Rate Maximizations for Reconfigurable Intelligent Surface-Aided Wireless Networks: A Unified Framework via Block Minorization-Maximization

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Abstract—The reconfigurable intelligent surface (RIS) has arose an upsurging research interest recently due to its promising outlook in 5G-and-beyond wireless networks. With the assistance of RIS, the wireless propagation environment is no longer static and could be customized to support diverse service requirements. In this paper, we will approach the rate maximization problems in RIS-aided wireless networks by considering the beamforming and reflecting design jointly. Three representative design problems from different system settings are investigated based on a proposed unified algorithmic framework via the block minorization-maximization (BMM) method. Extensions and generalizations of the proposed framework in dealing with some other related problems are further presented. Merits of the proposed algorithms are demonstrated through numerical simulations in comparison with the state-of-the-art methods.

Index Terms—Rate maximization, reconfigurable intelligent surface (RIS), power allocation, beamforming, reflecting design, minimization-maximization (MM), block successive upperbound minimization (BSUM) algorithm.

I. INTRODUCTION

Along with the worldwide deployment of 5G wireless networks, more stringent requirements (e.g., ultra-high reliability, capacity, and efficiency, as well as low latency) are anticipated to be fulfilled in a holistic fashion in the next-generation wireless networks [2], [3]. The existing technology trends in 5G networks (e.g., massive multiple-input multiple-output (MIMO) and millimeter wave communications) [4], [5], unfortunately, may be insufficient to meet such daunting demands since they will generally involve increased hardware costs and power consumptions. With the theoretical and experimental breakthroughs in micro electromechanical systems and metamaterials, reconfigurable intelligent surface (RIS), a.k.a. software-controlled metasurfaces [6] and intelligent reflecting surfaces [7], has recently been advocated as a powerful solution to enhance the spectrum efficiency and energy efficiency of wireless networks in a cost-effective way [7]–[9].

With the assistance of RIS, whose properties (e.g., scattering, absorption, reflection, and diffraction) are reconfigurable rather than static, the wireless environment is no longer an uncontrollable element, but can be customized to support diverse service requirements [10]. Considering an interference channel in the multi-user communication systems, where independent data streams are sent to some target receivers simultaneously, one classical goal for the system design is to suppress the inter-user interference and thus achieve a high system rate [11], [12]. As a crucial aspect of the RIS-aided wireless networks, rate maximization problems have received significant attention in different systems with interference channels, leading to the problems of designing the beamformers and configuring the elements of RISs simultaneously [1], [13]–[15].

In this paper, we focus on the rate maximization problems in RIS-aided wireless networks, and a unified algorithmic framework based on the block minorization-maximization (BMM) method [16], [17] is proposed. This framework is broadly applicable to diverse RIS-aided systems, where the specific minimization-maximization (MM) techniques are problem-tailored. Merits of the BMM algorithms are illustrated via three different representative system design problems with different design criteria, namely, weighted sum-rate (WSR) maximization for multi-hop RIS-aided multi-user multi-input single-output (MIMO) cellular networks, minimum rate (MR) maximization for RIS-aided multi-user MISO cellular networks, and sum-rate (SR) (i.e., the system capacity) maximization for RIS-aided MIMO device-to-device (D2D) networks.

WSR maximization for RIS-aided multi-user MISO cellular networks. Noticing that if no RIS is deployed in the network, this problem reduces to the classical WSR maximization, for which many algorithms have been proposed, two approaches were brought up in [14] based on the block coordinate descent (BCD) method (a.k.a. alternating minimization or Gauss-Seidel method) [18], where the design variables are partitioned into different blocks and are updated cyclically with the remaining blocks fixed. The first approach is to solve the beamforming block via the weighted minimum mean square error (WMMSE) method [19], [20] and the reflecting block via the Riemannian conjugate gradient (RCG) method [21]. Inheriting a double-loop nature, this approach may invoke many iterations to converge and result in high computational complexity. The other approach is through transforming the problem by the fractional programming (FP) [22] where the reflecting block is solved by MM with a carefully chosen stepsize for line search. This approach, however, relies on the manifold structure of the continuous phase constraint on RISs, making it lame in dealing with other system design problems.
like the case with RISs of discrete phase [23].

**MR maximization for RIS-aided multi-user MISO cellular networks.** Besides WSR, the MR metric, which is able to provide fairness among the multiple users in the network, is also worth considering. Generally, the MR objective in this case becomes nonsmooth. In [13], a similar problem under a multi-group multi-cast system setting was considered, where the authors aimed at solving two approximation problems. By convexifying the nonconvex unimodulus phase constraint on RISs, the first problem was tackled with a BMM algorithm where a second-order cone programming (SOCP) is invoked in each iteration. Besides, another approximation problem is obtained by smoothing the MR objective, based on which the per-iteration SOCP could be removed.

**SR maximization for RIS-aided MIMO D2D networks.** Apart from the MISO systems, the joint beamforming and reflecting design for SR maximization in MIMO systems is further investigated. We consider a MIMO D2D network, where a RIS is deployed to alleviate the co-channel interference among D2D pairs caused by the full frequency reuse [24].

To make it clear, main contributions of this paper are summarized in the following.

- A unified algorithmic framework via BMM for rate maximizations in RIS-aided networks by joint beamforming and reflecting design is presented. The proposed algorithms are of low signal processing complexity and are broadly applicable for a class of system design problems.
- To showcase the flexibility of the algorithm, three specific design cases are investigated covering various rate-related design criteria like WSR, MR, and SR, and diverse wireless system settings including MISO and MIMO.
- Merits of the proposed algorithmic framework are demonstrated both theoretically and empirically, while extensions and generalizations of the framework, e.g., in handling more general constraints and dealing with more general system configurations, are also demonstrated.

The rest of this paper is organized as follows. In Section II, we present the BMM method. Three system design problems, namely, WSR maximization for RIS-aided multi-user MISO networks, MR maximization for RIS-aided multi-user MISO networks, and SR maximization for RIS-aided MIMO D2D networks, are illustrated in Section III, Section IV, and Section V, respectively, with their convergence and complexity analyses given in Section VI. In Section VII, we further discuss several extensions and generalizations on RIS-aided system designs under the proposed algorithm. Section VIII provides simulation results, followed by conclusions in Section IX.

**Notations:** Boldface upper-case letters denote matrices, boldface lower-case letters denote column vectors, and italics stand for scalars. We denote by \( \mathbf{1} \) the all-one vector and by \( \mathbf{I} \) the identity matrices respectively. We denote the all-zero vectors and all-zero matrices uniformly by \( \mathbf{0} \). The real (complex) numbers are denoted by \( \mathbb{R} \) (\( \mathbb{C} \)), the \( N \)-dimensional real (complex) vectors are denoted by \( \mathbb{R}^N \) (\( \mathbb{C}^N \)), and the \( N \times N \)-dimensional complex matrices (Hermitean matrices) are denoted by \( \mathbb{C}^{N \times N} \) (\( \mathbb{H}^N \)). Superscripts \((\cdot)^*\), \((\cdot)^T\), \((\cdot)^H\), and \((\cdot)^{-1}\) denote the matrix conjugate, transpose, Hermitean, and inverse operations, respectively. \( \mathbf{x}_i \) denotes the \( i \)-th element of vector \( \mathbf{x} \), and \( [\mathbf{x}]_{-i} \) denotes \( \mathbf{x} \) with its \( i \)-th element replaced by zero. \( [\mathbf{X}]_{ij} \) denotes the \((i, j)\)-th element of matrix \( \mathbf{X} \), and \( [\mathbf{X}]_{i,\cdot} \) denotes the \( i \)-th column of matrix \( \mathbf{X} \). Given \( \mathbf{A}, \mathbf{B} \in \mathbb{H}^N \), \( \mathbf{A} \succeq \mathbf{B} (\mathbf{A} > \mathbf{B}) \) means \( \mathbf{A} - \mathbf{B} \) is a positive semidefinite (definite) matrix. \( j \) denotes the imaginary unit satisfying \( j^2 = -1 \). \( \Re(\cdot), \Im(\cdot), ||\cdot|| \), and \( \arg(\cdot) \) denote the real part, the imaginary part, the modulus, and the angle of a complex number, respectively. \( \odot \) and \( \odot \) denote the Kronecker product and the Hadamard product of two matrices respectively. \( \text{tr}(\cdot) \) and \( ||\cdot||_F \) denote the trace and the Frobenius norm of a matrix respectively. \( \text{diag}(\mathbf{x}) \) denote a diagonal matrix with entries of \( \mathbf{x} \) being on the diagonal.

## II. BLOCK MINORIZATION-MAXIMIZATION METHOD

In this section, we present the general scheme of the BMM method [16], [17], which can be regarded as a combination of the BCD method [18] and the MM method [17]. BCD is an optimization method aiming at finding a local optimum of the problem by optimizing along one variable block at a time while the other blocks are held fixed. Instead of solving for an exact variable update as in BCD, BMM solves a series of simpler surrogate problems w.r.t. one variable block each time via carrying out an inexact variable update. Specifically, consider the following maximization problem:

\[
\begin{align*}
\text{maximize} & \quad f(x_1, \ldots, x_m) \\
\text{subject to} & \quad x_i \in \mathcal{X}_i, \quad i = 1, \ldots, m, \\
\end{align*}
\]

where \( f : \prod_{i=1}^m \mathcal{X}_i \to \mathbb{R} \). To make the problem well-defined, we assume \( f \) is regular at every point in \( \prod_{i=1}^m \mathcal{X}_i \) and the level set \( \{ x_1, \ldots, x_m \mid f(x_1, \ldots, x_m) \geq f(x_1^{(t)}, \ldots, x_m^{(t)}) \} \) is compact [25], where \( x_i^{(t)} \)'s denote some given values. In the BMM method, different variable blocks are updated in a cyclic order. At the \( t \)-th iteration, the \( i \)-th variable block is updated by solving a maximization subproblem as follows:

\[
\begin{align*}
\text{maximize} & \quad f_i^t(x_i, x_{-i}^{(t)}) \\
\text{subject to} & \quad x_i \in \mathcal{X}_i, \\
\end{align*}
\]

where \( f_i^t(x_i, x_{-i}^{(t)}) \) is defined as a minorizing function of \( f \) w.r.t. \( x_i \) at iterate \( x_{-i}^{(t)} = (x_1^{(t)}, \ldots, x_{i-1}^{(t)}, x_{i+1}^{(t)}, \ldots, x_m^{(t)}) \). Suppose \( \mathcal{X}_1, \ldots, \mathcal{X}_m \) are convex, the generated sequence \( \{x_1^{(1)}, \ldots, x_m^{(t)}\} \) can be proved to converge to a stationary point of Problem (1) if the following mild assumptions hold for \( f_i^t(x_i, x_{-i}^{(t)}) \), \( i = 1, \ldots, m \) [16]:

\[
\begin{align*}
\text{(a)} & \quad f_i^t(x_i, x_{-i}^{(t)}) \text{ is continuous in } (x_i, x_{-i}^{(t)}) \\
\text{(b)} & \quad f_i^t(x_i^{(t-1)}, x_{-i}^{(t)}) = f(x_i^{(t-1)}, x_{-i}^{(t)}) \quad \forall x_i \in \mathcal{X}_i \\
\text{(c)} & \quad f_i^t(x_i, x_{-i}^{(t)}) \leq f_i^t(x_i, x_{-i}^{(t)}) \quad \forall x_i \in \mathcal{X}_i, \forall x_{-i} \in \mathcal{X}_- \\
\text{(d)} & \quad \nabla f_i^t(x_i, x_{-i}^{(t)}, d) |_{x_i = x_i^{(t-1)}} = \nabla f(x_i^{(t-1)}, d) |_{x_i = x_i^{(t-1)}} \\
\end{align*}
\]
where \( T_C(\Sigma^{(k)}) \) denotes the Boulingard tangent cone of \( \Sigma_C \) at \( x_C^{(k)} \) [26]. For minimization problems, a counterpart called block majorization-minimization (BMM), a.k.a. block successive upperbound minimization (BSUM) [16], can be applied where the maximization step of a minorizing function is replaced by a minimization step of a majorizing function. Minorizing and majorizing functions in BMM can be chosen in a flexible way [17] while a properly chosen one can make the updates easy and may lead to a faster convergence over iterations. In practice, the subproblems in BMM are applaudable if they are convex or have closed-form solutions.

