Formulation of Measurement Sampling Plan for Small and Medium Batch Products in Supply Chain

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Abstract. The article studies the supply chain of suppliers and manufacturers, considering the overall benefits of the supply chain, the measurement sampling plan is the small batch sampling plan with the smallest amount of sampling and the strongest discriminative ability. If the quality characteristic data is a measurement value, and the inspection workload is large and the inspection cost is high, it is recommended to adopt the measurement sampling plan based on the Laplace distribution. On the premise of ensuring quality, try to reduce the number of inspected samples as much as possible to reduce the workload and cost of inspection.

Keywords: Supply chain; Laplace distribution; small batch.

1. Introduction

The quality characteristics of products are transferred with the transfer of logistics. In the process of supply chain node enterprise transactions, there are often hidden information between manufacturers and suppliers. With the development of information technology, the focus of supply chain quality management has shifted. Product life cycle.

Usually, the method for manufacturers to inspect the quality level of the supply chain is to conduct random inspections on the quality of their products. Sampling inspection is to evaluate the quality of a batch of products by inspecting a part of a batch of products to determine whether they are qualified, or to take a part of samples from the production process for inspection to determine whether the process is stable.

According to the characteristics of sampling inspection, even if it is a product batch that is deemed qualified after sampling, it is impossible to guarantee that all the products that have not been sampled are qualified. That is to say, it must be admitted or allowed that there are still a small amount of non-standard products. Qualified product.

The purpose of this study is to try to reduce the possibility of this misjudgment to an acceptable range by using a measurement sampling plan, and to explore the feasibility of a measurement sampling plan for small and medium-sized products in the supply chain.

2. Research method

If the quality characteristic data of the inspected product is a measurement value, such as size, weight, strength, and composition, a measurement sampling plan can be used. It judges whether a batch of products are qualified or not based on the sample mean and sample variance of product quality indicators. It is better to know or estimate the distribution status of the overall quality data in advance during measurement and sampling.

Generally speaking, only when the quality data obeys a normal distribution can the measurement sampling plan be used. However, because we are studying multi-variety, small-batch products, the difference from conventional products is that it has the problem of small amount of data and tailing nature. The amount of data is small, so the sample size used for sampling is small, and the tailing of multi-variety and small-batch products is also small, and there will be large errors when using the
normal distribution. Therefore, this study decided to use the Laplace distribution to analyze the distribution status of the overall quality data.

2.1 Definition of Laplace distribution

The \( f(x | \mu, b) = \frac{1}{2b} e^{-\frac{|x-\mu|}{b}} \) distribution with a density function is called the Laplace distribution; \( \mu \) is the location parameter, and \( b \) is the scale parameter. The expectation of the Laplace distribution is \( \mu \), the variance is \( 2b^2 \), the skewness is 0, and the kurtosis is 3. The probability density of the Laplacian distribution looks very similar to the normal distribution. Draw the probability density diagram of the standard Laplace distribution (\( b=1 \)) and the standard normal distribution:

The 0.99 quantile of the standard Laplace distribution is 3.91, and the standard normal distribution is 2.32, which shows that the probability of an extremely large value for a random variable subject to the Laplace distribution is much greater than that of the normal distribution.

2.2 Sampling plan based on Laplace distribution

Assuming that the product quality is stable, the distribution status of its overall quality data should basically remain unchanged. At this time, if a fixed-size sample is drawn from such a population, the distribution of the sample mean should also be stable and obey the normal distribution, but the distribution of the sample should obey the Laplace distribution at this time, as shown in Figure 1.

![Figure 1 the Laplace distribution](image)

2.3 Model solving

Theoretical derivation can prove that the variance of the sample mean distribution should be the variance of the population distribution, that is, the sample standard deviation is \( \sigma_x = \sigma / \sqrt{n} \). \( n \) is the sample size.

