Chiral Bargmann-Wigner Equations for Spin-1 Massive Fields

T. B Watson and Z. E. Musielak
Department of Physics, University of Texas at Arlington, Arlington, TX 76019, USA
E-mail: timothy.watson@mavs.uta.edu; zmusielak@uta.edu

Abstract. The Bargman-Wigner equations are generalized to include chiral symmetry based on the irreps of the Poincaré group, and the chiral Bargmann-Wigner equations are derived for spin-1 massive fields. By specifying the chiral basis, the chiral Bargmann-Wigner equations are reduced to the Proca-like equation, which is coupled by chirality to an auxiliary equation for spin-0 massive field. The coupling is a new phenomenon whose physical implications for the Higgs field and dark matter are discussed.

1. Introduction

In the Bargmann-Wigner formalism [1], all possible relativistic equations are derived by using the unitary representations of the Poincaré group classified by Wigner [2,3]. A field of rest mass $m$ and spin $s \leq 1/2$ is described by a symmetric multispinor for which the Bargmann-Wigner (BW) equations are derived [1]. For special cases of $s = 1/2$, $s = 1$ and $s = 3/2$, the BW equations reduce to the Dirac [4], Proca [5] and Rarita-Schwinger [6] equations, respectively; however, for $s = 2$, see [7]. The BW equations are coupled first-order partial differential equation with the original Dirac operator $(\hat{D} = i\gamma^\mu \partial_\mu - m)$ acting on multispinor wavefunctions.

To generalize the Dirac equation, different modifications were made to the Dirac operator. Physical reasons for these modifications range from attempts to unify leptons and quarks [8-10] and account for the three families of elementary particles [11-15], to including ad hoc a pseudoscalar mass [16] and extending the Dirac equation to distances comparable to the Planck length [17]. The main procedures giving such generalizations have been identified as the Takuoka-Sen-Gupta [18-20], Weinberg-Tucker-Hammer [21-23] and Barut [11,12] formalisms [24]. Since each formalism leads to different forms of the Dirac equations and their operator $\hat{D}$, the corresponding modifications of the BW equations have also been accomplished [25].

Recently, we derived the Dirac equation with chiral symmetry (DECS) by using the irreducible representations (irreps) of the Poincaré group $\mathcal{P} = SO(3, 1) \otimes_s T(3+1)$, with $SO(3, 1)$ being a non-invariant Lorentz group of rotations and boosts and $T(3+1)$
Chiral Bargmann-Wigner Equations for Spin-1 Massive Fields

an invariant subgroup of spacetime translations, and the group including reversal of parity and time [3]. The derived DECS [26] can be written

\((i\gamma^\mu \partial_\mu - m e^{-2i\alpha\gamma^5})\psi = 0\),

where \(\alpha\) is the chiral angle and \(\psi\) represents a four-component spinor that transforms as one of the irreps of \(T(3 + 1) \in \mathcal{P}\) extended by parity, and each of its components satisfies the Klein-Gordon equation [27,28]. We demonstrated that the chiral rotation of a massive field is equivalent to an alternative choice of chiral basis, and that the Dirac equation is obtained by factorization of the Klein-Gordon equation if, and only if, a specific choice of chiral basis is selected [26].

The nonstandard factorizations of the Klein-Gordon equation resulting from different choices of chiral bases redistribute the mass available to the field between the left- and right-chiral components, which allows identifying the mass term in the DECS with pseudo-scalar mass. The presence of this pseudo-scalar mass results in pseudo-scalar Higgs that can be used to explain smallness of neutrino masses and also properties dark matter particles [26].

In this Letter, we extend the DECS beyond spin-1/2 particles and generalize the BW equations to include chiral symmetry. Our method to derive the chiral Bargmann-Wigner (CBW) equations is based on the irreps of the Poincaré group \(\mathcal{P}\). The derived CBW equations are valid for spin-1 massive fields. By specifying the chiral basis, the CBW equations reduce to the Proca-like equation [5] that is coupled to an auxiliary equation for spin-0 massive field is obtained. The resulting coupling is caused by chirality and it is a new phenomenon, whose physical implications are discussed.

