SELF-SIMILAR COLLAPSE OF A SELF-GRAVITATING VISCOUS DISK

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ABSTRACT

A self-similar solution for time evolution of isothermal, self-gravitating viscous disks is found under the condition that \( \alpha \equiv \alpha(H/r) \) is constant in space (where \( \alpha \) is the viscosity parameter and \( H/r \) is the ratio of a half-thickness to the radius of the disk). This solution describes a homologous collapse of a disk via self-gravity and viscosity. The disk structure and evolution are distinct in the inner and outer parts. There is a constant mass inflow in the outer portions so that the disk has flat rotation velocity, constant accretion velocity, and surface density decreasing outward according to \( \Sigma \propto r^{-1} \). In the inner portions, in contrast, mass is accumulated near the center owing to the boundary condition of no radial velocity at the origin, a strong central concentration being thereby produced; surface density varies according to \( \Sigma \propto r^{-5/3} \). Moreover, the transition radius separating the inner and outer portions increases linearly with time. The consequence of such a high condensation is briefly discussed in the context of formation of a quasar black hole.

Subject headings: accretion, accretion disks — black hole physics — galaxies: kinematics and dynamics — quasars: general — stars: formation

1. INTRODUCTION

Quasars (QSOs) are the most powerful objects that have ever existed in the universe. The emergence of quasars at high redshifts, \( z \lesssim 5 \), is thus crucial when considering the formation of astrophysical objects, notably of galaxies. The view is widely accepted that QSO phenomena result from the formation process of seed black holes at high redshifts, and that the basic equilibrium structure of accretion disks is now well understood, as long as we believe the standard model of star and galaxy formation.

Furthermore, this kind of study is, of course, of great importance when one investigates the physics of galaxy and star formation.

The basic equilibrium structure of accretion disks is now well understood, as long as we believe the standard model based on the \( \alpha \)-viscosity prescription (Shakura & Sunyaev 1973). Nevertheless, it is not easy to follow its dynamical evolution, mainly because the basic equations for the disks are highly nonlinear, especially when the disk is self-gravitating (e.g., Paczyński 1978; Fukue & Sakamoto 1992). To follow the nonlinear evolution of dynamically evolving systems, in general, the technique of self-similar analyses is sometimes useful. Several classes of self-similar disk solutions were known previously (Pringle 1974; Filipov 1984), but all of them considered a disk in a fixed, external potential.

We are now concerned with dynamical evolution of a self-gravitating disk in a time-evolving, self-consistent potential. As far as steady, nonviscous rotating disks are concerned, plenty of studies have so far been done. Mestel (1963) was the first to find a simple disk solution, in which physical quantities are integrated vertically with respect to the disk equatorial plane. Hayashi, Narita, & Miyama (1982) found two-dimensional, isothermal disk solutions with finite temperature (see also Toomre 1982 for stellar systems). Numerical steady solutions are calculated by several groups (Hachisu, Eriguchi, & Nomoto 1986; Bodo & Curir 1992; Hashimoto, Eriguchi, & Müller 1995).

Recently, we have found a simple analytical solution for a steady, self-gravitating, isothermal disk (Mineshige & Umemura 1996, hereafter Paper I) as an extension of the Mestel (1963) disk. However, little study has been done concerning dynamical evolution of self-gravitating viscous disks.

We, in the present study, seek a time-dependent, self-
similar solution for gravitational collapse of a rotation-supported, self-gravitating viscous disk.

When a disk is sufficiently cool, gravitational instability will occur (Toomre 1964), providing a source of disk viscosity (Paczynski 1978; Lin & Pringle 1987) or causing disk fragmentation (e.g., Bodenheimer, Tohline, & Black 1980). Several authors thus mainly discussed the consequence of gravitational instability in the context of fueling to active galactic nuclei (e.g., Shore & White 1982; Shlosman & Begelman 1987; Shlosman, Frank, & Begelman 1989), or (multiple) star formation (see Boss 1986; Myhill & Kaula 1992). We here adopt a rather distinct approach; we, in the present study, try to find an analytical solution for collapse of rotating, viscous disks, putting aside for the moment the stability argument.

