We explore a novel approach for building DNN training clusters using today’s commodity hardware. Our proposal, called TOPOOPT, co-optimizes the distributed training process across three dimensions: computation, communication, and network topology. TOPOOPT uses a novel alternating optimization technique and a group theory-inspired algorithm to find the best network topology and routing plan, together with parallelization strategy, for distributed DNN training. To motivate our proposal, we measure the communication patterns of distributed DNN workloads at a large online service provider. Experiments with a 12-node prototype demonstrate the feasibility of TOPOOPT. Simulations on real distributed training models show that, compared to similar-cost Fat-tree interconnects, TOPOOPT reduces DNN training time by up to 3×.

1 Introduction

Our society is rapidly becoming reliant on deep neural networks (DNNs). New datasets and models are invented frequently, increasing the memory and computational requirements for training. This explosive growth has created an urgent demand for efficient distributed DNN training systems.

Today’s DNN training systems are built on top of traditional datacenter clusters, with electrical packet switches arranged in a multi-tier Fat-tree topology [47]. Fat-tree topologies are traffic-oblivious fabrics, allowing uniform bandwidth and latency between server pairs. They are ideal when the workload is unpredictable and consists mostly of short transfers—two inherent properties of legacy datacenter workloads [49, 50, 54, 66, 67]. But Fat-tree networks are becoming a bottleneck for distributed DNN training workloads [57, 68, 76, 83, 98, 101, 131].

Previous work has focused on addressing this challenge by reducing the size of parameters to transmit through the network [48, 57, 58, 68, 72, 78, 81, 82, 90, 101, 118, 135] and developing techniques to discover faster parallelization strategies while considering the available network bandwidth [46, 48, 83, 101, 124]. These proposals co-optimize computation and communication as two important dimensions of distributed DNN training, but they do not consider the physical layer topology as an optimization dimension.

SiP-ML [87] recently demonstrated the benefits of 8 Tbps silicon photonics-based networks for distributed training workloads. While encouraging, the silicon photonics technology is not yet commercially available, which raises the question: “Can we build an optimized network topology for DNN training clusters using today’s commodity hardware?”

To answer this question, we analyze DNN training jobs from production clusters of a large-scale service provider with billions of users, which we call BigNET for anonymity. We demonstrate that training workloads do not satisfy standard assumptions about datacenter traffic that underlie the design of Fat-tree interconnects. Specifically, we show that (i) the communication overhead of large DNN training jobs increases dramatically as we increase the number of workers; and (ii) the traffic heatmap of DNN training jobs changes greatly depending on their parallelization strategies.

Motivated by these observations, we propose TOPOOPT, a DNN training system that co-optimizes network topology and parallelization strategy. To be precise, TOPOOPT creates dedicated partitions for each training job within the cluster, and jointly optimizes the topology and parallelization strategy of the job. To achieve this goal, we grapple with the algorithmic challenges of finding the best topology, such as how to navigate the large search space across computation, communication, and topology dimensions, and also with various operational challenges, such as which optical switching technologies match well with the traffic patterns of various DNN models.

In particular, we cast the topology and parallelization strategy co-optimization problem as an off-line alternating optimization framework. Our optimization technique alternates between optimizing the parallelization strategy and optimizing the network topology. It searches over the parallelization strategy space assuming a fixed topology, and feeds the traffic demand to a TOPOLOGYFINDER algorithm. The updated topology is then fed back into the parallelization strategy search algorithm. This alternating process repeats until the system converges to an optimized parallelization strategy and topology.

We demonstrate that finding an optimized network topology for DNNs with hybrid data and model parallelism is challenging because the ideal network topology needs to meet two goals simultaneously: (i) to complete large AllReduce transfers efficiently, and (ii) to ensure a small hop-count for Model Parallel transfers. To meet these goals, we propose a novel group theory-based technique, called TotientPerms. Our TotientPerms approach builds a series of AllReduce permutations that not only carry AllReduce transfers efficiently, but are also well-positioned to carry Model Parallel transfers and, hence, improve the overall training performance.

To demonstrate the feasibility of TOPOOPT, we build a 12-server testbed with NVIDIA A100 GPUs [37] and 100 Gbps NICs. Our evaluations with six representative DNN models (DLRM [21], CANDLE [4], BERT [129], NCF [74], ResNet50 [73], and VGG [121]) show that TOPOOPT...
reduces the training iteration time by up to $3 \times$ compared to a similar-cost Fat-tree. Moreover, we demonstrate that TopoOpt is, on average, $3.2 \times$ cheaper than an ideal full bisection bandwidth Fat-tree. Finally, we evaluate the impact of reconfiguration latency on performance and argue that commercially available reconfigurable optical switches are too slow for large-scale DNN workloads.

TopoOpt is the first system with entirely commodity hardware that co-optimizes topology and parallelization strategy and is currently being evaluated for deployment at BiGNet.

2 Characterizing DNN Workloads

Data parallelism. Data parallelism is a popular parallelization strategy, whereby a batch of training samples is distributed across training accelerators. Each accelerator holds a replica of the DNN model and executes the forward and backpropagation steps locally. In data parallelism, all accelerators synchronize their model weights during each training iteration. This step is commonly referred to as AllReduce and can be performed using various techniques, such as broadcasting [137], parameter servers [89], ring-AllReduce [3, 82, 125], tree-AllReduce [111], or hierarchical ring-AllReduce [126, 128].

Hybrid data and model parallelism. Pure data parallelism can become suboptimal for large training jobs because of the increasing cost of synchronizing model parameters across accelerators [21, 77, 83, 100, 102, 120]. In addition to data parallelism, large DNNs in BiGNET are distributed using a hybrid of data and model parallelism, where model parallelism happens when different parts of a DNN and its dataset are processed on different accelerators in parallel.

Types of data dependencies in DNN training. Each training iteration includes two major types of data dependencies. Type (1) refers to activations and gradients computed during the Forward and Backpropagation steps. This data dependency is required for each input sample. Type (2) refers to synchronizing the model weights across accelerators through the AllReduce step once a batch of samples is processed. Depending on the parallelization strategy, these data dependencies may result in local memory accesses or cross-accelerator traffic. For instance, in a hybrid data and model parallelization strategy, both type (1) and (2) result in cross-accelerator traffic, depending on how the model is distributed across accelerators. Given that type (1) is related to model parallelism, we refer to the network traffic created by type (1) as MP transfers. Similarly, we refer to the network traffic created by type (2) as AllReduce transfers. Note that AllReduce transfers do not strictly mean data parallelism traffic, as model parallelism can also create AllReduce transfers across a subset of training nodes (§3.3).\(^1\)

2.1 Production Measurements

We study traffic traces from hundreds of production DNN training jobs running on multiple clusters at BiGNET. We

\(^1\)We only consider transfers related to training because our servers have dedicated NICs for storage and other non-training traffic.
To demonstrate the impact of parallelization strategy on large embedding tables. Large embedding tables result in large AllReduce transfers. Hence, a common parallelization strategy for DLRMs is to use model parallelism to place each embedding table on one GPU and use data parallelism for the rest of the model [98]. Consider a simplified DLRM architecture with four embedding tables \( E_0, \ldots, E_3 \), each with embedding dimensions of 512 columns and \( 10^7 \) rows (total size 20 GB) distributed across 16 servers \( S_0, \ldots, S_{15} \). Following the parallelization strategy used at BIGNET, we place \( E_0 \) on \( S_0 \), \( E_1 \) on \( S_1 \), \( E_2 \) on \( S_8 \), and \( E_3 \) on \( S_{13} \), and replicate the rest of the model on all servers. This parallelization strategy creates a mix of MP and AllReduce traffic, shown in Figure 4. Each heatmap in 4a, 4b, and 4c corresponds to a different ring-AllReduce permutation, shown in Figures 5a, 5b, and 5c. Although all three heatmaps correspond to the exact same parallelization strategy and device placement, blue diagonal lines appear at different parts of the heatmaps, depending on the order of servers in the ring-AllReduce permutation. But MP transfers (green vertical and horizontal lines in each heatmap) are dictated by the parallelization strategy and device placement, and therefore remain at exactly the same spot in all three heatmaps. Hence, AllReduce transfers are mutable but MP transfers are not. We leverage this unique property of DNN training jobs in our TOPOLOGYFINDER algorithm (§3.3). Note that MP transfers in DLRM form one-to-many broadcast and many-to-one incast patterns to transfer the activation and gradients to all nodes because each server handling an embedding table needs to communicate with all other servers. The size of each AllReduce transfer in this example is 4 GB, whereas the size of MP transfers is 32 MB.

**CANDLE traffic pattern.** CANDLE is a family of DNN architectures used to predict the response of cancerous tumors to drug treatments, based on molecular features of tumor cells and drug descriptors [4, 8]. CANDLE models often contain several multilayer perceptrons (MLPs) for drug and cell features [4]. Consider a simplified CANDLE model with one drug MLP \( D_0 \) and one cell MLP \( C_0 \), each with a size of 4 GB. A common parallelization strategy is to distribute the model by replicating \( D_0 \) on four servers (e.g., \( \{S_0, S_1, S_2, S_3\} \)) and \( C_0 \) on another set of four servers (e.g., \( \{S_{12}, S_{13}, S_{14}, S_{15}\} \)). The rest of the model is replicated across all servers \( \{S_0, \ldots, S_{15}\} \). This parallelization strategy creates mostly AllReduce traffic with a few MP transfers, as shown in Figure 6. Similar to the...
DLRM experiments, we permute the order of servers in the ring-AllReduce communication according to Figure 5 and plot three different heatmaps in Figures 6a, 6b, and 6c. We confirm that the position of MP transfers remains fixed, while AllReduce transfers are mutable. We repeat the above experiment using tree-AllReduce and confirm the same takeaways hold (Appendix A).