![Figure 2 the distribution curve of the sample means](image)
To simplify the problem, we only discuss the case where the product specification has only upper and lower limits (SU or SL). When only the specification on-line SU is specified, the area of the overall distribution curve beyond the specification limit SU (the shaded part in Figure 1) is the probability of nonconforming product. Suppose the product is a qualified batch, then this probability is its qualified quality level $p_0$. Since we infer the quality of the population (lots) through samples, and for qualified lots, we should receive them with a high probability.

Assuming that $\alpha$ is still used to represent the producer's risk rate, $1-\alpha$ is the acceptance probability. This can set a limit $X_U$ for the distribution curve of the sample mean (Figure 2), so that the probability of the part other than $X_U$ is $\alpha$. At this time, we can use the sample mean $X$ actually obtained during the sampling inspection to compare with the $X_U$ that has been determined. If $X \leq X_U$, the batch of products can be judged to be a qualified batch. If $X > X_U$, it is judged as unqualified. We call $X_U$ the qualified judgment value.

In this way, determining the measurement and sampling plan is actually to determine the sample size $n$ and the qualified judgment value $X_U$. The value of $X_U$ can be obtained according to the following formula:

$$X_U = S_{SU} - K\sigma$$  \hspace{1cm} (1)

If only the specification lower limit SL is specified, then $X_L$ is obtained by the following formula:

$$X_L = S_{SL} + K\sigma$$  \hspace{1cm} (2)

Where: $\sigma \rightarrow$ the standard deviation of the overall distribution;

$X_L \rightarrow$ Only the qualified judgment value at the lower limit of the specification;

$K \rightarrow$ constant.

It can be seen that when $\sigma$ can be estimated, the determination of the sampling plan is actually to determine the two parameters $n$ and $K$. As with the counting standard sampling, $\alpha$, $\beta$, $p_0$, and $p_1$ must be determined first. If $\alpha=0.05$ and $\beta=0.10$ are still specified, the value of $(n, K)$ can be obtained directly according to the values of $p_0$ and $p_1$, and then the value of $(n, K)$ can be obtained according to the formula (1) or (2). $X_L$. At this time, the determination of the sampling plan is complete.

If $\alpha$ and $\beta$ are not the above values, there is no ready-made table to check the values of $n$ and $K$, and they must be calculated using the following formula:

$$n = (\frac{\sigma}{\epsilon})^2$$  \hspace{1cm} (3)

$$K = (p_0)$$  \hspace{1cm} (4)

Where: $\epsilon$ is the inverse function of Laplace distribution function ($X$); and is the inverse function of Laplace distribution function ($X$).

### 3. Applications

A certain factory of the Ministry of Aviation Industry mainly produces horizontal tail fins for aviation aircraft. Horizontal tail, also known as "flat tail". It refers to the wing surface that maintains the longitudinal (pitch) stability and control of the aircraft, and its importance is self-evident. The horizontal tails produced by this factory in recent years have stable quality and a small number of batches, but the full inspection is also time-consuming and labor-intensive, so the measurement-type sampling inspection is most suitable.
3.1 Quality parameter

The quality inspection of the product is divided into three levels: the shape of the horizontal tail, the position of the hole and the radius of the hole. The assembly requirements of the horizontal tail are strict. If the contour deviates too far, it will be deemed unqualified; at the same time, the position of the hole and the radius of the hole must not deviate too much, otherwise the assembly will not proceed normally. The values of the three indicators are as follows:

- Contour: \( \leq 150 \text{ cm} \), if the contour deviates too far, it will be unqualified
- The position of the holes: \( \leq 110 \text{ cm} \), the positions of the three holes cannot be shifted too much
- The radius of the hole: \( \leq 121 \text{ cm} \), the radius of the hole should not be too different

If any indicator exceeds the prescribed standard, the product is deemed unqualified.

Since the manufacturing index of the horizontal tail wing stipulates the upper and lower limit standards, it is necessary to consider the measurement and sampling plan with the upper and lower limits on both sides. Like counting standard sampling, measurement sampling also needs to specify the values of parameters such as \( \alpha \), \( \beta \), \( p_0 \), and \( p_1 \) in advance.

