Perturbative Pion Wave function in Coherent Pion-Nucleon Di-Jet Production

L. Frankfurt
School of Physics and Astronomy,
Tel Aviv University, 69978 Tel Aviv, Israel

G. A. Miller
Department of Physics, Box 351560
University of Washington
Seattle, WA 98195-1560
U.S.A.

M. Strikman
Department of Physics, Pennsylvania State University,
University Park, PA 16802, USA

Abstract

A perturbative QCD treatment of the pion wave function is applied to computing the scattering amplitude for coherent high relative momentum di-jet production from a nucleon.

I. INTRODUCTION

Consider a process in which a high momentum (∼ 500 GeV/c) pion undergoes a coherent interaction with a nucleus in such a way that the final state consists of two jets (JJ) moving...
at high transverse relative momentum ($\kappa_\perp > 1 \sim 2 \text{ GeV/c}$). In this coherent process, the final nucleus is in its ground state. This process is very rare, but it has remarkable properties [1]. The selection of the final state to be a $q\bar{q}$ pair plus the nuclear ground state causes the $q\bar{q}$ component of the pion dominate the reaction process. At very high beam momenta, the pion breaks up into a $q\bar{q}$ pair well before hitting the nucleus. Since the momentum transfer to the nucleus is very small (almost zero for forward scattering), the only source of high momentum is the gluonic interactions between the quark and the anti-quark. Because $\kappa_\perp$ is large, the quark and anti-quark must be at small separations—the virtual state of the pion is a point-like-configuration [2]. But the coherent interactions of a color neutral point-like configuration is suppressed by the cancellation of gluonic emission from the quark and anti-quark [3,2]. Thus the interaction with the nucleus is very rare, and the pion is most likely to interact with only one nucleon. For this coherent process, the forward scattering amplitude is almost (since the momentum transfer is not exactly zero) proportional to the number of nucleons, $A$ and the cross section varies as $A^2$. This reaction, in which there are no initial or final state interactions, is an example of (color singlet) color transparency [4,2]. This is the name given to a high momentum transfer process in which the normal strongly absorbing interactions are absent, and the nucleus is transparent. The term “suppression of a color coherent process” could also be used, because it is the quantum mechanical destructive interference of amplitudes caused by the different color charges of a color singlet that is responsible for the reduced nuclear interaction.

The forward angular distribution is difficult to observe, so one integrates the angular distribution, and the $A^2$ variation becomes $\approx A^{4/3}$. But the inclusion of the leading correction to this process, which arises from multiple scattering of the point-like configuration causes a further increase in the $A$-dependence [2]. Actually at sufficiently small $x_N = \frac{2\kappa_\perp^2}{s} \leq \frac{1}{2m_NR_A}$, the situation changes since the quark-antiquark system scatters off the collective gluon field of the nucleus. Since this field is expected to be shadowed, one expects a gradual disappearance of color transparency for $x \leq 0.01$ - this is the onset of perturbative color opacity [1]. Within the kinematical region of applicability of the QCD factorization theorem, the $A$
dependence of this process is given by the factor: $A^{4/3} \left[ G_A(x, Q^2)/G_N(x, Q^2) \right]^2$.

Our interest in this curious process has been renewed recently by experimental progress. The preliminary result from experiments comparing Pt and C targets is a dependence $\sim A^{1.55 \pm 0.05}$, qualitatively similar to our 1993 prediction. It is much stronger than the one observed for the soft diffraction of pions off nuclei (for a review and references see [2]), and it is qualitatively different from the behaviour $\sim A^{1/3}$ suggested in [3].

Since 1993 many workers have been able to make considerable progress in the theory related to the application of QCD to experimentally relevant observables, and we wish to incorporate that progress and improve our calculation. Our particular aim here is to use perturbative QCD to compute the relevant high-$\kappa_t, q\bar{q}$ component of wavefunction of the incident pion. We show here that QCD factorization holds for the leading term which dominates at large enough values of $\kappa_\perp$.

In the following we discuss the different contributions to the scattering amplitudes as obtained in perturbative QCD.

