1. INTRODUCTION

Lattice boom mobile cranes typically have a hoisting capacity of more than 1000 metric tonnes and a maximum hoisting radius of around 200 meters. Mobile cranes are set up with long, elastic lattice booms and acute-angled suspensions. Their fundamental motions are hoisting, slewing and luffing. The slender boom system has become more and more complex due to increasing load capacity.

Each working process causes dynamic forces on the supporting structure of a crane. In order to avoid security risks, the exact calculation of the boom system is an important task in the development of cranes. One main objective of the calculation is to reproduce the system’s realistic dynamic behaviour. According to current standards [3-5, 8, 9], stress calculations for mobile cranes are carried out using static approaches. The dynamic effects are considered in the calculation by means of special dynamic factors. The factors are often based on the experience of the crane manufacturers or, alternatively, they can be taken from tables in the standards. The cranes’ dynamic behaviour is analysed in several publications [10-12, 16, 21] and some previous articles have shown, that the calculation standards often describe the dynamic effects only approximately for the motions of slewing, luffing and hoisting grounded loads [2, 12, 15, 17]. For this reason, the standards also allow the use of other methods to calculate dynamic effects. One way to characterize the dynamic behaviour of cranes very accurately is to use the nonlinear dynamic finite element calculation. The main disadvantage of this method is the much higher computing time needed compared to a static calculation. Furthermore, there is no reasonable way to take the standards’ partial safety factors into consideration. These are the two main reasons why the dynamic analysis is very rarely used by crane manufacturers, even though it allows exact and reliable calculations. In order to achieve a more exact calculation of the dynamic behaviour of cranes without a disproportionate increase in computing time, special vibration models have been developed in a current research project. This paper presents a new vibration model for the process of hoisting suspended loads. The model is based on the response spectrum method and describes the dynamic effects in an exact way.

Keywords: mobile crane, dynamic calculation, finite element method, modal reduction, hoisting, vibration model.

2. BASICS

The stress calculation for cranes follows the rules of the European standard DIN EN 13001 [4, 5] or international standard ISO 8686 [8, 9]. The European standard refers to the standard DIN EN 13000 [3] for calculating mobile cranes. In the standards, the approach for considering dynamic loads is based on a rigid body kinetic analysis and uses quasi-static calculation methods. A previous project has shown that these methods are often inappropriate to reproduce the dynamic effects on mobile cranes. However, the standard ISO 8686 expressly permits the use of more advanced methods (calculations or tests) to evaluate the effects of loads and load
combinations, and the values of dynamic load factors, where it can be demonstrated that these provide at least equivalent levels of competence’ [8]. The European standards DIN EN 13001 and DIN EN 13000 contain similar remarks concerning other possible calculation methods. The nonlinear dynamic finite element calculation and the vibration model presented here are two of the aforementioned advanced methods.

In order to compensate uncertainties, the standards prescribe the use of partial safety factors. However, this paper does not consider any safety factors because only the accuracy of the loads generated with the vibration model and the standards is verified. The objective of the current research project is to depict the dynamic effects on cranes in an exact way and not to assess the quality of the safety factors. Nevertheless, partial safety factors can be considered in the vibration model presented here in the same way as stipulated in the calculation standard. The following sections describe the calculation standard and the basics of the newly developed vibration model.

2.1 DIN EN 13000

The standard DIN EN 13000 is currently used by crane manufacturers in Europe to do the stress calculation of mobile cranes. This standard refers to the guideline FEM 5.004 [6] for calculating loads and load combinations on the supporting structure. Unlike DIN EN 13001, it does not distinguish between hoisting grounded loads and hoisting suspended loads. In order to take the dynamic effects into consideration, a factor \( \varPhi \) is proposed for the process of hoisting suspended loads. This factor is dependent on the hoisting speed \( v_h \) and is calculated with the equation

\[
\varPhi = 1.1 + 0.133v_h.
\]

The minimum permissible value of \( \varPhi \) is 1.1, the maximum value is 1.3. For the proof of security, the weight force of the payload is multiplied by this factor and used in a static calculation.

