On the multiple depots vehicle routing problem with heterogeneous fleet capacity and velocity

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Abstract. This current manuscript concerns with the optimization problem arising in a route determination of products distribution. The problem is formulated in the form of multiple depots and time windowed vehicle routing problem with heterogeneous capacity and velocity of fleet. Model includes a number of constraints such as route continuity, multiple depots availability and serving time in addition to generic constraints. In dealing with the unique feature of heterogeneous velocity, we generate a number of velocity profiles along the road segments, which then converted into traveling-time tables. An illustrative example of rice distribution among villages by bureau of logistics is provided. Exact approach is utilized to determine the optimal solution in term of vehicle routes and starting time of service.

1. Introduction
In the academic literature, the vehicle routing problem (VRP) is one of the most challenging combinatorial optimization tasks and has become the topic of research for decades. VRP is one of the most studied among the combinatorial optimization problems, due both to its practical relevance and to its considerable difficulty [13]. An enormous number of researches have been devoted to studying the VRP since 1959, as it has been described by Dantzig & Ramser as a generalization of Travelling Salesman Problem, where the use of several vehicles are permitted in delivering products and all vehicles must return to the depot after visiting some customers [2]. In this effort, much attentions were mainly emphasized on the development of the problem features and assumptions leading to a huge number of problem variant formulations as well as various heuristic and metaheuristic algorithms advancement to address the problem. Since then VRP has been the heart of the transportation, distribution, and logistics management [10].

The capacitated vehicle routing problem (C-VRP) is known as the most basic and classical variant of VRP. In this case, the optimal routes from a depot to a set of customers with known finite demand should be sought in such a way they consist of the delivery of products conducted by vehicles with limited carrying capacity [6], [9], [14]. This variant is then extended to include the case where multiple depots are available to serve customers in which it is known as multiple depots vehicle routing problem (MD-VRP), see e.g., [11] and [15]; and one where each customer should be served in a specified time window, i.e., vehicle routing problem with time windows (VRP-TW), see for instance, [7], [8].

This present paper discusses MD-VRP-TW with heterogeneous fleet capacity and velocity. The unique feature of heterogeneous velocity is undertaken by generating a number of velocity profiles along the road segments, which then it is converted into several tables of traveling-time. In fact, in some part of modelling, we motivated by the work of [1].
2. Model of VRP
In general, VRP refers to a problem of seeking delivery routes for a fleet of vehicles of known capacities to supply a number of nodes of known locations and demands for a certain commodity, given a set of constraints. Constraints can include any combination of the following: each vehicle is originating and terminating at a depot upon completion of its route, each customer must be served by only one vehicle and each dispatched vehicle should visit at least one customer [5], [8], [12]. Routes for the vehicles should be generated to minimize some cost functions such as the total distance of trips and the cost of delivery.

The volume by [4] summarizes the most significant results for the VRP and its variants since 2000. They present significant methodological developments or unique approaches for addressing existing VRPs as well as novel problems that have introduced in the vehicle routing domain and underline new challenges for the field. In this section we present a brief overview regarding the VRP and its variants under consideration.

2.1. C-VRP
C-VRP is a VRP in which a fleet of identical vehicles with limited known capacity must service a number of customers with known in advance and may not be split demands for a single commodity from a depot at minimum traveling cost or distance needed to serve all the customers. That is, CVRP is VRP with the additional constraint that every vehicle must have uniform capacity of a single commodity, i.e., only the capacity restrictions for the vehicles are imposed.

Suppose there are \( n \) customers to be served and \( F \) is the set of all available vehicles. If \( x_{ij}^k \) denotes the decision variables, i.e., \( x_{ij}^k = 0 \) if there is no arc from customer \( i \) to customer \( j \) by vehicle \( k \), and \( x_{ij}^k = 1 \) otherwise; \( q_i \) is the demand of customer \( i \); and \( C \) is the capacity of vehicle, then the capacity restriction in a route is written as

\[
\sum_{i=1}^{n} q_i \sum_{j=1}^{n} x_{ij}^k \leq C, \quad \forall k \in F.
\]

It is conducted by [8] a literature review on the recent developments and publications involving the vehicle routing problem and its variants, including C-VRP and VRP-TW.

