Soft FSI Systematics for charmless strange $B^{\pm}$ Decays

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Abstract

New results going beyond those obtained from isospin and flavor symmetry and subject to clear experimental tests are obtained for effects of FSI in $B^{\pm}$ decays to final states containing neutral flavor-mixed mesons like $\omega$, $\phi$, $\eta$ and $\eta'$. The most general strong-interaction diagrams containing arbitrary numbers of quarks and gluons are included with the assumptions that any $q\bar{q}$ pair created by gluons must be a flavor singlet, and that there are no hairpin diagrams in which a final meson contains a $q\bar{q}$ pair from the same gluon vertex. The smallness of $K^-\eta$ suggests that it might have a large CP violation. A sum rule is derived to test whether the large $K^-\eta'$ requires the addition of an additional glueball or charm admixture. Further analysis from $D_s$ decay systematics supports this picture of FSI and raises questions about charm admixture in the $\eta'$.

The successful treatment of strong final state interactions involving neutral flavor-mixed mesons has a long history of successes going beyond isospin and flavor SU(3) symmetry back to the Alexander-Frankfurt-Harari-Iizuka-Levin-Okubo-Rosner-Scheck-Veneziano-Zweig rule [1–4], often abbreviated A...Z or OZI. Its first controversial prediction [1]

$$\sigma(K^-p \rightarrow \Lambda\rho^0) = \sigma(K^-p \rightarrow \Lambda\omega)$$

related final states in completely different isospin and flavor-SU(3) multiplets which would a priori be expected to have completely different final state interactions. The experimental confirmation of this prediction [5] indicates the existence of some dynamical symmetry that goes beyond isospin and flavor SU(3).

Our purpose is to identify this symmetry and develop its use to extend the standard isospin [6] and SU(3) [7,8] treatments of $B$ decays to include flavor-mixed final states cont-
taining ω, φ, η and η′ mesons not easily treated in SU(3). We also apply our new symmetry to otherwise unexplained Ds decay systematics [9,10].

A recent application of the OZI or A...Z rule to B decay predicts that [11]

\[ BR(B^± → K^±ω) = BR(B^± → K^±ρ^o) \]  

(1a)

A similar approach also including broken SU(3) symmetry gives the prediction

\[ \tilde{Γ}(B^± → K^±φ) \leq \tilde{Γ}(B^± → K^oρ^±) \]  

(1b)

where \( \tilde{Γ} \) denotes the theoretical partial width without phase space corrections, and the equality holds under the assumption of SU(3) flavor symmetry. The previous derivation justified the relation between different SU(3) amplitudes by a hand-waving asymptotic freedom argument enabling the final mesons to escape without final state interactions.

We show here that these relations hold even in the presence of strong final state rescattering via all possible diagrams involving quarks and gluons in which all quark-antiquark pairs created by gluons are flavor singlets and A...Z-forbidden disconnected “hairpin diagrams” are excluded. This is consistent with a large variety of experimental results and theoretical analyses for strong interaction three-point and four-point functions [1,4,12] expressed by the duality diagrams [3] of old-fashioned Regge phenomenology or the more modern planar quark diagrams in large \( N_c \) QCD [13,14] in which the leading Regge t-channel exchanges are dual to s-channel resonances [15,16]. This clear assumption leads to predictive power; e.g. eqs. (1) which can be tested with future experimental data.

We begin by exploiting known [17] flavor-topology [18] characteristics of charmless strange B± decays. The final states considered for B− decay all have the quark composition \( s\bar{u}q\bar{q} \) where \( q\bar{q} \) denotes a pair of the same flavor which can be \( uu, dd \) or \( ss \), and we do not consider the possibility of charm admixture in the final state. The \( q\bar{q} \) pair may come from a very complicated diagram involving many quarks and gluons. But there are only two possibilities for its origin illustrated by figs. 1 and 2 of ref. [17]: (1) It is created by gluons and must therefore be a flavor singlet denoted by \((q\bar{q})_1\); (2) The quark is a u quark from the
weak vertex and the pair can only be $u\bar{u}$. The decays are described by three parameters, an $s\bar{u}(q\bar{q})_1$ amplitude, a $K^{-}u\bar{u}$ amplitude and a relative phase. The one relation obtainable between the decays to four final states is shown below to be the sum rule:

$$\tilde{\Gamma}(B^± → K^±\eta') + \tilde{\Gamma}(B^± → K^±\eta) \leq \tilde{\Gamma}(B^± → K^±\pi^o) + \tilde{\Gamma}(B^± → \bar{K}^o\pi^±)$$

