Runaway core collapse and cluster survival: where are the parent clusters of ULXs?

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Accreting intermediate-mass black holes (IMBHs) have been proposed as an explanation for ultraluminous X-ray sources (ULXs). Runaway core collapse inside a massive cluster is a possible mechanism for IMBH formation. But if so, why are ULXs only rarely found associated with a cluster? We use a simple analytical approximation to show that rapid core collapse can occur in two physical regimes. For cluster masses $\sim 10^6 M_\odot$, an IMBH may be formed if the collapse timescale is $\lesssim 3$ Myr, as already well known; the parent cluster is expected to survive. For cluster masses $\sim 10^5 M_\odot$, an IMBH may result from a core collapse on even shorter timescales ($\approx 0.5$ Myr), when the protocluster is still embedded in optically thick gas. Most clusters in this latter regime are disrupted “explosively” as soon as the gas is ionized by the OB stars. We speculate that this process may produce isolated ULXs with masses up to a few $10^3 M_\odot$, surrounded by a loose OB association, and perhaps by a nebula, remnant of the expanding gas from the disrupted protocluster.

1. INTRODUCTION

Various alternative models have been proposed to explain the nature of ULXs in nearby galaxies. Inhomogeneous accretion flows may allow luminosities a factor of 10 in excess of the Eddington limit ($\dot{M}_\text{Edd}$). Moderate geometrical beaming ($\gamma_\text{geom}$) in the direction of the observer may boost the observed brightness by an order of magnitude. Thus, sources with an apparent isotropic X-ray luminosity up to a few $10^{39}$ erg s$^{-1}$ can be explained by stellar-mass black holes (BHs) with masses $\approx 5$–15 $M_\odot$, without the need to invoke new astrophysical mechanisms. However, these scenarios become problematic for ULXs with steady apparent X-ray luminosities $> 10^{40}$ erg s$^{-1}$: they require either strong beaming, such as a relativistic jet pointing towards the observer ($\gamma_\text{geom} \approx 4$), or a more massive accretor, with $M \approx 100$–1000 $M_\odot$ (IMBHs: $\gamma_\text{geom} \approx 6$). It is possible that ULXs are a mixture of different classes of objects. Observational and statistical arguments have been made against a predominance of beamed sources ($\gamma_\text{geom} \approx 4$). In other cases, circumstantial evidence in favor of IMBHs has been proposed from X-ray spectral and timing studies ($\gamma_\text{geom} \approx 6$).

One of the difficulties of the IMBH scenario is how to introduce a plausible formation mechanism. It has been suggested that IMBHs could be the remnants of very high-mass ($\sim 10^4 M_\odot$) zero-metallicity Population-III stars ($\gamma_\text{geom} \approx 4$). However, this is difficult to reconcile with the relative abundance of ULXs in young star-forming regions, especially in starburst and merging galaxies. Another possibility ($\gamma_\text{geom} \approx 6$) is that at least some ULXs may be the nuclei of smaller satellite galaxies accreted and tidally disrupted by a larger host ($\gamma_\text{geom} \approx 6$).

Alternatively, it has been proposed ($\gamma_\text{geom} \approx 4$, $\gamma_\text{geom} \approx 6$) that IMBHs could be formed in the dense cores of massive, young stellar clusters or super star clusters. Analytical approximations and numerical simulations have shown that, for a suitable range of cluster masses and densities, mass segregation and the Spitzer instability ($\gamma_\text{geom} \approx 6$) can lead to a runaway core collapse on a timescale shorter than the lifetime of its most massive stars ($\sim 3$ Myr). The same simulations also show that the mass of the collapsing core is $\sim 10^{-3}$ times the total mass of the cluster. Thus, this scenario could explain the formation of IMBHs with a mass up to $\sim 1000 M_\odot$.

