Testing single-photon wave packets by Hong-Ou-Mandel interference

Piotr Kolenderski and Konrad Banaszek

Abstract—We discuss characterization of single-photon wave packets by measuring Hong-Ou-Mandel interference with a weak coherent pulse. A complete multimode calculation is presented and effects of multiphoton terms in the coherent field as well as the impact of source and detection imperfections are discussed.

Index Terms—Two-photon interference, single photons, quantum information processing

I. INTRODUCTION

Single photons are elementary building blocks in optical implementations of quantum information processing, communication and cryptography [1]–[3]. Apart from the simplest protocols that encode and process information using just one photon, more elaborate schemes require interactions between photons. Experimentally, the most accessible way to realize such interactions is to resort to multiphoton interference occurring in linear optical networks. Despite the limited range of transformations that can be achieved this way, supplementing linear optics with auxiliary sources and feed-forward operations yields universal quantum information processing capabilities [4].

Realization of multiphoton interactions by means of linear optics places stringent requirements on the modal characteristics of single photons. Interference effects require spatio-temporal indistinguishability of the single-photon wave packets at the input of the linear-optics networks. A sufficient test for this condition is the Hong-Ou-Mandel interference effect on a 50/50 beam splitter [5], when two incident indistinguishable photons are always found in the same output path. One possible realization of such a test is to use two photons originating from independent generation events [6]. However, more insight into the modal structure of the generated photon can be gained by measuring two-photon interference with a suitably chosen collection of coherent pulse modes enabled the reconstruction of the complete single-photon density matrix in the spectral domain [8]. Here we extend previously used models to the fully multimode case and include the complete photon statistics of the coherent field.

II. BEAM SPLITTER TRANSFORMATION

We will consider here the spectral degree of freedom, corresponding to optical fields confined to single-mode fibres. A generalization including spatial degrees of freedom is straightforward. We assume that the single-photon wave packet is prepared with a probability \( p \), and is described by a density matrix \( \hat{\varrho}(\omega, \omega') \). Thus the quantum state of the field entering the beam splitter reads:

\[
\hat{\varrho}_a = (1 - p)|0\rangle\langle 0| + p \int d\omega \int d\omega' \varrho(\omega, \omega') \hat{a}^\dagger(\omega)|0\rangle\langle 0|\hat{a}(\omega')
\]

(1)

where \( \hat{a}(\omega) \) and \( \hat{a}^\dagger(\omega) \) are the annihilation and the creation operators for a frequency \( \omega \) at the corresponding input port of the beam splitter. The coherent state is prepared in a mode characterized by a spectral amplitude \( u(\omega) \), assumed to be normalized to one. The associated annihilation operator is therefore given by a superposition:

\[
\hat{b} = \int d\omega \ u^*(\omega)\hat{b}(\omega),
\]

(2)

where \( \hat{b}(\omega) \) are annihilation operators of monochromatic modes entering the second input port of the beam splitter. We will use the following notation for a coherent state with an amplitude \( \beta \) generated in the mode \( u(\omega) \):

\[
|\beta u(\omega)\rangle_b = \hat{D}_\beta(\beta)|0\rangle = \exp \left( -\frac{\beta^2}{2} + \int d\omega \ \beta u(\omega)\hat{b}^\dagger(\omega) \right)|0\rangle
\]

(3)

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where $\hat{D}_b(\beta) = \exp(\beta \hat{b}^\dagger - \beta^* \hat{b})$ is the displacement operator.

The total input density matrix is given by the tensor product
\[
\hat{\rho}_{\text{in}} = \hat{\rho}_a \otimes |\beta u(\omega)\rangle \langle \beta u(\omega)|.
\]
This expression is transformed to the output density matrix $\hat{\rho}_{\text{out}}$ represented in terms of the outgoing modes $\hat{c}(\omega)$ and $\hat{d}(\omega)$ leaving the beam splitter by substituting monochromatic field operators according to:
\[
\hat{a}(\omega) = \frac{\hat{c}(\omega) - \hat{d}(\omega)}{\sqrt{2}}, \quad \hat{b}(\omega) = \frac{\hat{c}(\omega) + \hat{d}(\omega)}{\sqrt{2}}
\]  
\hspace{1cm} (4)

We assume here a flat spectral characteristics of the beam splitter over the relevant frequency range.

