Bounds of the mass of $Z'$
and the neutral mixing angles
in general $SU(2)_L \times SU(2)_R \times U(1)$ models

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Abstract

We consider phenomenological constraints on the mass $M_{Z'}$ and the two mixing angles $\theta_R$ and $\xi$ of the neutral sector in a very general class of $SU(2)_L \times SU(2)_R \times U(1)$ models using electroweak data. We do not make any specific assumptions such as left-right symmetry or the Higgs structure. The analysis of the neutral sector has the advantage that it has relatively fewer parameters compared to the charged sector since the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements in the right-handed sector do not enter into the analysis, hence the number of various possibilities from a big parameter space is reduced. We utilize theoretical considerations on the masses of the gauge particles and the mixing angles. We combine the precision electroweak data from LEP I and the low-energy neutral-current experimental data to constrain the parameters introduced in the model. It turns out that $M_{Z'} > 400$ GeV, $-0.0028 < \xi < 0.0065$ with little constraint on $\theta_R$. In the left-right symmetric theory, $M_{Z'}$ should be larger than 900 GeV. With these constraints, we compare the values for $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$, $\sigma(e^+e^- \rightarrow b\bar{b})$ and $A_{FB}$ at LEP II with experimental values.

Key words: $SU(2)_L \times SU(2)_R \times U(1)$ models, $Z'$, mixing angles, electroweak data.
1 Introduction

The standard model with the $SU(3)_c \times SU(2)_L \times U(1)_Y$ gauge group has been successful in describing a wide range of high energy experimental data in collider experiments and in heavy quark decays. Almost all the experimental data can be explained within the standard model with surprising accuracy. However it is expected that there may be new physical phenomena beyond the standard model at the TeV scale. Theoretically there are implications that the standard model is an effective theory below 250 GeV, and it may be embedded in a larger theory. There has been enormous theoretical effort in extending the standard model such as the minimal supersymmetric standard model [1], models containing many Higgs particles [2], grand unified theories, and string-inspired models [3]. There are also experimental efforts to discover new physics effects such as supersymmetry and neutrino oscillation [4].

As a simple extension of the standard model, we consider the question whether the right-handed fermions can participate in the charged-current and the neutral-current interactions, and if they do, with what strength. This idea can be realized easily if we introduce the charged-current and the neutral-current interactions for the right-handed fermions by extending the gauge group. The simplest case arises when we choose the gauge group as $SU(2)_L \times SU(2)_R \times U(1)$ [5]. The left-handed fermions transform as doublets under $SU(2)_L$ and singlets under $SU(2)_R$, with the situation reversed for the right-handed fermions. The $U(1)$ factor is different from the $U(1)_Y$ in the standard model. The quantum number of $U(1)$ is proportional to $B - L$, so the model is called $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ in some literature. The standard model $U(1)_Y$ is obtained as a linear combination of the third component of $SU(2)_R$ and the $U(1)$ generator.

Since we introduce an additional $SU(2)$ gauge group, it implies that there exist three new gauge bosons: two charged and one neutral. Therefore there are two sets of gauge bosons: the $W^\pm_L(Z_L)$ belong to $SU(2)_L$ and are identical to the $W^\pm(Z)$ in the standard model, while $W^\pm_R(Z_R)$ of $SU(2)_R$ are new. There have been many theoretical and phenomenological studies of $SU(2)_L \times SU(2)_R \times U(1)$ models [5], and various constraints have been presented on the mass of the new charged gauge boson $W^\pm_R$ and the mixing angle $\zeta$ between the two sets of charged gauge bosons based on experimental data [6].

In this paper, we consider constraints on the parameters of the $SU(2)_L \times SU(2)_R \times U(1)$ models in a very general scheme. No assumptions are made about whether there is a left-right symmetry such that the Lagrangian is invariant under the interchange of the left-handed and the right-handed fermions. The left-right symmetry implies that the gauge couplings $g_L$ and $g_R$ of the $SU(2)_L$ and $SU(2)_R$ subgroups are equal, as well as the Yukawa couplings in
each sector. However, we release this assumption to include the possibility of left-right asymmetric models, in which \( g_L \neq g_R \).

In analyzing the charged sector, we have to carefully consider the origin of CP violation, and the Cabibbo-Kobayashi-Maskawa (CKM) matrix elements in the right-handed sector. There are many possibilities on the CKM matrix elements according to the assumptions whether CP is violated spontaneously or explicitly [7]. Leaving aside this complication in the charged sector, we pay attention to the neutral sector. At tree level, there is no flavor-changing-neutral-current process. Therefore various possibilities of how the CKM matrix elements arise and what their structures are both in the left- and the right-handed sector do not enter into the analysis of the neutral sector. Due to this fact, we can reduce specific model dependence in the analysis of the neutral sector.

Let us consider then which parameters are necessary in the neutral sector. There are three neutral gauge bosons: \( Z_L \) from \( SU(2)_L \), \( Z_R \) from \( SU(2)_R \) and \( B \) from \( U(1) \). These gauge fields mix to produce physical particles, one of which is massless. There are two heavy neutral particles: \( Z \) which is identical to \( Z \) in the standard model and a new particle \( Z' \). We need three mixing angles \( \theta_R \), \( \theta_W \) and \( \xi \) which describe the mixing between the gauge eigenstates and the mass eigenstates. The angle \( \theta_W \) is identical to the Weinberg mixing angle in the standard model. Therefore the new additional parameters in the neutral sector are two mixing angles \( \theta_R \), \( \xi \) and the mass \( M_{Z'} \) of the \( Z' \) particle.

We consider theoretical relations among the new parameters introduced in the \( SU(2)_L \times SU(2)_R \times U(1) \) model. The gauge boson masses have definite relations between the charged sector and its neutral counterpart. There are also relations between the small mixing angle \( \xi \) and the mass ratio \( M_Z^2/M_{Z'}^2 \), as there is a similar relation between the mixing angle \( \zeta \) and the ratio \( M_W^2/M_W'^2 \) in the charged sector. We utilize these relations along with the experimental data to constrain the parameters in the model.

The main points in this paper are to probe the parameter space spanned by \( \theta_R \), \( \xi \) and \( M_{Z'}^2 \) and to look for the region that is allowed by current experimental data. We first consider electroweak precision data from LEP I. Since LEP I performs the experiment at the \( Z \) peak, the corrections do not depend on \( M_{Z'}^2 \) explicitly. Therefore the LEP I data constrain only \( \theta_R \) and \( \xi \). We combine these bounds with the low-energy neutral-current data to constrain \( \theta_R \), \( \xi \) and \( M_{Z'} \). With these results, we predict the cross sections for \( e^+e^- \to \mu^+\mu^- \), \( e^+e^- \to b\bar{b} \) and the leptonic forward-backward asymmetry \( A_{FB}^L \) at LEP II energies and compare with experimental data.

This paper is organized as follows: In Section 2 we formulate the \( SU(2)_L \times SU(2)_R \times U(1) \) model. After reviewing the charged sector briefly, we describe in
detail the neutral gauge bosons and the interactions with them. We introduce three mixing angles explicitly to diagonalize the neutral gauge boson mass matrix. In obtaining the current interactions, we note the fact that \( M_{Z'} \gg M_Z \) and the mixing angle \( \xi \), which describes the mixing of the two massive gauge bosons, is expected to be small \( (\xi \sim M_Z^2/M_{Z'}^2) \). We express all the quantities to first order in \( \xi \) and \( M_Z^2/M_{Z'}^2 \), in order for the numerical estimates to be consistent. Using the information on the mixing, we obtain the current interactions. In Section 3 we consider theoretical relations between \( M_{Z'}, M_{W'}, \) and \( M_Z^2, M_W^2 \). We get the bounds for the mixing angles \( \zeta \) and \( \xi \) in terms of the mass ratios.

In Section 4, we use experimental data to constrain the parameters in the theory. There are many physical quantities observed at LEP I. The useful quantities in our analysis are the leptonic decay width \( \Gamma(Z \to \ell^+\ell^-) \) and the leptonic forward-backward asymmetry \( A_{FB}^l \). We combine the constraints obtained from LEP I with low-energy neutral-current data. For low-energy neutral-current interactions, we consider neutrino-electron scattering, neutrino-hadron scattering, polarized electron-hadron scattering and the atomic parity violation. With the bounds obtained from the combined data of LEP I and the low-energy data, we consider the cross sections \( \sigma(e^+e^- \to \mu^+\mu^-) \), \( \sigma(e^+e^- \to b\bar{b}) \) and the leptonic forward-backward asymmetry \( A_{FB}^l \) at LEP II energies. In Section 5, a summary of the analysis is given and a conclusion is presented.

2 \( SU(2)_L \times SU(2)_R \times U(1) \) model

2.1 General structure

Consider the theory based on the electroweak gauge group \( SU(2)_L \times SU(2)_R \times U(1) \). Such theory has been extensively investigated both as a simple generalization of the \( SU(2)_L \times U(1)_Y \) model and as possible intermediate stages in grand unified schemes such as \( SO(10) \). In constructing the \( SU(2)_L \times SU(2)_R \times U(1) \) theory, it is appealing to impose a discrete left-right symmetry so that parity is restored at a higher energy scale above the weak scale. This additional symmetry simplifies the structure of the Lagrangian. However, it is not required by the structure of the extended gauge group. Furthermore left-right symmetric theories encounter difficulties in the context of grand unified models or cosmology [8]. We do not impose the left-right symmetry from the outset so that our model includes a class of asymmetric left-right models, and we will work within the general framework of the \( SU(2)_L \times SU(2)_R \times U(1) \).

