Neutrino phenomenology of a high scale supersymmetry model

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Abstract

CP violation in the lepton sector, and other aspects of neutrino physics, are studied within a high scale supersymmetry model. In addition to the sneutrino vacuum expectation values (VEVs), the heavy vector-like triplet also contributes to neutrino masses. Phases of the VEVs of relevant fields, complex couplings and Zino mass are considered. The approximate degeneracy of neutrino masses $m_{\nu_1}$ and $m_{\nu_2}$ can be naturally understood. The neutrino masses are then normal ordered, $\sim 0.020$ eV, 0.022 eV, and 0.054 eV. Large CP violation in neutrino oscillations is favored. The effective Majorana mass of the electron neutrino is about 0.02 eV.

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I. INTRODUCTION

Neutrino physics, like leptonic CP violation, is an interesting topic in the current research of particle physics. Among other things, it might be the final place where experiments of particle physics will give definite results in the near future. The results will check various theoretical models about the fermion masses of the Standard Model (SM).

We proposed that supersymmetry (SUSY) can be the theory underlying the fermion masses in Refs. [3–5]. The basic idea is the following. It assumes a flavor symmetry. The flavor symmetry is broken after the sneutrinos obtain nonvanishing vacuum expectation values (VEVs). (In this way, SUSY is motivated.) These VEVs result in a nonvanishing neutrino mass. The empirical smallness of neutrino masses needs very large SM super partner masses to be understood which are about $10^{12}$ GeV. Thus, our SUSY is of high scale breaking [6–8].

A further natural assumption is that the flavor symmetry breaks softly. Namely the soft SUSY breaking masses of the sfermions do not obey the flavor symmetry either. The theoretical reason is that the soft masses are due to the supergravity effect which generically breaks any global symmetry. Soft breaking of the flavor symmetry implies that the lepton number violation due to sneutrino VEVs is explicit instead of being spontaneous. Therefore there is no any massless Nambu-Goldstone boson related to nonvanishing sneutrino VEVs. Actually the large masses of the model make the low energy effective theory just the SM via Higgs mass fine tuning, except for that we have an understanding of the hierarchical pattern of the charged lepton masses, or that of the SM Yukawa coupling constants.

To briefly review the model in a simple way, the SM is SUSY generalized. The flavor symmetry is $Z_3$ cyclic among the three generation $SU(2)_L$ lepton doublets $L_1, L_2$ and $L_3$. The other fields are trivial under $Z_3$. The $Z_3$ invariant combinations are $\sum_{i=1}^{3} L_i$ and $\epsilon_{\alpha\beta}(L_1^\alpha L_2^\beta + L_2^\alpha L_3^\beta + L_3^\alpha L_1^\beta)$ with $\alpha$ and $\beta$ denoting the $SU(2)_L$ indices. In terms of the following redefined lepton superfields, $L_e = \frac{1}{\sqrt{2}}(L_1 - L_2)$, $L_\mu = \frac{1}{\sqrt{6}}(L_1 + L_2 - 2L_3)$, $L_\tau = \frac{1}{\sqrt{3}}(\sum_i L_i)$, the above $Z_3$ invariant combinations are $L_\tau$ and $\epsilon_{\alpha\beta}L_e^\alpha L_\mu^\beta$, respectively. The superpotential is then

$$W \supset y_\tau \epsilon_{\alpha\beta}L_{\tau}^\alpha H_d^\beta E_\tau^c + \epsilon_{\alpha\beta}L_e^\alpha L_\mu^\beta(\lambda_\tau E_\tau^c + \lambda_\mu E_\mu^c) + \bar{\mu} \epsilon_{\alpha\beta} H_u^\alpha H_d^\beta,$$  \hspace{1cm} (1)

where $H_u$ and $H_d$ are the two Higgs doublets, the right-handed lepton singlet $E_\tau^c$ is defined as the one which couples to $L_\tau$, and $E_\mu^c$ is that orthogonal to $E_\tau^c$ and with a coupling to $L_e L_\mu$. 

\[ y_\tau, \lambda_\tau \text{ and } \lambda_\mu \text{ are coupling constants. (Note that considering the mixing between } L_\tau \text{ and } H_d \text{ gives the same form of the above superpotential \[4\].) It is seen that the electron is massless, because } E_e^c \text{ is always absent in the Lagrangian. This is true whenever SUSY is conserved, the nonvanishing electron mass is due to SUSY breaking (together with electroweak gauge symmetry and flavor symmetry breaking via loops). Note that all the coupling constants in our superpotential are assumed to be natural values, say typically } \sim 0.01 - 1, \text{ and the mass parameter } \bar{\mu} \text{ is taken to be large } \sim 10^{12} \text{ GeV. The SM fermion mass hierarchy is due to symmetries and their breaking.}

