A REMARK ON HIGHER TODD GENERA OF COMPLEX MANIFOLDS

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Abstract. Let $M$ be a compact complex manifold. In this paper, generalizing previous results due to Rosenberg and Block-Weinberger in the case of complex projective varieties, we show that the higher Todd genera of $M$ are bimeromorphic invariants.

Keywords: Complex manifold, bimeromorphic map, resolution of singularities, analytic K-homology, Todd genera.

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1. Introduction

Let $M$ be a smooth complex projective variety. Let $\text{Td}(M) \in H^*(M, \mathbb{Q})$ be the Todd genus of $M$. Let $\Gamma$ be a finitely generated discrete group and let $s : M \to B\Gamma$ be a continuous map into the classifying space of $\Gamma$. The higher Todd genera of $M$ are defined as the rational numbers

$$\{(\text{Td}(M) \cup s^*\alpha)[M], \, \alpha \in H^*(B\Gamma, \mathbb{Q})\}.$$

Equivalently,

$$\{\langle \alpha, s_*(\text{Td}_s(M)) \rangle, \, \alpha \in H^*(B\Gamma, \mathbb{Q})\}$$

with $\text{Td}_s(M) \in H_*(M, \mathbb{Q})$ denoting the Poincaré dual of $\text{Td}(M)$.

Jonathan Rosenberg [8], building on a well-established argument for proving the oriented homotopy invariance of the higher signatures, proved that if the assembly map $\beta : K_0(B\Gamma) \to K_0(C^*_r\Gamma)$ is rationally injective then the higher Todd genera are birational invariants. See Section 4 for definitions. Later Jonathan Block and Shmuel Weinberger proved this result unconditionally, i.e. without assuming the rational injectivity of $\beta$. See [3] and also [6] for a more analytic approach. These articles use in a crucial way the weak factorization theorem for birational maps, [1].

In this short note we have two goals in mind. On the one hand we extend these results from smooth complex projective varieties to compact complex manifolds, showing that the higher Todd genera are bimeromorphic invariants; on the other hand we give a direct proof that does not use the weak factorization theorem, even for smooth complex projective varieties, but relies instead on the notion of modification.
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2. Meromorphic maps and fundamental groups

We recall some definitions and properties that will play a central role in the paper.

Definition 1. Let $M$ and $N$ be two complex manifolds. A proper and surjective holomorphic map $f : M \to N$ is a proper modification if there exists a nowhere dense analytic subset $X \subset N$ such that $Y = f^{-1}(X)$ is a nowhere dense analytic subset of $M$ and $f|_{M \setminus Y} : M \setminus Y \to N \setminus X$ is a biholomorphism.

If $M$ and $N$ are compact then we will simply say that $f : M \to N$ is a modification.

Definition 2. Let $M$ and $N$ be two complex manifolds. A meromorphic map $f : M \dashrightarrow N$ is a map from $M$ to $P(N)$, the power set of $N$, such that

1. $\mathcal{G}(f)$, defined as the closure in $M \times N$ of $\{(x, y) \in M \times N$ such that $y \in f(x)\}$, is an irreducible analytic subvariety of $M \times N$;

2. The natural projection $p_M : \mathcal{G}(f) \to M$ is a proper modification.

The map is called bimeromorphic if also $p_N : \mathcal{G}(f) \to N$, the natural projection on $N$, is a modification.

Definition 1 and Definition 2 can be extended to the case in which $M$ and $N$ are complex analytic spaces, see [9]. Definition 2 implies the existence of a smallest analytic subset $Z \subset M$, usually called the set of points of indeterminacy of $f$, such that $f$ is defined and holomorphic on $M \setminus Z$. We shall denote the open set $M \setminus Z$ as $\text{Dom}(f)$. As $M$ is nonsingular and therefore in particular normal, a fundamental property is that the set of points of indeterminacy of $f$ has complex codimension at least 2, see [9] Th. 2.5. Clearly the composition of two modifications is still a modification and any modification is a bimeromorphic map. The following result is well known to the experts but as we could not find a quotable reference, we provide a proof for the benefit of the reader.

Proposition 2.1. Let $f : M \to N$ be a modification between two compact complex manifolds. Then $f_* : \pi_1(M) \to \pi_1(N)$ is an isomorphism.

