ARE LARGE X-RAY CLUSTERS IN THERMAL EQUILIBRIUM?

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ABSTRACT

We simulate the formation of a large X-ray cluster using a fully three-dimensional hydrodynamical code coupled to a particle-mesh scheme that models the dark matter component. We focus on a possible decoupling between electron and ion temperatures. We then solve the energy transfer equations between electrons, ions, and neutral particles without assuming thermal equilibrium between the three gases \((T_e \neq T_i \neq T)\). We self-consistently solve the chemical equations for a hydrogen-helium primordial plasma without assuming ionization-recombination equilibrium. We find that the electron temperature differs from the true dynamical temperature by 20% at the virial radius of our simulated cluster. This could lead to a marginal underestimation of the total mass in the outer regions of large X-ray clusters.

Subject headings: galaxies: clusters: general — hydrodynamics — intergalactic medium — methods: numerical — X-rays: galaxies

1. INTRODUCTION

Large X-ray clusters are well-defined cosmological objects that can provide useful constraints on theories of structure formation currently under discussion. It is believed that they are composed mainly of dark matter and X-ray–emitting gas. The physical conditions of the hot gas are rather extreme. Common values for the electron density range from \(10^{-3}\) to \(10^{-6}\) cm\(^{-3}\) in the core to \(10^{-7}\) to \(10^{-8}\) cm\(^{-3}\) in the outer regions. The electron temperature is about 10 keV, up to 15 keV for A2163, the hottest cluster observed so far (Arnaud et al. 1992).

Recently, Markevitch et al. (1996) studied the electron temperature profile of A2163 using different X-ray experiments. They measured \(T_e \approx 4\) keV at a radius corresponding roughly to the virial radius of the cluster \(r_{200} \approx 2.4\) Mpc h\(^{-1}\). Using the well-known hydrostatic equilibrium equation to derive the total mass distribution, they concluded that the total mass distribution is significantly steeper than the X-ray gas distribution. But, as shown by Schindler & Muller (1993) and Evrard, Metzler, & Navarro (1996), the hydrostatic equilibrium assumption is unlikely to hold for low-density regions. Moreover, at that radius, the electron density is assumed to be roughly a few \(10^{-5}\) cm\(^{-3}\); the timescale for electrons to reach thermodynamical equilibrium with ions is then about 4 Gyr, comparable to the merger timescale (Markevitch et al. 1996). Therefore, one can ask the following question: in the outer region of A2163, and more generally in any large X-ray cluster, is \(T_e = T_i \approx T\)? If \(T_e \neq T_i\), this could result in additional errors in the mass estimate arising from a departure from thermodynamical equilibrium between ions and electrons.

In Teyssier, Chèze, & Alimi (1997), as a first approach to this problem, we studied the collapse of a planar density perturbation, usually called a Zeldovich pancake, of comoving wavelength \(L = 16\) Mpc h\(^{-1}\). This rather formal case enabled us to use high-resolution one-dimensional simulations to test our hydrodynamical code, which solves a set of collisional processes such as energy exchange, nonequilibrium chemistry, shock heating, and electronic conduction.

We showed that a large region of the pancake \((\approx 1\) Mpc h\(^{-1}\)) does not recover thermodynamical equilibrium. Only the central part of the pancake recovers within a few percent \(T_e \approx T_i\). The strongest departure from thermodynamical equilibrium was found near the shock front, where \(T_e\) is 1 order of magnitude lower than \(T_i\).

For a real cluster, densities and temperatures much higher than those for pancakes are expected in the high-temperature, X-ray–emitting gas. In this paper, we intend to make a quantitative study of the thermodynamical history of the gas during hierarchical formation of a three-dimensional X-ray cluster embedded in a standard cold dark matter (CDM) cosmogony. We precisely solve the energy transfer equations between electrons, ions, and neutrals, in order to confirm or rule out the thermodynamical equilibrium assumption.

