Incoherent excitation of few-level multi-atom ensembles

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1. Introduction

The collective behaviour of quantum systems on interaction with a common radiation field was first studied by Dicke [1]. Since then, the studies regarding the collective interactions of an ensemble of few-level emitters with and via an environmental incoherent reservoir and pumped by external coherent or incoherent sources of electromagnetic light have been carried out intensively [1–6]. The investigation of such interactions has interested the scientific community because of the interesting effects and the possible application in the field of quantum communication and quantum information. The manipulation of collective fluorescence of an atomic sample via classical coherent fields and a heat bath has been shown in [7]. Further, it has been shown that the photon scattering by a collection of few-level emitters in incoherent environments leads to violation of the Cauchy–Schwarz inequality [8]. Thus, it can be realized that, contrary to general intuition, quantum features can be obtained from the interaction of quantum systems with a classical electromagnetic field (EMF) reservoir. This opens up a lot of possible applications in quantum information science, for example, entanglement between two arbitrary qubits has been shown to be generated when they interact with a common thermal bath [9–12]. Recently, other interesting developments like, stationary entanglement at high temperatures for two coupled, parametrically driven, dissipative harmonic oscillators [13] and room-temperature steady-state optomechanical entanglement on a chip [14], have been shown. Disentanglement versus decoherence of two qubits in thermal noise was investigated as well [15].

With such a motivation, in this paper, we investigate the quantum behaviour of a collection of three-level ladder emitters surrounded by an incoherent reservoir. We find that the steady-state distribution of radiators on the energy levels is not affected by the presence of the cross-damping terms caused by the interference of transition amplitudes. The collective effects drive the system into a final thermal steady state which is other than Boltzmann equilibrium distribution. The photon statistics changes from super-Poissonian to sub-Poissonian depending on the number of atoms in the sample, temperature and the mutual orientation of the induced dipole moments. In particular, we analysed the steady-state intensity and the normalized second-order correlation function for the light generated on the lower atomic transition. We found that a maximum in the steady-state intensity occurs for moderately large atomic samples with orthogonal transition dipoles. The physics behind such a behaviour is that the steady-state intensity of the emitted photons for the lower transition has a maximum due to the amplified spontaneous emission by the incoherent applied fields.
and the photon statistics is similar to that of a two-level ensemble.

The paper is organized as follows. In section 2, we consider the system of interest and obtain the exact steady-state solution of the master equation that describes the system. Using the solution, we arrive at the distribution of the emitters on the atomic states. Section 3 investigates the photon statistics of the spontaneously emitted photons as a function of the number of atoms, bath characteristics and orientation of atomic dipoles. The results are summarized in section 4.

2. Master equation and its exact steady-state solution

The basic element of our investigation is a sample of $N$ identical non-overlapping three-level ladder emitters that interacts with an environmental incoherent reservoir like thermal bath or broad-band incoherent lasers. The radiating atoms are located within a volume with less linear dimension compared to the relevant emission wavelengths $\{\lambda_{12}, \lambda_{23}\}$ (Dicke model). However, the obtained results apply to extended atomic samples as well where one atomic dimension is much larger than the relevant emission wavelength [16]. The incoherent reservoir induces transitions between the atomic levels with rates proportional to the mean incoherent photon number at corresponding atomic transitions. The excited atomic level $|1\rangle$ $(|2\rangle$) spontaneously decays to the state $|2\rangle$ $(|3\rangle$) due to the zero point fluctuation of the EMF, with a decay rate $2\gamma_1(2\gamma_2)$.

In the usual mean field, Born–Markov, dipole and rotating wave approximations, the interaction of the atomic sample with the surrounding incoherent bath is described by the master equation [2]

$$\frac{d}{df} \rho(t) + i[\omega_{12}S_{11} - \omega_{23}S_{33}, \rho] = -(1 + \tilde{n}_1)[S_{12}, \gamma_1(S_{21} + \gamma_2S_{12})\rho]$$
$$- (1 + \tilde{n}_2)[S_{23}, \gamma_2(S_{32} + \gamma_1S_{23})\rho]$$
$$- \tilde{n}_1[S_{21}, (\gamma_1S_{12} + \gamma_2S_{12})\rho]$$
$$- \tilde{n}_2[S_{12}, (\gamma_2S_{23} + \gamma_1S_{12})\rho] + \text{h.c.}$$, 

