On Heterotic/Type I Duality in $d = 8$ *

Kristin Förger
Centre de Physique Théorique, Ecole Polytechnique, F-91128 Palaiseau, France

Abstract. We discuss heterotic corrections to quartic internal $U(1)$ gauge couplings and check duality by calculating one-loop open string diagrams and identifying the $D$-instanton sum in the dual type I picture. We also compute $SO(8)^4$ threshold corrections and finally $R^2$ corrections in type I theory.

1 Introduction

Heterotic $SO(32)/$type I duality relates two string theories in ten dimensions [1, 2]. A field redefinition transforms the low energy effective action of the heterotic string into the one of the type I string [2, 3]

$$G^I_{\mu\nu} = \lambda^I G^{\text{het}}_{\mu\nu}, \quad \lambda^I = \frac{1}{\lambda^{\text{het}}}, \quad B^{R,I}_{\mu\nu} = B^{NS,\text{het}}_{\mu\nu}$$

(1)

where $G_{\mu\nu}$ is the metric, $B^{NS/R}_{\mu\nu}$ the antisymmetric tensor in the $NS$ or $R$ sector and $\lambda^I = e^{\varphi^{(10)}}$ the ten dimensional type I coupling. Compactifying to lower dimensions modifies the duality map due to the dependence on the volume of the compactification manifold.

Besides the equivalence of the BPS spectrum of the heterotic and type I string, non–trivial checks can be done for the BPS–saturated $F^4$ and $R^4$ couplings; see e.g. [4, 5, 6, 7] and [8] for gauge couplings in the context of heterotic/F-theory duality. The special feature of these couplings is that –on the heterotic side—they are protected from higher than one–loop corrections in the effective action thanks to supersymmetry. Thus they can be exactly calculated. These threshold corrections translate to perturbative open string amplitudes and non-perturbative BPS $D1$-instanton corrections on the type I side. Therefore by studying various examples of couplings one learns more about the rules of computing non-perturbative $D$-brane contributions (see e.g. [9]).

Toroidal compactification down to eight dimensions gives rise to 4 abelian gauge fields, corresponding to the components $G_{\mu I}$ and $B_{\mu I}$, where $\mu$ is the space-time index and $I$ labels the compact directions. The four real massless scalar fields parametrize the moduli space $\mathcal{O}(2,2)/\mathcal{O}(2) \times \mathcal{O}(2)$ of the torus $T^2$ (if no Wilson lines are switched on).

In the type I theory the $U(1)^2 \times U(1)^2$ gauge group is reduced to a diagonal $U(1)^2$, due to the twist operator $\Omega$ [10]. Similarly only two of the four massless scalar fields survive the projection.

* To appear in the proceedings of Quantum Aspects of Gauge Theories, Supersymmetry and Unification, Corfu, September 1998.

CPHT-PC696.1298
2 Heterotic string in eight dimensions

At string one–loop, the higher derivative couplings \( F^4, (F^2)^2, R^4 \) and \( (R^2)^2 \) and their respective CP-odd parts receive corrections in the effective string action. These constant contributions are highly fixed by ten–dimensional anomaly cancellation arguments. In the Green-Schwarz formalism they are calculated by (almost) holomorphic one loop string amplitudes, whose minimal number of external legs is fixed by saturating fermionic zero modes. The result for the amplitudes is summarized by the worldsheet \( \tau \) integral over a weight zero almost holomorphic function which is related to the elliptic genus [11, 12]. After compactification on \( T^2 \) these corrections become moduli dependent functions. It is believed that there are no space–time instanton effects, since the only supersymmetric soliton, the NS five–brane, cannot be wrapped around the torus.

