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To cite this article: Carlos Ríos et al 2016 J. Phys.: Conf. Ser. 720 012008

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The Emergent Universe and the Anomalies in the Cosmic Microwave Background

Carlos Ríos, Pedro Labraña, Antonella Cid
Departamento de Física, Universidad del Bio-Bío, Casilla 5-C, Concepción, Chile.
E-mail: crios@ubiobio.cl

Abstract. The existence of anomalies in the Cosmic Microwave Background (CMB) on large angular scales (power deficit at low multipoles and hemispherical asymmetry) could be related to a pre-inflationary period, previous to standard inflation. Recent studies show that a period of super-inflation in the context of emergent universes (EU) generates a suppression of CMB anisotropies on large scales. We also study the implications of an EU model in the asymmetry of the average temperature in opposite hemispheres of the sky.

1. Introduction

The standard inflationary paradigm (SIP) is successful in solving the problems of the standard cosmological model (SCM) as horizon problem, flatness problem, monopole problem and the origin of structures [1, 2, 17]. However, recent measurements of Planck collaboration have reported a power deficit in the low-l CMB power spectrum at \( l \lesssim 40 \) [4] that cannot be explained by the SIP.

In this work we use the Friedmann-Lemaître-Robertson-Walker (FLRW) metric:

\[
\begin{align*}
\frac{ds^2}{c^2} = dt^2 - a(t)^2 \left[ \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right]
\end{align*}
\]

where \( t \) is the cosmic time, \( a(t) \) is the scale factor, \( k \) is a constant related to the curvature of the spatial sections (positive for closed models, negative for open ones, zero for a flat Universe). Throughout this work we adopt units such that \( c = \hbar = 1 \). In the case of a homogeneous and isotropic Universe, the field equations are given by:

\[
\begin{align*}
H^2 + \frac{k}{a^2} &= \frac{8}{3} \pi G \rho \quad &\text{(2)}

\frac{\dot{a}}{a} &= -\frac{4\pi G}{3} (\rho + 3p) \quad &\text{(3)}
\end{align*}
\]

where \( H \) is the Hubble parameter defined by \( H(t) = \frac{\dot{a}(t)}{a(t)} \), \( \rho \) is the total energy density of the spacetime, \( p \) is the total pressure and \( G \) is the newtonian constant of gravitation. The Einstein equations also lead to the energy conservation equation:

\[
\dot{\rho} + 3H (\rho + p) = 0
\]
We suppose that the early Universe is dominated by a canonical scalar field \( \phi \), called inflaton, whose energy density and pressure are given by:

\[
\rho = \frac{1}{2} \dot{\phi}^2 + V(\phi) \tag{5}
\]

\[
p = \frac{1}{2} \dot{\phi}^2 - V(\phi) \tag{6}
\]

where \( V(\phi) \) is the scalar potential.

2. The Emergent Universe

The EU models suggest that the Universe had not origin but rather, the Universe has always existed [5, 6], initially in an Einstein static state which solves the problem of the initial singularity present in the SCM [24]. In an EU scenario, the evolution the scale factor can be modeled as follows:

\[
a(t) \simeq a_0 + Ae^{H_0 t} \tag{7}
\]

where \( a_0, A \) and \( H_0 \) are positive constants and \( t \) is the cosmic time. We note that this universe has a past that asymptotically tends to an Einstein Static State. To the future, the model approaches a de Sitter expansion phase.

With the scale factor described in (7), the normalized evolution of the Hubble parameter is shown in Figure 1.

![Figure 1](image-url)

**Figure 1.** Evolution of the Hubble parameter corresponding to the scale factor (7). We distinguish three periods in the evolution, a past that asymptotically tends to an Einstein Static State \((H \to 0)\), an era of super-inflation \((\dot{H} > 0)\) and eventually a state of de Sitter expansion \((H \to \text{Constant})\).

