On Principle of Inertia in Closed Universe

Han-Ying Guo$^{1,2}$

CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, China, and
Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing 100080, China

If our universe is asymptotic to a de Sitter space, it should be closed with curvature in $O(\Lambda)$ in view of $dS$ special relativity. Conversely, its evolution can fix on Beltrami systems of inertia in the $dS$-space without Einstein’s ‘argument in a circle’. Gravity should be local $dS$-invariant based on localization of the principle of inertia.

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I. INTRODUCTION

In classical physics, it is well known that for both Newton theory and Einstein’s special relativity the principle of inertia (PoI) with Galilean symmetry and Poincaré symmetry, respectively, plays an extremely important role as the benchmark of physics for defining physical quantities and introducing physical laws. But, in Einstein’s point of view, there is an ‘argument in a circle’ for the PoI as the benchmark.

Some eighty five years ago, Einstein claimed:

‘The weakness of the PoI lies in this, that it involves an argument in a circle: a mass moves without acceleration if it is sufficiently far from other bodies; we know that it is sufficiently far

* hyguo@itp.ac.cn
from other bodies only by the fact that it moves without acceleration. Are there at all any inertial systems for very extended portions of the space-time continuum, or, indeed, for the whole universe? We may look upon the principle of inertia as established, to a high degree of approximation, for the space of our planetary system, provided that we neglect the perturbations due to the sun and planets. Stated more exactly, there are finite regions, where, with respect to a suitably chosen space of reference, material particles move freely without acceleration, and in which the laws of the special theory of relativity, \( \cdots \), hold with remarkable accuracy. Such regions we shall call “Galilean regions.”\(^1\)

In fact, to avoid this ‘weakness’ is one of the main motivations for Einstein from special relativity to general relativity based on his equivalence principle and general principle of relativity as an extension of the special principle of relativity.

In general relativity, however, what is realized for the general principle of relativity is the principle of general covariance. Although it is always possible to analyze physics in terms of arbitrary (differentiable) coordinate systems at classical level, ‘the principle of covariance has no forcible content.’\(^2\) For the equivalence principle, it requires that physical quantities and laws are in ‘their familiar special-relativistic forms’ in local Lorentz frames.\(^2\) The symmetry for physical quantities and laws, however, is local $GL(4,R)$ or its subgroup $SO(1,3)$ without local translation in general. Thus, in ‘Galilean regions’, Poincaré symmetry of $PoI$ as the benchmark in special relativity is partially lost. These seem away from Einstein’s original intention and lead to the benchmark of physics with gravity is not completely in consistency with that in special relativity without gravity.

Recent observations show that our universe is accelerated expanding.\(^3, 4\) It is certainly not asymptotic to a Minkowski ($Mink$)-space, rather quite possibly asymptotic to a de Sitter ($dS$)-space with a tiny cosmological constant $\Lambda$. These present great challenges to the foundation of physics on the cosmic scale (see, e.g., \[5\]). In fact, it is the core of challenges: What are the benchmarks of physics on the cosmic scale? Are they consistent?

In view of the $dS$-invariant special relativity,\(^3, 7\) however, there is a $PoI$ of $dS$-invariance on $dS$-space with Beltrami systems of inertia (denoted $BdS$-space). Here we show that if the universe is asymptotic to a $dS$-space, it should be closed with a tiny curvature in the order of $\Lambda$, $O(\Lambda)$. Conversely, the evolution of the universe can fix on the Beltrami systems. Thus, the universe acts as the origin of the $PoI$ of $dS$-invariance without Einstein’s ‘argument in a circle’ so that the benchmark of physics on the cosmic scale should still be the $PoI$ of $dS$-invariance. Then, we explain that the benchmark of physics with gravity should be the localization of the $PoI$ of the $dS$ special relativity. Thus, the $PoI$ of $dS$-invariance and its localization should play the role of the consistent benchmarks of physics on the cosmic scale in the universe.

Actually, based on the principle of relativity\(^6, 7\) and the postulate on invariant universal constants, the speed of light $c$ and the curvature radius $R$\(^[8, 9]\), the $dS$ special relativity can be set up on the $BdS$-space. While Einstein’s special relativity is the limiting case of $R \to \infty$.

