Application of resonance demodulation method based on local characteristic-scale decomposition and spectral kurtosis in gearbox fault

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Abstract. Resonance demodulation technology has been widely used for its advantages on magnification and selection of fault information. However, the selection of filter parameters is affected by the subjectivity, and the analysis results are easily affected by the noisy signals under actual working conditions. To solve this problem, a method combining relative entropy and correlation coefficient is proposed to analyze and select Intrinsic Scale Components obtained by Local Characteristic-scale Decomposition, remove false components, and to some extent, de-noise. Then, after reconstructing the remaining components, we use spectral kurtosis analysis to find out the most easily detected frequency bands of fault features and analyze them by resonance demodulation, aiming to diagnose gearbox failures. Finally, we compare this method with that based on the empirical mode decomposition. The research shows that the method proposed in this paper can effectively diagnose the gearbox fault and is better than the empirical mode decomposition.

1. Introduction
The vibration signal of the gearbox has the characteristic of amplitude modulation and frequency modulation, and to extract modulation information in the vibration signal by the demodulation method can diagnose fault. Resonance demodulation, also known as envelope analysis, is commonly used to demodulate at present, which has the advantages of magnifying and selecting fault information [1]. But the results of the analysis are easily affected by the noises in the signals under the actual working conditions; In addition, in practical applications, the filtering parameters of the demodulation method need to be chosen artificially, which means it will be greatly influenced by the subjectivity, leading to inaccurate analysis results. Therefore, a suitable method must be selected to make up for the defects of the resonance demodulation method. The concept of spectral kurtosis is proposed by Dwyer [2,3], and its essence is to use the time-frequency analysis to reflect the kurtosis size of original signal in each spectrum section. It can effectively show the non-stationarity and detect transient components in the noise signals. With its sensitivity to the fault pulse, it can clearly point out the frequency band of the transient components, providing the basis for the selection of filter parameters.

In recent years, many scholars have used both spectral kurtosis and signal decomposition method to make up for the shortcomings of the resonance demodulation method. Zhang and others [4] applied the wavelet de-noising and demodulation method to the motor fault diagnosis, succeeding in the compound
fault diagnosis of motor. Cai and others [5] combined the empirical mode decomposition, (EMD) spectral kurtosis and resonance demodulation to diagnose the rolling bearing fault signal, thus overcoming the disadvantages of traditional envelope analysis while Zhou and others [6] did such diagnosis with the improved EMD algorithm and the resonance demodulation method. Cheng and others [7] used the Local Mean Decomposition (LMD) and spectral kurtosis analysis to get the best frequency band, and then used the resonance demodulation analysis to obtain the gearbox fault feature information. All these methods have achieved good results, but the effect of wavelet de-noising largely depends on the choice of basic functions and thresholds, which often requires designers to have rich experiences. EMD and LMD also have defects such as endpoint effect, frequency aliasing and large amount of computation.

In view of the above situation, this paper combines the LCD method, spectral kurtosis and demodulation analysis. LCD decomposition is proposed by Cheng, Yang and others [8] of Hunan University, based on the Intrinsic Time scale Decomposition (ITD). This method can divide a complex signal into a series of single component signals—Intrinsic Scale Component (ISC). Compared with other time-frequency decomposition methods, it has shorter operation time and more advantages in restraining endpoint effect and modal aliasing. Therefore, the combination of LCD method and spectral kurtosis to the gearbox fault diagnosis is more favorable to the accuracy of the diagnosis result.

However, as a new method, there are some defects in the LCD method, and the false component is one of them. As for the identification of false components, people ever used the naked eye relying on experience, but with the development of science and technology, it has been eliminated. At present, some research work has been done by scholars, and the correlation coefficient method [9] is more representative among them.

Correlation coefficient is the index to describe the correlation degree between variables. The value is [-1, 1], gamma >0, positive correlation, gamma <0, negative correlation, gamma =0, and the absolute value of gamma is greater, the correlation degree is higher. However, in some cases, the correlation coefficient between the ISC components is relatively close, which is easy to cause misjudgment.

In different cases, the selection of threshold is not the same. Ideally, the real components have absolute relationship with the original signal, while the false component is not. Actually, the problem of decomposition of the end effect will cause abnormal results in different degree, disturbing the choice of threshold for increasing the divergence value corresponding to the real components. So when dealing with different data, we need to adjust the threshold.

