gamma_v = 0\) and \(\gamma_s = 1\). However, for the physical system considered the fixed point is inaccessible since an initial point of the RG flow is always situated near the line \(\gamma_v = \gamma_s\). As shown in Fig. 2 there are possible three distinct types of the \(\rho(\xi)\) behavior for such initial points. Along the RG flow line a (Fig. 1) that crosses the curve \(d\) described by the equation \(1 + 6f(\gamma_v) + f(\gamma_s) = 0\) the resistance demonstrates the metallic behavior: \(\rho\) decreases as \(\xi\) grows. If we move along the RG flow line \(b\) which intersects the curve \(d\) twice, then the resistance develops the minimum and the maximum. At last, the resistance on the RG flow line \(c\) which has single crossing with the curve \(d\) has the maximum. Remarkably, in all three cases, the behavior of
the resistance is of the metallic type for relatively large $L$. The reason of this metallic behavior can be understood from the following arguments. At large $\xi$, the coupling $\gamma_\nu$ flows to large positive values whereas $\gamma_\nu \to -1$. Then, $\Gamma_+^+/\Gamma_++ \sim 1/\gamma_\nu \ll 1$ and the RG Eqs. (62) transforms into equations for the single valley system with the conductance equal $\sigma_{xx}/2$. The metallic behavior of this system is well-known.\[\]

V. COMPLETELY SYMMETRY BROKEN CASE

A. Effective action

At the long length scales $L \gg L_v$ the symmetry breaking term $S_{sb}$ becomes important. In the quadratic approximation it reads

$$S_{sb} = \frac{i z_0 \Delta_0}{2} \int d^2 \tau \left( \tau_2 - \tau_1 \right) \left[ \omega_{a_1 a_2} n_{1 \tau_1 \tau_2} \sigma \omega_{a_2 a_1} n_{2 \tau_2 \tau_1} \sigma \right].$$

Hence, the modes in $[Q^{\alpha \beta}_{mn,\tau_1 \tau_2}]_\nu$ with $\tau_1 \neq \tau_2$ acquire a finite mass of the order of $z_0 \Delta_0$. Therefore, they are negligible at long length scales $L \gg L_v$. As the result, the matrix $Q$ becomes diagonal matrix in the valley isospin space. The valley susceptibility remains constant under the action of the renormalization group on these length scales:

$$\frac{dz_0}{d\xi} = 0, \quad L \gg L_v.$$  (67)

Let us define

$$Q^{\alpha \beta}_j = \{ [Q^{\alpha \beta}_{11}]_+, [Q^{\alpha \beta}_{-1-1}]_+, [Q^{\alpha \beta}_{11}]_-, [Q^{\alpha \beta}_{-1-1}]_- \}.$$  (68)

Then the action $S = S_\sigma + S_F$ reads

$$S_\sigma = -\frac{\sigma_{xx}}{32} \sum_{j} \int d^2 \tau \left( \nabla Q_j \right)^2$$

and

$$S_F = \pi T \sum_{j,k} \sum_{\alpha \beta} \left( I^\alpha_n Q_j \hat{\Gamma}_{jk} \right)  \left( I_{-n}^\beta Q_k \right)$$

$$+ 4 \pi T^2 \sum_{\alpha \beta} \int d^2 \tau \eta Q_j,$$  (70)

where

$$\hat{\Gamma} = \left( \begin{array}{cccc}
\Gamma_+^+ - \Gamma_2 & \tilde{\Gamma}_++ & \tilde{\Gamma}_+^- & \Gamma_+^-
\Gamma_+^+ & \Gamma_+^+ - \Gamma_2 & \tilde{\Gamma}_+^- & \Gamma_+^-
\Gamma_+^- & \Gamma_+^- & \Gamma_+^+ - \Gamma_2 & \tilde{\Gamma}_+^+
\Gamma_+^- & \Gamma_+^- & \tilde{\Gamma}_+^+ & \Gamma_+^+ - \Gamma_2
\end{array} \right).$$  (72)

