A meta-analysis of geochronologically relevant half-lives: what’s the best decay constant?

Patrick Boehnke* and T. Mark Harrison

Department of Earth, Planetary, and Space Sciences, University of California, Los Angeles, CA, USA

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Twenty-first century advances in both the analytical procedures and instrumentation used in geochronology promise age accuracy better than ±1‰, but realizing this potential requires knowledge of decay constants (λ) that exceed this level. Given the paucity of improved recent measurements of λ, the community has experimented with hybrid methodologies utilizing data largely generated during the 1970s. In this article, we perform a systematic review of laboratory decay constant determinations relevant to geochronology (i.e. 87Rb, 147Sm, 176Lu, 230Th, 232Th, 235U, and 238U), focusing on methodological consistency. For radioisotopes for which multiple studies are available, results are combined through a random effects model to yield the best available values and associated uncertainties. Unfortunately, despite its vital role in modern geochronology, only one experimental determination of 238U decay met our criteria for consideration, significantly limiting the ability to assess its reliability. Thus, utilizing λ238 as an anchor for establishing other decay constants (e.g. 40K, 176Lu, and spontaneous 238U fission) places an unverified result at the core of geochronology. For geochronology to attain its greatest potential, more and better laboratory determinations of decay constants are required, along with a community methodology that permits us to continuously take advantage of new data.

Keywords: geochronology; geochemistry; Fish Canyon Tuff; decay constants

Although it would seem that utilization of the most accurate and precise decay constants (λ) would be of fundamental importance to geochronologists, as a community they have tended to value ongoing inter-comparison over periodic review and revision (e.g. Renne et al. 1994, 1998, 2010, 2011; cf. Begemann et al. 2001). The existing convention for geochronological decay constants, proposed by the Subcommission of Geochronology and Stratigraphy and ratified at the International Geological Congress in 1976 (Steiger and Jäger, 1977), has remained unmodified for nearly four decades. However, concerns have been increasingly raised that these canonical values are inconsistent with more recent measurements (e.g. Begemann et al. 2001; Mattinson 2000, 2010).

Because certain decay systems lend themselves to more precise measurement of λ than others, it has become commonplace to attempt to fix decay constants of poorly understood geochronological systems (e.g. spontaneous 238U fission, 40K, and 176Lu) to that of 238U alpha decay (λ238) (e.g. Hurford and Green, 1983; Kwon et al. 2002; Scherer et al. 2001) via concordant, coexisting minerals. In fact, establishing concordancy among coexisting phases has proven problematic in certain cases (e.g. Renne et al. 2010). More to the point, this approach puts a premium on ensuring that λ238 is precisely and accurately known when the value we use was generated more than 40 years ago using now obsolete analytical systems (Jaffey et al. 1971). Indeed, based on small discordances observed within apparently robust U-Pb zircon populations, Mattinson (2000) proposed that the more precisely determined λ238 be fixed at the value measured by Jaffey et al. (1971) and the corresponding λ235 be adjusted upward by ~0.1% to bring those ages into concordance (also see Schoene et al., 2006; Mattinson, 2010). In doing this, the uncertainty associated with λ235 was reduced to reflect only the analytical variance within those U-Pb ages. While this had the seemingly beneficial effect of improving the apparent accuracy of the U-Pb dating system (Schoene 2013), it assumes a priori knowledge of the source of the discordance between 207Pb/235U and 206Pb/238U ages.

Concerns regarding the Steiger and Jäger (1977) convention are not limited to uranium isotopes. For example, the 87Rb decay constant was re-measured by Kossert (2003) who found it to be 1.4% lower than the conventionally accepted value. This value was subsequently confirmed by an accumulation experiment (i.e. growth of 87Sr* in a purified 87Rb salt; Rotenberg et al. 2012). Furthermore, several now commonly utilized radiogenic systems that were not broadly used in 1977 (e.g. 147Sm and 176Lu, 176Hf) were not considered in the Steiger and Jäger (1977) recommendations.

