Abstract

The trace anomaly of matter in curved space generates an effective action for the conformal factor of the metric tensor in $D = 4$ dimensions, analogous to the Polyakov action for $D = 2$. We compute the contributions of the reparameterization ghosts to the central charges for $D = 4$, as well as the quantum contribution of the conformal factor itself. The ghost contribution satisfies the necessary Wess-Zumino consistency condition only if combined with the spin-2 modes, whose contributions to the trace anomaly we also discuss.
1. Introduction

In recent years there has been a great deal of interest in conformal field theory (CFT) in two dimensions. Most of this attention has focused on the strictly two-dimensional aspects of CFT, such as the existence of an infinite dimensional Virasoro algebra, characterized by a single number, the central charge. In critical string theory the matter central charges are cancelled by the \(-26\) contribution of the reparameterization ghosts, and the two dimensional metric of the world sheet decouples from the CFT matter system. Since the Einstein action in two dimensions is a topological invariant, the 2D metric has no (local) dynamics in critical string theory and may be neglected.

In non-critical string theory, the metric does not decouple and must itself treated as a dynamical degree of freedom on an equal basis with the matter in the full quantum theory. In this case the relationship of the central charge of the Virasoro algebra to the trace anomaly of 2D gravity becomes apparent. In the covariant approach to string theory initiated by Polyakov[1], the trace anomaly induces a non-local covariant effective action for 2D gravity which becomes local in the conformal parameterization (gauge),

\[ g_{ab}(x) = e^{2\sigma(x)} \bar{g}_{ab}(x). \]  

Here \( \bar{g}_{ab}(x) \) is a fiducial metric on a surface of fixed topology. The Polyakov-Liouville theory for \( \sigma(x) \) describes the dynamics of fluctuating random surfaces in two dimensions. In four dimensions, the quantum dynamics of the conformal factor may be studied in an analogous manner by constructing the effective action generated by the trace anomaly[4]. Conformal field theory techniques then provide information about the long distance behavior of four dimensional quantum gravity directly.

Proceeding by analogy with two dimensions, we begin by considering a CFT with matter fields, denoted generically by \( \phi_i \), which transform with definite conformal weights \( \alpha_i \), and described by the classical conformally invariant action,

\[ S_{cl}[e^{-\alpha_i \sigma} \phi_i; e^{2\sigma} \bar{g}_{ab}] = S_{cl}[\phi_i; \bar{g}_{ab}]. \]

At the quantum level this symmetry of the matter action is broken by the trace anomaly. The general form of the quantum trace anomaly of a CFT in curved space is:

\[ \langle T^{a} \rangle = \frac{2}{\sqrt{-g}} g_{ab} \frac{\delta S_{eff}[^{\phi_i = 0; g_{ab} = e^{2\sigma} \bar{g}_{ab}}]}{\delta g_{ab}} = bF + b'(G - \frac{2}{3} \Box R) + \zeta \frac{1}{3} \Box R \]

\[ \equiv \frac{1}{\sqrt{-g}} \frac{\delta}{\delta \sigma} \Gamma[^{\bar{g}_{ab}; \sigma}], \]

where the notation,

\[ F \equiv C_{abcd} C^{abcd} = R_{abcd} R^{abcd} - 2 R_{ab} R^{ab} + \frac{1}{3} R^2 \]
for the square of the Weyl tensor and

\[ G \equiv R_{abcd} R^{abcd} - 4 R_{ab} R^{ab} + R^2 \]  \tag{1.4}

for the Gauss-Bonnet integrand is employed. If non-zero background matter fields are considered, additional terms proportional to matter field operators, multiplied by the beta functions of the corresponding dimensionless couplings will appear in (1.2).

The coefficients \( b \) and \( b' \) have been calculated for free CFT’s in four dimensions[2]:

\[
\begin{align*}
    b &= \frac{1}{(4\pi)^2} \frac{1}{120} (N_S + 6 N_F + 12 N_V - 8) + b_G, \\
    b' &= -\frac{1}{(4\pi)^2} \frac{1}{360} (N_S + 11 N_F + 62 N_V - 28) + b'_G,
\end{align*}
\]  \tag{1.5}

where \( N_S, N_F, \) and \( N_V \) are the numbers of conformally coupled scalar, Dirac spinor, and vector gauge fields respectively. The additional contributions to these coefficients given in (1.5) are the quantum \( \sigma \) and gravitation/ghost contributions respectively which we discuss below. They are the direct analogues of the +1 and \(-26\) contributions to the central charge in the two dimensional Polyakov theory. Unlike \( b \) and \( b' \), the coefficient of the \( \Box R \) term of the anomaly is altered by the addition of a local \( R^2 \) term in the action, so that it must be treated as an additional renormalized coupling, and the \( \zeta \) coefficient is left undetermined.

An \( R^2 \) term in the anomaly is forbidden for CFT’s by the Wess-Zumino (WZ) consistency condition[3]. In the present context this is simply the statement that the variational relation (1.2) is integrable, i.e. that there exists an effective action functional \( \Gamma \), whose \( \sigma \) variation is the anomaly, such that the full effective action depends only on the complete metric in (1.1):

\[
S_{\text{eff}}[g_{ab} = e^{2\sigma} \bar{g}_{ab}] = S_{\text{eff}}[\bar{g}_{ab}] + \Gamma[\bar{g}_{ab}; \sigma]. \tag{1.6}
\]

By subjecting the background metric and conformal factor to the simultaneous transformation,

\[
\begin{align*}
    \bar{g}_{ab} &\rightarrow e^{2\omega} \bar{g}_{ab} \\
    \sigma &\rightarrow \sigma - \omega,
\end{align*}
\]  \tag{1.7}

which leaves the total metric unchanged, we immediately deduce from (1.6) that the functional \( \Gamma \) must satisfy the relation,

\[
\Gamma[\bar{g}; \sigma] = \Gamma[e^{2\omega} \bar{g}; \sigma - \omega] + \Gamma[\bar{g}; \omega], \tag{1.8}
\]

which is one form of the WZ condition. By taking two successive conformal variations of (1.6) in different orders, we arrive at a second form of the WZ condition:

\[
\Gamma[\bar{g}_{ab}; \sigma_1] - \Gamma[e^{2\sigma_2} \bar{g}_{ab}; \sigma_1] = \Gamma[\bar{g}_{ab}; \sigma_2] - \Gamma[e^{2\sigma_1} \bar{g}_{ab}; \sigma_2] \tag{1.9}
\]
From this relation, expanded to first order in $\sigma_1$ and $\sigma_2$, it follows that the local variation of the trace anomaly must yield a self-adjoint operator, symmetric in $\sigma_1$ and $\sigma_2$. One can easily check that the variation of $R^2$ yields a non-self adjoint operator (since $R \neq \Box R$), so that an $R^2$ term cannot appear in the conformal anomaly (1.2), if (1.6) is satisfied.