(1)

where the collective atomic operators $S_{\alpha\beta} = \sum_{j=1}^{N} a_{\alpha j}b_{\beta j}^{\dagger}$ describe the transitions between $|\alpha\rangle$ and $|\beta\rangle$ for $\alpha \neq \beta$ and populations for $\alpha = \beta$ and obey the commutation relation $[S_{\alpha\beta}, S_{\gamma\delta}^{\dagger}] = \delta_{\alpha\gamma}S_{\beta\delta} - \delta_{\beta\gamma}S_{\alpha\delta}$. Here, $\tilde{n}_1$ are the mean photon numbers that represent the intensity of incoherent pumping. For thermal bath, the mean thermal photon number is given by

$$\tilde{n}_1 = \frac{1}{\exp(h\omega_{i+1}) - 1},$$

where, $\omega_{i+1} = \omega_i - \omega_{i+1}$, $[i = 1, 2]$ and $\beta = (k_B T)^{-1}$ where $k_B$ is the Boltzmann constant and $T$ is the temperature of the bath. For incoherent pumping,

$$\tilde{n}_1 = \frac{R_{i+1}d_{i+1}^2}{\gamma_i^2/\hbar^2},$$

where $R_{i+1}$ describes the strength of the incoherent pumping. Furthermore, $2\gamma_1(2\gamma_2) = 4d_{12}^2/\omega_{12}(3\hbar c^2)$ is the single-atom natural line width. $\gamma_1 = \frac{\omega_{11}}{\omega_{12}}\cos \theta$ and $\gamma_2 = \frac{\omega_{22}}{\omega_{23}}\cos \theta$, with $\theta$ being the angle between the dipole moments $\vec{d}_{12}$ and $\vec{d}_{23}$, describe the interference (cross-damping) effects among the atomic transitions $|1\rangle \leftrightarrow |2\rangle$ and $|2\rangle \leftrightarrow |3\rangle$, and cannot be neglected for non-orthogonal dipole moments if $\Delta = |\alpha_2 - \alpha_3| \leq T_{\text{eff}} = N\gamma_1(1 + \tilde{n}_1(2)).$

The steady-state solution of the master equation, equation (1), is given by the relation

$$\rho_s = Z^{-1}e^{-\xi_1 S_{11} - \xi_2 S_{22}},$$

(2)

where for $\gamma_2 = \gamma_1 = 0$ (i.e. when $\vec{d}_{12} \perp \vec{d}_{23}$),

$$\xi_1 = \ln \left[1 + \frac{\tilde{n}_1}{\tilde{n}_1} \right],$$
$$\xi_2 = \ln \left[1 + \frac{\tilde{n}_2}{\tilde{n}_2} \right]$$

while for $\tilde{n}_1 = \tilde{n}_2 = \bar{n}$ and $\gamma_2 = \gamma_1 \neq 0,$

$$\xi_1 = -\xi_2 \equiv \bar{\xi} = \ln \left[1 + \frac{\bar{n}}{\bar{n}} \right].$$

Here $Z$ is chosen such that $\text{Tr} (\rho_s) = 1$. It is interesting to emphasize that the exact solution, i.e. equation (2), has a diagonal form while the master equation itself, equation (1), has non-diagonal terms which arise due to the two transitions $|1\rangle \rightarrow |2\rangle$ and $|2\rangle \rightarrow |3\rangle$ coupling. The solution of the master equation (1) was obtained by the direct substitution of equation (2) in the steady-state form of equation (1), noting that $e^{\xi_1 S_{11}}e^{-\xi_1 S_{22}} = S_{11}e^{\xi_1}$ and $e^{S_{ij}}S_{ij}e^{-S_{ij}} = S_{ij}e^{\xi_1}.

The steady-state expectation values of the atomic variables of interest can be calculated using the expression for the steady-state solution of the master equation, i.e. equation (2), and by making use of the symmetrical collective states $|N, n, m\rangle$ corresponding to the SU(3) algebra [2, 4, 5, 7, 8]. The meaning of the symmetrical collective state $|N, n, m\rangle$ is such that $n$ atoms are considered to be in the bare state $|1\rangle$, $m$ atoms in the state $|2\rangle$ and $N - n$ atoms in the bare state $|3\rangle$, where $N \geq n \geq 0$, $N \geq m \geq 0$. Using the SU(3) eigenstate properties of the bare state atomic operators we can immediately arrive at the expression for $Z$ (see, also [17]), i.e.

$$Z(\xi_1, \xi_2) = \frac{e^{-\bar{\xi}N}}{1 - e^{-\bar{\xi}}}[f(\xi_1 - \bar{\xi}) - e^{\bar{\xi}(1+N)}f(\xi_1)].$$

