DETECTION OF HOT GAS IN GALAXY GROUPS VIA THE THERMAL SUNYAEV–ZEL'DOVICH EFFECT

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ABSTRACT

Motivated by the observed shortfall of baryons in the local universe, we investigate the ability of high-resolution cosmic microwave background (CMB) experiments to detect hot gas in the outer regions of nearby group halos. We construct hot gas models with the gas in hydrostatic equilibrium with the dark matter and described by a polytropic equation of state. We also consider models that add entropy to the gas in line with constraints from X-ray observations. We calculate the thermal Sunyaev–Zel’dovich (tSZ) signal in these halos and compare it to the anticipated sensitivities of forthcoming tSZ survey experiments such as Antenna Cosmology Telescope, PLANCK, and South Pole Telescope. Using a multifrequency Wiener filter we derive tSZ detectability limits as a function of halo mass and redshift in the presence of galactic and extragalactic foregrounds and the CMB. We find that group-sized halos with virial masses below $10^{14} M_\odot$ can be detected at $z \lesssim 0.05$ with the threshold mass dropping to $(3–4) \times 10^{13} M_\odot$ at $z \lesssim 0.01$. The tSZ distortion of nearby group-sized halos can thus be mapped out to the virial radius by these CMB experiments, beyond the sensitivity limits of X-ray observations. These measurements will provide a unique probe of hot gas in the outer regions of group halos, shedding insight into the local census of baryons and the injection of entropy into the intragroup medium from nongravitational feedback.

Key words: cosmic microwave background – galaxies: clusters: general

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1. INTRODUCTION

Through the distinctive signatures that cosmological parameters, such as the baryon density, matter density, and spatial curvature have on the cosmic microwave background (CMB) anisotropy, recent measurements of the CMB temperature and polarization spectra (Nolta et al. 2009, and references therein) have placed precise constraints on these parameters, thereby establishing a cosmological model that is consistent with a wide range of astronomical observations (Dunkley et al. 2009, and references therein). Moreover, because the physics underlying the CMB anisotropy is to a good approximation linear on these scales, the CMB provides a powerful probe into the physics of the early universe and the primordial perturbations (Komatsu et al. 2009, and references therein).

Whereas on large angular scales the CMB anisotropy is fairly well described, apart from a late-time integrated Sachs–Wolfe (ISW) contribution (Sachs & Wolfe 1967), by the geometrical projection of inhomogeneities at the last scattering surface onto our celestial sphere, the so-called primary anisotropies, on smaller angular scales of around an arcminute CMB photons are more significantly affected by gravitational and scattering processes along the line of sight. These secondary anisotropies provide an effective probe of the physics of the low-redshift universe.

The most significant secondary anisotropy on scales of around a few arcminutes comes from inverse Compton scattering of CMB photons off hot gas in galaxy clusters along the line of sight, the so-called thermal Sunyaev–Zel’dovich (tSZ) effect (Sunyaev & Zeldovich 1970). The population of hot electrons in the cluster gas imparts energy to photons in the Rayleigh–Jeans part of the spectrum pushing them into the Wien tail. This distorts the thermal CMB spectrum by creating a deficit of photons at low frequencies and an excess of photons at high frequencies with no distortion at the tSZ null frequency, $ν \approx 218$ GHz. The galaxy cluster appears as a cold spot on the sky at frequencies below the tSZ null and as a hot spot above the tSZ null. The unique spectral signature of the tSZ effect will allow multifrequency, high-resolution CMB experiments to make a relatively clean separation of the tSZ effect from the primary CMB and other contaminants such as point sources and galactic foregrounds.

The utility of the tSZ effect as a cosmological probe arises from the fact that the Compton distortion of the CMB is a scattering effect, so that the central decrement toward a cluster is independent of the redshift of the cluster. Moreover the angular diameter distance flattens out at $z \sim 1$ so that the angular size of the cluster is approximately constant at high redshift. This means that the tSZ effect does not suffer from the strong redshift dimming of its optical and X-ray counterparts with the consequence that a microwave background tSZ cluster survey can detect a larger proportion of lower mass clusters out to higher redshifts. The tSZ effect can thus be used to construct galaxy cluster catalogs with a well-defined selection function, thereby providing a potentially powerful probe of cosmology through the evolution of the cluster abundance (for a review, see Carlstrom et al. 2002).

While most work has focused on the detection of the tSZ effect in galaxy clusters and its cosmological application, there is an intriguing possibility that high-resolution CMB experiments with sufficient sensitivity may be able to detect hot gas in lower mass galaxy or galaxy group halos. Whereas the central tSZ distortion scales roughly in proportion to the mass of the halo, the total tSZ flux, $S_{tSZ} \propto M^{5/3}/d_A^2$, has an additional dependence on the angular diameter distance of the halo. This means that nearby low-mass halos with a large angular size...
could produce a significant tSZ flux. This fact was exploited in Taylor et al. (2003) to study the possibility of detecting the tSZ effect in the halo of our neighboring galaxy, M31, with the upcoming PLANCK surveyor. In this paper, we focus on how well current and forthcoming CMB experiments will detect the tSZ effect in nearby galaxy and group halos. Such a detection will provide unique insights into the distribution of baryons and the properties of hot gas in the outskirts of galaxy and group halos.

Indeed, in a census of baryons in the local universe (Fukugita et al. 1998; Fukugita & Peebles 2004) it has been argued that most of the uncertainty in the baryon budget comes from the uncertainty in the mass in ionized plasma associated with groups of galaxies. Whereas the baryon fraction inferred from light element abundances (e.g., Steigman 2007), the CMB anisotropy (Dunkley et al. 2009), and the high-redshift Lyα forest (Kirkman et al. 2003) give a consistent baryon fraction of \( \Omega_b \approx 0.044 \), only about 10% of these baryons are observed to be in stars, hot gas in clusters, and neutral and molecular gas in galaxies at low redshift. Cool plasma in and around groups of galaxies indirectly detected through quasar absorption lines (Penton et al. 2000, 2004) makes up 20%–25% of the baryon budget, while the warm–hot gas residing in the filamentary large-scale structure is believed to account for another 30%–35% of the baryons (Dave et al. 2001; Cen & Ostriker 2006). The remaining 30%–40% of baryons is likely to be tied up in galaxy groups in a low-density, warm–hot plasma. Recent X-ray observations (Sun et al. 2009) that report average gas fractions, \( f_{\text{gas}} \approx 0.08 \), out to \( r_{500} \) in galaxy groups may have detected roughly half of these baryons. When combined with the fact that the gas fraction profiles inferred from these measurements are still rising at \( r_{500} \) (see also Vikhlinnin et al. 2006), this indicates that a large fraction, perhaps as much as 15%–20%, of baryons in the universe may be in the outskirts of galaxy halos and in the intragroup medium beyond \( r_{500} \). Recent evidence for this warm–hot component has come from quasar absorption lines in the ultraviolet (O vi; e.g., Tripp et al. 2000; Sembach et al. 2003; Tumlinson et al. 2005) and in X-rays (O vii and O viii; e.g., Fang et al. 2003) observed around galaxies and groups of galaxies, but these detections only map the two-dimensional gas distribution along a few lines of sight to allow a complete study of its spatial properties.

More detailed observations of this warm–hot component will yield important clues into the impact of galactic feedback on the distribution of gas in the intragroup medium. The relative dearth of hot gas observed in the central regions of galaxies and galaxy groups results in these halos being less X-ray luminous for a given temperature as compared to more massive galaxy clusters. The resulting break in the luminosity–temperature relation observed in X-ray clusters and groups (McCarthy et al. 2004) suggests that nongravitational processes that modify the entropy structure in the central regions of low-mass clusters and groups are responsible for departures from the cluster scaling relations expected in self-similar models. Several models have been proposed to explain the excess entropy (for a recent review, see Voit 2005), including preheating of infalling gas before it was shocked by the virial temperature, radiative cooling of low-entropy gas to form stars, and feedback from supernovae and active galactic nuclei that increased the temperature and reduced the density of gas in the central regions of galaxies and groups. While a combination of radiative cooling and feedback seems to reproduce the central entropy excess in low-mass clusters and groups, it is clear that improved measurements of the properties of hot gas in these halos are necessary to understand the exact details of entropy injection. Whereas X-ray observations lack sensitivity to the hot gas beyond \( r_{500} \), because the X-ray luminosity scales as the square of the (decreasing) gas density, the tSZ effect scales linearly with the density and thus provides a potentially more sensitive probe of the hot gas in the outskirts of galaxy and group halos. Moreover, the tSZ effect measures the projected gas pressure which is a complementary probe to the X-ray luminosity and can provide new constraints on models of the intracluster and intragroup medium. While the tSZ effect has been detected in a number of galaxy clusters by single dish and interferometric experiments (for a review, see Carlstrom et al. 2002), it has not yet been convincingly detected in any galaxy groups. However, a new generation of high-resolution CMB experiments with superior detector sensitivity offers the hope of detecting and constraining the properties of the hot gas in galaxy groups.

