Predicting the masses of baryons containing one or two heavy quarks

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The Feynman-Hellmann theorem and semiempirical mass formulas are used to predict the masses of baryons containing one or two heavy quarks. In particular, the mass of the $\Lambda_b$ is predicted to be $5620 \pm 40$ MeV, a value consistent with measurements.

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In a recent paper [1], the Feynman-Hellmann theorem [2,3] and semiempirical mass formulas [4,5,1] were used as tools enabling the prediction of the masses of a
number of as yet undiscovered mesons and baryons. The major point of the paper [1] is that one can exploit regularities in the pattern of known hadron masses to obtain estimates of the masses of unknown hadrons. Although results are obtained within the framework of the constituent quark model, no explicit Hamiltonian is used. Therefore, the results given in Ref. [1] are complementary to calculations which use specific potentials between quarks to calculate hadron masses.

In this brief report, we extend the considerations of Ref. [1] in order to (A) predict the mass of the $\Lambda_b$, (B) make slightly revised predictions of masses of other baryons containing one heavy quark, and (C) predict masses of baryons containing two heavy quarks. A number of authors have recently considered baryons [6–12] containing two heavy quarks in anticipation of future experiments which may discover these particles. Again, our work is complementary to that which uses a specific model, and provides predictions which do not depend upon any explicit Hamiltonian.

In Ref. [1], it was shown that, in the constituent quark model, the Feynman-Hellmann theorem can be applied to give useful information about the masses of mesons and baryons even in systems with relativistic kinematics. Under plausible assumptions, application of the Feynman-Hellmann theorem leads to the conclusion that the energy eigenvalues (excluding quark masses) of certain mesons and baryons are smooth, monotonically decreasing functions of the parameter $\mu$, given by

$$\mu^{-1} = \sum m_i^{-1},$$

where the $m_i$ are the masses of the constituent quarks. These results were found to hold for ground-state spin-1 mesons and spin-$\frac{3}{2}$ baryons even in the presence of the colormagnetic (spin-spin) interaction between pairs of quarks. On the other hand, the eigenenergies of spin-0 mesons and spin-$\frac{1}{2}$ baryons do not necessarily
decrease monotonically with increasing $\mu$.

The relation between the eigenenergy $E(12...)$ of a hadron containing quarks $q_1, q_2, ...$ and its mass $M(12...3)$ is

$$M(12...) = E(12...) + \sum_i m_i.$$  \hspace{1cm} (2)

The Feynman-Hellmann theorem is useful in allowing us to obtain estimates of $E(12...)$, not $M(12...)$. Therefore, we can obtain predictions about hadron masses $M(12...)$ from Eq. (2) only if we assume values for the quark masses.

However, we are not allowed to assume arbitrary values for the quark masses, as we can obtain constraints on the quark mass differences from the Feynman-Hellmann theorem. It was shown in Ref. [1] that the quark mass differences must satisfy the inequalities

$$m_s - m_q > M(K^*) - M(\rho) = 126 \pm 4 \text{ MeV},$$  \hspace{1cm} (3)

$$m_c - m_s > M(D^*) - M(K^*) = 1115 \pm 4 \text{ MeV},$$  \hspace{1cm} (4)

$$m_b - m_c > M(B^*) - M(D^*) = 3316 \pm 6 \text{ MeV},$$  \hspace{1cm} (5)

where the numbers in (3–5) come from the experimental values of the vector meson masses as given by the Particle Data Group [13].

The derivation of (3–5) was given in Ref. [1], but we briefly repeat it here for (3). From Eq. (2), we have

$$M(K^*) = m_q + m_s + E(qs)$$  \hspace{1cm} (6)

and

$$M(\rho) = 2m_q + E(qq).$$  \hspace{1cm} (7)

Subtracting (7) from (6) and noting from the Feynman-Hellmann theorem that $E(qs) < E(qq)$, we get the inequality in (3).
We can obtain stronger inequalities from spin-$\frac{3}{2}$ baryons in an analogous manner to the derivation of the inequality (3). In Ref. [1], the following inequality was derived:

$$m_s - m_q > M(\Sigma^*) - M(\Delta) = 153 \pm 4 \text{ MeV}, \quad (8)$$

where the numbers are from baryon masses given in Ref. [13]. This inequality for $m_s - m_q$ is stronger than the inequality given in (3). However, the analogous inequalities

$$m_c - m_s > M(\Sigma_c^*) - M(\Sigma^*) \quad (9)$$

and

$$m_b - m_c > M(\Sigma_b^*) - M(\Sigma_c^*) \quad (10)$$
do not immediately help us because the masses of the baryons $\Sigma_c^*$ and $\Sigma_b^*$ are not known experimentally.