In addition, a heavy vector-like SU(2)\(_L\) triplet field \( T(\bar{T}) \) with hypercharge 2(−2) needs to be introduced so as to make the Higgs mass realistic \[5, 6\]. This triplet field also contributes to neutrino masses. In terms of the redefined fields, the flavor symmetric superpotential relevant to the triplet \( T \) and \( \bar{T} \) fields is

\[
W \supset y^\nu \{ L_\tau H_d \} T + \lambda^1_\nu \{ L_e L_e + L_\mu L_\mu \} T + \lambda^2_\nu \{ L_\tau L_\tau \} T \\
+ \lambda^3_\nu \{ H_d H_d \} T + \lambda^4_\nu \{ H_u H_u \} \bar{T} + M_T T \bar{T}
\]

with \( M_T \) the mass \( \sim 10^{13} \) GeV. The braces denote that the two doublets form an SU(2)\(_L\) triplet representation.

The soft SUSY breaking terms in the Lagrangian are in general form which also break the flavor symmetry \[3–5\]. All the mass parameters of the model are taken to be about \( 10^{12} - 10^{13} \) GeV. The spontaneous gauge symmetry breaking of the SM occurs. Through fine tuning, the right electroweak vacuum is obtained. By including contribution due to the triplet field, this model can give reasonable neutrino spectrum and the mixing pattern, and predicted the right order of \( \theta_{13} \) \[4, 5\]. (The quark sector was considered in Ref. \[4\].)

Roughly speaking about the electroweak symmetry breaking. There are five scalar doublets, the mass parameters are all large \( \sim 10^{12} \) GeV. Eigenvalues of their mass-squared matrix are generically large. However, one of these values can be exceptional, because it is a difference between two large parameters. It is this difference that makes the fine-tuning possible. Whence the difference is tuned to be about \(- (100 \text{ GeV})^2\), correct electroweak symmetry breaking occurs. The corresponding eigenstate field is one superposition of the five doublets. It is the only light scalar doublet, and is just the SM Higgs field from the point of view of the low energy effective field theory. The SM Higgs gets a VEV is equivalent to that the original two Higgses and sleptons get their VEVs \[4, 5\].
II. COMPLEX COUPLINGS AND SNEUTRINO VEVs

In this paper, we will carefully consider CP violation of the lepton sector, and completely analyze the neutrino masses and mixing. In general, the coupling constants are complex, however, because of the flavor symmetry, many of them can be made real via field phase rotation. In the superpotential Eq. [1] for charged leptons, all the couplings can be adjusted to be real. On the other hand, in the superpotential Eq. [2] for neutrino masses, the couplings cannot be all taken real, as can be seen in the following way. The mass parameters \( \bar{\mu} \) and \( M_T \) are taken real, thus \( H_u \) and \( H_d \) always have opposite phases, and so do \( T \) and \( \bar{T} \). \( \lambda_2^\nu \) is real via rotating the phase of \( L_\tau \), \( \lambda_1^\nu \) is real via rotating \( H_u \) (or \( \bar{T} \)), \( y_\nu \) is real via \( E^c_\tau \), \( \lambda_\tau \) real via \( L_\nu L_\mu \) rotating, and \( \lambda_\mu \) real via \( E^c_\mu \). In such a phase convention, only \( y_\nu^\prime \), \( \lambda_1^\nu \) and \( \lambda_3^\nu \) can be complex. The \( \lambda_2^\nu \) term will contribute to the neutrino masses, which was omitted in our previous analysis [3].

In the soft SUSY breaking terms, the mass parameters and coupling constants are generally complex, and there is no enough freedom to rotate all of the phases away.

