Wireless Power Transfer Systems Optimization Using Multiple Magnetic Couplings

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Abstract: Multiple magnetic couplings used to increase the link distance in wireless power transfer systems (WPTTs) are not new. An efficient power transfer in conditions of an extended link distance requires a series connection of the intermediate coils. However, all four connections of the emitter and receiver coils are equally possible. This present paper conducts an extensive analysis of WPTTs utilizing three magnetic couplings. The type of connection of the emitter and receiver coils represented the criterion utilized for the WPTS optimization assessment. The first step requires the determination of the schematic of the sinusoidal equivalent circuit. Then, one synthesizes the functions describing the system performances (e.g., the amount of delivered active power or efficiency) by applying the entirely symbolic and or the hybrid symbolic-numerical formalism. The output of such functions consists of appropriate representation in the frequency domain, based upon Laplace state variable equations (SVE) or complex or Laplace modified nodal equations (MNE). The dependency of the WPTS performance on the number of magnetic couplings and their parameters included a study on resistive loss minimization. The minimization applies to the intermediate coils, whereas the outcomes are the active delivered power and the power transfer efficiency—the first study case aimed at a comparison between two distinct WPTSs: three magnetic couplings versus two. The second case of the study compared the WPTSs having a series connection of three magnetic couplings with those built with the emitter-receiver resonators in parallel. One determined the normalized sensitivities as frequency functions, which depend on circuit resistances, load resistance and the coupling factor between the second and the third coil. The optimization algorithms are suitable for computing optimal parameters of the given circuit to ensure maximum and minimum values of the performance value. Good simulation examples followed the proposed optimization techniques.

Keywords: multiple magnetic couplings; wireless power transfer systems (WPTSs); sensitivity; system performance; equivalent circuit

1. Introduction

Wireless transfer of electromagnetic energy (WTEE) is a new technology, which can bring electricity to consumers, whereas the system interconnection through cables is either impossible or impractical. The transfer distance requires enough electromagnetic field strength to allow for an efficient power transfer. Suppose both the transmitter and the receiver fulfill the condition of magnetic resonance. In such a case, the resonant bodies perform a much more efficient energy transfer than the non-resonant ones. However, the losses accounted for from the total transferred energy are high, around 30%. Despite this impediment, there are several significant applications of WTEE [1–9].

The literature presents several types of WTEE systems, yet the four-coil system (with three magnetic couplings) shows tremendous interest because it offers remarkable
features [1–4,10–18]. This system displays a high degree of implementation freedom regarding the alternative configuration and $S$ parameter values [5,19–24].

The WTEE system consists of a connection to the source-circuit one, the relay-circuits for extending the link distance-circuits two and three, and the load circuit four. Minimization of the relay-circuits losses represents the condition for an efficient power transfer [5,13,19–24]. Recent papers encourage the utilization of the high-temperature superconductors (HTS) for reducing the ohmic losses in the coils and to increase their quality factor. The implementation of the necessary system raises some problems to cool the HTS coils cryogenically. Therefore, one can design and implement a hybrid system consisting of HTS and copper-conductor coils [5]. There are biomedical applications with direct implantation of the load circuit and relay circuits within the same biological tissue [1,22,24]. Therefore, the dimension of the coils involved in the WPTS becomes a critical parameter. For such applications, it is essential to study the impact that individual relay-circuits have on the performance of the WPTS. The content of the present paper addresses the WPTS utilizing three magnetic couplings. The assessment of the WPTS optimization, the type of connection of the emitter and receiver coils, represented the criterion utilized for the WPTS optimization assessment.

The first step requires the determination of the schematic of the sinusoidal equivalent circuit. Then, one synthesizes the functions describing the system performances (e.g., the amount of delivered active power or efficiency) by applying the entirely symbolic or hybrid symbolic-numerical formalism. The representation for the output of such functions in the frequency domain is appropriate. [1,6,11–15].

To assess the dependency of the WPTS performance on the number of magnetic couplings and the parameter values of the magnetically coupled resonators [6,7,15–17], one can investigate the following:

- The effect of the intermediate circuits (coils) ohmic losses reduction on the values of delivered active power and the power transfer efficiency.
- How a WPTS with three magnetic couplings compare with those with two magnetic couplings.
- The performance of WPTSs with three magnetic couplings connected in series compared to those of WPTSs having emitter–receiver resonators in parallel.

