Characteristic Mode Decomposition Using the Scattering Dyadic in Arbitrary Full-Wave Solvers

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Abstract—Characteristic modes are formulated using the scattering dyadic, which maps incident plane waves to scattered far-fields generated by an object of arbitrary material composition. Numerical construction of the scattering dyadic using arbitrary full-wave electromagnetic solvers is demonstrated in examples involving a variety of dielectric and magnetic materials. Wrapper functions for computing characteristic modes in method-of-moments, finite-difference time domain, and finite-element solvers are provided as Supplementary Material.

Index Terms—Antenna theory, characteristic modes, computational electromagnetics, eigenvalues and eigenfunctions, scattering.

I. INTRODUCTION

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HARACTERISTIC mode analysis is a popular tool used in antenna design and the analysis of radiation mechanisms [1], [2], [3]. The most common numerical implementation of characteristic modes is via method-of-moments (MoM) system matrices as proposed by Harrington and Mautz [4]. As such, the characteristic mode decomposition was understood to be closely related—if not directly depending on [5]—MoM [6], a procedure for constructing system matrices out of prescribed integro-differential operators [7]. The fact that this is not necessarily the case was recently presented in [8], where an algebraic link between the transition matrix [9], [10] and an arbitrary system matrix was devised, referring back to the original, almost forgotten, definition of characteristic modes given by Garbacz, which is based on the perturbation matrix [11].

A close look at the existing eigenvalue problems producing characteristic fields [12] and currents [13] reveals that these scattering- and impedance-based formulations represent the same physical problem, only using different representations. This raises an important question of whether other representations exist and, if so, whether they have advantageous properties over existing approaches. This article addresses these questions utilizing a scattering dyadic [10], which affords the possibility for characteristic modes to be evaluated in arbitrary numerical methods, to be calculated for objects constructed from arbitrary materials, and to be reconstructed from far-field measured data.

An object’s scattering dyadic transforms incident plane waves to scattered far-fields. In this article, it is shown that the characteristic modes can be calculated directly from a matrix representation of the scattering dyadic that may be constructed from only the solution of a series of plane-wave scattering problems. Hence, the finite-element method (FEM) [14] or the finite-difference time-domain method (FDTD) [15] may be utilized, giving the opportunity to deal with arbitrary distributions of anisotropic dielectric and magnetic materials. An additional advantage is that the unknowns used in the underlying numerical method are decoupled from the degrees of freedom to express characteristic modes, i.e., a studied object can be described and efficiently evaluated with millions of unknowns, but the matrix representing the scattering dyadic used for characteristic mode eigenvalue decomposition can be significantly smaller, e.g., hundreds of degrees of freedom for practical problems.

The proposed method is versatile, simple to implement, yet general. It directly produces characteristic fields, which can be used for powerful far-field tracking as proposed in [16]. Furthermore, utilization of FDTD makes it possible to efficiently recover broadband characteristic mode data. Finally, since a discrete form of the scattering dyadic can also be assembled from bistatic measurements, there is a possibility to reconstruct the modal properties of realized antennas and scatterers.

This article is organized as follows. Section II describes the formulation of characteristic modes using the scattering dyadic along with high-level connections to formulations involving the transition matrix [8] and system (impedance) matrices [13]. In Section III, practical aspects of collecting necessary scattering dyadic data using MoM, FEM, and FDTD are discussed. In Section IV, example calculations involving PEC structures as well as those involving dielectric and magnetic materials are used to demonstrate the method’s efficacy and to draw general conclusions about the application of different solvers in calculating characteristic modes. Important aspects of the

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method are discussed in Section V. Section VI concludes
this article with discussion points and open problems. This
article is accompanied by Supplementary Material containing
wrappers for several contemporary EM simulation packages
(CST Studio Suite [17], Altair FEKO [18], Ansys HFSS [19],
and COMSOL Multiphysics [20]).

II. MODAL DECOMPOSITION OF SCATTERING DYADIC

Consider an object illuminated by an incident electric field \( E^i \) and the resulting scattered field \( E^s \).
Let the electric far-field \( F \) scattered by the object be defined as follows [21]:

\[
E^s(r) \approx F(\hat{r}) e^{-jkr} / r, \quad \text{as } r \to \infty, \quad (1)
\]

with the radial unit vector \( \hat{r} \), radial vector \( r = r \hat{r} \), wavenumber \( k \), and \( j \) being the imaginary unit.
The far-field scattering properties of the obstacle are fully characterized by a scattering dyadic \( S \), which maps the vector amplitude \( E_0 \) of an incident plane wave propagating in the \( \hat{r} \) direction into the far-field \( F \) in the \( \hat{r} \) direction [see Fig. 1(a)], i.e.,

\[
F(\hat{r}) = \frac{4\pi}{k} S(\hat{r}, \hat{r}') \cdot E_0(\hat{r}'), \quad (2)
\]

where the incident field is assumed to take the form

\[
E^i(r) = E_0(\hat{r}') e^{-jkr'}. \quad (3)
\]

