Wushu Routine Movement and Diagnosis Based on Deep Learning and Symmetric Difference Algorithm

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Abstract

Wushu is one of the traditional cultural symbols of the Chinese nation. It is also one of the most popular sports activities among the people. With the attention and love of contemporary people to sports activities, Wushu is also constantly developing and innovating. The requirements for professional martial arts routines of martial arts athletes are higher than ever. The development of martial arts has also made martial arts competitions more intense, and often a small detail of martial arts movements can determine the success or failure of the competition. Therefore, various Wushu teams pay more and more attention to the analysis and diagnosis of Wushu routines. It ensures that coaches and athletes can obtain more quantitative indicators of technical movement training. The analysis and diagnosis of martial arts routines are inseparable from the support of reliable science and technology and related algorithms. This article aims to study the analysis and diagnosis of martial arts routines based on deep learning and symmetric difference algorithm. It combines deep learning and symmetric difference algorithm to analyze and diagnose martial arts routines. The article concludes that the level of martial arts routines of martial arts athletes has the greatest influence on their martial arts competition performance, and its comprehensive influence index is as high as 4.3.

1. Introduction

As one of China’s traditional cultural symbols and one of the favorite sports, martial arts has been developing continuously in recent years. With the continuous development of martial arts and the increasingly fierce competition of modern martial arts, higher requirements are also put forward for the martial arts routines and movements of martial arts contestants. Nowadays, the effort and cost of martial arts athletes who want to improve their performance in martial arts sports are getting higher and higher. Therefore, how to make martial arts athletes achieve excellent results in martial arts competitions through the usual scientific guidance and training has become an important topic in modern martial arts sports training that needs to be studied urgently. Therefore, there are more and more studies on the analysis and diagnosis of martial arts routines. This is because the analysis and diagnosis of Wushu routine movements can be combined with some scientific theories and methods to diagnose and analyze Wushu athletes’ routine movements in Wushu sports training. In this way, the problems existing in the martial arts movements of martial arts athletes can be discovered in time. It further improves the scientific advice and guidance for the martial arts routine movement training of the martial arts athletes, so as to improve the martial arts competition level and performance of the martial arts athletes. The scientific diagnosis and analysis of Wushu routine movements usually refer to the characteristics of Wushu routine movements. It uses computer real-time video feedback technology, computer motion capture technology, and computer-aided action analysis to analyze and evaluate Wushu routine movements. It uses modern scientific and technological means to assist in training and guiding evaluation of martial arts routines. Scientific diagnosis and analysis of martial arts routines can effectively improve the training effect of martial arts. When diagnosing and analyzing martial arts
routines, we should pay more attention to the scientific diagnosis and analysis. Especially when analyzing and diagnosing some difficult martial arts routines, it is very necessary to combine the use of some scientific and technological means. This makes the diagnosis and analysis of martial arts routines more scientific and intuitive. In this article, the analysis and diagnosis of Wushu routine movements are mainly based on deep learning and symmetric difference algorithm.

The innovations of this study are as follows: (1) It provides a detailed introduction to deep learning and symmetric difference algorithms. Based on deep learning and symmetric difference algorithm, the analysis and diagnosis of Wushu routine movements are carried out. (2) It combines deep learning and symmetric difference algorithm to analyze and diagnose martial arts routines. It also draws a conclusion that it is helpful to provide scientific guidance for the training of martial arts athletes and improve their martial arts competition performance.

2. Related Works

Since deep learning and symmetric difference algorithm are two scientific technologies and algorithms with high application value, there are many researches related to deep learning and symmetric difference algorithm in academia. Among them, Litjens et al. mainly studied the application of deep learning in the fields of image classification, object detection, segmentation, and registration, especially in the analysis of medical images [1]. Chen et al. mainly studied the application of deep learning in hyperspectral remote sensing. He proposed a deep learning framework for hierarchically extracting deep features from hyperspectral remote sensing [2]. Shen et al. detailed the fundamentals of deep learning in his research. He also analyzed the utilization of deep learning in medical imaging [3]. He et al. research focused on the application of deep learning in the field of channel estimation in mmWave communications. He proposed a progressive performance analysis framework for channel estimation based on deep learning [4]. Fan et al. mainly studied the application of symmetric difference algorithm to reduce the difficulty of feed network design and cut the dependence of sum and difference beam pattern synthesis on centrosymmetric antenna arrays. He proposed a new way to generate a usual weight vector to generate these two beam patterns simultaneously for symmetric and asymmetric antenna arrays [5]. Ai et al. studied the application of deep learning in solving the problem of short-term spatiotemporal distribution prediction of shared bicycle system states. He proposed a deep learning method called convolutional long short-term memory networks to address both spatial and temporal dependencies [6]. Although these studies can provide this article with information related to deep learning and symmetric difference algorithms, the process of these studies is relatively complicated and the practicality is not strong enough. It takes a lot of time and labor costs, and the practicability is not strong enough.

