Improved capacitance model considering bottom-hole flowing pressure and interference between oil wells

Xuewu Wang1, Juan Wang2 and Zhizeng Xia1

Abstract
With the continuous production of oil wells, the reservoir properties, such as permeability and porosity, are changing accordingly, and the reservoir heterogeneity is also enhanced. This development is vulnerable to the problem of the one-way advance of injected water and low efficiency of water flooding. The interwell connectivity between injection and production wells controls the flow capacity of the subsurface fluid. Therefore, the analysis of interwell connectivity helps to identify the flow direction of injected water, which is of great significance for guiding the profile control and water plugging in the later stage of the oilfield. In this study, based on the principle of mass conservation, a capacitance model considering the bottom-hole flowing pressure was established and solved by using the production dynamic data of injection–production wells. Then, the validity of the capacitance model was verified by numerical simulation, and the influences of well spacing, compression coefficient, frequent switching wells, injection speed, and bottom-hole flowing pressure on interwell connectivity were eliminated. Finally, a practical mine technique for inversion of connectivity between wells using dynamic data was developed. The advantage of this model is that the production dynamic data used in the modeling process are easy to obtain. It overcomes the shortcomings of previous models and has a wider range of applications. It can provide a theoretical basis for the formulation of profile control and water-plugging schemes in the high-water-cut period.

Keywords
Interwell connectivity, improved capacitance model, bottom hole flowing pressure, interwell interference factor, frequent shut-ins

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Introduction

With continuous exploration and development in oil fields, the reservoir heterogeneity is aggravated, and injection water easily rushes in one direction, resulting in low water flooding efficiency and high water cut in oil wells (Han and Gu, 2014). Connectivity between injection and production wells controls the fluid flow capacity. Interwell connectivity analysis can help to determine the preferred flowing path of injection water and can also provide a theoretical basis for profile control and water plugging, which is of great significance to guide the recovery of residual oil in the middle and late stages of oilfield development (Dong et al., 2014; Shi et al., 2014; Zhu et al., 2012).

There are limitations in common methods of studying the connectivity. The cross-well lateral correlation method and logging data method are relatively simple to implement. However, for thin reservoirs or reservoirs with oil–water layer distribution, it is difficult to describe reservoir changes and identify the connectivity accurately (Satish et al., 1997; Xing and Xue, 2007). The production data analysis method mainly identifies the connectivity by observing the fluctuation between the injection and production curves (Qanbari and Clarkson, 2013; Wu et al., 2016). Although this method is simple and intuitive, it has a long interpretation period, high cost, and cannot evaluate the connectivity quantitatively. There are also human factors in the production data analysis method. When using the tracer test, pressure test, chromatographic fingerprinting, and well test analysis methods (Dmitry and Babadagli, 2010; Li, 2001; Serhat, 2003; Shui et al., 2011), the well must be shut off, which affects the normal production of the oilfield.

To overcome the shortcomings of the above methods, in the past 30 years, petroleum engineers worldwide have used dynamic testing data of multiple wells to study the interwell connectivity quantitatively. In the 1990s, most scholars mainly used statistical and correlation coefficient analysis methods to study the interwell connectivity. For example, applying probability theory, Alabert and Modot (1992) studied the effect of reservoir heterogeneity on the interwell connectivity. Malik et al. (1993) conducted a study of production dynamic data and static geological data, and they developed a comprehensive method to characterize reservoir connectivity. Using the Spearman coefficient of rank correlation analysis method, Refunjol (1996) proposed a method to identify the connectivity by a correlation coefficient. This method introduced the time lag of injected signals and analyzed the relationship between the injection well and adjacent production well. Using the Spearman coefficient of rank correlation analysis method, Soeriawinata and Kelkar (1999) proposed constructive and destructive interference of the injection signal based on a superposition principle, and they established the inversion model of multiple injection wells. Such methods regress an interwell connectivity coefficient, which is an index to characterize the correlation quantitatively between oil and water wells. It overcomes the shortcomings of common methods, but the established model is relatively simple and does not consider the time lag and attenuation of the injected signal. This kind of method does not consider complex situations, such as the shut-in of the well, and the inversion parameters have no actual physical significance, so it cannot be used for subsequent production optimization and cannot describe the interwell connection accurately.

