Excitation of the Dissipationless Higgs Mode in a Fermionic Condensate

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The amplitude mode of a fermionic superfluid, analogous to the Higgs Boson, becomes undamped in the strong coupling regime when its frequency is pushed inside the BCS energy gap. We argue that this is the case in cold gases due to the energy dispersion and nonlocality of the pairing interaction, and propose to use the Feshbach resonance regime for parametric excitation of this mode. The results presented for the BCS pairing dynamics indicate that even weak dispersion suppresses dephasing and gives rise to persistent oscillations. The frequency of oscillations extracted from our simulation of the BCS dynamics agrees with the prediction of the many-body theory.

The observation of resonance superfluidity in cold atomic Fermi gases\textsuperscript{1,2} at magnetically tunable Feshbach resonances\textsuperscript{3} opened new avenue of exploring the many-body phenomena. Similar to the earlier work on cold Bose gases which triggered studies of fascinating collective phenomena\textsuperscript{7,8,9,10,11,12}, fermionic pairing at Feshbach resonances\textsuperscript{7,8,9,10,11,12} presents new opportunities. In particular, the high degree of coherence of trapped atoms, and the possibility to control particle interaction \textit{in situ} on the shortest collective time-scale, the inverse Fermi energy\textsuperscript{4}, can facilitate exploring new regimes which are difficult to realize in solid state systems.

The theory of fermionic pairing predicts two principal collective modes intrinsic to the condensed state. One is the massless Bogoliubov-Anderson mode related to the order parameter phase dynamics. Being a Goldstone mode, it manifests itself in hydrodynamics in the same way as in Bose systems, and was probed recently in the experiments on gas expansion and oscillation in traps\textsuperscript{13,14,15,16}. In addition, there exists a second fundamental elementary excitation\textsuperscript{17,18,19}, related to the dynamics of the order parameter modulus $|\Delta|$. Notably, this excitation is unique to fermionic pairing and has no counterpart in Bose systems\textsuperscript{19}. This massive excitation, characterized by a finite frequency, is analogous to the Higgs Boson in particle physics. Like the latter it remained elusive, for a long time evading direct probes, although some indirect manifestations have been discussed\textsuperscript{20,21}. The main obstacle to the detection of the Higgs mode in superconductors is that it is essentially decoupled from the phase mode responsible for hydrodynamics and superfluidity.

In this work we propose to use the dynamical control of pairing interaction demonstrated in Refs.\textsuperscript{1,2} for parametric excitation of the Higgs mode. We argue that fermion superfluidity in the strong coupling regime realized near Feshbach resonance represents a distinct advantage, since in this case the Higgs mode is pushed inside the superconducting gap, $\hbar \omega < 2\Delta$, which eliminates damping due to coupling to quasiparticles. We demonstrate that this mode can be excited by a time-dependent pairing interaction, as illustrated in Fig. 1. In contrast, the BCS theory at weak coupling predicts the Higgs mode frequency right at the edge of the quasiparticle continuum, $\hbar \omega = 2\Delta\textsuperscript{19}$, which leads to collisionless damping of this mode\textsuperscript{22,23}.

The departure from the behavior at weak coupling arises from the change in the character of pairing interaction in the strong coupling regime, in particular due
to its finite spatial radius and frequency dispersion. Spatial nonlocality of pairing interaction is known to lead to discrete collective modes inside the BCS gap [17, 18]. Similarly, the energy dispersion of the pairing interaction and pairing amplitude $\Delta_p$ that becomes prominent at strong coupling [23], leads to discrete collective mode spectrum (see below). While the exact form of this dispersion is sensitive to the specifics of the strong coupling problem, it is established in the literature that, generally, both effects can occur near Feshbach resonance [8, 10, 11, 12, 24]. Although our understanding of the detailed microscopic picture may be hampered by the nonperturbative nature of the strong coupling problem, we shall see that within a simplified model used below the inequality $\hbar \omega < 2\Delta$ is fulfilled under very general conditions.

