User Authorization in a System with a Role-Based Access Control on the Basis of the Analytic Hierarchy Process

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Abstract

The problem of optimal authorization of a user in a system with a role-based access control policy is considered. The main criterion is to minimize the risks of permission leakage. The choice of the role for authorization is based on the analytic hierarchy process. The substantiation of a choice of criteria for formation of a hierarchy of the first level is given. An algorithm for calculating weight coefficients is presented, based on the quantitative characteristics of the role graph and not dependent on subjective expert evaluations. The complexity is estimated and the scalability of the proposed algorithm is discussed.

Keywords: role-based access control, user authorization, permission leakage, analytic hierarchy process.

1 Introduction

The basic idea of the role-based access control policy is that rights and privileges (we will call them permissions) are granted to the user not directly, but through assigning a certain role to the user. Together with the role, the user receives a certain set of permissions assigned to the given role. Each session of the user in the system requires his authorization with one or more allowed roles for him.

In this paper we consider the problem of selecting one or more roles in a situation where the list of permissions that a user must have is known, and a given set of permissions can be granted by several roles. The indicated task arises both at the stage of granting potentially possible roles to the user, and when authorizing in each specific session of work. In both cases, we will call the posed problem the authorization problem.
When designing a role-based access control, the system administrator must decide on the optimal organization for the requested access. Given that the security of information in computer systems is made up of confidentiality, accessibility and integrity, the following selection criteria for roles can be formulated:

1. Give the user as much permissions as possible - make the system as open as possible.
2. Give the user a minimum of permissions - make the system as closed as possible.

In connection with the mutual inconsistency of these criteria, the choice of the optimal version of granting roles to the user is a rather complex problem. In [1] it is proposed to consider the linear combination of the functions of "openness" (efficiency of information access) \( F_1 \) and "closedness" (the effectiveness of information confidentiality) \( F_2 \) as an objective function of the problem:

\[
F = c_1 F_1 + c_2 F_2.
\]

But with this approach, there is a significant difficulty in determining the quantitative characteristics of the weight coefficients \( c_1 \) and \( c_2 \), and the functions \( F_1 \) and \( F_2 \) themselves. In addition, the proposed criteria need to be expanded and detailed. In particular, such indicators as integrity and accessibility of information are not involved. In terms of the main channels of information leakage that can arise as a result of authorization, the criterion of "closedness" should be decomposed into components. In this situation, linear programming models for decision making become unacceptable. But the existence of several criteria and alternative solutions makes it possible to suppose the applicability of the analytic hierarchy process [2 3 4 5 6 7]. At the same time, it is important to determine the quantitative indicators for constructing a numerical scale of preferences for possible alternatives (in our case, roles that have the required permissions for the user).

## 2 Statement of the problem and criteria of the hierarchy

Let a role-based access control policy with hierarchical organization of multiple roles is implemented in the computer system. This means that on a set of roles the submission/domination ratio is given that generates a hierarchy of roles. The permissions between roles are distributed according to this hierarchy.

A new user \( u \) appears in the system which requires a set \( PU \) of permissions to work. It is
required to authorize this user for some role. There are two possible solutions. The first is to authorize the user for one of the existing roles. The second solution is to create a new role, specifically for this user. The second approach, of course, provides a higher level of security, but leads to a number of problems. First, it can lead to uncontrolled growth of many roles. Secondly, it requires solving additional tasks to determine the hierarchy of roles. Third, it can eliminate all the advantages of the role-based access control model if for each role there will be only one user authorized to it. In addition, allowing the modification of the hierarchical structure of multiple roles requires additional analysis of hidden channels of information leakage [8, 9]. The purpose of the proposed approach is to develop an algorithm that allows new users to be entered without changing the role structure of the system.

Let’s imagine the authorization problem in terms of the analytic hierarchy process. In the simplest case, it will be a task with one hierarchical level: the solution, the criteria for the hierarchy of level 1, and alternatives. Expected result is ranking of roles by degree of preference from the point of view of system administration. The hierarchy top or solution is the choice of a specific role for user authorization. Alternatives are the roles $r_1, r_2, ..., r_k$, which include in their list of permissions the required set $P_U$ of permissions.

Let us pass to the selection of criteria. At the initial stage of the model building, two opposite factors can be chosen: "openness" and "closedness" of the system. But, as noted earlier, this approach is too generalized and complex for quantitative estimates. Another option is to formulate criteria based on an analysis of possible channels of information leakage, leaving issues of integrity and accessibility beyond the scope of the review, focusing on confidentiality of information. We formulate a minimum risk requirement for solving this problem: the authorization of a new user should be carried out in such a way as to minimize the risks of information leakage. We indicate the main channels of information leakage that can arise as a result of authorization:

1. Obtaining additional permissions attributed to roles, but not intended for the user.
2. User authorization for additional roles subordinate to this role.

