Study of relationship between lateral slip coefficient and indicators of tire linear stiffness.

A U Abdulgazis
Crimean Engineering and Pedagogical University named after Fevzi Yakubov, Simferopol, Russia

at@kipu-rc.ru

Abstract. Based on the analysis of the recent study, the article delves into the unresolved problem of defining the influence of normal and lateral stiffness coefficients on the tire lateral slip. The authors propose a method to determine such influence of normal and lateral stiffness on the tire lateral slip coefficient. The obtained analytical expressions make it possible to calculate the lateral slip coefficients of the tires taking into account their design parameters, speed, normal and tangential reactions in the wheel road contact spot.

1. Problem formulation.
The effect of lateral slip of pneumatic tires, discovered in 1925 by G. Brulier, is the subject of multiple years of research conducted by many authors. This paper studies both linear and non-linear parts of the dependence of the lateral force on the wheel lateral slip angle. A large number of correcting coefficients and dependencies have been obtained, making it possible to take into account the influence of various operational factors on the lateral slip. However, well-known studies do not provide any analytical dependence linking the lateral slip coefficient to the radial and lateral stiffness coefficients of the tire, although this relationship is natural.

This article presents the results of a theoretical study that allows obtaining this relationship. The proposed analytical expressions make it possible to consider the indicators of various operational properties when studying the dynamics of a car.

2. Analysis of recent achievements and publications
The dependence proposed by Ya.M. Pevzner [1] is used in the study of the lateral slip

\[ P_y = K_y \cdot \delta; \]  

(1)

where \( K_y \) is the coefficient of tire lateral slip resistance or the slip coefficient; 
\( \delta \) is the lateral slip angle.

A simplified form of the dependence \( P_y(\delta) \), having linear, non-linear, and horizontal sections was proposed in [2].
Dependence (1) is linear and describes only a linear section $\overline{OA}$ of the lateral slip characteristic. For the non-linear part of this characteristic in the paper of D.A. Antonov [3] a correction factor $q$ is proposed; this factor being considered, expression (1) takes the form
\[
P_y = k_y \cdot q \delta; \tag{2}
\]
where $q$ is the total correction factor of the slip coefficient,
\[
q = \prod_{i=1}^{n} q_i; \tag{3}
\]
$q_i$ is the correction factor taking into account the influence of the $i$-th factor;
$n$ is the number of factors affecting the slip coefficient.

However, it is not one correcting factor of tire linear stiffnesses (normal $C_z$ and lateral $C_y$), although today empirical dependencies for their calculation already exist; those were obtained by E. Balakina [4]. Thus, the influence of normal $C_z$ and lateral $C_y$ stiffnesses on the lateral slip of an elastic tire is not presented in the known references.

**The purpose of the study** is to determine the influence of normal and lateral stiffness coefficients on the tire lateral slip.

To achieve this purpose, it is necessary to solve the problem of the tire stress-strain state in contact with the road, when the wheel is rolling and the side force effect is present. To solve this problem, a phenomenological approach in the theory of rolling a deformable wheel, proposed by M.A. Levin and N.A. Fufaev, is used [5].

The lateral force acting on the wheel (and the lateral response reaction of the road) causes tire lateral deformation.

**Figure 1.** Mechanism of formation of the lateral slip of an elastic tire under the action of lateral force [3].

Figure 1 shows the mechanism of formation of the lateral slip of an elastic tire under the action of lateral force.
This deformation is oscillatory in nature, since when a tire element comes into contact with the road, the contact spot gradually increases from zero to maximum, and then decreases from maximum to zero. Maximum lateral deformation of the tire:

\[ Y_{\text{max}} = \frac{P_y}{C_y}; \]  

(4)

where \( C_y \) is the tire lateral stiffness.

Time of maximum deformation \( Y_{\text{max}} \) of the tire element:

\[ T = \frac{L_k}{2V_0}; \]  

(5)

where \( V_0 \) is the linear velocity of the wheel axis;

\( L_k \) is the length of the spot of wheel contact with the road (see Figure 2),

\[ L_k = 2r_f \sin \frac{\alpha}{2} = 2r_f \sqrt{1 - \cos^2 \frac{\alpha}{2}}; \]  

(6)

\( r_f \) is the free radius of the wheel;

\( \alpha \) is the central angle corresponding to a chord equal to \( L_k \) (Figure 2)

Assuming that \( r_d \approx r_s \), determine the tire deformation:

\[ \Delta r = r_f - r_f = r_f (1 - \cos \frac{\alpha}{2}) = \frac{P_z}{C_z}; \]  

(7)

where \( C_z \) is the tire radial stiffness.

