Motion of particles on a $z = 2$ Lifshitz black hole background in 3+1 dimensions

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Abstract The object of study is the geodesic structure of a $z = 2$ Lifshitz black hole in 3+1 space–time dimensions, which is an exact solution to the Einstein-scalar-Maxwell theory. The motion of massless and massive particles in this background is researched using the standard Lagrangian procedure. Analytical expressions are obtained for radial and angular motions of the test particles, where the polar trajectories are given in terms of the ℘-Weierstraß elliptic function. It will be demonstrated that an external observer can see that photons with radial motion arrive at spatial infinity in a finite coordinate time. For particles with non-vanished angular momentum, the motion is studied on the invariant plane $\phi = \pi/2$ and, it is shown that bounded orbits are not allowed for this space–time on this plane. These results are consistent with other recent studies on Lifshitz black holes.

Keywords Lifshitz black holes · Causal Structure · Geodesics
1 Introduction

Motivated by generalizations to other areas of physics of the AdS/CFT correspondence (Maldacena 1998), i.e., the duality between the geometry of a d-dimensional anti-de Sitter (AdS) space and a d − 1-dimensional conformal field theory (CFT), Lifshitz space–times have received considerable attention. They represent gravity duals of non-relativistic systems that appear in condensed matter physics (Kachru et al. 2008; Hartnoll et al. 2010) with an anisotropic scaling symmetry $t \rightarrow \lambda^zt, x \rightarrow \lambda x$, where $z$ is the dynamical exponent accounting for the different scale transformation between the temporal and spatial coordinates. These space–times are described in 3+1 dimensions by the metrics

$$ds^2 = \ell^2 \left( -r^{2z} dt^2 + \frac{dr^2}{r^2} + r^2 dx^2 \right), \tag{1}$$

where $x$ represents a two-dimensional vector and the radial coordinate $r$ scales according to $r \rightarrow r/\lambda$. It is worth mentioning that for $z = 1$ the above metric reduces to the usual four-dimensional AdS metric. Black holes with asymptotic behavior given by these anisotropic scale invariant space–times represent gravity duals of condensed matter systems at finite temperature. Asymptotically Lifshitz black hole solutions have been reported in Balasubramanian and McGreevy (2009), Ayon-Beato et al. (2009), Mann (2009) and Dehghani and Mann (2010). Some thermodynamic aspects of these black holes have also been studied in Devecioglu and Sarioglu (2011), Myung and Moon (2012), and Myung (2012).

This paper focuses on the geodesic structure of a 3+1 dimensional black hole presented by Taylor (2008), the asymptotic behavior of which is given by Eq. (1) with dynamical exponent $z = 2$, which emerges as an exact solution of the Einstein-scalar-Maxwell theory (Taylor 2008; Pang 2010). Geodesic studies of the $z = 2$ topological Lifshitz black hole in 3+1 dimensions and of the $z = 3$ Lifshitz black hole in 2+1 dimensions, which has been found to be a solution to the New Massive Gravity theory, have been reported recently in Olivares et al. (2013) and Cruz et al. (2013), respectively. In this investigation the motion of massless and massive particles is studied in a similar black hole background using the standard Lagrangian procedure (Olivares et al. 2013; Cruz et al. 2013, 2005; Villanueva and Olivares 2013; Olivares and Villanueva 2013). The effective potential analysis provides the means to describe the motion of particles along null and time-like geodesics. The exact solutions of the geodesic equations are presented. The analytical expressions for the radial geodesics are given as expressions of the proper and coordinate times. Moreover, the equations for the angular motion of the test particle are provided through the $\wp$-Weierstraß function.

The paper is organized as follows: In Sect. 2 the geodesic equations are obtained for (massless and massive) particles in the space–time found in Taylor (2008) and Pang (2010). Then their radial and angular motions are examined. Finally, in Sect. 3 the results and conclusions are discussed.

