Decay Rate Difference in the Neutral B-System: \( \Delta \Gamma_{B_s} \) and \( \Delta \Gamma_{B_d} \)

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We review the theoretical status of the predictions for the decay rate differences in the neutral B-system. We find \( (\Delta \Gamma/\Gamma)_{B_s} = (12 \pm 5) \cdot 10^{-2} \) and \( (\Delta \Gamma/\Gamma)_{B_d} = (3 \pm 1.2) \cdot 10^{-3} \).

1 Introduction

Recently the width difference \( (\Delta \Gamma/\Gamma)_{B_s} \) of the \( B_s \) meson CP eigenstates was measured at the Tevatron by the CDF Collaboration:

\[
(\frac{\Delta \Gamma}{\Gamma})_{B_s} = 0.65^{+0.25}_{-0.33} \pm 0.01 .
\] (1)

This result can be compared with the Particle Data Group value

\[
(\frac{\Delta \Gamma}{\Gamma})_{B_s} < 0.54 \ (95\% \mathrm{C.L.}) .
\] (2)

In view of this new result it seems to be appropriate to update the theoretical numbers present in the literature, see e.g. Phenomenological aspects of the width difference will not be discussed in this letter, we refer the interested reader to e.g.

The calculation of \( \Delta \Gamma_{B_s} \) is performed in the framework of the heavy quark expansion (HQE), which offers the possibility to expand decay rates in powers of \( \Lambda_{QCD}/m_b \). In the case of \( (\Delta \Gamma/\Gamma)_{B_s} \), the leading contribution is parametrically of order \( 16\pi^2(\Lambda_{QCD}/m_b)^3 \).

\[
(\frac{\Delta \Gamma}{\Gamma})_{B_s} = \frac{\Lambda^3}{m_b^3} \left( \Gamma_3^{(0)} + \frac{\alpha_s}{4\pi} \Gamma_3^{(1)} + \ldots \right) + \frac{\Lambda^4}{m_b^4} \left( \Gamma_4^{(0)} + \frac{\alpha_s}{4\pi} \Gamma_4^{(1)} + \ldots \right) + \ldots
\] (3)

Each of this \( \Gamma_i^{(j)} \) consists of perturbative Wilson coefficients and non-perturbative matrix elements. The LO-result \( \Gamma_3^{(0)} (1/m_b^3) \) in the HQE, \( \alpha_s^0 \) in QCD and vacuum insertion approximation (VIA) for the matrix elements) was already calculated long time ago. Corrections of order \( 1/m_b \ (\Gamma_4^{(0)}) \) were calculated in and turned out to be unexpectedly large. Therefore terms of order \( 1/m_b^2 \ (\Gamma_5^{(0)}) \) should be determined in order to check the convergence of the HQE. \( O(\alpha_s) \) radiative corrections \( (\Gamma_3^{(1)}) \) were first calculated in and confirmed in. The non-perturbative matrix elements of local four-quark operators (which appear in \( \Gamma_3 \)) between
B-meson states have been determined within the framework of QCD sum rules [11] and lattice QCD [12, 13]. The overall normalization of these matrix elements is given by the $B_s$ decay constant, $f_{B_s}$. Since $\Delta \Gamma_{B_s}$ is proportional to $f_{B_s}^2$, already minor changes in the numerical value of the decay constant have a big impact on the final prediction for $\Delta \Gamma_{B_s}$. Recently unquenched lattice calculations were performed, which yielded large values for $f_{B_s}$ [14]. These numbers are in perfect agreement with recent sum rule determinations, see [15] and references therein. We will use the value which was quoted in LATTICE 2004 [16]

$$f_{B_s} = 245 \pm 30 \text{ MeV}.$$ (4)