**Figure 2.** Diagram for determining the parameters of the deformable wheel.

From equation (7) define:

\[ \cos \frac{\alpha}{2} = 1 - \frac{P_z}{C_z r_f}; \]  

(8)

Expression (5) with the account of (6) and (8) will take the form:

\[ T = \frac{r_f}{V_0} \sqrt{1 - \left(1 - \frac{P_z}{C_z r_f}\right)^2}; \]  

(9)

Equation of vibrations of a tire element:

\[ m_{\text{red}} \ddot{Y} + C_y Y = P_y; \]  

(10)

or

\[ \ddot{Y} + K_y Y = \frac{P_y}{m_{\text{red}}}; \]  

(11)
where \( m_{\text{red}} \) is the part of the vehicle’s mass reduced to the wheel;

\( K_j \) is the frequency of free vibrations of the tire element,

\[
K_j = \frac{c_y}{m_{\text{red}}};
\]  

(12)

The general solution of the second-order inhomogeneous differential equation with constant coefficients takes the form:

\[
\dot{Y} = A\cos\left(\frac{c_y}{m_{\text{red}}} t\right) + B\sin\left(\frac{c_y}{m_{\text{red}}} t\right);
\]  

(13)

Partial solution:

\[
Y_0 = \frac{P_y}{C_y};
\]  

(14)

Thus:

\[
Y = \dot{Y} + Y_0 = \frac{P_y}{C_y} + A\cos\left(\frac{c_y}{m_{\text{red}}} t\right) + B\sin\left(\frac{c_y}{m_{\text{red}}} t\right); \tag{15}
\]

Given the limit conditions (at \( t = 0; \dot{Y} = 0; Y = 0 \)), obtain the coefficients \( A \) and \( B \):

\[
A = -\frac{P_y}{C_y}; \tag{16}
\]

\[
B = 0; \tag{17}
\]

Equation (15) with the account (16) and (14) is transformed to the form:

\[
Y = \frac{P_y}{C_y} \left[ 1 - \cos\left(\frac{c_y}{m_{\text{red}}} t\right) \right]; \tag{18}
\]

\[
\dot{Y} = V_y = \frac{P_y}{C_y} \sqrt{\frac{c_y}{m_{\text{red}}}} \sin\left(\frac{c_y}{m_{\text{red}}} t\right); \tag{19}
\]

At \( t = T \) (at the point of maximum tire lateral deformation)

\[
V_y = \frac{P_y}{C_y} \sqrt{\frac{c_y}{m_{\text{red}}}} \sin\left[ \frac{c_y}{m_{\text{red}}} \cdot \frac{r_{fr}}{V_0} \sqrt{1 - \left(1 - \frac{P_y}{C_y r_{fr}}\right)^2} \right]; \tag{20}
\]

or

\[
V_y = \frac{P_y}{\sqrt{c_y} \sqrt{m_{\text{red}}}} \sin\left[ \frac{r_{fr}}{V_0} \sqrt{\frac{c_y}{m_{\text{red}}}} \left(1 - \left(1 - \frac{P_y}{C_y r_{fr}}\right)^2\right) \right]; \tag{21}
\]

Tangent of the lateral slip angle \( \delta \) of the tire
\[
tg \delta = \frac{V_x}{V_0} = \frac{P_y}{V_0/\sqrt{C_y} m_{red}} \sin\left[\frac{1}{V_0} \frac{C_y P_x}{C_z m_{red}} \left(2r_{fr} - \frac{P_x}{C_z}\right)\right]; \quad (22)
\]

Assuming that \(tg \delta \approx \delta\) for relatively small angles, obtain:
\[
\delta \cong \frac{P_y}{V_0/\sqrt{C_y} m_{red}} \sin\left[\frac{1}{V_0} \frac{C_y P_x}{C_z m_{red}} \left(2r_{fr} - \frac{P_x}{C_z}\right)\right]; \quad (23)
\]

Side slip resistance coefficient:
\[
K_y = \frac{P_y}{\delta} = \frac{V_0/\sqrt{C_y} m_{red}}{\sin\left[\frac{1}{V_0} \frac{C_y P_x}{C_z m_{red}} \left(2r_{fr} - \frac{P_x}{C_z}\right)\right]}; \quad (24)
\]

Expression (24) determines the dependence of the side slip resistance coefficient of the tire on the tire and car design and experimental parameters.

In [7], empirical dependences were obtained for determining the radial and lateral stiffness of car tires:
\[
C_z = 0.59067 P_z^{0.95}, \text{ N/mm}; \quad (25)
\]
\[
C_y = 0.67725 P_z^{0.80}, \text{ N/mm}; \quad (26)
\]

where \(P_z\) is the normal load on the tire, kg [7].

When using the international system of units SI, equations (25) and (26) will take the form:
\[
C_z = 67.5 P_z^{0.95}, \text{ N/m}; \quad (27)
\]
\[
C_y = 109.0 P_z^{0.80}, \text{ N/m}; \quad (28)
\]

where \(P_z\) is the normal tire load, N.

The approximation error for the dependences \(C_z(P_z)\) and \(C_y(P_z)\) falls in the range of 1-23% [7].

