Andreev reflections modulated by the breaking of space-inversion symmetry in graphene-typed materials

Xue-Si Li¹, Shu-Feng Zhang¹ and Wei-Jiang Gong¹

¹ College of Sciences, Northeastern University, Shenyang 110819, People’s Republic of China
² School of Physics and Technology, University of Jinan, Jinan, Shandong 250022, People’s Republic of China
E-mail: gwj@mail.neu.edu.cn

Keywords: Andreev reflection, space-reversal symmetry broken, graphene-typed materials

Abstract

We theoretically perform a comprehensive analysis about the influences of the space-inversion symmetry breaking in graphene-based materials on the Andreev reflections (AR) in the normal-metal/superconductor (NS) and NSN heterojunctions. It is found that in the NS junction, the AR can be suppressed or be enhanced by the enhancement of space-inversion symmetry breaking, depending on the relationship among the coherence parameters. Following this result, the AR properties in the NSN structure are evaluated. It is readily observed that the local AR can be weakened for low space-inversion symmetry breaking, and can be enhanced for high space-inversion symmetry breaking. Alternatively, the efficiency of the crossed AR can be improved to a great degree, with the increase of space-inversion symmetry breaking. One can therefore understand the special role of space-inversion symmetry in modulating the AR, especially for the enhancement of crossed AR.

1. Introduction

Heterojunctions between normal-metal (N) and superconducting (S) materials have attracted enormous attentions in the field of condensed matter physics [1–5]. The main reason is that Andreev reflection (AR) occurs within the superconducting gap $\Delta$, in which an electron in the normal metal is reflected at the NS interface as a hole accompanied by a Cooper pair entering into the superconductor [6]. This phenomenon exactly embodies the interplay between quantum coherence in the normal metal and the intrinsic coherence of the superconductivity [1]. Moreover, when two normal metals couple to one superconductor to build the NSN junction, one special AR mechanism, i.e. crossed Andreev reflection (CAR) has an opportunity to come into play, if the junction width is of the order of the superconducting coherence length [7], or even exceeds the coherence length, which has been reported recently [8, 9]. Interpretively, an electron in one normal metal that hits the NS interface forms Cooper pair in the superconductor by capturing the electron from the other normal metal [10–12]. In other words, via the CAR process, the two electrons of one Cooper pair are allowed to be spatially separated, realizing the splitting of Cooper pair. This nonlocal quantum effect can be considered to be one typical solid-state entanglement which is certainly important for quantum physics and quantum computation [13, 14].

During the past years, many groups have dedicated themselves to the investigation about the CAR process, for the purpose to achieve the high-efficiency Cooper-pair splitting [15–24]. And various Cooper-pair splitters have been designed, by considering different materials or structures. On the one hand, the quantum-dot (QD) Cooper-pair splitter has attracted much attention. As demonstrated in the previous reports [19–24], the strong Coulomb repulsion in the QDs benefits the CAR process. Another advantage of the QD Cooper-pair splitters consists in the independent level shift of respective QDs [20]. As a result, lots of theoretical researchers are encouraged to discuss the Andreev transport properties of the QD Cooper-pair splitters from different aspects [25–33]. For instance, the problems of coherence and entanglement of split Cooper pairs and their probing have been concerned [28–31], accompanied by the discussion about the noise correlations [25, 26] and Cooper pair microwave spectroscopy [27, 28]. In addition, the property of the metallic leads has been considered. When the
ferromagnetic leads are introduced to couple to the QDs, spin-polarized Andreev transport can be observed [34].

On the other hand, graphene-based Cooper pair splitters have received a lot of attentions, after the successful fabrication of graphene. Graphene has the advantage of the controllability of its-based mesoscopic circuits, and it can be superconducting by proximity to a superconductor on top of it [35–37]. This has attracted vast studies in the electronic transport through the NS heterostructures based on graphene [38–41]. It is also known that graphene has the linear dispersion relation and the conduction and valence bands touch each other at the Dirac point, making it a gapless semiconductor. Hence in graphene, the carrier type (electron: \( n \)-type; hole: \( p \)-type) and its density can be easily tuned in a controllable manner by local electrostatic gates or chemical doping. Many groups have demonstrated that the two-dimensional crystals, e.g. graphene, are possible areas for CAR processes where the magnitude of the CAR conductance can be enhanced in NSN hybrid structures. By building the NSN structure with the \( n \)-type and \( p \)-type graphene coupled to the superconducting part, channels for the electron transmission (ET) and local Andreev reflection (LAR) processes have been found to be seriously suppressed, leading to the enhancement of CAR process [42]. Other researches try to improve the CAR efficiency by proposing a graphene-based NSN spin valve [43]. This shows a spin-switch effect between pure CAR process and pure ET through the reverse of the magnetization direction in the ferromagnetic layers. Besides, a pure CAR process has been observed in a magnetized zigzag graphene nanoribbon junction with an even zigzag chain number [44].

It is well known that besides the physical real spin index in graphene, one additional intrinsic degree of freedom also affects the properties of graphene, due to the existence of two types of carbon atoms (labeled as A and B) in the honeycomb lattice of graphene. When A and B atoms are different in energy, the space-inversion symmetry is broken. The space-inversion symmetry breaking has some important applications in the fields of the wave physics and condensed matter physics, which has been discussed in different contexts such as aperiodic structures [45] and aperiodic structures with local symmetries [46]. According to the previous works, the broken inversion symmetry allows a valley Hall effect, where carriers in different valleys flow to opposite transverse edges when an in-plane electric field is applied. It can open a new possibility to the much desired electric generation and detection of valley polarization [47]. Also, it can be used to get a new phase, the Weyl semimetal can be obtained by breaking the space-inversion symmetry of the Dirac semimetal [48, 49]. In the past years, the effect of space-inversion symmetry breaking has been investigated in NS and NSN junctions, by considering its presence of the \( N \) part [50, 51]. However, more systematical research is still desirable.