While the decay constant convention succeeded in providing a touchstone for inter-comparison, it did not provide a mechanism that could either motivate further study or accommodate improved measurements as they came available. In this article, we propose a method for determining the best current value for decay constants by
analysing all published laboratory measurements collectively in a way that values studies of the highest quality. This general approach is referred to as meta-analysis and is commonly undertaken in the biomedical and social sciences where large numbers of investigations need to be evaluated (DerSimonian and Laird, 1986). Meta-analysis has also seen limited application in the physical sciences, perhaps the clearest example being in nuclear physics where elementary particle properties are updated and maintained through a community collaboration (Particle Data Group, 2012) using similar techniques to traditional meta-analysis (Baker and Jackson, 2013). Given the critical nature of decay constants to geochronology, we re-examine the conventional values here and point to refined results that emerge from meta-analysis.

1. Method

1.1. Study selection criteria

The quality of scientific investigations varies from one study to the next with some results ultimately proven unreliable (e.g. Baker and Jackson, 2013). Thus, simply obtaining weighted means of all collected data is inadvisable. Indeed, the first responsibility of meta-analysis is to identify poorly designed experiments or cases of under-reported results and exclude those studies lest they significantly bias the overall result (Baker and Jackson, 2013). Problematic cases of half-life measurements in natural samples could arise if the initial isotope composition was not ascertained or if the study did not describe the methodology in sufficient detail to permit an appropriate vetting. Thus, we propose the following criteria for inclusion in a meta-analysis of decay constants. A valid study must:

- (1) be peer-reviewed and widely accessible;
- (2) describe the radioactive sample in detail, including weight and elemental and isotopic compositions;
- (3) describe the experimental apparatus in sufficient detail to assess potential sources of analytical error;
- (4) yield results with an appropriate signal/noise ratio;
- (5) not be superseded by, or included in, later results from the same laboratory; and
- (6) include measures of uncertainty derived from presented data.

While we acknowledge qualitative limitations of several of our criteria, our approach is surely preferable to having no threshold for inclusion whatsoever. Indeed, our proposal is offered as the basis of a community discussion that would lead to a consensus model rather than as canon. Note that we only provide a discussion of those published measurements that meet the benchmark for inclusion. The specific bases for rejecting specific studies are provided in the online supplement (see http://dx.doi.org/10.1080/206814.2014.908420).

1.2. Statistical model

Since our goal is to combine the measurements of multiple studies, we first evaluate which model best fits the problem at hand. Weighted averages (or a fixed effect model) are commonly used in geochronology to calculate the best age based on repeated measurements, where weights reflect measurement uncertainties only. However, a weighted average is only strictly valid for the case of a single, homogenous population (i.e. MSWD ≈ 1). A more appropriate statistical model would permit an estimate of the overall study heterogeneity to adjust the weight of each study. This approach is called a random effects model (Cochran 1937), where weights are chosen based on both intra-study variance ($\sigma^2$) and the disagreement among studies ($\tau^2$). The weighting ($W$) for each study used in this work is given by

$$W = \frac{1}{\sigma_i^2 + \tau^2},$$

where $\sigma_i^2$ is the variance from each study and $\tau^2$ is the same for all studies within one decay system. To calculate the parameter $\tau^2$, we use the approach of Mandel and Paule (1970) where the difference of each study from the weighted average is compared to the uncertainty of that study. When the spread of studies is less than or equal to that expected from the experimental variance, then $\tau^2 = 0$ (thus returning a weighted mean). Note that when the dispersion is large, $\tau^2 > 0$. Our calculations were performed using the R statistical software (R Core Team 2013) with the Meta package (Schwarzer 2013).

1.3. Scaled errors versus added effect model

The Particle Data Group (2012) calculates a weighted average and associated uncertainty based on the scaled variances of each study, where the scale factor reflects study heterogeneity. This model differs from the additive random effects model in that the relative weighting between studies is unchanged when uncertainties are scaled. Baker and Jackson (2013) compared the two models by changing the weighting ($W$) to:

$$W' = \frac{1}{\sigma_i^2 + \theta \tau^2 + \sigma_i^2},$$

where $\theta$ is a free parameter that is determined by maximizing the log-likelihood function. This scaling reduces to an additive model when $\theta = 0$ and a multiplicative model when $\theta = 1$. Baker and Jackson (2013) evaluated 15 particle physics data sets and found that they were best fit by $\theta \approx 0.22$. This value is sufficiently close to 0 to conclude that the random effects model is superior to that of scaled uncertainties. We performed a similar test on the
three decay constant data sets ($^{87}\text{Rb}$, $^{176}\text{Lu}$, and $^{235}\text{U}$) that contained sufficient measurements to permit the calculation (Figure 1) and come to a similar conclusion to Baker and Jackson (2013). Thus, we chose to proceed using the traditional random effects model rather than pioneer a hybrid methodology that would have only marginal impact on the calculated decay constants and their associated uncertainties.

