The sneutrino resonance reactions $p\bar{p} \rightarrow \tilde{\nu} \rightarrow \ell^+\ell^- + X$, for $\ell = e, \mu, \tau$, in the MSSM without $R$-parity are considered. We perform a cross section analysis and show that present limits on some products of $R/P$ couplings in the sneutrino sector can be significantly improved in future upgraded Tevatron runs. Furthermore, we introduce CP-violating and CP-conserving $\tau$-spin asymmetries which are generated already at the tree-level in the reaction $p\bar{p} \rightarrow \tilde{\nu} + X \rightarrow \tau^+\tau^- + X$ if there is $\tilde{\nu}_\mu$-$\tilde{\nu}_\mu$ mixing and that vanish in the SM. We find, for example, that for muon-sneutrino masses in the range $150 \text{ GeV} \lesssim m_{\tilde{\nu}_\mu} \lesssim 450 \text{ GeV}$, these spin asymmetries reach $\sim 20$–$30\%$ and $\sim 10\%$ for mass splitting between the muon-sneutrino CP-odd and CP-even states at the level of $\Delta m \sim \Gamma$ and $\Delta m \sim \Gamma/4$, respectively, where $\Gamma$ is the $\tilde{\nu}_\mu$ width. Both the CP-violating and the CP-conserving spin asymmetries should be detectable in future Tevatron runs even for a heavy sneutrino with a mass $\lesssim 500 \text{ GeV}$. If detected, such asymmetries—being proportional to the mass splitting between the CP-even and CP-odd sneutrino states—may serve as a strong indication for the existence of the sneutrino mixing phenomenon.

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1. Introduction and working assumptions

The minimal supersymmetric standard model (MSSM) with an $R$-parity conserving superpotential possesses a very distinct prediction: superpartners must be produced in pairs and, as a consequence, the lightest supersymmetric particle is stable. This implies that direct production of sparticles is restricted to colliders with a c.m. energy at least twice the typical sparticle mass. Although the imposition of $R$-parity conservation on the MSSM Lagrangian was originally proposed in order to avoid fast proton decays \cite{1}, it is well known that if the $R_p$ Lagrangian violates only lepton number (or only baryon number), then the proton lifetime does not pose any phenomenological problem. Since in supersymmetry (SUSY) models, the supermultiplets of the lepton-doublet $\hat{L}$ and the down-Higgs doublet $\hat{H}_d$ have the same gauge quantum numbers, the $R / P$ lepton number violating operators are constructed simply by the replacement $\hat{H}_d \rightarrow \hat{L}$ \cite{2, 3}:

$$W_L = \frac{1}{2} \lambda_{ijk} \hat{L}_i \hat{L}_j \hat{E}_c^c + \lambda'_{ijk} \hat{L}_i \hat{Q}_j \hat{D}_c^c,$$  

where $\hat{L}$ and $\hat{Q}$ are the SU(2)-doublet lepton and quark superfields and $\hat{E}_c$ and $\hat{D}_c$ are the lepton and quark singlet superfield. Also, $i, j, k$ are flavor indices such that, in the pure leptonic operator in Eq. 1, $i \neq j$. The presence of $W_L$ drastically changes the phenomenology of the SUSY leptonic sector since it gives rise to the possibility of having $s$-channel slepton resonant formation in scattering processes, thus enabling the detection of sleptons with masses roughly up to the collider c.m. energy \cite{4, 5, 6, 7}.

In this work we focus specifically on the effects of the $R_p$ interactions in Eq. 1 on lepton-pair production processes at the Fermilab Tevatron, $p\bar{p} \rightarrow \ell^+\ell^- + X$, with $\ell = e, \mu, \tau$. In particular, in the presence of these new $R_p$ couplings there is an additional (apart from the SM) contribution to the total cross section coming from $s$-channel sneutrino resonances \cite{3, 8}. We will show below how the future Tevatron run with c.m. energy $\sqrt{s} = 2$ TeV and an integrated luminosity of $\mathcal{L} = 2$ fb$^{-1}$ \cite{9} is capable of significantly improving the existing limits on some products, $\lambda\lambda'$, of $R_p$ couplings through a detailed study of the total cross sections for $p\bar{p} \rightarrow \ell^+\ell^- + X$. 

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Apart from the possibility of constraining the $R_P$ couplings which enter the sneutrino resonant formation in $\ell$-pair production, we will also explore two new aspects of $\tilde{\nu}^\mu$ resonance at the Tevatron: the detection of $\tilde{\nu}^\mu - \tilde{\nu}^\mu$ mixing and CP-violation in the process $p\bar{p} \rightarrow \tau^+\tau^- + X$. Both phenomena may exist once $\lambda_{323},\lambda_{ijk}' \neq 0$ in Eq. 1. In a previous work [7], we have presented a detailed investigation of these two new issues for the process $\ell^+\ell^- \rightarrow \tau^+\tau^-$. We showed there that the sneutrino mixing phenomenon as well as the CP-violating effects, may be easily detected already at the CERN $e^+e^-$ collider LEP2 if indeed the pure leptonic sneutrinos $R_P$ couplings in Eq. 1 exist. In what follows, we will present an analogous investigation appropriate for the reaction $p\bar{p} \rightarrow \tau^+\tau^- + X$ at the Tevatron. In leptonic colliders, where the c.m. energy is fixed by the energy of the colliding leptons, in order to discover a new resonance particle one is forced to tune the c.m. energy to the new particle mass which is of course not possible if the particle mass is a priori unknown. The advantage of a hadron collider is that, due to the continuous energy distribution of the colliding partons, one can probe new resonances over a much wider range of the corresponding new particle mass. We will therefore show that, contrary to LEP2 where the new effects mentioned above can be detected only if the sneutrino mass lies within a $\sim 10$ GeV range of the LEP2 c.m. energy, at the Tevatron these effects can be detected over a $\sim 300$ GeV sneutrino mass range if $\tau$-polarization could be measured.

Sneutrino mixing phenomena have been gaining some interest recently [10, 11, 12]. The question of whether sneutrinos mix or not is of fundamental importance since this mixing is closely related to the generation of neutrino masses [10, 11]. Here we are interested in the detection of sneutrino mixing rather than in its origins. We therefore do not assume any specific model for it to occur. Instead, for a given sneutrino flavor $i = e, \mu, \tau$ we write $\tilde{\nu}^i = (\tilde{\nu}^i_+ + i\tilde{\nu}^i_-)/\sqrt{2}$ and simply assume that, due to some new short distance physics, there is a mass splitting between the new CP-even and CP-odd sneutrino mass eigenstates $\tilde{\nu}^i_+$ and $\tilde{\nu}^i_-$, respectively (we assume CP conservation in the mixing). In fact, it was found in [11] that, generically, $\Delta m_{\tilde{\nu}^i_+}/m_{\nu_i} \lesssim \text{few} \times 10^3$ is roughly required in order for the neutrino masses, $m_{\nu_i}$, to be within their present experimental upper limits. One therefore expects the mass splitting in the $\tilde{\nu}^\tau_\pm$ sector to be extremely small. But, for $\tilde{\nu}^\mu_\pm$ (and in particular for $\tilde{\nu}^\tau_\pm$, which is however not of our main interest here, insofar as the issue of mixing goes)
the mass splitting can be sizable enough to drive significantly large new CP-odd and CP-even asymmetries already at the tree-level in \( \tau \)-pair production at leptonic colliders \[7\] and at the Tevatron.

Let us now establish our working assumptions and conventions: here and throughout the rest of the paper we assume for simplicity and without loss of generality that the \( R/P \) couplings \( \lambda'_{ijk} \) and \( \lambda_{pip} \) for all allowed combinations of indices, except for \( \lambda_{323} \), are real (this assumption does not affect the calculations presented in this paper). As will be shown in section 3, the imaginary part of \( \lambda_{323} \) can be responsible for large tree-level CP-violating effects in the reaction \( p\bar{p} \rightarrow \tau^+\tau^- + X \).