2.2. VRP-TW
VRP-TW arises when a natural constraint for VRP is added, that is a customer can only receive a delivery or service in a specified window of time. By this constraint, a vehicle now has to visit a customer within a certain time frame. It means that we need a way to calculate the time a vehicle has to spend serving a customer and the travel time to the next customer on its route. More specifically, we must calculate the service time, the travel time and the total time.

If the time window of customer \( i \) is \([a_i, b_i]\) then vehicle may arrive before the time window opens but the customer cannot be served until the time windows open:

\[
a_i \leq y_i^k, \quad \forall k \in F,
\]

where \( y_i^k \) is a decision variable denotes the starting time to serve customer \( i \) by vehicle \( k \). It is not allowed to arrive after the time window has closed, which means that

\[
y_i^k + s_i \leq b_i, \quad \forall k \in F,
\]

where \( s_i \) is the known service time for customer \( i \).

VRP with time windows can be approached either by exacts, heuristics or metaheuristics methods. Readers may consult the survey paper of [3] for the comprehensive overview of the method.

2.3. MD-VRP
The case of MD-VRP is posed if a company may have more than one depots from which it can serve its customers. If the customers are clustered around depots, then a number of independent VRPs can be performed to serve customers. However, if the customers and the depots are intermingled then a MD-
VRP should be solved. In fact, an MD-VRP necessitates the assignment of customers to depots and a fleet of vehicles is organized at each depot basis. It means that each vehicle originates from one depot and is responsible for servicing customers assigned to that depot and return to the same depot at the end of their assigned tours as loosely can be expressed by

$$\sum_{j=1}^{n} x_{ij}^k \leq 1, \quad \forall k \in F_i, \forall i \in W,$$

and

$$\sum_{i=1}^{n} x_{ij}^k \leq 1, \quad \forall k \in F_j, \forall j \in W,$$

where $F_i$ denotes the set of all available vehicles at depot $i$ and $W$ is the set of all depots. Complete formulation of such a problem can be found, for instance, in [11].

3. Model of Interest

In this section we present our model of interest, namely time windowed multiple depots vehicle routing problem with heterogeneous capacity and velocity of fleet (MD-VRP-TW-HCV). To facilitate our model formulation, we define a number of sets, parameters and variables.

We denote by $W$, $V$, $N$, respectively the set of all depots with cardinality $m$, that of all customers with cardinality $n$ and that of all nodes, i.e., $N = W \cup V$. For clarity we define $W = \{1, 2, ..., m\}$ and $V = \{m + 1, m + 2, ..., m + n\}$. We define by $F_i$ the set of all available vehicles at depot $i$ and by $F$ the set of all available vehicles, i.e., $F = \cup_{i=1}^{m} F_i$, with cardinality $p$. We also define the following parameters:

$q_i$ : demand of customer $i$ (in unit),
$s_i$ : duration of service time for customer $i$ (in hour),
$[a_i, b_i]$ : time window of customer $i$,
$C_k$ : capacity of vehicle $k$ (in unit),
$c_k$ : fixed cost of vehicle $k$ (in currency unit),
$v_k$ : variable cost of vehicle $k$ (in currency unit per kilometer),
$d_{ij}$ : distance between node (depot or customer) $i$ and node $j$ (in kilometer),
$c_{ij}^k$ : traveling cost between node $i$ and node $j$ by using vehicle $k$, i.e., $c_{ij}^k = d_{ij} v_k$ (in currency unit),
$t_{ij}$ : travelling time between node $i$ and node $j$ (in hour),
$M$ : big positive constant (big-M),
$Z$ : the value of objective function.

For decision variables we have the following binary variable

$$x_{ij}^k = \begin{cases} 1; & \text{if vehicle } k \text{ travel from node } i \text{ to node } j \\ 0; & \text{otherwise,} \end{cases}$$

and $y_{ij}^k \in \mathbb{R}^+$ which defined by the starting time to serve customer $i$ by vehicle $k$, thus $y_{ij}^k \in [a_i, b_i]$. We search the routes such that minimize the total cost which includes both the vehicle fixed cost and the traveling cost

$$\min Z = \sum_{i=1}^{m+n} \sum_{j=1}^{m+n} \sum_{k=1}^{p} c_{ij}^k x_{ij}^k + \sum_{i=1}^{m+n} \sum_{j=m+1}^{m+n} \sum_{k=1}^{p} c_k x_{ij}^k,$$

subject to a set of constraints:

