Wave interactions with a porous and flexible cylindrical fish cage

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Abstract

The hydrodynamic response of a porous flexible circular-cylinder in regular waves was analytically studied. To simplify the problem, the bottom of the cylinder was considered as rigid porous plates. Small amplitude water wave theory and structural responses were assumed. The velocity potentials were solved using the Fourier-Bessel series expansion method and the least squares approximation method. The present study represents a preliminary step in the study of the fish cage.

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1. Introduction

Ocean fishing has gradually been replaced by ocean farming in the last 30 years, and more attention has been paid to the net cage fish farming system. The fish farms are forced to move into offshore area due to the shrinking availability of near shore sites and the increasing environmental impacts of aquaculture.

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To investigate the performance and reliability of the open ocean net cage, many studies have emphasized the mechanical performance of the nets by developing models of the fish net. The common fishage is constructed with flexible netting as a main structural element. It is important to understand the behaviour of such structures in different wave situations to maintain the water quality in the net cage for fish health and growth.

Cho and Kim [1] studied the oblique wave interaction with a submerged horizontal flexible membrane using the boundary element method and eigen function expansion method. Chan and Lee [2] analytically studied the scattering of surface waves by a flexible fishnet. The net was modeled as a porous, freely-flexible barrier and was assumed to behave similarly to a catenary. Bao et al. [3] considered a porous rigid circular cylinder with the bottom and cover semi-submerged in waves. A complicated dispersion relation was used to determine the eigenvalues in the interior region. A new component of damping caused by the porosity was found in the study. Park et al. [4] studied the wave excitation forces acting on an array of truncated porous circular cylinders based on the eigen function expansion approach method. Mandal et al. [5] theoretically investigated the interaction of surface waves with concentric truncated cylinder systems consisting of an inner rigid cylinder and an exterior porous and flexible cylinder. Zhao et al. [6] simulated the fluid field around a fishing plane using a porous model and Computational fluid dynamics (CFD) modeling method. Behera and Sahoo [7] studied the wave interaction with submerged horizontal flexible porous plate under the assumption of small amplitude water wave theory and structural response.

In the present study, the hydrodynamic response of a flexible fish net cylinder in regular waves is investigated in a finite water depth using linear wave theory. The fish net is modeled as a porous, freely-flexible cylinder that deforms like a one-dimensional beam. The bottom is assumed as a rigid and porous plate. By the application of the eigen function expansion method, the problems are converted into dual series relations and the least squares approximation is applied to obtain the potentials in and around the net.

2. Mathematical formulation

The problems are considered in the cylindrical polar coordinate system in water of finite depth $h$. The cylindrical fishnet of radius $a$ is porous, flexible and is of negligible thickness, as shown in Fig.1. The cylinder is assumed to follow two typical fishnet arrangements: (A) fixed at both ends and (B) fixed at the top, free at the end. The axis of the cylinder is situated at $r = 0$, with the $z$-axis oriented vertically upward and $z = 0$ is the free surface. The fluid occupies the region except the outer flexible porous cylinder. The notations $S_{net}$ and $S_{gap}$ represent the net portion and the gap of the system, with $S_{net} = (-d \leq z \leq 0)$ and $S_{gap} = (-h \leq z \leq -d)$. The fluid domain is divided into two regions, Region1: $(-h \leq z \leq 0, r \geq a)$ and Region 2: $(0 \leq z \leq 0, r \leq a)$. It is assumed that the fluid is inviscid and incompressible, its motion is irrotational, and the amplitude of the fluid oscillations is small. The flow is assumed to be harmonic in time with an angular frequency $\omega$ and a wave height $H$. Thus, the velocity potentials $\Phi_j(r, \theta, z, t)$ in the respective fluid regions are of the forms $\Phi_j(r, \theta, z, t) = Re[\phi_j(r, \theta, z)e^{-j\omega t}]$, where $\phi_j(r, \theta, z)$ is the spatial velocity potential that satisfies the Laplace equation given by:

$$\frac{\partial^2 \phi_j}{\partial r^2} + \frac{1}{r} \frac{\partial \phi_j}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \phi_j}{\partial \theta^2} + \frac{\partial^2 \phi_j}{\partial z^2} = 0, \quad j = 1, 2 \tag{1}$$

