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R-Parity Violation in Flavor Changing Neutral Current Processes and Top Quark Decays

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Abstract

We show that supersymmetric R-parity breaking ($R_p$) interactions always result in Flavor Changing Neutral Current (FCNC) processes. Within a single coupling scheme, these processes can be avoided in either the charge $+2/3$ or the charge $-1/3$ quark sector, but not both. These processes are used to place constraints on $R_p$ couplings. The constraints on the first and the second generations are better than those existing in the literature. The $R_p$ interactions may result in new top quark decays. Some of these violate electron-muon universality or produce a surplus of $b$ quark events in $t\bar{t}$ decays. Results from the CDF experiment are used to bound these $R_p$ couplings.

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1 Introduction

The Minimal Supersymmetric Standard Model (MSSM) \([1]\) with the gauge group 
\(G = SU(3)_c \times SU(2)_L \times U(1)_Y\) contains the Standard Model particles and 
their superpartners, and an additional Higgs doublet. In order to produce the 
observed spectrum of particle masses, the superpotential is given by

\[
\lambda_{ij} L_i E^c_j H + \lambda_{jk} H Q_j D^c_k + \lambda_{ij}^U U^c_i Q_j H' + \mu H H'
\]  

where \(L = \begin{pmatrix} N \\ E \end{pmatrix}\) and \(Q = \begin{pmatrix} U \\ D \end{pmatrix}\) denote the chiral superfields containing the 
lepton and quark \(SU(2)_L\) doublets and \(E^c, U^c\) and \(D^c\) are the \(SU(2)_L\) singlets, 
all in the weak basis. \(H\) and \(H'\) are the Higgs doublets with hypercharges \(-1\) and 
\(+1\) respectively. The \(SU(2)_L\) and \(SU(3)_c\) indices are suppressed, and \(i, j\) and \(k\) 
are generation indices. However, requiring the Lagrangian to be gauge invariant 
does not uniquely determine the form of the superpotential. In addition, the 
following renormalizable terms

\[
\lambda_{ijk} L_i L_j E^c_k + \tilde{\lambda}_{ijk} L_i Q_j D^c_k + \lambda_{ij}'' U^c_i D^c_j D^c_k
\]  

are allowed\(^\star\). Unlike the interactions of the MSSM, these terms violate lepton 
number and baryon number. They may be forbidden by imposing a discrete 
symmetry, \(R\)-parity, which is \((-1)^{3B+L+2S}\) on a component field with baryon 
number \(B\), lepton number \(L\) and spin \(S\). Whether this symmetry is realized 
in nature must be determined by experiment. If both lepton and baryon number 
violating interactions are present, then limits on the proton lifetime place 
stringent constraints on the products of most of these couplings. So, it is usually 
assumed that if \(R\)-parity is violated, then either lepton or baryon number 
violating interactions, but not both, are present. It is interesting that despite 
the large limits on the proton lifetime, some products of the \(R\)-parity violating 
couplings remain bounded only by the requirement that the theory remain perturbative \(^3\). If either \(L_i Q_j D^c_k\) or \(U^c_i D^c_j D^c_k\) terms are present, flavor changing 
neutral current (FCNC) processes are induced. It has been assumed that if only 
one \(R\)-parity violating (\(R_p\)) coupling with a particular flavor structure is non-
zero, then these flavor changing processes are avoided. In this single coupling

\(^\star\)A term \(\mu_i L_i H'\) is also allowed. This may be rotated away through a redefinition of the \(L\) 
and \(H\) fields \(^3\).
scheme \[4\] then, efforts at constraining \(R\)-parity violation have concentrated on flavor conserving processes \[5, 6, 7, 8, 9, 10\]. It is surprising that, even though individual lepton or baryon number is violated in this scheme, the constraints are rather weak.

In Section 2, we demonstrate that the single coupling scheme cannot be realized in the quark mass basis. Despite the general values the couplings may have in the weak basis, after electroweak symmetry breaking there is at least one large \(R\) coupling and many other \(R\) couplings with different flavor structure. Therefore, in the mass basis the \(R\)-parity breaking couplings cannot be diagonal in generation space. Thus, flavor changing neutral current processes are always present in either the charge 2/3 or the charge \(-1/3\) quark sectors. We use these processes to place constraints on \(R\)-parity breaking. We find constraints on the first and the second generations that are much stronger than existing limits.

The recent discovery of the top quark \[11, 12\] with the large mass of 176 GeV opens the possibility for the tree level decays \(t \rightarrow \tilde{l}_i^+ + d_k\) and \(t \rightarrow d_j^+ + d_k\) if \(R\)-parity is broken. If the \(R\) coupling are large enough, then these decay channels may be competitive with the Standard Model decay \(t \rightarrow b + W\). As no inconsistencies between the measured branching fractions and production cross-section of the top quark and those predicted by the Standard Model (SM) have been reported, limits on the branching fractions for the \(R\) decay channels may be obtained. Since the existing lower bound on the mass of the lightest slepton is \(\sim 45\) GeV \[13\], while the strong interactions of the squarks make it likely that the squarks are heavier than the sleptons, the decay \(t \rightarrow \tilde{l}_i^+ + d_k\) is more probable. In our analysis, we therefore assume that only the slepton decay channel is present. In Section 3 we analyse the \(R\) top decay channels to place constraints on the \(t \rightarrow \tilde{l}_i^+ + d_k\) coupling. For this reason, in this paper we assume that only the \(L\) terms \(L_iQ_jD_k^c\) are present. The conclusions of Section 2, however, are valid even if the \(L_iL_jE_k^c\) terms are also present. Constraints on products of couplings when both \(L\) interactions are present may be found in reference \[14\]. In Section 4 we summarize our results and compare them with limits existing in the literature.
2 Flavor Changing Neutral Current Processes

