Gluon poles and photon distribution amplitudes in Drell-Yan-like processes

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We calculate the hadron tensor related to the photon-induced Drell-Yan process, with incoming nucleon being transversally polarised. We predict the new single spin asymmetry which probes gluon poles and which is expressed in terms of photon (both chiral-odd and chiral-even) distribution amplitudes and chiral-odd nucleon function stemming from the nucleon transverse polarization.

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I. INTRODUCTION

In experimental studies, the single spin asymmetry (SSA) is a useful tool for such kind of investigations. In particular, the single transverse spin asymmetry gives a lot of information on the three-dimensional nucleon structure owing to the nontrivial connection between the nucleon transverse spin and the transverse momentum dependence of parton distributions (see, for example, [1–6]). Among the semi-inclusive reactions, the Drell-Yan-like (DY-like) processes play an unique role due to a possibility to uncover the gluon pole contributions and to study finest structure of hadron transverse polarization effects [7–12].

Moreover, there are several experimental projects which pursue the measurements with Drell-Yan-like processes, RHIC [13], COMPASS [14, 15] and future NICA [16, 17].

As well-known, the DY-like processes with an essential transverse polarization of nucleons give a possibility to explore the gluon poles in the twist-3 quark-gluon parton distribution functions (PDFs), denoted as $B^g$-PDFs in, for instance, [7–9]. In Ref. [10], based on the duality properties, the twist-3 $B^g$-PDF has been modelled by the corresponding convolution of two hadron distribution amplitudes. This model has allowed to study the gluon pole presence in more detail and it has resulted in the prediction of new SSAs related to the pion-induced DY process which are reachable for measurements by COMPASS [15].

Here, we propose a new study of the DY-like processes where the gluon poles lead in significant ways to new SSA observables. In particular, we would like to focus our attention on the photon-induced DY process where the twist-3 photon distribution amplitudes (photon DAs) can be probed. Let us emphasise that the understanding of hadron-like behavior of photons, which is encoded in the photon DA, still attracts the attention of community [18–22]. The attempt to study the leading twist-2 photon DA was implemented some years ago in [23].

In this paper we calculate the Drell-Yan hadron tensors related to the photon-nucleon processes with the essential spin transversity and “primordial” transverse momenta where the gluon poles have been shown up. We construct a new single spin asymmetry the existence of which entirely depends on the presence of gluon poles. We dwell on the case where one of photon DAs has been projected onto the twist-2 chiral-odd structure, $\langle \bar{\psi} \gamma^{\perp \perp} \psi \rangle$. We stress that the photon chiral-odd DA singles out the chiral-odd tensor combination in the nucleon matrix element with essential transverse polarization. Notice that, in this case, the access to the specific angular dependence of SSA (see Eqn. (18)) expressed in terms of the Collins-Soper angles has been opened only owing to the gluon pole presence. In other words, the experimental verification of SSA’s angular dependence predicted in the present paper can give evidences for the gluon pole probing and the hadron-like behaviour of photons.

II. KINEMATICS

Let us now go over to the discussion of kinematics. We consider the photon-induced Drell-Yan process with nucleon being transversely polarized at the regime where Bjorken fraction $x_B$ is close to 1, i.e. $x_B \rightarrow 1$:

$$\gamma(p_1) + N^{(i)}(p_2) \rightarrow \gamma'(q) + \bar{q}(K) + X(P_X)$$

$$\rightarrow \bar{q}(K) + \bar{\ell}(l_1) + \ell(l_2) + X(P_X).$$

The virtual photon producing the lepton pair ($l_1 + l_2 = q$) has a large mass squared ($q^2 \approx Q^2$) while its transverse momenta $q_\perp$ are small and integrated out.

The light-cone (Sudakov) decompositions supplemented with usual approximations take the forms [10]

$$P_1 = \frac{Q}{x_B \sqrt{2}} n^* - \frac{x_B P_{\perp 1}}{Q \sqrt{2}} n + P_1 \perp \approx \frac{Q}{x_B \sqrt{2}} n^* + P_1 \perp,$$

$$P_2 \approx \frac{Q}{y_B \sqrt{2}} n + P_2 \perp, \quad S \approx \frac{\lambda}{M_2} P_2 + S_\perp$$

for the real photon and nucleon momenta and nucleon spin.
vector;
\[ q = \frac{Q}{\sqrt{2}} n^* + \frac{Q}{\sqrt{2}} n + q_\perp, \quad q_\perp \ll Q^2, \]  
(3)
for the virtual photon momentum. The momenta \( P_1 \) and \( P_2 \) have the plus and minus dominant light-cone components, respectively.