The main difficulty of theoretical models invoking IMBH formation inside a cluster is that, in fact, most ULXs do not have a bright, massive cluster as their optical counterpart, apart from a few notable exceptions (e.g., a bright ULX in M 82 is associated with the star cluster MGG-11: $\gamma_\text{geom} \approx 4$). In some cases, such as the ULXs in the Cartwheel Galaxy ($\gamma_\text{geom} \approx 6$), the distance is such that we cannot draw firm conclusions on the significance of possible ULX-cluster coincidences. In the starbursting Antennae Galaxies, most ULXs are displaced by $\approx 300$ pc from their nearest star clusters ($\gamma_\text{geom} \approx 4$, $\gamma_\text{geom} \approx 6$). In the starburst galaxy NGC 7714, no optical counterparts have been found for the two brightest ULXs ($\gamma_\text{geom} \approx 4$, $\gamma_\text{geom} \approx 6$): there are no objects brighter than $M_V \sim M_B \sim -9$ within a few hundred pc of the two sources. This rules out young clusters with masses $\gtrsim 10^4 M_\odot$. In other cases (e.g., in NGC 5408: $\gamma_\text{geom} \approx 4$; in NGC 5204: $\gamma_\text{geom} \approx 6$; in NGC 4559: $\gamma_\text{geom} \approx 6$; in Holmberg IX: $\gamma_\text{geom} \approx 6$) the optical counterpart is thought to be an individual O or B star.

One explanation for the lack of ULX-cluster associations is that the accreting systems are runaway binaries ejected from a cluster. However, the kick velocities required to explain displacements of a few hundred pc would rule out BH masses $\gtrsim 20 M_\odot$ ($\gamma_\text{geom} \approx 4$). Hence, the ejection scenario is not consistent with IMBHs. An alternative possibility is that the parent cluster has already dispersed. Physical mechanisms leading to the expansion and disintegration of star clusters are tidal disruption, or mass loss via SN explosions and winds from evolved massive stars (e.g., $\gamma_\text{geom} \approx 4$, $\gamma_\text{geom} \approx 6$). However, it is doubtful that these processes
can entirely dissipate a massive cluster on timescales as short as $\sim 10^7$ yr.

In this paper, we use simple analytical approximations to compare the timescale for IMBH formation with characteristic timescales of cluster evolution. We show that for some density profiles, the collapse can already occur during the initial protocluster phase, when the stars are still embedded in optically-thick gas. We then show that for a range of masses and densities, the parent clusters may not survive the embedded phase, leading to the formation of an apparently isolated IMBH.

2. VELOCITIES AND DENSITIES IN A CLUSTER CORE

A useful definition of relaxation timescale for a cluster of mass $M$ is, from \cite{14}:

$$t_r = \frac{\sigma_3^3}{4\pi(3/2)^{3/2} G^2 \ln(\Lambda) nm^2}, \quad (1)$$

where $\sigma_3$ is the three-dimensional velocity dispersion, $n$ is the number density of stars, $m$ is the average stellar mass and $\ln(\Lambda)$ is the Coulomb logarithm. For processes related to core collapse, we can take the initial values of these quantities at the cluster center (e.g., \cite{11}); we obtain a central relaxation timescale $t_{rc}$:

$$t_{rc}(0) = \frac{3^{3/2} \sigma_1^3(0)}{4\pi(3/2)^{3/2} G^2 \ln(\Lambda) \rho(0)m(0)}, \quad (2)$$

where $\rho(0)$ is the initial mass density at the core and $\sigma_1(0)$ is the initial one-dimensional velocity dispersion. The initial average stellar mass $m(0)$ at the core may be larger than the average mass over the whole cluster, if there is initial mass segregation. For simplicity, we shall assume they are the same.

Assuming for simplicity that the system is virialized, we have an additional relation between density and velocity dispersion at the cluster center. For the Plummer model (\cite{28; 14}),

$$\sigma_3^3(0) = (1/2)^{3/2}(4\pi/3)^{1/2} G^{3/2} M^{1/2} \rho^{1/2}(0), \quad (3)$$

hence

$$\sigma_1(0) = 0.518 G^{1/2} M^{1/3} \rho^{1/6}(0). \quad (4)$$

Analogous expressions can be derived from the virial theorem for the King density profiles (\cite{27}). For example, for the King model with dimensionless central potential $W_0 = 2$ (\cite{28}),

$$\sigma_1(0) = 0.585 G^{1/2} M^{1/3} \rho^{1/6}(0), \quad (5)$$

and

$$\sigma_1(0) = 0.286 G^{1/2} M^{1/3} \rho^{1/6}(0), \quad (6)$$

for the more centrally concentrated $W_0 = 9$ profile.

