Full analysis of general non-standard $tbW$ couplings

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ABSTRACT

Possible non-standard couplings which could contribute to the $t \rightarrow bW$ process are studied based on the effective-Lagrangian approach. The corresponding effective Lagrangian consists of four kinds of dimension-6 effective operators, each of which has an independent coupling constant. In this analysis, all those couplings are treated as complex numbers and constraints on them are estimated by using recent experimental data from the LHC. We point out that the resultant constraints on those couplings are still not that strong because contributions from some couplings can work oppositely with each other.

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The top quark, the mass of which is about 173 GeV, is still the heaviest particle we can observe up to now although a new scalar indicating the Higgs boson, the last piece of the standard model, has been discovered [1, 2]. Studying this quark from various angles will be, therefore, a quite promising approach to new physics beyond the standard model [3, 4, 5]. In particular, precise analyses of the top-quark couplings could play a crucial role to reveal new-physics effects that might exist behind phenomena observed in collider experiments. We will soon have more information for those studies, considering that the Large Hadron Collider (LHC) has now re-started measuring the top-quark properties more precisely with $\sqrt{s} = 13$ TeV and a plan of luminosity upgrade [6].

In precision measurements, a sign of new-physics will appear in various observables as deviations from the standard-model predictions, unless new (non-standard) particles are directly discovered. Since those deviations in general arise through quantum loop effects of non-standard particles, the effective-Lagrangian procedure [7]–[10] is known as a useful way to describe such effects. This approach enables a model-independent analysis if we construct the effective Lagrangian using only the standard-model fields below the new-physics scale ($\Lambda$). The top-quark-decay process we focus on, $t \rightarrow bW$, is suitable for those studies because a top quark decays quickly within the perturbation region owing to its heavy mass [11, 12].

Although many authors have already studied top-decay processes in the effective-Lagrangian framework in order to probe possible new interactions [13]–[36], the non-standard couplings included there have been treated as real numbers, or as partially complex numbers, and/or only some couplings have been treated as free parameters at once fixing the others. In addition, it has not been unusual to adopt the linear approximation in those parameters, i.e., to neglect their quadratic (and higher-power) terms. Those limited analyses could be reasonable if the authors are implicitly considering some specific models. We cannot say however that they are the most satisfactory as purely model-independent studies. Therefore, in this short article, assuming all those non-standard couplings are complex numbers and contribute to the top-decay process at the same time, we estimate current constraints on them from recent experimental data without taking the linear approximation.
In our analysis, we assume that there exist no new particles at any energy less than $\Lambda$. Based on this assumption and adopting the notations of our previous work \[37, 38, 39\], the effective Lagrangian for $t \rightarrow bW$ is expressed as

$$\mathcal{L}_{tbW} = -\frac{1}{\sqrt{2}}g \left[ \bar{\psi}_b(x)\gamma^\mu (f_1^L P_L + f_1^R P_R) \psi_t(x) W^-_\mu(x) \\
+ \bar{\psi}_b(x)\frac{\sigma^{\mu\nu}}{M_W}(f_2^L P_L + f_2^R P_R) \psi_t(x) \partial_\nu W^-_\nu(x) \right], \quad (1)$$

where $g$ is the $SU(2)$ coupling constant, $P_{L/R} \equiv (1 \mp \gamma_5)/2$, and the coupling parameters $f^{L,R}_{1,2}$ are defined as

$$f^L_1 \equiv V_{tb} + C^{(3,33)}_{\phi q} \frac{v^2}{\Lambda^2}, \quad f^R_1 \equiv C^{33}_{\phi\phi} \frac{v^2}{2\Lambda^2}, \quad (2)$$

$$f^L_2 \equiv -\sqrt{2} C^{33}_{dW} \frac{v^2}{\Lambda^2}, \quad f^R_2 \equiv -\sqrt{2} C^{33}_{uW} \frac{v^2}{\Lambda^2},$$

with $v$ being the Higgs vacuum expectation value, $V_{tb}$ being the $(tb)$ element of Kobayashi–Maskawa matrix, and $C^{(3,33),33}_{\phi q,\phi\phi,dW,uW}$ being the parameters representing the contributions of the corresponding dimension-6 operators (see [9]). Among those parameters, we divide $f^L_1$ into the SM term and the rest (i.e., the non-SM term) as

$$f^L_1 \equiv f^{SM}_1 + \delta f^L_1, \quad (3)$$

where $f^{SM}_1 \equiv V_{tb}$ and $\delta f^L_1 \equiv C^{(3,33)}_{\phi q} v^2/\Lambda^2$. We then assume $f^{SM}_1(= V_{tb}) = 1$ and treat $\delta f^L_1$, $f^R_1$, and $f^{L,R}_{2}$ as complex numbers hereafter. As for the masses of the involved particles, we take as $m_t = 172.5$ GeV, $m_b = 4.8$ GeV and $M_W = 80.4$ GeV.

