VIOLATING THE BILOCAL INEQUALITY WITH SEPARABLE MIXED STATES IN THE ENTANGLEMENT-SWAPPING NETWORK

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Abstract. It has been showed that two entangle pure states can violate the bilocal inequality in the entanglement-swapping network, vice versa. What happens for mixed states? Whether or not are there separable mixed states violating the bilocal inequality? In the work, we devote to finding the mixed Werner states which violate the bilocal inequality by Particle Swarm Optimization (PSO) algorithms. Finally, we shows that there are pairs of states, where one is separable and the other is entangled, can violate the bilocal inequality.

1. Introduction and preliminaries

Quantum nonlocality, detected by violation of Bell inequalities, is a significant property of quantum mechanics [1]. Nowadays, it has become a key tool in the modern development of quantum information and its applications cover a variety of areas [2]: quantum cryptography [3]; complexity theory [4]; communication complexity [5]; Hilbert space dimension estimates [6].

Recently, generalizations of Bell’s theorem for more complex causal structures have attracted growing attention [7, 8, 9, 10, 11, 12, 13, 14, 15]. Some researchers have studied the characteristics of classical, quantum and post-quantum correlations in networks by constructing the network Bell inequalities and exploring their quantum violations [16, 17, 18, 19, 20, 21]. In this paper, we focus on the simplest nontrivial network Bell experiment, known as the bilocality scenario, with only three observers and two sources. It features two independent sources that each produce a pair of particles. The first pair $S_1$ is shared between observers Alice and Bob while the second pair $S_2$ is shared between Bob and another observer, Charles (see Fig.1). Consider that Alice receives measurement setting (or input) $x$, while Bob performs measurement $y$, and Charles $z$. After measuring the three parts, they obtain outcomes denoted $a$, $b$ and $c$, respectively. Bob’s measurement $y$ might correspond to a joint measurement on the two systems that he receives from each source. The correlations between the measurement outcomes of the three parties are described by the joint probability distribution $p(a, b, c|x, y, z)$.

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In our scenario, two independent sources distribute mixed states, $\rho_{AB}$ and $\rho_{BC}$, between three distant observers, Alice, Bob and Charles. The tripartite joint probability distribution $p(a, b, c|x, y, z)$ is 2-local if it can be written as

$$p(a, b, c|x, y, z) = \int \int d\lambda_1 d\lambda_2 q_1(\lambda_1) q_2(\lambda_2) p(a|x, \lambda_1) p(b|y, \lambda_1, \lambda_2) p(c|z, \lambda_2),$$

where $\lambda_1$ and $\lambda_2$ are the independent shared random variables distributed according to the densities $q_1(\lambda_1)$ and $q_2(\lambda_2)$, respectively.

It has been documented that the set of Bell-local (or 1-local) correlations can be fully characterized by linear Bell inequalities [22]. So how do we represent bilocality correlation sets? Fortunately, the nonlinear inequality in Ref [8] can be used to efficiently capture bilocality correlations. This nonlinear inequality is called bilocality inequality, which is satisfied by bilocality correlations but can be violated by non-bilocal correlations. Consider that Alice and Charles receive binary inputs, $x = 0, 1$ and $z = 0, 1$ and give binary outputs, denoted $a_x = \pm 1$ and $c_z = \pm 1$, respectively. The middle party Bob always performs joint measurement, denote Bob’s outcome by two bits $b_0 = \pm 1$ and $b_1 = \pm 1$. Then, the bilocality inequality can be written as:

$$S \equiv \sqrt{|I|} + \sqrt{|J|} \leq 2,$$

where

$$I \equiv \sum_{x,z} \langle a_x b_0 c_z \rangle = \langle (a_0 + a_1) b_0 (c_0 + c_1) \rangle,$$

$$J \equiv \sum_{x,z} (-1)^{x+z} \langle a_x b_1 c_z \rangle = \langle (a_0 - a_1) b_1 (c_0 - c_1) \rangle.$$

The bracket $\langle \cdot \rangle$ denotes the expectation value of many experimental runs.

