Electroweak Corrections to the Neutralino Pair Production at CERN LHC

A. I. Ahmadov\textsuperscript{1,2}\textsuperscript{*} and M. Demirci\textsuperscript{1}\textsuperscript{†}

\textsuperscript{1}Department of Physics, Karadeniz Technical University, 61080 Trabzon, Turkey
\textsuperscript{2}Department of Theoretical Physics, Baku State University, Z. Khalilov St. 23, AZ-1148, Baku, Azerbaijan

(Dated: May 7, 2014)

Abstract

We apply the leading and sub-leading electroweak (EW) corrections to the Drell-Yan process of the neutralino pair production at proton-proton collision, in order to calculate the effects of these corrections on the neutralino pair production at the LHC. We provide an analysis of the dependence of the Born cross-sections for $pp \to \tilde{\chi}_i^0 \tilde{\chi}_j^0$ and the EW corrections to this process, on the center-of-mass energy $\sqrt{s}$, on the $M_2-\mu$ mass plane and on the squark mass for the three different scenarios. The numerical results show that the relative correction can be reached the few tens of percent level as the increment of the center-of-mass energy, and the evaluation of EW corrections is a crucial task for all accurate measurements of the neutralino pair production processes.

PACS numbers: 11.30.Pb, 12.15.-y, 12.15.Lk, 12.60.Jv, 14.80.Ly

Keywords: Chargino sector; electroweak corrections; neutralino production

\textsuperscript{*}Electronic address: E-mail:ahmadovazar@yahoo.com
\textsuperscript{†}Electronic address: E-mail:mehmetdemirci@ktu.edu.tr

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I. INTRODUCTION

Supersymmetry (SUSY) [1–5] arose as a response to attempts by physicists to obtain a unified description of all fundamental interaction of nature and it is at present one of the most favoured ideas for new physics beyond the Standard Model (SM) [6, 7]. The realistic extension of the SM, the Minimal Supersymmetric Standard Model (MSSM) so that it is constructed by declaring the superpartners (sparticles) of the SM states, and declaring an additional Higgs doublet (higgsinos) which has opposite hypercharge according to Higgs doublet in the SM, so as to give separately masses to isospin up- and down-type chiral fermions and cancel the gauge anomalies [8, 9]. The MSSM contains a discrete symmetry known as R-parity [10–14] so that it ensures lepton and baryon number conservations. Assuming that conservation R-parity, the lightest supersymmetric particle (LSP) is definitely stable and this particle is the end product of any process involving sparticle in the final state. In most cases, the stable LSP is the lightest neutralino, which is one of the superpartners of the electroweak (EW) gauge bosons (gauginos) and the Higgs doublet (higgsinos), which mix to form four neutral (neutralinos $\tilde{\chi}^0_i$) and two charged (charginos $\tilde{\chi}^\pm_j$) mass eigenstates. The higgsino and gaugino decomposition of the neutralinos and charginos includes significant information about the SUSY-breaking mechanism and also plays an important role in the explanation of the relic density of the dark matter [15–18]. Thus a detailed study of the production of the lightest neutralino $\tilde{\chi}_1^0$ and the next-to-lightest neutralino $\tilde{\chi}_2^0$ at present and future experiments is so important that the neutralino sector can be help us to decide which kind of the supersymmetric models really exists in nature.

In the literature, some of the studies related to neutralino pair production in the MSSM as follows: The neutralino pair production via quark-antiquark annihilation at LHC was investigated in Ref. [19–21]. The neutralino and chargino pair production via gluon-gluon fusion were studied in Ref. [22, 23] in the framework of minimal supergravity (mSUGRA) scenario. Also, the neutralino pair production including the tree level contributions and the leading-log one loop radiative corrections were considered in Ref. [24]. The production of charginos, neutralinos, and sleptons in the direct channels $p\bar{p}/pp \rightarrow \tilde{\chi}_1^0\tilde{\chi}_j^0 + X$ at the hadron colliders Tevatron and LHC, via quark-antiquark annihilation was analyzed at the next-to-leading order in Ref. [25]. Focusing on the correlation of beam polarization, the gaugino pair production in unpolarized and polarized hadron collisions was studied in Ref. [26].
Moreover, the effects of the s-channel Higgs bosons exchange on the chargino and neutralino pair production in proton-proton collision in the following channels $p\bar{p}/pp \rightarrow \tilde{\chi}_i^0 \tilde{\chi}^0_j + X$ have been analyzed in Ref. [27].

We analyze the dependence of the Born cross-sections and the EW corrections on the SUSY parameters for the direct production of neutralino pair at the LHC energies. One of the important approach of our scenario consist of the mechanism the choosing of input parameters. We recover the Lagrangian parameters as direct analytical expressions of suitable physical masses without any constrained in the MSSM, in such a way that we essentially focus on the algebraically nontrivial inversion for the gaugino mass parameters, i.e., using tan $\beta$ and two chargino masses as input parameters, one can be obtained the other parameters, which are gaugino/higgsino mass parameters, neutralino masses and mixing matrix as outputs. We have not only taken into account the process $pp \rightarrow \tilde{\chi}_i^0 \tilde{\chi}^0_j$ at the Born level, but also logarithmic EW contributions to that process at the one-loop level. The overall Born level magnitude of the amplitudes is reduced by these EW corrections as an amount that could lie the few tens of percent level for the kinematical domain attainable at the LHC. Therefore, these corrections are important for the experimental and theoretical studies related to the production of neutralino pair at the LHC and the future colliders.

The remainder of this paper is organized as follows: In Section II, we present briefly definitions corresponding to the neutralino/chargino sector and our method for calculations. In Section III, the analytical expressions of the amplitudes and the cross-sections is given for subprocess $q\bar{q} \rightarrow \tilde{\chi}_i^0 \tilde{\chi}^0_j$. In Section IV, we provide the formulas of the leading and subleading EW logarithmic corrections for amplitudes of the subprocess $q\bar{q} \rightarrow \tilde{\chi}_i^0 \tilde{\chi}^0_j$ and in Section V, the numerical results for the cross-section and the EW corrections is given, and we discuss the dependence of the cross-section on the SUSY model parameters. Finally, our conclusions are given in section VI.

II. THE NEUTRALINO/CHARGINO SECTOR OF THE MSSM

The physical neutralino mass eigenstates $\tilde{\chi}_i^0$ ($i = 1,..,4$) are the combinations of the neutral gauginos $\tilde{B}, \tilde{W}^3$ and the neutral higgsinos $\tilde{H}_1^0, \tilde{H}_2^0$ in the MSSM. The soft SUSY-breaking terms in the Lagrangian include the following term $[8]$,

$$\mathcal{L} \supset -\frac{1}{2}(\psi_i^0)^T \mathcal{M} \psi_j^0 + h.c., \hspace{1cm} (2.1)$$


which is bilinear in the fermion fields $\psi^0_j = (-i\widetilde{B}, -i\widetilde{W}^3, \widetilde{H}^0_1, \widetilde{H}^0_2)^T$ with $j = 1, 2, 3, 4$. In the above relation, the neutralino mass matrix is given as

$$\mathcal{M} = \begin{pmatrix} M_1 & 0 & -m_Z c_\beta s_W & m_Z s_\beta s_W \\ 0 & M_2 & m_Z c_\beta c_W & -m_Z s_\beta c_W \\ -m_Z c_\beta s_W & m_Z c_\beta c_W & 0 & -\mu \\ m_Z s_\beta s_W & -m_Z s_\beta c_W & -\mu & 0 \end{pmatrix}, \quad (2.2)$$

which is symmetric. Here, $\mu$ and $M_1/M_2$ are the supersymmetric Higgsino mass parameter and the gaugino mass parameter related to the $U(1)/SU(2)$ subgroup, respectively, and $\tan \beta = v_2/v_1$ is the ratio of the vacuum expectation values of the two Higgs fields that break the EW symmetry. The mass parameters are possibly complex in CP noninvariant theories, in this case, by means of the reparametrization of the fields, the $M_2$ gaugino mass can be obtained as real and positive with no loss of generality in order that the two remaining nontrivial phases, which are reparametrization invariant, can be ascribed to $\mu$ and $M_1$ as follows: $\mu = |\mu| e^{i\phi_\mu}$ and $M_1 = |M_1| e^{i\phi_1}$ ($0 \leq \phi_1 < 2\pi$).

