Universal mechanism of dissipation in Fermi superfluids at ultra low temperatures

Mihail A. Silaev\textsuperscript{1,2}

\textsuperscript{1}Institute for Physics of Microstructures RAS, 603950 Nizhny Novgorod, Russia.
\textsuperscript{2}Department of Theoretical Physics, The Royal Institute of Technology, Stockholm, SE-10691 Sweden

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We show that the vortex dynamics in Fermi superfluids at ultra-low temperatures is governed by the local heating of the vortex cores creating the heat flux carried by non-equilibrium quasiparticles emitted by moving vortices. This mechanism provides a universal zero temperature limit of dissipation in Fermi superfluids. For the typical experimental conditions realized by the turbulent motion of \textsuperscript{3}He-B the temperature of vortex cores is estimated to be of the order 0.2\textit{T}_c. The dispersion of Kelvin waves is derived and the heat flow generated by Kelvin cascade is shown to have the value close to the experimentally observed.

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Vortex dynamics determines the most fundamental properties of superfluids and superconductors. The well-known textbook example is the motion of Abrikosov vortices under the action of the Lorentz force which determines the finite resistance of the type-II superconductors\textsuperscript{4}. Equally important is the vortex dynamics in the superfluids \textsuperscript{4}He and \textsuperscript{3}He where motion of quantized vortices mediates the mutual friction force between the normal and superfluid components originating from the scattering of normal excitations by moving vortex lines. The dissipative component of mutual friction provides the relaxation of superflow which is the key aspect in the theory of superfluid turbulence\textsuperscript{2}. Recently there has been a renewal interest in this field owing to the developing experimental techniques allowing to study in particular the decay of quantum turbulence at very low temperatures\textsuperscript{2}. In this regime the normal component of the fluid becomes extinct and the mutual friction can not provide the energy dissipation and the relaxation of the superflow. The state-of-art experiments\textsuperscript{4–6} demonstrate that the dissipation in the vortex motion remains finite at the lowest temperatures both in the Bose superfluid \textsuperscript{4}He and Fermi one \textsuperscript{3}He-B. The energy dissipated by turbulence is supposed to be released in the form of the thermal flux of non-equilibrium quasiparticles which recently has become accessible to direct measurements\textsuperscript{7,8}. These experimental results pose a challenging question about the fundamental nature of the dissipation in superfluids at ultra low and even zero temperatures.

In this Letter we propose the new mechanism which governs the vortex dynamics in Fermi superfluids at ultra-low temperatures. We show that even at zero temperature in the absence of the normal component the non-stationary dynamics of vortices is intrinsically dissipative as a result of relaxation processes inside the vortex core moving with the finite acceleration (which is a generic situation in superfluid turbulence). We address the case of typical superconductors and superfluid Fermi systems which can be described within the weak coupling Bardeen-Cooper-Sheriffer (BCS) theory. In particular such restrictions means that the critical temperature and energy gap are much less than the Fermi energy which allows the so-called quasiclassical description of the pairing state\textsuperscript{2}. Note that for the case of cold atomic gases this description is valid only in deep BCS limit.

![Diagram](image)

**FIG. 1:** (a) Container with the ideal gas of particles demonstrating the classical analog of vortex core. The temperature of gas is raised in case if $\dot{u} \neq 0$. The energy dissipation by the heat flux through the heat conducting wall is shown schematically. (b) Sketch of the vortex core and quasiparticle confinement due to Andreev reflection and the flow of non-equilibrium delocalized quasiparticles.

