Statistical Bias in D-Wave Qubits

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Abstract. Recent access to commercially available quantum computing capabilities offers the potential for testing a host of new algorithms that take advantage of fundamentally different processing approaches. One such computing resource is D-Wave’s quantum annealing computer, which employs an energy minimization-based computing mechanism to find solutions to problems stated in an Ising formulation. These solutions, however, must be interpreted in terms of Monte Carlo-style probabilistic occurrences and histograms of candidate solutions that presume the underlying statistics of the qubits are “fair.” This paper quantifies the underlying fairness of individual qubits on a single D-Wave machine, showing that there is a measurable bias in the resulting solution as a function of which qubits are used in the calculation. Further, an example problem is demonstrated, whereby the same problem formulation yields distinctly different solution probabilities by changing which physical qubits are employed.

1. Introduction

The field of quantum computing is advancing rapidly and the impact on national security cannot be understated. It is not a question of “if” but one of “when” will quantum computers be available to solve real-world problems. Quantum computing is the next generation technology that exploits the principles of quantum theory that describes the nature of matter and energy at the very smallest scales of space, time, and temperature. It is based on non-intuitive quantum-mechanical phenomena, such as superposition and entanglement, to create entirely new classes of computing machinery that, in time, will allow for exponentially more computing power, giving us the ability to solve a broad class of classically intractable problems. At this early stage of development it is critical to have the knowledge and understanding of the machinery and to begin developing the experience in creating algorithms customized to the quantum hardware. Often, quantum computers fall into two broad categories – gate-based and annealing. Programming of these different architectures are incompatible. That is, algorithms developed for one architecture, say annealing, are generally not applicable to gate-based architectures. This paper will focus on quantum annealing machines, specifically in the ability to identify the minimum energy solution to a formulated problem in the presence of a bias in the underlying qubit statistics. This becomes critical when annealing machines are used to solve real-world problems. [1] [2]

The technique of simulated annealing was first proposed using thermal fluctuations to solve optimization problems. [3] The core idea is to specify a system in which the problem to be optimized is represented by an “energy function” and then use thermal fluctuations of the optimization variables to search for globally minimal solutions. The thermal fluctuations allow the system to escape from local minima to reach the global minimum. Quantum effects have been found to play a very similar
role to thermal fluctuations and has allowed for the development of a new class of computing hardware based on adiabatic quantum annealing [4]. Quantum annealing is a technique that exploits a time-dependent Hamiltonian for a set of $N$ qubits by gradually decreasing quantum fluctuations [5]. The slow progression of the quantum energy state allows the system of qubits to traverse the energy barriers to find the global minimum of the high-dimension, complicated, energy function. The annealing process begins in a state with all qubits in superposition. The system is slowly evolved from the initial Hamiltonian to the desired problem Hamiltonian. If the rate of change of the Hamiltonian is slow enough, the system of qubits stays close to the ground state of the instantaneous Hamiltonian, essentially “tracking” the changing Hamiltonian. At the completion of the annealing, the goal of the system is to end with the qubits in the ground state of the desired problem Hamiltonian, representative of the overall minimum energy state and therefore global optimum. [6] [7] [8].

The problem Hamiltonian implements an Ising formulation. Let $q_i \in \{-1, +1\}$ represent the value of qubit $i$ then the energy, $E_{\text{Ising}}$ for a collection of qubits

$$Q = [q_1, q_2, \ldots, q_N]E_{\text{Ising}} = \sum_{i=1}^{N} h_i q_i + \sum_{i,j=1}^{N} J_{ij} q_i q_j$$

Current D-Wave machines [9] utilize superconducting loops in which the current flow direction represents the state of a qubit while controllable magnetic fields represent the bias ($h_i$) and coupling ($J_{i,j}$) terms. The annealing process then returns the vector of qubit values, $Q$, that minimizes Equation 1 That is, it returns $Q$ where

$$\arg\min_{Q} \{ \sum_{i=1}^{N} h_i q_i + \sum_{i,j=1}^{N} J_{i,j} q_i q_j \}$$

Inherent in this formulation is that the underlying ground state probabilities of the qubits, $q_i$, themselves are unbiased. That is, each qubit in the absence of an applied bias, $h_i$, will be equally likely to be observed as a $-1$ or $+1$ when measured; this (hopefully, time-invariant) state observation is comparable to a coin flip, where any deviation in distribution from a uniform 50% probability will be observable as a shift in solution values. Unlike a standard coin flip, however, statistical bias in a qubit may have a significant impact by driving otherwise equiprobable decisions in a specific direction.

