Black-box Smoothing: A Provable Defense for Pretrained Classifiers

Hadi Salman ¹  Mingjie Sun ²  Greg Yang ¹  Ashish Kapoor ¹  J. Zico Kolter ²

Abstract

We present a method for provably defending any pretrained image classifier against \( \ell_p \) adversarial attacks. By prepending a custom-trained denoiser to any off-the-shelf image classifier and using randomized smoothing, we effectively create a new classifier that is guaranteed to be \( \ell_p \)-robust to adversarial examples, without modifying the pretrained classifier. The approach applies both to the case where we have full access to the pretrained classifier as well as the case where we only have query access. We refer to this defense as black-box smoothing, and we demonstrate its effectiveness through extensive experimentation on ImageNet and CIFAR-10. Finally, we use our method to provably defend the Azure, Google, AWS, and ClarifAI image classification APIs. Our code replicating all the experiments in the paper can be found at: https://github.com/microsoft/blackbox-smoothing.

1. Introduction

Image classification using deep learning, despite its recent success, is well-known to be susceptible to adversarial attacks: small, imperceptible perturbations of the inputs that drastically change the resulting predictions (Szegedy et al., 2013; Goodfellow et al., 2015; Carlini & Wagner, 2017b). To solve this problem, many works proposed heuristic defenses that build models robust to adversarial perturbations, though many of these defenses were broken using more powerful adversaries (Carlini & Wagner, 2017a; Athalye et al., 2018; Uesato et al., 2018). This has led researchers to both strengthen empirical defenses (Kurakin et al., 2016; Madry et al., 2017) as well as to develop certified defenses that come with robustness guarantees, i.e., classifiers whose predictions are constant within a neighborhood of their inputs (Wong & Kolter, 2018; Raghunathan et al., 2018a; Cohen et al., 2019; Salman et al., 2019a). However, the majority of these defenses require that the classifier be trained (from scratch) specifically to optimize the robust performance criterion, making the process of building robust classifiers a computationally expensive one.

In this paper, we consider the problem of generating a robust classifier without retraining the underlying model at all. There are several use cases for such an approach. For example, a provider of a large-scale image classification API may want to offer a “robust” version of the API, but may not want to maintain and/or continually retrain two models that need to be evaluated and validated separately. Even more realistically, a user of a public vision API might want to use that API to create robust predictions (presuming that the API performs well on clean data), but may not have access to the underlying non-robust model. In both cases (which exemplify the full-access and query-access settings respectively), it would be highly desirable if one could simply apply an off-the-shelf “filter” that would allow practitioners to automatically generate a provably robust model from this standard model.

We propose a new approach to obtain a provably robust classifier from a fixed pretrained one, without any additional training or fine-tuning of the latter. This approach is depicted in Figure 1. The basic idea, which we call black-box smoothing\(^1\), is to prepend a custom-trained denoiser before the pretrained classifier, and then apply randomized smoothing (Cohen et al., 2019). Randomized smoothing is a certified defense that converts any given classifier \( f \) into a new smoothed classifier \( g \) that is characterized with a non-linear Lipschitz property (Salman et al., 2019a). When queried at a point \( x \), the smoothed classifier \( g \) outputs the class that is most likely to be returned by \( f \) under isotropic Gaussian perturbations of its inputs. Randomized smoothing requires that the underlying classifier \( f \) is robust to relatively large random Gaussian perturbations of the input, which is not the case for off-the-shelf pretrained models. By applying our custom-trained denoiser to the classifier \( f \), we can effectively make \( f \) robust to such Gaussian perturbations, thereby making it “suitable” for randomized smoothing.

Key to our approach is how we train our denoisers, which...
is not merely to reconstruct the original image, but also to maintain its original label predicted by $f$. Similar heuristics have been used before; indeed, some of the original adversarial defenses involved applying input transformations to “remove” adversarial perturbations (Guo et al., 2017; Liao et al., 2018; Prakash et al., 2018; Xu et al., 2018), but these defenses were soon broken by more sophisticated attacks (Athalye et al., 2018; Athalye & Carlini, 2018; Carlini & Wagner, 2017a). In contrast, the approach we present here exploits the certified nature of randomized smoothing to ensure that our defense is provably secure.

Our contribution is demonstrating for the first time, to the best of our knowledge, a simple yet effective method for converting any pretrained classifier into a provably robust one. This applies both to the setting where we have full access to the classifier, and to the setting where we only have query access to the classifier. We verify the efficacy of our method through extensive experimentation on ImageNet and CIFAR-10. We are able to convert pretrained ResNet-18/34/50 and ResNet-110, on CIFAR-10 and ImageNet respectively, into certifiably robust models; Our results are summarized in Tables 1 and 2 (details are in section 4)2. For instance, we are able to boost the certified accuracy of an ImageNet-pretrained ResNet-50 from 4% to: 31% for the query-access setting, and 33% for the full-access setting, under adversarial perturbations with $\ell_2$ norm less than 127/255. We also show the effectiveness of our method through real-world experiments on the Azure, Google, AWS, and ClarifAI image classification APIs. We are able to wrap these vision APIs with our method, leading to provably robust versions of these APIs despite having only query access to them.

2 Related Work

In the past few years, numerous defenses have been proposed to build adversarially robust classifiers. In this paper, we distinguish between robust training based defenses and input transformation based defenses. Although we use randomized smoothing, our defense largely fits into the latter category, as it is based upon a denoiser applied before the pretrained classifier.

2.1 Robust Training Based Defenses

Many defenses train a robust classifier via a robust training procedure, i.e., the classifier is trained, usually from scratch, specifically to optimize a robust performance criterion. We characterize two directions of robust training based defenses: empirical defenses and certified defenses.
Empirical defenses are those which have been empirically shown to be robust to existing adversarial attacks. The best empirical defense to date is adversarial training (Kurakin et al., 2016; Madry et al., 2017), in which a robust classifier is learned by training directly on adversarial examples. Although such defenses have been shown to be strong, nothing guarantees that a stronger, not-yet-known, attack would not “break” them. In fact, most empirical defenses proposed in the literature were later broken by stronger adversaries (Craiu & Wagner, 2017a; Athalye et al., 2018; Uséa et al., 2018; Athalye & Carlini, 2018). To put an end to this arms race, a few works tried to build certified defenses that come with formal robustness guarantees.

Certified defenses provide guarantees that for any input $x$, the classifier’s prediction is constant within a neighborhood of $x$. The majority of the training-based certified defenses rely on minimizing an upper bound of a loss function over all adversarial perturbations (Wong & Kolter, 2018; Wang et al., 2018a;b; Raghunathan et al., 2018a;b; Wong et al., 2018; Dvijotham et al., 2018a;b; Croce et al., 2018; Salman et al., 2019b; Gehr et al., 2018; Mirman et al., 2018; Singh et al., 2018; Gowal et al., 2018; Weng et al., 2018; Zhang et al., 2018a). However, these defenses are, in general, not scalable to large models (e.g. ResNet-50) and datasets (e.g. ImageNet). More recently, a more scalable approach called randomized smoothing was proposed as a probabilistically certified defense. Randomized smoothing converts any given classifier into another provably robust classifier by convolving the former with an isotropic Gaussian distribution. It was proposed by several works (Liu et al., 2018; Cao & Gong, 2017) as a heuristic defense without proving any guarantees. A few works afterwards were able to provide formal guarantees for randomized smoothing (Lecuyer et al., 2018; Li et al., 2019; Cohen et al., 2019).

Although, in theory, randomized smoothing does not require any training of the original classifier, in order to get non-trivial robustness results, the original classifier has to be custom-trained from scratch as shown in several papers (Lecuyer et al., 2018; Cohen et al., 2019; Salman et al., 2019a; Zhai et al., 2020). Lecuyer et al. (2018) experimented with stacking denoising autoencoders before deep neural networks (DNNs) to scale PixelDP to practical DNNs that are tedious to train from scratch. However, there are two key differences between this work and ours: 1) Lecuyer et al. (2018) trained the denoising autoencoder with only the reconstruction loss, as opposed to the classification-based stability loss that we discuss shortly; and 2) this past work further fine-tuned the classifier itself, whereas the central motivation of our paper is to avoid this step. Indeed, the denoising autoencoder in this prior work was largely intended as a heuristic to speed up training, and differs quite substantially from our application to certify pretrained classifiers.