We could approximate the Coulomb logarithm $\ln(\Lambda) \approx 7$–9 for the parameter range of interest here. More accurately, we use the definition of $\Lambda$ (\cite{14}, chapter 2), to obtain $\Lambda = 3 r_h \sigma_1^2/(GM) \approx 0.4 M/m$, where $r_h$ is the half-mass radius. Expressing $M \propto \rho^{-1/2}(0)\sigma_1^4(0)$, where the proportionality constant depends on the cluster model, we obtain:

$$\Lambda = \alpha G^{-3/2} \rho^{-1/2}(0)\sigma_1^3(0)m^{-1}(0). \quad (7)$$

Here, $\alpha = 2.9, 2.5, 17.2$ for the Plummer, $W_0 = 2$ and $W_0 = 9$ King profiles, respectively. As a first-order approximation, we can also substitute $\sigma_1^3(0)m^{-1}(0)$ from Eq. 2, so that:

$$\Lambda \approx \beta t_{rc}(0) \rho^{1/2}(0), \quad (8)$$

where $\beta = 0.022, 0.015, 0.131$ for the three selected models, respectively (in CGS units).

Finally, it can be shown that

$$r_h(0) = \gamma G^{-3/2} \rho^{-1/2}(0)\sigma_1(0), \quad (9)$$

where the constant $\gamma = 1.56, 1.17, 13.10$ for the three profiles.

Using Eqs. (2) and (7) one can now plot the central velocity dispersion as a function of central density, for a given central relaxation timescale: $\sigma_1(0) = f(\rho(0); t_{rc}(0))$. Using one of Eqs. (4/5/6), one can plot $\sigma_1(0) = g(\rho(0); M)$. From Eq. 9, one can plot $\sigma_1(0) = h(\rho(0); r_h(0))$. See also \cite{29}, in particular their Figure 1.4.

3. TIMESCALE FOR CLUSTER EVOLUTION

Numerical simulations (\cite{11}) for a variety of Plummer and King profiles have shown that the timescale for core collapse ($t_{cc}(0)$) in clusters with a broad mass spectrum is proportional to the central relaxation timescale, rather than to the relaxation timescale at the half-mass radius:

$$t_{cc}(0) \approx 0.15 t_{rc}(0), \quad (10)$$

also in agreement with the simulations of \cite{12}. The final core mass after the runaway collapse is found to be (\cite{11}):

$$M_{cc} \approx 0.002 M, \quad (11)$$

which is likely to evolve later into a BH with mass $M_{BH} \approx 0.001 M$, via direct collapse or SN explosion.

In the standard treatment, the timescale in Eq. (10) is compared with the stellar evolution timescale: core collapse and the subsequent formation of an IMBH are possible only if $t_{cc}(0) \lesssim 3$ Myr, typical lifetime of
the most massive stars on the main sequence (12). If the core collapse is not completed after 3 Myr, it will be stopped by strong mass losses from supergiant winds and SNe. By imposing this constraint, one can identify the regions in the \((\sigma_1(0), \rho(0))\) plane where core collapse is most likely to occur.

The situation becomes more complicated when the role of gas in the young clusters is taken into account. Both from an analytical approximation and from numerical simulations (e.g., 30, in particular Figure 1.21) it appears that centrally-concentrated clusters can achieve core collapse in < 1 Myr. If so, the runaway merger of the most massive stars occurs when most stars in the embedded cluster are still surrounded by a spherical cocoon, optically thick to the ionizing radiation (class 0/I protostars). The lifetime of a class I phase is \(\approx 1–3 \times 10^5\) yr for low-mass protostars (e.g., 34, 35, 36), and probably even longer for O stars, which spend \(\approx 13–15\%\) of their lifetime (\(\approx 3–5 \times 10^5\) yr) shrouded by an optically thick cocoon (33, 34). Taking into account that even “instantaneous” star formation in a young cluster is in fact spread out over \(\sim 10^5\) yr, a young cluster remains embedded in molecular gas for \(\approx 0.5–1 \times 10^6\) yr (e.g., 36). At these early ages, the remaining cold gas in the young cluster has a mass at least comparable to or larger than the mass in stars.