(2)

where $\bar{K}^o$ denotes $K^o$ for the $B^+$ decay and $\bar{K}^o$ for the $B^-$ decay, the equality holds in the flavor-SU(3) limit, the direction of the inequality follows from the assumption that SU(3) symmetry breaking will suppress the $s\bar{s}$ contribution to the singlet $(q\bar{q})_1$ and the result holds for any $\eta - \eta'$ mixing angle. These sum rules are of particular interest because of the large experimentally observed branching ratio [19] for $B^± → K^±\eta'$. A violation favoring $B^± → K^±\eta'$ can provide convincing evidence for an additional contribution [20] like a glueball, charm admixture [21–23] in the $\eta'$ wave function or an A...Z-violating hairpin diagram. Present data indicate a violation of between one and two standard deviations. If this violation is confirmed by better data, the best candidate is the charm admixture originally suggested by Harari [21] which still remains the only simple explanation for the anomalously large branching ratio for the apparently A...Z-violating cascade decay $\psi' → \eta\psi$ and the failure to observe the analogous cascade decays [24] $\Upsilon(nS) → \eta\Upsilon(1S)$. An A...Z-violating gluonic hairpin would contribute to all these cascades on the same footing.

In the kaon-vector (KV) system the ideal mixing of the $\omega - \phi$ system simplifies the treatment to give the equality (1a) and the simplified sum rule (1b).

In these $B^±$ decays all amplitudes arising from the $b → u\bar{u}s$ transition depend only upon a single sum of the color-favored and color-suppressed tree contributions. This simplification provides predictive power and allows crucial tests of the basic assumptions but does not arise in the neutral decays. Thus amplitudes derived here for charged decays are not simply related by isospin to amplitudes for neutral decays.

We now examine the dependence of these amplitudes on CKM matrix elements. The two $b$ quark decay topologies contributing to these decays, $b → c\bar{c}s$ and $b → u\bar{u}s$, depend upon two different products of CKM matrix elements. Their interference can give rise to
direct CP violation.

Assuming SU(3) flavor symmetry, the A...Z rule and a standard pseudoscalar mixing

\[ |\eta\rangle = \frac{1}{\sqrt{3}} \cdot (|P_u\rangle + |P_d\rangle - |P_s\rangle); \quad |\eta'\rangle = \frac{1}{\sqrt{6}} \cdot (|P_u\rangle + |P_d\rangle + 2 |P_s\rangle) \quad (3a) \]

\[ |\pi^o\rangle = \frac{1}{\sqrt{2}} \cdot (|P_u\rangle - |P_d\rangle) \quad (3b) \]

gives three types of contributions for the $B^-$ decay into a kaon and a pseudoscalar meson.

\[ B^- \rightarrow \bar{u}c\bar{c}s \rightarrow u\bar{s} + (q\bar{q})_1 \rightarrow |R\rangle \quad (4a) \]

\[ B^- \rightarrow \bar{u}u\bar{s}s \rightarrow |K^- P_u\rangle \quad (4b) \]

\[ B^- \rightarrow \bar{u}u\bar{s}s \rightarrow u\bar{s} + (q\bar{q})_1 \rightarrow |R\rangle \quad (4c) \]

where the state $|R\rangle$ is defined as

\[ |R\rangle \equiv \frac{1}{\sqrt{3}} \cdot (|K^- P_u\rangle + |K^- P_s\rangle + |K^o\pi^-\rangle) \quad (5a) \]

