Torus Formation in Neutron Star Mergers and Well-Localized Short Gamma-Ray Bursts

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ABSTRACT

Merging neutron stars (NSs) are hot candidates for the still enigmatic sources of short gamma-ray bursts (GRBs). If the central engines of the huge energy release are accreting relic black holes (BHs) of such mergers, it is important to understand how the properties of the BH-torus systems, in particular disc masses and mass and rotation rate of the compact remnant, are linked to the characterizing parameters of the NS binaries. For this purpose we present relativistic smoothed particle hydrodynamics simulations with conformally flat approximation of the Einstein field equations and a physical, non-zero temperature equation of state. Thick disc formation is highlighted as a dynamical process caused by angular momentum transfer through tidal torques during the merging process of asymmetric systems or in the rapidly spinning triaxial post-merger object. Our simulations support the possibility that the first well-localized short and hard GRBs 050509b, 050709, 050724, 050813 have originated from NS merger events and are powered by neutrino-antineutrino annihilation around a relic BH-torus system. Using model parameters based on this assumption, we show that the measured GRB energies and durations lead to estimates for the accreted masses and BH mass accretion rates which are compatible with theoretical expectations. In particular, the low energy output and short duration of GRB 050509b set a very strict upper limit of less than 100 ms for the time interval after the merging until the merger remnant has collapsed to a BH, leaving an accretion torus with a small mass of only $\sim 0.01 M_\odot$. This favors a (nearly) symmetric NS+NS binary with a typical mass as progenitor system.

Key words: stars: neutron; gamma-rays: bursts; hydrodynamics; relativity; equation of state

1 INTRODUCTION

Merger events of NS+NS or NS+BH binaries do not only belong to the strongest known sources of gravitational wave (GW) radiation, they are also widely favored as origin of the class of short, hard GRBs (Blinnikov et al. 1984; Paczynski 1986; Eichler et al. 1989; Paczynski 1991; Narayan et al. 1992). The recent first good localizations of short bursts, GRB 050509b, 050709, 050724, 050813, by the Swift and Hete satellites at redshifts between 0.160 and 0.722 (see Fox et al. 2003; Gehrels et al. 2003; Villasenor et al. 2003; and references therein) were interpreted as a possible confirmation of this hypothesis (Fox et al. 2003; Bloom et al. 2003; Horth et al. 2005; Lee et al. 2003), because the bursts have observational characteristics which are different from those of long GRBs, but which are in agreement with expectations for compact object mergers.

The central engines of such bursts are still poorly understood and observationally undetermined. But it seems unlikely that the energies required for typical short GRBs are set free during the dynamical phase of the merging of two NSs (Ruffert et al. 1994). The production of GRBs by the neutrino emitting, hot post-merger NS is also disfavored, because the high mass loss rates in a neutrino-driven wind, which is caused by neutrino energy deposition near the NS surface, rules out the production of high Lorentz factor outflow (Woosley & Baron 1992; Woosley 1993a). Instead, the long-time accretion of a BH formed from a transiently stable, supramassive or hypermassive merger remnant (for definitions of these terms, see Morrison et al. 2004) is a much more promising source (e.g., Woosley 1993a, Ruffert & Janka 1999; Popham et al. 1999; Rosswog et al. 2003; Lee et al. 2003), provided the BH is surrounded by a sufficiently massive accretion torus. Due to the geometry of the BH-torus system with relatively baryon-poor regions along the rotation axis, thermal energy release preferentially above the poles of the BH by the annihilation of neutrino-antineutrino ($\nu \bar{\nu}$) pairs (Jaroszynski 1993; Mochkovitch et al. 1993) can...
lead to collimated, highly relativistic jets of baryonic matter with properties in agreement with those needed to explain short GRBs (Aloy et al. 2005). Ultrarelativistic jets were found to develop if the rate of thermal energy deposition per unit solid angle is sufficiently large. Alternatively or in addition, magneto-hydrodynamic (MHD) field amplification in the rapidly spinning disc could drive polar jets, or magnetic coupling between the rotating BH and the girding accretion torus could tap the rotational energy of the BH (Blandford & Znajek 1977), and could help powering MHD-driven outflow (e.g., Brown et al. 2000; Drenkhahn & Spruit 2002).

Torus mass as well as BH mass and rotation are thus crucial parameters that determine the energy release from the merger remnant on the secular timescale of the viscous evolution of the BH-torus system. In this letter we present a first set of results from new three-dimensional (3D), relativistic smoothed particle hydrodynamics simulations of NS mergers with a physical equation of state (EoS), which aim at establishing the link between the remnant properties and those of the binary systems. Previous such attempts (see Baumgarte & Shapiro 2003 for a review) were either performed in the Newtonian limit (e.g., Ruffert & Janka 2001 and references therein; Rosewog et al. 2004), or with polytropic or otherwise radically simplified treatments of the high- and low-density EoSs and their temperature-dependent behaviour (e.g., Shibata et al. 2003; Shibata et al. 2003), or they considered disc formation by viscous or magnetic effects during the secular evolution of hypermassive NSs (Duez et al. 2004; Duez et al. 2005). Here we instead highlight torus formation in NS mergers as a dynamical process and argue that this is consistent with the recent observations of short GRBs.