III. WEIGHTED SUM-RATE MAXIMIZATION FOR RIS-AIrDED MULTI-USER MISO CELLULAR NETWORKS

A. System Model and Problem Formulation

We consider a multi-hop RIS-aided multi-user MISO downlink communication system, where the base station (BS) equipped with \( M \) antennas communicates with \( K \) users with single antenna in a circular region. We assume there are \( L \) cascaded RISs deployed in the system and the transmitted signal experiences \( I_k \) (\( I_k \leq L \)) hops on the RISs to arrive the \( k \)-th user. Denote \( W = [w_1, \ldots, w_K] \in \mathbb{C}^{M \times K} \) as the beamforming matrix with \( w_k \), \( k = 1, \ldots, K \) being the beamforming vector for the \( k \)-th user. Denote \( G_{i,0,1} \in \mathbb{C}^{N_i \times M} \), \( G_{i-1,i} \in \mathbb{C}^{N_i \times N_{i-1}} \), and \( \Theta_i = \text{diag}(\theta_i) \in \mathbb{C}^{N_i \times N_i} \), \( i = 1, \ldots, L \) as the channel matrix from the BS to the first RIS, the channel matrix from the \((i-1)\)-th RIS to the \( i \)-th RIS, and the phase shift matrix of the \( i \)-th RIS, respectively. Denote \( h_k \in \mathbb{C}^{N_i \times k} \) and \( h_k^2 \in \mathbb{C}^{M \times M} \) as the channel from the last RIS in the reflection channel to the \( k \)-th user and the direct channel from the BS to the \( k \)-th user, respectively. Then the received signal at the \( k \)-th user is given by

\[
y_k = w_k^H \sum_{i=1}^{L} (G_{i-1,i} \Theta_i) h_k^i + h_k^2 \]

where \( s_k \in \mathbb{C}^{1 \times K} \) are the \( K \) independent user symbols with zero mean and unit variance, and \( e_k \in \mathbb{C}^{N_i \times k} \) represents the additive white Gaussian noise for the \( k \)-th user. In this section, for the illustration simplicity, the system model is set up into a cascaded multi-hop signal transmission scenario, where only the direct transmission paths and the transmission paths through \( I_k \) RISs from the BS to the users have been considered. We will show in Section VII-E such a setup can be easily extended to a more general signal transmission scenario. This multi-hop relaying system model is classical [27], [28] and can be deployed to combat the propagation distance problem and to improve the coverage range. Specially, when \( I_1 = \cdots = I_k = 0 \), there only exists direct transmission paths, which reduces to the traditional system with no RIS deployed. Recently, a deep reinforcement learning approach was proposed for a similar multi-hop system design problem [29], whose performance highly relies on the carefully chosen initializations from some preliminary iterative algorithms.

Given the signal model, the signal-to-interference-plus-noise ratio (SINR) at the \( k \)-th user is computed as follows:

\[
\text{SINR}_k = \frac{|w_k^H \sum_{i=1}^{L} (G_{i-1,i} \Theta_i) h_k^i + h_k^2|}{\sum_{j,j \neq k} |w_j^H \sum_{i=1}^{L} (G_{i-1,i} \Theta_i) h_k^i + h_k^2|^2 + \sigma^2}.
\]

Our interest is to maximize the WSR of the system by jointly designing the beamforming matrix \( W \) and the phase shift matrices \( \{\Theta_i\} \). With the data rate (in nats per second per Hertz (nps/Hz)) at the \( k \)-th user defined by \( R_k = \log(1 + \text{SINR}_k) \), the WSR maximization problem is defined as follows:

\[
\text{maximize} \quad f_{\text{WSR}}(W, \{\Theta_i\}) = \sum_{k=1}^{K} \omega_k R_k \quad \text{(WSRMax)}
\]

subject to \( W \in \mathcal{W}, \Theta_i \in \mathcal{C}, \forall i = 1, \ldots, L \),

\( \omega_1, \ldots, \omega_K \) are the predefined nonnegative weights,

\[
\mathcal{W} = \left\{ W \mid \|W\|^2 \leq P \right\}
\]

denotes the transmit power limit constraint of the BS, and \( \mathcal{C}_i = \{ \Theta_i \mid \Theta_i = \text{diag}(\theta_i), \theta_i \in \mathbb{C}^{N_i}, ||\theta_i|| = 1, j = 1, \ldots, N_i \} \) represents the constant modulus constraint for the \( i \)-th RIS indicating that there is no energy loss for the signal when going through the RISs. Problem (WSRMax) is nonconvex and NP-hard [30]. In the following, we will develop a globally convergent algorithm via BMM for problem resolution.

B. The Update Step of \( W \)

We take beamforming variables \( \{w_i\} \) as one block. Given iterate \( \{W, \{\Theta_i\}\} \), the objective function \( f_{\text{WSR}} \) w.r.t. \( W \) is

\[
f_{\text{WSR}, W}(W) = \sum_{k=1}^{K} \omega_k \log(1 + \frac{|w_k^H h_k|^2}{\sum_{j,j \neq k} |w_j^H h_k|^2 + \sigma^2}), \quad (3)
\]

where \( h_k = \sum_{i=1}^{L} (G_{i-1,i} \Theta_i) h_k^i + h_k^2 \). Optimization with \( f_{\text{WSR}, W}(W) \) reduces to the classic WSR maximization problem, which is still nonconvex and NP-hard. We first introduce the following result, with which a quadratic minorizing function for \( f_{\text{WSR}, W}(W) \) can be constructed.

Proposition 1. The \( \log(1 + \frac{|x|^2}{y}) \) with \( x \in \mathbb{C} \) and \( y > 0 \) is minorized at \( (x, y) \) as follows:

\[
\log(1 + \frac{|x|^2}{y}) \geq -\frac{|x|^2}{y} \frac{2}{y^2} \left( y + |x|^2 \right) + \frac{2}{y} \Re(x^* x) + \log(1 + \frac{|x|^2}{y}) \geq \frac{|x|^2}{y} - \frac{|x|^2}{y^2}.
\]

Acknowledging that channel estimation is a nontrivial and crucial problem in RIS-aided system design, while we will assume perfect channel state information to be available through all this paper.

The natural logarithm is chosen since optimal solutions of all the rate maximization problems given later are irrelevant to bases of the log-functions.

\(^{1}\text{In this paper, underlined variables denote those whose values are given.}
\)^{2}\text{For \( f_{\text{WSR}, W}(W) \) has been used to represent \( f_{\text{WSR}, W}(W, W, \{\Theta_i\}) \) for notational simplicity. Similar simplifications will be adopted along this paper.}
Proof: The proof is deferred to Appendix A.

A pictorial illustration of the minorization procedure in Lemma 1 is demonstrated in Fig. 1. Based on Lemma 1, taking $w_k^H h_k$ as $x$ and $\sum_{j \neq k} |w_j^H h_k|^2 + \sigma^2$ as $y$, a minorizing function for $f_{\text{WSR, W}}$ is constructed as follows:

$$f'_{\text{WSR, W}}(W) = \sum_{k=1}^{K} \omega_k \left(-\alpha_k \sum_{j=1}^{K} |w_j^H h_k|^2 + 2\text{Re}(\beta_k w_k^H h_k^*)\right) + \text{const}_{w,k},$$

where

$$\alpha_k = \frac{\text{SINR}_k}{\sum_{j=1}^{K} |w_j^H h_k|^2 + \sigma^2}, \quad \beta_k = \frac{\text{SINR}_k}{w_k^H h_k^*},$$

and $\text{const}_{w,k} = \sum_{k=1}^{K} \omega_k (R_k - \text{SINR}_k - \alpha_k \sigma^2)$, in which $R_k$ and $\text{SINR}_k$ are calculated with the given $\{W, \{\Theta_i\}\}$. By rearranging the terms and ignoring the constant terms, we obtain the resultant convex subproblem for $W$ given by

$$\min_{W \in W} \sum_{k=1}^{K} \left( |w_k^H R w_k| - 2\text{Re}(w_k^H |Q|) \right),$$

where $R = \sum_{k=1}^{K} \omega_k \alpha_k h_k h_k^H$ and $|Q|_{i,k} = \omega_k \beta_k h_k^H$. Note that Problem (5) can be cast as a constrained weighted sum mean square error minimization problem and can be rewritten as

$$\min_{W \in W} \text{tr} \left( W^H R W \right) - 2\text{Re} \left( \text{tr}(W^H Q) \right).$$

Lemma 2. By solving the Karush-Kuhn-Tucker (KKT) system, the optimal solution to Problem (6) is given by

$$W^* = \begin{cases} R^{-1} Q & \text{if} \ |R^{-1} Q| \leq P \\ (R + \gamma I)^{-1} Q & \text{otherwise}, \end{cases}$$

where the variable $\gamma$ satisfies

$$\left\| (R + \gamma I)^{-1} Q \right\|_F = P,$$

and can be readily found via one-dimensional line search methods to meet

$$\sum_{n=1}^{N} \frac{\left( \left| L_{nn} \right| + \gamma \right)^2}{\sum_{n=1}^{N} \left( \left| L_{nn} \right| + \gamma \right)^2} = P,$$

where $V$ and $\Lambda$ are obtained from the eigendecomposition $R = VAV^H$.

