Realizing flavored leptogenesis: a reappraisal through special kinds of orthogonal matrices

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ABSTRACT: The parameterisation proposed by Casas and Ibarra in the year 2001 have shown promising role in the extraction of neutrino Yukawa coupling which is a basic ingredient of the seesaw mechanism generating neutrino mass. We pay special attention in establishing the crucial role of the Casas-Ibarra (CI) parameterisation in presence of two different orthogonal matrices, $R = O e^{iA}$ and $R = O e^A$ in order to investigate flavored leptogenesis. In the light of these two choices of the orthogonal matrix we examine the connection between the low energy and high energy CP violations along with certain interesting predictions on the low energy parameters namely, the lightest neutrino mass and the Dirac CP phase ($\delta$). Considering the right handed neutrino (RHN) mass window to be $10^8$ GeV, we show that Dirac phase leptogenesis is possible with the choices of these two orthogonal matrices. We choose a nearly degenerate spectrum for the RHN masses for having a successful leptogenesis. We also emphasize on presenting a range of the matrix elements of the skew symmetric matrix $A$. The results obtained in the present analysis underline the importance of understanding the status of CP violation in the low energy sector. We also discuss the phenomenological implications of these two case studies in the context of LFV considering the $\mu \rightarrow e\gamma$ decay process.
1 Introduction

The observation of neutrino oscillation stands for one of the windows facing towards new physics beyond the Standard Model (SM). In fact it is essentially the one found in the laboratory that has been established beyond doubt, thanks to the neutrino oscillation experiments [1–9]. In order to explain this oscillation phenomenon and it’s associated consequences it is imperative to include the right handed counter part of the SM neutrino to the SM fermion sector, which leads to make the SM neutrinos massive through the type-I seesaw mechanism. One appealing feature of the classical seesaw mechanism is that it can explain the observed baryon asymmetry of the Universe (BAU) via the mechanism of leptogenesis, as pointed out by Fukugita and Yanagida [10]. The predominance of matter over antimatter has been evidenced by many experimental observations [11, 12]. This excess of matter in the present Universe is quantified by a quantity called baryon to photon ratio ($\eta_B$), which has been reported by the recent Planck satellite experiments as [12, 13]

$$\eta_B = \frac{n_B - n_{\overline{B}}}{n_\gamma} = (6.02 - 6.18) \times 10^{-10}. \quad (1.1)$$
where, $n_B$, $n_{\bar{B}}$ and $n_\gamma$ are respectively the number densities of baryons, anti-baryons and photons.

Leptogenesis is a mechanism by which some lepton-number-violating processes [14–17] produce a lepton asymmetry which is subsequently converted into a baryon asymmetry through the non-perturbative $B + L$ violating but $B - L$ conserving sphaleron processes of the SM [18, 19]. Based on the type-I seesaw [15, 20–22] there has been proposed numerous frameworks (see for instance Ref. [23–31]) which are motivated by the studies of neutrino mass and a possible explanation for the BAU through the process of leptogenesis. Based on the temperature regime at which leptogenesis is supposed to take place, there exist several field theoretically consistent frameworks which in particular deal with the characteristic features of lepton asymmetry generation at a certain temperature [32–36]. Thermal leptogenesis is motivated by the scenario of lepton asymmetry production at a comparatively higher temperature which is in agreement with the type-I seesaw scale ($\mathcal{O}(10^{15})$). However, such a scenario predicts the RHN mass to be close to the GUT scale, which is difficult to probe at the LHC. In this context the low scale seesaw models can in principle explain the BAU through leptogenesis by the decay of a low scale (typically TeV scale) RHN. At such low scale the lepton asymmetry receives a resonant enhancement, a scenario termed as resonant leptogenesis which can be found in Refs. [37, 38].

As the lepton asymmetry parameter solely depends on the Dirac Yukawa coupling matrix, it is essential to construct the same containing the low energy observables explaining neutrino mass and mixing. Such Yukawa coupling matrix can be obtained with the help of a well known formalism known as Casas-Ibarra (CI) parameterisation [39]. There are various parameterisations available in the literature [39, 40]. Among them the most widely used parameterisation is the one mentioned in [39]. One aesthetic feature of such parameterisation is that, it can bridge the low energy inputs with high energy observations such as the estimation of the BAU. The Casas-Ibarra parameterisation introduces an orthogonal matrix (generally denoted as $R$) which in general play a key role in providing the high energy CP violation, while computing the CP asymmetry through leptogenesis. In principle this matrix is chosen to be complex by nature, considering the rotation angles to have both real and imaginary parts $^1$. The nature of this $R$ matrix can in principle play a decisive role in the context of leptogenesis and LFV as extensively studied in Ref. [53, 56]. Apart the influence of the orthogonal matrix on leptogenesis or LFV related calculations, there exist one more fascinating direction which is, it provides the connection between the low energy and high energy CP violation. In this regard there has been plenty of studies where such connection is discussed for different regimes of leptogenesis which solely depends on the RHN mass scale.