In the $dS$ special relativity, Beltrami coordinate systems\(^17\) with Beltrami time simultaneity are very similar to $Mink$-systems in Einstein’s special relativity. Namely, in the $BdS$-space geodesics are all straight world lines so that there is a $PoI$ with a law of inertia for free particles
and light signals. All these issues are transformed symmetrically under the fractional linear transformations with a common denominator (FLT's) of dS-group SO(1,4) in the Beltrami atlas chart by chart. It is significant that the Beltrami systems and their Robertson-Walker-like dS-counterpart with respect to proper-time simultaneity provide an important model. In this model, the dS-group as a maximum symmetry ensures that there are both the PoI and the cosmological principle on dS-space as two sides of a coin. On one side, there is the BdS-space with the PoI, while on the other there is a Robertson-Walker-like dS-space with the cosmological principle having an accelerated expanding closed 3-d cosmos $S^3$ of curvature in the order of $O(R^{-2})$. Since the both can be transformed each other explicitly by changing the simultaneity just like flip a coin, the Robertson-Walker-like dS-space displays as an origin of the PoI, while the PoI provides a benchmark of physics on the dS-space.

If the universe is asymptotic to a dS-space with $R \simeq (3/\Lambda)^{1/2}$. In view of the dS special relativity, the universe should be asymptotic to the Robertson-Walker-like dS-space in the model so that it should be closed and the deviation from flatness is in the order of $\Lambda, O(\Lambda)$. This is an important prediction more or less consistent with recent data from WMAP \[4\] and can be further checked.

Conversely, the asymptotic behavior of the universe should naturally pick up a kind of the Robertson-Walker-like dS-systems with such a ‘cosmic’ time $\tau$ that its axis coincides with the revolution time arrow of the real cosmic time $\tau_u$ in the universe. Since the ‘cosmic’ time $\tau$ in the Robertson-Walker-like dS-space is explicitly related to the Beltrami time $x^0$, the universe should also fix on a kind of Beltrami systems with $x^0$ transformed from the ‘cosmic’ time $\tau$. Therefore, via its evolution time arrow of $\tau_u$ picking up a ‘cosmic’ time $\tau$ on the Robertson-Walker-like dS-space, the universe should just act as an origin of such kind of Beltrami systems in which the PoI holds. Thus, there do exist the inertial systems in the universe and there is no Einstein’s ‘argument in a circle’ for the PoI.

In general relativity, there is no special relativity in dS-space. In the dS special relativity, there is no gravity in dS-space. How to describe gravity?

In the light of Einstein’s ‘Galilean regions’ \[1\], where his special relativity with full Poincaré symmetry should hold locally, the PoI should be localized. Therefore, in view of the dS special relativity, on spacetimes with gravity there should be local dS-frame anywhere and anytime so that the PoI of the dS special relativity should hold locally. If so, the localized PoI of the dS special relativity should be the benchmark of physics with gravity. This is in consistency with the role played by the PoI of the dS special relativity. We may further require that gravity have a gauge-like dynamics characterized by a dimensionless constant $g \simeq (\Lambda G \hbar/c^3)^{1/2} \sim 10^{-61}$ from the cosmological constant $\Lambda$ and the Planck length. A simple model has implied this should be the case \[23, 24, 25\].

This letter is arranged as follows. In sections 2, we argue why there is a PoI on dS-space and very briefly introduce the dS special relativity. In section 3, we introduce the relation between the PoI and the cosmological principle on dS-space as well as the cosmological meaning of dS special relativity. In section 4, we explain why the universe can fix on the Beltrami systems of inertia without Einstein’s ‘argument in a circle’. In section 4, we very briefly discuss that gravity should be based on localization of the dS special relativity with PoI and introduce
II. ON DE SITTER SPECIAL RELATIVITY

Is there special relativity with a PoI on dS-space?
Yes! Absolutely. This can be enlightened from two deferent but related angles [6, 7, 8, 9, 10, 11, 12].

Firstly, as is well known, weakening the Euclid fifth axiom leads to Riemann and Lobachevsky geometries on an almost equal footing with Euclid geometry. There is a physical analog via an inverse Wick rotation of 4-d Euclid space, Riemann sphere and Lobachevsky hyperboloid $E^4/S^4/L^4$, respectively. Namely, there should be two other kinds of the $dS/AdS$-invariant special relativity on an almost equal footing with Einstein’s special relativity [12]. In fact, there is a one-to-one correspondence between three kinds of geometries and their physical counterparts. We list the correspondence as follows:

| Geometry | Spacetime Physics |
|----------|--------------------|
| $E^4/S^4/L^4$ | $M^{1,3}/dS^{1,3}/AdS^{1,3}$ |
| ISO(4)/SO(5)/SO(1, 4) | ISO(1, 3)/SO(1, 4)/SO(2, 3) |
| Descartes, Beltrami atlas | Minkowski, Beltrami atlas |
| Points | Events |
| Straight line | Straight world line |
| Principle of Invariance | Principle of Relativity |
| Klein’s Erlangen Programm | Theory of Special Relativity |

Secondly, owing to Umov, Weyl and Fock [16], it can be proved that the most general form of the transformations among inertial coordinate systems

$$x'^i = f^i(x^i), \quad x^0 = ct, \quad i = 0, \cdots, 3,$$

which transform a uniform straight line motion, i.e. the inertial motion, in $F(x)$

$$x^a = v^a(t - t_0) + x_0^a, \quad v^a = \frac{dx^a}{dt} = \text{const.} \quad a = 1, 2, 3,$$

to a motion of the same nature in $F'(x')$, are of FLT-type.

