Solution to Trouble of $D_{sJ}$ Particles

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Abstract. Recent discovery of $D_{sJ}$ particles, which are considered to be a great trouble by experimentalists as well as theorists, had already been resolved by our potential model proposed some time ago by two of us (T.M. and T.M.), in which the Hamiltonian and wave functions are expanded in $1/(\text{heavy quark mass})$ respecting heavy quark symmetry.

Using our model, we explain how narrow states like $D_{sJ}$ can be realized, predict properties of $0^+$ and $1^+$ states of $B$ and $B_s$ heavy mesons, and interpret how global $SU(3)$ symmetry seems to be recovered for these $0^+$ and $1^+$ heavy mesons.

INTRODUCTION

Recent discovery of narrow meson states $D_{sJ}(2317)$ by BaBar [1], $D_{sJ}(2460)$ by CLEO [2] and the following confirmation of both states by Belle [3] has driven many theorists to explain these states since the former study of these states using a semi-relativistic potential model [4, 5] seems to fail to reproduce these mass values. This triggered a series of study on spectroscopy of heavy mesons again. To understand these states, an interesting explanation is proposed by Bardeen, Eichten, Hill and others [6, 7] who used an effective Lagrangian with heavy quark symmetry combined with chiral symmetry of light quarks but they cannot reproduce absolute mass values of these particles.

This problem of $D_{sJ}$ has been considered by many experimentalists as well as theorists to be a great trouble until recently. It turns out, however, that actually we have already predicted these masses within one percent accuracy at the first order of perturbation in $1/m_Q$ with $m_Q$ heavy quark mass some time ago [8] by using the same potential model as in [4, 5]. Since the data adopted at the time of publication [8] is obsolete, we are now refining calculations up to the second order [9]. The main difference between our treatment of the potential model and others is that we have taken into account negative energy states of a heavy quark in a bound state while others do not. Namely mass is

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expressed as an eigenvalue of a Hamiltonian of the heavy-light system, in which a bound state consists of a heavy quark and a light anti-quark and negative energy states appear in the intermediate states of a heavy quark in calculating an energy eigenvalue, that is missed in the former study. These negative-energy state contribution is not so small which is the main reason why people of [4, 5] could not get a right answer.

Figure 1 shows how our model generates masses step by step. At first chiral symmetry is intact with a limit of light-quark mass and scalar potential going to zero. At this stage no heavy quark mass correction is taken into account except for the lowest contribution, $m_Q$, to a heavy meson mass, i.e., heavy quark mass is finite even at this moment. Next chiral symmetry is broken by introducing light-quark mass and scalar potential. There is still heavy quark symmetry remained so that some states with partial angular momentum, $L = 0, 1$, between light quark and heavy quark are still degenerate. Finally all degeneracies are resolved by including $1/m_Q$ corrections. The similar scenario is adopted in [6] except for the first stage.

**OUR POTENTIAL MODEL**

$D_{3J}$ Particles

Our formulation using the Cornell potential is to expand Hamiltonian, energy, and wave function in terms of $1/m_Q$ and sets coupled equations order by order. The non-trivial differential equation is obtained in the zeroth order, which gives orthogonal set of eigenfunctions, and quantum mechanical perturbative corrections to energy and wave functions in higher orders are formulated. Consistency equation is given by

$$H \Psi_\ell = E_\ell \Psi_\ell,$$

$$(H_{-1} + H_0 + H_1 + \cdots) (\Psi_{\ell 0} + \Psi_{\ell 1} + \cdots) = (E_{\ell 0} + E_{\ell 1} + \cdots) (\Psi_{\ell 0} + \Psi_{\ell 1} + \cdots), \quad (1)$$

where integers of subscripts and superscripts denote order in $1/m_Q$. $H = H_{FWT} - m_Q$, and the Foldy-Wouthuysen-Tani transformation is operated on a heavy quark and the Hamiltonian.[8]

