EAdam Optimizer: How $\epsilon$ Impact Adam

Wei Yuan, Kai-Xin Gao

School of Mathematics, Tianjin University
Email: yuann@tju.edu.cn, gaokaixin@tju.edu.cn

Abstract

Many adaptive optimization methods have been proposed and used in deep learning, in which Adam is regarded as the default algorithm and widely used in many deep learning frameworks. Recently, many variants of Adam, such as Adabound, RAdam and Adabelief, have been proposed and show better performance than Adam. However, these variants mainly focus on changing the stepsize by making differences on the gradient or the square of it. Motivated by the fact that suitable damping is important for the success of powerful second-order optimizers, we discuss the impact of the constant $\epsilon$ for Adam in this paper. Surprisingly, we can obtain better performance than Adam simply changing the position of $\epsilon$. Based on this finding, we propose a new variant of Adam called EAdam, which doesn’t need extra hyper-parameters or computational costs. We also discuss the relationships and differences between our method and Adam. Finally, we conduct extensive experiments on various popular tasks and models. Experimental results show that our method can bring significant improvement compared with Adam. Our code is available at https://github.com/yuanwei2019/EAdam-optimizer.

1 Introduction

Deep neural network models have shown state-of-the-art performance in many application areas, such as image classification (He et al., 2016), natural language processing (Noda et al., 2015) and object detection (Redmon et al., 2016). As models becoming more and more complex, finding efficient training methods has attracted many researchers.

Among the algorithms for training deep networks, stochastic gradient descent (SGD) (Robbins and Monro, 1951) is one of the most widely used algorithms, due to its ease of implementation and low computational costs. However, SGD scales the gradient uniformly in all directions, which may lead to relatively-slow convergence and sensitivity to the learning rate (Ding et al., 2019; Luo et al., 2019). In order to solve this problems, several adaptive methods have been proposed, in which the adaptive moment estimation
of the past squared gradients is computed to scale the gradient of different parameters. Examples of these adaptive methods include AdaGrad (Duchi et al., 2011), AdaDelta (Zeiler, 2012), RMSprop (Tieleman and Hinton, 2012), and Adam (Kingma and Ba, 2014). In particular, Adam has been regarded as the default algorithm and widely used in many deep learning frameworks (Wilson et al., 2017).

Although Adam is popular in many areas, its generalization ability is likely worse than the SGD family (non-adaptive methods) (Wilson et al., 2017). What’s more, Reddi et al. (2018) focused on the non-convergence in Adam and elucidated how the exponential moving average causes it. So, many variants of Adam have been proposed to solve these problems, such as AMSGrad (Reddi et al., 2018), MSVAG (Balles and Hennig, 2017), RAdam (Liu et al., 2019), AdaBound (Luo et al., 2019) and Adabelief (Zhuang et al., 2020). These modifications usually have better performance compared to Adam.

As defined in Kingma and Ba (2014), let $g_t$ denote the gradient at the $t$-th timestep, $m_t, v_t$ denote the exponential moving averages of $g_t, g_t^2$, respectively and $\alpha$ denote the stepsize. Then the adaptive stepsizes in Adam can be seen as $\alpha/\sqrt{v_t}$. More details are given in Algorithm 1. Summarizing the variants of Adam, we find that most of these methods are focused on $v_t$. They all scale the factor $m_t$ by making different changes to $v_t$ and usually ignore the impact of $\epsilon$. The reason why adds $\epsilon$ to $\sqrt{v_t}$ is to keep the denominator from getting too small. It’s similar to the damping factor in second order optimization methods. However, powerful second-order optimizers (such as Hessian-Free optimization approach (Kiros, 2013) and natural gradient descent (Martens and Grosse, 2015)) usually require more sophisticated damping solutions to work well, and will may be completely fail without them (Martens and Grosse, 2015). Therefore, a direct question is whether $\epsilon$ has a great influence on the performance of Adam.

