The Reconstruction Quality Improvement of Single-Pixel Imaging via Modified Split-Bregman Iteration

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ABSTRACT A modified Split-Bregman iteration method is proposed to reconstruct the image of target in computational ghost imaging. Different from the conventional Split-Bregman iteration, each reconstruction result of Split-Bregman iteration is denoised by BM3D filter in the proposed, which can help reduce the noise of the final reconstructed result effectively. The method has high quality reconstruction effectiveness and is able to retain the edge and texture detail information well. The numerical simulation and experiment results verify the feasibility of the proposed method. Comparing with other methods qualitatively and quantitatively, our method has high performance in reconstructing image of target.

INDEX TERMS Ghost imaging, high reconstruction quality, iteration with filtering method, modified Split-Bregman iteration.

I. INTRODUCTION

Single-pixel imaging using a single-pixel detector in the imaging hardware may offer better detection efficiency, and lower noise than other imaging systems based on the pixilated detector array. Ghost imaging (GI), whose imaging architecture is essentially the same as the architecture of single-pixel imaging, can reconstruct the intensity information of targets based on the second order or higher order correlation of light field fluctuations [1]–[3]. The detection process of this non-local single-pixel imaging system is distinguished from the imaging process. Because of the advantages of non-lens imaging, single pixel imaging, and high robustness, GI has drawn great attention of researchers in the field of optical imaging, such as fluorescence imaging [4], [5], remote sensing [6], [7], X-ray imaging [8], [9], optical encryption [10], [11], three-dimensional imaging [12], [13], and so on.

The conventional ghost imaging (CGI) reconstruction algorithm, which is based on the correlation calculation between the measurements by the bucket detector and array detector, requires a great amount of measurements to guarantee the signal-noise-rate (SNR) of the reconstructed results. The huge number of measurements means low imaging frame rate. In order to obtain the high-quality results with fewer measurements, many researchers pay attention to the methods of improving the reconstruction quality of ghost imaging and reconciling the contradiction between the low sampling time and high SNR [14], [15]. In order to address the problem of low frame rate or low SNR, there are many different approaches, such as high-speed structured illumination [16], orthogonal sampling [17], and sub-pixel micro-scanning [18]. Moreover, the computational algorithms introduce competitive edges from the software perspective, due to ever-increasing processing power. One such example is applying the compressive sensing algorithm in the single-pixel imaging to reconstruct high quality intensity images of targets under the condition of sub-Nyquist sampling rate. Hence, the compressed sensing ghost imaging (CSGI) has the advantage of reconstructing high-quality results with few measurements [19]–[22]. Meanwhile, there is no reference path in the compressed ghost imaging system, which makes ghost imaging system much easier to be miniaturized and integrated.
The most important issue of CSGI is to find the efficient algorithm to solve the ill-posed problems. The image reconstruction algorithm of the compressed sensing can be divided into two main categories, which are the greedy algorithm and convex optimization algorithm respectively [23]–[27]. Most recently, with the wide application of image filtering method in the field of image processing and image denoising, the combination of the image filtering algorithm and compressed sensing algorithm has become one research focus. This kind of combination algorithm can reconstruct an image with higher quality than the other algorithms and is widely used in CSGI system [28], [29]. Inspired by the combination algorithm, this paper proposes a combination algorithm of the split Bregman iteration [30], [31] and BM3D filter [32], [33] to reconstruct target intensity images from the data measured by the bucket detector in the CSGI system. In the proposed algorithm, the split Bregman iteration is used to solve the L1-norm optimization problem. The BM3D filter is embedded in each iteration to denoise every reconstructed result. The denoised result is set as the original image of the next iteration, until the number of iterations reaches the maximum or the reconstruction result satisfies the iteration termination condition. The proposed method has the advantages of stable numerical solution process, fast convergence speed, easy realization, and high-quality reconstruction. Therefore, the proposed algorithm is suitable to apply in the CSGI system to reconstruct the image of targets.

