Abstract—Covert communication aims to shield the very existence of wireless transmissions in order to guarantee a strong security in wireless networks. In this work, for the first time we examine the possibility and achievable performance of covert communication in one-way relay networks. Specifically, the relay opportunistically transmits its own information to the destination covertly on top of forwarding the source’s message, while the source tries to detect this covert transmission to discover the illegitimate usage of the recourse (e.g., power, spectrum) allocated only for the purpose of forwarding source’s information. The necessary condition that the relay can transmit covertly without being detected is identified and the source’s detection limit is derived in terms of the false alarm and miss detection rates. Our analysis indicates that boosting the forwarding ability of the relay (e.g., increasing its maximum transmit power) also increases its capacity to perform the covert communication in terms of achieving a higher effective covert rate subject to some specific requirement on the source’s detection performance.

I. INTRODUCTION

Security and privacy are critical in existing and future wireless networks since a large amount of confidential information (e.g., location, credit card information, physiological information for e-health) is transferred over the open wireless medium [1]. Against this background, conventional cryptography and information-theoretic secrecy technologies have been developed to offer progressively higher levels of security by protecting the content of the message against eavesdropping [2]–[4]. However, these technologies cannot mitigate the threat to a user’s security and privacy from discovering the presence of the user or communication. Meanwhile, this strong security (i.e., hiding the wireless transmission) is desired in many application scenarios of wireless communications, such as covert military operations and avoiding to be tracked in vehicular ad hoc networks. As such, the hiding of communication termed covert communication or low probability of detection communication, which aims to shield the very existence of wireless transmissions against a warden to achieve security, has recently drawn significant research interests and is emerging as a cutting-edge technique in the context of wireless communication security [5], [6].

The fundamental limit of covert communication has been studied under various channel conditions, such as additive white Gaussian noise (AWGN) channel [7], binary symmetric channel [8], and discrete memoryless channel [9]. It is proved that $O(\sqrt{n})$ bits of information can be transmitted to a legitimate receiver reliably and covertly in $n$ channel uses as $n \to \infty$. This means that the associated covert rate is zero due to $\lim_{n \to \infty} O(\sqrt{n})/n \to 0$. Following these pioneering works on covert communication, a positive rate has been proved to be achievable when the warden has uncertainty on his receiver noise power [10], [11], the warden does not know when the covert communication happens [12], or an uniformed jammer comes in to help [13]. Most recently, [14] has examined the impact of noise uncertainty on covert communication by considering two practical uncertainty models in order to debunk the myth of this impact. In addition, the effect of the imperfect channel state information and finite blocklength (i.e., finite $n$) on covert communication has been investigated in [15] and [16], respectively.

In this work, for the first time we consider covert communication in the context of one-way relay networks. This is motivated by the scenario where the relay (R) tries to transmit its own information to the destination (D) on top of forwarding the information from the source (S) to D, while S forbids R’s transmission of its own message since the resource (e.g., power, spectrum) allocated to R by S is dedicated to be solely used on aiding the transmission from S to D. As such, R’s transmission of its own message to D should be kept covert from S, where S acts as the warden trying to detect this covert communication. We first identify the necessary condition that the covert transmission from R to D can possibly occur without being detected by S and then derive the detection limit at S in terms of the false alarm and miss detection rates under this condition. In addition, we analyze the achievable effective covert rate subject to a requirement on the detection performance at S. Our examination demonstrates a tradeoff between R’s ability to aid the transmission from S to D and R’s capability to conduct the covert communication.

II. SYSTEM MODEL

A. Considered Scenario and Adopted Assumptions

As shown in Fig. 1, in this work we consider a one-way relay network, in which S transmits information to D with the aid of R, since a direct link from S to D is not available. As mentioned in the introduction, S allocates some resource to R in order to seek its help to relay the message to D. However, in some scenarios R may intend to use this resource to transmit its own message to D as well, which is forbidden by S and thus should be kept covert from S. As such, in the considered system model S is also the warden to detect whether R transmits its own information to D when it is aiding the transmission from S to D.
We assume the wireless channels within our system model are subject to independent quasi-static Rayleigh fading with equal block length and the channel coefficients are independent and identically distributed (i.i.d.) circularly symmetric complex Gaussian random variables with zero-mean and unit-variance. We also assume that each node is equipped with a single antenna. The channel from S to R is denoted by $h_{sr}$ and the channel from R to D is denoted by $h_{rd}$. We assume R knows both $h_{sr}$ and $h_{rd}$ perfectly, while S only knows $h_{sr}$ and D only knows $h_{rd}$. Considering channel reciprocity, we assume the channel from R to S (denoted by $h_{rs}$) is the same as $h_{sr}$ and thus perfectly known by S. We further assume that R operates in the half-duplex mode and thus the transmission from S to D occurs in two phases: phase 1 (S transmits to R) and phase 2 (R transmits to D).