The neutralino mass matrix $\mathcal{M}$ can be diagonalized by one $4 \times 4$ unitary matrix $N$, which is sufficient to rotate from the gauge eigenstate basis $(\widetilde{B}, \widetilde{W}^3, \widetilde{H}^0_1, \widetilde{H}^0_2)$ to the mass eigenstate basis of the neutralino fields $\widetilde{\chi}^0_j$:

$$\mathcal{M}_D = N^T \mathcal{M} N = \sum_{j=1}^{4} m_{\widetilde{\chi}^0_j} E_j. \quad (2.3)$$

Therefore, the relation between physical and weak eigenstates can be extracted as $\chi^0_i = N_{ij} \psi^0_j$ with $i = 1, 2, 3, 4$. In order to determine $N$, the square of Eq. (2.3) obtaining

$$\mathcal{M}_D^2 = N^{-1} \mathcal{M}^+ \mathcal{M} N = \sum_{j=1}^{4} m_{\chi^0_j}^2 E_j, \quad (2.4)$$

where $(E_j)_{ik} = \delta_{ji} \delta_{jk}$. The neutralino mass eigenstates are expressed by

$$\widetilde{\chi}^0_j = \begin{pmatrix} \chi^0_j \\ \chi^0_{\bar{j}} \end{pmatrix}, \quad (2.5)$$

where $\chi^0_j$ denotes the two component Weyl spinor and $\widetilde{\chi}^0_j$ the four component Majorana spinor of the $j$th neutralino field. The application of projection operators leads to relatively compact analytic expressions for the mass eigenvalues $m_{\chi^0_1} < m_{\chi^0_2} < m_{\chi^0_3} < m_{\chi^0_4}$ [28].
mass eigenvalues \( m_{\chi_j} \) in the diagonal neutralino mass matrix \( \mathcal{M}_D \) are possibly chosen as positive and real by an appropriate definition of the unitary matrix \( N \). Rearranging Eq. (2.4) as follows,

\[
(\mathcal{M}^+ \mathcal{M}) N - NM_D^2 = 0
\]

(2.6)

and by solving this system of equations and by taking into account the following relation

\[
|N_{1j}|^2 + |N_{2j}|^2 + |N_{3j}|^2 + |N_{4j}|^2 = 1,
\]

(2.7)

the \( N_{ij} \) matrix's components are obtained. Also, the neutralino masses are obtained by solving the following characteristic equation,

\[
X^4 - aX^3 + bX^2 - cX + d = 0,
\]

(2.8)

where

\[
a = M_1^2 + 2\mu^2 + M_2^2 + 2m_Z^2,
\]

\[
b = (\mu^2 + m_Z^2)^2 + M_2^2(M_1^2 + 2\mu^2 + 2m_Z^2s_W^2) + 2M_1^2(\mu^2 + m_Z^2c_W^2) - 2\mu m_Z^2c_W M_2 \sin 2\beta \times \cos \phi_\mu - 2m_Z^2s_W M_1 \sin 2\beta \cos (\phi_\mu + \phi_1),
\]

\[
c = \mu^4 M_1^4 + \mu^2 m_Z^4 \sin 2\beta + M_1^2 m_Z^2 c_W (2\mu^2 + m_Z^2 c_W^2) +
\]

\[
M_2^2 (m_Z^4 s_W^4 + 2\mu^2 (m_Z^2 s_W^2 + M_1^2) + \mu^4) - 2\mu m_Z^2 s_W M_1 (\mu^2 + M_2^2) \sin 2\beta \cos (\phi_\mu + \phi_1) +
\]

\[
2m_Z^2 c_W M_2 [m_Z^2 M_1 s_W^2 \cos \phi_1 - \mu (\mu^2 + M_1^2) \cos \phi_\mu \sin 2\beta],
\]

\[
d = m_Z^4 c_W \mu^2 M_1^2 \sin 2\beta + 2m_Z^2 \mu^2 M_1 M_2 c_W (m_Z^2 s_W^2 \sin 2\beta \cos \phi_1 - \mu M_1 \cos \phi_\mu) +
\]

\[
\mu^2 m_Z^2 s_W M_2 \sin 2\beta (m_Z^2 s_W^2 \sin 2\beta - 2\mu M_1 \cos (\phi_1 + \phi_\mu)) + \mu^4 M_1^2 M_2^2.
\]

From solving Eq. (2.8), the exact analytic formulas of the neutralino masses are obtained as follows,

\[
m_{\chi_1}^2, m_{\chi_2}^2 = \frac{a}{4} - \frac{f}{2} + \frac{1}{2} \sqrt{\frac{r - w - \frac{p}{4f}}{f}},
\]

\[
m_{\chi_3}^2, m_{\chi_4}^2 = \frac{a}{4} + \frac{f}{2} + \frac{1}{2} \sqrt{\frac{r - w + \frac{p}{4f}}{f}},
\]

(2.9)

where

\[
f = \sqrt{\frac{r}{2} + w}, \quad r = \frac{a^2}{2} - \frac{4b}{3}, \quad w = \frac{q}{(3 \cdot 2^{1/3})} + \frac{(2^{1/3} \cdot h)}{3 \cdot q},
\]

\[
p = a^3 - 4ab + 8c, \quad q = (k + \sqrt{k^2 - 4h^3})^{1/3}
\]

(2.10)

\[
k = 2b^3 - 9abc + 27c^2 + 27a^2 d - 72bd, \quad h = b^2 - 3ac + 12d.
\]
The physical chargino mass eigenstates $\tilde{\chi}^\pm_i$ ($i=1,2$) are the combinations of the charged gauginos ($\tilde{W}^\pm$) and the charged higgsinos ($H^{\pm}_{2,1}$). In terms of two-component Weyl spinors, the chargino mass term in the SUSY Lagrangian can be expressed by [8]

$$\mathcal{L} \supset -\frac{1}{2} \begin{pmatrix} \psi^+ & \psi^- \end{pmatrix} \begin{pmatrix} 0 & \mathcal{M}_C^T \\ \mathcal{M}_C & 0 \end{pmatrix} \begin{pmatrix} \psi^+ \\ \psi^- \end{pmatrix} + \text{h.c.},$$  

(2.11)

which is bilinear in the two-component fermionic fields $\psi^j_\pm = (-i\tilde{W}^\pm_j, \tilde{H}^{\pm}_{2,1})^T$ with $j = 1, 2$.

The chargino mass matrix $\mathcal{M}_C$ is given as

$$\mathcal{M}_C = \begin{pmatrix} M_2 & \sqrt{2m_W}c_\beta \\ \sqrt{2m_W}s_\beta & |\mu|e^{i\phi_\mu} \end{pmatrix}. $$  

(2.12)

The matrix $\mathcal{M}_C$ is not symmetric, so it must be diagonalized by two different unitary matrices $V$ and $U$, which lead to the relation $U^*\mathcal{M}_C V^{-1} = \text{diag} \{m_{\tilde{\chi}^+_1}, m_{\tilde{\chi}^+_2} \}$, with the chargino mass eigenvalues:

$$m^2_{\tilde{\chi}^+_1,2} = \frac{1}{2} \left\{ M_2^2 + |\mu|^2 + 2m_W^2 \mp \left[ (M_2^2 - |\mu|^2 - 2m_W^2 \cos 2\beta)^2 + 8m_W^2 (M_2^2 c_\beta^2 + |\mu|^2 s_\beta^2 + M_2 |\mu| \sin 2\beta \cos \phi_\mu) \right]^{1/2} \right\}. $$  

(2.13)

The fundamental SUSY parameters $M_2$ and $\mu$ are possibly derived from these two chargino masses for given $\tan \beta$ [29, 30]. By taking appropriate sum and differences of the chargino masses in the Eq. (2.13), one can be derived the following equations for $M_2$ and $\mu$:

$$2M_2^2 = (m^2_{\tilde{\chi}^+_1} + m^2_{\tilde{\chi}^+_2} - 2m_W^2) \mp \sqrt{(m^2_{\tilde{\chi}^+_1} + m^2_{\tilde{\chi}^+_2} - 2m_W^2)^2 - \Delta_\pm}, $$  

(2.14)

$$2|\mu|^2 = (m^2_{\tilde{\chi}^+_1} + m^2_{\tilde{\chi}^+_2} - 2m_W^2) \pm \sqrt{(m^2_{\tilde{\chi}^+_1} + m^2_{\tilde{\chi}^+_2} - 2m_W^2)^2 - \Delta_\pm} $$  

