A Discussion on Simple Support Functions
Kandekar D.N.
Department of Mathematics, Dadapatil Rajale Arts & Science College, Adinathnagar.
Tal.: Pathardi, Dist.: Ahmednagar. (M.S.), India -414505.

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Abstract. Prediction about each and every incident happening in our daily life is impossible. But we can predict about some incidents. Probability is most helpful tool in predicting about outcomes or conclusions of such incidents. Such incidents happened in our life, always follow some known or unknown statistical probability distribution which may consist of simple or complicated probability density function. Therefore with help of probability distributions, we get some blurred idea about the functioning of incidents happening in our life. Using some commonly used probability distributions, we obtain conclusions which are helpful in decision making. Support functions viz. simple support functions are very useful in decision making. In this paper, we quote some results and applications regarding simple support function based on probability transformations.

1 Introduction

In this section, we give some preliminary background about belief function, plausibility function, commonality function and simple support function.

1.1 Frame of Discernment

Dictionary meaning of "Frame of Discernment" is frame of good judgment insight. The word "discern" means recognize or find out or hear with difficulty.

If frame of Discernment \( \Theta \) is

\[ \Theta = \{\theta_1, \theta_2, \ldots, \theta_n\} \]

then every element of \( \Theta \) is a proposition. The propositions of interest are in one-to-one correspondence with the subsets of \( \Theta \). The set of all propositions of interest corresponds to the set of all subsets of \( \Theta \), denoted by \( 2^\Theta \).

**Definition 1.1** If \( \Theta \) is frame of discernment, then a function \( m: 2^\Theta \rightarrow [0,1] \) is called basic probability assignment whenever

1. \( m(\emptyset) = 0 \).
2. \( \sum_{A \subseteq \Theta} m(A) = 1. \)

The quantity \( m(A) \) is called \( A \)'s basic probability number and it is a measure of the belief committed exactly to \( A \). The total belief committed to \( A \) is sum of \( m(B) \), for all proper subsets \( B \) of \( A \).

\[
Bel(A) = \sum_{B \subseteq A} m(B).
\]

(1)

If \( \Theta \) is a frame of discernment, then a function \( Bel: 2^\Theta \rightarrow [0,1] \) is called belief function over \( \Theta \) satisfying above condition (1).

**Theorem 1.1** If \( \Theta \) is a frame of discernment, then a function \( Bel: 2^\Theta \rightarrow [0,1] \) is belief function if and only if it satisfies following conditions

1. \( Bel(\emptyset) = 0. \)
2. \( Bel(\Theta) = 1. \)
3. For every positive integer \( n \) and every collection \( A_1, A_2, \ldots, A_n \) of subsets of \( \Theta \)

\[
Bel(A_1 \cup A_2 \cup \ldots \cup A_n) \geq \sum_{I \subset\{1,2,\ldots,n\}} (-1)^{|I|+1} Bel(\bigcap_{i \in I} A_i).
\]

(2)
Definition 1.2 A subset of a frame $\Theta$ is called a \textbf{focal element} of a belief function $Bel$ over $\Theta$ if $m(A)>0$. The union of all the focal elements of a belief function is called its \textbf{core}.

The quantity
$$Q(A) = \sum_{B \subseteq \Theta, A \cap B \neq \Phi} m(B)$$

is called \textbf{commonality number} for $A$ which measures the total probability mass that can move freely to every point of $A$. A function $Q: 2^\Theta \rightarrow [0,1]$ is called \textbf{commonality function} for $Bel$.

Degree of doubt : We have:
$$Dou(A) = Bel(\bar{A}).$$

The quantity
$$pl(A) = 1 - Dou(A) = \sum_{A \cap B \neq \Phi} m(B).$$

which expresses the extent to which one finds $A$ credible or plausible [3].

1.2 Simple Support Functions

We have embedding of sets as follows:
$$\{\text{simple support} \} \subseteq \{\text{separable support function} \} \subseteq \{\text{support function} \} \subseteq \{\text{belief function} \}$$

The class of support functions includes all those belief functions that can be obtained by coarsening the frame of a separable support functions.

