Determination of the admissible region of asteroids with data from one night of observation

Daniela Espitia Mosquera\(^{1}\) and Edwin Andrés Quintero Salazar\(^{1,2}\)
\(^{1}\) Grupo de Investigación en Astroingeniería Alfa Orión. Observatorio Astronómico, Facultad de Ciencias Básicas, Universidad Tecnológica de Pereira, Complejo Educativo La Julita, Risaralda, Colombia.

E-mail: \(^{1}\) daesmo95@utp.edu.co, \(^{2}\) equintero@utp.edu.co

Abstract. During the night of the discovery of an asteroid, a large number of images spaced in time, that represent an arc too short to propagate an orbit, are obtained. Initially, it is necessary to recover the body in the celestial vault to have more observations to determine its orbit. The first step in this process is to establish the admissible region, defined as the region in space where the object can be found. In this paper we present the calculation of the Admissible Regions from data from a single night observation, considering the geocentric and topocentric versions and restrictions such as belonging to the Solar System, the object does not belong to the Earth-Moon gravitational system, and the body is at a minimum distance from Earth. This procedure was applied in the calculation of the admissible regions of 2003 BH84, 3122 Florence, 3200 Phaethon, 555 Norma, 1738 Oosterhoff and 2006 SO375. The respective admissible regions were generated in their geocentric and topocentric variant, and the respective metric changes were made to visualize their geometric characteristics. It was found that the topocentric version generates a simpler geometry than the geocentric version, decreasing the re-observation area. It was identified that the logarithmic metric is appropriate for the study of regions near the Earth (NEO’s).

1. Introduction
The study of the minor bodies of the Solar System (asteroids and comets) is of high relevance for the astronomical community in general. The correct identification of this type of objects and the adequate treatment of the available data are fundamental for the anticipation of future collisions with Earth. In this way, knowing the orbit of the small bodies makes it possible in the future to know precisely the position of the bodies and anticipate an imminent risk to the Earth. Consequently, the study of minor bodies has been a subject of great interest to engineers, mathematicians and astronomers [1]. Subsequently, both instrumental and theoretical techniques have been developed for the acquisition of data and the design of mathematical models that allow scientific community to understand the behavior of these celestial bodies.

The technological advance in the field of astronomical instrumentation has made it possible to obtain a large number of images of the sky in a short period of time. Thus, when observatories around the world register new minor bodies, the first task is to track them. This allows for the recovery of the object in subsequent nights (procedure known as recovery). That is, the first task is to guarantee the re-observation of the asteroid during subsequent nights from the prediction of its location on the celestial vault. Because of this procedure, the set of available data increases in such a way that it is possible to construct a preliminary orbit.
When the sequence of observations belongs to the same object, or if it can be adjusted to a smooth curve (usually a low-degree polynomial), the sequence is called VSA (Very Short Arc) [2]. If a preliminary orbit cannot be determined or the Gauss differential correction procedure fails, the sequence of observations is called TSA (Too Short Arc) [3]. To carry out the recovery of the asteroid, it is necessary to first define the region in space in which it can be found (defined as the Admissible Region). For this, we have a set of observations that form a VSA with a record of \( N \) observations, generally spaced around 2 hours. From this set of observations, a linear adjustment is made by the method of least squares for the average observation time \( \bar{t} \). This results in the following data: 2 average angular coordinates (\( \alpha \) right ascension and \( \delta \) declination) on the celestial vault and two angular velocities [4]. This data set receives the name of attributable. By itself, the attributable does not permit the discovery of the radial distance \( \rho \) nor the speed in the reference period \( \dot{\rho} \) of the body in question. However, through the attributable, it is possible to extract information from the general characteristics of the object that allow the determination of possible values of \( \rho \) and \( \dot{\rho} \). To reduce the size of the region where these values are found, some restrictions are applied: the object belongs to the Solar System, the object is not an artificial satellite (if it is below the radius of the sphere of influence of the Earth), and finally, the object is not a shooting star (for which it is necessary to access the average of the apparent magnitudes [4, 5]).