To test the thermodynamical evolution of a realistic X-ray cluster, we have developed a three-dimensional hydrodynamical code, called HYDREL (hydrodynamique Euler-Lagrange), coupled to a particle-mesh (PM) scheme that describes the dark matter component. In § 2 we describe the physical processes involved in the primordial collisional plasma, the numerical methods we use to simulate the formation of X-ray clusters, and the initial conditions we choose. In § 3 we present our results, showing that significant departure from thermodynamical equilibrium can be obtained at a radius \(r \approx r_{200}\).\(^1\) We discuss the general properties of the simulated cluster, which has several average characteristics similar to those of the Coma cluster. Finally, in § 4, we discuss possible observational consequences of our work, such as an underestimation of the total mass in the outer regions of large X-ray clusters.

\(^1\) \(r_{200}\) is the radius of the sphere centered on the cluster center and containing a mean overdensity of \(\delta = 200\).
2. PHYSICAL AND NUMERICAL METHODS

In this paper, we intend to give a self-consistent description of the thermodynamical evolution of a cluster of galaxies embedded in an Einstein-de Sitter universe. We use the so-called standard CDM cosmogony, with $\Omega = 1$, $\Omega_b = 0.1$, and $h = 0.5$. This scenario is one of the most typical examples of the hierarchical clustering picture. We use the Bardeen et al. (1986) power spectrum to generate our initial Gaussian random field. In this section, we briefly review the physical processes that we study here. We also present our numerical algorithm, namely, the three-dimensional hydrodynamical code HYDREL, and finally we discuss the numerical parameters that we use for the specific realization presented here.

2.1. Physical Processes

The aim of this paper is to study the effect of several processes that we believe to be relevant in the primordial collisional plasma. We therefore focus on collisional processes and distinguish three thermodynamical species, namely, electrons, ions, and neutrals. Each species is assumed to be individually at the local thermodynamical equilibrium (LTE), but we allow where the subscripts design electrons, ions, and neutrals, respectively.

The LTE hypothesis is valid, since the isotropization timescale, $t_{\text{iso}}$, which drives the distribution function of a given species to a Maxwellian, is very small, even in the rather extreme conditions found in X-ray clusters. Indeed, from $n_e \approx n_i \approx 10^{-5} \text{ cm}^{-3}$ and $T_{e,i} \approx 10^7 \text{ K}$, we can deduce an estimate of $t_{\text{iso}} \approx 10^5 \text{ yr}$ for electrons and $t_{\text{iso}} \approx 10^9 \text{ yr}$ for ions (Spitzer 1962). On the other hand, as we outline in the introduction, the equipartition timescale between electrons and ions is quite long in the outer regions ($t_{\text{e,i}} \approx 10^9 \text{ yr}$). We must therefore solve the energy transfer equations between the three thermodynamical species. Note that in the case of X-ray clusters, the neutral component is of weak relevance, since the medium is fully ionized. However, our code was designed for a more general use, and therefore we follow self-consistently all chemical species.

For electrons and protons, the internal energy transfer per unit volume and per unit time is due to Coulomb collisions, and can be written (Spitzer 1962)

$$\frac{\delta \varepsilon}{\delta t} = -n_e n_p k(T_e - T_p) \frac{4(2\pi)^{1/2} e^4 m_e^{1/2} \ln \Lambda_{ep}}{m_p(k T_p)^{3/2}}, \quad (1)$$

where $T_{e,i} = m_e T_e + m_p T_p/c^2 + m_p$ is the reduced temperature of the two interacting particles and $\Lambda_{ep}$ is the Coulomb logarithm. This formula can be applied to other ions, with modifications to take into account their different atomic masses and charges (Spitzer 1962). We also compute the energy exchange rate between electrons and neutrals, using the classical “hard reflecting sphere” cross section

$$\sigma_{en} \approx 10^{-15} \text{ cm}^2 \quad (\text{Draine & Katz 1986}).$$

The energy exchange between ions and neutrals resulting from the resonant charge transfer interaction is derived by using the momentum transfer cross section of H $\text{I-He II}$ presented in Hunter & Kuran (1977) and Draine (1980).

