EFFECT OF WATER ON BIMODALITY OF AIR TEMPERATURE DISTRIBUTION FUNCTIONS AND CHANGES IN T-YEAR AIR TEMPERATURE VALUES IN HURBANOVO

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Variability and long-term trends in air temperature are in the interest of hydrologists in recent decades as they can affect the global hydrological cycle and water balance in river basins. The estimation of the T-year minimum and maximum mean daily air temperature is associated with the problem of selection of the distribution functions of these series. The histograms of the daily air temperature series manifest the two-peak (bimodal) shape in many stations all over the world. It is not feasible to describe the distribution function of bimodal distribution through single mean value and single standard deviation. In the present study the T-year values of air temperature are estimated for three temperature indexes in station Hurbanovo: 1. series of maximum mean daily air temperature of the year \( T_{d,\text{max}} \), 2. series of mean annual air temperature of the year \( T_c \), and 3. series of minimum mean daily air temperature of the year \( T_{d,\text{min}} \). Change in the theoretic distribution curve of the mean daily air temperature indexes is compared during two subperiods of the instrumental period 1872–2017. Generally, the changes in temperature design values are more marked in case of minimum temperatures, e.g. the value of 100-year minimum mean daily air temperature increased by more than 5°C. The 100-year maximum mean daily air temperature increased by 1.88°C in Hurbanovo.

KEY WORDS: air temperature, Hurbanovo, bimodal distribution, T-year values

Úvod

The knowledge of the extreme air temperature development is very important for hydrology and water balance in each country (Szolgay et al., 2002; 2007). The natural processes usually display the unimodal (single-peak) symmetric shape of the histogram (distribution function). In case of the air temperature it is valid e.g. for the series of mean annual air temperature, or seasonal air temperature series. In majority of stations over the world the distribution of mean daily air temperatures has bimodal (two peak) shape. Majority of stations in Slovakia belongs to this type as well. At station Hurbanovo (Fig. 1a), the histogram of mean daily air
temperature, as well as histograms of minimum/maximum daily air temperature, have bimodal shape. Fig. 1a demonstrates that one peak occurs in class of +2°C. The bimodality of the mean daily air temperature histograms is usually explained through different conditions during summer and winter seasons. We are of the opinion that the reason for occurrence of increased number of mean daily temperatures in span of 0 to +2°C is the amount of heat used in liquid/solid state transition in wider surroundings of the climatic station (in atmosphere, in soil, in rivers and lakes) and vice versa (Pekárová et al., manuscript). Due to it the temperature between 0° and +2°C occurs more often and they have higher frequency. In high mountain stations (e.g. Lomnický štít), where the mean annual air temperature is near to 0°C the both peaks merge into single peak and the histogram obtains single peak/unimodal shape (Fig. 1b).

Bimodality of the distribution functions of air temperature series complicates the possibility of selection of the theoretical distribution function of daily air temperature series and the estimation of the distribution of daily (minimum, mean, maximum) air temperatures in future in conditions of the expected atmosphere warming based on historical observations. At the same time the knowledge of the future development of temperature is in the center of interests of each country due to the increasing impact of increasing air temperature on society and economy (Melo et al., 2007; Melo and Lapin, 2000).

In this paper we focused on assessment of changes of the theoretical distribution functions of selected series of minimum and maximum (mean daily) air temperatures at station Hurbanovo during four 60-years periods: 1872–1931, 1901–1960, 1931–1990, 1958–2017.