(a). each customer is serviced exactly once by a vehicle and after visiting a customer it should be ensured that vehicle will visit the next customer:

$$\sum_{k=1}^{p} x_{ij}^k = 1, \quad \forall j \in V,$$
\[
\sum_{j=1}^{m+n} \sum_{k=1}^{p} x_{ij}^k = 1, \quad \forall i \in V,
\]

(b). an operated vehicle must start from a depot and must return to the same depot after completing the tour; it is not necessary to utilize all available vehicles:

\[
\sum_{j=m+1}^{m+n} x_{ij}^k \leq 1, \quad \forall k \in F_i, \forall i \in W,
\]

\[
\sum_{i=m+1}^{m+n} x_{ij}^k \leq 1, \quad \forall k \in F_j, \forall j \in W,
\]

(c). there is no trip conducted by a vehicle not belong to a depot:

\[
\sum_{j=m+1}^{m+n} x_{ij}^k = 0, \quad \forall k \not\in F_i, \forall i \in W,
\]

\[
\sum_{i=m+1}^{m+n} x_{ij}^k = 0, \quad \forall k \not\in F_j ; \forall j \in W,
\]

(d). route continuity:

\[
\sum_{i=1}^{m+n} x_{ir}^k = \sum_{j=1}^{m+n} x_{rj}^k, \quad \forall k \in F, \forall r \in V,
\]

(e). time window:

\[
a_i \leq y_i^k, \quad \forall k \in F, \forall i \in N,
\]

\[
y_i^k + s_i \leq b_i, \quad \forall k \in F, \forall i \in N,
\]

(f). sub-tour elimination:

\[
y_i^k + s_i + t_{ij}(y_j^k + s_i) - y_j^k \leq M(1 - x_{ij}^k), \quad \forall k \in F, \forall i \in N, \forall i \in V,
\]

(g). the capacity of any vehicle is not violated:

\[
\sum_{i=m+1}^{m+n} q_i \sum_{j=1}^{m+n} x_{ij}^k \leq C_k, \quad \forall k \in F,
\]

(h). there is no inter-depot route:

\[
x_{ij}^k = 0, \quad \forall k \in F; \forall i,j \in W.
\]

4. Model Implementation

4.1. Problem statement

As an illustrative example we consider a distribution problem of rice for the poor (RASKIN), where the bureau of logistics operates two depots (m = 2) to service eighteen villages as customers (n = 18). To show their indices, we denote \( W = \{1,2\} \) and \( V = \{3,4,...,20\} \). Each village may have different demand of rice and time window as depicted by Table 3 in the appendix. The duration of service time is known and is provided in the same table.

In the distribution process, the bureau operates a fleet consisting of six vehicle (p = 6). The first three vehicles belong to Depot 1 and the rest belongs to Depot 2. Thus, \( F_1 = \{1,2,3\} \) and \( F_2 = \{4,5,6\} \). Capacity, fixed cost and variable cost of each vehicle are given by Table 4 in the appendix. Distance between nodes is assumed to be symmetric and the data is provided in Table 5 in the appendix.

By having twenty nodes, in fact we have \( C(20,2) = 190 \) road segments with different condition and thus different expected velocity. However, for the sake of simplicity, we assume that there are only six different profiles of velocity applicable to all road segments as illustrated by Figure 1. In this figure, horizontal axis indicate the time span (in minute) and the vertical axis measures the velocity (in
kilometer per minute). In the first sub-figure, we can see that in the first five minutes, due to the road condition, vehicle can only travel 10 km/min and enhanced to 20 km/min during the next five minutes. Once distance between two nodes of certain profile is provided (see Table 4 in the appendix), then we can calculate the traveling time between them. Subsequently, a table of traveling time among nodes can be constructed (see Table 5 in the appendix).

![Velocity profiles](image1)

**Figure 1.** Velocity profiles

4.2. The optimal route

We implement the MD-VRP-TW-HCV model formulated in the previous section in Lingo 11 and run by using PC desktop of CPU 2.30 GHz and RAM 4 GB. Execution time was 1 minute and 3 seconds, providing the (global) optimal cost of 767,500 currency unit. Optimal distribution route of each vehicle is presented in Table 1 or Figure 2. It can be concluded from the table that all the vehicles should be operated to service all retailers. More particular, Vehicle 2 travels the most (39 km) to provide service to four retailers and Vehicle 6 the least for one retailer.

