Analysis of bending plates with mixed boundary conditions using generalized equations of finite difference method

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Abstract. Currently, plates with characteristics of high thermal insulation, sound insulation and strength are sufficiently and widely used in structural engineering. To describing the connect of the plates with other elements, it is possible to give many calculation schemes on the different views. In this paper, it is considered the problem of rectangular plates, each side of which are fixed with hinged support or fixed support. This paper deals with the discontinuous points of boundary conditions with the use generalized equations of finite difference method. The obtained results of the analysis of stress state in discontinuous area of boundary conditions are compared with the results of work of Smirnov V.A.

1. Introduction

To solve the problem of bending plates with mixed boundary conditions, the analysis were carried out using different methods. In the researches, it can be shown that the work of Asadi, Fariborz [1], which analyzed the dynamic of composite plate with using differential quadrature method, and the work of Mizusawa, Leonard [2] studied about the bucking of thin plate with using spline element method and the work of Ruocco, Fraldi [3] is presented a model of bucking of rectangular plates by using the method of separation of variables, and the work of Li, Narita [4] analyzed about vibration of laminated plates by using the Rayleigh- Ritz method and the work of Igo, Jan, Vladyslav, Andrey [5] introduced the theory of plates to solve the mixed problems using asymptotic methods.

In this paper, an approach of problem of rectangular plates with mixed boundary conditions by using generalized equations of finite difference method [6] is presented. The proposed method allow to write equations for boundary points without passing the edges of the plate and equations for points at the edges of the load without thickening the grid. In process of contruction of industrial buildings the calculation problem of bending plates with fixed or partially fixed sides (figure 1) will appear. The article show how to determine the bending moments that appears in cross sections for discontinuities of boundary conditions.

2. Theory background and examples

Example 1: figure 1 shows a square plate with the fixed edges by the dashed line and the hinged support by the double line. The plate subjected to along distributed load with dimensionless value \( p = 1 \). Dimensionless length of each side of plate is also unity. In this paper, based on the analysis of numerical methods [7-12] it is presented the grid division and calculation of plate according to figure 1. Calculation scheme is shown in figure 1 with the minimum number of divisions. To illustrate the
application of the generalized equations of finite difference method, they are written for the boundary points. Equations in general case are given in [6]. According to simplified into differential equations of fourth order for two differential equations of second order, they will be obtained in the dimensionless forms:

\[
\frac{\partial^2 m}{\partial \xi^2} + \frac{\partial^2 m}{\partial \eta^2} = -p; \quad (1)
\]

\[
\frac{\partial^2 w}{\partial \xi^2} + \frac{\partial^2 w}{\partial \eta^2} = -m; \quad (2)
\]

In which:

\(\xi, \eta\) - dimensionless coordinate (figure 1);

Applying the dimensionless values:

\[
m = \frac{M}{q_0a^2}; \quad M = \frac{M_x + M_y}{1 + \mu}; \quad p = \frac{q}{q_0}; \quad W = \frac{W.D}{q_0a^4};
\]

In which:

\(M_x, M_y\) - bending moments in the directions \(x, y\);

\(M\) - sum of bending moments in the system;

\(\mu\) - Poisson’s ratio;

\(W\) - deflection;

\(D\) - cylindrical stiffness;

\(a\) - characteristic size of plate;

\(q\) - distributed load;

\(q_0\) - fixed value \(q\).

It will be found the difference approximation of differential equation (1) by the generalized equations of finite difference method for boundary points \(ij\) as private case of equation (2.1.12) is given in [6] with changing of elements III; IV to I; II in which \(\omega = m; \quad \sigma = \delta = \beta = 0; \quad \tau_i = \tau = h = \xi = h = \eta; \quad \Delta m^i_j = -\xi^i_j + \eta^i_j\) where I, II - numbers of adjacent elements to point \(ij\) in figure 1. This is a point 00:

\[
\begin{align*}
\xi^i_j - h: & -m_{i+1,j}^{0} - 2\cdot m_{i,j}^{0} + m_{i-1,j}^{0} - \\
\eta^i_j - h: & -m_{i,j+1}^{0} - 2\cdot m_{i,j}^{0} + m_{i,j-1}^{0} + \\
& + h\Delta m_{i,j}^{0} = \\
& = -\frac{h^2}{2} \left( p_{ij} + \xi^i_j \right).
\end{align*}
\]

In which: \(m_{ij}^{0} = \frac{\partial m}{\partial \eta_{ij}}\).