Although the existence of sneutrino mixing is irrelevant for the purpose of the cross section analysis performed in the next section, for definiteness, total cross sections will be calculated in the \( \tilde{\nu}_i^\pm \) mass basis. The relevant couplings of the CP-even (\( \tilde{\nu}_i^+ \)) and the CP-odd (\( \tilde{\nu}_i^- \)) sneutrino mass eigenstates are then:

\[
d_j \tilde{\nu}_+^k d_k = i\lambda'_{ijk}/\sqrt{2} , \quad d_j \tilde{\nu}_-^k d_k = -\lambda'_{ijk} \gamma_5/\sqrt{2} ,
\]

and for \( \{i, p\} \neq \{2, 3\} \):

\[
\ell_p \tilde{\nu}_+^i \ell_p = i\lambda_{pip}/\sqrt{2} , \quad \ell_p \tilde{\nu}_-^i \ell_p = -\lambda_{pip} \gamma_5/\sqrt{2} .
\]

For \( i = 2, p = 3 \) we define \( \lambda_{323} \equiv (a + ib)/\sqrt{2} \), therefore:

\[
\tau \tilde{\nu}_+^\mu \tau = i(a - ib\gamma_5)/2 , \quad \tau \tilde{\nu}_-^\mu \tau = i(b + ia\gamma_5)/2 .
\]

As mentioned before, the CP-violating and CP-conserving asymmetries in the \( \tau \)-pair production channel are proportional to the possible mass splitting between \( \tilde{\nu}_+^\mu \) and \( \tilde{\nu}_-^\mu \). In section 3 we will specifically take \( \Delta m_{\tilde{\nu}_+^\mu} \lesssim \Gamma_{\tilde{\nu}_+^\mu} \), where throughout the paper we set \( \Gamma_{\tilde{\nu}_+^\pm} = 10^{-2} m_{\tilde{\nu}_+^\pm} \) for any sneutrino flavor \( i = e, \mu \) or \( \tau \). Indeed, if \( m_{\tilde{\nu}_+^\pm} > m_{\tilde{\chi}^\pm} \), \( m_{\tilde{\chi}^0} \) (\( \tilde{\chi}^+ \) and \( \tilde{\chi}^0 \) are the chargino and neutralino, respectively), then the two-body decays
\( \tilde{\nu}_{\pm} \to \tilde{\chi}^+ \ell, \tilde{\chi}^0 \nu \) are open and the corresponding partial widths are given by (see Barger et al. in [4]):

\[
\Gamma(\tilde{\nu}_{\pm} \to \tilde{\chi}^+ \ell) \sim \mathcal{O}\left[10^{-2}m_{\tilde{\nu}_{\pm}} \times \left(1 - \frac{m_{\tilde{\chi}^+}^2}{m_{\tilde{\nu}_{\pm}}^2}\right)^2\right], \quad (5)
\]

\[
\Gamma(\tilde{\nu}_{\pm} \to \tilde{\chi}^0 \nu) \sim \mathcal{O}\left[10^{-2}m_{\tilde{\nu}_{\pm}} \times \left(1 - \frac{m_{\tilde{\chi}^0}^2}{m_{\tilde{\nu}_{\pm}}^2}\right)^2\right]. \quad (6)
\]

In Eqs. 5 and 6 above we have omitted factors coming from the charginos and neutralinos mixing matrices, respectively, which should multiply the right hand side of Eqs. 5 and 6. Since these mixing factors depend on the MSSM parameters we assume for simplicity that at least for one chargino and one neutralino they are of order unity. This assumption does not change our predictions in section 2. In fact, if the mixing factors are much smaller than 1, then with a sneutrino mass splitting of the order of a GeV, our spin asymmetries in section 3 are even more useful for the detection of such a mass splitting. Assuming that at least one chargino and one neutralino has a mass below 100 GeV and summing the two partial widths, then for \( m_{\tilde{\nu}_{\pm}} \gtrsim 150 \text{ GeV} \), \( \Gamma_{\tilde{\nu}_{\pm}} = 10^{-2}m_{\tilde{\nu}_{\pm}} \) serves our purpose as it is a viable estimate even without taking into account the new \( R/P \) two-body decay modes which, when summed, can form a significant fraction of the total sneutrino width.

Our motivation for choosing the specific condition \( \Delta m_{\tilde{\nu}_{\pm}} \lesssim \Gamma_{\tilde{\nu}_{\pm}} \) for the \( \tilde{\nu}_{\pm} \) mass splitting is twofold:

1. In such a case the two \( \tilde{\nu}_{\pm}^\mu \) and \( \tilde{\nu}_{\pm}^\mu \) resonances will overlap and distinguishing between them becomes a non-trivial experimental task. Thus, the CP-conserving and CP-violating \( \tau \)-spin asymmetries presented in section 3 may provide a feasible alternative for establishing that \( m_{\tilde{\nu}_{\pm}^\mu} \neq m_{\tilde{\nu}_{\pm}^\mu} \).

2. With \( \Gamma_{\tilde{\nu}_{\pm}} = 10^{-2}m_{\tilde{\nu}_{\pm}} \), the choice \( \Delta m_{\tilde{\nu}_{\pm}} \lesssim \Gamma_{\tilde{\nu}_{\pm}} \) implies \( \Delta m_{\tilde{\nu}_{\pm}} / m_{\tilde{\nu}_{\pm}} << 1 \), as imposed by neutrino masses [11, 14].
We will show in section 3 that CP-odd and CP-even \( \tau \)-polarization asymmetries at the level of tens of a percent may arise in the reaction \( p\bar{p} \rightarrow \tau^+\tau^- + X \) within the muon-sneutrino mass range \( 150 \text{ GeV} \lesssim m_{\tilde{\nu}^\pm} \lesssim 450 \text{ GeV} \), even for a mass splitting as small as \( \Delta m_{\tilde{\nu}^\pm} = \Gamma_{\tilde{\nu}^\pm}/4 \). These asymmetries (for \( \Delta m_{\tilde{\nu}^\pm} = \Gamma_{\tilde{\nu}^\pm}/4 \)) are detectable at future Tevatron runs, with a statistical significance above 3\( \sigma \) throughout almost the entire mass range \( 150 \text{ GeV} \lesssim m_{\tilde{\nu}^\pm} \lesssim 450 \text{ GeV} \). If \( \Delta m_{\tilde{\nu}^\pm} \simeq \Gamma_{\tilde{\nu}^\pm} \) the effects are much more pronounced.

It is especially interesting that a large tree-level CP-violating effect may emanate in the reaction \( p\bar{p} \rightarrow \tau^+\tau^- + X \) and, in particular, may be detectable at the future runs of the Tevatron. Previous studies of CP-violating effects in \( \tau^+\tau^- \) final state, attributed to models beyond the SM such as multi-Higgs doublet model, SUSY, leptoquarks and Majorana \( \nu \), all involve one-loop exchanges of the new particles which generate a CP-violating electric dipole moment for the \( \tau \) (see [15] and references therein). These CP-odd effects are therefore much smaller than our tree-level effect.

The paper is organized as follows: in section 2 we discuss the limits on some of the \( R/P \) couplings attainable at future runs of the Tevatron. In section 3 we calculate our CP-violating and CP-conserving \( \tau \)-polarization asymmetries and discuss the numerical results and in section 4 we summarize.

2. Expected new limits on \( R/P \) couplings at the future upgraded Tevatron

In this section we perform a detailed investigation of the reaction \( p\bar{p} \rightarrow \ell^+\ell^- + X \) at the Tevatron. In the presence of the \( R/P \) couplings in Eq. 1, the cross section receives contributions from both the SM \( \hat{s} \)-channel \( \gamma, Z \) exchanges and from the new \( \hat{s} \)-channel sneutrino exchanges [8]. The interferences between the SM diagrams and the \( \hat{s} \)-channel \( \tilde{\nu}_i^{\pm} \) diagrams as well as between the \( \tilde{\nu}_i^+ \) and the \( \tilde{\nu}_i^- \) \( \hat{s} \)-channel exchange diagrams are proportional to the down-quarks masses and are therefore being neglected. As a consequence, the total hard cross section, \( \hat{\sigma}^T \), can be expressed as the incoherent sum of the SM and \( \tilde{\nu}_i^{\pm} \) parton-level cross sections: \( \hat{\sigma}^T = \hat{\sigma}_{\SM}^{jk} + \hat{\sigma}_{\tilde{\nu}_i^{\pm}}^{jk} \). For the SM, \( \hat{\sigma}_{\SM}^{jk} \equiv \sigma(q_j\bar{q}_k \rightarrow \gamma, \ Z \rightarrow \ell^+\ell^-) \) such that \( q = u \) or \( d \), i.e., up- or down-quark and \( j = k \), where \( j, k = 1, 2, 3 \) are flavor indices. For the \( \hat{s} \)-channel sneutrino case only down-quark annihilation contributes, since
sneutrino couplings to up-quarks are forbidden by gauge invariance. We, therefore, define: \( \hat{\sigma}_{\nu_{\pm}}^{jk} \equiv \sigma(d_j \bar{d}_k \to \tilde{\nu}_{\pm}^j \to \ell^+_p \ell^-_p) \), where we consider only flavor-diagonal lepton pair production and we explicitly keep the flavor indices \( j, k, p = 1, 2, 3 \) as, in principle, all combinations of the \( d_j \tilde{\nu}_{\pm}^i d_k \) and \( \ell_p \tilde{\nu}_{\pm}^i \ell_p \) couplings are present in the lagrangian of Eq. 1, with \( p \neq i \).

