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CONCRETE UNDER TRANSIENT THERMAL CONDITIONS:  
A STATE-OF-THE-ART REVIEW

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Abstract

Extensive research has been carried out over the past four decades on the behaviour of mechanically loaded concrete under transient thermal conditions. The purpose of this paper is to provide a concise review of the existing experimental and analytical works with a strong focus on the load-induced thermal strain (LITS) component. In order to eliminate ambiguities in definitions, the existing terms used to describe the strain components that develop in concrete under a transient thermal regime are compared and a clear definition of LITS and its components is given. The analysis of the existing experimental work shows that LITS is: mainly irrecoverable in terms of temperature; substantially reduced in case of stress-free preheating cycles; significantly influenced by the moisture flux in the temperature range 100-250°C; and, independent of aggregate type for temperatures up to about 400°C. Examination of the existing multiaxial test data demonstrates that LITS is the result of markedly anisotropic phenomenon and that experiments on concrete subjected to triaxial compression and transient temperatures above 250°C are needed. In the light of the experimental evidence, LITS seems to be mainly due to chemical and physical reactions taking place in the cement paste for temperatures up to about 400°C. By contrast, for higher temperatures, thermomechanical damage due to thermal incompatibility between cement paste and aggregates is believed to contribute significantly to the development of LITS. Moreover, the necessity for modelling explicitly the LITS component in the case of Heating-Cooling (HC) cycles is discussed. Finally, a review of the main existing uniaxial and multiaxial explicit LITS models is given, and the advantages and drawbacks of each model are outlined.

Nomenclature

\(a, b, c\)  Polynomial coefficients
\(E\)  Young’s modulus of elasticity of material
\(k_{tr}\)  Transient strain coefficient
\(T\)  Temperature
\(\bar{T}\)  Normalized temperature
\(V_a\)  Volume fraction of aggregates
\(\alpha\)  Thermal expansion coefficient expressing the ratio between the increment in free thermal strain and the increment in temperature
\(\alpha_T\)  Thermal expansion coefficient expressing the slope of the free thermal strain curve
\(\beta\)  Load-induced thermal strain function for multiaxial models
\(C_m\)  Triaxiality coefficient for multiaxial models
\(\Delta\varepsilon_{ela}\)  Increment in the elastic strain
\(\delta_{ij}\)  Kronecker symbol
\(\varepsilon\)  Strain
\(\varepsilon_{0,3}\)  Load induced thermal strain for load \(\sigma_i/\sigma_{u0} = 3\)
\(\varepsilon_{cr}\)  Creep strain
Introduction

Experimental evidence shows that mechanically loaded concrete specimens exhibit a significant quasi-instantaneous load-induced thermal strain, usually referred to as LITS, upon virgin heating. Accurate understanding and modelling of this phenomenon is crucial for a reliable assessment of the effects of thermal loads on concrete structures, particularly if a certain level of performance is required in the case of accidental loads such as fire. This is the case for nuclear structures, such as prestressed concrete pressure vessels (PCPVs). For example, the recent Fukushima accident, where reactors overheated due to a failure of the power station cooling system, clearly highlight the importance of considering a wide range of possible accidental conditions in designing and assessing nuclear structures. Moreover, other applications in civil engineering occur whenever concrete is loaded in compression under transient thermal conditions, for example concrete columns subjected to travelling fires [1].

Keywords

Concrete; Temperature; Fire; Load-induced thermal strain; Transient thermal creep; Thermal strain; Modelling;
LITS plays a key role in the behaviour of concrete structures subject to heating-cooling (HC) cycles combined with sustained compressive loads, both at a material and structural level. In the case of structural elements whose thermal deformation is partially or totally constrained, its effects are mainly beneficial during the first heating. This is because LITS develops in the loaded direction, thereby mitigating the additional compressive stresses appearing in such structures due to restraint to thermal expansion [3,4]. In addition, at a material level, LITS produces a stress redistribution mitigating concrete damage due to thermal incompatibility between cement paste (which tends to shrink considerably on heating, especially in the case of unsealed conditions and temperatures greater than 100 °C) and aggregates, which usually exhibit a thermal expansion [5].

By contrast, significant tensile stresses and associated cracking effects may also be related to this phenomenon. Since LITS is mainly irrecoverable in terms of temperature, thermal incompatibilities between the constituents of concrete may cause severe damage at a material level during the cooling phase. Similarly, constrained structures tend to contract and an unsafe level of tensile stresses may develop [6].

Despite these concerns, and considerable research efforts over the last forty years, a comprehensive up to date survey of LITS describing the state of knowledge and how it is best handled in analysis and design is lacking. Earlier attempts at such a survey are now either dated [4,7] or focussed on only few aspects of the phenomenon [6]. This paper aims to fill the gap by providing researchers and practitioners a comprehensive review of the topic, and by highlighting areas where further research is needed.

With these aims in mind, the first part of the paper outlines and discusses the experimentally demonstrated characteristics of LITS, as well as identifying gaps in knowledge. This leads to the second part of the paper that provides a discussion of the need to explicitly model LITS in numerical simulations, together with a critical review of the existing analytical models of LITS.

## 2 Transient thermal conditions and LITS

### 2.1 Experimental background

In order to evaluate the behaviour of concrete at high temperatures, various uniaxial test methods have been designed, among which two main types can be identified (Figure 2-1):

- Steady-state tests, where the material is subjected to a heat-then-load (HTL) regime: first the specimen is heated uniformly to a pre-defined temperature, then a mechanical load is applied, while the temperature of the material is kept constant. ($T=\text{constant}, \sigma \text{ varying}$).
- Transient tests, where the material is subjected to a load-then-heat (LTH) regime: first the specimen is mechanically loaded, then it is uniformly heated, while the mechanical load is kept constant. ($\sigma =\text{constant}, T \text{ varying}$).
In case of steady-state tests, the specimen is heated to a predefined temperature and then a specific load programme is applied. If a monotonically increasing load (or, equivalently, displacement) is applied until the strength of the material is reached, the stress-strain curve corresponding to a given temperature can be obtained. Usually, HTL tests are performed with a relatively high loading rate, so that development of delayed strain components, (e.g. basic creep, drying creep and shrinkage), during the loading phase can be neglected. These tests are useful for determining the “hot” mechanical parameters of the material, i.e. Young’s modulus $E(T)$, compressive strength $f_c(T)$ and ultimate strain $\varepsilon_u(T)$. Often, once a thermal equilibrium state is reached within the material, a pre-defined mechanical load is applied and kept constant over time. This approach allows evaluation of the “hot” Young’s modulus of the material – if a moderate compressive load is applied – and the time dependent strains occurring for a HTL regime (Figure 3-2). Similarly, relaxation tests may be performed, where an initial strain is imposed and kept constant over time, while the stress level is recorded. It is worth noting that such curves cannot be directly used to reproduce the most common structural problems, where the mechanical load can be modelled as constant while the temperature is varying over time [8]. In fact, it has been demonstrated that the strains occurring in case of HTL can be significantly higher than the ones occurring in case of LTH [9].

