On Far-infrared and Submillimeter Circular Polarization

B. T. Draine

Dept. of Astrophysical Sciences, Princeton University, Princeton, NJ 08544, USA; draine@astro.princeton.edu

Received 2021 August 20; revised 2021 November 7; accepted 2021 November 11; published 2022 February 16

Abstract

Interstellar dust grains are often aligned. If the grain alignment direction varies along the line of sight, the thermal emission becomes circularly polarized. In the diffuse interstellar medium, the circular polarization at far-infrared and submillimeter wavelengths is predicted to be very small, and probably unmeasurable. However, circular polarization may reach detectable levels in photodissociation regions viewed through molecular clouds, in infrared dark clouds, and in protoplanetary disks. Measurement of circular polarization could help constrain the structure of the magnetic field in infrared dark clouds, and may shed light on the mechanisms responsible for grain alignment in protoplanetary disks.

Unified Astronomy Thesaurus concepts: Infrared dark clouds (787); Interstellar dust (836); Radiative transfer (1335); Protoplanetary disks (1300)

1. Introduction

Since the discovery of starlight polarization more than 70 yr ago (Hall 1949; Hiltnner 1949), polarization has become a valuable tool for study of both the physical properties of interstellar dust and the structure of the interstellar magnetic field. Starlight polarization arises because initially unpolarized starlight becomes linearly polarized as a result of linear dichroism produced by aligned dust grains in the interstellar medium (ISM). While the physics of dust grain alignment is not yet fully understood, early investigations (Davis & Greenstein 1951) showed how spinning dust grains could become aligned with their shortest axis parallel to the magnetic field direction. Subsequent studies have identified a number of important physical processes that were initially overlooked (see the review by Andersson et al. 2015), but it remains clear that in the diffuse ISM the magnetic field establishes the direction of grain alignment, with the dust grains tending to align with their short axes parallel to the local magnetic field.

van de Hulst (1957) noted that if the magnetic field direction was not uniform, starlight propagating through the dusty ISM would become circularly polarized. This was further discussed by Serkowski (1962) and Martin (1972). The birefringence of the dusty ISM is responsible for converting linear polarization to circular polarization (Serkowski 1962; Martin 1972). The strength of the resulting circular polarization depends on the changes in the magnetic field direction and also on the optical properties of the dust.

Circular polarization of optical light from the Crab Nebula was observed by Martin et al. (1972). Circular polarization of starlight was subsequently observed by Kemp (1972) and Kemp & Wolstencroft (1972); the observed degree of circular polarization, $|V|/I \leq 0.04\%$, was small but measurable. As had been predicted, the circular polarization $V$ changed sign as the wavelength varied from blue to red, passing through zero near the wavelength $\sim 0.55\ \mu m$, where the linear polarization peaked (Martin & Angel 1976).

Because the circular polarization depends on the change in magnetic field direction along the line of sight, it can in principle be used to study the structure of the Galactic magnetic field. Data for 36 stars near the Galactic Plane suggested a systematic bending of the field for Galactic longitudes $80^\circ \leq \ell < 100^\circ$ (Martin & Campbell 1976). However, these studies do not appear to have been pursued, presumably because sufficiently bright and reddened stars are sparse.

In the infrared, circular polarization has been measured for bright sources in molecular clouds (Serkowski & Rieke 1973; Lonsdale et al. 1980; Dyck & Lonsdale 1981). Measurements of linear and circular polarization were used to constrain the magnetic field structure in the Orion molecular cloud OMC-1 (Lee & Draine 1985; Aitken et al. 2006).

Circular polarization has also been observed in the infrared ($K_s$ band) in reflection nebulae (Kwon et al. 2014, 2016, 2018), but in this case scattering is important (Fukushima et al. 2020). Scattering can convert linear to circular polarization, making interpretation dependent on the uncertain scattering geometry.

It was long understood that the nonspherical and aligned grains responsible for starlight polarization must emit far-infrared radiation, which would be linearly polarized. Observations of this polarized emission now allow the magnetic field direction projected on the sky to be mapped in the general ISM (see, e.g., Planck Collaboration et al. 2015a, 2015b; Fissel et al. 2016). Ground-based observations have provided polarization maps for high-surface-brightness regions at submillimeter frequencies (e.g., Dotson et al. 2010), and the Stratospheric Observatory for Infrared Astronomy (SOFIA) is providing polarization maps of bright regions in the far-infrared (e.g., OMC-1: Chuss et al. 2019).

Atacama Large Millimeter/submillimeter Array (ALMA) observations of millimeter and submillimeter emission from protoplanetary disks find that the radiation is often linearly polarized. Scattering may contribute to the polarization (Kataoka et al. 2015), but the observed polarization directions and wavelength dependence appear to indicate that a substantial fraction of the polarized radiation arises from thermal emission from aligned dust grains (Lee et al. 2021).

Previous theoretical discussions of circular polarization were mainly concerned with infrared and optical wavelengths where initially unpolarized starlight becomes polarized as a result of linear dichroism. In a medium with changing polarization direction, the resulting circular polarization is small because the linear polarization itself is typically only a few percent, and the
optical “phase shift” (between the two linear polarization modes) produced by the aligned medium is likewise small. At far-infrared wavelengths, however, the radiation is already substantially polarized when it is emitted, with linear polarizations of 20% or more under favorable conditions (Planck Collaboration et al. 2020). While absorption optical depths tend to be small at long wavelengths, the optical properties of the dust are such that phase shift cross sections at submillimeter wavelengths can be much larger than absorption cross sections, raising the possibility that a medium with changing alignment direction might exhibit measurable levels of circular polarization at far-infrared or submillimeter wavelengths.

The present paper discusses polarized radiative transfer in a medium with partially aligned nonspherical grains, including both absorption and thermal emission. We estimate the expected degree of circular polarization for emission from molecular clouds and protoplanetary disks. For nearby molecular clouds, the far-infrared circular polarization is very small, and probably unobservable. The circular polarization is predicted to be larger for so-called infrared dark clouds (IRDCs), although it is still small. For protoplanetary disks the circular polarization may be measurable, but will depend on how the direction of grain alignment changes in the disk.

The paper is organized as follows. The equations describing propagation of partially polarized radiation are presented in Section 2, and the optics of partially aligned dust mixtures are summarized in Section 3. Section 4 estimates the circularly polarized emission from molecular clouds, including IRDCs. Section 5 discusses the alignment of solid particles in stratified protoplanetary disks resembling HL Tau. If the grain alignment is due to dust-gas streaming, the emission may be circularly polarized. The results are discussed in Section 6, and summarized in Section 7.

