A Proposal for Detecting Superconductivity in Neutron Stars

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Abstract.— The recent burst GW170817 and GW190521 in the weak gravitational field limit [1, 2] study the modulation signal caused by reflection from SC. In this letter, we show that the GW reflection from neutron star can give rise to a modulation for the observed GW signals. In return, a direct observation of the modulation can provide one-stone-two-bird evidence: 1, it supports the existence of SC in NS; 2, it implies the GW reflection from SC. In this letter, we study the modulation signal caused by reflection from two systems: NSBH system and the BNS system. The modulation signal with small but non-negligible amplitude contributes additional frequency components apart from GW frequency. For real time observation in both NSBH and BNS cases, the modulation signal superimposes on the non-reflection sine signal, leading to non-negligible modification on the total amplitude and linear phase factor. The modulation tends to be stronger as the orbital angular velocity gets larger, except for some special inclination angle. We further show that the modulation can be probed by the Cosmic Explorer in a reasonable parameter region.

Introduction.— Superconductivity (SC) in neutron star (NS) has been predicted and discussed for a long time [2, 3]. Given high stellar density in NS, roughly $10^{14}g/cm^3$, the proton pairing is expected to take place since its critical temperature ($\sim 10^{10}K$) is larger than the typical NS temperature ($\sim 10^6K$ on the surface) in current universe [2, 4, 5]. In contrast, pairing of compressed electrons in NS is not excluded, due to its exponentially small critical temperature $T_c \approx 3 \times 10^3 \rho^{1/6} e^{-2.8\rho^{1/3}} [K]$ with the electron density $\rho$ typically of the order of $10^{12} g/cm^3$ in the neutron star [4]. Continuous efforts have been devoted to verifying SC in NS. One possible evidence comes from the observation of glitch (e.g., [6]). A widely accepted explanation attributes such phenomenon to relaxation of pinned vorticity of the neutron superfluid, which leads to angular momentum transfer in NS [7]. Under this framework, the role played by magnetic flux lines of SC proton and the dynamics in the NS rotation are detailedly studied [8–10]. Furthermore, Ref. [11] showed that the cooling procedure of the Cassiopeia A Neutron Star is 4% faster than conventional prediction [12] during the last ten years, whose cooling mechanism can be explained by enhanced neutrino emission during pairing formation of neutrons. Proton SC can significantly alter the modified Urca reaction rate which dominates cooling process before neutron pairing, matching the observed variance in cooling rate [13].

The observations of GW170817 [14], GW200105 and GW200115 [15] capture the emission of GW in a binary neutron star (BNS) system and neutron star-black hole (NSBH) systems, providing a possibility to connect the detection of SC in NS to GW observation. Following the study of GW reflection by SC film with electronic-Cooper pairs [1], we expect similar reflection can appear in a SC-accommodated NS with proton Cooper pairs. Protons in the SC ground state are delocalized. In the presence of weak incident GWs, paired protons protected by the SC gap maintain the zero-momentum state, leading to non-geodesic motion. On the other hand, normal-Cooper-paired electrons in the NS float with the GW induced space deformation. Such separation of positive and negative charges leads to Coulomb force which opposes the tidal force of the incoming GWs. As a result, the non-dissipative conductivity of mass current is greatly amplified by Coulomb interaction with a factor of $1.24 \times 10^{36}$ [16]. Ref. [17] describes a GW penetration process in SC, in which the GW strength exponentially decays with depth. Little energy of GW is lost during the penetration, implying GW is almost fully reflected.

The above discussion indicates that the GW reflection from the neutron star can give rise to a modulation for the observed GW signals. In return, a direct observation of the modulation can provide one-stone-two-bird evidence: 1, it supports the existence of SC in NS; 2, it implies the GW reflection from SC. In this letter, we study the modulation signal caused by reflection from two systems: NSBH system and the BNS system. The modulation signal with small but non-negligible amplitude contributes additional frequency components apart from GW frequency. For real time observation in both NSBH and BNS cases, the modulation signal superimposes on the non-reflection sine signal, leading to non-negligible modification on the total amplitude and linear phase factor. The modulation tends to be stronger as the orbital angular velocity gets larger, except for some special inclination angle. We further show that the modulation can be probed by the Cosmic Explorer in a reasonable parameter region.