### B. Transmission from Source to Relay (Phase 1)

In phase 1, the received signal at R is given by

$$y_r[i] = \sqrt{P_s} h_{sr} x_s[i] + n_r[i],$$  

where $P_s$ is the transmit power of source, $x_s$ is the transmitted signal by S satisfying $E[x_s[i]|x_s^i[i]] = 1$, $i = 1, 2, \ldots, n$ is the index of each channel use (n is the total number of channel uses in each phase), and $n_r[i]$ is the AWGN at relay with $\sigma_r^2$ as its variance, i.e., $n_r[i] \sim \mathcal{CN}(0, \sigma_r^2)$.

In this work, we consider that R operates in the amplify-and-forward mode and thus R will forward a linearly amplified version of the received signal to D in phase 2. As such, the forwarded (transmitted) signal by R is given by

$$x_r[i] = G y_r[i] = G(\sqrt{P_s} h_{sr} x_s[i] + n_r[i]),$$  

where the received signal is scaled by a scalar $G$. In order to guarantee the power constraint at R, the value of G must be chosen such that $E[x_r[i]|x_s^i[i]] = 1$, which leads to $G = 1/\sqrt{P_s h_{sr}^2 + \sigma_r^2}$.

In this work, we consider a fixed-rate transmission from S to D, in which this rate is denoted by $R_{sd}$. We also consider a maximum power constraint at R, i.e., $P_r \leq P_{r}^{\text{max}}$. As such, although R knows both $h_{sr}$ and $h_{rd}$ perfectly, transmission outage from S to D still incurs when $C_{sd}^* < R_{sd}$, where $C_{sd}^*$ is the channel capacity from S to D for $P_r = P_{r}^{\text{max}}$.

Then, the transmission outage probability is given by $\delta = P[C_{sd}^* < R_{sd}]$. In practice, the pair of $R_{sd}$ and $\delta$ determines the specific aid (i.e., the value of $P_{r}^{\text{max}}$) required by S from R, which relates to the amount of resource allocated to R by S as a payback. In this work, we assume both $R_{sd}$ and $\delta$ are predetermined, which leads to a predetermined $P_{r}^{\text{max}}$.

### III. Transmission Strategies at Relay

In this section, we detail the transmission strategies of R when it does and does not transmit its own information to D. We also determine the condition that R can transmit its own message to D without being detected by S with probability one, in which the probability to guarantee this condition is also derived.

#### A. Relay’s Transmission without Covert Message

In the case when relay does not transmit its own message (i.e., covert message) to Bob, it only transmit $x_r$ to D. Accordingly, the received signal at D is given by

$$y_d[i] = \sqrt{P_r} h_{rd} x_r[i] + n_d[i]$$  

where $P_r$ is the transmit power of $x_r$. The probability of transmission outage occurs (i.e., $y_d[i] < \mu \sigma_d^2$), where $\mu \sigma_d^2$ is the variance of the forwarded message, is given by

$$\gamma_d = \frac{P_r |h_{sr}|^2 P_r^* |h_{rd}|^2 G^2}{P_r |h_{rd}|^2 G^2 \sigma_r^2 + \sigma_d^2} = \frac{\gamma_1 \gamma_2}{\gamma_1 + \gamma_2 + 1},$$

where $\gamma_1 \triangleq (P_r |h_{sr}|^2) / \sigma_r^2$ and $\gamma_2 \triangleq (P_r^* |h_{rd}|^2) / \sigma_d^2$.

For the fixed-rate transmission, R does not have to adopt the maximum transmit power for each channel realization in order to guarantee a specific transmission outage probability. When the transmission outage occurs (i.e., $C_{sd}^* < R_{sd}$ occurs), R will not transmit (i.e., $P_r = 0$). When $C_{sd}^* \geq R_{sd}$, R only has to ensure $C_{sd} = C_{sd}^*$, where $C_{sd} = 1/2 \log_2 (1 + \gamma_d)$. Then, following (4) the transmit power of R when $C_{sd}^* \geq R_{sd}$ is given by

$$P_r^0 = \mu \sigma_d^2 / |h_{rd}|^2,$$

where

$$\mu \triangleq \frac{(P_s |h_{sr}|^2 + \sigma_s^2)(2^{|R_{sd}} - 1)}{[P_s |h_{sr}|^2 - \sigma_s^2(2^{|R_{sd}} - 1)].}$$