The linearized free surface boundary condition is:

$$\frac{\partial \phi_j}{\partial z} - \frac{\omega^2}{g} \phi_j = 0, \quad on \ z = 0, \ j = 1, 2 \tag{2}$$

The sea bed boundary condition is:

$$\frac{\partial \phi_j}{\partial z} = 0, \quad on \ z = -h, \ j = 1, 2 \tag{3}$$
The total velocity potential in Region 1 is given by:

$$\phi_1 = \phi_i + \phi_s$$  \hspace{1cm} (4)

The incident wave velocity potential given by:

$$\phi_i = -\frac{i g H}{2 \omega} \cosh k_o (z + h) \sum_{m=0}^{\infty} \mu_m J_m (k_o r) \cos m\theta$$  \hspace{1cm} (5)

The scattered potential $\phi_s^i$ in the outer region satisfies the radiation boundary condition:

$$\lim_{r \to \infty} (\frac{\partial \phi_s^i}{\partial r} - i k_o \phi_s^i) = 0$$  \hspace{1cm} (6)

Using Darcy’s law for flow past a porous structure, the boundary condition is given by:

$$\frac{\partial \phi_j}{\partial r} = i k_o G_j (\phi_j - \phi_i) + i \omega \eta \cos \theta \hspace{1cm} \text{on } r = a, -h \leq z \leq -d, j = 1, 2$$  \hspace{1cm} (7)

$$\frac{\partial \phi_{s_j}^i}{\partial z} = i k_o G_j (\phi_{s_j}^i - \phi_{s_j}^i), \quad \frac{\partial \phi_{s_j}^i}{\partial z} = \frac{\partial \phi_{s_j}^-}{\partial z} \hspace{1cm} \text{on } z = -d, r < a$$  \hspace{1cm} (8)

The porous-effect parameter $G_0, \ G_j$ is defined by Chwang (1983) and $k_o$ is the wave number, which is determined by the dispersion relation as:

$$\omega^2 = g k_o \tanh k_o h$$  \hspace{1cm} (9)

The continuity of the normal velocity and the pressures near the interfaces is:

$$\frac{\partial \phi_1}{\partial r} = \frac{\partial \phi_2}{\partial r} \hspace{1cm} \text{at } r = a, -h \leq z \leq 0$$  \hspace{1cm} (10)

$$\phi_1 = \phi_2 \hspace{1cm} \text{at } r = a, -h \leq z \leq -d$$  \hspace{1cm} (11)

Assuming that the transverse deflection of the porous and flexible cylinder is given as $\zeta(z,t) = \text{Re} [\eta(z) e^{-i \omega t}]$. Here, $\eta$ is the complex displacement amplitude of the cylinder defined by its own dynamical equation given by Mandal (2013):

$$r \frac{d^4 \eta}{dz^4} + \frac{\beta}{h} \frac{d^2 \eta}{dz^2} - \alpha \eta = \frac{2 i \omega}{\tau g h} \int_0^\pi (\phi_i - \phi_s) \cos (\pi - \theta) d\theta$$  \hspace{1cm} (12)

where $\gamma = EI / (m_i gh^3)$ is the non-dimensional uniform flexural rigidity of the net cylinder, $\beta = Q / (m_i gh^3)$ is the non-dimensional compressive force acting on the cylinder, $\alpha = \omega^2 / (gh^3)$, $\tau = m_i / (\rho h^2)$ is a non-dimensional
quantity related to structural density, $EI$ is the flexural rigidity of the cylinder, $m_s$ is the uniform mass per unit length and $\rho$ is the water density.