**Data used and methods**

For the statistical analysis of the air temperature we have used 146-years long series of mean daily air temperature from the Hurbanovo observatory (Fig. 2) of the period 1872–2017. The meteorological station at Hurbanovo (with its former name O Gyalla – Stará Šaľa) is a representative station for the relatively arid region of the Danubian lowland region (Petrovič, 1960; Konček, et al., 1974; Lapin, 2004; Faško et al., 2008; Halmová et al., 2015). The Hurbanovo station (latitude 47.9°N; longitude 18.2°E, m a.s.l.) ranks among the best meteorological stations in Central Europe providing sufficiently long, high quality, and homogenous observations. The meteorological observatory is situated in the large garden on the northern edge of the small city of Hurbanovo. Regular measurements of the air temperature [°C] and precipitation [mm] started in 1876 at this station. The period 1871–1875 was completed according to the Komarno station (distance of 20 km from Hurbanovo). The data are collected in the database of the Slovak hydro-meteorological institute (SHMI). The climatic station Hurbanovo (115 m a.s.l.) is the representative station of the southwestern region of Slovakia. The air temperature observations were carried on climatic schedule on 7 a.m. (T7), 2 p.m. (T12) and 9 p.m. (T19) of the middle local time in Slovakia. The daily mean of air temperature (Td) was recalculated according to formula: 

\[ T_d = (T_7 + T_12 + 2T_{19})/4 \]

Hurbanovo after the year 1921. The series of mean daily air temperature was used to prepare three 146-years series of following air temperature indexes in °C:

- \( T_{d,\text{max}} \) – series of maximum mean daily air temperature for each year (1 datum per year),
- \( T_d \) – series of mean annual air temperature (1 datum per year),
- \( T_{d,\text{min}} \) – series of minimum mean daily air temperature for each year (1 datum per year).

We can see the gradual increase of air temperature during the four periods at Hurbanovo in Table 1, where are the long-term characteristics of air temperature. The long-term mean air temperature increased by 0.64°C.

**Table 1. Climatic station Hurbanovo, long-term mean daily air temperature \( T_{d,\text{mean}} \), long-term minimum mean daily \( T_{d,\text{min}} \), and long-term maximum mean daily \( T_{d,\text{max}} \) air temperature during various periods in °C**

| Period       | \( T_{d,\text{mean}} \) | \( T_{d,\text{min}} \) | \( T_{d,\text{max}} \) |
|--------------|--------------------------|------------------------|------------------------|
| 1872–1900    | 9.11                     | −24.2                  | 28.7                   |
| 1901–1930    | 9.50                     | −21.3                  | 29.2                   |
| 1931–1960    | 9.77                     | −21.4                  | 30.2                   |
| 1961–1990    | 9.98                     | −16.7                  | 29.6                   |
| 1991–2018    | 10.75                    | −14.8                  | 32.0                   |

**Methods of T-year design values estimation**

The estimation of so called design values of air temperature occurrence or exceedance of certain probability is done by theoretical functions of probability density distribution (distribution functions) in meteorology and climatology. The 40-years series of observations can be applied for relatively credible estimation of extreme values with up to 100 years probability of exceedance/non exceedance. These values become to be used in several branches, e.g. in health care (waves of heat or extreme cold), in engineering, or in hydrology (Kyselý,
The selection of the theoretical distribution function is based on histograms. Histograms belong to basic statistical procedures of the graphical processing of the long time series. They allow showing of the frequency of values in individual classes during the selection of the theoretical frequency distribution function. If we want to assess the \(T\)-year design values of air temperature, we have to construct the distribution function for the given period. Histograms of the series of mean daily air temperature are bimodal in many cases (Fig. 3). The construction of the theoretical distribution function of mean daily air temperature with multimodal shape is a complicated problem. This problem can be avoided, e.g. by construction of the distribution curve for individual days of the year or individual months of the year. Such histograms are unimodal (single-peak), and in most of the cases the \(T\)-year design values can be estimated using the simple normal (Gaussian) distribution function. During the winter months of December to February the histograms are asymmetric, the temperatures near 0°C occur more frequently.

The other way how to solve this problem is to select each year single most extreme value of the selected characteristic, and to assess the \(T\)-year air temperature value using such processed time series. This approach was used in this study. The results are presented for the station Hurbanovo, because this station offers the longest homogeneous series of observation since the year 1872 and it gives us the opportunity to analyze the changes of \(T\)-year design temperatures in different time periods.

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**Fig. 1.** Histogram of the mean daily air temperature (\(T_d\)), a) in climatic station Hurbanovo, period 1872–2017, b) in climatic station Lomnický štít.

