Cyclotron parametric resonance in microwave-irradiated two-dimensional electron system

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In recent experimental studies of the microwave-irradiated ultraclean two-dimensional electron systems (2DESs) in magnetic field a puzzling colossal, narrow photoresistivity spike has been discovered in the vicinity of the 2nd cyclotron resonance harmonic $2\omega_c$. We suggest an explanation of this phenomenon in terms of the cyclotron parametric resonance in 2DES. We develop the hydrodynamic theory of 2D electron liquid in inhomogeneous microwave electric field with frequency $\Omega$. We demonstrate that excitation of virtual Bernstein magnetoplasma modes at $\Omega \approx 2\omega_c$ can strongly enhance an effective microwave field. That is why the fundamental mode of the parametric resonance is excited at very low microwave power. The resulting instability leads to the heating of 2DES and the photoresistivity spike.

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Unexpected features in the microwave (MW) response of the high-mobility 2DESs in the magnetic field $B$ have generated a lot of interest among condensed-matter physics community, for a review see [1]. Transport properties of these structures vary dramatically even at low MW power [2, 3]. In particular, the photoresistivity as a function of inverse magnetic field strongly oscillates with $\omega_c$ and this result still remains unclear. An attempt to explain this spike was made in Ref. [11], but its origin still remains unclear.

In this Letter, we propose a new approach to explain this phenomenon. Our approach developed within the hydrodynamic model of the 2DES is based on the parametric resonance (PR) phenomenon in the 2DES subject to inhomogeneous MW pumping. The PR in the 2DES can be understood using analogy with a simple pendulum whose length varies periodically with frequency $\omega_c$. The fundamental PR mode develops at $\Omega$ equal to the double eigenfrequency of the pendulum. In 2DES, $\omega_c$ acts as the eigenfrequency and the fundamental PR mode is excited at $\Omega \approx 2\omega_c$. In the PR regime, the amplitude of the electron velocity increases exponentially in time and 2DES becomes unstable. We show that this parametric instability takes place in the 2DES with sufficiently high mobility.

The electric field acting on electrons in 2DES can be evaluated within the linear response theory. At finite $qR_c$, nonlocal effects cause an anticrossing of classical magnetoplasmon mode and cyclotron resonance harmonics. Here, $q$ is the magnetoplasmon wave vector, $R_c = v_F/\omega_c$ is the electron cyclotron radius, $v_F$ is the Fermi velocity. This anticrossing splits the magnetoplasmon spectrum into the multiple the so-called Bernstein modes measured in 2DESs in Refs. [12–17]. Important feature of these modes is opening of the narrow gaps (the 'Bernstein gaps') in the plasma spectrum near $N\omega_c$, $N = 2, 3, ...$. If $\Omega$ falls in these gaps, inhomogeneous MW field can be strongly enhanced due to excitation of the virtual Bernstein modes.

The instability leads to the heating of 2DES. The heating destroys the GNMR, which samples exhibit in dark, and this results in the resistivity spike. It is worth to mention that PRs in 2DESs under different conditions were studied theoretically in Refs. [18–21].

We consider microwave-irradiated 2DES positioned in plane $z = 0$ and placed into the perpendicular magnetic field $B = (0, 0, B)$. We neglect the magnetic component of MW field and assume that the electric field is inhomogeneous on the cyclotron radius scale because of the metal contacts to the 2DES, which significantly modify MW field [22]. Electric field of MW pumping is defined as $E_0(r, t) = E_0(r) \cos \Omega t$, with an amplitude $E_0(r)$ dependent on the coordinates $r = (x, y)$. For simplicity we consider electric field $E_0(r)$ directed along $x$-axis and assume that it depends only on the $x$ coordinate.

The 2D electron liquid dynamics is described by the
where $\tau$ is the phenomenological relaxation time, and an effective field $\mathbf{E}_{\text{tot}}(x,t)$ is the pumping electric field $\mathbf{E}_0(x,t)$ screened (or anti-screened) by magnetoplasmons. We will discuss explicit form of this field later. We are looking for solutions of Eq. (1) for the high-mobility 2DES when $\Omega \tau \gg 1$, $\omega_c \tau \gg 1$ in the electric field $E_{\text{tot}}(x, \Omega) \cos \Omega t$.

The nonlinear term $\mathbf{V} \cdot \nabla \mathbf{V}$ in (1) plays central role in our approach. It can be interpreted as a nonlinear, local and instantaneous Doppler shift of the frequency of excitations described by Eq. (1).

