General Fault Admittance Method Solution for a Line-to-Line-to-Line-to-Ground Unsymmetrical Fault

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Authors’ contributions

This work was carried out in collaboration between both authors. Author JDS in collaboration with author JSJD, derived the equations for the symmetrical fault currents at the fault, developed the computer program, ran the study cases and drafted the paper. Author JSJD critiqued and edited the paper for publication. Both authors reviewed the results, read and approved the final manuscript.

ABSTRACT

Line-to-line-to-line-to-ground unsymmetrical faults involving ground are difficult to analyse using the classical fault analysis approach. This is because the classical solution approach requires knowledge of the connection of symmetrical component sequence networks in order to find the symmetrical fault currents. In the classical solution approach, the phase fault constraints are converted into symmetrical sequence constraints. The sequence networks are connected in a way that satisfies the constraints and the sequence currents determined. A consideration of the symmetrical component constraints for an unsymmetrical three-phase fault involving ground does not lead to an easy connection of the sequence networks. The connection of the positive, negative and zero sequence networks is difficult to deduce when the fault is unsymmetrical and involves ground. A classical solution is therefore difficult to find. In contrast, a solution by the general method of fault admittance matrix does not require prior knowledge of how the sequence networks are connected. It is therefore more versatile than the classical methods. The paper presents a procedure for solving a three-phase to ground unsymmetrical fault, for various fault admittances in each phase. Using the general fault admittance method, a computer program is developed to analyse an unsymmetrical three-phase earth fault on a power system with a delta-earthed-star connected transformer.

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1. INTRODUCTION

The paper presents a method for solving an unsymmetrical line-to-line-to-line-to-ground fault using the general fault impedance method [1-7]. The general fault admittance method differs from the classical approaches based on symmetrical components since it does not require prior knowledge of how the sequence components of currents and voltages are related. It is particularly suited to solve the unsymmetrical three-phase-to-ground fault as this type of fault does not have an established connection of sequence networks.

The classical approach requires knowledge of how the sequence components are related since the sequence networks must be connected in a prescribed way for a particular fault. The interconnection of the sequence networks for a particular fault is established after transforming the phase constraints arising out of the fault to symmetrical component constraints. For example for a line-to-ground short circuit fault the sequence currents are all equal, the sequence voltages summate to zero and therefore the sequence networks are connected in series. Similarly for a line-to-line-to-ground metallic fault the sequence voltages are equal, the sequence currents summate to zero and the sequence networks are connected in parallel. Thus the sequence currents and voltages at the fault are determined based on the interconnection on the sequence networks. This is followed by finding the symmetrical component currents and voltages in the rest of the network. Phase currents and voltages are found by transforming the respective symmetrical component values [8-13].

This paper discusses a procedure for simulating and solving an unsymmetrical line-to-line-to-line-to-ground fault that has some zero impedance faults. This fault is challenging to solve because of the need to derive the connection of the sequence networks from a transformation of the phase fault constraints. The solution is required to validate the versatility of the general fault admittance method to solve unbalanced faults.

2. BACKGROUND

A line-to-line-to-line-to-ground fault presents low value impedances, with zero value for direct short circuits or metallic faults, between the three phases and earth at the point of a fault in the network. In general, a fault may be represented as shown in Fig. 1. The admittances $Y_{ab}$, $Y_{bc}$, $Y_{cf}$ and $Y_{gf}$ are the inverses of the fault impedances in the respective phases and ground path.

The application of the fault admittance method to various types of faults, excluding the unsymmetrical fault considered in this work, has been presented in earlier works [1-6]. The faults studied include balanced three-phase fault, line-to-line faults, line-to-line-to-ground faults, and line-to-ground faults in reference and odd phases.
In the general case of the unsymmetrical fault considered in this work the phase fault impedances are not equal to each other i.e. $Y_{af} \neq Y_{bf} \neq Y_{cf}$. Furthermore the most challenging conditions of one of the phase fault admittances and that of the ground path $Y_{gf}$ being infinite are considered. The conditions of the zero impedance faults in one of the phases and the ground path involve infinite admittances that require simulation.

A systematic approach for using a fault admittance matrix in the general fault admittance method is given by Sakala and Daka [7]. The main steps of the method are given in sections 2.1 to 2.7.

