Gaussian Process Learning-Based Model Predictive Control for Safe Interactions of a Platoon of Autonomous and Human-Driven Vehicles

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Abstract—With the continued integration of autonomous vehicles (AVs) into public roads, a mixed traffic environment with large-scale human-driven vehicles (HVs) and AVs interactions is imminent. In challenging traffic scenarios, such as emergency braking, it is crucial to account for the reactive and uncertain behavior of HVs when developing control strategies for AVs. This paper studies the safe control of a platoon of AVs interacting with a human-driven vehicle in longitudinal car-following scenarios. We first propose the use of a model that combines a first-principles model (nominal model) with a Gaussian process (GP) learning-based component for predicting behaviors of the human-driven vehicle when it interacts with AVs. The modeling accuracy of the proposed method shows a 9% reduction in root mean square error (RMSE) in predicting a HV’s velocity compared to the nominal model. Exploiting the properties of this model, we design a model predictive control (MPC) strategy for a platoon of AVs to ensure a safe distance between each vehicle, as well as a (probabilistic) safety of the human-driven car following the platoon. Compared to a baseline MPC that uses only a nominal model for HVs, our method achieves better velocity-tracking performance for the autonomous vehicle platoon and more robust constraint satisfaction control for a platoon of mixed vehicles system. Simulation studies demonstrate a 4.2% decrease in the control cost and an approximate 1m increase in the minimum distance between autonomous and human-driven vehicles to better guarantee safety in challenging traffic scenarios.

I. INTRODUCTION

With the boom in autonomous vehicle (AV) development over the last decade, intelligent transportation systems such as inter-connected AV platooning technologies to improve traffic efficiency and safety have been actively studied [1]. A platoon of AVs can drive in a coordinated way, thus allowing small spaces between vehicles and still traveling safely at relatively high speeds [2]. With the use of vehicle-to-vehicle and vehicle-to-infrastructure technologies, vehicles in a platoon connect to each other and share real-time information such as speed, acceleration, and distance. This makes it possible to control vehicles in a platoon coordinately by synchronizing their maneuvers [3].

However, even with increased AV deployment on public roads, human-driven vehicles (HVs) are predicted to be predominant for decades [4]. Human drivers will inevitably need to interact with autonomous vehicles (AVs) and vice-versa. One example of this mixed traffic scenario is shown in Fig. 1 where a human-driven car drives behind an AV platoon. Despite numerous research on modeling interactions between AVs and HVs for car-following applications have been conducted, the majority of these models are not directly suitable for model-based control [5]. To achieve safe collaborations in mixed-traffic environments, it is important to develop the AV-HV interaction model that is easy to interpret and applicable for model-based control design to guarantee satisfactory goals.

Unlike AVs, even though many HVs are also equipped with sensors that can measure speed and distance, they do not communicate to share this information with other vehicles. AVs have the capabilities to measure their own states, and also can gather information about neighboring traffic. The modeling of HVs can therefore be made based on the measurements of the human-driven vehicle. Predictions of the human-driven vehicle made by the model can be shared with other AVs in the vehicle platoon to help improve the driving safety, fuel efficiency and riding comfort of the entire fleet [6].

A. Contributions

This paper proposes a learning-based MPC strategy for the longitudinal car-following control of a mixed vehicle platoon. The uncertainties of the human-driven vehicle are explicitly modeled in the MPC design to better guarantee safe interactions between AVs and HVs in challenging traffic scenarios such as emergency braking. The main contributions are summarized as follows:

- A new human-driven vehicle model that combines a first principles-based transfer function with a delay model of human behaviors in a velocity tracking setting [7], and a data-driven Gaussian Process (GP) model learned on data collected via a human-in-the-loop simulator. The
new model increases the modeling accuracy by 9% to the first principles model. The GP model is a function of the velocities of the human-driven vehicle and the autonomous vehicle it follows to provide velocity corrections to the predicted output of the first-principles model, along with a measure of uncertainty (variance) of the HV velocity prediction.

- A chance-constrained MPC strategy subject to acceleration, velocity, and safe distance constraints for the longitudinal car-following control of a mixed vehicle platoon. The safe distance between the human-driven vehicle and its front autonomous vehicle is adaptive to the variance estimations by the GP model.
- Simulation studies applying the proposed MPC strategy and a baseline MPC. The proposed MPC shows a 4.2% improvement in the control cost and a 1 m increase in the minimum distance between autonomous and human-driven vehicles to better guarantee safety in challenging traffic scenarios.

B. Related work

Model predictive control (MPC) is commonly used for vehicle platoon system control because it is a well-established optimal control method that explicitly considers constraints [8]. Different from a single or multiple AVs control, applying MPC to a mixed vehicle platoon control needs to satisfy the constraint subject to human-driven vehicle modeling. A growing number of studies have been conducted on the safe interactions of mixed traffic scenarios [9]. However, in most of these studies, human behaviors are either assumed to be the same in HV-AV interactions or uncertainties of HVs are difficult to be directly used for model-based control design. Traditional first principles-based models assume the reaction delay of human drivers to be fixed. The model parameters are identified from prior observations of a group of similar drivers to capture the general driving behaviors [10]. The accuracy of these models is moderate although they can be applied to interpret general human driving behaviors [6]. These models are parametric with limited parameters to capture the complex behaviors of human drivers.