When most of the cocoons dissipate, the cluster gas is quickly ionized (on a timescale of \(\sim 10^5\) yr) by the Lyman continuum photons emitted by the OB stars, and reaches a characteristic temperature \(\approx 10^4\) K. For typical densities \(\sim 10^3–10^6\) cm\(^{-3}\), this corresponds to pressures \(P/k_B \approx 10^7–10^{10}\) K cm\(^{-3}\), many orders of magnitude larger than the pressure of the ISM. As a consequence, the gas in the cluster expands “explosively” with a velocity of order of the sound speed, \(v \sim 10\) km s\(^{-1}\) (37, 38). In some young clusters in the Antennae, gas expansion velocities \(\approx 25–30\) km s\(^{-1}\) have been inferred (33, 40).

If the gravitational potential in the cluster is too shallow, the expanding gas becomes unbound; if the mass loss \(\gtrsim 50\%\), the whole cluster will dissipate on the same timescale. As a back-of-the-envelope estimate, this occurs when the gas expulsion speed is larger than the velocity dispersion at the half-mass radius: \(\sigma_3(r = r_h) \lesssim 10\) km s\(^{-1}\) (37). For typical Plummer and King profiles, this corresponds to a central velocity dispersion \(\sigma_1(0) \lesssim 15–25\) km s\(^{-1}\). A more accurate calculation of the exact value of the velocity dispersion threshold depends on the details of the gas and stellar distribution and is beyond the scope of this work. For simplicity, we shall take \(\sigma_1(0) < 20\) km s\(^{-1}\) as a condition for cluster disruption, and (conservatively) \(t = 5 \times 10^5\) yr for the lifetime of an embedded cluster. At the same time, clusters have to be more massive than \(\approx 10^4 M_\odot\) for the explosive disruption process to occur: smaller clusters do not contain enough O stars to ionize all the gas.

The analytical approximation outlined in Section 2 did not take into account the role of gas. A simple way to account for this effect is to substitute the total cluster mass \(M_{\text{tot}} = M + M_g\) for the stellar mass \(M\) in the virial theorem; \(f\) depends on the star formation efficiency and cluster age. For the young clusters we are dealing with, \(f \approx 1.5–3\). As a result, \(\sigma_1(0)\) in Eq. (4/5/6) will be multiplied by a factor \(1/f^{1/3}\). For example, taking \(f = 2\), we shall plot

\[
\sigma_1(0) = 0.65 G^{1/2} M^{1/3} \rho^{1/6}(0)
\]

for the Plummer model, and

\[
\sigma_1(0) = 0.36 G^{1/2} M^{1/3} \rho^{1/6}(0),
\]

for the \(W_0 = 9\) King profile, where \(\rho(0) \equiv f n(0)m(0)\) is now the total initial central density. With similar changes we also easily modify the other scaling relations in Section 2; for example \(\Lambda = f M/m \approx (\beta/f) \tau_{cc}(0) \rho^{1/2}(0)\), etc.