2 Geodesic structure

Let us consider the action of the Einstein-scalar-Maxwell theory, which is given by (Taylor 2008; Pang 2010)

$$I_{EsM} = \frac{1}{16\pi G} \int d^4x \sqrt{-g} \left( R - 2\Lambda - \frac{1}{2} \partial_\mu \varphi \partial^\mu \varphi - \frac{1}{4} e^{\varphi} F_{\mu\nu} F^{\mu\nu} \right), \tag{2}$$
Lifshitz black hole background in 3+1 dimensions

where $\Lambda$ is the cosmological constant, $\phi$ is a massless scalar field, and $F_{\mu \nu}$ corresponds to the Maxwell field. An analytical $z = 2$ asymptotically Lifshitz black hole solution to this theory with a flat transverse section is given by the metric (Amado and Faedo 2011; Brynjolfsson et al. 2010)

$$ds^2 = \ell^2 \left[ -r^4 \left( 1 - \frac{r^4}{r^4} \right) dt^2 + \frac{dr^2}{r^2 \left( 1 - \frac{r^4}{r^4} \right)} + r^2 \left( d\theta^2 + \theta^2 d\phi^2 \right) \right],$$

where the curvature radius of the Lifshitz black hole, $\ell$, is related to the cosmological constant $\Lambda = -6/\ell^2$. The event horizon is located at

$$r_+ = \left( \frac{8 \pi G \mathcal{M}}{\ell^2 V_2} \right)^{1/4},$$

where $\mathcal{M}$ is the mass of the corresponding black hole determined by the Euclidean action approach, and $V_2$ is the volume of two-dimensional spatial directions. While, the scalar and Maxwell fields are given by the expressions

$$e^{\tilde{\lambda} \phi} = \frac{1}{r^4} (\tilde{\lambda}^2 = 4), \quad F_{tr} = 2\sqrt{2} \ell r^3.$$

Myung and Moon (2012) determined the thermodynamical properties of this Lifshitz black hole and presented a stability analysis considering the scalar field perturbation of this black hole. In particular, it was found that the temperature $T_H$, the Bekenstein–Hawking entropy $S_{BH}$, the heat capacity $C$, and the Helmholtz free energy $F$ are given by

$$T_H = \frac{r_+^2}{2\pi}, \quad S_{BH} = \frac{\ell^2 V_2}{4G} r_+^2, \quad C = \frac{2\ell^2 V_2 r_+^2}{8G}, \quad F = -\frac{2\ell^2 V_2 r_+^4}{16\pi G}.$$

The curvature scalar, the principal quadratic invariant of the Ricci tensor and the Kretschmann scalar of this space–time are given by the following expressions

$$R = \frac{22}{\ell^2} + \frac{2r_+^4}{\ell^2 r^4},$$

$$R_{\mu \nu} R^{\mu \nu} = \frac{4 \left( 33r_+^8 + r_+^8 + 6r_+^4 r^4 \right)}{\ell^4 r^8},$$

$$R_{\mu \nu \rho \sigma} R^{\mu \nu \rho \sigma} = \frac{4 \left( 3\ell^2 r_+^8 + 2r^2 + 6\ell^2 r_+^4 r^4 + 23\ell^2 r_+^4 \right)}{\ell^6 r^8}.$$

These curvature invariants are regular on the event horizon, $r_+$, and therefore this surface only expresses a singularity of the coordinates used to define the metric (3). However, all these scalars diverge in $r = 0$, and thus this point corresponds to a curvature singularity.

In order to study the motion of test particles in the background (3), the standard Lagrangian approach is used (Cruz et al. (2005); Villanueva and Olivares (2013); Olivares and Villanueva (2013)). The corresponding Lagrangian is

$$2\mathcal{L} = \ell^2 \left( - \left( r_+^4 - r_+^4 \right) \dot{r}^2 + \frac{r_+^2 \dot{r}^2}{r^4 - r_+^4} + r^2 \left( \dot{\theta}^2 + \theta^2 \dot{\phi}^2 \right) \right) = -m \ell^2.$$

