Vortex-induced vibrational tristable energy harvester: Design and experiments

S Zhou1, J Li2, J Wang3, G Li3 and Q Wang4,5
1 School of Aeronautics, Northwestern Polytechnical University, Xi’an 710049, China
2 School of Power and Engineering, Northwestern Polytechnical University, Xi’an 710049, China
3 School of Chemical Engineering and Energy, Zhengzhou University, Zhengzhou 450001, China
4 Department of Civil and Environmental Engineering, Shantou University, Shantou 515063, China
5 Department of Mechanics and Aerospace Engineering, Southern University of Science and Technology, Shenzhen 518055, China
*Corresponding author
Emails: zhoushengxi@nwpu.edu.cn (S. Z); jlwang@zzu.edu.cn (J.W); wangq@sustc.edu.cn (Q. W)

Abstract. Wind energy harvesters have been widely studied for their great application potential to power small wireless sensors. Meanwhile, the unique dynamic characteristics of vibrational tristable energy harvesters have been theoretically and experimentally verified. More importantly, such vibrational tristable energy harvesters have excellent broadband energy harvesting performance under low-frequency and low-level excitations. This paper aims to develop a new kind of aeroelastic energy harvesters for enhancing wind energy harvesting performance. In detail, a vortex-induced vibrational tristable energy harvester is designed by using the magnetic force to realize the tristable configuration. A mathematical model of the presented harvester is provided. Experimental results verify that the presented vortex-induced vibrational tristable energy harvester performs better than the traditional linear vortex-induced vibration energy harvester.

1. Introduction
In the past 10 years, energy harvesting technique has attracted a lot of attentions from researchers in both industry and science. As we all well known that, it belongs to green energy and has a great application potential to solve the problem of energy supply for some wireless sensor networks and small portable electromechanical devices [1,2]. Therefore, many different energy harvesters were designed to harvest energy from environmental base vibrations, human motions, ocean waves, vehicle suspension, etc [3-7]. Meanwhile, aeroelastic energy harvesters were developed to power embedded micro electromechanical systems and sensors into bridges, high-altitude buildings, ducts, and son on [8-11]. The vortex-induced vibration energy harvester (VIVEH) is one of most popular aeroelastic energy harvesters, and it is a kind of linear resonant based energy harvester. In principle, the VIVEH will efficiently work when the vortex shedding frequency is near one of its natural frequencies. Goushcha et al [12] explored the driving mechanisms of the VIVEH based on the particle image velocimetry. Dai et al [13] and Zhang et al [14] presented analyse of different bluff bodies on the energy harvesting performance and the
dynamic response of VIVEHs. Weinstein et al [15] experimentally explored the energy harvesting performance of an aeroelastic piezoelectric beam in the real environments. Zhang et al [16] presented a governing model to solve the aero-electromechanical coupling problem existing in VIVEHs with different cross-sectional shapes. Recently, Zhou and Wang [17] designed a dual serial vortex-induced vibration energy harvesting system to enhance wind energy harvesting, which can generate higher output voltage and power than the traditional one in a wide wind speed range.

In order to improve energy harvesting performance in a wide frequency range, nonlinearities caused by the magnetic force or the geometric nonlinearity were brought to modify traditional linear energy harvesters under base excitations [18-22]. Especially, the tristable energy harvester (TEH) has an excellent performance subject to low-frequency and low-level excitations [23,24]. The TEH was demonstrated that its high-energy interwell oscillations could be realized under appropriate excitation conditions, which lead to large-amplitude output voltage [25-27]. In addition, the response mechanism of the TEH under random excitations was numerically and theoretically revealed [28,29]. For different harmonic and random excitations, a well-designed TEH performs better than the bistable energy harvester [23-29].

In order to improve the performance of the VIVEH, Nasser et al [30] designed a nonlinear monostable VIVEH based on the nonlinear magnetic force. Huynh and Tjahjowidodo [31] designed a bistable VIVEH and experimentally verified the performance enhancement. These results verified that the magnetic coupling has a positive influence on aeroelastic energy harvesters. Meanwhile, the trisbale configuration can greatly the performance of energy harvesters under base excitations, and it also has an excellent potential to improve the performance of aeroelastic energy harvesters.

This paper originally designed a vortex-induced vibrational tristable energy harvester (VIVTEH) for enhancing energy harvesting in a wide wind speed range. The paper is organized as follows: in section 2, the design and the mathematical model of the VIVTEH are illustrated. In section 3, the experiment setup is built and a comparison between the VIVTEH and the traditional VIVEH is provided. At last, key conclusions are addressed.