The scalar potential relevant to the electroweak symmetry breaking is

\[
V = (|\bar{\mu}|^2 + m_{\text{hs}}^2)|h_u|^2 + (|\mu|^2 + m_{\text{hs}}^2)|h_d|^2 + \frac{g^2 + g'^2}{8}(|h_u|^2 - |h_d|^2 - \bar{l}_\alpha\bar{l}_\alpha)^2
\]

\[
+ \frac{g^2}{4}[2|h_u|\bar{h}_u|^2 + 2(h_u^\dagger\bar{l}_\alpha)(\bar{l}_\alpha\bar{h}_u) + 2(h_d^\dagger\bar{l}_\alpha)(\bar{l}_\alpha\bar{h}_d)
\]

\[
- 2|h_d|^2(\bar{l}_\alpha\bar{l}_\alpha) + ((\bar{l}_\alpha\bar{l}_\beta)(\bar{l}_\beta\bar{l}_\alpha) - (\bar{l}_\alpha\bar{l}_\beta)(\bar{l}_\beta\bar{l}_\alpha)) + \frac{1}{2}\frac{M_{\text{dd}}^2}{m_{\text{d}}^2}h_d^\dagger h_d + H_d^\dagger H_d + B_{\mu\alpha}h_a\bar{h}_d + B_{\mu\alpha}h_a\bar{h}_d + \text{h.c.}
\]

(3)

where \( g \) and \( g' \) are SM gauge coupling constants. \( h_u \) and \( h_d \) denote the scalar components of \( H_u \) and \( H_d \), respectively, and \( \bar{l}_\alpha \)’s left-handed sleptons. \( m_{h_{(u,d)}^2}, m_{d_{(u,d)}^2}, m_{\alpha\beta}^2 \) and \( B_{\mu}, B_{\mu\alpha} \) are soft squared masses.

In considering CP violation of the scalar potential, the essential point lies in the soft bilinear terms where the mass parameters are complex. Field redefinition of \( h_d \) and \( \bar{l}_\alpha \) may remove phases of \( B_{\mu} \) and \( B_{\mu\alpha} \) respectively, however, the phases of \( m_{d_{(u,d)}^2} \) and off-diagonal terms of \( m_{\alpha\beta}^2 \) are still there. This means that after the electroweak symmetry breaking, Higgs and sneutrino VEVs are complex in general. (Previously we took all the VEVs real.) In the analysis, we still have the freedom to choose the VEV of Higgs field \( h_u \) to be real, and VEVs of the Higgs and the sneutrino fields are denoted as \( (v_u, v_d e^{i\delta_d}, v_\tau e^{i\delta_\tau}, v_{\nu_\mu} e^{i\delta_{\nu_\mu}}, v_{\nu_\tau} e^{i\delta_{\nu_\tau}) \) where the phases have been explicitly written down. These VEVs enter the lepton
mass matrices and thus contribute to CP violation in the leptonic mixing.

III. NEUTRINO MASSES

The sneutrino VEVs result in a nonvanishing neutrino mass,

\[
M_0^\nu = -\frac{a^2}{M_Z} e^{i\delta_Z} \begin{pmatrix}
 v_{l_e} v_{l_e} e^{2i\delta_{l_e}} & v_{l_e} v_{l_e} e^{i(\delta_{l_e} + \delta_{l_\mu})} & v_{l_e} v_{l_e} e^{i(\delta_{l_e} + \delta_{l_\tau})} \\
v_{l_\mu} v_{l_\mu} e^{i(\delta_{l_\mu} + \delta_{l_\tau})} & v_{l_\mu} v_{l_\mu} e^{2i\delta_{l_\mu}} & v_{l_\mu} v_{l_\mu} e^{i(\delta_{l_\mu} + \delta_{l_\tau})} \\
v_{l_\tau} v_{l_\tau} e^{i(\delta_{l_\tau} + \delta_{l_\mu})} & v_{l_\tau} v_{l_\tau} e^{i(\delta_{l_\tau} + \delta_{l_\mu})} & v_{l_\tau} v_{l_\tau} e^{2i\delta_{l_\tau}}
\end{pmatrix},
\]

(4)

where \(a = \sqrt{(g^2 + g'^2)/2}\), \(M_Z\) is the Zino mass which is the typical superpartner mass, and the phase of Zino mass term, \(\delta_Z\), is explicitly written. This is due to gauge interactions, it is natural realization of the type-I seesaw mechanism \cite{9} where the role of right-handed neutrinos is replaced by the Zino. In addition, the superpotential \cite{2} contributes following neutrino masses \cite{5},

\[
M_1^\nu = -\frac{\lambda_1^\nu v_u^2}{M_T} \begin{pmatrix}
 \lambda_1^\nu e^{\delta_{l_1}} & 0 & 0 \\
 0 & \lambda_1^\nu e^{\delta_{l_1}} & 0 \\
 0 & 0 & \lambda_2^\nu
\end{pmatrix},
\]

(5)

where the phase of coupling \(\lambda_1^\nu\) has been explicitly written. This part of neutrino mass generation is realization of the type-II seesaw mechanism \cite{10}.