2 NUMERICAL METHODS AND MODELS

We employ an improved version of our relativistic smoothed particle hydrodynamics (SPH) code (Oechslin et al. 2002; Oechslin et al. 2004), which solves the relativistic hydrodynamics equations together with the Einstein field equation in the conformally flat approximation (Isenberg & Nester 1984; Wilson et al. 1994). The code now allows for the use of a tabulated, non-zero temperature EoS and solves the energy equation in a form without explicit time derivatives of the metric elements on the RHS (c.f. Oechslin et al. 2002, eqn. A8). The properties of the stellar plasma are described by the non-zero temperature EoS of Shen et al. (1998a, b), which is used with baryonic density, internal energy and electron fraction (proton-to-baryon ratio) \( Y_e \) as input, and pressure and temperature as output. Since the backreaction of the neutrino emission on the stellar fluid is small on the timescales considered here, we ignore neutrinos in our models. As a consequence, the electron fraction \( Y_e \) needed by the EoS as an input remains constant on the Lagrangian SPH particles. Details on the numerical scheme will be presented in Oechslin et al. (2004).

We start our simulations shortly before the tidal instability sets in and follow the evolution with typically 400,000 SPH particles through merging and torus formation until either the collapse of the merger remnant sets in, or a quasi-stationary state has formed.

Initial conditions for our simulations are generated by placing two NSs in hydrostatic equilibrium on a circular orbit of a given orbital distance and by relaxing the configuration with the use of a small damping force into a circular orbital motion with an irrotational spin state. The orbital velocity is adjusted during this process in order to obtain the binary in orbital equilibrium. The initial electron fraction \( Y_e \) is obtained by requiring neutrinoless \( \beta \)-equilibrium for cold neutron star matter, i.e., we determine \( Y_e \) from the condition that the (electron) neutrino chemical potential \( \mu_{\nu_e} \) is equal to zero for a given density (and initial temperature \( T \approx 0 \)).

We consider a variety of models with different NS masses and mass ratios. Other unknown parameters like the NS spins and the EoS are kept fixed except for two models where thermal effects in the EoS are neglected. This is realized by reducing the EoS to the two-dimensional slice at \( T = 0 \) with density and \( Y_e \) as input and pressure and internal energy as output. This reduction has no influence as long as shocks are absent, i.e. as long as the fluid evolves adiabatically. In the presence of shocks, however, the \( T = 0 \) case corresponds to the extreme situation that a very efficient cooling mechanism extracts immediately the entropy and internal energy generated in shocks.

In Table 1 the key parameters characterizing our different models are summarized. We have investigated NS+NS binaries with mass ratios \( q \) between 0.55 and unity, roughly covering the theoretical range obtained by stellar population synthesis calculations (Bulik et al. 2003). Predictions of the mass distribution, however, are sensitive to parameters that govern single and binary star evolution and still contain significant uncertainties. A comparison with observations of galactic double NS binaries, which currently span a range of \( q \)-values between 0.67 and nearly unity (see the recent reviews by Lattimer & Prakash 2004 and Stairs 2004), is therefore difficult and is also hampered by the still small number of detected NS+NS systems.

In the following analysis and discussion of our models, we have synchronized the time axis to the time of maximal gravitational wave luminosity. This allows for a better comparison of the temporal evolution.

3 RESULTS OF MERGER MODELS

3.1 Dynamics of Merging

The evolution of a NS binary during the adiabatic inspiral is driven by gravitational radiation reaction. Once the binary becomes unstable to tidal forces, the hydrodynamic evolution sets in, leading to the tidal stretching of one or of both companions and to the formation of a central hypermassive NS surrounded by a thick, neutron-rich accretion torus. Depending on its mass and its rotation rate and the EoS, the hypermassive NS will collapse immediately or after a (short?) delay to a rapidly rotating BH (e.g. Morrison et al. 2004). The dynamics and post-merger structure largely depend on the mass ratio \( q \) of the binary. In asymmetric systems with \( q \) significantly smaller than 1, the less massive but slightly larger star is tidally disrupted and deformed into an elongated primary spiral arm, which is mostly accreted onto
the more massive companion. Its tail, however, contributes a major fraction to the subsequently forming thick disc/torus around a highly deformed and oscillating central remnant (see Fig. 6, panels (a) and (b)). In systems with $q \geq 1$ and nearly equally sized stars, both stars are tidally stretched but not disrupted and directly plunge together into a deformed merger remnant (see Fig. 6). In all models, excited radial and non-radial oscillations of the compact remnant periodically lead to mass shedding off the surface and to ejection of material into secondary tidal tails during the subsequent evolution. As illustrated in Figs. 6 and 7, this effect takes place both in symmetric and asymmetric cases, but is stronger in the latter because of the larger triaxial deformation of the post-merger object.

### 3.2 Disc Formation

If and when the compact post-merger object collapses to a BH (the secular evolution driven by GW radiation reaction, viscosity and MHD cannot be followed by 3D simulations), pressure support from the central remnant to the disc will vanish and matter will be prevented from infall mostly by rotational support. As a consequence, we distinguish future disc matter from the central remnant by demanding the specific angular momentum $j$ to be larger than the one associated with the innermost stable circular orbit (ISCO) of a Kerr BH with the gravitational mass and the spin parameter of the central remnant.

The ISCO can be analytically determined in the case of the Boyer-Lindquist Kerr metric [Bardeen et al. 1972]. In our case we shall use an approximate pseudo-Kerr metric (see, e.g., Grandclément et al. 2002) which is both isotropic and conformally flat, consistent with the coordinates we used for our numerical simulations. The ISCO can then be found among all circular orbits in the orbital plane by minimizing the specific angular momentum along the radial coordinate.

