Necessarily transient quantum refrigerator

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Abstract – We show that it is possible to construct a quantum absorption refrigerator that provides refrigeration only in the transient regime, by using three interacting qubits, each of which is also interacting with a local heat bath. Though the machine does not provide any effective cooling in the steady state, significant cooling of a qubit can be achieved much before the system reaches its steady state. We also demonstrate a scenario where the temperature of the qubit that has to be cooled, decreases monotonically to its minimum steady-state temperature. This facilitates bypassing of precise time control in cooling. We study the behaviour of the ratio of the heat currents of the machine for both the scenarios. The results remain qualitatively unchanged for different models of thermal baths. We also comment on the temporal behaviour of bipartite and multipartite quantum correlations present in the system, when transient cooling without steady-state cooling takes place.

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Introduction. – Study of the thermodynamic properties of microscopic quantum systems has been an active field of research in recent times [1–3]. Considerable efforts have been directed to develop and characterize quantum heat engines, and to determine whether “quantum” advantages can be obtained in these machines over their classical counterparts [4]. Quantum analogues of the well-known classical Carnot and Otto engines have been extensively studied [5,6], and implemented in laboratories using mesoscopic substrates [7], superconducting qubits [8], and ionic systems [9]. On the one hand, this has motivated researchers to test the laws of thermodynamics at the quantum-mechanical level, and to determine the efficiencies of quantum heat engines, analogous to those provided by the extensively studied classical heat engines [5,6]. On the other hand, a great deal of interest has been attracted towards building “small” quantum engines, like quantum refrigerators, which consist of only a few quantum levels. The energy required to drive the refrigerator is obtained from local heat baths attached to the subsystems constituting the refrigerator, so it is known as the absorption refrigerator [10–23]. Implementation schemes for such models in laboratories [24–26] have also been proposed, and realization of quantum absorption refrigerator in trapped-ion systems has been possible [27]. The motivation for studying such microscopic refrigerators from an information-theoretic perspective [17] lies, for example, in the facts that thermodynamics has a close connection with both classical and quantum information theory [28,29].

The phenomenon of steady-state cooling is a prime feature of a quantum absorption refrigerator constituted of only three qubits [10,12–17]. Here, the steady-state temperature of one of the qubits, called the “cold” qubit, is less than its initial temperature, and it, in general, can occur at large time in the dynamics. However, refrigeration at short time in these models, which can be more accessible in the experiments, remains a relatively less explored topic. Only recently, a few studies have addressed this issue [15,17,19], and pointed out the benefit of transient cooling over the steady-state cooling, by using uncorrelated product thermal states as well as states with coherence in the energy eigenbasis, as the initial states. It has been shown that the transient regime of such refrigerators may provide a better cooling, in the sense of attaining a lower temperature, as compared to that in its steady state, which highlights the importance of the study of the systems as they approach towards their equilibria. In situations where the time scale to attain the equilibrium is too high to implement, or where very fast cooling...
is required, transient cooling may emerge as the practical option to attain refrigeration. In this letter, we ask the following question: Can there exist a situation where the transient cooling (TC) is the “only” option for refrigeration to occur? In this scenario, the steady-state cooling (SSC) is either absent or negligible, while there is an appreciable amount of cooling in the transient times, which we refer to as the phenomenon of “TC without SSC”. This paper answers the question affirmatively.

Towards this aim, in what follows, we consider two different models of thermalization, namely, 1) the reset model [12] and 2) a Markovian thermal bath of harmonic oscillators [17], for a three-qubit self-contained quantum refrigerator attached to local heat baths, and demonstrate the TC without SSC phenomenon for both models. We point out substantial regions in the space of the qubit-bath interaction parameters where such a phenomenon takes place. We also study the cooling power and coefficient of performance of the machine, and comment on the dynamics of bipartite as well as multipartite quantum correlations present in the system when TC without SSC takes place. We also study a rather complementary scenario where a colder steady-state temperature is reached monotonically with time for both the types of baths.

