Selection of Reinforcing Fiber for High-strength Lightweight Concrete for 3D-Printing

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Abstract. The knowledge of cement concretes and the availability of raw materials allows them to be used for 3D-printing technology in construction. One of the ways to increase the strength and reduce the shrinkage of the extruded layers is the introduction of reinforcing fibers into the mixture. An important task is the selection of effective types and concentrations of fiber. In this work, research on the selection of such fibers for compositions of high-strength lightweight concrete on hollow microspheres has been carried out. The optimal content of fiber which provides an acceptable decrease in mobility and the greatest strength of concrete is 1.25...1.5 % of the mass of Portland cement. It was found that the studied concretes on hollow microspheres have a reliable connection at the «steel-concrete» interface. The adhesion strength to metal reinforcement with a diameter of 8...12 mm is 6.1...10.1 MPa with a relative deformation of 0.097...0.025 mm/mm. High-strength lightweight fiber-reinforced concrete has low air permeability parameters, corresponding to the high water permeability grade W20. Such concrete with an average density of 1400 kg/m³ has high frost resistance F₁₈₀₀.

1. Introduction

The increased interest in additive technologies, including in construction forms many multidisciplinary tasks. The knowledge of cement concretes and the availability of raw materials make it expedient to adapt mixtures for the technology of extrusion of products. However, the 3D-printing process using concrete is associated with some problems with extrudability, hydration caring, and reinforcement [1]. The use of fibers is one of the prescription methods for increasing the mechanical properties of extruded layers and for reducing the shrinkage deformations.

The influence of reinforcing fibers on the properties of concrete mixes and concrete is demonstrated [2]. Highly filled concrete mixtures with polypropylene fiber allow satisfactory extrudability (shear strength – 0.3...0.9 kPa) due to the use of cement in an amount of more than 30 % by weight and plasticizer up to 2% by weight of cement. In this case, the optimal amount of fiber is 1.2 kg/m³. The combination of steel and polypropylene fibers allows to improve the mechanical properties and also to control the rheology of the mixture (shear strength – 1.5...2.5 kPa) [3]. Also, the technological feature of the extrusion nozzle influences the properties of the printed fiber-reinforced concrete layer [4]. The use of a rectangular nozzle promotes the distribution of the reinforcing fibers in the concrete layer and their longitudinal orientation. The strength of fiber-reinforced concrete extruded
through such a printer nozzle is 15% greater than the strength of the same material printed by a circular nozzle of the extruder.

There are many factors to consider when choosing an effective concentration for extruded concrete for 3D-printing including fiber properties. The use of mathematical modeling in the development of building materials for various purposes is becoming increasingly important. According to researchers, [5...8] this may be due to the possibility of multivariate analysis for a wide range of control parameters of structure formation of building composites and also the establishment of new connections between control factors (including latent ones).

A large number of domestic and foreign works are devoted to the issues of experimental and model research of dispersed reinforcement of concrete [9...15]. Note that foreign interests are mainly focused on the issues of analytical and numerical research of dispersed-reinforced concrete by methods of fracture mechanics. This is fully disclosed in these papers [11 ... 15].

In this paper, the provisions of the theory of percolation and fractal geometry to develop an analytical model of chaotic reinforcement were used.

In the percolation theory [16, 17], it is shown that percolation occurs when the concentration of intersecting spheres is:

$$\frac{\pi}{6} D^3 n_V = B_{c,V},$$

where $B_{c,V}$ – average number of intersecting spheres ($B_{c,V} = 2.7 \pm 0.1$ [18]); $n_V$ – number of spheres with diameter $L_0$; $D$ – sphere diameter equal to fiber segment:

$$D = \frac{L_0}{k},$$

here $k$ – number of fiber parts.

The constant $B_{c,V}$ can be characterized by the number of centers of other spheres intersecting the selected sphere. The numerical value of $B_{c,V}$ shows that the number of centers of spheres located inside a sphere with a diameter $L_0$ should be more than two. It is also important to define the concept (the essence) of percolation for the conditions of the problem under consideration. The "matrix material – fiber" phase boundary affects the physical properties of the matrix material (cement stone). As a rule, a layer of matrix material with higher physical and mechanical properties as compared to the matrix material in the volume (far from the phase boundary) is formed. The matrix material in the boundary zone was named the film phase [17]. The contact of film phases leads to the appearance of synergistic effects which additionally increase the quality of the composite material. The absence of a film phase or the presence of contact between the surfaces of the fiber will lead to a decrease in the quality of the material. Moreover, direct contact of the surfaces of the fiber leads to the formation of structural defects which are aggregates of not wetted particles of the dispersed phase [17]. Thus, to solve the problem, we will assume that the percolation between two fibers occurs under the following condition: the fibers are located at a distance of intersection (association) of the film phases.