2.1 \( U(1)^2 \) int heterotic threshold corrections

Here we are mainly interested in threshold corrections to internal \( U(1) \) couplings. At tree level they are obtained by dimensional reduction of the \( R^2 \) term in the effective action. The internal gauge boson vertex operator can be read off from the \( \sigma \) model action. Therefore the vertex operator for \( G_{IJ} \) gauge bosons in the background gauge \( e^\mu \epsilon_{\mu k} X = -\frac{1}{2} F_{\mu \nu} X^\mu \) with \( F_{\mu \nu} = \) const in Green-Schwarz formalism is:

\[
V_{\text{gauge}} = \frac{i \pi}{\tau_2} F^A_{\mu \nu} Q^A : \left( \partial X^\mu X^\nu - \frac{1}{4} \gamma_{ab} S^a S^b \right) :,
\]

where \( A = \pm \) labels different \( U(1) \) charges \( Q^+ = \epsilon_I^+ \partial X^I = \sqrt{\frac{\tau_2}{\tau_1}} (1, U) A(\tau_1) \) and \( Q^- \), which is obtained from \( Q^+ \) by replacing \( U \) by \( \bar{U} \) and \( A = \left( \begin{smallmatrix} n_1 & -i \beta_1 \\ n_2 & i \beta_2 \end{smallmatrix} \right) \) is a \( GL(2, \mathbb{Z}) \) matrix. The complex structure and the Kähler modulus of the torus are defined in terms of the metric and NS-NS antisymmetric tensor as \( U = U_1 + i U_2 = (G_{12} + i \sqrt{\text{det} G})/G_{11} \) and \( T = T_1 + i T_2 = 2(b + i v/\sqrt{\text{det} G}) \), respectively.

The effective action which arises from string amplitudes with vertex operators (2) is:

\[
\mathcal{T}_{1-\text{loop}}^{\text{het}} = \frac{V(n^8)}{28 \pi^4} \int \frac{d^2 \tau}{\tau_2} \sum_{l,n \in \mathbb{Z}} e^{-\frac{\pi}{2} \left( n^I \tau - \tau I \right) (G + B)_{IJ} (n^J \tau - \tau J)} \int D S_0^a e^{-\pi/\tau_2 F_{\mu \nu}^A Q^A R_0^{\mu \nu} \mathcal{A}(\bar{q})},
\]

where \( R_0^{\mu \nu} = \frac{1}{2} \delta_{ab} \gamma_{\mu \nu} S_0^b \) and \( \mathcal{A}(\bar{q}) = \frac{E_2^b}{\bar{q}^2} \).

\[\text{1\ Supersymmetry relates even to odd spin structures. Since periodic Green-Schwarz fields } S^a \text{ are mapped to periodic NSR fields } \psi^\mu, \text{ CP-odd correlation functions of NSR currents are equivalent to CP-even correlation functions of Green-Schwarz currents due to a Riemann identity [12].}\]
where Poisson resummation turns the sum over winding modes $l$ into a sum over momenta $m$ and windings $n$, thereby transforming the charge insertions $Q^\pm$ into Narain momenta insertions $P_{R/L}$.

The $\tau$-integrals can be performed [8, 7, 15] by the unfolding trick [13] and techniques developed in [14]. Unfolding the integration region into the trivial orbit $A = 0$ the degenerate orbit $A = \begin{pmatrix} 0 & l \\ 0 & p \end{pmatrix}$, with $(j, p) \neq 0$ and the non-degenerate orbit $A = \begin{pmatrix} k & j \\ 0 & p \end{pmatrix}$ with $0 \leq j < k$ and $p > 0$ enables one to identify these contributions to tree-level, perturbative and non-perturbative corrections on the type I side. The full heterotic effective action can be expressed in terms of prepotentials [6, 8]. This is similar to the case of $N = 2$ supersymmetric string vacua in $d = 4$.

Integration of the non-degenerate orbit of (5) gives [15]

$$
\Delta^{\text{non-deg}}_{F_+^{l-1} F_-} \equiv \frac{\partial^{l-1}}{\partial s_+^{l-1}} \frac{\partial^l}{\partial s_-^l} \int \frac{d^2 \tau}{\tau_+^2} \sum_{A \in \text{GL}(2, \mathbb{Z})} e^{2\pi i T \det A - \frac{\tau^+}{2\tau_+} [(1, U) A (\tau_-, \tau_+)]^2} e^{-s_+ \frac{1}{2} Q^{+}} \tilde{A}(\bar{q}) \bigg|_{s=0},
$$

(5)

Poisson resummation turns the sum over winding modes $l, n$ into a sum over momenta $m$ and windings $n$, thereby transforming the charge insertions $Q^\pm$ into Narain momenta insertions $P_{R/L}$.

