Modulated Dust-Acoustic Wave Packets in an Opposite Polarity Dusty Plasma System

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Abstract The nonlinear propagation of the dust-acoustic bright and dark envelope solitons in an opposite polarity dusty plasma (OPDP) system (composed of non-extensive q-distributed electrons, iso-thermal ions, and positively as well as negatively charged warm dust) has been theoretically investigated. The reductive perturbation method (which is valid for a small, but finite amplitude limit) is employed to derive the nonlinear Schrödinger equation. Two types of modes, namely, fast and slow dust-acoustic (DA) modes, have been observed. The conditions for the modulational instability (MI) and its growth rate in the unstable regime of the DA waves are significantly modified by the effects of non-extensive electrons, dust mass, and temperatures of different plasma species, etc. The implications of the obtained results from our current investigation in space and laboratory OPDP medium are briefly discussed.

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Key words: dust-acoustic waves, opposite polarity, modulational instability, envelope solitons

1 Introduction

Now-a-days, the study of dusty plasma (DP) is one of the most rapidly growing branches in plasma physics due to their existence in space, viz., planetary rings,[1] cometary tails,[2] Jupiter’s magnetosphere,[2] lower part of the Earth’s atmosphere[1] and also in laboratory plasmas.[3–7] The DP have generally considered to be an ensemble of negatively charged dust grains, free electrons, and ions. However, the co-existence of opposite polarity (OP) dust grains in plasmas introduces a new DP model called OP DP (OPDP) whose main constituent species are positively and negatively charged warm massive dust grains.[2,6] The exclusive property of this OPDP, which makes it completely unique from other plasmas (viz., electron ion and electron-positron plasmas), is that the ratio of the size of positively charged dust grains to that of negatively charged dust grains can be smaller[8] or larger[9] or equal to unity.[9] There are three main processes by which dust grains become positively charged: (a) Secondary emission of electrons from the surface of the dust grains; (b) Thermionic emission induced by the radiative heating; (c) Photoemission in the presence of a flux of ultraviolet photons.[1,10]

The researchers have focused on wave dynamics, specifically, dust-acoustic (DA) waves (DAWs), dust-acoustic rogue waves (DARWs), and dust ion-acoustic waves (DIAWs) in understanding electrostatic density perturbations and potential structures (viz., shock, soliton, envelope solitons,[11–12] and rogue waves,[13–15]) in DP. Rao et al.[14] first theoretically predicted the existence of very low frequency DAWs (where the inertia is provided by the dust mass and restoring force is provided by the thermal pressure of electrons and ions) in comparison with the electron and ion thermal velocities and this theoretical prediction has been conclusively verified by Barken et al.[4] There is also direct evidence for the co-existence of both positively and negatively charged dust grains in different regions of space plasmas (viz., cometary tails,[2] upper mesosphere,[2] and Jupiter’s magnetosphere,[10] etc.) and laboratory devices (viz., direct current and radio-frequency discharges,[1] plasma processing reactors,[16] fusion plasma devices, and solid-fuel combustion products,[1] etc.). The novelty of this OPDP has attracted numerous authors[17–21] to investigate the linear and nonlinear propagation of electrostatic waves. Sayed and Mamun[2] studied the finite solitary potential structures that exist in OPDP. El-Taibany[17] examined the DAWs in inhomogeneous four component OPDP, and observed that only compressive soliton is created corresponding to fast DA mode.

In space and astro-physical situations, if the plasma species move very fast compared to their thermal velocities[22] then the Maxwellian distribution is no longer valid to explain the dynamics of these plasma species. For that reason, Tsallis proposed the non-extensive statistics,[23] which is the generalisation of Boltzmann-Gibbs-Shannon entropy. The importance of Tsallis statistics is that it can easily describe the long range interactions of the electron-ion in DP system.[13–15] The research regarding modulational instability (MI) of DAWs in nonlinear and dispersive mediums has been increasing significantly due to their existence in astrophysics, space physics[11–15] as well as in application in many laboratory situations.[11] A large number of researchers have used the nonlinear Schrödinger equation (NLSE), which governs the dynamics of the DAWs, to study the formation