The theoretical development that considers these restrictions has been widely used by the asteroid tracking community. In these investigations, the theoretical method establishes the set of physically acceptable orbits, according to a set of restrictions for the values (\( \rho, \dot{\rho} \)). As a result, a region is defined in the space known as the admissible region (AR), which is based on very short arc observations. This procedure was tested for the first time by Milani and Gronchi with an asteroid type NEA (Near Earth Asteroid) named 2003 BH84, of orbit type Apollo discovered on January 25, 2003 [4]. The method was only tested with the data on the night of its discovery and allowed the recovery of the body later and apply it to other studies. This method has been widely used in the study of space debris. For example, for optical observations of objects in Earth orbit [6], to establish the region of such debris using radar data [7] or even for synthetic data [8]. Additionally, methods have been developed that involve probability density functions that are associated with the calculation of the AR [9]. These methods are also developed as a starting element to define a probabilistic AR that is used to initialize a Bayesian tracking scheme [10]. The AR has been used conveniently, among other things, to calculate the probability of impact with the Earth [11]. This method has been tested with asteroids 2008 TC3 and 2014 AA. The AR has also been used to add new internal limits based on the geocentric hyperbolic movement of the immediate impactors [12], and in the study of distant objects in the Solar System [13].

In this paper the admissible regions of a sample of small bodies of the Solar System, corresponding to the NEA and MBA groups (Main Belt Asteroids), are calculated. For this, only a one-night observation, with data spaced on average of every two hours, were used. Most data used in this study come from the Astronomical Observatory of the Technological University of Pereira (MPC Code W63 [14]) with a MEADE LX200 GPS telescope and a SBIG STF8300M camera. How the topocentric correction [15] can reduce the region in which the asteroid being studied in several lunar distances is studied. Additionally, a metric change is made to study the geometric characteristics as well as establish which are more convenient for certain groups of asteroids. The results show that this methodology can be applied to calculate the admissible region of any type of asteroid that has not had any previous record in the database of the MPC (Minor Planet Center).
2. Methodology

The flowchart shown in Figure 1 illustrates the procedure followed for the calculation of the admissible region (AR). The process begins with a set of observations that are obtained from a small body during a night of observation. These data are adjusted by the method of linear least squares. This process results in a set of four variables known as attributable. Another necessary process is the calculation of the Earth state vector \((\mathbf{R}, \dot{\mathbf{R}})\) using the Poincaré interpolation. The result of both processes consists of six constant values that represent the input variables for the calculation of the admissible region (orange blocks in Figure 1).

![Flowchart](image)

Figure 1. Flowchart that synthesizes the calculation process of the admissible region.

After finding the AR, it is necessary to calculate the region where the artificial satellites of the Earth are. To accomplish this, a lower limit is established from two approximations: 1. Consider the terrestrial radio as the minimum distance at which the object can be placed and 2. Use the average of the apparent magnitudes to define a minimum distance at which the object has a significant size (blue blocks in Figure 1).

The next step is the calculation of the region of space where the orbits that belong to the Solar System are located. At this point, negative energies were considered for the geocentric approach. Conversely, for the topocentric correction, comets of long orbital period were excluded (violet blocks in 1).

Finally, the previous steps show a combination of possible values for \((\rho, \dot{\rho})\) in which the asteroid can be found (pink block in Figure 1). From this point, a metric change is made to show the geometric characteristics of the admissible region. A logarithmic metric is used for the geocentric approach and an exponential for the topocentric correction. In the following sections, the aforementioned process is explained in detail and is synthesized in the flowchart shown in Figure 1.

3. Input variables of the admissible region

The calculation of the AR uses as input data a series of parameters that are needed to be established. These data are represented by the attributable and the state vector of the Earth.
3.1. Finding the attributable

The determination of the AR starts from a set of astrometric data obtained during a night of observation, expressed as follows:

\[(t_i, \alpha_i, \delta_i) \quad i = 1, ..., m\]  

(1)

Where \(t\) represents the observation time, \(\alpha\) the right ascension, \(\delta\) the declination and \(m\) the total number of observations \((m \geq 2)\). From this, adjustments are made to the data to set to a linear model for a mean time \(\bar{t}\) from the \(t_i\) [5], of the form:

\[\xi(t) = \xi(\bar{t}) + \dot{\xi}(\bar{t})(t - \bar{t})\]  

(2)

Where \(\xi\) denotes the coefficient for \(\alpha, \delta\). This data set is grouped in a vector known as attributable \(A\), [16] which is defined as:

\[A = (\alpha, \delta, \dot{\alpha}, \dot{\delta}) \in [-\pi, \pi] \times \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \times \mathbb{R}^2\]  

(3)

The average value of the right ascension \((\alpha)\) and declination \((\delta)\), and the average of its temporal derivatives \((\dot{\alpha}, \dot{\delta})\) obtained in equation (3), correspond to 4 of the input variables of the main algorithm. The two remaining variables are given by the Earth state vector \((\mathbf{R}, \mathbf{R})\) and are obtained from the Poincaré interpolation.