We will compare roles on the specified factors: "additional permissions" and "subordinate roles". The advantage of this approach is that for both criteria there are quantitative estimates determined by a given hierarchy of roles - this is the power of the set of additional permissions provided to the user when authorizing for this role, and the power of the set of roles over
which this role is dominant. The smaller these values, the more preferable this role is from the standpoint of confidentiality. Then pairwise comparison matrices of the level of alternatives can be obtained numerically, without constructing a scale of preferences and attracting experts.

3 Filling the pairwise comparison matrix

Let user $u$ need to get a set $PU$ of permissions. We write out all the roles containing those permissions: $r_1, r_2, ..., r_k$. At this stage of the problem analysis, we will compare the roles based on the previously mentioned criteria:

1. $A$ is the set of additional permissions granted to the user when authorizing for this role. Quantitative estimate is $|A|$ (the power of the set $A$): the smaller $|A|$, the higher the level of confidentiality.

2. $B$ is the set of roles over which this role dominates. Quantitative estimate is $|B|$ (the power of the set $B$): the smaller $|B|$, the higher the level of confidentiality. The decision tree is shown in Fig.1.

![Figure 1: The hierarchy of the level 1, constructed according to two criteria](image)

According to the analytic hierarchy process [4], if the relative weight coefficients $w_a, w_b, w_{a1}, ..., w_{ak}, w_{b1}, ..., w_{bk}$ ($i = 1, ..., k$) are known, then the $i$-th role should be chosen with probability equal to $i$-th combined weight factor:

$$P(r_i) = w_a \times w_{ai} + w_b \times w_{bi}.$$  

To minimize the risks of information leakage, the user $u$ must be authorized for the role $r_i$ with the maximum value of $P(r_i)$. Once again, we emphasize that the calculation of the weight coefficients is carried out from the point of view of the system administrator.
To determine the relative weight coefficients, the pairwise comparison method [4] is used. For the top of the hierarchy (the solution $R$) and the criteria (in our case, these are the criteria $A$ and $B$) the pairwise comparison matrices are compiled. The dimension $k$ of the current pairwise comparison matrix $M$ is determined by the number of factors being compared (for the solution this is the number of criteria, for each criterion this is the number of roles). The diagonal elements $[M]_{ii}$ of the matrix $M$ are equated to 1, and the elements $[M]_{ij}$ show how many times the factor $i$ is more significant than the factor $j$. Then the columns of the pairwise comparison matrix are normalized, and the elements of the new normalized matrix $M^*$ are calculated by the formula:

$$[M^*]_{ij} = \frac{[M]_{ij}}{[M]_{1j} + \ldots + [M]_{kj}}.$$

Finally, the weight of each factor $w_i$ is calculated as the arithmetic mean of the corresponding row in the normalized matrix $M^*$:

$$w_i = \frac{1}{k} ( [M^*]_{i1} + \ldots + [M^*]_{ik} )$$

We consider first the pairwise comparison matrix $M_r$ for the top $R$ and the factors $A$ and $B$. We assume that the additional permissions obtained directly (factor $A$) create the danger of information leakage $s$ times more often than through the subordinate roles (factor $B$). This means that the factor $A$ in comparison with factor $B$ is $s$ times more preferable or more significant from the point of view of the attacker. Since our task is to obtain weights that reflect the administrator’s preferences, this value must be inverted, which means that the elements of the pairwise comparison matrix are $[M_r]_{ab} = 1/s$. As a result, the pairwise comparison matrix $M_r$ has the form:

$$M_r = \begin{pmatrix} 1 & 1/s \\ s & 1 \end{pmatrix}$$

Let’s move on to the next level of the hierarchy. The quantitative estimate for criterion $A$ is the value inverse to the power of the set of additional permissions provided to the user when authorizing for the selected role. Then the elements of the pairwise comparison matrix $M_a$ for determining the coefficients $w_{a1}, \ldots, w_{ak}$ will be found by the formula:

$$[M_a]_{ij} = \frac{1/dp_i}{1/dp_j} = \frac{dp_j}{dp_i},$$

here $dp_i$ is the number of permissions of role $r_i$, excluding the required set $PU$ of permissions. Note that if for some $i$ the equality $dp_i = 0$ holds, then user $u$ is authorized for the role $r_i$.
precisely, and this completes the calculations. Similarly, the quantitative estimate for criterion B is the value inverse to the power of the set of roles over which the selected role is dominant. To determine the coefficients $w_1, \ldots, w_k$, we construct the pairwise comparison matrix $M_b$, in which

$$[M_b]_{ij} = \frac{1/dr_i}{1/dr_j} = \frac{dr_j}{dr_i},$$

here $dr_i$ is the number of roles over which the role $r_i$ is dominant. To avoid zero values and, as a consequence, infinitely large values in weight coefficients, we assume that each role dominates itself, that is, the minimum value of the power of the set of subordinate roles will be equal to one. Thus, we assume that the dominance relation is reflexive (non-strict).

