THE EFFECT OF WEAK GRAVITATIONAL LENSING ON THE COSMIC MICROWAVE BACKGROUND ANISOTROPY: FLAT VERSUS OPEN UNIVERSE MODELS

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ABSTRACT

We have studied the effect of gravitational lensing on the cosmic microwave background (CMB) anisotropy in flat and open universes. We develop a formalism to calculate the changes on the radiation power spectrum induced by lensing in the Newtonian and synchronous-comoving gauges. The previously considered negligible contribution to the CMB radiation power spectrum of the anisotropic term of the lensing correlation is shown to be appreciable. However, considering the nonlinear evolution of the matter power spectrum produces only slight differences on the results based on linear evolution. The general conclusion for flat as well as open universes is that lensing slightly smooths the radiation power spectrum. For a given range of multipoles the effect of lensing increases with $\Omega$ but for the same acoustic peak it decreases with $\Omega$. The maximum contribution of lensing to the radiation power spectrum for $l \leq 2000$ is $\sim 5\%$ for $\Omega$ values in the range $0.1–1$.

Subject headings: cosmic microwave background — cosmology: theory — gravitational lensing — large-scale structure of universe

1. INTRODUCTION

Cosmic microwave background (CMB) temperature anisotropies, detected for the first time by Smoot et al. (1992) with the COBE-DMR experiment, are believed to be generated by the interaction of matter density perturbations and radiation to first order in perturbation theory. Numerical codes used to solve the linearized Einstein-Boltzmann coupled equations are able to calculate the radiation power spectrum with an accuracy better than 1\% (see, e.g., Sugiyama 1996; Seljak & Zaldarriaga 1996; Bond 1997). Nonlinear density perturbations make a small contribution through the Rees-Sciama effect that, except for the case of reionization, can be constrained to be $\lesssim 1\%$ (Martínez-González, Sanz & Silk 1992; Sanz et al. 1996; Seljak 1996a; Tulüe, Laguna, & Anninos 1996). However, the effect of gravitational lensing on the CMB anisotropies, not included in the numerical codes, may appreciably affect the radiation power spectrum.

Many groups have studied the lensing of the microwave photons using different analytical and numerical approaches (Blanchard & Schneider 1987; Cole & Efstathiou 1989; Sasaki 1989; Tomita & Watanabe 1989; Linder 1990a, 1990b; Cayón, Martínez-González, & Sanz 1993a, 1993b; Fukushige, Makino, & Ebisuzaki 1994; Seljak 1996b). They arrive at different conclusions about the importance of the effect: the result depends on the particular cosmological model considered and on the assumptions made in the calculation. Cayón et al. (1993a, 1993b) present the formalism to obtain the lensing of the microwave photons by the large scale matter distribution in a flat universe with null/non-null cosmological constant. However, they erroneously used the photon deflection angle instead of the photon angular excursion on the last scattering surface relative to its observed value, which leads to a factor of a few overestimate of the relative dispersion between two photons (Seljak 1996b; Muñoz & Portilla 1996). Some of the previous studies have used models that may not be a realistic representation of the large-scale structure observed (e.g., the models used in Fukushige et al. 1994). Another relevant ingredient of the calculations is to appropriately account for the evolution of matter density perturbations. Recently, Seljak (1996b) has done a relevant step in solving those shortcomings of previous studies. Based on a power spectrum approach he includes linear and nonlinear regimes of the matter evolution in realistic cosmological models and generalizes the formalism to open universes. However, results on the radiation power spectrum are not presented for open universes. Moreover, the nonlinear power spectrum evolution considered in that paper is not valid for spectral indexes $n < -1$ (in the case of cold dark matter for small scales) and for $\Omega < 1$ universes (Peacock & Dodds 1996).

In this paper we present a formalism to calculate the lensing effect in flat and open cosmological models and in two different gauges. Except for velocity and acceleration terms associated to the observer and the source that either do not contribute or the contribution is negligible, we show that the equations which provide the lensing effect are the same for the conformal Newtonian and synchronous-comoving gauges. Results for the effect of lensing on the radiation power spectrum are presented for cold dark matter (CDM) models with $0.1 \lesssim \Omega \lesssim 1$. We consider linear and nonlinear evolution for the matter power spectrum. The structure of the paper is as follows: in § 2 we describe the formalism to calculate the gravitational lensing effect. The results obtained for CDM open models are presented in § 3. Finally, the main conclusions are related in § 4.