In this paper, we first calculate the relative entropy of each component and the original signal, and make the preliminary judgment of the degree of correlation component and the original signal according to the set threshold. Then we calculate the correlation coefficient, and make a further step to determine the correlation degree between component and the original signal, reducing the error caused by threshold set according to the experience and removing false component more effectively.

2. Study on the method of removing false components in LCD

The vibration signal of gearbox often contains a lot of noises. At the same time, there will be some false components in the process of signal decomposition. We must use appropriate criteria to analyze ISC components, so as to select reasonable components. The commonly used range degree is Euclidean distance, relative entropy, normalized energy difference, correlation coefficient and so on. In this paper, the relative entropy and correlation coefficient are used to remove the false components in the signal.

2.1. Relative entropy

The relative entropy is also called mutual entropy, cross entropy, Kullback entropy, and KL divergence and so on. P (x) and Q (x) are two probability distributions of the value of X, and the relative entropy of P to Q is [10]:

$$D(P||Q) = \sum_{i=1}^{n} P(i) \ln \frac{P(i)}{Q(i)}$$ (1)
That is, calculating the mean value of the logarithmic difference between P and Q according to the probability P. Only if the sum of the probability P and Q is 1, and for any i to satisfy P (I) > 0, Q (I) > 0, formula (1) is defined. Any case of 0ln0 appears in formula (1), and its value is treated at 0.

The relative entropy has the following properties: (1) The relative entropy value is non-negative, i.e. \( D(P \| Q) \geq 0 \). (2) Although intuitively the relative entropy is a metric or a distance function, it is not actually a real measure or distance. Because the relative entropy is not symmetry, namely \( D(P \| Q) = D(Q \| P) \), when the relative entropy is used to analyze the similarity among data, it should be symmetric. That is:

\[
D(P, Q) = \frac{1}{2} \times (D(P \| Q) + D(Q \| P)) \tag{2}
\]

From above, we know that when the two random distributions are same, their relative entropy is zero. When the difference of two random distributions increases, their relative entropy will also increase.

2.2. Correlation analysis

Correlation analysis includes autocorrelation analysis and cross-correlation analysis. The autocorrelation function shows dependency relationships of signals between the value of one time and of the other time, and the cross-correlation function is a correlation statistic that expresses the dependence between two sets of data. The correlation function can describe the important information in the random signal. If the characteristic signal is a periodic signal, its correlation function is also periodic. When the characteristic signal is white noise, the autocorrelation function will be attenuated when the delay is large enough. The correlation analysis is used in this paper: the correlation between the components and the original signal is approximately equal to the autocorrelation of the components, and the correlation between the pseudo component and the original signal is very small. The cross correlation function is:

\[
R_{xy}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-T}^{T} x(t) y(t + \tau) dt \tag{3}
\]

According to the above theory of relative entropy and correlation coefficient, the smaller the relative entropy is, the closer the component is to the original signal. The higher the correlation coefficient is, the greater the correlation between the component and the original signal. The key is to set a threshold for both relative entropy and correlation coefficient. When the relative entropy is less than the set threshold and the correlation coefficient is greater than the set threshold, it is the real component and vice versa. The relative entropy of this chapter is normalized, i.e. the real relative entropy of component tends to 0, but the actual ISC component is affected by the end effect. According to the experience of previous studies, the relative entropy threshold is set at 0.1; in light of that when the correlation coefficient of the two signals is less than 0.3, it can be considered that the two signals have no relativity, we can set the threshold of correlation coefficient at 0.3.

3. Estimation resonance demodulation method based on LCD and spectral kurtosis

Using LCD method, a complex signal can be decomposed into a series of ISC components. By analyzing the ISC components, a reasonable component can be selected for reconfiguration, so as to reduce the false components produced in the decomposition process and the influence of noises in the signals.

Then the spectral kurtosis of the signal is calculated to determine the filter parameters required by the resonance demodulation technology, and the definition of spectral kurtosis is as follows:

For the Wold-Cramer decomposition of a non-stationary signal, it is assumed that Y (T) is a system response and X (T) is a signal, then the expression of Y (T) is [9]:

\[
Y(t) = \int_{-\infty}^{+\infty} e^{2\pi i f} H(t, f; \bar{\omega}) dX(f) \tag{4}
\]

In the formula, \( H(t, f; \bar{\omega}) \) is a time-varying transfer function of a system. It is changed over time
and can be regarded as a complex envelope of the signal \( Y(t) \).

\[ S_{2n}(t, f) \] is the 2n order instantaneous moment of the signal \( Y(t) \), and its expression is as follows:

\[
S_{2n}(t, f) \triangleq E\left\{ |H(t, f)\, dX(f)|^{2n} \right\} \, df = |dX(f)|^{2n} \, S_{2n}(t, f)
\]  

(5)

In the formula, \( S_{2n}(t, f) \) is the instantaneous moment of the 2n order; \( E \) is the expected operator.