In this work, we would like to focus on the NS and NSN junctions formed by graphene affected by the space-inversion symmetry breaking. Our motivation is that in comparison with the partial existence of space-inversion symmetry breaking in these kinds of junctions, introducing it in the whole junctions is more conformable with experimental requirements, because the structural complication of the former case inevitably leads to additional scattering behaviors and fabrication difficulties in experiment due to the different substrates. As is known, in a conventional NS junction, there does not only exist the electron reflection process but also the AR process. And it is also known that the AR process can be divided into retro-AR and specular-AR, according to whether the hole returns to the valence band or the conduction band, which is dependent on the relationship between the bias voltage and chemical potential [52, 53]. In the NS and NSN junctions with inversion-symmetry breaking, retro (specular)-LAR (CAR) also exist depending on the relationship among the bias voltage, chemical potential and the space-inversion symmetry breaking of graphene. The calculation results show that in the NS junction, the AR can be suppressed or be enhanced by the enhancement of space-inversion symmetry breaking, depending on the relationship among the coherence parameters. Following this result, the LAR and CAR properties in the NSN structure for both \( n \)-type and \( p \)-type are also evaluated. Alternatively, the efficiency of the CAR can be improved to a great degree, with the increase of space-inversion symmetry breaking. These results reflect the special effect of space-inversion symmetry breaking on the enhancement of the CAR process.

The rest of the paper is organized as follows. In section 2, we give the theoretical model and calculate the AR conductances with the scattering matrix method. In section 3, we investigate LAR conductance in the NS junction and the LAR and CAR conductances in the NSN junction of both \( nSn \) and \( nSp \) configurations. In section 4, we present a brief conclusion.

### 2. Model and formulate

The NS and NSN heterojunctions that we consider are illustrated in figures 1(a), (b). For the achievement of superconductivity, one can introduce the superconductor to be deposited on the top of the monolayer graphene due to the presence of proximity effect [34–36]. And the monolayer graphene is deposited on an appropriate substrate like SiC [57, 58], h-BN crystal [59], or some metal surface with a h-BN buffer layer [60, 61]. By this
To study quantum transport properties of the NS junction, we should employ the Bogoliubov–de Gennes (BdG) equation, which can be written as

\[
\begin{bmatrix}
H & \Delta(x) \\
\Delta^*(x) & -H
\end{bmatrix}
\begin{bmatrix}
u \\
v^*
\end{bmatrix}
= E
\begin{bmatrix}
u \\
v^*
\end{bmatrix},
\]

(1)

where \(u = (\psi_A, \psi_B)\) and \(v = (\psi_A^*, \psi_B^*)\) are two component spinors of electron and hole, respectively. In the presence of space-inversion symmetry breaking, \(H\) can be written as

\[
H = \hbar v_F (k_x \sigma_x + k_y \sigma_y) + m \sigma_z - \mu \mathbf{I} + U(x),
\]

(2)

where \(\sigma_x, \sigma_y, \sigma_z\) are the Pauli matrices, \(v_F\) is the Fermi velocity, and \(\mathbf{I}\) is the unit matrix. \(m\) is Dirac mass of charge carriers, which reflects the magnitude of space-inversion symmetry breaking. \(\mu\) corresponds to the chemical potential. Both the S order parameter \(\Delta\) and electrostatic potential \(U\) is spatially dependent, which can be expressed as

\[
\Delta(x) = \begin{cases} 
0 & \text{if } x < 0 \\
\Delta & \text{if } x > 0
\end{cases}, \\
U(x) = \begin{cases} 
0 & \text{if } x < 0 \\
-U_0 & \text{if } x > 0
\end{cases}
\]

(3)

The dispersion for quasiparticles at a given incident energy \(E\) and transverse wave vector \(q\) in the \(N\) region can be directly written as \(E_{n^+}^{+} = \pm \sqrt{\hbar^2 v_F^2 q^2 + m^2} - \mu\) for electron, whereas \(E_{n^-}^{-} = \pm \sqrt{\hbar^2 v_F^2 k^2 + m^2} + \mu\) for
hole, where \( k^2 = k_x^2 + q^2 \). As for the S region, the dispersion relation can be given by \( E_S = \pm \sqrt{\Delta^2 + (\pm \sqrt{m^2 + \hbar^2 \nu_F k_x^2} + \mu + U_b)^2} \).

Before the calculation about the AR process, it is necessary to present the band structure of this heterojunction. According to the dispersion relation, it is easy to find that the inversion-symmetry breaking gives rise to the appearance of band gap with width \( 2m \) at the Dirac point of graphene, and an extra band gap labeled X and Y in the S region, as shown in figure 1(c). Now we consider an electron of the conduction band with \( \sigma \) (\( \sigma \) denotes atom A) incidents from the left part, and it will be reflected as a hole in the conduction band with \( \sigma \), or in the valence band with \( \bar{\sigma} \) (\( \bar{\sigma} \) denotes atom B) \([51]\). According to the energy dispersion relation, for \( m < \mu \), it can be divided into two situations: (i) when \( 0 < E \leq \mu - m \), the hole will be retro-reflected, as shown in the top panel of figure 1(c); (ii) when \( E > \mu + m \), the hole can be specularly reflected, as shown in the bottom panel of figure 1(c).

Next, we continue to evaluate the AR properties with the help of scattering matrix method. The first step is to solve the wavefunctions of two parts, as shown in the appendix. Consider an electron incidents with energy \( E \) and wave vector \( q \), the quasiparticles can not run through the semi-infinite S region, thus the wavefunctions in the two regions can be respectively expressed as

\[
\Psi_f(r) = \Psi_n^{+}(r) + r_{ce}\Psi_n^{-}(r) + r_{ch}\Psi_p^{-}(r),
\]

\[
\Psi_n(r) = a\Psi_{n}^{+}(r) + b\Psi_{n}^{-}(r),
\]

where I, II denotes the N and S regions. \( \Psi_n^{\pm}(\Psi_p^{\pm}) \) are the eigenvectors for incoming and outgoing electrons (holes) of region N. \( r_{ce} \) and \( r_{ch} \) are electron reflection and AR coefficients, whereas \( a \) and \( b \) are coefficients of the right moving quasiparticle modes in region S. These coefficients are determined by the boundary conditions at the interface \( x = 0 \), i.e.

