Gauge invariant generalization of the 2D chiral Gross-Neveu model

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By means of the Lee-Shrock transformation we generalize the 2D Gross-Neveu (GN\(_2\)) model to a U(1) gauge theory with charged fermion and scalar fields in 2D (\(\chi U \phi_2\) model). The \(\chi U \phi_2\) model is equivalent to the GN\(_2\) model at infinite gauge coupling. We show that the dynamical fermion mass generation and asymptotic freedom in the effective four-fermion coupling persist also when the gauge coupling decreases. These phenomena are not influenced by the XY\(_2\) model phase transition at weak coupling. This suggests that the \(\chi U \phi_2\) model is in the same universality class as the GN\(_2\) model and thus renormalizable.

1. Introduction

Chiral symmetric strongly coupled lattice gauge theories with fermion and scalar matter fields, \(\chi\) and \(\phi\), contain unconfined fermions \(F = \phi^\dagger \chi\). Their mass \(m_F\) is generated dynamically in the phase with chiral symmetry breaking \([1,2]\) and such models can thus be considered as an alternative to the standard Higgs-Yukawa mechanism of fermion mass generation, provided they are nonperturbatively renormalizable \([3]\). Here we consider a 2D model of this type with continuous chiral symmetry, the \(\chi U \phi_2\) model, which in the strong coupling limit is equivalent to the chiral GN\(_2\) model. The observed scaling properties at large but finite gauge coupling indicate that the \(\chi U \phi_2\) model belongs also here to the universality class of the GN\(_2\) model. This would mean that the model is renormalizable and that the shielded gauge mechanism of fermion mass generation, proposed in \([3]\), works in 2D. A similar problem in 4D is addressed in ref. \([4]\) and in these proceedings \([5]\).

2. The model

The action of the \(\chi U \phi_2\) model consists of three parts:

\[
S_{\chi U \phi} = S_\chi + S_U + S_\phi
\]  

(1)

where

\[
S_\chi = \frac{1}{2} \sum_x \bar{\chi}_x \sum_{\mu=1}^2 \eta_{\mu x} (U_{x,\mu} \chi_{x+\mu} - U_{x-\mu,\mu} \chi_{x-\mu}) + a m_0 \sum_x \bar{\chi}_x \chi_x ,
\]

\[
S_U = \beta \sum_P (1 - \Re U_P) ,
\]

\[
S_\phi = -\kappa \sum_x \sum_{\mu=1}^2 (\phi_x^\dagger U_{x,\mu} \phi_{x+\mu} + H.c.) .
\]

Here \(\chi\) is a staggered fermion field with charge one, \(U_{x,\mu} \in U(1)\) is a compact abelian gauge field with coupling \(\beta = 1/a^2 g^2\) and \(\phi\) is a complex scalar field with charge one and constraint \(|\phi_n| = 1\). The mass term is introduced for technical reasons and the model is meant in the limit \(m_0 = 0\), where the action has a global U(1) chiral symmetry.

In the limiting case \(\beta = 0\) one can perform the Lee-Shrock-transformation \([3]\) in which the scalar and gauge fields are integrated out and a four fermion term appears. This leads to the following action \((r = I_1(2\kappa)/I_0(2\kappa))\):

\[
S_{4f} = - \sum_x \sum_{\mu=1}^2 \left( \frac{1 - r^2}{4r^2} \bar{\chi}_x \chi_x \bar{\chi}_{x+\mu} \chi_{x+\mu} \right) \frac{1}{G} \left( \bar{\chi}_x \chi_{x+\mu} - \bar{\chi}_{x+\mu} \chi_x \right)
\]
The four fermion coupling \( G(\kappa) \) is a function of \( \kappa \) which is shown in fig. 1. The transformed action (2) for \( am_0 = 0 \) is that of the chiral GN\(_2\) model. This model has a critical point at \( G = 0 \), where the fermion mass vanishes as

\[
\text{am}_F \rightarrow 0 \propto e^{-\pi F^2 \text{G}}.
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### 3. Generalized scaling behavior at \( \beta = 0 \)

We first verified the predicted scaling law (3) at \( \beta = 0 \). For \( am_0 = 0 \) the simulation is possible only for small lattices (\( \lesssim 64^2 \)) because of the slow convergence of the fermion matrix inversion.

Hence we 'somehow' had to extrapolate the masses measured at \( am_0 > 0 \) to \( am_0 = 0 \). It turned out that all examined schemes which tried to extrapolate for fixed \( \kappa \) to \( am_0 = 0 \) failed. But to perform the continuum limit \( \kappa \rightarrow \infty \) it is not necessary to set \( am_0 = 0 \) first. Also a combined limit \( \kappa \rightarrow \infty \) and \( am_0 \rightarrow 0 \) can lead to a continuum theory without bare mass.

Therefore we looked for a scaling law, in which \( am_0 > 0 \) is allowed for finite \( \kappa \) but \( am_0 \rightarrow 0 \) when \( \kappa \rightarrow \infty \). Such a generalized scaling law is provided by the SD-equations for the model [3] truncated to order \( O(\text{G}) \) [3]:

\[
N = am'_0 + \frac{4G}{V} \sum_k \frac{N}{\sum \nu F^2(\frac{1}{2} \sin(k_\nu a))^2 + N^2} \tag{4}
\]

\[
F_\mu = 1 + 2G^2 \sum_k \frac{F_\mu(\frac{1}{2} \sin(k_\nu a))^2}{\sum \nu F^2(\frac{1}{2} \sin(k_\nu a))^2 + N^2} \tag{5}
\]

These two coupled equations for functions \( N \) and \( F \), with \( am_F = N/F \), were solved numerically [7]. For infinite volume and small \( G \) their approximate analytic solution is

\[
N = am'_0 - \frac{8GN}{\pi F^2} \ln \left( \frac{2N}{\pi F} \right) \tag{6}
\]

\[
F = 1 + \frac{1}{2} \sqrt{1 + G}. \tag{7}
\]

For \( am_0 = 0 \) one obtains the scaling behavior (3) as \( G \rightarrow 0 \).