### 4. IMBHs WITH OR WITHOUT THEIR PARENT CLUSTERS

Putting together the timescale constraints discussed in Section 2 and 3, we identify two different regions in the parameter space, relevant to the formation of IMBHs from runaway core collapse:

- **the first regime** (region A in Figs. 1–4) is the one discussed by 12: core collapse on a timescale \(t_{\text{cc}}(0) \lesssim 3\) Myr, inside a super star cluster (mass \(\sim 10^5\) to a few \(10^6 M_\odot\)), giving rise to a compact remnant \(\gtrsim 500 M_\odot\). The parent cluster is massive enough to survive the initial gas expulsion; it may dissipate later, on much longer timescales, owing to mass loss in later stages of stellar evolution, or tidal interactions. Stellar populations near the brightest ULXs in nearby star-forming galaxies have typical ages \(\sim 10^7\) yr: if the accreting compact objects are IMBHs formed through this process, we expect them to be still contained inside their parent clusters. A likely example is the ULX associated to the super star cluster MGG-11 in M82 (13);

- **the second regime** (region B in Figs. 2–4) is for \(t_{\text{cc}}(0) \lesssim 0.5\) Myr, in clusters that satisfy the disruption condition discussed in Section 3: \(M \gtrsim 10^4 M_\odot\) and \(\sigma_1(0) < 20\) km s\(^{-1}\). A cluster in this region of the parameter space is unlikely to survive beyond its embedded phase: its stars will keep expanding freely with velocities \(\sim \sigma_3\). After \(10^7\) yr, this will result in an OB association with a diameter of \(\sim 200\) pc, difficult to distinguish from other surrounding star-forming regions. Not all the clusters in this regime are
suitable for IMBH formation: assuming that Eq. (11) holds, only the subset of exploding clusters with $M \gtrsim 5 \times 10^3 M_\odot$ are massive enough to produce BHs with $M \gtrsim 50 M_\odot$, required to explain the observed ULX luminosities.

The densest young star clusters in our galaxy (e.g., RU136, NGC 3606, the Arches) have total central densities $\sim 10^6 M_\odot$ pc$^{-3}$. Therefore, we took $\rho_{\text{max}}(0) = 10^6 M_\odot$ pc$^{-3}$ as a reasonable upper limit to the initial central density in our parameter space.

Core collapse is more likely to occur on short timescales for centrally concentrated profiles: for example, for the $W_0 = 9$ King profile (Fig. 2) rather than for the Plummer profile (Fig. 1), in agreement with the simulations of [30]. For a given cluster model, the parameter space available for runaway core collapse is enhanced when the average stellar mass is higher, that is when the initial mass function is top-heavy. Here we compare the initial conditions for $m = 0.5 M_\odot$ (Fig. 3) and $m = 3.0 M_\odot$ (Fig. 4), for the same $W_0 = 9$ King profile. For example, values of $m \approx 3.0 M_\odot$ have been inferred for the MGG-11 cluster in M 82, which is likely to contain an IMBH [12; 11].

Taking a $W = 9$ King profile with a typical central density $\sim 10^6 M_\odot$ pc$^{-3}$ (Figs. 2–4), we infer from our simple analytical approach that clusters with initial stellar masses $\sim 10^5 M_\odot$ may produce a collapsed core during their embedded phase, but may not survive the ionization/expulsion of their gas. The collapsed cores may later evolve into apparently isolated IMBHs with masses up to $\sim 200 M_\odot$. On the other hand, super star clusters with initial stellar masses $\sim 10^6 M_\odot$ may survive the embedded phase and produce IMBHs with masses up to $\sim 10^3 M_\odot$. Considering that the initial cluster mass function $dN \sim M^{-2} dM$, we also expect that IMBHs from clusters in region B should be an order of magnitude more numerous than those formed from clusters in region A.

5. CONCLUSIONS

We have used a simple analytical approximation to suggest that runaway core collapse in a young star cluster may occur in two distinct regimes. The first regime (extensively investigated with numerical simulations by [12] and [11]) is a collapse on a timescale $\approx 3$ Myr (main-sequence lifetime of the O stars) in a cluster of total initial mass $\sim 10^6 M_\odot$. Observed after $\sim 10^7$ yr, the outcome of the collapse is likely to be an accreting IMBH ($M_{\text{BH}} \sim 10^2 M_\odot$) inside a bright cluster. The second regime is a collapse on a shorter timescale ($\approx 0.5$ Myr), in a smaller cluster (initial mass $\sim 10^3 M_\odot$), during its initial embedded phase, when the most massive protostars are still shrouded by optically-thick cocoons. If the velocity dispersion in a cluster is smaller than the thermal velocity of the ionized gas, such a cluster does not survive beyond its embedded phase, because of the explosive loss of gas. Observing the system after $\sim 10^7$ yr, we may find an accreting IMBH ($M_{\text{BH}} \sim 10^2 M_\odot$) in a star-forming region or OB association, but apparently not inside any clusters.