However, we can obtain inequalities for $m_c - m_s$ and $m_b - m_c$ in MeV from (9) and (10) if we obtain theoretical estimates of the masses of $\Sigma_c^*$ and $\Sigma_b^*$. We do this using a semiempirical mass formula for the spin splittings among baryons containing a given quark content [1]. Using the known masses [13] of the $\Lambda_c$, $\Sigma_c$, and $\Lambda_b$ as input, we get from the semiempirical formula $M(\Sigma_c^*) = 2525 \text{ MeV}$ and $M(\Sigma_b^*) = 5855 \text{ MeV}$. Then we obtain the inequalities

$$m_c - m_s > M(\Sigma_c^*) - M(\Sigma^*) = 1140 \pm 20 \text{ MeV}, \quad (11)$$

$$m_b - m_c > M(\Sigma_b^*) - M(\Sigma_c^*) = 3340 \pm 60 \text{ MeV}, \quad (12)$$

where the errors are larger than for the previous inequalities because of uncertainties in the semiempirical mass formula [1] and in the experimental value of the mass of the $\Lambda_b$ [13].

In Ref. [1], the following quark masses were used for the discussion of baryons:

$$m_q = 300 \text{ MeV}, \ m_s = 475 \text{ MeV}, \ m_c = 1640 \text{ MeV}, \ \text{and} \ m_b = 4990 \text{ MeV}. \ \text{We}$$
see that these masses satisfy the inequalities (8), (11), and (12), and therefore are potentially good candidates for use with the Feynman-Hellmann theorem to enable us to obtain estimates of the masses of baryons with two heavy quarks. For a detailed discussion of how these quark masses were arrived at, see Ref. [1]. We use the same masses in this paper except that we take the mass of the $b$ quark to be 4985 MeV, five MeV lower than the value used in Ref. [1]. Thus, we use the quark mass values (in MeV):

$$m_q = 300, \quad m_s = 475, \quad m_c = 1640, \quad m_b = 4985.$$  \hspace{1cm} (13)

The small (5 MeV) difference in the value of $m_b$ comes about as follows: In Ref. [1], $m_b$ was obtained with the mass of the $\Lambda_b$ as input. However, the value of the $\Lambda_b$ mass, as given in the baryon table of the Particle Data Group [13], has the rather large error of $\pm 50$ MeV. Therefore, in this paper we choose not to use the $\Lambda_b$ mass as input but to obtain the value of $m_b$ from the vector mesons. This revised procedure allows us to predict the value of the $\Lambda_b$ mass. We use as input the quark masses $m_q = 300$ MeV, $m_s = 475$ MeV, and $m_c = 1640$ MeV as found in Ref. [1] for the baryons. We also use as input the observed masses [13] of the ground-state vector mesons $\rho, K^*, \phi, D^*, D_s^*, B^*, B_s^*, J/\psi,$ and $\Upsilon$. In addition, we use as input the predicted mass of the $B_c^*$, which in Ref. [1] was found to be 6320 MeV. Then, applying the Feynman-Hellman theorem as in Ref. [1], we fit the masses of the vector mesons with a monotonically decreasing function of the reduced mass $\mu$. The specific functional form has no theoretical significance. In practice, we use a three-parameter exponential curve similar to that used in Ref. [1], but with different parameters, as the parameters depend on the input quark masses. We vary the three parameters of the curve and the $b$ quark mass so as to get a best fit to the data. This procedure yields $m_b = 4985$ MeV (rounded to the nearest 5 MeV). We show the fit to the meson eigenenergies in Fig. 1.
We obtain predictions for unobserved spin-$\frac{3}{2}$ baryon masses as follows: We first use as input the observed masses $\Delta, \Sigma^*, \Xi^*$, and $\Omega$. Then we use the semiempirical mass formula of Ref. [1] (with slightly changed parameters because the parameters depend on the input quark masses) to obtain the masses of the $\Sigma_c^*$ and $\Xi_c^*$, using the observed masses of the $\Lambda_c, \Sigma_c, \Xi_c$ as input. We next subtract the quark masses of Eq. (13) from the masses of the spin-$\frac{3}{2}$ baryons $\Delta, \Sigma, \Xi, \Omega, \Sigma_c^*, \Xi_c^*$ [see Eq. (2)] to obtain the corresponding energy eigenvalues. We then fit a monotonically decreasing three-parameter exponential curve through these energies. The parameters of this curve are slightly different from the parameters used to fit the baryon eigenenergies in Ref. [1] because we take the $b$ quark mass to be 5 MeV less than in Ref. [1]. We then extrapolate the curve to give us the energy eigenvalues of unknown spin-$\frac{3}{2}$ baryons including the $\Sigma_b, \Omega_c^*$, and baryons containing two heavy quarks. We show this extrapolated curve in Fig. 2. Because we have to extrapolate rather a long way in $\mu$, we assign errors of as much as 100 MeV to the eigenenergies.