Flavor changing neutral current processes are more clearly seen by examining the structure of the interactions in the quark mass basis. In this basis, the $\tilde{\lambda}_{ijk}$ interactions are

$$\lambda'_{ijk}(N^m_i(V_{KM})_jlD^m_l-E^m_iU^m_j)D^m_k$$

where

$$\lambda'_{ijk} = \bar{\lambda}_{imn}U_{Ljm}D^*_{Rnk}$$

The superfields in Equation (3) have their fermionic components in the mass basis so that the Cabibbo-Kobayashi-Masakawa (CKM) matrix $V_{KM}$ appears explicitly. The rotation matrices $U_L$ and $D_R$ appearing in the previous equation are defined by

$$u_{Li} = U_{Lij}u^m_{Lj}$$
$$d_{Ri} = D_{Rij}d^m_{Rj}$$

where $q_i$ ($q^m_i$) are quark fields in the weak (mass) basis. Henceforth, all the fields will be in the mass basis and we drop the superscript $m$.

Unitarity of the rotation matrices implies that the couplings $\lambda'_{ijk}$ and $\bar{\lambda}_{ijk}$ satisfy

$$\sum_{jk} |\lambda'_{ijk}|^2 = \sum_{mn} |\bar{\lambda}_{imn}|^2$$

So any constraint on the $R_p$ couplings in the quark mass basis also places a bound on the $R_p$ couplings in the weak basis.

In terms of component fields, the interactions are

$$\lambda'_{ijk}[(V_{KM})_jl(\bar{\nu}_L\bar{d}_R\nu^i_L + \bar{d}_L\bar{d}_R \nu^i_L + (\bar{d}_R)^* (\nu^i_L)^c d^i_L) - \bar{e}_L d^i_R u^i_L - \bar{\nu}_L (\bar{d}_R)^* (\nu^i_L)^c u^i_L]$$

where $e$ denotes the electron and $\bar{e}$ it’s scalar partner and similarly for the other particles.

The contributions of the $R$-parity violating interactions to low energy processes involving no sparticles in the final state arise from using the $R_p$ interactions an even number of times. If two $\lambda'$ 's or $\lambda''$ 's with different flavor structure are non-zero, flavor changing low energy processes can occur. These processes are
considered in references [2] and [16], respectively. Therefore, it is usually assumed that either only one \( \lambda \) with a particular flavor structure is non-zero, or that the \( R \)-parity breaking couplings are diagonal in generation space. However, Equation (8) indicates that this does not imply that there is only one set of interactions with a particular flavor structure, or even that they are diagonal in flavor space. In fact, in this case of one \( \lambda'_{ijk} \neq 0 \), the CKM matrix generates couplings involving each of the three down-type quarks. Thus, flavor violation occurs in the down quark sector, though suppressed by the small values of the off-diagonal CKM elements. Below, we use these processes to obtain constraints on \( R \)-parity breaking, assuming only one \( \lambda'_{ijk} \neq 0 \).

It would be more natural to assume that there is only one large \( R_p \) coupling in the weak basis, i.e., only one \( \lambda_{ijk} \neq 0 \). As we have indicated, this generates many couplings with different flavor structure in the mass basis, e.g., many \( \lambda'_{imn} \)'s. It is possible that

\[ \lambda'_{imn} \simeq \bar{\lambda}_{ijk} V_{KMj} \delta_{kn} \tag{9} \]

This will be the case if, for example, the rotation to the mass basis occurs only for the charge \(+2/3\) quark sector. Then, in addition to the Feynman diagrams that contribute to the flavor changing neutral current processes when only one \( \lambda'_{ijk} \) is present, there are new contributions involving the \( \lambda'_{imn} (m \neq j, n = k) \) vertices. However, these new contributions interfere constructively with the operators that are present in the effective Lagrangian that is generated when there is only one non-zero \( \lambda'_{ijk} \). So if these more natural assumptions are made, any constraint found for \( \bar{\lambda}_{ijk} \) is slightly better than the constraint that is obtained when only one \( \lambda'_{ijk} \) is present.