Proof. Let $X$ and $Y$ be as in Def. 1. Thanks to [4] page 60 we can decompose $X$ as $X = \bigcup_{k=1}^\ell S_k$ such that

- $S_k$ is a complex submanifold of $N$ for each $k = 1, \ldots, \ell$ and $S_k \cap S_j = \emptyset$ whenever $j \neq k$,
- For each $k = 1, \ldots, \ell$ both $\overline{S_k}$ and $\overline{S_k} \setminus S_k$ are analytic subsets of $N$,
- If $S_j \cap \overline{S_k} \neq \emptyset$ and $S_j \neq S_k$ then $S_j \subset \overline{S_k}$ and $\dim(S_j) < \dim(S_k)$.

Without loss of generality we can assume that $S_1, \ldots, S_\ell$ are ordered in such a way that $\dim(S_i) \leq \dim(S_j)$ if $i \leq j$. It is easy to verify that the above properties imply that $S_1 \cup \ldots \cup S_k$ is closed in $N$ for each $k = 1, \ldots, \ell$. In particular $N \setminus (S_1 \cup \ldots \cup S_k)$ is a complex manifold, in fact it is an open subset of $N$, and $S_k$ is a closed complex submanifold of $N \setminus (S_1 \cup \ldots \cup S_{k-1})$. Moreover, as remarked above, we also know that the complex codimension of $S_k$ satisfies $\text{codim}_C(S_k) \geq 2$ for each $k = 1, \ldots, \ell$. This follows by the fact that $X$ is the set of point of indeterminacy of $f^{-1} : N \dashrightarrow M$. Clearly $Y$ has an analogous stratification in $M$ whose strata will be denoted with $T_1, \ldots, T_r$. Also in this case we will assume that $\dim(T_i) \leq \dim(T_j)$ if $i \leq j$ and as in the previous case we have that $T_1 \cup \ldots \cup T_k$ is closed in $M$ for each $k = 1, \ldots, r$. As $\text{codim}_C(S_k) \geq 2$ for any $k$ a well known application of Thom’s transversality theorem tells us that the inclusion $(N \setminus S_1) \hookrightarrow N$ induces an isomorphism $\pi_1(N \setminus S_1) \cong \pi_1(N)$. Consider now $S_2$. Since it is a closed submanifold of $N \setminus S_1$ we have that $N \setminus (S_1 \cup S_2)$ is still a (complex) manifold, and thus Thom’s transversality theorem tells us that the inclusion $N \setminus (S_2 \cup S_1) \hookrightarrow N \setminus S_1$ induces
an isomorphism $\pi_1(N \setminus (S_2 \cup S_1)) \cong \pi_1(N \setminus S_1)$. If we iterate this procedure at the $k$-th step we have $S_k$, which is a closed complex submanifold of $N \setminus (S_1 \cup \ldots \cup S_{k-1})$, and again Thom’s transversality theorem tells us that the inclusion $N \setminus (S_1 \cup \ldots \cup S_k) \hookrightarrow N \setminus (S_1 \cup \ldots \cup S_{k-1})$ induces an isomorphism $\pi_1(N \setminus (S_1 \cup \ldots \cup S_k)) \cong \pi_1(N \setminus (S_1 \cup \ldots \cup S_{k-1}))$. Finally after $\ell$-times we obtain that the inclusion $N \setminus (S_1 \cup \ldots \cup S_\ell) \hookrightarrow N \setminus (S_1 \cup \ldots \cup S_{\ell-1})$ induces an isomorphism $\pi_1(N \setminus (S_1 \cup \ldots \cup S_\ell)) \cong \pi_1(N \setminus (S_1 \cup \ldots \cup S_{\ell-1}))$. Composing all these maps and the corresponding isomorphisms we get that the inclusion $N \setminus X \hookrightarrow N$ induces an isomorphism $\pi_1(N \setminus X) \cong \pi_1(N)$. Moreover the same strategy applied to $M$ and $Y$ tells us that the inclusion $M \setminus Y \hookrightarrow M$ induces a surjective morphism $\pi_1(M \setminus Y) \rightarrow \pi_1(M)$. We remark that in this case we get a different result (in fact weaker as $\pi_1(M \setminus Y) \rightarrow \pi_1(M)$ is only an epimorphism) because, concerning the codimension of $Y$, we only know that $\text{codim}_{\mathbb{C}}(Y) \geq 1$. Therefore, at each step, Thom’s transversality theorem tells us only that the inclusion $M \setminus (T_1 \cup \ldots \cup T_k) \hookrightarrow M \setminus (T_1 \cup \ldots \cup T_{k-1})$ induces a surjective morphism $\pi_1(M \setminus (T_1 \cup \ldots \cup T_k)) \cong \pi_1(M \setminus (T_1 \cup \ldots \cup T_{k-1}))$. Finally let us now denote by $i$ and $j$ the inclusions $i : N \setminus X \hookrightarrow M$ and $j : M \setminus Y \hookrightarrow M$, respectively. We know that $(f \circ i)_* = (j \circ (f \setminus M \setminus Y))_*$. As $(f \circ (f \setminus M \setminus Y))_* : \pi_1(M \setminus Y) \rightarrow \pi_1(N)$ is an isomorphism and $i_* : \pi_1(M \setminus Y) \rightarrow \pi_1(M)$ is surjective we can conclude that $f_* : \pi_1(M) \rightarrow \pi_1(N)$ is an isomorphism as desired. 