For the sake of comparison, it is still stipulated that \( \alpha=0.05 \), \( \beta=0.1 \), and \( p_0=1\% \). As for \( p_1 \), it can be considered equal to 5% or 10%. It is important to note that the above parameters of \( \alpha \), \( \beta \), \( p_0 \) and \( p_1 \) are all for the entire horizontal tail, not for each individual index.

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3.2 Sampling plan development

According to the above mentioned, the product is considered qualified only when these three indicators are judged to be qualified. Therefore, the following relationship exists:

\[
\begin{align*}
\alpha' &= 1 - \alpha \\
\beta' &= \beta \\
(1-p_0') &= 1-p_0 \\
(1-p_1') &= 1-p_1
\end{align*}
\]

If \( \alpha=0.05 \), \( \beta=0.1 \), \( p_0=1\% \), \( p_1=5\% \), then through the above relationship, \( \alpha'=0.017 \), \( \beta'=0.464 \), \( p_0'=0.03 \), \( p_1'=0.017 \)

According to the statistical analysis of the data of multiple batches of the product in the past, the variances \( S_{H}^2 \), \( S_{M}^2 \) and \( S_{L}^2 \) of the three indicators of each sample are calculated as shown in the following table.

| Sample number | 1   | 2   | 3   | 4   | 5   |
|---------------|-----|-----|-----|-----|-----|
| \( S_{H}^2 \) | 8.5 | 10.7| 12.1| 9.3 | 7.2 |
| \( S_{M}^2 \) | 3.2 | 2.5 | 1.6 | 3.8 | 2.1 |
| \( S_{L}^2 \) | 1.5 | 0.9 | 1.8 | 1.6 | 1.4 |

In this way, according to the relationship \( \sigma^2 = D(X) = \sigma_0^2 \) of the overall variance of the normal distribution and the original variance, it can be obtained

\[
\begin{align*}
\sigma_H &= 3.46 \\
\sigma_M &= 1.62 \\
\sigma_L &= 1.20
\end{align*}
\]

After mastering the above data, at this time, the sample size of the measurement sampling can be calculated through the formulas (3) and (4) listed above.

\[
\begin{align*}
n &= \left( \frac{1}{\alpha} \right)^2 = \left( \frac{1}{\beta} \right)^2 = 37 \\
K &= (\alpha_0) = -2.8134 - = -2.2572
\end{align*}
\]
Furthermore, according to the relational expression of \( X_u = \pm S_u \pm K \sigma \), the qualified judgment value range of each index can be obtained:

\[
X_{uH} = \pm 50 \pm 2.2572 \times 3.46 = (-42.19, 42.19)
\]

\[
X_{uM} = \pm 10 \pm 2.2572 \times 1.62 = (-6.34, 6.34)
\]

\[
X_{uL} = \pm 2 \pm 2.2572 \times 1.20 = (-0.71, 0.71)
\]

In this way, the measurement and sampling plan for the three indicators is finally obtained, as shown in the following table:

| 1. indicator | Sample size n | Qualified judgment value \( X_u (cm) \) |
|--------------|---------------|------------------------------------------|
| 1. appearance | 37            | (-42.19, 42.19)                          |
| 1. location   |               | (-6.34, 6.34)                            |
| radius        |               | (-0.71, 0.71)                            |

### 4. Conclusion

In the context of economic globalization, quality competition does not stop between enterprises, but also extends to between supply chains and supply chains. In the supply chain of suppliers and manufacturers, considering the overall benefits of the supply chain and reducing the cost of the supply chain, the measurement sampling plan is the one with the smallest amount of sampling and the strongest discriminative ability among the small batch sampling plans.

In terms of the above application cases, the sample size \( n = 37 \) for measurement and sampling, the acceptance probability under the qualified quality level is high, that is, the risk rate of the producer is small. Similarly, when the quality of the product declines, for example, when the qualified product rate drops to 5%, the acceptance probability during measurement sampling is only 10%. Compared with the current common sampling plan, the Laplace sampling sampling plan by attributes has the advantages of reducing the amount of sampling and minimizing the expected total average cost. It can reduce the cost of the supply chain network and has a wide range of popularization and application values.

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