(2.15)

with

$$\Delta_\pm = 4 \left[ m^2_{\tilde{\chi}^+_1} m^2_{\tilde{\chi}^+_2} + m^4_W \cos 2\phi_\mu \sin^2 2\beta \pm 2m^2_W \cos \phi_\mu \sin 2\beta \times \sqrt{m^2_{\tilde{\chi}^+_1} m^2_{\tilde{\chi}^+_2} - m^4_W \sin^2 2\beta \sin^2 \phi_\mu} \right].$$

In the above equations, the upper (lower) signs correspond to $M_2 < |\mu|$ ($M_2 > |\mu|$) regime. Here, four solutions associated with different physical scenarios are occurred. For the $M_2 > |\mu|$ regime, the lightest chargino has a stronger higgsino-like component and thus it is mentioned as higgsino-like [30, 31]. The solution for the $|\mu| > M_2$ regime corresponds to the gaugino-like case, could be easily figured out by the following replacements: $M_2 \rightarrow |\mu|$.
and $\mu \rightarrow \text{sign}(\mu)M_2$ [31, 32]. The universality of the gaugino masses at the GUT scale, which leads to the relation,

$$M_1 = \frac{5}{3}M_2 \tan^2 \theta_W. \quad (2.16)$$

In this work, we take into account the gaugino/higgsino sector with the following assumptions: First, in order to obtain real mass eigenvalues, namely $\phi_1 = 0$ and $\phi_\mu = 0$. The signs among the mass parameters $M_1$, $M_2$ and $\mu$ are relative, which can be absorbed into phases $\phi_1$ and $\phi_\mu$ by redefinition of fields, and consequently, these mass parameters can be real and positive. Under the these assumptions, it is possible that there appear several scenarios for the choice of the parameters. On account of the fact that the SUSY parameters can be derived from the physical quantities, it is also possible that choose an alternative way to diagonalize the mass matrix, by using two chargino masses together with $\tan \beta$ as inputs. Moreover, there are several scenarios for the choice of two chargino masses and $\tan \beta$ [32]. The scenarios correspond to the choice of $\tan \beta$ as follow: scenario with small $\tan \beta$ ($\tan \beta \approx 1 \div 3$) and scenario with large $\tan \beta$ ($\tan \beta \approx 30 \div 70$) [33–36].

### III. CALCULATION OF THE CROSS SECTION

In this section, we present analytical expressions of amplitudes and the cross-section of the neutralino pair production. The neutralino pair production originates from quark-antiquark collision, is expressed by

$$q(p_1)\bar{q}(p_2) \rightarrow \tilde{\chi}^0_i(k_1)\tilde{\chi}^0_j(k_2), \quad (3.1)$$

where $p_1$, $p_2$, $k_1$ and $k_2$ represent the four momenta of the quark, antiquark, the two final state neutralinos, separately. The Mandelstam variables for subprocess are given by

$$\hat{s} = (p_1 + p_2)^2, \quad \hat{t} = (p_1 - k_1)^2, \quad \hat{u} = (p_1 - k_2)^2. \quad (3.2)$$

The relevant couplings of the supersymmetric particles for neutralino pair production are extracted from the following interaction Lagrangians [37] so that,

$$L_{Z\tilde{\chi}^0_i\tilde{\chi}^0_j} = \frac{g}{2 \cos \theta_W} Z_{\mu} \tilde{\chi}^0_i \gamma^\mu (O_{ij}^L P_L + O_{ij}^R P_R) \tilde{\chi}^0_j, \quad (3.3)$$

$$L_{Z\tilde{q}^0} = \frac{g}{\cos \theta_W} \bar{q} \gamma^\mu (L_q P_L + R_q P_R) q Z_{\mu}, \quad (3.4)$$

$$L_{q\tilde{\chi}^0} = \bar{q} \left( a_i^L (\tilde{q}_n) P_L + a_i^R (\tilde{q}_n) P_R \right) \tilde{\chi}^0_i \tilde{q}_n, \quad (3.5)$$
where $q$, $\tilde{q}_n$ and $\tilde{\chi}_i^0$ denote four-component spinor fields of the quark, squark and neutralino, respectively. Moreover, $g = e / \sin \theta_W$ is the weak coupling constant, $P_{R,L} = \frac{1}{2} (1 \pm \gamma^5)$. In the above Lagrangians, the relevant couplings $O''_{ij}, L_q, R_q$ and $a_i^{R,L}(\tilde{q}_n)$ are given by

$$O''_{ij} = O''_{Z} = \frac{1}{2} (N_{i3} N_{j4}^* - N_{i4} N_{j3}^*) \cos 2\beta - \frac{1}{2} (N_{i3} N_{j4}^* + N_{i4} N_{j3}^*) \sin 2\beta, \quad (3.6)$$

$$O''_{ij}^R = -O''_{ij}^*, \quad (3.7)$$

$$L_q = 2 I_3^q (1 - 2 \sin^2 \theta_W |Q_q|), \quad R_q = -2 \sin^2 \theta_W Q_q, \quad (3.8)$$

with $I_3^q, Q_q$ which are the isospin quantum number and charge of the various quarks, and

$$a_i^L(\tilde{u}_L) = -\frac{e}{3 \sqrt{2} s_W c_W} (N_{1i} s_W + 3 N_{2i} c_W), \quad a_i^L(\tilde{u}_R) = -\frac{e m_u}{\sqrt{2} m_W s_W s_\beta} N_{4i},$$

$$a_i^R(\tilde{d}_L) = -\frac{e}{3 \sqrt{2} s_W c_W} (N_{1i} s_W - 3 N_{2i} c_W), \quad a_i^L(\tilde{d}_R) = -\frac{e m_d}{\sqrt{2} m_W s_W c_\beta} N_{3i},$$

$$a_i^R(\tilde{d}_R) = -\frac{\sqrt{2} e}{3 c_W} N_{1i}^*, \quad a_i^R(\tilde{d}_L) = -\frac{e m_d}{\sqrt{2} m_W s_W c_\beta} N_{3i}^*. \quad (3.9)$$

One can note that the mixing matrices $N_{ij}$ control the higgsino and gaugino components of the neutralino in the $Z\tilde{\chi}_i^0 \tilde{\chi}_j^0$ and $q \bar{q} \tilde{\chi}_i^0$ coupling as shown in the Lagrangians.

FIG. 1: Feynman diagrams of the subprocess $q \bar{q} \to \tilde{\chi}_i^0 \tilde{\chi}_j^0$ to leading level.
The subprocess for neutralino pair production proceeds through $t$- and $u$-channel contributions due to exchange of the squarks, and $s$-channel contribution due to $Z$ boson exchange as shown in Fig. 1. The corresponding amplitudes for each diagram can be given as

$$T = T_s + T_i + T_u,$$

where

$$T_s = -\frac{e^2}{2\sin^2\theta_W\cos^2\theta_W} D_Z(\hat{s}) \bar{u}_s(k_1) \gamma^\mu \left[ O^ij_Z P_L - O^ij_Z P_R \right] \partial_j(k_2) \times \bar{\nu}(p_2) \gamma_\mu \left[ g_{\nu_i} + g_{A_q} \gamma_5 \right] u(p_1),$$

$$T_i = \sum_n \frac{1}{t - m_{\tilde{q}_n}^2} \bar{u}_i(k_1) \left[ a_{ij}^L(\tilde{q}_n) P_L + a_{ij}^R(\tilde{q}_n) P_R \right] u(p_1) \times \bar{\nu}(p_2) \left[ a_{ij}^L(\tilde{q}_n) P_R + a_{ij}^R(\tilde{q}_n) P_L \right] v_j(k_2),$$

$$T_u = -\sum_n \frac{1}{\bar{u}_j(k_2) \left[ a_{ij}^L(\tilde{q}_n) P_L + a_{ij}^R(\tilde{q}_n) P_R \right] u(p_1) \times \bar{\nu}(p_2) \left[ a_{ij}^L(\tilde{q}_n) P_R + a_{ij}^R(\tilde{q}_n) P_L \right] v_i(k_1),$$

where the label $n$ denotes the summation over the exchanged $\tilde{q}_L$ and $\tilde{q}_R$ squarks of the same flavor in the $t$- and $u$-channel, and $i, j$ denote the type of the final state neutralinos. After averaging over spins and colors in the initial state, the unpolarized differential cross-section is given by