3 TOPOOPT System Design

This section describes TOPOOPT, a novel system based on commodity optical devices that jointly optimizes DNN parallelization strategy and topology to accelerate today’s training jobs.

3.1 TOPOOPT Interconnect

A TOPOOPT cluster is a shardable interconnect where each server has $d$ interfaces connected to a core layer of $d$ optical switches, as shown in Figure 7. The optical switches enable TOPOOPT to shard the cluster into dedicated partitions for each training job. The size of each shard depends on the number of servers that the job requests. Given a DNN training job and a set of servers, TOPOOPT first finds the best parallelization strategy and topology between the servers off-line. Then, it reconfigures the optical switches to realize the target topology for the job.

Optical switching technologies. A wide range of optical switching technologies is suitable for a TOPOOPT cluster, including commodity available optical patch panels [43] and 3D-MEMS [6, 41], as well as futuristic designs such as Mordia [109], MegaSwitch [56], and Sirius [53, 59]. Table 1 lists the key characteristics of these technologies. TOPOOPT’s design is compatible with any of these technologies. For an immediate deployment in BigNet, this section focuses on commercially available optical patch panels [43, 114] and 3D MEMS circuit switches [6, 41]. Appendix B provides details about these devices. To guide future designs, we evaluate the performance of non-commercially available devices too ($\S$4).

Degree of each server. We denote the number of interfaces on each server (i.e., the degree of the server) by $d$. Typically, $d$ is the same as the number of NICs installed on the server. In cases where the number of NICs is limited, the degree can be increased using NICs that support break-out cables or the next generation of co-packaged optical NVLinks [12]. In our testbed, we use one 100 Gbps HPE NIC [29] with $4 \times 25$ Gbps interfaces to build a system with degree four ($d = 4$).

| Technology                     | Port-count | Reconfig. latency (minutes) | Insertion Loss (dB) | Cost /port |
|-------------------------------|------------|-----------------------------|---------------------|------------|
| Optical Patch Panels [43]     | 1008       | 10                          | 0.5                 | $100       |
| 3D MEMS [6, 41]               | 384        | 10                          | 1.5–2.7             | $520       |
| 2D MEMS [56, 109]             | 300        | 11.5                        | 10–20               | Not commercial |
| Silicon Photonics [87, 117]   | 256        | 900                         | 3.7                 | Not commercial |
| Tunable Lasers [53, 59]       | 328        | 3.8                         | 7–13                | Not commercial |
| RotorNet [95, 96]             | 64         | 10                          | 2                   | Not commercial |

Table 1: Comparison of optical switching technologies.

Host-based forwarding. In large-scale DNN training workloads, the degree of each server is likely smaller than the total number of neighbors with whom the server communicates during training. To ensure traffic is not blocked when there is no direct link between two servers, we use a technique called host-based forwarding, where hosts act as switches and forward incoming traffic toward destination servers. Previous work used similar technique at the ToR switch level [53, 95, 96]. Unlike switch-based interconnects, host-based forwarding introduces performance penalties, called bandwidth tax [95], due to extra forwarded traffic ($\S$4.5).

Target workload. The most suitable workload for a TOPOOPT cluster is a set of large DNN training jobs with hybrid data and model parallelism (or simply data parallelism). We assume the set of servers assigned to each job remains the same throughout the lifetime of the job, and the GPUs are not shared across multiple jobs.

Handing multiple jobs in a shared cluster. TOPOOPT accommodates multiple jobs through sharding, where each shard handles a different training task. To achieve this, TOPOOPT reconfigures the topology and splits the cluster into disjoint sets of servers, where each set is connected internally through host-based forwarding. Appendix C provides details on how TOPOOPT achieves sharding and dynamic job arrivals in shared clusters.

3.2 Co-optimizing Parallelization Strategy & Topology

The search space is too large. Finding the optimal parallelization strategy is an NP-complete problem [83], and adding network topology and routing makes the problem even harder. An extreme solution is to jointly optimize compute, communication, and topology dimensions using a cross-layer optimization formulation. Theoretically, this approach finds the optimal solution, but the search space quickly explodes, even at modest scales (e.g., six nodes [124]).

Naive approach. The other extreme is to optimize the network topology sequentially after the parallelization strategy has been found. While this approach is able to reconfigure the network to better match its traffic demand, the eventual combination of topology and parallelization strategy is likely to be sub-optimal in the global configuration space.

Our approach: alternating optimization. In TOPOOPT, we seek to achieve the best of both worlds. To make the problem tractable, we divide the search space into two planes: Comp. $\times$ Comm. and Comm. $\times$ Topo. We use an alternating optimization technique to iteratively search in one plane while keeping
the result of the other plane constant. Figure 8 illustrates our alternating optimization framework.

We use FlexFlow’s MCMC (Markov Chain Monte Carlo) search algorithm [83] to find the best parallelization strategy for a given network topology while considering the communication cost.

If the parallelization strategy improves the training iteration time, we feed it to the Comp. × Topo. plane to find the best network topology and routing using our TOPOFINDER algorithm. The discovered topology is then fed back into the Comp. × Comm. plane, which further optimizes the parallelization strategy and device placement based on the new topology. This optimization loop repeats until convergence or after k iterations, where k is a configurable hyper-parameter. The next section describes TOPOOPT’s TOPOFINDER algorithm inside the Comp. × Topo. plane.

### 3.3 TOPOFINDER Algorithm

**Prior proposals are inefficient for DNN workloads.** At first blush, finding a network topology seems straightforward: we just need to translate the parallelization strategy and device placement from the Comp. × Comm. plane into a traffic matrix and map the traffic matrix into circuit schedules. Several papers have used this technique for datacenter networks [56, 63, 67, 71, 87, 91–93, 109, 132]. The conventional wisdom in prior work is to allocate as many direct parallel transfers as possible to elephant flows and leave mice flows to take multiple hops across the network. In principle, this approach works well for datacenters but it leads to sub-optimal topologies for distributed DNN training. While the size of AllReduce transfers is larger than MP transfers, MP transfers have a higher communication degree than AllReduce in BigNet (Appendix D). Hence, the conventional approach leads to creating parallel direct links for carrying AllReduce traffic and forcing MP flows to have a large hop-count, thereby degrading the training performance.

**TOPOOPT’s novel technique.** In TOPOOPT, we seek to meet two goals simultaneously: (i) allocate ample bandwidth for AllReduce transfers, as the bulk of the traffic belongs to them, but (ii) ensure a small hop-count for MP transfers. We meet both goals by leveraging a unique property of distributed DNN training traffic, namely that the AllReduce part of the traffic matrix is mutable and can be split across multiple permutations (§2.2). Intuitively, this is because MP traffic is composed of network flows among nodes that contain different parts of a DNN model thus creating immutable data dependencies across these nodes, while AllReduce transfers contain network flows among nodes that handle the same part of the model, providing flexibility in the order of nodes participating in AllReduce. Consequently, if a group of servers is connected in a certain order, simply permuting the label of the servers gives another ordering that will finish the AllReduce operation with the same latency while potentially providing a smaller hop-count for MP transfers. Instead of selecting just one AllReduce order, we find multiple permutations for each AllReduce group and overlap their corresponding sub-topologies. In doing so we, not only serve the AllReduce traffic, but also decrease the hop-count for MP transfers.

**TOPOFINDER steps.** Algorithm 1 presents the pseudocode of our TOPOFINDER procedure. The algorithm takes the following inputs: N dedicated servers for the training job, each with degree d, as well a list of AllReduce and MP transfers (T_allreduce and T_MP) based on the parallelization strategy and device placement obtained from the Comp. × Comm. plane. The algorithm then finds the best topology (G) and routing rules (R) and returns them to the Comp. × Comm. plane for the next round of alternating optimization. Our algorithm consists of the following four steps.

**Step 1: Distribute the degree.** This step distributes the degree d between AllReduce and MP sub-topologies proportionally, based on their share of total traffic. We specifically start with AllReduce transfers and allocate at least one degree to the AllReduce sub-topology to ensure the network remains connected (line 2). The remaining degrees, if any, are allocated to the MP sub-topology (line 3).

**Step 2: Construct the AllReduce sub-topology and routing.** To find the AllReduce sub-topology, the algorithm iterates over every AllReduce group k and allocates degree d_k to each group proportionally based on the amount of traffic requires (line 6). Note that in hybrid data and model parallelism strategies, the AllReduce step can be performed across a subset of servers when a DNN layer is replicated across a few servers instead of all servers. Finding the set of all possible AllReduce orderings is non-trivial, since the number of possible permutations is O(n!), where n is the number of servers in group k. Inspired by group theory, we develop a technique to address this challenge, called TotientPerms, described next.

**Using group theory to find AllReduce permutations.** Given that ring-AllReduce is the dominant AllReduce collective in BigNet, we describe our TotientPerms technique on ring-AllReduce. Appendix E.1 explains how to extend our algorithm to other AllReduce communication collectives. For a ring-AllReduce group with n servers labeled S_0, ..., S_n-1, a straightforward permutation is (S_0 → S_1 → S_2 → ... → S_{n-1} → S_0). We denote this permutation by a ring generation rule as: S_i → S_{(i+1)} mod n. Since the servers form a ring, the index of the starting server does not matter. For instance, these two rings are equivalent: (S_0 → S_1 → S_2 → S_3 → S_0) and (S_1 → S_2 → S_3 → S_0 → S_1).

