DIPOLE MODE DETUNING IN THE NLC INJECTOR LINACS

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1 INTRODUCTION

A major consideration in the design of the accelerator structures in the injector linacs of the JLC/NLC[1] is to keep the wakefield effects within tolerances for both the nominal (2.8 ns) and alternate (1.4 ns) bunch spacings. One important multi-bunch wakefield effect is beam break-up (BBU), where a jitter in injection conditions of a bunch train is amplified in the linac; another is static emittance growth caused by structure misalignments.

The injector linacs comprise the prelinac, the 
+ drive linac, the 
− booster, and the 
+ booster. The first three will operate at S-band, the last one, at L-band. Compared to the main (X-band) linac, the wakes will tend to be smaller by a factor 1/64 and 1/512, respectively, for the S- and L-band linacs. This reduction, however,—especially for the S-band machines,—by itself, is not sufficient. Two ways of reducing the wake effects further are to detune the first pass-band dipole modes and to damp them. In this report our goal is to design the accelerator structures for the injector linacs using detuning alone, an option that is simpler than including damping. We will consider only the effects of modes in the first pass-band, whose strengths overwhelmingly dominate. The effects of the higher pass-band modes, however, will need to be addressed in the future.

Table 1: Machine properties of the injector linacs.

| Name          | \(E_0, E_f\) [GeV] | \(L\) [m] | \(\beta_0\) [m] | \(\zeta\) |
|---------------|--------------------|-----------|----------------|--------|
| Prelinac      | 1.98, 10.0         | 558       | 8.6            | 1/2    |
| \(e^+\) Drive | 0.08, 6.00         | 508       | 2.4            | 1/4    |
| \(e^-\) Booster | 0.08, 2.00       | 163       | 3.4            | 1/4    |
| \(e^+\) Booster | 0.25, 2.00        | 184       | 1.5            | 1      |

2 EMITTANCE GROWTH

2.1 Beam Break-Up (BBU)

In analogy to single-bunch BBU in a linac[2], multi-bunch BBU can also be characterized by a strength parameter, but one dependent on bunch number \(m\):

\[
\Upsilon_m = \frac{e^2NLs_m\beta_0}{2E_0} g(E_f/E_0, \zeta) \quad [m = 1, \ldots, M],
\]

with \(M\) the number of bunches in a train. The sum wake

\[
S_m = \sum_{i=1}^{m-1} W[(m-i)\Delta t] \quad [m = 1, \ldots, M],
\]

with \(W\) the transverse wakefield and \(\Delta t\) the time interval between bunches in a train. The wake, in turn, is given by a sum over the dipole modes in the accelerator structures:

\[
W(t) = \sum_{n=1}^{N_n} 2k_n \sin(2\pi f_n t/c) \exp(-\pi f_n t/Q_n),
\]

with \(t\) and \(N_m\) the number of modes; \(f_n, k_n,\) and \(Q_n\) are, respectively, the frequency, the kick factor, and the quality factor of the \(n^{th}\) mode. The function \(g(x)\) in Eq. (1) depends on the focusing profile in the linac. Assuming the beta function varies as \(\beta \sim E^\zeta\),

\[
g(x, \zeta) = \frac{1}{\zeta} \left( \frac{x^\zeta - 1}{x - 1} \right) \quad [\beta \sim E^\zeta].
\]

If \(\Upsilon_m\), for all \(m\), is not large, the linear approximation applies, and this parameter directly gives the (normalized) growth in amplitude of bunch \(m\). The projected (normalized) emittance growth of the bunch train then becomes (assuming, for simplicity, that, in phase space, the beam ellipse is initially upright) \(\delta\epsilon \approx \frac{1}{2} T_{rms,0}^2 y_0^2/\sigma_{0t}^2\), with \(T_{rms,0}\) the rms with respect to 0 of the strength parameter, \(y_0\) the initial bunch offset, and \(\sigma_{0t}\) the initial beam size. As jitter tolerance parameter, \(r_t\), we can take that ratio \(y_0/\sigma_{0t}\) that yields a tolerable emittance growth, \(\delta\epsilon_t\).