To compute accurate energy exchange rates, we solve the chemical equations without assuming ionization-recombination equilibrium. We consider only 6 chemical reactions, ionization and recombination for H $\text{I-He II}$, He $\text{I-He II}$, and He $\text{II-He III}$. This nonequilibrium approach is important behind shock fronts, where a description using the Saha equation would overestimate the ionization fraction, and would lead to wrong abundances and wrong energy exchange rates. We use the chemical reaction rates presented in Cen (1992).

In rich galaxy clusters, the gas temperature ranges from 1 to 10 keV. Line cooling is therefore negligible. We only take into account bremsstrahlung (Mewe, Lemen, & van den Oord, 1986) and Compton cooling by the cosmic background radiation (CBR) (Peebles 1993). These cooling processes are likely to slightly lower the electron temperature, and therefore enhance the departure from thermodynamical equilibrium.

In Teyssier et al. (1997), we also considered the influence of electronic conduction, assuming no transverse magnetic field. We showed that conduction is effective only in the very low density outer regions, where a thermal precursor preheats the gas ahead of the shock front (Zeldovich & Raizer 1966). The downstream flow properties were qualitatively similar to the nonconductive case, although the temperature decoupling between ions and electrons was slightly lowered. This led us to conclude that even assuming no magnetic field, electronic conduction might have no direct observational consequences. On the other hand, it has been shown that a small magnetic field ($\approx 1 \mu G$) does exist in the intracluster medium (Kim et al. 1990). Electronic conduction should then be efficiently suppressed. We therefore do not consider it in the present paper.

The presence of a magnetic field does not affect classical collisional energy exchange, since this is a purely local process. However, a magnetic field also has the well-known effect of introducing various plasma instabilities within the shock structure. These instabilities are believed to be responsible for a rapid, anomalous heating of electrons. Cargill & Papadopoulos (1988) proposed a mechanism for this strong, collisionless heating, based on the Buneman and ion acoustic instabilities. This mechanism explains why electrons are efficiently heated in young supernovae remnants, an observational fact that is unexplained by the classical collisional theory. However, Cargill & Papadopoulos (1988) showed that only 12% of the upstream kinetic energy can be converted into electron thermal energy. Consequently, after the collisionless shock front, complete ion-electron equipartition still relies on classical collisional processes. In the calculations we present in this paper, although we neglect the various plasma instabilities discussed here, we obtain rather low temperature differences, namely $T_e \geq T_i/5$.

This justifies the use of classical collision theory only.

Keeping in mind the physical assumption made above, let us now summarize the thermodynamical history of the intracluster gas. Ions and neutrals are shock heated through mergers or accretion shock waves. Electrons are less efficiently heated by shocks, as can be shown by using the Rankine-Hugoniot discontinuities relations. In fact, these relations state that a gas with mean molecular weight $\mu$ is heated by a shock front with velocity $D$ up to a postshock temperature given by

$$k T = \frac{3}{16} \mu D^2 \quad (2)$$

in the limit of a very high Mach number. Just after the compression front, the temperature of the electrons is therefore much lower than temperature of the ions, $T_e \approx (m_e/m_p) T_i$, where the factor on the right-hand side is on the order of $10^{-3}$. The plasma finally reaches thermody-
namical equilibrium \((T_e \approx T_f)\) after a few ion-electron energy exchange timescales \(t_{ei}\), given by equation (1)

\[
t_{ei} \approx \frac{T_e^{3/2}}{n_e} \text{ s.} \tag{3}
\]

The length of this so-called equipartition wave, where a significant departure from thermodynamical equilibrium is expected, can be estimated using \(L_{ei} \approx (1/4)\sqrt{D}D\). Under the rather extreme physical conditions encountered in large X-ray clusters, \(n_e \sim 10^{-5}\) and \(D \sim 1000\) km s\(^{-1}\), the equipartition mean free length is very extended, \(L_{ei} \approx 1.4\) Mpc \(h^{-1}\). Moreover, one can clearly see from equation (3) that the higher the gas temperature, and the lower the gas density, the larger this equipartition region will be. In Teyssier et al. (1997), we calculated more precisely the size of this region for a pancake of initial comoving wavelength \(L = 16\) Mpc \(h^{-1}\), and we found \(L_{ei} \approx 1\) Mpc \(h^{-1}\), with a total shocked region of 2 Mpc \(h^{-1}\).

