Second-harmonic generation and the conservation of spatiotemporal orbital angular momentum of light

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Light with spatiotemporal orbital angular momentum (ST-OAM) is a recently discovered type of structured and localized electromagnetic field. This field carries characteristic space–time spiral phase structure and transverse intrinsic OAM. Here, we present the generation and characterization of the second harmonic of ST-OAM pulses. We uncover the conservation of transverse OAM in a second-harmonic generation process, where the space–time topological charge of the fundamental field is doubled along with the optical frequency. Our experiment thus suggests a general ST-OAM nonlinear scaling rule, analogous to that in conventional OAM of light. Furthermore, we observe that the topology of a second-harmonic ST-OAM pulse can be modified by complex spatiotemporal astigmatism, giving rise to multiple phase singularities separated in space and time. Our study opens a new route for nonlinear conversion and scaling of light carrying ST-OAM, with the potential for driving other secondary ST-OAM sources of electromagnetic fields and beyond.

The orbital angular momentum (OAM) of light is a type of angular momentum associated with wavefront or phase vortices in the electromagnetic field. For a propagating paraxial wave, the longitudinal OAM of light means that the OAM is parallel to the averaged wavevector and propagation direction of the beam. OAM can be intrinsic or extrinsic (cf. a recent review in ref.7). Intrinsic OAM implies that the angular momentum is reference-frame-independent and can be described by an integer quantum number called (spatial) topological charge $\ell$. The intrinsic longitudinal OAM (here referred to as conventional OAM) of light has an OAM of $\ell h c / \lambda$ per photon and a spiral phase $e^{i\ell \Phi(x,y)}$ surrounding a phase singularity in the $x$–$y$ plane (Fig. 1a). Most studies over the past three decades have focused on conventional OAM, which has impacted many important applications including optical tweezers, super-resolution imaging, quantum and classical communication, and scatterometry-based surface metrology8–10. Very recently, time-varying OAM of light has been discovered3.

By contrast, a transverse OAM of light implies that the OAM is perpendicular to the averaged wavevector of the beam. This means that a spiral phase resides in space–time, for example, the $x$–$t$ plane (or, equivalently, the $x$–$z$ plane) in a simplified two-dimensional (2D) case (Fig. 1a), and is thus referred to as spatiotemporal orbital angular momentum (ST-OAM). By analogy with conventional OAM, we can designate an integer $\ell$ as the spatiotemporal topological charge to describe the space–time winding phase $e^{i\ell \Phi(x,x,t)}$. The scalar field carrying ST-OAM reads

$$E(x,y,z,t) \propto E_0(x,y,z,t) e^{i\ell \Phi(x,x,t)} e^{i(kz-\omega t)}$$

where $E_0$ is the scalar field envelope, $\omega$ is the optical frequency and $\Phi$ is the spatiotemporal phase. This transverse OAM of light was theoretically predicted11–13, and has been observed in filamentation7. Very recently, optical pulses with ST-OAM were experimentally realized in the linear regime for the first time14,15. Because this ST-OAM of light was only recently discovered with few experimental observations, many of its properties remain elusive. Therefore, the extent to which they can be described by an analogy with the conventional OAM of light remains unclear.

In nonlinear frequency conversion, the conventional OAM of light follows a simple scaling rule, where the $N$th harmonic has a topological charge $N\ell$, reflecting OAM conservation. This rule has been verified for second-harmonic generation (SHG)16–19, non-perturbative high-order ($N \gg 10$) harmonic generation20, and can be generalized to describe sum- and difference-frequency generation processes21. This rule only applies to scalar fields without spin angular momentum, otherwise total angular momentum conservation must be included22.

Here, we experimentally investigate the behaviour of ST-OAM pulses during the frequency upconversion process of SHG. By uncovering the conservation of ST-OAM in an SHG process from $\ell = 1$ to $\ell = 2$, where the space–time topological charge of the fundamental field is doubled along with the optical frequency, our experiment demonstrates a general ST-OAM nonlinear scaling rule—analogous to that describing the (spatial) topological charges in conventional OAM of light. We also investigate the effects of spatiotemporal astigmatism in SHG, which leads to non-conserved topological charges of the spiral phase structure, as well as the creation of multiple phase singularities separated in space–time.

