Comparative Analysis of Tunnel Seepage Field under Different Waterproof and Drainage System Using Analytical Methods

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Abstract

Tunnel seepage is an important factor affecting the progress and safety of tunnel construction. In this paper, the mining method tunnel construction in the water-rich weathered granite stratum is taken as the research object. Through the analytical calculation method, the distribution law of tunnel seepage field under different waterproof and drainage types is studied, and the comparative analysis is carried out. According to the analytical solution, the influencing factors of grouting parameters are proposed. The sensitivity of the tunnel seepage field to the variation of grouting parameters is analyzed. A novel waterproof and drainage system, and construction technology suitable for subway tunnels with large buried depth below groundwater level were proposed.

Keywords

Seepage Field, Waterproof, Analytical Solutions, Tunnel, Controlled Drainage

1. Introduction

The selection of waterproof and drainage type of tunnel should consider two aspects: groundwater environment protection and tunnel lining structure safety. Therefore, while focusing on the recovery of the groundwater environment in the later stage of tunnel construction, it is necessary to investigate the distribution law of external water pressure of tunnel lining under different waterproof and drainage types. It is also essential to select the best waterproof and drainage types according to the actual situation, in order to appreciate the environmental protection and the safety of tunnel construction engineering.
The calculation of tunnel seepage field is one of the key tasks in tunnel design and construction. Its core contents include the calculation of tunnel seepage flow and the calculation of external water pressure of lining. Tunnel seepage is an important indicator reflecting the impact of tunnel construction on the groundwater environment, which determines the parameters of tunnel waterproof and drainage structure. It is also an important factor affecting the progress and safety of tunnel construction. Lining external water pressure is an important parameter to determine the lining structure type. Lining under excessive water pressure is liable to produce excessive deformation, leakage, and other adverse phenomena, and even endanger tunnel safety. In order to reduce the negative effect of the groundwater environment in tunnel construction, it is necessary to ensure the long-term safety of the tunnel structure. The key is to coordinate the relationship between tunnel seepage and lining external water pressure. Nowadays, the commonly used calculation methods of tunnel seepage flow include empirical formula method, analytical method, and numerical method. The calculation methods of external water pressure of lining include reduction coefficient method, analytical method, and numerical method. The analytical method and numerical method used in this paper can simultaneously solve tunnel seepage flow and lining external water pressure.

Analytical method is the most commonly used method to calculate the tunnel seepage flow. By properly simplifying and assuming the engineering and hydrogeological conditions, a reasonable tunnel seepage model is established, and the analytical formula of the tunnel seepage flow and the external water pressure of the lining under the definite solution condition are given, which has high calculation accuracy. Nowadays, there are many research results about the analytical calculation method of tunnel seepage field worldwide. Based on the principle of image method, Harr [1] and Goodman et al. [2] obtained the calculation formula of seepage field distribution suitable for the deep buried high head tunnel, which has high precision and has been widely used. Bouvard and Pinto [3] assumed that the seepage direction around the tunnel was mainly radial, and deduced the distribution formula of water pressure around the tunnel with internal pressure. Rat [4] and Lei et al. [5] optimized the existing analytical formulas so that they could be applied to the calculation of the seepage field in shallow buried tunnels. El Tani [6] studied the problem of non-pressure circular hole seepage in a single-layer semi-infinite medium and gave an analytical solution to the problem. Kolymbas et al. [7] used the complex variable function to obtain the analytical solution of the seepage field applicable to the ground surface and tunnel circumference as a variable water head. Park et al. [8] compared the research results of El Tani and Kolymbas, and based on this, obtained the seepage analytical solution when the tunnel circumference was variable head and constant head respectively. Based on the literature [8], Tong Lei et al. 2011 (Chinese Language) solved the seepage problem of a semi-infinite aquifer lining tunnel by using the Fourier solution and flow continuity condition. According to the symmetry of the model, Huang et al. [9] analyzed the semi-infinite plane by
taking half of the structure, transformed the multi-connected domain problem into the simply connected domain problem, and completed the solution of the tunnel seepage field. Based on the steady-state flow control equation and conformal transformation, Zhu Chengwei et al. 2017 (Chinese Language) strictly deduced the analytical solution of the seepage field including grouting circle and lining tunnel applicable to arbitrary buried depth.

According to the above literature review, the existing research results of tunnel seepage field calculations are very rich. The influence of many factors, such as surrounding rock permeability coefficient, support parameters, grouting circle parameters and different waterproof and drainage conditions, are analyzed, but most of the results do not consider the influence of excavation damaged zone (EDZ) on seepage field. The physical and mechanical properties of the surrounding rock in the excavation damaged area are deteriorated, the permeability is enhanced, and the inflow of groundwater into the tunnel is intensified. The influence on the seepage field cannot be ignored, and further research is needed. In addition, there are few studies on the change of the seepage field and its sensitivity to various influencing factors in the process of mining method construction.

2. Methods and Materials

2.1. Tunnel Groundwater Disposal Method and Seepage Field Calculation Method

Nowadays, the disposal methods of groundwater in tunnel engineering can be divided into two types: fully sealed type and drainage type. The low head shallow buried tunnel usually adopts the fully sealed waterproof type, such as the provisions of the Chinese code for design of Metro (GB 50157-2003). The metro tunnel constructed by shield method and mining method adopts the fully sealed waterproof type, and the groundwater is completely sealed outside the lining through the sealing device or full section waterproof structure at the joint of lining segment. The seepage flow of tunnel with a fully sealed waterproof type is \( Q = 0 \), and the external water pressure of lining is calculated according to hydrostatic pressure, as follows

\[
F = \gamma \cdot H
\]

where \( H \) is the height of groundwater level (m); \( \gamma \) is the weight of water (kN/m\(^3\)); \( F \) indicates the water pressure along with the outside of the lining.