II. AMPLITUDE FOR $\pi N \to NJJ$

Consider the forward ($t = t_{\text{min}} \approx 0$) amplitude, $M$, for coherent di-jet production on a nucleon $\pi N \to NJJ$:

$$M(N) = \langle f, \kappa_\perp, x | \frac{1}{2} \hat{f} | \pi \rangle,$$

where $\hat{f}$ represents the soft interaction with the target nucleon. The initial $| \pi \rangle$ and final $| f, \kappa_\perp x \rangle$ states represent the physical states, which generally involve all manner of multi-quark and gluon components. Our notation is that $x$ is the fraction of the total longitudinal momentum of the incident pion, and $1 - x$ is the fraction carried by the anti-quark. The transverse momenta are given by $\vec{\kappa}_\perp$ and $-\vec{\kappa}_\perp$.

As discussed in the introduction, for large enough values of $\kappa_\perp$, only the $q\bar{q}$ components of the initial pion and final state wave functions are relevant in Eq. (1). This is because
we are considering a coherent nuclear process which leads to a final state consisting of a quark and anti-quark moving at high relative transverse momentum. The quark and anti-quark ultimately hadronize at distances far behind the target, and this part of the process is analyzed by the experimentalists using a well-known algorithm [5].

We continue by letting the $q\bar{q}$ part of the Fock space be represented by $|\pi\rangle_{q\bar{q}}$, then

$$|\pi\rangle_{q\bar{q}} = G_0(\pi)V_{eff}^{\pi}|\pi\rangle_{q\bar{q}}, \quad (2)$$

where $G_0(\pi)$ is the non-interacting $q\bar{q}$ Green's function evaluated at the pion mass:

$$\langle p_{\perp},y \mid G_0(\pi) \mid p'_{\perp},y' \rangle = \frac{\delta^{(2)}(p_{\perp} - p'_{\perp})\delta(y - y')}{m_{\pi}^2 - \frac{p_{\perp}^2 + m_q^2}{y(1-y)}}, \quad (3)$$

where $m_q$ represents the quark mass, $y$ and $y'$ represent the fraction of the longitudinal momentum carried by the quark; and the relative transverse momentum between the quark and anti-quark is $p_{\perp}$ and $V_{eff}^{\pi}$ is the complete effective interaction, which includes the effects of all Fock-space configurations. A similar equation holds for the final state:

$$|f,\kappa_{\perp},x\rangle_{q\bar{q}} = |\kappa_{\perp},x\rangle + G_0(f)V_{eff}^{f}|f,\kappa_{\perp},x\rangle_{q\bar{q}}, \quad (4)$$

$$\langle p_{\perp},y \mid G_0(f) \mid p'_{\perp},y' \rangle = \frac{\delta^{(2)}(p_{\perp} - p'_{\perp})\delta(y - y')}{m_f^2 - \frac{p_{\perp}^2 + m_q^2}{y(1-y)}}, \quad (5)$$

$$m_f^2 \equiv \frac{\kappa_{\perp}^2 + m_q^2}{x(1-x)}, \quad (6)$$

in which the first term on the right-hand-side of (4) is the plane-wave part of the wave function.

The use of the wave functions (2) and (4) in the equation (1) for the scattering amplitude yields

$$\mathcal{M}(N) = \frac{1}{2}(T_1 + T_2),$$

$$T_1 \equiv \langle \kappa_{\perp},x \mid \hat{f} \mid \pi \rangle, \quad T_2 \equiv \langle q\bar{q} \mid f,\kappa_{\perp},x \mid V_{eff}^{f}G_0(f)\hat{f} \mid \pi \rangle_{q\bar{q}}. \quad (7)$$

The term $T_2$ includes the effect of the final state $q\bar{q}$ interaction; this was not included in our 1993 calculation [1], but its importance was stressed in [6]. We shall first evaluate $T_1$, and then turn to $T_2$. 

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III. EVALUATION OF $T_1$

The wave function $| \pi \rangle_{q\bar{q}}$ is dominated by components in which the separation between the constituents is of the order of the diameter of the physical pion, but there is a perturbative tail which accounts for short distance part of the pion wave function. This perturbative tail is relevant here because we need to take the overlap with the final state which is constructed from constituents moving at high relative momentum. If we concentrate on those aspects it is reasonable to consider only the one gluon exchange contribution $V^g$ to $V^{\pi}_{\text{eff}}$ and pursue the Brodsky-Lepage analysis \cite{7} for the evaluation of this particular component. Their use of the light cone gauge $A^+ = 0$, simplifies the calculation. We also use their normalization and phase-space conventions.