Here, the dot refers to a derivative with respect to an affine parameter, $\tau$, along the trajectory, and, by normalization, $m = 0 \ (1)$ for massless (massive) particles. Since $(t, \phi)$ are cyclic coordinates, their corresponding conjugate momenta $(\Pi_t, \Pi_\phi)$ satisfy the following relations:
\[
\frac{d}{d\tau} \frac{\partial L}{\partial t} = \frac{d\Pi_t}{d\tau} = \frac{d}{d\tau} \left[ -\ell^2 (r^4 - r_+^4) i \right] = 0, 
\]

\[
\frac{d}{d\tau} \frac{\partial L}{\partial \dot{\theta}} = \frac{d\Pi_\theta}{d\tau} = \frac{d}{d\tau} \left[ \ell^2 r^2 \theta^2 \dot{\phi} \right] = 0. 
\]

Furthermore, the equation of motion associated with the $\theta$ coordinate becomes

\[
\frac{d}{d\tau} \frac{\partial L}{\partial \dot{\theta}} = \frac{d\Pi_\theta}{d\tau} = \frac{d}{d\tau} \left[ \ell^2 r^2 \theta^2 \dot{\phi} \right] = 0. 
\]

Conversely, the flat metric (3) admits the following Killing vectors field:

- the time-like Killing vector $\xi = \partial_t$ is related to the stationarity of the metric. The conserved quantity is given by
  \[
g_{\alpha\beta} \xi^\alpha u^\beta = -\ell^2 (r^4 - r_+^4) i = -\ell^2 \sqrt{E}. \tag{14}\]
  where $E$ is a constant of motion that cannot be associated with the total energy of the test particles because this space–time is asymptotically Lifshitz, and not flat.

- the space-like Killing vectors $\chi_0 = \partial_\phi$, $\chi_1 = \theta^{-1} \sin \phi \partial_\phi - \cos \phi \partial_\theta$, and $\chi_2 = \theta^{-1} \cos \phi \partial_\phi + \sin \phi \partial_\theta$, which are related to the axial symmetry of the metric. The conserved quantities are given by
  \[
g_{\alpha\beta} \chi_0^\alpha u^\beta = \ell^2 r^2 \theta^2 \dot{\phi} = C_0, \tag{15}\n\]
  \[
g_{\alpha\beta} \chi_1^\alpha u^\beta = \ell^2 r^2 (\sin \phi \dot{\theta} + \theta \cos \phi \dot{\phi}) = C_1, \tag{16}\n\]
  \[
g_{\alpha\beta} \chi_2^\alpha u^\beta = \ell^2 r^2 (-\cos \phi \dot{\theta} + \theta \sin \phi \dot{\phi}) = C_2, \tag{17}\n\]
  where $C_0$, $C_1$ and $C_2$ are constants associated with the angular momentum of the particles.

This last point implies that the motion can be restricted on an invariant plane, i.e., $\dot{\phi} = 0 \rightarrow \phi = \text{const.}$, which, for simplicity we set at $\phi = \pi/2$. Therefore, $C_0 = C_2 = 0$, so Eqs. (14) and (13, 16) lead to the following expressions

\[
i = \frac{\sqrt{E}}{(r^4 - r_+^4)}, \quad \dot{\theta} = \frac{L}{r^2}, \tag{18}\]

where $L = C_1/\ell^2$, which denotes the angular momentum about an axis normal to the invariant plane.

These relations together with Eq. (10) make it possible to obtain the following differential equations:

\[
\left( \frac{dr}{d\tau} \right)^2 = \frac{1}{r^2} \left( E - V_{\text{eff}} (r) \right), \tag{19}\n\]

\[
\left( \frac{dr}{dt} \right)^2 = \left( \frac{r^4 - r_+^4}{r^2} \right)^2 \left( \frac{E - V_{\text{eff}} (r)}{E} \right), \tag{20}\n\]

\[
\left( \frac{dr}{d\theta} \right)^2 = \frac{r^2}{L^2} \left( E - V_{\text{eff}} (r) \right) \tag{21},\n\]

where the effective potential, $V_{\text{eff}} (r)$, reads

\[
V_{\text{eff}} (r) = (r^4 - r_+^4) \left( m + \frac{L^2}{r^2} \right). \tag{22}\]
In the following sections, based on this effective potential, the geodesic structure of the space–time characterized by the metric (3) is analyzed.

