Research on Rough Entropy of Interval Set Include Order Rough Set

Qinghai Wang and Zhen Liu

1 Department of Computer Science, Qinghai Normal University, Xining 810008, Qinghai, China
2 D Nagasaki Institute of Applied Science, Nagasaki 851-0193, Japan
E-mail: wqhsucc@163.com

Abstract. In view of the characteristics that information incomplete system is difficult to be accurately expressed and described, based on the structure of interval containing ordered rough sets, this paper first gives the concept of granularity and roughness of interval rough sets through the inclusion relation between Boolean algebras. Then the roughness concept of interval set is defined, the structure and property of the measure satisfied are researched. By defining the element measure formula of interval set and the entropy measure of interval set, the rough entropy concept is further introduced in the rough interval set, and the theorem and conclusion satisfied by this measure are researched. For the measurement of roughness and roughness entropy of interval set rough set. For the measurement of roughness and roughness entropy of interval set rough set, the conclusion is that the uncertainty measure monotonically decreases as the granular size of interval set rough set becomes smaller, and finally the verification is carried out by an example.

1. Introduction

Granular computing is a research hotspot in recent years. Its research strategy is based on multi-granularity problem solving and information processing, aiming to provide a comprehensive, systematic and multi-angle theory for complex problems and machine learning solution. Its multi-level and multi-angle granular structure has been successfully applied in machine learning, data mining, pattern recognition and other fields [1, 2]. At present, the research on granular calculation model mainly includes rough set model, fuzzy set model, cloud model, quotient space theoretical model, interval analysis method and the combined model between them [3].

The rough set theory proposed by Pawlak has become an effective tool for processing and analyzing data uncertainty, incompleteness and ambiguity measurement. The rough set model provides numerical features such as approximate precision and roughness to describe its uncertainty measurement [1], which is of great significance for data intelligent analysis and processing. Uncertainty measurement is an important content of data analysis and rough set theory, which can reveal potential features in data. Therefore, various studies on uncertainty measurement of rough set model are proposed [4]. Liang et al. proposed an accuracy and approximate precision based on granularity [5]. The information entropy proposed by Shannon is also introduced into the rough set theory as an effective means of the uncertainty measure [6], and the rough entropy is widely used as the extension of the information entropy in the rough set model uncertainty.

In practical application, due to the incompleteness and complexity of information, it is difficult to accurately describe the extension of the concept on the theoretical domain with a set. Professor Yao yiyu of Regina university, Canada, proposed the concept of interval set [8], and studied the motivation.
of interval set generation, interval set algebra and incomplete information construction. By using the idea of upper and lower approximations of rough sets, the subsets of the domain are described by the lower and upper bounds, so the real extension of the set can be given.

Interval rough set is a new granular calculation model which combines interval set theory with rough set theory. Its essential idea is to extend the upper and lower approximations of rough sets to interval sets, and extend the classical upper and lower approximations to interval sets by using the uncertainty information reflected by interval sets. The interval set rough set inherits and extends the methods and techniques of rough set model, and it is an effective tool to study the dynamic change of set [9], also it is of great significance for the objective expression and accurate description of various entities in the real world [10].

This paper focuses on the construction of interval set theory including the sequence rough set model, the structural analysis of the roughness measure of the interval set rough set and the variation law between its intrinsic properties, and introduces the concept of granularity measurement in the interval set containing rough set. The variation law of rough entropy metrics of interval set rough sets is studied.

2. Interval Set Theory and Basic Operations

In the objective real world, in order to express the concepts of uncertainty, inaccuracy, ambiguity, etc, many researchers propose to expand the set theory, an interval set is a collection of data element sets.

**Definition 1.** Let \( A = [ A_l, A_u ] \) is a set of intervals, where \( A_l, A_u \) are arbitrary sets and \( A_l \subseteq A_u \). The interval set is represented by the upper and lower bounds set and is defined as follows: Let \( U \) be a finite set, \( 2^U \) be the power set of \( U \). Then the subset of \( 2^U \) on the interval set is:

\[
I(2^U) = \{ [ A_l, A_u ] | A_l, A_u \subseteq U, A_l \subseteq A_u \},
\]

which is called a closed interval set. The set of all interval sets on the closed interval is recorded as

\[
\mathcal{I}(2^U) = \{ [ A_l, A_u ] | A_l, A_u \subseteq U, A_l \subseteq A_u \},
\]

Explanation: Interval set \( \mathcal{A} = [ A_l, A_u ] \) is called a common set, where \( A_l = A_u \), especially, \( \phi = [ \emptyset, \emptyset ] \), where \( A_l = A_u = U \).

