iPOF: An Extremely and Excitingly Simple Outlier Detection Booster via Infinite Propagation

Sibo Zhu  
Brandeis University  
Waltham, MA, USA  
siboz@brandeis.edu

Handong Zhao  
Adobe Research  
San Jose, CA, USA  
hazhao@adobe.com

Hongfu Liu  
Brandeis University  
Waltham, MA, USA  
hongfuliu@brandeis.edu

ABSTRACT
Outlier detection is one of the most popular and continuously rising topics in the data mining field due to its crucial academic value and extensive industrial applications. Among different settings, unsupervised outlier detection is the most challenging and practical one, which attracts tremendous efforts from diverse perspectives. In this paper, we consider the score-based outlier detection category and point out that the performance of current outlier detection algorithms might be further boosted by score propagation. Specifically, we propose Infinite Propagation of Outlier Factor (iPOF) algorithm, an extremely and excitingly simple outlier detection booster via infinite propagation. By employing score-based outlier detectors for initialization, iPOF updates each data point’s outlier score by averaging the outlier factors of its nearest common neighbors. Extensive experimental results on numerous datasets in various domains demonstrate the effectiveness and efficiency of iPOF significantly over several classical and recent state-of-the-art methods. We also provide the parameter analysis on the number of neighbors, the unique parameter in iPOF, and different initial outlier detectors for general validation. It is worthy to note that iPOF brings in positive improvements ranging from 2% to 46% on the average level, and in some cases, iPOF boosts the performance over 3000% over the original outlier detection algorithm.

CCS CONCEPTS
• Information systems → Data mining; • Computing methodologies → Anomaly detection: Unsupervised outlier detection.

KEYWORDS
Outlier detection; K-NN; Score propagation

1 INTRODUCTION
Outlier detection, also known as anomaly detection, aims to identify the minority of data points with divergent characters from the majority. Due to its industrial value, there are numerous widely real-world applications of outlier detection, including credit card fraud, network intrusion, precision marketing, gene mutation, and so on. As an active research area, tremendous exploration has been taken to thrive outlier detection area with multiple practical settings to meet different real-world scenarios including supervised 1-3, unsupervised 4-6, even knowledge transferred settings 7-9.

Among the above settings, unsupervised outlier detection is the most challenging one due to the nature of no external guidance information. Many algorithms have been proposed based on the various assumptions on the differences between inliers and outliers. Generally speaking, these algorithms can be roughly divided into crisp or soft categories. The methods in the crisp category explicitly provide the outlier membership with zero or one indicators. The representative method is K-means++ 10 with user pre-defined $\sigma$ and $K$ denoting the numbers of outliers and clusters. It detects outliers and partitions the rest points into $K$ clusters, where the instances with large distances to the nearest centroid are regarded as outliers during the clustering process. COR 11, a variant of K-means++, conducts joint clustering and outlier detection in the partition space. Differently, the soft category calculates a continuous score for each data point as the degree of outlierness, where the high value of a data point indicates the high probability of being an outlier. Then top-$K$ data points with the largest scores are regarded as outlier candidates. In this category, various methods are put forward from different assumptions or aspects, including density-based LOF 12, COF 13, distance-based LODF 14, angle-based FABOD 15, ensemble-based isolation Forest (iForest) 16, eigenvector-based OPCA 17, cluster-based TONMF 18, deep learning-based anomalous event detection 19, and so on. More details on outlier detection can be found in the surveys 20-22.

In this paper, we consider the score-based unsupervised outlier detection category, where the Local Outlier Factor (LOF) is one of the most popular approaches. LOF measures the local deviation of a target data point with respect to its neighbors. In other words, the outlier score of the target data point in LOF depends on the local density of $K$ nearest neighbors, rather than the density of the target data point. From this view, we regard LOF as a one-round outlier factor propagation based on local neighborhood structure. It is interesting to see whether multi-round propagation will increase the gap between inliers and outliers, the final score will converge with infinite propagation, and the propagation will benefit other outlier detectors. In light of this, we propose infinite Propagation of Outlier Factor (iPOF) to address the above questions. iPOF is based on local neighborhood structure and assumes that inliers’ friends are more likely to be inliers, and inliers refuse to make friends with outliers. Here the friendship is defined based on the common neighbour. In Figure 2, $B$ and $D$ are the nearest neighbors of $F$; however, $B$ and $D$’s nearest neighbors do not include $F$, which means that $B$ and $D$ do not want to make friends with $F$. In light of this, we build a common neighborhood graph based on the local structure for the outlierness propagation. iPOF utilizes the outlier scores initialized by some outlier detector, such as LOF or other outlier detectors, and iteratively updates the outlier scores by averaging the scores of local neighbors. As a post-processing technique, iPOF can further boost the existing score-based outlier detectors via averaging propagation until convergence. Honestly, iPOF is extremely and excitingly simple, but quite effective. Extensive experiments demonstrate that iPOF enhances several outlier detectors.
where a bunch of labeled inlier and outlier samples is employed to

The major contributions of iPOF are summarized as follows:

- We propose infinite Propagation of Outlier Factor (iPOF), a performance-boosting technique for existing score-based outlier detection algorithms. iPOF employs the local neighborhood structure to propagate the outlier scores to converge the scores of outliers and inliers for better distinguishing.

- Technically, iPOF only requests nearest neighbor calculation and averaging, where the nearest neighbor information can be re-used from the density- or distance-based initial outlier detectors. Hence, iPOF is extremely and excitingly simple, time- & space-efficient and easy to implement in parallel.

- Experiments on 17 real-world datasets in diverse domains demonstrate the effectiveness of iPOF over numerous classical and recent outlier detectors. In some cases, iPOF brings in over 3000% improvements over the original detector.

The rest of this paper is organized as follows. Section 2 introduces the related work of outlier detection in terms of supervised, semi-supervised and unsupervised setting. Section 3 provides an illustrative example to deliver the idea of iPOF, and the core principle and algorithm of iPOF is elaborated in Section 4. We conduct comprehensive experiments on algorithmic comparison with other outlier detection algorithms and in-depth factor exploration in Section 5. Finally, Section 6 draws conclusion to the whole paper.