In the linear approximation, solutions of Eq. (1) for the forced oscillations of velocity $V_0 = (V_{0x}, V_{0y})$ can be written as $V_{ox}(x,t) = V_{sx}(x) \sin \Omega t$, $V_{oy}(x,t) = V_{cy}(x) \cos \Omega t$, where

$$V_{sx}(x) = \frac{eE_{\text{tot}}(x,\Omega)}{m(\Omega^2 - \omega_c^2)} V_{cy}(x) = V_{sx}(x) \frac{\omega_c}{\Omega}. \tag{2}$$

To find a nonlinear correction $\delta \mathbf{V}(x,t)$ to the velocity we substitute $\mathbf{V}(x,t) = V_0(x,t) + \delta \mathbf{V}(x,t)$ into Eq. (1) and derive the following system of equations for $\delta \mathbf{V} = (\delta V_x, \delta V_y)$:

$$\begin{cases} (\partial_t + 1/\tau) \delta V_x + V_{0x}' \delta V_x + (V_{0x} + \delta V_x) \delta V_x' - \omega_c \delta V_y = -V_{0x} V_{0x}' x \\
(\partial_t + 1/\tau) \delta V_y + V_{0y}' \delta V_x + (V_{0x} + \delta V_x) \delta V_y', + \omega_c \delta V_x = -V_{0x} V_{0y}' y \end{cases}, \tag{3}$$

where prime denotes derivative with respect to $x$.

The third and the forth terms in the left-hand side of Eqs. (3) as well as the right-hand side terms in these equations stem from the nonlinear term $\mathbf{V} \cdot \nabla \mathbf{V}$ in Eq. (1). The coefficients in the third and the forth terms in the left-hand side of Eqs. (3) periodically depend on time and cause the PR in the 2DES.

The variable $\delta V_y(x,t)$ can be excluded from Eqs. (3) and the obtained equation is linearized with respect to $\delta V_x$. We do not present an explicit form of this cumbersome equation.

Following the standard procedure of the PR theory solution for the fundamental PR mode at $\Omega \approx 2\omega_c$ can be written in the two-wave approximation as:

$$\delta V_x = e^{s_0 t} \left[ A(x) \cos \frac{\Omega t}{2} + B(x) \sin \frac{\Omega t}{2} \right], \tag{4}$$

where $s_0$ is the amplification coefficient, $s_0 \ll \Omega$. The instability occurs at $s_0 > 0$.

We substitute Eq. (4) into the linearized equation for $\delta V_x(x,t)$ to obtain the system of the linear equations for the coefficients $A(x), B(x)$. In this derivation, we neglect the terms containing high order frequency harmonics as well as the nonlinear terms with respect to the electric field amplitude.

Introducing new variables $\tilde{A} = V_{sx}(x)A$, $\tilde{B} = V_{sx}(x)B$ and neglecting the difference between $\Omega$ and $2\omega_c$ where it is possible we arrive to the following system equations:

$$\begin{pmatrix} -s - \frac{3V'_{sx}}{2} + \frac{V_{cy}}{\Omega} \partial_x \\ -3V''_{sx}/8 \end{pmatrix} \begin{pmatrix} \tilde{A} \\ \tilde{B} \end{pmatrix} = \delta \omega \begin{pmatrix} \tilde{A} \\ \tilde{B} \end{pmatrix}, \tag{5}$$

where the detuning $\delta \omega = \Omega/2 - \omega_c$ and $s = s_0 + 1/\tau$.

We exclude function $\tilde{B}$ from Eqs. (5) and simultaneously omit the terms small by the parameter $s/\Omega \ll 1$.

For simplicity we also consider ‘clean limit’, assuming that the dissipation is small: $s^2 \ll \delta \omega^2$. With these assumptions equation for $\tilde{A}(x)$ takes the form:

$$\begin{pmatrix} -\partial_x^2 - \frac{\Omega^2}{s^2} \left( \frac{4V'_{sx}}{V_{sx}}^2 - \delta \omega^2 \right) \end{pmatrix} \tilde{A}(x) = 0. \tag{6}$$

Eq. (6) is formally equivalent to the Schrödinger equation with an effective potential energy

$$U(x) = -\frac{\Omega^2}{s^2} \left( \frac{4V'_{sx}}{V_{sx}}^2 - \delta \omega^2 \right) \tag{7}$$

and zero effective energy. For solutions of this equation to exist, the denominator in Eq. (7) must be sufficiently small while the numerator to be positive.