Since the nature of interval sets is still set, it is obvious that new operations can be defined on interval sets.

**Definition 2.** Let \( U, 1, \cap, \cup, ^c \) are the normal sets intersection, union, difference and complement operation, \( A, B \) are arbitrary interval set, then the operation on the interval set can be defined as follows:

\[
A \cap B = \{ A_l \cap B_l, A_u \cap B_u | A_l, B_l \in A, B_l \in B \},
\]

\[
A \cup B = \{ A_l \cup B_l, A_u \cup B_u | A_l, B_l \in A, B_l \in B \},
\]

\[
A \setminus B = \{ A_l \setminus B_l, A_u \setminus B_u | A_l, B_l \in A, B_l \in B \},
\]

If \( A^c = U - A \) is the complement set of the set \( A \), then \( [ U - A_l, U - A_u ] = [ A_u^c, A_l^c ] \) is equivalent, and it is obvious from the above definition, \( \emptyset = [ \emptyset, \emptyset ] = [ U, U ] \).

**Theorem 1.** Let \( A, B, C \) be arbitrary interval sets, then there are

1. Idempotency, \( A \cap A = A \), \( A \cup A = A \);
2. Exchange law, \( A \cap B = B \cap A \), \( A \cup B = B \cup A \);
3. Combination law, \( ( A \cap B ) \cap C = A \cap ( B \cap C ) \), \( A \cup ( B \cup C ) = ( A \cup B ) \cup C \).
Obviously such \( A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \) and \( A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \)

(5) Absorption law, \( A \cap (A \cup B) = A \) and \( (A \cap B) \cup A = A \)

(6) De Morgan’s law, \( \neg (A \cap B) = \neg A \cup \neg B \) and \( \neg (A \cup B) = \neg A \cap \neg B \)

(7) The law of convergence, \( \neg \neg A = A \)

(8) The unit of the interval set operation is \( \emptyset, U \), such that \( \forall A \in I(2^U) \) and \( A \cup A = A \), \( A \cup [U, U] = [U, U] \).

Proof. Proof by Definition 2.

It can be seen from the above definition that the interval set is an extension of the set, but the operation property is still the operation between the sets. Compared with the classical set operation property, when the degraded interval set is used, the operations of \( \cap, \cup, \neg \) on the interval set degenerates into the operation of the classical set \( \emptyset, U, \neg \).

3. Inclusion Order on Interval Sets

Let \( A, B \in I(2^U) \) be arbitrary interval sets. Although the specific elements in \( A = \{A_1, A_2\} \) and \( B = \{B_1, B_2\} \) are not known, the inclusion relationship \( A \subseteq B \) in the interval set can be expressed by \( A \subseteq B \) and \( A \subseteq B \), so there is an inclusion order on the interval set.

Definition 3. Let the interval set \( A = \{A_1, A_2\} \) and \( B = \{B_1, B_2\} \), then the inclusion order \( \subseteq \) of the interval set can be defined as:

\[
A \subseteq B \iff A \subseteq B \land A \subseteq B \implies (\forall A \in [A_1, A_2] \exists B \in [B_1, B_2], A \subseteq B) \land (\forall B \in [B_1, B_2] \exists A \in [A_1, A_2], A \subseteq B),
\]

It can be seen from the above definition, for the \( A, B \in I(2^U) \), there are also similar conclusions \( A \subseteq B \iff (A \subseteq B) \land (B \subseteq A) \).

Theorem 2. Let \( A, B, C, D \in I(2^U) \) be arbitrary interval sets, then there are

\[
(\forall X \subseteq Y) \iff A \cap B = A \land A \subseteq B \iff A \cup B = B
\]

3. Inclusion Order on Interval Sets

(5) The inclusion relationship \( \subseteq \) of the inclusion sequence of definition 3 is a partial order relationship, \( \text{lub}(A, B) = A \cup B \), \( \text{glb}(A, B) = A \cap B \), where \( A, B \in I(2^U) \), and therefore \( (I(2^U), \subseteq) \) is the lattice, the algebraic system \( (I(2^U), \cup, \cap) \) is induced by the lattice \( (I(2^U), \subseteq) \), satisfying \( A \cap (B \cup C) = (A \cap B) \cup (A \cap C) \) and \( A \cup (B \cap C) = (A \cup B) \cap (A \cup C) \), then \( (I(2^U), \cup, \cap) \) is a complete distribution.