III. COINCIDENCE COUNT RATE

The probability of a click on a binary detector monitoring one of the outgoing beams is given by the expectation value of the following normally ordered operator:
\[
\hat{M}_c := \hat{\pi} - \xi \exp \left( -\eta \int d\omega \hat{c}^\dagger(\omega) \hat{c}(\omega) \right): \quad (5)
\]
for the beam $c$ and analogously for the beam $d$. In the above formula, $1 - \xi$ is the probability of a dark count, and $\eta$ is the quantum efficiency, assumed to be uniform across the spectrum of the detected fields. The coincidence count rate $R_C$ is given by the expectation value
\[
R_C = \text{Tr}(\hat{\rho}_{\text{out}} : \hat{M}_c \otimes \hat{M}_d :).
\]  
\hspace{1cm} (6)

This quantity can be conveniently expressed in terms of the expectation values of a two-parameter operator:
\[
\hat{Z}(\eta_c, \eta_d) := \exp \left( -\int d\omega [\eta_c \hat{c}^\dagger(\omega) \hat{c}(\omega) + \eta_d \hat{d}^\dagger(\omega) \hat{d}(\omega)] \right):
\]  
\hspace{1cm} (7)

Consequently:
\[
\hat{c}(\omega) \hat{Z}(\eta_c, \eta_d) = (1 - \eta_c) \hat{Z}(\eta_c, \eta_d) \hat{c}(\omega)
\]  
\hspace{1cm} (12)

and
\[
\hat{c}(\omega) \hat{Z}(\eta_c, \eta_d) \hat{c}^\dagger(\omega') =
\quad = (1 - \eta_c) \hat{Z}(\eta_c, \eta_d) + (1 - \eta_c)^2 \hat{c}^\dagger(\omega') \hat{Z}(\eta_c, \eta_d) \hat{c}(\omega)
\]  
\hspace{1cm} (13)

With the normally ordered expressions, the expectation value $\text{Tr}[\hat{\rho}_{\text{out}} \hat{Z}(\eta_c, \eta_d)]$ can be easily calculated to be equal to:
\[
\hat{Z}(\eta_c, \eta_d) =
\quad = (1 - \frac{1}{2}(\eta_c + \eta_d)p + \frac{1}{4}(\eta_c - \eta_d)^2 p|\beta|^2 T) e^{-\eta_c + \eta_d} \alpha^2 / 2
\]  
\hspace{1cm} (14)

where $T$ is the overlap of the single-photon wave packet with the mode function of the probe coherent state:
\[
T = \int d\omega \int d\omega' u^* (\omega) g(\omega, \omega') u(\omega').
\]  
\hspace{1cm} (15)

Substituting Eq. (14) into Eq. (8) we end up with the expression for the coincidence rate as a function of the overlap $T$ in the following form:
\[
R_C(T) = 1 - \xi \left( 2 - \eta p + \frac{1}{2} \eta^2 p|\beta|^2 T \right) e^{-\eta|\beta|^2 / 2} + \xi^2 (1 - \eta p) e^{-\eta|\beta|^2}.
\]  
\hspace{1cm} (16)

The coincidence count rate measured as a function of the delay between the single-photon wave packet and the coherent pulse provides information on the spectral characteristics of the single-photon wave packet.

IV. INTERFERENCE VISIBILITY

The mode overlap $T$ can be recovered from Eq. (16) by comparing the coincidence count rate with the case when the modes of the two incident beams are completely distinguishable, corresponding to $T = 0$. This can be quantified with the help of the visibility of the Hong-Ou-Mandel dip, defined as:
\[
V = \frac{R_C(0) - R_C(T)}{R_C(0)}
\]  
\hspace{1cm} (17)

After substitution of Eq. (16) into Eq. (17) we obtain that the visibility is proportional to the mode overlap $T$:
\[
V = c_f T
\]  
\hspace{1cm} (18)

where the proportionality constant $c_f$ is given by
\[
c_f = \frac{\xi \eta^2 p|\beta|^2 e^{-\eta|\beta|^2 / 2} \left[ 1 - \xi (1 - \eta p) e^{-\eta|\beta|^2} \right]}{2 \left[ 1 - \xi (1 - \eta p) e^{-\eta|\beta|^2 / 2} \right] \left( 1 - \xi e^{-\eta|\beta|^2} / 2 \right)}
\]  
\hspace{1cm} (19)

