Scaling law for the electromagnetic form factors of the proton

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Abstract

The violation of the scaling law for the electric and magnetic form factors of the proton is examined within the cloudy bag model. We find that the suppression of the ratio of the electric and magnetic form factors is natural in the bag model. The pion cloud plays a moderate role in understanding the recent data from TJNAF.

The description of the electromagnetic structure of the nucleon requires two independent form factors. The usual Sachs form factors fully characterize the charge and current distributions inside the nucleon. Electromagnetic probes interact not only with valence quarks confined inside a quark core by nonlinear, gluon dynamics but also with the pion field required by chiral symmetry. A full understanding of the electromagnetic structure of the nucleon is of fundamental importance.

Historically, experimental determination of the electric \(G_{Ep}\) and magnetic \(G_{Mp}\) form factors of the proton was mainly based on the Rosenbluth separation \(\Pi\) of the unpolarized differential cross section data. The results of various analyses from the early experiments are summarized in a simple scaling law,

\[ G_{Ep}(Q^2) = G_{Mp}(Q^2)/\mu_p = G_D(Q^2), \]  

(1)
for momentum transfers, \(Q\), up to several GeV. Here \(\mu_p\) is proton’s magnetic moment and \(G_D(Q^2)\) refers to the standard dipole form. However, from the Rosenbluth formula one sees that at large momentum transfer, the electric contribution to the cross section is kinematically suppressed relative to the magnetic contribution. Thus \(G_{Ep}\) can not be determined as accurately as \(G_{Mp}\) from such an analysis, especially at large \(Q^2\). In the literature, the ratios, \(\mu_p G_{Ep}(Q^2)/G_{Mp}(Q^2)\), obtained from different experiments are not consistent with each other, within the quoted errors.

With the advance of polarization technologies and the operation of high-duty electron machines, it is now possible to drastically reduce the systematic uncertainties in this ratio by direct measurement. That is, one can simultaneously measure the two components of the recoil proton polarization, \(P_T\) and \(P_L\), using a longitudinally polarized electron beam [2]. As the transverse component behaves as \(P_T \sim G_{Ep}G_{Mp}\), and the longitudinal component as \(P_L \sim G_{Mp}^2\), the ratio, \(\mu_p G_{Ep}/G_{Mp}\), can be determined directly from \(P_T/P_L\).

Precise data for the nucleon electromagnetic form factors sets a strong constraint for various quark models of the nucleon. It helps us to develop an understanding of the composite nature of the nucleon as well as its long range chiral structure. In our previous work, we studied the nucleon electromagnetic form factors in an improved cloudy bag model (CBM) [3], where the center-of-mass motion correction and relativistic effects were treated explicitly. As a result, the region of validity of the calculation of the electric and magnetic form factors in the model was extended to larger momentum transfer than had previously been possible. Here we focus on the ratio, \(\mu_p G_{Ep}(Q^2)/G_{Mp}(Q^2)\), and examine the mechanism for the violation of the scaling law, Eq. (1).

Let us start with the MIT bag model [4]. Under the static cavity approximation, the bag surface is spherical and all valence quarks are in the lowest eigenmode. The electric and magnetic form factors for the proton can be written as

\[
G_{Ep}(Q^2) = \int_0^R 4\pi r^2 dr j_0(Qr)[g^2(r) + f^2(r)], \tag{2}
\]

\[
G_{Mp}(Q^2) = 2m_N \int_0^R 4\pi r^2 dr \frac{j_1(Qr)}{Q}[2g(r)f(r)], \tag{3}
\]
where $R$ is the bag radius, $Q^2 = -q^2 = \vec{q}^2$ with $\vec{q}$ the three momentum of the photon in the Breit frame, and $j_l(x)$ refers to the spherical Bessel function. For a massless quark, the quark wave functions are given as $g(r) = N_S j_0(\omega_S r/R)$ and $f(r) = N_S j_1(\omega_S r/R)$, with $\omega_S = 2.04$ and $N_S^2 = \omega_S/8\pi R^3 j_0^2(\omega_S)(\omega_S - 1)$.

At small $Q^2$, the ratio $\mu_p G_{Ep}(Q^2)/G_{Mp}(Q^2)$ is determined by the electromagnetic root-mean-squared (r.m.s.) radius, which is defined as follows: $G_{E,M}(Q^2) \sim 1 - Q^2 \langle r^2 \rangle_{E,M}/6$, as $Q \to 0$. A direct evaluation gives

$$\langle r^2 \rangle_{Ep} = 4\pi N_S^2 \left( \frac{R}{\omega_S} \right)^5 \frac{\omega_S^2 (2\omega_S^3 - 2\omega_S^2 + 4\omega_S - 3)}{3(2\omega_S^3 - 2\omega_S + 1)},$$

$$\langle r^2 \rangle_{Mp} = \frac{8\pi N_S^2}{5\mu_p} \left( \frac{R}{\omega_S} \right)^6 \frac{\omega_S^2 (8\omega_S^3 + 10\omega_S^2 - 20\omega_S + 15)}{24(2\omega_S^3 - 2\omega_S + 1)}.$$ (4)

Note that $\mu_p = R(4\omega_S - 3)/12\omega_S(\omega_S - 1)$, thus the ratio $\langle r^2 \rangle_{Ep}/\langle r^2 \rangle_{Mp}$ is independent of $R$. It results in 1.36 using $\omega_S = 2.04$. This means that the charge r.m.s. radius is considerably larger than the magnetic r.m.s. radius. In other words, $G_{Ep}(Q^2)$ decreases faster than $G_{Mp}(Q^2)$ at small momentum transfers, so the suppression of $\mu_p G_{Ep}(Q^2)/G_{Mp}(Q^2)$, as $Q^2$ increases, is a natural consequence of the MIT bag model.