Now, we here focus on $t \rightarrow bW$ as mentioned and assume that it is the unique top-decay channel. The $W$-boson is produced there with one of the following helicities: $h = 0$ (longitudinal), $h = -1$ (left-handed), and $h = +1$ (right-handed), which means there are three kinds of helicity fraction corresponding to each helicity state. The analytical formulas of those partial decay widths are calculated by using Eq.(1) straightforwardly and we have confirmed that our formulas are the same as those presented in Ref.[22] but with their parameters $V_L, V_R,$ and $g_{L/R}$ being replaced by $f^{SM}_1 + \delta f^L_1$, $f^R_1$, and $-f^{L,R}_{2}$ in our notations. The total decay width is derived as the summation of the partial decay widths under the above assumption on the top-decay channel.
The corresponding $W$-boson helicity fractions have been measured in Tevatron and LHC experiments [40]. In this analysis, we take the following data as our input information [41]

$$F_L^t = 0.298 \pm 0.028\text{(stat.)} \pm 0.032\text{(syst.)},$$
$$F_0^t = 0.720 \pm 0.039\text{(stat.)} \pm 0.037\text{(syst.)},$$
$$F_R^t = -0.018 \pm 0.019\text{(stat.)} \pm 0.011\text{(syst.)},$$

and the total decay width of the top quark [42]

$$\Gamma_t = 1.36 \pm 0.02\text{(stat.)}^{+0.14}_{-0.11}\text{(syst.)}\text{ GeV},$$

(5)

to get constraints on $\delta f^L_1$, $f^R_1$ and $f^L/R_2$.

We are, however, going to utilize the total and partial decay widths instead of using the above $W$-boson helicity fractions directly. This is because the fraction, defined by the ratio of the partial width to the total width, could reproduce experimental results in the case that the numerator (i.e. partial width) and the denominator (i.e. total width) balance each other out, even if they are both out of experimentally-allowed ranges. Therefore, we derive the partial decay widths combining Eq.(4) and Eq.(5) as

$$\Gamma^t_{L^*} = 0.405 \pm 0.072 \text{ GeV},$$
$$\Gamma^t_{0^*} = 0.979 \pm 0.125 \text{ GeV},$$
$$\Gamma^t_{R^*} = -0.024 \pm 0.030 \text{ GeV},$$

(6)

and use them as input data in our analyses

As mentioned, we handle the real and imaginary parts of all the non-standard couplings independently and at the same time, that is, we are going to carry out a full eight-parameter analysis. More specifically, we compare our input data (5) and (6) with the corresponding formulas by varying each parameter in steps of 0.05, and explore the allowed parameter space. We express the results by presenting the maximum and minimum values of each parameter in the following.

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51Since it is not easy to handle an asymmetric error like this in the error propagation, we use $\Gamma^t = 1.36 \pm 0.02\text{(stat.)} \pm 0.14\text{(syst.)}\text{ GeV}$, the one symmetrized by adopting the larger (i.e., +0.14) in this systematic error, in the following calculation.

52The lower value of $\Gamma^t_{R^*}$ is set to be zero in the actual calculation because the decay width should not be a negative quantity.
At first, the result in the case that all the non-standard couplings are independent complex numbers is shown in Table 1. We there find that the constraints on each couplings are not very strong. Thus even if each coupling is large, the experimental data can be reproduced as a result of cancellations among the contributions from some of the couplings. In particular, the constraint on $\delta f_L^1$ is weaker than on the other couplings. It might seem strange that the contribution from the standard-model coupling $f_{1}^{\text{SM}}$ is diminished by its extended coupling $\delta f_L^1$ but we of course cannot get rid of such a possibility.

