Constraining Photon Mass by Energy-Dependent Gravitational Light Bending

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1 Introduction

Photons are particles mediating electromagnetic force. In the standard model of particle physics, they are zero-mass particles with a particular dispersion relation

\[ E^2 = p^2 c^2, \]

where \( E, p, \) and \( c \) are energy, momentum, and the speed of light, respectively. This property is closely related to the inverse square law of electrostatic force between two charges

\[ F = \frac{1}{4\pi\epsilon_0} \frac{q_1 q_2}{r^2}, \]

where \( F \) is the electric force between the two charges \( q_1 \) and \( q_2 \) with a separation of \( r \), and \( \epsilon_0 \) is the vacuum permittivity.

If a photon has a non-zero mass \( m \), the dispersion relation becomes

\[ E^2 = p^2 c^2 + m^2 c^4. \]

Also, the electrostatic potential takes on a Yukawa form of

\[ V = \frac{1}{4\pi\epsilon_0} \frac{q}{r} e^{-\mu r}, \]

accounting for the finite range of the force mediated by a non-zero mass particle. Electromagnetism with non-zero mass photon is also different, and can be described by

\[ \nabla \cdot \mathbf{E} = \frac{4\pi\rho}{c} - \mu^2 V, \]

\[ \nabla \times \mathbf{E} = -\frac{1}{c} \frac{\partial \mathbf{B}}{\partial t}, \]

\[ \nabla \cdot \mathbf{B} = 0, \]

\[ \nabla \times \mathbf{B} = \frac{4\pi}{c} \mathbf{J} + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} - \mu^2 \mathbf{A}, \]

where \( \mathbf{E}, \mathbf{B}, \rho, \mathbf{J}, V, \mathbf{A} \) and \( c \) are electric field, magnetic induction, charge density, current density, scalar potential, vector potential and the speed of light, respectively, while \( \mu^{-1} = \hbar/mc \) is the Compton wavelength of photons, with \( m \) denoting photon mass. Note that in equation (5) and equation (8), there are additional terms \( \mu^2 V \) and \( \mu^2 \mathbf{A} \), which vanish when photon mass \( m \) is zero, and then the above equations are just the Maxwell equations.

These effects caused by a non-zero photon mass can be used to constrain photon mass itself. The dispersion relation (3) means that non-zero mass photons with higher energies travel faster than lower energies ones. Therefore, measurements of the difference in arrival times of photons with different energies emitted simultaneously at the same place can be used to constrain photon mass. This has been done by analyzing gamma-ray burst data[2]. The key assumption...
is that photons of different energies were emitted simultaneously, however, in reality this is not guaranteed, and this will limit the accuracy of the final constraint on photon mass.

Deviations from the inverse square law of electrostatic force can also be constrained by torsion balance experiments in territory labs, which infer an upper limit on photon mass of $1.2 \times 10^{-5} g$[3]. This is an effective way to constrain photon mass. However, it is a small scale test, which would not be effective if the Compton length of photons $\mu^{-1} = \hbar/mc$ is large (say, as large as a galaxy). Experiments have also been done to constrain the $\mu^2 \Delta$ term in equation (3). With the estimation of $\Delta$, one can get a constraint on the Compton length of photons (also on photon mass)[1]. This method is limited by the accuracy of the estimation of $\Delta$. Some authors also mentioned the constraint on photon mass from the measurements of light bending by the sun[4-6], the estimated upper limit is about $10^{-40} g$. For a more detailed review on constraining photon mass, one may refer to e.g. [7].

In this paper, we try to investigate the constraints on photon mass at a cosmological scale with strong gravitational lensing data. In section 2, we present the equations used in our analysis. Results are given in section 3. We then do some discussion in section 4. Hereafter in this paper we use natural units ($c = \hbar = 1$) for simplicity.

## 2 Equations

The bending of light within a gravitational field can be considered as the scattering of photons in this gravitational field[4]. The gravitational field is treated classically as a background.

