Variable $G$ Correction for Dark Energy Model in Higher Dimensional Cosmology

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Abstract: In this work, we have considered $N (= 4 + d)$-dimensional Einstein field equations in which 4-dimensional space-time which is described by a FRW metric and that of the extra $d$-dimensions by an Euclidean metric. We have calculated the corrections to statefinder parameters due to variable gravitational constant $G$ in higher dimensional Cosmology. We have considered two special cases whether dark energy and dark matter interact or not. In a universe where gravitational constant is dynamic, the variable $G$-correction to statefinder parameters is inevitable. The statefinder parameters are also obtained for generalized Chaplygin gas in the effect of the variation of $G$ correction.

I. INTRODUCTION

Cosmological observations obtained by various cosmic explorations of supernova of type Ia [1], CMB analysis of WMAP data [2], extragalactic explorer SDSS [3] and X-ray [4] convincingly indicate that the observable universe is experiencing an accelerated expansion. Although the simplest and natural solution to explain this cosmic behavior is the consideration of a cosmological constant [5], however it leads to two relevant problems (namely the “fine-tuning” and the “coincidence” one). Recently new dynamical nature of dark energy are considered in the literature, at least in an effective level, originating from various fields, including a canonical scalar field (quintessence) [6], a phantom field, that is a scalar field with negative sign of the kinetic term [7], or the combination of quintessence and phantom in a unified model named quintom [8].

There are some numerous indications that $G$ can be varying and that there is an upper limit to that variation, with respect to time or with the expansion of the universe [9]. In this connection the most significant evidences come from the observations of Hulse-Taylor binary pulsar [10, 11], helio-seismological data [12], Type Ia supernova observations [1] and asteroseismological data coming from the pulsating white dwarf star G117-B15A [13]; all the above evidences combined lead to $|\dot{G}/G| \leq 4.10 \times 10^{-11} yr^{-1}$, for $z \lesssim 3.5$, thereby suggesting a mild variation on cosmic level [14]. On a more theoretical level, varying gravitational constant has some benefits too, for instance it can help alleviating the dark matter problem [15], the cosmic coincidence problem [10] and the discrepancies in Hubble parameter value [12]. In literature, a variable gravitational constant has been accommodated in gravity theories including the Kaluza-Klein [18], Brans-Dicke framework [19] and scalar-tensor theories [20].

From the perspective of new gravitational theories including string theory and braneworld models, (see [21] and references therein), there are a lot of speculations that there could be extra dimensions of space (6 extra dimensions of space in the string theories) besides there are no convincing evidences for their existence. The size of these dimensions (whether small as Planck scale or infinitely long like usual dimensions) is still open to debate. In literature, various theoretical models with extra dimensions have been constructed to account dark energy [22]. Previously some works on variable $G$ correction have been investigated [23] to find the statefinder parameters for several dark energy models. The main motivation of this work is to investigate the role of statefinder parameters [24] (which can be written in terms of some observable parameters) in higher dimensional cosmology assuming a varying gravitational constant $G$ for interacting, non-interacting and generalized Chaplygin gas models.

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II. BASIC EQUATIONS AND SOLUTIONS

We consider homogeneous and anisotropic $N$-dimensional space-time model described by the line element \[ ds^2 = ds_{FRW}^2 + \sum_{i=1}^{d} b_i^2(t) dx_i^2, \] where $d$ is the number of extra dimensions ($d = N - 4$) and $ds_{FRW}^2$ represents the line element of the FRW metric in four dimensions is given by \[ ds_{FRW}^2 = -dt^2 + a^2(t) \left[ \frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right], \]

where $a(t)$ and $b_i(t)$ are the functions of $t$ alone represents the scale factors of 4-dimensional space time and extra dimensions respectively. Here $k$ ($= 0$, $\pm 1$) is the curvature index of the corresponding 3-space, so that the above model of the Universe is described as flat, closed and open respectively.