Results

Generation and measurement of light with ST-OAM and its second harmonic. The generation of ST-OAM pulsed beams is depicted in Fig. 1b. A fundamental pulse at a central wavelength of $\lambda = 800$ nm from a Ti:sapphire amplifier is sent to a custom ST-OAM pulse shaper to generate light with ST-OAM of $\ell = 1$ (Methods). Unlike conventional OAM beams, which can be characterized by space-based methods such as fork holograms23, coherent diffractive imaging24 and structured apertures25, ST-OAM pulses require a space–time or equivalently space–frequency based characterization method. Figure 1c,d shows experimentally measured and reconstructed 3D intensity isosurface profiles of the fundamental and second-harmonic ST-OAM pulse.

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We used a Mach–Zehnder-like scanning interferometer to optically gate an ST-OAM pulse to fully characterize the ST-OAM of light, a method similar to that in ref. 1. Briefly, a long, 800-nm fundamental ST-OAM pulse of ~500 fs of $\ell = 1$ was interfered with a short, 800-nm fundamental Gaussian reference pulse of ~45 fs to form interference fringes. These fringes characterize the portion of the ST-OAM pulse gated by the more than 10 times shorter reference pulses. The delay-dependent fringes, where each delay is a time frame of 100 fs, are shown in Supplementary Fig. 2. The reconstructed spatiotemporal amplitude and phase of the fundamental ST-OAM pulse can be reconstructed computationally from the delay-dependent fringes, where each delay is a time frame of the reconstruction (Methods). Representative fringe patterns of ST-OAM pulses are shown in Supplementary Fig. 2. The reconstructed spatiotemporal amplitude and phase of the fundamental ST-OAM pulse are shown in Fig. 2a–c. Figure 2a shows the amplitude envelope, Fig. 2b the phase and Fig. 2c a complex field representation where the amplitude and phase are represented by brightness and hue, respectively. Our experimentally reconstructed spatiotemporal profiles clearly confirm the generation of a vortex-shaped ST-OAM pulse with a 2$\pi$ "azimuthal" phase swirl in space and time.

Second-harmonic fields were generated by sending the fundamental pulses through individual beta barium borate (BBO) crystals on two arms of a similar interferometer. We thus obtained a long, 400-nm second-harmonic ST-OAM pulse of an unknown, to-be-determined spatiotemporal topological charge $\ell$, and a short, 400-nm second-harmonic Gaussian reference pulse to form interference fringes. A second-harmonic ST-OAM pulse generated by a thin BBO crystal (20 $\mu$m in thickness) was experimentally reconstructed and is shown in Fig. 2d–f. The fringe patterns used for reconstruction are shown in Supplementary Fig. 2. The amplitude envelope profile of the second-harmonic pulse in Fig. 2d still looks close to a simple vortex shape, which shares the same topology with the fundamental ST-OAM pulse: a torus of genus close to one. Although the reconstructed phase profile in Fig. 2e shows two singularities, they are almost overlapped in the time domain and slightly dislocated in space by only ~200 $\mu$m, approximating a single phase singularity with 4$\pi$ azimuthal spiral phase in space–time. The small singularity separation could be attributed to a slight offset of the laser central wavelength to the designed spiral phase plate (SPP) wavelength and/or a slight amount of astigmatism of the initial Gaussian beam. Also, the generation of the fundamental ‘spatial’ OAM beams inside the pulse shaper is far from perfect—they are generated slightly off the Fourier plane of the 4–f pulse shaper because we relied on the beam double passage through the same SPP by retroreflection (Methods).

The 4$\pi$ spiral phase we observed indicates that the second-harmonic ST-OAM pulse has a confirmed spatiotemporal topological charge of $\ell = 2$ after frequency doubling. We therefore established a nonlinear scaling rule and uncovered that the spatiotemporal angular momentum is conserved during an SHG process.