In this paper, we will be investigating the underlying statistics of individual qubits in the absence of a driving bias. In digital systems, this would be much like measuring a difference in logical/binary decisions as the results of processing the inputs with two different identically configured sets of combinational logic. The core approach and description of our testing setup is described in Section 2. Results describing the “fairness” of the qubits in aggregate are provided in Section 3, followed by detailed analyses of fairness of individual qubits in Section 4 and the time dependence of the bias in Section 5. A concrete example of the impacts of this fairness in a chosen Ising formulation are provided in Section 6 to demonstrate how the bias can contribute to a non-optimal solution. Finally, conclusions and future work are described in Section 7.

2. Experimental Setup

Ideally, a broad survey of existing quantum annealing machines would allow for the identification of design or manufacturing variations in the annealing chips. Given the limited production volume of the D-Waves, as well as their general inaccessibility, a study of a single machine was undertaken. Here, the D-Wave 2000Q in Burnaby, Canada, was exercised. It was accessed via D-Wave Ocean SDK [10] and the latest DW_2000Q_2_1 solver [11] from custom Python scripts. This particular machine has a nominal 2048 qubits, of which 10 are inoperative, leaving 2038 individual qubits to evaluate. This paper will focus its investigation into the underlying probabilities of individual qubits on the Burnaby machine.

Since the D-Wave machine by nature computes a Monte Carlo-style probabilistic occurrence or histogram of candidate solutions, the D-Wave software automatically provides a mechanism to receive multiple “reads” for each Hamiltonian solution. Each read represents a complete annealing cycle of
setting the initial, empty Hamiltonian, annealing by transitioning to the problem Hamiltonian, then reading the resulting solution vector, $Q$. For this experiment multiple annealing cycles of the problem Hamiltonian are done. Each such “trial” includes specifying the same desired problem Hamiltonian and multiple reads.

If we assume no applied bias, $h_i = 0$ and no coupling to any other qubit, $J_{i,j} = 0$ for all $j$ then the qubit $q_i$ is in isolation and we would expect, ideally, the probability of measured qubit values to be identical. That is, $P(-1) = P(+1) = 0.5$. The question now becomes, is the D-Wave qubit a fair coin? Consider Figure 1. Here a single trial of 1000 reads of all 2048 qubits was executed with no bias and no coupling.

A few items of note. First, there are, on this particular machine, 10 “dead” qubits. That is, those 10 qubits are mapped out by D-Wave and are not to be used in any problem. The dead qubits are reported here as having a value of 0 (not from the allowed set of $q_i \in \{-1, 1\}$) and are visible as solid vertical bars in Figure 1. Second, notice that there is vertical structure to the read qubit values. That is, there is an apparent trend for some pixels to be more $-1$ (here colored gray) which manifests itself as a grayish vertical streak in Figure 1. It is this hint at a bias which we will pursue in the following sections.

![Figure 1](image)

**Figure 1.** Qubit values (horizontal axis) for a single “trial” of 1000 “reads” (vertical axis) with the qubit mean (bottom).

### 3. Qubits in Aggregate

Before investigating individual qubits, let's consider the fairness of all of the qubits on the D-Wave 2000Q chip. Multiple trials were run wherein the D-Wave Ocean SDK was used to execute the DW_2000Q_2_1 solver with no applied bias or couplings and N “reads”. Each read involves a clear,
load, anneal, read cycle with the expectation that the clear is complete and, therefore, subsequent trials from the annealing are independent. We will assume that each measurement of any given qubit, q$_i$, is statistically independent, that all qubits are independent, and that the underlying probabilities, P(−1, +1), are constant. Suppose we take N qubit measurements. If we measure $n_+$ values of +1 and $n_-$ values of −1 (where $n_+ + n_- = N$) and assume a uniform prior probability on [0,1], then the probability of getting $n_+$ + 1 values is

$$P(n_+ \mid r, N) = \begin{pmatrix} N \\ n_+ \end{pmatrix} r^{n_+} (1 - r)^{n_-}$$  \hspace{1cm} (3)$$

where $r$ is the probability of observing a +1 on the qubits. The best estimator $\hat{r}$, for the actual value of $r$ is

$$\hat{r} = \frac{n_+}{N}$$  \hspace{1cm} (4)$$

Assuming the probability of a +1 is near 0.5, then the standard deviation of error is roughly $\frac{1}{2\sqrt{N}}$ and the estimator of Equation 4 has a maximum error [13], $E$, where $|\hat{r} - r| < E$ given by

$$E = Zsp = \frac{Z}{2\sqrt{N}}$$  \hspace{1cm} (5)$$

where $Z$ is the Z value for the desired level of confidence and is the $1 - \frac{\alpha}{2}$ quantile of a standard normal distribution corresponding to the target error rate $\alpha$. This gives us an estimate of the true underlying probability, $r = P(+1)$.

