Optimization of spline bearing capacity considering the influence of assembly sequence

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Abstract. Splines have been widely using in aerospace and other fields. Their reliability has important impact on the capability of mechanical equipment. When a slot on the internal spline is combined with different teeth on the external spline due to machining errors, different tooth clearances are generated. Therefore, when the spline is assembled in different sequences, the loads on the spline teeth will be different, which can affect the bearing capacity of the spline. However, in the actual assembly process of the spline, this effect has not been considered. To overcome the issue as listed above, a method for optimizing spline assembly sequence is proposed. First of all, the calculation method of the tooth clearances depended on the spline assembly sequence is established. Then, a spline load distribution model is proposed, and the spline assembly sequence is optimized by the maximum clearance method. Compared with the traditional random assembly method, the results show that the mean value of the maximum nominal tangential force of the spline is reduced by 25%. The obtained results can be used to guide the high reliability design of the spline.

1. Introduction

Splines are multi-toothed parts used to transmit mechanical torque and are widely used in mechanical transmission systems [1]. It consists of two parts: internal spline and external spline. The internal spline and external spline are connected, and the torque is transmitted through the spline teeth and tooth slots. In an ideal situation, all the teeth of the spline can bear the load. Still, due to the inevitable machining errors, each tooth and the tooth slot randomly deviate from the ideal position by a certain distance. This makes the tooth clearances between the mating surfaces of the internal and external splines different. The tooth with the smallest clearance meshes first, and the teeth with larger clearances mesh in sequence until the load is entirely distributed on the spline teeth. Therefore, the smaller the clearances, the higher the pressure on the tooth and the more likely the tooth will break [2].

At the same time, due to the randomness of machining errors, when the slot is combined with the different teeth, different clearances will be generated, so that the load distribution on each tooth is depended on the assembly sequence [3]. Especially for the aero-engine splines, they are required to maintain a low failure rate in complex working environments. By optimizing the assembly sequence, the load-bearing capacity of splines can be improved without changing the parameters such as the size. Therefore, the research on the optimization of spline assembly sequence has important theoretical significance and practical value.

In recent years, there have been more studies on the load distribution on spline teeth [4-7], but on this basis, few studies considered the influence of assembly sequence on the spline load distribution. Hirota et al. [8-9] studied a new way of assembling spline joints, which allows the spline teeth to
slightly plastic deformation, realizes the tight fit between the spline teeth and the teeth slots in a simple way, and improves the torsional strength of the spline by increasing the combined length. Hu et al. [10] studied the nonlinear and vibration characteristics of splines under the conditions of the-side-fit and the-major-diameter-fit. They pointed out that the above two assembly methods would affect the vibration performance of spline system. Yu et al. [3] measured the clearances of spline in each assembly sequence, calculated the nominal tangential force distributed on each spline tooth, and selected the optimal assembly sequence of spline. In summary, the existing researches have not established a complete numerical model to calculate load on each tooth of the spline in different sequences. Although some considered the influence of the sequence, the calculation method of the tooth clearances was not given. It was unable to analyse the effect of clearance uncertainty on the load distributed law on the spline teeth.

Because of the deficiencies of the above models, this paper first analyses the calculation method of tooth clearances when spline is combined in different sequences. Then, based on the proposed method for judging the number of spline meshing teeth, the load distribution on each tooth is calculated. Finally, a simplified assembly sequence selection method is established. The difference of the loads on the spline teeth between the optimized sequence and the traditional random sequence are compared through a case. The results show that the optimization method can further improve the bearing capacity of spline.

2. Calculation of tooth clearances considering the influence of assembly sequence

When splines are combined in different sequences, the teeth and slots are assembled according to the corresponding position relationship. Randomly take a slot and a tooth as the starting point, and number each slot and tooth in clockwise order as \(1, 2, \ldots, n\). Furthermore, when a slot numbered \(1'\) is combined with a tooth numbered \(i'\), the combined order is \(i\). There are \(n\) assembly sequences for spline with \(n\) teeth. As shown in figure 1, the spline assembly sequence is 1 and 4 respectively.