and $P_u$, $P_d$ and $P_s$ denote the $u\bar{u}$, $d\bar{d}$ and $s\bar{s}$ components in the $\pi^o$, $\eta$ and $\eta'$ pseudoscalar mesons. This gives the result

\[ |R\rangle = \frac{1}{\sqrt{6}} \cdot |K^-\pi^o\rangle + \frac{1}{\sqrt{2}} \cdot |K^-\eta'\rangle + \frac{\xi}{\sqrt{2}} \cdot |K^-\eta\rangle + \frac{1}{\sqrt{3}} \cdot |K^o\pi^-\rangle \quad (5b) \]

where $\xi$ is a small parameter to introduce a $K\eta$ contribution which vanishes in the SU(3) symmetry limit with the particular mixing angle $\langle 3 \rangle$ as a result of a cancellation between the contributions from the $P_u$ and $P_s$ components in the $\eta$ wave function. A small but finite value of $\xi$ is suggested for realistic models by the $K\eta$ suppression observed in other experimental transitions $\langle 17,26 \rangle$ like decays of strong $K^*$ resonances known to proceed via an even parity $u\bar{s} + $ singlet state. The possibility of a relatively large $CP$ violation in a small $K\eta$ branching ratio is discussed below.

The description (5) of the final state also expresses the contribution both of the penguin diagram $\langle 26 \rangle$ and of other diagrams proportional to the $b \rightarrow c\bar{c}s$ vertex where the $c\bar{c}$ pair is annihilated via a final state interaction.
The $B^-$ transition into any kaon-pseudoscalar state $|f\rangle$ is then

$$\langle f | T | B^- \rangle = A\langle f | R \rangle + B\langle f | K^- p_u \rangle + C\sqrt{3} \cdot \langle f | R \rangle \cdot \langle R | K^- p_u \rangle \quad (6a)$$

$$\langle f | T | B^- \rangle = (A + C)\langle f | R \rangle + B\langle f | K^- p_u \rangle \quad (6b)$$

where $A$, $B$ and $C$ denote the amplitudes for the three transitions (4) and describe respectively:

(A). A penguin or other decay leading to the final state via $s\bar{u} + $ singlet and $|R\rangle$. This amplitude is proportional to the $b \rightarrow c\bar{s}s$ CKM matrix element.

(B). A tree decay via the state $K^- p_u$ followed only by final state interactions which do not annihilate the the initial $u\bar{u}$ pair. This amplitude is is proportional to the $b \rightarrow u\bar{u}s$ CKM matrix element.

(C). A tree decay followed by a final state rescattering which goes via the $s\bar{u} + $ singlet state to the final state $|R\rangle$. This amplitude denoted by $C\sqrt{3}$ is proportional to the $b \rightarrow u\bar{u}s$ CKM matrix element.

The relative magnitudes and strong phases of these amplitudes are model dependent. They are simply related to the isospin and SU(3) amplitudes conventionally used to treat $B \rightarrow K\pi$ decays and give no new information for these analyses [8]. The new ingredient introduced by flavor-topology analyses [18] is the inclusion of the neutral flavor-mixed meson states $K\eta$ and $K\eta'$ modes with the same amplitudes and same parameters. This allows the extension to the $K\eta$ and $K\eta'$ modes of any dynamical or phenomenological treatment of $B \rightarrow K\pi$ decays without introducing additional parameters.

Substituting the relations (5b) into (6b) then gives the relations

$$\langle K^0 \pi^- | T | B^- \rangle = \frac{A + C}{\sqrt{3}}; \quad \bar{\Gamma}(B^- \rightarrow K^0 \pi^-) = \frac{A^2 + C^2 + 2A \cdot C}{3} \quad (7a)$$

$$\langle K^- \pi^o | T | B^- \rangle = \frac{A + C}{\sqrt{6}} + \frac{B}{\sqrt{2}}; \quad \bar{\Gamma}(B^- \rightarrow K^- \pi^o) = \frac{A^2 + C^2 + 2A \cdot C}{6} + \frac{B^2}{2} + \frac{(A + C) \cdot B}{\sqrt{3}} \quad (7b)$$

