Scaling Properties of Optokinetic Nystagmus Amplitude Sequence

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ABSTRACT

Optokinetic nystagmus (OKN) is rhythmic eye movements, back and forth, with a slow and fast phase when the eyes are presented for full-field visual stimulus. OKN was recorded in a healthy subject for four conditions, stripes moving 30°/s left and right and 60°/s left and right. In this paper, surrogate data analysis was applied to test the Hurst exponents for increasing time horizons for the integrated OKN amplitude sequences statistically against 100 shuffled sequences (i.e., the serial dependency of the original data sequences is broken by changing the order of the sequence). The result shows that the pattern of the OKN amplitude sequence scales statistically different (p < 0.01) compared to random permutations of the same numbers for scaling shorter than 16 nystagmus components (slow and fast phases) for all four test conditions.

1. INTRODUCTION

When looking for identifiers for reduced functionality of a physiological system, we need to distinguish information pattern from random variations. The aim of this study was to investigate the possible presence of information pattern in, and the memory length of, the OKN amplitude sequence. Surrogate data analysis—using the Hurst exponent for increasing scaling length, as the statistical parameter—was applied to determine for how long, in terms of nystagmus phases, the statistical analysis showed significant differences between the integrated OKN amplitude sequence compared to the shuffled data, at 1% level.

1.1. Optokinetic Nystagmus

When presented with a moving image, the eyes respond with a movement in the same direction as the image, interrupted by quick resetting phases [1-3]. These reflexive, rhythmic eye movements, which are called optokinetic nystagmus (OKN), interact with the vestibulo-ocular reflex and the smooth pursuit...
function to hold objects steady on the retina. The optokinetic nystagmus amplitude (OKNA) sequence is the subsequent slow and fast phases of OKN amplitudes.

1.2. Hurst Exponent

The scaling coefficient $H$, the Hurst exponent, is a measure of memory of a time series with the three properties [4-8].

- A time-integrated white noise time series (i.e., a random walk series) has a Hurst exponent of $H = 0.5$.
- A Hurst exponent that is higher than 0.5 ($H > 0.5$) corresponds to a process which is trending (persistent), also known as positive autocorrelation.
- A Hurst exponent that is lower than 0.5 ($H < 0.5$) corresponds to a mean reverting process (anti-persistent) (i.e., negative autocorrelation).

1.3. Surrogate Data Testing

Surrogate data testing is a statistical technique that can be used to test if a sequence of data differs significantly from a random sequence; for example, a random permutation of the same data/numbers. New data sequences are generated from the original sequence by shuffling, randomly changing the order of the sequence (the same numbers but where the serial dependency is broken [9, 10]). A parameter is calculated from the original data sequence and tested statistically against the distribution of the parameters from the shuffled sequences. The null hypothesis $H_0$: Data$_{original} = \text{mean Data}_{surrogates}$, that the original data sequence belongs to the same distribution as the surrogates can then be rejected at, for example, 1% probability level.

2. MATERIAL AND METHODS

2.1. Subject

OKN was recorded from a healthy subject for four conditions: stripes moving 30°/s left and right and 60°/s left and right.

2.2. Recording Technique

Horizontal eye movements were recorded with two electrodes (Ag-AgCl skin electrodes), which were placed lateral to each eye, along with a reference electrode at the center of the forehead. The signal was amplified (10 s time constant and an upper cut-off frequency of 30 Hz) and digitized into a computer using 12 bit A/D resolution and 100 Hz sampling frequency (sampling time $\tau_s = 0.01$).

2.3. Optokinetic Stimulation and Registration

OKN was obtained by stimulating the visual field with 3.75° width vertical light stripes separated by 11.25° width dark stripes. A slit projector presented the stripes on the inside of a hemispherical screen (100 cm in diameter). The subjects were sitting in front of a screen in a darkened room with the head restrained. The subjects were instructed to not follow the stripes with the eyes but to focus their vision on the screen, allowing the optokinetic reflex to control the eye movements. Recordings were performed with the movement of the stripes at a velocity of 30°/s and at 60°/s, which are below and above the normal threshold for smooth pursuit function [11, 12]. Each recording lasted for 1 min. Figure 1 shows a 1 s recording of OKN.

2.4. Methods and Analysis

Analyzing methods for distinguishing random behavior from long-term correlation/memory in time series have been described (Mandelbrot and Van Ness [13]). It has been shown that the mean square
displacement exhibit scaling laws proportional to $\Delta t^{2H}$ [14].

$$\langle \Delta x^2 \rangle = \langle (x_i - x_{i-\Delta t})^2 \rangle - \Delta t^{2H}$$  \hspace{1cm} (1)

$\langle \Delta x^2 \rangle$ is the mean square displacement, $\Delta t$ is the time interval and $H$ is the Hurst exponent. If data are independent, i.e. no memory, the displacement will increase with the square root of time and $H = 1/2$.