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of the envelope solitons or rogue waves\textsuperscript{13–15} in DP. Bains et al.\textsuperscript{13} investigated the MI of the DAWs in non-extensive DP. Moslem et al.\textsuperscript{14} have studied the MI of the DAWs in three component DP in presence of the non-extensive electrons and ions, and have found that the threshold wave number ($k_\theta$) increases with $q$. Duan et al.\textsuperscript{19} have investigated the criteria for MI of the DAWs and the formation of envelope solitons in OPDP. Zagheer et al.\textsuperscript{20} have reported DARWs in a four component OPDP. Gill et al.\textsuperscript{21} have studied MI of DAWs in a four component OPDP, and have found that the presence of positive dust grains significantly modify the domain of the MI and localized envelope solitons. To the best knowledge of the authors, no attempt has been made to study the MI and corresponding dark and bright envelope solitons associated with the DAWs in a four component OPDP in presence of OP warm adiabatic dust grains. The aim of the present investigation is therefore to extend the work of Gill et al.\textsuperscript{21} by examining the conditions for the MI of the DAWs (in which inertia is provided by the OP warm dust masses and restoring force is provided by the thermal pressure of $q$-distributed electrons and iso-thermal ions) in four component OPDP.

The manuscript is organized as the following fashion: The governing equations of our plasma model are described in Sec. 2. The NLSE is derived in Sec. 3. The stability of DAWs is examined in Sec. 4. Envelope solitons is presented in Sec. 5. Finally, a brief discussion is provided in Sec. 6.

2 Model Equations

In this paper, we consider a collisionless, fully ionized, unmagnetized four component dusty plasma system composed of $q$-distributed electrons (charge $-e$, mass $m_e$), iso-thermal ions (charge $+e$, mass $m_i$) and inertial warm negatively charged dust grains (charge $q_1 = -z_1e$, mass $m_1$) as well as positively charged warm dust grains (charge $q_2 = +z_2e$, mass $m_2$), where $z_1$ ($z_2$) is the charge state of the negatively (positively) charged warm dust particles. The negatively and positively charged warm dust grains can be displayed by continuity and momentum equations, respectively, as:

\[
\frac{\partial n_1}{\partial t} + \frac{\partial}{\partial x}(n_1 u_1) = 0, 
\]

\[
\frac{\partial u_1}{\partial t} + u_1 \frac{\partial u_1}{\partial x} = \frac{z_1 e}{m_i} \frac{\partial \varphi}{\partial x} - \frac{1}{m_1 n_1} \frac{\partial p_1}{\partial x},
\]

\[
\frac{\partial n_2}{\partial t} + \frac{\partial}{\partial x}(n_2 u_2) = 0, 
\]

\[
\frac{\partial u_2}{\partial t} + u_2 \frac{\partial u_2}{\partial x} = \frac{z_2 e}{m_2} \frac{\partial \varphi}{\partial x} - \frac{1}{m_2 n_2} \frac{\partial p_2}{\partial x},
\]

where $n_1$ ($n_2$) is the density numbers of the negatively (positively) charged warm dust grains; $t$ ($x$) is the time (space) variable; $u_1$ ($u_2$) is the fluid speed of the negatively (positively) charged warm dust species; $e$ is the magnitude of the charge of the electron; $\varphi$ is the electrostatic wave potential; $p_1$ ($p_2$) is the adiabatic pressure of the negatively (positively) charged warm dust grains. The system is enclosed through Poisson’s equation as

\[
\frac{\partial^2 \varphi}{\partial x^2} = 4\pi e (n_i - n_e + z_1 n_1 - z_2 n_2),
\]

where $n_i$ and $n_e$ are, respectively, the ion and electron number densities. The quasi-neutrality condition at equilibrium can be written as

\[
n_e + zn_2 = n_i + zn_1 ,
\]