The full neutrino mass matrix is

\[
M^\nu = M_0^\nu + M_1^\nu.
\]

(6)

Note this is the full neutrino mass matrix of the model. It is due to tree level contribution of lepton number violation. The loop level contribution due to R-parity violation is negligible \cite{4}, because the sparticles in the loops are very heavy.

The physics analysis including \(\lambda_1^\nu\) is different from our previous one \cite{5}. We observe that it is natural to take that \(M_1^\nu\) is numerically dominant over \(M_0^\nu\), then there appears a degeneracy between the first two neutrinos. This roughly fits the neutrino spectrum obtained from neutrino oscillation experiments. This degeneracy is perturbed by \(M_0^\nu\) which also contributes neutrino mixing. Furthermore, it is interesting to note that inclusion of \(\lambda_1^\nu\) in certain cases does not really increase difficulty in the analysis because \(M_1^\nu\) is diagonal.

We rewrite \(M^\nu\) by adjusting the diagonal part \(M_1^\nu\) to be proportional to identity matrix,

\[
M^\nu = \tilde{M}_0^\nu + \tilde{M}_1^\nu,
\]

(7)
where
\[
\tilde{M}_0^\nu = -\frac{a^2}{M_Z e^{i\delta_Z}} \begin{pmatrix}
    v_{l_e} v_{l_e} e^{i2\delta_{l_e}} & v_{l_e} v_{l_\mu} e^{i(\delta_{l_e} + \delta_{l_\mu})} & v_{l_e} v_{l_\tau} e^{i(\delta_{l_e} + \delta_{l_\tau})} \\
    v_{l_\mu} v_{l_e} e^{i(\delta_{l_\mu} + \delta_{l_e})} & v_{l_\mu} v_{l_\mu} e^{2i\delta_{l_\mu}} & v_{l_\mu} v_{l_\tau} e^{i(\delta_{l_\mu} + \delta_{l_\tau})} \\
    v_{l_\tau} v_{l_e} e^{i(\delta_{l_\tau} + \delta_{l_e})} & v_{l_\tau} v_{l_\mu} e^{i(\delta_{l_\tau} + \delta_{l_\mu})} & v_{l_\tau} v_{l_\tau} e^{2i\delta_{l_\tau}} + \Delta \lambda e^{i(\delta_\lambda + \delta_Z)}
\end{pmatrix},
\]
and
\[
\tilde{M}_1^\nu = -\frac{a^2}{M_Z} \begin{pmatrix}
    \lambda'_1 e^{i\delta_{\lambda_1}} & 0 & 0 \\
    0 & \lambda'_1 e^{i\delta_{\lambda_1}} & 0 \\
    0 & 0 & \lambda'_1 e^{i\delta_{\lambda_1}}
\end{pmatrix},
\]
where \(\lambda'_1 = \frac{M_Z}{a^2} \frac{\lambda_1 \lambda_4 v_u^2}{M_T},\) \(\lambda'_2 = \frac{M_Z}{a^2} \frac{\lambda_2 \lambda_4 v_u^2}{M_T},\) and \(\Delta \lambda e^{i\delta_\lambda} = \lambda'_2 - \lambda'_1 e^{i\delta_{\lambda_1}}.\) Generally, \(M^\nu\) is complex, the phases make further analytical calculation difficult. For illustration and an easy analysis, and without losing generality about CP violation, we simply take \(\delta_{l_e} = 0\) and \(\delta_\lambda = -\delta_Z\) in the following. Then, up to an overall factor, \(\tilde{M}_0^\nu\) is a real symmetric matrix and can be diagonalized by an orthogonal matrix. It just needs diagonalizing \(\tilde{M}_0^\nu,\) because \(\tilde{M}_1^\nu\) is essentially an unit matrix which does not affect this diagonalization. By further assuming that \(v_{l_e}^2 + v_{l_\mu}^2 \ll v_{l_\tau}^2\) which is reasonable because \(v_{l_\tau} = \frac{v_1 + v_2 + v_3}{\sqrt{3}}\) which does not violate the \(Z_3\) flavor symmetry, it is found that \(\tilde{M}_0^\nu\) is diagonalized by,
\[
O_\nu \simeq \begin{pmatrix}
    \frac{v_{l_\mu}}{\sqrt{v_{l_e}^2 + v_{l_\mu}^2}} & \frac{v_{l_e}}{\sqrt{v_{l_e}^2 + v_{l_\mu}^2}} & \frac{v_{l_\tau}}{\sqrt{v_{l_\tau}^2 + \Delta \lambda}} \\
    \frac{v_{l_\mu}}{\sqrt{v_{l_e}^2 + v_{l_\mu}^2}} & \frac{v_{l_e}}{\sqrt{v_{l_e}^2 + v_{l_\mu}^2}} & \frac{v_{l_\tau}}{v_{l_\tau}^2 + \Delta \lambda} \\
    0 & -\frac{v_{l_\tau}}{v_{l_\tau}^2 + \Delta \lambda} & 1
\end{pmatrix},
\]
with eigenvalues
\[
\tilde{M}_0^{\nu \text{ diag}} \simeq -\frac{a^2}{M_Z} e^{-i\delta_Z} \begin{pmatrix}
    0 & 0 & \Delta \lambda \\
    0 & (v_{l_e}^2 + v_{l_\mu}^2) & 0 \\
    0 & 0 & v_{l_\tau}^2 + \Delta \lambda
\end{pmatrix}.
\]
In fact, \(O_\nu\) diagonalizes \(M^\nu,\)
\[
O_\nu^T M^\nu O_\nu = \tilde{M}_0^{\nu \text{ diag}} + \tilde{M}_1^\nu.
\]
Noticing that the diagonalized matrix is still complex, we further write that

