2022 Update on $\varepsilon_K$ with lattice QCD inputs

Sunghee Kim, Sunkyu Lee, Weonjong Lee, Jaehoon Leem and Sungwoo Park

$Lattice Gauge Theory Research Center, CTP, and FPRD, Department of Physics and Astronomy, Seoul National University, Seoul 08826, South Korea$

School of Physics, Korea Institute for Advanced Study (KIAS), Seoul 02455, South Korea

Computational Science and Engineering Team, Innovation Center, Samsung Electronics, Hwaseong, Gyeonggi-do 18448, South Korea.

Thomas Jefferson National Accelerator Facility, 12000 Jefferson Avenue, Newport News, VA 23606, USA

E-mail: wlee@snu.ac.kr

We present recent updates for $\varepsilon_K$ determined directly from the standard model (SM) with lattice QCD inputs such as $\hat{B}_K$, $|V_{cb}|$, $|V_{us}|$, $\xi_0$, $\xi_2$, $\xi_{LD}$, $f_K$, and $m_c$. We find that the standard model with exclusive $|V_{cb}|$ and other lattice QCD inputs describes only 65% of the experimental value of $|\varepsilon_K|$ and does not explain its remaining 35%, which leads to a strong tension in $|\varepsilon_K|$ at the $5.1\sigma \sim 3.9\sigma$ level between the SM theory and experiment. We also find that this tension disappears when we use the inclusive value of $|V_{cb}|$ obtained using the heavy quark expansion based on the QCD sum rule approach, although this inclusive tension is small ($\approx 1.4\sigma$) but keeps increasing as time goes on.

The 39th International Symposium on Lattice Field Theory,
8th-13th August, 2022,
Rheinische Friedrich-Wilhelms-Universität Bonn, Bonn, Germany

$^1$The SWME collaboration

$^*$Speaker
1. Introduction

This paper is an update from our previous reports [1–7]. Here, we present recent progress in determination of $|\varepsilon_K|$ with updated inputs from lattice QCD. Updated input parameters include $\bar{\rho}$, $\bar{\eta}$, exclusive $|V_{cb}|$, inclusive $|V_{cb}|$, $M_W$, and $M_t$.

Here, we follow the color convention of our previous papers [1–7] in Tables 1–8. We use the red color for the new input data which is used to evaluate $\varepsilon_K$. We use the blue color for the new input data which is not used for some obvious reason.

2. Input parameters: Wolfenstein parameters

In Table 1 (a), we present the most updated Wolfenstein parameters available in the market. As explained in Ref. [3, 7], we use the results of angle-only-fit (AOF) in Table 1 (a) in order to avoid unwanted correlation between $|\varepsilon_K|, |V_{cb}|$, and $(\bar{\rho}, \bar{\eta})$. We determine $\lambda$ from $|V_{us}|$ which is obtained from the $K_{\ell 2}$ and $K_{\ell 3}$ decays using lattice QCD inputs for form factors and decay constants as explained in Ref. [8]. We determine the $A$ parameter from $|V_{cb}|$.

| WP | CKMfitter | UTfit | AOF |
|----|-----------|-------|-----|
| $\lambda$ | 0.22475(25) | 0.22500(100) | 0.2249(5) |
| $\bar{\rho}$ | 0.1577(96) | 0.148(13) | 0.156(17) |
| $\bar{\eta}$ | 0.3493(95) | 0.348(10) | 0.334(12) |

(a) Wolfenstein parameters

Table 1: (a) Wolfenstein parameters and (b) QCD corrections: $\eta_{ij}$ with $i, j = c, t$.