Here, for the sake of notational and dimensional simplicity, the scattering dyadic \( S(\hat{r}, \hat{r}') \) contains a non-standard scaling factor \( -jk/(4\pi) \), see [10, eq. (4.29)]. Note that the case of \( \hat{r} = \hat{r}' \) corresponds to forward scattering and that the reciprocal systems exhibit the symmetry property

\[
S(\hat{r}, \hat{r}') = S^T(\hat{r'}, \hat{r}). \quad (4)
\]

For excitations consisting of a continuum of plane waves [10], with propagation directions covering the unit sphere, the scattered far-field is similarly given by

\[
F(\hat{r}) = \frac{4\pi}{k} \int_{4\pi} S(\hat{r}, \hat{r}') \cdot E_0(\hat{r}') \, d\Omega', \quad (5)
\]

where the integration limit \( 4\pi \) denotes the integration of \( \hat{r}' \) over the unit sphere. Notice that to keep the notation simple,
the same symbol \( E_0(\hat{r}') \) is used in (2) and (5), although their physical meaning is slightly different. In (5), the meaning is altered to the angular density with unit \( \text{Vm}^{-1}\text{sr}^{-1} \).

By (5), the scattering dyadic transforms an incident spectrum of plane waves described by the vector function \( E_0(\hat{r}') \) into the scattered far-field \( F(\hat{r}) \) analogous to how a transition matrix transforms regular spherical waves into outgoing spherical waves [10] or an admittance matrix transforms incident electric field into induced electric currents [6]. As such, an eigenvalue decomposition of the scattering dyadic can be used to generate eigenfunctions \( F_n \), which, when applied as incident fields, produce scattered fields with the same pattern, i.e.,

\[
\int_{4\pi} S(\hat{r}, \hat{r}') \cdot F_n(\hat{r}') \, d\Omega' = t_n F_n(\hat{r}). \quad (6)
\]

As will be discussed in Sections II-A and II-B, and Appendix B, these eigensolutions are alternative representations of characteristic modes obtained through transition [8] and impedance matrix [4] methods. In (6), the eigenvalues \( t_n \) are describing the relative scaling of incident and scattered fields. This concept is illustrated in Fig. 1(b), where an object is illuminated by an eigenexcitation consisting of a spectrum of plane waves with vector amplitudes \( F_n(\hat{r}') \), and the scattered field at all locations on the far-field sphere is a scaled copy of the incident plane-wave spectrum, i.e., \( t_n F_n(\hat{r}) \).

A. Relation to the Transition Matrix Decomposition

The eigenvalue problem in (6) closely resembles characteristic modes as defined by the transition matrix [16], [22]. In that definition, characteristic modes are produced by the eigenvalue problem

\[
T f_n = t_n f_n, \quad (7)
\]

where the vectors \( f_n \) represent spherical wave expansion coefficients, and the transition matrix \( T \) maps regular spherical waves onto outgoing spherical waves [10]. The equivalence of eigenvalue problems (6) and (7) is detailed in Appendix A and is based on a relation between the transition matrix and the scattering dyadic

\[
T_{\alpha\beta} = \int_{4\pi} \int_{4\pi} \phi_\alpha(\hat{r}) \cdot S(\hat{r}, \hat{r}') \cdot \phi_\beta(\hat{r}') \, d\Omega \, d\Omega', \quad (8)
\]

where \( \{\phi_\alpha\} \) is a basis of spherical harmonics (see Appendix A). This basis also connects modal far-fields \( F_n(\hat{r}) \) with the eigenvectors \( f_n \) via

\[
F_n(\hat{r}) = \sqrt{Z_0} \sum_{\alpha} f_{n,\alpha} \phi_\alpha(\hat{r}), \quad (9)
\]

with \( Z_0 \) being the free-space impedance.