2 RELATED WORK

In this section, we introduce the related work of outlier detection according to the label availability and highlight the differences between existing works and our proposed iPOF.

Supervised Outlier Detection. With enough labeled data, supervised outlier detection is a binary classification problem in essence, where a bunch of labeled inlier and outlier samples is employed to build a predictive model for new sample prediction. Different from the conventional classification problem, the challenge of supervised outlier detection lies in the imbalance of inlier and outlier class sizes. Many balancing techniques including sampling, re-weighting, deep representation learning can be used to tackle the supervised outlier detection.

Semi-supervised Outlier Detection. Semi-supervised learning uses both unlabeled and labeled data for particular tasks. It is worthy to note that only inlier samples are available in the training data in the setting of semi-supervised outlier detection. The most representative algorithm is one-class SVM, which seeks several support vectors to describe the boundary of the inlier class for the unseen new sample prediction. This idea is extended to the kernel version, and deep representation as well.

Unsupervised Outlier Detection. Among the diverse settings of outlier detection, the unsupervised scenario is the most challenging and practical one, that cannot be solved by conventional classification techniques. Tremendous and sustaining efforts have been devoted to making unsupervised outlier detection rich and interesting. Generally speaking, the existing algorithms in this category can be divided into two groups by the type of outlier scores. K-means and One-Class SVM provide the binary scores for samples to identify outlier candidates, while most other detectors calculate a continuous score to denote the degree of outlierness. Based on different assumptions, numerous score-based outlier detectors have been proposed including density-based DBSCAN, Local Outlier Factor (LOF), Connectivity-based Outlier Factor (COF), distance-based Local Distance-based Outlier Factor (LODF), frequent pattern-based Fp-outlier, angle-based ABOD. Moreover, some studies pursue outlier detection by representation learning as well from the subspace, low-rank and matrix-completion, respectively. Since the above basic outlier detectors are highly assumption-dependent, some ensemble outlier detector algorithms are put forward to alleviate the negative impact of assumption including ensemble-based iForest, Bi-Sampling Outlier Detection (BSOD), Feature Bagging, Locally Selective Combination of Parallel Outlier Ensembles (LSCP), Combination of outlier detectors by taking the median of the scores (Median), Combination of outlier detectors by taking the Average of Maximum (AOM) and Maximum of Average (MOA). Due to the superior performance of supervised learning over unsupervised learning, some studies transform the unsupervised outlier detection problem into a classification problem by artificially generating potential outliers. Typically, these algorithms first generate a set of potential outliers and treat them as labels. Then a classifier is trained for subsequent detection. Among them, the one-class classification method generates informative potential outliers based on the probability density function of the real data. The distribution-based artificial anomaly method exploits a heuristic to randomly change the value of one feature of an object by assuming that the boundary may be very close to the existing data. One-Class Random Forests makes use of classifier ensemble randomization principles for outlier generation procedure. Recently there is increasing attention focusing on generating fake outliers by min-max training with generative adversarial networks.

Different from the existing literature in the outlier detector area, our iPOF approach is a post-processing technique for boosting score-based outlier detector methods, rather than an outlier detector. iPOF is based on local neighborhood structure and assumes that an object is more likely to be an outlier if its neighbors are outliers as well, while an object is an inlier if it is surrounded by inliers. By literately averaging the outlier scores with neighbors, iPOF converges the scores of outliers and inliers for better distinguishing, hence increases the performance of unsupervised outlier detection.
Figure 2: iPOF on a 2D synthetic dataset. (a) shows the data distribution with three different density Gaussian clusters of 500 data points each and 150 outliers samples that are randomly landed out of these clusters; (b) shows the outlier score trend with respect to the iteration of iPOF on several inliers and one outlier that highlighted by different colorful circles in (a); and (c) demonstrates the AUC scores of iPOF during iterations, where iPOF initializes by LOF and increases the AUC score from 0.6709 to 0.9294. The improvement of iPOF over LOF on this synthetic dataset is up to 38%.

3 MOTIVATION

Unsupervised outlier detection employs some criterion to seek outlier candidates deviation from major normal points without external supervision. Usually, each data point gets a score calculated to identify the degree of outlierness, where top K points with the largest scores are regarded as outlier candidates. Tremendous efforts have been made in the outlier detection area based on different assumptions, including density, distribution, clustering, angle, and so on. Among these studies, Local Outlier Factor (LOF) is one of the most popular density-based methods, where outliers are identified by comparing the local density of the data point and its neighbors [4]. Then many variants including Connectivity-based Outlier Factor (COF) [45], Local Distance-based Outlier Factor (LODF) [52] follow to further improve the performance of LOF.

Here we take LOF as an example for further analysis that measures the local deviation of a target data point with respect to its neighbors. The outlier score of the target data point in LOF depends on the local density of K nearest neighbors, rather than the density of the target data point. From this view, we regard LOF as a one-round outlier factor propagation based on local neighborhood structure. It is interesting to see whether multi-round propagation will increase the gap between inliers and outliers, and the final score will converge with infinite propagation. In this paper, we are trying to answer the questions of whether the performance of these outlier detectors can be further improved via a simple average propagation strategy based on local neighborhood structure.

To validate the above conjecture, we conduct the experiments on a synthetic dataset. Figure 2 shows the data distribution, outlier scores of several selected data points, and the performance during the propagation of iPOF. The 2D dataset contains three Gaussian clusters of 500 objects, each with different densities and 150 outliers samples that are randomly landed out of these clusters. From Figure 2(b) the original outlier scores by LOF at the 0-th iteration mix the inlier and outlier samples. That is the intrinsic reason that LOF cannot well tackle this synthetic dataset. Instead, our iPOF method initializes with LOF value for each data point and propagates the scores through local neighborhood structure, which increases not only the gap between inliers and outliers but also narrows the score divergence of inliers from the same group. More technical details and properties will be provided in Section 4. There are two clear observations to validate the above point in Figure 2(b): (1) The gap among the red curve and other curves is enlarged along with the iteration. (2) The curves with the same color denoting the inliers from the same cluster converge together even they have different initial outlier scores. For instance, two green curves present two inliers in the same cluster, where one is the high-density central area, another is located in the boundary of the cluster with relatively low density. Through the propagation of iPOF, their final scores achieve the same. Finally, Figure 2(c) demonstrates the AUC performance of iPOF. It is appealing to see that initialized by LOF, iPOF increases the AUC score from 0.6709 to 0.9294, bringing in over 38% improvements on this synthetic dataset.

