Probing the electromagnetic fields in ultra-relativistic collisions with leptons from $Z^0$ decay and charmed mesons

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Ultra-relativistic heavy-ion collisions are expected to generate a huge electromagnetic field, $eB \approx 10^{18}$ Gauss, that induces a splitting of the directed flow, $v_1 = \langle p_x/p_T \rangle$, of charged particles and anti-particles. Such a splitting for charmed meson manifests even for neutral particle/anti-particle pairs ($D^0, \bar{D}^0$), hence being also a unique probe of the formation of the quark plasma phase. For the first time here we show that specific time evolution of the electromagnetic field may generate a $v_1(D^0) - v_1(\bar{D}^0)$ as huge as the one recently observed at LHC against early expectations.

Within this new research topic we point out a novel measurement: the directed flow $v_1$ of leptons from $Z^0$ decay and its correlation to the $D$ mesons one. Such a correlation between the $\Delta v_1$ of $l^+ - l^-$ and $D^0 - \bar{D}^0$ would provide a strong probe of the electromagnetic origin of the splitting and hence of the formation of a quark plasma phase with charm quarks as degrees of freedom. The case of the $v_1(l^\pm)$ presents features due to the peculiar form of the $p_T$ spectrum never appreciated before in the study of heavy-ion collisions. We specifically predict a sign change of the $\Delta v_1(p_T)$ of leptons at $p_T = 45$ GeV/c, that can be traced back to a universal relation of $\Delta v_1$ with the slope of the $p_T$ particle distribution and the integrated effect of the Lorentz force.

The ultra-relativistic heavy-ion collisions (uRHICs) at both RHIC and LHC have been a terrestrial tool to access the properties of the Hot QCD matter and have shown that such a matter expands hydrodynamically with a very small $\eta/s$, shear viscosity to entropy density ratio, close to a conjectured lower limit for strongly interacting matter [1]. At the current stage the description of the expanding dynamics in AA collisions appears to be quite solid which is opening a new frontiers in the study of uRHICs. The main novel aspects that are becoming accessible are those related to the impact of the strongest electromagnetic field [2, 3] and of the largest relativistic vorticity [4, 5] ever created in a physical system. This is triggering an intense studies in the off-central heavy ion collisions, such as the chiral magnetic effect (CME) [6–11], chiral magnetic wave (CMW) [12–15] and the splitting in the spin polarization [16–18] and the directed flow ($v_1$) of mesons [2, 3, 19, 20].

In the last decade it has been possible also to study the dynamics of heavy quarks (HQs), mainly charm, achieving a first phenomenological determination of their interaction with the QGP medium [21–23]. This opens the possibility to investigate by mean of HQs new aspects coming from the very early stage dynamics. In fact, as pointed out in Ref. [2], charm quark can provide a very suitable tool to explore the presence of the electromagnetic field thanks to their short formation time, $\tau_{\text{form}} \leq 0.08$ fm/c, that makes them present in the very early stage as gluons. Therefore they could be excellent probes of the huge electromagnetic field [2, 3, 24] as recently confirmed by a first measurement of $v_1$ of D mesons [25, 26].

A breakthrough will be to understand if the splitting very recently observed for $D^0$ and $\bar{D}^0$ has an electromagnetic origin. Here we point out for the first time that even the measurement by ALICE [25], much larger and even of opposite sign than the one predicted in Refs. [2, 3], can still be generated by the magnetic field, not with larger maximum strength but just damping slowly with a lifetime $\tau_B \approx 0.4$ fm/c. The main new measurement proposed in this Letter is the $v_1(p_T, y)$ of the leptons coming from $Z^0$ decays. Due to the fact that they interact with the electromagnetic field and not with the strong one, they are a most suitable tool to extract info on the e.m. field, especially because they can be separated from other sources of leptons and are generated in the pre-thermal equilibrium stage of QGP with a decay lifetime very similar to the charm formation time, i.e. when the e.m. field is expected to be about its maximum values. A further advantage will be the possibility to access a region of momenta very large, up to $p_T \approx 100$ GeV, exploiting the Lorentz force in a different kinematic region w.r.t. the case of charm quarks. The absolute measurement of $v_1(p_T, y)$ of $l^\pm$ and its correlation to the $D^0, \bar{D}^0$ will be smoking gun of the electromagnetic origin of the splitting.

Though the magnetic field at $t = 0$ can be evaluated with sufficient confidence in relativistic heavy ion collisions [27, 28], its time evolution is still largely an open question [19, 29–31]. To have a general study of the effects of electromagnetic fields, we include three typical configurations, where only $E_x$ and $B_y$ are included due to their dominance over other contributions.

