Noise Squeezing in a Nanomechanical Duffing Resonator

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We study mechanical amplification and noise squeezing in a nonlinear nanomechanical resonator driven by an intense pump near its dynamical bifurcation point, namely, the onset of Duffing bistability. Phase sensitive amplification is achieved by a homodyne detection scheme, where the displacement detector’s output, which has correlated spectrum around the pump frequency, is down converted by mixing with a local oscillator operating at the pump frequency with an adjustable phase. The down converted signal at the mixer’s output could be either amplified or deamplified, yielding noise squeezing, depending on the local oscillator phase.

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Micro/Nanoelectromechanical resonators allow a variety of applications such as sensing, switching, and filtering [1, 2, 3]. Improvement of force detection sensitivity is highly desirable for enhancing the performance of nanomechanical detectors. A possible technique to improve signal to noise ratio in such devices is to implement an on-chip mechanical amplification. Mechanical amplification and thermomechanical noise squeezing in microresonators have been achieved before using parametric amplification [4, 5]. In the present work, we demonstrate a novel scheme for mechanical amplification based on a bifurcating dynamical system, exploiting its high sensitivity to fluctuations near its bifurcation point [6, 7, 8, 9, 10, 11]. This amplification scheme has been used lately for quantum measurement of superconducting qubits [12]. In our case, we use the onset of bistability in a nanomechanical Duffing resonator as the bifurcation point. In a Duffing resonator, above some critical driving amplitude, the response becomes a multi-valued function of frequency in some finite frequency range, and the system becomes bistable with jump points in the frequency response [13, 14]. We employ this mechanism for the first time in nanomechanical resonators to demonstrate experimentally, high signal gain, phase sensitive amplification and noise squeezing. The system under study, coined as NanoMechanical Bifurcation Amplifier (NMBA), consists of a nonlinear doubly clamped nanomechanical PdAu beam, excited capacitively by an adjacent gate electrode. An intense pump signal drives the resonator near the onset of bistability, enabling amplification of a small signal in a narrow bandwidth. In a previous work we have demonstrated high intermodulation gain occurring in the same region [15].

The resonator length is \( l = 100 \mu m \), width \( w = 600 \text{ nm} \), and thickness \( t = 250 \text{ nm} \). The gap separating the doubly clamped beam and the stationary side electrode is \( d = 4 \mu m \) wide. The device is fabricated using bulk nanomachining process together with electron beam lithography [17].

The nonlinear dynamics of the fundamental mode of a doubly clamped beam driven by an external force per unit mass \( F(t) \) can be described by a Duffing oscillator equation for a single degree of freedom \( x \)

\[
\ddot{x} + 2\mu(1 + \beta x^2)\dot{x} + \omega_0^2(1 + \kappa x^2)x = F(t), \tag{1}
\]

where \( \mu \) and \( \beta \) are the linear and nonlinear damping constants respectively, \( \omega_0^2/2\pi \approx 500 \text{ kHz} \) is the resonance frequency of the fundamental mode, and \( \kappa \) is the cubic nonlinear constant. Generally, for resonators driven using a bias voltage applied to a side electrode, Eq. (1) should

![Diagram](image_url)
contain additional parametric terms $\delta$. In our case however, the prefactors of these parametric terms are at least one order smaller below threshold and thus negligible.

The relative importance of nonlinear damping can be characterized by the dimensionless parameter $p = 2\sqrt{3\mu/\kappa\omega_0}$, which is proportional to the relative phase $\varphi$ and the frequency of the bifurcation point. This indicates that the effect of nonlinear damping in the present case is relatively weak. To investigate nonlinear amplification of a small test signal, the resonator is driven by an applied force $F(t) = f_p \cos(\omega_p t) + f_s \cos(\omega_f t + \varphi)$, composed of an intense pump with frequency $\omega_p = \omega_0 + \sigma$, amplitude $f_p$, and a small force (called signal) with frequency $\omega_s = \omega_p + \delta$, relative phase $\varphi$, and amplitude $f_s$, where $f_s \ll f_p$ and $\sigma, \delta \ll \omega_0$. This is achieved by applying a voltage of the form $V = V_{dc} + v_p \cos(\omega_p t) + v_s \cos(\omega_f t + \varphi)$ where $V_{dc}$ is a dc bias (employed for tuning the resonance frequency) and $v_s \ll v_p \ll V_{dc}$. The resonator’s displacement has spectral components at $\omega_p, \omega_f$, and at the intermodulations $\omega_p \pm k\delta$ where $k$ is an integer. The one at frequency $\omega_s = \omega_p - \delta$ is called the idler component.