and we will refer to it as the correction factor, dependent on the parameters of the setup. In the idealized limit of $\eta = \xi = 1$ and $\alpha \to 0$ the correction factor is equal to one. In the first step, it is instructive to consider the case of no dark counts.
ξ = 1 and a small coherent amplitude, which allows one to apply an expansion in η|β|^2. One then obtains:
\[ c_f \approx 1 - \left( \frac{1}{2ηp} - \frac{1}{4} \right) η|β|^2 \] (20)
which shows that the correction factor decreases with the increasing coherent state effective amplitude. In the left panel of Fig. 1 we plot the correction factor \( c_f \) and the coincidence count rate \( R_C \) for \( T = 0 \) as a function of η|β|^2 and ηp under the assumption of no dark counts \( ξ = 1 \). Obviously, the coherent pulse amplitude needs to be non-zero to register any two-photon event at all, which results in a trade-off for its value.

The behavior of the correction factor \( c_f \) is qualitatively different in the presence of dark counts characterized by \( ξ < 1 \), as seen in the center and right panels of Fig. 1. This is because for very small coherent pulse amplitudes the interference dip becomes dominated by dark count events, and the visibility carries little information about the mode overlap. Consequently, for a fixed product ηp, there is a non-zero optimal coherent state amplitude that maximizes the correction factor \( c_f \). However, the correction factor never reaches the unit value in contrast to the regime free of dark counts. Fig. 1 shows that for detectors having 1% and 5% of dark counts the highest possible value of the correction factor is close to 0.8 and 0.7 respectively.

The statistical uncertainty of the determined overlap \( T \) is a function of both the correction factor, which attenuates the dependence of the measured visibility on \( T \), and the count rate of coincidence events. In a simple estimate, the relative uncertainty of \( T \) can be written as:
\[ \frac{ΔT}{T} \propto \frac{1}{c_f \sqrt{R_C(0)}} \] (21)
The expression on the right hand side of the above formula can be maximized with respect to η|β|^2 for fixed values of ξ and ηp. Results of this procedure are shown in Fig. 2 which provides guidelines for selecting the optimal amplitude of the probe coherent pulse. It is seen that the number of detected photons in the coherent pulse should be of the order of one.

V. CONCLUSIONS

The method of characterizing single-photon wave packets is very simple from the conceptual point of view. However, data obtained using an auxiliary coherent pulse to implement Hong-Ou-Mandel interference need to be corrected for multiphoton events. The optimal amplitude of the pulse has been found for a realistic range of parameters.

REFERENCES

[1] P. Kok, W. J. Munro, K. Nemoto, T. C. Ralph, J. P. Dowling, and G. J. Milburn, “Linear optical quantum computing with photonic qubits,” Rev. Mod. Phys., vol. 79, p. 135, 2007.
[2] N. Gisin, G. Ribordy, W. Tittel, and H. Zbinden, “Quantum cryptography,” Rev. Mod. Phys., vol. 74, p. 145, 2002.
[3] J. L. O’Brien, G. J. Pryde, A. G. White, T. C. Ralph, and D. Branning, “Demonstration of an all-optical quantum controlled-not gate,” Nature, vol. 426, no. 6964, pp. 264–267, Nov. 2003.
[4] E. Knill, R. Laflamme, and G. J. Milburn, “A scheme for efficient quantum computation with linear optics.” Nature, vol. 409, no. 6816, pp. 46–52, Jan 2001.
[5] C. K. Hong, Z. Y. Ou, and L. Mandel, “Measurement of subpicosecond time intervals between two photons by interference,” Phys. Rev. Lett., vol. 59, no. 18, pp. 2044–2046, Nov 1987.
[6] C. Santori, D. Fattal, J. Vuckovic, G. S. Solomon, and Y. Yamamoto, “Indistinguishable photons from a single-photon device.” Nature, vol. 419, no. 6907, pp. 594–597, Oct 2002.
[7] J. G. Rarity, P. R. Tapster, and R. Loudon, “Non-classical interference between independent sources,” 1997.
[8] W. Wasilewski, P. Kolenderski, and R. Frankowski, “Spectral density matrix of a single photon measured,” Phys. Rev. Lett., vol. 99, p. 123601, 2007.
[9] W. H. Louisell, Quantum Statistical properties of radiation. Willey, 1973.
Fig. 1. Contour plots of the correction factor $c_f$ (top row) and the coincidence count rate $R_C$ (bottom row) as a function of the product of the generation probability and the detector efficiency $\eta p$, and measured coherent state intensity $\eta |\beta|^2$. 