DYNAMICAL AND THERMAL EVOLUTION OF THE QUARK-NOVA EJECTA

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ABSTRACT

We explore the dynamical and thermal evolution of the ejected neutron star crust in a quark-nova explosion. Typical explosion energies and ejected crust masses result in relativistic ejection with Lorentz factors of a few to a few hundreds. The ejecta undergoes a rapid cooling and stretching resulting in break up into many small pieces (clumps) when the ejecta is only ~100 km from the explosion site. The number and size of the clumps depend on whether the breakup occurs in the liquid or solid phase. For these two cases, the clump number is \( \sim 10^3 \) (liquid phase) or \( \sim 10^7 \) (solid phase) and, at break up, are spherical (size \( \sim 10^4 \) cm; liquid phase) or needle shaped (\( \sim 10^4 \times 10^2 \) cm; solid phase).

Key words: dense matter – stars: mass loss – stars: neutron – supernovae: general

1. INTRODUCTION

In the quark-nova (QN) picture, (Ouyed et al. 2002; Keränen et al. 2005; hereafter ODD and KOJ, respectively) the \((u,d)\) core of a hybrid star\(^1\) that undergoes the phase transition to the \((u, d, s)\) quark phase shrinks in a spherically symmetric fashion to a stable, more compact strange matter configuration faster than the overlying material (the neutron-rich hadronic envelope) can respond, leading to an effective core collapse. The core of the neutron star is a few kilometers in radius initially, but shrinks to \(1–2\) km in a collapse time of about \(0.1\) ms [Lugones et al. 1994]. The gravitational potential energy released (plus latent heat of phase transition) during this event is converted partly into internal energy and partly into outward propagating shock waves which impart kinetic energy to the material that eventually forms the ejecta.

There are three previously proposed mechanisms for ejection of the outer layers of the neutron star (i.e., crust).

1. Unstable baryon to quark combustion leading to a shock-driven explosion (Horvath & Benvenuto 1988). More recent work, assuming realistic quark matter equations of state, argues for strong deflagration (Drago et al. 2007) that can expel surface material. In these models up to \(10^{-2} M_\odot\) can be ejected. These calculations focus on the microphysics and not on the effect of the global state of the resulting quark core which collapses prior to complete combustion (KOJ), leading to conversion only of the inner core \((\sim 1–2)\) km of the neutron star.

2. Neutrino-driven explosion where the energy is deposited in a thin (the densest) layer at the bottom of the crust above a gap separating it from the collapsing core (KOJ). For the neutrino-driven mechanism, the core bounce was neglected, and neutrinos emitted from the conversion to strange matter transported the energy into the outer regions of the star, leading to heating and subsequent mass ejection. Consequently, mass ejection is limited to about \(10^{-5} M_\odot\) (corresponding to the crust mass below neutron drip density) for compact quark cores of size \((1–2)\) km.

3. Thermal fireball-driven ejection which we consider for the present study. The fireball is inherent to the properties of the quark star at birth. The birth temperature was found to be of the order of \(10–20\) MeV since the collapse is adiabatic rather than isothermal (ODD; KOJ). In this temperature regime, the quark matter is in the superconducting color–flavor locked (CFL) phase (Rajagopal & Wilczek 2001) where the photon emissivity dwarfs the neutrino emissivity (Vogt et al. 2004; Ouyed et al. 2005). The average photon energy is \(\sim 3T\) for a CFL temperature \(T\); since the plasma frequency of CFL matter is \(\hbar \omega \sim 23\) MeV (e.g., Usov 2001), the photon emissivity is greatly attenuated as soon as the surface temperature of the star cools below \(T_\alpha = \hbar \omega/3 \sim 7.7\) MeV. To summarize, the thermal fireball is generated as the star cools from its birth temperature down to \(\sim 7.7\) MeV.