2.1 Bus Impedance Matrix Assembly

The sequence (Positive, negative and zero) bus impedance matrices are calculated and assembled in a $3n \times 3n$ matrix, where $n$ is the number of bus bars of the system.

2.2 Fault Admittance Calculation

The general fault admittance matrix $Y_f$ is calculated:

$$Y_f = \frac{1}{Y_{af} + Y_{bf} + Y_{cf}} \begin{bmatrix} Y_{af} (Y_{af} + Y_{bf} + Y_{cf}) & -Y_{af}Y_{bf} & -Y_{af}Y_{cf} \\ -Y_{af}Y_{bf} & Y_{bf} (Y_{af} + Y_{bf} + Y_{cf}) & -Y_{bf}Y_{cf} \\ -Y_{af}Y_{cf} & -Y_{bf}Y_{cf} & Y_{cf} (Y_{af} + Y_{bf} + Y_{cf}) \end{bmatrix}$$

(1)

2.3 Symmetrical Component Fault Admittance Matrix Determination

The general fault admittance matrix in equation (1) is transformed into its symmetrical component value. The resulting symmetrical component fault admittance matrix is:
where

\[
W = \begin{bmatrix}
    Y_{af} & Y_{bf} & Y_{cf} \\
    Y_{bf} & Y_{af} & Y_{cf} \\
    Y_{cf} & Y_{bf} & Y_{af}
\end{bmatrix}
\]

(2)

Note that for the general unsymmetrical fault with \(Y_{af} \neq Y_{bf} \neq Y_{cf}\) the symmetrical component matrix is not symmetrical since expressions for \(Y_{fs21} \neq Y_{fs12}\), \(Y_{fs31} \neq Y_{fs13}\) and \(Y_{fs23} \neq Y_{fs32}\). Consequently the expressions for the sequence currents and voltages do not lead to, or suggest, easy connection patterns of the sequence networks.

### 2.4 Symmetrical Component Currents in the Fault

Let the fault be at bus bar be \(j\). The symmetrical component currents in the fault are given by [7]:

\[
I_{j} = \begin{bmatrix}
    I_{j+} \\
    I_{j-} \\
    I_{j0}
\end{bmatrix} = Y_{j} \left( U + Z_{s} Y_{j} \right)^{-1} V_{j0}
\]

(3)

In which \(Z_{s}\) is the \(jj\)th component of the symmetrical component bus impedance matrix:

\[
Z_{s} = \begin{bmatrix}
    Z_{j+} & 0 & 0 \\
    0 & Z_{j-} & 0 \\
    0 & 0 & Z_{j0}
\end{bmatrix}
\]

The elements \(Z_{j+}\), \(Z_{j-}\), and \(Z_{j0}\) are the positive sequence, the negative sequence, and zero sequence impedances at the faulted bus bar respectively. Note that as the network is balanced the mutual terms are all zero.

In equation (3), \(V_{j0}\) is the prefault symmetrical component voltage at bus bar \(j\), the faulted bus bar:

\[
V_{j} = \begin{bmatrix}
    V_{j+} \\
    V_{j-} \\
    V_{j0}
\end{bmatrix} = \begin{bmatrix}
    V_{+} \\
    0 \\
    0
\end{bmatrix}
\]

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Where \( V \) is the positive sequence voltage before the fault. The negative and zero sequence voltages are zero because the system is balanced prior to the fault.

### 2.5 Phase Component Fault Currents Calculation

The phase currents in the fault are obtained by transformation:

\[
I_{fj} = [I_{ij}]
\]

\[
= TI_{fj}
\]

### 2.6 Symmetrical and Phase Component Bus Bar Voltages Calculations

The symmetrical component voltages at the faulted bus bar \( j \) are:

\[
V_{fj} = \begin{bmatrix}
V_{j+} \\
V_{j-} \\
V_{j0}
\end{bmatrix} = \left(U + Z_{ij} Y_{ij}\right) V_{ij}^0
\]

and the symmetrical component voltages at a bus bar \( i \) for a fault at bus bar \( j \) are:

\[
V_{ji} = \begin{bmatrix}
V_{i+} \\
V_{i-} \\
V_{i0}
\end{bmatrix} = V_{i0} - Z_{ij} Y_{ij} \left(U + Z_{ij} Y_{ij}\right) V_{ij}^0
\]

Where

\[
V_{ij}^0 = \begin{bmatrix}
V_{i+}^0 \\
0 \\
0
\end{bmatrix}
\]

are the symmetrical component prefault voltages at bus bar \( i \) [7]. The negative and zero sequence prefault voltages are zero.