B. Relation to the Impedance Matrix Decomposition

The more common method of generating the characteristic modes of a lossless scatterer considers the generalized eigenvalue problem [13]

\[
X \Lambda_n = \lambda_n R \Lambda_n, \quad (10)
\]
where \( \mathbf{X} = \text{Im}\{\mathbf{Z}\} \) and \( \mathbf{R} = \text{Re}\{\mathbf{Z}\} \) are the reactance and radiation components, respectively, of impedance matrix \( \mathbf{Z} \) evaluated via a MoM formulation of the electric-field integral equation (EFIE) \(^1\) [23], the values of \( \mathbf{I}_n \) are the vectors of expansion coefficients representing characteristic current densities \( \mathbf{J}_n \), and the values of \( \lambda_n \) are the eigenvalues related to \( t_n \) via \( 8 \)

\[
t_n = -\frac{1}{1 + j\lambda_n}.
\]

The connection between (6) and (10) is shown in Appendix B utilizing the scattering dyadic formulation (6) as a starting point. Appendix B also justifies the extraction of the elements of the scattering dyadic directly from the impedance matrix via

\[
S_{\gamma\gamma}(\mathbf{\hat{r}}, \mathbf{\hat{r}}') = -\frac{1}{\varepsilon_0} \mathbf{K}_\gamma(\mathbf{\hat{r}}) \mathbf{Z}^{-1} \mathbf{K}^H_\gamma(\mathbf{\hat{r}}'),
\]

where the matrix \( \mathbf{K}_\gamma(\mathbf{\hat{r}}) \) transforms a current vector \( \mathbf{I} \) into its \( \gamma \)-polarized far-field defined by (1) as \([24]\]

\[
F_\gamma(\mathbf{\hat{r}}) = \mathbf{K}_\gamma(\mathbf{\hat{r}}) \mathbf{I},
\]

with polar and azimuthal components \( \gamma = (\vartheta, \phi) \).

Formula (12) describes how to evaluate the scattering dyadic with a MoM solver. As such, it is an important side product of this article and, together with (11), serves as an independent validation of equality between (6) and (10). It is also worth mentioning that (12) defines the far-fields scattered by currents excited by plane waves and that these currents might be further processed to form a set of so-called characteristic basis functions \([25]\), which are sometimes employed to reduce the order of electromagnetic models.

### C. Evaluation of Characteristic Quantities

The eigenvectors \( \mathbf{F}_n \) in (6) represent both characteristic scattered far-fields and characteristic excitations; i.e., it is possible to extract all (modal) metrics by applying

\[
E_n(\mathbf{r}) = -j \frac{k}{4\pi t_n} \int_{4\pi} F_n(\mathbf{\hat{r}}') e^{-j\mathbf{\hat{r}}' \cdot \mathbf{r}} \, dQ',
\]

as a characteristic incident field impinging on the scatterer using any method of solving Maxwell’s equations.

For example, adopting (14), to obtain the modal current density within MoM reads

\[
\mathbf{I}_n = \mathbf{Z}^{-1} \mathbf{V}_n,
\]

where the excitation is constructed using

\[
\mathbf{V}_{n,i} = \int_{\mathbb{R}^3} \psi_i(\mathbf{r}) \cdot E_n(\mathbf{r}) \, dV
\]

and the incident field defined in (14). Hence, once the characteristic mode excitation \( \mathbf{F}_n \) is known, one additional run of the chosen solver must be executed to extract each characteristic mode’s field and current distributions.\(^3\) Similar relations may be used to extract modal quantities from scattering-based

\(^1\)Formula (10) can be applied for other formulations as well, identifying proper reactance and radiating operators \([8]\).

\(^3\)If induced field and current data are stored for each incident plane wave during the construction of the scattering dyadic, these additional solutions may be avoided (see Section III).
where the weighting matrix \( \mathbf{A} \) is a diagonal matrix with elements \( \ell_{q} \), and the eigenvectors \( \mathbf{F}_{n} \) are organized as

\[
\mathbf{F}_{n} = \begin{bmatrix} F_{n,0} \\ F_{n,p} \end{bmatrix},
\]

(22)

where the vectors \( F_{n,\gamma} \) contain the far-field coefficients \( F_{n,\gamma}^{p} \) of each mode collected at all quadrature points. By (2), the interpretation of each element of the matrix \( S_{\gamma\gamma'} = [S^{pq}_{\gamma\gamma'}] \) is that of the \( \gamma \)-polarized far-field amplitude at the observation direction \( \mathbf{r}_{p} \) when the scatterer is illuminated with a \( \gamma' \)-polarized plane wave propagating along the \( \mathbf{r}_{q} \) direction, where \( \gamma \in \{ \theta, \phi \} \) and \( \gamma' \in \{ \theta', \phi' \} \) represent the two choices for both incident and observed field polarizations (see Fig. 2).