Instead of using fixed-form parametric models for car-following controls, learning-based or data-driven modeling methods have been actively studied. Artificial Neural Network based approaches, including radial basis function networks [11], multilayer neural networks [12] and recurrent neural networks [13], were proposed to tackle the instantaneous reaction delay of human drivers. Other approaches to handle the stochastic uncertainties of human drivers are Gaussian Mixture Regression [14] and Hidden Markov Model [15]. These nonparametric methods are able to provide more accurate predictions of human drivers’ behaviors than the parametric models [16].

In the learning-based robotics control field, a non-parametric method, Gaussian Processes (GP) regression is commonly employed to model complex systems [17]. The GP-based methods are popular because they can directly provide an assessment of the model uncertainty together with predictions [18]. The system model is commonly represented by the sum of a fixed-form nominal model (parametric) and a GP model to learn the discrepancies between the nominal model and the true system behaviors [19]. Because the human driver’s behaviors have both deterministic and stochastic components [20], rather than utilizing either a fixed-form parametric model or a nonparametric modeling method, we model the human-driven vehicle as a combination of a traditional fixed-form model and a GP model.

The paper is organized as follows. Sec. II specifies the problem to be solved. Sec. III derives the modeling of human-driven vehicle for the car-following control. Sec. IV covers the development of the proposed Gaussian process learning-based MPC (GP-MPC). Sec. V shows simulation case studies of the GP-MPC and a baseline MPC. Sec. VI provides concluding remarks.

II. PROBLEM STATEMENT

This paper studies the longitudinal car-following control of a platoon of AVs followed by one human-driven vehicle. We want to achieve safe control of the mixed vehicle platoon by considering the HV modeling as a constraint in the control policy design for the AVs. Consider a mixed platoon of AVs and one human-driven vehicle, denoted by $A^{n_a}$ and $H$ respectively, with $n_a = \{1, 2, \ldots, N_a\}$ and $N_a$ represents the number of AVs shown in Fig. 1. The combination topology in the platoon of AVs is bidirectional, also known as the predecessor following. In this work, we assume that the leading vehicle $A^1$ plans the optimal maneuvers for AVs in the platoon. The position and velocity of $A^{n_a}$ at the current time step $k$ are denoted by $p_{k}^{n_a}$ and $v_{k}^{n_a}$, respectively. We consider the following kinematic model for the AVs as

$$v_{k+1}^{n_a} = v_{k}^{n_a} + T a_{k}^{n_a}, \quad (1a)$$

$$p_{k+1}^{n_a} = p_{k}^{n_a} + T v_{k}^{n_a}. \quad (1b)$$

Here $0 < T \ll 1$ is the sample time, and $a_{k}^{n_a}$ represents the acceleration of $A^{n_a}$.

In a vehicle platoon system, the control objective for each autonomous vehicle is to track the reference velocity determined by the leader vehicle. Besides the velocity tracking, each autonomous vehicle needs to keep a safe distance from the front vehicle at all time steps. This is also known as cooperative adaptive cruise control [21]. In a mixed vehicle platoon composed of AVs and HVs, it is critically important to model the behaviors of HVs in the controller design to guarantee safe control of the whole mixed platoon system.

We first estimate the behaviors of HVs by the sum of a first principles-based nominal model and a GP learning-based model. The GP models are used to correct discrepancies between the nominal model prediction and the actual behaviors of HVs and provide variance assessment of the modeling. An MPC strategy for a longitudinal car-following of a mixed vehicle platoon subject to acceleration, speed, and safe distance constraints is developed using the estimated HV model as a constraint. The uncertainties of the human-driven vehicle estimated by GP models are explicitly applied in the MPC design to guarantee safety in emergency braking scenarios.
III. HUMAN-DRIVEN VEHICLE MODELING

We model the human-driven vehicle as the sum of a first principles-based transfer function with delay (nominal model) and a Gaussian Process (GP) learning-based model. The nominal model is identified by the data of three drivers in different driving situations with an approximately 80% fit of actual HVs behaviors. The GP model improves the modeling accuracy by learning the discrepancies between the nominal model and the collected data. Besides accuracy improvements, the GP model quantifies the modeling variance that is important to guarantee the safe control of a mixed vehicle platoon.

A. First principles-based transfer function model

Considering the basic limitations and characteristics of the human cognitive and neuromuscular system, the human response can be modeled as a fixed-form transfer function with reaction delay as formulated in [7]:

\[
G_H(s) \approx \frac{1 + T_d s}{1 + 2\gamma T_w s + T_d^2 s^2} e^{-T_d s} = \frac{\hat{P}_H(s)}{\hat{P}_N(s)} \tag{2}
\]

where \(\hat{P}_H(s)\) and \(\hat{P}_N(s)\) represent Laplace transform of the velocity of the human-driven vehicle \(v^H_k\) and velocity of the autonomous vehicle in front \(v^N_k\), respectively. The \(T_d\) denotes the inherent cognitive and muscular delay of human drivers’ response. The parameters in \(G_H\) can be identified by the collected data. By applying a Padé approximation (second order) of the time delay component in the transfer function (2), a discrete form of the transfer function \(G(z)\) is derived as

\[
G(z) = \frac{V^H_k(z)}{V^N_k(z)} = \frac{b_1 z^3 + b_2 z^2 + b_3 z + b_4}{z^4 + c_1 z^3 + c_2 z^2 + c_3 z + c_4}. \tag{3}
\]