Now we turn to evaluation of the screened field $E_{\text{tot}}(x,t)$ and, consequently, $V_{sx}(x)$ and $U(x)$, see Eqs. (2) and (7). In the linear response theory, $E_{\text{tot}}(x,t)$ is determined by the equation

$$E_{\text{tot}}(x,t) = \cos \Omega t \int_{-\infty}^{+\infty} dq \frac{\epsilon(q, \omega)}{\epsilon(q) \epsilon(\Omega)} E_\Omega(q), \tag{8}$$

where $E_\Omega(q)$ is the Fourier transform of $E_\Omega(x)$, $\epsilon(q, \omega)$ is the dielectric function of 2DES which includes both frequency and spatial dispersion. Condition $\epsilon(q, \omega) = 0$ corresponds to the excitation of magnetoplasmons with a dispersion law $\omega_{mp}(q)$. If the function $\epsilon(q, \omega)$ is close to zero in sufficiently large interval of $q$, then virtual plasmons excite and the screened electric field $E_{\text{tot}}$ is enhanced (‘anti-screening’ effect).

We use the random phase approximation to find the dielectric function of 2DES $\epsilon(q, \omega)$. In the collisionless limit, at $q \ll k_F$, $\hbar \omega_c \ll 2\pi^2 k_B T \ll E_F$, where $h k_F$, $E_F$ are Fermi momentum and energy, $T$ is temperature, the function $\epsilon(q, \omega)$ is determined as $\epsilon(q, \omega) = 1 + \frac{2m}{\pi \hbar^2 V_{ee}(q)} \sum_{n=1}^{\infty} \frac{n^2 \omega_c^2 J_2^2(qR_c)}{n^2 \omega_c^2 - \omega^2 - i0\text{sgn}\omega}, \tag{9}$
where \( V_{ee}(q) = 2\pi e^2/\varkappa|q| \) is the Fourier transform of the 2D Coulomb potential, \( \varkappa \) is a background static dielectric constant, \( J_n(qR_c) \) is the \( n \)th order Bessel function of the first kind. Dissipation in 2DES will be included by introducing the phenomenological line broadening in the magnetoplasmon dispersion law.

The magnetoplasmon dispersion law is determined by the equation \( \varepsilon(q, \omega) = 0 \). Two lowest Bernstein branches of the dispersion law are shown in Fig.1. It is important to point out the opening of the Bernstein gap between \( \omega_0 \) and \( 2\omega_c \).

We consider the situation when the frequency \( \Omega \) is positioned in this gap and the pumping electric field is enhanced due to the excitation of the virtual Bernstein magnetoplasma modes. This effect is described by the factor \( \varepsilon^{-1} \) in Eq. (8), see also Fig.2.

We label the second and the first branches in the magnetoplasmon dispersion law as \( \Omega_\pm(q) \) respectively. To evaluate the screened electric field at the frequency \( \Omega \) close to \( 2\omega_c \), we retain only the terms with \( n = 1, 2 \) in Eq. (9).

At the next step, we estimate the wave vector \( q_0 \) corresponding to the maximum of the lower branch, \( \omega_0 = \Omega_-(q_0) \) and the gap \( \delta\omega_{gap} = 2\omega_c - \omega_0 \). In the lowest order in the small parameter \( qR_c \), we obtain

\[
q_0R_c = 4.5a_B/R_c, \quad \text{where} \quad a_B = h^2/\varkappa\epsilon e^2 \quad \text{is the effective Bohr radius.}
\]

For typical parameters of GaAs quantum wells (\( \varkappa = 7, \quad m = 0.067m_0, \quad n_s = 3 \times 10^{11} \text{cm}^{-2} \) and \( \omega_c/2\pi = 10^{11} \text{Hz} \)) we obtain \( a_B \approx 5.5 \text{nm} \), \( R_c \approx 0.37 \mu\text{m} \) and \( q_0R_c \approx 0.067 \ll 1 \) so that the parameter \( q_0R_c \) is really small. In the same approximation the gap is estimated as:

\[
\delta\omega_{gap} = 2\omega_c - \omega_0 \approx 11.4\omega_c \frac{a_B^2}{R_c^2} \approx 2.5 \times 10^{-3}\omega_c.
\]

Now we can estimate the screened electric field \( E_{tot}(x, t) \). In this estimate we include only the resonant part of the screened electric field, \( E_{res}^{res}(x, t) \), connected with the maximum of \( \varepsilon^{-1}(q, \omega) \) near \( q_0 \).