It would seem that the flavor changing neutral current processes may be rotated away by making a different physical assumption concerning which \( R_p \) coupling is non-zero. For example, while leaving the quark fields in the mass basis, Equation (3) gives

\[
W_{R_p} = \lambda'_{ijk} (N_i (V_{KM})_{jl} D_l - E_i U_j) D_c^k \\
= (\lambda'_{ijk} V_{KMj}) (N_i D_l - E_i (V_{KMP}^{-1}) U_p) D_c^k \\
= \bar{\lambda}_{ijk} (N_i D_j - E_i (V_{KMP}^{-1}) U_p) D_c^k \tag{10}
\]
Figure 1: $R_p$ contributions to $K^0-\bar{K}^0$ mixing with one $\lambda'_{ijk} \neq 0$. Arrows indicate flow of propagating left handed fields.

where

$$\tilde{\lambda}_{ijk} \equiv \lambda'_{imk}(V_{KM})_{mj} \quad (13)$$

With the assumption that the $\lambda'_{ijk}$ coefficients have values such that only one $\tilde{\lambda}_{ijk}$ is non-zero, there is only one interaction of the form $N_L D_L D^c$. There is then no longer any flavor violation in the down quark sector. In particular, there are no $R_p$ contributions to the processes discussed below. But now there are couplings involving each of the three up type quarks. So these interactions contribute to FCNC in the up sector; for example, $D^0-\bar{D}^0$ mixing. We use $D^0-\bar{D}^0$ mixing to place constraints on $R$-parity violation assuming only one $\tilde{\lambda}_{ijk} \neq 0$. Thus, there is no basis in which FCNC can be avoided in both sectors.

### 2.1 $K^0-\bar{K}^0$ Mixing

With one $\lambda'_{ijk} \neq 0$, the interactions of Equation (8) involve down and strange quarks. So, there are contributions to $K^0-\bar{K}^0$ mixing through the box diagrams shown in Figure (1).

Evaluating these diagrams at zero external momentum and neglecting the down quark masses, the following effective Hamiltonian is generated

$$\mathcal{H}^{S=2}_{R_p} = \frac{1}{128\pi^2} |\lambda'_{ijk}|^4 \left( \frac{1}{m^{2}_{\tilde{\nu}_i}} + \frac{1}{m^{2}_{d_Rk}} \right) ((V_{KM})_{j2}(V_{KM})_{j1})^2 (\bar{d}_L \gamma^\mu s_L)^2 \quad (14)$$

where $m_{\tilde{\nu}_i}$ is the sneutrino mass and $m_{d_Rk}$ is the right-handed down squark mass. As this operator is suppressed by the CKM angles, it is largest when $\lambda'_{ijk}$ is non-zero for $j = 1$ or $j = 2$.

The SM effective Hamiltonian is [17]

$$\mathcal{H}^{S=2}_{SM} = \frac{G_F^2}{4\pi^2} m_c^2 ((V_{KM})_{12}(V_{KM})_{11})^2 (\bar{d}_L \gamma^\mu s_L)^2 \quad (15)$$
where the CKM suppressed top quark contribution, the up quark mass, and QCD radiative corrections have been ignored. As the uncertainty in hadronic matrix elements of the Standard Model effective Hamiltonian are at most 40%, a conservative constraint on the \( R_p \) coupling is obtained by demanding that \( \mathcal{L}^{\Delta S=2}_{R_p} \leq 0.5 \mathcal{L}^{\Delta S=2}_{SM} \). This gives the constraint

\[
|\lambda'_{ijk}| \leq 0.08 \left( \frac{1}{z_i^2} + \frac{1}{w_k^2} \right)^{-\frac{1}{4}}
\]

where \( z_i = m_{\tilde{\nu}_i}/(100 \text{ GeV}) \) and \( w_k = m_{\tilde{d}_{Rk}}/(100 \text{ GeV}) \). This constraint applies for \( j = 1 \) or \( j = 2 \) and for any \( i \) or \( k \). The constraint for \( j = 3 \) is not interesting as the CKM angles suppress the \( R_p \) operator relative to the Standard Model operator.

### 2.2 \( B^0 - \bar{B}^0 \) Mixing

The \( R_p \) interactions also contribute to both \( B^0 - \bar{B}^0 \) mixing and \( B_s^0 - \bar{B}_s^0 \) mixing through box diagrams similar to those given in the previous section. As \( B_s^0 - \bar{B}_s^0 \) mixing is expected to be nearly maximal, it is not possible at present to place a constraint on any non-Standard Model effects that would add more mixing. However, \( B^0 - \bar{B}^0 \) mixing has been observed \[18\] with a moderate \( x_d \sim 0.7 \) \[13\]. As lattice QCD calculations predict \( B_K \sim 0.6 \) \[19\] and \( B_B \sim 1.2 \) \[20\], it is reasonable to expect that any \( R_p \) contributions to \( B^0 - \bar{B}^0 \) mixing should not exceed 50% of the amount expected from the Standard Model alone.

The effective Hamiltonian generated by these \( R_p \) processes is

\[
\mathcal{H}_{R_p} = \frac{1}{128\pi^2} |\lambda'_{ijk}|^4 \left( \frac{1}{m_{\tilde{\nu}_i}^2} + \frac{1}{m_{\tilde{d}_{Rk}}^2} \right) ((V_{KM})_{j3}(V_{KM})_{j1}^*)^2 (\bar{d}_L \gamma^\mu b_L)^2
\]

This is largest when \( \lambda'_{i3k} \) is non-zero.