Motivated by the above problem, we explored whether changing the update rule of $\epsilon$ in Adam would obtain better performance. An interesting finding is that we can obtain better performance than Adam simply changing the position of $\epsilon$ without extra hyper-parameters and computational costs. In particular, we make the following key contributions:

- Based on the above finding, we propose a new variant of Adam, called $\epsilon$-Adam (EAdam), in which $\epsilon$ is added to $v_t$ at every step before the bias correction and it is accumulated through the updating process (details are given in Section 3). We also discuss the difference of EAdam compared with Adam and show that Adam cannot achieve same performance as EAdam by tuning $\epsilon$ directly in experiments.

- We turn to an empirical study of EAdam on various popular tasks and models in image classification, natural language processing and object detection compared with Adam, RAdam and Adabelief. Experimental results demonstrate that our method outperforms Adam, and RAdam and has similar performance to Adabelief in all tasks and models. Compared these variants of Adam, EAdam can achieve better or similar results with simpler variation to Adam.
Algorithm 1 Adam algorithm

Require: $f(\theta)$: the objective function with parameters $\theta$

Require: $\alpha$: stepsize

Require: $\beta_1, \beta_2$: exponential decay rates

Require: $\theta_0$: initial parameters

$m_0 \leftarrow 0, v_0 \leftarrow 0, t = 0$

while convergence is not reached do

$t \leftarrow t + 1$

$g_t \leftarrow \nabla_\theta f_t(\theta_{t-1})$

$m_t \leftarrow \beta_1 m_{t-1} + (1 - \beta_1) g_t$

$v_t \leftarrow \beta_2 v_{t-1} + (1 - \beta_2) g_t^2$

$m_t' = m_t / (1 - \beta_1^t), v_t' = v_t / (1 - \beta_2^t)$

$\theta_t \leftarrow \theta_{t-1} - \alpha (m_t' / \sqrt{v_t'} + \epsilon)$

end while

2 Related work

There have been many optimizers proposed to train deep networks. Firstly, we summarize a few closely related works of the variants of Adam. MSVAG (Balles and Hennig, 2017) combines taking signs and variance adaptation into Adam. AMSGrad (Reddi et al., 2018) resolves the non-convergence in Adam based on the long-term memory of past gradients. RAdam (Liu et al., 2019) analyses the convergence issue in Adam and explicitly rectifies the variance of the learning rate. AdaBound (Luo et al., 2019) employs dynamic bounds on learning rate in adaptive methods and clips the extreme learning rate. Adabelief (Zhuang et al., 2020) considers to adapt the stepsize according to the belief in the current gradient direction and obtains good generalization performance. Of course, there are many other variants of Adam (such as NAdam (Dozat, 2016), AdamW (Loshchilov and Hutter, 2017), AdaMod (Ding et al., 2019) and TAdam (Ilboudo et al., 2020)), and we will not introduce birefly here.

Besides first-order methods, many optimization algorithms, such as quasi-Newton method (Le et al., 2011; Berahas et al., 2019), Hessian-Free optimization approach (Martens, 2010; Pan et al., 2017) and Kronecker-factored approximate curvature (KFAC) (Martens and Grosse, 2015; Grosse and Martens, 2016), have been presented and used for training deep networks. Second-order optimization algorithms can greatly accelerate convergence by using curvature matrix to correct gradient. However, the computation of curvature matrix and its inverse leads to heavy computational burden, and hence second-order optimizers are not widely used in deep learning.
3 Method

3.1 Algorithm

Notations: In this paper, we will use the following basic notations as defined in (Kingma and Ba, 2014).