The function \( \varepsilon(q, \omega) \) near \( q_0 \) can be represented as \( \varepsilon(q, \omega) = \varepsilon_0 + (q - q_0)^2/2M \), where \( \varepsilon_0 = \varepsilon(q_0, \Omega) \). To estimate the screened field \( E_{tot}^{res}(x, t) \) we also use the model expression for the pumping field, \( E_0(x) = \varphi_0/\sqrt{x^2 + l^2} \), where \( l \) is the characteristic length of the field inhomogeneity. This model describes the electric field induced by the thin metal contact [29]. The Fourier transformation of the function \( E_0(x) \) yields \( E_0(q) = 2\varphi_0K_0(ql) \), where \( K_0(ql) \) is the zeroth order modified Bessel function of the second kind. Using the asymptotics of \( K_0(ql) \) at \( ql \ll 1 \) we obtain the following expression for \( E_{tot}^{res}(x, t) \):

\[
E_{tot}^{res}(x, t) = \varphi_0 \sqrt{2M/\varepsilon_0} \ln(1.12/q_0l) \cos(q_0x) \times \exp(-|x|\sqrt{2M/\varepsilon_0}) \cos(\Omega t).
\]

The value of \( E_{tot}^{res} \) increases when \( \Omega \to \omega_0 \) and \( \varepsilon_0 \to 0 \), see Fig.2. However it saturates when \( |\omega_0 - \Omega| \) becomes less than \( 1/\tau^* \), where \( \tau^* \) is the relaxation rate associated with the smearing of the Bernstein gap. Therefore, the value of \( 1/\tau^* \) defines the maximal enhancement of \( E_{tot}^{res}(x, t) \).

An amplitude of \( E_{tot}^{res} \) takes zero values periodically in \( x \)-space resulting in the singular behaviour of the effective potential energy in Eq. (7) at points where \( E_{tot}^{res} = 0 \). One can estimate the value of the pumping potential \( \varphi_0 \) when the effective potential becomes attractive near these points, i.e. the numerator of the expression in Eq. (7) is positive. For the 2DES parameters listed above and \( \omega_c\tau^* \approx 10^3 \) we calculate \( \varepsilon_0 \approx 0.103, \quad M \approx 0.015/R_c^2 \). We find very small value for the pumping potential:\n
\( \varphi_0 \approx 1 \text{ mV} \). Equation (9) has solution when the condition \( \varphi_0 \geq 1 \text{ mV} \) is satisfied. It happens because the
denominator of the expression for $U(x)$ in Eq. (7) takes zero value while its numerator is positive. It leads to the asymptotic behaviour $U(x) \sim -1/\mu^2$ which corresponds to so-called 'fall to the centre' in quantum mechanics [30]. In this case solutions of Eq. (4) always exist.

One should point out that the instability develops at the distances from the contact of the order $\pi/q_0 \sim 50R_c \approx 18 \mu m$. This distance should be smaller than the sample's width. We suggest that in the narrow samples the anti-screening is not effective and the PR in such 2DES is never realized.

The parametric instability is developed in the samples with large width. However it can happen only in the high-mobility 2DES where the condition $\delta \omega_{gap} \tau^* \gg 1$ is satisfied. This condition defines the minimum frequency of the MW pumping. For 2DES with parameters listed above, $\mu = \epsilon \tau/m = 3 \times 10^7 \, cm^2/(Vs)$, and assuming $\tau^* \approx \tau$, we estimate this minimum frequency as:

$$\frac{\Omega_{\min}}{2\pi} = \frac{1}{2\pi} \sqrt{\frac{8}{11.4}} \left(\frac{v_F}{a_B}\right)^2 \frac{1}{\tau^*} \approx 160 \, GHz.$$

One can also estimate the minimum effective electron mobility needed for the instability to take place at typical frequency $\Omega/2\pi = 100 \, GHz$: $\mu_{\min} = \epsilon \tau^*/m \approx 13 \times 10^7 \, cm^2/(Vs)$.

In summary, we have developed the hydrodynamic theory of the cyclotron PR in 2D electron liquid irradiated by the inhomogeneous MW field of frequency $\Omega$. We use linear response theory with nonlocal dielectric function $\varepsilon(q, \omega)$ to find the screened microwave electric field acting on the 2D electrons. It is important that there is the Bernstein gap near $2\omega_c$ in the magnetoplasmon dispersion law. If $\Omega$ falls in this frequency gap and the gap width is larger than the effective relaxation rate, the screened electric field is enhanced due to small value of $\varepsilon(\Omega, q)$ in the gap. It is this effect that causes the parametric instability to occur in sufficiently wide samples at the very low pumping. The PR is developed at frequency $\Omega/2\omega_c \approx 1 - 5.7 a_B^2/R_c^2$ and leads to the 2DES heating which, in turn, destroys the temperature-dependent GNMR observed in these samples without irradiation. Therefore the narrow resistivity spike arises. This mechanism can explain the results of the recent experiments [31, 32].

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