The existence of the relation (1.6) has another important consequence. Consider the effective $\sigma$ theory defined by $\Gamma[\bar{g}; \sigma]$ and the conformally invariant measure $[\mathcal{D}\sigma]_g = [\mathcal{D}\sigma']_g$, where $\sigma' = \sigma - \omega$. We shall show now that the local Weyl invariance under (1.7) which leaves the metric $g$ invariant and the Wess-Zumino consistency condition imply that the total conformal anomaly of matter plus ghosts plus $\sigma$ vanishes identically.

Proceeding in analogy with the two dimensional case, we begin by considering the form of the generally covariant functional measure on the (cotangent) space of infinitesimal metric deformations, $\delta g_{ab}$. This is defined by the Gaussian normalization integral,

$$\int [\mathcal{D}\delta g]_g \exp \left\{ -\frac{i}{2} \langle \delta g, \delta g \rangle_g \right\} = 1$$  \hspace{1cm} (1.10)

The quadratic inner product on the space of metric fluctuations is defined by:

$$\langle \delta g, \delta g \rangle_g = \int d^4 x \, \sqrt{-g} \, \delta g_{ab} \, G^{abcd} \, \delta g_{cd}$$  \hspace{1cm} (1.11)

which makes use of the DeWitt supermetric[9],

$$G^{abcd} = \frac{1}{2} (g^{ac} g^{bd} + g^{ad} g^{bc} + C g^{ac} g^{bd})$$  \hspace{1cm} (1.12)

at the field “point” $g_{ab}$. The form of $G^{abcd}$ is determined up to the constant $C$ by the requirement that $G^{abcd}$ be ultralocal, i.e. free of derivatives of $g_{ab}$. This definition of the functional measure on the cotangent space of metrics is generally covariant, and depends only the full metric $g$. Hence it is unchanged by the Weyl transformation (1.7). It induces a covariant definition of the functional measure on the scalar conformal subspace of metrics parameterized by $\sigma$ in (1.1). Let us define the Jacobian $\mathcal{J}$ relating this $\sigma$ dependent covariant measure $[\mathcal{D}\sigma]_g$ induced by (1.10) and (1.11) above to the translationally invariant, $\sigma$ independent measure at the point $\bar{g}$ by the relation,

$$[\mathcal{D}\sigma]_g = \mathcal{J}[\bar{g}; \sigma][\mathcal{D}\sigma]_{\bar{g}}.$$  \hspace{1cm} (1.13)

The implication of the fact that $[\mathcal{D}\sigma]_g$ depends only on the full metric $g$ is that this Jacobian must satisfy:

$$\mathcal{J}[e^{2\omega} \bar{g}; \sigma - \omega] \mathcal{J}[\bar{g}; \omega] = \mathcal{J}[\bar{g}; \sigma].$$  \hspace{1cm} (1.14)

From this it is clear that the $\sigma$ theory can be defined with respect to the translationally invariant measure $[\mathcal{D}\sigma]_{\bar{g}}$ with a new action, $\Gamma_1 = \Gamma - i \ln \mathcal{J}$ which also satisfies the Wess-Zumino condition (1.8). Therefore $\Gamma_1$ must have the same form as $\Gamma$, but with different coefficients $b$ and $b'$ in (1.2). The action $\Gamma_1$ will be determined completely once
we prove that its conformal anomaly compensates the conformal anomaly of matter
fields (and ghosts). The simplest way to prove that is the following. Consider the result
of integrating out the $\sigma$ field:

$$e^{iW[\bar{g}]} = \int [D\sigma]_{\bar{g}} e^{iS_{eff}[\bar{g}_{ab}=e^{2\sigma}\bar{g}_{ab}]}$$

$$= e^{iS_{eff}[\bar{g}_{ab}]} \int [D\sigma]_{\bar{g}_{e^{2\sigma}}} e^{i\Gamma[\bar{g}_{ab};\sigma]} \tag{1.15}$$

$$= e^{iS_{eff}[\bar{g}_{ab}]} \int [D\sigma]_{\bar{g}} e^{i\Gamma_{1}[\bar{g}_{ab};\sigma]}.$$  

Now subject this functional to the transformation (1.7), using the WZ condition (1.8)
and translational invariance of the measure $[D\sigma]_{\bar{g}}$:

$$e^{iW[e^{2\omega}\bar{g}]} = e^{iS_{eff}[e^{2\omega}\bar{g}_{ab}]} \int [D\sigma^\prime]_{\bar{g}} e^{i\Gamma_{1}[e^{2\omega}\bar{g};\sigma^\prime]}$$

$$= e^{iS_{eff}[e^{2\omega}\bar{g}_{ab}]} e^{-i\Gamma[\bar{g};\omega]} \int [D\sigma]_{\bar{g}_{e^{2\omega}}} e^{i\Gamma[\bar{g};\sigma]} \tag{1.16}$$

$$= e^{iS_{eff}[\bar{g}_{ab}]} \int [D\sigma]_{\bar{g}} e^{i\Gamma_{1}[\bar{g}_{ab};\sigma]},$$

where the substitution, $\sigma^\prime = \sigma - \omega$ and (1.8) has been used. Therefore,

$$W[e^{2\omega}\bar{g}] = W[\bar{g}], \tag{1.17}$$

and the full effective action after integration over $\sigma$ is Weyl invariant.