The idea of the combined limit \( \kappa \rightarrow \infty \) and \( am_0(\kappa) \rightarrow 0 \) is to make \( am'_0 \) a function of \( \kappa \) in such a way, that eq. (6) is solvable and that \( \lim_{\kappa \rightarrow \infty} am'_0(\kappa) = 0 \). We choose (see fig. 2)

\[
am'_0 = \frac{1}{r} am_0(s) = (1 - s) \pi F^2 e^{-\pi F^2 s / 8G} \tag{8}
\]

with a free parameter \( s \) obeying \( 0 < s \leq 1 \). The generalized scaling law is then

\[
N = \frac{\pi F}{2} e^{-\pi F^2 s / 8G} \tag{9}
\]

and thus

\[
am_F = \frac{N}{F} = \frac{\pi F^2}{2} e^{\pi F^2 s / 8G} = \frac{am_0(G, s)}{rF(1 - s)}. \tag{10}
\]

Obviously, the ratio \( \text{am}_F \) does not vanish except if \( s = 1 \).

Fig. 3 shows, up to the factor \( Z = 1/rF \) which approaches 1 when \( G \rightarrow 0 \), the ratio of the measured fermion mass and the bare mass plotted against \( 1/G \). In this ratio the bare part \( am_0 \) of the fermion mass is 'divided out'. Calculations have been performed for \( s = 0.2, 0.3, 0.4, 0.5 \). It turned out to be very difficult to obtain sufficient data for larger values of \( s \), as much larger
lattices would be needed. The full lines show the corresponding ratio calculated using the SD-equations for finite lattice and those $am_0 > 0$ given by (8). Equation (10) is represented by horizontal dashed lines. Good agreement can be observed, especially for not too small $am_0$ and not too large $G$. Thus at $\beta = 0$ we have a suitable analytic description of the scaling behavior of the measured fermion mass, at least in the interval $0.2 \leq s \leq 0.5$.

4. Scaling behavior at $\beta > 0$

But how can the data be described for $\beta \gtrsim 0$? Fig. 4 shows the same ratio, but for masses measured at $\beta = 0.5$. As expected, at the same $G$ value the fermion masses are in general smaller than those for $\beta = 0$, as the gauge coupling becomes smaller. This is seen by comparison of the data with the dotted lines which indicate the ratios calculated by the SD-equations at $\beta = 0$.

The data suggest a simple modification of the scaling behavior at $\beta > 0$ with respect to $\beta = 0$. Indeed, we have found that a good agreement between the measured data and the SD-equations can be obtained when in these equations the parameter $s$ is replaced by a fitted value $s_{\text{fit}}$. The full lines in fig. 4 show the calculated ratio using
the SD-equations (4,5) with a bare mass which is given by (8) but using $s_{\text{fit}}$ instead of $s$.

Although the theoretical reasons are not well understood, the SD-equations modified in this way provide a suitable analytic description of the measured data also for $0 < \beta \lesssim 1$ in the range $0.2 \leq s \leq 0.5$.

The scaling law for $\beta \geq 0$ corresponding to equations (10) and (8) when using $s_{\text{fit}}$ is:

$$m_F = \frac{\pi}{2} D(s, \beta) e^{\frac{-\pi F^2 s}{8g_0}}$$  \hspace{0.5cm} (11)

with

$$D(s, \beta) = \frac{1-s}{1-s_{\text{fit}}}.$$  \hspace{0.5cm} (12)

For the examined interval $0.2 \leq s \leq 0.5$ we thus observe the same scaling law for $\beta = 0$ apart from the $G$-independent factor $D(s, \beta)$ with $D(s, 0) = 1$.

Thus we found some evidence that the $\chi U\phi_2$ model for $0 \leq \beta \lesssim 1$ and $am_0$ finite but approaching zero belongs to the same universality class as the $GN_2$ model with similar bare mass. A method of extrapolating to the $am_0 = 0$ ($s = 1$) limit has not yet been found, however.

Nevertheless, it is plausible that the scaling behavior remains consistent with (11), and thus with the scaling behavior of the $GN_2$ model, also when $s \to 1$.

A calculation of $am_F$ for $\beta > 1$ is very difficult because of its low values, and we could not study its scaling behavior any more. But looking at some bulk observables and bosonic masses, we found no indication that the properties of the $\chi U\phi_2$ model substantially change when $\beta$ is increased. We also found that the Kosterlitz-Thouless (KT) phase transition occurring at $\beta = \infty$, when the $\chi U\phi_2$ model reduces to the XY$_2$ model and decoupled fermions, gets weaker and dissolves at some finite $\beta$ when $\beta$ is increased. This ‘remnant’ of the KT phase transition does not influence the fermion mass in an appreciable way. These observations suggest that the scaling behaviour of $am_F$ is the same at small and large $\beta$, and that only the magnitude of $am_F$ decreases with increasing $\beta$. The continuum limit is approached at any $\beta < \infty$ by turning $\kappa \to \infty$.

If both above hypothetical extensions of our results are correct, then the $\chi U\phi_2$ model belongs to the universality class of the $GN_2$ model at all $\beta < \infty$ and is thus a renormalizable field theory in which the mass of an unconfined fermion is generated dynamically. It is thus an example of the shielded gauge mechanism proposed in [3].

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