| Vehicle | Distribution Route | Amount of distributed product | Distance Traveled |
|---------|--------------------|-------------------------------|-------------------|
| 1       | 1 → 5 → 6 → 14 → 12 → 1 | 338                           | 35                |
| 2       | 1 → 4 → 3 → 10 → 13 → 1 | 343                           | 39                |
| 3       | 1 → 16 → 7 → 8 → 19 → 1 | 285                           | 26                |
| 4       | 2 → 15 → 18 → 20 → 2    | 167                           | 43                |
| 5       | 2 → 17 → 9 → 2          | 150                           | 26                |
| 6       | 2 → 11 → 2             | 90                            | 4                 |
To inspect the fulfillment of demand and time window, let consider the distribution process undertaken by Vehicle 1. This vehicle delivers products to villages 5, 6, 14 and 12, respectively. The total demand of rice requested by these four villages is \(99 + 100 + 59 + 80 = 338\), as shown in Table 3 in the appendix and confirmed by optimal solution in Table 1. Even though Depot 1 is open since 6 am, Vehicle 1 can leave depot at 08.54 at the latest as it requires 6 minutes to reach village 5 and starting service at 9 am for one hour. The driver can have a waiting time or reserve time until 12.10 before proceeding to village 6, and so forth. Distribution task is completed by Vehicle 1 at 21.00 before returning to Depot 1. The time flow for Vehicle 1 may be depicted by Figure 3. Tables 8-12 in the appendix present the fulfillment of demand and time window for other vehicles.

**Table 2. Service time by Vehicle 1**

| Node | Time Windows   | Duration of service | Start time   | End time   | Time limit for leaving | Time to next node |
|------|----------------|---------------------|--------------|------------|------------------------|-------------------|
| (i)  | \([a_i, b_i]\) | \((s_i)\)           | \((y)\)      | \((y_f)\)  | \((t_{ij})\)           | \((t_{ij})\)      |
| 1    | 06.00 - 22.00 | 0                   | 06.00.00     | 06.00.00   | 08.54.00               | 6 min             |
| 5    | 09.00 - 14.00 | 1                   | 09.00.00     | 10.00.00   | 12.10.30               | 14 min 30 sec     |
| 6    | 10.00 - 15.00 | 1                   | 12.25.00     | 13.25.00   | 16.28.04               | 10 min 45 sec     |
| 14   | 16.00 - 21.00 | 1                   | 16.38.49     | 17.38.49   | 19.52.00               | 8 min             |
| 12   | 16.00 - 21.00 | 1                   | 20.00.00     | 21.00.00   | 21.11.40               | 48 min 20 sec     |
| 1    | 06.00 - 22.00 | 0                   | -            | -          | -                      | -                 |

![Figure 2. Distribution route](image)

**Figure 2. Distribution route**

![Figure 3. Time flow of Vehicle 1](image)

**Figure 3. Time flow of Vehicle 1**
5. Concluding Remark
In this paper we have presented a variant model of VRP, namely time windowed multiple depots VRP with heterogeneous capacity and velocity of fleet, where the last feature of homogeneous velocity is approached by characterizing the velocity profile of road segments. The model is formulated in the framework of mixed integer linear programming and an exact approached is employed to obtain the optimal solution. A model implementation of rice for the poor (RASKIN) distribution between depot and villages has been carried-out. A global optimal solution is reached in term of transportation route for vehicles including service time.

For problem involving more number of depots and customers, solution method based on heuristic and metaheuristic algorithms should be considered.

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Table 3. Demand, duration of service time and time windows

| Node (i) | Demand (q_i) | Service time (s_i) | Time Windows [a_i, b_j] |
|----------|--------------|--------------------|------------------------|
| 1 (depot)| 0            | 0                  | 06.00-22.00            |
| 2 (depot)| 0            | 0                  | 06.00-22.00            |
| 3        | 90           | 1                  | 07.00-12.00            |
| 4        | 96           | 1                  | 08.00-13.00            |
| 5        | 99           | 1                  | 09.00-14.00            |
| 6        | 100          | 1                  | 10.00-15.00            |
| 7        | 57           | 1                  | 11.00-16.00            |
| 8        | 68           | 1                  | 12.00-17.00            |
| 9        | 66           | 1                  | 13.00-18.00            |
| 10       | 95           | 1                  | 14.00-19.00            |
| 11       | 90           | 1                  | 15.00-20.00            |
| 12       | 80           | 1                  | 16.00-21.00            |
| 13       | 62           | 1                  | 17.00-21.00            |
| 14       | 59           | 1                  | 16.00-21.00            |
| 15       | 58           | 1                  | 07.00-11.00            |
| 16       | 82           | 1                  | 09.00-13.00            |
| 17       | 84           | 1                  | 11.00-15.00            |
| 18       | 52           | 1                  | 13.00-17.00            |
| 19       | 78           | 1                  | 15.00-19.00            |
| 20       | 57           | 1                  | 17.00-21.00            |