C. The Update Step of $\{\Theta_i\}$

In this section, we choose to update the $L$ phase shift matrices $\{\Theta_i\}$ successively. Given iterate $\{W, \{\Theta_i\}\}$, $f_{\text{WSR, W}}$ w.r.t. $\Theta_l$ for $l = 1, \ldots, L$ is given by

$$f'_{\text{WSR, W}}(\Theta_l) = \sum_{k=1}^{K} \omega_k \log \left( 1 + \frac{|w_k^H F_{k,l} \theta_l + w_k^H h_k^d|^2}{\sum_{l,j \neq k} |w_l^H F_{l,j} \theta_l + w_l^H h_l^d|^2 + \sigma^2} \right),$$

with $F_{k,l} = \prod_{i=1}^{L} (G_{i-1,l} \Theta_i) G_{i-1,l} \text{diag}(\prod_{i=1}^{L} (G_{i-1,l} \Theta_i) h_k^*)$. Based on Lemma 1, a minorizing function for $f'_{\text{WSR, W}}(\Theta_l)$ is constructed in the following way:

$$f'_{\text{WSR, W}}(\Theta_l) = \sum_{k=1}^{K} \omega_k \left(-\alpha_k \sum_{j=1}^{K} |w_j^H F_{k,l} \theta_l + w_j^H h_j^d|^2 \right) + 2\text{Re}(\beta_k w_k^H F_{k,l} \theta_l + \beta_k w_k^H h_k^d) + \text{const}_{w,l},$$

which can be further compactly rewritten as

$$f'_{\text{WSR, W}}(\Theta_l) = -\theta_l^H L_l \theta_l + \sum_{k=1}^{K} 2\omega_k \text{Re}(\theta_l^H F_{k,l}^H (\beta_k w_k^H)), \quad \theta_l = \text{const}_{\theta,l},$$

where $L_l = \sum_{k=1}^{K} \omega_k \alpha_k \sum_{j=1}^{K} |w_j^H F_{k,l} h_j|^2 + 2\text{Re}(\beta_k w_k^H h_k^d) + \text{const}_{\theta,l}$.

Optimizing $f'_{\text{WSR, W}}(\Theta_l)$ under $C_l$ is intricate, hence we further introduce the following two results to linearize $f'_{\text{WSR, W}}(\Theta_l)$:

Lemma 3 (331). Let $L, M \in \mathbb{R}^N$ such that $M \succeq L$. Then the function $x^H M x$ with $x \in \mathbb{C}^N$ is majorized at $x$ as follows:

$$x^H L x \leq x^H M x + 2\text{Re}(x^H (L - M) x) + x^H (M - L) x.$$

Lemma 4. Given $M = ||x||^2_2 I$ and $L = xx^H$ with $x \in \mathbb{C}^n$, it follows that $M - L \succeq 0$.

Proof: For any $y \in \mathbb{C}^N$, we can obtain $y^H (M - L) y = ||y||_2^2 - ||x||_2^2 > 0$, which completes the proof. ■

Applying Lemma 3 and Lemma 4 to the first term in (9), a linear minorizing function for $f'_{\text{WSR, W}}(\Theta_l)$ can be obtained as

$$f''_{\text{WSR, W}}(\Theta_l) = -2\text{Re}(\theta_l^H b_l) - N_l \theta_l - \theta_l^H (\lambda_l I - L_l) \theta_l + \text{const}_{\theta,l},$$

where

$$b_l = \sum_{k=1}^{K} \omega_k F_{k,l}^H (\alpha_k \sum_{j=1}^{K} |w_j^H F_{k,l} h_j|^2 - \beta_k w_k^H) + (L_l - \lambda_l I) \theta_l,$$

and $\lambda_l = \sum_{k=1}^{K} \omega_k \alpha_k \sum_{j=1}^{K} |w_j^H F_{k,l} h_j|^2$. Finally, noticing the second term in $f''_{\text{WSR, W}}(\Theta_l)$ is constant over $C_l$ and discarding the constants, the subproblem for $\Theta_l$ is given by

$$\min_{\Theta_l \in C_l} \text{Re}(\theta_l^H b_l).$$

5Note that w.l.o.g. we have assumed $L_l \succeq I$ holds for $k = 1, \ldots, K$.

6Note that $\alpha_k$ and $\beta_k$ have the same expressions as given in Section III-B, while the iterate value $\{W, \{\Theta_i\}\}$ may be different.
Ascent to the one discussed in Section III, except that there is only a complexity analyses deferred to Section VI.

Lemma 5 ([31]). Optimal solutions to Problem (11) can be obtained in closed-forms as $\theta^*_i = e^{\text{arg}(-b_i)}$.

In Problem (11), elements of $\theta_i$ are separable in the objective and the constraint, and hence can be updated in parallel.

In summary, based on BMM the variable blocks $W$ and $\{\Theta_i\}$ will be updated cyclically in closed-forms until some convergence criterion is met. The overall BMM algorithm is summarized in Algorithm 1 with its convergence and complexity analyses deferred to Section VI.

IV. MINIMUM RATE MAXIMIZATION FOR RIS-AIDED MULTI-USER MISO CELLULAR NETWORKS

A. System Model and Problem Formulation

The system model considered in this section is quite similar to the one discussed in Section III, except that there is only a single RIS with $N$ reflecting elements. We denote $G \in \mathbb{C}^{N \times M}$ and $\Theta \triangleq \text{diag}(\theta) \in \mathbb{C}^{N \times N}$ as the channel matrix from the BS to the RIS and the phase shift matrix of the RIS, respectively. The received signal at the $k$-th user is given by

$$y_k = w_k^H (G \Theta h_k^d + h_k^s) s_k + \sum_{j \neq k} w_j^H (G \Theta h_j^d + h_j^s) s_j + \epsilon_k,$$

and the SINR at the $k$-th user is accordingly computed as

$$\text{SINR}_k = \frac{|w_k^H (G \Theta h_k^d + h_k^s)|^2}{\sum_{j \neq k} |w_j^H (G \Theta h_j^d + h_j^s)|^2 + \sigma^2}.$$

Our design target is to maximize the MR of the system by jointly designing the beamforming matrix $W$ and the phase shift matrix $\Theta$. With the data rate at the $k$-th user defined by $R_k = \log(1 + \text{SINR}_k)$, the MR maximization problem is

$$\begin{align*}
\max_{W, \Theta} & \quad f_{MR}(W, \Theta) = \min_{k=1,\ldots,K} R_k \\
\text{subject to} & \quad W \in \mathcal{W}, \Theta \in \mathcal{C},
\end{align*}$$

where $\mathcal{W}$ is defined as before and $\mathcal{C} = \{\Theta | \Theta = \text{diag}(\theta), \theta \in \mathbb{C}^N, ||\theta||_1 = 1, \forall j = 1, \ldots, N\}$.

Problem (MRMax) is nonconvex and NP-hard. Like in the last section, a low-complexity and globally convergent BMM-based algorithm will be developed for problem resolution.

B. The Update Step of $W$

Given the iterate $(W, \Theta)$, the objective $f_{MR}$ w.r.t. $W$ is

$$f_{MR, W}(W) = \min_k \log \left(1 + \frac{|w_k^H h_k|}{\sum_{j \neq k} |w_j^H h_j|^2 + \sigma^2}\right),$$

where $h_k = G \Theta h_k^d + h_k^s$. For the pointwise minimum function $f_{MR, W}$, a minorizing function of it can be obtained based on the minorizing functions of $R_1, \ldots, R_k$ (see a proof given in [32]). With Lemma 1, a minorizing function for $f_{MR, W}$ can be constructed as follows:

$$f'_{MR, W}(W) = \min_k -\alpha_k \sum_{j=1}^K |w_j^H h_k|^2 + 2\text{Re}(\beta_h w_k^H h_k) + \text{const}_{w,k},$$

where $\text{const}_{w,k} = R_k - \text{SINR}_k - \alpha_k \sigma^2$. Then the subproblem for $W$ becomes a minimax problem given by

$$\begin{align*}
\min_{W \in \mathcal{W}} \max_{s \in \mathcal{S}} \sum_{k=1}^K s_k (\text{tr}(W^H R_k W) - 2\text{Re}(w_k^H q_k) - \text{const}_{w,k}),
\end{align*}$$

(13)

where $\mathcal{S} = \{s \in \mathbb{R}^K | 1^T s = c, s \geq 0\}$ with constant $c > 0$. Then solutions to Problem (12) can be obtained by solving Problem (13). The objective of Problem (13) is convex-concave in $W$ and $s$, and the constraint sets $\mathcal{W}$ and $\mathcal{S}$ are both nonempty compact and convex. Hence, a saddle point always exists for Problem (13) and then it can be swapped to be a maximin problem without affecting its solutions [33] as

$$\begin{align*}
\min_{W \in \mathcal{W}} \max_{s \in S} \sum_{k=1}^K s_k (\text{tr}(W^H R_k W) - 2\text{Re}(w_k^H q_k) - \text{const}_{w,k}).
\end{align*}$$

(14)

Problem (14) is a convex problem in variable $s$ with a simplex constraint $S$, which can be efficiently solved via many iterative algorithms. In this paper, we adopt the mirror ascent algorithm (MAA) [34], [35] outlined in the following.

Mirror Ascent Algorithm (MAA)

Input: function $h(s)$, initial feasible value of $s$.

Repeat
1. Calculate a subgradient $g \in \partial h(s)$ (the subdifferential of $h$ at $s$);
2. Update $s = \text{arg} \max_{s \in \mathcal{S}} \left\{ g^T s - \frac{1}{\gamma} D_{\phi}(s, s) \right\}$, with $D_{\phi}(s, s) = \psi(s) - \psi(s) - (\nabla \psi(s))^T (s - s)$;

Until the value of the objective function $h(s)$ converges.