The outline of this paper is as follows. In Section 2, we construct models of the hot gas in virialized halos of a given mass and redshift, over the mass range \( M_{\text{vir}} = 10^{13}–10^{15} M_\odot \), and redshift range \( z = 0–1.5 \). The models take into account constraints on the temperature, entropy injection, and gas fraction from X-ray observations. In Section 3, we utilize multifrequency filtering techniques to determine the detectability of hot gas in nearby halos with current and forthcoming microwave background experiments such as the Atacama Cosmology Telescope (ACT; Kosowsky 2006), the South Pole Telescope (SPT; Ruhl et al. 2004), and the PLANCK mission (The Planck Collaboration 2006). In Section 4, we summarize our findings and discuss how detection of the tSZ effect in group halos will constrain gas models and allow a measurement of physical parameters such as the entropy injection and baryon fraction in these halos. Throughout this paper, we assume a flat ΛCDM cosmological model with parameters \( h = 0.72, \Omega_m = 0.26, \Omega_b = 0.044, \Omega_{\text{de}} = 0.74, w_{\text{de}} = -1.0 \), and \( \sigma_8 = 0.8 \) that provide a best fit to the Wilkinson Microwave Anisotropy Probe (WMAP) five-year data (Dunkley et al. 2009).

2. HOT GAS HALO MODELS

Historically, the isothermal \( \beta \) model (Cavaliere & Fusco-Femiano 1976) has been used to describe the spatial distribution of hot gas in galaxy clusters, primarily to model the X-ray emission originating from thermal bremsstrahlung of the hot intracluster gas (Sarazin 1986). The \( \beta \) model provides a convenient analytical form that has been popular for X-ray surface brightness profile fitting. However, over the past decade it has been realized that X-ray observations of density profiles at large radii do not favor the \( \beta \) model: the isothermal model provides a poor description of the temperature profiles of intracluster gas, with observed temperature profiles declining at large radii and cooling flows observed in the central regions of some clusters (Vikhlinin et al. 2006). Furthermore, it has been shown that there is a fundamental incompatibility between the \( \beta \) model parameters fitted using X-ray data, and those fitted using data based on the tSZ effect (Hallman et al. 2007a; Atrio-Barandela et al. 2008), indicating that the simple \( \beta \) model is not sufficiently realistic to describe the observed cluster gas physics.

In this paper, we will study two different models for the hot gas in dark matter halos. Our first model, which we will refer to as the polytropic model, assumes that the hot gas follows a polytropic equation of state and traces the dark matter in the outskirts of the halo (e.g., Komatsu & Seljak 2001, 2002). As discussed in Komatsu & Seljak (2001) this model is in good agreement with observed X-ray surface brightness profiles and...
the mass–temperature relation above temperatures of a few keV. Our second model, which we refer to as the entropy model, builds upon the first and attempts to account for nongravitational feedback by adding entropy to the hot gas, similar to Voit et al. (2002). In both cases, the hot gas is in hydrostatic equilibrium with the underlying dark matter potential.

### 2.1. Dark Matter Halo

We define a dark matter halo at redshift $z$ with virial mass $M_{\text{vir}}$ and radius $r_{\text{vir}}$ to have a characteristic average density equal to $\Delta_{\text{vir}}(z)$ times the critical density

\[ \rho_c(z) = \frac{3 H^2(z)}{8 \pi G}. \]  

The Hubble parameter relative to its present value is given by

\[ \frac{H(z)}{H_0} = \left[ \frac{\Omega_m(1 + z)^3 + \Omega_{de}(1 + z)^{3(1+w_{de})}}{\Omega_m} \right]^{1/2}, \]  

where $\Omega_m$ and $\Omega_{de}$ represent the density of the matter and dark energy components, respectively, relative to the critical density today.

In the spherical collapse model (Peebles 1980) for an Einstein–de Sitter cosmology with $\Omega_m = 1$ and $\Omega_{de} = 0$, the virial collapse factor has a constant value $\Delta_{\text{vir}}(z) = 18\pi^2$. Using numerical simulations, Bryan & Norman (1998) found for $\Lambda$CDM cosmologies that the parametric form

\[ \Delta_{\text{vir}}(z) = 18\pi^2 + 82[\Omega(z) - 1] - 39[\Omega(z) - 1]^2, \]  

provides a good fit over a wide range of $\Lambda$CDM cosmological models. Once the characteristic average density is chosen, the virial mass and radius become uniquely related.

We assume that the dark matter density follows the self-similar Navarro–Frenk–White (NFW) profile (Navarro et al. 1997),

\[ \rho_{\text{dm}}(x) = \frac{\rho_s}{x(1 + x)^2}, \quad x \equiv r/r_s, \]  

where $r_s$ is the scale radius of the halo. By integrating the density profile and equating the mass found within the virial radius to $M_{\text{vir}}$, we obtain the density normalization

\[ \rho_s = \frac{M_{\text{vir}} c^3}{4 \pi r_{\text{vir}}^3 m(c)}, \]  

where the function

\[ m(x) = \ln(1 + x) - \frac{x}{1 + x} \]  

is the dimensionless dark matter mass profile. The concentration parameter is defined as $c = r_{\text{vir}}/r_s$ and we adopt the fitting formula

\[ c(M_{\text{vir}}, z) = \frac{7.5}{1 + z} \left[ \frac{M_{\text{vir}}}{M_{\text{vir}, 0}} \right]^{0.1}, \]

to describe the dependence on virial mass and redshift following Dolag et al. (2004). The dependence on cosmological parameters is captured by scaling the virial mass by the nonlinear mass $M_{\text{vir}, 0}$. In the spherical collapse model, the nonlinear mass is defined as the mass $M = (4/3) \pi \delta R^3$ enclosed within a sphere of radius $R$ for which the variance of the linear density field $\delta$, smoothed by a top-hat filter, equals the square of the critical overdensity threshold $\delta_c = 1.68$. For the chosen cosmology, the nonlinear mass is $M_{\text{vir}, 0} = 2.818 \times 10^{12} h^{-1} M_{\odot}$ at redshift $z = 0$.

The exact dependence of the concentration parameter on mass and redshift is still uncertain as various parameterizations have been used in the literature (e.g., Navarro et al. 1997; Seljak 2000; Bullock et al. 2001). However, as previously observed (Komatsu & Seljak 2002) we find that our results are insensitive to the exact choice of the concentration parameter. This is due to the fact that the tSZ effect is less sensitive to the central regions, where changes in the concentration parameter are most important.

During the formation of virialized halos, the collisionless dark matter undergoes violent relaxation and the collisional baryons get shockheated. According to the virial theorem, the internal energy of a virialized halo is twice its gravitational potential energy. From this relation, we can define a characteristic velocity dispersion for the dark matter and a virial temperature,

\[ T_{\text{vir}} = \frac{1}{3} G \mu m_p \frac{M_{\text{vir}}}{r_{\text{vir}}} \]  

\[ = 0.034 \left[ \frac{M_{\text{vir}}}{10^{12} M_{\odot}} \right]^{2/3} \left[ \frac{\Delta_{\text{vir}}(z)}{100} \right]^{1/3} \left[ \frac{H(z)}{H_0} \right]^{2/3} \text{keV} \]  

for the shockheated gas. We assume a fully ionized gas with a mean molecular weight $\mu = \frac{4}{3+5\xi_{HI}} = 0.588$ for a hydrogen mass fraction of $X_{HI} = 0.76$.