2. Design and operating principle

Fig. 1 shows the schematic diagram of the VIVTEH. The whole device is installed in a wind tunnel. Piezoelectric materials which are used to convert mechanical energy into electric energy are bonded on the clamped end of the substrate layer. In this study, a pure load resistance will be connected with the piezoelectric materials. At the free end, a bluff body is bonded to the substrate layer to produce the vortex shedding street, which will induce the vibration of the VIVTEH. The vibration direction of the harvester is perpendicular to the wind flow direction. Two small tip magnets are attached on the both sides of the bluff body, and the nonlinear magnetic force will be produced via the interaction between tip magnets and two external magnets. The tristable configuration can be obtained by adjusting the relative position between two tip magnets and two external magnets.

Based on Euler–Bernoulli beam theory, experimental identifications, etc., the traditional linear VIVEH can be described by the following mechanical and electrical governing equations:

\[ M \ddot{y} + C \dot{y} + Ky - \theta V = F_{IV} \]  

\[ C_p \dot{V} + \frac{V}{R} + \theta \dot{y} = 0 \]  

where \( M \) is the equivalent mass. \( C \) is the equivalent damping. \( K \) is the equivalent linear stiffness. \( \theta \) is the equivalent electromechanical coupling coefficient. \( C_p \) is the equivalent capacitance. \( R \) is the load resistance. \( y \) is the tip displacement in the bending direction of the harvester. \( V \) is the output voltage across \( R \). \( F_{IV} \) is the flow-induced force caused by the incoming wind force, which can be calculated by using the Lattice Boltzmann method (LBM). For the VIVTEH, Eq. (1) should be rewritten as:

\[ M \ddot{y} + C \dot{y} + \theta \dot{y} = F_{non} - \theta V = F_{VIV} \]
where $F_{\text{non}}$ is the equivalent nonlinear restoring force which is composed of the linear elastic force of the beam and the nonlinear magnetic force. It can be expressed by a polynomial, as follows:

$$F_{\text{non}} = K_0 + K_1 y + K_2 y^2 + \ldots + K_n y^n$$

where $K_0$, $K_1$, $K_2$, ..., $K_n$ are polynomial coefficients.

**Figure 1.** The schematic diagram of the VIVTEH.

### 3. Experimental validation

In this section, the experimental setup is built to verify the design as shown in Fig. 2. The experimental devices in this study are shown in Fig. 2(a). A wind tunnel with the diameter of 400 mm is used to produce wind flow to the harvester which is tested in the wind tunnel. The wind speed can be measured by a hot-wire anemometer (Testo Co., USA). The bluff body with the length of 90 mm and the diameter of 24 mm is made of the hard foam. The experimental data are collected by a computer based collection system. Three stable equilibrium positions of the VIVTEH are respectively shown in Fig. 2(b)-(d). The substrate layer (pure Aluminum) of the harvester has the dimensions of $200 \times 20 \times 0.5 \text{ mm}^3$. Two PZT-5A piezoelectric patches (Jiashi Co., China) have the same dimensions of $30 \times 20 \times 0.5 \text{ mm}^3$ are selected. Two tip magnets have the thickness of 1.5 mm and the diameter of 8 mm. Two external magnets have the thickness of 4.5 mm and the diameter of 30 mm.
Figure 2. (a) The experiment setup; (b) three stable equilibrium positions of the VIVTEH in the experiment.

Under a wide wind speed range, the experimental output voltage of the VIVTEH with different load resistances is shown in Fig. 3. It is found that the load resistance has a big influence on the output voltage and the dynamic responses of the VIVTEH. As the increasing of the load resistance, the output voltage is also increasing. It should be noted that the output voltage amplitude greatly depends on the property of piezoelectric materials of the harvester. However, this study mainly focuses on the design and the dynamic responses of the presented VIVTEH. The corresponding output power \( \left( \frac{V_{\text{amplitude}}}{\sqrt{2}} \right)^2 / R \) is shown in Fig. 4. Among all these five cases, the maximum output power is 36.45 \( \mu \)W when the load resistance is 100 k\( \Omega \). For the optimal load resistance leading to the maximum output power of a piezoelectric energy harvester, it can be calculated based on the angular vibration frequency of the harvester in this study) in Ref. [33].

![Figure 3. Output voltage of the VIVTEH with different load resistance.](image)

![Figure 4. Output power of the VIVTEH with different load resistance.](image)

In the case of the external magnets being removed, the VIVTEH will become a traditional VIVEH. To verify the energy harvesting enhancement, comparison of experimental output voltages of the VIVTEH and the VIVEH with open-circuit conditions is shown in Fig. 5. It is found that the output voltage of the former is obviously higher than that of the latter. In addition, the working wind speed range of the VIVTEH is much wider than that of the VIVEH. This indicates that the tristable
configuration not only improves the output voltage amplitude, but also greatly improves the working wind speed range of the VIV energy harvesters.