Noting $\gamma_1 < 1$, we have $1/2 \log_2 (1 + \gamma_1) > R_{sd}$ when $C_{sd} = C_{sd}^*$. As such, $P_r^0$ given in (5) is nonnegative. Following (4), we note that $C_{sd}^* \geq R_{sd}$ requires $|h_{rd}|^2 \geq \mu \sigma_r^2 / P_{r}^{\text{max}}$. As such, the transmit power of R without covert message is given by

$$P_r^0 = \begin{cases} \frac{\mu \sigma_r^2}{|h_{rd}|^2}, & |h_{rd}|^2 \geq \frac{\mu \sigma_r^2}{P_{r}^{\text{max}}}, \\ 0, & |h_{rd}|^2 < \frac{\mu \sigma_r^2}{P_{r}^{\text{max}}}. \end{cases}$$

#### B. Relay’s Transmission with Covert Message

In the case when R transmits the covert message to D on top of forwarding $x_o$, the received signal at D is given by

$$y_d[i] = \sqrt{P_r} G h_{rd} \sqrt{P_s} h_{sr} x_o[i] + \sqrt{P_r^\Delta} h_{rd} x_c[i] + n_d[i].$$

where $P_r^\Delta$ is the transmit power of R to forward $x_c$. The transmit power of R to forward $x_o$ under this case and $P_r^\Delta$ is the power of R used to transmit the covert message $x_c$, satisfying $E[x_c[i]|x_o^i[i]] = 1$. In this work, we assume $P_r^\Delta$ is fixed for all channel realizations. In general, the transmit power of a covert message is significantly less than that of the forwarded message, i.e., $P_r^\Delta << P_r$.
As such, here we assume D always first decodes \( x_s \) with \( x_c \) as interference. Following (7), the signal-to-interference-plus-noise ratio (SINR) for \( x_s \) is given by

\[
\gamma_d = \frac{P_s|h_{sr}|^2P_r^2|h_{rd}|^2G^2}{|P_r|h_{rd}^2G^2\sigma_s^2 + P_\Delta|h_{rd}|^2 + \sigma_d^2} = \frac{\gamma_3\gamma_3}{\gamma_3 + (\gamma_1 + 1)(\gamma_3P/\gamma_1^2 + 1)},
\]

where \( \gamma_3 \equiv (P_r^2|h_{rd}|^2)/\sigma_d^2 \). Then, when \( C_{sd} = R_{sd} \) we have

\[
P_r^1 = \mu P_\Delta + \frac{\mu \sigma_d^2}{|h_{rd}|^2},
\]

which requires \( C_{sd}^* \geq R_{sd} \) that leads to \( |h_{rd}|^2 \geq \mu \sigma_d^2/P_{\Delta}^{\max} \). Considering the maximum power constraint at R (i.e., \( P_r^1 + P_\Delta \leq P_{\Delta}^{\max} \)), R has to give up the transmission of the covert message. Therefore, R never transmits the covert message when it cannot support the transmission from S to D. In this case, we assume D always first decodes \( x_s \) with probability one (R could possibly transmit covert message when it cannot support the transmission from S to D). In this work, we first present the detection strategy adopted by S (i.e., Source) and then analyze the associated detection performance in terms of the false alarm and miss detection rates. Finally, the optimal detection threshold at S that minimizes the total error rate is examined.

### A. Binary Hypothesis Test

In phase 2 when R transmits to D, S is to detect whether R transmits the covert message \( x_c \) on top of forwarding S’s message \( x_s \) to D. In this section, we first focus on the case when C is guaranteed since R never transmits covert message when C is not met, as discussed in Section III-B. R does not transmit \( x_c \) in the null hypothesis \( H_0 \) while it does in the alternative hypothesis \( H_1 \). Then, the received signal at S in phase 2 is given by

\[
y_s[i] = \begin{cases} 
\sqrt{P_0} h_{rs} x_r[i] + n_s[i], & H_0, \\
\sqrt{P_1} h_{rs} x_r[i] + \sqrt{P_\Delta} h_{rs} x_c[i] + n_s[i], & H_1,
\end{cases}
\]

where \( n_s[i] \) is the AWGN at S with \( \sigma_s^2 \) as its variance. We note that neither \( P_0 \) nor \( P_1 \) is known at S since it does not know \( h_{rd} \), while the statistics of \( P_0 \) and \( P_1 \) are known since the distribution of \( h_{rd} \) is publicly known. The ultimate goal of S is to detect whether \( y_s \) comes from \( H_0 \) or \( H_1 \) in one fading block. As proved in [15], the optimal decision rule that minimizes the total error rate at S is given by

