Heterogeneity, quality, and reputation in an adaptive recommendation model

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Received 16 September 2010 / Received in final form 3 December 2010
Published online 6 January 2011 – © EDP Sciences, Società Italiana di Fisica, Springer-Verlag 2011

Abstract. Recommender systems help people cope with the problem of information overload. A recently proposed adaptive news recommender model [M. Medo, Y.-C. Zhang, T. Zhou, Europhys. Lett. \textbf{88}, 38005 (2009)] is based on epidemic-like spreading of news in a social network. By means of agent-based simulations we study a “good get richer” feature of the model and determine which attributes are necessary for a user to play a leading role in the network. We further investigate the filtering efficiency of the model as well as its robustness against malicious and spamming behaviour. We show that incorporating user reputation in the recommendation process can substantially improve the outcome.

1 Introduction

We live in an information-rich world with a vast number of sources competing for our attention \cite{2,3}. In addition to the old-fashioned information distribution systems, such as newspapers, which favor news of very general interest, recommender systems \cite{4–6} act as personalized information filters by analyzing users’ profiles and past activities. Techniques used to produce recommendations include correlation-based collaborative filtering \cite{5,7}, Bayesian clustering \cite{8}, probabilistic latent semantic analysis \cite{9}, matrix decomposition \cite{10}, and many others. However recent works show that similarity of past activities often plays a less important role than social influence and recommendations obtained purely by abstract mathematical analysis are valued less than those coming from our friends or peers \cite{11}. A new approach, social recommendation, has hence emerged to make direct use of social connections between members of a society \cite{12}. Examples of popular implementations of social recommender systems include blogger.com and delicious.com, where each user can select some other users as information sources and imports blog articles or URLs from them. In these systems, information favored by an individual user spreads to the user’s followers and, if favored again, to followers’ followers, resembling an epidemics or rumor spreading in a network \cite{13,14}.

A recently proposed news recommender model mimics the spreading process typical for social systems and combines it with an adaptive network of connections \cite{1}. In this model, when a user reads a news, she can either “approve” or “disapprove” it. If approved, the news spreads to followers of the given user (whom we refer to as leader). Each user has an evolving set of leaders (or, according to the terminology of the original paper, sources) and can become a leader for other users. Simultaneously with spreading of news, the leader-follower network evolves with time to best capture similarity of users. In \cite{1} they provide a detailed description of an agent-based approach which is used to assess model’s behaviour and test its performance.

Every recommendation method, if it is to be implemented in real applications, has to respect the heterogeneity of users. Users may differ, for example, by how often they use the recommender system, how broad are their interests, and how accurate they are in evaluation of recommended news. In this work we study the effects of introducing user heterogeneity in the above-described adaptive recommendation model. We show that when frequency of being active and evaluation noise vary among users, leaders with exceptionally high numbers of followers appear and a scale-free-like leadership structure emerges. Scale-free networks are observed in diverse systems \cite{15} and over the past two decades they attracted considerable attention. The mechanism of their emergence based on user heterogeneity in a social recommendation process is similar to the previously discussed “good get richer” phenomenon \cite{16}.

Heterogeneity also means that some users may try to intentionally misguide the system by providing wrong evaluations. We study whether the system is robust against such malicious behaviour, and if it can suppress low-quality content and promote high quality. While the
original adaptive recommender model already exhibits a notable resistance to malicious behaviour, we further improve it by introducing a simple measure of user reputation and employing a hybrid recommendation mechanism which combines similarities of users’ rating patterns with reputation (for a review of reputation systems, see [17]). We show that these changes enhance the filtering efficiency of the system and its robustness against various kinds of malicious behaviour, leaving its performance almost unchanged. The proposed combination of reputation and personalized recommendation hence seems as a promising candidate for real life applications.

