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Local relaxation and light-cone-like propagation of correlations in a trapped one-dimensional Bose gas

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Abstract
We describe the relaxation dynamics of a coherently split one-dimensional (1D) Bose gas in the harmonic approximation. A dephased, prethermalized state emerges in a light-cone-like evolution which is connected to the spreading of correlations with a characteristic velocity. In our description we put special emphasis on the influence of the longitudinal trapping potential and the finite size of the system, both of which are highly relevant in experiments. In particular, we quantify their influence on the phase correlation properties and the characteristic velocity with which the prethermalized state is established. Finally, we show that the trapping potential has an important effect on the recurrences of coherence which are expected to appear in a finite size system.

Keywords: non-equilibrium dynamics, thermalization, quantum gases, many-body physics

1. Introduction

The non-equilibrium dynamics of isolated quantum many-body systems has emerged as one of the recent frontiers in physics [1, 2]. The question of how and under which conditions such...
systems may relax towards stationary states, as well as the applicability of statistical mechanics to describe these states has been of theoretical interest for several decades [3–5]. Important and yet unsolved questions are, in particular, under which circumstances and on which time-scales many-body systems relax towards stationary states and whether these states can be described by statistical ensembles [4–11].

Recently it has become experimentally possible to address these questions with ultracold atomic gases [12, 13]. Experiments with optically or magnetically trapped ultracold atoms which are well isolated from the environment represent good candidates for tackling difficult problems of quantum field theory, as they allow to tune several parameters (dimensionality, interactions, temperature,...) and to probe many-body states in detail (see, e.g. [14–17]). In ultracold atom experiments, the trapping potential is an essential ingredient, which is known to strongly affect the thermodynamics of the system [18, 19]. To compare many-body calculations with experiments it is thus desirable to study also the effect of the trapping potential on the non-equilibrium dynamics.

In this work, we investigate the dynamics of a one-dimensional (1D) ultracold gas of bosons which is coherently split along the transverse direction, as in the experiments presented in [14, 20, 21]. Previous theoretical studies have described the relaxation of the average coherence between the two gases [22] as well as of the full distribution functions of interference contrast [23], and addressed the questions of dephasing beyond the harmonic approximation [24–26]. However, the influence of the trapping potential on the dynamics has not been discussed so far. Here we develop a model for the dynamics of coherence between the two gases in the presence of a trap. We use this model to describe the evolution of local correlation functions, the spreading of dephasing, and the recurrences of coherence expected in finite size systems.

The paper is organized as follows: in the first part, we review the description of the homogeneous system in the phononic approximation and show how the dephased state emerges locally and spreads in space with the speed of sound. In the second part, we consider the trapped system and derive the expression of the two-point relative-phase correlation function (PCF) between the two parts of the split system. Our model is in agreement with the experiments described in [21]. Finally, we discuss further implications of the trap on the recurrences of coherence that are expected to appear in a finite size system as well as the relevance of these recurrences for investigations of the dynamics beyond the harmonic approximation.

2. Model

We consider the dynamics of a gas of bosons that is confined in a radially symmetric harmonic trapping potential, with radial and longitudinal oscillation frequencies \( \omega_\perp \) and \( \omega \) respectively. The gas is in the quasi-1D regime [27] where the temperature \( k_\text{B}T \) and the chemical potential \( \mu \) are much less than \( \hbar \omega_\perp \), and \( \omega \ll \omega_\perp \). The gas is coherently split symmetrically along the radial direction (for experiments, see e.g. [14, 28]). The state of the system after splitting consists of two copies of the initial phase-fluctuating [27] condensate with (on average) \( N/2 \) atoms, where \( N \) is the number of atoms in the gas before splitting. We assume that there is no tunnel coupling between the gases after splitting.
The Hamiltonian of the problem is given by:

$$\hat{\mathcal{H}} = \int dz \sum_{j=1}^{2} \left( \frac{\hbar^2}{2m} \frac{\partial \hat{\psi}_j^\dagger}{\partial z} \frac{\partial \hat{\psi}_j}{\partial z} + \frac{g}{2} \hat{\psi}_j^\dagger \hat{\psi}_j^\dagger \hat{\psi}_j \hat{\psi}_j + (V(z) - \mu) \hat{\psi}_j^\dagger \hat{\psi}_j \right),$$  \hspace{1cm} (1)

with $\hat{\psi}_j$ the atomic field operator associated to each gas ($j = 1, 2$), $g = 2\hbar \omega_a a$ is the 1D interaction strength [29], $m$ is the atomic mass and $a$ is the 3D scattering length. For a homogeneous system in the Thomas–Fermi regime, the chemical potential is given by $\mu = g n_0^0$, with $n_0^0$ being the linear density in each gas [27]. We decompose the field operators into $\hat{\psi}_j = \sqrt{\hat{n}_j(z)} \exp \left[ i \hat{\theta}_j(z) \right]$, with $\hat{\theta}_j(z)$ and $\hat{n}_j(z)$ denoting the operators describing the phase and density of each quasi-condensate, respectively. This approach allows to employ a generalization of standard Bogoliubov theory to describe the system [30], as density fluctuations in the two individual gases are suppressed and can be considered as small corrections. In the quasi-condensate regime (reduced density fluctuations), we have the commutation relation $[\hat{n}_j(z), \hat{\phi}(z')] = i \delta(z - z') \delta_0$.