\[
T \equiv \begin{cases} 
D_1, & \tau \leq \rho_1, \\
D_0, & \tau > \rho_1,
\end{cases}
\]

where \( T = 1/n \sum_{i=1}^n |y_s[i]|^2 \), \( \tau \) is a predetermined threshold, \( D_1 \) and \( D_0 \) are the binary decisions that infer whether R transmits covert message or not, respectively. In this work, we consider infinite blocklength, i.e., \( n \to \infty \). As such, we have

\[
T = \begin{cases} 
P_0 h_{rs}^2 + \sigma_s^2, & H_0, \\
P_1 h_{rs}^2 + P_\Delta h_{rs}^2 + \sigma_s^2, & H_1,
\end{cases}
\]

### B. False Alarm and Miss Detection Rates

In this subsection, we derive S’s false alarm rate, i.e., \( \mathcal{P}(D_1|H_0) \), and miss detection rate, i.e., \( \mathcal{P}(D_0|H_1) \).

**Theorem 1:** When the condition C is guaranteed, the false alarm and miss detection rates at S are derived as

\[
\mathcal{P}_{FA} = \begin{cases} 
1, & \tau < \sigma_s^2, \\
1 - \kappa_1(\tau), & \sigma_s^2 \leq \tau \leq \rho_1, \\
0, & \tau > \rho_1,
\end{cases}
\]

\[
\mathcal{P}_{MD} = \begin{cases} 
0, & \tau < \rho_2, \\
\kappa_2(\tau), & \rho_2 \leq \tau \leq \rho_3, \\
1, & \tau > \rho_3,
\end{cases}
\]

where

\[
\rho_1 = [P_{\Delta}^{\max} - (\mu + 1)P_\Delta]|h_{rs}|^2 + \sigma_s^2,
\]

\[
\rho_2 = (\mu + 1)P_\Delta |h_{rs}|^2 + \sigma_s^2,
\]

\[
\rho_3 = P_{\Delta}^{\max} |h_{rs}|^2 + \sigma_s^2,
\]

\[
\kappa_1(\tau) \equiv \exp \left\{ \mu \sigma_d^2 \left[ \frac{1}{P_{\Delta}^{\max} - (\mu + 1)P_\Delta} - \frac{|h_{rs}|^2}{\tau - \sigma_s^2} \right] \right\},
\]

\[
\kappa_2(\tau) \equiv \exp \left\{ \mu \sigma_d^2 \left[ \frac{1}{P_{\Delta}^{\max} - (\mu + 1)P_\Delta} - \frac{|h_{rs}|^2}{\tau - \rho_2} \right] \right\}.
\]
Proof: Considering the maximum power constraint at R under $\mathcal{H}_0$ (i.e., $P_r^0 \leq P_r^{\text{max}}$) and following (6), (13), and (14), the false alarm rate under the condition $C$ is given by

$$P_{FA} = \mathbb{P} \left[ \frac{\mu \sigma_d^2}{|h_r|^2} |h_r|^2 + \sigma_s^2 > \tau | C \right]$$

$$= \begin{cases} 0, & \text{if } \frac{\mu \sigma_d^2}{|h_r|^2} |h_r|^2 + \sigma_s^2 \leq |h_r|^2 - \frac{\mu^2 \sigma_d^2}{\sigma_s^2}, \\ \frac{\mu \sigma_d^2}{|h_r|^2} & \text{if } |h_r|^2 - \frac{\mu^2 \sigma_d^2}{\sigma_s^2} \leq |h_r|^2 - \frac{\mu^2 \sigma_d^2}{\sigma_s^2}, \\ 1, & \text{if } |h_r|^2 - \frac{\mu^2 \sigma_d^2}{\sigma_s^2} \leq |h_r|^2 - \frac{\mu^2 \sigma_d^2}{\sigma_s^2}. \end{cases}$$

Then, substituting $F_{|h_r|^2}(x) = 1 - e^{-x}$ into the above equation (equation (15), the desired result is achieved.

