Structure Formation with Majoron Supermultiplet

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Abstract

We show that the late-decaying particle scenario may be realized in the supersymmetric singlet majoron model with the majoron scale $10 - 200$ TeV. The smajoron decaying into two neutrinos is the late-decaying particle with the mass $0.1 - 1$ TeV and the life-time $2 \times 10^3 - 8 \times 10^4$ seconds. The lower limit of the majorino mass is $4 - 40$ TeV in order to avoid the overclosure of the universe due to the decay-produced LSP. The muon neutrino and the tau neutrino can be used to explain the atmospheric and the solar neutrino deficit.

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The conventional cold dark matter (CDM) scenario for the structure formation predicts more power at small scales in a flat inflationary universe [1]. Among various possibilities to fix this problem, the late-decaying particle scenario with $\Omega = 1$ CDM is based on the idea that delaying the time of matter-radiation equality due to the relativistic decay-products increase the size of the scale starting to grow [2]. The question now is how this idea can be realized in a specific particle physics model. Until now a handful of models have been appeared. The first proposal was to use the 17 keV neutrino with the life-time around 1 yr [3]. More recently, the author and collaborators suggested a light axino which decays into the gravitino and the axion in the low-energy supersymmetry breaking scheme [4, 5]. Another possibility with a massive tau neutrino in a doublet majoron model with small majoron scale around 20 GeV is worked out in ref. [6]. In a slightly different context, another interesting suggestion was made in ref. [7] where a heavy tau neutrino producing an electron neutrino (plus majoron or familon) in the era of nucleosynthesis is used to fit the power spectrum.

The purpose of this paper is to propose a late-decaying particle scenario realized in the singlet majoron model [8] combined with supersymmetry. Conventional ways for fitting the power spectrum is to use heavy tau neutrinos. Most well-known is the mixed dark matter scenario with a tau neutrino of mass $\sim 30h^2$ eV as a hot dark matter component. Another way is to introduce a heavier tau neutrino of mass 1-10 MeV mentioned above [7]. This possibility can be realized in the singlet majoron model with the majoron scale $V_L \simeq 1$ TeV [9]. In these cases however one cannot reconcile both the solar and the atmospheric neutrino problem with the minimal number of neutrino species [9]. Introducing one sterile neutrino may provide a model where an unstable MeV Majorana tau neutrino can reconcile the CDM scenario with data on solar and atmospheric neutrinos as presented recently in ref. [11].

In this paper we do not explain the structure formation by a heavy neutrino. Therefore we can have three species of neutrinos with masses accounting for the deficit of solar and atmospheric neutrinos: $m_{\nu_{\mu}} \simeq 10^{-2} - 10^{-3}$ eV and $m_{\nu_{\tau}} \simeq 0.1$ eV [12]. In order to provide a good fit for the structure formation we invoke the late-decaying particle scenario

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1 For a reconciliation of all the problem one may introduce almost degenerate neutrinos of masses around a few eV in the mixed dark matter scenario [10].
with $\Omega = 1$ CDM. Our scenario uses a particle inherited in the supersymmetric singlet majoron model: the smajoron $s$ (the scalar partner of the majoron) which decays into a pair of tau neutrinos. Supersymmetric models are endowed with a natural candidate for CDM, namely, the lightest supersymmetric particle (LSP) when the R-parity is imposed for the proton stability [13]. In our scheme the LSP is assumed preferably to be the usual neutralinos. This fact imposes a constraint on the mass of the fermion partner of the majoron ($J$), called the majorino ($\psi$), since the majorino decay produces at least one LSP.

The late-decaying particle scenario assumes a long-lived massive relic particle $X$ which dominates once the energy density of the universe and then decays into relativistic particles. Let the mass of $X$ be $m_X$, the life-time $\tau_X$ and the ratio of the relic number density to the entropy density $Y_X$. These relativistic remnants are red-shifted away to delay the time of the usual matter-radiation equality. At the time of the new matter-radiation equality, the length scale $\lambda_{EQ}$ characterizing the evolution of fluctuation spectrum is given by

$$\lambda_{EQ} \approx 30 (\Omega h^2)^{-1/2} \theta^{1/2} \text{Mpc}$$

with

$$\theta = 1 + \frac{b}{0.67} \left( \frac{\tau_X}{\text{sec}} \right)^{2/3} \left( \frac{m_X Y_X}{\text{MeV}} \right)^{4/3}.$$  \hspace{1cm} (1)