A difference equation of the Auto Regressive structure with exogenous input (ARX) model is derived from the discretized transfer function as

\[
v^H_{k+1} = -c_1 v^H_k - c_2 v^H_{k-2} - c_3 v^H_{k-3} - c_4 v^H_{k-4} + b_1 v^N_k + b_2 v^N_{k-2} + b_3 v^N_{k-3} + b_4 v^N_{k-4},
\
= f\left(v^H_{k-1}, v^H_{k-2}, v^H_{k-3}, v^H_{k-4}, v^N_{k-1}, v^N_{k-2}, v^N_{k-3}, v^N_{k-4}\right),
\tag{4}
\]

Here, \(v^H_{k-1}\) and \(v^N_{k-1}\) represent the velocity of the human-driven vehicle and the velocity of the last vehicle in the autonomous vehicle platoon respectively. In our previous work, the transfer function (3) was identified by the mean values of the collected data comprising different drivers in different driving situations, it models the HVs with approximately 80% accuracy [10]. However, in order to better control the human in-loop platooning systems, we need to improve the modeling accuracy of the human-driven vehicle. More importantly, we want to quantify the modeling variance of HVs that can be explicitly considered in the safe distance constraint design. Gaussian Process (GP) models [22] were used to estimate discrepancies between the ARX nominal model (4) and the observed input-output behaviors of the human-driven vehicle and also to provide quantification of the modeling uncertainties.

B. Gaussian process regression

Gaussian process (GP) regression is one of the most commonly employed machine learning techniques in learning-based control [17]. Consider \(m\) input data points \(\mathbf{a} = [a_1, \ldots, a_m]^T \in \mathbb{R}^{n_a \times m}\) and the corresponding measurements \(\mathbf{d} = [d_1, \ldots, d_m]^T \in \mathbb{R}^{n_d \times m}\), related through an unknown function \(d_k = g(a_k) + \omega_k; \omega_k \sim N(0, \Sigma^\omega)\), where \(\omega_k\) is i.i.d. Gaussian noise with \(\omega_k \sim N(0, \Sigma^\omega)\) and \(\Sigma^\omega = \text{diag}([\sigma^\omega_1, \ldots, \sigma^\omega_d])\). The function \(g(\cdot)\) can be identified by the observed input-output dataset

\[
D_m = \{\mathbf{a} = [a_1, \ldots, a_m]^T, \mathbf{d} = [d_1, \ldots, d_m]^T\}. \tag{5}
\]

Assume each output dimension of \(d_k\) is independent of each other given the input \(a_k\). For each dimension \(M \in \{1, \ldots, n_d\}\) of the function output \(d_k\), specifying a GP with zero mean and prior kernel \(k^M(\cdot, \cdot)\), the measurement data \([d_k]_M\) is normally distributed as \([d_k]_M \sim N(0, K^M_{aa} + \sigma^2_{M})\), where \(K^M_{aa}\) is the Gram matrix of the data points using the kernel function \(k^M(\cdot, \cdot)\) on the input locations \(a\), i.e., \([K^M_{aa}]_{ij} = k^M(a_i, a_j)\). The choice of kernel functions \(k^M(\cdot, \cdot)\) is specified by a prior knowledge of the observed data. In this work, we use the Squared Exponential (SE) kernel like many other GP learning-based robotic control applications [18].

\[
K^M(a, a) = \sigma^2_M \exp\left(-\frac{1}{2} (a - a)^T L^{-1}_M (a - a)\right), \tag{6}
\]

where \(a\) represents new data points where predictions are made, \(\sigma^2_M, L_M \in \mathbb{R}^{n_a \times n_a}\) are hyperparameters that are optimized by the log marginal likelihood function [23]. The GP models with the optimized hyperparameters are the trained models by using the observed dataset [22].

In the output dimension \(M\), the joint distribution of the observed output \([d]_M\), and the prediction output \([d]_M\) at new data points \(a\) is \(P\left([d]_M, [d]_M | a, a\right)\), and

\[
\begin{bmatrix}
[d]_M \\
[d]_M
\end{bmatrix} \sim N \left( \begin{bmatrix}
0 \\
K_{aa} + \sigma^2_M K_{aa}^M
\end{bmatrix}, \begin{bmatrix}
K_{aa}^M & K_{aa}^M \\
K_{aa}^M & K_{aa}^M + \sigma^2_M K_{aa}
\end{bmatrix} \right), \tag{7}
\]

where \([K_{aa}^M]_{ij} = k^M(a_i, a_j), K_{aa}^M = (K_{aa})^T\), and \(K_{aa} = k^M(a, a)\). The posterior distribution of \([d]_M\) conditioned on the observed data can be derived from (7) as \(P\left([d]_M | [d]_M, a, a\right) = N \left( \mu^M(a), \Sigma^M(a) \right)\) by following equations in [23]

\[
\begin{align}
\mu^M(a) &= K_{aa}^M (K_{aa} + \sigma^2_M)^{-1} [d]_M, \tag{8a} \\
\Sigma^M(a) &= K_{aa} - K_{aa}^M (K_{aa} + \sigma^2_M)^{-1} K_{aa}. \tag{8b}
\end{align}
\]