The scalar field potential in a standard EU could be given by [6]:

\[
V(\phi) = \frac{2}{\kappa a_0^2} (e^{B\phi} - 1)^2 \tag{8}
\]

where \( \kappa = 8\pi G \) and \( B \) is a constant, see Figure 2.
The evolution of the scalar field is determined by the Klein-Gordon equation:

$$\ddot{\phi} + 3H \dot{\phi} + V' = 0$$  \(9\)

We are interested in the evolution of potential \(\phi(t)\) in the flat region of the scalar field potential (see Figure 2). Then, if we consider that the potential is approximately constant \(V \approx V_0\), we have

$$\ddot{\phi} + 3H \dot{\phi} \approx 0$$  \(10\)

Using the Eqs. (2), (3), (5) and (6) we find that:

$$\dot{H} = \frac{k}{a^2} - \frac{\kappa \dot{\phi}^2}{2}$$  \(11\)

If we make the change of variable \(X = \frac{k\dot{\phi}^2}{2}\) and using the Eq. (10), the Eq. (11) becomes in

$$\dot{X} + 6HX = 0$$  \(12\)

On other hand, using the Eqs. (2), (5) and (11) we find that

$$\dot{H} = -\frac{2}{3}X - H^2 + \frac{\kappa V_0}{3}$$  \(13\)

We can write Eqs. (12) and (13) as an autonomous dynamical system using that \(k = +1\) for an EU models [5, 6]. The directional field of the dynamical system is shown in Figure 2. We can note that the system of Eqs. (12) and (13) has an attractor for \(H = \sqrt{\frac{\kappa V_0}{3}}\) and \(\dot{\phi} = 0\), i.e. the system tends from the Einstein Static State to a de Sitter State. Then, we can model the Hubble parameter \(H(t)\), giving one scale factor \(a(t)\), getting well the form of \(\dot{\phi}\) using Eq. (10). From here on, we will consider and approximately constant potential or \(\dot{\phi} \propto a^{-3}\).
Figure 3. Directional field of the dynamical system and three numerical solutions. The instability point in this system is \(X = \frac{\xi V_0}{2}, H = 0\) and the attractor point is \(X = 0, H_0^2 = \frac{\xi V_0}{3}\). In this graph \(X = \frac{\xi \dot{\phi}}{2}\) and \(V_0 = \frac{3}{\kappa}\).

3. Primordial Perturbations

The CMB temperature is approximately the same in all directions in the sky. The first measurements of CMB by Penzias and Wilson in 1960s [26] showed no evidence for anisotropies in this temperature. The COBE satellite first measured anisotropies in the CMB temperature in 1992 [25]. Since then the satellites WMAP [3] and Planck [4] have measured there anisotropies with very high precision.

The scalar perturbations in the geometry of Friedmann-Lemaître-Robertson-Walker in the newtonian gauge, can be written as:

\[
 ds^2 = (1 + 2\Phi)dt^2 - (1 - 2\Phi)a(t)^2d\vec{x}^2
\]  

where \(\Phi\) is the Newtonian gravitational potential [17]. We use a Mukhanov variables [7, 8] defined by

\[
v_k = a \left( \delta \phi_k + \frac{\phi'}{H} \Phi_k \right)
\]  

where \(\delta \phi_k\) are the perturbations of the canonical inflation field, \(\phi'\) denote derivate with respect to the conformal time \(\eta\) defined by \(dt = a(\eta)d\eta\) and \(h = \frac{\dot{H}}{H}\). If we consider that \(\frac{\ddot{z}}{z} \approx \frac{\ddot{z}}{\dot{z}}\) where \(z = \frac{\phi'}{H}\) and the scalar factor given in (7), the perturbed equation in the Fourier space is given by

\[
v''_k + \left( k^2 - \frac{\ddot{z}}{z} \right) v_k = 0
\]  

whose solution is [9]:

\[
v_k = \frac{1}{\sqrt{2k}} \left[ \frac{e^{-ik\eta}}{1 - e^{a_0 H_0 \eta}} \right] 2 F_1(x_2 - 3, x_3 - 3; x_1; e^{a_0 H_0 \eta})
\]
where \( _2F_1 \) is a hypergeometric function, and

\[
\begin{align*}
 x_1 & = 1 - 2i\chi \\
x_2 & = 2 - i\chi - \sqrt{1 - \chi^2} \\
x_3 & = 2 - i\chi + \sqrt{1 - \chi^2} \\
\chi & = \frac{k}{a_0H_0}
\end{align*}
\]

