ESTIMATE OF THE TOTAL MECHANICAL FEEDBACK ENERGY FROM GALAXY CLUSTER-CENTERED BLACK HOLES: IMPLICATIONS FOR BLACK HOLE EVOLUTION, CLUSTER GAS FRACTION, AND ENTROPY

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ABSTRACT

The total feedback energy injected into hot gas in galaxy clusters by central black holes can be estimated by comparing the potential energy of observed cluster gas profiles with the potential energy of non-radiating, feedback-free hot gas atmospheres resulting from gravitational collapse in clusters of the same total mass. Feedback energy from cluster-centered black holes expands the cluster gas, lowering the gas-to-dark-matter mass ratio below the cosmic value. Feedback energy is unnecessarily delivered by radio-emitting jets to distant gas far beyond the cooling radius where the cooling time equals the cluster lifetime. For clusters of mass \( \left(4 - 11\right) \times 10^{14} M_{\odot} \), estimates of the total feedback energy, \((1 - 3) \times 10^{63} \text{ erg}\), far exceed feedback energies estimated from observations of X-ray cavities and shocks in the cluster gas, energies gained from supernovae, and energies lost from cluster gas by radiation. The time-averaged mean feedback luminosity is comparable to those of powerful quasars, implying that some significant fraction of this energy may arise from the spin of the black hole. The universal entropy profile in feedback-free gaseous atmospheres in Navarro–Frenk–White cluster halos can be recovered by multiplying the observed gas entropy profile of any relaxed cluster by a factor involving the gas fraction profile. While the feedback energy and associated mass outflow in the clusters we consider far exceed that necessary to stop cooling inflow, the time-averaged mass outflow at the cooling radius almost exactly balances the mass that cools within this radius, an essential condition to shut down cluster cooling flows.

Key words: black hole physics – galaxies: clusters: general – galaxies: clusters: intracluster medium – X-rays: galaxies: clusters

1. INTRODUCTION

Massive black holes in the cores of group and cluster-centered galaxies become active when a tiny mass of cluster gas is accreted, causing large amounts of “feedback” energy to be deposited in the surrounding cluster gas. The energy provided by occasional feedback events, in the form of jets containing supra-thermal and relativistic particles, increases the entropy of both nearby and very distant cluster gas. Feedback jets create expanding, shock-driving cavities in the hot cluster gas, increasing its entropy and delaying its flow toward the black hole by radiation losses. In the absence of feedback, the loss of entropy by thermal X-radiation is accompanied by a slow cooling inflow toward the black hole which, if unabated over time, would concentrate a mass of centrally cooled gas in or near the central black hole far exceeding limits set by stellar velocities and other observations. For example, intermittent cavities formed from feedback events of energy \(10^{59}/2 \times 10^{50} \text{ yr at 10 kpc can arrest the currently observed gas density and temperature profiles (and low central cooling) in the Virgo cluster for} \sim 3 \text{ Gyr (Mathews 2009). Including more distant feedback events at 50 kpc can maintain the observed gas profiles and low cooling rate for several additional Gyr. Entropy and cosmic rays delivered to the cluster gas by feedback episodes cause gas to flow out in the cluster potential, offsetting the cooling inflow. In general, however, central black holes are unable to provide the optimal or minimum feedback energy that must be deposited at every radius in the cluster gas just sufficient to shut down black hole accretion of locally cooling gas. Instead, black holes typically overreact in a clumsy fashion, depositing much more energy than the minimum required, much of it in distant regions of the cluster where the radiative cooling time exceeds the age of the cluster. After a few \(10^8 \text{ yr following a feedback event most of the feedback energy converts to potential energy, as the entire gaseous cluster atmosphere adiabatically expands outward (Mathews & Brighenti 2008). Consequently, the increased cluster gas entropy is necessarily related to a reduction of the cluster gas density relative to the local dark matter.}

We discuss here an approximate estimation of the total feedback energy received by cluster gas during the cluster lifetime by comparing gas potential energy profiles in observed clusters with that of idealized gas density distributions resulting from “adiabatic” gravitational collapse into the cluster halo in the absence of radiative cooling and associated feedback. We show that this energy, \(\sim 10^{63} \text{ erg}\), far exceeds the energy lost by radiation during the cluster lifetime and consequently the minimum energy required merely to stop the cooling inflow. The collective energy from all supernovae also provides a negligible fraction of the total feedback energy. Since only 10% of the hot baryonic gas in massive clusters cools to form stars, star formation can also be ignored in our estimate of the global energetics of the cluster gas where we seek an overall accuracy of \(\sim 25\%.\)

Estimates of the total feedback energy from cluster-centered black holes are possible for two reasons. First, the potential energy of cluster gas in hydrostatic equilibrium in a fixed dark halo potential can be found by integrating outward from the cluster center. Second, to a good approximation, the formation of dark halos and their gravitational potential proceeds from the inside outward as the size of the virialized region in the dark halo increases with time. In view of this latter point, it is possible to...
estimate the increase in potential energy of the cluster gas due to feedback without knowing when the feedback occurred.

We begin by estimating the current gas density, temperature, mass, entropy, and potential energy profiles expected at zero redshift in the absence of radiative losses, star formation, and black hole feedback, referred to as the “adiabatic” cluster atmosphere. Then, assuming hydrostatic equilibrium, potential energy profiles are estimated from gas distributions in observed clusters having the same total mass. When comparing the difference between the potential energy evaluated at the same mass of cluster gas with and without feedback, we find that hot gas in the idealized adiabatic atmosphere must expand significantly to resemble cluster gas profiles currently observed. The total feedback energy associated with this expansion, \( \sim (1-3) \times 10^{63} \text{erg} \), comfortably exceeds energies of the most powerful known individual feedback events. Central to our feedback energy estimate is the inverse relationship between increasing cluster gas entropy and decreasing gas fraction, the ratio of gas to total cluster densities, which is lowered by a global expansion of the cluster gas.

If this feedback energy is released during periods of radiatively efficient central accretion with \( \sim 0.1 \) of the accreted mass returned as feedback energy to the cluster gas, we find that the final black hole masses in large clusters would exceed those observed. Alternatively, the feedback energy may be provided from the rotational energy of rapidly spinning central black holes.

