Effect of optomechanical coupling on squeezed-spin states

N Aggarwal\textsuperscript{1}, B Joshi\textsuperscript{2} and A B Bhattacheryee\textsuperscript{3}

\textsuperscript{1}Department of Physics and Astrophysics, University of Delhi, Delhi-110007, India
\textsuperscript{2}Department of Science & Technology, Technology Bhavan, New Mehrauli Road, New Delhi – 110016, India
\textsuperscript{3}School of Physical Sciences, Jawaharlal Nehru University, New Delhi-110067, India

E-mail: bhattach@mail.jnu.ac.in

Abstract. We investigate the effect of optomechanical coupling on the squeezed-spin states for a Bose-Einstein Condensate embedded within the lossless optomechanical cavity for the three special cases of initial states of cavity field, namely, a coherent state, a squeezed vacuum state and a squeezed state. We show that the radiation pressure or pondermotive force acting on the cavity end mirror plays a significant role in producing the atomic-squeezed states by producing squeezed states of the cavity field which is then transferred to the condensate. We further show that the maximum spin-squeezing along the x-direction is obtained in the presence of optomechanical coupling for the initial cavity field prepared in the amplitude squeezed state, whereas, squeezing along the y-direction reaches a maximum value in the absence of optomechanical coupling for the initial coherent cavity field. We also study the additional effect of nonlinear atomic interaction on spin-squeezing.

1. Introduction
Spin squeezing [1–3] has attracted considerable attention in both the theoretical and experimental zones for over a decade. In order to improve the measurements precision in experiments [2–5] and to study the correlations and entanglement between particles [6–8], spin squeezed states are considered to be a very useful quantum resource. The phenomenon of spin-squeezing in collective spin system has also grabbed great interest due to its application in atomic clocks for reducing quantum noise [2, 3, 9] and quantum information [6, 10]. Note that different definitions of spin squeezing can be used and the most widely used spin squeezing parameters were proposed by Kitagawa and Ueda in [1] and by Wineland in [2, 3]. For decades, squeezed states of electromagnetic field have also been the very popular topic of research [11–13]. It can be produced by exploiting the optomechanical coupling between the light and movable mirror, which is also termed as ponderomotive squeezing [14]. Pondermotively generated squeezed light has been studied both theoretically [14] and experimentally [15,16]. Furthermore, spin squeezing in atomic ensembles can be produced via interaction between light and the atom, which involves the transferring of squeezing from light to atoms [3, 17]. The production of atomic-squeezed states in a two-component Bose-Einstein Condensates (BECs) via nonlinear atom-atom interaction has also been studied [6, 18, 19]. The spin-squeezed states in a BEC can be experimentally realized [18,20], which can be used to detect weak forces [21]. Motivated by these interesting developments in this field, we propose an optomechanical system consisting of an elongated cigar-shaped gas of two-level BEC atoms interacting with a single mode of a lossless optical

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cavity with one movable mirror. We study this model to generate the atomic-squeezed states in both the x- and y- directions for the three special cases of initial states of cavity field, namely, a coherent state, a squeezed vacuum state and a squeezed state. We further discuss how the pondermotive force acting on the end mirror as a result of optomechanical coupling helps in generating atomic-squeezed states and how this squeezing can be coherently controlled by the light-mirror coupling. We also show the additional effect of nonlinear atom-atom interaction on spin-squeezing.

2. The Model
We consider an optomechanical system consisting of a lossless Fabry-Perot optical cavity with one heavy fixed mirror and another highly reflecting end mirror which is light and allowed to move freely, as shown in Figure 1. In addition, we have an elongated cigar-shaped gas of \( N \) BEC atoms of \( ^{87}\text{Rb} \) having two different hyperfine levels \( |F=1,m_f=-1\rangle \) and \( |F=2,m_f=1\rangle \) with mass \( m \) and transition frequency \( \omega_a \). In the two-mode approximation, each atomic mode is associated with an annihilation operator \( c_j \) \((j=1,2)\). The cloud of BEC atoms interacts with a single quantized optical cavity mode having frequency \( \omega_c \) of an optical cavity of length \( L \). A single-mode quantized cavity field is equivalent to a single quantum-mechanical harmonic oscillator of unit mass. The external pump laser drives the optomechanical system through the fixed cavity mirror. Note that the movable mirror is considered as a single quantum-mechanical oscillator of frequency \( \omega_m \) and mass \( M \), which can be realized experimentally by using a bandpass filter in the detection loop such that the other mechanical degrees of freedom arising from the radiation pressure can be neglected [22]. The simplest model of such an optomechanical system is provided by the following Hamiltonian in the dipole approximation \((\hbar=1\) throughout the paper) [23,24]:

\[
H = \omega_a J_z + \omega_c a^+ a + \omega_m b^+ b + \frac{\gamma}{N} J_z^2 + \frac{g_0}{\sqrt{N}} (a + a^+) J_x + \varepsilon a^+ a (b + b^+) ,
\]

(1)

Figure 1. (color online) Schematic representation of the system involving Bose-Einstein Condensate inside a lossless optical cavity driven by an external field. Here, one of the cavity mirrors is movable.