Three-qubit quantum absorption refrigerator. –

We consider a quantum absorption refrigerator consisting of three qubits [10,15] labelled as “1”, “2”, and “3”. The first qubit, “1”, represents the qubit which is to be cooled, while “2” and “3” behave as the refrigerator. Describing the qubits in terms of standard Pauli representations, the eigenvectors of \( \sigma_z \), forms the computational basis, the free Hamiltonian of the three-qubit system can be written as \( \hat{H}_{loc} = \frac{1}{2} \sum_{i=1}^{3} E_i \sigma_i^z \), where the ground and excited state energies of the qubit \( i \) are given by \( -E_i \) and \( \frac{kE}{2} \), respectively, \( k \) being a constant having the dimension of energy, and \( E_i \) being dimensionless numbers.

The coupling between the qubits is represented by a three-body interaction Hamiltonian,

\[
\hat{H}_{int} = kg(|010\rangle\langle 011| + h.c.),
\]

with \( kg \) being the corresponding interaction strength, \( g \) being a dimensionless number. Each of the qubits is considered to be interacting with a heat bath at absolute temperature \( T_1 \), where \( T_1 \leq T_2 \leq T_3 \). The third qubit is coupled with the hottest bath, while the bath associated with the second qubit is considered to be at room temperature. The interaction between the qubits is switched on at time \( t = 0 \), such that \( kg \geq 0 \) for \( \tilde{t} > 0 \). All the qubits are initially in a thermal equilibrium state with their respective baths, and the initial state of the three-qubit system is chosen to be a thermal product state given by \( \rho_0 = \rho_0^1 \otimes \rho_0^2 \otimes \rho_0^3 \), with \( \rho_0^i = Z_i^{-1} \exp(-\beta_i kE_i \sigma_i^z /2) \). Here, \( Z_i = \text{Tr} (\exp(-\beta_i kE_i \sigma_i^z /2)) \) is the partition function corresponding to the qubit \( i \), and \( \beta_i = (kg T_i)^{-1} \), \( k_B \) being the Boltzmann constant.

When the interaction between the qubits is turned on, the time-evolved state \( \rho(t) \) of the three-qubit system at \( \tilde{t} > 0 \) is obtained as a solution of the quantum master equation (QME) \( \frac{d\rho}{dt} = -\frac{i}{\hbar} [\hat{H}_{loc} + \hat{H}_{int}, \rho] + \frac{\hbar}{k} \Phi(\rho) \), where the operator \( \Phi \) dictates the dynamics of the system due to the effect of the local heat baths coupled to the system. One can rewrite the QME in dimensionless variables and parameters as

\[
\frac{d\rho}{dt} = -i[H_{loc} + H_{int}, \rho] + \frac{k}{\hbar} \Phi(\rho),
\]

where \( t = kt/\hbar \), \( H_{loc} = \tilde{H}_{loc}/k \), and \( H_{int} = \tilde{H}_{int}/k \). The explicit form of the dynamical term \( \Phi(\rho) \) in terms of dimensionless variables depends on the type of the heat baths, and will be determined in subsequent discussions. Let us also introduce the dimensionless temperature \( T = \tilde{T} k_B/k \), so that the initial state of the \( i \)-th qubit becomes \( \rho_0^i = \frac{1}{Z_i} \exp(-E_i \sigma_i^z /2T_i) \).

