Nondeterministic Model Updating of Laser Spot Welded Structure via Response Surface Methodology

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Abstract. This paper presents the application of response surface method in finite element model updating, as an approach for improving the accuracy and efficiency of the finite element model of a laser spot welded structure, to reflect the physical responses of the structure. The procedures of implementation for response surface in model updating such as sampling method, selecting the significant updating parameters and constructing a quadratic polynomial response surface are discussed. Initially, the finite element model of the structure was developed using CQUAD4 shell element alongside CWELD element connectors to represent laser spot weld joints. Then, NASTRAN SOL 103 was used to calculate the dynamic behaviour of the model (natural frequencies and mode shapes). The experimental modal analysis was then conducted under free-free boundary conditions via LMS SCADAS to obtain experimental data. The minimisation of the discrepancies of the finite element model was based on objective function that was formed by the residuals between finite element and experimental natural frequencies. Results show that response surface method was efficient to be used in finite element model updating since it is capable to improve the accuracy of the finite element model.

1. Introduction
In recent years, many structural engineering problems have become more laborious to be solved due to the high demands towards complex and unique design of the structure, involving many sophisticated numerical analysis and simulation software to be used for design analysis. One of the most predominant and practical numerical analysis tools is the finite element method. However, the finite element model of a structure is normally constructed based on the engineering assumptions of the structural design whereby it may not truly represent all the aspects of an actual structure [1]–[4]. These may result in a highly disagreement between the predicted result of the finite element model and the actual structure. Discrepancies that are generated from the finite element model basically originated from the uncertainties in simplifying assumptions of the structural geometry, material properties and mechanical joints. It is often required to optimise the finite element model by adjusting the uncertain parameters that may improve the prediction results or responses of an actual structure [5], [6].

Since initial finite element models are always not in good agreement with the actual structure, finite element model updating is such a procedure that can be used to alter or modify the uncertain parameters that appear so that a more realistic model can be achieved. Theoretically, finite element model updating is an inverted problem in which the dynamic characteristic is used to identify and modify the uncertain parameters of finite element model [7]. Previous studies by Ren et al stated that setting up of an objective
function, selecting updating parameters and applying robust optimisation algorithm are the crucial steps in model updating [8]. Moreover, Mottershead et al have stated that, finite element model updating is not only emphasizing the satisfactory degree of accuracy of predicted results with actual structure, but the updated parameters must also maintain the physical meaning of the structure so that it can mimic the actual structure accurately and reliably [9].

Basically, there are two methods of model updating which are, direct based model updating and iterative based model updating [9]. Direct based model updating methods are commonly known as one step procedure as it directly updates the finite element model to reduce the discrepancies with experiment result without considering the physical meaning of the updated parameters. Meanwhile, iterative based model updating methods are involved with the identification of sensitivity of responses to the updated parameters so as to preserve the physical meaning of the structure. Many previous works had involved with the use of iterative based model updating by coupling with sensitivity analysis [10]–[14]. Boscato et al stressed that, the sensitivity based model updating required complicated constructions of sensitivity matrices because the finite element models should be tuned and recomputed iteratively during optimisation process [15]. Furthermore, for a large structure, a finite element model contains a very large degree of freedoms and since iterative method is a repeated procedure, it may cause convergence difficulty and also contribute to high computational time [16]. These issues make all the stated methods as not efficient and unpractical to be used for the complex jointed structure such as laser spot welded structure. Therefore, alternative approaches are explored to obtain more efficient method that can be used to improve the correlation of the initial finite element model to the experimental data by using response surface based model updating.

Response surface method is an approach to replace a finite element model by an approximate meta-model. Originally, meta-model is a simplified model of an actual model of finite element model and has been replaced statically to represent the input and output relation. Thus, response surface is much efficient, fast running and has low computational cost since only few yet important parameters are involved in predicting the responses [17], [18]. In the finite element model updating, once the response surface of a structure has been constructed, the process is reduced to the task of finding the smallest value on the response surface and the parameters that correspond to the smallest value are selected to update the model. The aim of this paper is to present a procedure of finite element model updating based on response surface method and to obtain better correlation between the predicted and measured results. In this work, the laser spot welded structure has been used since it contains complex joint connector and it also involves with many uncertainties.