In equation (6), \( Z_{ij} \) gives the \( ij^{th} \) components of the symmetrical component bus impedance matrix, the mutual terms for row \( i \) and column \( j \) (corresponding to bus bars \( i \) and \( j \)):

\[
Z_{ij} = \begin{bmatrix}
Z_{ij+} & 0 & 0 \\
0 & Z_{ij-} & 0 \\
0 & 0 & Z_{ij0}
\end{bmatrix}
\]

The phase voltages in the fault, at bus bar \( j \), and at bus bar \( i \) are then obtained by transformation:
2.7 Currents in Lines, Transformers and Generators

The symmetrical component currents in a line, transformer or generator between bus bars \(i\) and \(j\) are given by:

\[
\begin{bmatrix}
V_{\text{fs}ij} \\
V_{\text{fs}ij} \\
V_{\text{fs}ij}
\end{bmatrix}
= TV_{\text{fs}ij}
\text{ and }
\begin{bmatrix}
V_{\text{fs}ij} \\
V_{\text{fs}ij} \\
V_{\text{fs}ij}
\end{bmatrix}
= TV_{\text{fs}ij}
\tag{7}
\]

\[
I_{\text{fs}ij} = \begin{bmatrix} I_{\text{fs}ij+} \\ I_{\text{fs}ij} \\ I_{\text{fs}ij0} \end{bmatrix} = Y_{\text{fs}ij} \left( V_{\text{fs}ij} - V_{\text{fs}ij} \right) = \left( V_{\text{fs}ij} - V_{\text{fs}ij} \right)
\tag{8}
\]

where

\[
Y_{\text{fs}ij} = \begin{bmatrix} Y_{\text{fs}ij+} & 0 & 0 \\ 0 & Y_{\text{fs}ij} & 0 \\ 0 & 0 & Y_{\text{fs}ij0} \end{bmatrix}
\]

is the symmetrical component admittance of the branch between bus bars \(i\) and \(j\). The phase currents in the branch are found by transformation [7]:

2.8 Symmetrical Component Topological Considerations

Recent works on topological considerations by Gandelli et al. [14], and Della Torre et all [15], are interesting in that unsymmetrical faults may be represented by voltage sources at the fault point that reflect the unbalances in the fault impedances. This provides a possible extension to power system analysis based on the matrix methods.

3. LINE-TO-LINE-TO-LINE-TO-GROUND UNSYMMETRICAL FAULT SIMULATION

The current in the fault is given by equation (3). This requires inversion of the term \(U + Z_{\text{fij}} Y_{\text{fj}}\). In the general form of the solution the inversion is carried out within the computer program in the course of the solution. In the simplified form of the solution the full expression of the symmetrical component fault admittance matrix is directly substituted in equation (3), and the inversion of the matrix carried out to obtain expressions for \(I_{\text{fij}}\).

The simplified solution is:

\[
I_{\text{fij}} = \frac{V^0}{\Delta} \begin{bmatrix} I_{\text{fij}+} \\ I_{\text{fij}0} \\ I_{\text{fij}10} \end{bmatrix}
\tag{10}
\]
Where

\[
\Delta = 1 + Z_{sij}Y_{\beta 3} + Z_{sij}Y_{\beta 22} + Z_{sij}Y_{\beta 11} + Z_{sij}Z_{sij0} \left( Y_{\beta 31}Y_{\beta 33} - Y_{\beta 32}Y_{\beta 31} \right) \\
+ Z_{sij}Z_{sij} \left( Y_{\beta 21}Y_{\beta 22} - Y_{\beta 21}Y_{\beta 21} \right) + Z_{sij}Z_{sij0} \left( Y_{\beta 22}Y_{\beta 33} - Y_{\beta 23}Y_{\beta 32} \right) \\
+ Z_{sij}Z_{sij}Z_{sij0} \left[ Y_{\beta 11} \left( Y_{\beta 22}Y_{\beta 33} - Y_{\beta 23}Y_{\beta 32} \right) + Y_{\beta 12} \left( Y_{\beta 22}Y_{\beta 31} - Y_{\beta 21}Y_{\beta 31} \right) \\
+ Y_{\beta 13} \left( Y_{\beta 21}Y_{\beta 32} - Y_{\beta 22}Y_{\beta 31} \right) \right],
\]

