Deterministic secure communication without using entanglement

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We show a deterministic secure direct communication protocol using single qubit in mixed state. The security of this protocol is based on the security proof of BB84 protocol. And it can be realized with today’s technology.

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Quantum key distribution (QKD) is a protocol which is provably secure, by which private key bit can be created between two parties over a public channel. The key bits can then be used to implement a classical private key cryptosystem, to enable the parties to communicate securely. The basic idea behind QKD is that Eve cannot gain any information from the qubits transmitted from Alice to Bob without disturbing their states. First, the no-cloning theorem forbids Eve to perfectly clone Alice’s qubit. Secondly, in any attempt to distinguish between two non-orthogonal quantum states, information gain is only possible at the expense of introducing disturbance to the signal [1].

Based on the postulate of quantum measurement [2] and no-cloning theorem [3], different QKD protocols are presented [4-7]. However, these types of cryptographic schemes are usually nondeterministic. In Ref.[8], K. Boström and T. Felbinger presented a protocol, which allows for deterministic communication using entanglement. The basic idea of the ping-pong protocol is that one can encode the information locally on an EPR pair, but it has a nonlocal effect. In this paper, we show a secure communication protocol which is a deterministic secure direct communication protocol using single qubit in mixed state.

This protocol is based on the property that non-orthogonal quantum states cannot be reliably distinguished [2]. One cannot simultaneously measure the polarization of a photon in the vertical-horizontal basis and simultaneously in the diagonal basis. It is well known one can prepare a photon in states \(|0\rangle, |1\rangle\) or \(|\varphi_0\rangle, |\varphi_1\rangle\) in its polarization degree of freedom, where

\[
|\varphi_0\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle),
\]

\[
|\varphi_1\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle).
\]

Denoting that \(i\sigma_y = |0\rangle\langle 1| - |1\rangle\langle 0|\), it can be obtained:

\[
i\sigma_y|0\rangle = -|1\rangle, \quad i\sigma_y|1\rangle = 0,
\]

and

\[
i\sigma_y|\varphi_0\rangle = |\varphi_1\rangle, \quad i\sigma_y|\varphi_1\rangle = -|\varphi_0\rangle.
\]

Suppose Alice want to obtain some information from Bob. First Alice selects state \(|0\rangle\) or \(|\varphi_0\rangle\) randomly with the probability \(\frac{1}{2}\) every time. For an external person without Alice’s a prior knowledge, this qubit appears to be in a mixed state \(\rho_0\):

\[
\rho_0 = \frac{1}{2}|0\rangle\langle 0| + \frac{1}{2}|\varphi_0\rangle\langle \varphi_0|.
\]

Then Alice sends this qubit to Bob. Bob decides either to perform the operation \(i\sigma_y\) on the travel qubit to encode the information ‘1’ or do nothing, i.e., to perform the operation \(I = |0\rangle\langle 0| + |1\rangle\langle 1|\) to encode the information ‘0’. Then Bob sends the travel qubit back to Alice. Alice performs a measurement on this back qubit to gain the information Bob encoded. After Alice’s decoding measurement, she tells Bob she has received the back qubit through the public channel by one bit (This can be called as Alice’s receipt. ). In this protocol, there are two communication modes, ‘message mode’ and ‘control mode’. By default, Bob and Alice are in message mode and the communication

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is described as above. With probability $c$, Bob switches the message mode to control mode. In Control Mode. Instead of his encoding operation, Bob replaces the qubit he receives from Alice with a qubit that he completely randomly prepares in the state $|0\rangle$, $|1\rangle$, $|\varphi_0\rangle$ or $|\varphi_1\rangle$ and sends this qubit to Alice. Alice performs her decoding measurement and in 50% of the case she uses the basis as Bob used. After Alice announces her receipt of the qubit, Bob announces that this has been a control run and he tells Alice which state he prepared. If Alice used the same basis as Bob and if she found a state different from Bob prepared then Eve is detected and the communication stops. This protocol can be described explicitly like this:

(1). Alice prepares one qubit in state $|0\rangle$ or $|\varphi_0\rangle$ randomly with record.
(2). Alice sends this qubit to Bob.
(3). Bob receives the travel qubit. He decides to the message mode (4m) or the control mode (4c) by chance.

(4c). Control mode. Bob replaces the qubit he receives from Alice with a qubit that he randomly prepares in the state $|0\rangle$, $|1\rangle$, $|\varphi_0\rangle$ or $|\varphi_1\rangle$ and sends this qubit to Alice. Alice performs her decoding measurement. After Alice announces her receipt of the qubit, Bob announces that it is a control run this time and he tells Alice which state he prepared. If Alice used the same basis as Bob and if she found a state different from Bob prepared then Eve is detected and the communication stops. Else, Alice sends next qubit to Bob.

(4m). Message mode. Bob performs an operation on the travel qubit to encode information. He encodes the bit ‘0’ using by the operation $I$ and the bit ‘1’ by the operation $i\sigma_y$. Then Bob sends this travel qubit back to Alice. Alice measures the qubit to gain the message Bob encoded and sends her receipt to Bob through public channel.