An important parameter of the proposed method is the number of plane waves \( 2N_{q} \) (counting two polarizations) used to construct the matrix \( \mathbf{S} \). The parameter \( N_{q} \) controls the overall precision of this discrete representation of the scattering dyadic. For the particular case of Lebedev quadrature [27], a sufficient number of plane waves used to represent the scattering dyadic of a scatterer of electrical size \( ka \) can be estimated from

\[
N_{q} \geq \frac{4}{3} \left( ka + 2\sqrt{ka} + 1 \right)^{2},
\]

(23)

based on the maximum number of spherical vector waves that can accurately be integrated via this quadrature rule [28]. Note that, for Lebedev quadrature, the number of quadrature points \( N_{q} \) must be selected from a specific set of integers, so the above rule should be understood as taking the lowest member of that set satisfying the inequality. Regardless of the chosen quadrature rule and meshing, the precision of the characteristic mode decomposition (20) for lossless structures can be verified by checking the condition \( |2n + 1| = 1 \).

\[
|2n + 1| = 1.
\]

(24)

The matrix \( \mathbf{S} \) can be filled by repeated excitation of a structure with plane waves in directions \( \mathbf{r}_{q} \) described by the selected quadrature rule and observing the scattered fields at locations \( \mathbf{r}_{p} \), also based on the chosen quadrature rule. Because the scattered fields at all selected directions can generally be computed with negligible cost compared with that of solving the underlying plane-wave scattering problem, the overall computational cost of populating the matrix \( \mathbf{S} \) scales linearly with the parameter \( N_{q} \). Nevertheless, parallelization can be readily employed in frequency domain solvers for simultaneous evaluation of multiple frequency points and FDTD for simultaneous evaluation of incident plane-wave excitations. Frequency domain methods may also be efficiently implemented using direct solvers that reuse matrix inversion or factorization (e.g., LU decomposition) for multiple plane-wave excitations. This approach necessarily becomes infeasible as the system matrix becomes very large and where iterative solutions must be used instead, in which case only partial solution reuse may be possible [29].

The following subsections outline particular considerations for using three common numerical methods, MoM, FEM, and FDTD, to numerically compute the matrix \( \mathbf{S} \), as well as a short discussion of applying far-field modal tracking using scattering dyadic data.

### A. Method of Moments

Apart from rare exceptions [16], [30], [31], MoM is the only method used for characteristic mode decomposition. Despite the ease of using the system matrix approach to compute characteristic modes using MoM data [13], MoM can also be utilized in the calculation and decomposition of the scattering dyadic. The advantage of utilizing MoM in the scattering dyadic approach to characteristic modes is that it is independent of which MoM formulation, e.g., EFIE, magnetic field integral equation (MFIE), combined field integral equation (CFIE), Poggio-Miller-Chang-Harrington-Wu-Tsai method (PMCHWT), or multilevel fast multipole algorithm (MLFMA) [32], is used, since only plane-wave scattering problems must be solved, and no decomposition of system (impedance) operators is necessary.

When surface equivalence methods are employed, the calculation of the matrix \( \mathbf{S} \) via MoM excels in precision and computational time. As compared with the contemporary approaches [33], [34], the form of the decomposition is always the same, see (20), no matter how the equivalence is formulated. For dielectric and magnetic materials, it can be combined with FEM, where MoM serves as a radiation boundary condition [32].

A drawback of MoM lies mainly in dealing with inhomogeneous materials, which is infeasible using the surface equivalence principle. Volumetric MoM is well suited to dealing with inhomogeneous media, but its computational cost scales rapidly for electrically large problems, though this can be somewhat mitigated through the use of FFT acceleration [32].

### B. Finite-Difference Time-Domain Method

In FDTD, the scattering dyadic is determined by illumination of the object with time-domain pulses from different directions. Absorbing boundary conditions, e.g., perfectly matched layers (PML), are used to mimic free space, and a near to far-field transformation is used to evaluate the scattered far-field [32]. Because broadband incident pulses can be used to produce scattering dyadic data over broad bandwidths, the FDTD affords the ability to compute broadband characteristic mode data with a number of calculations scaling with the parameter \( N_{q} \) (as opposed to \( N_{q} \) times the number of desired frequency points, as occurs in frequency domain solvers). FDTD is matrix-free, and time stepping is easily accelerated by graphics processing units (GPUs) [35].

Drawbacks of FDTD include numerical dispersion producing directional phase shifts in the scattering dyadic, low-order approximation of curved surfaces, increased computational time for narrow resonances due to ringing effects, and limited possibilities to accelerate computations for multiple excitations without the use of parallelization.

### C. Finite-Element Method

Similar to MoM, FEM solvers may be used to construct the matrix \( \mathbf{S} \) by solving plane-wave excitation problems in the frequency domain. FEM excels in the study of arbitrarily shaped inhomogeneous scatterers, but similar to FDTD, the
solution domain in FEM calculations must be terminated either by absorbing boundary conditions, e.g., PML, or by a boundary surface modeled by MoM [32]. The latter approach removes the computational burden associated with conventional absorbing boundary conditions and modeling of a truncated free-space region surrounding the scatterer. Regardless of which approach is taken, the numerical accuracy of scattering dyadic data produced by FEM is dependent not only on the discrete representation of the scatterer itself but also on the treatment of the open boundary. In large problems, the rapid growth of the size of the FEM system matrix necessitates the use of iterative solvers, which lowers the efficiency of repeated evaluation of scattering from individual plane-wave excitations [29].