Transient tests consist of loading a specimen (by a given stress or strain) and applying a particular temperature programme over time. In order to ensure a uniform temperature field throughout the specimen and to simulate different service and accidental thermal loads, the heating rate should be in the range of $0.1 – 10 \, ^\circ C/min$ [4]. If a monotonically increasing thermal load is applied under constant stress, the results are often expressed by plotting the strain $\varepsilon_{heat}(T)$ developed during the heating phase as a function of the temperature of the material, for different load levels (Figure 2-4). The applied stress is usually expressed as a percentage of the ambient compressive strength $\sigma_{u0}$ of the material. These types of test are particularly meaningful since they effectively reproduce the design conditions of concrete structures subjected to permanent loads together with
expected temperature cycles or unpredictable accidental thermal loads. On the other hand, transient tests where the total deformation is kept constant after the application of the mechanical stress can be designed. Such tests are particularly interesting in assessing the behaviour of structural elements such as columns which may be restrained during heating and therefore subjected to constant strain and varying stresses (due to restrained thermal expansion) during the rise in temperature.

2.2 Definition of FTS and LITS

Existing research shows that the thermal strain of concrete is significantly reduced when a constant compressive load is applied while heating. Such a reduction has been commonly seen as an additional thermal strain component due to the presence of a constant stress occurring in the loaded direction.

Over the past 40 years, a number of terms have been used to describe this thermo-mechanical strain and its components, including LITS, transient creep (TC), transient strain (TS) and transient thermal creep (TTC). Since the physical meaning of these terms has been interpreted differently by various authors, this section aims at clearly defining LITS and its components.

When heated, concrete is generally subjected to a volumetric expansion. The strain in an unloaded specimen during the heating process is mainly due to two coupled phenomena: thermal expansion $\varepsilon_{th}$, related to the thermal expansion of the aggregates, and shrinkage strain $\varepsilon_{sh}$, due to a loss of water in the cement paste. In general, these two mechanisms result in a global volumetric expansion, usually called stress-independent strain or Free Thermal Strain (FTS) $\varepsilon_0$ (Figure 2-2). Several studies investigating the behaviour of concrete in the case of stress-free thermal load have demonstrated that concrete’s expansion behaviour is highly non-linear with temperature [3,10,16]. Figure 4-1 shows that such FTS is very sensitive to the content and nature of aggregates used. For simplicity, the strains in the longitudinal direction only are shown in Figure 2-2, Figure 2-3 and Figure 2-5.

![Figure 2-2 Schematisation of FTS ($\varepsilon_0$) developing in a mechanically unloaded specimen subjected to a thermal load. Only the longitudinal component is shown.](image)

If the test is repeated with the only difference being a constant compressive mechanical load applied before heating the specimen – LTH – the strain which develops during the heating phase is different from the FTS measured in the unloaded specimen – usually referred to as control specimen. This difference in strain can be seen as an additional temperature dependent strain component, occurring in presence of a compressive stress.
state, hereby referred to as LITS ($\varepsilon_{\text{lits}}$). Although different names have been used to refer to LITS, the same physical definition has been adopted by many authors [3,6,10,11].

The strain components that develop in concrete specimens subjected to constant compressive load are schematically represented in Figure 2-3, where $\varepsilon_{\text{tot}}$ is the total strain of the specimen and $\varepsilon_{\text{ela},0}$ is the elastic strain measured at ambient temperature immediately after loading the specimen.

Figure 2-3 Definition of $\varepsilon_{\text{lits}}$: schematisation of the strain components developing in a concrete specimen subjected to (a) stress-free heating (b) mechanical loading (c) heating under constant mechanical load.

According to this definition, LITS may be experimentally estimated as follows:

$$\varepsilon_{\text{lits}} = \varepsilon_{\text{tot}} - \varepsilon_0 - \varepsilon_{\text{ela},0} = \varepsilon_{\text{tot}} - \varepsilon_{\text{th}} - \varepsilon_{\text{sh}} - \varepsilon_{\text{ela},0}$$  \hspace{1cm} (2-1)

Where $\varepsilon_{\text{tot}}$ and $\varepsilon_{\text{ela},0}$ are measured in the loaded specimen, while the FTS $\varepsilon_0$ is measured in an unloaded control specimen, subjected to the same thermal load of the loaded one.

Often, the results of LTH tests performed for different stress levels are expressed by plotting the strain that develops during the heating phase $\varepsilon_{\text{heat}}$, usually referred to as thermal strain, against the average temperature of the specimen $T$, where $\varepsilon_{\text{heat}}$ is evaluated as:

$$\varepsilon_{\text{heat}} = \varepsilon_{\text{tot}} - \varepsilon_{\text{ela},0}$$  \hspace{1cm} (2-2)

According to the definition above, $\varepsilon_{\text{lits}}$ may be seen as the difference between the strain developing during the heating phase $\varepsilon_{\text{heat}}$ and the FTS $\varepsilon_0$:

$$\varepsilon_{\text{lits}} = \varepsilon_{\text{heat}} - \varepsilon_0$$  \hspace{1cm} (2-3)

Thus, if on the same chart the values of $\varepsilon_{\text{heat}}$ for different stress levels are plotted together with the unloaded control specimen behaviour (for which $\varepsilon_{\text{heat}} = \varepsilon_0$), a curve expressing LITS as a function of temperature is produced. For each stress level it is the
difference between the curve expressing $\varepsilon_{\text{heat}}$ for that particular stress level and the curve obtained for the unloaded control specimen (Figure 2-4).

Figure 2-4 Total thermal strain measured during a LTH test, expressed as a function of temperature, for different load levels. Aggregate: basalt. Heating rate: 1°C/min. Adapted from [12].

It is important to underline that in several studies LITS has been defined as the difference between the total mechanical strain - stress dependent strain - and the instantaneous elastic deformation $\varepsilon_{\text{ela}}(T)$, thus not including any increment in the elastic strain due to a temperature rise, as shown from Figure 2-5 [13–15].

The notation $\varepsilon_{\text{LITS}}$ is used here to refer to LITS according to this second definition:

$$\varepsilon_{\text{LITS}} = \varepsilon_{\text{tot}} - \varepsilon_0 - \varepsilon_{\text{ela}}(T)$$ (2-4)
3 Experimentally demonstrated characteristics of LITS

The first experimental evidence of LITS emerged during the 1960s as a result of experiments by Johansen & Best [17] and Hansen & Eriksson [9]. Over the last five decades, a range of further experimental studies have documented the existence and the main features of the LITS phenomenon. A thorough review of the investigations carried out until the mid-1980s was produced by Khoory, Grainger, & Sullivan [7], while several interesting partial reviews have been produced since. This section brings the survey of experimental work up to date by organizing, comparing and discussing the main outputs of the experimental research in order to identify the key features of LITS, without strictly adhering to the chronological sequence of different works.

Firstly, the experimentally demonstrated effects of temperature on elasticity and creep are discussed in order to define the different strain components of LITS. Then, the main parameters influencing LITS are analysed in the light of the experimental evidence. In particular, the dependence of LITS on the hygral, thermal and mechanical loading conditions is considered, as well as the influence of concrete material properties.

Finally, a critical discussion of the experimentally demonstrated characteristics of LITS is presented, with a view of identifying its physical origins.

3.1 Components of LITS

According to the definition of LITS reported in (2-1), it includes a contribution due to the variation of the elastic strain with temperature, and another due to the development of basic and drying creep, which are accelerated by a rise in temperature. Therefore, in order to understand the mechanisms underlying LITS and to define its components, the main effects of temperature changes on the elastic and creep strains for steady state conditions need to be outlined first.
3.1.1 Temperature and modulus of elasticity

It is well known that the elastic stiffness of concrete decreases with increasing temperatures [18–27]. Such degradation of the elasticity can be attributed to physical and chemical changes in concrete microstructure.