The illustrative example demonstrated in Figure 2 shows that LOF can further gain significant improvements via simple local neighborhood average propagation. It is worthy to note that the local neighborhood structure is pre-calculated through the LOF, which indicates that our iPOF method boosts the performance only via iterative averaging. In the next section, we will provide more technical details and properties of our proposed iPOF method.
4 METHODOLOGY

Generally speaking, outlier detectors predict outlier scores under various assumptions in terms of density, distribution, clustering, angle, and so on. Our proposed infinite Propagation of Outlier Factor (iPOF) based on local neighborhood structure, assuming that inliers’ friends are more likely to be inliers, and inliers refuse to make friends with outliers; thus, outliers’ friends are more likely to be outliers. Here the friendship is defined based on the common nearest neighbour, which means that the direct neighborhood relationship is not friendship.

Before illustrating the algorithm of iPOF, we first introduce the concept of common neighbor set with an illustrative example. Figure 1 shows a toy example of common neighbor with six blue inlier nodes and one red outlier node, where \( k = 2 \) denotes that we calculate 2 nearest neighbors for every node. The directed edges denote the common neighborhood relationship. For instance, the node \( B \) and \( D \) are 2 nearest neighbors of \( F \), which indicates the edges are directed from \( F \) to \( B \) and \( D \). The common neighbor set \( N \) of a target data point includes all the nodes pointing to the target one. Here \( k \) is used to construct the network and does not mean that there are at most \( k \) elements in the set. Given the fixed network derived by the local neighbors, iPOF iteratively updates the outlier score of each data point via simply averaging its top \( K \) points scores in its common neighbor sets. In the case of Figure 1, we first initialize the outlier score of each node by some outlier detectors, which leads that node \( F \) has a relatively high score, and others have relatively low scores. Then we apply iPOF to propagate the outlier scores. Note that the in-degree of node \( F \) is zero, which means its score will not get affected by other inlier nodes during the propagation. Instead, there exist several paths among these blue inlier nodes. With proper propagation, the outlier scores of these inliers will converge to the same value. Note that \( k \) denotes the number of neighbors, while \( K \) is the number of common neighbors for the propagation in iPOF. Note that \( k < K \) and \( k \) can be relatively large to incorporate more common neighbors, while \( K \) is the key parameter to control the propagation. Table 1 shows the notations and descriptions used in this paper.

Given a dataset containing \( n \) data points \( x_l, 1 \leq l \leq n \), we first initialize the outlier score \( s_l^{(0)} \) of each data point via some existing outlier detector. Then, we calculate the \( k \)-nearest neighbors based on the original data features for each data point and obtain the common neighbor sets, based on which our proposed iPOF propagates the outlier scores with top-\( K \) points in the common neighbor sets as follows:

\[
s_j^{(t+1)} = \left( \sum_{j \in N_K(x_l)} s_j^{(t)} + s_j^{(t)} \right) / |N_K(x_l)| + 1,
\]

where \( N_K(x_l) \) includes top-\( K \) points in the common neighbor set of \( x_l \), \( t \) is the index of iteration. The outlier score of each point is iteratively updated by averaging the scores of common neighbors until convergence of all data points’ scores. Therefore, the core idea of iPOF is the local average to converge outlier and inlier scores so that, in general, the outliers have high scores and inliers have low scores during the propagation. By this means, the gap between inliers and outliers can be further improved to distinguish them. The network constructed by local common neighbors naturally determines the outlier score flow among these data points and makes the outlier scores along a path equal when iPOF achieves convergence. By choosing top-\( K \) common neighbors, iPOF splits up the network into several connected communities and the outliers. In general, iPOF is extremely and excitationly simple. We summarize the whole process of iPOF in Algorithm 1. For convergence, it is easy to understand that iPOF is guaranteed to converge with any initialization, when the data points in a same connected community have the same score.

5 EXPERIMENTAL RESULTS

In this section, we provide comprehensive experiments of iPOF, including algorithmic performance and in-depth exploration. We compare iPOF with 12 classical and recent unsupervised outlier detectors on 17 datasets in terms of quantitative performance and execution time. We also provide the parameter analysis on the number of neighbors and different initial outlier detectors in iPOF.

5.1 Experimental Settings

Datasets. To fully evaluate our iPOF algorithm, numerous data sets in different domains are employed. They include the hospital patients data such as BreastW, Arrhythmia, Cardio, Pima, and Ver tebral; handwritten digits data like Mnist, Optdigits, and Pendigits; also satellite image data like Satellite and Satimage-2, and other multivariate data. These data sets are used in [8, 9, 15, 21, 27, 35, 37].

| Algorithm 1: Infinite Propagation of Outlier Factor (iPOF) |
|---|
| **Input:** Data points \( x_l, 1 \leq l \leq n; \) Number of common neighbors for propagation \( K \).  |
| **Output:** Outlier scores: \( s_l^{(t)} \), \( 1 \leq l \leq n \). |
| 1. Initialize the outlier score \( s_l^{(0)} \) for each data point with some outlier detector;  |
| 2. Build \( K \)-nearest common neighbor graph;  |
| 3. Set \( t = 0 \);  |
| 4. while not converge do  |
| \( t = t + 1 \);  |
| For \( x_l \), update the outlier score \( s_l^{(t)} \) by Eq. (1);  |
| end |