The $\tau$-integrals can be performed [8, 7, 15] by the unfolding trick [13] and techniques developed in [14]. Unfolding the integration region into the trivial orbit $A = 0$ the degenerate orbit $A = \begin{pmatrix} 0 & l \\ 0 & p \end{pmatrix}$, with $(j, p) \neq 0$ and the non-degenerate orbit $A = \begin{pmatrix} k & j \\ 0 & p \end{pmatrix}$ with $0 \leq j < k$ and $p > 0$ enables one to identify these contributions to tree-level, perturbative and non-perturbative corrections on the type I side. The full heterotic effective action can be expressed in terms of prepotentials [6, 8]. This is similar to the case of $N = 2$ supersymmetric string vacua in $d = 4$.

Integration of the non-degenerate orbit of (5) gives [15]

$$
\Delta^{\text{non-deg}}_{F_+^{l-1} F_-} \equiv \frac{\partial^{l-1}}{\partial s_+^{l-1}} \frac{\partial^l}{\partial s_-^l} \int \frac{d^2 \tau}{\tau_+^2} \sum_{A \in \text{GL}(2, \mathbb{Z})} e^{2\pi i T \det A - \frac{\tau^+}{2\tau_+} [(1, U) A (\tau_-, \tau_+)]^2} e^{-s_+ \frac{1}{2} Q^{+}} \tilde{A}(\bar{q}) \bigg|_{s=0},
$$

where

$$
\begin{align*}
 b(s_+, s_-) &= \left( p - \frac{i(s_+ + s_-)}{2\pi \sqrt{2} T_2 U_2} \right)^2 + \frac{s_+ s_-}{2\pi^2 T_2 U_2} \\
 \tilde{T}(s_+, s_-) &= T_1 - \frac{i}{p} T_2 \left[ \sqrt{b(s_+, s_-) + \frac{i(s_+ - s_-)}{2\pi \sqrt{2} T_2 U_2}} \right] \\
 \tilde{U}(s_+, s_-) &= \frac{1}{k} \left( j + p U_1 - i U_2 \left[ \sqrt{b(s_+, s_-) + \frac{i(s_+ - s_-)}{2\pi \sqrt{2} T_2 U_2}} \right] \right).
\end{align*}
$$

(7)

The worldsheet instanton corrections $\Delta^{\text{non-deg}}$ are exponentially suppressed. They can be expressed in terms of Hecke operators:

$$
\Delta^{\text{non-deg}}_{F_+^{l-1} F_-} = \frac{\pi^4}{2} \sum_{N=1}^{\infty} \left( \frac{T_2}{U_2} \right)^{2-l} \left( D^{l, q_T}_T \right)^{N^{l-1}} H_N[D^l_U, \mathcal{A}](U) \right)
$$

$$
+ \left( \frac{U_2}{T_2} \right)^{2-l} D^{l, \tilde{q}_T}_T \frac{1}{N^{l-1}} H_N[D^{l-1}_U, \mathcal{A}](\tilde{U})
$$

(8)

The Hecke operator acts on a modular form $\phi_w$ of weight $w$ as $H_N[\phi_w](U) = \sum_{k, p > 0} \sum_{0 \leq j < k} k^{-w} \phi_w(U)$ with the complex structure $U = \frac{i}{\sqrt{k}} [16]$. 

\[ \text{for } l = 0, \ldots, 4 \text{ with coupling} \]

$$
\Delta_{F_+^{l-1} F_-} = \frac{\partial^{l-1}}{\partial s_+^{l-1}} \frac{\partial^l}{\partial s_-^l} \int \frac{d^2 \tau}{\tau_+^2} \sum_{A \in \text{GL}(2, \mathbb{Z})} e^{2\pi i T \det A - \frac{\tau^+}{2\tau_+} [(1, U) A (\tau_-, \tau_+)]^2} e^{-s_+ \frac{1}{2} Q^{+}} \tilde{A}(\bar{q}) \bigg|_{s=0},
$$

(4)
where \( q_T = e^{2 \pi i T} \) with \( T = NT \). This rewriting is one step towards the understanding of semi-classical \( D \)-instanton calculus in the dual type I picture.