Utilizing injection and production data, Albertoni and Lake (2003) established a multivariate linear regression model for balancing injection and production, and they introduced a diffusion filter to process injection signals, which can be used to solve the problem of interwell connectivity. Yousef et al. (2005) built a capacitance model (CM) of interwell
connectivity based on material balance, and they introduced the time constant to characterize the time lag of the injected signal. In the case of great fluctuation of the bottom-hole flowing pressure (BHFP), the connectivity coefficient between wells can be inverted by using pressure data together, and more-accurate connectivity can be obtained. Yousef et al. (2009) improved the CM, and the improved model introduced the connectivity coefficient and time constant between wells, which can identify the interwell connectivity more accurately. Daniel et al. (2009) improved the CM and optimized the research of interwell connectivity of large-scale reservoirs. Kaviani et al. (2012) estimated the interwell connectivity in the case of unmeasured fluctuating bottom-hole pressures. For cases in which pressures are unavailable, as often occurs with legacy assets two enhancements of the CM, the segmented CM and the compensated CM, were described. Ballin et al. (2012) compared the connectivity coefficients calculated by the CM and streamline simulation and proposed the idea of combining the CM and streamline simulation. However, the aforementioned CM did not consider the interference between oil wells and the reasonable influence space range of injection and production wells.

Recently, Kaviani et al. (2014) proposed that there is a statistical side of CM performance involving the number of sampling data, sampling interval, and amount of noise that affects the accuracy of the connectivity results. A sensitivity analysis was done of the accuracy of the expected results before applying the CM to field data. Soroush et al. (2014) evaluated interwell connectivity in cases of changing skin and frequent production interruptions. Based on the superposition principle, they developed a method called “multiwell-compensated CM” that is tolerant to changes caused by external factors, such as shut-ins and workovers, that cause production rate changes that are not caused by injection rate fluctuations. Yang et al. (2015) introduced the interference factor and the influence coefficient matrix of injection and production wells to improve the model. In general, the present research on the connectivity between wells is not comprehensive. The model does not take into account the time lag and attenuation of injection signals in the propagation process, and the interaction between multiple oil wells, the influence of well location, compression coefficient, injection rate, bottom-hole pressure, and other parameters of the interwell connectivity are also not considered (Liu et al., 2019a, 2019b; Luo et al., 2018). Earlier methods, in some cases, may give misleading connectivity results and inaccurate rate predictions.

In this study, based on previous research results and the principle of mass conservation, using production and pressure data of wells, the interference coefficient between oil wells was introduced, and the improved CM was established. One can invert the unknown parameters in the model by fitting the actual dynamic data, and then the connectivity between wells can be identified. The CM was verified by numerical simulation as well. This model eliminates the influence of different well spacings, compression coefficients, frequent switching wells, bottom-hole pressure fluctuations, and injection rates on the connectivity index. The model overcomes the shortcomings of previous models and has a wider scope of application. It can quantify the connectivity and the waterflooding response time quickly. This method is simple and practical, and it is easy to obtain the production data used in the modeling. The calculation results can provide a theoretical basis for the profile control and water plugging in the high-water-cut period. Finally, a practical technique using dynamic data to invert interwell connectivity was developed.
Establishment of CM

The BHFP fluctuates in reservoirs. Introducing the BHFP data, one can establish a CM that combines production data with pressure data. Considering the interference between oil wells, the coefficient $\nu_{kj}$ between oil wells is introduced into the CM. The inversion parameters in the CM include the connectivity coefficient $\lambda_{ij}$ between interwells, the time constant $\tau$, and the interference coefficient $\nu_{kj}$ between oil wells. $\lambda_{ij}$ characterizes the fluid flow capacity between injection and production wells, $\tau$ characterizes the time lag of the injected signal, and $\nu_{kj}$ characterizes the interaction between oil wells. Obviously, under the same differential pressure, the bigger the connectivity coefficient and the smaller the time constant, the earlier the water breakthrough.