2.1 Null geodesics

In order to study the motion of massless particles, let us consider the effective potential (22) with \( m = 0 \), so it can be expressed by

\[
V_n(r) = L^2 \left( \frac{r^4 - r_+^4}{r^2} \right). \tag{23}
\]

A typical graph of this effective potential is shown in Fig. 1, for the arbitrary value of \( L \neq 0 \). From this plot it is easy to see that photons cannot escape to spatial infinity, but that the confined orbits are also forbidden in this space–time.

2.1.1 Radial motion

The radial motion for photons is characterized by the vanishing angular momentum, \( L = 0 \). Thus, the effective potential (23) is \( V_{nr} = 0 \), and therefore only photons with radial motion can escape to spatial infinity. Thus, the radial Eq. (19) reduces to

\[
\dot{r}^2 = \frac{E}{r^2}, \tag{24}
\]

so, an elementary integration leads to

\[
\tau (r) = \pm \frac{R_0^2}{2\sqrt{E}} \left[ \left( \frac{r}{R_0} \right)^2 - 1 \right], \tag{25}
\]

where \( R_0 \) denotes the radial distance of the massless particle when \( \tau = 0 \). This result is in accordance with previous works dealing with other Lifshitz space–times: Cruz et al. (2013); Villanueva and Vásquez (2013); Maeda and Giribet (2011).

Now, integrating Eq. (20) an explicit expression for the coordinate time is easily obtained:

\[
t(r) = \pm \frac{1}{4r_+^2} \ln \left( \frac{(r^2 - r_+^2) \left( R_0^2 + r_+^2 \right)}{(r^2 + r_+^2) \left( R_0^2 - r_+^2 \right)} \right). \tag{26}
\]
Fig. 2 Plot of the radial motion of massless particles. Particles moving to event horizon, \( r_+ \), cross in a finite proper time, but an external observer will see that photons take an infinite (coordinate) time to do it. Here the values \( r_+ = 2 \), \( R_0 = 5 \), and \( E = 10^4 \) have been employed (in geometrized units).

In the asymptotic region, \( r \to \infty \), we obtain the limit
\[
\lim_{r \to \infty} t(r) = \frac{1}{4r_+^2} \ln\left(\frac{R_0^2 + r_+^2}{R_0^2 - r_+^2}\right).
\]  

(27)

This fact has been reported in other Lifshitz black holes (Olivares et al. 2013; Cruz et al. 2013), and seems to express a behavior characteristic of this kind of space–time (Villanueva and Vásquez (2013)). Figure 2 shows this situation graphically.

2.1.2 Angular motion

Massless particles with non-vanished angular momentum, \( (L \neq 0) \), comply with the effective potential given by
\[
V_{na}(r) = \frac{L^2}{2} \left(\frac{r^4 - r_+^4}{r^2}\right);
\]  

(28)

thus, the turning point is located at
\[
R_L = \sqrt{\frac{E}{2L^2} + \sqrt{\frac{E^2}{4L^4} + r_+^4}}.
\]  

(29)

Now, using Eq. (21), the quadrature is obtained
\[
\theta(r) = -\int_{R_L}^{r} \frac{1}{\sqrt{(R_L - r)(r + R_L)(r + i\rho)(r - i\rho)}} \, dr,
\]  

(30)

where the roots of the fourth-degree polynomial inside the radical is given by
\[
\rho = \sqrt{-\frac{E}{2L^2} + \sqrt{\frac{E^2}{4L^4} + r_+^4}}.
\]  

(31)

In order to integrate Eq. (30), \( u = R_L - r \) is set, and after a brief manipulation, we obtain the polar trajectory of massless particles,
\[
r(\theta) = R_L - \frac{1}{4\rho \left(\sqrt{u_1u_2u_3} \theta; g_2, g_3\right) + \frac{\rho}{3}}.
\]  

(32)
Fig. 3 Plot of the angular motion of massless particles. The solid line represents the regular orbit performed by the photons, while the dashed line is the analytic continuation of that orbit. Here, the initial condition $\theta = 0$ when $r = R_L$, together with the values $L = 1$, $r_+ = 2$, $R_L = 8$ (in geometrized units), has been employed.