Table 2: Characteristics of outlier detection datasets

| Dataset | #Size | #Dim | #Outlier | Outlier% | Outlier object |
|---|---|---|---|---|---|
| Arrhythmia | 452 | 274 | 66 | 14.6% | Affected patients |
| BreastW | 683 | 9 | 239 | 35.0% | Malignant cases |
| Cardio | 1831 | 21 | 176 | 9.6% | Patients |
| Glass | 214 | 9 | 9 | 4.2% | Glass identification |
| Ionosphere | 351 | 33 | 126 | 35.9% | Radar data |
| Mammography | 11183 | 6 | 260 | 2.3% | Calcification classes |
| Mnist | 7603 | 100 | 700 | 9.2% | Handwritten digits |
| Optdigits | 5126 | 64 | 150 | 2.9% | Handwritten digits |
| Pendigits | 6870 | 16 | 156 | 2.2% | Handwritten digits |
| Pima | 768 | 8 | 268 | 34.9% | Patients |
| Satellite | 6435 | 36 | 2036 | 31.6% | Satellite images |
| Satimage-2 | 214 | 9 | 9 | 4.2% | Satellite images |
| Shuttle* | 4909 | 9 | 351 | 7.1% | Starfish points |
| Speech | 3686 | 400 | 61 | 1.7% | Accents |
| Vertebral | 240 | 6 | 30 | 12.5% | Patients |
| Vowels | 1456 | 12 | 50 | 3.4% | Speakers |
| Wine | 129 | 13 | 10 | 7.8% | Wine |

Note: * means that we randomly sample 10% data points from the whole set.
Table 3: AUC results of iPOF and other detectors on 17 datasets

| Datasets (outliers%) | iPOF   | LOF    | COF | FABOD | SOD  | MGAAL  | SGAAL  | iForest | LSCP | FB    | Median | AOM  | MOA  |
|---------------------|--------|--------|-----|-------|------|--------|--------|---------|------|-------|--------|------|------|
| Arrhythmia (15%)    | 0.6284 | 0.7319 | 0.7094 | 0.2315 | 0.7201 | 0.5000 | 0.5000 | 0.3267 | 0.5975 | 0.7953 | 0.7744 | 0.7699 | 0.7714 |
| BreastW (35%)       | 0.5481 | 0.4106 | 0.4666 | N/A   | 0.9374 | 0.9883 | 0.9882 | 0.0150 | 0.5479 | 0.2358 | 0.6606 | 0.5095 | 0.4943 |
| Cardio (9.6%)       | 0.6564 | 0.5968 | 0.5641 | 0.4126 | 0.5617 | 0.3778 | 0.3746 | 0.2396 | 0.5441 | 0.5966 | 0.8660 | 0.9029 | 0.8905 |
| Glass (4.2%)        | 0.9222 | 0.7827 | 0.7572 | 0.1463 | 0.7702 | 0.7035 | 0.7038 | 0.3707 | 0.5268 | 0.8244 | 0.7089 | 0.7415 | 0.7881 |
| Ionosphere (36%)    | 0.8859 | 0.9023 | 0.9103 | 0.0720 | 0.8948 | 0.4270 | 0.4273 | 0.1549 | 0.6327 | 0.8758 | 0.9193 | 0.9038 | 0.8972 |
| Mammaryography (3.32%) | 0.8575 | 0.6709 | 0.6863 | N/A   | 0.7977 | 0.7380 | 0.7429 | 0.3072 | 0.7189 | 0.8099 | 0.8229 | 0.8259 |
| Mushroom (9.2%)     | 0.9232 | 0.5824 | 0.5543 | 0.2435 | 0.5902 | 0.4993 | 0.5000 | 0.2224 | 0.5683 | 0.6518 | 0.8220 | 0.8484 | 0.8379 |
| Optdigits (3%)      | 0.9676 | 0.6154 | 0.5720 | 0.4893 | 0.5637 | 0.5512 | 0.5833 | 0.2776 | 0.6088 | 0.5722 | 0.4521 | 0.5865 | 0.5347 |
| Pendigits (2.27%)   | 0.9562 | 0.5256 | 0.5314 | 0.3208 | 0.6753 | 0.3855 | 0.7186 | 0.2445 | 0.5878 | 0.4632 | 0.6688 | 0.8196 | 0.7731 |
| Pima (35%)          | 0.4910 | 0.4937 | 0.4800 | 0.4028 | 0.5775 | 0.4983 | 0.4983 | 0.3374 | 0.4419 | 0.5318 | 0.6962 | 0.6816 | 0.6852 |
| Satellite (32%)     | 0.6602 | 0.5221 | 0.5031 | N/A   | 0.5913 | 0.5007 | 0.5000 | 0.3152 | 0.5481 | 0.5448 | 0.5717 | 0.6069 | 0.5997 |
| Satimage-2 (1.2%)   | 0.8192 | 0.5303 | 0.5513 | 0.1529 | 0.7921 | 0.5002 | 0.5000 | 0.9351 | 0.7048 | 0.5346 | 0.9707 | 0.9925 | 0.9879 |
| Shuttle* (7%)       | 0.9737 | 0.6173 | 0.6051 | N/A   | 0.6199 | 0.5007 | 0.5007 | 0.1659 | 0.4528 | 0.6191 | 0.5878 | 0.7082 | 0.6737 |
| Speech (1.65%)      | 0.6435 | 0.5492 | 0.6000 | 0.2822 | 0.5270 | 0.4494 | 0.4513 | 0.5391 | 0.5162 | 0.5072 | 0.4791 | 0.4984 | 0.4999 |
| Vertebral (12.5%)   | 0.2084 | 0.4906 | 0.5078 | 0.6390 | 0.4273 | 0.5762 | 0.5786 | 0.6232 | 0.4127 | 0.5019 | 0.3222 | 0.3663 | 0.3575 |
| Vowels (3.4%)       | 0.8802 | 0.9467 | 0.8716 | 0.0168 | 0.9074 | 0.0341 | 0.0346 | 0.2929 | 0.8944 | 0.9399 | 0.9373 | 0.9468 | 0.9445 |
| Wine (7.7%)         | 0.9958 | 0.9361 | 0.8899 | 0.0185 | 0.6753 | 0.5420 | 0.5420 | 0.1857 | 0.9916 | 0.9899 | 0.8605 | 0.8739 | 0.9067 |
| AVG                 | 0.7657 | 0.6450 | 0.6318 | 0.2680 | 0.6841 | 0.5160 | 0.5379 | 0.3266 | 0.5985 | 0.6414 | 0.7121 | 0.7463 | 0.7329 |