We start with the extended gauge group \( SU(2)_L \times SU(2)_R \times U(1) \) which breaks down to \( SU(2)_L \times U(1)_Y \) at the energy scale \( v_R \), much larger than the weak
scale. The remaining gauge group is identified as that of the standard model and it finally cascades down to $U(1)_{em}$. The covariant derivative is defined as

$$D_\mu = \partial_\mu + ig_L W^a_L \mu T_{La} + ig_R W^a_R \mu T_{Ra} + ig_1 B_\mu S,$$

(1)

where $T^a_{L,R}$ are $SU(2)_{L,R}$ generators and $S$ is the $U(1)$ generator. $g_L$, $g_R$ and $g_1$ are the gauge coupling constants for the corresponding gauge groups. The representations of quarks and leptons under the gauge group $SU(2)_L \times SU(2)_R \times U(1)$ are given as

$$q'_L = \begin{pmatrix} u' \\ d' \end{pmatrix}_L \sim (2, 1)^{1/6}, \quad q'_R = \begin{pmatrix} u' \\ d' \end{pmatrix}_R \sim (1, 2)^{1/6},$$

$$l'_L = \begin{pmatrix} \nu' \\ e' \end{pmatrix}_L \sim (2, 1)^{-1/2}, \quad l'_R = \begin{pmatrix} \nu' \\ e' \end{pmatrix}_R \sim (1, 2)^{-1/2}.$$ (2)

Under the strong gauge group $SU(3)_c$, quarks transform as triplets while leptons transform as singlets. The primed fields in Eq. (2) denote gauge eigenstates rather than mass eigenstates.

In order to invoke spontaneous symmetry breaking, we introduce the scalar field

$$\Phi = \begin{pmatrix} \phi_1^0 \\ \phi_1^+ \\ \phi_2^- \\ \phi_2^0 \end{pmatrix} \sim (2, \overline{3})^0,$$ (3)

which acquires the vacuum expectation value (VEV)

$$\langle \Phi \rangle = \begin{pmatrix} k \\ 0 \\ 0 \end{pmatrix}.$$ (4)

In general $k$, $k'$ can be complex. This VEV generates fermion masses in the Yukawa sector after the spontaneous symmetry breaking.

We need to include additional scalar fields into our theory to implement the symmetry breaking pattern $SU(2)_L \times SU(2)_R \times U(1)_S \rightarrow SU(2)_L \times U(1)_Y \rightarrow U(1)_{em}$. There are a number of choices for how these scalars transform under the full symmetry group. However we choose the simplest case which involves
two doublet fields,

\[
\begin{pmatrix}
\chi_L \\
\chi_R
\end{pmatrix} \sim (2,1)^{1/2},
\chi_R = \begin{pmatrix}
\chi_R^0 \\
\chi_R
\end{pmatrix} \sim (1,2)^{1/2}
\]  

(5)

which acquire the real VEVs

\[
\langle \chi_L \rangle = \begin{pmatrix} 0 \\ v_L \end{pmatrix}, \quad \langle \chi_R \rangle = \begin{pmatrix} 0 \\ v_R \end{pmatrix}.
\]  

(6)

Though \( \chi_L \) is not necessary for the desired structure of the symmetry breaking, we introduce it anyway along with \( \chi_R \) so that our model can accommodate left-right symmetric models.

The Lagrangian for the scalar fields is given by

\[
L_{\text{scalar}} = \text{tr} \left( (D^\mu \Phi)^\dagger D^\mu \Phi \right) + (D^\mu \chi_L)^\dagger D^\mu \chi_L + (D^\mu \chi_R)^\dagger D^\mu \chi_R - V(\Phi, \chi_L, \chi_R),
\]  

(7)

where the covariant derivatives acting on the scalar fields are defined as

\[
D^\mu \Phi = \partial^\mu \Phi + ig_L W^a_{L \mu} T^a \Phi - ig_R W^a_{R \mu} T^a \Phi, \\
D^\mu \chi_{L,R} = \partial^\mu \chi_{L,R} + ig_{L,R} W^a_{L,R \mu} T^a \chi_{L,R}.
\]  

(8)

The scalar potential \( V(\Phi, \chi_L, \chi_R) \) is constructed such that the spontaneous symmetry breaking occurs and the minimum of the potential produces the desired VEVs [9]. We will not go further into the detail of how this is possible. After the spontaneous symmetry breakdown, the kinetic energy in Eq. (7) induces gauge boson masses.

We can also have a variation of the contents of the scalar fields. For example, we can introduce a scalar triplet in order to produce Majorana neutrino mass. In addition to producing Majorana neutrino masses, the scalar triplet can affect the gauge boson mass matrix. However, the contribution of the triplet for the gauge boson masses amounts to effectively changing the VEVs of the existing scalar fields, hence not affecting the analysis of the gauge sector.
2.2 Charged gauge bosons

After the spontaneous symmetry breakdown, the scalar Lagrangian in Eq. (7) generates gauge-boson masses. The mass-squared matrix for the charged gauge bosons can be written as

\[
M^2_{W^\pm} = \begin{pmatrix}
  g_L^2(v_L^2 + K^2)/2 & -g_L g_R k^* k' \\
  -g_L g_R k k'^* & g_R^2(v_R^2 + K^2)/2
\end{pmatrix} \equiv \begin{pmatrix}
  M^2_L & M^2_{LR} e^{i\alpha} \\
  M^2_{LR} e^{-i\alpha} & M^2_R
\end{pmatrix},
\]

(9)

where \( K^2 = |k|^2 + |k'|^2 \) and \( \alpha \) is the phase of \( k^* k' \).

We introduce the mixing angle \( \zeta \) between the two gauge eigenstates defined by

\[
\tan 2\zeta = -\frac{2M^2_{LR}}{M^2_R - M^2_L}.
\]

(10)

Then the eigenvalues can be written in terms of \( \zeta \) as

\[
M^2_W = M^2_L \cos^2 \zeta + M^2_R \sin^2 \zeta + M^2_{LR} \sin 2\zeta,
\]

\[
M^2_{W'} = M^2_L \sin^2 \zeta + M^2_R \cos^2 \zeta - M^2_{LR} \sin 2\zeta,
\]

(11)

and the corresponding eigenvectors are

\[
\begin{pmatrix}
  W^+ \\
  W'^+
\end{pmatrix} = \begin{pmatrix}
  \cos \zeta & e^{-i\alpha} \sin \zeta \\
  -\sin \zeta & e^{-i\alpha} \cos \zeta
\end{pmatrix} \begin{pmatrix}
  W^+_L \\
  W^+_R
\end{pmatrix}.
\]

(12)

The \( W^+ \) and \( W'^+ \) fields are the physical charged gauge bosons in the \( SU(2)_L \times SU(2)_R \times U(1) \) theory. For \( v_R \gg v_L, |k|, |k'| \), the mass \( M^r_W \) (\( M_W \)) is almost the mass of the \( W_R \) (\( W_L \)) particle since the \( W_L-W_R \) mixing angle \( \zeta \) is small.

The two parameters \( M^r_W \) and \( \zeta \) are those appearing in the charged sector and they are restricted by a number of low-energy phenomenological constraints along with the CKM matrix elements in the right-handed sector. Conservative numerical estimates for the bounds lie in the range \[6\]

\[
M^r_W > 300 \text{ GeV}, \quad |\zeta| < 0.075.
\]

(13)

These bounds can differ depending on the form of the CKM matrix in the right-handed sector and the type of the right-handed neutrino. (For reference,
see Tables I and II in Ref. [6].) The quoted bounds in Eq. (13) are the weakest bounds and, in other specific cases, the bounds get tighter. For example, for the left-right symmetric case, the bound for the $Z'$ mass can go up as high as 1.4 TeV. These constraints have been obtained from the $K_L-K_S$ mass difference, $B_d\bar{B}_d$ mixing, semileptonic $b$ decays and the neutrinoless double beta decay.

2.3 Neutral gauge bosons

The Lagrangian in Eq. (7) also produces the masses of the neutral gauge bosons after the spontaneous symmetry breakdown. In a similar way, we can construct the mass-squared matrix for the neutral gauge bosons. The diagonalization of the mass matrix in the neutral sector is more complicated since there are three neutral gauge particles $W_{L3}$, $W_{R3}$ and $B$ and the mass matrix becomes a $3 \times 3$ matrix. The mass-squared matrix for the neutral gauge bosons is given by

$$M^2 = \begin{pmatrix} g_L^2(v_L^2 + K^2)/2 & -g_L g_R K^2/2 & -g_L g_1 v_L^2/2 \\ -g_L g_R K^2/2 & g_R^2(v_R^2 + K^2)/2 & -g_R g_1 v_R^2/2 \\ -g_L g_1 v_L^2/2 & -g_R g_1 v_R^2/2 & g_1^2(v_L^2 + v_R^2)/2 \end{pmatrix}. \quad (14)$$

The mass-squared matrix in Eq. (14) is a real symmetric $3 \times 3$ matrix. In order to diagonalize it, we need a real orthogonal matrix which can be parameterized in terms of three Euler-type angles. We introduce $\theta_R$, $\theta_W$ and $\xi$ as three mixing angles in diagonalizing the mass matrix. The derivation of obtaining the mass eigenstates in terms of the gauge eigenstates and the introduction of three mixing angles are presented in Appendix in detail.

The mass eigenstates can be expressed in terms of the gauge eigenstates as

$$A^\mu = \sin \theta_W W_{L3}^\mu + \cos \theta_W \sin \theta_R W_{R3}^\mu + \cos \theta_W \cos \theta_R B^\mu,$$

$$Z^\mu = \cos \xi \cos \theta_W W_{L3}^\mu + (\sin \xi \cos \theta_R - \cos \xi \sin \theta_W \sin \theta_R) W_{R3}^\mu$$

$$- (\cos \xi \sin \theta_W \cos \theta_R + \sin \xi \sin \theta_R) B^\mu,$$

$$Z'^\mu = - \sin \xi \cos \theta_W W_{L3}^\mu + (\cos \xi \cos \theta_R + \sin \xi \sin \theta_W \sin \theta_R) W_{R3}^\mu$$

$$+ (\sin \xi \sin \theta_W \cos \theta_R - \cos \xi \sin \theta_R) B^\mu. \quad (15)$$

The mixing angles $\theta_R$ and $\theta_W$ are defined as

$$\sin \theta_R = \frac{g_1}{\sqrt{g_1^2 + g_R^2}}, \quad \sin \theta_W = \frac{g_1 g_R}{\sqrt{g_1^2 g_R^2 + g_L^2 g_1^2 + g_R^2 g_1^2}}. \quad (16)$$
As noted in Appendix, the mixing angle $\theta_W$ between the $A$ and the $Z$ fields corresponds to the Weinberg mixing angle in the standard model. The mixing angle $\xi$ is defined in Appendix and it is very small.