Thus, the WZ condition implies that the total trace anomaly must vanish. In other
words, as a consequence of the absence of diffeomorphism anomalies (which is the real
physical content of (1.6)), a form of local conformal invariance survives in the quantum
theory.

To proceed further, we notice that the $\Gamma$ satisfying the WZ condition may be found
explicitly, by directly integrating (1.2)[5]:

$$\Gamma[\bar{g};\sigma] = 2b^\prime \int d^4x \sqrt{-\bar{g}} \sigma \bar{\Delta}^{\sigma} + \int d^4x \sqrt{-\bar{g}} \left[ b^\prime \bar{F} + b^\prime (\bar{G} - \frac{2}{3} \bar{\square} R) \right] \sigma$$

$$- \frac{\zeta}{36} \int d^4x \sqrt{-\bar{g}} R^2 |_{\bar{g} = e^{2\sigma}\bar{g}_{ab}} \tag{1.18}$$

where $\Delta_4$ is the Weyl covariant fourth order operator acting on scalars:

$$\Delta_4 = \square^2 + 2R^a_{\ b} \nabla_a \nabla_b - \frac{2}{3} \square R + \frac{1}{3} (\nabla^a R) \nabla_a. \tag{1.19}$$

As in two dimensions, the $\sigma$ dependence of the non-local anomaly-induced action
becomes local in the conformal parameterization (1.1). However, it corresponds to a fully
covariant but non-local action,

$$- \frac{1}{8b^\prime} \int [bF + b^\prime (G - \frac{2}{3} \square R)] \frac{1}{\Delta_4} [bF + b^\prime (G - \frac{2}{3} \square R)] - \frac{\zeta}{36} \int R^2. \tag{1.20}$$
This action plays a role similar to the WZ action of low energy pion physics, as realized in the Skyrme model, for example. That is, it can be interpreted as an effective action at low energies, which describes all modifications to the Ward-identities due to the presence of the quantum trace anomalies. The Ward identity corresponding to the local Weyl transformation (1.7) in the effective theory guarantees that the total trace anomaly vanishes at the CFT fixed point. Contrary to the higher derivative Skyrme model, however, when the $\sigma$ action is treated as a quantum action, we find that it is ultraviolet renormalizable. Hence its renormalization group beta functions may be studied in ordinary flat space perturbation theory. The $\zeta$ coupling is infinitely renormalized, and therefore, would in general be expected to contribute an $R^2$ term to the anomaly, proportional to $\beta(\zeta)$. Since WZ consistency requires that there be no such term, general coordinate invariance of the effective theory requires that this $\beta$ function vanish, i.e. that we sit at a fixed point of the $\zeta$ coupling. In [4] we found in flat space perturbation theory that $\zeta$ has an infrared stable perturbative fixed point at

$$\zeta = 0. \quad (1.21)$$

In [4] the fiducial metric was taken to be flat, and the residual conformal Killing symmetries of flat spacetime appeared to play a crucial role in the Ward identities and fixed point condition (1.21). It now becomes evident that the fixed point conditions on $\zeta$ and the other couplings of the $\sigma$ action actually derives from the WZ condition of eq. (1.6), i.e. from the requirement that the full effective action depend only on the combination $g_{ab} = e^{2\sigma} \tilde{g}_{ab}$, for an arbitrary background satisfying the full field equations of the theory. The condition (1.21) fixes the arbitrariness present in $\zeta$, the coefficient of the local $R^2$ term in the effective action.

The effective action (1.18) involves four derivatives of $\sigma$ and raises the problem of unitarity, known to plague local higher derivative theories. However, recall that the Einstein action in its covariant form appears to contain a negative metric scalar degree of freedom, but that this “degree of freedom” is removed by the reparameterization ghosts in a covariant framework[6]. In the quartic action (1.18) there will remain one additional scalar dilaton degree of freedom not cancelled by the constraints. In order to settle the unitarity issue a study of the coordinate reparameterization constraints, derived from the energy momentum tensor of the $\sigma$ theory must be undertaken, to determine if the negative norm states drop out of the physical spectrum, as in $c > 25$ non-critical string theories[7]. The results of this investigation will be reported in a separate publication.

Once the action (1.20) or (1.18) is added to the Einstein-Hilbert action, it describes dynamics of the conformal part of the metric tensor which is quite different from the classical Einstein theory, where $\sigma$ is completely constrained by the classical equations of motion. In particular, the conformal field $e^\sigma$, which has classical scaling dimension one, receives an anomalous scale dimension $\alpha$ from the $\sigma$ loops, given by the quadratic relation,

$$\alpha = \frac{1 - \sqrt{1 - \frac{4}{Q^2}}}{\frac{2}{Q^2}}. \quad (1.22)$$
where

\[ Q^2 \equiv 16\pi^2(\zeta - 2b') = -32\pi^2b', \quad (1.23) \]

by (1.21). The anomalous scaling of the conformal factor leads to anomalous scaling of the Ricci scalar under conformal transformations as well. This observation we argued in [4] is relevant to the issue of the effective cosmological “constant” of the low energy limit of quantum gravity. The fact that the \( \zeta = 0 \) fixed point is infrared stable is essential for any application of the effective \( \sigma \) theory to long distance physics in four dimensions. The contributions to \( Q^2 \) from the \( \Delta_4 \) anomaly and reparameterization ghosts are required for any such application as well. From eqs. (1.5) and (1.23) the possibility that \( Q^2 \) could be less than or equal to four remains open. From the anomalous scaling relation, (1.22) we observe that the value \( Q^2 = 4 \) is a critical value (corresponding to \( c = 1 \) in two dimensions) at which the theory could exhibit a phase transition with qualitatively new phenomena. Thus, it would be very interesting if the quantum \( \sigma \), ghost and gravitational contributions to this central charge could be calculated reliably, for eventual application to observations of large scale structure in the universe. In this letter we describe how these missing contributions quoted in (1.5) above may be obtained.
2. The Quantum $\sigma$ Contribution to the Central Charges

We consider first the contribution of the quartic operator $\Delta_4$ itself to the coefficients $b$ and $b'$. This is equivalent to finding the modification of the effective action $\Gamma \to \Gamma_1$ due to the shift from the coordinate invariant measure on $\sigma$ induced by the definitions (1.10) and (1.11) to the translationally invariant measure $[\mathcal{D}\sigma]_{\bar{g}}$, in the language of the introduction. Notice that it makes no sense to define the Jacobian of this transformation in (1.13), without reference to the operator $\Delta_4$, since it is the trace anomaly of this operator which determines the difference $\Gamma_1 - \Gamma$ and the full effective action $W[\bar{g}]$ in (1.15). This operator in four dimensions is the precise analog of the scalar Laplacian in two dimensions which contributes to the central charge like one additional scalar degree of freedom, shifting $c_m - 26 \to c_m - 25$.