The NSBH system.— In the NSBH model, GW is assumed to be effectively emitted at the center of mass (COM) of the system. The COM is near the black hole (BH) center ($S$ in Fig. 1), as the mass of the BH ($M_{BH}$) is usually several times larger than that of the NS ($M_{NS}$). The source is placed at $S$ for simplification, whose deviation doesn’t affect the final conclusion. Also, NS is assumed to rotate around $S$ in a circular motion without eccentricity. By taking Newtonian orbital approximation, the orbital radius $R_{orb} = (GM_{BH}/\omega_a)^{2/3}$, where...
The NS (earized as Maxwell-like equations for GW propagation [18]. The NS (N in Fig. 1) plays the role of “spherical mirror” of GW. Since the typical radius of NSs ($r_{NS} \approx 10 \text{ km}$) is much smaller than the wavelength of GW discussed here, isotropic reflection is considered. Following the quadrupole approximation of GW, frequency of the GW is given by $f_{GW} = 2 \omega_{a}/c$, and it is sufficient to focus on one GW polarization component to study the waveform. Therefore, the total signal received at local time $t_{obs}$ is $E_{tot}(t_{obs}) = E_{dir}(t_{obs}) + E_{ref}(t_{obs})$, with $E_{ref}$ and $E_{dir}$ denoting the reflected and non-reflected parts respectively.

The received non-reflected GW takes the form

$$E_{dir}(t_{obs}) = \frac{\mathcal{E}_0}{|SO|} e^{-i\phi_0(t_{obs})},$$

where $\mathcal{E}_0 = 4GR_{orb}^2M_{NS}\omega_a^2/c^4$ is the initial amplitude, $\phi_0(t) = 2\pi f_{GW}t - k_{GW}|SO|$ with $k_{GW}$ being the GW wave vector, and $|\cdot|$ denotes the distance between two points. Similarly, the GW incidents on the NS with amplitude $\mathcal{E}_0/R_{orb}$. After being scattered by the NS, amplitude of the outgoing GW which propagates a distance $l$ becomes $\mathcal{E}_0/r_{NS}/(lR_{orb})$.

Due to the reflection at different points of spacetime, the NS orbital motion leads to an additional phase modulation from the reflected GW. With the practical condition $|SO| \gg R_{orb}$, $|NO| \approx |SO| - R_{orb} \sin \theta \cos(\omega_a t')$.

Here $t'$ is a function of $t_{obs}$ with $\omega_a t'$ showing the reflection position for reflection signal received at $t_{obs}$. By setting initial position of NS on $H$, $t'$ can be determined by solving the equation $R_{orb} \sin \theta [1 - \cos(\omega_a t')] = c(t_{obs} - t')$ with $c$ denoting the speed of light. In the Newtonian limit, variation of the difference between $t$ and $t'$ can be neglected and $t'$ can be replaced by $t$. As a result, the leading term of $E_{ref}$ is

$$E_{ref}(t_{obs}) \approx \gamma \frac{\mathcal{E}_0}{|SO|} e^{-i\phi_0(t_{obs})} - k_{GW}R_{orb}] e^{-i\kappa \cos(\omega_a t_{obs})},$$

where the two coefficients $\gamma = r_{NS}/R_{orb}$ and $\kappa = k_{GW}R_{orb}\sin \theta$, with $\theta$ denoting the inclination angular of observer in Fig. 1. By applying Fourier transformation to $E_{ref}(t_{obs})$, the reflection part in frequency domain (denote as $E_{ref}(f)$) has a dominant contribution at $f_{GW}$ as well as other components at $f_{GW}/2$, $3f_{GW}/2$, $2f_{GW}$, $\cdots$ (Fig. 2). This can be viewed from the expansion of $\exp[i\kappa \cos(\omega_a t_{obs})]$, where $\kappa \cos \theta (\omega_a t_{obs})$ can be rewritten as a summation of $\kappa n \cos (\omega_a t_{obs})$ terms with $m, n \in \mathbb{N}$. As a result, only discrete frequency components at integer times of $\omega_a/2\pi$ show up. Magnitude of the exponent term is suppressed as $n$ gets larger, leading to the main contribution from $f_{GW}$, $f_{GW}/2$, $3f_{GW}/2$.
channels. Those additional peaks lead to distinct differences in the observation result.