\[
\tilde{M}_0^{\nu} + \tilde{M}_1^{\nu} = e^{i\delta_{\lambda_1}^Z} M_0^{\nu} + e^{i\delta_{\lambda_2}^Z} M_1^{\nu},
\]

with

\[
\begin{pmatrix}
e^{i\delta_{\lambda_1}^Z} & 0 & 0 \\
0 & e^{i\delta_{\lambda_2}^Z} & 0 \\
0 & 0 & e^{i\delta_{\lambda_2}^Z}
\end{pmatrix}
\]

(13)

the neutrino masses in our model are

\[
m_{\nu_1} = \frac{a^2}{M_Z} \lambda_1',
\]

\[
m_{\nu_2} \approx \frac{a^2}{M_Z} [\lambda_1' + (v_{l_e}^2 + v_{l_\mu}^2) \frac{\Delta \lambda}{v_{l_r}^2 + \Delta \lambda} \cos(\delta_{\lambda_1} + \delta_Z)],
\]

\[
m_{\nu_3} = \frac{a^2}{M_Z} \sqrt{\lambda_2^2 + v_{l_r}^4 + 2\lambda_2' v_{l_r}^2 \cos \delta_Z},
\]

(14)

with the phases

\[
\beta_1 \approx \delta_{\lambda_1},
\]

\[
\beta_2 = \arctan \frac{v_{l_e}^2 \sin \delta_Z}{\lambda_2' + v_{l_r}^2 \cos \delta_Z}.
\]

(15)

It is clear that \(\nu_1\) and \(\nu_2\) are almost degenerate with a mass \(\approx \frac{a^2}{M_Z} \lambda_1'\). Their mass splitting is about \(\frac{a^2}{M_Z} (v_{l_e}^2 + v_{l_\mu}^2) \frac{\Delta \lambda}{v_{l_r}^2 + \Delta \lambda} \cos \delta_Z\). \(\Delta \lambda\) and \(\Delta \lambda\) have the same order of magnitude by definition, and we take \((v_{l_e}^2 + v_{l_\mu}^2) \approx \frac{\lambda_1'}{10}\). According to neutrino oscillation experiments [12], \(\Delta m_{12}^2 = 8.0 \times 10^{-5} \text{eV}^2\) and \(|\Delta m_{23}^2| = 2.4 \times 10^{-3} \text{eV}^2\), this model typically gives that

\[
m_{\nu_1} \approx 2.0 \times 10^{-2} \text{eV}, \quad m_{\nu_2} \approx 2.2 \times 10^{-2} \text{eV}, \quad m_{\nu_3} \approx 5.4 \times 10^{-2} \text{eV}.
\]

(16)

Naturally the phases in above formulae are \(O(1)\). This makes us to take all the cosines to be \(O(1)\) for simplicity in estimating the neutrino masses. And \(m_{\nu_3}\) is numerically fixed by choosing \(\lambda_2'\) and \(v_{l_r}^2\).