Note: (1) The performance of iPOF in this table is based on LOF as the first stage detector, (2) N/A indicates the out-of-memory error.
result on BreastW and excels other outlier detectors by a large margin. The ensemble outlier detectors FB and Median achieve the best performance on Arrhythmia, Ionosphere and Pima, respectively. Unfortunately, we can also see that FABOD returns extremely worst outlier candidates on Ionosphere, Vowels and Wine. iForest cannot work well on BreastW. These phenomena demonstrate that the assumptions of outlier algorithms are crucial to the success of effective detection. The unsupervised outlier detection setting also increases the task difficulty since we can only apply the default parameters. Usually, the ensemble outlier detectors outperform the basic outlier detectors due to their robustness by fusion mechanism that alleviates the dependence on algorithmic dependence. (2) Our iPOF algorithm achieves 9 out of 17 best results with the highest average performance as well. iPOF obtains 0.9676, 0.9562 and 0.9737 on Optdigits, Pendigits and Shuttle*, that outperforms other basic and ensemble outlier detectors by around 30%, 10% and 30%, respectively. Although iPOF also holds the assumption that inliers’ core neighbors are inliers and outliers’ core neighbors are outliers, the network built by common neighbors is robust and reliable in general. (3) Recall that the performance of iPOF in Table 3 is based on LOF as the initialization. If we take the comparisons between iPOF and LOF, there are significant improvements in 15 out of 17 datasets. Figure 6(a) provides a better visualization on the comparisons between iPOF and LOF on these 17 datasets with \( K = 20 \). Therefore, LOF is not the termination point of outlier detection. iPOF starts from LOF and employs the local neighbor structure to further enhance the outlier detection performance by simply averaging propagation. Admittedly, iPOF degrades the detection performance on some datasets. We conjecture that the parameter \( K \) to choose the propagation neighbors plays a crucial role in whether there exist paths between outliers and inliers. In Table 3 we set \( K = 10 \) at the default. In the following subsection, we provide the parameter analysis and show the improvement on these datasets.

Figure 3 shows the outlier score trend with respect to the iterations of iPOF on Glass and Mammography, with tracks of several inliers and outliers that highlighted by blue and red color over iterations. they are separated after iPOF converges. Since the core idea of iPOF is to propagate the scores via averaging the local common neighbors, it is also normal to have the scores of some outliers decrease as well. Figure 4 demonstrates the local neighborhood relationship on Optdigits and Vertebral, where red and blue nodes represent inlier and outlier samples, respectively. For better visualization, only the neighborhood relationship of outliers are shown in Figure 4. We can see that iPOF brings in +57% and -57% improvements on these two datasets. Based on the local homogeneity assumption of iPOF, the positive or negative boost results from the local neighbor structure. On Optdigits, outliers’ neighbors are outliers and inliers’ neighbors are inliers; on the contrary, some neighbors of outlier samples in Vertebral are inliers, which leads to the negative propagation. In the unsupervised scenarios, algorithmic assumptions are crucially important and directly determine the ultimate performance. We do not claim that iPOF is helpful to boost basic outlier detectors on all datasets. Fortunately, iPOF brings in the positive improvements on the average level to most data sets. We will provide more solid experiments in the next subsection of in-depth factor exploration.

Table 4 shows the running time of iPOF and other detectors. Generally speaking, iPOF is simple and fast. The most time-consuming part of iPOF is to build the common neighbor network based on data features in the first stage, which is pre-calculated in some density or distance-based outlier detectors and can be directly used in iPOF. Moreover, some techniques including sampling [12], clustering [7] can be applied to quickly calculate the neighbor matrix. The second propagation stage just iteratively averages the outlier scores of top-\( K \) common neighbors. The time complexity of the propagation stage is \( O(nK) \), which can be further accelerated by parallel computing. It is worthy to note that the input of the second stage is an \( n \times 1 \) outlier score vector, rather than the original data matrix \( n \times d \), where \( d \) is the number of feature dimensions. Therefore, iPOF is also space efficient and has \( O(n) \) in space complexity, which is essential to store the outlier score for every data point.

5.3 Factor Exploration

In this subsection, we provide an in-depth exploration of iPOF in terms of parameter analysis and initial outlier detectors.
We provide the parameter analysis on $K$ Therefore, there is only one parameter $K$ is the key factor in determining the outlier score propagation. In Figure 5, where $K$ plays a crucial role in outlier detection, especially in unsupervised settings. We can see that the performance of LOF increases with an increasing $K$ on $Satellite$, while it decreases on $Mnist$. On $COF$, iPOF also enjoys the dramatic improvements over LOF around 83%, 50%, 70%, 55% on four different detectors LOF, COF, FABOD and SOD in Figure 6. So far we use LOF as the default initialization of our iPOF in the above experiments, iPOF can improve LOF by 18.71% on average AUC performance. Different Outlier Detectors as Initialization. To demonstrate more experimental results, we set $K_0$ to construct the common neighbor network and select $K$ to outlier score propagation in iPOF. In fact, $K$ is used to illustrate the concept of a common neighbor network. We can set $K = n$ to get a fully connected network. Instead, the number of neighbors for propagation $K$ is the key factor in determining the outlier score propagation. Therefore, there is only one parameter $K$ in our iPOF algorithm. We provide the parameter analysis on $K$ in iPOF on 3 real-world datasets $Mnist$, $Optdigits$ and $Speech$, with tracks of initial LOF detector’s performance and converged iPOF that highlighted by red and blue color.