**Regular rings.** To reduce the search space of all possible permutations, we find the ring generation rule for all regular
AllReduce permutations between the servers. Note that each
Algorithm 1 TOPOLOGYFINDER pseudocode
1: procedure TOPOLOGYFINDER(N, d, TReduce, TMP)
   // Input N: Number of dedicated training servers for the job.
   // Input d: Degree of each server.
   // Input TReduce: AllReduce transfers.
   // Input TMP: MP transfers.
   // Output G: Topology to give back to the Comp. × Comm. plane.
   // Output R: Routing rules to give back to the Comp. × Comm. plane.
   // Distribute depth d between AllReduce and MP sub-topologies.
   \[ \text{d}_{\text{AllReduce}} = \max(1, \left\lceil \frac{\text{max}(\text{AllReduce}\text{\# of transfers})}{\text{max}(\text{AllReduce}\text{\# of transfers}) \times \text{network diameter}} \right\rceil) \]
2: \[ \text{d}_{\text{AllReduce}} = d - \text{d}_{\text{AllReduce}} \]
3: \[ \text{d}_{\text{MP}} = d - \text{d}_{\text{AllReduce}} \]
4: \[ G_{\text{AllReduce}} = \{ \} \]
5: for each AllReduce group \( k \) with set of transfers \( T_{\text{Reduce}} \) do
   \[ \text{Assign degree } d_k \text{ to group } k \text{ according to its total traffic} \]
6: \[ d_k = \frac{\text{max}(T_{\text{Reduce}})}{\text{AllReduce\# of transfers}} \]
7: \[ d_{\text{AllReduce}} = d_{\text{AllReduce}} - d_k \]
8: \[ G_{\text{AllReduce}} = G_{\text{AllReduce}} \cup \text{SelPermutations}(N, d_k, P_k) \]
9: \[ \text{If } d_{\text{AllReduce}} <= 0 \text{ then} \]
10: break
\[ G = G_{\text{AllReduce}} \cup G_{\text{MP}} \]
\[ G = G_{\text{AllReduce}} \cup G_{\text{MP}} \]
11: Compute routes on \( G_{\text{AllReduce}} \) using the coin change algorithm [52]
12: \[ R = \text{CoinChangeMod}(N, G) \]
13: \[ \text{Construct the MP sub-topology } G_{\text{MP}} \]
14: \[ G_{\text{MP}} = \{ \} \]
15: for \( i : j : d_{\text{MP}} \) do
   \[ \text{Find a maximum weight matching according to } T_{\text{MP}} \]
16: \[ g = \text{BlossomMaximumWeightMatching}(T_{\text{MP}}) \]
17: \[ G_{\text{MP}} = G_{\text{MP}} \cup G \]
18: \[ \text{Reduce the amount of demand in each link } l \text{ in graph } g \]
19: \[ \text{Combine the AllReduce and MP topologies} \]
20: \[ R := \text{ShortestPath}(G, R) \]

Figure 9: Example of TOPOOPT’s topology and traffic matrix.

is the union of permutations selected in line 9.

Coin-change routing. Consider servers \( S_i \) and \( S_j \) that need to exchange AllReduce transfers but do not have a direct edge between them. We use a modified version of the classical coin change problem [52] to find an efficient routing path (line 12). In classical coin change, the goal is to find the minimum number of coins that would sum to a certain total value. Our ring generation rules enable us to treat the routing problem similarly. In particular, the \( p \) values of AllReduce permutations that have been selected in the AllReduce sub-topology are the coin values, and the difference between server \( i \) and \( j \) indices \((j-i) \mod n\) is the target total value that we want to achieve. The only difference is that our problem runs with modulo \( n \) arithmetic, as the server IDs wrap around in the ring structure. Appendix E.1 lists the pseudocode of TotientPerms, SelPermutations, and CoinChangeMod methods.

Step 3: Construct the MP sub-topology. Given that MP transfers are immutable, we use the classical Blossom maximum weight matching algorithm [62] to find the best connectivity between servers with MP transfers (line 15). We repeat the matching algorithm until we run out of degrees. To increase the likelihood of more diverse connectivity across server pairs, we divide the matching algorithm by two (line 18). In general, division by two can be replaced by a more sophisticated function with a diminishing return.

Step 4: Final topology and routing. Finally, we combine the MP and AllReduce sub-topologies and compute k-shortest path routes for MP transfers (lines 19 and 20).

Example. We use the DLRM model in Figure 4 distributed across 16 servers each with six NICs (\( d = 6 \)) as an example. Instead of choosing one of the AllReduce permutations in Figure 5, TOPOOPT combines the three ring-AllReduce permutations to load-balance the AllReduce transfers while providing a short hop-count for MP transfers. Figure 9 illustrates TOPOOPT’s topology and traffic matrix and shows a more balanced traffic matrix than Figure 4.

4 Large-scale Simulations
This section evaluates the performance of a large-scale TOPOOPT interconnect.
4.1 Methodology & Setup

We implement two simulators to evaluate TOPOOPT. We will release our codebase and all related data and scripts online.

**FlexNet simulator.** We augment FlexFlow’s simulator [27] to be network-aware and call it FlexNet. Given a DNN model and a batch size, FlexFlow’s simulator explores different parallelization strategies and device placements to minimize iteration training time. The output of this simulator is a task graph describing the set of computation and communication tasks on each GPU and their dependencies. But the current implementation of FlexFlow ignores the network topology entirely by assuming servers to be connected in a full-mesh interconnect. Our FlexNet simulator extends the FlexFlow simulator and enables it to consider multiple networks, including Fat-trees, TOPOOPT, and expander networks. Moreover, FlexNet implements our alternating optimization framework (§3) to find an optimized network topology and routing rules for TOPOOPT.

**FlexNetPacket simulator.** FlexFlow’s simulator only provides coarse-grind estimation of training iteration time, because it does not simulate individual packets traversing through a network. Extending FlexNet to become a packet-level simulator is computationally infeasible, because FlexFlow generally requires thousands of MCMC iterations to converge. To faithfully simulate per-packet behavior of network switches, buffers, and multiple jobs sharing the same fabric, we build a second event-based packet simulator, called FlexNetPacket, on top of htsim [7]. FlexNetPacket takes the output of FlexNet (i.e., the optimized parallelization strategy, device placement of each operator, optimized network topology, and routing rules) and simulates several training iterations. The link propagation delay is set to 1 µs throughout this section.

**Simulated network architectures.** We simulate distributed training clusters with \( n \) servers equipped with four NVIDIA A100 GPUs [37]. We vary \( n \) in different experiments and simulate the following network architectures:

- **TOPOOPT.** A TOPOOPT interconnect where each server is equipped with \( d \) NICs, each with bandwidth \( B \) connected via a flat layer of optical devices. At the beginning of each job, the topology is reconfigured based on the output of our alternating optimization framework (§3) and remains unchanged throughout the entire training job. Both OCS and patch panels are suitable for this architecture.

- **OCS-reconfig.** To study the impact of changing the network topology within training iterations, we simulate a reconfigurable TOPOOPT interconnect. We only rely on commercially available Optical Circuit Switches (OCSs) for this design and assume the reconfiguration latency is 10 ms. Given that FlexFlow’s parallelization strategy search is not aware of statically reconfigurable networks, following prior work [87], we measure the traffic demand every 50 ms and adjust the circuits based on a heuristic algorithm to satisfy the current traffic demand as much as possible. We also enable host-based forwarding such that the communication is not blocked even when a direct link is not available (Appendix E.3).

- **Ideal Switch.** An ideal electrical switch that scales to any number of servers, where each server is connected to the switch via a link with \( d \times B \) bandwidth. For any pair of \( d \) and \( B \), no network can communicate faster than this ideal case. In practice, the Ideal Switch can be approximated with a full-bisection bandwidth Fat-tree where the bandwidth of each link is \( d \times B \).

- **Fat-tree.** To compare the performance of TOPOOPT to that of a similar-cost Fat-tree architecture, we simulate a full bisection bandwidth Fat-tree where each server has one NIC and the bandwidth of each link is \( d \times B \), where \( B \) is lower than \( B \) and is selected such that Fat-tree’s cost is similar to TOPOOPT (§4.2).

- **Oversub. Fat-tree.** This is a 2:1 oversubscribed Fat-tree interconnect, where the bandwidth of each link is \( d \times B \) but half of the links in the ToR uplink layer are omitted.

- **SiP-ML [87].** SiP-ML is a futuristic DNN training cluster with several Tbps bandwidth per GPU. While having a Tbps network is certainly a plus, our goal is to compare the algorithmic contributions of TOPOOPT and SiP-ML. Hence, to make an apples-with-apples comparison, we allocate \( d \) wavelengths, each with bandwidth \( B \), to each SiP-ML GPU and follow its SiP-Ring algorithm to find a topology with a reconfiguration latency of 25 µs. Appendix F elaborates on our modifications to SiP-ML.

- **Expander [122, 130].** Finally, we simulate a fabric where each server has \( d \) NICs with bandwidth \( B \) interconnected via an Expander topology.

**DNN Workloads.** We simulate six real-world DNN models: DLRM [21], CANDLE [4], BERT [61], NCF [74], ResNet50 [73], and VGG [121]. List 1 (Appendix D) provides details about model configurations and batch sizes used in this paper.

**Parallelization strategy.** We use FlexNet’s topology-aware parallelization strategy search for Ideal Switch, Fat-tree, Oversub. Fat-tree, SiP-ML, and Expander networks. For TOPOOPT, we use FlexNet’s alternating optimization framework to find the best parallelization strategy jointly with topology, where the final parallelization strategy is either hybrid or pure data-parallel. We use ring-AllReduce and distributed parameter server [89] as default AllReduce communication collectives between servers and within servers, respectively. Each data point averages 5–10 simulation runs.
4.2 Cost Analysis

We begin our evaluations by comparing the cost of various network architectures. Details about the cost of each component used in each architecture are given in Appendix G.