2. ESTIMATE OF TOTAL FEEDBACK ENERGY

A key element in our estimate is the assertion that adiabatic (non-radiating, non-feedback) gaseous cluster atmospheres can be fit with the same properly normalized Navarro–Frenk–White (NFW) profile as the dark matter. Among the many computations of galaxy cluster formation that include both baryons and dark matter, surprisingly few have analyzed in detail the deep density and entropy profiles in dark halos and adiabatic gas described in the cosmological cluster formation calculations of Eke et al. (1998) and in particular Faltenbacher et al. (2007). The cluster gas entropy can be characterized with \( S_g = \frac{\sigma}{\sqrt{\rho}} \), where \( \rho \) is the gas density and \( \sigma = \frac{3kT}{(\mu m_p)} \)^{1/2} is the thermal velocity dispersion. By analogy, Faltenbacher et al. define a corresponding dark matter entropy \( S_{dm} = \frac{\sigma}{\rho}^{3/2} \) for which \( \rho \) is the dark matter density and \( \sigma \) is the three-dimensional velocity dispersion of collisionless dark matter particles. The detailed cosmological cluster calculations of Faltenbacher et al. (2007) using GADGET2 reveal that, apart from a small central gaseous core, the gas density and entropy profiles in the adiabatic case share identical, appropriately scaled NFW profiles regardless of cluster mass. Beyond the central core where \( S_g > S_{dm} \), both \( S_g \) and \( S_{dm} \) vary as power laws, \( \propto r^{-1.21} \), but with slightly different normalizations, \( S_g / S_{dm} = 0.71 \pm 0.18 \). However, as discussed by Faltenbacher et al., the final zero-redshift gas configuration in the gas contains residual subsonic macroscopic velocities which, if damped, would bring the gas and dark matter entropies into near perfect agreement, \( S_g / S_{dm} \approx 1 \). We adopt the assumption, implicit in the discussion of Faltenbacher et al., that the two entropies are in fact equal but the gas remains undamped because the appropriate physical damping mechanisms are absent from GADGET2. The artificial viscosity in this code that damps the accretion shock may require an unphysically long time to damp the residual subsonic velocity field. Assuming \( S_g \approx S_{dm} \) is also more consistent with the smaller \( \sim 5\% \) component of turbulent energy found for relaxed clusters in recent cosmological simulations using the ENZO code (Vazza et al. 2011). Finally, cluster virial masses are determined from observed gas pressure profiles without allowing for undamped kinetic energy and its associated pressure. This assumption (which underestimates the virial mass) is also consistent with \( S_g \approx S_{dm} \). While it is comforting that the radial profiles of adiabatic gas and dark matter are nearly identical, this may nonetheless be surprising due to the very different nature of their dissipative mechanisms.\(^1\)

More relevance to our discussion is the similar NFW shape of adiabatic gas and dark matter density profiles. The assumption \( S_g \approx S_{dm} \) ensures that the adiabatic gas has experienced the same dissipative history as the dark matter.

The radial dark matter NFW distribution is shaped by all entropy-producing dissipations that occurred during both smooth accretion of diffuse dark matter as well as inhomogeneous accretion of smaller groups, clusters, and filaments that merged into the final cluster potential. The entropy increases that accompanied the formation of these smaller structures are also embedded in the final NFW structure. Furthermore, the results of Faltenbacher et al. (2007) indicate that all dissipative information about the cluster merger history is also encoded in the NFW gas density profiles in idealized adiabatic baryon atmospheres formed by gravity alone.

When computing purely gravitational (adiabatic) collapse in cosmological calculations without radiation or feedback, a controversy has arisen in recent years concerning differences in the dissipation between dark matter and baryons in cluster cores where \( S_g > S_{dm} \) (recently reviewed by Borgani & Kravtsov 2009; Springel 2010; Vazza 2011). In mesh-based calculations the baryons are found to have large, well-resolved central density cores which are larger than those computed with smoothed particle hydrodynamics. Possible origins for this discrepancy have been proposed and discussed by Mitchell et al. (2009) and Vazza (2011). For our purposes here we assume two limiting adiabatic gas density profiles following pure gravitational baryonic collapse into dark halos: core and no core.

2.1. Adiabatic Cluster Gas Atmospheres without Cores

Consider first the “no core” case in which baryons and dark matter suffer the same dissipation so the post-collapse gas density profile is identical to that of the total density but scaled down by the universal baryon fraction \( f_b = 0.17 \), i.e., \( \rho = f_b \rho_i \) (e.g., Faltenbacher et al. 2007), where \( \rho_i \) is the total density dominated by dark matter. We seek the idealized radial structure of cluster gas formed without feedback or radiative losses and which has evolved to the current time. The temperature profile can be found by integrating the equation of hydrostatic equilibrium:

\[
\frac{d \theta}{dr} = \frac{\theta}{\rho} \frac{d \rho}{dr} - \frac{1}{g},
\]

\(^1\) In adiabatic cluster simulations gases of different entropies evidently mix in the cluster core, raising the total gas entropy, particularly in grid-based computations. However, beyond the core we expect the density of gas and dark matter to share the same appropriately normalized NFW profile. This profile similarity has been verified in many calculations including the Santa Barbara cluster, an average of 12 different structure formation codes using identical cosmologies and initial conditions (Frenk et al. 1999). In more recent adiabatic simulations the effective dark matter temperature profile \( T_d(r) = (\mu m_p/3k) \sigma^2 \) has been shown to be identical (within 10%) to the gas temperature profile \( T_g(r) \) (e.g., Host et al. 2009; ZuHone 2011).
where \( \theta = kT/\mu m_p \) and \( g = GM(r)/r^2 \). The self-gravitation of the gas is implicitly included in the total mass \( M(r) \) that includes both dark matter and gas. The total matter density and mass profile in a cluster with virial mass \( M_v \) are given by the usual NFW relations,

\[
\rho_c = \frac{\rho_0 \Delta_c}{y(1+y)^2} \quad \text{and} \quad M(r) = M_c \frac{f(y)}{f(c)},
\]

where \( y = r/r_c \) is the concentration and

\[
f(y) = \ln(1+y) - \frac{y}{(1+y)\Delta_c} = \Delta_c c^3/3 f(c).
\]

The radius where the local cluster density is \( \Delta \) times larger than the critical density \( \rho_c = 3H^2/8\pi G = 9.24 \times 10^{-30} \text{ g cm}^{-3} \) is

\[
r_\Delta = (3M_c/4\pi \Delta \rho_c)^{1/3}
\]

and the virial radius is \( r_v = r_\Delta \), where \( \Delta = 178(\Omega_m)^{0.45} = 103 \) (Eke et al. 1998) when \( \Omega_M = 0.3 \). For nearby clusters we assume \( H = 70 \text{ km s}^{-1} \text{ Mpc}^{-1} \). Finally, we adopt a convenient concentration–\( M_c \) relation \( c = 9(M_c/10^{14} M_\odot)^{-0.172} \) for low-redshift clusters (Buote et al. 2007).