It has been shown that the refrigerator can work as an autonomous machine without any external control and drawing resource only from the attached heat baths if the dimensionless energies of the qubits satisfy \( E_1 + E_3 < E_2 \) [10]. We demonstrate our results under this condition. We further set \( T_1 = T_2 \), so that the qubit 1 is initially at the room temperature. Note that the initial state, \( \rho_0 \), is diagonal in the eigenbasis of \( \tilde{H}_{loc} \), and the off-diagonal elements emerging in the evolved state due to \( H_{int} \) are \( \rho \neq 0 \) [10] and its Hermitian conjugate. We will use models for thermal baths that do not generate coherence between the eigenbasis elements of the individual qubits, thereby keeping the form of \( \rho(t) \) unchanged. Such a form of \( \rho(t) \) leads to diagonal local density matrices corresponding to each qubit, \( \rho^i(t) = \text{Tr}_{jk}(\rho(t)) \), \( j,k \neq i \), \( i,j,k \in \{1,2,3\} \), which allows one to define a local temperature for each qubit at every time instant \( t \), according to the form of \( \rho_0^i \). Here, \( \rho(t) \) is obtained as the solution of eq. (2). We want to clarify here that the term “temperature” used in the literature of quantum absorption refrigerators, is a manifestation of the relative populations of the ground/excited states of the respective qubits. Thus, “cooling qubit 1” means increasing the population of (i.e., fidelity of being in) the state \( |1\rangle \), in cases where the reduced states of the qubits are not an equilibrium state, defining a temperature is not possible [19]. Note here that the dimensionless transient temperature, \( T_i(t) \), of qubit 1 is a function of the system as well as the qubit-bath interaction parameters. If \( T_i(t) < T_1 \) at some specific value of \( t \), successful cooling of the qubit “1” is achieved with the help of the refrigerator. Let us denote the temperature of the cold qubit corresponding to the steady state of the system (\( \partial \rho/\partial t = 0 \)) by \( T_1^* \). We call a situation to be of SSC if \( T_1^* < T_1 \), while \( T_1(t) < T_1 \) represents a case of TC when the value of \( t \) is such that the steady state has not yet been reached. Note that a SSC is always accompanied by a TC, while the reverse is not true.
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Fig. 1: (a) Variation of $T_c(t)$ as a function of $t$ for qubit 1, with the values of the qubit-bath interaction parameters as $p_1 = \frac{10^{-2}}{3}$, $p_2 = 10^{-6}$, $p_3 = 10^{-1}$, and with the values of the system parameters as $E_1 = 1$, $E_2 = 100$, $T_1 = T_2 = 1$, $T_3 = 100$, and with $g = 10^{-2}$ (continuous curve) and $5 \times 10^{-3}$ (dashed curve). (b) Projection plot of $T^*_c$ as functions of $p_1$ and $p_2/p_1$ for $g = 10^{-2}$. The continuous lines correspond to $T^*_1 = 0.9$, $0.95$, $0.99$ (from bottom to top). There exists a significant region on the $(p_1,p_2)$-plane (the region above the line corresponding to $T^*_1 = 0.99$), where $T^*_1 \approx 1$. This situation is “rich” in the first quadrant of the region considered over the $(p_1,p_2)$-plane, which we mark by “R”. (c) Projection plot of $T_{\text{min}} = \min T_c(t)$ as functions of $p_1$ and $p_2/p_1$. In R, $T_{\text{min}}$ has a significantly low value compared to $T_1$. Qualitatively similar results are found for different values of $g$. (d) Variation of $T_{\text{min}}$ as a function of $T_c$. The values of the system-bath interaction parameters $p_1$, $p_2$, $p_3$ and the system parameter $g$ corresponding to the curves are $p_1 = 10^{-3.5}$, $p_2 = 10^{-6}$, $p_3 = 10^{-1}$, $g = 10^{-2}$ (red continuous curve), $p_1 = 10^{-3.5}$, $p_2 = 10^{-6}$, $p_3 = 10^{-1}$, $g = 5 \times 10^{-3}$ (green dashed curve with crosses), $p_1 = 10^{-4}$, $p_2 = 10^{-7}$, $p_3 = 10^{-1}$, $g = 10^{-2}$ (blue dashed curves with filled squares), and $p_1 = 10^{-4}$, $p_2 = 10^{-7}$, $p_3 = 10^{-1}$, $g = 5 \times 10^{-3}$ (black dotted curve with empty squares). All quantities are dimensionless.