It is written (3) for point 10 in figure 1 taking into account the symmetry, boundary conditions

\(h = \frac{1}{4}; \quad m_{20} = m_{00} = 0:\)

\[\begin{align*}
-h^i_{0,j} - h^\eta_{0,j} - 2h^i_{0,j} - 2\xi^i_{0,j} + 2\eta^i_{0,j} + h\Delta m_{0,j}^{0} = -\frac{1}{4},
\end{align*}\]
In which: \( \Delta m_{10}^\phi = m_{10}^\phi - \mu m_{10}^\phi = \frac{1}{h}( - m_{00} + \imath m_{10} + \mu m_{10} - m_{20} ) = \frac{1}{h} \cdot \mu m_{10} \).

(5)

In these equations:

\[ \mu m_{00} = m_{10} + 0; \imath m_{10} = m_{10} - 0 = m_{20} = 0; \]

(6)

\[ m_{10}^\phi = \frac{\partial m_{10}}{\partial \eta_{10}}; \imath m_{10}^\phi = m_{10}^\phi + 0; \] \[ \mu m_{10}^\phi = m_{10}^\phi + 0. \]

(7)

Taking into account of symmetry the equations are written for other points with \( h = \frac{1}{4} \) and \( m_{00} = 0 \) taking into (6) and (7). For points 01, 11, 02 in figure 1 we use generalized equations of finite difference method is given in [6] according to the numbers (2.2.6). With writing (2.2.6) from [6] we take into account:

\[ \imath p_{0} = \mu p_{0} = \mu p_{0} = \nu p_{0} = p_{0}; \imath - \mu \Delta m_{0}^\phi = \mu - \nu \Delta m_{0}^\phi = \Delta m_{0}^\phi; \tau = h; \omega = m; \]

(8)

\[ \imath m_{i+1,j} + \mu m_{i-1,j} + \]

\[ + \imath m_{i+1,j+1} - 2( \imath m_{ij} + \mu m_{ij}) + \mu m_{i+1,j+1} + \]

\[ + \mu m_{i+1,j+1} - 2( \mu m_{ij} + \nu m_{ij}) + \nu m_{i+1,j+1} + \]

(9)

\[ \imath m_{i+1,j} + \mu m_{i+1,j} + \]

\[ +2h( \Delta m_{0}^\phi + \Delta m_{0}^\phi) = -2h^2 . p_{0}. \]

For above-mentioned points 01, 02, 11, equation (9) is written taking into account the symmetry; \( p=1, h=1; m_{00}=m_{20}=0; \)

point 01: \( 2m_{01} - 4m_{00} + m_{02} = -\frac{1}{4^2} \)

(10)

point 02: \( 4m_{01} - 4m_{00} = -\frac{1}{4^2} \)

(11)

point 11: \( 2m_{01} + 2m_{01} = 4m_{20} = -\frac{1}{4^2} \)

(12)

To determine deflections \( w \), since the differential equations (1) and (2) are different from each other only by the designations, it is sufficient to change \( m, \mu \) in (3) and (9) to \( w, \mu \) and it will put \( \Delta w_{0}^\phi = \Delta w_{0}^\phi = 0; \) it will be obtained:

\[ \imath w_{i+1,j} - \]

\[ -h. \imath w_{0}^\phi - 2. \imath w_{0}^\phi + \imath w_{i+1,j+1} - \]

\[ -h. \mu w_{0}^\phi - 2. \mu w_{0}^\phi + \mu w_{i+1,j+1} + \]

\[ + \mu w_{i+1,j+1} = \frac{h^2}{2}( \imath m_{0} + \mu m_{0} ). \]