Case A - A standard approach, developed in several papers [2, 19, 32], has been based on the solution of the Maxwell equations for the field generated by the spectators in the overlapping region assuming this is made by matter in equilibrium with a constant conductivity.
$\eta_t = 0.023 \text{ fm}^{-1}$ IQCD calculations [33–35]. The drawback is to assume a finite conductivity associated to the QGP matter even before the collisions, hence strongly damping the maximum value of $B_y$ ($\sim 50$ times) [28] w.r.t. the vacuum estimate.

Case B - The large gap between the maximum initial $B_y(t = 0) = B_0$ field with and without assuming a conducting medium has led other authors to consider a parametrization that at least at initial time agrees with the large value expected in the vacuum. We label this case as $eB_y(x, y, \tau) = -B(\tau)\rho_B(x, y)$ with $\rho_B(x, y) = \exp[-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}]$ [36] and a $B(\tau) = eB_0/(1 + \tau^2/\tau_B^2)$ as developed in Refs [9, 10] with $B_0$ given by in vacuum estimate. $eE_x$ is then determined by solving the Faraday’s Law $\nabla \times \mathbf{E} = -\partial \mathbf{B}/\partial t$:

$$eE_x(t, x, y, \eta_S) = \rho_B(x, y) \int_0^{\eta_S} d\eta B'(t, \eta) \frac{t}{\cosh \eta}$$

where $\eta_S$ is space-time rapidity. We will see that such a field even if much larger than case A still does not lead to a prediction in agreement with the large splitting observed by ALICE at LHC [25], indeed it even leads to a $\Delta v_1 = v_1(D^0) - v_1(D^\pm)$ splitting of opposite sign.

Case C - A third case for the magnetic field $B_y$ starts from the same initial value and the space distribution of case B, but has a slower time evolution of the field: $B(\tau) = eB_0/(1 + \tau^2/\tau_B)$.

In this study we focus on 5.02 TeV Pb+Pb collisions at 20-30% centrality, that corresponds to impact parameter $b = 7.5$ fm, due to the recent measurements by ALICE of non-zero directed flow splitting between $D^0$ and $\overline{D}^0$ [26]. Though we choose this system to illustrate the effect, our conclusions are pretty general. In Fig. 1 we show the time evolution of $eB_y(t)$ and $eE_x(t)$ at $x_T = 0$ and $\eta_S = 1$ for the three cases discussed above. $eB_y$ is chosen to be negative as in the experimental convention.

For cases B and C, the parameters are found to be $eB_0 = 73 m_Z^2$, $\sigma_x = 3$ fm and $\sigma_y = 4$ fm. The $\tau_B$ has a large uncertainty but we fix it to be the same for case B and C and equal to a standard value $\tau_B = 0.4$ fm/c. It is seen that in case B the time derivative of the $B_y(t)$ is such to generate a $E_x \simeq B_y$ already at $t \simeq 0.6$ fm/c, while for case C the slower time evolution leads, due to the Faraday’s Law, to a quite smaller $E_x$ that essentially remains always smaller than $B_y$, even if the initial values of case B and C are equal and they remain so up to time $t \approx 1$ fm/c.

The space-momentum distribution of leptons from $Z^0$ decay can be found by the phase space distribution of $Z^0$. We use the experimental measurements [37, 38] to construct a parametrization of the transverse momentum and rapidity dependence of $Z^0$. It is seen in Fig. 2 that $Z^0$ generated by a Monte Carlo simulation with such parametrization does agree quite well with the CMS measurements shown by black dots [37]. The $x_T$ dependence of $Z^0$ is given by the binary collisions of colliding nuclei, and position are given by $t = \tau_{Z^0}\cosh y$ and $z = \tau_{Z^0}\sinh y$ with $\tau_{Z^0} = 1/2m_{Z^0} = 0.0011$ fm/c.

In Fig. 2 for the leptons decayed from $Z^0$ with mean decay proper time $1/2.495 \text{ GeV}^{-1}$, we use both analytical and Monte Carlo methods to generate their spectra. The exact matching between these further validates our approach.