2. Bargmann-Wigner equations with chiral symmetry

We may extend the usefulness of DECS beyond spin-1/2 particles by using multispinors considered in the Bargmann-Wigner formalism [1]. We begin by observing that the at-rest solutions of Eq. (1) may be written as

\(\psi^{(\pm)} = \omega^{(\pm)}(\alpha)e^{\mp imt}\),

where \((+) \in \{(1), (2)\}\) indicate positive energy solutions and \((-) \in \{(3), (4)\}\) indicate negative energies. Then, these spinors take the form

\[
\begin{align*}
\omega^{(1)}(\alpha) & = \begin{bmatrix} \cos \alpha & 0 \\ i \sin \alpha & 0 \end{bmatrix}, & \omega^{(2)}(\alpha) & = \begin{bmatrix} 0 & \cos \alpha \\ i \sin \alpha & 0 \end{bmatrix}, \\
\omega^{(3)}(\alpha) & = \begin{bmatrix} i \sin \alpha & 0 \\ \cos \alpha & 0 \end{bmatrix}, & \omega^{(4)}(\alpha) & = \begin{bmatrix} 0 & i \sin \alpha \\ \cos \alpha & 0 \end{bmatrix},
\end{align*}
\]

(3)
It is easily to verify that spinors $\omega^{(1)}$ and $\omega^{(3)}$ are $\frac{1}{2}$ eigenstates of the spin projection operator $\hat{S}^3$ and $\omega^{(2)}$ and $\omega^{(4)}$ are similarly $-\frac{1}{2}$ eigenstates. Moreover, all of these states have a definite propagation mass.

We may now define the general set of positive energy multispinors of spin-1 from the tensor product of the spinors of spin-1/2 as follows

$$\omega^{(1,1)}(\alpha, \beta) = \omega^{(1)}(\alpha) \otimes \omega^{(1)}(\beta),$$

$$\omega^{(1,2)}(\alpha, \beta) = \omega^{(2,1)}(\alpha, \beta) = \omega^{(1)}(\alpha) \otimes \omega^{(2)}(\beta) + \omega^{(2)}(\alpha) \otimes \omega^{(1)}(\beta),$$

and

$$\omega^{(2,2)}(\alpha, \beta) = \omega^{(2)}(\alpha) \otimes \omega^{(2)}(\beta),$$

where $\beta$ is the chiral angle associated with the chiral basis of the second bispinor of our representation. It must be noted that different chiral angles can be paired together in a single spinor as they are all valid eigenstates of momentum and spin as can be seen by observing these multispinors are eigenstates of the following spin operator (with indices included for clarity)

$$(\hat{S}^3)_{\mu^\nu} = (\hat{S}^3)_\mu^\nu \delta_{\nu}^\mu + (\hat{S}^3)_{\nu}^\mu \delta_{\mu}^\nu,$$

and satisfy their respective at-rest DECS

$$(\gamma^0 - e^{-2i\alpha})_{\mu}^{\nu \omega^{(+,+)}(\alpha, \beta) = 0},$$

and

$$(\gamma^0 - e^{-2i\beta})_{\nu}^{\mu \omega^{(+,+)}(\alpha, \beta) = 0},$$

for $(+, +) \in \{(1, 1), (1, 2), (2, 2)\}$. We may then move from the rest frame to an arbitrary momentum state by boosting each spinor

$$\omega^{(+,+)}_{\mu \nu}(\alpha, \beta; p^\mu) = \Lambda(\alpha, p^\mu)^\mu_{\mu'} \Lambda(\beta, p^\nu)^\nu_{\nu'} \omega^{(+,+)}_{\mu' \nu'}(\alpha, \beta).$$