The relationship between modulus of elasticity and temperature has been proved to be highly sensitive to the type of aggregates used. For example, if normal weight aggregates are used, $E$ decrease more rapidly with increasing temperatures than in case of lightweight concrete [4,20].

The elastic modulus is also connected to the load-temperature history path. In particular, if concrete is loaded during the heating process, i.e. is subjected to a LTH regime, it develops a higher stiffness than if it is subjected to HTL regime [19,21]. Moreover, the original strength of concrete, the water to cement ratio, the type of cement and the stress level – as long as it remains in the range of 0.1 to 0.3 $f_c$ – have been shown to have little or no influence on the relationship [4].

In case of HTL regimes, the curve expressing the modulus of elasticity as a function of temperature may be obtained by evaluating the initial tangents of the isothermal constitutive curves.

For LTH regimes, such a curve can be obtained by measuring the elastic expansion-contraction developed during a quasi-instantaneous unloading-loading cycle performed at different temperature levels (Figure 3-1). In this way, [13] showed that the elastic strain increment developed during transient heating, $\Delta \varepsilon_{\text{ela}}(T)$ is mainly irrecoverable, in terms of temperature, for various types of concrete including ordinary concrete and high-performance concrete. In Figure 3-1, the instantaneous elastic strain $\varepsilon_{\text{ela}}(T)$ is represented by the length peaks of the total strain evolution.

![Figure 3-1 Temperature and total strain evolution for a high performance concrete specimen subjected to an HC cycle in LTH regime. The peaks of strain evolution curves are due to quasi-instantaneous unloading-loading of the specimen [15].](image.png)
3.1.2 Temperature and creep strain

Several reports have shown that the short-term creep behaviour of unsealed specimens, (those allowed to exchange moisture with the ambient atmosphere), for steady state conditions, is a function of temperature. In particular, the magnitude of creep strains increases rapidly as temperature increases [18,19,23,24,28]. Creep tests at constant, high temperatures are usually performed on unsealed specimens, since practical issues arise in imposing sealed condition for temperatures higher than 100 °C [29], especially for uniaxial stress states [30]. Figure 3-2 shows that the effect of temperature on creep strains grows rapidly for temperatures higher than 300-400 °C.

If LTH regimes reproducing the actual accidental fire situations are taken into account, the isothermal creep contribution to the total strain developed during the heating phase is negligible with regard to the LITS. This is because the time needed for appreciable creep strains to develop is usually much greater than the duration of the transient phase, even for heating rates slower than expected in case of real heating situations [4,24,31–33]. However, in the case of a LTH regime with long-term high temperature conditions, i.e. structures subjected to severe and prolonged accidental conditions, creep strains that develop after the initial transient phase cannot be disregarded.

3.1.3 Definition of the LITS components

The experimentally measured magnitude of LITS exceeds the expected creep and elastic strain increments due to the increase in temperature during HTL tests [4,23,24,31–33]. This leads to the definition of an additional strain component, which occurs during first heating under load, known as TC [34] or TS [6,19,23,31] – see Figure 3-3. In this work, the term TS is used:

$$\varepsilon_{lits} = \Delta \varepsilon_{eta} + \varepsilon_{cr} + \varepsilon_{ts}$$ (3-1)

It is important to underline that it is almost impossible to design tests allowing transient strain to be measured directly [19].

For unsealed specimens, TS is itself composed of two different strain components: drying creep $\varepsilon_{d,cr}$, due to the occurrence of an accelerated drying process on heating, and a moisture-flux-independent component TTC $\varepsilon_{ttc}$:
The existence of the TTC component $\varepsilon_{ttc}$ is proven by the fact that TS develops even in case of sealed specimen, i.e. in absence of drying creep [9]. Thus, for sealed conditions, one obtains $\varepsilon_{ts} = \varepsilon_{ttc}$:

$$\varepsilon_{lits} = \Delta \varepsilon_{ela} + \varepsilon_{cr} + \varepsilon_{dcr} + \varepsilon_{ttc}$$  \hspace{1cm} (3-2)

In Table 3-1, an overview of the definition adopted by different authors to refer to different strain components is provided.

| Strain component studied | Term used | Works |
|--------------------------|-----------|-------|
| LITS                     | LITS      | [3,4,6,7,11,35–37] |
|                          | TTC       | [38]  |
|                          | TTS       | [10]  |
|                          | INCREASE IN CREEP | [39] |
| TS                       | TS        | [19]  |
|                          | TC        | [29,40]| |
| TTC                      | TC        | [41]  |
| LITS*                    | TTC       | [13,15,42] |
|                          | TTS       | [37]  |

Table 3-1 Strain components studied (according to the definition adopted in this work) and terms used to refer to them by different authors.
3.2 Hygral conditions

3.2.1 Experiments and sealing conditions
It has commonly been assumed that the hygral conditions of concrete in structures
where LITS is of significance are better represented by unsealed than by sealed concrete
specimens [4,43]. For example, as experimentally demonstrated by Hornby & Grainger
[44], this is true of the top-cap of PCPVs, since the moisture vapour present in such
areas can migrate relatively easily to the atmosphere through the numerous steel
cement-interfaces in case of severe thermal loads. For this reason, studies on the
transient state behaviour of concrete at temperatures higher than 100°C have focused
mainly on unsealed conditions, while all tests on sealed specimens have been
performed at temperatures lower than 100°C [9,17,45–47]. One exception to this
general situation is an unusual experimental study performed on specimens whose
boundary conditions may be interpreted as “partially sealed”, due to the presence of
steel plates preventing moisture from evacuating through most of the external surfaces,
[11].

3.2.2 Time independency and irrecoverability
LITS is commonly regarded as a quasi-instantaneous strain component, i.e. as a time-
independent phenomenon [23]. This assumption is based on the fact that it mostly
occurs during the transient state phase, and the time needed for it to fully develop after
the temperature has been stabilized is generally relatively little compared to the usual
duration of isothermal creep. This is particularly true for unsealed specimens subjected
to temperatures higher than 250 °C, where the drying process is completed (see section
3.2.3).

Several studies have demonstrated that LITS developed on first heating is mostly
irrecoverable on cooling and that if the material is reheated under sustained load, no
appreciable additional LITS occurs for temperature levels lower than the maximum
temperature reached during first heating [10,11,15,39,42,45,46,48]. Figure 3-4 shows
the temperature and strain histories of an unsealed concrete specimen, subjected to a
relative humidity of 50% and several HC cycles, demonstrating this point clearly.

Despite normal assumptions, it is worth noting that, in reality LITS is a time-dependent
phenomenon. In fact, the time needed for LITS to fully develop has been found to be
strictly related to the hygral boundary conditions for low temperatures. For example,
Fahmi et al. [39] demonstrated that when concrete is exposed to saturated air, i.e. is
prevented from exchanging moisture with the ambient atmosphere, LITS needs much
more time to develop than in case of 50% relative humidity. Moreover, in the case of
100% relative humidity, even though most of LITS develops upon first heating, several
HC cycles up to 60 °C under load are needed in order for LITS to reach a limit value
(see Figure 3-4). These results suggest that moisture flux plays a key role on the time
for LITS to develop and, consequently, on its occurrence upon secondary heating
phases.
3.2.3 Moisture content and LITS

In case of unsealed conditions, the initial moisture content has been found to have little or no influence on the development of LITS for temperatures higher than 250 °C [3], while it has been found to influence concrete behaviour at lower temperatures [11].