$$\langle K^- \eta | T | B^- \rangle = \frac{B}{\sqrt{3}} + \frac{\xi(A + C)}{\sqrt{2}};$$
\[ \Gamma(B^- \to K^- \eta) = \frac{B^2}{3} + \frac{\xi^2(A^2 + C^2 + 2A \cdot C)}{3} + \frac{2\xi(A + C) \cdot B}{\sqrt{6}} \]  
(7c)

\[ \langle K^- \eta' \vert T \vert B^- \rangle = \frac{A + C}{\sqrt{2}} + \frac{B}{\sqrt{6}}; \quad \Gamma(B^- \to K^- \eta') = \frac{A^2 + C^2 + 2A \cdot C}{2} + \frac{A^2}{6} + \frac{(A + C) \cdot B}{\sqrt{3}} \]  
(7d)

Direct CP-violation asymmetries are obtained from interference between the $A$ amplitude and the $B$ and $C$ amplitudes which have different weak phases.

\[ \Gamma(B^- \to \bar{K}^\circ \pi^-) - \Gamma(B^+ \to K^\circ \pi^+) = \frac{2A \cdot (C - \bar{C})}{3} \]  
(8a)

\[ \Gamma(B^- \to K^- \pi^0) - \Gamma(B^+ \to K^+ \pi^0) = \frac{2A \cdot (C - \bar{C})}{6} + \frac{A \cdot (B - \bar{B})}{\sqrt{3}} \]  
(8b)

\[ \Gamma(B^- \to K^- \eta) - \Gamma(B^+ \to K^+ \eta) = \frac{2\xi A \cdot (B - \bar{B})}{\sqrt{6}} \]  
(8c)

\[ \Gamma(B^- \to K^- \eta') - \Gamma(B^+ \to K^+ \eta') = \frac{2A \cdot (C - \bar{C})}{2} + \frac{A \cdot (B - \bar{B})}{\sqrt{3}} \]  
(8d)

where an overall phase convention is defined in which the $A$ amplitude has the same phase for $B^-$ and $B^+$ decays and $\bar{B}$ and $\bar{C}$ respectively denote the $B$ and $C$ amplitudes for $B^+$ decays.

The $A$ amplitude is dominated by the penguin and expected to be much larger than the $B$ and $C$ amplitudes. Thus $\Gamma(B^- \to K^- \eta)$ is expected to be much smaller than for the other decays. However, to first order in the small parameter $\xi$ and the small ratios $B/A$ and $C/A$,

\[ \frac{\Gamma(B^- \to K^- \eta) - \Gamma(B^+ \to K^+ \eta)}{\Gamma(B^- \to K^- \eta) + \Gamma(B^+ \to K^+ \eta)} \approx \frac{3\xi A \cdot (B - \bar{B})}{\sqrt{6}B^2} \]  
(9)

This is of order $(\xi A/B)$ while the analogous relative asymmetries for other decay modes are of order $(B/A)$ and $(C/A)$. Thus even though the signal for CP violation (8c) may be small for $B^+ \to K^+ \eta$, the signal/background ratio (9) may be more favorable. An exact theoretical calculation of $\xi$ is not feasible. A good estimate from future data may enable a choice between different decay modes as candidates for observation of direct CP violation.

Higher resonances can be incorporated into the final state rescattering with simplifications from $C, P$ Bose symmetry and flavor $SU(3)$ symmetry. Since the vector-pseudoscalar
states have opposite parity, the next higher quasi-two-body final states allowed by conservation laws are the vector-vector s-wave and d-wave states. These can be incorporated by using models for the decays of a scalar resonance into these channels and inputs from polarization measurements and branching ratios for the vector-vector states.

The same approach can be used to treat vector-pseudoscalar final states. We label corresponding quantities by subscripts $V_P$ and $K V$ for $K^*$-pseudoscalar and kaon-vector decays respectively. Expressions for the $K \rho$ and $K \pi$ final states have opposite parity, the next higher quasi-two-body final states allowed by conservation laws are the vector-vector s-wave and d-wave states. These can be incorporated by using models for the decays of a scalar resonance into these channels and inputs from polarization measurements and branching ratios for the vector-vector states.