We note that the form of $\Lambda(\alpha, p^\mu)$ differs non-trivially from the form when $\alpha = 0$. We omit a more detailed discussion of this form as, for our purposes, the existence of such a transformation is sufficient. We next define the symmetric and anti-symmetric positive energy multispinors and find

$$\Omega^{(+,+)}_{\mu \nu}(\alpha, \beta; p^\mu) = \frac{1}{2}(\omega^{(+,+)}_{\mu \nu}(\alpha, \beta; p^\mu) + \omega^{(+,+)}_{\nu \mu}(\beta, \alpha; p^\mu)), $$

and

$$\tilde{\Omega}^{(+,+)}_{\mu \nu}(\alpha, \beta; p^\mu) = \frac{1}{2}(\omega^{(+,+)}_{\mu \nu}(\alpha, \beta; p^\mu) - \omega^{(+,+)}_{\nu \mu}(\beta, \alpha; p^\mu)), $$

such that the full symmetric and anti-symmetric positive energy solutions are given by

$$\psi^{(+)}_{\mu \nu}(\alpha, \beta; x^\mu) = \sum_{(+, +)} \int C^{(+,+)}_{1}(p^\mu) \Omega^{(+,+)}_{\mu \nu}(\alpha, \beta; p^\mu) e^{-ip_\mu x^\mu} d^3p, $$

and

$$\tilde{\psi}^{(+)}_{\mu \nu}(\alpha, \beta; x^\mu) = \sum_{(+, +)} \int C^{(+,+)}_{2}(p^\mu) \tilde{\Omega}^{(+,+)}_{\mu \nu}(\alpha, \beta; p^\mu) e^{-ip_\mu x^\mu} d^3p, $$
Chiral Bargmann-Wigner Equations for Spin-1 Massive Fields

where the sum is over \((+,+) \in \{(1, 1), (1, 2), (2, 2)\}\).

Similar arguments for negative energy solutions lead to the construction of the negative energy multispinors. Notationally this is achieved via a simple index substitution \((+,+) \rightarrow (-,-) \in \{(3,3),(3,4),(4,4)\}\) and flipping the sign of the exponential. With this complete set of states, we may identify the most general symmetric and antisymmetric multispinors

\[
\Psi_{\mu\nu}(\alpha, \beta; x^\mu) = a_1 \psi^{(+)}_{\mu\nu}(\alpha, \beta, x^\mu) + b_1 \psi^{(-)}_{\mu\nu}(\alpha, \beta, x^\mu),
\]

and

\[
\tilde{\Psi}_{\mu\nu}(\alpha, \beta; x^\mu) = a_2 \tilde{\psi}^{(+)}_{\mu\nu}(\alpha, \beta, x^\mu) + b_2 \tilde{\psi}^{(-)}_{\mu\nu}(\alpha, \beta, x^\mu)
\]

that satisfy the following four coupled equations

\[
[i\gamma^\mu \partial_\mu - m \cos(\alpha - \beta)e^{-i(\alpha+\beta)\gamma^5/\mu/\nu}] \Psi_{\mu\nu}(\alpha, \beta; x^\mu) = [-im \sin(\alpha - \beta)\gamma^5 e^{-i(\alpha+\beta)\gamma^5/\mu/\nu}] \tilde{\Psi}_{\mu\nu}(\alpha, \beta; x^\mu),
\]

and

\[
[i\gamma^\mu \partial_\mu - m \cos(\alpha - \beta)e^{-i(\alpha+\beta)\gamma^5/\mu/\nu}] \tilde{\Psi}_{\mu\nu}(\alpha, \beta; x^\mu) = [-im \sin(\alpha - \beta)\gamma^5 e^{-i(\alpha+\beta)\gamma^5/\mu/\nu}] \Psi_{\mu\nu}(\alpha, \beta; x^\mu),
\]

where the summed indices have been combined for brevity.

