Calculation, evaluation and limit values of the mean residual life of technogenic-hazardous objects

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Abstract. Indicator of residual life of technogenic-hazardous objects is determined for the hazardous operational life duration. Calculation formulas, estimates, and limit values are proved for the determined indicator.

1. Introduction

Let there be a \((\tau, \tau + t)\) as some assigned hazardous interval of the exploitation time of a technogenic-hazardous object when its failure can bring about accidents and catastrophes. The value \(\tau\) exceeds the length of the “running-in” period, and \(\tau + t\) is the time value which is less than the time onset of “aging” [1], [2], [3].

Estimation of the item life value within the interval \((\tau, \tau + t)\) by such conventional indicators as “gamma-percentile life” and “mean life” is not correct, since the duration determined by the former indicator includes the running-in period, whereas the duration determined by the latter one encompasses, in addition, the “aging” period [4], [5]. Therefore a new indicator of item life is required, free of the above-mentioned disadvantages, as well as its calculation and evaluation.

2. Definition

Let us define as the “mean residual life” of an object over time \(\tau\) for the duration \(t\), the following value:

\[
R_r(\tau) = E(\zeta_r(\tau)),
\]

where the right-hand member of the equation is the mathematical expectation of a random variable censored from above by the duration \(t\):

\[
\zeta_r(\tau) = \begin{cases} 
\zeta - \tau, & \text{if } \zeta \in (\tau, \tau + t); \\
\tau, & \text{if } \zeta \geq \tau + t,
\end{cases}
\]

where \(\zeta\) is the object non-failure operation time.

To calculate the indicator (1), we have determined the following formula:
\[ R_i(\tau) = \frac{1}{P(\tau)} \int_0^\tau P(\tau + x) \, dx, \]  
(2)

where \( P(\cdot) \) is the probability of the object failure-free operation during the time given in brackets.

For example, for the exponential distribution law \( P(t) = \exp(-\lambda t) \) where \( \lambda > 0 \) is a constant, according to (2), one will find

\[ R_i(\tau) = \frac{1}{\lambda} (1 - P(t)). \]  
(3)

Let us note that from formula (3) it follows that

\[ R_i(0) = R_i, \]

where \( R_i \) is the truncated mean residual life [7], which amounts to

\[ R_i = \int_0^t P(x) \, dx. \]

Substituting probability \( P(x) \) for a conditional \( \frac{P(\tau + x)}{P(\tau)} \), one will have formula (2).

3. **Evaluation of the object mean residual life**

Let us prove the following.

Theorem 1. For objects whose failure rate as a function of time increases monotonically, the following estimate of indicator \( R_i(\tau) \) is interpolated:

\[ R_i(\tau) > R_i(\tau_0), \]  
(4)

where \( \tau_0 > \tau \).

In other words, the value of mean residual life for a later term \( R_i(\tau_0) \) can be taken as an estimate of the indicator \( R_i(\tau) \) for an earlier term.

Proof. Since [3]

\[ P(t) = \exp\left( -\int_0^t \lambda(u) \, du \right), \]
(5)

where

\[ \lambda(u) = \frac{P'(u)}{P(u)} \]
(6)

– is the failure rate, thus, according to (2),

\[ R_i(\tau) = \int_0^\tau \exp\left( -\int_0^\tau \lambda(u) \, du \right) \, dx. \]
(7)

By partial derivative, one will obtain
As the failure rate increases, the content of the square bracket in the right-hand side is negative. Consequently, 
\[
\frac{\partial R_i(\tau)}{\partial \tau} < 0,
\]
thus indicator \( R_i(\tau) \) as a function of time \( \tau \) decreases monotonously. Therefore, if \( \tau_0 > \tau \) there follows (4), which proves the theorem.

Theorem 2. For objects whose failure rate as a function of time decreases monotonically, the following estimate of indicator \( R_i(\tau) \) is extrapolated:
\[
R_i(\tau) > R_i(\tau_0),
\]
where \( \tau_0 < \tau \).

In other words, if we know the value of the mean residual life for an early term \( R_i(\tau_0) \), then we can take this value for an estimate of indicator \( R_i(\tau) \) for a later term.