It might be noted in this context that Shu (1977) found the self-similar solution for gravitational collapse of an isothermal sphere. Saigo & Hanawa (1996) discussed the effects of rotation. We extend these works to incorporate the effects of mass accretion via viscosity. We derive self-similar solutions in §2 and then discuss the formation of a primordial quasar black hole in §3.

2. SELF-SIMILAR, SELF-GRAVITATING DISK

2.1. Basic Equations for Self-similar Variables

We start with the time-dependent version of the height-averaged equations for isothermal accretion disks (cf. Honma, Matsumoto, & Kato 1991; Narayan & Yi 1994):

\[ \frac{\partial \Sigma}{\partial t} + \frac{1}{r} \frac{\partial}{\partial r} (r \Sigma V_r) = 0 , \]

\[ \frac{\partial V_r}{\partial t} + V_r \frac{\partial V_r}{\partial r} = - \frac{\partial \Sigma}{\partial r} \frac{GM_r}{r^2} + \frac{V_r^2}{r} , \]

\[ \frac{\partial (r V_\phi)}{\partial t} + V_r \frac{\partial (r V_\phi)}{\partial r} = \frac{1}{r} \frac{\partial}{\partial r} \left[ \nabla \Sigma r^3 \frac{\partial \Omega}{\partial r} \right] . \]

Here \( \Sigma = 2 \rho H \) is the surface density, \( H \) is the half-thickness of the disk, \( \Omega = V_\phi / r \), \( c_s \) is the sound velocity (which is constant by assumption), \( M_r \) is the mass of a disk within a radius \( r \), and we approximated a potential to be \( \sim -GM_r/r \). This is a good approximation if a \( \Sigma(r) \) profile is steeper than \( 1/r \) (see Appendix). We prescribe kinematic viscosity as

\[ v = \alpha c_s H = \alpha \left( \frac{H}{r} \right) c_s r , \]

with \( \alpha \) being the viscosity parameter, because we will find later that self-similar solutions exist if \( \alpha' \equiv \alpha (H/r) \) is constant in space. From now on, therefore, we assume \( \alpha' \) (instead of \( \alpha \)) to be constant. For vertically self-gravitating disks, \( H \) is determined as

\[ H = \frac{c_s}{\sqrt{2\pi G \rho}} = \frac{c_s^2}{2 \pi G \Sigma} , \quad \rho = \frac{\Sigma}{2H} = \frac{\pi G \Sigma^2}{c_s^2} . \]

To proceed, it is convenient to rewrite mass conservation (eq. [1]) using \( M_r(t, t) \):

\[ \frac{\partial M_r}{\partial t} + V_r \frac{\partial M_r}{\partial r} = 0 , \quad \frac{\partial M_r}{\partial r} = 2 \pi r \Sigma . \]

Now we introduce the following self-similar variables

\[ x \equiv \frac{r}{c_s t} , \quad \Sigma(r, t) = \frac{c_s}{2 \pi G t} \sigma(x) , \quad M_r(r, t) = \frac{c_s t}{G} m(x) , \]

\[ \rho(r, t) = \frac{\sigma^2(x)}{4 \pi G t^2} , \quad H(r, t) = \frac{c_s t}{\sigma(x)} , \quad V_r(r, t) = c_s u(x) , \]

\[ V_\phi(r, t) = c_s \phi(x) , \quad j(x) \equiv x v = \frac{1}{c_s} \frac{r V_\phi(r, t)}{t} . \]

Note that derivatives are transformed according to

\[ \frac{\partial}{\partial t} \rightarrow - \frac{x}{t} \frac{\partial}{\partial x} + \frac{\partial}{\partial r} \frac{x}{r} \frac{\partial}{\partial x} , \]

for the transformation, \( (r, t) \rightarrow (x, t' = t) \). Since all the time derivatives with respect to \( t' \) disappear if we use self-similar variables (eq. [7]), we hereafter write \( d/dx \) instead of \( \partial/\partial x \).