where $n_0$, $n_{20}$, $n_{e0}$, and $n_{10}$ are the equilibrium number densities of the iso-thermal ions, positively charged warm dust grains, $q$-distributed electrons, and negatively charged warm dust grains, respectively. Now, in terms of normalized variables, namely, $N_1 = n_1/n_{10}$, $N_2 = n_2/n_{20}$, $U_1 = u_1/C_{d1}$ (with $C_{d1}$ being the sound speed of the negatively charged warm dust grains); $U_2 = u_2/C_{d1}$, $\phi = e2_1/k_n T_1$ (with $T_1$ being the temperature of the iso-thermal ion); $T = w_{pd1}$ (with $\omega_{pd1}$ being the plasma frequency of the negatively charged warm dust grains): $X = x/\lambda_{pd1}$ (with $\lambda_{pd1}$ being the Debye length of the negatively charged warm dust grains); $C_{d1} = (z_1 k_b T_1/m_1)^{1/2}$, $\omega_{pd1} = (4\pi e^2 z_1 n_{10}/m_1)^{1/2}$, $\lambda_{pd1} = (k_b T_1/4\pi e^2 z_1 n_{10})^{1/2}$; $p_1 = p_{10}(n_1/n_{10})^\gamma$ (with $p_{10}$ being the equilibrium adiabatic pressure of the negatively charged warm dust grains and $\gamma = (N + 2)/N$, where $N$ is the degree of freedom, for one-dimensional case, $N = 1$ so that $\gamma = 3$; $p_{10} = n_{10} k_b T_1$ (with $T_1$ being the temperature of the negatively charged warm dust grains and $k_b$ is the Boltzmann constant); $p_2 = p_{20}(n_2/n_{20})^3$ (with $p_{20}$ being the equilibrium adiabatic pressure of the positively charged warm dust grains) and $p_{20} = n_{20} k_b T_2$ (with $T_2$ being the temperature of the positively charged warm dust particles). After normalization, the governing equations (1)–(5) can be written as

\[
\frac{\partial N_1}{\partial T} + \frac{\partial}{\partial X}(N_1 U_1) = 0,
\]

\[
\frac{\partial U_1}{\partial T} + U_1 \frac{\partial U_1}{\partial X} + 3\sigma_1 N_1 \frac{\partial N_1}{\partial X} = \frac{\partial \phi}{\partial X},
\]

\[
\frac{\partial N_2}{\partial T} + \frac{\partial}{\partial X}(N_2 U_2) = 0,
\]

\[
\frac{\partial U_2}{\partial T} + U_2 \frac{\partial U_2}{\partial X} + 3\sigma_2 N_2 \frac{\partial N_2}{\partial X} = -\alpha \frac{\partial \phi}{\partial X},
\]

where $\alpha = (\mu_1 + \beta - 1)N_e - \mu_1 N_1 + N_1 - \beta N_2$, (11)

where $\sigma_1 = T_1/z_1 T_1$, $\sigma_2 = m_1 T_2/z_2 m_2 T_1$, $\alpha = m_1 z_2/m_2 z_1$, $\beta = z_2 n_{20}/z_1 n_{10}$, and $\mu_1 = n_{01}/z_1 n_{10}$. It may be noted here that we have considered for our numerical analysis $n_1 > n_{20}$, $n_{10} > n_{20}$, and $T_e, T_i > T_1, T_2$. The number densities of the non-extensive $q$-distributed\textsuperscript{15} electron can be given by the following normalized equation

\[
N_e = [1 + (q - 1)\delta e^{(q+1)/2(q-1)}] ,
\]

where $\delta = T_i/T_e$ (with $T_e$ being the temperature of the non-extensive $q$-distributed electron and $T_i > T_1$) and $q$ is the non-extensive parameter describing the degree of non-extensivity, i.e., $q = 1$ indicates the Maxwellian distribution, whereas $q < 1$ refers to the super-extensivity,
and the opposite condition \( q > 1 \) corresponds to the subextensivity.\(^{[15]}\) The number densities of the iso-thermally distributed\(^{[15]}\) ion can be represented as

\[
N_i = \exp(-\phi) .
\]