Definition 1.3 If $s$ is the degree of support for $A$, where $0 \leq s \leq 1$, then the degree of support for $B \subset \Theta$ is given by
$$S(B) = \begin{cases} 0 & \text{if } B \text{ does not contain } A \\ s & \text{if } B \text{ contains } A \text{ but } B \neq \Theta. \\ 1 & \text{if } B = \Theta. \end{cases}$$

The function $S: 2^\Theta \rightarrow [0,1]$ thus defined is called a \textbf{simple support function} focused on $A$.

If $S$ is a simple support function focused on $A$, then $S$ is a belief function with basic Probability numbers $m(A) = S(A)$, $m(\emptyset) = 1 - S(A)$ and $m(B) = 0$ for all other $B \subset \Theta$ [3].

1.3 Bernoulli’s Rule of Combination

Suppose one body of evidence has the precise effect of supporting $A \subset \Theta$ to the degree $s_1$, while the another entirely separate body of evidence has the precise effect of supporting $A$ to the degree $s_2$, then degree of supporting $A$ two bodies of evidence together is $1 - (1 - s_1)(1 - s_2)$ with $m(\emptyset) = (1 - s_1)(1 - s_2)$.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{berno.png}
\caption{Bernoulli’s Rule of Combination}
\end{figure}
The relation between the weight of evidence, \( w \), precisely and unambiguously supporting \( A \) and the degree of support \( s \) for \( A \) in the corresponding simple function is

\[
s = 1 - e^{cw},
\]

(7)

Where \( c \) is a negative constant (figmentary taken as \( c = -1 \)). Notice that the weight of evidence can take any non-negative value, including 0 and infinity. Evidence of zero weight produces a degree of support zero, while evidence of infinity weight produces a degree of support one or infinity.\[3\]

1.4 Heterogeneous Evidence

![Figure 2: Heterogeneous Evidence](image)

Suppose \( A \cap B \neq \emptyset \) with \( S_1 \) is a simple support function focused on \( A \) with \( S_1(A) = s_1 \) and \( S_2 \) is a simple support function focused on \( B \) with \( S_2(B) = s_2 \) then

\[
S(C) = \begin{cases} 
0 & \text{if } C \text{ does not contain } A \cap B \\
 s_1 s_2 & \text{if } C \text{ contains } A \cap B \text{ but neither } A \text{ nor } B \\
 s_1 & \text{if } C \text{ contains } A \text{ but not } B \\
 s_2 & \text{if } C \text{ contains } B \text{ but not } A \\
1 - (1 - s_1)(1 - s_2) & \text{if } C \text{ contains both } A \text{ and } B \\
1 & \text{if } C = \emptyset
\end{cases}
\]

for all \( C \subseteq \Theta \).

In next section, we will study particular cases obtained by our new probability transformations and corresponding properties.

2 Simple Support Function with Probability Transformations

2.1 Some Transformations based on Probabilities

**Definition 2.1** If \( |A| \) and \( |\Theta| \) are cardinalities of set \( A \) and the frame of discernment \( \Theta \) and \( S = \frac{|A|}{|\Theta|} \) or probability mass function of some discrete probability distribution for subset \( A \) of \( \Theta \)[2], is the degree of support for \( A \) where \( 0 \leq s \leq 1 \) then the degree of support for \( B \subseteq \Theta \) is given by

\[
S(B) = \begin{cases} 
0 & \text{if } B \text{ does not contain } A \\
 s & \text{if } B \text{ contains } A \text{ but } B \neq \emptyset, \Theta \\
1 & \text{if } B = \emptyset
\end{cases}
\]

(9)

The function \( S : 2^\Theta \rightarrow [0,1] \) thus defined is a simple support function focused on \( A \).

**Notes :-**

1. If \( s \) is a simple support function focused on \( A \), then \( s \) is a belief function with basic Probability numbers \( m(A) = S(A), m(\emptyset) = 1 - S(A) \) and \( m(B) = 0 \) for all other \( B \subseteq \Theta \).

