Symmetry interaction and many-body correlations

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Abstract

It is shown how many-body correlations involving the symmetry potential naturally arise in the molecular dynamics CoMD-II model. The effect of these correlations on the collision dynamics at the Fermi energies is discussed. Small level of correlations for systems of moderate asymmetry are able to produce large effects also in simple observables like the charge distributions. The comparison with predictions based on EOS static calculations is also discussed.

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I. INTRODUCTION

Great efforts have been made in the last decades in trying to extract information on the behavior of the symmetry energy as a function of the density. Semiclassical microscopic approaches predict that the isotopic composition of cluster productions, at Fermi or higher energies, induced by nucleus-nucleus collisions lead to the definition of well suited observables able to highlight the degree of ”stiffness” of the symmetry interaction [1]. A wide class of phenomenological effective two and three-body interactions are used in semi-classical calculations (see for examples the class of Skyrme interactions [2] and Gogny forces) for which the density of symmetry interaction is usually expressed as: 

$$\varepsilon = e_a \rho F(\rho/\rho_0) \beta^2$$

(the $\beta^4$ dependence is usually considered negligible). $\beta = \frac{\rho_n - \rho_p}{\rho}$ where $\rho_n$, $\rho_p$ and $\rho$ are the neutron, proton and total densities respectively. $\rho_0$ is the ground state (g.s.) density. These results are consistent with in-medium EOS calculations using microscopic effective interactions (see as a typical example ref. [3, 5, 6]). At density lower than 1-1.5 $\rho_0$, $e_a F$ is usually a positive monotonic increasing function of the density, i.e. it produces in most of the cases a repulsive behavior as a function of the density. In semiclassical transport models the above expressions are used locally to generate the single particle potentials (see as an example BUU calculations [3]).

The calculations based on realistic effective interactions using variational techniques and cluster expansion methods start from quite complex two and three body interactions. As suggested also from Energy Density Functional theories [7], the one-body total density functional can represent the effect of rather complex interactions and correlations quite difficult to be managed in time dependent approaches. However, we note that the presence of the $\beta^2$ variable, as the leading factor determining the iso-vectorial dynamics (see as an example ref. [5, 6]), does not take into account correlations able to describe the different dynamics between neutron-neutron ($nn$), proton-proton ($pp$) and neutron-proton ($np$) couples.

In fact, it is easy to verify that the $\beta^2$ dependence implies that the probability to find a $np$ couple in a small volume $V$ (local description) is just the product of the probability to find a $pp$ and a $nn$ couple in the same volume. With this respect the $\beta^2$ dependence of the symmetry interaction resembles a kind of mean-field approximation concerning the two-body iso-vectorial dynamics (which in the following we indicate with I.M.F.A.). At the Fermi energies, however, many-body correlations involving also the isospin degree of
freedom are necessary to describe the cluster formation process; a closer investigation on the behavior of the effective interaction including such correlations in a time dependent picture would be desirable. In this energy range, in fact, the formation of clusters and the related dependence on the symmetry potential, are the results of the many-body fast evolution leading the investigated systems in regions of density and excitation far from the g.s. one. The study of observables having a strict link with this fast changes can give the opportunity to probe the density functional dependence of the microscopic interaction. In sect II we discuss some details concerning the symmetry interaction as treated in our Constrained Molecular Dynamics Model-II (CoMD-II). In section 2, as an example, we illustrate results of the performed calculations for the $^{40}\text{Cl} + ^{28}\text{Si}$ at 40 MeV/nucleon. The effects of correlations on the two-body iso-vectorial dynamics are also discussed. Section 3 contains the conclusive remarks.

II. SYMMETRY ENERGY AND THE COMD-II MODEL

As it is well known the symmetry energy contribution to the total energy include a kinetic term and a potential one. In CoMD-II model the kinetic part is naturally included through the constraint related to the Pauli principle acting on the phase space of the neutron and proton fluids. The increasing of the kinetic term with the increasing of the charge/mass asymmetry is therefore included in our calculations. In particular in ref. it is shown how the constraint is able to produce for finite systems Fermi like distribution during all the dynamical evolution of the investigated system.