3.2. Poincaré interpolation

In this study, the geocentric and topocentric variant are used to calculate the AR [15]. These variants determine how the Earth state vector \((\mathbf{R}, \mathbf{R})\) is calculated. In the geocentric approach it is considered that the registered observations are made from the center of the Earth. Whereas, the topocentric approach considers the position of the observer [17]. Therefore, the state vector calculation is the sum of the heliocentric position of the center of the Earth and the geocentric position of the observer, as shown in equation (4)

\[\mathbf{R} = \mathbf{R}_\oplus + \mathbf{P}_{\text{obs}}\]  

(4)

And, for the velocity vector:

\[\dot{\mathbf{R}} = \dot{\mathbf{R}}_\oplus + \dot{\mathbf{P}}_{\text{obs}}\]  

(5)

Where \(\mathbf{R}_\oplus\) indicates the heliocentric position of the Earth and \(\mathbf{P}_{\text{obs}}\) the position of the observer. For the calculation of \(\mathbf{P}_{\text{obs}}\), it is essential to know the parallax constants of the observatory that makes the observations (available in the web page of the Minor Planet Center, MPC, https://www.minorplanetcenter.net/iau/lists/ObsCodesF.html). The calculation of the position and speed of the observer should be made using the same interpolation shown in equation (2) [18, 19]. For example, the linear variant of the Poincaré interpolation is recommended for when there is a short observation interval [5].

4. Determining the admissible region

To obtain a reliable preliminary orbit, the data obtained during a night of observation of a small body may be insufficient. When applying the classical methods to the set of observations to find a preliminary orbit, these methods do not converge [20]. This set of observations that does not generate an initial orbit is known as TSA. In addition to this, the attributable vector (3) calculated from a TSA, does not yield the values of \(\rho\) and \(\dot{\rho}\) on its own. However, in [4] a method is proposed that allows for the restriction of such values if they are assumed, among other restrictions. Two \textit{a priori} assumptions are that the object belongs to the Solar System
and the object is not a terrestrial satellite. Thus, given an attributable, the AR is defined as the set of all possible pairs \((\rho, \dot{\rho})\) that satisfy such conditions [11].

To restrict that set of values of \(\rho\) and \(\dot{\rho}\), we assume a small body \(B\) that belongs to the Solar System and moves around the Sun with a heliocentric position \(\mathbf{r}\). This small body is observed from Earth \(E\) with a radius vector \(\mathbf{R}\), known for some instant of time. These restrictions include: the object belongs to the Solar System, the object is not a satellite of the Earth, and the object is beyond a minimum distance from Earth.

4.1. Belonging to the Solar System

In order to restrict the fact that the small body belongs to the Solar System, one of the following two heliocentric energy conditions must be fulfilled:

\[
E(\rho, \dot{\rho}) \leq 0, \quad E(\rho, \dot{\rho}) \leq -k^2/(2a_{max})
\]  

(6)

However, the last condition was used to perform topocentric correction with \(a_{max} = 100\) AU which indicates, in addition to the above, that the object is not a comet of a too long period [5]. Due to the robustness of the method, a dynamic two-body model is used. Its energy is established as:

\[
E(\rho, \dot{\rho}) = \frac{1}{2}\|\dot{\mathbf{r}}(\rho, \dot{\rho})\|^2 - k^2 \frac{1}{r(\rho)}
\]  

(7)

Where \(E(\rho, \dot{\rho})\) denotes the heliocentric energy of two bodies with \(k^2 = 0.01720200895\) which is known as the Gaussian gravitational constant [5, 21].

