Process planning solution for increasing the productivity and robustness of work schedules for batch production processes

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Abstract. This paper analyses the issue of ensuring enhanced productivity and robustness to work schedules designed for multi-operational batch production processes, when a fixed, limited number of workstations can be used for production. The productivity of a batch production schedule for a specific quantity of items is measured by the length of its cycle time, while the robustness of the schedule is given by the size and distribution of operational slack times that are ensured during the production process run. It is mathematically demonstrated that a simple process planning solution may be the key for meeting the above mentioned objectives, if it is possible to be applied in practice in combination with the lot streaming strategy for the production flow.

1. Introduction
Batch production is a manufacturing strategy used for manufacturing large quantities of items in periodical shares consisting in small-sizes, medium-sized or large-sized production batches [1, 2]. The units in each batch are moved from one workstation to another by using special transportation means. The entire production batch can be transferred at once in case of small-sized batches, or, in the cases of small-sized, medium-sized or large-sized batches, a fraction of production batch termed transfer batch is transported between the workstations of consecutive operations [1-3].

The two manufacturing flow strategies most used today for batch production are production batch flow strategy (BPF) and transfer batch flow strategy (BTF).

BPF is the manufacturing flow strategy that implies the transfer of the entire production batch from preceding operation with index number \( k \) to succeeding operation with index number \( k+1 \) [1-3]. The strategy is frequently used for small-sized batches.

BTF is the manufacturing flow strategy that implies the transfer of fractions of production batch known as transfer batches from operation \( k \) to operation \( k+1 \) [1-3]. This strategy may be used whatever the size of production batch \( N \), by adopting a size of transfer batch \( N_t \) smaller than that of production batch: \( N_t < N \) [1-3]. BTF strategy is frequently termed lot streaming strategy [4-10].

In order to deliver the transfer batches from preceding operation \( k \) to succeeding operation \( k+1 \) so that no idle times or work-item shortages appear in operation \( k+1 \), a minimum start-to-start lag time is necessary to be ensured for operations in case of BTF strategy [1, 2, 4, 9]. Assuming that setup times of operations are negligible with respect to their run times, the minimum start-to-start lag time \( D_{k,k+1} \) of consecutive operations \( k \) and \( k+1 \) can be computed as follows [2]:

\[ D_{k,k+1} = \frac{N_t}{N} \times D_{k,k+1} \]
Influence of operation times pattern on the productivity and robustness of production schedule

We assume that three operation time patterns may result after the planning stage of a multi-operational production process:

1. A pattern with random arrangement of process operation run time values (T\text{uk});
2. A pattern with decreasing values of the process operation run times (T\text{uk});
3. A pattern with increasing values of the process operation run times (T\text{uk}).

The productivity and robustness of a batch production schedule will be further investigated by taking into account only the operations time pattern in the production process. It is considered that a production process which needs a specific work quantity may be divided during the process planning stage in several operations with different operation time values.

The productivity of a batch production process that produces a quantity of N items can be measured by the length of its production cycle time (T_{cN}), so that the shorter the production cycle time, the higher the productivity of the process.

In the case of BPF strategy, minimum length of batch production cycle time can be calculated with equation (1.2), while for BTF strategy minimum length of batch production cycle time can be estimated with equation (1.3) [1, 2]. In both cases, setup times of operations are considered negligible with respect to operation run times.

\[ T_{cN} = N \cdot \sum_{k=1}^{n} T_{uk}, \text{for BPF} \]  \[ T_{cN} = N \cdot \sum_{k=1}^{n} T_{uk} + (N - N_t) \cdot \sum_{k=1}^{n} (T_{uk} - T_{uk+1})^+, \text{for BTF} \]

where: \( n \) is the total number of operations in the manufacturing process; \( (T_{uk} - T_{uk+1})^+ \) is the positive difference of run times of consecutive operations \( k \) and \( k+1 \) (time difference is taken into account only if \( T_{uk} > T_{uk+1} \)); \( T_{un+1} \) is nil run time of an additional operation with index number \( n+1 \) that is added at the end of manufacturing process for calculation purpose in equation (1.3).