Table 4. Capacity, fixed cost and variable cost

| Vehicle (k) | Capacity (G_k) | Fixed Cost (G_k) | Variable Cost (v_k) |
|-------------|----------------|------------------|---------------------|
| 1           | 550            | 120,000          | 1,000               |
| 2           | 450            | 100,000          | 1,000               |
| 3           | 300            | 97,000           | 1,000               |
| 4           | 275            | 95,000           | 1,000               |
| 5           | 195            | 92,500           | 1,000               |
| 6           | 100            | 90,000           | 1,000               |

Table 5. Distance between nodes

| Node | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|
| 1    | 0 | 14| 20| 10| 1  | 3 | 20| 2  | 8  | 10 | 3  | 25 | 16 | 15 | 15 | 17 | 2  | 4  | 17 |
| 2    | 0 | 23| 25| 22| 25 | 9  | 18 | 11 | 7  | 2  | 14 | 12 | 21 | 13 | 6  | 12 | 19 | 24 | 15 |
| 3    | 0 | 21| 12| 18 | 24 | 15 | 8  | 3  | 23 | 18 | 4  | 20 | 11 | 15 | 18 | 14 | 5  |    |    |
| 4    | 0 | 16| 10| 5  | 4  | 11 | 7  | 16 | 5  | 22 | 6  | 22 | 21 | 10 | 11 |    |    |    |    |
| 5    | 0 | 4 | 20| 22 | 5  | 19 | 5  | 3  | 6  | 25 | 16 | 13 | 19 | 1  | 5  |    |    |    |    |
| 6    | 0 | 12| 16| 6  | 19 | 16 | 10 | 8  | 3  | 15 | 16 | 8  | 21 | 7  | 25 |    |    |    |    |
| 7    | 0 | 4 | 19| 19 | 13 | 11 | 3  | 15 | 2  | 13 | 12 | 23 | 14 | 5  |    |    |    |    |    |
| 8    | 0 | 25| 2 | 4  | 10 | 25 | 16 | 11 | 3  | 16 | 21 | 1 | 23 |    |    |    |    |    |    |
| 9    | 0 | 20| 12| 13 | 12 | 22 | 19 | 11 | 3  | 16 | 18 | 24 |    |    |    |    |    |    |    |
| 10   | 0 | 18| 13| 4  | 25 | 6  | 4  | 24 | 21 | 16 | 6  |    |    |    |    |    |    |    |    |
| 11   | 0 | 18| 4 | 16 | 7  | 22 | 7  | 18 | 23 | 21 |    |    |    |    |    |    |    |    |    |
| 12   | 0 | 19| 2 | 7  | 9  | 22 | 4  | 10 | 12 |    |    |    |    |    |    |    |    |    |    |    |
| 13   | 0 | 11| 17| 25 | 19 | 1 | 14 | 17 |    |    |    |    |    |    |    |    |    |    |    |    |
| 14   | 0 | 2 | 3 | 18 | 8  | 23 | 8  |    |    |    |    |    |    |    |    |    |    |    |    |    |
| 15   | 0 | 18| 10| 10 | 22 | 24 |    |    |    |    |    |    |    |    |    |    |    |    |    |    |    |
Table 6. Condition of road segment between nodes