- $f(\theta)$: the objective function with parameters $\theta$
- $g_t = \nabla_\theta f(\theta)$: the gradient of $f(\theta)$ at the $t$-th step
- $\alpha$: stepsize (learning rate), the default setting is 0.001
- $m_t, v_t$: exponential moving average of $g_t, g_t^2$, respectively
- $\beta_1, \beta_2 \in (0, 1)$: the exponential decay rates of moving averages, typically set as $\beta_1 = 0.9$ and $\beta_2 = 0.999$
- $\epsilon$: a small positive constant, and the typical value is $10^{-8}$

**EAdam:** The Adam algorithm and our algorithm are summarized in Algorithm 1 and Algorithm 2 respectively, in which the differences are marked in blue and the operations for vectors are element-wise. Our method mainly change the update rule of $\epsilon$, so we call this proposed method EAdam. For the standard Adam, $\epsilon$ is added to the factor $\sqrt{v_t}$ after the bias correction step, that is $\sqrt{v_t} + \epsilon$. So the adaptive stepsize in Adam is $\alpha/\sqrt{v_t + \epsilon}$. For EAdam, $\epsilon$ is added to $v_t$ at every step before the bias correction, it is accumulated through the updating process, and finally the adaptive stepsize in EAdam is $\alpha/\sqrt{v_t}$. Compared with Adam, the change in our method is simple but it is efficient without extra computation cost and hyper-parameters setting. More discussion is given in the following subsection. What’s more, EAdam shows better performance compared with Adam in experiments (results are given in Section 4).

3.2 Comparison with Adam

We will here analyse how the change in EAdam affects the update of parameters $\theta$. Let $g_t$ be the gradient of the objective function at timestep $t$. According to update formulas in Algorithm 1 and Algorithm 2, $v_t^{(Adam)}$ and $v_t^{(EAdam)}$ can be expressed by the gradients at all previous timesteps as follows

$$v_t^{(Adam)} = (1 - \beta_2) \sum_{i=1}^{t} \beta_2^{t-i} g_i^2,$$

$$v_t^{(EAdam)} = (1 - \beta_2) \sum_{i=1}^{t} \beta_2^{t-i} g_i^2 + \frac{1 - \beta_2^2}{1 - \beta_2} \epsilon.$$

After the bias correction step, we have

$$\hat{v}_t^{(Adam)} = \frac{1 - \beta_2}{1 - \beta_2^t} \sum_{i=1}^{t} \beta_2^{t-i} g_i^2 = G_t,$$
Algorithm 2 EAdam algorithm. This algorithm is proposed based on the Adam algorithm and the differences from standard Adam are shown in blue. The default settings are same as Adam, where $\beta_1 = 0.9, \beta_2 = 0.999, \epsilon = 10^{-8}$.

**Require:** $f(\theta)$: the objective function with parameters $\theta$

**Require:** $\alpha$: stepsize

**Require:** $\beta_1, \beta_2$: exponential decay rates

**Require:** $\theta_0$: initial parameters

$m_0 \leftarrow 0, v_0 \leftarrow 0, t = 0$

while convergence is not reached do

$t \leftarrow t + 1$

$g_t \leftarrow \nabla \theta f_t(\theta_{t-1})$

$m_t \leftarrow \beta_1 m_{t-1} + (1 - \beta_1) g_t$

$v_t \leftarrow \beta_2 v_{t-1} + (1 - \beta_2) g_t^2$

$v_t \leftarrow v_t + \epsilon$

$m_t = m_t / (1 - \beta_1^t), v_t = v_t / (1 - \beta_2^t)$

$\theta_t \leftarrow \theta_{t-1} - \alpha (m_t / \sqrt{v_t})$

end while

$v_t^{(EAdam)} = 1 - \beta_2 \sum_{i=1}^t \beta_{2}^{t-i} g_i^2 + \frac{1}{1 - \beta_2} \epsilon = G_t + \frac{1}{1 - \beta_2} \epsilon,$

where $G_t = \frac{1 - \beta_2}{1 - \beta_2} \sum_{i=1}^t \beta_{2}^{t-i} g_i^2$, $\epsilon$ is a small positive constant in EAdam. Then the adaptive stepsize in Adam and EAdam can be seen as

$$\frac{\alpha}{\sqrt{G_t + \epsilon'}} = \frac{\alpha}{\sqrt{G_t + \epsilon/(1 - \beta_2)}}$$

(3.1)

where $\epsilon'$ is the small positive constant in Adam.