Here we focus on the case where a QN goes off in isolation, neglecting any interaction with the surroundings, and the crust is ejected by a thermal fireball (i.e., Case 3 above). The remainder of this paper is presented as follows. In Section 2, we discuss the energetics of the QN and the resulting fireball and ensuing crust ejection. Section 3 deals with the hydrodynamical and thermal evolution of the ejecta. We conclude this paper in Section 4.

2. FIREBALL GENERATION AND CRUST EJECTION

The QN ejecta, which is the left-over crust of the parent neutron star, is initially in the shape of a shell and is imparted with energy from the QN explosion. This energy released during the QN explosion \((E_{QN,53} ; \text{in units of } 10^{53} \text{ erg})\) is a combination of baryon to quark conversion energy and gravitational energy release due to contraction. Also, the rapid contraction creates a gap between the surface of the collapsed quark matter core and the inner edge of the remaining hadronic matter. Before entering the CFL phase at \(T \sim 10–20\) MeV, the QS cools mainly by neutrino emission; as shown in Keränen et al. (2005), a significant fraction of neutrinos escape through the crust. Once the QS core enters the CFL phase, the energy is released in photons (see Vogt et al. 2004; Ouyed et al. 2005). Then at that point thermalization leads to the creation of a \((e^+e^-)\) fireball with a temperature of \(10–20\) MeV. Thus, the initial state for crust ejection is a QS core surrounded by a fireball in the gap

\(^1\) As outlined in KOJ, the initial state for the QN is that of a deleptonized neutron star with a \((u,d)\) core. In the QN, the two-step process, neutron to \((u,d)\), then \((u,d)\) to \((u, d, s)\) is crucial. In our case, the neutrinos come from weak reactions at the edge of the \((u,d)\) core and can leak out easily into the surrounding cooler and deleptonized envelope where they can deposit energy. This is significantly different from phase conversion in a proto-neutron star stage where neutrino transport is slower (of the order of seconds) because of the hot and lepton-rich matter.
(1 km) in turn surrounded by the initially stationary neutron star crust.

The general picture is that the fireball from the QN acts as a piston at the base of the crust, i.e., the QS fireball expands approximately adiabatically while pushing the overlaying crust, and cooling fairly rapidly. Using the relativistic force equation, a simple estimate for the timescale to accelerate the crust to the Lorentz factor of a few is \( \sim 0.1-1 \) ms. The force is due to the fireball pressure \( F = PA \), where \( P = (a/3)T^4 \), \( T \sim 10\text{–}20 \text{ MeV} \) and \( A \) is the shell’s area. The acceleration timescale is only a factor of a few smaller than the free-fall timescale. As the crust moves outward, the fireball cools approximately adiabatically, with the temperature \( T \) proportional to \( V^{-1/3} \) with \( V \) being the volume between the quark star surface and the inner edge of the crust. Initially, the temperature decreases slowly with \( R(t) \) then later it decreases as \( 1/R(t) \), where \( R(t) \) is the radial position of the crust. Furthermore, the final kinetic energy of the crust for a Lorentz factor of 10 is about \( 2 \times 10^{50} \text{ erg} \) for a \( 10^{-4} M_\odot \) crust. Thus the energy needed to eject the crust is less than 1% of fireball energy.

The energy input to the crust is mainly kinetic and only a small fraction is thermal since there is no strong shock in this scenario. During the acceleration, the crust can respond on timescales \( \Delta R/c_s \), where \( \Delta R \) is the crust thickness (about 1 km), and \( c_s \) is the speed of sound in the crust. The majority of the crust is degenerate with \( c_s = c/\sqrt{3} \). The response timescale is \( \sim 5 \times 10^{-6} \) s, about 1/100 of the acceleration timescale. Thus no strong shock will be generated except in the outermost nondegenerate (i.e., low sound speed) layers (the mass contained in the nondegenerate layers is small \( < 10^{-6} M_\odot \)). The bulk of the crust is thus accelerated smoothly by the pressure from the fireball. In comparison, a normal SN has the energy of a strong shock is generated. In the QN case, most of the crust is thus accelerated by the pressure from the fireball. In comparison, a normal SN has the energy of the fireball. In comparison, a normal SN has the energy of the fireball. In comparison, a normal SN has the energy of