2.2. Input Transformation Based Defenses

These defenses try to remove the adversarial perturbations from the input by transforming the input before feeding it to the classifier. Many such defenses have been proposed (but later broken) in previous works (Guo et al., 2017; Meng & Chen, 2017; Xu et al., 2018; Liao et al., 2018). Guo et al. (2017) proposed to use traditional image processing, e.g. image cropping, rescaling, and quilting. Meng & Chen (2017) trained an autoencoder to reconstruct clean images. Xu et al. (2018) used color bit depth reduction and spatial smoothing to reduce the space of adversarial attacks. Liao et al. (2018) trained a classification-guided denoiser to remove adversarial noise. However, all these defenses were broken by stronger attacks (Warren et al., 2017; Carlini & Wagner, 2017a; Athalye et al., 2018; Athalye & Carlini, 2018). To the best of our knowledge, all existing input transformation based defenses are empirical defenses. In this work, we present the first input transformation based defense that provides provable guarantees.

2.3. Defending Against General $\ell_p$ Threat Models

Throughout the paper, we focus on defending against $\ell_2$ adversarial perturbations, although nothing in theory prevents our method from being applied to other $\ell_p$ threat models. In fact, as long as randomized smoothing works for other $\ell_p$ norms (which has been shown in recent papers (Li et al., 2019; Lee et al., 2019; Dvijotham et al., 2020)), our method automatically works. The only change would be the way our denoisers are trained; instead of training via Gaussian noise augmentation, the denoisers shall be trained on data corrupted with noise that is sampled from other distributions (e.g. Laplace distribution for $\ell_1$ threat models (Li et al., 2019)).

3. Black-box Smoothing

In this section, we describe our proposed defense, black-box smoothing, for provably defending pretrained classifiers. This contrasts with the common setting of directly training the classifier to optimize the performance of randomized smoothing, which we refer to within this paper using the term white-box smoothing. We first introduce some background on randomized smoothing. We refer the reader to Cohen et al. (2019) and Salman et al. (2019a) for a more detailed description of this technique.

3.1. Background on Randomized Smoothing

Given a classifier $f$ mapping inputs in $\mathbb{R}^d$ to classes in $\mathcal{Y}$, the randomized smoothing procedure converts the base classifier $f$ into a new, smoothed classifier $g$. Specifically, for input $x$, $g$ returns the class that is most likely to be returned by the base classifier $f$ under isotropic Gaussian

White-box Smoothing

Randomized smoothing is a powerful and general method for certifying input transformations. It is based on the following randomized smoothing procedure. Consider a base classifier $f : \mathbb{R}^d \rightarrow \mathbb{R}^m$ and a loss function $\ell : \mathbb{R}^m \rightarrow \mathbb{R}$, e.g. $\ell$ is the 0-1 loss for classification. For input $x$ and label $y$, the randomized smoothing procedure converts the base classifier $f$ into a new, smoothed classifier $g$. Specifically, for input $x$, $g$ returns the class that is most likely to be returned by the base classifier $f$ under isotropic Gaussian

"Provable" defenses for defending against adversarial perturbations from the input by transforming the input before feeding it to the classifier. Many such defenses have been proposed (but later broken) in previous works (Guo et al., 2017; Meng & Chen, 2017; Xu et al., 2018; Liao et al., 2018). Guo et al. (2017) proposed to use traditional image processing, e.g. image cropping, rescaling, and quilting. Meng & Chen (2017) trained an autoencoder to reconstruct clean images. Xu et al. (2018) used color bit depth reduction and spatial smoothing to reduce the space of adversarial attacks. Liao et al. (2018) trained a classification-guided denoiser to remove adversarial noise. However, all these defenses were broken by stronger attacks (Warren et al., 2017; Carlini & Wagner, 2017a; Athalye et al., 2018; Athalye & Carlini, 2018). To the best of our knowledge, all existing input transformation based defenses are empirical defenses. In this work, we present the first input transformation based defense that provides provable guarantees.

2.3. Defending Against General $\ell_p$ Threat Models

Throughout the paper, we focus on defending against $\ell_2$ adversarial perturbations, although nothing in theory prevents our method from being applied to other $\ell_p$ threat models. In fact, as long as randomized smoothing works for other $\ell_p$ norms (which has been shown in recent papers (Li et al., 2019; Lee et al., 2019; Dvijotham et al., 2020)), our method automatically works. The only change would be the way our denoisers are trained; instead of training via Gaussian noise augmentation, the denoisers shall be trained on data corrupted with noise that is sampled from other distributions (e.g. Laplace distribution for $\ell_1$ threat models (Li et al., 2019)).
noise perturbations of \( x \), i.e.,
\[
g(x) = \arg \max_{c \in \mathcal{Y}} \mathbb{P}[f(x + \delta) = c] \tag{1}
\]
where \( \delta \sim \mathcal{N}(0, \sigma^2 I) \).

where the noise level \( \sigma \) controls the tradeoff between robustness and accuracy. Cohen et al. (2019) presented a tight robustness guarantee for the smoothed classifier \( g \) and gave an efficient algorithm based on Monte Carlo sampling for the prediction and certification of \( g \).

**Robustness guarantee for smoothed classifiers** The robustness guarantee of the smoothed classifier is based on the Neyman-Pearson lemma (Cohen et al., 2019). The procedure is as follows: suppose that when the base classifier \( f \) classifies \( \mathcal{N}(x, \sigma^2 I) \), the class \( c_A \) is returned with probability \( p_A = \mathbb{P}(f(x + \delta) = c_A) \), and the “runner-up” class \( c_B \) is returned with probability \( p_B = \max_{c \neq c_A} \mathbb{P}(f(x + \delta) = c) \). The smoothed classifier \( g \) is robust around \( x \) within the radius
\[
R = \frac{\sigma}{2} (\Phi^{-1}(p_A) - \Phi^{-1}(p_B)) , \tag{2}
\]
where \( \Phi^{-1} \) is the inverse of the standard Gaussian CDF. When \( f \) is a deep neural network, computing \( p_A \) and \( p_B \) accurately is not practical. To mitigate this problem, Cohen et al. (2019) used Monte Carlo sampling to estimate some \( p_A \) and \( p_B \) such that \( p_A \approx p_A \) and \( p_B \geq p_B \) with arbitrarily high probability. The certified radius is then computed by replacing \( p_A, p_B \) with \( p_A, p_B \) in Equation 2.

### 3.2. Black-box Smoothing via Image Denoising
Randomized smoothing gives a framework for certifying a classifier \( f \) without any restrictions on the classifier itself. However, naively applying randomized smoothing on a standard-trained classifier gives very loose certification bounds (as verified by our experiments in section 4). This is because standard classifiers, in general, are not trained to be robust to Gaussian perturbations of their inputs. To solve this problem, previous works on randomized smoothing use Gaussian noise augmentation (Cohen et al., 2019) and adversarial training (Salman et al., 2019a) to train the underlying classifier.

We propose a general method called black-box smoothing, where the goal is to certify existing pretrained classifiers using randomized smoothing without modifying those classifiers. We identify two common scenarios for using black-box smoothing: 1) we have complete knowledge and full access to the pretrained classifiers (e.g. API service providers). In this setting, we can back-propagate gradients efficiently through the pretrained classifiers; 2) we only have query access to the pretrained classifiers (e.g. API users).

Our method avoids training the base classifier \( f \) using Gaussian noise augmentation, but instead, uses an image denoising pre-processing step before passing inputs through \( f \). In our setting, denoising is aimed at removing the Gaussian noise used in randomized smoothing. More concretely, we do this by augmenting the classifier \( f \) with a custom-trained denoiser \( D_\theta : \mathbb{R}^d \to \mathbb{R}^d \). Thus, our new base classifier is defined as \( f \circ D_\theta : \mathbb{R}^d \to \mathcal{Y} \).