3. Wushu Routine Movements and Diagnostic Methods

3.1. Deep Learning. Deep learning is one of the branches of machine learning. It refers to learning the inherent laws and representation levels of sample data through artificial neural networks, so that machines can have the ability to analyze and learn like humans. It can recognize the ultimate goal of data such as text, images, and sounds [7, 8]. It can also be said that deep learning is a complex machine learning algorithm. It has more powerful functions and functions in speech and image recognition. The deep learning technology architecture is shown in Figure 1 [9].

As can be seen from Figure 1, in the deep learning technology architecture, the Internet and computer technology are two more important technologies. Deep learning has been widely used in data mining, machine learning, natural language processing, and speech and image recognition. Compared with the staged learning of traditional algorithms in computer vision, the end-to-end learning framework is a significant innovation of deep learning technology [10–12]. Under the framework of deep learning algorithm, it does not need to manually design features to perform feature extraction operations. It only needs to input data and output results through training. Deep learning can choose the number of layers of the network according to the needs of the task. In theory, it can be mapped to any function, so deep learning can solve many complex problems. Deep learning utilizes hierarchical learning of multi-layer neural networks, which implements a nonlinear mapping from input to output. At present, the application of deep learning in computer vision mainly focuses on three aspects: One is that the result of the middle layer or layers of the trained network is used as a feature extractor, which replaces the traditional feature extraction algorithm. The second is to add new layers or reduce some layers in the existing network, which is fine-tuned. The third is for a specific task, which builds a new network structure and runs it from scratch [13, 14]. The end-to-end deep learning framework is shown in Figure 2.

If we want to represent a complex function, if the complex function is directly represented by a single-layer network model, the number of parameters of the entire network is very huge. However, if it uses a 5-layer model to represent complex functions, each layer only needs to represent a simple function, then the final number of parameters may not be many. In the case of accomplishing the same expressive ability, the number of learning parameters is greatly reduced [15, 16]. The process of deep learning hierarchical representation of complex functions is shown in Figure 3.

All in all, whether it is from the human brain’s visual mechanism or its mathematical nature, the performance of deep-level features is far superior to that of shallow-level models. This makes the classification and recognition effect with this feature also very good. In this process, a deep model with multiple hidden layers is a method, and feature extraction is the purpose of deep learning. The deep learning network maps the features of the samples in the original
space to a new feature space through layer-by-layer feature transformation, which makes the classification or prediction more accurate. Compared with the traditional method of constructing features by artificial rules, the features learned using deep learning combined with big data can better describe the intrinsic information of the data [17].

3.2. Symmetric Difference Algorithm. The symmetric difference algorithm, also known as the three-frame method, is an algorithm for tracking and monitoring moving targets. It is a method of taking the intermediate frame as the symmetry when the object is moving fast. In order to divide the moving objects from the stationary background, we differentiate two adjacent source images. Due to the vital difference between picture frames, the shape contour of the moving target in the middle frame can be better detected by the "AND" of the symmetrical difference of three consecutive frames of images. The basic algorithm flow is described as follows: First, four time-domain feature definitions [20] are required, and the mean feature is defined as

\[
\mu_i = \frac{1}{N} \sum_{n=1}^{N} x_{i,n}.
\]  

The feature extraction process of the symmetric difference algorithm is as follows: First, four time-domain feature definitions [20] are required, and the mean feature is defined as

\[
\mu_i = \frac{1}{N} \sum_{n=1}^{N} x_{i,n}.
\]  

The principle of the symmetric difference algorithm is shown in Figure 4.
It defines the variance feature as
\[ \sigma_i^2 = \frac{1}{N} \sum_{n=1}^{N} (x_i - \mu_i)^2. \] (2)