**Obr. 1.** Histogram súboru priemerných denných teplôt vzduchu (\(T_d\)), a) v klimatickej stanici Hurbanovo, za obdobie 1872 – 2017, b) v klimatickej stanici Lomnický štít.

**Fig. 2.** Mean daily air temperature (dark blue line) in station Hurbanovo (1872–2017) and 4-year moving average of daily values (light green line, right axis).

**Obr. 2.** Priebeh priemerných denných teplôt vzduchu (tmavomodrá čiara) v stanici Hurbanovo (1872 – 2017) a priebeh 4-ročných kľuzových priemerov denných hodnôt (svetlo-modrá čiara, mierka je na osi vpravo).
Results

T-year design values of different air temperature indexes in Hurbanovo

The basic assumption for the estimation of the distribution functions is the homogeneity and stationarity of the measured time series. It is generally valid that the air temperature series are not stationary, but they are influenced by multiannual variability and the long-term trend of increase (Brázdil et al., 1996; Hansen et al., 2010; Pekárová et al., 2011; Labudová et al., 2015, GISTEMP Team, 2019). The long-term development of the global air temperature over the world is plotted in Fig. 4 according to Hansen et al. (2010), the station Hurbanovo is not included in his set of stations. More information can be found at http://www.shmu.sk/sk/?page=2049&id=975.

The development of time series of maximum, mean and minimum daily temperature at Hurbanovo is in Fig. 5. In Fig. 5b we can see that the mean annual temperature series at Hurbanovo corresponds with the worldwide development of the global temperature during the period 1880–2017. During the period 1930–1985 the trend of mean daily temperature was negligible at Hurbanovo, but it has got significant increasing trend since the year 1985. Therefore the air temperature series are not stationary at Hurbanovo. In order to partially eliminate this fact, we

![Graph showing the development of air temperature series at Hurbanovo](image)

**Fig. 3.** Change of the distribution curve of the mean daily air temperature in station Hurbanovo. Full line with points – old 30-years period 1872–1901, dashed line – new 30-years period 1988–2017.

**Obr. 3.** Zmena distribučnej čiary priemerných denných teplôt vzduchu v stanici Hurbanovo. Plná čiara s bodmi – staré 30-ročné obdobie 1872 – 1901, prerušovaná čiara – nové 30-ročné obdobie 1988 – 2017.

![Graph showing global mean temperature estimates](image)

**Fig. 4.** Atmosphere temperature index, 1880 to present, differences to base period 1951–1980. The solid black line with dots is the global annual average and the solid red line is the five-year lowess smooth. [This is an update of Fig. 9a) in Hansen a kol. (2010) The black line with the points is the global annual average and the red line represents the five year filter].

**Obr. 4.** Index teploty atmosféry, odchýlky od obdobia 1951 – 1980. Čierna čiara s bodmi je globálny ročný priemer a červená čiara znázorňuje päťročný filter. [Aktualizácia obrázka 9a) v práci Hansen et al. (2010)].
have subdivided the data series of the whole period 1872–2017 into four 60-years subperiods: 1. 1872–1931; 2. 1901–1960; 3. 1931–1990 and 4. 1958–2017 and we estimated the parameters of theoretical distribution functions for all subperiods. We are aware of the fact that even the 60-years periods do not fulfill the condition of stationarity, but if we want to estimate the 200-year design values of air temperature, we need to apply the longer series.

We used software STATGRAPHICS PLUS for the statistical processing of the series. The graphical presentation of changes in statistical characteristics of three given series (T_{d,min}, T_d and T_{d,max}) is made using Box-and-Whisker plots (Fig. 6 left) for the whole period and for the four subperiods (subperiods are labeled as 1, 2, 3, 4). These plots demonstrate the significant statistical data of the time series. The plots divide data into 4 equal frequency regions. The rectangles include 50 percent of data; the mean value is plotted as vertical line inside the rectangle. The left horizontal line represents the first quartile, the right one the upper quartile. Individual points (small rectangles) show the temperatures exceeding certain limits (dubious extremes). The plots show the gradual shift to higher temperatures.