One determined the normalized sensitivities as frequency functions, which depend on circuit resistances, load resistance $R_L$ and on the coupling factor $k_{23}$ between the second and the third coil. The dependency on the frequency of the normalized sensitivities may decide on the critical aspect represented by the parameters $R_{12}$, $R_{23}$, $R_L$ and $k_{23}$ (see Figures 1 and 2). The mechanisms in place, aiming to maximize both the delivered active power and the transfer efficiency, may include the following:

- Increasing the number of magnetic couplings of the resonant circuits.
- Adjusting the connections of these circuits and their electrical parameters.

There is a constraint: the same value(s) of the frequency correspond to the maximum power and efficiency.

This paper investigates the consequences of losses on each relay-circuit of typical four-coil WPTSs, whereas special attention is dedicated to analyzing the following two critical parameters: power transfer and efficiency.

The structure of the paper is the following: Section 2 consists of a thorough circuit analysis, concluded with the presentation of expressions regarding the parameters of interest. Section 3 contains comparisons between simulation and theoretical results together with their analyses. Section 4 contains the overall conclusions.
2. Wireless Power Transfer Systems with Multiple Magnetic Couplings

The equivalent circuit of a WPTS with four magnetically coupled coils appears in Figure 1, whereas all four resonators are series-connected, as shown in Figure 2, whereas the first circuit (transmitter circuit) and the fourth circuit (load circuit) are parallel-connected. In the last setup (Figure 2), the second and the third circuits (intermediate circuits) are series-connected.

There are no magnetic couplings other than the adjacent circuits in the configurations from Figures 1 and 2 are subjected to analysis. The transmitter is circuit one (powered by a voltage source), whereas $R_i$ comprises the voltage source’s losses, and resistor $R_{L1}$ explains the losses encountered by $L_1$ and $C_1$. The relay circuits two and three develop losses in their passive components, modeled by $R_{L2}$ and $R_{L3}$.

The load circuit (Figure 2) has the load resistance $R_L$, whereas the resistor $R_{L4}$ accounts for the losses developed by the passive components $L_4$ and $C_4$. All the analyzed circuits are resonant at the same angular frequency $ω_0$, expressed further by $ω_0 = \frac{1}{\sqrt{L_1C_1}} = \frac{1}{\sqrt{L_2C_2}} = \frac{1}{\sqrt{L_3C_3}} = \frac{1}{\sqrt{L_4C_4}}$.

The simulation of the circuits from Figures 1 and 2 appealed to the entirely symbolic, partially symbolic, and numeric form using the following programs: ACAP (Analogue Circuit Analysis Program), CSAP (Circuit Symbolic Analysis Circuit) and SYSEG (i.e., Symbolic State Equation Generation) [1,6,11–15,25–27].
These programs work in conjunction with the more well-known software packages MAPLE and SPICE, whereas MATLAB proved the most suitable for the graphical plots.

Equation (1) denotes $P_i$ the active power generated by the voltage source $v_i$, and $P_{RL}$ the active power delivered to the load:

$$P_i = \text{Re}(E_i \cdot I_i^*), \quad P_{RL} = R_L \cdot |I_L|^2.$$  \hspace{1cm} (1)

The efficiency of the active power transmission is as follows:

$$\eta_{21} = \frac{100 P_{RL}}{P_i}.$$ \hspace{1cm} (2)

The maximum active power delivered to the load, according to the Thevenin’s Theorem is as follows:

$$P_{RL,\text{Thef, max}} = \frac{|V_{\text{Thef}}|^2}{4R_{L,\text{Thef}}}.$$ \hspace{1cm} (3)

The normalized sensitivities of active power delivered to the load $P_{RL}$ and of the transmission efficiency $\eta_{21}$ defined with respect to any parameter $x$ of the circuits represented in Figures 1 and 2 are as follows:

$$S^P_{x} = \frac{\partial P_{RL}}{\partial x} \cdot \frac{x}{P_{RL}}, \quad S^\eta_{x} = \frac{\partial \eta_{21}}{\partial x} \cdot \frac{x}{\eta_{21}}.$$ \hspace{1cm} (4)

In a similar manner to that in Equation (4), one can compute any normalized sensitivity.

For the equivalent circuit represented in Figure 1, one neglects the cross-coupling effect between non-adjacent resonators. By introducing the notations $R_1 = R_0 + R_{L1}$ and $R_4 = R_{L4} + R_0$, the harmonic complex regime equations considering the four coils at resonance are as follows:

\[
\begin{align*}
R_1 I_5 + j\omega M_{12} I_2 &= E_i \\
-\omega M_{10} I_2 + R_{L2} I_2 + j\omega M_{23} I_3 &= 0 \\
j\omega M_{23} I_3 + R_{L3} I_3 + j\omega M_{34} I_4 &= 0 \\
j\omega M_{34} I_4 + R_{L4} I_4 &= 0
\end{align*}
\]  \hspace{1cm} (5)