As in Einstein’s special relativity, the principle of relativity implicates that there is a metric in inertial systems on 4-d spacetime with signature $\pm 2$ and it is invariant under a transformation group with ten parameters including spacetime ‘translations’, boosts and space rotations. Thus, these 4-d spaces are maximally symmetric, i.e. $Mink/dS/AdS$ of zero, positive or negative constant curvature, invariant under group $ISO(1, 3)/SO(1, 4)/SO(2, 3)$, respectively. As for invariant universal constants, in addition to the speed of light $c$ there is another invariant constant $R$, the radius of $dS/AdS$-spaces. Therefore, the $dS/AdS$ special relativity can be set up based on the principle of relativity and the postulate on invariant universal constants [8, 9].
The $dS$-space as a 4-d hyperboloid $\mathcal{H}_R$ can be embedded in a 1+4-d Mink-space, $\mathcal{H}_R \subset M^{1,4}$:

$$\mathcal{H}_R : \quad \eta_{AB} \xi^A \xi^B = -R^2, \tag{2.3}$$
$$ds^2 = \eta_{AB} d\xi^A d\xi^B, \tag{2.4}$$

where $\eta_{AB} = \text{diag}(1, -1, -1, -1, -1)$, $A, B = 0, \ldots, 4$.

On the hyperboloid, a kind of uniform great ‘circular’ motions of a particle with mass $m_R$ can be defined by a conserved 5-d angular momentum:

$$\frac{d\mathcal{L}^{AB}}{ds} = 0, \quad \mathcal{L}^{AB} := m_R (\xi^A \frac{d\xi^B}{ds} - \xi^B \frac{d\xi^A}{ds}). \tag{2.5}$$

For the particle, there is an Einstein-like formula:

$$-\frac{1}{2R^2} \mathcal{L}^{AB} \mathcal{L}_{AB} = m_R^2, \quad \mathcal{L}_{AB} := \eta_{AC} \eta_{BD} \mathcal{L}^{CD}. \tag{2.6}$$

Obviously, the eqns (2.3), (2.4), (2.5) and (2.6) are invariant under linear transformations of $dS$-group $SO(1, 4)$. For a massless particle or a light signal with $m_R = 0$, similar motion can be defined as long as the proper time $s$ is replaced by an affine parameter $\lambda$.

Via a ‘gnomonic’ projection without antipodal identification, $\mathcal{H}_R$ becomes the $BdS$-space with a Beltrami atlas [8, 9] chart by chart. In the charts $U_{\pm 4}$, for instance,

$$x^i|_{U_{\pm 4}} = R \xi^i / \xi^4, \quad i = 0, \ldots, 3; \tag{2.7}$$
$$\xi^4|_{U_{\pm 4}} = (\xi^0^2 - \sum_{a=1}^{3} \xi^a^2 + R^2)^{1/2} \geq 0, \tag{2.8}$$

there are condition from (2.3) and $BdS$-metric from (2.4)

$$\sigma(x) = \sigma(x, x) := 1 - R^{-2} \eta_{ij} x^i x^j > 0, \tag{2.9}$$
$$ds^2 = [\eta_{ij} \sigma(x)^{-1} + R^{-2} \eta_{ik} \eta_{jk} x^i x^j \sigma(x)^{-2}] dx^i dx^j, \tag{2.10}$$

where $\eta_{ij} = \text{diag}(1, -1, -1, -1)$. Under FLT’s of $SO(1, 4)$ sending an event $A(a^i)$ to the origin

$$x^i \rightarrow \tilde{x}^i = \pm \sigma(a)^{1/2} \sigma(a, x)^{-1} (x^i - a^i) D_j^i, \quad D_j^i = L_j^i + R^{-2} \eta_{jk} a^k a^l (\sigma(a) + \sigma(a)^{1/2} L_l^i)^{-1} L_j^l, \tag{2.11}$$
$$L := (L_j^i)_{i,j=0,\ldots,3} \in SO(1, 3),$$

(2.9) and (2.10) are invariant. Thus, inertial systems and inertial motions transform among themselves, respectively.

