Research on constitutive model of hyperelastic-viscoelastic-plastoelastic material of rubber waveform generator

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Abstract. Considering the hyperelastic property, frequency dependency and amplitude dependency of the rubber material was independent, the hyperelastic-viscoelastic-plastoelastic model was used for characterizing the dynamic properties of the rubber material. Shear specimens firstly were made, and shear tests were carried out. The test values of the dynamic shear modules and damping were obtained based on static and dynamic shear tests. The theoretical values were obtained by the theoretical calculation. By minimizing the relative errors between the test values and theoretical values, the hyperelastic-viscoelastic-plastoelastic constitutive model parameters of the rubber material were obtained.

1. Introduction
The rubber waveform generator is a key component of the gun dynamic recoil test machine, which plays an important role in the transmission and conversion of the impact energy [1]. The structure of the waveform generator is shown in figure 1. The waveform generator mainly uses rubber material. The load-displacement relationship of the rubber material shows the hyperelastic property in the static deformation process. The rubber material shows the frequency dependency and amplitude dependency during the dynamic deformation, apart from the hyperelastic property [2-4]. In this paper, the material constitutive model of the waveform generator was studied by means of the theoretical calculation and material tests.

Figure 1. Waveform generator.

2. Constitutive model theory of hyperelastic-viscoelastic-plastoelastic material
The rubber material has the hyperelastic property, frequency dependency and amplitude dependency during the dynamic deformation. These properties are generally assumed to be independent [5-7]. Therefore, the hyperelastic-viscoelastic-plastoelastic model was used for characterizing the rubber material.
material, as shown in figure 2. The model consisted of three parts. The first part was the hyperelastic model, used to describe the static elastic property. The second part was the viscoelastic model, used to describe the frequency dependency of the dynamic property. The third part was the plastoelastic model, used to describe the amplitude dependency of the dynamic property [8].

2.1. Hyperelastic model
During the equation derivation process, the rubber material was in the uniaxial stress and small strain state. In order to simplify the theoretical calculation, the linear elastic model was used to replace the hyperelastic model. Where $G_{\infty}$ was the quasi-static shear modulus of the linear elastic model.

2.2. Hyperelastic-viscoelastic model
As shown in figure 2, some Maxwell units were used in parallel to represent the viscoelastic model of the rubber material. $G_{vi}$ and $t_{si}$ were the shear modulus and relaxation time of the $i$ Maxwell unit. $i=1,2,\ldots,N$, $N$ was the unit number.

2.3. Plastoelastic model
As shown in figure 2, some plastoelastic units were used in parallel to represent the plastoelastic model of the rubber material. $G_{pj}$ and $k_{pj}$ were the shear modulus and yield strain of the $j$ plastoelastic unit. $j=1,2,\ldots,M$, $M$ was the unit number.

2.4. Hyperelastic-viscoelastic-plastoelastic model
Summing the calculation results of the hyperelastic model and the plastoelastic model in the frequency domain, the total dynamic shear modulus $G_{dy}$ and damping $d$ of hyperelastic-viscoelastic-plastoelastic model can be expressed as

$$G_{dy} \approx G_{dy}^{ep} + G_{dy}^{ve}, \quad d \approx \frac{G_{dy}^{ep} d^{ep} + G_{dy}^{ve} d^{ve}}{G_{dy}}$$

3. Shear tests of rubber material
Using the constitutive model in figure 2, the model parameters can be calculated by shear tests of the rubber material. Firstly the shear specimens were made, and then the static and dynamic shear tests were carried out. The linear elastic model can be obtained by the static shear tests. The shear modulus and viscous coefficient of the viscoelastic model, the shear modulus and yield strain of the plastoelastic model can be obtained by the dynamic shear tests.
3.1. Shear specimens preparation
The schematic diagram of the shear specimen was shown in figure 3. The material hardness of the shear specimen were HA70, HA80 and HA90, respectively.

![Shear specimen](image)

**Figure 3.** Shear specimen.

3.2. Static shear test
Static shear tests were carried out on the electronic universal testing machine. Through the tests, the load-displacement change curves of shear specimens were obtained, as shown in figure 4.

![Load-displacement curves](image)

**Figure 4.** Load-displacement curves.

The quasi-static shear modulus was calculated as follows

\[ G_\sigma = \frac{\tau_{\sigma\theta}}{\kappa_{\sigma\theta}} \]  

(2)

\[ \tau_{\sigma\theta} = \frac{F_{\sigma\theta}}{2A} \]

(3)

Where \( \tau_{\sigma\theta} \) was the shear stress. \( \kappa_{\sigma\theta} \) was the shear strain. \( 2A \) was the shear area of the rubber block. \( t_0 \) was the thickness of the rubber block.

So the quasi-static shear modulus of the shear specimens with three hardness were respectively 1.6084MPa, 2.1399MPa and 2.9930MPa.

3.3. Dynamic shear test
Dynamic shear tests were carried out on the electrohydraulic servo fatigue testing machine. The maximum dynamic load was 20kN, the frequency range was from 0.1 to 45Hz, and the displacement excitation precision was 0.01mm.

During the test process, the displacement excitation \( x(t) \) with a certain amplitude and frequency was applied to the shear specimen. \( x(t) = X_0 \sin(w_0t) \). \( X_0 \) and \( w_0 \) were the amplitude and the angular frequency of the displacement excitation. The load signal \( F(t) \) was measured by the force sensor. The displacement signal \( x(t) \) was measured by the displacement sensor.

