Online tuning of artificial neural networks using para-model algorithm - A preliminary study

Loïc MICHEL

Abstract
In this preliminary work, we explore the possibilities of using a model-free based control law in order to adjust synaptic weights in artificial neural networks. In the supervised learning context, we consider the problem of tuning the weights as a feedback control tracking problem where the control algorithm adjusts the weights online according to the input-output of the neural network. Simulation results illustrate first properties of our proposed approach.

1 Introduction
The model-free control methodology, originally proposed 10 years ago by Fliess & Join [1], has been designed to control a priori any "unknown" dynamical system in a "robust" manner, and is referred to as a self-tuning controller in [2]. This control law has been widely and successfully applied to control many nonlinear processes in several fields like biology, mechanics, electronics... See, e.g., the references in [1], [3], [4] and the references therein for an overview of the applications. This control law can be considered as an extended PI control and the performances are really satisfactory taking into account that no explicit model is a priori given since the control is only based on input & output signals. Recently, a modified version, the "para-model" control, that can be considered as a derivative-free & model-free control law, has been proposed by the author [5] and has been successfully experimentally validated in the case of the control of a highly nonlinear magnetic process [6].

In [7], the authors show connexions between adaptive control and optimization methods and therefore they highlight a certain equivalence to use tools from the adaptive control field to solve problems in the machine learning field. In this line of thinking, the motivation of this work is to propose a strategy to tune neural networks using the recent (derivative-free and model-free) para-model algorithm in the context of supervised learning and therefore, we consider such training as a tracking control problem. The paper is organized as follow.

*Centrale Nantes, Laboratoire des Sciences du Numérique de Nantes (LS2N) UMR 6004 CNRS, Nantes, France (loic.michel@ec-nantes.fr).
Section 2 reviews the para-model approach. Section 3 presents the application of the para-model control to train neural networks and numerical simulations are presented to illustrate the first observed properties. Section 4 gives some concluding remarks.

2  Principle of the para-model control

We consider a nonlinear SISO dynamical system to control:

\[
\begin{align*}
&u \mapsto y, \\
&\dot{x} = f_{nl}(x, u) \\
&y = Cx
\end{align*}
\]

where \( f_{nl} \) is the function describing the behavior of a nonlinear system and \( x \) is the state vector; the para-model control is an application \((y^*, y) \mapsto u\) whose purpose is to control the output \( y \) of (1) following an output reference \( y^* \). In simulation, the system (1) is controlled in its "original formulation" without any modification / linearization.

2.1 Definition of the closed-loop

Consider the control scheme depicted in Fig. 1 where \( C_\pi \) is the proposed para-model controller, and \( K_{in}, K_{out} \) are positive gains.

\[\text{Figure 1: Proposed para-model scheme to control a nonlinear system.}\]

2.2 Definition of the para-model algorithm

For any discrete moment \( t_k, k \in \mathbb{N}^* \), one defines the discrete controller \( C_\pi \) as an integrator associated to a numerical series \((\Psi_k)_{k \in \mathbb{N}}\) such as symbolically:

\[
C^{(K_p, K_i, k_{\alpha}, k_{\beta})}_\pi : (y, y^*) \mapsto u_k = \Psi_k \cdot \int_0^t K_i(y_{k-1}^* - y_{k-1}) \, d\tau
\]

with the recursive term:

\[
\Psi_k = \Psi_{k-1} + K_p(k_{\alpha}e^{-k_{\beta}k} - y_{k-1}),
\]
where: $y^*$ is the output reference trajectory; $K_p$ and $K_I$ are real positive tuning gains; $e_{k-1} = y^{*}_{k-1} - y_{k-1}$ is the tracking error; $k_\alpha e^{-k_\beta k}$ is an initialization function where $k_\alpha$ and $k_\beta$ are real constants; practically, the integral part is discretized using e.g. Riemann sums.

We define the set of the $C_\pi$-parameters of the controller as the set of coefficients \( \{K_p, K_i, k_\alpha, k_\beta\} \)\(^1\)

### 3 Application to the training of neural networks

#### 3.1 Problem statement

In the context of supervised learning, let’s consider a neural network described as a (static) ”black-box” model $E$:

$$E(x_1, x_2, \cdots, x_n, y, W_1, W_2, \cdots, W_q) = 0 \quad (3)$$

that is composed of $n$ inputs $x_1, x_2, \cdots, x_n$; an output $y$; $q$ synaptic weights $W_1, W_2, \cdots, W_q$ and a sigmoid activation function that defines the output of each node.

Given training data $x_1^{\text{train}}, x_2^{\text{train}}, \cdots, x_n^{\text{train}}$ and $y^{\text{train}}$ associated respectively to the inputs and to the output of $E$, we assume that the algorithm (2) updates directly (possibly through a filter) each synaptic weight $\text{such as:}$

$$W_i = C_\pi^{\{K_p, K_i, k_\alpha, k_\beta\}}(y, y^{\text{train}}), \quad i = 1, \cdots, q \quad (4)$$

and therefore allows ”configuring” the neural network (updates of the $W_i$ for all $i = 1, \cdots, q$) in such manner that asymptotically, we get the output $y$ ”as close as possible” to $y^{\text{train}}$.