We have also calculated the basic statistical characteristics of the selected time series of Hurbanovo station for the whole period and four subperiods: 1. 1872–1931; 2. 1901–1960; 3. 1931–1990 and 4. 1958–2017. The results are summarized in Table 2.

We have also tested the suitability of the Gaussian (normal) distribution for estimation of the probability distribution of the series. The statistical tests have shown, that for estimation of the T-year values of selected temperature indexes it is possible to apply the normal probability distribution. The right plots on Fig. 6 show the probability distribution of particular indexes, but only for three periods – the bold line for the whole period 1872–2017, the thin line for the oldest subperiod 1872–1931, and the middle line for the newest subperiod 1958–2017.

From the theoretical distribution functions we have estimated the T-year values of selected air temperature indexes for selected periods in station Hurbanovo. We have to notice that for the minimum temperatures we are interested in bottom/left end of the distribution curve – probability of non exceedance 0.01 %, and for the maximum temperatures we are interested in upper/right end of the distribution curve – probability of exceedance 0.99 % (in case of 100-year design values). Results of the basic statistical characteristics of studied three indexes and T-year temperatures (50-, 100- and 200-year values) are in Table 2.

**Conclusions and discussion**

Variability of extreme air temperatures is in scope of interest of climatologists, hydrologists and the whole society in last decades, because it can impact the global...
hydrological cycle and energetic balance of the Earth. From the frequency analysis of the air temperature observations at Hurbanovo during last 146 years it follows that the statistically significant change has occurred of the distribution curve of the mean daily air temperature (Fig. 3).

Table 2. Basic statistical characteristics of the selected temperature series of the station Hurbanovo for the whole period and 4 subperiods: 1. 1872–1931; 2. 1901–1960; 3. 1931–1990 and 4. 1958–2017

|             | \(T_{d,\text{min}}\) | \(\bar{T}_d\) | \(T_{d,\text{max}}\) |
|-------------|---------------------|-------------|---------------------|
| 1872–1977   |                     |             |                     |
| Mean        | −12.00              | 10.00       | 27.07               |
| Std. Dev.   | 4.58                | 0.88        | 1.61                |
| 50          | −21.41              | 30.38       |                     |
| 100         | −22.65              | 30.81       |                     |
| 200         | −23.80              | 31.22       |                     |
| 1. 1872–1931| −13.11              | 9.52        | 26.43               |
| Mean        | 4.92                | 0.64        | 1.35                |
| Std. Dev.   | −23.21              | 29.20       |                     |
| 100         | −24.55              | 29.56       |                     |
| 200         | −25.78              | 29.90       |                     |
| 2. 1901–1960| −12.52              | 9.75        | 26.79               |
| Mean        | 4.66                | 0.71        | 1.54                |
| Std. Dev.   | −22.09              | 29.96       |                     |
| 100         | −23.36              | 30.38       |                     |
| 200         | −24.52              | 30.76       |                     |
| 3. 1931–1960| −11.81              | 9.97        | 27.01               |
| Mean        | 4.43                | 0.71        | 1.39                |
| Std. Dev.   | −20.91              | 29.86       |                     |
| 100         | −22.11              | 30.24       |                     |
| 200         | −23.22              | 30.58       |                     |
| 4. 1958–2017| −10.34              | 10.55       | 27.69               |
| Mean        | 3.62                | 0.82        | 1.61                |
| Std. Dev.   | −17.78              | 31.00       |                     |
| 100         | −18.76              | 31.44       |                     |
| 200         | −19.67              | 31.84       |                     |

In the present study the \(T\)-year values of air temperature were estimated for three temperature indexes in station Hurbanovo: 1. series of maximum mean daily air temperature of the year \(T_{d,\text{max}}\); 2. series of mean annual air temperature of the year \(\bar{T}_d\); and 3. series of minimum mean daily air temperature of the year \(T_{d,\text{min}}\). Change in the theoretic distribution curve of the mean daily air temperature indexes were compared during two subperiods of the instrumental period 1872–2017. The 100-year minimum mean daily air temperature \(T_{d,\text{min}}\) was −24.55°C in the span 1872–1931, and −18.76°C in the span 1958–2017, respectively. The 100-year maximum mean daily air temperature \(T_{d,\text{max}}\) was 30.81°C in the span 1872–1931, and 31.44°C in the span 1958–2017, respectively. Generally, the changes in temperature design values are more marked in case of minimum temperatures, e.g. the value of 100-year minimum mean daily air temperature increased by more than 5°C. The 100-year maximum mean daily air temperature increased by 1.88°C in Hurbanovo.