Necessarily transient cooling. – We demonstrate our results via a simple “reset model” [12] of thermalization, where at every time instant, the state of qubit $i$ is left unchanged with a non-zero probability, while with the rest of the probability, the qubit is reset to the initial thermal state $\rho_0$. Hence the operator $\Phi$, in this case, is given by

$$\Phi(\rho) = \sum_{i=1}^{3} \bar{\rho}_i (\varphi_i(\rho) - \rho),$$

where $\{\bar{\rho}_i\}$ are the probability densities per unit time, and $\varphi_i(\rho) = \rho_0 \otimes T_i(\rho(t))$. Subject to eq. (2), one can now introduce the dimensionless qubit-bath interaction parameter, $p_i = \frac{\bar{p}_i}{\epsilon}$, thus resulting in a dimensionless second term in the right-hand-side of eq. (2). The dynamical term $\Phi(\rho)$ can be written in the usual Lindblad form as

$$\Phi(\rho) = \sum_{i=1}^{3} q_{i} \sqrt{\alpha_{i\beta}} L_{\alpha\beta}^{i} \rho L_{\alpha\beta}^{-i} - \frac{1}{2} \left(L_{\alpha\beta}^{i} L_{\alpha\beta}^{-i} \rho \right),$$

where $\sqrt{\alpha_{i\beta}} L_{\alpha\beta}^{i}$ are the Lindblad operators via which the environment couples with the system, and are given by $L_{00}^{i} = \frac{1}{2}(1 + i \sigma_x)$, $L_{01}^{i} = \frac{1}{2}(1 - i \sigma_x)$, $L_{11}^{i} = \frac{1}{2}(1 + i \sigma_y)$, $L_{01}^{i} = \frac{1}{2}(1 - i \sigma_y)$, $L_{10}^{i} = \frac{1}{2}(1 + i \sigma_z)$, $L_{01}^{i} = \frac{1}{2}(1 - i \sigma_z)$, with $q_i$ being the corresponding rates, $q_{0i}^{i} = q_{0i}^{0} = p_0 r_a$, and $q_{1i}^{i} = q_{1i}^{0} = p_0 (1 - r_a)$. As it is evident that each collapse operator $L_{\alpha\beta}^{i}$ corresponds to the transitions between the states of the $i$-th qubit only, it is clear that the baths locally interact with the respective qubits, without affecting the total system.

Solving the QME, the steady-state temperature and the transient temperature of the cold qubit as a function of time can be computed for fixed values of the system parameters. For specific values of the probabilities, $\{p_i\}$, transient temperature lower than the steady-state temperature ($T_c(t) < T_1$) has been shown to occur [15,17]. This implies that the refrigerator can be more effective in the transient domain compared to being in the steady state. In this paper, we show that there exist regions in the space of system parameters as well as qubit-bath interaction parameters, where the steady-state temperature is almost equal or larger than the initial temperature (of qubit 1) and where there is a span of time in the transient regime at which the temperature is smaller than the initial temperature. Therefore, cooling of qubit 1 here is necessarily transient, and we denote such situations with the phrase “TC without SSC”. Here, “almost equal” is a numerical statement, and in this paper, we have used it to indicate that the corresponding (dimension-free) temperatures have a difference not exceeding $10^{-2}$. This definition of equality is motivated by practical concerns: experimental and numerical. In an actual experiment, it is only possible to check equality between two numbers to within a finite precision. The same is true for checking the equality numerically.