Finally, we obtain the unitary matrix \(U_\nu\) which diagonalizes \(M_\nu\),

\[
U_\nu^T M_\nu U_\nu = -\frac{a^2}{M_Z} \begin{pmatrix} m_{\nu_1} & 0 & 0 \\
0 & m_{\nu_2} & 0 \\
0 & 0 & m_{\nu_3} \end{pmatrix},
\]

(17)

with \(P\) being the pure phase matrix appearing in Eq. [13].
IV. CHARGED LEPTON MASSES

From Eq. (1), the charged lepton mass matrix is obtained. Considering the sneutrino and Higgs VEVs are complex, it is

\[
M^l = \begin{pmatrix}
0 & \lambda_\mu v_\mu e^{i\delta_\mu} & \lambda_\tau v_\tau e^{i\delta_\tau} \\
0 & \lambda_\mu v_\mu e^{i\delta_\mu} & \lambda_\tau v_\tau e^{i\delta_\tau} \\
0 & 0 & y_\tau v_d e^{i\delta_v\tau}
\end{pmatrix}.
\]  

(19)

Here the electron mass is neglected. In this model, the electron mass would be a loop contribution of SUSY breaking terms which also break the flavor symmetry and the electroweak symmetry \[3, 4\]. \(M^l\) in the above equation basically fixes the mixing due to charged leptons with a precision of \(m_e/m_\mu\). It is standard to find the unitary matrix \(U_l\) which diagonalizes \(M^l M^l\),

\[
U_l^\dagger M^l M^l U_l = \begin{pmatrix}
m_e^2 & 0 & 0 \\
0 & m_\mu^2 & 0 \\
0 & 0 & m_\tau^2
\end{pmatrix}.
\]

(20)

It can be expressed as

\[
U_l = P_l O_l,
\]

where

\[
P_l = \begin{pmatrix}
e^{i\delta_\mu} & 0 & 0 \\
0 & e^{i\delta_\tau} & 0 \\
0 & 0 & e^{i\delta_v}\n\end{pmatrix},
\]

(21)

\[
O_l \simeq \begin{pmatrix}
-\nu_{l_e} & \nu_{l_\mu} & \nu_{l_\tau} y_\tau v_d & \lambda_\tau v_\tau \\
\sqrt{v_{l_e}^2 + v_{l_\mu}^2} & \sqrt{v_{l_\mu}^2 + v_{l_\tau}^2} & y_\tau v_d & \lambda_\tau v_\tau \\
\nu_{l_e} & \sqrt{v_{l_\mu}^2 + v_{l_\tau}^2} & \sqrt{v_{l_\mu}^2 + v_{l_\tau}^2 + \lambda_\tau^2 (v_{l_\mu}^2 + v_{l_\tau}^2)} & \lambda_\tau v_\tau \\
0 & \nu_{l_\mu} & \sqrt{v_{l_\mu}^2 + v_{l_\tau}^2 + \lambda_\tau^2 (v_{l_\mu}^2 + v_{l_\tau}^2)} & \lambda_\tau v_\tau \\
0 & 0 & -\lambda_\tau \sqrt{v_{l_\mu}^2 + v_{l_\tau}^2} & y_\tau v_d \\
0 & \nu_{l_\mu} & \sqrt{v_{l_\mu}^2 + v_{l_\tau}^2 + \lambda_\tau^2 (v_{l_\mu}^2 + v_{l_\tau}^2)} & y_\tau v_d
\end{pmatrix}.
\]

(22)

V. LEPTON MIXING MATRIX

The lepton mixing matrix is \(V = U_l^\dagger U_\nu\). It is obtained that \(\nu_e - \nu_\mu\) mixing is

\[
V_{e2} = \frac{v_{l_\mu}^2 - v_{l_\tau}^2}{v_{l_\mu}^2 + v_{l_\tau}^2} e^{-i\frac{\delta_v}{2}}.
\]

(23)
The $\nu_\mu - \nu_\tau$ mixing is
\[ V_{\mu 3} = \frac{2v_\mu v_\mu v_\tau}{\sqrt{v^2_\mu + v^2_\mu (v^2_\tau + \Delta \lambda)}} \frac{y_\tau v_d}{\sqrt{y^2_v v^2_\mu + \lambda^2_\tau (v^2_\mu + v^2_\mu)}} e^{-i \frac{\delta_\mu}{2}} \]  
\[ \lambda_\tau \sqrt{v^2_\mu + v^2_\mu} \frac{v_\tau}{\sqrt{y^2_v v^2_\mu + \lambda^2_\tau (v^2_\mu + v^2_\mu)}} e^{-i \delta_{\mu d} - i \frac{\delta_\mu}{2}}. \]  
(24)