To solve Problem (14) via MAA, we define input function

$$h(s) = \min_{W \in \mathcal{W}} \sum_{k=1}^K s_k (\text{tr}(W^H R_k W) - 2\text{Re}(w_k^H q_k) - \text{const}_{w,k}),$$

the subgradient $g_w$ can be computed as

$$[g_w]_k = \text{tr}(X^H R_k X) - 2\text{Re}(X^H q_k) - \text{const}_{w,k}$$
with $X = \arg \min_{W \in W} \sum_{k=1}^{K} s_k \left( \text{tr}(W^H R_k W) - 2\text{Re}(w_k^H q_k) \right)$, which is readily solved based on Lemma 2. With the simplex space $S$, we can choose

$$\varphi(s) = \begin{cases} \sum_{k=1}^{K} s_k \log s_k & s \in S \\ +\infty & \text{otherwise}, \end{cases}$$

and set stepsize $\gamma = r \frac{1}{\lambda}$ with constant $r > 0$ at the $t$-th iteration, which leads to the following closed-form update rule:

$$s = c \frac{\mathbf{x} \otimes e^{-\gamma s_\theta W}}{1^T (\mathbf{x} \otimes e^{-\gamma s_\theta W})}.$$  

(15)

**C. The Update Step of $\Theta$**

Given the iterate $(\mathbf{W}, \Theta)$, the objective $f_{\text{MR}}$ w.r.t. $\Theta$ is

$$f_{\text{MR}, \Theta}(\Theta) = \min_k \log \left( 1 + \frac{|w_k^H F_k \theta + w_k^H h_k^d|^2}{\sum_{j \neq k} |w_j^H F_k \theta + w_j^H h_j^d|^2 + \sigma^2} \right),$$

where $F_k = \text{Gdiag}(h_k^d)$. Then based on Lemma 1, a minimizing function for $f_{\text{MR}, \Theta}$ can be constructed as follows:

$$f'_{\text{MR}, \Theta}(\Theta) = \min_k -\theta^H L_k \theta - \alpha_k \sum_{j=1}^{K} \left( 2\text{Re}(\theta^H w_j^H F_k \theta) + \beta_k w_j^H h_j^d \right) + |h_k^d w_j|^2 + 2\text{Re}(\beta_k w_k^H F_k \theta + \beta_k w_k^H h_k^d) + \text{const}_{w,k},$$

which can be further rewritten as

$$f'_{\text{MR}, \Theta}(\Theta) = \min_k -\theta^H L_k \theta - \alpha_k \sum_{j=1}^{K} \left( 2\text{Re}(\theta^H F_k^H w_j^H h_k^d) + |h_k^d w_j|^2 \right) + 2\text{Re}(\beta_k w_k^H F_k \theta + \beta_k w_k^H h_k^d) + \text{const}_{w,k},$$

where $L_k = \alpha_k \sum_{j=1}^{K} w_j^H w_j^H F_k$. According to Lemma 3 and Lemma 4, a piecewise linear minimizing function for $f'_{\text{MR}, \Theta}$ is further obtained as follows:

$$f''_{\text{MR}, \Theta}(\Theta) = \min_k -2\text{Re}(\theta^H b_k) + \text{const}_{\theta,k},$$

where

$$b_k = \alpha_k \sum_{j=1}^{K} F_k^H w_j^H h_j^d - \beta_k F_k^H w_k + (L - \lambda I) \theta$$

and $\text{const}_{\theta,k} = K \lambda + \theta^H (L - \lambda I) \theta + \alpha_k \sum_{j=1}^{K} |h_j^d w_j|^2 - 2\text{Re}(\beta_k h_k^H w_k) - \text{const}_{w,k}$ with $\lambda = \alpha_k \sum_{j=1}^{K} \|w_j^H F_k\|^2$.

Then the subproblem for $\Theta$ is given by

$$\max_k 2\text{Re}(\theta^H b_k) + \text{const}_{\theta,k}.$$  

(16)

**Algorithm 2** The BMM Algorithm for Problem (MRMax).

**Input:** $\{h_i^d\}, \{h_i^d\}, G, P, \sigma^2$, initial feasible values of $\mathbf{W}$ and $\Theta$.

**Repeat**
1. Update $\mathbf{W}$ by solving Prob. (14) via MAA;
2. Update $\Theta$ by solving Prob. (16) via MAA;

**Until** the value of the objective function converges.

where $C_{\text{relaxed}} = \{ \theta \ | \ \theta \in \mathbb{C}^N, |\theta|, 1 \leq v, j = 1, \ldots, N \}.$

**Proof:** The detailed proof is given in Appendix B.

We can swap the order of minimization and maximization as before, and Problem (17) is equivalently transformed to

$$\max_k \sum_{k=1}^{K} s_k (2\text{Re}(\Theta^H b_k) + \text{const}_{\theta,k}),$$

which can be solved via MAA with the input function

$$h(s) = \min_{\Theta \in C_{\text{relaxed}}} \sum_{k=1}^{K} s_k (2\text{Re}(\Theta^H b_k) + \text{const}_{\theta,k}).$$

Then the corresponding subgradient $g_\theta$ can be calculated as

$$[g_\theta]_k = 2\text{Re}(x^H b_k) + \text{const}_{\theta,k},$$

where $x$ is computed from the following equation

$$x = \text{diag} \left( \arg \min_{\Theta \in C_{\text{relaxed}}} \sum_{k=1}^{K} 2s_k \text{Re}(\Theta^H b_k) \right),$$

where the last line can be proved in a similar way as Lemma 5. The update rules in MAA are chosen the same as (15).

In summary, to solve the MR maximization problem via BMM, the two variable blocks, i.e., $\mathbf{W}$ and $\Theta$, will be updated cyclically until some convergence criterion is met. The overall algorithm is summarized in Algorithm 2 with its convergence and complexity analyses given in Section VI.

**V. SUM-RATE MAXIMIZATION FOR RIS-AIDED MIMO DEVICE-TO-DEVICE NETWORKS**

**A. System Model and Problem Formulation**

In this section, the RIS-aided MIMO D2D system is considered, where $K$ transceivers pairs transmit multiple data streams. We assume the $k$-th ($k = 1, \ldots, K$) transmitter and receiver are equipped with $M_k$ and $M_k'$ antennas. Let $H_{r,j}^i \in \mathbb{C}^{M_k \times M_k'}$, $H_{r}^R \in \mathbb{C}^{M_k \times N}$, and $G_j \in \mathbb{C}^{N \times M_k'}$ be the direct channel between the $i$-th receiver and the $j$-th transmitter, the channel in the reflection link between the RIS and the $i$-th receiver, and the channel between the $j$-th transmitter and the RIS, respectively. Other notations are defined the same as in previous sections. Then the received signal at the $k$-th receiver is given by

$$y_k = (H_k^R \Theta G_k + H_{k,j}^R) W_k s_k$$

(desired signal)

$$+ \sum_{j \neq k} (H_{i,j}^R \Theta G_j + H_{k,j}^R) W_j s_j + e_k,$$

(interference plus noise)

**Proposition 6.** A saddle point exists for Problem (16) and can be obtained by solving the following relaxed problem

$$\min_{\Theta \in C_{\text{relaxed}}} \max_{s \in S} \sum_{k=1}^{K} s_k (2\text{Re}(\Theta^H b_k) + \text{const}_{\theta,k}),$$

(17)
where $W_k \in \mathbb{C}^{M_k \times d_k}$ denotes the beamforming matrix, $s_k \in \mathbb{C}^{d_k}$ is the symbol vector for the $k$-th transceiver pair, and $e_k \sim \mathcal{CN}(0, \sigma^2)$ represents the noise.

The target is to maximize the SR of the system by jointly designing the beamforming matrices $\{W_i\}$ and the phase shift matrix $\Theta$. The data rate at the $k$-th receiver is defined as

$$R_k = \log \det (I + W_k^H (H_k^\prime \Theta G_k + H_k^d) W_k^H) \times (H_k^\prime \Theta G_k + H_k^d) W_k^H \Theta^{-1}$$

where the interference-plus-noise term is

$$T_k = \sum_{j,j \neq k}(H_k^\prime \Theta G_j + H_k^d) W_j^H (H_k^\prime \Theta G_j + H_k^d) + \sigma^2 I.$$ 

Then the SR maximization problem is formulated as follows:

$$\max_{\{W_i\}, \Theta} f_{\text{SR}}(\{W_i\}, \Theta) = \sum_{k=1}^K R_k$$  

subject to $W_i \in W_i, \forall i = 1, \ldots, K, \Theta \in C$, where the transmit power constraint for the $i$-th transmitter is

$$W_i = \left\{W_i \mid \|W_i\|_F^2 \leq P_i\right\}.$$ 

Problem (SRMax) is nonconvex and NP-hard. In the following, a BMM-based algorithm is developed for problem resolution.

B. The Update Step of $\{W_i\}$

Given the iterate $\{\{W_i\}, \Theta\}$, the $f_{\text{SR}}$ w.r.t. $\{W_i\}$ is

$$f_{\text{SR},\Theta}(\{W_i\}) = \sum_{k=1}^K \log \det (I + W_k^H H_k^H W_k^H),$$

where $H_k^d = H_k^\prime \Theta G_k + H_k^d$.

Proposition 7. Function $\log \det (I + X^H Y^{-1} X)$ with $X \in \mathbb{C}^{M \times N}$ and $Y > 0$ is minimized at $(X, Y)$ as follows:

$$\log \det (I + X^H Y^{-1} X) \geq -\text{tr}((Y + XX^H)^{-1} XH_k^d W_k^H W_k X)$$

$$+ 2\Re(\text{tr}(I + XX^H Y^{-1} X)) + \log(I + XX^H Y^{-1} X) - \text{tr}(XX^H Y^{-1} X).$$

Proof: The proof is given in Appendix C.

Based on Lemma 7, taking $H_{k,k} W_k$ as $X$ and $T_k$ as $Y$, a minimizing function for $f'_{\text{SR},\Theta}$ is computed as follows:

$$f'_{\text{SR},\Theta}(\{W_i\}) = \sum_{k=1}^K \left(-\text{tr}(A_k \sum_{j=1}^K H_{k,j} W_j W_j^H H_{k,j}^d) + 2\Re(\text{tr}(B_k H_{k,k} W_k))\right) + \text{const}_w,$$

where $A_k = (T_k + H_{k,k} W_k W_k^H H_{k,k}^d)^{-1} H_{k,k} W_k B_k$ with $B_k = (I + W_k^H H_k^H W_k^H) (T_k + H_{k,k} W_k) W_k^H H_k^H (T_k + H_{k,k} W_k)^{-1}$, and $\text{const}_w = \sum_{k=1}^K |R_k - \text{tr}(W_k^H H_k^H W_k^H H_k^d)| - \sigma^2 \text{tr}(A_k)$. Ignoring the constant terms, we obtain the resultant subproblem for $W_k$ as follows:

$$\min_{W_k \in \mathbb{C}^{M_k \times d_k}} \text{tr}(W_k^H R_k W_k) - 2\Re(\text{tr}(W_k^H Q_k)).$$

Problem (18) is separable over different elements in $\Theta$ and can be solved in parallel via Lemma 2.

Algorithm 3 The BMM Algorithm for Problem (SRMax).

Input: $\{H_i^e\}, \{H_i^d\}, \{G_j\}, \{P_i\}, \sigma^2$, initial feasible values of $\{W_i\}$ and $\Theta$.

Repeat
1. Update $\{W_i\}$ successively by solving Prob. (18) via Lemma 2.
2. Update $\Theta$ by solving Prob. (21) via Lemma 5.
Until the value of the objective function converges.

C. The Update Step of $\Theta$

Given iterate $\{\{W_i\}, \Theta\}$, $f_{\text{SR},\Theta}$ w.r.t. $\Theta$ is given in Eq. (19).