The ensemble of \( N \) atoms is described using the picture of a collective spin operators such that

\[
J_x = \frac{c_1^+ c_2 + c_2^+ c_1}{2}, \quad J_y = \frac{c_1^+ c_2 - c_2^+ c_1}{2i} \quad \text{and} \quad J_z = \frac{c_1^+ c_1 - c_2^+ c_2}{2} \quad [25],
\]

satisfying the commutation relations \( [J_x,J_y] = 2J_z \) and \( [J_x,J_z] = \mp J_y \) where \( J_x = c_1^+ c_2 \) and \( J_z = c_1^+ c_1 \). The Hilbert space is spanned by symmetric Dicke states \( |J,M\rangle \) where \( M = -J,-J+1,...,J-1,J \) with the total spin length \( J = N/2 \) [26]. Moreover, the operators act on these states as

\[
J_\pm |J,M\rangle = \sqrt{J(J+1)-M(M \pm 1)} |J,M \pm 1\rangle, \quad J_\pm |J,M\rangle = M |J,M\rangle \quad \text{and}
\]
\[ J^2 | J, M > = J(J+1) | J, M > \]. The lowering (raising) operator is given by \( a (a^\dagger) \) such that \([a, a^\dagger] = 1\). The annihilation and creation operators for the mechanical oscillator are denoted by \( b \) (\( b^\dagger \)) satisfying the commutation relation \([b, b^\dagger] = 1\). The effective nonlinear atomic interaction strength and atom-photon coupling strength are represented by \( \gamma \) and \( g_0 \) respectively. Origin of last term in the Hamiltonian is the result of radiation pressure exerted by the intra-cavity photons on the mirror such that the nonlinear dispersive coupling between the cavity field and the oscillating mirror is denoted by \( \varepsilon \).

The optomechanical Hamiltonian given by equation (1) can be rewritten in the interaction picture as [27]:

\[
H = \frac{\gamma}{N} J_x^2 + \varepsilon a^\dagger a (b e^{-i\omega_0 t} + b^\dagger e^{i\omega_0 t}) + \frac{g_0 a}{\sqrt{N}} [J_x e^{i(\omega_0 - \omega_1) t} + J_x e^{-i(\omega_0 + \omega_1) t}] \\
+ \frac{g_0 a^\dagger}{\sqrt{N}} [J_x e^{i(\omega_0 + \omega_1) t} + J_x e^{-i(\omega_0 - \omega_1) t}].
\]

(2)

Now, we treat the mechanical mode operator \( b \) semiclassically so that it can be replaced by its real scalar quantity \( \beta \) with \(|\beta| > 1\). Therefore, for \( \omega_a = \omega_x \), the resulting simplified Hamiltonian in the rotating wave approximation becomes:

\[
H = \frac{\gamma}{N} J_x^2 + g(t) a^\dagger a + \frac{g_0 a}{\sqrt{N}} [a J_x + a^\dagger J_x]
\]

(3)

where \( g(t) = 2\varepsilon \beta \cos(\omega_0 t) \) is the modified mirror-photon coupling, which is related to the time modulated frequency of the cavity arising due to the motion of the cavity boundary [28]. In the next section, we study the generation of spin squeezing when an ensemble of BEC atoms held within the optomechanical cavity interacts with the single-mode quantized cavity field (harmonic oscillator) through the interaction Hamiltonian (3). We first focus our attention in studying the effect of pondermotive force on spin-squeezing, i.e., \( \gamma = 0 \). Later we discuss the effect of atom-atom interaction (\( \gamma \)) on atomic-squeezing.