Using this criterion, we are now able to identify the disc matter and disc mass for the various models considered. Their very different merger dynamics is reflected in the different evolution of the disc mass (see Fig. 7). The rapid rise at $t \approx 0$, which depends strongly on the mass ratio. On the other hand, the further evolution after merging is similar in all cases.

![Figure 1. Disc formation in models with non-zero temperature EoS and $M_{\text{sum}} = 2.8 M_\odot$. Plotted is the disc mass as inferred from our disc mass criterion (see text). The final disc mass as given in Table 1 is read off at about 6 ms. Clearly visible is the rapid rise at $t \approx 0$, which depends strongly on the mass ratio. On the other hand, the further evolution after merging is similar in all cases.](image-url)
for all models in Fig. 2. We find a rapid rise of the disc mass with increasing $q$ values below unity, and a flattening for $q$-values below $\sim 0.8$. The disc mass increases from about $0.05 M_\odot$ at $q = 1$ to about $0.26 M_\odot$ at $q = 0.55$.

We checked the consistency of our disc mass determination by comparing the obtained masses with the amount of matter residing outside the equatorial radius of the remnant $r_{\text{rem}}$. We determine $r_{\text{rem}}$ in a radial density profile by locating the point between the steep density gradient at the remnant surface and the slower fall-off in the disc. In all cases, we found $M_{\text{gas}}(r > r_{\text{rem}}) > M_{\text{gas}}(j > j_{\text{ISCO}}) \equiv M_{\text{disc}}$, suggesting that gas pressure still plays an important role in the quasi-stationary phase. Our disc masses are therefore lower limits, since pressure support will not completely vanish even after BH formation.

In Fig. 2 we have color-coded for $T = 0$ and a $T \neq 0$ model in red the material that belongs to the future disc, whereas the material that currently fulfills the disc criterion is shown in yellow. By definition, these two attributes coincide at the end of the simulation. We see that the future disc matter originates mainly from the spiral arm tips and from the stellar surface. This indicates that the formation of a post-merger disc is linked to the presence of spiral arms. Indeed, the yellow tips of these tidal tails in Fig. 2 suggest angular momentum transfer from the central remnant to the spiral arm tips via tidal torques. This hypothesis is confirmed by a postprocessing analysis of Model C1216 where we integrated the torque exerted by gravitational and pressure forces from the compact deformed remnant on the yellow mass elements in the primary spiral arm as a function of time. The result for the total angular momentum evolution agrees very well with the simulation result (Fig. 4).

An interesting effect can also be seen by comparing models with $T = 0$ and $T \neq 0$ EoSs (Fig. 3). In Fig. 3 we show the disc mass evolution of the cold Models C1216 and C1315 together with their non-zero temperature counterparts S1216 and S1315. Obviously, the first increase in disc mass, which can be associated with the cold primary spiral arm, is unaffected by temperature effects. Thermal pressure effects during the following evolution, however, reduce the further increase in disc mass in the non-zero temperature case, whereas in the cold case the disc mass increases by an additional $\sim 0.07 M_\odot$. We determine two possible reasons for this difference. On the one hand, the core of the cold merger remnant contracts more strongly due to the absence of thermal pressure. Therefore, more gravitational binding energy is converted into rotational energy and the rotation rates become higher, supporting the formation of spiral arms. On the other hand, the shock-heated matter from the collision region between the two NSs forms a halo of hot matter which engulfs the compact remnant-torus system in the $T \neq 0$ case (Figs. 3 4 5). This halo may damp the oscillations of the remnant and the development of large deviations from axisymmetry.

![Figure 2](image2.png)

**Figure 2.** Disc masses versus mass ratio $q$ for all non-zero temperature runs.

![Figure 3](image3.png)

**Figure 3.** Same as in Fig. 2 but comparing the differences between $T = 0$ EoS and non-zero temperature EoS for two different mass ratios $q$.

![Figure 4](image4.png)

**Figure 4.** Total angular momentum of a blob of matter in the primary spiral arm of Model C1216. The dashed line represents the results of the simulation, the solid line those of a postprocessing analysis.
4 CONSTRAINTS FROM OBSERVED GRBS

In this section we shall discuss how the properties of the GRB engine may be constrained from the recent observations of four short-duration GRBs. To this end we shall make the assumption that the bursts originated from ultrarelativistic jets whose acceleration was mainly driven by thermal energy, provided by the annihilation of neutrino-antineutrino ($\nu\bar{\nu}$) pairs in the vicinity of post-merger BH-torus systems (see, e.g., Ruffert & Janka 1999, Setiawan et al. 2004). Although we will refer to this scenario in our discussion because numerical models provide at least order of magnitude information of involved parameters, our arguments are more general and apply basically also to models that consider MHD-powered GRB jets.

4.1 BH-Torus System Parameters

Linking the measured apparent (i.e., isotropic-equivalent) energy $E_{\gamma,\text{iso}}$ of a short GRB to the energy output from the central engine and the mass $M_{\text{acc}}$ accreted by the BH (during the phase when the neutrino emission from the accretion torus is sufficiently powerful to drive the jets) involves a chain of efficiency parameters corresponding to the different steps of physical processes between the energy release near the BH and the gamma-ray emission at $\sim 10^{14}$ cm:

$$E_{\gamma,\text{iso}} = f_1 f_2 f_3 f_4^{-1} f_4 M_{\text{acc}} c^2 .$$

Here $f_1$ denotes the efficiency at which accreted rest mass energy can be converted to neutrino emission, $f_2$ is the conversion efficiency of neutrinos and antineutrinos by annihilation to $e^\pm$ pairs, $f_3$ is the fraction of the $e^\pm$-photon fireball energy which drives the ultrarelativistic outflow with Lorentz factors $\Gamma > 100$ as required by GRBs, and $f_4$ is the fraction of the energy of ultrarelativistic jet matter which can be emitted in gamma rays in course of dissipative processes that occur in shocks when optically thin conditions are reached.