It is convenient to define the Collins-Soper frame, \( i.e. \) the center-mass-system of lepton pair, as \( [24] \)
\[ \hat{\tau} = \frac{q}{Q}, \quad \hat{x} = \frac{q_\perp}{Q}, \quad \hat{z} = \frac{y_B}{Q} \tilde{P}_1 - \frac{y_B}{Q} \tilde{P}_2, \]  
(4)
where \( \tilde{P}_1 = P_1 - q/(2\gamma_B) \) and \( \tilde{P}_2 = P_2 - q/(2\gamma_B) \) \( [32] \). Moreover, we have
\[ \sqrt{2} n^* = \hat{\tau} + \frac{Q_\perp}{Q} \hat{x}, \quad \sqrt{2} n = \hat{\tau} - \frac{Q_\perp}{Q} \hat{z}. \]  
(5)

### III. HADRON TENSOR

The processes (1) are studied in the kinematics with a large \( Q^2 \), so we apply the factorization theorem to get the corresponding hadron tensor factorized in the form where the hard part is convoluting with the soft part. All important steps of our factorization approach can be found, for instance, in \( [7–9] \) (see also the references therein).

It is natural to study the role of observables at twist-3 level by exploring of different associated asymmetries. Any single spin asymmetries (SSAs) can be presented in the following symbolical form
\[ A \sim d\sigma^{(1)} - d\sigma^{(1)} \sim \mathcal{L}_{\mu\nu} H_{\mu\nu}, \]  
(6)
where \( \mathcal{L}_{\mu\nu} \) is the corresponding leptonic tensor and \( H_{\mu\nu} \) stands for the hadronic tensor. For our purposes, it is enough to consider the case of unpolarized leptons which leads to the real lepton tensor and, therefore, the hadron tensor \( H_{\mu\nu} \) has to be real one too. As for any DY-like processes, the hadron tensor includes at least two non-perturbative blobs associated with different dominant (plus and minus) light-cone directions. For the process (1), the hadron tensor involves the upper non-perturbative blob (see Figs. 1 and 2) which corresponds to the matrix elements with the spin transversity:
\[ \langle P_2, S_\perp | \bar{\psi} \gamma^\perp \psi | S_{\perp}, P_2 \rangle \overset{\mathcal{F}}{\sim} e^{-L_\perp S} P_2 \tilde{h}, \]  
(7)
Notice that the upper blob corresponds to the dominant minus light-cone direction and the dominant antiquark momentum \( k_2 \) is defined along the minus direction too. In more general case, the function \( \tilde{h}(y) \) can be expressed through the corresponding moments of the transverse momentum dependent functions.

In contrast to the usual DY-process (with two initial nucleons) with the twist-3 \( b^V \)-PDF \( [7] \), in the photon-induced DY case the lower non-perturbative blob splits into two photon distribution amplitudes: the one of twist-2, \( \langle \bar{\psi} \gamma^\perp \psi \rangle \), and the second one of twist-3, \( \langle \bar{\psi} \gamma^\perp \gamma^\perp \psi \rangle \) (in the similar way as for the pion-induced DY-process \( [10] \)).

Generally speaking, the hadron tensor involves the contributions from two different kinds of amplitude interference, see Figs. 1 and 2 together with the mirror contributions. Moreover, the hadron tensor term which arises from such an interference contains the contributions from both the “standard” and “non-standard” diagrams as shown in Fig. 1 \( [33] \). As was explained in \( [25] \), in this case the gluon propagator involves the transverse gluons only, \( i.e. \) with the gluon propagator numerator \( d_{ab}(\ell) = \varepsilon_{ab}^\perp \). Thus, it is easy to see, by simple \( \gamma \)-algebra, that for the lower blobs the convolution combination of \( \langle \bar{\psi} \sigma^\perp \psi \rangle \) and \( \langle \bar{\psi} \gamma^\perp \psi \rangle \) as presented in Fig. 1 does not contribute to the hadron tensor at all.