In the case of a fixed edge, the deflection and slope at the edges of the cylinder are zero:

$$\eta = 0, \frac{d\eta}{dz} = 0 \quad \text{on } r = a \quad (13)$$

In the case of a free edge, the shear force and the bending moment near the edges of the cylinder are zero:

$$\frac{d^2\eta}{dz^2} = 0, EI \frac{d^3\eta}{dz^3} + Q \frac{d\eta}{dz} = 0 \quad \text{on } r = a \quad (14)$$

3. Method of solution

By using separation of variables and applying the free surface, bottom boundary, and radiation conditions, the velocity potential in Eq.(4) is obtained as:

$$\phi_1 = \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} A_{mn} K_m(k_n r) \cos[k_n(z+h)] \cos m\theta \quad (15)$$

In Eq.(15), $K(\bullet)$ is the modified Bessel function of the second kind of order $m$. $k_n$ is determined by the following dispersion relations:

$$\left\{ \begin{array}{l}
k_n = -ik_0 \quad \text{if } n = 1 \\
\omega = -k_n \tan k_n h \quad \text{if } n \geq 2
\end{array} \right. \quad (16)$$

Proceeding in a similar manner, and with the usage of Eq.(8), the velocity potential in Region 2 is obtained as

$$\phi_2 = \sum_{m=0}^{\infty} \sum_{n=1}^{\infty} B_{mn} I_m(\lambda_n r) f_n(z) \cos m\theta \quad (17)$$

Where $I_m(\bullet)$ is modified Bessel function of first kind, $\lambda_n$ are the roots of

$$\lambda_n \sinh \lambda_n(h-d) \left[ \frac{\omega^2}{g} \cosh \lambda_n d - \lambda_n \sinh \lambda_n d \right] = ik_0 G_1 \left[ \frac{\omega^2}{g} \cosh \lambda_n h - \lambda_n \sinh \lambda_n h \right] \quad (18)$$

in the upper half complex plane of $\lambda$.

The vertical eigenfunction is given by:

$$f_n(z) = \left\{ \begin{array}{l}
(\lambda_n \cosh \lambda_n z + \frac{\omega^2}{g} \sinh \lambda_n z) \sinh \lambda_n(h-d) \quad -d \leq z \leq 0 \\
(\frac{\omega^2}{g} \cosh \lambda_n d - \lambda_n \sinh \lambda_n d) \cosh \lambda_n(h+z) \quad -h \leq z \leq -d
\end{array} \right. \quad (19)$$

Substituting Eqs.(15) and (17) into Eq.(10), and using the orthogonal characteristics of $\cos m\theta$ over $0 \leq \theta \leq 2\pi$, the continuity of velocity at $r = a$ can be expressed as:

$$\frac{-igH}{2\omega} \cosh k_{j0} \frac{k_{j0} \mu_0 Y_j'(k_{j0} a) + \sum_{n=1}^{\infty} k_n A_{mn} K_m(k_n a) \cos[k_n(z+h)]}{\cosh k_{j0} h} \quad (20)$$

$$= \sum_{n=1}^{\infty} B_{mn} \lambda_n \mu_0 Y_n'(\lambda_n a) f_n(z) \quad at -h \leq z \leq 0$$

Multiply $\cos[k_n(z+h)]$ ($p = 1, \ldots, N$) on both side of Eq.(20) respectively, and then integrate from $-h$ to $0$, we get the following equations:
\[ A_{mp} K'_m(k_p a) \text{Int}_{11}(p) + \delta_{1p} \beta_k J'_m(k_0 a) \text{Int}_{11}(p) \]
\[ = \sum_{n=1}^{\infty} B_{mn} \lambda_n \text{I}_n(\lambda_n a) \text{Int}_{12}(p, n) \]
\[ A_{mp} = \frac{\sum_{n=1}^{\infty} B_{mn} \lambda_n \text{I}_n(\lambda_n a) \text{Int}_{12}(p, n) - \delta_{1p} \beta_k J'_m(k_0 a) \text{Int}_{11}(p)}{k_p K'_m(k_p a) \text{Int}_{11}(p)} \]

With
\[ \text{Int}_{11}(p) = \int_{-h}^{0} \cos^2[k_p(z+h)] \, dz \quad \text{Int}_{12}(p, n) = \int_{-h}^{0} f_n(z) \cos[k_n(z+h)] \, dz \]
\[ \beta_n = -\frac{igH_i}{2\omega \cosh k_h} \]

\( \delta \) is Kronecker delta function.