We first clarify that we have $\rho_2 < \rho_3$ due to $P_r^{\text{max}} > (\mu + 1)P_{\Delta}$ as discussed after (11). Then, considering the maximum power constraint at R under $\mathcal{H}_1$ (i.e., $P_r^1 + P_{\Delta} \leq P_r^{\text{max}}$) and following (10), (13), and (14), the miss detection rate under the condition $C$ is given by

$$P_{MD} = \mathbb{P} \left[ \left( \frac{\mu \sigma_d^2}{|h_r|^2} + (1 + \mu)P_{\Delta} \right) |h_r|^2 + \sigma_s^2 < \tau | C \right]$$

$$= \begin{cases} 0, & \text{if } \frac{\mu \sigma_d^2}{|h_r|^2} |h_r|^2 + \sigma_s^2 \leq |h_r|^2 - \frac{\mu^2 \sigma_d^2}{\sigma_s^2}, \\ \frac{\mu \sigma_d^2}{|h_r|^2} & \text{if } |h_r|^2 - \frac{\mu^2 \sigma_d^2}{\sigma_s^2} \leq |h_r|^2 - \frac{\mu^2 \sigma_d^2}{\sigma_s^2}, \\ 1, & \text{if } \frac{\mu \sigma_d^2}{|h_r|^2} \leq |h_r|^2 - \frac{\mu^2 \sigma_d^2}{\sigma_s^2}. \end{cases}$$

Then, substituting $F_{|h_r|^2}(x) = 1 - e^{-x}$ into the above equation we achieve the desired result in (16).

We note that the false alarm and miss detection rates given in Theorem 1 are functions of the threshold $\tau$ and we examine how S sets the value of it in order to minimize its total error rate. Specifically, the total error rate of the detection at S is defined as

$$\xi \triangleq P_{FA} + P_{MD},$$

which is used to measure the detection performance at S.

C. Optimization of the Detection Threshold at Source

In this subsection, we examine how S optimally sets the value of $\tau$ aiming to minimize $\xi$. To this end, we first determine a preliminary constraint on $P_{\Delta}$ and the bounds on the optimal $\tau$ in the following theorem.

Theorem 2: R’s transmit power of the covert message $P_{\Delta}$ should satisfy

$$P_{\Delta} \leq P_{\Delta}^U \triangleq P_r^{\text{max}}/2(\mu + 1)$$

in order to guarantee $\xi > 0$ and when (20) is guaranteed the optimal $\tau$ ($\tau^*$) at S that minimizes $\xi$ should satisfy $\rho_2 \leq \tau^* \leq \rho_1$.

Proof: When $\rho_1 < \rho_2$ that requires $P_{\Delta} > P_r^{\text{max}}/2(\mu + 1)$ as per Theorem 1, following (15) and (16), we have

$$\xi = \begin{cases} 1, & \tau \leq \sigma_s^2, \\ 1 - \kappa_1(\tau), & \sigma_s^2 < \tau < \rho_1, \\ \kappa_2(\tau), & \rho_1 \leq \tau \leq \rho_2, \\ 1, & \tau \geq \rho_3. \end{cases}$$

This indicates that S can simply set $\tau \in [\rho_1, \rho_2]$ to ensure $\xi = 0$ when $P_{\Delta} > P_r^{\text{max}}/2(\mu + 1)$, i.e., S can detect the covert transmission without being detected with probability one under this condition. As such, $P_{\Delta}$ should satisfy (20) in order to guarantee $\xi > 0$.

When $P_{\Delta} \leq P_r^{\text{max}}/2(\mu + 1)$, i.e., $\rho_2 < \rho_1$, following (15) and (16), we have

$$\xi = \begin{cases} 1, & \tau \leq \sigma_s^2, \\ 1 - \kappa_1(\tau), & \sigma_s^2 < \tau < \rho_2, \\ \kappa_2(\tau), & \rho_2 \leq \tau < \rho_1, \\ 1, & \tau \geq \rho_3. \end{cases}$$

This demonstrates that $\xi$ is a decreasing function of $\tau$ and thus we have $\tau^* = \rho_2$ when $\sigma_s^2 < \tau < \rho_2$.

For $\rho_1 \leq \tau < \rho_3$, we derive the first derivative of $\xi$ with respect to $\tau$ as

$$\frac{\partial \xi}{\partial \tau} = -\frac{\mu \sigma_d^2 |h_r|^2}{(\tau - \sigma_s^2)^2} \kappa_1 < 0.$$}

This proves that $\xi$ is an increasing function of $\tau$ and we would have $\tau^* = \rho_1$ when $\rho_1 \leq \tau < \rho_3$.

Noting that $\xi$ is a continuous function of $\tau$, we can conclude that $\tau^*$ should satisfy $\rho_2 \leq \tau^* \leq \rho_1$, no mater what is the value of $\xi$ for $\rho_2 < \tau < \rho_1$.