For the high-water head deep-buried tunnel, if the fully sealed waterproof type is adopted, it is likely to cause the excessive external water pressure of the lining, resulting in the lining deformation, damage and leakage of water and other adverse consequences. The Chinese code for the design of railway tunnels (TB 10003-2005) stipulates that: the mountain tunnel constructed by the mining method adopts the drainage method to deal with the groundwater. By setting up the permeable cushion, blind pipe or drainage pipe behind the lining, the groundwater in the surrounding rock will be guided to the outlet set at the foot.
of the lining wall, and the external water pressure of the lining will be

$$F = 0$$

According to the analysis above, the full drainage groundwater treatment method will cause great waste of groundwater resources, and will adversely affect the human settlements and ecological environment around the tunnel, which runs counter to the current advocated environmental protection principle. Therefore, the full drainage waterproof and drainage type is gradually replaced by other environmentally friendly groundwater disposal methods, such as controlled and limited drainage waterproof and drainage type.

The controlled drainage and limited drainage waterproof and drainage system can effectively reduce the seepage flow of the tunnel by means of surrounding rock grouting and other means. At the same time, controlled drainage measures are adopted to significantly reduce the water pressure acting on the lining with less drainage cost, so as to make the tunnel project more economical and environmentally friendly. In this paper, the seepage field of a tunnel under three different waterproof types and drainage system namely: controlled and limited drainage, full drainage, and fully sealed, is calculated and studied.

### 2.2. Study on the Analytical Calculation of the Seepage Field of Controlled Drainage and Limited Drainage Tunnel

The controlled drainage and limited drainage waterproof and drainage system are mostly applied to the deeply buried tunnel in the water-rich stratum with a head height of more than 60 m. Considering that there are many factors that affect the seepage field of controlled drainage and limited drainage tunnel, it is necessary to select appropriate methods to study its variation law and parameter sensitivity. At present, the commonly used calculation methods of tunnel seepage field include an analytical method, numerical method, and empirical analysis method. Although the analytical method must be based on a large number of assumptions. Compared with numerical simulation and other methods, the analytical method can be more economical and convenient to solve the tunnel seepage field, which is convenient for sensitivity analysis of various parameters, and has the advantages of simplicity and clear concept. Therefore, in this section, the analytical method is used to study the seepage problem of controlled drainage and limited drainage tunnel.

#### 2.2.1. Computational Models and Basic Assumptions

The simplified calculation model of the seepage field of controlled and limited drainage tunnel is shown in Figure 1. It is assumed that the research object is a saturated, homogeneous, continuous and isotropic semi-infinite rock mass medium, and $H$ is the water head of the stratum surface. A tunnel with an external radius of $r_L$ is excavated inside the rock mass, and the distance between the center of the tunnel and the surface of the stratum is $h$. The permeability coefficient of surrounding rock is $k_S$; the inner radius of the lining is $r_0$; the permeability coefficient is $k_L$; the grouting circle radius is $r_G$. The permeability coefficient is...
$k_\omega$ and the areas I, II, and III represent the surrounding rock, grouting circle, and lining, respectively.

Based on the above conditions, the following assumptions are made: 1) the surrounding rock, grouting circle and lining are homogeneous, continuous and isotropic; 2) the rock mass and water are incompressible, the groundwater supply is sufficient and the water level is stable; 3) the seepage flow is steady, and the motion law obeys Darcy’s law; 4) the buried depth of the tunnel is far greater than the radius, that is, $h > r_\omega$. The direction of the seepage in the grouting circle and the lining is mainly radial; 5) the zero potential planes are located in the dotted line in Figure 1.

2.2.2. Governing Equations and Boundary Conditions

Due to the isotropy of the medium, according to the conservation of mass and Darcy’s law, the two-dimensional steady-state groundwater seepage field around the tunnel meets the Laplace equation. In the $z$-plane, the seepage differential equation in the region I is expressed in the rectangular coordinate system as follows:

$$\frac{\partial^2 \phi_s}{\partial x^2} + \frac{\partial^2 \phi_s}{\partial y^2} = 0 \tag{3}$$

$$\phi_s = \frac{P}{\gamma_\omega} + Y \tag{4}$$

where: $\phi_s$ is the total head of surrounding rock, which is the sum of the pressure head and the location head; $P$ is the pore water pressure; $\gamma_\omega$ is the unit weight of water; $Y$ is the position of the waterhead; in this coordinate system $Y = y + h$.

The differential equations of seepage in region II and III can be expressed in polar coordinates as follows:

$$\frac{\partial^2 \phi_\omega}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial \phi_\omega}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2 \phi_\omega}{\partial \theta^2} = 0 \tag{5}$$
where $\phi_i$ is the total waterhead of grouting circle; $\phi_e$ is the total head of lining; $\rho$ is the polar path; $\theta$ is the polar angle.