**Spatiotemporal astigmatism and modal analysis.** We also investigated a case with strong spatiotemporal astigmatism by generating second-harmonic ST-OAM pulses using a thick (1 mm) BBO crystal, as shown in Fig. 2g–i. Unlike the thin BBO case, the second-harmonic pulse from a thick BBO crystal has a different topology in the pulse profile: there are two amplitude holes (two phase singularities) close to the centre, with two outer lobes (a case resembling a torus of genus two). The phase profile in Fig. 2h shows two phase singularities, each with 2$\pi$ spiral phase, giving rise to 4$\pi$ accumulated phase. This again demonstrates ST-OAM conservation in an SHG process, although the topology is not conserved. The two phase singularities are dissociated in both time (by ~50 fs) and space (by ~500 $\mu$m). We note that this separation is not surprising—a similar deformation was described in an earlier simulation1, and such separation of phase singularities in an SHG process was predicted and presented in a recent simulation21. The use of a thicker BBO crystal might lead to spatiotemporal astigmatism by introducing phase mismatch and group velocity mismatch. The phase mismatch can narrow the frequency conversion bandwidth, and the group velocity mismatch can stretch the second-harmonic pulse duration. Both these effects would contribute to the final distortion of the second-harmonic ST-OAM pulses in space and time, causing a change in topology of the vortices. Detailed investigations of such complex processes in thick nonlinear crystals are beyond the scope of this work. Further studies are needed to explore the propagation effects of ST-OAM pulses in nonlinear dispersive media.

**Fig. 1 | Second-harmonic ST-OAM pulse generation and characterization.** a, Schematic of conventional OAM of light, ST-OAM of light and its SHG. The averaged OAM L (purple arrows) in a conventional OAM pulse is parallel to the mean wavevector k, (green arrows), and thus named longitudinal OAM, while the averaged OAM in an ST-OAM pulse is perpendicular to k, and thus transverse. b, In our experiment, fundamental ST-OAM pulses of topological charge $\ell = 1$ are generated by a custom pulse shaper. Second-harmonic ST-OAM pulses are generated in BBO crystals and characterized by interference with reference pulses. c,d, Experimentally reconstructed 3D intensity isosurface profile of the fundamental ST-OAM pulse (c) and its second harmonic (d). M, mirror; DM, dichroic mirror; CM, chirped mirror; CL, cylindrical lens; BS, beamsplitter; TFP, thin-film polarizer; QWP, quarter-wave plate; SPP, spiral phase plate; G, grating; BBO, beta barium borate crystal; D, detector.
To estimate how much spectral power is located at the expected frequency, we performed a modal decomposition of the ST-OAM pulses in the SHG process. The OAM mode spectra are extracted by Fourier transformation over the experimentally reconstructed phase profiles along the azimuthal direction on the $x$-$t$ plane for normalized polar coordinates $r_{\text{avg}}$. Unlike a conventional OAM beam where the $x$ and $y$ axes are equivalent, ST-OAM pulses need to be scaled in the $x$ axis and the $t$ ($z$) axes to obtain the mode spectrum correctly (Methods). The insets in Fig. 3 show how the origins in the polar coordinates and scaling factors are defined in each case to deal with a different number of phase singularities, where the solid red lines are overlapped with amplitude envelope profiles taken from Fig. 2a–i for reference. The mode spectra are shown for the fundamental field, $\ell = 1$ for the second-harmonic pulse generated using a thin BBO crystal in Fig. 3b, for the second-harmonic pulses generated using a thick BBO crystal in Fig. 3c. As expected, the peak of the ST-OAM spectrum is located at $\ell = 1$ for the fundamental field and mainly at $\ell = 2$ for its second-harmonic field.

To estimate how much spectral power is located at the expected charge number, each mode spectrum was integrated along the radial coordinate. For the fundamental field, ~94.5% of the total spectral power is located at $\ell = 1$ (Fig. 3a). For the second-harmonic field generated using a thin BBO crystal in Fig. 3b, the majority (~80.9%) of the total power is at $\ell = 2$ (Fig. 3b), reflecting the conservation of ST-OAM in the SHG process. However, in the case of a thick BBO crystal, we observed severe degradation or increased impurity of the modal spectrum, where only ~57.8% of the total power is at $\ell = 2$ and a notable d.c. peak appears (Fig. 3c). This result indicates that the second-harmonic ST-OAM field is distorted after generation and propagation through the thicker crystal, due to spatiotemporal astigmatism.

