Construction of elastoplastic shallow panels manufactured by stretching from sheet metal

N V Dedov* and V N Isytkina
Samara State Technical University, Samara, Russian Federation

* nikolai_dedov@mail.ru

Abstract. In the article, in a geometrically and physically nonlinear formulation, the problem of the stress-strain state of shallow shells on a rectangular plane under pressure loading is solved. The solving equations of the theory of flexible shallow shells are obtained on the basis of the Henki-Ilyushin deformation theory of plasticity and the equations of the geometrically non-linear theory of shells, which contain quadratic terms with respect to the angles of rotation of the normals to the median surface. The geometrical parameters of the median surface are taken in the initial undeformed state. The equations are written in a form that makes it possible to carry out numerical calculations of problems with allowance for geometric and physical nonlinearities or only with geometric nonlinearity, to take into account unloading, compressibility of the material and secondary plastic deformations. The results of numerical calculations of critical loads, unloading zones and secondary plastic deformations are presented. Recommendations are given on the choice of geometrical parameters when designing shallow shells.

1. Introduction
In papers [1, 2] the elastic stability of cladding panels manufactured on presses is considered. The relationship between uniformly applied load and deflection in the center of the panel is taken according to [3]. Recommendations on the choice of design parameters of panels are given.

The stress-strain state during elastoplastic deformations of rectangular plates loaded with pressure was studied for various materials [4].

In [5] the elastoplastic deformation of the shells of revolution is studied with allowance for geometric nonlinearity. The values of the upper critical load are obtained. Defined unloading area and the secondary plastic deformation. The values of the lower critical load were not determined.

In [6–8] a numerical study of the stability and supercritical deformations of elastic and elastoplastic plates and shallow shells was carried out. The upper and lower critical loads with loss of stability are determined.

For the numerical implementation of nonlinear problems that represent significant difficulties, an incremental approach is considered in [9] when linear equations are obtained on the basis of nonlinear equations containing as an unknown the increments of the desired functions the methods for solving which are well developed. V.V. Petrov proposed a method of successive loads, which makes it possible to reduce the solution of a nonlinear problem to a sequence of quasilinear solutions [10].
During operation shallow panels are loaded with external pressure and large deflections and elastoplastic deformations occur which makes it necessary to take them into account when designing structures.

Therefore, it is important to develop methods for the study of nonlinear bending and supercritical elastoplastic deformations of thin-walled shell structures taking into account geometric and physical nonlinearities.

2. Formulation of the problem

In this paper elastoplastic bending and the stability of flexible shallow shells are considered with allowance for unloading. Studies of the elastoplastic stress-strain state during bending and after loss of stability are considered in the case of loading pressure shallow panels fixed on a rectangular plane. The mathematical model of the problem of bending shells of elastoplastic material involves taking into account the geometric and physical nonlinearity, unloading, secondary plastic deformations and compressibility of the material.

When studying the behavior of the shell of an elastoplastic material in the process of active deformation with simple loading or close to it the dependence of the components of the stress deviator on the components of the strain deviator in the form of \([11,12]\) is used

\[
\sigma = 3K\varepsilon_0 + \frac{2}{3}\left(\varepsilon_{ij} - \varepsilon_0\delta_{ij}\right), \quad i, j = 1, 2, 3
\]

where \(K = \frac{E}{3(1-2\nu)}\) is bulk compression module, \(\varepsilon_0 = \frac{\varepsilon_{11} + \varepsilon_{22} + \varepsilon_{33}}{3}\) is mean strain.

The dependence of stress intensity on strain intensity is taken as

\[
\sigma_i = 3G\left(1 - \omega_i\right)\varepsilon_i
\]

where \(\omega_i = f\left(\varepsilon_i\right)\) is the function of A.A. Ilyushin. This is some analytical function of the strain intensity which is nonzero only beyond the elastic limit. As a condition for the onset of plasticity the von Mises condition is taken according to the intensity of stresses or strains

\[
\sqrt{3}\left(3\sigma_1^2 - \sigma_2^2 - \sigma_3^2 + \sigma_{12}^2 + \sigma_{13}^2 + \sigma_{23}^2\right) = 2\sigma_T,
\]

\[
\varepsilon_i = \frac{\sqrt{3}}{3}\left(\varepsilon_{11} - \varepsilon_{22}\right)^2 + \left(\varepsilon_{22} - \varepsilon_{33}\right)^2 + \left(\varepsilon_{33} - \varepsilon_{11}\right)^2 + 6\left(\varepsilon_{12}^2 + \varepsilon_{13}^2 + \varepsilon_{23}^2\right) = 2\varepsilon_T.
\]

The dependence of stress intensity on strain intensity can be established from tensile tests. In this case, the curve \(\sigma_i\left(\varepsilon_i\right)\) can be obtained from the stretching diagram by decreasing the values along the abscissa axis by \(3/2 (1 - \nu)\) times.

