Construction of isometric outlines of surfaces using their superimposed orthogonal projections

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Abstract. The article solves the geometric task of constructing isometrics of the contour of spatial forms. The problem is solved by special construction of orthogonal projections of especial lines. These lines turn out to be contour lines of intersecting surfaces when they are further transformed into axonometry. Methods are considered to construct isometric contour of curved surfaces with finding their anchor points by superimposing projections of the same name. This simplifies the task of determining the coordinates of anchor points in a complex drawing.

The drawing is always an essential element in the development of technical architectural or design projects. Graphic construction of complex drawings and visual representations of spatial forms is performed [1, 2, 3, 4, 5, 6].

The theoretical basis for the construction of technical, architectural drawings is laid in the course "Descriptive Geometry". The dominant imaging methods are the method of central projection and its special case, it is a method of parallel projection. Using these methods, you can obtain visual graphic models of specific objects: perspective or axonometry [7, 8]. As a rule, perspective and axonometry are graphically aligned according to the specified parameters of a complex drawing. The construction of visual images of spatial forms of objects is associated with the construction of newly formed contours of details [9,10]. This can be difficult, especially when you want to depict mutually intersecting curved surfaces. As it seems to the authors, this issue is not sufficiently represented in the well-known educational literature on descriptive geometry [11, 12, 13].

Therefore, it is relevant in this article to consider some ways of constructing newly formed contours of objects. This will allow you to construct visual images quite simply and accurately.

The graphic task is to construct a visual image of two intersecting curved surfaces. The proposed solution to the problem can be applied to the surfaces of machine-building plan parts. Technical drawings of their images are made on complex drawings based on parallel projection. In this case, it is logical to use visual images in our solution, which are constructed using the method of parallel projection. In other words, we will consider the solution of the problem using such types of drawings as "Complex drawing" and "Axonometry". An axonometric drawing is graphically constructed from a projection complex drawing. The shape of the part can be different. It can be polyhedral or curved. It is set parametrically. Axonometric projections are based on coordinates taken from a complex drawing.

Note that the geometric constructions of axonometric drawings are quite well known [7, 8, 9, 10]. However, this can be difficult when it is necessary to construct anchor points of outline generators of intersecting curved surfaces. We focus our attention on this point of solving the problem.
Let's consider the solution of a specific problem: a complex drawing of two mutually intersecting surfaces is given: a cone of rotation and a sphere. Let's formulate the task: to build a visual image of them. We consider isometry as a visual image.

**Figure 1.** Construction of a line of the intersection of a cone and a sphere.

In picture 1(on the left) two intersecting surfaces of rotation: a cone and a sphere are shown in the complex drawing. As you can see in the drawing, the line of intersection of these surfaces is constructed using the cross-section method. The decision to use a horizontal level plane and intersecting a level plane passing through the apex of the cone. As a result of this method of solving the problem, anchoring points 1,2,3 are determined. Reference points are the points where the outline’s generatrixes of one surface intersect with the surface of another body.

And how to solve this problem in isometry?

Obviously, if we solve this problem using the method of horizontal sections of the sphere, the images of circles will be accompanied by distortions. Such a graphical solution in axonometry will be very difficult and time-consuming. Therefore, the axonometric image of the line of intersection of the cone and sphere is determined by another method, which is used in practice.

We use coordinates to transform the points of this line from a complex drawing to an isometry of intersecting surfaces, using two coordinate systems, orthogonal and axonometric. It is clear that this line will belong to both surfaces at the same time. Figure 1 (right).

It is easy to predict that the outline generatrixes of these surfaces on the isometry and the outline of these surfaces on the complex drawing are images of different lines, outside of the projection connection. It follows that the reference points are built on the outline generatrixes of the complex drawing will no longer be the reference points for the generatrixes of the axonometric drawing.

Hence the problem arises – to determine the missing outline generatrixes in the axonometric drawing. Moreover, then use these outline generatrixes to construct reference points of the intersection of the outline generatrixes of the cone with the surface of the sphere and the outline generatrix of the sphere with the surface of the cone.

To solve the problem, we apply the following algorithm:
special contour lines are drawn using well-known geometric methods in a complex drawing. These lines will coincide projectionally with the axonometric outline of the consideration intersecting surfaces; next, we will find the intersection points of the projections of these special contour lines with the intersection line of the surfaces built on the complex drawing. These are the reference points for axonometry; these points are projected onto axonometric outlines using coordinates.

As the experience of solving such problems shows, the process of manipulation of the transfer of generatrixes from one drawing to another, establishing a projection correspondence between reference points on axonometric outline generatrixes of surfaces and their projections on a complex drawing can be difficult. To the solution of such tasks requires a special approach.