For the degenerate orbit we obtain

\[
\Delta_{F^4 - F^l}^{\text{deg}} = \frac{c_0}{\pi} \frac{U^3}{T_2^2} \sum_{(j,p) \neq (0,0)} \frac{(j - p U)^{4-l}(j - p \bar{U})^l}{|j - p U|^l},
\]

where \( c_0 = 504 \) originates from \( \bar{E}_2 = \sum_n c_n \bar{q}^n \). The sum is taken over winding modes \( j, p \). Setting \( n_1 = n_2 = 0 \) corresponds to vanishing winding numbers. Only Kaluza-Klein momenta contribute to \( \Delta_{\text{deg}} \), which will be identified with type I perturbative corrections in the next section. For \( l = 2 \) the result can be expressed in terms of the generalized Eisenstein function \( E(U, 3) = \sum_{(j,p) \neq 0} U^3_2 \zeta(6) |j - p U|^6 \). [17]

### 3 Type I in eight dimensions

In the following we calculate two kinds of one-loop gauge threshold corrections: First corrections w.r.t. the internal gauge group \( U(1)^2_{\text{int}} \) and second corrections including discrete Wilson lines that break the gauge group to \( SO(8)_4 \). Finally we discuss type I \( R^2 \) corrections.

#### 3.1 \( U(1)^2_{\text{int}} \) type I threshold corrections

Using the duality map (1) one can verify that the degenerate orbit of the heterotic threshold corrections corresponds to one loop terms in the type I effective action:

\[
T_{\text{het}}^{\text{deg}} = \frac{V^{(8)} T_2}{2^8 \pi^4} t_8 F^4 - F^l \Delta_{F^4 - F^l}^{\text{deg}} (T_2, U) \leftrightarrow T_{1-\text{loop}}^l.
\]

\( V^{(8)} t_8 \) is invariant under this transformation, and the factor \( e^{2 \phi U} \) which arises from \( U_{\text{int}}^{-2} \) which contracts internal indices of the gauge kinetic term, altogether resulting in a \( \lambda_0^0 \) coupling. In the following we will check this by an independent calculation of the type I one-loop corrections to these couplings.

One loop open string amplitudes consist in summing over oriented and unoriented surfaces with and without boundaries like the torus (\( T \)) and Klein bottle (\( K \)) for the closed string sector and the annulus (\( A \)) and Möbius strip (\( M \)) for the open string sector, which have Euler number \( \chi = 0 \).

The vertex operator of the gauge fields coincides with the one of the type IIB theory:

\[
V_{\text{gauge}} = G_{\mu I} : (\partial X^I - \frac{1}{4} k_\sigma \bar{S}^a \gamma_{\alpha \beta} L^\sigma \bar{S}^b) (\partial X^\mu - \frac{1}{4} k_\nu \bar{S}^a \gamma_{\nu \mu} S^b) e^{ikX} :.
\]
There is no contribution from the torus diagram $T$ since the sixteen fermionic zero modes cannot be saturated at the level of four derivative terms. The remaining amplitudes can be written as:

$$\mathcal{I}_{1\text{-loop}}^{I} = \frac{1}{2} V^{(8)} \sum_{\sigma} \rho_{\sigma} \int_{0}^{\infty} dt \left( \frac{1}{(2\pi t)^2} \right)^{4} \left( \sum_{p \in \ell_2} e^{-\pi t |p|^2 / 2} \right) Z(\tau_{\sigma}) \int \prod_{i=1}^{4} d^2 z_i \left( \prod_{i=1}^{4} V_{\text{gauge,}i} \right)$$