The mechanism of the ultra-low temperature energy dissipation in Fermi superfluids has a very transparent classical physics analogy. As will be discussed below the vortex core can be viewed as an insulating box containing a gas of monoatomic molecules which represent localized vortex core quasiparticles shown in Fig.\textsuperscript{1}(a). Let us assume that the oscillating center of mass motion of the box is produced characterized by the velocity $u(t) = u_0 \Theta(t)$, where $\Theta(t)$ is a step function with a period $t_p$ larger than the thermalization time of the gas. Then each step change of the velocity will result in the temperature rise which can be calculated from the energy conservation law to be $\Delta T_{\text{loc}} = \frac{4u_0^2}{c_v}$, where $c_v$ is the specific heat per unit mass. Averaging over the time interval much larger than the period we obtain the linear temperature growth in time $T_{\text{loc}}(t) = \frac{4u_0^2}{c_v t_p} t$. Furthermore, the temperature growth will certainly stop at some point regulated by the finite heat conductivity of the container walls.
balance of the heating by box acceleration and the heat release through the heat conducting walls will set the stationary value of the temperature $T_{\text{loc}}$ of the particles localized inside the box. The heat flow through the container walls determines the rate of the energy dissipation in the system.

At temperatures considerably lower than $T_c$ vortex dynamics in clean type-II superconductors and Fermi superfluids is determined by the kinetics of localized excitations bound in the vortex cores driven out of equilibrium by vortex motion\textsuperscript{2}. In Fermi superfluid containing vortices the profile of the order parameter $\Delta (\mathbf{r})$ near the vortex core produces a potential well where localized states with a discrete spectrum exist. The quasiparticle confinement results from the subsequent Andreev reflections transforming particles to the holes and vice versa [Fig.1b]. The localized states correspond to energies $|\epsilon_{\text{loc}}| < \Delta_0$ where $\Delta_0$ is bulk value of the gap. The spectrum has the so called anomalous branch\textsuperscript{10} whose energy varies from $-\Delta_0$ to $+\Delta_0$ and crosses $\epsilon = 0$. The anomalous energy branch has topological origin\textsuperscript{11} and exists for all kinds of quantized vortices in the weak coupling Fermi superfluids which allows for the quasiclassical description. In model axially symmetric singly-quantized vortices the anomalous branch states are characterized by the particle angular momentum projection on the vortex axis $\mu$. For low energies $|\epsilon_{\text{loc}}| \ll \Delta_0$, the anomalous branch is $\epsilon_{\text{loc}} = -\hbar \omega_0 \mu$. The separation between the levels with neighboring angular momenta $\mu$ and $\mu + 1$ is $\omega_0 \sim \Delta_0/(\hbar k_F \xi)$ where $k_F$ is the Fermi momentum and $\xi$ is the coherence length. In the weak coupling limit the interlevel distance is small $\omega_0 \ll \Delta_0$ therefore the angular momentum $\mu$ can be considered as classical variable commuting with the corresponding angle $\theta_p$, characterizing the direction of quasiparticle motion in plane perpendicular to the vortex axis.

Our results remains valid with some modifications for the non-axisymmetric vortices existing in multicompontent Fermi superfluids like $^3\text{He}$. In this case one should assume the angular dependence of bound states energy $\epsilon_{\text{loc}} = \epsilon_{\text{loc}}(\mu, \theta_p)$\textsuperscript{11-13}. Note also that in general the singly-quantized vortices have two anomalous energy branches corresponding to different spin states of bound fermions\textsuperscript{14}. The kinetic theory that we will use allows to treat each of the anomalous branches in separate. Therefore below we omit the spin indices, which finally reduces the full Bogoliubov-de Gennes $4 \times 4$ matrix theory to the effective scalar theory of the core fermions.

The key idea of the present work is that the ensemble of quasiparticles localized within the moving vortex core is in some sense analogous to the particle gas confined in a moving box. Below we will show that the local heating of the localized quasiparticles due to the vortex acceleration can lead to the temperature growth which is compensated by the heat flow out of the vortex cores in the form of the non-equilibrium delocalized quasiparticles.