As an example of TC without SSC, let us consider the dynamics of the temperature of qubit 1 when the values of the qubit-bath interaction parameters are chosen such that $p_2 < p_1,p_3$ (see fig. 1(a)). The temperature of the cold qubit (qubit “1”) decreases at first, reaches a minimum, and then increases to attain a steady state at a temperature $T^*_1 \approx T_1 = 1$, so that cooling in the steady state is negligible, while substantial cooling occurs in the transient regime. Therefore, the three-qubit system in the example represents a necessarily transient quantum absorption refrigerator. There exists substantial region in the space of the qubit-bath interaction parameter, where, if the experimentalist finds herself/himself forced
to work in, due to may be some practical limitations in the laboratory technology, the only way to have a refrigerator, within the reset model, is to consider a necessarily transient cold qubit (see figs. 1(b), (c)). Here, for the ease of representation, we have chosen the qubit-bath interaction parameters to be of the form \( p_1 = 10^{-2} \), \( p_2 = 10^{-(x+y)} \), \( p_3 = 10^{-(x-y)} \), where \( x, y \geq 0 \), such that \( \max\{p_1, p_2, p_3\} = p_3 \), and \( p_1 \leq 1 \), so that the hierarchy \( p_2 < p_1, p_3 \) is maintained. For obtaining a typical dynamics profile, we always focus on the range \( 0 \leq t \leq 10^5 \). We consider \( t = 10^5 \) to be large time, and in all the graphs shown in this paper, steady state of the system is achieved for \( t \leq 10^5 \). It is evident from figs. 1(a)–(c) that better cooling is achieved when \( g \) is high, \( x \) is high, and \( y \) is low. Lower \( T_{\text{min}} \) with increase in \( g \) (fig. 1(a)) is expected as the cooling occurs for the three-body interaction with specific initial bias, and higher value of the interaction strength provides better cooling. As is seen from figs. 1(b), (c), for a fixed value of \( y \), \( T_{\text{min}} \) is lower if \( x \) is higher, as \( \{p_i\} \) varies linearly with \( 10^{-2} \), whereas for a fixed value \( x \), \( T_{\text{min}} \) is lower if \( y \) is lower, as \( p_3 \) is the strongest bath coupling parameter and it varies linearly with \( 10^{-9} \). The minimum achievable temperature of qubit 1 during a necessarily transient cooling depends also on the temperature of the hottest bath \( T_3 \). We observe that \( T_{\text{min}} \) monotonically decreases with increase in \( T_3 \), and finally reaches a saturation (fig. 1(d)).

Note here that in the case of a good absorption refrigerator with SSC, it is desired that the object to be cooled should be well-insulated. Hence, the coupling of the first (cold) qubit with the environment should be taken to be small. It is also needed that the intermediate qubit (qubit 2) interacts with the environment strongly, to dissipate heat quickly, implying a high value of the interaction parameter corresponding to qubit 2 and its environment. In contrast, in the situation reported in our paper, the insulation of the qubit 2 from its attached heat bath is the best among the three qubits (i.e., \( p_2 \) is minimum), while the same for the qubit 3 is the worst (i.e., \( p_3 \) is maximum). Since the intermediate qubit is the one dissipating energy into the environment in the described set-up, a weak coupling of this qubit with the heat-bath may lead to a high steady-state temperature, while transient cooling can still be achieved in this regime due to the interaction between the qubits driving them from their thermal equilibrium with their respective heat bath.

Cooling that bypasses time control. In scenarios where TC occurs without SSC, one has to freeze the dynamics to achieve \( T_{\text{min}} \), as evident from fig. 1(a). Freezing the dynamics can potentially be achieved, e.g., by periodic driving. However, we are exclusively considering here a dynamics without requiring external control. In pointing towards a mechanism to get around this difficulty, we now describe a situation achievable by the three-qubit absorption refrigerator where the cold qubit temperature monotonically decreases to its minimum temperature at the steady state, i.e., \( T_{\text{min}} = T_1^* \) (see fig. 2 for an example).

Here, qubit-bath interaction parameters satisfy \( p_1 < p_2, p_3 \) (which can be achieved from the forms of \{\( p_i \)\} by simply interchanging \( p_1 \) and \( p_2 \)). Note that unlike previous studies [15], damped coherent dynamics is observed here without any oscillation in the transient temperature \( T_c(t) \). This bypasses the challenge of precise control needed for freezing the dynamics at a particular time to achieve the cooling. Also, the scenario is such that the more one waits, the colder the qubit s/he gets. The value of \( T_c^* \) is found to increase with increasing \( g \). We point out here that the choice of the value of \( g \) is important in this case, as initial oscillation of \( T_c(t) \) is possible if the qubit-bath interaction parameters are chosen to be in the form of \{\( p_i \)\}, depending on the values of \( g \).