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2. FORMALISM

2.1. Geodesics in the Conformal Newtonian Gauge

We will consider the propagation of photons from recombination to the present time, the universe being a perturbed Friedmann model with a dust \((p = 0)\) matter content. We shall not consider a cosmological \(\Lambda\) term, but the generalization to include \(\Lambda \neq 0\) is very easy. For scalar perturbations, the metric in the conformal Newtonian gauge is given in terms of a single potential \(\phi(t, x)\) as follows:

\[
\begin{align*}
 ds^2 &= a^2(t)[-(1 + 2\phi)dt^2 + (1 - 2\phi)\gamma^{-2}\delta_{ij}dx^idx^j], \\
 &\gamma = 1 + \frac{k}{4}|x|^2, \quad (1)
\end{align*}
\]

we take units such that \(c = 8\pi G = a_0 = 2H_0^{-1} = 1\) and \(k/(4|1 - \Omega|) = 0\), \(-1\), \(+1\) denote the flat, open, and closed Friedmann background universe. The gravitational potential satisfies the Poisson equation:

\[
\nabla^2 \phi = \frac{1}{2} \rho_s a^2 \delta, \quad (2)
\]

where \(\delta\) is the density perturbation. The Green’s function associated to the previous equation can be found in the literature (D’Eath 1976; Traschen & Eardley 1986). We are interested in the effect of gravitational lensing on high multipoles \((l \sim 10^3)\) of the CMB. Only the smaller scales are contributing to such effect, so curvature will show related to the angular distance. In fact, the Green’s function on such scales can be approximated by

\[
G(x, x') \approx -\frac{1}{4\pi} |x - x'|_\Omega^{-1}, \quad (3)
\]

where the distance between the two points \(x = \lambda n, x' = \lambda' n'\) (being \(n\) and \(n'\) two unit vectors in the directions of observation) is given by the equation \((\lambda \approx \lambda')\)

\[
|x - x'|_\Omega \approx [s^2 + s'^2 - 2ss' \cos \alpha]^{1/2},
\]

\[
s \equiv \frac{\lambda}{1 - (1 - \Omega)\lambda^2}, \quad \cos \alpha \equiv n \cdot n'. \quad (4)
\]

On the other hand, after a straightforward calculation, the geodesic equation associated to the metric (1) gives the following equation for the vector \(s' \equiv k'/k = dx'/dt\)

\[
\frac{ds'}{dt} = k\gamma^{-1} \left[ (s \cdot s)s' - \frac{1}{2} \gamma^2 x^i \right] - 2k\gamma x^i
\]

\[
+ 2 \left[ \frac{d\phi}{dt} + 2\nabla^2 \phi \cdot s \right] s' - 2\gamma^2 (\nabla \phi)^i. \quad (5)
\]

Assuming a perturbation scheme ("weak lensing"), this equation can be integrated in the form

\[
x = \lambda n + \epsilon, \quad (6)
\]

where \(n\) is the direction of observation and \(\lambda\) is the distance to the photon for the background metric, i.e.,

\[
\lambda = \tau_0 - \tau \quad (k = 0),
\]

\[
\lambda = (1 - \Omega)^{-1} \tanh \left[ (1 - \Omega)(\tau_0 - \tau) \right] \quad (k = -1). \quad (7)
\]

The perturbation \(\epsilon\) can be decomposed in a term parallel to \(n\) and a term orthogonal to such a direction \(s\). The last term satisfies the following differential equation when parameterized by \(\lambda\)

\[
\frac{d^2\lambda_\perp}{d\lambda^2} + \frac{k}{2\gamma} \left[ -\lambda \frac{d\lambda_\perp}{d\lambda} + \lambda_\perp \right] = -2\nabla_\perp \phi, \quad (8)
\]

where \((\nabla_\perp \phi)^j \equiv (\delta^j - n^j n) d\phi/dn^i\). The solution to the previous equation with the initial conditions: \(\lambda_\perp(\lambda = 0) = 0 = \frac{d\lambda_\perp}{d\lambda}(\lambda = 0)\) is

\[
\lambda_\perp = 2 \int_0^\lambda d\lambda' W(\lambda, \lambda') \nabla_\perp \phi(x = \lambda n), \quad (9)
\]

where \(W(\lambda, \lambda')\) is a window function

\[
a(\lambda) = \frac{(1 - \lambda)^2}{1 + k\lambda^2/4}, \quad W(\lambda, \lambda') = (\lambda - \lambda') \frac{1 + k\lambda'^2/4}{1 + k\lambda^2/4}. \quad (10)
\]