Now, by substituting Eqs. (12) and (13) into Eq. (11), and extending up to the third order in \( \phi \), we can obtain

\[
\frac{\partial^2 \phi}{\partial X^2} + \beta N_2 - N_1 = \beta - 1 + \gamma_1 \phi + \gamma_2 \phi^2 + \gamma_3 \phi^3 + \cdots ,
\]

(14)

where

\[
\gamma_1 = [(\beta + \mu_1 - 1)(q + 1)\delta + 2\mu_1]/2 ,
\]

\[
\gamma_2 = [(\beta + \mu_1 - 1)(q + 1)(3 - q)\delta^2 - 4\mu_1]/8 ,
\]

\[
\gamma_3 = [(\beta + \mu_1 - 1)(q + 1)(q - 3)(3q - 5)\delta^3 + 8\mu_1]/48 .
\]

The left hand side of Eq. (14) is the contribution of electron and ion species.

### 3 Derivation of the NLSE

We will use the reductive perturbation method (RPM) to derive the NLSE for studying the MI of the DAWs in OPDP. Now, the stretched co-ordinate\(^{[15]}\) can be defined as

\[
\xi = \epsilon(X - V_y T) ,
\]

\[
\tau = \epsilon^2 T ,
\]

where \( V_y \) is the envelope group velocity and \( \epsilon \) is a small but real parameter. The dependent variables\(^{[15]}\) can be written as

\[
N_1 = 1 + \sum_{m=1}^{\infty} \epsilon^{(m)} \sum_{l=-\infty}^{\infty} N_1^{(m)}(\xi, \tau) \exp(i l \Upsilon) ,
\]

(17)

\[
U_1 = \sum_{m=1}^{\infty} \epsilon^{(m)} \sum_{l=-\infty}^{\infty} U_1^{(m)}(\xi, \tau) \exp(i l \Upsilon) ,
\]

(18)

\[
N_2 = 1 + \sum_{m=1}^{\infty} \epsilon^{(m)} \sum_{l=-\infty}^{\infty} N_2^{(m)}(\xi, \tau) \exp(i l \Upsilon) ,
\]

(19)

\[
U_2 = \sum_{m=1}^{\infty} \epsilon^{(m)} \sum_{l=-\infty}^{\infty} U_2^{(m)}(\xi, \tau) \exp(i l \Upsilon) ,
\]

(20)

\[
\phi = \sum_{m=1}^{\infty} \epsilon^{(m)} \sum_{l=-\infty}^{\infty} \phi_l^{(m)}(\xi, \tau) \exp(i l \Upsilon) ,
\]

(21)

where \( \Upsilon = kX - \omega T \) and \( k (\omega) \) is the carrier wave number (frequency). The derivative operators in the above equations are considered as follows:

\[
\frac{\partial}{\partial T} \rightarrow \frac{\partial}{\partial \tau} - \epsilon V_y \frac{\partial}{\partial \xi} + \epsilon^2 \frac{\partial}{\partial \tau} ,
\]

(22)

\[
\frac{\partial}{\partial X} \rightarrow \frac{\partial}{\partial \xi} + \epsilon \frac{\partial}{\partial \xi} ,
\]

(23)

Now, by substituting Eqs. (15)–(23) into Eqs. (7)–(10), and Eq. (14) and collecting power term of \( \epsilon \), the first order approximation \( (m = 1) \) with the first harmonic \( (l = 1) \) provides the following relation

\[
 ik N_1^{(1)} - i \omega U_1^{(1)} = 0 ,
\]

(24)

\[
 ik U_1^{(1)} - i \omega N_1^{(1)} = 0 ,
\]

(25)

\[
 ik U_{21}^{(1)} - i \omega N_{21}^{(1)} = 0 ,
\]

(26)

\[
 ik \delta N_{21}^{(1)} + i k \alpha \phi_1^{(1)} - i \omega U_{21}^{(1)} = 0 ,
\]

(27)

\[
 \beta N_{21}^{(1)} - N_{11}^{(1)} - k^2 \phi_1^{(1)} - \gamma_1 \phi_1^{(1)} = 0 ,
\]