Equation (6) now becomes

\[ m + (u - x) \frac{dm}{dx} = 0 \quad \text{and} \quad \frac{dm}{dx} = x \sigma , \]

yielding a simple relation between \( m \), \( \sigma \), and \( u \); \( m = x \sigma (x - u) \). With this in mind, equations (1)–(3) can be modified as

\[ (u - x) \frac{1}{\sigma} \frac{d\sigma}{dx} + \frac{du}{dx} + \frac{u - x}{x} = 0 , \]

\[ 2 \frac{d\sigma}{dx} + (u - x) \frac{du}{dx} - \frac{u - x}{x} v^2 = 0 , \]

\[ j + (u - x) \frac{dj}{dx} = \alpha' \frac{1}{\alpha x} \frac{d}{dx} \left[ \sigma x \left( 2j + x \frac{dj}{dx} \right) \right] . \]

2.2. Solution in a Slow Accretion Limit

In the limit of slow accretion (\( v \gg 1, \sigma \gg 1, |u| \ll 1 \)), equation (11) gives

\[ v = \sigma^{1/2} (x - u)^{1/2} , \quad j = \sigma^{1/2} x (x - u)^{1/2} , \]

leading to

\[ \frac{d \ln j}{d \ln x} = 1 + \frac{1}{2} \frac{x - u}{x} \left( \frac{du}{d \ln x} + \frac{1}{2} \frac{d \ln \sigma}{d \ln x} \right) . \]

Note that from equation (10) we derive

\[ \frac{d \ln \sigma}{d \ln x} = \frac{1}{x - u} \frac{du}{d \ln x} - 1 . \]

Inserting equation (15) into equation (14), we have

\[ \frac{d \ln j}{d \ln x} = \frac{1}{2} + \frac{x}{2} \frac{x - u}{x - u} = \frac{2x - u}{2(x - u)} . \]

After some algebra, we obtain

\[ u = \frac{2x}{2x - u} \frac{d}{dx} \left( \frac{sxj}{x - u} \right) . \]

With the help of the expressions for \( j \) (eq. [13]) and \( \sigma \) (eq. [15]), we finally derive an ordinary differential equation for \( u(x) \):

\[ \frac{du}{dx} = \frac{2x - u}{u^2} \left( 4x^2 - 6ux + 3u^2 \right) - \frac{1}{\alpha' (x - 3u)x} . \]

Equation (18) can easily be integrated numerically for an appropriate boundary condition; \( u = 0 \) at \( x = 0 \) if we
assume no central object (such as a black hole). Once \( u = u(x) \) is obtained, we can derive \( \sigma = \sigma(x) \) by integrating equation (15) for a given \( \sigma_0 \equiv \sigma(x = 1) \). The results of the integration are displayed in Figure 1 for different values of \( \alpha' \). The azimuthal velocity is derived from equation (13).

Note that each physical quantity is a rather smooth function of \( x \). We generally find \( du/dx < 0 \); that is, \( u(x) \) is a monotonically decreasing function of \( x = r/c_\text{t} \). Furthermore, physical quantities, such as \( u \) and \( \sigma \), are power-law functions of radius in the limits of \( x \gg \alpha' \) and \( x \ll \alpha' \).

In the limit of large \( x \) (\( \gg \alpha' \)), mass accretion is induced by viscosity. Two terms on the right-hand side of equation (18) are balanced with each other (while \( du/dx = 0 \)). We find

\[
\begin{align*}
  u &\approx -2\alpha' \left(1 - \frac{9}{11} \frac{x}{\alpha'}\right), \\
  \sigma &\approx \sigma_0 \left(\frac{x}{\alpha'}\right)^{-5/3} \left(1 + \frac{8}{11} \frac{x}{\alpha'}\right), \\
  v &\approx (3\sigma_0)^{1/2} \left(\frac{x}{\alpha'}\right)^{-1/3} \left(1 + \frac{1}{11} \frac{x}{\alpha'}\right), \\
  \dot{m} &\approx 2\alpha'\sigma_0 \left(\frac{x}{\alpha'}\right)^{1/3} \left(1 - \frac{1}{11} \frac{x}{\alpha'}\right).
\end{align*}
\]

Note that \( u \) (and therefore \( V_r \)) is not proportional to \( \alpha' \), indicating that mass inflow is not controlled by viscosity but is regulated by the inner boundary condition of \( V_r = 0 \) at \( r = 0 \). Mass is thus being accumulated continuously near the origin.