### 2.2. Polytropic Model

We first consider a model where the gas follows a polytropic equation of state, $P_g \propto \rho_g^\gamma$, with a polytropic index $\gamma$. The gas density and temperature profiles can be parameterized as

\[ \rho_g(x) = \rho_{g,0} y_{\text{poly}}(x), \]  

\[ T_g(x) = T_{g,0} y_{\text{poly}}^{-1}(x), \]

where $y_{\text{poly}}(x)$ is the dimensionless gas density profile and the coefficients $\rho_{g,0} \equiv \rho_{g}(0)$ and $T_{g,0} \equiv T_{g}(0)$ are two boundary conditions. Assuming that the gas is in hydrostatic equilibrium with an NFW potential, we obtain the analytical solution (e.g., Komatsu & Seljak 2001)

\[ y_{\text{poly}}(x) = \left[ 1 - B f(x) \right]^{1/(\gamma-1)} \]  

\[ f(x) = 1 - \frac{\ln(1 + x)}{x} \]  

\[ B = \left( \frac{\gamma - 1}{\gamma} \right) \left( \frac{3 T_{\text{vir}}}{T_{g,0}} \right) \left[ \frac{c}{m(c)} \right]. \]

For a given mass and redshift, there are three free parameters: the polytropic index $\gamma$, the central gas temperature $T_{g,0}$, and the central gas density $\rho_{g,0}$. In Komatsu & Seljak (2001, 2002) these parameters were specified by requiring that the gas density profile matches the dark matter density profile in the outer parts of the halo. This assumption is known to be in good agreement with adiabatic hydrodynamic simulations, but it remains unclear how valid it is for radiative simulations which include cooling, star formation, and feedback. We will take an alternative
approach, choosing appropriate values for the free parameters such that they are in agreement with hydrodynamic simulations, are complimentary with other recent semi-analytical models, and are flexible enough for us to apply them to our nonpolytropic model.

We set the polytropic index to be $\gamma = 1.2$ as suggested by hydrodynamic simulations and used in other recent semi-analytical models (see Ostriker et al. 2005, and references therein). This value is also consistent with the range of values used by Komatsu & Seljak (2001, 2002), who parameterized the polytropic index as a function of the concentration parameter and found that it varies only weakly with $c$. In reality, the effective polytropic index $\gamma_{\text{eff}} \equiv d \ln P_e / d \ln \rho_e$ is likely to be a function of radius. Toward the center of the halo where cooling is more efficient because of the higher densities, the temperature can decrease, resulting in $\gamma_{\text{eff}} < 1$. In the outskirts of the halo beyond the virial radius where shockheating is less efficient, the effective polytropic index will approach the characteristic value $\gamma_{\text{eff}} = 1.62$ for the intergalactic medium (IGM; Hui & Gnedin 1997). Furthermore, nongravitational feedback can also change the equation of state. In our second model, we consider a profile where the effective polytropic index is scale dependent.

Halos found in adiabatic hydrodynamic simulations are generally well described by the virial theorem. The temperature at the virial radius is close to the virial temperature and the central and average temperature are found to be slightly higher (e.g., Frenk et al. 1999; Rasia et al. 2004). Therefore, we choose to equate the temperature at the virial radius, $T_{\text{g,c}}$, to $T_{\text{vir}}$, and this fixes the central temperature as

\[
T_{\text{g,0}} = T_{\text{vir}} + 3T_{\text{vir}} \left( \frac{\gamma - 1}{\gamma} \right) \left[ \frac{c f(\gamma)}{m(c)} \right].
\]

Our chosen values for $T_{\text{g,0}}$ and $T_{\text{g,c}}$ are also consistent with the range of values used by (Komatsu & Seljak 2001, 2002).

We normalize the density profile by fixing the mass of hot gas within the virial radius, $r_{\text{vir}}$, and compare the gas fractions predicted by this model to X-ray constraints within $r_{\text{500}}$. X-ray observations of hot clusters have demonstrated that the cumulative gas fraction within $r_{\text{500}}$ approaches a constant value that lies in the range $f_{\text{gas,500}} \approx 0.10-0.16$ independent of cluster mass (Sun et al. 2009; LaRoque et al. 2006). Furthermore, there is good evidence that $f_{\text{gas}}(< r_{\text{200}})$ converges to the universal baryon fraction $f_b$ (Vikhlinin et al. 2006; Sanderson et al. 2003; McCarthy et al. 2007). These results agree with cosmological hydrodynamic simulations, which also suggest that for massive clusters, there is very little evolution of the gas fraction with redshift within $r_{\text{vir}}$ (Kay et al. 2004, 2007; Eke et al. 1998; Ettori et al. 2004; Kravtsov et al. 2005).

Observational constraints on the gas fraction in the cooler halos of galaxies and groups of galaxies are much weaker than the cluster measurements. However, there is evidence for a decrease of $f_{\text{gas}}(< r_{\text{500}})$ with decreasing halo mass. It also appears that $f_{\text{gas}}(< r_{\text{200}})$ does not approach the universal baryon fraction $f_b$ (Sanderson et al. 2003; see also McCarthy et al. 2007). The main process that changes the gas fraction while maintaining the baryon fraction is condensation of cold, low-entropy gas into stars. The lower value of $f_{\text{gas}}$ for smaller halos is consistent with the higher stellar fraction measured for lower mass halos (Lin et al. 2003; Gonzalez et al. 2007) combined with the fact that lower mass halos are more sensitive to nongravitational heating which can expel gas from their shallower potentials. There are few observational constraints on the evolution of the gas fraction with redshift for cooler systems, but simulations suggest that there is little evolution of the baryon fraction from $z = 1$ to $z = 0$ down to the galactic scales of $M_{\text{vir}} \approx 10^{12} M_{\odot}$ (Crain et al. 2007).

Taking these uncertainties into account, we assume that the baryon fraction within the virial radius is given by the cosmic fraction $\Omega_b/\Omega_m$, but allow for a fraction $f_{\text{gas}} = 0.1$ of baryons in the form of stars. We also assume that $f_{\text{gas,vir}}$ is redshift independent (for $z \lesssim 1$) for all halo masses ($M_{\text{vir}} = 10^{13}-10^{15} M_{\odot}$) that we consider. The central gas density is then given by

\[
\rho_{g,0} = \frac{(1 - f_{\text{gas}}) \Omega_b}{4 \pi r_{\text{vir}}^3} \frac{M_{\text{vir}}}{y_{\text{poly}}(x) x^2 dx}.
\]

Note that Komatsu & Seljak (2001, 2002) chose to fix the gas fraction at the virial radius to the cosmic value, without any explicit allowance for stars. In general, the integrated gas mass and pressure within the virial radius from our model agrees well with theirs with typical differences of order 10%. We note that this simple parameterization does indeed fit X-ray observations as can be seen from Figure 1.

Now that the density and temperature of the gas are completely specified for the polytropic model, we can write down additional quantities that are relevant to tSZ and X-ray observations. The electron pressure and entropy are given as

\[
P_e = n_e k T_e = P_{e,0} y_{\text{poly}},
\]

and

\[
S_e = T_e n_e^{-2/3} = S_{e,0} y_{\text{poly}}^{-5/3},
\]

where $P_{e,0} = n_{e,0} k T_{e,0}$ and $S_{e,0} = T_{e,0} n_{e,0}^{-2/3}$ are the central electron pressure and entropy, respectively, and the central electron number density and temperature are $n_{e,0}$ and $T_{e,0} = T_{\text{g,0}}$, respectively. The electron number density $n_e$ is calculated from the gas density assuming a fully ionized gas, and the electron temperature is assumed to be equal to the gas temperature.

### 2.3. Entropy Model

Observations indicate that in the inner regions of low-temperature clusters there is excess entropy above the predictions of the self-similar model (Ponman et al. 1999; Lloyd-Davies et al. 2000; Finoguenov et al. 2007). Together with the observed departure from the simple scaling relations suggested by purely gravitational physics (Arnaud & Evrard 1999), this means that nongravitational effects should be included when modeling the ICM gas distribution. Various authors have suggested that the ICM was heated by some energy input, e.g., via star formation, supernova (SN) explosions or active galactic nucleus (AGN) feedback. For example, Voit et al. (2003) showed that preheating can explain the entropy profiles of groups. This model was also studied (Reid & Spergel 2006) in the context of tSZ observations. Ostriker et al. (2005) constructed a model including various nongravitational processes that could be used to match the observed scaling relations of clusters (see also Bode et al. 2007).