Specifically, LITS developed for temperatures up to 250°C in initially moist and air-dried specimens has been found to be comparable, while a significant reduction has been measured in the case of oven-dried specimens [3]. Moreover, it was observed that above 250°C the evolution of the LITS curve does not significantly depend on the initial water content [3], as shown in Figure 3-6. These results are consistent with the main output of tests performed to evaluate the effect of preheating on LITS, outlined in previous sections, and with the results of stress-free-heating tests aimed at evaluating moisture loss as a function of the temperature. In fact, different authors found that the water content of stress-free specimens does not depend on the initial moisture for temperatures higher than 250 °C, meaning that the drying process is completed at that temperature level, i.e. the evaporable (free) water has disappeared (Figure 3-5).
Figure 3-5 Weight loss as a function of temperature for different initial moisture contents. Rate of heating 5 °C/min [19]

3.3 Thermal load

3.3.1 Effect of temperature

Of all the parameters influencing the development of LITS, temperature is the most significant. The development of LITS with the temperature is highly nonlinear: in general the LITS coefficient, i.e. the slope of the LITS curve, gradually increases with the temperature. However, this is only a general trend and there are various details to consider. By analysing the global trend of the LITS curve for LTH tests up to elevated temperatures (Figure 3-7), various authors have found that LITS starts developing significantly only after 100°C [10,11]. Such a sharp start to LITS development is probably linked to the significant drying of the cement matrix that occurs at temperatures between 100°C and 200°C (Figure 3-5). Khoury et al [3] found that, even though the LITS coefficient generally increases with temperature, for both air dried and initially moist concrete there is a minimum at about 150 °C – maybe due to the conclusion of the drying process. They also noted a slight reduction between 500°C and 600°C, possibly related to the Ca(OH)₂ content (Figure 3-6).
3.3.2 Heating rate and LITS

For heating rates lower than 5°C/min, the heating rate has been found to have little influence on the LITS behaviour of small specimens subjected to LTH tests [3,19,23,34], while for higher heating rates, a significant influence on FTS and LITS has been demonstrated. Such dependency is due to the appearance of substantial thermal gradients through the specimens, owing to structural effects [23]. Specifically, unloaded specimens exploded as a result of the relatively fast thermal contraction and of the strength degradation of the external layer of the material. Similarly, premature failure has been observed for loaded specimen, stress levels greater than 60% of the compressive strength [23].

3.3.3 Preheating cycles

Hansen & Eriksson [9] showed that if concrete is first subjected to a HC cycle up to 60°C and 100°C without being loaded, and then subjected to a LTH test, LITS develops to a lesser extent upon the second heating to the previously attained temperature. Since their specimens were submerged in water, the recorded LITS did not include drying shrinkage effects, meaning that a preheating cycle inhibits the development of the TS component. Similarly, LITS of unsealed specimens has been found to be strongly reduced in the case of stress-free preheating cycles [10,14,15,45,48], as shown from Figure 3-7. In addition, Mindeguia et al. [10] demonstrated that the development of LITS is slightly influenced by the duration of the preheating period. In particular, when the constant temperature phase of the preheating cycle is prolonged, LITS needs higher temperatures to start developing, and its magnitude decreases (see Figure 3-7).

Together, these studies suggest that, for practical purposes, it can be assumed that LITS is activated only when the temperature is higher than the pre-heating temperature. These results provide interesting insights into the assessment of PCPVs of AGRs, where the hydration process taking place during the early age of concrete curing may lead to the development of significant endogenous heat and, consequently, to relevant rises in temperature in absence of load.
3.4 Mechanical load

3.4.1 Influence of loading levels

As shown in Figure 2-4 and Figure 3-11, LITS has been shown to be reasonably proportional to the compressive load level when this is limited to 30–40% of the compressive strength [3,10,19]. This proportionality led to the concept of normalized or specific LITS [10], defined as the LITS due to a unit of stress, similar to the concept of specific basic creep, based on the well-known proportionality between the latter and applied stress for moderate stress levels.

3.4.2 LITS in the unloaded direction and multiaxiality

Most LITS test have been performed in uniaxially loaded concrete specimen, while a small number of experiments involved biaxial and triaxial compression, therefore reproducing conditions more representative of the actual stress states of many heated concrete members.

3.4.2.1 LITS in the unloaded direction for uniaxial tests

A few works have shown that an expansive LITS strain appears in the unloaded direction in the case of uniaxial compression [10,11,25]. This leads to the definition of LITS Poisson’s ratio $\nu_{\text{LITS}}$, a coefficient analogous to the Elastic Poisson’s ratio $\nu$ and defined as the ratio of transverse to axial LITS strain. Yet, the evolution of LITS in the unloaded direction with the temperature is not proportional to LITS in the loaded direction. From the analysis of the results obtained by Kordina et al. [25], shown in Figure 3-8, it can be inferred that the temperature at which LITS in the unloaded direction starts becoming significant, i.e. the temperature at which the thermal strain in the unloaded direction deviates from the FTS curve, decreases with the stress level. In particular for stress levels equal to 20%, 40% and 60%, substantial lateral LITS develops starting from about 400°C, 250°C and ambient temperature respectively. Such results are in quite good agreement with the ones obtained by Petkovski & Crouch [11], where for a load level equal to 44% LITS in the unloaded direction was found to
develop significantly above 100°C. Moreover, this trend is also confirmed by the results obtained by Mindeguia et al. [10] for a load level of 20%, proving the existence of significant LITS in the unloaded direction after 400°C. The authors argued that this expansion is related to the anisotropic crack pattern induced in the specimen: for temperatures higher than 400 °C, cracks parallel to the loading direction were observed, in accordance to results obtained by Ehm & Schneider [22].

![Figure 3-8 Total thermal deformation in the loaded and unloaded direction during uniaxial LTH test. Load levels 0%, 20%, 40%, 60 %. [25]](image)

3.4.2.2 LITS in the loaded and unloaded directions for biaxial tests

LTH test involving biaxial stress states have been performed at the University of Braunschweig [22,25,49] and and the Univesity of Sheffield [11].

Figure 3-8, Figure 3-10 and Table 3-2, produced from the thermal strain curves obtained by Kordina et al. [25], show that, in the loaded direction, the magnitude of LITS obtained for biaxial compression is comparable with the one obtained for uniaxial tests, for load levels in the range 20-60% and temperatures up to 600 °C. Similar results have been obtained Petkovski & Crouch [11] for a load level of 44% and temperatures up to 250 °C (see Figure 3-9).

According to the results obtained by Kordina et al. [25], summarized in Table 3-2, the LITS in biaxial tests in the unloaded direction is always significantly higher than twice the lateral expansion measured in uniaxial tests. This phenomenon has been confirmed by the tests performed by Petkovski & Crouch [11]. According to these findings, a Poisson’s ratio higher than the one evaluated for uniaxial compression has to be defined to express the relationship between LITS in the loaded and unloaded direction in case of biaxial tests. In other words, \( v_{\text{LITS}} \) depends on the triaxiality of the stress state. It is worth noting that, in the biaxial tests performed by Petkovski & Crouch [11], smaller LITS were recorded, particularly in the unloaded direction, probably due to the presence of partially sealed conditions obstructing the moisture evacuation and, therefore, limiting
the drying creep component of LITS. Consequently, in [11] \( v_{\text{LTS}} \) at 250°C was found to be 0.34 for uniaxial compression and 0.37 for biaxial compression, while Kordina et al. [25] previously measured a \( v_{\text{LTS}} \) of 0.46 in biaxial tests at the same temperature.