3. Relaxation of the homogeneous Bose gas

3.1. Hamiltonian and phononic excitations

We start by studying the case of an homogeneous gas with $V(z) = 0$. We quickly recall the theoretical analysis introduced in [31] and derive the time evolution of the two-point relative-PCF which can be used to characterize the decay of coherence locally. The situation of two quasi-condensates in thermal equilibrium has been addressed in [32]. Here we discuss the relaxation of the PCF after the coherent splitting described above and explain the local equilibration observed recently in the experiment of [21].

We introduce the relative phase and relative density between the two gases through $\hat{\phi} = \hat{\theta}_1 - \hat{\theta}_2$ and $\hat{n} = (\hat{n}_1 - \hat{n}_2)/2$, with the commutation relation $[\hat{n}(z), \hat{\phi}(z')] = i \delta(z - z')$. This is motivated by interference experiments which give direct access to the relative phase field. The common (center-of-mass) degrees of freedom are described by the operators $\hat{\phi}_c = (\hat{\theta}_1 + \hat{\theta}_2)/2$ and $\hat{n}_c = \hat{n}_1 + \hat{n}_2$. Inserting the definition of $\hat{\psi}_j$ in equation (1) and neglecting third order terms ($\sim (\partial^2 \phi)^2 \hat{n}$), we obtain a quadratic Hamiltonian for the phase and density operators. Assuming a symmetric splitting (equal density in each gas after splitting), the dynamics of the relative and the common degrees of freedom decouple, as shown in [23]. We obtain the following Hamiltonian governing the evolution of the relative phase and density operators:

$$\hat{\mathcal{H}} = \int dz \left[ -\frac{\hbar^2 n_0}{4m} (\partial_z \hat{\phi})^2 + g \hat{n}^2 \right].$$ \hspace{1cm} (2)
We rewrite this Hamiltonian in the more usual form of a Luttinger liquid [33]

\[
\hat{H} = \frac{\hbar c}{2} \int dz \left[ \frac{K}{\pi} \left( \partial \hat{\phi} \right)^2 + \frac{\pi}{K} \hat{n}^2 \right],
\]

with the speed of sound \( c = \sqrt{\frac{\hbar n_0}{m}} \), the Luttinger parameter \( K = \frac{\hbar n}{2 \sqrt{\frac{\hbar n}{m^2 g n_0}}} \) and \( n_0 \) the 1D peak density. The Luttinger liquid model is the long wavelength approximation of the many-body Hamiltonian equation (1) and allows to describe the phononic (long wavelength) excitations of the system. This approximation is valid for the description of typical ultracold atom experiments, where optical methods with an imaging resolution corresponding to a few times the healing length are used to probe the excitations in the system.

Considering periodic boundary conditions for a system of size \( L \), we expand the field operators as [31]

\[
\hat{n}(z, t) = \frac{1}{\sqrt{L}} \sum_{k \neq 0} \hat{n}_k(t) e^{ikz}, \quad \hat{\phi}(z, t) = \frac{1}{\sqrt{L}} \sum_{k \neq 0} \hat{\phi}_k(t) e^{ikz},
\]

with the expansion coefficients given by:

\[
\hat{n}_k(t) = \sqrt{\frac{n_0 S_k}{2}} \left( \hat{b}_k(t) + \hat{b}_k^\dagger(t) \right), \\
\hat{\phi}_k(t) = \frac{1}{i \sqrt{2n_0 S_k}} \left( \hat{b}_k(t) - \hat{b}_k^\dagger(t) \right).
\]

Here \( \hat{b}_k^\dagger \) and \( \hat{b}_k \) are the creation and annihilation operators for an elementary excitation with momentum \( \hbar k \) in the relative degrees of freedom \( (k = p \times 2\pi/L \) with \( p \) integer different than 0; note that the sum expands over both positive and negative \( k \). The commutation relation for the expansion coefficients reads \( \left[ \hat{n}_k, \hat{\phi}_k^\dagger \right] = \left[ \hat{n}_k, \hat{\phi}_k \right] = i \). The structure factor in the phononic regime is given by

\[
S_k = \frac{\hbar |k|}{2mc} = \frac{|k|K}{\pi n_0},
\]

and is related to the \( (u_k, v_k) \) Bogoliubov coefficients used in previous works [30, 32] through \( \sqrt{S_k} = (u_k - v_k)^{-1} \).