Then a minorizing function for $f_{\text{SR},\Theta}$ can be constructed based on Lemma 7 as in Eq. (20), which is written compactly as

$$f'_{\text{SR},\Theta}(\Theta) = -\sum_{k=1}^K \text{tr}(A_k (\Theta^T \Theta) K_2 + (\Theta^T \Theta) K_1)$$

$$+ 2\Re(\text{tr}(N \Theta (\Theta^T \Theta))) + \text{const}_\Theta,$$

where

$$K_1 = H_{r,k}^H A_k H_{r,k}, K_2 = \sum_{j=1}^K G_j W_j W_j^H G_j^H,$$

$$N = \sum_{k=1}^K G_k W_k B_k H_{r,k} - \sum_{k=1}^K G_j W_j W_j^H H_{d,k,j}^H A_k H_{r,k},$$

and $\text{const}_\Theta = \sum_{k=1}^K \left(-\text{tr}(A_k (H_{d,k,j} W_j W_j^H H_{d,k,j}^d)) + 2\Re(\text{tr}(B_k H_{d,k,j} W_k)))\right) + \text{const}_w$. After some mathematical manipulation, the $f'_{\text{SR},\Theta}(\Theta)$ can be further rewritten as

$$f'_{\text{SR},\Theta}(\Theta) = -\Theta^H L \Theta + 2\Re(\text{diag}(N)^T \Theta^T) + \text{const}_\Theta,$$

with $L = (\sum_{k=1}^K K_1) \otimes K_2^T$.

Based on Lemma 3, a linear minorizing function for $f'_{\text{SR},\Theta}$ can be further obtained as follows:

$$f''_{\text{SR},\Theta}(\Theta) = -2\Re(\Theta^T b) - N\lambda - \Theta^H (\lambda I - L) \Theta + \text{const}_\Theta,$$

where $b = \text{diag}(\Theta^H (L - \lambda I)^H - N)$ with $\lambda$ being the largest eigenvalue of $L$. Then the subproblem for $\Theta$ is given by

$$\min_{\Theta \in \mathbb{C}^{M \times N}} \text{Re}(\Theta^T b).$$

Problem (21) is separable over different elements in $\Theta$ and can be solved in parallel via Lemma 5.

In summary, based on BMM the variable blocks $\{W_i\}, \Theta$ will be updated cyclically until some convergence criterion is met. The overall algorithm is summarized in Algorithm 3 with its convergence and complexity analyses given in Section VI.

VI. CONVERGENCE AND COMPLEXITY ANALYSIS

A. Convergence Analysis

Theorem 8. Every limit point generated by Algorithm 1, Algorithm 2, or Algorithm 3 is a stationary/KKT point of Prob. (WSRMax), Prob. (MRMax), or Prob. (SRMax), respectively.

Proof: The detailed proof is given in Appendix D. □
\[
\begin{align*}
    f_{SR, \Theta}(\Theta) &= \sum_{k=1}^{K} \log \det \left( I + W_k^H \left( H_k^T (\Theta^T \otimes 1) G_k + H_k^T \right)^H \left( \sum_{j \neq k} \left( H_k^T (\Theta^T \otimes 1) G_j + H_k^T \right) W_j \right) \right. \\
    &\quad \left. \times W_j^H \left( H_k^T (\Theta^T \otimes 1) G_k + H_k^T \right)^H + \sigma^2 I \right)^{-1} \left( H_k^T (\Theta^T \otimes 1) G_j + H_k^T \right) W_j \right).
\end{align*}
\]

Further majorizing the objective in (5) based on Lemma 3 and Lemma 4, then the \( W \)-block subproblem in WSR maximization becomes
\[
\text{minimize} \quad \| W - \Pi \|_F^2,
\]
where
\[
[\Pi]_{i,k} = \left[ Q \right]_{i,k} - \left( R - \sum_{k=1}^{K} \omega_k \alpha_k \| h_k \|^2 \right) W_k.
\]

By solving the KKT system of Problem (22), the optimal solution can be obtained in closed-form as follows:
\[
W^* = \begin{cases} 
    \Pi & \text{if } \| \Pi \|_F^2 \leq P \\
    \sqrt{\frac{P}{\| \Pi \|_F}} K & \text{otherwise}
\end{cases}
\]

Therefore, with an additional majorization step, a cheaper result is obtained leading to a lower per-iteration computational complexity. When we apply the closed-form updating rule of \( W \), the inverse and eigendecomposition operations of \( R \) are avoided and the per-iteration complexity becomes \( O(K^2 M^2) \). However, in practice, whether a further majorization step like this brings better convergence property or not depends on the specific problem and the characterization of the inner-loop residual error, in that it solves a looser surrogate problem to trade for a closed-form solution and, hence, it may requires more iterations for convergence compared with its counterpart.

\begin{section}
\textbf{B. General Power Constraint for The \( W \)-Block}
\end{section}

Beyond the total power constraint, the proposed algorithm can also handle more general power constraints [36] like
\[
\text{tr}(\Omega_j W W^H) \leq P_j, \quad \forall j = 1, \ldots, J,
\]
where \( \Omega_1, \ldots, \Omega_J \succeq 0 \) are application-oriented. This general power constraint reduces to the total power limit considered in the previous sections when \( J = 1 \) and \( \Omega_1 = I \). It is also commonly used to model a more realistic case that each antenna is equipped with an independent power amplifier, where we can limit the per-antenna power by setting \( J = M \) and \( \Omega_j \) to be a diagonal matrix with the \( j \)-th diagonal element being one while the other elements being zeros.
Proposition 9. By solving the KKT system, the optimal solution to Problem (6) with the general power constraints in (23) can be obtained in the following way

$$ W^* = \begin{cases} R^{-1}Q & \text{if } \|R^{-1}Q\Omega_j^2\|_F^2 \leq P_j, \forall j = 1, \ldots, J \\ (R + \sum_{i=1}^j \gamma_i \Omega_i)^{-1}Q & \text{otherwise}, \end{cases} $$

where the variables $\gamma_1, \ldots, \gamma_J$ satisfies

$$ \| (R + \sum_{i=1}^j \gamma_i \Omega_i)^{-1}Q \Omega_j^2 \|_F^2 = P_j, \forall j = 1, \ldots, J. $$

Proof: It can be proved similarly as in Proposition 2. \hfill \blacksquare

C. A Serial Update Scheme for The $\Theta$-Block

Besides updating the elements in each phase shift matrix in parallel, a serial update can also be conducted. Consider Problem (WSRMax), we can rewrite $f_{\text{WSR}, \Theta_1}$ w.r.t. $\Theta_j$ as

$$ f'_{\text{WSR}, \Theta_j}(\Theta_j) = -2\text{Re}\left(\Theta_j^* b_j^* \right) \Theta_j^* - L_j \Theta_j^* - \text{Re}\left(Re(\Theta_j^* b_j^*) - \sum_{j=1}^K \omega_j \gamma_j \Theta_j \right) + \text{const}_{\theta,\ell}, $$

where

$$ b_j^* = \sum_{k=1}^K \omega_k F_{k,l}^H (\sigma_k w_j^H \bar{h}_k^0 - \beta_k w_k) + L_j \Theta_j^*. $$

Then the subproblem w.r.t. $\Theta_j$ is given by

$$ \text{minimize } \text{Re}(\Theta_j^* b_j^*), $$

which can be solved easily via Lemma 5.

Based on the above formulation, the phase shift matrices $\{\Theta_j\}$ will be updated in series with the elements in each phase shift matrix updated also in series. Such update rule can also be applied to Problem (MRMax) and Problem (SRMMax).

D. Discrete Phase for The $\Theta$-Block

Along the paper, we have taken a continuous-phase scheme for the phase shift matrices, while for Problem (WSRMax) and Problem (SRMMax), a discrete-phase scheme [23], which is more practical for hardware implementation, can also be considered which is defined as follows:

$$ D_1 = \{\Phi_1 \mid \Phi_1 = \text{diag}(\theta_j), \theta_j \in \mathbb{C}^{N_i}, \|\theta_j\|_1 = 1, \text{ang}(\theta_j) \in \Phi_1, \forall j = 1, \ldots, N_i\}, $$

where $\Phi_1$ denotes the set of fixed angles that is achievable for the $i$-th RIS. Under the discrete-phase scheme, the closed-form solutions given in Lemma 5 should be modified to be

$$ [\theta_j^*]_i = e^{j\text{arg}\min_{\phi_i \in \Phi_1} \| \phi_i + \text{ang}(b_j^i) \|_2}, \forall j = 1, \ldots, N_i, $$

which can be efficiently implemented on hardwares leveraging on a look-up table.

E. WSR Maximization for General-Topology Multi-Hop RIS-Aided Multi-User MISO Cellular Networks

In Section III, a simplified model where only the direct transmission paths from the BS to the users and the reflection transmission paths through cascaded $J_1, \ldots, J_K$ RISs to the users have been considered. However, the developed BMM algorithm can be easily extended for system designs with general-topology [37]. To get a taste of it, let us first consider a feed-forward “fully connected” double-RIS system (we neglect any feed-backward signals which are in general much weaker when received). We adopt a similar system setting as used in Section III and denote the channel between the BS and the first RIS, the channel between the BS and the second RIS, and the channel between the first RIS and the second RIS as $G_{0,1}$, $G_{0,2}$, and $G_{1,2}$, respectively. Besides, we denote the channel between the first RIS and the $k$-th user, the channel between the second RIS and the $k$-th user, and the direct channel from the BS to the $k$-th user as $h_{k,1}^0$, $h_{k,2}^0$, and $h_k^0$, respectively. Then the SINR at the $k$-th user is given by

$$ \text{SINR}_k = \frac{|w_k^H h_k^0|^2}{\sum_{j,k \neq j} |w_j^H h_j^0|^2 + \sigma^2}, $$

where $h_k = G_{0,1} \Theta_1 h_{k,1}^0 + G_{0,2} \Theta_2 h_{k,2}^0 + h_k^0$. Following the general idea of the proposed BMM algorithms, given the iterate $\{W, \Theta_1, \Theta_2\}$, we optimize $W$, $\Theta_1$, and $\Theta_2$ cyclically. It is easy to verify that $f_{\text{WSR}, W}(\Theta_1)$ takes exactly the same mathematical form as Eq. (3) and hence is omitted. By defining $F_{k,1} = G_{0,1} \text{diag}(G_{1,2} \Theta_2 h_{k,2}^0 + h_{k,1}^0)$ and $h_{k,1}^0 = G_{0,2} \Theta_2 h_{k,2}^0 + h_{k,1}^0$, we obtain the objective $f_{\text{WSR}, W}(\Theta_1)$ in the following way

$$ f_{\text{WSR}, W}(\Theta_1) = \sum_{k=1}^K \text{log} \left(1 + \frac{|w_k^H F_{k,1} \Theta_1 + w_k^H h_k^0|^2}{\sum_{j,k \neq j} |w_j^H F_{k,1} \Theta_1 + w_j^H h_k^0|^2 + \sigma^2}\right), $$

which shares the same form of expression as Eq. (7). Similar result applies to the objective $f_{\text{WSR}, W}(\Theta_2)$. Therefore, the problem of the feed-forward “fully connected” double-RIS system design can be addressed readily following the same algorithmic procedure as introduced in Section III.