In the following we instead describe how the symmetry interaction is included in the model. From a general microscopic point of view the symmetry potential can be introduced by assuming that, in $S$ waves (see the following), the two body interaction for triplet isospin states ($T=1$) $\Upsilon^1$ is repulsive as compared to the more attractive singlet states ($T=0$) $\Upsilon^0$ i.e. $\Upsilon^1 > \Upsilon^0$ (both $\Upsilon^1$ and $\Upsilon^0$ are negative). These are well known facts established from low energy nucleon-nucleon scattering processes, from studies concerning the binding energy of the deuteron and the ones related to binding energies of isobars nuclei.

According to the properties of the many-body wave function describing the system, the triplet interaction is experienced by $(N^2 + Z^2 + NZ)/2$ couple of states, while the singlet one involves $NZ/2$ couples. $N$ and $Z$ indicate the number of neutrons and protons respectively.
for a system with mass number $A$. By using a Skyrme interaction framework, the above considerations are equivalent to use a symmetry microscopic interaction having the following structure:

$$V^\tau = \frac{a_0}{2\rho_0} \sum_{i \neq j=1}^{A} (2\delta_{\tau_i \tau_j} - 1)\delta(\mathbf{r}_i - \mathbf{r}_j)$$

(1)

with $a_0 = (\Upsilon^1 - \Upsilon^0)/2 = 72MeV$. $\rho_0$ is the ground state (g.s.) density. $\mathbf{r}_i$ indicates the generic nucleonic coordinate. The above assumptions generate also an attractive iso-scalar interaction $V$ related to the contribution $A(3/8\Upsilon^1 + 1/8\Upsilon^0)$ [9, 10].

By performing the convolution of eq.(1) with the nucleonic wave packets, we obtain the following expression for the total symmetry energy:

$$U^\tau = \frac{a_0}{2\rho_0} \beta_M$$

(2)

$$\beta_M = \rho^{nn} + \rho^{pp} - 2\rho^{np}$$

(3)

$$\rho^{aa'} = \sum_{i \neq j} \rho_{i,j}$$

(4)

with $a$ and $a'$ equal to $n$ or $p$ being $n$ and $p$ the ensemble of neutrons and protons respectively. $\rho_{i,j}$ represents the normalized two-body Gaussian overlap integral typical of molecular dynamics approaches. The above expressions represent the contribution related to the leading two-body contribution to the symmetry interaction. We note that, due to the character of our molecular dynamics approach, the elementary quantities appearing in the above expressions are the overlap integrals between the different couples. Usually, as suggested from realistic in medium variational calculations the above expression is implemented by introducing form factors which are able to take into account the effects of more complex in medium interactions quite difficult to be managed in time dependent approaches. In all the time dependent semiclassical approaches [1], however, this form factors are introduced as phenomenological correction factors whose parameters should be definitely obtained through the comparison with the experimental data [1].

For clarity, in the following we report the expressions obtained in the so called Non-Local (N.L.) approximation which is essentially valid for non light systems in compact configurations. The following expressions are in fact more simple than the ones used in the calculations being able to easily enlighten the roles played by the leading factor $\beta_M$ and by the form
In particular, the N.L. approximation is easily obtained by substituting the generic overlap integral $s_j = \sum_{i \neq j} \rho_{i,j}$ for the $j$-th particle appearing in the complete expression, with the average value related to the $A$ particles of the system $s_j \simeq s/2A$. $s$ according to eq. (9) is associated to the total overlap integral.