The next step is to add back the quark masses to obtain the masses of these spin-$\frac{3}{2}$ baryons. Lastly, we use the semiempirical mass formula to obtain the masses of spin-$\frac{1}{2}$ baryons, including the $\Lambda_b, \Sigma_b, \Omega_c$, and baryons with two heavy quarks. In Table I we give the masses of baryons containing one or two heavy quarks. Masses of known baryons used as input are marked with $^a$, and masses already predicted in Ref. [1] are marked with $^b$. In some of the cases marked with $^b$, our present predictions differ by 10 MeV from those given in Ref. [1]. These differences are well within our estimated errors.

Note from Table I that we predict that the mass of the $\Lambda_b$ is $5620 \pm 40$ MeV. (In Ref. [1] the observed mass of the $\Lambda_b$ was used as input.) We also predict in Table I that the mass of the $\Omega_c$ is $2710 \pm 30$ MeV. (This is the same mass found in
Ref. [1], but here the error is smaller.) Both these masses are consistent with the values $5641 \pm 50$ MeV and $2710 \pm 5$ MeV respectively given by the Particle Data Group [13].

The predicted baryon masses given in Table I satisfy an inequality derived by Bagan et al. [14], which says that if quarks 1, 2, and 3 are ordered according to increasing mass, then

$$M(123) \leq M(113) + M(112) - M(111). \tag{14}$$

If we compare the masses of baryons with two heavy quarks given in Table I with masses calculated with a potential model [12], we find that our predicted masses are from 20 to 130 MeV larger.

Before ending this paper, we say a few words about meson masses. In Ref. [1], in order to obtain best fits to the data on mesons and baryons separately, different input quark masses were used for mesons and baryons. The quark masses used for mesons were (in MeV):

$$m_q = 300, \quad m_s = 440, \quad m_c = 1590, \quad m_b = 4920. \tag{15}$$

Comparing these masses with the ones in Eq. (13), we see that the differences range from 0 (for $m_q$) to 65 MeV (for $m_b$). Although there is no good theoretical reason why effective constituent quark masses need to be the same in mesons and baryons, it is economical in parameters to be able to use the same masses.

We immediately see that we cannot use the quark masses of Eq. (15) for the baryons, as these mass differences do not satisfy the baryon inequality (8). In confirmation, we have verified numerically that we obtain poor agreement with known baryon masses when using the quark mass set of Eq. (15). We also find that using an average of the quark masses of Eqs. (13) and (15) is unsatisfactory,
although these average masses do satisfy all the baryon inequalities (8), (11), and (12). However, as we see from Fig. 1, if we use the quark masses of Eq. (13) as input, we get reasonable agreement with the experimental vector meson masses [although the fit is not statistically as good as with the masses of Eq. (15)]. As we have stated, we used the predicted mass of the $B_c^*$ from Ref. [1] as input. However, all other predictions of the masses of as yet undiscovered mesons are the same as those given in Ref. [1] within the errors given in that paper. We conclude that the quark masses of Eq. (13) are suitable to use for both mesons and baryons in applications involving the Feynman-Hellmann theorem.