The $K^* P$ system differs from $K P$ because the relative phase of the the strange and nonstrange contributions of the $\eta$ and $\eta'$ are reversed [9,20]. The analogs of eqs. (5b -8) are thus

$$|R_{VP} \rangle = \frac{1}{\sqrt{6}} \cdot |K^{*-\pi^o}\rangle - \frac{1}{3\sqrt{2}} \cdot |K^{*-\eta}\rangle + \frac{2}{3} \cdot |K^{*-\eta'}\rangle + \frac{1}{\sqrt{3}} \cdot |K^{*o\pi^-}\rangle$$

$$\langle f | T | B^- \rangle = (A_{VP} + C_{VP})(f | R) + B_{VP}(f | K^{*-\pi^o})$$

$$\tilde{\Gamma}(B^- \to \bar{K}^{*o\pi^-}) = \frac{A_{VP}^2 + C_{VP}^2 + 2A_{VP} \cdot C_{VP}}{3} + \frac{B_{VP}^2}{2} + \frac{(A_{VP} + C_{VP}) \cdot B_{VP}}{\sqrt{3}}$$

$$\tilde{\Gamma}(B^- \to K^{*-\pi^o}) = \frac{B_{VP}^2}{3} + \frac{4(A_{VP}^2 + C_{VP}^2 + 2A_{VP} \cdot C)}{9} + \frac{4(A_{VP} + C_{VP}) \cdot B}{3\sqrt{3}}$$

$$\tilde{\Gamma}(B^- \to K^{*-\eta}) = \frac{B_{VP}^2}{18} + \frac{2A_{VP} \cdot C_{VP}}{3} + \frac{B_{VP}^2}{6} - \frac{(A_{VP} + C_{VP}) \cdot B}{3\sqrt{3}}$$

$$\tilde{\Gamma}(B^{\pm} \to K^{*\pm \eta'})$$

$$\tilde{\Gamma}(B^- \to K^{*o\pi^-}) - \tilde{\Gamma}(B^+ \to K^{*o\pi^+})$$

$$\tilde{\Gamma}(B^- \to K^{*-\pi^o}) - \tilde{\Gamma}(B^+ \to K^{*+\pi^o}) = \frac{2A_{VP} \cdot (C_{VP} - \bar{C}_{VP})}{3}$$

Here $\Gamma(B^- \to K^{*-\eta'})$ is expected to be much smaller than the other decay widths. The reversal of $\eta'/\eta$ ratio has been suggested as a test for the presence of an additional
component in the \( \eta' \) with a quantitative prediction based only on the \( A_{VP} \) amplitude. A relation which also includes the contributions from \( B_{VP} \) and \( C_{VP} \) is

\[
\tilde{\Gamma}(B^\pm \to K^{\ast\pm} \eta') = (1/3) \cdot \tilde{\Gamma}(B^\pm \to K^{\ast\mp} \pi^\pm) - (1/3) \cdot \tilde{\Gamma}(B^\pm \to K^{\ast\mp} \pi^0) + \frac{B_{VP}^2}{3} \tag{15}
\]

We now note one other case which also suggests that A...Z allowed transitions via a \( q\bar{q} + \) singlet intermediate state may be a general feature of final state interactions which warrants further investigation. Such a transition can produce the dramatic change in color suppression noted \[9\] between \( D \) and \( D_s \) decays which differ only by the flavor of a spectator quark. Since they must involve annihilation of the spectator quark they occur only for the color-favored \( D^o \) decays and color-suppressed \( D_s \) decays and not vice versa. They can thus compensate for the color-suppression not observed in the data for the \( D_s \) decays into \( K^{\ast\mp} \bar{K} \) and \( K^{\ast\mp} \bar{K}^* \) modes relative to color-favored \( \phi\pi \) and \( \phi\rho \), without affecting the definite color suppression seen in VP and \( VV \) \( D^o \) decays.