By solving system Equation (5), one obtained the following:

\[
\begin{align*}
I_5 &= I_2 = \frac{E_i ((R_{12} M_{13}^2 + R_{34} M_{23}^2) \omega_0^2 + R_{L2} R_{L3} R_{L4})}{M_{12}^2 M_{13}^2 \omega_0^4 + (M_{12}^2 R_{L2} R_4 + M_{13}^2 R_{L3} R_1 + M_{23}^2 R_{L4} R_3) \omega_0^2 + R_{L1} R_{L3} R_{L4}} \\
I_2 &= -j\omega M_{10} E_i (R_{L3} R_4 + \omega_0^2 M_{23}^2) \\
I_3 &= \frac{-j\omega M_{10} M_{13}^2 M_{23} R_4}{M_{12}^2 M_{34}^2 \omega_0^4 + (M_{12}^2 R_{L2} R_4 + M_{34}^2 R_{L3} R_1 + M_{23}^2 R_{L4} R_3) \omega_0^2 + R_{L1} R_{L3} R_{L4}} \\
I_4 &= I_3 = \frac{j\omega M_{10} M_{13}^2 M_{34} E_i}{M_{12}^2 M_{34} \omega_0^4 + (M_{12}^2 R_{L2} R_4 + M_{34}^2 R_{L3} R_1 + M_{23}^2 R_{L4} R_3) \omega_0^2 + R_{L1} R_{L3} R_{L4}}
\end{align*}
\]  \hspace{1cm} (6)

The complex input impedance, seen from the source output (see Figure 1), is equal to $Z_{in} = \frac{E_i}{I}$, the complex impedance of circuit two reflected in circuit one is equal to $Z_{12} = \frac{(j\omega M_{12}) I_2}{I_5}$, the complex impedance of circuit three reflected in circuit two is equal to $Z_{23} = \frac{I_3}{I_5}$, whereas the complex impedance of circuit four reflected in circuit three, is equal
to $Z_{34} = \frac{j \omega M_{34} I_3}{L_3}$. The detailed expressions appear in relation to Equation (7a–d) as follows:

\[
Z_{32} = \frac{j \omega M_{12} L_2}{L_2} = \frac{\omega_0 M_{12}^2 (R_{13} R_4 + \omega_0^2 M_{34}^2)}{R_{13} R_4 + \omega_0^2 M_{34}^2 + R_4 \omega^2 M_{23}^2} \tag{7a}
\]

\[
Z_{23} = \frac{j \omega M_{23} L_3}{L_2} = \frac{\omega_0^3 M_{12}^2 R_4}{R_{13} R_4 + \omega_0^2 M_{34}^2} \tag{7b}
\]

\[
Z_{34} = \frac{j \omega M_{34} L_4}{L_3} = \frac{\omega_0^2 M_{34}^2}{R_4} \tag{7c}
\]

Consequently, the equivalent circuit of the four-coil WPTS, with $Z_{12}$ being the reflected impedance in circuit one, seen by the voltage source, is given in Figure 3.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig3.png}
\caption{Equivalent circuit.}
\end{figure}

Solving the circuit in Figure 3, the current $I_2$ has the expression from Equation (6).

The overall system efficiency requires the computation of the power transferred to circuit four ($P_{RL} = R_l \cdot |I_2|^2$) and the power delivered by the source ($P_1 = Re\{E_i I_2^*\} = (R_1 + Z_{12}) \cdot |I_2|^2$), yielding to Equation (8).

\[
\eta_{21} = 100 \frac{P_{RL}}{P_1} = 100 \frac{R_1}{R_1 + Z_{12}} \left(\frac{|I_2|^2}{|I_2|^2}\right) \tag{8}
\]

By introducing the expressions given by Equation (7a–d) into the relation defining efficiency (8), the concluding relationship utilized to express the efficiency became the following:

\[
\eta_{21} = 100 \frac{R_1}{R_1 + Z_{12}} \cdot \left(\frac{\omega_0^2 M_{34}^2}{(R_{12} M_{23}^2 + R_4 M_{23}^2) \omega^2 + R_{12} R_4 R_4}\right) \tag{9}
\]

A similar procedure described by Equations (5)–(9) can subject WPTSs configured with parallel-series-series-parallel (p-s-s-p) connections, represented by the equivalent circuit from Figure 2. However, such expressions in the entirely symbolic form are much more complicated.