### 1.4. Interpreting study heterogeneity

As noted earlier, reporting a mean and standard deviation for combined data sets does not incorporate the fundamentally important knowledge of study disagreement beyond that expected from internal statistics (Higgins et al. 2009). As an intuitive aid to the calculations, we present a ‘forest’ plot (Ioannidis et al. 2008) for each decay constant analysis that permits visual inspection of study heterogeneity. Also included is a summary statistic, $I^2$, which is the percentage of the variance of the combined result from the disagreement between the studies that can be assigned to each study (Higgins and Thompson 2002).

In biomedical meta-analysis, random effects models are interpreted as including the possibility of a difference in the effect under observation among the various studies (Higgins et al. 2009). Given that decay constants are, literally, physical constants, this interpretation is inappropriate for our case. Instead, we favour the view that individual study uncertainties have been underestimated leading to standard weighted averages with underestimated uncertainties (Zhang 2006; Rukhin 2009).

### 1.5. Statistical model limitations

As this is, to our knowledge, the first use of a random effects model in geochemistry, we discuss some potential limitations of the approach. The first is that our uncertainty calculation is only statistically optimal if the data is normally distributed (Rukhin et al. 2000). Given the low number of included studies overall ($\leq9$), this is generally not independently verifiable. However, DerSimonian and Kacker (2007) note that the Mandel and Paule (1970) approach is generally more robust than other mechanisms (e.g. weighted average). The second potential limitation is that the generally low number of experimental determination of decay constants does not permit tests for bias in the published studies. An example of such is the publication bias, where new results that differ substantially from established values be suppressed from publication. However, we feel this is unlikely to be a significant problem as half-life studies always produce a result (i.e. there is no null effect), and, as evidenced by the highly variable reports of the $^{176}\text{Lu}$ half-life (Figure 2(C)), workers in this field appear unfazed about publishing disparate values. Finally, we wish to note here that a meta-analysis is only as good as the published literature and in several cases is clearly limited by the available studies.

### 1.6. Comparisons with previous compilations

Although there have been numerous summaries of half-lives and associated nuclear data (e.g. nuclear data sheets), the methodologies utilized in their compilation are highly variable. In some cases, a weighted average is used (e.g. $^{51}\text{V}$), and in others, a single best value (e.g. $^{53}\text{Mn}$) is reported (Junde 2009). It is generally unclear why such ad hoc selections are made. In other compilations, such as Holden (1989), uncertainties are increased arbitrarily to compensate for procedural errors or the effect of significant corrections with difficult to quantify effects. In the analysis that follows, we emphasize consistency in both statistical methodology and earlier described criteria for study inclusion.

### 2. Results

#### 2.1. Introduction

The preferred decay constant emerging from our meta-analysis of each geochronological system is given in Table 1. The included studies meet the criteria set forth in this section and in only one case ($^{238}\text{U}$) was there an insufficient number of investigations to perform a meta-analysis. We now examine each system in detail, discuss the level of agreement among studies, and present the calculated value of $\lambda$ and its variance. The uncertainties stated in the following correspond to standard uncertainties ($k = 1$).
Given the large number of accumulation, decay counting, and natural age comparison studies published of the $\beta^-$ decay of $^{87}\text{Rb}$ to $^{87}\text{Sr}$, we begin with this geochronometric system (see supplementary materials of Rotenberg et al. 2012). However, only three of the 32 studies (Kossert 2003; Neumann and Huster 1974; Rotenberg et al. 2012) meet the criteria given in Section 1.1 (see online supplement for excluded studies). These studies are split between decay counting (Neumann and Huster 1974; Kossert 2003) and accumulation (Rotenberg et al. 2012; note that Davis et al. 1977 was not used for reasons discussed in the supplementary materials). Note that we utilize a random effects model to re-analyse the results of Rotenberg et al. (2012) as our analysis requires symmetric uncertainties; we calculate a half-life of $49.579 \pm 0.026 \text{ Ga}$.