These parameters are constrained to some degree by numerical and analytic work, but their values are still rather uncertain and might vary strongly with time-dependent and system-dependent conditions of the source. "Typical" values from merger and accretion simulations are: $f_1 \sim 0.05$ (Lee et al. 2005, Setiawan et al. 2004), $f_2 \sim 0.001 \ldots 0.01$ (Ruffert & Janka 1999, Setiawan et al. 2004), $f_3 \sim 0.1$, $f_4 \sim 0.01 \ldots 0.05$, and $f_4 < 0.2$ from estimates for internal shock models (Daigne & Mochkovitch 1998, Kobayashi & Sari 2001, Guetta et al. 2001, and references therein).

The cited values for $f_2$ and $f_3$ were determined by relativistic hydrodynamical calculations of the formation and propagation of jets driven by thermal energy deposition around BH-torus systems (Aloy et al. 2005). These simu-
Figure 6. Characteristic evolutionary phases of Model C1216 (left panels) and Model S1216 (right panels). Panels (a) and (b) show the merging phase with a primary spiral arm forming. Plotted is every 10th SPH particle around the equatorial plane. The matter of the two stars is represented by green and blue particles, respectively. Particles that end up in the disc at the end of the simulation are marked in red and the ones that currently fulfill the disc criterion (see Sect. 3.2) are plotted in yellow. Panels (c) and (d) display the situation $\sim 2\, \text{ms}$ after merging, when a secondary spiral arm appears. Panels (e) and (f) show the time when a quasi-stationary torus has formed. Note that the particle density is not strictly proportional to the density of the fluid because the particles near the initial stellar boundaries are attributed a smaller particle mass. These particles preferentially end up in the torus so that the particle density there appears enlarged.
tions revealed that about 10–30% of the $e^+e^-$-pair plasma fireball energy are used for hydrodynamically accelerating the baryonic matter in the jets to ultrarelativistic velocities. Typical jet half-opening angles were found to be between 10° and 15°, corresponding to $f_1 \sim 1.5 \times 10^{-2} ... 3.4 \times 10^{-2}$ (Aloy et al. 2005, Janka et al. 2005). Two of the four well-localized short GRBs indeed provide hints for this degree of collimation (see Fox et al. 2005, Berger et al. 2005) predicted by the theoretical work.

Because of the lack of detailed information about how the factors $f_1$ to $f_4$ and $f_0$ depend on the properties of NS+NS binaries and their relic BH-torus systems, we made the simplifying and bold assumption that the set of parameter values is the same for all cases, $(f_1, f_2, f_3, f_0, f_4) = (0.1, 0.01, 0.1, 0.01, 0.1)$, and applied Eq. (1) to the four observed short GRBs, using the measured isotropic-equivalent gamma-energies of Table 2. Doing so, we obtained the estimates listed in that table for the accreted mass $M_{\text{acc}}$. Assuming further that the duration $t_\gamma$ of the GRB (at the location of the redshifted source) is an upper bound for the duration of the accretion of $M_{\text{acc}}$ by the BH, $t_{\text{acc}} \lesssim t_\gamma$ (cf. Aloy et al. 2005), we can deduce lower limits for the average mass accretion rates,

$$M_{\text{acc}} \gtrsim \frac{M_{\text{acc}}}{t_\gamma}.$$  

These are also listed in Table 2 and plotted versus $M_{\text{acc}}$ in Fig. 4. We obtain in all cases values for the torus masses in the range of those determined in this work (Fig. 4) and values for the mass accretion rates in the ballpark of theoretical expectations for post-merger BH accretion. Simulations of the latter find mass accretion rates of typically between several $0.1 M_\odot \, \text{s}^{-1}$ and several $M_\odot \, \text{s}^{-1}$ (e.g., Ruffert & Janka 1999, Setiawan et al. 2004, Lee & Ramirez-Ruiz, 2002, Lee et al. 2003). Of course, the numerical values for $M_{\text{acc}}$ and $M_{\text{acc}}$ in Table 2 are very uncertain and have to be taken with caution. Shifts by factors of a few are easily possible because of probable system-dependent variations of the efficiency factors $f_1$ to $f_4$ and of the jet collimation factor $f_0$. Nevertheless, it is very encouraging that the simple and straightforward calculations based on Eqs. (1) and (2) lead to reasonable numbers in all cases.

GRB 050509b, for example, is particularly interesting because of its very short intrinsic duration of $t_\gamma \sim 33$ ms and an extraordinarily low isotropic energy of a few $10^{50}$ erg if it is located at $z = 0.225$ (Gehrels et al. 2005). Such a low-energy event can be explained by the accretion of discs in the lower range of masses found here for symmetric NS+NS binaries, even when rather conservative assumptions are made

\[1\] Note that the (initial) torus mass may be larger than the accreted mass that we determine from the observed GRB properties. A part of the torus may fall into the BH without producing a release of energy which is strong enough to power ultrarelativistic GRB-jets. Some part of the torus mass will also be ejected by winds driven by neutrino energy deposition (see e.g., Ruffert et al. 1994, Rosswog & Liebendörfer 2003, viscous heating, or MHD effects (see, e.g., Daigne & Mochkovitch 2002), and some part of the torus mass may be lost equatorially due to the outward transport of angular momentum by viscous shear in the (magnetized) accretion torus.

\[2\] The development of the neutrino-driven wind is not seen in current NS+NS merger simulations, because it requires a suitable treatment of neutrino transport, whereas the present models describe neutrino effects with a trapping scheme at best, which releases neutrinos from the stellar medium according to their local production rates, properly reduced due to a finite diffusion timescale (see Ruffert et al. 1994 and Rosswog & Liebendörfer 2003).