The classical Einstein and cosmological terms are soft by comparison to the quartic term in $\Gamma$, so that they cannot affect the contribution of the $\sigma$ field to $b$ and $b'$. Hence, we consider the free $\sigma$ action which is conformally invariant when treated as a classical action since $\Delta_4$ transforms covariantly under (1.1):

$$\Delta_4 = e^{-4\sigma} \bar{\Delta}_4. \quad (2.1)$$

The standard Seeley-deWitt heat kernel expansion does not apply to this fourth order operator. However, a variety of other methods have appeared in the literature to treat fourth order operators. The general algorithm for the $a_2$ coefficient in the heat kernel expansion of a minimal fourth order operator of the form,

$$\hat{F} = \Box^2 \hat{1} + \hat{D}^{(\mu\nu)} \nabla_\mu \nabla_\nu + \hat{H}^{\mu} \nabla_\mu + \hat{P} \quad (2.2)$$

is[8]:

$$a_2(\hat{F}) = \frac{1}{(4\pi)^2} \text{tr} \left\{ \frac{1}{90} (R_{abcd} R^{abcd} - R_{ab} R^{ab}) \hat{1} + \frac{1}{6} \hat{R}_{ab} \hat{R}^{ab} + \frac{1}{36} R^2 \hat{1} - \hat{P} - \frac{1}{6} \hat{D}^{ab} R_{ab} + \frac{1}{12} \hat{D} R + \frac{1}{48} \hat{D}^2 + \frac{1}{24} \hat{D}_{ab} \hat{D}^{ab} \right\}. \quad (2.3)$$

In this algorithm $\hat{1}$ denotes the unit matrix for the field upon which the operator $\hat{F}$ acts and $\hat{R}$ is the commutator of covariant derivatives in the given representation:

$$\hat{1}_B^A \phi^B \equiv \phi^A$$
$$\left( \nabla_a \nabla_b - \nabla_b \nabla_a \right) \phi^A \equiv \hat{R}_B^{A} \phi^{B}$$
$$\hat{D} \equiv \hat{D}^{\mu} \phi^A. \quad (2.4)$$

Terms proportional to $\Box R$ have been neglected here in (2.3).

The scalar operator $\Delta_4$ appearing in (1.19) is of this type with $\hat{D}^{ab} = 2 R^{ab} - \frac{2}{3} R g^{ab}$, $\hat{H}^{a} = \frac{1}{3} (\nabla^a R)$, and $\hat{P} = \hat{R} = 0$. Applying the general formula (2.3) to this case yields the anomaly coefficient in the form of (1.2) with the values of $b$ and $b'$ equal to $-8$. 

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and $-28$ respectively in scalar units, as quoted in (1.5) above. In the basis of $F$ and $G$ used in (1.2) the coefficient of $R^2$ vanishes as required by the WZ condition for the conformally covariant operator (1.19) obeying (2.1). By using this WZ condition we may check the result for the $b'$ coefficient independently by the zeta function method, as follows.

Consider the conformally flat and maximally symmetric $S_4$ with radius $H^{-1}$ and vanishing Weyl tensor, $F = 0$. The spectrum of $\Delta_4$ on this space is $n(n+1)(n+2)(n+3)H^4$ with the scalar degeneracy of $\frac{1}{3}(n+1)(n+\frac{3}{2})(n+2)$. This yields the zeta function,

$$\zeta_4(s) = \frac{1}{3} \left( \frac{\mu}{H} \right)^{4s} \sum_{n=1}^{\infty} \frac{(n+1)(n+\frac{3}{2})(n+2)}{n^s(n+1)^s(n+2)^s(n+3)^s}. \tag{2.5}$$

The value of $\zeta_4(0)$ determines the scaling behavior of the operator $\Delta_4$. It may be determined by employing a binomial expansion for each of the three factors in the denominator of the form $(n+p)^s$, interchanging the sums generated by this expansion with the $n$ sum, and performing the $n$ sum in terms of the Riemann zeta function $\zeta_R$, whose analytic properties at $s = 0$ are well known. The result of this calculation is:

$$\zeta_4(0) = \frac{1}{3} \zeta_R(-3) + \frac{3}{2} \zeta_R(-2) + \frac{13}{6} \zeta_R(-1) + \zeta_R(0) - \frac{1}{6} = -\frac{38}{45}. \tag{2.6}$$

To this we must add the one from the excluded mode at $n = 0$, and multiply the result by two to account for the fact that the fourth order operator has twice the scaling behavior under $\mu \frac{d}{d\mu}$ as a second order operator to obtain the result $\frac{28}{90}$, or $-28$ in scalar units, as stated in (1.5). The same result is obtained more rapidly by considering a general Einstein space,

$$R_a{}^b = \Lambda \delta_a{}^b, \tag{2.7}$$

which otherwise has no particular symmetry. Then the Riemann tensor squared survives as an independent contribution to the anomaly coefficient $a_2$. The quartic operator $\Delta_4$ factorizes into the product of two second order operators in this case: $\Box (\Box - \frac{R}{6})$. Since the heat kernels of these scalar operators are well known, the $b$ and $b'$ coefficients of $\Delta_4$ may be determined by simply adding the results for these two second order operators. Again the same results are obtained.