In two years’ time the next 30-years period of observations 1990–2020 will be available and we shall have a chance to update the analysis and to compare the development in different periods. Our results indicate that the temperature characteristics of the new period will differ significantly from the period 1961–1990, which was quite stable and calm in our region. The period 1931–1960 was more extreme. We suppose that even the 60-years period is not sufficient enough for estimation of \(T\)-year values. Therefore it is necessary to use the longest possible series for estimation of the \(T\)-year values and not to settle with the assumption of the permanent temperature increase (Charvátová and Sršetík, 1995, 2004; Charvátová, 2008, Mörner, 2018). It is interesting that according to the study of Gasparini et al. (2015) the low temperatures have more unfavorable impact on human mortality than high temperatures.

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Fig. 6. Graphical presentation of differences between series: upper $T_{d\text{,min}}$, center $T_d$, and below $T_{d\text{,max}}$: left: Box and Whisker plot for the whole period and 4 subperiods; right: distribution densities for the whole period and two 60-years subperiods 1. 1901–1960 and 2. 1958–2017.

Obr. 6. Grafické znázornenie rozdielov radov: hore $T_{d\text{,min}}$, v strede $T_d$; a dole $T_{d\text{,max}}$: vľavo: Box a Whisker obrázok za celé obdobie a 4 podobdobia; vpravo: hustoty rozdelenia za celé obdobie a dve 60-ročné obdobia 1. 1901 – 1960 a 2. 1958 – 2017.
VPLYV VODY NA BIMODALITU DISTRIBUČNÝCH FUNKCIÍ TEPLOTY VZDUCHU A ZMENY N-ROČNÝCH HODNÔT TEPLOTY VZDUCHU V HURBANOVE

Prírodné procesy majú zvyčajne jednovrcholový symetrický tvar histogramu (distribučnej funkcie). V prípade teploty vzduchu to platí napr. pre rady priemerných ročných teplot vzduchu, alebo rady sezónnych teplot. Vo väčšine stanic na svete však nadobúda rozdele niechenných hodnôt teploty vzduchu bimodálny tvar. K týmto staniciam patrii však aj vľačina stanie na Slovensku. Napr. v stanici Hurbanovo (obr. 1a), histogram priemerných denných teplot vzduchu, ale aj histogramy minimálnych/maximálnych denných teplot majú dvojvrcholový/bimodálny tvar.

Z obrázku 1a je zreťažiace vidieť, že jeden vrchol sa vyskytuje v tride okolo 2 °C. Bimodalita histogramu rady priemerných denných teplot vzduchu sa zvyčajne vysvetľuje rôznymi podmienkami za letné a zimné obdobie. My sa domnievame, že pričinou zvýšeného vyskytu denných teplot okolo 0 až 2 °C je množstvo tepla, ktoré je potrebné na zmenu skupenstva vody v súčelí okoli klimatickej stanice (v atmosfére, v rieke a okolí klimatickej stanice (v atmosfére, v rieke a okolí klimatickej stanice).

V bimodálnom tvarom histogramu však často výskytu okolo 0 °C. Teplota okolo 0 °C sa tým pádom vyvyšuje častejšie, má vľačušiu početnosť. Vo vysokošorovských stanicich (Lomnický štít), kde je priemerná ročná teplota vzduchu okolo 0 °C tiež dvoj vrcholy splývajú do jedného vrcholu a histogram nadobúda jednovrhový/ unimodálny tvar (obr. 1b).