2. Here \( s \) can be obtained by probability mass function or probability density function of distribution.
### 2.2 Bernoulli’s Rule of Combination

Suppose one body of evidence has the precise effect of supporting $A \subseteq \Theta$ to the degree $s_1$, while the another entirely separate body of evidence has the precise effect of supporting $A$ to the degree $s_2$, then degree of supporting $A$ obtained by pmf of first distribution $s_1 = p_1(A)$, while another entirely separate body of evidence has precise effect of supporting $A$ obtained by pmf of second distribution $s_2 = p_2(A)$. Then degree of supporting $A$ based on two bodies of evidence together i.e. total evidence is

$$s = 1 - (1 - s_1)(1 - s_2) = 1 - (1 - p_1(A))(1 - p_2(A))$$

$$= s_1 + s_2(1 - s_1) = p_1(A) + p_2(A)(1 - p_1(A))$$

$$= s_2 + s_1(1 - s_2) = p_2(A) + p_1(A)(1 - p_2(A)).$$

with $m(\Theta) = (1 - s_1)(1 - s_2) = (1 - p_1(A))(1 - p_2(A))$.

**Remarks:**

1. $A$’s degree of support on the total evidence is greater than its degree of support on either of the separate bodies of evidence. Indeed the deficit from unity of the degree of support provided by one of the bodies of evidence is reduced by a proportion equal to the degree of support by the other body of evidence.

2. Bernoulli’s rule of combination is a special case of Dempster’s rule of combination. This can be justified as: Let simple support function $S_1$ has basic probability numbers $m_1(A) = s_1 = p_1(A)$ and $m_1(\Theta) = 1 - s_1 = 1 - p_1(A)$ and simple support function $S_2$ has basic probability numbers $m_2(A) = s_2 = p_2(A)$ and $m_2(\Theta) = 1 - s_2 = 1 - p_2(A)$. The only upper right hand rectangle of measure $(1 - s_1)(1 - s_2) = (1 - p_1(A))(1 - p_2(A))$ fails to be committed to $A$. the orthogonal sum $S = S_1 \oplus S_2$ has probability numbers

$$m(A) = 1 - (1 - s_1)(1 - s_2) = 1 - (1 - p_1(A))(1 - p_2(A))$$

and

$$m(\Theta) = (1 - s_1)(1 - s_2) = (1 - p_1(A))(1 - p_2(A))$$

$S$ is a simple support function $A$ focused on with

$$S(A) = 1 - (1 - s_1)(1 - s_2) = 1 - (1 - p_1(A))(1 - p_2(A))$$

### 2.3 The Weight of Evidence

The degree of support for the various propositions discerned by a frame $\Theta$ ought to be determined by the weights of the items of evidence attesting to those various propositions. For evidence underlying a simple support function, if the evidence points precisely and unambiguously to a single subset $A \subseteq \Theta$, then the degree of support for $A$ ought to be completely determined by the weight of that evidence i.e. it must be a function of that weight.

The weight $w$ of the evidence pointing to $A$ can take any non-negative value, the degree of support $s = p(A)$ must be between zero and one. Hence we require a function $g: [0, \infty) \to [0,1]$ such that $s = g(w)$. The function $g$ is closely determined by Bernoulli’s rule of combination with the intuitive idea that weights combine additively.

If $w_1$ and $w_2$ be the weights of evidence underlying the simple support function $S_1$ and $S_2$ then the weight of evidence underlying their orthogonal sum will be $w_1 + w_2$.

i.e. $g(w_1) = s_1 = p_1(A)$ and $g(w_2) = s_2 = p_2(A)$ then

$$g(w_1 + w_2) = 1 - (1 - s_1)(1 - s_2) = 1 - (1 - p_1(A))(1 - p_2(A)).$$

If we set $f(w) = 1 - g(w)$ then

$$f(w_1) = 1 - s_1 = 1 - p_1(A), \quad f(w_2) = 1 - s_2 = 1 - p_2(A)$$

$$\Rightarrow f(w_1 + w_2) = f(w_1)f(w_2).$$

Hence $f$ must be exponential.
As \( f(w) = e^{cw} \) and \( g(w) = 1 - e^{cw} \), where \( c \) is constant. 

As \( f: [0, \infty] \to [0,1] \), the constant \( c \) must be negative.

Also as \( g(w) = 1 - e^{cw} \Rightarrow s = p(A) = 1 - e^{cw} \).

