A chiral quark model study of $Z^+(4430)$ in the molecular picture

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We investigated the bound state problem of the S wave charged $D_1D^*$ ($D_1^*D^*$) system in a chiral quark model by solving the resonating group method equation. Our preliminary study does not favor the molecular assumption of $Z^+(4430)$. On the contrary, if $Z^+(4430)$ is really a molecule, its partner with opposite $G$-parity should also exist and probably may be found in the $\pi^+\eta_c(2S)$, $J/\psi\eta_c(2S)$, or $\psi^*\eta_c(2S)$ channel. For the bottom systems, we found the existence of both $I^G=1^+$ and $I^G=2^-$ is possible. Whether the existence of these two structures is supported awaits experimental confirmation from other collaborations.

Since the announcement of $Z^+(4430)$, lots of discussions in various pictures have appeared, which include a tetraquark state \cite{10,20}, a resonance or a molecule in the $D_1D^*$ ($D_1^*D^*$) channel \cite{21,22,23,24,25}, a baryonium state \cite{26}, a threshold cusp \cite{27} and a radial excited $c\bar{s}$ state \cite{28}. In Ref. \cite{29}, the bottom analogs of $Z^+(4430)$ were studied. Besides the spectroscopy, there were discussions about its production \cite{30,31,32,33,34} and decay \cite{35}. The $\pi\psi'$ scattering is studied in Ref. \cite{36}.

The molecular picture is widely used because $Z^+(4430)$ is close to the threshold of $D_1^*D^*$ or $D_1D^*$ system. In Ref. \cite{28}, the calculation at hadron level indicates that it is possible to get a bound state in the $D_1^*D^*$ or $D_1D^*$ system with appropriate parameters and to interpret $Z^+(4430)$ as a molecule. The QCD sum rule study \cite{24} and a quark model calculation \cite{25} also favor the $D_1D^*$ molecule interpretation. However, a recent calculation on the lattice indicates such an interpretation is probably problematic \cite{37}.

To help to understand this charged state further, we present our preliminary study from a chiral quark model (CHQM) \cite{38} and an extended chiral quark model (ECHQM) \cite{39} calculation in this article. The former model includes $\sigma$ and $\pi$ exchange interactions between light quarks. The later model is an extended version of the former one by including the vector meson exchanges. We investigate the bound state problem of the S-wave $D_1^*D^*$ or $D_1D^*$ system by solving the resonating group method (RGM) equation \cite{40}. In previous studies, this approach has successfully reproduced the energies of the light quark baryonium states, the binding energy of the deuterons and the NN scattering phase shifts. When using it to study the system of a light meson and a light baryon \cite{41}, the resulting phase shifts are also in agreement with the experimental data. With this model, we have preliminarily studied the bound state problem of two S-wave heavy mesons in Ref. \cite{42} and \cite{43}. The results are roughly consistent with similar studies at hadron level \cite{44,45}. We here intend to explore whether or not the model can be used to the case of orbitally excited mesons.

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I. INTRODUCTION

In recent years, a series of heavy quark hadrons with unexpected properties were observed one by one, from $D_{sJ}(2317)$ \cite{1,2,3}, $X(3872)$ \cite{4,5,6,7}, $Y(4260)$ \cite{8,9,10}, $X(3940)$ \cite{11}, and $Y(3940)$ \cite{12,13}, to the newly observed $Y(4140)$ \cite{14}. These near threshold mesons stimulated the interpretations beyond the quark model. For the interesting hidden charm XYZ states, the interpretations include tetraquark or molecular states, hybrid charmonia, cusps, and threshold effects. However, it is not excluded that these XYZ states are still mesons dominated by $cc$ components. On the contrary, the observation of charged charmonium-like states is a surprising issue because such states contain at least four quarks.

The Belle Collaboration announced a distinct peak $Z^+(4430)$ in the $\pi^+\psi'$ invariant mass distribution in the decay $B \to K\pi^+\psi'$ in Ref. \cite{15}. The mass and width are $M = 4433 \pm 4(\text{stat}) \pm 2(\text{syst})$ MeV and $\Gamma = 45^{+18}_{-13}(\text{stat})^{+30}_{-19}(\text{syst})$ MeV, respectively. The minimum quark content is $c\bar{c}\bar{u}\bar{d}$. Very recently, a little heavier and broader $Z^+(4430)$ is obtained in Belle’s reanalysis based on the same data sample \cite{17}. However, the experimental data from the BaBar collaboration do not provide significant evidence for the existence of $Z^+(4430)$ \cite{17}.