The Einstein’s field equations for the above non-vacuum higher dimensional space-time symmetry are

\[
3 \left( \frac{\dot{a}^2 + k}{a^2} \right) = \frac{\ddot{D}}{D} - \frac{d^2 \dot{b}^2}{8 b^2} + \frac{d \dot{b}^2}{8 b^2} + 8\pi G \rho, \tag{3}
\]

\[
2 \frac{\dot{a}^2}{a} + \frac{\dot{a}^2 + k}{a^2} = \frac{\dot{a} \dot{D}}{a D} + \frac{d^2 \dot{b}^2}{8 b^2} - \frac{d \dot{b}^2}{8 b^2} - 8\pi G p, \tag{4}
\]

and

\[
\frac{\ddot{b}}{b} + 3 \frac{\dot{a} \dot{b}}{a b} = -\frac{\dot{D} \dot{b}}{D b} + \frac{\dot{b}^2}{b^2} - \frac{8\pi G p}{2}. \tag{5}
\]

where $\rho$ and $p$ are energy density and isotropic pressure of the fluid filled in the universe respectively. We choose, $D^2 = b^d(t)$, so we have $\frac{\dot{D}}{D} = \frac{d \dot{b}}{2 b}$ and $\frac{\ddot{D}}{D} = \frac{d^2 \dot{b}^2}{4 b^2}$. Hence the equations (1), (2) and (3) become

\[
3 \left( \frac{\dot{a}^2 + k}{a^2} \right) = \frac{d \ddot{b}}{2 b} - \frac{d^2 \dot{b}^2}{4 b^2} - \frac{2d \dot{b}^2}{8 b^2} + \frac{d \dot{b}^2}{8 b^2} + 8\pi G \rho, \tag{6}
\]

\[
2 \frac{\dot{a}^2}{a} + \frac{\dot{a}^2 + k}{a^2} = \frac{\dot{a} \dot{D}}{a D} + \frac{d^2 \dot{b}^2}{8 b^2} - \frac{d \dot{b}^2}{8 b^2} - 8\pi G p, \tag{7}
\]

and

\[
\frac{\ddot{b}}{b} + 3 \frac{\dot{a} \dot{b}}{a b} = -\frac{\dot{D} \dot{b}}{D b} + \frac{\dot{b}^2}{b^2} - \frac{8\pi G p}{2}. \tag{8}
\]

Defining $H_1 = \frac{\dot{a}}{a}$, $H_2 = \frac{\dot{b}}{b}$ we have from (6) to (8),

\[
3H_1^2 + \frac{k}{a^2} - \frac{d}{2} \dot{H}_2 + \left( -\frac{d}{8} - \frac{d^2}{8} \right) H_2^2 = 8\pi G \rho, \tag{9}
\]

\[
3H_1^2 + \frac{k}{a^2} + 2\dot{H}_1 - \frac{d}{2} H_1 H_2 + \left( \frac{d}{8} - \frac{d^2}{8} \right) H_2^2 = -8\pi G p. \tag{10}
\]
\[ \dot{H}_2 + 3H_1 H_2 - \frac{d}{2} H_2^2 = - \frac{8\pi G p}{2}. \]  

(11)

Now consider the universe is filled with the dark matter (with negligible pressure) and dark energy. Assuming \( p = \omega \rho_x \), \( \rho = \rho_m + \rho_x \), where \( \rho_m \) and \( \rho_x \) are the energy densities of dark matter and dark energy respectively, \( \omega \) is the equation of state parameter for dark energy. Note that \( \omega \) is a dynamical time dependent parameter and will be useful in later calculations.

Now eliminating \( \dot{H}_1 \), \( \dot{H}_2 \) from (9), (10) and (11) we have,

\[ 24k = a^2 \left( -24H_1^2 - 12dH_1 H_2 - (d - 1)dH_2^2 + 16\pi G (4\rho_m + (4 - d\omega)\rho_x) \right) \]  

(12)

This equation can be written as

\[ (d + 3)^2 H^2 + 3H_1^2 - \frac{d(d - 1)}{2} H_2^2 + \frac{12k}{a^2} = 32\pi G \rho_m + (4 - d\omega)8\pi G \rho_x \]  