The resulting GP model estimation \(d(\cdot)\) of the unknown function \(g(\cdot)\) is obtained by vertically concatenating the individual output dimension \(M \in \{1, \ldots, n_d\}\) as

\[
d(a) \sim N \left( \mu^d(a), \Sigma^d(a) \right), \tag{9}
\]

with \(\mu^d(a) = [\mu^1(a), \ldots, \mu^{n_d}(a)]^T \in \mathbb{R}^{n_d}\) and \(\Sigma^d(a) = \text{diag}\left(\Sigma^1(a), \ldots, \Sigma^{n_d}(a)\right)\).
C. The proposed ARX+GP model

We propose to model the human-driven vehicle by using a GP model to correct the prediction by the nominal ARX model \((4)\). This gives us the following ARX+GP model:

\[
v_H^k = \sum_{i=1}^{4} c_i v_H^{k-i} + \sum_{i=1}^{4} b_i v_N^{k-i} = f(\cdot) \quad (\text{see} \ [4]) \tag{10a}
\]

\[
\hat{v}_H^k = v_H^k + g(v_H^{k-1}, v_N^{k-1}) \tag{10b}
\]

Here, \(\hat{v}_H^k\) represents the GP compensated predicted velocity of the human-driven vehicle. The model \(f(\cdot)\) is the ARX nominal model identified in \((4)\) with parameters \(b_i, c_i\) via system identification (see Sec. III-D), and \(g(\cdot)\) is a GP model to learn the discrepancies between the nominal model and the system model (the observed system behavior data).

The GP model is a function of the human-driven vehicle velocity \((v_H)\) and the velocity of the last AV in the platoon \((v_N)\) in front of the human-driven vehicle.

**Training the GP model.** With \((10b)\), by using data points from previously collected measurements of velocity states \(v_H^j\) and \(v_N^j\), the system discrepancies are estimated by a GP model \(d(\cdot)\) as

\[
d_{j-1} = \hat{v}_H^j - v_H^j = g(a_{j-1}) \tag{11}
\]

where the discrepancy state is defined as \(a_{j-1} = (v_H^{j-1}, v_N^{j-1})\). The \(\hat{v}_H^j\) and \(v_H^j\) represent measured velocities in the previously collected data and predictive velocities derived from the nominal model \((4)\), respectively.

In each experiment, the collected one-dimensional input-output data at all time steps are prepared by \((11)\) as a discrepancy data set

\[
\mathcal{D}_m = \{a = [a_1, \cdots, a_{j-1}, \cdots, a_m]^T, \quad g = [g_1, \cdots, g_{j-1}, \cdots, g_m]^T\} \tag{12}
\]

with \(m\) being the total time steps in an experiment. The training process of GP models can be found in [23].

D. Human-driven vehicle model

The data for the transfer function with delay model identification and GP model estimation was collected by three drivers driving in three different driving scenarios (9 data sets) to follow a platoon of AVs in a Unity simulation environment [10]. The three drivers were distracted by being requested to perform a cognitive task (multiple choice algebra questions) when they drove the vehicle. The detailed steps of the data collection experiments can be found in [10, Sec. V.B]. The collected data were processed to calculate the mean values of all data points, then the calculated mean values were used to identify the transfer function of \((2)\) as

\[
G_h(s) = \frac{1 + 6.96s}{1 + (0.65)(4.76)s + (4.76)^2s^2}e^{-0.512s} \tag{13}
\]

By following \((3)\) and \((4)\) to reformulate \((13)\), the parameters of the ARX nominal model \((4)\) were calculated as \(c_1 = -3.0227, c_2 = 3.3543, c_3 = -1.6329, c_4 = 0.3014, b_1 = 0.0063, b_2 = -0.0303, b_3 = 0.0495,\) and \(b_4 = -0.0254\).

Fig. 2: The test result on one testing data set for the ARX+GP model shows a better fit to the actual velocity curves. The modeling accuracy of the ARX+GP model provides a 9% reduction in root mean square error (RMSE) in predicting the HV’s velocity compared to the nominal model.

The nominal model \((4)\) provides an approximately 80% fit of the behaviors of human drivers shown in [10].

To further estimate the discrepancy between the identified nominal model \((4)\) and the actual HV behaviors as GP models, we used 20% evenly picked data points in six data sets for training and three data sets for testing purposes. There are two main reasons to use a limited amount of data (13% of the total collected data in 9 data sets) for training the GP model. If we train the GP model with a big number of data points, it is computationally expensive to use the ARX+GP model in the prediction loop of MPC. And most of the collected data points are with the velocity of 10m/s, 15m/s, and 20m/s, and we tested that GP models will not improve too much by using a big number of data points due to the limited diversity of the collected data.

The training data sets were prepared by following \((11)\) and \((12)\). One of the testing results of the trained GP model is shown in Fig. 2. In Fig. 2, simulation results of the ARX nominal model and ARX+GP model together with two times the standard derivation (2\(\sigma\)) estimated by the GP model were plotted. The actual velocity of HV was plotted together to show the accuracy improvement by combining the GP model to the ARX nominal model. The root mean square error (RMSE) of the nominal model to the actual velocity data is 1.96 on average with three testing data sets, and the average RMSE of the ARX+GP model is 1.12. The modeling accuracy of the ARX+GP model gains an approximate 9% improvement compared to the ARX nominal model.