(28)

where \( \lambda = 3\sigma_1 \) and \( \theta = 3\sigma_2 \). Now, these equations can be reduced to the following pattern

\[
N_1^{(1)} = \frac{k^2}{S} \phi_1^{(1)} ,
\]

(29)

\[
U_1^{(1)} = \frac{\omega k}{S} \phi_1^{(1)} ,
\]

(30)

\[
N_{21}^{(1)} = \frac{nk^2}{A} \phi_1^{(1)} ,
\]

(31)

\[
U_{21}^{(1)} = \frac{\omega nk}{A} \phi_1^{(1)} ,
\]

(32)

where \( A = \omega^2 - \theta k^2 \) and \( S = \lambda k^2 - \omega^2 \). Therefore, the dispersion relation for the DAWs can be written as

\[
\omega^2 = \frac{G \pm \sqrt{G^2 - 4HM}}{2H} ,
\]

(33)

where \( G = (\lambda k^2 + \theta k^2 + \theta_1 + \lambda_1 \gamma_1 + \alpha_\beta + 1) \), \( H = (k^2 + \gamma_1)k^2 \), and \( M = k^2(\alpha \lambda k^2 + \theta_1 \gamma_1 \lambda + \theta + \alpha_\beta \lambda) \). The condition \( G^2 > 4HM \) must be satisfied in order to obtain real and positive values of \( \omega \). Normally, two types of DA modes exist, namely, fast \( (\omega_f) \) and slow \( (\omega_s) \) DA modes according to the positive and negative sign of Eq. (33). Now, we have studied the dispersion properties by depicting \( \omega \) with \( k \) in Figs. 1 and 2 which clearly indicates that \( (a) \) The fast DA mode exponentially increases with \( k \) for its lower range, but a saturation starts after a certain value of \( k \); \( (b) \) The value of \( \omega_f \) increases exponentially with the increasing values of \( z_2 \) for fixed value of \( z_1, n_{20}, \) and \( n_{10} \) (see in Fig. 1); \( (c) \) On the other hand, the slow DA mode linearly increases with \( k \); \( (d) \) The \( \omega_s \) decreases with the increase of \( z_2 \) for the fixed value of \( z_1, n_{20}, \) and \( n_{10} \) (see in Fig. 2). This result agrees with the result of previous published work."\(^{[6,18]}\) It is important to mention that in fast DA mode, both positive and negative warm dust species oscillate in phase with electrons and ions. Whereas in slow DA mode, only one of the inertial massive dust components oscillate in phase with electrons and ions, but the other species are in anti-phase with them.\(^{[15]}\) Next, with the help of second-order \( (m = 2 \) with \( l = 1) \) equations, we obtain the expression of \( V_y \) as

\[
V_y = \frac{F_1}{2\omega k(2A + \alpha_\beta S)} ,
\]

\[
F_1 = \alpha_\beta \theta k_2 S^2 + \alpha_\beta \lambda \gamma_1 k^2 + \omega^2 A^2 + \alpha_\beta \lambda S^2 - 2A^2 S^2 - SA^2 .
\]
Finally, we get
\begin{align}
N_{10}^{(2)} &= C_0|\phi_1^{(1)}|^2, \\
U_{10}^{(2)} &= C_7|\phi_1^{(1)}|^2, \\
N_{20}^{(2)} &= C_8|\phi_1^{(1)}|^2, \\
U_{20}^{(2)} &= C_9|\phi_1^{(1)}|^2, \\
\phi_2^{(2)} &= C_{10}|\phi_1^{(1)}|^2,
\end{align}

