Current noise in long diffusive SNS junctions in incoherent MAR regime.

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Spectral density of current fluctuations at zero frequency is calculated for a long diffusive SNS junction with low-resistive interfaces. At low temperature, \( T \ll \Delta \), the subgap shot noise approaches linear voltage dependence, \( S = (2/3R)(eV + 2\Delta) \), which is the sum of the shot noise of the normal conductor and voltage independent excess noise. This result can also be interpreted as the 1/3-suppressed Poisson noise for the effective charge \( q = e(1+2\Delta/eV) \) transferred by incoherent multiple Andreev reflections (MAR). At higher temperatures, anomalies of the current noise develop at the gap subharmonics, \( eV = 2\Delta/n \). The crossover to the hot electron regime from the MAR regime is analyzed in the limit of small applied voltages.

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During the last decade, considerable attention has been focused on the study of current fluctuations in mesoscopic systems, especially on shot noise which reflects the discrete nature of the electron charge and correlations between electrons. Whereas in normal ballistic systems with tunnel barriers the spectral density \( S \) of the shot noise at zero frequency approaches full Poissonian value \( S_P = 2eI \) mesoscopic diffusive wires shorter than the inelastic scattering length produce Poisson noise suppressed by a factor 1/3. The universality of this fact was demonstrated within different theoretical models, and confirmed experimentally. The inelastic scattering could change the suppression factor, which was used in experiments as a probe of the electron relaxation.

In hybrid normal-superconducting (NS) systems, the current transport at subgap voltages, \( eV < \Delta \), is associated with the transfer of an elementary charge \( 2e \), instead of \( e \), due to Andreev reflection of quasiparticles from the NS interface, which converts two electrons in the normal metal to a Cooper pair in the superconductor. As shown theoretically and observed in experiments, such doubling of the elementary charge leads to doubling of the subgap shot noise in NS junctions.

A more pronounced enhancement of the shot noise is expected in superconducting tunnel and SNS junctions whose subgap conductivity involves multiple Andreev reflections (MAR). In this case, the effective transferred charge \( q = (N_A + 1)e \) increases along with the number \( N_A \sim 2\Delta/eV \) of Andreev reflections of quasiparticles which overcomes the energy gap \( 2\Delta \) by elementary steps \( eV \). As a result, the shot noise spectral density \( S(V) \) at low voltages, \( eV \ll \Delta \), should greatly exceed the Poissonian value and approach a constant level, which can be estimated as \( S(0) = 2qV/R = 4\Delta/R \) for ballistic junctions with normal resistance \( R \), and as

\[
S(0) = 4\Delta/3R
\]

for diffusive junctions taking into account the 1/3-suppression factor. Alternatively, the enhanced noise may be interpreted as “thermal” noise generated by nonequilibrium quasiparticles within the whole subgap region \( |E| < \Delta \), which is a characteristic property of the MAR regime. The experimental observation of multiply enhanced shot noise was reported for NbN-based tunnel junctions and several types of SNS junctions.

Theoretical analysis of the shot noise in short junctions, with length \( d \) smaller than the coherence length \( \xi_0 = (hD/\Delta)^{1/2} \) in the superconductor, was done in Refs. 2 and 3. In such systems, the quantum coherence between the electrons and retroreflected holes extends over the entire junction, which leads to the ac Josephson effect and to non-ohmic \( I-V \) characteristic. Nevertheless, the estimate \( q \sim (N_A + 1)e \) for the effective transferred charge holds, although the zero-bias spectral density of the shot noise differs from Eq. (1). Along with the dc current, the shot noise reveals subharmonic gap structure (SGS), i.e., steps in \( S(V) \) or \( dS/dV \) at \( V = 2\Delta/ne \) \((n = 1, 2, \ldots)\). A semiclassical model of the shot noise in ballistic point contacts was proposed in Ref. 10.

In this paper we analyze the current noise in long diffusive SNS junctions, \( d > \xi_0 \). When the bias voltage is much larger than the Thouless energy \( E_{Th} = hD/d^2 \ll \Delta \) (\( D \) is the diffusion coefficient), then the size of the coherent proximity regions near the NS interfaces, \( \xi_E = (hD/E)^{1/2} \), is much smaller than the junction length \( d \) at all relevant energies \( E > eV \). In this case, the Josephson effect is suppressed, and the subgap current transport can be quantitatively described in terms of incoherent MAR similar to the quasiclassical theory for ballistic systems (OTBK). The shot noise in the incoherent MAR regime was analyzed in Ref. 2 for a diffusive SNS junction with a tunnel barrier inside the normal metal, assuming the barrier to dominate the junction resistance. Under these conditions, the shot noise is generated by tunneling electrons and can therefore be calculated within the tunnel approach. If the
resistance of the normal metal exceeds the resistance of possible tunnel barriers within the junction, shot noise emerges due to impurity scattering. In this case, it can be calculated within a Langevin approach, if we neglect the contribution of the small proximity regions in the vicinity of the interfaces. Following Ref. [19], in which the Langevin equation was applied to the current fluctuations in a diffusive NS junction, we derive an expression for the current noise spectral density in SNS junctions at zero frequency in terms of the nonequilibrium population numbers \( n_e^{e,h}(E, x) \) of electrons and holes within the normal metal, \( 0 < x < d \),