For photons that are propagated from recombination, \(\lambda_\perp = [(1 - (1 + \Omega z)^{-1})]^{-1}[(1 - (1 - \Omega)(1 + \Omega w_z)^{-1})]^{-1} \approx 10^3\), to the observer, \(\lambda_0 = 0\), the lensing vector \(\beta\) is defined in the usual way (see Fig. 1):

\[
\beta \equiv n - \frac{x_r - x_o}{|x_r - x_o|}, \quad (11)
\]

so we find \(\beta = -(1/\lambda) \lambda_\perp(\lambda_0)\), and the final result, taking into account equations (9) and (10), is

\[
W(\lambda) = (1 - \lambda) \frac{1 - (1 - \Omega)\lambda}{1 - (1 - \Omega)\lambda^2}, \quad (12)
\]

because \(\lambda_\perp \approx 1\) for \(\Omega = 10^2\).

The lensing vector for a flat universe has been given by Kaiser (1992). For the open case, Pyne & Birkinshaw (1996) and Seljak (1996b) have used a window function \(W\) that agrees after a straightforward calculation with our equation (12).

\[
\text{Fig. 1.—Diagram describing the geometry and the angles involved in lensing calculations on the CMB. Note that } \epsilon \equiv (x_r - x_o)/|x_r - x_o| \text{ as appears in eq. (11).}
\]
2.2. Geodesics in the Conformal Synchronous-Comoving Gauge

Once we have obtained the expression for the trajectory of the photon in the conformal Newtonian gauge, it is easy to calculate everything in the conformal synchronous-comoving gauge. The infinitesimal transformation connecting both gauges is

\[
\tau' = \tau + \epsilon^0(\tau, x), \quad x' = x + \epsilon^i(\tau, x),
\]

with

\[
\epsilon^0 = \frac{2}{a^3 \rho_b} \frac{\partial}{\partial \tau} (a \phi), \quad \epsilon^i = \frac{2y^2}{a^3 \rho_b} \nabla (a \phi).
\]

The expressions for \( \epsilon^0 \) and \( \epsilon^i \) can be obtained by taking into account that in the synchronous-comoving gauge one has zero velocity \((\dot{v}^i = 0)\), i.e.,

\[
u^0 = (1 - \epsilon^0) v^0, \quad \nu^i = -\epsilon^i v^0, \quad \nu^i = -\epsilon^i \cdot \frac{\partial}{\partial x},
\]

and the metric has the following components: \( g_{00} = -a^2(\tau), g_{ij} = 0 \). So,

\[\epsilon^0 = \frac{\partial}{\partial \tau} \epsilon^0 = \phi, \quad \epsilon^0_j - \gamma^{-1} \epsilon_j = 0.\]

and integrating the last equations we get the result mentioned above for \((\epsilon^0, \epsilon^i)\). Moreover, the metric in the conformal synchronous-comoving gauge reads

\[ds^2 = a^2(\tau) \left[ -dt^2 + \gamma^{-2} \left( 1 - 2\phi - 2 \frac{\dot{a}}{a} \epsilon^0 + k \gamma^{-1} \epsilon \cdot \epsilon \delta_{ij} - \epsilon_{ij} - \epsilon_{ij} \right) dx^i dx^j \right].\]

This last expression for \( k = 0 \) agrees with the one given by Sachs & Wolfe (1967). By changing the gauge, the new lensing vector \( \beta' \) is given by an equation similar to (11), so we obtain

\[\beta' = \beta - \frac{1}{1 + v^0} (\epsilon_{1a} - \epsilon_{10}) - \frac{d\ln a}{dt} \epsilon_{0a},\]

where \( \epsilon^0, \epsilon \) are given by equation (13). The velocity of the fluid in the conformal Newtonian gauge is given by \( v = -\dot{v} \epsilon/\dot{\epsilon} \), from which \( \epsilon = -(a/\dot{a}) \dot{v} \int f D \ln D/\ln a, D(a) \) being the growing mode. Taking this into account one can easily understand that the new terms appearing in equation (17) can be interpreted as Doppler contributions at recombination and at the observer and an acceleration term at the observer. For a flat model \((k = 0)\), we explicitly have

