Instance Analysis for the Error of Three-pivot Pressure Transducer Static Balancing Method for Hydraulic Turbine Runner

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Abstract. The working principle, process device and test procedure of runner static balancing test method by weighting with three-pivot pressure transducers are introduced in this paper. Based on an actual instance of a V hydraulic turbine runner, the error and sensitivity of the three-pivot pressure transducer static balancing method are analysed. Suggestions about improving the accuracy and the application of the method are also proposed.

1. Introduction
Static balancing test should be accomplished to turbine runners during shop tests after a new runner has been assembled or an old runner has been repaired by repair welding according to relative rules[1,2]. Presently, four methods are mainly applied to the static balancing test of hydraulic turbine runner, including the methods of balancing with steel ball on plate, balancing on hydro static bearing[3], balancing with measuring rod and balancing with three-pivot pressure transducers[4]. References [3] and [5] detailedly analysed the error and sensitivity of the methods of balancing with steel ball on plate and balancing on hydro static bearing respectively. Additionally, reference [3] analysed the method of balancing with measuring rod and raised doubt about its actual reliability. However, researches about the error and sensitivity analyses for the method of balancing with three-pivot pressure transducers for hydraulic turbine runner are still rare. In this circumstances, combining an actual instance of a V hydraulic turbine runner, the error and sensitivity analyses of the three-pivot pressure transducer static balancing method are conducted in this paper.

2. Working principle of runner static balancing method by weighting with three-pivot pressure transducers
2.1. Working principle
The three-pivot pressure transducer static balancing method is based on the principle of plane moment balance. As seen in Figure 1, suppose that the forces at the three vertexes of an equilateral triangle are $F_A$, $F_B$ and $F_C$ respectively, which are normal to the paper plane and inward. On the other hand, there exists an unknown force $F$ in the plane(referring to the weight deviation), which is normal to the paper plane and outward. In this case, for the plane to remain static without rotation, the moments of the four forces about x and y axes are both 0. Assuming $F$ is in the first quadrant, equation (1) comes into existence.

$$
\begin{align*}
F_X + F_C \cos 30^\circ - F_B \cos 30^\circ &= 0 \\
F_Y - F_A + (F_B + F_C)\sin 30^\circ &= 0
\end{align*}
$$

(1)

By virtue of equation (1), $F$ and $\alpha$(the weight deviation angle) can be obtained.
Then the unbalanced moment \( M \) can be calculated by

\[
M = F \times R
\]

(3)

Where \( R \) is the distribution circle radius of three pressure transducers.

\[
F = \left( F_x^2 + F_y^2 \right)^{1/2}
\]

\[
\alpha = \arctan \left( \frac{F_x}{F_y} \right)
\]

(2)

2.2. The process device and test procedure

Take the Francis turbine runner as an example, the common test process device of the three-pivot pressure transducer static balancing method is shown in Figure 2.

The steps of the test procedure are as follows:

① Three pressure transducers are disposed on the balance plate according to Figure 1.

② The runner is placed according to Figure 2. The runner weight is born by the four jacks firstly. As the runner stabilized, the jacks lower slowly and the runner weight is born by the three transducers finally.

③ According to Figure 1, \( F_A, F_B \) and \( F_C \) can be read from the three transducers. Furthermore, the unbalanced moment \( M \) and its position can be calculated from equations (1)-(3).

④ The weight balancing for the runner is conducted by virtue of the calculation results in step ③ (including rough counter weight and fine counter weight).

3. Parameters of runner and process device

3.1. The main parameters of \( V \) runner

The runner weight: \( m = 178982 \text{ kg} \); 

The rotational speed of the turbine: \( n = 125 \text{ rpm} \); 

The static balancing has to meet the requirements defined by ISO 1940/1 Grade G6.3 and the corresponding allowable residual unbalanced moment \( E_r \leq 86.14 \text{ kg·m} \).

3.2. The main parameters of the process device

The balance plate of the process device is located by applying the spigot at the water outlet end of runner crown. The sizes of the spigot and the balance plate, as well as their fit tolerance, are required to be 1380H7/g6. The concentricity of the spigot(\( \delta_1 \)) is 0.05mm. The balance plate weight(\( m_b \)) is 2000kg. The levelness(\( \delta_3 \)) is required to be 0.05mm/m and the installation accuracy should meet the requirement of the eccentricity(\( \delta_2 \)) being less than 0.05mm.