where $\wp = \wp(y; g_2, g_3)$ is the $\wp$-Weierstraß function, and $g_2$ and $g_3$ are the so-called Weierstraß invariants given by

$$g_2 = \frac{1}{4} \left( \frac{\alpha^2}{3} - \beta \right), \quad g_3 = \frac{1}{16} \left( \gamma + \frac{2}{27} \alpha^3 - \frac{\alpha \beta}{3} \right).$$  \hspace{1cm} (33)

The other constants are

$$\alpha = \frac{1}{u_1} + \frac{1}{u_2} + \frac{1}{u_3}, \quad \beta = \frac{1}{u_1 u_2} + \frac{1}{u_1 u_3} + \frac{1}{u_2 u_3}, \quad \gamma = \frac{1}{u_1 u_2 u_3}. \hspace{1cm} (34)$$

with,

$$u_1 = 2 R_L, \quad u_2 = R_L + i \rho, \quad u_3 = R_L - i \rho. \hspace{1cm} (35)$$

2.2 Time-like geodesics

In this section the motion of massive particles, $m = 1$, is computed, so the effective potential is given by

$$V_t (r) = \left( r^4 - r_+^4 \right) \left( 1 + \frac{L^2}{r^2} \right). \hspace{1cm} (36)$$

This is illustrated in Fig. 4 for radial ($L = 0$) and non-radial ($L \neq 0$) particles.

2.2.1 Radial motion

In this case the effective potential (36) becomes

$$V_t (r) = \left( r^4 - r_+^4 \right); \hspace{1cm} (37)$$
Fig. 4  Plot of the effective potential of massive particles. This graph shows that, independently of the angular momentum, particles cannot escape to spatial infinity. Here the values $r_+ = 2$, $L = 0$ for radial motion, and $L = 4$ for polar motion (in geometrized units) were employed.

Fig. 5  Plot of the radial motion of massive particles. Particles moving to event horizon, $r_+$, cross into a finite proper time, but an external observer will see that particles take an infinite (coordinate) time to do so. Here, $r_+ = 2$, $r_0 = 5$ and $E = 609$ (in geometrized units) have been used.

Thus, the turning point is located at

$$r_0 = \left( E + r_+^4 \right)^{1/4}.$$  \hspace{1cm} (38)

Therefore, using Eqs. (37) and (19), the proper time of the radial massive particles in terms of the radial coordinate $r$ can be obtained explicitly, which results in

$$\tau (r) = \frac{1}{2} \arccos \left( \frac{r_0^2}{r^2} \right).$$ \hspace{1cm} (39)

It is interesting to note that massive particles take a finite proper time, $\tau_+ \equiv \tau (r = r_+)$, to cross the event horizon, which depends on the initial distance, $r_0$. Also, the analytic continuation of the motion in the region $r < r_+$ means that it takes a finite proper time $\tau_0 \equiv \tau (r = 0) = \pi/4$ to reach the singularity, which turns out to be independent of the initial distance $r_0$. This is a novel result because it does not agree with those obtained in previous works (Olivares et al. 2013; Cruz et al. 2013), where the time to the singularity depends on the initial distance and the parameters of space–time through the event horizon (see Fig. 5).
On the other hand, Eq. (37) into Eq. (20) yields the coordinate time directly as a function of \( r \),

\[
t( r) = \frac{1}{4r^2} \left[ \arccosh \left( \frac{r_0^4 - r^4}{r_0^2 (r^2 - r^2_+)} \right) - \arccosh \left( \frac{r_0^4 + r^4_+ r^2}{r_0^2 (r^2 + r^2_+)} \right) \right].
\]

(40)

In Fig. 5 we plot the functional relations (39) and (40), which show us that the physics is essentially the same as Einstein’s space–times (S, SdS, SAdS, etc.).