To sum up, the disk structure and evolution are distinct in the inner and outer parts. The transition radius \( r_\text{tr} \) separating these two parts increases linearly with time, because \( r_\text{tr} \approx \alpha' c_\text{s} t \propto t \) for a fixed \( \alpha' \) (eq. [7]). We thus assume \( r_\text{tr} = 0 \) initially; in other words, we consider the later evolution of the disk with \( \Sigma \propto r^{-1} \) everywhere. (This is the situation postulated in Paper I.) As matter accretes toward the center, the \( \Sigma \) profile changes from inside.

Now we recover physical variables from self-similar ones using equation (7): we obtain

\[
\begin{align*}
  V_r &\approx -2\alpha' c_s, \\
  \Sigma &\approx \Sigma_0 \left(\frac{r}{r_0}\right)^{-1}, \\
  V_r &\approx (2\pi G\Sigma_0 r_0)^{1/2}, \\
  \dot{M} &\approx 4\pi c_s r_0 \Sigma_0 \left(\frac{r}{r_0}\right)^{-1}.
\end{align*}
\]

at large \( r/t \) (\( \gg \alpha' c_s \)), and

\[
\begin{align*}
  V_r &\approx -2c_s \left(\frac{r}{r_0}\right)^{1/2} - 1, \\
  \Sigma &\approx \Sigma_0 \left(\frac{r}{r_0}\right)^{-5/3} \left(\frac{t}{t_0}\right)^{2/3}, \\
  V_r &\approx c_s \left(\frac{r}{r_0}\right)^{-1/3} \left(\frac{t}{t_0}\right)^{1/3}, \\
  \dot{M} &\approx 4\pi r_0 \Sigma_0 r_0 c_s \left(\frac{r}{r_0}\right)^{1/3} \left(\frac{t}{t_0}\right)^{-1/3}.
\end{align*}
\]

Note that \( \sigma_0 \) represents the ratio of disk radius to height at \( x = 1 \) (see eq. [7]), or the initial ratio of gravitational energy to thermal energy of the disk, \( V_r^2/c_s^2 \) (eq. [21]). The model parameters of the self-similar solutions are \( \alpha' \), \( c_s \) (or temperature), and \( \sigma_0 \).

Figure 2 plots the time evolution of a self-gravitating disk. Clearly, there are two regimes as mentioned previously (cf. Fig. 1). The radius separating the outer and inner parts is increasing linearly with time. Hence, if we follow a disk evolution at a fixed \( r \), we see that \( V_r \) is initially constant and then decreases at \( t > r/\alpha' c_s \). Accordingly, mass-inflow rate also decreases with time, causing a rapid growth of \( \Sigma \) and \( M_\alpha \). Note that since \( H/r \approx (x\sigma)^{-1} \) (eq. [7]), \( H/r \) is constant.
3. DISCUSSION

3.1. Summary of the Self-similar Solution

We have derived a self-similar solution for time evolution of an isothermal, self-gravitating viscous disk in the slow accretion limit. Disk structure changes from the inner to the outer parts. For example, surface density is scaled as $r^{-5/3}$ in the inner part, while it is $r^{-1}$ in the outer. This interface gradually moves outward in proportion to $t$. In this solution density increases monotonically with the time at the center. The mass profile near the center is

$$M_x(r) = \int_0^r 2\pi \Sigma(r) dr \approx 3 \times 10^8 \left( \frac{r}{r_0} \right)^{1/3} \left( \frac{t}{t_0} \right)^{2/3} \frac{r_{pc}}{T_4^{1/3}} M_\odot .$$  

Although this yields a diverging $M_x$, the increase of $M_x$ should be terminated in a realistic situation, when the outer disk is depleted with gas.

As claimed first by Mestel (1963) and also in Paper I, the thin-disk approximation breaks down at radii comparable to the thickness. In fact, the present solution gives diverging $\Omega$ and $x$ as $x$ approaches zero, which suggests that the solution does not represent a physical situation at $x \ll 1$. Moreover, since we assume steady mass input toward the center, the central mass condensation increases at any time. Once a central object forms from a central mass condensation, gravity is dominated by this object at sufficiently small radii, where we may adopt a solution for a point-mass potential.