Hence, we are left with the only interference which generates the hadron tensor shown in Fig. 2 which refers to the “standard” diagram. Note that the “non-standard” diagram vanishes in this case.

We are now in position to discuss the derivation of hadron tensor generated by the diagram in Fig. 2. As mentioned, we adhere the terminology and the collinear factorization procedure as described in \( [7] [9] \). The dominant quark and gluon momenta \( k_1 \) and \( \ell \) lie along the plus direction while, as above-mentioned, the dominant antiquark momentum \( k_2 \) – along the minus direction (see Figs. 1 and 2). That is the behaviours of parton momenta on the light-cone satisfy the conditions:
\[ k_1 \sim \left( P_1^+, \frac{\Lambda_1^2}{P_1^+}, \Lambda_1 \right), \quad k_2 \sim \left( \frac{\Lambda_2^2}{P_2^+}, P_2^+, \Lambda_2 \right), \]  
(8)
where \( \Lambda \) characterize the hadron typical scales.

Before factorization, this diagram leads to the following contribution (all prefactors are included in the integration measures)

\[
\mathcal{W}_{\mu\nu} = \int (d^4k_1) (d^4k_2) \delta^{(4)}(k_1 + k_2 - q)
\times \int (d^4\ell) \mathcal{D}_{\alpha\beta}(\ell) \text{tr} \left[ \gamma_\alpha \sigma^{\perp+} \gamma_\sigma \sigma^{\perp-} \right] 
\times \Phi^{[\gamma^1]}(k_2) \Phi^{[\gamma^2]}(k_2; \ell) \Phi^{[\gamma^1]}(k_1) \delta \left( (P_1 - k_1)^2 \right) + \\
\text{(Fierz projection replacement: } [\sigma^{\perp+}] \leftrightarrow [\gamma^1]), \tag{9}
\]

where

\[
\mathcal{D}_{\alpha\beta}(\ell) = \Delta(\ell) d_{\alpha\beta}(\ell), \quad \Delta(\ell) = \frac{1}{\ell^2 + i0}, \tag{10}
\]

and

\[
\Phi^{[\gamma^1]}(k_2) = \sum_X \int (d^4\eta_2) e^{-i k_2 \eta_2} 
\times \langle P_2, S \parallel \text{tr} \left[ \bar{\psi}(0) P X \psi(\eta_2) \sigma^{-1} \right] \parallel S, P_2 \rangle 
\]

and

\[
\Phi^{[\gamma^2]}(k_2; \ell) = i P_1^+ e^{+}_{(\lambda)} \Phi^{[\gamma]}(k_2, \ell) = \\
\int (d^4\eta_1) e^{i (P_1 + k_2 - \ell) \eta_1} \langle 0 \parallel \bar{\psi}(\eta_1) \sigma^{+1} \psi(0) \parallel P_1 \rangle,
\]

\[
\Phi^{[\gamma^1]}(k_1) = e^{+}_{(\lambda)} \Phi^{[\gamma]}(1)(k_1) = \\
\int (d^4\xi) e^{-i k_1 \xi} \langle P_1 \parallel \bar{\psi}(\xi) \gamma^1 \psi(0) \parallel 0 \rangle. \tag{13}
\]

In Eqns. (12) and (13) the factors which include the quark condensate \( \langle \bar{\psi} \psi \rangle \), the magnetic susceptibility \( \chi \), and the non-perturbative constant \( f_{\gamma} \) are absorbed in the definitions of \( \phi^{(\gamma)}_\gamma \) (see [20] for details).

In Eqn. (9) we write explicitly the \( \delta \)-function which shows that the quark operator with the momentum \( P_1 - k_1 \) corresponds to the on-shell fermion. Moreover, since we here deal with the only “standard” diagram, within the contour gauge \( [\epsilon^+; -\infty]_{A^+} = 1 \) (see, [17,20]) which leads to the local axial gauge \( A^+ = 0 \), the gluon propagator in Eqn. (9) contains contributions of two contractions: \( A^+ A^- \) and \( A^- A^- \). In other words, the numerator \( d^{\mu\nu}(\ell) \) in Eqn. (10) receives the contributions from \( d^{+\perp}(\ell) \) and \( d^{\perp\perp}(\ell) \) combinations.