In the dynamic shear tests, the displacement excitation amplitude \( X_0 \) was 0.1mm, and the displacement excitation frequency \( w_0 \) ranged from 1Hz ~ 30Hz. The load and displacement signals at different frequencies were measured respectively. Due to the damping effects of the rubber material,
the load response lagged behind the displacement excitation. So the hysteresis loop was formed, with the horizontal axis of displacement and the vertical axis of load. Take the shear specimen with the hardness H70 as an example, when the displacement excitation frequency was 5Hz, the load response curve was obtained. The displacement excitation and load response curves were shown in figure 5. The hysteresis loop was shown in figure 6.

![Figure 5. Displacement excitation and load response curves.](image)

Figure 5. Displacement excitation and load response curves.

According to the hysteresis loop, the dynamic stiffness $K_d$ and damping angle $\delta$ can be calculated. The equation was as follows.

$$K_d = \frac{F_0}{X_0}, \quad \delta = \sin^{-1}\left(\frac{U_s}{F_0X_0\pi}\right)$$

(4)

Where $F_0$ was the load response. $X_0$ was the displacement excitation amplitude. $U_s$ was the load-displacement hysteresis loop area. According to the equation (4), the dynamic stiffness $K_d$ of the shear specimen was 485.8N / mm and the damping angle $\delta$ was 7.629° at this frequency.

The load $F(t)$ was converted to the shear stress $\tau(t)$, and the displacement $x(t)$ was converted to the shear strain $\kappa(t)$. Take the shear stress as the horizontal axis, and the shear stress as the vertical axis, the hysteresis loop also can be formed. Based on the hysteresis loop, the dynamic shear modulus was 3.3972, and the damping was 0.0996. Similarly, the dynamic shear modulus and damping of the rubber material at the other hardness and frequencies can be calculated, as shown in figure 7 and figure 8.

![Figure 7. Change curves of dynamic shear modulus with frequency.](image)

Figure 7. Change curves of dynamic shear modulus with frequency.
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4. Constitutive model parameters determination of rubber material

Through the static and dynamic tests, the test values of the dynamic shear modulus and damping of the rubber material at different hardness and frequencies were obtained. According to the equations (1), the theory values of the two physical quantities at different hardness and frequencies also can be calculated. The Constitutive model parameters of the rubber material were obtained by minimizing the relative errors between the test values and the calculated values of the dynamic shear modulus and damping.

$$\psi = \sum_{i=1}^{m} \left( \frac{G_{dy,i} - \bar{G}_{dy,i}}{d_{dy,i}} \right)^2 + \sum_{i=1}^{m} \left( \frac{d_{dy,i} - \bar{d}_{dy,i}}{G_{dy,i}} \right)^2$$  \hspace{1cm} (5)

Where $G_{dy,i}$ and $d_{dy,i}$ were the test values of the dynamic shear modulus and damping. $\bar{G}_{dy,i}$ and $\bar{d}_{dy,i}$ were the theoretical values of the dynamic shear modulus and damping. $m$ was the dynamic shear test number.

According to the equation (5), the constitutive model parameters of the rubber material were obtained, as shown in table 1. Where $G_\infty$ was the quasi-static shear modulus of the linear elastic model, $G^{ve}_{dy,i}$ and $t_i$ were the shear modulus and relaxation time of the $i$ Maxwell unit. $i=1,2,3$. $G^{re}_{dy,i}$ and $k_{sy,i}$ were the shear modulus and yield strain of the $j$ plastoelastic unit, $j=1,2,3$.

| Hardness | $G_\infty$ | $t_{i1}$ | $t_{i2}$ | $t_{i3}$ | $G_{dy,1}^{ve}$ | $G_{dy,2}^{ve}$ | $G_{dy,3}^{ve}$ |
|----------|------------|-----------|-----------|-----------|----------------|----------------|----------------|
| HA70     | 1.6084     | 0.8646    | 0.0066    | 0.0001    | 0.0751         | 0.1133         | 0.0004         |
| HA80     | 2.1399     | 0.5559    | 0.02      | 0.01      | 0.0931         | 0.061          | 0.15           |
| HA90     | 2.9930     | 0.939     | 0.02      | 0.0096    | 0.0671         | 0.1322         | 0.1015         |

| Hardness | $\kappa_{i1}$ | $\kappa_{i2}$ | $\kappa_{i3}$ | $G_{dy,1}^{re}$ | $G_{dy,2}^{re}$ | $G_{dy,3}^{re}$ |
|----------|----------------|----------------|----------------|----------------|----------------|----------------|
| HA70     | 0.0760         | 0.0108        | 0.0096        | 0.1898         | 0.7695         | 0.2026         |
| HA80     | 0.0550         | 0.0001        | 0.0100        | 0.1798         | 0.8668         | 1.5331         |
| HA90     | 0.0817         | 0.0084        | 0.0001        | 0.1219         | 1.9263         | 0.1055         |

5. Conclusions

This article mainly completed the following work:

(1) Assuming that the hyperelastic property, frequency dependency and amplitude dependency of rubber material, the hyperelastic-viscoelastic-plastoelastic model was used for characterizing the dynamic properties of the rubber material.

(2) Through the theoretical calculation and shear tests, the hyperelastic-viscoelastic-plastoelastic constitutive model parameters of rubber material were obtained.

Figure 8. Change curves of damping with frequency.
This study provided a theoretical and methodological reference for the establishment of the rubber material constitutive model. It also laid a research foundation for the dynamic characteristics of the rubber waveform generator and the gun dynamic recoil test machine.

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