In our integrations of Equation (1) using the observed gas density, we consider two clusters, each of which are composites of two very similar clusters selected from the sample of nearby relaxed clusters observed with the Chandra telescope by Vikhlinin et al. (2006). For our purposes we choose massive clusters in which the full impact of feedback energy can be absorbed and which do not presently have huge cavities that dominate and disrupt the cluster gas profiles observed. Of particular interest are the NFW parameters determined for these clusters and values of the observed cluster gas fraction \( f_b = \rho_b/\rho_c \) plotted in Figures 3–14 of Vikhlinin et al. To avoid being distracted by spurious observational errors and to improve the quality of our estimates, we combine the mean properties of two pairs of clusters that share nearly the same virial mass: (A133 + A383) and (A478 + A1413). The two averaged clusters are subsequently referred to as composite clusters 1 and 2, respectively. Relevant observed NFW properties of all four clusters and the two composite clusters are listed in Table 1. Each composite cluster is chosen to have values of \( r_{500} \) and \( M_{500} \) that are averages of the two observed clusters. Composite clusters 1 and 2 have virial masses \( M_c = 4.34 \times 10^{14} \) and \( M_c = 11.3 \times 10^{14} M_\odot \), respectively.

Our first integrations of Equation (1) are for adiabatic “no core” versions of the two composite clusters using \( \rho(r) = f_b \rho_c(r) \) for the gas density. We choose an initial temperature at some very small cluster radius and seek solutions for which the gas entropy \( S = \theta/\rho^{1/3} \) varies like a self-similar power law at large radius \( S \propto r^{q-2} \), similar to entropy profiles in detailed adiabatic gravitational collapse computations (e.g., Tozzi & Norman 2001; Voit 2005). This is a well-defined initial value problem with unique solutions. By varying the initial temperature, an exact value and solution can be found for which \( S \propto r^q \), where \( q \approx 1.2 \) remains constant over a large range of cluster radius near the virial radius and beyond. The value of \( q \) emerges naturally from the solution as an eigenvalue that is not imposed in advance.

Solid lines in Figures 1 and 2 show adiabatic post-collapse profiles of gas temperature, entropy, and density for composite clusters 1 and 2, respectively, found by integrating Equation (1).

### Table 1

| Cluster   | \( M_{500} \) (10^{14} M_\odot) | \( r_{500} \) (kpc) | \( T_{500} \) (keV) | \( M_c \) (10^{14} M_\odot) | \( r_v \) (kpc) |
|-----------|---------------------------------|---------------------|---------------------|-----------------------------|----------------|
| A133      | 3.17                            | 1007                | 3.67                | ...                         | ...           |
| A383      | 3.06                            | 944                 | 4.37                | ...                         | ...           |
| Cluster 1 | 3.11                            | 974                 | 4.02                | 4.34                        | 1950          |
| A478      | 7.68                            | 1337                | 7.36                | ...                         | ...           |
| A1413     | 7.57                            | 1299                | 6.81                | ...                         | ...           |
| Cluster 2 | 7.63                            | 1319                | 7.08                | 11.3                        | 2682          |

**Notes.**

a Cluster data from Vikhlinin et al. (2006).
b Cluster mass and radius at overdensity \( \Delta = 500 \).
c Density weighted mean temperature of cluster gas.
d Cluster mass and radius at virial radius.

Figure 1. Two composite atmospheres for composite cluster 1: a purely adiabatic virialized gaseous atmosphere without central core (solid lines) and an observed atmosphere in a cluster of the same mass (dotted lines). Upper panel: gas temperature profiles. Center panel: gas entropy profiles. The dash-dotted line shows the adiabatic entropy \( S_{ad} \) estimated from the observed entropy at the same radius (see Section 4). Lower panel: gas density profiles. Points show observed gas densities for A133 and A383 from Vikhlinin et al. (2006) which fit the dotted line. The total cluster density profile \( \rho_c(r) \) is shown with a dashed line. The cooling radius for cluster 1, 98 kpc, corresponds to \( \log r_{cool}/r_{vir} = -1.30 \).

Figure 2. Identical to Figure 1 but for composite cluster 2 with observed gas densities for A478 and A1413. The cooling radius for cluster 2, 120 kpc, corresponds to \( \log r_{cool}/r_{vir} = -1.35 \).
The hydrostatic post-collapse cluster gas atmospheres in Figures 1 and 2 have a broad temperature maximum near $r/r_v = 0.1$ and distant entropy profiles $S \propto r^{1.3}$ that are very similar to the Tozzi–Norman computation. Our simple approximation closely resembles sophisticated cluster gas profiles computed in detailed cosmological simulations that include mergers of smaller bound systems and gas inflowing in filaments (e.g., Borgani & Kravtsov 2009). Gas and dark matter in these merging subsystems and filaments have already experienced some dissipation when they enter the cluster virial radius. Apart from this, we consider no other type of hypothetical ad hoc pre-heating.

### 2.2. Observed Cluster Gas Atmospheres without Cores

Next, we perform similar integrations using the observed cluster gas density profiles $\rho_{\text{obs}}(r)$, approximately including the gas self-gravity in the total NFW mass distribution. The data points shown with open circles in Figures 1 and 2 show observed gas densities found from $\rho_{\text{obs}}(r) = f_g(r)\rho_c(r)$ for both clusters in each composite cluster at radii for which $f_g$ is determined by Vikhlinin et al. In addition we show (with filled circles) several additional approximate values measured directly from the $\rho_{\text{obs}}(r)$ profiles plotted by Vikhlinin et al. (Closed and open circles have sizes roughly comparable to observational errors.)

The mean observed gas density profiles of our two composite clusters are fit with an analytic curve

$$\rho_{\text{obs}}(r) = \frac{f_g\rho_c}{y^{1-\alpha}(y_0 + y)^2},$$

where $y = c(r/r_v)$. By design, $\rho_{\text{obs}}(r)$ asymptotically approaches $f_g\rho_c(r) = 0.17\rho_c(r)$ as the radius continues beyond the observed region, i.e., $r \gtrsim 0.5r_v$. As the cluster gas conserves baryons during feedback expansion, regions of $f_g < f_b$ in the cluster gas observed within $\sim 0.5r_v$ must be compensated by regions of $f_g > f_b$ in more distant cluster gas. However, due to the increased volume available in the outer regions of the clusters, we expect the excess $f_g - f_b$ to be much less than $f_b$ and difficult to observe. Profiles for $\rho_{\text{obs}}(r)$ are shown with dotted lines in the bottom panels of Figures 1–4 where we take $y_0 = 1.8$ and 1.5 for cluster 1 and 2, respectively, and $\alpha = 0.05$ for both composite clusters.