$$\frac{d\sigma(q\bar{q} \rightarrow \chi^0_i\chi^0_j)}{dt} = \frac{1}{16\pi s^2} \frac{1}{34} \left( \frac{1}{2} \right)^{\delta_{ij}} \left( M_{\tilde{s}\tilde{s}} + M_{\tilde{i}\tilde{i}} + M_{\tilde{u}\tilde{u}} - 2M_{\tilde{s}\tilde{t}} + 2M_{\tilde{s}\tilde{u}} - 2M_{\tilde{u}\tilde{t}} \right),$$

where the factors $\frac{1}{34}$, $\frac{1}{4}$ and $(\frac{1}{2})^{\delta_{ij}}$ come from averaging over color, spin in the initial state and the final identical particle factor, respectively. The squares of the amplitudes can be obtained and summed over final states using standard trace techniques. Therefore, we obtain the following equations,

$$M_{\tilde{s}\tilde{s}} = \frac{e^4}{4\sin^4\theta_W\cos^4\theta_W} \left| D_Z(\hat{s}) \right|^2 (L_q^2 + R_q^2) \left\{ O^ij_Z O^ij_Z^* [(m_{\chi^0_i} - \tilde{u})(m_{\chi^0_j} - \tilde{u}) + (m_{\chi^0_i} - \tilde{t})(m_{\chi^0_j} - \tilde{t}) - m_{\chi^0_i} m_{\chi^0_j} \delta(O^ij_Z^2 + O^ij_Z^*)] \right\},$$

$$M_{\tilde{i}\tilde{i}} = \sum_{k,l} \frac{1}{(\tilde{t} - m_{\tilde{q}_k}^2)(\tilde{t} - m_{\tilde{q}_l}^2)} \left\{ [a_{ij}^L(\tilde{q}_k)a_{ij}^L(\tilde{q}_l) + a_{ij}^R(\tilde{q}_k)a_{ij}^R(\tilde{q}_l)] [a_{ij}^L(\tilde{q}_k)a_{ij}^L(\tilde{q}_l) + a_{ij}^R(\tilde{q}_k)a_{ij}^R(\tilde{q}_l)] \right\} (m_{\chi^0_i} - \tilde{t})(m_{\chi^0_j} - \tilde{t}),$$

$$M_{\tilde{u}\tilde{u}} = \sum_{k,l} \frac{1}{(\tilde{t} - m_{\tilde{q}_k}^2)(\tilde{t} - m_{\tilde{q}_l}^2)} \left\{ [a_{ij}^L(\tilde{q}_k)a_{ij}^L(\tilde{q}_l) + a_{ij}^R(\tilde{q}_k)a_{ij}^R(\tilde{q}_l)] [a_{ij}^L(\tilde{q}_k)a_{ij}^L(\tilde{q}_l) + a_{ij}^R(\tilde{q}_k)a_{ij}^R(\tilde{q}_l)] \right\} (m_{\chi^0_i} - \tilde{t})(m_{\chi^0_j} - \tilde{t}),$$

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In the above equations, the following abbreviation is used

\[
M_{\tilde{u}\tilde{u}} = \sum_{k,l} \frac{1}{(\tilde{u} - m_{\tilde{q}_k}^2)(\tilde{u} - m_{\tilde{q}_l}^2)} \left\{ \left( a_i L^*(\tilde{q}_k) a_j^L(\tilde{q}_l) + a_i R^*(\tilde{q}_k) a_j^R(\tilde{q}_l) \right) \left( a_i L^*(\tilde{q}_l) a_j^L(\tilde{q}_k) + a_i R^*(\tilde{q}_l) a_j^R(\tilde{q}_k) \right) \right\}
\]

(3.15)

\[
M_{\tilde{t}\tilde{u}} = \sum_{k,l} \frac{1}{(\tilde{t} - m_{\tilde{q}_k}^2)(\tilde{t} - m_{\tilde{q}_l}^2)} \left\{ \left( \frac{1}{2} a_i L^*(\tilde{q}_k) a_j^L(\tilde{q}_l) a_j^R(\tilde{q}_k) a_i R^*(\tilde{q}_l) + a_i R^*(\tilde{q}_k) a_j^R(\tilde{q}_l) a_j^L(\tilde{q}_k) \right) \right\}
\]

(3.16)

\[
M_{\tilde{s}\tilde{u}} = \sum_{k} \frac{e^2}{2\sin^2\theta_W \cos^2\theta_W (\tilde{s} - m_{\tilde{q}_k}^2)} \left\{ \left( L_q a_i L^*(\tilde{q}_k) a_j^L(\tilde{q}_k) O_{Z}^{ij*} - R_q a_i R^*(\tilde{q}_k) a_j^R(\tilde{q}_k) O_{Z}^{ij} \right) \right\}
\]

(3.17)

\[
M_{\tilde{s}\tilde{t}} = \sum_{k} \frac{e^2}{2\sin^2\theta_W \cos^2\theta_W (\tilde{t} - m_{\tilde{q}_k}^2)} \left\{ \left( R_q a_i R^*(\tilde{q}_k) a_j^R(\tilde{q}_k) O_{Z}^{ij*} - L_q a_i L^*(\tilde{q}_k) a_j^L(\tilde{q}_k) O_{Z}^{ij} \right) \right\}
\]

(3.18)

In the above equations, the following abbreviation is used

\[
D_Z(\tilde{s}) = \frac{1}{\tilde{s} - m_{Z^0}^2 + im_{Z^0}\Gamma_{Z^0}}
\]

(3.19)

for propagator of the boson $Z^0$. We get $m_{Z^0} = 91.1876$ GeV and the width of the boson $Z^0$ by $\Gamma_{Z^0} = 2.499947$ GeV. To obtain the final cross-section, we use the basic parton model expression of the hadron-hadron collision \(h_1(p_1)h_2(p_2) \rightarrow \chi_0(k_i)\chi_0(k_j)\) [38, 39] which is

\[
\frac{d\sigma}{d\cos\theta} = \frac{1}{2} \sum_{q_1q_2} \int dx_1dx_2 \ x_1G_{q_1/h_1}(x_1, Q) \ x_2G_{q_2/h_2}(x_2, Q) \ \frac{d\hat{\sigma}(q_1q_2 \rightarrow \chi_0\chi_0)}{d\hat{t}},
\]

(3.20)

where \(G_{q_1/h_1}(x_1, Q) \ (G_{q_2/h_2}(x_2, Q))\) is the distribution function of parton \(q_1 \ (q_2)\) in the hadron \(h_1 \ (h_2)\) at the factorization scale \(Q\). We fix the factorization scale to the average mass of
the final state particles, \( Q = (m_{\tilde{\chi}_i^0} + m_{\tilde{\chi}_j^0})/2 \). Taking the \( h_1 h_2 \)-center-of-mass system as the Lab-system, the Lab-momentums of the produced \( \tilde{\chi}_i^0 \) and \( \tilde{\chi}_j^0 \) are \([40]\)

\[
k_i^\mu = (E_i, k_T, k_i\cos\theta), \quad k_j^\mu = (E_j, -k_T, k_j\cos\theta),
\]

(3.21)

where their transverse momentums are clearly just opposite such that \( k_T = k_T_i = -k_T_j \), while their transverse energies \( E_{T_i} = \sqrt{k_T^2 + m_{\tilde{\chi}_i^0}^2}, \quad E_{T_j} = \sqrt{k_T^2 + m_{\tilde{\chi}_j^0}^2} \) are used to define \( x_{T_{i,j}} = 2E_{T_{i,j}}/\sqrt{s} \). Moreover, the momentums of the incoming partons are expressed by

\[
p_1 = \frac{\sqrt{s}}{2}(x_1, 0, 0), \quad p_2 = \frac{\sqrt{s}}{2}(x_2, 0, 0),
\]

(3.22)

\[
p_0 = \frac{\sqrt{s}}{2}(x_1 + x_2) = E_i + E_j, \quad p_3 = \frac{\sqrt{s}}{2}(x_1 - x_2) = (k_i\cos\theta_i + k_j\cos\theta_j),
\]

(3.23)

which lead to

\[
x_1 = \frac{1}{2}[x_{T_i}e^{y_i} + x_{T_j}e^{y_j}] = \frac{M}{\sqrt{s}}e^y,
\]

(3.24)

\[
x_2 = \frac{1}{2}[x_{T_i}e^{-y_i} + x_{T_j}e^{-y_j}] = \frac{M}{\sqrt{s}}e^{-y},
\]