\[
\Psi_f(r)|_{x=0^-} = \Psi_n(r)|_{x=0^+}.
\]

After solving this equation, we can obtain the electron reflection and LAR coefficients, i.e. \( R_{ce} \) and \( R_{ch} \), written as

\[
R_{ce} = |r_{ce}|^2, \quad R_{ch} = \left| \frac{e^{iq\phi}\cos \alpha'}{e^{i\phi} \cos \alpha} \right||r_{ch}|^2.
\]

\( \phi \) and \( \phi' \) are parameters caused by the breaking of space-inversion symmetry, and the specific expressions are shown in the appendix. And then, the AR conductance can be obtained with

\[
G_{AR}(eV) = G_0 \int_{0}^{\alpha_{c}} R_{ch} \cos \alpha' d\alpha',
\]

where the ballistic conductance \( G_0 = 4e^2N(eV)/h \), and \( N(E) = \frac{W(E + E')}{\pi\hbar v_F \sqrt{(E + E')^2 - m^2}} \) denotes the number of available channels for a sheet of gapped graphene sample of width \( W \). In the whole scattering process, there is a critical angle of incidence \( \alpha_{c} \) above which the waves of the LAR, the CAR in nSn configuration and the ET in nSp configuration become evanescent waves and have no contribution to the transport (see the appendix for more details), it can be defined by

\[
\alpha_{c} = \arcsin \left( \frac{\sqrt{(E - \mu)^2 - m^2}}{\sqrt{(E + \mu)^2 - m^2}} \right).
\]

With respect to the NSN heterojunction, we consider the width of region S to be \( L \), as shown in figure 1(b). And then, two boundary conditions form at the interfaces \( x = 0 \) and \( x = L \), respectively:

\[
\Psi_f(r)|_{x=0^-} = \Psi_n(r)|_{x=0^+},
\]

\[
\Psi_n(r)|_{x=L^-} = \Psi_n(r)|_{x=L^+}.
\]

The quasiparticles can run through the finite length S region, thus the wavefunctions in the three regions can be expressed as

\[
\Psi_f(r) = \Psi_n^{+}(r) + r_{ce}\Psi_n^{-}(r) + r_{ch}\Psi_p^{-}(r),
\]

\[
\Psi_n(r) = a\Psi_{n}^{+}(r) + b\Psi_{n}^{-}(r) + c\Psi_{p}^{-}(r) + d\Psi_{p}^{+}(r),
\]

\[
\Psi_p(r) = t_{ce}\Psi^{+}(r) + t_{ch}\Psi^{-}(r).
\]

Region III denotes the right N region, \( r_{ce} \) and \( r_{ch} \) are electron reflection and LAR coefficients, respectively. And \( t_{ce} \) and \( t_{ch} \) are ET and CAR coefficients. \( c \) and \( d \) are coefficients of the left moving quasiparticle modes in the S region.

Region III has an opportunity to be set as n-type or p-type, by applying gate voltages in different manners. If it is p-type, the corresponding dispersion relation for electron and hole can be written as

\[
E_p^{\pm} = \pm \sqrt{\hbar^2 \nu_F k_x^2 + m^2 + \mu} \quad \text{and} \quad E_p^{h\pm} = \pm \sqrt{\hbar^2 \nu_F k_x^2 + m^2 - \mu}.
\]

Meanwhile, the wavefunctions also have
their alternative forms, as shown in the appendix. As a result, the electron reflection and LAR coefficients $R_{ee}$ and $R_{eh}$, and the ET and CAR coefficients $T_{ee}$ and $T_{eh}$ can be given as

$$R_{ee} = |r_{ee}|^2, \quad R_{eh} = \left| \frac{e^{i\alpha'}\cos\alpha'}{e^{i\alpha}\cos\alpha} \right| |r_{eh}|^2,$$

$$T_{ee} = \begin{cases} |t_{ee}|^2, & nSn \\ \left| \frac{e^{i\alpha'}\cos\alpha'}{e^{i\alpha}\cos\alpha} \right| |t_{ee}|^2, & nSp \end{cases}$$

$$T_{eh} = \begin{cases} \left| \frac{e^{i\alpha'}\cos\alpha'}{e^{i\alpha}\cos\alpha} \right| |t_{eh}|^2, & nSn \\ e^{-i\alpha} \cos\alpha \left| t_{eh} \right|^2, & nSp \end{cases}$$

And then, we are allowed to directly write out the LAR, CAR conductances, i.e.

$$G_{\text{LAR}}(eV) = G_0 \int_0^{\alpha_e} R_{eh} \cos\alpha'_n d\alpha'_n,$$

$$G_{\text{CAR}}(eV) = G_0 \int_0^{\alpha_e} T_{eh} \cos\alpha'_n d\alpha'_n$$

3. Numerical results and discussion

Following the theory in the above section, we proceed to study in detail the AR conductance of NS junction, and LAR and CAR conductances of NSN junction in both nSn and nSp configurations via the scattering matrix method. Prior to calculation, relevant parameters are taken to be $\hbar = 1$ and $\nu_e = 1$ during the whole calculation process. For the system’s temperature, we consider it to be zero in the context.

3.1. NS heterojunction

In the NS heterojunction, the AR characteristics can be well differentiated according to the dispersion relation. To be specific, when $m < \mu$, the retro-AR process will occur if $0 < E \mu - m$, whereas the specular-AR will take place if $E > \mu + m$. On the other hand, when $m \mu$, there only exists specular-AR if $E > \mu + m$, but the retro-AR will be forbidden.