Observationally, ULXs have been explained as accreting IMBHs. Runaway core collapse is a possible mechanism of IMBH formation in young stellar environments. However, most ULXs are not coincident with a cluster, though they are often associated with OB stars. We have suggested that the existence of two possible regimes of core collapse may explain this puzzle. The ULX associated with the M 82 cluster MGG-11 may be an example of core collapse in the first regime, when the parent cluster survives. Most of the other ULXs in nearby star-forming galaxies, with an OB companion and X-ray luminosities up to $\sim 10^{40}$ erg s$^{-1}$, may have been formed in the other regime, when the parent cluster evaporates explosively.

The parameter space allowing IMBH formation from a runaway core collapse is more extended for a top-heavy stellar mass function. It has often been claimed that the stellar initial mass function is top-heavy (higher fraction of high-mass stars) in some starburst galaxies, although the issue is still controversial [42 and references therein]. If this is the case, we speculate that it may explain why ULXs seem to be more often found in starburst galaxies or in environments where star formation is triggered by galactic mergers or tidal interactions.

Finally, if many ULXs were indeed formed in the core of a long-dissolved protocluster, we may want to search for clues of that disruption event. For example, after $\sim 10^7$ yr, the expanding shell or cloud of gas with a mass $\sim 10^5 M_\odot$ will have a characteristic radius $\sim 100$ pc and density $\sim 1$ cm$^{-3}$. This gas may now be X-ray photoionized by the accreting IMBH. We speculate that at least some of the ionized nebulae discovered around nearby ULXs [23] may be related to the disruption of the same protocluster where the BH was formed.

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Figure 1: Parameter space for the initial central density $\rho(0)$ and one-dimensional velocity dispersion $\sigma_1(0)$, in a cluster with a Plummer profile, with average stellar mass $m = 1M_\odot$. We have plotted $\sigma_1(0)$ as a function of $\rho(0)$ at fixed core-collapse timescale $t_{cc}(0)$ (solid blue lines), and at fixed total stellar masses $M$ (dashed green lines). We have taken $t_{cc}(0) = 0.15t_{rc}(0)$. A red box (marked with “A”) identifies the region of the parameter space where the runaway core collapse occurs in $< 3$ Myr and the cluster is massive enough to produce an IMBH ($M_{BH} \approx 10^{-3}M > \sim 50M_\odot$). Finally, we have divided the parameter space into two subsets with $\sigma_1(0) > 20$ and $< 20$ km s$^{-1}$. Clusters above the threshold are more likely to survive their embedded phase; clusters below the threshold (but more massive than $\sim 10^4 M_\odot$) are more likely to evaporate, turning into OB associations. Finally, we have chosen an initial density $\rho(0) = 10^7 M_\odot$ pc$^{-3}$ as a plausible upper limit, based on the values estimated for the densest clusters in our and nearby galaxies.
Figure 2: As in Figure 1, for a King profile of concentration index $W_0 = 9$. Here there are two distinct regions of the initial parameter space where runaway core collapse can occur: region A is for massive clusters which are expected to survive bound, and region B for less massive systems which are expected to evaporate. We speculate that IMBHs formed from region-B clusters may explain a population of ULXs with X-ray luminosities $L_x \approx 10^{40}$ erg s$^{-1}$ found in nearby star-forming galaxies but not associated to any present-day cluster.

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Figure 3: As in Figure 2, for an average initial stellar mass $m = 0.5M_\odot$. The initial parameter space available for runaway core collapse and IMBH formation is much reduced.

Figure 4: As in Figure 2, for an average initial stellar mass $m = 3.0M_\odot$. The initial parameter space available for runaway core collapse and IMBH formation is enhanced for a top-heavy stellar mass function.