In this basis, the Hamiltonian equation (3) takes the diagonal form

\[
\hat{H} = \frac{\hbar c}{2} \sum_{k \neq 0} \left[ \frac{K}{\pi} k^2 \hat{\phi}_k^\dagger \hat{\phi}_k + \frac{\pi}{K} \hat{n}_k^\dagger \hat{n}_k \right] + \frac{\hbar \pi c}{2K} \hat{n}_0^\dagger \hat{n}_0 \\
= \sum_{k \neq 0} \hbar \omega_k \hat{b}_k^\dagger \hat{b}_k + \frac{\hbar \pi c}{2K} \hat{n}_0^\dagger \hat{n}_0,
\]

with \( \omega_k = c|k| \). We have made explicit the term corresponding to the \( k = 0 \) mode which accounts for the global phase diffusion. The dynamics can thus be reduced to that of a set of uncoupled harmonic oscillators. It follows from the absence of coupling between modes with different momentum \( k \) that any momentum occupation numbers \( \left\{ \hat{b}_k^\dagger \hat{b}_k \right\} \) that are initially imposed on the
system will be conserved. Equation (7) therefore strikingly shows the integrability of the system.

3.2. Initial conditions and equations of motion.

We assume the coherent splitting process to be instantaneous (fast with respect to the interaction energy, $t_{\text{split}} \ll h/\mu = 2\pi \xi_{\text{split}}/c$) so that it can be described like a beam splitter in optics. Consequently the local distribution of atoms in each small region of the quasi-condensate (of size $\sim \xi_{\text{split}} = h/mc$) is binomial, with the respective minimum uncertainty relative-phase distribution. In that way, the coherent splitting copies the phase fluctuations of the initial quasi-condensate into both parts of the split system and the relative density fluctuations are given by the local shot noise. In terms of elementary excitations, this means:

$$\langle \hat{\phi}_k^\dagger \hat{\phi}_k \rangle |_{t=0} = \frac{1}{2 \xi_{\text{split}}^2 n_0}, \quad \langle \hat{n}_k \hat{n}_k \rangle |_{t=0} = \frac{\xi_{\text{split}}^2 n_0}{2},$$

(8)

with $\xi_{\text{split}}^2$ being the number squeezing parameter (in the following we will take $\xi_{\text{split}}^2 = 1$ unless specified). A state where the fluctuations are initialized with an excess of relative-density fluctuations and a lack of relative-phase fluctuations is a non-equilibrium state far away from the thermal equilibrium state of the Hamiltonian equation (7) at a temperature $T$ where:

$$\langle \hat{\phi}_k^\dagger \hat{\phi}_k \rangle |_{\text{th}} = \frac{2}{\lambda_T k_T}, \quad \langle \hat{n}_k \hat{n}_k \rangle |_{\text{th}} = \frac{k_B T}{2g},$$

(9)

with $\lambda_T = h^2 n_0/mk_B T$. The atom number fluctuations caused by the splitting correspond to the introduction of energy into the system due to the interactions. This energy manifests itself in additional excitations in the density quadrature (rather than in the phase quadrature) with respect to the thermal equilibrium (compare equations (8) and (9)).

In terms of the elementary excitations, the initial conditions of equation (8) lead to an approximately thermal-like form of the occupation numbers that reads

$$\langle \hat{b}_k^\dagger \hat{b}_k \rangle = \frac{k_B T_{\text{eff}}}{\hbar \omega_k},$$

(10)

with the effective temperature given by $k_B T_{\text{eff}} = \xi_{\text{split}}^2 n_0 g / 2$. The fast splitting process thus equally distributes the energy $k_B T_{\text{eff}}$ in the different modes of the system [23].

The equations of motions for the operators $\hat{\phi}_k(t)$ and $\hat{n}_k(t)$ read

$$i\hbar \frac{d\hat{\phi}_k}{dt} = \left[ \hat{\phi}_k, \hat{H} \right] = \frac{\hbar c}{2 K} \left( -2i\hat{n}_k \right),$$

$$i\hbar \frac{d\hat{n}_k}{dt} = \left[ \hat{n}_k, \hat{H} \right] = \frac{\hbar c}{2} K k^2 \left( 2i\hat{\phi}_k \right),$$

(11)

and lead to harmonic oscillator equations for $\hat{n}_k$ and $\hat{\phi}_k$, with the angular oscillation frequency $\omega_k = ck$. With the initial conditions equation (8) and neglecting the initial phase fluctuations $\langle \langle \hat{\phi}_k^\dagger \hat{\phi}_k \rangle \rangle |_{t=0} \approx 0$, which is valid as soon as $t > h/\mu$, we obtain the evolution of the expansion coefficients:
These equations show the oscillation between the density and phase quadratures for the phononic excitations.