The meta-analysis (Figure 2(A)) shows that there is no excess variability between the studies (i.e. $I^2 = 0 \%$). Given that the agreement between studies utilizing both decay counting and accumulation is very good, we conclude that the half-life of $^{87}\text{Rb}$ is $49.579 \pm 0.026 \text{ Ga}$ (i.e. $\lambda = 1.398 \times 10^{-11} \text{ year}^{-1}$).

### 2.3. $^{147}\text{Sm}$

Five of the sixteen studies of the $\alpha$ decay of $^{147}\text{Sm}$ to $^{143}\text{Nd}$ (Wright et al. 1961; Donhoffer 1964; Gupta and MacFarlane 1970; Kossert et al. 2009; Su et al. 2010) meet the criteria given in Section 1.1. While they all utilized some form of decay counting, three used a liquid scintillation detector (Wright et al. 1961; Donhoffer 1964; Kossert et al. 2009); Gupta and MacFarlane (1970) used an ionization chamber, and Su et al. (2010) used a silicon barrier detector. As shown in Figure 2(B), there is excellent agreement between studies and, in this case, the random effects model collapses to a fixed effect model (i.e. $I^2 = 0 \%$). Given that the agreement between studies utilizing both decay counting and accumulation is very good, we conclude that the half-life of $^{147}\text{Sm}$ is $106.44 \pm 0.6 \text{ Ga}$ (i.e. $\lambda = 6.515 \times 10^{-12} \text{ year}^{-1}$).
2.4. $^{176}\text{Lu}$

Our meta-analysis of the $\beta^-$ decay of $^{176}\text{Lu}$ to $^{176}\text{Hf}$ included nine published studies (Prodi et al. 1969; Komura et al. 1972; Norman 1980; Sguigna et al. 1982; Gehlke et al. 1990; Dalmasso et al. 1992; Grinyer et al. 2003; Nir-El and Haquin 2003; Kossert et al. 2013), all of which were by decay counting. This decay system is particularly challenging to measure experimentally because of the low and variable energy of the escaping electron. Several studies have attempted to circumvent this difficulty by measuring one or more of the emergent $\gamma$ rays emitted during decay (e.g. Grinyer et al. 2003). These studies are further limited by self-absorption and attenuation effects, coincidence summing ambiguities, and the fact that not every decay emits a $\gamma$ (and thus knowing the probability of a $\gamma$ of a certain energy being produced is key; Ott et al. 2012). In light of these difficulties, it is unsurprising that there is significant scatter among these studies (Figure 2(C)) that is, above that expected from the stated uncertainties. Indeed, virtually all of the overall error ($I^2 = 97\%$) derives from inter-study heterogeneity with a calculated half-life for $^{176}\text{Lu}$ of $37.49 \pm 0.88$ Ga (i.e. $\lambda = 1.85 \times 10^{-11}$ year$^{-1}$).

2.5. $^{230}\text{Th}$

Two counting studies of the $\alpha$ decay of $^{230}\text{Th}$ to $^{226}\text{Ra}$ (Attree et al. 1962; Meadows et al. 1980) meet our criteria for inclusion. While there is excellent agreement (Figure 2(D)) between the two studies ($I^2 = 0\%$), their limited number and similar methodologies leave open the possibility that future variance could be documented. We calculate a half-life for $^{230}\text{Th}$ of $75,375 \pm 290$ years (i.e. $\lambda = 9.196 \times 10^{-5}$ year$^{-1}$).

2.6. $^{232}\text{Th}$

The $\alpha$ decay of $^{232}\text{Th}$ to $^{228}\text{Ra}$ has been measured six times but only three of those studies (Farley 1960; Macklin and Pomerance 1956; Senftle et al. 1956) meet our criteria for inclusion (see online supplement). As with the other decay systems, one study (Farley 1960) is significantly more precise than the rest, limiting our ability to internally verify the result (Figure 2(E)). The half-life we calculate for $^{232}\text{Th}$ is $14.13 \pm 0.13$ Ga (i.e. $\lambda = 4.92 \times 10^{-11}$ year$^{-1}$).