This factorization is instructive for a different reason. It shows that the quartic action may be regarded as containing a conformally coupled scalar mode from $\Box - \frac{R}{6}$ which has a negative a kinetic term, and a minimally coupled scalar from $\Box$ with a positive kinetic term. Nevertheless, the first operator contributes to both $b$ and $b'$ like one ordinary conformally coupled scalar, while it is the second operator which gives the negative contributions ($-9$ and $-29$ in scalar units) that makes the full contribution of $\Delta_4$ negative. The first mode is similar to the Liouville kinetic term in two dimensions with $c > 25$, or the scalar mode of Einstein gravity whose propagator must be cancelled by the ghosts if the theory is to be unitary. The minimally coupled scalar mode with positive kinetic term has no analog in two dimensions or the Einstein theory, and is responsible for the unusual behavior of this theory in the infrared, discussed in [4].
3. The Ghost Contribution to the Central Charges

Having determined the quantum contribution of the $\sigma$ field itself to the anomaly coefficients, we turn now to the more difficult problem of the ghost contributions to the central charge(s). If in the definition of the inner product (1.11) and covariant measure (1.10) we decompose the general metric deformation in the form,

$$\delta g_{ab} = h_{ab}^\perp + (L\xi)_{ab} + (2\sigma + \frac{1}{2} \nabla \cdot \xi) g_{ab}$$ (3.1)

where $L$ is the conformal Killing operator, mapping vectors into traceless, symmetric tensors, and defined by:

$$(L\xi)_{ab} = \nabla_a \xi_b + \nabla_b \xi_a - \frac{1}{2}(\nabla \cdot \xi) g_{ab},$$ (3.2)

leads to the following Jacobian of the change of variables to $h^\perp$, $\xi$, and $\sigma$:

$$J = \det ^\frac{1}{2} (L^\dagger L),$$ (3.3)

where $L^\dagger$ is the Hermitian adjoint of $L$ as defined by the inner product (1.11). Explicitly,

$$(L^\dagger L)^a_b = -2(\delta^a_b \Box + \frac{1}{2} \nabla^a \nabla_b + R^a_b).$$ (3.4)

The prime in (3.3) indicates that the zero modes of $L$ must be excluded from $J$ and treated separately.

A word about the relationship of this ghost determinant to that obtained by ordinary Fadeev-Popov gauge fixing is in order. In the standard approach one fixes a gauge by requiring that the space of deformations spanned by $h^\perp$ satisfy some linear condition of the form,

$$\mathcal{F}^b h^\perp_{ab} = 0,$$ (3.5)

where $\mathcal{F}$ is independent of the general gauge field to be integrated, in this case the metric. One can easily show [10] that in gauges of this form, the Jacobian of the change of variables above becomes

$$J = \det ^\frac{1}{2} (\mathcal{F} \circ \mathcal{F}^\dagger) \det (\mathcal{F} \circ L)$$ (3.6)

instead of (3.3). The determinant to the first power is the ordinary Fadeev-Popov determinant in the gauge (3.5), while the other determinant is a field independent normalization factor which may be removed outside of the functional integral over metrics. It is only when the $h^\perp$ components of the metric are required to satisfy the field dependent condition,

$$(L^\dagger h^\perp)_a = -2\nabla^b h^\perp_{ab} = 0$$ (3.7)
that the Jacobian is given by (3.3). This choice of coordinates on the configuration space of metrics is the natural, orthogonal one with respect to the inner product (1.11), but in the more standard language of gauge-fixing, it corresponds to a non-linear field dependent condition on the metric deformations \( h^\perp \). The signal of this is that the determinant in (3.3) appears to the one-half power.

Although no particular gauge choice is preferred over any other, the choice of the coordinates on field space (gauge) satisfying (3.7) is very useful in the present context. This is because of the transformation properties of the operators \( L \) and \( L^\dagger \) under the substitution (1.1):

\[
L = e^{2\sigma} L e^{-2\sigma}, \quad L^\dagger = e^{-4\sigma} L^\dagger e^{2\sigma}
\]

Hence \( L^\dagger L \) is the product of two operators each of which transform covariantly under a local conformal transformation. Using the heat kernel definition for the determinant and these transformation properties we find[6]:

\[
-\frac{1}{2} \delta \ln \text{det}'(L^\dagger L) = \text{Tr'} \frac{1}{2} \int_\epsilon^{\infty} \frac{ds}{s} e^{-sL^\dagger L} = \text{Tr'} \int_\epsilon^{\infty} ds \left\{ -2\delta\sigma L^\dagger L e^{-sL^\dagger L} + 2L^\dagger \delta\sigma L e^{-sL^\dagger L} - L^\dagger L \delta\sigma e^{-sL^\dagger L} \right\}
\]

\[
= \text{Tr'} \int_\epsilon^{\infty} ds \left\{ -3\delta\sigma L^\dagger L e^{-sL^\dagger L} + 2\delta\sigma L L^\dagger e^{-sL^\dagger L} \right\},
\]

\[
= \text{Tr'} \int_\epsilon^{\infty} ds \frac{d}{ds} \left\{ 3\delta\sigma e^{-sL^\dagger L} - 2\delta\sigma e^{-sL^\dagger L} \right\}
\]

\[
= -3\text{Tr'} \delta\sigma e^{-L^\dagger L} + 2\text{Tr'} \delta\sigma e^{-\epsilon L^\dagger L},
\]

where the cyclic property of the trace has been used repeatedly, and the lower limit of the proper time heat kernel has been regulated by \( \epsilon \), to be taken to zero in the end. Because of the explicit appearance of \( \text{Tr'} \) over the subspace of nonzero modes of \( L \), the upper limit of the evaluation of the integral in \( s \) vanishes and only the lower limit survives in (3.9).