Suppose we perform $T$ trials each of $M$ annealing reads and extract the values of all available qubits (2038 on this machine). This would provide us with $N = T \cdot M \cdot 2038$ qubit measurements. Using Equations 4 and 5 will provide an estimate of the fairness of the qubits across the entire chip. Ideally, $P(+1) = r \approx \hat{r} = 0.5$. In this case a very large number of measurements are available, therefore the accuracy should be very high. We will choose a confidence level of 99.999%, or $Z = 4.4172$ in Equation 5.

| Table 1. Trial results for 2000Q aggregate qubits with confidence level of 99.999% |
|---|---|---|---|---|
| $T$ | $M$ | $E$ | $\hat{r}$ | $\hat{r} \pm E$ |
| 50 | 10000 | 6.9188e-05 | 0.50038 | 0.50031 – 0.50044 |

Table 1 shows the results of 50 trials each with 10000 anneals. Each anneal of each trial provides 2038 measurements of a qubit or $N=1,019,000,000$ measurements. The resulting estimate of $P(+1)$ is 0.50038 and we are 99.999% certain that the real value lies between 0.50031 and 0.50044. In terms of coin flips, the Burnaby 2000Q is nearly fair, much better than the 1% bias of a traditional coin flip [14]. In terms of problem Hamiltonian and solution accuracy, however, this may be insufficient. To help quantify that impact, the next two sections seek to quantify the variation of individual qubits within the population of 2038 on the chip, while the following section evaluates time dependence of the potential bias.

If the 2000Q were perfectly unbiased the number of qubits read as −1 should exactly equal those read as+1. Noise in the system, however, would preclude such an exact reading. An estimate of the noise can therefore be had by looking at the sum of all qubits for a given trial.

4. Individual qubit fairness
Rather than aggregating all $N = T \cdot M \cdot 2038$ qubit measurements into a single fairness measure, let's instead look at the fairness of individual qubits. Here we run $T$ trials of $N$ reads each but compute the fairness only over the measurements for a select few bits.
Table 2. Qubit fairness for individual bits with 2000000 measurements, confidence level of 99.999

| Bit | \( n_+ \)  | \( n_- \)  | E    | \( \hat{p} \) | \( \hat{p} \pm E \) |
|-----|------------|------------|------|--------------|------------------|
| 0   | 945033     | 1054967    | 0.0016 | 0.4725      | 0.4710 – 0.4741  |
| 100 | 945033     | 1076563    | 0.0016 | 0.4617      | 0.4602 – 0.4633  |
| 1234| 1032539    | 967461     | 0.0016 | 0.5147      | 0.5147 – 0.5178  |
| 1965| 852570     | 1147430    | 0.0016 | 0.4263      | 0.4247 – 0.4278  |

Table 2 shows the results of 200 trials of 10000 anneals which yields 2000000 measurements of each qubit. While the qubits in aggregate are quite fair \( (P(\neg 1) \approx Pr(+1) \approx 0.5) \), the individual qubits are not, themselves, fair. Consider, for instance, qubit 0. Here the estimate of the probability \( P(+1) \) is 0.4725 with a 99.999\% confidence interval of 0.4710 to 0.4741. Qubit 1234, on the other hand, has an estimated probability \( P(+1) \) of 0.5163 with a 99.999\% confidence interval of 0.5147 to 0.5178. Clearly these two qubits are biased: qubit 0 biased toward -1, qubit 1234 biased toward +1. Beyond these randomly selected qubits, a histogram of the bias from 50\% is shown in Figure 2.

Figure 2. Histogram of the sums of all qubit values for each of 100,000 reads of the chip with Gaussian fit \( \mu =-1.23, \sigma =45.4 \)

5. Fairness time dependence
Section 4 showed that over very long time periods, individual qubits demonstrate a bias. Unfortunately, when problems are submitted a relatively small number of annealing cycles are used to identify a solution – significantly less than the 2 million used in the generation of Table 2. The behaviour of a qubit bias over shorter time frames, therefore, may impact the individual annealing runs and, therefore, the return problem solutions. To help quantify that time variance, Table 3 captures 10 successive trials of bit 100, with each trial consisting of 10000 reads.
Table 3. Trial results for bit 100 with 10000 reads per trial, confidence level of 99.0

| Bit | $n_+$ | $n_-$ | $\hat{r}$ | $\hat{r} \pm E$ |
|-----|-------|-------|-----------|-----------------|
| 1   | 5001  | 4999  | 0.5001    | 0.4872 – 0.5130 |
| 2   | 5407  | 4593  | 0.5407    | 0.5278 – 0.5536 |
| 3   | 4232  | 5768  | 0.4232    | 0.4103 – 0.4361 |
| 4   | 4998  | 5002  | 0.4998    | 0.4869 – 0.5127 |
| 5   | 4820  | 5180  | 0.4820    | 0.4691 – 0.4949 |
| 6   | 5561  | 4439  | 0.5561    | 0.5432 – 0.5690 |
| 7   | 4845  | 5155  | 0.4845    | 0.4716 – 0.4974 |
| 8   | 4614  | 5386  | 0.4614    | 0.4485 – 0.4743 |
| 9   | 4636  | 5264  | 0.4636    | 0.4507 – 0.4765 |
| 10  | 4676  | 5324  | 0.4676    | 0.4547 – 0.4805 |

Although a small set of observations, these results suggest a time varying nature to the biases, given each of the short-term metrics are mostly non-overlapping. We anticipate that this time variance may lend itself to a future calibration process that reduces the bias, yet it also indicates that the coin flip “fairness” is non-stationary.