The expression of the 2n order spectral moment \( S_{2n}(t, f) \) for the definition of \( Y(t) \) is as follows:

\[
S_{2n}(f) \triangleq E\left\{ S_{2n}(t, f) \right\}
\]  

(6)

The expression of the four order spectral cumulant \( C_{4f}(f) \) is as follows:

\[
C_{4f}(f) = S_{4f}(f) - 2S_{2f}^2(f), f \neq 0
\]  

(7)

Therefore, the spectral kurtosis can be defined as:

\[
K_{r}(f) \triangleq \frac{C_{4f}(f)}{S_{2f}^2(f)} = \frac{S_{4f}(f)}{S_{2f}^2(f)} - 2
\]  

(8)

In the actual gearbox fault diagnosis, the signals collected include the vibration signal and the noise signal, that is:

\[
C(t) = x(t) + N(t)
\]  

(9)

Among them: \( C(t) \) is the acquisition signal; \( x(t) \) is the vibration signal of the gearbox; \( N(t) \) is a noise signal.

The spectral kurtosis of the gearbox acquisition signal \( C(t) \) can be expressed as:

\[
K_{r}(f) = K_{x}(f) / \left[ 1 + r(f) \right]^2
\]  

(10)

\( r(f) \) is a noise to signal ratio, and its value is equal to \( S_{N}(f) / S_{G}(f) \), \( S_{N}(f) \) and \( S_{G}(f) \) are respectively the power spectral density of noise and gearbox vibration signals.

From formula (10), the spectral kurtosis value \( K_{r}(f) \) of the acquisition signal is inversely proportional to the noise signal \( r(f) \), and when the value of \( r(f) \) is very small, the value of \( K_{r}(f) \) is very large. Therefore, analyzing of each frequency band of the signal through spectral kurtosis and comparing the size of spectral kurtosis of each frequency band, we can find the best frequency band to get fault feature information.

To sum up, the steps of the resonance demodulation method based on LCD and spectral kurtosis are as follows:

The signal \( C(t) \) is decomposed by LCD. When the correlation coefficient between the ISC component and the original signal is less than 0.1, the decomposition is stopped.

After calculating the relative entropy between the ISC component and the original signal, we calculate the correlation coefficients again, remove the false components and reconstruct the remaining components.

The spectral kurtosis of the reconstructed signal is calculated, and the spectral kurtosis of the reconstructed signal is obtained, and the filter parameters required by the resonance demodulation technology are determined.

The resonance demodulation analysis of the reconstructed signal is carried out, and the fault features of the gearbox are extracted and the fault type of the gear box is determined.
4. Simulation and example calculation

4.1. Simulation analysis

In order to verify the validity of the resonance demodulation method based on the LCD spectrum and kurtosis analysis, we construct the FM amplitude modulation simulation signal according to the characteristics of gearbox vibration frequency and amplitude, with meshing frequency at 385 Hz and the modulation frequency at 7 Hz. First of all, with the original termination condition of LCD method as the stopping basis, the signal is conducted by LCD decomposition. Then according to the relative entropy and the correlation coefficient of ISC component and simulation signal, we remove false component. The simulation signal is shown in formula (11).

$$x(t) = \left(1 + 2\sin(14\pi t)\right) \times \sin(770\pi t + 12\sin(14\pi t))$$  \hspace{1cm} (11)

A white noise sequence with a mean value of 0 and a variance of 1 is added to \(X(T)\), and figure 1 is its time domain waveform.

![Figure 1. The time domain waveform of the simulation signal.](image)

The 11 ISC components and residuals obtained after the LCD decomposition of the simulation signal are shown in figure 2. The relative entropy value of each ISC component and the simulation signal is calculated and the relative entropy is normalized. The results are shown in table 1. From table 1, we can see the relative entropy of each ISC component and the simulation signal, and draw a conclusion that the relationship between ISC3, ISC4 and ISC5 is closely related to the simulation signal, and the relative entropy is at least one order of magnitude different from other components. By calculating the
correlation coefficient between the two, the results are shown in table 1. It can be seen that the correlation coefficient of ISC5 and simulation signal is less than 0.3. It can be found that it is not related to the simulation signal, that is, it is a false component. The ISC3 and ISC4 are reconstructed, as shown in figure 3, and the correlation coefficient of the reconstructed signal and X (T) is calculated to be 0.9308. Therefore, we decompose the signal by LCD method, and remove the false components by relative entropy and correlation analysis, which can realize signal de-noising and retain the function of original signal basically.