(13)

where \( H \) is the Hubble parameter defined by \( H = \frac{1}{a^2} (3H_1 + dH_2) \). This can be written as

\[ \Omega_1 + \frac{3}{(d + 3)^2} \Omega_2 - \frac{d(d - 1)}{2} \Omega_k = \frac{4}{d + 3} \Omega_m + \frac{4 - d\omega}{d + 3} \Omega_x \]  

(14)

where \( \Omega_1 = \frac{H^2}{17} \), \( \Omega_2 = \frac{H^2}{17} \) are dimensionless parameters and \( \Omega_m = \frac{8\pi G \rho_m}{(d + 3)H^2} \), \( \Omega_x = \frac{8\pi G \rho_x}{(d + 3)H^2} \) are fractional density parameters, \( \Omega_k = \frac{k}{a^2 H^2} \) is another dimensionless parameter, represents the contribution in the energy density from the spatial curvature and \( \Omega \) is the total density parameter. Now solving (9), (10) and (11) we have the solutions of \( \dot{H}_1 \) and \( \dot{H}_2 \) as

\[ \dot{H}_1 = \frac{1}{24} \left( -48H_1^2 + d(d - 1)H_2^2 - \frac{24k}{a^2} + 8\pi G (4\rho_m - (-4 + (d + 12)\omega)\rho_x) \right) \]  

(15)

and

\[ \dot{H}_2 = \frac{2}{d} \left( 3H_1^2 - \frac{1}{d} d(d + 1)H_2^2 + \frac{3k}{a^2} - 8\pi G (\rho_m + \rho_x) \right) \]  

(16)

The deceleration parameter \( q = -1 - \frac{\dot{H}}{H^2} \) is given by in terms of dimensionless parameters

\[ q = -1 - \frac{3}{d + 3} \Omega_k + \frac{d}{8} \Omega_2 + \frac{3}{2} \Omega_m + \frac{12 + 12\omega + d\omega}{8} \Omega_x \]  

(17)

and the derivative of deceleration parameter is obtained as

\[ \dot{q} = \frac{3}{d + 3} H \Omega_k \left( 2\sqrt{\Omega_1 - 2(q + 1)} \right) + \frac{d}{8} \Omega_2 + \frac{3}{2} \Omega_m + \frac{12 + 12\omega + d\omega}{8} \Omega_x + \frac{d + 12}{8} \dot{\omega} \Omega_x \]  

(18)

Also from (14) we obtain the expression of the total density parameter in the form

\[ \Omega = \frac{4}{d + 3} \frac{\rho}{\rho_{cr}} - \frac{d\omega}{d + 3} \Omega_x - \frac{12}{(d + 3)^2} \Omega_k - \frac{3}{(d + 3)^2} \Omega_1 + \frac{d(d - 1)}{2} \Omega_2 \]  

(19)

Now define the critical density,

\[ \rho_{cr} = \frac{3H^2}{8\pi G(t)} \]  

which gives after differentiation \( \dot{\rho}_{cr} = \rho_{cr} \left( \frac{2\dot{H}}{H} - \frac{\dot{G}}{G} \right) \)  

(20)

which implies

\[ \dot{\rho}_{cr} = -H \rho_{cr} (2(1 + q) + \Delta G) \]  

(21)
where, $\Delta G \equiv \frac{G}{G'}$, $\dot{G} = HG'$ (prime denotes differentiation with respect to $x \equiv \ln a$). The benefit of the previous rule $\dot{G} = HG'$ relates the variations in $G$ with respect to time $\dot{G}$ and the expansion of the universe $G'$. Differentiating (19) we have

$$\dot{\Omega} = \frac{4}{d+3} \frac{\dot{\rho}}{\rho_{cr}} + \frac{4H(2(1+q) + \Delta G)}{(d+3)^2} \frac{\rho}{\rho_{cr}} - \frac{24H}{(d+3)^2} \Omega_k(\sqrt{\Omega_0} - (q+1)) - \frac{3}{(d+3)^2} \Omega_1 + \frac{d(d-1)}{2} \Omega_2 - \frac{d}{d+3} \omega \dot{\Omega}_x - \frac{d}{d+3} \omega \dot{\Omega}_x$$  (22)