As the sum of differences of run times in equation (1.3) is smaller than the sum of run times of operations, it appears from equations (1.2) and (1.3) that the productivity ensured by BTF strategy is always higher than that of BPF strategy, whatever the size of transfer batch. Under these conditions, our research will further focus only on the batch production processes using the BTF strategy, in order to find a solution for increasing the productivity and robustness of their work schedules.

The productivity of a batch production schedule for a specific size of production batch (N) is generally given by the value of \( T_{cN} \). However, in the case the production follows BTF strategy, the productivity of the schedule may be given also by the length of production cycles of transfer batches that compose the production batch. Let \( T_{cNl} \) be the length of production cycle time for transfer batch with index number \( l \) during production.

The robustness of a batch production schedule is given by the size and distribution of operational slack times that are ensured for the work on transfer batches during the run of production process following BTF strategy. The larger the slack times provided for transfer batches in the production schedule, the higher the robustness of batch production process schedule will be. Moreover, knowing that at the end of the execution of the batch production process may generally appear problems induced by the wear of machines and tool, the workers fatigue and boredom states, etc., which lead to the extension of run times of operations for last transfer batches, the robustness of a batch production schedule is given by the size of operational slack times for last transfer batches. Therefore, the larger the slack times provided for the last transfer batches, the higher the robustness of a batch production schedule will be.

The productivity and robustness of a batch production schedule will be further investigated by taking into account only the operations time pattern in the production process. It is considered that a production process which needs a specific work quantity may be divided during the process planning stage in several operations with different operation time values.

2. Influence of operation times pattern on the productivity and robustness of production schedule

We assume that three operation time patterns may result after the planning stage of a multi-operational production process:

- A pattern with increasing values of the process operation run times (T\text{uk});
- A pattern with decreasing values of the process operation run times (T\text{uk});
- A pattern with random arrangement of process operation run time values (T\text{uk}).
The following sequence of positive increasing numbers: \( a_1, a_2, a_3, \ldots, a_n \), will be considered, where \( n \) is the number of operations in manufacturing process.

Because terms \( N_i \sum_{k=1}^{n} T_{n k} \) and \((N - N_i)\) of equation (1.3) do not depend on the process operations run time pattern, the influence of the operation run times pattern will be studied only for the term \( \sum_{k=1}^{n} (T_{n k} - T_{n k+1})^+ \) in equation (1.3).

2.1. Influence on the productivity of the work schedule for entire production batch

2.1.1. Case I. Increasing values of the process operation run times
This hypothesis implies: \( T_{u n} = a_1, T_{u 2} = a_2, \ldots, T_{u m} = a_n \). Thus, the term \( \sum_{k=1}^{n} (T_{n k} - T_{n k+1})^+ \) value is as presented in equation (2.1).

\[
\sum_{k=1}^{n} (T_{n k} - T_{n k+1})^+ = \left( T_{n 1} - T_{n 2} \right) + \left( T_{n 2} - T_{n 3} \right) + \ldots + \left( T_{n m} - T_{n m+1} \right) = T_{u c} = a_g \quad \text{[min]} \tag{2.1}
\]

2.1.2. Case II. Decreasing values of the process operation run times
This hypothesis implies: \( T_{u n} = a_n, T_{u 2} = a_1, \ldots, T_{u m} = a_1 \). Thus, the term \( \sum_{k=1}^{n} (T_{n k} - T_{n k+1})^+ \) value is as presented in equation (2.2).

\[
\sum_{k=1}^{n} (T_{n k} - T_{n k+1})^+ = \left( T_{n 1} - T_{n 2} \right) + \left( T_{n 2} - T_{n 3} \right) + \ldots + \left( T_{n m} - T_{n m+1} \right) = T_{u c} = a_g \quad \text{[min]} \tag{2.2}
\]