The concentration $n_V$ determines the volumetric coefficient of reinforcement of the material $\mu$:

$$n_V = \frac{N_f}{V_0} = \frac{\mu}{4 \pi d_i^2 \ell_i},$$

where $N_f$ – number of fibers; $V_0$ – composite volume; $d_i, L_0$ – fiber diameter and length.

After transforming formula (1), we get:

$$B_{c,V} = \frac{2}{3 k^3} \left( \frac{L_0}{d_i} \right)^2,$$

Due to the implementation of technological operations, the actual fiber length $L$ in the composite may change (bending, fiber splitting). Then we introduce the coefficient of fiber tortuosity $\xi$:
Formula (7) shows the natural relationship between the coefficients $k$ and $\xi$. Fiber bend ($\xi$) can be characterized as the appearance of an additional part that increases the probability of contact between other fibers. An increase in this possibility naturally leads to a decreasing number of fibers. In a first approximation the relationship between the coefficients $k$ and $\xi$ can be represented as:

$$k = \frac{\text{const}}{\xi^2},$$

(8)

The values of $\mu$, calculated according to the presented formula (Figure 1), are minimal and sufficient to ensure the formation of an infinite cluster in a composite material (Shklovsky – De Gennes models). For real objects, it is important to take into account the mechanical and physical-mechanical properties of the substance of fiber.

![Figure 1. Dependence $\mu = (L_0/d_l)$](image)

The purpose of this study is the theoretical and experimental establishment of the effective concentrations of reinforcing additives for high-strength lightweight concrete (HSLWC).

2. Material and methods

Portland cement CEM I 42.5 N produced by «Lipetsk cement» was used as a binder. The mineral part of the composition includes fractionated quartz sand fr. 0.16...0.63 mm, micro-silica MK-85, and quartz flour with a fraction of less than 0.1 mm. Hollow glass microspheres «ForeSphere 3000» were used as a functional filler to reduce the average density of concrete. The average particle size of
microspheres is 30 microns, bulk density – 270...300 kg/m³, hydrostatic compression strength – up to 405 bar. To reduce water demand and increase the mobility of the concrete mixture plasticizer «Melflux 1641F» on a polycarboxylate basis was used. The ratio of the components was selected in accordance to achieve an average density of high-strength lightweight concrete of 1400 kg/m³ by [19].

The concrete mobility was determined by the spread diameter of the flow on a shaking table. The study of the physical and mechanical properties of concrete (flexural and compressive strength) was carried out on samples-prisms 40×40×160 mm at the age of 1 day after the heat humidity treatment using a servo-hydraulic system «Advantest 9».

3. Results and discussion
To calculate the range of variation of the reinforcement coefficient the value of the constant in formula (8) was taken to be 25. Three possible options were considered:

1. fibers have a constant length in the material ξ = 1.0. In this case, the value of the coefficient k = 12.5. That is, the fiber has 12.5 parts that can contact other fibers;
2. fibers have shorter lengths by 1.5 times in the material (ξ = 1.5) and the number of fiber parts is k = 8.8;
3. fibers have shorter the length by 2 times in the material (ξ = 2.0) and the number of fiber parts is k = 6.3.

The simulation results are presented in Table 1. It can be seen that an infinite cluster is formed at \( L_0/d_l > 600 \) with an acceptable reinforcement coefficient µ when using rigid non-bending fibers. For flexible fibers, the \( L_0/d_l \) ratio can be significantly reduced.

| \( B_{c,V} \) | 50 | 100 | 200 | 300 | 400 | 500 | 600 |
|----------------|----|-----|-----|-----|-----|-----|-----|
| ξ = 1.0 and k = 12.5 | 2.6 | 76.2 | 19.0 | 8.5 | 4.8 | 3.0 | 2.1 |
| 2.7 | 79.1 | 19.8 | 8.8 | 4.9 | 3.2 | 2.2 |
| 2.8 | 82.0 | 20.5 | 9.1 | 5.1 | 3.3 | 2.3 |
| ξ = 1.5 and k = 8.8 | 2.6 | 47.9 | 12.0 | 3.0 | 1.3 | 0.75 | 0.48 | 0.33 |
| 2.7 | 49.7 | 12.4 | 3.1 | 1.4 | 0.78 | 0.50 | 0.35 |
| 2.8 | 51.6 | 12.9 | 3.2 | 1.4 | 0.81 | 0.52 | 0.36 |
| ξ = 2.0 and k = 6.3 | 2.6 | 9.5 | 2.4 | 0.60 | 0.26 | 0.15 | 0.10 | 0.07 |
| 2.7 | 9.9 | 2.5 | 0.62 | 0.27 | 0.15 | 0.10 | 0.07 |
| 2.8 | 10.3 | 2.6 | 0.64 | 0.28 | 0.16 | 0.10 | 0.07 |

The presented calculations show that the volumetric reinforcement coefficient naturally depends on the geometric characteristics of the fibers. The optimal concentration should be 0.54 ± 0.19 % for the variation range of the fibers \( L_0/d_l \in (400, 600) \) and the fibers having moderate flexibility \( \xi = 1.5 \).