The different options concerning the form factors describe different degrees of stiffness. For the system under study, $^{40}\text{Cl} + ^{28}\text{Si}$ at 40 MeV/nucleon, and for compact configurations, $s$ is well approximated by the one-body density $\rho$. This value produces a total strength factor which is in substantial agreement with the ones obtained by EOS in medium calculations (see the following). We note that the positive form factors $F'(s)$ are such that $F'(s) \frac{s}{s_{g.s.}}$ has the same functional form like the $F(u)$ proposed in [3, 4, 5]. They have been used extensively in BUU calculations. The factor $\beta_M$ already introduced in eq. (3), instead arises naturally from the many-body approach. It takes into account explicitly, through the last negative term, that the microscopic two-body interaction in the isospin singlet states is more attractive than the one related to triplet states.

For consistency and clarity, before showing the results of our calculations, we want now evaluate the previous expressions in I.M.F.A.. For this purpose we assume, for simplicity, $A, N, Z \gg 1$, and we rewrite $\beta_M$ as:

$$\beta_M = N^2 \bar{\rho}^{nn} + Z^2 \bar{\rho}^{pp} - 2NZ\bar{\rho}^{np}. \tag{10}$$

where: $\bar{\rho}^{aa'} = \rho^{aa'}/II'$ with $a$ and $a'$ equal to $n$ or $p$ and correspondingly $I$ and $I'$ equal to $N$ or $Z$. $\bar{\rho}^{aa'}$ represents the average overlap integral per couple of nucleons.

I.M.F.A. is now easily obtained by supposing $\bar{\rho}^{aa'} = \frac{1}{A^2} \sum_{i \neq j} \rho_{i,j} \equiv \bar{\rho} = \frac{2}{4A} \rho$ to be independent on $a$ and $a'$. This is a rather strong approximation in our framework, and it is equivalent
to assume same average distances \( d_{a,a'} \) between nucleons in \( nn, pp \) and \( np \) couples. This means that correlations producing differences in the two-body iso-vectorial dynamics \( \tilde{\rho}^{a,a'} \) are averaged through the overlap integral \( \tilde{\rho} \), so that:

\[
\beta_M \rightarrow \frac{3}{4} \frac{\rho}{\rho^2} (N - Z)^2 = \frac{3}{4} \rho^2 V \frac{(\rho_n - \rho_p)^2}{\rho^2}.
\]

In this case we obtain that the interaction depends locally only through one-body densities, it is positively defined and the \( \beta^2 \) dependence is restored. Taking into account that for the present system, in compact configurations, \( s \cong \rho \), by substituting the right hand of the above relation in eq. (4) and dividing by the volume \( V \), we obtain a precise correspondence with the symmetry potential density \( \varepsilon^7 \) used in ref.[3] (see also section 1) if \( e_a = 27 MeV \). This corresponds to an \( S_0 \) value [3] of about 40 MeV.

So that, unambiguously, the microscopic interaction defined through eq. (1) is able to generate a symmetry term strictly equivalent, in the spirit of the I.M.F.A., to the expression obtained through EOS calculations like the one reported in ref.[5].

III. RESULTS OF CALCULATIONS

In this section, we illustrate the results obtained with our model for the system \( ^{40}Cl + ^{28}Si \) at 40 MeV/nucleon.

For moderately asymmetric systems, self-consistent calculations, including the ones for the searching of the g.s. configurations, produce a negative value of \( \beta_M \) (see eq. (3)). This result is due to the combined actions of the more attractive force, for the isospin singlet states (which is experienced only for \( np \) couples), and the repulsive effect of the Coulomb interaction for \( pp \) couples. The Pauli principle also plays its role in the symmetry interaction. In our model calculations it becomes dominant for light systems. These correlations tend to increase the average neutron-proton overlap integral \( \tilde{\rho}^{np} \) (decreasing the related average distance \( d_{np} \)) and to decrease the overlap \( \tilde{\rho}^{nn} \) and \( \tilde{\rho}^{pp} \) related to the \( nn \) and \( pp \) couples (deuteron effects). As an example, for the system under study and for an impact parameter \( b=4 \text{ fm} \), in Fig. 1(a) we show, as a function of time, the ensemble average overlap integral \( \overline{\sigma} \) for the three investigated cases (\( Stiff1, Stiff2, Soft \)) related to the total system. During the time interval 18-60 fm/c about 95\% of the total system is formed by a compact cluster having, for the different options, the same mass and the same charge/mass asymmetry within 1\%. In this way, the hot compact system, in all the three cases, experiences almost adiabatic compression and decompression processes from which the density dependence of
FIG. 1: For the system $^{40}Cl + ^{28}Si$ at 40 MeV/nucleon and $b = 4$ fm, we show: (a) average overlap integral $\bar{s}$, (b) average symmetry potential per nucleon as a function of time for the different options describing the symmetry potential (see eqs.(6-8)), (c) charge distribution for $Z > 1$ obtained after 650 fm/c.