(5). When all of Bob’s information is transmitted, this communication is successfully terminate.

Security proof. The basic idea behind QKD is the fundamental proposition: Eve can not gain any information from the qubits transmitted from Alice to Bob with out disturbing their state [1]. Consider that $|\varphi\rangle$ and $|\phi\rangle$ are non-orthogonal quantum states Eve is trying to obtain information about. Without loss of generality that the process she uses to obtain information is to unitarily interact the state with an ancilla prepared in a standard state $|e_0\rangle$. Assuming that this process does not disturb the states, then one obtains:

\[
U|\varphi\rangle|e_0\rangle = |\varphi\rangle|e_1\rangle, \\
U|\phi\rangle|e_0\rangle = |\phi\rangle|e_2\rangle.
\]

To acquire information about the different state, Eve would like $|e_1\rangle$ and $|e_2\rangle$ different. Since the inner products are preserved under unitary transformations, it must be that

\[
<\varphi|\phi> <e_0|e_0> = <\varphi|\phi> <e_2|e_3>.
\]

Since $|\varphi\rangle$ and $|\phi\rangle$ are non-orthogonal, then it has

\[
<e_2|e_3> = <e_0|e_0> = 1,
\]

which implies that $|e_2\rangle$ and $|e_3\rangle$ must be identical. That distinguishing two non-orthogonal states would at least disturb one of them.

To gain information, Eve has to know which operation Bob performed. First, she can attack the travel qubit in the line $A \rightarrow B$. And perform a measurement to acquire Bob’s information in line $B \rightarrow A$. Or she can take another strategy that she only performs a measurement after Bob’s operation to gain Bob’s information. No matter what strategy Eve uses, she has to attack the qubit in the line $B \rightarrow A$. We can see that our protocol in control mode is as the same as the BB84 protocol’s detection of Eve’s eavesdropping. Many works have been accomplished of the security proof of the BB84 protocol [7, 9]. Eve’s any attempt to eavesdrop the information will give a detection probability $d > 0$. Taking into account the probability $c$ of a control run, the effective transmission rate is $r = 1 - c$. The probability of Eve’s eavesdropping one message transfer without being detected is [8]

\[
s(c,d) = \frac{1-c}{1-c(1-d)},
\]

where $d(I_0)$ is the detection probability in the control mode. After $n$ protocol run, the probability to successfully eavesdrop $I = nI_0(d)$, the probability to successfully eavesdrop becomes
Instead of transmitting the message directly to Alice, Bob will take a random sequence of bits from a secret random number generator. After a successful transmission, the random sequence is used as a shared secret key between Bob and Alice. Bob and Alice can choose classical privacy amplification protocols, which make it very hard to decode parts of the message with only some of the key bits given. So Eve has virtually no advantage in eavesdropping only a few bits. When Eve is detected, the transfer stops. Then Eve has nothing but a sequence of nonsense random bits.

In contrast to quantum key distribution protocol BB84 [4], our protocol provides a deterministic transmission of bits. It is possible to communicate the message directly from Bob to Alice. Essentially, this protocol is a special case of BB84 protocol. The essence of this protocol is that the communicators can freely select the message mode and the control mode. In BB84 protocol, when Alice and Bob want to transform a bit message, it need about 4n qubit. On the other hand, comparing with the ‘ping-pong’ protocol, we use a single qubit to realize the deterministic secure direct communication instead of using entanglement. Also, there may be a denial-of-service (DoS) attack in the line A → B [14]. But any method of message authentication can protect the protocol against man-in-the-middle attacks with a reliable public channel.

In order to be practical and secure, a quantum key distribution scheme must be based on existing—or nearly existing—technology [5]. Experimental quantum key distribution was demonstrated for first time by Bennett, et al [16]. Since then, single photon source have been studied in recent years and a great variety of approaches has been proposed and implemented [17-22]. Today, several groups have shown that quantum key distribution is possible, even outside the laboratory. In principle, any two-level quantum system could be used to implement quantum cryptography (QC). In practice, all implementations have relied on photons. The reason is that their decoherence can be controlled and moderated. The technological challenges of the QC are the questions of how to produce single photons, how to transmit them, how to detect single photons, and how to exploit the intrinsic randomness of quantum processes to build random generators [23]. Considered the experimental feasibility, our protocol needs a single photon source and some linear optical elements and a single-photon detector. Recently, the full implementation of a quantum cryptography protocol using a stream of a single photon pulses generated by a stable and efficient source operating at room temperature was reported [24]. The single pulses are emitted on demand and the secure bit rate is 7700bits/s. And quantum logic operations using linear optical elements can be realized with today’s technology [25]. The implementation of the single-photon detection technology for quantum cryptography have been reported [26] and the values of $\sigma_x, \sigma_y,$ and $\sigma_z$ of a polarization qubit on a single photon can be ascertained [27]. Considered the experimental feasibility, this protocol can be realized with today’s technology. It is explained that when this paper was completed, we see the protocol [28] presented by Deng et al, which also is a secure direct communication, using Einstein-Podolsky-Rosen pair block.

