On the choice of reference in sensor offset calibration

Raj Thilak Rajan*

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Abstract

Sensor calibration is an indispensable feature in any networked cyberphysical system. In this paper, we consider a sensor network plagued with offset errors, measuring a rank-1 signal subspace, where each sensor collects measurements under additive zero-mean Gaussian noise. Under varying assumptions on the underlying noise covariance, we investigate the effect of using an arbitrary reference for estimating the sensor offsets, in contrast to the ‘mean of all the unknown sensor offsets’ as a reference. We show that the mean reference yields an efficient estimator in the mean square error sense. If the underlying noise is homoscedastic in nature, then the mean reference yields a factor 2 improvement on the variance, as compared any arbitrarily chosen reference within the network. Furthermore, when the underlying noise is independent but not identical, we derive an expression for the improvement offered by the mean reference. We demonstrate our results using the problem of clock synchronization in sensor networks, and present directions for future work.

Index terms—Sensor networks, Blind calibration, Parametric constraints, Cramér-Rao bounds

*The authors are with the Faculty of Electrical Engineering, Mathematics and Computer Science, Delft University of Technology, 2628CD Delft, The Netherlands (e-mail: r.t.rajan@tudelft.nl). This work is partially funded by the European Leadership Joint Undertaking (ECSEL JU), under grant agreement No. 876019, the ADACORSA (Airborne Data Collection on Resilient System Architectures) project, and by the Dutch-PIPP (Partnerships for Space Instruments Applications Preparatory Programme), funded by NWO (Netherlands Organisation for Scientific Research) and NSO (Netherlands Space Office).
1 Introduction

Sensors play a vital role in the burgeoning fields of internet of things (IoT) [1], networked cyberphysical systems [2], and Wireless Sensor Networks (WSN) [3], with diverse applications e.g., environmental monitoring [4], remote sensing [5] and space systems [6], to name a few. Network-wide sensor calibration is a ubiquitous challenge in these applications, which is quintessential for accurately measuring the underlying signal of interest [7, 8]. Under the assumption that the ground truth lies in a known lower-dimensional subspace of the measurement subspace, the gains of the sensors can be uniquely determined using blind calibration techniques [9, 10, 11]. However, in the absence of a known reference, the estimation of offsets is typically infeasible, and thus leads to an ill-posed problem.

In practice, in the absence of an external reference, one (or many) of the sensors within the network is chosen as a reference to make the system identifiable [12]. More recently, in pursuit of an efficient estimator in the mean square error sense, the mean of the calibration parameters (e.g., offsets) is proposed as a reference in various applications, e.g., in antenna calibration [13], in clock synchronization [14], and in sensor calibration of air-quality networks [15]. Although the mean reference yields optimality for various data models, it is unclear on how much improvement is achieved in comparison to chosen reference(s) in the network.

Contributions: In this paper, we consider a sensor network plagued with offset errors, where each sensor collects measurements, and measures an underlying signal subspace of rank-1. If the underlying Gaussian noise covariance is wide-sense stationary across the measurements, we show that the mean reference is the optimal reference for estimating the unknown sensor offsets in the network. Furthermore, when the Gaussian noise is i.i.d, we show that the mean reference offers a factor 2 improvement in the variance of any unbiased estimator w.r.t. any arbitrarily chosen sensor reference in the network. Finally, when the Gaussian noise is independent across the sensors, but not identical, we derive an expression for the improvement offered by the mean reference in contrast to any other sensor within the network.

Notation and properties: The Kronecker product is indicated by $\otimes$, the transpose operator by $(\cdot)^T$ and $\equiv$ denotes equality by definition. $\mathbf{1}_N$ and $\mathbf{0}_N$ denote a column vector of ones and zeros respectively. $\mathbf{I}_N$ is an identity matrix, and $\text{diag}(\cdot)$ represents a diagonal matrix. We frequently use the properties,

$$\text{vec}(\mathbf{ABC}) = (\mathbf{C}^T \otimes \mathbf{A})\text{vec}(\mathbf{B}) \quad (1)$$
$$\text{vec}(\mathbf{A} \otimes \mathbf{B})(\mathbf{C} \otimes \mathbf{D}) = (\mathbf{AC} \otimes \mathbf{BD}) \quad (2)$$

where $\text{vec}(\cdot)$ indicates vectorization and $\mathbf{A}, \mathbf{B}, \mathbf{C}$ and $\mathbf{D}$ are matrices of appropriate dimensions. We extensively use the Sherman-Morrison identity

$$(\mathbf{A} + \mathbf{bb}^T)^{-1} = \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{b}(1 + \mathbf{b}^T\mathbf{A}^{-1}\mathbf{b})^{-1}\mathbf{b}^T\mathbf{A}^{-1}, \quad (3)$$

where $\mathbf{A}$ is an invertible square matrix, and $\mathbf{b}$ is a vector of appropriate dimensions.
1.1 Preliminaries