Corollary 2.2. Let $\phi : M \rightarrow N$ be a bimeromorphic map between two compact complex manifolds. Then $\phi$ induces an isomorphism $\phi_* : \pi_1(M) \rightarrow \pi_1(N)$.

Proof. We use the notations of Def. 2. Let $\pi : L \rightarrow G(f)$ be a resolution of $G(f)$. Then $p_M \circ \pi : L \rightarrow M$ and $p_N \circ \pi : L \rightarrow N$ are both modifications. Now the statement is an immediate consequence of Prop. 2.1. 

3. The Levy-Riemann-Roch Theorem

We begin by recalling some fundamental facts about modifications.

Theorem 3.1. Let $p : L \rightarrow M$ be a proper modification of complex manifolds. Then

(i) $p_* \mathcal{O}_L = \mathcal{O}_M$;

(ii) $R^k p_* \mathcal{O}_L = 0$ for $k > 0$.

Proof. See [9] Cor. 1.14 and Prop 2.14, respectively. 

We also recall a particular version of Levy’s Riemann-Roch theorem [7]. Let $M$ be a compact complex manifold. Consider $K_0^{\text{hol}}(M)$, the Grothendieck group of coherent analytic sheaves on $M$. Let $K_0^{\text{top}}(M)$ be the topological K-homology of $M$. Then there exists a homomorphism of abelian groups $\alpha_M : K_0^{\text{hol}}(M) \rightarrow K_0^{\text{top}}(M)$ such that, in particular, the following holds:

if $f : M \rightarrow N$ is a proper holomorphic map and $f_! : K_0^{\text{hol}}(M) \rightarrow K_0^{\text{hol}}(N)$ is the direct image homomorphism provided by Grauert’s theorem, then

\[(3.1) \quad f_*(\alpha_M[\mathcal{O}_M]) = \alpha_N(f_! [\mathcal{O}_M]).\]

Since $M$ is a smooth complex manifold, the image of $\alpha_M[\mathcal{O}_M] \in K_0^{\text{top}}(M)$ under the isomorphism

$K_0^{\text{top}}(M) \rightarrow KK_0(C(M), \mathbb{C})$

is precisely $[\mathcal{F}_M]$, the analytic K-homology class associated to the operator $\overline{\partial} + \overline{\partial}^f$ over $M$. See [2], p. 35.

Notation: we set $K_0^{\text{an}}(M) := KK_0(C(M), \mathbb{C})$.

Thus, if $p : L \rightarrow M$ is a modification we have

\[p_*(\alpha_L[\mathcal{O}_L]) = \alpha_M[p_* \mathcal{O}_L] = \alpha_M[p_* \mathcal{O}_L] = \alpha_M[\mathcal{O}_M] \text{ in } K_0^{\text{top}}(M)\]
where the first equality comes from (3.1), the second from (ii) of Theorem 3.1 and the third from (i) of Theorem 3.1. Summarizing, if \( p : L \to M \) is a modification of compact complex manifolds, then

\[
(3.2) \quad p_*(\alpha_L[\mathcal{O}_L]) = \alpha_M[\mathcal{O}_M] \quad \text{in} \quad K^\text{top}_0(M)
\]

\[
(3.3) \quad p_*[\overline{\mathcal{O}}_L] = [\overline{\mathcal{O}}_M] \quad \text{in} \quad K^\text{an}_0(M).
\]

These equalities will be crucial in what follows.