3.3. Relative-PCF

To study the decay of coherence between the two quasi-condensates and the evolution of correlations in the system, we consider the two point relative-PCF defined as [31]:

\[ C(z, z', t) = \frac{\langle \hat{\psi}_1'(z)\hat{\psi}_2^\dagger(z')\hat{\psi}_1(z')\hat{\psi}_2(z) \rangle}{\langle |\hat{\psi}_1(z)|^2 \rangle \langle |\hat{\psi}_2(z')|^2 \rangle} \]
\[ \approx \exp \left( -\frac{1}{2} \left\langle \Delta \phi_{zz'}(t)^2 \right\rangle \right). \]  

(13)

In the last step we have used the fact that the fluctuations are Gaussian, which is a consequence of the quadratic Hamiltonian equation (3). Here, \( \left\langle \Delta \phi_{zz'}(t)^2 \right\rangle \equiv \left( \hat{\phi}(z, t) - \hat{\phi}(z', t) \right)^2 \) denotes the phase variance between two points z and z' of the relative phase field. Using equation (12), the time evolution of the phase variance is given by

\[ \left\langle \Delta \phi_{zz'}(t)^2 \right\rangle = \frac{\pi^2 n_0}{LK^2} \sum_{k \neq 0} \frac{1}{k^2} \left( \frac{e_z}{2} \right)^2 \left( 1 - \cos(kz) \right). \]  

(14)

The first term in the sum equation (14) represents the growth and subsequent oscillations in the amplitude of the phase fluctuations as they get converted from the initial density fluctuations. The factor \( 1/k^2 \) in the amplitude reflects the \( 1/k \) scaling of the excitation occupation numbers associated with the equipartition of energy induced by the fast splitting, equation (10). The term in parenthesis in the sum corresponds to the spatial fluctuations.

We will discuss the physical interpretation of this expression in more details in the next two paragraphs, considering first the dephased (prethermalized) state and second the dynamics governing the emergence of this state.

3.4. Prethermalized state

We first focus on the dephased state of the system where the two-point relative-phase variance can be approximated by its long time limit, taking \( \sin^2(\omega_0 t) = 1/2 \). Approximating the sum by an integral, we obtain

\[ \left\langle \Delta \phi_{zz'}^2 \right\rangle = \frac{4mg}{\hbar^2} \times \frac{1}{2} \times \frac{|E|}{2}, \]

(15)
where we have used \( \int \frac{dk}{2\pi} \frac{1 - \cos(kz)}{k^2} = \frac{\pi}{2} \). In the relaxed (dephased) state, the PCF is thus given by

\[
C(\bar{z}) = \exp \left( -\frac{1}{l_0^2} \right), \quad \text{with} \quad l_0 = \frac{2\hbar^2}{mg}.
\] (16)

The correlation length \( l_0 \) does not depend on the detail of the system (such as the density) but only on the 1D coupling constant, and is in that sense universal. The scaling comes from the equipartition of energy between the modes during the splitting.

### 3.5. Multimode phase diffusion

It is interesting to discuss the role of the correlation length \( l_0 \) in the context of dephasing, linking our study with previous results obtained on phase diffusion in 3D BEC [34–36] and quasi-condensates [22]. The decay of the coherence factor \( \Psi(t) \equiv e^{i\phi(z,t)} \) due to the lowest mode \( (k = 0 \text{ term in equation (7))} \) is given by:

\[
\Psi(t) \bigg|_{k=0} \propto e^{-t/\tau_0^2},
\] (17)

with the characteristic time \( \tau_0 \) known from the phase diffusion in 3D BEC [34–36]: \( \tau_0 = \frac{\hbar}{g} \sqrt{\frac{L}{\pi \eta_0}} \).

On the other hand, the decay of coherence due to the contribution of all other \( (k \neq 0) \) modes is given by [22]

\[
\Psi(t) \bigg|_{k \neq 0} \propto e^{-t/\tau},
\] (18)

with the dephasing time \( \tau = 8K^2/\pi^2 n_0 c \) (note that our definition of \( K \) differs by \( 1/2 \) with respect to [22]). The dephasing will therefore be dominated by 1D effects as soon as \( \tau < \tau_0 \), which correspond to the condition

\[
l_0 = \frac{1}{\xi_n^2} < \frac{2\hbar^2}{mg} < \frac{L}{2},
\] (19)

where we explicitly introduced the squeezing parameter.

Recent studies on the role of atom–atom interactions in interferometers indicate that low-dimensional geometries could be favourable for interferometry [37]. However, 1D effects will limit this coherence time if the prethermalized correlation length is smaller than half the system size, whereas they can be neglected with respect to the 3D phase diffusion if \( l_0 > L/2 \). The latter is, for example, the case in experiments that are working with low atom numbers, high number squeezed states or not too elongated traps [38, 39]. Equation (19) can be re-formulated in terms of experimentally controllable parameters, by expressing the amount of number squeezing required for a beam splitter so that the interferometer will not be limited by multimode dephasing:

\[
\xi_n^2 < \frac{\xi_n^2}{\lim} = \frac{2\hbar}{m\omega_\perp a L}.
\] (20)

We used \( g = 2\hbar \omega_\perp a_L \) (in the quasi-1D regime) to express the number squeezing parameter in terms of measurable quantities (radial trap frequency \( \omega_\perp \) and system length \( L \)). Equation (20) provides a criterion for evaluating the possible sensitivity of elongated condensates working...
close to the 1D regime for applications in interferometry. We illustrate this criterion in figure 1 where we represent the required number squeezing (in dB) for an interferometer to be limited by 3D phase diffusion rather than 1D (multimode) effects, as a function of the physical parameters \( \omega_\perp, L \) controlling the 1D-ness of the system. We finally note that in the homogeneous case, condition equation (20) does not involve the linear density of the system as the dephasing time due to the \( k = 0 \) mode and that due to the \( k \neq 0 \) modes have the same dependence \( \propto n_0^{-1/2} \). In practical applications, however, the system size and the peak density of the trapped atoms are mutually related, thus leading to an implicit density dependence of the condition equation (20).