3. Input parameters: $|V_{cb}|$

In Table 2 (a) and (b), we present recently updated results for exclusive $|V_{cb}|$ and inclusive $|V_{cb}|$ respectively. In Table 2 (a), we summarize results for exclusive $|V_{cb}|$ obtained by various groups: HFLAV, BELLE, BABAR, FNAL/MILC, LHCb, and FLAG. Results from LHCb comes from analysis on $B_s \rightarrow D_s^+ \ell \bar{\nu}$ decays which are not available in the $B$-factories. Since results for $B_s$ decay channels have poor statistics, we drop out them here without loss of fairness. The rest of results for exclusive $|V_{cb}|$ have comparable size of errors and are consistent with one another within 1.0$\sigma$. In addition, we find that the results are consistent between the CLN and BGL analysis, after the clamorous debates [3, 14].

In Table 2 (b), we present recent results for inclusive $|V_{cb}|$. The Gambino group has reported updated results for inclusive $|V_{cb}|$ in 2021. There are a number of attempts to calculate inclusive $|V_{cb}|$ in lattice QCD, but they belong to a category of exploratory study rather than that of precision measurement yet [15].
\[ \varepsilon_K \text{ with lattice QCD inputs} \]
Weonjong Lee

| channel   | value    | method | ref       | source          |
|-----------|----------|--------|-----------|-----------------|
| ex-comb   | 39.25(56)| CLN    | [16] p115e223 | HFLAV-2021      |
| \( B \to D^\ast \ell \bar{\nu} \) | 39.0(2)(6)(6) | CLN    | [17] erratum p4 | BELLE-2021      |
| \( B \to D^\ast \ell \bar{\nu} \) | 38.9(3)(7)(6) | BGL    | [17] erratum p4 | BELLE-2021      |
| \( B \to D^\ast \ell \bar{\nu} \) | 38.40(84) | CLN    | [18] p5t2   | BABAR-2019       |
| \( B \to D^\ast \ell \bar{\nu} \) | 38.36(90) | BGL    | [18] p5t1   | BABAR-2019       |
| \( B \to D^\ast \ell \bar{\nu} \) | 38.40(78) | BGL    | [14] p27e76 | FNAL/MILC-2022  |
| \( B_s \to D_s^\ast \ell \bar{\nu} \) | 41.4(6)(9)(12) | CLN    | [19] p15   | LHCb-2020        |
| \( B_s \to D_s^\ast \ell \bar{\nu} \) | 42.3(8)(9)(12) | BGL    | [19] p15   | LHCb-2020        |
| ex-comb   | 39.48(68) | comb   | [8] p145   | FLAG-2021       |

(a) Exclusive \(|V_{cb}|\) in units of \(10^{-3}\).

| channel   | value    | ref       | source          |
|-----------|----------|-----------|-----------------|
| kinetic scheme | 42.16(51) | [20] p1   | Gambino-2021   |
| kinetic scheme | 42.00(64) | [8, 21] p145 | FLAG-2021     |
| 1S scheme  | 41.98(45) | [16] p110e208 | HFLAV-2021  |

(b) Inclusive \(|V_{cb}|\) in units of \(10^{-3}\).

Table 2: Results for (a) exclusive \(|V_{cb}|\) and (b) inclusive \(|V_{cb}|\). The p115e223 is an abbreviation for Eq. (223) in page 115. The p5t2 is an abbreviation for Table 2 in page 5.

4. Input parameter \(\xi_0\)

The absorptive part of long distance effects on \(\varepsilon_K\) is parametrized into \(\xi_0\).

\[
\xi_0 = \frac{\text{Im} A_0}{\text{Re} A_0}, \quad \xi_2 = \frac{\text{Im} A_2}{\text{Re} A_2}, \quad \text{Re} \left( \frac{\varepsilon'}{\varepsilon} \right) = \frac{\omega}{\sqrt{2} |\varepsilon_K|} (\xi_2 - \xi_0). \tag{1}
\]

There are two independent methods to determine \(\xi_0\) in lattice QCD: the indirect and direct methods. The indirect method is to determine \(\xi_0\) using Eq. (1) with lattice QCD results for \(\xi_2\) combined with experimental results for \(\varepsilon'/\varepsilon, \varepsilon_K,\) and \(\omega\). The direct method is to determine \(\xi_0\) directly using the lattice QCD results for \(\text{Im} A_0,\) combined with experimental results for \(\text{Re} A_0\).