The dominant contribution to \( B^0 - \bar{B}^0 \) mixing in the Standard Model is \[21\]

\[
\mathcal{H}^{\Delta S=2}_{SM} = \frac{G_F^2 m_t^2}{4\pi^2} ((V_{KM})_{33}(V_{KM})_{31}^*)^2 G(x_t)(\bar{d}_L \gamma^\mu b_L)^2
\]

where \( x_t = m_t^2/m_W^2 \), and

\[
G(x) = \frac{4 - 11x + x^2}{4(x - 1)^2} - \frac{3x^2 \ln x}{2(1 - x)^3}
\]

\[16\]
Figure 2: $\mathcal{R}_p$ contribution to $K^+ \to \pi^+ \nu \bar{\nu}$ with one $\lambda'_{ijk} \neq 0$.

For a top mass of 176 GeV, $G(x_t) = 0.54$.

This gives the constraint

$$|\lambda'_{ijk}| \leq 0.77 \left( \frac{1}{z_i^2} + \frac{1}{w_k^2} \right)^{-\frac{1}{4}}$$

with $z_i$ and $w_k$ as previously defined.

In addition to inducing $B^0 - \bar{B}^0$ mixing, these interactions also contribute to the $b \to s + \gamma$ amplitude. However, with reasonable values for squark and sneutrino masses, the constraint is significantly weaker than that found from the top quark analysis.

### 2.3 $K^+ \to \pi^+ \nu \bar{\nu}$

The tree level Feynman diagram in Figure (2) generates an effective Hamiltonian which contributes to the branching ratio for $K^+ \to \pi^+ \nu \bar{\nu}$. Using a Fierz rearrangement, a straightforward evaluation of this diagram gives

$$\mathcal{H}_{B_p} = \frac{1}{2 m_{\tilde{d}_R}^2} |\lambda'_{ijk}|^2 (V_{KMj1}^* V_{KMj2})(\bar{s}_L \gamma^\mu L_L)(\bar{\nu}_L \gamma_\mu \nu_L)$$

(21)

There is also a Standard Model contribution to this decay [21]. This is an order of magnitude lower than the existing experimental limit. To obtain a bound on the $\mathcal{R}_p$ coupling, we shall assume that the $\mathcal{R}_p$ effects dominate the decay rate.

As the matrix element for this semi-leptonic decay factors into a leptonic
and a hadronic element, the isospin relation
\[
\langle \pi^+(p) | \bar{s} \gamma_\mu d | K^+(k) \rangle = \sqrt{2} \langle \pi^0(p) | \bar{s} \gamma_\mu u | K^+(k) \rangle
\] (22)
can be used to relate \( \Gamma[K^+ \to \pi^+ \nu \bar{\nu}] \) to \( \Gamma[K^+ \to \pi^0 \nu e^+] \). The effective Hamiltonian for the neutral pion decay channel arises from the spectator decay of the strange quark. It is
\[
H_{\text{eff}} = \frac{4G_F}{\sqrt{2}} V_{KM12}^* (\bar{s}_L \gamma_\mu u_L) (\bar{\nu}_L \gamma_\mu e_L)
\] (23)
So in the limit where the lepton masses can be neglected,
\[
\frac{\Gamma[K^+ \to \pi^+ \nu_i \bar{\nu}_i]}{\Gamma[K^+ \to \pi^0 \nu_e + \nu_e]} = \left( \frac{|\lambda'_{ijk}|^2}{4G_F m^2_{d_{jk}}} \right)^2 \left( \frac{|V_{KM11} V_{KM22}|^2}{|V_{KM12}|^2} \right)
\] (24)
This ratio is valid for \( i = 1, 2, or 3 \), since in the massless neutrino and electron approximation, the integrals over phase space in the numerator and denominator cancel. So using \( BR[K^+ \to \pi^+ \nu \bar{\nu}] \leq 5.2 \times 10^{-9} \) \( (90\% \text{ CL}) \) and \( BR[K^+ \to \pi^0 \nu e^+] = 0.0482 \) \( (13) \), the constraint is
\[
|\lambda'_{ijk}| \leq 0.012 \left( \frac{m_{d_{jk}}}{100 \text{ GeV}} \right) (90\% \text{ CL})
\] (25)
for \( j = 1 \) or \( j = 2 \). Using \( |V_{KM13}| \geq 0.004 \) \( (13) \) and \( |V_{KM23}| \geq 0.03 \) \( (13) \), a conservative upper bound for \( \lambda'_{3jk} \) is
\[
|\lambda'_{3jk}| \leq 0.52 \left( \frac{m_{d_{jk}}}{100 \text{ GeV}} \right) (90\% \text{ CL})
\] (26)

2.4 \( D^0-\bar{D}^0 \) Mixing

If there is only one \( \tilde{\lambda}_{ijk} \) in the mass basis, then from Equation (12) it is clear that flavor changing neutral current processes will occur in the charge +2/3 quark sector. Rare processes such as \( D^0-\bar{D}^0 \) mixing, \( D^0 \to \mu^+ \mu^- \) and \( D^+ \to \pi^+ l^+ l^- \), for example, may be used to place tight constraints on \( \tilde{\lambda}_{ijk} \). For illustrative purposes, in this section we will consider \( D^0-\bar{D}^0 \) mixing.