Gisin et al [23] showed that pairs of pure states can violate bilocality inequality if and only if the two pure states are entangled. Naturally, we ask the question: what kind of mixed states can violate the bilocality inequality? Whether or not are there separable mixed states violating the bilocal inequality? In the work, we devote to finding the mixed Werner states which violate the bilocal inequality by Particle Swarm
Optimization (PSO) algorithms. Finally, we shows that there are pairs of states, where one is separable and the other is entangled, can violate the bilocal inequality.

2. Arbitrary pairs of Werner state

Let us first consider that Bob performs measurement $b_y$ has trace zero. That is, $b_0 = \sum_{ij} m_{ij} \sigma_i \otimes \sigma_j$, $b_1 = \sum_{ij} n_{ij} \sigma_i \otimes \sigma_j$ (with $i, j \in \{1, 2, 3\}$) and eigenvalues of $b_{0(1)}$ have to lie in $[-1, 1]$. $\sigma_i$ denotes the Pauli matrices. Then, $a_i$ and $c_i$, $i = 0, 1$, are Hermitian operators with eigenvalues $\in [-1, 1]$. In particular, they can be expressed as: $a_i = x_i \sigma_1 + x_{i2} \sigma_2 + x_{i3} \sigma_3$ and $c_i = y_i \sigma_1 + y_{i2} \sigma_2 + y_{i3} \sigma_3$, where $\overrightarrow{x_i} = (x_{i1}, x_{i2}, x_{i3})$ and $\overrightarrow{y_i} = (y_{i1}, y_{i2}, y_{i3})$ are four unit-length vectors.

Every bipartite state involving two qubits can be written in the form

\[
\rho = \frac{1}{4}(I \otimes I + \mathbf{r} \cdot \vec{\sigma} \otimes I + I \otimes \mathbf{s} \cdot \vec{\sigma} + \sum_{m,n=1}^{3} t_{mn}^{AB} \sigma_m \otimes \sigma_n)
\]

(2.1)

where, in the notation of [24], $I$ is the $2 \times 2$ identity operator, $\{\sigma_i\}_{i=1}^{3}$ are Pauli matrices, $\mathbf{r}$ and $\mathbf{s}$ are Bloch vectors in $\mathbb{R}^3$, and $\mathbf{r} \cdot \vec{\sigma} = \sum_{i=1}^{3} r_i \sigma_i$, $t_{mn} = \text{Tr}(\rho \sigma_m \otimes \sigma_n)$ forms a matrix denoted $T_{\rho}$.

Let

\[
\rho_{AB} = \frac{1}{4}(I \otimes I + \overrightarrow{m_A} \cdot \vec{\sigma} \otimes I + I \otimes \overrightarrow{m_B} \cdot \vec{\sigma} + \sum_{i,j=1}^{3} t_{ij}^{AB} \sigma_i \otimes \sigma_j)
\]

(2.2) be the state shared by Alice and Bob. Similarly we express $\rho_{BC}$, the state shared by Bob and Charles, which can be written in the form $\rho_{BC} = \frac{1}{4}(I \otimes I + \overrightarrow{m_B} \cdot \vec{\sigma} \otimes I + I \otimes \overrightarrow{m_C} \cdot \vec{\sigma} + \sum_{i,j=1}^{3} s_{ij}^{BC} \sigma_i \otimes \sigma_j)$.

Substituting in Eq. (1.2) and Eq. (1.3) we obtain

\[
I = \sum_{k=1}^{3} (x_{ok} + x_{1k}) \sum_{i=1}^{3} t_{ki}^{AB} \left(\sum_{j=1}^{3} m_{ij} \right) \sum_{l=1}^{3} s_{kl}^{BC} (y_{0l} + y_{1l})
\]

(2.3)

and

\[
J = \sum_{k=1}^{3} (x_{ok} - x_{1k}) \sum_{i=1}^{3} t_{ki}^{AB} \left(\sum_{j=1}^{3} n_{ij} \right) \sum_{l=1}^{3} s_{kl}^{BC} (y_{0l} - y_{1l}).
\]