The equilibrium distribution function of Fermi quasiparticles has the form $f(0) = \tanh(\epsilon/2T)$. The delocalized quasiparticles with energies $|\epsilon| > \Delta_0$ have the temperature $T = T_{\text{del}}$ fixed by the heat bath. We assume that $T_{\text{del}} \to 0$ which allows to neglect the concentration of delocalized particles so that $f_{\text{del}}^{(0)}(\epsilon > \Delta_0) = 1$ and $f_{\text{del}}^{(0)}(\epsilon < -\Delta_0) = -1$. On the other hand as we will see below the localized particles have the different temperature $T = T_{\text{loc}}$ which can be much larger $T_{\text{loc}} \gg T_{\text{del}}$.

We start with a kinetic equation\textsuperscript{12} for the distribution function of localized quasiparticles $f = f(\mu, \theta_p)$ which depends on time $t$ and a pair of canonical variables $(\mu, \theta_p)$

$$-\omega_0 \left[ (\mathbf{p} \times \mathbf{u}) \cdot \mathbf{z} \right] \frac{\partial f}{\partial \epsilon} + \frac{\partial f}{\partial t} + \frac{\partial f}{\partial \theta_p} \theta_p + \frac{\partial f}{\partial \mu} \dot{\mu} = St(f). \quad (1)$$

Here the energy spectrum of localized particles $\epsilon_{\text{loc}} = -\hbar \omega_0 \mu$ plays the role of the classical Hamiltonian governing the dynamics of the canonical variables so that $\theta_p = -\omega_0$ and $\dot{\mu} = 0$. The source of non-equilibrium is the first term in Eq. (1) which is generated by the vortex motion with the velocity $\mathbf{u} = (u_x, u_y)\mathbf{y}$.

The collision integral (CI) in the r.h.s. of this equation can be determined by different scattering processes which depend on the particular system where the Fermi superfluidity is considered. We will consider the universal mechanism which inherently exists in any Fermi liquid - the mutual scattering of quasiparticles. In $^3\text{He}$ and ultra-cold Fermi gases the collision integral is determined ultimately by this mechanism.

The general form of the particle-particle CI has rather complicated form\textsuperscript{15}. Therefore we use the model forms of the CI which features however the important properties of the exact CI. We are interested in the relaxation of localized quasiparticle with the energy $|\epsilon| < \Delta_0$ and momentum $\mathbf{p}$. It collides with a particle characterized by $\epsilon_1, \mathbf{p}_1$ and they scatter into states $\epsilon_2, \mathbf{p}_2$ and $\epsilon_3, \mathbf{p}_3$. During this process the energy and momentum are conserved $\epsilon + \epsilon_1 = \epsilon_2 + \epsilon_3$ and $\mathbf{p} + \mathbf{p}_1 = \mathbf{p}_2 + \mathbf{p}_3$. Due to the Galilean invariance the CI integral vanishes when all quasiparticles are moving with a net velocity $\mathbf{w}$ having the equilibrium distribution function $f^{(0)} = \tanh(|\epsilon - \mathbf{p} \cdot \mathbf{w}|/2T)$.

In general, there are three types of scattering processes which contribute to the particle-particle CI which we consider in separate.

(i) Collision of localized and delocalized quasiparticles when $|\epsilon|, |\epsilon_1| < \Delta_0$ and at the same time $|\epsilon_2, \epsilon_3| > \Delta_0$. This is the basic scattering process determining the force acting on the vortex from the heat bath\textsuperscript{16}. For $T_{\text{del}} \to 0$ the probability of this collision is negligibly small and for $T_{\text{del}} = 0$ the exact calculation shows that the corresponding part of the CI is zero.

(ii) The scattering involving only localized quasiparticles $|\epsilon|, |\epsilon_1|, |\epsilon_2|, |\epsilon_3| < \Delta_0$. In this case the collision time approximation can be used with account of the Galilean invariance in the system of localized particles\textsuperscript{16}

$$St_{\text{loc}}(f) = \frac{1}{\tau_{\text{loc}}} [f^{(0)}(\epsilon - \mathbf{p} \cdot \mathbf{w}) - f]. \quad (2)$$

