Abstract—The conventional approach for designing reinforced concrete members is based on load and resistance factors. In spite of the fact that, the load and resistance parameters are random variables, constant values are designated to them during the design. However, accounting these factors as constants, will ultimately lead to unsafe and uneconomical designs. Safe designs of structures require the appropriate recognition of parameters and their uncertainties. This may be achieved by clarifying the effective design parameters and applying risk-based design methods. The main purpose of this paper is the reliability-based design of reinforcement concrete beams under the simultaneous effects of bending, shear and torsion. For this purpose, a computer program is developed, and the most usual sections are selected for studying. Rectangular sections with tension rebars (singly reinforced), rectangular sections with tension and compression rebars (doubly reinforced) and T-shape sections are designed based on probabilistic methods. An appropriate tool for reliability calculations is selected based on the pros and cons of different methods. Evaluation of load and resistance factors for all mentioned beams is conducted. The steel usages under specific safety levels are determined. Hence, a method for the economic and fully probabilistic design of concrete beams is proposed.

Keywords—reinforced concrete beam; reliability-based design; monte-carlo simulation; safety factor; load and resistance factors

I. 1. INTRODUCTION

Using new designing methods based on reliability is a relatively old issue in civil engineering [1] and a lot of researches on reliability based design have been conducted because the existing uncertainties in designing parameters and the use of constants in regulations sometimes overshadow the design. So, using reliability methods based on the limit state governing the problem and paying attention to statistical parameters for each designing variable such as mean, standard deviation and statistical distribution in the various regulations of designing have been followed in various countries. For example, in [2] the shear force of reinforced concrete for a reinforced concrete beam in ACI regulations using a new reliability method is calculated. It has been indicated that various factors are effective in the failure of concrete beam including the ratio of the span length over the effective depth, the ratio of longitudinal steel, support conditions, loading conditions, and the type of materials. In (3), authors investigated the reliability of a concrete beam. The studied beam was designed under the effect of shear-bending-torsion. The failure level for three states of shear, bending, and torsion was considered according to the first order reliability method (FORM) and indicated the fact that the safety index is dependent to the rate of live load, the rate of effective parameters in beam strength as well as the uncertainty of the model.

In [5], authors studied risk management in the reliability issues of concrete infrastructures. In [6], authors investigated the reliability indices in beams and columns obtained for ACI-318 regulation and the designed structures. In [7], authors studied the punching shear using the reliability method in a flat slab. In [8], a research in order to obtain the reliability index to show shear and reinforcement of bridges concrete was carried out. In [9], authors studied the reliability of a concrete bridge in which the intended beam was under the effect of moment and shear forces. In [10], the concrete safety index was investigated with regard to uncertainty parameters in various material. In [11], Greek seismic regulations were investigated focusing at the calibration of the regulations for the beam-column and beam members. In [12], authors studied the impact of the calibration of regulations and the theory of probability in designing concrete bridges performance.

In accordance with the recommendations made in [4] and other statistical studies carried out by researchers such as [13, 14], the safety index for dead load + live load is 3 for bending and 2 for shear and torsion that in the present study the interval of the safety index to the proposed value is studied. The aim of this study is to investigate the safety index of Iranian concrete regulations for three limit states of shear, bending and torsion and also for the combination of the above limit states. The present study, suggesting the limit relations, has investigated the limit functions of regulation and has calculated the safety level and coefficients for all proposed states.

According to these studies, in the present paper after determining the factors of uncertainty in a beam affected by torsion and based on effective limit functions in designing, the
safety level of Iranian concrete regulations has been estimated using the Monte-Carlo method. To achieve this goal, the existing parameters of uncertainty have been recognized and its random production method is with a high number of about 1500000 and is based on statistical parameters such as mean, standard deviation and statistical distributions. Then, various limit states such as shear, torsion and bending have been considered and the safety level and the factors addressed in the regulations have been estimated.

II. DESIGNING OF CONCRETE BEAMS UNDER BENDING, SHEAR AND TORSION CONDITIONS

In order to calculate the safety index, we should calculate the limiter functions which show the structural efforts. The relations of bending calculating, shear and torsional efforts are presented in the following. The resistant moment of the section is given in (1), the shear force of the section in (2) and the torsion moment of the section in (3).

\[ M_r = A_f (d - \frac{A_s f_s}{1.7 f_d}) \]  
\[ V_r = 0.2 \phi b_d d f_c + \phi A_s d \frac{d}{s} \]  
\[ T_r = 2 \phi A_s A_f \frac{f_s}{s} \]