(2.4)

We can see that from Eq. (2.3) and Eq. (2.4) the value of $S$ depends largely on $t_{ij}^{AB}, s_{ij}^{BC}, \overrightarrow{x_i}, \overrightarrow{y_i}$ and $m_{ij}, n_{ij}$. We seek to find the maximum value of the bilocality inequality (1.1) for arbitrary pairs of mixed states using particle swarm optimization (PSO). According to PSO, we can find $m_{ij}, n_{ij}$ such that $S^{\text{max}} > 2$ when $\rho_{AB}$ and $\rho_{BC}$ both are entangled states. This means exists two different entangled states violation of the bilocality inequality. Now let’s think about an entangled state $\rho_{AB}$ distributes to Alice and Bob, and another separable state $\rho_{BC}$ distributes to Bob and
Charles. In this case, can we get two states that violate the standard bilocality inequality \[ \text{(1.1)} \]?

In particular, consider the case where Alice-Bob, as well as Bob-Charles, share a noisy Bell state, a so-called Werner state, the best-known class of mixed states. For qubits, the Werner state is given by

\[
\rho = p \phi^+ | \phi^+ \rangle + (1 - p) I/4,
\]

where \( | \phi^+ \rangle = \frac{1}{\sqrt{2}} (| 00 \rangle + | 11 \rangle) \) is the singlet state, \( I \) is the \( 4 \times 4 \) identity matrix and \( p \in [0, 1] \). The Werner state is entangled if and only if \( p > 1/3 \). The Werner state is pure only when \( p = 1 \).

We consider a specific quantum implementation of the bilocality experiment illustrated in Fig.1. The both sources emit pairs of qubits corresponding to Werner states. \( \rho_{AB} = p | \phi^+ \rangle \langle \phi^+ | + (1 - p) I/4 \) and \( \rho_{BC} = q | \phi^+ \rangle \langle \phi^+ | + (1 - q) I/4 \) (with \( p, q \in [0, 1] \)).

Let’s transform Werner state into something similar to Eq.\[ \text{(2.1)} \]. We can get that

\[
\rho_{AB} = 1/4 (I \otimes I + p \sigma_1 \otimes \sigma_1 - p \sigma_2 \otimes \sigma_2 + p \sigma_3 \otimes \sigma_3).
\]

Similarly, we express \( \rho_{BC} \).

We can rewrite \( I \) and \( J \), substituting in Eq.\[ \text{(2.3)} \] and Eq.\[ \text{(2.4)} \], we obtain

\[
I = \left[ (x_{01} + x_{11}) p \left( \sum_{i=1}^{3} m_{1i} \right) - (x_{02} + x_{12}) p \left( \sum_{i=1}^{3} m_{2i} \right) + (x_{03} + x_{13}) p \left( \sum_{i=1}^{3} m_{3i} \right) \right]
\times \left[ q (y_{01} + y_{11}) - q (y_{02} + y_{12}) + q (y_{03} + y_{13}) \right]
\]
\[
= pq \left[ (x_{01} + x_{11}) \left( \sum_{i=1}^{3} m_{1i} \right) - (x_{02} + x_{12}) \left( \sum_{i=1}^{3} m_{2i} \right) + (x_{03} + x_{13}) \left( \sum_{i=1}^{3} m_{3i} \right) \right]
\times \left[ (y_{01} + y_{11}) - (y_{02} + y_{12}) + (y_{03} + y_{13}) \right].
\]

\[ \text{(2.5)} \]

\( I \) can be expressed as \( I = pq I' \), where

\[
I' = \left[ (x_{01} + x_{11}) \left( \sum_{i=1}^{3} m_{1i} \right) - (x_{02} + x_{12}) \left( \sum_{i=1}^{3} m_{2i} \right) + (x_{03} + x_{13}) \left( \sum_{i=1}^{3} m_{3i} \right) \right]
\times \left[ (y_{01} + y_{11}) - (y_{02} + y_{12}) + (y_{03} + y_{13}) \right].
\]

\[ \text{(2.6)} \]