The total cross section \( \sigma^T = \sigma(pp \to \ell^+\ell^- + X) \) can also be subdivided into the SM and the \( \tilde{\nu}_{\pm} \) parts as \( \sigma^T = \sigma^T_{SM} + \sigma^T_{\tilde{\nu}_{\pm}}. \) Each part is then given in terms of the parton luminosities \( dL_{jk}(\tau)/d\tau \) [16]:

\[
\sigma^T_{SM}; \; \sigma^T_{\tilde{\nu}_{\pm}} = \sum_{j,k} \int_{\tau_-}^{\tau_+} d\tau \frac{dL_{jk}(\tau)}{d\tau} \hat{\sigma}_{\nu_{\pm}}^{jk} (\hat{s} = \tau s),
\]

where \( \sqrt{\hat{s}} \) and \( \sqrt{s} \) are the c.m. energies of the \( q\bar{q} \) and \( p\bar{p} \) systems, respectively. For later use, the integration over the variable \( \tau \) is carried out by imposing lower and upper cuts \( \tau_- \) and \( \tau_+ \), respectively, corresponding to lower (\( M_{\ell\ell}^- \)) and upper (\( M_{\ell\ell}^+ \)) cuts on the \( \ell^+\ell^- \) invariant mass \( M_{\ell\ell} \equiv \sqrt{\hat{s}}. \) Also:

\[
\frac{dL_{jk}(\tau)}{d\tau} = \frac{C_{jk}}{1 + \delta_{jk}} \int_\tau^1 \frac{dx_j}{x_j} \left[ f_{j/p}(x_j)f_{k/\bar{p}}(\tau/x_j) + (j \leftrightarrow k) \right] ,
\]

where the color factor \( C_{jk} \) arises from summing and averaging over initial colors and for our processes \( C_{jk} = 1/3. \) For the distribution functions, \( f_{j/p}(x_j), \) we use the CTEQ4M parametrization [17].

The SM parton-level cross section is given by (neglecting the quarks and leptons masses):

\[
\hat{\sigma}_{SM}^{jk} = \frac{\pi \alpha^2}{3\hat{s}} \left( s_W^4 c_W^4 \beta_Z \right)^{-1} \times \left\{ 4Q_q^2 s_W^4 c_W^4 \beta_Z - 2Q_q s_W^2 c_W (1 - x_Z)(g_L + g_R)(g_L^2 + g_R^2) + (g_L^2 + g_R^2)((g_L^2)^2 + (g_R^2)^2) \right\} ,
\]

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where $\beta_Z \equiv (1-x_Z)^2 + x_Z^2(\Gamma_Z/m_Z)^2$ and $x_Z \equiv m_Z^2/\hat{s}$. Also, $g_L = s_W^2 - 1/2$, $g_R = s_W^2$ and $g_{L,R}^q = T_{L,R}^3 - s_W^2 Q_q$. Here $s_W$ is the sine of the weak mixing angle $\theta_W$, $Q_q$ is the charge of $q$ and $T_{L,R}^3$ are the appropriate $z$-components of the weak isospin of the quark $q$.

For the $\hat{s}$-channel sneutrino contribution it is useful to write the hard cross section in the form:

$$\hat{\sigma}^{jk}_{\tilde{\nu}} = v_{ip}^{jk} \times g^i(\hat{s}) ,$$  \hspace{1cm} (10)

where:

$$\sqrt{v_{ip}^{jk}} = |\lambda_{pip}| \lambda'_{ijk} , \quad g^i(\hat{s}) = \frac{\hat{s}}{32\pi |\hat{s} - m_{\tilde{\nu}}^2 + im_{\tilde{\nu}}\Gamma_{\tilde{\nu}}|^2} .$$  \hspace{1cm} (11)

In this section we assume $m_{\tilde{\nu}} \equiv m_{\tilde{\nu}^+} = m_{\tilde{\nu}^+}$ and $\Gamma_{\tilde{\nu}} \equiv \Gamma_{\tilde{\nu}^+} = \Gamma_{\tilde{\nu}^+}$. Therefore, through the present section starting with Eq. 10, we drop the subscript $\pm$, then resurrect it in the next section. As mentioned above, we set $\Gamma_{\tilde{\nu}} = 10^{-2} m_{\tilde{\nu}}$. It is then clear from Eqs. 7, 8 and 10 that the $\tilde{\nu}^i$ contribution to the total cross section, i.e., $\sigma_{\tilde{\nu}^i}$, can be expressed as a sum over the $R_P$ couplings $v_{ip}^{jk}$ with corresponding weights $c_{jk}$ as:

$$\sigma_{\tilde{\nu}^i}^T = \sum_{jk} c_{jk} v_{ip}^{jk} ,$$ \hspace{1cm} (12)

where:

$$c_{jk} \equiv \int_{\tau_-}^{\tau_+} d\tau \frac{d\mathcal{L}_{jk}(\tau)}{d\tau} g^i(\hat{s}) .$$  \hspace{1cm} (13)

In what follows we have employed an upper cut on the $\ell^+\ell^-$ system invariant mass of $M_{\ell\ell}^+ = 500$ GeV. Also, the $c_{jk}$’s are calculated with the three lower cuts $M_{\ell\ell}^- = 50$, 100 and 150 GeV. The third lower cut, i.e., $M_{\ell\ell}^- = 150$ GeV, is useful for removing most of the SM contribution via the $\hat{s}$-channel $Z$ exchange. We will show below that such a cut
can substantially improve the limits on the $R_P$ couplings at the future Tevatron Run II through an analysis of the reaction $p\bar{p} \to \ell^+\ell^- + X$.

In Tables 1, 2 and 3 we list the presently allowed upper limits on the products $\lambda\lambda'$ relevant for $e, \mu$ and $\tau$- pair production processes, respectively. We combine the limits on individual $R_P$ couplings \[2\] with existing limits on products of $\lambda\lambda'$ \[18, 19\], where in Tables 1–3 we write the more stringent ones, \textit{i.e.}, coming either from the individual or the $\lambda\lambda'$ bounds \[20\]. Evidently, from the numbers in Tables 1–3, two useful conclusions can be drawn:

1. As expected, the limits on flavor changing couplings, \textit{i.e.}, $j \neq k$, relevant for $e$ and $\mu$-pair production are much tighter, thus their contribution to the corresponding cross sections is negligible.

2. In the $\mu$ and $\tau$-pair production case the $R_P$ couplings associated with $\tilde{\nu}_e$ are negligible compared to the other sneutrino flavor, \textit{i.e.}, $\tilde{\nu}_\tau$ for the $\mu\mu$ case and $\tilde{\nu}_\mu$ for $\tau\tau$ production.

These two distinct features come in extremely handy later on when we discuss the attainable limits on the products $\lambda\lambda'$ at the future Tevatron Run II.

We will focus here only on the $\mu\mu$ and $\tau\tau$ production channels. In these cases our investigation simplifies since, as noted above, to a good approximation it is sufficient to consider an exchange of only one sneutrino flavor. Production of $ee$ at the Tevatron for one sneutrino flavor exchange was explored in \[6\]. There, only the $s$-channel tau-sneutrino was investigated and only its coupling to the valence $d$-quark, \textit{i.e.}, $\lambda'_{311}$, was considered by assuming $\lambda'_{3jk} = 0$ for $j, k \neq 1$. Although, for the case of $ee$ production, the generalization to $s$-channel exchanges of two sneutrino flavors is straightforward, we will not discuss it here.