We firstly let $\epsilon' = \epsilon = 10^{-8}$, then we want to analyse the differences of stepsizes when using Adam and EAdam to train deep networks. At the begin of training, the elements in $G_t$ are far larger than $\epsilon'$ and $\epsilon$, the stepsizes in Adam and EAdam can all approximated as $\alpha / \sqrt{G_t}$. In this case, the stepsize is determined by $G_t$. Then, the elements in $G_t$ may become small and $\epsilon'$ or $\epsilon$ can affect the elements in $G_t$. In this case, the stepsize is determined by $G_t$ and $\epsilon'$ ($\epsilon$). It easy to see that this case happens earlier in EAdam because $\epsilon$ is added to $G_t$ rather than $\sqrt{G_t}$. Finally, the elements in $G_t$ may become far smaller than $\epsilon'$ or $\epsilon$, and the stepsizes become

$$\frac{\alpha}{\epsilon'}, \frac{\alpha}{\sqrt{\epsilon/(1 - \beta_2)}}$$

in Adam and EAdam, respectively. In this case, EAdam takes smaller stepsize than Adam. A possible intuitive comparison is given in Figure 1. When $\theta_t$ is close to the global minimum point, EAdam takes smaller step (as shown by 3) can avoid the cases as
shown by ② and ③. This may be an explanation why EAdam achieves better performance than Adam.

From Eq. (3.1), we can see that EAdam essentially adds a constant times of $\epsilon$ to $G_t$ before the square root operation. However, this operation is not equivalent to adding a fixed constant $\epsilon'$ to $\sqrt{G_t}$. In other words, we can’t find a fixed constant $\epsilon'$ such that $\sqrt{G_t} + \epsilon' = \sqrt{G_t} + \epsilon/(1 - \beta_2)$, where $\epsilon$ is known, for the following reasons.

If we let $\sqrt{G_t} + \epsilon' = \sqrt{G_t} + \epsilon/(1 - \beta_2)$ where $\epsilon$ is known. Then, we have

$$\epsilon' = \sqrt{G_t} + \epsilon(1 - \beta_2) - \sqrt{G_t}. \quad (3.2)$$

Because $G_t$ is constantly updated, $\epsilon'$ is also adjusted based on $G_t$ in the iterative process. Therefore, $\epsilon'$ is not fixed. From this interpretation, the change in EAdam can be seen as adopting an adaptive $\epsilon$ rather than a constant in Adam.

To sum up, we give some intuitive comparisons and explanations for EAdam in this subsection. However, analyzing the reasons why EAdam performances better in theory may be difficult and it is worthy to be further studied.

4 Experiments

In this section, we perform a thorough evaluation of our EAdam optimizer against popular and latest optimization methods including Adam (Kingma and Ba, 2014), RAdam (Liu et al., 2019) and Adabelief (Zhuang et al., 2020) on different deep learning tasks. We focus on these following tasks: image classification on CIFAR-10 and CIFAR-100 (Krizhevsky et al., 2009) with VGG11 (Simonyan and Zisserman, 2014), ResNet18 (He et al., 2016) and DenseNet121 (Huang et al., 2017), language modeling on Penn Treebank (Mitchell et al., 1993) with LSTM (Ma et al., 2015) and object detection on PASCAL
VOC (Everingham et al., 2010) with Faster-RCNN + FPN (Lin et al., 2017; Ren et al., 2015). The setup for each task is summarized in Table 1. We mainly use the official implementation of AdaBelief and refer to the hyper-parameter tuning in Zhuang et al. (2020) for image classification and language modeling. We mainly refer to the setting in (Yong et al., 2020) for object detection. Experimental results are given in the following subsection.