Prior to modeling the sensor calibration problem, we briefly state the theoretical lower bound on the variance of any unbiased estimator under parametric constraints in Theorem 1, and in Theorem 2 we give the conditions for an optimal constraint set to yield an efficient estimator.

**Theorem 1** (Constrained Cramér Rao Bound (CCRB)). Consider a consistent set of $k$ continuously differentiable constraints on $\theta$ i.e., $c(\theta) = 0$, and let $C(\theta) = \partial c(\theta)/\partial \theta^T$ be the gradient matrix which is full row rank, then the Constrained Cramér Rao lower Bound (CCRB) on the variance of any unbiased estimator exists, and is bounded by

$$
\mathbb{E}\{(\hat{\theta} - \theta)(\hat{\theta} - \theta)^T\} \equiv \Sigma_\theta \geq U(U^T F U)^{-1} U^T, \tag{4}
$$

where $U$ spans the null space of the gradient matrix $C(\theta)$, and the Fisher information matrix (FIM) is given by $F = -\mathbb{E}\{\partial^2 \log p(y; \theta)/\partial \theta^2\}$, where $p(.)$ is the p.d.f. of the measurements $y$, which meets the regularity conditions [16].

*Proof.* [17, Theorem 1.]

**Theorem 2** (Optimal constraint set). Any feasible set of linearly independent vectors which span the nullspace of the FIM forms an optimal constraint set.

*Proof.* Let the spectral decomposition of the FIM be $F = V \Lambda V^T$ where $\Lambda$ is a diagonal matrix containing the non-zero eigenvalues, and let $V$ contain the corresponding eigenvectors. Now, consider a constraint matrix $\bar{C}$, such that the orthogonal basis for the null space of $\bar{C}$ spans the range of $V$, then substituting for $U$ with $V$ in (4),we have

$$
\text{Tr}(\Sigma_\theta) = \text{Tr}[V(V^T F V)^{-1} V^T] \equiv \text{Tr}(\Lambda^{-1}) = \text{Tr}(F^{-1})
$$

where we substitute for $F = V \Lambda V^T$ to obtain (a), exploit the cyclic nature of the trace operator, and the orthonormal property of $V$. Observe that the unconstrained FIM yields the lowest achievable variance for any unbiased estimator, and thus $\bar{C}$ is an optimal constraint set [13, 14].

2 Data Model

2.1 Blind sensor calibration

Consider a network of $N$ sensor nodes, where each sensor collects $K \geq N$ measurements. Let the true signal impinging on the $N$ sensors be given by a $K \times N$ matrix $S = [s_1, s_2, \ldots, s_N]$, where the ground-truth $S$ lies in a lower dimensional subspace, say $r$, such that $r < N$. Furthermore, let $\Gamma$ be a known $N \times N$ projection matrix of rank $r$, which spans the orthogonal complement of the signal subspace, such that $\Gamma^T = 0$, then using (1), we have

$$
(\Gamma \otimes I_K) \text{vec}(S) = \Gamma s = 0, \tag{5}
$$
which is the underlying premise for blind calibration in sensor networks [9, 10, 7, 11].

We now consider a scenario where the $N$ sensors are plagued with offset errors, and the measurements are corrupted with noise. The measurements of the $n$th sensor ($1 \leq n \leq N$) are denoted by $y_n = s_n + \theta_n + \eta_n$, where $s_n$ is the true signal of length $K$, $\theta_n$ is the unknown sensor offset and $\eta_n$ is the stochastic noise on the measurements. Extending for all $N$ sensors we have the following data model

$$y = s + H\theta + \eta,$$  

(6)