It was also shown in \[6\] that a potential discovery of sneutrino resonance formation in the Tevatron is possible through a study of the $\ell\ell$ invariant mass distribution $d\sigma^T/dM_{\ell\ell}$, wherein the sneutrino will unveil itself as a resonant peak. However, in what follows we wish to take a different approach than that presented in \[6\], in a further attempt to
Table 1: The present upper limits on the products $\lambda'\lambda$ relevant for the parton level reactions $d_j\bar{d}_k \rightarrow \tilde{\nu}^i \rightarrow e^+e^-$ and $d_j\bar{d}_k \rightarrow \tilde{\nu}^i \rightarrow e^+e^-$. $j, k = 1, 2, 3$ are flavor indices and $n_i \equiv m_\tilde{\nu}_i/[100 \text{ GeV}]$ (see [18]).

| $jk$ | $d_j\bar{d}_k \rightarrow \tilde{\nu}^i \rightarrow e^+e^-$ |
|------|--------------------------------------------------|
| 11   | $(\tilde{\nu}^\mu) \lambda_{12i} \lambda_{2jk}/n_\mu^2$ | $4.5 \times 10^{-3}$ |
| 12, 21 | $(\tilde{\nu}^\mu) \lambda_{12i} \lambda_{2jk}/n_\mu^2$ | $6.0 \times 10^{-3}$ |
| 13, 31 | $(\tilde{\nu}^\mu) \lambda_{12i} \lambda_{2jk}/n_\mu^2$ | $2.5 \times 10^{-8}$ |
| 22   | $(\tilde{\nu}^\tau) \lambda_{13i} \lambda_{3jk}/n_\tau^2$ | $4.6 \times 10^{-5}$ |
| 23, 32 | $(\tilde{\nu}^\tau) \lambda_{13i} \lambda_{3jk}/n_\tau^2$ | $9.0 \times 10^{-3}$ |
| 33   | $0.018$ | $0.012$ |

To illustrate the magnitude of the sneutrino cross section as compared to the SM one, for $\mu\mu$ and $\tau\tau$ production, we plot in Figure 1 the ratio:

\[ R_{\text{sneu}} \equiv \frac{\sigma^T_{\tilde{\nu}^i}}{\sigma^T_{\text{SM}}}, \quad (14) \]

where for the $\mu\mu$ case, $i = \tau$ and for $\tau\tau$ production, $i = \mu$. We show the ratio $R_{\text{sneu}}$ for a $p\bar{p}$ c.m. energy of $\sqrt{s} = 2$ TeV (which is assumed throughout the rest of the paper) and for three lower cuts on the $\ell\ell$ invariant mass, $M_{\ell\ell} = 50, 100$ and $150$ GeV, where we constrain the $R_P$ couplings. In particular, we will show that the study of total cross sections may also be very useful for obtaining limits on some of the $R_P$ couplings. For the purpose of bounding the $R_P$ couplings we will focus later on $\mu$-pair production. However, it should be emphasized at this point that for $\tau$-pair production, our total cross section analysis (with cuts on the $\tau\tau$ invariant mass) for constraining the $R_P$ couplings may prove to be more useful than a study of $d\sigma^T/dM_{\tau\tau}$. The reason is that it will be experimentally easier to determine the $\tau\tau$ production rate above some value of $M_{\tau\tau}$ than to reconstruct the $\tau\tau$ invariant mass on an event by event basis.
Table 2: The same as Table 1 for the parton level reactions $d_jd_k \to \tilde{\nu}^i \to \mu^+\mu^-$ and $d_jd_k \to \tilde{\nu}^i \to \mu^+\mu^-$ (see [18]).

| $jk$ | $(\tilde{\nu})$ | $\lambda_{212}^j\lambda_{1jk}^i/n_c^2$ | $(\tilde{\nu})$ | $\lambda_{232}^j\lambda_{3jk}^i/n_c^2$ |
|------|------------------|---------------------------------|------------------|---------------------------------|
| 11   | $1.8 \times 10^{-6}$ | $6.0 \times 10^{-3}$ | \text{---} | \text{---} |
| 12, 21 | $3.8 \times 10^{-7}$ | $3.8 \times 10^{-7}$ | \text{---} | \text{---} |
| 13, 31 | $2.4 \times 10^{-6}$ | $2.4 \times 10^{-6}$ | \text{---} | \text{---} |
| 22   | $1.0 \times 10^{-3}$ | \text{0.012} | \text{---} | \text{---} |
| 23, 32 | $5.5 \times 10^{-6}$ | $5.5 \times 10^{-6}$ | \text{---} | \text{---} |
| 33   | $3.5 \times 10^{-6}$ | \text{0.016} | \text{---} | \text{---} |

Table 3: The same as Table 1 for the parton level reactions $d_jd_k \to \tilde{\nu}^e \to \tau^+\tau^-$ and $d_jd_k \to \tilde{\nu}^\mu \to \tau^+\tau^-$ (see [18]).

| $jk$ | $(\tilde{\nu}^e)$ | $\lambda_{313}^j\lambda_{1jk}^i/n_c^2$ | $(\tilde{\nu}^\mu)$ | $|\lambda_{323}^j\lambda_{2jk}^i/n_\mu^2|$ |
|------|------------------|---------------------------------|------------------|---------------------------------|
| 11   | $1.1 \times 10^{-6}$ | $5.4 \times 10^{-3}$ | \text{---} | \text{---} |
| 12, 13 | $6.0 \times 10^{-5}$ | $5.4 \times 10^{-3}$ | \text{---} | \text{---} |
| 21   | $1.1 \times 10^{-4}$ | \text{0.011} | \text{---} | \text{---} |
| 22   | $6.0 \times 10^{-5}$ | \text{0.011} | \text{---} | \text{---} |
| 23   | $6.0 \times 10^{-4}$ | \text{0.011} | \text{---} | \text{---} |
| 31   | $1.1 \times 10^{-4}$ | \text{0.013} | \text{---} | \text{---} |
| 32   | $9.9 \times 10^{-4}$ | \text{0.022} | \text{---} | \text{---} |
| 33   | $2.1 \times 10^{-6}$ | \text{0.022} | \text{---} | \text{---} |
Figure 1: The ratio $R_{sneu} \equiv \frac{\sigma^T_{\tilde{\nu}_i}}{\sigma^T_{SM}}$, as a function of the $\tau$-sneutrino mass for the reaction $p\bar{p} \rightarrow \mu^+\mu^- + X$ for: $M_{\mu\mu}^- = 50$ GeV (lower dashed line), $M_{\mu\mu}^- = 100$ GeV (lower long-dashed line), $M_{\mu\mu}^- = 150$ GeV (lower solid line), and as a function of the $\mu$-sneutrino mass for the reaction $p\bar{p} \rightarrow \tau^+\tau^- + X$ for: $M_{\tau\tau}^- = 50$ GeV (upper dashed line), $M_{\tau\tau}^- = 100$ GeV (upper long-dashed line), $M_{\tau\tau}^- = 150$ GeV (upper solid line). We use $M_{\ell\ell}^+ = 500$ GeV, $\sqrt{s} = 2$ TeV, $\Gamma_{\tilde{\nu}_i} = 10^{-2} m_{\tilde{\nu}_i}$ and $m_{\tilde{\nu}_i} = m_{\tilde{\nu}_i}$. $M_{\ell\ell}^+$ and $M_{\ell\ell}^-$ are upper and lower cuts on the dilepton invariant mass, respectively. Maximal allowed values of the products $\lambda\lambda'$ are used for any given sneutrino mass on the horizontal axis, i.e., scaled as $m_{\tilde{\nu}_i}/[100 \text{ GeV}]$ (see Tables 2 and 3). See also [20].