In Fig. 2 the modeling accuracy improvement can be easily viewed as the ARX+GP model fits the curves of the actual velocity data much better. This is especially true for the time steps during 200-600, 800-1200, and 1400-1800 when the velocities are around 10m/s or 15m/s.

IV. CONTROLLER DESIGN

A. System model

In this section, a GP learning-based MPC for a mixed vehicle platoon comprising of \(n_a\) number of AVs and one HV is developed. The modeling of \(n_a\) number of AVs is
shown in (14a) and (14b). We assume autonomous vehicles are deterministic and can measure (and communicate) their states without any error, i.e., \( \Sigma(v_{n_2}^a) = 0 \). By applying (10b), the human-driven vehicle model is
\[
\begin{align*}
\dot{v}_k^H &= v_k^H + d(v_{k-1}^H, v_{k-1}^a) , \\
p_{k+1}^H &= p_{k}^H + T v_k^H , \\
&= p_{k}^H + T v_k^H + T d(v_{k-1}^H, v_{k-1}^a) ,
\end{align*}
\] (13a)
\[\Sigma\]
\[
\begin{align*}
\mu_{k+1}^H &= \mu_{k}^H + T v_k^H + T \mu_d(v_{k-1}^H, v_{k-1}^a) ,
\end{align*}
\] (14)
with the initial value \( \mu_{0}^H = v_{0}^H \). The propagation equation for the variance of position is
\[
\Sigma_{k+1}^H = \Sigma(k_H) + \Sigma(T v_k^H) + \Sigma(T d(v_{k-1}^H, v_{k-1}^a)) ,
\] (15a)
\[
\begin{align*}
\mu_{k+1}^H &= \mu_{k}^H + T^2 \Sigma_d , \\
\Sigma_{k+1}^H &= \Sigma(k_H) + T^2 \Sigma_d ,
\end{align*}
\] (15)
with the initial value \( \Sigma(k_0^H) = 0 \). The equation (15) neglects the covariance between \( p_{k}^H \) and \( v_{k}^H \).

B. Safe distance chance constraint

To guarantee a safe distance between vehicles in the mixed platoon, a constraint of distance between AVs is designed with a constant value \( \Delta \) as \( p_{k}^{n-1} - p_{k}^{H} > \Delta \). Due to the stochastic feature of the HV model, a chance constraint of the distance between the last autonomous vehicle and the human-driven vehicle is designed to be satisfied as
\[
\Pr(p_{k}^{a} - p_{k}^{H} > \Delta) \geq \phi_{\text{def}} ,
\] (16)
where \( \phi_{\text{def}} \) represent the defined satisfaction probabilities. We can reformulate the distance constraint \( \lambda \) following a single half-space constraint \( \lambda^{H} := \{ x | h^T x \leq b \} , h \in \mathbb{R}^n , b \in \mathbb{R} \)
\[
\begin{align*}
\begin{bmatrix}
-1 & 1
\end{bmatrix}
\begin{bmatrix}
p_{k}^{a} \\
p_{k}^{H}
\end{bmatrix}
\leq -\Delta ,
\end{align*}
\] (17)
where \( h = \begin{bmatrix}
-1 & 1
\end{bmatrix} \) , \( x := \begin{bmatrix}
p_{k}^{a} \\
p_{k}^{H}
\end{bmatrix} \) , and \( b = -\Delta \). In [19], a method of tightened constraint on the state mean is derived as
\[
\lambda^{H} (\Sigma_{k}^{H}) := \{ x | h^T x \leq b - \phi^{-1}(\phi_{\text{def}}) \sqrt{h^T \Sigma_{k}^{H} h} \} ,
\] (18)
where \( \phi^{-1} \) represents the inverse of the cumulative distribution function (CDF). In our case,
\[
\Sigma_{k}^{H} := \begin{bmatrix}
\Sigma_{k}^{p_{k}^{a}} & \Sigma_{k}^{p_{k}^{H}} \\
\Sigma_{k}^{p_{k}^{a}} & \Sigma_{k}^{H}
\end{bmatrix}
= \begin{bmatrix}
0 & 0 \\
0 & \Sigma_{k}^{H}
\end{bmatrix} .
\] (19)

Here, there is no covariance between the position states of the last AV and the HV and \( \Sigma_{k}^{p_{k}^{a}} = 0 \). The position variance of the human-driven vehicle \( \Sigma_{k}^{p_{k}^{H}} \) is calculated in (13d). The "tightened" position constraint is obtained by substituting (17) and (19) to (18) as
\[
p_{k}^{a} - p_{k}^{H} \geq \Delta + \phi^{-1}(\phi_{\text{def}}) \sqrt{\Sigma_{k}^{p_{k}^{H}}} .
\] (20)
The chance constraint of distance (16) is approximated as a deterministic equation with customer-defined satisfaction probabilities \( \phi_{\text{def}} \) in (20). There is an extra component \( \phi^{-1}(\phi_{\text{def}}) \sqrt{\Sigma_{k}^{p_{k}^{H}}} \) added to the fixed distance constraint \( \Delta \). The distance constraint between the HV and AVs changes according to the uncertainty estimations of the HV by the GP model.