The eigenvalues for the mass eigenstates are given by

\[
\begin{align*}
M_A^2 &= 0, \\
M_Z^2 &\approx \frac{1}{2} g_L^2 (K^2 + v_L^2) - \frac{g_L^2}{2 c_W^2} \left( \frac{c_R^2 K^2 - s_R^2 v_L^2}{v_R^2} \right), \\
M_{Z'}^2 &\approx \frac{1}{2} (g_1^2 + g_R^2) v_R^2 + \frac{1}{2} (g_R^2 c_R^2 K^2 + g_1^2 s_R^2 v_L^2),
\end{align*}
\]

(17)

where $c_i = \cos i, s_i = \sin i$ with $i = \theta_W, \theta_R$. The massless field $A$ corresponds to the photon field. $Z'$ is the heaviest since its mass is proportional to $v_R$ ($v_R \gg v_L, k, k'$). Note that, at order $O(1/v_R^2)$, $M_Z$ decreases while $M_{Z'}$ increases compared to the leading values. This is a general feature of quantum mechanics that, if there is mixing in a two-level system, the mixing widens the energy gap between the unperturbed states.

In summary the neutral gauge sector is described by three mixing angles, one of which is the Weinberg mixing angle. And there is the mass $M_{Z'}$ of the new heavy particle. These parameters $\theta_R, \xi$ and $M_{Z'}$ are going to be restrained from experimental data.

2.4 Current interactions

Here we write down the current interactions in the $SU(2)_L \times SU(2)_R \times U(1)$ model. Though we will not consider the charged-current interactions, we list them for completeness. Now that we have physical, mass eigenstates for the gauge bosons, we can write down the interaction of fermions with gauge particles. First we write down the fermion fields in terms of mass eigenstates. After the symmetry breakdown, fermions get masses and the mass eigenstates are related to the gauge eigenstates by the following unitary transformations

\[
\begin{align*}
&u'_L = S_u u_L, \quad u'_R = T_u u_R, \quad d'_L = S_d d_L, \quad d'_R = T_d d_R,
\end{align*}
\]

(18)

where the primed fields are gauge eigenstates and the unprimed fields are mass eigenstates. Let $V_L = S_u^\dagger S_d$ and $V_R = T_u^\dagger T_d$. $V_L$ is the usual CKM matrix in the standard model and $V_R$ is its counterpart in the right-handed sector.

The charged-current interaction is given by
\[ L_{CC} = \frac{1}{\sqrt{2}} (\pi, \pi, \bar{t}) \left[ W^+ \left( -g_L \cos \zeta V_L P_- - g_R \sin \zeta e^{i\alpha} V_R P_+ \right) \right. \]
\[ + W'_+ \left( g_L \sin \zeta V_L P_- - g_R \cos \zeta e^{i\alpha} V_R P_+ \right) \left[ \begin{array}{c} d \\ s \\ b \end{array} \right] + \text{h.c.} \] \tag{19}

where \( P_\pm = (1 \mp \gamma_5)/2 \) are the left- and right-handed projection operators respectively. The leptonic charged-current interactions can be written in a similar way. Since the analysis on \( \zeta \) and \( M_{W'} \) in the charged sector has been extensively investigated in Ref. [6], we will not consider it here any more.

In the neutral-current interaction, there are no flavor-changing neutral currents at tree level. Therefore the CKM matrices \( V_L \) and \( V_R \) do not appear and the Glashow-Illiopoulos-Maiani mechanism still works in the \( SU(2)_L \times SU(2)_R \times U(1) \) model. The neutral-current interaction is written as

\[ L_{NC} = -e \overline{\psi} A \left[ (T_{L3} + S)P_- + (T_{R3} + S)P_+ \right] \psi \]
\[ -e \overline{\psi} A' \left[ (g_L c_W c_\xi T_{L3} - g_R (c_R s_W c_\xi + s_R s_\xi) S) P_- \right. \]
\[ + (g_R (c_R s_\xi - s_R s_W c_\xi) T_{R3} - g_L (c_R s_W c_\xi + s_R s_\xi) S) P_+ \] \]
\[ -e \overline{\psi} A' \left[ (g_L c_W s_\xi T_{L3} + g_R (c_R s_W s_\xi - s_R c_\xi) S) P_- \right. \]
\[ + (g_R (c_R c_\xi + s_R s_W s_\xi) T_{R3} + g_L (c_R s_W s_\xi - s_R c_\xi) S) P_+ \] \tag{20}

Here \( \psi \) denotes fermion fields and \( c_\xi = \cos \xi \) and \( s_\xi = \sin \xi \).

From Eq. (20), the electric charge operator \( Q \) can be obtained by looking at the coupling of fermions with the photon. It is given by

\[ Q = T_{L3} + T_{R3} + S. \] \tag{21}

And we also introduce the electromagnetic coupling constant \( e \), and the relations among the coupling constants are given by

\[ e = g_L s_W = g_R s_R c_W = g_1 c_R c_W = \frac{g_1 g_R g_L}{\sqrt{g_1^2 g_R^2 + g_L^2 g_1^2 + g_R^2 g_L^2}}. \] \tag{22}

The current coupled to the \( Z \) field to first order in \( \xi \) is given by

\[ J^\mu_Z = \frac{e}{s_W c_W} \overline{\psi} \gamma^\mu \left[ T_{L3} - s_W^2 Q + \xi s_W \left( t_R (T_{L3} - Q) + (t_R + \frac{1}{t_R}) T_{R3} \right) \right] \psi. \] \tag{23}
where \( t_R = s_R/c_R \). And the current coupled to \( Z' \) to zeroth order in \( \xi \) is

\[
J_{Z'}^\mu = \frac{e}{c_W} \bar{\psi} \gamma^\mu \left( t_R(T_{L3} - Q) + (t_R + \frac{1}{t_R})T_{R3} \right) \psi.
\] (24)

Since those processes mediated by \( J_{Z'}^\mu \) will be already suppressed by \( 1/M_{Z'}^2 \), we do not include the term proportional to \( \xi \).

3 Theoretical considerations

Before we use experimental data to constrain the parameters in our model, it is useful to consider the structure of the theory. There are a few interesting relations between the masses of the gauge bosons. When the mixing angles \( \zeta \) and \( \xi \) are small, we can estimate these mixing angles in terms of the mass ratios of the gauge bosons. Though the bounds obtained from theoretical considerations may not be helpful in obtaining strong constraints on the parameters, it is worthwhile to notice the interwoven structure of the theory and it will give rough estimates of the parameters.

3.1 Masses of \( Z' \) and \( W' \)

The exact masses of \( W' \) and \( Z' \) are expressed in terms of mixing angles with VEVs as in Eqs. (11), (93). Since we assume that \( v_R \gg k, k', v_L \), we can express the masses in a power series with respect to \( 1/v_R^2 \) (or equivalently in powers of \( \xi \)). The approximate masses are given by

\[
M_{W'}^2 \approx \frac{1}{2} g_R^2 v_R^2 \left( 1 + \frac{K^2}{v_R^2} \right), \quad M_{Z'}^2 \approx \frac{1}{2} \left( g_1^2 + g_R^2 \right) v_R^2 \left( 1 + \frac{c_R^2 K^2 + s_R^2 v_L^2}{v_R^2} \right).
\] (25)

The ratio of these masses is given by

\[
\frac{M_{Z'}^2}{M_{W'}^2} \approx \frac{1}{c_R^2} \left[ 1 + \frac{s_R^2}{v_R^2} \left( s_R^2 v_L^2 - (1 + c_R^2)K^2 \right) \right],
\] (26)

which leads to the relation

\[
M_{W'}^2 = M_{Z'}^2 c_R^2 + O(K^2/v_R^2).
\] (27)

From Eq. (27), we conclude that \( M_{Z'} \) is larger than \( M_{W'} \).
This is the relation between $M^2_W$ and $M^2_Z$. It is similar to the relation between $M^2_W$ and $M^2_Z$ in the standard model. If we define the $\rho$ and $\rho'$ parameters in the $SU(2)_L \times SU(2)_R \times U(1)$ model as $\rho \equiv M^2_W / M^2_Z c^2_W$ and $\rho' \equiv M^2_{W'} / M^2_Z c^2_R$ at tree level, they are close to 1 up to order $O(K^2/v^2_R)$. That is,

$$\rho \approx \rho' = 1 + O(K^2/v^2_R). \tag{28}$$

In the standard model, since the scalar field has a larger symmetry than the original $SU(2)_L$ symmetry, this extra symmetry requires $\rho = 1$. This is called the custodial symmetry. In the $SU(2)_L \times SU(2)_R \times U(1)$ model, there are two kinds of custodial symmetries when we construct the scalar fields as we have done. The relation $\rho' \approx 1$ is the result of the additional custodial symmetry. The correction to $\rho = \rho' = 1$ is due to the mixing $\xi$ of order $O(K^2/v^2_R)$ between the two massive neutral states.

From the relation $M_{W'} \approx M_{Z} c_R$, we only know that $M_{Z'} > M_{W'}$. In the special case with the left-right symmetry ($g_L = g_R$), we can have a stronger bound. Using the relation in Eq. (22), when $g_L = g_R$, $\sin \theta_R = \tan \theta_W$. Using the value $\sin^2 \theta_W = 0.231$, we get $\theta_W = 28.7^\circ$ and $\theta_R = 33.2^\circ$, resulting in $\cos \theta_R < \cos \theta_W$. Therefore we get the bound

$$M_{Z'} = \frac{M_{W'}}{\cos \theta_R} > \frac{M_{W'}}{\cos \theta_W}. \tag{29}$$

If we use a conservative lower bound for $M_{W'}$ is given by $M_{W'} > 300$ GeV according to Ref. [6], the bound for $M_{Z'}$ is given by

$$M_{Z'} > 340 \text{ GeV}. \tag{30}$$

However, for the left-right symmetric case, the stringent bound [10], $M_{W'} > 1.4 - 2.5 \text{ TeV}$, comes from the $K_L-K_S$ mass difference $\Delta m_K$ though this limit strongly depends on certain theoretical assumptions. In this case the lower bound for $M_{Z'}$ can be as large as $1.6 - 2.9 \text{ TeV}$.