(3)

Here,

$$f(\xi) = 1 - \frac{1 - e^{\xi}}{1 - e^{-\xi}}.$$

Both the collective steady-state populations on the bare atomic states and their mutual correlations can be calculated from the relations

$$\langle S_{11}(\xi_1, \xi_3) \rangle = (-1)^{k_1+k_2}Z^{-1}\frac{\partial^{k_1+k_2}}{\partial \xi_1^{k_1}\partial \xi_3^{k_2}}Z(\xi_1, \xi_3),$$

(4)

with $|S_{22}\rangle \equiv N - \langle S_{11}\rangle - \langle S_{33}\rangle$, $\langle k_1, k_2 = 0, 1, 2, \ldots \rangle$. In particular, for $N = 1$, one obtains

$$\langle S_{11}\rangle = \frac{\tilde{n}_1}{n_2(1 + \tilde{n}_1)},$$
$$\langle S_{22}\rangle = \frac{\tilde{n}_2}{n_2(1 + \tilde{n}_2)},$$
$$\langle S_{33}\rangle = \frac{\tilde{n}_1}{n_1(1 + \tilde{n}_1)}.$$  

(5)
and it can be observed that their corresponding ratios are in accordance with the equilibrium Boltzmann distribution.

In general, for any $N$, the population distribution of a collection of atoms is given by

$$\langle S_{11} \rangle_s = \frac{\eta_1^{-N}}{Z(\xi_1, \xi_3)(\eta_1 - 1)} \times \left[ \frac{(N+1)(\eta_1 \eta_2)^{N+1} - N(\eta_1 \eta_2)^{N+2} - \eta_1 \eta_2}{(1 - \eta_1 \eta_2)^2} \right],$$

$$\langle S_{33} \rangle_s = \frac{\eta_2^{-N}}{Z(\xi_1, \xi_3)(\eta_2 - 1)^2} \times \left[ \frac{N + (\eta_1 \eta_2)^{N+2} - \eta_1 \eta_2(1 - (N + 2) \eta_2 + N - (N + 1) \eta_2)}{(1 - \eta_1 \eta_2)^2} \right],$$

with $(\langle S_{22} \rangle_s = N - \langle S_{11} \rangle_s - \langle S_{33} \rangle_s$ and $\eta_i = \bar{n}_i/[1 + \bar{n}_i], [i = 1, 2].$ In this case, for larger $N$, the collective interaction between the atoms drives the system into a thermal steady state away from a Boltzmann distribution [8].

Now, we consider the following limiting cases of the applied incoherent field and the size of the system.

(i) For a system with $\eta_1 = 0, \eta_2 \neq 0$, we find that

$$\langle S_{11} \rangle_s = 0,$$

$$\langle S_{33} \rangle_s = \frac{N - (N + 1) \eta_2 + \eta_2^{N+1}}{(1 - \eta_2)(1 - \eta_2^{-N+1})},$$

i.e. we recovered the well-known results for a two-level $(2 \leftrightarrow 3)$ atomic sample [3].

(ii) If $\eta_1 \neq 0, \eta_2 = 0$, or when there is no external incoherent pumping, $\eta_1 = \eta_2 = 0$, then $\langle S_{11} \rangle_s = 0$ and $\langle S_{33} \rangle_s = N.$

The system is entirely in the ground state.

(iii) For a weak incoherent bath $\eta_1 < 1$ and a large sample $N \gg 1$ such that $\{\eta_1 \eta_2 \eta_1^{N} \eta_1^{N+2}\} \rightarrow 0$, we obtain,

$$\langle S_{11} \rangle_s = \frac{\eta_1 \eta_2}{1 - \eta_1 \eta_2},$$

$$\langle S_{22} \rangle_s = \frac{\eta_2}{1 - \eta_2},$$

$$\langle S_{33} \rangle_s = N - \frac{\eta_2}{1 - \eta_2} - \frac{\eta_1 \eta_2}{1 - \eta_1 \eta_2}. \quad (7)$$

(iv) On application of a strong incoherent field ($\eta_1 \rightarrow 1$), we obtain,

$$\lim_{\eta_1, \eta_2 \rightarrow 1} \frac{\langle S_{11} \rangle_s}{N} = \lim_{\eta_1, \eta_2 \rightarrow 1} \frac{\langle S_{22} \rangle_s}{N} = \lim_{\eta_1, \eta_2 \rightarrow 1} \frac{\langle S_{33} \rangle_s}{N} = \frac{1}{3},$$

i.e. the atomic levels are equally populated (see figure 1).