**Momentum density and energy density flux.** We next investigated the experimentally extracted momentum density and energy density flux of the fundamental and its second-harmonic ST-OAM pulses to better understand the SHG process, including spatiotemporal astigmatism. Our ST-OAM pulses, as quasi-monochromatic, near-paraxial fields with linear polarization, can be described by a scalar field $E = E_0 e^{i\Phi}$ (see Supplementary Information for details). The canonical momentum density in vacuum $P$ (also called scalar optical current in free space), with the dimension of energy per unit time and per unit area, is then defined as the expectation value of the intensity-weighted momentum operator,

$$P \propto \frac{1}{c} \text{Im}(E^* \nabla E) = \frac{1}{\omega} |E_0|^2 \nabla \Phi$$

(refs. 3,21,22). It has been shown that the canonical momentum density can describe the local phase gradient in an arbitrary structured field and is much more suitable for ST-OAM field.
to describe the momentum and angular momentum properties of a free-space light field than the kinetic Poynting vector\(^7\). This momentum density can be further used to extract a single quantity, the normalized expectation value of the ST-OAM per pulse, \(\langle \ell \rangle = \omega \int |\mathbf{E} \times \mathbf{D}| / \int |\mathbf{E}|^2 dV\), following ref. \(^3\). Adopting these definitions and assuming unity refractive index in air, Fig. 4a–c shows our experimentally measured spatiotemporal canonical momentum density for a fundamental pulse (Fig. 4a) and its second-harmonic pulse generated using thin and thick BBO crystals (Fig. 4b and 4c, respectively). The modulus of the momentum density vectors (\(\mathbf{P}\)), the length of the yellow arrows in the figures) are scaled by \(1/\epsilon_0\) and by \(1/\epsilon_0\) in the horizontal and vertical axes for better visualization (Methods). The momentum density of a fundamental ST-OAM pulse follows the gradient of the spatiotemporal phase and forms a vortex around the singularity. For a second-harmonic pulse in a thin BBO crystal, the momentum density is no longer uniformly distributed, but it still forms a vortex. In this case, we obtain the ratio of the normalized expectation value of ST-OAM per pulse as \(\langle \ell (2\omega) \rangle / \langle \ell (\omega) \rangle = 2.09\), the scenario of a near-perfect SHG process. However, for the case in a thick BBO crystal, the momentum density is redistributed following the distorted amplitude profile, resulting in the reduced ratio \(\langle \ell (2\omega) \rangle / \langle \ell (\omega) \rangle = 1.36\). The smaller expectation value of ST-OAM per pulse is consistent with the appearance of a notable d.c. peak in the OAM modal spectrum observed in Fig. 3c, similarly reflecting the degradation and reduced mode purity due to spatiotemporal astigmatism in a thick crystal.

To further understand the physical mechanism responsible for this astigmatism, we experimentally extracted the energy density flux in the ST-OAM pulses inside the BBO crystals. We note that energy flows in electromagnetic waves are generally described by the well-known Poynting vector, which coincides with the local distributions of optical energy per unit time and per unit area inside the dispersive BBO crystals. This energy density flux \(\mathbf{J}\) can be described as the transverse \((x, y)\) and longitudinal \((\zeta; z, t)\) contributions and can be written as \(\mathbf{J} \propto \int |\mathbf{E}|^2 (\nabla \cdot \mathbf{E} - \nabla \times \mathbf{D})\), following refs. \(^{10,25}\), where \(\nabla \cdot \mathbf{E}\) is the transverse gradient and \(\nabla \times \mathbf{D}\) is the dimensionless group velocity dispersion of the material (Methods). In Fig. 4d–f, the modulus of the energy density flux (\(|\mathbf{J}|\), the length of the cyan arrows in the figures) is presented without rescaling, so we can observe its orientation directly. In a fundamental ST-OAM pulse, the energy density flux forms a saddle-shaped antivortex structure around the singularity, given the positive GVD at this wavelength of a BBO crystal. As a fundamental ST-OAM pulse propagates through a thin BBO crystal, the SHG process, as well as a small amount of dispersion, leads to minor energy redistribution. By contrast, in the case of a thick BBO crystal, larger dispersion can substantially reshape the ST-OAM pulses, leading to strong spatiotemporal astigmatism in the second-harmonic ST-OAM pulse. In Fig. 4d–f, our extracted energy density flux in the BBO crystal shows a tendency to reshape the ST-OAM pulse into a two-lobe structure upon continued propagation, resulting in the higher degree of distortion observed for the pulse emerging from the thick BBO crystal. It is worth mentioning that the canonical momentum density in air (Fig. 4a–c) represents the non-trivial optical power distributions of the fields generated after the nonlinear conversion, while the energy density flux in media (Fig. 4d–f) represents the non-trivial optical power distributions of the fields inside the dispersive BBO crystals.