Assuming the denoiser \( D_\theta \) is effective at removing Gaussian noise, our framework is characterized to classify well under Gaussian perturbation of its inputs. Our black-box smoothing procedure, illustrated in Figure 1, is then formally defined as taking the majority vote using this new base classifier \( f \circ D_\theta \):
\[
g(x) = \arg \max_{c \in \mathcal{Y}} \mathbb{P}[f(D_\theta(x + \delta)) = c] \tag{3}
\]
where \( \delta \sim \mathcal{N}(0, \sigma^2 I) \).

Our defense can be seen as a form of image processing, where we perform input transformations before before classifying. As opposed to previous works that also used image denoising as an empirical defense (Gu & Rigazio, 2014; Liao et al., 2018; Xie et al., 2019; Gupta & Rahtu, 2019), our method gives provable robustness guarantees. Our denoisers are not intended to remove the adversarial noise, which could lie in some obscure high-dimensional subspace (Tramr et al., 2017) and is computationally hard to find (Carlini & Wagner, 2017b). Our denoisers are only needed to “remove” the Gaussian noise used in randomized smoothing. In short, we effectively transform the problem of adversarial defense to the problem of Gaussian denoising; the better the denoising performance, in terms of the custom objectives we will mention shortly, the more robust the resulting smoothed classifier.

### 3.3. Training the Denoiser \( D_\theta \)
The effectiveness of black-box smoothing highly depends on the denoisers we use. For each noise level \( \sigma \), we train a separate denoiser. The noise level \( \sigma \) allows users to choose the desired robustness/accuracy trade-off; as \( \sigma \) increases, the robustness of the resultant classifier increases while its standard accuracy decreases. In this work, we explore two different objectives for training the denoiser \( D_\theta \): 1) the mean squared error objective (MSE), and 2) the stability objective (\( \text{STAB} \)).

**MSE objective:** this is the most commonly used objective in image denoising. Given an (unlabeled) dataset \( \mathcal{S} = \{x_i\} \) of clean images, a denoiser is trained by minimizing the reconstruction objective, i.e., the MSE between the original image \( x_i \) and the output of the denoiser \( D_\theta(x_i + \delta) \), where \( \delta \sim \mathcal{N}(0, \sigma^2 I) \). Formally, the loss is defined as follows,
\[
L_{\text{MSE}} = \mathbb{E}_{x_i} \| D_\theta(x_i + \delta) - x_i \|^2 \tag{4}
\]
This objective allows for training $D_\theta$ in an unsupervised fashion. In this work, we focus on the additive white Gaussian noise denoising, which is one of the most studied discriminative denoising models (Zhang et al., 2017; 2018b).

**Stability objective:** the MSE objective turns out not to be the best way to train denoisers for our goal. We desire the pretrained classifier to be effective at recognizing the denoised images. However, the MSE objective does not actually optimize for this goal. Thus, we explore another objective that explicitly considers classification under Gaussian noise. Specifically, given a dataset $S = \{(x_i, y_i)\}$, we train a denoiser $D_\theta$ from scratch with the goal of classifying images corrupted with Gaussian noise:

$$L_{\text{Stab}} = \mathbb{E}_{S, \delta} \ell_{\text{CE}}(F(D_\theta(x_i + \delta)), f(x_i))$$

where $\delta \sim \mathcal{N}(0, \sigma^2 I)$,

where $F(x)$, that outputs probabilities over the classes, is the soft version of the hard classifier $f(x)$ (i.e., $f(x) = \arg \max_{c \in \mathcal{Y}} F(x)$), and $\ell_{\text{CE}}$ is the cross entropy loss. We call this new objective the stability objective\(^4\), and we refer to it as Stab in what follows\(^5\). This objective can be applied both in the full-access and the query-access settings of the pretrained classifier:

- **Full-access pretrained classifiers:** since we have full access to the pretrained classifier in hand, we can backpropagate gradients through the classifier to optimize the denoisers using Stab. In other words, we train denoisers from scratch to minimize the classification error using the pseudo-labels given by the pretrained classifier.

- **Query-access pretrained classifiers:** since we only have query access to these classifiers, it is difficult to use their gradient information. We get around this problem by using (pretrained) surrogate classifiers\(^6\) as proxies for the actual classifiers we are defending. More specifically, we train the denoisers to minimize the stability loss of the surrogate classifiers. It turns out that training denoisers in such a fashion can transfer to unseen classifiers.

We would like to stress that when using the stability objective, the underlying classifier is fixed. In this case, the classifier can be seen as providing high-level guidance for training the denoiser.

Combining the MSE and Stability Objectives: We explore a hybrid training scheme which connects the low-level\(^6\)

\(^4\)Note that the stability objective used here is similar in spirit to, but different in context from, the stability training used in Zheng et al. (2016); Li et al. (2019)

\(^5\)We also experiment with the classification objective which uses the true label $y_i$ instead of $f(x_i)$. Details are shown in Appendix E.

\(^6\)For ImageNet, we experiment with standard-trained ResNet-18/34/50 as surrogate models. For CIFAR-10, we experiment with 14 standard-trained models listed in Appendix A.

image denoising task to high-level image classification. Inspired by the “pretraining+fine-tuning” paradigm in machine learning, denoisers trained with MSE can be good initializations for training with Stab. Therefore, we combine these two objectives by fine-tuning the MSE-denoisers using Stab. We refer to this as Stab+MSE.

In this work, we experiment with all the above mentioned ways of training $D_\theta$. A complete overview of the objectives used for different access types and classifiers is given in Table 3. Note that we experiment with MSE and Stab+MSE for all the classifiers, but we do Stab-training from scratch only on CIFAR-10 due to computational constraints.

### 4. Experiments

In this section, we present our experiments to robustify pretrained ImageNet and CIFAR-10 classifiers. We use two recent denoisers: DnCNN (Zhang et al., 2017) and MemNet (Tai et al., 2017)\(^7\). For all experiments, we compare with two baselines: 1) certifying pretrained classifiers without stacking any denoisers before them (denoted as “No denoiser”), and 2) certifying classifiers trained via Gaussian noise augmentation (Cohen et al., 2019), i.e., white-box smoothing (denoted as “White-box”).

For a given (classifier $f$, denoiser $D_\theta$) pair, the certified radii of the data points in a given test set are calculated using Equation 2 with $f \circ D_\theta$ as the base classifier. The certification curves are then plotted by calculating the percentage of the data points whose radii are larger than a given $\ell_2$-radius. In the following experiments, we only report the results for $\sigma = 0.25$,\(^8\) and we report the best curves over the denoiser architectures mentioned above. For the complete results using other values of $\sigma$, we refer the reader to Appendix B. The compute resources and training time

\(^7\)We also experiment with DnCNN-wide, a wide version of DnCNN which we define (more details in Appendix A). Note that we do not claim these are the best denoisers in the literature. We just picked two common denoisers. A better choice of denoiser might lead to improvements in our results.

\(^8\)We use the same $\sigma$ for training the denoiser and for certifying the corresponding denoised classifier via Equation 2.
for our experiments are shown in Table 4. Note that we can train denoisers on different datasets in reasonable time. For more details on the architectures of the classifiers/denoisers, training/certification hyperparameters/procedure, etc., we refer the reader to Appendix A.

4.1. Certifying Full-Access Pretrained Classifiers

In this experiment, we assume that the classifier to be defended is known and accessible by the defender, but the defender is not allowed to train or fine-tune this classifier.

For CIFAR-10, this classifier is a pretrained ResNet-110 model. The results are shown in Figure 2a. Attaching a denoiser trained on the stability objective (STAB) leads to better certified accuracies than attaching a denoiser trained on the MSE objective or only finetuned on the stability objective (STAB+MSE). All of these substantially surpass the “No denoiser” baseline; we achieve an improvement of 49% in certified accuracy (over the “No denoiser” baseline) against adversarial perturbations with $\ell_2$ norm less than 64/255 (see Table 2 for more results).