2. Experimental Modal Analysis (EMA)

In the experimental work, the laser spot welded structure which consists of two sub-components: hat-shape plate and flat plate as shown in Figure 1 were tested. The sub-components of the structure were connected by twenty laser spot welds with nominal 5 mm diameter for each and were separated by 60 mm apart. The material used to fabricate the sub-components of the structure are cold rolled mild steel sheets with overall dimension of 560 mm length, 110 mm wide and 15 mm thickness.

The experimental modal analysis was performed to the laser spot welded structure by using impact hammer and roving accelerometers method so as to measure the dynamic characteristic of the structure such as natural frequencies and mode shapes. The frequency bandwidth of interest was set from 1 – 1000 Hz. Prior to measuring the dynamic characteristic of the structure, the structure was set up under free-free boundary conditions whereby four sets of rubber band and strings were used to hang the structure from the specially designed clamps by attaching the strings to the holes on the structure and the other ends of the strings to the clamps (Figure 2). Finally, all the data obtained were processed using LMS SCADAS data acquisition system.
Figure 1. Laser spot welded structure

Figure 2. The structure is suspended using rubber bands and strings

Figure 3. Geometry of a laser spot welded structure

3. Finite Element Modelling of Laser Spot Welded Structure
The finite element model under investigation is a laser spot welded structure that has been constructed so as to replicate component of car body-in-white such as pillar part. In the pre-processing stage of the finite element modelling, the MSC PATRAN was used to develop the finite element model. The structure was modelled using CQUAD4 elements (shell) and the model was meshed into 3 mm meshing size based on the suitability of the meshes size recorded from mesh convergence test (Figure 4). The CWELD element connectors as shown in Figure 5 then were used to represent as laser spot welds and to connect the hat-shape plate and flat shape plate sub-components as recommended from previous studies [19]–[21]. The material properties used in the finite element modelling was based on nominal properties of mild steel as shown in Table 1.

| Component               | Property            | Value | Unit  |
|-------------------------|---------------------|-------|-------|
| Hat shape plate         | Young's Modulus     | 210   | GPa   |
|                         | Poisson's Ratio     | 0.30  | Unitless |
|                         | Mass Density        | 7700  | kg/m³ |
| Flat shape plate        | Young's Modulus     | 210   | GPa   |
|                         | Poisson's Ratio     | 0.30  | Unitless |
|                         | Mass Density        | 7700  | kg/m³ |
| Laser spot weld (CWELD) | Young's Modulus     | 210   | GPa   |
|                         | Poisson's Ratio     | 0.30  | Unitless |
|                         | Mass Density        | 7700  | kg/m³ |

Table 1. Material properties of the laser spot welded structure [21]
In this research, normal mode analysis was run using MSC NASTRAN with SOL 103 solver that was used to identify the natural frequencies and mode shapes of the finite element model. The equation of motion that has been used to discretise the system to a finite element model, also known as 2nd order differential equation is given as

$$\mathbf{M}\ddot{x}(t) + \mathbf{C}\dot{x}(t) + \mathbf{K}x(t) = f(t)$$  \hspace{1cm} (1)

where \(\mathbf{M}\), \(\mathbf{C}\) and \(\mathbf{K}\) are symmetric matrices of mass, damping and stiffness. Meanwhile \(\dot{x}\), \(\ddot{x}\) and \(x\) represent the vector of accelerations, velocities and displacement respectively and \(f(t)\) is vector of external forces. It has been found that, the damping value of the structure can only be determined experimentally. In this study, the experimental frequency response function (FRF) data as shown in Figure 6 indicates that the laser spot welded structure tends to be undamped. The value of damping is less than one percent and it can be considered lightly damped [20]. Therefore, the effect of damping can be theoretically neglected in finite element modelling. As a result, for the undamped free vibration analysis, the equation (1) can be simplified as

$$\mathbf{M}\ddot{x}(t) + \mathbf{K}x(t) = 0$$  \hspace{1cm} (2)