In Table 3 (a), we summarize experimental results for \(\text{Re} A_0\) and \(\text{Re} A_2\). In Table 3 (b), we summarize lattice results for \(\text{Im} A_0\) and \(\text{Im} A_2\) calculated by RBC-UKQCD. In Table 3 (c), we summarize results for \(\xi_0\) which is obtained using results in Table 3 (a) and (b).

Here, we use results of the indirect method for \(\xi_0\) to evaluate \(\varepsilon_K\), since its systematic and statistical errors are much smaller than those of the direct method.

5. Input parameters: \(\hat{B}_K, \xi_{LD},\) and others

In FLAG 2021 [8], they report lattice QCD results for \(\hat{B}_K\) with \(N_f = 2, N_f = 2 + 1,\) and \(N_f = 2 + 1 + 1\). Here, we use the results for \(\hat{B}_K\) with \(N_f = 2 + 1\), which is obtained by taking an
\( \varepsilon_K \) with lattice QCD inputs

Weonjong Lee

| parameter | method | value | Ref. | source |
|-----------|--------|-------|------|--------|
| Re \( A_0 \) | exp | \( 3.3201(18) \times 10^{-7} \) GeV | [22, 23] | NA |
| Re \( A_2 \) | exp | \( 1.4787(31) \times 10^{-8} \) GeV | [22] | NA |
| \( \omega \) | exp | 0.04454(12) | [22] | NA |
| \( |\sigma_K| \) | exp | \( 2.228(11) \times 10^{-3} \) | [24] | PDG-2021 |
| Re \( (\varepsilon'/\varepsilon) \) | exp | \( 1.66(23) \times 10^{-3} \) | [24] | PDG-2021 |

(a) Experimental results for \( \omega \), Re \( A_0 \) and Re \( A_2 \).

| parameter | method | value (GeV) | Ref. | source |
|-----------|--------|-------------|------|--------|
| Im \( A_0 \) | lattice | \( -6.98(62)(144) \times 10^{-11} \) | [25] p4t1 | RBC-UK-2020 |
| Im \( A_2 \) | lattice | \( -8.34(103) \times 10^{-13} \) | [25] p31e90 | RBC-UK-2020 |

(b) Results for Im \( A_0 \), and Im \( A_2 \) in lattice QCD.

| parameter | method | value | ref | source |
|-----------|--------|-------|-----|--------|
| \( \xi_0 \) | indirect | \( -1.738(177) \times 10^{-4} \) | [25] | SWME |
| \( \xi_0 \) | direct | \( -2.102(472) \times 10^{-4} \) | [25] | SWME |

(c) Results for \( \xi_0 \) obtained using the direct and indirect methods in lattice QCD.

**Table 3**: Results for \( \xi_0 \). Here, we use the same notation as in Table 2.

average over the four data points from BMW 11, Laiho 11, RBC-UKQCD 14, and SWME 15 in Table 4 (a).

| Collaboration | Ref. | \( \hat{B}_K \) |
|--------------|------|----------------|
| SWME 15      | [26] | 0.735(5)(36)   |
| RBC/UKQCD 14 | [27] | 0.7499(24)(150) |
| Laiho 11     | [28] | 0.7628(38)(205) |
| BMW 11       | [29] | 0.7727(81)(84)  |
| FLAG 2021    | [8]  | 0.7625(97)     |

(a) \( \hat{B}_K \)

**Table 4**: (a) Results for \( \hat{B}_K \) and (b) other input parameters.

The dispersive long distance (LD) effect is defined as

\[
\xi_{LD} = \frac{m'_{LD}}{\sqrt{2}\Delta M_K}, \quad m'_{LD} = -\text{Im} \left[ \mathcal{P} \sum_C \langle \bar{K}^0 | H_w | C \rangle \langle C | H_w | K^0 \rangle \right] / m_{K^0} - E_C
\]  