(12)

where the sum is taken over one-loop surfaces $\sigma = A, M, \mathcal{K}$ with relative weights $\rho_{\mathcal{K}} = 1, \rho_{A} = N^2$ and $\rho_{M} = -N$, and $N$ is the Chan-Paton charge which takes the value $N = 32$ for the gauge group $SO(32)$. The factor $(2\pi t)^{-4}$ arises from momentum integration and $V^{(8)}$ is the uncompactified volume in type I units. The open string oscillator sum is $Z(\tau_{\sigma}) = \frac{1}{\eta^{4}(\tau_{\sigma})} \sum_{s_{\alpha}=2,3,4} \frac{1}{2} s_{\alpha} \theta^{4}_{\alpha}(0, \tau_{\sigma})$ with GSO projection signs $s_{3} = -s_{2} = -s_{4} = 1$ and modular parameters $\tau_{A} = \frac{2}{3}, \tau_{M} = \frac{i t + 1}{2}, \tau_{\mathcal{K}} = 2it$. The torus partition function $I_{2}$ is now restricted to the Kaluza-Klein momenta $m_{1}$ and $m_{2}$ and $p^{2} = p_{I} G^{I\mu} p_{\mu} = \frac{1}{2i \pi \chi_{\mathcal{G}}} |m_{1} + m_{2} U|^{2}$. This reflects the fact that for the open string the perturbative duality group is reduced to $SL(2, \mathbb{Z})_{U}$.

Contraction of the leftmoving fermions $S^{a_{\gamma_{ab}} \nu} S^{b}$ contributes four derivatives to the amplitude and using a Riemann identity one finds $Z(t) G_{\mathcal{I}}^{4}(t) = -\frac{i}{2} t^{2}$ which is independent of worldsheet coordinates. Poisson resummation gives:

$$\mathcal{I}_{1\text{-loop}}^{I} = \frac{V^{(8)} T_{2}}{2^{8} t_{8} F_{+}^{4} F_{l}^{4}} \int \frac{dt}{t^{6}} \sum_{j, p \neq 0} e^{-\pi t^{2} (j-pU)^{2}} \frac{T_{2}^{2}}{U_{2}} (j-pU)^{4-1}(j-pU)^{1} \frac{1}{2} \left( N^{2} - N + 2^{4} \right)$$

(13)

where $\frac{1}{2} \left( N^{2} - N + 2^{4} \right) = c_{0}$. Integration over $t$ thus reproduces the corresponding heterotic coupling $\Delta_{F_{+} F_{-}}^{\text{log}}(T_{2}, U)$.

### 3.2 $SO(8)^{4}$ Type I Threshold Corrections

This is the orientifold example of Sen [19], for which heterotic-F theory duality can be checked explicitly [8]. We switch on discrete Wilson lines $a_{1}^{I} = \frac{1}{2}(0,4,1,4)$ and $a_{2}^{I} = \frac{1}{2}(0,1,4,1)$ which break the $SO(32)$ gauge group to $SO(8)^{4}$. The internal $U(1)^{2}$ gauge group cannot be enhanced for this particular choice of Wilson lines [2] and the underlying prepotential is trivial. I.e. in this case the corresponding gauge couplings $\Delta_{F_{+} F_{-}}^{\text{log}}$ vanish identically [8].

We apply the background field method of [20, 4] to calculate type I one-loop threshold corrections. The expression for the one-loop amplitude reads:

$$\mathcal{I}_{1\text{-loop}}^{I} = \frac{i V^{(8)}}{2} \sum_{\sigma, \nu} \rho_{\sigma} \int \frac{dt}{t} \left( \frac{1}{(2\pi t)^{4}} \right)^{4} \sum_{\alpha_{I}, \alpha_{I} + \ell_{2}} e^{-\pi t p^{2} / 2} \frac{1}{\eta^{12}(\tau_{\sigma})} \frac{i}{2} \theta_{\nu}^{4} B_{t} \theta_{\nu}^{4} \left( \frac{i \sigma t}{2}, \tau_{\sigma} \right) \sum_{\alpha} \frac{1}{2} s_{\alpha} \theta_{\alpha}^{3}(0, \tau_{\sigma})$$

(14)
where \( F = BQ \) is the background gauge field and \( Q \) a generator of the Cartan subalgebra and \( q_i \) the corresponding charge. The non-linear function \( \epsilon_\sigma \) can be expanded as \( \epsilon_\sigma \simeq q_\sigma B + O(B^3) \) with \( q_\sigma^j = (q_i + q_j) \) and \( q_\sigma^M = 2q_i \).