These findings suggest that, in case of biaxial stress states, the LITS in the loaded and unloaded direction cannot be predicted by a mere superposition of the lateral expansions measured for uniaxial compression, that is concrete subjected to LTH regime exhibits a markedly anisotropic behaviour.

Figure 3-9 LITS in the loaded and unloaded direction during uniaxial, biaxial and hydrostatic LTH test. Load level 44% [11].

Figure 3-10 LITS in the loaded and unloaded direction during biaxial LTH test. Load levels 0%, 20%, 40%, 60%. Deduced from the thermal strain curves obtained by Kordina et al. [25]
| Load  | Temperature [°C] | LITS loaded direction [%o] | LITS unloaded direction [%o] |
|-------|------------------|----------------------------|-----------------------------|
|       |                  | Uniaxial                   | Biaxial                     | Uniaxial                   | Biaxial                     |
| 20%   | 200              | -0.80                      | -0.73                       | 0.01                       | 1.54                        |
|       | 400              | -2.01                      | -1.61                       | 0.31                       | 3.54                        |
|       | 600              | -6.23                      | -6.95                       | 1.75                       | 4.63                        |
| 40%   | 200              | -1.46                      | -1.66                       | 0.16                       | 2.55                        |
|       | 400              | -2.90                      | -2.73                       | 1.40                       | 5.11                        |
|       | 600              | -11.58                     | -9.28                       | 4.33                       | 8.52                        |
| 60%   | 200              | -3.07                      | -4.42                       | 1.23                       | 3.47                        |
|       | 400              | -3.92                      | -5.76                       | 2.92                       | 6.20                        |
|       | 600              | -13.73                     | -15.48                      | 7.28                       | 16.77                       |

Table 3-2 LITS for different load levels and temperatures, in the loaded and unloaded directions, for uniaxial and biaxial tests. Adapted from [25].

3.4.2.3 LITS in triaxial tests
To the authors’ knowledge, the development of LITS in case of multiaxial stress states has been experimentally analysed only by Petkovski & Crouch [11]. They found that, in case of hydrostatic stress states, LITS starts developing significantly for temperatures above 100-120 °C and around 250°C it reaches values close to two thirds of the LITS measured in case of uniaxial tests and biaxial tests in the loaded direction (see Figure 3-9).

However, to date there have been no LTH tests involving triaxial stress states and transient temperatures above 250 °C.

3.5 Concrete properties

3.5.1 Cement paste and LITS
It has been demonstrated that LITS occurs in hardened cement paste [3,10,45] and it is 2-3 times higher than when LITS is measured in concrete with aggregates [3]. Accordingly, different studies have shown that LITS is larger for high performance concretes, characterized by low water/cement ratios [14,15]. These results suggest that LITS is a phenomenon taking place in the cement paste, and is somehow restrained by the aggregates, i.e. that it is proportional to the cement content.

3.5.2 Aggregates and LITS
Several studies shown that unlike what happens for the FTS, the nature of aggregates has little or no influence on LITS for moderately high temperatures, while it becomes an influencing parameter for extremely high temperatures. The temperature at which the LITS curves for different aggregates start to slightly deviate from each other has been found to be in the range 300 °C to 450 °C [3,10,16].
As a consequence, Khoury et al. [3] stated that a ‘master’ LITS curve exists, expressing LITS as a function of temperature for a given stress/strength ratio, preheating condition, heating rate and curing regime, regardless of the type of concrete.

3.5.3 Influence of age of concrete at loading and heating

Parrot [48] found that the magnitude of LITS occurring in unsealed specimens in the case of LTH regimes is quite sensitive to the instants at which concrete is loaded and heated. In particular, LITS was found to decrease with both the age of concrete at loading, and the time between loading and heating. However, it is worth noting that the tests only explored such sensitivity for ages of less than 1 year. In fact, Figure 3-11 shows that for ages at between 1 and 9 years, LITS does not vary significantly [3,12]. Therefore, LITS can be seen as an age-independent phenomenon for scenarios involving mature concrete.

![Figure 3-11](image)

3.6 Discussion on the physical origins of LITS

Despite the number of experimental and theoretical works that have been published over the last four decades on the subject, the complexity of the mechanisms underlying LITS have prevented the elucidation of a unique and universally accepted explanation of the phenomenon. For this reason, this section outlines and discusses the various theories of the physical mechanisms causing LITS, in the light of the existing experimental evidences.

The fact that the strains that develop during transient conditions are much higher than the expected strain due to isothermal creep, accelerated drying creep and increment in elastic strain, suggests that other mechanisms, causing TTC, exist during heating under sustained load.

At low temperatures, concrete degradation due to thermal incompatibility between aggregates (which tend to expand) and cement paste (which tends to shrink) is unlikely to be one of the causes of TTC, since it has been experimentally proved that TTC takes place to a higher degree in pure hardened cement paste [45,46], and does not depend on the nature of aggregates, when those are included in the mixture.
Similarly, the development of cracks due to thermal gradients is unlikely to be the main mechanism leading to TTC, considering that TTC develops even for very slow heating rates, i.e. in case of negligible thermal gradients [9].

Instead, many scholars hold the view that, for low temperatures, TTC is mainly due to physical disintegration and chemical reactions taking place in the cement paste in conjunction with a rapid internal mass transfer in the porous media, while the thermal incompatibility between cement and aggregates significantly influences the development of TTC for high temperatures [10,23,26].

For temperatures up to 300-400 °C, a temperature rise might cause an increase of the rate of chemical decomposition of the cement paste (mostly due to the dehydration of the C-S-H), together with a rearrangement of the water molecules of the cement gel, which may occur even in case of sealed conditions, i.e. in absence of drying. A possible explanation of TTC in this temperature range is that, when a simultaneous sustained compressive load is applied, such temperature-dependent material transformations lead to a strain in the loaded direction [23].

By contrast, for temperatures higher than 300-400°C, the development of crack patterns has been documented even for slow heating rates, whereas TTC has been proved to increase sharply and to depend on the aggregate type. An implication of these observations may be that the thermal incompatibility between aggregates and cement causes cracking, i.e. a thermomechanical damage, therefore contributing to the development of TTC for this temperature range [3,10].

4 LITS Models

This section reviews the literature on uniaxial and multiaxial LITS models. Since FTS is the other component of strain that develops during heating $\varepsilon_{\text{heat}}$ (see section 2.2), and is therefore a crucial contribution to be implemented in a material model, a concise review of the most common FTS models is firstly given. Afterwards, a discussion of the necessity of explicitly including LITS in concrete behaviour laws is presented. Finally, the main existing uniaxial and multiaxial models are outlined and critically discussed.

4.1 FTS models

FTS, $\varepsilon_0(T)$, is commonly modelled simply as a function of temperature, $T$, and obtained from stress-free heating tests directly. Usually it is expressed by functions involving a limited number of parameters such as low-order polynomials of temperature.