The equation (2) can be solved by assuming the harmonic solution in the form of

$$x = \phi \sin\omega t$$  \hspace{1cm} (3)

where \(\omega\) and \(\phi\) are the mode shape and natural frequency of the system. If the differentiation of the assumed harmonic solution is performed and substituted in equation (2), the equation of motion yields and simplified to the following

$$(\mathbf{K} - \omega^2\mathbf{M})\phi = 0$$  \hspace{1cm} (4)

The natural frequencies and mode shapes of the laser spot welded structure can be predicted by solving the equation (4) using finite element commercial software such as MSC NASTRAN.
4. Response Surface Methodology for Finite Element Model Updating

Finite element model updating based on response surface methodology is an approach to acquire the global approximations of the structural response that contains objectives and constraints established from functional evaluations at various points in the design space. Basically, finite element model updating based on response surface methodology often involves a combination of experimental strategies, mathematical, probability and statistical methods and much useful for the engineers to optimising structures based on the research interest [22].

4.1. Latin hypercube sampling method

In order to develop a response surface that will serve as a meta-model for the finite element model, one of the basic process is to calculate the initial predicted response features at various points in the parameter space by performing experiment at each of the points. The values that are featured in the experiment ran across the parameter domain fit with a response surface. The term of experimentation herein refers to either physical experiments or computer experiments that will serve as sampling. Latin hypercube sampling (LHS) is a method of sampling that can be used to sample the design space by bounding the upper lower limits of each of the design variable [23]. However, this method is performed by generating random sample points and ensuring that all portions of the design space are represented in the same probability trend. This method operates by subdividing the sample space into smaller regions and evenly distributed sampling points by ensuring a good coverage of the random parameter [24].

4.2. Response surface regression

It is essential to construct a response surface form that can represent structural response accurately. The selected response surface form should be capable of attaining surfaces that meet specific smoothness requirements of an application. In the case of finite element model updating in structural dynamics, polynomials are popular forms representing a response surface because the calculations are simple and the resulting function is closed-form algebraic expression [25]. The first step in response surface methodology is to find a suitable approximation for the true functional relationship between y and the set of its independent variables of x. Usually, a low-order polynomial in some regions of the independent variable is employed. If the response is well modelled by a linear function of the independent variables, the approximating function is the first order model represented as:

\[ y = \beta_0 + \sum_{i=1}^{n} \beta_i x_i \]  

Since the real world problems are usually very complicated, linear estimation may not perform well in
providing a good representation of the objective function. If a curvature appears in the system, then quadratic model will be:

\[ y = \beta_0 + \sum_{i=1}^{n} \beta_i x_i + \sum_{i=1}^{n} \beta_{ii} x_i^2 + \sum_{i=1}^{n} \sum_{j=i+1}^{n} \beta_{ij} x_i x_j + \sum_{i=1}^{n} \sum_{j=i+1}^{n} \sum_{k=i+1}^{n} \beta_{ijk} x_i x_j x_k \]  

(6)

Equation (5) and equation (6), y is the response variable, \( \beta_0 \) is the constant term, \( \beta_i \) is the coefficient of the linear term, \( \beta_{ii} \) is the coefficient of quadratic single term, \( \beta_{ij} \) is the coefficient of the quadratic cross product term, \( \beta_{ii} \) is the coefficient of the cubic single term, \( \beta_{ij} \) is the coefficient of the cubic two cross product terms, and \( \beta_{ijk} \) is the coefficient of the cubic three cross product terms. The \( x_i, x_j, x_k \) terms represent the independent variables.

4.3. Finite element model updating

Model updating is an approach to improve the correlation of finite element model and the test structure by correcting the invalid assumptions of the model to an acceptable level of accuracy. In the structural dynamics, the structural response are often eigen-solutions related to such as natural frequencies and mode shapes. In this research, natural frequencies are employed as objective response. Therefore, the optimised objective function is formulated in terms of the residuals between analytical and measured natural frequencies and can be expressed as

\[ J = \sum_{i=1}^{n} W_i \left( \frac{\lambda_{i,\text{exp}}^\text{fe}}{\lambda_i^\text{fe}} - 1 \right)^2 \]  

(7)

where, \( \lambda_{i,\text{exp}}^\text{fe} \) is the \( i \)-th experimental eigenvalue and \( \lambda_i^\text{fe} \) is the \( i \)-th predicted eigenvalue from the finite element model and \( n \) is the number of eigenvalues involved in the updating procedure.