Table 1: The allowed maximum and minimum values of non-standard-top-decay couplings in the case that all the couplings are dealt with as free parameters.

| $\delta f_L^1$ | $f_1^R$ | $f_2^L$ | $f_2^R$ |
|----------------|---------|---------|---------|
| Re($\delta f_L^1$) | Re($f_1^R$) | Im($f_1^R$) | Re($f_2^L$) | Im($f_2^L$) | Re($f_2^R$) | Im($f_2^R$) |
| Min. | $-2.55$ | $-1.55$ | $-1.30$ | $-1.30$ | $-0.65$ | $-0.65$ | $-1.20$ | $-1.20$ |
| Max. | $0.55$ | $1.55$ | $1.30$ | $1.30$ | $0.65$ | $0.65$ | $1.20$ | $1.20$ |

Having these results, we then have considered the cases where $\text{Re}(\delta f_L^1) = 0$ and also $\text{Re}(\delta f_L^1) = \text{Im}(\delta f_L^1) = 0$, and performed the same estimation for each case. Their results are shown in Table 2 and Table 3. As we see there, if the standard $V-A$ interaction, i.e., the $f_1^{\text{SM}}$ term, is not affected by $\delta f_L^1$, constraints on the remaining couplings get a bit stronger. Moreover, it is remarkable that the allowed region of $\text{Re}(f_2^R)$ has become largely asymmetric and the upper limits are both zero, which seems to indicate that a negative $\text{Re}(f_2^R)$ (in our notation) is favored.

Table 2: The allowed maximum and minimum values of non-standard-top-decay couplings in the case that all the couplings are dealt with as free parameters except for $\text{Re}(\delta f_L^1)$ being set to be zero.

| $\delta f_L^1$ | $f_1^R$ | $f_2^L$ | $f_2^R$ |
|----------------|---------|---------|---------|
| Im($\delta f_L^1$) | Re($f_1^R$) | Im($f_1^R$) | Re($f_2^L$) | Im($f_2^L$) | Re($f_2^R$) | Im($f_2^R$) |
| Min. | $-1.20$ | $-1.10$ | $-1.10$ | $-0.50$ | $-0.55$ | $-0.95$ | $-1.00$ |
| Max. | $1.20$ | $1.05$ | $1.10$ | $0.55$ | $0.55$ | $0.00$ | $1.00$ |

Note that the results have an error of about 0.05 because of our computational precision.
Table 3: The allowed maximum and minimum values of non-standard-top-decay couplings in the case that all the couplings are dealt with as free parameters except for $\text{Re}(\delta f^L_1)$ and $\text{Im}(\delta f^L_1)$ both being set to be zero.

|          | $f^R_1$ | $f^L_2$ | $f^R_2$ |
|----------|---------|---------|---------|
|          | Re($f^R_1$) | Im($f^R_1$) | Re($f^L_2$) | Im($f^L_2$) | Re($f^R_2$) | Im($f^R_2$) |
| Min.     | $-1.10$ | $-1.10$ | $-0.50$ | $-0.55$ | $-0.95$ | $-0.45$ |
| Max.     | $1.05$ | $1.10$ | $0.55$ | $0.55$ | $0.00$ | $0.45$ |

Some comments should be mentioned on what we have obtained: The above asymmetric result is not surprising because $\text{Re}(f^R_2)$ produces the only term which can interfere with the standard-model coupling even when the $b$-quark is treated as a massless particle, that is, we have a term proportional to this coupling in $\Gamma_{t,0,R}^L$. Therefore, the sign of $\text{Re}(f^R_2)$, if any, could be determined from the measurable decay widths and/or $W$-boson helicity fractions in the near future. On the other hand, let us not forget that an error around 0.05 is included in our calculations, concerning the upper (and lower) bound. Finally, all the allowed parameter spaces contain the standard-model solution, i.e., $\delta f^L_1 = f^R_1 = f^L_2 = 0$, which means there is no new-physics signal yet in the quantities studied here.\[4\]

To summarize, we have studied possible non-standard $tbW$ interactions and found that the present data are consistent with the standard-model predictions but there is some non-negligible space left for possible non-standard couplings, too. We have derived the maximum and minimum values of those couplings allowed by the present experimental data of the total and partial decay widths by varying all the couplings independently at the same time.

To be more specific, the conceivable non-standard-top-decay couplings are classified into eight types if we treat all the coupling constants as complex numbers. In that case, the allowed regions of those couplings are not that small yet because cancellations could happen between the contributions originated from those couplings. On the other hand, if we assume that $f^L_1$ does not include any non-standard contribution, the resultant constraints on the other non-standard couplings, espe-

\[4\] Some non-vanishing contributions to these parameters are also made via standard-model radiative corrections, see [43] [44].
cially $f_2^R$, become a bit stronger, although their allowed ranges are not such tiny
that we can drop their quadratic terms easily. These results tell us that we should
be very careful when taking the linear approximation on those non-standard $tbW$
couplings.

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