The deviation feature is defined as
\[ \text{ske}_i = \frac{1}{N \sigma_i} \sum_{n=1}^{N} (x_i - \mu_i). \] (3)

Finally, the kurtosis feature is defined as
\[ \text{kur}_i = \frac{1}{N \sigma_i^2} \sum_{n=1}^{N} (x_i - \mu_i)^4, \] (4)
where \( N \) represents the length of the action sequence segmentation, \( x_i \) and \( n \) represent the value of the \( i \)th dimension feature in the feature vector at the \( n \)th time. After the FFT transformation is performed on the signal, the frequency domain features of the signal, namely FFT coefficients, frequency domain entropy and energy, etc., are extracted.

It defines the FFT coefficients as
\[ X_i(k) = \sum_{n=1}^{N} x_i \cdot e^{-2\pi j}. \] (5)

The frequency domain threshold is defined as
\[ H = -\sum_{k=1}^{N} X_i. \] (6)

It defines the energy value as
\[ E = \frac{1}{N} \sum_{k=1}^{N} x_i. \] (7)

During the calculation, the FFT coefficients need to be normalized first. Since the frequency of human actions is low in daily behavior, we select the first 10 FFT coefficients as features. To sum up, the above basic features have a total of 6-dimensional data. It then extracts 4-dimensional time-domain features and 12-dimensional frequency domain features from each dimension of the existing basic features [21] and concatenates these features together to form a 96-dimensional feature vector. In order to eliminate the influence of the dimension and value range of the features of different dimensions in the feature vector, it is necessary to normalize the features of different dimensions. It maps the features of each dimension between 0 and 1. This process is as follows:
\[ y_i = \frac{x_i - \min(f_i)}{\max(f_i)} \] (8)
where \( \min(f_i) \) and \( \max(f_i) \) represent the minimum and maximum values of the \( i \)th dimension feature, respectively, and \( x_i \) represents the value of the \( i \)th dimension feature. \( y_i \) indicates the value of the \( i \)th dimension feature after the normalization operation. Next, linear discriminant analysis is performed by projecting the sample data into a low-dimensional space with good class separability. This thus avoids over fitting and reduces computational complexity. Since the sample data are subjected to linear discriminant analysis, it maximizes the ratio of the interclass distance and the intraclass distance of the sample data in a low-dimensional space, thus improving the separability of the sample data [22]. The intraclass divergence \( S_{ii} \) and interclass
The divergence $S_B$ of action feature vectors are expressed as follows:

$$S_w = \sum_{i=1}^{C} \sum_{j=1}^{n} (x_{ij} - \mu_i).$$  

$$S_B = \sum_{i=1}^{C} (\mu_i, \mu).$$  

where the feature vector corresponding to the ith sample in the JTH action category in the action sample set $x^j$. Mean of all eigenvectors in the ith action class in $\mu_i$. According to the definition of linear discriminant analysis, the goal of linear discriminant analysis is to obtain the maximum value of the following formula by solving the projection vector $W$ [23].

$$J(W) = \frac{WS_BW}{W^T S_w W}.$$  

where

$$W^T S_w = 1.$$  

Figure 5: Flowchart of the symmetric difference algorithm.
And the Lagrangian function corresponding to the above formula is as follows:

\[ L_p = -\frac{1}{2} W^T S_w W + \frac{1}{2} \gamma S_w. \]  

(13)

According to the KKT condition, we can get

\[ S_w^{-1} = S_w W \gamma W. \]  

(14)

The number of eigenvalues selected can also be calculated by the contribution rate of the eigenvalues:

\[ \sum_{i=1}^{d} \gamma_i = \sum_{i=1}^{k} \gamma_i. \]  

(15)

where \( K \) represents the number of eigenvalues, \( \gamma_i \) represents the \( i \)th eigenvalue, and \( d \) represents the contribution rate of the feature. The eigenvalue \( \gamma \) and the eigenvector \( W \) satisfy the following relationship:

\[ \gamma S_w W = S_w W. \]  

(16)

The basic flow of the symmetric difference algorithm is shown in Figure 5.