Solving equations (27) and (28) together, define:
\[
C_y = 1.615 P_z^{0.15} C_z; \quad (29)
\]

With the expressions (28) ... (29) and taking the assumption that:
\[
P_z = m_{red} g; \quad (30)
\]

Transform equation (24) to the form:
\[
K_y = \frac{3.33 V_0 P_{z^{0.9}}}{\sin\left(\frac{5.629 V_0 P_z}{r_{fr} P_z^{0.074}}\right)} \cdot \frac{N}{\text{rad}}, \quad (31)
\]

where \(V_0\) is the linear velocity of the wheel axis, m/s

\(P_z\) is the normal load on the car, N;

\(r_{fr}\) is the free radius of the wheel, m.
Figure 3 shows the graphs of the dependence \( K_y(P_z) \) for the tire 7.5-20 \((r_f = 0.464; P_{z_{\text{max}}} = 12 \text{ kN})\) at various values of the linear velocity \( V_0 \) of the wheel axis. The analysis of these graphs shows that increase in \( V_0 \) and \( P_z \) results in \( K_y \) increase as well.

\[ K_y = \frac{V_0 \sqrt{C_y P_z / g}}{\sin \left( \frac{1}{V_0} \sqrt{\frac{C_y}{C_z} \left( \frac{2r_{fr} - P_z}{C_z} \right)} \right)} \left( 1 - \frac{V_0 \delta \sqrt{C_y P_z / g}}{P_{z_{\text{max}}} \sin \left( \frac{1}{V_0} \sqrt{\frac{C_y}{C_z} \left( \frac{2r_{fr} - P_z}{C_z} \right)} \right)} \right) ; \quad (32) \]

When a tangential force is applied at the point of contact of the wheel with the road, expression (32) takes the form:

\[ K_y = \frac{V_0 \sqrt{C_y P_z / g}}{\sin \left( \frac{1}{V_0} \sqrt{\frac{C_y}{C_z} \left( \frac{2r_{fr} - P_z}{C_z} \right)} \right)} \times \left( 1 - \frac{V_0 \delta \sqrt{C_y P_z / g}}{\sqrt{\frac{\phi^2 P_z^2 - R_z^2 \sin \frac{1}{V_0} \sqrt{\frac{C_y}{C_z} \left( \frac{2r_{fr} - P_z}{C_z} \right)}}} \right) ; \quad (33) \]

After substituting expression (31) in equation (3) get:

\[ K_y = \frac{3.33 V_0 P_z^{0.9}}{\sin \left( \frac{5.629}{V_0 P_z^{0.95}} \frac{r_{fr}}{P_z^{0.95}} \right) 0.0074} \times \left( 1 - \frac{3.33 V_0 P_z^{0.9}}{\sqrt{\frac{\phi^2 P_z^2 - R_z^2 \sin \frac{1}{V_0} \sqrt{\frac{C_y}{C_z} \left( \frac{2r_{fr} - P_z}{C_z} \right)}}} \right) \cdot N_{\text{rad}}^{-1} ; \quad (34) \]

At \( R_z = 0 \), expression (34) will be simplified to:

\[ K_y = \frac{3.33 V_0 P_z^{0.9}}{\sin \left( \frac{5.629}{V_0 P_z^{0.95}} \frac{r_{fr}}{P_z^{0.95}} \right) 0.0074} \times \left( 1 - \frac{3.33 V_0}{\phi P_z^{0.9} \sin \left( \frac{5.629}{V_0 P_z^{0.95}} \frac{r_{fr}}{P_z^{0.95}} \right) 0.0074} \right) ; \quad (35) \]
Thus, the obtained analytical expressions (32) - (35) make it possible to consider the lateral slip coefficient of tires \( K_y \) taking into account their design parameters, speed, normal and tangential reactions of the road at the contact point, as well as slip angle \( \delta \).

3. Conclusions
1. As a result of the study, a method was proposed for defining the influence of normal and lateral stiffness on the tire lateral slip coefficient.
2. The obtained analytical expressions make it possible to calculate the lateral slip coefficients of the tires taking into account their design parameters, speed, normal and tangential reactions in the wheel contact spot with the road.

References
[1] Ya.M. Pevzner Lateral slip of a car / Ya.M. Pevzner // Automotive Motor, 1939. - No. 4.- p. 51.
[2] Issues of dynamics of braking and operation processes of brake systems of cars / [B. B. Genbom, G.S. Gudz, V.A. Demyanyuk et al.]; edited by B.B. Genbom. - Lviv: High school, 1974. - 234 p.
[3] D.A. Antonov Theory of stability of multi-axle vehicles / D.A. Antonov. - M.: Mechanical Engineering, 1978.- 216 p.
[4] E.V. Balakina Improving the stability of the movement of a wheeled vehicle in braking mode based on a pre-design selection of the parameters of the chassis elements: abstract of the doctoral dissertation in Tech. Sci.: spec. 05.05.03 - Wheeled and tracked vehicles / E.V. Balakina. - Volgograd, 2011. - 40 p.
[5] M.A. Levin Theory of deformable wheel rolling/ M.A. Levin, N.A. Fufaev. - M.: Nauka, 1989. - 272 p.
[6] Car tire operation/ Ed. by V.I. Knoroz. - M.: Transport, 1976. - 238 p.
[7] V.A. Petrushov Power balance of the car / V.A. Petrushov, V.V. Moskovkin, A.N. Evgrafov. - M.: Mechanical Engineering, 1984. - 160 p.