We take $M_{\ell\ell}^+ = 500$ GeV. We include all combinations of $\{j,k\}$ in $\lambda'$, from valence and sea down-quarks (that is, for $\mu\mu(\tau\tau)$ production all combinations of $\lambda_{232}\lambda'_{ijk}(|\lambda_{323}|\lambda'_{2jk})$ are included). Also, for any $\{j,k\}$ we take the sneutrino mass-dependent maximal allowed values of the corresponding $R_{\lambda'}$ coupling, i.e., scaled as $m_{\tilde{\nu}_i}/[100 \text{ GeV}]$ (see Tables 2 and 3). Therefore, for any sneutrino mass value on the horizontal axis, Figure 1 represents the largest possible value of $R_{sneu}$ that might be measured at $\sqrt{s} = 2$ TeV. As expected, with $M_{\ell\ell}^- = 50$ GeV where the SM $s$-channel $Z$ resonance dominates, we find that $R_{sneu} \sim 10^{-2}$ for sneutrino masses between 100 and 500 GeV. However, the sneutrino contribution is
much more pronounced if the lower invariant mass cut is set to $M_{\ell\ell} = 100$ or $150$ GeV. In the latter case, i.e., $M_{\ell\ell} = 150$ GeV, the $s$-channel $Z$ resonance contribution is practically removed and it is possible to have $R^{\text{snu}} > 1$ over almost the entire sneutrino mass range 150–500 GeV. Therefore, these reactions may indeed lead to a discovery of a new scalar resonance, in particular, a sneutrino. For example, we see from Fig. 1 that with a lower cut of $M_{\ell\ell} = 150$ GeV ($\ell = \tau$ or $\mu$), one can expect the total cross sections for $\tau^+\tau^-$ or $\mu^+\mu^-$ pair production to increase by a factor of $\sim 50$ or 20, respectively, if the sneutrino mass is $\sim 200$ GeV, i.e., a spectacular discovery.

Let us now illustrate how a study of total cross sections for $p\bar{p} \to \ell^+\ell^- + X$ can be used to constrain some of the $R_P$ couplings. Even in the most general case in which all combinations of the products $\lambda\lambda'$ are taken into account, one can take advantage of the information given in Tables 1–3 combined with the corresponding numerical values of the $c_{jk}$’s (defined in Eq. 13) to greatly simplify the analysis. The simplest example perhaps is the reaction $p\bar{p} \to \mu^+\mu^- + X$ which is also the easiest to measure. In $\mu$-pair production, as already mentioned above, to a good approximation only $\tilde{\nu}^\tau$ with flavor diagonal couplings to $d\bar{d}$, $s\bar{s}$ and $b\bar{b}$ contributes. Moreover, owing to the very small probability of finding a $b$-quark in the proton with momentum fraction $x_b$, $b\bar{b}$ annihilation is negligible compared to the valence $d\bar{d}$ and sea $s\bar{s}$ fusion. We are therefore left with only two relevant $R_P$ couplings: $\sqrt{v_{11}^{11}} = \lambda_{332}\lambda'_{311}$ and $\sqrt{v_{22}^{22}} = \lambda_{332}\lambda'_{322}$. In fact, although the present limits allow $v_{22}^{22} = 4v_{11}^{11}$ (see Table 3), we still find that the valence $d\bar{d}$ contribution dominates over the sea $s\bar{s}$ annihilation due to larger probability distributions (calculated from Eq. 13). Nonetheless, for completeness we include in our analysis below both the $d\bar{d}$ and the $s\bar{s}$ contributions.

The statistical significance of the deviation of the total cross section from the SM cross section is defined by:

$$N_{\text{SD}} = \frac{|\sigma^T - \sigma^T_{\text{SM}}|L}{\sqrt{\sigma^T L}},$$

which in our case, i.e., $\sigma^T = \sigma^T_{\nu} + \sigma^T_{\text{SM}}$ and $i = \tau, p = 2$ (see Eq. 10), simplifies to:
Figure 2: The attainable 1σ-limit contours on the $R_P$ coupling products $\lambda_{232}\lambda_{311}'$ and $\lambda_{232}\lambda_{322}'$, for the reaction $p\bar{p} \rightarrow \mu^+\mu^- + X$ at the Tevatron Run II with c.m. energy $\sqrt{s} = 2$ TeV and an integrated luminosity of $L = 2$ fb$^{-1}$. For illustration we choose $m_{\tilde{\nu}_\tau}^\pm = m_{\tilde{\nu}_\tau}^\mp = 200$ GeV and take $M_{\mu\mu}^+ = 500$ GeV. We use: $M_{\mu\mu}^- = 50$ GeV (dotted-dashed line) [21], $M_{\mu\mu}^- = 100$ GeV (dashed line) and $M_{\mu\mu}^- = 150$ GeV (solid line). See also caption to Figure 1.

\[ N_{SD} = \frac{\sum_{jk} c_{jk} v_{r2}^{jk}}{\sqrt{\sum_{jk} c_{jk}^2 v_{r2}^{jk}}} \sqrt{\mathcal{L}} \]  
\[ \text{dd,ss only} \]  
\[ = \frac{c_{11} v_{r2}^{11} + c_{22} v_{r2}^{22}}{\sqrt{c_{11}^2 v_{r2}^{11} + c_{22}^2 v_{r2}^{22} + \sigma_{SM}^2}} \sqrt{\mathcal{L}}, \]  

(16)

where $\mathcal{L}$ is the integrated luminosity at the Tevatron. Using Eq. (16) we can now plot contours of $N_{SD}$ limits on the two $R_P$ couplings $\sqrt{v_{r2}^{11}} = \lambda_{232}\lambda_{311}'$ and $\sqrt{v_{r2}^{22}} = \lambda_{232}\lambda_{322}'$. In Figure 2 we show the 1σ-limit contours corresponding to the three lower $\mu\mu$ invariant mass cuts $M_{\mu\mu}^- = 50$, 100 and 150 GeV, again taking $M_{\mu\mu}^+ = 500$ GeV. For illustration we choose $m_{\tilde{\nu}_\tau}^\pm = 200$ GeV and a total integrated luminosity of $L = 2$ fb$^{-1}$ appropriate for the Tevatron Run II. The values at the end points of the $x$ and $y$ axes in Figure 2 are the presently allowed upper limits on the coupling products $\lambda_{232}\lambda_{311}'$ and $\lambda_{232}\lambda_{322}'$. 

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respectively, for \( m_{\tilde{\nu}^\pm} = 200 \text{ GeV} \). We observe that, due to larger valence \( d \)-quark probability functions, a significant improvement over the present limits may be obtained for the \( \mathcal{R}_{p} \) product \( \lambda_{232} \lambda'_{311} \) \[21\]. As expected, the effect is much more pronounced when the SM “background” is removed, \( i.e., \delta_{\bar{\mu}} = 150 \text{ GeV} \), where the attainable 1\( \sigma \) limits are: \(-0.003 \lesssim \lambda_{232} \lambda'_{311} \lesssim 0.003 \) and \(-0.011 \lesssim \lambda_{232} \lambda'_{322} \lesssim 0.011 \), for a \( \tau \)-sneutrino mass of 200 GeV. For \( \lambda_{232} \lambda'_{311} \) this limit is about one order of magnitude better than the current limit and for \( \lambda_{232} \lambda'_{322} \) it provides an improvement by about a factor of 4 over the present limit.

3. Sneutrino mixing and \( \tau \)-polarization asymmetries in \( p\bar{p} \rightarrow \tau^+\tau^- + X \)

In this section we explore the possibility of studying detailed properties of the sneutrino sector in the MSSM with \( \mathcal{R}_{p} \). In particular, we discuss the reaction \( p\bar{p} \rightarrow \tau^+\tau^- + X \) proceeding via muon-sneutrino exchange. Therefore, throughout the rest of the section we drop the sneutrino index \( \mu \) and denote \( m_{\tilde{\nu}^\pm} \equiv m_{\pm}, \Gamma_{\tilde{\nu}^\pm} \simeq \Gamma_{\tilde{\nu}^\pm} \equiv \Gamma \) and set \( \Gamma = 10^{-2}m_{-} \).

We focus on two issues:

1. Detection of muon-sneutrino mixing, \( i.e., \) probing a possible mass splitting between the muon-sneutrino CP-even (\( \tilde{\nu}_+ \)) and CP-odd (\( \tilde{\nu}_- \)) states.

2. The possibility of having large CP-violating signals at the Tevatron, which, in the presence of the \( \mathcal{R}_{p}\hat{s} \)-channel \( \tilde{\nu}_\pm \) exchanges, are generated already at the tree-level if there is a non-vanishing mass splitting \( \Delta m \equiv m_{+} - m_{-} \neq 0 \).

As demonstrated below, these two effects can be studied at the Tevatron through measurements of some specific CP-violating and CP-conserving \( \tau \)-double-spin asymmetries which were suggested in \[7\] for the reaction \( \ell^+\ell^- \rightarrow \tau^+\tau^- \) appropriate to leptonic colliders. Note that spin asymmetries in \( p\bar{p} \rightarrow \ell^-\ell^+ + X \) can be measured only for \( \ell = \tau \); the electron or the muon polarization is not accessible in such high energy collider experiments.