Table 1: Summaries of the models used in our experiments.

| Dataset       | Network Type   | Architecture                     |
|---------------|----------------|----------------------------------|
| CIFAR-10      | Deep Convolutional | VGG16, ResNet18, DenseNet121    |
| CIFAR-100     | Deep Convolutional | VGG16, ResNet18, DenseNet121    |
| Penn Treebank | Recurrent       | 1,2,3-Layer LSTM                |
| PASCAL VOC    | Deep Convolutional | Faster-RCNN + FPN               |

4.1 Image classification

Figure 2: The curves of testing accuracy with epochs for Adam, RAdam, Adabelief and EAdam on CIFAR-10 and CIFAR-100. The models we used here are VGG11, ResNet18 and DenseNet121.
Firstly, we consider the image classification task on CIFAR-10 and CIFAR-100 datasets. For all experiments in this part, we all use the default parameters of Adam for all methods ($\alpha = 10^{-3}, \beta_1 = 0.9, \beta_2 = 0.999, \epsilon = 10^{-8}$). We set the weight decay as $5e-4$ of all methods. We train all models for 200 epochs with batch-size 128 and multiply the learning rates by 0.1 at the 150-th epoch. Results are summarized in Figure 2 and Table 2.

Table 2: Test accuracy for VGG11 ResNet18 and DenseNet121 on CIFAR-10 and CIFAR-100.

| Dataset  | Model        | Adam  | RAdam | Adabelief | EAdam  |
|----------|--------------|-------|-------|-----------|--------|
| CIFAR-10 | VGG11        | 88.95 | 89.54 | 91.66     | 91.45  |
| CIFAR-10 | ResNet18     | 92.88 | 94.26 | 94.85     | 94.99  |
| CIFAR-10 | DenseNet121  | 93.55 | 94.97 | 95.69     | 95.61  |
| CIFAR-100| VGG11        | 55.33 | 58.84 | 67.88     | 67.45  |
| CIFAR-100| ResNet18     | 73.79 | 73.56 | 76.68     | 76.29  |
| CIFAR-100| DenseNet121  | 74.46 | 76.96 | 79.64     | 78.77  |

**VGG11**: The accuracy curves on VGG11 are shown in the first column of Figure 2. We can see that EAdam outperforms Adam and RAdam and has similar performance to Adabelief on both datasets. Compared with Adam, our method has both faster convergence and better performance. As shown in Table 2, the test accuracy of EAdam is improved about 2.5% and 12% than Adam on CIFAR-10 and CIFAR-100, respectively. It is also obvious that EAdam outperforms RAdam. AdaBelief is a latest variant of Adam and shows good generalization as in the SGD family with fast convergence as adaptive methods. EAdam can keep similar performance to it without change to $g^2$ in Adam.

**ResNet18**: The accuracy curves on ResNet18 are shown in the second column of Figure 2. As we except, EAdam also has both faster convergence and higher test accuracy than Adam. EAdam achieves about approximately 2% and 2.5% improvement on CIFAR-10 and CIFAR-100 in terms of test accuracy as shown in Table 2. Compared with RAdam, EAdam converges slower than RAdam in the early training. But when the learning rates are decayed at the 150-th epoch, EAdam begins to outperform RAdam. On the CIFAR-10 dataset, EAdam has similar performance to AdaBelief. On the CIFAR-100 dataset, EAdam has similar convergence to AdaBelief but the testing accuracy is slightly lower.

**DenseNet121**: The accuracy curves on ResNet18 are shown in the third column of Figure 2. We can see that the overall performance of each method on DenseNet121 is similar to that on ResNet18. On the CIFAR-100 dataset, the improvement of EAdam relative to Adam becomes more significant than that on ResNet18, which is enhanced with more than 4% in the test accuracy. In addition, the testing accuracy of Adam reduces about 0.9% than Adabelief on the CIFAR-100 dataset. To sum up, the change of $\epsilon$ in Adam is efficient, and experimental results validate the fast convergence and good
generalization performance of EAdam.