For the observed hot gas atmospheres in clusters 1 and 2, Equation (1) is solved in the same manner as before but with $d\ln \rho_{\text{obs}}/dr$ on the right-hand side. The corresponding hydrostatic temperature $T(r)$ and entropy $S(r)$ profiles are shown with dotted lines in the upper and central panels of Figures 1 and 2 for each composite cluster. Both the temperature maximum and the somewhat flatter entropy profile that asymptotically approach the adiabatic profile resemble typical cluster observations (e.g., Vikhlinin et al. 2006; Pratt et al. 2010). The total bolometric X-ray luminosity within the virial radius $L_x(r_v) = (3.83 \times 10^{44}, 1.58 \times 10^{45})$ erg s$^{-1}$, respectively, for composite clusters (1, 2) are consistent with the $L_x-M$ scaling relation for observed clusters (e.g., Maughan 2007).

While the two cluster atmospheres plotted in Figures 1 and 2—adiabatic and observed—are final zero-redshift gas distributions, we can also regard them as initial and final configurations of the cluster gas before and after feedback energy is deposited. Of particular interest is the potential energy of the gas at various radii in the two atmospheres that enclose the same integrated gas mass $M_g(r)$. The potential energy (PE) of gas in the cluster is

$$\text{PE}(r) = \int_0^r \phi(r')\rho 4\pi r'^2dr,$$  

where the NFW potential

$$\phi = -\frac{GM_c}{r} \ln \left(1 + y \right)$$

also satisfies $g = -d\phi/dr$. In evaluating the potential $\phi(r)$, we again approximate the self-gravity of the gas by using the total cluster mass, and we ignore the small adiabatic expansion of the dark matter as gas moves out in the cluster potential. The NFW potential is expected to be valid within about twice the virial radius (Cuesta et al. 2008; Tavio et al. 2008).

Note that the potential energy profile is determined by an integration out from the cluster center. Furthermore, to a good approximation, the cluster dark matter density and potential profiles remain fixed with time as the size and mass of the virialized region increases with time, i.e., the mass distribution in virialized dark halos forms from the inside out. The inside-out character of halo formation is apparent, for example, from

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**Figure 3.** Two equilibrium atmospheres for composite cluster 1: a purely adiabatic virialized gaseous atmosphere with central core (solid lines) and an observed atmosphere of the same mass (dotted lines). Panels are identical to Figure 1.

**Figure 4.** Identical to Figure 3 but for composite cluster 2 with observed gas densities for A478 and A1413.
In a second series of similar calculations, we consider adiabatic collapse atmospheres in which the baryons experience an additional central dissipation that produces a density core. For this purpose, we adopt the recent mesh-based purely gravitational cluster collapse computations of Vazza (2011). We create a core from the no core density profile by flattening the central gas density so that \( d \log \rho / d \log r \approx -2 \) at \( r/r_v \approx 1.4 \) as in Figure 3 of Vazza (2011). However, to satisfy the stability requirement of radially increasing entropy, the gas density in the core cannot be perfectly flat. We find positive \( dS/dr \) when the core gas density gradually decreases as \( \rho \propto r^{-0.3} \) and this solution is shown as a solid line in the bottom panels of Figures 3 and 4. In our approximation, baryons in the core region are simply removed without a corresponding increase in \( f_g \) beyond the core region. Corrections of this sort that must occur for baryon conservation, and which are not apparent in the mesh-based calculations, are not important for our estimate here since the baryon fraction in the cored adiabatic atmosphere is essentially \( f_b = 0.17 \) at large cluster radii. Finally, we assume that adiabatic cored density profiles are a universal function, scaling with \( r/r_v \), for clusters of different virial masses. Performing similar integrations of the hydrostatic Equation (1), temperature and entropy profiles corresponding to the cored density profile are shown as solid lines for composite clusters 1 and 2 in the upper two panels of Figures 3 and 4 where it is seen that the entropy has a slowly sloping entropy “floor,” as expected.

Performing the same procedure as before, three pairs of equal gas mass \( M_g(r) \) locations in the two cluster gas atmospheres—adiabatic “core” collapse and observed—are listed at the bottom of Table 2. When baryonic cores are present, the potential energy increases \( \Delta PE \) are only slightly less than if the core is completely ignored.

### 2.4. Further Observational Implications

Both of the starless adiabatic cluster atmospheres we consider—with and without cores—have local gas fractions that agree with the cosmic baryon value \( f_b = 0.17 \) near \( r_v \). However, the large expansion of cluster gas caused by feedback reduces the local gas fraction throughout the observed post-feedback cluster. The observed gas fraction at the most distant density observations near \( r \approx 0.4 r_v \) is \( f_g = 0.082 \), only about half the cosmic value. This shortfall is typical of all massive clusters. The average value of the gas fraction at \( r_{500} \approx 0.5 r_v \) for clusters having mean temperatures greater than 4 keV is \( f_g(r_{500}) \approx 0.12 \pm 0.02 \) (McCarthy et al. 2007). When feedback-energized cluster gas expands out beyond \( r_{500} \), baryon conservation requires that \( f_g > f_b \) somewhere beyond \( r_{500} \), although baryon excesses have not yet been observed. Nevertheless, it is possible that nearly baryonically closed systems in which \( \Delta PE \ll |PE(r_v)| \) and \( f_g(r_v) \approx 0.17 \) may exist in some potential energy due to feedback when integrated to the same gas mass, shown as \( \Delta PE \) in Table 2, is huge, \( (1–3) \times 10^{62} \) erg, greatly exceeding the most energetic known individual bipolar feedback events \( \lesssim 10^{52} \) erg (McNamara et al. 2005; Guo & Mathews 2010a), which are sufficient to convert cool-core to non-cool-core clusters (Guo & Mathews 2010b). The mean luminosity of this enormous feedback energy, if spread over a typical cluster lifetime \( t_d \approx 7 \) Gyr, is \( L_{db} \approx 4 \times 10^{45} |PE|/10^{63} \) erg s\(^{-1}\), comparable to continuously active quasars.

### Figure 1 of Diemand et al. (2007)

Figure 1 of Diemand et al. (2007) who plot the accumulated density within many Lagrangian mass zones with time as a dark halo grows in size and mass. In view of this, our estimate of the change in PE due to non-gravitational feedback, assuming an unchanging total NFW mass distribution, is independent of the time when the feedback occurred. As the dark halo grows, the global outflow of cluster gas driven by feedback energy can relocate the accretion shock beyond the instantaneous virial radius. But we do not consider extremely distant feedback events \( (r \gtrsim 1.5–2 r_v) \) that energize gas before it reaches the accretion shock and where the NFW potential may no longer apply. This restriction on feedback seems reasonable, particularly since the virial radius continuously increases with time as a dark halo grows in size and mass. In view of this, our estimate of the change in PE due to non-gravitational feedback, assuming an unchanging total NFW mass distribution, is independent of the time when the feedback occurred. As the dark halo grows, the global outflow of cluster gas driven by feedback energy can relocate the accretion shock beyond the instantaneous virial radius. But we do not consider extremely distant feedback events \( (r \gtrsim 1.5–2 r_v) \) that energize gas before it reaches the accretion shock and where the NFW potential may no longer apply. This restriction on feedback seems reasonable, particularly since the virial radius continuously increases with time as a dark halo grows in size and mass.