A common way to express $\varepsilon_0$ as a polynomial of a temperature is to express it as a function of a thermal expansion coefficient $\alpha$:

$$\varepsilon_0 = \alpha(T - T_0)$$  (4-1)

Where $T_0$ is a reference temperature and $\alpha$ is a polynomial expressed as a function of temperature. For normal weight concrete with siliceous aggregates, Lie [50] proposed expressing $\alpha$ as a linear function of the temperature, thus obtaining a parabolic FTS strain curve:
\[
\alpha = (a \times T + b) \quad (4-2)
\]

1. Where the calibration of parameters \(a\) and \(b\) leads to:

\[
\alpha = (0.008T + 6)10^{-6} \quad (4-3)
\]

2. Another interesting function, designed to fit the FTS curve for temperatures above 600°C, where a stabilisation of the thermal strain has been experimentally observed [51], has been proposed by Nielsen, Pearce, & Bicanic [52] and Pearce et al. [29]. According to this formulation, a coefficient \(\alpha_T\) is defined as the ratio of strain rate \(\varepsilon_0\) to the temperature variation rate \(\dot{T}\), i.e. the slope of the FTS curve, therefore having a different physical meaning to the \(\alpha\) coefficient defined in (4-1):

\[
\dot{\varepsilon}_0 = \alpha_T \dot{T} \quad (4-4)
\]

3. For quartzite normal weight concrete, \(\alpha\) was defined as:

\[
\left\{
\begin{array}{l}
\alpha_T = \frac{6 \times 10^{-5}}{7 - T} \quad \text{for } 0 \leq \dot{T} \leq 6 \\
\alpha_T = 0 \quad \text{for } \dot{T} > 6
\end{array}
\right. \quad (4-5)
\]

4. Where:

\[
\bar{T} = \frac{T - 20 \, ^\circ C}{100 \, ^\circ C} \quad (4-6)
\]

5. Figure 4-1 Experimental FTS vs temperature for various concretes [51] and the curve obtained by the FTS model exposed in [52]

6. **4.2 Implicit and explicit uniaxial LITS models**

7. The inclusion of LITS in uniaxial constitutive laws for concrete is fundamental to reliably analysing the behaviour of structures subjected to transient fire conditions [53].
For this reason, several implicit and explicit models of LITS have been formulated over the past decades.

The term explicit has come to be used to refer to models where the LITS component (or the TS component, when a creep component is included) is explicitly formulated in the constitutive law, in addition to the instantaneous stress-related strain [41]. For explicit models, the following strain decomposition is generally adopted:

\[ \varepsilon_{tot}(\sigma, T, t) = \varepsilon_0(T) + \varepsilon_\sigma(\sigma, T) + \varepsilon_{cr}(\sigma, T, t) + \varepsilon_{ts}(\sigma, T) \]  

(4-7)

Where \( \sigma \) is stress, \( T \) temperature, \( t \) time, \( \varepsilon_0 \) FTS, \( \varepsilon_\sigma \) the instantaneous stress-related strain, \( \varepsilon_{cr} \) is the isothermal creep strain and \( \varepsilon_{ts} \) is the TS. It is worth noting that the instantaneous stress-related strain \( \varepsilon_\sigma(\sigma, T) \) may be modelled as stress history dependent, i.e. as an elastoplastic component, so that different values of \( \varepsilon_\sigma \) may be obtained for the same stress state but different stress histories. Since it is impossible to experimentally uncouple the creep strain and TS, in some explicit models they are considered as a single creep component, denoted as \( \varepsilon_{cr^*} \):

\[ \varepsilon_{tot}(\sigma, T, t) = \varepsilon_0(T) + \varepsilon_\sigma(\sigma, T) + \varepsilon_{cr^*}(\sigma, T, t) \]  

(4-8)

Where:

\[ \varepsilon_{cr^*}(\sigma, T, t) = \varepsilon_{cr}(\sigma, T, t) + \varepsilon_{ts}(\sigma, T) \]  

(4-9)

On the other hand, in implicit models the instantaneous stress-related strain \( \varepsilon_\sigma \) and the LITS \( \varepsilon_{lits} \) (or TS \( \varepsilon_{ts} \)) are modelled as a single mechanical elastoplastic strain component \( \varepsilon_m \):

\[ \varepsilon_{tot}(\sigma, T, t) = \varepsilon_0(T) + \varepsilon_m(\sigma, T) + \varepsilon_{cr}(\sigma, T, t) \]  

(4-10)

The main disadvantage of the implicit models is that they are unable to capture the difference between HTL and LTH regimes, since they always implicitly consider the TS component, i.e. they are suitable only for LTH conditions. Moreover, they present drawbacks even in case of LTH conditions involving mechanical unloading of the material. This is because they treat TS as a fully reversible strain component, since the unloading stiffness is defined as the tangent of the mechanical stress-strain curve, which implicitly includes TS [41]. Such limitations result in an overestimation of the real unloading elastic stiffness (see Figure 4-2). It can be concluded that implicit models can accurately describe the behaviour of concrete in the case of LTH regimes where the temperature increases and the stress is constant, but for all the other possible stress-temperature paths, explicit models are needed.
Figure 4-2 Stress-strain curves for implicit and implicit models at 500°C [41].

4.3 Uniaxial explicit LITS models

4.3.1 Anderberg and Thelandersson’s TS model

Anderberg and Thelandersson [19] proposed a uniaxial constitutive model based on the decomposition of the total strain expressed in equation (4-7). The instantaneous stress-strain model, allowing evaluation of $\varepsilon_\sigma$ at high temperatures, and the isothermal creep component $\varepsilon_{cr}$, were calibrated from the results of HTL tests. By contrast, the transient strain model was formulated and calibrated in order to fit the difference between the total thermal strain $\varepsilon_{etot}$ measured in LTH tests and the instantaneous and creep strains $\varepsilon_\sigma$ and $\varepsilon_{cr}$, evaluated for transient conditions according to the models formulated for steady state regime. Since the TS evaluated this way was found to be nearly proportional to the FTS for the analysed concrete mixture (quartzite aggregates) and temperatures up to 500 °C (see Figure 4-3), a uniaxial model expressing the TS as linearly proportional to the applied stress level and the thermal strain was formulated:

$$\varepsilon_{ts} = k_{tr} \frac{\sigma}{\sigma_{u0}} \varepsilon_{th} \quad (4-11)$$

Where $\sigma_{u0}$ is the compressive strength of the material at ambient temperature and $k_{tr}$ is a material parameter, which was found to vary between 1.8 and 2.35 to fit different experimental results.

However, the authors recognized that, even though the approximation of TS given by the model at temperatures around and above 550 °C may be acceptable for some practical purposes, such linear correlation between thermal strain and TS is not suitable for accurately describing the TS at temperatures above 550 °C. In addition, the assumption of a linear proportionality between TS and FTS seems to contrast with experimental results obtained for different types of concrete, where for decreasing water to cement ratios the FTS decreased while the TS increased [13–15].
4.3.2 Nielsen’s LITS model

Nielsen et al. [52] proposed modelling LITS as linearly proportional to the applied stress level and to the temperature:

$$\varepsilon_{lits} = \frac{\sigma}{\sigma_{u0}} a(T - T_0)$$  \hspace{1cm} (4-12)

Where the coefficient $a$ was calibrated as $a = 3.8 \times 10^{-5} \circ C^{-1}$ in order to best-fit the LITS curve proposed in [36] - see equations (4-15), (4-16) and (4-17) - in the temperature range 20 to 500 °C;

Such LITS formulations can be seen as a modified Anderberg and Thelandersson TS model, where the linear coefficient of thermal expansion $a(T)$ remains constant with temperature and is included, with the coefficient $k_2$, in the coefficient $a$. The main advantage of this model is the possibility of calibrating the LITS coefficient $a$ independently of the thermal expansion coefficient $a(T)$. Actually, such independency is suggested by the fact that FTS is strongly influenced by the nature of aggregates, while LITS mainly takes place in the cement paste (see sections 3.5.1, 3.5.2 and 3.6).