The next step is to perform the factorization procedure for the hadron tensor. We are not going to discuss details of factorization stages (the comprehensive description of factorization can be found in many papers, see, for example, [26,29]).

By referring to [10], in the limit \( x_B \rightarrow 1 \), we finally derive the following expression for the gauge invariant hadron tensor

\[
\Phi^{(\gamma)}_\gamma(y, x_2; k_2^+, k_2^-) \approx \Phi^{(\gamma)}_\gamma(y, k_2^+, k_2^-) + O \left( \frac{\epsilon}{P_1}, \frac{k_2^+}{P_1} \right), \tag{16}
\]

where \( \Phi \) has a role of “spectator” function. Justification of the separability in Eqn. (16) can be demonstrated by using of
the effective Lagrangian method in order to calculate the non-
perturbative photon-vacuum matrix element of twist-2 quark
operators which corresponds to $\Phi^{(2)}_Y$, see Appendix A

Finally, we focus on discussion of the pole contribution
1/$\lambda_{21}$ in Eqn. [14]. Similarly as in Ref. [10], the gluon pole,
which has to be treated within the contour gauge framework,
can be described as [7–9]

$$\frac{1}{x_2 - x_1} \text{e}^{-\frac{1}{x_2 - x_1 - \epsilon}}. \quad (17)$$

The complex prescription originates from the corresponding
integral representation of theta function (see, [8] for details).
This complex prescription in (17) generates the imaginary part
in order to compensate the complex $i$ in parametrization of the
photonic-vacuum matrix element [12] and which leads to real
part on the l.h.s. of (6).

**IV. SINGLE SPIN ASYMMETRY**

We now construct the single spin asymmetry observable.
Within the Collins-Soper frame [6][24], performing the invariant
integration in [14] and contracting the leptonic and hadron
tensors, we finally obtain (we remind that $\mathcal{L}_{\mu\nu}$ corresponds
the unpolarized lepton tensor)

$$\frac{4 \mathcal{L}_{\mu\nu} \bar{F}_{\mu\nu}(x)}{Q^2} = \Phi^{(1)}_Y(x_B) \Phi^{(2)}_Y(\bar{x}_B) \bar{f}_1(x_B), \quad (18)$$

$$\times \left( e^{(2)}_{V} \cdot e^{(2)}_{A} \right)_P L_{\perp} \times \sin^2 \theta \cos 2\varphi, \quad (19)$$

where we use Eqn. (16) for $\Phi^{(2)}_Y$ and $\bar{f}_1$ stands for the corresponding
integration over $\bar{k}^+_2$ (cf. Eqn. [14]).

Therefore, we predict a new asymmetry which reads (cf. [13])

$$S_T = \frac{S_{\perp} L_{\perp} \sin(\phi_S + \phi) C_{UT}^{\phi_S}}{f_1(y_B) H_1(x_B)}, \quad (19)$$

$$D_{[\theta, \phi]} = \frac{2 \sin^2 \theta \cos 2\varphi}{1 + \cos^2 \theta}, \quad (20)$$

where $C_{UT}^{\phi_S} = \Phi^{(1)}_Y(x_B) \Phi^{(2)}_Y(\bar{x}_B) \bar{f}_1(x_B) \bar{f}_1(y_B)$ and $H_1(x_B)$ stem from the unpolarized cross-section and they parameterize the following matrix elements:

$$\langle P_2 | \bar{\psi} \gamma^- \psi | P_1 \rangle \bar{f}_1(y), \quad (19)$$

$$\langle P_1 | \bar{\psi} \gamma^5 q | P_1 \rangle \bar{f}_1(x) \sim \bar{f}_1(x). \quad (21)$$

The new SSA defined by Eqn. (19) constitutes the main result
of our study.

**V. CONCLUSION**

We derive the gauge invariant photon-induced DY hadron
tensor with the essential spin transversity and “primordial”
transverse momenta. In the paper, we focus on the case
where one of photon distribution amplitudes has been pro-
jected onto the chiral-odd twist-2 combination which singles
out the chiral-odd parton distribution inside nucleons.