### Parameter Analysis on $K$.

In Figure 4 we introduce $k$ to construct the common neighbor network and select $K$ to outlier score propagation in iPOF. In fact, $k$ is used to illustrate the concept of a common neighbor network. We can set $k = n$ to get a fully connected network. Instead, the number of neighbors for propagation $K$ is the key factor in determining the outlier score propagation. Therefore, there is only one parameter $K$ in our iPOF algorithm. We provide the parameter analysis on $K$ in iPOF on 3 real-world datasets $Mnist$, $Optdigits$ and $Speech$ in Figure 5 where $K$ varies from 5 to 100 and the performance of LOF is added as a baseline. Generally speaking, $K$ plays a crucial role in outlier detection, especially in unsupervised settings. We can see that the performance of LOF increases with an increasing $K$ on $Mnist$, while it decreases on $Optdigits$ and $Speech$. The performance of iPOF goes up and then decreases with an increasing $K$. On $Optdigits$ and $Speech$, there exist the significant performance drop when $K$ is larger than 40. These phenomena result from that a large $K$ involves the path between inliers and outliers and propagates the high scores of outliers to inliers. Based on the above experimental results, we recommend a small $K$ in iPOF, such as $K = 10$ or $K = 20$ for effective propagation.

## Table 4: Running time of iPOF and other detectors by second

| Datasets (size)         | iPOF | LOF | COF | FABOD | SOD | MGAAL | SGAAL | iForest | LSCP | FB | Median | AOM | MOA |
|------------------------|------|-----|-----|-------|-----|-------|-------|---------|------|-----|--------|-----|-----|
| Arrhythmia (452)       | 0.09 | 0.19| 0.10| 0.39  | 1.03| 3.93  | 10.43 | 1.58    | 0.26 | 0.26| 0.67  | 1.86| 1.85|
| BreastW (683)          | 0.41 | 0.42| 0.01| 0.60  | N/A | 3.71  | 8.35  | 1.12    | 0.19 | 0.27| 0.47  | 0.38| 0.40|
| Cardio (1831)          | 0.97 | 1.07| 0.10| 3.94  | 1.74| 5.09  | 15.68 | 2.00    | 0.25 | 3.27| 0.82  | 2.90| 2.93|
| Glass (214)            | 0.08 | 0.09| 0.01| 0.07  | 0.73| 3.51  | 7.41  | 0.86    | 0.17 | 0.15| 0.03  | 0.07| 0.07|
| Ionosphere (351)       | 0.20 | 0.21| 0.01| 0.17  | 0.83| 3.59  | 8.53  | 1.10    | 0.20 | 0.36| 0.09  | 0.23| 0.24|
| Mammography (11183)    | 7.95 | 8.26| 0.31| 167.05| N/A | 59.38 | 158.64| 12.27   | 0.56 | N/A | 2.91  | 11.05| 11.35|
| Mnist (7603)           | 4.60 | 14.65| 10.05| 105.25| 14.71| 37.24 | 120.66| 10.67   | 1.17 | 217.69| 82.40| 177.34| 203.69|
| Optdigits (5126)       | 2.32 | 5.01| 2.69| 47.58 | 6.26| 16.94 | 67.63 | 6.65    | 0.65 | 63.43| 20.74| 64.73 | 69.05|
| Pendigits (6870)       | 6.48 | 6.87| 0.39| 60.96 | 4.47| 23.42 | 95.84 | 7.39    | 0.51 | 13.04| 2.41  | 19.22| 20.81|
| Pima (768)             | 0.27 | 0.28| 0.01| 0.67  | 1.03| 3.83  | 11.31 | 1.09    | 0.20 | 0.55 | 0.06  | 0.48 | 0.54|
| Satellite (6435)       | 4.07 | 5.03| 0.96| 57.74 | N/A | 21.57 | 104.96| 7.33    | 0.50 | 24.35| 6.73  | 24.67| 30.12|
| Satimage-2 (214)       | 3.29 | 4.13| 0.84| 48.25 | 4.52| 18.27 | 78.42 | 6.19    | 0.52 | 22.49| 6.75  | 22.97| 27.56|
| Shuttle* (4909)        | 5.92 | 6.10| 0.18| 28.15 | 13.56| 54.89 | 4.92  | 0.37    | 0.21 | 7.06 | 1.25  | 8.81 | 10.55|
| Speech (5868)          | 1.81 | 11.47| 9.66| 48.67| 13.19| 18.76 | 73.61 | 12.33   | 1.73 | 227.41| 76.60| 211.37| 227.59|
| Vertebral (240)        | 0.07 | 0.08| 0.01| 0.08  | 0.81| 3.43  | 8.49  | 1.20    | 0.17 | 0.15 | 0.03  | 0.07 | 0.08|
| Vowels (1456)          | 0.54 | 0.57| 0.03| 2.43  | 1.41| 4.33  | 14.20 | 1.56    | 0.22 | 1.68 | 0.25  | 1.46 | 1.54|
| Wine (129)             | 0.03 | 0.04| 0.01| 0.04  | 0.74| 3.62  | 7.48  | 0.92    | 0.16 | 0.09 | 0.02  | 0.04 | 0.04|
| **AVG**                | 2.30 | 3.79| 1.49| 35.65 | 3.96| 14.35 | 49.80 | 4.66    | 0.46 | 36.53| 11.87| 32.20| 35.62|

Note: the first column denotes the score propagation of iPOF, while the second column is the total running time of iPOF with LOF as the initial detector.
The simple assumption of iPOF cannot capture all the scenarios and outlier detectors as initialization. Fortunately, iPOF works well on generally average level. Table 5 shows average AUC performance of iPOF with different initial outlier detectors on 17 datasets with $K = 10$ and $K = 20$, where iPOF brings the positive improvements ranging from 2% to 46%.