The interactions in Equation (12) generate box diagrams identical to those discussed in the previous sections if both the internal sneutrino (neutrino) propagators are replaced with slepton (lepton) propagators and the external quarks
lines are suitably corrected. Using the same approximations that we re made
earlier, the $R_p$ effects generate the following effective Hamiltonian

$$H_{R_p} = \frac{1}{128\pi^2} |\tilde{\lambda}_{ijk}|^4 \left( \frac{1}{m_{\tilde{l}_i}} + \frac{1}{m_{\tilde{d}_R}} \right) \left( (V_{KM})_{2i} (V_{KM})_{1j}^* \right)^2 (\bar{\ell}_L \gamma^\mu u_L)^2$$  \hspace{1cm} (27)

$$\equiv G(\tilde{\lambda}_{ijk}, m_{\tilde{l}_i}, m_{\tilde{d}_R}) (\bar{\ell}_L \gamma^\mu u_L)^2$$  \hspace{1cm} (28)

In the vacuum saturation approximation, the $R_p$ effects contribute an amount

$$(\Delta m)^{th} = m_{D_L} - m_{D_S} = \frac{2}{3} f_D^2 m_D Re G(\tilde{\lambda}_{ijk}, m_{\tilde{l}_i}, m_{\tilde{d}_R})$$  \hspace{1cm} (29)

to the $D_L - D_S$ mass difference. With $f_D = 200$ MeV \cite{23}, $m_D = 1864$ MeV \cite{13}, and $|(\Delta m)^{exp}| \leq 1.32 \times 10^{-10}$ MeV \cite{13}(90%CL), the constraint on $\tilde{\lambda}_{ijk}$ for $j = 1$ or $j = 2$ is

$$|\tilde{\lambda}_{ijk}| \leq 0.16 \left( \left( \frac{100 \text{ GeV}}{m_{\tilde{l}_i}} \right)^2 + \left( \frac{100 \text{ GeV}}{m_{\tilde{d}_R}} \right)^2 \right)^{-\frac{1}{4}}$$  \hspace{1cm} (30)

3 Top Quark Decay

In the Standard Model, the dominant decay mode for the top quark is

$$t \to b + W$$  \hspace{1cm} (31)

with a real $W$ gauge boson produced. This has a partial decay width

$$\Gamma[t \to W + b] = \frac{G_F^3 m_t^3}{8\pi \sqrt{2}} |V_{tb}|^2 (1 - x_W^2)(1 - 2x_W^4 + x_W^4)$$  \hspace{1cm} (32)

where $x_W = m_W/m_t$. The $b$ quark mass has been neglected.

The $R$-parity violating interactions (see Equation (8) with $j = 3$) $\lambda_{ijk}^c \bar{\ell}^{\dagger}_i L d^{\dagger}_R t_L$ contribute to the decay $t_L \to \tilde{l}^+_i + d_R$ at tree level \cite{24}, if kinematically allowed. This is possible only if there exist sleptons lighter than the top quark. The partial width for this process is

$$\Gamma[t \to \tilde{l}^+_i + d_k] = \frac{|\lambda_{ijk}^c|^2 m_t (1 - y_i^2)^2}{32\pi}$$  \hspace{1cm} (33)
with \( y_i \equiv m_i/m_t \) \[24\]. The mass of the down type quark has been neglected. If this is the only non-zero \( R \)-parity coupling, the two top quark decay channels are \( t \to b + W \) and \( t \to d_{Rk} + \tilde{l}_i^+ \), with branching fractions \( 1 - x \) and \( x \), respectively.

We assume that the Lightest Supersymmetric Particle (LSP), denoted by \( \tilde{\chi}_0 \), is neutral and that the real slepton decays with 100\% branching fraction to the \( \tilde{\chi}_0 \) and a lepton. The presence of a non-zero \( R \)-parity breaking coupling implies that the \( \tilde{\chi}_0 \) is no longer stable \[1\]. The two dominant decays are \[24\] \( \tilde{\chi}_0 \to \nu_i + b + \bar{d}_k \) and \( \tilde{\chi}_0 \to \bar{\nu}_i + \bar{b} + d_k \). The LSP decays inside the detector if \[4\]

\[
|\lambda_{i3k}'| \geq 6 \times 10^{-5} \sqrt{\gamma} \left( \frac{100\, GeV}{m_{\tilde{d}_{Rk}}} \right)^2 + \left( \frac{100\, GeV}{m_{\tilde{b}_R}} \right)^2 \left( \frac{100\, GeV}{m_{\tilde{\chi}_0}} \right)^{5/2}
\]

where \( \gamma \) is the Lorentz boost factor of \( \tilde{\chi}_0 \). For this decay chain to be kinematically allowed, we require that \( m_{\tilde{\chi}_0} \geq m_b \) for \( k = 1 \) or \( k = 2 \), and \( m_{\tilde{\chi}_0} \geq 2m_b \) for \( k = 3 \). Using the previous equation, the maximum lower bound on \( \lambda_{i3k}' \) such that the LSP decays inside the detector is 0.0003 \( \times \sqrt{\gamma} \) for \( k = 3 \), and 0.002 \( \times \sqrt{\gamma} \) for \( k = 1 \) or \( k = 2 \); all for 300 \( GeV \) squark masses. We shall assume that \( \lambda_{i3k}' \) is larger than this value so that the LSP decays within the detector.