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One can estimate the Lorentz factor\(^2\) of the QN ejecta by

\[
\Gamma_1 = \frac{\eta E_{\text{QN}}}{m_{\text{ej,53}} c^2} \approx 50 \frac{\eta_{0.1} E_{\text{QN,53}}}{m_{\text{ej,4}}} ,
\]

where \( \eta_{0.1} \) is the efficiency of energy transfer from the QN to the ejecta’s kinetic energy in units of 0.1, and the ejecta mass, \( m_{\text{ej,4}} \), is given in units of \( 10^{-4} M_\odot \). Mass ejection in the QN scenario depends on details of energy transfer to the crust. Best estimates of ejected mass from existing calculations are \( 10^{-5} M_\odot \) to \( 10^{-2} M_\odot \). We adopt a rounded, fiducial, ejected mass of \( 10^{-4} M_\odot \) for the remainder of this paper. Heavier mass ejection would lead to mildly or nonrelativistic ejecta.

3. EVOLUTION OF THE QUARK-NOVA EJECTA

For ejecta masses higher than \( 10^{-4} M_\odot \), the corresponding initial density and size (i.e., of the neutron star crust) just before ejection are \( \rho_0 > 10^{10} \text{ g cm}^{-3} \) and \( \Delta r_0 > 0.025 r_0 \), respectively (e.g., Datta et al. 1995), where \( r_0 \) is the radius of the parent neutron star before the QN explosion. Also, the ejecta’s birth temperature, \( T_0 \), is estimated to be of the order of 10 MeV. Using general volume expansion \( (V = V_0 r/r_0)^{\alpha_v} \), mass conservation implies that the density of the ejecta will be \( \rho = \rho_0 (r/r_0)^{-\alpha_v} \). We define \( \alpha_v \) by a power-law dependence of \( V \) on \( r \), with different \( \alpha_v \) describing different physical situations, e.g., constant volume, adiabatic, etc.. At this density and temperature, the ejecta is relativistic degenerate, so that the Fermi energy evolves with radius as \( \epsilon_F \propto r^{1/3} \propto (L/r_0)^{-\alpha_v/3} \) (see Appendix A for details).

The total internal energy, \( U \), in the ejecta at birth can be estimated to be \( U_0 \sim 5 \times 10^{57} \text{ erg} \). Here, the ejecta mass is given in units of \( 10^{-4} M_\odot \), its density in units of \( 10^{10} \text{ g cm}^{-3} \), and \( \mu_e = 2 \) is the mean molecular weight dominated by the degenerate electrons. The corresponding heat equation describing the thermal evolution of the ejecta can then be written as

\[
\frac{dU}{dt} = -L_\gamma - P \frac{dV}{dt} = -L_\gamma - \frac{2}{3} U \frac{d\ln V}{dt} ,
\]

where all quantities are expressed in the ejecta’s frame. Since the ejecta is expanding relativistically, in its frame we can approximate \( \frac{dU}{dt} \approx \frac{dL_\gamma}{dt} \), with \( r \) in this case being the distance to the star from the ejecta. This allows the equation above to be recast into

\[
\frac{r^{-2\alpha_v/3}}{V} \frac{dU}{dr} \approx -L_\gamma c ,
\]