When cross-damping effects are considered [18] then the corresponding expressions for the population distributions can be obtained with the help of equation (2) in the limit $\eta_1 = \eta_2 \equiv \eta = \bar{n}/[1 + \bar{n}].$ The mean value of the inversion operator $(S_t = S_{11} - S_{33})$, can be evaluated using the expression

$$\langle S_{11}^k \rangle_l = (-1)^k Z^{-1} \frac{\partial^k}{\partial \xi^k} Z(\xi), \quad \{ k = 1, 2, \ldots \}. \quad (8)$$

![Figure 1](image)
the positive and negative frequency parts of the amplitude of the EMF operator $E$ at the space-point $r$, and $\cdots \cdots$ means normal ordering. In the far-zone limit of experimental interest, i.e. $r = \infty$, $\rho_{12} \approx \lambda_{12}$, one can express the first- and second-order correlation functions via the collective atomic operators. Taking then the long-time limit of equation (10) and making use of equation (2) the steady-state coherence properties of the generated EMF will be investigated in the next subsections.

3.1. Photon statistics of distinguishable photons

Let us consider that the atomic transitions $|1\rangle \rightarrow |2\rangle$ and $|2\rangle \rightarrow |3\rangle$ have dipole moments orthogonal to each other ($d_{12} \perp d_{23}$). Then the emitted photons from the corresponding transitions can be distinguished by their polarizations and frequencies and can be detected by single- or two-photon detectors, respectively. For this case, the normalized second-order coherence function can be defined as follows:

$$g_{ij}^{(2)}(0) = \frac{\langle J_i^* J_j^* J_j J_i \rangle}{\langle J_i^* J_j^* J_j J_i \rangle}, \quad (i, j = 1, 2),$$

where for brevity we have set $J_1 = S_{21}$ and $J_2 = S_{32}$, and $\langle J_i^* J_j \rangle$ can be used to quantify the intensity of the emitted light from the transition $i$. The quantity $g_{ij}^{(2)}(0)$ can be interpreted as a measure of the probability of detecting one photon emitted in transition $i$ and another photon emitted in transition $j$ simultaneously and its value determines the nature of the emitted photons. $g_{ij}^{(2)}(0) < 1$ characterizes sub-Poissonian; $g_{ij}^{(2)}(0) > 1$, super-Poissonian; and $g_{ij}^{(2)}(0) = 1$, Poissonian photon statistics of the emitted EMF. Anti-correlation or correlation of the emitted light occurs when $g_{ij}^{(2)}(0)$ is smaller or larger than unity, respectively. To evaluate these atomic correlation functions, we can use the SU(3) eigenstate properties of the bare state atomic operators [2, 4, 5, 7, 8] and the exact steady-state solution, i.e. equation (2).

Firstly, we evaluate the fluorescent steady-state intensities of light emitted on $|1\rangle \rightarrow |2\rangle$ and $|2\rangle \rightarrow |3\rangle$, with the help of the following relations obtained from the master equation (1):

$$G_1^{(1)}(0) \propto \langle S_{12} S_{21} \rangle = \frac{n_1}{1 - n_1} \left( N - \langle S_{33} \rangle_S - 2 \langle S_{11} \rangle_S \right),$$

$$G_2^{(1)}(0) \propto \langle S_{23} S_{32} \rangle = \frac{n_2}{1 - n_2} \left( \langle S_{11} \rangle_S + 2 \langle S_{33} \rangle_S - N \right).$$

Equations (12) and (13) were obtained from the steady-state form of the corresponding equations for $\langle S_{11} \rangle$ and $\langle S_{33} \rangle$ using also the commutation relations $[S_{12}, S_{32}] = S_{11} - S_{22}$ and $[S_{23}, S_{32}] = S_{22} - S_{33}$ as well as the relation $\langle S_{11} \rangle + \langle S_{22} \rangle + \langle S_{33} \rangle = N$. Consider the following limiting cases for $G_1^{(1)}(0)$ and $G_2^{(1)}(0)$:

(i) if a weak incoherent field is applied to the sample, i.e. $\{n_1, n_2\} \rightarrow 0$, gives $G_1^{(1)}(0) = G_2^{(1)}(0) = 0$;