Here an essential difference from the two dimensional case manifests itself in the appearance of the tensor operator \( LL^\dagger \) whose kernel is infinite dimensional, being spanned by all transverse, traceless graviton mode fluctuations. Unlike for \( D = 2 \), where the zero modes of \( LL^\dagger \) are countable by the Riemann-Roch theorem, and their conformal variations may be added explicitly to (3.9), in \( D = 4 \) these modes cannot be “counted” without some action over the transverse, traceless degrees of freedom \( h^\perp \). Equivalently, if we exclude these modes by restriction to the non-zero mode space of \( LL^\dagger \), then the conformal variation of the ghost operator \( L^\dagger L \) in (3.9) is necessarily non-local, and violates WZ consistency by itself. The ghost operator cannot yield a coordinate invariant effective action unless it is combined with the action for the physical graviton modes. As in the case of determining the Jacobian (1.13), this is another illustration of the fact that the anomaly coefficients (central charges) cannot be fixed by kinematic
considerations of functional measures or mode “counting” alone. Information from the differential operator(s) appearing in the Lagrangian, i. e. dynamical information about the theory is required.

To see how this works in zeta function regularization, let the two terms in the last line of (3.9) be represented by $-3\zeta(0|L^\dagger L)\delta\sigma$ and $+2\zeta(0|LL^\dagger)\delta\sigma$ respectively, for global conformal variations. The zeta function is evaluated over the non-zero mode subspaces of each operator. Since $LL^\dagger$ annihilates the trace part of the metric variation, the only tensors in the general decomposition (3.1) which survive in the second term of (3.9) are precisely those which are in the range of $L$. Since there is a one-to-one correspondence between eigenvectors $\xi^{(\lambda)}$ of $L^\dagger L$ and eigentensors of $LL^\dagger$ in the range of $L$ by the relation:

$$ (LL^\dagger)(L\xi^{(\lambda)}) = L(L^\dagger L)\xi^{(\lambda)} = \lambda(L\xi^{(\lambda)}) ,$$  

(3.10)

it follows that their zeta functions are equal:

$$ \zeta(0|L^\dagger L) = \zeta(0|LL^\dagger) .$$  

(3.11)

If $L$ and $L^\dagger$ have a finite number of zero modes these must be added explicitly to the zeta functions to find the full conformal variation of the the two terms in (3.9). Then, using (3.11), (3.9) becomes:

$$ -\frac{1}{2}\delta \ln \det(L^\dagger L) = -\zeta(0|L^\dagger L) - 3N_0(L^\dagger L) + 2N_0(LL^\dagger) ,$$  

(3.12)

in four dimensions. In $D$ dimensions the corresponding formula has $\frac{D}{2} + 1$ and $\frac{D}{2}$ replacing the 3 and 2 respectively. Since $N_0(LL^\dagger)$ is infinite for $D = 4$, this formula is meaningless as it stands, and the last term can be defined only by specifying some action for the transverse, traceless modes, which leads to its own zeta function definition of $N_0(LL^\dagger)$ from the corresponding differential operator for these modes.

The algorithm for the evaluation of the $a_2$ coefficient of the general (non-minimal) vector operator has been given by Barvinsky and Vilkovisky [8]. Applying their general formula for the operator,

$$ M^a_b = \Box \delta^a_b - \lambda \nabla^a \nabla_b + P^a_b ,$$  

(3.13)

viz.

$$ a_2(M) = \frac{1}{48\pi^2} \left\{ -\frac{11}{60}G + \frac{1}{8}\gamma^2 + \frac{1}{4}\gamma + \frac{4}{5} R_{ab} R^{ab} + \frac{1}{16}\gamma^2 + \frac{1}{4}\gamma + \frac{7}{20} R^2 ight. $$

$$ + \frac{1}{4}\gamma^2 + \gamma^2 R_{ab} P^{ab} + \frac{1}{8}\gamma^2 + \frac{3}{4}\gamma - \frac{3}{2} P_{ab} P^{ab} $$

$$ \left. + \frac{1}{8}\gamma^2 + \frac{1}{4}\gamma - \frac{1}{2} \right\} R P + \frac{1}{16} \gamma^2 P^2 ,$$  

(3.14)
to the case of $L^\dagger L$ in (3.4) with $\gamma = \frac{\lambda}{1-\lambda} = -\frac{1}{3}$ and $P_{ab} = R_{ab}$, we obtain the result:

$$a_2(L^\dagger L) = \frac{1}{(4\pi)^2} \left\{ -\frac{11}{180} R_{abcd} R^{abcd} + \frac{37}{135} R_{ab} R^{ab} + \frac{19}{108} R^2 \right\}, \quad (3.15)$$

which has a non-vanishing $R^2$ contribution when expressed in the basis of (1.2). This is because $a_2(L^\dagger L)$ is not the conformal variation of $\ln \det(L^\dagger L)$ because of the extra contribution of $N_0(LL^\dagger)$ in (3.12). The contribution proportional to $R^2$ in (3.15) should cancel against the regularized contribution from the transverse, traceless modes $h^\perp$ of a classical conformally invariant theory, when expressed in the basis (1.2).
4. The Transverse Spin-2 Contribution to the Central Charges

In order to illustrate in a concrete example how the WZ condition works and the $R^2$ contribution to the ghost anomaly in (3.15) is cancelled, we consider the Weyl-squared action for the graviton degrees of freedom. In the present context, this action has the advantage of being classically conformally invariant, so that the WZ condition may be checked explicitly. Of course, use of this higher derivative action for the graviton modes leads to perturbative non-unitarity, about which we have nothing new to add. The restriction to the one-loop contribution of this action is equivalent to imposing a self-duality constraint on the graviton degrees of freedom[13]. Our calculation will also provide an independent check on those that have appeared in the literature on the Weyl-squared theory[15].