### 3.2 Relation between mixing angles and mass ratios

We can set bounds for the mixing angles $\zeta$ and $\xi$ in terms of the mass ratios of the gauge bosons. Masso [11] has derived the important bound in the charged sector

$$|\zeta| < \frac{M^2_W}{M^2_{W'}}. \tag{31}$$
for the left-right symmetric case with $g_L = g_R$. Langacker and Sankar \[6\] generalized this bound to the asymmetric case with $g_L \neq g_R$. We briefly derive the relation and generalize the bound for the mixing angle in the neutral sector.

The mixing angle $\zeta$ in the charged sector is given in Eq. (10). For large $v_R$ ($v_R^2 \gg v_L^2, K^2$), the masses in Eq. (10) can be approximated as

$$M_L^2 = \frac{g_L^2}{2} (v_L^2 + K^2) \approx M_W^2, \quad M_R^2 = \frac{g_R^2}{2} (v_R^2 + K^2) \approx \frac{g_R^2}{2} v_R^2 \approx M_W^2.'$$ (32)

And the mixing is given by $M_{LR}^2 = -g_L g_R |k^* k'|$. Therefore $\zeta$ can be approximately written as

$$\zeta \approx -\frac{M_{LR}^2}{M_R^2} \approx \frac{g_L g_R |k^* k'|}{M_W^2}. \quad (33)$$

Using the Schwarz inequality $-K^2 < 2|k^* k'| < K^2$, we get

$$-\frac{g_R M_W^2}{g_L M_W^2} \ll \zeta < \frac{g_R M_W^2}{g_L M_W^2}. \quad (34)$$

This is the result obtained by Langacker and Sankar, which is a generalization from the left-right symmetric case.

We can similarly obtain an inequality for the mixing angle $\xi$ in the neutral sector. The mixing angle $\xi$ is defined in Eq. (91). The masses appearing in $\xi$ can be approximated as [See Eq. (90).]

$$M_Z^2 = \frac{g_L^2}{2 c_W^2} (K^2 + v_L^2) \approx M_Z^2,$$

$$M_{\tilde{Z}}^2 = \frac{1}{2} (g_1^2 + g_R^2) (v_R^2 + c_R^2 K^2 + s_R^2 v_L^2) \approx \frac{g_1^2 + g_R^2}{2} v_R^2 \approx M_{\tilde{Z}}^2. \quad (35)$$

And the mixing $M_{ZZ}^2$ is given by

$$M_{ZZ}^2 = -\frac{1}{2} g_L g_R c_R (K^2 - t_R^2 v_L^2) = -\frac{1}{2} s_W \frac{g_L^2}{2 c_W^2} (K^2 - t_R^2 v_L^2). \quad (36)$$

Therefore, for large $v_R$, we have

$$\xi \approx s_W \frac{g_L^2}{t_R^2 c_W^2} (K^2 - t_R^2 v_L^2) \frac{1}{M_Z^2}. \quad (37)$$

13
We use the following relation

\[-t^2_R (K^2 + v^2_L) < K^2 - t^2_R v^2_L < K^2 + v^2_L\]  

(38)

to have the inequality for \(\xi\) as

\[-s_W t_R \frac{M^2_Z}{M^2_{Z'}} < \xi < s_W \frac{M^2_Z}{t_R M^2_{Z'}}.\]  

(39)

This is the generalization of the inequality for the mixing angle \(\xi\) in the neutral sector.

### 3.3 \(\rho\) parameter

The \(\rho\) parameter, defined as \(\rho \equiv M^2_W/M^2_Z \cos^2 \theta_W\), can be expressed in terms of mixing angles and gauge boson mass ratios. Since we have obtained the bounds for the mixing angles, we can set bounds for the \(\rho\) parameter. In the \(SU(2)_L \times SU(2)_R \times U(1)\) model, the masses \(M_W\) and \(M_Z\) are given in Eqs. (11) and (93) as

\[M^2_W = M^2_L \cos^2 \zeta + M^2_R \sin^2 \zeta + M^2_{LR} \sin 2\zeta,\]

\[M^2_Z = M^2_Z \cos^2 \xi + M^2_{\tilde{Z}} \sin^2 \xi + M^2_{\tilde{Z}Z} \sin 2\xi,\]  

(40)

where \(\zeta\) and \(\xi\) are defined in Eqs. (10) and (91).

Since \(M^2_R\) and \(M^2_{\tilde{Z}}\) are of order \(v^2_R\), the mixing angles are small and can be approximately written as

\[\zeta \approx -\frac{M^2_{LR}}{M^2_R}, \quad \xi \approx -\frac{M^2_{\tilde{Z}Z}}{M^2_Z}.\]  

(41)

Using this expression, we can write Eq. (40) to first order in the mixing angles as

\[M^2_W \approx M^2_L + \zeta M^2_{LR}, \quad M^2_Z \approx M^2_{\tilde{Z}} + \xi M^2_{\tilde{Z}Z}.\]  

(42)

Therefore the \(\rho\) parameter can be written as

\[\rho - 1 \approx \zeta \frac{M^2_{LR}}{M^2_L} - \xi \frac{M^2_{\tilde{Z}Z}}{M^2_Z}.\]
\[
\approx -\xi g_L g_R |k'^*k'|/M_W^2 + \xi g_L g_R c_R (K^2 - t_R^2 v_L^2)/(2c_W).
\] (43)

The correction to the \(\rho\) parameter at tree level is proportional to the small mixing angles \(\zeta\) in the charged sector and \(\xi\) in the neutral sector.

We can get the bound of \(\rho - 1\) in Eq. (43) using the same technique in Sec.3.2. Using the inequalities

\[-K^2 < 2|k'^*k'| < K^2, \quad -t_R^2(K^2 + v_L^2) < K^2 - t_R^2 v_L^2 < K^2 + v_L^2,\] (44)

we obtain the relation

\[-(1 + c^2_R s^2_R)\left(-\frac{v_L}{s_R c_R}\right)^2 M_Z^2 < \rho - 1 < (1 + c^4_R)\left(-\frac{v_L}{s_R c_R}\right)^2 M_Z'^2.\] (45)

In deriving Eq. (45), we use the tree-level relations \(M_Z^2 = M_Z'^2 \cos^2 \theta_W\), \(M_{Z'}^2 = M_{Z'}^2 \cos^2 \theta_R\) since the corrections to the tree-level values give higher-order corrections.

Though Eq. (45) gives a rough estimate on the bounds of the \(\rho\) parameter, it is not helpful to use this inequality in numerical estimates since the bounds in Eq. (45) may be overestimated for large \(t_R\). Because of this, we do not expect that the consideration on \(\rho\) from Eq. (45) gives a useful bound for the mass bound of \(Z'\). Therefore we exclude the \(\rho\) parameter in analyzing the LEP I data.

### 4 Phenomenological constraints

We consider phenomenological constraints on the parameters in the \(SU(2)_L \times SU(2)_R \times U(1)\) model. We employ various phenomenological inputs such as LEP I data, low-energy neutral-current data to constrain the parameters \(\theta_R\), \(\xi\) and \(M_{Z'}\). First we obtain the constraints on \(\theta_R\) and \(\xi\) using the LEP I data. With these bounds, we constrain \(\theta_R\), \(\xi\) and an additional parameter \(M_{Z'}\) to satisfy the low-energy neutral-current data. We present the prediction for LEP II with the constraints obtained from the combined constraints from LEP I and low energy data.

We probe all the allowed values of \(\xi\), \(\theta_R\) and \(M_{Z'}\), which satisfy the experimental bounds. However, there are some special regions in the parameter space from theoretical considerations. First of all, we consider the case with \(\xi = 0\). This can be achieved by setting \(K = t_R v_L\). As can be seen in Eq. (36), this is the case where the mixing in the mass matrix represented by \(M_{ZZ}^2\) is zero.
This fine tuning seems arbitrary, but it is not. Recall that the scalar doublet $\chi_L$, which has the VEV $v_L$, is introduced for the theory to include left-right symmetry easily, but it is not necessary to attain the desired structure of the theory. Therefore we can vary $v_L$ along with $\theta_R$ as we want in order to make such a fine tuning. In this case, the remaining parameters $\theta_R$ and $M_Z'{}^2$ are not constrained by LEP I data since the corrections are proportional only to the mixing angle $\xi$. Therefore in this limit, we can evade the precision electroweak LEP I data and the remaining two parameters are constrained from other experimental data.

Of course, the value of $v_L$ is not completely arbitrary when we consider the mechanism why the left-handed neutrinos are so light. For example, if the neutrino is of Majorana type, in order for the seesaw mechanism to work such that the left-handed neutrino mass becomes very small, $v_L$ cannot be large. However, since there are many variations in introducing the right-handed neutrino, the possibility for this fine tuning is still robust.

The second interesting case is the limit of the left-right symmetry with $g_L = g_R$. The left-right symmetric model has been extensively investigated by many authors [16]. Therefore we can compare our results with previous analyses. In our version, this left-right symmetric model puts a definite relation between the mixing angles $\theta_R$ and $\theta_W$. From Eq. (22), with $g_L = g_R$, the relation between $\theta_R$ and $\theta_W$ is given by

$$\sin \theta_R = \tan \theta_W.$$  \hspace{1cm} (46)

In this case the parameter space is spanned effectively by the two parameters $\xi$ and $M_Z'{}^2$. We will consider this parameter space also in the following analysis. As it turns out, the combined results of the LEP I data and the low-energy neutral-current data raise the lower bound for $M_Z'$ in the left-right symmetric theory. This point will be discussed in detail when we consider the low-energy neutral-current data.