Solving for \( w \), we get

\[
    w = \frac{1}{c} \log(1 - s) = \frac{1}{c} \log(1 - p(A)).
\]

**Note:** The weight of evidence can take any non-negative value, including zero and infinity. Evidence of zero weight produces a degree of support zero while evidence of infinite weight produces a degree of support one or certainty.

### 2.4 Heterogeneous Evidences

The essential feature of combining evidence that points towards a proposition \( A \) with evidence that points towards a different but compatible proposition \( B \) is that the total evidence provides a support not only for \( A \) and \( B \) separately, but also for the conjunction \( A \cap B \).

Suppose \( A \cap B \neq \emptyset \) and we wish to combine \( S_1 \) and \( S_2 \) where \( S_1 \) is a simple support function focused on \( A \) with \( S_1(A) = s_1 = p_1(A) \) and \( S_2 \) is a simple support function focused on \( B \) with \( S_2(B) = s_2 = p_2(B) \).

By Dempster's rule, \( A \cap B \) is supported to the extent \( s_1 s_2 = p_1(A) p_2(B) \). By Figure 1, the orthogonal sum \( S = S_1 \oplus S_2 \) has basic probability numbers

\[
    m(A \cap B) = s_1 s_2 = p_1(A) p_2(B)
\]

\[
    m(A) = s_1(1 - s_2) = p_1(A)(1 - p_2(B))
\]

\[
    m(B) = s_2(1 - s_1) = p_2(B)(1 - p_1(A))
\]

Therefore

\[
    S(C) = \begin{cases} 
    0 & \text{if } C \text{ does not contain } A \cap B \\
    p_1(A)p_2(B) & \text{if } C \text{ contains } A \cap B \text{ but neither } A \text{ nor } B \\
    p_1(A) & \text{if } C \text{ contains } A \text{ but not } B \\
    p_2(B) & \text{if } C \text{ contains } B \text{ but not } A \\
    1 - (1 - p_1(A))(1 - p_2(B)) & \text{if } C \text{ contains both } A \text{ and } B \text{ but } C \neq \emptyset \\
    1 & \text{if } C = \emptyset
    \end{cases}
\]

for all \( C \subset \Theta \). In above, we assumed that neither \( A \) nor \( B \) contains the other. If \( A \) is subset of \( B \) then the evidence is not so heterogeneous and the expression for the orthogonal sum simplifies to

\[
    S(C) = \begin{cases} 
    0 & \text{if } C \text{ does not contain } A \\
    p_1(A) & \text{if } C \text{ contains } A \text{ but not } B \\
    1 - (1 - p_1(A))(1 - p_2(B)) & \text{if } C \text{ contains both } A \text{ and } B \text{ but } C \neq \emptyset \\
    1 & \text{if } C = \emptyset
    \end{cases}
\]

Here we listed results which are dependent on two simple support functions focusing on subset \( A \). In next section, we generalize these results for more than two simple support functions focused on subset \( A \) [1].

### 3. Generalizations of Above Results

#### 3.1 Bernoulli's Rule of Combination

Suppose one body of evidence has the precise effect of supporting \( A \subset \Theta \) to the degree \( s_1 \), while the another entirely separate bodies of evidence has the precise effect of supporting \( A \) to the degree...
$s_2$ and $s_3$, then degree of supporting $A$ obtained by pmf of first distribution $= p_1(A)$, while another entirely separate bodies of evidence has precise effect of supporting $A$ obtained by pmf of second and third distribution $s_2 = p_2(A)$ and $s_2 = p_3$. Then degree of supporting $A$ based on three bodies of evidence together i.e. total evidence is

$$s = 1 - (1 - s_1)(1 - s_2)(1 - s_3) = 1 - (1 - p_1(A))(1 - p_2(A))(1 - p_3(A))$$

$$s = s_1 s_2 (1 - s_3) + s_2 (1 - s_1) s_3 (1 - s_2) + p_1(A) + p_2(A)(1 - p_1(A)) + p_3(A)(1 - p_1(A))(1 - p_2(A))$$