(3.25)

\[
\hat{s} = M^2 = (p_1 + p_2)^2 = x_1x_2s = \frac{s}{4}[x_{T_i}^2 + x_{T_j}^2 + 2x_{T_i}x_{T_j}\cosh(\Delta y)].
\]

(3.26)

Using Eq. (3.20), the expression the differential cross-section in terms of the overall center-of-mass rapidities of the two jets is obtained as follows,

\[
\frac{d\sigma}{dy_idy_jdk_{T_{i,j}}} = x_1x_2 \sum_{q_1q_2} G_{q_1/b_1}(x_1, Q)G_{q_2/b_2}(x_2, Q) \frac{d\hat{\sigma}(q_1q_2 \to \tilde{\chi}_i^0\tilde{\chi}_j^0)}{dt}.
\]

(3.27)

**IV. ELECTROWEAK LOGARITHMIC CORRECTIONS ON THE AMPLITUDES OF THE SUBPROCESSES \( q\bar{q} \to \tilde{\chi}_i^0\tilde{\chi}_j^0 \) AT ONE-LOOP**

In the TeV range such terms reach the several percent level and be easily measurable at future hadron colliders whose experimental accuracy should be at the few permille level. Actually, the logarithmic contributions to the amplitudes may reach the few tens of percent level at the high energy which is reached at the LHC and the validity of the simple one-loop approximation must be seriously questioned \([41]\). From this point of view, If the high energy behaviour of the amplitudes for neutralino pair production at proton-proton collision is considered, one-loop EW corrections should be kept in view. Since the nonlogarithmic one-loop contributions come into view to reach at the few percent level, which is also the
level of the expected experimental accuracy, it may be adequate to disregard these difficult
to figure out effects in the neutralino pair production processes at the LHC energies.

We now present the formulas of the leading and subleading EW logarithmic corrections
for amplitudes of the subprocess \( q\bar{q} \to \tilde{\chi}_i^0\tilde{\chi}_j^0 \), are included in Refs. [41, 42]. At the one-loop
level, these corrections can be separated into three types of terms as follows: Renormal-
ization Group (RG) terms, Universal terms and Non-Universal terms (angular and process
dependent terms).

(a) Renormalization Group (RG) terms: The RG contributions represent the linear loga-
rithms [43], which are produced by the running of the gauge coupling constants, which
are known and can be calculated in a straightforward way. These terms are obtained
by introducing in Born amplitude the running couplings \((g, g')\) of the \( SU(2) \otimes U(1) \)
according to the asymptotic MSSM \( \beta \)-functions are defined as:

\[
\tilde{\beta}_0 = \frac{3}{4} C_A - \frac{n_g}{2} - \frac{n_h}{8} = -\frac{1}{4} \quad \text{and} \quad \tilde{\beta}'_0 = -\frac{5}{6} n_g - \frac{n_h}{8} = -\frac{11}{4} \tag{4.1}
\]

with

\[
g^2(s) = \frac{g^2(\mu^2)}{1 + \beta_0 g^2(\mu^2) \ln(\frac{s}{\mu^2})}, \quad g'^2(s) = \frac{g'^2(\mu^2)}{1 + \beta'_0 g'^2(\mu^2) \ln(\frac{s}{\mu^2})}, \tag{4.2}
\]

where \( C_A = 2, n_g = 3, n_h = 2 \) in the MSSM and \( g = e/s_W, g' = e/c_W \). These terms
correspond to the subleading logarithmic (SL) RG corrections just like in the case of
the SM, but now with the MSSM particle spectrum contributing. At the one-loop,
these contributions only appear from higgsino components \((N_3, N_4)\) produced through
\( Z^0 \) exchange in the s-channel for subprocess \( q\bar{q} \to \tilde{\chi}_i^0\tilde{\chi}_j^0 \). In that case, they are written
as

\[
T^{RG} = -\frac{1}{4\pi^2} \left( g^4\tilde{\beta}_0 dT_s \frac{d\hat{s}}{dg^2} + g'^4\tilde{\beta}'_0 dT_s \frac{d\hat{s}}{dg'^2} \right) \ln(\hat{s}/\mu^2), \tag{4.3}
\]

where \( T_s \) is the s-channel amplitude and \( \mu \) is a reference scale defining the numerical
values of \( g, g' \). Applying this procedure to the amplitudes, by means of the substitutions
are given as;

\[
\frac{e^2 L_q}{s_W c_W^2} \to -\frac{2I_{3u}^3}{4\pi^2} \left( g^4\tilde{\beta}_0 + g'^4\tilde{\beta}'_0 [1 - 2|Q_q|] \right) \ln(\hat{s}/\mu^2), \tag{4.4}
\]

\[
\frac{e^2 R_q}{s_W c_W^2} \to \frac{2Q_q}{4\pi^2} \left( g'^4\tilde{\beta}'_0 \right) \ln(\hat{s}/\mu^2). \tag{4.5}
\]
Universal electroweak (EW) terms: These are process-independent terms, which appear as correction factors to the Born amplitude. Also called “Sudakov” terms, these terms appear to be typically of the form

\[ 2 \ln \left( \frac{\hat{s}}{m_W^2} \right) - \ln^2 \left( \frac{\hat{s}}{m_W^2} \right) \]

and in a covariant gauge are generated by diagrams of vertex (initial/final triangles) and of box type. They are specific of the quantum numbers and chirality of each external particle line and consist of “Yukawa” and “gauge” contributions associated to this line. In addition, they depend on the type of interaction and on the energy. The universal EW terms appearing in \( q\bar{q} \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^0 \) can be separated into two group: the contributions associated with external quark (initial) and neutralino (final) lines as given:

External quark line of chirality \( a = L, R \): The quark lines correspond to a definite chirality \( a \), since all quarks other than third family quarks are taken as massless as far as the kinematics are concerned. The sum of amplitudes for the subprocess \( q\bar{q} \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^0 \) is \( T_a^{ij} \) which is defined by adding indices \((a, i, j)\) to Eq. (3.10). The contribution from external quark line of chirality to \( T_a^{ij} \) is written as

\[ T_a^{ij} \cdot (c_a^{q\bar{q}}), \]  

(4.6)

where \( a \)-index refers to exchanged \( q_L \) and \( q_R \) quarks, and \((i, j)\) describes type of the final neutralinos. The factor in Eq. (4.6), \( c_a^{q\bar{q}} \) is given as

\[ c_a^{q\bar{q}} = c_{gauge,a}^{q\bar{q}} + c_{Yukawa,a}^{q\bar{q}}, \]  

(4.7)

where the gauge term is

\[ c_{gauge,a}^{q\bar{q}} = \frac{\alpha}{8\pi} \left[ I_{qa}(I_{qa} + 1) + \frac{Y_{qa}^2}{4c_W^2} \right] \left[ 2 \ln \left( \frac{\hat{s}}{m_W^2} \right) - \ln^2 \left( \frac{\hat{s}}{m_W^2} \right) \right], \]  

(4.8)

while the Yukawa term is defined as

\[ c_{Yukawa,a}^{q\bar{q}} = -\frac{\alpha}{16\pi s_W^2} \ln \left( \frac{\hat{s}}{m_W^2} \right) \left\{ \left[ \frac{m_t^2}{m_W^2 s_\beta^2} + \frac{m_b^2}{m_W^2 c_\beta^2} \right] \delta_{aL} \right. \]

\[ \left. + 2 \left[ \frac{m_t^2}{m_W^2 s_\beta^2} \delta_{I_{qa},1/2} + \frac{m_b^2}{m_W^2 c_\beta^2} \delta_{I_{qa},-1/2} \right] \delta_{aR} \right\}, \]  

(4.9)

and only this term appears for bottom and top quarks since masses of the other quarks can be neglected. In Eq. (4.8), \( I_{qa} \) is the full weak isospin of the quark with chirality \( a \), and \( Y_{qa} \) is the hypercharge which is defined as \( Y_{qa} = 2(Q_{qa} - I_{qa}^3) \). Consequently, the amplitude of the \( q\bar{q} \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^0 \) with this contribution can be written as

\[ T_{\text{one-loop}}^{ij} = [1 + c_a^{q\bar{q}}] T_a^{ij}. \]  

(4.10)
External neutralino line of chirality $b = L, R$ : The contribution from external neutralino line of chirality to $T_b^{ij}$ may be written as

$$
\sum_k \left[ T_b^{ik} \cdot c_b^{\tilde{\chi}_i^0 \tilde{\chi}_j^0} + T_b^{kj} \cdot c_b^{\tilde{\chi}_k^0 \tilde{\chi}_i^0*} \right].
$$