Ratio of heat currents. The rate of heat flow between qubit \( i \) (\( i \in \{1,2,3\} \)) and the corresponding bath at any time \( t \), called heat current, is given by \( Q_i(t) = \text{Tr}[H_i^S(t)\rho(t) - \text{Tr}[\rho(t)]] \), \( H_i \) being the local Hamiltonian of the \( i \)th qubit. The coefficient of performance (COP) of the three-qubit absorption refrigerator [10,11,16,30] is \( \varepsilon = Q^S_1/Q^S_3 \), where \( Q^S_i \) is the heat current of qubit \( i \) in steady state \( \rho^S \), given by \( Q_i = \text{Tr}[H_i^S(t)\rho^S - \text{Tr}[\rho^S]] \). Cooling of qubit 1 is indicated by a flow of heat from the bath to the qubit, which is reflected by a positive value of \( Q_1(t) \). We study how the ratio of the heat currents approach the value of COP at steady state. Figure 3 shows this ratio as functions of time with different values of \( g \), for both the cases when TC without SSC occurs, or SSC takes place with transient temperature monotonically decreasing to the steady-state temperature. Please note that this ratio can have a higher value at transient times compared to the steady-state value. But if we want to achieve the higher transient value, we have to introduce an external device to switch off the inter-qubit interaction at that point of time. The work thus performed by the external device should be taken into account while defining COP in transient regime. However, we do not take into account this work done in any of our numerical calculations, hence COP is only well-defined for steady state of refrigerator.

 Behaviour of quantum correlations. We now comment on the properties of bipartite and multipartite
correlations in the parameter space where necessarily TC takes place. As bipartite quantum correlations, we focus on logarithmic negativity (LN) [31], denoted by $\mathcal{L}$, from the entanglement-separability domain, and quantum discord (QD) [32,33], denoted by $D$, from the information-theoretic domain. In the case of multipartite correlations, we consider the tripartite total mutual information, given by $I_{\text{tot}}$ [34], monogamy scores [35] corresponding to LN and QD, and the genuine concurrence [36] detected by an entanglement witness, $W$ [37] when $W > 0$.

Since the initial state of the dynamics is a product state, both LN and QD in bipartition $i$: rest, $i = 1, 2, 3$, are zero in all bipartitions for the three-qubit state at $t = 0$. As the system evolves in time, one expects generation of bipartite quantum correlations in different bipartitions of the three-qubit system at $t > 0$. This is indeed the case for both LN and QD. However, the generated LN decays to zero quickly as time progresses, while the decay of QD is much slower, the common trait of both the measures being the extremely low values throughout the dynamics. On the other hand, in case of multiparty quantum correlations, the value of $I_{\text{tot}}$ increases with $t$ at first, reaches a maximum, and then decreases to attain a saturation value close to zero at high $t$, the overall value being very low during the dynamics. We also find the value of $W$ to be never positive during a typical evolution of the three-qubit state, indicating that no genuine tripartite entanglement is generated during the evolution. Note also that since no coherence is generated outside the subspace spanned by $|010\rangle$ and $|101\rangle$ during the dynamics in the present scenario, all the two-qubit reduced density matrices obtained from the three-qubit time-evolved state are diagonal in the computational (product) basis. This implies that in a certain bipartition, say, $1: 23$, the values of monogamy score corresponding to LN and QD are given by $\mathcal{L}_{1,23}$ and $D_{1,23}$, respectively, the features of which has already been discussed. Therefore, TC without SSC is associated with generation of no, or very low bipartite and multipartite quantum correlations during the dynamics of the system.

Here, “low” values of a quantum correlation measure is with respect to the maximum possible value of that measure when considering arbitrary quantum states.