The fourth order tensor operator for the linearized Weyl squared action may be put into the standard form of (2.2) with[12]:

\[
(\hat{D}^\mu_\nu)^{ab}_{\ cd} = (-\frac{2}{3} R g^{\mu\nu} + 2 R^{\mu\nu}) \hat{1}^{ab}_{\ cd} + \frac{4}{3} R^{ab} \delta_c^{(\mu} \delta_d^{\nu)} + 4 g^{\mu\nu} R^a_c R^b_d - 4 \delta^a_c \delta^b_d (R^\nu)^b
\]

\[
(\hat{P}^\mu_\nu)^{ab}_{\ cd} = \frac{1}{3} (R^2 - R^{\mu\nu}) \hat{1}^{ab}_{\ cd} - \frac{4}{3} RR^a_c R^b_d - \frac{4}{3} R^{ab} R_{cd} - \frac{4}{3} R R^a_c \delta^b_d
\]

\[
(\hat{R}^\mu_\nu)^{ab}_{\ cd} = R^a_c \delta^b_d + R^b_c \mu \nu \delta^a_d
\]

In order to use the algorithm (2.3), we must extend the quartic tensor operator acting on transverse traceless tensors, spanned by $h^\perp_{ab}$ to act on all traceless symmetric tensors, i.e. the union of the spaces spanned by $h^\perp_{ab}$ and $(L\xi)^{ab}$. This we may do by multiplying the expressions for $\hat{D}^T$ and $\hat{P}^T$ from the left and right by the traceless projector $\hat{1}^{(TF)}$ given by:

\[
(\hat{1}^{(TF)})^{ab}_{\ cd} = \frac{1}{2} (\delta^a_c \delta^b_d + \delta^a_d \delta^b_c - \frac{1}{2} g^{ab} g_{cd}).
\]

Then we must subtract from the $a_2$ of the resulting tensor operator the four vector modes of the operator $\mathcal{M}_V$ we have added to the transverse tensors. $\mathcal{M}_V$ is determined by the property,

\[
\mathcal{M}_T(L\xi) = L(\mathcal{M}_V)\xi,
\]

which in the case that $\mathcal{M}_T$ is given by the standard form (2.2) with the expressions in (4.1) is again of the standard form (2.2) with:

\[
(\hat{D}^\mu_\nu)^a_b = (-\frac{2}{3} R g^{\mu\nu} + 2 R^{\mu\nu}) \delta^a_b - R^a(\mu \nu)
\]

\[
(\hat{P}^\mu_\nu)^a_b = \frac{3}{2} R^{ac} R_{bc} - \frac{2}{3} R R^a_b
\]

\[
(\hat{R}^\mu_\nu)^a_b = R^a_b \mu \nu
\]
In these formulae, we have systematically neglected all terms involving derivatives of the Riemann tensor and its contractions, so that $\Box R$ contributions to the anomaly have been dropped. Then, the $a_2$ coefficient for the extended tensor operator $\mathcal{M}_T^{(TF)}$ and the vector operator $\mathcal{M}_V$ may be computed using the general quartic algorithm (2.3). After a straightforward but tedious exercise in index contraction, we obtain

$$a_2(\mathcal{M}_T^{(TF)}) = \frac{1}{(4\pi)^2} \left\{ \frac{21}{10} R_{abcd} R^{abcd} + \frac{2417}{1080} R_{ab} R^{ab} - \frac{455}{432} R^2 \right\}. \quad (4.5)$$

and

$$a_2(\mathcal{M}_V) = \frac{1}{(4\pi)^2} \left\{ - \frac{11}{90} R_{abcd} R^{abcd} - \frac{781}{360} R_{ab} R^{ab} + \frac{131}{144} R^2 \right\}. \quad (4.6)$$

Subtracting this last quantity and the result for the vector ghost operator $L^\dagger L$ of (3.15) above from (4.5) yields finally:

$$a_2(F) = a_2(\mathcal{M}_T^{(TF)}) - a_2(\mathcal{M}_V) - a_2(L^\dagger L) = \frac{1}{(4\pi)^2} \left\{ \frac{199}{30} F - \frac{87}{20} G \right\}, \quad (4.7)$$

thus verifying explicitly the cancellation of $R^2$ in the basis (1.2), as required by WZ consistency. The coefficients in (4.7) agrees with the results for the Weyl Lagrangian obtained previously by Fradkin and Tseytlin[15].

As yet another check of these calculations we may use the $\zeta$ function technique on the maximally symmetric Euclidean $S^4$ with radius $H^{-1}$. Decomposing the general vector field $\xi$ on which $L^\dagger L$ acts into its transverse and longitudinal components, we have:

$$(LL^\dagger)_a^b (\xi_b^\perp + \nabla_b \psi) = 2(-\Box - \frac{R}{4}) \xi_a^\perp + 3 \nabla_a (-\Box - \frac{R}{3}) \psi \quad (4.8)$$
on $S_4$. The transverse vector operator has eigenvalues $(n+4)(n-1)H^2$ with degeneracy $n(n+\frac{3}{2})(n+3)$. This leads to the zeta function

$$\zeta_{\perp}(s|L^\dagger L) = \left( \frac{\mu}{H} \right)^{2s} \sum_{n=2}^{\infty} \frac{n(n+\frac{3}{2})(n+3)}{(n+4)^s(n-1)^s}, \quad (4.9)$$

where the ten zero modes (Killing vectors of $S_4$) at $n = 1$ are excluded from the sum. Analytically continuing this zeta function to $s = 0$ by the technique already described above in connection with (2.5) yields:

$$\zeta_{\perp}(0|L^\dagger L) = \zeta_R(-3) + \frac{15}{2} \zeta_R(-2) + \frac{33}{2} \zeta_R(-1) + 10 \zeta_R(0) = -\frac{191}{30} \quad (4.10)$$
in agreement with ref. [12]. The scalar operator $-\Box - 4H^2$ has the same eigenvalue spectrum as the transverse vector operator on $S_4$ with the scalar degeneracy, $\frac{1}{3}(n+
1)(n + \frac{3}{2})(n + 2). The five zero modes (conformal Killing vectors of $S_4$) at $n = 1$ are excluded. Thus the total number of zero modes of $L\dagger L$ on the sphere is:

$$N_0(L\dagger L) = 15$$

(4.11)

The constant mode with negative eigenvalue at $n = 0$ is not in the spectrum of $L\dagger L$, since the gradient of a constant vanishes identically. The scalar’s $\zeta$ function,

$$\zeta_S(s|L\dagger L) = \frac{1}{3}(\mu H)^{2s} \sum_{n=2}^{\infty} \frac{(n + 1)(n + \frac{3}{2})(n + 2)}{(n + 4)^s(n - 1)^s},$$

