Bounding the flavor-violating $Hbs$ vertex from the $B \to X_s\gamma$ decay

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Abstract. The nondiagonal $Hbs$ coupling within the context of an effective Yukawa sector that comprises $SU_L(2) \times SU_Y(1)$-invariant operators of up to dimension six is studied. The recent experimental result on $B \to X_s\gamma$ with hard photons is employed to constrain the $Hbs$ vertex, with which the branching ratio for the $B_s \to \gamma\gamma$ decay is estimated. It is found that the $B_s \to \gamma\gamma$ decay can reach a branching ratio of the order of $4 \times 10^{-8}$.

1. Introduction

Suppressed observables such as $B \to X_s\gamma$, has been measured with good accuracy, showing no deviations from the standard model (SM) [1]. This means that this observable can provide stringent constraints on physics beyond the electroweak scale. We are interested in studying the flavor violating transitions $b \to s\gamma$ and $b \to s\gamma\gamma$ mediated by a SM-like Higgs boson within the context of extended Yukawa sectors that incorporates $SU_L(2) \times SU_Y(1)$ invariants of up to dimension six, which is enough to induce, in a model independent manner, the presence of flavor and CP violation. Our main goal is to use the experimental data on the $B \to X_s\gamma$ decay to constrain the flavor violating $Hbs$ vertex. Then we will use these results to predict the branching ratio for the $B_s \to \gamma\gamma$ transition.

2. The effective Yukawa sector

An effective Yukawa sector that generates flavor violating effects in the quark sector is:

$$
L_{eff}^Y = - Y_{ij}^d(Q_i \Phi d_j) - \frac{\alpha_{ij}^d}{\Lambda^2} (\Phi^\dagger \Phi)(Q_i \Phi d_j) - Y_{ij}^u(Q_i \Phi^\dagger u_j) - \frac{\alpha_{ij}^u}{\Lambda^2} (\Phi^\dagger \Phi)(Q_i \Phi^\dagger u_j) + H.c., \tag{1}
$$

where $Y_{ij}$, $Q_i$, $\Phi$, $d_i$ and $u_i$ stand for the components of the Yukawa matrix, the left-handed quark doublet, the Higgs doublet and the right-handed quark singlets of down and up type, respectively. The $\alpha_{ij}$ are the components of a $3 \times 3$ general matrix, which parametrize the details of the underlying physics and $\Lambda$ is the new physics scale.
After spontaneous symmetry breaking, in the unitary gauge, the diagonalized Lagrangian is:

\[
\mathcal{L}^Y_{\text{eff}} = -\left(1 + \frac{g}{2m_W}H\right)\left(D\mathcal{M}_aD + \bar{U}\mathcal{M}_aU\right) - H\left(1 + \frac{g}{4m_W}H\left(3 + \frac{g}{2m_W}H\right)\right)
\times \left(D\Omega^aP_RD + \bar{U}\Omega^aP_RU + H.c.\right),
\]

where the \(\mathcal{M}_a\) (\(a = d, u\)) are the diagonal mass matrix and \(D = (\bar{d}, \bar{s}, \bar{b})\) and \(U = (\bar{u}, \bar{c}, \bar{t})\) are vectors in the flavor space. The \(\Omega^a\) are matrices defined in the flavor space through the relation: \(\Omega^a = (1/\sqrt{2})(v/\Lambda)^2V_L^a\alpha^aV_R^a\). In general, \(\Omega^e\) \(\neq \Omega^e\) and the Higgs boson couples to fermions through both scalar and pseudoscalar components. As a consequence, the flavor violating coupling \(\mathcal{H}_{d, u}\) has the most general renormalizable structure of scalar and pseudoscalar type given by \(-i(\Omega_{ij}P_R + \overline{\Omega}^j_L P_L)\).

3. Constraint on \(Hbs\) from \(B \to X_s\gamma\)

The leading contribution to \(B \to X_s\gamma\) decay with a hard photon is dominated by the \(b \to s\gamma\) process [1, 2]. We calculate the contribution of the flavor violating \(Hbs\) coupling to the \(b \to s\gamma\) and \(b \to sg\) decays (see Fig. 1) and study their implications for the \(B \to X_s\gamma\) process.