The lower and upper bounds on $\tau^*$ given in Theorem 2 significantly facilitate the numerical search for $\tau^*$ at S. Then, following Theorem 2 and (22), $\tau^*$ can be obtained through

$$\tau^* = \arg \min_{\rho_2 \leq \tau \leq \rho_1} [1 - \kappa_1(\tau) + \kappa_2(\tau)].$$

Substituting $\tau^*$ into (22), we obtain the minimum value of $\xi$ as

$$\xi^* = 1 - \kappa_1(\tau^*) + \kappa_2(\tau^*).$$

V. OPTIMIZATION OF EFFECTIVE COVERT RATE

In this section, we examine the effective covert rate achieved in the considered system subject to a covert requirement.

A. Effective Covert Rate

As discussed in Section III-B, R can only transmit the covert message without being detected by S with probability one under the condition $C$. As such, a positive covert rate is only achievable under this condition. When the covert message is transmitted by R, D first decodes $x_0$ and subtracts the corresponding component from its received signal $y_d$ given in (7). Then, the effective received signal used to decode the covert message $x_c$ is given by

$$\tilde{y}_d[i] = \sqrt{P_{\Delta} |h_c|^2 x_c[i]} + \sqrt{P_i |h_c|^2 G n_c[i] + n_d[i]}.$$
As such, following (10) the SINR for $x_c$ is
\[ \gamma_c = \frac{P_\Delta (\eta |h_{sr}|^2 + 1) |h_{rd}|^2}{\mu P_\Delta |h_{rd}|^2 (\eta |h_{sr}|^2 + \mu + 1) \sigma_d^2}, \]
where $\eta = \frac{P_c}{\sigma_d^2}$. Then, the covert rate achieved by $R$ is $R_c = \log_2 (1 + \gamma_c)$. We next derive the effective covert rate, i.e., averaged $R_c$ over all realizations of $|h_{rd}|^2$, in the following theorem.

**Theorem 3:** The achievable effective covert rate $\overline{R}_c$ by $R$ is derived as a function of $P_\Delta$ given by
\[
\overline{R}_c = \frac{1}{2 \ln 2} \exp \left\{ -\frac{\mu \sigma_d^2}{P_{\text{max}} - (\mu + 1) P_\Delta} \right\} \times \ln \left( \beta_1 - \frac{\beta_2}{\alpha_2} \right) + e^{\beta_1 \alpha_2} e^{\beta_2 \alpha_1} \left( \frac{\beta_1}{\alpha_2} - \frac{\beta_2}{\alpha_1} \right) \right\},
\]
(28)
where
\[
\begin{align*}
\beta_1 &\triangleq [\eta |h_{sr}|^2 + (\mu + 1)][P_{\text{max}} - (\mu + 1) P_\Delta], \\
\beta_2 &\triangleq \left\{ \eta |h_{sr}|^2 + (\mu + 1) \right\} P_{\text{max}} - (\mu + 1) P_\Delta, \\
\alpha_1 &\triangleq \mu P_\Delta P_{\text{max}} - (\mu + 1) P_\Delta, \\
\alpha_2 &\triangleq \mu P_\Delta P_{\text{max}} - (\mu + 1) P_\Delta,
\end{align*}
\]
and the exponential integral function $Ei(\cdot)$ is given by
\[ Ei(x) = -\int_{-x}^{\infty} e^{-t} t dt, \quad [x < 0]. \]
(29)

**Proof:** A positive covert rate is only achievable under the condition $C$ and thus $\overline{R}_c$ is derived by
\[
\overline{R}_c = \int_{0}^{\infty} \frac{R_c f(|h_{rd}|^2) d|h_{rd}|^2}{P_{\text{max}} - (\mu + 1) P_\Delta} \times \ln \left( \frac{\beta_1 + \alpha_1 x}{\beta_2 + \alpha_2 x} \right) e^{-x} dx,
\]
(30)
where $\frac{\alpha}{\beta}$ is achieved by exchanging variables (i.e., setting $x = |h_{rd}|^2 \mu \sigma_d^2/[P_{\text{max}} - (\mu + 1) P_\Delta]$). Then we solve the integral in (30) with the aid of [17, Eq. (4.337.1)] and achieve the result given in (28).

Based on Theorem 3, we note that $\overline{R}_c$ is not an increasing function of $P_\Delta$, since as $P_\Delta$ increases $R_c$ increases but $\overline{R}_c$ (i.e., the probability that the condition $C$ is guaranteed) decreases.

**B. Maximization of $\overline{R}_c$ with the Covert Constraint**

A covert transmission normally requires $\xi \geq 1 - \epsilon$, where $\epsilon \in [0, 1]$ is predetermined to specify the covert constraint. In practice, it is impossible to know $\xi$ at $R$ since the threshold $\tau$ adopted by $S$ is unknown. In this work, we consider the worst-case scenario where $\tau$ is optimized at $S$ to minimize $\xi$. As such, the covert constraint can be rewritten as $\xi^* \geq 1 - \epsilon$. Then, following Theorem 2 the optimal value of $P_\Delta$ that maximizes $\overline{R}_c$ subject to this constraint can be obtained through
\[
P_\Delta^* = \arg \max_{0 \leq P_\Delta \leq P_2} \overline{R}_c
\]
(31)
s.t. $\xi^* \geq 1 - \epsilon$.