4 Consistency of the pairwise comparison matrices

It is known that in the case when the pairwise comparison matrix is ideally consistent: $[M]_{ij} = [M]_i \times [M]_j$ (for any $i, j, k$), the columns of the corresponding normalized matrix are obtained the same [4]. Then the weight coefficients are $w_i = [M^*]_{ij}$ for any $j$, and for their calculation it is sufficient to normalize only one column of the pairwise comparison matrix.

The pairwise comparison matrices $M_r$, $M_a$, $M_b$ were filled by us without involving the mechanism of expert evaluations, and their ideal consistency follows in an obvious manner from the filling algorithm itself. Indeed, based on the hierarchy of roles and the distribution of permissions, we attributed to the compared alternatives (roles) some numerical characteristics that are convenient to represent in vector form:

$$P = \begin{pmatrix} 1/dp_i \\ \vdots \\ 1/dp_k \end{pmatrix}, \quad R = \begin{pmatrix} 1/dr_i \\ \vdots \\ 1/dr_k \end{pmatrix}.$$ 

Then, the elements of the matrices $M_a$ and $M_b$ were calculated as the ratios of the corresponding coordinates of these vectors. Obviously, the following equalities hold:

$$[M_a]_{is} \times [M_a]_{sj} = \frac{dp_s}{dp_i} \times \frac{dp_j}{dp_s} = \frac{dp_j}{dp_i} = [M_a]_{ij},$$

$$[M_b]_{is} \times [M_b]_{sj} = \frac{dr_s}{dr_i} \times \frac{dr_j}{dr_s} = \frac{dr_j}{dr_i} = [M_b]_{ij}.$$ 

The ideal consistency of the matrix $M_r$ follows from the fact that the two-dimensional reciprocal matrix is always ideally consistent. But in this case the vectors of the weight coefficients $W_a$
and $W_b$ are none other than the normalized vectors $P$ and $R$. Indeed:

$$w_{ai} = \frac{[M_a]_{ij}}{[M_a]_{1j} + \ldots + [M_a]_{kj}} = \frac{dp_j/dp_i}{dp_j/dp_1 + \ldots + dp_j/dp_k} = \frac{1/dp_i}{1/dp_1 + \ldots + 1/dp_k},$$

$$W_a = \begin{pmatrix} w_{a1} \\ \vdots \\ w_{ak} \end{pmatrix} = \frac{P}{||P||} = \begin{pmatrix} 1/dp_1 \\ \vdots \\ 1/dp_k \end{pmatrix}.$$

$$w_{bi} = \frac{[M_b]_{ij}}{[M_b]_{1j} + \ldots + [M_b]_{kj}} = \frac{dr_j/dr_i}{dr_j/dr_1 + \ldots + dr_j/dr_k} = \frac{1/dr_i}{1/dr_1 + \ldots + 1/dr_k},$$

$$W_b = \begin{pmatrix} w_{b1} \\ \vdots \\ w_{bk} \end{pmatrix} = \frac{R}{||R||} = \begin{pmatrix} 1/dr_1 \\ \vdots \\ 1/dr_k \end{pmatrix}.$$

The vector of weight coefficients of the level 1 is obtained by normalizing the first column of the matrix $M_r$:

$$W = \begin{pmatrix} w_a \\ w_b \end{pmatrix} = \begin{pmatrix} 1/(1 + s) \\ s/(1 + s) \end{pmatrix}.$$