In figure 2, we present the curves of the AR conductance of NS heterojunction. The superconducting order parameter is considered to be the energy unit by setting $\Delta = 1$. In order to study the properties of the retro-AR and specular-AR, we would like to pay attention to two cases, respectively, i.e. $\mu = 10\Delta$ and $\mu = 0.1\Delta$. It is evident that retro-AR will be dominant in the former case, and specular-AR will make leading contribution in the latter case. The results of $\mu = 10\Delta$ are shown in figures 2(a), (b) and those of $\mu = 0.1\Delta$ are in figures 2(c), (d), when the space-inversion symmetry breaking is taken into account with the difference mainly lies in that the decline amplitude of retro-AR is much larger than that of specular-AR. In figures 2(a) and (c) show the dependence of the AR conductance (i.e. $G_{\text{AR}}$) on the space-inversion symmetry breaking $m$ for different bias voltage $eV$. And figures 2(b) and (d) show the relation between the bias voltage $eV$ and $G_{\text{AR}}$ for different values of $m$. It can be seen that in both the images of retro-AR and specular-AR, the magnitude of $G_{\text{AR}}$ decreases with the increase of $m$, independent of the value of $eV$, as shown in figures 2(a) and (c). The difference mainly lies in that the decline amplitude of retro-AR is much larger than that of specular-AR. In figure 2(b), it shows that the curve of $G_{\text{AR}}$ decreases monotonically with the increment of the bias voltage $eV$ for our model. However, for the specular-AR, the situation becomes different. In figure 2(d), it shows that the AR conductance magnitude first increases and then slightly decreases with the increase of $eV$.

These results reflect two aspects. One is that $G_{\text{AR}}$ always decreases with the increase of $m$, independent of the type of AR. The other consists in that $G_{\text{AR}}$ is much more sensitive to the change of $m$ for the retro-AR, in comparison with the specular-AR. These two facts can be explained by paying attention to the band structure of our system shown in figure 1(c). For the retro-AR, the incident electron and the reflected hole are both from the conduction band. When increasing $m$, one can see the decrease of the density of states of the hole in the conduction band for the invariance of $\mu$ and $eV$. As a result, $G_{\text{AR}}$ of retro-AR decreases. Similarly, the hole is reflected to the valence band for specular-AR. When the values of $eV$ and $\mu$ are fixed and $m$ increases, the density of states of hole in the valence band decreases, thus $G_{\text{AR}}$ of specular-AR decreases. In addition, the chemical potential $\mu$ is set to be only $0.1\Delta$ for specular-AR dominates, thus the density of states of the hole in the valence band changes little with the development of $m$. Accordingly, $G_{\text{AR}}$ of specular-AR is less sensitive to the variation of $m$ compared with retro-AR.
In Figure 3, we plot the AR conductance as a function of the ratio of the space-inversion symmetry breaking of graphene to the chemical potential $m/\mu$ for different bias voltages. In this case, $\mu$ is taken to be the energy unit, i.e. $\mu = 1$, the other parameters are the function of $\mu$. As shown in the figure, the value of $m/\mu$ is varied from 0.01 to 100, in the cases of different $eV$. According to the energy dispersion in the $N$ region, the retro-AR occurs when $m/\mu < 1/(1 + eV/m)$, and the specular-AR comes into being as $m/\mu > 1/(eV/m − 1)$. Besides, for the case of $1/(1 + eV/m) < m/\mu < 1/(eV/m − 1)$, there will emerge one band gap which certainly suppresses the AR process. For the retro-AR, it can be seen that the conductance magnitude decreases with the increase of $m$, and the smaller $G_{AR}$ corresponds to the higher bias voltage. Furthermore, $G_{AR}$ decreases more rapidly than the case in Figure 2 with the increase of $m$, except for the zero bias voltage case. This phenomenon is easy to understand. With the increment of $m$, $eV$ increases accordingly but $\mu$ remains unchanged, which accelerates the decrease of the density of states of the hole in the conduction band. If $m$ is further increased, $G_{AR}$ will decrease and finally reach zero at the position of $m/\mu = 1/(1 + eV/m)$, because the density of states of the hole in the conduction band decreases to zero.

However for the specular-AR, the situation is quite different. For the case of zero-bias voltage, the specular-AR is eliminated. In the presence of the bias voltage, $G_{AR}$ first increases and then almost stabilizes at its maximum, which originates from the increase of the density of states of the hole in the valance band along with
the increment of $m$. And when the value of $m$ is significantly larger than $\mu$, the density of states almost remains unchanged with the increase of $m$. Moreover, one can find that in the cases of $eV/\Delta = 0.5$ and $0.8$, the maximal $G_{\text{AR}}$ is able to arrive at $1.0$ at the large-$m$ limit. However if $eV/\Delta = 1.0$, the value of $G_{\text{AR}}$ only reaches $0.65$ in this process. It means that $G_{\text{AR}}$ first increases and then decreases by increasing $eV$ for large space-inversion symmetry breaking, which is consistent with figure 2(d). It can also be found that the variation of $G_{\text{AR}}$ of the specular-AR is contrast to the result in figure 2(c). This means that the trend of the specular-AR is determined by the relationship among the bias voltage, chemical potential and the inversion-symmetry breaking.

3.2. NSN heterojunction

In this subsection, we continue to investigate the AR scattering of the NSN heterojunction, by considering two configurations, i.e. $nSn$ and $nSp$. In the conventional NSN junction, there exists two kinds of ARs, i.e. LAR and CAR, in which the electron enters into the $S$ region accompanied by the appearance of one hole in the same and the other metal side, respectively. In this part, we only consider the case of in the energy unit of $\mu = 1$, because in this case the conductance versus the space-inversion symmetry breaking is more sensitive. We first pay attention to the $nSn$ configuration and investigate the LAR and CAR conductances versus the space-inversion symmetry breaking, for different bias voltages. The results are shown in figure 4(a), (b) respectively. In this junction, according to the energy dispersion relationship, one can know that the retro-LAR and specular-LAR occur at the same situation with those in the NS junction, respectively. In figure 4, it can be obviously seen that both the curves of $G_{\text{LAR}}$ and $G_{\text{CAR}}$ show oscillation behaviors in the retro-AR process, but these oscillations almost disappear in the specular-AR process. To be specific, in figure 4(a) the curve of $G_{\text{LAR}}$ shows the same variation behavior in figure 3, expect for the oscillation and a lower value, because it can be viewed as one NS junction with finite length of the $S$ region. Next in figure 4(b), we can see that for the retro-CAR, $G_{\text{CAR}}$ decreases with the increase of $m$ in the presence of the bias voltage. It is because the density of states of the hole in the conduction band decreases with the increase of $m$. However, the value of $m$ has an apparent effect on $G_{\text{CAR}}$ at the case of zero-bias voltage. When $m/\mu = 0.85$, the maximum of $G_{\text{CAR}}$ occurs at the position of $eV/\Delta = 0$, almost equal to 0.12, as marked in blue circle. We choose the maximum of $G_{\text{CAR}}$ and calculate the

![Figure 4](image-url)

Figure 4. (a) LAR and (b) CAR conductances in the NSN heterojunction as a function of $m/\mu$ in the $nSn$ configuration. Inset of (b) shows the coefficients of reflection ($R_e$ and $R_h$) and transmission ($T_e$ and $T_h$) with the increment of the incident angle $\alpha$. Parameters: $\mu = 1$, $U = 5\mu$, $\Delta = 10m$, $L = 0.5\xi$. 