5. Results and Discussion

In this study, the dynamic characteristic of interest which is natural frequencies of the laser spot welded structure were successfully obtained using the finite element method and experimental modal analysis. CQUAD4 shell elements were used to model the structure, and CWELD element connectors were implemented to represent as laser spot welds. Initially, MSC NASTRAN SOL 103 was used to calculating the predicted dynamic characteristic of the structure. Meanwhile, SOL 200 was used for updating the initial finite element model using response surface method in light of the experiment results. Tables 2 shows the results of natural frequencies obtained using the above-mentioned methods.

Table 2 shows the comparison of results between the initial finite element model and experiment modal analysis of the first six vibration modes. An enormous total error of 21.38 percent was identified in the initial predicted model with 2\textsuperscript{nd} mode was contributed to the largest error. The other noticeable modes that have a high contribution to the total error were observed in the 3\textsuperscript{rd} and 6\textsuperscript{th} modes with 3.69 percent and 3.67 percent respectively. It has been found that, these relative errors occurred due to the inability of the initial finite element model to mimic the actual structure accurately. This is because, the finite element model was constructed based on engineering assumptions on geometry, material properties, and spot weld joints [26]. Therefore, some improvement on the assumptions in the initial finite element model must be done to accurately represent the structure.

Finite element model updating using response surface method was successfully applied to the initial finite element model with the minimisation of the natural frequencies in a light of measured natural frequencies as stated in the objective function. As tabulated in Table 2, a great improvement was achieved with the total error showing a huge decrement from 21.38 percent to 14.54 percent. The results in Table 2 also show that the updating has led to a reduction in the individual error of every vibration mode particularly for the 5\textsuperscript{th} mode, from 3.17 percent to 1.86 percent.
Table 2. Comparison between the measured, predicted and updated results for the structure

| Mode | I. EMA (Hz) | II. FE (Hz) | III. Error between I & II (%) | IV. RSM (Hz) | V. Error between I & IV (%) |
|------|-------------|-------------|-------------------------------|--------------|-----------------------------|
| 1    | 503.83      | 515.57      | 2.33                          | 512.32       | 1.69                         |
| 2    | 555.53      | 583.48      | 5.03                          | 575.58       | 3.61                         |
| 3    | 572.48      | 593.58      | 3.69                          | 589.36       | 2.95                         |
| 4    | 632.32      | 654.40      | 3.49                          | 645.51       | 2.09                         |
| 5    | 643.03      | 663.44      | 3.17                          | 654.99       | 1.86                         |
| 6    | 664.89      | 689.26      | 3.67                          | 680.53       | 2.35                         |
| Total Error | 21.38     |             |                               | 14.54        |                             |

6. Conclusion
In this paper, an application of the finite element model updating based on response surface methodology of the laser spot welded structure is presented. The response surface using quadratic polynomial was constructed using the finite element so as to improve the initial prediction response such as natural frequencies. On top of that, the Latin hypercube was successfully used to develop the replacement model of the initial finite element model of the structure. In the part of finite element modelling, CQUAD shell element and CWELD element connectors were implemented by assuming the material properties of mild steel to represent as hat-flat shape plate and laser spot welds respectively. After that, the initial prediction was optimising using finite element model updating based on response surface methodology. By replacing the original finite element model to the response surface form, the model updating process becomes efficient and capable to reduce the discrepancies of the initial finite element model.

Acknowledgments
The authors wish to acknowledge the Malaysia Ministry of Higher Education (MOHE), Research Management Institute (RMI) and Institute of Graduate Studies of Universiti Teknologi MARA (UiTM) for providing financial support for this research through the fundamental research grant scheme (FRGS) 600-IRMI/FRGS 5/3 (172/2019). They would also like to extend their sincere gratitude for the support and help given by Prof. Hadariah Bahron, all SDAV members and Assoc Prof Dr Hjh Siti Akmar Abu Samah.

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