It should be noted that the main difficulty of the analytic hierarchy process lies precisely in the search for each level of the hierarchy of positive weight coefficients $w_1, \ldots, w_k$ satisfying the normalization condition: $w_1 + \ldots + w_k = 1$. The formulas used to calculate the weights are based on the following fact: the weight vector is the normalized eigenvector of an ideally consistent pairwise comparison matrix corresponding to its maximal eigenvalue equal to the dimension of the matrix $k$ [4]. An ideal consistency of the pairwise comparison matrix is possible, for example, if experts fill only the first row of the matrix, and the remaining elements are calculated based on the relation: $[M]_{ij} = [M]_{1j}/[M]_{1i}$ [3]. In practice, with the classical "element-by-element" formation of pairwise comparison matrices by experts, one cannot count on its consistency. Nevertheless, the weight vector in the analytic hierarchy process is also equal in this case to the eigenvector corresponding to the maximum eigenvalue, which for the inconsistent matrix is already strictly greater than $k$. But the equality used is heuristic, which means that the application of the analytic hierarchy process in the case of an inconsistent pairwise comparison matrix contains a "model" error in computing the weight vector. To evaluate it, the author of the method proposes to use a special numerical index - the "consistency index" [4]. Unfortunately, the value of this index allows us to judge the magnitude of the resulting "model" error only indirectly [3].
In this approach, to solving the authorization problem, there is no need to form pairwise comparison matrices, and the analytic hierarchy process, in addition to reducing the complexity, is deprived of the main source of criticism, a "model" error that arises from the inconsistency of expert judgments.

5 Algorithm of selecting the role for user authorization

We formally write down the algorithm for solving the authorization problem. As before, we assume that the additional permissions received by the user directly create the danger of information leakage $s$ times more often than through authorization for subordinate roles. Let the set of permissions which is necessary for user $u$ in the current session is set by the set $PU$. The permissions of role $r$ are accessible through the field (set) $r.p$, and the list of subordinate roles is accessible through the field (set) $r.r$. The algorithm steps can be written in the following form.

1. Create a set $RU$ of roles that are available to user $u$ for authorization, according to the rule: if $PU \subseteq r_i.p$, then $r_i \in RU$.

2. Each role $r_i \in RU$ is put in correspondence with the value $dp_i$ equal to the number of additional permissions of the role $r_i$: $dp_i = |r_i.p| - |PU|$. If there is a role for which $dp = 0$, then user $u$ must be authorized for this role, and the algorithm’s work ends.

3. Each role $r_i \in RU$ is put in correspondence with the value $dr_i$ equal to the number of roles over which the role $r_i$ is dominant: $dr_i = |r_i.r|$ (providing $r_i \in r_i.r$).

4. Calculate the coordinates of the weight vectors of the level of alternatives (vectors $W_a$ and $W_b$):

   $$w_{ai} = \frac{1/dp_i}{1/dp_1 + \ldots + 1/dp_k}, \quad w_{bi} = \frac{1/dr_i}{1/dr_1 + \ldots + 1/dr_k},$$

   (here $k = |RU|$).

5. Calculate the coordinates of the weight vector of the level of criteria (vector $W$):

   $$w_a = \frac{1}{1 + s}, \quad w_b = \frac{s}{1 + s}.$$

6. Calculate for each role $r_i \in RU$ the probability of its selection by the system administrator to authorize the user $u$:

   $$P(r_i) = w_a \times w_{ai} + w_b \times w_{bi}.$$
7. Find the role $r_{\text{max}} \in RU$ such that

$$P(r_{\text{max}}) = \max_{r_i \in RU} P(r_i).$$

8. Authorize user $u$ for the role $r_{\text{max}}$.

We estimate the algorithmic complexity of the presented algorithm.

**Statement.** The complexity of the algorithm for solving the authorization problem based on the analytic hierarchy process is $O(\max(n \times m, n^2))$, here $n$ is the number of roles, $m$ is the number of permissions.

**Proof.** We denote the complexity of the $i$-th step of the algorithm by $T_i$ ($i = 1, ..., 8$), and the complexity of the preparatory stage by $T_0$. Then the total complexity of the algorithm $T$ is calculated by the formula: $T = T_0 + T_1 + ... + T_8$. Notice that $|PU| \leq m$, $|RU| \leq n$.

For calculations, it is necessary to compute the powers of the sets $r_i.p$ and $r_i.r$ for each $r_i$ ($i = 1, ..., n$) in the system. We denote the complexity of these operations by $T_p$ and $T_r$. Obviously, regardless of the implementation of the data structure that defines the hierarchy of roles, $T_p = O(n \times m)$ and $T_r = O(n^2)$. The complexity of calculating the power of the set $PU$ also doesn’t cause difficulties: $T_{PU} = O(m)$. Thus, $T_0 = O(n \times m) + O(n^2) + O(m) = O(\max(n \times m, n^2))$.