New J. Phys. 21 (2019) 083017 X-S Li et al
dependence of reflection and transmission coefficients on the incident angle, as shown in the inset of figure 4(b). It shows that $R_{ee}$ increases, and the rest coefficients decrease by increasing $\alpha$, and the maximal value of $T_{eh}$ reaches 0.12 approximately. On the other hand, the value of $G_{CAR}$ is relatively small and can be ignored for the specular-CAR process. In this process, it is proven that the ET and LAR are dominant, while the electron reflection and CAR are suppressed.

Next, we would like to evaluate the LAR and CAR conductances in the $nSp$ configuration by supposing the parameters to be identical with figure 4. The corresponding results are shown in figure 5. In such a case, the energy dispersion relationship suggests that the retro-(specular-)LAR occur at the same situation with the retro-AR (specular-AR) in the NS junction. While for the CAR, there only exists the specular-CAR, without the restriction of critical angle and bias voltage, but the retro-CAR is forbidden completely. Further more, it is important to note that in the band gap region $1/(1 + eV/m) < m/\mu < 1/(eV/m - 1)$, the ET and the LAR are forbidden, there only exists the CAR and the electron reflection. In this case, the incident electrons have a chance to be completely converted into CAR. Firstly, in figure 5(a) we see that $G_{LAR}$ behaves in a similar way with the $nSn$ configuration (See figure 4(a)). However, $G_{CAR}$ exhibits an alternative result in comparison with the case of $nSn$ junction, as shown in figure 5(b). One can see that the curve of $G_{CAR}$ exhibits the oscillatory behavior throughout all the values of $m$ in the presence of the bias voltage, indicating that there only exists one type of hole transmission, i.e. specular-CAR. While for the case of zero-bias voltage, the specular-CAR is eliminated after $m > \mu$. In addition, it shows that with the increment of $m$, $G_{CAR}$ first increases with oscillations and then decreases to zero for a large space-inversion symmetry breaking in the presence of the bias voltage. The maximum value of $G_{CAR}$ is able to reach 0.83 when $eV/\Delta = 0.3$ and $m/\mu = 0.42$ in the band gap region, as marked in blue circle. In view of this result, we choose the maximum point of $G_{CAR}$ and present the dependence of reflection and transmission coefficients on the incident angle, as shown in the inset of figure 5(b). It can be readily found that there only exist the electron reflection and hole refraction, i.e. $R_{ee}$ and $T_{eh}$ in the scattering process, because the incident angle exceeds the critical angle. It is worth mentioning that in this case, the maximum of $T_{eh}$ has an opportunity to arrive at 1.0 for an appropriate angle. This indicates that the incident electrons can be switched to the CAR without any electron reflection.

Figure 5. (a) LAR and (b) CAR conductances in the NSN junction as a function of $m/\mu$ in $nSp$ configuration. Inset of (b) shows the coefficients of the reflection ($R_{ee}$ and $R_{eh}$) and transmission ($T_{ee}$ and $T_{eh}$) with the enhancement of the incident angle $\alpha$. Parameters: $\mu = 1$, $U = 5\mu$, $\Delta = 10m$, $L = 0.5\xi$. 
We then perform the investigation about the influence of the length of S region, i.e. $L$, on the property of $G_{\text{CAR}}$ in the $nSn$ and $nSp$ junctions, as shown in figure 6. For describing the role of space inversion symmetry, the results of $m = 0$ are presented as well. Figures 6(a), (b) are the results of the $nSn$ configuration in the cases of $m = 0$ and $m = 1.1$, whereas figures 6(c), (d) correspond to the $nSp$ case. With respect to the bias voltage, its value is taken to be $eV = 0$ in figures 6(a), (b) and $eV = \mu - m$ in figures 6(c), (d), so the critical angle $\alpha_s$ is $\frac{\pi}{2}$ and 0, respectively. Under this condition, $G_{\text{CAR}}$ is allowed to reach its maximum in these two junctions. By comparing the results in this figure, the effects of $m$ can be well differentiated, in addition to $L$. Firstly, for the junction of $nSn$, we can see that with the increase of $L$, the curve of $G_{\text{CAR}}$ oscillates seriously, and the CAR conductance reaches its maximum in the vicinity of $L \approx 0.5\xi$. Thereafter, the conductance amplitude decays rapidly. With respect to the effect of $m$, it is only to magnify the amplitude of the conductance. In the case of $m/\mu = 1.1$, the conductance maximum is about three times the case of $m = 0$, which is consistent with the result of figure 4(b). On the other hand, for the $nSp$ junction shown in the bottom panel, the effect of $m$ is relatively distinct. It shows that in the case of $m = 0$, the amplitude of $G_{\text{CAR}}$ decreases monotonically by increasing $L$, accompanied by the weak oscillation of the conductance curve. Instead in the case of $m = \mu/1.1$, the conductance amplitude first grows up with the increment of $L$ until $L \approx 1.0\xi$, then it falls down slowly with the further lengthening of $L$. What is important is that the value of $G_{\text{CAR}}$ in the $m = \mu/1.1$ case is far larger than the $m = 0$ case, and it can almost be close to 1.0 near the position of $L = \xi$, and when $L = 5\xi$, it can be 0.5 approximately. Thus, we know that the breaking of the space-inversion symmetry is an important mechanism for enhancing the CAR process. Furthermore, in the two junctions the effects of finite $m$ exhibit notable difference. As shown in figure 6(b), the maximum value of $G_{\text{CAR}}$ can arrive at 0.18, in the presence of the breaking of space-inversion symmetry, for the $nSn$ junction, but in this process the CAR conductance amplitude can almost be equal to 1.0 in the $nSp$ junction. Therefore, the role of space-inversion symmetry breaking is dependent on the junction configuration.