\[\beta' \simeq \beta - \frac{1}{2} \frac{v^0 - v_0}{(1 + z_f)T + a_0},\]

where the linear gravitational potential, \( \phi(x) \), is time-independent and \( v_0 = -\frac{1}{2} \nabla \phi \), \( v = -\frac{1}{2} (1 + z_f)^{-1} \nabla \phi \), and \( a_0 = (d\ln a/\dot{a}) \). The ratio of these terms, as they appear in equation (18), to the angular scale is negligible \((v_0 \) is given by the Doppler velocity respect to the CMB and \( a_0 \) can be estimated from our local infall toward either the Virgo cluster or the Great Attractor). A similar reasoning can be applied to open universes. Therefore, the lensing vector \( \beta' \) in the synchronous-comoving gauge (that is the appropriate one from the point of view of the observations) is approximately given by \( \beta \), as defined by equation (12).

2.3. The Influence of Weak Gravitational Lensing on the C_l's

The correlation function \( C(\theta) \) including gravitational lensing is calculated as the average

\[C(\theta) = \langle \Delta(n + \beta(n)) \Delta(n' + \beta(n')) \rangle, \quad (19)\]

where \( \Delta(n) \) is the temperature anisotropies field, \( n \) and \( n' \) are two directions such that \( n \cdot n' = \cos \theta \). By introducing two-dimensional Fourier components of the temperature anisotropies \( \Delta \) and assuming that the anisotropies, \( \Delta \), and the lensing vector, \( \beta \), are uncorrelated, we obtain

\[C(\theta) = \frac{1}{2\pi} \int_0^\infty dq dP \delta(q) \langle J_q(q) \rangle \delta(\theta - \theta(q)) \]

where \( J_q \) is the Bessel function, \( v \equiv |n - n' + \beta(n) - \beta(n')| \) and \( P_\Delta \) is the two-dimensional power spectrum of the radiation field: \( \langle \Delta \Delta \delta \rangle = P_\Delta(q) \delta(q - q') \). On the other hand, assuming weak gravitational lensing, i.e., on the average the relative lensing vector is very small as compared to the angle \( \theta \), we can make a series expansion in the previous equation obtaining

\[C(\theta) - C(\theta) = \frac{1}{2\theta^2} \left[ Q_{ij} \frac{\partial^2 C(\theta)}{\partial \theta^2} \right] \delta^2(\theta) \]

(21)

\[Q_{ij} \text{ is the bending correlation matrix} \]

\[Q_{ij} = \langle \beta(n) - \beta(n') \rangle \langle \beta(n) - \beta(n') \rangle \]

(22)

and it can be decomposed into the trace and an anisotropic component

\[Q^T = \frac{1}{2} \sigma^2(\theta), \quad \xi(\theta) \equiv Q_{ij} \frac{\partial^2(\theta)}{\partial \theta^2} - \sigma^2 \]

(23)

where \( \sigma(\theta) \) is the bending dispersion and \( \xi(\theta) \) is the anisotropic correlation (\( \xi(\theta) \) corresponds to \( C_{\theta,2}(\theta) \) in Seljak 1996b). Therefore, equation (21) can be rewritten as

\[C(\theta) - C(\theta) = \frac{\sigma^2}{2} \left[ \frac{d^2 C(\theta)}{d \theta^2} + \frac{1}{\theta} \frac{d C(\theta)}{d \theta} \right] + \frac{\xi}{2} \left[ \frac{d^2 C(\theta)}{d \theta^2} - \frac{1}{\theta} \frac{d C(\theta)}{d \theta} \right]. \]

(24)

On the other hand, taking into account the expansion

\[C(\theta) = \frac{1}{4\pi} \sum_{l} (2l + 1) C_l P_l(\cos \theta) \]

(25)

and the approximation \( P_l(\cos \theta) \approx J_l(\theta) \) for \( l > 1 \), we get

\[C_l \approx 2\pi \int_0^\infty d\theta \theta J_l(\theta) J_l(\theta) \]

(26)

