ON THE ORIGIN OF OPTICAL RADIATION DURING THE IMPULSIVE PHASE OF FLARES ON dMe STARS.

I. DISCUSSION OF GAS DYNAMIC MODELS

E. S. Morchenko

Physics Faculty, M. V. Lomonosov Moscow State University, Moscow, Russia;
e-mail: morchenko@physics.msu.ru

Original article submitted August 8, 2019; accepted for publication December 18, 2019

In connection with a published critique, the author justifies the use of a motionless homogeneous plane layer of pure hydrogen plasma that is near local thermodynamic equilibrium (LTE) for analyzing the characteristics of the radiation from a chromospheric condensation of thickness \( \Delta z_m = 10 \) km in a gas dynamic model of stellar flares. It is shown that the shock-wave model of flares proposed by Belova and Bychkov, as opposed to the model of Kostyuk and Pikel’ner, has irremovable internal defects owing to exclusion of the interaction between a thermal wave (temperature jump) and a non-stationary radiative shock. In particular, this model (a) does not make it possible to increase the geometric thickness of a chromospheric condensation owing to divergence of the fronts of the thermal and shock waves during impulsive heating, (b) cannot provide heating of the chromospheres of red dwarfs over significant distances, and (c) predicts \( H_\alpha \) line profiles in conflict with observational data.

It is argued that: (a) the shock-wave model by Belova and Bychkov represents a development of the kinematic model of solar flares (Nakagawa et al.) and its application to dMe stars, specifically: a study of the radiative response of the chromosphere of a red dwarf to impulsive heating in the simplest gas dynamic statement of the problem (a thermal wave is excluded, a stationary approach is used); (b) in terms of the Kostyuk and Pikel’ner model, the regions behind the stationary shock fronts do not correspond to a chromospheric condensation with time-varying thickness but to zones in which the plasma relaxes to a state of thermal equilibrium. It is emphasized that the separation of the Kostyuk and Pikel’ner model into “thermal” and “shock-wave” components is fundamentally impossible.

Keywords: red dwarf stars: flares: impulsive heating: gas dynamic models: optical radiation

1. Introduction. In a recent article Belova and Bychkov [1] calculated the profiles of plane-parallel stationary radiative shock waves propagating in the chromosphere of a red dwarf in the direction of the photosphere (“downward”) at a velocity of 30–100 km/s (referred to as the shock-wave model of stellar flares below). They took into account the difference in the heating of the atom-ion and electron components of the plasma behind a stationary shock front [2,3] \( T_{oi} > T_e \), where \( T_{oi} \) is the atom-ion temperature and \( T_e \) is the electron temperature). Their calculations [1] neglect the influx of energy from non-thermal electrons (heating power \( P_e = 0 \)), thermal conductivity (classical thermal flux \( F_c = 0 \)), gravitational acceleration \( (g = 0) \), and the time-independent power of the heating.

\[1\] A constant energy input is a necessary condition for “downward” propagation of a shock wave in the chromosphere of the Sun and dMe stars over a long time (see Ref. 4).
sources that maintain the stationary state of the unperturbed chromosphere \((Q = 0)\). The effect of the hot post-shock plasma on the degree of ionization of the cold gas ahead of the front (precursor) is neglected; until the passage of the shock wave the plasma is uniform. The calculations \[1\] were performed in a reference system associated with the discontinuity (viscous jump). The magnetic field \((H_0 \text{ from } 0 \text{ to } 5 \text{ G})\) is directed perpendicular to the velocity \((u_0)\) of the unperturbed gas; the frozen-in condition holds not only at the viscous jump but also in the flow over the entire duration of non-stationary plasma cooling.

Based on these calculations the authors \[1\] have shown that under the conditions of the chromospheres of dMe stars “the gas behind the front of a stationary shock wave remains transparent in the optical continuum…” \[1\] and it was concluded that during a flare the “emission in [hydrogen] lines is determined by a shock wave in layers above the photosphere, while black-body radiation comes from the photosphere that is heated by a flux of suprathermal particles” \[1\]. Belova and Bychkov also assume \[1\] that “the model of a shock wave propagating in the chromospheric gas may be applicable to explaining the radiation from a [flaring] hydrogen plasma that is transparent in frequencies of the continuum spectrum.”

Belova and Bychkov have also criticized (see \[5\], \[6\] and \[1\]) an article by Katsova et al. \[7\] that discusses the results of a study of the response of a red-dwarf chromosphere to impulsive heating by a beam of accelerated electrons with a power-law spectrum (low-energy cutoff at \(E_{10} = 10 \text{ keV},\) spectral index \(\gamma = 3\) (hard spectrum), energy flux at the upper boundary of the flare region of \(F_0 = 10^{12} \text{ erg/cm}^2\text{s},\) heating duration 10 s, and a rectangular impulse). Thus, in \[5\] the authors \[1\] pointed out that Katsova et al. \[7\] are using a “quasi-stationary approximation” for calculating the atomic level populations \((n_k, \text{ where } k \text{ is the principal quantum number})\) in terms of which the values of \(n_k\) are uniquely specified by instantaneous value of the temperature \(T_{ai} = T_e = T\) (see Eq. (1) of this paper), while “for emission behind the front of a [stationary] shock under the conditions of the chromospheres of stars in late spectral classes, the populations of the discrete levels of a hydrogen atom are determined by instantaneous temperature and electron density \(n_e\), but also depend on the entire prehistory of the process, beginning with heating at the shock front” \[5\]. Belova and Bychkov pointed out \[1\] that “for computing the absorption coefficient the authors \[7\] use calculations . . . that are valid for stellar atmospheres under of thermodynamic equilibrium,” “while the situation behind the shock front [propagating at a constant velocity] is not only out of equilibrium, but also non-stationary.” Finally, in \[6\] they \[1\] noted that Katsova et al. \[7\] use a model of the hydrogen atom consisting of just two levels (+ continuum).

Based on solving a system of balance equations for elementary processes \[3\] it has been shown \[8\] that the Menzel factors for the atomic levels of a gas in a motionless homogeneous plane layer with \(T_{ai} = T_e\) corresponding \[7\] to a chromospheric condensation\(^2\) of thickness \(\Delta z_m = 10 \text{ km}\) differ little from unity, and the emission from such layer is transparent in the optical continuum. This fact was viewed \[8\] as an important argument in favor of the viewpoint of Grinin and Sobolev \[11\] on the formation of the quasi-black-body radiation observed at the brightness maximum of powerful

\(^2\)The dense cold formation between the thermal wave front (temperature jump) and the relaxation zone of the plasma to a state of thermal equilibrium behind the front of a non-stationary shock wave (see Fig. 1).
Figure 1. “Profiles of the density, temperature, and velocity of the gas at different times” (Katsova et al. [7]). Here \( n \equiv n_{H} \) is the total density of hydrogen atoms and protons and \( \xi \) is the Lagrangian coordinate:
\[
d\xi = -n_{H}dz
\]
where \( z \) is the height above the photosphere. The concentration \( 10^{15} \) should be read as \( 10^{16} \). The rectangles distinguish the range of values of \( \xi \) corresponding to the relaxation region of the plasma at three points of time. The arrow denotes heating of the gas directly behind the shock front \( (T_{ei} = T_{e} [9]) \).