4.2 Language Modeling

![Perplexity curves for Adam, RAdam, Adabelief, and EAdam on Penn Treebank.](image)

Figure 3: The curves of test perplexity with epochs for Adam, RAdam, Adabelief and EAdam on Penn Treebank. The models we used here are 1, 2, 3-layer LSTM.

We next conduct an experiment on the language modeling task. We consider the LSTM network on the Penn Treebank dataset. For all experiments in this part, we set the weight decay as $5e^{-4}$ of all methods and train all models for 200 epochs with batch-size 20 and multiply the learning rates by 0.1 at the 100-th epoch and 145-th epoch. Other related parameters are set as $\alpha \in \{10^{-4}, 10^{-3}, 10^{-2}, 10^{-1}\}$, $\beta_1 = 0.9$, $\beta_2 = 0.999$, $\epsilon \in \{10^{-16}, 10^{-12}, 10^{-8}\}$. Results are summarized in Figure 3 and Table 3.

Table 3: Test perplexity (lower is better) on Language Modeling for the 1, 2, 3-Layer LSTM on Penn Treebank.

| Dataset   | Model      | Adam | RAdam | Adabelief | EAdam |
|-----------|------------|------|-------|-----------|-------|
| Treebank  | 1-Layer LSTM | 84.42 | 88.30 | 84.24     | 84.46 |
| Treebank  | 2-Layer LSTM | 67.36 | 73.46 | 66.58     | 66.32 |
| Treebank  | 3-Layer LSTM | 64.29 | 69.98 | 60.93     | 61.13 |

Perplexity curves are shown in Figure 3. We can see that RAdam performs worse than Adam, Adabelief and EAdam. For the 1-layer model, the performance of EAdam is similar to Adam and Adabelief. For the 2-layer and 3-layer LSTM models, EAdam outperforms Adam and has similar performance to Adabelief. As shown in Table 3, Adabelief achieves the lowest perplexity on the 1-layer and 3-layer LSTM model but the perplexity of EAdam is close to Adabelief, and EAdam achieves the lowest perplexity on 2-layer LSTM model. In a word, EAdam also can keep fast convergence and good accuracy in the language modeling task.
4.3 Object detection

Finally, we do object detection experiments on the PASCAL VOC dataset. The model we used here is pre-trained on ImageNet and the pre-trained model is from the official website. We train this model on the VOC2007 and VOC2012 trainval dataset (17K) and evaluate on the VOC2007 test dataset (5K). We use the MMDetection (Chen et al., 2019) toolbox as the detection framework. The official implementations and settings are used for all experiments. The model we used is Faster-RCNN + FPN. The backbone is ResNet50. We set the weight decay as $10^{-4}$ of all methods. We train the model for 4 epochs with batch-size 2 and multiply the learning rates by 0.1 at the last epoch. Other parameters are set as $\alpha = 10^{-4}, \beta_1 = 0.9, \beta_2 = 0.999, \epsilon = 10^{-8}$ for all methods. Results are summarized in Figure 4 and Table 4. As we except, EAdam outperforms Adam, RAdam and has similar performance to AdaBelief. These results also illustrates that our method is still efficient in object detection tasks.

Table 4: The mAP (higher is better) on PASCAL VOC using Faster-RCNN+FPN.

| Method   | Adam   | RAdam | Adabelief | EAdam |
|----------|--------|-------|-----------|-------|
| mAP      | 71.47  | 76.58 | 81.02     | 80.62 |

![Detection examples using Faster-RCNN + FPN trained on PASCAL VOC.](image)

5 Conclusions

In this paper, we discussed the impact of the constant $\epsilon$ in Adam and found that a simple change could bring significant improvement. So we proposed the EAdam algorithm, which is a new variant of Adam. To our knowledge, EAdam is the first variant of Adam.
based on the discussion of $\epsilon$. We also gave some intuitive explanations of the relationships and differences between EAdam and Adam. Finally, we validated the benefits of EAdam on extensive experiments. Of course, the exact reason why $\epsilon$ impact the performance of Adam greatly is still mysterious for us. What’s more, can EAdam still outperform Adam on other DNNs or more complex large-scale training tasks, such as image classification on ImageNet and object detection on COCO? These deserve to be discussed and verified. This paper is only the first draft, we will further enrich and improve the analysis and experiments in the next version.