CM of one injection and one production well

The water wells, oil wells, and pores of the reservoir are a complete system. The difference between injection and production affects the reservoir pressure. Based on the hypothesis that the comprehensive compressibility coefficient of the reservoir is very small and constant, and that there is no fluid inflow or outflow in pore volume $V_p$, the material balance equation of pore volume under the condition of one injection and one production well is established (Bera and Mandal, 2015)

$$C_t V_p \frac{d\bar{p}}{dt} = I(t) - Q(t) \quad (1)$$

According to the seepage formula of plane radial flow (Feng et al., 2018), the production of the oil well after water breakthrough is as follows

$$Q = \frac{2\pi kh\Delta p}{\ln \frac{r_o}{r_w}} \left( \frac{kr_o}{\mu_o} + \frac{kr_w}{\mu_w} \right) \quad (2)$$

There is a linear relationship between liquid production and bottom-hole pressure in oil wells

$$Q = J \times (\bar{p} - p_{wf}) \quad (3)$$

A system of simultaneous equations of equations (2) and (3) is as follows

$$J = \frac{2\pi kh}{\ln \frac{r_o}{r_w}} \left( \frac{kr_o}{\mu_o} + \frac{kr_w}{\mu_w} \right) \quad (4)$$

Combining equations (1), (3), and (4) and eliminating the mean reservoir pressure, one obtains

$$\tau \frac{dQ}{dt} + Q(t) = I(t) - \tau J \frac{dp_{wf}}{dt} \quad (5)$$

In equation (5), the time constant $\tau$ is equal to $\frac{C_t V_p}{J^2}$, which characterizes the time lag and attenuation of injected signals (Lin, 2012).
The integrating factor method (Lin et al., 2013) is used to solve equation (5). The integral factor is

\[ f(t) = e^{\int_{t_0}^{t} dt} = e^t \] (6)

Taking the initial conditions \( t = t_0, Q(t) = Q(t_0) \) into equation (5), one can deduce the basic CM of one injection and one production well (Yousef, 2005)

\[ Q(t) = Q(t_0)e^{t_0-t} + \frac{e^{t_0}}{\tau} \int_{t_0}^{t} e^{t_0-\xi} I(\xi) d\xi + J \left[ p_{wfi}(t_0)e^{t_0-t} - p_{wfi}(t) + \frac{e^{t_0}}{\tau} \int_{t_0}^{t} e^{t_0-\xi} p_{wfi}(\xi) d\xi \right] \] (7)

Equation (7) shows that the liquid production of oil wells consists of three parts. The first is the initial liquid production, which is contributed by its own productivity. The second is the injection signal, which represents the influence of injection on production. This is the main research focus of this study. The third part is the liquid production caused by changing the BHFP. In the CM, only \( \tau \) is an unknown parameter. The value of each part is only determined by \( \tau \).

### Multiwell discrete model considering interwell interference

The total production of a well in a reservoir is usually affected by multiple injection wells. Therefore, the CM of one injection and one production well should be extended to the multiwell model. Based on the CM of one injection and one production well, the weighting coefficient \( \lambda_{ij} \) is introduced to represent the distribution of water injection in multiple oil wells. When the superposition principle (Lin et al., 2017) is applied, the material balance formula between one production well and multiple injection wells (Yousef et al., 2009) is

\[ \sum_{i=1}^{I} C_{ij} V_{p_{ij}} \frac{d\bar{p}_{ij}}{dt} = \sum_{i=1}^{I} \lambda_{ij} I_i(t) - \sum_{i=1}^{I} Q_{ij}(t) \] (8)

Eliminating the average formation pressure \( \bar{p}_{ij} \), one obtains

\[ \sum_{i=1}^{I} \tau_{ij} \frac{dQ_{ij}(t)}{dt} + \sum_{i=1}^{I} Q_{ij}(t) = \sum_{i=1}^{I} \lambda_{ij} I_i(t) - \frac{dP_{wfi}}{dt} \sum_{i=1}^{I} \tau_{ij} J_{ij} \] (9)

In equation (9), \( \tau_{ij} \) is equal to \( C_i V_p / J_{ij} \) (Lin, 2012). For each injection and production well, there are specific values of connectivity coefficient and time constant. This shows that the change of BHFP has an effect on the calculation results of connectivity.