Additionally, Figure 2a plots the certified accuracies of a ResNet-110 classifier trained using Gaussian data augmentation (Cohen et al., 2019), represented by the “White-box” curve. Although our method does not modify the underlying standard-trained ResNet-110 model, our STAB-denoiser achieves similar certified accuracies as the white-box smoothing model.

For ImageNet, we apply our method to PyTorch-pretrained ResNet-18/34/50 classifiers. We assume that we have full access to these pretrained models. The results are shown in Figure 3. STAB+MSE performs better than MSE, and again, both of these substantially improve over the “No denoiser” baseline; we achieve an improvement of 29% in certified accuracy (over the “No denoiser” baseline) against adversarial perturbations with $\ell_2$ norm less than 127/255 (see Table 1 for more results). Note that we do not experiment with STAB denoisers on ImageNet as it is computationally expensive to train those denoisers, instead we save time by fine-tuning a MSE-denoiser on the stability objective (STAB+MSE).

4.2. Certifying Query-Access Pretrained Classifiers

In this experiment, we assume that the classifier to be defended only allows query access.

For CIFAR-10, this classifier is a pretrained ResNet-110 model. The results are shown in Figure 2b. Similar to the full-access setting, STAB leads to better certified accuracies than both STAB-MSE and MSE. Note that in this setting, STAB and STAB-MSE are both trained with 14 surrogate

---

Table 4. Average training statistics for our denoisers. We run our experiments on NVIDIA P100 and V100 GPUs. The “+” sign refers to the additional epochs/time incurred over the MSE baseline.

| Trained Denoiser | Objective | Compute | #Epochs | Sec/Epoch | Total Time (HR) |
|------------------|-----------|---------|---------|-----------|-----------------|
| CIFAR-10/DsCNN   | MSE       | 1xP100  | 90      | 31        | 0.78            |
|                  | STAB+MSE  | 1xP100  | +20     | 57        | +0.32           |
|                  | ResNet-110| 1xP100  | 600     | 59        | 9.80            |
| CIFAR-10/DsCNN-WIDE | MSE       | 1xP100  | 90      | 122       | 3.05            |
|                  | STAB+MSE  | 1xP100  | +20     | 135       | +0.75           |
|                  | ResNet-110| 1xP100  | 600     | 156       | 26.00           |
| CIFAR-10/McMNet  | MSE       | 1xP100  | 90      | 85        | 2.13            |
|                  | STAB+MSE  | 1xP100  | +20     | 118       | +0.66           |
|                  | ResNet-110| 1xP100  | 600     | 125       | 20.83           |
| imagenet/DsCNN   | MSE       | 4xV100  | 5       | 6320      | 8.78            |
|                  | STAB+MSE  | 4xV100  | +20     | 6500      | +36.11          |
|                  | ResNet-18 | 4xV100  | +20     | 6900      | +38.33          |
|                  | STAB+MSE  | 4xV100  | +20     | 7620      | +42.33          |
|                  | ResNet-34 | 4xV100  | +20     | 8720      | +45.73          |
|                  | STAB+MSE  | 4xV100  | +20     | 7620      | +42.33          |
| Vision-APIs/DsCNN | Same as ImageNet | | | | |

---

Figure 2. Certifying a ResNet-110 CIFAR-10 classifier with (a) full-access and (b)(c) query-access using various denoisers. $\sigma = 0.25$. Table 4.
models (in order to transfer to the query-access ResNet-110). See Appendix A for details of these surrogate models.

It turns out that for STAB-MSE, only one surrogate CIFAR-10 classifier (namely Wide-ResNet) is also sufficient for the denoiser to transfer well to ResNet-110 as shown in Figure 2c, whereas for STAB, more surrogate models are needed. A detailed analysis of the effect of the number of surrogate models on the performance of STAB and STAB+MSE is deferred to Appendix C.

Overall, we achieve an improvement of 38% in certified accuracy (over the “No denoiser” baseline) against adversarial perturbations with $\ell_2$ norm less than 64/255. It turns out the certified accuracies obtained for query-access ResNet-110 are lower than those obtained for full-access ResNet-110 (see Figure 2 and Table 2), which is expected as we have less information in the query-access setting.

For ImageNet, we again consider PyTorch-pretrained ResNet-18/34/50 classifiers, but now we treat them as “black-box” models. The results are shown in Figure 4, and are similar to the CIFAR-10 results, i.e., attaching a STAB+MSE-denoiser trained on surrogate models leads to a more robust model than attaching a MSE-denoiser. Note that here for STAB+MSE, we fine-tune a MSE-denoiser on the stability objective using only one surrogate model due to computational constraints, and also because, as observed on CIFAR-10, the performance of STAB+MSE is similar whether only 1 or 14 surrogate models are used. The exact surrogate models used are shown in Figure 4. For example, in Figure 4c, the black-box model to be defended is ResNet-50, so ResNet-18/34 are used as surrogate models. Note that using either model to fine-tune the denoiser leads to roughly the same certified accuracies. We achieve an improvement of 27% in certified accuracy (over the “No denoiser” baseline) against adversarial perturbations with $\ell_2$ norm less than 127/255 (see Table 1 for more results).

4.3. Perceptual Performance of Denoisers

We note that although the certification results of stability trained denoisers (STAB and STAB+MSE) are better than the MSE-trained ones, the actual denoising performance of the former does not seem to be as good as the latter. Figure 5 shows an example of denoising a noisy image of an elephant (noise level $\sigma = 1.0$). The reconstructed image

![Figure 3. Certifying ResNet-18/34/50 ImageNet classifiers with full-access using various denoisers. $\sigma = 0.25.$](image3.png)

![Figure 4. Certifying ResNet-18/34/50 ImageNet classifiers with query-access using various denoisers. $\sigma = 0.25.$ Note how fine-tuning the denoisers by attaching surrogate classifiers maintains the high certification accuracy as the full-access setting.](image4.png)

![Figure 5. Different denoising performance for different denoisers (noise level $\sigma = 1.00$). Note that, although the STAB+MSE denoiser (trained on ResNet-18) leads to strange artifacts as compared to the MSE-denoiser, it gives better certification results as shown in Figure 3.](image5.png)
using stability-trained denoisers has some strange artifacts. For more examples, see Appendix F.

5. Defending Public Vision APIs

We demonstrate that our approach can provide certification guarantees for commercial classifiers. We consider four public vision APIs: Azure Computer Vision API\(^\text{10}\), Google Cloud Vision API\(^\text{11}\), AWS Rekognition API\(^\text{12}\), and Clarifai API\(^\text{13}\), the models of which are not revealed to the public. Previous works have demonstrated that the Google Cloud Vision API is vulnerable to adversarial attacks (Ilyas et al., 2018; Guo et al., 2019). In this work, we demonstrate for the first time, to the best of our knowledge, a certifiable defense for online APIs. Our approach is general and applicable to any API with no knowledge whatsoever of the API’s underlying model. Our defense only requires query-access to these APIs.

We focus on the classification service of each API.\(^\text{14}\) Given an input image, each API returns a sorted list of related labels ranked by the corresponding confidence scores. The simplest way to build a classifier from the information is to define the classifier’s output as the label with the highest confidence score among the list of labels returned.\(^\text{15}\) In this work, we adopt this simple strategy of obtaining a classifier from these APIs.

To assess the performance of our method on these APIs, we aggregate 100 random images from the ImageNet validation set and certify their predictions across all four APIs. We use 100 Monte-Carlo samples per data point to estimate the certified radius using Equation 2. We experiment with \(\sigma = 0.25\) and with two types of denoisers: an MSE-DnCNN trained on the full ImageNet, and a STAB+MSE DnCNN trained with ResNet-18/34/50 as surrogate models. We also compare to the “No denoiser” baseline, which refers to applying randomized smoothing directly on the APIs.