2. Polarized Radiative Transfer

2.1. Refractive Index of a Dusty Medium

Aligned dust grains result in linear dichroism—the attenuation coefficient depends on the linear polarization of the radiation. Linear dichroism is responsible for the polarization of starlight—initially unpolarized light from a star becomes linearly polarized as the result of polarization-dependent attenuation by aligned dust grains.

We adopt the convention that the electric field $E \propto \text{Re}[e^{ikz - \omega t}]$ for a wave propagating in the $+z$ direction, where $k = \omega/c = 2\pi/\lambda$, is the wavevector in vacuo, and $m(\omega)$ is the complex refractive index of the dusty medium. For radiation polarized with $E || \hat{e}$, the complex refractive index is

$$m_j = 1 + mj' + imj''.$$  (1)

The real part $m_j'$ describes retardation of the wave, relative to propagation in vacuo. The phase delay $\phi$ varies as

$$d\phi_j = \frac{2\pi}{\lambda} m_j' = n_d C_{\text{pha},j},$$  (2)

where $n_d$ is the number density of dust grains, and $C_{\text{pha},j}$ is the “phase shift” cross section of a grain. The imaginary part $m_j''$ describes attenuation of the energy flux $F$:

$$d \ln F = -\frac{4\pi}{\lambda} m_j'' = -n_d C_{\text{ext},j},$$  (3)

where $C_{\text{ext},j}$ is the extinction cross section.

2.2. Transfer Equations for the Stokes Parameters

Consider a beam of radiation characterized by the usual Stokes vector $S \equiv (I, Q, U, V)$. The equations describing transfer of radiation through a dichroic and birefringent medium with changing magnetic field direction have been discussed by Serkowski (1962) and Martin (1974).1 The discussions have assumed that the aligned grains polarize the light by preferential attenuation of one of the polarization modes, with circular polarization then arising from differences in propagation speed of the linearly polarized modes.

For submicron particles, scattering is negligible at far-infrared wavelengths, because the grain is small compared with the wavelength. However, the grains are themselves able to radiate, and aligned grains will emit polarized radiation.

Let the direction of the static magnetic field $B_0$ be

$$\hat{b} \equiv \frac{B_0}{|B_0|} = (\hat{n} \cos \Psi + \hat{e} \sin \Psi) \sin \gamma + \hat{z} \cos \gamma$$  (4)

where $\hat{n}$ and $\hat{e}$ are unit vectors in the N and E directions, $\hat{z} = \hat{n} \times \hat{e}$ is the direction of propagation, and $\sin \gamma = 1$ if $\hat{b}$ is in the plane of the sky.

Let $\hat{x}$ and $\hat{y}$ be orthonormal vectors in the plane of the sky, with $\hat{x}$ parallel to the projection of $B_0$ on the plane of the sky (see Figure 1):

$$\hat{x} = \hat{n} \cos \Psi + \hat{e} \sin \Psi$$  (5)

$$\hat{y} = -\hat{n} \sin \Psi + \hat{e} \cos \Psi.$$  (6)

If the dust grains are partially aligned with their short axes tending to be parallel to $B_0$, we expect $C_{\text{ext},y} > C_{\text{ext},x}$. At long wavelengths ($\lambda > 10 \mu m$) we also expect $C_{\text{pha},y} > C_{\text{pha},x}$. We assume that the dust grains themselves have no overall chirality, hence circular dichroism and circular birefringence can be neglected so long as the response of the magnetized plasma is negligible, which is generally the case for $\nu \gtrsim 30$ GHz.

Following the notation of Martin (1974), define

$$\delta \equiv n_d \frac{(C_{\text{ext},y} + C_{\text{ext},x})}{2} = \frac{2\pi}{\lambda} (m_y'' + m_x'')$$  (7)

$$\Delta \sigma \equiv n_d \frac{(C_{\text{ext},y} - C_{\text{ext},x})}{2} = \frac{2\pi}{\lambda} (m_y'' - m_x'')$$  (8)

$$\Delta \epsilon \equiv n_d \frac{(C_{\text{pha},y} - C_{\text{pha},x})}{2} = \frac{2\pi}{\lambda} (m_y' - m_x').$$  (9)

If scattering is neglected, the propagation of the Stokes parameters is given by

---

1 Our axes $\hat{x}$ and $\hat{y}$ correspond, respectively, to axes 2 and 1 in Martin (1974).
2 Equation (10) conforms to the IEEE and IAU conventions for the Stokes parameters (Hamaker & Bregman 1996): $Q > 0$ for $E$ along the N–S direction, $U > 0$ for $E$ along the NE–SW direction, $V > 0$ for “right-handed” circular polarization ($E$ rotating in the counterclockwise direction as viewed on the sky).
A spinning grain develops a magnetic moment from the Magnus effect (if it has unpaired electrons and the Rowland effect (if it has a net charge). For submicron grains, the resulting net magnetic moment is large enough that the Larmor precession period in the local interstellar magnetic field is short.

Figure 1. Angle $\Psi$, and directions $\hat{x}$, $\hat{y}$.

\[
d\frac{I}{d\zeta} = \begin{pmatrix} \Delta\sigma \cos 2\Psi & \Delta\sigma \cos 2\Psi & 0 \\ \Delta\sigma \sin 2\Psi & 0 & \Delta\sigma \cos 2\Psi \\ 0 & -\Delta\varepsilon \sin 2\Psi & \Delta\varepsilon \cos 2\Psi \end{pmatrix} \begin{pmatrix} I - B(T_d) \end{pmatrix},
\]

(10)

where $B(T_d)$ is the intensity of blackbody radiation for dust temperature $T_d$. Equation (10) differs from Martin (1974) only by replacement of $I$ by $(I - B)$ on the right-hand side to allow for thermal emission (see also Reissl et al. 2016). It is apparent that Equation (10) is consistent with thermal equilibrium blackbody radiation, with $dS/d\zeta = 0$ for $S = (B, 0, 0, 0)$.

3. Optical Properties of the Dust

We now assume that the grains can be approximated by spheroids. Draine & Hensley (2021a) found that observations of starlight polarization and far-infrared polarization appear to be consistent with dust with oblate spheroidal shapes, with axial ratio $b/a \approx 1.6$ providing a good fit to observations. Observations of the diffuse ISM are consistent with

\[
\frac{\delta}{n_H} = \frac{\tau}{N_H} \approx 6.5 \times 10^{-27} \left( \frac{\lambda}{\text{mm}} \right)^{-1.8} \text{cm}^2\text{H}^{-1}
\]

(11)

for $100 \mu\text{m} \lessgtr \lambda \lessgtr 1 \text{cm}$ (Hensley & Draine 2021; Draine & Hensley 2021b).