3.1. Relativistic Expansion of a Degenerate Ejecta

We solve for the heat equation (Equation (3)) for the cases of adiabatic and isothermal expansion of the QN ejecta in the shape of a spherical shell. The expansion is relativistic and assumed to occur at constant speed. The analysis is in the ejecta’s frame. In the adiabatic case, \( L_\gamma = 0 \) which leads to \( U/U_0 = (r/r_0)^{-2\alpha_v/3} \). Using the expression for \( \epsilon_F \) (with \( \epsilon_{F,0} = \epsilon_F(\rho_0) \)) given in Appendix A, we arrive at

\[
\frac{r}{r_0} = \left(1 + \frac{5\pi^2}{12} \frac{(k T_0)}{\epsilon_{F,0}} \right)^{3/\alpha_v} ,
\]

which shows that the temperature goes to zero in finite time corresponding to a radius

\[
\frac{r}{r_0} = \left(1 + \frac{5\pi^2}{12} \frac{(k T_0)}{\epsilon_{F,0}} \right)^{3/\alpha_v} .
\]

Clearly, the radius is too close to the origin which means that in the adiabatic case the ejecta would cool almost immediately and would continue to expand at zero temperature, with \( U = (3/5) N \epsilon_F \). In reality, the ejecta will solidify when it cools below the melting temperature of iron (see Section 3.2 below).

In the isothermal case, \( T = T_0 \), the heat equation leads to

\[
U/U_0 = \left(1 - A_{avv} \left(1 - \frac{r}{r_0}\right)^{-2\alpha_v/3} \right) ,
\]

with \( A_{avv} \sim 6.11 \frac{\mu_e}{\rho_0} \frac{c^2}{T_0} \frac{T_i^4}{m_4} \rho_0^{1/3} \). \( r_0 \) is the radius in units of 10\( ^6 \) cm, and \( T_{0.10} \) is the temperature in units of 10 MeV. The internal energy \( U \) goes to zero at a finite radius

\[
\frac{r}{r_0} = \frac{1}{1 + \frac{1}{A_{avv}}} ,
\]
which corresponds to \( r \sim 1.15r_0 \) for \( \alpha_e \sim 9/4 \). The corresponding ejecta density at this radius is \( \sim 10^6 \) gm cm\(^{-3} \). For \( T_0 \sim 0.1 \) MeV, we find \( r \sim 17r_0 \) for \( \alpha_e \sim 9/4 \), with a corresponding ejecta density \( \sim 10^8 \) gm cm\(^{-3} \).

In summary, applying the heat equation to the relativistically expanding ejecta shows an almost instantaneous loss of the internal energy. This mainly due to the PdV work done by the ejecta, as it expands or due to its rapidly increase in area which leads to efficient cooling.

### 3.2. Liquid to Solid Transition

In the early stages the QN ejecta, with a temperature in the tens of MeV, resembles a hot molten plasma. Because it is degenerate it cools extremely rapidly (see Appendix A). As the ejecta moves outward, it expands and cools undergoing a liquid to solid transformation—we will refer to the corresponding radius as the solidification radius \( r_s \). The state of ions (in this case iron nuclei) can be conveniently specified by the Coulomb plasma parameter (e.g., Potekhin et al. 1999).

\[
\Xi = \frac{0.26Z^2}{T_{\text{keV}}} \left( \frac{\rho_b}{A} \right)^{1/3} \approx 1640 \left( \frac{\rho_b}{T_{\text{keV}}} \right)^{1/3},
\]

where \( Z \) and \( A \) are the ion charge number and the atomic weight of iron. Since here \( \Xi > 1 \) prior to solidification, the shell material constitutes a strongly coupled liquid. Solidification occurs for \( \Xi > \Xi_m = 172 \) which implies a solidification temperature of \( T_s \sim 9.5 \rho_b^{1/3} \) keV (see also de Blasiis 1995). For example, using equation above and for \( \alpha_e \sim 9/4 \), an ejecta born at \( \sim 10 \) MeV and \( \rho_b = 10^{10} - 10^{11} \text{ gm cm}^{-3} \), will cool to below \( T \sim 10 \) keV and solidify when it reaches a radius of \( \sim 3r_0-10r_0 \). The solidification radius is then \( r_s \sim 30-100 \) km, at which point the ejecta’s density is \( \rho_e > 10^8 \) g cm\(^{-3} \) (see Equation B.1 in Appendix B). The corresponding ejecta density at this radius is \( \sim 10^8 \) gm cm\(^{-3} \).