Here $\mathbf{w}$ is the ‘drag velocity’ which determines the evolution of distribution function. In the absence of collisions
with delocalized states it coincides with the vortex velocity $\mathbf{w} = \mathbf{u}$. The collision time depends on the temperature of localized quasiparticles and for $T_{\text{loc}} \ll \Delta_0$ was estimated to be $\tau_{\text{loc}}^{-1} \sim \tau_n^{-1}(T_{\text{loc}}/\Delta_0)^4$ where $\tau_n^{-1} \sim \Delta_0^2/E_F$ is the normal state scattering rate at $T = T_c$. Importantly this part of CI involving only localized particles \(^{(2)}\) conserves the energy of localized particles

$$\int_{-\Delta_0}^{\Delta_0} cS_{\text{loc}}(f)\, dc = 0. \quad (3)$$

\text{(iii) Collision of two localized quasiparticles with production of localized and delocalized ones.} In this case we assume that $|\epsilon_1|, |\epsilon_1|, |\epsilon_2| < \Delta_0$ and $|\epsilon_1| > \Delta_0$. During such processes the non-equilibrium delocalized quasiparticles are created which carry the energy flow out of the vortex cores shown schematically in Fig.1(b). This mechanism of energy losses is similar to the heat flow through the heat conducting wall in the gas container discussed above [see Fig.\(^{11}\)].

To calculate the probability of scattering event \(\text{(iii)}\) one needs to take into account the contributions from states with $\epsilon_3 > \Delta_0$ and $\epsilon_3 < -\Delta_0$. We will consider in detail only the first case and the second one can be treated analogously. We can put $f^{(0)}(\epsilon_3) = 1$ which means that the final delocalized state is empty. The scattering probability is determined by the factor $F(\epsilon, \epsilon_1, \epsilon_2) = [1 - f(\epsilon)]\{1 - f(\epsilon_1)\}[1 + f(\epsilon_2)]$ which is the product of the initial states occupational numbers and the probability of the final localized state to be empty. To obtain the total scattering probability we should integrate over the energies $\epsilon_1, \epsilon_2$ taking into account the energy conservation requirement $-\Delta_0 < \epsilon_2 < \epsilon^*$ where $\epsilon^* = \epsilon_1 + \epsilon - \Delta_0$ which yields in particular $\Delta_0 > \epsilon_1 > -\epsilon$. Then we obtain the following contribution to CI

$$S_{\text{loc}}(f) = (\tau_n\Delta_0^2)^{-1} \int_{-\epsilon}^{\epsilon^*} \epsilon_1 \int_{-\Delta_0}^{\Delta_0} \epsilon_2 F(\epsilon, \epsilon_1, \epsilon_2) \quad (4)$$

where the prefactor can be determined from exact form of the CI. The contribution from the states with $\epsilon_3 < -\Delta_0$ yields the same result Eq.(3) up to the numerical prefactor which depends on the particle-hall asymmetry. Note that in contrast to the Eq.\(^{12}\) the part of the CI \(^{14}\) does not conserve the energy of localized quasiparticles and therefore provides the energy flow out of the vortex cores.

Having in hand the expressions for the CI we can find the corrections to the distribution functions generated by the vortex motion. At first we find the anisotropic corrections taking into account only the localized part of the CI \(^{12}\) $S(f) = S_{\text{loc}}(f)$. Our key idea is to reduce the non-equilibrium distribution in Galilean invariant system of localized quasiparticles the accelerated motion of the vortex is needed $\dot{u} \neq 0$. The solution of the kinetic equations can be found in the usual way. Substituting the ansatz $f = f^{(0)}(\epsilon) - (p\mathbf{u})\frac{\partial f^{(0)}}{\partial \epsilon} + f^{(1)}$ to the Eq.(1) we obtain the expression for the non-equilibrium correction:

$$f^{(1)} = \tau_{\text{loc}} \left( \gamma_\parallel p \cdot \dot{u} + \gamma_\perp [p \times \dot{u}] \cdot \mathbf{z}_v \right) \frac{\partial f^{(0)}_{\text{loc}}}{\partial \epsilon} \quad (5)$$

where $\mathbf{z}_v$ is a unit vector determined by the direction of vorticity, $\gamma_\parallel = 1/\{1 + (\omega_0\tau_{\text{loc}})^2\}$ and $\gamma_\perp = \omega_0\tau_{\text{loc}}\gamma_\parallel$.

