Uncertainty evaluation on aperture area measurement for radiometric traceability

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Abstract. In computer vision applications one of important steps, when the aim of the system is to obtain a measurement result, is the uncertainty of measurement evaluation. Involving image processing and analysis for measurement requires a specific study of all input quantities and its contribution for expanded uncertainty. In this paper will be described all contributions and how each one affect the measurement result, including contributions from algorithms of image processing. Finally this work compares the uncertainty obtained with a previous result from Monte Carlo simulation for analysis validation.

1. Introduction
Circular aperture as standards are used in radiometric and photometric measurements as part of traceability chain, usually built of black anodized aluminum, electroformed and covered with nickel, brass, among other materials. This standard has a circular shape, with reduced thickness. On some models, the region of the hole has its edge sharpened at an angle of 45º.

The purpose of calibrating a circular aperture is to determine the area of its hole. This result is then used in determining the value of the solid angle formed by the light passing through it to reach a detector. The area measurement can be performed by radiometric methods [1], by coordinate measuring machines [2], or non-contact measurement by image processing.

Brazil’s National Metrology Institute (Inmetro) developed a calibration system for circular apertures [3] using computer vision in order to provide traceability for radiometric measurements and researches. In this study the measurement procedure is described and also the measurement uncertainty estimation, focused in validation of the mathematical model for the area measurement described with its uncertainty measurement estimated using GUM method.

The measured results are obtained using a numerical fitting from a matrix equation. The aim of this paper is to present an approach to the uncertainty estimation using the ISO GUM as reference. Each variable was studied, quantified and analyzed separately, as well as their sensitivity coefficients, the combined uncertainty and expanded measurement uncertainty.
2. Measuring method

The measurement of the circular aperture is performed on a system consisting of a microscope with a CCD camera and an \(xy\) displacement table for moving the mensurand. The table displacements are measured by a HeNe laser interferometer.

The measurement of the circular aperture consists of 12 images in circular shape spaced of approximately 30°, each saved in bitmap format. The values corresponding to the \(x\) and \(y\) laser coordinates are saved in text format for each of the 12 positions around the aperture. For each image, the edge points \(i\), in pixels unit \((P_k(i))\) are multiplied by the pixel length \((l_{pk}(i))\) and added to the lasers in \(x\) and \(y\) axes \((L_k(i))\), resulting in each coordinate point \((C_k(i))\), in the direction \(k\) as shown in equation (1), already in length unit.

\[
C_k(i) = L_k(i) + \left(P_k(i) \cdot l_{pk}(i)\right)
\]  

In order to evaluate the aperture radius the coordinates for all the edges, \(C_x(i)\) and \(C_y(i)\), in millimetres, are used. These coordinates are grouped into two matrices \(K\) and \(J\), shown respectively by equations (2) and (3), for circular fitting and radius estimation.

\[
K = \begin{bmatrix}
    \sum_{i=1}^{n}[c_x(i)^2 + c_y(i)^2] & \sum_{i=1}^{n}[c_x(i)^2 + c_y(i)^2] \cdot c_x(i) & \sum_{i=1}^{n}[c_x(i)^2 + c_y(i)^2] \cdot c_y(i) \\
    \sum_{i=1}^{n}[c_x(i)^2 + c_y(i)^2] \cdot c_x(i) & \sum_{i=1}^{n}c_x(i)^2 & \sum_{i=1}^{n}c_y(i)^2 \\
    \sum_{i=1}^{n}[c_x(i)^2 + c_y(i)^2] \cdot c_y(i) & \sum_{i=1}^{n}c_y(i)^2 & \sum_{i=1}^{n}c_y(i)^2 \\
\end{bmatrix}
\]

\[
J = \begin{bmatrix}
    \sum_{i=1}^{n}[c_x(i)^2 + c_y(i)^2] \\
    \sum_{i=1}^{n}c_x(i) \\
    \sum_{i=1}^{n}c_y(i) \\
\end{bmatrix}
\]

The coordinates corresponding to the center of the aperture in \(x\) and \(y\) directions, \(G\), as well as the radius \(R\), are obtained by solving the matrix equations shown in (4) and (5), respectively.

\[
G = K^{-1} \cdot J
\]

\[
R = \sqrt{4 \cdot G_{11}^2 + G_{21}^2 + G_{31}^2} / 2 \cdot G_{11}
\]
Five measurements were performed using this procedure, and the average result for the area measurement is 7.0992 mm$^2$ with a standard deviation of 0.0018 mm$^2$.