The dissipationless BCS dynamics [22, 25] and the possibility to realize it in cold gases [26] attracted much attention recently [24, 27, 28]. These investigations, with the exception of Ref. [24], focused on the case of pairing interaction which is constant in the entire fermion energy band, concluding [29, 30] that several interesting dynamical states, synchronized and desynchronized (or dephased), can be realized by a sudden change in the interaction strength (see the phase diagram in Ref. [29]).

In contrast, as we shall see below, the dephased behavior is suppressed in the strong coupling regime when due to the energy dispersion of the pairing interaction the Higgs mode falls inside the BCS energy gap. Under these conditions an undamped Higgs mode can be excited upon a sudden change in interaction. By analyzing the limit when the interaction dispersion disappears we show how the different regimes of Ref. [29] are recovered. This correspondence suggests an interpretation of the dephased oscillations discussed in Refs. [22, 29, 30] as a manifestation of the Higgs mode, algebraically dephased at $\hbar \omega = 2\Delta$.

We shall analyze the pairing dynamics in a spatially uniform system using the pseudospin representation [25] of the BCS problem in which spin $1/2$ operators $s_p^x = s_p^x + i s_p^y$ describe Cooper pairs $(p, -p)$:

$$\mathcal{H} = - \sum_p 2\epsilon_p s_p^z - \sum_{pq} \lambda_{pq}(t) s_p^- s_q^+,$$  \hspace{1cm} (1)

where $\epsilon_p$ is the free particle spectrum. The interaction $\lambda_{pq}(t)$ that models the energy dispersion at strong coupling is taken in the form of a sum of a dispersing and nondispersing parts

$$\lambda_{pq}(t) = \frac{g(t)}{\nu_F} (a_1 + a_2 f_p f_q), \quad f_p = \frac{\gamma}{\sqrt{\gamma^2 + \epsilon_p^2}},$$  \hspace{1cm} (2)

where the dimensionless parameter $g(t)$ specifies the interaction time-dependence, the constants $a_{1,2} \geq 0$ satisfy $a_1 + a_2 = 1$, and $\nu_F$ is the density of states at the Fermi level. The second term in (2) features dispersion on the energy scale $\gamma$. Our motivation for choosing the model (2) was two-fold. Firstly, the form (2) is general enough to provide insight into the role of different features, such as the energy dispersion (which is controlled by the parameter $\gamma$) and separability (which is absent unless $a_1$ or $a_2$ vanishes). Secondly, our numerical method utilized the rank two form of (2), allowing for substantial speedup that could not be implemented for a more general interaction $\lambda_{pq}$. In addition, the model (2) is physically motivated by the theory of BCS pairing in the simultaneous presence of a retarded and non-retarded interaction [31].

Within the mean-field approximation, the dynamical equations derived from Eq. (1) assume a Bloch form:

$$\frac{dr_p}{dt} = 2b_p \times r_p, \quad b_p = - (\Delta^x_p, \Delta^y_p, \epsilon_p),$$  \hspace{1cm} (3)

where $r_p = 2\langle s_p \rangle$ are Bloch vectors, and the effective magnetic field $b_p$ depends on the pairing amplitude $\Delta_p$. The latter is defined self-consistently:

$$\Delta_p = \Delta^x_p + i \Delta^y_p = \sum_q \frac{\lambda_{pq}(t)}{2} r_{q}^+, \quad r_{p}^+ = r_p^x + i r_p^y.$$  \hspace{1cm} (4)

The interaction time dependence of interest is a step-like change from the initial value $g_i$ to the final value $g$. Without loss of generality, the phase of the order parameter can be chosen equal to zero, allowing us to consider only the $x$-component of the pairing amplitude, $\Delta_p = \Delta^x_p$. As an initial state we take the paired ground state

$$r_p^x(0) = \frac{\Delta_i^x}{\sqrt{(\Delta_i^x)^2 + \epsilon_p^2}}, \quad r_p^z(0) = \frac{\epsilon_p}{\sqrt{(\Delta_i^x)^2 + \epsilon_p^2}}.$$  \hspace{1cm} (5)

The equilibrium energy-dependent amplitude $\Delta_p$ is determined by the self-consistency equation

$$\Delta_p = \frac{1}{2} \sum_q \frac{\lambda_{pq}}{\sqrt{\epsilon_q^2 + \Delta_q^2}}.$$  \hspace{1cm} (6)

in which $\lambda_{pq}$ is given by (2) with the parameter values $g_i$ and $g$ for the initial and final state. The corresponding equilibrium pairing gap values, $\Delta_i^x$ and $\Delta_f$, are found by numerically solving the integral equation (6). Throughout the paper we use the equilibrium value of the pairing gap at the Fermi level, $\Delta_F$, at the final coupling $g$ as a natural energy scale to parameterize the dynamics.