where $\Omega_1$ and $\Omega_2$ are given by

$$\Omega_1 = H \left[ \Omega_1^{1/2} \Omega_k - 3 \Omega_1^2 - \frac{3d}{2(d+3)} \Omega_1^{1/2} \Omega_2^{1/2} + \frac{d(d-1)}{8} \Omega_1^{1/2} \Omega_2 - (d+3) \omega \Omega_1^{1/2} \Omega_x - \frac{9}{d+3} \Omega_1 \Omega_k + \frac{d}{2} \Omega_1 \Omega_2^{1/2} \right]$$  (23)

$$\dot{\Omega}_2 = H \left[ \frac{12}{d} \Omega_1^{3/2} - \frac{3}{d+3} \Omega_1^{3/2} \Omega_2^{1/2} - \frac{d+1}{2} \Omega_1^{1/2} \Omega_2 - \frac{3d}{2(d+3)} \Omega_1^{3/2} \Omega_2 + \frac{d(d+7)}{8(d+3)} \Omega_1^{1/2} \Omega_2^{1/2} + \frac{12}{d} \Omega_1^{1/2} \Omega_k \right.$$  

$$- \frac{9}{d+3} \Omega_1^{1/2} \Omega_2^{1/2} \Omega_2 - \frac{4(d+3)}{d} \Omega_1^{1/2} \Omega_2^{1/2} \Omega_2 \Omega_x + \Omega_1^{1/2} \Omega_2^{1/2} \Omega_2 \left( 4 \Omega_m + (4 + 3 \omega) \Omega_x \right) \right]$$  (24)

The trajectories in the $\{r, s\}$ plane corresponding to different cosmological models depict qualitatively different behaviour. The statefinder diagnostic along with future SNAP observations may perhaps be used to discriminate between different dark energy models. The above statefinder diagnostic pair for cosmology are constructed from the scale factor $a$. The statefinder parameters are given by [24]

$$r = \frac{\ddot{a}}{aH^2}, \quad s = \frac{r - 1}{3(q - 1/2)}$$

Now we obtain the expressions for $r$ and $s$ as follows

$$r = \frac{d^2 \Omega_2}{32} - \frac{d \Omega_2}{8H} + (12 + (d+12) \omega) \Omega_x \left( -\frac{3}{8} + \frac{3}{4} \Omega_m - \frac{3}{4(d+3)} \Omega_k + \frac{d}{16} \Omega_2 - \frac{1}{8} \frac{\dot{\Omega}_x}{H} \right) - \frac{3}{d+3} \Omega_k (3 \Omega_m - 3 + 2 \Omega_1^{1/2})$$

$$- \frac{3d}{4(d+3)} \Omega_2 \Omega_k + \frac{3d}{8} \Omega_2 (2 \Omega_m - 1) + 4 \Omega_2^2 - \frac{3}{2} \Omega_m \frac{\dot{\Omega}_x}{H} + \frac{9}{2} \Omega_m + \frac{1}{32} (12 + (d+12) \omega)^2 \Omega_x^2 \right] - \frac{d+12}{8} \frac{\dot{\Omega}_x}{H} \Omega_x + 1$$  (25)

$$s = \frac{8(d+3)}{3[d(d+3) \Omega_2 - 24 \Omega_k + (d+3) (12 + (d+12) \omega) \Omega_x]} \left[ \frac{d^2 \Omega_2}{32} - \frac{d \Omega_2}{8H} \right.$$

$$+(12 + (d+12) \omega) \Omega_x \left( -\frac{3}{8} + \frac{3}{4} \Omega_m - \frac{3}{4(d+3)} \Omega_k + \frac{d}{16} \Omega_2 - \frac{1}{8} \frac{\dot{\Omega}_x}{H} \right) - \frac{3}{d+3} \Omega_k (3 \Omega_m - 3 + 2 \Omega_1^{1/2})$$

$$- \frac{3d}{4(d+3)} \Omega_2 \Omega_k + \frac{3d}{8} \Omega_2 (2 \Omega_m - 1) + 4 \Omega_2^2 - \frac{3}{2} \Omega_m \frac{\dot{\Omega}_x}{H} + \frac{9}{2} \Omega_m + \frac{1}{32} (12 + (d+12) \omega)^2 \Omega_x^2 \right] - \frac{d+12}{8} \frac{\dot{\Omega}_x}{H} \Omega_x \right]$$  (26)