Here, one use a matrix notation for external particle is one member of mixed states. The amplitude of the subprocess $q\bar{q} \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^0$ involves the neutral higgsino components $(N_{3i}, N_{4i})$ produced through $Z^0$ exchange in the $s$-channel, but it only involves the neutral gaugino $(\tilde{W}_3)$ component $(N_{2i})$ produced through squark exchange in the $t$- and $u$-channels. In addition to this, the logarithmic contributions for higgsino $s$-channel amplitude $(T_s)$ involve both the “higgsino, gauge” and “higgsino, Yukawa” parts, whereas for the gaugino $t$- and $u$-channels amplitudes $(T_t$ and $T_u$), only include the “gaugino, gauge” part. Thus, these contributions may be written as

$$
c_b^{\tilde{\chi}_i^0 \tilde{\chi}_j^0} = c_{\text{higgsino, gauge},b}^{\tilde{\chi}_i^0 \tilde{\chi}_j^0} + c_{\text{higgsino, yuk},b}^{\tilde{\chi}_i^0 \tilde{\chi}_j^0} + c_{\text{gaugino, gauge},b}^{\tilde{\chi}_i^0 \tilde{\chi}_j^0},
$$

where

$$
c_{\text{higgsino, gauge},b}^{\tilde{\chi}_i^0 \tilde{\chi}_j^0} = \frac{\alpha(1 + 2 \beta)}{32 \pi^2 W^2 c_W^2} \left[ 2 \ln \left( \frac{\hat{s}}{m_W^2} \right) - \ln^2 \left( \frac{\hat{s}}{m_W^2} \right) \right]
\times \left[ (N_{3i}^* N_{3j} + N_{3i} N_{3j}^*) \delta_{bL} + (N_{3i} N_{3j}^* + N_{3i}^* N_{3j}) \delta_{bR} \right],
$$

$$
c_{\text{higgsino, yuk},b}^{\tilde{\chi}_i^0 \tilde{\chi}_j^0} = - \frac{3 \alpha}{16 \pi^2 m_W^2} \left[ \ln \left( \frac{\hat{s}}{m_W^2} \right) \right]
\left\{ \frac{m_t^2}{s_{\beta}} (N_{4i}^* N_{4j} \delta_{bL} + N_{4i} N_{4j}^* \delta_{bR})
\right. + \left. \frac{m_b^2}{c_{\beta}} (N_{3i}^* N_{3j} \delta_{bL} + N_{3i} N_{3j}^* \delta_{bR}) \right\},
$$

and

$$
c_{\text{gaugino, gauge},b}^{\tilde{\chi}_i^0 \tilde{\chi}_j^0} = - \frac{\alpha}{4 \pi^2 W} \left[ \ln^2 \left( \frac{\hat{s}}{m_W^2} \right) \right]
\left[ N_{2i} N_{2j} P_L + N_{2i}^* N_{2j}^* P_R \right].
$$

One sees from these contributions, the $[2 \ln(\hat{s}/m_W^2) - \ln^2(\hat{s}/m_W^2)]$ combination can also be found in the higgsino components, and $[-\ln^2(\hat{s}/m_W^2)]$ term in the gaugino components. The leads to an additional potential check of the assumed supersymmetric nature of the interactions of neutralinos which can be achieved by a measurement of the production rate of the four neutralinos [44]. Consequently, this contribution to the amplitude of the $q\bar{q} \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^0$ can be carried out as

$$
T_{\text{one-loop}}^{ij} = \sum_k \left[ \delta_{ij} + \delta_{jk} \cdot c_b^{\tilde{\chi}_i^0 \tilde{\chi}_j^0} + \delta_{ki} \cdot c_b^{\tilde{\chi}_k^0 \tilde{\chi}_i^0*} \right] T_b^{ij}.
$$
(c) Angular and process dependent terms: They only consist in residual terms arising from the quadratic logarithms $\ln^2 t$ or $\ln^2 u$ produced by box diagrams containing $Z^0, W^\pm$ and $\gamma$ gauge boson internal lines, where $t = -\frac{s}{2}(1 - \cos(\theta))$ and $u = -\frac{s}{2}(1 + \cos(\theta))$, $\theta$ being the scattering angle. There are only few such diagrams and they have been all clearly calculated. The diagrams with internal $Z$ lines can be disregarded, because their contributions become orthogonal to the Born terms and cannot interfere with them.

For amplitude of the each subprocess, we can write,

$$T_{\text{One-loop EW}}^{ij} = \left[ 1 + c_{a}^{q\bar{q} \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^0} \right] T_{a}^{ij}, \quad (4.17)$$

where $c_{a}^{q\bar{q} \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^0}$ includes all of the contributions given above. We have exactly calculated these three types of contributions for SL logarithmic accuracy. The total cross-section including the EW corrections reads

$$\sigma = \sigma_0 + \Delta \sigma = \sigma_0(1 + \delta), \quad (4.18)$$

where $\sigma_0$ is the Born level cross-section, $\Delta \sigma$ is the full electroweak contribution to cross-section and $\delta$ is the EW relative correction.
V. NUMERICAL RESULTS AND DISCUSSION

In this section, we present a detailed numerical study of the neutralino pair production process $pp \rightarrow q\bar{q} \rightarrow \tilde{\chi}_1^0\tilde{\chi}_j^0$ at the LHC energies with special emphasis on effects of the EW logarithmic contributions, which are so important thereby can reach the few tens of percent level at the high energy. Focusing on the lightest neutralino $\tilde{\chi}_1^0$ is likely to be the LSP and the next-to-lightest neutralino $\tilde{\chi}_2^0$, we investigate the relevant processes $pp \rightarrow \tilde{\chi}_1^0\tilde{\chi}_1^0$, $pp \rightarrow \tilde{\chi}_2^0\tilde{\chi}_2^0$ and $pp \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0$, can be the most dominant neutralino pair production processes. In our numerical calculations, we just limit the values of $M_1$, $M_2$ and $\mu$ to be real and positive, and we set $\tan \beta = 45$, $m_{\tilde{u}_L} = 998.56$ GeV, $m_{\tilde{u}_R} = 999.36$ GeV, $m_{\tilde{d}_L} = 1000.31$ GeV, $m_{\tilde{d}_R} = 1001.77$ GeV. In addition, we fix the chargino masses as $m_{\tilde{\chi}_1^+} = 85.99$ GeV and $m_{\tilde{\chi}_2^+} = 206.00$ GeV for higgsino and gaugino-like scenarios, and $m_{\tilde{\chi}_1^+} = 87.89$ GeV and $m_{\tilde{\chi}_2^+} = 204.09$ GeV for mixture-case. When using Eqs. (2.14) and (2.15) with given chargino masses, there appear three different cases to choices of the parameters $\mu$ and $M_2$, as mentioned previously, these are the higgsino-like, the gaugino-like and mixture-case respectively.

- In the higgsino-like case, we obtain $M_2 = 150$ GeV, $\mu = 120$ GeV, $M_1 = 75.309$ GeV and by inserting the values of $M_2$, $\mu$ and $M_1$ into Eq. (2.9), the neutralino masses are obtained by

$$m_{\tilde{\chi}_1^0} = 57.45 \text{ GeV}, m_{\tilde{\chi}_2^0} = 99.00 \text{ GeV}, m_{\tilde{\chi}_3^0} = 136.05 \text{ GeV}, m_{\tilde{\chi}_4^0} = 204.91 \text{ GeV}.$$  

- In the gaugino-like case, we have $M_2 = 120$ GeV, $\mu = 150$ GeV, $M_1 = 60.247$ GeV and by inserting the values of $M_2$, $\mu$ and $M_1$ into Eq. (2.9), the neutralino masses are obtained by

$$m_{\tilde{\chi}_1^0} = 52.26 \text{ GeV}, m_{\tilde{\chi}_2^0} = 90.05 \text{ GeV}, m_{\tilde{\chi}_3^0} = 165.56 \text{ GeV}, m_{\tilde{\chi}_4^0} = 203.49 \text{ GeV}.$$  

- Finally, In mixture case we take $M_2 = \mu = 135$ GeV so obtained as $M_1 = 67.78$ GeV and also by inserting the values of $M_2$, $\mu$ and $M_1$ into Eq. (2.9), the neutralino masses are obtained by

$$m_{\tilde{\chi}_1^0} = 55.95 \text{ GeV}, m_{\tilde{\chi}_2^0} = 95.22 \text{ GeV}, m_{\tilde{\chi}_3^0} = 150.79 \text{ GeV}, m_{\tilde{\chi}_4^0} = 202.40 \text{ GeV}.$$  

In the numerical calculations, we use the MSTW2008 parton distribution functions [45] for the quark distribution inside the proton and set the factorization scale to the average final
state mass. For each scenario given above, we have numerically evaluated the hadronic Born cross-sections $\sigma_0$ of the process $pp \rightarrow \tilde{\chi}_1^0\tilde{\chi}_j^0$ (for only $u,d$ quarks and $i,j = 1,2$), the EW logarithmic contributions $\Delta\sigma$ to this process and the relative corrections $\delta$, as a function of the center-of-mass energy from Fig. 2 to Fig. 4, the $M_2-\mu$ mass parameters from Fig. 5 to Fig. 7 and the squark mass from Fig. 8 to Fig. 10, and differential cross-section as a function of the neutralino pair transverse momentum $k_T$ from Fig. 11 to Fig. 13. In these figures, we use the following abbreviations: GL, gaugino-like; HL, higgsino-like; MC, mixture-case.