4.2 Constraints from the low-energy, short-duration GRB 050509b

If further observations substantiate the link between NS+NS mergers and short GRBs, this may set constraints on the post-merging evolution and the nuclear EoS. On the one hand, the hypermassive NS should escape the collapse to a BH for a sufficiently long time so that an accretion torus can form by the described dynamical processes. On the other hand, the collapse of the hot, neutrino radiating NS should not be delayed too much, because neutrino energy deposition in the surface-near layers of the NS will lead to a massive baryonic wind (Duncan et al. 1986, Woosley 1993, Qian & Woosley 1996, Thompson et al. 2001), which can seriously endanger the subsequent formation of a GRB jet or fireball.

The mass loss rate of the neutrino radiating neutron star due to this nonrelativistically expanding outflow of baryonic matter was estimated by (Woosley 1993, Qian & Woosley 1996) and is approximately given by (see Heger et al. 2005)

$$M_{\text{wind}} \approx 8.5 \times 10^{-3} \frac{L_{\nu_e+\bar{\nu}_e,53}}{R_{30}^{5/3}} \frac{M_3}{M_\odot} \frac{L_5}{10^{53} \text{erg s}^{-1}} \, \text{s}^{-1},$$

where $L_{\nu_e+\bar{\nu}_e,53}$ is the luminosity of electron neutrinos plus antineutrinos in units of $10^{53}$ erg s$^{-1}$, $R_{30}$ the NS radius normalized to 30 km, and $M_3$ the NS mass in units of $3 M_\odot$. This mass loss rate can be up to $\sim 10^{-2} M_\odot \, \text{s}^{-1}$ for the neutrino luminosities found in NS+NS merger simulations and the masses and radii of the compact post-merger object (see Ruffert & Janka 2001, Rosswog & Ramirez-Ruiz 2002, Rosswog & Liebendörfer 2003). Upon colliding with each other, the two merging neutron stars heat up by shocks and compression to central temperatures of several 10 MeV (Figs. 6, 8). Within only a few milliseconds after the final plunge, the neutrino emission reaches luminosities of $10^{53}$ erg s$^{-1}$ or higher (for more information, see Ruffert et al. 1993, Ruffert & Janka 2001, Rosswog & Liebendörfer 2004, Rosswog & Ramirez-Ruiz 2003), and the post-merger object should start losing mass in the neutrino-driven wind. Equation (3), however, does not take into account the effects of the very rapid rotation of the neutron star on the wind. This question is still unexplored, but the rapid spinning of the remnant born in a NS+NS merger event may lead to changes of the functional
Figure 7. Same as panels (a) & (c) and (b) & (d), respectively, of Fig. 6 but for Model S1414. The primary spiral arm is absent in the symmetric binary case and the post-merging (secondary) arms are smaller than in the asymmetric models.

Figure 8. Four stages of the NS+NS merger Model S1216. The surface is chosen to correspond to a density of $10^{10}\,\text{g}\,\text{cm}^{-3}$, the temperature distribution is visible color coded in the octant cut out from the three-dimensional mass distribution. Temperatures up to 30 MeV are reached at the center only milliseconds after the two neutron stars have merged. Time is given in the lower left corner of the panels, and length is measured in units of 1.47 km (top right corner of each panel).
Torus Formation in Neutron Star Mergers

A corresponding mass limit for the wind halo around the newly formed BH-torus system can be derived by the following arguments. In order for the baryon loading to remain sufficiently low so that acceleration to Lorentz factors above a value $\Gamma_0$ (100 or more) is possible, the mass of the halo swept up by two relativistic GRB jets, $M_{\text{swept}}$, is constrained by the condition $E_{\text{kin}}(\Gamma > \Gamma_0)/(M_{\text{swept}} c^2) > \Gamma_0$. Here $E_{\text{kin}}(\Gamma > \Gamma_0)$ is the kinetic energy of matter which can accelerate to Lorentz factors larger than $\Gamma_0$ in the two polar jets that expand away from the BH in the axial direction. For a jet collimation factor $f_4$, this kinetic energy is linked to the isotropic equivalent $\gamma$-energy of the GRB by $E_{\text{kin}} = E_{\gamma, \text{iso}} f_3 f_4$, when $f_3$ is again the efficiency for the conversion of kinetic energy to gamma rays. On the other hand, the wind mass swept-up by the jets is given as $M_{\text{swept}} = f_0 \alpha M_{\text{wind}}$, where $M_{\text{wind}} \approx M_{\text{wind,NS}}$ is the mass of the wind for a lifetime $t_{\text{NS}}$ of the neutron star, and $\alpha < 1$ is a fudge factor which accounts for the fact that the wind from a rapidly rotating neutron star might be less dense along the rotation axis than calculated from Eq. 4. Combining all, we get the condition

$$t_{\text{NS}} \lesssim \frac{E_{\gamma, \text{iso}}}{\Gamma_0 f_4 f_3 \alpha M_{\text{wind}} c^2}.$$  