The smooth temperature profile on the right and the narrow transition zone after it follow from including the term \( F_{c} \) (\( \kappa_{e} \propto T_{e}^{5/2} \) and \( \kappa_{e} \) is the electron thermal conductivity). “Positive values of the velocity correspond to removal of plasma from the star’s surface. The photosphere lies to the left. The dashed curve is the initial model [of the atmosphere]” [7].
stellar flares (the blue component of the optical continuum) near the photosphere. Thus, the critical comments of Belova and Bychkov ([5, 6] and [11]) regarding the work of Katsova et al. [7] apply substantially to the article by Morchenko [8], as well. Also the astrophysical conclusion [1] about the source location of the blue continuum of flares in stellar photospheres which literally regenerated [1] the concept of Gordon and Kron [12], does not agree with the point of view of Refs. 11 and 8.

In this same article [8] it was noted that the heating of the atom-ion component (of the gas) along the Hugoniot adiabatic and the electron component along the Poisson adiabatic [2, 3] is not only inherent to stationary shocks, but also to non-stationary ones. As a result, immediately behind the shock front [7] in the region referred to in [9] as the “relaxation zone [of the plasma] to a state of thermal equilibrium,” \( T_{ni} \gg T_e \) from the beginning (this situation was first noted by Kosovich: see section 5 of Ref. 9). The neglect of this temperature difference was a fundamental shortcoming of the gas dynamic models of stellar [7] and solar [13] flares in the opinion of the author [8, 13].

A possibility of attaining a concentration \( n_H = 3 \times 10^{16} \text{ cm}^{-3} \) (the value near the photosphere [11]) owing to radiative cooling of the gas behind the front of a plane-parallel stationary shock (propagating “downward” in the chromosphere of a red dwarf) was discussed in [8] and the dissertation [15] (assuming [15] no effect of the radiation field of the heated layers lying near the photosphere). This is one of a “set” of waves in the approach [11]. The author showed that: (a) when the frozen-in condition for the magnetic field holds over the entire radiative cooling time, an increase in the gas density by two orders of magnitude is not possible; the corresponding rise in the magnetic pressure \( p_m \) by \( 10^4 \) times halts ([15], pp. 90–91) the plasma compression; (b) when there is no coupling between the changes in \( n_H \) and \( p_m \), the increase in \( n_H \) from \( 3.9 \times 10^{14} \text{ cm}^{-3} \) to \( 3 \times 10^{16} \text{ cm}^{-3} \) [8] corresponding to the hypothetical strong radiative cooling means that the gas flows out of the viscous jump by a small distance \( \Delta l_1 \sim 0.5 \text{ km} \) [8] and for weaker cooling (to \( n_H \approx 10^{15} \text{ cm}^{-3} \)), \( \Delta l \) increases to \( \Delta l_2 \sim 10 \text{ km} \) [15] (results within the framework of a homogeneous plane layer model [3]). Given the smallness [13] \( \Delta l_1 \) compared to the linear sizes of the sources of the blue continuum of the flares on AD Leo (dM4.5e) determined by Lovkaya [16] in the black-body approximation [4] in [8] it was stated that the “gas radiating behind a stationary shock front (propagating in the direction of the photosphere of a red dwarf) is unable to generate the [quasi-]black-body radiation . . . at the brightness maximum of stellar flares.” [5] These results were examined in [15] as yet another argument in favor of the viewpoint [11].

The differences between the model of a stationary radiative shock wave and the model [7] were noted in [8] and [15]. Thus, it was mentioned [8] that a system of equations for one-dimensional gas dynamics in partial derivatives was introduced in [7], while the article of Belova et al. [17] (used by the authors [11]) examines a system of ordinary differential equations (ODE) with detailed

3The term “black body” is applied in [8] to the blue component of the optical continuum of powerful flares. In reality, this component of the radiation is quasi-black-body [11].

4Based on the estimates of the areas of the flares at the brightness maxima (~ \( 10^{18} \text{ cm}^2 \)) in a plane layer model.

5The radiative recombination time \( t_r \) of a dense chromospheric gas is low ([11], p. 355) and the picture of radiative cooling behind the shock front is independent of the choice of reference frame.
accounting for elementary processes in the post-shock plasma. Also in \[15\] the neglect of thermal conductivity \( F_c = 0 \) responsible \[18\] for the transfer of energy in the high-temperature region of flares (initially heated by a beam of non-thermal electrons to \( T_c \sim 10^7 \text{ K}, T_i \sim 10^6 \text{ K} \) \[19\]) was noted (in \[1\] (p. 236), the ion temperature \( T_i \) of the gas is referred to as atom-ion one). In addition, the increase in the geometric thickness \( \Delta z \) of the chromospheric condensation \[7\] (referred to as c.c. in the following text) during impulsive heating was noted (a non-stationary radiative shock wave \[7\] precedes a temperature jump moving at a subsonic speed). Belova and Bychkov \[1\] in criticizing the paper of Katsova et al. \[7\] recall (see the section “Discussion”) the first result in \[8\], and ignore the remarks \[8, 15\] on the differences in the models \[7, 17\].

Finally, the origin of the \( \text{H}_\alpha \) line profile with a blue asymmetry in the wings in the spectrum of a flare on UV Ceti (dM5.6e) (Eason et al. \[20\]) is discussed in Refs. 3, 8, and 13. Thus, it was noted that: (a) the model profile with a Doppler core (half width \( \Delta \lambda_D = 0.9 \text{ Å} \) \[20\]) and Stark wings (\( \lg n_e = 14.75 \) \[20\]) is similar to the \( \text{H}_\alpha \) line profile (because the parameter \( b_{32} \) \[3\] \( \ll 1 \)); (b) the deviation from a Stark profile for fitting of the right wing of the line (see Fig. 9 of Ref. 20) may be caused \[21\] by neglecting the contribution of electron broadening to the formation of significantly opaque (optically depth \( \gg 1 \)) wing of \( \text{H}_\alpha \); (c) the Doppler core of \( \text{H}_\alpha \) \[20\] is shifted as a unified whole to the left \[8\]. In \[13\] it is stated that this kind of profile can be generated by the gas behind a shock front which propagates “upward” in the partially ionized chromosphere of a red dwarf (on the basis \[8\] that in a laboratory reference system the core of a line in the plasma behind a shock front should be “shifted” in the direction of motion of the front).