4. Bimeromorphic invariance

We begin this section by explaining what we mean by bimeromorphy invariance of the higher Todd genera. To this end we first recall the Novikov conjecture on the oriented homotopy invariance of the higher signatures. Let \( N \) and \( M \) be oriented smooth compact manifolds. If \( \Gamma \) is a finitely generated discrete group and \( r : N \to B\Gamma \) is a continuous map, then the higher signatures of \( N \) are the collection of numbers

\[
\left\{ \int_N L(N) \wedge r^\ast \alpha, \quad \alpha \in H^*(B\Gamma, \mathbb{Q}) \right\}
\]

By homotopy invariance of these numbers we mean the following: given an orientation preserving homotopy equivalence \( M \xrightarrow{f} N \), the following equality

\[
\int_N L(N) \wedge r^\ast \alpha = \int_M L(M) \wedge (r \circ f)^\ast \alpha
\]

holds for any \( \alpha \in H^*(B\Gamma, \mathbb{Q}) \).

Consider now two compact complex manifolds \( M \) and \( N \) and a bimeromorphism \( f : M \dasharrow N \). When we try to follow the above formulation in order to define the bimeromorphym invariance of the higher Todd genera we face the problem that \( f \), in contrast with the Novikov case, is not everywhere defined. We could define the bimeromorphic invariance of the Todd higher genera as follows: if \( s : M \to B\Gamma \) and \( r : N \to B\Gamma \) are two continuous maps such that \( s = r \circ f \) on the dense open subset of \( M \) where \( f \) is defined, then

\[
\int_N \text{Td}(N) \wedge r^\ast \alpha = \int_M \text{Td}(M) \wedge s^\ast \alpha
\]

holds for any \( \alpha \in H^*(B\Gamma, \mathbb{Q}) \). This is how birational invariance for the higher Todd genera is formulated for example in [6] and, implicitly, in [8] [3], in the context of smooth projective varieties. Recall that this invariance, which holds without additional hypothesis on \( \Gamma \), is proved in these papers using in a fundamental way the weak factorization theorem [1].

In this article we follow a more general formulation, based on Corollary 2.2. So our goal in the rest of the paper is to reformulate in a more general way the bimeromorphic invariance of the higher Todd genera and to establish it for smooth complex manifolds. Let \( s : M \to B\Gamma \) be any continuous map. By bimeromorphic invariance of the higher Todd genera

\[
\left\{ \int_M \text{Td}(M) \wedge s^\ast [c], \quad [c] \in H^*(B\Gamma, \mathbb{Q}) \right\}
\]

we mean the equality

\[
(4.1) \quad \int_M \text{Td}(M) \wedge s^\ast [c] = \int_N \text{Td}(N) \wedge r^\ast [c]
\]

for each \( [c] \in H^*(B\Gamma, \mathbb{Q}) \), where \( r : N \to B\Gamma \) is any continuous map such that

1. \( s(p) = r(f(p)) \) for some \( p \in \text{Dom}(f) \),
2. \( r \circ f : \text{Dom}(f) \to B\Gamma \) is homotopic to \( s|_{\text{Dom}(f)} : \text{Dom}(f) \to B\Gamma \) with a homotopy fixing \( p \).
It is clear that this definition of bimeromorphic invariance is stronger than the one in \[6\] \[8\] \[3\], in that it allows a larger set of compatible maps into \(BT\). Consequently, the birational invariance or more generally the bimeromorphic invariance of the higher Todd genera proved in this article is a stronger invariance-property compared to the one established, for smooth projective varieties, in \[6\] \[8\] \[3\].

The following result is the crucial step in establishing the bimeromorphic invariance of the higher Todd genera.

**Theorem 4.1.** Let \(f : M \to N\) be a bimeromorphic map between two compact complex manifolds and let \(Z\) be any \(K(\Gamma, 1)\) space, \(\Gamma\) any discrete finitely generated group. Let \(s : M \to BT\) be any continuous map. We have the following properties:

1. For any arbitrarily fixed \(p \in \text{Dom}(f)\) there exists a continuous map \(r_p : N \to Z\) such that \(s(p) = r_p(f(p))\) for some \(p \in \text{Dom}(f)\).