After combining Eq. (1) and Eq. (2), the real part of total signal $E_{\text{tot}}$ can be expressed in a compact form

$$\text{Re}\left[E_{\text{tot}}(t_{\text{obs}})\right] = \frac{E_0}{|SO|} A(t_{\text{obs}}) \cos[\phi_0(t_{\text{obs}}) - \phi(t_{\text{obs}})],$$

where

$$A(t) = \sqrt{1 + 2\gamma \cos\left[k_{GW}R_{\text{orb}} - \kappa \cos(\omega_{\alpha}t)\right] + \gamma^2},$$

$$\tan \phi(t) = \frac{\gamma \sin\left[k_{GW}R_{\text{orb}} - \kappa \cos(\omega_{\alpha}t)\right]}{1 + \gamma \cos\left[k_{GW}R_{\text{orb}} - \kappa \cos(\omega_{\alpha}t)\right]}.$$

As a consequence of $E_{\text{ref}}$, the explicit modulations appear in the relative amplitude $A(t)$ and the non-linear phase $\phi(t)$ of $\text{Re}[E_{\text{tot}}]$. Both modulations are at the same order of magnitude of $\gamma$, whose effect can be viewed from the standard deviations of the two quantities,

$$\sigma_A = \sqrt{\langle A^2 \rangle - \langle A \rangle^2}$$

$$\sigma_\phi = \sqrt{\langle \phi^2 \rangle - \langle \phi \rangle^2},$$

where $\langle \cdot \rangle = \int_0^{T_{\text{orb}}} \cdot \, dt / T_{\text{orb}}$ with $T_{\text{orb}}$ being the orbital period. It is clear from Eq. (4) that, the larger $\gamma$ and $\kappa$, the more significant fluctuations. And these two quantities can be rewritten as $\gamma = \tau_{NS}(\omega_{\alpha}^2 / G M_{BH})^{1/3}$ and $\kappa = 2 \sin \theta(G M_{BH} \omega_{\alpha})^{1/3} / c$. In the parameter region where our assumptions are valid, order of magnitude of $M_{BH}$ varies in a relatively limited range compared with $\omega_{\alpha}$. It can be observed from Fig. 3 that the fluctuation becomes stronger as $\omega_{\alpha}$ getting larger for both $A$ and $\phi$. And the increment of $M_{BH}$ enhances $\sigma_A$ while doesn’t obviously affect $\sigma_\phi$. It should be mentioned that the discussions here are valid for most range of $\theta$ except for the limiting case $\theta \to 0$, where $\kappa \to 0$ leading to flat amplitude and phase variation.

The **BNS system**.— In the BNS case, two NSs, $N_1$ and $N_2$ in Fig. 4, are assumed to have equal mass, then the GW source is right at the COM of the BNS system. The same weak field limit and relation between length scales are considered. Correspondingly, the orbital radius $R_{\text{orb}} = (G M_{NS}/4 \omega_{\alpha}^2)^{1/3}$. We use $\bar{E}_{\text{ref}}^1$ and $\bar{E}_{\text{ref}}^2$ to denote the two reflected GWs received by the observer, whose expressions take the form of Eq. (2). Interference between the two reflection signals happens. The reflection signal received at $t_{\text{obs}}$ is

$$\bar{E}_{\text{ref}}(t_{\text{obs}}) = \bar{E}_{\text{ref}}^1(t_{\text{obs}}) + \bar{E}_{\text{ref}}^2(t_{\text{obs}})$$

$$= \frac{\tilde{E}_0}{|SO|} 2 \cos[\kappa \cos(\omega_{\alpha}t)] e^{-i[\phi_0(t_{\text{obs}}) - k_{GW}R_{\text{orb}}]},$$

where $\tilde{E}_0 = 8 G R_{\text{orb}}^2 M_{NS} \omega_{\alpha}^2 / c^4$, $\tilde{\gamma} = \tau_{NS} / R_{\text{orb}}$ and $\tilde{\kappa} = k_{GW} R_{\text{orb}} \sin \theta$. The $2 \cos[\kappa \cos(\omega_{\alpha}t)]$ term caused by orbital motion modulates the amplitude of the GW, which is different from Eq. (2) of the NSBH system. As discussed before, frequency components in addition to $f_{GW}$ appear with the composition effect of orbital motion, which is captured by $\tilde{\kappa} \cos(\omega_{\alpha}t)$ term. Different from pre-
vious situation, here the secondary contribution comes from $2f_{GW}$ and zero frequency components. This can be understood from the $\pi$–rotational symmetry of the system, and effectively the periodicity of the system is reduced from $2\pi/\omega_\alpha$ to $\pi/\omega_\alpha$. Also, in the expansion of the $2\cos[\kappa \cos(\omega_\alpha t)]$ term, even orders of $\cos(\omega_\alpha t)$ survive and can be absorbed in the exponent, giving $\exp(\pm i 2n\omega_\alpha t)$ terms with $n \in N$.