| Node | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|
| 1    | -3 | 2 | 3 | 1 | 3 | 4 | 2 | 2 | 1 | 5 | 6 | 4 | 2 | 5 | 2 | 6 | 5 |
| 2    | -1 | 6 | 3 | 4 | 4 | 3 | 3 | 6 | 5 | 3 | 5 | 6 | 2 | 5 | 3 | 6 | 6 |
| 3    | -5 | 3 | 6 | 4 | 6 | 1 | 2 | 1 | 5 | 4 | 3 | 2 | 2 | 4 | 1 | 4 |
| 4    | -4 | 2 | 5 | 4 | 5 | 1 | 1 | 2 | 3 | 3 | 3 | 1 | 5 | 2 | 2 |
| 5    | -6 | 6 | 4 | 2 | 6 | 3 | 4 | 1 | 6 | 6 | 2 | 6 | 3 | 2 | 4 |
| 6    | -5 | 5 | 5 | 5 | 6 | 6 | 1 | 1 | 1 | 1 | 4 | 2 | 2 | 6 | 2 |
| 7    | -4 | 4 | 1 | 3 | 2 | 5 | 5 | 2 | 4 | 2 | 6 | 6 |
| 8    | -3 | 5 | 3 | 3 | 5 | 4 | 3 | 6 | 2 | 5 | 4 | 3 |
| 9    | -2 | 6 | 5 | 5 | 6 | 6 | 3 | 2 | 3 | 1 |
| 10   | -1 | 1 | 5 | 1 | 4 | 1 | 6 | 6 | 4 | 6 |
| 11   | -1 | 6 | 2 | 4 | 5 | 1 | 1 |
| 12   | -2 | 4 | 3 | 6 | 4 | 1 | 4 |
| 13   | -4 | 2 | 2 | 2 | 3 | 3 |
| 14   | -6 | 2 | 3 | 2 | 6 | 2 |
| 15   | -3 | 2 | 4 | 1 | 6 |
| 16   | -3 | 3 | 3 |
| 17   | -6 | 4 | 1 |
| 18   | -3 | 2 |
| 19   | -1 |
| 20   | - |

Table 7. Traveling time between nodes (10⁻² hour)

| Node | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
|------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|
| 1    | 00 | 45 | 65 | 37 | 10 | 18 | 57 | 13 | 41 | 45 | 23 | 81 | 55 | 54 | 48 | 55 | 54 | 57 | 48 |
| 2    | 00 | 00 | 74 | 82 | 62 | 75 | 33 | 60 | 39 | 32 | 50 | 45 | 74 | 51 | 26 | 41 | 70 | 30 | 55 |
| 3    | 00 | 09 | 59 | 45 | 55 | 80 | 55 | 41 | 23 | 74 | 60 | 20 | 57 | 47 | 75 | 55 | 53 | 23 |
| 4    | 00 | 50 | 45 | 16 | 57 | 23 | 19 | 43 | 28 | 57 | 27 | 62 | 29 | 71 | 67 | 45 | 47 |
| 5    | 00 | 24 | 30 | 60 | 69 | 27 | 55 | 23 | 18 | 30 | 82 | 57 | 47 | 55 | 10 | 23 |
| 6    | 00 | 45 | 55 | 26 | 62 | 59 | 40 | 30 | 18 | 55 | 50 | 41 | 67 | 32 | 75 |
| 7    | 00 | 20 | 58 | 63 | 43 | 10 | 42 | 16 | 55 | 13 | 51 | 45 | 78 | 80 |
| 8    | 00 | 66 | 12 | 24 | 37 | 75 | 50 | 39 | 19 | 57 | 67 | 07 | 66 |
| 9    | 00 | 65 | 45 | 47 | 45 | 69 | 70 | 42 | 21 | 57 | 53 | 77 |
| 10   | 00 | 61 | 51 | 19 | 81 | 25 | 20 | 80 | 74 | 50 | 30 |
| 11   | 00 | 61 | 24 | 57 | 37 | 65 | 29 | 61 | 74 |
| 12   | 00 | 63 | 13 | 31 | 37 | 35 | 20 | 35 | 40 |
| 13   | 00 | 38 | 59 | 75 | 63 | 10 | 45 | 64 |
| 14   | 00 | 14 | 23 | 55 | 41 | 78 | 41 |
| 15   | 00 | 33 | 45 | 35 | 71 | 80 |
| 16   | 00 | 29 | 72 | 57 | 19 |
| 17   | 00 | 42 | 48 | 20 |
| 18   | 00 | 43 | 31 |
| 19   | 00 | 74 |
| 20   | 00 |