**Discussion**

The second-harmonic ST-OAM pulse generated using a thin BBO crystal can be described by a simple theory. In this simple scenario with perfect phase matching, an undepleted pump approximation, and assuming a lossless medium, the second-harmonic field is proportional to the square of the complex input fundamental field, namely

\[
E^{(2\omega)}(x, y, \zeta) = E_0 e^{i(2\omega t)} \phi(x, y, \zeta) \propto (E^{(\omega)}(x, y, \zeta))^2 \propto (e^{i(\omega t)})^2 \propto e^{i(2\omega t)}
\]

Here, \(\zeta \equiv z - v_g t\) is the space coordinate in the moving reference frame of a pulse travelling with group velocity \(v_g\), and the definition of the scalar field follows equation (1). As a result, \(e^{i(2\omega t)} = e^{i(\omega t)}\), and we obtain \(E^{(2\omega)} = 2E^{(\omega)}\). This indicates that the space–time topological charge of the fundamental field is doubled along with the optical frequency. In this scenario, the field profile of the second-harmonic wave should mimic the fundamental pulse. Because the pulse duration and the beam size of a fundamental pulse can be shortened and reduced in a perfect SHG process due to its quadratic response, this partially explains the fact that the second-harmonic ST-OAM pulse appears thinner when plotting intensity isosurface profiles as shown in Figs. 1 and 2. The nonlinear relationship between the electric field amplitudes of a fundamental and its second-harmonic field means...
that the amplitude profile of the ST-OAM pulse changes during frequency doubling. Beyond the simple scenario, the SHG process is further complicated by additional factors in addition to phase mismatches, such as group velocity mismatch, intrapulse group velocity dispersion, absorption or losses, and pump depletion. A rigorous theoretical study of a final second-harmonic ST-OAM pulse profile must take the above factors into account and is the subject of further detailed investigation, which is beyond the scope of this manuscript.

Importantly, our demonstration of ST-OAM nonlinear conversion and ST-OAM conservation in SHG shows that a new approach comprising secondary sources with ST-OAM can be realized experimentally in upconverted and probably downconverted electromagnetic waves. For example, it would be of interest to investigate stimulated OAM Raman scattering, harmonic spin–orbit angular momentum cascades or entangled photons generated by parametric processes with ST-OAM pulses. It is also possible that other matter waves such as electron vortex beams or electron bunches with ST-OAM could be generated in the future through photoelectron generation (photoemission) or other means that could improve accelerator technologies and electron microscopy.

In summary, we report the experimental observation of a second-harmonic ST-OAM of light and its space–time topological charge conservation during frequency doubling from $\ell' = 1$ to $\ell' = 2$. The charge of the ST-OAM pulse, corresponding to the space–time spiral phase structure, was observed to double in an SHG process. Our finding also confirms that the spatiotemporal phase singularities in ST-OAM of light can be interpreted as space–time topological charges carrying transverse OAM—the term coined in analogy to the spatial topological charges in a conventional OAM of light carrying longitudinal OAM. In this analogy, both types of OAM follow the same charge conservation and nonlinear scaling rules. We also find that the topological structure of the space–time phase swirl in a second-harmonic field may not be conserved. An SHG process can generate additional phase singularities, depending on spatiotemporal astigmatism due to group velocity mismatch or the phase-mismatch condition in a dispersive medium. SHG is the foundation of any nonlinear optics textbook. Our work thus opens a new avenue of light carrying ST-OAM in nonlinear conversion and scaling, and it further suggests the possibilities of driving secondary ST-OAM sources, from electromagnetic waves, mechanical or acoustic waves, to matter waves.