On “local” and “global” master equations. The model described above presents a local approach of deriving the QME, as it is assumed that each bath is responsible for transition between the eigenstates of the corresponding qubit, and the dissipator $\Phi(p)$ in eq. (2) is a sum of the local dissipators of each bath. This description works fine in this weak-coupling limit, i.e., $g \ll E_I$, as reported in the literature [38]. However, for higher values of inter-qubit interaction $g$, the eigen-spectrum of the total Hamiltonian $H_{\text{loc}} + H_{\text{tot}}$ differs significantly from the spectrum of the free Hamiltonian $H_{\text{loc}}$. Hence, each bath takes part in the transition between the eigenstates of the composite system. Physically, this means that due to a higher coupling $g$, the interaction between the qubits is also mediated between the baths. In such cases, it is necessary to use the “global” master equation. In the succeeding section, we use this global master equation to describe a Markovian qubit-bath interaction, when the inter-qubit interaction is not negligible any more, i.e., $g \sim E_I$.

However, there has recently been considerable debates on whether the local master equation is thermodynamically consistent [39]. In particular, it has been demonstrated that in such a local description, the rate of total entropy production can become negative, thus potentially violating the second law of thermodynamics, whereas the global master equation remains thermodynamically consistent. But there exist counterarguments that this violation is of the order of terms neglected during the derivation of the local master equation [40, 41]. Also, several studies on exactly solvable models of quantum heat engines [38, 42] have reported that in the weak-coupling regime, the local master equation keeps good account of the thermodynamic properties as well as quantum states and heat currents, whereas the global approach fails to do so. For a relatively strong inter-qubit coupling regime, the global approach is in good agreement with the exact results but the local approach breaks down. Thus, the “local” and “global” descriptions act complementary to each other and remain valid in respective regimes [42, 43]. For intermediate values of $g$, a master equation have been proposed that reduces to the local and global master equations, respectively for weak and ultrastrong values of $g$ [21, 44].
of the mode $k$ of the bath corresponding to qubit $i$, and the $b_{i,k}^\dagger$ ($b_{i,k}$) is the bosonic mode creation (annihilation) operator. The interaction between the qubit and its bath is modelled by a coupling between the spin degrees of freedom of the system with the momentum modes of the bath, $H_{sb} = \sqrt{\gamma_i}(c_i^x \sigma_i^x + c_i^y \sigma_i^y) \otimes \sum_k g_k^i (\eta_k^i - \eta_k^i)$, so that the total system-bath interaction Hamiltonian is given by $\hat{H}_{sb} = \sum_i H_{sb}$. Here, $c_i^\alpha \in \mathbb{R}$, where we set $c_i^0 = 0$, and one can absorb $c_i^\alpha$ into $\sqrt{\gamma_i}$. The total Hamiltonian describing the system consisting of the three qubits and their respective baths is then given by $\hat{H}_{\text{tot}} = (H_{\text{loc}} + H_{\text{int}}) + H_b + H_{sb}$.

The “global” QME in the present case can be derived by going over to the interaction picture, and successive application of the Born-Markov and rotating wave approximations [30]. The dynamical term in the QME in this case has the Lindblad form

$$\Phi(\rho) = \sum_{i,\omega} \tilde{\gamma}_i(\omega) \left[ \mathcal{L}_i^\omega \rho \mathcal{L}_i^{\omega\dagger} - \frac{1}{2} \{ \mathcal{L}_i^\omega \mathcal{L}_i^{\omega\dagger}, \rho \} \right],$$