Strong correlation between the signal and the idler, occurring near the edge of the bistability region, could be exploited for phase sensitive amplification and noise squeezing $\delta$. This is achieved by a homodyne detection scheme, where the displacement detector’s output is down converted by mixing with a LO operating at frequency $\omega_p$ with an adjustable phase $\phi_{LO}$ and phase locked to the pump. The mixer’s output (IF port) has a spectral component at frequency $\delta$, which is proportional to the phasor sum of the signal and the idler, yielding phase sensitive amplification, controlled by $\phi_{LO}$. In the notation of Ref. [15], the displacement $x(t)$ is given by

$$x(t) = \frac{1}{2}A(t)e^{i\omega_p t} + \text{c.c.},$$

where $A(t) = a_p + a_s e^{i\delta t} + a_s^* e^{-i\delta t}$ is a slowly varying function (relative to the time scale $1/\omega_p$) and the complex numbers $a_p, a_s$ and $a_s^*$ are the pump, signal and idler spectral components of $A(t)$ respectively. Suppose that the LO voltage is given by $V^{LO}(t) = V^{LO}_0 \cos(\omega_f t + \phi_{LO})$ and the mixer’s output is given by $V_{LO} = Mx(t)V^{LO}(t)$ where $M$ is a constant term depending on the optical detector’s sensitivity, amplification and the mixing factor. After passing through a low pass filter (LPF), the output signal is

$$\frac{1}{4}MV_{LO}^0[A(t)e^{-i\phi_{LO}} + \text{c.c.}].$$

The measured quantity is the amplitude $R(\delta)$ of the spectral component of the output signal at frequency $\delta$. $R(\delta)$ depends on the LO phase $\phi_{LO}$ and is given by

$$R(\delta) = \frac{1}{2}MV_{LO}^0|a_p e^{-i\phi_{LO}} + a_s^* e^{i\phi_{LO}}|.$$
FIG. 2: (Color online) Typical frequency response curves for various excitation voltages \( V_p \). The inset shows hysteresis response for \( V_p = 90 \text{mV} \).

where \( v_p/v_s = 25 \). Measurements of \( \Delta \) vs. frequency are shown in Fig. 3(c) for four pump amplitudes (related to lines (1)-(4) in Fig. 3(b)). The response of the frequency upward (downward) sweep is depicted with black (green) line. For \( v_p = 50 \text{mV} \) (Fig. 3(c)-1) the frequency sweep is contained within the monostable region and consequently the value of \( \Delta \) is relatively small. For \( v_p = 70 \text{mV} \) (Fig. 3(c)-2), \( v_p = 90 \text{mV} \) (Fig. 3(c)-3), and \( v_p = 110 \text{mV} \) (Fig 3(c)-4), on the other hand, the frequency sweeps cross the bistability region and two peaks are seen for \( \Delta \), corresponding to the upward and downward frequency sweeps. These peaks originate from the high signal amplification in the jump points of the pump response. Note that in this case the width of the hysteresis loop (which is the distance between the peaks) is smaller relative to the case when the pump is the only excitation.

We now turn to investigate the resonator response to pump and noise. First, the bifurcation point \( (B_p) \) is located. A frequency response of the beam, excited by the pump (without noise) in the vicinity of \( B_p \) is shown in Fig. 4(a). In the next step, the pump frequency is fixed to the bifurcation point and we add white noise to the excitation having spectral density \( S_{\text{noise}}^{1/2} = 1\text{mV}/\sqrt{\text{Hz}} \).

The measured spectrum taken around the pump frequency (see Fig. 4(b)) demonstrates strong amplification occurring in this region, a manifestation of the noise rise phenomenon [22]. There is a good agreement between the theoretical fit (\( \delta^{-3} \) dependence) [8] to the experimental data for \( \delta > 50 \text{Hz} \). For smaller values of \( \delta \) the model breaks down due to high order terms.

Noise squeezing is demonstrated in Fig. 5 where \( \delta = 10 \text{Hz} \), \( S_{\text{noise}}^{1/2} = 1\text{mV}/\sqrt{\text{Hz}} \) and \( S_x^{1/2} \) is plotted vs. the LO phase \( \phi_{\text{LO}} \). Here the sweep time is 6 s and the resolution bandwidth is 2 Hz. The blue line demonstrates the case where the pump is in the vicinity of the bifurcation point, whereas the green line demonstrates the case where the pump is far from the bifurcation point. The noise amplitude amplification is about 6. The deamplified (squeezed) quadrature is below the measurement noise floor, hence it can’t be measured.

To summarize, the nonlinear region of nanomechanical resonators could be exploited for both signal amplification and noise reduction which could be useful for detection of weak forces. A possible application for our noise squeezing scheme is sensitive mass detection [21], which can be achieved by operating close to the bifurcation point and adjusting \( \phi_{\text{LO}} \) to maximize the mass.
FIG. 4: (Color online) (a) Pump response near $B_p$. Upward and downward sweeps are seen in black and green respectively. (b) Averaged spectrum response for pump and noise excitation. The input noise spectral density is $1\text{mV}/\sqrt{\text{Hz}}$. Circles indicate the experimental data, whereas a theoretical fit is seen as a blue line.

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