Then degree of supporting $A$ obtained by pmf of first distribution, while another entirely separate bodies of evidence has precise effect of supporting $A$ obtained by pmf of second and third distribution $s_2 = p_2(A)$ and $s_2 = p_3$. Then degree of supporting $A$ based on three bodies of evidence together i.e. total evidence is

$$m(\Theta) = (1 - s_1)(1 - s_2)(1 - s_3) = (1 - p_1(A))(1 - p_2(A))(1 - p_3(A))$$

Remarks:-

1. $A$’s degree of support on the total evidence is greater than its degree of support on either of the separate bodies of evidence. Indeed the deficit from unity of the degree of support provided by one of the bodies of evidence is reduced by a proportion equal to the degree of support by the other bodies of evidence.

2. Bernoulli’s rule of combination is a special case of Dempster’s rule of combination. This can be justified as: Let simple support function $S_1$ has basic probability numbers $m_1(A) = s_1 = p_1(A)$ and $m_1(\Theta) = 1 - s_1 = 1 - p_1(A)$, simple support function $S_2$ has basic probability numbers $m_2(A) = s_2 = p_2(A)$ and $m_2(\Theta) = 1 - s_2 = 1 - p_2(A)$ and simple support function $S_3$ has basic probability numbers $m_3(A) = s_3 = p_3(A)$ and $m_3(\Theta) = 1 - s_3 = 1 - p_3(A)$. The only upper right hand parallelopiped of measure $(1 - s_1)(1 - s_2)(1 - s_3) = (1 - p_1(A))(1 - p_2(A))(1 - p_3(A))$ fails to be committed to $A$. The orthogonal sum $S = S_1 \oplus S_2 \oplus S_3$ has probability numbers

$$m(A) = 1 - (1 - s_1)(1 - s_2)(1 - s_3) = 1 - (1 - p_1(A))(1 - p_2(A))(1 - p_3(A))$$

and

$$m(\Theta) = (1 - s_1)(1 - s_2)(1 - s_3) = (1 - p_1(A))(1 - p_2(A))(1 - p_3(A))$$

$S$ is a simple support function focused on $A$ with

$$S(A) = 1 - (1 - s_1)(1 - s_2)(1 - s_3) = 1 - (1 - p_1(A))(1 - p_2(A))(1 - p_3(A))$$

Thus Bernouilli’s rule of combination is generalized for $k$ simple support functions as follows:

Suppose one body of evidence has the precise effect of supporting $A \subset \Theta$ to the degree $s_1$, while the another entirely separate bodies of evidence has the precise effect of supporting $A$ to the degree $s_2, s_3, \ldots, s_k$ then degree of $A$ obtained by pmf of first distribution $= p_1(A)$, while another entirely separate bodies of evidence has precise effect of supporting $A$ obtained by pmf of second and third up to $k^{th}$ distribution $s_2 = p_2(A), s_3 = p_3(A), \ldots, s_k = p_k(A)$. Then degree of supporting $A$ based on $k$ bodies of evidence together i.e. total evidence is

$$s = 1 - (1 - s_1)(1 - s_2)(1 - s_3) \cdots (1 - s_k)$$

$$s = 1 - (1 - p_1(A))(1 - p_2(A))(1 - p_3(A)) \cdots (1 - p_k(A))$$

$$s = s_1 + s_2 (1 - s_1) + s_3 (1 - s_1)(1 - s_2) + \cdots + s_k (1 - s_1)(1 - s_2) \cdots (1 - s_{k-1})$$

$$s = p_1(A) + p_2(A)(1 - p_1(A)) + p_3(A)(1 - p_1(A))(1 - p_2(A)) + \cdots + p_k(A)(1 - p_1(A))(1 - p_2(A)) \cdots (1 - p_{k-1})$$

$$s = p_2(A) + p_3(A)(1 - p_2(A)) + p_3(A)(1 - p_3(A))(1 - p_2(A)) + \cdots + p_k(A)(1 - p_3(A))(1 - p_2(A)) \cdots (1 - p_{k-2})$$

$$s = p_3(A) + p_4(A)(1 - p_3(A)) + p_4(A)(1 - p_4(A))(1 - p_3(A)) + \cdots + p_k(A)(1 - p_4(A))(1 - p_3(A)) \cdots (1 - p_{k-3})$$

$$;$$
1. A’s degree of support on the total evidence is greater than its degree of support on either of the separate bodies of evidence. Indeed the deficit from unity of the degree of support provided by one of the bodies of evidence is reduced by a proportion equal to the degree of support by the other bodies of evidence.