Also, the µ values given in table 1 have a good agreement with the estimates made in the works [2, 4, 9, 10].

3.1. Rheological properties
The use of dispersed reinforcing additives is an effective solution that increases the strength of concrete. However, the introduction of fibers has some limitations caused by the decrease in the flow of the mixture. This general dependence is also typical for concrete mixtures of high-strength
lightweight concrete (Figure 2). An increase in the content of fiber regardless of the type leads to a decrease in the spread diameter from 230 to 170 mm (by 35 %). The least impact is made by fiberglass. With an equal fiber concentration, the spreading diameter of the mixture is reduced to 210 mm (by 9.5%). For basalt and polypropylene fibers it is decreased to 200 mm (by 15 %) and 190 mm (by 21 %) respectively.

![Figure 2. Dependence between concrete mix mobility and fibers type and content](image1)

![Figure 3. Dependence between the average density of high-strength lightweight concrete and fibers type and content](image2)

3.2. Physical and mechanical properties

It is obvious that a decrease in the mobility of the concrete mixture contributes to a decrease in workability and leads to worse compaction of the mixture during molding. Figure 3 shows that the average concrete density is decreased within the range of 15...25 kg/m³ in proportion to the change in mobility in the investigated fiber concentrations.

Thus, the features of the change in the mobility of the mixture and the average density of concrete from the fiber content allow concluding that the use of additional criteria for choosing the optimal fiber concentrations is advisable. Such criteria should take into account the effect of fibers on the physical and mechanical properties of concrete. The range of variation of the content of fibers 1.0...1.5 % of the mass of Portland cement depending on the fiber material makes it possible to limit the decrease in the mobility of the concrete mixture by 10...15 %. It is important to note that these values are in good agreement with theoretical estimates of the volumetric coefficient of concrete reinforcement (m³/m³).

The change in the mechanical properties of high-strength lightweight concrete with the introduction of fibers is shown in figure 4. Varying the content of the dispersed additive has shown that the compositions with polypropylene fibers have the greatest change in flexural and compressive strength. An increase in the number of fibers to 2 % of the mass of Portland cement allows an increase in flexural strength by 60.0 % and in compressive strength by 15.5 %. The use of fiberglass and basalt fibers in an amount of up to 2.5 and 3.5% of the mass of Portland cement, respectively, provides an increase in flexural strength up to 6.5 MPa and compressive strength up to 47 MPa. At the same time for compositions without fibers $R_{fl} = 5$ MPa and $R_{con} = 43$ MPa. But the use of basalt fiber does not increase the compressive strength of concrete.
Thus, the most effective disperse additive for controlling the strength of high-strength lightweight concrete for 3D-printing is polypropylene fiber, which provides an increase in the bending and compressive strength. The optimum content of such fiber is 1.25 ... 1.75%.

3.3. Deformation properties

The elastic modulus is the most important material characteristic used in the design of reinforced concrete structures. Studies of determination of the elastic modulus and Poisson's ratio were carried out on the composition of fiber-reinforced concrete with a fiber concentration of 1.25% in comparison with the composition without fiber. Diagrams of deformation are shown in figure 5.

Analysis of the deformation diagrams shows that the use of fibers leads to a decrease in the relative longitudinal deformations and an increase in the lateral deformations. This indicates a change like material destruction. From table 2 it can be seen that the elastic modulus of the concretes understudy is 15.3 GPa. Hence, these compositions have a lower deformation under equal load (20 MPa). It ensures a change in the elastic modulus in the range of 4...5 %. Naturally, an increase in lateral deformations for these compositions is fixed in an increase of Poisson's ratio by 28...31 %. Poisson's ratio for high-strength lightweight fiber-reinforced concrete (HSLWFC) is 0.36. It should be noted that the Poisson's
ratio for the control composition of the HSLWC has a value of 0.28. This is significantly higher than the values for traditional heavy and lightweight concrete (0.15...0.2).