Realistically, there may be two or three zones in a disk. Before forming an object, a self-gravitating disk has two zones (as mentioned in the previous section). After the formation of a central object, in contrast there are three zones; the innermost region is dominated by a point-mass potential, and the other two zones are dominated by self-gravity of the disk.

Since $M > 0$, the mass of the central object is continuously increasing with time. The transition radius between the innermost and the inner region again increases linearly with time (Paper I).

Self-similar solutions assume that heating and cooling rates have the same radial dependence (see eq. [4] in Paper I). A flat temperature distribution is the result of this assumption. This is a reasonable approximation, at least in the outer regions: when we balance viscous heating and radiative cooling rates in a thin-disk approximation, we find

$$c_s \propto r^{-1/12} \sim r^{-5/12},$$

depending on the optical depth of the disk and opacity sources (Paper I). This relatively flat temperature profile results from the fact that for $\Sigma \propto r^{-1}$ (as in the outer parts) the potential is logarithmic and thus has a weak radial dependence. At $x \ll 1$, in contrast, this approximation may break down, since the potential has stronger radial dependence. The isothermal approximation may not be justified at the innermost region at later times ($r \ll c_s t$).

A self-gravitating disk is locally stable if

$$Q \equiv \frac{c_s K}{\pi G \Sigma} \gtrsim 1 ,$$

as long as the effects of viscosity and radial mass inflow are ignored (Toomre 1964). Here $K$ is epicyclic frequency and $\kappa = 2^{1/2} \Omega$ for $\Omega \propto R^{-1}$. If we simply apply this criterion to the present model, we find $Q \approx 2^{1/2}(H/R)^{1/3}$ at $x \gtrsim 1$ (eqs. [5] and [21]), indicating that the disk is stable for $H/R \gtrsim \frac{1}{2}$.

If $H/R$ is small, gravitational instability will set out, making the disk turbulent, thickening the disk (Paczyński 1978). However, this is a very naive picture, and a more sophisticated stability analysis, similar to that of Christodoulou et al. (1995a, 1995b) but including the effects of disk viscosity and radial gas inflow, is needed and should be the subject of future work.
3.2. Formation of a Quasar Black Hole

When $M_r$ exceeds a critical value at some radius,

$$M_{\text{crit}}(r) = \left(\frac{r}{10^{5.4}} \text{ cm}\right)^{2/3} M_\odot,$$

(27)

the cloud will start to collapse due to a general relativistic instability (Shapiro & Teukolsky 1983, p. 508), resulting in the formation of a black hole. Equation (27) gives a critical mass (for a given radius) for spherical supermassive stars, while we are now concerned with the evolution of a rotation-supported disk. Nevertheless, we employ the argument concerning spherical stars in order to see qualitative effects of general relativity, since a thin-disk approximation breaks down anyway near the center as mentioned above, and since a solid analysis of a collapsing self-gravitating disk based on the general relativistic formulation is not available at this moment.

With this in mind, we discuss the fate of a rotationally supported, viscous disk with a mass of $\sim 10^6 M_\odot$, a temperature of $\sim 10^4$ K, and a size of several parsecs. In the present picture, such a relatively high disk temperature is preferable, since otherwise the disk will stay molecular rather than ionized. The accretion timescale is inversely proportional to the temperature, and hence it may exceed the age of the universe for a molecular disk with $x < 0.01$ (e.g., eq. [1] in Sasaki & Umemura 1996), unless alternative mechanisms, such as gravitational torque, remove the disk angular momentum. There are several possible mechanisms for heating the disk. First, if the formation of primordial hydrogen molecules proceeds more slowly than the dynamical collapse, gas will not cool below $\sim 10^4$ K. This may occur if residual free electrons recombine quickly due to density enhancement, thereby suppressing the formation of a sufficient amount of H$^-$ ions, which help to make hydrogen molecules (see Hutchins 1976; Palla, Salpeter, & Stahler 1983). Second, if the universe was reionized through first-generation stars or objects, the disk will be effectively heated by strong UV background radiation (e.g., Sasaki & Umemura 1996). Finally, if star formation occurs within the disk itself, the disk material can be photoionized by stars.