Expanding the integrand to the order \( O(B^4) \) gives

\[
\mathcal{I}_{1-\text{loop}} = -\frac{V^{(8)} B^4}{2} \sum_{\sigma,ij} \rho_\sigma \int \frac{dt}{t} \left[ \epsilon^4 \sum_{a_\sigma^j} e^{-\pi t p_j G_{ij} p_j / 2} \right]
\]

After Poisson resummation and changing variables from the direct channel to the closed string transverse channel \( l = 1/t \) for the annulus and \( l = 1/(4t) \) for the Möbius strip, one finds:

\[
\mathcal{I}_{1-\text{loop}} = -i \frac{V^{(8)} B^4}{2\pi} \sum_w \frac{T_2}{w! G_{ij} w^2} \left[ \sum_{ij} e^{2\pi i (a_i + a_j) t w^4} (q_i + q_j)^4 - \sum_i e^{4\pi i a_i t w^4} (2q_i)^4 \right]
\]

Evaluating the sum leads to

\[
\mathcal{I}' = i \pi V^{(8)} B^4 \left[ 4 \ln \left[ T_2 U_2 |\eta(U)|^4 \right] \sum_{i<j;(i,j)=(k,k')} (q_i + q_j)^4 \right.
\]

\[
+ 2 \sum_{k=2}^4 \ln \left[ T_2 U_2 \left| \frac{\theta_k(U)}{2\eta(U)} \right|^2 \right] \sum_{(i,j)=(1,k)} (q_i + q_j)^4 \right]
\]

(17)

where \( 1 = \{1, \ldots, 4\}, 2 = \{5, \ldots, 8\}, 4 = \{9, \ldots, 12\}, 3 = \{13, \ldots, 16\} \). We omitted some moduli independent constant which appears after regularization of the logarithmic divergence [13].

In the T-dual type I’ picture this example corresponds to placing four seven branes at each of the four fixed points. The above result are the threshold corrections to \( \text{Tr} F^2_{SO(8),k} \text{Tr} F^2_{SO(8),k'} \) for \( k, k' = 1, \ldots, 4 \) coming from open strings stretched between the branes sitting on the same fixed point or different fixed points. On the heterotic side they translate to the degenerate orbit of the coupling [8].

### 3.3 D–instanton contribution

Heterotic worldsheet instantons that appear in the non-degenerate orbit of the heterotic amplitude, are ‘dual’ to type I BPS \( D \)–instantons which are wrapped \( D1 \) branes on the spacetime torus. The way to count all inequivalent ways in which a torus can cover \( N \)-times the space-time torus is characterized by the transformations matrix \( A \) of the non-degenerate orbit where \( kp = N \) is the instanton number [5].

The classical instanton saddle point is the exponent of the Born-Infeld action of the wrapped \( D1 \)-brane [5, 6, 7]

\[
S_{BI} = \int d^2\sigma \frac{1}{\lambda_I} \sqrt{\text{det} \hat{G}_t - i \int \hat{B}_t}.
\]

(18)

In the presence of \( U(1) \) background fields the square root also includes gauge fields \( \hat{F} \) from \( G_{\mu t} \) whereas the Wess-Zumino coupling contains \( B_{\mu t} \) RR gauge fields.
Fluctuations around the classical instanton saddle point are taken into account by the elliptic genus for \( N \) \( D \)-1-branes, which is equivalent to the one of a single \( D \)-1-brane wrapped \( N \) times over \( T^2 \). This counting procedure is captured by the action of the Hecke operator on the elliptic genus \( H_N[A](U) \) [see also eq. (8)].