In a fully three-dimensional environment, shock waves interact in a very complicated pattern. Hierarchical merging means here that small, low-temperature substructures merge together, leading to strong shocks propagating in a low-density environment, especially in the outer regions of X-ray clusters. Consequently, we need a three-dimensional hydrodynamical code that can self-consistently solve the gas dynamic equations with the different collisional processes previously mentioned.

### 2.2. Numerical Schemes

The choice of our numerical method is dictated by the specific regions of clusters we are interested in. These regions are likely to be far from thermodynamical and chemical equilibrium. First, the low value of the gas density results in rather long collisional timescales. Second, these regions are not relaxed, having high bulk velocities and strong shock waves, making nonequilibrium phenomena dominant. We therefore use an Eulerian hydrodynamical code (Kang et al. 1994) to simulate the formation of a large cluster of galaxies.

Our code, called HYDREL, has been presented in great detail in its one-dimensional version in Teyssier et al. (1997). Here we briefly recall its main characteristics, and also detail the specific three-dimensional features. HYDREL is based on an operator-splitting algorithm, and solves the different thermo- and hydrodynamical equations in three consecutive steps. The first step, called the gravity step, solves the Poisson equation. This calculates the gravitational potential deduced from the gas and dark matter density fields. Dark matter particles are displaced during this step with a classical particle-mesh (PM) scheme (Hockney & Eastwood 1981), developed by Alimi & Scholl (1993) first on the Connection Machine, then implemented on Cray-YMP. The equations of motion in this PM code are solved in comoving coordinates, and the Green function takes into account aliasing effects and minimizes anisotropies of the force. The time integrator of the PM has, however, been modified; it is now based on a predictor-corrector scheme. This allows both great accuracy and variable time stepping, which is impossible with the classical leap-frog scheme. The second step is the adiabatic hydrodynamical step. This solves the hydrodynamical equations using directional splitting and a staggered mesh. Shock waves are treated using the pseudoviscosity method (Von Neumann & Richtmyer 1950). We use for that purpose a viscous tensor, not a viscous pressure. This tensorial formulation (Tscharnuter & Winkler 1979; Mihalas & Weibel Mihalas 1984; Stone & Norman 1992; Chieze et al. 1997) is of great importance for cosmological flows. We therefore now describe the main features of our tensorial pseudoviscosity in three dimensions.

This approach relies on the assumption that dissipation in shock waves is correctly described by the Navier-Stokes equations. For each direction \(i = 1, 2, \text{ and } 3\), we use a diagonal stress tensor, whose coordinates \(\sigma_i\) are proportional to the diagonal terms of the shear tensor,

\[
\sigma_i = \frac{P}{c_s} \Delta x \left( \frac{\partial u_i}{\partial x_i} - \frac{1}{3} \nabla \cdot \mathbf{u} \right),
\tag{4}
\]

where \(\nabla \cdot \mathbf{u}\) is the divergence of the velocity field, \(c_s\) is the local sound speed, and \(P\) is the thermal pressure. Nondiagonal terms in the usual Navier-Stokes stress tensor are dropped here, in order to avoid spurious entropy generation due to unresolved turbulent motions. Indeed, in the last formula, \(\Delta x\) is the mesh size. For a real viscous fluid, this term is replaced by the mean free path \(l\) of the gas particles, which is orders of magnitude lower than the cell size. The term “pseudoviscosity” reflects the artificial enhancement of the mean free path \((l \rightarrow \Delta x)\) as a result of the finite resolution of the grid. The viscous stress exerts on each fluid element a net force given by

\[
F_i = -\frac{\partial}{\partial x_i} \sigma_i.
\tag{5}
\]

Note that the three components of the pseudoviscous force differ in general. A viscous pressure, to the contrary, would always be isotropic. Note also that in the case of an homologous flow \((\sigma_i = 0)\), there is no dissipation in the flow (Mihalas & Weibel Mihalas 1984). A viscous pressure would not satisfy this fundamental physical requirement. Because shock waves result in a contraction of the fluid elements that they cross, the pseudoviscous force acts only when the criterion

\[
\frac{\partial u_i}{\partial x_i} \leq \frac{1}{3} \nabla \cdot \mathbf{u} < 0
\]

is fulfilled.