2. Bernoulli’s rule of combination is a special case of Dempster’s rule of combination. This can be justified as: Let simple support function \( S_1 \) has basic probability numbers \( m_1(A) = s_1 = p_1(A) \) and \( m_1(\Theta) = 1 - s_1 = 1 - p_1(A) \), simple support function \( S_2 \) has basic probability numbers \( m_2(A) = s_2 = p_2(A) \) and \( m_2(\Theta) = 1 - s_2 = 1 - p_2(A) \) and simple support function \( S_k \) has basic probability numbers \( m_k(A) = s_k = p_k(A) \) and \( m_k(\Theta) = 1 - s_k = 1 - p_k(A) \). The only upper right hand parallelepiped of measure \( (1 - s_1)(1 - s_2)(1 - s_3) \cdots (1 - s_k) \) fails to be committed to \( A \). the orthogonal sum \( S = S_1 \oplus S_2 \oplus \cdots \oplus S_k \) has probability numbers

\[
\begin{align*}
    m(A) &= 1 - (1 - s_1)(1 - s_2)(1 - s_3) \cdots (1 - s_k) = 1 - (1 - p_1(A))(1 - p_2(A))(1 - p_3(A)) \cdots (1 - p_k(A)) \\
    m(\Theta) &= (1 - s_1)(1 - s_2)(1 - s_3) \cdots (1 - s_k) = (1 - p_1(A))(1 - p_2(A))(1 - p_3(A)) \cdots (1 - p_k(A)).
\end{align*}
\]

Thus we got generalized Bernoulli’s rule of combination for \( k \) simple support functions.

### 3.2 The Weight of Evidence

The degree of support for the various propositions discerned by a frame \( \Theta \) ought to be determined by the weights of the items of evidence attesting to those various propositions. For evidence underlying a simple support function, if the evidence points precisely and unambiguously to a single subset \( A \subset \Theta \), then the degree of support for \( A \) ought to be completely determined by the weight of that evidence i.e. it must be a function of that weight.

The weight \( w \) of the evidence pointing to \( A \) can take any non-negative value, the degree of support \( S = p(A) \) must be between zero and one. Hence we require a function \( g: [0, \infty] \to [0,1] \) such that \( S = g(w) \). The function \( g \) is closely determined by Bernoulli’s rule of combination with the intuitive idea that weights combine additively.

If \( w_1, w_2, \ldots, w_k \) be the weights of evidence underlying the simple support function \( S_1, S_2, \ldots, S_k \) respectively then the weight of evidence underlying their orthogonal sum will be \( w_1 + w_2 + \cdots + w_k \)

i.e. \( g(w_1) = s_1 = p_1(A), g(w_2) = s_2 = p_2(A), \ldots, g(w_k) = s_k = p_k(A) \) then
If we set \( f(w) = 1 - g(w) \) then
\[
f(w_j) = 1 - s_j = 1 - p_j(A), j = 1, 2, ..., k
\]
Hence \( f \) must be exponential. \( f: [0, \infty) \rightarrow [0,1] \), the constant \( c \) must be negative.
As \( g(w) = 1 - e^{cw} \Rightarrow s = p(A) = 1 - e^{cw} \).
Solving for \( w \), we get
\[
w = \frac{1}{c} \log(1 - s) = \frac{1}{c} \log(1 - p(A)).
\]

