A Multi-Objective Optimization Algorithm for Truss Structures

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Abstract. For the optimization of multi-objective discrete variable structure optimization in civil engineering, a new improved multi-objective ant colony optimization algorithm for truss structures has been proposed in this paper. The new improved algorithm established two sets, including sets of feasible solutions and non-feasible solutions, and “the repeated solutions” was replaced with “the special solutions” to acquire the Pareto optimal front-end of the multi-objective problems. This algorithm can not only make the multi-objective ant colony algorithm better solve multi-objective problems under discrete variables, but also realize the successful application of multi-objective ant colony algorithm in truss structure. The present paper proposes a new improved multi-objective ant colony optimization algorithm for truss structures, which can provide a good performance in program design, arithmetic speed and generality of the proposed method. It is also a simple and practical, and suitable for projects in the future.

1. Introduction

Multi-objective optimization design of structures in civil engineering: under a given load or environmental conditions and within the constraints of the geometric relationship of the structure or other factors, the design variables are selected and multiple objective functions are established to finally solve a finite solution set, i.e.: a noninferior solution set for multi-objective optimization design problems¹². In civil engineering, the design variables of structural multi-objective optimization problems are often discrete, and in many multi-objective evolutionary algorithms, multi-objective ant colony algorithm can better meet the requirements in this respect. Although many researchers have proposed algorithms such as: Multiple Objective Ant-Q Algorithm, Ant Algorithm for Bi-criterion Optimization Problem, Multi Colony for Bi-criterion Optimization Problem, Pareto Ant Colony Optimization and Multiple Ant Colony System³⁸. However, it is undeniable that these algorithms are far from satisfying people’s requirements. Especially for the structural optimization design in civil engineering, the existing multi-objective ant colony algorithm cannot fully meet the engineering requirements in terms of performance and function. Therefore, how to propose a more effective multi-objective ant colony algorithm has been the goal that the researchers have been striving for. This
paper summarizes and analyzes many scholars’ researches on multi-objective ant colony algorithm, and proposes an “improved multi-objective ant colony algorithm”.

2. Multi-objective ant colony algorithm

Pareto Ant Colony Optimization (hereinafter referred to as P-ACO) was first proposed by Doerner et al. [9-10] in 2001, mainly to solve the problem of multi-objective portfolio selection, and achieved very good effects. Since the ant colony algorithm was proposed, how to improve the convergence speed of the algorithm and ensure that the multi-objective result is close to the Pareto front-end without depending on the weight of the objective function is the goal that many researchers have dreamed of. Although the existing P-ACO algorithm has been successfully applied in other multi-objective combinatorial problems such as portfolios, its versatility and practicability are far from satisfying people’s requirements. The main reasons are as follows:

1) The traditional single-objective ant colony algorithm has incomplete convergence, premature algorithm, and difficulty in demonstrating convergence. For P-ACO algorithm, these problems are also inevitable.

2) When the optimization algorithm solves the actual problem, the time for iterating out the new solution is often shorter, and the feasibility of judging the new solution is relatively complicated. How to minimize or avoid the production of a large number of repeated solutions, and expand the range of the solutions and greatly improve the efficiency of the algorithm as the number of iterations increases is currently a major problem facing the optimization algorithm.

3) For the problem of structural optimization design based on discrete variables, the current P-ACO cannot be applied to practical problems, and needs further improvement to meet the requirements of civil engineering.

2.1 Improved multi-objective ant colony algorithm

In order to better adapt to the multi-objective structure optimization design problem, it is necessary not only to reduce the occurrence of repeated solutions in the iterative process, but also to improve the mass of the new solution. To this end, based on the P-ACO algorithm, this paper puts forward the concept of “adding alternative repeated solutions”. The specific methods are as follows:

1) In the P-ACO algorithm, establish a “feasible solution set, FP” and “non-feasible solution set, NFP”, record each new solution one by one, and compare with the new solution generated from each iteration (NC > 1). If the new solution already exists, it will be replaced by a “special solution”. On the contrary, there is no effect.