The hydrodynamical step we now describe is divided into two substeps for each direction; first, the gas dynamic equations are solved in a Lagrangian way, and then we remap the new flow variables from the perturbed Lagrangian grid to the fixed Eulerian one. The Lagrangian step uses a predictor-corrector time solver, which ensures second-order accuracy in time. The Eulerian step uses the Van Leer (1977) advection scheme, which ensures second-order accuracy in space.

The third step is the dissipative step. This solves all collisional processes, namely, chemical reactions, energy exchange between the three thermodynamical processes, and cooling. Because these processes are driven by rather stiff equations, we use an individual time stepping for each cell. This allows us to compute new energies and abundances with high accuracy without slowing down the whole simulation. Chemical reactions and the other collisional processes are strongly coupled equations. To ensure stability, we solve the system of chemical and thermodynamical equations using the “fully implicit” method. The large hydrodynamical time step is controlled by the Courant con-
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Fig. 1.—Top left panel: Gas density in atomic mass per cm$^3$. Top right panel: Temperatures of ions (solid line) and electrons (dashed line). Bottom left panel: Abundances of hydrogen species; fraction of H I (solid line) and H II (dashed line). Bottom right panel: Abundances of helium species; fractions of He I (solid line), He II (dashed line), and He III (dotted line). The different quantities are taken along a line of sight that crosses the cluster center.

condition, and the small sub-time steps are controlled by the relative variations of the chemical and thermodynamical variables in each cell. In the case of strong cooling ($t_{\text{cool}} < t_{\text{dyn}}$), the pressure can be dramatically underestimated. We therefore impose the condition that during the dissipative step, the total pressure does not vary by more than 10%. This method works well in general, and allows a very good accuracy in the chemical calculations. However, for large clusters of galaxies, which are the focus of this paper, cooling is not efficient because of the high temperature of the intracluster gas ($T \geq 1$ keV) and the relatively low densities that we obtain in our simulation.

2.3. Initial Conditions

We now present the initial conditions used to simulate a rich cluster of galaxies embedded in a CDM cosmogony. We use a comoving box size of 25 Mpc $h^{-1}$, with periodic boundary conditions. It would have been better to use a larger box, but we have to compromise between large-scale power and spatial resolution. This choice is similar to that of Anninos & Norman (1996), and thus we can compare our results to their calculations. We use 128$^3$ grid points, and the same number of dark matter particles. The numerical force is 50% of the true gravitational force at a scale of roughly 1.5 cells. This gives an effective resolution of 300 kpc $h^{-1}$. This also corresponds to the hydrodynamical resolution, as shown by extensive tests (Teyssier et al. 1997).

We start our simulation at redshift $z_i = 50$. The initial abundances are taken from Peebles (1993) for our chosen value of $\Omega_b = 0.1$. The gas temperature is initially uniform, and we use the relation $T(z) = (1 + z)^2 1.37 \times 10^{-2}$ K, which states that the gas temperature is strongly coupled to the CBR up to $z = 200$, and then evolves adiabatically up to $z_i$. Dark matter particles are initially uniformly distributed on the grid, and then displaced using the Zeldovich approximation. The initial baryon density field is assumed to be equal to the initial Gaussian random density field. We impose a 3 $\sigma$ peak of length scale 4 Mpc $h^{-1}$ at the center of the box, using the Hoffman-Ribak (1991) algorithm. We reach the final epoch (defined as the epoch at which the linear rms is equal to 1 at 8 Mpc $h^{-1}$) with approximately 350 time steps, controlled by the Courant condition. The energy conservation, as defined by Cen (1992), was less than 0.8% during the whole run.