![FIGURE 1. The schematic of CSGI system.](image)

II. THEORETICAL FRAMEWORK OF MODIFIED SPLIT-BREGMAN ITERATION

Fig. 1 shows the schematic of the CSGI system. In the CSGI system, the beam emitted from the laser source is expanded by a beam expander and illuminates a digital mirror device (DMD) controlled by the computer. M pre-generated patterns \( S = \{s^{(1)}, s^{(2)}, \ldots, s^{(M-1)}, s^{(M)}\} \) are sent from the computer to the DMD, and the DMD modulates the spatial distribution of the light according to the patterns. Suppose that the intensity \( I_0 \) of light illuminated on the DMD is uniform everywhere. The spatial distribution of the light is \( I = \{I_0s^{(1)}, I_0s^{(2)}, \ldots, I_0s^{(M-1)}, I_0s^{(M)}\} = \{I^{(k)}(x, y)|i = 1, \ldots, M\} \). Here, \( I^{(k)}(x, y) \) represents the spatial distribution in the \( k \)th measurement, and \((i, j)\) is the spatial transverse coordinates. The modulated light emits through the optical transmitting antenna and illuminates the targets. The reflectivity distribution of the targets is denoted as \( T(i, j) \). In the \( k \)th measurement, the spatial distribution of the light intensity reflected by the targets is expressed as:

\[
\hat{I}^{(k)}(i, j) = T(i, j)I^{(k)}(i, j) \tag{1}
\]

The total reflected light is received by the optical receiving antenna and collected by a bucket detector with no spatial resolution. The measurement of the bucket detector in the \( k \)th measurement is:

\[
y^{(k)} = \sum_{(i,j)\in T} T(i, j)I^{(k)}(i, j) \tag{2}
\]

In the image reconstruction algorithm of the CGI, the reconstruction process calculates the 2nd order correlation between the measurements of the bucket detector and the spatial distribution of the light. The reconstruction result of this method is relatively low in SNR and the contrast ratio. Generally, the compressed sensing algorithms are used to improve the reconstruction efficiency.

In compressed sensing algorithms, the whole \( M \) speckle patterns are denoted in matrix form. Each speckle pattern is expressed as a row vector and placed in matrix \( \mathbf{I} \) in the order of the spatial distribution modulation. The numbers of pixels in the horizontal direction and vertical direction are \( p \) and \( q \) respectively. The number of elements in each vectorization of speckle pattern is \( N = p \times q \). The matrix of speckle distribution is:

\[
\mathbf{I} = \begin{bmatrix}
I^{(1)}(1,1) & \cdots & I^{(1)}(p,q) \\
\vdots & \ddots & \vdots \\
I^{(M)}(1,1) & \cdots & I^{(M)}(p,q)
\end{bmatrix}_{M \times N} \tag{3}
\]

Similar to the matrix of the speckle distribution, the reflectivity distribution of targets can be expressed as a column vector \( \mathbf{x} \):

\[
\mathbf{x} = \begin{bmatrix}
T(1,1) \\
\vdots \\
T(p,q)
\end{bmatrix}_{N \times 1} \tag{4}
\]

According to (2), the \( M \) measurements of bucket detector is:

\[
\mathbf{y} = \begin{bmatrix}
y^{(1)} \\
y^{(2)} \\
\vdots \\
y^{(M)}
\end{bmatrix}_{M \times 1} = \begin{bmatrix}
\sum_{i}^{p} \sum_{j}^{q} T(i, j)I^{(1)}(i,j) \\
\vdots \\
\sum_{i}^{p} \sum_{j}^{q} T(i, j)I^{(M)}(i,j)
\end{bmatrix} = \mathbf{I} \mathbf{x} \tag{5}
\]

In (5), the column vector \( \mathbf{y} \) is the measurements of the bucket detector and the speckle distribution matrix \( \mathbf{I} \) are pre-generated by the computer. The column vector \( \mathbf{x} \) is requested...
to be solved or approximately solved based on the vector \( y \) and matrix \( I \), which is also the essence of CSGI. The goal of CSGI is to reconstruct high-quality results through fewer measurements. Hence, the number of measurement \( M \) is always much smaller than the number of pixels \( N \). The results in (5) is an underdetermined system of equation. The solution is not unique and is difficult to be obtained. According to the theory of the compressed sensing, the method of implicit regularization can be utilized to solve (5), such as iterative methods of algebraic recovery and unconstrained L2-norm regularization methods. These methods are easy to be implemented, but the potential problem is that these methods may over-regularize the results and lose the edge information of the imaging results. In the presented method in this paper, the L1-norm of image gradient, namely the total variation (TV), is regularized to solve (5) in order to guarantee the high quality of reconstruction result and enhance the protection of edge information.