For a pair of events $(A(a^i), X(x^i))$

$$\Delta_R^2(a, x) = R^2 [\sigma^2(a, x) - \sigma(a) \sigma(x)] \geq 0 \tag{2.12}$$

is invariant under (2.11). Thus, the pair is time-like, null, or space-like, respectively.

The Beltrami light-cone at an event $A$ with running points $X$ is

$$\mathcal{F}_R := R \{ \sigma(a, x) - [\sigma(a) \sigma(x)]^{1/2} \} = 0. \tag{2.13}$$
At the origin $a^i = 0$, it is just Minkowskian $\eta_{ij}x^ix^j = 0$.

Under the ‘gnomonic’ projection, the uniform great ‘circular’ motions are projected as a kind of inertial motions along geodesics. In fact, the geodesics are Lobachevsky-like straight world lines and vise versa. A time-like geodesic, along which a particle with mass $m_R$ moves, is equivalent to

$$\frac{dp^i}{ds} = 0, \quad p^i := m_R\sigma(x)^{-1}\frac{dx^i}{ds} = C^i = \text{const.} \tag{2.14}$$

Under certain initial condition it is just a straight world line with respect to $w = w(s)$

$$x^i(w) = c^iw + b^i. \tag{2.15}$$

A light signal moves along a null geodesic with an affine parameter $\lambda$ can be written as

$$\frac{dk^i}{d\lambda} = 0, \quad k^i := \sigma^{-1}(x)\frac{dx^i}{d\lambda} = \text{const.} \tag{2.16}$$

It can also be expressed as a straight line.

From both (2.14) and (2.16), it follows that the coordinate velocity components are constants

$$\frac{dx^a}{dt} = v^a = \text{const.}, \quad a = 1, 2, 3. \tag{2.17}$$

Thus, the both motions of free particles and light signals are indeed of inertia as in (2.2), chart by chart. Namely, the law of inertia holds on the $BdS$-space. This together with the principle of relativity is just the PoI in the $BdS$-space.

For such a free massive particle a set of conserved quantities $p^i$ in (2.14) and $L^{ij}$ can be defined as a pseudo 4-momentum vector and a pseudo 4-angular-momentum, respectively

$$L^{ij} = x^ip^j - x^j p^i, \quad \frac{dL^{ij}}{ds} = 0. \tag{2.18}$$

In fact, $p^i$ and $L^{ij}$ constitute the conserved 5-d angular momentum in (2.5). And the Einstein-like formula (2.6) becomes a generalized Einstein’s formula

$$E^2 = m^2_Rc^4 + p^2c^2 + \frac{c^2}{R^2}j^2 - \frac{c^4}{R^2}k^2, \tag{2.19}$$

with energy $E = p^0$, momentum $p^a, p_a = \delta_{ab}p^b$, ‘boost’ $k^a, k_a = \delta_{ab}k^b$ and 3-angular momentum $j^a, j_a = \delta_{ab}j^b$. For a massless particle or a light signal with $m_R = 0$, similar issues hold so long as the proper time is replaced by an affine parameter.

If we introduce the Newton-Hooke constant $\nu$ and take $R$ as $R \simeq (3/\Lambda)^{1/2}$,

$$\nu := cR \simeq c(3/\Lambda)^{-1/2}, \quad \nu^2 \sim 10^{-35} s^{-2}. \tag{2.20}$$

It is so tiny that all experiments at ordinary scales cannot distinguish the $dS$ special relativity from Einstein’s one.

In order to make sense of inertial motions and these observables for an inertial observer $O_I$ rested at the spacial origin of the Beltrami system, the simultaneity should be defined. Similar
to Einstein’s special relativity, two events \( A \) and \( B \) are simultaneous if and only if their Beltrami temporal coordinate values \( x^0(A) \) and \( x^0(B) \) are equal:

\[
a^0 := x^0(A) = x^0(B) =: b^0.
\]

It is called the Beltrami simultaneity and defines a 1+3 decomposition of the \( BdS \)-metric \( (2.10) \)

\[
ds^2 = N^2(dx^0)^2 - h_{ab}(dx^a + N^a dx^0)(dx^b + N^b dx^0)
\]

with lapse function, shift vector, and induced 3-geometry on \( \Sigma_c \) in the chart, respectively,

\[
N = \{\sigma_{\Sigma_c}(x)[1 - (x^0/R)^2]\}^{-1/2},
N^a = x^0 x^a [R^2 - (x^0)^2]^{-1},
\]

\[
h_{ab} = \delta_{ab}\sigma_{\Sigma_c}^{-1}(x) - [R\sigma_{\Sigma_c}(x)]^{-2}\delta_{ac}\delta_{bd} x^c x^d,
\]

where \( \sigma_{\Sigma_c}(x) = 1 - (x^0/R)^2 + \delta_{ab} x^a x^b / R^2 \). In particular, at \( x^0 = 0 \), \( \sigma_{\Sigma_c}(x) = 1 + \delta_{ab} x^a x^b / R^2 \), 3-hypersurface \( \Sigma_c \) is isomorphic to an \( S^3 \) in all Beltrami coordinate charts.