(2)

As explained in Refs. [3], there are two independent methods to estimate \( \xi_{LD} \): one is the BGI estimate [31], and the other is the RBC-UKQCD estimate [32, 33]. The BGI method is to estimate
Weonjong Lee

\[ \varepsilon_K \text{ with lattice QCD inputs} \]

| Collaboration | \( N_f \) | \( m_c(m_c) \) | Ref. |
|---------------|----------|----------------|------|
| FLAG 2021    | 2 + 1    | 1.275(5)       | [8]  |
| FLAG 2021    | 2 + 1 + 1 | 1.278(13)      | [8]  |

(a) \( m_c(m_c) \) [GeV]

| Collaboration | \( M_t \) | \( m_t(m_t) \) | Ref. |
|---------------|----------|----------------|------|
| PDG 2019      | 172.9(4) | 163.08(38)(17) | [35] |
| PDG 2021      | 172.76(30) | 162.96(28)(17) | [24] |
| PDG 2022      | 172.69(30) | 162.90(28)(17) | [30] |

(b) \( m_t(m_t) \) [GeV]

| source  | error (%) | memo     |
|---------|-----------|----------|
| \(|V_{cb}|\) | 49.7      | exclusive|
| \(\eta_{ct}\) | 20.7      | \(c - t\) Box |
| \(\bar{\eta}\) | 13.3      | AOF       |
| \(\eta_{cc}\) | 8.7       | \(c - c\) Box |
| \(\xi_{LD}\) | 2.1       | RBC-UKQCD |
| \(\tilde{\rho}\) | 2.1       | AOF       |
| \(\hat{B}_K\) | 1.7       | FLAG      |

(b) Error budget for \(|\varepsilon_K|^\text{SM}\)

Table 5: Results for (a) charm quark mass and (b) top quark mass.

Table 6: (a) \( M_t \) history (b) error budget.

6. Quark masses

In Table 5, we present the charm quark mass \( m_c(m_c) \) and top quark mass \( m_t(m_t) \). From FLAG 2021 [8], we take the results for \( m_c(m_c) \) with \( N_f = 2 + 1 \), since there is some inconsistency among the lattice results of various groups with \( N_f = 2 + 1 + 1 \). For the top quark mass, we use the PDG 2022 results for the pole mass \( M_t \) to obtain \( m_t(m_t) \).
Weonjong Lee

\[ \varepsilon_K \text{ with lattice QCD inputs} \]

| Source   | \( M_W \) (GeV) | Ref. |
|----------|-----------------|------|
| SM-2022  | 80.356(6)       | [30] |
| CDF-2022 | 80.4335(94)     | [36] |
| PDG-2021 | 80.377(12)      | [24] |
| PDG-2019 | 80.379(12)      | [35] |
| PDG-2018 | 80.385(15)      | [37] |

(b) Table of \( M_W \)

In Table 6 (a), we plot top pole mass \( M_t \) as a function of time. Here we find that the average value drifts downward a little bit and the error shrinks fast as time goes on, thanks to accumulation of high statistics in the LHC experiments. The data for 2020 is dropped out intentionally to reflect on the absence of Lattice 2020 due to COVID-19.

7. \( W \) boson mass

In Fig. 7 (a), we plot \( M_W \) (\( W \) boson mass) as a function of time. The corresponding results for \( M_W \) are summarized in Table 7 (b). In Fig. 7 (a), the light-green band represents the standard model (SM) prediction, the red circles represents the PDG results, and the brown cross represents the CDF-2022 result. The upside is that the CDF-2022 result is the most precise and latest experimental result for \( M_W \). The downside, however, is that it has a 6.9\( \sigma \) tension from that of SM-2022 (the standard model prediction). Here, we use the SM-2022 result for \( M_W \) to evaluate \( \varepsilon_K \).