### Table 2

| \( r/r_v \) | \( \Omega_g(r)^a \) | \( \Delta \Omega_g \) |
|---|---|---|
| Cluster 1: No Core |
| ad,b | 1.000 | 7.382 | 42.06 |
| ob,b | 1.862 | 7.383 | 32.37 |
| ad | 0.501 | 4.459 | 30.38 |
| ob | 0.933 | 4.453 | 24.17 |
| ad | 0.269 | 2.486 | 19.57 |
| ob | 0.501 | 2.476 | 16.15 |

Cluster 2: No Core

| \( r/r_v \) | \( \Omega_g(r)^a \) | \( \Delta \Omega_g \) |
|---|---|---|
| ad | 1.000 | 19.22 | 204.6 |
| ob | 1.549 | 19.20 | 171.6 |
| ad | 0.501 | 11.23 | 142.4 |
| ob | 0.776 | 11.19 | 122.4 |
| ad | 0.324 | 7.362 | 103.2 |
| ob | 0.501 | 7.317 | 90.26 |

Cluster 1: Core

| \( r/r_v \) | \( \Omega_g(r)^a \) | \( \Delta \Omega_g \) |
|---|---|---|
| ad | 1.000 | 7.221 | 40.46 |
| ob | 1.549 | 7.275 | 32.12 |
| ad | 0.501 | 4.299 | 23.79 |
| ob | 0.891 | 4.284 | 23.58 |
| ad | 0.282 | 2.448 | 18.74 |
| ob | 0.501 | 2.476 | 16.15 |

Cluster 2: Core

| \( r/r_v \) | \( \Omega_g(r)^a \) | \( \Delta \Omega_g \) |
|---|---|---|
| ad | 1.000 | 19.00 | 200.7 |
| ob | 1.549 | 19.19 | 171.6 |
| ad | 0.501 | 11.01 | 138.5 |
| ob | 0.758 | 10.96 | 120.7 |
| ad | 0.357 | 7.325 | 101.3 |
| ob | 0.501 | 7.317 | 90.26 |

Notes.

^a For each cluster/core combination, the integrated gas mass is chosen to be nearly the same for each pair of “ob” and “ad” solutions.

^b “ad” and “ob” designate cluster atmospheres based on the adiabatic and observed gas density profiles.

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fossil groups where the total mass of gas cooling near the central black hole is much less than in the more massive clusters considered here (Mathews et al. 2005).

The final pair of entries for each of the four cluster atmospheres in Table 2 are designed for the majority of current cluster observations that extend only to \( r_{500} \approx 0.5 r_{c} \). Values of \( \Delta P E(\text{rad}) \) with \( r_{c} \approx 0.3 r_{c} \) in Table 2 represent the minimum feedback energy consistent with clusters observed to \( \sim 0.5 r_{c} \). This minimum feedback energy—about \((3-13) \times 10^{62} \text{ erg}\) from Table 2—would be correct if \( f_{b} \) increases abruptly to \( f_{b} = 0.17 \) just beyond the outermost gas density observed. While a few recent cluster X-ray observations with the Suzaku telescope extend to the virial radius and beyond (e.g., George et al. 2009), these data may be more difficult to interpret. Detailed calculations of the cosmological evolution of massive clusters often indicate significant undamped subsonic gas velocities beyond about \( r_{500} \approx 0.5 r_{c} \), which may degrade the assumption of hydrostatic equilibrium (Evrard et al. 1996). Baryonic clumps or compressions occur in this same outer region, causing the observed gas density to appear too high and the entropy too low (Nagai et al. 2007a, 2007b).

2.5. Influence of Stellar Baryons

In our estimates of the feedback energy, we have ignored the small fraction of baryons in our clusters that convert to stars. Nevertheless, it is interesting to compare the mass of stellar baryons to the total baryonic gas mass that flows out at large cluster radii due to feedback energy. The mean cluster stellar mass fraction \( f_{s} \) decreases with cluster mass as \( f_{s} \propto M_{*}^{-0.35} \) (Andreon 2010) but this variation has considerable scatter. For our clusters 1 and 2 with mass log \( M_{200} = 14.56 \) and 14.96 in \( M_{\odot} \), the stellar fraction, \( f_{s} \approx 0.008-0.015 \) (from Figure 7 of Andreon 2010), is dominated by cosmic scatter. Adopting a mean value for this range of cluster mass, \( f_{s} \approx 0.010 \), the total stellar mass within \( 0.5 r_{c} \) in clusters (1, 2) is \((0.26, 0.65) \times 10^{13} M_{\odot} \). By comparison, from Table 2 the total mass flowing out beyond \( 0.5 r_{c} \) for (core, no core) versions of cluster 1 is \((1.823, 1.983) \times 10^{13} M_{\odot} \) and (3.69, 3.91)\( \times 10^{13} M_{\odot} \) for cluster 2. The fraction of outflowing gas mass consumed by star formation, \( \sim 0.13-0.17 \), is another small correction that we neglect here.

2.6. Specific Feedback Energy

Our values of \( \Delta P E(r_{c}) = (8.34, 29.1) \times 10^{62} \text{ erg} \) for cored clusters (1, 2) translate into mean specific feedback energies \( \Delta P E/(0.17 M_{*}) = (5.65, 7.57) \times 10^{35} \text{ erg m}^{-1} \) when applied to the total gas mass within \( r_{c} \) in the original adiabatic clusters that extends out to \((1.5, 1.8) r_{c} \), in the post-feedback atmosphere. The specific feedback energy can also be expressed as \((3.6, 4.8) \text{ keV particle}^{-1} \). These values appear comparable to those estimated from the \( L_{X}-T \) scaling relation for clusters by Wu et al. (2000), \( \sim 1-3 \text{ keV particle}^{-1} \). However, Wu et al. only consider particles in the mass of observed gas within \( r_{200} \), our gas masses are larger by \((2.0, 1.8) \) so our estimated feedback energies are more than twice as large as those of Wu et al. By comparison, our specific feedback energies are considerably less than those of McCarthy et al. (2007), \( \gtrsim 10 \text{ keV particle}^{-1} \), probably because they use an extrapolation procedure for the observed gas density \( \rho_{\text{obs}}(r) \) quite different from our Equation (5).