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TABLE I. Masses of baryons containing one or two heavy quarks. In column 3 we show predictions for ground-state spin-1/2 baryons (denoted by $M_A$ in the first row of this column) whose first two quarks have an antisymmetric spin wave function. In column 4 we show predictions for ground-state spin-1/2 baryons (denoted by $M_S$) with symmetric spin wave functions in the first two quarks. In column 5 we show predictions for the ground-state spin-3/2 baryons (denoted by $M^*$).

| Name                  | Quark content | $M_A$ (MeV)    | $M_S$ (MeV)    | $M^*$ (MeV)  |
|-----------------------|---------------|----------------|----------------|--------------|
| $\Lambda_c, \Sigma_c, \Sigma^*_c$ | $qqc$         | 2285 ± 1$^a$   | 2453 ± 3$^a$   | 2520 ± 20    |
| $\Xi_c, \Xi'_c, \Xi^*_c$ | $qsc$         | 2468 ± 3$^a$   | 2580 ± 20      | 2650 ± 20    |
| $\Omega_c, \Omega^*_c$ | $ssc$         | —              | 2710 ± 30$^b,c$| 2770 ± 30$^b$|
| $\Lambda_b, \Sigma_b, \Sigma^*_b$ | $qqb$         | 5620 ± 40      | 5820 ± 40$^b$ | 5850 ± 40$^b$|
| $\Xi_b, \Xi'_b, \Xi^*_b$ | $qsb$         | 5810 ± 40$^b$ | 5950 ± 40$^b$ | 5980 ± 40$^b$|
| $\Omega_b, \Omega^*_b$ | $ssb$         | —              | 6060 ± 50$^b$ | 6090 ± 50$^b$|
| $\Xi_{cc}, \Xi^*_{cc}$ | $ccq$         | —              | 3660 ± 70      | 3740 ± 70    |
| $\Omega_{cc}, \Omega^*_{cc}$ | $ccs$         | —              | 3740 ± 80      | 3820 ± 80    |
| $\Xi_{cb}, \Xi'_{cb}, \Xi^*_{cb}$ | $qcb$         | 6990 ± 90      | 7040 ± 90      | 7060 ± 90    |
| $\Omega_{cb}, \Omega'_{cb}, \Omega^*_{cb}$ | $scb$         | 7060 ± 90      | 7090 ± 90      | 7120 ± 90    |
| $\Xi_{bb}, \Xi^*_{bb}$ | $bbq$         | —              | 10340 ± 100    | 10370 ± 100  |
| $\Omega_{bb}, \Omega^*_{bb}$ | $bbs$         | —              | 10370 ± 100    | 10400 ± 100  |

$^a$Masses of known baryons used as input.

$^b$Masses also predicted in Ref. [1]. The masses in this table sometimes differ
from the masses of Ref. [1] by up to 10 MeV because of slightly different input parameters. These differences are well within our estimated errors.

This prediction agrees with the value $2710 \pm 5$ MeV given in the full listings of the Particle Data Group [13]. We did not use the Particle Data Group value as input because the $\Omega_c$ was omitted from their summary baryon table.

Figure captions

FIG. 1. Fit to the energy eigenvalues of vector mesons with an exponential curve. Open circles denote mesons whose eigenergies are obtained from experiment by subtracting the quark masses of Eq. (13), and the solid circle denotes an eigenergy obtained from the mass of the $B^*_c$ predicted in Ref. [1].

FIG. 2. Energy eigenvalues of baryons with spin $\frac{3}{2}$. Open circles denote baryons whose eigenergies are obtained from experiment [13] by subtracting the quark masses of Eq. (13), triangles denote eigenergies obtained with the aid of the baryon semiempirical mass formula of Ref. [1], and solid circles denote predicted eigenergies obtained from an exponential curve by extrapolation.
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