Here $b$ is the fraction of the relativistic energy density from the $X$ decay. The shape of the power spectrum is set by the value of $\kappa = (\Omega h)\theta^{-1/2}$. The best fit requires $\kappa = 0.2 - 0.3$ [14]. Therefore the late-decaying particle scenario with $\Omega = 1$ fixes the basic relation

$$\left( \frac{\tau_X}{\text{sec}} \right) \left( \frac{m_X Y_X}{\text{MeV}} \right)^2 \approx 0.55 b^{-3/2} \left( h^2 / \kappa^2 - 1 \right)^{3/2}.$$  \hspace{1cm} (2)

Since the particle $X$ decays after the nucleosynthesis, their energy density should be less than that of one neutrino at the time of the nucleosynthesis. It gives the restriction

$$\frac{m_X Y_X}{\text{MeV}} < 0.107.$$  \hspace{1cm} (3)

Then eq. (2) puts the lower limit on the life-time; $\tau_X > 114 b^{-3/2}$, e.g., for $h = 0.5$ and $\kappa = 0.3$. What makes the late-decaying particle scenario different from the others is the existence of an extra small scale corresponding to the horizon at the first matter-radiation equality. It is

$$\lambda_{EQ1} \approx 80 \left( \frac{\text{keV}}{m_X Y_X} \right) \text{kpc}.$$  \hspace{1cm} (4)
Further investigation is needed to confirm or exclude the existence of such a scale.

The supersymmetric singlet majoron model in its simplest form assumes the presence of one extra singlet which couples to three right-handed neutrinos \[15\]. In this model the extra singlet as well as the right-handed sneutrinos develop non-vanishing vacuum expectation values of order of the supersymmetry breaking scale \(m_{3/2} \sim 1\) TeV as it is the only scale appearing in the scalar potential with soft-terms. Although this kind of model is appealing in its minimality, it cannot provide a solution to the dark matter problem since the non-vanishing vacuum expectation value of the right-handed sneutrino breaks the R-parity and thus destabilizes the LSP. Therefore one would like to consider other kind of supersymmetric singlet majoron models where the lepton number breaking scale \(V_L\) is a free parameter \[16\]. The class of models we are considering have the following superpotential in addition to that of the minimal supersymmetric standard model:

\[
W = h_{ij} L_i H N_j + f_{ija} N_i N_j S_a + W'(S_a)
\]

with three families \(N_i\) of right-handed neutrinos and arbitrary number of singlets \(S_a\) \[17\]. The whole superpotential is invariant under lepton number, spontaneous-breaking of which is provided by the model-dependent superpotential \(W'\). Apart from being larger than \(m_{3/2}\) the lepton number breaking scale \(V_L\) is taken as a free parameter. It is important to observe that, whenever \(V_L > m_{3/2}\), the right-handed sneutrinos do not require a vacuum expectation value so that the R-parity is not broken \[18\]. Therefore the LSP can be a candidate for cold dark matter. The masses of the smajoron and the majorino are expected to be smaller or equal to the supersymmetry breaking scale \(m_{3/2}\) \[16, 17\]. Our late-decaying particle scenario will fix the masses given the scale \(V_L\).

In the models under consideration, strong bounds on \(V_L\) were found due to the fact that the relic density of smajorons (majorinos) can provide excessive energy density of the universe either at the time of nucleosynthesis or at present \[16, 17\]. Smajorons (majorinos) usually decouple from the thermal bath when they are relativistic to result in \(Y_{s,\psi} \simeq 10^{-3}\). The relic number density however can be further suppressed due to the self-couplings among the majoron supermultiplet, which was the key observation in ref. \[18\]. In order to see this, we consider the trilinear coupling of the majoron supermultiplet \(\Phi\)
to a heavy field $Z$ in the Higgs superpotential; $\frac{1}{2} \Phi^2 Z + \frac{1}{2} M_Z Z^2$. The trilinear coupling is absorbed into the (effective) mass $M_Z$. The six-dimensional operators arise in the D-term of $(\Phi \bar{\Phi})^2/4M_Z^2$ due to the tree-level exchange of $Z$. In calculating this one can use the supermultiplet formalism since it involves no supersymmetry breaking effect. And we neglect contributions due to supersymmetry breaking. It leads to the following interaction in components:

$$\mathcal{L}_{\text{eff}} \sim \frac{1}{4M_Z^2} \left[ s^2 (\partial_\mu J)^2 + J^2 (\partial_\mu s)^2 \right] + \frac{1}{4M_Z^2} J^2 \bar{\psi}_M i\gamma_\mu \partial^\mu \psi_M$$

(6)

where $\psi_M$ denotes the majorana spinor of the majorino. The effective mass $M_Z$ encodes the model-dependence of the interaction strength. These interactions are effective even after the decoupling of the smajoron and the majorino from the thermal bath. Non-relativistic smajorons (majorinos) can follow its thermal distribution to yield the suppressed relic number density for the values of $V_L$ lower than $10^6$ GeV.