In (1) to (3), \( M_r \) is the section moment of resistance, \( V_r \) is the shear moment of resistance and \( T_r \) is the torsional moment of resistance. Also, the parameters of \( A_f \), \( f_c \), \( f_s \), \( b \) and \( d \) are the depth of section, width of section, the concrete compressive strength, tensile strength of the steel used and the amount of steel required in the section, respectively. Using (1) to (3) separately for the states of shear, bending and torsion based on forces applied on the sections, we have calculated the nominal values of the used steel with regulations [15]. In order to bending design in all sections, the compressive steel was used. In the case of L-shape and T-shape sections, the proposed equations have been used by regulations to obtain the section moment of resistance, while the torsion moment was not calculated. The factors considered in the concrete regulations of Iran to increase the load factor and reduce the resistance for designing are \( \gamma_0=1.25 \) and \( \gamma_1=1.50 \) in limit state for the dead load and live load respectively. As well as the reduction factors of steel and concrete strengths in designing stage at limit state are \( \phi_S=0.85 \) and 0.60 respectively. The needed statistical information to analyze the reliability are provided in Table I.

III. FORMULATION OF THE LIMIT FUNCTION AND SAFETY INDEX

For designing all possibilities of concrete beams under the simultaneous efforts of bending, shear and torsion, the limit functions must meet the following conditions:

\[ M_r \leq M_{x_i} \]
\[ V_r \leq V_{i} \]
\[ T_r \leq T_{i} \]  

In (4), the limit values are forces that the beam can tolerate. To estimate the safety index, first the amount of steel required of section for three mentioned forces is calculated and then using Hasofer-Lind equation [9-15], safety index is calculated according to (5).

\[ \beta = \frac{\mu_s - \mu_t}{\sqrt{\sigma_s^2 + \sigma_t^2}} \]

In this study, the safety index is studied for different limit states in the concrete beams with different aspects using Monte Carlo technique.

| Random variable | Nominal value | Density Function | Mean | Standard deviation |
|-----------------|---------------|------------------|------|--------------------|
| \( f'_c \) (MPa) | 21            | Normal           | 19.3 | 0.18               |
| \( f'_s \) (MPa) | 420           | Normal           | 317  | 0.12               |
| Dimension (mm)  |               |                  |      |                    |
| \( b \)         | Normal        | b                | b/10 |                    |
| \( h \)         | Normal        | h                | h/17 |                    |
| \( d \)         | Normal        | d                | d/15 |                    |
| Area (mm²)      |               |                  |      |                    |
| \( A_f \)       | Normal        | A_f              | A_f  | 0.03               |
| \( A_s \)       | Normal        | A_s              | A_s  | 0.03               |
| Loading         |               | Normal Gamble    | 1.05D| 0.1                |
| Dimension (mm)  |               |                  |      | 0.40-0.25          |

A. Safety level of shear-torsion

Based on the previous studies, by limiting the equation (6) to a specified value, the combination of simultaneous effect of shear-torsion can be achieved in various sections of concrete.

\[ \frac{V_r}{b_d d} + \frac{2 T_r (b_d + d)}{(b_d)^2} < 0.25 \phi f'_c \]

The safety level estimation of Iranian concrete regulations for shear and torsion is as follows: The amount of the shear steel is calculated according to the considered section and also (2). The calculating method is such that by assuming the certain amount of the dead load and placing the parameter ratio=(1-t)/t and changing the value of t from 0.4 to 1, the amount of live load gets calculated. The amount of shear load is obtained from adding the live load to the dead load and finally, with respect to (2) the required amount of shear steel is obtained. Applying the value of the moments acting on the beam (the moment of dead load + the moment of live load) and (3), the required torsional steel is calculated. The method of calculating the total moments exerted on the beam is similar to those of shear state and is practical by considering the proposed ratio in [3] for the amount of t. The limiting function considered for solve the problem by take the formulas presented in regulations for shear, torsion and the combination of shear and torsion is as in (7) to (9), respectively.
In the above equations, the index S is the load and moment exerted on the section (demand), r is the resistance level of the section (capacity) against acting loads. In sections 3-2 to 6-2, different levels of regulations are presented for various combinations of forces.

B. The safety level of bending-torsion

To investigate the safety level of Iranian regulations, the amount of shear and bending steel is calculated using (1) to (3). After calculating the amount of the moment of resistance exerted on the section and the amount of the applied loads, a limiting function such as in (10) and (11) is considered.

\[
M_r = A_f \left( \frac{d \cdot \frac{A_f}{1.7 t b_d}}{b_d} \right) M_c
\]

\[
G = \left( \frac{M_c}{I} + \frac{2T_t (b_d + d)}{(b_d)^2} \right) \frac{M_c}{I} + \frac{2T_t (b_d + d)}{(b_d)^2}
\]

In the equations above, the effects resulted from the loads (live+dead) is shown with index S, and the resistance level of the section against the exerted loads according to the characteristics of the constructed section is shown by index r.