As an example we write the \( F_4^+ \) coupling of (6) as D-instanton sum:

\[
\langle F_4^+ \rangle_{\text{inst}} = \frac{1}{\sqrt{\text{det}(G + F)}} \sum_j e^{-S_{\mathcal{M}}( \mathcal{U})} \tag{19}
\]

where \( \sqrt{\text{det}(G + F)} = kT^2 |p + \frac{i\sigma}{2\pi \sqrt{2T^2 U^2}}| \).

In analogy to semiclassical instanton calculations the correlation function in an instanton background is obtained by saturating fermionic zero modes and integrating over the moduli space of instantons. In this case the instanton moduli space is provided by the heterotic matrix string model [21], describing a worldsheet \( O(N) \) two dimensional gauge theory. In the infrared limit this gauge theory flows to a \((8,0)\) supersymmetric \( S_N \times \mathbb{Z}_N^2 \) orbifold conformal field theory. The elliptic genus for \( S_N \) symmetric orbifolds is naturally described by the action of the \( N^{\text{th}} \) Hecke operator on the elliptic genus [22].

### 3.4 One-Loop corrections to \( R^2 \)

Let us consider the \( CP \)-even \( R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \) coupling in eight dimensions. On the heterotic side the one-loop correction vanishes, whereas the type I correction is non-trivial as we will show in the following. The graviton vertex operator in the zero ghost picture is:

\[
V_{\text{grav}} = \epsilon_{\mu\nu} \left( \partial X^\nu - \frac{1}{4} \hat{S}_a^{\gamma\rho} S_a^{\gamma\rho} \partial_{k} k_\rho \right) \left( \partial X^\mu - \frac{1}{4} \hat{S}_a^{\gamma\mu} S_a^{\gamma\mu} \partial_{k} k_\sigma \right) e^{ikX} \tag{20}
\]

Although the kinematic structure of a two point graviton amplitude vanishes due to the on shell constraints we can still calculate its four derivative gravitational coupling \( \Delta_{\text{grav}}^I \). A three point amplitude which e.g. includes a modulus and two gravitons will then give a non-vanishing kinematic structure multiplied by the derivative of the same coupling with respect to the modulus \( \partial U \Delta_{\text{grav}}^I \) [23].

But we get non-vanishing contributions from \( K, A \) and \( M \). In order to extract the order \( O(k^4) \) term of the amplitude, we have to contract the eight fermions since otherwise we will get a zero result due to Riemann identities. For non-oriented surfaces there are additional contractions between chiral and anti-chiral fermions \( \langle \psi(z) \psi(\bar{w}) \rangle_\sigma = G_F(z, I_\sigma(w)) \) with the involution \( I_A(w) = I_M(w) = I_K = -\frac{i}{2} = 1 - \bar{w} \) and \( G_F(z, w) = \frac{1}{2} \theta_1(z-w, \tau) \theta_1(z-w, \tau)^* \).

Using the same Riemann identity as before and taking the sum over worldsheets \( \sigma \) gives for the type I one loop correction to \( R^2 \):

\[
\Delta_{\text{grav}}^I = \frac{V^{(8)} T_2}{2^8 \pi^5} \Gamma(3) c_0 \frac{T_2^3}{T^2} \sum_{j,p \neq 0} \frac{1}{|j - pU|^6} \tag{21}
\]
where the coefficient $c_0 = N^2 - N + \frac{1}{4}$ arises after taking the sum over worldsheets $\sigma$. The coupling coincides with the one of $F^2 \bar{F}^2$. In the decompactification limit $\Delta^I_{\text{grav}}$ disappears, in agreement with heterotic-type I duality in ten dimensions [3].

Duality relates this term to a one–loop correction to $R^2$ on the heterotic side. Since such a term does not exist, we conclude that on the type I side it is a combination of $R^2$ and $\Delta F^2 \bar{F}^2$, which corresponds to the heterotic $R^2$ term. In particular this combination is such that no one–loop correction to $R^2$ is predicted on the heterotic side. A similar observation was recently made with the $R^2$ correction in the duality of heterotic–type IIA [24].

Acknowledgements: I am grateful to C. Angelantonj, C. Bachas, E. Kiritsis for interesting discussion and S. Stieberger for collaboration and comments. I would like to thank the organizers of the TMR meeting.