The force acting on the moving vortex from the ensemble of localized quasiparticles can be calculated substituting the solution \(^{\text{13}}\) to the conventional expression:

$$\mathbf{F}_{\text{loc}} = -M_\parallel \dot{u} - M_\perp [\mathbf{z}_v \times \dot{u}] \quad , \quad (6)$$

where the masses are determined by

$$M_{\parallel, \perp} = \frac{\pi \hbar N}{2} \left( \int_{-\Delta_0}^{\Delta_0} \tau_{\text{loc}} \gamma_{\parallel, \perp} \frac{\partial f^{(0)}_{\text{loc}}}{\partial \epsilon} \, dc \right)_{\text{F.S.}} \quad (7)$$

where $N$ is the particle concentration and the brackets denote the Fermi surface averaging. The first term in the Eq.\(^{\text{13}}\) is determined by the inertial vortex mass $M_\parallel$ which in the limit $\tau_{\text{loc}} = \infty$ coincides with the expression obtained in collisionless regime.\(^{15}\) This inertial term does not lead to the dissipation, that is to the transformation of the vortex kinetic energy to the heat. On the contrary the second term in the Eq.\(^{\text{13}}\) can provide the dissipation. It is easy to see that for the circular vortex motion the period-averaged work of force $\mathbf{F}_{\text{loc}}$ is non-zero due to the second term. In case of non-axially symmetric vortices the expressions for masses $M_{\parallel, \perp}$ become more involved and take into account the interplay between bound fermions and the modes of rotational vortex core motion.\(^{17}\)

Let us consider the generic example of the single vortex line motion in the self-induced superfluid velocity field described in terms of the propagation of Kelvin waves (KW) along the vortex line. KW play a key part in the relaxation of superfluid turbulence at low temperatures. The KW cascade characterized by the universal spectrum\(^{18,20}\) carries the turbulent energy from the main length scale of the vortex line separation to essentially lower length scales.

In the local induction approximation (LIA) valid for the large Kelvin wavelength the motion of vortex line\(^{21}\) is determined by the linear tension energy, Magnus force $F_M = -\kappa N [\mathbf{z}_v \times \mathbf{u}]$ where $\kappa = 2\pi \hbar/m$ is a vorticity and the force \(\mathbf{U}\) exerted on the vortex line by the localized excitations

$$m_\parallel \ddot{U} + m_\perp [\mathbf{z}_v \times \dot{U}] + [\mathbf{z}_v \times \dot{U}] = \nu_\perp U_{zz} \quad (8)$$

where $\mathbf{U}(z, t)$ is the vortex displacement, $\nu_\parallel = \kappa \Lambda/4\pi$ and $\Lambda = \ln(\xi/l_c)$ where $l_c$ is the cutoff parameter, $m_{\parallel, \perp} = M_{\parallel, \perp} / (\pi N)$. The dispersion relation following from the Eq.(5) has the form

$$\omega_K = \pm k^2 \nu_\parallel + (k^2 \nu_\parallel)^2 (m_\perp - m_\parallel) \quad . \quad (9)$$

The above expression can be extrapolated to the smaller wavelengths\(^{22}\) by setting the cutoff parameter $l_c = 1/k$.\(^{23}\)
Note that the corrections to the KW frequency of the same type as given by Eq. \( \ref{eq:1} \) are necessarily taken into account to break the integrability of LIA and launch the KW cascade (see e.g. Refs. \cite{11,12}). Thus the corrections given by Eq. \( \ref{eq:1} \) in principle have significant influence on the theory of quantum turbulence.