3. RESULTS

The clusters we obtained at $z = 0$ have average characteristics (mass, size, and temperature) similar to those observed for the Coma cluster. We define the cluster center as the cell of maximum X-ray emissivity. We then define the virial
radius $r_{200}$ of the cluster as the radius of the sphere centered on the cluster center and containing a mean overdensity of $\delta = 200$. We find in this way $r_{200} = 1.6$ Mpc $h^{-1}$. The total mass embedded in this radius is $M_{200} = 9.8 \times 10^{14}$ $M_\odot h^{-1}$.

We plot in Figure 1 the gas density, temperatures, and ionization fraction along a line of sight that crosses the center of the cluster. We can define here three characteristic regions in the vicinity of the cluster: (1) the cluster itself, which has recovered thermodynamical and chemical equilibrium; (2) a large nonequilibrium region, where ion and electron temperatures differ significantly; and (3) the cold, unshocked intercluster medium.

Note that the accretion shock is relatively steep, and that the gas is very quickly ionized. We did not consider here any ionizing background, although it is strongly suggested by the Gunn-Peterson effect, but ionization by shock waves turns out to be efficient enough, as soon as the intracluster medium is considered. The equipartition front, where $T_e$ gradually reaches $T_e$ has a thickness of approximately 1.5 Mpc $h^{-1}$. This nonequilibrium region stands mainly outside the virial radius of the cluster. The shock front, which marks the beginning of the nonequilibrium region, is located at roughly $2r_{200}$ from the cluster center. The temperature decoupling between ions and electrons is at a maximum just after the shock front, but is always greater than $T_e \approx T_i/3$ (except “inside” the shock front). This justifies a posteriori our assumption that plasma instabilities can be neglected here (see § 2.1).

In Figure 2, we plot gray-scale images of the dark matter density contrast, the gas density contrast, and the electron and ion temperatures. Note that the filamentary structures clearly converge toward the cluster. Gas and dark matter isocontours have similar elliptical shapes, with an axis ratio of 2:1. They are both relatively smooth. On the other hand, the temperature isocontours show very complicated patterns, with several shock waves propagating in different directions. Note that the electron temperature appears to be smoother than the ion temperature. The hottest regions are located not in the center of the cluster, but in the outer regions, where strong shock heating occurs. This explains why the strongest temperature decoupling is located mainly in the low-density outer regions of the cluster.

In Figure 3 we plot the spherically averaged density and temperature profiles. The radius is expressed in units of $r_{200}$. The innermost point corresponds to our resolution limit in the computation of the gravitational force. Note
that gas and dark matter density profiles are very similar. We see no evidence of core radii in any mass distribution. Moreover, both density profiles are well fitted by a power law, $\rho \propto r^{-9/4}$. This is in good agreement with the findings of Anninos & Norman (1996), who studied the influence of numerical effects on gas and dark matter density profiles using higher resolution simulations. The electron and ion temperature profiles again show clearly that a large non-equilibrium region extends from $r_{200}/2$ to $2r_{200}$. In order to quantify the error that one observer does between the X-ray temperature $T_e$ and the true dynamical temperature $T \approx (T_e + T_i)/2$, we plot the ratio $(T - T_i)/T$ as a function of radius. The maximum departure from thermodynamical equilibrium is located at $r_{200}$ and is about 20%. We also calculate the ratio between the bulk kinetic energy and the internal energy of the gas. This ratio is equal to unity at $r_{200}$, showing that hydrostatic equilibrium is also not recovered in this region. Therefore, the hydrostatic equilibrium assumption and the thermodynamical equilibrium assumption are both valid in the central region of the cluster ($r < r_{200}/2$), but are both violated in the outer regions of the cluster ($r > r_{200}/2$).