D. Far-Field Tracking

Tracking is a set of postprocessing techniques for studying and interpreting modal quantities across discrete frequency points. Many procedures exist (see [36] for a detailed reference list), typically based on correlation between eigenvectors at two adjacent frequency points. Since the eigenvectors in (20) are themselves the modal far-fields, far-field tracking is, similar to [16], adopted here using

\[
\max_n \left| \tilde{F}_n(k_q) \begin{bmatrix} \Lambda & 0 \\ 0 & \Lambda \end{bmatrix} \tilde{F}_n(k_{q+1}) \right| \quad (25)
\]

where the characteristic far-fields are assumed to be normalized to unit radiated power, \(k_q\) and \(k_{q+1}\) are the wavenumbers corresponding to the \(q\)th and \((q+1)\)th frequency points, and \(n\) denotes the index of a chosen characteristic far-field for which the most similar one obtained by searching over the index \(n\). Note that the weighting matrix \(\Lambda\) is introduced in (25), so that the maximized quantity represents the inner product of modal far-fields over the unit sphere.

IV. EXAMPLES

Here, we present examples illustrating the practical calculation of characteristic modes using the scattering dyadic formulation and several numerical methods, namely, MoM, FEM, and FDTD, as implemented in CST Studio Suite [17], Altair FEKO [18], Ansys HFSS [19], COMSOL Multiphysics [20], and Antenna Toolbox for MATLAB (AToM) [37]. Templates and scripts for these solvers may be found in the Supplementary Material [38]. The examples are selected to demonstrate agreement among differing numerical methods, agreement with analytic results, and the analysis of a practical structure involving both dielectric and metallic materials. Throughout all examples, with an exception of COMSOL Multiphysics, the number of quadrature points \(N_q\) is fixed across electrical size \(ka\) and set lower than the conservative estimate given in (23). Evaluations within the COMSOL environment used (23) at every frequency point. The selected number of quadrature points differs between implementations in different solvers to reduce the computational cost, though in all cases it is listed next to the corresponding results. A detailed study of accuracy and quadrature settings is presented later as part of the discussion in Section V.

A. PEC Plate

As a first example, we consider a perfectly electrically conducting (PEC) rectangular plate of dimension \(\ell \times \ell/2\) (\(\ell = 15\) cm). Because of the structure’s simplicity, it is straightforward to analyze using most full-wave solvers. Characteristic modes are calculated via (20) using matrices \(\tilde{S}\) obtained from each method. Modal significances \(|t_n|\) and characteristic angles for modes with low modal significance \(|t_n| \ll 1\) calculated by (26) is highly sensitive to small numerical errors, leading to jumps. These errors can largely be avoided by a slight reformulation \(\alpha_n = \arg(t_n) \in [\pi/2, 3\pi/2]\) from all calculations are shown in Figs. 3 and 4, respectively. All methods show good agreement in modal significance and characteristic angle, with slight deviation in FEM results at higher frequencies.

Calculation of the eigenvectors \(\tilde{F}_n\) and eigenvector \(t_n\) requires only numerical computation of the matrix \(\tilde{S}\). In non-MoM solvers, however, the corresponding characteristic fields or current densities must be obtained by illuminating the structure with multiple plane-wave excitations described by the vectors \(\tilde{F}_n\) (see Section II-C). An example of this calculation is shown in Fig. 5, where characteristic mode current distributions are reconstructed using FDTD simulation data. When compared with characteristic currents and characteristic far-fields obtained using the conventional EFIE MoM (10) (not shown), the results are visually indistinguishable.

B. Structure Involving Dielectric and Metallic Elements

A significant advantage of the scattering dyadic formulation of characteristic modes is its applicability to structures.
Fig. 4. Characteristic angles $\alpha_n$ of a rectangular PEC plate with dimensions $\ell \times \ell/2$ for $\ell = 15$ cm. Data are shown for calculations using the system matrix formulation (10) (AToM, dashed lines) and the scattering dyadic formulation (20) using MoM (FEKO, $N_q = 50$), FDTD (CST, solid lines, $N_q = 110$), and FEM [HFSS, $\odot$, $N_q$ given by (23)].

Fig. 5. Characteristic far-fields $F_n(\hat{r})$ (top) and current densities $J_n$ (bottom) for the three modes of highest modal significance for a rectangular PEC plate with dimensions $\ell \times \ell/2$ for $\ell = 15$ cm evaluated at 0.9 GHz using 110 Lebedev quadrature points in FDTD. The polarization and amplitude of far-fields (top) are displayed at the Lebedev quadrature points by arrows. The colormap displays the far-field amplitude. The corresponding characteristic currents (real-valued) on the plate are similarly represented in the bottom.