2 Description of the model

We first briefly recall the original adaptive recommendation model introduced in [1]. The system consists of $U$ users. Each user is connected to $L$ other users (to whom we refer as the user’s leaders); in the network representation this corresponds to a monopartite directed network with $U$ nodes of fixed in-degree $L$. Evaluation of news (or a different kind of content) $e$ by user $i$, $e_{ij}$, is either $+1$ (liked), $-1$ (disliked) or 0 (not rated yet). Similarity of reading tastes of users $i$ and $j$, $s_{ij}$, is estimated by comparing past users’ assessments. If $i$ and $j$ evaluated $N_{ij}$ common news and agreed in $A_{ij}$ cases, their similarity can be measured in terms of the overall probability of agreement

$$s_{ij} = \frac{A_{ij}}{N_{ij}} \left( 1 - \frac{1}{\sqrt{N_{ij}}} \right)$$

where the term in parentheses disadvantages user pairs with small overlap $N_{ij}$ (their similarity estimates, albeit possibly high, are prone to statistical fluctuations). If $N_{ij} = 0$ then $s_{ij}$ is undefined and put equal to a small positive value $s_0$. Apart from their ratings, no other information about users is assumed by the model.

Propagation of news is governed by their recommendation scores. We denote as $R_{io}$ the recommendation score of news $o$ for user $i$. When news $o$ is introduced to the system by user $i$ at time $t_o$, its initial recommendation score is $R_{io}(t_o) = s_{ij}$ for users $j$ who are followers of $i$ and it is zero for the others (i.e., it cannot be recommended to them yet). In this way, the news is passed from user $i$ to $i$’s followers. If this news is later liked by one of users $j$ who received it, it is similarly passed further to this user’s followers, and so on. A user may receive the same news from multiple leaders – recommendation scores are summed up in that case, reflecting that a news liked by several leaders is more likely to be liked by this user too. To allow fresh news to be accessed fast, recommendation scores are exponentially damped with time. In this way, novelty of news fades with an exponential law [18]. By combining the described processes, we have the formula for the recommendation score

$$R_{io}(t) = (1 - \delta_{t, t_o}) \sum_{j \in L_i} s_{ij} \delta_{t, t_{o,j}},$$

Here $L_i$ is the set of leaders of user $j$ and $\lambda \in (0, 1]$ is the damping factor. The term $\delta_{t, t_o}$ is one when user $i$ liked news $o$ and zero otherwise. Similarly, the term $1 - \delta_{t, t_{o,j}}$ equals one only when user $j$ has not rated news $o$ yet. For user $j$ at time $t$, news are recommended according to their current score $R_{io}(t)$ (the higher, the better). Note that the described damping mechanism is different from the one proposed in [1] where the damping factor was additive and the damping occurred only if too many news were recommended to a user. Our motivation for decreasing scores always is that news lose their novelty regardless of being recommended or not, and that the multiplicative factor keeps $R_{io}(t)$ positive, hence even old news can be in principle read by users if there are no relevant fresh news with higher recommendation scores. Since the spreading of a news over a long path may take long time, recommendation scores decreasing with time not only enhance novelty in the system but also promote news that come from the local neighborhood, effectively working as a local news filter.

Starting from an initial random network configuration (random assignment of leaders to users), connections are periodically rewired to drive the system to an optimal state where users with high similarity (taste mates) are directly connected. In this way the topological evolution of the network and the dynamics of the network’s nodes becomes invariably linked, as in other adaptive co-evolutionary networks [19]. Thus the updating procedure is an important part of the model. Some simple methods are:

1. **Global rewiring.** Leaders are selected using all currently available information: for each user $i$, $L$ leaders with the highest similarity values $s_{ij}$ are selected. This is the best performing method but it is also computationally expensive as it requires computation of all $U(U - 1)/2$ similarity values.

2. **Random rewiring.** For each user, the leader with the lowest similarity value is replaced with a randomly chosen user (if this user is even less similar, no replacement occurs). This is the simplest possible method but its rate of convergence to the optimal state is, as we will see, very slow.

3. **Local rewiring.** For each user $i$, the leader with the lowest similarity value is replaced with the most similar user among leaders of $i$’s leaders (hence we are exploring $i$’s neighborhood within the distance of two). This mechanism is based on the simple observation that two users who share a common neighbor are likely to be similar (for more sophisticated methods for link prediction in networks based on propagation of trust/similarity, see [20, 21]). Computational cost of this method scales as $O(U L^2)$ and hence as long as $L^2 < U$ (a mild constraint, since $L$ is small), this method is faster than global rewiring.