C. GP learning-based model predictive control

A GP learning-based MPC (GP-MPC) strategy for the longitudinal car-following control of a mixed vehicle platoon comprising \( N_{\text{a}} \) number of AVs and one HV shown in Fig. 1 is developed as
\[
\begin{align*}
\min \sum_{i=k}^{k+N} \| v_{i+1}^1 - v_{i+1}^{ref} \|_{Q_1}^2 \\
+ \sum_{i=k}^{N_{\text{a}}} \sum_{i=k}^{k+N} \| v_{i+1}^a - v_{i+1}^{ref} \|_{Q_2}^2 \\
+ \sum_{i=k}^{N_{\text{a}}} \| \alpha_{i+1}^a \|_{R}^2 
\end{align*}
\] (21a)
with \( \mathbb{V} = \{ v_{1}^1, v_{1}^{a}, v_{1}^{H}, p_{1}^{a}, \mu_{1}^{H}, \Sigma_{1}^{p_{1}^{H}}; a_{1}^{a} \} \) subject to
\[
\begin{align*}
v_{i+1}^{a} &= v_{i}^{a} + T a_{i}^{a} , \\
p_{i+1}^{a} &= p_{i}^{a} + T v_{i}^{a} , \\
v_{i}^{H} &= -c_{1} v_{i}^{H} - c_{2} v_{i-1}^{H} - c_{3} v_{i-2}^{H} \\
&- c_{4} v_{i-1}^{a} + b_{1} v_{i-1}^{H} + b_{2} v_{i-2}^{H} \\
&+ b_{3} v_{i-1}^{H} + b_{4} v_{i-2}^{H} \\
\mu_{i+1}^{H} &= \mu_{i}^{H} + T v_{i}^{H} + T \mu_{i}^{a} , \\
\Sigma_{i+1}^{H} &= \Sigma_{i}^{H} + T^2 \Sigma_{i}^{a} \\
p_{i+1}^{a} - p_{i}^{H} \geq \Delta , \\
v_{i}^{min} \leq v_{i}^{a} \leq v_{i}^{max} , \\
\alpha_{i}^{a} \leq \alpha_{i}^{a} \leq \alpha_{max} , \\
v_{i}^{1} = v_{i}^{1} , \\
v_{i}^{a} = v_{i}^{a} , \\
v_{i}^{H} = v_{i}^{H} , \\
p_{i}^{a} = p_{i}^{a} \\
p_{i}^{H} = p_{i}^{H} , \\
\Sigma_{i}^{p_{i}^{a}} = 0 .
\end{align*}
\] (21)
The current time step is $k$, and the system (hardware or simulation) starts at time $k = 0$. In the MPC prediction horizon, the prediction horizon time step starts at $i = 1$, and variable values at $i = 0$ are initialized with measurements shown in the right sides of (21k), (21h), (21m), and (21n). In the cost function (21a), differences between the reference velocity and velocities of the leading AV, differences between velocities of the neighboring AVs, and control inputs are weighted by three positive weights $Q_1$, $Q_2$, and $R$ respectively. In the equality constraint (21b), the (21b) and (21c) are the model of AVs defined in (1a) and (1b), the (21d) is the ARX nominal model of the HV specified in (4), the (21e) and (21f) are mean and variance propagation equations of the HV position derived in (14) and (15d) respectively. The inequality constraints include the velocity (21j), acceleration (21k), and safe distance (21l) and (21m).

V. SIMULATION AND EXPERIMENTS

This section shows simulation case studies of a mixed vehicle platoon of two autonomous vehicles being trailed by one human-driver vehicle applying the MPC strategy developed in Sec IV-C and a baseline MPC. All simulations were implemented in MATLAB R2022a on a computer running Ubuntu 20.04. Our implementation can be found at: https://github.com/CL2-UWaterloo/GP-MPC-of-Platooning.

A. Simulation setup

At the beginning of all simulations, the initial velocities of all vehicles were set to 0. The sample time was set as $T = 0.1s$, and the prediction horizon of both MPC was $N = 10$. The weights of the MPC cost function were tuned as $Q_1 = Q_2 = 5$ and $R = 10$. The maximum and minimum acceleration were set as $a_{\text{max}} = 5m/s^2$ and $a_{\text{min}} = -5m/s^2$ respectively, and the maximum and minimum velocity were defined as $v_{\text{max}} = 35m/s$ and $v_{\text{min}} = -35m/s$ respectively. We initialized the leader autonomous vehicle at $p = 0$, the second AV was $\Delta = 20m$ behind the leader AV, and the human-driven vehicle was $\Delta = 20m$ behind the follower AV. The satisfaction probability of the chance constraint in (16) was set to be $p_{\text{sat}} = 0.95$.

Besides the proposed GP-MPC, a baseline MPC, referred to as nominal MPC was implemented for comparison purposes. In the baseline MPC, the prediction loop uses the nominal HV model of (21d) and (21e) without the third GP component. The distance constraint between the HV and the follower AV given is $\Delta$ without the second adaptive component in (21h). There is no position variance propagation by (21f) in the nominal MPC. In all simulations for the nominal MPC and GP-MPC, the simulated human-driven vehicle model is the ARX+GP model derived in (13a) and (13b).

B. Simulation results

We first conducted a constant velocity tracking simulation with the reference velocity $v_{\text{ref}} = 20m/s$ given to the leader autonomous vehicle for the proposed GP-MPC. The simulation results are shown in Fig. 3. From top to bottom plots, they show the constant reference velocity tracking, the distance between vehicles, and applied acceleration inputs respectively. We can see in the top plot of the velocity tracking, the velocities of the HV between $4 - 5s$ are noisy. This is because of the limited diversity of the collected data for the GP model training discussed in Sec. III-D the trained GP used in the simulated HV model thus can not provide high-quality discrepancy corrections when $v_H = 0 - 10m/s$ at $4 - 5s$. The quantitative analysis of the simulation results by metrics were conducted and summarized in Tab. I.

