Weak localisation, hole-hole interactions and the “metal”-insulator transition in two dimensions

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A detailed investigation of the metallic behaviour in high quality GaAs-AlGaAs two dimensional hole systems reveals the presence of quantum corrections to the resistivity at low temperatures. Despite the low density ($r_s > 10$) and high quality of these systems, both weak localisation (observed via negative magnetoresistance) and weak hole-hole interactions (giving a correction to the Hall constant) are present in the so-called metallic phase where the resistivity decreases with decreasing temperature. The results suggest that even at high $r_s$ there is no metallic phase at $T=0$ in two dimensions.

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Since the claimed observation of metallic behaviour in strongly interacting two-dimensional (2D) systems over 5 years ago, experimentalists have tried to provide data from which an understanding of the conduction processes in high quality 2D systems can be obtained. Initial studies of these new strongly interacting systems revealed that the resistivity data can be “scaled” over a wide range of temperatures indicating the presence of a true phase transition between insulating and metallic states. Following this an empirical formula for $\rho(T)$ has been put forward which fits all the available experimental data of the metallic states. This formula describes a saturation of the resistivity as the temperature is reduced, giving a finite resistance at $T=0$, further testifying the existence of a 2D metallic state. Despite these results the nature of the metallic state and whether it really persists to the zero of temperature remains unclear. Early theoretical and experimental studies of weakly interacting systems (low $r_s$) revealed that the presence of any disorder would give rise to logarithmic corrections to the conductivity. Since these corrections become increasingly important as $T$ is reduced the question of what happens to the metallic behaviour as $T \to 0$ in 2D systems remains.

This paper reports the observation of both weak localisation and weak hole-hole interactions in the “metallic” phase of a high quality 2D GaAs hole system. First we demonstrate that the system studied here exhibits all of the characteristics previously associated with the 2D “metal”-insulator transition. Magnetoresistance measurements are then used to extract the logarithmic corrections to the Drude conductivity at low temperatures. The data show that: (1) the anomalous exponential decrease of resistivity with decreasing temperature in the metallic phase is not due to quantum interference or strong interaction effects, (2) phase coherence is preserved in the metallic regime with evidence for normal Fermi liquid behaviour, and (3) hole-hole interactions provide a localising correction to the conductivity.

The sample used here is a gated, modulation doped GaAs quantum well grown on a (311)A substrate. Four terminal magnetoresistance measurements were performed at temperatures down to 100mK using low frequency (4 Hz) ac lockin techniques and currents of 0.1-5 nA to avoid electron heating. The hole density could be varied in the range 0 - $3.5 \times 10^{11}$ cm$^{-2}$, with a peak mobility of $2.5 \times 10^5$ cm$^2$V$^{-1}$s$^{-1}$. Only the heavy hole ($|M_j|=3/2$) subband is occupied, although there is some mixing between light and heavy hole bands for $|k| > 0$.

Figure 1(a) shows the temperature dependence of the $B=0$ resistivity $\rho$ plotted for carrier densities close to the transition, from $p_c = 3.2 - 5.6 \times 10^{10}$ cm$^{-2}$. Strongly localised behaviour is observed at the lowest carrier densities, with $\rho$ taking the familiar form for variable range hopping: $\rho(T) = \rho_{\text{VRH}} \exp[(T/T_{\text{VRH}})^{-m}]$, with $m=1/2$ far from the transition and $m=1/3$ close to the transition. A transition from insulating to metallic behaviour occurs as the carrier density is increased, with a critical density of $p_c = 4.6 \times 10^{10}$ cm$^{-2}$ ($r_s = 12$) at the transition. Above this critical density the resistivity drops markedly as the temperature is reduced, although it is difficult to see this drop on the logarithmic axis of Fig. 1(a).

