Gravitational physics with antimatter

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Abstract
The production of low-energy antimatter provides unique opportunities to search for new physics in an unexplored regime. Testing gravitational interactions with antimatter is one such opportunity. Here a scenario based on Lorentz and CPT violation in the Standard-Model Extension is considered in which anomalous gravitational effects in antimatter could arise.

Keywords
Antimatter · Gravity · Lorentz violation

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1 Introduction

Antimatter is a regime in which numerous predictions of our current best description of nature, the Standard Model of particle physics together with Einstein’s General Relativity, remain untested. As such, experiments with antimatter provide an opportunity to detect new physics, and are required to place our existing theories on an experimental foundation in this regime.

One such property that deserves experimental testing is the interaction of antimatter with gravity. Within our present theories, CPT symmetry, which has passed all tests to date [1] indicates that the properties of an antiparticle system should be the same as those of a particle system. Thus, CPT symmetry suggests that a pair of antiparticles should interact gravitationally in the same manner as a pair of particles. However, arguing from CPT symmetry alone provides no information about the gravitational interaction of an antiparticle with a particle. Moreover, there exist ways in which CPT could be violated [2,3,4,5], some of which have never been tested. Of these untested potential violations, some could be large relative to existing bounds [6].

Numerous attempts have been made to place indirect limits on the degree to which the behavior of an antimatter test particle in a gravitational field may differ from that of a matter
particle [7]. However, such considerations are inherently model dependent, and it is still possible to find models that predict an appreciable difference in the gravitational response of matter and antimatter. One such suggestion is a Supergravity inspired model involving vector and scalar partners to the graviton [8]. In the remainder of these proceedings, the possibility that such differences can be produced by violations of Lorentz and CPT symmetry in the context of the Standard-Model Extension (SME) [3,4] is outlined. This work was done in collaboration with Alan Kostelecký [6,9].

2 Standard-Model Extension

The SME is a general theoretical framework for testing Lorentz and CPT symmetry. It contains the Standard Model and General Relativity along with arbitrary coordinate-independent Lorentz violation in the form of an effective quantum field theory [3,4]. Searches for Lorentz violation within the SME are motivated in part by the fact that although General Relativity and the Standard Model provide an excellent description of nature at our present low energies, a single quantum-consistent theory at the Planck scale is lacking. Ideally experiment would guide the development of the underlying theory; however, direct experiments at the Planck scale are infeasible. Lorentz and CPT violation offer the possibility of detecting suppressed Planck-scale effects with existing technology [10] and have been shown to be possible within the context of numerous candidates for the underlying theory.

A large number of experimental searches for Lorentz violation have been performed in the context of the minimal SME. In the Minkowski-spacetime limit those tests include experiments with electrons [11,12,13,14,15,16], protons and neutrons [17,18], photons [19,20,21], mesons [22,23], muons [24,25], neutrinos [26,27], and the Higgs [28]. Gravitational searches have also begun [29] based on investigations of the pure gravity sector [30] and gravitational couplings in the matter sector [6,9]. While the experimental searches performed thus far have revealed no compelling experimental evidence for Lorentz violation, much remains unexplored. For example, only about half of the coefficients for Lorentz violation in the minimal SME involving light and ordinary matter (protons, neutrons, and electrons) have been investigated experimentally, and other sectors remain nearly unexplored.

3 Relativistic Theory

Here we consider the QED limit of the SME with gravitational couplings [4]. The general geometric framework assumed is Riemann-Cartan spacetime, which contains the Riemann curvature tensor $R_{\kappa \lambda \mu \nu}$ as in General Relativity and allows for a nonzero torsion tensor $T_{\lambda \mu \nu}$, the effects of which have been tightly constrained by a recent re-interpretation of bounds on SME coefficients as bounds on torsion [31]. The basic non-gravitational fields are the photon $A_\mu$ and the Dirac fermion $\psi$, while the spin connection $\omega_{\mu}^{\ ab}$ and the vierbein $e^a_{\ \mu}$ are taken as fundamental gravitational objects.

The action of the minimal SME can be expanded in the following way:

$$S = S_G + S_{\psi} + S'.$$

The action of the pure-gravity sector is provided by the first term $S_G$. It contains the dynamics of the gravitational field and can also contain coefficients for Lorentz violation in that
sector \([4,30]\). The second term provides the action for the fermion sector, which takes the form

\[
S_{\psi} = \int d^4x \left[ \frac{1}{2} i e \bar{e}_a \gamma^\mu \bar{\psi} (\gamma^\mu - c_{\mu\nu} e^\nu - \psi + \ldots) \hat{D}_\mu \psi - e \bar{\psi} (m + a_\mu e^\mu \gamma^\mu + \ldots) \right].
\]  \(2\)

The symbols \(a_\mu\) and \(c_{\mu\nu}\) are coefficient fields for Lorentz violation of the minimal fermion sector included here as a sample of such terms. The ellipsis here contains additional coefficients fields for Lorentz violation. In general, these fields vary with position and differ for each species of particle. Note also that \(a_\mu\) is CPT violating, while \(c_{\mu\nu}\) is CPT preserving and can be taken as traceless. For additional discussion of the fermion-sector action, see Ref. \([4]\).