There are analogue theorems to those 1 and 2, for such objects whose lifetime is determined by a discrete number of non-failure operations to failure [4].

4. The limit value of the mean residual life of the object

Let us note that the period of exploitation \( t \) is short for some objects [5-7]. For example, for missiles, \( t \) is the duration of their flying if launched at a moment in time \( \tau \) [8-9]. In such cases we can assume that during the exploitation \((\tau,\tau+t)\) the failure rate is a constant; whence the following is valid.

Theorem 3. Let it be that the object failure rate during exploitation \((\tau,\tau+t)\) is a constant, i.e.
\[
\lambda(\tau+x) \equiv z,
\]
where \( x \in (0,t) \), \( z > 0 \) is a constant. Then the mean residual life over time \( \tau \) for the duration \( t \) amounts to
\[
R_i(\tau) = \frac{1}{z} (1 - \exp(-z\tau)).
\]

Proof. To prove this, let us apply formula (7), taking into consideration condition (8). Hence we obtain
\[
R_i(\tau) = \int_0^t \exp(-zx)dx.
\]

Integrating the right side above, one will find the sought formula (9).

In particular, with \( z = \lambda > 0 \), according to formula (9), we get formula (3) for an object whose lifetime distribution follows exponential law.

Some types of technogenic-hazardous objects such as, for example, nuclear power plants, have a long period of exploitation, and a steady-state failure rate [10-15]. Let us prove, for such type of objects, the following.
Theorem 4. Let the object failure rate meet a condition

$$\lim_{\tau \to \infty} \lambda(\tau) = z > 0.$$  \hspace{1cm} (10)

Then the limit value of the mean residual life over time $\tau$ for the duration $t$ is as follows:

$$\lim_{\tau \to \infty} R_t(\tau) = \frac{1}{z}(1 - \exp(-zt)).$$  \hspace{1cm} (11)

Proof. According to formula (2), we have

$$\lim_{t \to \infty} \lim_{\tau \to \infty} R_t(\tau) = \frac{1}{P(\tau)} \int_{\tau}^{\infty} P(u)du.$$  \hspace{1cm} (12)

Hence the indicator $R_t(\tau)$ as a function of time $\tau$ ($\tau \to \infty$) has an indeterminate form $0/0$. When converting this indeterminate form according to L'Hopital's rule, one will have

$$\lim_{\tau \to \infty} R_t(\tau) = \lim_{\tau \to \infty} \frac{P(\tau + t) - P(\tau)}{P'(\tau)}.$$  \hspace{1cm} (13)

Applying formula (6) to (12), one will have

$$\lim_{\tau \to \infty} R_t(\tau) = \lim_{\tau \to \infty} \frac{1}{\lambda(\tau)} \left(1 - \frac{P(\tau + t)}{P(\tau)}\right).$$  \hspace{1cm} (14)

Since according to (5)

$$\frac{P(\tau + t)}{P(\tau)} = \exp\left(-\int_{\tau}^{\tau+t} \lambda(\tau + x)dx\right),$$

then

$$\frac{P(\tau + t)}{P(\tau)} = \exp\left(-\int_{0}^{\infty} \lambda(\tau + x)dx\right).$$

Consequently, taking into account condition (10), one will find

$$\lim_{\tau \to \infty} \frac{P(\tau + t)}{P(\tau)} = \exp(-zt).$$

Taking account of the obtained limit and condition (10) in equation (13), one will have the sought formula (11), which was to be proved.

5. Example
Let there be

$$\lambda(\tau) = 0.0005 + \exp(-10\tau).$$

The limit value of indicator $R_t(\tau)$ is to be found for $t = 2000h$ and $\tau \to \infty$.

Solution. Since

$$\lim_{\tau \to \infty} \lambda(\tau) = 0.0005,$$
then, according to (11), one will find the limit value that was sought:

$$\lim_{t \to \infty} R(t) = 1260 \text{h}.$$ 

6. Conclusions

Thus the indicator of the residual life for technogenic-hazardous objects has been determined. Calculation formulas, estimates, and limit values have been proved for the determined indicator.

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