Previous studies of decompressed neutron star crust show that the material heats up as it expands because the matter fuses into heavier elements and releases energy, which could prevent crystallization. The relevant calculations for expanding quark-nova ejecta have been performed in detail in Jaikumar et al. (2007). That work considers \( r \)-process in quark-nova ejecta with and without \( \beta \)-decay heating. In particular, Section 3 in that paper discusses the decompression of the ejecta. As can be seen from Figures 5 and 6 in Jaikumar et al. (2007), the ejecta does not reheat above 10 keV (see peak in temperature at time of a few milliseconds). The temperature prior to \( \beta \)-decay reheating is low enough for crystallization and break up to occur. However, \( \beta \)-decay reheating which peaks at a few milliseconds after ejection (see Figures 5 and 6 in Jaikumar et al. 2007) might remelt the crystallized ejecta. Even so, the melting temperature and the temperature of the reheated ejecta may be similar: this would lead to crystallization at some later time. Finally, we note that those calculations did not include radiative losses, so they provide an upper limit on the ejecta reheating temperature, thus increasing the likelihood of crystallization.

### 3.3. Ejecta Breakup

Regardless of when crystallization occurs, the relativistic expansion causes rapid breakup into small chunks because of the inability of causal communication laterally in the shell. Specifically, as the spherical shell expands radially outward relativistically, it also stretches laterally relativistically.\(^3\) Since the lateral expansion of the matter in the shell is limited by the speed of sound, \( c_s \), adjacent patches of the shell separate leading to breakup into small clumps.

The breakup depends on the existence of surface tension in the expanding shell. For liquid or solid iron (our case), interionic forces (mediated by the electrons) provide the tension. Such a tension does not exist in a gas. e.g., for SN ejecta where the shock accelerates the gas beyond the sound speed the lack of surface tension means the particles simply expand away from each other. An analogy is that of a bursting balloon versus expanding dust. For the innermost SN ejecta, the density may be high enough that it behaves like a liquid rather than a gas. In that case, it is likely that the sound speed (which increases inward) is lower than the expansion speed in the inner parts (which decreases inward; \( v \propto r \)). If the SN ejecta is still a liquid at the radius where the expansion speed exceeds the speed of sound then it will be subject to this lateral breakup mechanism.

The size of the clumps depends on whether the breakup occurs in the liquid or solid phase. We discuss the solid phase case first; the liquid case differs in having a much larger breakup strain \( \psi \). As the ejecta solidifies at \( r_s \), a strain rapidly builds up inside as the ejecta continues to expand relativistically. Defining the breaking strain of solid iron to be \( \psi \sim 10^{-3} \) (e.g., see Section 13.6 of Halliday & Resnick 1988) then the ejecta breakup occurs at roughly a radius of \( \psi \sim 1 \) \( r_s \) (see Appendix C). The typical chunk size at birth is \( c_s \Delta r \) where \( \Delta r = (r_b - r_s)/c \), or,

\[
\Delta r_c \sim 186 \text{ cm } \xi_0 \rho_{b,100},
\]

where we defined \( \xi_0 = \psi^{1/6} \). Here \( \psi \) and the breakup radius are given in units of \( 10^{-3} \) and 100 km, respectively, while the density at solidification radius is in units of \( 10^8 \text{ gm cm}^{-3} \).

Table 1 lists the properties of the clumps/chunks for the two cases of breakup occurring in early liquid or later solid phase. If the shell breaks into clumps during the liquid phase, there are two possibilities.

### 1. The chunks will breakup into smaller pieces when they solidify. Because of the degeneracy of the ejecta, the cooling is so rapid that even if initial breakup occurs in the liquid phase it will be immediately followed by another breakup governed by the smaller breaking strain \( \psi \) of the solid. Since the breakup radius is hardly changed the solid chunk size should be independent of whether clumping first took place during the liquid phase or not.