When the buried depth of the tunnel is far greater than the radius, the water level boundary has little influence on the seepage field around the tunnel, so the seepage path can be simplified as an axisymmetric form, and it is approximately considered that the waterhead is equal where the radius around the tunnel is the same. Then, the initial boundary conditions are as follows

$$\phi_{ \rho=0} = H + h_0$$

$$\phi_{ \rho=\rho_0} = h_0$$

$$\phi_{ \rho=\rho_0} = \phi_{ \rho=\rho_2} = h_L$$

$$\phi_{ \rho=\rho_0} = h_G$$

where $h_0$ is the internal waterhead of the lining; $h_L$ is the waterhead at the junction of the lining and the excavation damaged zone; $h_G$ is the waterhead at the junction of the excavation damaged zone and the surrounding rock.

### 2.2.3. Analytical Calculation of the Seepage Field of the Surrounding Rock

The analytical problem of the seepage field in the surrounding rock area in the computation model belongs to the problem of the semi-infinite plane orifice, which can be effectively converted and solved by using the conformal transformation of complex function. The region I in $z$-plane is mapped to $\zeta$ plane according to the projection transformation Equation (11), and a circular region with an outer diameter of 1 and an inner diameter of $\alpha$ is obtained. As shown in Figure 2, $\phi$ and $\beta$ are the polar diameter and polar angle of the polar coordinate system in the $\zeta$ plane, respectively.

$$z = \omega(\zeta) = -iA\frac{1+\zeta}{1-\zeta}$$

where $A = h(1-\alpha^2)/h(1+\alpha^2)$; $\alpha$ is calculated by the following formula.

![Figure 2. Conformal transformation region.](image)
According to the conformal transformation principle of complex function, the transformed potential function also satisfies Laplace equation, and Equation (3) can be rewritten as in \( \xi-\eta \) coordinate system as follows

\[
\frac{\partial^2 \phi}{\partial \xi^2} + \frac{\partial^2 \phi}{\partial \eta^2} = 0
\]  

According to the boundary conditions, the total head acting on the circle with radius \( \varphi \) in \( \zeta \) plane can be expressed as the following Fourier form [7]

\[
\phi_\xi = C_1 + C_2 \ln \varphi + \sum_{n=1}^{\infty} \left( C_3 \varphi^n + C_4 \varphi^{-n} \right) \cos n\beta
\]  

where \( C_1, C_2, C_3, \) and \( C_4 \) are undetermined constants determined by boundary conditions.

For the polar coordinate system in \( \zeta \) plane, Equations (7) and (10) can be rewritten as

\[
\phi_{S(\varphi=\alpha)} = H + h
\]  

\[
\phi_{L(\varphi=\alpha)} = h_G
\]

Substituting Equation (15) into Equation (14) can obtain

\[
\phi_{S(\varphi=\alpha)} = C_1 + \sum_{n=1}^{\infty} \left( C_3 + C_4 \right) \cos n\beta = H + h
\]

Compare the left and right sides of the equation

\[
C_1 = H + h , \ C_3 = -C_4
\]

Then substitute Equation (16) into Equation (14), and combine Equation (18) to get

\[
\phi_{S(\varphi=\alpha)} = H + h + C_2 \ln \alpha + \sum_{n=1}^{\infty} C_3 \left( \alpha^n - \alpha^{-n} \right) \cos n\beta = h_G
\]

Since the buried depth of the tunnel is larger than the radius, \( i.e. \ h > r_G \) it can be approximately considered that the waterhead is the same where the radius around the tunnel is the same, \( i.e. \ h_G \) is a constant. Therefore, from Equation (19), we can get

\[
C_2 = \frac{h_G - H - h}{\ln \alpha} , \ C_3 = 0
\]

Equation (14) can be rewritten as

\[
\phi_\xi = H + h + \frac{\ln \varphi}{\ln \alpha} \left( h_G - H - h \right)
\]

According to the integral of Equation (21), the seepage flow at the interface of region I and region II is as follows

\[
Q_S = k_\varsigma \int_0^{2\pi} \frac{\partial \phi_\xi}{\partial \varphi} \varphi \mathrm{d} \beta = \frac{2\pi k_\varsigma}{\ln \alpha} \left( h_G - H - h \right)
\]
2.2.4. Analytical Calculation of the Seepage Field of the Grouting Circle and the Lining

Since the direction of seepage in the grouting circle and the lining is mainly radial, the seepage in the grouting circle and lining can be simplified as an axisymmetric constant seepage problem, the continuous differential equation of seepage flow is simplified from Equations (5) and (6) to

\[ \frac{\partial^2 \phi_G}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial \phi_G}{\partial \rho} = 0 \] (23)

\[ \frac{\partial^2 \phi_L}{\partial \rho^2} + \frac{1}{\rho} \frac{\partial \phi_L}{\partial \rho} = 0 \] (24)

Combined with the boundary condition Equation (8), Equation (9) and Equation (10), the solutions of Equations (23) and (24) are as follows

\[ \phi_G = \frac{h_t \ln r_G - h_o \ln r_e}{\ln (r_G / r_e)} + \frac{h_G - h_e}{\ln (r_G / r_e)} \ln \rho \] (25)

\[ \phi_L = \frac{h_o \ln r_G - h_t \ln r_e}{\ln (r_G / r_e)} + \frac{h_e - h_o}{\ln (r_G / r_e)} \ln \rho \] (26)