where $y = [y_1^T, y_2^T, \ldots, y_N^T]^T$, $s = [s_1^T, s_2^T, \ldots, s_N^T]^T$, $\theta = [\theta_1, \theta_2, \ldots, \theta_N]^T$, $H \equiv I_N \otimes 1_K$, and $\eta = [\eta_1^T, \eta_2^T, \ldots, \eta_N^T]^T$. Furthermore, let the noise on the system (6) be zero mean Gaussian, where the covariance function is homogeneous i.e., wide sense stationary across the $K$ measurements. Then exploiting the assumption (5), multiplying (6) by a known $\bar{\Gamma}$, and rearranging the terms we have

$$z \sim \mathcal{N}(\bar{\Gamma}(y - H\theta), \bar{\Gamma}(\Sigma \otimes I_K)\bar{\Gamma}),$$  

(7)

where $\Sigma$ is a $N \times N$ covariance matrix. The Fisher information (FIM) of the unknown offsets for (7) is straightforward [16], and is given by

$$F = H^T\bar{\Gamma}(\Sigma \otimes I_K)^{-1}\bar{\Gamma}H = K\bar{\Gamma}\Sigma^{-1}\bar{\Gamma},$$  

(8)

where we substitute for $\bar{\Gamma} = \Gamma \otimes I_K$, use the relation (2), and exploit the idempotency of the symmetric projection matrix. Now, since $\bar{\Gamma}$ is rank deficient by $N - r$, the FIM is also rank deficient. Let $C$ span the nullspace of the FIM, then from Theorem 2, the optimal parametric constraint set is given by $C^T\theta = d$, where $d$ is a known response vector of length $N - r$. If sufficient data is collected $K \geq N$ and if the covariance structure is known, then $C$ can be designed, which would lead to a data-driven reference.

### 2.2 Single source

In following section, we look at a special case of (7), where the sensors measure an identical signal e.g., densely deployed sensor array measuring air quality [18]. In these scenarios, the signal subspace for the $k$th measurement ($\forall k \leq K$) across all $N$ nodes, spans a column vector of ones i.e., $1_N$. Subsequently, the projection matrix takes the form of the centering matrix $\Gamma \equiv P = I_N - N^{-1}1_N 1_N^T$, and subsequently (7) simplifies to

$$z \sim \mathcal{N}(\bar{P}(y - H\theta), \bar{P}(\Sigma \otimes I_K)\bar{P}),$$  

(9)

where $\bar{P} = P \otimes I_K$, and the corresponding FIM from (8) is

$$F = K\bar{P}\Sigma^{-1}\bar{P},$$  

(10)

Observe that $P$, and subsequently $F$, is rank deficient by $r = 1$, which is intuitively expected, since at least 1 sensor reference is needed to uniquely estimate all the sensor offsets.
2.3 Constraints

A direct solution to estimate the sensor offset is to arbitrarily assume one of the sensors as a reference, which without a loss of generality, we choose as sensor 1. The corresponding parametric constraint, the gradient vector and the orthonormal bases for the nullspace of the gradient vector are given as

\[
\theta_1 = 0, \quad c_1 = [1, 0_{N-1}^T]^T, \quad U_1 = \begin{bmatrix} 0_N^T \cr I_{N-1} \end{bmatrix},
\]

(11)

respectively, where we use the subscript 1 to denote a *single* reference. Alternatively, the mean of all unknown offsets could be used as a reference, as has been proposed in various applications [13, 14, 15]. The constraint, the corresponding gradient and the bases for the nullspace of the gradient are given as

\[
\frac{1}{N} \sum_{n=1}^{N} \theta_n = 0, \quad c_2 = N^{-1} I_N, \quad U_2 = \begin{bmatrix} -1_N^T \cr I_{N-1} \end{bmatrix},
\]

(12)

respectively, where the subscript 2 is used to denote the *mean* reference. Increasing the number of constraints would further improve the performance of any estimator, however, we limit our discussion in this paper to a single reference.

3 Optimal constraint

We now aim to compare the performance of any constrained unbiased estimator for (9), under varying assumptions of the covariance \( \Sigma \). More concretely, let \( \Sigma_1 \) and \( \Sigma_2 \) denote the lower bound on the variance of any unbiased estimator under the constraints (11) and (12) respectively, then using (4), we aim to evaluate and analyze

\[
\begin{align*}
\text{Tr}(\Sigma_1) &= \text{Tr}\left( (U_1^T F U_1)^{-1} \right) \\
\text{Tr}(\Sigma_2) &= \text{Tr}\left( (U_2^T F U_2)^{-1} U_2^T U_2 \right)
\end{align*}
\]

(13a)

where we exploit the cyclic nature of the trace operator, and the property \( U_1^T U_1 = I \) in (13a). To this end, we have the following theorem.