The two sets of values for the parameters of the equivalent circuits (Figures 1 and 3), considered for simulation, are as follows:

Set 1:
- $R_{12} = 0.0545 \, \Omega$, $R_{13} = 0.0925 \, \Omega$, $R_4 = 30.0 \, \Omega$.
- $C_1 = 0.15784 \, \text{e} - 06 \, \text{F}$, $C_2 = 0.315685 \, \text{e} - 06 \, \text{F}$, $C_3 = C_1$; $C_4 = C_2$. 
\[ L_1 = 63.76e^{−6} \text{H}, \quad L_2 = 32.6e^{−0.6} \text{H}, \quad L_3 = L_1, \quad L_4 = L_2. \]

\[ R_i = 1.5 \Omega, \quad R_{1i} = 0.0925 \Omega, \quad R_{4i} = 0.0545 \Omega, \quad k_{12} = 0.65. \]

\[ e_i(t) = 170.0\sqrt{2}\sin(\omega t) \text{V}. \]

\[ k_{34} = k_{12}, \quad k_{23} = 0.035. \]

\[ f_0 = 50.2 \text{kHz}. \]

\[ M_{12} = k_{12} \cdot \sqrt{L_1 \cdot L_2}, \quad M_{23} = k_{23} \cdot \sqrt{L_2 \cdot L_3} \quad \text{and} \quad M_{34} = k_{34} \cdot \sqrt{L_3 \cdot L_4} \]

Set 2:

The values of the parameters \( R_{12}, R_{13}, R_l \) and \( k_{23} \) are reduced five times, which leads to the following:

\[ R_{12} = 0.0109 \Omega; \quad R_{13} = 0.0185 \Omega; \quad R_l = 6.0 \Omega \quad \text{and} \quad k_{23} = 0.007. \]

The parameters are modified as stated above to analyze the variation of the following parameters: how the efficiency and the transmitted power change when the ohmic resistances of the three coils, the load resistance and the coupling factor reduces.

The numerical results for the parameters of the four magnetically coupled coils (Figures 1 and 2) required the application of the software ANSYS EXTRACTOR Q3D [9], in the following conditions:

- The frequency is equal to \( f = f_0 = 50.2 \text{kHz}; \)
- Coil one to two distance = coil three to four distance: \( d_{12} = d_{34} = 5 \text{ cm}; \)
- Coil two to three distance \( d_{23} = 30 \text{cm}; \)
- The coils one and three are identical, as well as the coils two and four.

The results of the computations performed using the above-mentioned software programs encompassed the following quantities:

- \( P_{RL} \) —the load delivered active power;
- \( P_1 \) —the active power generated by the source \( e_i; \)
- \( \eta_{21} \) —the load power transmission efficiency;
- The normalized sensitivity of \( P_{RL} \) with respect to the parameters \( R_{12}, R_{13}, R_l \) and \( k_{23}; \)
- The normalized sensitivity of \( \eta_{21} \) with respect to the parameters \( R_{12}, R_{13}, R_l \) and \( k_{23}. \)

3. Results

The results appear in graphical form, in which the computed quantities are functions of the frequency. There are the following two categories of comparisons regarding the results:

- The circuit from Figure 1, versus the circuit from Figure 2.

One compared all the graphical plots with those of WPTSs with only two resonators (with a single magnetic coupling \( k_{23} \)) and with the series–series (ss) connection of the two coils. The magnetic couplings \( k_{12} \) and \( k_{34} \) are strong and \( k_{23} \) is weak. The obtained efficiency is superior to that of two magnetic couplings systems for the considered parameter values.

The following notations are in use:

- \( \text{ss-1mc-n} \) —WPTSs with two coils with series–series connection, one magnetic coupling and nominal values of the parameters.
- \( \text{ss-3mc-n} \) —WPTSs with the four coils connected in series with three magnetic couplings and nominal values of the parameters.
- \( \text{pssp-3mc-n} \) —WPTSs having the emitter coil in parallel and the receiver coil in parallel, and the intermediate circuits three and four, each connected in series and with nominal values of the parameters.

In all the simulations, the WPTS fulfilled the resonance condition.

Figure 4a–d represent the dependency on the frequency of the active power delivered to the load \( P_{RL} \) as follows:

- Figure 4a — \( \text{ss-3mc} \) and \( \text{ss-1mc} \) connections for nominal values of the parameters;
• Figure 4b—ss-3mc and ss-1mc connections and the values for resistances $R_{L2}$ and $R_{L3}$ (i.e., connection ss-3mc), and the values for resistances $R_{L1}$ and $R_{L2}$ (i.e., connection ss-1mc), reduced five times;
• Figure 4c—pssp-3mc and ss-1mc connections and nominal values of the parameters;
• Figure 4d—pssp-3mc and ss-1mc connections and the values for resistances $R_{L2}$ and $R_{L3}$ (i.e., for connection pssp-3mc), and values for resistances $R_{L1}$ and $R_{L2}$ (i.e., for connection ss-1mc), reduced five times.