Now consider the structural region. Substituting Eqs. (15) and (17) into (12) and integration, we obtain
\[ \eta(z) = \sum_{j=1}^{4} C_j e_j(z) + D \cosh[k_0(z+h)] + \sum_{n=1}^{\infty} E_n \cos[k_n(z+h)] + \sum_{n=1}^{\infty} F_n f_n(z) \]

With
\[ E_n = \frac{RA_m K_1(k_p a)}{\gamma k_n^4 - \beta k_n^2 - \alpha} \quad F_n = \frac{-RB_m L_1(\lambda_n a)}{\gamma \lambda_n^4 + \beta \lambda_n^2 - \alpha} \quad R = -\frac{\pi \omega i}{\tau gh^2} \quad e_j(z) = e^{p_j z} \quad D = \frac{R\beta_1 L_1(k_0 a)}{\gamma k_0^4 + \beta k_0^2 - \alpha} \]

where \( p_n \) represents the roots of the characteristic equation, \( \gamma p_n^4 + \beta p_n^2 - \alpha = 0 \).

The integrating constants, \( C_i \, (i = 1,\ldots,4) \), will be found later.

Considering Eq. (11) for the gap region on \( r = a \), using the orthogonal characteristics of \( \cos m \theta \) yields:
\[ \frac{-igH_i \cosh k_0(z+h)}{2\omega \cosh k_h} J_m(k_0 a) + \sum_{n=1}^{\infty} A_{mn} K_m(k_n a) \cos[k_n(z+h)] \]
\[ = \sum_{n=1}^{\infty} B_{mn} \text{I}_n(\lambda_n a) f_n(z) \quad \text{at} -h \leq z \leq -d \]

Similary, from Eq. (7), we obtain:
\[ \sum_{n=1}^{\infty} B_{mn} \lambda_n \text{Y}_n(\lambda_n a) f_n(z) = ik_0 G_0 \left[ \sum_{n=1}^{\infty} B_{mn} \text{I}_n(\lambda_n a) f_n(z) + \frac{igH_i \cosh k_0(z+h)}{2\omega \cosh k_h} J_m(k_0 a) \right] + i\omega \delta_{lm} \cos \theta \quad \text{on} r = a, -d \leq z \leq 0 \]

According Eqs. (27), (28) and (25), let us define \( H_m(z) \) on \( r = a \):
\[ \sum_{n=1}^{\infty} B_{mn} \lambda_n \text{Y}_n(\lambda_n a) f_n(z) = ik_0 G_0 \left[ \sum_{n=1}^{\infty} B_{mn} \text{I}_n(\lambda_n a) f_n(z) + \frac{igH_i \cosh k_0(z+h)}{2\omega \cosh k_h} J_m(k_0 a) \right] + i\omega \delta_{lm} \cos \theta \quad \text{on} r = a, -d \leq z \leq 0 \]
\[ H_m(z) = \left( \sum_{n=1}^{\infty} B_{mn} \frac{I_m(\lambda_n, a)f_n(z) + \frac{ig\mu}{2i\omega} \cosh k_n(z+h) J_m(k,n)}{\cosh k_nh} - \sum_{n=1}^{\infty} A_{mn} K_n(k_n, a) \cos[k_n(z+h)] \right) \delta_{mn} \cos \theta \quad -d \leq z \leq 0 \] (29)

The relations in Eq.(29) can be rewritten as:

\[ H_m(z) = \sum_{n=1}^{\infty} B_{mn} v_{mn}(z) + w_m(z) = 0 . \] (30)

Truncating to \( N \) terms of \( H(z) \):

\[ H_m(z) = \sum_{n=1}^{N} B_{mn} v_{mn}(z) + w_m(z) . \] (31)

Then, using the least squares approximation method on Eq.(31) yields:

\[ \int_{-h}^{0} |H_m(z)|^2 \, dz = \min . \] (32)

Truncating \( m \) to \( M \) terms, a system of equations for the determination of \( A_{mn} \) is obtained:

4. Conclusion

The hydroelastic problem between a fishnet cage and surface wave was investigated in this paper. The cage was simplified as a porous, flexible circular cylinder with a rigid porous bottom. The cylinder was expected to deform like a one-dimensional beam. The fluid domain was divided into two regions. By applying the eigenfunction expansion method, the problem was converted into dual series relations, and the least squares approximation was then applied to obtain the potentials in and around the cylinder.

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