If a top quark decays through this \( R \)-parity violating process, the final state will contain one lepton, at least one \( b \) quark and missing transverse energy. The two novel features of this decay channel are that it spoils lepton universality and, when \( k = 3 \), produces a surplus of \( b \) quark events. Both of these signatures can be used to test the strength of \( R \)-parity violation.

The CDF collaboration reconstructs \( t\bar{t} \) quark events from observing: (1) dilepton (electron or muon) events coming from the leptonic decays of both the \( W \)’s; or (2) one lepton event arising from leptonic decay of one \( W \) and jets from the hadronic decay of the remaining \( W \) boson. CDF also requires a \( b \)-tag in the lepton+jets channel. If the lightest slepton has a mass between 50 and 100 \( GeV \), then the kinematics of the decay \( \tilde{l}_i \to \tilde{\chi}_0 + l_i \) will be similar to that of the leptonic decay of the \( W \) boson. A slepton of mass less than 45 \( GeV \) is ruled out by the LEP limit on the \( Z \) decay width \[13\]. If the slepton mass is close to the top mass, then the \( b \) quark produced in the top decay via this channel will have less energy than the \( b \) quark from the top decay via the SM channel. Also, the lepton from the slepton decay will have more energy than the lepton from the \( W \) decay. These will affect the lepton and the \( b \) quark detection efficiencies.
Although these decay channels will be present for any slepton lighter than the top quark, for the purpose of obtaining a constraint, we shall assume that there is a slepton with a mass in the range given above. The presence of the $R$-parity violating coupling will then contribute signals to all of these channels.

We assume that the $i = 1$ coupling is non-zero. However, all that is required is that the slepton in the generation with the non-zero coupling have a mass in the range quoted above, i.e., if $\lambda'_{13k} \neq 0$ then we require $50 \text{GeV} < m_{\tilde{e}} < 100 \text{GeV}$, and if $\lambda'_{23k} \neq 0$ then we require $50 \text{GeV} < m_{\tilde{\mu}} < 100 \text{GeV}$. Assuming also that the CDF data is consistent with lepton universality, the constraints we obtain for $\lambda'_{13k}$ and $\lambda'_{23k}$ are identical.

In the $k = 1, 2$ cases, two $b$ quarks are always produced in a $t \bar{t}$ event. In the $k = 3$ case, the LSP decays into $\bar{b}b\nu_i$ or $\bar{b}b\bar{\nu}_i$. Thus, four or six $b$ quarks may be produced if one or both of the top quarks decay through the $R$-parity breaking channel; this possibility must be treated separately.

### 3.1 $\lambda'_{i3k}, k \neq 3$

The branching fraction for the di-electron event is

$$BR[t \bar{t} \to ee + X] = x^2 + L^2(1-x)^2 + 2Lx(1-x)$$

with $L =$ leptonic branching fraction of $W$, approximately $1/9$. The first term arises from both top quarks decaying via the $R$-parity violating interaction; the second is the Standard Model contribution; and the third is the contribution from one top quark decaying through the $R$-parity breaking channel and the other top quark decaying through the Standard Model channel. The other branching fractions are

$$BR[t \bar{t} \to \mu \mu + X] = L^2(1-x)^2$$

$$BR[t \bar{t} \to \mu e + X] = 2(1-x)^2L^2 + 2x(1-x)L$$

$$BR[t \bar{t} \to \mu + \text{jets}] = 2(1-x)^2L(1-3L)$$

$$BR[t \bar{t} \to e + \text{jets}] = 2(1-x)^2L(1-3L) + 2x(1-x)(1-3L)$$

The factor of $1 - 3L$ is the hadronic branching fraction of the $W$ boson. We have also assumed that the branching fraction for $\tilde{l} \to l + \chi^0$ is close to one. We
are ignoring leptonic events produced from the Standard Model decay of the W boson into $\tau\nu_\tau$.

Two independent constraints on the $R_\rho$ interactions may be obtained from the top quark data. CDF has observed the $t\bar{t}$ cross section to be $\sigma(t\bar{t})_{\text{exp}} = 6.8^{+3.6}_{-2.4}$ pb \cite{12}. The QCD calculation \cite{25} gives the value $\sigma(t\bar{t})_{\text{th}} = 4.79^{+0.67}_{-0.41}$ pb for $m_t = 176 GeV$.