\[
S = \frac{2}{R} \int_0^d dx \int_{-\infty}^{\infty} dE \left[ n^e (1 - n^e) + n^h (1 - n^h) \right].
\]

(2)

The electric current through the junction is given by

\[
I = \frac{d}{2eR} \int_{-\infty}^{\infty} dE \, \partial_x (n^e - n^h).
\]

(3)

The population numbers obey the diffusion equation,

\[
D \frac{\partial^2 n}{\partial x^2} = I_e(n),
\]

(4)

with the inelastic collision term \( I_e \). At \( |E| > \Delta \), the boundary populations \( n_{0,d}(E) = n(E, x) \big|_{x=0,d} \) are local-equilibrium Fermi functions, \( n_0^e(E) = n_F(E) \), \( n_0^h(E) = n_F(E \pm eV) \) (we use the potential of the left electrode as the energy reference level). At subgap energies, \( |E| < \Delta \), the boundary conditions should be modified in accordance with the mechanics of complete Andreev reflection which equalizes the electron and hole population numbers at a given electrochemical potential and blocks the net probability current through the NS interface.\[2]

\[
n_0^e(E) = n_0^h(E), \quad n_0^e(E - eV) = n_0^h(E + eV), \\
n_0^f(E) + n_0^{h_f}(E) = 0, \quad n_0^f(E - eV) + n_0^{h_f}(E + eV) = 0,
\]

(5)

where \( n_{0,d}^e \) are the boundary values of the electron and hole probability flows \( \partial n/\partial x \).

In the absence of inelastic scattering, the population numbers are linear functions of \( x \), \( n^{e,h}(E, x) = n^{e,h}(E) + xn^{e,h}(x) \), which results in the recurrences for boundary populations and diffusive flows within the subgap region,

\[
n_0^{e,h}(E - eV) - n_0^{e,h}(E + eV) = \mp 2n_e^{e,h}(E \mp eV), \\
n_0^{e,h}(E - eV) = n_0^{e,h}(E + eV).
\]

(6)

According to Ref. [3], these recurrences are equivalent to the problem of “current” and “voltage” distribution in an equivalent network in energy space. In the present approximation, which assumes the contribution of the proximity effect and the normal scattering at the interfaces to be negligibly small, this network consists of a series of resistances of unit value \( R \) connected periodically, at the energies \( E_k = E + keV \), with the distributed “voltage source” \( n_F(E) \) (see Fig. 1). The “potentials” \( n_k \) of the network nodes with even numbers \( k \) represent equal electron and hole populations \( n_0^{e,h}(E + keV) \) at the left NS interface, whereas the potentials of the odd nodes describe equal boundary populations \( n_0^{e,h}(E + keV \mp eV) \) at the right interface. The “currents” \( I_k \) entering \( k \)-th node are related to the probability currents \( n_i(E_k) \) as

\[
I_k(E) = -dn_i(E_{k-1})(\text{odd } k) \quad \text{and} \quad I_k(E) = dn_i(E_k)(\text{even } k),
\]

(7)

and represent partial electric currents transferred by the electrons and holes across the junction, obeying Ohm’s law in energy space, \( I_k = n_{k-1} - n_k \). Within the gap, \( |E_k| < \Delta \), i.e., at \( -N_- < k < N_+ \), \( N_+(E) = \text{Int}[\frac{(\Delta \mp E)/eV}{1} + 1] \), the nodes are disconnected from the reservoir due to complete Andreev reflection and therefore all currents flowing through the subgap nodes are equal.