Initially, at the length scale of the order of $L_v$, the coupling $\Gamma_+^+ = \Gamma_2$ and $\Gamma_+^- = \Gamma_+^-$. However, more general structure (72) is consistent with the renormalization group. It is worthwhile to mention that if the matrix $\tilde{\Gamma}$ is diagonal then the theory (69) and (71) would include four copies of the singlet $U(1)$ theory studied in Refs. [78]. The action (71) corresponds to the following low energy part of the electron-electron interaction Hamiltonian:

$$H_{\text{int}} = \frac{1}{2} \int d\tau \sum_{\sigma \sigma', \tau \tau'} \rho^\sigma_\tau \left[ \left( \Gamma_\sigma \right)^{\sigma \sigma'}_{\tau \tau'} + \Gamma_\epsilon \left( t^\sigma \right)^{\sigma \sigma'}_{\tau \tau'} \left( t^\epsilon \right)^{\epsilon \epsilon'}_{\tau \tau'} \right] \rho^\epsilon_\tau$$

$$\rho^\sigma_\tau = \bar{\psi}^\tau \psi^\sigma$$  (73)

In order to have the invariance under the global rotations

$$Q_j \to e^{i \tilde{\chi}} Q_j e^{-i \tilde{\chi}}, \quad \tilde{\chi} = \sum_{\alpha \beta} \chi^\alpha n_\alpha^\beta,$$  (74)

the following relation has to be fulfilled

$$z + \Gamma_2 - \Gamma_+^+ - \tilde{\Gamma}_++ = \Gamma_+^- + \tilde{\Gamma}_+^-.$$  (75)

B. Perturbative expansions

As above, in order to resolve the constraints $Q_j^2 = 1$, we shall use the “square-root” parameterization for each $Q_j$: $Q_j = W_j + \Lambda \sqrt{1 - W_j^2}$. Then, the propagators are defined by the theory (69) and (71) as

$$\langle [w_{n_{a_1 a_2}}(q)]_j (w_{n_{a_1 a_2}}^* q) \rangle_k = \frac{32}{\sigma_{xx}} \hat{D}_{jk},$$  (76)

$$\hat{D} = \delta^{a_1 a_3} \delta^{a_2 a_4} \delta_{n_{a_1 a_2}} \delta_{n_{a_3 a_4}} \left[ \delta_{n_1 n_3} D_q(\omega_1) + \frac{32 \pi T}{\sigma_{xx}} \tilde{\Gamma} \delta^{a_1 a_3} \right] \times D_q(\omega_1) \hat{D}_q^{\gamma}(\omega_1),$$  (77)

where

$$[\hat{D}_q^{\gamma}(\omega_n)]^{-1} = q^2 + \frac{16}{\sigma_{xx}} (z - \Gamma) \omega_n.$$  (78)

The conservation of the $z$-components of the total spin, $\sum_{\sigma \tau} \sigma \rho^\tau_\sigma$, and the total valley isospin, $\sum_{\sigma \tau} \tau \rho^\tau_\sigma$, implies (see Eq. (68)) that $z_s = 2 \Gamma_+^+ + 2 \Gamma_+^-$ and $z_v = 2 \tilde{\Gamma}_+^+ + 2 \tilde{\Gamma}_+^-$. Since, for $L \gg L_v$ both $z_s$ and $z_v$ are not renormalized, we obtain

$$\frac{d \tilde{\Gamma}_+^+}{d \xi} = \frac{d \Gamma_+^+}{d \xi} = \frac{d \tilde{\Gamma}_+^-}{d \xi} = \frac{d \Gamma_+^-}{d \xi} = 0.$$  (79)