Before concluding, it is necessary for us to compare our results with those of partial existence of space-inversion symmetry breaking in the NS and NSN junctions, despite the complications of their experimental realization. It is known that except the case of $m = 0$ in the $N$ part, the space-inversion symmetry breaking is also allowed to only exist in the $S$ part. In figure 7, we present the numerical results of three cases, i.e. $m_N = m$ and $m_S = 0$ for Case I, $m_N = m$ and $m_S = 0$ for Case II, and $m_N = m_S = m$ for Case III, i.e. our case (The sub-indexes of $m$ correspond to the cases of partial existence of space-inversion symmetry breaking in the junctions). The relevant parameters are set to $\Delta = 1$, $\mu = 10\Delta$ and $L = 0.5\xi$. For the NS junction, it can be seen in figure 7(a) that in these three cases, the amplitudes of $G_{\text{CAR}}$ decrease with the increase of space-inversion symmetry breaking (i.e. $m/\mu$). Meanwhile, their difference can be clearly observed. In Case I, the AR amplitude is smaller than the other two cases, in the region of $m/\mu \leq 0.97$. For $G_{\text{CAR}}$ in Case III, it is larger than Case I, except for the points that $m/\mu = 0$ and $m/\mu = 0.97$. Also when $m/\mu > 0.7$, it is more sensitive to the increase.

Figure 6. CAR conductances through the NSN junction as a function of the S region length $L/\xi$ in nSn configuration with $eV = 0$ for (a) $m = 0$ and (b) $m = \mu/1.1$, and in nSp configuration with $eV = \mu - m$ for (c) $m = 0$ and (d) $m = \mu/1.1$. Parameters: $\mu = 1$, $\Delta = 1/11$. 

![Figure 6](https://example.com/figure6.png)
of $m$, due to the rapid decrease of the AR amplitude. Next with respect to the NSN junction of $nSn$ configuration, it shows in figure 7(b) that the LAR properties exhibit much difference for the three cases. In Case I, the amplitude of $G_{LAR}$ decreases monotonously, whereas it shows oscillation in Case II following the increment of $m/\mu$. However in Case III, the LAR conductance seems to be independent of the increment of $m$ in the region of $m < 0.8$. As for the CAR results, one can find that with the enhancement of space-inversion symmetry breaking, they all first go up and then fall down. It seems that in Case II, the CAR enhancement is coincident with Case III before $m/\mu = 0.9$, though it is advantageous to the other cases. What is notable is that the CAR ability in Case I is weaker than others. Therefore, it can be found that space inversion symmetry breaking in the $N$ part tends to only suppress the LAR behaviors in the NS and NSN junctions, whereas in the $S$ part it plays an alternative role. And in our considered systems, all these results can be observed. We can then conclude that our systems are more helpful for understanding the role of space inversion symmetry breaking in modulating the AR behaviors.

4. Summary

In Summary, we have theoretically investigated the influences of the space inversion-symmetry breaking in graphene-based materials on the ARs in the NS junction and the NSN heterojunction with its $nSn$ and $nSp$ configurations. It has been found that in the NS junction, the AR can be suppressed or be enhanced by the enhancement of space-inversion symmetry breaking, depending on the relationship among the coherence parameters. Following this result, the LAR and CAR properties in the NSN structure for both $n$-type and $p$-type are also evaluated. We readily observe that the LAR can be weakened for low inversion-symmetry breaking, and can be enhanced for high inversion-symmetry breaking. As for the CAR, it can be greatly enhanced to a great degree, with the increase of inversion symmetry breaking. One can therefore understand the special role of spatial inversion-symmetry in modulating the AR properties.

Our obtained results have also been analyzed by comparing the cases of space-inversion symmetry breaking in respective parts of the NS and NSN junctions, respectively. It has shown that our considered NSN junction is advantageous to the case of space-inversion symmetry breaking only in the $N$ part, in driving the CAR. Besides,
due to the higher experimental feasibility, it can be believed that our considered NSN junction is a promising candidate of the Cooper-pair splitter.

Acknowledgments

This work was supported by the Liaoning BaiQianWan Talents program, the National Natural Science Foundation of China (Grant No. 11747122), the Natural Science Foundation of Shandong Province (Grant No. ZR2018PA007), and the Doctoral Foundation of University of Jinan (Grant No. 160100147).

Appendix

In the appendix, we aim to give the eigenvectors in the N and S regions, respectively.

For the N region, via solving the BdG equation, we can obtain the wave vector of it. When the incidence angle $\alpha$ is below $\alpha_c$, the wave functions with energy $E$ and transverse wave vector $q$ are given by

$$\Psi^e_{n}(\mathbf{r}) = \begin{pmatrix} \pm e^{i \phi + i \sin \alpha} \\ 1 \\ 0 \\ 0 \end{pmatrix} \exp(\pm ik_x x + i q y),$$

$$\Psi^h_{n}(\mathbf{r}) = \begin{pmatrix} 0 \\ 0 \\ \mp e^{i \phi' + i \sin \alpha'} \\ 1 \end{pmatrix} \exp(\mp i \sigma k'_x x + i q y),$$

$$\Psi^e_{p}(\mathbf{r}) = \begin{pmatrix} \pm e^{-\sigma \phi' + i \cos \alpha} \\ 1 \\ 0 \\ 0 \end{pmatrix} \exp(\pm i \sigma k'_x x + i q y),$$

$$\Psi^h_{p}(\mathbf{r}) = \begin{pmatrix} 0 \\ 0 \\ \mp e^{-\phi + i \cos \alpha} \\ 1 \end{pmatrix} \exp(\mp i \cos \alpha k'_x x + i q y),$$