2.1.3. Case III. Random arrangement of process operation run time values
In order to obtain this random arrangement, the insertion method will be used. Two hypotheses will be taken into account, as follows:
- a random arrangement obtained by relocating a member of an increasing operation run times pattern: \( a_1, a_2, a_3, \ldots, a_{k-1}, a_{k+1}, a_k, a_{k+1}, \ldots, a_{k-1} \), where \( 2 \leq j \leq k-1 \) and \( a_{k-j} < a_{k+1} \). Thus, the term \( \sum_{k=1}^{n} (T_{n k} - T_{n k+1})^+ \) value is as presented in equation (2.3).

\[
\sum_{k=1}^{n} (T_{n k} - T_{n k+1})^+ = \left( a_1 - a_2 \right) + \ldots + \left( a_{k-1} - a_{k-j} \right) + \left( a_{k-j} - a_k \right) + \ldots + \left( a_n - 0 \right) = a_g + a_{k-j} - a_k \quad \text{[min]} \tag{2.3}
\]

- a random arrangement obtained by relocating a member of a decreasing operation run times pattern: \( a_n, a_{n-1}, a_{n-2}, \ldots, a_k, a_{k+1}, a_{k+1}, a_{k+2}, \ldots, a_1 \), where \( 2 \leq j \leq k-1 \) and \( a_{k+1} < a_{k+1} \). Thus, the term \( \sum_{k=1}^{n} (T_{n k} - T_{n k+1})^+ \) value is as presented in equation (2.4).

\[
\sum_{k=1}^{n} (T_{n k} - T_{n k+1})^+ = \left( a_n - a_{n-1} \right) + \left( a_{n-1} - a_{n-2} \right) + \ldots + \left( a_{k-j} - a_k \right) + \left( a_k - a_{k+1} \right) + \ldots + \left( a_{k-j} - a_{k+1} \right) + \left( a_{k+1} - a_{k+2} \right) + \ldots + \left( a_1 - a_2 \right) + \left( a_2 - 0 \right) = a_n + a_{k-j} - a_{k+1} \quad \text{[min]} \tag{2.4}
\]

Analysing the above data, it can be observed that the batch production cycle time, \( T_{cN} \), has the minimum value for increasing or decreasing operation times pattern, as can be seen in equation (2.5).

\[
T_{cN}^{\min} = N_i \sum_{k=1}^{n} T_{n k} + (N - N_i) T_{u k}^{\max} \quad \text{[min]} \tag{2.5}
\]

where \( T_{u k}^{\max} \) represents the longest run time of the production process operations.
As a consequence, a solution for enhancing the productivity of the work schedule of a multi-operational batch production process is to design the process with an increasing or decreasing operation times pattern.

2.2. Influence on the productivity of the work schedules for transfer batches

As follows, the influence of the increasing and decreasing values of the process operation run times on the transfer batches production cycle time will be checked. The transfer batch production cycle time is the time interval between the start of the batch manufacturing (in first process operation) and the end of the batch manufacturing (in last process operation).

In order to demonstrate the influence given by the operation run times pattern on the productivity of the work schedules for transfer batches, the following transfer batches will be considered: 1, 2, …, l, …, n, which are forming the manufacturing batch \( (n_l = N/N_t) \).

2.2.1. Case I. Increasing values of the process operation run times

This type of process operation run times pattern is exemplified in the Gantt chart presented in figure 1. The production cycle times for the first three transfer batches in the production batch are computed by using equations (2.6) ÷ (2.8).