Fig. 6. Characteristic angles $\alpha_n$ of an air-filled PEC mobile device model. Data are shown for calculations using the system matrix formulation (10) (AToM) and by (20) with the scattering dyadic evaluated according to Section III-A using FEKO and Section III-B using CST. In terms of characteristic angles $\alpha$, shown in Fig. 6, the observed agreement between the methods is excellent. The dielectric-filled structure was analyzed using only the scattering dyadic method. Within FEKO, a surface equivalent treatment of the dielectric material was used for this calculation. Although no reference data are available in for this example, the observed agreement of characteristic angles from two different computational schemes gives assurance of the validity of the results. The overall effect of the dielectric filling is a lowering of sharp resonant frequencies by $\approx 30\%$ in Fig. 7 ($\{0.50, 1.0\}$ GHz) as compared with Fig. 6 ($\{0.68, 1.4\}$ GHz). These resonances are predominantly governed by the current density on the rim and are, therefore, strongly affected by the presence of the dielectric.

C. Multilayer Spheres

In order to obtain a comparison with analytical data, two spherical multilayer structures are studied. The first is a

containing arbitrary material distributions.\footnote{Lossy materials can be introduced to characteristic mode calculations, but at the cost of losing far-field orthogonality and complication of the interpretation of modal quantities [16]. For simplicity, here, we only consider lossless materials.}
layered dielectric sphere consisting of four layers of thickness 0.25 $a$ with the relative permittivities of $\varepsilon_r = \{3, 5, 8, 2\}$ (inner layer to outer layer). The second structure contains layers with the relative permeabilities of $\mu_r = \{3, 1, 8, 1\}$. Eigenangles $\alpha_n$ of tracked modes for the dielectric and dielectric/magnetic spheres are depicted in Figs. 8 and 9, respectively. The dashed lines correspond to analytical Mie series solutions, and the solid lines are from FDTD simulations in CST Studio Suite, and the markers are from FEM simulation in Altair FEKO. The square markers at $ka \approx 3.54$ were evaluated 10% off the position of the equidistant frequency point ($ka \approx 3.55$) due to the instability of FEKO’s solver at the original frequency.

Within the results for the dielectric/magnetic layered sphere, the features of note include significantly lower modal resonance frequencies compared with the dielectric layered sphere and a wideband resonance near $ka = 3.5$. The fact that the characteristic modes for these general structures are known analytically means that these examples validate the accuracy of the methods proposed in this work and can serve as benchmark problems for other characteristic mode calculation techniques that may be developed in the future.

V. DISCUSSION

The scattering dyadic method proposed in this article has several notable advantages over the classical characteristic mode decomposition based on system (impedance) matrices. First, the matrix decomposition is a standard eigenvalue problem, as opposed to the generalized eigenvalue problem utilized in the impedance-based decomposition [4]. Second, the rank of the scattering dyadic matrix (i.e., the number of calculated modes) can easily be controlled by the selected number of quadrature points. The type of the quadrature and its degree controls the precision (see Fig. 10). In this respect, the

5The diagonal transition matrix of layered spheres may be constructed analytically using the Mie series [10], and this matrix may, in turn, be used to obtain the characteristic values via (7).

Fig. 8. Characteristic angles $\alpha_n$ of a multilayered dielectric sphere (illustrated in the top left of image). The layers have a thickness of 0.25 $a$ and a relative permittivity of $\{3, 5, 8, 2\}$ (from the most central layer to the most outward). Data are shown for calculation using Mie series (dashed lines) and the scattering dyadic formulation (20) using FDTD (CST, solid lines, $N_q = 74$) and FEM (FEKO, * MoM used as radiation boundary condition, $N_q = 38$).

Fig. 9. Characteristic angles $\alpha_n$ of a multilayered dielectric-magnetic sphere (illustrated in the top left of image). The layers have a thickness of 0.25 $a$, a relative permittivity of $\{1, 5, 1, 2\}$, and the relative permeability of $\{3, 1, 8, 1\}$ (from the most central layer to the most outward). Data are shown for calculation using Mie series (dashed lines) and the scattering dyadic formulation (20) using FDTD (CST, solid lines, $N_q = 74$) and FEM (FEKO, * MoM used as radiation boundary condition, $N_q = 38$).