Note added in proof: It has come to our attention that, after our manuscript was submitted and the preprint was posted online in an open repository Research Square, an independent experiment that explored similar light science using a different experimental approach was reported in a preprint online (https://arxiv.org/pdf/2012.10806v1).

Online content
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**Fig. 4 | Experimentally extracted spatiotemporal momentum density and energy density flux of ST-OAM pulses.**

| Fundamental ($\ell' = 1$) | Second harmonic ($\ell' = 2$) (thin BBO) | Second harmonic ($\ell' = 2$) (thick BBO) |
|--------------------------|----------------------------------------|----------------------------------------|
| a                        | b                                      | c                                      |
| Canonical momentum density | Energy density flux in air              | Energy density flux in a dispersive medium (BBO crystals), which is proportional to both phase gradient and material dispersion as defined in refs. 27,28, shown in a fundamental ST-OAM field (d) and its second-harmonic field generated by a thin BBO crystal (e) and a thick BBO crystal (f). The modulus of the canonical momentum density (the length of the yellow arrows) is scaled by 1/$c\ell_0$ and 1/x₀ along the horizontal and vertical axes, respectively, for better visualization. The modulus of the energy density flux (the length of the cyan arrows) is not scaled. The ST-OAM pulses propagate from left to right. |
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Methods

ST-OAM pulse generation. A stretched (full-width at half-maximum pulse duration of ~500 fs) optical pulse at a central wavelength of ~800 nm from a regenerative Ti:sapphire amplifier with 1-kHz repetition rate (KMLabs Wyvern HE) was first split into two paths before entering into a homemade Mach-Zehnder-type scanning interferometer. One path was used to generate a short (~45 fs) Gaussian reference pulse and the other a long (~500 fs) ST-OAM pulse. The reference Gaussian pulses were compressed using dispersive chirped mirrors (Ultrafast Innovations) with multiple bounces to shorten the pulse duration. The ST-OAM pulses were generated by a custom pulse shaper we designed that consisted of a reflective grating (600 grooves mm⁻¹), a cylindrical lens (f = 25 cm), a multi-faceted spatial phase plate (SPP, HoloOpt, 16 steps per phase wrap) and a high-reflec-tance end mirror for retroreflection. To generate fundamental ST-OAM pulses of the spatiotemporal topological charge ℓ ∈ 1, a SPP with a designed spatial topological charge ℓ = 0.5 at the design wavelength of 790 nm was used. Upon retroreflection at the end mirror, the pulse passed through the SPP twice and generated the desired spatial topological charge ℓ = 0.5 – (–0.5) = 1. Note that an SPP is a non-reciprocal optical component, and a mirror reflection changes the sign of the ℓ. Thus, the double-passing of the SPP from the opposite direction makes the ℓ, charge addition possible, when the two passes are very close. A Fourier transform in the spectral domain of the pulse shaper converts a spatially chirped beam with a spatial spiral phase into a pulsed beam with a spatiotemporal spiral phase. BBO crystals with thickness 20 mm (thick BBO), cut and oriented to phase-match type-I SHG, are used to generate the second-harmonic fields at a wavelength of ~400 nm, which are placed 1 mm (thick BBO) away from the spatially chirped beam with a spatial spiral phase. The spatially chirped beam with a spatial spiral phase into a pulsed beam with a spatiotemporal phase. BBO crystals with thickness 20 mm (thick BBO), cut and oriented to phase-match type-I SHG, are used to generate the second-harmonic fields at a wavelength of ~400 nm, which are placed 1 mm (thick BBO) away from the spatially chirped beam with a spatial spiral phase. The spatially chirped beam with a spatial spiral phase into a pulsed beam with a spatiotemporal spirald wave packet with transverse orbital angular momentum. It has been used over the past decade in applications such as filamentation and light bullets propagating in dispersive media. It is defined as J (x, y, z, t) = ℓ |E(x, y, z, t)|² ( Ψ(x, y, z, t) − GVD(Ω(x, y, z, t)) + transversal and longitudinal terms separated as in refs. 4, 5, where Ψ(x, y, z, t) is the transverse gradient and GVD is the dimensionless group velocity dispersion of the material. The modulus of the energy density flux vector |J| (the length of the yellow arrows in Fig. 4a–c) is scaled by 1/λ₀, x₀ = 1.25 mm, on the x axis, and by 1/λ₀, t₀ = 300 fs, on the ζ axis for better visualization.