From the equation (7), the values of \(\mathbf{r}, \dot{\mathbf{r}}\) are the position and heliocentric velocity of \(B\), respectively, as shown in equation (8)

\[
\mathbf{r} = \mathbf{R} + \rho \hat{\rho} \quad \dot{\mathbf{r}} = \dot{\mathbf{R}} + \dot{\rho} \hat{\rho} + \rho \left(\dot{\alpha} \hat{\rho}_\alpha + \dot{\delta} \hat{\rho}_\delta\right)
\]  

(8)

with:

\[
\hat{\rho} = (\cos \alpha \cos \delta, \sin \alpha \cos \delta, \sin \delta), \quad \hat{\rho}_\alpha = (-\sin \alpha \cos \delta, \cos \alpha \cos \delta, 0), \quad \hat{\rho}_\delta = (-\cos \alpha \sin \delta, -\sin \alpha \sin \delta, \cos \delta)
\]

Now, by taking the equations (8), the square norms of the heliocentric position and velocity are calculated, resulting in:

\[
\mathbf{r}^2 = \rho^2 + \epsilon_5 \rho + \epsilon_0 \\
\|\dot{\mathbf{r}}(\rho, \dot{\rho})\|^2 = \dot{\rho}^2 + \epsilon_1 \dot{\rho} + \epsilon_2 \rho^2 + \epsilon_3 \rho + \epsilon_4
\]  

(9)

Where the coefficients \(\epsilon_0\) to \(\epsilon_5\) are:

\[
\epsilon_0 = \|\mathbf{R}\|^2, \quad \epsilon_3 = 2\dot{\alpha} \hat{\mathbf{R}} \cdot \hat{\rho}_\alpha + 2\dot{\delta} \hat{\mathbf{R}} \cdot \hat{\rho}_\delta, \\
\epsilon_1 = 2\dot{\mathbf{R}} \cdot \hat{\rho}, \quad \epsilon_4 = \|\dot{\mathbf{R}}\|^2, \\
\epsilon_2 = \dot{\alpha}^2 \cos^2 \delta + \dot{\delta}^2 = \eta^2, \quad \epsilon_5 = 2\mathbf{R} \cdot \hat{\rho}
\]  

(10)

To determine the outermost limit of the AR, and therefore the pairs \((\rho, \dot{\rho})\), it is necessary to know the largest value of the variable \(\rho\), or, the maximum distance limit to which the body can
be found. The expressions (9) are replaced in equation (7) and the first condition of equation (6) is considered, so that:

$$2E_\odot(\rho, \dot{\rho}) = \dot{\rho}^2 + \epsilon_1 \dot{\rho} + W(\rho) - \frac{2k^2}{\sqrt{S(\rho)}} \leq 0$$

(11)

Where $S(\rho) = \rho^2$ and $W(\rho) = \epsilon_2 \rho^2 + \epsilon_3 \rho + \epsilon_4$. In general, to obtain real solutions, the discriminant $\dot{\rho}$ must be zero or positive, like this:

$$\frac{\epsilon_1^2}{4} - W(\rho) + \frac{2k^2}{\sqrt{S(\rho)}} \geq 0$$

(12)

The polynomial $W(\rho)$ is replaced in equation (12) resulting in the expression

$$\frac{2k^2}{\sqrt{S(\rho)}} \geq \epsilon_2 \rho^2 + \epsilon_3 \rho + \epsilon_4 - \frac{\epsilon_1^2}{4}$$

(13)

To simplify the expression (13), $\gamma = \epsilon_4 - \epsilon_1^2/4$ ($\gamma \geq 0$) and $P(\rho) = \epsilon_2 \rho^2 + \epsilon_3 \rho + \gamma$ are considered. Now, introducing the first condition established in (6), an inequality is obtained that, when squared on both sides, results in [4]:
Where $\mu = \frac{1}{328900.5614}$ [4]. Using $\|\dot{\rho}(\rho, \dot{\rho})\| = \dot{\rho}^2 + \rho^2 \eta^2$, where $\eta = \sqrt{e_2}$ corresponds to the proper motion, equation (15) is reduced to:

$$\rho^2 + \rho^2 \eta^2 - 2k^2 \frac{\mu}{\rho} \geq 0$$

Which implies that:

$$\dot{\rho}^2 \geq F(\rho), \quad F(\rho) = \frac{2k^2 \mu}{\rho} - \eta^2 \rho^2$$  \hspace{1cm} (16)