![Figure 1. Spline in different assembly sequences.](image)

(a) Assembly sequence is 1, (b) assembly sequence is 4.

2.1. Calculation of assembly interference considering machining errors

For an ideal error-free spline, when an internal spline and an external spline are assembled, each tooth fits into the slot, and the tooth clearances are zero, as shown in figure 2.

![Figure 2. Error-free softs and teeth assembly.](image)
For the spline teeth and slots in the two-dimensional plane, according to the error definition [11], the positional relationship can be expressed by the indexing error and the slot width/tooth thickness error. For the convenience of calculation, the positive direction of the indexing error is defined as the direction of movement along the external spline relative to the internal spline. On the basis of figure 2, if the tooth has a positive indexing error $\delta_{sz}$, the actual tooth thickness is $S_r$, and the basic tooth thickness is $S$, the positions of the tooth and slot are shown in figure 3. The dotted line represents the contour of the slot without error.

![Figure 3](image_url)

**Figure 3.** Assemble the tooth with machining errors and the slot without error.

When only the indexing error of tooth is considered, the interference of the tooth invading the slot is

$$I_{s,1} = \delta_{sz}$$

(1)

When only the tooth thickness error is considered, the interference is

$$I_{s,2} = \frac{S_r - S}{2}$$

(2)

Therefore, when the indexing error and the tooth thickness error are both considered, the interference is

$$I_s = \delta_{sz} + \frac{S_r - S}{2}$$

(3)

At the same time, if the tooth is error-free, the indexing error of the slot in figure 4 is negative. When only the indexing error is considered, the interference is

$$I_{e,1} = -\delta_{ez}$$

(4)

When only the slot width error is considered, the interference is
\[ l_{E,2} = \frac{E - E_r}{2} \] (5)

When the indexing error and the slot width error are both considered, the interference is

\[ l_E = -\delta_{EZ} + \frac{E - E_r}{2} \] (6)

Therefore, if there are errors between the spline tooth and the tooth slot, the interference is

\[ l = l_S + l_E = \delta_{SZ} - \delta_{EZ} + \frac{S_r - E_r}{2} \] (7)

If the interference calculated by equation (7) is negative, there is tooth clearance between the mating surface of the spline. For any two spline teeth, if the indexing errors of the teeth are \( \delta_{SZ,i} \) and \( \delta_{SZ,j} \), the indexing errors of the tooth slots are \( \delta_{EZ,i} \) and \( \delta_{EZ,j} \), the actual tooth thicknesses are \( S_{r,i} \) and \( S_{r,j} \), the actual slot widths are \( E_{r,i} \) and \( E_{r,j} \), the positional relationship is shown in figure 5(a). Then the interferences of tooth \( i \) and tooth \( j \) are

\[ l_i = \delta_{SZ,i} - \delta_{EZ,i} + \frac{S_{r,i} - E_{r,i}}{2} \] (8)

\[ l_j = \delta_{SZ,j} - \delta_{EZ,j} + \frac{S_{r,j} - E_{r,j}}{2} \] (9)

![Figure 5. Assembly of internal spline and external spline with errors.](image)

(a) Position with interference; (b) position after moving according to the maximum interference

2.2. Calculation of tooth clearances in different assembly sequences

If the values of \( l_i \) and \( l_j \) are negative, there is no interference between the tooth and the tooth slot, the tooth clearance is respectively

\[ a_i = -l_i = \frac{1}{2} (E_{r,i} - S_{r,i}) - \delta_{SZ,i} + \delta_{EZ,i} \] (10)

\[ a_j = -l_j = \frac{1}{2} (E_{r,j} - S_{r,j}) - \delta_{SZ,j} + \delta_{EZ,j} \] (11)
If the interference \( i \) or \( j \) contains positive value, there is interference. When combining the spline in the manner shown in figure 2, the external spline needs to be moved by a distance \( x \) in the negative direction. The position after movement is shown in figure 5(b). Where \( x \) is the maximum interference, i.e.

\[
x = \max(l_i, l_j)
\]