Since, both $\tilde{\Gamma}_+^+$ and $\Gamma_+^-$ coincides at the length scales $L \sim L_v$ and they are not renormalized we shall not distinguish $\Gamma_+^+$ and $\Gamma_+$ from here onwards. If we introduce $\gamma_s$ and $\gamma_v$ such that $\Gamma_+ = \Gamma_+^+ + \gamma_s + \gamma_v$ then both $\gamma_s$ and $\gamma_v$ coincide with the corresponding couplings of the previous sections at the length scales $L \sim L_v$.\[\]
C. One-loop approximation

Evaluating the conductivity with the help of Eq. 23 in the one-loop approximation, we find

\[ \sigma'_{xx}(i\omega_n) = \sigma_{xx} + \frac{2^8 \pi^2}{D \sigma_{xx}} \int p^2 T \sum_{\omega_m > 0} \min \left\{ \frac{\omega_m}{\omega_n}, 1 \right\} \]

\[ D_p(\omega_m + \omega_n) D_p(\omega_m) \sum_j \left( \tilde{D}_p(\omega_m) \right)_{jj}. \quad (80) \]

Hence,

\[ \sigma'_{xx}(i\omega_n) = \sigma_{xx} + \frac{2^8 \pi^2}{D \sigma_{xx}} \int p^2 T \sum_{\omega_m > 0} \min \left\{ \frac{\omega_m}{\omega_n}, 1 \right\} \]

\[ D_p(\omega_m + \omega_n) D_p(\omega_m) \left[ D^s_p(\omega_m) - 2\gamma_\nu \tilde{D}^t_p(\omega_m) - \gamma_s \tilde{D}^t_s(\omega_m) \right] \quad (81) \]

where

\[ \left[ \tilde{D}^t_q(\omega_n) \right]^{-1} = q^2 + \frac{32}{\sigma_{xx}}(\Gamma_{++} + \tilde{\Gamma}_{++})\omega_n. \quad (82) \]

Performing the analytic continuation to the real frequencies in Eq. (81), we find

\[ \sigma'_{xx} = \sigma_{xx} - \frac{2^8 \pi^2}{D \sigma_{xx}} \int p^2 \int_0^{\infty} d\omega D^2_p(\omega) \left[ D^s_p(\omega) - 2\gamma_\nu \tilde{D}^t_p(\omega) - \gamma_s \tilde{D}^t_s(\omega) \right]. \quad (83) \]

As in the previous Section, in order to compute \( z' \) we evaluate the thermodynamic potential in the one-loop approximation. The result is

\[ T^2 \frac{\partial \Omega \langle T \rangle}{\partial T} = 8TN_c \sum_{\omega_n > 0} \omega_n \left[ 1 + \frac{8}{\sigma_{xx}} \int p \left( \frac{1 + \gamma_\nu}{2} \tilde{D}^t_p(\omega_n) \right) \right] \]

\[ + (1 + \gamma_\nu) \tilde{D}^t_p(\omega_n) - 2D_p(\omega_n) \right] \]. \quad (84) \]

Hence, we obtain

\[ z' = z + \frac{16}{\sigma_{xx}}(\Gamma_2 - \Gamma_{++}) \int p D_p(0). \quad (85) \]

We mention that the results (79), (83), and (85) can be obtained with the help of the background field procedure applied to the action (69)-(71).

D. One loop RG equations

Equations (79), (83) and (85) allow us to derive the following one-loop results for the renormalization group functions which determine the \( T = 0 \) behavior of the physical observables with changing the length scale \( L \):

The renormalization group equations (86)-(89) constitute one of the main results of the present paper. We mention that the length scale \( l \) involved in \( x = \ln L/l \) is now of the order of \( L_0 \) and Eqs. (86)-(89) describe the system at the long length scales \( L \gg L_0 \).