Then the seepage flow at the interface between area II and area III is

\[ Q_G = k_G \int_0^{2\pi} \frac{\partial \phi_G}{\partial \rho} \rho d\theta = \frac{2\pi k_G}{\ln (r_G / r_e)} (h_G - h_e) \] (27)

The seepage flow inside the lining is

\[ Q_L = k_L \int_0^{2\pi} \frac{\partial \phi_L}{\partial \rho} \rho d\theta = \frac{2\pi k_L}{\ln (r_G / r_e)} (h_e - h_o) \] (28)

2.2.5. Simultaneous Solution

According to the continuous condition of interlayer seepage flow, the seepage flow between layers is equal, that is

\[ Q = Q_S = Q_G = Q_L \] (29)

By substituting Equation (22), Equation (27), and Equation (28) into Equation (29), the waterhead at the junction of grouting circle and surrounding rock is

\[ h_G = \frac{B (H + h) - h_o \ln \alpha}{B - \ln \alpha} = \frac{B (H + h) + h_o E}{B + E} \] (30)

where \( B = (k_s / k_G) \ln (r_G / r_e) + (k_s / k_L) \ln (r_e / r_o) \); \( E = \ln \left( \frac{h + \sqrt{h^2 - r_G^2}}{r_G} \right) \);
\( C = (k_s / k_G) \ln (r_G / r_e) \); \( D = (k_s / k_L) \ln (r_e / r_o) \); the external waterhead of the lining is

\[ h_L = \frac{D (H + h) - h_o (\ln \alpha - C)}{B - \ln \alpha} = \frac{D (H + h) + h_o (C + E)}{B + E} \] (31)

Substituting Equation (30) for Equation (22), the tunnel seepage flow is

\[ Q = \frac{2\pi k_s (H + h - h_o)}{B - \ln \alpha} = \frac{2\pi k_s (H + h - h_e)}{B + E} \] (32)
Equation (32) is the analytical solution of the seepage flow of controlled and limited drainage tunnel. The flow is positive in the inflow tunnel and negative in the outflow tunnel. The combined (4), (30) and (31) can obtain the external water pressure of the grouting circle and the lining, respectively.

\[
P_G = \gamma_w \left[ \frac{B(H + h) + h_0 E}{B + E} - y - h \right] \tag{33}
\]

\[
P_L = \gamma_w \left[ \frac{D(H + h) + h_0 (C + E)}{B + E} - y - h \right] \tag{34}
\]

### 2.2.6. Factors Affecting the Grouting Parameters

Nowadays, considering environmental protection, the lining structure strength, and other factors, for deep buried tunnels with the waterhead greater than 60 m, the groundwater treatment method of controlled and limited drainage waterproof type and drainage system is often adopted, such as Yuan Liangshan tunnel in China and Qing Han subsea tunnel in Japan (Zhang Chengping et al. 2007, Chinese). The stratum grouting reinforcement is a widely used water blocking method in the controlled and limited drainage waterproof type and drainage system (Liu Zhichun and Wan Liangyong 2015, Chinese). Therefore, the key problem in the study of the “controlled and limited drainage tunnel seepage field is to determine the influence of the variation of the grouting circle parameters on the tunnel seepage field. In this section, the influence factors of the grouting parameters are put forward and based on the analytical solution and the influence factors, the sensitivity of seepage field of controlled and limited drainage tunnel to the variation of the grouting parameters is analyzed, so as to provide a reference for the judicious determination of the grouting parameters.

Taking the deeply buried tunnel with controlled and limited drainage limited waterproof type and drainage system as the engineering background, for simplified analysis, assuming that the water level is located on the ground surface \((H = 0)\) and the lining waterhead is 0, then the Formulas (31) and (32) can be simplified as

\[
h_L = \frac{\left( \frac{k_S}{k_L} \ln \frac{r_L}{r_0} \right) h}{F_G + \left( \frac{k_S}{k_L} - 1 \right) \ln \frac{r_L}{r_0} + G} \tag{35}
\]

\[
Q = \frac{2\pi k_S h}{F_G + \left( \frac{k_S}{k_L} - 1 \right) \ln \frac{r_L}{r_0} + G} \tag{36}
\]

where \(G = \ln \left[ \left( h + \sqrt{h^2 - r_0^2} \right) / r_0 \right] \); \(F_G = (a - 1) \ln b\), is defined as the influence factor of grouting parameters, of which \(a = k_S/k_G\), which is the ratio of the permeability coefficient between the surrounding rock and the grouting circle, reflecting the impermeability of the grouting circle; \(b = r_S/r_L\), which is the ratio of the grouting circle radius to the tunnel external radius, reflecting the thickness of...
the grouting circle. The variation law of $F_G$ with $a$ and $b$ is shown in Figure 3. $F_G$ increases linearly with the increase of $a$ and increases logarithmically with the increase of $b$.

In practical engineering, considering the principle of economy, the adjustable range of the permeability coefficient of the grouting circle is much larger than the thickness of the grouting circle. Therefore, parameter $a$, namely, the impermeability of the grouting circle, is the main control factor affecting the size of $F_G$, while the thickness of the grouting circle is the secondary control factor, and its significance is more reflected in the role of the stratum reinforcement. In conclusion, when determining the reasonable parameters of the grouting circle, if only considering the water blocking effect of the grouting circle, the minimum value of the permeability coefficient of the grouting circle, i.e. the maximum value of the parameter $a$, should be determined first, and then the corresponding parameter $b$, i.e. the thickness of the grouting circle, should be determined.