4. Conclusions
In this paper, the vortex-induced vibrational tristable energy harvester is presented based on the magnetic coupling to enhance wind energy harvesting. In addition, we propose a mathematical model to describe the presented harvester. Experimental results verify that the output voltage and the working wind speed range of the vortex-induced vibrational tristable energy harvester is respectively higher and wider than those of the traditional linear vortex-induced vibration energy harvester. This verifies the efficiency of the new design. In the future work, the dynamic analysis and the optimization design of the vortex-induced vibrational tristable energy harvester will be explored.

Acknowledgments: This project has been supported by the National Natural Science Foundation of China (Grant Nos. 11802237 and 51606171), and the Fundamental Research Funds for the Central Universities (Grant No. G2018KY0306).

References
[1] Ulukus S, Yener A, Erkip E, Simeone O, Zorzi M, Grover P and Huang K 2015 IEEE. J. Sel. Area. Comm. 33 360-81
[2] Yang Z, Zhou S, Zu J and Inman D J 2018 Joule 2 642-97
[3] Wu N, Wang Q and Xie X 2015 Appl. Ocean. Res. 50 110-18
[4] Fan K, Tan Q, Zhang Y, Liu S, Cai M and Zhu Y 2018 Appl. Phys. Lett. 112 123901
[5] Zou H, Zhang W, Li W, Wei K, Gao Q, Peng Z and Meng G 2017 Energ. Convers. Manage. 148 1391-98
[6] Zhou S, Cao J, Erturk A and Lin J 2013 Appl. Phys. Lett. 102 173901
[7] Abdelkareem M A, Xu L, Ali M K A, Elagouz A, Mi J, Guo S, Liu Y and Zuo L 2018 Appl. Energ. 229 672-99
[8] Zhao L, Tang L and Yang Y 2013 Smart Mater. Struct. 22 125003
[9] Bryant M and Garcia E 2011 J. Vib. Acoust. 133 011010
[10] Wang J, Zhou S, Zhang Z, Yurchenko D 2019 Energ. Convers. Manage. 181 645-52
[11] Akaydin H D, Elvin N and Andreopoulos Y 2012 Smart Mater. Struct. 21 025007
[12] Goushcha O, Elvin N and Andreopoulos Y 2014 Appl. Phys. Lett. 104 021919
[13] Dai H, Abdelkefi A, Yang Y and Wang L 2016 Appl. Phys. Lett. 108 053902
[14] Zhang B, Song B, Mao Z, Tian W and Li B 2017 Energy 133 723-36
[15] Weinstein L A, Cacan M R, So P M and Wright P K 2012 Smart Mater. Struct. 21 045003
[16] Wang J, Li G, Zhang M, Zhao G, Jin Z, Xu K and Zhang Z 2018 Energ. Source. Part A. 40 2903-13
[17] Zhou S and Wang J 2018 AIP Adv. 8 075221
[18] Cottone F, Vocca H and Gammaitoni L 2009 Phys. Rev. Lett. 102 080601
[19] Daqaq M, Masana R, Erturk A and Quinn D 2014 *Appl. Mech. Rev.* 66 040801
[20] Chen L and Jiang W 2015 *J. Appl. Mech.* 82 031004
[21] Chen L, Jiang W, Panyam M and Daqaq M F 2016 *J. Vib. Acoust.* 138 061007
[22] Huang D, Zhou S and Litak G 2019 *Commun. Nonlinear Sci.* 69 270-86
[23] Zhou S, Cao J, Inman D, Lin J, Liu S and Wang Z 2014 *Appl. Energ.* 133 pp 33-9
[24] Kim P, Son D and Seok J 2016 *Appl. Phys. Lett.* 108 243902
[25] Zhou S and Zuo L 2018 *Commun. Nonlinear Sci.* 61 271-84
[26] Panyam M and Daqaq M 2017 *J. Sound Vib.* 386 pp 336-58
[27] Yan B, Zhou S, Litak G 2018 *Int. J. Bifurcat. Chaos.* 28 1850092
[28] Li H, Qin W, Lan C, Deng W and Zhou Z 2015 *Smart Mater. Struct.* 25 015001
[29] Xu P, Jin Y, Zhang Y 2019 *Appl. Math. Comput.* 346 352-62
[30] Naseer R, Dai H, Abdelkefi A and Wang L 2017 *Appl. Energ.* 203 142-53
[31] Huynh B H and Tjahjowidodo T 2017 *Mech. Syst. Signal Pr.* 85 1005-19
[32] Alhadidi A H, Abderrahmane H and Daqaq M F 2016 *Physica D.* 337 30-42
[33] Erturk A and Inman D J 2011 *Piezoelectric Energy Harvesting* (Chichester: Wiley)