4. CONCLUSION

In this paper, we studied the formation of a rich X-ray cluster. We found that the total mass embedded in the virial radius ($r_{200} = 1.6$ Mpc $h^{-1}$) is equal to $10^{15} M_\odot h^{-1}$. We also found that the density profiles of gas and dark matter are both well fitted by a $r^{-9/4}$ power law. We therefore reach conclusions similar to those of Anninos & Norman (1996), using similar initial conditions. We studied more specifically the thermodynamical history of the intracluster gas. We found that a significant decoupling between electron and ion temperatures occurs between $r_{200}/2$ and the shock front, located at roughly $2r_{200}$. The maximum departure is found at $r_{200}$ and reaches 20%. Therefore, the usual assumption of thermodynamical equilibrium between ions and electrons breaks down in this region. We also confirmed that the hydrostatic equilibrium assumption is not valid in the outer regions of the simulated cluster. These two errors can both lead to underestimations of the gravitational mass in the outer regions of X-ray clusters ($r > r_{200}/2$). These results could be carefully extrapolated to the case of A2163. As mentioned in the introduction, at the virial radius of A2163, Markevitch et al. (1996) measured a temperature of 4 keV, roughly twice the value we found here for our simulated cluster. This could lead to a thermodynamical decoupling of 50%, which means that the (observed) electron temperature underestimates by a factor of 2 the true dynamical temperature. Therefore, for A2163, the error in the mass estimate due to a departure from thermodynamical equilibrium could be as large as a factor of 2. Further studies are required to confirm these conclusions, using other initial conditions.

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REFERENCES

Alimi, J.-M., & Scholl, H. 1993, Int. J. Mod. Phys. C, 4, 197
Anninos, P. A., & Norman, M. L. 1996, ApJ, 459, 12
Arnaud, M., Hughes, J. P., Forman, W. J., Jones, C., Lachieze-Ray, M., Yamashita, K., & Hatsukade, I. 1992, ApJ, 390, 345
Bardeen, J. M., Bond, J. R., Kaiser, N., & Szalay, A. S. 1986, ApJ, 300, 15
Cargill, P. J., & Papadopoulos, K. 1998, ApJ, 329, L29
Cen, R. Y. 1992, ApJS, 78, 341
Chieze, J.-P., Teyssier, R., & Alimi, J.-M. 1997, ApJ, 484, 40
Draine, B. T. 1980, ApJ, 246, 1045
Draine, B. T., & Kaiz, N. 1986, ApJ, 306, 655
Evrard, A. E., Metzler, C. A., & Navarro, J. F. 1996, ApJ, 469, 494
Hockney, R. W., & Eastwood, J. W. 1981, Computer Simulations Using Particles (New York: McGraw-Hill)
Hoffman, Y., & Ribak, E. 1991, ApJ, 380, 5
Hunter, G., & Kuryan, M. 1977, Proc. R. Soc. London, 353, 575
Kang, H., Ostriker, J. P., Cen, R., Ryu, D., Hernquist, L., Evrard, A. E., Bryan, G. L., & Norman, M. L. 1994, ApJ, 430, 83
Kim, K.-T., Kronberg, P. P., Dewdney, P. E., & Landecker, T. L. 1990, ApJ, 355, 29
Mihalas, D., & Weibel Mihalas, B. 1984, Foundations of Radiation Hydrodynamics (New York: Oxford Univ. Press)

Markovitch, M., Mushotzky, R., Inoue, H., Yamashita, K., Furuzawa, A., & Tawara, Y. 1996, ApJ, 456, 437
Mewe, R., Lemen, J. R., & van den Oord, G. H. J. 1986, A&AS, 65, 511
Peebles, P. J. E. 1993, Principles of Physical Cosmology (Princeton: Princeton Univ. Press)
Schindler, S., & Muller, E. 1993, A&A, 272, 137
Spitzer, L. 1962, The Physics of Fully Ionized Gases (New York: Interscience)
Stone, J. M., & Norman, M. L. 1992, ApJS, 80, 753
Teyssier, R., Chieze, J.-P., & Alimi, J.-M. 1997, ApJ, 480, 36
Tscharnutter, W.-M., & Winkler, K.-H. 1979, Comput. Phys. Commun., 18, 171
Van de Weygaert, R., & Bertschinger, E. 1995, MNRAS, submitted (astro-ph/9507024)
Van Leer, B. 1977, J. Comput. Phys., 23, 276
Von Neumann, J., & Richtmyer, R. D. 1950, J. Appl. Phys., 21, 232
Zeldovich, Ya. B., & Raizer, Yu. P. 1966, Physics of Shock Waves and High-Temperature Hydrodynamic Phenomena (New York: Academic)