**iPOF on Deep Outlier Detection Methods.** As an outlier detection booster, we have demonstrated how iPOF helps on the conventional outlier detector methods so far. Actually, iPOF can be also applied on the deep outlier detection methods as well. Usually, the new representations are learned with the deep unsupervised outlier detection methods, where iPOF can build the common neighbor graph based on the learned representation, rather than the original feature space. Figure 7 shows the iPOF performance on recent deep unsupervised outlier detection methods REPEN [38] and RDP [46] with different $K$ values. We can see that iPOF boosts the performance of RDP on glass, mnist, satellite and satimage-2 by a large margin. However, the performance on optdigits with $K = 10$ drops a lot. This might result from the impact of the number of common neighbors. Recall that iPOF is an extremely simple method with only one parameter.

6 CONCLUSION

In this paper, we considered the unsupervised outlier detection problem and proposed the algorithm Infinite Propagation of Outlier Factor (iPOF), an extremely and excitingly simple outlier detector via infinite propagation. Under the assumption of local homogeneity that one data point has the neighbors of the same class, iPOF iteratively repeated the propagation process until convergence of outlier and inlier scores for better distinguishing. To achieve this, we first initialized iPOF with existing score-based outlier detectors. With awareness of neighborhood connectivity of each data point, iPOF then updated the outlier score of each instance by averaging the outlier factors of all its connected neighbors. Extensive experimental results demonstrated the effectiveness and efficiency of iPOF significantly over numerous classical and state-of-the-art unsupervised outlier detection and ensemble algorithms.

**REFERENCES**

[1] Charu C. Aggarwal. 2013. Outlier analysis. Springer science & business media. (2013).
[2] Charu C. Aggarwal and Saket Sathe. 2015. Theoretical foundations and algorithms for outlier ensembles. SIGKDD Explorations 17 (2015), 24–47.
[3] Shikha Agrawal and Jitendra Agrawal. 2015. Survey on anomaly detection using data mining techniques. Procedia Computer Science 60 (2015), 708–713.
[4] Sameet Akcay, Amir Atapour-Abarghouei, and Toby P Breeckon. 2018. Ganomaly: Semi-supervised anomaly detection via adversarial training. In Proceedings of Asian Conference on Computer Vision 622–637.
[5] Tharindu R Bandaragoda, Kai Ming Ting, David Albrecht, Fei Tony Liu, and Jonathan R Wells. 2014. Efficient anomaly detection by isolation using nearest neighbour ensembles. In Proceedings of IEEE International Conference on Data Mining Workshop 696–705.
[6] Markus M Breunig, Hans-Peter Kriegel, Raymond T Ng, and Jörg Sander. 2000. LOF: identifying density-based local outliers. In Proceedings of ACM SIGMOD record Vol. 29. 93–104.
[7] Maria Rosa Brito, Edgar Chavez, Adolfo J. Quirox, and Joseph E. Yukich. 1997. Connectivity of the mutual K-Nearest-Neighbor graph in clustering and outlier detection. Statistics & Probability Letters 35, 1 (1997), 33–42.
[8] Raghavendra Chalapathy and Sanjay Chawla. 2019. Deep learning for anomaly detection: A survey. arXiv preprint arXiv:1901.03407 (2019).
Raghavendra Chalapathy, Aditya Krishna Menon, and Sanjay Chawla. 2018. Anomaly detection using one-class neural networks. arXiv preprint arXiv:1802.06360 (2018).

Sanjay Chawla and Aristides Gionis. 2013. K-means+: A unified approach to clustering and outlier detection. In Proceedings of SIAM International Conference on Data Mining.

Chesner Deis, Simon Bernard, Caroline Petitjany, and Laurent Heutte. 2013. One class random forests. Pattern Recognition 46, 12 (2013), 3490–3506.

Sahubshing A Dudani. 1976. The distance-weighted K-Nearest-Neighbor rule. IEEE Transactions on Systems, Man, and Cybernetics 4 (1976), 325–327.

Sarah M Erfani, Sutharshan Rajasegarar, Shanika Karunasekera, and Christopher Leckie. 2016. High-dimensional and large-scale anomaly detection using a linear one-class SVM with deep learning. Pattern Recognition 58 (2016), 121–134.

Martin Ester, Hans-Peter Kriegel, Jörg Sander, Xiaowei Xu, et al. 1996. A density-based algorithm for discovering clusters in large spatial databases with noise. In Knowledge Discovery and Data Mining, Vol. 96. 226–231.

Wei Fan, Matthew Miller, Sal Stolfo, Wenke Lee, and Phil Chan. 2004. Using artificial anomalies to detect unknown and known network intrusions. Knowledge and Information Systems 6, 5 (2004), 507–527.

Chandan Gastam, Ramesh Balaji, K Sudharsan, Aruna Tiwari, and Kapil Abuja. 2019. Localized multiple kernel learning for anomaly detection: one-class classification. Knowledge-Based Systems 165 (2019), 241–252.

Prasanta Gogoi, Bhogeswar Borah, and Dhruba Bhattacharyya. 2010. Anomaly detection of intrusion data using supervised & unsupervised approach. Journal of Convergence Information Technology 5, 1 (2010), 95–110.

Nicó Gómez, Marius Kloft, Konrad Rieck, and Ulf Brefeld. 2013. Towards supervised anomaly detection. Journal of Artificial Intelligence Research 46 (2013), 235–262.

Manish Gupta, Jing Gao, Chun C Aggarwal, and Jiawei Han. 2014. Outlier detection for temporal data: A survey. IEEE Transactions on Knowledge and Data Engineering 26, 9 (2014), 2250–2267.

Zengyou He, Xiaofei Xu, Zhexue Joshua Huang, and Shengchun Deng. 2005. FP-outlier: Frequent pattern based outlier detector. Computer Science and Information Systems 2, 1 (2005), 103–118.

Kathryn Hempstead, Elie Frank, and Ian H Witten. 2008. One-class classification by combining density and class probability estimation. In Proceedings of Joint European Conference on Machine Learning and Knowledge Discovery in Databases.

Ramakrishnan Kannan, Hyenkyun Woo, Charu C Aggarwal, and Haesun Park. 2017. Outlier detection for text data. In Proceedings of SIAM International Conference on Data Mining.