Isolating either multispinor yields

\[
[\partial^\mu \partial_\mu + m^2]_{\mu\nu} \Psi_{\mu\nu}(\alpha, \beta; x^\mu) = 0,
\]

and

\[
[\partial^\mu \partial_\mu + m^2]_{\mu\nu} \tilde{\Psi}_{\mu\nu}(\alpha, \beta; x^\mu) = 0,
\]

which demonstrates that each element of our multispinors satisfy the Klein-Gordon equation. These are new BW equations with chiral symmetry (CBW equations) for spin-1 massive fields, and they reduce to the BW equations when \(\alpha = \beta\) in which case \(\tilde{\Psi}\) vanishes. This shows that chiral symmetry requires additional equation for \(\tilde{\Psi}\). The effects of this additional equation on the Proca equation [5] are now considered and discussed.

3. New insights into the Proca Equation

The CBW equations allows us to identify representations of spin-1 fields consistent with those degrees of freedom observed in the DECS. Using [29], we relate these representations to those known in the particle physics. Let \(\hat{\mathcal{C}} = i\gamma^2\gamma^0\) and \(\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]\), then the Clifford basis is the following set of ten symmetric matrices \(\{\gamma^\mu\hat{\mathcal{C}}, \sigma^{\mu\nu}\hat{\mathcal{C}}\}\), and the six anti-symmetric matrices \(\{\gamma^\mu\gamma^5\hat{\mathcal{C}}, i\gamma^5\hat{\mathcal{C}}, \hat{\mathcal{C}}\}\) that allow expanding the multi-spinors in this basis, and obtain

\[
\Psi = mA_\sigma\gamma^\sigma\hat{\mathcal{C}} + \frac{1}{2}F_{\sigma\tau}\sigma^{\sigma\tau}\hat{\mathcal{C}},
\]
Chiral Bargmann-Wigner Equations for Spin-1 Massive Fields

and

\[ \Psi = \rho e^{-i\theta \gamma^5 \hat{C}} + m B_\sigma \gamma^\sigma \gamma^5 \hat{C}, \]  

(22)

where \( \rho \) is a scalar field, \( \theta \) is a scalar parameter, \( A^\mu \) and \( B^\mu \) are vector fields, and \( F^{\mu\nu} \) is an antisymmetric tensor.

Writing the CBW equations in matrix form, we find the four linearly independent combinations

\[ i\partial_\mu \left( \gamma^\mu \Psi + (\gamma^\mu)^T \Psi \right) - m \cos (\alpha - \beta) \left\{ \Psi, e^{-i(\alpha + \beta)\gamma^5} \right\} \]

\[ -im \sin (\alpha - \beta) \left[ \Psi, \gamma^5 e^{-i(\alpha + \beta)\gamma^5} \right] = 0, \]  

(23)

\[ i\partial_\mu \left( \gamma^\mu \tilde{\Psi} + (\gamma^\mu)^T \tilde{\Psi} \right) - m \cos (\alpha - \beta) \left\{ \tilde{\Psi}, e^{-i(\alpha + \beta)\gamma^5} \right\} \]

\[ -im \sin (\alpha - \beta) \left[ \tilde{\Psi}, \gamma^5 e^{-i(\alpha + \beta)\gamma^5} \right] = 0, \]  

(24)

\[ i\partial_\mu \left( \gamma^\mu \Psi - \Psi(\gamma^\mu)^T \right) + m \cos (\alpha - \beta) \left\{ \Psi, e^{-i(\alpha + \beta)\gamma^5} \right\} \]

\[ +im \sin (\alpha - \beta) \left\{ \tilde{\Psi}, \gamma^5 e^{-i(\alpha + \beta)\gamma^5} \right\} = 0, \]  

(25)

and

\[ i\partial_\mu \left( \gamma^\mu \tilde{\Psi} - \tilde{\Psi}(\gamma^\mu)^T \right) + m \cos (\alpha - \beta) \left\{ \tilde{\Psi}, e^{-i(\alpha + \beta)\gamma^5} \right\} \]

\[ +im \sin (\alpha - \beta) \left\{ \Psi, \gamma^5 e^{-i(\alpha + \beta)\gamma^5} \right\} = 0. \]  