3. Results and Discussions

3.1. Numerical Simulation Verification of the Analytical Solution

3.1.1. Numerical Computation Model and Parameters

In order to further verify the correctness of the analytical solution in this paper, the finite element numerical simulation software ABAQUS is used for modeling and solutions, and the results of the analytical solution and the numerical solution are compared and verified. The geometric parameters of tunnel and the permeability coefficient of model materials are shown in Table 1, and the following two methods are used for verification analysis: 1) with the permeability coefficient of lining as a fixed value, the results of numerical and analytical solutions of seepage field under different tunnel buried depth are compared, and the influence of the variation of buried depth on the computation results is analyzed; 2) with the buried depth of the tunnel as a fixed value, the analytical and numerical results of seepage field under different permeability coefficient of the lining are compared, and the influence of the variation of the permeability coefficient of the lining on the calculation results is analyzed.

![Figure 3](image.png)

**Figure 3.** Variation of the grouting parameter influence factor $F_G$ with the parameters $a$ and $b$. 
Table 1. Geometric parameters of the tunnel and the permeability coefficient of the model material.

| Working conditions | r/cm | r0/cm | rG/cm | h/m | H/m | h0/m | kL/(cm/s) | kG/(cm/s) | kL/(cm/s) |
|--------------------|------|-------|-------|-----|-----|-------|------------|------------|------------|
| A                  | 3    | 2.4   | 6     | 100 | 0   | 0     | 1.0 × 10⁻⁴ | 1.0 × 10⁻⁵ | 1.0 × 10⁻⁶ |
| B                  | 3    | 2.4   | 6     | 100 | 0   | 0     | 1.0 × 10⁻⁴ | 1.0 × 10⁻⁵ | 1.0 × 10⁻⁴ - 1.0 × 10⁻⁷ |

According to the simplified analytical computation model of the tunnel seepage field (Figure 1), the numerical computation model of the tunnel seepage field is established as shown in Figure 4. The transverse width of the model is taken as 10 times the maximum burial depth, 1000 m in total, and the longitudinal depth is taken as 4 times the maximum burial depth, 400 m in total. The model includes three materials: the surrounding rock, the grouting circle, and the lining. The Mohr-coulomb constitutive model is used for the surrounding rock and the grouting circle, and the elastic constitutive model is used for the lining. The physical and mechanical parameters of materials are shown in Table 2. In the table, \( \rho \) is the density, \( C \) is the cohesion, \( \phi \) is the friction angle, \( E \) is the elastic modulus and \( \mu \) is the Poisson’s ratio. The four-node plane strain pore pressure solid element (CPE4P) is used for mesh generation.

3.1.2. Simulation Results and Comparative Verification

Figure 5 depicts the comparison between the analytical solution and the numerical solution of the tunnel seepage field under different buried depths. Figure 6 depicts the comparison between the analytical solution and the numerical solution of the tunnel seepage field under different lining permeability coefficients. In Figure 5(b) and Figure 6(b), the top, middle and bottom limit of the numerical solutions refer to the external waterhead values of the top, middle, and bottom of the lining respectively. According to the distribution of the three values in the figure, for example in this paper, when the buried depth of the tunnel is greater than 30 m, the influence of the water level boundary on the seepage field around the tunnel is very small. It can be approximately considered that the waterhead of the same radius around the tunnel is the same, which is consistent with the assumption when the analytical solution is derived in this paper. At the same time, it can be seen from Figure 5(b) that with the decrease of the tunnel buried depth, the difference between the top limit, the middle value and the bottom limit of the numerical solution gradually increases. For shallow buried tunnels, the influence of the water level boundary on its surrounding seepage field is not negligible. Taking the tunnel depth \( h = 10 \) m as an example, the external waterhead of the lining obtained from the solution in this paper is 7.33 m, which is more consistent with the median value (7.31 m) of the numerical solution, while there are obvious differences with the top and bottom limits (7.93 m, 6.68 m) of the numerical solution. Therefore, the waterhead distribution around the shallow tunnel needs to be determined by other methods.
Figure 4. Numerical computation model of the tunnel seepage field.

Figure 5. Comparison of the analytical and numerical solutions of the seepage field of the tunnel under different buried depths. (a) Seepage flow rate; (b) External waterhead of the lining.

Table 2. Physical and mechanical parameters of model materials.

| Materials         | ρ (kg/m³) | c (kPa) | φ/° | E (GPa) | μ   |
|-------------------|-----------|---------|-----|---------|-----|
| Surrounding rock  | 2000      | 500     | 35  | 1.5     | 0.25|
| Grouting circle   | 2300      | 500     | 40  | 2       | 0.25|
| Lining            | 2500      | -       | -   | 30      | 0.2 |
As depicted in Figure 5 and Figure 6, the tunnel seepage flow and the external waterhead of the lining increase approximately linearly with the increase of the buried depth of the tunnel. With the decrease of the permeability coefficient of the lining, the seepage flow of the tunnel decreases and the external waterhead of the lining increases. The analytical solution of the shaft method is suitable for solving the seepage field of the deep tunnel with high accuracy. Based on the analytical solution of the shaft method and conformal transformation, this paper optimizes the seepage calculation in the actual semi-infinite surrounding rock area. Through the above comparative analysis, it can be seen that for the deeply buried tunnel in the semi-infinite plane, the analytical solution in this paper is in good agreement with the analytical solution and numerical solution of the shaft method, which further verifies the reliability of the analytical solution in this paper. Compared with the shortcomings of the numerical methods, such as complex modeling and computation time consuming, the analytical solution in this paper is simple and practical, which is convenient for the prediction of seepage field and parameter analysis of deep-buried tunnel.
3.2. Sensitivity Analysis of Seepage Field to Grouting Parameters in Controlled Drainage and Limited Drainage Tunnel