where
\begin{align*}
C_0 &= \frac{k^2(2\omega_k V_0 + \lambda k^2 + \omega^2) - C_{10} S^2}{S^2(V_0^2 - \lambda)}, \\
C_7 &= \frac{C_0 V_0 S^2 - 2\omega k^3}{S^2}, \\
C_8 &= \frac{\alpha^2 k^2(2\omega_k V_0 + \theta k^2 + \omega^2) + \alpha C_{10} A^2}{A^2(V_0^2 - \theta)}, \\
C_9 &= \frac{C_0 V_0 A^2 - 2\omega^2 k^3}{A^2}, \\
C_{10} &= \frac{F_4}{F_5}, \\
F_4 &= 2\gamma_2 A^2 S^2 (V_0^2 - \lambda)(V_0^2 - \theta) + A^2 k^2 (2\omega_k V_0 + \lambda k^2 + \omega^2) \\
&\quad \times (V_0^2 - \theta) - \gamma_2 \alpha^2 k^2 S^2 (V_0^2 - \lambda) \\
&\quad \times (2\omega_k V_0 + \theta k^2 + \omega^2), \\
F_5 &= A^2 S^2 [\alpha \beta (V_0^2 - \lambda) + (V_0^2 - \theta) - \gamma_1 (V_0^2 - \theta)(V_0^2 - \lambda)].
\end{align*}

Now, we can obtain the standard NLSE from the third harmonic \((m = 3, l = 1)\) modes with the help of Eqs. (29)–(44) which can be written as
\begin{equation}
i \frac{\partial \Phi}{\partial t} + P \frac{\partial^2 \Phi}{\partial \xi^2} + Q|\Phi|^2 \Phi = 0,
\end{equation}
where \(\Phi = \phi_1^{(1)}\) for simplicity and the dispersion \((P)\) and nonlinear \((Q)\) coefficient are, respectively, written as
\begin{align*}
P &= \frac{F_6}{2\omega A S k^2 (A^2 + \alpha \beta S^2)}, \\
Q &= \frac{F_7}{2\omega k^2 (A^2 + \alpha \beta S^2)},
\end{align*}
where
\begin{align*}
F_6 &= 4\alpha \beta \omega^2 k^2 V_0^2 S^3 + 4\lambda \omega V_0 A^3 k^3 + 4k V_0 \omega^3 A^3 \\
&\quad + 2\alpha \beta \omega^2 A^2 S^3 + \alpha \beta^2 S^4 k^4 + \lambda S k^2 A^3 \\
&\quad + \alpha \beta S^3 \omega^4 + \alpha \beta \theta A k^2 S^3 - 4\alpha \beta k V_0 \omega^3 S^3 \\
&\quad - 4\alpha \beta \omega V_0 k^2 A^3 S^3 - 4\omega^2 k^2 V_0^2 A^3 - \lambda^2 A^3 k^4 \\
&\quad - 2\omega^2 k^2 A^3 - \alpha \beta A k^2 V_0^2 S^3 \\
&\quad - 5k^2 V_0^2 S^3 - A^3 k^4 - 3A^4 S^3, \\
F_7 &= 3\gamma_3 A^2 S^2 + 2\gamma_2 C_5 A^2 S^2 + 2\gamma_2 C_{10} A^2 S^2 \\
&\quad - 2\omega C_2 A^2 k^3 - 3\alpha \beta \omega C_4 S^4 k^3 - 2\omega C_{10} A^2 k^3 \\
&\quad - 2\alpha \beta \omega C_6 S^3 k^3 - \lambda C_6 A^2 k^4 - \phi_3 A^2 k^4 \\
&\quad - \alpha \beta C_3 S^4 k^4 - \alpha \beta C_6 \omega^2 k^5 S^2 - \lambda C_6 A^2 k^4
\end{align*}
\[- C_q \omega^2 A^2 k^2 - \alpha \beta C_q S^2 k^4 = - \alpha \beta C_q \omega^2 k^2 S^2. \]

4 Stability DAWs

The DAWs are modulationally stable against external perturbation when \( P/Q < 0 \). On the other hand, when \( P/Q > 0 \), the DAWs are modulationally unstable against external perturbation. When \( P/Q \rightarrow \pm \infty \), the corresponding value of \( k (= k_c) \) is called the critical or threshold wave number \( (k_c) \) for the onset of MI. The variation of \( P/Q \) with \( k \) for \( \mu_1 \) and \( \alpha \) are shown in Figs. 3 and 4, respectively, which clearly indicate that (a) The value of \( k_c \) increases with the increase of \( n_{i0} \) for fixed value of \( z_1 \) and \( n_{10} \); (b) On the other hand, \( k_c \) value decreases with the increase of \( n_{10} \) for fixed value of \( m_1, z_2, \) and \( z_1 \). The growth rate \( (\Gamma) \) of the modulationally unstable region for the DAWs (when \( P/Q > 0 \) and \( \tilde{k} < \tilde{k}_c = (2Q|\hat{\Phi}_0|^2/P)^{1/2} \)) can be written as\[^{[6,11-12,15,18]} \]

\[ \Gamma = |P| \tilde{k}^2 \left( \frac{\tilde{k}_c^2}{k^2} - 1 \right)^{1/2}. \tag{46} \]