Figure 10 compares the interconnect cost across various network architectures as the number of servers is increased. We estimate the cost of Ideal Switch with a full-bisection Fat-tree of the same bandwidth. We make the following observations. First, using OCSs for TopoOPT is more expensive (1.33×, on average) than patch panels. Note that OCSs can be used in both TopoOPT and OCS-reconfig interconnects. Second, the cost of TopoOPT (blue curve) overlaps with the Fat-tree (yellow curve). This is intentional, because having a cost-equivalent architecture enables us to compare the performance of TopoOPT to a cluster at the same price point. Third, the ratio of Ideal Switch’s cost to TopoOPT’s cost is 3.2× on average. Finally, the most and least expensive fabrics are SiP-ML and Expander, respectively, and this section shows that they both perform worse than TopoOPT for certain workloads.

We acknowledge that estimating the cost of networking hardware is challenging because prices are subject to significant discounts with bulk orders. Assuming all components in this analysis are subject to the same bulk order discounts, the relative comparison across architectures remains valid. As a point of comparison, we compute the cost of a cluster with 4,394 servers (k = 26 Fat-tree) by following the cost model in Sirius [53]—where discount cost trends are considered—and with 50% discounts for patch panel costs. For a cluster at this scale, the cost of full-bisection bandwidth Fat-tree (which approximates our Ideal Switch baseline) relative to the cost of TopoOPT changes from 3.0× to 3.6×, indicating that our estimates are reasonable. Additionally, a TopoOPT cluster incurs lower energy cost than Fat-trees, as optical switches are passive devices.

4.3 Performance Comparison for Dedicated Clusters

Figure 11a compares the training iteration time of CANDLE distributed on a dedicated cluster of 128 servers with a server degree of four (d = 4). We vary the link bandwidth (B) on the x-axis. The figure shows that Ideal Switch, TopoOPT, and SiP-ML architectures achieve similar performance because the best parallelization strategy for CANDLE at this scale is mostly data parallel, with few MP transfers. The OCS-reconfig architecture performs poorly because it uses the instantaneous demand as the baseline to estimate the future traffic to schedule circuits. This estimation becomes inaccurate during training, in particular when the current AllReduce traffic is about to finish but the next round of AllReduce has not started. Finally, the Expander architecture has the worst performance, as its topology is not optimized for DNN workloads. Averaging across all link bandwidths, compared to Fat-tree interconnect, TopoOPT improves the training iteration time of CANDLE by 2.8×; i.e., the ratio of CANDLE’s iteration time on Fat-tree to TopoOPT is 2.8. This is because TopoOPT’s servers have more raw bandwidth, resulting in faster completion time.

Figures 11b and 11c show the training iteration times for VGG and BERT. The trends are similar to CANDLE, as these models have similar communication degree requirements. Compared to Fat-tree, on average, TopoOPT improves the iteration time of VGG and BERT by 2.8× and 3×, respectively.

The cases of DLRM and NCF are more interesting, as they have more MP transfers than the other DNNs. As shown in Figures 11d and 11e, TopoOPT’s performance starts to deviate from Ideal Switch, especially for NCF, because of using host-based forwarding for the many-to-many MP transfers (§4.4 and §4.5). For DLRM (and NCF), TopoOPT is 2.8× (and 2.1×) faster compared to Fat-tree, while Ideal Switch further improves the training iteration time by 1.3× (and 1.7×) compared to TopoOPT. SiP-ML performs poorly, and even when we increase the link bandwidth, its training iteration time stays flat. This happens because MP transfers in DLRM and NCF require several circuit reconfiguration to meet the traffic demand.

Finally, Figure 11f shows most architectures achieve similar training iteration time for ResNet50 since it is not a communication-heavy model. The Expander architecture performs poorly when the link bandwidth is lower than 100 Gbps because the topology does not match the AllReduce traffic pattern. We repeat this simulation with d = 8 and observe a similar performance trend (Appendix H).

4.4 Impact of All-to-all Traffic

This section evaluates the impact of all-to-all traffic patterns on TopoOPT’s performance. In particular, TopoOPT’s host-based forwarding approach incurs bandwidth tax that is exacerbated by all-to-all and many-to-many communication patterns. This tax is defined as the ratio of the traffic volume in the network (including forwarded traffic) to the volume of logical communication demand. Hence, the bandwidth tax for a full bisection bandwidth Fat-tree topology is 1 since hosts...
do not act as relays for each other.

Consider a DNN model with \( R \) bytes of AllReduce traffic and \( A \) bytes of all-to-all traffic, distributed on a full bisection bandwidth topology with total network bandwidth \( NB_T \) (i.e., number of servers multiplied by the bisection bandwidth). The training iteration time of this DNN is: \[ T_F = \frac{R}{NB_T} + \frac{A}{NB_T} + C_h, \]
where \( C_h \) is the computation time of the model with batch size \( bs \).

Now suppose the same DNN is distributed on a TopoOPT topology with total network bandwidth \( NB_T \). In this case, assuming the entire AllReduce traffic is carried on Totient-Perms with direct links, the training iteration time becomes \[ T_T = \frac{R}{NB_T} + \frac{A}{NB_T} + C_{hs}, \]
where \( C_{hs} \) represents the slow-down factor that all-to-all transfers create in the network, due to host-based forwarding. The value of \( C_{hs} \) depends on the amount of bandwidth tax and routing strategy (§4.5).

Increasing the amount of all-to-all traffic \( (A) \) results in increasing the iteration time for both \( T_F \) and \( T_T \). But when \( NB_F \) and \( NB_T \) are equal, TopoOPT’s performance degrades faster because of the \( \alpha \) factor in the numerator. To quantify this behavior concretely, we distribute a DLRM training task with 128 embedding tables on a cluster with 128 servers. We choose large embedding tables and distribute each table on each server, creating worst-case all-to-all traffic.

Figure 12 compares the training iteration times of TopoOPT, Ideal Switch, and Fat-tree as the batch size is increased. The top x-axis lists the ratio of all-to-all to AllReduce traffic for each batch size value given on the bottom x-axis. As shown in Figure 12a, when the batch size is 128 and \( d = 4 \), TopoOPT’s performance matches that of Ideal Switch, while Fat-tree is a factor of 2.7 slower. This result agrees with the performance gains in Figure 11d, as the batch sizes are the same.

Increasing the batch size increases \( A \), which in turn increases the training iteration times in all three architectures. As predicted by Eq. (1), TopoOPT’s iteration time increases faster. Specifically, when the batch size is 2048 and all-to-all traffic is 80% of AllReduce traffic, TopoOPT performs poorly, and the iteration time is a factor of 1.1 higher than that of the Fat-tree architecture. Increasing the server degree \( d \) mitigates the problem, as shown in Figure 12b. Note that increasing the batch size does not always result in faster training time [87, 105, 119]. Moreover, publicly available data suggest that 2048 is the largest batch size for training DLRM [98]. In their workload, the number of columns in the embedding tables and the number of servers were smaller ((92, 16) vs. (128, 128), respectively). Hence, the DLRM workload we evaluate contains more all-to-all traffic compared to the state-of-the-art model used in industry.

### 4.5 Impact of Host-based Forwarding

Two factors impact the performance of host-based forwarding in TopoOPT: bandwidth tax and routing strategy.

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2For clarity of presentation, this formulation assumes no overlap between communication and computation stages and no competing traffic.
as a lower bound. We leave optimizing the routing strategy in TOPOOPT to future work.

### 4.6 Performance Comparison for Shared Clusters

We now compare the performance of different network architectures when the cluster is shared across multiple DNN jobs. Following prior work [94, 110], we run a series of simulations where 40% of the jobs are DLRM, 30% are BERT, 20% are CANDLE, and 10% are VGG16. We change the number of active jobs to represent the load on the cluster.

Figure 16 compares the average and 99%-tile iteration time at different loads for a cluster with 432 servers where \(d = 8\) and \(B = 100\) Gbps. SiP-ML does not support multiple jobs; hence, we omit it in this experiment. We omit OCS-reconfig and Expander networks since they both have a poor performance in this setting. Instead, we add the Oversub. Fat-tree interconnect to demonstrate the impact of congestion on Fat-tree topologies. Figure 16a shows that TOPOOPT improves the average iteration time by 1.7× and 1.15×, compared to the Fat-tree and Oversub Fat-tree architectures, respectively. We observe a similar trend for the tail iteration completion times, depicted in Figure 16b. Averaging across all load values on the x-axis, TOPOOPT improves the tail training iteration time by 3× and 1.4× compared to Fat-tree and Oversub Fat-tree architectures, respectively.

### 4.7 Impact of Reconfiguration Latency

Figure 17 shows the training iteration time of DLRM and BERT in the same setting as in Figure 11 while sweeping the reconfiguration latency of OCSs in OCS-reconfig from 1 \(\mu\)s to 10 ms. We find host-based forwarding is challenging when the network is reconfigurable, as the circuit schedules need to account for forwarding the traffic while the topology reconfigures. The horizontal blue line corresponds to TOPOOPT’s iteration time; it remains constant as it does not reconfigure the network topology. The purple line corresponds to OCS-reconfig with host-based forwarding (same as OCS-reconfig evaluated in Figure 11), denoted by OCS-reconfig-FW. For the orange line, we disable host-based forwarding (similar to SiP-ML) and call it OCS-reconfig-noFW.

We find enabling host-based forwarding when the topologies reconfigures within a training iteration is not always beneficial. For DLRM (Figure 17a), OCS-reconfig-FW achieves better performance than OCS-reconfig-noFW, as DLRM has all-to-all MP transfers which benefit from host-based forwarding. However, for BERT (Figure 17b), enabling forwarding increases the chance of inaccurate demand estimation and imposes extra bandwidth tax, therefore increases the iteration time of OCS-reconfig-FW by a factor of 1.4 compared to OCS-reconfig-noFW.