where $\Psi^e_{n(p)}$ and $\Psi^h_{n(p)}$ are the n-(p-)type N region wavefunctions traveling along the $\pm x$ directions for electron and hole, respectively. Here

$$\phi = \arcsinh(m^2 \sqrt{(E + \mu)^2 - m^2}),$$

$$\phi' = \arcsinh(m^2 \sqrt{(E - \mu)^2 - m^2}),$$

$$\alpha = \arcsin \left( \frac{\hbar y q}{(\mu + E)^2 - m^2} \right),$$

$$\alpha' = \arcsin \left( \frac{\hbar y q}{(\mu - E)^2 - m^2} \right),$$

$$k = \sqrt{(\mu + E)^2 - m^2}, k_x = k \cos \alpha,$$

$$k' = \sqrt{(\mu - E)^2 - m^2}, k'_x = k' \cos \alpha',$$

$$q = k \sin \alpha = k' \sin \alpha',$$

$$\sigma = \text{sign}(\mu - E),$$

where $\alpha$ is the incident and reflection angle of electrons in region I, and also the refraction angle of electrons (holes) in region III for nSn (nSp) configuration. $\alpha'$ is the reflection angle of holes in region I and also the refraction angle of electrons in region III for nSp configuration. $k$ ($k'$) is the wave vector for n-(p-)type election and p-(n-)type hole, the transverse vector $q$ remains unchanged in scattering process satisfies the Snell Descartes, and $\sigma$ indicates whether the holes in region I belong to the conduction band or the valence band.

When $\alpha > \alpha_c$, the wave vector of holes in region I and elections in region III in nSp configuration $k'_x$ is an imaginary number, and the corresponding scattering angle $\alpha'$ is a complex angel. Thus the wave functions in the three regions are still expressed as equation (15), except for the expressions of $\Psi^e_{n}$ and $\Psi^e_{p}$, which can be written as the evanescent wave [42].
\[
\Psi_{n}^{\pm}(\mathbf{r}) = \begin{bmatrix}
0 \\
0 \\
-\text{i}q e^{\text{i}q/r} e^{-\text{arccosh}(q/k')}
\end{bmatrix} \exp(\sqrt{q^2 - k'^2} x + \text{i}q y),
\]
\[
\Psi_{p}^{\pm}(\mathbf{r}) = \begin{bmatrix}
1 \\
0 \\
\text{i}q e^{\text{i}q/r} e^{-\text{arccosh}(q/k')}
\end{bmatrix} \exp(-\sqrt{q^2 - k'^2} x + \text{i}q y).
\]

In the S region, the four excited modes with energy \(E\) and wave vector \(q\) can be written as
\[
\Psi_{S}^{\pm}(\mathbf{r}) = \begin{bmatrix}
u_{m} \\
u_{m} e^{\text{i}\theta_{m}} \\
u_{m} e^{\text{i}\theta_{m}^+}
\end{bmatrix} \exp(\text{i}\xi_{S}^{\pm} x + \text{i}q y),
\]
\[
\Psi_{S}^{\pm}(\mathbf{r}) = \begin{bmatrix}
u_{m} \\
u_{m} e^{\text{i}\theta_{m}^+} \\
u_{m} e^{\text{i}\theta_{m}^-}
\end{bmatrix} \exp(-\text{i}\xi_{S}^{\mp} x + \text{i}q y),
\]
\[
\Psi_{S}^{\pm}(\mathbf{r}) = \begin{bmatrix}
u_{m} \\
u_{m} e^{\text{i}\theta_{m}^+} \\
u_{m} e^{\text{i}\theta_{m}^-}
\end{bmatrix} \exp(\text{i}\xi_{S}^{\pm} x + \text{i}q y),
\]
\[
\Psi_{S}^{\pm}(\mathbf{r}) = \begin{bmatrix}
u_{m} \\
u_{m} e^{\text{i}\theta_{m}^+} \\
u_{m} e^{\text{i}\theta_{m}^-}
\end{bmatrix} \exp(-\text{i}\xi_{S}^{\mp} x + \text{i}q y).
\]

And the parameters are
\[
u = e^{\text{i}q/2}, \quad \nu = e^{-\text{i}q/2},
\]
\[
\beta = \begin{cases} 
\text{arccos}(E/\Delta) & \text{if } E < \Delta \\
-\text{i}\text{arccos}(E/\Delta) & \text{if } E > \Delta
\end{cases}
\]
\[
m_{1/2}^{\pm} = \sqrt{m \pm \sqrt{E^2 - \Delta^2 + U_0 + \mu}},
\]
\[
m_{1/2}^{\pm} = \sqrt{m \pm \sqrt{E^2 - \Delta^2 + U_0 + \mu}},
\]
\[
\theta_{S}^{\pm} = \arcsin(\sqrt{(\pm \sqrt{E^2 - \Delta^2 + U_0} - m^2)}),
\]
\[
\theta = \theta_{S}^{\pm}, \quad \theta = \pi - \theta_{S}^{\pm}.
\]

where \(\theta_{S}^{\pm}\) and \(\theta_{S}^{\pm}\) are the scattering angles for electronlike and holelike quasiparticles.