This is the expressions for $\{r, s\}$ parameters in terms of fractional densities of dark energy model in higher dimensional cosmology for closed (or open) universe where the derivative of the density parameters i.e., $\dot{\Omega}_1$ and $\dot{\Omega}_2$ are given in equation (23) and (24). Now in the following subsections, we shall analyze the statefinder parameters for the non-interacting and interacting dark energy models.
A. Non-interacting Dark Energy Model

In this subsection we study the model of non-interacting case where the dark energy and dark matter do not interact with each other. We assume that dark matter and dark energy are separately conserved. So the continuity equation for cold dark matter is $\dot{\rho}_m + (d + 3)H \rho_m = 0$ and for dark energy is $\dot{\rho}_x + (d + 3)H(1 + \omega)\rho_x = 0$. So solving (22) for two different cases we have the expressions of $\dot{\Omega}_m$ and $\dot{\Omega}_x$ as:

$$\dot{\Omega}_m = \frac{1}{2(d + 3)(d + 3 + d\omega)} \left[ -6\Omega_1 + d(d - 1)(d + 3)^2\Omega_2 + 2 \left( 24H(q + 1 - \Omega_1^{1/2})\Omega_k + 4H((d + 3)(2q - 1) - d + \Delta G) 
+ d(2q + \Delta G + 2)\omega + d(d + 3)\omega^2)\Omega_m - d(4H(2q + 2 + \Delta G)\omega + (d + 3)\dot{\omega})\Omega_x \right) \right]$$  

(27)

$$\dot{\Omega}_x = \frac{1}{2(d + 3)(d + 3 + d\omega)} \left[ -6\Omega_1 + d(d - 1)(d + 3)^2\Omega_2 + 2 \left( 24H(q + 1 - \Omega_1^{1/2})\Omega_k - (d + 3)(4(d + 3)H(\omega + 1)\Omega_m 
+ (-4H(2q + 2 + \Delta G) + d\dot{\omega})\Omega_x) \right) \right]$$  

(28)

In the equations (25) and (26), we have calculated the general expressions of the statefinder parameters $\{r, s\}$. In this non-interacting dark energy model, the above parameters are also same where the $\dot{\Omega}_m$ and $\dot{\Omega}_x$ are given by the equations (27) and (28).

B. Interacting Dark energy Model

In this subsection we study the model of interacting case where the dark energy and dark matter are interact with each other. These models describe an energy flow between the components i.e. they not separately conserved. According to recent observational data of Supernovae and CMB the present evolution of the Universe permit the energy transfer decay rate proportional to present value of the Hubble parameter. Many authors have widely studied this interacting model. In Pavon and Zimdahl [27] state that the unknown nature of dark energy and dark matter make no contradiction about their mutual interaction. In Zhang and Olivers et al [28] showed that the theoretical interacting model are consistent with the type Ia supernova and CMB observational data.

Here we assume that the dark energy and dark matter are interacting with each other, so the continuity equations of dark matter and dark energy become

$$\dot{\rho}_m + (d + 3)H \rho_m = Q$$  

(29)

and

$$\dot{\rho}_x + (d + 3)H(1 + \omega)\rho_x = -Q$$  

(30)

where $Q$ is is the interacting term which is a arbitrary function. This interacting term determine the direction of the energy flow both sides of the dark matter and dark energy. In general this term can be choose as a function of different cosmological parameters like Hubble parameter and dark energy or dark matter density. in this work we choose $Q = (d + 3)\delta H \rho_x$, where $\delta$ is a couple constant. The positive $\delta$ represents the energy transfer from dark energy to dark matter. If $\delta = 0$ the above model transfer to non-interacting case. Here negative $\delta$ is not considered as it can violate the thermodynamical laws of the universe. So the $\dot{\Omega}_m$ and $\dot{\Omega}_x$ are given by the equations

$$\dot{\Omega}_m = \frac{1}{2(d + 3)(d + 3 + d\omega)} \left[ -6\Omega_1 + d(d - 1)(d + 3)^2\Omega_2 + 2 \left( 24H(q + 1 - \Omega_1^{1/2})\Omega_k + 4H((d + 3)(2q - 1) - d + \Delta G) 
+ d(2 + \Delta G + 2q)\omega + d(d + 3)\omega^2)\Omega_m + (4H(-d(2q + 2 + \Delta G)\omega + (d + 3)\delta(d + 3 + 2d\omega)) - d(d + 3)\dot{\omega})\Omega_x \right) \right]$$  