The metallic behaviour can be seen more clearly in the “scaled” data shown in Fig. 1(b). Each $\rho(T)$ trace was individually scaled along the $T$-axis in order to collapse all the data onto one of two separate branches. It has been suggested that the ability to scale the data both in the strongly localised and metallic branches is evidence for a phase transition between insulating and metallic states in a 2D system. More recently Pudalov et al. have shown that $\rho(T)$ in the metallic regime is well described by:

$$\rho(T) = \rho_0 + \rho_1 \exp[-T_o/T]$$

Fig. 1(c-e) shows the temperature dependent resistivity for three different carrier densities in the metallic regime, with fits to Eqn. (1) shown as dashed lines. In Fig. 1(c), at a density close to the transition, saturation of the resistivity is just visible at the lowest temperatures (100mK). As the density is increased and we move further into the metallic regime this saturation becomes visible at higher temperatures until at the highest density $\rho(T)$ saturates below 350mK. The empirical formula
(1), which characterises the metallic behaviour observed in all 2D systems therefore dictates a saturation of $\rho(T)$ as $T \to 0$. Although different from the scaling analysis of Ref. [1] and shown in Fig. 1(b), it is still consistent with the existence of a 2D metal-insulator transition because $\rho(T)$ remains finite as $T \to 0$. Early studies of weakly interacting, disordered 2D systems ($r_s \sim 4$) demonstrated that both weak localisation and weak electron-electron interactions caused a logarithmic reduction of the conductivity as $T \to 0$. More recently it has been shown that the same interaction effects occur in slightly less disordered samples ($r_s \sim 6$) that exhibit “metallic behaviour”, at high carrier densities, far from the metal-insulator transition. However neither the scaling analysis in Fig. 1(b) nor the empirical Eqn. (1) address what has happened to these logarithmic corrections near the metal-insulator transition, and whether the conductivity remains finite as $T \to 0$.

We now turn to one of two main results of this paper. Figure 2 shows the temperature dependence of the $B=0$ resistivity (left hand panel) and magnetoresistance (right hand panel) at different densities on both sides of the “metal”-insulator transition. In Fig. 2(a) we are just on the insulating side of the transition. The left hand panel shows that $\rho(T)$ is essentially $T$-independent down to 300mK and then increases by 2.5% as the temperature is further reduced. This weak increase in the resistivity has been previously taken as evidence for weak localisation and weak electron-electron interaction effects. It is however not possible to determine the precise origins of this weak increase in resistivity solely from the $B=0$ data, and we therefore look at the magnetoresistance shown in the right hand panel of Fig. 2(a). A characteristic signature of weak localisation is a strong temperature dependent negative magnetoresistance, since the perpendicular magnetic field breaks time reversal symmetry, removing the phase coherent back-scattering. As observed previously there is no evidence of weak localisation for temperatures down to 300mK in these high quality samples. However, as $T$ is lowered below 300mK a strong negative magnetoresistance peak develops as phase coherent effects become important, mirroring the small increase in the resistivity at $B=0$.

Increasing the carrier density brings us into the metallic regime (Fig. 2(b)) where the exponential drop in the resistivity with decreasing temperature predicted from Eqn. (1) starts to become visible. The upturn in $\rho(T)$ marked by the arrow has moved to lower temperatures and the negative magnetoresistance in the right hand panel has become less pronounced. Further increasing the density (Fig. 2(c)) causes the metallic behaviour to become stronger, with the upturn in $\rho(T)$ moving to even lower temperatures, until at $p_s = 5 \times 10^{10}$ cm$^{-2}$ the upturn is no longer visible within the accessible temperature measurement range. However, the magnetoresistance still exhibits remnants of the weak localisation temperature dependent peak at $B=0$. The weak localisation is therefore always present and is neither destroyed in the metallic regime, nor is it “swamped” by the exponential decrease in resistivity with decreasing temperature. Instead what can clearly be seen in the left hand panel of Fig. 3, is that the upturn in $\rho(T)$ due to weak localisation marked by the arrows, moves to lower $T$ as the carrier density is increased. This is not surprising since as we move further into the metallic regime both the conductivity and therefore the mean free path ($l \propto \sigma/\sqrt{\rho_s}$) increase, such that the weak localisation corrections are only visible at lower temperatures (larger $l_0$).