The point now is that smajorons (majorinos) can decay after the ear of nucleosynthesis without causing any problem if their relic number $Y_{s,\psi}$ is smaller than $10^{-7}$. Then for a suitable range of values for $V_L$ and $m_s$ ($m_\psi$) the condition (6) can be fulfilled to realize the late-decaying particle scenario. The supersymmetrized vertex of majoron-neutrino-neutrino also contains the coupling of smajoron(majorino)-neutrino-(s)neutrino. This will be the main decay mode of smajoron (majorino) in our case. The cosmological role of majorinos is different from that of smajorons in that majorinos produce LSP’s which may overclose the universe if relic number density of majorinos is too much. As it turns out, our scheme does not allow the majorino to be the LSP contrary to the case in ref. [18]. As we will see, smajorons producing two neutrinos can properly delay the time of matter-radiation equality for suitable choices of $V_L$ and $m_s$. Given $V_L$ and $m_s$ the lower limit for the majorino mass has to be put in order to reduce the relic density in a sufficient amount.

It is now straightforward to calculate the life-time and the relic number density of smajorons (majorinos) in terms of $m_s$ ($m_\psi$) and $V_L$. In certain models the five-dimensional self-interaction among the majoron supermultiplet are present to cause the decay of the smajoron into two majorons [17]. Since it leads too fast decay for our purpose, we assume the absence of this decay mode. Then producing a pair of tau neutrinos is the dominant
decay mode of the smajoron with the life-time,
\[ \tau_s = 1.65 \times 10^2 \left( \frac{V_L}{10 \text{ TeV}} \right)^2 \left( \frac{1 \text{ TeV}}{m_s} \right) \left( \frac{0.1 \text{ eV}}{m_\nu} \right)^2 \text{ sec}. \] (7)

From the life-time limit following eq. (3), the preferable value of the scale \( V_L \) is bigger than roughly 10 TeV for \( m_s \simeq 1 \text{ TeV} \).

In order to determine the relic number density we compare the interaction rate with the expansion rate \( H \) of the universe. From the above expression (6) one obtains the following interaction rate [19];

\[ \Gamma_{int} = n_s\langle \sigma v_{rel} \rangle = \frac{m_s^3}{(2\pi x_s)^{3/2}} e^{-x_s} \sigma_0 (1 + x_s^{-1}) \]

where \( \sigma_0 = \frac{9m_s^2}{128\pi M_Z^4} \). (8)

Here \( x_s \equiv m_s/T_J \) and \( T_J \) is the majoron temperature in terms of which the majoron follows the equilibrium distribution. The majoron temperature is related to the photon temperature \( T : T_J = a(T)T \) with \( a(T) = [g_{ss}(T)/g_{ss}(T_D)]^{1/3} \) where \( T_D \) is the decoupling temperature of the smajoron out of the (photon) thermal bath and \( g_{ss} \) is the effective degrees of freedom contributing the entropy density. Taking the final decoupling temperature \( T_D' \) smaller than the top-quark mass, the reference value of \( a(T_D') \) is \( a = 0.72 \).

The relic population of the smajoron is given by

\[ Y_s = \frac{45a^3}{(2\pi)^{5/2}} \frac{\langle \sigma v_{rel} \rangle}{g_{ss}(T_D')} x_s^{3/2} e^{-x_s} \] (9)

where \( x_s \) is determined by \( \Gamma_{int} = H = 1.66g_s^{1/2}T^2/M_{Pl} \). The condition for the successful structure formation (2) can now be analyzed in terms of \( m_s, \tau_s \) and \( Y_s \) given above.

Let us now turn to the question of the majorino mass. As mentioned earlier the usual neutralinos form \( \Omega = 1 \text{ CDM} \) into which majorinos can decay. Then the relic density of the majorino should be suppressed in order for the decay-produced neutralinos not to overclose the universe. The number density \( Y_\psi \) is also calculated by equating the interaction rate for the majorino to the expansion rate of the universe. The interaction in eq. (B) gives

\[ \Gamma_{int} = \frac{2m_\psi^3}{(2\pi x_\psi)^{3/2}} e^{-x_\psi} \sigma_0 x_\psi^{-1} \]

where \( \sigma_0 = \frac{3m_\psi^2}{32\pi M_Z^4} \). (10)
The relic number density $Y_\psi$ is two times $Y_s$ with $x_s$ replaced by $x_\psi$. For the computations our reference values are $h = 0.5$ and $\kappa = 0.3$. The whole analysis is not sensitive to the allowed variation of these values. The condition for the secondary neutralinos not to overclose the universe reads

$$Y_\psi \leq 1.36 \times 10^{-11} \left(h/0.5\right)^2 \left(\frac{60\text{ GeV}}{m_{\chi^0}}\right)^2 \quad (11)$$

where $m_{\chi^0}$ is the mass of the LSP. As one can see below the majorino should be heavier than a few TeV in order to meet the above restriction.