2.7. $^{235}\text{U}$

For the $\alpha$ decay of $^{235}\text{U}$ to $^{231}\text{Th}$, four (Fleming et al. 1952; White et al. 1965; Jaffey et al. 1971; Deruytter and Wegener-Penning 1974) studies are available for examination (Section 1.1). While Jaffey et al. (1971) noted a ‘slight source of drift’ in the $\lambda_{235}$ measurements, the $p$-value of $\sim 0.06$ that they ascertained is not significant at the currently accepted level of 0.05 (Fisher 1925). Because $^{231}\text{Th}$ has a half-life of 26 hours, simple accumulation studies are not possible. However, the availability of highly enriched $^{235}\text{U}$ and its short half-life relative to $^{238}\text{U}$ suggests the potential for decay counting experiments of superior precision and accuracy. Unfortunately, we detect study heterogeneity ($I^2 \sim 45\%$) with the weighting dominated by Jaffey et al. (1971) result. Thus, while we are able to calculate a half-life of $702.5 \pm 5.2$ Ma (i.e. $\lambda = 9.867 \times 10^{-13}$ year$^{-1}$), the need for more experiments to further verify and refine the $^{235}\text{U}$ decay constant (Figure 2(F)) is clear.

2.8. $^{238}\text{U}$

The $\alpha$ decay of $^{238}\text{U}$ to $^{234}\text{Th}$ is pivotal to current geochronological practice. Schön et al. (2004) reviewed experimental determinations of $\lambda_{238}$ but did not apply objective criteria for study inclusion when calculating recommended values. Unfortunately, only the study of Jaffey et al. (1971) meets the criteria given in Section 1.1. Three of the published works (Kovarik and Adams 1938; Schiedt 1935; Steyn and Strelov 1960) do not address whether $^{234}\text{U}/^{238}\text{U}$ was in equilibrium, while the Kienberger (1949) study did not provide adequate analytical details to assess the reliability of their measurement. Although previous authors have expressed concern over the paucity of reliable $U$ decay studies (Mattinson 2000; Begemann et al. 2001; Schön et al. 2004), we suggest that this limitation is far more significant than previously expressed. In order to perform a meta-analysis or any verification of the accepted value, a priority for the geochronological community must be support of new, high precision and accuracy measurements.

3. Discussion

3.1. Revision of accepted values

After eliminating studies for which insufficient details are provided (Section 1.1), we have combined the results in a random effects model that weights studies by both their objective criteria for study inclusion when calculating recommended values. Unfortunately, only the study of Jaffey et al. (1971) meets the criteria given in Section 1.1. Three of the published works (Kovarik and Adams 1938; Schiedt 1935; Steyn and Strelov 1960) do not address whether $^{234}\text{U}/^{238}\text{U}$ was in equilibrium, while the Kienberger (1949) study did not provide adequate analytical details to assess the reliability of their measurement. Although previous authors have expressed concern over the paucity of reliable $U$ decay studies (Mattinson 2000; Begemann et al. 2001; Schön et al. 2004), we suggest that this limitation is far more significant than previously expressed. In order to perform a meta-analysis or any verification of the accepted value, a priority for the geochronological community must be support of new, high precision and accuracy measurements.
data ($^{238}\text{U}$) or insufficient data on branched decay ($^{40}\text{K}$) to meet the requirements of meta-analysis.

### 3.2. Uranium decay constants

The decay of U to Pb has grown to be the most widely used decay system for geochronology (Schoene 2013). Despite its popularity, decay constants for $^{238}\text{U}$ and $^{235}\text{U}$ have not been re-measured since Jaffey et al. (1971), although analytical capabilities have vastly improved over the intervening 43 years. The high intrinsic precision of ID-TIMS U-Pb zircon dating (e.g. Schoene 2013) has attracted users of other decay systems to calibrate their parent decay rate to that of $^{238}\text{U}$ (e.g. Hurford and Green 1983; Mattinson 2000; Kwon et al. 2002). As noted earlier, we are only able to substantiate use of one $^{238}\text{U}$ decay study for meta-analysis and four for $^{235}\text{U}$. It is at least arguable that until the $^{238}\text{U}$ decay constant is re-measured through laboratory experiment, other geochronological systems should be calibrated to $^{235}\text{U}$ rather than $^{238}\text{U}$ (i.e. hold $\lambda_{235}$ constant and adjust $\lambda_{238}$ to achieve concordance of robust U-Pb zircon standards).