Figure 6 shows the certified accuracies for all the APIs using both STAB+MSE and MSE. Both denoisers outperform the baseline. Note how the certification results are in general best for stability-trained denoisers, although these were trained on surrogate models, and had no access to underlying models of the vision APIs. For details of the surrogate models used and for more results on these APIs, see Appendix D.

We believe this is only a first step towards certifying public machine learning APIs. Here we restrict ourselves to 100 noisy samples due to budget considerations, however, one can always obtain a larger certified radius by querying more noisy samples (e.g. 1k or 10k) (as verified in Appendix D). Our results also suggest the possibility for machine learning service providers to offer robust versions of their APIs without having to change their pretrained classifiers. This can be done by simply wrapping their APIs with our custom-trained denoisers.

6. Conclusion and Future Work

In this paper, we presented a simple defense called black-box smoothing that can convert existing pretrained classifiers into provably robust ones without any retraining or fine-tuning. We achieve this by prepping a custom-trained denoiser to these pretrained classifiers. We experimented with different strategies for training the denoiser and obtained significant boost over the trivial application of randomized smoothing on pretrained classifiers. We are the first, to the best of our knowledge, to show that we can provably defend online vision APIs.

There is still plenty of room for improving our method. For example, training denoisers that can get as good certified accuracies as white-box smoothing is still not fully achieved in our paper. We are able to achieve almost as good results as white-box smoothing in the full-access setting, but not in the query-access setting as shown in Figures 2b and 4. Finding methods that can train denoisers to close the gap between black-box smoothing and white-box smoothing remains a valuable future direction.
Black-box Smoothing: A Provable Defense for Pretrained Classifiers

References

Athalye, A. and Carlini, N. On the robustness of the cvpr 2018 white-box adversarial example defenses. arXiv preprint arXiv:1804.03286, 2018.

Athalye, A., Carlini, N., and Wagner, D. Obfuscated gradients give a false sense of security: Circumventing defenses to adversarial examples. arXiv preprint arXiv:1802.00420, 2018.

Cao, X. and Gong, N. Z. Mitigating evasion attacks to deep neural networks via region-based classification. In Proceedings of the 33rd Annual Computer Security Applications Conference, pp. 278–287. ACM, 2017.

Carlini, N. and Wagner, D. Adversarial examples are not easily detected: Bypassing ten detection methods. In Proceedings of the 10th ACM Workshop on Artificial Intelligence and Security, pp. 3–14. ACM, 2017a.

Carlini, N. and Wagner, D. Towards evaluating the robustness of neural networks. In 2017 IEEE Symposium on Security and Privacy (SP), pp. 39–57. IEEE, 2017b.

Cohen, J. M., Rosenfeld, E., and Kolter, J. Z. Certified adversarial robustness via randomized smoothing. arXiv preprint arXiv:1902.02918, 2019.

Croce, F., Andriushchenko, M., and Hein, M. Provable robustness of relu networks via maximization of linear regions. arXiv preprint arXiv:1810.07481, 2018.

Dvijotham, K., Gowal, S., Stanforth, R., Arandjelovic, R., O’Donoghue, B., Uesato, J., and Kohli, P. Training verified learners with learned verifiers. arXiv preprint arXiv:1805.10265, 2018a.

Dvijotham, K., Stanforth, R., Gowal, S., Mann, T., and Kohli, P. A dual approach to scalable verification of deep networks. UAI, 2018b.

Dvijotham, K. D., Hayes, J., Balle, B., Kolter, Z., Qin, C., Gyorgy, A., Xiao, K., Gowal, S., and Kohli, P. A framework for robustness certification of smoothed classifiers using f-divergences. In International Conference on Learning Representations, 2020.

Gehr, T., Mirdamadi, M., Drachsler-Cohen, D., Tsankov, P., Chaudhuri, S., and Vechev, M. Ai2: Safety and robustness certification of neural networks with abstract interpretation. In 2018 IEEE Symposium on Security and Privacy (SP), pp. 3–18. IEEE, 2018.

Goodfellow, I. J., Shlens, J., and Szegedy, C. Explaining and harnessing adversarial examples. ICLR, 2015.

Gowal, S., Dvijotham, K., Stanforth, R., Bunel, R., Qin, C., Uesato, J., Mann, T., and Kohli, P. On the effectiveness of interval bound propagation for training verifiably robust models. arXiv preprint arXiv:1810.12715, 2018.

Gu, S. and Rigazio, L. Towards deep neural network architectures robust to adversarial examples. arXiv preprint arXiv:1412.5068, 2014.

Guo, C., Rana, M., Cisse, M., and Van Der Maaten, L. Countering adversarial images using input transformations. arXiv preprint arXiv:1711.00117, 2017.

Guo, C., Gardner, J. R., You, Y., Gordon Wilson, A., and Q. Weinberger, K. Simple black-box adversarial attacks. arXiv preprint arXiv:1905.07121, 2019.

Gupta, P. and Rahtu, E. Ciid defence: Defeating adversarial attacks by fusing class-specific image inpainting and image denoising. In The IEEE International Conference on Computer Vision (ICCV), October 2019.

Ilyas, A., Engstrom, L., Athalye, A., and Lin, J. Black-box adversarial attacks with limited queries and information. In Dy, J. and Krause, A. (eds.), Proceedings of the 35th International Conference on Machine Learning, volume 80 of Proceedings of Machine Learning Research, pp. 2137–2146, Stockholmsmssan, Stockholm Sweden, 10–15 Jul 2018. PMLR.

Kurakin, A., Goodfellow, I., and Bengio, S. Adversarial machine learning at scale. arXiv preprint arXiv:1611.01236, 2016.

Lecuyer, M., Atlidakis, V., Geambasu, R., Hsu, D., and Jana, S. Certified robustness to adversarial examples with differential privacy. arXiv preprint arXiv:1802.03471, 2018.

Lee, G.-H., Yuan, Y., Chang, S., and Jaakkola, T. Tight certificates of adversarial robustness for randomly smoothed classifiers. In Advances in Neural Information Processing Systems 32, pp. 4911–4922, 2019.

Li, B., Chen, C., Wang, W., and Carin, L. Certified adversarial robustness with additive noise. In Advances in Neural Information Processing Systems 32, pp. 9459–9469, 2019.

Liao, F., Liang, M., Dong, Y., Hu, X., and Zhu, J. Defense against adversarial attacks using high-level representation guided denoiser. In The IEEE Conference on Computer Vision and Pattern Recognition (CVPR), June 2018.

Liu, X., Cheng, M., Zhang, H., and Hsieh, C.-J. Towards robust neural networks via random self-ensemble. In Proceedings of the European Conference on Computer Vision (ECCV), pp. 369–385, 2018.

Madry, A., Makelov, A., Schmidt, L., Tsipras, D., and Vladu, A. Towards deep learning models resistant to adversarial attacks. arXiv preprint arXiv:1706.06083, 2017.
Black-box Smoothing: A Provable Defense for Pretrained Classifiers

Meng, D. and Chen, H. Magnet: a two-pronged defense against adversarial examples. In Conference on Computer and Communications Security (CCS), 2017.

Mirman, M., Gehr, T., and Vechev, M. Differentiable abstract interpretation for provably robust neural networks. In International Conference on Machine Learning, pp. 3575–3583, 2018.

Prakash, A., Moran, N., Garber, S., DiLillo, A., and Storer, J. Deflecting adversarial attacks with pixel deflection. In The IEEE Conference on Computer Vision and Pattern Recognition (CVPR), June 2018.

Raghunathan, A., Steinhardt, J., and Liang, P. Certified defenses against adversarial examples. International Conference on Learning Representations (ICLR), arXiv preprint arXiv:1801.09344, 2018a.

Raghunathan, A., Steinhardt, J., and Liang, P. S. Semidefinite relaxations for certifying robustness to adversarial examples. In Advances in Neural Information Processing Systems, pp. 10877–10887, 2018b.

Salman, H., Li, J., Razenshteyn, I., Zhang, P., Zhang, H., Bubeck, S., and Yang, G. Provably robust deep learning via adversarially trained smoothed classifiers. In Advances in Neural Information Processing Systems, pp. 11289–11300, 2019a.