FIG. 2: The cross-sections of the process $pp \rightarrow \tilde{\chi}_1^0\tilde{\chi}_1^0$ at tree level, the EW corrections and the relative corrections as a function of the center-of-mass energy $\sqrt{s}$.

In Figs. 2 to 4, we present the dependence of the Born level cross-sections, the EW corrections and the relative corrections on the center-of-mass energy. These figures indicate that both Born level cross-sections and EW corrections increase slowly and smoothly with increasing the center-of-mass energy from 7 TeV to 14 TeV for each scenario. Furthermore, the relative corrections increase by about 2 factor as the increment of the center-of-mass energy from 7 TeV to 14 TeV. It implies that EW contributions to the amplitudes fairly depend on the center-of-mass energy. As shown in Fig. 2, the cross-section of the process $pp \rightarrow \tilde{\chi}_1^0\tilde{\chi}_1^0$ in the higgsino-like scenario is larger than the mixing scenario and the gaugino-like scenario in magnitude as about 42 and 65 percent, respectively. At center-of-mass energy 7 TeV (14 TeV), the EW corrections to this process increase the Born cross-section by around 2.4% (4%) in the higgsino-like scenario, 0.6% (1.2%) in the gaugino-like scenario,
FIG. 3: The cross-sections of the process $pp \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_2^0$ at tree level, the EW corrections and the relative corrections as a function of the center-of-mass energy $\sqrt{s}$.

FIG. 4: The cross-sections of the process $pp \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_2^0$ at tree level, the EW corrections and the relative corrections as a function of the center-of-mass energy $\sqrt{s}$.

0.7% (1.3%) in the mixture-case scenario. Furthermore, it can be seen from Fig. 3 that the cross-section of the process $pp \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_2^0$ in the gaugino-like scenario is larger than the mixing scenario and the higgsino-like scenario in magnitude as about 12 and 47 percent, respectively. The EW corrections to this process increase the Born cross-section by around
2%, 0.9% and 1.2% (3.5%, 1.7% and 2.1%) in the higgsino-like, the gaugino-like and the mixture-case scenario at center-of-mass energy 7 TeV (14 TeV), respectively. Finally, in Fig. 4, the cross-section of the process \( pp \rightarrow \tilde{\chi}_2^0\tilde{\chi}_2^0 \) in the gaugino-like scenario is larger than the mixture-case scenario and the higgsino-like scenario in magnitude as around 80 percent and 5 times, respectively. For center-of-mass energy 7 TeV (14 TeV), the EW corrections to this process increase the Born cross-section by around 5.2% (9.4%) in the higgsino-like scenario, 18% (32%) in the gaugino-like scenario, 15% (26%) in the mixture-case scenario.

In Table I we document a numerical survey over our scenarios for LHC center-of-mass energies of 7 TeV and 14 TeV. One can deduce from above analysis and this table that

**TABLE I:** The cross-sections (in fb) for the neutralino pair production processes at Born-level, the EW contributions to these processes and the relative correction for each scenario. Here the relative correction \( \delta \) is \( \Delta \sigma / \sigma_0 \) ratio as percent.

| \( \sigma \) [fb] | \( \sqrt{s} \) [TeV] | Higgsino-like | | Gaugino-like | | Mixture-case |
|------------------|-----------------|---------------|-----------------|-----------------|-----------------|
| \( pp \rightarrow \tilde{\chi}_1^0\tilde{\chi}_1^0 \) | | \( \sigma_0 \) | \( \Delta \sigma \) | \( \delta \) [%] | \( \sigma_0 \) | \( \Delta \sigma \) | \( \delta \) [%] | \( \sigma_0 \) | \( \Delta \sigma \) | \( \delta \) [%] |
| 7                | 357.05          | 8.73          | 2.44            | 217.42          | 1.37            | 0.63            | 252.46          | 1.86            | 0.74            |
| 14               | 800.63          | 33.49         | 4.18            | 478.73          | 5.61            | 1.17            | 564.47          | 7.61            | 1.35            |
| \( pp \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0 \) | | 13.13          | 0.26           | 1.96            | 19.61           | 0.18           | 0.91            | 17.27           | 0.20           | 1.17            |
| 7                | 31.57           | 1.09          | 3.46            | 45.88           | 0.76           | 1.66            | 41.04           | 0.87           | 2.11            |
| 14               | 7.42            | 0.39          | 5.24            | 35.79           | 6.42           | 17.94           | 19.59           | 2.89           | 14.74           |
| \( pp \rightarrow \tilde{\chi}_2^0\tilde{\chi}_2^0 \) | | 19.93          | 1.86           | 9.36            | 89.46           | 28.69          | 32.07           | 50.19           | 13.14          | 26.18           |

the cross-section of the process \( pp \rightarrow \tilde{\chi}_1^0\tilde{\chi}_1^0 \) in the higgsino-like scenario is usually larger than others. Thus, one can say that this process is the most dominant for neutralino pair production processes. In particular, the cross-section of the process \( pp \rightarrow \tilde{\chi}_1^0\tilde{\chi}_1^0 \) in the higgsino-like scenario, appears in the range of 0.357 (\( \Delta \sigma = 0.009 \)) to 0.80 (\( \Delta \sigma = 0.03 \)) pb and should be observable at LHC. Furthermore, for process \( pp \rightarrow \tilde{\chi}_2^0\tilde{\chi}_2^0 \) in the gaugino-like scenario, the cross-section appears in the range of 0.036 (\( \Delta \sigma = 0.006 \)) to 0.089 (\( \Delta \sigma = 0.03 \)) pb. Moreover, as one sees from Table I, the EW corrections to processes \( pp \rightarrow \tilde{\chi}_2^0\tilde{\chi}_2^0 \) are significant and increase the Born cross-section by around 18% (32%) in the gaugino-like scenario and 15% (26%) in the mixture-case scenario for center-of-mass energy 7 TeV.
(14 TeV). One notes that the EW corrections are less than for the other processes. These results imply that the relative corrections increase by about 2 factor with increasing of the center-of-mass energy from 7 TeV to 14 TeV.

The neutralino/chargino masses and mixing matrices depend on the $M_2$ and $\mu$ mass parameters, therefore one can be obtained significant information from the dependence of the cross-section of the neutralino pair production on these parameters. Accordingly, we

![Graphs of cross-sections](image)

**FIG. 5:** The cross-section of the process $p\bar{p} \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0$ (a) at tree level, (b) the EW correction and (c) the relative correction as functions of $M_2$ and $\mu$ for $\sqrt{s} = 8$ TeV.

evaluate the Born level cross-sections, the EW corrections and the relative corrections as functions of $M_2$ and $\mu$ in the range from 100 to 1000 GeV in steps of 50 GeV for $\sqrt{s} = 8$
FIG. 6: The cross-section of the process $pp \rightarrow \tilde{\chi}^0_1 \tilde{\chi}^0_2$ (a) at tree level, (b) the EW correction and (c) the relative correction as functions of $M_2$ and $\mu$ for $\sqrt{s} = 8$ TeV.