\[
I_{fji} = Y_{\beta 11} + Z_{sij} \left( Y_{\beta 11}Y_{\beta 22} - Y_{\beta 12}Y_{\beta 21} \right) + Z_{sij0} \left( Y_{\beta 11}Y_{\beta 33} - Y_{\beta 13}Y_{\beta 31} \right) \\
+ Z_{sij}Z_{sij0} \left[ Y_{\beta 11} \left( Y_{\beta 22}Y_{\beta 33} - Y_{\beta 23}Y_{\beta 32} \right) + Y_{\beta 12} \left( Y_{\beta 22}Y_{\beta 31} - Y_{\beta 21}Y_{\beta 31} \right) \\
+ Y_{\beta 13} \left( Y_{\beta 21}Y_{\beta 32} - Y_{\beta 22}Y_{\beta 31} \right) \right],
\]

\[
I_{fj} = Y_{\beta 21} + Z_{sij} \left( Y_{\beta 21}Y_{\beta 33} - Y_{\beta 23}Y_{\beta 31} \right) + Z_{sij0} \left[ Y_{\beta 21} \left( Y_{\beta 22}Y_{\beta 33} - Y_{\beta 23}Y_{\beta 32} \right) + Y_{\beta 22} \left( Y_{\beta 22}Y_{\beta 31} - Y_{\beta 21}Y_{\beta 31} \right) \\
+ Y_{\beta 23} \left( Y_{\beta 21}Y_{\beta 32} - Y_{\beta 22}Y_{\beta 31} \right) \right],
\]

\[
I_{fj0} = Y_{\beta 31} + Z_{sij} \left( Y_{\beta 22}Y_{\beta 31} - Y_{\beta 21}Y_{\beta 32} \right) + Z_{sij0} \left[ Y_{\beta 31} \left( Y_{\beta 22}Y_{\beta 33} - Y_{\beta 23}Y_{\beta 32} \right) + Y_{\beta 32} \left( Y_{\beta 22}Y_{\beta 31} - Y_{\beta 21}Y_{\beta 31} \right) \\
+ Y_{\beta 33} \left( Y_{\beta 21}Y_{\beta 32} - Y_{\beta 22}Y_{\beta 31} \right) \right].
\]

The expressions for \( I_{fji} \), \( I_{fj} \), and \( I_{fj0} \) are useful for validating the results obtained using the general form of the solution in equation (3).

4. COMPUTATION OF THE LINE-TO-LINE-TO-LINE-TO-GROUND UNSYMMETRICAL FAULT

Equations (1) to (10) are used in a computer simulation program to solve an unsymmetrical fault on a general power system. The simulation is applied on a simple power system comprising of three bus bars to solve for a line-to-line-to-line-to-ground unsymmetrical fault. The parameters and connection arrangements for the simple system used are the same as in reference [6]. The power system is taken to be at no load before the occurrence of a fault, and therefore voltages of 1.0 per unit are used at the bus bars and in the generator.

The line-to-line-to-line-to-ground unsymmetrical fault is at bus bar 1, the load bus bar. The unsymmetrical fault is described by the fault impedances 0 \( \Omega \), j0.1 \( \Omega \), and j0.2 \( \Omega \) in the a, b, and c phases respectively and 0 \( \Omega \) in the ground path. Note that the admittances in the a phase and ground path are infinite and have to be simulated.

The fault impedances are input in each phase and in the ground path by inputting the fault type value of 4 in the computer program. The zero impedances that need to be simulated are input in the a phase and the ground path. Note that the fault type descriptor of 4 in reference [6] refers to the fact that individual phase and ground path impedances are input, and does not describe the fault itself.
The initial fault impedance values, in the a phase and ground path, are of the order of $10^{-3}$ Ω. This value of impedance is much smaller than any of the system sequence impedances at the fault point. The sequence fault currents are calculated for the initial values. A second set of values of the fault impedances in the a phase and the ground path, obtained by multiplying the initial value by a factor of $10^{-1}$, is used to calculate the second values of sequence fault currents. The absolute value of the change in the positive sequence current is compared against a tolerance, say of $10^{-6}$, and if smaller the solution is considered converged. If the absolute value of the change is larger than the tolerance, the fault impedance is again reduced and the next value of symmetrical fault currents calculated. The iterative process is carried out to its conclusion; until either convergence or non-convergence is obtained.