Bimodalita distribučných funkcii radov teplot komplikuje možnosti odvodenia teoretickej distribučnej funkcie uvedených denných radov údajov a tým aj odhad rozdele niechenných (minimálnych, priemerných, maximál-
ných) teplot vzduchu v budúcnosti pri očakávanom oteplení atmosféry na základe historických pozorovaní. Pri tom poznanie budúceho vývoja týchto hodnôt stojí v centre záujmu každej krajiny. V tomto článku sme sa zamerávali na zhodnotenie zmien teoretických distribučných čiar pre rady vybraných indexov minimálnych a maximálnych (priemerných denných) teplot vzduchu v stanici Hurbanovo za štyri 60-ročné časové obdobia: 1872 – 1931, 1901 – 1960, 1931 – 1990, 1958 – 2017. Pri štatistickej analýze teploty vzduchu sme použili 146-ročný rad priemerných denných teplot vzduchu z observatória v Hurbanove (obr. 2) za obdobie 1872 – 2017. Z radu priemerných denných teplot vzduchu sme zostavili tri 146-ročné rady teplotných indexov v °C: 

\[ T_{d,max} \] – rad maximálnych priemerných denných teplot vzduchu v roku (1 údaj za rok),  
\[ T_a \] – rad priemerných ročných teplot vzduchu (1 údaj za rok),  
\[ T_{d,min} \] – rad minimálnych priemerných denných teplot vzduchu v roku (1 údaj za rok).

Základným predpokladom odhadu distribučných funkcií je homogénnosť a stacionárnosť nameraných časových radov. Vo všetkých prípadoch platí, že rady teplot vzduchu sú stacionárne, ale sú ovplyvnené viacročnou variabilitou a dlhodobým rastúcim trendom. Dlhodobý vývoj globálnej teploty vzduchu podľa Hansena a kol. (2010) na svete je znázornený na obr. 4 (v súbore stanic podľa Hansena nie je zahrnutá stanica Hurbanovo).

Časový priebeh radu maximálnej dennéj, priemernej a minimálnej dennéj teploty v Hurbanove je znázornený na obr. 5. Z obrázku 5b vidieť, že priebeh ročnej teploty v Hurbanove za obdobie 1880 – 2017 korešponduje s dlhodobým vývojom globálnej teploty vo svete. Napr. zaťaž čo v období 1930 – 1985 bol trend radu priemernej teploty v Hurbanove zaradený do priemernej teploty v roku 1985 má výrazný rastúci trend. Rady teplot vzduchu v Hurbanove teda nie sú stacionárne. Aby sme aspoň štáti eliminovali tento fakt, rozdelení sme súbor údajov za celé obdobie 1872 – 2017 na štyri 60-ročné podobnosti: 1. 1872 – 1931; 2. 1901 – 1960; 3. 1931 – 1990 a 4. 1958 – 2017 a aj pre ne sme odparametre teoretických distribučných funkcií. Uvedomujeme si, že ani 60-ročné obdobia nesplňa podmienku stacionárnosti, ale keďže chceme odvodiť 200-ročné náhradné hodnoty teploty vzduchu, je potrebné použiť dlhší rad.

Pri štatistickom spracovaní sme použili softvér STATGRAPHICS PLUS. Na grafické znázornenie zmien štatistických charakteristik daných troch radov \( T_{d,min}, T_{a} \) a \( T_{d,max} \) pre celé obdobie a v štyroch podobobiach (podobobiá sú značené 1, 2, 3, 4) sme použili grafy Box-and-Whisker (obr. 6). Tento graf zobrazuje dôležité štatistické údaje o rade hodnôt. Graf zobrzuje údaje na štyri rovnaké oblasti frekvencie. Obdĺžníky obklopujú 50 percent údajov; stredná hodnota je vykreslená ako vertikálna čiara vo vnútri obdĺžnika. Lavička (dolná) horizontálna čiara znázorňuje prvú kvartil, pravá čiara znázorňuje horný kvartil. Jednotlivé body na grafie znázornené maľými štvorcikmi, znázorňujú teploty, ktoré prekráčajú isté hranice (podozrivé extrémy). Z grafov vidieť postupný prechod k vyšším hodnotám teplot.