### 3.3. Geochronology Data Group

Since decay constants are of foundational importance to geochronology, it is imperative that the values in use be the best available. This requires continuing assessment. That there has not been an update to the decay constant convention of 1976 underscores the lack of benefit that the geochronology community has gained from increasingly sophisticated experimental work. We propose the formation of an international group that regularly examines published experimental decay constant measurements, combines them through meta-analysis, and publishes the community consensus. Whether this arises directly from international cooperation among geochronologists or led by International Council for Science organizations is a matter for discussion informed by past experience.

### 3.4. Age of Fish Canyon Tuff

The Fish Canyon Tuff (FCT) is a voluminous ignimbrite sheet erupted over a relatively short period during the late Oligocene within the San Juan volcanic field, southern Colorado (Lipman et al. 1970). It appears to satisfy many of the criteria for an ideal dating standard material in that the rock is easily accessible and has a broad assemblage of unaltered and datable modal and accessory minerals, including plagioclase, sanidine, biotite, hornblende, titanite, apatite, and zircon. Steven et al. (1967) found that the phenocryst assemblage plotted on a 27.9 ± 0.7 Ma K-Ar isochron (recalculated using the decay constants and isotopic abundances of Steiger and Jaeger 1977). On this basis, Naeser et al. (1981) proposed apatite and zircon from the FCT as a fission track dating standard (cf. Galbraith 1986), and these and other phases, including sanidine, have been investigated for use as inter- and intra-laboratory standards using the $^{40}\text{Ar}/^{39}\text{Ar}$, (U-Th-Sm)/He, and U-Pb methods (Lipman et al. 1970, 1997; Hurford and Hammerschmidt 1985; Cebula et al. 1986; Carpena and Mailhe 1987; Oberli et al. 1990, 2002; Renne et al. 1994, 1998, 2011, 2010; Baksi et al. 1996; Lanphere and Baadsgaard 1997, 2001; Reiners and Farley 1999; Bachmann et al. 2000, 2002, 2007; Lanphere and Dalrymple 2000; Schmitz and Bowring 2001; Reiners et al. 2002; Dazé et al. 2003; Schmitz et al. 2003; Spell and McDougall 2003; Tagami et al. 2003; Lanphere 2004; Jourdan and Renne 2007; Dobson et al. 2008; Kuiper et al. 2008; Schwarz et al. 2011; Kohn et al. 2014).

Because of the difficulty of quantitatively extracting Ar from sanidine, even at temperatures well above melting, its age is usually determined by comparison with primary K-Ar standards (e.g. Jourdan and Renne 2007). In recent years, numerous decay constants for $^{40}\text{K}$ have been proposed which are mutually inconsistent outside of their respective quoted uncertainties (e.g. Kuiper et al. 2008; Renne et al. 2010). As an additional uncertainty, there remains an unexplained discrepancy between a 27.5 Ma Rb-Sr isochron (Lanphere and Baadsgaard 2001) and the proposed U-Pb zircon age of 28.5 Ma (Schmitz and Bowring 2001).