![Fig. 3](image1.png)

The variation of \( P/Q \) with \( k \) for different values of \( \mu_1 \), along with \( \alpha = 1.2, \beta = 0.07, \delta = 0.3, \sigma_1 = 0.0001, \sigma_2 = 0.001, q = 1.8, \) and \( \omega_f \).

![Fig. 4](image2.png)

The variation of \( P/Q \) with \( k \) for different values of \( \alpha \), along with \( \beta = 0.07, \delta = 0.3, \mu_1 = 1.4, \sigma_1 = 0.0001, \sigma_2 = 0.001, q = 1.8, \) and \( \omega_f \).

Now, we have graphically shown how the \( \Gamma \) varies with \( \tilde{k} \) for different values of \( \alpha \) and \( q \) in Figs. 5–7. It is obvious from Figs. 5–7 that (a) Within three limits of \( q \) (\( q = 1, q = +ve, \) and \( q = -ve \)), the maximum value of \( \Gamma \) increases with the increases in the value of \( z_2 \) for fixed values \( z_1, m_1, \) and \( n_2 \) (via \( \alpha \)); (b) So, the effects of the \( \alpha \) on the maximum value of the growth rate is independent from the various limits of \( q \). The physics of this result is that the nonlinearity, which leads to increase the maximum value of the growth rate of DAWs, increases with the increase in the values of \( \alpha \).

![Fig. 5](image3.png)

The variation of \( \Gamma \) with \( \tilde{k} \) for different values of \( \alpha \) (when \( q = 1.0 \)), along with \( \beta = 0.07, \delta = 0.3, \mu_1 = 1.4, \sigma_1 = 0.0001, \sigma_2 = 0.001, \Phi_0 = 0.5, k = 0.5, \) and \( \omega_f \).

![Fig. 6](image4.png)

The variation of \( \Gamma \) with \( \tilde{k} \) for different values of \( \alpha \) (when \( q = 1.5 \)), along with \( \beta = 0.07, \delta = 0.3, \mu_1 = 1.4, \sigma_1 = 0.0001, \sigma_2 = 0.001, \Phi_0 = 0.5, k = 0.5, \) and \( \omega_f \).

![Fig. 7](image5.png)

The variation of \( \Gamma \) with \( \tilde{k} \) for different values of \( \alpha \) (when \( q = -0.6 \)), along with \( \beta = 0.07, \delta = 0.3, \mu_1 = 1.4, \sigma_1 = 0.0001, \sigma_2 = 0.001, \Phi_0 = 0.5, k = 0.5, \) and \( \omega_f \).

The effects of non-extensivity of the electrons on the MI growth rate can be observed from Figs. 8 and 9, and it is obvious from these figures that (a) The maximum value of \( \Gamma \) decreases (decreases) with the increase in the values...
of \( q \) for the limits of \( q > 1 \) (\( q < 1 \)); (b) So, the variation of \( \Gamma \) with respect to \( k \) is independent on the sign of the \( q \). This result agrees with the result of previous published work.[18]

\[ \psi(\xi, \tau) = \left[ \psi_0 \text{sech}^2 \left( \frac{\xi - U\tau}{W} \right) \right]^{1/2} \times \exp \left\{ \frac{i}{2P} \left[ U\xi + \left( \Omega_0 - \frac{U^2}{2} \right) \tau \right] \right\}, \]  