We predict a new single transverse spin asymmetry which
is a novel phenomenon and which are associated with the
spin transversity and the nontrivial specific $\varphi$-angular
dependence. This asymmetry can be treated as a signal of the
gluon pole presence integrally with the study of both chiral-
odd and chiral-even photon distribution amplitudes.

We emphasize that the possibility to study different SSAs
induced by chiral-odd distribution functions/amplitudes
appears only owing to the gluon pole presence. Thus, the pro-
posed angular dependence of SSA can furnish the implicit
evidences for the gluon pole observation in experiments.

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**Appendix A: Demonstration of Separable Ansatz**

In order to demonstrate the separable ansatz (16), it is
instructive to estimate the non-perturbative matrix element
which corresponds to the distribution amplitude within the
frame of some effective model. A concrete realization of the
model is not important for our purposes. So, we begin with
the matrix element representing $\Phi^{(2)}_Y$ of the leading twist in
the interaction representation. We have

$$M = \int (d^4 \eta) e^{i(\xi - k_2 - \eta)} \langle 0 | T \bar{\psi}(0) \sigma^{+\perp} \psi(\eta) \rangle | 0 \rangle \quad (A.1.1)$$

$$= \int (d^4 \eta) e^{i(\xi - k_2 - \eta)} \int (d^4 \xi) e^{-iP_1 \xi}$$

$$\times \langle 0 | T \bar{\psi}(0) \sigma^{+\perp} \psi(\eta) \rangle \frac{\delta \delta}{\delta M(\xi)} | 0 \rangle, \quad (A.2)$$
where $S$-matrix is given by the interaction action:

$$S_{\text{eff}}^M = g_M \int (d^4z) M(z) \psi(z) \Gamma_M \psi(z). \quad \text{(A.3)}$$

Here, $M$ denotes the relevant meson which in our case is hadron-like photon. Note that, within the effective frame, the coupling constant $g_M$ can be calculated with a help of Fourier transformations, we obtain that

$$M = \text{tr} \left[ \sigma^{\perp \perp} S(\ell - k_2) \Gamma_M S(\ell - k_2 - P_1) \right]. \quad \text{(A.4)}$$

In the paper, we study the hadron-like behaviour of photon, therefore we can choose $\Gamma_M = \sigma^{\perp \perp}$.

Provided $|\vec{E}_\perp^2| \sim |\vec{E}_\perp \cdot \vec{K}_\perp^\perp| \sim |\vec{K}_\perp^2| \sim O(1)$, we can make an approximation

$$\varphi((\vec{E}_\perp - \vec{K}_\perp^2)^2) = \frac{1}{-2k_2^2 - (\vec{E}_\perp - \vec{K}_\perp^2)^2/\ell^+ + ie}.$$  

With this choice and within the frame [2], we have

$$M = \text{tr} \left[ \sigma^{\perp \perp} (\ell^+ \gamma^-) \sigma^{\perp \perp} (\ell^+ - P_1^+) \gamma^- \right] = \text{tr} \left[ \sigma^{\perp \perp} \gamma^- \sigma^{\perp \perp} \gamma^- \right] \left[ -2k_2^2 - (\vec{E}_\perp - \vec{K}_\perp^2)^2/\ell^+ + ie \right]. \quad \text{(A.5)}$$

Let us now focus on the typical function which can be extracted from Eqn. (A.5)

$$\varphi((\vec{E}_\perp - \vec{K}_\perp^2)^2) = \frac{1}{[2k_2] \left[ 1 - \frac{(\vec{E}_\perp - \vec{K}_\perp^2)^2}{2k_2 \ell^+} + ... \right]} \approx \frac{1}{2k_2} \left[ 1 - \frac{\vec{E}_\perp^2 - \vec{K}_\perp^2}{2k_2 \ell^+} + ... \right] \approx \frac{1}{2k_2} \left[ 1 - \frac{\vec{E}_\perp^2 - \vec{K}_\perp^2}{2k_2 \ell^+} + ... \right] \left[ 1 + \frac{\vec{K}_\perp^2}{2k_2 \ell^+} + ... \right]. \quad \text{(A.7)}$$

This chain of approximations supports the separable ansatz.

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