2.2 Vibration model

Some publications have shown, that the dynamic effects during the process of slewing and hoisting grounded loads can be depicted by the static approach with great precision if suitable quasi-static loads are applied [13, 14, 20]. The following paragraph contains a new calculation method for the process of hoisting suspended loads. This vibration model offers a more exact method to generate quasi-static loads than the regulation proposed by the standard. Consequently, the dynamic effects can be described with greater accuracy in a static calculation.

The model is based on the response spectrum method. It uses a linearized approach of the nonlinear equation of motion and the method of modal reduction. This approach is very similar to the calculation method described in [13, 20] for the process of slewing.

To reproduce the cranes’ dynamic behaviour, the vibration model has to replace the equation of motion

\[
M \ddot{u} + D \dot{u} + K(t)u = r.
\]

In Eq. (2) \( M, D \) and \( K(t) \) are the mass, damping and stiffness matrices. The vector \( r \) contains the externally applied loads and \( u, \dot{u}, \ddot{u} \) are the vectors of displacement, velocity and acceleration [1]. To take the damping into consideration, we assume Rayleigh damping of the type

\[
D = \alpha M + \beta K \ (\alpha, \beta \in \mathbb{R}).
\]

The vector \( r \) in (2) only contains the inertia force \( m_p a_p \) acting on the payload, where \( m_p \) is the payload’s mass and \( a_p \) is the acceleration of the load. The external forces on all other degrees of freedom are zero. In a first step the equation is linearized so that the stiffness matrix is only calculated in the initial state. The model is consequently based on the assumption that \( K(t) \) remains constant throughout the whole working process.

The matrices \( M, D \) and \( K(0) \) are symmetric and positive definite and thus can be diagonalised with the eigenvectors of the autonomous conservative system. A modal transformation into the modal coordinates \( q \) is performed using the equation

\[
u = \phi q,
\]

where \( \phi \) is the modal matrix containing the mass-normated eigenvectors as columns. This modal transformation results in the mass-normated equation of motion

\[
\ddot{q} + D_{mod} \dot{q} + \Omega q = \phi^T r.
\]

In this equation the matrix \( \Omega = \phi^T K \phi \) contains the squares of the eigenfrequencies and \( D_{mod} = \phi^T D \phi \) is the matrix of modal damping. As the matrices \( \Omega \) and \( D_{mod} \) are diagonal, the differential equations in (5) are decoupled and can be solved analytically. Consequently, there is no need for any numerical method to solve these equations of motion so that the proposed method needs only slightly more computing time than the calculation methods proposed by the standards.

The cranes’ vibrations caused by the working process contain only few eigenfrequencies. This is why the method of modal reduction can be used as a very exact approximation. With this method, only a certain number of \( n \) equations is considered in (5). Consequently, even less time is needed for the calculation.

In a next step, an approximate solution of (2) is calculated using the analytical solution of (5). The solution of the physical displacements related to the \( m \)-th modal coordinate is computed using the equation

\[
u_m(t) = \varphi_m q_m(t),
\]

which contains the eigenvector \( \varphi_m \) and the modal displacement \( q_m \). The superposition

\[
u(t) = \sum_{m=1}^{n} \varphi_m q_m(t)
\]

results in an approximate solution of \( u(t) \). This solution describes the dynamic displacements of every degree of freedom of the finite element model.

The final stress calculation should still be based on the results of static calculations, as specified in the standards. For this reason, appropriate quasi-static loads
have to be generated that reflect the worst state of elastic deformation in the same way as the dynamic calculation. The nodal forces related to the $m$-th modal coordinate $R_{Q,m}$ are calculated with the equation

$$R_{Q,m} = M \phi \ddot{q}_m, \quad (8)$$

which contains the maximum value $\ddot{q}_{m,\text{max}}$ of the modal acceleration $\ddot{q}_m(t)$.