### Table 2. Deformation properties of HSLWC

| Concrete type | $\sigma$ (MPa) | $\varepsilon_{lons}$ ($\cdot 10^{-6}$) | $\varepsilon_{lats}$ ($\cdot 10^{-6}$) | $E$ (GPa) | $E'$ (GPa) | $\mu$ | $\mu'$ (%) |
|---------------|----------------|--------------------------------------|--------------------------------------|-----------|-----------|-------|-----------|
| HSLWC         | 20.0           | 1375                                 | 390                                  | 14.6      | 100       | 0.28  | 100       |
| HSLWFC        | 1305           | 470                                  | 15.3                                 | 105       | 0.36      | 128   |

Notes: $\sigma$ – load, $\varepsilon_{lons}$ – longitudinal deformations, $\varepsilon_{lats}$ – lateral deformations, $E$ – elastic modulus, $E'$ – relative change in elastic modulus from HSLWC values, $\mu$ – Poisson's ratio, $\mu'$ – relative change in Poisson's ratio from HSLWC values.

Interaction between new types of concrete and reinforcement is important in the design and manufacture of reinforced concrete structures, including 3D-printing. This interaction is assessed by the strength of the adhesion of concrete with reinforcement during pulling out. The strength of adhesion can be varied in the range of 12…24 MPa for concrete classes B15…B30.

An important condition for lightweight concrete is providing a value of adhesion with reinforcement similar to heavy concrete.

The assessment of the interaction of high-strength lightweight fiber-reinforced concrete with metal reinforcement with a diameter of 8, 10, and 12 mm was carried out in the value of the maximum load and tensile deformation. Compositions of high-strength lightweight concrete with an average density of 1400 kg/m$^3$ were studied in comparison with high-strength fine-grained heavy concrete (Table 3).

### Table 3. Results of reinforcement pull-out test of concrete

| Concrete type | $R_{com}$ (MPa) | $R_{sp}$ (MPa) | $L$ (mm/mm) | $R_{adh}$ (MPa) | $R_{adh}/R_{com}$ |
|---------------|-----------------|----------------|-------------|-----------------|-------------------|
|               | $\varnothing$8 A-I |                |             |                 |                   |
| Heavy concrete| 90.0            | 37.5           | 0.162       | 7.0             | 0.055             |
| HSLWC         | 55.0            | 39.3           | 0.009       | 4.0             | 0.061             |
| HSLWFC        | 62.0            | 44.3           | 0.097       | 6.1             | 0.088             |
|               | $\varnothing$10 A-III |          |             |                 |                   |
| Heavy concrete| 90.0            | 37.5           | 0.027       | 11.0            | 0.086             |
| HSLWC         | 55.0            | 39.3           | 0.016       | 6.2             | 0.095             |
| HSLWFC        | 62.0            | 44.3           | 0.030       | 9.6             | 0.137             |
|               | $\varnothing$12 A-III |          |             |                 |                   |
| Heavy concrete| 90.0            | 37.5           | 0.025       | 10.3            | 0.081             |
| HSLWC         | 55.0            | 39.3           | 0.015       | 5.5             | 0.085             |
| HSLWFC        | 62.0            | 44.3           | 0.025       | 10.1            | 0.144             |

Notes: $R_{com}$ – compressive strength, $R_{sp}$ – specific strength, $R_{adh}$ – adhesion strength, $L$ – maximum deformation, $\varnothing$ – reinforcement diameter.

Analysis of obtained results shows that the adhesion strength of high-strength heavy concrete metal to reinforcement in the considered range of reinforcement diameters is 7.0...10.3 MPa. At the same time for high-strength lightweight concrete $R_{adh} = 4.0...6.2$ MPa (by 43...46 % less than for high-strength heavy concrete) and for high-strength light fiber-reinforced concrete $R_{adh} = 6.1...10.1$ MPa (by 2.0...13 % less than for high-strength heavy concrete). However, the relative adhesion strength (the adhesion strength of concrete with reinforcement divided to the compressive strength of concrete $R_{adh}/R_{com}$) shows the opposite result.

Thus, studies show that the developed high-strength lightweight concretes on hollow microspheres are capable of providing a reliable contact connection at the “steel reinforcement – concrete” interface. The adhesion strength of high-strength lightweight fiber-reinforced concrete with metal reinforcement with a diameter of 8...12 mm is 6.1...10.1 MPa and relative deformations of 0.097...0.025 mm/mm.
3.4. Operational properties

The functional purpose of the composition of high-strength lightweight fiber-reinforced concrete on hollow microspheres is the manufacture of reinforced concrete structures both using traditional technology (prefabricated or monolithic) and also 3D-printing. It requires an assessment of the reliability of this material under the influence of external factors. These factors include the physical and chemical effect of the operating environment, the destructive effect of which depends on the absorbing capacity of the material.