Reducing the reconfiguration latency all the way to 1 \(\mu\)s enables OCS-reconfig-noFW to match the performance of TOPOOPT. However, OCS-reconfig-FW still suffers from inaccurate demand estimations. Although fast reconfigurable switches are not yet commercially available, they are going to be essential in elastic scenarios where the cluster is shared across multiple jobs and servers join and leave different jobs unexpectedly, or when large, high-degree communication dominates the workload. We believe futuristic fast reconfigurable switches, such as Sirius [53], are well-suited for this setting. Finding a parallelization algorithm that is aware of reconfigurability within training iterations is a challenging and exciting future research problem.

### 5 Prototype

**Testbed setup.** We build a prototype to demonstrate the feasibility of TOPOOPT. Our prototype includes 12 ASUS ESC4000A-E10 servers and a G4 NMT patch panel [43]. Each
server is equipped with one A100 Nvidia GPU [37] (40 GB of HBM2 memory), one 100 Gbps HP NIC [29], and one 100 Gbps Mellanox ConnectX5 NIC. Our HP NICs are capable of supporting $4 \times 25$ Gbps interfaces using a PSM4 transceiver with four breakout fibers [9], enabling us to build a TOPOOPT system with degree $d = 4$ and $B = 25$ Gbps. We use RoCEv2 for communication, and enable DCB [20] and PFC on these interfaces to support a lossless fabric for RDMA. We build a completely functional TOPOOPT prototype with our patch panel (Figure 18). We compare TOPOOPT’s performance with two baselines: (i) Switch 100Gbps, where the servers are connected via 100 Gbps links to a switch, and (ii) Switch 25Gbps, where the servers are connected via 25 Gbps links to a switch. The Switch 100Gbps baseline corresponds to the Ideal Switch case in our simulations.

**Distributed training framework.** We use FlexFlow’s training engine [26], based on Legion’s parallel programming system [30], to train four DNN models: ResNet50 [73], BERT [61], VGG16 [121], and CANDLE [4]. For DLRM, we use Facebook’s implementation from [21]. Since our prototype is an order of magnitude smaller in scale than our simulation setup, we use smaller model and batch sizes.

**Modifications to NCCL.** By default, the NCCL communication library [36] assumes all network interfaces are routable from other interfaces. This assumption is not ideal for TOPOOPT because we have a specific routing strategy to optimize training time. We modify NCCL to understand TOPOOPT’s topology and respect its routing preferences. Moreover, we integrate our TotientPerms AllReduce permutations into NCCL and enable it to load-balance parameter synchronization across multiple ring-AllReduce permutations.

**RDMA forwarding.** Implementing TOPOOPT with today’s RDMA NICs requires solving an engineering challenge, because the RDMA protocol assumes a switch-based network. Packet processing and memory access in RDMA protocol are offloaded to the NIC, and a RoCEv2 packet whose destination IP address is different from that of the host is assumed to be corrupted. Therefore, the NIC silently drops forwarded packets. To address this issue, we collaborated with engineers at Marvell who developed the firmware and driver of our HNICs. Our solution uses a feature called network partitioning (NPAR) which enables the NIC to separate host-based forwarding traffic from direct traffic, and uses the Linux kernel to route them (details in Appendix I). Our conversations with Marvell indicate that updating the firmware and the driver enables the NIC to route forwarded RoCEv2 packets, thereby bypassing the kernel entirely.

**Training performance.** Figure 19 demonstrates that TOPOOPT’s training throughput (samples/second) is similar to our Switch 100 Gbps baseline for all models. The performance of Switch 25Gbps baseline is lower because its available bandwidth is lower than TOPOOPT. Figure 20 shows the time-to-accuracy of training ResNet50 on ImageNet [60]. As the figure indicates, TOPOOPT achieves a similar performance to the Switch 100Gbps case.

**Impact of all-to-all traffic.** Similar to Section 4.4, we evaluate the impact of all-to-all MP traffic on our RDMA-forwarding enabled testbed by measuring the average iteration time across 320 iterations of a DLRM job distributed in our testbed. We vary the amount of all-to-all traffic by changing the batch size. To create worst-case traffic, we increase the embedding dimensions by $128 \times$ relative to the state-of-the-art [21] (model details are in List 1 Appendix D). Figure 21 shows the training iteration time for various batch sizes. This result is consistent with Figure 12, but since the bandwidth tax in our 12-server testbed is smaller than our 128-server cluster in simulations, TOPOOPT performs better relative to the switch-based architectures for a given all-to-all to AllReduce traffic ratio. For instance, for batch size 512, the ratio of all-to-all to AllReduce is 78%, and the training iteration time with TOPOOPT is $1.6 \times$ better than the 25 Gbps Switch baseline. Our GPUs could not handle batch sizes higher than 512.

6 Discussion

**Storage and control plane traffic.** BIGNet’s training clusters consist of custom-designed servers, each with eight GPUs, eight dedicated NICs for training traffic (GPU NICs), and four additional NICs for storage and other traffic (CPU NICs). Other companies, such as Facebook and NVIDIA, have similar server architectures [11, 98]. TOPOOPT only considers GPU NICs as server degree and partitions the network that is dedicated for training traffic. The CPU NICs are connected through a separate fabric to carry storage and other control plane traffic.

**Handling scale.** A flat TOPOOPT cluster with OCSs scales to 384 servers and a TOPOOPT cluster with patch panels scales to 1,000 servers. Assuming each server has 8 GPUs, these
clusters can host 3,072 and 8,000 GPUs, respectively. Given that our DNN jobs run on fewer than 1,000 workers (Figure 1a), there is no immediate need to create a hierarchy of switches. To further scale a TOPOOPT cluster, we create a hierarchical interconnect by placing the servers under ToR switches and connecting ToR switches to the optical switch layer, similar to previous work [53, 70, 71, 96]. Another option is to build a Clos topology using a hierarchy of optical switches and patch panels. We leave exploring these options to future work.

**Supporting dynamic scheduling and elasticity.** Others have demonstrated the benefits of dynamically choosing the training servers for elastic training jobs [94, 110]. Our target use case in BIGNET is to leverage TOPOOPT for the vast number of long-lasting training jobs that do not change dynamically. In cases where elasticity is required, instead of using patch panels, we use OCSs (or other fast reconfigurable optical switches) to change the servers participating in a job quickly. Note that dynamically changing the set of servers participating in a job while keeping both the topology and the parallelization strategy optimal requires augmenting the optimization space with an additional dimension, making the problem even more challenging. We leave this to future work.

**Handling failures.** Unlike SiP-ML’s single ring topology [87], a single link failure does not disconnect the graph in TOPOOPT. Upon a fiber failure, TOPOOPT can temporarily use a link dedicated to MP traffic to recover an AllReduce ring. In case of permanent failures, TOPOOPT reconfigures the patch panel to swap ports and recover the failed connection.

**Supporting multi-tenancy.** To support multi-tenancy [138, 139], TOPOOPT can leverage NVIDIA’s MIG [39] to treat one physical server as multiple logical servers in its topology.

**TotientPerms in Fat-trees.** Although our TotientPerms technique is well-suited for reconfigurable optical interconnects, it may be of independent interest for Fat-tree interconnects as well since load-balancing the AllReduce traffic across multiple permutations can help with network congestion.

**TOPOOPT’s limitations.** TOPOOPT’s approach assumes the traffic pattern does not change between iterations. However, this assumption may not hold for Graphic Neural Network (GNN) models [116] or Mixture-of-Expert (MoE) models [79]. In addition, we plan to extend TOPOOPT by bringing its demand-awareness design within training iterations. This is an open research question, and as shown in Section 4.7, we will need fast-reconfigurable optical switches, as well as a more sophisticated scheduling algorithm. Another limitation of TOPOOPT is that a single link failure within a AllReduce ring causes the full ring to become inefficient for AllReduce traffic. A fast optical switch also addresses this problem by quickly reconfiguring the topology.

7 Related Work

**Optimizing DNN training.** To address the increasing computation and network bandwidth requirements of large training jobs, a plethora of frameworks have been proposed [5, 46, 57, 68, 76, 78, 83, 84, 101, 104, 107, 112, 113, 118, 124, 131, 134, 142]. These frameworks distribute the dataset and/or DNN model across accelerators while considering the available network bandwidth, but unlike TOPOOPT, they do not consider the physical layer topology as an optimization dimension. Specifically, Blink [131] builds fast collectives for distributed ML, but it needs a physical topology to generate spanning trees. Zhao et al. [143] study the optimal topology for collective communication operations, which does not apply for general MP traffic. In addition, several methods have been proposed to quantize and compress the gradients to reduce the amount of communication data across servers [48, 55, 140]. While all these approaches are effective, they are designed for data parallel strategies and do not consider the large amount of data transfers caused by model parallel training. Wang et al. [133] compare the performance of Fat-trees and BCube topologies for distributed training workloads and highlight several inefficiencies in Fat-trees. However, unlike TOPOOPT, their proposed approach does not co-optimize topology and parallelization strategy.

**DNN parallelization strategies.** Data and model parallelism are widely used by today’s DNN frameworks (e.g., TensorFlow [44], PyTorch [42], MXNet [18]) to parallelize training across multiple devices. Recent work has also proposed automated frameworks (e.g., FlexFlow [83], ColocRL [97], MERLIN [38]) finding efficient parallelization strategies by searching over a comprehensive space of potential strategies. These frameworks rely on and are optimized for the conventional Fat-tree interconnects. TOPOOPT proposes a new approach to building DNN training systems by jointly optimizing network topology and parallelization strategy.