The TV is defined as \( TV(x) = ||\nabla x||_1 \). (5) can be solved through an unconstrained optimization process:

\[
\arg \min_x TV(x) + \frac{\lambda}{2} \|Ix - y\|_2^2,
\]

where \( \lambda \) is the regularization parameter, and \( ||\cdot||_1 \) is the L1-norm. As shown in (6), there is a TV functional in the optimization process. Due to the nonlinear and non-differentiable of the TV functional, a stable and efficient algorithm is required to solve this optimization problem. Therefore, the split Bregman iteration is adopted to optimize (6).

The split Bregman iteration can be used to solve the optimization problems as (6). As an efficient algorithm with fast convergence speed and less computational resource consumption, this algorithm is widely used in image de-blurring and non-blind image deconvolution. The process of the split Bregman iteration is the iteration of the following equations [31]:

\[
x^{k+1} = \min_x TV(x) + \frac{\lambda}{2} \|Ix - y\|_2^2 + \frac{\alpha}{2} \|x^k - x - b^k\|,
\]

\[
v^{k+1} = \max (x^{k+1} + b^k, 0),
\]

\[
b^{k+1} = b^k + x^{k+1} - v^{k+1},
\]

\[
y^{k+1} = y^k + v^{k+1} - Ix^{k+1},
\]

where vector \( b \) and \( v \) are auxiliary variables in the Bregman iteration, and both original values are set as 0. \( k \) is the number of iterations. \( \lambda \) and \( \alpha \) are regularization parameters, which are chosen as 0.6 and 1 respectively. Generally, the regularization parameters of Split-Bregman allow to be slightly tuned in the algorithm if needed, but it is generally quite robust. The function \( \max(x, y) \) represents getting the larger value of \( x \) and \( y \). For (7), the stable double conjugate gradient algorithm is utilized to obtain the solution more quickly. In the split Bregman iteration, an iteration maximum is set. When the number of iterations reaches the maximum, the calculation stops and outputs the results. The iteration maximum is set as 140.

In each iteration, a preliminary reconstruction result can be obtained, and there is some noise in the results. In order to further improve the quality of each iteration, the result \( x^k \) needs to be denoised after every iteration. Here, the BM3D algorithm is used to denoise the result \( x^k \) in each iteration. The BM3D algorithm aggregates 2D image blocks with similar structures into the 3D images, and the 3D images are denoised through collaborative filtering. Although the BM3D algorithm [32] costs some computational resource in searching and matching similar blocks, the algorithm still has outstanding filtering performance. Especially, it can preserve the edge and texture details of reconstruction results.

In this paper, the size of the reference block in the BM3D algorithm is selected as \( 8 \times 8 \), the sliding step is 3, and the length of the searching neighborhood for full-search block-matching is 35. The image similarity is measured by the Euclidean distance between the reference block and similar block. The threshold for the block-distance is set as 3500. In order to obtain the better denoised result, the wavelet BIOR1.5 is used for the 2-dimensional transform and the wavelet HAAR is used for the transform of the 3rd dimension, in the collaborative filtering process of basic estimate step. The discrete cosines transform is used for 2-dimensional transform and the wavelet HAAR is used for the transform of the 3rd dimension in the collaborative filtering process of the final estimate step. Through the combination between the split Bregman iteration and BM3D filtering, the high-quality reconstruction can be obtained.

III. NUMERICAL SIMULATION RESULTS OF GHOST IMAGING WITH MODIFIED SPLIT-BREGMAN ITERATION METHOD

The numerical simulations are presented to verify the effectiveness of the proposed method. In the numerical simulations, the target is chosen as a binary image and grayscale image respectively. The simulation is performed in Matlab R2016a. The CPU of the computer is Intel(R) Core(TM) i7-8550(2.00GHz), the memory is 8.00GB and the operating system is 64-bit win10.

A. NUMERICAL SITUATION OF BINARY IMAGE

Firstly, the target is chosen as a binary image which contains 4 characters “ZSTU”. The total number of image pixels is \( N = 64 \times 64 \). The reflectivity of the pixels in the area where there are no characters is set as 1. Other pixels in the image is set as 0. The spatial distribution of speckles is chosen as the Hadamard matrix. The Hadamard matrix is that using Hadamard matrix can bring two advantages, which are perfect reconstruction in principle and measurement reduction [34], [35]. The number of measurements in the simulation is 1000, which is corresponding to the sample rate \( \beta = 0.244 \). The numerical simulation results are shown in Fig. 2.