Table 8. Service time by Vehicle 2

| Node | Time Windows \([a_i, b_i]\) | Duration of service \((s_i)\) | Start time \((y_{i1}^f)\) | End time \((y_{i2}^f)\) | Time limit for leaving | Time to next node \((t_{i1})\) |
|------|----------------|----------------|----------------|----------------|----------------|----------------|
| 1    | 06.00 - 22.00 | 0             | 06.00.00       | 06.00.00       | 07.50.00       | 22 min         |
| 4    | 08.00 - 13.00 | 1             | 08.12.00       | 09.12.00       | 10.10.05       | 5 min 20 sec   |
| 3    | 07.00 - 12.00 | 1             | 10.15.25       | 11.15.25       | 15.25.01       | 24 min 20 sec  |
| 10   | 14.00 - 19.00 | 1             | 15.49.21       | 16.49.21       | 19.48.40       | 11 min 20 sec  |
| Node | Time Windows | Duration of service | Start time | End time | Time limit for leaving | Time to next node |
|------|--------------|---------------------|------------|----------|------------------------|------------------|
| 13   | 17.00 - 21.00 | 1                   | 20.00.00   | 21.00.00 | 21.27.15               | 32 min 45 sec    |
| 1    | 06.00 - 22.00 | 0                   | -          | -        | -                      | -                |

**Table 9. Service time by Vehicle 3**

| Node | Time Windows | Duration of service | Start time | End time | Time limit for leaving | Time to next node |
|------|--------------|---------------------|------------|----------|------------------------|------------------|
| 1    | 06.00 - 22.00 | 0                   | 06.00.00   | 06.00.00 | 08.45.00               | 33 min           |
| 16   | 09.00 - 13.00 | 1                   | 09.18.00   | 10.18.00 | 11.32.24               | 8 min            |
| 7    | 11.00 - 16.00 | 1                   | 11.40.24   | 12.40.24 | 15.03.24               | 12 min 15 sec    |
| 8    | 12.00 - 17.00 | 1                   | 15.15.39   | 16.15.39 | 17.56.00               | 4 min            |
| 19   | 15.00 - 19.00 | 1                   | 18.00.00   | 19.00.00 | 21.45.30               | 14 min 30 sec    |
| 1    | 06.00 - 22.00 | 0                   | -          | -        | -                      | -                |

**Table 10. Service time by Vehicle 4**

| Node | Time Windows | Duration of service | Start time | End time | Time limit for leaving | Time to next node |
|------|--------------|---------------------|------------|----------|------------------------|------------------|
| 1    | 06.00 - 22.00 | 0                   | 06.00.00   | 06.00.00 | 08.33.00               | 24 min 24 sec    |
| 15   | 07.00 - 11.00 | 1                   | 09.03.36   | 10.03.36 | 13.16.07               | 21 min 15 sec    |
| 17   | 13.00 - 17.00 | 1                   | 13.37.22   | 14.37.22 | 19.41.40               | 18 min 20 sec    |
| 20   | 17.00 - 21.00 | 1                   | 20.00.00   | 21.00.00 | 21.27.00               | 33 min           |
| 2    | 06.00 - 22.00 | 0                   | -          | -        | -                      | -                |
| 2    | 06.00 - 22.00 | 0                   | 06.00.00   | 06.00.00 | 08.33.00               | 30 min 36 sec    |

**Table 11. Service time by Vehicle 5**

| Node | Time Windows | Duration of service | Start time | End time | Time limit for leaving | Time to next node |
|------|--------------|---------------------|------------|----------|------------------------|------------------|
| 2    | 06.00 - 22.00 | 0                   | 06.00.00   | 06.00.00 | 10.35.36               | 24 min 24 sec    |
| 17   | 11.00 - 15.00 | 1                   | 11.00.00   | 12.00.00 | 16.47.20               | 12 min 40 sec    |
| 9    | 13.00 - 18.00 | 1                   | 17.00.00   | 18.00.00 | 21.36.48               | 23 min 12 sec    |
| 2    | 06.00 - 22.00 | 0                   | -          | -        | -                      | -                |

**Table 12. Service time by Vehicle 6**

| Node | Time Windows | Duration of service | Start time | End time | Time limit for leaving | Time to next node |
|------|--------------|---------------------|------------|----------|------------------------|------------------|
| 2    | 06.00 - 22.00 | 0                   | 06.00.00   | 06.00.00 | 14.52.40               | 7 min 20 sec     |
| 11   | 15.00 - 20.00 | 1                   | 15.00.00   | 16.00.00 | 21.52.40               | 7 min 20 sec     |
| 2    | 06.00 - 22.00 | 0                   | -          | -        | -                      | -                |