(ii) large samples, $N \gg 1$, with fixed $\{n_1, n_2\} < 1$, have

$$G_1^{(1)}(0) = \frac{n_1}{1 - n_1} \left[ \frac{n_2}{1 - n_2} - \frac{n_1 n_2}{1 - n_1 n_2} \right],$$

$$G_2^{(1)}(0) = \frac{n_2}{1 - n_2} \left[ N - \frac{2 n_2}{1 - n_2} - \frac{n_1 n_2}{1 - n_1 n_2} \right].$$

(iii) in the strong field limit $\{n_1, n_2\} \rightarrow 1$ and fixed $N$, we find that

$$G_1^{(1)}(0) = G_2^{(1)}(0) = \frac{N}{12} (3 + N).$$

One can observe here that for larger atomic systems and moderate strengths of incoherent excitation the first-order correlation function $G_1^{(1)}(0)$ does not depend on $N$ while $G_2^{(1)}(0)$ increases linearly with $N$, i.e. $G_2^{(1)}(0) \approx n_3 N$. In the limit of intense incoherent pumping the radiated fluorescence intensities in both atomic transitions scale as $N^2$, similar to the superradiance phenomenon [1, 2]. Figure 2 depicts these intensities as a function of the incoherent pumping strength. An interesting result here is that $G_2^{(1)}(0)/N^2$ shows a maximum for lower pumping intensities. From equation (13), $G_2^{(1)}(0)$ can be written as $\bar{\eta}_2 \rho_2(S_{33}) = \langle S_{22} \rangle$, the value of $G_2^{(1)}(0)$ increases with the pumping parameter $\eta$. After a certain value, the value of $G_2^{(1)}(0)$ decreases due to the rapidly decreasing nature of $\langle S_{33} \rangle_S - \langle S_{22} \rangle_S$, and hence exhibits a maximum due to amplified spontaneous emission by the external incoherent excitation.

We now shall investigate the coherence properties of the light emitted on $|2\rangle \rightarrow |3\rangle$ atomic transition. For $N = 2$, the coherence factor $g_{22}^{(2)}(0)$ changes from unity (coherent light) to values less than 1 (i.e. it exhibits sub-Poissonian photon statistics, see figure 3). Hence, the emitted light possesses quantum features. For a moderately large atomic system, the fluorescent field generated on this particular atomic transition has partial coherent properties because $g_{22}^{(2)}(0) < 2$. The light
statistics of a large sample behaves as follows. For a weak bath ($\eta < 1$) it is incoherent since $\lim_{N \to \infty} G^{(2)}_\eta(0) = 2$, showing the super-Poissonian statistics of photons, while for an intense incoherent reservoir ($\eta = 1$), it is partially coherent, since

$$\lim_{\eta \to 1} G^{(2)}_\eta(0) = \frac{8(N - 1)(N + 4)}{5N(3 + N)} \to \frac{8}{5}, \quad \text{when } N \gg 1.$$ 

It should be noted here that the minimum for the coherence factor $G^{(2)}_\eta(0)$, shown in figure 3, occurs near the value of $\eta$ for which $G^{(2)}_\eta(0)$ is maximum, leading to the emission of quasi-coherent light. Therefore, there occurs an enhancement of the multiparticle spontaneous emission corresponding to the maximum of $G^{(2)}_\eta(0)$ and quasi-coherent light emission corresponding to the minimum of $G^{(2)}_\eta(0)$ due to the surrounding incoherent reservoir and multi-level structure of the emitters in the system. The incoherent pumping scheme developed here for orthogonal dipoles can be useful in higher frequency domains due to the absence of good coherent sources. As can be seen from our results, one can obtain quasi-coherent light via incoherent pumping.

### 3.2. Photon statistics of indistinguishable photons

When decay interference effects are accounted, i.e. for near parallel dipoles ($\vec{d}_{\alpha} \parallel \vec{d}_{\beta}$), the second-order correlation function can be represented as follows:

$$g^{(2)}(\eta) = \frac{(\langle J_1^+ + J_2^+ \rangle^2)(J_1 + J_2)^2}{(\langle J_1^+ + J_2^+ \rangle)(J_1 + J_2)^2}. \quad (17)$$