**Theorem 3** (Optimal reference). Consider an network of \( N \) sensors, where each sensor collects \( K \) measurements based on the data model (9), then the following statements hold.

(a) Heteroscedasticity: The optimal reference for estimating the unknown sensor offsets is the mean reference (12)

(b) Homoscedasticity: If \( \Sigma = \sigma^2 I \), the optimal reference for estimating the unknown offsets, outperforms any arbitrarily chosen reference in the network, by a factor 2.
(c) Independent, but not identical: In the special case of
\[ \Sigma = \text{diag}(\sigma_1^2, \sigma_2^2, \ldots, \sigma_N^2), \] (14)
the variance of the optimal unbiased estimator outperforms any arbitrarily
chosen sensor reference by
\[ NK^{-1} \frac{\sum_{n=1,n\neq i}^{N} \sigma_n^2}{\sigma_i^2 + \sum_{n=1,n\neq i}^{N} \sigma_n^2}, \] (15)
where \( \sigma_i^2 \) is the variance of the chosen sensor reference.

**Proof.** (Proof of Theorem 3(a)) Observe from (10) and (12) that
\[ \mathbf{F} \mathbf{c}_2 = \sigma^{-2} \mathbf{K} \mathbf{P} \Sigma^{-1} \mathbf{c}_2 = \mathbf{0}, \]
since \( \mathbf{1}_N \) spans the nullspace of the centering matrix, and hence using Theo-
rem 2, the mean (12) is the optimal reference. \( \square \)

In the following sections, we present the proofs of Theorem 3(b) and Theo-
rem 3(c).

### 3.1 Homoscedasticity

**Proof.** (Proof of Theorem 3(b)) The optimal reference for estimating the sensor
offsets is the mean of all the unknown offsets as proved in Theorem 3(a). Hence,
it suffices to show
\[ \delta = \text{Tr}(\Sigma_2)/\text{Tr}(\Sigma_1) = 0.5, \] (16)
for a homoscedastic system, where \( \text{Tr}(\Sigma_1) \) and \( \text{Tr}(\Sigma_2) \) are given in (13). When
\( \Sigma = \sigma^2 \mathbf{I} \), the FIM in (10) simplifies to \( \mathbf{F} = \sigma^{-2} \mathbf{K} \mathbf{P} \). Now, substituting this
FIM and \( \mathbf{U}_1 \) from (11) in (13), the CCRB for a single reference is given by
\[ \text{Tr}(\Sigma_1) = \frac{\sigma^2}{K} \text{Tr} \left[ (\mathbf{U}_1^T \mathbf{P} \mathbf{U}_1)^{-1} \right] = \frac{\sigma^2}{K} \text{Tr}(\mathbf{P}^{-1}) \] (17)
where we define
\[ \mathbf{P} = \mathbf{I}_{N-1} - N^{-1} \mathbf{1}_{N-1} \mathbf{1}_{N-1}^T. \] (18)
Let \( \mathbf{a} = 1/\sqrt{N} \), then using Sherman-Morrison identity (3), and the properties
\( \mathbf{a}^T \mathbf{a} = N^{-1}(N - 1) \) and \( \mathbf{a} \mathbf{a}^T = N^{-1}(\mathbf{1}_{N-1} \mathbf{1}_{N-1}^T) \), we have \( \Psi^{-1} = \mathbf{I}_{N-1} + \mathbf{1}_{N-1} \mathbf{1}_{N-1}^T \) and following immediately (17) is
\[ \text{Tr}(\Sigma_1) = \frac{\sigma^2}{K} \text{Tr}[\mathbf{I}_{N-1} + \mathbf{1}_{N-1} \mathbf{1}_{N-1}^T] = \frac{2\sigma^2}{K}(N - 1). \] (19)
Along similar lines, the CCRB for the mean reference (12) using (13b), is given by
\[ \text{Tr}(\Sigma_2) = \frac{\sigma^2}{K} \text{Tr} \left[ (\mathbf{U}_2^T \mathbf{U}_2)^{-1} \mathbf{U}_2^T \mathbf{U}_2 \right] = \frac{\sigma^2}{K}(N - 1), \] (20)
where we substitute for the FIM from (10), use the properties \( \mathbf{P} \mathbf{U}_2 = \mathbf{U}_2 \), and
\[ \mathbf{U}_2^T \mathbf{U}_2 = \mathbf{P}^{-1} = \mathbf{I}_{N-1} + \mathbf{1}_{N-1} \mathbf{1}_{N-1}^T. \] (21)
Finally, from (19) and (20), we have (16), and hence proved. \( \square \)
Figure 1: Figure (a) shows an estimate of (16) i.e., \( \hat{\delta} = \frac{\text{MSE}_2}{\text{MSE}_1} \), for varying \( K \) and \( N \), illustrating the factor 2 improvement using the mean reference in contrast to any single reference in the network, under i.i.d Gaussian noise. Under the assumption of independent but not identical Gaussian noise, (b) and (c) show that the estimator performance, for varying \( K \) (fixed \( N \)) and varying \( N \) (fixed \( K \)) respectively. The estimators achieve the respective CCRBs, validating the derived lower bounds.