(4.12)

evaluated by the same technique at $s = 0$ yields:

$$\zeta_S(0|L\dagger L) = \frac{1}{3}\zeta_R(-3) + \frac{5}{2}\zeta_R(-2) + \frac{37}{6}\zeta_R(-1) + 5\zeta_R(0) = -\frac{271}{90}. \quad \text{(4.13)}$$

The quartic operator $\mathcal{M}_T$ factorizes on $S_4$ into the product, $(-\square + 2H^2)(-\square + 4H^2)$ with eigenvalues $H^4n(n+1)(n+2)(n+3)$, and tensor degeneracy $\frac{5}{6}(n-1)(n+1)(n+2)(n+3)$ giving:

$$\zeta_\perp(s|\mathcal{M}_T) = \frac{5}{6}(\mu H)^{4s} \sum_{n=2}^{\infty} \frac{(n - 1)(n + 4)(2n + 3)}{n^s(n + 1)^s(n + 2)^s(n + 3)^s},$$

(4.14)

with the result,

$$\zeta_\perp(0|\mathcal{M}_T) = \frac{164}{9}, \quad \text{(4.15)}$$

which may be obtained as well by taking the sum of the zeta functions at $s = 0$ of the two factorized second order operators. Using this last value of $164/9$ as the regulated $2N_0(LL\dagger)$ appearing in eq. (3.12), we obtain the total scaling behavior of the Weyl theory on the maximally symmetric space $S_4$ (where $F = 0$):

$$-\frac{1}{2} \delta \ln \det(L\dagger L) = \frac{191}{30} + \frac{271}{90} - 3 \cdot 15 + \frac{164}{9} = -\frac{87}{5}, \quad \text{(4.16)}$$

which gives:

$$b'_C = -\frac{1}{4\pi^2} \frac{87}{200}, \quad \text{(4.17)}$$

in perfect agreement with (4.7). In units of $Q^2$ this gives $\frac{87}{10} = 8.7$ for the spin-2 contribution to the central charge appearing in the anomalous scaling relation (1.22) of the effective $\sigma$ theory.

We have considered the Weyl-squared Lagrangian primarily as a good illustrative example of the WZ condition. However, it is far from clear that this is the appropriate action to use for computing the contributions of the transverse, traceless graviton degrees
of freedom to the central charges of the effective \( \sigma \) theory in the infrared, which is the regime we may hope to apply this theory to four dimensional physics. If we attempt to use the Einstein theory for this purpose we run into several well-known difficulties. The first problem is that loop calculations in quantum gravity, as in non-Abelian gauge theory, make sense only if the the background field equations are satisfied. However, the field equations of the Einstein theory obscure the WZ condition, since they imply that any \( R^2 \) term in the anomaly is indistinguishable from \( R_{ab}R^{ab} \), on shell. One possible way out of this difficulty is to use the conformal off-mass shell extension of Fradkin and Vilkovisky[11]. More serious, of course, is the fact that the Einstein theory is non-renormalizable. Hence it is not clear that we can trust any one-loop calculation of \( a_2 \) coefficients in the Einstein theory for the \( Q^2 \) and anomalous scaling behavior we find in the effective \( \sigma \) theory, (which is renormalizable).

If we simply ignore these difficulties, the one loop calculation in the Einstein theory reduces to computing the \( a_2 \) coefficient of the Lichnerowicz laplacian,

\[
(M_T)^{ab}_{cd} = \Box \delta^{ab}_{cd} + 2 R^a_c b_d \quad (4.18)
\]

over transverse, traceless tensors. In order to use the standard algorithm for general second order operators, we again extend \( M_T \) to act on all traceless tensors, by multiplying it on both sides by the traceless projector, (4.2). Then we subtract from the \( a_2 \) of this operator the four vector modes of the operator \( M_V \) we have added to the transverse tensors. In this case \( M_V \), determined from (4.3) is given by:

\[
(M_V)^a_b = \Box \delta^a_b + R^a_b, \quad (4.19)
\]

Then the standard algorithm gives:

\[
a_2(M_T^{(TF)}) = \frac{21}{20}(R_{ab}cR^{abcd} - R_{ab}R^{ab})
\]

\[
-a_2(M_V) = \frac{11}{180}R_{abcd}R^{abcd} - \frac{43}{90}R_{ab}R^{ab} - \frac{2}{9}R^2. \quad (4.20)
\]

for the Einstein theory at one-loop order. When the ghost contribution of (3.15) is subtracted from the sum of these, and the vacuum Einstein equations, \( R_{ab} = Rg_{ab}/4 \) are used, we find:

\[
b_G = \frac{1}{(4\pi)^2} \frac{611}{120} \quad \text{and} \quad b'_G = -\frac{1}{(4\pi)^2} \frac{1411}{360} \quad (4.21)
\]

These results agree with the standard results for the Einstein theory with the conformal part removed[12]. This gives a contribution to \( Q^2 \) of 1411/180 \( \approx 7.9 \), which curiously does not differ much from that obtained in the Weyl-squared theory, \textit{viz.} 8.7

In either case, it is noteworthy that the total \( b \) and \(-b'\) coefficients are positive, and dominated by the ghost + graviton contributions, which add with the same sign
as the matter contributions. This is different from the two dimensional result that the matter and ghosts contribute to the central charge with opposite sign. Inspection of the quartic action (1.18) reveals that the ghosts behave the same way in two and four dimensions. It is the matter which behaves differently. Namely, in two dimensions the matter contributes to a negative kinetic term for the Liouville theory, but in four dimensions the matter contribution to the fourth order action is positive in Euclidean signature. Although a general proof is lacking, it seems that $Q^2$ is always positive. As already remarked above, both the Einstein and Weyl squared calculations are not entirely free of problems, so that we regard the issue of the correct contribution of the spin-2 modes to the value of $Q^2$ in the infrared as still open, and deserving of further study.

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