**ORCID iDs**

Shu-Feng Zhang @ https://orcid.org/0000-0001-5510-7591

**References**

[1] Andreev A F 1964 Sov. Phys. JETP 19 1228
[2] Hekking F W J and Nazarov Y V 1993 Phys. Rev. Lett. 71 1625
[3] Bandin A I 2009 Rev. Mod. Phys. 77 935
[4] Bergeret F S, Volkov A F and Efetov K B 2005 Rev. Mod. Phys. 77 3132
[5] de Jong M M and Beenakker C W J 1995 Phys. Rev. Lett. 74 1657
[6] Andreev A F 1964 Zh. Eksp. Teor. Fiz. 46 1823
[7] Mohammadpour H and Asgari A 2011 Phys. Lett. A 375 1339
[8] Chen W, Shi D N and Xing D Y 2015 Sci. Rep. 5 7607
[9] Herrera W J, Levy Yeyati A and Martín-Rodero A 2009 Phys. Rev. B 79 014520
[10] Deutscher G and Feinberg D 2000 Appl. Phys. Lett. 76 487
[11] Falci G, Feinberg D and Hekking F W J 2002 Europhys. Lett. 54 235
[12] Binkman A A and Golubov A A 2006 Phys. Rev. B 74 214512
[13] Morten J P, Huertas-Hernando D, Belzig W and Brataas A 2008 Phys. Rev. B 78 224515
[13] Recher P, Sukhorukov E V and Loss D 2001 Phys. Rev. B 63 165314
[14] Lesovik G B, Martin T and Blatter G 2001 Eur. Phys. J. B 24 287
[15] Hofstetter L, Csonka S, Nygård J and Schönnerberger C 2009 Nature 461 960
[16] Chen Z, Wang B, Xing D Y and Wang J 2004 Appl. Phys. Lett. 85 2553
[17] Golubev D S and Zaikin A D 2007 Phys. Rev. B 76 184510
[18] Futterer D, Governale M, Pala M G and König J 2009 Phys. Rev. B 80 035405
[19] Herrmann I G, Portier F, Roche P, Yeyati A L, Kontos T and Strunk C 2010 Phys. Rev. Lett. 104 026801
[20] Hofstetter L, Csonka S, Baumgartner A, Fulop G, d’Holllosy S, Nygård J and Schönnerberger C 2011 Phys. Rev. Lett. 107 136801
[21] Das A, Ronen Y, Heiblum M, Mahalu D, Kreinin A V and Shtrikman H 2012 Nat. Commun. 3 1165
[22] Schindele J, Baumgartner A and Schönnerberger C 2012 Phys. Rev. Lett. 109 157002
[23] Fülop G, d’Holllosy S, Baumgartner A, Malik P, Guzenko V A, Madsen M H, Nygard J, Schönnerberger C and Csonka S 2014 Phys. Rev. B 90 235412
[24] Tan Z B, Cox D, Nieminen T, Lahteenmaki P, Golubev D, Lesovik G B and Hakonen P J 2015 Phys. Rev. Lett. 114 096602
[25] Eldridge J, Pala M G, Governale M and König J 2010 Phys. Rev. B 82 184507
[26] Hiltscher B, Governale M, Spletstosser J and König J 2011 Phys. Rev. B 84 155403
[27] Burnet P, Herrera W J and Yeyati A L 2011 Phys. Rev. B 84 115448
[28] Chevallier D, Rech J, Jonckheere T and Martin T 2011 Phys. Rev. B 83 125421
[29] Rech J, Chevallier D, Jonckheere T and Martin T 2012 Phys. Rev. B 85 035349
[30] Cottet A 2012 Phys. Rev. B 86 075107
[31] Cottet A, Kontos T and Yeyati A L 2012 Phys. Rev. Lett 108 166803
[32] Rozhkov A V and Arovas D P 2000 Phys. Rev. B 62 66678
[33] Trocha P and Weymann I 2015 Phys. Rev. B 91 235424
[34] Bayandin K V, Lesovik G B and Martin T 2006 Phys. Rev. B 74 085326
[35] Trocha P and Bärnás J 2014 Phys. Rev. B 89 245418
[36] Trocha P and Weymann I 2015 Phys. Rev. B 91 235424
[37] Heersche H B, Jarillo-Herrero P, Oostinga J B, Vandersypen L M K and Morpurgo A F 2007 Nature 446 56
[38] Fagas G, Tkachov G, Pfund A and Richter K 2005 Phys. Rev. B 71 224510
[39] Beenakker C W J 2008 Rev. Mod. Phys. 80 1337
[40] Beenakker C W J 2006 Phys. Rev. Lett. 97 067007
[41] Benjamin C and Pachos J K 2008 Phys. Rev. B 78 235403
[42] Zareyan M, Mohammadpour H and Moghadam A G 2008 Phys. Rev. B 78 193406
[43] Asano Y, Yoshida T, Tanaka Y and Golubov A A 2008 Phys. Rev. B 78 014514
[44] Cayssol J 2008 Phys. Rev. Lett. 100 147001
[45] Linder J, Zareyan M and Sudbo A 2009 Phys. Rev. B 80 014513
[46] Wang J and Liu S 2012 Phys. Rev. B 85 035402
[47] Macia E 2012 Rep. Prog. Phys. 75 036502
[48] Kalozoumis P A, Morfonios C, Diakonos F K and Schmelcher P 2014 Phys. Rev. Lett. 113 050403
[49] Xiao D, Yao W and Niu Q 2007 Phys. Rev. Lett. 99 236809
[50] Halzé G B and Balents L 2012 Phys. Rev. B 85 035103
[51] Li X S, Zhang S F, Sun X R and Gong W J 2018 New J. Phys. 20 103005
[52] Majidi L and Zareyan M 2011 Phys. Rev. B 83 115422
[53] Benjamin C and Pachos J K 2008 Phys. Rev. B 78 235403
[54] Beenakker C W J 2006 Phys. Rev. Lett. 97 067007
[55] Heersche H B, Jarillo-Herrero P, Oostinga J B, Vandersypen L M K and Morpurgo A F 2007 Nature 446 56
[56] Du X, Skachko I and Andrei E Y 2008 Phys. Rev. B 77 184507
[57] Halterman K and Alidoust M 2018 Phys. Rev. B 98 134510
[58] Zhou S Y, Gweon G H, Fedorov A V, First P N, de Heer W A, Lee D H, Guinea F, Castro Neto A H and Lanzara A 2007 Nat. Mater. 6 770
[59] Varchon F et al 2007 Phys. Rev. Lett. 99 126805
[60] Giovannetti G, Khomyakov P A, Brocks G, Kelly P J and van den Brink J 2007 Phys. Rev. B 76 073103
[61] Zhou S, Gweon G, Fedorov A, First P, de Heer W, Lee D, Guinea F, Castro Neto A H and Lanzara A 2007 Nat. Mater. 6 770
[62] Giovannetti G, Khomyakov P A, Brocks G, Kelly P J and Van den Brink J 2007 Phys. Rev. B 76 073103