Similar to Voit et al. (2002; see also Balogh et al. 2006; Younger & Bryan 2007), we allow for an additive term $\Delta \rho_{\text{inj}}$ to the polytropic entropy profile that incorporates the combined effect of nongravitational processes, such as feedback from supernovae and galactic nuclei, and radiative cooling and star
formation. $S_{\text{inj}}$ is constant with radius so that
\[
S = S_0 y_{\text{ent}}^{-5/3} + S_{\text{inj}},
\] (17)

The addition of an entropy term modifies the temperature profile according to
\[
T_{\text{g}}(x) = T_{g,1} y_{\text{ent}}^{-1}(x) + T_{\text{inj}} y_{\text{ent}}^{2/3},
\] (18)

so that the central temperature is now $T_{\text{g}}(0) = T_{g,1} + T_{\text{inj}}$. Here $T_{\text{inj}}$ is the amount of injected thermal energy per particle, and relates to the amount of injected entropy via
\[
S_{\text{inj}} = 100 \left( \frac{T_{\text{inj}}}{1 \text{ keV}} \right) \left( \frac{n_{e,0}}{10^{-3} \text{ cm}^{-3}} \right)^{-2/3} \text{ keV cm}^2.
\] (19)

The form of heating proposed here is effective at increasing the temperature in the inner regions of the halo but has little effect in the outskirts of the halo for all but the lowest mass halos. The additional entropy due to the nongravitational heating term breaks the self-similarity of the cluster physics. This very simple model provides a good phenomenological description that is adequate for our purposes in spite of its shortcomings.

We assume that the gas remains in hydrostatic equilibrium with the dark matter, which yields an implicit equation for the modified profile $y_{\text{ent}}$, which we solve numerically. For a given mass and redshift, there are four free parameters in this model: the index $\gamma$ that characterizes the polytropic part of the temperature, the central gas temperature, the central gas density, and the amount of injected energy $T_{\text{inj}}$. In general, the gas will not retain a polytropic equation of state and its profile will be altered. However, we want to continue to associate the polytropic-like terms with virialization and shockheating, thus we keep $\gamma = 1.2$ and normalize the polytropic term in the temperature equation just like in the polytropic model. That means we set the value at the virial radius to be equal to the virial temperature, which fixes the constant $T_{g,1}$ via
\[
T_{g,1} = \frac{T_{\text{vir}}}{y_{\text{ent}}^{-1}(c)}.
\] (20)

Note that we implicitly assume that the gas has first settled in the gravitational potential setup by the dark matter before being redistributed by the injection of entropy.

We normalize the gas density by assuming that the gas pressure at the virial radius remains unchanged by the entropy injection, i.e., $P_{\text{gas}}(r_{\text{vir}}) = P_{\text{gas}}^{\text{poly}}(r_{\text{vir}})$. This means that we assume that the gas settles back into a pressure-balanced hydrostatic equilibrium at the virial radius after the energy injection. Note that with our choices, the entropy model reduces to the polytropic model when $T_{\text{inj}}$ reaches zero. The resulting electron temperature, density, pressure, and entropy profiles for the polytropic model and entropy injection model are shown in the Appendix, where they are discussed in more detail.

In order to adjust the amount of heating $T_{\text{inj}}$, we construct a fitting function for $T_{\text{inj}}$ such that $S_{\text{inj}} \approx 100 \text{ keV cm}^2$ is independent of halo mass and redshift. It has been shown that this form and amount of feedback reproduces the observed scaling relations of clusters (McCarthy et al. 2004); see Voit (2005) for a detailed discussion of the feedback energy available from physical processes such as supernovae or AGN heating.

In Figure 2, we plot the entropy, $S_e$ at 0.1 $r_{\text{vir}}$ against virial mass for the different models. It can be seen that the entropy model has more central entropy in group halos. This results from the feedback that heats the gas and flattens the density profile by pushing more gas from the center to the outskirts, thereby breaking the scale invariance of the entropy–mass relation. We also note that the level of feedback chosen in our entropy injection model provides an entropy floor of $S \approx 100–200 \text{ keV cm}^2$ which agrees with entropy profiles derived from X-ray observations (see Ponman et al. 1999; Lloyd-Davies et al. 2000) and simulations (see Finoguenov et al. 2003) of galaxy clusters and groups.

An additional constraint on the entropy injection parameter comes from the X-ray luminosity–temperature relation. The
Figure 2. Entropy at 0.1 $r_{\text{vir}}$ against halo mass, $M_{\text{halo}}/M_\odot$, for the polytropic model (solid curve) and the entropy model (dot-dashed curve), showing a break in the scaling of the core entropy scaling as suggested by X-ray observations.

Figure 3. Integrated X-ray luminosity within $r_{500}$ against emission-weighted gas temperature, $T_{\text{gas}}$, in the polytropic model (solid curve) and the entropy model (dot-dashed curve). The data points were obtained by McCarthy et al. (2004) from a compilation of Chandra and XMM-Newton data.

X-ray luminosity within radius $r_X$ is

$$L_X(<r_X) = 1.41 \times 10^{35} \text{ erg s}^{-1} \left( \frac{n_e,0}{10^{-3} \text{ cm}^{-3}} \right)^2 \left( \frac{T_e,0}{\text{keV}} \right)^{1/2} \times \left( \frac{r_s}{\text{kpc}} \right)^3 \times 4\pi \int_0^{r_X/r_s} \gamma_X(x_p) x_p dx_p,$$

where the projected radius is $r_p = x_p r_s = \sqrt{r^2 - l^2} r_s$. In the case of the entropy model, the integrand $\gamma_X$ is given in terms of $\gamma_{\text{gas}}$, the dimensionless gas profile, as

$$\gamma_X(r) = \sqrt{\gamma_{\text{gas}}(r) + (T_{\text{inj}}/T_e,0)\gamma_{\text{gas}}^{14/3}(r)},$$

while in the case of the polytropic model $\gamma_X$ is simply obtained by setting $T_{\text{inj}} = 0$ in the above expression.

In Figure 3, we plot the integrated X-ray luminosity within $r_{500}$ against the emission-weighted electron temperature. We see that the entropy injection model has significantly lower X-ray luminosity for group halos, resulting in the observed break of the scale invariant $L_X-T_X$ relation on group scales.

3. MULTIFREQUENCY FILTERING OF THE HALO tSZ SIGNAL

We now study the significance with which the halo tSZ signal from nearby groups and galaxies can be detected with forthcoming multifrequency CMB experiments. Microwave maps contain not only the tSZ, but also a host of contaminants including primary CMB anisotropies, kinetic SZ, microwave emission from galactic dust, infrared and radio point sources, over and above the detector noise of the experiment. The importance of utilizing the tSZ as a cosmological probe has prompted several
authors to develop specialized techniques for detecting galaxy clusters through the tSZ effect. Proposed techniques include the maximum entropy method (Hobson et al. 1998), fast independent component analysis (Maino et al. 2002), matched filter analysis (Herranz et al. 2002), wavelet filtering (Pierpaoli et al. 2005), and Wiener filtering (Tegmark & Efstathiou 1996). Map filtering, in general, utilizes both the spatial and frequency information to separate galactic foreground emission and extragalactic point source contamination from the primary CMB and tSZ signals.

For the purposes of this investigation, which involves assessing the level of detection of a tSZ halo, we utilize a simple multifrequency Wiener filtering technique as described in Tegmark & Efstathiou (1996) to separate the halo tSZ signal from other components. The method allows us to determine the level of residual noise in the filtered maps and a signal-to-noise ratio for each halo. Foreground and noise subtraction is done in harmonic space which allows one to exploit the fact that contaminants such as the CMB, galactic dust, and extragalactic point sources have power spectra that differ from that of the tSZ effect. We also include a contribution from the tSZ background that has the same frequency dependence as the halo tSZ signal. The contamination arising from the superposition of tSZ sources along the line of sight has the potential to reduce detectability and distort observable properties of the halo tSZ signal (Holder et al. 2007).

3.1. Multifrequency Wiener Filter

We assume that we have microwave sky maps \( d_i(r) \) at pixel position \( r \) for \( M \) different frequencies. The signal in each map originates from \( N \) components \( s_j(r) \) such as the primary CMB anisotropy, tSZ sources, galactic foregrounds, and extragalactic point sources, so that

\[
\Delta T(r, \nu) = \sum_j f_j(\nu) s_j(r),
\]

where \( f_j(\nu) \) is the frequency dependence of the \( j \)th component. In addition to these components, each map contains detector noise \( n_j(r) \) which we treat as random in each pixel. Then we can write our observation as

\[
d_i(r) = F_{ij} s_j(r) + n_i(r),
\]

where the \( M \times N \) frequency response matrix \( F \) is defined as \( F_{ij} = \int w_i(\nu) f_j(\nu) d\nu \). It is convenient to absorb the beam response factor into the definition of the pixel noise (Knox 1995) which allows us to set the response coefficients \( w_i \) to unity.