In the received signal, $\tilde{E}_{ref}$ superimposes on the non-reflection signal, giving rise to the following result for real part of the total signal

$$\text{Re}[\tilde{E}_{tot}(t_{obs})] = \frac{E_0}{|SO|} \tilde{A}(t_{obs}) \cos[\phi_0(t_{obs}) - \tilde{\phi}(t_{obs})],$$

where

$$\tilde{A}(t) = \left(1 + 4\tilde{\gamma} \cos(k_{GW} \tilde{R}_{orb}) \cos[\kappa \cos(\omega_\alpha t)] \right) + \frac{\gamma^2 \cos[\kappa \cos(\omega_\alpha t)]^2}{1 + 2\gamma \cos(k_{GW} \tilde{R}_{orb}) \cos[\kappa \cos(\omega_\alpha t)]}^{1/2},$$

and

$$\tan \tilde{\phi}(t) = \frac{2\tilde{\gamma} \sin(k_{GW} \tilde{R}_{orb}) \cos[\kappa \cos(\omega_\alpha t)]}{1 + 2\gamma \cos(k_{GW} \tilde{R}_{orb}) \cos[\kappa \cos(\omega_\alpha t)]},$$

The relative amplitude $\tilde{A}(t)$ and non-linear phase $\tilde{\phi}(t)$ in $\text{Re}[E_{tot}]$ show explicit modulation in the presence of $\tilde{E}_{ref}$. Standard deviation $\sigma_{\tilde{A}}$ and $\sigma_{\tilde{\phi}}$ can be defined in the same way as Eq. (5). Also, the modulation in $\tilde{A}(t)$ and $\tilde{\phi}(t)$ is in the order of $\tilde{\gamma}$. Both $\sigma_{\tilde{A}}$ and $\sigma_{\tilde{\phi}}$ exhibit increasing tendencies with larger $\omega_\alpha$ and $M_{NS}$ as shown in Fig. 6. Those non-negligible fluctuations cause distinguishable modifications compared with the non-reflection model. Last, we note that when the direction of GW being received is perpendicular to the orbital plane of the binary system ($\theta \to 0$), it leads to vanishing $\kappa$ thus vanishing modulation. At this situation the reflected signal is not observable.

Signal distinguishability.— In this section we estimate the practical ability for the modulated signal caused by GW reflection to be distinguishable in experiment. As the discussion in [19], two waveforms $h_1$ and $h_2$ are distinguishable in principle if

$$||\delta h|| \equiv \sqrt{\langle h_1 - h_2 | h_1 - h_2 \rangle} \gtrsim 1$$

is satisfied, where the noise-weighted inner product is given by

$$\langle h_1 - h_2 | h_1 - h_2 \rangle = 4 \text{Re} \int_0^\infty \frac{|h_1(f) - h_2(f)|^2}{S(f)} df,$$

and $S(f)$ is the instrument noise spectrum. In the comparison of the reflection model with the non-reflection one, difference between two waveforms is given by $E_{ref}$ ($\tilde{E}_{ref}$). For the estimation of Cosmic Explorer [20], the distance $|SO| = 100$ Mpc, $M_{BH} = 10M_\odot$ and $M_{NS} = 1.4M_\odot$. In the region we are interested in, $||\delta h||$ increases with GW frequency, and the modulation signal is experimentally distinguishable when $f_{GW}$ is larger than the labelled frequencies in Fig. 7 for the two systems. And signals with $f_{GW} \lesssim 1Hz$ are also expected to be distinguishable in the future setup.

Conclusion.— To conclude, we provide a proposal to give a possibility of detecting SC in NSs based on the reflection of GWs by superconductor. In both NSBH and BNS systems, reflected GW signals contribute negligibly to the total signal, and modulate the amplitude and phase term at the order of ratio of NS radius to orbital radius. The modulation is captured by the fluctuations of the amplitude and the linear phase, which increases with the angular velocity. These results are applied to a large range of inclination observation angles except for the special case $\theta \to 0$. Lastly, we show that such modulations caused by the GW reflection are detectable with Cosmic Explorer in a reasonable parameter region. The experimental results, after comparing with our theoretical predictions, can provide evidence to support the existence of SC as well as GW reflection of neutron stars.

Y. G., J. Y., and Z. Z. contribute equally to this study.

\begin{thebibliography}{9}
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This factor can be calculated as $e^2/4\pi\epsilon_0Gm_p^2$ by making an analogy to Eq. (96) in Ref. [1], where $G$ is the gravitational constant and $m_p$ is the mass of proton.