Table 1. The Kullback-Leibler divergence and correlation coefficient between each ISC component and adding noise x(t) of the simulation signal.

|        | ISC1 | ISC2 | ISC3 | ISC4 | ISC5 |
|--------|------|------|------|------|------|
| Relative entropy | 1    | 0.4182 | 0.0129 | 0.0222 | 0.1991 |
| Correlation coefficient | 0.1507 | 0.2213 | 0.6144 | 0.6639 | 0.0200 |

Figure 3. Time-domain waveform of the reconstructed signal.

Figure 4. Spectral kurtosis of the simulation signal.

Next, we calculate the spectral kurtosis of reconstructed signal. Figure 4 is the spectral kurtosis diagram of reconstructed signal. The result shows that the highest spectral kurtosis is 1.8, the central frequency is 426.6667 Hz, and the bandwidth is 170.6667 Hz, that is, the signal to noise ratio of 0~170.6667 Hz is the largest zone. The selected filter parameters are used to filter the reconstructed signal, and then the signal after the filter is demodulated by resonance demodulation. The envelope spectrum is shown as figure 5. From figure 5, we can see clearly the frequency 7.362 Hz and its frequency doubling that is similar to the amplitude modulation frequency 7 Hz of the analog signal. We can conclude that the demodulation method based on LCD and spectral kurtosis effectively extract the modulation information of the simulation signal.
4.2. Example analysis

In this paper, QPZZ-II rotary machinery vibration analysis and fault diagnosis experimental platform is used to simulate gearbox failure experiment. The experimental device is shown in figure 6. Two accelerometer sensors are used to collect signals, which are arranged in X direction and Y direction respectively. A total of experiments under normal gear state and three failure states were carried out. The data of gear crack and wear failure were analyzed in this paper. The time-domain and frequency domain waveform diagram of the fault data is shown in figure 7. Experimental parameters: the number of large gear teeth is 75, the number of small gear teeth is 55, the speed of the motor is 360 r/min when the signal is collected, the number of sampling points is 102400, and the sampling frequency is 10240 Hz. In order to make the effectiveness of the method more general, the data of small rotating frequency of 10 Hz is used for crack failure, while the wear fault is diagnosed by the data of small rotating frequency 6 Hz. The meshing frequency of them is 550 Hz and 330 Hz respectively.

Figure 5. Envelope spectrum of the simulation signal.

Figure 6. Experimental facility.
Figure 7. Time-domain waveform and frequency domain waveform of failure data. (a) crack fault and (b) wear fault.

The fault signals of gear with crack LCD decomposition results is shown in figure 8, and relative coefficient and calculation of each component and the original signal relative entropy is shown in table 2. According to the threshold of correlation coefficient, ISC1, ISC4, ISC5 component were selected to reconstruct, but the relative entropy of ISC1 exceeds the set threshold value, which means it is false component that should be removed. To reconstruct the components ISC4 and ISC5, as shown in figure 9, and to calculate the steep spectrum degree of signal reconstruction, we obtain spectral kurtosis, as shown in figure 10. We know that the spectral kurtosis maximum value is 485.8, the center frequency is 4048.1875 Hz, bandwidth is 426.125 Hz, namely (0~426.125 Hz) SNR maximum signal. The reconstructed signal is filtered by the above parameters, and then the signal after the filter is demodulated by resonance demodulation. The envelope spectrum is shown in figure 11. From the results of resonance demodulation analysis, we can see that there is a significant frequency impact component 9.312 Hz and its frequency doubling. This is approximate to crack fault gearbox vibration signal characteristic frequency 10Hz and frequency doubling, and the diagnostic error is 6.88%, within a reasonable range.
Table 2. The Kullback-Leibler divergence and correlation coefficient between each ISC component and adding noise x(t) of the crack fault signal.

|              | ISC1 | ISC2 | ISC3 | ISC4 | ISC5 |
|--------------|------|------|------|------|------|
| Correlation coefficient | 0.3804 | 0.2370 | 0.2093 | 0.3564 | 0.3271 |
| Relative entropy   | 0.1220 | 0.0475 | 0     | 0.0123 | 0.0186 |
| ISC6 ISC7 ISC8 ISC9 ISC10 |      |      |      |      |      |
| Correlation coefficient | 0.2359 | 0.2355 | 0.2201 | 0.1505 | 0.1156 |
| Relative entropy   | 0.1838 | 0.9842 | 0.7607 | 1     | 0.5521 |

Figure 9. Time-domain waveform of the crack fault signal.