8. Results for \( \varepsilon_K \)

In Fig. 1, we show results for \(|\varepsilon_K|\) evaluated directly from the standard model (SM) with lattice QCD inputs given in the previous sections. In Fig. 1 (a), the blue curve represents the theoretical evaluation of \(|\varepsilon_K|\) obtained using the FLAG-2021 results for \( \hat{B}_K \), AOF for Wolfenstein parameters, the [FNAL/MILC 2022, BGL] results for exclusive \(|V_{cb}|\), results for \( \xi_0 \) with the indirect method, and the RBC-UKQCD estimate for \( \xi_{LD} \). The red curve in Fig. 1 represents the experimental results for \(|\varepsilon_K|\). In Fig. 1 (b), the blue curve represents the same as in Fig. 1 (a) except for using the 1S scheme results for the inclusive \(|V_{cb}|\).

Our results for \(|\varepsilon_K|^{\text{SM}}\) and \( \Delta\varepsilon_K \) are summarized in Table 8. Here, the superscript \( ^{\text{SM}} \) represents the theoretical expectation value of \(|\varepsilon_K|\) obtained directly from the SM. The superscript \( ^{\text{Exp}} \) represents the experimental value of \(|\varepsilon_K| = 2.228(11) \times 10^{-3} \). Results in Table 8 (a) are obtained using the RBC-UKQCD estimate for \( \xi_{LD} \), and those in Table 8 (b) are obtained using the BGI estimate for \( \xi_{LD} \). In Table 8 (a), we find that the theoretical expectation values of \(|\varepsilon_K|^{\text{SM}}\) with lattice QCD

---

(a) History of \( M_W \) (\( W \) boson mass).

(b) Table of \( M_W \)

Table 7: (a) \( M_W \) history (b) table of \( M_W \).

---

In Fig. 1, we show results for \(|\varepsilon_K|\) evaluated directly from the standard model (SM) with lattice QCD inputs given in the previous sections. In Fig. 1 (a), the blue curve represents the theoretical evaluation of \(|\varepsilon_K|\) obtained using the FLAG-2021 results for \( \hat{B}_K \), AOF for Wolfenstein parameters, the [FNAL/MILC 2022, BGL] results for exclusive \(|V_{cb}|\), results for \( \xi_0 \) with the indirect method, and the RBC-UKQCD estimate for \( \xi_{LD} \). The red curve in Fig. 1 represents the experimental results for \(|\varepsilon_K|\). In Fig. 1 (b), the blue curve represents the same as in Fig. 1 (a) except for using the 1S scheme results for the inclusive \(|V_{cb}|\).

Our results for \(|\varepsilon_K|^{\text{SM}}\) and \( \Delta\varepsilon_K \) are summarized in Table 8. Here, the superscript \( ^{\text{SM}} \) represents the theoretical expectation value of \(|\varepsilon_K|\) obtained directly from the SM. The superscript \( ^{\text{Exp}} \) represents the experimental value of \(|\varepsilon_K| = 2.228(11) \times 10^{-3} \). Results in Table 8 (a) are obtained using the RBC-UKQCD estimate for \( \xi_{LD} \), and those in Table 8 (b) are obtained using the BGI estimate for \( \xi_{LD} \). In Table 8 (a), we find that the theoretical expectation values of \(|\varepsilon_K|^{\text{SM}}\) with lattice QCD
With lattice QCD inputs

Weonjong Lee

1.5 2 2.5 3
1σ 2σ 3σ 4σ 5σ 6σ

(a) Exclusive $|V_{cb}|$ (FNAL/MILC 2021, BGL)

(b) Inclusive $|V_{cb}|$ (HFLAV 2021, 1S scheme)

**Figure 1:** $\varepsilon_K$ with (a) exclusive $|V_{cb}|$ (left) and (b) inclusive $|V_{cb}|$ (right) in units of $1.0 \times 10^{-3}$.

inputs (with exclusive $|V_{cb}|$) has $5.12\sigma \sim 3.93\sigma$ tension with the experimental value of $|\varepsilon_K|^\text{Exp}$, while there is no tension with inclusive $|V_{cb}|$ (obtained using heavy quark expansion and QCD sum rules). We also find that the tension with inclusive $|V_{cb}|$ is small but keeps increasing with respect to time.