Table 1 contains the maximum values of the active power transferred to the load, the maximum efficiency and their frequencies for all configurations and connections under scrutiny.

| Connection Type | the Coordinates of the Maximum of PRL | the Coordinates of the Maximum of the $\eta_{21}$ |
|-----------------|--------------------------------------|-----------------------------------------|
| 1mc-ss-n        | (50.257 kHz, 94.921 W)               | (50.339 kHz, 0.5266 %)                 |
| 1mc-ss-m        | (50.246 kHz, 104.632 W)              | (50.315 kHz, 0.55362 %)                |
| 3mc-ss-n        | (84.82 kHz, 233.754 W)               | (50.177 kHz, 58.645 %)                 |
| 3mc-ss-m        | (84.82 kHz, 259.546 W)               | (50.17 kHz, 87.938 %)                  |
| 3mc-pssp-n      | (66.085 kHz, 628.092 W)              | (84.957 kHz, 66.607 %)                 |
| 3mc-pssp-m      | (66.054 kHz, 855.222 W)              | (85.035 kHz, 84.968 %)                 |

From Figure 4a–d, one can observe that the largest value of the active power delivered to the load is $P_{RL,pssp,3mc,m,max} = 855.22$ W at a frequency $f_{PRL,pssp,3mc,m,max} = 66.054$ kHz, whereas the maximum transmission efficiency value is equal to $\eta_{21,ss,3mc,m,max} = 87.938\%$ corresponding to the frequency $f_{\eta_{21,ss,3mc,m,max}} = 50.17$ kHz.
Figure 4. The dependencies on the frequency of the active power delivered to the load $P_RL$ for (a) Connections ss-3mc-n and ss-1mc-n; (b) Connections ss-3mc-m and ss-1mc-m, whereas the values of the resistances $R_2$ and $R_3$ for connection ss-3mc and the values of the resistances $R_1$ and $R_2$ for connection ss-1mc, are reduced five times; (c) Connections pssp-3mc-n and ss-1mc-n; (d) Connections pssp-3mc-m and ss-1mc-m, when the values of the resistances $R_2$ and $R_3$ for connection pssp-3mc and the values of the resistances $R_1$ and $R_2$ for connection ss-1mc, are reduced five times.

4. Comments

- The maximum values of $P_{RL}$ and of $\eta_{21}$ correspond to the connection pssp-3mc-m and 3mc-ss-m, respectively, but at the following different frequencies: $f_{RL,pssp,3mc,m,max} = 66.054$ kHz and $f_{RL,ss,3mc,m,max} = 50.017$ kHz.
- It is obvious that the maximum values of $P_{RL}$ and of $\eta_{21}$ obtained for any connection type of WPTS occurred for the cases of reduced resistance values, given the diminished the Joule–Lenz losses.
- Figure 5a–d represent the dependency on the frequency of the power transfer efficiency as follows:
  - Figure 5a—ss-3mc and ss-1mc connections for nominal values of the parameters.
  - Figure 5b—ss-3mc and ss-1mc connections and the values for resistances $R_2$ and $R_3$, (i.e., connection ss-3mc), and the values for resistances $R_1$ and $R_2$ (i.e., connection ss-1mc), reduced five times.
  - Figure 5c—pssp-3mc and ss-1mc connections and nominal values of the parameters.
  - Figure 5d—pssp-3mc and ss-1mc connections and the values for resistances $R_2$ and $R_3$, (i.e., for connection pssp-3mc), and values for resistances $R_1$ and $R_2$ (i.e., for connection ss-1mc), reduced five times.
- After assessing the graphical representation from Figure 5a–d, it results that:
  - The frequencies values corresponding to max efficiency for connection ss-1mc-n, ss-3mc-n, ss-1mc-m and ss-3mc-m (Figure 5a,b) are the same, approximately, 50.25 kHz, close to the resonance frequency $f_0 = 50.2$ kHz.
  - The frequency values corresponding to the max efficiency for connections pssp-3mc-n and pssp-3mc-m (Figure 5c,d) are very close: $f_{\eta_{21},pssp,3mc,n,max} = 84.957$ kHz and $f_{kHz,\eta_{21},pssp,3mc,m,max} = 85.035$ kHz.
  - The splitting frequency occurs for all types of connections and appear in all the previously defined Figures 4 and 5.
  - The first step in the calculation of the resonance frequency for each type of WPTS is the computation of the input impedance $Z_{in} = \frac{S}{j}$ of the equivalent circuit. Then, the resonance frequency results from the condition $Im(Z_{in}) = 0$. Table 2 contains the values of $P_{RL}$ and $\eta_{21}$ for each resonance frequency.
According to Table 2, one can observe that the values of $P_{RL}$ and of $\eta_{21}$, corresponding to the resonance frequencies for all the WPTSs analyzed, are much smaller than those shown in Figures 4a–d and 5a–d.