Discretizing equation (9), one obtains the discrete CM of multiple injection and production wells

\[ Q_j(n) = \lambda_p Q(n_0)e^{\frac{t_0-n}{\tau}} + \sum_{i=1}^{4} \lambda_{ij} \sum_{m=n_0}^{m=n} \frac{\Delta n}{\tau_{ij}} I_{ij}(m) + V_j \left[ p_{wfi}(n_0)e^{\frac{n_0-n}{\tau_{ij}}} - p_{wfi}(n) + p'_{wfi}(n) \right] \] (10)
There exists interference between production wells that affects production. The working system of surrounding oil wells affects the production of target oil wells. The interference coefficient $\nu_{kj}$ between production wells is introduced into the CM. Based on the pressure superposition principle, the BHFP of production wells is superposed, and the third item in equation (10) is extended to multiple wells. Finally, the multiwell discrete CM considering the interwell interference is obtained

$$Q_j(n) = \hat{\lambda}_P Q(n_0) e^{n_0-n} + \sum_{i=1}^{I} \hat{\lambda}_{ij} \hat{t}'(n) + \sum_{k=1}^{K} \nu_{kj} \left[ p_{wfj}(n_0) e^{n_0-n} - p_{wfkj}(n) + p'_{wfj}(n) \right]$$

(11)

Here, $\nu_{kj}$ characterizes the influence of changing the BHFP of $k$ well on the production of target $j$ well. If the BHFP of the surrounding wells remains constant, the value of the last term of equation (11) is 0.

This model consists of three parts. The first is the production capacity of the oil well before water injection. The second is the liquid production caused by multiple water injections. The third is the liquid production caused by the change of BHFP of surrounding production wells.

**Solution of CM**

**Solution algorithm**

In the CM, there are two undetermined coefficients for each well pair. They are connectivity coefficients and time constants. If all time constants are known, the CM becomes linear, and then one can calculate the connectivity coefficient by multiple linear regression (MLR) (Breiman and Friedman, 2010). It is necessary to measure the minimum squared error between actual and calculated production when using MLR. The minimum squared error is

$$\min \left\{ \sum_{n=1}^{N} \left[ Q_j(n) - \hat{Q}_j(n) \right]^2 \right\}$$

(12)

Transforming the problem into an optimization problem for solution (Pate et al., 2014), the solving process is subject to constraints $Q_j(n) = \hat{Q}_j(n)$. Adding constraints and introducing Lagrange multipliers to equation (12) change the objective function into equation (13)

$$\min \left\{ \sum_{n=1}^{N} \left[ \bar{Q}_j(n) - \bar{Q}_j(n) \right]^2 - 2\mu_j(\bar{Q}_j - \bar{Q}_j) \right\}$$

(13)

If the derivative of each parameter is equal to zero, for each well, $I + K + 2$ linear equations are generated. Iterative and nonlinear optimization methods are used to invert the connectivity coefficient and time constants.
The optimization steps are: (1) giving an initial value of a time constant, (2) calculating the connection coefficient and time constant by mean squared error algorithms, and (3) giving another time constant, until the optimal value of the objective function is determined.

The MATLAB optimization mode is used to execute the optimization process. The number of iterations can be set according to the requirement (Mairal et al., 2009).

One can obtain two parameters, connectivity coefficient $\lambda_{ij}$ and time constant $s_{ij}$. Here, $\lambda_{ij}$ is a parameter to characterize the connectivity between injection wells and production wells in a reservoir quantitatively. The larger the value, the better the connectivity between wells. The time constant $s_{ij}$ can be used to characterize the loss of the injected signal. The larger the value, the greater the loss.

**Solution example**

In a group of five injection and four production wells, the well pattern is five-point, the reservoir is under the condition of constant wellbore pressure (CWP), the well spacing is 300 m, and the wells’ dynamic data are known. The CM was established. An initial time constant is given in the CM, which can be estimated by actual geological parameters to ensure the accuracy and rapidity of the solution. Then, the model is transformed into an optimization problem for solution, using MLR to find the minimum square error between the actual and calculated production. Iterative and nonlinear optimization methods are used in the process to ascertain the connectivity coefficient and time constants.

The fitting image of real and calculated liquid production is shown in Figure 1. The blue line represents the calculated liquid production data, and the black line represents the real liquid production data. The real and calculated productions have higher fitting precision.

The results of the connectivity coefficient and time constant are shown in Table 1.

Using the data of Table 1, the connection coefficient plan and time constant plan were drawn, as shown in Figure 2. The length of the arrow in the plan represents the magnitude of the connectivity degree.

This shows that the connectivity coefficient between I01-P01, I05-P02, I01-P03, and I04-P03 is small. It is inferred that there is a fault, which is consistent with the actual situation. The time constant on two sides of the fault is much larger than that on the same side of the fault.