According to Equations (35) and (36), the influencing factors of seepage field of the controlled drainage and limited drainage tunnel include the grouting parameters (the permeability coefficient and the thickness of the grouting circle), the lining parameters (the lining permeability coefficient and the thickness), the tunnel section size and the location parameters (the inner radius and the depth of tunnel), and the tunnel seepage flow and the lining external waterhead decrease with the increase of grouting parameter $F_G$. In order to explore the sensitivity of the tunnel seepage field to the variation of the grouting parameters under different conditions, based on the analytical solution in this paper, the variation law of the seepage flow and the lining external waterhead with the influence factors of the grouting parameters under different lining permeability coefficient, the tunnel buried depth and the section size are analyzed with examples. The calculation parameters and the initial values of the tunnel seepage field are shown in Table 3.

As shown in Figure 7, the influence curves of grouting parameter influence factor $F_G$ on tunnel seepage field under different lining permeability coefficient, the tunnel buried depth, and the section size conditions are depicted respectively. The abscissa of the curve is $F_G$, and the ordinate is the absolute value of the tunnel ratio seepage flow, the variation value of the lining external waterhead and the initial seepage flow $Q_0$, and the initial external waterhead of the lining $h_{L0}$ respectively, where $Q_0$ and $h_{L0}$ are the tunnel seepage flow and the lining external waterhead when $F_G = 0$.

From the curve slope variation of Figure 7, it can be seen that the sensitivity of tunnel seepage field to the grouting parameter variations decreases with the increase of $F_G$, which can be roughly divided into the sensitive stage, the transitional stage and the insensitive stage. For example, in this paper, when $F_G$ increases from 0 to 100, both the seepage flow and the lining external waterhead

| Example | $r$/m | $r_0$/m | $h$/m | $k_d$/ks | $H$/m | $h_0$/m | $F_G$ | $Q_0$/(m³·m⁻¹·d⁻¹) | $h_{L0}$/m |
|---------|------|--------|-------|--------|------|--------|------|----------------|----------|
| 1       | 3    | 2.4    | 100   | 100    | 0    | 0      | 0 - 600 | 2.05          | 84.16    |
|         |      |        |       |        | 1000 | 0      | 0      | 0.24          | 98.15    |
|         |      |        |       |        | 40   | 0      | 0      | 0.85          | 34.88    |
| 2       | 3    | 2.4    | 70    | 100    | 0    | 0      | 0 - 600 | 1.45          | 59.72    |
|         |      |        |       |        | 100  | 0      | 0      | 2.05          | 84.16    |
| 3       | 2    | 2.4    | 100   | 100    | 0    | 0      | 0 - 600 | 2.05          | 84.16    |
| 4       | 2    | 2.4    | 100   | 100    | 0    | 0      | 0 - 600 | 2.05          | 84.16    |
| 5       | 2    | 2.4    | 100   | 100    | 0    | 0      | 0 - 600 | 2.05          | 84.16    |
| 6       | 2    | 2.4    | 100   | 100    | 0    | 0      | 0 - 600 | 2.10          | 86.42    |
Figure 7. Influence factor $F_G$ of the grouting parameters on the tunnel seepage field under different lining permeability coefficient, the buried depth and the tunnel section size.
(a) The different permeability coefficient of the lining; (b) Different burial depths; (c) Different tunnel section size.
are greatly reduced, and the seepage field of the tunnel presents a strong sensitivity. However, with the continuous increase of $F_G$, the variation of the seepage field of the tunnel tends to be gentle. In the practical engineering application, according to the design limit value of the tunnel seepage flow and the lining external waterhead, the influence factors $F_{G-1}$ and $F_{G-2}$ of the corresponding grouting parameters can be calculated by using the analytical solution in this paper, and then the reasonable permeability coefficient and the thickness of the grouting circle can be determined according to the methods seen previously.

As shown in Figure 7(a), the sensitivity of the tunnel seepage field to the variation of the grouting parameters decreases with the decrease of the lining permeability coefficient. Combined with the initial seepage flow and the lining external waterhead in Table 3, it can be seen that with the increase of the impermeability of the lining, the main role of the grouting circle gradually changes from reducing the seepage flow to reducing the external waterhead of the lining. It can be seen from Figure 7(b) that the sensitivity of the seepage field of the deeply buried tunnels to the variation of grouting parameters is very little affected by the variation of the buried depth. However, for the tunnels with greater buried depth, the greater the initial seepage flow and the external waterhead of the lining, the more obvious the improvement effect of the grouting circle on the seepage field of the tunnel is.

As shown in Figure 7(c), for the tunnels with $r_0 = 2.4$ m, $r_L = 3$ m, and $r_0 = 4.8$ m, $r_L = 6$ m, the sensitivity of the seepage field to the variation of the grouting parameters is basically the same, while for the tunnels with $r_0 = 4$ m, $r_L = 3$ m, the sensitivity of the seepage field to the variation of grouting parameters is relatively weak. It can be seen that the sensitivity of the seepage field of the deeply buried tunnel to the variation of the grouting parameters is little affected by the variation of tunnel section size, while the variation of the lining thickness ($r_0/r_L$) will have a more obvious impact on it. The sensitivity of the tunnel seepage field to the variation of grouting parameters decreases with the increase of $r_0/r_L$.