2.2.2 Angular motion

In the case of massive particles with non-vanished angular momentum, we have that effective potential is given by

\[
V_{ta}( r) = (r^4 - r^4_+ ) \left( 1 + \frac{L^2}{r^2} \right);
\]

(41)

thus, we can write Eq. (21) as

\[
\left( \frac{dr}{d\theta} \right)^2 = \frac{(r_L^2 - r^2)(r^2 - r^2_1)(r^2 - r^2_2)}{L^2}.
\]

(42)

Here \( r_L \) is the turning point given by the relation

\[
r^2_L = u_0 - \frac{L^2}{3},
\]

(43)

and \( r_1, r_2 \) are two complex quantities (without physical meaning) given by

\[
r^2_j = u_j - \frac{L^2}{3}, \quad (j = 1, 2)
\]

(44)

with

\[
u_n = \sqrt{\frac{\eta^2_2}{3}} \cos \left[ \frac{1}{3} \arccos \sqrt{\frac{27 \eta^2_3}{\eta^2_2} + \frac{2n \pi}{3}} \right], \quad (n = 0, 1, 2)
\]

(45)

where the \( \eta \)'s are given by

\[
\eta_2 = 4 \left( E + r^4_+ + \frac{L^4}{3} \right) \quad (> 0)
\]

(46)

\[
\eta_3 = -4 \left( \frac{2L^6}{27} + \frac{L^2}{3} (E + r^4_+) - L^2 r^4_+ \right) \quad (< 0).
\]

(47)

Therefore, after a little algebraic manipulation in Eq. (42), the polar trajectory of the massive particles in terms of the \( \wp \)-Weierstraß function is obtained, which results in

\[
r( \theta) = \sqrt{r^2_L - \frac{1}{4\wp \left( \frac{2}{L} \gamma_1 \gamma_2 \gamma_3 \theta; \ g_2, g_3 \right) + \frac{\sigma}{3}}},
\]

(48)

where the Weierstraß invariants are given by

\[
g_2 = \frac{1}{4} \left( \frac{\alpha^2}{3} - \frac{\beta}{3} \right), \quad g_3 = \frac{1}{16} \left( \frac{\gamma}{3} + \frac{2}{27} \frac{\alpha^3}{3} - \frac{\alpha \beta}{3} \right).
\]

(49)
Fig. 6  Polar trajectory for massive particles. The solid line represents the regular orbit performed by the test particles, while the dashed line is the analytic continuation of that orbit. Here the initial condition $\theta = 0$, when $r = r_L$, together with the values $L = 1$, $r_+ = 2$, $r_L = 8$ and $E = 4 \times 10^3$ (in geometrized units) has been employed with

\[
\alpha = \frac{1}{y_1} + \frac{1}{y_2} + \frac{1}{y_3}, \quad \beta = \frac{1}{y_1 y_2} + \frac{1}{y_1 y_3} + \frac{1}{y_2 y_3}, \quad \gamma = \frac{1}{y_1 y_2 y_3}.
\] (50)

Also, we have that

\[
y_1 = r_{L}^{2}, \quad y_2 = r_{L}^{2} - r_{+}^{2}, \quad \text{and} \quad y_3 = r_{L}^{2} - r_{+}^{2}.
\] (51)

In Fig. 6 we plot the polar trajectory (48).

3 Final remarks

In this paper the geodesic structure of a Lifshitz black hole that is a solution to the Einstein-scalar-Maxwell theory in $3 + 1$ space–time dimensions with critical exponent $z = 2$ was examined. Using the Lagrangian procedure radial and angular motions of massless and massive test particles were studied. Analytical expressions for the proper time and coordinate time as a function of the radial coordinate for the strictly radial motion were obtained and the same analysis was provided for the polar trajectories. In Fig. 2, the proper and coordinate time for radial photons was depicted. The graphic shows that an external observer can see that it takes a finite coordinate time, $t_1$, for the photons to reach the asymptotic region, but an infinite proper time to do it. This result is consistent with previous studies dealing with Lifshitz space–times: a topological black hole of $3 + 1$ dimensions with critical exponent $z = 2$ (Olivares et al. 2013), in $2 + 1$ dimensions and $z = 3$ (Cruz et al. 2013), generalized for $D$-dimensions, and with an arbitrary critical exponent $z$ (Villanueva and Vásquez 2013).

