Propagation and scattering effects in temporal metastructures

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Abstract. Electromagnetic scattering typically occurs when a change in the material properties is perceived by the propagating wave, that inevitably splits into a reflected and refracted wave to maintain the continuity of the field components at the interface between the two media. However, such a scattering phenomenon occurs also when the entire media suddenly switches its properties to other values at a certain instant of time, realizing the so-called temporal interface. After a temporal interface, a couple of waves, one reflected and one transmitted, starts to propagate in the new media with the same wavelength but at a different frequency. Exploiting the analogies and differences between spatial and temporal interfaces, in this contribution we present the temporal counterparts of conventional electromagnetic devices based on dielectric slabs and a cascade of them, i.e., the multilayered structures. We discuss about the analysis and design strategies for synthetizing the desired scattering response in both transmission and reflection and present the possible families of devices based on multi-switched temporal metamaterials that can be conceived.

1. Introduction

Metamaterials and metasurfaces are a recognized platform for achieving unprecedented light-matter interactions. It has been demonstrated that spatial modulation of the constitutive parameters of a metamaterial, or the surface properties of metasurfaces, is able to tailor the phase-fronts of the interacting electromagnetic waves, i.e., the spatial frequencies of the wave, realizing some revolutionizing devices, such as invisibility cloaking [1,2], filtering or beam-steering [3–6] and focusing structures [3,7,8]. Introducing a modulation over time increasing the degrees of freedom, allowing to extend the control also to the temporal frequencies. It has been demonstrated that time-varying profiles of material properties can induce artificial non-reciprocity[9–14], generate/control harmonics [15], and control Doppler shift [16,17], thanks to the synthetic motion of a medium in one specific direction [11,12,18–20]. Time-varying metasurfaces also demonstrated to be able to induce frequency translation [19,21–23] metasurfaces.

Recently, metamaterials whose electromagnetic properties suddenly changes over time have been also investigated for inducing instantaneous frequency conversion of the propagating waves. The sudden change of the medium properties realizes the temporal counterpart of the well-known spatial interface...
between two different media, generating two scattered waves, one backward- and one forward-propagating [24–26].

Here we present our recent investigation on propagation and scattering from a multi-switched temporal metamaterial, whose properties change several times during the wave propagation. According to the number of switching, the temporal duration and material properties, we can realize several different temporal devices, such as temporal impedance matching slabs, Fabry-Perot cavities, transparent multilayered structures, and amplifying devices, just to name a few.

2. Temporal metamaterial slab devices
A temporal metamaterial slab can be implemented by switching two times the properties of a uniform homogeneous medium in the whole space, as shown in Figure 1.

![Figure 1. A temporal metastructure composed by a single temporal metamaterial slab applied for a period $\Delta t$.](image)

![Figure 2. Transmission and reflection coefficient as a function of the temporal thickness of slab when it is designed to act as (a) an impedance matching slab and (b) a Fabry-Perot resonator.](image)

It realizes the temporal version of a dielectric slab between two semi-infinite media, i.e., a temporal slab [27], which allows designing both a perfect matching temporal slab, as proposed in [28], and a temporal Fabry-Perot cavity [27]. In Fig. 1 we report a representation of a single temporal metamaterial slab of refractive index $n_{\text{slab}}$ that is present in the whole space where the wave is propagating for the period $\Delta t$. Before and after such a temporal slab, the whole space is filled by a medium whose refractive indices are $n_a$ and $n_b$, respectively. The two consecutive temporal interfaces, $n_a$-$n_{\text{slab}}$ and $n_{\text{slab}}$-$n_b$, scatter four wave contributions whose amplitudes are defined by the Morgenthaler coefficients [16], [17]. They are combined into the overall reflection/transmission coefficients considering the phase delay introduced by the slab [18]:

\[
\begin{align*}
\Gamma_i & = e^{i 2 \xi \omega \Delta t} \left( \tau_i \rho_i' + \tau'_i \rho_i e^{-i 2 \xi \omega \Delta t} \right) \\
T_i & = e^{i 2 \xi \omega \Delta t} \left( \tau_i' \rho_i + \tau_i \rho_i' e^{-i 2 \xi \omega \Delta t} \right)
\end{align*}
\]

(1)

where $\tau, \tau', \rho, \rho'$ are the Morgenthaler coefficients at the two interfaces, $\omega_0$ is the illumination frequency, and $\xi$ that is nothing but the time-dilation factor related to the change in the speed of the wave after the first temporal boundary [17]. Equation (1) give us the possibility to design temporal Fabry-Perot cavities and impedance-matching coating for half-period and quarter-period temporal slabs, respectively, as shown in Fig. 2 where the scattering coefficients are reported.

3. Temporal multi-layered devices
Temporal multi-layered structures consist of a cascade of different media over time that, in analogy with what it can be achieved in conventional spatial multi-layer structures to realize several families of different devices.
In Fig. 3, we report some of our designs of multilayered temporal metamaterials that consists of m temporal slabs applied in cascade over time.

![Figure 3](image)

Figure 3. A temporal metastructure composed by a multilayered structure consisting of a cascade of M different temporal metamaterial slabs, each applied for a specific time.

The higher number of dielectric slabs, each with its refractive index and application time, enables the design of further electromagnetic devices based on temporal metamaterials by engineering the scattering response from the structure. As shown in [19], the overall scattering coefficients can be derived in terms of transfer matrix $\text{TM}$ that considers the scattering contribution of all temporal slabs:

$$
\begin{pmatrix}
E_{\theta}^t \\
E_{\phi}^t
\end{pmatrix}
= \left[ \text{MM}_1 \prod_{m=1}^{M} \text{DM}_{m+1} \text{MM}_{m+1} \right]
\begin{pmatrix}
E_{\theta}^e \\
0
\end{pmatrix}
= \text{TM}_M
\begin{pmatrix}
E_{\theta}^e \\
0
\end{pmatrix}
$$

(2)

where MM and DM identify the matching matrices and delay matrices, respectively. Again, by engineering the phase delay between two consecutive interfaces, we can design different classes of devices [19]. In Fig. 4, we report the scattering coefficient of two multilayered device: the first is a transparent device which exhibits zero reflection and unitary transmission at the design frequency (Fig. 4a); the second is an amplifying device for both the reflected and transmitted waves at the design frequency (Fig. 4b). More details on the design and numerical verification of the performances will be presented at the conference.

Conclusions

In this contribution, we have presented the possibilities offered by temporal metamaterial in realizing several types of devices. Exploiting the analogies and differences between spatial and temporal interfaces, in this contribution we have presented the temporal counterparts of conventional electromagnetic devices based on dielectric slabs and a cascade of them, i.e., the multilayered structures. We have discussed about the analysis and design strategies for synthesizing the desired scattering response in both transmission and reflection and present the possible families of devices based on multi-switched temporal metamaterials that can be conceived.

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