### 3.3. Study on the Computation of the Seepage Field of Full Drainage and Guide Tunnel

In order to avoid excessive water pressure on the lining structure, a large number of mountain tunnels used to adopt the drainage-based groundwater treatment measures in the past. By setting up circular and longitudinal drainage blind pipes between the initial support and the secondary lining, the groundwater seepage was collected and drained from the internal drainage ditch of the tunnel. In this section, the analytical computation model of the full drainage tunnel seepage field is established, and the analytical solution of the full drainage tunnel seepage field is obtained by analogy with the analytical solution of controlled and limited drainage tunnel seepage field, and the comparative analysis of the full drainage and controlled and limited drainage tunnel seepage field is carried out by using the analytical solution.
3.3.1. Analytical Solution of Seepage Field in the Full Drainage and Guide Tunnel

The main difference between the full drainage guide and controlled and limited drainage is that the full drainage guide does not take the grouting drainage-controlled measures. Compared with the simplified computation model of the seepage field of controlled and limited drainage tunnel, the simplified computation model of the seepage field of the full drainage tunnel is established as shown in Figure 8. The difference between the computation model of seepage field of the full drainage and the controlled drainage and limited drainage tunnel is that the grouting circle is changed into excavation damaged zone, the radius of excavation damaged zone is \( r_E \), the permeability coefficient is \( k_E \), and other parameters are consistent with the basic assumptions.

Considering the strong similarity between the two models, the control equation, boundary conditions and the solution process of the analytical solution of the seepage field of the controlled drainage and limited drainage type tunnel are analogized, and \( H = h_0 = 0 \) is taken, and the external waterhead and seepage flow of the full drainage and guide tunnel lining can be obtained.

3.3.2. Comparison of Seepage Field between the Full Drainage and Controlled Drainage and Limited Drainage Tunnels

Through the calculation example, the difference between the tunnel seepage flow and the lining external water head under the two water proof types and drainage system is analyzed. The geometric parameters and the material permeability coefficient in the calculation example are shown in Table 4. Figure 9 depicts the comparison of the seepage field of the controlled drainage and limited drainage and the full drainage and guide tunnels under different buried depths. Figure 10 depicts the comparison of the seepage field of controlled drainage and limited drainage and the full drainage and guide tunnels under different lining permeability coefficients.
Figure 9. Comparison of the seepage field of the water controlled and limited drainage and the full drainage and guide tunnels under different buried depths. (a) Seepage flow rate; (b) External waterhead of the lining.

Table 4. Tunnel geometric parameters and material permeability coefficient geometric.
When the shotcrete of the initial support has a certain impermeability and no seepage occurs, the initial support in the composite lining of the full drainage and guide tunnel still needs to bear a large water pressure after the groundwater level recovers to a certain height and is stable. It can be seen from Figure 9 that when the tunnel is 60 m below the water level, the seepage flow and the lining external waterhead of the full drainage and guide tunnel are 4.81 m³/(d·m) and 30.56 m respectively, and the seepage flow and the lining external waterhead of the controlled drainage and limited drainage tunnel are 2.44 m³/(d·m) and 15.48 m respectively, with a difference of 2.37 m³/(d·m) and 15.08 m respectively. With the increase of the buried depth, the seepage flow and the lining external waterhead of the two types of tunnels increase gradually. According to the linear
slope, the increase rate of the full drainage waterproof type is greater than that of the controlled drainage and limited drainage. When the tunnel is 150 m below the water level, the seepage flow and the lining external waterhead of the full drainage tunnel are 10.59 m³/(d∙m) and 67.29 m, respectively. And the seepage flow and the lining external water head of the controlled drainage and limited drainage tunnel are 5.70 m³/(d∙m) and 36.20 m, respectively. The volume of seepage flow and the lining external waterhead are 5.70 m³/(d∙m) and 36.20 m, respectively, with a difference of 4.89 m³/(d∙m) and 31.09 m. Compared with the burial depth of 60 m, it increases significantly.

It can be seen from Figure 10 that when kS/kL = 10, the seepage flow and the lining external waterhead of the full drainage and guide tunnel are 12.00 m³/(d∙m) and 15.25 m respectively, and the seepage flow and external head of lining of the controlled drainage and limited drainage tunnel are 4.88 m³/(d∙m) and 6.20 m respectively, with a difference of 7.12 m³/(d∙m) and 9.05 m respectively. As the permeability coefficient of shotcrete decreases, the seepage flow of the two tunnel types decreases gradually, and the lining external waterhead increases gradually. The decrease rate of the seepage flow and the increased rate of the lining external waterhead are gradually slowed down. The influence of the impermeability of shotcrete on the seepage field of the full drainage and guide tunnel is more significant. When kS/kL = 100, the seepage flow and the lining external waterhead of the full drainage and guide tunnel are 5.06 m³/(d∙m) and 64.28 m respectively. And, the seepage flow and the lining external waterhead of the water controlled drainage and limited drainage tunnel are 3.13 m³/(d∙m) and 39.80 m, respectively, with a difference of 1.93 m³/(d∙m) and 24.48 m. Compared with kS/kL = 10, the difference of the seepage flow is significantly reduced, and the difference of the lining external water head is significantly increased.