The calculation of the next-to-leading order QCD radiative corrections to the Wilson coefficient functions for $\Delta \Gamma_{B_s}$ was a very important step in gaining a reliable theoretical prediction. First the renormalization scale dependence will be reduced compared to the leading order prediction - unfortunately it turned out that for $\Delta \Gamma_{B_s}$ the remaining scale dependence is still quite large. Second, the inclusion of $O(\alpha_s)$ corrections is necessary for a satisfactory matching of the Wilson coefficients to the matrix elements. The unphysical renormalization scheme dependence has to cancel between the Wilson coefficients and the matrix elements. Since in the Wilson coefficients this scheme dependence arises first at NLO one has to go beyond LO in order to obtain reliable predictions. Moreover, the consideration of subleading QCD radiative effects was of conceptual interest for the construction of the HQE, since one could show hereby explicitly the infrared safety of the HQE in that order. For powerlike IR divergencies the cancellation was already shown in [17]. The result in [9] was the first complete calculation of perturbative QCD effects beyond the leading logarithmic approximation to spectator effects in the HQE for heavy hadron decays. Currently NLO-QCD corrections to spectator effects are known for the lifetime ratios of heavy hadrons [18, 19, 20] and for $\Delta \Gamma_{B_d}$ and the semileptonic CP-asymmetries [21, 10].

## 2 Theoretical prediction of $\Delta \Gamma_{B_s}$

### 2.1 Preliminaries

The nature of the weak interaction leads to the fact that the physical eigenstates of the neutral B mesons are linear combinations of the flavor eigenstates

$$B_H := p B + q \overline{B},$$ (5)

$$B_L := p B - q \overline{B}.$$ (6)

Three measurable quantities can be deduced from this particle-antiparticle mixing:

$$\Delta M := M_H - M_L,$$ (7)

$$\Delta \Gamma := \Gamma_L - \Gamma_H,$$ (8)

$$a_{fs} = -2 \left( \left| \frac{q}{p} \right| - 1 \right).$$ (9)

$a_{fs}$ describes CP asymmetries in flavor specific B decays, which are often called semi-leptonic CP asymmetries. This quantity is discussed e.g. in [21, 10]. In the following we restrict
ourselves to $\Delta \Gamma$. The decay rate difference can be expressed as the matrix element of the transition operator $T$

$$\Delta \Gamma = -\frac{1}{m_{B_s}}\langle B_s | T | B_s \rangle,$$

(10)

which consists of a double insertion of the $\Delta B = 1$ effective Hamiltonian

$$T = \text{Im} \int d^4x T [H_{\text{eff}}(x), H_{\text{eff}}(0)].$$

(11)

Formally one performs now a operator product expansion for the transition operator, graphically one matches the $\Delta B = 1$ double insertion to a $\Delta B = 2$ insertion.

2.2 Leading order

In LO in the HQE the matching equation is described by fig. 1. The l.h.s. of fig. 1 corresponds to the double insertion of the effective $\Delta B = 1$ Hamiltonian. By calculating this loop diagram one obtains the r.h.s., which consists of the Wilson coefficient $c_{B_s}^{LO}$ and a four-quark $\Delta B = 2$ operator. One can express the transition operator in the following form

$$T = -G_F \frac{m_b}{2\pi} \langle B_s | \bar{Q} \cdot \left( \frac{b_i s_i}{m_b} V - A \cdot \left( \frac{b_j s_j}{m_b} V - A \right) \right) | B_s \rangle + O(\frac{1}{m_b}).$$

(12)

with the Wilson coefficients $F$ and $F_S$ ($z = m_c^2/m_b^2$) and the following $\Delta B = 2$-operators

$$Q = \left( \bar{b}_i s_i \right)_{V-A} \cdot \left( \bar{b}_j s_j \right)_{V-A}$$

(13)

$$Q_S = \left( \bar{b}_i s_i \right)_{S-P} \cdot \left( \bar{b}_j s_j \right)_{S-P}$$

(14)

The color-rearranged operators which arise during the calculation have been eliminated via

$$\tilde{Q} = Q$$

(15)

$$\tilde{Q}_S = -Q_S - \frac{1}{2} Q + O(\alpha_s) + O\left( \frac{1}{m_b} \right)$$

(16)

The matrix elements of $Q$ and $Q_S$ can be parametrized in terms of the decay constant $f_{B_s}$ and bag parameters $B$ and $B_S$.

$$\langle B_s | Q | B_s \rangle = \frac{8}{3} f_{B_s}^2 M_{B_s}^2 B$$

(17)

$$\langle B_s | Q_S | B_s \rangle = -\frac{5}{3} f_{B_s}^2 M_{B_s}^2 \left( \frac{M_{B_s}}{m_b + m_s} \right)^2 B_S$$

(18)