Another case study is an emergency braking simulation for the nominal MPC and GP-MPC. The reference velocity $v_{\text{ref}}$ for the leading AV was set to $20m/s$ and changed to $10m/s$ at $t = 15s$. Similar to the constant velocity tracking simulation, we plotted the velocity tracking results, the distance between vehicles, and applied acceleration inputs from top to bottom shown in Fig. 4 and Fig. 5 for the nominal MPC and GP-MPC respectively. In the top plots of Fig. 4 and 5 when a braking happens at $t = 15s$, we saw immediate deceleration of the leader and follower autonomous vehicle. However, the human-driven vehicle accelerated for approximately 1s before started to decelerate at $t = 17s$. The main reason is the human-driven vehicle is modeled by the data collected when the drivers were distracted explained in Sec. III-D.
In fact, even if drivers are attentive, the response delays are smaller but still much bigger than the AVs. The response time difference can cause accidents and make the consideration of uncertainties of the human-driven vehicles meaningful.

We summarized two simulation performance metrics, accumulative control cost and minimum relative distance between vehicles, as a Table I to better show the performance improvements of the proposed GP-MPC compared to the nominal MPC. The accumulative control cost was calculated by a modified cost function based on (21a) for the vehicle platoon simulations as

\[
\sum_{k=1}^{299} (\|v_{k}^1 - v_{ref}^1\|^2_{Q_1} + \|v_{k}^2 - v_{ref}^2\|^2_{Q_2}) + \sum_{k=1}^{299} (\|a_{k}^1\|^2_{R} + \|a_{k}^2\|^2_{R}) + \sum_{k=1}^{300} (\|v_{k}^{FH}\|^2_{Q_1} + \|v_{k}^{FH}\|^2_{Q_2}).
\] (22)

In Tab. I, on average the constant velocity tracking and the emergency braking simulation results, the total control cost of the full trajectory for the GP-MPC is 4.2% lower than the nominal MPC. In both the constant velocity tracking and the emergency braking simulations, the minimum distances between vehicles are the distances between the follower autonomous vehicle and the human-driven vehicle denoting as $p^{FH}$. For the constant velocity tracking simulation, $p^{FH}$ of the GP-MPC (29.80m) is slightly bigger than the $p^{FH}$ of the nominal MPC (29.68m). In our proposed GP-MPC, the extra distances to be added to the distance constraint specified in (21h) are proportional to the GP uncertainty assessments. When the traffic is more challenging such as in an emergency braking scenario, bigger extra distances are added to the distance constraint in (21h) at all time steps to better guarantee safety. We can see the $p^{FH}$ of the GP-MPC (22.18m) is 1m bigger than the $p^{FH}$ of the nominal MPC (21.12m).

C. Summary of simulation results

In the two simulation case studies, the proposed GP-MPC shows a lower control cost than a baseline MPC for the longitudinal car-following control of a platoon of autonomous vehicles interacting with a human-driven vehicle. And by integrating the uncertainty assessment of the human-driven vehicle modeling to the distance constraint, the minimum distance between vehicles is larger in the GP-MPC for

### TABLE I: Simulation performance metrics

| Controller    | Velocity tracking Cost | Min Distance  | Emergency braking Cost | Min Distance |
|---------------|------------------------|---------------|------------------------|--------------|
| Nominal MPC   | 54125                  | 29.68m        | 71429                  | 21.12m       |
| GP-MPC        | 52206                  | 29.80m        | 68078                  | 22.18m       |
challenging traffic scenarios.

VI. CONCLUSIONS

Summary. In this paper, we study the problem of safe control of a platoon of autonomous vehicles interacting with a human-driven vehicle in longitudinal car-following scenarios. To better predict the behaviors of the human-driven vehicle when it interacts with AVs, we modeled the human-driven vehicle as the sum of a first-principles model and a Gaussian process learning-based model. Exploiting the properties of this model, we designed a model predictive control strategy for a platoon of AVs to ensure the probabilistic safety of the mixed vehicle platoon. We evaluated our method by comparing it to a baseline MPC in simulation case studies. Our results show that the proposed method achieves better velocity-tracking performance and more robust constraint satisfaction control for a platoon of mixed vehicles in challenging traffic scenarios.

Limitations and future work. There are many limitations of our work in this paper that deserve future work. First, the diversity of the collected data for modeling the human-driven vehicle was limited. Second, the Gaussian process model was trained offline, a more computationally efficient method such as spares GP or local GP is needed for applications on real vehicles. Further, regarding the values of distance constraints in the controller design, we did not investigate the boundary ability of the proposed GP-MPC. This can be an interesting future work to study the minimum distance constraint needed for the proposed method to guarantee safe control. Last but not least, developing a decentralized GP-MPC policy will be meaningful for real vehicle platooning control applications.

ACKNOWLEDGMENT

This work was supported in part by Magna International and the Canada Natural Sciences and Engineering Research Council Discovery Grant.