4. Wushu Routine Movement Analysis and Diagnosis Experiment

4.1. Experimental Design and Main Steps. The experimental subjects selected for this movement analysis experiment on martial arts routines are the outstanding martial arts routine athletes and competitors of the Beijing team preparing for the 10th National Games. The Beijing team has a total of 9 members, including 6 male athletes and 3 female athletes. There are 4 competitors in total, including 1 male athlete from the Hunan team, 1 male athlete from the Shaanxi team, 1 male athlete from the Shanghai team, and 1 female athlete from the Jiangsu team. It combines deep learning and symmetric difference algorithm to analyze and diagnose multiple martial arts routines of experimental subjects. It uses a connection of horizontal shooting and overhead shooting to analyze the athletes on-site (Figure 6).

4.2. Mathematical Analysis on the Scores of Women and Men in the Preliminaries of Martial Arts Routines. The 10th National Games martial arts routines are set to 6 items of self-selected Changquan, self-selected Nanquan and Nandao all-around, self-selected knife and stick all-around, self-selected sword and gun all-around, self-selected Taijiquan and sword all-around, and pair practice. According to the participating events of the female athletes of the Beijing team, this experiment only conducts statistical analysis on the four items of self-selected Changquan, self-selected Nanquan and Nandao all-around, self-selected knife and stick all-around, and self-selected Taijiquan and sword all-around. The results are shown in Table 1.

If the arrangement of the Wushu routine medals of the 10th National Games is ignored, the technical scores of the top 12 athletes in each small event will be unilaterally viewed. From Table 1, it is not difficult to find that in each event competition, the correlation between action quality, drill level, difficulty score, and final performance has different mathematical characteristics. From the analysis in Table 1, it can be concluded that the final performance of martial arts athletes in the Changquan event has no correlation with the score of the sport quality and difficult movement. There was a moderate correlation (\( r = 0.60 \)) between the final score and the score of the drill level. While the final performance and the score of difficult movement had no correlation (\( r = 0.48 \)), the end score and the score of the drill level have a high correlation (\( r = 0.89 \)). In the Cudgel event, there was a moderate correlation (\( r = 0.76 \)) between the final performance of athletes and the score of movement quality. However, the final score had no correlation with the scores of difficult movements and drill levels (\( r = 0.37 \)). It can be seen that in the Nanquan competition, the quality of movements and the technique of difficult movements are the dominant factors for victory. In the Nandao event, there was no correlation between the final performance of athletes and the score of movement quality (\( r = 0.51 \)). In the end, there is no correlation between the final score and the score of the difficult action, but the final score and the score of the exercise level are highly correlated (\( r = 0.86 \)). It can be seen that in the competition of the Nandao project, the skill of the drill level is the leading factor in winning. Figure 7 shows the correlation and mathematical value of the impact of drill level on the results of martial arts competitions.

As can be seen from Figure 7, the correlation index of drill level to martial arts competition is as high as 4.4, its lowest value also exceeds 2.5, and the mathematical value is as high as 6.8. This shows that the level of exercise has a greater impact on martial arts competition.

4.3. Comparison of the Relevant Score Data of the Top 12 Athletes in Each Individual Event. The results of the comparison of the relevant score data of the top 12 women’s athletes in each individual event are shown in Table 2.

From the data in Table 2, in terms of movement quality, the top 12 athletes in Changquan, Daoshu, Cudgel, Taijiquan, Nanquan, and Nandao have all achieved full marks. Only the highest separation score of Tai Chi sword athletes is
The difference between the highest score and the lowest score among different items tends to be obvious. The highest difference between Taijiquan athletes is 0.30 points, and the highest difference between Changquan, Cudgel, and Nanquan athletes is 0.20 points. The difference between the highest score of Daoshu, Taijijian, and Nandao athletes is only 0.10 points. In terms of difficult movements, the top 12 athletes in Changquan, Daoshu, Cudgel, Taijiquan, and Taijijian have all achieved full marks. Only the all-around athletes of Nanquan and Nandao are 0.10 points away from the highest separation score. The difference between the highest scores of Nanquan athletes is 0.30 points. The difference between the highest scores of the Cudgel and Tai Chi swordsmen was 0.15 points, while the top 12 athletes in the Nandao single event all had the same difficulty score.