The CP-violating and CP-conserving \( \tau \)-double-spin asymmetries are derived at the parton level, \( i.e., q_j\bar{q}_k \rightarrow \tau^+\tau^- \), following the same line of arguments as introduced in our previous work \[7\] and are then “dressed” with the parton distribution functions. In
the rest frame of \( \tau^- \) we define the basis vectors: \( \vec{e}_z \propto -(\vec{p}_{qj} + \vec{p}_{\bar{q}k}), \ \vec{e}_y \propto \vec{p}_{qj} \times \vec{p}_{\bar{q}k} \) and \( \vec{e}_x = \vec{e}_y \times \vec{e}_z \). For the \( \tau^+ \) we use a similar set of definitions such that \( \vec{e}_x, \vec{e}_y, \vec{e}_z \) are related to \( \vec{e}_x, \vec{e}_y, \vec{e}_z \) by charge conjugation. We then introduce the following \( \tau^+\tau^- \) double-polarization operator with respect to each of the coordinate directions defined above:

\[
\hat{\Pi}_{mn} \equiv \frac{N(\uparrow_m \uparrow_n) - N(\uparrow_m \downarrow_n) - N(\downarrow_m \uparrow_n) + N(\downarrow_m \downarrow_n)}{N(\uparrow_m \uparrow_n) + N(\uparrow_m \downarrow_n) + N(\downarrow_m \uparrow_n) + N(\downarrow_m \downarrow_n)},
\]

where \( m, n = x, y, z \). For example, \( N(\uparrow_x \uparrow_y) \) stands for the number of events in which \( \tau^+ \) has spin +1 in the direction \( x \) in its rest frame and \( \tau^- \) has spin +1 in the direction \( y \) in its rest frame. The spin vectors of \( \tau^+ \) and \( \tau^- \) are therefore defined in their respective rest frames as: \( \vec{s}^+ = (\vec{s}_x, \vec{s}_y, \vec{s}_z) \) and \( \vec{s}^- = (\vec{s}_x, \vec{s}_y, \vec{s}_z) \) and \( \hat{\Pi}_{mn} \) is calculated at the parton level in the \( q_j\bar{q}_k \) c.m. frame by boosting \( \vec{s}^+ \) and \( \vec{s}^- \) from the \( \tau^+ \) and \( \tau^- \) rest frames to the \( q_j\bar{q}_k \) c.m. frame.

It is easy to verify that the \( \hat{\Pi}_{mn} \)'s possess the following transformation properties under the operations of CP and that of the naive time reversal \( T_N \) [22]:

\[
\text{CP}(\hat{\Pi}_{mn}) = \hat{\Pi}_{mn} \quad \text{for all } m, n ,
\]

\[
T_N(\hat{\Pi}_{mn}) = -\hat{\Pi}_{mn} \quad \text{for } m \text{ or } n = y \text{ and } m \neq n ,
\]

\[
T_N(\hat{\Pi}_{mn}) = \hat{\Pi}_{mn} \quad \text{for } m, n \neq y \text{ and for } m = n = x, y, z .
\]

We can therefore define:

\[
\hat{A}_{mn} = \frac{1}{2} (\hat{\Pi}_{mn} - \hat{\Pi}_{nm}), \quad \hat{B}_{mn} = \frac{1}{2} (\hat{\Pi}_{mn} + \hat{\Pi}_{nm}),
\]

such that \( \hat{A}_{mn} \) are CP-odd (\( \hat{A}_{mm} = 0 \) by definition) and \( \hat{B}_{mn} \) are CP-even. Also, \( \hat{A}_{xy}, \hat{A}_{zy}, \hat{B}_{xy} \) and \( \hat{B}_{zy} \) are \( T_N \)-odd while \( \hat{A}_{xz}, \hat{B}_{xz}, \hat{B}_{xy} \) and \( \hat{B}_{zy} \) are \( T_N \)-even.

In the SM, as expected, there is no CP violation at the tree-level and we find that only the CP-even asymmetries \( \hat{B}_{xz}, \hat{B}_{yy} \) and \( \hat{B}_{zz} \) are non-zero at the tree-level, where in fact,
for either $u\bar{u}$ or $d\bar{d}$ annihilation, $\hat{B}_{xx} = -\hat{B}_{yy}$ and $\hat{B}_{zz} = 1$ \[6\]. However, in the $s$-channel $\tilde{\nu}_\pm$ exchange case, the CP-violating asymmetry $\hat{A}_{xy}$, being a $T_N$-odd quantity, is also non-zero already at the tree-level. In particular, for any given flavor combination $\{j, k\}$ in $d_j\bar{d}_k \to \tilde{\nu}_\pm \to \tau^+\tau^-$, $\hat{B}_{xx}$, $\hat{B}_{yy}$, $\hat{B}_{zz}$ and $\hat{A}_{xy}$ are given by \[7\] (recall that $\lambda_{323} = (a+ib)/\sqrt{2}$):

$$\hat{A}_{xy} = \left( \frac{2ab}{a^2 + b^2} \right) \frac{D_-}{D_+}, \quad \hat{B}_{xx} = \hat{B}_{yy} = \left( \frac{a^2 - b^2}{a^2 + b^2} \right) \frac{D_-}{D_+}, \quad \hat{B}_{zz} = -1,$$

where:

$$D_\pm \equiv |\pi_+|^2 \pm |\pi_-|^2, \quad \pi_\pm = \left( s - m_\pm^2 + im_\pm \Gamma \right)^{-1}.$$  

While the presence of a non-vanishing tree-level CP-nonconserving asymmetry in $p\bar{p} \to \tau^+\tau^- + X$ is a unique outcome of sneutrino resonance formation, as was mentioned above, the CP-even spin-asymmetries receive contributions from the SM too. However, it is also possible to construct a CP-even $\tau$-spin asymmetry which is sensitive only to the $s$-channel $\tilde{\nu}_\pm$ exchanges and is identically zero in the SM. This CP-conserving asymmetry is defined as \[7\], $\hat{B} \equiv (\hat{B}_{xx} + \hat{B}_{yy})/2$. Obviously, at tree-level, $\hat{B} = 0$ in the SM (due to $\hat{B}_{xx} = -\hat{B}_{yy}$) and $\hat{B} = \hat{B}_{xx} = \hat{B}_{yy}$ for the sneutrino case. Thus, a measurement of $\hat{B}$ substantially different from zero will be a strong indication for the existence of new physics in $p\bar{p} \to \tau^+\tau^- + X$, in the form of new non-vanishing $s$-channel scalar exchanges and will provide explicit information on the new $\tau\tilde{\nu}_\pm^\tau\tau$ coupling. The use of $\tau$-spin asymmetries as a probe of new physics was also suggested in \[23\],\[24\], e.g., in the decay $H^0 \to \tau^+\tau^-$. Note that the observable $\hat{B}_{zz}$, which in the SM simply translates to the spin correlation $<\vec{s}_+ \cdot \vec{s}_->$ that was suggested in \[23\] for the decay $H^0 \to \tau^+\tau^-$, may also be useful in distinguishing between the SM vector-boson exchanges and the sneutrino scalar exchanges. However, since $\hat{B}_{zz} = -1$ for the pure $\tilde{\nu}_\pm$ exchange case, a measurement of $\hat{B}_{zz}$, will be insensitive to the couplings $a$ and $b$ in $\lambda_{323}$.