The first part of this paper (I) contains a comparative analysis of the approaches \[7\] and \[1\]. In section 2: (a) it is argued that a “set” of stationary radiative shock waves \[1\] cannot ensure simultaneous fulfillment of the heat balance condition (between \( P_e \) and the radiative energy losses behind the shock front) and an increase in the thickness \( \Delta z \) of the c.c. (cold gas ahead a thermal wave) — the divergence effect between the wave fronts — during impulsive heating, as in the model \[7\]; (b) it is demonstrated that the idea ( \[5, 6, 1\] ) of the formation of a c.c. owing to radiative cooling of the plasma in isolation from the thermal wave is not based on the fundamental article of Kostyuk and Pikel'ner \[22\], so that the critique of Belova and Bychkov ( \[5, 6, 1\] ) regarding the paper of Katsova et al. \[7\] performed within the framework of the approach of Ref. 22 (see Refs. 7 and 3) is incorrect in the author’s opinion; (c) the use \[8\] of a fixed homogeneous plane layer of pure hydrogen plasma for an approximate analysis of the radiative characteristics of the high-density region \[7\] of thickness \( \Delta z_m \) (instantaneous picture) is justified. In particular, it is noteworthy that the claims \[8\] that the Menzel factors of this layer are close to unity and of its transparency in the continuum beyond the Balmer jump do not contradict Ref. 23 for gas dynamic modeling by Allred et al. \[24\] (the populations of atomic levels are non-stationary explicitly).

In section 3 of this article it is shown that the “set” of shock waves \[1\], as opposed to models of the type of Ref. 7, cannot heat the chromosphere of a red dwarf at significant distances. In addition, \[6\] it is easy to confirm this by drawing two vertical segments from the divisions corresponding to wavelengths of 6562 and 6564 Å in Fig. 7a of Ref. 20 and by comparing the areas on the left and right sides from the Doppler profile.
it is argued that: (a) the calculations of Belova and Bychkov [1] represent a development of the
kinematic model of solar flares (Nakagawa et al. [25]) and its application to dMe stars, specifically:
a study of the radiative response of the chromosphere to impulsive heating in the simplest gas
dynamic statement of the problem (a thermal wave is excluded and a stationary approach is used);
(b) as in [25], the profiles of the \( \text{H}_\alpha \) line in the model [1] conflict [22] with spectral observations; (c)
from the standpoint of the model [22], the regions behind the stationary shock fronts [1] correspond
not to the c.c., with time-varying thickness (in [7] from \( \sim 1 \) to \( \sim 10 \) km), but to zones in which
the plasma relaxes to a state of thermal equilibrium; (d) the conclusion of Refs. 8 and 15 that it
is impossible to generate (quasi-)black-body radiation (at the maximum brightness of stellar flares)
behind the front of one of these waves (velocity \( u_0 = 60 \text{ km/s} \) [8]) is confirmed by the calculations of
Ref. 1. It is emphasized that the separation (Belova and Bychkov [1]) of the model of Kostyuk and
Pikel’ner [22] (basis of modern gas dynamic program packages modeling the secondary processes in
solar and stellar flares) into “thermal” and “shock-wave” components is fundamentally impossible.

In the second part of this article (in preparation) it is argued that both the blue and red
elements of the optical continuum of stellar flares are formed near the photosphere [11] and the
point of view [1] according to which the source of “hot” quasi-black-body radiation is localized in the
photosphere of a red dwarf conflicts with observational data. Also: (a) the effect of the radiation
field of heated near photospheric layers (at the maximum of the brightness of flares) on gas dynamic
processes taking place in the overlying layers of the chromosphere is discussed in more detail than
in [3, 8] and [15]; (b) the fundamental possibility is discussed of the appearance and enhancement
of \( \text{HeI} \) lines (e.g., [20]) in the thermal relaxation zone [7, 9] (with increasing \( T_e \) of the gas behind
the front of a non-stationary chromospheric shock wave owing to elastic collisions of electrons with
atoms and ions \((T_{ni} \gg T_e)\); and the origin of the \( \text{H}_\alpha \) line profiles with a blue asymmetry in the
wings is discussed considering interpretations that differ from Ref. 13.

2. Chromospheric condensation in the model of Ref. 7 and the gas behind a stationary
shock front.

2.1. We begin by noting two fundamental differences in the calculations [7] from the approach [1]:
(a) the system of gas dynamic equations [7] includes the power of heating by a beam of ac-
celerated electrons \( P_e(\xi) \), the classical thermal flux \( F_c \) (Fourier law), gravitational acceleration
\((g = \text{const})\), and the cooling function \( L(T) \) used over the entire range of the plasma temperature.
In the calculations [1] \( P_e = 0 \), \( F_c = 0 \), and \( g = 0 \). Primary attention is devoted to radiative cooling;
(b) the shock wave [7] is non-stationary (it propagates in the chromosphere of a red dwarf in
the direction of increasing density, i.e., “downward”); in deep layers of the chromosphere the wave
transforms into a sound perturbation of a discontinuous type ( [22], p. 594). In the approach of
Ref. 1 the shock velocity \( u_0 = \text{const} \) and the pre-shock gas is uniform (a so-called stationary shock
wave). The problem of accounting for the density gradient in the chromosphere is solved palliatively
in Ref. 1; they consider a range of values of \( u_0 \) from 30 to 100 km/s and obtain a corresponding
“set” of profiles of stationary radiative shock waves.

The difference “a” shows up in that in the model of Ref. 7 the thermal wave is responsible

\footnote{For high shock velocities corresponding [26] to the high fluxes \( F_0 \) in the non-thermal electron beams.}
for production of the compression wave ahead of itself, and after some time (at the heating of the denser plasma) it becomes a shock; “the thermal front then acts as a piston pushing the gas” [9] (a so-called temperature wave of the second kind [27]). As a result of the emission from the post-shock gas of the non-stationary shock wave, a layer of dense cold plasma appears ahead the temperature jump with a typical flat profile of $T$ (Fig. 1): the radiative cooling ceases when the loss of energy owing to radiation $L(T)$ becomes comparable to the energy influx $P_e$ from the thermal wave (the transition of the shock wave in the stationary regime [9]).

During impulsive heating, the fronts of the thermal and shock waves diverge (the temperature jump moves at a subsonic speed [7, 27]), so that the geometric thickness $\Delta z$ of the cold gas (after passage of the shock wave), the c.c., increases. (It is clear from Fig. 1 how the region of elevated density is established at an ever higher range of values of $\xi$.) As a result, closer to the end of heating, the width of the thermal relaxation zone $\Delta l_1 \sim 0.5 \text{ km}$ [8] turns out to be small compared to $\Delta z_m$.

The authors [1] assume that the “set” of stationary radiative shocks in the chromosphere of a red dwarf exists independently of the thermal wave ($P_e = 0$). As a result, the regions behind the fronts of the shock waves [1] in the calculations of Ref. 7 correspond formally to the zones where the plasma relaxes to a state of thermal equilibrium, with a sharp increase in $n_H$ owing to radiative cooling (see Fig. 1).