2) This “special solution” is a solution in the front-end of the current iterative solution of the random selection algorithm, selecting its surrounding solutions in a specific order, and the solution satisfies the condition that not existing in “feasible solution set, FP” and “non-feasible solution set, NFP”, and such a solution is defined as the current “special solution”. This solution will be used to replace the repeated solution that occurs during this iterative process.

3) The weight of the release concentration set by each ant colony for different objective functions in the P-ACO algorithm is cancelled, and the improved P-ACO algorithm adopts the classic AS mode as the selection mode.

2.2 Flow chart of multi-objective optimization algorithm for truss structures
Figure 1 Truss structural multiple objective optimal design flow chart

2.3. Multi-objective algorithm convergence

If the number of iterations tends towards infinitude, it can converge the global optimal solution of the multi-objective problem with probability 1, i.e., the Pareto front-end $F^*$. For a finite number of vectors $V_1^*$, $V_2^*$, ..., and $V_N^*$ included in the discrete variable problem, the mathematical description is:

$$ P(\lim_{t \to \infty} F^* \in P(t)) = 1 $$

Where, $P(t) = P_{\text{current}}(t)$.

In reference to the proof method of the convergence of the MOCEA algorithm, the author found that if the “improved multi-objective ant colony algorithm” proposed herein can satisfy the two conditions of convergence, this algorithm will satisfy the basic convergence requirements of the multi-objective algorithm. The specific proof is as follows:

1. $\forall F^*, F' \in I, F'$ can be obtained by $F$.

Proof: For the method proposed herein, the generated repeated solutions needs to be replaced by “special solutions” in each iterative process. That is to say: within a limited space, when the number of iterations satisfies certain conditions, the algorithm can finally search out all the solutions in the space,
which satisfies the requirements of condition (1) by repeatedly using the process of replacing “repeated solutions” by “special solutions”.

(2) The $P_{current}(t)$ order solved by the algorithm is monotone, that is, the following conditions are met for $\forall t$:

$$\min\{\phi(F(t+1)) \mid F(t+1) \in P(t+1)\} \leq \min\{\phi(F(t)) \mid F(t) \in P(t)\}$$

Proof: At the end of each iterative process, the $P_{current}(t)$ monotone solved by the proposed algorithm herein approaches the requirement of Pareto front-end $F^*$. The main reason is that $P_{current}(t)$ generated in the previous iteration by this algorithm will combine with the solution generated by the $t+1$ iteration to form new $P_{current}(t+1)$. That is: of $P_{current}(t)$ and the solution generated by the $t+1$ iteration, the one closer to the solution of Pareto front-end $F^*$ will be retained after recombination. This update method satisfies the problem that the algorithm is monotone, and satisfies the requirement of condition (2). Therefore, the improved P-ACO algorithm proposed herein satisfies the convergence condition of the multi-objective algorithm.

3. 25-bar transmission illustrative example

Optimal design of 25-bar transmission tower structure. As shown in Figure 2, the model includes 10 nodes and 25 bars. The elastic modulus of each bar is $E = 68.96GPa$, and the volume weight is $\rho = 2715.08N/m^3$. The grouping situation of each bar and the allowable stress value of each bar are shown in Table 1. The load conditions are shown in Table 2. The lower limit of the cross-sectional area of each bar is $0.645cm^2$, and the set of section variables is shown in Table 3 (area unit: cm2). The main control parameter values in the ant colony algorithm are shown in Table 4.