### 2.5. The Algorithm

The algorithm was applied to the integrated—the cumulative sum—of the nystagmus amplitude sequence series, $A_i$, adjusted for the mean.

$$\{x\}_i = \sum_{i=1}^{n} (A_i - \text{mean } A_i)$$  \hspace{1cm} (2)

where

$$\text{mean } A_i = \sum_{i=1}^{n} \frac{A_i}{n}$$ \hspace{1cm} (3)

The time series, $x_i$, represents the integrated OKNA sequence.

First, we calculated the mean square displacement $\langle \Delta x^2 \rangle$ in measure of number of nystagmus phases.

$$\langle \Delta x^2 \rangle_k = \left\{ \sum_{i=1}^{k+n} (x_{i+k} - x_i)^2 \right\}_{k=1}^{\beta=54}$$ \hspace{1cm} (4)

Then, from the scaling properties, we found the slope, $s_j$.

$$s_j = \frac{1}{2} \log \left( \frac{\langle \Delta x^2 \rangle_k}{\log (k)} \right)$$ \hspace{1cm} (5)

The scaling, the Hurst exponents for 50 time scales, was then calculated using the method of least square to fit straight lines for 1 to $n$ ($n = 4$ to 54) nystagmus phases.

### 2.6. Statistical Hypothesis Testing - Surrogate Data Analysis

The Hurst exponents for increasing time horizons for the integrated original OKNA sequences were statistically tested against 100 shuffled sequences (i.e., the serial dependency of the original data sequences is broken by changing the order of the sequence) [9, 10].

The statistical $Z$ score is

$$Z = \frac{H_{\text{orig}} - \text{mean } H_{\text{surr}}}{\text{SD } H_{\text{surr}}}$$ \hspace{1cm} (6)
where $H_{\text{orig}}$ is the Hurst exponent of the original integrated OKNA sequence, mean $H_{\text{surr}}$ is the mean value for the 100 surrogates

$$\text{mean } H_{\text{surr}} = \frac{1}{n} \sum_{i=1}^{n} H_{\text{surr}}^i$$  

(7)

and SD $H_{\text{surr}}$ is the standard deviation of the distributed Hurst exponents from the 100 surrogate data sets.

$$\text{SD } H_{\text{surr}} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (H - \text{mean } H_{\text{surr}})^2}$$  

(8)

3. RESULTS

The Hurst exponent parameters for the 50 time scales, $H_{\text{orig}}$, were tested against the 50 time scales of the 100 shuffled surrogates. The null hypothesis $H_0$: $H_{\text{orig}} = \text{mean } H_{\text{surr}}$ is rejected when $Z > 2.576$, which states that $H_{\text{orig}}$ is outside the 99% confidence interval of the shuffled distribution ($p < 0.01$) (Figure 2).

Surrogate data analysis shows that the pattern of the OKNA sequence analyzed in this study is statistically different ($p < 0.01$) compared to the random permutations of the same numbers for scaling shorter than 16 nystagmus phases for left 30°/s, <53 phases for right 30°/s, <38 phases for left 60°/s and <35 phases for right 60°/s.

4. DISCUSSION

Earlier studies [3, 15] have discussed the inter-relation between the fast and slow phases of nystagmus. One study found a lower correlation between the amplitude of the slow phase and the following fast phase compared to the correlation between the fast phase and the following slow phase, which was interpreted as the eye is not determined by the previous slow phase but is free to move voluntarily in order to focus on an object of interest [15]. This study has focused on the sequence of following slow and fast phases and found that the physiological regulating system keeps memory beyond subsequences of nystagmus beats. The pattern of information and memory is essential when looking for identifiers for reduced functionality of a physiological system.

Figure 2. The Hurst exponent parameter, $H_{\text{orig}}$, tested against 100 shuffled surrogates. The null hypothesis $H_0$: $H_{\text{orig}} = \text{mean } H_{\text{surr}}$ is rejected when $Z > 2.576$, which states that $H$ is outside the 99% confidence interval of the shuffled distribution ($p < 0.01$).
5. CONCLUSION

Testing the displacement of the integrated OKNA sequences from passive (subcortical) stare full-field OKN stimulation for four conditions, stripes moving 30°/s left and right and 60°/s left and right from a healthy subject as a function of time span in measure of number of nystagmus beats/phases, shows that the amplitude of the fast and slow nystagmus phases scales statistically different (p = 0.01) compared to random permutations of the same numbers for scaling shorter than 16 nystagmus components for all four test conditions. It will be interesting to see if this will be affected for conditions related to vertigo symptoms.

CONFLICTS OF INTEREST

The author declares no conflicts of interest regarding the publication of this paper.

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