We assume that the noise has zero mean \( \langle n_i \rangle = 0 \) with covariance

\[
\langle \tilde{n}_i(\ell) \tilde{n}_i^*(\ell') \rangle = (2\pi)^3 \delta(\ell - \ell') \tilde{N}_{ij}(\ell).
\]

We will consider the case of white noise, for which \( \tilde{N}_{ij}(\ell) \) is constant. We assume that the signal and foreground components have means

\[
\langle s_j(r) \rangle = A_i
\]

covariance

\[
\langle (\tilde{s}_i(\ell) - A_i)(\tilde{s}_j^*(\ell') - A_j) \rangle = (2\pi)^3 \delta(\ell - \ell') \Delta_s(\ell).
\]

The condition that the signal and noise components are uncorrelated ensures that the signal covariance matrix, \( \tilde{S}_j(\ell) = \delta_{ij} \tilde{C}_{s,ij} \), and noise covariance matrix, \( \tilde{N}_{ij}(\ell) = \delta_{ij} \tilde{N}_{t,ij} \), are diagonal. In what follows, we will drop the tilde on harmonic space quantities.

The most general linear estimator \( \hat{s} \) of the signal can be constructed from the data \( d \) as

\[
\hat{s}(r) = \int W(r - r') d(r') d^2 r',
\]

where \( W \) is an \( N \times M \) weight matrix. We use the flat sky approximation and work in harmonic coordinates. Some signals, e.g., the primordial CMB, have a zero mean, \( A_i = 0 \). For a signal with a nonzero mean, the condition of an unbiased estimator requires that

\[
\langle \hat{s}_i(r) \rangle = \int \sum_j W_{ij}(\ell) s_j(\ell) d^2 \ell = A_i,
\]

where we have used Parseval’s theorem.

The residual error in the maps from the noise and foregrounds is given by

\[
\langle \Delta_i(r)^2 \rangle = \langle (\hat{s}_i(r) - s_i(r))^2 \rangle = \int \Delta^2_s(\ell) d^2 \ell,
\]

where

\[
\Delta^2_s(\ell) = \sum_j \{(W_i F - I)_{ij}\}^2 S_{ij} + \sum_{j=1}^{M} |W_{ij}|^2 N_{ij}.
\]

The first term accounts for the contamination of the desired signal by other components, while the second term measures the detector noise. A nonzero mixing arises when two or more components have similar frequency dependence. Requiring that the residual error is minimized we derive the Wiener filter weights

\[
W_i = S_i F^T (F S_i F^T + N_i)^{-1}.
\]

To set the threshold for detection, we compare the mean tSZ signal to the residual noise and define the signal-to-noise ratio as

\[
S/N = \frac{\langle \hat{s}_{tSZ}(r) \rangle}{\langle \Delta_{tSZ}(r)^2 \rangle^{1/2}} = \frac{\int \hat{s}_{tSZ}(\ell) d\ell}{\left[ \int \Delta^2_{tSZ}(\ell) d\ell \right]^{1/2}},
\]

where the recovered signal is given by

\[
\hat{s}_{tSZ}(\ell) = \sum_{j=1}^{M} W_{tSZ,j}(\ell) s_{tSZ}(\ell).
\]

The signal-to-noise ratio depends on the range of \( \ell \) over which the integration is performed. We chose \( \ell_{\text{min}} \) and \( \ell_{\text{max}} \) to give the maximum signal to noise over this \( \ell \) range— in practice this would correspond to applying the appropriate high-pass and low-pass filters to the recovered map.

In the above derivation, we have assumed a perfect knowledge of the frequency response of the instrument to the various components. In practice, the bandpasses are only known to within some error, which results in leakage of flux between components when applying the multifrequency filter to separate components. Following Church et al. (2003) we have computed the mixing matrix that quantifies the leakage between components due to an imperfect knowledge of the bandpass error, using a
Figure 4. tSZ temperature distortion profile (black curves, decreasing from left to right) and integrated flux profile (blue curves, increasing from left to right) plotted against $r/r_{\text{vir}}$ for the polytropic model (solid curves) and the entropy model (dotted curves). The panels from top to bottom show the tSZ distortion and integrated flux profiles for halo masses $M_{\text{vir}} = 10^{13}, 10^{14},$ and $10^{15} M_\odot$, respectively. The sharp falloff that is visible in the temperature distortion profile for the polytropic model is a result of the Gaussian smoothing described in the text and occurs at larger radius for the entropy injection model.

(A color version of this figure is available in the online journal.)

five-component model that includes the CMB, the tSZ signal, galactic dust, radio point sources, and infrared point sources. In the case of both the ACT and PLANCK experiments, we have found that the error resulting from the leakage of other components into the tSZ component dominates the reconstruction error from the multifrequency Wiener filter, $\Delta_{\text{tSZ}}(\ell)$, on large scales, $\ell \lesssim 300$. Most of this error is due to the CMB which has significant large-scale power and the largest mixing component with the tSZ. On smaller scales, however, where practically all of the signal to noise accumulates for the tSZ cluster and group detections, the bandpass leakage error is subdominant to the reconstruction error for both the ACT and PLANCK experiments. This permits us to ignore the bandpass leakage error when studying the detection of individual tSZ clusters and groups.

3.2. Halo Thermal Sunyaev–Zel’dovich Signal

The upscattering of microwave background photons by hot electrons in the halo results in a projected Compton profile

$$\nu_{\text{comp}}(r_p) = \nu_c 0 \times 2 \int_0^{r_{\text{max}}/r_s} \nu_{\text{tSZ}}(r) dl,$$

where the central Compton distortion is

$$\nu_c 0 = 8.0 \times 10^{-3} \left( \frac{n_e 0}{10^{-3} \text{ cm}^{-3}} \right) \left( \frac{T_e 0}{\text{keV}} \right) \left( \frac{r_s}{\text{Mpc}} \right),$$

and the projected radius is $r_p = x_p r_s = \sqrt{r^2 - l^2} r_s$. In the case of the entropy model, the projected pressure is given by

$$\nu_{\text{tSZ}}(r) = \sqrt{\nu_{\text{gas}}(r) + (T_{\text{inj}}/T_e 0) \nu_{\text{gas}}(r)},$$

where $\nu_{\text{gas}}$ is the dimensionless gas profile, while in the polytropic model the corresponding expression is obtained by setting $T_{\text{inj}} = 0$.

The resulting tSZ temperature distortion is given by

$$\Delta T(\theta) = j(x) T_{\text{cmb}} \nu_{\text{comp}}(\theta),$$

where $\theta = r_p/d_A$, the CMB temperature is $T_{\text{cmb}} = 2.73 \text{ K}$, and the frequency dependence is given by

$$j(x) = \frac{x (e^x + 1)}{(e^x - 1)^4} - 4, \quad x = \nu/56.9 \text{ GHz}.$$  

The projected temperature distortion profiles are shown in Figure 4. We note that the polytropic and entropy injection models are almost indistinguishable for the most massive halos. In Figure 4, we also show the cumulative tSZ flux within radius $r_{\text{tSZ}}$, which is given by

$$S_{\text{tSZ}}(< r_{\text{tSZ}}) = \nu_{\text{cmb}} g(x) \times 2\pi \int_0^{r_{\text{tSZ}}/r_s} \nu_{\text{comp}}(x_p) x_p dx_p,$$

where

$$g(x) = j(x) \frac{x^4 e^x}{(e^x - 1)^2}, \quad \nu_{\text{cmb}} = 0.27 \text{ GJy}.$$  

While the tSZ signatures of the largest clusters are nearly identical, it is clear from Figure 4 that the tSZ temperature distortions of less massive group halos are sensitive to feedback effects even for models which have the same tSZ flux at the virial radius.
The relevant quantities in harmonic space for determining the
detectability of a given tSZ halo are the Bessel transform of the
halo tSZ distortion
\[
s_{\text{tsz}}(\ell) = 2\pi \int_0^{\theta_{\max}} \left[ \Delta T(\theta)/j(\theta) \right] J_0(\ell \theta) \theta d\theta,
\]
where \(J_0\) is the Bessel function of order zero, and the tSZ halo
power spectrum,
\[
C_{\ell,\text{tsz}} = 2\pi \int_0^{\theta_{\max}} \left[ \Delta T(\theta)/j(\theta) \right]^2 J_0(\ell \theta) \theta d\theta,
\]
which enters the reconstruction error if the filter weights are not
diagonal in the tSZ component i.e., \((WF)^{-1}_{1,1} \neq \delta_{1,1}\).