Fig. 10. Panes show average error in (a) magnitude and (b) phase of the eigenvalues $t_n$. First 25 eigenvalues (ordered with respect to decreasing modal significance) are evaluated using scattering dyadic formulation (20) and system matrix decomposition (10). The eigenvalues $t_n$ were first converted to eigenvalues $s_n = 2t_n + 1$ [8], and the error in absolute value was evaluated as $\frac{1}{N} \sum_{n=1}^{25} |s_n| - 1$ and in phase as $\frac{1}{N} \sum_{n=1}^{25} \min(\pi - \Delta_n, \Delta_n)$ with $\Delta_n = s_n^{\text{MoM}} - s_n^{\text{FEM}}$ and $s_n^{\text{FEM}}$ representing data evaluated via (10). Both evaluations (10) and (20) are dependent on system matrix, while only (20) depends on degree of the quadrature $N_q$. Therefore, data from (10) serve as a reference. The estimate (23) is depicted as the solid dashed lines. The values of $N_q$ are taken only from the set of allowed Lebedev quadrature degrees. The model used in this calculation is the metallic rim from Fig. 6 discretized with 1212 basis functions and with the ground plane removed from one of the quadrants (to enrich the characteristic mode spectrum and to break the symmetries).

Lebedev quadrature [27] excels in number of points required and in knowledge of how many spherical waves are taken into account. As an alternative to Lebedev quadrature with the estimate (23), other quadrature rules enabling reuse of integration points, e.g., [39], would, however, allow for iterative decomposition based on checking for predefined error in, e.g., modal significance. These computational aspects accelerate characteristic mode decomposition.

An additional benefit of the scattering dyadic approach is that, similar to (7), the eigenvectors are themselves the
characteristic far-fields, used often for tracking the eigenvalues and eigenvectors [16], [40]. Therefore, it is not necessary to evaluate the characteristic far-fields from surface current densities, as is required in the classical impedance decomposition. The advanced tracking routines based on scattering matrix interpolation and rational fitting [16] can be used to mitigate the computational burden of frequency-domain solvers.

A unique feature is a principal possibility to decompose datasets provided by a bistatic measurement (including phase) [41] or datasets generated by co-simulation of full-wave and circuit designers. Other possibilities include investigation of objects above infinite ground planes (filled by stratified media), periodic structures, or specific scenarios addressed with Inagaki modes [42].

VI. Conclusion

In this work, the scattering dyadic is used to define the characteristic mode decomposition. The final formulation is independent of the choice of electromagnetic solver, and it can easily be implemented in many academic and commercial software packages, as it requires only the solution of plane-wave scattering problems. Wrappers for MoM, FEM, and FDTD in several commercial software packages are available as Supplementary Material [38].

The method is general and valid for arbitrary materials of arbitrary distribution. Lossy materials can be treated as well; however, this comes at the cost of losing modal far-field orthogonality.

The eigenvectors produced by the scattering dyadic method simultaneously represent the characteristic far-fields as well as characteristic excitations. Other characteristic quantities often used for antenna design, such as modal surface currents, can be reconstructed by exciting the obstacle with the selected characteristic excitation. This also implies that other non-standard modal quantities, such as modal near fields, may be directly calculated using the same process, which may be advantageous in systems involving inhomogeneous dielectrics.

Calculation of characteristic modes using the scattering dyadic may be accelerated in several ways. The advanced tracking algorithms utilizing interpolation and fitting can be used in the frequency domain to significantly reduce the number of frequency points required to reconstruct modal data over broad bandwidths. In the time domain, the entire frequency sweep can be recovered from a single simulation through the proper choice of the excitation signal. Finally, as the number of plane waves used to construct the scattering dyadic grows, an adaptive quadrature rule reusing quadrature points from the previous iteration may provide significant computational speed-up.

The proposed approach also opens up several future areas of work in characteristic modes. Use of the scattering dyadic makes it possible to obtain characteristic data from co-simulation of full-wave and linear circuit designs, since this framework only demands an input-output relation between impinging plane waves and scattered far-fields. In addition, the scattering dyadic can, in principle, be constructed using bistatic scattering data, allowing for the reconstruction of characteristic numbers and far-fields from laboratory measurements. Other open topics, such as the characteristic modes of patch antennas over layered media, infinite ground planes, or periodic environments, are also possible to explore with the scattering dyadic methodology.

APPENDIX A

EQUVALENCE OF T-MATRIX AND SCATTERING DYADIC EIGENVALUE PROBLEMS

Alternatively to the description via scattering dyadic (2) and plane waves, the incident field can be represented as a sum of spherical vector waves

\[ E^i(r) = k\sqrt{\mu_0} \sum a_n u_n^{(1)}(kr) \] (27)

and the scattered far-field as a sum of vector spherical harmonics

\[ F(\hat{r}) = \sqrt{\mu_0} \sum f_n \phi_n(\hat{r}), \] (28)

where

\[ \phi_n(\hat{r}) = (-1)^{n+\tau} A_n(\hat{r}), \] (29)

see [10, Appendix C] for details.