Studies of samples of high-strength lightweight fiber-reinforced concrete, hardening under normal conditions (Figure 7), showed that the kinetics of water absorption of such concrete obeys the classical exponential law of absorption:

\[ W = W_{\text{max}} \left( 1 - e^{-bt} \right), \]  

where \( W_{\text{max}} \) – maximum water absorption; \( b \) – the average size of open capillary pores, \( t \) – time.

The obtained approximation dependences of the kinetics of water absorption of compositions with fiber after exposition for 384 hours show lower sorption properties. The maximum water absorption of fiber-reinforced concrete is 4.76 % which is 21.0 % less than the control composition. In this case, we can note similar indicators of the average size of open capillary pores.

Figure 6. Kinetics of water absorption of high-strength lightweight concrete

A feature of the structure of lightweight concrete on hollow microspheres is a closed pore structure formed by particles of the hollow filler of the true spherical shape. This creates the preconditions for the low permeability of the material. The water-resistance grade can be a parameter for assessing the durability of concrete. The results of determining water-resistance by an accelerated method for air permeability on a series of samples of high-strength lightweight concrete are presented in Table 4.

| Material   | Air permeability parameter, \( a \) (\( \text{sm}^3/\text{s} \)) | Resistance to air penetration, \( m \) (\( \text{sm}^3/\text{s} \)) | Permeability grade |
|------------|-------------------------------------------------|-------------------------------------------------|------------------|
| HSLWC      | 0.0005                                          | 2050                                            | W20              |
| HSLWFC     | 0.0014                                          | 730                                             | W20              |
Table 4 shows that each type of lightweight concrete on hollow microspheres has low air permeability parameters, corresponding to a high grade for water permeability W20.

Frost resistance is one of the most important properties of concrete. This characteristic like water resistance directly depends on the features of the material structure. To determine the minimum frost resistance grade, the number of freezing and thawing cycles by the accelerated method was set until the appearance of defects on the surface of the samples.

According to [21], high-strength lightweight concrete on hollow microspheres has a frost resistance grade of at least $F_{1300}$ according to the accelerated third method. By the proposed recipe, samples of high-strength lightweight fiber-reinforced concrete, hardening under heat humidity treatment were used for freezing and thawing cycles after 7 days in the normal hardening chamber. The results of frost resistance tests are presented in Table 5.

| Parameter                  | Value |
|----------------------------|-------|
| $q$                        | 27    |
| $\Delta m / m$, %          | 0.62  |
| $X_{av}^I$, MPa             | 48.8  |
| $\sigma^I$, MPa             | 4.0   |
| $X_{av}^{II}$, MPa          | 48.4  |
| $\sigma^{II}$, MPa          | 3.0   |
| $V_m$, %                   | 0.1   |
| $0.9X_{min}^I$, MPa         | 43.8  |
| $X_{min}^{II}$, MPa         | 48.3  |
| Frost resistance            | $F_{1800}$ |

Notes: $q$ – number of freeze/thaw cycles, $\Delta m / m$ – relative mass change, $X_{av}^I$, $X_{av}^{II}$ – average strength of samples of control and main series, $\sigma^I$, $\sigma^{II}$ – standard deviation for the control and main series of samples, $V_m$ – the coefficient of variation, $X_{min}^I$, $X_{min}^{II}$ – minimum strength of samples of control and main series.

The frost resistance test results show that high-strength lightweight fiber concrete is more resistant to consistently freezing and thawing than such concrete without fiber. The frost resistance grade increases to $F_{1800}$.

4. Conclusion
The study of the influence of the type and number of fibers on the strength of high-strength lightweight fiber-reinforced concrete made it possible to establish that the maximum increase in the strength of the developed composite is observed after the introduction of polypropylene fiber. The optimal content of such fiber which provides an acceptable decrease in mobility and the greatest strength of concrete is $1.25…1.5$ % of the mass of Portland cement. The obtained value coincides with the range of volumetric reinforcement predicted by the developed model of chaotic reinforcement based on the percolation theory.

It was found that the studied concretes on hollow microspheres have a reliable connection at the «steel-concrete» interface. The adhesion strength to metal reinforcement with a diameter of $8…12$ mm is $6.1…10.1$ MPa with a relative deformation of $0.097…0.025$ mm/mm.

High-strength lightweight fiber-reinforced concrete has low air permeability parameters, corresponding to the high water permeability grade W20. Such concrete with an average density of $1400$ kg/m$^3$ has high frost resistance $F_{1800}$. 
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