In addition to $Z^+(4430)$, Belle Collaboration recently observed two more charged resonance-like structures in the $\pi^+\chi_{c1}$ invariant mass distribution in $B \to K\pi^+\psi'$ decays \cite{18}. The mass and width for the first structure are

$$M_1 = 4051 \pm 14(\text{stat})^{+20}_{-41}(\text{syst}) \text{ MeV} \quad (1)$$
$$\Gamma_1 = 82^{+21}_{-17}(\text{stat})^{+47}_{-22}(\text{syst}) \text{ MeV} \quad (2)$$

while the values for the second one are

$$M_2 = 4248^{+44}_{-29}(\text{stat})^{+180}_{-35}(\text{syst}) \text{ MeV} \quad (3)$$
$$\Gamma_2 = 177^{+54}_{-35}(\text{stat})^{+318}_{-61}(\text{syst}) \text{ MeV.} \quad (4)$$

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For the system studied, the orbitally excited heavy mesons are $D_1$ and $D'_1$. These two $J^P=1^+$ mesons are mixed from $^3P_1$ and $^1P_1$ states

\[
|D_1\rangle = \cos \theta |^{3}P_1\rangle + \sin \theta |^{1}P_1\rangle,
|D'_1\rangle = - \sin \theta |^{3}P_1\rangle + \cos \theta |^{1}P_1\rangle. \tag{5}
\]

The mixing angle \(\theta\) = -54.7° or 35.3° may be deduced with the mass of the heavy quark going into infinity. In this article, we adopt the widely adopted value \(\theta = -54.7^\circ\).\[24, 46\].

In the molecular picture, the flavor wave function of $Z^+(4430)$ reads

\[
Z^+(4430) = \frac{1}{\sqrt{2}} \left( D_1^0 D^{*+} + D^{*0} D'_1 \right) \tag{6}
\]
or

\[
Z^+(4430) = \frac{1}{\sqrt{2}} \left( D_1^0 D^{*+} + D^{*0} D'_1 \right). \tag{7}
\]

The quantum numbers are $I^GJ^P=1^+(0,1,2)^-$. As a preliminary study, we consider only interactions involving color-singlet mesons. The one-meson exchange potentials between the heavy mesons are induced by the meson exchange between light quarks.

This paper is organized as follows. After the introduction, we present our model in Sec. II. Then in Sec. III we present our results and discussions.

II. THE CHIRAL QUARK MODEL

The Hamiltonian for the heavy quark meson-antimeson system in the chiral quark model has the form [38, 39]

\[
H = \sum_{i=1}^{4} T_i - T_G + V^{OGE} + V^{conf} + \sum_M V^M \tag{8}
\]

where $T_i$ is the kinetic term of the $i$th quark or antiquark and $T_G$ is the kinetic energy operator of the center of mass motion. $M$ is the exchanged meson between light quarks. In the chiral quark model, one of the sources for the constitute quark mass is the coupling with chiral fields which come from the spontaneous vacuum breaking. Because the breaking has small effects on the generation of the constituent mass of the heavy quarks, the coupling of the $\sigma$ meson and the heavy quarks should be weak. As a result, the possible flavor-singlet meson exchange interactions between heavy quarks and between a heavy quark and a light quark have small contributions and so we ignore them.

The potential induced by the one-gluon-exchange (OGE) interaction reads

\[
V_{q\bar{q}}^{OGE} = g_{q\bar{q}} q_{\bar{q}} F^c_{\bar{q}} \cdot F^c_{\bar{q}} \left\{ \frac{1}{r} - \frac{\pi}{2} \hat{s}^3(r) \right\} \left( \frac{1}{m_{q}} + \frac{1}{m_{\bar{q}}} \right)
+ \frac{4}{3} \delta \left( \sigma_{\bar{q}} \cdot \sigma_{q} \right) \right\} + V^{1s}_{OGE} \tag{9}
\]