We can see from these figures that the Born cross-sections increase with decreasing $M_2$ and any value of $\mu$ for each process. In particular, cross-section reaches maximal values in the region $M_2 \lesssim 200$ GeV into the scan region. The maximum values of the relative correction are obtained in the region $\mu \lesssim 500$ GeV and $M_2 = 2\mu + 50$ (and $+100$) GeV for processes $pp \rightarrow \tilde{\chi}^0_1 \tilde{\chi}^0_1$ and $pp \rightarrow \tilde{\chi}^0_1 \tilde{\chi}^0_2$, whereas in the region $\mu > M_2$ for process $pp \rightarrow \tilde{\chi}^0_2 \tilde{\chi}^0_2$. For example, it can reach about 9.5%, 0.8% at $\mu = 300$ GeV and $M_2 = 650$ GeV for $pp \rightarrow \tilde{\chi}^0_1 \tilde{\chi}^0_1$, $\tilde{\chi}^0_1 \tilde{\chi}^0_2$, respectively, while 21.3% for $pp \rightarrow \tilde{\chi}^0_2 \tilde{\chi}^0_2$ at $\mu = 650$ GeV and $M_2 = 300$ GeV. Furthermore, one can note that the EW
FIG. 7: The cross-section of the process $pp \rightarrow \tilde{\chi}_0^0 \tilde{\chi}_0^0$ (a) at tree level, (b) the EW correction and (c) the relative correction as functions of $M_2$ and $\mu$ for $\sqrt{s} = 8$ TeV.

correction for $pp \rightarrow \tilde{\chi}_0^0 \tilde{\chi}_0^0$ is larger than the remaining ones. From these figures we can see that the EW correction strongly depend on the $M_2$ and $\mu$ mass parameters.

In Figs. 8 to 10, we show the dependence of the Born level cross-sections, the EW corrections and the relative corrections on the squark mass for each scenario at $\sqrt{s} = 8$ TeV. Here, there appear the same dominant scenarios as in the dependence of the cross-sections on the center-of-mass energy. The EW corrections are not sensitive according to increment of the squark mass as shown from these figures. It can be seen from Fig. 8 that the EW corrections to $pp \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0$ increase the Born cross-section by around 2.4%, 0.6% and 0.7%
FIG. 8: The cross-sections of the process $pp \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0$ at tree level, the EW corrections and the relative corrections as a function of the squark mass at center-of-mass energy $\sqrt{s} = 8$ TeV.

FIG. 9: The cross-sections of the process $pp \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_2^0$ at tree level, the EW corrections and the relative corrections as a function of the squark mass at center-of-mass energy $\sqrt{s} = 8$ TeV.

in the higgsino-like, the gaugino-like and the mixture-case scenarios, respectively, for all values of the squark mass. As seen in Fig. 9, the EW corrections to $pp \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_2^0$ increase the Born cross-section by around 2%, 0.9% and 1.2% in the higgsino-like, the gaugino-like and the mixture-case scenarios, respectively, for all values of the squark mass. Finally, the EW
corrections to $pp \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0$ increase the Born cross-section by around 5.2%, 18% and 15% in the higgsino-like, the gaugino-like and the mixture-case scenarios, respectively, for all values of the squark mass as shown in Fig. 10. These results imply that the EW corrections to $pp \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0$ are larger than the others and the relative corrections are not affected by increasing of the squark mass from 400 GeV to 2000 GeV.

Finally, in Figs. 11 to 13, we display the dependence of the differential cross-sections for the process $pp \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0$ as a function of the neutralino pair transverse momentum $k_T$ at rapidity $y_i = y_j = 0$. It is seen from these figures that the differential cross-sections reach a maximum value at around $k_T = 450$ GeV and then decrease with increasing $k_T$ in the range of 450 to 2500 GeV. The differential cross-sections at Born-Level decrease in the range between about $10^{-9}$ to $10^{-14}$ fb/GeV$^2$ and the differential cross-sections of the processes with EW corrections decrease in the range between about $10^{-10}$ to $10^{-15}$ fb/GeV$^2$ with the increment of $k_T$. It should be noted that the dependence of the differential cross-section of the processes on the neutralino pair transverse momentum $k_T$ is dominated by one of the processes, $pp \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0$ in the gaugino-like scenario appears in the value $4.7 \times 10^{-9}$ fb/GeV$^2$. The relative correction for $pp \rightarrow \tilde{\chi}_1^0\tilde{\chi}_2^0$ decrease from 2.5% to 1.97%, 0.68% to 0.67% and 0.78% to 0.76% in the higgsino-like, the gaugino-like and the mixture-case scenario as the increment of the transverse momentum from 300 to 2500 GeV, respectively.
FIG. 11: The differential cross-sections of the process $pp \rightarrow \tilde{\chi}^0_1 \tilde{\chi}^0_1$ at tree level, the EW corrections and the relative corrections as a function of the neutralino pair transverse momentum $k_T$ at center-of-mass energy $\sqrt{s} = 7$ TeV.

FIG. 12: The differential cross-sections of the process $pp \rightarrow \tilde{\chi}^0_1 \tilde{\chi}^0_2$ at tree level, the EW corrections and the relative corrections as a function of the neutralino pair transverse momentum $k_T$ at center-of-mass energy $\sqrt{s} = 7$ TeV.
FIG. 13: The differential cross-sections of the process \( pp \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0 \) at tree level, the EW corrections and the relative corrections, as a function of the neutralino pair transverse momentum \( k_T \) at center-of-mass energy \( \sqrt{s} = 7 \text{ TeV} \).

The relative correction for \( pp \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0 \) decrease from 2.0% to 1.7%, 0.94% to 0.83% and 1.2% to 1.0% in the higgsino-like, the gaugino-like and the mixture-case scenario with the increasing the transverse momentum from 300 to 2500 GeV, respectively. The relative correction for \( pp \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_2^0 \) decrease from 5.22% to 4.99%, 17.8% to 17.1% and 14.6% to 13.7% in the higgsino-like, the gaugino-like and the mixture-case scenario as the increment of the transverse momentum from 300 to 2500 GeV, respectively. These results show that the EW corrections are sensitive to the transverse momentum.

VI. CONCLUSION

In this paper, we have considered EW corrections for the neutralino pair production processes in proton-proton collisions at the LHC. In the description, we have taken into account the process \( pp \rightarrow \tilde{\chi}_i^0 \tilde{\chi}_j^0 \) at the tree level as a first choice, leading and SL contributions for these processes at the one-loop level as a second choice. These corrections are significant for the theoretical and experimental studies relating to the neutralino pair productions via the proton-proton collisions at the LHC and the future colliders, since they can be reach the few tens of percent level at the high energy. We have given detail illustrations for the
dependence of the cross-sections of the processes $pp \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0$, $\tilde{\chi}_1^0 \tilde{\chi}_2^0$, $\tilde{\chi}_2^0 \tilde{\chi}_2^0$, on the center-of-mass energy, $M_2-\mu$ mass parameters and squark mass for three different scenarios.

The numerical results show that the EW corrections significantly increase the Born cross-section in the dependence of the processes on the center of mass energy. In particular, the relative correction for $pp \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_2^0$ reaches about 30% in the gaugino-like scenario. Moreover, we can see that the EW correction strongly depend on the $M_2$ and $\mu$ mass parameters. The maximum values of the relative correction are obtained in the region $\mu \lesssim 500$ GeV and $M_2 = 2\mu + 50$ (and +100) GeV for processes $pp \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_1^0$, $\tilde{\chi}_1^0 \tilde{\chi}_2^0$, and in the region $\mu > M_2$ for process $pp \rightarrow \tilde{\chi}_2^0 \tilde{\chi}_2^0$. However, the squark mass dependence of the cross-sections for each scenario decrease with increasing of the squark mass from 400 GeV to 1000 GeV, but the EW corrections are not affected by increasing of the squark mass. Finally, the dependence of the differential cross-sections for the process on the neutralino pair transverse momentum $k_T$ shows that the relative corrections decrease as the increment of the transverse momentum from 300 to 2500 GeV.

It should be underlined that there appear sizeable EW corrections to the neutralino production, which significantly increase the extracted bounds on the gaugino masses from the negative search for these particles at the LHC. To our opinion these results imply an interesting complementarity between the future LHC measurements, the related neutralino pair measurements at a future Linear Collider. We hope our results will be help for investigations and analysis the different neutralino decay channels, gaugino and higgsino production in the LHC and future hadron colliders.

**Acknowledgments**

This work is supported by TUBITAK under grant number 2221(Turkey). One of the authors A. I. Ahmadov is grateful for financial support Baku State University Grant “50+50”. 
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