Table 2. The load transferred power and the efficiency in conditions of resonance.

| Connection Type | Resonance Frequency (kHz) | Power $P_{RL}$ (W) | Efficiency $\eta_{21}$ (%) |
|-----------------|----------------------------|--------------------|---------------------------|
| ss-1mc-n        | 50.199                     | 94.774             | 0.525                     |
| ss-1mc-m        | 50.2                       | 104.484            | 0.552                     |
| ss-3mc-n        | 39.07                      | 50.352             | 0.2976                    |
| ss-3mc-m        | 39.069                     | 56.305             | 0.3147                    |
| ss-3mc-m        | 50.2                       | 28.338             | 87.933                    |
| ss-3mc-m        | 84.967                     | 259.188            | 1.469                     |
| pssp-3mc-n      | 38.865.209                 | 1.116              | 11.239                    |
| pssp-3mc-n      | 65.209                     | 585.212            | 3.8966.895                |
| pssp-3mc-n      | 84.589                     | 101.671            | 0.1864                    |
| pssp-3mc-m      | 38.856                     | 0.00921            | 0.893.962                 |
| pssp-3mc-m      | 65.313                     | 156.537            |                           |
| pssp-3mc-m      | 84.386                     | 0.947              |                           |
Figure 5. The dependence on the frequency $\eta_{21}$ from the source to the load for (a) Connections ss-3mc-n and ss-1mc-n; (b) Connections ss-3mc-m and ss-1mc-m, whereas the values of the resistances $R_{L2}$ and $R_{L3}$ for connection ss-3mc and the values of the resistances $R_{L1}$ and $R_{L2}$ for connection ss-1mc, are reduced five times; (c) Connections pssp-3mc-n and ss-1mc-n; (d) Connections pssp-3mc-m and ss-1mc-m, when the values of the resistances $R_{L2}$ and $R_{L3}$ for connection pssp-3mc and the values of the resistances $R_{L1}$ and $R_{L2}$ for connection ss-1mc, are reduced five times.

Figure 6a–d present the following normalized sensitivity regarding the active power transferred to the load $P_{RL}$, as a function of coils resistances $R_{L2}$ and $R_{L3}$:

- Figure 6a—$S_{PRL}^{ss-3mc}$ for connections ss-3mc-n, pssp-3mc-n and ss-1mc-n;
- Figure 6b—$S_{PRL}^{ss-3mc}$ for connections ss-3mc-m, pssp-3mc-m and ss-1mc-m, whereas the values of the resistances $R_{L2}$ and $R_{L3}$ for connections ss-3mc and pssp-3mc-m, and the values of the resistances $R_{L1}$ and $R_{L2}$ for connection ss-1mc, are five times reduced;
- Figure 6c—$S_{PRL}^{ss-3mc}$ for connections ss-3mc-n, pssp-3mc-n and ss-1mc-n;
- Figure 6d—$S_{PRL}^{ss-3mc}$ for connections ss-3mc-m, pssp-3mc-m and ss-1mc-m, whereas the values of the resistances $R_{L2}$ and $R_{L3}$ for connections ss-3mc and pssp-3mc-m, and the values of the resistances $R_{L1}$ and $R_{L2}$ for connection ss-1mc, are five times reduced.

Figure 7a–d presents the following normalized sensitivity of the active power transmission efficiency $\eta_{21}$ as a function of coils resistances $R_{L2}$ and $R_{L3}$:

- Figure 7a—$S_{\eta_{21}}^{ss-3mc}$ for connections ss-3mc-n, pssp-3mc-n and ss-1mc-n;
- Figure 7b—$S_{\eta_{21}}^{ss-3mc}$ for connections ss-3mc-m, pssp-3mc-m and ss-1mc-m, whereas the values of the resistances $R_{L2}$ and $R_{L3}$ for connections ss-3mc and pssp-3mc-m, and the values of the resistances $R_{L1}$ and $R_{L2}$ for connection ss-1mc, are five times reduced;
- Figure 7c—$S_{\eta_{21}}^{ss-3mc}$ for connections ss-3mc-n, pssp-3mc-n and ss-1mc-n;
- Figure 7d—$S_{\eta_{21}}^{ss-3mc}$ for connections ss-3mc-m, pssp-3mc-m and ss-1mc-m, whereas the values of the resistances $R_{L2}$ and $R_{L3}$ for connections ss-3mc and pssp-3mc-m, and the values of the resistances $R_{L1}$ and $R_{L2}$ for connection ss-1mc, are five times reduced.