Let $\hat{b}$ be a “special” direction in space for grain alignment: the short axis $\hat{a}_1$ of the grain may be preferentially aligned either parallel or perpendicular to $\hat{b}$. For grains in the diffuse ISM, $\hat{b}$ is the magnetic field direction, and the short axis $\hat{a}_1$ tends to be parallel to $\hat{b}$. In protostellar disks, however, other alignment mechanisms may operate, and $\hat{b}$ may not be parallel to the magnetic field.

We approximate the grains by oblate spheroids, spinning with short axis $\hat{a}_1$ parallel to the angular momentum $\hat{J}$. For oblate spheroids, the fractional alignment is defined to be

\[
f_{\text{align}} = \frac{3}{2} \left( (\hat{a}_1 \cdot \hat{b})^2 - \frac{1}{2} \right),
\]

(12)

where $(\ldots)$ denotes averaging over the grain population. If $\hat{J} \parallel \hat{b}$, then $f_{\text{align}} \rightarrow 1$; if $\hat{J}$ is randomly oriented, then $f_{\text{align}} = 0$; if $\hat{J} \perp \hat{b}$, then $f_{\text{align}} \rightarrow -\frac{1}{2}$.

The “modified picket fence approximation” (Draine & Hensley 2021a relates $\delta$, $\Delta\sigma$, and $\Delta\varepsilon$ to $f_{\text{align}}$ and the angle $\gamma$:

\[
\delta = n_d f_{\text{align}} \sin^2 \gamma \left( \frac{C_{\text{abs},b} - C_{\text{abs},a}}{2} \right),
\]

(13)

\[
\Delta\sigma = n_d f_{\text{align}} \sin^2 \gamma \left( \frac{C_{\text{abs},b} - C_{\text{abs},a}}{2} \right),
\]

(14)

\[
\Delta\varepsilon = n_d f_{\text{align}} \sin^2 \gamma \left( \frac{C_{\text{pha},b} - C_{\text{pha},a}}{2} \right). 
\]

(15)

In the Rayleigh limit (grain radius $a \ll \lambda$), we have (Draine & Lee 1984):

\[
C_{\text{abs},j} = \frac{2\pi V}{\lambda} \frac{\epsilon_j}{|1 + (\epsilon - 1) L_j|^2},
\]

(16)

\[
C_{\text{pha},j} = \frac{\pi V}{\lambda} \left( (\epsilon_1 - 1)[1 + L_j(\epsilon_1 - 1)] + \epsilon_j^2 L_j \right),
\]

(17)

where $\epsilon(\lambda) \equiv \epsilon_1 + i\epsilon_2$ is the complex dielectric function of the grain material, and $L_0$ and $L_1 = (1 - L_0)/2$ are dimensionless “shape factors” (van de Hulst 1957; Bohren & Huffman 1983) that depend on the axial ratio of the spheroid. Draine & Hensley (2021b) have estimated $\epsilon(\lambda)$ of astrodust for different assumed axial ratios.

Figure 2 shows the dimensionless ratios $\Delta\sigma/\delta$ and $\Delta\varepsilon/\delta$ for oblate astrodust spheroids with porosity $\mathcal{P} = 0.2$, $b/a = 1.6$ ($L_0 = 0.464$, $L_1 = 0.268$), and $f_{\text{align}} = 0.5$, for the case where the magnetic field is in the plane of the sky ($\sin \gamma = 1$). The relatively high opacity that enables “astrodust” to reproduce the observed far-infrared emission and polarization also implies that $\epsilon_1$ has to be fairly large at long wavelengths (Draine & Hensley 2021b). This causes $\Delta\varepsilon/\delta$ to be relatively large, as seen in Figure 2. For $\lambda \gtrsim 70 \mu\text{m}$, oblate astrodust grains with $b/a = 1.6$ have

\[
\frac{\Delta\sigma}{\delta} \approx 0.38 f_{\text{align}} \sin^2 \gamma
\]

(18)

\[
\frac{\Delta\varepsilon}{\delta} \approx 9.0 \left( \frac{\lambda}{\text{mm}} \right)^{0.7} f_{\text{align}} \sin^2 \gamma.
\]

(19)

Equations (18) and (19) neglect the weak dependence of $\delta$ on $f_{\text{align}}$ and $\gamma$ (see Equation (13)). Equations (18) and (19) are shown in Figure 2 for $f_{\text{align}} \sin^2 \gamma = 0.5$.

4. Circular Polarization from Interstellar Clouds

4.1. Grain Alignment

A spinning grain develops a magnetic moment from the Barnett effect (if it has unpaired electrons) and the Rowland effect (if it has a net charge). For submicron grains, the resulting net magnetic moment is large enough that the Larmor precession period in the local interstellar magnetic field is short.
compared with the timescales for other mechanisms to change the direction of the grain’s angular momentum $J$. The rapid precession of $J$ around the local magnetic field $B_0$ and the resulting averaging of grain optical properties establishes $B_0$ as the special direction for grain alignment—grains will be aligned with their short axes preferentially oriented either parallel or perpendicular to $B_0$.

Paramagnetic dissipation, radiative torques, or systematic streaming of the grains relative to the gas will determine whether the grains align with their short axes preferentially parallel or perpendicular to $B_0$. Although the details of the physics of grain alignment are not yet fully understood, it is now clear that grains in diffuse and translucent clouds tend to align with short axes $\hat{a}_1$ tending to be parallel to $B_0$, i.e., with $f_{\text{align}} > 0$ (see Equation (12)).

If the dust grains are modeled by oblate spheroids with axial ratio $b/a = 1.6$, a mass-weighted alignment fraction $f_{\text{align}} \approx 0.5$ can reproduce the highest observed levels of polarization of both starlight and far-infrared emission from dust in diffuse clouds (including diffuse molecular clouds) (Draine & Hensley 2021a).

In dark clouds, the fractional polarization of the thermal emission is generally lower than in diffuse clouds. The lower fractional polarization may indicate lower values of $f_{\text{align}}$ within dark clouds, but it could also result from a nonuniform magnetic field in the cloud, with the overall linear polarization fraction reduced by beam-averaging over regions with different polarization directions.

If the reduced values of linear polarization are due to systematic changes in magnetic field direction along the line of sight, the emission from the cloud could become partly circularly polarized. We now estimate what levels of circular polarization might be present.

### 4.2. Nearby Molecular Clouds

Planck has observed linearly polarized emission from many molecular clouds. To estimate the levels of circular polarization that might be present, we consider one illustrative example, in the “RCrA-Tail” region in the R Corona Australis molecular cloud (see Figure 11 in Planck Collaboration et al. 2015a). The polarized emission in this region has a number of local maxima. One of the polarized flux maxima coincides with a total emission peak near $(\ell, b) \approx (-0^\circ9, -18^\circ7)$, with total intensity $I(353 \text{ GHz}) \approx 4 \text{ MJy sr}^{-1}$ and linear polarization fraction $p \approx 2.5\%$.