### 3.6. Local relaxation and light-cone effect

We now discuss the full expression equation (14) which characterizes the decay of coherence as a result of the superposition of many phononic modes. Mathematically, expression equation (14) is the fourier decomposition of a trapezoid with a siding edge at \( \xi_c = 2ct \), so that the phase variance (and thus the PCF) exhibits a two step feature. This two-step feature can physically be interpreted as follows: for a given time \( t \), short wavelength modes will grow in amplitude and linearly increase the phase variance up to a distance \( \bar{z}_c = 2ct \). Beyond that point the growth in amplitude of longer wavelength modes with \( 2\pi/k > \bar{z}_c \) exactly compensates the decrease in amplitude of the shorter wavelength modes with \( 2\pi/k < \bar{z}_c \), leading to a constant phase variance. This is illustrated in figure 2.
Quantitatively, computing the derivative of the phase variance with respect to time, we find

\[ \frac{\partial}{\partial t} \left< \Delta \phi_{zz}^2(t) \right> = \frac{2c}{l_0} \times \Theta(2ct - \bar{z}), \]

with \( \Theta(x) \) being the Heaviside step function (\( \Theta(x) = 1 \) if \( x > 0 \) and 0 otherwise) and \( l_0 = 2\hbar^2/mg \) the prethermalized correlation length introduced in equation (19). The phase randomizes with a constant rate up to the point where \( \bar{z} = 2ct \). Beyond \( \bar{z} \), modes with a wavelength larger than \( \bar{z} \) would be needed for a further randomization of the phase. However, while these long-wavelength modes grow in amplitude for \( \bar{z} > \bar{z} \), modes with shorter wavelength start to decrease again in amplitude. Overall, this leads to a constant phase variance beyond \( \bar{z} \).

Figure adapted with permission from [21].

Figure 2. Visualization of the light-cone-condition for \( c = 1 \) mm s\(^{-1} \) and \( t = 5 \) ms. The relative phase of the system is randomized by a superposition of many modes (solid lines, the first seven modes are represented). Initially, the contribution of all these modes grows in amplitude (arrows), leading to a linear increase in the variance of the phase (bottom plot). For the correlation function, this corresponds to the establishment of thermal correlations up to \( \bar{z} = 2ct \). Beyond \( \bar{z} \), modes with a wavelength larger than \( \bar{z} \) would be needed for a further randomization of the phase. However, while these long-wavelength modes grow in amplitude for \( \bar{z} > \bar{z} \), modes with shorter wavelength start to decrease again in amplitude. Overall, this leads to a constant phase variance beyond \( \bar{z} \).
quantum many-body systems [41–45], and the local relaxation picture was first introduced in a general way in the context of the Bose–Hubbard model [46]. Here, we provide a connection to the relaxation of a Luttinger liquid, finding a similar result as the one obtained numerically for the Bose–Hubbard model [46].

We finally note that while the squeezing parameter $\xi_n^2$ influences the final relaxed state (influence on the correlation length $l_c$), it does not modify the velocity of correlations, i.e. the time after which the relaxed state is reached.

\[ \text{Figure 3.} \] \( (a) \) Time-evolution of the relative-phase correlation function for \( c = 1.8 \) mm s\(^{-1}\). \( (b) \) Time evolution of the relative-phase variance \( \langle \Delta \phi_{cc}^2 \rangle \). For a given evolution time, the phase variance grows linearly up to a distance \( z_c = 2ct \). Beyond that sharp cross-over point the phase variance is constant, revealing the persisting long-range phase coherence in the system. \( (c) \) Derivative of the phase variance visualizing how the front of correlations travels through the system.
4. Relaxation of the trapped system

4.1. Relative-PCF

We now investigate the dynamics of coherence between the two gases in the case of a longitudinal trapping potential $V(z) = \frac{1}{2}m\omega^2z^2$ (see equation (1)). Under the same approximations as for the homogeneous case (neglecting third order terms in the fluctuations and considering only phononic modes), the Hamiltonian for the relative degrees of freedom can be written as

$$\hat{H} = \int dz \left[ -\frac{\hbar^2 n_0(z)}{4m} \left( \partial_z \hat{\phi} \right)^2 + g\hat{n}^2 \right],$$

(22)

where the density profile $n_0(z)$ solves the Gross–Pitaevski equation

$$\left[ -\frac{\hbar^2}{2m} \Delta + V(z) - \mu \right] \sqrt{n_0(z)} = 0.$$  

(23)

The Hamiltonian thus takes the form of a non-homogeneous Luttinger Liquid with spatially dependent velocity $c(z) = \sqrt{n_0(z)g/m}$ and Luttinger parameter $K(z) = \frac{\hbar}{2} \sqrt{\frac{n_0(z)}{mg}}$.