References

Lukas Balles and Philipp Hennig. Dissecting adam: The sign, magnitude and variance of stochastic gradients. arXiv preprint arXiv:1705.07774, 2017.

Albert S Berahas, Majid Jahani, and Martin Takáč. Quasi-Newton methods for deep learning: Forget the past, just sample. arXiv preprint arXiv:1901.09997, 2019.

Kai Chen, Jiaqi Wang, Jiangmiao Pang, Yuhang Cao, Yu Xiong, Xiaoxiao Li, Shuyang Sun, Wansen Feng, Ziwei Liu, Jiarui Xu, Zheng Zhang, Dazhi Cheng, Chenchen Zhu, Tianheng Cheng, Qijie Zhao, Buyu Li, Xin Lu, Rui Zhu, Yue Wu, Jifeng Dai, Jingdong Wang, Jianping Shi, Wanli Ouyang, Chen Change Loy, and Dahua Lin. MMDection: Open mmlab detection toolbox and benchmark. arXiv preprint arXiv:1906.07155, 2019.

Jianbang Ding, Xuancheng Ren, Ruixuan Luo, and Xu Sun. An adaptive and momental bound method for stochastic learning. arXiv preprint arXiv:1910.12249, 2019.

Timothy Dozat. Incorporating nesterov momentum into adam. 2016.

John Duchi, Hazan Elad, and Singer Yoram. Adaptive subgradient methods for online learning and stochastic optimization. Journal of Machine Learning Research, 12(Jul): 2121–2159, 2011.

Mark Everingham, Luc Van Gool, Christopher K. I. Williams, John Winn, and Andrew Zisserman. The pascal visual object classes (voc) challenge. International Journal of Computer Vision, 88(2):303–338, 2010.

Roger Grosse and James Martens. A kronecker-factored approximate fisher matrix for convolution layers. In International Conference on Machine Learning, pages 573–582, 2016.

Kaiming He, Xiangyu Zhang, Shaoqing Ren, and Jian Sun. Deep residual learning for image recognition. In Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition, pages 770–778, 2016.
Gao Huang, Zhuang Liu, Van Der Maaten Laurens, and Kilian Q Weinberger. Densely connected convolutional networks. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, pages 4700–4708, 2017.

Wendyam Eric Lionel Ilboudo, Taisuke Kobayashi, and Kenji Sugimoto. Tadam: A robust stochastic gradient optimizer. *arXiv preprint arXiv:2003.00179*, 2020.

Diederik P Kingma and Jimmy Ba. Adam: A method for stochastic optimization. In *International Conference on Learning Representations*, 2014.

Ryan Kiros. Training neural networks with stochastic Hessian-free optimization. In *International Conference on Learning Representations*, 2013.

Alex Krizhevsky, Geoffrey Hinton, et al. Learning multiple layers of features from tiny images. 2009.

Quoc V Le, Jiquan Ngiam, Adam Coates, Abhik Lahiri, Bobby Prochnow, and Andrew Y Ng. On optimization methods for deep learning. In *Proceedings of the 28th International Conference on International Conference on Machine Learning*, pages 265–272, 2011.

Tsung-Yi Lin, Piotr Dollár, Ross Girshick, Kaiming He, Bharath Hariharan, and Serge Belongie. Feature pyramid networks for object detection. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, pages 2117–2125, 2017.