For an assumed dust temperature $T_d \approx 15 \text{ K}$, the observed intensity $I(353 \text{ GHz}) = 4 \text{ MJy sr}^{-1}$ implies $\tau(353 \text{ GHz}) \approx 1.3 \times 10^{-4}$. For diffuse ISM dust (see, e.g., Hensley & Draine 2021), this would correspond to $A_V \approx 5 \text{ mag}$.

For simple assumptions about the angle $\Psi$ characterizing the projection of the magnetic field on the sky, we can obtain approximate analytic solutions to the radiative transfer Equation (10), valid for $\tau \ll 1$ (see Appendix A). Define $d^{\tau} \equiv \delta z$. Suppose that $T_p(\Delta\sigma/\delta)$, and $(\Delta\epsilon/\delta)$ are constant, and assume that the magnetic field direction has a smooth twist along the line of sight, with $\Psi$ varying linearly with $\tau'$ as $\tau'$ varies from $0$ to $\tau$:

$$\Psi = \Psi_0 + \alpha\tau', \quad \alpha \equiv \frac{\Delta\Psi}{\tau}. \quad (20)$$

For $\tau \ll 1$, the linear and circular polarization fractions are then (see Appendix A)

$$p \approx \left(\frac{\Delta\sigma}{\delta}\right)^2 \left[1 - \cos(2\Delta\Psi)\right]^{1/2} \quad (21)$$

$$\frac{V}{I} \approx \left(\frac{\Delta\sigma}{\delta}\right)^2 \left[\frac{\Delta\epsilon}{2\Delta\Psi}\right] \frac{\tau}{1 - \sin(2\Delta\Psi)} \quad (22)$$

Equations (21–22) are for the special case of an isothermal medium with a uniform twist in the alignment direction.

If we assume diffuse cloud dust properties (Equations (18), (19)) but with $f_{\text{align}} \sin^2\gamma = 0.075$ and a twist angle $\Delta\Psi = 90^\circ$, we can reproduce the observed polarization $p \approx 2.5\%$ in the RCrA-Tail region. With these parameters, Equation (22) predicts circular polarization $V/I \approx 7 \times 10^{-7}(\lambda/850 \mu\text{m})^{-1.3}$, far below current sensitivity limits. It is clear that measurable levels of circular polarization in the far-infrared will require much larger optical depths $\tau$.

### 4.3. Infrared Dark Clouds

Typical giant molecular clouds (GMCs), such as the Orion Molecular Cloud, have mass surface densities resulting in $A_V \approx 10 \text{ mag}$ of extinction, and are therefore referred to as “dark clouds.” However, in the inner Galaxy, a number of clouds have been observed that appear to be “dark” (i.e., opaque) even in the mid-infrared. These IRDCs have dust masses per area an order of magnitude larger than “typical” GMCs. Because of the much larger extinction in IRDCs, the circular polarization may be much larger than in normal GMCs.

The “Brick” (G0.253+0.016) is a well-studied IRDC (Carey et al. 1998; Longmore et al. 2012). With an estimated mass $M > 10^5 M_\odot$ and high estimated density ($n_H > 10^4 \text{ cm}^{-3}$), the Brick appears to be forming stars (Marsh et al. 2016; Walker et al. 2021), although with no signs of high-mass star formation.
It has been mapped at 70–500 μm by Herschel Space Observatory (Molinari et al. 2016) and at 220 GHz by the Atacama Cosmology Telescope (ACT; Guan et al. 2021). Polarimetric maps have been made at 220 GHz by ACT, and at 850 GHz by the Caltech Submillimeter Observatory (CSO; Dotson et al. 2010).

The NE region at (ℓ, b) = (16°, 2°) has 4(600 GHz) ≈ 5000 MJy sr\(^{-1}\) (Molinari et al. 2016) and 4(220 GHz) ≈ MJy sr\(^{-1}\) (Guan et al. 2021). For an assumed dust temperature \(T_d \approx 20\ \text{K}\), this indicates optical depths \(\tau(600\ \text{GHz}) \approx 0.05\), \(\tau(220\ \text{GHz}) \approx 0.005\). Astrodust would then have \(\tau(850\ \text{GHz}) \approx 0.09\), and \(\tau(353\ \text{GHz}) \approx 0.014\) — about 100 times larger than in the RCrA molecular cloud.

The fractional polarization is expected to be approximately independent of frequency in the submillimeter. At 220 GHz, Guan et al. (2021) report a linear polarization of 1.8% at 220 GHz for the NE end of the cloud, (ℓ, b) ≈ (16°, 2.5°) (Yilun Guan 2021, private communication). The CSO polarimetry suggests a similar fractional polarization at 850 GHz.

While this fractional polarization is relatively small compared with the highest values (~20%) observed by Planck in diffuse clouds, it is still appreciable, requiring significant grain alignment in a substantial fraction of the cloud volume (i.e., not just in the surface layers of the IRDC). The inferred average magnetic field direction \(\Psi \approx 20°\) (Guan et al. 2021) differs by ~60° from the \(\Psi \approx 80°\) field direction indicated by the 220 GHz polarization outside the cloud, demonstrating that the magnetic field in this region is far from uniform.

As a simple example, we suppose, as we did for the RCrA-Tail region above, that the projected field rotates by \(\Delta \Psi = 90°\) from the far side of the Brick to the near side.

We calculate the circular polarization at 850 GHz (350 μm) for the estimated total optical depth \(\tau(850\ \text{GHz}) = 0.09\) of the Brick. We use the estimated properties of astrodust in the diffuse ISM, with \(f_{\text{align}} \sin^2 \gamma = 0.075\) to approximately reproduce the ~1.8% polarization observed for the Brick.

Figure 3 shows the polarization state of the radiation as it propagates through the cloud from \(\tau' = 0\) to \(\tau' = \tau\). The fractional polarization \(p\) starts off at ~2.9%, dropping to ~1.8% at \(\tau' = \tau\) as the result of the assumed magnetic field twist of \(\Delta \Psi = 90°\).