(26)

Then, substituting the matrix expansions of the multispinors and exploiting the linear independence of the basis matrices, we obtain our constraints. Among these are the requirement that the fields \( \rho \) and \( B_\mu \) vanish and \( \theta = 0 \) unless we enforce \( \theta = \pi/2 - \alpha - \beta \). With this condition, we define the rotated fields

\[ \begin{bmatrix} A'_\mu \\ iB'_\mu \end{bmatrix} = \begin{bmatrix} \cos (\alpha - \beta) & \sin (\alpha - \beta) \\ -\sin (\alpha - \beta) & \cos (\alpha - \beta) \end{bmatrix} \begin{bmatrix} A_\mu \\ iB_\mu \end{bmatrix} \]  

(27)

and summarize the set of constraint equations as

\[ F_{\sigma\tau} \cos (\alpha + \beta) + \frac{1}{2} \partial^\rho F^{\mu\nu} \epsilon_{\mu\nu\sigma\tau} \sin (\alpha + \beta) - (\partial_\sigma A'_\tau - \partial_\tau A'_\sigma) = 0, \]

\[ \partial_\rho B'_\tau - \partial_\tau B'_\rho = 0, \]

\[ \partial^\sigma F_{\sigma\tau} + m^2 A'_\tau \cos (\alpha + \beta) = 0, \]

\[ \frac{1}{2} \partial^\rho F^{\mu\nu} \epsilon_{\mu\nu\sigma\tau} + m^2 A'_\tau \sin (\alpha + \beta) = 0, \]

\[ \partial^\mu A'_\mu + i\rho \sin (2\alpha - 2\beta) = 0, \]

\[ \partial^\mu B'_\mu + \rho \cos (2\alpha - 2\beta) = 0, \]

\[ \partial_\sigma \rho - m^2 B'_\sigma = 0. \]  

(28)
Taking the divergence of the first of these constraints and eliminating explicit dependence on $F_{\mu\nu}$ and $B'_\mu$, we find the basic equations governing the fields reduce to three coupled equations of $A'_\mu$ and $\rho$:

\begin{align}
\partial^{\nu}(\partial_{\nu}A'_\mu - \partial_{\mu}A'_\nu) + m^2 A'_\mu &= 0 , \\
\left[\partial^{\mu}\partial_{\mu} + m^2 \cos(2\alpha - 2\beta)\right] \rho &= 0 , \\
\partial^{\mu}A'_\mu + i\rho \sin (2\alpha - 2\beta) &= 0 .
\end{align}

(29)

It is evident that the vanishing of $\rho$ is equivalent to the reduction of these equations to the Proca equation [5].

A more symmetric form of these equations is obtained by the definition of the constant $\kappa \equiv \sqrt{m^2 \cos 2(\alpha - \beta) - \mu^2}$ (where $\mu$ appears as a free parameter), and the scalar field $\varphi$ defined in terms of the divergence of the vector field $\varphi \equiv \kappa^{-1}\partial^{\mu}A'_\mu$. The constraint equations then reduce to

\begin{align}
\left[\partial^{\nu}\partial_{\nu} + m^2\right] A'_\mu &= +\kappa \partial_{\mu}\varphi , \\
\left[\partial^{\nu}\partial_{\nu} + \mu^2\right] \varphi &= -\kappa \partial^{\mu}A'_\mu ,
\end{align}

(30)

where the first equation is the Proca-like equation and the second is the auxiliary equation for $\varphi$. Note that $m$ and $\mu$ are generally not equal and correspond to the masses of the vector and scalar fields respectively. In the Lorentz gauge $\partial^{\mu}A'_\mu = 0$, the scalar wavefunction $\varphi = 0$ and the Proca-like equation becomes the Proca equation [5]. The coupling between the spin-1 and spin-0 massive fields is a new phenomena whose physical implications are now discussed.