\[
V_{q\bar{q}}^{1s} = \frac{1}{4} g_{q\bar{q}} q_{\bar{q}} F^c_{\bar{q}} \cdot F^c_{\bar{q}} \left( \frac{3}{m_{q} m_{\bar{q}}} \right) \left( \frac{1}{r^3} \cdot (\sigma_{\bar{q}} + \sigma_{q}) \right) \tag{10}
\]

where $F^c_{\bar{q}} = \frac{\lambda}{2}$ for quarks and $F^c_{\bar{q}} = -\frac{\lambda^*}{2}$ for antiquarks and $m_q (m_{\bar{q}})$ is the light (heavy) quark mass. The linear confinement potential inside the color-singlet meson is

\[
V_{q\bar{q}}^{conf} = -4 F^c_{\bar{q}} \cdot F^c_{\bar{q}} \left( g_{q\bar{q}} q_{\bar{q}}^r + g_{\bar{q}q} q_{\bar{q}}^{0} \right).
\]

There are similar expressions for $V^{OGE}_{q\bar{q}}$ and $V^{conf}_{q\bar{q}}$. Because we preliminarily ignore the possible hidden color contributions, we do not need $V^{OGE}_{q\bar{q}}$ and $V^{conf}_{q\bar{q}}$.

For the meson exchange potentials between two light quarks, we have [38, 39]

\[
V_{w_2}^{\sigma}(r_{ij}) = -C(g_{ch}, m, \Lambda) X_1(m, \Lambda, r_{ij}), \tag{11}
\]

\[
V_{w_2}^{\pi}(r_{ij}) = C(g_{ch}, m, \Lambda) \frac{m_{q}}{2 m_{q} m_{q}} X_2(m_{p}, \Lambda, r_{ij}) \times [\sigma(i) \cdot \sigma(j)] \left[ \tau_{a}(i) \tau_{a}(j) \right], \tag{12}
\]

\[
(a = 1, 2, 3)
\]

\[
V_{w_2}^{\rho}(r_{ij}) = C(g_{chv}, m_{p}, \Lambda) \left\{ X_1(m_{p}, \Lambda, r_{ij}) + \frac{m_{p}^2}{6 m_{q} m_{q}} \right\}
\times \left( 1 + \frac{f_{che} m_{q}}{g_{che} M_{N}} + \frac{f_{che} m_{q}}{g_{che} M_{N}^2} \right)
\times X_2(m_{p}, \Lambda, r_{ij}) [\sigma(i) \cdot \sigma(j)] \left[ \tau_{a}(i) \tau_{a}(j) \right], \tag{13}
\]

\[
V_{w_2}^{\omega}(r_{ij}) = C(g_{chv}, m, \Lambda) \left\{ X_1(m, \Lambda, r_{ij}) + \frac{m_{q}^2}{6 m_{u}} \right\}
\times \left( 1 + \frac{f_{che} 2 m_{u}}{g_{che} M_{N}} + \frac{f_{che} 2 m_{u}}{g_{che} M_{N}^2} \right)
\times X_2(m, \Lambda, r_{ij}) [\sigma(i) \cdot \sigma(j)] \right\}. \tag{14}
\]

where

\[
C(g_{ch}, m, \Lambda) = \frac{g_{ch}^2}{4 \pi} \Lambda^2 - m^2, \tag{15}
\]

\[
X_1(m, \Lambda, r) = Y(m r) - \frac{\Lambda}{m} Y(\Lambda r), \tag{16}
\]

\[
X_2(m, \Lambda, r) = Y(m r) - \left( \frac{\Lambda}{m} \right)^3 Y(\Lambda r), \tag{17}
\]

\[
Y(x) = \frac{e^{-x}}{x}. \tag{18}
\]

We do not present the tensor term and the spin-orbital term in the meson exchange potentials since we consider
TABLE I: Three sets of model parameters. Other meson masses are: $m_\pi = 138$ MeV, $m_\rho = 775.8$ MeV, and $m_\omega = 782.6$ MeV.

|        | $\chi_{QM}$ | $E\chi_{QM}$ |
|--------|--------------|--------------|
| Set 1  | 0.5          | 0.45         |
| Set 2  | 0.45         | 0.45         |
| Set 3  | 0.45         | 0.45         |

-only S-wave meson-meson interactions. Here we use the same cutoff $\Lambda$ in describing various meson interactions. Its value is around the scale of chiral symmetry breaking ($\sim 1$ GeV).

