We present a new Skyrme-Hartree-Fock-Bogoliubov nuclear-mass model in which the contact pairing force is constructed from microscopic pairing gaps of symmetric nuclear matter and neutron matter calculated from realistic two- and three-body forces, with medium-polarization effects included. With the pairing being treated more realistically than in any of our earlier models, the rms deviation with respect to essentially all the available mass data falls to 0.581 MeV, the best value ever found within the mean-field framework. Since our Skyrme force is also constrained by the properties of pure neutron matter this new model is particularly well-suited for application to astrophysical problems involving a neutron-rich environment, such as the elucidation of the r-process of nucleosynthesis, and the description of supernova cores and neutron-star crusts.

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With a view to their astrophysical application in neutron-rich environments, we have developed a series of nuclear-mass models based on the Hartree-Fock-Bogoliubov (HFB) method with Skyrme and contact-pairing forces, together with phenomenological Wigner terms and correction terms for the spurious collective energy. All the model parameters are fitted to essentially all the available mass data.

Model HFB-9 [1] and all later models constrained the underlying Skyrme force to fit the energy-density curve of neutron matter, as calculated by Friedman and Pandharipande [2] for realistic two- and three-nucleon forces. In the latest of our published models, HFB-16 [3], we imposed a comparable constraint on the contact pairing force. Instead of postulating a simple functional form for its density dependence, as is usually done, we constructed the pairing force by solving the HFB equations in uniform matter and requiring that the resulting gap reproduce exactly, as a function of density, the microscopic $^1S_0$ pairing gap calculated with realistic forces. In that preliminary study we assumed that the pairing strength for neutrons (protons) depended only on the neutron (proton) density, as suggested by Duguet [4], and chose for this microscopic reference gap the one calculated for pure neutron matter without medium effects [5]. We obtained thereby what was at the time our best-ever fit to the mass data, the rms deviation for our usual data set of 2149 measured masses of nuclei with $N$ and $Z \geq 8$ [6] being 0.632 MeV. On the other hand, the mass fits were much worse if we chose reference pairing gaps calculated with medium effects taken into account.

Here we show that it is possible to obtain excellent mass fits even when the pairing force is constrained to microscopically calculated gaps in which medium effects have been included. The essential step is to impose the additional constraint of asymmetric nuclear matter pairing, thereby allowing the neutron and proton pairing strengths each to depend on both the neutron and proton densities.

**The HFB-17 mass model.** With this generalization of our earlier pairing model, Eq. (3.3) of Ref. [3] for the pairing strength is replaced by

$$v_{\pi q}[\rho_n, \rho_p] = -8\pi^2 \left(\frac{\hbar^2}{2M_q^* (\rho_n, \rho_p)}\right)^{3/2} \times \left(\int_0^{\mu_q + \varepsilon_A} \frac{d\xi}{\sqrt{\xi - q^2}} \sqrt{q^2 + \Delta_q (\rho_n, \rho_p)^2}\right)^{-1}$$

where $\Delta_q (\rho_n, \rho_p)$ is the corresponding pairing gap of asymmetric nuclear matter calculated microscopically, $M_q^* (\rho_n, \rho_p)$ is the effective nucleon mass and $\varepsilon_A$ is the pairing cutoff. The chemical potential $\mu_q$ is approximated by $\mu_q = \hbar^2 k_F^2 / (2M_q^*)$, where $k_F = (3\pi^2 \rho_n)^{1/3}$ is the Fermi wave number.

For the reference microscopic gap we use the recent Brueckner calculations of Cao et al. [7], which are based on realistic two- and three-nucleon forces. All these calculations include the effect on the interaction of medium polarization, and are performed both with and without self-energy corrections. Ideally, we should have used the former gaps, which imply an effective mass $M_q^*$ different from $M$. However, for complete consistency in Eq. (1) our Skyrme force would then have been required to reproduce the same $M_q^*$. Satisfying this further constraint in addition to all the others that we have already imposed seems to be impossible within the framework of the conventional Skyrme forces used here. Thus if one wishes to retain the self-energy corrections the best that one can do is to relax this constraint on the effective mass, but we found that this option was incompatible with good mass fits. We thus take the gaps calculated without self-energy effects, and then for consistency set $M_q^* = M$ in

Skyrme-Hartree-Fock-Bogoliubov nuclear mass formulas: Crossing the 0.6 MeV threshold with microscopically deduced pairing

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Eq. (1)]. This was the choice that led to the excellent mass fits reported below.