The difference “b” shows up in that in the approach of Ref. 1, the accounting for the radiative losses of the gas behind a shock front is substantially more precise than in the model of Ref. 7. This result is attained through substantial simplifications in the gas dynamic part of the problem statement [1] compared to that of Ref. 7: the thermal wave is excluded and a stationary approach is used (see section 10 in [28]). In addition, in [1], $T_{ai}$ and $T_e$, the concentrations of the plasma components, and the relative populations of the HI levels ($\nu_k = n_k/n_H$) are found as functions of $t$ by solving a system of ODE for $\frac{d\nu_k}{dt} \neq 0$, the ionization states of the gas components, the internal energy of the plasma, and the derivative $dT_e/dt$ (Eqs. (32), (40), (41), (57), and (89) in [17]) and a number of auxiliary algebraic equations. Here $t$ is the time since the time a given element of the gas intersected the shock front. The shock is stationary, so $\bar{l} = u(t)dt$, where $\bar{l}$ is the distance from the viscous jump, and $u(t)$ is the velocity of the gas in the reference system associated to the discontinuity at time $t$. (A detailed derivation of the system of equations for calculating the profile of a stationary radiative shock (under the conditions of the partially ionized chromospheres of dMe stars) including the effect of the radiation field of the heated near photospheric layers is given in Ref. 15, pp. 22-28, 71-90.)

Given these remarks, we have the following differences in the non-stationary cooling of the

---

8 An increase in $n_H$ by two orders of magnitude behind the front of the downward moving (toward the photosphere) shock is discussed in Ref. 7. At the same time, an increase in the concentration by a factor of $\approx 77 \sim 100$ is “hidden away” in the estimate of $\Delta l_1$ [8].

9 The calculations of Ref. 17 used [18] a two-level approximation: it assumes that $\nu_2(t) = 4\nu_1(t)[1+\Omega_{21}^{-1}]^{-1} \cdot \exp(-\Delta E_{12}/k_b T_e)$, where $\Omega_{21} = q_{21} n_e/2 A_{21}$. Here $\Delta E_{12}$ is the excitation energy of the second level relative to the ground one, $k_b$ is the Boltzmann constant, $q_{21}$ is the electron impact de-excitation coefficient for the hydrogen atom, and $A_{21}$ is the effective spontaneous transition probability.
plasma behind the shock fronts in the models discussed here:

(a) Ref. 7: the radiative cooling is caused by the inequality of the heating and cooling functions owing to the sharp rise in $T$ and $n_H$ of the plasma; a reduction in the gas temperature and a rise in its density owing to emission continue until a thermal balance is established between the influx of energy from the non-thermal electrons and loss owing to radiative cooling;

(b) Ref. 1: the plasma parameters corresponding to the cold gas are specified by the choice of the final step in $t$. The increase in plasma density is limited only by the pressure rise of the magnetic field and the influx of energy from “photospheric emission” with a temperature $< 6 \cdot 10^3 \text{ K}$.

Thus, the “set” of shock waves cannot ensure simultaneous satisfaction of the condition for heat balance and an increase in the thickness of the c.c., as occurs in Ref. 7.

2.2. Belova and Bychkov [1] expound their concept of the calculations of the type (see their critique of the work of Allred et al. [21], in Refs. 6 and 1) in the following way: “in [7], a hypothesis of “chromospheric condensation” is formulated, according to which black-body radiation originates from a region of size about 10 km located at a height of roughly 15000 km (so in [1]) and formed by gas that has been isobarically compressed by radiative cooling behind a shock front to a temperature of about 9000 K and density of about $10^{15} \text{ cm}^{-3}$.” Likewise, in justification of their views, the authors [1] introduce a citation from the abstract of the pioneering article of Kostyuk and Pikel’ner [22]: “a temperature jump that is primarily associated with thermal conductivity propagates through the gas. A shock wave propagates ahead of the temperature jump, the wave heats and compresses the gas. The velocity of motion decreases with depth.”

Here Belova and Bychkov [1] omit the following sentence from the abstract for that article: “in this paper, the viscosity is neglected, and heating by the shock does not appear explicitly, but it is estimated from the Hugoniot adiabatic curve.” Thus, “the lower front of the motion” in the calculations of Ref. 22 “is not a shock wave...” ( [22], p. 594). Therefore, in [22], the divergence of the fronts of the thermal and shock waves during a time of impulsive heating could not be described, as it is in Ref. 7, where an artificial (quadratic) viscosity $\omega$ [7, 29] is introduced; the authors [22] hoped to “solve the problem with viscosity, at a velocity downward to 100 km/s,” later [22]. This conclusion is confirmed on comparing the geometric thickness $\Delta z$ of the region in which the $H\alpha$ line emission (the conditions in the solar chromosphere) is predominantly localized; in Ref. 22 it is 8 km [30] (pulse duration 100 s, “an illusion of continuity” caused by poor time resolution [1]), while in Ref. 14 it goes to $\sim 10$ km over 10 s of impulsive heating (a single elementary flare burst, EFB).

The authors [1] also do not consider the following comment of Kostyuk and Pikel’ner [22] (p. 593) regarding the qualitative picture of processes in the region of the optical burst (in $H\alpha$): “the gas heated by the shock cools sooner than the temperature jump reaches it,” so that “the temperature has a minimum between two heated regions.” Also [22]: (a) the shock speed (“downward”) depends on the energy flux $F_0$ in the beam of accelerated electrons, the radiative energy loss per cm$^2$ of the region ahead the thermal wave (the cold radiating gas), the density of the unperturbed chromosphere, and the plasma temperature in the hot region (at the jump); (b) part of the c.c. is similar [22] to the flare element in the model of Brown [31], where the energy influx from the non-thermal electrons is balanced by radiative losses in $H\alpha$. 
In addition, the simplified representation [1] about a temperature jump ahead of which a non-stationary radiative shock propagates does not fully reflect the complicated interaction of the thermal and shock waves in the calculations of Ref. 7. Thus, it is noted [7] that “over a certain time the thermal wave ‘amplifies’ the shock wave.”

Therefore, the claim by Belova and Bychkov [1, 5, 6] that the formation of the c.c. is caused exclusively by radiative cooling (“in a separation” from the thermal wave) is not based on the Kostyuk-Pikel’ner model [22]. (We recall that in Ref. 1 \( P_e = 0, F_c = 0, g = 0, \) and \( Q = 0 \).) For this reason, the critique [1, 5, 6] regarding the paper of Ref. 7 in terms of this model is not correct.

2.3. Referring to the book [32], Katsova et al. [7] assume that in sufficiently dense layers of the unperturbed chromosphere, where the optical depth in the resonance transition \( \tau_{cr} = 10^6 \) [7], the populations of hydrogen levels are determined by the Boltzmann formula

\[
\frac{n_2}{n_1} = 4 \cdot \exp \left( -\frac{\Delta E_{12}}{T_{eV}} \right) = f(T),
\]

(1)

where \( T_{eV} \) is the gas temperature in electron-volts. Here [7] the degree of ionization of the plasma \( x^* = n_e/n_H \) [7] and \( n_2 \) are found by solving the system of equations (1) and (2):

\[
n_1 n_e q_1 + n_2 n_e q_2 = n_e^2 \tilde{\alpha},
\]

(2)

where \( q_1 \) and \( q_2 \) are the electron-impact ionization coefficients of the atom from levels 1 and 2, and \( \tilde{\alpha} \) is the total coefficient of spontaneous photorecombination to all levels except the first (an approximate accounting for \( L_c \) radiation scattering). \( n_H \) and \( T \) are assumed to be specified [10].

