Study of Pulse Scattering from Two-Dimensional Non-Gaussian Rough Ocean Surfaces

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Abstract. In this paper the analytical expression of two-frequency mutual coherence function of the scattered wave from two-dimensional non-Gaussian rough ocean surfaces is derived first by the Kirchhoff approximation. The Gram-Charlier distribution instead of the conventional Gaussian distribution is utilized in carrying out the ensemble average operation in order to represent the asymmetric characteristics of ocean surfaces. Then the scattered pulse for arbitrary incident pulse is derived utilizing the two-frequency mutual coherence function for wide-sense stationary uncorrelated scattering channel. Finally the two-frequency mutual coherence function and the scattered pulse with a Gaussian modulated or rectangular incident pulse from two-dimensional non-Gaussian rough ocean surfaces for different wind speed are numerically simulated, and the pulse broadening phenomenon is discussed.

1. Introduction

Study of pulse scattering from two-dimensional rough surfaces is of great theoretical and engineering significance. By investigating the propagation of pulse waves in random media, Bello and Ishimaru found that the propagation characteristics of pulse waves can be described by the two-frequency mutual coherence function (MCF)\cite{1-3}. Based on this idea Ishimaru first studied the pulse scattering from one-dimensional rough surfaces utilizing Kirchhoff approximation (KA)\cite{4}. In this method the height distribution of rough surfaces is needed, and conventionally it is assumed to satisfy the Gaussian distribution for simplicity. However for two-dimensional rough ocean surfaces due to the influence of the wind the crests of the waves are tilted towards the wind direction, i.e., the ocean surfaces exhibit asymmetry, which causes the height distribution of ocean surfaces to deviate from the conventional Gaussian distribution.

In this paper the Gram-Charlier distribution which can describe the asymmetry of ocean surfaces is utilized to derive the two-frequency MCF of scattered wave from two-dimensional rough ocean surfaces, and the numerical simulation is performed and discussed.

2. Two-frequency mutual coherence function from two-dimensional rough ocean surfaces

Let two plane waves with different frequency impinge upon an \( L \times L \) two-dimensional rough ocean surface from the same incident direction, the incident fields are defined as

\[ \tilde{E}_{i,\nu} = \hat{\nu} E_{\nu} \exp\left(-j\hat{\nu} \cdot \hat{r}\right) \]

(1)
\[ E_{s,p} = \hat{p} E_0 \exp\left(-j \hat{k}_s \cdot r\right) \tag{2} \]
where a time factor of the form \( \exp\left(j \omega t\right) \) is understood, \( \hat{p} \) and \( E_0 \) are the unit polarization vector and amplitude of the incident electric field, \( \hat{k}_s \) and \( \hat{k}'_s \) are the propagation vector of the two incident waves with different frequency, which can be expressed as
\[
\hat{k}_s = k \sin \theta \cos \phi \hat{x} + k \sin \theta \sin \phi \hat{y} - k \cos \theta \hat{z} \\
\hat{k}'_s = k' \sin \theta \cos \phi \hat{x} + k' \sin \theta \sin \phi \hat{y} - k' \cos \theta \hat{z}
\]

In accordance with the Kirchhoff Approximation\[5\], the far-zone scattered field can be expressed as
\[
E_{s,p} = C E_0 \int f_{p,q} \exp\left[j\left(\hat{k}_s - \hat{k}_s\right) \cdot r\right] \, dvdy \tag{3} \\
E_{s,p} = C' E_0 \int f_{p,q} \exp\left[j\left(\hat{k}_s - \hat{k}'_s\right) \cdot r\right] \, dvdy \tag{4}
\]
where the incident polarization is denoted by \( p \) and the receiving polarization by \( q \), the expression of \( f_{p,q} \) can be found in reference \[5\], \( C = -jk \exp\left(-jkR\right)/4\pi R \), \( C' = -jk' \exp\left(-jk'R\right)/4\pi R \), and \( R \) is the range from the center of the illuminated area to the point of observation. \( \hat{k}_s \) and \( \hat{k}'_s \) are the propagation vector of the scattered waves, which can be represented by
\[
\hat{k}_s = k \sin \theta \cos \phi \hat{x} + k \sin \theta \sin \phi \hat{y} + k \cos \theta \hat{z} \\
\hat{k}'_s = k' \sin \theta \cos \phi \hat{x} + k' \sin \theta \sin \phi \hat{y} + k' \cos \theta \hat{z}
\]

Similar to the scattering coefficient, the two-frequency MCF can be defined as
\[
\Gamma_{q,p} = \frac{4\pi R^2}{A_0 E_0^2} \left\langle \left(E_{s,p} E_{s,p}^* \right)-\left(E_{s,p} \right)\left(E_{s,p}^* \right) \right\rangle \tag{5}
\]
where \( \langle \rangle \) is the ensemble average operator, \( * \) is the symbol for complex conjugate and \( A_0 = L^2 \) is the illuminated area.