We want to draw attention to the issue of gauge invariance. The pion wave function is not gauge invariant, but the sum of two-gluon exchange diagrams for the pion transition to $q\bar{q}$ is gauge invariant. This is because only the imaginary part of the scattering amplitude survives in the sum of diagrams, and because two exchanged gluons are vector particles in a color singlet state as a consequence of Bose statistics. So, in this case, conservation of color current has the same form as conservation of electric current in QED. We also note, that in the calculation of hard high-momentum transfer processes, the $q\bar{q}$ pair in the non-perturbative pion wave function should be considered on energy shell. Corrections to this enter as an additional factor of $\frac{1}{\kappa L}$ in the amplitude.

We define the non-perturbative part of the momentum space wave function as

$$\psi(l_{\perp}, y) \equiv \langle l_{\perp}, y | \pi \rangle_{q\bar{q}}.$$ \hspace{1cm} (8)

We use the one-gluon exchange approximation to the exact wave function of Eq. (2) to obtain an approximate wave function, $\chi$, valid for large values of $k_{\perp}$.

$$\chi(k_{\perp}, x) = -4\pi C_F \frac{1}{m_{\pi}^2} \int_0^1 dy \int \frac{d^2l_{\perp}}{(2\pi)^3} V^g(k_{\perp}, x; l_{\perp}, y) \psi(l_{\perp}, y)$$ \hspace{1cm} (9)

with
\[ V^g(k_\perp, x; l_\perp, y) = -4\pi C_F \alpha_s \frac{\bar{u}(x, k_\perp)}{\sqrt{x}} \gamma_\mu \frac{u(y, l_\perp)}{\sqrt{y}} \frac{\bar{v}(x, -k_\perp)}{\sqrt{1-x}} \gamma_\nu \frac{v(1-y, -l_\perp)}{\sqrt{1-y}} y^{\mu\nu} \]

\[
\times \left[ \frac{\theta(y-x)}{y-x} \frac{1}{m^2_\pi - \frac{k^2_\perp - m^2_\pi}{x} - \frac{l^2_\perp + m^2_\pi}{1-y}} \right], \quad (10)
\]

and \( C_F = \frac{n_c^2 - 1}{2n_c} = \frac{4}{3} \). The range of integration over \( l_\perp \) is restricted by the non-perturbative pion wave function \( \psi \). Then we set \( l_\perp \) to 0 everywhere in the spinors and energy denominators and evaluate the strong coupling constant at \( k^2_\perp \):

\[ \alpha_s(k^2_\perp) = \frac{4\pi}{\beta \ln \frac{k^2_\perp}{\Lambda^2}}, \quad (11) \]

where \( \beta = 11 - \frac{2}{3} n_f \). Then

\[ V^g(k_\perp, x; l_\perp, y) \approx -4\pi C_F \alpha_s(k^2_\perp) \frac{x(1-x)}{x(1-x)y(1-y)} V^{BL}(x, y) \quad (12) \]

where \( V^{BL}(x, y) \) is the Brodsky-Lepage kernel:

\[ V^{BL}(x, y) = 2 \left[ \frac{\theta(y-x)}{x-y} x(1-y) + \frac{\Delta}{x-y} + (x \rightarrow 1-x, y \rightarrow 1-y) \right], \quad (13) \]

with the operator \( \Delta \) defined by \( \frac{\Delta}{x-y} \phi(x) = \frac{\phi(x) - \phi(y)}{x-y} \). This kernel includes the effects of vertex and quark mass renormalization. The net result for the high \( k_\perp \) component of the pion wave function is then

\[ \chi(k_\perp) = \frac{4\pi C_F \alpha_s(k)}{k^2_\perp} \int_0^1 dy V^{BL}(x, y) \frac{\phi(y, k^2_\perp)}{y(1-y)}, \quad (14) \]

where

\[ \phi(y, q^2_\perp) \equiv \int \frac{d^2 l_\perp}{(2\pi)^3} \theta(q^2_\perp - l^2_\perp) \psi(l_\perp, y). \quad (15) \]

The quark distribution amplitude \( \phi \) can be obtained using QCD evolution [7]. Furthermore, the analysis of experimental data for virtual Compton scattering and the pion form factor performed in [13,14] shows that this amplitude is not far from the asymptotic one for \( k^2_\perp \geq 2 - 3 \text{ GeV}^2 \)

\[ \phi(x) = a_0 x(1-x), \quad (16) \]
where \( a_0 = \sqrt{3} f_\pi \) with \( f_\pi \approx 93 \text{ MeV} \).