(13)

\[ \imath w_{i+1,j} + \mu w_{i+1,j} + \]

\[ + \imath w_{i+1,j+1} - 2( \imath w_{ij} + \mu w_{ij}) + \mu w_{i+1,j+1} + \]

\[ + \mu w_{i+1,j+1} - 2( \mu w_{ij} + \nu w_{ij}) + \nu w_{i+1,j+1} + \]

\[ \imath w_{i+1,j} + \mu w_{i+1,j} + \]

\[ +2h( \Delta w_{0}^\phi + \Delta w_{0}^\phi) = -2h^2 . m_{0}. \]

Put \( \Delta w_{0}^\phi = \Delta w_{0}^\phi = 0, \mu w_{0}^\phi = 0, \) it will be obtained according to (13) and (14):
We have 7 equations (10)–(12), (15)–(18), 7 unknowns $m_{01}$, $m_{11}$, $m_{02}$, $m_{10}$, $w_{01}$, $w_{11}$, $w_{02}$. From solutions of these equations it will be obtained:

$$m_{01} = -0.0497; \quad m_{11} = 0.0057; \quad m_{02} = 0.0298; \quad m_{10} = 0.0455; \quad w_{01} = 0.000777; \quad w_{11} = 0.0014; \quad w_{02} = 0.0021.$$

At the point of contact of different boundary conditions, the values of moment ($m_{10}$) sharply increase.

**Figure 1.** Calculation scheme of bending plate with mixed boundary conditions and grid of coordinate lines.

Example 2– plate with other boundary conditions. Shaded sites- fixed edges and double sites- fixed hinged support (figure 2).

**Figure 2.** Calculation scheme of bending plate according to V.A. Smirnov [12].
Based on the theory background of example 1, it will be obtained the calculation results of example 2 in the following Table 1.

The solution of this research is found on four grids changed from one into another is given in the table and compared with the solutions of V.A. Smirnov [12] received on grid 8x8.

| Value | 4x4 | 8x8 | 16x16 | 32x32 | According to [12] 8x8 | Inaccuracy |
|-------|-----|-----|-------|-------|----------------------|------------|
| \(w_{02}\) | 0.00171 | 0.00141 | 0.00134 | 0.00131 | 0.00142 | 7.74% |
| \(m_{10}\) | 0.0407 | 0.0367 | 0.0359 | 0.0356 | 0.0368 | 3.26% |
| \(l_{m_{10}}\) | -0.0782 | -0.0586 | -0.0547 | -0.0535 | -0.0603 | 11.28% |

Summary

Practical coincidence of the results allows to recommend the proposed algorithm for practical use of design process in construction for real structures.

References

[1] E Asadi, S J Fariborz 2012 Free vibration of composite plates with mixed boundary *Archive of applied mechanics* 755-766.
[2] T Mizusawa J, W Leonard 1990 Vibration and buckling of plates with mixed boundary conditions *Engineering structures* 12 (4) 285-290.
[3] E Ruocco, M Fraldi 2012 An analytical model for the buckling of plates under mixed boundary conditions *Engineering structures* 38 78-88.
[4] J Li, Y Narita 2013 Vibration suppression for laminated composite plates with arbitrary boundary conditions *Mech Compos Mater*. 49 519–530. https://doi.org/10.1007/s11029-013-9368-9.
[5] Igor Andrianov, Jan Awrejcewicz, Vladyslav Danishevs'ky, Andrey Ivankov 2014 *Asymptotic methods in the theory of plates with mixed boundary conditions* (John Wiley & Sons, Ltd).
[6] Gabbasov R F, Gabbasov A R, Filatov V V 2008 *Numerical formulation of discontinuity problems of structural mechanics* (Moscow, ACB Publisher).
[7] Bate K, Wilson E 1982 *Numerical analysis methods and the finite element method* (Structural publisher, Moscow) 447.
[8] Dlugach M I 1964 The grid method in the mixed plane problem of the theory of elasticity (Naukova Dumka, Kiev) 260.
[9] Dlugach M I 1966 *Some problems of applying the grid method to the calculation of plates and shells* (In the book. Electronic computers in structural mechanics, Moscow, Structural publisher) 555-560.
[10] Kiselyov V A 1973 *Calculation of plates* (Moscow, Structural publisher) 151.
[11] Bakhvalov N S 1973 *Numerical methods* (Moscow, Science) 632.
[12] Smirnov V A *Calculation of plates of complex shape* (Moscow, Structural publisher) 300.