The expression \( \ref{eq:1} \) demonstrates that even in the absence of the heat bath the non-trivial dynamics of localized excitation can provide the finite decrement which can be estimated as \( \tau_{K}^{-1} = m_{\perp} \omega_{K}^{2} \sim \tau_{loc} (\omega_{k}/\omega_{0})^{2} \). The decrement depends on the temperature \( T_{loc} \) and can be finite even if the heat bath temperature is zero \( T_{dhel} = 0 \).

To calculate the temperature \( T_{loc} \) we use the method analogous to the one employed in\cite{22} with the only difference that the energy flow out of the vortex core is determined by the collisions described by the CI \( \ref{eq:1} \) and not by the collisions with phonons. Let us substitute the distribution function \( \ref{eq:1} \) into the Eq. \( \ref{eq:1} \) and take the average, by quasiparticle momentum direction over the Fermi surface and time \( t \). From the first term in the Eq. \( \ref{eq:1} \) we obtain

\[
\left\langle \omega_{0} [\mathbf{p} \times \mathbf{u} \cdot \mathbf{z} \cdot \frac{\partial f}{\partial p} \right\rangle_{FS} = \alpha \frac{d^{2} f}{d \epsilon^{2} \epsilon} (0) \left\langle \gamma_{\perp} \right\rangle_{FS},
\]

where \( \alpha = \left( u_{x} u_{y} - u_{y} u_{x} \right)/2 \) and \( \gamma_{\perp} = \left\langle \gamma_{\perp} \right\rangle_{FS} \). We have neglected the term containing the time average \( \langle \dot{u} \rangle \), assuming the periodic vortex motion. Also we multiply and integrate the Eq. \( \ref{eq:1} \) by energy. Making use of the energy conserving property of localized CI \( \ref{eq:1} \) we obtain the equation which determines the temperature \( T_{loc} \)

\[
\int_{-\Delta_{0}}^{\Delta_{0}} \epsilon S_{t-a}(f) d\epsilon = \alpha \gamma_{\perp} \int_{-\Delta_{0}}^{\Delta_{0}} \frac{d^{2} f_{0}}{d \epsilon^{2}} d\epsilon \quad (10)
\]

The obtained equation has the simple physical meaning expressing the balance between the heat produced by the accelerated vortex motion and the energy flow out of the vortex core carried by the emitted delocalized quasiparticles.

The further analytical analysis can be carried out assuming that \( T_{loc} \ll \Delta_{0} \). By the order of magnitude we have \( \omega_{0} \sim \tau_{n}^{-1} \sim \Delta_{0} / (k_{F} \epsilon) \) therefore \( \omega_{0} T_{loc} \ll 1 \) and \( \gamma_{\perp} \ll \omega_{0} T_{loc} \). Taking into account the temperature dependence of the collision time \( \tau_{loc} \) we obtain the simplified heat balance equation in the form

\[
\alpha \epsilon \gamma_{\perp} (c) V^{3} = (\Delta_{0}/T_{loc}) e^{-\Delta_{0}/T_{loc}},
\]

where \( V_{c} = \Delta_{0}/p_{F} \) is the critical superfluid velocity.

To estimate the value of the vortex core temperature \( T_{loc} \) which can be realized in the conditions of quantum turbulence in \( ^{3}\text{He-B} \) we evaluate the factor \( \alpha \) in \( \ref{eq:1} \) assuming that the vortex dynamics is regulated by the KW cascade. Introducing the complex field \( W = U_{x} + i U_{y} \) we obtain \( \alpha = \left\langle i W^{*} W + c_{c} c_{c} \right\rangle_{FS} / 4 \). The time average can be expressed through the ‘kelvon’ occupational number \( n_{k} = L(|W_{k}|^{2}) \) where \( W = \kappa^{-1/2} \sum_{k} W_{k}(t) e^{i k z} \) and \( L \sim l^{-2} \) is the vortex line length per unit volume, \( l \) is the vortex line separation. To obtain the estimation of \( \alpha \) we can use the spectrum of kelvons \( n_{k} = \kappa^{-3} (k_{l})^{-\gamma} \) where \( \gamma = 2/5 \) or \( \gamma = 2/3 \) according to Refs. \cite{11,12} correspondingly. We have omitted the dimensionless prefactors which are not important in our derivation. The above scaling law results in the estimation \( \alpha \sim k_{c}^{3} (k_{l})^{-\gamma} \) where \( k_{c} \) is the cutoff wave number of the KW cascade\cite{22}. The heat balance equation \( \ref{eq:1} \) can be rewritten in the form

