HEAVY QUARK PRODUCTION:
THEORY VS. EXPERIMENT ♣

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Heavy quark production has been for years a closely scrutinized process, as it has represented one of the very few instances in which experimental measurements and next-to-leading order (NLO) QCD predictions seemed to be at variance. Such a disagreement appeared surprising, as one expects perturbative QCD to be able to handle heavy quark predictions well, due to the (relatively) large scale set by their mass and to their small hadronization corrections. Recently, new theoretical developments and better use of higher-quality non-perturbative information have however greatly reduced the disagreement in bottom production in $p\bar{p}$ collisions, to the point that it does not appear significant anymore. At the same time, comparisons for top and charm production appear successful, pointing once again to a healthy status for this sector of QCD.

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1 It should however be noted that, while the large heavy quark mass does indeed make accurate calculations possible, at the same time it raises the bar: heavy quark production rates become calculable and they are a real prediction of QCD, contrary to the light quark case. Recalling that the NLO corrections predict a large increase with respect to the leading order rates, it should not come as a surprise if this calculation does not yet manage to describe perfectly the measurements.

2 Recent comparisons to new preliminary bottom data from the Tevatron Run II show an even better agreement. I am not reviewing here $ep$ and $\gamma\gamma$ collisions, where discrepancies still seem to exist, but where the accuracy of both data and theoretical predictions is also lower.
Top - The top quark was discovered at the Tevatron during Run I, and it has recently been once again observed in the first Run II data. Its total cross section, now measured at the slightly higher centre-of-mass energy $\sqrt{S} = 1960$ GeV, can therefore be compared to new predictions. While NLO corrections [2] for the total cross section were already available at the time of the top discovery, improvements of the last few years include the implementation of the resummation of soft-gluon (threshold) effects to next-to-leading logarithmic (NLL) accuracy [3], and the determination of parton distribution function sets (PDFs) with associated errors [4]. Figure 1 displays in the left panel the theoretical predictions and corresponding uncertainties [5] for the top total cross section at the Tevatron Run II, i.e. $p\bar{p}$ collisions at $\sqrt{S} = 1960$ GeV. For each PDF set the crosses correspond to the prediction obtained by setting the renormalization and factorization equal to 1/2, 1 or 2 times the central scale. The rectangle around each cross represents instead the uncertainty related to the given PDF set. It is apparent from the right panel of figure 1 that the experimental uncertainties are nowadays still much larger than the theoretical ones, and that the latter are dominated by the PDFs uncertainty, due to our poor knowledge of the gluon distribution function at large $x$. This points to a situation where not a better calculation, but rather more accurate experimental inputs will be needed in order to perform a more compelling comparison for this observable.

Bottom - Bottom production in $p\bar{p}$ collisions at UA1 and the Tevatron [6] has been for years an example of a possible discrepancy between theory and data, by factors of 2 to 3. Many explanations for the discrepancy...
were proposed, from conventional ones (like the importance of higher orders or small-\(x\) contributions) to a more exotic one involving supersymmetric particles production \[7\].

A careful analysis of the way the data are extracted and/or theoretical predictions are evaluated shows however that some common practices need to be revised in the light of today's desired accuracy. Bottom quarks are not observed as free quarks, as they hadronize into \(H_b\) hadrons before decaying to other bottomless final states. The \(b \rightarrow H_b\) transition cannot be described by perturbative QCD, and it is usually parameterized by convolving the momentum distribution of the quark with a phenomenological function - usually extracted from \(e^+e^-\) data - which accounts for the degradation of its momentum in the hadronization process. The bottom quark not being directly observed, there is however no way to determine uniquely its 'fragmentation': when measuring the \(H_b\) cross section we only observe the result of both a perturbative (hard gluon emissions) and a non-perturbative (soft gluons) degradation of the initial quark momentum. Hence, in the theoretical description the two 'steps' must be properly matched, and no single function can be a tool good for all purposes and instances. Data presented at the unphysical \(b\) quark level, the result of a deconvolution performed on real \(H_b\) hadron data, may therefore be tainted by the use of an improper phenomenological parameterization of the fragmentation effects. Fortunately, the CDF Collaboration has also published data for the \(B^+\) mesons, real observable objects, allowing for a safer comparison between theory and experiment.

Ref. \[8\] implements these considerations by properly matching perturbative (FONLL, i.e. full massive fixed order calculation to NLO accuracy plus resummation of \(\log(p_T/m)\) terms to NLL accuracy) and non-perturbative physics, extracting the relevant (i.e. moments around \(N = 5\)) experimental
input from $e^+e^-$ data employing the same kind of perturbative calculation (and the same parameters) which will then be used to calculate the cross section at the Tevatron. The results of this analysis are shown in fig. 2. The data and the theoretical predictions are compatible within the uncertainties.