The resulting 850 GHz circular polarization \(V/I\) is small, only ~0.025%. Measuring such low levels of circular polarization will be challenging. For \(\Delta \epsilon/\delta \propto \lambda^{0.7}\) (see Figure 2) and the absorption coefficient \(\delta \propto \lambda^{-1.8}\) (see Equation (11)), the circular polarization from an IRDC is expected to vary as \(V/I \propto \lambda^{-1.1}\). For the adopted parameters \(\Delta \Psi = 90°, \tau(850\ \text{GHz}) = 0.09\), \(f_{\text{align}} \sin^2 \gamma = 0.075\), the Brick would have

\[
\frac{V}{I} \approx 0.025\% \left(\frac{350 \ \mu\text{m}}{\lambda}\right)^{1.1},
\]

for 70 μm ~\(\lambda \lesssim 1\) cm. While much larger than for normal GMCs, this estimate for the circularly polarized emission from the Brick is small, and measuring it will be challenging.

4.4. A Photodissociation Region Seen through a Molecular Cloud

The warm dust surrounding an embedded HH region may allow measurement of circular polarization at wavelengths as short as ~20 μm. We consider a cloud with optical depth \(\tau(353\ \text{GHz}) = 4 \times 10^{-4}\), somewhat greater than the R Corona Australis cloud example considered in Section 4.2, but small compared with the Brick.

The far edge of the cloud is assumed to contain warm dust in a photodissociation region (PDR) with optical depth \(\tau(\lambda)\). The PDR is assumed to contribute 10% of the total column density through the molecular cloud, with dust heated to \(T_1 = 80\ \text{K}\). The dust in the rest of the molecular cloud is cold, \(T_2 = 15\ \text{K}\).

We assume the dust in the PDR to be moderately aligned, with \(f_{\text{align}} \sin^2 \gamma = 0.1\), whereas for the dust in the rest of the molecular cloud we take \(f_{\text{align}} \sin^2 \gamma = 0.05\).

Using the analytic approximation for the “two-zone” model in Appendix B, we find the fractional linear polarization \(p\) and circular polarization \(V/I\) shown in Figure 4. At \(\lambda \lesssim 100\ \mu\text{m}\), the polarization is the combination of polarized emission from the warm dust in the PDR and dichroic absorption by the cool dust. At longer wavelengths, \(\lambda > 300\ \mu\text{m}\), dichroic absorption is minimal, and we see the sum of the polarized emission from the warm and cool regions. The polarization angle rotates as the ratio of warm emission to cool emission drops with increasing wavelength. The features at \(10 < \lambda < 30\ \mu\text{m}\) arise from the strong silicate absorption bands at 10 and 18 μm.

The circular polarization reaches \(V/I = 0.02\%\) at \(\lambda = 20\ \mu\text{m}\) but declines as \(\sim \lambda^{-1.1}\) at longer wavelengths.

5. Circular Polarization from Protoplanetary Disks

Protoplanetary disks can have dust surface densities well in excess of IRDCs, raising the possibility that \(\tau\) may be large enough to generate measurable circular polarization if the grains are locally aligned and the alignment direction varies along the optical path.
Figure 4. Polarization of dust continuum from a molecular cloud with a PDR on the far side. The magnetic field in the cold cloud is assumed to have a systematic twist along the line of sight, with a twist angle $\Delta \psi = 60^\circ$. The dust is assumed to be partially aligned, with $f_{\text{align}} \sin^2 \gamma = 0.1$ for the warm dust ($T_1 = 80$ K) in the PDR, and $f_{\text{align}} \sin^2 \gamma = 0.05$ for the cold dust ($T_2 = 15$ K) in the rest of the cloud.

5.1. Grain Alignment in Protoplanetary Disks

Gas densities in protoplanetary disks exceed interstellar gas densities by many orders of magnitude. The observed thermal emission spectra from young protoplanetary disks appear to require that most of the solid material be in particles with sizes that may be as large as $\sim$mm (Beckwith & Sargent 1991; Natta & Testi 2004; Draine 2006), orders of magnitude larger than the submicron grains in the diffuse ISM.

The physics of grain alignment in protoplanetary disks differs substantially from the processes in the diffuse ISM. One important difference from interstellar clouds is that in protoplanetary disks the Larmor precession period for the grain sizes of interest is long compared with the time for the grain to undergo collisions with a mass of gas atoms equal to the grain mass (Yang 2021). With Larmor precession no longer important, the magnetic field no longer determines the preferred direction for grain alignment. Instead, the “special” direction may be either the local direction of gas-grain streaming—in which case, $\hat{b} \parallel \nabla \nu_{\text{drift}}$—or perhaps the direction of anisotropy in the radiation field—in which case, $\hat{b} \parallel \hat{\sigma}$. Whether grains will tend to align with short axes $\hat{a} \parallel \hat{r}$ parallel or perpendicular to $\hat{b}$ (i.e., $f_{\text{align}} > 0$ or $f_{\text{align}} < 0$) is a separate question.

5.1.1. Alignment by Radiative Torques?

Radiative torques resulting from outward-directed radiation provide one possible mechanism for grain alignment. Starlight torques have been found to be very important for both spin-up and alignment of interstellar grains (Draine & Weingartner 1996, 1997; Weingartner & Draine 2003; Lazarian & Hoang 2007a). With millimeter-sized grains, both stellar radiation and infrared emission from the disk may be capable of exerting systematic torques large enough to affect the spin of the grain. However, the radiation pressure $\sim L_* / 4\pi R^2 c \approx 5 \times 10^{-9} (L_* / L_\odot) (100 \text{ au}/R)^2 \text{ erg cm}^{-3}$ is small compared with the gas pressure $\sim 8 \times 10^{-3} (m/10^{10} \text{ cm}) (T/100 \text{ K}) \text{ erg cm}^{-3}$. If the grain streaming velocity exceeds $\sim 10^{-4} c_s$, where $c_s$ is the sound speed, systematic torques exerted by gas atoms may dominate radiative torques. Studies of realistic grain geometries are needed to clarify the relative importance of gaseous and radiative torques.

5.1.2. Alignment by Grain Drift?

The differential motion of dust and gas in three-dimensional disks has been discussed by Takeuchi & Lin (2002). Grains well above or below the midplane will sediment toward the midplane, with $v_{\text{drift}} \parallel \hat{z}_{\text{disk}}$, where $z_{\text{disk}}$ is height above the midplane. Dust grains close to the midplane will be in near-Keplerian orbits, but will experience a “headwind,” with $v_{\text{drift}} \parallel \hat{\phi}$. Vertical and azimuthal drift velocities will in general differ, with different dependences on grain size and radial distance from the protostar.

Gold (1952) proposed grain drift relative to the gas as an alignment mechanism. For hypersonic motion, Gold concluded that needle-shaped particles would tend to align with their short axes perpendicular to $v_{\text{drift}}$; Purcell (1969) analyzed spheroidally shaped grains, finding that significant alignment requires hypersonic gas-grain velocities if the grains are treated as rigid bodies. The degree of grain alignment of spheroidal grains is increased when dissipative processes within the grain are included (Lazarian 1994), but the degree of alignment is small unless the streaming is supersonic.