4. Physical implications

The two main results of this Letter are the chiral Bargmann-Wigner (CBW) equations for spin-1 massive fields and the Proca-like equation with its required auxiliary equation for spin-0 massive fields. There are several physical implications of these results.

The degrees of freedom introduced by the choice of chiral basis allowed by Poincaré invariance admit an asymmetry to the defined representations of the considered spin-1 massive fields described by the multispinors, thereby allowing for generalization of the BW equations to include chiral symmetry. As a result two CBW equations are obtained, one for $\Psi$ and the other for $\tilde{\Psi}$, and they are reduced to the original BW equations when $\tilde{\Psi} = 0$, which is equivalent to the case when chirality is neglected. Thus, the main physical implication of the CBW equations is the presence of the additional field described by the multispinor $\tilde{\Psi}$.

To explore this additional field in detail, we specified the chiral basis in such a way that the CBW equations reduce to the Proca-like equation that is coupled to a spin-0 massive field. We demonstrated that the asymmetry of the defined representations manifests itself physically as the coupled scalar and vector fields, with the total spin being consistent with our choice of representations. By committing to a specific chiral basis one fixes the coupling between these fields and so restricts the system to the same
total number of degrees of freedom as in the case when the chiral bases coincide. The coupling is described by the coefficient $\kappa$ that depends on the chiral angles $\alpha$ and $\beta$ as well as on masses $m$ and $\mu$ of the scalar and vector fields, respectively.

The coupling between the scalar and vector fields caused by the presence of chiral symmetry is a new phenomenon reported in this Letter. While the fundamental spin-1 massive fields describing bosons $W^\pm$ and $Z^0$ are well-known in the Standard Model (SM) [29], the only fundamental scalar field in the model is the Higgs field $[30,31]$, with a strong evidence for a elementary massive particle of spin-0 and positive parity $[32]$. Therefore, let us identify the scalar field coupled to the vector field is the Higgs field and explore the physical implications which follow from this supposition. Since our results show that the physical properties of the scalar field are such that its wavefunction $\varphi$ is proportional to the divergence of the vector field, with $\kappa$ being the proportionality coefficient representing chirality, this implies that the Higgs field must be related to the divergence of the vector wavefunction and that chirality of spin-1 massive elementary particles place the dominant role in this relationship. To determine the validity of this statement further theoretical studies and potential experimental verifications are necessary but both out the scope of this Letter.

Another possibility is the existence of a scalar massive field representing the currently unexplained dark matter (DM) $[33,34]$ and its physical properties that are likely to be described by a spin-0 massive elementary particle $[35,36]$, whose existence has not yet been verified experimentally $[37,38]$. The presence of such DM field and its possible coupling to spin-1 massive fields of ordinary matter (OM) through chirality would allow both OM and DM to be coupled. Moreover, as recently shown, in the nonrelativistic limit, DM may have both scalar $[39]$ and vector $[40]$ components that could be coupled by chirality. Further studies of these interesting phenomena are necessary and they will be described elsewhere.

Finally, let us point out that the form of the dependence of the coupling $\kappa$ on the chiral angles $\alpha$ and $\beta$ illustrates an important result of our derivation that we postulate holds in all physical systems, namely that only differences in chiral bases are experimentally observable.

5. Conclusions

The original Bargman-Wigner equations are generalized by taking into account chiral symmetry for spin-1 massive fields. The generalization is based on the irreps of the Poincaré group. By specified the chiral bases, the derived chiral Bargmann-Wigner equations are reduced to the Proca-like equation, which is shown to be coupled by chirality to an auxiliary equation for spin-0 massive field. The physical implications of this coupling are discussed in the context of the scalar field to be either the Higgs field or a scalar massive field describing dark matter. In both cases, new and interesting results are likely to be obtained after more detailed investigations are performed.
6. References

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