**CM validation**

Several factors were considered for the interwell connectivity in the improved CM, such as the time lag and attenuation of injected signals, interaction between oil wells, well spacing, compression coefficient, formation permeability, and BHFP. Therefore, whether the improved CM can overcome the shortcomings of previous algorithms needed to be verified.

A numerical simulation method was used to realize the validation process (Rai et al., 2015). First, a conceptual well group model was established with CMG software, which can simulate different types of reservoir and different production systems. Then, the interwell connectivity of the conceptual well group was inverted with the CM. The inversion results of connectivity could verify whether the improved algorithm is reasonable.

**Conceptual well group with one injector and one producer**

The CMG numerical simulation software was used to establish the conceptual model of one injection and one production well. The reservoir is unsaturated and under the
CWP condition. There is oil–water two-phase flow in the reservoir. The compressibility coefficients of oil and water are set to be the same, and rock compressibility is minimal. Under such conditions, it is feasible to change the comprehensive compressibility coefficients in CMG, and the fluid saturation does not affect the value of comprehensive compressibility coefficients (Yin et al., 2016; Yousefi et al., 2018). The parameters in the model are shown in Table 2, and the data of water injection are shown in Figure 3.
Effect of compression coefficient on connectivity. In the conceptual well group model composed of an injection and a production well, the oil and the water wells are connected, and the injection and production are balanced. The connectivity coefficient between the injection

![Figure 2. Connectivity index diagram. (a) Connectivity coefficient diagram and (b) time constant diagram.](image)

| Parameter                  | Value         |
|----------------------------|---------------|
| Porosity ($\phi$)          | 0.18          |
| Horizontal permeability (mD)| 40            |
| Vertical permeability (mD) | 4             |
| Compressibility (1/MPa)    | $3 \times 10^{-4}$ |
| Grid                       | $31 \times 31 \times 5$ |
| $D_x \times D_y \times D_z$ (m) | $40 \times 40 \times 6$ |

![Figure 3. Water injection data diagram.](image)
and production wells should be equal to one. In CMG, different compression coefficients represent different homogeneous reservoirs. One can obtain the production data under different compression coefficients by numerical simulation. The simulation was run for 80 months. When the production data were applied to inverse the connectivity index between wells in the CM, the relationship curve between the compression coefficient and connectivity coefficient were obtained (Figure 4); then, the change of connectivity index was analyzed.

When the compression coefficients change, the connectivity coefficient inverted by the CM is constant. The improved CM eliminates the influence of the compression coefficient on the connectivity index.

According to the definition of time constant \( \tau = C_t V_p / J \) (Lin, 2012), one can calculate a value that is the correction value of the time constant. When the comprehensive compression coefficient was changed in CMG, a group of production data was obtained by

![Figure 4. Related curve between compression coefficient and connectivity coefficient.](image)

![Figure 5. Related curve between time constant and compression coefficient.](image)
numerical simulation. The production data in the CM were applied to invert a set of estimated time constants. Drawing the estimated and corrected values of time constants on the same graph revealed the relationship between the time constant and the comprehensive compression coefficient (Figure 5).

The relationship between the time constant and comprehensive compression coefficient is linear, which conforms to the definition of the time constant. Although the trend of the estimated and corrected values is consistent, there is a slight deviation. This deviation is caused by the numerical approximation in the process of solving the CM. For larger time constants, the error becomes minimal.

**Influences of permeability on connectivity.** Changing the formation permeability of the conceptual well group model composed of an injection and a production well resulted in a group of production data being obtained by numerical simulation. Using the improved CM to invert the connectivity of homogeneous reservoirs with different permeabilities revealed the relation between the connectivity coefficient and formation permeability (Figure 6).

The permeability does not affect the inversion result of the connectivity coefficient in the CM. The value of the connectivity coefficient is always equal to one. The improved CM eliminates the influence of permeability on the connectivity and can be used to invert connectivity in different permeability formations.

**Effect of well spacing on connectivity.** By changing the well spacing between injection and production wells in the conceptual well group model composed of an injection and a production well, it was possible to design 30 numerical simulation schemes. The numerical simulation production data in the CM were applied to invert the connectivity, revealing the connectivity coefficients under different well spacings (Figure 7).

Figure 7 shows that the connectivity coefficient is constant when the well spacing is small. When the well spacing increases, the connectivity calculation results fluctuate. Therefore, the improved CM is not suitable for inverting the connectivity when the well spacing is too large. One can optimize the range of well spacing of the application of the CM by numerical simulation.