It is emphasized here that due to the quantum decay interference the atomic transitions are indistinguishable. The correlation function, in this case, is detected by a two-photon detector. Equation (17) contains off-diagonal terms that cannot be represented in equation (17) via those atomic correlations that can be evaluated with the solution obtained in equation (2). However, we can represent the correlation functions entering equation (17) via those atomic correlations that can be evaluated with the steady-state solution in equation (2). Thus, using the master equation, equation (1), we can show that

$$G^{(2)}(\eta) \propto \langle J_1^+ + J_2^+ \rangle_J(J_1 + J_2)_s = -\frac{\eta}{1 - \eta} \langle S_z \rangle_s,$$

$$G^{(2)}(\eta) \propto \langle J_1^+ + J_2^+ \rangle_J^2(J_1 + J_2)_s^2 = \frac{\eta^2}{(1 - \eta)^2} \left[ 1 + \frac{N}{1 - \eta} \langle S_z \rangle_s + 2\langle S_z^2 \rangle_s \right]. \quad (18)$$

and, thus, $g^{(2)}(\eta)$ can be written as

$$g^{(2)}(\eta) = \left[ \frac{(1 + \eta)(1 - \eta)}{(1 - \eta)^2} \langle S_z \rangle_s + 2\langle S_z^2 \rangle_s \right] \frac{(\eta^2 N)^3}{(S_z^2)^3}, \quad (19)$$

with

$$\langle S_z \rangle_s = \frac{a(\eta, N)\eta^3 + b(\eta)\eta^{N+1} + c(\eta, N)}{(1 - \eta)(1 - \eta^2)(1 - \eta^{N+1})(1 - \eta^{N+2})} + \frac{2\eta(1 + 3\eta)^3}{1 - \eta^2} \langle S_z \rangle_s. \quad (20)$$

Here,

$$a(\eta, N) = (3 + N)^3 - (2 + N)(4 + N)\eta + N(6 + N)\eta^3 - (1 + N)(5 + N)\eta^2,$$

$$b(\eta) = 1 + 4\eta - 8\eta^3 - 5\eta^4,$$

$$c(\eta, N) = N^2 - (N^2 - 1)\eta - (N^2 - 4)\eta^2 + (N^2 - 9)\eta^3.$$
for fixed $\eta < 1$ and large samples, $N \gg 1$, one obtains
\begin{equation}
G^{(2)}(0) \approx 2(nN)^2,
\end{equation}
and, thus, $g^{(2)}(0)$ shows super-Poissonian photon statistics since in this case $\lim_{N \to \infty} g^{(2)}(0) = 2$ (see equations (21) and (24)). Partial coherence features occur for $\eta = 1$ and $N \gg 1$ because $\lim_{N \to \infty} g^{(2)}(0) = 8/5$ (see equation (23)).

The second-order correlation function for a three-level system with near parallel dipoles behaves similar to that for a two-level sample [3]. This can be seen also by introducing new atomic operators, i.e. $S^+ = \sqrt{2}(S_{23} + S_{12}), S^- = \sqrt{2}(S_{23} - S_{12})$ and $S_z = S_{11} - S_{13}$ obeying the commutation relations for SU(2) algebra: $[S^+, S^-] = 2S_z$ and $[S_z, S^\pm] = \pm S^\pm$. For equal decay rates, the master equation (1) can be represented via new operators as follows:
\begin{equation}
\frac{d}{dt} \rho = -\frac{1}{2}(1 + n)[S^+ \rho, S^-] - \frac{1}{2}n[S^+, S^+ \rho] + \text{h.c.},
\end{equation}
and looks like the master equation describing two-level atoms [3]. Hence, there is no amplified steady-state spontaneous emission for such a system.

4. Summary

The interaction of an ensemble of ladder-type emitters with an environmental incoherent reservoir is investigated. The steady-state solution of the master equation and steady-state population distributions for the system are obtained and it is shown that collective effects force the system away from the Boltzmann-like thermodynamic equilibrium for systems with more than one atom. Particularly, the ground-state emitters obey the Bose–Einstein statistics. We analysed the photon statistics of the emitted light under different conditions. The emitted EMF in the case of one- or two-atom sample emitting distinguishable photons, or a single-atom emitting indistinguishable photons exhibit quantum properties. In case of atoms, emitting distinguishable photons, for larger samples, amplified steady-state spontaneous emission of quasi-coherent light occurs. Therefore, the investigated model can be useful in higher frequency domains as a source of quasi-coherent light. Furthermore, the first- and second-order coherence functions do not exhibit any critical behaviours, i.e. discontinuities or abrupt changes proper to phase transition phenomena. Finally, the steady-state expectation values of any atomic variables of interest do not depend on spontaneous decay rates.

Rydberg atoms possessing almost equidistant energy levels and embedded inside a cavity with a low quality factor are suitable candidates to test some of the results described here [24]. With suitable cavity parameters one can avoid the difficulties connected with the condition, $d_{12} \parallel d_{23}$ [25].

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