The interaction between a quark and an antiquark is related to that between two quarks through the relation $V_{q\bar{q}} = G_M V_{qq}$, where $G_M$ is the G-parity of the exchanged meson.

By calculating the RGM matrix elements and solving the RGM equation for the bound state problem, one gets the energy of the system and the relative motion wave function, from which one deduces the binding energy $E_0 = M_{Q\bar{q}} + M_{q\bar{q}} - M_{sys}$. If $E_0$ is positive, the system is bound.

For the model parameters, we take the values determined in the previous investigations $^{38, 39}$. The harmonic-oscillator width parameter $b_u=0.5$ fm for $\chi_{QM}$ and $b_u=0.45$ for $E\chi_{QM}$. The up (down) quark mass $m_u(d)=313$ MeV. The coupling constant $g_{ch}=2.621$ is derived from the measured $NN\pi$ coupling constant $g_{NN\pi}/\sqrt{4\pi} = 13.67$. The masses of $\pi$, $\rho$, and $\omega$ are taken to be the experimental values, whereas $\sigma$ meson mass is adjusted to fit the binding energy of the deuteron. In $E\chi_{QM}$, we use two sets of values. We present the above parameters in Table I. The parameters in the OGE and the confinement potentials can be derived from the masses of the ground state baryons and the heavy mesons. In fact, their values do not give effects to the binding energy of the meson-antimeson system when we ignore the hidden color contributions $^{42}$. So we do not present their values here. For the charm quark masses, we take $m_c=1430$ MeV $^{17}$ and 1870 MeV $^{48}$ to see the heavy quark mass dependence of the binding energy. For bottom quark, we use $m_b=4720$ MeV $^{49}$ and 5259 MeV $^{48}$. In our calculation, we take two values for the cutoff $\Lambda=1000$ MeV and $\Lambda=1500$ MeV.

III. RESULTS AND DISCUSSIONS

We first study the S-wave $D_1\bar{D}^*$ system. We illustrate the diagonal meson exchange matrix elements of the Hamiltonian in the generator coordinate method (GCM) calculation for different angular momenta in Fig. I with the parameters Set 3 in Table I $m_c=1870$ MeV and $\Lambda=1500$ MeV. One finds the dominate contributions

![Figures](a)(b)(c)
come from the \( \sigma \) and \( \pi \) meson exchange interactions. The \( J=0 \) system is more attractive than \( J=1, 2 \) systems. However, we do not find a binding solution in this system with the parameters presented in the former section. For the S-wave \( D_1D^* \) system, the meson exchange GCM matrix elements are illustrated in Fig. 2. The \( J=2 \) system is the most attractive one. But the system is also unbound. So our preliminary calculation does not support the interpretation that \( Z^+(4430) \) is an S-wave \( D_1D^* \) or \( D'_1D^* \) bound state.

The \( D_1D^* \) or \( D'_1D^* \) can also form a \( G=- \) system with the flavor wave function

\[
Z' = \frac{1}{\sqrt{2}} (D_1^0D^{++} - D^{*0}D_1^+ \) \quad (19)
\]

or

\[
Z' = \frac{1}{\sqrt{2}} (\bar{D}_1^0D^{++} - \bar{D}^{*0}D_1^+). \quad (20)
\]

We found these systems are also unbound with our parameters. The attractive force in this case is a little stronger than that in the \( G=+ \) case. One observes this feature by comparing the GCM matrix elements for \( J=0 \) case in Fig. 3 with those in diagram (a) of Fig. 1.

The bottom analogs have better chances to form molecular states because the kinetic term in the Hamiltonian has relatively small contributions. We do get binding solutions with the parameters in the former section. Table II gives the binding energy and the root-mean-square (RMS) radius for the \( B_1B^* \) and \( B'_1B^* \) systems. The system is unbound for \( J=2 \), \( B_1B^* \) and \( J=0, 1 \), \( B'_1B^* \). In that table, we present both the results for the \( G=+ \) case and those for the \( G=- \) case. A little deeper bound states appear in the later case.