4. **Hybrid rewiring.** Random rewiring is used in 10% of cases and local rewiring is used in the others. This rewiring mimics the natural evolution of communities where users search for friends among friends of friends.
(local rewiring) but also casual encounters occur and may lead to long-term relationships (random rewiring).

While the first three methods were already studied in [1], the last one is novel.

For numerical tests of the model, we use the agent-based framework described in [1]. Taste of user $i$ is represented by a $D$-dimensional binary vector $t_i$ and attributes of news $\alpha$ by a $D$-dimensional binary vector $a_\alpha$. Each vector has a fixed number, $D_A$, of elements equal one (active tastes) and all remaining elements equal zero. We always set the system so that all mutually different user taste vectors are present exactly once, hence $U = (D_A^D)$. This also means that taste vectors of two users differ at least in two elements. Opinion of user $i$ about news $\alpha$ is based on the overlap of the user’s taste vector with the news’s attribute vector

$$\Omega_{i\alpha} = (t_i, a_\alpha)$$

where $(\cdot, \cdot)$ is a scalar product of two vectors. If $\Omega_{i\alpha} \geq \Delta_\alpha$, user $i$ likes news $\alpha$ ($\varepsilon_{i\alpha} = +1$), otherwise she dislikes it ($\varepsilon_{i\alpha} = -1$). The value $\Delta_\alpha$ is an approval threshold of user $i$; the higher it is, the more demanding the user is.

Simulation runs in discrete time steps. In each step, an individual user is active with probability $p_A$. When active, the user reads and evaluates the top $R$ news from her recommendation list and with probability $p_S$ submits a new news with attributes identical to the user’s tastes. To save computational time, the network of connections is rewired every ten time steps. Finally to measure the system’s performance, we use approval fraction which is the ratio of approvals to all assessments and tells us how often users are satisfied with the news they get recommended, and average differences which is the average number of vector elements in which users differ from their leaders and tells us how well the network has adapted to users’ tastes.

### 2.1 Rewiring performance

Since the aforementioned hybrid rewiring method is new, we begin this study with its comparison to the previously known methods. For simplicity we assume a homogeneous setting of users with identical values of $\Delta_i$, $p_A$, and $p_S$. Figure 1 shows that all methods are able to gradually improve both approval fraction and average differences. Apart from local rewiring, the other three methods slowly approach the optimal assignment of leaders with average differences equal two. This ability to converge is due to a gradually increasing pool of commonly evaluated news which allows for precise similarity estimates and, eventually, the optimal assignment of leaders. Since $p_A$ is small, the amount of available information grows slowly and employing the rewiring more often would not make the convergence much faster. Note that for each user there are $N = D_A(D - D_A)$ possible optimal leaders who differ exactly in two taste elements, the optimal state is unique only if $L = N$. If $L < N$ there are different possible optimal states which are equivalent in term of global properties of the system. Initial conditions and users’ dynamics determine the particular equilibrium state of the system. If $L > N$ (which is not our case, however), average differences are greater than two even in the optimal state.

By contrast, local rewiring reaches only a sub-optimal assignment of leaders (the degree of sub-optimality strongly depends on the ratio between the number of optimal leaders to the total number of users, and also on the particular realization of system’s evolution). This is because if the network’s evolution once stops in a sub-optimal state, there is no means to escape from it with local rewiring; if user’s best taste mates are at that moment out of the second-order neighborhood, they can never be reached. In other words, the effectiveness of local rewiring is limited by the current network’s topology, which completely determines the pool of candidate leaders for each user. Unlike other rewiring methods, such pool is very small compared to the whole network ($\leq L^2$ users) and it changes slowly in time. This trapping in a sub-optimal state is hence similar to the trapping of greedy optimization algorithms in a local minimum.