### 4.1 Constraints from LEP I data

In the analysis of the LEP I data, we follow the method employed by Altarelli et al. [12]. It is equivalent to the analysis using oblique parameters $S$, $T$ and $U$ [13]. The basic observables in this analysis are the mass ratio $M_W/M_Z$, the leptonic decay width $\Gamma_\ell$, the leptonic forward-backward asymmetry $A_{FB}^{\ell}$. From these quantities, we can obtain dynamically significant corrections $\Delta r_W$, $\Delta \rho$ and $\Delta k$, which contain small effects to be disentangled. First $\Delta r_W$ is defined
from $M_W/M_Z$ by the relation

$$
(1 - \frac{M_W^2}{M_Z^2}) \frac{M_Z^2}{M_W^2} = \frac{\pi \alpha(M_Z)}{\sqrt{2} G_F M_Z^2 (1 - \Delta r_W)}.
$$

(47)

Here $\alpha(M_Z)$ is fixed to the value 1/128.87.

In order to define $\Delta \rho$ and $\Delta k$, we first write the coupling of the $Z$ particle to charged leptons in the form $\bar{\ell} \gamma^\mu (g_V - g_A \gamma_5) \ell$. Then the physical quantities $\Gamma_\ell$ and $A_{FB}^\ell$ can be parameterized by the effective vector and axial-vector couplings $g_V$ and $g_A$ as

$$
\Gamma_\ell = \frac{G_F M_Z^3}{6\pi\sqrt{2}} (g_V^2 + g_A^2) \left( 1 + \frac{3\alpha}{4\pi} \right), \quad A_{FB}^\ell(\sqrt{s} = M_Z) = \frac{3g_V^2 g_A^2}{(g_V^2 + g_A^2)^2}.
$$

(48)

Here $\Gamma_\ell$ stands for the inclusive partial width $\Gamma(Z \to \ell\ell + \text{photons})$. $\Delta \rho$ and $\Delta k$ are defined as

$$
g_A = -\frac{1}{2} \left( 1 + \frac{\Delta \rho}{2} \right), \quad \frac{g_V}{g_A} = 1 - 4(1 + \Delta k)s_W^2,
$$

(49)

where $s_W^2$ is evaluated at tree level, given by

$$
s_W^2 c_W^2 = \frac{\pi \alpha(M_Z)}{\sqrt{2} G_F M_Z^2},
$$

(50)

with $c_W^2 = 1 - s_W^2$. ($s_W^2 = 0.231184$ for $M_Z = 91.187$ GeV.)

We express $\Delta \rho$, $\Delta r_W$ and $\Delta k$ in terms of the following combinations:

$$
\epsilon_1 = \Delta \rho,
$$

$$
\epsilon_2 = c_W^2 \Delta \rho + \frac{s_W^2 \Delta r_W}{c_W^2 - s_W^2} - 2s_W^2 \Delta k,
$$

$$
\epsilon_3 = c_W^2 \Delta \rho + (c_W^2 - s_W^2) \Delta k.
$$

(51)

These variables $\epsilon_i$ are used in analyzing new physics effects beyond the standard model. In our case, the additional corrections are expressed in terms of the mixing angle $\xi$, $\theta_R$. Note that the ratio of the masses $M_Z^2/M_Z'$ does not enter the analysis for LEP I data since the LEP I experiment is performed at the $Z$ peak.

Note that only the parameter $\epsilon_2$ depends on $\Delta r_W$, hence on $M_W^2/M_Z^2$ as shown in Eq. (47). The ratio $M_W^2/M_Z^2$ is related to the $\rho$ parameter and the approximate relation for the $\rho$ parameter is given in Eq. (43). There are two kinds
of uncertainties in this expression. First, it depends on the information of the charged sector which needs an independent analysis. Secondly, it is difficult to express the term belonging to the neutral sector in terms of the parameters $\xi$, $\theta_R$ and $M_Z$. For these reasons, we avoid using $\epsilon_2$, and we will consider constraints only in the $(\epsilon_1, \epsilon_3)$ parameter space.

\[ \epsilon_1 = \epsilon_1^{\text{SM}} + \Delta \rho_{LR}, \quad \epsilon_3 = \epsilon_3^{\text{SM}} + c_W^2 \Delta \rho_{LR} + (c_W^2 - s_W^2) \Delta k_{LR}, \] (52)

where

\[ \Delta \rho_{LR} = -\frac{2 \xi s_W}{t_R}, \quad \Delta k_{LR} = \xi s_W \left[ \frac{1}{t_R} + \frac{1}{2s_W^2} (t_R - \frac{1}{t_R}) \right]. \] (53)

The quantities $\epsilon_1^{\text{SM}}, \epsilon_3^{\text{SM}}$ are the contributions from the standard model. In the numerical analysis, we use the values of $\epsilon_1^{\text{SM}}$ and $\epsilon_3^{\text{SM}}$ including the electroweak radiative corrections as encoded in ZFITTER [14]. The experimental values for $\epsilon_1$ and $\epsilon_3$ are given by

\[ \epsilon_1 = (2.6 - 5.3) \times 10^{-3}, \quad \epsilon_3 = (1.0 - 4.5) \times 10^{-3}, \] (54)

which can be calculated from Ref. [15]
Using the variables $\epsilon_1$ and $\epsilon_3$, we can put constraints on the parameters $\xi$ and $\theta_R$. The constraints on $\xi$ and $\theta_R$ are shown in Fig. 1. The solid curves represent the constraint by $\epsilon_3$ and the dashed curves represent the constraint by $\epsilon_1$. From the overlapping region of these two bounds in Fig. 1, we obtain

$$-0.0028 < \xi < 0.0065,$$

while there is little constraint on $\theta_R$.

Note that these bounds are for all possible values of $\theta_R$. We can easily get bounds for $\xi$ at some specific values of $\theta_R$. For example, the left-right symmetric model corresponds to $\theta_R \approx 33^\circ$ ($\sin \theta_R = \tan \theta_W$). In this case, the bound for $\xi$ is $-0.0005 < \xi < 0.0026$. As shown in Fig. 1, the characteristic of the relation between $\theta_R$ and $\xi$ is that positive (negative) values of $\xi$ are preferred for small (large) $\theta_R$.

Fig. 2. Behavior of $(\epsilon_1, \epsilon_3)$ as $\xi$ and $\theta_R$ are varied. The ellipses are obtained from experimental data. The solid curve corresponds to 95% CL, the long-dashed curve to 90% CL and the short-dashed curve to 1$\sigma$ level. The straight lines represent the behavior of $(\epsilon_1, \epsilon_3)$ with $t_R = 0.1, 0.5, 1.0, 1.5, 2.0, 3.0$ and 20.0 respectively, starting from the lower left line to the clockwise direction. Positive values of $\xi$ only are shown.
We can also consider the behavior of \((\epsilon_1, \epsilon_3)\) as we vary \(\xi\) and \(\theta_R\). It is shown in Fig. 2 for the Higgs mass \(M_H = 100\) GeV. The ellipses are bounds from LEP I data. They represent 95\% CL, 90\% CL, and 1\(\sigma\)-level estimates respectively as we go inside. The converging point corresponds to the standard model value. The straight lines show the behavior of \(\epsilon_1\) and \(\epsilon_3\) as we vary \(\theta_R\). As we increase \(\theta_R\), the lines move in the clockwise direction. We show the behavior for positive values of \(\xi\) only. As we increase \(\xi\), the values of \(\epsilon_1\) and \(\epsilon_3\) deviate further away from the standard model value. For negative values of \(\xi\), the direction is reversed. As we vary the Higgs mass \(M_H\) from 100 GeV to 1 TeV, this qualitative feature does not change though the point representing the standard model moves downward a little bit. In order for the point \((\epsilon_1, \epsilon_3)\) to approach the central region of the data, \(\xi\) should be negative for large values of \(\theta_R\) or \(\xi\) should be positive for small values of \(\theta_R\). We can understand this behavior clearly in Fig. 1, as already pointed out.

In the special case for \(\xi = 0\), there is no deviation from the standard model since all the corrections are proportional to \(\xi\). Therefore with this fine tuning \((K = t_R v_L)\), the standard-model prediction remains intact. The constraint on \(\xi\), with \(\theta_R\) varied, from LEP I will be combined with the low-energy neutral-current data to further constrain all the three parameters \(\xi\), \(\theta_R\) and \(M_{Z'}\) in Sec. 4.2.

4.2 Constraints from low-energy neutral-current data

Now we consider the low-energy neutral-current interactions such as \(\nu e \to \nu e\), \(\nu N\) scattering, and \(e_{L,R}N \to e_{L,R}X\) in which both \(Z\) and \(Z'\) can participate. As far as the neutral-current interactions involving neutrinos are concerned, note that there is one striking difference in the \(SU(2)_L \times SU(2)_R \times U(1)\) model compared to the standard model. It is the existence of the right-handed neutrino. There are a few possibilities to include the right-handed neutrino, such as a heavy Dirac neutrino, medium-mass neutrino, light neutrino, or heavy Majorana neutrino. The Majorana-type neutrino provides an interesting way to make the left-handed neutrino light via the seesaw mechanism \cite{16}.

We include both left-handed and right-handed neutrinos in the analysis. However, if we keep the corrections to first order in \(\xi\) or \(M_Z^2/M_{Z'}^2\), the right-handed neutrino does not contribute to the physical observables irrespective of the types of right-handed neutrinos. In order to illustrate this point, consider the process \(\nu_e e \to \nu_e e\). The effective Hamiltonian for this process is of the form

\[
H_{\text{eff}} = \frac{G_F}{\sqrt{2}} \bar{\nu} \left( g_L^\nu \gamma^\mu (1 - \gamma_5) + g_R^\nu \gamma^\mu (1 + \gamma_5) \right) \nu
\]
The value of $g_{R}^{\nu}$ in the standard model is zero and it starts from the first order in $\xi$ or $M_{2}^{Z}/M_{2}'^{Z}$. In the matrix element squared, the nonzero contribution has an even number of left-handed and right-handed currents due to helicity conservation. The matrix element squared for any process from Eq. (56) depends on $g_{R}^{\nu}$, hence of second order in $\xi$ or $M_{2}^{Z}/M_{2}'^{Z}$. Therefore to first order in $\xi$ or $M_{2}^{Z}/M_{2}'^{Z}$, we can safely disregard the contribution from the right-handed neutrino.