From equations (24) and (26)

\[C_l - C_l = - \frac{1}{4\pi} \sum_{l} (2l + 1) \frac{d^2 C_l}{d \theta^2} J_l(\theta) \times \left[ \sigma^2(\theta) J_l(\theta)^2 - \xi(\theta) J_l(\theta)^2 \right], \quad (l > 1) \]

(27)

The next step is the calculation of the dispersion and correlation of the lensing vector as a function of the power spectrum \( P(a, k) \) defined by

\[\langle \delta_c(a) \delta^*_c(a) \rangle \equiv P(a, k) \delta^2(k - k'). \]

(28)
From equation (12) one can obtain
\[ \langle \beta_i \beta_j \rangle = 4 D_i D_j \int_0^1 d\lambda \frac{W(\lambda)}{\lambda} \int_0^1 d\lambda' \frac{W(\lambda')}{\lambda'} C_0(\lambda, \lambda', r), \] (29)
where \( D_i \equiv (\delta_i - n^i n) d/dn^i \) and \( C_0(\lambda, \lambda', r) \) is the correlation of the gravitational potential at two different times. If one assumes Limber's approximation (see also Kaiser 1992), i.e., only a small region \( r \) with \( \lambda' \approx \lambda \) is contributing, the previous equation can be approximated by
\[ \langle \beta_i \beta_j \rangle = 8 D_i D_j \int_0^1 d\lambda \left[ \frac{W(\lambda)}{\lambda} \right]^2 \int_0^\infty drr \times (r^2 - \theta^2 \sin^2) \left[ 1 - (1 - \Omega) \lambda^2 \right] C_0(\lambda, r). \] (30)
Notice that the correlation depends only on a single time and \( s \) is given by equation (4). Introducing the power spectrum \( P_\phi(a, k) \), the last expression becomes
\[ \langle \beta_i \beta_j \rangle = 2 \pi D_i D_j \int_0^1 d\lambda \left[ \frac{W(\lambda)}{\lambda} \right]^2 \left[ 1 - (1 - \Omega) \lambda^2 \right] \times \int_0^\infty dkk^{-1} P_\phi(\lambda, k) J_0(ks) \]. (31)
\( P_\phi(a, k) \) is given by the Poisson equation (2) for scales \( k^2 \gg 12(1 - \Omega) \)
\[ P_\phi(a, k) \approx \left( \frac{6 \Omega}{a} \right)^2 k^{-4} P(a, k), \] (32)
where \( P(a, k) \) is the power spectrum associated to the matter perturbations. In the linear regime: \( P(a, k) = D(a)P(k), D(a) \) being the growing mode normalized to the present time (see Peebles 1980).

Finally, calculating the derivatives that appear in equation (31) and applying equations (22) and (23) one can obtain
\[ \sigma^2(\theta) = \frac{72 \Omega^2}{\pi} \int_0^\infty dk k \int_0^1 d\lambda \left[ \frac{W(\lambda)}{\lambda} \right]^2 \frac{P(a, k)}{1 - (1 - \Omega) \lambda^2} \times \left[ 1 - J_0 + \frac{1}{2} \sin^2 \theta J_0 - \sin \frac{\theta}{2} J_2 \right], \] (33)
\[ \zeta(\theta) = \sigma^2 + \frac{36 \Omega^2}{\pi} \int_0^\infty dk k \int_0^1 d\lambda \left[ \frac{W(\lambda)}{\lambda} \right]^2 \frac{P(a, k)}{1 - (1 - \Omega) \lambda^2} \times \left[ (\cos \theta - 3)(1 - J_0 - J_2) - \sin^2 \theta J_0 \right], \] (34)
where the argument of the Bessel functions \( J_0 \) and \( J_2 \) is \( ks \).

Notice that the behavior of \( \sigma(\theta) \) and \( \zeta^{1/2}(\theta) \) for small \( \theta \) is linear:
\[ \frac{\sigma(\theta)}{\theta} \rightarrow a \theta, \quad \frac{\zeta^{1/2}(\theta)}{\theta} \rightarrow b \theta, \quad b \approx \frac{a}{\sqrt{2}}, \] (35)
as will be shown by the numerical calculations presented in the next section.