Salman, H., Yang, G., Zhang, H., Hsieh, C.-J., and Zhang, P. A convex relaxation barrier to tight robustness verification of neural networks. In Advances in Neural Information Processing Systems, pp. 9832–9842, 2019b.

Singh, G., Gehr, T., Mirman, M., Püschel, M., and Vechev, M. Fast and effective robustness certification. In Advances in Neural Information Processing Systems, pp. 10825–10836, 2018.

Szegedy, C., Zaremba, W., Sutskever, I., Bruna, J., Erhan, D., Goodfellow, I., and Fergus, R. Intriguing properties of neural networks. arXiv preprint arXiv:1312.6199, 2013.

Tai, Y., Yang, J., Liu, X., and Xu, C. Memnet: A persistent memory network for image restoration. In Proceedings of the IEEE international conference on computer vision, pp. 4539–4547, 2017.

Tramr, F., Papernot, N., Goodfellow, I., Boneh, D., and McDaniel, P. The space of transferable adversarial examples. arXiv preprint arXiv:1704.03453, 2017.

Uesato, J., O’Donoghue, B., Oord, A. v. d., and Kohli, P. Adversarial risk and the dangers of evaluating against weak attacks. arXiv preprint arXiv:1802.05666, 2018.

Wang, S., Chen, Y., Abdou, A., and Jana, S. Mixtrain: Scalable training of formally robust neural networks. arXiv preprint arXiv:1811.02625, 2018a.

Wang, S., Pei, K., Whitehouse, J., Yang, J., and Jana, S. Efficient formal safety analysis of neural networks. In Advances in Neural Information Processing Systems, pp. 6369–6379, 2018b.

Warren, H., Wei, J., Chen, X., Carlini, N., and Song, D. Adversarial example defenses: Ensembles of weak defenses are not strong. arXiv preprint arXiv:1706.04701, 2017.

Weng, T.-W., Zhang, H., Chen, H., Song, Z., Hsieh, C.-J., Boning, D., Dhillion, I. S., and Daniel, L. Towards fast computation of certified robustness for ReLU networks. In International Conference on Machine Learning, 2018.

Wong, E. and Kolter, Z. Provable defenses against adversarial examples via the convex outer adversarial polytope. In International Conference on Machine Learning (ICML), pp. 5283–5292, 2018.

Wong, E., Schmidt, F., Metzen, J. H., and Kolter, J. Z. Scaling provable adversarial defenses. Advances in Neural Information Processing Systems (NIPS), 2018.

Xie, C., Wu, Y., Maaten, L. v. d., Yuille, A. L., and He, K. Feature denoising for improving adversarial robustness. In The IEEE Conference on Computer Vision and Pattern Recognition (CVPR), June 2019.

Xu, W., Evans, D., and Qi, Y. Feature squeezing: Detecting adversarial examples in deep neural networks. In 25th Annual Network and Distributed System Security Symposium, NDSS, 2018.

Zhai, R., Dan, C., He, D., Zhang, H., Gong, B., Ravikumar, P., Hsieh, C.-J., and Wang, L. Macer: Attack-free and scalable robust training via maximizing certified radius. In International Conference on Learning Representations, 2020.

Zhang, H., Weng, T.-W., Chen, P.-Y., Hsieh, C.-J., and Daniel, L. Efficient neural network robustness certification with general activation functions. In Advances in Neural Information Processing Systems, pp. 4939–4948, 2018a.

Zhang, K., Zuo, W., Chen, Y., Meng, D., and Zhang, L. Beyond a Gaussian denoiser: Residual learning of deep CNN for image denoising. IEEE Transactions on Image Processing, 26(7):3142–3155, 2017.

Zhang, K., Zuo, W., and Zhang, L. Ffdnet: Toward a fast and flexible solution for CNN based image denoising. IEEE Transactions on Image Processing, 2018b.

Zheng, S., Song, Y., Leung, T., and Goodfellow, I. Improving the robustness of deep neural networks via stability training. In Proceedings of the ieee conference on computer vision and pattern recognition, pp. 4480–4488, 2016.
A. Experiments Details

In this appendix, we include details of all the experiments conducted in our paper.

A.1. Denoiser Models

The denoisers used in this paper are:

1. DnCNN (Zhang et al., 2017) https://github.com/cszn/DnCNN/tree/master/TrainingCodes/dncnn_pytorch.

2. DnCNN-Wide: a wide version of the DnCNN architecture; specifically convolutional layers with a width of 128 instead of 64. Check the code for more details.

3. MemNet (Tai et al., 2017) https://github.com/tyshiwo/MemNet.

We run experiments with all of these denoisers on CIFAR-10. For ImageNet, we stick to DnCNN only due to GPU memory constraints. Note that we do not claim these are the best denoisers in the literature. We just picked these common denoisers. A better choice of denoisers might lead to improvements in our results.

Training details Here we include the training details of each denoiser used in our paper. The results in the paper are reported for the best denoisers over all the hyperparameters (architectures, optimizer, and learning rate) summarized in the following table. Also, for each architecture, the reported training time of each model is averaged over all the instances of training this architecture with various optimizers and learning rates.

Table 5. Average training statistics for our denoisers. We run our experiments on NVIDIA P100 and V100 GPUs. For Stab+MSE, the “+” sign refers to the additional epochs/total time incurred over the MSE baseline since Stab+MSE is basically fine-tuning MSE denoisers.

| Trained Denoiser          | Objective | Compute | #Epochs | Optimizer          | Learning Rate | Sec/Epoch | Total Time (Hr) |
|---------------------------|-----------|---------|---------|--------------------|---------------|-----------|-----------------|
| CIFAR-10/DnCNN            | MSE       | 1 × P100| 90      | Adam (Adam, SGD)   | 1e − 3        | 31        | 0.78            |
| Stab+MSE ResNet-110       | MSE       | 1 × P100| +20     | AdamThenSGD        | (1e−4, 1e−5)  | 57        | +0.32           |
| Stab ResNet-110           |           | 1 × P100| 600     | AdamThenSGD        | See Below     | 59        | 9.80            |
| CIFAR-10/DnCNN-wide       | MSE       | 1 × P100| 90      | Adam (Adam, SGD)   | 1e − 3        | 122       | 3.05            |
| Stab+MSE ResNet-110       | MSE       | 1 × P100| +20     | AdamThenSGD        | (1e−4, 1e−5)  | 135       | +0.75           |
| Stab ResNet-110           |           | 1 × P100| 600     | AdamThenSGD        | See Below     | 156       | 26.00           |
| CIFAR-10/MemNet           | MSE       | 1 × P100| 90      | Adam (Adam, SGD)   | 1e − 3        | 85        | 2.13            |
| Stab+MSE ResNet-110       | MSE       | 1 × P100| +20     | AdamThenSGD        | (1e−4, 1e−5)  | 118       | +0.66           |
| Stab ResNet-110           |           | 1 × P100| 600     | AdamThenSGD        | See Below     | 125       | 20.85           |
| ImageNet/DnCNN            | MSE       | 4 × V100| 5       | Adam               | 1e − 4        | 6220      | 8.78            |
| Stab+MSE ResNet-18        | MSE       | 4 × V100| +20     | Adam               | 1e − 5        | 6500      | +36.11          |
| Stab+MSE ResNet-34        | MSE       | 4 × V100| +20     | Adam               | 1e − 5        | 6900      | +38.33          |
| Stab+MSE ResNet-50        | MSE       | 4 × V100| +20     | Adam               | 1e − 5        | 7620      | +42.33          |

Note that for Stab training, we find that using AdamThenSGD leads to significantly better performance than using only one of them. For this setting, we basically use Adam with a learning rate of 1e − 3 for 50 epochs, then use SGD with the following settings:

1. SGD with learning rate that starts at 1e − 4 and drops by a factor of 10 every 100 epochs.
2. SGD with learning rate that starts at 1e − 3 and drops by a factor of 10 every 100 epochs.
3. SGD with learning rate that starts at 1e − 4 and drops by a factor of 10 every 200 epochs.
4. SGD with learning rate that starts at 1e − 3 and drops by a factor of 10 every 200 epochs.
5. SGD with learning rate that starts at 1e − 2 and drops by a factor of 10 every 200 epochs.
6. SGD with learning rate that starts at 1e − 3 and drops by a factor of 10 every 400 epochs.