In contrast to experimental studies of high carrier density hole gas quantum well samples there are no signs of weak anti-localisation in these low density samples. This is perhaps to be expected since recent theoretical work has predicted that the magnetoresistance behaviour is determined by the degree of heavy-hole/light-hole mixing at the Fermi energy, which is characterised by the parameter $k_F a/\pi$, where $a$ is the width of the quantum well. In our sample the carrier concentration is small, such that $k_F a/\pi \ll 1$, and only negative magnetoresistance is expected.

In Fig. 3 (a) we fit the temperature dependent magnetoconductance data to the formula of Hikami et al.:

$$\Delta \sigma(B) = \frac{-e^2}{\pi h} \left[ \Psi \left( \frac{1}{2} + \frac{B_\phi}{B} \right) - \Psi \left( \frac{1}{2} + \frac{B_0}{B} \right) \right]$$  \hspace{1cm} (2)

where $\Psi(x)$ is the digamma function, $B_0$ and $B_\phi$ are characteristic magnetic fields related to the transport scattering rate and the phase relaxation rate. We obtain $\sigma_{xx}$ by matrix inversion of $\sigma_{xx}$ and $\rho_{xy}$. Using Eqn. (2) we fit the experimental data just on the metallic side of the transition ($p_s = 4.7 \times 10^{10}$ cm$^{-2}$) for different temperatures as shown in Fig. 3(a). These fits are in good agreement with the experimental data, and from this it is possible to extract the fitting parameter $B_\phi$ and thus the phase relaxation time $\tau_\phi$.

Figure 3(b) shows the temperature dependence of the phase breaking rate, $1/\tau_\phi$, for three different densities on both sides and close to the “metal”-insulator transition. The phase breaking rate falls approximately linearly with decreasing temperature for all three traces. The linear dependence agrees well with that predicted for disordered enhanced hole-hole scattering, where $1/\tau_\phi \sim 2k_B T/(\hbar k_F l)$. This phase breaking mechanism should only depends on $k_F l$ and not on the carrier density, mobility or interaction strength. It is therefore particularly noteworthy that the phase breaking rates in these low density p-GaAs samples, with $2.5 < k_F l < 5$, are almost identical to those found in n-type silicon MOSFETs, with $k_F l \sim 1$, despite a factor of 20 difference in the carrier densities (see data in Fig. 3(b)). This agreement with scattering limited electron lifetime suggests that the electron states are only mildly perturbed by the strong interactions and essentially remain Fermi liquid like.

Another important feature of these results is that there is little variation in $\tau_\phi$ with density and in particular there is no dramatic change in $\tau_\phi$ as we cross from insulating ($p_s = 4.5 \times 10^{10}$ cm$^{-2}$) to metallic behaviour.
There is therefore no reflection of the exponential decrease of $\rho(T)$ with decreasing temperature in the phase breaking rate. This implies that whatever mechanism is causing metallic behaviour does not suppress weak localisation as originally believed and is further evidence that the system is behaving as a Fermi liquid. Since all models of the resistivity in the metallic phase predict that the exponential drop saturates at low temperatures, our data shows that localisation effects will again take over as $T \to 0$.