In Fig. 2 (a), we show the time evolution of $\Delta \varepsilon_K$ starting from 2012 till 2022. In 2012, $\Delta \varepsilon_K$ was $2.5\sigma$, but now it is $5.05\sigma$ with exclusive $|V_{cb}|$ (FNAL/MILC-2022, BGL). In Fig. 2 (b), we

---

| $|V_{cb}|$ | method | reference | $|\varepsilon_K|^\text{SM}$ | $\Delta \varepsilon_K$ |
|----------|--------|-----------|-----------------|----------------|
| exclusive | BGL    | BELLE 2021| 1.518 ± 0.180   | 3.93\sigma   |
| exclusive | CLN    | BELLE 2021| 1.532 ± 0.171   | 4.07\sigma   |
| exclusive | BGL    | BABAR 2019| 1.441 ± 0.166   | 4.72\sigma   |
| exclusive | CLN    | BABAR 2019| 1.446 ± 0.161   | 4.86\sigma   |
| exclusive | BGL    | FNAL/MILC 2021 | 1.446 ± 0.154 | 5.05\sigma |
| exclusive | CLN    | HFLAV 2021 | 1.566 ± 0.142   | 4.63\sigma   |
| inclusive | kinetic| Gambino 2021 | 2.041 ± 0.168  | 1.12\sigma   |
| inclusive | 1S     | HFLAV 2021  | 2.008 ± 0.160   | 1.37\sigma   |

(a) RBC-UKQCD estimate for $\xi_{LD}$

| $|V_{cb}|$ | method | reference | $|\varepsilon_K|^\text{SM}$ | $\Delta \varepsilon_K$ |
|----------|--------|-----------|-----------------|----------------|
| exclusive | BGL    | FNAL/MILC 2021 | 1.494 ± 0.157  | 4.66\sigma   |
| exclusive | CLN    | HFLAV 2021   | 1.614 ± 0.145   | 4.22\sigma   |

(b) BGI estimate for $\xi_{LD}$

**Table 8:** $|\varepsilon_K|$ in units of $1.0 \times 10^{-3}$, and $\Delta \varepsilon_K = |\varepsilon_K|^\text{Exp} - |\varepsilon_K|^\text{SM}$.
show the time evolution of the average $\Delta \varepsilon_K$ and the error $\sigma_{\Delta \varepsilon_K}$ during the period of 2012–2022.

At present, we find that the largest error ($\approx 50\%$) in $|\varepsilon_K|^{\text{SM}}$ comes from $|V_{cb}|$. Hence, it is essential to reduce the errors in $|V_{cb}|$ as much as possible. To achieve this goal, there is an on-going project to extract exclusive $|V_{cb}|$ using the Oktay-Kronfeld (OK) action for the heavy quarks to calculate the form factors for $B \rightarrow D^{(*)}\ell\nu$ decays [38–44].

A large portion of interesting results for $|\varepsilon_K|^{\text{SM}}$ and $\Delta \varepsilon_K$ could not be presented in Table 8 and in Fig. 2 due to lack of space: for example, results for $|\varepsilon_K|^{\text{SM}}$ obtained using exclusive $|V_{cb}|$ (FLAG 2021), results for $|\varepsilon_K|^{\text{SM}}$ obtained using $\xi_0$ determined by the direct method, and so on. We plan to report them collectively in Ref. [45].