Clearly, we cannot take full advantage of recent analytical developments permitting high-precision U-Pb (and $^{40}\text{Ar}/^{39}\text{Ar}$) geochronology using obsolete decay constants. We recalculated the Rb-Sr and U-Pb ages for the FCT of Lanphere and Baadsgaard (2001) and Schmitz and Bowring (2001) using the $\lambda$ values of Table 1 and arrived at 27.8 ± 0.2 Ma and 28.5 ± 0.1 Ma, respectively. While this narrows the gap, the discrepancy is still larger than the assigned uncertainties. A further complication of using U-Pb zircon ages to date volcanic eruption of is that magmatic zircons can predate eruption by >500 ka (Reid et al. 1997; see review in Schmitt 2011). Indeed, while the U-Pb zircon analyses of Schmitz and Bowring (2001) yielded MSWD = 0.7 and were interpreted as a single age population, increasingly precise U-Pb measurements led to steadily increasing MSWDs. The data of Bachmann et al. (2007) yield an age spread with MSWD = 3 while the recent data of Wotzlaw et al. (2013) yield an MSWD = 64. This appears to underscore the conclusion of Reid et al. (1997) that zircons can form in magmas and quantitatively retain radiogenic Pb for many hundreds of thousands of years. One approach then would be to assume that the youngest $^{207}\text{Pb}/^{235}\text{U}$ zircon age best approximates eruption and then apply our calculated decay constant. Although the data we note earlier (i.e. Schmitz and Bowring 2001; Bachmann et al. 2007; Wotzlaw et al. 2013) are associated with very different measurement blank and analytical qualities, they present a range of
minimums from 28.0 to 28.2 Ma, which is consistent with the Rb-Sr isochron age (Lanphere and Baadsgaard 2001), which, by its nature, is free of pre-eruption memory (cf. Halliday et al. 1989). Note that this range is younger than ages proposed for $^{40}$Ar/$^{39}$Ar sanidine using essentially ad hoc $^{40}$K decay constants (Kuiper et al. 2008; Renne et al. 2010) but agrees with the Fish Canyon sanidine age determined via $^{40}$Ar/$^{39}$Ar inter-calibration using standards whose ages were determined by ab initio K-Ar dating (i.e. absolute measurements of both K and Ar) coupled with the conventional $^{40}$K decay constant and branching ratio (Jourdan and Renne 2007).

4. Conclusion

We compiled, systematically analysed, and assigned uncertainties for decay constant (and thus half-life) measurements of $^{87}$Rb, $^{147}$Sm, $^{176}$Lu, $^{230}$Th, $^{232}$Th, $^{235}$U, and $^{238}$U. The range of agreement between different experimental values varies from excellent (i.e. $^{147}$Sm) to poor (i.e. $^{176}$Lu). In the case of $^{235}$U, only one study met the criteria for inclusion and thus caution should be taken using its half-life as a basis for widespread comparison to other geochronometers. For the purposes of age comparison, it would appear that, given the larger number of studies and their good agreement, the $^{235}$U decay constant should instead form the basis of any scheme to adjust decay constants in light of results from natural samples. However, rather than leading to diminished uncertainties associated with $\lambda_{235,238}$, this proposal would increase the $\lambda_{235}$ uncertainty by more than 50% of that reported by Jaffey et al. (1971).

Attempts to inter-calibrate geochronologic systems using coexisting phases from the FCT have proven problematic due to contrasting petrologic and kinetic controls. Our analysis suggests that an eruption age between 28.0 and 28.2 Ma is consistent with both current knowledge of decay constants and the contrasting behaviour of the various mineral systems used (e.g. retention of daughter product at magmatic temperatures).

In this article, we emphasize introduction of a methodology for systematically combining published decay constant data over advocating specific new decay constant values. Indeed, our analysis leads to the conclusion that numerous additional experimental decay constant investigations are required before any new decay constant convention for a geochronologically significant nuclide be adopted.

Clearly, current knowledge of $\lambda$ for many radiometric systems is the limiting parameter to achieving absolute age accuracies of better than ca. 1%. While much useful research can be undertaken knowing only relative age differences within a single decay system (e.g. Schoene et al. 2012), to truly achieve ±0.1%, inter-decay system, absolute time accuracy requires significantly improved experimental determinations of $\lambda$. This cannot occur if we continue to anchor twenty-first century geochronology to often incomplete, 1970s-era nuclear physics reports. What our community appears to lack is the confidence to directly acquire fundamental new knowledge of decay constants with significantly improved precision and accuracy using the best contemporary facilities and to continuously refine knowledge of decay constants using objective measures of combination. The moment calls for a forging of new relationships across disciplinary boundaries to attain the next generation of geochronological accuracy that will drive future scientific breakthroughs.

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Supplemental data

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