Fig. 2(a) is the original image of the target which is a binary image. Fig. 2(b) is the reconstructed image with the presented method. Fig. 2(b) shows that the target image can
be reconstructed with high quality utilizing the proposed algorithm. There is rarely noise in the reconstructed image. Furthermore, this method can retain the edge information while denoising.

In order to quantitatively evaluate the reconstruction capability of the proposed method, the peak signal to noise ratio (PSNR) and structural similarity (SSIM) of reconstruction image are calculated respectively. PSNR and SSIM can be expressed as:

\[
\text{PSNR} = 10 \times \log_{10}(\frac{\text{MAX}^2}{\text{MSE}}),
\]

\[
\text{SSIM}(x, y) = \frac{(2\mu_x\mu_y + c_1)(2\sigma_{xy} + c_2)}{\mu_x^2 + \mu_y^2 + c_1(\sigma_x^2 + \sigma_y^2 + c_2)}.
\]

In (11), MAX is the maximum possible value of the image pixels. In the simulation, the maximum value of image pixels is 1, therefore \(\text{MAX}^2 = 1\). MSE is the mean square error of the image. For the target image with the pixel number of \(N = n \times m\), the MSE is defined as:

\[
\text{MSE} = \frac{1}{nm} \sum_{i=1}^{m} \sum_{j=1}^{n} |O(i, j) - R(i, j)|^2,
\]

where \(O(i, j)\) represents the original image, \(R(i, j)\) represents the reconstructed image, and \(i, j\) represent the horizontal and vertical coordinates of pixels respectively.

In (12), \(\mu_x\) and \(\mu_y\) are the means of total pixels in the original image and reconstructed image respectively. \(\sigma_x^2\) and \(\sigma_y^2\) are the variances of the original image and reconstructed image. \(\sigma_{xy}\) is the covariance of the original image and the recovered image. \(c_1\) and \(c_2\) are two small positive numbers, which are used to avoid dividing by 0. When the SSIM is calculated, \(c_1\) and \(c_2\) are selected as \(10^{-4}\) and \(9 \times 10^{-4}\). By calculation, the PSNR of the reconstructed image with the proposed method is 16.835 and the SSIM is 0.8848.

### B. NUMERICAL SITUATION OF GRAYSCALE IMAGE

It can be concluded from the visual effect and the quantitative calculation results of the numerical simulation that the proposed method can successfully reconstruct the intensity distribution of the binary image in quality. In order to verify the effectiveness of the algorithm when the targets are more complex, several grayscale images as the targets are chosen. The targets are “circles-bright-dark”, “camera”, and “house” respectively. Meanwhile, the reconstructed results with the proposed algorithm are compared with some other common ghost imaging algorithms to verify its better performance. The compared algorithms are the CGI algorithm in [36], differential ghost imaging (DGI) algorithm in [37], orthogonal matching pursuit (OMP) algorithm in [26], alternating direction method of multipliers (ADMM) algorithm in [38], conventional Split-Bregman (CSB) algorithm in [39], projected Landweber regularization with guide filter (PLRG) algorithm in [28] and Total Variation Augmented Lagrangian Alternating Direction (TVAL3) algorithm in [40]. The reconstructed results of above algorithms are shown in Fig. 3.