The quasi-static load for every degree of freedom $i$ is generated with the equation

$$R'_{Q,i} = \sqrt{\sum_{m=1}^{n} R_{Q,m}^2} \cdot (9)$$

This method to generate quasi-static loads is also used in a similar way in the field of civil engineering to calculate the effect of earthquakes on high-rise buildings [18]. In a static calculation these loads depict the worst state of elastic deformation in a much more accurate way than the loads based on the calculation standard.

3. NUMERICAL ANALYSIS

The nonlinear dynamic finite element calculation is the most realistic calculation method for cranes. For this reason, the results of the vibration model and the calculation standard are evaluated by comparing them with the results obtained with the dynamic calculation method. To verify the accuracy of the model, the comparison takes various accelerations and velocities of the hoisting drive into account. The following sections describe the modelling of the analysed cranes and the calculation results.

3.1 Modelling

The analysis of the dynamic behaviour is based on two cranes with a maximum hoisting capacity of around 500 and 1000 metric tonnes. In order to investigate the applicability of the vibration model for different lattice boom systems, the evaluation comprises different complexities of the boom system and various crane set-ups (see Figure 1). Furthermore, different hoisting radii of every crane configuration are analysed.

The method of finite tower elements is applied to model the lattice boom structure [7]. This modelling method replaces each lattice boom component by a beam element with equivalent stiffness and mass. The payload’s position is chosen close to the ground. In the

numerical investigations no damping is considered because the objective of the project is not to analyse the influence of damping on the supporting structure. However it would be possible to consider the damping of the structure as presented above (3).

The finite element program NODYA is used to carry out the calculations. This software was developed for the dynamic and static calculation of lattice boom mobile cranes and provides special elements for the crane calculation such as a rope element [12].

The calculation comprises the phases of acceleration and constant velocity as shown in Figure 2. At the beginning, there is a linear increase in the velocity of the payload followed by the phase of constant velocity. In the calculations three different velocities (0.7 rad/s, 1.4 rad/s, 2.1 rad/s) and three different accelerations (0.42 rad/s², 0.56 rad/s², 0.7 rad/s²) of the hoisting drum are considered. Furthermore, the investigation comprises different values for the rope reeving, resulting in different hoisting parameters of the payload. A simulation time of 60s was chosen to detect the maximum modal acceleration. Numerical investigations have shown that a number $n = 10$ of modal coordinates is sufficient for the calculation of all analysed boom systems.

![Figure 2. Variation of acceleration and velocity over time](image)

3.2 Results

All of the comparisons collate the results obtained using the vibration model and the dynamic finite element analysis.

In a first step, the results of the vertical displacements of different nodes are considered. Figure 3 shows the time course of the vertical displacement of the payload for a crane of set-up d) depicted in Figure 1 by way of example. The considered crane configuration M72D42L72 (72 meters main boom, 42 meters derrick boom and 72 meters luffing boom) was analysed with a hoisting radius of 120 meters. In the upper diagram, the displacement result obtained by the vibration model after superposition (7) of all considered modes is compared with the results of the dynamic finite element calculation. The result of the vibration model duplicates the results of the nonlinear dynamic finite element method in a good approximation. Both the amplitudes and the frequencies of the two signals are almost exactly matched. The second diagram of Figure 3 shows all 10 modal components $u_1 \text{ to } u_{10}$ of the physical displacement solution (6). It becomes very clear that the vibration is influenced mainly by one modal coordinate. The

Figure 1. Analysed crane configurations: a) M-configuration (only main boom), b) MD-configuration (main and derrick boom), c) ML-configuration (main and luffing boom), d) MDL-configuration (main, derrick and luffing boom)
Figure 3. Displacements of the payload (M72D42L72) top: dynamic calculation and vibration model, bottom: displacements of every modal coordinate in physical coordinates

Figure 4. Variation of stresses over time, comparison of static calculation methods and nonlinear dynamic finite element calculation (M60L87, hoisting radius: 76 m)

easily be seen that the static approach in the calculation standard leads to a much too conservative approximation of the dynamic stress. This diagram clearly shows that the quasi-static loads generated with the vibration model can reproduce the dynamic effects of the crane in a very accurate way. However, the vibration model’s results are also slightly conservative and consequently still enable a safe sizing of cranes.