For the scattering $\nu e \rightarrow \nu e$, only the $Z$ and $Z'$ particles participate in the interactions. The relevant effective Hamiltonian at low energy can be written in the form

$$H^{\nu e} = \frac{G_{F}}{\sqrt{2}} \overline{\nu} \gamma^{\mu} (1 - \gamma_{5}) \nu \bar{e} \gamma_{\mu} (g_{V}^{\nu e} - g_{A}^{\nu e}) e,$$  

(57)

neglecting the contribution from the right-handed neutrino as discussed above. We can obtain $g_{V}^{\nu e}$ and $g_{A}^{\nu e}$ to first order in the small mixing angle $\xi$ and the mass ratio $M_{2}^{Z}/M_{2}'^{Z}$. Note that $\xi$ is of order $O(M_{2}^{Z}/M_{2}'^{Z})$, therefore we have to keep both terms in order to be consistent. The coupling constants $g_{V}^{\nu e}$ and $g_{A}^{\nu e}$ are written as

$$g_{V}^{\nu e} = - \frac{1}{2} + 2 s_{W}^{2} + \xi s_{W} \left[\left(\frac{1}{2} + 2 s_{W}^{2}\right) t_{R} - \frac{1}{2 t_{R}}\right] + \frac{M_{2}^{Z}}{M_{2}'^{Z} s_{W}} (s_{W}^{2} t_{R} - \frac{1}{2}),$$

$$g_{A}^{\nu e} = - \frac{1}{2} - \frac{\xi s_{W}}{2} (t_{R} - \frac{1}{t_{R}}) + \frac{M_{2}^{Z}}{2 M_{2}'^{Z} s_{W}} s_{W}^{2}.$$  

(58)

In Eq. (58), those terms independent of $\xi$ and $M_{2}^{Z}/M_{2}'^{Z}$ are the values from the standard model.

For neutrino-hadron scattering, the relevant effective Hamiltonian can be written as

$$H^{\nu N} = \frac{G_{F}}{\sqrt{2}} \overline{\nu} \gamma^{\mu} (1 - \gamma_{5}) \nu$$

$$\times \sum_{i} \left[ \epsilon_{L}(i) \overline{q}_{i} \gamma^{\mu} (1 - \gamma_{5}) q_{i} + \epsilon_{R}(i) \overline{q}_{i} \gamma^{\mu} (1 + \gamma_{5}) q_{i} \right],$$

(59)

where $\epsilon_{L,R}(i)$ are given by

$$\epsilon_{L}(u) = \frac{1}{2} - \frac{2}{3} s_{W}^{2} + \frac{s_{W} t_{R}}{3} \left[ \xi (1 - 2 s_{W}^{2}) - \frac{1}{2} \frac{M_{2}^{Z}}{M_{2}'^{Z} s_{W}} t_{R} \right],$$

$$\epsilon_{R}(u) = \frac{1}{2} - \frac{2}{3} s_{W}^{2} - \frac{s_{W} t_{R}}{3} \left[ \xi (1 - 2 s_{W}^{2}) + \frac{1}{2} \frac{M_{2}^{Z}}{M_{2}'^{Z} s_{W}} t_{R} \right].$$
The coefficients \( C \) gives the value \( 18 \).

The recent measurement and the analysis of the weak charge for the Cs atom gives the value [18]

\[
\Delta Q_W = 0.79 \pm 1.06.
\]
Table 1

| Quantity | Experiments | SM prediction |
|----------|-------------|---------------|
| $\epsilon_L(u)$ | $0.328\pm0.016$ | $0.3461\pm0.0002$ |
| $\epsilon_L(d)$ | $-0.440\pm0.011$ | $-0.4292\pm0.0002$ |
| $\epsilon_R(u)$ | $-0.179\pm0.013$ | $-0.1548\pm0.0001$ |
| $\epsilon_R(d)$ | $-0.027^{+0.077}_{-0.048}$ | $0.0775\pm0.0001$ |
| $g_V^{\nu e}$ | $-0.041\pm0.015$ | $-0.0395\pm0.0005$ |
| $g_A^{\nu e}$ | $-0.507\pm0.014$ | $-0.5064\pm0.0002$ |
| $C_{1u}$ | $-0.216\pm0.046$ | $-0.1885\pm0.0003$ |
| $C_{1d}$ | $0.361\pm0.041$ | $0.3412\pm0.0002$ |
| $C_{2u} - \frac{1}{2}C_{2d}$ | $-0.03\pm0.12$ | $-0.0488\pm0.0008$ |
| $Q_W$ | $-72.41 \pm 1.05$ | $-73.20 \pm 0.13$ |

Values of the model-independent neutral-current parameters, compared with the standard model predictions for $M_Z = 91.1867$ GeV ($M_H = M_Z$) [17].

All the experimental values for the observables and the standard model prediction are tabulated in Table 1. We use the ten physical observables listed in Table 1 to constrain $\xi$, $\theta_R$ and $M_{Z'}$ at 95% CL. As shown in Eqs. (58), (60) and (62), since there are two kinds of terms proportional to $\xi$ and $M_Z^2/M_{Z'}^2$, and since $\xi$ can be either positive or negative, the contribution of these terms may be partially cancelled. If the relative sign of these two terms in a quantity is opposite, both parameters $|\xi|$ and $M_Z^2/M_{Z'}^2$, can be large without exceeding experimental bounds. Due to this fact, the bounds obtained by the low-energy neutral-current data alone are not useful. In order to obtain useful bounds, we first constrain the parameters $\xi$ and $\theta_R$ from LEP I data, and then we look for constraints on $\xi$, $\theta_R$ and $M_{Z'}$ satisfying the low-energy neutral-current data.

We vary $\theta_R$ freely since there is little constraint from the LEP I data. However, since the bounds for $\xi$ depend on $\theta_R$, we vary $\xi$ according to the relation with $\theta_R$ as shown in Fig. 1. We also vary $M_{Z'}$ and look for the values of the set ($\theta_R$, $\xi$, $M_{Z'}$) which satisfy the low-energy neutral-current data. In order to show the bounds in a transparent way, we consider the bounds in two-parameter spaces ($\theta_R$, $M_{Z'}$) and ($\xi$, $M_{Z'}$). We do not show the bounds in the ($\xi$, $\theta_R$) space since the low-energy data do not put additional constraints on the bounds from LEP I.

First, we show the bounds in the ($\theta_R$, $M_{Z'}$) space in Fig. 3. As can be seen in Fig. 3, the low-energy data do not constrain $\theta_R$ either. However, we obtain
Fig. 3. Bounds on $\theta_R$ and $M_{Z'}$ satisfying the low-energy neutral-current data and the LEP I constraints at 95% CL. The region above the curve is allowed.

The smallest lower bound occurs at $\theta_R = 55^\circ - 60^\circ$ and the bound increases when $\theta_R$ is away from this region. The left-right symmetric case corresponds to $\theta_R = 33^\circ$ and the lower bound at the point is about 900 GeV. In another

$$M_{Z'} > 400 \text{ GeV}. \quad (66)$$

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parameter space spanned by \((\xi, M_{Z'})\), the result is shown in Fig. 4.

In summary, the analysis using the combined data from LEP I and the low-energy experiments shows that the bounds are given as

\[-0.0028 < \xi < 0.0065, \quad M_{Z'} > 400 \text{ GeV},\]  

(67)

with little constraint on \(\theta_R\).

Before finishing this analysis, there is one technical comment about dealing with the accuracy of experimental data. We fit the parameters to the experimental data at 95% CL. If we analyze the data at 90% CL, there are more stringent limits on the parameters. The LEP I data include the standard model value at 90% CL, so there is no problem in fitting the parameters at 90% CL for LEP I data. However, for low-energy neutral-current data, the standard model value for \(\epsilon_R(u)\) only is outside the 90%-CL estimates. Therefore at 90% CL, the most severe constraint comes from \(\epsilon_R(u)\) and in this case, the bounds are given by

\[\theta_R > 72^\circ, \quad -0.0014 < \xi < 0.0003, \quad 800 \text{ GeV} < M_{Z'} < 8.3 \text{ TeV}.\]  

(68)

In this case, the left-right symmetric case is disfavored. However, we need more data to obtain precise central values of the experimental data and to decrease experimental errors in order to draw a definite conclusion.

4.3 Comparison with LEP II data

We consider the total cross section and the forward-backward asymmetry for \(e^+e^- \rightarrow \mu^+\mu^-\) at \(s > M_Z^2\). The differential cross section is written as

\[\frac{d\sigma}{d \cos \theta} = \frac{1}{32\pi s} \left( S(1 + \cos^2 \theta) + 2A \cos \theta \right),\]  

(69)

where \(\theta\) is the angle between the incoming electron and the outgoing muon in the center-of-mass frame. The total cross section is given by

\[\sigma = \sigma_F + \sigma_B = \frac{1}{12\pi s} S(1 + \Delta_\ell),\]  

(70)

where \(\Delta_\ell\) is the one-loop electroweak correction. And the forward-backward asymmetry can be expressed as

\[A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} = \frac{3A}{4S}.\]  

(71)
We can express $\sigma$ and $A_{FB}$ to first order in $\xi$ and $M_Z^2/M_{Z'}^2$. The coupling of the charged leptons with $Z$ and $Z'$ are obtained to first order in $\xi$ in the limit of small mixing angle $\xi$. If we write the leptonic current coupled to $Z$ as $-(e/2s_Wc_W)\ell\gamma^\mu(g_{\nu Z} - g_{\nu Z}\gamma_5)\ell$, $g_{\nu Z}$ and $g_{\nu Z}$ are given by

$$g_{\nu Z} = g_{\nu} + \xi s_W(t_R - \frac{1}{2t_R}), \quad g_{\nu Z} = g_A + \xi \frac{s_W}{2t_R}. \quad (72)$$

Here $g_{\nu} = -\frac{1}{2} + 2s_W$ and $g_A = -\frac{1}{2}$ are the couplings in the standard model. Similarly, if we write the coupling with $Z'$ as $-(e/2s_Wc_W)\ell\gamma^\mu(g_{\nu Z'} - g_{\nu Z'}\gamma_5)\ell$, the coupling constants are given by

$$g_{\nu Z'} = s_W(t_R - \frac{1}{2t_R}), \quad g_{\nu Z'} = \frac{s_W}{2t_R}. \quad (73)$$