3.3 Heterogeneous Evidences
The essential feature of combining evidence that points towards a proposition \( A_1 \) with evidence that points towards a different but compatible propositions \( A_2, A_3, ..., A_k \) is that the total evidence provides a support not only for \( A_1, A_2, ..., A_k \) separately, but also for the conjunction \( A_1 \cap A_2 \cap ... \cap A_k \).
Suppose \( A_1 \cap A_2 \cap ... \cap A_k \neq \emptyset \) and we wish to combine \( S_1, S_2, ..., S_k \) where \( S_j \) is a simple support function focused on \( A_j \) with \( S_j(A) = s_j = p_j(A), j = 1, 2, ..., k \).
By Dempster’s rule, \( A_1 \cap A_2 \cap ... \cap A_k \) is supported to the extent
\[
S = S_1 \oplus S_2 \oplus ... \oplus S_k
\]
has basic probability numbers
\[
m(A_1 \cap A_2 \cap ... \cap A_k) = s_1 s_2 ... s_k = p_1(A_1)p_2(A_2) ... p_k(A_k)
\]
\[
m(A_j) = s_j + s_1(1 - s_j) + s_2(1 - s_j)(1 - s_j) + ... + s_{j-1}(1 - s_j)(1 - s_j) ... (1 - s_{j-1})(1 - s_j)
\]
\[
+ s_{j+1}(1 - s_j) ... (1 - s_{j-1})(1 - s_j) + ... + s_k(1 - s_j) ... (1 - s_{j-1})(1 - s_j) ... (1 - s_k)
\]
\[
= p_1(A_j) + p_2(A_j) ... (1 - p_j(A_j)) + p_2(A_j) ... (1 - p_k(A_j)) + ... + p_{j-1}(A_j) ... (1 - p_k(A_j)) + ... + p_{j+1}(A_j) ... (1 - p_k(A_j)) + ... + p_k(A_j)
\]
\[
+ p_{j+1}(A_j) ... (1 - p_k(A_j)) + ... + p_k(A_j)
\]
and \( m(\emptyset) = (1 - s_1)(1 - s_2) ... (1 - s_k) = (1 - p_1(A))(1 - p_2(B)) ... (1 - p_k(A_k)) \).
Therefore
\[
S(\mathcal{C}) = \begin{cases} 
0 & \text{if } \mathcal{C} \text{ does not contain } A_1 \cap A_2 \cap ... \cap A_k \\
p_1(A_1)p_2(A_2) ... p_k(A_k) & \text{if } \mathcal{C} \text{ contains } A_1 \cap A_2 \cap ... \cap A_k \text{ but not } A_j, j = 1, 2, ... \\
\prod p_j(A_j) & \text{if } \mathcal{C} \text{ contains subsets } A_j \\
p_j(A_j) & \text{if } \mathcal{C} \text{ contains } A_j \text{ but not } A_i, i \neq j, j = 1, 2/ ... , k \\
1 - (1 - p_1(A_1))(1 - p_2(A_2)) ... p_k(A_k) & \text{if } \mathcal{C} \text{ contains } A_j, j = 1, 2, ... , k \text{ but } \mathcal{C} \neq \emptyset \\
1 & \text{if } \mathcal{C} = \emptyset.
\end{cases}
\]
for all \( \mathcal{C} \subset \Theta \). In above, we assumed that neither \( A_j \) nor \( A_i, i \neq j \) contains the other. If for some \( j, A_j \) is subset of \( A_i, i \neq j \) then the evidence is not so heterogeneous and the expression for the orthogonal sum simplifies to
\[
S(\mathcal{C}) = \begin{cases} 
0 & \text{if } \mathcal{C} \text{ does not contain subset } A_j, j = 1, 2, ... , k \\
p_j(A_j) & \text{if } \mathcal{C} \text{ contains } A_j \text{ but not } A_i, i \neq j \\
1 - (1 - p_1(A_1))(1 - p_2(A_2)) ... p_k(A_k) & \text{if } \mathcal{C} \text{ contains } A_j, j = 1, 2, ... , k \text{ but } \mathcal{C} \neq \emptyset \\
1 & \text{if } \mathcal{C} = \emptyset.
\end{cases}
\]
4. Conclusion

In this paper, we have quoted simple generalizations i.e. derived formulae in terms of probability for simple support function, Bernoulli’s rule of combination, the weight of evidence and heterogeneous evidence. Though these properties seem quite simple but having characteristic of getting lots of information in one stroke because experts opinions are based on probability mass function of discrete probability distributions. Also we take advantage of appropriate discrete probability distributions depending upon situation.

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