Although this model may be useful for some practical purposes, its linearity does prevent it from capturing the sharp increase in LITS that typically occurs at high temperatures –above around 400-500 °C.

4.3.3 Diederichs’ LITS model

With a view to conveniently fitting the highly nonlinear curve expressing LITS as a function of the temperature, Diederich’s model expresses LITS as a third order polynomial of the temperature [54]:

$$\varepsilon_{lits} = \frac{\sigma}{\sigma_{u0}} [a_1(T - T_0) + a_2(T - T_0)^2 + a_3(T - T_0)^3]$$  \hspace{1cm} (4-13)
Where the coefficients $T_0$, $a_1$, $a_2$ and $a_3$ were calibrated to fit experimental data provided by Diederichs [55] as follows:

\[
\begin{align*}
T_0 &= 20 \, ^\circ C \\
a_1 &= +0.0412 \times 10^{-3} \, ^\circ C^{-1} \\
a_2 &= -1.72 \times 10^{-7} \, ^\circ C^{-1} \\
a_3 &= +3.3 \times 10^{-10} \, ^\circ C^{-1}
\end{align*}
\]

Such models involve more parameters than the Nielsen’s model but allows an accurate fit to experimental LITS curves, particularly for temperatures above 400-500°C.

### 4.3.4 Terro’s LITS model

Terro [36] formulated a LITS model in the light of the LITS “master” curve identified by Khoury et al. [3]. The proposed expression takes into account the experimentally demonstrated dependency of LITS on the volume fraction of aggregates $V_a$ and is valid for stress levels up to 30% of the compressive strength:

\[
\varepsilon_{lits} = \varepsilon_{0,3} \times \left(0.032 + 3.226 \frac{\sigma_i}{\sigma_{u0}}\right) \frac{V_a}{0.65}
\]

Where $\sigma_i$ is the initial compressive stress before heating, and $\varepsilon_{0,3}$ is the value of $\varepsilon_{lits}$ when $\sigma_i/\sigma_{u0} = 3$.

For concrete with calcareous and lightweight aggregates, $\varepsilon_{0,3}$ is expressed by a fourth order polynomial of the temperature:

\[
\varepsilon_{0,3} = a_0 + a_1 T + a_2 T^2 + a_3 T^3 + a_4 T^4
\]

Where:

\[
\begin{align*}
a_0 &= -43.87 \times 10^{-6} \\
a_1 &= 2.73 \times 10^{-6} \\
a_2 &= 6.35 \times 10^{-8} \\
a_3 &= -2.19 \times 10^{-10} \\
a_4 &= 2.77 \times 10^{-13}
\end{align*}
\]

While for LITS of Thames gravel, which departs from the master curve for temperatures above 400°C, a fifth order polynomial of the temperature is defined:

\[
\varepsilon_{0,3} = b_0 + b_1 T + b_2 T^2 + b_3 T^3 + b_4 T^4 + b_5 T^5
\]

Where:

\[
\begin{align*}
b_0 &= -1625.78 \times 10^{-6} \\
b_1 &= 58.03 \times 10^{-6} \\
b_2 &= 0.6364 \times 10^{-6} \\
b_3 &= 3.6112 \times 10^{-9} \\
b_4 &= 9.2796 \times 10^{-12} \\
b_5 &= 8.806 \times 10^{-15}
\end{align*}
\]
This model is suitable for modelling the highly nonlinear evolution of TS with temperature, similarly to Diederichs model, with the advantage of including the effect of the volume fraction of aggregate $V_a$.

### 4.3.5 Discussion

It is worth noting that the calibration of the coefficients defining the LITS curves is a crucial modelling phase.

This is because such parameters do not have an intuitive physical meaning and the polynomial curves are sensitive to them. In this regard, for example, it may be verified that increasing by 5% the coefficients $a_0$, $a_1$, $a_2$, $a_3$ and $a_4$ defined in the Terro’s model produces respectively a LITS variation of 1%, 1%, 9%, 11% and 18%, considering a temperature of 600°C and load level of 20% the cold compressive strength.

This is because such parameters do not have an intuitive physical meaning and the polynomial curves are sensitive to them. For the presented models, the parameters have been calibrated so as to fit the behaviour of the typical concrete mixes used in the nuclear industry four decades ago. Therefore, the best way to accurately represent the thermomechanical properties of current and future concrete mixes, is to re-calibrate them via LTH tests. However, if no experimental data are available, it is reasonable to use the coefficients initially provided by the different authors. In particular, if temperatures beyond 400°C are examined, the authors of this paper recommend using the curves provided by Terro [36], since they can reproduce the highly nonlinear LITS development, they have been obtained considering a number of different concrete mixes, and they take into account the influence of the volume fraction of aggregate $V_a$.

### 4.4 Multiaxial explicit models

#### 4.4.1 Thelandersson’s LITS model

Thelandersson [31] proposed a multiaxial generalisation of the uniaxial model presented in [19], conceived as an extension of isotropic, temperature dependent, linear-elastic behaviour. If $E$ and $\nu$ are treated as time dependent, the time differentiation of the isotropic thermoelastic law gives:

$$\dot{\varepsilon}_{ij} = \frac{1}{E}\hat{\sigma}_{ij} - \frac{\nu}{E}\hat{\sigma}_{kk}\delta_{ij} + \alpha \hat{\delta}_{ij}\hat{T} + (y_1\sigma_{kk}\hat{\sigma}_{ij} + y_2\sigma_{ij})\hat{T}$$  \hspace{1cm} (4-20)

Where:

$$\begin{align*}
y_1 &= \frac{d}{dT}\left(-\frac{\nu}{E}\right) \\
y_2 &= \frac{d}{dT}\left(\frac{1+\nu}{E}\right)
\end{align*}$$  \hspace{1cm} (4-21)

The strain component defined by the last term of (4-20) can be seen as a thermomechanical strain representing the LITS strain in case of transient regime:

$$\dot{\varepsilon}_{ij}^{LITS} = (y_1\sigma_{kk}\hat{\sigma}_{ij} + y_2\sigma_{ij})\hat{T}$$  \hspace{1cm} (4-22)
The material parameters $\gamma_1$ and $\gamma_2$ describe the proportionality between stress level, temperature variation and thermomechanical strain rate, and for practical purposes they can be calibrated by fitting the experimental results of LTH tests. Accordingly, the total strain can be written as:

$$\dot{\varepsilon}_{ij} = \frac{1 + v}{E} \dot{\sigma}_{ij} - \frac{v}{E} \dot{\sigma}_{kk} \delta_{ij} + \alpha \delta_{ij} \dot{T} + \dot{\varepsilon}^{lits}_{ij}$$

(4-23)

The author proved that this model captures the general trend of the experimentally estimated thermal strain curves. However, when the combinations of stress and temperature approach crushing failure, the model does not provide accurate results.

### 4.4.2 de Borst & Peeters’ LITS model

A modified version of the thermomechanical strain component defined by Thelandersson has been formulated by de Borst & Peeters [56]. Here the LITS strain component is incorporated in a concrete constitutive model that includes a smeared crack model. According to this model, the general strain rate decomposition is defined in matrix notation as:

$$\dot{\varepsilon}_{tot} = \dot{\varepsilon}_{cra} + \dot{\varepsilon}_{ela} + \dot{\varepsilon}_{0} + \dot{\varepsilon}_{lits}$$

(4-24)

Where $\dot{\varepsilon}_{tot}$ is the total strain rate tensor, $\dot{\varepsilon}_{cra}$ the crack strain rate tensor, $\dot{\varepsilon}_{ela}$ the elastic strain rate tensor, $\dot{\varepsilon}_{0}$ the thermal strain rate tensor, and $\dot{\varepsilon}_{lits}$ the LITS rate tensor.