C. The safety level of bending-shear

Previously the amount of the required steel has been calculated. Also, the amount of the forces acting on the section and the strength of the section has been obtained for three modes of shear, torsion and bending. For a combination of the bending and shear, the following limiting function is used.

\[
G = \left( \frac{v_s}{b_d} \frac{M_c}{I} - \frac{v_y}{b_d} + \frac{M_c}{I} \right)
\]

D. The safety level of bending-torsion-shear

To estimate the safety level of Iranian regulations for the three modes of shear, bending and torsion, the following limiting function are considered.

\[
G = \left( \frac{M_c}{I} + \frac{2T_t (b_d + d)}{(b_d)^2} \right) \frac{M_c}{I} + \frac{2T_t (b_d + d)}{(b_d)^2} + \frac{v_y}{b_d}
\]

In the limiting function above, three states of shear, bending and torsion have been observed simultaneously and by considering the limit state above, the safety level was estimated. To investigate the influence of dead load on the safety level, the numerical value of dead load was changed.

IV. RESULTS

To investigate the safety index of concrete beams, a section with given geometrical specifications was considered as shown in Figure 1. The extensive load of 25kN/m was applied on the beam. The live load is t*qD and the beam sections were L-shape, T-shape and rectangular. Choosing the beam was in such a way that the section area of the T-shape beam was two times bigger than the section area of the L-shape beam and for rectangular section was as d·b·w. For example, the total section and designing table have been presented. Designing of the beam, at first, has been done in accordance with Iranian concrete regulations.

Table II. The nominal used values

| b_d (mm) | d (mm) | L (mm) | M_L=ratio*5bd+1-t/t=1 |
|----------|--------|-------|------------------------|
| 400      | 200    | 2000  | 420                    |
| 200      | 400    | 2000  | 420                    |

A. The safety level of regulations for states of torsion-shear

The safety index has been calculated considering the limiting function and statistical values given in Table I. The calculation method is as follows. Nominal values such as the dimensions of the beam, the amount of shear-torsional steel, the amounts of exerted loads on the section, the concrete compressive stress and the tension of steel fluidity and etc., were considered as random variables according to Table I. Also, using the statistical values obtained such as the mean, standard deviation and statistical distribution, we have calculated the exact amount of shear force acting on the beam and the moment of torsion. The method of estimating the values of the moment of torsion and the tolerable shear force of the section was the extensive production of random numbers per each cycle written in MATLAB [16]. The Monte Carlo Method was used in procedures, which at the beginning of the cycle the values of each variable for each cycle of production, the amount of the moment and resistance (R) were calculated and at the end of each cycle the amount of load exerted on the section (S), the total load acted on the section and the moment of shear will be calculated. The safety indices in the section for various load ratios are estimated using (5).
In Figure 2, the impact numbers of Monte Carlo periodic cycles on the numerical value of the safety index are shown. This figure shows the safety index of Iranian concrete regulations for torsion and shear per different speeds. The considered section to investigate the number of Monte Carlo cycles is rectangular.

By considering Figure 2, it is clear that for all load values the recommended safety level of Iranian concrete regulations is higher than the suggested number of 2 for shear and torsion (20). Safety index reaches its maximum in number of 0.6. Also, speeding up the cycle to the number of 15000 estimates the safety level with a good approximation and for higher speeds the safety index showed little changes. As is seen in the figure, the calculations for the number of rounds to 10 times, i.e., up to 150,000 rounds have been done, but the safety index has not changed much. So, in follow-up research the number of simulated rounds has been limited to 15,000. As can be seen from the figure, reducing the live load (increasing the numerical value of $t$) has led to a decline in the safety index that is likely because of the loss of influence of the live load factor (factor of 1.5).

B. The safety level of bending-torsion

Using the formulas presented earlier, the safety index is calculated for loading combination of bending-torsion. In order to present and compare the results better, bending levels have been shown with separated lines and the torsional levels with dotted lines and the combination of the considered states with continuous lines; the rectangular sections are shown with blue color, L-shape section with red color, and T-shape section with green color. In the graph shown below the safety index for each section has been presented.

According to Figure 3, it is clear that the regulation level of Iranian concrete for torsion is from 2.20 to 2.70, and the average obtained is 2.48. Also, the regulation level of Iranian concrete for bending is between 3 and 4 and the average obtained is 3.8. The regulation level of Iranian concrete is for bending and torsion is between the values of bending and torsion, and the average obtained is 2.90 for bending-torsion. The maximum of safety index for combination of shear-bending occurs at $t=0.8$.