2.7. Energetics of Radiation Loss and Supernovae

We assume that the energy and entropy lost by radiation can be ignored. To verify this, consider the total energy radiated by the two composite clusters within the cooling radius \( E_{\text{rad}} \approx L_{c}(r_{\text{cool}} M_{c}) \) during the cluster lifetime \( t_{\text{cl}} = 7 \text{ Gyr} \). For composite clusters (1, 2), we find \( r_{\text{cool}} = (97.7, 119.8) \text{ kpc}, \) \( L_{c}(r_{\text{cool}}) = (1.36, 4.96) \times 10^{44} \text{ erg} \text{ s}^{-1} \), and \( E_{\text{rad}} = (0.30, 1.1) \times 10^{42} \text{ erg} \). The total radiated energy for both clusters is a small fraction of the change in potential energy \( \Delta P E \) listed in Table 2, particularly those values of the total \( \Delta P E(r_{c}) \) evaluated at the virial radius in the adiabatic solutions. Radiation losses, while not a major factor in the overall energy budget for our clusters, are nevertheless essential in driving the central cooling accretion that creates the feedback energy. The cooling radius \( \log(r_{\text{cool}} M_{c}) \approx -1.3 \) is small compared to the more extended regions in the bottom panels of Figures 1–4 where the cluster density is observed to be significantly below the maximum cosmic baryon density, \( \rho_{\text{obs}} < f_{b} \rho_{b} \). Feedback energy from cluster-centered black holes has obviously caused huge outflows, removing cluster gas from regions far beyond the cluster cooling radius.

Feedback from Type II and Ia supernovae can also be ignored. For a single-population Salpeter initial mass function, about 0.007 Type II supernova events occur per solar mass of gas formed into stars. Consider the total mass of gas observed in the composite observed clusters (1, 2), extrapolated to the virial radius, about \( M_{*}(r_{c}) = (4.7, 13.9) \times 10^{13} M_{\odot} \). If 10% of the baryons in this mass forms into stars, the total feedback energy from SNI, each of energy 10^{51} \text{ erg}, is of order \( E_{\text{SNI}} \approx (0.3, 0.9) \times 10^{42} \text{ erg} \) which is substantially less than the \( \Delta P E(r_{c}) \) in Table 2 evaluated at radius \( r_{c} \) in the pre-feedback gas. If the average iron abundance in cluster gas is about 0.3 solar, the total mass of iron within \( r_{c} \) in the clusters is roughly \( 0.3 \times 0.0017 \times M_{*}(r_{c}) = (2.4, 7.1) \times 10^{10} M_{\odot} \), which 0.0017 is the fraction of iron by mass in the solar photosphere. If all the iron is created in Type Ia supernovae, each having energy 10^{51} \text{ erg} and providing 0.7 \( M_{\odot} \) in iron, the total feedback energy from Type Ia supernovae cannot exceed about \( E_{\text{SNI}} \approx (0.3, 1.0) \times 10^{62} \text{ erg} \), which are also less than \( \Delta P E(r_{c}) \) in Table 2.

While active galactic nucleus (AGN) feedback dominates cluster energetics, supernovae may nevertheless contribute 10%–20% of the total feedback energy. Nagai et al. (2007a, 2007b) computed a variety of gaseous atmospheres in clusters including supernovae of all types but without AGN feedback, as commonly assumed. In their clusters the baryon fraction and entropy profiles are in reasonably good agreement with those observed by Vikhlinin et al. (2006) outside the central region, \( r > 0.2 r_{c}/r_{c} \). But central overcooling in \( r \lesssim 0.2 r_{c}/r_{c} \) is a serious problem. At zero redshift in the models of Nagai et al. about 40% of the baryons within \( r \approx 0.5 r_{c} \) are in the form of a centrally concentrated mass of stars and cold gas. The mass of this central concentration of radiatively cooled baryons causes the gas density and temperature to peak up near the cluster centers unlike the observations. A large amount of AGN feedback energy, similar to that estimated here, is essential to remove this overcooling gas before it forms into stars and relocate it to distant regions of the cluster. The outward flow of cluster gas that results from the creation of X-ray cavities is described by Mathews & Brighenti (2008) and Mathews (2009). AGN feedback has just begun to be included in recent cosmological cluster calculations where the overcooling problem is greatly alleviated (Teyssier et al. 2010; Puchwein et al. 2010; McCarthy et al. 2010).

2.8. Feedback Stops Cooling Flows within the Cooling Radius

The approximate rate that mass cools in the two composite clusters in the absence of feedback can be estimated from the
decreases because of feedback expansion from radiation. In this limiting case the mass of gas within any radius $r$ flow of cluster gas due to feedback, ignoring energy losses due to overzealous, overreaching efforts to feed back to the cluster atmosphere the energy acquired from gas cooling in their immediate vicinity—drive huge flows of gas out beyond the virial radius, but in the process necessarily provide enough mass outflow within the relatively small cooling radius to shut down the large cooling inflow that would otherwise occur in this critical central region. Although we do not consider the detailed time evolution of the initial adiabatic cluster gas profile as it transforms into the gas density profiles observed today, we imagine that this occurs in a quasi-steady manner, as explained above, in which feedback energy is widely distributed as PE in the cluster gas. By this means, the feedback outflow always nearly balances the radiative inflow, avoiding any large central gas concentration (and eventual overdensity due to star formation) or other excursions very far from the gas density profiles currently observed.

2.9. Feedback Production of Cosmic Rays

It is likely that feedback consists of jets and jet-produced cavities that are filled mostly with cosmic rays. If so, it is interesting to compare the total feedback energy in our composite clusters, $\sim 10^{59}$ erg, with that expected from other cluster cosmic-ray sources. For simplicity, consider proton cosmic rays that have cluster lifetimes comparable to the cluster age and compare the total (proton) cosmic-ray energy from feedback shocks to the much stronger accretion shock that produced the underlying entropy gradients in cluster atmospheres. From Table 2, the total potential energy of gas within the virial radius in clusters 1 and 2 is $|PE| \sim 10^{59}$ erg so the thermal energy is $E_\text{th} \sim 0.5 \times 10^{59}$ erg by the virial theorem. The virial temperature and entropy in the cluster gas are acquired in the accretion shock as the cluster formed. Typically, about 10% of the shock energy is converted to cosmic rays, so the total energy in cosmic rays created as the cluster formed is $E_{\text{cr}} \sim 0.1 E_\text{th} \sim 0.5 \times 10^{59}$ erg. Therefore, the cosmic-ray energy from feedback and cluster accretion are comparable.