the symmetry interaction stored in the system can be investigated.

From Fig. 1(a) we can see that the maximum value of $\bar{s}$, $s_{\text{max}}$, is reached in about 30 fm/c. The Soft case shows the lower $s_{\text{max}}$ value, the Stiff1 the higher one (about 1.25 times the $s_{\text{g.s.}}$ value). Accordingly, the average total potential (not shown) displays the less attractive behavior for the Soft case. The reasons of these differences can be understood by looking at Fig. 1(b) where we show the corresponding average symmetry potential per nucleon $\overline{U}_A$. $\overline{U}_A$ shows, in fact, an average repulsive effect as a function of the density for the Soft case (it increases with the density), while the Stiff1 case gives rise to an attractive effect.

The Stiff2 case shows an attractive behavior and intermediate values concerning the strength. The differences observed around 18-60 fm/c play an important role in to determine the later evolution of the system. In Fig.1(c) we show the charge distributions evaluated for the three different options after 650 fm/c (main fragments almost cold). The rather high sensitivity of this simple observable to the different shapes of the form factors is clearly evident. In particular, in our calculations we have verified that $\tilde{\rho}^{np} = \tilde{\rho}^{pn} \equiv (1 + \alpha)\left(\frac{\tilde{\rho}_{pp} + \tilde{\rho}_{nn}}{2}\right) \equiv \tilde{\rho}$ with $\alpha \simeq 0.15$. Therefore, at low asymmetry, a small correlation effect is enough to produce a negative value of $\beta_M$, due to the structure of eq. (2). Moreover the strength
of $\mathcal{U}_A$ is rather high, of the order of -1.5 MeV/nucleon for $t = 0 \, fm/c$ at normal density. Therefore, according to these results, we observe that also a correlation value of the order of 1%-2% can induce non negligible effects on the symmetry term. This strongly suggests that the effects discussed in this section can be still present by using other kinds of approaches and effective interactions and that they refer to a rather general aspect of the symmetry term due to its intrinsic structure.

Concerning the negative sign, we would like to precise that $\beta_M$ shows always a repulsive character with the asymmetry of the system. This repulsive effect follows an approximate parabolic dependence with respect to the $\chi = N - Z$ variable. In particular:

$$\beta_M \sim \tilde{\rho}\{\chi^2[1 + \alpha/2] - \alpha A^2/2\}$$  \hspace{1cm} (11)

where the correlation coefficient $\alpha$ is a function of $N,Z$ and $\tilde{\rho}$. $\alpha$ has to be evaluated through self-consistent calculations. This means that $\beta_M$ contains, independently on its sign, the right behavior necessary to explain the differences in the binding energies of isobars nuclei. For systems with mass around 68 units, the $\beta_M$ factor becomes positive for relevant "limiting" asymmetries restoring therefore the repulsive behavior with the density of the asy-stiff cases here investigated. However, we stress that the negative sign of $\beta_M$ has relevant consequences in to determine the effects of the different options for $U^r$ on the dynamics of the investigated processes. Contrary to the uncorrelated case, the so called Stiff parameterizations (in spite of this change of sign, we retain the same nomenclature to indicate the different options) show an average decreasing behavior with the density. For the system under study an overall description, as a function of time, of the different factors which define the symmetry interaction per nucleon is shown in fig.2.