For $e^+e^- \rightarrow \mu^+\mu^-$, $A$ and $S$, to first order in the mixing angle $\xi$ and $M_Z^2/M_{Z'}^2$, are given as

$$A = \frac{e^4}{4s_W^4c_W^4} \frac{s^2}{(s - M_Z^2)^2} g_{\nu}^2 g_A^2 + \frac{e^4}{2s_W^2c_W^2} \frac{s}{s - M_Z^2} g_{\nu}^2 + \xi \left[ \frac{e^4}{2s_W^4c_W^4} \frac{s^2}{(s - M_Z^2)^2} g_{\nu} g_A (g_{\nu} t_R + \frac{g_{\nu} - g_A}{2t_R}) + \frac{e^4}{2s_W^2c_W^2} \frac{s}{s - M_Z^2} g_A \right] - \frac{s}{M_Z^2} \left[ \frac{e^4}{8s_W^2c_W^4} \frac{1}{t_R} + \frac{e^4}{8s_W^2c_W^4} \frac{s}{s - M_Z^2} (g_{\nu} t_R + \frac{g_{\nu} - g_A}{2t_R}) \right], \quad (74)$$

and

$$S = e^4 + \frac{e^4}{16s_W^4c_W^4} \frac{s^2}{(s - M_Z^2)^2} (g_{\nu}^2 + g_A^2)^2 + \frac{e^4}{2s_W^2c_W^2} \frac{s}{s - M_Z^2} g_{\nu}^2 + \xi \left[ \frac{e^4}{4s_W^4c_W^4} \frac{s^2}{(s - M_Z^2)^2} (g_{\nu}^2 + g_A^2) (g_{\nu} t_R + \frac{g_{\nu} - g_A}{2t_R}) \right] + \frac{e^4}{s_W^2c_W^2} \frac{s}{s - M_Z^2} g_{\nu} (t_R - \frac{1}{2t_R}) \right] - \frac{s}{M_Z^2} \left[ \frac{e^4}{2s_W^2c_W^2} (t_R - \frac{1}{2t_R})^2 + \frac{e^4}{8s_W^2c_W^4} \frac{s}{s - M_Z^2} (g_{\nu} t_R + \frac{g_{\nu} - g_A}{2t_R})^2 \right]. \quad (75)$$

For $e^+e^- \rightarrow b\bar{b}$, we can similarly parameterize the cross section. The quark current coupled with $Z$ is given by $-(e/2s_Wc_W)\bar{b}\gamma^\mu(b_{\nu Z} - b_{\nu Z}\gamma_5)b$ where

$$b_{\nu Z} = g_{\nu} - \xi s_W(t_R - \frac{1}{2t_R}), \quad b_{\nu Z} = g_A + \xi s_W \frac{1}{2t_R}, \quad (76)$$
Table 2
Comparison of the LEP II data, the standard model (SM), and the $SU(2)_L \times SU(2)_R \times U(1)$ (LR) model. The cross sections and the forward-backward asymmetries at different energies are listed in each case.

| $\sqrt{s}$ | 130.12 GeV | 136.08 GeV | 183 GeV |
|------------|-------------|-------------|---------|
| $A_{FB}^I$ | SM          | 0.70        | 0.68    | 0.57    |
|            | LR model    | 0.60 – 0.81 | 0.57 – 0.81 | 0.34 – 0.79 |
|            | Experiments | 0.55 ± 0.13 | 0.76 ± 0.09 | 0.60 ± 0.05 |
| $\sigma^\mu$ | SM         | 8.5         | 7.3     | 3.45    |
|            | LR model    | 6.93 – 8.52 | 5.78 – 7.32 | 2.07 – 3.46 |
|            | Experiments | 7.6 ± 1.4   | 10.4 ± 1.6 | 3.46 ± 0.38 |
| $\sigma^b$  | SM          | 3.96        |         |         |
|            | LR model    | 3.59 – 3.97 |         |         |
|            | Experiments | –           | –       | 4.6 ± 0.9 |

and $g_V^b = -\frac{1}{2} + \frac{2}{3}s^2_W$, and $g_A^b = -\frac{1}{2}$ are the standard model values.

The total cross section is given by

$$\sigma_b = \frac{1}{12\pi s} S_b (1 + \Delta_b), \quad (77)$$

where

$$S_b = \frac{1}{9} \frac{e^4}{16s_Wc_W} \frac{s^2}{(s-M_Z^2)^2} (g_V^{b2} + g_A^{b2})^2 + \frac{e^4}{6s_Wc_W} \frac{s}{s-M_Z^2} g_V^{b2}$$

$$+ \xi \left[ \frac{e^4}{4s_Wc_W} \frac{s^2}{(s-M_Z^2)^2} (g_V^{b2} + g_A^{b2}) \left( -\frac{g_V^b}{3} + \frac{g_A^b}{2t_R} \right) \right]$$

$$- \frac{e^4}{3s_Wc_W} \frac{s}{s-M_Z^2} g_V^{b2} \left( \frac{t_R}{3} + \frac{1}{2t_R} \right)$$

$$- \frac{s}{M_Z^2} \left[ \frac{e^4}{6c_W} \left( \frac{t_R}{3} + \frac{1}{2t_R} \right)^2 + \frac{e^4}{8s_Wc_W} \frac{s}{s-M_Z^2} \left( -\frac{g_V^b}{3} + \frac{g_A^b}{2t_R} \right) \right]. \quad (78)$$

And the QCD correction factor in $\Delta_b$ is given by

$$\Delta_{QCD} = 1.2 \frac{\alpha_s(\sqrt{s})}{\pi} - 1.1 \left( \frac{\alpha_s(\sqrt{s})}{\pi} \right)^2 + \ldots. \quad (79)$$
We compare our results with those of the OPAL Collaboration [19]. The accuracy of the experimental data from LEP II is not as good as that from LEP I, but it will be improved as more data will be accumulated. We find that the bounds using the LEP II data do not give more severe bounds obtained from LEP I. Instead, we calculate the cross sections and the forward-backward asymmetry at LEP II using the bounds obtained from the combined data of LEP I and the low-energy data.

The results are shown in Table 2. The cross sections and the forward-backward asymmetry at different energies are listed. The results from the $SU(2)_L \times SU(2)_R \times U(1)$ (LR) model and the standard model (SM) are shown along with the current experimental data. The standard model values, quoted in Table 2, include the effects of the electroweak radiative corrections and the QCD corrections. Most of the values in our model are within $1\sigma$ of the experimental values, but the comparison will be useful after the experimental results are more refined.

5 Conclusion

We have studied constraints on the neutral sector in the $SU(2)_L \times SU(2)_R \times U(1)$ model. We introduce three mixing angles $\xi$, $\theta_R$ and $\theta_W$ to diagonalize the neutral gauge boson mass matrix. Here $\theta_W$ is identified as the Weinberg angle and we use the remaining two mixing angles and the heavy neutral gauge boson mass $M_{Z'}$ to describe new physics effects from the neutral sector in the model. Since $\xi$ and $M_Z^2/M_{Z'}^2$ are small parameters, we calculate all the corrections to the standard model in various processes to first order in these small parameters and fit to experimental data.

First we use the LEP I data to constrain $\xi$ and $\theta_R$ without any information on $M_{Z'}$ since the LEP I energy is at the $Z$ peak. There is little constraint on $\theta_R$, but $\xi$ is bounded by $-0.0028 < \xi < 0.0065$ for all $\theta_R$. Note that the bound for $\xi$ varies for different values of $\theta_R$ as shown in Fig. 1. With the constraints obtained from the LEP I data, we find the bounds for $\xi$ and $M_{Z'}$ which simultaneously satisfy the low-energy neutral-current data. The combined bounds at 95% CL are given as

$$-0.0028 < \xi < 0.0065, \quad M_{Z'} > 400 \text{ GeV.} \quad (80)$$

The bound for the mixing angle $\xi$ in the neutral sector is more severe compared to the bound for the mixing angle $\zeta$ in the charged sector, $|\zeta| < 0.075$. We also consider other experimental results such as the LEP II data in the context of the $SU(2)_L \times SU(2)_R \times U(1)$ model.
The lower bound for the $Z'$ mass is 400 GeV when we combine the LEP I data and the low-energy neutral-current data. In the case of the left-right symmetric model with $g_L = g_R \ (\theta_R = 33^\circ)$, $M_{Z'} > 900$ GeV. On the other hand, we have considered the relation between $M_{W'}$ and $M_{Z'}$ in Sec. 3.1. In the left-right symmetric case, it is given as

$$M_{Z'} = \frac{M_{W'}}{\cos \theta_R} > \frac{M_{W'}}{\cos \theta_W}, \quad (81)$$

using the fact that $\cos \theta_R < \cos \theta_W$ for $\theta_R = 33^\circ$. If we accept the assumptions and the result in Ref. [10] for the left-right symmetric theory, we can get a more severe bound

$$M_{Z'} > 1.6 \text{ TeV} \quad (82)$$

for $M_{W'} > 1.4$ TeV. Note that this result is obtained from an independent information on $M_{W'}$, while the bound from the analysis of the neutral sector is $M_{Z'} > 900$ GeV. However, when we consider the bound in Eq. (82), there is a caveat that the bound $M_{W'}$ from the charged sector depends on many assumptions.

There has been a search for an additional $Z'$ particle irrespective of the detailed structure of the theory. From the search for the process $Z' \rightarrow \mu^+ \mu^-$ at Fermilab [20], the bound for $M_{Z'}$ at 95% CL is $M_{Z'} > 412$ GeV. This is the result independent of the experimental data considered here. However, it is interesting to note that the lower bounds for $M_{Z'}$ in both cases are similar.

Amaldi et al. and Costa et al.[21] considered the model with an additional $U(1)$ from string-inspired models. They obtained the limits on the $Z'$ mass larger than 325 GeV, and the mixing angle corresponding $\xi$ in our model should satisfy $|\xi| < 0.05$ considering low-energy data. Cho et al. [22] have considered the additional neutral particle $Z'$ in the context of the supersymmetric $E_6$ models. Their results are based on the heavy $Z'$ from an additional $U(1)$ gauge group. Therefore care should be taken in comparing their results with ours.