The thermal energy profiles in our adiabatic, pre-feedback cluster models (upper panels in Figures 1–4) change very little after receiving the enormous feedback energies we consider. In our scenario almost all of the thermal energy created when feedback energy is initially deposited subsequently becomes potential energy as the cluster gas expands; this is the energy evolution described in Mathews & Bright (2008). More likely, most of the feedback energy is supplied from the central black holes as jets of supra-thermal cosmic rays that drive the shocks that increase the cluster gas entropy. When feedback is...
mostly in the form of cosmic rays, the net thermal energy of the cluster gas is subsequently reduced by cluster expansion more than it is initially increased by shock waves, and the net effect of cavity production is to cool, not heat, the cluster gas (Mathews & Brighenti 2008). AGN feedback energy and cavity production in cosmological calculations are assumed to be in the form of ultrahot thermal gas. This injection of thermal energy results in a small net increase in the total thermal energy of the cluster gas even after it has fully expanded. Since we do not explicitly consider the non-thermal cosmic-ray component here, a very small increase (rather than a decrease) in the post-feedback temperature profiles appears in Figures 1–4. In either case, any small change in the thermal energy of the cluster gas can be neglected in our estimates of the total feedback energy.

2.10. Locations of Feedback Energy Deposition and Storage

It is important to recognize the distinction between the location in cluster gas where entropy-increasing feedback energy is deposited and where it is ultimately stored as potential energy which is what we address here. Evidently, most feedback heating occurs in shock waves moving away from advancing jets and expanding X-ray cavities. X-ray observations of these shocks generally indicate modest Mach numbers, \( M \lesssim 2 \), in which the gas entropy is increased only by \( \lesssim 1.2 \). The lower density of recently shock-heated gas results in an adiabatic, buoyant outward flow in the cluster that stops when the entropy of the heated region matches that of the ambient atmosphere. However, the entropy increases with cluster radius as \( S \propto r^{1.2} \), mostly due to dissipation acquired during cosmic accretion. Consequently, gas heated with a \( M \sim 2 \) shock can only rise by a factor of \( \sim 1.2 \) in cluster radius where its feedback energy is stored as potential energy. Conversely, if the reduced gas fraction observed at \( 0.5r_e \sim 1 \) Mpc were due to shock-heating events at 50 kpc (where X-ray cavities can be observed), the entropy difference \( \Delta S \sim 36 \) would require shocks with Mach numbers \( M \sim 17 \) in which the post-shock temperature would be increased by about 95. Shocks this strong have not been observed in cluster gas. Furthermore, it is likely that there is not enough cluster gas within 50 kpc which, when continuously heated this much with a filling factor of unity, can buoyantly supply the much larger mass of high entropy, feedback-heated gas observed at 1 Mpc. In addition, mixing instabilities are likely to dilute the entropy of heated gas during their long buoyant journey to \( r \sim 1 \) Mpc. Clearly, long range buoyant outflow is a losing proposition. The conclusion we draw from this is that feedback energy stored at some radius always exceeds the radius where the energy was initially deposited, but the difference between these two radii is unlikely to be very large.

3. BLACK HOLE ACCRETION OR SPIN ENERGY?

The time-averaged mechanical feedback power \( L_{fb} \) required to lift the adiabatic cluster atmospheres beyond the virial radius \( r_v \) can be estimated from \( L_{fb} \sim \Delta PE/t_{cl} \), where \( t_{cl} = 7 \) Gyr is a typical age for large clusters. For composite clusters (1, 2) the feedback luminosity with and without initial cores is \( L_{fb} \sim (0.38, 1.32) \times 10^{46} \) and \( L_{fb} \sim (0.44, 1.50) \times 10^{46} \) erg s\(^{-1} \), respectively. These luminosities are comparable to those of powerful quasars continuously active during the cluster lifetime.

What are the implications of these enormous feedback powers for the mass of cluster-centered black holes? The conversion of accretion energy into black hole mass is more straightforward for luminous quasars in which the (thin) accretion disk can be directly observed. In this case, the total bolometric feedback energy radiated by an active black hole is related to the mass of the black hole by \( E_{rad} = \epsilon_r M_{bh} c^2 \) with \( \epsilon_r \sim 0.1 \) (e.g., Davis & Laor 2010). While low-redshift cluster-centered black holes generally have bolometric luminosities very much less than the estimated \( L_{fb} \), it is unclear if this also applies at higher redshift when most of the feedback energy may have been created.

Suppose we adopt a mechanical feedback efficiency \( \epsilon_{mb} \) such that the change in cluster gas potential energy is related to mass accretion by

\[
\Delta PE = \epsilon_{mb} \Delta M_{bh} c^2.
\]

If we identify the total mass accreted with the mass of the black hole, \( \Delta M_{bh} \approx M_{bh} \), and assume \( \epsilon_{mb} \approx 0.1 \), the resulting black hole masses for cluster 1, \( M_{bh} \approx (4.6-5.4) \times 10^9 \, M_\odot \), are similar to those observed in cluster-centered galaxies, e.g., \( M_{bh} \approx 6.4 \times 10^9 \, M_\odot \) in M87 at the center of the Virgo cluster (Gebhardt & Thomas 2009). However, the corresponding black hole masses for the more massive cluster 2, \( M_{bh} \approx (1.5-1.6) \times 10^{10} \, M_\odot \), exceed those observed. This large black hole mass cannot be reduced by invoking the energy contributed to the cluster gas from accreting black holes in bulges of non-central satellite galaxies in the cluster. Almost all observed X-ray cavities and shock waves are associated with cluster-centered black holes, not those in orbiting satellite galaxies. Moreover, the cluster center is where the virialized cluster gas is densest and has the shortest radiative cooling time. However, for cluster masses \( M_c \gtrsim 10^{14} \, M_\odot \), such as we consider here, the mass of the central galaxy (and therefore also its black hole) increases very little with increasing cluster virial mass \( M_c \) (Lin & Mohr 2004). The mass of the central black hole in cluster A478 (in composite cluster 2) estimated from the bulge luminosity of the cluster-centered galaxy, \( 5.8 \times 10^9 \, M_\odot \) (McNamara et al. 2011), is considerably less than the mass estimated from \( M_{bh} \approx \Delta PE(r_e)/(\epsilon_{mb} c^2) \approx 1.6 \times 10^{10} \, M_\odot \) with \( \epsilon_{mb} = 0.1 \) for the cored cluster 2. Finally, the mechanical accretion efficiency \( \epsilon_{mb} \) cannot be much larger than 0.1 and is likely to be much smaller.