The analysis shows that our scenario prefers relatively large values for the smajoron and the majorino mass. As varying the smajoron mass $m_s$ from 10 GeV to 1 TeV, we get the values of the majoron scale and the majorino mass as in Table 1. The presented majorino masses are for the decay-produced LSP (with $m_{\chi^0} = 60$ GeV) to form $\Omega = 1$ CDM. Varing the mass of the LSP from 20 to 100 GeV, these values increase by a factor of 0.8 or 1.2, respectively.

As can be seen, the majorino should be heavier than the smajoron by one to three orders of magnitude. The discrepancy becomes larger for smaller smajoron masses (or for smaller $V_L$), which requires unpleasant tuning of the masses. We did not show the values for the smajoron mass larger than 1 TeV since they need the majorino mass too far from the supersymmetry breaking scale $m_{3/2} \sim 1$ TeV. Admitting tuning of two orders between the smajoron and the majorino mass the preferable values are $m_s \simeq 0.1 - 1$ TeV and $m_\psi \simeq 8 - 36$ TeV which requires $V_L \simeq 12 - 36$ TeV.

When the effective interaction becomes stronger ($M_Z < V_L$) the tuning becomes weaker. For instance, when the mass $M_Z$ is pushed down up to $M_Z = V_L/10$ we get the results in Table 2. Again for $m_s \simeq 0.1 - 1$ TeV we have $V_L \simeq 73 - 220$ TeV and $m_\psi \simeq 4 - 20$ TeV. Therefore an acceptable late-decaying particle scenario can be realized without too much tuning among the parameters.

For the cases with $m_s = 0.1 - 1$ TeV, the smajoron life-time falls in the range $\tau_s = 2 \times 10^3 - 8 \times 10^4$ sec. Therefore we have the extra small scale $\lambda_{EQ1} \simeq 3 - 21$ kpc.

In conclusion, the late-decaying particle scenario with the smajoron in the supersymmetric singlet majoron model with $V_L \simeq 10 - 200$ TeV is suggested. One also obtains
Table 1: The selected values of the majoron scale $V_L$, the smajoron mass $m_s$ and the lower limit of the majorino mass $m_\psi$ realizing the late-decaying scenario. The taken values for the parameters are $h = 0.5$, $\kappa = 0.3$ and $M_Z = V_L$.

| $V_L/1\text{ TeV}$ | 4.02 | 7  | 8.63 | 10  | 12  | 13.8 | 20  | 30  | 36.4 |
|---------------------|------|----|------|-----|-----|------|-----|-----|------|
| $m_s/100\text{ GeV}$ | 0.1  | 0.322 | 0.5 | 0.681 | 1 | 1.34 | 2.89 | 6.7 | 10 |
| $m_\psi/1\text{ TeV}$ | 1.93 | 4.04 | 5.34 | 6.5 | 8.31 | 10 | 16.4 | 28.1 | 36.4 |

Table 2: Same as Table 1 but $M_Z = V_L/10$.

| $V_L/1\text{ TeV}$ | 24.2 | 40 | 52.4 | 65.0 | 73.2 | 100 | 138 | 170 | 223 |
|---------------------|------|----|------|-----|------|-----|-----|-----|-----|
| $m_s/100\text{ GeV}$ | 0.1 | 0.285 | 0.5 | 0.782 | 1 | 1.91 | 3.72 | 5.71 | 10 |
| $m_\psi/1\text{ TeV}$ | 0.982 | 1.92 | 2.75 | 3.66 | 4.29 | 6.5 | 10 | 13.2 | 19 |

the lower bound for the majorino mass since its decay-products contains at least one LSP which may cause the overclosure of the universe. The required values for the smajoron and the majorino mass are $m_s \simeq 0.1 - 1 \text{ TeV}$ and $m_\psi \simeq 4 - 40 \text{ TeV}$. Contrary to the other scenarios with a heavy tau neutrino, our suggestion can explain both the solar and the atmospheric neutrino deficit by choosing the appropriate neutrino masses.

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