Also, note that for Stab+MSE ResNet-110, we sweep over the product of the two sets of optimizers {Adam, SGD}, and learning rates {1e−4, 1e−5}.

Please refer to our code for further details!
A.2. Pretrained Classifiers

Our method presented in the paper works on any pretrained classifier. For the sake of demonstrating its effectiveness, we use standard CIFAR-10 and ImageNet neural network architectures.

On CIFAR-10, we train our own versions of the classifiers found in the following repository https://github.com/kuangliu/pytorch-cifar, namely, we train:

- ResNet-110
- Wide-ResNet-28-10
- Wide-ResNet-40-10
- VGG-16
- VGG-19
- ResNet-18
- PreActResNet-18
- GoogLeNet
- DenseNet-121
- ResNetXt29_2x64d
- MobileNet
- MobileNet-V2
- SENet-18
- ShuffleNet-V2
- EfficientNet-B0.

We train these classifiers in a standard way with data augmentation (random horizontal flips and random crops). We train each model for 300 epochs using SGD with an initial learning rate of 0.1 that drops by a factor of 10 each 100 epochs. We provide these pretrained models and code to train them in the repository accompanying this paper.

On ImageNet, we use PyTorch’s ResNet-18, ResNet-34, and ResNet-50 pretrained ImageNet models from the following link https://pytorch.org/docs/stable/torchvision/models.html.

A.3. Certification Details

In order to certify our (denoiser, classifier) pairs, we use the CERTIFY randomized smoothing algorithm of Cohen et al. (2019).

For CERTIFY, unless otherwise specified, we use \( n = 10,000, n_0 = 100, \alpha = 0.001 \). Note that in (Cohen et al., 2019), \( n = 100,000 \), which leads to better certification results. Due to computational constraints, we decrease this by a factor of 10. All our results can be improved by increasing \( n \).

In all the above settings, we report the best models over all the hyperparameters we mentioned.

A.4. Source code

Our code and trained denoisers/classifiers can be found in the the following GitHub repository: https://github.com/microsoft/blackbox-smoothing. The repository also includes all our training/certification logs, which allows for easy replication of all our results!

---

16Note that when we train with STAB or STAB+MSE in the query-access setting in the main paper, we use all these models as surrogate models, excluding ResNet-110, since this is the pretrained model that we assume we only have query access to.
B. Detailed Experimental Results

In this part, we show more detailed experimental results of black-box smoothing on ImageNet and CIFAR-10 (More pretrained classifiers and more noise levels).

B.1. Our Best Certified Accuracies over a Range of $\ell_2$-Radii

Here we show the best certified accuracy we get at various $\ell_2$-radii. The results are shown in Table 6, Table 7, Table 8 and Table 9. Note that in both full-access and query-access settings, we outperform the baseline without denoisers\footnote{Table 6 and Table 9 are the same as Table 1 and Table 2 in the main paper, respectively.}.

Table 6. Certified top-1 accuracy of ResNet-50 on ImageNet at various $\ell_2$ radii (Standard accuracy is in parenthesis).

| $\ell_2$ RADIUS (ImageNet) | 0.25 | 0.5 | 0.75 | 1.0 | 1.25 | 1.5 |
|-----------------------------|------|-----|------|-----|------|-----|
| White-box smoothing (COHEN ET AL., 2019) (%) | (70)62 | (70)52 | (62)45 | (62)39 | (62)34 | (50)29 |
| No denoiser (baseline) (%) | (49)32 | (12)2 | (12)2 | (0)0 | (0)0 | (0)0 |
| Black-box smoothing (Query Access) (%) | (69)48 | (56)31 | (56)19 | (34)12 | (34)17 | (30)4 |
| Black-box smoothing (Full Access) (%) | (67)50 | (60)33 | (60)20 | (38)14 | (38)11 | (38)6 |

Table 7. Certified top-1 accuracy of ResNet-34 on ImageNet at various $\ell_2$ radii (Standard accuracy is in parenthesis).

| $\ell_2$ RADIUS (ImageNet) | 0.25 | 0.5 | 0.75 | 1.0 | 1.25 | 1.5 |
|-----------------------------|------|-----|------|-----|------|-----|
| White-box smoothing (COHEN ET AL., 2019) (%) | (60)50 | (53)44 | (53)39 | (53)33 | (53)28 | (42)22 |
| No denoiser (baseline) (%) | (44)26 | (5)2 | (5)1 | (0)0 | (0)0 | (0)0 |
| Black-box smoothing (Query Access) (%) | (65)47 | (53)32 | (53)18 | (34)12 | (34)18 | (34)3 |
| Black-box smoothing (Full Access) (%) | (64)47 | (55)32 | (55)19 | (35)12 | (35)18 | (16)4 |

Table 8. Certified top-1 accuracy of ResNet-18 on ImageNet at various $\ell_2$ radii (Standard accuracy is in parenthesis).

| $\ell_2$ RADIUS (ImageNet) | 0.25 | 0.5 | 0.75 | 1.0 | 1.25 | 1.5 |
|-----------------------------|------|-----|------|-----|------|-----|
| White-box smoothing (COHEN ET AL., 2019) (%) | (56)47 | (48)36 | (48)31 | (48)26 | (35)22 | (35)19 |
| No denoiser (baseline) (%) | (37)18 | (5)1 | (5)1 | (0)0 | (0)0 | (0)0 |
| Black-box smoothing (Query Access) (%) | (60)42 | (50)26 | (50)14 | (28)7 | (28)5 | (28)3 |
| Black-box smoothing (Full Access) (%) | (61)42 | (52)29 | (52)16 | (35)10 | (35)6 | (35)4 |

Table 9. Certified accuracy of ResNet-110 on CIFAR-10 at various $\ell_2$ radii (Standard accuracy is in parenthesis).

| $\ell_2$ RADIUS (CIFAR-10) | 0.25 | 0.5 | 0.75 | 1.0 | 1.25 | 1.5 |
|-----------------------------|------|-----|------|-----|------|-----|
| White-box smoothing (COHEN ET AL., 2019) (%) | (77)59 | (77)45 | (63)31 | (65)21 | (45)18 | (45)13 |
| No denoiser (baseline) (%) | (10)7 | (9)3 | (9)0 | (16)0 | (16)0 | (16)0 |
| Black-box smoothing (Query Access) (%) | (81)45 | (68)20 | (21)15 | (21)13 | (16)11 | (16)10 |
| Black-box smoothing (Full Access) (%) | (72)56 | (62)41 | (62)28 | (44)19 | (42)16 | (44)13 |
B.2. Certifying Full-Access CIFAR-10 Classifiers

Figure 7 shows the complete results for certifying CIFAR-10 pretrained classifiers with full access at various noise levels \( \sigma \in \{0.12, 0.25, 0.50, 1.00\} \). We can see that using denoisers trained with stability objective (STAB) can get close certification results to white-box smoothing. The results for \( \sigma = 0.25 \) are the same as Figure 2a in the main text.

![Figure 7](image)

Figure 7. Results for certifying a ResNet-110 CIFAR-10 classifier with full-access using various methods.

B.3. Certifying Query-Access CIFAR-10 Classifiers

Figure 8 shows the full results for certifying CIFAR-10 pretrained classifiers with query access at various noise levels \( \sigma \in \{0.12, 0.25, 0.50, 1.00\} \). Compared the full access setting, there is still a noticeable gap between black-box smoothing and the white-box smoothing. The results for \( \sigma = 0.25 \) are the same as Figure 2b in the main text. Note that for \( \sigma = 0.50 \), MSE is better than STAB for small \( \ell_2 \)-radii, but for this range of radii, one would practically choose models with smaller noise level, say \( \sigma = 0.12 \), as the certified accuracies of the latter are higher in this range of radii.