The height between the theoretical gravity center of the runner and the mounting plane of pressure transducers(\( h \)) is 791mm.
The measurement range, non-linearity error and sensitivity of each pressure transducer are 100T, 0.02%FS and 2.85mV/V respectively. The distribution circle radius of three pressure transducers(R) is 690mm and the resolution of the corresponding weight display instrument is 0.01uV/V.

4. Error analysis
It can be seen from the working principle and process of the three-pivot pressure transducer static balancing method analysed in section 2, errors are mainly caused by the following factors:

4.1. The accuracy error caused by the pressure transducer $E_1$
As the non-linearity error of each pressure transducer is 0.02%FS, the accumulative error of the three transducers can be regarded as the repeated measurement error[6], which equals to $\sqrt{0.02\%^2 + 0.02\%^2 + 0.02\%^2} = 0.0245\%$ FS. Consequently, the weight measurement error $\Delta m=100T\times 0.0245\%=24.5kg$. In addition, the distribution circle radius of three pressure transducers $R$ is 690mm. Hence, the unbalanced moment error can be calculated as: $E_1 = \Delta m \times R \times 10^{-3} = 16.90kg\cdot m$.

4.2. The error caused by the deviation between the gravity center of the runner and the distribution circle center of three pressure transducers $E_2$
As mentioned in section 3.2, the concentricity of the spigot at the water outlet end of runner crown($\delta_1$) is 0.05mm and the eccentricity of the balance plate($\delta_2$) is less than 0.05mm according to the installation accuracy requirement. In this case, the maximum deviation between the theoretical gravity center of the runner and the distribution circle center of three pressure transducers $\mu_1=0.075mm$, which is calculated according to $\mu_1=\delta_2+\delta_1/2$, as seen in Figure 3.
As a result, the unbalanced moment error can be calculated as: $E_2 = m \times \mu_1 \times 10^{-3} = 13.42kg\cdot m$.

![Figure 3. Calculation schematic diagram for the error caused by the deviation between the gravity center of the runner and the distribution circle center of three pressure transducers.](image)

4.3. The error caused by the levelness of the runner $E_3$
By virtue of Figure 4, $\mu_2 = h \times \tan \alpha$ according to the plane geometry knowledge and $\tan \alpha = \delta_1$ (the levelness of the runner, which is 0.05mm/m). In this case, the unbalanced moment error caused by the levelness of the runner can be calculated as: $E_3 = m \times \mu_2 \times 10^{-3} = m \times h \times \delta_1 \times 10^{-3} = 7.08kg\cdot m$.

4.4. The error caused by the levelness of the balancing plate $E_4$
As illustrated in Figure 5, $\mu_4 = R - R \cos \beta = R(1 - \cos \beta)$ and $\tan \beta = \delta_3$ (the levelness of the balancing plate, which is 0.05mm/m). As $1 - \cos \beta = 2 \sin^2 \beta/2$, when $\beta$ is quite small, $\sin(\beta/2) \approx \tan(\beta/2) = \tan \beta/2 = \delta_3/2$ and hence $\mu_4 = R \times \delta_3^2/2$. In this case, the unbalanced moment error caused by the levelness of the balancing plate can be calculated as: $E_4 = m_b \times \mu_4 \times 10^{-3} = m_b \times R \times \delta_3^2/2 \times 10^{-3} = 1.725 \times 10^4 kg\cdot m$. 
In conclusion, the total error of the balancing system can be calculated by summing the above four errors: $E_5 = E_1 + E_2 + E_3 + E_4 = 37.4 \text{ kg} \cdot \text{m}$. And the ratio of $E_5$ to the allowable residual unbalanced moment $E_r$ is: $E_5 / E_r = 37.4 / 86.14 \times 100\% = 43.42\%$.

5. Sensitivity analysis

For the steel ball on plate based method, the sensitivity of the static balancing test system is represented by the vertical distance between the gravity center of the runner and the center of the balancing plate. It is calculated from the deflection of the runner band when a weight block with standard mass is placed to the runner band\(^5\).

For the three-pivot pressure transducer static balancing method, as the contact between the transducers and the runner is rigid, a weight block placed at any position of the runner will not result in the descend of the runner band. Therefore, the sensitivity calculation method mentioned in reference [5] is not applicable to the three-pivot pressure transducer static balancing method.