Due to periodicity of the network, the partial currents obey the relationship \( I_k(E) = I_m[E + (k - m)eV] \), and the boundary population \( n_0 \) is related to the node potentials \( n_k \) as \( n_0(E + keV) = n_k(E) \). This allows us to integrate the over energy in Eqs. (6) and (7) to an elementary interval \( 0 < E < eV \),

\[
I = \frac{1}{eR} \int_0^{eV} dE \sum_{k=-\infty}^{\infty} I_k,
\]

(7)

\[
S = \frac{2}{R} \int_0^{eV} dE \sum_{k=-\infty}^{\infty} 2n_k(1 - n_k) + \frac{1}{3}I_k^2.
\]

(8)

\begin{figure}[h]
\centering
\includegraphics[width=0.8\textwidth]{fig1.png}
\caption{Equivalent MAR network in energy space in the limit of negligibly small normal reflection and proximity regions at the NS interfaces.}
\end{figure}

The “potentials” of the nodes outside the gap, \( |E_k| > \Delta \), are equal to local-equilibrium values of the Fermi function, \( n_k(E) = n_F(E_k) \) at \( k \geq N_+ \), \( k \leq -N_- \). The partial currents flowing between these nodes,

\[
I_k = n_F(E_{k-1}) - n_F(E_k), \quad k > N_+ \quad k < -N_- \quad (9)
\]

are associated with thermally excited quasiparticles. The subgap currents may be calculated by Ohm’s law for the series of \( N_+ + N_- \) subgap resistors,

\[
I_k = \frac{n_+ - n_-}{N_+ + N_-}, \quad -N_- < k < N_+ \quad (10)
\]

where \( n_\pm(E) = n_F(E \pm N_\pm) \). From Eqs. (6) and (10) we obtain the current spectral density in Eq. (7) as
\[ \sum_{k=-\infty}^{\infty} I_k = 1, \] which results in Ohm’s law, \( V = IR \), for the net electric current through the junction. This conclusion is closely related to our disregarding the proximity effect and the normal scattering at the interface. Actually, both of these factors lead to the appearance of SGS and excess or deficit currents in the \( I-V \) characteristic, with the magnitude increasing along with the interface barrier strength and the ratio \( \xi_0/d \).

The subgap populations can be found as the potentials

\[ n_k = n_- - (n_- - n_+) \frac{N_- + k}{N_+ + N_-}. \tag{11} \]

By making use of Eqs. (3)-(11), the net current noise can be expressed through the sum of the thermal noise of quasiparticles outside the gap,

\[
S_\gamma = \frac{4T}{3R} \left\{ 2 \left[ n_F(\Delta) + n_F(\Delta + eV) \right] + \left[ \frac{eV}{T} + \ln \frac{n_F(\Delta + eV)}{n_F(\Delta)} \right] \coth \frac{eV}{2T} \right\}, \tag{12}
\]

and the subgap noise,

\[
S_\Delta = \frac{2}{3R} \int_0^{eV} dE(N_+ + N_-)[f_+ - f_- + 2(f_+ + f_-)], \tag{13}
\]

At low temperatures, \( T \ll \Delta \), the thermal noise \( S_\gamma \) vanishes, and the total noise coincides with the subgap shot noise, which takes the form

\[
S = \frac{2}{3R} \int_0^{eV} dE(N_+ + N_-) = \frac{2}{3R} (eV + 2\Delta), \tag{14}
\]

of 1/3-suppressed Poisson noise \( S = (2/3)qI \) for the effective charge \( q = e(1 + 2\Delta/eV) \). At \( V \to 0 \), the shot noise turns to a constant value \( 4\Delta^3/3R \) in Eq. (1), which is identical to the result of Ref. 1 and therefore seems to be universal for the incoherent MAR regime in long diffusive junctions and independent of the shot noise mechanism. At finite voltages, this quantity plays the role of the “excess” noise, i.e. the voltage-independent addition to the shot noise of a normal metal at low temperatures [see Eq. (1) of Fig. 1(a)]. Unlike short junctions, where the excess noise is proportional to the excess current \( I_0 \) in our system the excess current is small and has nothing to do with large excess noise.

Results of numerical calculation of the noise at finite temperature are shown in Fig. 1. While the temperature increases, the noise approaches its value for normal metal structures \( 3 \) with additional Johnson-Nyquist noise coming from thermal excitations. In this case, the voltage-independent part of current noise may be qualitatively approximated by the Nyquist formula \( S(T) = 4T^*/R \) with the effective temperature \( T^* = T + \Delta(T)/3 \). The most remarkable phenomenon at nonzero temperature is the appearance of steps in the voltage dependence of the derivative \( dS/dV \) at the gap subharmonics \( eV = 2\Delta/n \) [Fig. 2(b)], which reflect discrete transitions between the quasiparticle trajectories with different numbers of Andreev reflections. The magnitude of SGS decreases both at \( T \to 0 \) and \( T \to T_c \), which resembles the behavior of SGS in the \( I-V \) characteristic of long ballistic SNS junction with perfect interfaces within the OTBK model. A small “residual” SGS in current noise, similar to the one in the \( I-V \) characteristic \( 3 \) should occur at \( T \to 0 \) due to normal scattering at the interface or due to proximity effect [see comments to Eq. (13)].