3. Preparation of atomic-squeezed states

Here, we study the preparation of spin-squeezed states for some special cases of initial cavity field by numerically solving the Schrodinger’s equation for the interaction Hamiltonian. To assess the effect of pondermotive squeezing on the generation of spin-squeezing, we first discuss the situation of an initially coherent intracavity field and then that of an initially squeezed vacuum and squeezed cavity field. The collective spins is assumed to be initially prepared in the Dicke state \(|J, M > = -J >\) with \( M = \langle J_x > \). Also, \( < J_x(0) > = < J_x > = 0, \Delta J_x(0) = \Delta J_y(0) = J/2 \) and \( \Delta J_z(0) = 0 \). The square root of the variance of any operator \( \Delta \) is denoted by \( \Delta \Delta = \sqrt{<A^2> - <A>^2} \). The position and momentum quadratures of the single-mode quantized cavity field (harmonic oscillator) can be written in terms of cavity field operators \( a \) and \( a^\dagger \) as \( x(t) = [a(t) + a^\dagger(t)]/\sqrt{2} \) and \( p(t) = [a(t) - a^\dagger(t)]/i\sqrt{2} \) respectively. We can characterize the squeezing of the harmonic oscillator by the parameters \( \chi_x(t) = \Delta x(t)/x_0 \) or \( \chi_p(t) = \Delta p(t)/x_0 \), with \( x_0 [x_0 = \Delta x \) (coherent state)] denoting the zero-point amplitude. The condition \( \chi_x < 1 \) represents amplitude squeezing and \( \chi_p < 1 \) represents momentum squeezing.

The squeezing parameter that we are considering here is given by Kitagawa and Ueda [1] and is defined as \( \zeta_s(t) = (\sqrt{2/J}) \Delta J_\perp \), where the subscript \( \perp \) refers to an axis perpendicular to the mean.
angular momentum $<J>$ where the minimum value of $\Delta J$ is obtained. The parameter $\zeta_S(t)$ indicates the degree of quantum correlations among the elementary spins and $\zeta_S < 1$ signifies atomic squeezing. In this view, the squeezing parameters considered are defined as follows:

Figure 2. (color online) Squeezing parameter $\zeta_S(t)$ versus scaled time ($\omega_m t$) in absence of cavity-mirror coupling $\epsilon = 0$ (solid line) and for two different values of mirror-photon coupling with $\epsilon = 0.2 \omega_m$ (dashed line) and $\epsilon = 0.4 \omega_m$ (dot dashed line) by assuming $|\psi(0)\rangle \approx |J, -J \rangle$ ($J = 10$) and the oscillator to be initially prepared in a coherent state with an average of one quanta where $<x(0)> \neq 0$, $<p(0)> \neq 0$ and $\chi_0(0) = \chi_p(0) = 1$.

(a): Plot of $\zeta_{S_x}(t)$ as a function of time. (b): Plot of $\zeta_{S_y}(t)$ as a function of time. The other parameters used are $g_0 = 0.8 \omega_m$, $\gamma = 0$ and $\beta = 10$.

Figure 3. (color online) Squeezing parameter $\zeta_S(t)$ versus scaled time ($\omega_m t$) in absence of cavity-mirror coupling $\epsilon = 0$ (solid line) and for two different values of mirror-photon coupling with $\epsilon = 0.2 \omega_m$ (dashed line) and $\epsilon = 0.4 \omega_m$ (dot dashed line). We assume $|\psi(0)\rangle \approx |J, -J \rangle$ ($J = 10$) and the oscillator is initially prepared in an amplitude squeezed vacuum state where $<x(0)> \approx 0$, $<p(0)> \approx 0$ and $\chi_0(0) = \chi_p(0) = 0.3807$. (a): Plot of $\zeta_{S_x}(t)$ as a function of time. (b): Plot of $\zeta_{S_y}(t)$ as a function of time. The other parameters used are $g_0 = 0.8 \omega_m$, $\gamma = 0$ and $\beta = 10$. 
\[ \xi_{s,x}(t) = \sqrt{\frac{2}{J}} \Delta J_x, \quad (4) \]
\[ \xi_{s,y}(t) = \sqrt{\frac{2}{J}} \Delta J_y. \quad (5) \]

The state vector for the combined spin ensemble held within the optomechanical cavity and the single-mode quantized cavity field can be given as:

\[ |\Psi_1(t)\rangle = \sum_{n,M} C_{n,M}(t) |n\rangle |M\rangle, \quad (6) \]

where we use \( |M\rangle > |J, M\rangle \) since \( J \) is a constant of motion for our Hamiltonian and \( C_{n,M}(t) \) represent the time-dependent coefficients. The harmonic-oscillator eigen states are represented by \( |n\rangle \) with \( a^\dagger |n\rangle = \sqrt{n+1} |n+1\rangle \). Furthermore, the harmonic oscillator and BEC are assumed to be uncorrelated for the initial state of the system such that the state vector can be written as a direct product