Figure 3 depicts the evolution of such a disk (solid lines) and the critical line for a gravitation instability (dotted line) in the ($\log r$-$\log M_r$) diagram. As time goes on, the disk becomes more and more condensed at the center, thereby increasing its mass within a fixed radius. The mass profile is $M_r \propto r^{1/3}$ (eq. [25]) according to the self-similar solution, while the critical value gives $M_{\text{crit}} \propto r^{1/3}$ (eq. [27]). The solid line should cross the dotted line at

$$r_{\text{crit}} \approx 10^{11.8} \left(\frac{t}{t_0}\right)^2 \frac{r_p^3}{T_k^{2/3}} \text{ cm},$$

(28)

$$M_r(r_{\text{crit}}) \approx 10^{4.3} \left(\frac{t}{t_0}\right)^{4/3} \frac{r_p^3}{T_k^{2}} M_\odot.$$

We get a condensation of $\sim 10^3 M_\odot$ on a timescale of $\sim 0.1 t_0 \sim 10^4$ yr.

The estimates above are optimistic, however, since it takes $r_0/c_s \sim 10^3$ yr to $r_0/(\alpha' c_s) = 10^6 (\alpha'/0.1)^{-1}$ yr for accreting gas to reach the center and thereby establish a self-similar evolution of the disk. We thus safely conclude that within a timescale of $\sim 10^5 (\alpha'/0.1)^{-1}$ yr a central region with a mass of $10^4-10^5 M_\odot$ could become unstable, which may give rise to a protoquasar black hole at high redshifts. Again, a general relativistic study of a collapsing rotating disk is necessary to conclude whether this scenario can work or not.

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APPENDIX

SELF-GRAVITY UNDER A THIN-DISK APPROXIMATION

The most straightforward expression for the potentials under a thin-disk approximation is

$$\Psi(r) = 2G \int_0^\pi d\theta \int_0^{r_0} \frac{\Sigma(R) dR}{(r^2 + R^2 - 2rR \cos \theta)^{1/2}}$$

(A1)
(e.g., Mestel 1963), where \( r_0 \) denotes the size of the disk and we ignored vertical mass distribution in the disk. After some algebra, we have

\[
\frac{d\Psi}{dr} = G(I_1 + I_2 + I_3),
\]

(A2)

where \( I_1, I_2, \) and \( I_3 \) represent the Keplerian term, finite contributions from the mass within \( R \), and the mass beyond \( R \), respectively, and are

\[
I_1 = \frac{1}{r^2} \int_0^r 2\pi R \Sigma(R) dR, \quad I_2 = 2\pi \sum_{k=1}^{\infty} \alpha_{2k} \left[ \frac{(2k + 1)}{r^{2k + 2}} \int_0^r R^{2k+1} \Sigma(R) dR - \Sigma(r) \right]
\]

(A3)

\[
I_3 = 2\pi \sum_{k=1}^{\infty} \alpha_{2k} \left[ \Sigma(r) - 2kr^{2k-1} \int_0^r \frac{\Sigma(R)}{R^{2k}} dR \right],
\]

with

\[
\alpha_{2k} = \frac{1}{\pi} \int_0^\pi P_{2k}(\cos \theta) d\theta = \left[\frac{(2k)!}{(2^k k!)^2}\right]^{-1/2}
\]

(A4)

\( P_{2k} \) is the Legendre function; see eq. [24] of Mestel 1963. When \( \Sigma(r) = \Sigma_0 r_0/r \), in particular, we find

\[
\frac{d\Psi}{dr} = \frac{2\pi G \Sigma_0 r_0}{r} \left[ 1 + \sum_{k=1}^{\infty} \alpha_{2k} \left( \frac{r}{r_0} \right)^{2k} \right].
\]

(A5)

We understand, hence, that if the \( \Sigma(r) \) profile is steeper than \( 1/r \), we may approximate the gravitational attraction force by \( -GM_*/r^2 \) except near the outer edge.

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