\[
\begin{align*}
\text{FIG. 2. Spectral density } S \text{ of current noise vs voltage (a) and its derivative } dS/dV \text{ vs inverse voltage (b) at different temperatures. Dashed line shows the result for normal metal junction at } T = 0.3T_c. \end{align*}
\]

The role of inelastic scattering is to suppress the nonequilibrium of the subgap electrons created by MAR. Within the relaxation time approximation, the nonequilibrium distribution of the subgap quasiparticles holds as soon as the inelastic relaxation time \( \tau(E) \) at the characteristic energies \( E \sim \Delta \) is larger than the diffusion time through the junction, \( \tau_d(V) \sim (2\Delta/eV)^2 D/\Delta \). Thus, in this model, the enhanced shot noise (as well as the SGS in the \( I-V \) characteristic \( 3 \)) can be observed if the applied voltage is sufficiently large, \( eV > 2\Delta W_{e/2}^{-1/2} \), where \( W_{e} = E_{Th} \tau_{e}(\Delta)/\hbar \); at smaller voltages, \( S(V) \) should decrease and approach the thermal noise level. The noise temperature is equal to the physical temperature \( T \) if the inelastic scattering is dominated by electron-phonon interaction (assuming that the phonons are in equilibrium with the electron reservoir). If the electron-electron (e-e) scattering dominates, the noise temperature may exceed the temperature \( T \) of the electron reservoir if this temperature is small, \( T \ll \Delta \) (hot electron regime). The reason is that at low temperature, the subgap electrons are well decoupled from the reservoir (electrons outside the gap) due to weak energy flow through the gap edges.

In order to quantitatively analyze this situation, we consider the small voltage limit, \( eV \ll \Delta \), in equation Eq. (1), taking into account the e-e collision term. Due to weak spatial dependence of the population numbers at small voltage, they can be replaced by their boundary
values, \( n_{\text{eff}}^{e,h}(E) \approx n_{\text{eff}}^{e,h}(E) \equiv n(E) \), in the e-e collision integral \( I_{ee}(n) \). This allows us to easily include the collision term in the recurrences of Eq. (13). Within the same approximation, these recurrences are to be considered as differential relations, which results in the diffusion equation

\[
D_E \frac{\partial^2 n}{\partial E^2} = I_{ee}(n). \tag{15}
\]

where \( D_E = (eV)^2 \gamma e / h \) is the effective diffusion coefficient in energy space. The finite resistance \( R_{NS} \) of the NS interfaces, which partially blocks quasiparticle diffusion, can be taken into consideration by renormalization of the diffusion coefficient, \( D_E \rightarrow D_E[1 + (d/\xi_0)(R_{NS}/R)^2]^{-1} \) (see Ref. 13).

Equation (15) describes the crossover from the “collisionless” MAR regime to the hot electron regime as function of the parameter \( D_E \gamma e \Delta(E) / \Delta^2 \). In the hot electron limit, \( \Delta^2 \gg D_E \gamma e \Delta(E) \), the collision integral dominates in Eq. (15), and therefore the approximate solution of the diffusion equation is the Fermi function with a certain effective temperature \( T_0 \ll \Delta \). The value of \( T_0 \) can be found from Eq. (15) integrated over energy within the interval \( (-\Delta, \Delta) \) with the weight \( E \), taking into account the boundary conditions \( n(\pm \Delta) = \bar{n}_F(\pm \Delta) \) and neglecting the exponentially small derivative \( \partial n / \partial E \) at the gap edges. At zero temperature of the reservoir, we obtain an asymptotic equation for \( T_0 \),

\[
(eV)^2 W_e \exp(\Delta/T_0) = T_0 \Delta(1 + T_0/\Delta), \tag{16}
\]

which shows that the effective temperature of the subgap electrons decreases logarithmically with decreasing voltage. The noise of the hot subgap electrons is given by the Nyquist formula with temperature \( T_0 \),

\[
S(V) = (4T_0/R) \left[ 1 - 2 \exp(-\Delta/T_0) \right], \tag{17}
\]

where the last term is due to the finite energy interval available for the hot electrons, \( |E| < \Delta \). Equations (16), (17) give a reasonably good approximation to the result of the numerical solution of Eq. (15).

In summary, we have calculated current noise in a long diffusive SNS structure with low-resistive interfaces at arbitrary temperatures. Whereas the I-V characteristic is approximately described by Ohm’s law, the current noise reveals all characteristic features of the MAR regime: “giant” enhancement at low voltages, pronounced SGS, and excess noise at large voltages. In the limit of strong electron-electron scattering, the junction undergoes crossover to the hot electron regime, with the effective temperature of the subgap electrons decreasing logarithmically with the voltage.

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20 The unit values of the network resistances are provided by the normalization of “currents” \( I_n \) (note that it differs from the definition in Ref. 4 by the factor \( R^{-1} \)).