### 2. The ambient temperature remains high enough to keep the clumps liquid as they expand. We note that as the density drops due to expansion, the crystallization temperature also drops \( T_m \propto r^{-2/3} \) (since \( \rho \propto r^{-2} \)). There is a delicate competition between the dropping ambient temperature and the dropping crystallization temperature.

\(^3\) The lateral separation velocity for 2 points separated by \( \Delta \theta \) is \( c \Delta \theta \) (see Appendix C).

### Table 1

Clump Chunk Properties at Breakup for Liquid (L) and Solid (S; see Appendix C) Cases

| Case | \( \psi \) | \( \Delta r_c \) (cm) | \( \theta \) (rad) | \( m_c \) (gm) | \( N_c \) |
|------|----------|-------------------|-----------------|--------------|------|
| S    | \( 10^{-3} \) | 190               | 2 \times 10^{-5} | \( 1.7 \times 10^{19} \) | 2.5 \times 10^{7} |
| L    | \( 10^{-1} \) | \( 1.9 \times 10^{3} \) | 2 \times 10^{-3} | \( 1.7 \times 10^{23} \) | 2.5 \times 10^{7} |
3.4. Expansion of Broken Ejecta

Beyond the breakup radius, \( r_b \), the chunks remain in contact with each other within the relativistically expanding ejecta as a whole. This is because the pieces expand in volume, filling up the space between them, and causing the density of each piece to continuously decrease, until they reach the zero pressure iron density \( (\rho_P \sim 10^6 \text{ g cm}^{-3}) \), at which point they stop expanding. If we define a filling factor to be, \( f_b = 1 \) at \( r_b \), then it will remain unity until some separation radius \( r_{\text{sep}} \). This radius of separation can be found from Equation (B3). With \( \rho = \rho_P \) and our fiducial values this radius is

\[
\frac{r_{\text{sep}}}{r_0} \sim 1300, \tag{10}
\]

which implies that the chunks remain closely packed until the ejecta as a whole reaches \( r_{\text{sep}} \sim 10^6 \text{ cm} \). This is an upper limit for the separation radius since we neglected the effects of the ions (see Appendix B).

An estimation of the cross-section area extended by each chunk just before separation is given by \( N_c \Delta r_{\text{c,sep}}^2 = \theta_{\text{B}}^2 \Delta r_{\text{c,sep}}^2 \), or, \( \Delta r_{\text{c,sep}} \sim 3 \times 10^4 \text{ cm } \theta_0^2 \) (see Appendix C for details). Furthermore, their length can be found, from Equation (B2), to be \( \Delta r_{\text{ejecta,sep}} \sim 5 \times 10^6 \text{ cm } m_{\text{ejecta}}^{-4/3} \). Hence, we expect the chunk’s length is roughly a hundred times its width for Case 1 or equal to its width for Case 2. In the observer’s frame the ratio of length to width will be contracted by a factor of \( 1/\Gamma \). For Case 1, the chunks thus resemble what we will refer to as “iron needles,” reminiscent of the subjet model of Toma et al. (2005) in the context of GRBs. Finally, as the chunks expand radially outward to a radius \( r_{\text{out}} > r_{\text{sep}} \), we can associate a filling factor \( f_{\text{out}} = \frac{r_{\text{out}}^2}{r_{\text{sep}}^2} \).

4. SUMMARY AND CONCLUSION

In this paper, we investigate the thermal and dynamic evolution of a relativistically expanding iron-rich shell from a QN explosion. The QN produces a photon fireball which acts as piston to eject and accelerate the crust of the parent neutron star to mildly relativistic speeds. We find that the shell rapidly cools and breaks up into numerous \( (10^2-10^4) \) chunks because of the rapid lateral expansion. Breakup may occur while the material is in liquid or solid phase; if liquid the chunks are nearly spherical; if solid they are needle shaped moving parallel to their long axis.