It is also interesting to get bounds on these parameters from other experiments such as $B$ decays. Cho and Misiak [23] have considered the decay rate for $b \rightarrow s \gamma$ in the $SU(2)_L \times SU(2)_R \times U(1)$ models. Their conclusion is that though QCD corrections diminish the difference between this model and the standard model, but for reasonable ranges of parameters, the decay rates can be distinguished and used to probe for new physics beyond the standard model. Babu et al. [24] have considered the same process including the effect of the Higgs particle exchange and obtained the result $-0.015 < \frac{\xi}{\Sigma} < 0.003$ and $M_H >$ a few GeV. However, in the decay $b \rightarrow s \gamma$, only the charged gauge
bosons contribute to the process. In order to consider the new physics effects from the neutral sector, it may be interesting to consider decays such as $b \rightarrow s \ell^+ \ell^-$. The search for bounds on the parameters in the $SU(2)_L \times SU(2)_R \times U(1)$ model in $B$ decays such as $B \rightarrow X_s \ell^+ \ell^-$ or $B \rightarrow X_s \nu \nu$ is in progress [25].

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Appendix: Diagonalization of the Neutral Gauge Boson Masses

Here we show in detail how to diagonalize the neutral gauge boson mass matrix. In order to obtain physical gauge boson states, we have to diagonalize the mass matrix in Eq. (14). Since the mass-squared matrix is a real, symmetric $3 \times 3$ matrix, we need a real, orthogonal matrix to diagonalize it. That is, we have to find three Euler angles parameterizing the orthogonal matrix which diagonalizes the mass matrix. There are two ingredients to facilitate the diagonalization. First we assume that the VEV $v_R$ is much larger than other VEVs such as $v_L$, $|k|$, $|k'|$. And since the electromagnetic $U(1)_{em}$ remains unbroken, there remains one massless field which corresponds to the photon field.

As a first step, consider the mass matrix in Eq. (14) in the limit $v_R \rightarrow \infty$ and neglect small terms compared to $v_R$. Then the mass-squared matrix is written as

$$M^2 \approx \begin{pmatrix} 0 & 0 & 0 \\
0 & g_R^2 v_R^2 / 2 & -g_R g_1 v_R^2 / 2 \\
0 & -g_R g_1 v_R^2 / 2 & g_1^2 v_R^2 / 2 \end{pmatrix}. \quad (83)$$

We can diagonalize the lower right $2 \times 2$ block matrix by defining the following fields:

$$Z_1 = B \cos \theta_R + W_{R3} \sin \theta_R, \quad \hat{Z} = -B \sin \theta_R + W_{R3} \cos \theta_R. \quad (84)$$
where

$$\frac{g_R}{\sqrt{g_1^2 + g_R^2}} = \cos \theta_R, \quad \frac{g_1}{\sqrt{g_1^2 + g_R^2}} = \sin \theta_R. \quad (85)$$

If we rewrite the mass matrix in the basis of $W_{L3}, Z_1$ and $\hat{Z}$, the biggest VEV, $v_R$, appears only in the lower right end.

Now we use the fact that one of the eigenvalues of the matrix in Eq. (14) is zero, which corresponds to the photon field. It is straightforward to obtain the eigenvector with the eigenvalue 0. It is given by

$$A = \frac{g_1 g_R W_{L3} + g_L \sqrt{g_1^2 + g_R^2} Z_1}{\sqrt{g_1^2 g_R^2 + g_L^2 g_1^2 + g_L^2 g_R^2}}. \quad (86)$$

Let us define another field $\tilde{Z}$ which is orthogonal to the photon field $A$:

$$A = \sin \theta_W W_{L3} + \cos \theta_W Z_1, \quad \tilde{Z} = \cos \theta_W W_{L3} - \sin \theta_W Z_1, \quad (87)$$

where

$$\frac{g_1 g_R}{\sqrt{g_1^2 g_R^2 + g_L^2 g_1^2 + g_L^2 g_R^2}} = \sin \theta_W, \quad \frac{g_L \sqrt{g_1^2 + g_R^2}}{\sqrt{g_1^2 g_R^2 + g_L^2 g_1^2 + g_L^2 g_R^2}} = \cos \theta_W. \quad (88)$$

The mixing angle $\theta_W$ corresponds to the Weinberg mixing angle in the standard model if we identify $Z_1$ as the neutral $Z$ in the standard model. Actually $\theta_W$ is equal to the Weinberg mixing angle in the limit $v_R \rightarrow \infty$ and the correction is of order $O(v_L^2/v_R^2)$.

In the basis of $A, \tilde{Z}$ and $\hat{Z}$, the mass-squared matrix takes a simple form. Nonzero terms appear only in the lower right $2 \times 2$ block. The mass-squared matrix looks like

$$M^2 = \begin{pmatrix}
0 & 0 & 0 \\
0 & M_Z^2 & M_{\tilde{Z}Z}^2 \\
0 & M_{Z\tilde{Z}}^2 & M_{\hat{Z}}^2
\end{pmatrix}, \quad (89)$$

where

$$M_Z^2 = \frac{1}{2} \frac{g_L^2}{c_W^2} (K^2 + v_L^2), \quad M_{\tilde{Z}}^2 = \frac{1}{2} (g_1^2 + g_R^2) (v_R^2 + c_R^4 K^2 + s_R^4 v_L^2),$$

$$M_{\hat{Z}}^2 = \frac{1}{2} (g_1^2 + g_R^2) (v_R^2 + c_R^4 K^2 + s_R^4 v_L^2),$$

$$M_{Z\tilde{Z}}^2 = \frac{1}{2} g_L \sqrt{g_1^2 + g_R^2} (v_R^2 + c_R^4 K^2 + s_R^4 v_L^2),$$

$$M_{\tilde{Z}\hat{Z}}^2 = \frac{1}{2} g_L \sqrt{g_1^2 + g_R^2} (v_R^2 + c_R^4 K^2 + s_R^4 v_L^2).$$
\[ M_{ZZ}^2 = -\frac{1}{2} \frac{g_L}{c_W} g_R c_R (K^2 - t_R^2 v_L^2), \] (90)

where \( t_R = s_R/c_R \). Note that \( v_R \) appears only in \( M_{ZZ}^2 \). Therefore we expect that the mixing angle to diagonalize this matrix is small.

Now the diagonalization of the remaining \( 2 \times 2 \) can be done in a similar way to diagonalize the charged gauge boson masses. We introduce the mixing angle as

\[ \tan 2\xi = -\frac{2M_{ZZ}^2}{M_{Z}^2 - M_{Z}^2}. \] (91)

Finally the physical neutral gauge bosons \( Z \) and \( Z' \) can be written as

\[ Z = \tilde{Z} \cos \xi + \hat{Z} \sin \xi, \quad Z' = -\tilde{Z} \sin \xi + \hat{Z} \cos \xi. \] (92)

The field \( Z \) corresponds to the \( Z \) gauge boson in the standard model and the field \( Z' \) is a new field which is more massive than the \( Z \) particle. The corresponding mass eigenvalues are

\[ M_Z^2 = M_{Z}^2 \cos^2 \xi + M_{Z}^2 \sin^2 \xi + M_{ZZ}^2 \sin 2\xi, \]
\[ M_{Z'}^2 = M_{Z}^2 \sin^2 \xi + M_{Z}^2 \cos^2 \xi - M_{ZZ}^2 \sin 2\xi. \] (93)

In the limit \( v_R \gg v_L, |k|, |k'| \), these are approximately given by

\[ M_Z^2 \approx \frac{1}{2} \frac{g_L^2}{c_W^2} (K^2 + v_L^2) - \frac{g_L^2}{2c_W^2} (c_R^2 K^2 - s_R^2 v_L^2) v_R, \]
\[ M_{Z'}^2 \approx \frac{1}{2} (g_1^2 + g_R^2) v_R^2 + \frac{1}{2} (g_R^2 c_R^2 K^2 + g_1^2 s_R^2 v_L^2). \] (94)

Note that with a fine tuning \( K = t_R v_L \), the mixing term \( M_{ZZ}^2 \) becomes zero, hence \( \xi = 0 \). This is one of the cases we consider in constraining the remaining parameters.

In summary, the physical mass eigenstates \( A, Z \) and \( Z' \) fields can be written as a linear combination of the gauge eigenstates \( W_{L3}, W_{R3} \) and \( B \). Since the \( 3 \times 3 \) mass-squared matrix is a real, symmetric matrix, we need an orthogonal matrix to diagonalize it. In the prescription described above, we find three mixing angles which are Euler angles to parameterize the orthogonal matrix. The physical fields can be written as

\[ A = s_W W_{L3} + c_W s_R W_{R3} + c_W c_R B, \]
\begin{align}
Z &= c_W c_\xi W_{L3} + (c_R s_\xi - s_W s_R c_\xi) W_{R3} - (s_W c_R c_\xi + s_R s_\xi) B, \\
Z' &= -c_W s_\xi W_{L3} + (c_R c_\xi + s_W s_R s_\xi) W_{R3} + (s_W c_R s_\xi - s_R c_\xi) B,
\end{align}

(95)

where \(c_\xi = \cos \xi\) and \(s_\xi = \sin \xi\).

It is useful to verify the result by taking the limit \(g_L = g_R\). In this case \(\sin \theta_R = \tan \theta_W\) and \(\cos \theta_R = \sqrt{\cos^2 \theta_W / \cos \theta_W}\). Using these relations, and taking the limit \(\xi \to 0\), Eq. (95) becomes

\begin{align}
A &= s_W(W_{L3} + W_{R3}) + \sqrt{\cos 2\theta_W} B, \\
Z &\approx c_W W_{L3} - s_W t_W W_{R3} - t_W \sqrt{\cos 2\theta_W} B, \\
Z' &\approx \frac{\sqrt{\cos 2\theta_W}}{c_W} W_{R3} - t_W B.
\end{align}

(96)

This coincides with the symmetric result of Ref. [9]

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