### III. PRINCIPLE OF INERTIA AND COSMOLOGICAL PRINCIPLE AS TWO SIDES OF A COIN

On the \( dS \)-space, there is an important relation between the \( PoI \) and the cosmological principle. It is just like two sides of a coin.

In fact, for an observer rest at spacial origin \( x^a = 0 \) of Beltrami system, there is another simultaneity: the \( proper-time simultaneity \) with respect to an ideal clock’s proper-time \( \tau \). It is easy to see that the proper-time \( \tau \) is explicitly related to the Beltrami time \( x^0 \):

\[
\tau := \tau_R = R \sinh^{-1}(R^{-1}\sigma_{\Sigma_c}^{-1}(x)x^0).
\]

Thus, the \( proper-time simultaneity \) can be defined as: all events \( X(x^i) \) are simultaneous with respect to the observer if and only if their proper time are equal. Namely,

\[
x^0 \sigma_{\Sigma_c}^{-1/2}(x, x) =: \xi^0 = R \sinh(R^{-1}\tau) = \text{constant}.
\]

In fact, these events are comoving with the observer, who now becomes a comoving one \( O_C \) with respect to all these events. The line-element on a simultaneous hypersurface \( \Sigma_\tau \) now is

\[
dl^2 = -ds_{\Sigma_\tau}^2,
\]

where

\[
ds_{\Sigma_\tau}^2 = R_{\Sigma_\tau}^2 \cdot dl^2_{\Sigma_\tau,0},
R_{\Sigma_\tau}^2 := \sigma_{\Sigma_\tau}^{-1}(x, x) = 1 + (\xi^0/R)^2,
\]

\[
\sigma_{\Sigma_\tau}(x, x) := 1 + R^{-2}\delta_{ab} x^a x^b > 0,
\]

\[
dl_{\Sigma_\tau,0}^2 := \{\delta_{ab}\sigma_{\Sigma_\tau}^{-1}(x) - [R\sigma_{\Sigma_\tau}(x)]^{-2}\delta_{ac}\delta_{bd} x^c x^d\} dx^a dx^b.
\]
It is clear that this simultaneity is directly related to the cosmological principle on the $dS$-space. In fact, if the proper time $\tau$ is taken as a temporal coordinate for the observer $O_C$, the $BdS$-metric (2.10) becomes as a Robertson-Walker-like $dS$-metric with $\tau$ being a ‘cosmic’ time and an accelerated expanding 3-d cosmos isomorphic to $S^3$:

$$ds^2 = d\tau^2 - dl^2 = d\tau^2 - \cosh^2(R^{-1}\tau)dl^2_{\Sigma_0}.$$ (3.5)

It is important that two kinds of simultaneity relate the $BdS$-metric (2.10) with the PoI and the Robertson-Walker-like $dS$-metric (3.5) with the cosmological principle. They do make sense in two types of measurements: the Beltrami simultaneity is for those of the inertial observer $O_I$ relevant to the PoI and the proper time simultaneity for those of the comoving observer $O_C$ concern ‘cosmic’ effects of all distant stars and cosmic objects except the cosmological constant as test stuffs. Thus, on the $dS$-space there is a kind of inertial-comoving observers $O_{I-C}$ who play two roles with apparatus having two different types of time scales and relevant rulers. What should be done for them from their comoving observations to another type of measurements is to switch off the ‘cosmic’ time $\tau$ with the ‘cosmic’ rule and on the Beltrami time $x^0 = ct$ with the Beltrami rule, respectively, and vise versa. Namely, if the observers as comoving ones, $O_C$, on (3.5) would change their measurements from the proper-time simultaneity to the Beltrami time one according to the relation (3.1), they become inertial ones $O_I$, for whom the PoI makes sense, and vise versa.

Actually, for the $dS$-space this provides a very meaningful model like a coin with two sides. On one side, there is the PoI on the $BdS$-space (2.10) together with the law of inertia on inertial systems with respect to a set of inertial observers $O_I$. On another side, the Robertson-Walker-like $dS$-space (3.5) displays the cosmological principle with respect to a set of comoving observers $O_C$. In other words, the ‘cosmic’ background of the Robertson-Walker-like $dS$-space (3.5) supports the PoI on the $BdS$-space (2.10). And conversely, the PoI provides a benchmark of physics related to ‘cosmic’ observations.