We note that this is a two-dimensional optimization problem that can be solved by efficient numerical searches. Specifically, for each given $P_\Delta$, $\xi^*$ should be obtained based on (25) where $\tau^*$ is also numerically searched. We note that the numerical search of $P_\Delta^*$ and $\tau^*$ is efficient since their lower and upper bounds are explicitly given.

**VI. NUMERICAL RESULTS**

In this section, we first examine the detection performance at $S$ (i.e., Source) under the condition $C$. Then, the impact of some system parameters on the achievable effective covert rate subject to a specific covert constraint is investigated.

In Fig. 2, we plot the false alarm rate $P_{FA}$, miss detection rate $P_{MD}$, and total error rate $\xi$ versus the threshold $\tau$, in which the adopted system parameters guarantee condition $C$ and $P_\Delta \leq P_2$. As expected, we observe that $\xi > 0$ due to the guaranteed condition $C$ and $P_\Delta \leq P_2$, which means that covert transmission is possible (not being detected with probability one) under this condition. We observe that the minimum value of $\xi$ is achieved when $\rho_2 \leq \tau \leq \rho_1$, which verifies the correctness of our Theorem 2.

In Fig. 3, we plot the minimum total error rate $\xi^*$ versus the covert transmit power $P_\Delta$, which is achieved through searching the optimal threshold $\tau^*$ as per (25). In this figure, we first observe that $\xi^*$ monotonically decreases as $P_\Delta$ increases, which demonstrates that the covert transmission becomes easier to be detected when more power is used. As such, the covert constraint $\xi^* \geq 1 - \epsilon$ determines a maximum possible value of $P_\Delta$, which is significantly less than $P_2^*$ since we have $\xi = 0$ when $P_\Delta = P_2^*$ but we normally require $\xi > 0.5$ in practice [16]. This can facilitate the search of the optimal value of $P_\Delta$ as per (31) by significantly reducing the feasible region of $P_\Delta$. We also observe that $\xi^*$ increases as $P_{\text{max}}^r$ increases. This shows that covert transmission becomes easier (i.e., the detection probability of covert transmission at $S - \xi^*$ decreases) as the desired performance of the normal transmission increases (i.e., the transmission outage probability decreases as $P_{\text{max}}^r$ increases for a fixed $R_{sd}$).

In Fig. 4, we plot the effective covert rate $\overline{R}_c$ versus $P_\Delta$, in which we also show the maximum possible value of $P_\Delta$ determined by the covert constraint $\xi^* \geq 1 - \epsilon$ (denoted by $P_\Delta^*$ and marked by red circle in this figure). We first observe that $\overline{R}_c$ is not be a monotonically increasing function of $P_\Delta$ without the constraint $\xi^* \geq 1 - \epsilon$. This is due to the fact that as $P_\Delta$ increases the probability to guarantee the condition...

Authorized licensed use limited to: UNSW Library. Downloaded on August 27,2021 at 12:38:51 UTC from IEEE Xplore. Restrictions apply.
\( C \) (i.e., \( P_c \)) decreases while the covert rate \( R_c \) increases. In addition, we observe that \( R_c \) without \( \xi^* \geq 1 - \epsilon \) increases as \( \left| h_{sr} \right|^2 \) increases. This is due to the fact that as \( \left| h_{sr} \right|^2 \) increases \( \mu \) as given in (5) decreases, which leads to that \( P_c \) increases, i.e., the probability that R can conduct covert transmission increases (although the covert rate \( R_c \) does not change). Finally, we observe that \( P_{\Delta}^c \) increases as well when \( \left| h_{sr} \right|^2 \) increases. As such, following the last two observations we can conclude that the achievable effective covert rate with the constraint \( \xi^* \geq 1 - \epsilon \) increases as \( \left| h_{sr} \right|^2 \) increases. Intuitively, this is due to that as \( \left| h_{sr} \right|^2 \) increases R has a higher chance to support the transmission of \( x_b \) and perform covert transmission, resulting in that from S’s point of view the possible transmit power range of R used to transmit \( x_b \) increases (i.e., transmit power uncertainty increases).