Under these expansions, the object’s scattering characteristics are represented by a transition matrix \( T \) mapping incident (regular) wave coefficients to scattered (outward) wave coefficients via

\[ f = T a. \] (30)

It is this transition matrix that forms the definition of characteristic modes originally proposed by Garbacz [22]

\[ T f_n = t_n f_n, \] (31)

where, by (28), (27), and (30), the eigenvectors \( f_n \) represent both characteristic incident and scattered fields.

The T-matrix may be represented in terms of the scattering dyadic \( S(\hat{r}, \hat{r}') \) via [10]

\[ T_{\alpha\beta} = \int_{4\pi} \int_{4\pi} \phi_\alpha^*(\hat{r}) \cdot S(\hat{r}, \hat{r}') \cdot \phi_\beta(\hat{r}') \, d\Omega \, d\Omega'. \] (32)

Using relations (28) and (32), the eigenvalue problem in (31) may be written as

\[ \frac{1}{\sqrt{\mu_0}} \int_{4\pi} \phi_\alpha^*(\hat{r}) \cdot S(\hat{r}, \hat{r}') \cdot F_n(\hat{r}') \, d\Omega \, d\Omega' = t_n f_n, \] (33)

for all indices \( \alpha \). Because of the orthogonality of the chosen basis [10, Appendix C-4]

\[ \int_{4\pi} \phi_\alpha^*(\hat{r}) \cdot \phi_\beta(\hat{r}) \, d\Omega = \delta_{\alpha\beta}, \] (34)

the right-hand side of (33) may be rewritten using (28) to give

\[ \int_{4\pi} \phi_\alpha^*(\hat{r}) \cdot \int_{4\pi} S(\hat{r}, \hat{r}') \cdot F_n(\hat{r}') \, d\Omega' \, d\Omega = t_n \int_{4\pi} \phi_\alpha^*(\hat{r}) \cdot F_n(\hat{r}) \, d\Omega, \] for all \( \alpha \). (35)
The basis $\{\phi_n\}$ is complete for vector functions over the unit sphere [10, Appendix C-4], which implies the equality of the integrands in the above expression, i.e.,
\[
\int_{4\pi} S(\hat{r}, \hat{r}') \cdot F_n(\hat{r}') \, d\Omega' = t_n F_n(\hat{r}).
\] (36)

**APPENDIX B**

**RELATION OF CHARACTERISTIC MODES AND IMPEDANCE MATRIX**

The conversion of the eigenvalue equation (6) into a matrix eigenvalue problem can also be done using MoM applied to the EFIE [6]. Within that paradigm, the interaction of the scatterer eigenvalue problem can also be done using MoM applied to the interaction of the scatterer.

Relation (41) together with (37) and (2) can be used to evaluate the components of scattering dyadic as
\[
S_{Y,Y}(\hat{r}, \hat{r}') = -\frac{1}{Z_0} K_{Y,\gamma}(\hat{r}) Z^{-1} K_{Y,\gamma}^H(\hat{r}')
\] (42)
where the invertibility of the matrix $Z$ is assumed.

To demonstrate the equivalence between the eigenvalue problems (6) and (10), consider a characteristic mode calculated from (6) defined by a modal far-field $F_\eta$. This modal far-field can be written in terms of a MoM representation of the modal current $I_\eta$ as
\[
F_{\gamma,\eta}(\hat{r}) = K_{\gamma}(\hat{r})I_\eta.
\] (43)

Similar to the construction of single plane-wave excitations in (40) and (41), the relations in (14), (16), and (39) can be combined to write the MoM representation of the characteristic excitation for this mode as
\[
V_n = -\frac{1}{Z_0 t_n} \sum_{\gamma} \int_{4\pi} K_{H,\gamma}^H(\hat{r}') F_{\gamma,\eta}(\hat{r}') \, d\Omega'
= -\frac{1}{Z_0 t_n} \sum_{\gamma} \int_{4\pi} K_{H,\gamma}^H(\hat{r}') K_{\gamma}(\hat{r}') I_\eta \, d\Omega'
\] (44)
where the final right-hand side is obtained by substitution of (43). For lossless structures, the integrated power radiated in all directions corresponds to the total real power dissipated by the current, i.e.,
\[
\int_{4\pi} \frac{1}{Z_0} \sum_{\gamma} \int_{4\pi} K_{H,\gamma}^H(\hat{r}') K_{\gamma}(\hat{r}') \, d\Omega I = i^HRI.
\] (45)

Because the above relation holds for any current $I$, we have
\[
V_n = -\frac{1}{t_n} RI_n,
\] (47)
which, when combined with (37), gives
\[
ZI_n = -\frac{1}{t_n} RI_n.
\] (48)
Rearranging and substitution of (11) yield the typical characteristic mode eigenvalue problem in (10).

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