![Fig. 1. (Color online) Comparison of rewiring mechanisms for $D = 14$, $D_A = 6$, $L = 10$, $R = 3$, $p_A = 0.05$, $p_S = 0.02$, $\Delta = 3$, $\lambda = 0.9$.](image-url)
Methods’ convergence rates differ significantly, with global and random rewiring being the fastest and slowest, respectively. Notably, the hybrid method converges almost as fast as the global one (the relation between system’s convergence rate and the percentage of randomness used in the hybrid rewiring is shown in Fig. 2). We employ a randomness of 10% to have both fast convergence at the beginning of the evolution and reasonable time to get to the ground state.

3 Heterogeneity and leadership

In real social networks there are people with different profiles. In this section we study the effects of usage frequencies and judgment abilities on the leader-follower network. Activity frequencies \( p_A \) are drawn from a power-law distribution

\[
P(p_A) \sim p_A^{-\gamma}, \quad p_A \in [\eta, 1].
\]

In this way we obtain a very diverse set of activity frequencies which mimics the observed scale-free patterns of human behaviour [22]. Exponent \( \gamma \) can be tuned to obtain a desired percentage of highly active users. In our simulations we set \( \eta = 0.01 \) and \( \gamma = 2 \) which implies that 10% of users have \( p_A > 10\eta \). For the sake of simplicity we assume \( p_S = p_A/10 \) (that is, a user who is often online also has a high submitting rate). This assumption gets on well with real life experience: high usage users are also the ones who contribute most to the functioning of the system by introducing hot news. The second source of user heterogeneity lies in diverse levels of errors present in their evaluations. We model this feature by generalizing equation (3) to

\[
\Omega_{\alpha} = (t_i, a_\alpha) + u \cdot x_i
\]

where \( u \) is a random value drawn at each assessment from the uniform distribution with domain \([-1, 1]\) and \( x_i \) is the fixed magnitude of evaluation errors for user \( i \), distributed uniformly in \([0, X]\).

Figure 3 illustrates the impact of heterogeneity on the system. The upper panel shows the time evolution of the network. Compared to the original homogeneous case (which is shown with a dotted line), convergence to the optimal state is lost and the evolution itself is so slow...
that the system can be considered to stay in a quasi-steady and sub-optimal state. Moreover, as shown in the bottom panel, the out-degree distribution (recall that a user’s out-degree is equal to the number of the user’s followers) becomes very broad. The initial part of the distribution can be fitted by a power law with exponent approximately 1.5. A similar distribution arises also when global rewiring is used, though it is narrower than in the case of hybrid rewiring (the corresponding power-law exponent rises to approximately 2.0). This suggests that the emergence of a scale-free leadership structure is related to self-organization in the society [23] and that a centralized control favors more homogeneous resulting states.

System dynamics can be explained by the presence of users who have high usage frequencies and, in turn, also much more evaluations of news than the average. At the beginning of the evolution, a large overlap of users’ rating histories favors the formation of links (this feature does not depend on the term in parentheses in (1)), and high usage users are obviously in advantage: they quickly attract many followers and become hubs of the network. Then if two taste-mates are linked to different hubs, even as time runs further they rarely evaluate the same news and their high similarity remains undiscovered: connections with high usage users are not abandoned and the network is trapped in a sub-optimal state and cannot evolve further. A high submitting rate for high usage users magnifies this phenomenon, as there are much more evaluations of news than the average. At the beginning of the evolution, a large overlap of users’ rating histories favors the formation of links (this feature does not depend on the term in parentheses in (1)), and high usage users are obviously in advantage: they quickly attract many followers and become hubs of the network.

Figure 4 reports how both usage frequency and evaluation noise affect user’s out-degree. As explained above, highly active users and precise users have on average more followers than other users. Note that active but imprecise users, as well as precise but lazy ones, cannot be popular leaders as opposed to the few who posses both features. These exceptional users attract a large number of followers, allowing for the scale-free leadership structure to emerge. This behavior is similar to the “good get richer” mechanism [16] which explains a scale-free network structure on the basis of intrinsic fitness values of nodes.