We integrate Eqs. (3) using the Runge-Kutta method of the 4-th order with a time step adjusted to achieve sufficient precision of the calculation. In our simulation we use $N = 10^4, 10^5$ equally spaced energy states within bandwidth $W$, $-W/2 < \epsilon_p < W/2$, with the level spacing much smaller than all other energy scales in the problem.

We analyze the quantity $\Delta_F(t)$ which at long times oscillates between the maximum and minimum values
The energy-independent interaction. Should the dephasing occur, the asymptotic values would coincide, \( \Delta_+ = \Delta_- \).

In contrast to the above, for the interaction (2) the dephased behavior is suppressed. Instead, as illustrated in Fig. 2 we observe non-decaying periodic oscillations for a wide range of initial states, both for the initial states close to the normal state \( (g_i \ll g) \) as well as for the initial states near equilibrium \( (g_i \approx g) \). At increasing \( g_i \) there is a critical point at which the asymptotic pairing amplitude \( \Delta_\pm \) becomes zero.

To understand the origin of the oscillatory behavior for the dispersive interaction, Eq. (2), we develop perturbation theory near the point \( g_i = g \). Linearizing the Bloch equations and taking a harmonic variation of the pairing amplitude, \( \delta \Delta_p^{x,y}(t) \propto e^{-i\omega t}\delta \Delta_p^{x,y} \), we find two collective modes for the \( x \) and \( y \) components of \( \Delta_p \) corresponding to the order parameter amplitude and phase variation (see Ref. [19]). The amplitude (Higgs) mode with frequency \( \omega \) obeys the integral equation

\[
\delta \Delta_p^{x,y} = \frac{1}{2} \sum_q \frac{\lambda_{pq} \delta \Delta_p^{x,y}}{\sqrt{\epsilon_q^2 + \Delta_q^2}} \frac{\epsilon_q^2}{\sqrt{\epsilon_q^2 + \Delta_q^2 - \omega^2/4}},
\]

where \( \Delta_p \) is the equilibrium gap obtained from Eq. (6).

The equation for \( \delta \Delta_p^x \) (the phase mode) is similar to Eq. (7) except for the denominator of the second fraction which is \( \epsilon_q^2 - \omega^2/4 \). As expected from Goldstone theorem, the equation for \( \delta \Delta_p^y \) is solved by \( \omega = 0 \).

To find the frequency \( \omega \) of the Higgs mode, we note that for the interaction \( \lambda_{pq} \) given by (2), which is an operator of rank two, Eq. (7) turns into an algebraic equation involving a \( 2 \times 2 \) determinant. Solving it we find that for \( a_2 > 0 \) the frequency \( \omega \) lies within the BCS gap, as illustrated in Fig. 3. To gain more insight, let us consider a separable interaction, \( a_1 = 0 \), \( a_2 = 1 \), which yields

\[
1 = \frac{g}{24\nu} \sum_q \frac{f_q^2}{\sqrt{\epsilon_q^2 + \Delta_q^2}} \frac{\epsilon_q^2}{\sqrt{\epsilon_q^2 + \Delta_q^2 - \omega^2/4}},
\]

where \( \Delta_q \propto f_q \). Balancing the factors under the sum in order to obtain unity on the left hand side, and noting that without the second factor Eq. (8) would be identical to Eq. (6), it is easy to see that \( \omega < 2\Delta_F \), i.e. the Higgs mode is discrete.