Lazarian & Hoang (2007b) discussed mechanical alignment of subsonically drifting grains with “helicity,” arguing that helical grains would preferentially acquire angular momentum parallel or antiparallel to $v_{\text{drift}}$; internal dissipation would then cause the short axis to tend to be parallel to $v_{\text{drift}}$. Lazarian & Hoang (2007b) based their analysis on a simple geometric model of a spheroidal grain with a single projecting panel. More realistic irregular geometries have been considered by Das & Weingartner (2016) and Hoang et al. (2018). However, these studies all assumed Larmor precession to be rapid compared with the gas-drag time, and are therefore not directly applicable to protoplanetary disks.

It appears possible that, averaged over the ensemble of irregular grain shapes, the net effect of gas-grain streaming in protoplanetary disks may be (1) suprathermal angular momenta tending to be perpendicular to $v_{\text{drift}}$, and (2) the tendency of grains to align with short axes perpendicular to $v_{\text{drift}}$. Below, we consider the consequences of this conjecture.

5.2. The HL Tau Disk as an Example

ALMA has observed a number of protoplanetary disks (e.g., Andrews et al. 2018). HL Tau remains one of the best-observed cases: it is nearby ($\sim$140 pc), bright, and moderately inclined...
The optical depth in the disk is large, with beam-averaged $\tau(3.1 \text{ mm}) \approx 0.13$ at $R \approx 100 \text{ au}$. Given that the dust is visibly concentrated in rings, and the possibility that there may be additional unresolved substructure, the actual optical depth of the emitting regions at 100 au is likely to be larger.

The polarization in HL Tau has been mapped by ALMA at 870 $\mu$m, 1.3, and 3.1 mm (Kataoka et al. 2017; Stephens et al. 2017). The observed polarization patterns show considerable variation from one frequency to another, complicating interpretation. Both intrinsic polarization from aligned grains and polarization resulting from scattering appear to be contributing to the overall polarization. Yang et al. (2019) and Mori & Kataoka (2021) argue that polarized emission makes a significant contribution to the polarization, at least at 3.1 mm.

The 3.1 mm polarization pattern is generally azimuthal (Stephens et al. 2017). If due to polarized emission, this would require that the radiating dust grains have short axes preferentially oriented in the radial direction. The alignment mechanism is unclear.

Kataoka et al. (2019) favor radiative torques, with the grain’s short axis assumed to be parallel to the radiative flux, in the radial direction. This would be consistent with the observation that the linear polarization tends to be in the azimuthal direction. If radiative torques are responsible for grain alignment in protoplanetary disks, then we do not expect the thermal emission from the disk to be circularly polarized, because the grains in the upper and lower layers of the disk will tend to have the same alignment direction as the grains near the midplane. If there is no change in the direction of the grain alignment along the optical path, there will be no circular polarization.

Here we instead suppose that grain alignment is dominated by gas-grain streaming due to systematic motion of the dust grains relative to the local gas. If we define $\vec{b} \parallel \vec{v}_{\text{drift}}$, we can apply the discussion above. As discussed above, we conjecture that the irregular grains align with short axes tending to be perpendicular to $\vec{v}_{\text{drift}}$, thus $f_{\text{align}} < 0$.

As before, let $\gamma$ be the angle between the line of sight and $\vec{b}$, and let $\Psi$ be the angle (relative to north) of the projection of $\vec{b}$ on the plane of the sky. For illustration, we take the disk to have the major axis in the E–W direction (see Figure 5), with inclination $i$. Thus vertical drifts correspond to $\Psi = 0$ and $180^\circ$. The treatment of radiative transfer developed above for magnetized clouds can be reapplied to protoplanetary disks—the only difference is that if the grains align with their short axis tending to be perpendicular to $\vec{v}_{\text{drift}}$ then $f_{\text{align}} < 0$, implying $\Delta \sigma < 0$ and $\Delta \epsilon < 0$.

The direction and magnitude of $\vec{v}_{\text{drift}}$ will vary with height in the disk. $\vec{v}_{\text{drift}}$ may be approximately normal to the disk plane for grains that are falling toward the midplane, whereas $\vec{v}_{\text{drift}}$ will be azimuthal for grains near the midplane, with Keplerian rotation causing them to move faster than the pressure-supported gas disk. Thus, grain orientations may vary both vertically and azimuthally. With $\Psi$ varying along a ray, the emerging radiation may be partially circularly polarized.

The observed linear polarization of a few percent suggests that $|\Delta \sigma/\delta| \approx 0.2 - 1.0$, which is consistent with the analytic approximation (Equations (28)–(30)) also being plotted. The analytic approximation is seen to provide fair accuracy, even though $\tau_2 = 0.2$ is not small.

The circular polarization $V/I$ is quite accurate, but in Figure 5(c), the analytic approximation slightly overestimates the linear polarization fraction. However, the analytic approximations were developed for $\tau < 1$, and here the total optical depth $\tau_1 + \tau_2 + \tau_3 = 0.3$ is not small.

For this model, the linear polarization varies from 1.4% to 3.2% around the disk, with average value $\sim 2.5\%$. The linear polarization tends to be close to the azimuthal direction, with largest values on the major axis, and smallest values along the minor axis of inclined disk (see Figure 5).

The predicted circular polarization $|V/I|$ is small but perhaps detectable, with $V/I$ varying from positive to negative from one quadrant to another (see Figure 5), with maxima $|V/I| \approx 0.2\%$ (see Figure 5(d)).

Stephens et al. (2017) mapped $V$ over the HL Tau disk at 3.3 mm, 1.3 mm, and 870 $\mu$m. The 3.3 mm $V$ map does not appear to show any statistically significant detection, with upper limits $|V/I| \lesssim 1\%$. At 1.3 mm and 870 $\mu$m the NW side of the major
axis may have $V/I \approx -1\%$, but whether this is real rather than an instrumental artifact remains unclear. In any event, the likely importance of scattering at these shorter wavelengths will complicate interpretation.

6. Discussion

For typical molecular clouds we conclude that the circular polarization will be undetectably small at the far-infrared and submillimeter wavelengths where the clouds radiate strongly. Probing the magnetic field structure in such clouds using circular polarization is feasible only at shorter infrared wavelengths where the extinction is appreciable, using embedded infrared sources (stars, protostars, or PDRs).