The final portion of the action, \(S'\), contains the dynamics associated with the coefficient fields for Lorentz violation and is responsible for spontaneous Lorentz-symmetry breaking. Through spontaneous symmetry breaking, the coefficient fields for Lorentz violation are expected to obtain vacuum values. Thus, it is possible to express an arbitrary coefficient field for Lorentz violation \(t_{\lambda \mu \nu \ldots}\) as \(t_{\lambda \mu \nu \ldots} \mapsto \tilde{t}_{\lambda \mu \nu \ldots}\), where \(t_{\lambda \mu \nu \ldots}\) is the corresponding vacuum value and \(\tilde{t}_{\lambda \mu \nu \ldots}\) represents the fluctuations about that vacuum value. In general these fluctuations contain both the massless Nambu-Goldstone modes \([32]\) and massive modes \([33]\) associated with Lorentz-symmetry breaking. The necessary tools to analyze fermion experiments in the presence of gravity and Lorentz violation may be developed without specifying \(S'\) \([9]\). For the \(a_\mu\) case those results are obtained in Ref. \([6]\) and are summarized in the following section.

4 Non-Relativistic Analysis

One can proceed from the fully relativistic action \([4]\) toward experimental analysis at a number of different levels \([9]\). Relativistic quantum-mechanical investigations can be performed by obtaining the relativistic quantum hamiltonian from action \([2]\). Doing so involves a redefinition of the fermion field followed by the usual Euler-Lagrange procedure. The resulting hamiltonian can be used to obtain the non-relativistic physics via a Foldy-Wouthuysen transformation. The classical lagrangian can then be obtained by inspection of the non-relativistic hamiltonian, which allows one to replace \(S_{\psi}\) in Eq. \((1)\) with

\[
S_u = \int d\tau \left( -m \sqrt{-g_{\mu\nu} u^\mu u^\nu - a_\mu u^\mu} \right),
\]  \(3\)

for the purposes of analyzing classical point-particle experiments at leading order in Lorentz violation associated with \(a_\mu\) and leading order in the metric fluctuation.

A generic investigation of spontaneous breaking performed by requiring coordinate independence of the physics and geometric compatibility provides the form of the fluctuations in the Nambu-Goldstone limit. Under such circumstances, the fluctuations, can be written in terms of the metric fluctuation and the vacuum value. In the case of \(a_\mu\) for example \([6]\),

\[
\tilde{a}_\mu = \frac{1}{2} \alpha \sigma^\nu h_{\nu \mu} - \frac{1}{4} \alpha \sigma^\nu \sigma_{\nu \mu} h_{\mu \nu}
\]  \(4\)

in harmonic coordinates, where \(\alpha\) is determined by the strength of the coupling to gravity. Similarly, by restricting to theories for which Newton’s third law holds, it is possible to find the Lorentz-violating contributions to the metric fluctuation, which in the \(a_\mu\) case take the form \(h_{00} \supset 4\alpha \sigma_0 U / m\), in terms of the Newtonian potential \(U\).
5 Experiments

Numerous suggestions for experiments that can achieve sensitivity to $\mathfrak{F}_\mu$ are considered in Refs. [6,9], and several experimental constraint are obtained on combinations of the 12 $\mathfrak{F}_\mu$ coefficients for ordinary matter. The possibility of scenarios in which the CPT odd effects of $\mathfrak{F}_\mu$ considered explicitly in that work could cancel against CPT even Lorentz-violating effects within the SME is also noted. Under such circumstances gravitational tests with antimatter could disentangle the situation.

Several proposals have been made to perform gravitational tests with neutral antimatter or neutral combinations of particles and antiparticles. These include tests with antihydrogen, antineutrons, muonium, and positronium.

Tests with antineutrons are mentioned in the proposal of Ref. [34]. Such tests are of potential interest here since they could provide a clean bound on neutron coefficients. However they are technically challenging at present due to issues associated with controlling the particles.

The proposal of Ref. [35] suggests that interferometric methods could be used to obtain sensitivity to the gravitational acceleration of a positronium atom. Note that since positronium consists of a particle and its antiparticle, $\mathfrak{F}_\gamma$ effects cancel completely providing a clean bound on CPT even effects. Muonium interferometry [36] could offer the analogous sensitivity for muon coefficients.

The development of cold antihydrogen provides unique opportunities to perform tests with neutral antimatter including some unique tests of Lorentz and CPT symmetry. A proposal already exists for tests of Lorentz symmetry based on antihydrogen spectroscopy [12]. Various ideas also exist for measuring the gravitational acceleration of antihydrogen including tests involving atom traps [38], atom interferometry [34,39], and free fall from ion traps [40]. A suggestions has also been made to perform such tests in space, which would provide further improved sensitivity [40]. Such experiments have the potential to provide sensitivity to combinations of electron and proton coefficients.

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