With $\Gamma_0 = 100$, $f_4 \sim 0.1$, $M_{\text{wind}} = 10^{-3} M_\odot s^{-1}$ (this assumption is on the low side of reasonable numbers from Eq. 5), and $E_{\gamma, \text{iso}}$ from Table 2, we find for GRB 050509b (the lowest-energy case and thus the one providing the strongest limits) that $t_{\text{NS}} < 1$ ms, assuming $\alpha = 1$. This means that due to the low energy of GRB 050509b even a tiny amount of baryonic pollution by swept-up wind matter would have prevented ultrarelativistic motion of the jets. The neutrino radiating merger remnant therefore must have collapsed to a BH essentially immediately after the binary NS merging and the onset of the neutrino-driven wind. Since it cannot be excluded that the density of the neutrino-driven wind along the rotation axis is much lower than estimated by Eq. 3, a more conservative estimate with $\alpha \sim 0.1$ yields $t_{\text{NS}} < 100$ ms.

If GRB 050509b originated from a NS+NS merger, this result therefore suggests that the merger remnant was a hypermassive object, which was transiently stabilized by its rapid and differential rotation (Bauswein et al. 2004), but had a very short lifetime. Referring to the analysis by Morrison et al. (2004) – see their Table 2 – this can either mean (i) that, if the high-density EoS was stiff, the system

has started to radiate neutrinos with high luminosities (see the discussion in Ruffert & Janka 1999, Ruffert & Janka 1999, Rosswog & Ramirez-Ruiz 2004, Rosswog & Ramirez-Ruiz 2004, and find information about the neutrino emission from such accretion tori in Sotani et al. 2004 and Lee et al. 2005, and references therein). This wind does not provide such a hazard for GRB-urable outflows, because it originates from outside the innermost stable circular orbit, and angular momentum conservation prevents it from filling the axial regions above the poles of the BH. In fact, Rosswog et al. 2003 and Rosswog & Ramirez-Ruiz 2003 consider the pressure of such a “wind envelope” as helpful for the collimation of a $\nu \bar{\nu}$-annihilation-driven axial jet (see Levinson & Eichler 2000), an effect which turned out to be secondary for getting collimated, highly-relativistic outflow from BH-torus systems in recent hydrodynamical jet simulations (Aloy et al. 2004).

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3 Note that neutrino energy deposition will later on also drive a baryonic wind off the accretion torus, after the latter was heated by viscous dissipation of rotational energy and

dependence of $\dot{M}$ on the NS properties and may also imply a strong pole-to-equator difference of the mass-loss rate.

The nonrelativistic wind will create an extended baryonic halo that surrounds the merger remnant, before the compact central part undergoes collapse to a BH and the outer, high-angular momentum matter assembles in a torus around the BH. A jet launched from such a system and expanding into the pre-existing wind halo may then sweep up so much mass – even if the halo has a low density and small mass – that the high Lorentz factor outflow needed for GRBs ($\Gamma > 100$) is prevented. This was, for example, seen in the type-A models of Aloy et al. 2005. A rather fast collapse of the relic hot, supramassive or hypermassive remnant to a BH is therefore required, if the cloud of baryonic matter produced by the neutrino-driven wind from the transiently stable NS shall not become such a hazard for the GRB3.

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Figure 9. Estimated accreted masses, $M_{\text{acc}}$, and lower bounds for the average mass accretion rates, $M_{\text{acc}}$, for the four well-localized short GRBs of Table 2.

Table 2. Estimated accreted masses, $M_{\text{acc}}$, and lower bounds for the average mass accretion rates, $M_{\text{acc}}$, assuming post-merger BH-torus systems being the energy sources of the recently observed, well-localized short GRBs. The estimates are based on Eqs. 1 and 2. The quantity $E_{\gamma, \text{iso}}$ is the isotropic-equivalent $\gamma$-ray burst energy (corrected for the cosmological redshift $z$ of the burst), and $t_\gamma$ is the GRB duration at the source, computed as $t_\gamma = t_\nu_0/(1 + z)$ from the measured 90%-inclusive interval of high-energy emission. The observational data were taken from Fox et al. 2005.

| GRB      | $z$   | $t_\gamma$ | $E_{\gamma, \text{iso}}$ | $M_{\text{acc}}$ | $M_{\text{acc}}$ |
|----------|-------|------------|--------------------------|------------------|------------------|
|          |       | (sec)      | (ergs)                   | (M_\odot)        | (M_\odot)        |
| 050509b  | 0.225 | 0.633      | $4.5 \times 10^{48}$     | 2.5 $\times 10^{-3}$ | 0.08              |
| 050709   | 0.160 | 0.060      | $6.9 \times 10^{49}$     | 3.8 $\times 10^{-2}$ | 0.6              |
| 050724   | 0.258 | 2.4        | $4.0 \times 10^{50}$     | 2.2 $\times 10^{-1}$ | 0.09             |
| 050813   | 0.722 | 0.35       | $6.5 \times 10^{50}$     | 3.6 $\times 10^{-1}$ | 1.0              |
mass of the merging binary was unusually high compared to the known galactic double neutron stars, which all possess combined masses in the range of \( M_{\text{sum}} = M_1 + M_2 = 2.6-2.8 \, M_\odot \) (Stairs 2004); or, alternatively, (ii) that the nuclear equation of state is so soft that it could not support the merger remnant despite of its rapid differential rotation and thermal pressure; or (iii) that a very efficient mechanism was at work which destroyed the differential rotation of the compact remnant and extracted a sizable amount of its angular momentum on the required short timescale. This could, for example, happen by hydrodynamical effects and mass shedding and/or by strong gravitational wave emission (Shibata et al. 2005).