After movement, the tooth clearance is respectively

\[
a_i = x + \frac{1}{2}(E_{r,i} - S_{r,i}) - \delta_{SZ,i} + \delta_{EZ,i}
\]

\[
a_j = x + \frac{1}{2}(E_{r,j} - S_{r,j}) - \delta_{SZ,j} + \delta_{EZ,j}
\]

Assuming that the interference of teeth \( i \) is the largest, then bring \( x = l_i \) into equation (13) to obtain

\[
a_i = 0
\]

Equation (15) shows that at least one clearance of the spline with the side-fit is zero, which is consistent with the actual situation. If the interference is negative, the equation (13) and equation (14) still hold.

For the spline with \( n \) teeth, the tooth slots and teeth are numbered as shown in figure 1. The indexing errors of the tooth slots are set to \( \delta_{EZ,i} = \left[ \delta_{EZ,1}, \ldots, \delta_{EZ,j}, \ldots, \delta_{EZ,n} \right]^T \), the indexing errors of the teeth is \( \delta_{SZ,i} = \left[ \delta_{SZ,1}, \ldots, \delta_{SZ,j}, \ldots, \delta_{SZ,n} \right]^T \), the width of slots are \( E_r = \left[ E_{r,1}, \ldots, E_{r,j}, \ldots, E_{r,n} \right]^T \), and the tooth thicknesses are \( S_r = \left[ S_{r,1}, \ldots, S_{r,j}, \ldots, S_{r,n} \right]^T \). According to the positional correspondence, when the assembly sequence is \( i \), the numbers of teeth matching the slots \( 1', 2', \ldots, n' \) is \( i', \ldots, n', i', \ldots, (i - 1)' \). Therefore, when the assembly sequence is \( i \), the clearances calculated according to equation (13) are

\[
\hat{a}^i = \left[ \begin{array}{c}
\max\left( \delta_{SZ,i} - \delta_{EZ,i} + \frac{S_{r,i} - E_{r,i}}{2} \right) + \frac{1}{2}(E_{r,1} - S_{r,1}) + \delta_{EZ,1} - \delta_{SZ,1} \\
\vdots \\
\max\left( \delta_{SZ,i} - \delta_{EZ,i} + \frac{S_{r,i} - E_{r,i}}{2} \right) + \frac{1}{2}(E_{r,n-i+1} - S_{r,1}) + \delta_{EZ,n-i+1} - \delta_{SZ,n} \\
\max\left( \delta_{SZ,i} - \delta_{EZ,i} + \frac{S_{r,i} - E_{r,i}}{2} \right) + \frac{1}{2}(E_{r,n-i+2} - S_{r,1}) + \delta_{EZ,n-i+2} - \delta_{SZ,1} \\
\vdots \\
\max\left( \delta_{SZ,i} - \delta_{EZ,i} + \frac{S_{r,i} - E_{r,i}}{2} \right) + \frac{1}{2}(E_{r,n} - S_{r,1}) + \delta_{EZ,n} - \delta_{SZ,1} 
\end{array} \right]
\]

The clearances under each assembly sequence are

\[
\hat{A} = [\hat{a}^1, \ldots, \hat{a}^i, \ldots, \hat{a}^n]
\]

3. Calculation of load distribution on spline teeth
When spline is in different assembly sequences, the load distribution on each tooth is different. And if the maximum load distributed on the tooth is the minimum value of all sequences, the spline in this sequence has the maximum bearing capacity. Therefore, to optimize assembly sequence of spline combination, it is necessary to calculate the loads distributed on the teeth based on known tooth clearances.

In this paper, when calculating the load distribution on the spline teeth, it is assumed that the tooth is completely fitted with the slot, the influences of the axial and radial deformation of the tooth and the slot are not considered.