\[ |\Psi_1(0)\rangle = \left( \sum_n C_n(0) |n\rangle \right) |\psi(0)\rangle, \quad (7) \]

where, \( |\psi(0)\rangle = |J = N/2,M = -N/2\rangle \). The initial harmonic oscillator number state coefficients are given by \( C_n(0) \) [29]. The basis of harmonic-oscillator states are also to be truncated of course by determining an adequate number of states until the result is not changing. The equation of motion for Hamiltonian (3) is evaluated using Schrodinger’s equation and is given as:

\[
\dot{C}_{n,M}(t) = \frac{i}{\hbar} \left[ C_{n,M}(t) \right] + \frac{g_0}{2\sqrt{N}} \left[ \sqrt{n} \sqrt{J(J+1)-M(M+1)} \right] C_{n-1,M+1}(t) \\
+ \frac{g_0}{2\sqrt{N}} \left[ \sqrt{n+1} \sqrt{J(J+1)-M(M-1)} \right] C_{n+1,M-1}(t). \quad (8)
\]

The value of \( J \) is considered to be 10. Interaction Hamiltonian couples only those states where \( n+M \) is always a constant. The time-dependent state vector for the system can be written as a sum over all the possible eigen states. In order to obtain \( |\Psi_1(t)\rangle \), time-dependent coefficients \( C_{n,M}(t) \) are evaluated using equation (8) with the help of Mathematica 9.0. Thus, squeezing parameters defined in equations (4) and (5) can be evaluated with time using this state vector of the system. Experimentally, the frequency of mechanical oscillator may vary from \( 2\pi \times 100\text{Hz} \) [30], \( 2\pi \times 10\text{kHz} \) [31] to \( 2\pi \times 73.5\text{MHz} \) [32]. In an optomechanical system, the mirror-photon coupling rate can take the values as low as \( 2\pi \times 2.7\text{Hz} \) [33] to as high as \( 2\pi \times 4 \times 10^5\text{Hz} \) [34]. The intracavity field interacting with the cloud of BEC held within the optomechanical cavity may have a coherent coupling strength of \( 2\pi \times 5.86\text{kHz} \) [35] (2\( \times \) 10.9MHz [36]). Moreover, in our calculations, if the frequency of mechanical resonator is varied from \( 2\pi \times 100\text{Hz} \) to \( 2\pi \times 10\text{kHz} \), then, the time will come out to be nearly of the order of milliseconds. There have been many experimental and theoretical papers in which the time evolution of spin-squeezing parameter is usually taken to be small (nearby of the order of milliseconds) [3, 19, 37–39]. At short times, it should be possible to avoid the effects of damping and thermal noise such that the substantial amount of spin-squeezing can be achieved at an earlier time.

In figure 2, we plot \( \xi_s(t) \) versus scaled time (\( \omega_n t \)) in the absence of cavity-mirror coupling \( \varepsilon = 0 \) (solid line) and for two different values of mirror-photon coupling \( \varepsilon, \varepsilon = 0.2\omega_n \) (dashed line) and \( \varepsilon = 0.4\omega_n \) (dot dashed line). Here, we assume the harmonic oscillator to be initially prepared in a coherent state. Figure 2(a) shows that even though we start with a coherent state, squeezing along the x-direction increases significantly with the increase in light-mirror coupling. This is due to the fact that
the radiation pressure force acting on the end mirror as a result of coupling between the optical field and the mirror motion generates squeezing in harmonic oscillator, known as pondermotive squeezing \[14\], which is then transferred to the condensate atoms via interaction Hamiltonian. However, spin-squeezing along the y-direction deteriorates with the increase in mirror-photon coupling and shows the maximum amount of squeezing for \( \varepsilon = 0 \) (see figure 2(b)). Thus, squeezing in spins along both the x- and y- directions can be significantly altered by choosing the appropriate value of optomechanical coupling. Further, we observe that spins in the x-direction remains squeezed for sufficiently large times (persistent spin-squeezing), with relatively much higher degree of persistent spin-squeezing for \( \varepsilon = 0.4 \omega_m \).

**Figure 4.** (color online) Squeezing parameter \( \zeta_x(t) \) versus scaled time \( \omega_m t \) in absence of cavity-mirror coupling \( \varepsilon = 0 \) (solid line) and for two different values of mirror-photon coupling with \( \varepsilon = 0.2 \omega_m \) (dashed line) and \( \varepsilon = 0.4 \omega_m \) (dot dashed line) by assuming \( |\psi(0)\rangle = |J, -J > \) (\( J = 10 \)) and the oscillator to be initially prepared in an amplitude squeezed state where \( <x(0)> \neq 0 \), \( <p(0)> = 0 \) and \( \chi_x(0) = 0.3904 \). (a): Plot of \( \zeta_{S_x}(t) \) as a function of time. (b): Plot of \( \zeta_{S_y}(t) \) as a function of time. The other parameters used are \( g_0 = 0.8 \omega_m \), \( \gamma = 0 \) and \( \beta = 10 \).