Although the presented model is based on physical arguments, most of these are in reality more complicated and so would require more detailed studies and the help of numerical simulations. For example, the process of clumping, crystallization, and breakup of the ejecta, would require better knowledge of the ambient conditions surrounding the ejecta.

If verified by numerical studies, the crust ejection and subsequent breakup into chunks has interesting astrophysical implications. First, the ejected crust offers conditions for successful r-process nucleosynthesis (Jaikumar et al. 2007) and might act as the second r-process site discussed in the literature (e.g., Kratz et al. 2007). The additional energy from the QN, following a supernova, provides a plausible explanation for hypernovae (super-luminous supernovae; Leahy & Ouyed 2008). Finally, we are currently studying the interaction of the chunks with the Wolf-Rayet envelope in the context of Gamma-ray bursts progenitors. Our findings, so far, suggest that this interaction can account for optical and X-ray precursor activity. While the above phenomena are indirect evidence of the QN, we are exploring the observational signatures of an isolated neutron star converting to a quark star via the QN and the resulting MeV fireball. We hope to isolate distinct signatures that might confirm the QN scenario.

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APPENDIX A

FERMI GAS

The Fermi energy for a relativistic and a non-relativistic gas is \( \epsilon_F \simeq 1.413 \times 10^{-5} \text{ erg } \rho_{0/3}^{1/3} \simeq 8.8 \text{ MeV } \rho_{0/3}^{1/3} \) and \( \epsilon_F \simeq 1.219 \times 10^{-4} \text{ erg } \rho_{0/3}^{1/3} \simeq 76.2 \text{ MeV } \rho_{0/3}^{1/3} \), respectively (e.g., Shapiro & Teukolsky 1983, p24). The internal energy, \( u \), per particle is

\[
u = \frac{3}{5} \epsilon_F \left[ 1 + \frac{5 \pi^2}{12} \left( \frac{kT}{\epsilon_F} \right)^2 \right], \tag{A1}
\]

with a corresponding pressure \( P = (2/3)(u/v) \), where \( v \) is the volume per particle. In the relativistic degenerate regime, \( P = \kappa_c \rho^{4/3} \) with \( \kappa_c = 1.244 \times 10^{34} \text{ cm}^{-4} \text{ erg}^{-1/3} \); in the non-relativistic regime \( P = \kappa_m \rho^{5/3} \) with \( \kappa_m = 9.91 \times 10^{41} \text{ cm}^{-5/3} \). The corresponding sound speed is \( c_{s,c} \simeq 2.57 \times 10^{17} \rho_{0/3}^{1/6} \text{ cm s}^{-1} \) and \( c_{s,m} \simeq 2.26 \times 10^6 \rho_{0/3}^{1/3} \text{ cm s}^{-1} \), respectively. The transition from relativistic to nonrelativistic degeneracy occurs at density \( \rho_c \simeq 2 \times 10^6 \text{ g cm}^{-3} \). Finally, the transition from Fermi–Dirac to Boltzmann statistics occurs at the degeneracy temperature \( kT \sim \epsilon_F \).

APPENDIX B

EVOLUTION OF EJECTA THICKNESS AND DENSITY

We approximate the pressure in the ejecta by degenerate electron gas pressure neglecting the contribution from the non-relativistic ions. The ejecta thickness increases in time at the speed of sound \( d(\Delta r) = c_{s,c} dt \) with \( dt = dr/c \) in the ejecta’s frame. It is straightforward to show that the ejecta’s density quickly drops below the transition density \( \rho_0 \sim 2 \times 10^6 \text{ g cm}^{-3} \) so that most of the ejecta thickness expansion occurs during the non-relativistic degenerate phase. That is, \( d(\Delta r) \sim 2.26 \times 10^6 \rho_{0/3}^{1/3} dt \) with \( \rho = m_{\text{ejecta}}/(4 \pi r^2 \Delta r) \). Combining these equation gives