The projection of the RG flow for Eqs. (86)-(88) on the \( \gamma_\nu - \gamma_s \) plane is shown in Fig. 3. There exits the line of the fixed points that is described by the equation

\[ 2\gamma_\nu + \gamma_s = 1. \]

If the initial point has large \( \gamma_\nu \) or \( \gamma_s \) then the RG flow line crosses the curve that is determined by the condition \( 1 + 2f(\gamma_\nu) + f(\gamma_s) = 0 \). Therefore, the \( \rho(\xi) \) dependence along the RG flow line develops the minimum
and will be of the insulating type as is shown in Fig. 4.

VI. DISCUSSIONS AND CONCLUSIONS

The renormalization group equations discussed above describe the $T = 0$ behavior of the observable parameters with changing of the length scale $L$. At finite temperatures $T \gg \sigma_{xx}/(zL^2)$, where $L_{\text{sample}}$ is the sample size, the temperature behavior of the physical observables can be found from the RG equations stopped at the inelastic length $L_{\text{in}}$ rather than at the sample size. Formally, it means that one should substitute $\xi_T = \frac{1}{2} \ln \sigma_{xx}/(zT^2)$ for $\xi$ in the RG equations with $\xi_T$ obeying the following equation:

$$\frac{d\xi_T}{d\xi} = 1 - \frac{1}{2} \frac{d\ln z}{d\xi}. \quad (90)$$

Having in mind Eq. (90), we find that the $T$-behavior of the resistivity at $B = 0$ is described by Eqs. (36) and (37) for $T \gg \Delta_v$ and Eqs. (22) and (61) with interchanged $\gamma_v$ and $\gamma_s$ for $T \ll \Delta_v$. In what follows, we assume that $\Delta_v < T_{\text{max}}^{(I)}$ where $T_{\text{max}}^{(I)}$ denotes the temperature of the maximum point that appears in $\rho(T)$ according to the RG Eqs. (36) and (37). Our assumption is consistent with the experimental data in Si-MOSFET where, for example, the valley splitting is of the order of hundreds of $mK$ and $T_{\text{max}}^{(I)}$ is about several Kelvin. Then, depending on the initial conditions at $T \sim 1/\tau$ two types of the $\rho(T)$ behavior are possible as is shown in Fig. 5. The curve $a$ represents the typical $\rho(T)$ dependence that was observed in transport experiments on two-valley 2D electron systems in Si-MOSFET and n-AlAs quantum well. Surprisingly, the other behavior with the two maximum points is possible, as illustrated by curve $b$ in Fig. 5. So far, this interesting non-monotonic $\rho(T)$ dependence has been neither observed experimentally nor predicted theoretically. At very low temperatures $T \ll \Delta_v$, the metallic behavior of $\rho(T)$ wins even in the presence of the valley splitting.

In the presence of the sufficiently low parallel magnetic field $\Delta_v < T_{\text{max}}^{(I)}$, the $\rho(T)$ behavior of three distinct types is possible as plotted in Fig. 6. In all three cases, the $\rho(T)$ dependence has the maximum point at temperature $T = T_{\text{max}}^{(I)}$ and is of the insulating type as $T \rightarrow 0$. As follows from Fig. 2 in the intermediate temperature range, when $T$ is between $\Delta_s$ and $\Delta_v$, the metallic (curve $a$), insulating (curve $b$) and nonmonotonic (curve $c$) types of the $\rho(T)$ behavior emerge. As a result, there has to exist the $\rho(T)$ dependence with two maximum points in the presence of $B$.

For high magnetic fields such that $\Delta_s > T_{\text{max}}^{(I)}$, the maximum point at $T = T_{\text{max}}^{(I)}$ is absent, and two types of the $\rho(T)$ behavior are possible as is shown in Fig. 4. If $T_{\text{max}}^{(II)} < \Delta_v$, then the dependence of the resistivity is monotonic and insulating, see the curve $a$ in Fig. 4. Here, $T_{\text{max}}^{(II)}$ denotes the temperature of the maximum point that appears in the resistivity in accord with the RG Eqs. (62) and (63). In the opposite case $T_{\text{max}}^{(II)} > \Delta_v$, a typical $\rho(T)$ dependence is illustrated by the curve $b$ in Fig. 4. Therefore, if the valley splitting is sufficiently large, i.e., $\Delta_v > T_{\text{max}}^{(II)}$, then the monotonic insulating behavior of the resistivity appears in the parallel magnetic field which corresponds to $\Delta_s \sim T_{\text{max}}^{(II)}$. This is the case for the experiments on the magnetotransport in Si-MOSFET. However, if the valley splitting is small, $\Delta_v < T_{\text{max}}^{(II)}$, then the maximum point of the $\rho(T)$ dependence survives even in high magnetic fields but shifts down to lower temperatures.