The thermal dust emission from so-called IRDCs in the inner Galaxy—such as the Brick—can show appreciable levels of linear polarization, demonstrating both that there is appreciable grain alignment and that the magnetic field structure in the cloud, while showing evidence of rotation, is relatively coherent. IRDCs have large enough column densities that the resulting circular polarization may reach detectable levels. For one position on the Brick and plausible assumptions concerning the field, we estimate a circular polarization $|V/I| \approx 0.025\%$ at 850 GHz. If the circular polarization can be detected and mapped in IRDCs, it would provide constraints on the three-dimensional magnetic field structure. Unfortunately, the predicted $V/I$ is small, especially at longer wavelengths (we expect $V/I \propto \lambda^{-1.1}$), and detection will be challenging.

Protoplanetary disks may offer the best opportunity to measure circular polarization at submillimeter wavelengths. If there are significant changes in the direction of grain alignment between the dust near the midplane and dust well above and below the midplane, linear dichroism and birefringence will produce circular polarization. Alignment processes in protoplanetary disks remain uncertain, but we suggest that grain drift may cause the grains near the midplane to be aligned with long axes preferentially in the azimuthal direction, while grains

![Figure 5.](image-url)
above and below the midplane may be aligned with long axes
tending to be in the vertical direction (normal to the disk). If the
grains are small enough that scattering can be neglected, we
calculate the linear and circular polarization that would be
expected for such a model. A characteristic quadrupole pattern
of circular polarization is predicted for this kind of grain
alignment (see Figure 5). Equation (30) can be used to estimate
the circular polarization at wavelengths $\lambda \gtrsim 100 \mu m$ where
thermal emission is strong and the grains may be approximated
by the Rayleigh limit.

We present a simple example to show the linear and circular
polarization that might be present in protoplanetary disks, such
as the disk around HL Tau. This example is not being put
forward as a realistic model for HL Tau, but simply to illustrate
the possible circular polarization from dust aligned by
streaming in a stratified disk. If observed, this would help
clarify the physical processes responsible for grain alignment
in protoplanetary disks. Absence of this circular polarization
would indicate that the preferred direction for grain alignment
in high-altitude regions is the same as the preferred direction
near the midplane, or else that grain alignment occurs only in
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
the midplane, or only in the upper layers. If circular
polarization is detected and mapped in a protoplanetary disk,
\[ Q_1 = -\left( \frac{\Delta \sigma}{\delta} \right)_1 \cos(2\Psi_1) I_1 \]  
(B3)

\[ U_1 = -\left( \frac{\Delta \sigma}{\delta} \right)_1 \sin(2\Psi_1) I_1 \]  
(B4)

\[ V_1 = 0 \]  
(B5)

\[ I_2 \approx I_1 e^{-\tau_2} + B(T_d2)(1 - e^{-\tau_2}) \]  
(B6)

\[ Q_2 \approx Q_1(1 - \tau_2) + A\tau_2(1 - \tau_2) \frac{\sin(2\Psi_2) - \sin(2\Psi_1)}{2\Delta\Psi} \]  
(B7)

\[ U_2 \approx U_1(1 - \tau_2) + A\tau_2(1 - \tau_2) \frac{\cos(2\Psi_2) - \sin(2\Psi_1)}{2\Delta\Psi} \]  
(B8)

\[
V_2 \approx \left( \frac{\Delta \epsilon}{\delta} \right)_2 Q_1 \left[ \tau_2 \frac{\cos 2\Psi_2 - \cos 2\Psi_1}{2\Delta\Psi} - \frac{\tau_2}{2} \frac{\sin 2\Psi_2 - \sin 2\Psi_1}{2\Delta\Psi} \right]
- \left( \frac{\Delta \epsilon}{\delta} \right)_2 A \left[ \tau_2 \frac{1 - \tau_2}{2\Delta\Psi} \right] \left( \frac{\tau_2}{2} \right) \right]
+ \left( \frac{\Delta \epsilon}{\delta} \right)_2 A \frac{\tau_2}{2\Delta\Psi} \sin 2\Delta\Psi
+ \left( \frac{\Delta \epsilon}{\delta} \right)_2 A \frac{\tau_2}{2\Delta\Psi} \left( 1 - \cos 2\Delta\Psi \right)
- 2\Delta\Psi \sin 2\Delta\Psi \right] + O(\tau_2^2). \]  
(B9)

**Appendix C**

**Three Zone Model**

Suppose the dust is located in three zones, with dust temperatures \( T_{d1}, T_{d2}, \) and \( T_{d3} \). The aligned dust grains have \( \Psi = \Psi_1 \) for \( 0 < \tau < \tau_1 \), \( \Psi = \Psi_2 \) for \( \tau_1 < \tau < \tau_1 + \tau_2 \), and \( \Psi = \Psi_3 \) for \( \tau_1 + \tau_2 < \tau < \tau_1 + \tau_2 + \tau_3 \). Suppose all \( \tau_j < 1 \).

Define

\[ I_1 \equiv B(T_{d1})[1 - e^{-\tau}]e^{-\tau_2} \approx B(T_{d1})r_1 \left[ 1 - \frac{1}{2} \tau_1 \right] e^{-\tau_1} \]  
(C1)

\[ I_2 \equiv B(T_{d2})[1 - e^{-\tau}]e^{-\tau_2} \approx B(T_{d2})r_2 \left[ 1 - \frac{1}{2} \tau_2 \right] e^{-\tau_2} \]  
(C2)

\[ I_3 \equiv B(T_{d3})[1 - e^{-\tau}] \approx B(T_{d3})r_3 \left[ 1 - \frac{1}{2} \tau_3 \right] \]  
(C3)

If \( S = (0, 0, 0, 0) \) for \( \tau = 0 \), then the radiation emerging from layer 3 has

\[ I \approx I_1 + I_2 + I_3 \]  
(C4)

\[ Q \approx -\left( \frac{\Delta \sigma}{\delta} \right)_1 \cos(2\Psi_1) I_1 - \left( \frac{\Delta \sigma}{\delta} \right)_2 \cos(2\Psi_2)[I_2 - \tau_3(I_1 + I_2)] \]  
(C5)

\[ U \approx -\left( \frac{\Delta \sigma}{\delta} \right)_1 \sin(2\Psi_1) I_1 - \left( \frac{\Delta \sigma}{\delta} \right)_2 \sin(2\Psi_2)[I_2 - \tau_3(I_1 + I_2)] \]  
(C6)

\[ V \approx \left( \frac{\Delta \epsilon}{\delta} \right)_1 \left( \frac{\Delta \sigma}{\delta} \right)_1 \sin(2\Psi_2 - 2\Psi_1) \tau_3 I_1 \]  
(C7)