REFERENCES

[1] J. Guanetti, Y. Kim, and F. Borrelli, “Control of connected and automated vehicles: State of the art and future challenges,” Annual reviews in control, vol. 45, pp. 18–40, 2018.

[2] M. Martínez-Díaz, C. Al-Haddad, F. Soriguera, and C. Antoniou, “Platooning of connected automated vehicles on freeways: a bird’s eye view,” Transportation research procedia, vol. 58, pp. 479–486, 2021.

[3] M. Aramrattana, A. Habibovic, and C. Englund, “Safety and experience of other drivers while interacting with automated vehicle platoons,” Transportation research interdisciplinary perspectives, vol. 10, p. 100381, 2021.

[4] Y. Rahmati, M. Khajeh Hosseini, A. Talebpour, B. Swain, and C. Nelson, “Influence of autonomous vehicles on car-following behavior of human drivers,” Transportation research record, vol. 2673, no. 12, pp. 367–379, 2019.

[5] A. Sadat, S. Casas, M. Ren, X. Wu, P. Dhawan, and R. Urtasun, “Perceive, predict, and plan: Safe motion planning through interpretable semantic representations,” in European Conference on Computer Vision. Springer, 2020, pp. 414–430.

[6] L. Guo and Y. Jia, “Inverse model predictive control (IMPC) based modeling and prediction of human-driven vehicles in mixed traffic,” IEEE Transactions on Intelligent Vehicles, vol. 6, no. 3, pp. 501–512, 2020.

[7] C. C. Macadam, “Understanding and Modeling the Human Driver,” Vehicle System Dynamics, 2003.

[8] S. Yu, M. Hirche, Y. Huang, H. Chen, and F. Allgöwer, “Model predictive control for autonomous ground vehicles: A review,” Autonomous Intelligent Systems, vol. 1, no. 1, pp. 1–17, 2021.

[9] C. M. Tampère, “Human-kinetic multiclass traffic flow theory and modelling. With application to advanced driver assistance systems in congestion,” Ph.D. dissertation, Delft University of Technology, 2004.

[10] M. Pirani, Y. She, R. Tang, Z. Jiang, and Y. V. Pant, “Stable interaction of autonomous vehicle platoons with human-driven vehicles,” in 2022 American Control Conference (ACC). IEEE, 2022, pp. 633–640.

[11] S. Panwai and H. Dia, “Neural agent car-following models.” IEEE Transactions on Intelligent Transportation Systems, vol. 8, no. 1, pp. 60–70, 2007.

[12] A. Khodayari, A. Ghaffari, R. Kazemi, and R. Braunstingl, “A modified car-following model based on a neural network model of the human driver effects,” IEEE Transactions on Systems, Man, and Cybernetics-Part A: Systems and Humans, vol. 42, no. 6, pp. 1440–1449, 2012.

[13] J. Morton, T. A. Wheeler, and M. J. Kochenderfer, “Analysis of recurrent neural networks for probabilistic modeling of driver behavior,” IEEE Transactions on Intelligent Transportation Systems, vol. 18, no. 5, pp. 1289–1298, 2016.

[14] S. Lefèvre, Y. Gao, D. Vasquez, H. E. Tseng, R. Bajcsy, and F. Borrelli, “Lane keeping assistance with learning-based driver model and model predictive control,” in 12th International Symposium on Advanced Vehicle Control, 2014.

[15] T. Qu, S. Yu, Z. Shi, and H. Chen, “Modeling driver’s car-following behavior based on hidden markov model and model predictive control: A cyber-physical system approach,” in 2017 11th Asian Control Conference (ASCC). IEEE, 2017, pp. 114–119.

[16] S. Lefèvre, C. Sun, R. Bajcsy, and C. Laugier, “Comparison of parametric and non-parametric approaches for vehicle speed prediction,” in 2014 American Control Conference. IEEE, 2014, pp. 3494–3499.

[17] L. Brunke, M. Greiff, A. W. Hall, Z. Yuan, S. Zhou, J. Panerati, and A. P. Schoellig, “Safe learning in robotics: From learning-based control to safe reinforcement learning,” Annual Review of Control, Robotics, and Autonomous Systems, vol. 5, 2021.

[18] L. Hewing, J. Kabzan, and M. N. Zeilinger, “Learning-based model predictive control: Toward safe learning in control,” Annual Review of Control, Robotics, and Autonomous Systems, vol. 3, pp. 269–296, 2020.

[19] L. Hewing, J. Kabzan, and M. N. Zeilinger, “Cautious model predictive control using Gaussian process regression,” IEEE Transactions on Control Systems Technology, vol. 28, no. 6, pp. 2736–2743, 2019.

[20] X. Chen, L. Li, and Y. Zhang, “A markov model for headway/spacing distribution of road traffic,” IEEE Transactions on Intelligent Transportation Systems, vol. 11, no. 4, pp. 773–785, 2010.

[21] V. Milanés, S. E. Shladover, J. Spring, C. Nowakowski, H. Kawazoe, and M. Nakamura, “Cooperative adaptive cruise control in real traffic situations,” IEEE Transactions on intelligent transportation systems, vol. 15, no. 1, pp. 296–305, 2013.

[22] J. Wang, “An intuitive tutorial to Gaussian processes regression,” arXiv preprint arXiv:2009.10862, 2020.

[23] C. E. Rasmussen and C. K. I. Williams, Gaussian processes in machine learning. MIT Press, 2006.