In Fig. 3 (a)-(c), the original image of different targets in the simulation are shown on the left side. The pixel number of each image is \(64 \times 64\). Similar to the simulation of the binary image, the numbers of measurements are 200, 1000 and 2000. The measurement matrix is the Hadamard matrix. On the right side of each column of Fig. 3 (a)-(c), the reconstructed results of different algorithms with different measurements are shown. Every row of Fig. 3 (a)-(c) represents the simulation results of each algorithm. And every row of Fig. 3 (a)-(c) represents the results with a certain number of measurements for all the algorithms. As Fig. 3 shows, the reconstructed results with CGI are the worst of all. The SNR and contrast ratio of result are rather low. The edge and texture detail information of the targets can hardly be observed from the reconstructed image. The reconstructed results with DGI are better than CGI. However, the noise level of the reconstructed image is still high. Especially, when there are relatively few measurements, the information of original image cannot be observed from the reconstructed results of CGI and DGI. The OMP algorithm is better than the CGI algorithm. Compared with the CGI algorithm, the OMP algorithm improves the SNR and contrast ratio, although the results are still blurry. Compared with CGI, DGI and OMP algorithm, the ADMM algorithm and CSB algorithm have better reconstruction quality. The details of the images result are clearer. The high noise level is still the weakness of the two algorithms. The reconstructed results with the PLRG method, which is also based on the iteration filter strategy, have less noise. But the reconstructed results with PLRG lose a lot of detail and edge information of original image. In contrast, the reconstructed results with TVAL3 algorithm and our algorithm have higher contrast ratios and SNRs. The edge and texture detail of the results are well remained. When there is few measurements, such as 1000, the image reconstruction ability of the proposed algorithm is stronger and the visual effect is better than the TVAL3 algorithm. From the visual effect of the reconstructed image, the algorithm in this paper has high quality performance of imaging reconstruction.

These 8 algorithms with different measurements are also be quantitatively compared by calculating the PSNR and SSIM,
shown in Fig. 4. The blue bars in Fig. 4 represent the PSNR index and the red bars represent the SSIM index. It can be observed that the quality judgement indexes based on the proposed algorithm have superior performance. The proposed method has higher PSNR and SSIM than other methods under different measurement numbers. Meanwhile, the complexity of the scene also influences the reconstruction quality. The image of “circle-bright-dark” has the simplest structure, which results in that all of the methods have the high PSNR and SSIM with few measurements, such as 200. The PSNR and SSIM of the proposed method reach 22.648 and 0.822 respectively. The images of “Cameraman” and “House” are more complex. When there are 200 measurements, all the algorithms can hardly get high quality reconstruction, and thus the values of PSNR and SSIM with different algorithm are relatively low and close to each other. When there are more measurements, the reconstruction abilities of algorithms are revealed. The proposed algorithm has almost the highest values in the comparison of 1000 and 2000 measurements. The quantitative calculated results (as shown in Fig. 4) are in agreement with the qualitative visual effect (as shown in Fig. 3).

IV. EXPERIMENT RESULTS OF GHOST IMAGING WITH MODIFIED SPLIT-BREGMAN ITERATION METHOD

An experiment is conducted based on the schematic of the CSGI system as Fig. 1 shows. In the experiment, the laser source is a single mode fiber laser (VLSS-1064-M) whose wavelength is 1064nm and the pulse power is 300mW. The light beam is expanded by a beam expander (BE05-1064), and illuminates on a DMD (DPLLCR6500EVM). The DMD modulates the speckle distribution utilizing the Hadamard matrix. The dimension of the modulation matrix is set to $64 \times 64$. A camera lens is chosen as the transmitting antenna. The diameter of the camera lens is 50mm and the focus length of camera lens is 100mm. The target is placed 3m away from the transmitting antenna. And the distance between the DMD and transmitting antenna is 10cm. Therefore, according to the experiment parameters, the amplification factor of the imaging system can be calculated as 0.033 [41]. Light reflected from the targets is collected by a single pixel detector (DET36A2). The detector is connected to a data acquiring card (NI-PXI-5154) to measure the total light intensity. The bandwidth of the data acquiring card is 2GHz and the sampling frequency is 5GHz/s.
In the experiment, the number of total measurements is similar to the numerical simulations, which are 200, 1000 and 2000 respectively. The target scene is shown on the left side of Fig. 5. There are three targets in the field of view, which are two white foam boards and a plaster model of horsehead. One of foam boards is solid, and the middle area of the other one is hollowed out. Hence, the base of the plaster model of the horse head is visible through the first white foam plate. The reconstructed results of different algorithms are shown on the right side of Fig. 5. The pictures on different rows represent the reconstructed results of each algorithm with different measurement numbers. It can be observed that the result with CGI and DGI algorithms are badly fuzzy. When the number of measurements is relatively small, there is no target information obtained from the results with the two algorithms. The reconstructed results of OMP and PLRG algorithms have good denoising ability, but the problem is that the reconstructed results of the two algorithms are remarkably smoothed. Some details of the original image are lost. The reconstructed results of ADMM and CSB
algorithms can remain the edge and detail information. The noise, however, decreases the quality of reconstructed results. Comparing with other algorithms, TVAL3 and the proposed algorithm are both more powerful in edge-preserving while smoothing the image. And the proposed algorithm has higher reconstruction precision. It can be observed that there are many patches in the horse face and foam board of the reconstructed results with the TVAL3 algorithm, which is not consistent with the real target. Therefore, the visual effects of the reconstructed images show that the proposed method has the highest reconstruction performance, which not only removes the noise better, but also retains more edge and detail information.