Based upon the above data, the following general equation can be used to calculate the production cycle time for the transfer batch with index number \( l \):

\[
T_{cN_l} = \sum_{k=1}^{n-1} D_{k,k-1} + N_l \cdot T_u = N_l \cdot \sum_{k=1}^{n-1} T_{uk} + N_l \cdot T_{u_1} = N_l \cdot \sum_{k=1}^{n} T_{uk} \quad [\text{min}] 
\]  

(2.6)

\[
T_{cN2} = T_{cN1} + N_l \cdot T_{u_2} - N_l \cdot T_{u_1} > T_{cN1} \quad [\text{min}] 
\]  

(2.7)

\[
T_{cN3} = T_{cN2} + N_l \cdot T_{u_3} - N_l \cdot T_{u_2} = T_{cN1} + (3-1) \cdot N_l \cdot (T_{u_3} - T_{u_1}) > T_{cN2} \quad [\text{min}] 
\]  

(2.8)

Thus, the transfer batches production cycle times ranking is: \( T_{cN1} < T_{cN2} < \ldots < T_{cNl} < \ldots < T_{cNn_t} \). Hence, it results that the minimum transfer batch production cycle time can be calculated as shown in equation (2.10).

\[
T_{cN_l}^{\text{min}} = N_l \cdot \sum_{k=1}^{n} T_{uk} \quad [\text{min}] 
\]  

(2.10)
2.2.2. Case II. Decreasing values of the process operation run times

This type of process operation run times pattern is exemplified in the Gantt chart presented in figure 2. The production cycle times for the first three transfer batches in the production batch are computed by using equations (2.11) ÷ (2.13).

\[ T_{c_{N1}} = \sum_{k=1}^{n-1} D_{k,k-1} + N_t \cdot T_{u_1} = N_t \cdot T_{u_1} - (N - N_t) \cdot T_{u_2} + N_t \cdot T_{u_2} - (N - N_t) \cdot T_{u_3} + \ldots + N \cdot T_{u_{n-1}} - (N - N_t) \cdot T_{u_n} + N_t \cdot \sum_{k=1}^{n} T_{u_k} + (N - N_t) \cdot (T_{u_1} - T_{u_2}) \]  

\[ T_{c_{N2}} = T_{c_{N1}} + N_t \cdot T_{u_1} - N_t \cdot T_{u_1} < T_{c_{N1}} \]  

\[ T_{c_{N3}} = T_{c_{N2}} + N_t \cdot T_{u_1} - N_t \cdot T_{u_1} = T_{c_{N1}} + (3-1) \cdot N_t \cdot (T_{u_n} - T_{u_n}) < T_{c_{N2}} \]  

Based upon the above data, the following general equation can be used to calculate the production cycle time for the transfer batch with index number \( l \):

\[ T_{c_{Nl}} = T_{c_{N1}} + (l-1) \cdot N_t \cdot (T_{u_1} - T_{u_1}) = N_t \cdot \sum_{k=1}^{n} T_{u_k} + (N - N_t) \cdot (T_{u_1} - T_{u_1}) + (l-1) \cdot N_t \cdot (T_{u_n} - T_{u_n}) \]  

For the last transfer batch, the cycle time is as follows:

\[ T_{c_{Nn}} = N_t \cdot \sum_{k=1}^{n} T_{u_k} + (N - N_t) \cdot (T_{u_1} - T_{u_1}) + (N - N_t) \cdot (T_{u_n} - T_{u_n}) = N_t \cdot \sum_{k=1}^{n} T_{u_k} \]  

Thus, the transfer batches production cycle times ranking is: \( T_{c_{N1}} > T_{c_{N2}} > \ldots > T_{c_{Nl}} > \ldots > T_{c_{Nn}} \).

Hence, it results that the minimum transfer batch production cycle time can be calculated as shown in equation (2.16).

\[ T_{c_{Nl}}^{\text{min}} = N_t \cdot \sum_{k=1}^{n} T_{u_k} \]  

As can be seen in equations (2.9) and (2.14) and figures 1 and 2, the transfer batches have identical production cycle times, with the pattern of these times reversed for batches. In both cases, the minimum cycle time for production of a transfer batch was the same, see equations (2.10) and (2.16). In conclusion, the analysed patterns give the same productivity for the schedule of batch production process.
2.3. Influence on the robustness of the work schedules for transfer batches

The slack time of a work schedule for the transfer batch \( j \) at operation \( k \) represents the time interval between the end of the batch manufacturing at operation \( k \) and the beginning of the same batch manufacturing at operation \( k+1 \).