Equation (14) represents the high relative momentum part of the pion wave function. Using the asymptotic function (16) in Eq. (14) leads to an expression for \( \psi(k, x) \propto x(1 - x)/k^2 \) which is of the factorized form used in Ref. [1].

To compute the amplitude \( T_1 \), it is necessary to specify the scattering operator \( \hat{f} \). For high energy scattering the operator \( \hat{f} \) changes only the transverse momentum and therefore in the coordinate space representation \( \hat{f} \) depends on \( b^2 \). The transverse distance operator \( \vec{b} = (\vec{b}_q - \vec{b}_{\bar{q}}) \) is canonically conjugate to \( \vec{\kappa}_\perp \). At sufficiently small values of \( b \), the leading twist effect and the dominant term at large \( s \) arises from the diagrams when pion fragments into two jets as a result of interactions with the two-gluon component of gluon field of a target, see Figure 1.

The perturbative QCD determination of this interaction involves a diagram similar to the gluon fusion contribution to the nucleon sea-quark content observed in deep inelastic scattering. One calculates the box diagram for large values of \( \kappa_\perp \) using the wave function of the pion instead of the vertex for \( \gamma^* \to q\bar{q} \). The application of QCD factorization theorem leads [2,8,9] to

\[
\hat{f}(b^2) = is\pi^2 b^2 \left[ x_N G_N(x_N, \lambda/b^2) \right] \alpha_s(\lambda/b^2),
\]

in which \( x_N = 2\kappa_\perp^2/s \) where \( G_N \) is the gluon distribution function of the nucleon, and \( \lambda(x = 10^{-3}) = 9 \) according to Frankfurt, Koepf, and Strikman, [10]. Accounting for the difference between the pion mass and mass of two jet system requires us to replace the target gluon distribution by the the skewed gluon distribution. The difference between both distributions is calculable in QCD using the QCD evolution equation for the skewed parton distributions [11,12].

The most important effect shown in Eq. (17) is the \( b^2 \) dependence which shows the diminishing strength of the interaction for small values of \( b \). In the leading order approximation it is legitimate to rewrite \( \sigma \) in the form:

\[
\hat{f}(b^2) = is\sigma_0 \frac{b^2}{\langle b_0^2 \rangle},
\]

where \( \sigma_0 = \sqrt{3} f_\pi \).
in which the logarithmic dependence on $b^2$ is neglected. Our notation is that $\langle b^2_0 \rangle$ represents the pionic average of the square of the transverse separation, and

$$\frac{\sigma_0}{\langle b^2_0 \rangle} \approx \frac{\pi^2}{3} \alpha_s (\kappa^2) [x_N G_N(\text{skewed}) (x_N, \kappa^2)].$$

(19)

The use of Eq. (18) allows a simple evaluation of the scattering amplitude $T_1$ because the $b^2$ operator acts as $-\nabla^2_{\kappa \perp}$. Using Eqs. (2) and (18) in Eq. (7), leads to the result:

$$T_1 = -4i \frac{\sigma_0}{\langle b^2 \rangle} \frac{4\pi C_F \alpha_s (\kappa^2)}{\kappa^4} \left(1 + \frac{1}{\ln \frac{\kappa^2}{\Lambda^2}}\right) a_0 x(1 - x).$$

(20)

This is, except for the small $\frac{1}{\ln \frac{\kappa^2}{\Lambda^2}}$ correction ($\kappa^2 \approx 2$ GeV and $\Lambda \approx 0.2$) arising from taking $-\nabla^2_{\kappa \perp}$ on $\alpha_s$, is of the same form as the corresponding result of our 1993 paper. The amplitude of Ref. [3] varies as a Gaussian in $\kappa^\perp$.