(18) (19) (20) (21)

It is important to note that this solution tends to the standard solution \( v_k \approx e^{-ik\eta} \sqrt{2k} \) in the limit of short wavelengths \( (k >> a_0H_0) \). The spectrum of primordial perturbations is defined by the following expression [16]:

\[
P_{\delta\phi} = \frac{k^3}{2\pi^2} \left| \frac{\partial v_k}{\partial a} \right|^2
\]

(22)

In the context of the EU, we need to take the limit \( \eta \to 0 \), finally we get:

\[
P_{\delta\phi} = \frac{H_0^2}{\pi^2} \frac{\chi^2}{\Gamma(x_1)\Gamma(x_2)\Gamma(x_3)\Gamma(x_4)}
\]

(23)

In the limit of small scales \( (k >> a_0H_0) \) the standard result \( P_{\delta\phi} = \left( \frac{H_0}{2\pi} \right)^2 \) is recovered from Eq. (23), see ref. [16].

4. Anomalies in the CMB

The WMAP and Planck satellites measured a lack of observed power in the lowest CMB multipoles of the power spectrum [3, 4]. This anomaly can be explained in the context of the EU, because this model naturally generates a suppression in the power spectrum of the primordial fluctuation at large scales (or low multipoles).

On the other hand, the data collected by the WMAP satellite [11, 14] and subsequently by the Planck satellite [15] indicated a difference between the CMB temperature fluctuations at two opposite hemispheres of the sky separated approximately in the ecliptic. A convenient way to model this asymmetry is by introducing a dipole in the sky using the following relation [19, 21]:

\[
P^{1/2}_R(k, \vec{x}) = \left( 1 + A(k) \frac{\vec{P} \cdot \vec{x}}{x_{ls}} \right) P^{1/2}_R(k)
\]

(24)

where

\[
P_R(k) \approx \left( \frac{H}{\dot{\phi}} \right)^2 P_{\delta\phi}|_{k=aH}
\]

(25)

is the spectrum of curvature perturbations evaluated in the horizon crossing condition \( k = aH \) [17], \( \phi \) is the inflaton scalar field, \( \vec{P} \) is the unit vector in the direction of dipolar modulation, \( \vec{x} \) is a spatial vector, \( x_{ls} \) is the distance to the last scattering surface and \( A(k) \) is the amplitude of modulation.
5. Effective Power Spectrum

In analogy with the standard inflationary paradigm, where \( P_R \approx \frac{1}{\epsilon_{inf}} P_{\delta \phi} \) and \( \epsilon_{inf} < 0.008 \) is the standard slow-roll parameter [13], we define a new parameter \( \epsilon^* \), such that:

\[
\epsilon^*(\chi) = \frac{1}{2} \left( \frac{\dot{\phi}(\chi)}{H(\chi)} \right)^2 + \epsilon_{inf}
\]  

so that the spectrum of curvature perturbation is represented by:

\[
P_R \approx \left( \frac{1}{\kappa a_0^2 H_0^2 \chi^2 (1 + \chi)^4} + 0.008 \right)^{-1} P_{\delta \phi}
\]

where we used the Klein-Gordon equation and we have considered that in the pre-inflationary phase \( V \approx \text{Constant} \).