With black symbols in the tree columns of panels (a), (b),(c) we plot $\mathcal{U}_A$, the form factors $F'$ and the leading factor $\overline{\beta_M}$ respectively. The three rows are referred to the three options Stiff1, Stiff2, and Soft respectively. The vertical dashed line enlighten the time around which the average density reaches the maximum value. From the figure it is possible to see how the interrelated behavior of the factors $F'$ and $\beta_M$ determines the final evolution of the symmetry potential for the different options. The red symbols on the left column indicate the $\overline{\mathcal{U}_A}$ values for a system having the same mass of the $^{40}Cl + ^{28}Si$ system but a charge/mass asymmetry $\frac{N-Z}{A}$ of about 0.22. These results have been obtained, as a first order approximation, by
FIG. 2: In the generic panels (ai),(bi),(ci) are reported the average symmetry potentials per nucleon $U_A$, the form factors $F'$ and the leading average factor $\beta_M$ respectively. The index i=1,2,3 are referred to the different options for the symmetry potential Stiff1, Stiff2 and Soft respectively. The black symbols concern the system under study. The red symbols represent results for a system having the same total mass and 27 units of charge.

supposing that the average overlap integral (per nucleon couples) $\bar{\rho}$ for this new system follows the same time behavior as the one associated to the $^{40}$Cl+$^{28}$Si system and applying eq. (11) (full self-consistent calculations are in progress). These estimations show clearly that for the stiff cases, around 25 fm/c, $U_A$ becomes close to zero and the behavior changes from an attractive action to a repulsive one as a function of the density. This change of behavior concerns only the stiff cases analyzed in the present work and not the soft option which produces always a repulsive action. This transition, therefore, could be considered as a ”fingerprint” of a finite value for the correlation coefficient $\alpha$ in the case of a stiff behavior of the symmetry interaction.

We conclude this section by showing in Fig.3 the same quantities as plotted in Fig.1 but referred to our self consistent calculations obtained with the I.M.F.A. discussed at the end of the previous section. It is clearly evident that the further constraint reflecting the absence of iso-vectorial two-body correlations ($\alpha = 0$) has noticeable effects. We observe a quite reduced strength of the symmetry interaction which now assume positive values (with an order of magnitude valuable also in the framework of the Liquid Drop Model) and a substantial repulsive effect for all the three options. The final results on the charge
Non-vanishing correlations in the neutron-proton dynamics related to the symmetry interactions, and obtained with the semiclassical CoMD-II model, strongly affect the dynamics of heavy ion collisions at the Fermi energies. These many-body correlations suggest that the leading factor responsible for the microscopic two-body iso-vectorial dynamics can have a structure different from the usual $\beta^2$ dependence suggested by EOS calculations like the ones reported in ref. 5. At moderate asymmetries these effects can produce noticeable change in the dynamics of the heavy ion collisions. In particular, the Stiff options here investigated show in the low density region (contrary to the I.M.F.A. case discussed in this paper) an average attractive behavior as a function of the density. Preliminary calculations also suggest that the strength of this attractive effect decreases with the asymmetry of the system and that, for values larger than some limiting value $\beta_c$ ($|\beta_c| > 0.22$), the repulsive behavior is restored.

We think that the definitive answer about the real entity of such correlations and their change with the system asymmetry should be obtained through further deep investigations on the behavior of different isospin observables to be measured experimentally. This subject
can have a certain relevance in the isospin physics studied through heavy ion collisions. In fact, it has been shown that time dependent semiclassical approaches including such correlations can give quite different results with respect the ones based on the I.M.F.A. This, therefore, allows for different conclusion about the estimation of the ”stiffness” degree of the symmetry interaction when comparing calculations with experimental data. Finally we remark that, the large effects produced also through a small degree of correlations, as predicted by our model calculations, suggest that, in the quite sophisticated static EOS calculations, the inclusion of variational parameters having a charge/mass asymmetry dependence (as performed in studies related to pairing forces [7, 12]) should be investigated in more detail.

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