The $\kappa^\perp$ dependence: $\frac{d\sigma(\kappa^\perp)}{d\kappa^\perp} \propto \frac{1}{\kappa^\perp}$ follows from simple reasoning. The probability to find a pion at $b \leq \frac{1}{\kappa^\perp}$ is $\propto b^2$, while the square of the total cross section for small dipole-nucleon interactions is $\propto b^4$. Hence the cross section of productions of jets with sufficiently large values of $\kappa^\perp$ is $\propto \frac{1}{\kappa^\perp}$ leading to a differential cross section $\propto \frac{1}{\kappa^\perp^2}$. Similar counting can be applied to estimate the $\kappa^\perp$ dependence for diffraction of a nucleon into three jets.

**IV. OTHER AMPLITUDES**

So far we have emphasized that the amplitude we computed in 1993 is calculable using perturbative QCD. However there are four different contributions which occur at the same order of $\alpha_s$. The previous term in which the interaction with the target gluons follows the gluon-exchange represented by the potential $V^g$ in the pion wave function has been denoted by $T_1$. But there is also a term, in which the interaction with the target gluons occurs before the action of $V^g$ is denoted as $T_2$, see Figure 2. However, the two gluons from the nuclear target can also be annihilated by the exchanged gluon (color current of the pion wave function). This amplitude, denoted as $T_3$, is shown in Figure 3. The sum of diagrams when one target gluon is attached before the potential $V^g$ and a second after the potential $V^g$, see E.g. Figure 4, corresponds to an amplitude, $T_4$.
We briefly discuss each of the remaining terms $T_2, T_3, T_4$. Their detailed evaluation will appear in a later publication. However we state at the outset that each of these amplitudes is suppressed by color coherent effects, and that each has the same $\kappa_{-4}$ dependence. The existence of the term $T_2$, which uses an interaction that varies as $b^2$, caused Jennings & Miller \[6\] to worry that the value of $M_N$ might be severely reduced due to a nearly complete cancellation. Our preliminary and incomplete estimate obtained by neglecting the term arising from differentiation of the potential, $V^g$ finds instead enhancement.

The $T_3$ or meson-color-flow term arises from the attachment of both target gluons to the gluon appearing in $V^g$ as well as the sum of diagrams where one target gluon is attached to potential $V^g$ in the pion wave function and another gluon is attached to a quark. This term is suppressed by color coherent destructive interference caused by the color neutrality of the $qar{q}g$ intermediate state. Thus this term has a form which is very similar to that of $T_1$ and $T_2$. The $T_4$ term arises from the sum of diagrams when one target gluon interacts with a quark in the pion wf before exchange by potential $V^g$ and second gluon interacts after that. The sum of these diagrams seems to be $O$ because, for our kinematics, the $q$ and $\bar{q}$ in the initial and final states are not causally connected in these diagrams. The mathematical origin of this near $0$ arises from the sum of diagrams having the form of contour integral: $\int d\nu \frac{1}{(\nu-a-i\epsilon)(\nu-b-i\epsilon)}$, which vanishes because one can integrate using a closed contour in the lower half complex $\nu$-plane.

V. SUMMARY DISCUSSION

The purpose of this paper has been to show how to apply leading-order perturbative QCD to computing the scattering amplitude for the process: $\pi N \rightarrow JJ$. The high momentum component of the pion wave function, computable in perturbation theory is an essential element of the amplitude. Another essential feature of our result (20) is the $\sim \frac{1}{\kappa_{-4}}$ dependence of the amplitude manifest in a $\kappa_{-8}$ behavior of the cross section. This feature needs to be observed experimentally before one can be certain that the experiment \[5\] has verified the
prediction of Ref. [1].

VI. ACKNOWLEDGEMENTS

It is a pleasure to dedicate this work to Prof. Kurt Haller, who has been recently interested in color transparency [15], and who has long been interested in the fundamentals of QCD as applied to light-cone physics [16]. Happy birthday, Kurt, and our best wishes for many more to come.

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FIGURES

Figure 1. Contribution to $T_1$. The high momentum component of the pion interacts with the two-gluon field of the target. Only a single diagram of the four that contribute is shown.

Figure 2. Contribution to $T_2$. The high momentum component of the final $q\bar{q}$ pair interacts with the two-gluon field of the target. Only a single diagram of the four that contribute is shown.

Figure 3. Contribution to $T_3$. The exchanged gluon interacts with the two-gluon field of the target. Only a single diagram of the several that contribute is shown.

Figure 4. Contribution to $T_4$. A gluon interacts with a quark and another with the exchanged gluon. Only a single diagram of the several that contribute is shown.