Ďalej sme vypočítali základné štatistické charakteristiky vybraných teplotných radov zo stanice Hurbanovo pre cele obdobie a v štyroch podobobiach: 1. 1872 – 1931; 2. 1901 – 1960; 3. 1931 – 1990 a 4. 1958 – 2017. Výsledky sú zhrnuté v tabuľke 2.

Následne sme testovali vhodnosť použitia Gaussovo (normálneho) rozdelenia na odhad rozdelenia pravdepodobnosti výskytu daných radov. Zo štatistických testov vyplýva, že na odhad \( N \)-ročných hodnôt radov vybraných teplotných indexov je možné použiť normálne rozdelenie pravdepodobnosti. Na právych grafoch obrázku 6 sú vykreslené rozdelenia pravdepodobnosti pre jednotlivé indexy, ale len pre tri časové obdobia – najhľadnejšie časové obdobie 1872 – 2017, pre najstarší rad - 1. obdobie 1872 – 1931, a pre najnovší rad - 4. obdobie 1958 – 2017.

Z teoretických distribučných funkcií sme odhodali \( N \)-ročné hodnoty vybraných indexov teploty vzduchu v stanici Hurbanovo pre dané obdobie. Tresta upozorniť, že pre minimálny teploty nás zaujíma dolný/ľavý koniec distribučnej čiary – pravdepodobnosť podkoľkovania 0,01 %, a pre maximálny teploty horný/pravý koniec distribučnej čiary – pravdepodobnosť prekročenia 0,99 % (pre 100-ročné náhradné hodnoty). Výsledky základných štatistických charakteristik daných troch indexov a \( N \)-ročné teploty (50-, 100- a 200-ročné hodnoty) sú uvedené v tabuľke 2.

Variabilita extrémnych teplot vzduchu je v posledných desaťričoch v centre záujmu klimatológov, hydrológov, i celej spoločnosti, ako mať vplyv na celo svetový hydrologický cyklus a energetickú rovnováhu na Zemi. Z frekvenčnej analýzy meraní teploty vzduchu v klimatickej stanici Hurbanovo za plynulých 146 rokov vyplýva, že došlo k štatisticky významnej zmene distribučnej čiary radu priemerných denných teplot vzduchu (obr. 3).

V príspevku sme sa sústredili na zostrojenie teoretických distribučných čiar radov minimálnych a maximálnych priemerných denných teplot vzduchu (jedna hodnota za rok) za účelom odhadu ich \( N \)-ročných náhradných hodnôt. V tejto práci prezentujeme výsledky z klimatickej stanice Hurbanovo. Napr. 100-ročná minimálna priemerná denná teplota radu \( T_{d,min} \) v období 1872 – 1931 bola –24,55 °C a v období 1958 – 2017 –18,76 °C. Storočná maximálna priemerná denná teplota radu \( T_{d,max} \) v období 1872 – 1931 bola 30,81 °C a v období 1958 – 2017 31,44 °C. Vo všetkých období zároveň sa výrazne zmenili návrhové hodnoty teplot vzduchu, podľa prípade minimálnych teploty nastali v prípade minimálnych teploty (obr. 3).

Celá práca je v celej spoločnosti, nakoľko môže mať vplyv na celosvetový hydrologický cyklus a energetickú rovnováhu na Zemi. Zo štatistických charakteristík daných troch indexov a \( N \)-ročné teploty tančia s údajmi zdejších 146 rokov.
regioné obdobie netypicky pokojné. Obdobie 1931 – 1960 bolo extrémnejšie. Domnievame sa, že aj 60-ročné obdobie je nepostačujúce na N-ročných hodnôt. Preto na určenie N-ročných teplôt je potrebné použiť čo najdlhšie rady pozorovaní a neuspokojiť sa s predpokladom trva-
lých rastov teploty vzduchu (Charvátová a Střeštík, 1995; 2004; Charvátová, 2008; Mörner, 2018). Zo štúdie Gasparrini a kol. (2015) vyplýva, že nízke teploty majú nepriaznivejší vplyv mortalitu človeka, než vysoké teploty.

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