In Figures 5 and 6, the results of the crane component’s utilization is shown for the different considered calculation methods. The utilization is defined as the ratio of the compressive stress and the limit of the compressive design stress. Figure 5 shows the utilization of the different components for the considered crane configuration M90D36. In the diagram shown here, the maximum utilization is located in the derrick boom. The vibration model can accurately reproduce the results of the dynamic finite element calculation in every lattice boom section. Conversely, the calculation standard cannot describe all components of the crane with the same precision. The standard’s approach can depict the low level utilization in a good approximation whereas the utilization of the derrick boom is overestimated.

The overview in Figure 6 shows a comparison of the higher lifting capacity crane in different crane configurations. Each bar of the chart depicts the lattice boom section with maximum utilization in the analysed crane configuration. This diagram shows the results for two
analysed hoisting radii respectively. The purpose of this diagram is to compare the outcome of the calculation methods relating to different complexities of the boom system. The results presented in this figure confirm those of the other charts. In almost every crane configuration, the utilization calculated using the approach in the calculation standard leads to higher values than if the nonlinear dynamic finite element calculation is used. A dependency of the inaccuracy on any crane configuration is not apparent. Contrary to the calculation standard, the vibration model follows the results of the nonlinear dynamic finite element calculation very accurately. Most of the results are only slightly too conservative or even equal to these results. This diagram shows clearly that the linearized approach of the vibration model is applicable to different boom systems of mobile cranes.

4. CONCLUSION

This paper presents a vibration model that reproduces the dynamic behaviour of lattice boom mobile cranes during the process of hoisting suspended loads. The model enables an effective analysis of the dynamic behaviour of boom systems with any kind of configuration. Thanks to linearized formulations and the method of modal reduction, the computing time decreases compared to the dynamic finite element calculation whilst maintaining a similar accuracy. Furthermore, it is possible to consider the partial safety factors stipulated in the calculation standards.

This paper compares the results of the calculation standard, the vibration model and the dynamic finite element calculation. The approach in the standard often leads to great inaccuracies compared to the dynamic finite element calculation. One important aspect is that the vibration model presented here reproduces the dynamic effects in a more exact manner than the methods commonly used by the standards. For this reason, the use of the new calculation method could lead to improvements in crane safety in future.

Measurements are currently being carried out to obtain further comparisons regarding the accuracy of the vibration model. In future, it will also be necessary to develop a model that depicts the dynamic effects of loader cranes during the process of hoisting suspended loads. The forces on the supporting structure of loader cranes during this motion are fundamentally different from those on mobile cranes. Another important objective is to consider the combination of the working processes hoisting and slewing in an additional model and to develop a further model which describes the dynamic effects during luffing motions of mobile cranes.

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ИЗРАЧУНАВАЊЕ ДИНАМИЧКОГ ПОНАШАЊА ПОКРЕТНИХ КРАНОВА СА РЕШЕТКАСТОМ СТРЕЛОМ ТОКОМ ПОДИЗАЊА ТЕРЕТА

М. Столцнер, М. Клебергер, В. Гинтер, Ј. Фотнер

Покретни кранови са решеткастом стрелом се највише користе за подизање тешких терета са великим радијусом кретања. Радом крана стварају се динамичке сили које делују на његову структуру. Како би се обезбедила сигурност крана потребно је извршити точан прорачун за систем стреле. Стандарди прорачуна укључују посебне динамичке факторе који се односе на динамичка оптерећења. Већи број истраживања је показало да стандарди често описују динамички утицаји само приближно. Да би се прецизно израчунало динамичко понашање, без повећања времена израчунавања, развијени су посебни вибрациони модели. Рад приказује нови вибрациони модел процеса подизања суспендованог терета. Модел се базира на методи спектра одговора и тачно описује динамичке ефekte.