More theoretical progress in the bottom production sector has taken place for the heavy quark fragmentation function. Large-$N$ moments are not needed to describe hadroproduction, but they are well measured in $e^+e^-$ collisions, and have an enhanced sensitivity to non-perturbative contributions. All finite order perturbative calculations, and even finite logarithmic accuracy resummations, are bound to fail eventually when the endpoint region is approached. An ‘all-logs’ calculation can however be performed in the large-$\beta_0$ limit [9]. Including infinitely many orders in the strong coupling this so-called Dressed Gluon Exponentiation (DGE) result is necessarily divergent, as perturbative QCD series are only asymptotic. The divergence manifests itself in terms of poles (“renormalons”) in the Borel transform, which becomes non-invertible unless a regularization prescription is supplied. The ambiguity of the regularization procedure is related to the missing higher twist terms, whose functional form can therefore now be inferred. Hence one is left with a perturbative calculation and with the matching power corrections which build up its complementary non-perturbative function. The full result can be written as a convolution of the two terms, $D(x) = D_{PT}(x) \otimes D_{NP}(x)$, neither independent of the other. Being the functional form for $D_{NP}$ suggested by the ambiguity, there is no need to resort to a phenomenological model, and only a few parameters have to be fixed by using the data. The fragmentation function $D(x)$ resums effects at the scale $m/N$, and it is furthermore universal. It can be convoluted with a specific coefficient function in order to describe a given process, e.g. $e^+e^-$. Figure 3 shows how well the data can be described just by fitting the
non-perturbative function

\[ D_N^{NP} \left( \epsilon_1, \epsilon_2, \frac{(N-1)\Lambda_{QCD}}{m} \right) = \exp \left\{ -\epsilon_1 \frac{(N-1)\Lambda_{QCD}}{m} - \epsilon_2 \left( \frac{(N-1)\Lambda_{QCD}}{m} \right)^2 \right\} \]

(1)

to a very limited set of moments. The leading non-perturbative effect, a shift to the left of the perturbative distribution controlled by \( \epsilon_1 \Lambda_{QCD} \), is compatible with the expectation that this be a typical hadronic scale, i.e. \( \sim 300 \) MeV. It is also worth mentioning how, fitting also \( \Lambda_{QCD} \) in order to check for consistency, this does indeed return a value fully compatible with other determinations of the strong coupling.

**Charm** - A QCD calculation for bottom production should also be able to reproduce charm data, given the replacement \( m_b \rightarrow m_c \) and a different non-perturbative part, since the quark \( \rightarrow \) meson transition is quantitatively different.

To verify this, the same theoretical framework which gives a fairly good description of the \( B^+ \) CDF data at the Tevatron has been checked against new charm data from the Tevatron Run II [10]. In this case [11] the determination of the non-perturbative function was slightly more involved, as LEP data were not available for all the states (\( D^0, D^+, D^* \) and \( D_s \)) measured in hadronic collisions. Effective fragmentation functions were therefore constructed from the one for \( D^* \), which is accurately measured in e\(^+\)e\(^-\), modeling the \( V \rightarrow P \) decays and employing a perturbative QCD model [12] to parameterize fragmentation into vector (\( V \)) and pseudoscalar (\( P \)) mesons with one free parameter only. The results for \( D^0 \) are shown in figure 4 (those for \( D^+ \) and \( D^{*+} \) are qualitatively identical). The FONLL calculation, com-
plemented by non-perturbative information properly extracted from $e^+e^-$
data, is clearly able to describe the CDF data within the uncertainties.

Last but not least, the resummed calculation of ref. [9] and the form it
predicts for the non-perturbative power corrections, given in eq. (1), can be
tested against $D^*$ fragmentation data in $e^+e^-$ collisions. Figure 4 shows
in the right panel the ALEPH data and the curve given by the same non-
perturbative parameters fitted to $B$ meson data and listed in fig. 3 the
only change having been the replacement of the bottom mass by the charm
mass in both the perturbative and the non-perturbative components of the
fragmentation function. The fairly good description of the data, at this stage
to be taken only as a preliminary indication [13], shows that the scaling of
the non-perturbative effects with the heavy quark mass is indeed correctly
predicted, and that from the numerical point of view the hadronization
effects are pretty similar in the $B$ and the $D$ sector, though a more refined
analysis should certainly be performed.

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