The transverse compression coefficient in the elastic region is constant and beyond the elastic limit it increases approaching \(\nu = 0.5\). The value of \(\nu\) at different levels of deformations can be determined directly from tensile experiments or use the dependency

\[
\nu = \frac{1}{2}\left[1 - \frac{E}{E_0}\left(1 - 2\nu_0\right)\right],
\]

where \(\nu_0\) - the coefficient of transverse compression in the elastic region.

Defined unloading area and the secondary plastic deformation. According to A.A. Ilyushin on elastic unloading have

\[
\sigma'_i = 3K\left(\varepsilon'_0 - \varepsilon_0\right)\delta_{ij} + 2G\left[\left(\varepsilon'_0 - \varepsilon_0\right) - \left(\varepsilon'_0 - \varepsilon_0\right)\delta_{ij}\right], \quad i, j = 1, 2, 3.
\]
Here the strokes indicate the stresses and strains at the corresponding point at the end of the active deformation process.

According to the Masing principle at the first loading the onset of plasticity in a plane with coordinates $\sigma_i - \varepsilon_i$ is determined by quantities $\sigma_T$ and during unloading the onset of plasticity in a plane with coordinates $\overline{\sigma}_i - \overline{\varepsilon}_i$ is determined by the values $2\sigma_T$.

The beginning of the coordinates $\overline{\sigma}_i, \overline{\varepsilon}_i$ is taken at the point corresponding to the end of the active loading. The dependence of the stress intensity on the intensity of deformation during elastic and secondary plastic deformations is shown in Figure 1.

Figure 1. Dependence of stress intensity on strain intensity with active loading (OAN section) and during elastic unloading and secondary plastic deformations (NPR section)

If under active loading the areas of elastic deformations are preserved in the body then when unloading, the Masing principle cannot be extended to these areas. Plastic deformations in these areas during unloading will appear at stresses $\sigma_T$ and not than $2\sigma_T$ as follows from the Masing principle.

In [13,14], a generalization is given A. A. Ilyushin's theory of unloading in an elastic region to the case of plastic deformation in the process of unloading (area of secondary plastic deformations) - part of the NPR diagram in Figure 1

\[
\sigma_{ij} - \sigma_{ij} = \frac{2}{3} \frac{\sigma}{\varepsilon_i} \left( \varepsilon_{ij} - \varepsilon_{ij} \right) \quad i, j = 1, 2, 3
\]  
(7)

\[
\overline{\sigma}_{ij} = \sigma'_{ij} - \sigma_{ij}, \quad \overline{\varepsilon}_{ij} = \varepsilon'_{ij} - \varepsilon_{ij}, \quad i, j = 1, 2, 3
\]  
(8)

where components of the tensor of elastoplastic stresses and strains at the end of the active loading section, $\sigma_{ij}, \varepsilon_{ij}$ - components of the tensor of elastoplastic stresses and strains during unloading and secondary plastic deformations, $\overline{\varepsilon}_{ij} = \varepsilon'_{11} + \varepsilon'_{22} + \varepsilon'_{33}$ - mean strain, $\overline{\sigma}_{ij} = \sigma'_{11} + \sigma'_{22} + \sigma'_{33}$ - mean stress, $\overline{\varepsilon}_o = f(e)$ - the Ilyushin function, $\overline{\sigma}_o = 3K\overline{\varepsilon}_o$ - connection between the components of the stresses and strains spherical tensor.