Since the elements in the completely allocated lattice \( (I(2^U), \cup, \cap) \) don’t necessarily have a complement, they don’t constitute a complement and thus they don’t constitute a Boolean algebra.

4. Proposal of Interval Set Rough Set

In the approximate space \( (U, R) \) of the rough set, the equivalence relation \( R \) can uniquely determine the partition \( U/R \) on a \( U \), and this partition can induce a Boolean algebra \( B = U / S | S \subseteq U / R \).

Obviously such \( B \) contains the empty set \( \emptyset \) and the equivalence class of \( R \), and for the intersection, union and complement closure of the set, the rough set model can be induced under \( B \).
Definition 4. Let \((U, R)\) be an approximate space, \(B\) be a Boolean algebra induced by \(R\), for \(\forall A \subseteq U\), the lower and upper approximations of set \(A\) are \(A_\ell = \{X \in B \mid X \subseteq A\} \subseteq B, A^\prime = \{X \in B \mid X \subseteq A\}\) rough set called A. Obviously there are \(RA = A_\ell, \overline{RA} = A^\prime\).

Definition 5. Let \((U(2^n), R)\) be an approximate space, and \(B\) be a Boolean algebra induced by \(R\). For \(\forall A \subseteq I(2^n)\), and thus \(A_{\ell}\), \(\overline{A}\), \(R_{\downarrow}\), \(R_{\downarrow}\), \(R_{\uparrow}\), \(R_{\uparrow}\), \(R_{\downarrow}\), \(R_{\downarrow}\) are called the interval set rough set, otherwise it is a definable concept, and its approximate set is \((A, A)\). The boundary of the interval set \(A\) is \(\text{Bnd}_R = \overline{R}(A) - \overline{R}(A)\).

Theorem 4. Let \(A, B \subseteq I(2^n)\) be arbitrary interval sets, then there are

\[
\begin{align*}
(1) & RA \subseteq \overline{RA} ; \\
(2) & \overline{R}(A \cap B) = \overline{RA} \cap \overline{RB}, \overline{R}(A \cup B) = \overline{RA} \cup \overline{RB} ; \\
(3) & \overline{R} A \cup \overline{B} \subseteq \overline{R} (A \cup B) , \overline{R} (A \cap B) \subseteq \overline{R} A \cap \overline{R} B ; \\
(4) & \overline{R} (A^\prime) = \overline{R} (A^\prime) ; \\
(5) & \text{If } A \subseteq B \text{, then } \overline{R} (A) \subseteq \overline{R} (B) , \overline{R} (A) \subseteq \overline{R} (B) ; \\
(6) & \overline{R} [U, U] = \overline{R} [U, U] = [U, U] , \overline{R} [\emptyset, \emptyset] = [\emptyset, \emptyset] ;
\end{align*}
\]

Proof. (1), (2), (5), (6) are directly provable by definition 5.

(3) Let the interval sets \(A = \{A_1, A_2\} \subseteq I(2^n)\) for \(R\). For \(A \cup B = \{X \subseteq A \cup B\} \subseteq \{X \subseteq A \cup B\} \cup \{X \subseteq A \cup B\} \subseteq \{X \subseteq A \cup B\} \cup \{X \subseteq A \cup B\}\), and \(\overline{R} (A \cup B) = \{X \subseteq A \cup B\} \subseteq \{X \subseteq A \cup B\} \subseteq \{X \subseteq A \cup B\} \subseteq \{X \subseteq A \cup B\}\), similarly, \(\overline{R} (A \cap B) \subseteq \overline{R} A \cap \overline{R} B\) can be proved.

(4) For \(\overline{R} (A^\prime) = \overline{R} (U - A) = \overline{R} (U - A, U - A) = \overline{R} (A^\prime, A^\prime) = \overline{R} (A^\prime, A^\prime) = (\overline{R} (A^\prime, A^\prime))\) can be proved.

5. Granularity Concept of Interval Set Rough Sets

Definition 6. Let \(R_1, R_2\) be the equivalence relations on \(U\). For \(\forall x \in U\), there are \([x]_{R_1} \subseteq [x]_{R_2}\), then \((I(2^n), R_1)\) is thinner than \((I(2^n), R_2)\), or \((I(2^n), R_2)\) is coarser than \((I(2^n), R_1)\).