We study the evolution of both lepton and $D_0$ meson by standard Langevin equations [23, 39–46] but including Lorentz force [2, 3]:

$$dx_i = \frac{p_i}{E} dt,$$

$$dp_i = -\Gamma p_i dt + \rho_i \sqrt{2D} dt + q(E_i + \epsilon_{ijk} v_j B_k) dt,$$

where the diffusion is related to the drag by $D = \Gamma ET$, and $\rho_i$ is randomly sampled from a normal distribution with $\langle \rho_i \rangle = 0$ and $\langle \rho_i \rho_j \rangle = \delta_{ij}$. For charm quarks, the

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{(Color online) Time evolution of $-eB_y$ and $-eE_x$ at $x_T = 0$ and space-time rapidity $\eta_S = 1$ in 5.02 TeV Pb+Pb collision at impact parameter $b = 7.5$ fm for three cases. See text for details.}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{(Color online) Normalized spectra of $Z^0$ boson and lepton as a function of transverse momentum $p_T$ from Monte Carlo simulation in 5.02 TeV Pb+Pb collisions, compared to the CMS measurements [38] of $Z^0$ and an analytical calculation of lepton from $Z^0$ decay.}
\end{figure}
drag coefficient is determined by the study of the D meson observables \[47-51\]. We instead include only Lorentz force in Eq. (3) for leptons from Z⁰ decay due to their negligible interaction with the medium.

In Fig. 3, we show the directed flow splitting between \(D⁰\) and \(\overline{D⁰}\) as a function of pseudorapidity for the three cases of the e.m. field evolution in 20-30% centrality 5.02 TeV Pb+Pb collisions. It is found that \(\Delta_\eta/v_1/d\eta\) is -0.004 for case A, -0.42 for case B with \(\tau_B = 0.4\) fm/c, and 0.44 ± 0.15 for case C with the interval of values corresponding to \(\tau_B = 0.4 ± 0.1\) fm/c. Compared to ALICE measurements \[26\] of \(\Delta_\eta/v_1/d\eta\) = 0.49 ± 0.18, we find that only case C with \(\tau_B\) in the range of 0.3-0.5 fm/c can lead to a correct prediction. In particular, it is surprising that for case B and C even starting with the same initial values of \(B_0\) and similar magnitude up to \(t ≈ 1\) fm/c one can have completely opposite predictions for \(d\Delta_\eta/v_1/d\eta\).

Such a feature is however not accidental, and even if this is the result of a quite involved dynamics we can catch the key features determining the above result by mean of suitable approximations. Discarding charm-bulk interaction, we assume also that the electromagnetic term \(qB_0\tau_B\) is a small perturbation of the charm (or lepton) energy. We look for the shift in \(p_x\) of a charm (lepton) of momentum \(p_T, y, z\) at the initial transverse position \(r_0\). The average value of it comes from the integral over the participant space region of \(\Delta p_x(r_0)\):

\[
\overline{\Delta p_x}(p_T, y, z) = \int d^2r_0 \rho(r_0) \Delta p_x(r_0, p_T, y, z),
\]

where \(\rho(r_0)\) is the initial distribution of the particles in transverse plane at the formation time \(t_0\). Assuming that \(y_z = \eta_s\) and evaluating the shift of each particle as the propagation along straight lines from Eq. (2), we can write:

\[
\Delta p_x(r_0, p_T, y_z) = qE_x(t, \vec{r}(t), y_z) - qv_zB_y(t, \vec{r}(t), y_z),
\]

where \(\vec{r}(t) = \vec{r}_0 + \Delta \vec{r}(t)\) gives the position of the particle at time \(t\). One can write the average \(p_x\) shift of the particle more explicitly as:

\[
\overline{\Delta p_x}(p_T, y_z) = \int_0^{\infty} dt \int dx_0 dy_0 \rho(x_0, y_0) \int \frac{d\phi}{2\pi} q \left[ \tanh y_z B(t, y_z) + \int_0^{y_s} d\eta B'(t, \eta) \frac{t}{coshn} \right] 
\]

\[
\rho_B \left[ x_0 + \frac{p_T cosh \phi}{m_T cosh y_z} (t - t_0), y_0 + \frac{p_T \sinh \phi}{m_T cosh y_z} (t - t_0) \right] \tag{4}
\]

We can find some general features of the integration over \(x_0, y_0\) and \(\phi\) without knowing the forms of \(\rho(x_0, y_0)\). The integration depends only on the factor \(\gamma = \frac{1}{E_T} (\frac{t}{cosh y_z} - \tau_0)\), and it should be quite uniform in the region of \(\gamma < R \sim \sqrt{m_T}\) and decreases fast outside. Because of these features, we can replace the integration by a step function \(K(1 - \gamma/R)\), where \(K\) is some constant depending on the specific forms of \(\rho\) and \(p_B\). Hence integrated by parts, Eq. (4) can be further simplified to:

\[
\overline{\Delta p_x}(p_T, y_z) \propto q \int_0^{y_s} \frac{d\eta}{cosh \eta} \left[ \tau_2 B(\tau_2) - \tau_1 B(\tau_1) \right] \tag{5}
\]

with \(\tau_1 = \frac{\gamma cosh y_z}{cosh \eta}\) and \(\tau_2 = \frac{(\gamma + R_m + p_T) cosh y_z}{cosh \eta}\), and \(\tau_{1,2}\) can be treated as the formation time and the escape time out of the electromagnetic field of the particle.