It was also found that the massless and massive particles (see Fig. 5) cross the event horizon, $r_+$, in a finite proper time. However, the external observer can see that it takes the photon an infinite time to reach the horizon, which is analogous to the situation that occurs with the Einstein space–times. Angular motions are described by the polar trajectories of the particles.
in terms of the \( \wp \)-Weierstraß function, the results of which are depicted in Fig. 3 for photons and in Fig. 6 for massive particles. The behavior is analogous for both types of particles, where the motion is studied on the invariant plane \( \phi = \pi/2 \), and it is observed that bounded orbits are not allowed for this space–time on this plane. Similar behavior has recently been reported in the literature for other asymptotically Lifshitz black holes (Olivares et al. 2013; Cruz et al. 2013).

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References

Amado, I., Faedo, A.F.: Lifshitz black holes in string theory. JHEP 1107, 004 (2011). arXiv: 1105.4862

Ayon-Beato, E., Garbarz, A., Giribet, G., Hassaine, M.: Lifshitz black hole in three dimensions. Phys. Rev. D 80, 104029 (2009)

Balasubramanian, K., McGreevy, J.: An analytic Lifshitz black hole. Phys. Rev. D 80, 104039 (2009)

Brynjolfsson, E.J., Danielsson, U.H., Thorlacius, L., Zingg, T.: Holographic superconductors with Lifshitz scaling. J. Phys. A 43, 065401 (2010). arXiv: 0908.2611

Cruz, N., Olivares, M., Villanueva, J.R.: Geodesic structure of Lifshitz black holes in 2+1 dimensions. Eur. Phys. J. C 73, 2485 (2013). arXiv: 1305.2133

Cruz, N., Olivares, M., Villanueva, J.R.: The geodesic structure of the Schwarzschild anti-de sitter black hole. Class. Quantum Grav. 22, 1167–1190 (2005). arXiv: 0408016

Dehghani, M.H., Mann, R.B.: Lovelock-Lifshitz black holes. JHEP 1007, 019 (2010)

Devecioglu, D.O., Sarioglu, O.: On the thermodynamics of Lifshitz black holes. Phys. Rev. D 83, 124041 (2011). arXiv: 1103.1993

Hartnoll, S.A., Polchinski, J., Silverstein, E., Tong, D.: Towards strange metallic holography. JHEP 1004, 120 (2010)

Kachru, S., Liu, X., Mulligan, M.: Gravity duals of Lifshitz-like fixed points. Phys. Rev. D 78, 106005 (2008)

Maeda, H., Giribet, G.: Lifshitz black holes in Brans-Dicke theory. JHEP 1111, 015 (2011). [arXiv: 1105.1331]

Maldacena, J.M.: The large N limit of superconformal field theories and supergravity. Adv. Theor. Math. Phys. 2, 231 (1998)

Mann, R.B.: Lifshitz topological black holes. JHEP 06, 075 (2009)

Myung, Y.S., Moon, T.: Quasinormal frequencies and thermodynamic quantities for the Lifshitz black holes. Phys. Rev. D 86, 024006 (2012). arXiv: 1204.2116

Myungs, Y.S.: Phase transitions for the Lifshitz black holes. Eur. Phys. J. C 72, 2116 (2012). arXiv: 1203.1367

Olivares, M., Rojas, G., Vásquez, Y., Villanueva, J.R.: Particles motion on topological Lifshitz black holes in 3+1 dimensions. Astrophys. Space Sci. 347, 83–89 (2013). arXiv: 1304.4297

Olivares, M., Villanueva, J.R.: Massive neutral particles on heterotic string theory. Eur. Phys. J. C 73, 2659 (2013). arXiv: 1311.4236

Pang, D.W.: Conductivity and diffusion constant in Lifshitz backgrounds. JHEP 1001, 120 (2010)

Taylor, M.: Non-relativistic holography (2008). arXiv: 0812.0530

Villanueva, J.R., Olivares, M.: On the null trajectories in conformal Weyl Gravity. J. Cosmol. Astropart. Phys. 1306, 040 (2013). arXiv: 1305.3922

Villanueva, J.R., Vásquez, Y.: About the coordinate time in Lifshitz space–times. Eur. Phys. J. C 73, 2587 (2013). arXiv: 1309.4417