6. Qubit fairness impacts

Sections 4 and 5 shown that individual qubits have a time-dependent bias. The question is what impact does that bias have on a given solution. To investigate this, consider an extremely simple two variable binary minimization problem of the form

$$E = 0.1q_1 - 0.1q_2 + 2.4q_1q_2$$

(6)

Wherein $q_1 \in \{0, 1\}$. For the two qubits, $q_1$, $q_2$, there are four possible solution energies to Equation 6 enumerated in Table 4.

Table 4. Energy solutions for simple 2-variable binary minimization problem

| $q_1$ | $q_2$ | $E$  |
|-------|-------|------|
| 0     | 0     | 0.0  |
| 0     | 1     | -0.1 |
| 1     | 0     | 0.1  |
| 1     | 1     | 2.4  |

Three of the solutions are closely spaced while the fourth is significantly different. The quantum annealing technique that the D-Wave exploits ideally should find the minimum energy solution no matter how closely spaced. Noise and interference in the system, as well as a bias in the qubits being used will impact the identified solutions. Therefore, a large number of “reads” were performed in order to ensure the minimum energy solution is found.

Consider a single trial finding the minimum $E$ for Equation 6 in which 100 reads of the solution are performed using two physical qubits 635 and 639 for the logical qubits $q_1$ and $q_2$. The selection of these specific qubits are not specifically relevant: they were simply chosen by D-Wave’s automatic embedding algorithm “minorminer” as being suitable for the two-variable problem. Ideally, we would see 100% or 1000 returned solutions of (0, 1). Instead, the return solution distribution is as seen in Table 5.
Table 5. Energy solution distribution using 635 for \(q_1\) and 639 for \(q_2\)

| Solution \((q_1, q_2)\) | E   | Count | Percent |
|------------------------|-----|-------|---------|
| \((0,1)\)             | -0.1| 62    | 62%     |
| \((0,0)\)             | 0.0 | 33    | 33%     |
| \((1,0)\)             | 0.1 | 5     | 5%      |

In Table 5, the minimum energy solution is found roughly 60% of the time with non-optimal solutions found the other 40%. Direct measurement of the qubit bias, as per Section 4, for those two qubits immediately before and after the above minimization run showed that qubit 635 (i.e., the physical qubit for \(q_1\)) was relatively small, with \(\hat{r} = 0.499\), while qubit 639 (i.e., the physical qubit for \(q_2\)) had a net bias toward 0 with \(\hat{r} = 0.470\). This bias in physical qubit used to represent \(q_2\) has shifted the result distribution of non-optimal solutions. As an additional demonstration, consider the effects of swapping the physical qubits used. The results are dramatically improved as shown in Table 6.

Table 6. Energy solution distribution using 639 for \(q_1\) and 635 for \(q_2\)

| Solution \((q_1, q_2)\) | E   | Count | Percent |
|------------------------|-----|-------|---------|
| \((0,1)\)             | -0.1| 83    | 83%     |
| \((0,0)\)             | 0.0 | 15    | 15%     |
| \((1,0)\)             | 0.1 | 2     | 2%      |

Here, the slight bias toward 0 of qubit 639 had much less impact on the solution since \(q_1 = 0\) is part of the optimal, global minimum solution. It is noteworthy that in this very simple case there was no trial in which the annealing failed to find the minimum energy solution. In problems involving more than two qubits, however, instances were observed in which a trial of thousands of runs failed to find the known lowest energy solution.

7. Conclusions and Future Work

These preliminary experiments demonstrate a number of methods to quantify fairness in quantum annealing computational systems. Through this work, we have shown that the individual qubits exhibit a measurable bias within the larger population of qubits, suggesting that effort must be taken to either understand and incorporate the amount of that bias into our Ising formulations or to perform a time-dependent calibration of the qubits used to minimize or select biases that minimize differences in the solutions. The observed bias appears to be relatively small when taken as a large population, yet conditioning upon individual qubits or short-term evaluations of a qubit each exhibit material variations. Ongoing work is being performed to isolate and quantify the actual behaviours underlying these qubit biases, ultimately aiming to translate that sensitivity analysis into an actionable design guide for the programmer.

8. References

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