C. Safety level of bending-shear state

Figure 4 shows the regulation safety level of Iranian concrete for shear-bending. Figure 4 also shows the regulation level for limit state of shear-bending. The regulation level in the combination of the two limit states will skew towards shear. As seen in the figure, the regulation level at shear and bending modes are near to each other. Shear and bending levels and the combination of the two states have been shown with dotted points, dotted lines and line respectively. To present and compare the results better the results of shear levels have been shown with dotted points, the combination of the considered states has been shown with lines, and the rectangular sections have been shown with blue color, the slab section with red color, and T-shape section with green color.

D. The safety level of bending-torsion-shear

Combination of the shear, bending and torsional states has been done using existing formulas. Figure 5 shows the maximum safety index for the three states above related to T-shaped section that is 3.05.
E. Estimation of the regulation safety factors for limit states.

Safety factors calculated with the safety index (shear-torsion-bending)

In the following section, the factors of regulations are estimated. In order to estimate the factors, only the data of the rectangular section have been used. To obtain the safety factor, at first a wide range of factors are used. Then, using the existing formulas in the Iranian concrete regulations and the applied factors, the nominal values of steel are calculated. Next, the safety index of the considered limit functions are calculated without the use of any of the factors existed in the regulations and by using Monte Carlo cycle. In Figures 5 to 8, the coefficients obtained from the Monte Carlo technique and the intended safety index are presented. As it can be seen from the figures, the estimated safety factors have a direct influence on the safety index and they affect the fluctuations of the safety index. Also, it can be seen that the increase of the amount of live load and dead load, and consequently reducing the resistance reduction factors (steel, concrete) increases the safety index while the increase is linear.

Figure 8 illustrates this important issue that at the states of shear and torsion, Iranian concrete regulations are close to torsion and at the states of torsion and bending, Iranian concrete regulations are near to torsion. As can be seen in figure 8, the increase of resistance reduction factor ($\phi_C, \phi_S$) reduces the safety index, because the safety index is directly affected by loads than resistance and resistance parameters, and the more the effect of the load, the higher the safety level; Therefore, to investigate the importance of these two factors in the safety index, the increase of resistance factors (that leads to a decrease in safety index) and the increase of load factors (that leads to an increase in safety index) have been studied and the safety levels and their affectability from the factors are compared and discussed.

Figures 6, 7 and 8 indicate that with respect to regulation formulas, the effect of factors in safety index at states of shear and torsion is relatively lower than load factors and on the contrary, at the bending state the Iranian regulation level concrete is more affected by resistance reduction factors; and at the combinational state of torsion-bending (the index is more affected by torsion) compared with the state of shear and bending (the index is more affected by shear), the regulation level is more affected by the increase of resistance reduction factors and the increase in load at torsional state could not affect the safety index and the resistance factors have been more effective on safety index at state of bending-torsion. The slopes of drawn lines are indicative of the fact.
The following conclusions can be obtained:

- At bending state and its combination with torsion, the regulation level is more affected by resistance reduction factors and the increase of these factors decreases the safety index further.

- According to the dimensions and load given, the regulation level is affected by shear steel. Certainly, the change of dimensions leads to the changes in used steel and the rate of affectability from shear because the size and composition of load directly affect the safety level.

- The regulation level of Iranian concrete at general state of the combination of bending-torsion-shear at $t=0.7, 0.9, & 1$ is affected by the increasing of load factors and at $t=0.4-7$ the regulation level is affected by resistance reduction factors; therefore the increase of live load to dead load directly affects the safety level in all regulation states.

- For an increase of the amount of $t=0.7, 0.9, & 1$ in the state, the shear is effective and consequently the increase in load factors have more power for increasing the safety index, because in these states the increase of the effect of the live load factor has affected the safety level.

- The absolute values of the slope of the drawn lines are the power of the factors in changing the numerical value of the safety index.

V. CONCLUSION

This study investigated the safety level of Iranian concrete regulations for limited combinations of torsion-shear-bending and obtained the regulation safety level of Iranian designs using the Monte Carlo technique. The basic results are as follows:

- The safety index obtained for bending, shear, and torsional state is set between 2 and 3 (considering that in shear state the safety level is higher than at the bending state, and in the bending state it is higher than at the torsional state).

- The combinations of limit states for shear, torsion and bending are near to the safety index of shear and in load combinations reaches to its maximum level which indicates that the increase level of the live load factor and dead load factor are at their maximum level.

- An increase in the section area will increase the safety level of regulations; because at the amount of fixed load the increase in concrete results to the reduction of resistance factors that directly affect the safety index.

- At all calculated load states, the regulation level is set higher than the recommended safety index, which indicates that factors existing in regulations may result to an unsafe design. Certainly, changing of the dimensions of the beam and also the rate of exerted loads will affect the safety index. Therefore, a design based on regulation principles results in changing the safety in different conditions; so the accurate design must be done with careful study and clarification of the utilization goals according to the lifetime of the considered structure.

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