In Tables 1 and 2, $J^P$ stands for total spin and parity, $M_0$ lowest degenerate mass, $c_1/M_0$ the first order correction, $M_{\text{calc}}$ calculated value of mass, and $M_{\text{obs}}$ observed mass.
In Tables 1 and 2, we give our refined first order calculations both of $D$ and $D_s$ heavy mesons with various quantum numbers. Especially $0^+$ and $1^+$ states should be compared with experiments, i.e., $D_0^0 (2308)$, $D_1^0 (2427)$, $D_{sJ} (2317)$, and $D_{sJ} (2460)$.[1, 2] The calculated masses, $M_{\text{calc}}$, are within one percent of accuracy compared with the observed masses, $M_{\text{obs}}$. In the calculations, we have used values of parameters listed in Table 3.

### 0$^+$ and 1$^+$ States of $B$ and $B_s$

In Tables 4 and 5, we give our refined calculations both of $B$ and $B_s$ heavy mesons with various quantum numbers. For $B$ and $B_s$ mesons, unfortunately there are only a few data. We predict the mass of several excited states which have not yet been observed. Among them, calculated masses of $0^+$ and $1^+$ states for $B_s$ mesons are below $BK / B^* K$ thresholds the same as the case of $D_s$ mesons. Therefore, their decay modes to $B(0^- / 1^-) + K / K^*$ are kinematically forbidden, and the dominant modes are the pionic decay:

$$B_s (0^+) \rightarrow B_s (0^-) + \pi, \quad B_s (1^+) \rightarrow B_s (1^-) + \pi.$$  (2)

The decay widths of these states are expected to be narrow as for those of $D_s$ mesons, since these decay modes violate the isospin invariance.
TABLE 3. Most optimal values of parameters.

| Parameters | $\alpha_s$ | $a$ (GeV$^{-1}$) | $b$ (GeV) | $m_{u,d}$ (GeV) | $m_s$ (GeV) | $m_c$ (GeV) | $m_b$ (GeV) |
|------------|------------|-----------------|------------|----------------|------------|------------|------------|
|            | 0.2620     | 1.937           | 0.07091    | 0.00803        | 0.09270    | 1.040      | 4.513      |

TABLE 4. $B$ meson mass spectra (units are in MeV).

| State ($^{2s+1}L_J$) | $k$ | $J^P$ | $M_0$ | $c_1/M_0$ | $M_{\text{calc}}$ | $M_{\text{obs}}$ |
|-----------------------|-----|-------|-------|-----------|--------------------|------------------|
| $^1S_0$               | -1  | 0$^-$ | 5255  | $0.373 \times 10^{-2}$ | 5275             | 5279             |
| $^3S_1$               | -1  | 1$^-$ | 5042  | $0.994 \times 10^{-2}$ | 5307             | 5325             |
| $^3P_0$               | 1   | 0$^+$ | 5546  | $0.882 \times 10^{-2}$ | 5595             | -                |
| "$^3P_1$"            | 1   | 1$^+$ | 1.453 | $10^{-2}$           | 5627             | -                |
| "$^1P_1$"            | -2  | 1$^+$ | 5596  | $1.235 \times 10^{-2}$ | 5665             | -                |
| $^3P_2$               | -2  | 2$^+$ | 1.413 | $10^{-2}$           | 5675             | -                |
| $^3D_1$               | 2   | 1$^-$ | 5802  | $1.723 \times 10^{-2}$ | 5902             | -                |
| "$^3D_2$"            | 2   | 2$^-$ | 1.870 | $10^{-2}$           | 5911             | -                |

The similar decay modes are predicted for $B(0^+)$ and $B(1^+)$ mesons.

$$B(0^+) \to B(0^-) + \pi, \quad B(1^+) \to B(1^-) + \pi,$$

but with broad decay widths, the same as $D(0^+)$ and $D(1^+)$ since isospin invariance is not broken and there is no threshold.

These higher states of $B$ and $B_s$ mesons might be observed in Tevatron/LHC experiments in near future by analyzing above decay modes. We hope that our framework is confirmed in the forthcoming experiments.