Assuming VIA for the matrix elements, which corresponds to setting the bag parameters equal to one, we get with $z = 0.085, m_b = 4.2$ GeV, $m_s = 0.1$ GeV, $V_{cb} = 40.1 \cdot 10^{-3}$, $m_b = 4.8$ GeV

$$\left( \frac{\Delta \Gamma}{\Gamma} \right)_{B_s} = O(30)\%.$$  

(19)
2.3 Next-to-Leading QCD corrections

If one dresses the diagrams of fig. 1 with one gluon in all possible ways one gets the NLO QCD correction to the Wilson coefficients. At this level one has to take the $\alpha_s$-corrections to eq. (10) into account. The NLO-QCD calculation was performed in [9] and [10] and gives a sizeable reduction of the LO result:

$$\left(\frac{\Delta \Gamma}{\Gamma}\right)_{B_s} = \mathcal{O}(24\%) . \quad (20)$$

Unfortunately it turned out that the reduction of the renormalization scale dependence was not very pronounced. An improvement of this point would require the calculation of $\alpha_s^2$-corrections, which will be a hard endeavour, even though the three-loop anomalous dimensions of the effective Hamiltonian are now known [22].

2.4 Lattice evaluation of the matrix elements

Since the scheme dependence is now visible due to the NLO QCD calculation, a next step to obtain a reliable prediction for $\Delta \Gamma_{B_s}$ is the inclusion of lattice predictions for the bag parameters instead of VIA. Now the unphysical scheme dependence, which can be numerically large cancels up to effects of order $\alpha_s^2$. With [12]

$$B = 0.87 \pm 0.06 \quad (21)$$
$$B_S = 0.84 \pm 0.05 \quad (22)$$

one obtains again a reduction of the final number:

$$\left(\frac{\Delta \Gamma}{\Gamma}\right)_{B_s} = \mathcal{O}(20\%) . \quad (23)$$

2.5 Power corrections

Till now we were setting the momentum of the light quark in the B-meson to zero. Power corrections ($\equiv 1/m_b$-corrections) can be obtained by expanding the transition operator in powers of the light quark momentum. In addition one has to take the $1/m_b$-corrections to eq. (10) into account. The calculation of the $1/m_b$-corrections was performed first in [7]. In this order of the HQE operators of dimension 7 appear. Some of these operators can be rewritten with the help of e.o.m. to dimension 6 operators, the remaining operators have to be estimated by VIA. Once again we get a sizeable reduction of our prediction:

$$\left(\frac{\Delta \Gamma}{\Gamma}\right)_{B_s} = \mathcal{O}(12\%) . \quad (24)$$

The power corrections turn out to be the most important corrections and at the same time the least well known ones. In order to improve our knowledge about this corrections several tasks have to be completed:

- Test the HQE expansion: the calculation of $1/m^2$-corrections is under way [5].
Matrix elements of dimension 7 operators: As was noted already in [10] some of these power suppressed operators can be obtained from the lattice evaluation in [12]. Despite this progress it is still necessary to have a reliable determination of the remaining dimension 7 operators.

QCD corrections to power corrections: with a lattice determination of the dimension 7 operators at hand it might be worthwhile to calculate $\Gamma^{(1)}_4$.

2.6 The Final number

We have here the very special situation that all corrections have a negative sign and are quite sizeable. Moreover we have an additional source of uncertainty. So far we were actually only calculating $\Delta \Gamma_{B_s}$, the ratio $(\Delta \Gamma/\Gamma)_{B_s}$ can be obtained in different ways (A, B and C)

$$(\Delta \Gamma)^A_{B_s} = \Delta \Gamma_{B_s} \tau_{B_s/d} = \frac{G_F^2}{12\pi} m_b^2 V_{cb}^2 \tau_{B_s/d} f_{B_s}^2 K,$$

$$(\Delta \Gamma)^B_{B_s} = \Delta \Gamma_{B_s} \frac{1}{\Gamma_{B_s}} \tau_{B_s/d} f_{B_s}^2 K,$$

$$(\Delta \Gamma)^C_{B_s} = \Delta \Gamma_{B_s} \frac{\Delta M_{B_s}}{\Delta M_{B_d}} \Delta M_{B_d} \tau_{B_s} = \frac{\pi}{2 M_W^2} \frac{\Delta M_{B_d}}{M_{B_d}} \frac{m_b^2 V_{tb}^2 \xi^2 \tau_{B_s}}{(V_{tb} V_{td})^2 \eta_{B S_0(x_t)}} K,$$

with

$$K = M_{B_s} V_{cs}^2 [F(Q) + F_S(Q_S)].$$

Unfortunately we have here a similar situation like in the case of the missing charm puzzle [23], that different normalizations lead to big numerical effects. Method C, which was used e.g. in [10] tends to give values which are about 25% smaller than method B, which was used e.g. in [9]. In this letter we were using method B, for future estimates we suggest to use method A, see [8].