**DNN training infrastructures and schedulers.** Several training infrastructures have been proposed recently, including NVIDIA DGX SuperPOD [11], TPU cluster [10], and supercomputers [1]. All these systems assume non-reconfigurable network topologies, such as Fat-tree, Torus, and other traffic-oblivious interconnects. TOPOOPT is the first DNN system to use commodity reconfigurable interconnects to accelerate DNN jobs. Gandiva [136], Themis [94], Tiresias [69], BytePS [84, 107], and Pollux [110] seek to improve the utilization of GPU clusters through scheduling algorithms. These approaches are complementary to ours, and many of their techniques can be applied to a TOPOOPT cluster.

**Optical Interconnects.** Several papers have demonstrated the benefits of optically reconfigurable interconnects for datacenters [51, 53, 56, 59, 63, 67, 91–93, 95, 96, 109]. These designs lead to sub-optimal topologies for distributed DNN traffic. Similarly, traffic oblivious interconnects, such as RotorNet [95, 96], are a great fit for datacenter workloads, but they are not suitable for DNN training jobs characterized by repetitive traffic demands. Hybrid electrical/optical datacenter proposals [63, 132] can be used to route AllReduce traffic through the optical fabric and MP flows through a standard electrical Fat-tree network. But hybrid clusters are not cost effective and suffer from many problems, including TCP ramp-up inefficiencies [99], segre-
8 Conclusion
We present TOPOOPT, a novel network interconnect to build DNN training clusters. We design an alternating optimization algorithm to explore the large space of \(\text{Computation} \times \text{Communication} \times \text{Topology}\) strategies for a DNN workload, and demonstrate TOPOOPT obtains up to 3x faster training iteration time than a Fat-tree.

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A Tree-AllReduce and other AllReduce permutations

Section 2 established that we can manipulate the traffic of a ring-AllReduce collective by permuting the labeling of servers in the AllReduce group. Here, we illustrate how to use the same technique on another AllReduce algorithm, called tree-AllReduce.

In the tree-AllReduce algorithm, the servers are connected logically to form a tree topology. The AllReduce operation happens by first running a reduce operation to the root node with recursive halving, followed by a broadcast to the rest of the cluster with recursive doubling [127].

A common instantiation of tree-AllReduce is the double binary tree (DBT) algorithm described in [115]. In this algorithm, the first step is to create a balanced binary tree for the nodes. The properties of balanced binary trees guarantee that one half of the nodes will be leaf-nodes, and the other half will be in-tree; thus, a second binary tree is constructed by flipping the labeling of the leaf and in-tree nodes. This way, each node (except the root in both trees) has the same amount of communication requirement for the AllReduce operation described in the last paragraph, and bandwidth-optimally is achieved. Figure 23a shows an example where in the first binary tree, the in-tree nodes are even, and the leaf nodes are odd, while the second tree flips the labeling.

Essentially, the DBT itself is an example of permuting the node labeling to achieve an AllReduce operation with balanced communication load. We also note that we can permute the labeling for the entire set of nodes for a pair of DBT to create a new pair of trees that can perform the AllReduce operation at the same speed. Figures 23b and 23c illustrate two other possible double binary trees, and their corresponding traffic demand matrix for the DLRM and CANDLE example shown in Figures 22 and 24 (§2). Arbitrary permutations can be used, and to limit the cases, we could simply consider the cyclic permutations in the modular space as described in TotientPerms.

In general, all AllReduce operations can be described as a directed graph $G = (V,E)$ where $V$ is the set of nodes in the cluster, and $E$ denotes data dependencies. The permutable property says that every graph $G' = (V,E')$ that is isomorphic to $G$ can perform the AllReduce operation equally well, where the homomorphism between $G$ and $G'$ is described by the symmetric group on $V$ (generally denoted $S(V)$ in group theory).

B Commercially Available Patch Panels and Optical Circuit Switches

Optical patch panels. A patch panel is a device to facilitate connecting different parts of a system. For instance, electrical patch panels are used in recording studios and concert sound systems to connect microphones and electronic instruments on demand [40]. Fiber optic patch panels are commonly used for cable management, and has been proposed in recent datacenter topology designs [141]. Reconfigurable optical patch panels are a new class of software-controlled patch panels and are already commercialized at scale [114]. For instance, Telescent offers 1008 duplex ports with insertion loss less than 0.5 dB and cost ≈$100K ($100/port) [86, 114]. Reconfiguration is performed using a robotic arm that grabs a fiber on the transmit side and connects it to a fiber on the receive side [86]. However, the reconfiguration latency of optical patch panels is several minutes [43]. Note that reliability is of utmost concern for operation in unmanned locations, and for example Telescent NTMTM patch panels have been certified to NEBS Level 3 and have over 1 billion port hours in operation [31].

3D MEMS-based Optical Circuit Switches (OCSs). An OCS uses tiny mirrors to change the direction of light, thereby reconfiguring optical links. The largest optical circuit switch on the market has 384 duplex ports with ≈10 ms reconfiguration latency and is available for $200K ($520/port) [41]. However, the optical loss of these switches is 1.5–2.7 dB [2]. Compared to patch panels, OCSs have the following disadvantages: (i) each port is five times more expensive; (ii) their insertion loss is higher; and (iii) their port-count is three times lower. The main advantage of OCSs is that their reconfiguration latency is four orders of magnitude faster than patch panels.
of servers required by each job. This design allows each server to participate in two independent topologies. Hence, when a set of servers uses one topology for a training job, TOPOOPT pre-provisions the next topology, optimized for the next task by reconfiguring look-ahead ports. Once all the servers for the new job are ready, TOPOOPT immediately flips to the new topology by reconfiguring the corresponding $1 \times 2$ switches.

### D Model Configurations and Transfer Sizes

List 1 summarizes the parameters that we used in our simulation and testbed. Model parameters and batch sizes are selected based on common values used in BigNet for simulations. For prototype, we reduce parameter values and batch sizes to fit the models in our 12-node cluster.

#### List 1: DNN models used in our simulations and testbed.

| Model  | Batch/GPU: | #Trans. blks: | Hidden layer: | Seq. length: | #Attn. heads: | Embed. size: | Dense layer size: | #Dense feat. layer: | #Dense layer: | Batch/GPU: |
|--------|------------|---------------|---------------|-------------|--------------|-------------|-----------------|------------------|-------------|------------|
| **VGG:** | $64$ ($\S 4.3$, $\S 4.6$), $20$ ($\S 5$) | $12$ ($\S 4.3$), $6$ ($\S 4.6$, $\S 5$) | $128$ ($\S 4.3$), $256$ ($\S 4.6$), $32768$ ($\S 5$) | $16$ ($\S 4.3$), $6$ ($\S 4.6$), $16$ ($\S 5$) | $512$ ($\S 4.3$, $\S 4.6$, $\S 5$) | $2048$ ($\S 4.4$), $256$ ($\S 4.6$), $[64 \cdots 512]$ ($\S 5$) | $2048$ ($\S 4.3$), $256$ ($\S 4.6$), $1024$ ($\S 5$) | $16$ ($\S 4.3$, $\S 4.6$, $\S 5$) | $16$ ($\S 4.3$), $128$ ($\S 4.4$), $12$ ($\S 5$) | $128$ ($\S 4.3$), $20$ ($\S 5$) |
| **ResNet50:** | $128$ ($\S 4.3$), $20$ ($\S 5$) | $8$ ($\S 4.3$), $6$ ($\S 4.6$, $\S 5$) | $512$ ($\S 4.3$, $\S 4.6$, $\S 5$) | $16$ ($\S 4.3$, $\S 4.6$, $\S 5$) | $4096$ ($\S 4.3$), $4096$ ($\S 4.6$, $\S 5$) | $16$ ($\S 4.3$, $\S 4.6$, $\S 5$) | $4096$ ($\S 4.3$), $4096$ ($\S 4.6$, $\S 5$) | $16$ ($\S 4.3$, $\S 4.6$, $\S 5$) | $16$ ($\S 4.3$, $\S 4.6$, $\S 5$) | $128$ ($\S 4.3$), $20$ ($\S 5$) |
| **BERT:** | $16$ ($\S 4.3$, $\S 4.6$), $2$ ($\S 5$) | $8$ ($\S 4.3$, $\S 4.6$, $\S 5$) | $16$ ($\S 4.3$, $\S 4.6$, $\S 5$) | $16$ ($\S 4.3$, $\S 4.6$, $\S 5$) | $16$ ($\S 4.3$, $\S 4.6$, $\S 5$) | $16$ ($\S 4.3$, $\S 4.6$, $\S 5$) | $16$ ($\S 4.3$, $\S 4.6$, $\S 5$) | $16$ ($\S 4.3$, $\S 4.6$, $\S 5$) | $16$ ($\S 4.3$, $\S 4.6$, $\S 5$) | $128$ ($\S 4.3$), $20$ ($\S 5$) |
| **DLRM:** | $256$ ($\S 4.4$), $256$ ($\S 4.6$), $[64 \cdots 512]$ ($\S 5$) | $8$ ($\S 4.3$, $\S 4.6$, $\S 5$) | $1024$ ($\S 4.3$), $256$ ($\S 4.6$), $32768$ ($\S 5$) | $16$ ($\S 4.3$, $\S 4.6$, $\S 5$) | $256$ ($\S 4.3$), $32768$ ($\S 5$) | $16$ ($\S 4.3$, $\S 4.6$, $\S 5$) | $256$ ($\S 4.3$), $32768$ ($\S 5$) | $16$ ($\S 4.3$, $\S 4.6$, $\S 5$) | $16$ ($\S 4.3$, $\S 4.6$, $\S 5$) | $256$ ($\S 4.3$), $32768$ ($\S 5$) |
| **CANDLE:** | $128$ ($\S 4.3$), $10$ ($\S 5$) | $8$ ($\S 4.3$, $\S 4.6$, $\S 5$) | $16$ ($\S 4.3$, $\S 4.6$, $\S 5$) | $16$ ($\S 4.3$, $\S 4.6$, $\S 5$) | $16$ ($\S 4.3$, $\S 4.6$, $\S 5$) | $16$ ($\S 4.3$, $\S 4.6$, $\S 5$) | $16$ ($\S 4.3$, $\S 4.6$, $\S 5$) | $16$ ($\S 4.3$, $\S 4.6$, $\S 5$) | $16$ ($\S 4.3$, $\S 4.6$, $\S 5$) | $128$ ($\S 4.3$), $20$ ($\S 5$) |
| **NCF:** | $32$ ($\S 4.3$), $32$ ($\S 4.3$) | $8$ ($\S 4.3$) | $4096$ ($\S 4.3$), $4096$ ($\S 4.6$, $\S 5$) | $16$ ($\S 4.3$, $\S 4.6$, $\S 5$) | $16$ ($\S 4.3$, $\S 4.6$, $\S 5$) | $16$ ($\S 4.3$, $\S 4.6$, $\S 5$) | $16$ ($\S 4.3$, $\S 4.6$, $\S 5$) | $16$ ($\S 4.3$, $\S 4.6$, $\S 5$) | $16$ ($\S 4.3$, $\S 4.6$, $\S 5$) | $128$ ($\S 4.3$), $20$ ($\S 5$) |