In what follows, we will therefore discuss only the CP-odd $\hat{A}_{xy}$ and the CP-even $\hat{B}$ spin-asymmetries. Let us comment first on how to measure these spin-asymmetries.
Since the $\tau$ spins are not directly observable, the $\tau$ decay products should be used as spin analyzers (see [23, 24, 25, 26]). In particular, following [23], since in the $\tau^-\tau^+$ c.m. frame our asymmetry $\hat{A}_{xy}$ corresponds to the spin correlation $\hat{p}_{\tau^-} \cdot (\vec{s}^- \times \vec{s}^+)$ (i.e., the operator $O_2$ in [23]), in the case of the two-body decays $\tau^{\pm} \to \pi^\pm\nu_\tau(\bar{\nu}_\tau)$, $\hat{A}_{xy}$ can be translated to the correlation $\hat{p}_{\tau^-} \cdot (\hat{p}_{\pi^+}^* \times \hat{p}_{\pi^-}^*)$, where $\hat{p}_{\tau^-}$ is the flight direction of $\tau^-$ in the $\tau^-\tau^+$ c.m. frame and $\hat{p}_{\pi^\pm}^*$ is the flight direction of $\pi^\pm$ in the corresponding $\tau^\pm$ rest frames, respectively. Thus, in order to measure $\hat{A}_{xy}$ one can trace the $\tau^\pm$ spins by measuring the flight direction of their decay products in their respective rest frames or equivalently by measuring the triple correlation $\langle \hat{p}_{\tau^-} \cdot (\hat{p}_{\pi^+}^* \times \hat{p}_{\pi^-}^*) \rangle$. Note that similar correlations between the $\tau^\pm$ spins and their decay products can be obtained for the other combinations of the $\tau^\pm$ two-body decays (see Table II in [23]). In the same way, $\hat{B}$ is proportional to spin correlation $\vec{s}^- \cdot \vec{s}^+ = (\hat{p}_{\tau^-} \cdot \hat{p}_{\tau^-}^*)$ (this corresponds to the operator $(O_3 - O_4)$ in [23]) and can be translated to a correlation between the $\tau^\pm$ decay products, e.g., in the case of $\tau^\pm \to \pi^\pm\nu_\tau$, $\langle O_3 - O_4 \rangle \propto \langle (\hat{p}_{\pi^+}^* \cdot \hat{p}_{\pi^-}^*) - (\hat{p}_{\tau^-} \cdot \hat{p}_{\tau^-}^*) \rangle$ [23].

It is important to note that these two asymmetries change sign around $\sqrt{s} \sim m_-$ (see [7]). In fact, roughly $\hat{A}_{xy}(\sqrt{s} = m_- - \delta m) \sim -\hat{A}_{xy}(\sqrt{s} = m_- + \delta m)$ and similarly for $\hat{B}$, for a given mass shift $\delta m$. Of course, this does not introduce any difficulty in a leptonic collider where the c.m. energy of the colliding leptons is fixed. However, for the Tevatron, after folding in the parton luminosities (see Eqs. 7 and 8) and integrating over $\sqrt{s}$ (or equivalently $M_{\tau\tau}$), due to this change in sign, the effects become too small to be of any measurable consequences. To bypass this problem we suggest here a two-step measurement; at the first stage one would have to identify the sneutrino resonant peak by measuring the $\tau\tau$ invariant mass distribution $d\sigma^T/dM_{\tau\tau}$. Then, once the sneutrino mass is known, the asymmetries $\hat{A}_{xy}$ and $\hat{B}$ may be multiplied by the sign of $(M_{\tau\tau} - m_-)$ to account for the relative minus sign as one goes from $M_{\tau\tau} < m_-$ to $M_{\tau\tau} > m_-$:

$$\hat{A}_{xy} \to \text{sgn}(M_{\tau\tau} - m_-)\hat{A}_{xy}, \quad \hat{B} \to \text{sgn}(M_{\tau\tau} - m_-)\hat{B}.$$  \hspace{1cm} (24)

The corresponding CP-odd and CP-even asymmetries for the overall reaction $p\bar{p} \to \tau^+\tau^- + X$, $A_{xy}$ and $B$, are then given by:
Figure 3: The maximal values of $A_{xy}$ and $B$ as a function of the lighter CP-odd muon-sneutrino mass $m_-$, for three mass-splitting values $\Delta m = \Gamma$ (solid line), $\Delta m = \Gamma/4$ (dashed line) and $\Delta m = \Gamma/10$ (dotted-dashed line). $M_{\tau\tau}^{-} = 150$ GeV, $M_{\tau\tau}^{+} = 500$ GeV, $\sqrt{s} = 2$ TeV and $\Gamma = 10^{-2}m_-$. 

From Eq. 22 we observe that both $\hat{A}_{xy}$ and $\hat{B} \propto D_-/D_+$, where the proportionality factors do not depend on the absolute magnitude of the couplings $a$ and $b$ in $\lambda_{323}$ but rather on any function of their ratio $f(a/b)$. As in [4], without loss of generality, we will assume that $a$ and $b$ are positive and study the asymmetries as a function of the ratio $r \equiv b/(a + b)$. Thus $r$ can vary between $0 \leq r \leq 1$, where the lower and upper limits of $r$ are given by $b = 0$ and $a = 0$, respectively. One can immediately observe that $\hat{A}_{xy}$ and $\hat{B}$ complement each other as they probe opposite ranges of $r$. For $\hat{A}_{xy}$ the maximal value $D_-/D_+$ is obtained when $r = 1/2$ ($a = b$) and the largest positive $\hat{B}$ possible is $\hat{B} = D_-/D_+$ when
Figure 4: The scaled statistical significance, $N_{SD}/\sqrt{L}$, for both $A_{xy}$ and $B$, as a function of $m_-$. As in Figure 3, $A_{xy}$ and $B$ are evaluated at their maximal values. For the Tevatron runs II, III the statistical significance, $N_{SD}$, is obtained by multiplying the values on the $y$ axis by $\sqrt{L} = \sqrt{2}$, $\sqrt{L} = \sqrt{30}$, respectively. See also caption to Figure 3.

$r = 0 \ (b = 0)$. Also, at $r = 1 \ (a = 0)$, $\hat{B} = -D_-/D_+$, thus reaching its maximum negative value.

In Figure 3 we plot the maximal values of $A_{xy}$ and $B$, corresponding to the maxima $\hat{A}_{xy} = \hat{B} = D_-/D_+$, as a function of the lighter muon-sneutrino mass $m_-$. We take $M_{\tilde{\tau} \tilde{\tau}} = 150$ GeV, $M_{\tilde{\tau} \tilde{\tau}}^+ = 500$ GeV and the mass splitting values $\Delta m = \Gamma$, $\Gamma/4$, $\Gamma/10$ (recall that $\Gamma = 10^{-2}m_- \sqrt{\epsilon}$). Also, for completeness we include all $\{j, k\}$ flavor combinations of the annihilating down quarks with their corresponding $R_P$ couplings (see Table 3). Evidently, $A_{xy}$ and $B$ can reach $\sim 20$–30% throughout the entire mass range, $150$ GeV $\lesssim m_- \lesssim 450$ GeV, if $\Delta m = \Gamma$, and $\sim 10$–13% even with a smaller splitting of $\Delta m = \Gamma/4$.

The statistical significance, $N_{SD}$, with which $A_{xy}$ or $B$ can be detected at the Tevatron, is given by $N_{SD} = \sqrt{N}|A|\sqrt{\epsilon}$ [23], where $A = A_{xy}$ or $B$ are given in Eq. [23]. $N = (\sigma_{\tilde{\nu}_\pm}^T + \sigma_{SM}^T) \times L$ is the total number of $p\bar{p} \to \tau^+\tau^- + X$ events and $L$ is the integrated
luminosity at the Tevatron. \( \epsilon \) is the combined efficiency for the simultaneous measurement of the \( \tau^+ \) and \( \tau^- \) spins which, therefore, depends on the efficiency for the spin analysis and also on the branching ratios of the specific \( \tau^+ \) and \( \tau^- \) decay channels that are being analyzed. The simplest examples perhaps are the two-body decays \( \tau^\pm \to \pi^\pm \nu_\tau \) and \( \tau^\pm \to \rho^\pm \nu_\tau \), although 3-body decays may also be useful \[23, 24, 26\]. For these two-body decays the decay density matrix is \[23, 24, 26\] \[ (1 \mp \alpha_X \hat{p}_{X^\pm} \cdot \vec{s}^\pm) d\Omega_{X^\pm}/4\pi, \] where \( X^\pm = \pi^\pm \) or \( \rho^\pm \) and \( \alpha_X \) is the spin analyzing quantity which determines the sensitivity for measuring the \( \tau^- \) spin via the momentum, \( \hat{p}_{X^\pm} \), of its decay product. In particular, \( \alpha_\pi = 1 \) and \( \alpha_\rho = 0 \) \[23, 24, 26\]. Now, since in practice one measures the momenta of the \( \tau \) decay products in order to trace its spin, to determine the expected statistical significance one has to multiply the asymmetries \( A_{xy} \) and \( B \) by the quality of the spin analysis in a given decay scenario of the \( \tau^-\tau^+ \) pair. Thus, for example, if \( \tau^- \to \rho^- \nu_\tau \) and \( \tau^+ \to \pi^+ \nu_\tau \), \( A_{xy} \) and \( B \) are suppressed by \( \alpha_\rho \times \alpha_\pi = 0.456 \) (and \( |A_{xy}|^2 \) and \( |B|^2 \) are suppressed by \( (\alpha_\rho \times \alpha_\pi)^2 = 0.456^2 \)). Therefore, when all combinations of the above \( \tau^+ \), \( \tau^- \) two-body decay channels are taken into account the combined efficiency is given by \( \epsilon = \sum_{X_1,X_2} \text{Br}_{X_1} \text{Br}_{X_2} (\alpha_{X_1} \alpha_{X_2})^2 \sim 0.03 \), where \( \text{Br}_{X_i} \) is the branching ratio for \( \tau \) to decay to \( X_i = \pi \) or \( \rho \). \[23\]. We will adopt this number henceforward \[23\].