![Diagrams](image)

**Figure 1.** Diagrams contributing to the \(b \to s\gamma\) transition. The \(b \to sg\) process occurs via the same type of diagrams.

The total theoretical contribution to the \(b \to s\) transition is given by the sum of the SM contribution and the new physics effect induced by the \(Hbs\) vertex: \(\mathcal{M}_T = \mathcal{M}_SM + \mathcal{M}_{NP}\). To get a bound for the \(\Omega_{bs}\) parameter, we use the discrepancy between the theoretical prediction within the SM and the experimental measurement [3]:

\[
R_{\text{EXP} - \text{SM}} = \frac{\Gamma_{\text{EXP}} - \Gamma_{\text{SM}}}{\Gamma_{\text{SM}}} = \frac{\Gamma_{\text{EXP}}(B \to X_s\gamma)}{\Gamma_{\text{SM}}(B \to X_s\gamma)} - 1,
\]

where \(\Gamma_{\text{EXP}}\) is the experimental decay width of the \(B \to X_s\gamma\) transition and \(\Gamma_{\text{SM}}\) is the theoretical prediction of the SM. Explicitly, \(R_{\text{EXP} - \text{SM}} = 0.117 \pm 0.113\). To constrain the \(Hbs\) vertex, we will assume that the SM prediction plus the \(Hbs\) contribution, coincides with the experimental value. Working out at leading order, the SM contribution is:

\[
\mathcal{M}_{SM}(b \to s\gamma) = -\frac{\alpha^2}{4\sqrt{s}^2m_W^2}\frac{C_{\gamma}^{eff}(m_b)\bar{s}(p_s)\sigma_{\mu\nu}\epsilon^{*\mu}(q, \lambda)q^\nu(m_sP_L + m_bP_R)b(p_b)},
\]

with an effective Wilson coefficient \(C_{\gamma}^{eff}(m_b) = 0.689C_7(m_W) + 0.087C_8(m_W)\), which already contains the QCD contribution at the \(m_b\) scale [2].

The new physics contribution is:

\[
\mathcal{M}_{NP}(b \to s\gamma) = -\frac{Q_b\alpha}{16\pi s_W m_W}\left(0.089 + \frac{0.087}{Q_b}\right)\bar{s}(p_s)\sigma_{\mu\nu}\epsilon^{*\mu}(q, \lambda)q^\nu(\Omega_{bs}P_L + \Omega_{bs}P_R)b(p_b),
\]

where \(Q_b\) is the electric charge of \(b\), \(s_W\) is the sine of the weak angle and \(\mathcal{F}\) is the loop function given by \(\mathcal{F} = \frac{3}{2} + x\sqrt{x^2 - 4x}\text{sech}^{-1}\left(\frac{2}{x}\right) + \frac{(2 - 3x + x^2 + x^2 - 2x^3)}{2(x - 1)}\ln(x),\) where \(x = m_b^2/m_H^2\).

The problem of finding a bound for the \(\Omega_{bs}\) parameter reduces now to solve a quadratic equation. The physical solution corresponds to that for which the allowed values for \(\Omega_{bs}\) satisfy the \(|A_{SM}|^2 > |A_{NP}|^2\) condition, which implies that \(|\Omega_{bs}|^2 < (0.7 - 6.8) \times 10^{-3}\) for a Higgs mass in the range \(115\text{ GeV} < m_H < 200\text{ GeV}\) [1].
4. The $B_s \rightarrow \gamma \gamma$ decay

The $Hbs$ effective vertex induces the flavor violating process $b \rightarrow s\gamma\gamma$ at the one-loop level (see Fig. 2). The contribution to $b \rightarrow s\gamma\gamma$ occurs through two sets of Feynman diagrams, each given a finite and gauge invariant contribution. The first set of diagrams (see Fig. 2-a) includes box diagrams, reducible diagrams characterized by the one-loop $bs\gamma$ coupling and reducible diagrams composed by the one-loop $b-s$ bilinear coupling. Henceforth we will refer to this set of graphs as box-reducible diagrams. The second set of diagrams is characterized by the SM one-loop $H^*\gamma\gamma$ coupling, where $H^*$ represents a virtual Higgs boson (see Fig. 2-b). These type of graphs will be named Higgs-reducible diagrams.