5. RESULTS AND DISCUSSIONS

The values of the simulated fault impedances that give accurate values of sequence currents are given in Table 1. Case 1 in the table is for a resistive fault, case 2 is for a resistive and inductive fault while case 3 is for an inductive fault. The resistive fault impedance gives a better convergence, since the current tolerance is met by a relatively higher fault impedance, than for the other two cases. Note that in cases 2 and 3 the current tolerances are reduced, otherwise there is no convergence.

The results obtained from the computer program are listed in Table 2. A summary of the transformer phase currents is shown in Fig. 2.

Table 1. Solution Convergence Characteristics

| Case | Fault impedance | Phase a | Ground path | Current tolerance | Current difference |
|------|-----------------|---------|-------------|-------------------|--------------------|
| 1    | (r+j0)          | 5×10^{-6} | 5×10^{-6}   | 1×10^{-6}         | 5.7×10^{-9}        |
| 2    | (r+jx)          | (5+j5)×10^{-8} | (5+j5)×10^{-8} | 1×10^{-6} | 5.8×10^{-7}        |
| 3    | (0+jx)          | j5×10^{-9}  | j5×10^{-9}  | 1×10^{-7}         | 5.8×10^{-7}        |

5.1 Fault Simulation Impedances

Table 2(a) shows the value of impedances which were used to describe the unsymmetrical fault. The values of resistances in Phase a and in the ground path are generated from the program and show the values of resistances which met the convergence criteria in the positive sequence currents.

5.2 Symmetrical Component Fault Admittance Matrix

Table 2 (b) shows the symmetrical component fault admittance matrix obtained from the program. The matrix is complex and shows the effect of the resistances used to simulate the zero fault impedances in the a phase and the ground path. Note that the matrix is nearly symmetrical indicating the major influence of the zero fault impedances in the a phase and the ground path.
Table 2. Simulation Results - Unbalanced Fault Study

General Fault Admittance Method – Delta-star Transformer Model

a) Fault Impedances

\[
Z_f = \begin{bmatrix}
Z_{af} \\
Z_{bf} \\
Z_{cf}
\end{bmatrix} = \begin{bmatrix}
0.5 \times 10^{-6} \\
0.1 \\
0.2
\end{bmatrix} \Omega \\
\Rightarrow Z_{af} = 0.5 \times 10^{-6} \Omega
\]

b) Symmetrical Component Fault Admittance Matrix

\[
Y_f = 3.3333 \times 10^4 \begin{bmatrix}
1 & 0.9991 + j0.000225 & 1 + j0.0003375 \\
1 + j0.000225 & 1 & 1 + j0.0003375 \\
1 + j0.0003375 & 1 + j0.0003375 & 1
\end{bmatrix}
\]

c) Thévenin’s Symmetrical Component Matrix at Faulted Bus Bar

\[
Z_{ij} = \begin{bmatrix}
0.5 & 0 & 0 \\
0 & 0.5 & 0 \\
0 & 0 & 0.8125
\end{bmatrix}
\]

d) Symmetrical Component Fault Currents

| Current Component | Simplified Method | General Method |
|-------------------|-------------------|---------------|
|                  | Magnitude/Angle   | Magnitude/Angle |
| +ve               | 1.8928/-89.9998°  | 1.8928/-89.9998° |
| -ve               | 0.1642/-63.6184°  | 0.1642/-63.6183° |
| 0                 | 0.1083/245.4993°  | 0.1083/245.4993° |

e) Fault Current in Phase Components

| Phase Component | Magnitude/Angle |
|-----------------|-----------------|
| a               | 1.9386/-89.1700° |
| b               | 1.6730/148.0775° |
| c               | 1.4682/31.0152°  |
| Ground          | 0.3248/245.4993° |