The dispersion relation of a photon of energy $E$ and mass $m$ in the gravitation field of an object with mass $M$ is

$$2\left(\frac{GM}{b}\right)^2 = \frac{1 - \cos \theta}{\left(2 + \frac{m^2}{E^2 - m^2}\right)^2} \cdot \frac{1}{\left(1 - \cos \theta\right) \ln(1 - \cos \theta) - \frac{2}{7}(1 - \cos \theta)^2},$$

where $G$ is the gravitational constant, $b$ is the impact parameter and $\theta$ is the deflection angle. When $\theta$ is small and in the extreme relativistic limit $E \gg m$, this equation can be approximated as

$$\theta = \theta_E \left(1 + \frac{m^2}{2E^2}\right),$$

where $\theta_E = \frac{4GM}{b}$ is the Einstein radius of the gravitating object. This approximation is good enough even for small $E/m$ (see Figure 1).

It is convenient to define a relative deviation

$$\Delta \theta \equiv \frac{\theta - \theta_E}{\theta_E}.$$

## 3 Results

From equation (9) and (10), it can be seen that the deflection angle of a photon in a gravitational field depends on its energy (or frequency). The relation between the relative deviation of deflection angle from the Einstein radius, $\Delta \theta \equiv (\theta - \theta_E)/\theta_E$, and the ratio, $E/m$, is shown in Figure 1. The relative deviation can be calculated by using equation (9) and (10).

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1C.S. Kochanek, E.E. Falco, C. Impey, J. Lehar, B. McLeod, H.-W. Rix, [http://www.cfa.harvard.edu/glensdata/](http://www.cfa.harvard.edu/glensdata/)
Table 1. The first column is the name of image pairs (e.g., PMN0134-0931 A-B means the image A and B in the source PMN0134-0931). The second and third column are the difference of right ascension and difference of declination between the two images of a pair in the infrared band, while the fourth and fifth column are those in the radio band. We denote the IR image separation with $\phi_2$ and the radio image separation with $\phi_1$. Since the lensing patterns at different bands are geometrically similar, $\phi_1/\theta_1 = \phi_2/\theta_2$, and we have

$$\Delta \theta = \frac{\theta_1 - \theta_2}{\theta_2} = \frac{\phi_1 - \phi_2}{\phi_2}.$$  (12)

Using equation (10), $m^2 = 2E^2 \Delta \theta$ can be calculated, which scatter around 0 (see Figure 2). The standard deviation is $7.59 \times 10^{-37} g^2$, which corresponds to an upper limit of $8.71 \times 10^{-39} g$ on photon mass.

As can be seen from Figure 1, equation (10) is a good approximation of equation (9) when $\Delta \theta^{-1} = 2E^2/m^2 > 8$. The result above is consistent with this condition.

![Distribution of $2E^2\Delta \theta$. The abscissa is the index of image pairs. The horizontal line is the line corresponds to $2E^2\Delta \theta = 0$.](image)

**Figure 2: Distribution of $2E^2\Delta \theta$. The abscissa is the index of image pairs. The horizontal line is the line corresponds to $2E^2\Delta \theta = 0$.**

### 4 Discussion

In this paper, strong gravitational lensing data are used to infer an upper limit of photon mass, $8.71 \times 10^{-39} g$.

As shown in Figure 2, the $2E^2\Delta \theta$ values inferred from the lensing data are roughly consistent with 0, but with large error bars. There are several reasons. The uncertainty in the determination of image position may be the dominant factor.

Since $m^2 = 2E^2\Delta \theta$, more accurate observation with lower frequencies can help to improve this measurement. To improve the accuracy, longer VLBI (Very Long Baseline Interferometry) base line is needed, which is possible with space VLBI (e.g., VSOP project in Japan). However, the improvement may be limited.

As mentioned before, in the constraints from gamma-ray burst data, non-simultaneous emissions of photons of different energies will affect the final result. In the current study, there are also problems. In a quasar, radiation in different energy bands may come from different regions. This may affect the constraint on photon mass. This error is intrinsic and will exist no matter how accurate the instruments are.