Ref. [7] calculates pairing gaps only for symmetric nuclear matter, $\Delta_{SM}(\rho = \rho_n + \rho_p)$, and pure neutron matter, $\Delta_{NM}(\rho_n)$. Since we need the pairing gaps for arbitrary asymmetry we adopted the interpolation ansatz

$$\Delta_q(\rho_n, \rho_p) = \Delta_{SM}(\rho)(1 - |\eta|) \pm \Delta_{NM}(\rho_q) \frac{\rho_q}{\rho},$$

(2)

where $\eta = (\rho_n - \rho_p)/\rho$ and the upper (lower) sign is to be taken for $q = n(p)$; we have also assumed charge symmetry, i.e., $\Delta_n(\rho_n, \rho_p) = \Delta_p(\rho_p, \rho_n)$. This expression ensures that for symmetric nuclear matter, $\Delta_q(\rho/2, \rho/2) = \Delta_{SM}(\rho)$ and for neutron matter $\Delta_n(\rho, 0) = \Delta_{NM}(\rho)$ and $\Delta_p(\rho, 0) = 0$.

Because of Coulomb effects, and a possible charge-symmetry breaking act only on protons.

Results. The foregoing model, labeled HFB-17, was fitted to the above data set of 2149 measured nuclear masses [6]; in making this fit we followed our recently adopted strategy [10] of dropping the Coulomb-exchange term. The resulting parameter set, labeled BSk17, is given in Table I (definitions of these parameters can be found in [8]).

The deviations (data-theory) between all the 2149 measured masses of our data set and the new predictions are shown graphically in Fig. 2; no deviation exceeded 2.8 MeV. The rms and mean values of these deviations are shown in the first two lines of Table II; with an rms deviation of 0.581 MeV this is the most accurate mass model ever achieved within the mean-field framework. The next six lines of this table refer to various subsets of our data set. We stress that all the 2149 data points to which we make our fit are taken from the 2003 Atomic Mass Evaluation (AME) [6]. However, a considerable amount of mass data has accumulated in the meantime, but since these new measurements have not been subjected to the same scrutiny that went into the 2003 AME we have excluded them from our fit. Nevertheless, it is of interest to compare these new data with our model, and we do this in lines 9 to 12 of Table II for two sets of measurements, Refs. [11] and [12]. It is remarkable that our model agrees better with these new data than with the fitting data.

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We have pointed out in Ref. [14] that only slight changes in the quality of the mass fit can be accompanied by considerable changes in the overall pairing strength, as measured by the spectral pairing gaps (defined in Eq. (5) of Ref. [14]). Now throughout our entire mass-model project we have been concerned not only with nuclear
success rate of 47% on the spins and 72% on the parities. For the 90 odd nuclei we apply the Nordheim rule [15]. For the 90 odd-A and odd-odd nuclei we obtain a global success rate of 47% on the spins and 72% on the parities. We have determined the ground-state spins and parities of odd nuclei using the HFB-17 single-particle level scheme obtained in the EFA, as described above. For odd-A nuclei, the spin and parity are assumed to be those of the single nucleon of the last filled orbit, while for odd-odd nuclei we apply the Nordheim rule [15]. For the 90 spherical odd-A nuclei with quadrupole deformation parameter $\beta_2 \leq 0.05$, 91% of the experimental spins [16] are correctly predicted, while for the 717 deformed ones with $\beta_2 > 0.16$, only 41% are correctly determined. For all the 1582 odd-A and odd-odd nuclei we obtain a global success rate of 47% on the spins and 72% on the parities (we assume a spherical configuration for $\beta_2 \leq 0.16$). The most important ground state properties predicted by the HFB-17 model, including spins and parities, have been tabulated for all the 8508 nuclei with $8 \leq Z \leq 110$ between the proton and neutron drip lines.

Table III shows the macroscopic parameters (infinite and semi-infinite nuclear matter) calculated for the force BSk17 (for the definition of these parameters see, for example, Ref. [2]). This table also shows the values of these parameters for force BSk16, the force underlying mass model HFB-16, and it will be seen that in this respect there is very little difference between the two forces. This is hardly surprising given that the macroscopic parameters depend entirely on the Skyrme force, and it is in the pairing channel that we have introduced the principal modifications. (Note that the values of the isoscalar effective mass $M^*_s$ and symmetry energy $J$ were imposed.) It will be seen that in both models the isovector effective mass $M^*_v$ is found to be smaller than $M^*_s$ at the saturation density $\rho_0$, implying thereby that the neutron effective mass $M^*_n$ is larger than the proton effective mass $M^*_p$ in neutron-rich matter. Such an isovector splitting of the effective mass is consistent with measurements of isovector giant resonances [17], and has been confirmed in several many-body calculations with realistic forces [18, 19].