The energy density flux in dispersive media was used and introduced in ref. 28 to characterize the propagation of ultrashort laser pulses with spatiotemporal coupling. It has been used over the past decade in applications such as filamentation and light bullets propagating in dispersive media. It is defined as J (x, y, z, t) = ℓ |E(x, y, z, t)|² ( Ψ(x, y, z, t) − GVD(Ω(x, y, z, t)) + transversal and longitudinal terms separated as in refs. 4, 5, where Ψ(x, y, z, t) is the transverse gradient and GVD is the dimensionless group velocity dispersion of the material. The modulus of the energy density flux vector |J| (the length of the yellow arrows in Fig. 4a–c) is scaled by 1/λ₀, x₀ = 1.25 mm, on the x axis, and by 1/λ₀, t₀ = 300 fs, on the ζ axis for better visualization.

Amplitude and phase reconstruction. Both fundamental and second-harmonic field amplitudes |E₀(x, y, t)| and phase Ψ(x, y, t) were measured using a Mach-Zehnder-type scanning interferometer. When scanning the time delay between a short reference pulse and a long ST-OAM pulse, the reference pulse serves as an optical delay line. When scanning the time delay between a short reference pulse and a long ST-OAM pulse, the reference pulse serves as an optical delay line. Amplitude and phase reconstructions. Both fundamental and second-harmonic field amplitudes |E₀(x, y, t)| and phase Ψ(x, y, t) were measured using a Mach-Zehnder-type scanning interferometer. When scanning the time delay between a short reference pulse and a long ST-OAM pulse, the reference pulse serves as an optical delay line. When scanning the time delay between a short reference pulse and a long ST-OAM pulse, the reference pulse serves as an optical delay line. To reconstruct the phase, we first applied 1D Fourier transform to the |x|/x₀ and |y|/y₀ cross-sections show two peaks that are the front and rear ends of the pulse. We then define the separations along the x axis for better visualization.

Experimental extraction of momentum density and energy density flux. The free-space canonical momentum density in vacuum or in air, also called the scalar optical current or wave current, is defined as the expectation value of the intensity-weighted momentum operator: P(x, y, z, t) = ∇·E |E|² = |E₀(x, y, z, t)|² ³ (Ψ(x, y, z, t) − GVD(Ω(x, y, z, t)) + transversal and longitudinal terms separated as in refs. 4, 5, where Ψ(x, y, z, t) is the transverse gradient and GVD is the dimensionless group velocity dispersion of the material. The modulus of the energy density flux vector |J| (the length of the yellow arrows in Fig. 4a–c) is scaled by 1/λ₀, x₀ = 1.25 mm, on the x axis, and by 1/λ₀, t₀ = 300 fs, on the ζ axis for better visualization.

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Data availability

The datasets utilized to prepare the data presented in this manuscript are available free of charge from the corresponding author under reasonable request.

References

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Author contributions

C.-T.L. conceived the project. G.G. conducted and designed the experiment. C.-T.L. and G.G. both analysed the data. M.M.M. and H.C.K. proposed the research thrust, supervised the research, helped in the preparation of the manuscript, and applications. All authors contributed to the discussion and writing of the manuscript.

Competing interests

M.M.M. and H.C.K. have a financial interest in KMLabs. The other authors declare no competing interests.

Additional information

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