Case (i) might be rejected as unlikely. Case (ii) tends to favor the “softest” of the EoSs surveyed by Morrison et al. (2004) which support nonrotating NSs only up to a gravitational mass around \( 1.65 \, M_\odot \). Such EoSs, however, are excluded by the recent accurate measurement of a mass of \( 2.1 \, M_\odot \) for the millisecond pulsar PSR J0751+1807 (Nice et al. 2005) and by other neutron stars with masses beyond \( 2 \, M_\odot \) (see, e.g., the review by Lattimer & Prakash 2004). An EoS which is sufficiently stiff to fulfill this observational constraint can also ensure the transient stability of the merger remnant of binary NS systems with a canonical mass of \( 2.6-2.8 \, M_\odot \) (see the discussion of the critical mass for hypermassive NSs by Shibata et al. 2005). Therefore case (iii) seems to offer a plausible scenario for GRB 050509b. The NS+NS binary had a typical mass and the merger remnant was well above the maximum mass of stable, rigidly rotating neutron stars. The redistribution of angular momentum by non-axisymmetric hydrodynamic interaction and gravitational radiation (Shibata et al. 2003) or by viscosity and MHD effects (Shapiro 2004) has then driven the hypermassive object to gravitational instability on a timescale of \( \lesssim 100 \, \text{ms} \). The short accretion timescale \( (t_{\text{acc}} < t_\gamma) \) suggests that the remaining torus mass was small. This is consistent with our conclusion that little mass was accreted by the BH, which we have drawn from the low energy output of GRB 050509b (Table 2). And it is consistent with the requirement coming from optical limits that little radiating material was ejected (Hjorth et al. 2005a), e.g. by neutrino-driven winds or by mass shedding due to viscous transport of angular momentum within the accretion torus. In view of the torus formation models presented here, this points to a symmetric or nearly symmetric NS+NS binary with a typical mass as the source of GRB 050509b.

5 DISCUSSION

Although our analysis of the four well-localized short GRBs was based on a number of unsettled assumptions, we found that the measured GRB energies and durations lead to estimates for \( M_{\text{acc}} \) and \( M_{\text{acc}} \) that are roughly consistent with expectations from theoretical models of NS merging and post-merging evolution. However, in spite of this amazing result we think that it would be premature to claim that relic BH-torus systems from binary NS mergers and their associated neutrino emission can power all short GRBs.

Short GRBs have durations between a few milliseconds and about two seconds, and they might differ in their energy output by a factor of 100 or more. Observations also reveal large differences with respect to their lightcurves and spectral hardness ratios. This wide range of short GRB properties leaves plenty of room for the possibility that different compact binaries contribute to the observed events. For example, not only NS+NS but also NS+BH mergers can lead to BH-torus systems with sizable torus masses and favorable properties for GRB production. Even the physical processes which play a dominant role in the energy release of the central engine and jet formation might differ between the events. In fact, it might turn out that this is needed to account for the observed diversity of short GRBs and their afterglows. Flare-like events followed by a very rapid decay were, for example, observed in case of GRB 050724 hundreds of seconds after the initial GRB. This was interpreted as a consequence of long-time activity of the central engine, which might point to a partial disruption and gradual accretion of a NS by a BH (Barthelmy et al. 2005). Episodic and long-lasting mass transfer from the NS to the BH was found to be possible for certain NS/BH mass ratios in combination with special properties of the NS equation of state (see e.g., Kluźniak & Lee 1998; Portegies Zwart 1998; Lee & Kluźniak 1999; Janka et al. 1999; Rosswog et al. 2004; Davies et al. 2005).

The rather uncertain estimates of the merger rates also allow for the possibility that not all NS+NS/BH mergers produce GRBs (see, e.g., Guetta & Piran 2005, taking into account that the hydrodynamic models by Aloy et al. 2003 yield jet semi-opening angles of \( \lesssim 10^\circ \) instead of a value of \( \sim 1^\circ \) as found by Guetta & Piran with the assumption that all mergers make GRBs).

We therefore refrain (unlike others, see, e.g., Lee et al. 2004; Rosswog & Ramirez-Ruiz 2004) from making predictions of the distribution of GRB properties on grounds of current simulations of NS+NS or NS+BH mergers. Such attempts are seriously hampered by the large number of unknowns in the theoretical picture and in particular their relation to the properties of compact object mergers:

- The distributions of binary parameters (NS and BH masses and spins, binary mass ratios) are highly uncertain. Theoretically, because of our incomplete understanding of the binary formation and evolution and of the supernova explosions which terminate the lives of massive stars and give birth to neutron stars (Bulik et al. 2003). And observationally, because of the rather limited sample of known NS+NS or NS+BH binaries (e.g., Lattimer & Prakash 2004, Stairs 2004). The latter aspect is particularly bothersome in view of the fact that only a minor fraction (of order 10\%?) of NS+NS/BH mergers might be able to produce GRBs and might be needed to account for the observed short bursts.