![Figure 6. Related curve between connectivity coefficient and permeability.](image)
Conceptual well group with five injectors and four producers

A conceptual well group model was established with five injectors and four producers to simulate frequent shut in and pressure fluctuation conditions. The well pattern is 5\times 4, and the well location distribution is shown in Figure 2. The reservoir is unsaturated, and there is oil–water two-phase flow in the reservoir. The compressibility coefficients of oil and water are set to be the same. Rock compressibility is minimal. Under such conditions, it is feasible to change the comprehensive compressibility coefficients in CMG, and the value of compressibility coefficients is not affected by fluid saturation. The values of the parameters of this model are shown in Table 3.

The simulation was run for 80 months. Calculating the connectivity coefficient of this basic scheme, the actual well production and that calculated with the CM had higher fitting precision. The data in Table 4 are the connection coefficients of the basic scheme, which can be divided into three categories. The connectivity coefficient between central injection wells and production wells is 0.25, that between corner injection wells and adjacent production wells is 0.33, and that between corner injection wells and distant production wells is 0.17.

As shown in Figure 8, the connectivity between oil and water wells can be visually expressed by arrows. The arrow points from the injector to the producer. The longer the arrow, the greater the connectivity coefficient.

**Figure 7.** Related curve between connectivity coefficient and well spacing.

**Table 3.** Parameter setting of the numerical model.

| Parameter                  | Value          |
|----------------------------|----------------|
| Porosity (\(\varphi\))    | 0.18           |
| Horizontal permeability (mD) | 40             |
| Vertical permeability (mD)   | 4              |
| Compressibility (1/MPa)    | \(3 \times 10^{-4}\) |
| Grid                      | \(31 \times 31 \times 5\) |
| \(D_x \times D_y \times D_z\) (m) | \(80 \times 80 \times 12\) |

**Conceptual well group with five injectors and four producers**

A conceptual well group model was established with five injectors and four producers to simulate frequent shut in and pressure fluctuation conditions. The well pattern is 5\times 4, and the well location distribution is shown in Figure 2. The reservoir is unsaturated, and there is oil–water two-phase flow in the reservoir. The compressibility coefficients of oil and water are set to be the same. Rock compressibility is minimal. Under such conditions, it is feasible to change the comprehensive compressibility coefficients in CMG, and the value of compressibility coefficients is not affected by fluid saturation. The values of the parameters of this model are shown in Table 3.

The simulation was run for 80 months. Calculating the connectivity coefficient of this basic scheme, the actual well production and that calculated with the CM had higher fitting precision. The data in Table 4 are the connection coefficients of the basic scheme, which can be divided into three categories. The connectivity coefficient between central injection wells and production wells is 0.25, that between corner injection wells and adjacent production wells is 0.33, and that between corner injection wells and distant production wells is 0.17.

As shown in Figure 8, the connectivity between oil and water wells can be visually expressed by arrows. The arrow points from the injector to the producer. The longer the arrow, the greater the connectivity coefficient.
Effect of a producer is frequent shut-in on connectivity. The improved CM is effective when producers shutting in frequently. Changing the water wells’ working system and making them shut in frequently, this is scheme A, and the production data of oil wells were recalculated with numerical simulation. Then, the CM was used to invert the connectivity. The $\lambda_{ij}$ cross plot of the basic scheme and frequent-shut-in scheme is shown in Figure 9. The dotted line in Figure 9 is the calculation result of the basic scheme. The points represent the connectivity calculation results of the frequent-shut-in scheme.

One can conclude that the connectivities when producers shutting in frequently are very close to those of the basic scheme. This shows that the improved CM eliminates the influence of frequently shutting in wells on the connectivity. When one frequently shuts in the wells, one can accurately invert the connectivity with the improved CM.

Effect of BHFP on connectivity. Kaviani et al. (2012) put forward a segmented CM that can be used where unknown bottom-hole pressures change during the analysis interval, providing a means to increase the CM tolerance to common field conditions. However, when the pressure changes frequently, overfitting may occur, in which case it is not possible to get satisfactory connectivity results. However, the model was established considering the BHFP fluctuation, and the validity of the model was verified by numerical simulation.