Theorem 5. \(\Psi_1 = (I(2^n), R_1)\) is thinner than \(\Psi_2 = (I(2^n), R_2)\) \(\iff \forall x \in U, [x]_{R_1} \subseteq U_{x \in I(2^n)} [y]_{R_1}\) .

Proof: necessity. For \(\forall x \in U\), on the one hand, \([x]_{R_1} = U_{y \in I(2^n)} [y]_{R_1} \subseteq U_{y \in I(2^n)} [y]_{R_1} ;\) on the other hand, \(y \in [x]_{R_2}\), there are \([y]_{R_1} \subseteq [y]_{R_2}\), so there are \(U_{y \in I(2^n)} [y]_{R_1} \subseteq [y]_{R_2}\).

Adequacy: for \(\forall x \in U\), \([x]_{R_1} \subseteq U_{y \in I(2^n)} [y]_{R_1} ;\) so \(x \in U_{y \in I(2^n)} [y]_{R_1} ;\) that is \(\exists y \in [x]_{R_1}\) makes \(x \in [y]_{R_1}\), and thus \([x]_{R_1} \subseteq [y]_{R_1}\), that is \(\Psi_1 = (I(2^n), R_1)\) is thinner than \(\Psi_2 = (I(2^n), R_2)\).

Theorem 6. \(\Psi_1 = (I(2^n), R_1)\) is thinner than \(\Psi_2 = (I(2^n), R_2)\) \(\iff B_{R_1} \subseteq B_{R_2}\) (where \(B_{R_1}, B_{R_2}\) are respectively Boolean algebras induced by \(R_1, R_2\)).

Proof: necessity. Let \(A \in B_{R_1}\), that is \(A = U_{x \in I(2^n)} [x]_{R_1} \subseteq X\) \(U\), \(A \in B_{R_2}\), that is \(A = U_{x \in I(2^n)} [x]_{R_2} \subseteq U\) \(X\), so \(A \in B_{R_2}\).
Adequacy: for $\forall x_i \in B_i \subseteq B$, so $[x_i]_B = \bigcup_{y \in Y} [y]_B$, where $Y \subseteq U$, that is $\exists y \in Y$ makes $x \in [y]_B$, and thus $[x_i]_B = [y]_B \cap [x]_B$, that is $\Psi_1 = (I(2^U), R_1)$ is thinner than $\Psi_2 = (I(2^U), R_2)$.

**Theorem 7.** Let $\forall A \subseteq I(2^U)$, the approximate space $\Psi_1 = (I(2^U), R_1)$ is thinner than $\Psi_2 = (I(2^U), R_2)$, then there are $R_1, A \subseteq R_2, A$ and $R_1, A \subseteq R_2, A$

**Proof.** According to Theorem 5 knows that $\Psi_1 = (I(2^U), R_1)$ is better than $\Psi_2 = (I(2^U), R_2)$ and therefore $B_i \subseteq B_i$, $R_1 = \bigcup \{X \in B_i \mid X \subseteq A\}$ $R_2 = \bigcup \{X \in B_i \mid X \subseteq A\}$ and $R_1 = \bigcup \{X \in B_i \mid X \subseteq A\}$, $R_2 = \bigcup \{X \in B_i \mid X \subseteq A\}$, similar to the certificate $R_1 = \bigcup \{X \in B_i \mid X \subseteq A\}$.

Theorem 7 shows that as the knowledge classification of the approximate space is refined, the lower approximation of the interval set $A$ rises monotonously, and the upper approximation decreases monotonically.

### 6. Roughness Measurement of Interval Set Roughness

The interval set rough set is an extension of the traditional rough set, and its roughness can be defined by the roughness of the conventional rough set.

**Definition 7.** Let the lower and upper approximations of the interval set $A$ in the Boolean algebra $B_\infty$ induced by the equivalence relation $R$ be $R_\infty = \bigcap R_\infty$, $R_\infty = \bigcap R_\infty$ and $R_\infty = \bigcap R_\infty$, the roughness of the interval set

$$\rho_{B_\infty}(A) = 1 - \frac{|R_\infty|}{|R_\infty|} = 1 - \frac{|R_\infty|}{|R_\infty|}, \quad (6)$$

**Theorem 8.** Let $\forall A \subseteq I(2^U)$, the approximate space $\Psi_1 = (I(2^U), R_1)$ is thinner than $\Psi_2 = (I(2^U), R_2)$, then there are $\rho_{B_\infty}(A) \leq \rho_{B_\infty}(A)$.