By evaluating the information presented in Figures 6 and 7, one can draw the following interesting conclusions:
• The values of the normalized sensitivities $S_{RL}^{P_R}$ and $S_{RL}^{P_{RL}}$ are relatively small (less than 0.225) around the resonance frequency 50.0 kHz, an observation for which is valid for the connections ss-3mc-n, pssp-3cm-n and ss-1mc-n, respectively, for the connections ss-3mc-m, pssp-3cm-m and ss-1mc-m (see Figure 6a–c);

• The largest value of $S_{RL}^{P_R}$ is equal to 0.2211 at 66.0 kHz, for the connection pssp-3mc-n (see Figure 6a), whereas the smallest value is equal to 0.000688 recorded at 70.0 kHz, for the connection ss-1mc-m (see Figure 6b);

• The largest value of $S_{RL}^{P_{RL}}$ is equal to 251.0 recorded at 50.0 kHz, for the connection ss-3mc-n (see Figure 7a), and is equal to 96.9 at 48.0 kHz for connection ss-3mc-m (see Figure 6b). The absolute values of the sensitivities $S_{RL}^{P_{RL}}$ corresponding to the other connections are very small, less than 0.0036 (see Figure 7a,b);

• The absolute values of the sensitivities $S_{RL}^{P_{RL}}$ for connections ss-3mc-n and ss-3mc-m have maximum values equal to 12.23 at 48.0 kHz (see Figure 6b), and equal to 5.526 at the same frequency of 48.0 kHz respectively. For the other connections, the normalized sensitivities have been very small values: 0.00093–0.057 (see Figure 7c,d).

Figure 8a–d present the normalized sensitivity $P_R$ and the efficiency $\eta_2$ as functions of load resistance $R_L$ and of the coupling coefficient $k_{23}$, respectively.

• Figure 8a—$S_{RL}^{P_R}$ for connections ss-3mc-n, ss-3mc-m, pssp-3mc-n and pssp-3mc-m
• Figure 8b—$S_{RL}^{P_{RL}}$ for connections ss-3mc-n, ss-3mc-m, pssp-3mc-n and pssp-3mc-m
• Figure 8c—$S_{RL}^{P_{RL}}$ for connections ss-3mc-n, ss-3mc-m, pssp-3mc-n and pssp-3mc-m
• Figure 8d—$S_{RL}^{P_{RL}}$ for connections ss-3mc-n, ss-3mc-m, pssp-3mc-n and pssp-3mc-m

Figure 8a,c show that the dependencies of sensitivities $S_{RL}^{P_R}$ ($S_{RL}^{P_{RL}}$) on frequency are almost identical for all the connection type of WPTSs with magnetic couplings, with a maximum absolute value of 0.9786 recorded at $f = 50$ kHz for the connections ss-3mc-n and ss-3mc-m. Similarly, the maximum absolute value is equal to 2.159 at $f = 88$ kHz for connections pssp-3mc-n and pssp-3mc-m.

The maximum absolute value of the normalized sensitivity regarding the efficiency as a function of load resistance $S_{RL}^{P_{RL}}$ has a very large value of 4.041e04 at $f = 48.0$ kHz recorded for the connection pssp-3mc-n (see Figure 8b), whereas the sensitivity with respect to the coupling value $S_{k_{23}}^{P_{RL}}$ is even larger, with a recorded value of 2.464e05 at $f = 48.0$ kHz for the connection pssp-3mc-n, (see Figure 8d).