The $\nu_e - \nu_\tau$ mixing is
\[ V_{e 3} \approx \frac{v^2_\mu - v^2_\mu}{\sqrt{v^2_\mu + v^2_\mu}} \frac{v_\tau}{v^2_\mu + \Delta \lambda} e^{-i \frac{\delta_\mu}{2}}. \]  
(25)

Experimental data for best values of these mixings are $|V_{e 2}| \approx 0.54$, $|V_{\mu 3}| \approx 0.65$, and $|V_{e 3}| \approx 0.15$. Obviously, taking $v_\mu \approx 2v_\mu$, $|V_{e 2}|$ is in agreement with data. The value of $v_\tau$ is taken to be larger and still in the natural range, $v_\tau \approx 3v_\mu$. Choosing $\Delta \lambda \approx 0.3v^2_\mu$, it is easy to get $|V_{e 3}| \approx 0.3|V_{e 2}|$.

For $|V_{\mu 3}|$, there are two terms in Eq. (24), neglecting the first term for simplicity, this mixing would be maximal if $\lambda_\tau \sqrt{v^2_\mu + v^2_\mu} = y_\tau v_d$, namely $\lambda_\tau \approx 0.8$. Of course, a smaller $\lambda_\tau$ is more natural. Therefore this model slightly favors the atmospheric neutrino angle to be in the first octant.

The important CP violation in neutrino oscillations is given through the invariant parameter $J$,

\[ Im(V_{i \lambda} V_{j \rho} V^\ast_{i \lambda} V_{j \rho}) = J \sum_{\kappa, \delta} \epsilon_{ijk} \epsilon_{\lambda \rho \delta}, \]  
(26)

and

\[ J \approx \frac{2v_\mu v_\mu v_\tau (v^2_\mu - v^2_\mu) \lambda_\tau y_\tau v_d}{(y^2_v v^2_\mu + \lambda^2_\tau (v^2_\mu + v^2_\mu))(v^2_\mu + v^2_\mu)^2(v^2_\mu + \Delta \lambda)} \sin \delta \approx 0.04 \sin \delta, \]  
(27)

\[ \delta = - \delta_{\mu d}. \]

$\delta_{\mu d}$ is expected to be large, namely $|\sin \delta| \approx 0.1 - 1$. This agrees with current preliminary experimental results.

VI. MAJORANA NEUTRINO MASS

The effective Majorana mass in the neutrinoless double beta decay is
\[ \langle m \rangle_{ee} = |m_{\nu_1} V_{e 1}^2 + m_{\nu_2} V_{e 2}^2 + m_{\nu_3} V_{e 3}^2|. \]  
(28)
In this work, it is
\[ \langle m \rangle_{ee} = |m_{\nu_1}|V_{e1}|^2 + m_{\nu_2}|V_{e2}|^2 + m_{\nu_3}|V_{e3}|^2 e^{i(\delta_1 - \beta_2)} | \simeq 0.02 \text{ eV}. \] (29)

In the above formula, the \( V_{e3} \) term has a Majorana phase dependence, which is negligibly small anyway.

VII. DISCUSSIONS

Like gauge theories which are used to describe the elementary particle interactions, SUSY is used for fermion masses. Our model is the minimal SUSY SM with a vector-like triplet field extension, but SUSY breaks at a high scale and the R-parity (lepton number) is not required. The sneutrino VEVs result in a neutrino mass which is suppressed by the Zino mass. This is a nice realization of the type-I seesaw mechanism which, even does not need to introduce any right-handed neutrino. The triplet field is originally for the realistic Higgs mass. However, it also contributes to neutrino masses through a type-II seesaw mechanism. The Zino related seesaw mechanism results in only one massive neutrino. By including the triplet contribution, the neutrino masses can be realistic. Compared to our previous studies \([4, 5]\), a more natural pattern for neutrino masses is obtained.