3.1 Establishment of load distribution model
When the internal and external spline mesh, the teeth and slots will be deformed. For the convenience of calculation, this paper only considers the deformation within the elastic range, so the nominal tangential force and corresponding deformation of the spline teeth obey Hooke's law. According to the reference [12], the single tooth stiffness of internal and external spline is calculated as $K_i$ and $K_e$. The matched tooth and slot are equivalent to two spring systems in series, as shown in figure 6. The meshing stiffness $K$ is

$$K = \frac{K_i \cdot K_e}{K_i + K_e} \quad (18)$$

![Figure 6. Equivalent calculation model of spline tooth stiffness.](image)

For a spline with $n$ teeth, sort the tooth clearances $a_1, a_2, \ldots, a_n$ from small to large as $a_{(1)}, a_{(2)}, \ldots, a_{(n)}$, the corresponding spline teeth are numbered 1, 2, ..., $n$ in order. To associate the series spring system established above with spline, the parallel spring system as shown in figure 7 is established.

![Figure 7. Equivalent mechanical model of spline.](image)

Each spring corresponds to a pair of meshing tooth and slot. The initial distance from load $F$ to each spring is the initial clearance of the spline, and the springs are numbered as 1, 2, ..., $n$. The distance $b$ is the common deformation of all compressed springs under the action of load $F$, that is, the common shape variable of each spline tooth. If the action point of the load on the spline tooth is at the indexing circle $D$, and the torsional load transmitted by the spline is $M$, then the load $F$ on the parallel spring system can be expressed as
\[ F = \frac{2M}{D} \]  

(19)

\( F \) is the sum of the nominal tangential forces carried on the teeth of the spline.

Therefore, under the action of torsional load \( F \), the number of springs that are compressed represents the number of teeth that the spline is engaged in, and the load distribution on each spring represents the load on the tooth.

3.2. Calculation of meshing teeth

According to the equivalent mechanical model of the spline, when the load \( F \) has just contacted the second spring, the deformation of the first spring is \( a_{(2)} - a_{(1)} \); when the load \( F \) has just touched the third spring, the deformation of the first spring is \( a_{(3)} - a_{(1)} \), and the deformation of the second spring is \( a_{(3)} - a_{(2)} \); By analogy, when the load \( F \) just contacts the \( i \)-th spring, the deformation of the \( i \)-th spring is \( a_{(i)} - a_{(1)} \). Considering that the spline tooth stiffness is different due to the difference in tooth shape, the meshing stiffness of each spline tooth numbered \( 1, 2, ..., n \) is \( K_1, K_2, ..., K_n \), that is, the spring stiffness are \( 1, 2, ..., nK K K \). According to Hooke's law, when the load \( F \) just contacts the \( (j+1) \)-th spring, the sum of the front \( j \) springs is

\[ F_{k,j} = K_1\left(a_{(j+1)}-a_{(1)}\right) + K_2\left(a_{(j+1)}-a_{(2)}\right) + \ldots + K_j\left(a_{(j+1)}-a_{(j)}\right) \]  

(20)

When the load \( F \) is acting, it is assumed that \( j(1 \leq j < n) \) springs are compressed. If the top \( j \) springs are compressed to the same length as the \( (j+1) \)-th spring, the sum of the front \( j \) springs’ elastic forces must be greater than or equal to the load \( F \), as shown in equation (21).

\[ K_1\left[a_{(j+1)} - a_{(1)}\right] + K_2\left[a_{(j+1)} - a_{(2)}\right] + \ldots + K_j\left[a_{(j+1)} - a_{(j)}\right] \geq F \]  

(21)

Bring \( j \) from 1 to \( n-1 \) into equation (21), first satisfy the value of \( j \) of the inequality relationship, which is the number of springs that are compressed under the action of load \( F \). If each \( j \) substituted does not satisfy the magnitude relationship of equation (21), then \( n \) springs are compressed. Put equation (19) into equation (21), and the value of \( j \) is the number of meshing teeth of spline under torsion load \( M \).