| Bar type | Bar No. | Allowable compressive stress | Allowable tensile stress | Bar type | Bar No. | Allowable compressive stress | Allowable tensile stress |
|----------|---------|-----------------------------|--------------------------|----------|---------|-----------------------------|--------------------------|
| 1        | 1-5     | -242.04                     | 275.90                   | 3        | 3-6    | -242.04                     | 275.90                   |
| 2        | 1-5, 6  | -79.94                      | 275.90                   | 5        | 3-6, 5 | -46.62                      | 275.90                   |
| 3        | 2-5, 7  | -119.36                     | 275.90                   | 7        | 3-5, 7 | -46.62                      | 275.90                   |
| 4        | 3-4, 5  | -242.04                     | 275.90                   | 8        | 3-4, 5 | -76.44                      | 275.90                   |

Note: The allowable stress unit is: $MPa$.

| Working condition | Load node No. | Load component | Working condition | Load node number | Load component |
|-------------------|---------------|---------------|-------------------|------------------|---------------|
|                   | $P_x$         | $P_y$         |                   | $P_z$            |               |
| 1                 | 1             | 4.45          | 0                 | 4.45            | 0             |
| 2                 | 2             | 4.45          | 0                 | 4.45            | 0             |
|                   | 3             | 2.225         | 0                 | 2.225           | 0             |

Note: The load unit is: $kN$.

| Serial No. | Area | Serial No. | Area | Serial No. | Area | Serial No. | Area |
|------------|------|------------|------|------------|------|------------|------|
| 1          | 0.6452 | 2         | 1.2903 | 3         | 1.9355 | 4         | 2.5806 | 5         | 3.2258 |
| 6          | 3.8710 | 7         | 4.5161 | 8         | 5.1613 | 9         | 5.8064 | 10        | 6.4516 |
| 11         | 7.0968 | 12        | 7.3419 | 13        | 8.3871 | 14        | 9.0322 | 15        | 9.6774 |
Table 4. Control parameters of multi-objective ant colony optimization

| Parameters | α  | β  | ρ  | Q  | m  | NC_max |
|------------|----|----|----|----|----|--------|
| Numerical value | 1  | 0.2| 0.1| 1  | 30 | 200    |

The multi-objective optimization results solved by this algorithm are within the range of [2.06369, 87.17564] and [0.235643, 1032.933]. The Pareto front-end consists of 429 kinds of solutions, and the calculation result is obviously better than the 232 Pareto front-end solutions within the range of [977.39, 0.2363] and [99.87, 2.0281] solved by the multi-objective immune algorithm. Among them, the blue in Figure 3 is the Pareto front-end solution solved by the multi-objective immune algorithm, and the red is the result calculated by the algorithm. The comparison shows that the distribution of the Pareto front-end searched by this algorithm is more wide and uniform, the Pareto front-end is more complete and diverse, and the solving speed is faster and the constraints are less, etc. Therefore, the multi-objective ant colony optimization algorithm based on discrete variables in truss structure proposed herein has a good application prospect in practical engineering.

4. Conclusion

In order to better meet the needs of practical engineering, this paper proposes an improved multi-objective P-ACO algorithm based on multi-objective ant colony algorithm, i.e.: a multi-objective optimization algorithm for truss structures. The analysis and calculation of multiple illustrative examples further prove the feasibility and practicability of the algorithm. The detailed analysis conclusions are as follows:

1) The multi-objective optimization algorithm for truss structures proposed herein is an efficient multi-objective optimization algorithm. It shows good optimization effects in solving multi-objective section optimization of truss structures, and multi-objective section optimization of transmission tower structures.

2) Compared with the traditional multi-objective optimization algorithm in solving the multi-objective section optimization problems of truss structures and transmission tower structures, the proposed algorithm not only has the same advantages as solving multi-objective functions, but also takes a relatively short time to obtain a satisfactory Pareto front-end, and the types of solutions are more diverse.
(3) This algorithm shows a good superiority in solving structural multi-objective optimization design problems, thus further proves its application value in practical engineering, and provides theoretical basis and design ideas for multi-objective topology, shape and layout optimization of the truss structure (transmission tower) in the future.

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