\[
\frac{\Delta r}{\Delta r_0} \simeq \left( 1 + 355B \left( \frac{r}{r_0} \right)^{1/3} - 1 \right)^{3/4}, \tag{B1}
\]

where \( B = m_{\text{ejecta}} r_{0/3}^{-4} \Delta r_{0/3}^{4/3}/\Delta r_{0/4}^{4/3} \) with the ejecta’s thickness at birth \( \Delta r_{0/4} \) given in units of \( 10^4 \text{ cm} \), corresponding to \( \rho_0 \sim 10^{10} \text{ g cm}^{-3} \) (e.g., Datta et al. 1995). For \( r \gg r_0 \),

\[
\Delta r \sim 2.6 \times 10^4 \text{ cm } m_{\text{ejecta}}^{-1/4} \Delta r_{0/4}^{1/4}. \tag{B2}
\]
Mass conservation then gives (for \( r \gg r_0 \))
\[
\frac{\rho}{\rho_0} \sim \left( \frac{r_0}{r} \right)^{9/4} \quad .
\]

**APPENDIX C**

**EJECTA BREAK-UP**

A spherical shell expanding radially at a speed of light gives two points separated by \( \Delta \theta \) a lateral separation velocity \( c \Delta \theta \). This causes a strain to rapidly build up leading to break up. Here we discuss the case that the shell has solidified prior to break up (see Table 1 for the liquid case). A typical breaking strain of iron is \( \psi \sim 10^{-3} \) (e.g., Halliday & Resnick 1988, Section 13.6). The matter in the ejecta expands at the speed of sound \( (c_s) \) yielding a the break-up radius of
\[
 r_b = r_s \left( 1 + \psi \left\{ \frac{\beta c}{G_{cs}} m_{\text{ejecta}} \right\}^{-1/4} \right) \approx r_s (1 + \psi) ,
\]
where \( r_s \) is the solidification radius and \( \beta c \) is the velocity of the ejecta.

The typical chunk size at birth is \( c_s \Delta \tau \) where \( \Delta \tau = (r_b - r_s)/c \), or,
\[
 \Delta r_c \sim 186 \text{ cm } \psi \sim 16 \psi r_{b,100} = 186 \text{ cm } \xi_b r_{b,100} ,
\]
where we have defined \( \xi_b = \psi \sim 16 \psi \). Here \( \psi \) and the break-off radius are given in units of \( 10^{-3} \) and 100 km, respectively, while the density at solidification radius is in units of \( 10^8 \text{ g cm}^{-3} \).

The angle subtended (measured from the QS) by a broken chunk of the ejecta at birth (i.e., at the break-up radius \( r_b \)) is
\[
 \theta_c = \Delta r_c/r_b \sim 2 \times 10^{-5} \text{ rad } \xi_b .
\]
The corresponding solid angle is then
\[
 \Omega_c = \pi \theta_c^2 \sim 5 \times 10^{-10} \text{ sr } \xi_b^2 ,
\]
The corresponding mass of the broken-off chunk is then roughly,
\[
 m_c \equiv \frac{\Omega_\text{c}}{4\pi} m_{\text{ejecta}} \sim 1.7 \times 10^{19} \text{ gm } \xi_b^2 m_{\text{ejecta}}^{-4} .
\]
The ejecta breakup only depends on the stress in the two dimensions due to the ejecta’s lateral expansion being independent of its thickness. Thus, the chunk mass is linearly dependent on the ejecta mass as a more massive ejecta would be thicker. The total number of chunks can be estimated as
\[
 N_c = \frac{\theta_c^2}{\theta_b^2} \sim 2.5 \times 10^7 \frac{\xi_b^2 \theta_b^0.1}{\xi_b^2} ,
\]
where the collimation angle in the ejecta frame is \( \theta_{b,c} = \Gamma \theta_b \).

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