(5)

where the Lindblad operators $\{\mathcal{L}_i^\omega\}$, corresponding to the incoherent jumps between different eigenstates of the composite system having energy gap $\omega$ and satisfying $\mathcal{L}_i^{\omega\dagger} = \mathcal{L}_i^{-\omega}$, can be explicitly determined by decomposing the system degrees of freedom in the system-bath interaction term (for qubit $i$ in the present case) in terms of the eigenoperators of the system Hamiltonian, $\hat{H}_{\text{loc}} + \hat{H}_{\text{int}}$ [17]. We assume that each bath corresponding to qubit $i$ is described by the Ohmic spectral function, $J_i(\omega) = \alpha_i \omega \exp(-\omega/\Omega)$, where $\alpha_i$ is the dimensionless coupling strength defining the qubit-bath coupling. We further assume that the “cut-off frequency”, $\Omega$, has a high enough value, leading to a negligible memory time of the bath ($\sim \Omega^{-1}$), so that the Markovian approximation is a valid one. The incoherent transition rates, $\{\tilde{\gamma}_i(\omega)\}$, obtained in terms of the spectral function of the baths as [30] $\tilde{\gamma}_i(\omega) = \int_0^\infty |J_i(\omega)|^2 \rho(\omega, \tilde{\beta}) d\omega$, when $\omega > 0$ and $\tilde{\gamma}_i(\omega) = \int_0^\infty |J_i(\omega)|^2 \rho(\omega, \tilde{\beta}) d\omega$, when $\omega < 0$, $f(\omega, \tilde{\beta}) = \{ \exp(h \tilde{\beta} \omega) - 1 \}^{-1}$ represents the Bose-Einstein distribution. One can go over to the dimensionless form of eq. (2) by introducing the dimensionless variables $\gamma_i(\omega) = \frac{\hbar}{\beta} \gamma_i(\omega)$, and $J(\omega) = \frac{\hbar}{\beta} J(\omega)$. Note here that for the rotating wave approximation to be a valid one, in the present case, one has to consider a parameter space where a typical time-scale of the system is much smaller than the dissipation time, implying $\min\{E_i, g\} \gg \max\{\gamma_i\}$ [17].

The model of thermalization described above exhibits all the characteristic traits of necessarily transient cooling phenomenon, as well as the instance of monotonic decrease to steady-state temperature, along with behaviour of the COP and generation of very low quantum correlations, similar to the reset model. However, in contrast to the reset model, with increasing $g$, the value of $T_1^\ast$ increases, and eventually crosses $T_1$, thereby moving over to a region where a steady-state heating of the cold qubit takes place (see fig. 4) along with TC. In this scenario, the necessity of a transient refrigeration of the cold qubit is pressing, and is obtainable at a sufficiently low time, as shown in the figure. Note that in the previous model, we were unable to find any range of parameters where $T_1^\ast > T_1$, which is observed in this model.

A word on the occurrence of the steady-state heating in the case of the harmonic-oscillator heat bath, choosing $\alpha_1 = 10^{-4}$, $\alpha_2 = 10^{-5}$, $\alpha_3 = 10^{-3}$, for different values of $g$, where the system parameters are chosen as $E_1 = E_3 = 1$, $T_1 = T_2 = 1$, $T_3 = 2$. All quantities plotted are dimensionless.

![Fig. 4: Variation of $T_1(t)$ against $t$ in the case of harmonic-oscillator heat bath, choosing $\alpha_1 = 10^{-4}$, $\alpha_2 = 10^{-5}$, $\alpha_3 = 10^{-3}$, for different values of $g$, where the system parameters are chosen as $E_1 = E_3 = 1$, $T_1 = T_2 = 1$, $T_3 = 2$. All quantities plotted are dimensionless.](image-url)
any effective cooling in the steady state of the system. We demonstrate the results for two different models of thermalization, namely, a reset model, and an infinite harmonic-oscillator bath model for the thermal reservoirs. We also show that a steady-state heating is possible in the latter case, which emphasizes the necessity of transient cooling in order to obtain refrigeration. We also comment on the performance of the machine as well as the generation of bipartite and multipartite quantum correlations in the system when the refrigerator is working in the regime of necessarily transient cooling. Our results remain qualitatively the same for two different models of thermalization, one operating in the weak coupling, while the other in the strong-coupling limit. However, the potential of this finding to be generic for any appropriate model of thermalization in the three-qubit quantum absorption refrigerator set-up is a topic requiring further investigation.

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