This difficulty can be alleviated if a substantial fraction of the required mechanical feedback energy is supplied by the rotational energy of central black holes which, when magnetically coupled to accretion disks, may form powerful feedback jets (Blandford & Znajek 1977). The maximum available energy from a rotating black hole, \( 0.29M_{bh} c^2 = 5.3 \times 10^{52}[M_{bh}/(10^9 \, M_\odot)] \) erg, is large enough to account for the gas depletion in both clusters 1 and 2. Recent models of the cosmological co-evolution of galaxies and their black holes are consistent with the assumption that “radio-mode” jet feedback derives from black hole spin and low-luminosity, advection-dominated central accretion ( Sikora et al. 2007; Fanidakis et al. 2011). There is considerable evidence for mechanical outflow from active galaxies and quasars (e.g., Crenshaw et al. 2003), but most relevant to our feedback estimates here are powerful FRII radio sources in luminous quasars that can transport enormous energies to great distances in the cluster gas (e.g., Mullin et al. 2008). Recently, Singal et al. (2011) have argued that the evolution of quasar luminosities at optical and radio frequencies are strongly correlated since redshift \( z \approx 3 \) with a significantly higher ratio than optical evolution. In all likelihood, most of the radio-mode feedback energy from cluster-centered black holes probably occurred at earlier times. While spin energy is an attractive hypothesis, it must be reconciled with the approximately isotropic ejection of feedback jet energy currently observed in local galaxy clusters such as Virgo and Perseus and the rather
Once the gas density profile of a cluster is known, the gas mass per unit radius is obtained by multiplying the density profile by the cluster radius. In the absence of feedback from cluster centers requiring energies of \( \gtrsim 10^{46} \) erg, relatively localized feedback events are required to account for the observed depletion in cluster gas entropy and drive it out in the cluster potential, lowering the gas fraction, are either too weak or too old to retain evidence of increased gas temperature due to the most recent feedback event. Conversely, clusters for which \( \tilde{S}_{ad}(r) \) is comparable to \( S_{ad}(r) \) show evidence of recent feedback-related increases in gas temperature, and \( \tilde{S}_{ad}(r) - S_{ad}(r) \) is a measure of the location, energy, and age of these feedback events.

5. CONCLUSIONS

By considering the difference between gas density profiles in clusters created by non-radiating gravitational collapse and similar observed clusters, we estimate the total feedback energy received by the gas during the cluster lifetime. This estimate is sensitive to the precise time of the feedback events, but on average, feedback energy must be widely distributed in cluster radius. Individual feedback events produce jets and cavities and associated shock waves that heat the cluster gas. The heated, high-entropy gas expands, ultimately leading to an expansion of the entire cluster atmosphere in which the feedback energy is stored as potential energy. The final cluster gas entropy profile is increased by feedback energy and the gas fraction \( f_g(r) \) profile is reduced below the cosmic baryonic value \( f_b = 0.17 \).

For clusters having masses \( M_c \gtrsim 10^{14} M_\odot \), the total estimated feedback energy required to account for the observed depletion of cluster gas and the enhanced entropy profile is \( \gtrsim 10^{43} \) erg, considerably in excess of cluster gas energy gained from supernovae or lost by radiation. Although enormous, this energy exceeds the largest known energy released in single, most powerful feedback events, \( \lesssim 10^{42} \) erg. The most likely source of this energy is feedback from central black holes in cluster-centered elliptical galaxies. When averaged over a typical cluster lifetime, about 7 Gyr, the mean mechanical luminosity, \( \sim 10^{46} \) erg s\(^{-1} \), is comparable to that of powerful quasars. Such a large sustained luminosity may require energy creation not just from black hole accretion but also its spin energy.

Immediately following a feedback energy event, we expect a significant local increase in gas temperature and thermal energy. But after a cluster sound-crossing time, the cluster expands and the temperature returns to a rather flat profile near the virial temperature required to support the cluster gas. The sound-crossing time \( t_{500} \) in our two composite clusters \((1, 2)\) is \( t_{500} \approx (4.6, 3.5) \times 10^8 \) yr. In view of the insensitivity of the gas temperature profile to the feedback energy after time \( \sim 10^8 \) yr, changes in the cluster gas entropy \( S(r) = T(r)/\rho(r)^{2/3} \) arise almost entirely from changes in the gas density profile. After \( \sim 10^8 \) yr, transient feedback events that increase the local gas temperature evolve by expansion.
into density reductions as thermal energy converts to potential energy, retaining approximately the same global $T(r)$. This explains the strong anticorrelation between excess entropy and reduced gas fraction in galaxy groups and clusters (Sun et al. 2009). Inspired by Pratt et al. (2010), we also demonstrate that the entropy profile observed in any relaxed cluster $S(r)$, when multiplied by a factor containing the gas fraction $f_g(r)$, recovers the universal adiabatic gas entropy profile expected in the absence of feedback, $S_{ad}(r) = S(r)[f_b/f_g(r)]^{2/3} \propto r^{-2}$, with small deviations related to recent feedback events.

Most of the feedback energy and entropy are delivered to very distant regions in cluster hot gas atmospheres, far beyond the cooling radius, where they have little or no effect on reducing the rate at which gas cools near the central black hole, the presumed source of feedback energy. In the absence of feedback, the observed cluster gas is expected to cool at a rate $M_{cl}(r_{cool})$ within the cooling radius $r_{cool}$, the cluster radius at which the cooling time equals the typical cluster age $t_{cl}$. For our clusters, $r_{cool} \approx 100 \ kpc$. Cooling flows near the central black hole can be greatly reduced or stopped if the mass of cluster gas that flows out across $r_{cool}$ during the cluster lifetime $t_{cl}$, driven by feedback, is approximately equal to the inflowing mass due to radiation losses $M_{cl}(r_{cool})t_{cl}$. For the clusters we consider it gratifying that the cluster gas mass flows in both directions at $r < r_{cool}$ during $t_{cl}$ to drastically reduce cooling near the black hole where little or no cold or cooling gas is observed. Nevertheless, only a small fraction of the total feedback energy, less than 1%, is delivered and stored within $r_{cool}$.

Finally, it is significant that the average mechanical feedback power $L_{fb} \sim 10^{46} \ erg \ s^{-1}$ implied by observed cluster gas fractions $f_g < f_b = 0.17$ is very substantially higher than estimates of $L_{fb}$ from observations of X-ray cavities, $\sim 10^{59} \ erg$ (Rafferty et al. 2006; McNamara et al. 2011). This discrepancy may be attributed to the difficulty of detecting X-ray cavities at distances exceeding about 50–70 kpc from cluster centers, particularly at higher redshifts, where most of the feedback energy is delivered and stored.

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