Where $F(\rho) > 0$ for $0 < \rho < \rho_0 = \sqrt{(2k^2 \mu)/\eta^2}$. However, the condition defined by equation (15) is significant within the sphere of influence of the Earth. If not, $B$ is dominated by the Sun [5] would be concluded. To restrict the fact that the object is too close to the observer, it is necessary to introduce the condition:

$$\rho \geq R_{SI}$$  \hspace{1cm} (17)

Where $R_{SI} = 0.0100044$ AU [4] is known as the radius of the sphere of influence of the Earth $R_{SI}$ (which is equivalent to the Hill Sphere radius for Earth [22]). In order to exclude orbits that are dominated by Earth, equations (15) or (17) are used. For this, the parameter $\rho_0$ is calculated, and from this, two possible situations are identified. If $\rho_0 \leq R_{SI}$, then the AR (in its internal part) is defined by (16) and intercepted in $\rho = \rho_0$. Conversely, if $\rho_0 > R_{SI}$, then the region that contains the orbits controlled by the Earth is formed by the straight-line segment $\rho = R_{SI}$ and the two arcs of the curve $\dot{\rho}^2 = F(\rho)$ where $0 < \rho < R_{SI}$. This curve is symmetric with respect to $\dot{\rho} = 0$.

However, the initial value of the variable $\rho$ has two connotations. When the geocentric approach is considered, the initial value of $\rho$ is the radius of the Earth, which means $\rho \geq R_{\text{earth}} \sim 4.2 \times 10^{-5}$ AU. On the other hand, when the topocentric correction is used, the variable $\rho$ starts from a value that is not close to the observer, determined as described below.

4.3. Internal limit for the Admissible Region in the topocentric correction

According to the aforementioned, the value of the internal limit of the AR can be studied from the point of view of an observer located in the center of the Earth (geocentric version) or considering its position on it (topocentric version). In the latter, it is necessary to consider the second condition given in (6) and include an additional restriction in order to discard those orbits that correspond to meteors that are too small to become meteorite sources [11]. In fact, when topocentric correction is used, the condition $\rho \geq R_{\text{earth}}$ has no meaning [4].

4.3.1. Limit for very small objects

In these investigations [4, 5], an alternative condition is proposed to set a lower limit to the distance $\rho$. The condition consists in considering that the object is not a shooting star. This is one of the conditions that most influences the reduction of the AR shown here. This condition is introduced considering the absolute magnitude and setting a maximum for it, like this:

$$H(\rho) \leq H_{max}$$  \hspace{1cm} (18)

For the case in which there are values of apparent magnitude, the absolute magnitude $H$ is obtained from the average of apparent magnitudes $h$:

$$H = h - 5 \log_{10} \rho - x(\rho)$$  \hspace{1cm} (19)

By combining equations (18) and (19), we get [4]:

$$\log_{10} \rho \geq \frac{h - H_{max} - x_0}{5} := \log_{10} \rho_H$$  \hspace{1cm} (20)
The AR can reduce its size using the condition (18) without making its geometry more complicated. For example, for $\rho_H \geq R_{\text{earth}}$ the same considerations shown in section 4.2 are contemplated. However, when performing the topocentric correction, the case $\rho_H \geq R_{SI}$, to calculate the condition defined by the equation (15) was unnecessary; therefore, the geometry became much simpler. In this study, for the calculations was considered that $x_0 = 0$ [4, 5].

5. Results

The procedure described in the previous section was implemented in MATLAB functions and was applied in the calculation of the AR of the following 6 asteroids: Three asteroids type NEA (2003 BH84, and the PHAs 3200 Pheaton and 3122 Florence), two asteroids MBA (1738 Oosterhoff and 555 Norma) and finally the asteroid type Hilda 2006 SO375. Predominantly, data registered by the Astronomical Observatory of the Technological University of Pereira (OAUTP, Code MPC W63) were used. Some supplemented observations were registered from the observatories 809 and 705 (see Table 1). The algorithms developed and written in MATLAB function, and the data used in this article, are available in https://observatorioastronomico.utp.edu.co/astrometria/determination-of-the-admissible-region-of-asteroid.html.

For the recovery of asteroids, the AR must be sampled following a procedure called triangulation. The sampling process requires that the AR uses a metric that optimizes the visualization of its geometric characteristics. Considering previous information, in this study, the AR of the asteroids under study were plotted using two metrics [5] in order to establish which one is the most appropriate, according to the different groups to which the objects belong.