The nonstrange vector-pseudoscalar modes in both \( D \) and \( B \) decays already present other puzzles \[9\] which surprise theorists and provide interesting opportunities for future investigations. The role of \( G \) parity has been pointed out with reference to the \( \eta\pi, \eta'\pi, \eta\rho, \) and \( \eta'\rho \) for the \( D_s \) decays where four channels with different parities and \( G \) parities are not mixed by strong final state interactions \[9\]. The same is also true for \( B \) and \( B_s \) decays. For the \( VP \) decays, which have a definite odd parity, there are still two channels. One has odd \( G \)-parity like the pion and couples to \( \rho\pi \); the other has exotic even \( G \) and couples to \( \omega\pi, \eta\rho, \) and \( \eta'\rho \). This even-\( G \) state does not couple to any \( q\bar{q} \) state containing no additional gluons. It therefore does not couple to any single meson resonances, nor to the state produced by an annihilation diagram with no gluons emitted by the initial state before annihilation \[9\]. We now note that the coupling of the even-\( G \) state is A...Z-forbidden in the present model also for annihilation diagrams with additional gluons present because of cancellation between contributions from the \( uu \) and \( dd \) components of the \( \omega, \eta, \) and \( \eta' \) wave functions, whereas the two contributions add in the \( D_s \to \rho\pi \). Here the presence of charm in the \( \eta' \) wave function may be significant because of the generally overlooked contribution
of the “backward” weak diagram $s \rightarrow c\bar{ud}$.

Comparison of corresponding $D_s$ and $D$ decays into final states containing the $\eta$ and $\eta'$ mesons have been suggested \cite{25} as a means to test for the breaking of the nonet picture by additional flavor singlet components.

Further information which may provide important clues to this complicated four-channel system may be obtained from angular distributions in the $K\bar{K}\pi$ modes, including the $K^*\bar{K}$ and $K\bar{K}^*$ modes which decay into $K\bar{K}\pi^o$. The VP $K^*\bar{K}$ and $K\bar{K}^*$ modes are not individually eigenstates of $G$ parity and have unique p-wave angular distributions for the vector-pseudoscalar states. The $G$-parity eigenstates are coherent linear combinations of the two with opposite phase. They have opposite relative parity in the $K\bar{K}$ system, even though they are not produced from the same resonance. This relative parity can be observed as constructive or destructive interference in the kinematic region in the Dalitz plot where the two $K^*$ bands overlap. In a region where $s$ and $p$ waves dominate the angular distribution of the $K\bar{K}$ momentum in the $K\bar{K}$ center of mass system of the $K\bar{K}\pi$ final state relative to the pion momentum, one $G$ eigenstate will have a $\sin^2 \theta$ distribution, the other will have a $\cos^2 \theta$ distribution and interference between the two $G$ eigenstates can show up as a forward-backward asymmetry.

A general theorem from CPT invariance \cite{11} shows that all observed CP asymmetries must cancel when the data are summed over all final states or over any set of final states which are eigenstates of the strong-interaction S-matrix. Any CP asymmetry arising in a given channel must be canceled by an opposite CP asymmetry in some other channels. In the case of the model described by eqs. (8), this can occur only if there is a definite relation between the $A \cdot C$ and $A \cdot B$ interference terms. Any total CP asymmetry arising in a finite set of final states indicates significant strong interaction rescattering between these states and others outside the set; e.g. vector-vector or multiparticle final states. This casts doubt on theoretical estimates of direct $CP$ violation which do not include such rescattering.

Other attempts to estimating soft strong effects on CP violation in weak decays \cite{24} have used Regge phenomenology with parameters obtained from fits \cite{24} to total cross section
data. These fits are unfortunately highly controversial and unreliable [13]. Better fits to the same data with completely different parameters [13,12] have been obtained by using the physics input described above [14]. A recent Regge phenomenology calculation [28] for $B \to K\pi$ decays using PDG parameters [24] shows neither a dominant effect of order unity nor an insignificant effect of order 1%. Further improvement seems unlikely. In contrast the approach presented here uses well defined physics input subject to experimental tests. If these tests are successful they can lead the way to a considerable simplification in future treatments of FSI.

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