- NS+NS/BH merger models require much more work. Relativistic simulations of compact object mergers are

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4 Simulations and analytic studies of NS+BH mergers have so far been carried out only in Newtonian gravity and disregarding the probable rotation of the BH. A Newtonian treatment, however, does not reproduce the correct mass-radius relation of a relativistic neutron star, for example. Current predictions of the merger evolution and of the dynamics of disc formation and accretion in such extremely relativistic systems can therefore not be considered as reliable and conclusive, neither quantitatively nor qualitatively.
needed for different (non-zero temperature) nuclear equations of state and different parameters of the binary systems. A softer nuclear EoS, for example, might lead to a relation between binary parameters and torus masses that differs from the results of this work, which are based on the rather stiff EoS of Shen et al. (1998a,b). NS+NS merger models must be consistently evolved from the last stages of the progenitor system, through the moment of BH formation, to the subsequent accretion and jet production phase, including all the potentially relevant physics like neutrino transport and magnetic fields. Only then, for example, will the question be answered under which circumstances the neutrino-driven wind from the transiently stable hypermassive merger remnant represents an unsurmountable obstacle for GRB jets.

- The complex sequence of physical processes from the energy release by the merger remnant to the stage of GRB production involves large uncertainties, expressed by the product of efficiency factors $f_1$ to $f_4$ and jet collimation factor $f_3$, which appears in Eq. (1). Each of these factors can contain still unknown dependences on the parameters of the NS+NS/BH binary and of its relic BH-torus system. Improved simulations of jet formation by energy release around post-merger BH-torus systems are necessary to clarify these dependences.

- Since the compact proto-BHs found in our NS+NS merger simulations have significant angular momentum ($\alpha_{\text{rem}}$ is between 0.64 and 0.91 for the considered mergers of irrotational binaries; Table 1), the energy release during the accretion is likely to be boosted by Kerr effects. Not only neutrino emission and neutrino-antineutrino annihilation may play a role, but also energy release through magnetic processes, e.g., by the Blandford-Znajek (1977) mechanism (see, for example, Lee et al. 2003).

Much more theoretical work and numerical modeling are therefore needed before truly meaningful and reliable theoretical predictions of GRB properties and their link to NS+NS/BH merger parameters will become possible. Such predictions need to involve, in particular, detailed models of the structure of the collimated ultrarelativistic outflows from the merger remnants and assumptions about the production of the GRB emission at large distances. Detailed consideration of the relevant physics is clearly beyond the scope of this paper. A first, still very crude attempt to employ such calculations of jets from BH-torus systems for predicting the distributions of observable GRB properties has been attempted recently by Janka et al. (2003).

6 SUMMARY AND CONCLUSIONS

We have shown that nonradial oscillations and triaxial deformation and the associated tidal torques can mediate efficient angular momentum transfer in the compact remnant within the first milliseconds after the merging of two NSs. Applying different criteria, we estimated the mass of the torus which will survive the future collapse of the central, dense object to a BH. For the considered rotational systems and the employed relatively stiff nuclear EoS of Shen et al. (1998a,b), we determined lower disc mass limits of $1 - 2\%$ of the (baryonic) system mass for a NS-NS (gravitational) mass ratio around $q = 1$, increasing to about $9\%$ for systems with $q$ around $0.55$. Though the disc mass is more sensitive to the parameter $q$, we also observed a mild inverse relation between torus mass and binary system mass. For the same value of $q$, slightly higher torus masses were found when $M_{\text{sum}} = M_1 + M_2$ was lower (Fig. 2). Non-zero temperature effects turned out to be important and to reduce the disc mass by several $10\%$ compared to the values for the $T = 0$ case.

The disc masses obtained in our simulations confirm the viability of post-merging BH-torus systems as central engines of short GRBs, and they support the theoretical possibility that thermal energy deposition above the poles of the BH by $\nu\bar{\nu}$ annihilation could be the primary energy source of short GRBs. The ultrarelativistic outflow of matter powered by this energy deposition was found in hydrodynamic simulations to be highly collimated in jets with semi-opening angles between about $10^\circ$ and $15^\circ$ (Aloy et al. 2005). This prediction seems to be supported by the observations of two of the four recently detected well-localized, short hard GRBs (Fox et al. 2003).

Using typical parameters that describe the multi-step process from the energy release of the central engine to the observed GRB in this scenario, we derived estimates of the accreted masses and mass accretion rates (see Table 2) from the measured GRB energies and durations, using Eqs. (1) and (2). For all cases we found values for $M_{\text{acc}}$ and $\dot{M}_{\text{acc}}$ that are in the ballpark of the expectations from our NS+NS merger models and from models of BH accretion in post-merging BH-torus systems. Employing the same assumptions about the GRB engine, the extraordinarily low-energetic and short-duration GRB 050509b can be used to set strict limits for the lifetime of the post-merger hypermassive neutron star. It must have collapsed to a BH in much less than 100 ms with only little matter ($\sim 0.01 M_\odot$) remaining in a torus around the BH to partially produce GRB-viable outflow. This favors a symmetric or nearly symmetric NS+NS binary as progenitor system, with a mass typical of the known galactic double neutron stars.

Knowledge of the binary system parameters, which could be obtained from measurements of the gravitational wave chirp signal during the inspiral, might allow one to deduce interesting information about the properties of the neutron star equation of state from future short GRBs with determined distances. Constraining the EoS of NS matter by lifetime arguments for the supramassive or hypermassive remnant of NS+NS mergers, however, will require detailed relativistic models of the NS merging and of the postmerging evolution for different non-zero temperature EoSs, including the effects of magnetic fields (Shibata et al. 2004). GRB 050509b provides particularly strict limits in such an analysis, but may be a rare event where an exceptionally low-energy GRB with very short duration could be well localized.

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