In Ref. 7 it is stated that the heating and cooling functions \( (P_e \text{ and } L) \), classical thermal flux \( F_c \), pressure \( p \), and energy \( \varepsilon \) were calculated “simultaneously with the degree of ionization of the hydrogen plasma” [11]. For this reason, on one hand the relation of \( n_e \) and \( n_H \) in the c.c. should be determined from Eqs. (1) and (2), and on the other, a balance [33] between the energy influx from the thermal wave \( (\xi > \xi_0 = E_{10}^2/2a, \text{ where } a \text{ is a function of } E_{10} \text{ and } n_H \) [34]) and the radiative losses in the H\(_\alpha\) line \( (L_{H_\alpha}) \) must be ensured: a “manifestation” of the flare element of Brown [31] in the model [22].

Following Ref. 33 we check fulfillment of these statements in Ref. 7. We begin with the fact that 0.8 s have passed from the beginning of impulsive heating (the fronts of the thermal and shock waves have not separated too strongly); we do not examine [33] the plasma layers on reaching the temperature jump since in this part of the c.c. there should be movements such as an “upward” expansion [31].

Solving the system of Eqs. (1) and (2) yields \( x^* = \left(1 + \tilde{\alpha} (1 + f(T)) / (q_1 + q_2 f(T)) \right)^{-1} \); it is clear that \( x^* = x^*(T) \). \( T \approx 9000 \text{ K} \) \( (T_{eV} \approx 0.78 \text{ eV}) \), so that \( f(T) \approx 7.7 \cdot 10^{-6} \ll 1 \) (i.e., \( n_2 \ll n_1 \)) and \( x^* \approx 0.03 \). Here as in Refs. 7 and 14, the coefficient \( \tilde{\alpha} \) was calculated according to Seaton,

\[\text{It should be noted that Eqs. (1) and (2) contradict one another: the Saha equation should be used (as in Ref. 22) instead of Eq. (2), neglecting the effect of the radiation field of the photosphere of a red dwarf.} \]

\[\text{In the model [7], in the upper layers of the chromosphere the quantity } x^* \text{ was “replaced by the analogous quantity” } x, \text{ defined within the framework of the “one level + continuum” model of an atom.} \]
but \( q_1 \) and \( q_2 \), according to Johnson \[35\] \((q_1 \ll q_2)\) [12] From Eqs. (1) and (2) we also find that \( n_2 \approx (1 - x^*)n_Hf(T) \). A Lagrange variable of \( \xi_1 = 10^{20} \text{ cm}^{-2} \) corresponds to \( n_H \approx 6 \cdot 10^{15} \text{ cm}^{-3} \) (see Fig. 1), whence \( n_2 \approx 4.5 \cdot 10^{10} \text{ cm}^{-3} \).

In turn \[7\],

\[
P_e(\xi)/n_H = 0.7x^*P_1(\xi) + 0.3P_1(\xi),
\]

where \( P_1(\xi) \) is the Syrovatskii and Shmeleva function \[34\] (the so-called CEA approach \[26\]); for \( \gamma = 3, E_{20} \to \infty \) (\( E_{20} \) is the upper boundary of the accelerated electron spectrum), \( \xi \geq \xi_0 \)

\[
P_1(\xi) = 2^{-3.5}\pi F_0E_{10}^0\xi^{-1.5} \cdot \xi^{-6} \approx 1.9 \cdot 10^{-9} \text{ erg/s}.\]

From Eq. (3) we finally obtain \( P_e(\xi_1) \approx 6.1 \cdot 10^{-10}n_H \).

On the other hand, the radiative energy loss in the \( \text{H}_0 \) line \[7\] is:

\[
L_{\text{H}_0} \approx 0.14n_2x^*q_{23}n_H\text{Ry};
\]

taking \( q_{23} \approx 5 \cdot 10^{-8} \text{ cm}^3/\text{s} \) (the electron-impact excitation coefficient for the atom) \[35\], we have \( L_{\text{H}_0} \approx 1.9 \cdot 10^{-10}n_H \). (We neglect the contribution of \( L_{\text{L}_0} \) \[33\] since the escape probability for a resonance photon outside of the c.c. is low.)

The absence of a divergence between \( P_e(\xi_1) \) and \( L_{\text{H}_0} \) by orders of magnitude serves as a quantitative confirmation that the calculations of Ref. 7, as opposed to those of Belova and Bychkov \[11\] which exclude a thermal wave, have been done \[7\] within the framework of the model of Kostyuk and Piklen'ner \[22\].

2.4. The authors of Ref. 7 analyze the characteristics of the emission from the c.c., with time-varying thickness (in particular, for \( \Delta z_m \)), using the results of Grinin and Sobolev \[11\] for a fixed homogeneous plane layer of pure hydrogen plasma with a source function \( S_\nu = B_\nu(T) \). Thus, Katsova et al. \[7\] set up a correspondence between the “densifications” \[7\] and the “set” of plane layers (instantaneous pictures \[13\] while simultaneously correcting \( x^* \) by the equilibrium value \( x^{eq} \) for fixed \( n_H, \Delta z, \) and \( T \) (the populations of the atomic levels of the gas in the c.c. correspond to the Boltzmann formula in view of Eq. (1) and the above discussed features of the problem statement in Ref. 7; for \( T \approx 9000 \text{ K}, n_H = 2 \cdot 10^{15} \text{ cm}^{-3} \), and \( \Delta z_m x^{eq} \approx 0.15 \)).

Numerical solution of the system of balance equations for the elementary processes \[3\] shows \[8\] that within a motionless homogeneous plane layer of thickness \( \Delta z_m \) with \( T_{ai} = T_e \) (in Ref. 7 the cooled and, thus, single-temperature \[8\] gas behind the thermal relaxation zone) the Menzel factors of the plasma differ little from unity (i.e., \( S_\nu \approx B_\nu(T) \)): given the high density of the gas, its low temperature, and high optical depth in spectral lines (for the resonance transition at the center of the layer \( \tau_{\text{L}_0} \approx 3 \cdot 10^7 \) — see Eq. (53) in Ref. 3) the radiative processes turn out to be secondary \[3\] compared to collisional ones. In particular, for \textit{conservative scattering} the average escape probability over the layer for a resonance photon beyond the confines of the plasma (in the case of a symmetric model profile of the spectral line with a Doppler core and Holtsmark wings) is \( \theta_{\text{L}_0} \sim 10^{-6} \ll 1 \) \[3 \& 8\] (recall that \( \theta_{\text{L}_0} \) obtained with an approximate analytic solution \[37\] of

\[12\] In general, in the range of \( T \) from \( 10^4 \) K to \( 2.5 \cdot 10^4 \) K of astrophysical interest, \( q_2 \) \textit{increases} \[36\] as the principal quantum number gets larger.

\[13\] With this approach, information on the distribution of the velocities, “both upward and downward” (\[22\], p. 596), in the c.c. is lost; the motion of the layers set up in Ref. 7 in accordance with “downward” “densifications” is ignored \[33\].
the radiative transfer equation, for \( \tau_{L_\alpha} \gg 1 \) turns out to be close to the escape probability for a photon beyond the confines of the plasma without scattering from the center of the layer). The smallness of \( \theta_{L_\alpha} \) also implies the secondary importance of including the motion of the layer as a unified whole in the direction toward the photosphere when solving the system of steady state equations \([3]\) (but without calculating the H\(_\alpha\) line profile \([22, 30]\)).