### VII. Conclusion

This work examined covert communication in one-way relay networks over Rayleigh fading channels. Specifically, we analyzed S’s detection limit of the covert transmission from R to D in terms of the total error rate. We also determined the maximum achievable effective covert rate subject to \( \xi^* \geq 1 - \epsilon \). Our examination shows that covert communication in the considered relay networks is feasible and a tradeoff between the achievable effective covert rate and R’s performance of aiding the transmission from S to D exists.

### Acknowledgments

This work was supported in part by the National Natural Science Foundation of China (Nos. 61472190 and 61501238), the Australian Research Council’s Discovery Projects (D-P150103905), and the open research fund of National Mobile Communications Research Laboratory, Southeast University, China (Nos. 2013D02 and 2017D04).

### References

[1] M. Bloch and J. Barros, Physical-Layer Security: From Information Theory to Security Engineering, 1st ed. Cambridge, UK: Cambridge University Press, 2011.

[2] S. Yan, X. Zhou, N. Yang, B. He, and T. D. Abhayapala, “Artificial-noise-aided secure transmission in wiretap channels with transmitter-side correlation,” IEEE Trans. Wireless Commun., vol. 15, no. 12, pp. 8286–8297, Dec. 2016.

[3] J. Hu, F. Shu, and J. Li, “Robust synthesis method for secure directional modulation with imperfect direction angle,” IEEE Commun. Lett., vol. 20, no. 6, pp. 1084–1087, Jun. 2016.

[4] J. Hu, S. Yan, F. Shu, J. Wang, J. Li, and Y. Zhang, “Artificial-noise-aided secure transmission with directional modulation based on random frequency diverse arrays,” IEEE Access, vol. 5, pp. 1658–1667, Jan. 2017.

[5] B. A. Bash, D. Goeckel, D. Towsley, and S. Guha, “Hiding information in noise: fundamental limits of covert wireless communication,” IEEE Commun. Mag., vol. 53, no. 12, pp. 26–31, Dec. 2015.

[6] M. Bloch, “Covert communication over noisy channels: A resolvability perspective,” IEEE Trans. Inf. Theory, vol. 62, no. 5, pp. 2334–2354, May 2016.

[7] B. A. Bash, D. Goeckel, and D. Towsley, “Limits of reliable communication with low probability of detection on AWGN channels,” IEEE J. Sel. Areas Commun., vol. 31, no. 9, pp. 1921–1930, Sep. 2013.

[8] P. H. Che, M. Bakhshi, and S. Jaggi, “Reliable deniable communication: Hiding messages in noise,” in Proc. IEEE Int. Symp. Inf. Theory, Jul. 2013, pp. 2945–2949.

[9] L. Wang, G. W. Wornell, and L. Zheng, “Fundamental limits of communication with low probability of detection,” IEEE J. Sel. Areas Commun., vol. 20, no. 2, pp. 349–350, Jun. 2005.

[10] S. Lee, R. J. Baxley, M. A. Weimauer, and B. T. Walkenhorst, “Achieving undetectable communication,” IEEE J. Sel. Topics Signal Process., vol. 9, no. 7, pp. 1195–1205, Oct. 2015.

[11] D. Goeckel, B. A. Bash, S. Guha, and D. Towsley, “Covert communications when the warden does not know the background noise power,” IEEE Commun. Lett., vol. 20, no. 2, pp. 236–239, Feb. 2016.

[12] B. A. Bash, D. Goeckel, and D. Towsley, “LPD communication when the warden does not know when,” in Proc. IEEE Int. Symp. Inf. Theory, Jul. 2014, pp. 606–610.

[13] T. V. Sobers, B. A. Bash, D. Goeckel, S. Guha, and D. Towsley, “Covert communication with the help of an uninformed jammer achieves positive rate,” in Proc. Asilomar Conf. on Signals, Syst., and Comput., Nov. 2015, pp. 625–629.

[14] B. He, S. Yan, X. Zhou, and V. K. N. Lau, “On covert communication with noise uncertainty,” IEEE Commun. Lett., vol. 21, no. 4, pp. 941–944, Apr. 2017.

[15] K. Shahzad, X. Zhou, and S. Yan, “Covert communication in fading channels under channel uncertainty,” in Proc. IEEE VTC Spring, Jun. 2017, pp. 1–5, arXiv:1703.02169.

[16] S. Yan, B. He, Y. Cong, and X. Zhou, “Covert communication with finite blocklength in AWGN channels,” in Proc. IEEE ICC, May 2017, pp. 1–6, arXiv:1701.08891.

[17] I. S. Gradshteyn and I. M. Ryzhik, Table of Integrals, Series, and Products, 7th ed. San Diego, CA, USA: Academic Press, 2007.