The first metric used is defined by the function:

$$f(\rho) = 1 - \exp\left(-\frac{\rho^2}{2s^2}\right)$$

(21)

Where $s = \rho_{\text{max}}$.

Finally, the second metric used is defined by:

$$f(\rho) = \log_{10}(\rho)$$

(22)

For the case of the geocentric approach, the metric defined by equation (22) was used. This metric shows the qualitative characteristics of the AR to be evidenced in the regions close to the Earth. Conversely, for topocentric correction, both metrics were used in order to evaluate their suitability when graphically presenting the geometric characteristics of the calculated AR.

Figure 2 shows the ARs calculated for the small bodies under study in the geocentric approach. In this case, the metric shown by the equation (22) was used. As shown in the Figure 2, due to the change of metrics, is it possible to see the characteristics corresponding to the geocentric energy curve. On the other hand, Figures 3 and 4 show the metrics (21) and (22) respectively, applied to the topocentric correction. For both cases, the calculation of the equation (15) was not necessary, thus reducing the complexity of the AR and in the same way the search area of the asteroid in the future.

For the verification of the ARs calculated and shown in Figures 2 to 4, the actual position of the respective object was calculated for the average time $\bar{t}$ (see last two columns of Table 1), using data obtained from the ephemeris simulator of NASA (available at: https://ssd.jpl.nasa.gov/horizons.cgi). In Figures 2 to 4 the actual positions of all the tested objects are effectively within the limits of the calculated ARs are observed.

As seen in Figures 3 and 4, the calculated ARs have a simpler geometry than those presented in Figure 2. This is because no additional geocentric approach was used. In this way, the
Figure 2. AR of different asteroids with geocentric approach using the metric defined by equation (22) for: (a) 3122 Florence, (b) 3200 Phaethon, (c) 2003 BH84, (d) 1738 Oosterhoff, (e) 555 Norma, (f) 2006 SO375.

Figure 3. AR of different asteroids with topocentric correction using the metric defined by equation (21) for: (a) 3122 Florence, (b) 3200 Phaethon, (c) 2003 BH84, (d) 1738 Oosterhoff, (e) 555 Norma, (f) 2006 SO375.
Figure 4. AR of different asteroids with topocentric correction using the metric defined by equation (22) for: (a) 3122 Florence, (b) 3200 Phaethon, (c) 2003 BH84, (d) 1738 Oosterhoff, (e) 555 Norma, (f) 2006 SO375.

Table 1. Values obtained for test asteroids.

| Asteroid     | ∆t              | Type         | $S_{max}$ (AU) | $\eta$ (°/day) | Obs. Code | Real position |
|--------------|-----------------|--------------|----------------|----------------|-----------|---------------|
| 3122 Florence| 1 h 44 min      | NEA/PHA      | 0.0863         | 0.0849         | W63       | 0.0475, 0.0011 |
| 3200 Phaethon | 2 h 04 min      | NEA/PHA      | 0.2147         | 0.2157         | W63       | 0.0859, -0.0110 |
| 2003 BH84    | 1 h 37 min      | NEA          | 4.4507         | 4.4617         | 809       | 1.9928, 0.0065 |
| 1738 Oosterhoff | 1 h 24 min     | MBA          | 1.8371         | 1.7940         | W63       | 1.0104, 0.0081 |
| 555 Norma    | 1 h 34 min      | MBA          | 3.3571         | 3.2767         | W63       | 2.2395, 0.0131 |
| 2006 SO375   | 1 h 23 min      | Hilda        | 7.5751         | 7.5199         | 705       | 2.2756, -0.0044 |

The definition of AR that best fits the group of asteroids studied in this study is that given by the set:

$$ D(A) = \left\{ (\rho, \dot{\rho}) : \rho \geq \rho_H, E_\odot \leq -\frac{k^2}{2a_{max}} \right\} $$