Furthermore, we can extend our algorithm to the case of solving general-topology multi-hop RIS-aided, a.k.a. multi-RIS, system designs equipped with finite $L$ RISs. Assume there are $P_{k,n}$ reflection paths from the BS to the $k$-th user that go through $n$ ($n = 1, \ldots, L$) RISs, within which we denote by $\mathcal{R}_{k,n}^p = \{p_{k,1}, \ldots, p_{k,n}\}$ with $p = 1, \ldots, P_{k,n}$ the $p$-th path, where the index of the $i$-th intermediate RIS is denoted by $p_{k,i}$. (We also define $p_{k,0} = 0$ for any $k$ and $p$, which refers to the BS.) Note that there is a chance that $P_{k,n} = 0$ for some $n$ in practice. The system model considered in Section III can be seen as a special case of this general-topology system where
with and the scheme. Interested readers are referred to [40] for details. The BMM method may suffer from a slow convergence, so a general acceleration strategy is needed for this class of algorithms. In this section, we provide numerical experiments to corroborate our theoretical results with the codes available at 

https://github.com/zepengzhang/RateMaxRIS-BMM

Already given the mathematical form above which resembles the aforementioned double-RIS system, such a general topology multi-hop RIS-aided system designs can be addressed readily via the algorithm proposed in Section III-C.

F. SINR Maximizations for RIS-Aided Wireless Networks

Beyond the rate maximization problems, another commonly used system performance metric is SINR [38]. The proposed BMM method is also applicable in solving many SINR maximization problems in RIS-aided wireless network designs, e.g., weighted sum-SINR maximizations, minimum SINR maximizations, and sum-SINR maximizations. As for the rate maximizations where the minorizing functions for the objectives are constructed by minorizing individual rate functions for different users, the construction of minorizing functions for the SINR-based objectives can be done in a similar way. Taking the system considered in Section III and Section IV as an example, the minorizing functions for individual SINRs can be obtained based on Lemma 10 (given in Appendix A). And then the SINR maximization problems can be addressed by optimizing the resultant surrogate problems following similar procedures as for rate maximizations under the proposed BMM algorithmic framework.

A. Simulations for RIS-Aided MISO Cellular Networks

1) System Settings: Under a three-dimensional Cartesian coordinate system, we consider a multi-user MISO system where the BS located at \((0,0,10)\)m communicates with \(K\) users assisted by a RIS at \((d,0,10)\)m. The \(K\) users are randomly distributed in a circle centered at \((d,30,0)\)m with radius of 10m. The antennas at the BS are arranged as a uniform linear array with spacing of \(\frac{\lambda}{2}\), and the passive reflecting elements at the RIS is arranged as a uniform planar array with spacing of \(\frac{\lambda}{2}\), where \(\lambda\) is the wavelength. We assume that the channel fading is frequency flat and adopt the Rician fading model for all channels. To make an example, the channel from the BS to the RIS is modeled as

\[
G_{0,1} = \sqrt{\frac{\kappa_G(d)}{K_G+1}} (\sqrt{K_G G_{\text{LoS},0,1} + G_{\text{NLoS},0,1}}),
\]

where \(\kappa_G(d)\) is the distance-dependent path loss, \(K_G \in [0,\infty)\) is the Rician factor, \(G_{\text{LoS},0,1}\) is the line-of-sight (LoS) component, and \(G_{\text{NLoS},0,1}\) is the non-line-of-sight (NLoS) components. Specifically, the distance-dependent path loss is computed as \(\kappa_G(d) = T_0 \left( \frac{d}{d_0} \right)^{-\gamma} \) with the path-loss at the reference distance \(d_0 = 1\)m being \(T_0 = -30\)dB and \(\gamma\) denoting the path loss exponent, the LoS component is computed as the product of the array responses at the two sides, and the NLoS component is modeled by Rayleigh fading, with \(G_{\text{NLoS},0,1} = C N(0,1)\). The other channels are modeled similarly as for \(G_{0,1}\). The path loss exponents and the Rician factors for channels \(G_{i-1,i}\), \(\{h^s_i\}\), and \(\{h^d_i\}\) set as \(\gamma_\text{LoS} = 2.2\), \(K_G = 3\), \(\gamma_\text{NLoS} = 3.5\), \(K_d = 0\), and \(K_r = 2.8\), \(K_r = 3\). Besides, we have considered the noise power spectrum density of \(-169\)dBm/Hz and the transmission bandwidth of 240kHz. In the following, if not specified, we will assume \(P = 0\)dBm, \(\sigma^2 = 1\), \(K = 4\), \(M = 4\), \(N = 100\), and \(d = 200\)m. Moreover, all the simulation curves are averaged over 100 independent channel realizations.

2) Performance Evaluations: Four benchmarks are considered for the WSR maximization as introduced in Section III: i) WMMSE-based BCD method [14] in which case the \(W\)-block is tackled by WMMSE [20] and the \(\Theta\)-block is tackled via RCG; ii) FP-based BCD method [14] in which case the problem is tackled via FP aided by the prox-linear update for the \(W\)-block and an MM step for the \(\Theta\)-block; iii) Random Phase in which case the phase shift matrix \(\Theta\) is randomly initialized and \(W\) is optimized via MM; iv) Without RIS, i.e., no RIS is used and \(W\) is optimized via MM.

The convergence behaviors of the proposed BMM algorithm and the above benchmarks are investigated under the single-hop scenario. The comparisons in terms of the outer iteration numbers are depicted in Fig. 2, while the convergence times of different approaches with different number of elements at RIS and those with different number of antennas at BS are showcased in Fig. 3 and Fig. 4, respectively. It can be observed that the serial BMM and the parallel BMM acquire similar outer iteration numbers to converge, which is better or comparable w.r.t. the benchmark methods. Moreover, the parallel BMM method consistently acquires the lowest CPU computation times to converge in all the simulations cases.
We further investigate the benefits of introducing multiple RISs, especially in combating the large path loss in long-distance propagations. As for the system setting, the second RIS at $(\frac{d}{2}, 0, 10)\text{m}$ is deployed in the two-hop system and an another RIS at $(\frac{d}{4}, 0, 10)\text{m}$ is included for the three-hop system. Besides, we assume the additional RISs are also equipped with $N$ elements. The achievable WSRs for different systems as distance increases are showcased in Fig. 5, in which the BMM approach (parallel BMM is used hereafter) and the benchmarks converge to numerically similar WSR for the single-hop scenario, and the benefits of introducing multiple RISs to improve WSR is obvious.

For the MR maximization problems, two benchmarks are considered: i) SOCP-based BMM method [13] where the BMM method is applied with the resultant SOCP subproblems solved via off-the-shelf scripting language CVX [42]; ii) Approximation-based BMM method [13] where the pointwise minimum function is first smoothened with the log-sum-exp function and the approximation problem is tackled via BMM. The convergence behaviors of the proposed BMM algorithm along with these two benchmarks are demonstrated in terms of outer iteration numbers in Fig. 6, where the proposed BMM algorithm enjoys the fastest convergence speed with the highest achievable MR among these three methods. Moreover, the convergence time of different approaches with different number of users is depicted in Fig. 7. We can easily observe that the proposed BMM algorithm acquires the lowest CPU times for convergence in all cases.

B. Simulations for RIS-Aided MIMO D2D Networks

1) System Settings: We consider a MIMO D2D system where $K$ transceiver pairs transmit multiple data streams with the assistance of a RIS located at $(d, 30, 0)\text{m}$. The transmitters and the receivers are randomly distributed in two circles with radius of $10\text{m}$ that are centered at $(0, 0, 10)\text{m}$ and $(d, 30, 0)\text{m}$, respectively. The antennas at the transmitters and the receivers
are arranged as uniform linear arrays with spacing of $\frac{\lambda}{2}$. The number of antennas of each transmitter and receiver is set uniformly to be $M$, and the dimension of symbol vectors is set as $K$. The channels are modeled in a fashion similar to that for the MISO system in Section VIII-A1, and hence we omit the details here.

2) Performance Evaluations: In this section, we will investigate the performance loss of considering more practical constraints in joint beamforming and reflecting design for SR maximization in the specified MIMO D2D system. Particularly, we consider the per-antenna power constraint introduced in Section VII-B, which is more realistic due to the fact that each antenna is usually equipped with an independent power amplifier. Besides, considering hardware limitation, instead of the continuous-phase shift, we further impose a 2-bit discrete-phase constraint. Therefore, four variants of the proposed BMM algorithms are implemented: i) BMM with total power limit and the continuous-phase constraint; ii) BMM (2-bit) with total power limit and the 2-bit discrete-phase constraint; iii) BMM (per-antenna) with per-antenna power limit and the continuous-phase constraint; iv) BMM (2-bit & per-antenna) with per-antenna power limit and the 2-bit discrete-phase constraint. Convergence behaviors of these four variants of BMM are demonstrated in Fig. 8 and performance comparisons by considering more practical constraints is depicted in Fig. 9.

IX. CONCLUSIONS

In this paper, we have considered the joint beamforming and reflecting design problem for rate maximization problems in RIS-aided wireless networks. A unified algorithmic framework based on the block minorization-maximization (BMM) method has been developed. Merits of the algorithm have been showcased via three system design problems, where problem-tailored low-complexity and globally convergent algorithms are proposed. Benefits of the BMM algorithms in comparison with existing algorithms have been demonstrated numerically.
We have also shown that many more RIS-aided system design aspects can be addressed by the unified BMM method, leaving much space to explore for future research.

APPENDIX

Refer to the attached Supplementary Materials.

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Supplementary Materials for “Rate Maximizations for Reconfigurable Intelligent Surface-Aided Wireless Networks: A Unified Framework via Block Minorization-Maximization”

Zepeng Zhang and Ziping Zhao

A. Proof for Proposition 1

Proof: The function \( \log(z) \) with \( z > 0 \) is concave and hence can be majorized by its linear expansion around \( \bar{z} \) as follows:

\[
\log(z) \leq \log(\bar{z}) + \frac{1}{\bar{z}}(z - \bar{z}).
\]

By substituting \( z \) with \( \frac{y}{y + |x|^2} \), we get

\[
\log\left(1 + \frac{|x|^2}{y}\right) \geq \log\left(1 + \frac{|\bar{x}|^2}{y}\right) - \frac{y + |\bar{x}|^2}{y}\left(\frac{y}{y + |\bar{x}|^2} - \frac{y}{y + |x|^2}\right)
\]

\[
= \log\left(1 + \frac{|\bar{x}|^2}{y}\right) - \frac{y + |\bar{x}|^2}{y}\left(\frac{|\bar{x}|^2}{y + |\bar{x}|^2} - \frac{|\bar{x}|^2}{y + |x|^2}\right)
\]

\[
= \log\left(1 + \frac{|\bar{x}|^2}{y}\right) - \frac{|\bar{x}|^2}{y} + \left(1 + \frac{|\bar{x}|^2}{y}\right)\frac{|\bar{x}|^2}{y + |\bar{x}|^2}
\]

(24)

where the equality is attained at \((x, y) = (\bar{x}, \bar{y})\).