Since machining errors have little effect on the stiffness of spline teeth, in order to facilitate the calculation, it is considered that the stiffness of each spline tooth is the stiffness under the ideal tooth shape. Then the inequality for calculating the number of teeth engaged is simplified to

\[ K\left[ja_{(j+1)} - \sum_{i=1}^{j} a_{(i)}\right] \geq \frac{2M}{D}, (1 \leq j < n) \]  

(22)

3.3. Calculation of nominal tangential force of spline teeth

According to the conclusion in 3.1 and 3.2, when the number of teeth engaged is \( j \), according to the mechanical balance relationship, the sum of the elastic force generated by elastic deformation of each tooth and slot is equal to the external load, that is

\[ \left(a_{(j)} - a_{(1)} + b\right) + K_2\left(a_{(j)} - a_{(2)} + b\right) + \ldots + K_{j-1}\left(a_{(j)} - a_{(j-1)} + b\right) + K_jb = \frac{2M}{D} \]  

(23)

From above
\[
\begin{align*}
\frac{2M}{D} + \sum_{p=1}^{j} K_p a(p) & \quad \sum_{p=1}^{j} K_p - a(j) \\
&= \frac{2M}{D} + \sum_{p=1}^{j} K_p a(p) \quad \sum_{p=1}^{j} K_p - a(i) \\
&= K_i \left[ \frac{2M}{D} + \sum_{p=1}^{j} K_p a(p) \right] - K_i a(i) \\
&= 2M + \frac{1}{j} K \sum_{p=1}^{j} a(p) - K a(i) \\
&= \frac{2M}{jD} + \frac{1}{j} K \sum_{p=1}^{j} a(p) - K a(i) \\
&= 2M + \frac{1}{j} K \sum_{p=1}^{j} a(p) - K a(i) \\
\end{align*}
\]

The nominal tangential force on any tooth \( i \) \((i=1, 2, \ldots, j)\) is

\[
F_i = K_i \left( a(j) - a(i) \right) + K_i b
\]

\[
= K_i \left[ \frac{2M}{D} + \sum_{p=1}^{j} K_p a(p) \right] - K_i a(i)
\]

If the effect of tooth shape difference on spline stiffness is not considered, the nominal tangential force on any tooth \( i \) \((i=1, 2, \ldots, j)\) is

\[
F_i = \frac{2M}{jD} + \frac{1}{j} K \sum_{p=1}^{j} a(p) - K a(i)
\]

Because most of the cracks first appear on the most loaded teeth, the key is to calculate the maximum load on the tooth. When the clearance \( a(i) = a(1) \) is the minimum value, \( F_i \) has the maximum value, that is

\[
F_{\text{max}} = \frac{2M}{jD} + \frac{1}{j} K \sum_{p=1}^{j} a(p) - K a(1)
\]

4. Effect of assembly sequence on load distribution

4.1. Selection of optimized spline assembly sequence

For the deterministic optimization method of spline assembly sequence, Yu et al. [3] have already made a part of the explanation, the calculation process is shown in figure 8. In addition, combined with the calculation method of the spline tooth clearances, it can be further improved: for the spline with \( n \) teeth, the tooth clearances can be calculated according to equation (17); the minimum value among the \( n \) maximum nominal tangential forces is selected; the spline assembly sequence corresponding to the minimum value is the optimal assembly sequence.
The clearances in assembly sequence \( i \):
\[ a_i = [a_{i1}, a_{i2}, \ldots, a_{in}] \]

Calculate the maximum nominal tangential force
\[ F[i] = F_{\text{max}} \]

The optimal combination order of spline is \( i=j \)

Figure 8. The process of optimizing the sequence of spline combination through the deterministic optimization method and the maximum clearance method.

However, for a large number of splines, if the load distributed on the spline teeth in different assembly sequences is calculated one by one, the efficiency will be extremely low. Therefore, a simplified selection method of spline assembly sequence is proposed.