**Figure 5.** (color online) Squeezing parameter \( \zeta_x(t) \) versus scaled time \( \omega_m t \) in the absence of optomechanical coupling \( \varepsilon = 0 \) for two different values of nonlinear atomic interaction strength with \( \gamma = 0.2 \omega_m \) (solid line) and \( \gamma = 0.4 \omega_m \) (dashed line) by assuming \( |\psi(0)\rangle = |J, -J > \) (\( J = 10 \)) and the oscillator to be initially prepared in a coherent state with an average of one quanta where \( <x(0)> \neq 0 \), \( <p(0)> = 0 \) and \( \chi_x(0) = \chi_p(0) = 1 \). (a): Plot of \( \zeta_{S_x}(t) \) as a function of time. (b): Plot of \( \zeta_{S_y}(t) \) as a function of time. The other parameters used are \( g_0 = 0.8 \omega_m \) and \( \beta = 10 \).
Figure 3 illustrates the time evolution of spin-squeezing parameter for the oscillator to be initially prepared in an amplitude squeezed vacuum state. As before, we observe that higher value of mirror-photon coupling favors spin-squeezing in the x-direction while lower value of $\varepsilon$ favors spin-squeezing along the y-direction. However, no squeezing along x-direction is observed in the absence of cavity-mirror coupling. Further note that the persistent squeezing along x-direction is observed for high $\varepsilon$, whereas, persistent and high squeezing is attained in the y-direction for $\varepsilon = 0$. We further show the variation of $\zeta_S(t)$ with time in figure 4 by considering the oscillator to be initially prepared in an amplitude squeezed state. These figures illustrate a similar kind of behavior as in the previous cases but more important is the fact that, in the presence of optomechanical coupling, initially squeezed state is able to produce a higher degree of persistent spin-squeezing in the x-direction as compared to previous cases. This is due to the pondermotive force acting on the movable mirror (cavity boundary) that helps in producing further squeezing in cavity mode, which gets drained away and then transferred to the spins. However, maximum squeezing along y-direction is attained in the absence of optomechanical coupling for the initial coherent state. This implies that the atomic spin-squeezing along both the x- and y- directions can be coherently controlled by the mirror-photon coupling. In principle, it is evident that spin-squeezing in two-level atoms can also be generated from nonlinear atomic interaction [6, 18, 19]. Thus, keeping other parameters same as before, we have also examined the effect of nonlinear atom-atom interaction $\gamma$ on spin-squeezing in the absence of optomechanical coupling $\varepsilon = 0$ for the initial coherent cavity field (figure 5). We have observed a respective decrease and increase in the value of squeezing parameter $\zeta_{S,x}(t)$ and $\zeta_{S,y}(t)$ with increase in $\gamma$. In a realistic optomechanical system, decay of the cavity mode and thermal noise acting on the mechanical resonator will deteriorate the pondermotive squeezing and consequently the atomic spin-squeezing. However, significant squeezing can still be obtained if pondermotive interaction predominates over thermal noise [40], which can be achieved by using a large enough cavity finesse and mechanical quality factor such that substantial squeezing can still be achieved during the initial dynamics.

4. Conclusion
We have shown the effect of mirror-photon coupling on the generation of squeezed-spin states for a BEC confined in a lossless optomechanical cavity. In particular, we have studied the squeezing in spins for the three special cases- initial cavity field in coherent state, squeezed vacuum state and squeezed state. We have observed that the spin-squeezing in both the x- and y- directions can be coherently controlled by choosing the appropriate value of optomechanical coupling. Persistent spin-squeezing can also be generated by the proper choice of the system parameters. The pondermotive force acting on the movable mirror as a result of coupling between the optical field and the movable mirror generates squeezing in the optical cavity field which gets transferred to the condensate atoms due to a Tavis-Cummings type interaction and results in squeezed-spin states. We found that, even if the initial state of the cavity field is coherent, still a high degree of squeezing is generated and transferred to the atoms, with maximum squeezing along the y-direction in the absence of optomechanical coupling. Maximum amount of spin-squeezing along the x-direction is observed in the presence of optomechanical coupling for the initial cavity field prepared in the amplitude squeezed state. Such correlated-particle states have applications in entanglement detection as well as high precision metrology.

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