4 Quality and reputation

Until now it was only the overlap between user’s tastes and news’s attributes what distinguished a liked news from a disliked one. Now we shall amend the rating process by another important factor, intrinsic quality of news. To this end, we assign a real-valued quality $Q_{\alpha}$ to each submitted news and generalize equation (3) to the form

$$
\Omega_{\alpha} = Q_{\alpha} \cdot (t_i, a_{\alpha}).
$$

Quality of news is chosen when the news enters the system and does not change with time\(^1\). We draw $Q_{\alpha}$ from the normal distribution with mean 1 and standard deviation 1/2 (normal distribution is chosen to have only a small number of exceptionally good or bad news); when $Q_{\alpha}$ lies out of the range [0; 2], the draw is repeated. Figure 5 shows how news of different qualities propagate over the network. Remarkably, the recommender system has a high filtering efficiency: high-quality news spread to many users while low-quality news perish quickly. Saturation of the number of readers for high-quality news is mainly due to the damping factor $\lambda$. We remark that the

\(^{1}\) The quality factor in (6) transforms the overlap from integer to real value, resulting in a smoother dependence of system behaviour on approval threshold $\Delta$. Introduction of $Q_{\alpha}$ hence makes simulation results more robust and easier to analyze.
spreading of a news in the system can be compared to a branching process \cite{25} of the number of the news’ readers. News’ propagation stops only when there are no users who could read or like it. Such a cascade can either die out quickly (when the news is liked by few) or invade a finite fraction of the system (when it is liked by many).

Once we have introduced the concept of news quality to our simulations, it is straightforward to use it to investigate system vulnerability to malicious behaviour. We introduce two different kinds of malicious users to our system: (a) users with non-informative ratings (either rating at random, always liking, or always disliking), (b) spammers who intentionally introduce low-quality content. Non-informative users are easily taken care of by the system because their similarity values with normal users are small and they are soon disconnected from the network. In particular, all-like and all-dislike users have high mutual similarity and hence they form small separate communities. Our adaptive system is thus robust against malicious users of this kind.

With respect to spammers, the system is rather robust to their actions because a single low-quality news introduced by a spammer spreads only to a limited number of spammer’s followers and as soon as they dislike the news, the news is removed from the system without affecting a large number of users. Alas, spammers can submit a large amount of worthless content and hence even a limited impact of each individual low-quality news can contribute to substantial discomfort of users. One could further argue that when followers of spammers dislike their low-quality news, spammers’ similarity values suffer and soon they are left with no followers. However, as we shall soon see, spammers can easily mask themselves by reasonably rating other news and hence keep their followers. At the same time, users submitting high-quality content are not rewarded with high popularity in the original model.

Instead of studying spammers and providers of good content, we pose a more general question: if the quality of submitted news differs from one user to another, what is the relation between the quality of news posted by a user and this user’s out-degree? To simulate users with different submitting abilities we simply assume that each user has assigned a quality \( Q_i \) and a news takes its quality from the user who submits it. In this way we obtain a system where some users always introduce low-quality content (spammers) and others who submit high-quality news (good sources).

We introduce reputation as a tool to discriminate users. Reputation systems, already widely used in successful commercial online applications, represent an important class of decision support tools that can help reduce risk when engaging in interactions on the Internet and also encourage good behaviour \cite{17}. Reputation itself is a measure of trustworthiness based on referrals or ratings from other members of a community \cite{26,27}. In our case, we introduce the reputation score of user \( i \) as

\[
 r_i = \frac{\sum_{\alpha \in I_i} l_\alpha}{|I_i|} \left( 1 - \frac{1}{\sqrt{|I_i|}} \right) \quad (7)
\]

where \( I_i \) is the set of news introduced by \( i \) and \( l_\alpha \) is the fraction of all users\(^2\) who liked news \( \alpha \); when \( I_i = 0 \) (no news submitted by this user), we set \( r_i = 0 \). Using user similarity and reputation, we set the strength of the link coming from user \( j \) to user \( i \) as

\[
 s'_{ij} = ms_{ij} + (1 - m)r_j \quad (8)
\]

where \( m \) is a mixing parameter which sets the weight of similarity and reputation in the recommendation process (notice that \( s'_{ij} \) is not symmetric). This mechanism differs from the traditional popularity-based recommendation in replacing the object’s popularity with that of the author as well as in using a spreading mechanism in a social adaptive network. When \( m = 1 \), we recover the original reputation-free model, when \( m = 0 \), recommendation is based purely on reputation, and submitters of news of general interest are favoured by achieving a high value of \( l_a \).