**IV. ARE THERE ANY INERTIAL SYSTEMS FOR THE WHOLE UNIVERSE?**

‘Are there at all any inertial systems for very extended portions of the space-time continuum, or, indeed, for the whole universe? ‘[1] For Einstein, the answer seems to be negative unless for the ‘Galilean regions’. However, in view of the $dS$ special relativity, the answer is positive!

Actually, the universe does fix on a kind of inertial systems in the following manner. Firstly, if the universe is accelerated expanding and asymptotic to a $dS$, its fate should be the Robertson-Walker-like $dS$-space (3.5). This is very natural in view of the $dS$ special relativity. Secondly, the time direction and the homogeneous space of the universe tend to the ‘cosmic’ time and the 3-d cosmos as an accelerated expanding $S^3$ of the Robertson-Walker-like $dS$-space, respectively. These set up the directions of the ‘cosmic’ time axis and the spacial axes for the Robertson-Walker-like $dS$ up to some spacial rotations in all them transformed each other by $dS$-group. Thirdly, by means of the important relation between the $BdS$-metric (2.10) and the Robertson-Walker-like $dS$-metric (3.5) by changing the simultaneity, or just simply via the relation (3.1)
between the Beltrami time $x^0$ and the ‘cosmic’ time $\tau$, the directions of the axes of the Beltrami systems can be given. In fact, the Beltrami time axis is related to the ‘cosmic’ time axis in the Robertson-Walker-like $dS$-space, while the spacial axes of the Robertson-Walker-like $dS$-metric (3.3) are just the Beltrami spacial ones in the $BdS$-metric (2.10). Thus, the evolution of the universe does fix on the Beltrami inertial systems.

It is important that such a way of determining the Beltrami systems of inertia is completely different from the way of Einstein [1]. Actually, the gravitation in the universe does not explicitly play any roles here and there is nothing related to Einstein’s ‘argument in a circle.’

In the Beltrami systems, there are two universal constants, $c$ and $R$. In order to set up the Beltrami systems, it is also needed to determine their values concretely. However, it is clear that as inertial-frames the Beltrami systems do not depend on their concrete values unless they are related to observations in the universe. In this case, their values should be given by two independent experiments or observations. Note that these constants are supposed to be invariant and universal approximately. So, the speed of light $c$ may still be taken as that in Einstein’s special relativity, which is just a limiting case $R \rightarrow \infty$ of the $dS$ special relativity. Thus, this also fixes on the origin of the Beltrami systems since the Beltrami light cone (2.13) at the origin is just Minkowskian. As for the value of $R$, it may also be given by $R \simeq (3/\Lambda)^{1/2}$ with the $\Lambda$ being taken in the precise cosmology nowadays. Furthermore, the re-scaling of the curvature radius $R$ may lead to the conformal extension and compactification of the $dS$-space together with that of the Mink-space and the $AdS$-space [14].

It is also clear and important that although the temporal axis of such kind of Beltrami systems can be fixed on by the evolution of the universe in the above manner, the symmetry among all Beltrami systems is still of the $dS$-group so long as the cosmological effects are not be taken into account. Otherwise, the symmetry should be reduced to the group $SO(4)$ for the comoving observations in the universe. This may shed light on the inconsistency between the principle of relativity and the cosmological environment (see, e.g. [17]).

Further, different kinds of PoI together with relevant inertial-frames in all possible kinematics, such as Einstein’s special relativity, Newton mechanics, Newton-Hooke mechanics [13] and so on can be viewed as certain contractions in different limits of $c$ and $R$, respectively. Therefore, the origin of all these PoI should be inherited from the PoI in the $BdS$-space and in this sense they can also be set up by the evolution of the universe.

In conclusion, the Beltrami systems of inertia and their contractions does exist in the universe. Their coordinate axes can be fixed on by the cosmic time’s arrow of the universe via the Robertson-Walker-like $dS$-space, to which the universe is asymptotic. This is independent of the gravitational effects. In this sense, for the PoI in the $dS$ special relativity and all other kinds of PoI as its contractions, there is no longer Einstein’s ‘argument in a circle’ [1].

Of course, in the universe except at its fate as a $dS$-space, there is gravity anywhere and anytime. How to take into account the gravitational effects and what should be done for the PoI? What is the benchmark of physics with gravity?
V. GRAVITY AND LOCALIZED PRINCIPLE OF INERTIA

In view of the $dS$ special relativity, there is no gravity in the $dS$-space. The ‘gravitational effects’ in the $dS$-space with coordinate atlas other than the Beltrami one should be a kind of non-inertial effects. Temperature and entropy in the static $dS$-system are just this case in analogy with the Rindler space in view of Einstein’s special relativity in the $Mink$-space$^{[11]}$. Thus, the $dS$-space does not like a black hole.