Our study with the chiral quark model approach does not support the existence of an S wave molecule in the \( D_1D^* \) and \( D'_1D^* \) systems. This result is inconsistent with our similar study at hadron level [23]. However, in the case of \( DD^* \) system, we got consistent conclusions with these two approaches [42, 44]. A possible reason for the present inconsistency is due to the different approximations in getting the potentials. As a first step to derive the potential, one writes out the quark-quark (or meson-meson) scattering matrix in momentum space.

The denominator of the propagator for a meson reads

\[
p^2 - m^2 + i\epsilon = p_0^2 - \mathbf{p}^2 - m^2 + i\epsilon \quad \text{where} \quad p_0 (\mathbf{p}) \quad \text{is the four(three)-momentum} \quad \text{and} \quad m \quad \text{is the meson mass}. \quad \text{The approximation} \quad p^2 \rightarrow -\mathbf{p}^2, \quad \text{i.e.} \quad p_0 \sim 0, \quad \text{is adopted in the chiral quark model approach, whereas the possible large} \quad p_0 \quad \text{is considered for the hadron level calculation [23].} \quad \text{In the later approach, the principal integration is always assumed if} \quad p_0 \quad \text{is larger than the meson mass} \quad m \quad \text{when we get the coordinate-space potentials. In the present case, the large} \quad p_0 \quad \text{is around} \quad 3m_{\pi} \quad \text{while the large} \quad p_0 \quad \text{is about} \quad m_{\pi}+7 \ \text{MeV in the} \quad DD^* \quad \text{case. Probably it is this} \quad p_0 \quad \text{around} \quad 3m_{\pi} \quad \text{leads to inconsistent conclusions for the studies using these two approaches.} \quad \text{We reanalyzed the binding energies of the} \quad D_1D^* \ (D'_1D^* \) \quad \text{system at hadron}.

\[\text{FIG. 2: The meson exchange GCM matrix elements for} \quad J=0 \quad \text{(a)}, \quad J=1 \quad \text{(b)}, \quad \text{and} \quad J=2 \quad \text{(c)} \quad D_1D^* \quad \text{system. The used parameters are the same as those for} \quad D_1D^*.\]
level with the approximation $p_0 \sim 0$. As expected, we did not find a binding solution, which indicates the important role of $p_0$. However, we need the experiments to judge which approximation is correct. The comparison of model predications with experimental measurements may finally answer the puzzle.

Although our model calculation does not support the assumption that $Z^+(4430)$ is a molecule, such an interpretation is still possible. To get a more conclusive result in a future investigation, the following effects may be included. First, the hidden-color configuration and a larger model space may have contributions and can be considered. Secondly, the coupling with D wave interaction is probably not negligible and may be studied. Thirdly, the different approximation in deriving the coordinate space potential may be investigated in detail. In addition, our model neglects the contribution from $\sigma$ exchange interaction between two heavy quarks or between a heavy quark and a light quark. Although the coupling constant $g_{QQ\sigma}$ is expected to be small, the value may have big effects because no mass factor in the $\sigma$ potential can suppress the contribution. This is also an open question one may discuss.

According to the GCM matrix elements, which roughly reflect the force between the two mesons, if $Z^+(4430)$ can be identified as a $D_1 \bar{D}^*$ or $D_1' \bar{D}^*$ molecular state, a $G=-$ state around 4430 MeV should also exist. One expects that such a state may be searched for in the $\pi^+\eta_c(2S)$, $J/\psi\pi^+\pi^0$, or $\psi^*\pi^+\pi^0$ channel.

In short summary, we have studied the bound state problem of the S wave $D_1 \bar{D}^*$ ($D_1' \bar{D}^*$) system in a chiral quark model. Our preliminary calculation does not favor the assumption that $Z^+(4430)$ is an S wave molecule. On the contrary, once $Z^+(4430)$ ($G=+$) may be identified as a $D_1 \bar{D}^*$ ($D_1' \bar{D}^*$) molecule, its partner with $G=-$ should also exist. When we move on to the bottom analogs, the existence of the charged $B_1 \bar{B}^*$ ($B_1' \bar{B}^*$) molecules with $G=+$ and $G=-$ are both possible. Such states can probably be found in the $\pi \Upsilon(2S)$, $\pi \eta_c(2S)$, $\Upsilon(1S)\pi^+\pi^0$, and $\Upsilon(1S)\pi^+\pi^0$ channels in future measurements.

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