Finally we address the role of electron-electron (hole-hole) interactions in the 2D metallic phase - the second important result from this paper. Unlike weak localisation, interactions not only affect the $B = 0$ resistivity, but also cause a correction to the zero field resistance:

$$\frac{\Delta R_H}{R_H} = -2 \frac{\Delta \sigma_I}{\sigma}$$

By measuring the low field Hall effect it is thus possible to distinguish between weak localisation and interaction effects. Figure 4(a) shows the Hall resistivity $\rho_{xy}$ measured on the metallic side of the transition (i.e. where the zero field resistivity shows an exponential drop with decreasing temperature as shown in Fig. 2(c)). The data reveal a small, but significant, decrease of the Hall slope with increasing temperature. Whilst a series of different temperatures traces from 100-700mK were taken only three of these traces are presented for clarity. Upon closer investigation this small decrease of the Hall slope is found to vary as $\log(T)$. We extract the interaction correction to the zero field conductivity, $\Delta \sigma_I$ from the temperature dependent Hall data using equation (3). Figure 4(b) shows a plot of the interaction correction for different carrier densities on both sides of the transition. All the data collapses onto a single line, clearly demonstrating a $\log(T)$ dependence of $\Delta \sigma_I$, which reduces the conductivity to zero as $T \to 0$.

Logarithmic corrections to the Hall resistivity have previously been observed in studies of interaction effects in high density electron systems. It is perhaps surprising that results observed in, and derived from, weakly interacting systems apply to our system where interactions are strong and $r_s > 10$. Nevertheless, we find reasonable agreement between the magnitude of the logarithmic corrections due to interactions in our system and those predicted by Altshuler et al. (within a factor of 2). As with the phase coherent effects this logarithmic correction due to hole-hole interactions is independent of whether we are in the insulating or metallic phase and is present despite the exponential drop in resistivity. This is the first proof that electron-electron interactions are not responsible for the 2D “metal”-insulator transition observed in high mobility (low $E_F$) systems.

In summary we have presented a comprehensive study of localisation and interaction effects in a high mobility two-dimensional hole gas sample that shows all the signatures of a $B = 0$ “metal”-insulator transition. The results clearly demonstrate that neither phase coherent effects nor electron-electron interactions are responsible for the apparent 2D metal-insulator transition. Both of these effects are present in the metallic regime and both give rise to localising corrections to the conductivity at low temperatures. The importance of phase coherent effects in studies of the metallic behaviour in high mobility 2D systems has not previously been recognised because the mean free path in these systems is large, so that weak localisation is only observable at very low, often inaccessible, temperatures. Nevertheless we clearly observe negative magnetoresistance characteristic of weak localisation in the so-called metallic phase. There is no suppression of phase coherent effects in the metallic regime. Instead the weak localisation simply moves to lower temperatures as we go further into the metallic regime. Most importantly we demonstrate that the metallic behaviour is not due to hole-hole interactions, because these also cause a logarithmic localising correction to the conductivity. These results strongly suggest that the metallic behaviour must be a finite temperature effect, and that as $T \to 0$ the old results of scaling theory and weak electron-electron interactions remain valid - there is no genuine 2D metallic phase.

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FIG. 1. (a) Temperature dependence of the resistivity at densities from $p_s = 3.2 - 5.6 \times 10^{10}$ cm$^{-2}$. (b) The same data, scaled to collapse onto metallic and insulating branches. (c) Temperature dependence of the resistivity in the metallic regime at carrier densities of 4.8, (d) 8.6 and (e) $17 \times 10^{10}$ cm$^{-2}$. The dashed lines show the fit to Eqn. (1).

FIG. 2. (a-d) The left hand panels show resistivity at $B=0$ versus temperature data, illustrating the transition from insulating to metallic behaviour as the density increases. The right hand panels show the corresponding magnetoresistance traces for temperatures of 147, 200, 303, 510, 705 and 910mK.
FIG. 3. (a) The magnetoconductivity, $\sigma_{xx}$, just on the metallic side of the transition for temperatures of 147, 200, 303, 510 and 705 mK. (b) A plot of $1/\tau_\phi$ versus temperature for densities close to the “metal”-insulator transition. Solid symbols are data obtained from this study; open symbols are data from Si MOSFETs, Ref. [15]

FIG. 4. (a) Hall resistivity for different temperatures at a density of $4.7\times10^{10}$ cm$^{-2}$ (b) the logarithmic correction to the conductivity for densities close to the transition.