3.2 Independent, but not identical

We now consider the underlying noise (14), where without loss of generality, we assume \( \sigma_1^2 \leq \sigma_2^2 \leq \ldots \leq \sigma_N^2 \). Following immediately, sensor 1 with the lowest variance is an appropriate reference for calibration. To this end, the constraints (11) hold, and therefore proving Theorem 3(c) is equivalent to showing

\[
\text{Tr}(\Sigma_1) - \text{Tr}(\Sigma_2) = NK^{-1} \frac{\sigma_1^2 \sum_{n=2}^{N} \sigma_n^2}{\sigma_1^2 + \sum_{n=2}^{N} \sigma_n^2}.
\]

(22)

Proof. (Proof of Theorem 3(c)) Given (14), substituting for the FIM (10) and
where we use the cyclic property of the trace, and substitute \( \Psi \) and using (21), we have
\[ \Phi = \left( \begin{array}{c} 1 \\ \Psi \\ \Sigma^{-1} \end{array} \right) \]
then (23) further simplifies to
\[ \Phi^{-1} \left( \begin{array}{c} 1 \\ \Psi \\ \Sigma^{-1} \end{array} \right) \]
where in (a) we recollect the definition of \( \Psi \) from (18), and in (b) we introduce \( \Phi = \Psi \Sigma^{-1} \Psi \), and apply (3) to yield (c). Observe that \( \Phi^{-1} = \Psi^{-1} \Sigma \Psi^{-1} \), and using (21), we have \( \Phi^{-1} 1_{N-1} = N \Psi^{-1} \Sigma 1_{N-1} \) and \( 1_{N-1} \Phi^{-1} 1_{N-1} = N^2 \Sigma 1_{N-1} \Sigma 1_{N-1} \). Now, consider the definition
\[ \Omega = \Sigma - N_{N-1} = \left( \begin{array}{c} 1 \\ \Sigma \end{array} \right) \]
then (23) further simplifies to
\[ \text{Tr}(\Sigma) = K^{-1} \text{Tr} \left[ \Psi^{-1} \Omega \Psi^{-1} \right] \]
where we use the cyclic property of the trace, and substitute \( \Psi^{-1} \Omega^{-1} = N 1_{N-1} 1_{N-1}^T + \Psi^{-1} \). Now, the CCRB for the mean reference (12) from (13b) is,
\[ \text{Tr}(\Sigma_2) = \text{Tr} \left[ U_2 (U_2^T F U_2)^{-1} U_2^T \right] = \text{Tr} \left[ (U_2^T F U_2)^{-1} \Psi^{-1} \right] \]
\[ = K^{-1} \text{Tr} \left[ \left( \Sigma^{-1} + \sigma_1^{-2} 1_{N-1} 1_{N-1}^T \right)^{-1} \Psi^{-1} \right] \]
\[ = K^{-1} \text{Tr} \left[ \Omega \Psi^{-1} \right], \]
where we apply (3) in the final step. Subtracting (26) from (25), we have
\[ \text{Tr}(\Sigma_1) - \text{Tr}(\Sigma_2) = NK^{-1} \text{Tr} \left[ \Omega 1_{N-1} 1_{N-1}^T \right], \]
and substituting for \( \Omega \) from (24), and further manipulations yield (22), and hence proved.