**Proof.** It is known from Theorem 7 that if the approximate space $\Psi_1 = (I(2^U), R_1)$ is thinner than $\Psi_2 = (I(2^U), R_2)$, then $R_1 \subseteq R_2$ and $R_1 \subseteq R_2$. Theorem 8 shows that as the knowledge classification of the approximate space is refined, the lower approximation of the interval set $A$ rises monotonously, and the upper approximation decreases monotonically.

**Example 1.** Let $U = \{x_1, x_2, x_3, x_4\}$, interval set $= \{\{x_1, x_2, x_3\}, \{x_1, x_2, x_3, x_4\}\}$, equivalence relation $R_1 = \{\{x_1, x_2\}, \{x_1, x_3, x_4\}\}$, $R_2 = \{\{x_1, x_2\}, \{x_1, x_3, x_4\}\}$.

It can be known from the above-mentioned equivalence relation: $R_1 \subseteq R_2 \subseteq R_1$, therefore, there is a $B_1 \supseteq B_2 \supseteq B_3$ between the Boolean algebra $B_1, B_2, B_3$.

$$B_1 = \{\emptyset, \{x_1, x_2\}, \{x_1, x_3, x_4\}, \{x_2, x_3, x_4\}, \{x_1, x_2, x_3, x_4\}\},$$

$$B_2 = \{\emptyset, \{x_1, x_2\}, \{x_1, x_3, x_4\}, \{x_2, x_3, x_4\}, \{x_1, x_2, x_3, x_4\}\},$$

$$B_3 = \{\emptyset, \{x_1, x_2\}, \{x_1, x_3, x_4\}, \{x_2, x_3, x_4\}, \{x_1, x_2, x_3, x_4\}\}.$$
The induced is established, then \( A \subseteq B \) are \( \overline{RA} = (\overline{RA} \cap \overline{RA} \cap \overline{RA} \cap \overline{RA}) \), where
\[
\overline{RA} = U \{ X \in B | X \subseteq A \} = \{ x_j \},
\]
\[
\overline{RA} = U \{ X \in B | X \subseteq A \} = \{ x_j \},
\]
\[
\overline{RA} = U \{ X \in B | X \subseteq A \} = \{ x_j \},
\]
\[
\overline{RA} = U \{ X \in B | X \subseteq A \} = \{ x_j \},
\]
\[
\rho_{B_k} (A) = 1 - \frac{\overline{RA} \cap \overline{RA} \cap \overline{RA} \cap \overline{RA}}{\overline{RA} \cap \overline{RA} \cap \overline{RA} \cap \overline{RA}} = 1 - \frac{1}{5} = \frac{4}{5}.
\]

The lower and upper approximations of interval set \( A \) in \( B_{k} \) are \( R_{A} = (R_{A} \cap R_{A} \cap R_{A} \cap R_{A}) \), where
\[
R_{A} = U \{ X \in B | X \subseteq A \} = \{ x_j \},
\]
\[
R_{A} = U \{ X \in B | X \subseteq A \} = \{ x_j \},
\]
\[
R_{A} = U \{ X \in B | X \subseteq A \} = \{ x_j \},
\]
\[
R_{A} = U \{ X \in B | X \subseteq A \} = \{ x_j \},
\]
\[
\rho_{B_k} (A) = 1 - \frac{\overline{RA} \cap \overline{RA} \cap \overline{RA} \cap \overline{RA}}{\overline{RA} \cap \overline{RA} \cap \overline{RA} \cap \overline{RA}} = 1 - \frac{1}{5} = \frac{4}{5}.
\]

The lower and upper approximations of interval set \( A \) in \( B_{k} \) are \( R_{A} = (R_{A} \cap R_{A} \cap R_{A} \cap R_{A}) \), where
\[
R_{A} = U \{ X \in B | X \subseteq A \} = \{ x_j \},
\]
\[
R_{A} = U \{ X \in B | X \subseteq A \} = \{ x_j \},
\]
\[
R_{A} = U \{ X \in B | X \subseteq A \} = \{ x_j \},
\]
\[
R_{A} = U \{ X \in B | X \subseteq A \} = \{ x_j \},
\]
\[
\rho_{B_k} (A) = 1 - \frac{\overline{RA} \cap \overline{RA} \cap \overline{RA} \cap \overline{RA}}{\overline{RA} \cap \overline{RA} \cap \overline{RA} \cap \overline{RA}} = 1 - \frac{1}{5} = \frac{4}{5}.
\]

It is verified by an example that if \( R_{1} \subseteq R_{2} \subseteq R_{3} \), the \( B_{k} \subseteq B_{k} \subseteq B_{k} \) of the Boolean algebra thus induced is established, then \( \rho_{B_{k}} (A) \leq \rho_{B_{k}} (A) \leq \rho_{B_{k}} (A) \) is established.