We denote the complexity of solving the problem of checking the inclusion of one set in another ($PU \subseteq r_i.p$) by $T_{\subseteq}$. Then $T_1 = nT_{\subseteq}$, $T_2 = O(n)$, $T_3 = O(n)$, $T_4 = O(n)$, $T_5 = O(\text{const})$, $T_6 = O(n)$, $T_7 = O(n)$, $T_8 = O(\text{const})$.

The complexity $T_{\subseteq}$ depends on the way the set is implemented. Obviously, $T_{\subseteq} \geq O(m)$. We show that we can choose a way to form the set, which allows us to obtain equality for the implementation of the operation ”is contained”. Indeed, the universal set for the sets $PU$ and $r_i.p$ is the set of all permissions. If the set is given by a binary vector (array) of dimension $m$, in which the $j$-th coordinate is 1, if the permission $p_j$ belongs to the set, and 0 otherwise, then the ”is contained” operation is realized as follows: $PU \subseteq r_i.p$ if and only if $([PU]_i = 0) \lor ([PU]_i = 1) \land ([r_i.p]_i = 1))$. The complexity of this test is $O(m)$. Therefore, $T_{\subseteq} = O(m)$ and $T_1 = O(n \times m)$.

As a result, we get: $T = O(\max(n \times m, n^2)) + O(n \times m) + O(n) + O(n) + O(\text{const}) + O(n) + O(n) + O(\text{const}) = O(\max(n \times m, n^2))$. Q.E.D.

**Note.** It should be noted that the main complexity of the algorithm lies in the preparatory stage: $T_0 = O(\max(n \times m, n^2))$. If the powers of the sets $r_i.p$, $r_i.r$ and $PU$ are known in
advance, then $T = O(n \times m)$. In this case, again, the complexity is determined by the decision of the choice of roles that are "suitable" for authorizing the user $u$. In the case when this question is solved in advance, the complexity of the algorithm becomes linear: $T = O(n)$.

6 Scalability

In the approach presented, two criteria are used to estimate the confidentiality of information. Obviously, the method is scalable and can easily be extended to any number of criteria.

We add to the proposed set of criteria factors to estimate the accessibility and integrity of information. It can be a "total set of permissions" that includes the total number of permissions of the current role. The more the power of such a set, the more preferable this role is in terms of accessibility. As for integrity, to take into account this component of information security, it will be necessary to introduce the criterion of the same name and estimates the roles, rather attributed to them sets of permissions, pairwise for the integrity of information. If we try, as for the three previous criteria, to obtain a quantitative estimate, then we can single out a set of permissions that are potentially dangerous in terms of integrity. Their total count for the current role will give the desired numerical value. The lower this indicator, the more preferable the role.

In addition to the criteria generated by the requirements of information security, one can consider the factor characterizing the costs of management. The criterion of "manager costs" is determined by the cost of the role $r_i$ in managing subordinate roles: $(dm_i + 1)^\alpha \lambda^\alpha$, here $\alpha$ and $\lambda$ are some constants, $dm_i$ is the number of roles directly subordinate to the role $r_i$. Obviously, the role for which this value is minimal is preferable.

Here it should be noted again that for the presented criteria there are quantitative estimates determined by a given dominance relation on the set of roles and allowing to fill the pairwise comparison matrices automatically.

In determining the weights of the criteria level, there is some subjectivity since, even with the requirement of the ideal consistency of the matrix $M_r$, it is necessary to determine the degree of preference of the first criterion in comparison with the others (to form the first row of the matrix). These quantities are configurable parameters of the algorithm. Their variation and recalculation of probabilities can give the system administrator additional information about
the stability of the formed hierarchy of many roles against changes in external conditions.

7 Conclusion

As can be seen from the proposed methodology, the analytic hierarchy process allows to significantly automate the process of selecting a role for user authorization in order to obtain the necessary permissions. However, the recommendations of the method should not be taken as a final solution. It is more reasonable to arrange a list of suitable roles by not increasing the probability of choice. Then the decision should be made by the system administrator, based on the meaning of the permissions themselves and additional restrictions on the ownership of role or permission that may exist in the system. Thus, the proposed approach is a decision support system.

It should also be noted that the use of the analytic hierarchy process proposed in the article is based only on the properties of the system itself, without external subjective expert estimates. Such use is new, not previously used. And this approach allows us to run the analysis of the system in an automatic mode, not only when authorizing a new user in the system, but also to check the security of the system as a whole.

The estimation of the method complexity carried out in the article allows to speak about the absence of difficulties in the program implementation of this approach and high speed.

In conclusion, we note that the analysis of hierarchical structures existing in the information system allows obtaining additional information on the distribution of access rights and requires the development of mathematical models of security taking into account the hierarchy of their elements [8, 9, 11].

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