3. RESULTS

With the formalism presented in the previous section, we have calculated the dispersion of lensing \( \sigma(\theta) \) and the anisotropic term of the correlation of lensing \( \zeta^{1/2}(\theta) \) as given by equations (33) and (34). We assume a CDM model with a primordial Harrison-Zel'dovich spectrum, a Hubble parameter \( h = 0.5 (H = 100 \ h \ km \ s^{-1} \ Mpc^{-1}) \) and flat and open universe models. The radiation power spectrum not including lensing is normalized to the 2 yr COBE-DMR map as given by the analysis of Cayón et al. (1997) (this normalization does not appreciably change with the 4 yr data). However, since the lensing effect is generated by small scales, much less than 100 Mpc, it might be more sensible to use the normalization \( \sigma_s = 0.6, 1, 1.4 \) for universes with \( \Omega = 1, 0.3, 0.1 \) following Viana & Liddle (1996). This normalization is based on the cluster abundance (see also White, Efstathiou & Frenk 1993; Eke, Cole, & Frenk 1996).

For the nonlinear evolution of the power spectrum we use the recently improved fitting formula given by Peacock & Dodds (1996). That formula is based on the Hamilton et al. (1991) scaling procedure to describe the transition between linear and nonlinear regimes. It accounts for the correction introduced by Jain, Mo, & White (1995) for spectra with \( n < -1 \) and applies to flat as well as to open universes.

In Figure 2 it is shown the relative dispersion \( \sigma(\theta)/\theta \) and the anisotropic term \( \zeta^{1/2}(\theta)/\theta \) for three values of the density parameter \( \Omega = 1, 0.3, 0.1 \). Linear and nonlinear matter evolutions have been considered for comparison. Discrepancies between the two regimes can be noticed at scales \( \theta < 3' \). At scales \( \theta > 6' \) \( \sigma(\theta)/\theta \) as well as \( \zeta^{1/2}(\theta)/\theta \) are below 20% being slightly larger as \( \Omega \) increases. Also, notice that \( \zeta^{1/2}(\theta)/\theta \) at \( \theta > 10' \) is below 20% in all cases (as expected from the considerations made in the previous section). Therefore, the anisotropic term should in principle be considered when calculating the distortions on the radiation power spectrum contrary to the isotropic approximation often made in the literature (we confirm this statement below). Lensing becomes negligible at angular scales above a few degrees.

![Fig. 2.](image)
The effect of lensing can be as much as \( \approx 2\% \) for multipoles \( l \leq 1000 \) and \( \approx 5\% \) for \( l \leq 2000 \). Therefore, if one wants to compute the radiation power spectrum for a particular cosmological model with an accuracy better than 1% such effect should be considered. Bending of the microwave photons due to the large-scale structure should be considered when analyzing data provided by future very sensitive CMB experiments (e.g., COBRAS/SAMBA).

4. CONCLUSIONS

A formalism has been developed to calculate the lensing effect on the primary CMB radiation power spectrum. This formalism provides an expression for the lensing vector in flat and open universes that is approximately the same in both the conformal Newtonian and comoving-synchronous gauges. In particular, we give the window function \( W \) for open models in terms of the distance to the photon from the observer.

The influence of gravitational lensing on the \( C_\ell \)'s has been obtained in terms of the bending dispersion \( \sigma \) and anisotropic bending correlation \( \xi \). It is found that the contribution of \( \xi \) to the lensing distortion of the radiation power spectrum is smaller than that of \( \sigma \). However, this contribution is not negligible and should be considered in the calculation of the radiation power spectrum with lensing.

We use the recently improved fitting formula for the evolution of the nonlinear matter power spectrum, which provides an accuracy better than 12% for the scales considered (Peacock & Dodds 1996). This improvement over previous works (Peacock & Dodds 1994; Jain et al. 1995) generates a larger lensing dispersion at small scales for open models, as compared with Seljak (1996a, 1996b). In spite of this, the contribution of nonlinear evolution to the distortion of the radiation power spectrum is negligible.

For flat as well as open universes, the effect of lensing is to slightly smooth the primary radiation power spectrum of the CMB. For a given range of multipoles the relative change of \( C_\ell \) due to lensing increases with \( \Omega \). However, for the same acoustic peak it decreases with \( \Omega \). The maximum contribution of lensing to the radiation power spectrum for \( l \leq 2000 \) is \( \approx 5\% \) for \( \Omega \) values in the range 0.1–1. Therefore, the effect of lensing should be considered in analyses of CMB anisotropy data provided by future very sensitive experiments.

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