In Figure 4 we scale out the luminosity factor from the theoretical prediction, by plotting \( N_{SD}/\sqrt{L} \) — where \( L \equiv L/(1 \text{fb}^{-1}) \) has no units — corresponding to \( A_{xy} \) and \( B \) at their maximal values at the upgraded Tevatron with \( \sqrt{s} = 2 \text{TeV} \), as a function of \( m_- \). We choose the same values for \( \Delta m \) and for \( M_{\tau\tau}^\pm \) as in Figure 3 and we again include all \( \{j,k\} \) flavor combinations in \( d_j d_k \) fusion. We see that with \( \Delta m = \Gamma, 4.2 \lesssim N_{SD}/\sqrt{L} \lesssim 1.8 \) within the mass range \( 155 \text{GeV} \lesssim m_- \lesssim 450 \text{GeV} \). Also, \( 1.8 \lesssim N_{SD}/\sqrt{L} \lesssim 0.8 \) for \( \Delta m = \Gamma/4 \), within the same muon-sneutrino mass range. Thus, for example, at the Tevatron Run II with \( L = 2 \text{fb}^{-1} \) both the CP-violating and the CP-conserving spin asymmetries may be detected with a sensitivity above \( 3\sigma \) over the mass range \( 155 \text{GeV} \lesssim m_- \lesssim 400 \text{GeV} \) if \( \Delta m = \Gamma \). These asymmetries are detectable, at the Tevatron Run III with \( L = 30 \text{fb}^{-1} \), with a statistical significance above \( 3\sigma \) over the entire mass range \( 155 \text{GeV} \lesssim m_- \lesssim 450 \text{GeV} \) for \( \Delta m = \Gamma/4 \). In fact, for this case, even if the splitting
Figure 5: \(N_{SD}/\sqrt{L}\) for \(A_{xy}\) and \(B\) as a function of \(r \equiv b/(a+b)\). The cases \(\Delta m = \Gamma\) and \(\Delta = \Gamma/4\) are illustrated. See also captions to Figures 3 and 4.

is as small as \(\Delta m = \Gamma/10\), a 3\(\sigma\) detection of the CP-odd and CP-even spin asymmetries is feasible in Run III within the \(\sim 150\) GeV mass range \(155 \text{ GeV} \lesssim m_{-} \lesssim 300\) GeV.

Finally, Figure 5 shows the dependence of \(N_{SD}/\sqrt{L}\), for \(A_{xy}\) and \(B\), on the ratio \(r\), where as in Figures 3 and 4, all \(\{j, k\}\) flavor combinations are included and \(M_{\tau\tau}^{-} = 150\), \(M_{\tau\tau}^{+} = 500\) GeV, respectively. For illustration we set \(m_{-} = 200\) GeV and chose \(\Delta m = \Gamma\) and \(\Delta m = \Gamma/4\) for both \(A_{xy}\) and \(B\). Recall that for the Tevatron Runs II, III the statistical significance, \(N_{SD}\), is obtained by multiplying the values on the \(y\) axis by \(\sqrt{L} = \sqrt{2}\), \(\sqrt{30}\), respectively. We therefore observe that a simultaneous measurement of \(A_{xy}\) and \(B\) can cover practically the entire range of \(r\) with a statistical sensitivity above 3\(\sigma\) at the Tevatron Run II, if \(\Delta m = \Gamma\). At the Tevatron Run III, the entire range of \(r\) is covered with at least 3\(\sigma\) standard deviations even for \(\Delta m = \Gamma/4\).

4. Summary and conclusions
To summarize, we found that if the data do not deviate from the SM, then existing limits on some products of the $R_P$ couplings $\lambda$ and $\lambda'$ can be substantially improved in future runs of the Tevatron through a study of total cross sections for the reactions $p\bar{p} \rightarrow \ell^+\ell^- + X$. As an example, we have considered $\mu^+\mu^-$ pair production channel and found that at the Tevatron Run II the existing limits on the two specific $R_P$ coupling products $\lambda_{232}\lambda'_{311}$ and $\lambda_{232}\lambda'_{322}$ can be improved by a factor of $\sim 10$ and $\sim 4$, respectively.

We have also introduced CP-violating and CP-conserving $\tau$-double-spin asymmetries and applied them to the process $p\bar{p} \rightarrow \tau^+\tau^- + X$. We have shown that two of these spin asymmetries are unique in their ability to distinguish between the CP-odd and CP-even muon-sneutrino mass eigenstates in $p\bar{p} \rightarrow \tilde{\nu}_\tau^\mu + X \rightarrow \tau^+\tau^- + X$. Both asymmetries arise already at the tree-level and can become large, of the order of tens of a percent. Although the $\tau$ spins have not been measured at the Tevatron yet, the large CP-odd and CP-even effects that are possible in the future runs of the Tevatron may motivate the experimentalists for a serious attack on the problem. Only after an experimental setup for measuring the $\tau$ spins is established will the spin asymmetries suggested in this work become observable with a sensitivity above $3\sigma$ at future runs of the Tevatron provided that the efficiencies for detecting the $\tau$ spin approach $\sim 10\%$. In such an optimistic scenario, the above $3\sigma$ signal is possible throughout almost the entire muon-sneutrino mass range $150$ GeV $\lesssim m_{\tilde{\nu}^\mu} \lesssim 450$ GeV. We have shown that such significant CP-violating and CP-conserving signals can arise even for a mass splitting between the CP-odd and CP-even $\tilde{\nu}_\tau^\mu$ states below their corresponding widths, i.e., $\Delta m < \Gamma$. They may therefore serve as extremely powerful probes of the sneutrino mixing phenomenon.

As far as CP-violation is concerned, it is especially gratifying that such a large CP-nonconserving effect may arise in $\tau$-pair production at the Tevatron and may be searched for in the near future.

We acknowledge partial support from U.S. Israel BSF (G.E. and A.S.) and from the U.S. DOE contract numbers DE-AC02-98CH10886(BNL), DE-FG03-94ER40837(UCR). G.E. thanks the Israel Science Foundation and the Fund for the Promotion of Research
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The working hypotheses for the future upgraded Tevatron runs are that in Run II and Run III the integrated luminosity will be \(2 \text{ fb}^{-1}\) and \(30 \text{ fb}^{-1}\), respectively, with a modest increase of the center of mass energy to \(2 \text{ TeV}\). See e.g., Report No. FERMILAB-PUB-96/082 (1996), edited by D. Amidei and R. Brock.

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We neglect the effects of electron-sneutrino exchanges in section 3 since the upper limits on the couplings \(d_j \bar{\nu}_\pm^e d_k\) are much smaller than \(d_j \bar{\nu}_\pm^\mu d_k\). Besides, as mentioned above, the rough limit \(\Delta m_{\bar{\nu}_\pm} / m_{\nu_e} \lesssim \text{few} \times 10^3\) obtained in [11], implies that electron-sneutrino mixing is expected to be much smaller than muon-sneutrino mixing.

Note that, in some instances, the bound \(\Delta m_{\bar{\nu}_i} / m_{\nu_i} \lesssim \text{few} \times 10^3\) found in [11] is relaxed by about an order of magnitude if one uses the laboratory bound for \(m_{\nu_i}\). Given the state of the theoretical constructs we do not think that this is unreasonable and that evasion of this bound can be ruled out.

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[21] We caution that the contour shown in Figure 2 for $M_{\mu\mu} = 50$ GeV is unlikely to be accessible to experiment as for this case $R_{\text{sneu}} < 10^{-2}$. Such a small deviation from the SM will be very difficult to verify experimentally. The experimental problem should be significantly alleviated for $M_{\mu\mu} = 100$ GeV and $M_{\mu\mu} = 150$ GeV for which $R_{\text{sneu}} > 0.1$ and $R_{\text{sneu}} > 1$, respectively.

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