In simulations we draw users’ quality values \( Q_i \) from the same distribution that we used for news’ quality values \( Q_\alpha \) before. As shown in Figure 6, in the original setting \((m = 1)\) user’s number of followers does not depend on the user’s quality – a feature that has been discussed above. When \( m \) is significantly less than 1, reputation of users plays an important role and users with low values of \( Q_i \) can be left with no followers (when \( m \leq 0.7 \)). Moreover, as the introduction of the reputation system causes news’ qualities to affect the recommendation scores, the relative size of cascades in news propagation is magnified. Therefore, the similarity-reputation hybrid mechanism increases the filtering capability of the system. When \( m = 0 \), leaders are selected and news are recommended purely according to reputation. As a result, recommended news are diverse and of high quality but not personalized for each individual user. Thus when the role of reputation is too big \((m \) is too small) users’ satisfaction decreases. This is reported in Figure 7 where, when \( m \) is small, approval fraction is lower than in the original model. At \( m \approx 0.7 \) we observe a behaviour which is similar to a second order phase transition: approval fraction suddenly stops to grow and remains practically constant until \( m = 1 \). This stationarity of approval fraction, while somewhat surprising, in fact makes our system easier to tune: all values of \( m \) between 0.7 and 1.0 are equally good (with respect to approval fraction) and hence we can freely decide how much we want to suppress users providing low-quality content (cf. Fig. 6).

5 Conclusion

After the advent of Web 2.0, many on-line resource-sharing websites have been developed and their popularity grows steadily. Some of them (delicious.com, douban.com, and others) recently introduced social recommendation

\(^2\)It is also possible to define \( l_a \) using only the users who rated news \( \alpha \). Numerical simulations show that using the former definition better distinguishes users with different \( Q_i \).
where users can recommend content to others and in turn receive recommendations for themselves. Fast growth of online communities [28] and users' preference for recommendations from friends [11] make social recommendation a promising way to better organize and deliver online resources and to enhance users' experience as well as social contacts.

The news recommender model introduced in [1] and further analyzed and improved in this work mimics spreading processes in adaptive social networks. It makes use both of users submitting new content as well as of other users rating that content and deciding its future fate in the system. We studied the behaviour and performance of this model in artificial computer simulations. We proposed a new method for the network's adaptation. This method is almost as efficient as global optimization using all available information, yet it is computationally much less expensive. Investigation of user heterogeneity showed that users’ personalities strongly influence the properties of the resulting leader-follower network and give rise to a “good get richer” mechanism which was suggested in previous theoretical studies of complex networks [16]. Our simulations show that popularity of individual leaders is very broadly distributed; it can be partially described by a power law with exponent around 1.5. We further studied model’s resistivity against reckless and malicious behaviour of users. Although the original model is already rather resistant to such users, we showed that when user reputation is introduced and recommendations are obtained by mixing this reputation with user similarity, power of malicious users can be further lowered and diffusion of good contents in the system enhanced.

Agent-based models similar to the one studied here can contribute greatly to our understanding of social systems [29] as they allow us to study the effect of each individual model’s assumption on the simulation outcome. The drawback is that the complexity of assumptions can be such that it is hard to make a link between the model and the modeled system. In addition to our efforts to make results robust with respect to the assumptions, it still would be beneficial to have direct empirical input for user behaviour. We envision a real implementation of the studied recommendation model as an ideal source of this kind of information, serving as a useful tool for users and a unique living laboratory for researchers.

We acknowledge stimulating discussions with C.-H. Yeung. This work was partially supported by the Future and Emerging Technologies programmes of the European Commission FP7-ICT-2007 (project LiquidPublication, grant No. 213360) and FP7-COSI-ICT (project QLectives, grant No. 231200).

**Fig. 6.** (Color online) Number of followers vs. user’s quality (upper panel) and number of readers vs. news’s quality (bottom panel) for different values of $m$. Parameters values as in Figure 5.

**Fig. 7.** (Color online) Stationary values of the approval fraction for different values of $m$. Simulation parameters as in Figure 5.

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