In order to describe gravity, we would like to recall Einstein’s description on ‘Galilean regions’ first. In these finite regions, ‘the laws of the special theory of relativity, · · ·, hold with remarkable accuracy.’$^{[1]}$ Namely, all gravitational effects can be ignored on Einstein’s ‘Galilean regions’ in such a way that his special relativity with full Poincaré symmetry should hold locally. This is because all these regions are finite. Although in practice, it may still be regarded as global symmetry approximately with remarkable accuracy.

If there are two such kind of ‘finite regions’ of full local Poincaré invariance at different but nearby positions, how to pass from one to another?

According to Einstein, there should be gravity in-between these ‘regions’. Therefore, in order to transit from one to another, some curved spacetime with gravity in-between should be passed. In other words, in order to connect these ‘regions’ together, some gravitational field as interaction should be taken into account. Since there is local Poincaré symmetry in these ‘regions’, in order to transit in-between, the spacetime with gravity should also be of local Poincaré symmetry! Otherwise, it cannot be consistently transited from one ‘region’ to another if Poincaré symmetry cannot be maintained locally in the course of transition. For any number of such ‘finite regions’, it is the same.

This may also be seen from another angle in terminology of differential geometry. Each of finite ‘Galilean regions’ is essentially a portion of a $Mink$-space with Poincaré symmetry isomorphic to an $R^4$, so that there are intersections among these $Mink$-spaces with different ‘finite regions’ at different positions. And the transition functions on these intersections should also be valued in Poincaré symmetry. Further, these $Mink$-spaces with ‘finite regions’ may be viewed as tangent spaces at different positions of a curved manifold as the spacetime with gravity and the transition functions are valued in local Poincaré symmetry.

Thus, it seems to be the core of Einstein’s idea on gravity that the theory of gravity should be based on the localization of his special relativity with Poincaré group as full symmetry anywhere and anytime on some curved spacetimes. For the sake of definiteness, we name this principle as the localized $PoI$ with full local symmetry or the principle of localization. Mathematically, this indicates that spacetimes with gravity might be such a kind of manifolds that on them the $Mink$-space with (local) full Poincaré symmetry should be as a kind of tangent spaces anywhere and anytime in the universe. If so, the $PoI$ as a benchmark should be localized on the spacetimes with gravity and this should be in consistency with the case of the $Mink$-space as a free spacetime where gravity might be ignored.

But, in general relativity, it is not really the case as was mentioned at beginning.

Due to the asymptotic behavior of the universe and in the light of Einstein’s ‘Galilean regions’ as well as in view of the $dS$ special relativity, we may require that gravity in the universe should
be based on the localization of the $dS$ special relativity with localized PoI in local $dS$-frame anywhere and anytime in the universe. Further, its dynamics should also be properly of local $dS$-invariance characterized by a dimensionless constant $g \simeq (\Lambda G \hbar/c^3)^{1/2} \sim 10^{-61}$ from the cosmological constant $\Lambda$ and the Planck length (see, e.g., [18, 19]). If so, the benchmark of physics is either the PoI on the $dS$-space as a free space on the cosmic scale or its localization with local $dS$-invariance anywhere and anytime in the universe. In addition, the evolution of the universe can also fix on the local inertial frames of $dS$-invariance in the same manner as the case without gravity or where gravitational effects can be ignored at very high accuracy.

A simple model for the $dS$-gravity has implied that these points should work.

In fact, from Cartan connection 1-form $\theta^{ab} = B^{ab} j dx^j \in \mathfrak{so}(1,3)$ and Lorentz frame 1-form $\theta^a = e^a_j dx^j$ on Riemann-Cartan manifold of Einstein-Cartan theory [20, 21, 22], it follows a kind of connections valued at $dS$-algebra [23, 24, 25]

$$B := B_j dx^j, \quad B^a_j := (B^{AB} j)_{A,B=0,...,4} = \begin{pmatrix} B^{ab}_{jk} & R^{-1}e^a_j \\ -R^{-1}e^b_j & 0 \end{pmatrix} \in \mathfrak{so}(1,4).$$  \hspace{1cm} (5.1)

The curvature valued at $dS$-algebra reads:

$$\mathcal{F}_{jk} = (\mathcal{F}^{AB})_{jk} = \begin{pmatrix} F^{ab}_{jk} + 2R^{-2}e^a_{jk} & R^{-1}T^a_{jk} \\ -R^{-1}T^b_{jk} & 0 \end{pmatrix} \in \mathfrak{so}(1,4),$$  \hspace{1cm} (5.2)

where $e^a_{bjk} = \frac{1}{2} (e^a_j e_{bk} - e^a_k e_{bj}), e_{bj} = \eta_{ab} e^a_j, F^{ab}_{jk}$ and $T^a_{jk}$ are curvature and torsion of Cartan connection.