\[ \Phi(\xi, \tau) = \left[ \psi_0 \text{tanh}^2 \left( \frac{\xi - U\tau}{W} \right) \right]^{1/2} \times \exp \left\{ \frac{i}{2P} \left[ U\xi - \left( \frac{U^2}{2} - 2PQ\psi_0 \right) \tau \right] \right\}, \]  

where \( \psi_0 \) indicates the envelope amplitude, \( U \) is the traveling speed of the localized pulse, \( W \) is the pulse width which can be written as \( W = \sqrt{2P/Q}/\psi_0 \), and \( \Omega_0 \) is the oscillating frequency for \( U = 0 \). The bright (by using Eq. (47)) and dark (by using Eq. (48)) envelope solitons are depicted in Figs. 10 and 11, respectively.

5 Envelope Solitons

The bright (when \( P/Q > 0 \)) and dark (when \( P/Q < 0 \)) envelope solitonic solutions, respectively, can be written as[24–27]

\[ \Phi(\xi, \tau) = \left[ \psi_0 \text{sech}^2 \left( \frac{\xi - U\tau}{W} \right) \right]^{1/2} \times \exp \left\{ \frac{i}{2P} \left[ U\xi + \left( \Omega_0 - \frac{U^2}{2} \right) \tau \right] \right\}, \]

\[ \Phi(\xi, \tau) = \left[ \psi_0 \text{tanh}^2 \left( \frac{\xi - U\tau}{W} \right) \right]^{1/2} \times \exp \left\{ \frac{i}{2P} \left[ U\xi - \left( \frac{U^2}{2} - 2PQ\psi_0 \right) \tau \right] \right\}, \]

where \( \psi_0 \) indicates the envelope amplitude, \( U \) is the traveling speed of the localized pulse, \( W \) is the pulse width which can be written as \( W = \sqrt{2P/Q}/\psi_0 \), and \( \Omega_0 \) is the oscillating frequency for \( U = 0 \). The bright (by using Eq. (47)) and dark (by using Eq. (48)) envelope solitons are depicted in Figs. 10 and 11, respectively.

6 Discussion

We have studied an unmagnetized realistic space dusty plasma system consists of \( q \)-distributed electrons, isothermal ions, positively charged warm dust grains as well as negatively charged warm dust grains. The RPM is used to derive the NLSE. The results that have been found from our investigation can be summarized as follows:

(i) The fast DA mode increases exponentially with \( z_2 \) for fixed value of \( z_1, n_{20}, \) and \( n_{10} \) (via \( \beta \)). On the other hand, the slow DA mode linearly decreases with the increase of \( z_2 \) for the fixed value of \( z_1, n_{20}, \) and \( n_{10} \) (via \( \beta \)).

(ii) The DAWs is modulationally stable (unstable) in the range of values of \( k \) in which the ratio \( P/Q \) is \( P/Q < 0 \) (\( P/Q > 0 \)).

(iii) The value of \( k_c \) increases with the increase of \( n_{10} \) for fixed value of \( z_1 \) and \( n_{10} \) (via \( \mu_1 \) and for fast mode). On the other hand, \( k_c \) value decreases with the increase of \( m_2 \) for fixed value of \( m_1, z_2, \) and \( z_1 \) (via \( \alpha \) and for slow mode).
(iv) The value of $\Gamma$ increases with $\alpha$ for fixed value of $q$ (within three ranges of $q$, namely, $q > 1$, $q = 1$, and $q < 1$). So, the variation of $\Gamma$ with $\alpha$ is independent of possible values of $q$.

(v) The maximum value of $\Gamma$ decreases (decreases) with the increase in the values of $q$ for the limits $q > 1$ ($q < 1$). So, the growth rate is independent on the sign of the $q$.

The results of our present investigation will be useful in understanding the nonlinear phenomena both in space (viz., Jupiters magnetosphere,[2] upper mesosphere, and comets tails,[2] etc.) and laboratory (viz., direct current and radio-frequency discharges, plasma processing reactors, fusion plasma devices,[1] and solid-fuel combustion products,[1] etc.) plasma system containing $q$-distributed electrons, iso-thermal ions, negatively and positively charged massive warm dust grains in OPDP medium.

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