The results from above came following the assumption of very strong magnetic couplings; $k_2$ and $k_4$ are very strong, whereas the magnetic coupling $k_3$ was weak. For some values of the system parameters (given above), we obtained a higher efficiency than the one corresponding to the two magnetic couplings system.
Figure 6. The dependences on frequency of the normalized sensitivity of the active power in relation with the resistances $R_L$ and $R_L$ of the circuits (coils) 2 and 3: (a) $S_{RL1}$ for connections ss-3mc-n, pssp-3mc-n and ss-1mc-n; (b) $S_{RL2}$ for connections ss-3mc-m, pssp-3mc-m and ss-1mc-m, when the values of the resistances $R_L$ and $R_L$ for connections ss-3mc and pssp-3mc-m, and the values of the resistances $R_L$ and $R_L$ for connection ss-1mc, respectively, are five times reduced; (c) $S_{RL3}$ for connections ss-3mc-n, pssp-3mc-n and ss-1mc-n; (d) $S_{RL4}$ for connections ss-3mc-m, pssp-3mc-m and ss-1mc-m when the values of the resistances $R_L$ and $R_L$ for connections ss-3mc and pssp-3mc-m, and for the values of the resistances $R_L$ and $R_L$ for connection ss-1mc, respectively, are five times reduced.
Figure 7. The dependences on frequency of the normalized sensitivity of the active power transmission efficiency in relation with the resistances RL2 and RL3 of circuits (coils) 2 and 3: (a) $S_{\text{RL2}}^{\text{RL2}^\text{RL2}}$ for connections ss-3mc-n, pssp-3mc-n-m and ss-1mc-n-m; (b) $S_{\text{RL2}}^{\text{RL2}^\text{RL2}}$ for connections ss-3mc-m, pssp-3mc-m and ss-1mc-m, when the values of the resistances RL2 and RL3, for connections ss-3mc and pssp-3mc-m, and for the values of the resistances RL1 and RL2 for connection ss-1mc, respectively, are five times reduced; (c) $S_{\text{RL3}}^{\text{RL3}^\text{RL3}}$ for connections ss-3mc-n, pssp-3mc-n and ss-1mc-n; (d) $S_{\text{RL3}}^{\text{RL3}^\text{RL3}}$ for connections ss-3mc-m, pssp-3mc-m and ss-1mc-m when the values of the resistances RL2 and RL3, for connections ss-3mc and pssp-3mc-m, and for the values of the resistances RL1 and RL2 for connection ss-1mc, respectively, are five times reduced.
5. Conclusions

The WPTS with three magnetic couplings allows us to extend the distance between the emitter and receiver, leading to more significant values for the maximum load delivered active in conditions of increased power transmission efficiency, compared to WPTSs with a single magnetic coupling. The demand for an optimal structure of a WPTS required the analysis of several configurations of magnetically coupled coils. For each configuration, two sets of electrical parameters completed the simulation scenario. The first step consisted of the assembling of the equivalent scheme of the WPTS operating in conditions of sinusoidal excitation.

The next step involved the representation of the WPTS outcomes, more precisely the power transferred to the load, its efficiency in fully symbolic, and it is in symbolic-numeric.
form. In such a way, the final form of the WPTS appears expressed as a function of frequency.

The WPTS classified as ss-3mc-m, with the four coils connected in series with three magnetic couplings, provided the highest values of power transferred to the load: 

\[ P_{\text{BL, psp, 3mc, m, max}} = 865.043 \text{ W}, \quad f_{\text{BL, psp, 3mc, m, max}} = 86.054 \text{ kHz}; \]

for the efficiency:

\[ \eta_{\text{21, psp, 3mc, m, max}} = 91.023\%, \quad f_{\text{21, psp, 3mc, m, max}} = 85.094 \text{ kHz}; \]

As a remark the maximum values are obtained at different frequencies.

The reduction in the resistance values (i.e., decreasing the loss due to the Joule–Lenz effect) determines an increase in the load transferred power and of the efficiency, regardless of the connection type. Once confronted with the decision to maximize the efficiency or the load output power, the WPTS designer must compromise. Maximum efficiency demands a loss reduction on circuit two, whereas the maximum load transferred power means to minimize the loss on circuit three. For both of the types of WPTS circuits analyzed, the maximum efficiency occurs in the close neighborhood of the resonance frequency. Moreover, the frequency splitting phenomena represents a constant for all power–frequency and efficiency–frequency characteristics.

The absolute values of the normalized sensitivities calculated versus the load resistance and the coupling coefficient show small values in around the resonance frequency, no matter the type of circuit chosen for the WPTS. This remark is valid regarding the load transferred active power and efficiency. However, the absolute values of the sensitivities are higher in the condition of nominal resistances than for load resistances reduced five times. The load resistance value \( R_L \) and the magnetic coupling coefficient of the circuits two and three \( k_{23} \) are critical for the process of wireless power transfer process performances. Such affirmation results from the way, the normalized sensitivities associated with the efficiency depend on \( R_L \) and \( k_{23} \).

The working frequency is the resonance frequency \( \omega_0 \), neglecting the impedances corresponding to the coils and the capacitors placed in the series connection.

The computation of the power transferred to the load, \( P_L \), and of the efficiency, \( \eta_L \), performed entirely symbolic, in a system containing magnetically connected coils, leads to optimizing these parameters. Then, the function of each parameter determines the maximum values for the performance parameters. Usually, the optimum is the computed function of the coupling coefficient, load resistance and frequency. Then, one extracted the parameters that ensure the optimum values for the performance.

As future developments, there is a consideration for designing WPTSs with multiple magnetic coupling with maximum power and efficiency at the same frequency.

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