Notably, as Fig. 3 illustrates, the frequency obtained from Eq. (7) coincides with the frequency of oscillations in \( \Delta_F(t) \) obtained by simulating BCS dynamics at \( g \approx g_i \), proving that the observed excitation is indeed the Higgs mode. Furthermore, for \( g \) away from \( g_i \) the frequency extracted from \( \Delta_F(t) \) varies with \( g \), decreasing below the value at \( g \approx g_i \) and approaching zero at \( g_i \ll g \) and \( g_i \gg g \) (see Fig. 3b). This indicates unharmonicity of the Higgs mode that sets on at a large amplitude of oscillations.

To test these ideas further, we considered the regime when the Higgs mode is strictly inside the quasiparticle continuum, which can be realized in the model (2) with

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**Figure Legends**

**FIG. 2:** Long-time behavior of \( \Delta_F(t) \), the pairing amplitude at the Fermi level, oscillating between \( \Delta_+ \) and \( \Delta_- \). Shown are two examples of \( \Delta_\pm \) as a function of the initial state for a non-separable (circles) and a separable (squares) interaction (2). Parameters used: \( a_1 = a_2 = 0.5 \), \( g = 0.43 \), \( \Delta_F/W = 0.016 \), \( \gamma/W = 0.01 \), and \( a_1 = 0 \), \( a_2 = 1 \), \( g = 0.61 \), \( \Delta_F/W = 0.005 \), \( \gamma/W = 0.01 \), respectively. Inset: Linear fit of a sample trace \( \Delta_F(t) \) vs. \( t^{-1/2} \) used to extract \( \Delta_\pm \).

**FIG. 3:** (a): The Higgs mode frequency obtained from the simulation with \( g_i \approx g \) (circles) and from Eq. (7) (solid line) as a function of the dispersion parameter \( \gamma \). The quasiparticle energy minimum (dashed line) lies above the collective mode frequency (parameters of the simulation: \( g = 0.61 \), \( a_1 = a_2 = 0.5 \)). (b): Frequency of the Higgs mode as a function of the initial state for non-separable and separable interactions with the same parameters as in Fig 1. The frequency changes away from \( g_i = g \) as the amplitude of oscillations increases (Fig 1), indicating unharmonicity of the Higgs mode.
the second term of a repulsive sign, $a_2 < 0$. In this case Eq.\([7]\) has no real-valued solution in the region $\omega \leq 2\Delta$. Simulating the BCS dynamics near $g_i \approx g$ we find that $\Delta(t)$ exhibits exponentially decaying oscillations of the form $e^{-\gamma t} \cos(\omega t + \phi)$ corresponding to a complex-valued frequency $\omega$. For $a_2 = 0$ the collective mode frequency $\omega = 2\Delta_F$ lies at the edge of the quasiparticle continuum. This property was linked to algebraic Landau damping of this mode in Refs.\([22,28]\).

The discrete Higgs mode makes the BCS dynamics undamped for $g$ near $g_i$ even for weakly dispersing interaction $\Delta_{pq}$. It is interesting to connect this behavior to the dephased BCS dynamics found in the case of constant interaction. This is illustrated (Fig.\(4\)) by the dynamics at weakly dispersing interaction $\gamma \gg \Delta_F$, where we observe that the region of dephased dynamics shrinks, with the onset of dephasing shifting towards small $g < g_i$. While the oscillation amplitude $\frac{1}{2}(\Delta_+ - \Delta_-)$ is now finite, it remains small due to dephasing in the transient region (see Fig.\(5b\)). This behavior is consistent with the Higgs mode approaching the quasiparticle continuum boundary.

In conclusion, we have shown that the energy dispersion of pairing interaction leads to quenching of dephasing of the BCS dynamics, making the Higgs mode of the pairing amplitude discrete. Parametric control of interaction in the strong coupling regime near a Feshbach resonance of cold atoms can be used to excite this mode.

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\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{dephasing.png}
\caption{Quenching of dephasing for weakly dispersing interaction. Asymptotic values of the pairing amplitude $\Delta_\pm$ for a dispersing (circles) and non-dispersing (red line) interaction for different initial states. The onset of dephasing is marked by arrows. Parameters used: $a_1 = a_2 = 0.5, g = 0.33, \Delta_\ell/W = 0.02, \gamma/W = 0.1$, and $a_1 = 1, a_2 = 0, g = 0.33, \Delta_\ell/W = 0.05$, respectively.}
\end{figure}

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