As seen in Fig. 3 the energy-density curve of neutron matter for force BSk17 is identical to the realistic curve of

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**TABLE II: Rms ($\sigma$) and mean ($\bar{\epsilon}$) deviations between data [6] and HFB-17 predictions.** The first pair of lines refers to all the 2149 measured masses $M$, the second pair to the masses $M_{nr}$ of the subset of 185 neutron-rich nuclei with $S_n \leq 5.0$ MeV, the third pair to the 1988 measured neutron separation energies $S_n$, and the fourth pair to 1868 measured beta-decay energies $Q_\beta$. The fifth and six pairs correspond to the deviation with respect to the recently measured masses of Ref. [11] and [12], respectively. The seventh pair shows the comparison with the 782 measured charge radii [13], and the last line shows the calculated neutron-skin thickness of $^{208}$Pb. Note that units for energy and length are MeV and fm respectively.

|                      | HFB-16 | HFB-17 |
|----------------------|--------|--------|
| $\sigma(2149 M)$ [6] | 0.632  | 0.581  |
| $\bar{\epsilon}(2149 M)$ [6] | -0.001 | -0.019 |
| $\sigma(M_{nr})$ [6]  | 0.748  | 0.729  |
| $\bar{\epsilon}(M_{nr})$ [6] | 0.161  | 0.119  |
| $\sigma(S_n)$ [6]     | 0.500  | 0.506  |
| $\bar{\epsilon}(S_n)$ [6] | -0.012 | -0.010 |
| $\sigma(Q_\beta)$ [6] | 0.559  | 0.583  |
| $\bar{\epsilon}(Q_\beta)$ [6] | 0.031  | 0.022  |
| $\sigma(434 M)$ [11]  | 0.484  | 0.363  |
| $\bar{\epsilon}(434 M)$ [11] | -0.136 | -0.092 |
| $\sigma(142 M)$ [12]  | 0.516  | 0.548  |
| $\bar{\epsilon}(142 M)$ [12] | -0.070 | 0.172  |
| $\sigma(R_c)$ [13]    | 0.0313 | 0.0300 |
| $\bar{\epsilon}(R_c)$ [13] | -0.0149 | -0.0114 |
| $\theta(208$Pb)       | 0.15   | 0.15   |

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**TABLE III: Macroscopic parameters for forces BSk16 and BSk17.** The first twelve lines refer to infinite nuclear matter, the last two to semi-infinite nuclear matter. Note that units for energy and length are MeV and fm respectively.

|                      | BSk16 | BSk17 |
|----------------------|-------|-------|
| $a_v$                | -16.053 | -16.054 |
| $\rho_0$             | 0.1586 | 0.1586 |
| $J$                  | 30.0  | 30.0  |
| $M^*_p/M$            | 0.80  | 0.80  |
| $M^*_v/M$            | 0.78  | 0.78  |
| $K_v$                | 241.6 | 241.7 |
| $L$                  | 34.87 | 36.28 |
| $G_0$                | -0.65 | -0.69 |
| $G'_0$               | 0.51  | 0.50  |
| $G_1$                | 1.52  | 1.55  |
| $G'_1$               | 0.44  | 0.45  |
| $\rho_{\text{frm}}/\rho_0$ | 1.24   | 1.24   |
| $a_{sf}$             | 17.8  | 17.9  |
| $Q$                  | 39.0  | 38.1  |

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**FIG. 3: (Color online) Energy per neutron (MeV) as a function of density (fm$^{-3}$) of neutron matter for BSk17 and for the calculations of Ref. [2] (FP) and A18+δv+UIX$^*$ of Akmal et al. [20].**
Ref. [2] up to the supernuclear density of 0.4 fm$^{-3}$, as is the case with all our other forces that have been fitted to $J = 30$ MeV. It is to be noted that unlike Ref. [17] we have not had to resort to a second $t_3$ term in the Skyrme force in order to simultaneously fit neutron matter and obtain the correct sign for the isovector splitting of the effective mass. Fig. 3 also shows the energy-density curve given by the realistic calculation $A_{18} + \delta v + \text{UIX}^*$ of Akmal et al. [20]. There is more physics in this calculation than in the one of FP, but we are unable to fit this curve without degrading the quality of the mass fit. However, there have been some very recent indications that this curve might be too steep [21].

Fig. 4 shows the potential energy per particle in each of the four two-body spin-isospin ($S, T$) channels as a function of density for symmetric nuclear matter; we give results for both BSk17 and Brueckner-Hartree-Fock (BHF) calculations with realistic two- and three-nucleon forces [22]. A fair agreement between BSk17 and the realistic calculations in all states can be seen; note particularly that the deviation in the (1,1) channel is much less marked than in Ref. [3], mainly because there we compared with older BHF calculations.

Conclusions. We have described a new Skyrme-HFB nuclear-mass model, HFB-17, in which the contact pairing force is constructed from microscopic pairing gaps of symmetric nuclear matter and neutron matter calculated from realistic two- and three-body forces, with medium-polarization effects included. In this way the rms deviation with respect to essentially all the available mass data has been reduced, for the first time with a mean-field model, below 0.6 MeV. Given also the constraint imposed on the Skyrme force by microscopic calculations of neutron-matter, this new model is particularly well adapted to astrophysical applications involving a neutron-rich environment, such as the elucidation of the r-process of nucleosynthesis, and the description of supernova cores and neutron-star crusts.

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