**ORCID iDs**

B. T. Draine @ https://orcid.org/0000-0002-0846-936X

**References**

Aitken, D. K., Hough, J. H., & Chrysostomou, A. 2006, MNRAS, 366, 491

Andersson, B.-G., Lazarian, A., & Vaillancourt, J. E. 2015, ARA&A, 53, 501

Andrews, S. M., Huang, J., Pérez, L. M., et al. 2018, ApJL, 809, L41

Beckwith, S. V. W., & Sargent, A. I. 1991, ApJ, 381, 250

Bohren, C. F., & Huffman, D. R. 1983, Absorption and Scattering of Light by Small Particles (New York: Wiley)

Carey, S. J., Clark, F. O., Egan, M. P., et al. 1998, ApJ, 508, 721

Chuss, D. T., Andersson, B. G., Bally, J., et al. 2019, ApJ, 872, 187

Das, I., & Weingartner, J. C. 2016, MNRAS, 457, 1958

Davis, L. J., & Greenstein, J. L. 1951, ApJ, 114, 206

Dotson, J. L., Vaillancourt, J. E., Kirby, L., et al. 2010, ApJS, 186, 406

Draine, B. T. 2006, ApJ, 636, 1114

Draine, B. T., & Hensley, B. S. 2021a, ApJ, 919, 65

Draine, B. T., & Hensley, B. S. 2021b, ApJ, 909, 94

Draine, B. T., & Lee, H. M. 1984, ApJ, 285, 89

Draine, B. T., & Weingartner, J. C. 1996, ApJL, 470, 551

Draine, B. T., & Weingartner, J. C. 1997, ApJ, 480, 633

Dyck, H. M., & Lonsdale, C. J. 1981, in Infrared Astronomy; Proc. of the IAU Symp. 96 (Dordrecht: Reidel Publishing), 223

Fissel, L. M., Ade, P. A. R., Anglè, F. E., et al. 2016, ApJ, 824, 134

Fukushima, H., Yajima, H., & Umemura, M. 2020, MNRAS, 496, 2762

Gold, T. 1952, MNRAS, 112, 215

Guan, Y., Clark, S. E., Hensley, B. S., et al. 2021, ApJ, 920, 6

Hall, J. S. 1949, Sci, 109, 166

Hamaker, J. P., & Bregman, J. D. 1996, A&AS, 117, 161

Hensley, B. S., & Draine, B. T. 2021, ApJ, 906, 73

Hiltner, W. A. 1949, Natur, 163, 283

Hoang, T., Cho, J., & Lazarian, A. 2018, ApJL, 852, 129

Kataoka, A., Muto, T., Moronese, M., et al. 2015, ApJ, 809, 78

Kataoka, A., Okuzumi, S., & Tazaki, R. 2019, ApJL, 874, L6

Kataoka, A., Tsukagoshi, T., Pohl, A., et al. 2017, ApJL, 844, L5

Kemp, J. C. 1972, ApJL, 175, L35

Kemp, J. C., & Wolfstencroft, R. D. 1972, ApJL, 176, L115

Kwon, J., Nakagawa, T., Tamura, M., et al. 2018, ApJ, 856, 1

Kwon, J., Tamura, M., Hough, J. H., et al. 2014, ApJL, 795, L16

Kwon, J., Tamura, M., Hough, J. H., Nagata, T., & Kusakabe, N. 2016, ApJ, 152, 67

Lazarian, A. 1994, MNRAS, 268, 713

Lazarian, A., & Hoang, T. 2007a, MNRAS, 378, 910

Lazarian, A., & Hoang, T. 2007b, ApJL, 669, L71

Lee, C.-F., Li, Z.-Y., Yang, H., et al. 2021, ApJ, 910, 75

10
Lee, H. M., & Draine, B. T. 1985, ApJ, 290, 211
Longmore, S. N., Rathborne, J., Bastian, N., et al. 2012, ApJ, 746, 117
Lonsdale, C. J., Dyck, H. M., Capps, R. W., & Wolk, 1980, ApJL, 238, L31
Marsh, K. A., Ragan, S. E., Whitworth, A. P., & Clark, P. C. 2016, MNRAS, 461, L16
Martin, P. G. 1972, MNRAS, 159, 179
Martin, P. G. 1974, ApJ, 187, 461
Martin, P. G., & Angel, J. R. P. 1976, ApJ, 207, 126
Martin, P. G., & Campbell, B. 1976, ApJ, 208, 727
Martin, P. G., Illing, R., & Angel, J. R. P. 1972, MNRAS, 159, 191
Molinari, S., Schisano, E., Elia, D., et al. 2016, A&A, 591, A149
Mori, T., & Kataoka, A. 2021, ApJ, 908, 153
Natta, A., & Testi, L. 2004, in ASP Conf. Ser. 323, Star Formation in the Interstellar Medium: In Honor of David Hollenbach, ed. D. Johnstone et al. (San Francisco, CA: ASP), 279
Okuzumi, S., & Tazaki, R. 2019, ApJ, 878, 132
Reissl, S., Wolf, S., & Brauer, R. 2016, A&A, 593, A87
Serkowski, K. 1962, AdA&A, 1, 289
Serkowski, K., & Rieke, G. H. 1973, ApJL, 183, L103
Stephens, I. W., Yang, H., Li, Z.-Y., et al. 2017, ApJ, 851, 55
Takeuchi, T., & Lin, D. N. C. 2002, ApJ, 581, 1344
van de Hulst, H. C. 1957, Light Scattering by Small Particles (New York: John Wiley & Sons)
Walker, D. L., Longmore, S. N., Bally, J., et al. 2021, MNRAS, 503, 77
Weingartner, J. C., & Draine, B. T. 2003, ApJ, 589, 289
Yang, H. 2021, ApJ, 911, 125
Yang, H., Li, Z.-Y., Stephens, I. W., Kataoka, A., & Looney, L. 2019, MNRAS, 483, 2371
Planck Collaboration, Ade, P. A. R., Aghanim, N., et al. 2015a, A&A, 576, A104
Planck Collaboration, Ade, P. A. R., Aghanim, N., et al. 2015b, A&A, 576, A106
Planck Collaboration, Aghanim, N., & Akrami, Y. 2020, A&A, 641, A12
Purcell, E. M. 1969, Phy, 41, 100
Reissl, S., Wolf, S., & Brauer, R. 2016, A&A, 593, A87
Takeuchi, T., & Lin, D. N. C. 2002, ApJ, 581, 1344
van de Hulst, H. C. 1957, Light Scattering by Small Particles (New York: John Wiley & Sons)
Walker, D. L., Longmore, S. N., Bally, J., et al. 2021, MNRAS, 503, 77
Weingartner, J. C., & Draine, B. T. 2003, ApJ, 589, 289
Yang, H. 2021, ApJ, 911, 125
Yang, H., Li, Z.-Y., Stephens, I. W., Kataoka, A., & Looney, L. 2019, MNRAS, 483, 2371