The runner manufacturer pointed out that, the minimum unbalanced moment of the runner $G_{\text{min}}$ can be calculated by the resolution of the pressure transducers and the digital weight display, and the sensitivity of the method can be represented by $G_{\text{min}}$. Presetting an unbalanced moment for the runner after the final test, $G_{\text{min}}$ can be verified by comparing the theoretical value and the actual measured value of this unbalanced moment.

The minimum weight $W_{\text{min}}$ that can be accurately measured and displayed by the digital weight display is 1kg. The runner manufacturer pointed out that the display is unstable in unit digits in actual application due to the environment factors, but is stable in tens digits. Therefore, $W_{\text{min}}$ equals to 10kg. Assuming the unbalanced mass is located at the point with the longest distance from the center line of the runner, $G_{\text{min}}$ can be calculated as: $G_{\text{min}} = W_{\text{min}} \times R = 10 \times 690 \times 10^{-3} = 6.90 \text{ kg} \cdot \text{m}$.

A 4kg weight block was placed at the point with the distance of 1.7m from the center line on the runner assembling joint after the final balancing test. The theoretical value of the unbalance moment is 6.80kg·m(4kg×1.7m) which is close to $G_{\text{min}}$ and the measured value is 7.59kg·m. The error ratio between the theoretical value and the actual measured value of the unbalance moment equals to $(7.59 - 6.8) / 6.8 \times 100\% = 11.62\%$, which is less than the ratio of the system error to the allowable residual unbalanced moment(43.42%). Consequently, the three-pivot pressure transducer static balancing method can basically recognize the $G_{\text{min}}$ of 6.90kg·m.

6. Conclusion and discussion

6.1. Judgment of test results

Considering that the system error of the three-pivot pressure transducer static balancing method is considerably large, the final residual unbalanced moment after fine balancing $E_r$ is supposed to be less than the difference between $Er$ and $E_5$ (satisfying inequality (4)) instead of just less than $Er$ during site acceptance tests, Where $Er$ is the allowable residual unbalanced moment according to the contract and $E_5$ is the system error based on the theoretical analyses in section 4.
The test result is $E_f = 39.8 \text{ kg} \cdot \text{m}$, which is less than $E_r - E_s = 86.14 - 37.4 = 48.74 \text{ kg} \cdot \text{m}$. Consequently, inequality (4) is satisfied and the static balancing test of the V runner is considered to pass the acceptance.

### 6.2. Further improvement of measurement accuracy

On the basis of above analyses, the test system error of the three-pivot pressure transducer static balancing method mainly results from the machining and installation errors of the process device, as well as the transducer error. The accuracy of the transducer applied in the test system has already reached 0.02%FS and is difficult to be further improved. However, in the respect of machining accuracy, the related requirement for the levelness according to Chinese standards\(^{[1,5]}\) is 0.02mm/m, while the requirement of the manufacturer for the levelness is 0.05mm/m. The concentricity deviation between the process device and the runner is required to be 0.07mm according to Chinese standards\(^{[1,5]}\), while 0.075mm according to the requirement of the manufacturer. Obviously, the requirements of the manufacturer are below the Chinese national standards. Therefore, the machining and installation accuracies of the manufacturer have some room for improvement and the measurement accuracy of the test system is hopefully to be improved.

### 6.3. Discussion

The principle of the three-pivot pressure transducer static balancing method discussed in this paper is similar to that of the method of balancing with measuring rod mentioned in reference [3]. By virtue of reference [3], the system error of the measuring rod based method may reach 30-40% of the allowable residual unbalanced moment, which is basically in accordance with the result of the three-pivot pressure transducer static balancing method analysed in this paper. Moreover, similar to the doubt about the actual reliability of the measuring rod based method raised in reference [3], the actual reliability of the three-pivot pressure transducer static balancing method is supposed to be investigated. Although, the three-pivot pressure transducer static balancing method has been repeatedly adopted by several hydraulic turbine manufacturers\(^{[4]}\), the relevant international standard\(^{[7]}\) has not covered this method in the static balancing parts yet. This method should be researched and further improved by the hydraulic turbine industry and manufacturers. In this case, the relevant standards can be developed.

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