f) Symmetrical Component Voltages at Faulted Bus Bar

| Voltage Component | Magnitude/Angle |
|-------------------|-----------------|
| +ve               | 0.1536/-0.0012°  |
| -ve               | 0.0821/206.3817° |
| 0                 | 0.0880/155.4993° |

g) Phase Voltages at Faulted Bus Bar

| Phase Component | Magnitude/Angle |
|-----------------|-----------------|
| a               | 0.0000/267.26°  |
| b               | 0.1673/238.0776° |
| c               | 0.2936/121.0955° |
h) Post fault Voltages at Bus Bars

| Bus Bar Number | Phase a | Phase b | Phase c |
|----------------|---------|---------|---------|
|                | Magnitude/Angle | Magnitude/Angle | Magnitude/Angle |
| 1              | 0.0000/267.26º | 0.1673/238.0776º | 0.2936/121.0955º |
| 2              | 0.5303/-1.4861º | 0.7682/238.0776º | 0.6200/118.4801º |
| 3              | 0.7257/31.0768º | 0.7682/-89.1837º | 0.7449/148.1087º |

i) Post fault Currents in Lines

| Line no. | SE Bus | RE Bus | Phase a | Phase b | Phase c |
|----------|--------|--------|---------|---------|---------|
|          | Magnitude/Angle | Magnitude/Angle | Magnitude/Angle |
| 1        | 2      | 1      | 1.9368/-89.1700º | 1.6730/148.0775º | 1.4682/31.0952º |
| 1        | 1      | 2      | 1.9386/90.8300º | 1.6730/-31.9225º | 1.4682/211.0952º |

j) Post fault Currents in Transformers

| Trans. no. | SE Bus | RE Bus | Phase a | Phase b | Phase c |
|------------|--------|--------|---------|---------|---------|
|            | Magnitude/Angle | Magnitude/Angle | Magnitude/Angle |
| 1          | 3      | 2      | 3.1728/-62.8448º | 2.6802/177.2974º | 2.9636/65.4º |
| 1          | 2      | 3      | 1.9386/90.8300º | 1.6730/-31.9225º | 1.4682/211.09º |

k) Link Currents in Delta Connection at Sending End

| Trans. no. | SE Bus | RE Bus | Phase a | Phase b | Phase c |
|------------|--------|--------|---------|---------|---------|
|            | Magnitude/Angle | Magnitude/Angle | Magnitude/Angle |
| 1          | 3      | 2      | 1.6730/148.0775º | 1.4682/31.0952º | 1.9386/-89.1700º |

l) Post fault Currents in Generators

| Gen. no. | SE Bus | RE Bus | Phase a | Phase b | Phase c |
|----------|--------|--------|---------|---------|---------|
|          | Magnitude/Angle | Magnitude/Angle | Magnitude/Angle |
| 1        | 4      | 3      | 3.1728/-62.8448º | 2.6802/2.9636º | 2.9636/65.4967º |

m) Generator Neutral Current

| Magnitude/Angle |
|-----------------|
| 0.0000/237.9946º |

5.3 Sequence Impedances at the Faulted Bus Bar

The Thevenin’s self-sequence impedances of the network seen from the faulted bus bar are given in Table 2(c). The diagonal terms are zero and in agreement with the theoretical values. The self-sequence impedances at the faulted bus bar obtained from the program are equal to the theoretical values.

5.4 Symmetrical Component and Phase Currents in the Fault

The symmetrical component fault currents obtained from the program using equations (3) and (10) are given in Table 2(d). The two sets of results are practically equal. The results are consistent with theory in that all three components are present.

The fact that the sequence currents from the simplified form and the general form are equal is an important result in that the simplified expressions for the sequence fault currents may be
used instead of carrying out the inversion within the computer program. Derivation of the simplified expressions is a major contribution of this work.

The phase fault currents, Table 2 (e), are highest in the phase a, the phase with the least fault impedance. The next largest current is phase b, which had the second lowest fault impedance. Phase c, with the largest fault impedance has the least fault current. The neutral current is calculated so that it can be compared to the neutral current on the star side of the transformer.

5.5 Symmetrical Component and Phase Voltages in the Fault and at the Bus Bars

Table 2 (f) gives the symmetrical component voltages at the fault point. All the sequence voltages are present. The positive sequence value is a lot larger than the negative and zero sequence values, while the latter are of the same order of magnitude. The values do not lead to a simple interpretation of the connection of the sequence networks. This is expected given that the fault impedances are not balanced.