The sharp cutoff in the Bessel transform, combined with the
fact that the tSZ distortion has not fallen to zero at \(\theta_{\max}\),
results in ringing of the profile spectrum and power spectrum in \(\ell\)-space.
We therefore smooth the density profile using a Gaussian profile so that
\(\rho_{\text{gas}}(r) \to \rho_{\text{gas}}(r) e^{-r^2/\xi^2}\), where \(\xi\) is chosen
to ensure that the total gas mass is unchanged. The smoothed
profile falls off sharply after \(r_{\max}\), so in practice we integrate
out to \(\rho_{\text{smooth}}\) that is a few times larger than \(r_{\max}\), which suffices
to remove the ringing. The resulting spectra have slightly more
\((\sim\text{few percent})\) power on intermediate and large scales with the
power going smoothly to zero at large \(\ell\).

In Figure 5, we compare the tSZ halo spectrum \(s_{\text{tsz}}(\ell)\) in the
polytropic and entropy injection models for halos of mass
\(M_{\text{vir}} = 10^{13} M_\odot\) at redshift \(z = 0.01\), mass \(M_{\text{vir}} = 10^{14} M_\odot\)
at redshift \(z = 0.1\), and mass \(M_{\text{vir}} = 10^{15} M_\odot\) at redshift
\(z = 1.0\). In the largest mass halo, we note that both our gas
models produce nearly identical spectra due to the fact that the
heating is small relative to the thermal energy of the hot gas.
For the lower mass halos, we observe that the polytropic model
produces a larger amplitude tSZ signal due to the higher density
of gas in the central regions of the halo. The temperature increase
that occurs in the entropy injection models is insufficient to
compensate for the reduced density, which results in a tSZ profile
with smaller amplitude for larger entropy injection. The trend
is monotonic suggesting that, for experiments with sufficient
sensitivity and frequency coverage to extract the tSZ halo signal,
the amount of entropy injection may be measurable from these
tSZ observations.

### 3.3. Foreground Components

We assume that the spatial and frequency dependence of each
foreground component can be written as a product
\[
\Delta T_i^2(\nu, \ell) = A^2_i f_i(\nu) C_{\ell,i},
\]
where \(f_i(\nu)\) measures the average frequency dependence of a component, \(C_{\ell,i}\) is its spatial power spectrum, and \(A_i\) is its amplitude.

Note that \(f_i(\nu)\) gives the frequency dependence of the rms fluctuations in thermodynamic temperature referenced to the CMB blackbody. We normalize the frequency term \(f_i(\nu)\) to be unity at \(\nu = 56.9\, \text{GHz}\) and the spatial term \(C_{\ell,i}\) to be unity at \(\ell = 2\) so that the units are absorbed into the overall amplitude \(A_i\). We now consider models for each of the foreground components in turn.

#### 3.3.1. Galactic Dust Emission

We model the frequency dependence of thermal galactic dust
emission as
\[
f_{\text{dust}}(\nu) = c(\nu) c_\nu(\nu) \frac{x_{\text{dust}}^3}{e^{x_{\text{dust}}} - 1}, \quad x_{\text{dust}} = h\nu/k_B T_{\text{dust}},
\]
where \(\alpha\) is the emissivity index, and
\[
c(\nu) = \left(\frac{2 \sinh \frac{\nu}{c}}{\nu}\right)^2 \quad \text{and} \quad c_\nu(\nu) = \frac{1}{\nu^2} \frac{1}{2k} \left(\frac{hc}{k_B T_{\text{cmb}}(\nu)}\right)^2
\]
convert antenna temperature to thermodynamic temperature and
specific intensity to antenna temperature, respectively. In our
model, we assume an emissivity index \(\alpha = 1.7\) and a dust
temperature \(T_{\text{dust}} = 18\, \text{K}\) (Tegmark et al. 2000; Draine &
Lazarian 1999; Ponthieu et al. 2005).

We model the spatial power spectrum of the thermal dust component as a power law
\[
C_{\ell,\text{dust}} = (\ell/\ell_*)^{-\beta},
\]
where $\beta$ is the power-law index. We set $\beta = 3$ which was the value derived from an analysis of the DIRBE maps (Wright 1998). We fix the amplitude of the galactic dust emission to be $A = 10.2 \, \mu K$ at 56.9 GHz.

An analysis of the FIRAS and DIRBE data sets (Schlegel et al. 1998; Finkbeiner & Schlegel 1999) has provided evidence for two dust components with different temperatures and emissivities. We account for uncertainty in the emissivity by introducing a residual dust component with the same spatial power spectrum but with scatter, $\delta \alpha$, in the emissivity index. We choose $\delta \alpha = 0.3$ as suggested by the analysis of Finkbeiner et al. (1999), which is consistent with the results of Draine & Lazarian (1999). The top left and top right panels of Figure 6 display the power spectra of the dust and residual dust components, respectively.

### 3.3.2. Radio and Infrared Point Sources

We consider two point source populations: radio sources e.g., blazars, and infrared point sources e.g., early dusty galaxies. We model radio sources using a fit to the WMAP Q-band ($\nu_o = 41$ GHz) data (Bennett et al. 2003):

$$dN \over dS_\nu = N_o \left( {S_\nu \over S_o} \right)^{2.3},$$

where $N_o = 80 \, \text{deg}^{-2}$ and $S_o = 1 \, \text{mJy}$. Since we are mostly concerned with fluxes at the mJy level, the slope of the distribution was altered to $-2.3$ from the fiducial slope of $-2.7$ (White & Majumdar 2004). In the case of infrared point sources, we use the fit (Borys et al. 2003) to SCUBA observations (Holland et al. 1999) at $\nu_o = 350$ GHz, given by

$$dN \over dS_\nu = N_o \left( {S_\nu \over S_o} \right)^{3.3} \left( {S_\nu \over S_o} + (S_\nu \over S_o)^{3.3} \right)^{-1},$$

where $N_o = 1.5 \times 10^4 \, \text{deg}^{-2}$ and $S_o \approx 1.8 \, \text{mJy}$. The sources fluxes were extrapolated to the frequencies of the various CMB experiments using a power law, $S_\nu \propto (\nu/\nu_o)\alpha$. We use a spectral index $\alpha = 0$ for radio sources and $\alpha = 2.5$ for infrared sources (White & Majumdar 2004).

The power spectra of the point sources is calculated from

$$C_l = \left( {dB \over dT} \right)^{-2} \int_0^{S_{\text{cut}}} dN \over dS_\nu \int_0^{S_{\text{cut}}} dN \over dS_\nu S_\nu^2 dS_\nu,$$

where $dB/dT$ is the derivative of the Planck spectrum and $S_{\text{cut}}$ is the imposed flux cut, which we assume to be 5 mJy for ACT and SPT (White & Majumdar 2004) and 250 mJy for PLANCK (Vielva et al. 2001). We assume that the point sources are spatially uncorrelated on the sky, thus the power is constant at all multipoles. The power spectra of infrared and radio point sources are displayed in the bottom left and bottom right panels of Figure 6, respectively.

### 3.3.3. Cosmic Microwave Background

The cosmic microwave background anisotropy has a constant frequency dependence with reference to the blackbody temperature so that $f_{\text{cmb}}(\nu) = 1$. The CMB power spectrum, $C_l$, was calculated using the CAMB software package\(^4\) using the WMAP five-year best-fit cosmological model. The lensed CMB angular power spectrum is shown in Figure 6.

### 3.3.4. tSZ background

The projection of tSZ sources of varying mass and redshift along the line of sight creates a diffuse tSZ background which can contaminate halo tSZ observables. To account for the contamination of the foreground halo signal by background clusters, we model the tSZ background statistically by using its power spectrum. Ideally, one would utilize a simulated map of tSZ halos to study the contamination due to projection.