7. Rough Entropy Measurement of Interval Set Rough Sets
The interval set rough set is an extension of the traditional rough set, and its roughness can be defined by the roughness of the conventional rough set.

In order to compensate for the lack of roughness measurement, this paper introduces a set interval rough set entropy concept and metrics.

**Definition 8.** Let \( R \) be the equivalence relation on \( U \), and \( U / R = \{ X_1, X_2, ..., X_m \} \), then the interval set induced by \( R \) under \( I(2^U) \) is \( A_{\overline{2}} = [A_{\overline{2}}, A_{\overline{2}}] \), where \( A_{\overline{2}} = \emptyset \), \( A_{\overline{2}} = X_i \), \( i \in [2, m] \), \( k = 1, 2, ..., m \), and \( A_{\overline{2}} \) is the interval set of \( R \). For the sake of simplicity, note \( A_{\overline{2}} \) is \( A_{\overline{2}} \).

**Definition 9.** The length of interval set \( A = [A_i, A_j] \) is the number of elements it contains, ie \( |A| = ||A_i, A_j| = 2^{k-1}| \). Let \( R \) be the equivalence relation on \( U \), and \( U / R = \{ X_1, X_2, ..., X_m \} \), then the entropy of interval set \( [\emptyset, U] \) on \( R \), where \( A_{\overline{2}}, i = 1, 2, ..., m \), is the set of intervals induced by \( R \) at \( I(2^U) \).

For the traditional rough set, the entropy of the equivalence relation (attribute) decreases monotonously with the fine representation of the equivalence relation (attribute), and the same conclusion is obtained for the entropy of the equivalence relation (attribute) of the interval set rough set.

**Theorem 10.** Approximate space \( \Psi_i = (I(2^U), R_i) \) is thinner than \( \Psi_i = (I(2^U), R_i) \), then there are \( E(R_i) \neq E(R_i) \).
Proof. It is known that \( R_1 \subseteq R_2 \) is obtained, and \( A \ , \ B \in I(2^U) \) is set by \( R_1 \) and \( R_2 \) respectively. The number of elements included in \( A \) is \( n_1, n_2, \ldots, n_m \), and the number of elements included in \( B \) are respectively \( r_1, r_2, \ldots, r_k \), and \( \sum n = \sum r \), because any element in \( A \) contains in \( B \) , so for any \( r_j = \sum n_j \), where \( s = 1, 2, \ldots, t \), there are \( 2^i g_j \geq \sum(2^i g_n) \), so there are \( \sum s = \sum g(t) \), that is: \( \sum n = \sum g(t) \log_2 |g| \geq \sum n = \sum g \log_2 |A| \), and therefore \( E(R_1) \leq E(R_2) \).

It can be seen from Theorem 10 that if \( R_1 \subseteq R_2 \subseteq R_3 \), there is \( E(R_1) \leq E(R_2) \leq E(R_3) \). The implication is that the thinner the equivalence relation (attribute), the smaller the granularity of the interval set, the higher the precision of knowledge, and the smaller the entropy of the uncertainty measure characterizing its information, and vice versa.

**Definition 10.** Let \( R \) be the equivalence relation on \( U \), and interval set \( A = \{ A_i, A_j \} \), then the rough entropy of interval set \( A = \{ E(A) = \rho \_a(A) = \rho \_a(A) \geq \sum(2^i g) \log_2 |g| \log_2 |A| \} \), where \( A \subseteq U \) is the interval set induced by \( R \) at \( I(2^U) \).

**Theorem 11.** Approximate space \( \Psi_1 = (I(2^U), R) \) is thinner than \( \Psi_2 = (I(2^U), R) \), \( A \subseteq \Psi \) is the interval set, then there are \( E_k(A) \leq E_k(A) \).

It can be proved by Theorem 8 and Theorem 10.