The phase voltages at the fault point are given in Table 2(g). They are zero in phase a, which has a zero fault impedance, 17% in phase b and 29% in phase c. Note that the fault impedances in Phases b and c are $j0.1\Omega$ and $j0.2\Omega$ respectively.

5.6 Symmetrical Component and Phase Voltages at the Bus Bars

The phase voltages at the bus bars are given in Table 2 (h). Bus bar 1 is the faulted bus bar and its voltage is zero. The phase voltages at bus bar 2 are nearly equal being 53.0%, 58.1% and 62.0% of the prefault values in the a, b and c phases respectively.

The phase voltages at bus bar 3 are nearly balanced with magnitudes of 72.6%, 76.8% and 74.5% of the prefault values in the a, b and c phases respectively. The phase voltages at bus bar 3 lead the phase voltages at bus bar 2 by 33.1º, 26.9º and 30.0º in the a, b and c phases respectively.

5.7 Phase Currents in Transmission Line

The phase currents in the transmission line, Table 2 (i), are equal to the currents in the fault. Note that the current at the receiving end of the line is by convention considered as flowing into the line, rather than out of it.

5.8 Phase Currents in the Transformer

The currents in the Transformer are given in Table 2 (j) and shown in Fig. 2. The currents on the line side are equal to the currents in the line, after allowing for the sign change due to convention. The currents in the transformer windings satisfy the ampere-turn balance requirements of the transformer.

The magnitudes of the currents at the sending end of the transformer, the delta connected side, are $\sqrt{3}$ times the magnitudes of the currents in the phase windings, although not a phase-to-phase correspondence. While the magnitude in phase a on the delta connected side is $\sqrt{3}$ times the current magnitude on the star connected side, the magnitude in the b
phase on the delta connected side is $\sqrt{3}$ times the value of the current magnitude in the $c$ phase. Similarly the magnitude of the current in the $c$ phase on the delta connected side is $\sqrt{3}$ times the current magnitude in the $b$ phase. The phase fault currents flowing from the generator are equal to the phase currents into the transformer.

The results also include the currents in the links of the delta connection, Table 2 (k). They have been included to show how the ampere turn balances are satisfied in the transformer.

### 5.9 Phase Currents in the Generator

Table 2 (l) shows the phase currents in the generator. They are in agreement with the currents entering the transformer.

The neutral current in the generator is zero, Table 2(m), which is expected as it is connected to the delta side of a delta earthed star transformer.

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**Fig. 2. Transformer Currents for a Line-to-Line-to-Line-to-Ground Unsymmetrical Fault**

### 6. CONCLUSIONS

A line-to-line-to-line-to-ground unsymmetrical fault has been solved using the general fault admittance method; with very small values to represent zero fault impedances. This type of fault is difficult to solve using the classical symmetrical components approach based on the connection of the sequence component networks at the fault point. The difficulty arises because the phase and symmetrical component constraints do not suggest a simple connection of the sequence component networks.

A major result is the fact that the simplified form of the solution, in which the matrix $\mathbf{U} + Z_{af} \mathbf{Y}_{fa}$ is inverted outside of the computer program to obtain the sequence currents, gives the same results as the general form of the solution in which the matrix is inverted within
the program. The simplified expressions for the sequence currents may therefore be used without requiring inversion.

The results show that the phase voltages on the delta side of a delta-earthed-star connected transformer, with the fault on the star side, are nearly balanced. This is consistent with the effect that a delta-star connected transformer has on unbalanced loads on the star side.

The line-to-line-to-line-to-ground unsymmetrical fault is interesting for studying the delta-earthed-star transformer arrangement. The currents and voltages are nearly balanced on both sides of the transformer. The current in the neutral of the transformer equals the ground current at the fault. Phase shifts of nearly 30° between the voltages on the delta side to those on the star connected side are shown, consistent with theory. The results give an insight in the effect that a delta-earthed star transformer has on a power system during line-to-line-to-line-to-ground unsymmetrical faults.

The work confirms the main advantage of the general fault admittance method which is that knowledge of the interconnection of symmetrical component networks at the fault point is not required for calculating the sequence currents and voltages.

COMPETING INTERESTS

Authors declare that there are no competing interests.

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