In terms of training level, the highest score difference of the top 12 athletes in each individual event is also less than 0.25 points. From the above overall analysis, the competition of modern competitive Wushu routine competition is more intense. The battle between the masters can only be decided in a small link. Taijiquan and Taijijian all-round athletes should strive to improve their performance in terms of movement quality. Nanquan and Nandao all-around athletes should strive to improve their performance in difficult movements, and the level of exercise is a factor that athletes in each event should pay attention to for improvement. The influence and correlation of the practice level at this time on the highest score is shown in Figure 8:

![Figure 7: Exercise-level correlations and mathematical values. (a) Correlation. (b) Mathematical value.](image-url)

It can be seen from Figure 8 that the influence index of drill level on the highest score in martial arts competition is as high as 4.3, while the correlation index is as high as 4.2. It can be seen that the exercise level has a great influence and correlation on the highest score in the martial arts competition.

To sum up, after analyzing and diagnosing martial arts routines by combining deep learning and symmetric difference algorithm, it is concluded that the biggest factor affecting martial arts routines and competition performance is the athlete’s martial arts routines drill level, and the comprehensive influence index is 4.3. This conclusion is of
certain significance to provide some guidance and suggestions for martial arts athletes in their usual martial arts routines.

5. Discussion

With the continuous improvement in China’s economy and sports, Wushu has also been greatly developed. Today’s martial arts competition is very different from the past, and its main manifestation is that in the martial arts competition, the requirements for the martial arts routines of martial arts athletes are getting higher and higher. If martial arts athletes want to achieve good results in martial arts competitions, they must improve their martial arts routines through continuous scientific and effective training. And in this process, it is very important to find out the problems in their martial arts routines in time.

By analyzing and diagnosing Wushu routine movements, we can find out the existing problems through the analysis of Wushu routine movements of Wushu athletes. This makes it convenient for martial arts coaches to provide some scientific establishments for countless athletes, so as to improve their martial arts level. The analysis and diagnosis of Wushu routine movements cannot be separated from certain technical support. This article mainly studies the analysis and diagnosis of Wushu routine movements based on deep learning and symmetric difference algorithm.

In order to achieve the research purpose, this article combines deep learning and symmetric difference algorithm to analyze and diagnose Wushu routines. The experiment analyzes and diagnoses the martial arts routine movements and results of the Beijing Wushu Team in the 10th National Games. It is found that the most important influence on martial arts athletes’ martial arts routines and martial arts competition performance is the martial arts athletes’ usual practice level of martial arts routines. It has an influence index of 4.3 on the level of martial arts routines and martial arts competition performance of martial arts athletes.

6. Conclusions

Based on deep learning and symmetric difference algorithm, the analysis and diagnosis of Wushu routine movements in this article draw the following conclusions: The most influential factor on the martial arts routines and martial arts competition performance of martial arts athletes is the practice level of martial arts athletes’ martial arts routines, and the influence index is as high as 4.3. This conclusion has great reference significance for improving the martial arts routine movement level and martial arts performance of martial arts athletes. However, due to the limitations of various aspects, the research in this article also has certain shortcomings. For example, research methods and research angles are not comprehensive and innovative. This article believes that it will be better in the future, and there will be more excellent researches related to the analysis and diagnosis of martial arts routines.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest with any financial organizations regarding the material reported in this manuscript.

References

[1] G. Litjens, T. Kooi, B. E. Bejnordi et al., “A survey on deep learning in medical image analysis,” *Medical Image Analysis*, vol. 42, no. 9, pp. 60–88, 2017.
[2] Y. Chen, Z. Lin, Z. Xing, W. Gang, and Y. Gu, “Deep learning-based classification of hyperspectral data,” *Ieee Journal of Selected Topics in Applied Earth Observations and Remote Sensing*, vol. 7, no. 6, pp. 2094–2107, 2017.
[3] D. Shen, G. Wu, and H.-I. Suk, “Deep learning in medical image analysis,” *Annual Review of Biomedical Engineering*, vol. 19, no. 1, pp. 221–248, 2017.
[4] H. He, C.-K. Wen, S. Jin, and G. Y. Li, "Deep learning-based channel estimation for beamspace mmWave massive MIMO systems," IEEE Wireless Communications Letters, vol. 7, no. 5, pp. 852–855, 2018.

[5] X. Fan, J. Liang, X. Zhao, Y. Zhang, and H. C. So, "Optimal synthesis of sum and difference beam patterns with a common weight vector for symmetric and asymmetric antenna arrays," IEEE Transactions on Antennas and Propagation, vol. 68, no. 10, pp. 6982–6996, 2020.