Acknowledgments

We thank Jon Bailey, Yong-Chull Jang, Stephen Sharpe, and Rajan Gupta for helpful discussion. We thank Guido Martinelli for providing us the most updated results of the UTfit Collaboration in time. The research of W. Lee is supported by the Mid-Career Research Program (Grant No. NRF-2019R1A2C2085685) of the NRF grant funded by the Korean government (MSIT). W. Lee would like to acknowledge the support from the KISTI supercomputing center through the strategic support program for the supercomputing application research (No. KSC-2018-CHA-0043, KSC-2020-CHA-0001). Computations were carried out in part on the DAVID cluster at Seoul National University.

References

[1] SWME Collaboration, W. Lee, J. Kim, Y.-C. Jang, S. Lee, J. Leem, C. Park, and S. Park, 
2021 update on $\varepsilon_K$ with lattice QCD inputs, PoS LATTICE2021 (2021) 078, [2202.11473].

BABAR experimental results.

[2] Refer to Table 6 (b) for more details.
Weonjong Lee

\[ \varepsilon_K \text{ with lattice QCD inputs} \]

[2] **LANL-SWME** Collaboration, J. Kim, S. Lee, W. Lee, Y.-C. Jang, J. Leem, and S. Park, *PoS LATTICE2019* (2019) 029, [1912.03024].

[3] J. A. Bailey *et al.*, *Phys. Rev.* **D98** (2018) 094505, [1808.09657].

[4] J. A. Bailey, Y.-C. Jang, W. Lee, and S. Park, *Phys. Rev.* **D92** (2015) 034510, [1503.05388].

[5] J. A. Bailey *et al.*, *PoS LATTICE2018* (2018) 284, [1810.09761].

[6] Y.-C. Jang, W. Lee, S. Lee, and J. Leem, *EPJ Web Conf.* **175** (2018) 14015, [1710.06614].

[7] J. A. Bailey, Y.-C. Jang, W. Lee, and S. Park, *PoS LATTICE2015* (2015) 348, [1511.00969].

[8] **Flavour Lattice Averaging Group (FLAG)** Collaboration, Y. Aoki *et al.*, *Eur. Phys. J. C* **82** (2022), no. 10 869, [2111.09849].

[9] J. Charles *et al.*, *Eur. Phys. J. C* **41** (2005) 1–131, [hep-ph/0406184]. updated results and plots available at: http://ckmfitter.in2p3.fr.

[10] M. Bona *et al.*, *JHEP* **10** (2006) 081, [hep-ph/0606167]. Standard Model fit results: Summer 2016 (ICHEP 2016): http://www.utfit.org.

[11] **UTfit** Collaboration, M. Bona *et al.*, [2212.03894].

[12] A. J. Buras and D. Guadagnoli, *Phys. Rev.* **D78** (2008) 033005, [0805.3887].

[13] J. Brod and M. Gorbahn, *Phys. Rev.* **D82** (2010) 094026, [1007.0684].

[14] **Fermilab Lattice, MILC** Collaboration, A. Bazavov *et al.*, *Eur. Phys. J. C* **82** (2022), no. 12 1141, [2105.14019].

[15] A. Barone, A. Jüttner, S. Hashimoto, T. Kaneko, and R. Kellermann, *Inclusive semi-leptonic B_{(s)} mesons decay at the physical b quark mass, PoS LATTICE2022* (2023) 403, [2211.15623].

[16] **HFLAV** Collaboration, Y. S. Amhis *et al.*, *Eur. Phys. J. C* **81** (2021), no. 3 226, [1909.12524].

[17] **Belle** Collaboration, E. Waheed *et al.*, *Phys. Rev. D* **100** (2019), no. 5 052007, [1809.03290]. [Erratum: Phys.Rev.D 103, 079901 (2021)].

[18] **BaBar** Collaboration, J. P. Lees *et al.*, *Phys. Rev. Lett.* **123** (2019), no. 9 091801, [1903.10002].