2. \(r_p \circ f : \text{Dom}(f) \to BT\) is homotopic to \(s|_{\text{Dom}(f)} : \text{Dom}(f) \to BT\) with a homotopy fixing \(p\).

For any continuous map \(r : N \to Z\) satisfying the two properties listed above we have

\[
s_*[\overline{M}] = r_*[\overline{N}] \quad \text{in} \quad K_0^n(Z)
\]

where we recall that

\[
K_0^n(Z) = \lim_{\longrightarrow} X \subset Z, X\text{compact} K_0^n(X)
\]

**Proof.** Let \(p \in \text{Dom}(f) \subset M\) be an arbitrarily fixed point. By Corollary \[2.2\] we know that \(f_* : \pi_1(M, p) \to \pi_1(N, f(p))\) is an isomorphism. Let us consider the morphism \(\pi_1(N, f(p)) \to \pi_1(BT, \text{s}(p))\) equal to \(s_* \circ f_*^{-1}\). By \[5\] Prop. 1B.9, pg 90 we know that there exists a continuous map \(r_p : N \to BT\) sending \(f(p)\) into \(s(p)\) and unique up to homotopies fixing \(f(p)\), such that

\[
f_* \circ (r_p)_* = s_*
\]

as morphisms from \(\pi_1(M, p)\) to \(\pi_1(BT, \text{s}(p))\). By construction we have \(r_p(f(p)) = s(p)\) and the morphism \(r_p * \circ f_* : \pi_1(M, p) \to \pi_1(Z, s(p))\) equals \(s_* : \pi_1(M, p) \to \pi_1(Z, s(p))\). Thus \[5\] Prop. 1B.9 tells us that \(r_p \circ f : \text{Dom}(f) \to BT\) is homotopic to \(s|_{\text{Dom}(f)} : \text{Dom}(f) \to BT\) with a homotopy fixing \(p\). This proves the first part of the theorem. We tackle now the second part. Let \(r : N \to BT\) any continuous map satisfying the two properties listed above. By the very definition of bimeromorphic map \(f : M \to N\) we know that there exists a compact irreducible analytic subvariety \(G(f)\) of \(M \times N\) and a pair of modifications

\[
p_M : G(f) \to M \quad \text{and} \quad p_N : G(f) \to N
\]

induced by the natural projections of \(M \times N\) onto the first and second factor respectively. Let \(b : B \to G(f)\) be a resolution of \(G(f)\). By composing \(b\) with \(p_M\) and \(p_N\) we obtain a pair of modifications

\[
M \xrightarrow{\beta_M} B \xrightarrow{\beta_N} N.
\]

Let \(q\) be a point in \(B\) such that \(b(q) = p\), where \(p \in \text{Dom}(f)\) and \(r(f(p)) = s(p)\). By the assumptions on \(r\) we know that \(r_* \circ f_* : \pi_1(M, p) \to \pi_1(Z, s(p))\) coincides with \(s_* : \pi_1(M, p) \to \pi_1(Z, s(p))\). On the other hand, by definition, \(f : \text{Dom}(f) \to N\) equals \(\beta_N \circ (\beta_M|_{\text{Dom}(f)})^{-1} : \text{Dom}(f) \to N\). Thus we can conclude that the following two morphisms of groups coincide

\[
(s \circ \beta_M)_* : \pi_1(B, q) \to \pi_1(Z, s(p)) \quad \text{and} \quad (r \circ \beta_N)_* : \pi_1(B, q) \to \pi_1(Z, s(p)).
\]

Indeed

\[
r_* \circ f_* = s_* \Leftrightarrow r_* \circ (\beta_N)_* \circ (\beta_M)_*^{-1} = s_* \Leftrightarrow (r \circ \beta_N)_* = (s \circ \beta_M)_*.
\]

Prop. 1B.9 in \[5\] allows us to conclude that \(s \circ \beta_M : B \to Z\) and \(r \circ \beta_N : B \to Z\) are homotopic with a homotopy fixing \(q\). Consequently, using \(\[3.3\]\) and the homotopy invariance of K-homology, we have the following equalities in \(K_0(Z)\):

\[
r_*(\overline{N}) = r_*(\beta_N_*(\overline{B})) = s_*(\beta_M_*(\overline{B})) = s_*(\overline{M}).
\]
Corollary 4.2. The higher Todd genera
\[ \{(\text{Td}(M) \cup s^* \alpha)[M], \alpha \in H^*(B\Gamma, \mathbb{Q})\} \]
are bimeromorphic invariants of \( M \) in the sense of (4.1).