Based on the above analysis, no matter the amount of groundwater loss or the water pressure on the composite lining of the tunnel, the water controlled and limited drainage tunnel is smaller than the full drainage and guide tunnel, and with the increase of the tunnel buried depth, the gap between the two types is more obvious. When the construction quality of the shotcrete is poor and the permeability is strong, the seepage volume of the full drainage tunnel is much larger than that of the water control and limited drainage tunnel, which will lead to the loss of a large number of groundwater resources. When the construction quality of the shotcrete is good and the permeability is weak, the seepage flow of the full drainage guide tunnel is gradually close to the seepage flow of the water control and limited drainage tunnel. However, the lining external waterhead is obviously larger than the lining external water head of the water blocking and limited drainage tunnel, which will cause adverse effects on the stability of the tunnel lining structure. By comparison, it can be concluded that the full-drainage groundwater disposal method is to obtain a certain reduction effect of the lining external waterhead at the cost of more groundwater resources, and only when the groundwater level falls to the bottom of the tunnel and does not recover, can the lining external waterhead be 0 by means of the full-drainage method, which
is bound to produce a great negative effect on the groundwater environment. The treatment method of the controlled drainage and limited drainage is to obtain a significant reduction effect of the lining external waterhead at the cost of fewer groundwater resources. Therefore, for tunnels with large buried depth below the water level, the controlled drainage and limited drainage waterproof type and drainage system should be given priority.

3.4. Research on the Computation of the Seepage Field of the Fully Sealed Tunnel

The fully sealed waterproof structure, also known as an all-inclusive waterproof structure, can completely block the seepage of groundwater by laying waterproof board in the whole section between the initial support and the secondary lining in the composite lining without setting drainage blind pipe. The full sealed waterproof type is commonly used in tunnels with small buried depth and high waterproof requirements below the water level, such as subway tunnels. Due to the inability to discharge water, the lining structure of such tunnels needs to bear large external water pressure. This section analyzes the seepage field of the fully sealed tunnel based on the above analytical formula and proposes a full-inclusive control waterproof type and drainage system, which is suitable for a subway tunnel with large buried depth below the water level.

3.4.1. An Analytical Study on the Seepage Field of the Fully Sealed Tunnel

The seepage flow of the fully sealed tunnel without grouting is approximately 0 m³/(d·m), and the external waterhead of the lining is approximately equal to the static waterhead. Then, according to Equations (31) and (32), the seepage flow of the fully sealed tunnel after grouting circle and the size of the external waterhead of the lining are calculated. Similarly, it is considered that the lining permeability coefficient \( k_L \to 0 \). The above analysis shows that for the fully sealed tunnel, it is not effective to reduce the external water pressure of the lining by grouting, and the change of the parameters in the EDZ will not affect the seepage field.

3.4.2. The Full-Inclusive Control Waterproof and Drainage System

In actual project construction, when the subway tunnel passes through mountains or is buried deep below the water level, the fully sealed waterproof type must be adopted according to relevant requirements. At this time, not only the tunnel lining structure will bear huge external water pressure, but also the risk of rupture and failure of the waterproof board due to extrusion will be greatly increased. Once the waterproof board fails, long-term seepage may lead to leakage inside the tunnel and affect the normal operation of the tunnel. Taking into account the above situation, based on the fully sealed waterproof type, a full-inclusive control waterproof and drainage system is proposed, which is suitable for a subway tunnel with large buried depth below the water level.

The full-inclusive control waterproof type and drainage system are based on the fully sealed waterproof type. The circular and longitudinal drainage blind
pipes are set between the waterproof board and the secondary lining to collect the groundwater seepage that breaks through the waterproof board and discharge it from the drainage ditch inside the tunnel. The structure is shown in Figure 11. By setting drainage blind pipe, the tunnel will be converted from the fully sealed waterproof type to the full drainage waterproof type and drainage system in case of waterproof board failure. According to the previous analysis, the full drainage waterproof type and drainage system are not as good as the controlled water and limited drainage waterproof type and drainage system in terms of groundwater environmental protection and reduction of the lining external water pressure. Therefore, the full-inclusive control waterproof type and drainage system also needs to reinforce the surrounding rock within a certain range of the tunnel by grouting, and further transform the full-drainage and guide waterproof type and drainage system into the controlled water and limited drainage waterproof type and drainage system, so as to minimize the impact of waterproof board failure on the tunnel structure and groundwater environment. Although the grouting circle has no effect on improving the seepage field of the fully sealed tunnel under the condition that the waterproof board does not leak, once the waterproof measures fail, the grouting circle can give full play to the role of blocking water and sharing the external water pressure.

4. Conclusion

In this paper, based on the complex variables and the theory of seepage mechanics, the analytical solutions for seepage field of tunnels with controlled drainage were derived and the sensitivity of seepage field to the variation of grouting parameters was analyzed using the proposed analytical solutions. The seepage fields of tunnels with different drainage systems, including controlled and full...
drainages, were compared and analyzed by the analytical solutions. The seepage flow and water head on lining of tunnel without drainage were calculated and the effects of grouting circle and excavation damaged zone on the seepage field of tunnel without drainage were analyzed using the analytical solutions. A novel waterproof and drainage system, which is suitable for subway tunnels with large buried depth below groundwater level, was proposed.

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Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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