In addition, to interesting $T$-dependences of the resistivity, the theory predicts strong renormalization of the electron-electron interaction with temperature. In order to characterize this renormalization, we consider the ratio $\chi_v/\chi_s$ of valley and spin susceptibilities. In Figure 8 we present the schematic dependence of $\chi_v/\chi_s$ on $T$ for a fixed valley splitting but with varying spin splitting. At high temperatures, $T \gg \Delta_v, \Delta_s$ the ratio of the susceptibilities equals unity, $\chi_v/\chi_s = 1$. At low temperatures,
the temperature behavior of such physical observables as the resistivity, spin and valley susceptibilities in 2D electron liquid with two valleys in the MIT vicinity and in the presence of both the parallel magnetic field and the valley splitting. First, we found that the metallic behavior of the resistivity at low temperatures survives in the presence of only the parallel magnetic field or the valley splitting. If both the spin splitting and the valley splitting exist then the metallic $\rho$-dependence crosses over to insulating one at low temperatures. Second, we have predicted the existence of the novel, nonmonotonic dependence of resistivity at zero and finite magnetic field in which the $\rho(T)$ has two maximum points. It would be an experimental challenge to identify this novel regime.

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**APPENDIX A: “COOPERON” CONTRIBUTION TO THE CONDUCTANCE**

We start from the standard equation for the “cooperon”

$$C_{\sigma_1,\sigma_2,\tau_1,\tau_2}^{\sigma_3,\sigma_4,\tau_3,\tau_4}(q) = \delta^{\sigma_1,\sigma_2} \delta_{\tau_1,\tau_2} \delta^{\sigma_3,\sigma_4} \delta_{\tau_3,\tau_4} \frac{1}{2\pi\nu\tau_i} + I_{\sigma_1,\sigma_2,\tau_1,\tau_2}(q) C_{\sigma_3,\sigma_4,\tau_3,\tau_4}^{\sigma_1,\sigma_2,\tau_1,\tau_2}(q) \quad (A1)$$

where the “impurity ladder” is given as

$$I_{\sigma_1,\sigma_2,\tau_1,\tau_2}(q) = \frac{1}{2\pi\nu\tau_i} \int_p G_{\sigma_1,\sigma_2,\tau_1,\tau_2}^R(p_+) G_{\sigma_3,\sigma_4,\tau_3,\tau_4}^A(p_-) \quad (A2)$$

Here, $p_\pm = p \pm q/2$ and the impurity averaged Green functions are given as

$$[G_{\sigma_1,\sigma_2,\tau_1,\tau_2}^{R(A)}(p)]^{-1} = \delta^{\sigma_1,\sigma_2} \delta_{\tau_1,\tau_2} \left[ \frac{p^2}{2m_e} - \mu + \Delta_v \text{sgn} \sigma_1 \right. + \Delta_s \text{sgn} \tau_i \pm i/2\tau_i \]. \quad (A3)$$

Performing integration, we find for $q \to 0$

$$I_{\sigma_1,\sigma_2,\tau_1,\tau_2}^{\sigma_3,\sigma_4,\tau_3,\tau_4}(q) = \delta^{\sigma_1,\sigma_3} \delta_{\tau_1,\tau_3} \delta^{\sigma_2,\sigma_4} \delta_{\tau_2,\tau_4} \left[ 1 - Dq^2\tau_i - i\Delta_v \tau_i (\text{sgn} \sigma_3 - \text{sgn} \sigma_1) - i\Delta_s \tau_i (\text{sgn} \tau_3 - \text{sgn} \tau_1) \right]. \quad (A4)$$