Proof. Thanks to Th. 4.1 we know that \( s_*[\overline{M}] \in K_0^\text{an}(B\Gamma) \) is a bimeromorphic invariant, in that \( s_*[\overline{M}] = r_*[\overline{N}] \) in \( K_0^\text{an}(B\Gamma) \). Thus \( \text{Ch}_*(s_*[\overline{M}]) \in H_*(B\Gamma, \mathbb{Q}) \) is a bimeromorphic invariant. But \( \text{Ch}_*(s_*[\overline{M}]) = s_*(\text{Ch}_*(\overline{M})) \) and \( \text{Ch}_*(\overline{M}) = \text{PD}(\text{Td}(M)) \), with PD denoting Poincaré duality, from which the Corollary follows.

While writing this short note we realized that the proof of [2, Proposition 7.1] contains a mistake: in Lemma 7.2 we cannot conclude that \( \ell = \pi \circ r \) up to homotopy, as \( \tilde{M} \) and \( \pi^* \tilde{V} \) are isomorphic as coverings but not as principal bundles. In that formulation [2, Proposition 7.1] remains unproved. Our goal here is to provide a different version of [2, Proposition 7.1], in line with the formulation of bimeromorphic invariance of Th. 4.1 above. Note also that Th. 4.1 allows us to reformulate and prove [2, Proposition 7.1] in the more general framework of complex analytic spaces.

Proposition 4.3. Let \( \psi : V \rightarrow W \) be a bimeromorphic map between compact and irreducible complex analytic spaces with \( \dim(\text{sing}(V)) = \dim(\text{sing}(W)) = 0 \). Assume that there exist resolutions \( \pi : M \rightarrow V \) and \( \rho : N \rightarrow W \) such that both maps \( \pi_* : \pi_1(M) \rightarrow \pi_1(V) \) and \( \rho_* : \pi_1(N) \rightarrow \pi_1(W) \) are isomorphisms. Let \( \phi : M \rightarrow N \) be the bimeromorphic map induced by \( \pi, \psi \) and \( \rho \). Let \( Z \) be any \( K(\Gamma, 1) \) space, \( \Gamma \) any discrete finitely generated group and let \( s : W \rightarrow Z \) be any continuous map. We have the following properties:

For any arbitrarily fixed \( p \in \text{Dom}(\phi) \) there exists a continuous map \( r_p : W \rightarrow Z \) such that

1. \( s(\pi(p)) = r_p(\rho(\phi(p))) \) for some \( p \in \text{Dom}(\phi) \),
2. \( r_p \circ \rho \circ \phi : \text{Dom}(\phi) \rightarrow Z \) is homotopic to \( s \circ \pi|_{\text{Dom}(\phi)} : \text{Dom}(\phi) \rightarrow Z \) with a homotopy fixing \( p \).

For any continuous map \( r : W \rightarrow Z \) satisfying the two properties listed above we have

\[ s_*[\overline{\text{rel}}^V] = r_*[\overline{\text{rel}}^W] \] \hspace{1cm} (4.3)

in \( K_0^\text{an}(Z) \).

Proof. The first part of the proposition follows immediately by the first part of Th. 4.1. We tackle now the second part. Combining [2, Th. 4.1] and Th. 4.1 we have

\[ s_*[\overline{\text{rel}}^V] = s_*(\pi_*[\overline{M}]) = r_*[\rho_*[\overline{N}]] = r_*[\overline{\text{rel}}^W]. \]

Consequently, if we define the homological Todd class as

\[ \text{Td}_*(V) := \text{Ch}_*[\overline{V}] \] \hspace{1cm} in \( H_*(V, \mathbb{Q}) \)

then the higher Todd genera in this singular setting

\[ \{(\alpha, s_*(\text{Td}_*(V))) : \alpha \in H^*(B\Gamma, \mathbb{Q})\} \]

are bimeromorphic invariants in the sense of Prop. 4.3.

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