To be numerically natural, let us return back to the original superpotential in the beginning. The couplings are assumed to be taken natural values. The field VEVs are mainly fixed by the soft parameters in the Lagrangian, in addition to those in the superpotential. To fit the lepton spectrum and mixing, we take \( v_{\nu_e} \simeq 1 \text{ GeV}, v_{\nu_\mu} \simeq 2 \text{ GeV}, v_{\nu_\tau} \simeq 6 \text{ GeV}, v_d \simeq 10 \text{ GeV}, \) and \( v_u \simeq 228 \text{ GeV}. \) Note \( v_{\nu_e} \) does not break the flavor symmetry, it is natural that its value is more close to \( v_d. \) And the large \( v_u/v_d \) ratio is for explaining the top quark mass \([4]\). When \( \lambda_1', \lambda_2' \) and \( v_{\nu_\tau}^2 \) are in the same order, the correct neutrino spectrum is obtained. In terms of parameters in the superpotential, we have \( M_{\tilde{Z}} \simeq 3 \times 10^{11} \text{ GeV}, M_T \simeq (1 - 10)M_{\tilde{Z}}, \) and \( \lambda's \simeq (0.01 - 0.1). \)

It is necessary to check the reliability of our approximation in estimating the neutrino masses. That approximation about the phases can be good when the quantities appear in the mass formulae are hierarchical, say if \( \lambda_1' \gg v_{\nu_e}^2 + v_{\nu_\mu}^2. \) As it has been seen that this is indeed the case for \( m_{\nu_2}. \) In \( m_{\nu_3} \) (Eq. (14)), \( \lambda_2', \lambda_1' \text{ and } v_{\nu_\tau}^2 \) are of the same order. This allows us to look at an extreme case where the phase is \( \pi. \) In this case, there is a possibility of
inverted neutrino mass hierarchy, namely a very small $m_{\nu_3}$. But this is achieved through a large cancellation between $\lambda'_3$ and $\nu_t^2$. Although this is possible, it is unnatural.

The physics of neutrinos in this work is quite different from that in Refs. [4, 5]. This is mainly due to the triplet. In Ref. [4], we introduced a singlet, the neutrino mass matrix $M_{11}^\nu$ was that with only the 33 matrix element nonvanishing. And in [5], the triplet replaced the singlet for the Higgs mass in the beginning, however, in the neutrino mass analysis, we took $\lambda'_1$ to be zero which essentially was the same as that for the singlet case. Taking $\lambda'_1$ to be zero was actually unreasonable because our principle is to treat all the basic couplings close to $0.01 - 1$. As a result, in Refs. [4, 5], there was always one massless neutrino. That led to that the Majorana mass $\langle m \rangle_{ee}$ is about $10^{-3}$ eV. In addition, in [5] it was wrong to say CP violation is small in the lepton sector.

VIII. SUMMARY

In summary, in the model of high scale SUSY for understanding the fermion mass hierarchies, we have studied CP violation in the lepton sector, and other aspects of neutrino physics in detail. In the analysis, the phases of the Higgs and sneutrino VEVs, and contribution of the $\lambda'_1$ term in superpotential (2), have been included. This analysis is more complete than previous consideration. The neutrino mass matrix, and the charged lepton one, are fixed by the model. Its specific feature is the triplet contribution, the approximate degeneracy of neutrinos $\nu_1$ and $\nu_2$ can be naturally explained.

This model could not predict exact values of the fermion masses because of the flavor symmetry breaking as well as SUSY breaking. However, the principle we follow is that all the coupling constants should be in the natural parameter range which is about $(0.01 - 1)$. Taking triplet contribution dominant, and inputting relevant experimental data on leptons, we obtain that (i) $m_{\nu_1} \simeq 0.020$ eV, $m_{\nu_2} \simeq 0.022$ eV, $m_{\nu_3} \simeq 0.054$ eV. This normal ordering neutrino spectrum is to be checked in JUNO experiment [14]. (ii) CP violation in neutrino oscillation most probably is large. There have been some experimental hint on this [15]. CP violation in neutrino oscillations is a great study task experimentally [16]. (iii) The effective Majorana neutrino mass in the neutrinoless double beta decay is about 0.02 eV, it is within the detection ability of future measurements [17]. (iv) $\theta_{23}$ is slightly favored being in the first octant. (v) The electron neutrino mass to be measured in $\beta$ decays is about 0.02 eV. This
is, however, still one order of magnitude lower than the future limit of direct measurements \[18\]. (vi) The sum of three neutrino masses is close to \( \sum m_\nu \simeq 0.1 \text{ eV} \). If the standard cosmology is correct, astrophysics measurements on the cosmic microwave background has constrained this sum to be \(< 0.15 \text{ eV} \) \[19\]. It is interesting to note that a recent analysis showed the sum is about \( \sim 0.11 \text{ eV} \) \[20\]. Most of the above predictions are close to their experimental limits, therefore, this model will soon be checked experimentally.

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