4 Simulations : Clock synchronization

We consider a sensor network of \( N \) clocks, where each clock is plagued with an offset, and collects \( K \) measurements [19]. Let \( t = [t_1, t_2, \ldots, t_K] \) denote the true time over \( K \) instances, and let \( T_i = [T_{i1}, T_{i2}, \ldots, T_{iK}] \) be the \( K \) measurements at the \( i \)th clock, then we have \( T_i = t + \theta_i 1_K + \eta_i \), where \( \theta_i \) and \( \eta_i \) are
the clock offset and the stochastic noise plaguing the \(i\)th clock respectively. Let \(T = [T_1, T_2, \ldots, T_N]^T\) populate all the time measurements from the \(N\) sensors, then observe that the data model (9) holds where \(y = vec(T)\). Subsequently, to estimate the unknown sensor offsets, we aim to minimize

\[
\hat{\theta} = \arg\min_{\theta} \|WP(y - H\theta)\|^2_2 \quad \text{s.t. } c^T\theta = d \tag{28}
\]

where \(W\) is a weighting matrix, \(c\) is the gradient of the parametric constraint and \(d\) is the corresponding response vector. The cost function has a closed form solution for a non-empty constraint set \(c\), such that \([H^TP^T \quad c^T]^T\) is full rank [20]. The estimator (28) under constraints (11) and (12) are denoted by \(\hat{\theta}_1\) and \(\hat{\theta}_2\) respectively. Observe that, with \(W = \Sigma^{-1/2}\), (28) yields the optimal estimator i.e., the minimum variance unbiased estimator for (9), independently for the respective constraints (11) and (12).

We perform experiments to validate the derived lower bounds, using the performance of the optimal estimators (28) under parametric constraints (11) and (12), for varying assumptions on the noise covariance matrix. The performance metric used is the Mean Square Error (MSE) i.e., 

\[
\text{MSE}_i = \frac{1}{N_{\text{exp}}} \text{Tr}[(\theta - \hat{\theta}_i)(\theta - \hat{\theta}_i)^T], \quad \text{where } N_{\text{exp}} = 1000 \text{ is the number of Monte Carlo runs, and } i = 1 \text{ or } i = 2 \text{ indicate the single reference and the mean reference respectively.}
\]

In case of \(\Sigma = \sigma^2I\), where \(\sigma^2 = 10^{-3}\), we perform a Monte carlo experiment by linearly varying both the number of sensors and measurements from 10 to 100. We evaluate an estimate of (16) i.e., \(\delta = \text{MSE}_2/\text{MSE}_1\) for each parameter set. Figure 1(a) visually illustrates the factor 2 improvement, which is invariant over the number of measurements or sensors, as stated in Theorem 3(b).

When the underlying noise is independent, but not identical (14), we consider two independent simulations i.e., a fixed number of sensors \(N = 5\), and increasing number of measurements from \(K = 10\) to 100 in Figure 1(b), and a fixed number of measurements \(K = 100\), and varying the number of sensors from \(N = 10\) to 100 in Figure 1(c). Note that the estimated MSEs meet the CCRBs as expected. Furthermore, observe from (25) and (26), that for a fixed \(N\), the CCRB linearly decreases with increasing \(K\). However, for varying \(N\) with a fixed \(K\), the distribution of variances impacts the performance of the estimator. In this particular case, we observe from Figure 1(c) that as \(N\) increases, the difference \(\text{Tr}(\Sigma_1) - \text{Tr}(\Sigma_2)\) in (22) approaches 0. If the probability distribution of the variances is available, then further deductions can be made on the improvement offered by the mean reference, using (22).

5 Conclusion
In this paper, we investigated the effect of the choice of reference in offset estimation of a sensor network, where sensor measurements are plagued with zero-mean Gaussian noise. We show that using the mean of all the unknown offsets i.e., the mean reference as a parametric constraint, we can outperform any arbitrarily chosen sensor reference within the network. In the particular case of
homoscedasticity, this improvement is by a factor 2, which is not readily intuitive at the outset of the problem formulation. When the underlying Gaussian noise is independent, but not identical, we derive an expression for the improvement offered by the mean reference. In the future, we aim to investigate the optimal constraint set for other data models and probability distributions, including a Bayesian inference framework, where the choice of the prior would play a key role in the estimator performance.

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