Substituting (3) and (4) into (9) and letting \( \bar{v} = \hat{k}_s - \hat{k}'_s \), \( \bar{v}' = \hat{k}'_s - \hat{k}'_s \), we have
\[
\Gamma_{q,p} = CC^* f_{q,p} f_{q,p}^* \int \exp\left[j\left(\bar{v} \cdot x + \bar{v}' \cdot y - \bar{v} \cdot x' - \bar{v}' \cdot y'\right)\right] \\
\cdot \left\langle \left\{ \exp\left[j(\bar{v} \cdot z + \bar{v}' \cdot z')\right]\right\} \right\rangle \left\langle \left\{ \exp\left[-j(\bar{v} \cdot z')\right]\right\} \right\rangle \, dvdydv'dy' \tag{6}
\]

To carry out the ensemble average operation we must make an assumption about the height distribution of rough ocean surfaces. Conventionally the Gaussian distribution is assumed for simplicity. However, due to the influence of the wind the crests of the waves are tilted towards the wind direction. To describe the asymmetry of ocean surfaces, the Gram-Charlier distribution\[6\] is used in this paper, which is represented by
\[
P(z) = \frac{1}{\sigma \sqrt{2\pi}} \exp\left(-\frac{z^2}{2\sigma^2}\right) \left[1 + \frac{\lambda z}{6\sigma} \left(\frac{z^2}{\sigma^2} - 3\right)\right] \tag{7}
\]
where \( \lambda = \mu^2 / \sigma^2 \) is called the skewness coefficient, \( \sigma \) and \( \mu \) are the variance and third-order moment of ocean surfaces, respectively. By the method of characteristic function, we can calculate the ensemble average in (6) and finally we can get
\[
\Gamma_{q,p} = CC^* f_{q,p} f_{q,p}^* \Phi \Phi_s \int \Psi_s(x,y) \exp\left[j\left(\frac{v_x + v'_x}{2} x + \frac{v_y + v'_y}{2} y\right)\right] \, dvdy \tag{8}
\]
where
\[
\Phi = A_0 \text{sinc} \left[\left(\frac{v_x - v'_x}{2}\right)L\right] \text{sinc} \left[\left(\frac{v_y - v'_y}{2}\right)L\right] \tag{9}
\]
\[
\Phi_s = \exp\left[-\frac{\sigma^2}{2}(v_x^2 + v'_x^2) - \frac{\sigma^2}{6}(v_y^2 + v'_y^2)\right] \tag{10}
\]
\[
\Psi_k(x, y) = \exp\left[ v_x v_y \rho(x, y) - \frac{1}{2} S_x(x, y)(v_x^2 + v_y^2) \right] - 1
\]  

(11)

In (11) \( \rho(x, y) \) is the normalized correlation function of ocean surfaces and can be obtained by the inverse Fourier transform of the ocean surfaces roughness spectrum \( W(k_x, k_y) \), i.e.

\[
\rho(x, y) = \frac{1}{(2\pi)^2 \sigma} \int W(k_x, k_y) \exp\left[ j(k_x x + k_y y) \right] dk_x dk_y
\]  

(12)

\( S_x(x, y) \) is called the skewness function, which represents the asymmetric characteristics of ocean surfaces and is defined in polar coordinates as[5]

\[
S_x(\xi, \phi) = \frac{\xi^3 \cos \phi}{s_0^3} \exp\left( -\frac{\xi^3}{s_0^3} \right)
\]  

(13)

From (13) it can be shown that the skewness function changes sign as the wind direction changes 180 degrees, i.e., there is a sign difference between upwind and downwind direction. In addition, it vanishes in the crosswind direction \( (\phi = 90^\circ) \), which means there is no effect of skewness in the crosswind direction.