In most workloads observed in BigNet, the size of AllReduce transfers is larger than the size of MP transfers for each iteration. This is because in most cases it would not be worthwhile if MP transfers are as large as AllReduce transfers. Consider the DLRM example in Section 2.2 with 20 GB embedding tables with double-precision floating parameters. If we were to distribute this embedding table using

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**Figure 25:** Active & Look-ahead ports for high reconfiguration latency.

**Figure 26:** Sharding TOPOOPT cluster for two jobs.

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### C Handling Sharding and Dynamic Job Arrivals in Shared Clusters

Section 3 mentioned how TOPOOPT can support multiple job sharing the cluster through sharding; here we provide a detailed example of how sharding works. Figure 26 shows how a TOPOOPT cluster is sharded for training two jobs together. In this scenario, the optical switches are configured in a way such that the green part (Server 1, 2 and their corresponding links) is completely disjoint from the red part (Server $n-1$, server $n$). Such complete isolation ensures each job gets their dedicated resources, and benefits the performance (especially the tail latency) as shown in Section 4.6.

To start a job with $k$ servers, we need to reconfigure the interconnection between these $k$ servers before the job starts. This can be done quickly when OCSs are used, but when patch panels are used, there could be several minutes of delay before the job can start. To address this challenge, we use a look-ahead approach to pre-provision the next topology while current jobs are running. More specifically, we use a simple $1 \times 2$ mechanical optical switch [108] at each server’s interface to choose between Active vs. Look-ahead ports. These $1 \times 2$ switches are inexpensive ($\$25$) and have $0.73$ dB optical loss measured in our prototype. This is because, unlike optical splitters [15], that incur $3$ dB loss, these switches choose where to send light between their two output ports. We then connect the two ends of each $1 \times 2$ switch to different patch panels, as shown in Figure 25. As a result, a TOPOOPT cluster with $n$ servers, each with $d$ interfaces, has $2d$ patch panels where each interface is split into two parts: Active and Look-ahead. At any point in time, only one end of each $1 \times 2$ switch is participating in the active topology; the other end is pre-provisioning the topology for the next job. Since the topology and parallelization strategy are calculated off-line, we already know the sequence of job arrivals and the number
data parallelism, each server would need to send and receive 37.5 GB of data for the AllReduce operation. On a 100 Gbps fabric this would take 3 seconds by itself, where as if we put it on one server, it would only need to transfer 32 MB/server (assume we have a per-server batch size of 8192, then MP and TotientPerms cluster as $G$ of vertices $p$ adding theorem of cyclic groups, $Z$). Consider the integer modulo $p$.

Proof. Consider the integer modulo $N$ group with addition $Z_N^+ = \{0, 1, \ldots , N-1\}$. $Z_N^+$ is a cyclic group. By fundamental theorem of cyclic groups, $p$ is a generator of $Z_N^+$ if and only if $\gcd(p,N) = 1$. Hence we can cover the entire $Z_N^+$ by repeatedly adding $p$ to itself.

Now consider the graph $G_{Z_N^+,p} = (V_{Z_N^+},E_p)$ where the set of vertices $V_{Z_N^+} = Z_N^+$ and $E_p = \{a \times p, (a+1) \times p\} \subseteq V_{Z_N^+} \times Z_N^+$ \ 	ext{a} \in Z_N^+\}. \text{The set } E_p \text{ forms a cycle on } G_{Z_N^+,p}. \text{Now denote our cluster as } G = (V,E) \text{ where } V \text{ is defined as above and } E \text{ represents a set of directed links. Then } G_{Z_N^+,p} \text{ is isomorphic to } G, \text{ hence following the rule in } E_p \text{ we can define a valid ring in } G. \text{Furthermore, since } \forall p_i \neq p_j \text{ we can guarantee that } (0, p_i) \in E_p, \text{ and } (0, p_j) \notin E_p, \text{ each } p_i \text{ is guaranteed to describe a unique ring.} \]

E.1 TOPOMETRYFINDER

We first provide the mathematical foundation of the ring permutation rule.

Theorem 1 (Ring Generation). For a cluster of $N$ nodes $V = \{S_0, S_1, \ldots , S_{N-1}\}$, all integer numbers $p < N$, where $p$ is co-prime with $N$ (i.e. $\gcd(p,N) = 1$) represent a unique ring AllReduce permutation rule.

Proof. Consider the integer modulo $N$ group with addition $Z_N^+ = \{0, 1, \ldots , (N-1)\}$. $Z_N^+$ is a cyclic group. By fundamental theorem of cyclic groups, $p$ is a generator of $Z_N^+$ if and only if $\gcd(p,N) = 1$. Hence we can cover the entire $Z_N^+$ by repeatedly adding $p$ to itself.

Now consider the graph $G_{Z_N^+,p} = (V_{Z_N^+},E_p)$ where the set of vertices $V_{Z_N^+} = Z_N^+$ and $E_p = \{(a \times p, (a+1) \times p) \in V_{Z_N^+} \times Z_N^+, a \in Z_N^+\}$. The set $E_p$ forms a cycle on $G_{Z_N^+,p}$. Now denote our cluster as $G = (V,E)$ where $V$ is defined as above and $E$ represents a set of directed links. Then $G_{Z_N^+,p}$ is isomorphic to $G$, hence following the rule in $E_p$ we can define a valid ring in $G$. Furthermore, since $\forall p_i \neq p_j$ we can guarantee that $(0, p_i) \in E_p$, and $(0, p_j) \notin E_p$, each $p_i$ is guaranteed to describe a unique ring.

Algorithms 2, 3 and 4 list the detailed pseudocodes of sub-modules in Algorithm 1, namely TotientPerms, CoinChangeMod and SelPermutations.

Algorithm 2 TotientPerms pseudocode

1: procedure TotientPerms($N$, $k$)
   2: \> Input $N$: Total number of nodes
   3: \> Input $k$: AllReduce group size
   4: \> Output $P_k$: Set of permutations for AllReduce group of size $k$
   5: \> Initially, $P_k$ is empty
   6: $P_k = \{\}$
   7: \> This loop runs $\phi(p)$ times, where $\phi$ is the Euler Totient function, $\phi(p) = |\{k < p: \gcd(k,p) = 1\}|$
   8: \> one can also restrict $p$ to be prime only
   9: for $p \leq k$, $\gcd(p,k) == 1$ do
   10: \> one_perm = []
   11: \> for $i$ in 0 to $N/k$ do
   12: \> one_perm += [i + j \times p for $j$ in 0 to $k$]
   13: \> $P_k +=$ one_perm
   14: return $P_k$

Algorithm 3 CoinChangeMod pseudocode

1: procedure CoinChangeMod($N$, $G$)
   2: \> Input $N$: Total number of nodes
   3: \> Input $G$: Network Topology
   4: \> Output $R$: Routings
   5: \> $R$ is the routing result
   6: $R = \{\}$
   7: \> Acquire the set of “coins” from the topology,
   8: \> which are the choices of Algorithm 4
   9: $C = \text{GetCoins}(G)$
   10: for $i \in \{1, N-1\}$ do
   11: \> $\text{curr_dist}$ denotes the “distance” of a value
   12: \> (node distance) counted by number of “coins”
   13: \> $\text{curr_dist}[i] =$ $\infty$
   14: \> $\text{curr_bt}$ record a back-trace of “coins” to
   15: \> get to a value (node distance)
   16: \> $\text{curr_bt}[i] =$ $\infty$
   17: for $c \in C$ do
   18: \> $\text{curr_dist}[c] = 0$
   19: \> $\text{curr_bt}[c] =$ $c$
   20: while $\text{curr_dist}$ has at least one $\infty$ in it do
   21: for $i \in \{1, N-1\}$ do
   22: \> $\text{new_dist}[i] =$ $\text{curr_dist}[i]$
   23: \> $\text{new_bt}[i] =$ $\text{curr_bt}[i]$
   24: for $c \in C$ do
   25: \> if $\text{curr_dist}[(i-c) \mod N] < \text{new_dist}[i]$ then
   26: \> $\text{new_dist}[i] =$ $\text{curr_dist}[(i-c) \mod N] + 1$
   27: \> $\text{new_bt}[i] =$ $c$
   28: \> $\text{curr_dist} =$ $\text{new_dist}$
   29: \> $\text{curr_bt} =$ $\text{new_bt}$
   30: \> Construct the routing for each node distance from the back-trace
   31: $R =$ GetRouteSeq($\text{curr_bt}$)
   32: return $R$

To extend our approach to other AllReduce algorithms, one way is to generalize TotientPerms (Algorithm 2) so that the $E_p$ described in theorem 1 simply represents a permutation which we apply to the original node labeling, while keeping the edge relation, to create an isomorphic graph that describes the new AllReduce topology.