(31)
\[ \dot{\Omega}_x = \frac{1}{2(d + 3)(d + 3 + d \omega)} \left[ -6\dot{\Omega}_1 + d(d - 1)(d + 3)^2 \dot{\Omega}_2 + 2 \left( 24H(q + 1 - \Omega_1^{1/2})\Omega_k \right. \
\left. - (d + 3)(4H(d + 3)(1 + \omega)\Omega_m + (-4H(2q + 2 + \Delta G) + 4(d + 3)H\delta + d\dot{\omega})\Omega_x) \right) \right] \] (32)

In the equations (25) and (26), we have calculated the general expressions of the statefinder parameters \( \{r, s\} \). In this interacting dark energy model, the above parameters are also same where the \( \dot{\Omega}_m \) and \( \dot{\Omega}_x \) are given by the equations (31) and (32).

### III. GENERALIZED CHAPLYGIN GAS

It is well known to everyone that Chaplygin gas provides a different way of evolution of the universe and having behaviour at early time as pressureless dust and as cosmological constant at very late times, an advantage of generalized Chaplygin gas (GCG), that is it unifies dark energy and dark matter into a single equation of state. This model can be obtained from generalized version of the Born-Infeld action. The equation of state for generalized Chaplygin gas is

\[ p_x = -\frac{A}{\rho^2} \] (33)

where \( 0 < \alpha < 1 \) and \( A > 0 \) are constants. Inserting the above equation of state (33) of the GCG into the non-interacting energy conservation equation we have

\[ \rho_x = \left[ A + \frac{B}{(a^{3b}d^{\alpha+1})} \right] d\dot{\Omega}_x \] (34)

where \( B \) is an integrating constant.

\[ \omega = -A \left( A + \frac{B}{(a^{3b}d^{\alpha+1})} \right)^{-1} \] (35)

Differentiating (35) we have

\[ \frac{\dot{\omega}}{H} = -(d + 3)AB(1 + \alpha) \frac{1}{(a^{3b}d^{\alpha+1})} \left( A + \frac{B}{(a^{3b}d^{\alpha+1})} \right)^{-2} \] (36)

Now putting (36) in (25) and (26), we have

\[ r = \frac{d^2}{32} \frac{\Omega^2}{\Omega_x} \frac{d\dot{\Omega}_x}{8H} - \frac{9}{2} \Omega_m + \frac{9}{2} \Omega^2_m - \frac{3}{2} \Omega_k - \frac{6}{d + 3} \Omega^{1/2}_k - \frac{3d}{d + 3} \Omega_2 \Omega - \frac{3d}{8} \Omega_2 \]

\[ s = -\frac{9}{2} \frac{d\Omega_2}{8H} - \frac{3d\Omega_2}{d + 3} + \frac{9}{2} \Omega^2_m + \frac{3}{8} \frac{B(d + 12)\Omega_x}{A(a^{3b}d^{\alpha+1}) + B} \left[ \frac{d^2}{32} \frac{\Omega^2}{\Omega_x} \frac{d\dot{\Omega}_x}{8H} - \frac{9}{2} \Omega_m + \frac{4\Omega^2_m}{2} + \frac{3}{8} \frac{\Omega_k}{A(a^{3b}d^{\alpha+1}) + B} \right] \] (37)
Now putting (41) in (25) and (26), we have

\[ \dot{\Omega} = \frac{1}{3} \left( \frac{d + 12}{A + (a^3 b^d)^{-1}} \right)^2 \Omega_x \]

These are the expressions for the Chaplygin gas model in higher dimensional Cosmology, where \( \Omega_m \) and \( \Omega_x \) are given by the equations (27) and (28).