Using the form of \(B(\tau)\) from case B and C and \(\tau_0 \sim 0.1\) fm/c, we can find that \(\overline{\Delta p_x}(y_z > 0)\) of charm quarks is positive for case C and negative for case B and so will be \(\Delta_\eta\). In the full calculation one should take into account the interaction with the medium (for charm quarks) and the not small \(qB_0\tau_B\), that induce some further modulation w.r.t. Eq. (5) for low \(p_T\) particles. However Eq. (5) catches the main feature and predicts correctly the sign and even the magnitude with reasonable accuracy, as the results of the simulations in Fig.3 show. Eq. (5) enlighten our understanding showing that the sign of \(\Delta_\eta\) is not determined by the initial maximum value of \(cB_0\), but by its time evolution. It includes implicitly the effect \(E_z\) directly related to the \(B_y\) time derivative: a slower time evolution leads to a smaller electric field \(E_z\), inducing a positive sign of \(\Delta_\eta\). In the following, we will focus our attention on the proposal of a new measurement: the \(v_1\) of the leptons from the \(Z^0\) decay. We will consider case C that is the only one able to account for the observed positive splitting of \(\Delta v_1^D = v_1(D^0) - v_1(\overline{D^0})\).

In the left of Fig. 4 we show the splitting \(\Delta v_1^D = v_1(t^+) - v_1(t^-)\) between positively and negatively charged leptons for case C. It is seen that \(d\Delta v_1^D/d\eta\) is negative at \(p_T \sim 40\) GeV/c, hence opposite to the one of the charm quarks which is already not trivially expected. Even more relevant is the observation that \(d\Delta v_1^D/d\eta\) changes abruptly sign around \(p_T \sim 45\) GeV/c reaching a peak value comparable to the one observed and here predicted for \(d\Delta v_1^D/d\eta\) in the region \(p_T \sim 3 - 10\) GeV/c. This sign change for \(l^\pm\) is quite surprising and not expected if one relates the flow to the direct effect of the Hall and Faraday drift on one single particle. In this Letter we indeed
clarify for the first time that the role of the slope of particle spectra can strongly affect the flow sign. We find that differential $\Delta v_1$ is not a direct measure of $\Delta p_x$, but relates also to the spectra of charged particles. In the specific case of the lepton from $Z^0$ decays one has unique shape of the spectrum, very different from the particle spectra of hadrons in uRHICs, as shown in Fig. 2. This induces a new feature in the directed flow never appreciated before that we explain by mean of Fig. 5.

We have found two key aspects that determine the splitting $\Delta v_1$ of D mesons and leptons. The sign of the splitting is determined by the time evolution of the magnetic field more than its initial strength, see Eq. (5). Furthermore the $\Delta v_1^D$ and $\Delta v_1^\ell$ can have an opposite sign and $|\Delta v_1^D| < |\Delta v_1^\ell|$ for any $p_T$, see Eq. (7), even if both are generated by the same e.m. field, due to the peculiar and more flat momentum distribution of leptons. We propose for the first time in uRHICs the measurement of the $v_1$ of leptons from $Z^0$ decay to confirm the electromagnetic origin of the $\Delta v_1^D$ and have a novel constraint on e.m. strength and time evolution. The experimental search for the above relation for both heavy flavors and leptons can give us a proof of the strength and time evolution of electromagnetic field in the early stage of uRHICs. This work should trigger extended studies that include also advancement in the physics that can affect the evolution of the electromagnetic fields, but also the extension to the $b$ quark and a thorough study as a function of energy and centrality. However the scope is much wider than the understanding of heavy flavor dynamics and merely the generation of the electromagnetic field in the early stage. In fact providing an independent novel probe of the e.m. field is a key aspect that can trigger a breakthrough in the new ongoing search for the CME and CVE [6–11], CMW [12–15], and splitting in the A.
polarization [16–18]. Finally, we highlight that if the splitting $\Delta v^p = v_1(D_0) - v_1(\overline{D}^0)$ for neutral particles has an electromagnetic origin, it provides also a direct probe of the existence of the deconfined phase with the charged charm quarks as degrees of freedom. This represents an absolutely new and unique probe allowing to access the deconfinement as a function of the flavor, a key and open question of the understanding of the QCD phase transition [52, 53].

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