The indexing errors of spline and the tooth thickness/slot width errors follow the normal distribution [13]. From equation (10), the tooth clearances also follow the normal distribution. Use equation (10) to calculate the minimum value of the tooth clearances in the assembly sequence \( 1, i, \ldots, n \), which is recorded as
\[
L_{\text{min}} = [a_{\text{min}}^1, a_{\text{min}}^i, \ldots, a_{\text{min}}^n]
\] (28)

Let the minimum value of \( L_{\text{min}} \) be \( l_{\text{min}} \), the maximum value be \( l_{\text{max}} \). According to the distribution characteristics of normal distribution "middle height, two ends bottom", when the minimum clearance is \( l_{\text{max}} \), compared with \( l_{\text{min}} \), the dispersion of the clearances is smaller. So the load is more evenly distributed on each tooth, and the maximum load on the tooth is smaller. Therefore, when the minimum tooth clearance is \( l_{\text{max}} \), the spline is most likely in the optimal assembly sequence. The method of selecting spline assembly sequence according to \( l_{\text{max}} \) is recorded as the maximum clearance method. The process of optimizing the sequence of spline through the maximum clearance method is shown in figure 8.

4.2. Comparison of load distribution on teeth with different assembly sequences
In the actual process of assembling the spline, the influence of the sequences on the load distribution on the teeth is not considered, internal and external spline are assembled together randomly. So according to the traditional way of assembling splines, the sequences obeys the uniform distribution.

The basic parameters of the spline are shown in table 1 and table 2, the torque applied to the spline is 3000 N·m, 6000 N·m, 9000 N·m, and 12000 N·m. 200000 sets of data are randomly obtained within the tolerance range of the spline fitting dimension. Each set of data is composed of the tooth thickness/slot width errors and indexing errors. The deterministic optimization method, the maximum clearance method, and the traditional random combination method are used to calculate the probability density of the maximum nominal tangential force on the teeth. The results are shown in figure 9.

### Table 1. Basic parameters of spline.

| Number of teeth $n$ | Standard pressure angle $\alpha_D$ | Module $m$ | Contact length $l$ | Tolerance grade | Fit category |
|---------------------|-----------------------------------|------------|--------------------|-----------------|-------------|
| 40                  | 30°                               | 2mm        | 40mm               | 4               | H/If        |

### Table 2. Conventional mechanical properties of materials.

| Material           | Yield strength $\sigma_{0.2}$ | Tensile strength $\sigma_b$ | Modulus of elasticity | Poisson’s ratio $\mu$ |
|--------------------|--------------------------------|-----------------------------|-----------------------|-----------------------|
| External spline    | Brass 575MPa                   | 750MPa                      | 112GPa                | 0.34                  |
| Internal spline    | 20Cr2Ni4A 1292MPa              | 1483MPa                     | 207GPa                | 0.29                  |

**Figure 9.** Probability density of maximum nominal tangential force.
It can be seen from figure 9 that when the above three combination methods are used to assemble splines, the maximum nominal tangential force follows a lognormal distribution. When the splines are randomly combined, the range and average value of the nominal tangential forces are significantly higher than when the splines are combined using the deterministic optimization method and the maximum clearance method. When the torque is 3000 N·m, the range of load distribution obtained by the maximum clearance method is about 1/2 of the traditional method, and the average load is about 3/4 of the random way. Therefore, by optimizing the assembly sequence, the bearing capacity of the spline is improved.

On the other hand, as the torque increases, the maximum load on the teeth optimized by the maximum clearance method gradually approaches the deterministic method. When the torsional load is 9000 N·m or 12000 N·m, the maximum clearance method can fit the load distribution law obtained by the deterministic optimization method well.

5. Conclusion

(1) The influence of machining errors on the spline assembly position was analysed, and the positional relationship between teeth and slots in different assembly sequences was studied. The clearances in different assembly sequences were expressed by an analytical formula.

(2) An equivalent model of spline load distribution was established. The number of spline meshing teeth was predicted by an iterative method, and the load distribution on each spline tooth was calculated. The model showed that the number of spline meshing teeth was related to the clearances, tooth stiffness, torque, and basic parameters of the spline. The maximum nominal tangential force was related to the minimum clearance.

(3) The maximum clearance method was proposed to select spline assembly sequence. By optimizing the sequence of splines, the range of load distribution is about 1/2 of the traditional method, and the average load is about 3/4 of the random way. When the torque was large, the errors between the two optimization methods was minimal.

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