References

1. E. Witten, Nucl. Phys. B 443 (1995) 85, hep-th/ 9503124
2. J. Polchinski, E. Witten Nucl. Phys. B 460(1996) 525
3. A. Tseytlin, Nucl. Phys. B 467 (1996) 383; Phys. Lett B 367 (1996) 84
4. C. Bachas, E. Kiritsis, Nucl. Phys. Proc. Suppl. 55 B (1997) 194, hep-th/9611205
5. C. Bachas, C. Fabre, E. Kiritsis, N. A. Obers, P. Vanhove, Nucl. Phys. B 509 (1998) 33, hep-th/9707126; C. Bachas, Talk at STRINGS’97 (Amsterdam, June 16-21) and HEP-97 (Jerusalem, August 19-26), hep-th/9710102
6. E. Kiritsis and N. Obers, J. High Energy Phys. 10 (1997) 4, hep-th/9709058
7. M. Bianchi, E. Gava, F. Morales, K.S. Narain, D-String in Unconventional Type I Vacuum Configurations, hep-th/9811013
8. W. Lerche and S. Stieberger, Prepotential, Mirror Map and F-Theory on K3, hep-th/9804176; W. Lerche, S. Stieberger, N. P. Warner, Quartic Gauge Couplings from K3 Geometry, hep-th/9811228
9. I. Kostov. P. Vanhove, Matrix String Partition Function, hep-th/9809130
10. M. Bianchi, G. Pradisi, A. Sagnotti, Nucl. Phys. B 376(1992) 365
11. A. Schellekens, N. Warner, Phys. Lett B 177 (1986) 317; Phys. Lett. B 181 (1986) 339; Nucl. Phys. B 287 (1987) 87; W. Lerche, B.E.W. Nilsson and A.N. Schellekens, Nucl. Phys. B 289 (1987) 609; W. Lerche, B.E.W. Nilsson, A.N. Schellekens and N.P. Warner, Nucl. Phys. B 299 (1988) 91; W. Lerche, A.N. Schellekens and N.P. Warner, Phys. Rep. 177 (1989) 1
12. W. Lerche, Nucl. Phys. B 308 (1988) 102
13. L. Dixon, V. Kaplunovsky and J. Louis, Nucl. Phys. B 355 (1991) 649
14. K. Förger and S. Stieberger, Nucl. Phys. B 514 (1998) 135, hep-th/9709004
15. K. Förger and S. Stieberger, to appear
16. J.P. Serre, Cours d’Arithmetique, PUF Paris 1970
17. A. Terras, Harmonic Analysis on Symmetric Spaces and Applications (1985) Springer-Verlag
18. I. Antoniadis, E. Gava, K. S. Narain, T. R. Taylor, Nucl. Phys. B 455 (1995) 109
19. A. Sen, Nucl. Phys. B 475 (1996) 562
20. C. Bachas, M. Porrati, Phys. Lett B 296 (1992) 77, hep-th/9209032; C. Bachas, C. Fabre, Nucl. Phys. B 476 (1996) 418, hep-th/9605028
21. D. A. Lowe, Phys. Lett B 403 (1997) 243, hep-th/9704041; P. Horava, Nucl. Phys. B 505 (1997) 84, hep-th/9705055; S. J. Rey, Nucl. Phys. B 502 (1997) 170, hep-th/9704158; D. Kabat, S. J. Rey, Nucl. Phys. B 508 (1997) 535, hep-th/9707099
22. R. Dijkgraaf, E. Verlinde, H. Verlinde, Nucl. Phys. B 500 (1997) 43, hep-th/9703030; R. Dijkgraaf, G. Moore, E. Verlinde, H. Verlinde, Comm. Math. Phys. 185(1997) 197; E. Gava, J. F. Morales, K. S. Narain, G. Thompson, Nucl. Phys. B 528 (1998) 95, hep-th/9801128
23. I. Antoniadis, C. Bachas, C. Fabre, H. Partouche, T. R. Taylor, Nucl. Phys. B 489 (1997) 160
24. C. Kounnas, private discussion.