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Table 1: Separation of lensing images in different bands from CASTLE database.

| Image pairs | Separation of IR image | Separation of Radio image | Relative separation (Δθ) |
|-------------|------------------------|---------------------------|--------------------------|
|             | RA(arcsec) | Dec(arcsec) | RA(arcsec) | Dec(arcsec) |             |
| PMNJ0134-0931[8] A-B | 0.082±0.003 | 0.156±0.003 | 0.07918±0.00146 | 0.15069±0.00219 | -0.034±0.028 |
| PMNJ0134-0931 A-C | 0.539±0.003 | 0.415±0.003 | 0.53962±0.00120 | 0.41471±0.00151 | 0.000462±0.0069 |
| B0218+357[9] A-B | 0.307±0.003 | 0.126±0.003 | 0.30920±0.00014 | 0.12740±0.00014 | 0.0077±0.013 |
| MG0414+0534[10] A1-B | 0.600±0.003 | 1.942±0.003 | 0.5876±0.0003 | 1.9341±0.0003 | -0.00550±0.0021 |
| MG0414+0534 A2-B | 0.732±0.003 | 1.549±0.003 | 0.7208±0.0003 | 1.5298±0.0003 | -0.01292±0.0025 |
| MG0414+0534 B-C | 1.342±0.003 | 1.650±0.003 | 1.3608±0.0003 | 1.6348±0.0003 | 0.000098±0.0020 |
| B0712+472[11] A-B | 0.052±0.004 | 0.146±0.007 | 0.051±0.010 | 0.160±0.010 | 0.084±0.11 |
| B0712+472 A-C | 0.808±0.005 | 0.648±0.004 | 0.806±0.010 | 0.670±0.010 | 0.0119±0.015 |
| B0712+472 A-D | 1.186±0.007 | 0.463±0.004 | 1.163±0.010 | 0.460±0.010 | -0.0176±0.013 |
| B0739+366[12] A-B | 0.222±0.004 | 0.485±0.004 | 0.2217±0.0001 | 0.4910±0.0001 | 0.0100±0.011 |
| J1004+1229[13] A-B | 0.267±0.003 | 1.516±0.003 | 0.2633±0.0010 | 1.5172±0.0017 | 0.000354±0.0030 |
| B1152+200[14] A-B | 0.936±0.003 | 1.246±0.003 | 0.935±0.005 | 1.248±0.005 | 0.000641±0.0052 |
| B1359+154[15] A-B | 0.483±0.007 | 1.253±0.009 | 0.49020±0.00003 | 1.25240±0.00003 | 0.00152±0.0085 |
| B1359+154 A-C | 0.323±0.007 | 1.640±0.003 | 0.31126±0.00003 | 1.66956±0.00003 | 0.1604±0.0046 |
| B1359+154 A-D | 0.957±0.008 | 1.357±0.008 | 0.96257±0.00003 | 1.36864±0.00003 | 0.00766±0.0069 |
| B1359+154 A-E | 0.627±0.013 | 1.129±0.011 | 0.60876±0.00003 | 1.14296±0.00003 | 0.00275±0.013 |
| B1359+154 A-F | 0.426±0.016 | 0.951±0.028 | 0.42220±0.00003 | 0.96377±0.00003 | 0.0097±0.031 |
| B1422+231[16] A-B | 0.385±0.003 | 0.317±0.003 | 0.387±0.005 | 0.320±0.005 | 0.0069±0.017 |
| B1422+231 B-C | 0.336±0.003 | 0.750±0.003 | 0.332±0.005 | 0.750±0.005 | -0.00198±0.010 |
| B1422+231 B-D | 0.984±0.004 | 0.802±0.003 | 0.939±0.005 | 0.810±0.005 | -0.0231±0.068 |
| B1555+375[17] A-C | 0.417±0.014 | 0.013±0.008 | 0.412±0.001 | 0.028±0.001 | -0.0102±0.038 |

a. The IR image C corresponds to D in Radio observation.
b. The radio observation is at a frequency of 15GHz.
c. The radio observation is at a frequency of 8GHz.
d. The radio observation is at a frequency of 1.7GHz.