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I. REFERENCES:

[1] Nielsen M. A. and Chuang I. L. 2000 Quantum Computation and Quantum Information (Cambridge University Press, Cambridge, UK)

[2] Peres A. 1998 Phys. Lett. A 128, 19; Duan L.-M. and Guo G.-C. 1998 Phys. Rev. Lett., 80, 4999-5002

[3] Dieks D. 1982 Phys. Lett. A, 92, 271-272; Wootters W. K. and Zurek W. H. 1982 Nature, 299, 802-803; Barnum H., Caves C. M., Fuchs C. A., Jozsa R. and Schumacher B. 1996 Phys. Rev. Lett., 76, 2818-2821; Mor T. 1998 Phys. Rev. Lett., 80, 3137-3140

[4] Bennett C. H. and Brassard G. 1984, in proceedings of the IEEE International Conference on Computers, Systems and Signal Processing, Bangalor, India, (IEEE, New York), p175-179.

[5] Ekert A. 1991 Phys. Rev. Lett. 67, 661

[6] Bruss D. 1998 Phys. Rev. Lett. 81, 3018

[7] Lo H. and Chau H. F. 1999 Science, 283, 2050

[8] Boström K. and Felbinger T. 2002 Phys. Rev. Lett. 89, 187902
[9] Shor P. W. and Preskill J. 2000 Phys. Rev. Lett., 85, 441; Lo H. K., arXiv:quant-ph/0102138; Biham E., Boyer M., Boykin P. O., Mor T. and Roychowdhury V 1999 arXiv:quant-ph/9912053.
[10] Schneier B. 1996 Applied Cryptography (Wiley, New York), 2nd ed.
[11] Holevo A. S. 1973 Statistical problems in quantum physics. In Gisiro Maruyama and Jurii V. Prokhorov, editors, Proceedings of second Japan-USSR Symposium on Probability Theory, pages 104-109, Springer-Verlag, Berlin.
[12] Kraus K. 1983 States, Effects, and Operations (Spinger-Verlag, Berlin); Barnum H., Nielsen M. A. and Schumacher B. 97 arXiv e-print quantum-ph/9702049.
[13] Shannon C. E. 1949 Bell Syst. Tech. J. 28, 656.
[14] Cai Q.-Y. 2003 Phys. Rev. Lett., 91,109801.
[15] Brassard G., Lutkenhaus N., Mor T. and Sanders B. C. 2000 Phys. Rev. Lett., 85, 1330.
[16] Bennett C. H., Bessette F., Brassard G., Salvail L. and Smolin J. 1992 J. Cryptology 5, 3.
[17] de Martini F., di Guisippe G., and Marrocco M. 1996 Phys. Rev. Lett., 76, 900.
[18] Broui R., Beveratos A., Poizat J-Ph. and Grangier P. 2000 Phys. Rev. A, 62, 063817.
[19] Lounis B. and Moerner W. E. 2000 Nature (London) 407, 491.
[20] Treussart F., Clouqueur A., Grossman C. and Roch J.-F. 2001 Opt. Lett. 26, 1504.
[21] Michler P., Imamouglu A., Mason M. D., Garson P. J., Strouse G. E. and Buratto S. K. 2000 Nature (London) 406, 968.
[22] Kim J., Benson O., Kan H. and Yamamoto Y. 1999 Nature (London) 397, 500.
[23] Gisin N., Ribordy G., Tittel W. and Zbinden H. 2002 Rev. Mod. Phys. 74, 145.
[24] Beveratos A., Broui R., Gacoin T., Villing A., Poizat J.-P. and Grangier P. 2002 Phys. Rev. Lett. 89, 187901.
[25] Knill E., Laflamme R. and Milburn G. J. 2001 Nature (London) 409, 46 ; Franson J. D., Donegan M. M., Fitch M. J., Jacobs B. C. and Pittman T. B. 2002 Phys. Rev. Lett., 89, 137901.
[26] Ribordy G., Gautier J. D., Zbinden H. and Gisin N. 1998 Appl. Opt. 37, 2272-2277 ; Bourennane M., Gibson F., Karlsson A., Hening A., Jonsson P., Tsegaye T., Ljunggren D. and Sundberg E. 1999 Opt. Express 4, 383; Bethune D. and Risk W. 2000 IEEE J. Quantum Electron. 36, 340 ; Hughes R., Morgan G. and Peterson C. 2000 J. Mod. Opt. 47, 533; Ribordy G., Gautier J.-D., Gisin N., Guinnard O. and Zbinden H. 2000 J. Mod. Opt. 47, 517.
[27] Vaidman L., Aharonov Y. and Albert D. Z. 1987 Phys. Rev. Lett. 58, 1385 ; Schulz O., Stenhoel R., Weber M., Engler B. G., Kursiefer C. and Weinfurter H. 2003 Phys. Rev. Lett. 90, 177901.
[28] Deng F.-G., Long G. L., and Liu X.-S. 2003 Phys. Rev. A 68 042317.