Similarly, we express \( J = pq J' \), where

\[
J' = \left[ (x_{01} - x_{11}) \left( \sum_{i=1}^{3} n_{1i} \right) - (x_{02} - x_{12}) \left( \sum_{i=1}^{3} n_{2i} \right) + (x_{03} - x_{13}) \left( \sum_{i=1}^{3} n_{3i} \right) \right]
\times \left[ (y_{01} - y_{11}) - (y_{02} - y_{12}) + (y_{03} - y_{13}) \right].
\]

\[ \text{(2.7)} \]

So we can get

\[
S = \sqrt{pq} (\sqrt{|I'|} + \sqrt{|J'|}).
\]

\[ \text{(2.8)} \]
Denote $S' = \sqrt{I} + \sqrt{J}$, then our goal is to maximize $S'$ with the Bloch vector $\vec{x}_0, \vec{x}_1, \vec{y}_0, \vec{y}_1$, and $n_{ij}, m_{ij}$ (with $i, j \in \{1, 2, 3\}$). Next we use particle swarm optimization (PSO) algorithm as a approach to calculate $S_{\text{max}}$.

3. Experimental scheme and results

Particle swarm optimization (PSO) algorithms are outstandingly successful for non-convex optimization. PSO is a 'collective intelligence' strategy from the field of machine learning that learns via trial-and-error and performs as well as or better than simulated annealing and genetic algorithms. We have shown that PSO also delivers an autonomous approach to design an optimal measurement strategy. Here the optimal measurement means that the measurement can measure more quantum states with violation of the bilocality inequality.

The first step of the protocol is the initialization of a population: a set of lists \{\vec{x}_0, \vec{x}_1, \vec{y}_0, \vec{y}_1, m_{ij}, n_{ij}\} of measurement, $\forall i, j \in \{1, 2, 3\}$, corresponding the algorithm chromosomes, is randomly generated. The fitness is given by $S'$.

To search for $\rho_{\text{opt}}$, the PSO algorithm models a 'swarm' of $\sharp$ 'particles' $p^{(1)}, p^{(2)}, \ldots, p^{(\sharp)}$ that move in the search space $\mathcal{P}_N$. In this paper, $\sharp$ take values 30. A particle's position $\rho^{(i)} \in \mathcal{P}_N$ represents a candidate policy for measurement $\varphi$, which is initially chosen at random. Furthermore, $p^{(i)}$ remembers the best position, $\hat{\rho}^{(i)}$, it has visited so far (including its current position). In addition, $p^{(i)}$ communicates with other particles in its neighborhood $\mathcal{N}^{(i)} \subseteq \{1, 2, \ldots, \#\}$. We adopt the common approach to set each $\mathcal{N}^{(i)}$ in a pre-defined way regardless of the particles' positions by arranging them in a ring topology: for $p^{(i)}$, all particles with maximum distance $r$ on the ring are in $\mathcal{N}^{(i)}$. In iteration $t$, the PSO algorithm updates the position of all particles in a round-based manner as follows.

(i) Each particle $p^{(i)}$ samples $\hat{S}'(\rho^{(i)})$ of its current position with $K$ trial runs.

(ii) $p^{(i)}$ re-samples $\tilde{S}'(\hat{\rho}^{(i)})$ of its personal-best policy $\hat{\rho}^{(i)}$, and the performance of $\hat{\rho}^{(i)}$ is taken to be the arithmetic mean $\hat{S}'(\hat{\rho}^{(i)})$ of all sharpness evaluations.

(iii) Each $p^{(i)}$ update $\hat{\rho}^{(i)}$ if $\hat{S}'(\hat{\rho}^{(i)}) > S'(\hat{\rho}^{(i)})$ and

(iv) communicates $\hat{\rho}^{(i)}$ and $\tilde{S}'(\hat{\rho}^{(i)})$ to all members of $\mathcal{N}^{(i)}$.