3. Uncertainty Evaluation
The evaluation of the measurement uncertainty for the circle area uses equation (6) as reference.

$$A = \pi r^2$$  \hspace{1cm} (6)

To determine the measurement uncertainty the mathematical model needs to be expanded and evaluated on how each input variable affects the aperture area value $A$ [4-5]. As described before, the aperture area is measured from the number of fitted points around the aperture, which, in this case, results in 7161 fitted points. These points are positions in the $xy$ plane with its respective coordinates measured in millimeters by $C_x(i)$ and $C_y(i)$, with $i = 1, 2, 3, ..., 7161$. Each coordinate $C(i)$ is determined as a function of the laser displacement ($L$), the pixel position ($P$) and the pixel’s length ($l_p$), as previously show by Equation (1). For the uncertainty evaluation of the output quantity (area), each of the six input quantities of equations (7) and (8) are analysed.

$$C_x(i) = L_x(i) + \left( P_x(i) \cdot l_{p(x)}(i) \right)$$  \hspace{1cm} (7)

$$C_y(i) = L_y(i) + \left( P_y(i) \cdot l_{p(y)}(i) \right)$$  \hspace{1cm} (8)

The last step of the uncertainty evaluation is the combination of all the contributions previously calculated and the repeatability of the measurements obtained by the standard error of the standard deviation, calculated from 5 measurements.
A summary of all uncertainty values obtained can be seen in Table 1.

| name               | distrib. | $u(x_i)$ unit | $v_i$ | $c_i = \partial dl/\partial x_i$ | $u_i(dl)/\text{mm}^2$  |
|--------------------|----------|--------------|------|-------------------------------|----------------------|
| Laser Displacement – X axis | R        | 0,0001 mm    | 1000 | 5,16.10$^{-7}$ mm            | 8,87.10$^{-22}$       |
| Laser Displacement – Y axis | R        | 0,0001 mm    | 1000 | 5,67.10$^{-4}$ mm            | 1,07.10$^{-15}$       |
| Pixel – X axis     | N        | 0,00003 mm   | 1000 | 173 mm                       | 6,73.10$^{-6}$        |
| Pixel – Y axis     | N        | 0,00003 mm   | 1000 | 20,1 mm                      | 9,09.10$^{-8}$        |
| Edge detection – X axis | R        | 2            | 1000 | 7,09.10$^{-7}$ mm$^2$        | 6,70.10$^{-13}$       |
| Edge detection – Y axis | R        | 2            | 1000 | 3,04.10$^{-7}$ mm$^2$        | 1,23.10$^{-13}$       |
| Repeatability      | N        | 1,78.10$^{-3}$ mm$^2$ | 4    | 1                            | 3,15.10$^{-6}$        |
For the average result of 7.099 mm$^2$ for the aperture area, with a coverage factor of $k = 2.066$ and a coverage probability of 95.45%, the expanded uncertainty is 0.007 mm$^2$.

The evaluation of measurement uncertainty for this application was also conducted by using Monte Carlo method [3], resulting in an expanded uncertainty of 0.0064 mm$^2$, showing consistency with this development.

4. Conclusions

The study shows a significant gain regarding the knowledge of the system and the importance of each contribution to the uncertainty of the developed method. It was possible to observe the difference in the influence of each axes. The measurement procedure proposed uses more movements in the $x$ axis of the machine, making the uncertainties of the variables related to this axis a greater contribution to the evaluation of the final expanded uncertainty.

It was also possible, using this methodology, to present a procedure that can be applied in other mathematical models in the form of evaluation of the measurement uncertainty in mathematical models involving matrix equations.

By comparing the results from this paper with results previously obtained by the Monte Carlo method, it was possible to validate the data and processing methodology of the variables.

5. References

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