Following the notations of Petrov et al [47], this Hamiltonian can be re-written as

$$\hat{H} = \frac{\hbar}{2\pi} \int dz \left[ v_n (n\hat{n})^2 + v_j (\partial_z \hat{\phi})^2 \right],$$

(24)

with the density and phase velocities given by $v_n = c/K$ and $v_j = cK$.

For a weakly interacting gas in the Thomas–Fermi regime, the density profile is an inverted parabola

$$n_0(z) = n_0 \left( 1 - \frac{z^2}{R^2} \right),$$

(25)

with the Thomas–Fermi radius $R = \sqrt{2}c_0/\omega$ given by peak density of the gas via $c_0 = \sqrt{gn_0/m}$ and the peak density determined from the atom number [27]. In the Thomas–Fermi regime, the velocity $v_n$ reduces to $v_n = 2g/\pi\hbar$ and is therefore independent on the spatial coordinate.

Proceeding as for the homogeneous system, we expand the relative-phase and relative-density fluctuations as

$$\hat{\phi}(z, t) = \sum_{j>0} \hat{\phi}_j(t)f_j(z), \quad \hat{n}(z, t) = \sum_{j>0} \hat{n}_j(t)f_j(z).$$

(26)

The equations of motion read

$$\partial_t \hat{\phi}(z, t) = -\pi v_n \hat{n}(z, t)$$

$$\pi \partial_t \hat{n}(z, t) = -\partial_z \left[ \pi v_j(z) \partial_z \hat{\phi}(z, t) \right].$$

(27)

Inserting the expansion equation (26) and using the expression of $n_0(z)$ in the Thomas–Fermi regime, we find that the eigenfunctions of the problem satisfy the equation
where $x = z/R$ and $f_j'$ (resp. $f_j''$) denotes the first (resp. second) spatial derivative of $f_j$ with respect to $x$. The solutions are well known and can be expressed in terms of Legendre polynomials $P_j$:

$$f_j(z) = \sqrt{j + 1/2} P_j(x) \text{ with } x = z/R, \quad \omega_j = \omega_j \sqrt{(j + 1)/2}.$$  

(29)

The initial conditions for the mode occupation numbers after splitting for the trapped system are a priori different than for the homogeneous system (equation (8)). This is due to the fact that the amount of noise is proportional to the local density of atoms and is thus lower at the edges of the cloud than in the center. In the quantum optics picture, this would be the analogue of a local beam splitter. More specifically, the fluctuations are given by

$$\langle \hat{n}(z) \hat{n}(z') \rangle = \frac{\xi^2_n(z) n_0(z)}{2} \delta(z - z')$$

$$\langle \hat{\phi}(z) \hat{\phi}(z') \rangle = \frac{1}{2 \xi^2_n(z) n_0(z)} \delta(z - z').$$  

(30)

where we introduced the local squeezing factor $\xi^2_n(z)$. It can be shown that the corresponding mode occupation numbers $\langle \hat{n}^j \hat{n}^j \rangle_{\omega_0}$ for the trapped system are close to that for the homogeneous system (see equation (8)) and only differ slightly for the first two modes $j = 1, 2$ [48]. As this small difference does not influence the final result, we will assume for clarity the same occupation for all modes, with initial density fluctuations given by

$$\langle \hat{n}^j \hat{n}^j \rangle_{\omega_0} = \frac{n_0}{2R^2}.$$  

(31)

With this choice of initial conditions, the dephased state has the same effective temperature $T_{\text{eff}} = n_0 g/2k_B$ as for the homogeneous system. These initial conditions correspond to mode occupation numbers $\langle \hat{b}^\dagger_j \hat{b}_j \rangle = k_n T_{\text{eff}} / \hbar \omega_j$ which are the same as in thermal equilibrium [47] but with an effective temperature $T_{\text{eff}}$.

Finally, combining equations (29) and (31) we obtain for the two-point phase variance:

$$\langle \Delta \phi(z,z')^2 \rangle = \frac{n_0 \pi^2 v^2}{2R} \sum_{j=1}^{\infty} \frac{\sin(\omega_j t)^2}{\omega_j^2} \left[ f_j(z) - f_j(z') \right]^2.$$  

(32)

The expression has a similar form as the one found for the homogeneous system (equation (14)). Again, the time dependent term denotes the oscillation of energy between density and phase fluctuations, and the position-dependent term reflects the spatial phase fluctuations.