3. Pulse scattering from two-dimensional rough ocean surfaces

For wide-sense stationary uncorrelated scattering channel (WSSUS), the scattered power can be expressed with the incident power and the two-frequency MCF as[4]

\[
P_s(t) = \frac{A_0}{4\pi R} \frac{1}{2\pi} \left| \Gamma(\omega_1) P(\omega_2) \exp\left( j\omega_2 t \right) \right|^2 \, d\omega_2
\]  

(14)

where

\[
P(\omega_2) = \int P(t) \exp(-j\omega_2 t) \, dt
\]

For the incident Gaussian modulated pulse with unit amplitude as follows

\[
E_i(t) = \exp\left( j\omega_1 t - \frac{t^2}{2T^2} \right)
\]  

(15)

the incident power can be calculated as

\[
P_i(\omega_2) = T \sqrt{\frac{\pi}{2}} \exp\left( -\frac{\omega_2^2 T^2}{8} \right)
\]  

(16)

And for the incident rectangular pulse with unit amplitude as follows

\[
E_i(t) = \begin{cases} 
\exp(j\omega_1 t) & |t| \leq T \\
0 & |t| > T 
\end{cases}
\]  

(17)

the incident power can be calculated as

\[
P_i(\omega_2) = \text{sinc} \left( \frac{\omega_2 T}{2} \right)
\]  

(18)

By substituting (16) or (18) into (14), the scattered power with an incident Gaussian modulated pulse or rectangular pulse can be obtained.

4. Numerical simulation and analysis

The variation of backscattering two-frequency MCF from two-dimensional rough ocean surfaces with frequency difference and azimuth angle is calculated for HH polarization in the 12-14 GHz frequency range. The semi-empirical sea-spectrum model[7] is utilized in this paper and the incident angle is \( \theta_i = 30^\circ \). The result is displayed in Figure 1(a) and Figure 1(b), where the wind speed is \( v = 5\text{m/s} \) and \( v = 10\text{m/s} \), respectively. It can be shown that as the frequency difference increases, the two-frequency MCF drops from the maximum value to zero, which means the coherence between the two waves with different frequency declines. However, for the upwind direction (azimuth \( \phi = 0^\circ \)) and downwind
direction ($\phi = 180^\circ$) there is an obvious difference of the two-frequency MCF, which means the Gram-Charlier distribution can represent the asymmetric characteristics of ocean surfaces and can therefore be utilized to describe the ocean surfaces properly. In addition, the two-frequency MCF declines more slowly in the crosswind direction ($\phi = 90^\circ$) whereas in other directions it declines quickly. To show the difference between the upwind and downwind directions further, the variation of two-frequency MCF with the azimuth angle for frequency difference equalling zero and wind speed equalling 10m/s is displayed in Figure 2 and the measured data are also displayed for comparison. In accordance with the definition of the two-frequency MCF it will degenerate into the scattering coefficient when the frequency difference becomes zero. From Figure 2 it is clear that there is about a 1.5dB difference in backscattering coefficient between the upwind and downwind directions, and the calculated result is in good agreement with the measured data[8].

![Variation of backscattering two-frequency MCF with frequency difference and azimuth angle.](a) $v = 5\text{m/s}$, (b) $v = 10\text{m/s}$.](b)

![Variation of backscattering coefficient with azimuth angle.](Figure 2. Variation of backscattering coefficient with azimuth angle for $v=10\text{m/s}$ and $\theta_i = 30^\circ$.)

The scattered pulses with Gaussian modulated and rectangular incident pulses are investigated for the upwind direction, and the comparison between the incident pulses and scattered pulses with amplitude normalized in the forward scattering direction is illustrated in Figure 3, where $T = 1\text{ns}$. To remove the influence of range $R$, the scattered power $P_s(t)$ in (14) is multiplied by a factor of $4\pi R^2/A_0$. It is shown that for either the Gaussian modulated incident pulse or the rectangular incident
pulse the scattered pulses are broadened compared with the incident pulses, which is called the pulse broadening phenomenon, and the extent of broadening is almost invariant with the wind speed.

![Pulse shape comparison](image)

Figure 3. Comparison of scattered pulses and incident pulses for the upwind direction. (a) the incident pulse is a Gaussian modulated pulse, (b) the incident pulse is a rectangular pulse.

5. Conclusion
In this paper the analytical expression of two-frequency MCF of the scattered wave from two-dimensional non-Gaussian rough ocean surfaces is derived by the Kirchhoff approximation. In carrying out the ensemble average operation the height distribution of rough ocean surfaces is assumed to satisfy the Gram-Charlier distribution instead of the conventional Gaussian distribution, therefore the asymmetric characteristics of ocean surfaces can be described properly. By numerical simulation it is concluded that the two-frequency MCF differs in the upwind direction and downwind direction, and it declines more slowly in the crosswind direction than in other directions. Finally the scattered power is expressed with the incident power and the two-frequency MCF for wide-sense stationary uncorrelated scattering channel, and the numerical simulation indicates that for either the Gaussian modulated incident pulse or the rectangular incident pulse the scattered pulses are broadened and the extent of broadening is almost invariant with the wind speed.

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