The amplitude for the $b \rightarrow s\gamma\gamma$ decay is:

$$\mathcal{M}^{\mu\nu} = \frac{\alpha g}{8\pi m_W} F_0 \bar{u}_s(p_s) (\Omega_{bs} \sigma_R + \Omega_{bs}^* \sigma_L) \frac{q^\mu q^\nu - q_1^\mu q_2^\nu}{2k_1 \cdot k_2 - m_H^2 + i m_H \Gamma_H} u_b(p_b),$$  

with

$$F_0 = \frac{8 m_W^2}{2 k_1 \cdot k_2} \left[ 3 + \frac{2 k_1 \cdot k_2}{2 m_W^2} + 6 m_W^2 \left( 1 - \frac{2 k_1 \cdot k_2}{2 m_W^2} \right) C_0(1) \right] - Q_t^2 N_c \frac{8 m_t^2}{2 k_1 \cdot k_2} \left( 2 + (4 m_t^2 - 2 k_1 \cdot k_2) C_0(2) \right),$$

where $C_0(1) = C_0(0,0,2 k_1 \cdot k_2, m_W^2, m_W^2)$ and $C_0(2) = C_0(0,0,2 k_1 \cdot k_2, m_t^2, m_t^2, m_t^2)$ are the Passarino-Veltman scalar functions, $m_t$ is the top quark mass, $Q_t$ is the top quark charge and $N_c = 3$ is the color factor.

According to the static quark approximation [4], we can compute the decay width $\Gamma(B_s \rightarrow \gamma\gamma)$ starting from $\Gamma(b \rightarrow s\gamma\gamma)$, where it is assumed that the three-momenta of the $b$ and $s$ quarks vanish in the rest frame of the $B_s$ meson. In this approximation, the $B_s$ meson decays into two photons emitted with energies $m_{B_s}/2$ and the product $k_1 \cdot k_2 = m_{B_s}/2$, where $m_{B_s} = m_b + m_s$ is the $B_s$-meson mass. The decay width for the $B_s \rightarrow \gamma\gamma$ process arising from the new physics effects encoding in $B_{NP}$ has the following form

$$\Gamma(B_s \rightarrow \gamma\gamma) = f_{B_s}^2 \left( \frac{m_{B_s}^3}{16\pi} \right)^2 |B_{NP}|^2,$$

where

$$B_{NP} = \frac{\alpha^3 \Omega_{bs}}{4\pi^3 s_W m_W^2 m_H^2} F_0.$$  

We show in Fig. 3 the branching ratio for the $B_s \rightarrow \gamma\gamma$ process. From this figure, it can be appreciated that the contribution induced by the Higgs-reducible graphs is approximately 2 orders of magnitude larger than those generated by the box-reducible graphs in the range of a Higgs mass of 115 GeV $< m_H < 200$ GeV.

5. Conclusions

We have estimated the $Hbs$ coupling strength from the branching ratio for the $B \rightarrow X_s\gamma$ process. The effective parameter $\Omega_{bs}$ was bounded by using the discrepancy between the respective theoretical and experimental central values of the branching ratios. This constraint was used to bound the Higgs-mediated flavor violating $B_s \rightarrow \gamma\gamma$ decay and we found that its branching ratio is less than $10^{-8}$ in the Higgs mass interval ranging from 115 GeV to 200 GeV. Our results for the branching ratio are 2 orders of magnitude smaller than the current experimental bound imposed by the Belle Collaboration.

1 As in Refs. [4], we will use the constituent mass for the strange quark $m_s = m_K = 0.497$ GeV.
Figure 2. (a) Contribution of the box and reducible diagrams to the $b \to s \gamma \gamma$ decay. (b) Contribution of the SM one-loop induced $H^* \gamma \gamma$ vertex to the $b \to s \gamma \gamma$ decay.

Figure 3. The branching ratio of the $B_s \to \gamma \gamma$ decay for the Higgs-reducible contribution (solid line) and box-reducible contribution (dashed line) as a function of the Higgs mass.

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