We have determined the power spectrum of curvature perturbations of our model generated during super-inflation period, but we must include the SIP. To get this, we follow [12] and [9]. A power spectrum that include both stages of evolution of the Universe is:

\[
P_R = \frac{\bar{A}}{e^4} \left( \frac{k}{k_0} \right)^{n_s-1} \frac{\chi^2 \Gamma[x_1] \Gamma[x_1^*]}{\Gamma[x_2] \Gamma[x_2^*] \Gamma[x_3] \Gamma[x_3^*]}
\]  

where \( k_0 \) is the pivot scale, \( \bar{A} \) is a constant and \( n_s \) is the standard spectral index. Which can be written conveniently [9]:

\[
P_R = \frac{A}{e^4} \left( \frac{k}{a_0 H_0} \right)^2 \left( 1 + \frac{k}{a_0 H_0} \right)^{-2} \left( \frac{k}{k_0} \right)^{n_s-1}
\]

The temperature power spectrum is obtained using the CLASS code [22, 23]. We use the following values for the constants \( A = 2.05 \times 10^{-9} \) and \( a_0 H_0 = 0.0002 \text{Mpc}^{-1} \). The value of \( k_0 = 0.002 \text{Mpc}^{-1} \) is used by WMAP and \( n_s = 0.967 \) is reported by the same satellite [18].

The temperature power spectrum in terms of the \( C_\ell \) [19] , in our model can be seen in Figure 4.

![Figure 4. Temperature power spectrum in terms of the \( C_\ell \). The dashed line represent the SCM and the solid line is the EU scenario. The points show the Planck data.](image-url)
We can also observe the evolution of the power spectrum $P_R(k)$ in terms of the scale in Figure 5.

![Figure 5](image-url)

**Figure 5.** Normalized Power spectrum of curvature perturbations using EU (Solid line). It can be appreciated its dependence on the scale. The dashed line represent the scale invariant Power spectrum.

### 6. Hemispherical Asymmetry

Other anomaly that survives, even after the accurate observations of the Planck satellite [20] is a hemispherical asymmetry on the CMB power spectrum modeled by (24). Following [12] it can be shown that the modulation amplitude $A(k)$ satisfies:

$$|A(k)| \lesssim 0.02 \left| \frac{n_s(k) - 1}{2} \right| |1 - \epsilon(k)|$$

where $\epsilon$ is defined by

$$\epsilon \equiv - \frac{\dot{H}}{H^2}$$

and the spectral index $n_s(k)$ is obtained analytically by its definition:

$$n_s - 1 = \frac{d \ln P_R}{d \ln k}$$

the evolution of the modulation amplitude is shown in Figure 6.

![Figure 6](image-url)

**Figure 6.** Evolution of the modulation amplitude as function of $k$. For small scales, Planck reports a value of $A = 0.07 \pm 0.02$. 

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Standard inflation predicts $n_s - 1 \approx 0$, then the modulation amplitude $A \approx 0$, but super-inflation in the context of EU indicates that the modulation amplitude depends on the scale (see Figure 6) then it is possible to explain that Planck reported $A = 0.07 \pm 0.02$ value $l \leq 40$ and $A \approx 0$ to $l \approx 500 - 600$ [10].

7. Conclusions

The Cosmic Microwave Background have several anomalies that survive even after the Planck’s observations. One of these anomalies is the low power that has the power spectrum curvature of perturbation $\mathcal{P}_R(k)$ in the lowest multipoles [4]. This anomaly can be explained in the context of the EU, because our model naturally generates a suppression in the power spectrum curvature of perturbation of the CMB at low multipoles. This suppression can be observed in Figure 5. On the other hand, Planck’s observation show a Hemispherical Asymmetry in the CMB [20] unexplained when we use the SIP, because the standard inflation predicts $n_s - 1 \approx 0$ then $n_s$ is constant and the modulation amplitude $A \approx 0$, but super-inflation in the context of EU indicates that the modulation amplitude depends on the scale then it is possible to explain that Planck reported $A = 0.07 \pm 0.02$ value $l \leq 40$ and $A \approx 0$ to $l \approx 500 - 600$ [10].

Acknowledgments

We thank to Dirección de Investigación of Universidad del Bío Bío for GI121407/VC (AC, PL, CR) and 141407 3/R (PL), to CONICYT for FONDECYT N° 11110507 (AC).

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