2.5. In the article of Allred et al. \([24]\), the response of the chromosphere of the Sun and dMe stars to impulsive heating by a beam of non-thermal electrons has been modeled by combined solution of the equations of radiative gas dynamics (plane-parallel approximation; \( g \neq 0 \)), the equations for non-stationary \([39]\) atomic level populations, and the radiative transfer equation. For the questions discussed here the important fact (\([23]\), pp. 9, 10) is that for fluxes \( F_0 \) equal to \( 5 \cdot 10^{11} \) erg/cm\(^2\)s (the Sun’s chromosphere) and \( 10^{13} \) erg/cm\(^2\)s (dMe stars; referred to below as model F13), the populations of hydrogen levels in chromospheric condensations are close to the LTE values \(\text{after some time}\) (see table 2 in Ref. 23) after the onset of impulsive heating, except for the upper part (a thickness of \( \sim 1 \) km) for each condensation (zone behind the non-stationary shock front), where the populations of levels with \( k = 1, 2 \) deviate significantly from their equilibrium values.

From a physical standpoint these results are caused by the following:

(a) including in the conservation law of the internal energy of the gas \([24]\): the divergence of the radiative flux \( \partial F_r / \partial z \) (\( z \) is the height per unit mass) and the power of the plasma heating by the beam of accelerated electrons (enters with the opposite sign) which ensure fulfillment of the condition for thermal balance, and the divergence of the thermal conductivity flux (same sign as \( \partial F_r / \partial z \));

(b) the high density \([23]\) of the gas in the chromospheric condensations, their substantial geometric thickness (\( \sim \) several tens of km along the vertical \([23]\)) and the comparatively low \( T \) of the plasma ahead the thermal wave (in the F13 model \( T \sim 10^4 \) K \([23]\)).

The motionless homogeneous plane layer of thickness \( \Delta z_m \) with \( n_{H} = 2 \cdot 10^{15} \) cm\(^{-3}\), \( T \approx 9300 \) K produces radiation with an optical depth at a wavelength of 4170 \( \text{Å} \) \( \tau_{4170} \approx 0.02 \) \([3, 8]\). On the other hand, in Ref. 23, \( \approx 2 \) s after the onset of heating (calculation with \( F_0 = 10^{13} \) erg/cm\(^2\)s, \( \gamma = 3, E_{10} = 37 \) keV) \( \tau_{4170} \approx 0.5 \) in \( \text{the lower part of} \) a c.c. of thickness \( \sim 20 \) km c \( n_{e_{\text{max}}} \approx 5 \cdot 10^{15} \) cm\(^{-3}\) at \( T \sim 10^4 \) K.

Thus, the conclusion of Ref. 8 regarding the transparency of the gas \( \tau_{4170} \ll 1 \) in a layer with parameters corresponding to the c.c. \([7]\) (model F12) \text{does not conflict} with the results of Ref. 23 obtained by gas dynamic modeling (the populations of the atomic levels are non-stationary explicitly) and is, thereby, positive.

3. Shock-wave model of stellar flares. The characteristic time for recombination radiation of an optically \textit{thin} gas behind a stationary shock front (one of the “set” of waves in the approach of Ref. 1) is

\[
t_r \sim (\alpha \cdot n_e)^{-1},
\]

\( 14 \) We note that the definition of c.c. given in Ref. 9 differs from that used in Ref. 23.

\( 15 \) The inconsistency of Ref. 14 regarding the nature of white flares on dMe stars with the results of RADYN in the section on the value of the energy flux \( F_0 \) was first pointed out by Kowalski \([10]\).
where $\alpha$ is the spontaneous radiative recombination coefficient summed over the levels $k = 2, \ldots, k_{\text{max}}$ ($k_{\text{max}}$ is given by the Inglis-Teller criterion). Thence, for $n_e = 10^{14} \text{cm}^{-3}$, $T_e = 10^4 \text{K}$ and using the Kramers approximation, we obtain $t_r \approx 4 \cdot 10^{-2} \text{s}$.

On the other hand, the time for evolution of the c.c. [7] (the cold gas ahead a thermal wave) is $t_h \approx 9 \text{s}$ (see Fig. 1), comparable to the duration of all the impulsive heating, 10 s. Furthermore, after heating ceases ($t > 10$ s) the shock [7] is rapidly damped [9]. Thus, the time of existence and penetration depth into the chromosphere of the non-stationary shock are first of all determined by the presence of an influx of energy from accelerated electrons (through the thermal wave), i.e., the duration of the impulsive heating [4, 7]. Remembering that the duration of the line emission in flares ranges from several minutes and longer [11] and given that a flare consists of a “set” of elementary events (for example, Ref. 20), we find that $t_h \sim 10$ s is a good agreement between theory and observations. Thus, without a constant influx of energy from outside, the “set” of shock waves [1] cannot exist for a long time.

Finally, multiplying $t_r$ by $u_0 = 100 \text{km/s}$, we find that over the radiative cooling time, the corresponding shock wave [1] travels a path of $\approx 4 \text{km}$. Therefore, the “set” of shock waves [1] cannot heat the chromosphere of a red dwarf over significant distances.

For these reasons it is impossible to agree with the claim of Belova and Bychkov [1] that the “model of a shock wave propagating in the gas of the chromosphere can be used to explain ... emission of the hydrogen plasma that is transparent at continuum frequencies.” The component of the emission from stellar flares that is transparent in the continuum is mainly formed in the c.c., but not only [1] in the zone behind the front of the non-stationary shock wave where the plasma relaxes to a state of thermal equilibrium.

We note that the above comments against the approach of Ref. 1 are known in the scientific literature [4, 10, 42] as applied to the kinematic model of solar flares (Nakagawa et al. [25]), where it is proposed that a “set” of stationary shock waves that heat and compress the gas ($g \neq 0$, single-temperature post-shock heating (with $T_{\text{ai}} = T_e$), the cooling function with a power law dependence on $T$) and, thereby, intensify the H$_\alpha$ radiation [22], propagates in the chromosphere. In fact, the calculations of Ref. 1 represent a development of the model of Ref. 25 and its application to dMe stars, specifically: a study of the radiative response of the chromosphere of a red dwarf to impulsive heating in the simplest gas dynamic formulation of the problem (a thermal wave is excluded and a stationary approach [28] is used).