![Figure 8](image)

Figure 8. Results for certifying a ResNet-110 CIFAR-10 classifier with query-access using various methods.
B.4. Certifying Full-Access ImageNet classifiers

Figure 9, Figure 10 and Figure 11 show the complete results for certifying ImageNet pretrained classifiers with full access at various noise levels $\sigma \in \{0.25, 0.50, 1.00\}$. The results for $\sigma = 0.25$ are the same as Figure 3 in the main text. Notice that $\text{STAB+MSE}$ is better than $\text{MSE}$ for all cases. It can be seen that with larger noise level $\sigma$, the gap between black-box smoothing and white-box smoothing becomes larger.

Figure 9. Results for certifying a ResNet-18 ImageNet classifier with full-access.

Figure 10. Results for certifying a ResNet-34 ImageNet classifier with full-access.

Figure 11. Results for certifying a ResNet-50 ImageNet classifier with full-access.
B.5. Certifying Query-Access ImageNet classifiers

Figure 12, Figure 13 and Figure 14 show the full results for certifying ImageNet pretrained classifiers with query access at various noise levels $\sigma \in \{0.25, 0.50, 1.00\}$. The results for $\sigma = 0.25$ are the same as Figure 4 in the main text. In general, STAB+MSE outperforms MSE. Notice that similar to the case of full-access setting, the gap becomes larger between black-box smoothing and white-box smoothing.

![Figure 12](image1.png)

(a) $\sigma = 0.25$

![Figure 13](image2.png)

(b) $\sigma = 0.50$

![Figure 14](image3.png)

(c) $\sigma = 1.00$

*Figure 12.* Results for certifying a ResNet-18 ImageNet classifier with query-access.

*Figure 13.* Results for certifying a ResNet-34 ImageNet classifier with query-access.

*Figure 14.* Results for certifying a ResNet-50 ImageNet classifier with query-access.
C. More Surrogate Models, Better Transfer

Here we demonstrate that when certifying pretrained classifiers with query access, we can get better certification results if we use more surrogate models when training the denoisers. The comparisons are shown in Figure 15 on CIFAR-10 for STAB and STAB+MSE. We can see that in the case of STAB, using 14 surrogate models is important to generalize to an unseen ResNet-110 classifier especially for large \( \sigma \). The results for \( \sigma = 0.25 \) are the same as Figure 2c in the main text.

\[ t_2 \text{ radius} \]

\[ \sigma = 0.12 \]

\[ \sigma = 0.25 \]

\[ \sigma = 0.50 \]

\[ \sigma = 1.00 \]

Figure 15. Results for certifying a ResNet-110 CIFAR-10 classifier with query-access using various number of surrogate models with STAB and STAB+MSE.


D. Vision APIs Detailed Results

In this appendix, we present more detailed results of black-box smoothing on the four Vision APIs we consider in this paper.

D.1. Comparison between Stab+MSE and MSE objectives

Figure 16, Figure 17, Figure 18 and Figure 19 show the comparison between the certification results of Stab+MSE and MSE objectives for various vision APIs, surrogate models (ResNet-18/34/50), and noise levels $\sigma \in \{0.12, 0.25\}$. Observe that the performance of the Stab+MSE objective either roughly matches or outperforms the MSE objective\(^\text{18}\).

\(^{18}\)Figure 6 in the main text is generated by plotting, for each API, the best certification curve for Stab+MSE across the three surrogate models presented here, along with the MSE curve, and for $\sigma = 0.25$. 

---

**Figure 16.** The certification results of the Azure API using Stab+MSE (across various surrogate models) and MSE denoisers.

**Figure 17.** The certification results of the Google Cloud Vision API using Stab+MSE (across various surrogate models) and MSE denoisers.

**Figure 18.** The certification results of the Clarifai API using Stab+MSE (across various surrogate models) and MSE denoisers.

**Figure 19.** The certification results of the AWS API using Stab+MSE (across various surrogate models) and MSE denoisers.
D.2. More Monte Carlo Samples, More Robustness

Here we empirically demonstrate that by using more Monte Carlo samples per image in black-box smoothing, we get better certification bounds. Figure 20 reports the certified accuracies (over 100 samples of the ImageNet validation set), after applying black-box smoothing to the Azure Vision API, over a range of $\ell_2$-radii, and with 1000 vs. 100 Monte Carlo samples. Indeed, we notice that more samples lead to higher certified accuracies (i.e. more robust versions of the Azure API).

![Figure 20. The certification results, with 1000 vs. 100 Monte Carlo samples per image, of the Azure API with an MSE-denoiser.](image)
E. Stability vs Classification Objectives

The stability objective, introduced in subsection 3.3, is similar to another objective, namely the classification objective, which we will refer to as CLF. For this objective the loss function, previously defined in Equation 5, is now defined as the cross entropy loss between the output probabilities and the true labels $y_i$'s (as opposed to the pseudo-labels $f(x_i)$'s generated by the pretrained classifier $f$):

$$L_{\text{CLF}} = \mathbb{E}_{\mathcal{S}, \delta} \ell_{\text{CE}}(F(D_\theta(x_i + \delta)), y_i)$$

(6)

where $\delta \sim \mathcal{N}(0, \sigma^2 I)$.

Note that in the main text, we stick with the stability objective, since, as we show in the following subsections, the performance of the stability objective is comparable to that of the classification objective (and sometimes slightly better).

Also note that, for training the denoisers with classification objectives, we use the same hyperparameters as the stability objective shown in Table 5.

E.1. CIFAR-10

Figure 21 compares the performance of denoisers trained with STAB against denoisers trained with CLF on CIFAR-10. (a) and (b) show the full-access and query-access settings, respectively. Observe that the performance of the classification objectives is comparable to that of the stability objective.

Figure 21. Comparison between stability objective and classification objective on a ResNet-110 CIFAR-10 classifier.

E.2. ImageNet

Figure 22 compares MSE-trained denoisers fine-tuned on the stability objective (STAB+MSE) against MSE-trained denoisers fine-tuned on the classification objective (CLF+MSE) on ImageNet ResNet-18/30/50 in the full-access setting. Again, the performance of the classification objectives is comparable to that of the stability objective.

Figure 22. Comparison between stability objective and classification objective on ResNet-18/34/50 ImageNet classifiers.
E.3. Vision APIs

Figure 23, Figure 24, Figure 25 and Figure 26 show the performance of fine-tuning using the stability objective (STAB+MSE) and fine-tuning using the classification objective (CLF+MSE) on four vision APIs, with $\sigma \in \{0.12, 0.25\}$. We notice that STAB+MSE and CLF+MSE have largely the same performance.

Figure 23. The certification results of the Azure API with denoisers trained with STAB+MSE vs. CLF+MSE.

Figure 24. The certification results of the Google API with denoisers trained with STAB+MSE vs. CLF+MSE.

Figure 25. The certification results of the Clarifai API with denoisers trained with STAB+MSE vs. CLF+MSE.

Figure 26. The certification results of the AWS API with denoisers trained with STAB+MSE vs. CLF+MSE.
F. Denoising Examples on ImageNet

Figure 27. Performance of the various ImageNet denoisers on noisy images (first row) of standard deviation of 0.12, 0.25, 0.5, and 1.0 respectively from left to right.
Figure 28. Performance of the various ImageNet denoisers on noisy images (first row) of standard deviation of 0.12, 0.25, 0.5, and 1.0 respectively from left to right.
Figure 29. Performance of the various ImageNet denoisers on noisy images (first row) of standard deviation of 0.12, 0.25, 0.5, and 1.0 respectively from left to right.
Figure 30. Performance of the various ImageNet denoisers on noisy images (first row) of standard deviation of 0.12, 0.25, 0.5, and 1.0 respectively from left to right.
Figure 3. Performance of the various ImageNet denoisers on noisy images (first row) of standard deviation of 0.12, 0.25, 0.5, and 1.0 respectively from left to right.