Depending on the expected closed-loop transient dynamic, a possible choice of the $C_\pi$-parameters is to consider e.g.:

$$K_{pq+1} < K_{pq}, \quad K_{i q+1} < K_{i q}, \quad k_{\alpha q+1} = k_{\alpha q}, \quad k_{\beta q+1} = k_{\beta q}$$

to obtain a good dynamic response regarding changes of the model $E$ and the rejection of external disturbances, like changes in the training data set.

\(^1\)An interesting property that has been observed with para-model control throughout the studied applications is the relative flexibility of the $C_\pi$-parameters \( \{K_p, K_i, k_\alpha, k_\beta\} \) to obtain good tracking performances while ”prototyping” the proposed $C_\pi$ control law on a new process. In particular, we highlight the case of the experimental validation \( \text{[6]} \) for which no representative model of the testbed was available and the control has been tested under several working conditions using indeed the $C_\pi$-parameters adjusted for the corresponding simplified simulation.

\(^2\)Since the neural network does not include any internal dynamic, a first order filter may be added to each $W_i$ in order to include a dynamic regarding the proper use of the $C_\pi$ controllers.
3.2 Simple example of training

To illustrate our proposed training strategy, consider a three-node network depicted in Fig. 2 with two inputs $x_1$ and $x_2$ and an output $y$.

![Figure 2: Example of simple neural network defined by $E : (x_1, x_2) \rightarrow y$.](image)

We apply the strategy (4) to determine the weights $W_1, W_2, \cdots, W_6$ given the training values $x_{1\text{train}}, x_{2\text{train}}$ and $y_{\text{train}}$ (the latter corresponds to the output reference). A first order filter is added to include a dynamic to each controller.

Simulation results

To illustrate some first properties, the following tests have been performed considering the initial set of training data $x_1^{\text{train}} = 0.2$, $x_2^{\text{train}} = 0.6$ and $y^{\text{train}} = 0.55$. The $C_{\pi}$-parameters have not been optimized regarding the transient response and the $W_i$ are bounded such as $W_i \leq 1$ for all $i = 1 \cdots 6$. The simulation time-step is of $10^{-5}$ s.

**Short-term behavior** Figure 3 shows respectively the evolution of the weights $W_i$ and the controlled output $y$, that *a priori* remains close to $y^{\text{train}}$, according to the iterations.

**Online modifications of the training data** Figure 4 shows respectively the evolution of the weights and the controlled output $y$, when the network is subjected to arbitrary changes of the training data.

**Online modifications of the network topology** Figure 5 shows respectively the evolution of the weights and the controlled output $y$, when the network is subjected to an arbitrary change of its topology.
Figure 3: Evolution of the weights $W_i$ and the controlled output $y$ according to the iterations when subjected to the initial set of training data.
Figure 4: Evolution of the weights $W_i$ and the controlled output $y$ according to the iterations when subjected first to the changes $x_1^{train} = 0.15$, $x_2^{train} = 0.7$ at $k = k_1$ and then $y^{train} = 0.6$ at $k = k_2$. 
Figure 5: Evolution of the weights $W_i$ and the controlled output $y$ according to the iterations when subjected to a modification of the neural network topology (setting $W_4 = 0$) at $k = k_1$.

4 Conclusion and perspectives

This paper presented an application of the model-free based control methodology in the field of artificial neural networks. Encouraging preliminary results show interesting tracking performances taking into account online modifications of the training set as well as modifications of the topology of the studied net-
work. Further work will include a complete stability & dynamical performances study as well as investigations regarding the application of our proposed algorithm to large scale neural networks and specific neural networks, like e.g. the dynamical memory network model proposed in [8]. Considering also other tuning algorithms, that could be used simultaneously with our proposed algorithm to train neural networks, future work aims at investigating such interactions and to derive more general convergence and stability conditions.

References

[1] M. Fliess and C. Join, "Model-free control", Int. J. Control, vol. 86, issue 12, pp. 2228-2252, Jul. 2013.

[2] K.J. Åström and P.R. Kumar, "Control: A perspective", Automatica, vol. 50, issue 1, pp. 3-43, 2014.

[3] H. Abouaïssa, M. Fliess, C. Join, "On ramp metering: towards a better understanding of ALINEA via model-free control", Int. J. Control, vol. 90, issue 5, pp. 1018-1026, Jul. 2016.

[4] H. Abouaïssa, O. A. Hasan, C. Join, M. Fliess, D. Defer, "Energy saving for building heating via a simple and efficient model-free control design: First steps with computer simulations", 21st International Conference on System Theory, Control and Computing (ICSTCC), Oct. 2017.

[5] L. Michel, "A para-model agent for dynamical systems", preprint arXiv, Mar. 2018.

[6] L. Michel, O. Ghibaudo, O. Messal, A. Kedous-Lebouc, C. Boudinet, F. Blache, A. Labonne, "Model-free based digital control for magnetic measurements", preprint arXiv, Mar. 2017.

[7] J.E. Gaudio, T.E. Gibson, A.M. Annaswamy, M.A. Bolender, E. Lavretsky, "Connections between adaptive control and optimization in machine learning", preprint arXiv, Apr. 2019.

[8] C. Federer, J. Zylberberg, "A self-organizing short-term dynamical memory network", Neural Networks, vol. 106, pp. 30-41, Oct. 2018.