[6] Y. Ai, Z. Li, M. Gan, Y. Zhang, and Y. Ju, "A deep learning approach on short-term spatiotemporal distribution forecasting of dockless bike-sharing system," Neural Computing & Applications, vol. 31, no. 39, pp. 1–13, 2019.

[7] J. Lee, "Integration of digital twin and deep learning in cyber-physical systems: towards," Smart Manufacturing, vol. 38, no. 8, pp. 901–910, 2020.

[8] G. Suryanarayana, K. Chandran, O. I. Khalaf, Y. Alotaibi, A. Alsufyani, and S. A. Alghamdi, "Accurate magnetic resonance image super-resolution using deep networks and Gaussian filtering in the stationary wavelet domain," IEEE Access, vol. 9, Article ID 71406, 2021.

[9] M. F. Dixon, N. G. Polson, and V. O. Sokolov, "Deep learning for spatio-temporal modeling: dynamic traffic flows and high frequency trading," Applied Stochastic Models in Business and Industry, vol. 35, no. 3, pp. 788–807, 2019.

[10] S. Ketu and P. K. Mishra, "India perspective: CNN-LSTM hybrid deep learning model-based COVID-19 prediction and current status of medical resource availability," Soft Computing, vol. 26, no. 2, pp. 645–664, 2022.

[11] H. Lu, Q. Liu, X. Liu, and Y. Zhang, "A survey of semantic construction and application of satellite remote sensing images and data," Journal of Organizational and End User Computing, vol. 33, no. 6, pp. 1–20, 2021.

[12] Y. Zhao, H. Li, S. Wan et al., "Knowledge-aided convolutional neural network for small organ segmentation," IEEE journal of biomedical and health informatics, vol. 23, no. 4, pp. 1363–1373, 2019.

[13] R. Subha, A. Haldorai, and A. Ramu, "An optimal approach to enhance context aware description administration service for cloud robots in a deep learning environment," Wireless Personal Communications, vol. 117, no. 4, pp. 3343–3358, 2021.

[14] T. Young, D. Hazarika, S. Poria, and E. Cambria, "Recent trends in deep learning based natural language processing [review article]," IEEE Computational Intelligence Magazine, vol. 13, no. 3, pp. 55–75, 2018.

[15] G. B. Goh, N. O. Hodas, and A. Vishnu, "Deep learning for computational chemistry," Journal of Computational Chemistry, vol. 38, no. 16, pp. 1291–1307, 2017.

[16] N. Majumder, S. Poria, A. Gelbukh, and E. Cambria, "Deep learning-based document modeling for personality detection from text," IEEE Intelligent Systems, vol. 32, no. 2, pp. 74–79, 2017.

[17] J. Sandberg and Y. Barnard, "How can deep learning advance computational modeling of sensory information processing?" vol. 25, no. 1, pp. 15–36, 2018, https://arxiv.org/abs/1810.08651.

[18] Y. Dong and J. Li, "Recent progresses in deep learning based acoustic models," IEEE/CAA Journal of Automatica Sinica, vol. 4, no. 3, pp. 396–409, 2017.

[19] Y. Jian, J. Ni, and Y. Yang, "Deep learning hierarchical representations for image steganalysis," IEEE Transactions on Information Forensics and Security, vol. 12, no. 11, pp. 2545–2557, 2017.

[20] S. P. Raja, "Multiscale transform based secured joint efficient medical image compression-encryption using symmetric key cryptography and EBCOT encoding technique," International Journal of Waves, Multiresolution and Information Processing, vol. 17, no. 5, pp. 1907–1917, 2019.

[21] Z. Zheng, M. Fu, W.-Q. Wang, S. Zhang, and Y. Liao, "Localization of mixed near-field and far-field sources using symmetric double-nested arrays," IEEE Transactions on Antennas and Propagation, vol. 67, no. 11, pp. 7059–7070, 2019.

[22] M. Solaïmani, "Spectra of PT-symmetric fractional Schrödinger equations with multiple quantum wells," Journal of Computational Electronics, vol. 19, no. 4, pp. 1416–1425, 2020.

[23] Y. Jin and J. Dan, "Symmetric positive semidefinite FDTD subgridding algorithms for arbitrary grid ratios without compromising accuracy," IEEE Transactions on Microwave Theory and Techniques, vol. 65, no. 12, pp. 5084–5095, 2017.