\(^4\) CAMB: http://www.camb.info.
effects, but we defer this investigation to a future publication. This map would also take into account the contamination from hot gas outside collapsed structures, though Hernández-Monteagudo et al. (2006) have shown that this component does not significantly contribute to the tSZ power spectrum.

The frequency dependence of the tSZ background is the same as that of the tSZ halo signal given in Equation (26). The power spectrum of the tSZ background is computed following Komatsu & Seljak (2002) over the mass range $10^{12} - 10^{16} \, M_\odot$. We also allow for an uncertainty in the tSZ background which we conservatively model as arising from background halos smaller than $5 \times 10^{14} \, M_\odot$. The top left and top right panels of Figure 6 display the power spectra of the tSZ background and residual tSZ background.

### 3.4. Detector Noise and Experimental Specifications

We model the detector noise as an additional sky signal (Knox 1995) with power spectrum

$$N_{\ell,i} = \sigma_{\ell,i}^2 \ell (\ell+1)$$

in a given frequency band, $i$. In this case, each of the sky signals is not convolved with the experimental beam. We assume that the experimental beam is a Gaussian of width $\theta_{b,i}$ so that the full width at half-maximum (FWHM) is given by FWHM$_i = \sqrt{8 \ln 2} \theta_{b,i}$. The inverse noise weight

$$w_{i}^{-1} = \sigma_{\ell,i}^2 \times \theta_{b,i}^2$$

is defined as the noise variance per pixel times the pixel area in steradians.

We consider three nominal experiments, a shallow all sky tSZ survey by the PLANCK surveyor,\(^5\) a 200 deg$^2$ tSZ survey by the ACT,\(^6\) and a 4000 deg$^2$ survey by the SPT.\(^7\) We also consider a wider ACT survey (over 4000 deg$^2$) and a deeper SPT survey (over 200 deg$^2$) where we have rescaled the pixel noise using a fixed total integration time. Specifications for the various experiments are listed in Table 1 (Kosowsky 2006; Ruhl et al. 2004). The top left panel of Figure 6 displays the detector noise power spectra for the ACT experiment.

### 4. DETECTABILITY OF THE HALO TSZ SIGNAL

We now study the detectability of tSZ halos in our mass and redshift range for the polytropic and entropy injection models presented above. In Figure 7, we plot the minimum detectable halo mass, or threshold mass, as a function of redshift for the ACT experiment using a detection significance of $S/N = 3$ and $S/N = 5$. Similar plots for the PLANCK and SPT experiments are presented in Figures 8 and 9, respectively.

At high redshift ($z \sim 1$), the ACT experiment reaches a threshold mass of $2 \times 10^{14} \, M_\odot$ for a signal-to-noise ratio of 5 and sky coverage of 200 deg$^2$, which is similar to the completeness limit presented in (Sehgal et al. 2007). The threshold mass at high redshift is similar for the SPT experiment but larger ($\sim 10^{15} \, M_\odot$) for the all-sky PLANCK survey which has larger pixel noise. At low redshift, however, it is interesting to note that the threshold mass for ACT, in the case of the entropy injection model, drops below $10^{14} \, M_\odot$ at $z \approx 0.05$ and as low as $4 \times 10^{13} \, M_\odot$ at $z \approx 0.005$ ($\approx 20$ Mpc), and is similar in the case of the SPT-deep survey. Although the threshold mass at low redshift is higher for the SPT wide (4000 deg$^2$) survey and the all-sky PLANCK survey, it is still possible to detect group-sized halos below $10^{14} \, M_\odot$ at $z < 0.02$. This indicates that the tSZ effect in nearby group-sized halos can be detected with multifrequency observations that reach pixel sensitivities of a few $\mu$K, and that the detectability of these halos can be improved with longer integration times.

For the polytropic model, the detection levels are more optimistic because the tSZ signal is larger due to the gas being more concentrated; however, as discussed in the previous section, this model is less realistic, particularly for low-mass clusters and groups. To determine the significance with which one can distinguish the entropy model from the polytropic model via measurements of the tSZ distortion in these halos, we computed the $\chi^2$ statistic as a function of virial mass. We observe that the information gained from group halos is of the same order of magnitude as the information gained from larger clusters, because the larger differences in the tSZ spectra that results from the increased impact of the entropy injection in lower mass halos compensates the larger residual noise. For group halos detected with higher significance these measurements will provide useful joint constraints on the gas fraction and level of entropy injection in these halos. It is important to note that in our polytropic and entropy models we have assumed that all the baryons not in the form of stars are in the form of gas. If this turned out not to be the case and the gas fraction is in fact lower than what we have assumed, the significance of detections quoted here will be lower due to the reduced tSZ signal.

It is interesting to quantify the yield of galaxy groups and low-mass clusters that are detectable in these surveys. We calculate the yield by integrating the cluster abundance from the minimum survey threshold mass to a cutoff mass of $2 \times 10^{14} \, M_\odot$, which is roughly the minimum mass quoted for the detection of clusters in upcoming tSZ cluster surveys e.g., Sehgal et al. (2007). In choosing this cutoff mass, our aim is to quantify the additional

\(^5\) PLANCK: http://www.rssd.esa.int/index.php?project=planck.
\(^6\) ACT: http://www.physics.princeton.edu/act/.
\(^7\) SPT: http://pole.uchicago.edu/.

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Table 1

| Experiment | $\nu$ (GHz) | $\sigma_{p,i}$ ($\mu$K pixel$^{-1}$) | $\theta_{b,i}$ (°) |
|------------|-------------|-----------------------------------|-------------------|
| PLANCK: all-sky | 100.0 | 4.5 | 0.18 |
| | 143.0 | 5.5 | 0.13 |
| | 217.0 | 11.8 | 0.092 |
| | 353.0 | 39.3 | 0.083 |
| | 545.0 | 401.3 | 0.083 |
| ACT: 200 deg$^2$ (4000 deg$^2$) | 145.0 | 2.0 (8.9) | 0.028 |
| | 215.0 | 5.2 (23.3) | 0.018 |
| | 280.0 | 8.8 (39.4) | 0.015 |
| SPT: 4000 deg$^2$ (200 deg$^2$) | 95.0 | 9.1 (2.0) | 0.026 |
| | 150.0 | 13.4 (3.0) | 0.017 |
| | 219.0 | 41.2 (9.2) | 0.012 |
| | 274.0 | 71.4 (16.0) | 0.009 |
| | 345.0 | 583.9 (130.6) | 0.007 |

Note. In addition to the nominal ACT and SPT surveys, the specifications for a wider ACT survey and deeper SPT survey are also listed, where we have assumed a fixed total integration time in rescaling the pixel noise.
yield of tSZ halos, over and above the yield of more massive clusters, in these surveys. The anticipated number of detectable galaxy groups and low-mass clusters are given in Table 2 for the different surveys. We note that the mass function is steep so the yield is very sensitive to the minimum and maximum mass limits; consequently, the numbers quoted here should only be taken as a rough guide to the anticipated yields. We observe that all surveys will yield a reasonable number of detectable halos below the mass cutoff at the $S/N = 5$ level, with the numbers increasing significantly for halos detected at the lower significance of $S/N = 3$, though the contamination will be higher at this level. It is generally the case that the deep surveys produce a higher yield than the wide surveys for a given model, presumably because the mass function is so steep in this mass range. This trend is reversed in the case of the SPT wide survey at the $S/N = 3$ level, however, because the increased sky area is sufficient to compensate for the reduced sensitivity. We also note that the yields for the polytropic model are larger than those for the entropy model due to the larger signal in the polytropic model.