Lemma 10. The function \( \frac{|z_1|^2}{z_2} \) with \( z_1 \in \mathbb{C} \) and \( z_2 > 0 \) is minorized at \((z_1, z_2)\) as follows:

\[
\frac{|z_1|^2}{z_2} \geq \frac{2}{z_2} \text{Re}(z_1^*z_1) - \frac{|z_1|^2}{z_2^2} z_2.
\]

Proof: For \( z_1 \in \mathbb{C} \) and \( z_2 > 0 \), we have

\[
\left(\frac{z_1}{\sqrt{z_2}} - \frac{z_1\sqrt{z_2}}{z_2}\right)^*\left(\frac{z_1}{\sqrt{z_2}} - \frac{z_1\sqrt{z_2}}{z_2}\right) \geq 0.
\]

Expanding the above formula and rearranging the terms lead to the above result.

Remark 11. Lemma 10 reduces to the case of minorizing the convex “quadratic-over-linear” function by the first order Taylor expansion when \( z_1 \in \mathbb{R} \) and \( z_2 > 0 \) [17].

Applying the results in Lemma 10 to the last line of Eq. (24) with \( z_1 = x \) and \( z_2 = y + |x|^2 \), we can conclude that

\[
\log\left(1 + \frac{|x|^2}{y}\right) \geq \log\left(1 + \frac{|\bar{x}|^2}{y}\right) - \frac{|\bar{x}|^2}{y} + \left(1 + \frac{|\bar{x}|^2}{y}\right)\left(\frac{2}{y + |\bar{x}|^2}\text{Re}(\bar{x}^*x) - \frac{|\bar{x}|^2}{(y + |\bar{x}|^2)^2}\left(y + |x|^2\right)\right)
\]

\[
= -\frac{|\bar{x}|^2}{y(y + |x|^2)}(y + |x|^2) + \frac{2}{y}\text{Re}(\bar{x}^*x) + \log\left(1 + \frac{|\bar{x}|^2}{y}\right) - \frac{|\bar{x}|^2}{y},
\]

where the equality is attained when \((x, y) = (\bar{x}, \bar{y})\).

B. Proof for Proposition 6

Proof: The objective of Problem (17) is concave-convex in \( \Theta \) and \( s \), and constraint sets \( \mathcal{C}_{\text{related}} \) and \( \mathcal{S} \) are both nonempty compact and convex. Therefore, a saddle point \((\Theta^*, s^*)\) exists for Problem (17) [33, Corollary 37.6.2]. In the following, we will verify that \( \Theta^* \in \mathcal{C} \) always hold by contradiction, with which the proof is completed.

Suppose \( \Theta^* \notin \mathcal{C} \), i.e., in the interior of \( \mathcal{C}_{\text{related}} \), then there must exist some element of \( \Theta^* = \text{diag}(\theta^*) \), say \( [\theta^*]_j \), such that \( |[\theta^*]_j| < 1 \). If the \( j \)-th element of \( b_k \) is nonzero, we can always reset the phase of \( \theta^*_j \) to be aligned with the \( j \)-th element of \( -b_k \) and increase its modulus by a small amount without violating feasibility. Then the objective of Problem (17) will be pulled down from the side of \( \theta \), which contradicts with the saddle point nature of \((\Theta^*, s^*)\). In case the \( j \)-th element of \( b_k \) is zero, the optimal \( [\theta^*]_j \) may be non-unique (and thus the saddle point is non-unique), but we can always modulate the modulus of \( \theta^*_j \) to find an optimal \( [\theta^*]_j \) on the boundary. Therefore, we can conclude that the saddle point (or at least one saddle point) of Problem (17) naturally satisfies \( \Theta^* \in \mathcal{C} \), which means there must exist a saddle point for Problem (16) that can be obtained by solving Problem (17).
C. Proof for Proposition 7

Proof: This proof is intrinsically parallel to that of Proposition 1, based on which the result is extended to the matrix domain. The concave function \( \log \det(Z) \) with \( Z \succ 0 \) can be majorized by its linear expansion around \( Z \) as follows:

\[
\log \det(Z) \leq \log \det(Z) + \text{tr}(Z^{-1}(Z - Z))
\]

By substituting \( Z \) with \( (I + X^H Y^{-1} X)^{-1} \), we get

\[
\log \det(I + X^H Y^{-1} X) \geq \log \det(I + X^H Y^{-1} X) - \text{tr}\left( (I + X^H Y^{-1} X)(I + X^H Y^{-1} X)^{-1} - I \right)
\]

where the equality is attained at \( (X, Y) = (\bar{X}, \bar{Y}) \). The Woodbury matrix identity gives

\[
(I + X^H Y^{-1} X)^{-1} = (I - I^{-1} X^H Y X^{-1} X)^{-1} X
\]

and hence

\[
\log \det(I + X^H Y^{-1} X) \geq \log \det(I + X^H Y^{-1} X) - \text{tr}\left( (I + X^H Y^{-1} X)(I + X^H Y^{-1} X)^{-1} - I \right)
\]

\[
= \log \det(I + X^H Y^{-1} X) - I
\]

\[
- \text{tr}\left( (I + X^H Y^{-1} X)(I + X^H Y^{-1} X)^{-1} - I \right)
\]

\[
\geq \log \det\left( I - I^{-1} X^H Y X^{-1} X \right)
\]

Since \( I + X^H Y^{-1} X \succ 0 \), it admits a unique positive semidefinite square root \( (I + X^H Y^{-1} X)^{1/2} \) [43, Theorem 7.2.6]. Let \( Z_1 = X(I + X^H Y^{-1} X)^2 \) and \( Z_2 = Y + X^H X \), we have

\[
(Z_2^{-1/2} Z_1 - Z_2^{-1/2} Z_2^{-1/2} Z_1) H_2^{-1} Z_2^{-1/2} Z_1 - Z_2^{-1/2} Z_2^{-1/2} Z_2^{-1/2} Z_1 \geq 0,
\]

which means

\[
Z_2^{-1/2} Z_1 - Z_2^{-1/2} Z_2^{-1/2} Z_2^{-1/2} Z_1 = Z_2^{-1/2} Z_2^{-1/2} Z_2^{-1/2} Z_1.
\]

Then we can conclude that

\[
\log \det(I + X^H Y^{-1} X) \geq \log \det(I + X^H Y^{-1} X) - \text{tr}(X^H Y^{-1} X) + 2 \operatorname{Re}\left( \text{tr}(X^H Y^{-1} X) X^H Y X^H Y^{-1} X) \right)
\]

\[
- \text{tr}(X^H Y^{-1} X) X^H Y X^H Y^{-1} X)
\]

where the equality is attained when \( (X, Y) = (\bar{X}, \bar{Y}) \), and the proof is completed by rearranging the terms.

D. Proof for Theorem 8

Proof: In the following, we will give the convergence proof for Algorithm 1 which is designed for Problem (WSRMax), and the convergence properties for Algorithm 2 and Algorithm 3 can be proved similarly. Besides, it can be verified that the convergence properties also hold for other cases that BMM has been applied to in Section VII.

The convergence proof partly hinges on the proof for BSUM in [16]. The sequence \( \{W(t), \{\Theta(t)\}\}_{t \in \mathbb{N}} \) generated by Algorithm 1 lies in a compact set and, hence, it has a limit point \( \{W(\infty), \{\Theta(\infty)\}\} \). Then we can get

\[
f_{WSR,W}'(W(\infty), W(\infty), \{\Theta(\infty)\}) \geq f_{WSR,W}'(W, W(\infty), \{\Theta(\infty)\}), \quad \forall W \in \mathcal{W}, \tag{25}
\]

and

\[
f_{WSR,\Theta,l}'(\Theta(\infty), W(\infty), \{\Theta(\infty)\}) \geq f_{WSR,\Theta,l}'(\Theta, W(\infty)), \quad \forall \Theta_l \in \mathcal{C}, \quad l = 1, \ldots, L. \tag{26}
\]

We first define the real counterparts of \( W \) and \( h_k \) in (3) as follows:

\[
\tilde{W} = [\tilde{w}_1, \ldots, \tilde{w}_K], \quad \tilde{h}_k = \begin{bmatrix} \text{Re}(h_k) & \text{Im}(h_k) \\ \text{Im}(h_k) & -\text{Re}(h_k) \end{bmatrix}, \quad k = 1, \ldots, K,
\]

and then the constraint set for \( \tilde{W} \) derived from \( \mathcal{W} \) is

\[
\tilde{\mathcal{W}} = \left\{ \tilde{W} \mid \|\tilde{W}\|_F^2 \leq P \right\}.
\]

With the above definitions, the counterpart functions of \( f_{WSR,W} \) and \( f_{WSR,\Theta,l}' \) with real variables can be constructed as follows:

\[
g_{\tilde{W}}(\tilde{W}) = \sum_{k=1}^{K} \omega_k \log(1 + \frac{\|\tilde{w}_k^T \tilde{h}_k\|_2^2}{\sum_{j,j\neq k}^{K} \|\tilde{w}_j^T \tilde{h}_j\|_2^2 + \sigma^2}),
\]

\[
g_{\tilde{W},h}'(\tilde{W}) = \sum_{k=1}^{K} \omega_k (-\alpha_k \sum_{j=1}^{K} \|\tilde{w}_j^T \tilde{h}_k\|_2 + 2 \beta_k \|\tilde{w}_k^T \tilde{h}_k\|_2) + \text{const}_w,
\]

where \( \bar{X} \in \mathcal{S} \).
where $\alpha_k$ and $\text{const}_w$ are defined as in (4) while $\beta_k = \frac{\text{SINR}}{||w_k h_k||^2}$. Given $(W^{(\infty)}, \{\Theta_i^{(\infty)}\})$, it is straightforward that the Problem (WSRMax) w.r.t. $W$ given by

$$\maximize_{W \in W} f_{\text{WSR}, W}(W, W^{(\infty)}, \{\Theta_i^{(\infty)}\})$$

(27)

is equivalent to the following problem with real variable $\tilde{W}$:

$$\maximize_{\tilde{W} \in \tilde{W}} g_{\tilde{W}}(\tilde{W}, W^{(\infty)}, \{\Theta_i^{(\infty)}\}),$$

(28)

and the corresponding surrogate problem for $W$ given by

$$\maximize_{W \in W} f'_{\text{WSR}, W}(W, W^{(\infty)}, \{\Theta_i^{(\infty)}\})$$