Finally, it is observed that for the study of asteroids located in regions close to the Earth, the metric defined by equation (22) is the most appropriate. This is because this metric highlights the points near the observer’s position. Consequently, the metric given by equation (22) is suitable for the study of those small bodies that may become possible impactors, given their proximity to the Earth. Conversely, the results obtained suggest that when it is required to study objects located in the outermost zone of the AR, the topocentric correction and the metric defined by equation (21) should be used.
6. Conclusion
The asteroids studied in this investigation, belonging to different groups of the Solar System, were located within the limits of the calculated admissible regions. This shows the versatility of the method used to determine the AR, since, considering the short time interval of the observational data it was possible to estimate a search area to relocate that small body on later nights. The results obtained show that the logarithmic metric facilitates the visualization of the geometry of the interior zones of the AR. Therefore, the logarithmic metric is the most suitable for the recovery of small bodies that are in the vicinity of the planet Earth. Conversely, for the recovery of asteroids that are farthest from the vicinity of the Earth, the metric that presented a better performance was the exponential. Despite the above, both metrics permitted the observation of the effects of the restriction of the possible values for $\rho, \dot{\rho}$, which delimit admissible region.

The geocentric approach generated a more complex geometry for the admissible region. This situation increases the search area that be observed again to find the asteroid. Meanwhile, the topocentric correction, due to the calculation of the apparent magnitudes of the object, permitted the exclusion of bodies dominated by terrestrial gravity. This situation significantly reduces the search area of the asteroid making the recovery easier, greatly simplifying the calculations after the determination of the admissible region.

Acknowledgments
The authors express their gratitude to the Technological University of Pereira for their financial support for the execution of the research project "Methodology for the Generation of Ephemeris and the Refining of the Orbit of Small Bodies of the Solar System of Recent Discovery”, code 3-18 -9.

References
[1] Cellino A and Dell’Oro A 2011 Memorie della Societa Astronomica Italiana 82 300
[2] Milani A, Gronchi G F, Knežević Z, Sansaturio M E and Arratia O 2005 Icarus 179 350–374
[3] Gronchi G F 2004 Proceedings of the International Astronomical Union 2004 293–303
[4] Milani A, Gronchi G F, Vitturi M d and Knežević Z 2004 Celestial Mechanics and Dynamical Astronomy 90 57–85
[5] Milani A and Gronchi G 2009 Theory of Orbit Determination (Cambridge University Press)
[6] Maruskin J M, Scheeres D J and Alfriend K T 2009 Journal of Guidance, Control, and Dynamics 32 194–209
[7] Tommei G, Milani A and Rossi A 2007 Celestial Mechanics and Dynamical Astronomy 97 289–304
[8] DeMars K J, Jah M K and Schumacher P W 2012 IEEE Transactions on Aerospace and Electronic Systems 48 2028–2037
[9] DeMars K J and Jah M K 2013 Journal of Guidance, Control, and Dynamics 36 1324–1335
[10] Hussein I I, Roscoe C W, Schumacher Jr P W and Wilkins M P 2014 Probabilistic admissible region for short-arc angles-only observations Tech. rep. AIR FORCE RESEARCH LAB KIHEI MAUI HI DETACHMENT 15
[11] Spoto F, Del Vigna A, Milani A, Tommei G, Tanga P, Mignard F, Carry B, Thuillot W and David P 2018 Astronomy & Astrophysics 614 A27
[12] Valk S and Lemaitre A 2006 Proceedings of the International Astronomical Union 2 455–464
[13] Farnocchia D, Chesley S R, Milani A, Gronchi G F and Chodas P W 2015 Orbits, Long-Term Predictions, Impact Monitoring (University of Arizona Press) pp 815–834
[14] Villarraga S J, Salazar E A Q and Galvis J A A 2017 Téccienca 12 43–50
[15] Curtis H D 2013 Orbital mechanics for engineering students (Butterworth-Heinemann)
[16] Milani A, Sansaturio M E and Chesley S R 2001 Icarus 151 150–159
[17] Danby J 1962 Fundamentals of celestial mechanics (Macmillan)
[18] Poincaré H 1906 Bulletin Astronomique, Serie I 23 161–187
[19] Villarraga S J and Quintero-Salazar E A 2016 Revista de la Academia Colombiana de Ciencias Exactas, Físicas y Naturales 40 43–52
[20] Virtanen J, Muinonen K and Bowell E 2001 Icarus 154 412–431
[21] Barbosa J 2009 *Elementos de astronomía de posición* Coleccion textos (Universidad Nacional de Colombia. Facultad de Ciencias) ISBN 9789587192643

[22] Von Bloh W, Bounama C and Franck S 2005 *A Comparison of the Dynamical Evolution of Planetary Systems* (Springer) pp 287–300