Let us first consider the solution of this problem in relation to the surface of a sphere. It is known that a sphere in isometry and in a complex drawing has an outline in the form of circles of the same size (a circle of the same diameter), but there is no projection connection between the points of these circles. This is clearly seen in figure 2. The frontal projection of the hemisphere, the rectangular coordinate system assigned to it, and the degenerated frontal projection of the axonometry plane are given on the left. In figure 2 on the right, its isometry is plotted from the given frontal projection of the hemisphere. The construction of axonometry is performed graphically, without determining the numerical values of distortion indicators, based on the projection relationship. The method for constructing axonometry based on the frontal projection of the surface is borrowed from the textbook descriptive geometry edited by prof. N. F. Chetverukhin. [8]

The outline of a sphere in orthogonal isometry is the orthogonal projection of the axial section of the sphere. Building it in figure 2 is not difficult. You only need to know the value of the radius of the sphere, and an axonometric outline on the isometry should be built with a compass.

A sphere's frontal projection in figure 2 coincides with the degenerated projection of the axonometry plane. Note that in this case, if necessary, it could serve as a line of the visible contour in the complex drawing for building an axonometric outline. It follows that the points of these contours, including random ones, the reference points A, B, are in a projection connection and are easily mutually determined. Note that the main meridional sections are plotted on axonometry specifically to convey the depth of the shape.

**Figure 2.** Construction a hemisphere according to its complex drawing.

Let's follow the construction of axonometry in figure 3, where the same hemisphere is defined in the complex drawing in the traditional way. Its equator is located in the horizontal plane of the level, and the coordinate axis Oz is perpendicular to this plane. In this case, it is more convenient to build axonometry of the hemisphere with the determination of numerical values of distortion indicators.
Despite the different construction methods, axonometric images of equal-sized hemispheres will be congruent, and therefore we will limit our discussion to one drawing of the axonometry of the hemisphere. This drawing is already built in figure 2. Note that, unlike the previous example, the line of the visible contour for building an axonometric outline is not present in the orthogonal drawing. Due to the fact that there was no need to portray it. But now, this makes it impossible to establish the relationship between the reference points. In other words, there is no contour in the orthogonal drawing that can be used to establish a relationship with the reference points of the outline in axonometry. It follows that to solve the main problem, it is necessary to supplement the complex drawing with this special contour line and find reference points for the axonometric outline.

To solve this problem, we will use the concept of "wrapping projecting surface. Under the concept of "enveloping projecting surfaces" we will understand a set of projecting lines parallel to the axonometric direction and touching the surface of the hemisphere straight lines. The line of contact between the wrapping surface and the surface of the hemisphere will be a special contour line. For fig.3 the complex drawing of the hemisphere is based on the size of the hemisphere of the drawing in figure 1. We show the direction of the axonometric projectioning of S. It is known that for orthogonal isometry, the angle of projectioning is 35 degrees, and the direction of the axonometric projectioning is located at an angle of 45 degrees to the coordinate axes. Then we find the point M where the projecting ray S touches the surface of the hemisphere. The construction is performed using the rotation method. As you can see in figure 3, the defined contour line should be the axial circle of the sphere. It lies in a plane with an angle of 35° and is projected onto the horizontal plane of projections by an ellipse. Our task is to build it.

**Figure 3.** Construction of anchor points of axonometric outlines based on superimposed projections.

We will not build a frontal projection of the semicircle due to the lack of necessity. On the horizontal projection of this semicircle, we apply the horizontal projection of the intersection line of the given surfaces of the complex drawing from figure 1. Then find the intersection points C and D. These points
are obviously the reference points of the intersection of the axonometric outline of the hemisphere with the surface of the cone. The found points are reprojected to the isometry. Note that this solution is moved from the main complex drawing of figure 1 to figure 3 for easy reading of the problem solution.

Let's consider a possible solution of the problem in relation to the surface of a cone. The graphic solution is compatible with the complex drawing in figure 1. The tangent line of the wrapping projecting surface and the surface of the rotation cone will specially constructed generatrixes. To construct them, we draw a projecting ray S in accordance with the axonometric direction of projection, similar to the previous solution. Then we find the intersection point N of the projecting ray S with the plane of the base of the cone. Tangentially to this base of the cone, we draw straight lines. Let's define the tangent points E and F. We draw the generatrixes SE and SF, which are outline generatrixes in the axonometric drawing. As can be seen from figure 1, the horizontal projections of these cone generatrixes and the horizontal projection of the intersection line of the specified surfaces of the cone and hemisphere are superimposed on the horizontal projection of the complex drawing. We find the intersection points A and B. We take them as the reference points of the intersection of the axonometric outlines of the hemisphere with the surface of the cone.

In conclusion, we state that as a result of the geometric construction of isometric projections of curved surfaces, it is possible to solve the problem of determining the missing outline generatrixes in an axonometric drawing by constructing special contour lines in a complex drawing. Using them as superimposed on orthogonal projections of the line of intersection of surfaces allows you to build geometrically accurate reference points on axonometric outlines of curved lines.

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