(v) Each particle $p^{(i)}$ determines the sharpest policy $\Lambda^{(i)} = \max_{j \in \mathcal{N}^{(i)}} \hat{\rho}^{(i)}$ found so far by any one particle in $\mathcal{N}^{(i)}$ (including itself) and

(vi) moves to

\begin{equation}
(3.1) \quad \rho^{(i)} \leftarrow \rho^{(i)} + \omega \delta^{(i)}, \quad \delta^{(i)} \leftarrow \delta^{(i)} + \beta_1 \xi_1 (\hat{\rho}^{(i)} - \rho^{(i)}) + \beta_2 \xi_2 (\Lambda^{(i)} - \rho^{(i)}).
\end{equation}

The arrows indicate that the right value is assigned to the left variable. The damping factor $\omega$ assists convergence, and $\xi_1, \xi_2$ are uniformly-distributed random numbers from
the interval $[0, 1]$ that are re-generated each time Eq (3.1) is evaluated. The 'exploitation weight' $\beta_1$ parametrizes the attraction of a particle to its personal best position $\hat{\rho}^{(i)}$, and the 'exploration weight' $\beta_2$ describes attraction to the best position $\Lambda^{(i)}$ in the neighborhood. To improve convergence, we bound each component of $\omega \delta^{(i)}$ by a maximum value of $\nu_{\text{max}}$. The userspecified parameters $\omega, \beta_1, \beta_2$ and $\nu_{\text{max}}$ determine the swarm’s behavior. Tests indicate that $\omega = 0.8$, $\beta_1 = 0.5$, $\beta_2 = 0.5$, and $\nu_{\text{max}} = 0.2$ result in the highest probability to find an optimal policy.

After 500 iterations using PSO algorithm, we get $S_{\text{max}} = 4.0642$. 

![Fig. 2. Convergence of the $S_{\text{max}}$](image)

At this time, the corresponding measurements of Alice, Charles and Bob can be expressed as:

\[ \vec{x}_0 = (-0.9122, -0.1869, 0.3647), \]
\[ \vec{x}_1 = (0.3321, -0.8910, 0.3095), \]
\[ \vec{y}_0 = (-0.7915, -0.3305, 0.5140), \]
\[ \vec{y}_1 = (-0.5737, 0.5731, -0.5851), \]

\[ M = (m_{ij}) = \begin{pmatrix} -0.1258 & -0.1882 & -0.2448 \\ 0.3078 & 0.4614 & 0.5996 \\ 0.1740 & 0.2606 & 0.3390 \end{pmatrix}, \]
\[ N = (n_{ij}) = \begin{pmatrix} -0.4062 & -0.5048 & -0.5051 \\ -0.2797 & -0.3474 & -0.3476 \\ -0.0049 & -0.0060 & -0.0060 \end{pmatrix}. \]
In order to $S^{\text{max}} > 2$, we consider the following expression:

\begin{equation}
\sqrt{pq}(\sqrt{|I'|} + \sqrt{|J'|}) > 2.
\end{equation}

We thus get $pq \in [(\frac{2}{1.0642})^2, 1] = [0.2422, 1]$. This shows that we get a violation of our inequality (1.1) for $pq \in [0.2422, 1]$. The most interesting case is when $p = \frac{3.2}{4.1294}$ and $q = \frac{1}{3.1}$, as satisfied by the scope of the $pq$. In this case, $\rho_{AB}$ is an entangled state, $\rho_{BC}$ is a separable state and they are mixed states.

The result of this paper is to select the appropriate measurement to find the mixed Werner states which violate the bilocal inequality. Specifically, entangled state and separable state can violate the bilocality inequality (1.1) if $p \in (\frac{1}{3}, 1]$ and $q \in [0, \frac{1}{3}]$. Similarly, when $\rho_{BC}$ is an entangled state and $\rho_{AB}$ is a separable state, we can get $p$ and $q$, such that they violate the bilocality inequality. An evaluation of the relevant semidefinite program guarantees that a violation of bilocality is obtained whenever $p = q \geq 83\%$ \cite{29}. However, we provide a better scope of $pq$ and find particular conclusion.

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