E.2 Bounding maximum hop count with TotientPerms

In this section, we argue that fitting a geometric sequence for choosing permutation provides an approximately $O(d \sqrt{n})$ bound for the maximum diameter of a cluster with $n$ nodes and
As mentioned in Section 3.3, our goals are: (i) have enough bandwidth for large transfer demands; (ii) while also minimize the latency of indirect routing for nodes that do not have a direct link between them.

To achieve this goal in a reconfigurable interconnect, we propose a utility function that finds a balance between the two goals by maximizing the number of parallel links between high demand nodes but with a diminishing return. More formally, assume a network topology is represented by graph $G = (V,E)$ and each node has degree $d$. We define $L(i,j)$ to be the number of parallel links between node-pair $(i,j)$. Let $T(i,j)$ be the amount of unsatisfied traffic demand, we define

$$Utility(G) = \sum_{(i,j) \in E} T(i,j) \times Discount(L(i,j))$$

The Discount function can be defined in different ways; in Algorithm 5 as well as Algorithm 1’s MP construction, we use

$$Discount(l) = \sum_{x=1}^{l} 2^{-x}$$

to reduce the utility of additional links exponentially. One can also explore other discount scaling, such as linear or factorial functions.

When the fabric is reconfigurable (as in OCS-reconfig), we collect the unsatisfied traffic demand every 50 ms and run Algorithm 5 to decide the new network topology. After the new topology is computed, we pause all the flows for 10 ms representing the reconfiguration delay of the OCS, apply the new topology, and then resume the flows that has one or more corresponding physical links across the flow source and destination. The two-edge replacement algorithm from OWAN [85] in line 21 ensures the topology is connected, when we enable host-based forwarding.

## Modifications to SiP-ML

Since SiP-ML’s SiP-Ring proposal is based on a physical ring topology, its reconfiguration algorithm has several constraints about wavelength allocation for adjacent nodes. Given that TopoOpt’s physical topology is not a ring, directly applying SiP-Ring’s optimization using their original C++ code have resulted SiP-ML to perform extremely poorly in our setup. To give SiP-ML a leg up, we observe that its formulation tries to optimize a utility function very similar to Equation 1 without the Discount part (i.e. Discount = 1), but with an ILP. While an ILP gives the optimal solution, its runtime makes it prohibitive for the amount of simulation parameters we explore. Therefore, we substitute the ILP with Algorithm 5 with Discount = 1 which is a heuristic that tries to achieve a similar goal.

Note that SiP-ML paper has another design called SiP-OCs, which is more similar architecturally to TopoOpt. In the SiP-ML paper, SiP-OCs is proposed as a one-shot reconfiguration approach due to the long reconfiguration latency of 3D-MEMS based OCSs.

## Cost of Network Components

| Link bandwidth | Transceiver ($) | NIC ($) | Electrical switch port ($) | Patch panel port ($) | OCS port ($) | 1 x 2 switch ($) |
|----------------|----------------|--------|-----------------------------|---------------------|--------------|------------------|
| 10 Gbps        | 20 [14]        | 185 [32] | 94 [22]                    | 100 [43]            | 520 [41]     | 25 [108]         |
| 25 Gbps        | 39 [16]        | 185 [32] | 144 [24]                  | 100 [43]            | 520 [41]     | 25 [108]         |
| 40 Gbps        | 75 [17]        | 354 [72] | 144 [22]                  | 100 [43]            | 520 [41]     | 25 [108]         |
| 100 Gbps       | 99 [13]        | 678 [34] | 187 [23]                  | 100 [43]            | 520 [41]     | 25 [108]         |
| 200 Gbps       | 198 [13]       | 815 [35] | 374 [23]                  | 100 [43]            | 520 [41]     | 25 [108]         |

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$^{2}$200 G transceivers and switch ports are estimated as $2 \times 100$ G cost.
Table 2 lists the cost of network components we use in Section 4.2, namely NICs, transceivers, fibers, electrical switches, patch panels, and optical switches. The cost of transceivers, NICs, and electrical switch ports is based on the lowest available prices in official retailer websites [19, 28]. Note that for 200 Gbps, we use more 100 Gbps ports and fibers, since it was the less expensive option compared to high-end 200 Gbps and 400 Gbps components, or their price was not even available. To estimate the cost of electrical switch ports, we consider Edgecore bare metal switches with L3 switching and maximum number of ports to amortize the per port cost. The cost of NICs are taken from Mellanox ConnectX series and considered two 2-port NICs as a 4-port NIC. We obtain the cost of patch panel, OCS, and 1×2 optical switch directly from their suppliers, Telescent [43] and Polatis [41] (with 40% discount). The cost of transceivers match the number reported in Sirius [53] as well.

To compute the network cost of Fat-tree and Ideal Switch, we consider number of nodes in a full bisection bandwidth Fat-tree. For example, a standard k = 8 Fat-tree has 80 switches with 64 ports, or 640 switch ports in total, in addition to 1 NIC per host and one transceiver per NIC and switch port. A TOPOOPT system of 128 nodes with degree d uses 128 × d NICs and transceivers, but 128 × 2 × d patch panel ports because of the look-ahead design. Note that the cost of optical components stays constant as link bandwidth increases, an inherent advantage of optics. Following prior work, we estimate the cost of fiber optics cables as 30 cents per meter [67] and select each fiber’s length from a uniform distribution between 0 and 1000 meters [144]. We calculate the cost of TOPOOPT based on 2d patch panels and 1×2 switches at each link to support its look-ahead design (§C). OCS-reconfig’s cost is based on d OCSs connected to all servers in a flat topology.

H Impact of Server Degree on TOPOOPT’s Performance

Figure 27 shows the same setting as Figure 11 except that each server has a degree of eight (d = 8). The results show a similar trend: even though per server bandwidth has increased, the behavior of different network architectures remains consistent.

Next we do a sensitivity analysis regarding the impact of server degree d on TOPOOPT’s performance. Specifically, we vary the degree of each server in TOPOOPT for two link bandwidths: 40 Gbps and 100 Gbps. Figure 28 shows the trend for different DNN models. Both DLRM and CANDLE are network-heavy; therefore, they benefit more from the additional bandwidth obtained by increasing d. CANDLE’s improvement is almost linear as degree goes up, as the strategy is closer to data parallel and the amount of bandwidth available to AllReduce operation increases linearly as well. In the case of DLRM, we observe a super-liner scaling when B = 100 Gbps. This is because DLRM has one-to-many and many-to-one MP transfers which require a low hop count in the topology. As we increase d, TOPOLOGYFINDER is able to find network topologies with much lower diameter, consequently benefiting the performance by both increasing bandwidth and reducing hop-count for MP transfers. Finally, BERT is mostly compute bound at higher bandwidth; hence, increasing the server degree and bandwidth per node has marginal impact on its iteration time.

I Enabling Host-based Forwarding in RDMA

To support a multihop TOPOOPT interconnect using host-based forwarding, we enable RDMA RoCEv2 forwarding on all our HP NICs. RoCEv2 is an implementation of RDMA on top of UDP/IP protocol, by utilizing a particular UDP port (4791) and encapsulating an InfiniBand (IB) data packet. Hence, each RoCEv2 packet can be routed with their source and destination IP addresses. However, host-based forwarding is challenging in RDMA protocol, as the packet processing and memory access are offloaded to the NIC, and host does not have access to individual packets. More precisely, if a packet’s IP destination IP address does not match the NIC’s IP address, RDMA engine silently drops the packet.

To address this issue, we collaborated with engineers from
This enables the upper layer software to consider if the traffic that needs to be forwarded uses MAC address of if2 and hence it is delivered to the host networking stack instead of going to the NIC's RDMA engine.

However, for the connection between server A and D, we set the iproute and arp tables on server A and D to dictate which port the traffic should go out, as well as the proper MAC address of the next server in the forwarding chain. In this case, the packets are delivered to the kernel. Then on server B and C, we set the tc flower rules to forward the packets to the next server with the proper MAC address. In these tc flower rules, we look-up the final destination IP and assert the routing that was computed by our algorithm.

Walk-through of an example for a packet going from server A to server D. In Figure 29, the RDMA engine of server A assumes server D is connected on the third port. It uses the kernel’s routing tables for destination MAC address, which is set to the MAC address of if2 of second port on server B. Therefore, a packet which starts as an RDMA packet of server A is treated as an Ethernet packet when it arrives at server B, and goes to server B’s kernel. In the kernel, based on the packet’s final destination IP of server D, server B redirects the packet to the fourth port, with destination MAC address set to if2 of server C. In this connection, the packet is treated as a normal Ethernet packet. Finally, on server C, the kernel rewrites the destination MAC address to that of if1 on the third port of server D, and redirects it to that port. In this connection, the outgoing Ethernet packet is considered as an RDMA packet due to the destination MAC address. For the reverse connection from server D to A, the same process happens in reverse, to support a bidirectional connection.

With these forwarding rules, we construct logical RDMA connections between all pairs of servers. Upper layer communication libraries such as NCCL requires all-to-all connectivity, and they will utilize these connections. We also modified NCCL to be topology-aware, as certain pairs of servers are only connected through specific ports.

Compared to native point-to-point RDMA, this approach takes a performance penalty. Our experiments indicates the overhead is negligible when the amount of forwarded traffic is small. Currently our NICs support TCP forwarding offload. With firmware and driver modifications or future verion of the NICs, they will also support RDMA forwarding offload. This will further reduce the overhead of our approach.