The comparison of the two-point relative-PCF for the trapped and homogeneous system is shown in figure 4, for the same value of the peak density $n_0$. The general behaviour is similar to the homogeneous system, with a correlation function being exponential up to a characteristic distance beyond which partial long-range order remains. However, three differences appear between the two systems. First, the effective correlation length $\lambda_{\text{eff}}$ corresponding to the $1/e$
value of $C(z)$ is slightly lower than in the homogeneous case (see equation (19)), due to the presence of the Legendre polynomials in the sum equation (32). Second, the long-range order plateaus appear at slightly shorter separations because the effective correlation length is lower, and because of a smaller velocity of correlations, as we will see in the next paragraph. Finally, the correlation function drops to zero at the edge of the cloud corresponding here to $R \approx 56$ micron.

4.2. Light-cone effect

In the homogeneous case, the light-cone-like dynamics of correlations directly comes from the Lorentz invariant form of the equation for the relative phase field. Such a light-cone condition is not obvious for the trapped system, where the speed of sound depends on the local density in the gas. Numerically computing the second derivative of the phase variance from equation (32), we find a well-defined front of correlations, the position of which linearly evolves in time. From this, we determine the characteristic velocity by extracting the position of the correlation front for various evolution times. The procedure is illustrated for three different atom numbers (peak densities) in figure 5.

For evolution times corresponding to a distance travelled by the quasi-particles approaching the edge of the cloud at $\pm R$, the dynamics is dominated by the finite size effect (bending of the correlation function, see figure 4) and the position of the front slightly deviates from the linear scaling in time. This is illustrated by indicating the separation corresponding to half the Thomas–Fermi radius in figure 5 (horizontal lines). However, exploring this region experimentally would be very hard as it would require a very high number of realizations in
order to obtain a smooth correlation function in that range of separations (more than $10^4$ realizations typically [21]).

In typical experiments, the gases are rather in the quasi-1D regime than perfectly 1D (i.e. $\mu, k_B T \lesssim \hbar \omega$). The effect of the radial extension of the cloud can be integrated out, leading to a density profile in the longitudinal direction that is modified with respect to the Thomas–Fermi regime [49]. Treating the density profile as an inverted parabola with a modified peak density and an effective radius, we can use equation (32) to compute the dynamics of phase fluctuations and determine the position of the front of correlations numerically in the quasi-1D regime. The results are presented in figure 6 and show a lower value for the speed of correlations ($\sim 10\%$) for the same atom number, due to the slightly lower value of the radius ($\sim 4\%$) in the quasi-1D regime.

**Figure 5.** Position of the front of correlation for the trapped system for different atom numbers: 3000 (red), 6000 (green) and 9000 (blue); the radial and longitudinal trap frequencies are $\omega_r/2\pi = 1400$ Hz and $\omega_r/2\pi = 7$ Hz, respectively. The solid line is a linear fit for times $<10$ ms to extract the characteristic velocity. The horizontal dot-dashed lines indicate half the Thomas–Fermi radius, $R/2$.

**Figure 6.** Comparison of the velocity of correlations for the homogeneous system, the Thomas–Fermi regime (TF), and the quasi-1D regime.
regime than in the purely 1D Thomas–Fermi regime. We emphasize that the lower velocity found for the quasi-1D system than for the Thomas–Fermi cloud is due to the more pronounced finite size effect captured in the Legendre polynomials \( P_j(z/R) \) in equation (32), and not by the change in density between the two regimes (the latter would rather lead to a higher velocity of correlations in the quasi-1D regime because \( n_0 |_{\text{quasi--1D}} > n_0 |_{\text{TF}} \) by about 10%). Therefore, this difference could not have been captured by a homogeneous system calculation, which would have overestimated the velocity of correlations. Our trapped calculation for the quasi-1D regime is in good agreement with experiments [21].

5. Recurrences of coherence

In the final part of this article, we study the effect of the trapping potential on the recurrences of coherence which are expected to occur when the phase quadrature completes one oscillation. Full recurrences manifest themselves in a PCF that returns to its initial value \( \bar{C} = 1 \) for all \( \bar{z} \), meaning that the initial state is re-established. These recurrences of coherence show similarities with the many-body revivals in optical lattices [50, 51], the characterization of which is known to be strongly affected by an external trapping potential [51].

Moreover, investigating possible recurrences of coherence is of great importance to characterize the long-time evolution of the system in detail, where physics beyond the harmonic approximation is expected to lead to thermalization. Possible mechanisms leading to this thermalization may be described, for example, by nonlinear terms present in the 1D Hamiltonian and effectively leading to phonon–phonon scattering [26] or by integrability breaking terms due the non-perfect 1D-ness of the system [52, 53]. Understanding the exact form and the scaling of these recurrences with the system parameters in the harmonic approximation is thus important to distinguish genuine many-body effects (quasi-particle relaxation) from the integrable dynamics.

In the purely harmonic approximation, the energy initially introduced by the splitting process in the density quadrature oscillates in time between the relative phase and relative density fluctuations. In the homogeneous system, this corresponds to full recurrences of phase coherence at times \( t_{\text{rev}} = L/2c \), where the value of the two-point PCF comes back to 1, i.e. where the initial state is re-established, see figure 7. The recurrences of coherence emerge in the form of an inverse light-cone-like evolution, symmetric to the dephasing leading to the prethermalized state. This is illustrated by the dashed black lines in figure 7(a), which shows the position of the front where long-range phase coherence starts.