In Figure 11, we compare detection curves for the ACT experiment to the distribution of nearby groups in the UZC-SRSS2 Group Catalog (USGC; Ramella et al. 2002) and a group catalog compiled from the Sloan Digital Sky Survey.
Table 2

| Experiment | Model     | S/N = 3 | S/N = 5 |
|------------|-----------|---------|---------|
| ACT Deep   | Entropy   | 1520    | 696     |
| ACT Deep   | Polytropic| 3570    | 813     |
| ACT Wide   | Entropy   | 307     | 23      |
| ACT Wide   | Polytropic| 1371    | 87      |
| SPT Wide   | Entropy   | 5487    | 131     |
| SPT Wide   | Polytropic| 15856   | 759     |
| SPT Deep   | Entropy   | 2323    | 706     |
| SPT Deep   | Polytropic| 3845    | 1465    |
| PLANCK     | Entropy   | 653     | 227     |
| PLANCK     | Polytropic| 1110    | 418     |

We have investigated the detectability of tSZ groups using an analytic prescription for the hot gas in these halos. The models that we studied were based on hot gas being in hydrostatic equilibrium with the dark matter halo, and described by a polytropic equation of state, or an equation of state modified to include an entropy injection term. We have found that the entropy models are distinguishable from the polytropic models via measurements of their tSZ distortion, even in low-mass clusters and galaxy groups. While these models provide a useful starting point to evaluate the detectability of tSZ groups, an improved analysis will include a more realistic treatment of the gas distribution as provided by high-resolution cosmological simulations, which we intend to pursue in a forthcoming paper.

Another issue that we have only partially addressed here, through the inclusion of an tSZ background contaminant, is
the confusion caused by the superposition of tSZ distortions from hot gas in halos along the line of sight (see, e.g., Holder et al. 2007; Hallman et al. 2007b). Here again a large volume cosmological simulation will help us to quantify the impact of the tSZ background on the detection of group halos and their recovered flux. By taking advantage of the fact that nearby group halos produce a more extended tSZ signal, we aim to mitigate the impact of the tSZ background by devising algorithms to separate the group signal from the tSZ emission at smaller angular scales produced by higher redshift clusters. Finally, the combination of maps of the various foreground contaminants with simulated tSZ maps will allow us to undertake a more accurate treatment of the foreground contamination. While we have been relatively conservative in our modeling of the foreground contaminants, we have not included effects such as the clustering of infrared point sources, which could turn out to be a significant contaminant in the extraction of tSZ halos (Righi et al. 2008). We have also not included the kinetic SZ effect as a possible contaminant, because it has the same frequency dependence as the primary CMB but a much smaller amplitude on the relevant angular scales. For the same reason, we have not attempted to detect the halo via its kinetic SZ signal, though we note that a detection of the kinetic SZ signal could be enhanced by cross-correlation with optical or X-ray observations of the halo or the tSZ signal.

Prospects for detection of tSZ clusters have been studied previously in the case of PLANCK (Malte Schäfer & Bartelmann 2007; Melin et al. 2006), ACT (Pace et al. 2008; Sehgal et al. 2007), and SPT (Melin et al. 2006), but these studies have mainly focused on the statistics of tSZ detections above the mass completeness limit of the respective surveys. Pace et al. (2008) found that ACT could detect tSZ halos down to $6 \times 10^{13} h^{-1} M_{\odot}$ fairly independent of redshift, whereas we have found that these halos become hard to detect at higher redshifts. The analysis of Pace et al. (2008) only included the CMB as a foreground, so it is conceivable that the inclusion of point source foregrounds would degrade their forecasts for low-mass halos at high redshift, as we have found to be the case in our analysis. We also note that the analysis of Malte Schäfer & Bartelmann (2007) found that PLANCK could detect halos of mass $6 \times 10^{13} M_{\odot}$ below $z = 0.1$ when they included the CMB and all galactic foregrounds. In our analysis, we have emphasized that smaller halos, with $M \simeq (3-4) \times 10^{13} M_{\odot}$, could be detected at $z \lesssim 0.01$.

The detection of hot gas in these galaxy groups and low-mass clusters with the upcoming set of tSZ survey experiments will provide an interesting probe of galaxy formation and its effect on the distribution and state of the hot gas, which is most prominent in these halos. A measurement of the tSZ distortion at the virial radius of these halos will set a joint constraint on the level of entropy injection and the baryon fraction, which can be compared to the predictions from galaxy formation models. In particular, a measurement of the baryon fraction in the outskirts of galaxy groups and low-mass clusters will provide a unique update to the baryon census in the local universe.

While the upcoming generation of tSZ experiments has been designed to carry out blind surveys of galaxy clusters, a targeted survey of galaxy groups already detected in optical and X-ray observations would yield a very interesting set of objects to study. The measurement of a diffuse signal on the scale of tens of arcminutes will be challenging though, and carefully planned and executed observations will be necessary to control systematic effects and produce high fidelity maps of the tSZ distortion in these halos. In combination with existing optical and X-ray observations of these halos, these measurements will enhance our knowledge of the physics of galaxy formation and its effect on the intragroup medium.

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APPENDIX

GAS PROFILES

We present here the radial profiles of the electron temperature, density, pressure, and entropy for the polytropic model and entropy injection model (see Figures 12–15). For the higher mass halos ($\sim 10^{15} M_{\odot}$), the entropy injection model profiles are very similar to the corresponding polytropic model profiles, which indicates that the distribution of gas in large clusters is fairly insensitive to the injection of entropy. For lower mass halos, the imposed heating is much more effective, resulting in a higher electron temperature especially in the central parts of the halo (see Figure 13). This reflects the fact that feedback effects are more significant for galaxy and group-sized halos, raising the temperature above the shockheated infall value. The density profiles of the polytropic and entropy injection models are significantly different for the low halo mass range, demonstrating that the entropy injection has a more marked effect on group-sized halos, pushing gas into the outer regions of the halo and flattening the density profile. There is much more hot gas in the inner halo regions in the polytropic model which produces levels of X-ray emission in galaxy-sized halos that are in violation of observational constraints, as we discuss below. Similarly, the imposed heating only significantly alters the pressure profiles for galaxy and group-sized halos (see Figure 14), greatly lowering the central electron pressure. The

![Figure 11. Minimum detectable mass, $M_{\text{Min}}$, of tSZ halos as a function of redshift, $z$, for the ACT experiment compared to a compilation of groups (plotted as crosses) from the USGC (Ramella et al. 2002) and a galaxy group catalog (plotted as dots) based on the SDSS (Yang et al. 2007). The SDSS groups are only shown up to a redshift of $z = 0.1$. The minimum detectable mass is plotted for an entropy model with $S/N = 5$ (solid curve) and $S/N = 3$ (dashed curve). (A color version of this figure is available in the online journal.)](image-url)
Figure 12. Electron number density profiles, \(n_e \, (10^{-3} \text{ cm}^{-3})\), plotted against \(r/r_{\text{vir}}\) for the polytropic model (solid curves) and entropy model (dot-dashed curves). The panels from top to bottom show the density profiles for halo masses \(M_{\text{vir}} = 10^{13}, 10^{14},\) and \(10^{15} \, M_\odot\), respectively.

Figure 13. Electron temperature profiles, \(T_e \) (keV), plotted against \(r/r_{\text{vir}}\) for the polytropic model (solid curves) and the entropy model (dot-dashed curves). The panels from top to bottom show the temperature profiles for halo masses \(M_{\text{vir}} = 10^{13}, 10^{14},\) and \(10^{15} \, M_\odot\), respectively.
Figure 14. Electron pressure profiles, $P_e$ ($10^{-3}$ keV cm$^{-3}$), plotted against $r/r_{\text{vir}}$ for the polytropic model (solid curves) and the entropy model (dot-dashed curves). The panels from top to bottom show the pressure profiles for halo masses $M_{\text{vir}} = 10^{13}$, $10^{14}$, and $10^{15}$ $M_\odot$, respectively.

Figure 15. Entropy profiles, $S_e$ (keV cm$^2$), plotted against $r/r_{\text{vir}}$ for the polytropic model (solid curves) and the entropy model (dot-dashed curves). The panels from top to bottom show the entropy profiles for halo masses $M_{\text{vir}} = 10^{13}$, $10^{14}$, and $10^{15}$ $M_\odot$, respectively.
heating term was modeled such that the pressure was unaltered at the virial radius, where we do not expect feedback to have an effect even for the lowest mass halos. The entropy injection model was constructed to have significantly more entropy than the polytropic model in the inner regions for the low-mass halos, where the injected energy input is significant relative to the gravitational binding energy of the halo.

The cumulative X-ray luminosity profiles for the two models are shown in Figure 16. While the integrated luminosity profiles are similar for high-mass halos, the X-ray luminosity in the polytropic model is significantly higher for low-mass halos, in violation of the upper limits on the diffuse X-ray emission from hot gas in nearby galaxy halos like M31 (Taylor et al. 2003; Takahashi et al. 2004). The entropy model does not violate these constraints though, as the reduced central density in this model lowers the X-ray luminosity in the inner regions despite the increased temperature.

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