Recovery of Global $SU(3)$ for $0^+$ and $1^+$ Heavy Mesons

When one carefully looks at Tables 1 and 2, one notices that it looks that the global $SU(3)$ flavor symmetry is recovered when $J^P = 0^+$ and $1^+$ for $D(s)$ mesons compared with $0^-$ and $1^-$. The magnitude of $SU(3)$ symmetry breakdown for $D(s)(0^-)$ and $D(s)(1^-)$ mesons is given by (units in MeV),

$$M(D(c\bar{u})) \approx M(D(c\bar{d})) \approx M(D(c\bar{s})) - 100,$$

while the symmetry seems to be recovered for $D(0^+)$ and $D(1^+)$ mesons,

$$M(D(c\bar{u})) \approx M(D(c\bar{d})) \approx M(D(c\bar{s})) - (9 \sim 33).$$

For instance, mass difference among members of a multiplet $(D(0^-), D_s(0^-))$ seems to be larger compared with that among those of $(D(0^+), D_s(0^+))$. These equations are both for observed and calculated masses. What causes this recovery of $SU(3)$ symmetry? Is this just an accidental or is there a deep meaning for this? We study this
phenomenon in details[11] and have found that this is a general phenomenon occurring for heavy mesons, \((Q\bar{q})\), as well as heavy baryons like \((QQq)\).

This is explained by drawing a figure of mass gap between degenerate states, \(M_0(0^-) = M_0(1^-) (L = 0)\) and \(M_0(0^+) = M_0(1^+) (L = 1)\). These degenerate masses are written as \(M_0 = m_Q + C_0(m_q)\), i.e., a function of light quark mass \(m_q\). Hence defining

\[
\Delta M(m_\mu) = M_{D0}(0^+) - M_{D0}(0^-), \quad \Delta M(m_s) = M_{D_{0,0}}(0^+) - M_{D_{0,0}}(0^-),
\]

these mass gaps have the same values for \(D\) and \(B\) mesons. This mass gap function has monotonous decreasing tendency so that \(\Delta M(m_\mu) > \Delta M(m_s)\) for \(m_\mu < m_s\).

\[
\Delta M(m_s) - \Delta M(m_\mu) = -91 \text{ MeV},
\]

which almost cancels mass difference between \(D(c\bar{u})\) \((D(c\bar{d}))\) and \(D(c\bar{s})\) given by Eq.(4). Equation (6) is the origin of recovery of \(SU(3)\) invariance, which does not depend on whether the light-heavy system is \(Q\bar{q}\) or \(QQq\) when \(q = u, d, s\). That is, this is a general phenomenon not peculiar to \(D(0^\pm)\) and \(D(1^\pm)\). Relation between this phenomenon and constituent quark mass will be discussed elsewhere.[11]

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| TABLE 5. \(B_s\) meson mass spectra (units are in MeV). |
|---|---|---|---|---|---|---|
| State \((2s+1L_J)\) | \(k\) | \(J^P\) | \(M_0\) | \(c_1/M_0\) | \(M_{calc}\) | \(M_{obs}\) |
| \(1S_0\) | -1 | 0\(^-\) | 5375 | 0.287 \times 10^{-2} | 5391 | 5369 |
| \(3S_1\) | -1 | 1\(^-\) | 5489 | 0.989 \times 10^{-2} | 5424 | - |
| \(3P_0\) | 1 | 0\(^+\) | 5575 | 0.945 \times 10^{-2} | 5627 | - |
| \("3P_1\) | 1 | 1\(^+\) | 5571 | 1.531 \times 10^{-2} | 5781 | - |
| \("3P_2\) | -2 | 1\(^+\) | 5715 | 1.147 \times 10^{-2} | 5791 | - |
| \(3D_1\) | -2 | 2\(^+\) | 5822 | 1.322 \times 10^{-2} | 5931 | - |
| \("3D_2\) | 2 | 2\(^-\) | 5940 | 2.018 \times 10^{-2} | 5940 | - |