An inconsistency of the H$_\alpha$ line profile in the model of Ref. 25 with observational data was noted in the papers by Canfield and Athay [43] and Kostyuk and Pikel’ner [22]: according to the calculations of Ref. 43, the profile “has a strong, shifted central reversal, so that in Ref. 43 it was necessary to assume turbulent motions at velocities of 40–70 km/s” [22]. From a physical standpoint these defects of the profile are caused, on one hand by the large optical depth in the center of the H$_\alpha$ line core and, on the other, by the significant velocity of the gas behind the stationary shock front in the laboratory coordinate system: $u_1 < 0.75u_0$ (we neglect radiative cooling). Therefore, the H$_\alpha$ line profiles in the approach of Ref. 1 contradict the observational data.\footnote{The optical depth in the resonance transition in the radiative cooling region of the gas behind a shock...}
It was also noted [22] (p. 598) that in the approach [22] the Hα profile is a “combination” [22] of the profile corresponding to the gas ahead a heat wave (the Brown model [31]: a symmetric profile with a deep reversal in the center [44]) and a line profile in the kinematic model [25]. Since the “maximum of one profile is superimposed to the minimum of the other ... a strong reversal is not obtained” [22]. In other words, the Hα line will have a symmetric core with a shallow “dip” in the center and a red asymmetry of the wings [45].

The above remarks imply that the model of Kostyuk and Pikel’ner [22] lying at the basis of modern gas dynamic program packages for modeling secondary processes in solar and stellar flares does not in principle allow “splitting” (Belova and Bychkov [1]) into “thermal” and “shock wave” components; otherwise it “degenerates” into the model of Nakagawa et al. [25]. Therein lies the main difficulty.

Nevertheless, calculating the profiles of stationary radiative shocks is of interest [15] from the standpoint of support of views [11] regarding the nature of the blue component of the optical continuum at the brightness maximum of powerful flares on dMe stars, since it is precisely in the thermal relaxation zone the plasma density nH increases sharply owing to radiative cooling. The conclusion [8] that the quasi-black-body radiation cannot be generated in flares by post-shock gas of one of a “set” of shock waves [1] (velocity u0 = 60 km/s) under the conditions of the chromospheres of red dwarf stars is based on the optical depth in the Lα line (∼ 10⁷ [3]) and the hydrogen atom concentration in the ground state (2 · 10¹⁶ cm⁻³, the hypothetical strong radiative cooling regime) in the region where the gas radiates. This conclusion does not use the approximation of stationary populations behind the shock front, so it is correct. It is now confirmed [1] by direct calculation [17]

4. Additional comments. We emphasize that, as opposed to Refs. 11 and 8, in Refs. 46 and 23, interpretations are given not only of the blue but also of the NUV components of the continuum spectrum during the impulsive phase of stellar flares, as well as of the energy distribution over wavelengths of 3646 – 3730 Å. For this, Kowalski [46] uses a “composite” model of a perturbed chromosphere of a red dwarf that includes a c.c. (flux F13) and stationary layers with T ∼ 9000 – 12000 K and a thickness of a few hundred km lying ahead of the front of a non-stationary shock. These layers are heated [23] by high energy (Ee ≫ E₁₀) electrons from a beam with a falling power-law spectrum — a mechanism pointed out in Ref. 22 (p. 594).

The difficulties with the F13 approach owing to the so-called reverse current problem (e.g., Ref. 19) are noted in Ref. 23. In addition, the number of electrons with Ee ≫ E₁₀ = 37 keV [46] is substantially lower than with a kinetic energy near E₁₀.

The authors of Ref. 23 believe that stationary layers can make a significant contribution to the continuum spectrum of flares if the optical thickness in the c.c. is ≪ 1. It is clear that this point of front [1] propagating at a velocity of u₀ = 60 km/s, τLα ∼ 10⁷ [3] (the value of τLα is reckoned from the viscous jump).

17It is necessary, however, to caution against attempts to treat Ref. 1 as a calculation of an improved cooling function that can be used in the complete system of gas dynamic equations [7] instead of the corresponding function L(T): the theory of stationary radiative shock waves is an independent area of radiative gas dynamics.
view differs from that discussed in Refs. 11 and 8. At the same time, a comparison of the theoretical (for different radiation mechanisms) and observed color indices implies \[17\] that the best agreement is obtained for a combination of “short-lived [quasi-]black-body radiation near the maxima of a flare and the long-lived radiation from a hydrogen plasma with temperatures and densities somewhat higher than in the unperturbed chromosphere.”

Belova and Bychkov \[1\] assume that their calculations “... do not confirm the hypothesis advanced in Ref. 7 of a bright optically thick, in the continuum, “chromospheric condensation,” formed during a flare by radiative cooling of the post-shock gas.” As noted above, the regions behind the fronts of the stationary shocks \[1\] in the model of Ref. 7 correspond formally to zones in which the plasma relaxes to a state of thermal equilibrium (see Fig. 1) and not to a dense cold gas ahead a thermal wave (a c.c. of thickness $\Delta z_m$). For this reason, the calculations \[1\] cannot serve as a replacement for the analysis \[8\] of the characteristics of the continuous optical radiation from a layer with parameters corresponding to a “densification” \[7\], just as they cannot supplement the selection \[13, 15\] of errors by Katsova \[48\] in Refs. 7 and 14.

5. Discussion. The analysis in sections 2 and 3 of this article makes it possible to discuss in more detail than in \[8\] and \[13\], the statement of the problem in \[3\] and the results obtained there. The authors of Ref. 3 calculated the spectrum of the radiation from a homogeneous plane layer of pure hydrogen plasma with $T_{ai} \neq T_e$ passing through the front of a stationary shock wave (one of the “set” of waves in the approach of Ref. 1). The layer was assumed to be motionless, since the post-shock gas moves at a subsonic speed relative to the viscous jump; no precursor was included. The physical parameters of the layer ($T_{ai}$, $T_e$, and $\tau L_\alpha$) were chosen on the basis of a calculation \[17\] of the profile of a stationary radiative shock under the conditions of the atmospheres of the variable stars of the type $\alpha$ Cet (the density of the gas ahead of the front is $n_0 = 10^{12}$ cm$^{-3}$, $u_0 = 60$ km/s). A range of $n_H$ from $3 \cdot 10^{14}$ cm$^{-3}$ to $3 \cdot 10^{16}$ cm$^{-3}$ was examined.

Reference 3 is a study of the influence of the post-shock inequality \[2\] for the atom-ion and electron temperatures of the plasma on the formation of the emission spectrum of this kind of layer. We note that from the standpoint of the theory of stationary radiative shock waves these calculations are abstract: this is indicated by the invariance of the values of the atom-ion and electron temperatures in the confines of the layer \[3\] and by the solution of the balance equations for the elementary processes involving the atomic level populations.

The author \[8\] has confirmed that the results of Ref. 3 are correct even in the single-temperature case, \ie, when $T_{ai} = T_e$. In particular, the following remain unchanged: (a) Eq. (71) for the full intensity of the radiation in the Balmer series lines including the fact that for certain shifts in frequency from the center, the wings of the lines are “immersed” (Eason et al. \[20\], p. 1167) in the continuum (here the asymmetry of the wings was neglected for simplicity); (b) the criterion for applicability in the calculations of a symmetric model profile with a Doppler core and Holtsmark wings (Eq. (49) in Ref. 3).