2.2 Misalignments

If the structures in the linac are (statically) misaligned with respect to a straight line, the beam at the end will have an increased projected emittance. If we have an ensemble of misaligned linacs then, to first order, the distribution in emittance growth at the end of these linacs is given by an exponential distribution \(\exp[-\delta\epsilon/(\langle\delta\epsilon\rangle)]/(\delta\epsilon)\), with

\[
\langle\delta\epsilon\rangle = e^2NL_0(x_0)_{rms}S_{rms} \sqrt{\frac{N_0\beta_0}{2}} h(E_f/E_0, \zeta)
\]
with $L_o$ the structure length, $(x_a)_{rms}$ the rms of the structure misalignments, $S_{rms}$ the rms of the sum wake with respect to the average, and $N_s$ the number of structures; the function $h$ is given by (again assuming $\beta \sim E^3$):

$$h(x, \zeta) = \sqrt{\frac{1}{\zeta x} \left( \frac{x^2 - 1}{x - 1} \right)} [\beta \sim E^3]. \quad (6)$$

Eq. 5 is valid assuming the so-called betatron term in the equation of motion is small compared to the misalignment term. We can define a misalignment tolerance:

$$x_{at} = (x_a)_{rms} \sqrt{\delta \epsilon_t / (\delta \epsilon_c)}$$

where $\delta \epsilon_t$ is the tolerance in emittance.

We are also interested in the tolerance to cell-to-cell misalignments caused by fabrication errors. A structure is built as a collection of cups, one for each cell, that is brazed together, and there will be errors, small compared to the cell dimensions, in the straightness of each structure. To generate a wake (for a beam on-axis) in a structure with cell-to-cell misalignments we use a perturbation approach based on the eigenmodes of the unperturbed structure.

### 3 WAKEFIELD DESIGN

Reducing emittance growth requires reducing the sum wake. In the main (X-band) linac of the NLC, the strategy to do this is to use Gaussian detuning to generate a fast Gaussian fall-off in the wakefield envelope; in particular, we will aim for a uniform distribution, as a collection of cups, one for each cell, that is brazed together, and there will be errors, small compared to the cell dimensions, in the straightness of each structure. To generate a wake (for a beam on-axis) in a structure with cell-to-cell misalignments we use a perturbation approach based on the eigenmodes of the unperturbed structure.

#### 3.1 Optimization

The cells in a structure are coupled to each other, and to obtain the wakefield we need to solve for the eigenmodes of the system. We obtain these numerically using a double-band circuit model applied to representative cell geometries, and then using interpolation. Here we consider structures of the disk-loaded type, with rounded irises. The iris and cavity radii are adjusted to give the correct fundamental mode frequency and the desired synchronous dipole mode frequency. Therefore, cell $m$ can be specified by one free parameter, the synchronous frequency (of the first dipole mode pass-band). The $3\pi/4$ S-band structure consists of 102 cells with a cell period of 3.94 cm, iris thickness of 0.584 cm, and cavity radius $\sim 4.2$ cm; the Q due to wall losses (copper) $\sim 14,500$. Fig. 1 shows the first two dispersion curves of representative cell geometries (for iris radii from 1.30 to 2.00 cm). The plotting symbols give the OMEGA2 results, the curves, those of the circuit program.

![Figure 1](image_url)  
Figure 1: The dispersion curves of the first two dipole bands of representative cells in a $3\pi/4$ structure.

We will consider a uniform input (synchronous) frequency distribution, but with a slanting top. This leaves us with 3 parameters to vary: the (relative) shift in average frequency (from a nominal 4.012 GHz) $\delta \bar{f}$, the (relative) width of the distribution $\Delta \bar{f}$, and the tilt parameter $\alpha$ ($-1 \leq \alpha \leq 1$, with $\alpha = 1$ giving a right triangle distribution with positive slope). Varying these parameters we calculate $S_{rms0}$ and $S_{rms}$ for the coupled modes, and for both...
bunch train configurations, and we optimize. We find that a fairly optimal case consists of $\delta f = -2.3\%$, $\Delta \delta f = 5.8\%$, and $\alpha = -0.20$, where $S_{rms,0} = S_{rms} = 0.004$ MV/nC/m².