In [56], the LITS component has been obtained by modifying the expression proposed in [31]. In particular, the material coefficients $\gamma_1$ and $\gamma_2$ have been expressed as:

$$\gamma_1 = -v_{lits} \frac{k_{tr} \alpha}{\sigma_{u0}}$$

(4-25)

$$\gamma_2 = (1 + v_{lits}) \frac{k_{tr} \alpha}{\sigma_{u0}}$$

(4-26)

In the above equations $v_{lits}$ is a material parameter, analogous to the elastic Poisson modulus $v$, allowing a description of the strain in the unloaded directions. Moreover, such coefficients allow the behaviour to be modelled as isotropic and LITS in the unloaded direction to be modelled as proportional to the term $k_{tr} \alpha / \sigma_{u0}$, similarly to that suggested for the loaded direction in [1]:

$$\varepsilon^{lits}_{ij} = \frac{k_{tr} \alpha}{\sigma_{u0}} (-v_{lits} \sigma_{kk} \delta_{ij} + (1 + v_{lits}) \sigma_{ij}) \dot{T}$$

(4-27)

#### 4.4.3 Pearce’s LITS model

Recently, Pearce et al. [29] also proposed an extension of Thelandersson’s model, where the total strain is decomposed as (ignoring conventional creep strains and plastic strains):

$$\dot{\varepsilon}_{tot} = \dot{\varepsilon}_{ela} + \dot{\varepsilon}_{0} + \dot{\varepsilon}_{lits}$$

(4-28)
Where the loss of strength and stiffness of the material due to the mechanical processes were modelled by a thermal and mechanical damage model.

With regard to the third component, they modified the expression (4-27) by substituting the term $k_t \alpha$ with a generic function of the temperature $\beta(T)$, aimed at fitting the experimental data by eliminating the dependency of LITS on the FTS:

$$\dot{\varepsilon}_{i j}^{\text{LITS}} = \frac{\beta(T)}{\sigma_{u 0}} \left(-v_{l t s} \sigma_{k k} \delta_{i j} + (1 + v_{l t s}) \sigma_{i j}\right) \dot{T} \quad (4-29)$$

In particular, they defined $\beta(T)$ by imposing the same LITS curve obtained by the bi-parabolic model proposed in [32] in case of uniaxial load.

4.4.4 Discussion

Of above models, the one proposed by Pearce et al. [29] allows the more accurate modelling of the uniaxial LITS curve while also containing material parameters with an intuitive physical meaning. Following the same approach suggested by the authors, it is theoretically possible to implement whichever uniaxial law (see 4.3.1, 4.3.2, 4.3.3 and 4.3.4) in 3D by adequately modifying the formulation of the generic function of the temperature $\beta(T)$ defined in equation (4-29).

The major limitation of all the above triaxial models is represented by the hypothesis of isotropic development of LITS, e.g. by the analogy with isotropic elasticity. Indeed, such an assumption prevents capturing the marked anisotropic development of LITS discussed in section 3.4.2 which, for temperatures above 250°C, is not even fully understood experimentally yet.

5 Summary and conclusions

In this paper, firstly, a review of the experimentally demonstrated characteristic of LITS was given. This section classified the main testing methodologies in order to identify the most appropriate experimental procedures for reproducing transient and steady-state loading conditions. Then, the two main components of thermal strain measurable under constant load, i.e. FTS and LITS, were defined. The experimentally identified characteristics of the FTS were discussed, while an analysis of the effects of temperature on elasticity and isothermal creep strain was necessary prior to formulating the definition of LITS components. Once the different contributions had been identified, the effects of different parameters on the development of LITS and its physical origins were outlined in the light of experimental evidence. In particular, it was found that LITS is:

- a quasi-instantaneous strain component in the case of unsealed conditions,
- strongly nonlinear and mostly irrecoverable with respect to temperature,
- reduced by stress-free preheating cycles,
- an age-independent phenomenon for ages at loading higher than one year and temperatures up to about 450 °C,
• almost proportional to the compressive stress level,
• independent of the initial moisture content for unsealed conditions and temperatures above 250 °C,
• independent on the heating rate for heating rates lower than 5°C/min,
• significantly influenced by moisture flux in the range 100-250 °C,
• independent on the nature of aggregates for temperatures up to about 400 °C,
• a strongly anisotropic phenomenon,
• mainly due to chemical and physical reactions taking place in the cement paste for temperatures up to about 400°C, while also due to thermal incompatibility between aggregate and cement paste for higher temperatures.

It was also found that new triaxial LTH tests, involving temperatures higher than 250°C, are needed to understand and reliably model the behaviour of concrete in such loading conditions.

Secondly, this study reviewed the mathematical models of LITS available in the literature. The main differences between implicit and explicit LITS models were examined. In particular, the necessity of using explicit models when modelling HC cycles and when the stress is varying over time was underlined. Then, the main features of the existing uniaxial and multiaxial models were presented with a view to outlining their strengths and limitations. In this regard, it was highlighted that:

• Anderberg and Thelandersson’s model cannot be applied to a wide range of concrete mixtures, since the assumption of LITS dependency on the thermal expansion coefficient \( \alpha \) has been proved to be unacceptable in general.
• Nielsen’s model is quite simple and allows LITS to be modelled regardless of free thermal expansion. However, it cannot capture the typical sharp increase in LITS for temperatures higher than 400 °C, a significant limitation for fire analyses.
• Diederich’s and Terro’s models both allow accurate fits to experimental curves, while involving a higher number of parameters than earlier models.
• Pearce’s multiaxial LITS models combines the main advantages of Thelandersson’s and De Borst & Peeters’s models: it involves material parameters having an intuitive physical meaning and it allows fitting of LITS curves regardless of the coefficient \( \alpha \). However, due to their isotropic nature, such models cannot describe the strongly anisotropic development of LITS.

The following conclusions can be drawn from the present study:

• In recent years, there has been an increasing interest in the modelling of LITS. This is due to the necessity of assessing the fire resistance of concrete structures such as nuclear pressure vessels, containers for chemicals, water towers or reservoirs, silos, and building structures in fire (particularly travelling fire) conditions.
• Earlier experimental work on LITS necessarily focused on uniaxial loading conditions. This provided a basis for understanding the behaviour of framed structures but also left many questions about the multiaxial behaviour of bulk concrete structures open. In recent years, there has been an increasing interest in this area.
• Even though more recent tests on multiaxial loading conditions have begun to fill the gaps in knowledge, more experimental research is required to understand the triaxial behaviour of concrete for temperatures higher than 250°C.

• The experimental knowledge is reflected in the available analytical models: the uniaxial models are more comprehensive and firmly linked to experimental evidence than the multiaxial ones. In this regard, most of the multiaxial models estimate the triaxial behaviour by the assumption of isotropic development of LITS, which is in contrast with the experimental evidence. Therefore, further research should be carried out to develop multiaxial models able to capture the anisotropic behaviour of heated concrete.

• Most of the available models have been formulated and calibrated for concrete mixes used in the 1970-1980s. As a consequence, it would be advisable to re-calibrate them via new tests performed on appropriate modern concrete mixes.

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