In the trapped system, the incommensurate ratios between the excitation frequencies \( \omega_j \) lead to partial recurrences at times which strongly differ from the ones in the homogeneous case (see figures 7(b) and (d)). Contrary to the homogeneous case, no recurrence is observed at early times (compare (a) and (b)). In contrast, the strongest recurrence is observed at much longer times, where the interference of the different modes is the most favourable (at 202 ms for the parameters of figure 7). In that case, visualizing the different mode amplitudes in figure 8 reveals that this recurrence corresponds to a point in time slightly before the lowest \( j = 1 \) mode completes its third oscillation (at time \( t = 3\pi/\omega_1 \approx 71 \text{ ms} \) for \( \omega = 2\pi \times 7 \text{ Hz} \)), while the second \( j = 2 \) mode almost completes its fifth oscillation. Even at longer evolution times, full
Recurrence ($\bar{z} = z_1$) cannot be observed for the trap system because of the incommensurate ratio of the mode frequencies $\omega_j$.

For a more intuitive illustration of the recurrences, we show in figure 9 the time evolution of the mean squared contrast (integrated over a region of length $L$), $C^2(t)$, which is a simple measure of coherence in the system [20]. It can be calculated from a double integration of the PCF, and is directly accessible in experiments from the interference patterns integrated over a size $L$. Here again, we observe the more complex structure of the dynamics due to the trap, with a clear shift of the recurrence time with respect to the homogeneous case, and a different form of the recurrence.

We finally note that in typical atom chip experiments, anharmonicity of the longitudinal trapping potential might arise because of irregular current flow in the wires [54], which modifies

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**Figure 7.** Time evolution of the relative-phase correlation function for the homogeneous (left, a and c) and trapped (right, b and d) systems. The color-scale indicates the degree of correlation (red: high correlation, blue: low correlation). The top row illustrates the relaxation to the prethermalized state with the speed of sound as characteristic velocity for the decay of correlations (dashed line is $\bar{z} = 2ct$). In the homogeneous case, the initial state is re-established at times which are multiples of the system length divided by the characteristic velocity. In the trapped case, the recurrences are only partial and the more complex structure is due to the incommensurate ratios of the mode frequencies $\omega_j$.

In this time window (0–300 ms), the strongest recurrence is observed at 202 ms ($\omega/2\pi = 7$ Hz).
the longitudinal potential. These irregular variations in the trapping potential strongly depend on the particular atomchip used; in particular they depend on how close the cloud is from the atomchip [55]. Potential modulations of length scale $x$ are exponentially suppressed with distance $d$ by a factor $e^{-2d/x}$. Consequently only large-scale modulations become important. One can measure those very accurately using the magnetic field microscope [56]. Moreover techniques to reduce the fragmentation have been demonstrated [57].

How much will the dephasing and rephasing dynamics be influenced by the anharmonicity of the trap? Comparing the box potential, which is very anharmonic, to the harmonic trap gives

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**Figure 8.** Visualization of the first six mode amplitudes $\left(\sin\left(\frac{\omega t}{\omega_0}\right)/\omega_0\right)^2$ contributing to the phase variance: (a) homogeneous system, (b) trapped system. For the trap system, the strongest recurrence of coherence appears when most of the modes complete their oscillation at about the same time, which occurs around 202 ms for the parameters of figure 7 where $\omega/2\pi = 7$ Hz. (c) Zoom around the recurrence time.
a good estimate. From the calculation in our paper we see that the dephasing dynamics is only very minor affected, the rephasing and the recurrences significantly. Adding a small anharmonicity will lead to minor changes in the details of the revivals. This however strongly depends on a particular experiment, and the measured trapping potential can be used in numerical simulations of the dephasing, the rephasing and the recurrence.

6. Conclusion

We have theoretically studied the dynamics of a coherently split 1D Bose gas in the harmonic approximation, considering phononic excitations and taking into account a longitudinal trapping potential. We showed that the relaxation to the prethermalized dephased state is local and that the dephasing spreads in time through the system in a light-cone-like evolution with a characteristic velocity. Including the trapping potential in the model, we showed that the trap has small effects at short evolution times, but important effects at long evolution times (compared to the dephasing time-scale). More precisely, the trapping potential does not affect
the nature of the dephasing which is still local, but affects the velocity of correlations. At longer evolution times, the trap has an important effect on the recurrences of coherence due to the rephasing of elementary excitations in a finite size system. In our present work we restricted ourselves to the Luttinger liquid model, which treats the phonons as non-interacting quasi-particles. At long times, much longer than the dephasing time, we expect that physics beyond the Luttinger liquid approximation, such as scattering or decay of phonons, will emerge, which will lead to further relaxation and thermalization [58–61]. Also in this case it will be important to take the trapping potential into account when comparing experimental data to the theoretical models.

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