Furthermore, the PSNR and SSIM of the reconstructed images utilizing different methods are calculated based on the experiment data. The curves of the PSNR and SSIM of reconstructed results with measurement numbers are shown in Fig. 6.

![Fig. 6. The curves of reconstruction quality of different algorithms with measurement numbers. (a) The relationship between the PSNR and the number of measurements. (b) The relationship between the SSIM and the number of measurements.](image)

In Fig. 6, Different colors of curves represent different reconstruction algorithms. It can be seen that the number of measurements affects the reconstruction quality of all the algorithms. When the number of measurements is small, the PSNR and SSIM of reconstructed results are all relatively low and the PSNR and SSIM of the proposed algorithm is higher than other algorithms. With the increase of measurement numbers, the PSNR and SSIM indexes of the proposed algorithm improve rapidly and remain optimal. When the measurement number increases to a certain value, the improvement of the reconstruction quality becomes insignificant, and the PSNR and SSIM curves of all reconstruction methods tend to be stable. After the curve tends to be stable, the PSNR and SSIM indexes of the proposed method are much higher than other reconstruction methods.

V. DISCUSSIONS AND CONCLUSIONS

A. DISCUSSIONS ABOUT BACKGROUND NOISE, THE NUMBER OF ITERATIONS AND COMPLEXITY

The experimental and numerical simulation results demonstrate the effectiveness of the proposed algorithm. Through comparing with other algorithms, the reconstruction quality of the proposed algorithm is the highest. In this section, the influence of background noise and iteration numbers in the combination of the split Bregman iteration and BM3D filter algorithm is discussed.

In order to analyze the influence of background noise, the Gaussian white noise is added in the processing of numerical simulation of reconstructing the image “house”. The level of background noise is measured by noise-signal-ratio $R$. $R$ is defined as:

$$R = \frac{\langle N_b \rangle}{\langle y \rangle},$$

where $\langle N_b \rangle$ is the mean value of background noise and $\langle y \rangle$ is the mean value of the measurements by the bucket detector. The relationship between the PSNR and SSIM of different methods and noise-signal-ratio $R$ is shown in Fig. 7.

As Fig. 7 shows, noise has a great impact on the reconstruction quality of different methods. With the increase of noise, the PSNR and SSIM of various methods will decrease. When the noise is relatively small, the PSNR and SSIM of the reconstruction results with the proposed method are higher than those of other methods. With the noise level increasing, the two indexes of the proposed method will decrease and remain, higher than the values of other algorithms under the same noise condition.

On the other hand, it can be concluded that, the algorithms based on the iteration filter, such as PLRG and proposed algorithm, has excellent noise resistant ability, as their PSNR and SSIM decrease relatively slow. But with the noise level increasing, the PSNR and SSIM of TVAL3, ADMM and CSB algorithms decrease rather rapidly. When the noise level $R$ is set to 0, the PSNR and SSIM of TVAL3 is higher than the values of PLRG. But when the noise level is over 0.05, these quality indexes of TVAL3 are quite lower than those of PLRG.