Figure 10. Spectral kurtosis of the crack fault signal.

Figure 11. Envelope spectrum of the crack fault signal.

The wear of gear fault signal LCD decomposition results is shown in figure 12. The calculation of
the relative entropy and correlation coefficient of each component and the original signal is shown in table 3. According to the set threshold of relative entropy and correlation coefficient, component of ISC1, ISC2, ISC3 and ISC4 are selected to reconstruct, and the reconstructed signal is shown in figure 13. Spectrum of reconstructed signal was calculated, and spectral kurtosis is shown in figure 14. The spectral kurtosis maximum value is 15.5, the center frequency is 4474.3125 Hz, bandwidth is 1278.375 Hz, namely (0~1278.375 Hz) SNR maximum signal. The reconstructed signal is filtered by the above parameters, and then the signal after the filter is demodulated by resonance demodulation. The envelope spectrum is shown in figure 15. From the results of resonance demodulation analysis, we can see that there is a significant frequency impact component 304 Hz and its frequency doubling. It is similar to the characteristic frequency 330 Hz of the vibration signal of worn fault gearbox and its frequency doubling, and the diagnostic error is 7.88%.

Figure 12. Results by using LCD of the wear fault signal.

Table 3. The Kullback-Leibler divergence and correlation coefficient between each ISC component and adding noise x(t) of the wear fault signal.

|       | ISC1   | ISC2   | ISC3   | ISC4   | ISC5   |
|-------|--------|--------|--------|--------|--------|
| Relative entropy | 0.4223 | 0.3331 | 0.4180 | 0.0292 | 1      |
| Correlation coefficient | 0.0374 | 0.0779 | 0      | 0.3937 | 0.1163 |

Figure 13. Time-domain waveform of the wear fault signal.
5. Conclusion
In this paper, the correlation coefficients and relative entropy are used to discriminate the components after LCD decomposition, and the false components are removed, and then the resonance demodulation analysis is carried out. By signal simulation, we can see the frequency 7.362 Hz and its frequency doubling that is close to the amplitude modulation frequency of the simulation signal 7 Hz.

Through analyzing the experimental data, there is a significant frequency impact component 304 Hz and its frequency doubling. It is similar to the characteristic frequency 330 Hz of the vibration signal of the worn fault gearbox, and the diagnostic error is 7.88%.

Through the above analysis, the correlation coefficient and relative entropy can be considered to have a good effect on the removal of false components after LCD decomposition.

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References
[1] Mcfadden P D and Smith J D 1984 Vibration monitoring of rolling element bearings by the high-frequency resonance technique- a review Tribol. Int. 17(1) 3-10
[2] Dwyer R 1983 Detection of non-Gaussian signals by frequency domain Kurtosis estimation IEEE Int. Conf. on ICASSP Acoustics, Speech, and Signal Processing (Boston, USA) 8 607-10
[3] Antoni J and Randall R B 2006 The spectral kurtosis: A useful tool for characterizing non-stationary signals Mech. Syst. Signal Pr. 20 282-307
[4] Zhang X X and Liu Z X 2010 Application of resonance demodulation and wavelet denosing in
fault diagnosis of induction motors Elect. Mach. Cont. 14 66-70

[5] Cai Y P, Li A H and Shi L S 2011 Roller bearing fault detection using improved envelope spectrum analysis based on emd and spectrum kurtosis J. Vib. Shock 30(2) 167-72

[6] Zhi Z, Zhu Y S and Zhang Y Y 2013 Adaptive fault diagnosis of rolling bearings based on EEMD and demodulated resonance J. Vib. Shock 32 76-80

[7] Cheng J S and Yang Y 2012 Application of spectral kurtosis approach based on local mean decomposition (lmd) in gear fault diagnosis J. Vib. Shock 31(18) 20-3+54

[8] Cheng J S and Zheng J D 2012 A nonstationary signal analysis approach-the local characteristic-scale decomposition method J. Vib. Eng. 25(2) 215-20

[9] Zhang J F and Huang Z C 2006 Discriminant approach to the machinery or sensor fault based on the k-l divergence J. Mech. Strength 28 670-73

[10] Han Z H and Zhu X X 2012 A false component identification method of EMD based on kullback-leibler divergence Proc. CSEE 32 112-17