**Example 2.** From Example 1, we know that \( \rho \_a(A) = \frac{3}{5}, \rho \_b_2(A) = \frac{7}{8} \).

\( R =\{ A_i, A_j \} = 1 \); we can find the interval set induced by \( R_1 \ , \ R_2 \ , \ R_3 \) as follows:

\( A_1 = \{ \emptyset, \{ x, x_2 \} \} \), \( A_2 = \{ \emptyset, \{ x, x_2 \} \} \), \( A_3 = \{ \emptyset, \{ x_3 \} \} \)

\( A_4 = \{ \emptyset, \{ x_2 \} \} \)

\( B_1 = \{ \emptyset, \{ x, x_2 \} \} \), \( B_2 = \{ \emptyset, \{ x_3 \} \} \)

\( C_1 = \{ \emptyset, \{ x, x_2 \} \} \), \( C_2 = \{ \emptyset, \{ x_3 \} \} \)

That is \( E(R_1) = \frac{10}{3} \leq E(R_2) = \frac{17}{3} \leq E(R_3) = 8 \)

Then, \( E_k(A) = 2 \leq E_k(A) = \frac{119}{24} \leq E_k(A) = 8 \).

The rough entropy of the interval set is the product of the entropy of the equivalent relationship (property) and the roughness of the rough set, which is known from the theorem 11, the equivalence relation (property) is thinner, the composition is smaller, the knowledge granularity is smaller, so the roughness and entropy value is smaller, i.e. the rough entropy is lower, and vice versa; As an extension of ordinary set, interval set abstracts more objective and complex real world. Under the induction of equivalence relation (attribute), it can measure the uncertainty of interval set by rough entropy.

8. Conclusion

In this paper, the concept of interval set rough set is firstly defined by the concept of interval set rough set, and the structure and properties of interval set rough set are given. Then, the inclusion relation between Boolean algebras induced is proposed by the inclusion relation between equivalence relation, so as to give the concept of granularity fine and roughness of interval set rough set. It is conclude that roughness and rough entropy are monotonically decreasing in the granularity of the coarse set of the interval set, and finally the verification is done by way of example. The concept of interval rough
set defined in this paper lays a foundation for further research on the uncertainty and reduction algorithm of interval rough set.

9. Acknowledgments.
Project of Qinghai Provincial Science and Technology department (2017-ZJ-768); Next Generation Internet Innovation Project (NGII20160504); Project of Chunhui Planning of Ministry of Education (Z2015051); National Social Science Project (17XTQ013); Key Project of Qinghai Normal University Teaching Research (qhnujy2015102).

10. References
[1] Z.Pawlak, Rough Sets, International Journal of Information and Computer Sciences 11 (1982) 341-356.
[2] G.Y.Wang, H.Yu, A Survey on Rough Set Theory and Applications, Chinese Journal of computers 12(7) (2009) 1229-1246.
[3] G.Y.Wang, D.Y.Li, Y.Y.Yao, et al, Cloud model and particle calculation, science Press, Beijing, 7(2012) ,pp. 77-87.
[4] S.F.Wang, G.F.Wu, J.G.Pan, Study on knowledge granularity and relationship between knowledge granularities based on rough sets, Computer Engineering and Applications 43(14) (2007) 38-41.
[5] J.Y.Liang, J.Wang, Y.H.Qian, A new measure of uncertainty based on knowledge granulation for rough sets, Information Sciences 179 (4) (2009) 458-470.
[6] X.B.Ma, H.R.Ju, X.B.Yang, J.J.Song, Multi-cost based decision-theoretic rough sets in incomplete information systems, Journal of Nanjing University (Natural Science) (02) (2015) 335-342.
[7] W.B.Qian, B.R.Yang, Z.Y.Xu, C.S.Zhang, Efficient Incremental Updating Algorithm for Core Attribute Based on Information Entropy, Pattern recognition and artificial Intelligence 26 (1) (2013) 42-49.
[8] Y.Yao, Interval-set and Interval-set algebras, Proceedings of the 8th IEEE International conference on Cognitive Information (2009) 307-314.
[9] Y.Y.Yao, Y.Zhao, Attribute reduction in decision-theoretic rough set models, Information Sciences 178 (2008) 3356-3373.
[10] S.L.Ma, D.Y.Ye, Research on Computing Minimum Entropy Based Attribute Reduction via Stochastic Optimization Algorithms, Pattern recognition and artificial Intelligence 25 (1) (2012) 96-104.