Liyuan Liu, Haoming Jiang, Pengcheng He, Weizhu Chen, Xiaodong Liu, Jianfeng Gao, and Jiawei Han. On the variance of the adaptive learning rate and beyond. *arXiv preprint arXiv:1908.03265*, 2019.

Ilya Loshchilov and Frank Hutter. Decoupled weight decay regularization. *arXiv preprint arXiv:1711.05101*, 2017.

Liangchen Luo, Yuanhao Xiong, Yan Liu, and Xu Sun. Adaptive gradient methods with dynamic bound of learning rate. In *International Conference on Learning Representations*, 2019.

Xiaolei Ma, Zhimin Tao, Yinhai Wang, Haiyang Yu, and Yunpeng Wang. Long short-term memory neural network for traffic speed prediction using remote microwave sensor data. *Transportation Research Part C: Emerging Technologies*, 54:187–197, 2015.

James Martens. Deep learning via Hessian-free optimization. In *International Conference on Machine Learning*, pages 735–742, 2010.

James Martens and Roger Grosse. Optimizing neural networks with kronecker-factored approximate curvature. In *International Conference on Machine Learning*, pages 2408–2417, 2015.
P. Marcus Mitchell, Ann Marcinkiewicz Mary, and Santorini Beatrice. Building a large annotated corpus of English: The penn treebank. *Computational Linguistics*, 19(2): 313–330, 1993.

Kuniaki Noda, Yuki Yamaguchi, Kazuhiro Nakadai, Hiroshi G. Okuno, and Tetsuya Ogata. Audio-visual speech recognition using deep learning. *Applied Intelligence*, 42(4):722–737, 2015.

Wenyong Pan, Kristopher A Innanen, and Wenyuan Liao. Accelerating Hessian-free Gauss-Newton full-waveform inversion via l-BFGS preconditioned conjugate-gradient algorithm. *Geophysics*, 82(2):R49–R64, 2017.

Sashank J. Reddi, Satyen Kale, and Sanjiv Kumar. On the convergence of adam and beyond. In *International Conference on Learning Representations*, 2018.

Joseph Redmon, Santosh Divvala, Ross Girshick, and Ali Farhadi. You only look once: Unified, real-time object detection. In *Proceedings of the IEEE Conference on Computer Vision and Pattern Recognition*, 2016.

Shaoqing Ren, Kaiming He, Ross Girshick, and Jian Sun. Faster r-cnn: Towards real-time object detection with region proposal networks. In *Advances in Neural Information Processing Systems*, pages 91–99, 2015.

Herbert Robbins and Sutton Monro. A stochastic approximation method. *The annals of mathematical statistics*, pages 400–407, 1951.

Karen Simonyan and Andrew Zisserman. Very deep convolutional networks for large-scale image recognition. *arXiv preprint arXiv:1409.1556*, 2014.

Tijmen Tieleman and Geoffrey Hinton. Lecture 6.5-rmsprop: Divide the gradient by a running average of its recent magnitude. *COURSERA: Neural Networks for Machine Learning*, 4(2):26–31, 2012.

Ashia C Wilson, Rebecca Roelofs, Mitchell Stern, Nati Srebro, and Benjamin Recht. The marginal value of adaptive gradient methods in machine learning. In *Advances in Neural Information Processing Systems*, pages 4148–4158, 2017.

Hongwei Yong, Jianqiang Huang, Xiansheng Hua, and Lei Zhang. Gradient centralization: A new optimization technique for deep neural networks. *arXiv preprint arXiv:2004.01461*, 2020.

Matthew D Zeiler. Adadelta: an adaptive learning rate method. *arXiv preprint arXiv:1212.5701*, 2012.

Juntang Zhuang, Tommy Tang, Sekhar Tatikonda1, Nicha Dvornek, Yifan Ding, Xenophon Papademetris, and James S. Duncan. Adabelief optimizer: Adapting step-sizes by the belief in observed gradients. *arXiv preprint arXiv:2010.07468*, 2020.