(29)

is equivalent to the following problem with real variable $\tilde{W}$:

$$\maximize_{\tilde{W} \in \tilde{W}} g'_{\tilde{W}}(\tilde{W}, W^{(\infty)}, \{\Theta_i^{(\infty)}\}).$$

(30)

From Eq. (25), we know that $W^{(\infty)}$ is a global maximizer of Problem (29), then $\tilde{W}^{(\infty)}$ is a global maximizer of Problem (30) as well due to the equivalence between Problem (30) and Problem (29), indicating that

$$\nabla g_{\tilde{W}}(\tilde{W}, W^{(\infty)}, \{\Theta_i^{(\infty)}\}; d_w) |_{\tilde{W}=\tilde{W}^{(\infty)}} \leq 0, \forall d_w \text{ s.t. } \tilde{W}^{(\infty)} + d_w \in \tilde{W}.$$  

It can be verified that $\nabla g'_{\tilde{W}}(\tilde{W}, W^{(\infty)}, \{\Theta_i^{(\infty)}\}) = \nabla g_{\tilde{W}}(\tilde{W}, W^{(\infty)}, \{\Theta_i^{(\infty)}\})$, in that $g'_{\tilde{W}}(\tilde{W})$ is a minorizing function of $g_{\tilde{W}}(\tilde{W})$ according to Proposition 1. Therefore, we have

$$\nabla g_{\tilde{W}}(\tilde{W}, W^{(\infty)}, \{\Theta_i^{(\infty)}\}; d_w) |_{\tilde{W}=\tilde{W}^{(\infty)}} \leq 0, \forall d_w \text{ s.t. } \tilde{W}^{(\infty)} + d_w \in \tilde{W},$$

which means $\tilde{W}^{(\infty)}$ is a stationary point of Problem (28). Then $W^{(\infty)}$ is also a stationary point of Problem (27) because of the equivalence between Problem (27) and Problem (28).

We further define the real counterparts of $\tilde{\Theta}_l$, $w_j^T h_k^d$, $w_j^T F_{k,l}$ in (7), and $b_l$ in (10) as follows:

$$\tilde{\Theta}_l = \begin{bmatrix} \text{Re}(\Theta_l) \\ \text{Im}(\Theta_l) \end{bmatrix}, \quad \tilde{h}_j = \begin{bmatrix} \text{Re}(w_j^T h_k^d) \\ \text{Im}(w_j^T h_k^d) \end{bmatrix}, \quad \tilde{f}_j = \begin{bmatrix} \text{Re}(w_j^T F_{k,l}) \\ -\text{Im}(w_j^T F_{k,l}) \end{bmatrix}, \quad \tilde{b}_l = \begin{bmatrix} \text{Re}(b_l) \\ \text{Im}(b_l) \end{bmatrix}, \quad j = 1, \ldots, K,$$

and then the constraint set for $\tilde{\Theta}_l$ is

$$\tilde{C}_l = \left\{ \tilde{\Theta}_l | \tilde{\Theta}_l \in \mathbb{R}^{2N_i}, \left[\tilde{\Theta}_l\right]_j^2 + \left[\tilde{\Theta}_l\right]_{j+N_i}^2 = 1, \ j = 1, \ldots, N_i \right\}.$$  

With the above definitions, the counterpart functions of $f_{\text{WSR}, \Theta_i}$ and $f'_{\text{WSR}, \Theta_i}$ with real variable can be constructed as follows:

$$g_{\tilde{\Theta}_l}(\tilde{\Theta}_l) = \sum_{k=1}^K \omega_k \log \left(1 + \frac{\left[\tilde{f}_k \tilde{\Theta}_l + \tilde{h}_k^d\right]_2^2}{\sum_{j,k \neq k} \left[\tilde{f}_j \tilde{\Theta}_l + \tilde{h}_j^d\right]_2^2 + \sigma^2} \right)$$

$$g'_{\tilde{\Theta}_l}(\tilde{\Theta}_l) = -2\tilde{\Theta}_l \tilde{b}_l + \text{const}_{\tilde{\Theta}_l},$$

where $\text{const}_{\tilde{\Theta}_l} = \text{const}_{\Theta_l} - N_i \lambda_l - \tilde{\Theta}_l^H (\lambda_l I - L_l) \tilde{\Theta}_l$ represents the constant parts in (10). Given $(W^{(\infty)}, \{\Theta_i^{(\infty)}\})$, it is straightforward that the Problem (WSRMax) w.r.t. $\Theta_i$ given by

$$\maximize_{\Theta_i \in \tilde{C}_l} f_{\text{WSR}, \Theta_i}(\Theta_i, W^{(\infty)}, \{\Theta_i^{(\infty)}\})$$

(31)

is equivalent to the following problem with real variable $\tilde{\Theta}_l$

$$\maximize_{\tilde{\Theta}_l \in \tilde{C}_l} g_{\tilde{\Theta}_l}(\tilde{\Theta}_l, W^{(\infty)}, \{\Theta_i^{(\infty)}\}),$$

(32)

and the corresponding surrogate problem for $\Theta_i$ given by

$$\maximize_{\Theta_i \in \tilde{C}_l} f'_{\text{WSR}, \Theta_i}(\Theta_i, W^{(\infty)}, \{\Theta_i^{(\infty)}\})$$

(33)

is equivalent to the following problem with real variable $\tilde{\Theta}_l$

$$\maximize_{\tilde{\Theta}_l \in \tilde{C}_l} g'_{\tilde{\Theta}_l}(\tilde{\Theta}_l, W^{(\infty)}, \{\Theta_i^{(\infty)}\}).$$

(34)
where $\Gamma$ (together with (35) and (36), we can conclude that from Eq. (26), we know that $\Theta$ as well due to the equivalence between Problem (34) and Problem (33), indicating that

$$
\nabla g_{\theta_i}(\tilde{\theta}_i, W^{(\gamma)}, \{\Theta_i^{(\infty)}\}; d_{\theta,i}) \bigg|_{\theta_i = \tilde{\theta}_i^{(\infty)}} \leq 0, \quad \forall d_{\theta,i} \in T_{C_i}(\tilde{\theta}_i^{(\infty)}),
$$

It can be verified that $\nabla g_{\theta_i}(\tilde{\theta}_i, W^{(\gamma)}, \{\Theta_i^{(\infty)}\}) = \nabla g_{\theta_i}(\tilde{\theta}_i, W^{(\gamma)}, \{\Theta_i^{(\infty)}\})$, in that $g_{\theta_i}^{*}(\tilde{\theta}_i)$ is a minorizing function of $g_{\theta_i}(\tilde{\theta}_i)$ according to Proposition 1, Lemma 3, and Lemma 4. Therefore, we have

$$
\nabla g_{\theta_i}^{*}(\tilde{\theta}_i, W^{(\gamma)}, \{\Theta_i^{(\infty)}\}; d_{\theta,i}) \bigg|_{\theta_i = \tilde{\theta}_i^{(\infty)}} \leq 0, \quad \forall d_{\theta,i} \in T_{C_i}(\tilde{\theta}_i^{(\infty)}),
$$

which means $\tilde{\theta}_i^{(\infty)}$ is a stationary point of Problem (32). Then $\Theta_i^{(\infty)}$ is also a stationary point of Problem (31) because of the equivalence between Problem (31) and Problem (32).

Similarly, by repeating the above argument for the other $\{\Theta_i\}$ blocks, we can conclude that $(W^{(\gamma)}, \{\Theta_i^{(\infty)}\})$ is a coordinate-wise maximum of Problem (WSRMax). Therefore, $(W^{(\gamma)}, \{\Theta_i^{(\infty)}\})$ is a stationary point of Problem (WSRMax) as well since the objective function of Problem (WSRMax) is regular at the limit point.

Considering that the power limit constraint set $W$ is convex, Eq. (25) implies that $W^{(\gamma)}$ satisfies

$$
0 \leq \gamma + \|W^{(\gamma)}\|_F^2 - P \leq 0,
$$

where $\gamma$ is the Lagrange multiplier.

**Lemma 12.** Liner independence constraint qualification (LICQ) [44] holds everywhere on set

$$
C_i = \{\Theta_i = \text{diag}(\theta_i) \mid \theta_i \in \mathbb{C}^{N_i}, ||\theta_i||_1 = 1, \forall j = 1, \ldots, N_i, \forall i = 1, \ldots, L
$$

and set

$$
D_i = \{\Theta_i = \text{diag}(\theta_i) \mid \theta_i \in \mathbb{C}^{N_i}, ||\theta_i||_1 = 1, \text{ang}([\theta_i]_j) \in \Phi_i, \forall j = 1, \ldots, N_i, \forall i = 1, \ldots, L
$$

**Proof:** The constraint $\Theta_i \in C_i$ can be rewritten as

$$
[c_i(\theta_i)]_j = ||\theta_i||_1 - 1 = 0, \quad j = 1, \ldots, N_i,
$$

while the constraint $\Theta_i \in D_i$ can be rewritten as

$$
[d_i(\theta_i)]_j = \prod_{\phi \in \Phi_i} ([\theta_i]_j - e^{j \phi}) = 0, \quad j = 1, \ldots, N_i.
$$

The gradients $\nabla [c_i(\theta_i)]_1, \ldots, \nabla [c_i(\theta_i)]_{N_i}$ meet the LICQ evidently since $[\theta_i]_j$ appears only at the $j$-th entry of $\nabla [c_i(\theta_i)]_j$,

while the same goes for $\nabla [d_i(\theta_i)]_1, \ldots, \nabla [d_i(\theta_i)]_{N_i}$, through which the proof is completed.

The constraint set $C_i$ meet the LICQ according to Lemma 12, then Eq. (26) implies that $\Theta_i$ satisfies

$$
||\Theta_i||_1 = 0, \quad ||\Theta_i||_i = 1, \quad \forall i \neq j, \quad i, j = 1, \ldots, N_i,
$$

$$
\nabla f_{\text{WSR}, \Theta_i}(\Theta_i, W^{(\gamma)}, \{\Theta_i^{(\infty)}\}) \bigg|_{\Theta_i = \Theta_i^{(\infty)}} = 0, \quad \sum_{i=1}^{N_i} \sum_{j=1, j \neq i}^{N_i} [\Gamma]_{ij} |\theta_i|_{ij} + \sum_{i=1}^{N_i} [\Gamma]_{ii} (|\theta_i|_{ii} - 1) = 0
$$

(36)

where $\Gamma$ is the Lagrange multiplier. Similarly, by repeating the above argument for the other $\{\Theta_i\}$ blocks and putting them together with (35) and (36), we can conclude that $(W^{(\gamma)}, \{\Theta_i^{(\infty)}\})$ is a KKT point of Problem (WSRMax).

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The Wirtinger derivative is adopted for differentials of complex variables [45].