The first method is to compare the ratio of theoretically predicted values for the numbers of events found in two channels with the experimentally observed ratio. For example, $\sigma(t\bar{t})_{\text{th}} \times BR[t\bar{t} \to \mu + \text{jets}] \times \int Ldt \times \text{(detection efficiencies)}$ is the number of $\mu$ + jets events that should have been observed where $\int Ldt$ is the integrated luminosity. This theoretical prediction contains uncertainties in both the value for the $t\bar{t}$ production cross section and in the lepton and the $b$ quark detection efficiencies. In comparing the ratio

$$\frac{\sigma(t\bar{t})_{\text{th}} \times BR[t\bar{t} \to e + \text{jets}]}{\sigma(t\bar{t})_{\text{th}} \times BR[t\bar{t} \to \mu + \text{jets}]}$$ (40)

the uncertainties in the $t\bar{t}$ cross section cancel. The $b$-detection efficiencies also cancel. If the electron and the muon detection efficiencies in the lepton + jets channel are equal, these uncertainties will also cancel. The only remaining errors are statistical. The CDF collaboration reported observing 37 $b$-tagged events in the lepton + $\geq 3$ jets channel. In this set there were 50 $b$-tags, with a background of 22 $b$-tags. A conservative estimate for the background in the 37 events is 22. This leaves 15 $t\bar{t}$ events in the lepton + jets channel. Since no inconsistencies with electron-muon universality have been reported, a central value of 7 $\mu$ + jets and 7 $e$ + jets events will be assumed. This leads to

$$\frac{BR[t\bar{t} \to e + \text{jets}]_{\text{th}}}{BR[t\bar{t} \to \mu + \text{jets}]_{\text{th}}} = \frac{\#(e + \text{jets events})}{\#(\mu + \text{jets events})} = 1^{+a}_{-b}$$ (41)

Inserting the theoretical predictions for the branching ratios leads to the constraint $x < L a/(1 + L a)$, where $a$ is the uncertainty in the previous ratio. In this case, $a = b = 1/\sqrt{7}$. This gives $x < 0.077$ at 95%CL which leads to

$$|\lambda'_{13k}| \leq 0.41 \ (95\% \ CL)$$ (42)

for $k = 1$ or $k = 2$ and a slepton of mass 100 GeV.

A similar analysis may be performed for the dilepton channels. In principle these channels should lead to a good constraint since a non-zero $\lambda'_{13k}$ coupling
will lead to an excess of electrons observed in the di-electron channel over the
number of muons observed in the di-muon channel. However at present only a
small number of dilepton events have been observed and an interesting constraint
cannot be obtained.

In the other method we will compare the number of events produced in a
given channel with the theoretical expectation. The number of produced events
is $\sigma[t\bar{t}]_{th} \times BR[t \to l + \text{jets}]_{th} \times \int L dt$. Here $\sigma[t\bar{t}]_{th}$ is the production cross section
calculated in perturbative QCD for the assumed top quark mass of 176 GeV. We
will use the fact that the number of experimentally observed events in any given
channel is consistent with, within experimental errors, the number expected in
the standard model. The actual number of events detected depends upon the
detection efficiency. We will use the number of observed events in any channel
to determine the statistical accuracy with which the rate in that channel is
measured, and then constrain the strength of the $R_p$ terms by requiring that the
rate is not changed by more than the error.

This leads to the constraint

$$\frac{BR[tt \to l + \text{jets}, x]_{th}}{BR[tt \to l + \text{jets}, x = 0]_{th}} = \frac{\sigma[t\bar{t}]_{exp}}{\sigma[t\bar{t}]_{th}}$$\hspace{1cm}(43)$$

within theoretical and experimental errors. Using the theoretical and experi-
mental values for the production cross sections \[^{12, 25}\] leads to

$$\epsilon^2 \leq \frac{BR[tt \to l + \text{jets}, x]_{th}}{BR[tt \to l + \text{jets}, x = 0]_{th}} \leq 1 + d$$\hspace{1cm}(44)$$

with $\epsilon = 0.9$ and $d = 1.37$. The constraint on $x$ is then

$$x \leq \min \left(1 - \epsilon, \frac{1 - 2L - \sqrt{(1 - 2L)^2 - 4Ld(1 - L)}}{2(1 - L)}\right)$$\hspace{1cm}(45)$$

The first entry is the constraint from the $\mu + \text{jets}$ channel and the second entry
is from the $e + \text{jets}$ channel. For these values of $\epsilon$ and $d$, the constraint is
$x \leq 0.1$. For a 100 GeV slepton this translates into the constraint

$$|\lambda'_{13k}| \leq 0.46$$\hspace{1cm}(46)$$

for $k = 1$ or $k = 2$.\text{13}
3.2 $\lambda'_{133}$

For this coupling the analysis of the previous section must be modified in the lepton + jets channel since the $b$-detection efficiencies no longer cancel. This is because in the $R$-parity breaking decay channel three $b$ quarks are produced. To correct for this, introduce the function $P(k, n)$ that gives the probability that, given that $n$ $b$ quarks are produced, $k$ of them are detected. Then the number of observed single $b$ quark events expected in the $e+$jets channel is

$$
\#(e + \text{jets events}) = \left(2(1 - x)^2 L(1 - 3L)P(1, 2) + 2x(1 - x)(1 - 3L)P(1, 4)\right) \times \mathcal{N}
$$

(47)

where

$$
\mathcal{N} \equiv \int L dt \times \sigma(t\bar{t})_{th}
$$

(48)

With $P(1, 2) \leq P(1, n)$ for $n \geq 2$, then

$$
\#(e + \text{jets events}) \geq \left(2(1 - x)^2 L(1 - 3L) + 2x(1 - x)(1 - 3L)\right) P(1, 2) \times \mathcal{N}
$$