Fabian Keller, Emmanuel Müller, and Klemens Böhm. 2012. HICs: High Contrast Subspaces for Density-Based Outlier Ranking. In Proceedings of IEEE International Conference on Data Engineering. 1037–1048.

Hans-Peter Kriegel, Peer Kröger, Erich Schubert, and Arthur Zimek. 2009. Outlier detection in axis-parallel subspaces of high dimensional data. In Proceedings of Pacific-Asia Conference on Advances in Knowledge Discovery and Data Mining.

Hans-Peter Kriegel, Matthias Schubert, and Arthur Zimek. 2008. Angle-based outlier detection in high-dimensional data. In Proceedings of ACM SIGKDD International Conference on Knowledge Discovery and Data Mining. 444–452.

Donghyun Kwon, Hyunjoo Kim, Jinoh Kim, Sang C Suh, Ikkyun Kim, and Hans-Peter Kriegel. 2008. Angle-based outlier detection for temporal data: A survey. IEEE Transactions on Knowledge and Data Engineering 20, 1 (2008), 235–262.

Aleksandar Lazarevic and Vinip Kumar. 2005. Feature bagging for outlier detection. In Proceedings of ACM SIGKDD International Conference on Knowledge Discovery and Data Mining. 157–166.

Yuh-Jye Lee, Yi-Ren Yeh, and Yu-Chiang Frank Wang. 2012. Anomaly detection via online oversampling principal component analysis. IEEE Transactions on Knowledge and Data Engineering 25 (2012), 1460–1470.

Dan Li, Dacheng Chen, Jonathan Goh, and See-Kiong Ng. 2018. Anomaly detection with generative adversarial networks for multivariate time series. arXiv preprint arXiv:1809.04758 (2018).

Wei Li, Guodong Wu, and Qian Du. 2017. Transferred deep learning for anomaly detection in hyperspectral imagery. IEEE Geoscience and Remote Sensing Letters 14, 5 (2017), 597–601.

Fei Tony Liu, Kai Ming Ting, and Zhi-Hua Zhou. 2008. Isolation forest. In Proceedings of ACM SIGKDD International Conference on Data Mining. 413–422.

Hongfu Liu, Jun Li, Yue Wu, and Yun Fu. 2019. Clustering with outlier removal. IEEE Transactions on Knowledge and Data Engineering (2019).

Hongfu Liu, Yuchao Zhang, Bo Deng, and Yun Fu. 2016. Outlier detection via sampling ensemble. In Proceedings of IEEE International Conference on Big Data.

Yezheng Liu, Zhe Li, Chong Zhou, Yuanchun Jiang, Jianshan Sun, Meng Wang, and Xiangnan He. 2019. Generative adversarial active learning for unsupervised outlier detection. IEEE Transactions on Knowledge and Data Engineering (2019).

Mengye Ren, Wenyuan Zeng, Bin Yang, and Ruqiang Yu. 2018. Learning to reweight examples for robust deep learning. arXiv preprint arXiv:1803.00502 (2018).

Volker Roth. 2005. Outlier detection with one-class kernel fisher discriminants. In Advances in Neural Information Processing Systems. 1169–1176.

Mohammad Sabokrou, Mohammad Khalooei, Mahmood Filety, and Elias Adeli. 2018. Adversarially learned one-class classifier for novelty detection. In Proceedings of IEEE Conference on Computer Vision and Pattern Recognition. 3379–3388.

Saket Sathe and Chara C. Aggarwal. 2016. LODES: Local density measures spectral outlier detection. In Proceedings of SIAM International Conference on Data Mining.

Fan Tang, Zhixiang Chen, Ada Wai-Chee Fu, and David W Cheung. 2002. Enhancing effectiveness of outlier detections for low density patterns. In Proceedings of Pacific-Asia Conference on Knowledge Discovery and Data Mining.

Hu Wang, Guansong Pang, Chunhua Shen, and Congbo Ma. 2019. Unsupervised Representation Learning by Predicting Random Distances. International Joint Conference on Artificial Intelligence (2019).

Drausin Wulsin, Justin Blanco, Ram Mani, and Brian Litt. 2010. Semi-supervised anomaly detection for EEG waveforms using deep belief nets. In Proceedings of SIAM International Conference on Machine Learning and Applications.

Dan Xu, Elisa Ricci, Yan Yan, Jingkuan Song, and Nicola Sebe. 2015. Learning deep representations of appearance and motion for anomalous event detection. arXiv preprint arXiv:1510.00553 (2015).

Zeping Yang and Daqi Gao. 2013. Classification for imbalanced and overlapping classes using outlier detection and sampling techniques. Applied Mathematics & Information Sciences 7, 1 (2013), 375–381.

Weiren Yu, Zhengming Ding, Chunjing Hu, and Hongfui Liu. 2019. Knowledge Reused Outlier Detection. IEEE Access (2019), 45763–45772.

Housam Zennati, Chuan Sheng Foo, Bruno Lecoutot, Gaurav Maneck, and Vijay Ramaseshan Chandrashekar. 2018. Efficient gan-based anomaly detection. arXiv preprint arXiv:1802.06222 (2018).

Ke Zhang, Marcus Hutter, and Huidong Jin. 2009. A new local distance-based outlier detection approach for scattered real-world data. In Proceedings of Pacific-Asia Conference on Knowledge Discovery and Data Mining.

Handong Zhao and Yun Fu. 2015. Dual-regularized multi-view outlier detection. In the International Joint Conference on Artificial Intelligence.

Yue Zhao, Maciej K. Hrynewicz, Zain Nasrullah, and Zheng Li. 2018. LSCP: Locally selective combination in parallel outlier ensembles. CoRR (2018). arXiv:1812.01358.

Qing Zhao, Zain Nasrullah, and Zheng Li. 2019. PyOD: A Python toolbox for scalable outlier detection. Journal of Machine Learning Research (2019). http://jmlr.org/papers/v20/19-011.html