---

18The fact that the homogeneous layer \[3\] generates a continuous spectrum that is close to a black-body spectrum (see Fig. 2 of Ref. 3) indicates that the difference between $T_{ai}$ and $T_e$ \[2, 8\] had no significant influence on the formation of the emission spectrum of this kind of layer.
We emphasize that the c.c. in the model [7] – a single-temperature layer (cold radiating gas ahead of a thermal wave) – and a two-temperature layer [3] are “unified” only by the formal closeness of $\Delta z_m$ to $\Delta l_2$ or to the layer thickness [3] of 10 km.

Further, a number of corrections and refinements to bring the article [8] into line with [13] are given.

The author thanks Drs. Yu. A. Fadeyev and N. N. Chugai for useful comments made during discussions of the basic results of Refs. 3 and 8 and the dissertation [15] during the Astrophysics Seminar at the Institute of Astronomy of the Russian Academy of Sciences (INASAN).

References

1. O. M. Belova and K. V. Bychkov, Astrophysics 62, 234 (2019). doi: 10.1007/s10511-019-09577-4 [http://astro.asj-oa.am/4213/1/267%2D.pdf]

2. S. B. Pikel’ner, Izv. Krymsk. Astrofiz. Obs. 12, 93 (1954).

3. E. Morchenko, K. Bychkov, and M. Livshits, Astrophys. Space Sci. 357, 119 (2015). arXiv: 1504.02749.

4. B. V. Somov, Proc. Phys. Inst. Acad. Sci. USSR 110, 57 (1979).

5. O. M. Belova and K. V. Bychkov, Astrophysics 60, 200 (2017). doi: 10.1007/s10511-017-9475-8

6. O. M. Belova and K. V. Bychkov, Astrophysics 61, 101 (2018). doi: 10.1007/s10511-018-9519-8

7. M. M. Katsova, A. G. Kosovichev, and M. A. Livshits, Astrophysics 17, 156 (1981). doi: 10.1007/BF01005196 [http://astro.asj-oa.am/3339/1/285.pdf]

8. E. S. Morchenko, Astrophysics 59, 475 (2016). doi: 10.1007/s10511-016-9450-9

9. A. G. Kosovichev, Bull. Crimean Astrophys. Obs. 75, 6 (1986).

10. B. V. Somov, Soviet Astron. Lett. 6, 312 (1980).

11. V. P. Grinin and V. V. Sobolev, Astrophysics 13, 348 (1977). doi: 10.1007/BF01006610

12. K. C. Gordon and G. E. Kron, Publ. Astron. Soc. Pacif. 61, 210 (1949).

13. E. S. Morchenko, arXiv: 1710.08008 [astro-ph.SR] (2017).

14. M. A. Livshits, O. G. Badalyan, A. G. Kosovichev, and M. M. Katsova, Solar Phys. 73, 269 (1981).

15. E. S. Morchenko, Candidate’s Dissertation, Phys. Math. Sci., M. V. Lomonosov Moscow State University (2017). [https://istina.msu.ru/download/60621005/1ii1MmG:rWMu8TaN-UseXLbrIXpeFahwKUYQ/]

16. M. N. Lovkaya, Astron. Rep. 57, 603 (2013). doi: 10.1134/S1063772913080040

17. O. M. Belova, K. V. Bychkov, E. S. Morchenko, and B. A. Nizamov, Astron. Rep. 58, 650 (2014). doi: 10.1134/S1063772914090029

18. O. P. Shmeleva and S. I. Syrovatskii, Solar Phys. 33, 341 (1973).

19. B. V. Somov, S. I. Syrovatskii, and A. R. Spektor, Solar Phys. 73, 145 (1981).
20. E. L. E. Eason, M. S. Giampapa, R. R. Radick, et al., Astron. J. 104, 1161 (1992).

21. V. V. Sobolev and V. P. Grinin, Astrophysics 38, 15 (1995). doi:10.1007/BF02113956 [http://astro.asj-oa.am/1372/1/1995%2D1(33).pdf]

22. N. D. Kostyuk and S. B. Pikel’ner, Soviet Astron. 18, 590 (1975).

23. A. F. Kowalski and J. C. Allred, Astrophys. J. 852, 61 (2018).

24. J. C. Allred, A. F. Kowalski, and M. Carlsson, Astrophys. J. 809, 104 (2015).

25. Y. Nakagawa, S. T. Wu, and S. M. Han, Solar Phys. 30, 111 (1973).

26. M. K. Druett and V. V. Zharkova, Astron. Astrophys. 610, A68 (2018).

27. P. P. Volosevich, S. P. Kurdyumov, L. N. Busurina, and V. P. Krus, USSR Comput. Math. Math. Phys. 3, 204 (1963). doi:10.1016/0041-5553(63)90131-9

28. S. A. Kaplan, Interstellar Gas Dynamics, Pergamon Press, Oxford (1966).

29. A. G. Kosovichev and Yu. P. Popov, USSR Comput. Math. Math. Phys. 19, 168 (1979). doi:10.1016/0041-5553(79)90107-1

30. N. D. Kostyuk, Soviet Astron. 20, 206 (1976).

31. J. C. Brown, Solar Phys. 31, 143 (1973).

32. V. V. Ivanov, Transfer of Radiation in Spectral Lines, U.S. Government Printing Office, Washington (1973).

33. M. A. Livshits, Soviet Astron. 27, 557 (1983).

34. S. I. Syrovatskii and O. P. Shmeleva, Soviet Astron. 16, 273 (1972).

35. L. C. Johnson, Astrophys. J. 174, 227 (1972).

36. R. E. Gershberg and E. E. Shnol, Izv. Krymsk. Astrofiz. Obs. 50, 122 (1974).

37. V. V. Ivanov, Soviet Astron. 16, 91 (1972).

38. V. V. Ivanov, Astrophysics 52, 24 (2009). doi:10.1007/s10511-009-9054-8

39. A. N. McClymont and R. C. Canfield, Astrophys. J. 265, 483 (1983).

40. A. F. Kowalski, PhD thesis, University of Washington (2012).

41. R. E. Gershberg, Astrophysics 13, 310 (1977). doi:10.1007/BF01001231

42. G. H. Fisher, R. C. Canfield, and A. N. McClymont, Astrophys. J. 289, 434 (1985).

43. R. C. Canfield and R. G. Athay, Solar Phys. 34, 193 (1974).

44. R. C. Canfield, Solar Phys. 34, 339 (1974).

45. K. Ichimoto and H. Kurokawa, Solar Phys. 93, 105 (1984).
46. A. F. Kowalski, Proc. IAU Symp. 320, 259 (2016). arXiv:1511.05085 [astro-ph. SR].

47. B. E. Zhilyaev, Ya. O. Romanyuk, O. A. Svyatogorov, et al., Astron. Astrophys. 465, 235 (2007).

48. M. M. Katsova, Soviet Astron. 25, 197 (1981).