\[
(\Delta_{0}/T_{loc}) e^{-\Delta_{0}/T_{loc}} \sim (k_{c} \xi)^{3-\gamma} (\xi/l)^{\gamma} \quad (12)
\]

Further we need to estimate the value of the cutoff wave number \( k_{c} \) of the KW cascade. This can be obtained in the usual way by comparing the large-scale energy flow in a Kolmogorov cascade\cite{22} \( E_{t} \sim k^{4} l^{-4} \) to the value of the energy dissipated by the KW cascade \( P_{kw} = l^{-2} \int k_{c}^{3} (k_{l})^{-\gamma} \xi_{K} d\xi \) where \( \xi_{K} = \omega_{K} / k_{c} \). From the equation \( P_{kw} = E_{t} \) we obtain the cutoff parameter

\[
k_{c} \sim \xi^{-1} (\xi/l)^{\beta} (\Delta_{0}/T_{loc})^{\beta} \quad (13)
\]

where \( \beta = (4 - \gamma)^{-1} \). Substituting the obtained value of \( k_{c} \) to the \( \ref{eq:1} \) and taking into account that for the typical experiments in \( ^{3}\text{He-B} \) \( \xi / l \approx 10^{-4} \) we obtain the temperature \( T_{loc} \approx 0.1 \Delta_{0} \approx 0.2 T_{c} \). The value of the cutoff parameter is \( k_{c} \sim 0.2 \xi^{-1} \) corresponding to the Kelvin wavelength \( \lambda = 2 \pi / k_{c} \sim 30 \xi \) which is much larger than the core size \( \xi \) and therefore can be reached by the KW cascade. Thus we obtain that at temperatures lower than \( 0.2 T_{c} \) the dynamics of the vortices in \( ^{3}\text{He-B} \) should be determined by the local heating of the vortex cores rather than by the interaction with the heat bath.

It is interesting also to compare the power of the energy losses predicted by the theory with the experimentally observed values of the energy flux out of the turbulent region generated by the propagating vortex front in \( ^{3}\text{He-B} \) at the temperatures down to \( 0.2 T_{c} \). We use the typical \( ^{3}\text{He} \) parameters \( \xi = 10^{-6} \text{cm} \), \( (k_{p} \xi) \sim 10^{2} \), \( \Delta_{0} \sim 10^{-25} J \), \( \Delta_{0} \sim 10^{-6} \) and assume \( f_{loc} \approx 0.1 \Delta_{0} \) to obtain that \( P_{kw} \approx 10^{-10} (k_{c} \xi)^{3-\gamma} W/cm^{2} \). In the experiment the typical value of the turbulent region is \( \sim 1 \text{cm} \) the heat flux is \( J_{F} \approx 10^{-10} (k_{c} \xi)^{3-\gamma} W/cm^{2} \) which is close to the experimentally observed value if \( (k_{c} \xi) 

In conclusion we have studied the dynamics of quantized vortices in ultra cold Fermi superfluids. The new mechanism is suggested of how the energy can be transferred to the ensemble of localized quasiparticles resulting in the local temperature rise inside the vortex cores and dissipation due to the escape of hot quasiparticles into delocalized states. The force acting on vortex is derived and shown to provide the finite decrement of Kelvin waves which is determined by the vortex core temperature. It would be interesting to investigate the dissipative vortex dynamics under the action of this force in more detail in large-scale numerical simulations.
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