When the BHFP of well P1 is changed in the basic scheme in CMG, this becomes scheme B. The pressure data of scheme B are shown in the Figure 10.

Table 4. Connection coefficients of the basic scheme.

|    | P1  | P2  | P3  | P4  |
|----|-----|-----|-----|-----|
| 11 | 0.33| 0.33| 0.17| 0.17|
| 12 | 0.33| 0.17| 0.33| 0.17|
| 13 | 0.25| 0.25| 0.25| 0.25|
| 14 | 0.17| 0.33| 0.17| 0.33|
| 15 | 0.17| 0.17| 0.33| 0.33|

Figure 8. Connectivity under basic scheme.
With numerical simulation, the liquid production of scheme B was obtained. Then, the improved CM was used to invert the connectivity. The liquid production fitting curves between the numerical simulation and CM are shown in Figure 11, and the fitting precision is high.

Because of the high fitting precision of liquid production, the interwell connectivity coefficient calculated by the CM is reliable (Qin et al., 2019; Rafael et al., 2018; Wang et al., 2019).

The $\lambda_{ij}$ cross plot of the basic and pressure fluctuation schemes is shown in Figure 12. The dotted line represents the calculation result of the basic scheme, and the points represent the connectivity calculation results of the pressure fluctuation scheme. Figure 12 shows that the inverted connectivity coefficient of the pressure fluctuation scheme basically falls on the dotted line. The calculation results are consistent with those of the basic scheme.

Thus, when the BHFP fluctuates, the improved CM can accurately calculate the connectivity coefficient. It shows that the CM established in this study has a wide applicability to the BHFP fluctuation condition.
Conclusions

1. A multiwell discrete CM considering the well interference and BHFP was established. The model is helpful for identifying the flow direction of injection water, which is of great
significance to guide the profile control and water plugging in the later period of oilfield
development. Moreover, the model has strong practicability, overcomes some shortcomings of previous models, and has a wider range of application.

2. The connection coefficient $\lambda_{ij}$ and time constant $\tau_{ij}$ were inverted in the CM by using the MATLAB optimization mode. Here, $\lambda_{ij}$ is a parameter to characterize quantitatively the connectivity between injection and production wells. The larger the value, the stronger the connectivity between wells. Also, $\tau_{ij}$ is a quantitative characterization of the injected signal loss. The larger the value, the greater the loss.

3. The CM was verified by a numerical simulation method. The applicability of the CM was extended. The improved model eliminates the effects of well spacing, compression coefficient, permeability, frequent shutting off of wells, and BHFP fluctuation on the connectivity results. It was concluded that the connectivity between wells is determined by reservoir geological characteristics, physical properties, and the relative position of wells. It has nothing to do with production dynamic data and pressure data.

4. The time constant is related to the connectivity coefficient. The larger the time constant, the smaller the connectivity coefficient.

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Appendix I

Notation

\[ C_{ij} \text{ comprehensive compressibility between well } i \text{ and well } j \text{ (MPa}^{-1}\text{)} \]
\[ C \text{ comprehensive compressibility (MPa}^{-1}\text{)} \]
\[ h \text{ reservoir thickness (m)} \]
\[ I \text{ injection volume (m}^3\text{/d)} \]
\[ J \text{ production index (m}^3\text{/d/MPa)} \]
\[ K \text{ permeability (mD)} \]
\[ n \text{ time step (dimensionless)} \]
\( m \) time step (dimensionless)
\( p_{wf} \) bottom-hole pressure (MPa)
\( Q \) liquid production (m³/d)
\( r_0 \) drainage radius (m)
\( r_w \) wellbore radius (m)
\( t \) time (month)
\( V_p \) pore volume (m³)
\( V_{p_{ij}} \) pore volume between well \( i \) and well \( j \) (m³)
\( \Delta p \) differential pressure (MPa)
\( \lambda_{ij} \) interwell connectivity coefficient (dimensionless)
\( \lambda_p \) the constant value in the initial output term (dimensionless)
\( \mu_0 \) viscosity of crude oil (mPa·s)
\( \mu_w \) viscosity of water (mPa·s)
\( \tau \) time constant (d)
\( \tau_p \) the value of the time constant in the initial output term (d)
\( v_{kj} \) the influence of changing the BHFP of well \( k \) on the production of target well \( j \) (m³/d/MPa)
\( \phi \) porosity (dimensionless)