(49)

These approximations will give a conservative limit for $\lambda'_{133}$. The analysis of the previous section may now be carried out with the following restrictions:

(i) In comparing the ratio of the numbers of events detected in two channels with the theoretical prediction, the inequality in Equation (49) indicates that only upper limit in Equation (41) may be used;

(ii) In comparing the number of events detected in a channel with the theoretically predicted value for that channel, only the upper bound in Equation (44) may be used in the $e+$jets channel, and either limit may be used in the $\mu+$ jets channel. With these caveats, a conservative limit on the branching fraction for $t \to b + \tilde{t}_i^+$ is then

$$
x \leq \min \left( \frac{La}{1 + La}, 1 - \epsilon, \frac{1 - 2L - \sqrt{(1 - 2L)^2 - 4Ld(1 - L)}}{2(1 - L)} \right)
$$

(50)

For the errors quoted in the previous section, the result is

$$
|\lambda'_{133}| \leq 0.41 \ (95\% \ CL)
$$

(51)

As the $R$-parity breaking decay channels produce three $b$ quarks, then for moderate values of $\lambda'_{133}$ or $\lambda'_{233}$, semi-leptonic events containing four and six $b$ quarks
should be observable at the Tevatron. The non-observance of these events should provide the strongest test for the $R$-parity breaking couplings $\lambda'_{133}$ or $\lambda'_{233}$. If limits on the branching fractions for the $t\bar{t}$ pair to decay into these excess $b$ quark channels are known, then the $R$-parity branching fraction $x$ is constrained. Namely,

1. $BR[t\bar{t} \to X + \geq 3b's] \leq B_1 \Rightarrow x \leq \left( 1 - \sqrt{1 - B_1} \right)$

2. $BR[t\bar{t} \to X + \geq 3b's + 2e] \leq B_2 \Rightarrow x \leq \frac{\sqrt{L^2 + B_2(1 - 2L) - L}}{1 - 2L}$

3. $BR[t\bar{t} \to X + \geq 6b's + 2e] \leq B_3 \Rightarrow x \leq \sqrt{B_3}$

4. $BR[t\bar{t} \to X + \geq 3b's + e] \leq B_4 \Rightarrow x \leq \frac{1}{2} \left( 1 - \frac{2B_4}{1 - 3L} \right)$

This constrains $|\lambda'_{133}|$. To constrain $|\lambda'_{233}|$, interchange $e$ with $\mu$ in the previous equations.

The constraints on $|\lambda'_{133}|$ and $|\lambda'_{233}|$ found in this section are comparable to those obtained from examining $\mathcal{R}_p$ contributions either to $Z \to b\bar{b}$ and $Z \to l^+l^-$ decays [9] or to forward-backward asymmetry measurements in $e^+e^-$ collisions [10]. We have engaged in this exercise to illustrate how comparable $\mathcal{R}_p$ constraints may be obtained from analysing top quark decays even though the experimental and theoretical errors are still large. These processes will provide much better tests of $R$-parity violation once more top quark decays are seen.

4 Summary

In this paper we have argued that $R$-parity breaking interactions always lead to flavor changing neutral current processes. It is possible that there is a single $\mathcal{R}_p$ coupling in the charge $+2/3$ quark sector. But requiring consistency with electroweak symmetry breaking demands that $\mathcal{R}_p$ couplings involving all the charge $-1/3$ quarks exist. That is, a single coupling scheme may only be possible in either the charge $2/3$ or the charge $-1/3$ quark sector, but not both. As a result, flavor changing neutral current processes always exist in one of these
Table 1: Constraints on $|\lambda'_{ijk}|$ from: (a) $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ (90\% CL); (b) top quark decay (95\% CL); (c) atomic parity violation and $eD$ asymmetry (90\% CL) \cite{5}; (d) $\nu_\mu$ deep-inelastic scattering (95\% CL) \cite{5}; (e) partial $Z^0$ decay width (95\% CL) \cite{9}; (f) $\nu_e$ mass (90\% CL) \cite{6}. All limits are for 100 GeV sparticle masses.

The $R$-parity breaking interactions lead to the top quark decay $t \rightarrow \tilde{\ell}_i + d_k$, if the slepton is lighter than the top quark. Some of the new top quark decays spoil electron-muon universality or result in $t\bar{t}$ events with more than 2 $b$ quarks. At present, the CDF collaboration has not reported any inconsistencies with lepton universality or reported any events with more than 2 $b$ quarks. These decays also lower the branching fractions for Standard Model top quark decays. These observations are used to constrain some $R_p$ couplings.

A list of the known constraints on the $\lambda'_{ijk}$ couplings is presented in Table (1). Although several of these couplings are constrained by different low energy processes, we have only listed the smallest known upper limit.

The tightest constraint is on $|\lambda'_{ijk}|$ for $j = 1, 2$ and any $i$ and $k$. This comes from the rare decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$. With the exception of $\lambda'_{333}$, the constraints
on the third quark generation couplings are only of order $e/\sin \theta_w$. Once more top quark decays are observed the signatures discussed in this paper will more tightly constrain these couplings.

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