The number of iterations has a certain influence on the results of reconstruction. Generally, the more iterations, the more noise is filtered out in each iteration, and the higher quality of reconstruction is achieved. Fig. 8 shows the PSNR and SSIM changing with the number of iterations. Different colors of the curves represent different noise level.
As the Fig. 8 (a) shows, the PSNR of the reconstruction results improves with the increased number of iterations. When the number of iterations is relatively small, the PSNR improves faster as the number of iterations increases. When the number of iterations reaches a certain level, the growth rate of PSNR gradually decreases and becomes stable. At this point, simply increasing the number of iterations will not significantly improve the PSNR. In addition, when the noise is relatively large, such as the noise signal ratio $R = 0.3$, the PSNR curve remains almost unchanged with the increase of the number of iterations. The Fig. 8 (b) shows the relationship between the SSIM and the number of iterations. The relationship between the SSIM and the number of iterations is similar to the trajectory of the PSNR. When the number of iterations is relatively small, the SSIM increases significantly with the number of iterations. If the number of iterations reaches a certain level, the curve tends to flatten. Increasing the number of iterations cannot lead to the improvement of the SSIM, especially under the low noise level condition. As the blue curves Fig. 8 (b) shows, increasing iteration numbers will not increase the value of SSIM, but it will bring a slight decrease in SSIM corresponding to the noise-signal-ratio $R = 0$. In addition, it can be observed from the curves of SSIM under different noise levels that the higher the noise is, the more iterations are needed for the SSIM to be stabilized. Therefore, the proposed method needs more iterations to achieve the optimal reconstruction effect under high noise condition.

Based on the analysis of Fig. 8, the following conclusions of the proposed method can be obtained. (1) When the number of iterations is low, the quality of reconstruction result is rapidly improved by increasing the number of iterations. (2) When the number of iterations is high, the quality of reconstruction result is reduced with increasing the number of iterations. (3) There is a saturation point of the number of iterations and after reaching saturation point, if the number of iterations is further increased, on one hand, it will only bring more computational overhead and cannot improve the quality of reconstruction; on the other hand, it may reduce the quality of reconstruction. When the method proposed in this paper is utilized, the level of background noise needs to be evaluated to select the appropriate number of iterations to obtain high-quality reconstruction results with a small amount of computing cost.

In our proposed algorithm, (7)-(10) and BM3D algorithm will be executed once in each iteration. The maximum time complexity of (7)-(10) is $O(MN)$, where $M$ is measurement times and $N$ in the total pixel number of the image. The time complexity of BM3D algorithm is $O(N)$ [32]. Therefore, the time complexity of the proposed algorithm is $O(MN)$.

In order to verify the time complexity of the proposed algorithm, the numerical simulation is preformed based on the image “house”. In the numerical simulation, the image
“house” is resized into different scales and the measurement numbers is also changed. The execution time is recorded under the condition of different target sizes and measurement numbers. The numerical simulation result is shown in Fig. 9. As Fig. 9 shows, the graph representing the relationship between the execution time and the number of target total pixels and the number of measurements is approximately an inclined plane, which means that the execution time changes with the total pixels number linearly, and the execution time also changes with the measurement numbers linearly. Therefore, the numerical simulation results of image “house” verify the time complexity analysis above. Namely, the time complexity of the proposed algorithm is \( O(MN) \). Furthermore, the typical execution time can be obtained from the Fig. 9. The corresponding execution time is obtained by selecting the number of measurements and the number of pixels. For instance, when the number of target pixels is \( 128 \times 128 \) and the number of measurements is 1000, the execution time is 25.373s.

**B. CONCLUSIONS**

Based on computational ghost imaging system, the combination of the split Bregman iteration and BM3D filtering algorithm is proposed. The main structure of the algorithm is the split Bregman iteration, which has fast convergence speed and high running efficiency. Meanwhile, inspired by iterative filtering, the BM3D filtering is coupled to the split Bregman iteration. And the results of each iteration are filtered by the BM3D algorithm, which finally obtains high-quality reconstruction results. The effectiveness of the proposed algorithm is verified by theoretical analysis and experiment results. It is compared qualitatively and quantitatively with the CGI, DGI, OMP, ADMM, CSB, PLRG and TVAL3 methods. The image quality of the reconstruction with the proposed method in this paper is higher than others. The reconstruction of the proposed method not only has low image noise, but also remains the edge and texture detail information well. Finally, the influence of background noise and the number of iterations on the reconstruction with the proposed method are analyzed. Through the analysis, it can be found that the increase of noise can reduce the reconstructed quality of all the methods, but the methods based on iterative filtering have great anti-noise capacity. The proposed method and PLRG, which are both based on iterative filtering, can reconstruct better results when the noise level is relatively high. The increase of the iteration number will significantly improve the quality of reconstruction when the number of iterations is small. In addition, there is a saturation point of the number of iterations. When the number of iterations exceeds this saturation point, increasing the number of iterations will not help the quality improvement of reconstruction, and the quality of reconstruction tends to be stable.

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