Putting everything together and estimating the dominant errors we get

$$(\Delta \Gamma)^{A}_{B_s} = \left( \frac{f_{B_s}}{245 \text{MeV}} \right)^2 [0.234 B S(m_b) - 0.086 + 0.008 B(m_b)]$$

$$= (12 \pm 5) \%.$$  

3 Theoretical prediction of $\Delta \Gamma_{B_d}$

In principle the calculation of $\Delta \Gamma_{B_d}$ proceeds in the same way, but one has to keep in mind that in this case different CKM structures contribute with a similar strength (order $\lambda^6$ in the Wolfenstein parameter $\lambda$), while in the case of $\Delta \Gamma_{B_s}$ the contribution of two internal charm
quarks is leading by two powers of $\lambda$.

\[
\begin{array}{|c|c|c|}
\hline
\text{internal quarks} & \Delta\Gamma_{B_s} & \Delta\Gamma_{B_d} \\
\hline
uu & \lambda^8 & \lambda^6 \\
uu & \lambda^6 & \lambda^6 \\
cu & \lambda^6 & \lambda^6 \\
cc & \lambda^4 & \lambda^6 \\
\hline
\end{array}
\] (28)

The $uu$ and the $cc$ contribution can be taken from the $\Delta\Gamma_{B_s}$-calculation, while the $uc$ and $cu$ contributions have to calculated anew. $\Gamma_3^{(0)}$ was calculated in [6], $\Gamma_4^{(0)}$ was calculated in [24] and $\Gamma_3^{(1)}$ was calculated in [21] and [10].

\[
\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_d} = (3 \pm 1.2) \cdot 10^{-3}.
\] (29)

4 Outlook for $\Delta\Gamma_{B_s}$

What do we expect for the future? First we are waiting eagerly for the D0 number for $\Delta\Gamma_{B_s}$ and we of course expect much smaller errors in the future. From the theory side we have the following what-to-do-list

- Calculation of $1/m_b^2$-corrections: $\Gamma_5^{(0)}$
- Lattice determination of dimension 7 operators for $\Gamma_4$
- Calculation of $\alpha_s$-corrections to the $1/m_b$corrections: $\Gamma_4^{(1)}$
- Calculation of $\alpha_s^2$-corrections to the leading term: $\Gamma_3^{(2)}$

It was shown in [25] that new physics effects can not enhance $\Delta\Gamma_{B_s}$ compared to the standard model value. If after all these efforts the central experimental und theoretical numbers stay at their current values, this would probably be a signal of local quark-hadron duality violation. The operator product expansion of the transition operator is based on the duality assumption. Little is known in QCD about the actual numerical size of duality-violating effects. Experimentally no violation of local quark-hadron duality in inclusive observables of the $B$-meson sector has been established so far. Comparison of experiment and theory for $\tau(B^+)/\tau(B_d)$ supports the duality assumption, but in that case we have only one heavy charm in the intermediate state, compared to two charm quarks in the $\Delta\Gamma_{B_s}$-case. In [26] it has been shown that for $\Delta\Gamma_{B_s}$ local duality holds exactly in the simultaneous limits of small velocity ($\Lambda_{QCD} \ll m_b - 2m_c \ll m_b$) and large number of colours ($N_c \to \infty$). In this case

\[
\left(\frac{\Delta\Gamma}{\Gamma}\right)_{B_s} = \frac{G_F^2 m_b^3 f_{B_s}^2}{4\pi} |V_{cs}V_{cb}|^2 \sqrt{2 - 4 \frac{m_c}{m_b}} \tau_{B_s} \approx 0.18.
\] (30)

It is interesting that the numerical value implied by the limiting formula [30] appears to be quite realistic.
Acknowledgments

I would like to thank the organizers of FPCP2004 for the invitation and the financial support, M. Beneke, G. Buchalla, C. Greub and U. Nierste for the pleasant collaboration and Fermilab and DFG for financial support, while calculating the $1/m^2$-corrections.

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