The results of our numerical calculations are shown in Fig. 1, where $R = 0.8$ fm is used. The long-dashed line is the ratio in the static MIT bag model. Two lowest curves are respectively electric and magnetic form factors of the proton within the MIT bag model. The experimental data are from the recent measurement at TJNAF [2]. Note that the naive MIT bag model for the nucleon is only sensible in the region of very small momentum transfers. The ratio in a static bag calculation deviates from the data very quickly as $Q^2$ increases. The singularity in the long-dashed curve comes from a node in $G_{Mp}(Q^2)$, which is related to the sharp bag surface, and will be extremely sensitive to small admixtures in the nucleon ground state wave function. Such admixtures may be induced by residual one-gluon exchange or meson exchange interactions [3]. More sophisticated models, such as the color dielectric and soliton bag models [4], would also help to remove the zero (or at least move it to larger $Q^2$), thus removing the singularity in the ratio.

As the bag model is inherently an independent particle model, the spurious center-of-mass
motion must be removed in a realistic calculation. We have incorporated this correction by constructing momentum eigenstates using the Peierls-Thouless projection method [3]. Such wave functions are consistent with Galilean translational invariance. In addition, since the static solution of the quark wave function is spherical, the calculated form factors are trustworthy only at very small momentum transfer. To account for relativistic effects we use the prescription proposed by Licht and Pagnemma [6]. The technical details for implementing such corrections can be found in Ref. [3]. The resulting ratio for the quark core of the bag model, after including the center-of-mass motion corrections and relativistic effects, is plotted as the solid curve in Fig. 1. The prominent zeros in the form factors are forced out much further and do not occur in the momentum region we are studying. Compared with the recent data, the improvement over the naive MIT bag model is significant, in particular, away from the region of small momentum transfers.

In the cloudy bag model the physical nucleon is a superposition of a quark core and a quark core plus a meson cloud [7]. We keep only the dominant pion terms and treat them in the one loop approximation. Inside the pion loop, the intermediate baryon is either a \( N(939) \) or a \( \Delta(1232) \). Thus the electromagnetic form factors receive contributions from the pion current in addition to the direct photon coupling to the confined quarks in the core. For more details and the explicit expressions for the form factors, we refer the readers to Eqs.(36-40) and Eqs.(A5,A6) in Ref. [3].

In our calculations the \( \pi NN \) coupling constant is taken to be \( f_{\pi NN}^2 = 0.0791 \). Other coupling constants are fixed by SU(6) symmetry. The resulting ratios in the full calculations are presented in Fig. 2. Here the solid and long-dashed curves are respectively for massless and 10 MeV quarks in the nucleon bag, using a bag radius of \( R = 0.8 \) fm. Clearly the ratio is insensitive to the quark mass variations. The addition of the pion cloud for \( R = 0.8 \) fm increases the ratio by roughly 20\% towards the recent data. In the same figure, the dot-dashed and dashed curves have similar meanings but with \( R = 0.7 \) fm. This is about the smallest bag radius which allows a perturbative treatment of the pion cloud. The recent data indicate that a smaller bag radius is preferred.
One feature of our calculation is that the decline of the ratio of the Sachs form factors slows down as $Q^2 > 2$ Gev$^2$. The corresponding ratio of the Pauli and Dirac form factors $Q^2F_{2p}/F_{1p}$ shows a rapid rise at small $Q^2$ (below 2 GeV$^2$) and seemingly become flat at large $Q^2$. This is consistent with the prediction of pQCD, which claims $F_{1p} \sim 1/Q^4$ and $F_{2p} \sim 1/Q^6$, but is not obvious in other nucleon models \cite{10-13}.

In summary, we have studied the ratio, $\mu_p G_{Ep}(Q^2)/G_{Mp}(Q^2)$, in an improved quark model. We find that the sharp decline of the ratio as $Q^2$ increases is natural in a quark bag model, so the empirical scaling law for the nucleon electromagnetic form factors is unavoidably violated. The deviations of the ratio, $\mu_p G_{Ep}(Q^2)/G_{Mp}(Q^2)$ from unity can be understood semi-quantitatively within the cloudy bag model.

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FIG. 1. The ratio, $\mu_p G_{Ep}(Q^2)/G_{Mp}(Q^2)$, calculated from the quark core of the bag model. The solid curve is the result after the center-of-mass motion correction and a semi-classical relativistic correction. The data are from the recent measurement at TJNAF [2].
FIG. 2. The ratio, $\mu_p G_{Ep}(Q^2)/G_{Mp}(Q^2)$, from full calculations in the improved cloudy bag model. Results with different bag radius and quark masses are also presented. The data are from the recent measurement at TJNAF [2].