The condition of the onset of plasticity during unloading and secondary plastic deformations in the coordinates we write in the form

\[\overline{\sigma}_o = \frac{3}{3} \overline{\varepsilon}_o \]

\[f(e) = \overline{\sigma}_o = 3K\overline{\varepsilon}_o \]
\[ \sigma_i = \frac{\sqrt{2}}{2} \left( (\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + 6(\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{31}^2) \right) = 2\sigma_T, \quad (9) \]

\[ \sigma_T = \frac{\sqrt{2}}{3} \left( (\sigma_{11} - \sigma_{22})^2 + (\sigma_{22} - \sigma_{33})^2 + (\sigma_{33} - \sigma_{11})^2 + 6(\sigma_{12}^2 + \sigma_{23}^2 + \sigma_{31}^2) \right) = 2\sigma_T, \quad (10) \]

where \( \sigma_i \) - is the stress intensity, \( \sigma_T \) - is the strain intensity.

As is known, when building an algorithm for the calculation of elastoplastic continuum it is not enough to use only the tensor of total deformations. In most modern works studying elastoplastic deformation strain measures are introduced that are associated separately with elastic and separately with plastic deformation.

With the active loading of thin-walled structures from an elastoplastic material the stresses are represented

\[ \sigma_{ij} = \sigma_{ij}' + \Delta \sigma_{ij}, \quad i,j = 1,2,3, \quad (11) \]

where \( \sigma_{ij}' \) - the stresses arising in the shell if the shell material obeyed Hooke's law under true deformations, \( \Delta \sigma_{ij} \) - value stress taking into account the deviation of the diagram of the shell material from Hooke's law.

The stresses during unloading and in the area of secondary plastic deformations are written using the V.V. Moskvitin theorem in the form

\[ \sigma_{ij} = \sigma_{ij}' - \sigma_{ij} \quad i,j = 1,2,3, \quad (12) \]

or, extracting the elastic part of the stresses in the form

\[ \sigma_{ij} = \sigma_{ij}' + \Delta \sigma_{ij}, \quad i,j = 1,2,3, \quad (13) \]

where \( \Delta \sigma_{ij} = \sigma_{ij}' - \sigma_{ij} - \sigma_{ij}' \) - additional stresses taking into account the deviation of the diagram of the shell material from Hooke's law the presence of unloading and secondary plastic deformations.

Based on the relationships for stresses you can imagine the forces and moments in the cross section of the shell

\[ T_{ij} = \int_{-0.5h}^{0.5h} \sigma_{ij} \, dz = T_{ij}' + \Delta T_{ij}, \quad M_{ij} = \int_{-0.5h}^{0.5h} \sigma_{ij} z \, dz = M_{ij}' + \Delta M_{ij} \quad (14) \]

\[ N_i = N_i' + \Delta N_i, \quad i,j = 1,2, \]

where \( T_{ij} \) - the forces acting in the middle surface of the shell, \( M_{ij} \) - bending and torsional moments, \( N_i \) - transverse forces.

In the absence of external loads applied in the shell plane along the x, y axes the main system of differential equations of the theory of thin elastoplastic shells taking into account unloading, secondary plastic deformations and compressibility of the material will take the form

\[ D \nabla^2 \nabla^2 w + \bar{F}(\phi) = L(w,\phi) + G(\Delta M_{ij}) + p \quad (15) \]

\[ \frac{1}{Eh} \nabla^2 \nabla^2 \phi + \bar{N}(w) = K(w) + H(\Delta T_{ij}). \quad (16) \]

To represent the resolving system of differential equations in a dimensionless form, dimensionless geometric and physical parameters were used.
\[ \lambda = \frac{a}{b}, \quad x = \frac{x}{a}, \quad y = \frac{y}{b}, \quad w = \frac{w}{h}, \quad K_1 = \frac{4a^2}{R_1 h}, \quad K_2 = \frac{4b^2}{R_2 h}, \]

\[ \bar{q} = \frac{q}{E h^3}, \quad p = \frac{16 p b^4}{E h^6}, \quad \sigma_{ij} = \frac{b^2}{E h^2}. \]

The system of differential equations is constructed in a form convenient for solving geometrically and physically nonlinear problems in an elastic, nonlinear elastic or elastoplastic region.

3. Numerical calculation method

In this paper the solution of nonlinear equations is carried out by the method of general iteration [6]. The value of the roots of the system of algebraic equations in the process of iteration is formed by the formula

\[ x_i^n = x_i^{n-1} + \alpha_{cx} \left( x_i^n - x_i^{n-1} \right), \quad (17) \]

where \( \alpha_{cx} \) - numerical coefficients that must be determined so that the iteration process converges as quickly as possible. When \( \alpha_{cx} = 1 \) we have the usual method of iteration which for the systems of nonlinear equations considered here begins to diverge already at small values of the load parameter \( \bar{p} \).