The total action of the model with source may be taken as

$$S_T = S_{GY,M} + S_m,$$  \hspace{1cm} (5.3)

where $S_m$ is the action of source and $S_{GY,M}$ the Yang-Mills-like action of gravity:

$$S_{GY,M} = \frac{1}{4g^2} \int_M d^4x e \text{Tr}_{dS}(\mathcal{F}_{jk} \mathcal{F}^{jk})$$

$$= \int_M d^4x e \left[ \chi(F + 2\Lambda) - \frac{1}{4g^2} F^{ab}_{jk} F_{ab}^{jk} + \frac{\chi}{2} T^a_{jk} T^a_{jk} \right].$$  \hspace{1cm} (5.4)

Here $e = \det(e^a_j)$, a dimensionless constant $g$ should be introduced as usual in gauge theory to describe the self-interaction of the gravitational field, $\chi$ a dimensional coupling constant related to $g$ and $R$, and $F = \frac{1}{2} F^{ab}_{jk} e^a_j$ the scalar curvature of Cartan connection, the same as the action in Einstein-Cartan theory. In order to make sense in comparison with Einstein-Cartan theory, we take $\chi = 1/(8\pi G)$ and $g^{-2} \simeq 3\chi \Lambda^{-1}$ with $\hbar = c = 1$. In fact, $g^2 \simeq G\hbar c^{-3}\Lambda$.

Although the gravitational field equation now should be of Yang-Mills type, this model does pass the observation tests in solar-scale and there are simple cosmic models having ‘Big Bang’. But, different from general relativity, there are ‘energy-momentum-like tensors’ for gravity from the $F^2$ and $T^2$ terms as a kind of the ‘dark stuffs’ in the action (5.4). In fact, by means of the relation between Cartan connection $B^{ab}_{jk}$ and Ricci rotational coefficients $\gamma^{ab}_{jk}$, we may pick up Einstein’s action from Einstein-Cartan’s action $F$, and the rest terms in (5.4) are all ‘dark
stuffs’ in view of general relativity. Thus, this model should provide an alternative framework for the cosmic data analysis.

In this model, there is the cosmological constant $\Lambda$ from local $dS$-symmetry so that it is not just a ‘dummy’ constant at classical level as in general relativity. In fact, this model can be viewed as a kind of $dS$-gravity in a ‘special gauge’ and the 4-dimensional Riemann-Cartan manifolds should be a kind of 4-dimensional umbilical manifolds that there is local $dS$-spacetime together with ‘gauged’ $dS$-algebra anywhere and anytime (see, e.g. [18, 19, 24]).

It is interesting that the model is renormalizable [26] with an $SO(5)$ gauge-like Euclidean action having a Riemann sphere as an instanton. Thus, quantum tunneling scenario may support $\Lambda > 0$. For the gauge-like gravity, asymptotic freedom may indicate that the coupling constant $g$ should be very tiny and link the cosmological constant $\Lambda$ with the Planck length $\ell_P$ properly, since both the $\Lambda$ and Planck scale as fixed points provide an infrared and an ultraviolet cut-off, respectively.

We will explain these issues in detail elsewhere.

VI. CONCLUDING REMARKS

In physics of the last century, symmetry, localization of symmetry and symmetry breaking play very important roles. For the cosmic scale physics without or with gravity, it should be also the case. In view of the $dS$ special relativity and in the light of Einstein’s ‘Galilean regions’, the PoI with maximum symmetry and its localization should still play a central role as the benchmarks of physics in the large scale.

If the universe is asymptotic to a $dS$-space, it should be asymptotic to a slightly closed Robertson-Walker-like $dS$-space, which closely relates to the $BdS$-space with the PoI. Therefore, the evolution of the universe also supports the PoI on the $BdS$-space and fix on the Beltrami systems without Einstein’s ‘argument in a circle’. Thus, the PoI of the $dS$ special relativity is a benchmark of physics on the cosmic scale when gravity can be ignored.

We may require that on the spacetimes with gravity there should be locally the PoI with local inertial frames of full $dS$-symmetry anywhere and anytime. Then, the evolution of the universe can also fix on these local inertial frames. A simple model for the $dS$-gravity has implied these requirements.

Thus, the PoI of the $dS$ special relativity and its localization are consistent benchmarks of physics without or with gravity in the universe.

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