The temperature splitting does not change the “cooperon” contribution to the RG equations in the one-loop approximation. Finally, we remind that we do not consider above the contribution to the RG equations from the particle-particle (Cooper) channel. It can be shown (see Appendix) that neither the spin splitting nor the valley splitting does not change the “cooperon” contribution to the RG equations in the one-loop approximation. Therefore, the Cooper-channel contribution to the RG equations discussed above can be taken into account by the substitution of $1 + 2$ for $1$ in the square brackets of Eqs. (36), (62) and (86). The Cooper-channel contribution does not change qualitative behavior of the resistivity, and the valley and spin susceptibilities discussed above.

To summarize, we have obtained the novel results on
Next, solving Eq. (A1), we obtain

\[ C^{\sigma_1, \sigma_2 \tau_1 \tau_2}(q) = \delta^{\sigma_1 \sigma_2} \delta^{\tau_1 \tau_2} \frac{1}{2 \pi \nu \tau_1} \left[ Dq^2 \right] \]  \tag{A5}

\[ + i \Delta \langle \text{sgn} \sigma_3 - \text{sgn} \sigma_1 \rangle + \Delta \langle \text{sgn} \tau_3 - \text{sgn} \tau_1 \rangle \] \tag{A6}

The interference correction to the conductance is given as

\[ \delta \sigma_{xx} = -\frac{D}{\tau_1} \int_p G^R(-p)_{\sigma_3 \sigma_4 \tau_3 \tau_4} G^A_{\sigma_4 \sigma_1 \tau_4 \tau_1}(-p) \times G^R_{\sigma_1 \sigma_2 \tau_1 \tau_2}(p) G^A_{\sigma_2 \sigma_1 \tau_2 \tau_1}(p) \int \frac{d^2q}{(2\pi)^2} C^{\sigma_2 \sigma_3 \tau_3 \tau_2 \tau_1}(q). \] \tag{A7}

Using the result:

\[ \int \frac{d^2q}{(2\pi)^2} C^{\sigma_2 \sigma_3 \tau_3 \tau_2 \tau_1}(q) = \frac{4 \pi \nu \tau_1^3 \delta^{\sigma_1 \sigma_2} \delta^{\tau_1 \tau_2} \delta^{\tau_3 \tau_4}}{1 + \Delta_3^2 \tau_1^2 (\text{sgn} \sigma_2 - \text{sgn} \sigma_3)^2 + \Delta_2^2 \tau_2^2 (\text{sgn} \tau_2 - \text{sgn} \tau_3)^2}, \] \tag{A8}

we find

\[ \delta \sigma_{xx} = -\sum_{\sigma_1, \sigma_2} \delta^{\sigma_1 \sigma_2} \delta^{\tau_1 \tau_2} \left[ 1 + \frac{1}{2 \pi \nu \tau_1^2} Dq^2 \right] \left( \text{sgn} \sigma_1 - \text{sgn} \sigma_2 \right)^2 + \Delta^2 \tau_2^2 (\text{sgn} \sigma_2 - \text{sgn} \sigma_3)^2 \] \[ + \Delta_2^2 \tau_2^2 (\text{sgn} \tau_1 - \text{sgn} \tau_2)^2 \] \[ + i \Delta_1 \langle \text{sgn} \sigma_1 - \text{sgn} \sigma_2 \rangle + i \Delta_2 \langle \text{sgn} \tau_1 - \text{sgn} \tau_2 \rangle \] \[ = -\frac{4}{\pi} \int \frac{dq}{q}. \] \tag{A8}

Therefore, neither the spin splitting nor the valley splitting affect the Cooper channel (interference) contribution to the conductance.

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