2.3.1. Case I. Increasing values of the process operation run times

This type of process work schedule is exemplified the Gantt chart presented in figure 3 and the calculation of the slack times of transfer batches in process operations is presented in equations (2.17) ÷ (2.19).

\[
RT^k_{Nt_1} = 0 \quad [\text{min}] \\
RT^k_{Nt_2} = N_j \cdot (T_{uk+1} - T_{uk}) \quad [\text{min}] \\
RT^k_{Nt_l} = N_j \cdot (l-1) \cdot (T_{uk+1} - T_{uk}) \quad [\text{min}] \\
\]

Thus, the transfer batches slack times ranking is: \( RT^k_{Nt_1} \leq RT^k_{Nt_2} \leq RT^k_{Nt_l} \leq \ldots \leq RT^k_{Nt_{Nt}} \).

2.3.2. Case II. Decreasing values of the process operation run times

This type of process work schedule is exemplified the Gantt chart presented in figure 4 and the calculation of the slack times of transfer batches in process operations is presented in equations (2.20) ÷ (2.22).

\[
RT^k_{Nt_1} = \ldots \\
RT^k_{Nt_2} = \ldots \\
RT^k_{Nt_l} = \ldots \\
\]

Thus, the transfer batches slack times ranking is: \( RT^k_{Nt_1} \geq RT^k_{Nt_2} \geq RT^k_{Nt_l} \geq \ldots \geq RT^k_{Nt_{Nt}} \).
Thus, the transfer batches slack times ranking is: $RT_{N_{t_i}}^k \geq RT_{N_{t_{i-1}}}^k \geq RT_{N_{t_{i-2}}}^k \geq ... \geq RT_{N_{t_1}}^k$.

As can be seen in equations (2.19) and (2.22) and figures 3 and 4, for both cases the transfer batches slack times are identical, with the pattern of these times reversed for the transfer batches.

However, an extension of the run time for the last transfer batch in the first operation, $\Delta T_{N_{t_3}}^1$, as shown in figures 5 and 6, has different consequences on the cycle times of both the transfer batch and production batch depending on the operation run times pattern.

Figure 5. Schedule of transfer batches in the case of increasing values of the process operation run times, when an extension of the first operation occurs.

Figure 6. Schedule of transfer batches in the case of decreasing values of the process operation run times, when an extension of the first operation occurs.
In the case of increasing values of the process operation run times and when the extension of the run time for the last transfer batch in the first operation is smaller than the slack \( RT_{N/3}^1 > \Delta T_{N/3}^1 \), as seen in figure 5, the length of the cycles will not be modified for the last transfer batch and for the entire production batch as compared with those calculated with equations (2.9) and (1.3) and graphically presented in figure 1.

In the case of decreasing values of the process operation run times, any extension of the run time of the last transfer batch in first operation \( \Delta T_{N/3}^1 \), as shown in figure 6, will extend the length of cycles for the last transfer batch and the entire production batch as compared with those calculated with equations (2.14) and (1.3) and graphically presented in figure 2.

As a consequence, a solution for enhancing the robustness of the work schedule of a multi-operational batch production process is to design the process with an increasing operation run times pattern because this pattern ensures larger slack times for the last transfer batches within the production batch.

3. Conclusions
This paper studied multi-operational batch production processes and the need to ensure high productivity and robustness for the work schedules of these processes. It was mathematically demonstrated that a favourable design of the processes, with increasing operation run times pattern, will minimize the production cycle time for the batch and will maximize the robustness of the work schedule of the process, in combination with the application of lot streaming strategy in the production flow.

4. References
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