Again inserting the equation of state (33) of the GCG into the interacting energy conservation equation (30) we have

\[ \rho_x = \left[ \frac{\Omega}{\delta + 1} + \frac{B}{(\delta + 1)(a^3 b^d)^{(\delta + 1)(\alpha + 1)}} \right]^{\frac{\delta + 1}{\delta + 1}} \]  

(39)

and

\[ \omega = -A \left( \frac{\Omega}{\delta + 1} + \frac{B}{(\delta + 1)(a^3 b^d)^{(\delta + 1)(\alpha + 1)}} \right)^{-1} \]  

(40)

Differentiating (40) we have

\[ \frac{\dot{\Omega}}{H} = -(d + 3)AB(1 + \alpha) \left( \frac{A}{(\delta + 1)(a^3 b^d)^{(\delta + 1)(\alpha + 1)}} + \frac{B}{(\delta + 1)(a^3 b^d)^{(\delta + 1)(\alpha + 1)}} \right)^{-2} \]  

(41)

Now putting (41) in (25) and (26), we have

\[ r = \frac{d^2}{32} \Omega_x^2 - \frac{d \dot{\Omega}_x}{8H} - \frac{9}{2} \Omega_m + \frac{9}{2} \Omega_\omega^2 - \frac{3}{2} \Omega_k - \frac{6}{d + 3} \Omega_x \Omega_k - \frac{3d}{d + 3} \Omega_2 \Omega_k - \frac{3d}{8} \Omega_2 - \frac{9}{d + 3} \Omega_k \Omega_m + \frac{3d}{4} \Omega_2 \Omega_m - \frac{A(d + 12)(1 + \delta)}{A + B(a^3 b^d)^{-(1 + \alpha)(1 + \delta)}} \]  

(42)

\[ s = \frac{1}{3d \Omega_2 - \frac{72}{d + 3} \Omega_k + 36 \Omega_m - 36 + 3 \left( -12 + \frac{A(d + 12)(1 + \delta)}{A + B(a^3 b^d)^{-(1 + \alpha)(1 + \delta)}} \right) \Omega_x} \left[ \frac{d^2}{32} \Omega_x^2 - \frac{d \dot{\Omega}_x}{8H} - \frac{9}{2} \Omega_m + \frac{9}{2} \Omega_m - \frac{3}{2} \Omega_m + \frac{9}{d + 3} \Omega_k \right] \]  

(43)

These are the expressions for the \( \{r, s\} \) parameters in terms of fractional densities for interacting case of generalized Chaplygin gas model in higher dimensional Cosmology, where \( \Omega_m \) and \( \Omega_x \) are given by the equations (31) and (32).
IV. CONCLUSIONS

In this work, we have considered $N (= 4 + d)$-dimensional Einstein field equations in which 4-dimensional space-time is described by a FRW metric and that of the extra $d$-dimensions by an Euclidean metric. We have calculated the corrections to statefinder parameters $\{r, s\}$ and deceleration parameter $q$ due to variable gravitational constant $G$ in higher dimensional Cosmology. These corrections are relevant because several astronomical observations provide constraints on the variability of $G$. We have first assumed that the dark energy do not interact with dark matter. Next we have considered the dark energy and dark matter are not separately conserved i.e., they interact with each other with a particular interacting term in the form $Q = (d + 3)\delta H_{\rho_\gamma}$ where $\delta$ is a couple constant. In both the cases, the statefinder parameters have been found in terms of the dimensionless density parameters as well as EoS parameter $\omega$ and the Hubble parameter. An important thing to note is that these are the $G$-corrected statefinder parameters and they remain geometrical parameters as previous. Because, the parameter $\Delta G$ is a pure number and is independent of the geometry. Finally we have analyzed the above statefinder parameters in terms of some observable parameters for the non-interacting and interacting cases when the universe is filled with generalized Chaplygin gas. These dynamical statefinder parameters may generate different stages of the anisotropic universe in higher dimensional Cosmology if the observable parameters are known for interacting and non-interacting models.

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