Static boundary conditions can be expressed by ratios containing displacements, rotation angles, forces and moments. At each edge of the shell the number of boundary conditions is four. Unlike a simple iterative method \( \alpha_{cx} \neq 1 \) the total iteration method allows a proper choice of the correction coefficient \( \alpha_{cx} \) to achieve rapid convergence of the iterative process at high load parameter \( \bar{p} \).

To solve a specific problem a system of nonlinear differential equations must be integrated with the appropriate boundary conditions on the contour of the shallow shell.

Figure 2 for the physically and geometrically nonlinear case shows the dependence of the load on the deflection in the center for a square cylindrical panel \( h/b = 0.005 \) with different values of curvature \( k_2 \). The edges of the cylindrical panel are pinched. Stability of a square cylindrical panel for the nonlinearity considered here appears at \( k_2 \geq 52 \).

![Figure 2. Dependence \( \bar{p}(W_0) \) for a square cylindrical panel \( h/b = 0.05 \) with pinched edges](image)

With an increase in the ratio \( h/b \) the influence of physical nonlinearity on critical loads and deflection increases significantly. Numerical calculations have shown that for a square panel pinching...
curved edges increases stiffness in the subcritical state and the upper critical load. For rectangular panels $\lambda \leq 0.67$ stiffness and upper critical load increase as a result of pinching of rectilinear edges.

From Figure 2 it is seen that when designing structures it is advisable to set relatively large values of the curvature parameters. The shell with a curvature $k_2 = 48$ does not lose stability has a gentle initial part of the diagram $P(W)$, i.e. has low rigidity.

Figure 3 for the same cylindrical panels with hingedly supported edges shows the dependence of the load on the deflection in the center $P(W)$ for different values $k_2$ of curvature. Loss of stability of the panels occurs at smaller parameters of curvature than in the case of pinching edges. Loss of stability of a square cylindrical panel for the nonlinearity considered here and the edges fixed appears at $k_2 \geq 32$.

To avoid loss of stability of the panels and the appearance of supercritical deformations, it is necessary that the operating pressure be a certain part of the upper critical load and not exceed the values of the lower critical load. Initial shape irregularities can significantly reduce the theoretical values of the upper critical load. Therefore, when assigning working pressures, it is necessary to take into account the values of the initial shape irregularities and enter experimental coefficients.

![Figure 3. Dependence for a square cylindrical panel $h/b = 0.05$ with hinged edges](image)

![Figure 4. Development of zones of plasticity 1, unloading 2 and secondary plastic deformations 3 through the thickness of a rigidly embedded square spherical panel $a/b = 0.1$, $k_1 = k_2 = 16$ with deflections in the center: a) $w_0 = 0.5$, b) $w_0 = 1.0$, v) $w_0 = 1.75$](image)
Calculations are carried out for a rigidly clamped spherical panel on a rectangular plan loaded with uniformly distributed pressure from material with mechanical characteristics $G = 0.75 \times 10^5$ MPa, $G_1 = 0.148 \times 10^5$ MPa, $\nu = 0.3$.

Figure 4 shows the zones of plastic deformations 1, unloading 2 and secondary plastic deformations 3 through the thickness of a rigidly embedded spherical panel during deflections $w_0 = 0.5$, $w_0 = 1.0$, $w_0 = 1.75$.

At deflection of the spherical panel in the center $w_0 = 1.75$, almost the entire section of the panels is in a plastic state. The unloading zone 2 and the zone of secondary plastic deformation 3 is located inside the spherical panel. The elastic zone is not marked with a number.

4. Conclusion

The proposed method allows one to study the class of problems of plates and shallow shells that are important in practical and theoretical applications for bending and supercritical deformations. Taking into account the geometric and physical nonlinearities, the development of zones of plastic deformation, unloading and secondary plastic deformations is traced, which is important for the assessment of structural damage. The results of numerical calculations indicate the appearance of zones of secondary plastic deformation, which are realized even with a single transverse loading of shallow panels with unloading. In the case of cyclically hardening materials with elastoplastic deformations, it is possible to solve a nonlinear problem with a variable yield strength and allow for adaptability. Therefore, the use of the theory of geometric nonlinearity in the elastoplastic deformation of plates and shells is necessary.

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