In Fig. 2 we show the dependence of $S_{rms,0}$ on $\delta f$ and $\Delta \delta f$ near the optimum.

Figure 2: $S_{rms,0}$ [MV/nC/m²] vs. $\delta f$ and $\Delta \delta f$ near optimum, for $\Delta t = 2.8$ ns (solid) and 1.4 ns (dashes).

In Fig. 3 we display, for the optimal case, the frequency distribution (a), the kick factors (b), and the envelope of the wake (c). The dashed curves in (a) and (b) give the synchronous (input) values. The plotting symbols in (c) give $|W|$ at the bunch positions for the alternate (1.4 ns) bunch train configuration. In (b) we see no spikes, thanks to the fact that the synchronous point is near $\pi$, and, serendipitously, $f_0 < f_x$ for cell geometries near the beginning of the structure, $f_0 > f_x$ for those near the end. (Note that for the optimized $2\pi/3$ structure, for which $f_0 > f_x$ for all cell geometries, there is such a spike, and consequently $S_{rms,0}$ is 5 times larger than here.) From (c) we note that many of the earlier bunches have wakes with amplitudes significantly below the wake envelope.

Figure 3: Results for the optimal $3\pi/4$ structure.

### 3.2 Frequency Errors

Errors in cell manufacturing will result in frequency errors. In Fig. 4 we give $S_{rms,0}$ and $S_{rms}$, when a random error component is added to the input synchronous frequencies of the optimal distribution (each plotting symbol, with its error bars, represents 400 seeds). With a frequency spacing of $\sim 8 \times 10^{-4}$, an rms frequency error of $1 \times 10^{-4}$ is a relatively small perturbation, and for the 1.4 ns bunch spacing its effect is small, whereas for the 2.8 ns spacing it is not. The reason is that in the former case the beam sits on the half-integer resonance (which is benign), while in the latter case it sits on the integer (which is not). As to the effect in a linac, let us distinguish two types of errors: “systematic random” and “purely random” errors; by the former we mean errors, random in one structure, that are repeated in all structures of the linac; by the latter we mean random also from structure to structure. We expect the effect of a purely random error, of say, $10^{-3}$ (which we think is achievable) to be similar to a systematic random error of $10^{-4}/\sqrt{N_t}$. $N_t = 140, 127, 41$ in, respectively, the prelinac, the $e^+$ drive linac, and the $e^-$ booster; therefore the appropriate abscissas in the figure become $0.8, 0.9, and 1.6 \times 10^{-5}$. At these points, for the 2.8 ns spacing, we see that $S_{rms,0}$ is only a factor $2 \pm 1, 2 \pm 1, 3 \pm 2$ times larger than the error-free result.

Figure 4: The effect of random frequency errors.

### 4 TOLERANCES

To obtain tolerances we performed particle tracking using LIAR and compare the results with the analytical formulas given in Sec. 2. We take $\delta \epsilon_t = 10\%$ as acceptable. For BBU the tightest tolerance is for the $e^+$ booster, where $r_t$ is $3.8 (2.2)$ analytically, $5.5 (3.0)$ numerically, for $\Delta t = 2.8 (1.4)$ ns. For misalignments the tightest tolerance is for the prelinac, where $x_{at}$ is $2.9 (4.6)$ mm analytically, $3.2 (4.8)$ mm numerically. (For the other machines these tolerances are $\gtrsim 10$ times looser.) Purely random machining errors, equivalent to $10^{-3}$ frequency errors, will tighten these results by $50\%-100\%$, but they are still very loose.

Finally, what is the random, cell-to-cell misalignment tolerance? Performing the perturbation calculation mentioned earlier for 1000 different random structures, we find that $S_{rms} = 0.27 \pm 0.12 (0.032 \pm 0.003)$ MV/nC/m² for $\Delta t = 2.8 (1.4)$ ns. We again see the effect of the integer resonance on the 2.8 ns option result. For the prelinac the cell-to-cell misalignment tolerance becomes $40 (600) \mu$m for the 2.8 (1.4) ns configuration.

We thank T. Raubenheimer and attendees of the NLC linac meetings at SLAC for comments and suggestions.

### 5 REFERENCES

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