[19] **LHCb** Collaboration, R. Aaij *et al.*, *Phys. Rev. D* **101** (2020), no. 7 072004, [2001.03225].

[20] M. Bordone, B. Capdevila, and P. Gambino, *Phys. Lett. B* **822** (2021) 136679, [2107.00604].

[21] P. Gambino, K. J. Healey, and S. Turczyk, *Phys. Lett. B* **763** (2016) 60–65, [1606.06174].

[22] T. Blum *et al.*, *Phys. Rev.* **D91** (2015) 074502, [1502.00263].
\( \varepsilon_K \) with lattice QCD inputs

Weonjong Lee

[23] Z. Bai et al. Phys. Rev. Lett. 115 (2015) 212001, [1505.07863].

[24] Particle Data Group Collaboration, P. Zyla et al. PTEP 2020 (2020), no. 8 083C01.

[25] RBC, UKQCD Collaboration, R. Abbott et al. Phys. Rev. D 102 (2020), no. 5 054509, [2004.09440].

[26] B. J. Choi et al. Phys. Rev. D 93 (2016) 014511, [1509.00592].

[27] T. Blum et al. Phys. Rev. D 93 (2016) 074505, [1411.7017].

[28] J. Laiho and R. S. Van de Water PoS LATTICE2011 (2011) 293, [1112.4861].

[29] S. Durr et al. Phys. Lett. B705 (2011) 477–481, [1106.3230].

[30] Particle Data Group Collaboration, R. L. Workman and Others PTEP 2022 (2022) 083C01.

[31] A. J. Buras, D. Guadagnoli, and G. Isidori Phys.Lett. B688 (2010) 309–313, [1002.3612].

[32] N. Christ et al. Phys.Rev. D88 (2013) 014508, [1212.5931].

[33] N. Christ et al. PoS LATTICE2013 (2014) 397, [1402.2577].

[34] J. Brod, M. Gorbahn, and E. Stamou Phys. Rev. Lett. 125 (2020), no. 17 171803, [1911.06822].

[35] M. Tanabashi et al. Phys. Rev. D98 (2018) 030001. http://pdg.lbl.gov/2019/.

[36] CDF Collaboration, T. Aaltonen et al. Science 376 (2022), no. 6589 170–176.

[37] C. Patrignani et al. Chin. Phys. C40 (2016) 100001. https://pdg.lbl.gov/ .

[38] T. Bhattacharya, B. J. Choi, R. Gupta, Y.-C. Jang, S. Jwa, S. Lee, W. Lee, J. Leem, S. Park, and B. Yoon PoS LATTICE2021 (2021) 136, [2204.05848].

[39] LANL-SWME Collaboration, S. Park, T. Bhattacharya, R. Gupta, Y.-C. Jang, B. J. Choi, S. Jwa, S. Lee, W. Lee, and J. Leem PoS LATTICE2019 (2020) 050, [2002.04755].

[40] LANL/SWME Collaboration, T. Bhattacharya, B. J. Choi, R. Gupta, Y.-C. Jang, S. Jwa, S. Lee, W. Lee, J. Leem, and S. Park PoS LATTICE2019 (2020) 056, [2003.09206].

[41] T. Bhattacharya et al. PoS LATTICE2018 (2018) 283, [1812.07675].

[42] J. A. Bailey et al. EPJ Web Conf. 175 (2018) 13012, [1711.01786].

[43] J. Bailey, Y.-C. Jang, W. Lee, and J. Leem EPJ Web Conf. 175 (2018) 14010, [1711.01777].

[44] LANL-SWME Collaboration, J. A. Bailey, Y.-C. Jang, S. Lee, W. Lee, and J. Leem Phys. Rev. D 105 (2022), no. 3 034509, [2001.05590].

[45] SWME Collaboration, J. Bailey, J. Kim, S. Lee, W. Lee, Y.-C. Jang, J. Leem, S. Park, et al. in preparation.