In this work we attempt to reappraise the aforementioned connection from a different perspective, considering a special construction of the orthogonal matrix $R$, as can be found in the literature [40, 56], which makes the CI parameterisation as a whole very predictive. We study the deterministic nature of these two $R$ matrices, in the context of having a

$^1$Although some early literature assumed this matrix to be real, for the purpose of making the connection of low and high energy CP violation strong, see e.g., Ref. [41–55].
viable Leptogenesis parameter space. According to this special construction, the $R$ matrix can further be expressed as an exponential of a skew-symmetric matrix, the elements of which needs a deeper evaluation. This article primarily aims to emphasize the role of the low energy CP phases in high energy CP violation, assuming the Dirac CP phase being the only source of CP violation in the low energy sector. However, a detailed study in this regard have become essential after the announcement made by T2K [57] in the recent year. We try to highlight the importance of the Dirac CP phase for the given choices for the orthogonal matrix, making the CI parameterisation different from the usual one \(^2\). The characteristic features of these two different $R$ matrices lead to interesting phenomenologies which can be verified in the low energy experiments. Especially the predictions on the low energy phase can be probed in the neutrino factories. The constraint on a parameter which we would call as the skew symmetric matrix element (SSME) \(^3\) which is explicitly involved in the CI parametrisation can be probed through the different LFV experiments.

We have chosen the canonical type-I seesaw mechanism having three RHNs. The scale of the leptogenesis is decided by the mass scale of the RHNs, the decay of which is supposed to source the lepton asymmetry. Taking into account the leptogenesis constraints on the relevant parameters we show the predictions on low energy parameters which may have relevance in the upcoming neutrino oscillation experiments. In addition we explore here the role of the Dirac CP phase in bringing a non-zero lepton asymmetry considering the RHN mass scale to be around \(10^8\) GeV. The observational bound on $\eta_B$ introduces a limit on the range of the Dirac CP phase, the lightest neutrino mass and the SSME. We consider a nearly degenerate spectrum for the RHN masses to account for the case of leptogenesis. The required amount of mass degeneracy is found to differ by an order of magnitude for each cases of the orthogonal matrix considered here.

This article is organized as follows. In Section 2 we discuss about different kinds of CI parameterisations we are going to explore. Here we also provide a brief discussion on the neutrino Yukawa Lagrangian and the neutrino mass and mixing parameters. Section 3 is provided with the required prescriptions for the calculation of lepton asymmetry. Section 4 is kept for the detailed phenomenology as obtained owing to the choices of these two parameterisations. In Section 5 we discuss the evolution of RHN population and lepton asymmetry with the help of Boltzmann Equations. In Section 6 we discuss the required parameter space which are tightly constrained in view of LFV data. Finally we highlight the conclusions of our analysis in Section 7.

2 Type-I seesaw and Casas-Ibarra parameterisation

Extension of the SM fermion sector by a pair of RHNs (the minimalistic scenario [59]) are no more a choice but has become essential to explain the tiny neutrino mass through the type-I seesaw mechanism. In this mechanism the heavy RHN couples with the SM

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\(^2\)In the usual CI parameterisation the orthogonal matrix $R$ is parameterised as a complex orthogonal matrix, as can be found in [39].

\(^3\)The elements $(a, b, c)$ of this skew symmetric matrix $A$ were also named as leptogenesis CP-violation (CPV) parameter as can be found in the Ref. [58]
lepton and the Higgs doublet through Yukawa like interaction. The coupling governing such interaction serves as the key role in offering the tiny neutrino masses through the canonical type-I seesaw mechanism. The same Yukawa coupling also governs the necessary interactions violating CP symmetry and thereby potentially explaining the origin of matter-antimatter asymmetry via the process of leptogenesis [10]. To obtain the neutrino Yukawa coupling from a model independent perspective Casas-Ibarra parameterisations have been playing promising role. For such example related to this please refer to [60–63].

In this section we briefly describe the type-I seesaw mechanism having three RHNs, a framework where all the three active neutrinos get tiny but non-zero masses. We also detail the lepton mixing matrix and the low energy neutrino observables subsequently.

2.1 Type-I seesaw and neutrino mass

The Yukawa Lagrangian generating the light neutrino mass through the type-I seesaw can be cast into,

\[ -\mathcal{L} = Y^\nu \ell_i \nu^c L H N_{Ri} + M_R (N_{Ri})^c N_{Ri} + \text{h.c.} \]  

(2.1)

with, \( \ell, i \) being respectively the flavor and generation indices for three generations of leptons and RHNs. The above Lagrangian generates a Majorana mass matrix for the left-handed neutrinos of the form:

\[ m_\nu = (Y_\nu v)^T M^{-1}_\text{diag} (Y_\nu v), \]  

(2.2)

where, \( M_\text{diag} \equiv \text{diag}(M_1, M_2, M_3) \) and \( v \) being the SM Higgs VEV. The light Majorana neutrino mass matrix \( m_\nu \) can be diagonalised approximately by the unitary PMNS matrix \( U_{\text{PMNS}} \approx U \), as follows

\[ U^\dagger m_\nu U^* = m_\text{diag}, \]  

(2.3)

where, \( m_\text{diag} \equiv \text{diag}(m_1, m_2, m_3) \), with the mixing matrix \( U \) having the following form,

\[ U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & -c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} U_M, \]  

(2.4)

where, we define \( c_{ij} = \cos \theta_{ij}, \ s_{ij} = \sin \theta_{ij} \) as the sin and cos of the three mixing angles for three lepton generations and \( \delta \) as the leptonic Dirac CP phase. The diagonal matrix \( U_M = \text{diag}(1, e^{i\alpha}, e^{i(\beta + \delta)}) \) contains the undetermined Majorana CP phases \( \alpha, \beta \). One can express the neutrino mass eigenvalues as \( m_\nu^{\text{diag}} = \text{diag}(m_1, \sqrt{m_2^2 + \Delta m_{21}^2}, \sqrt{m_3^2 + \Delta m_{31}^2}) \) for normal hierarchy (NH) and \( m_\nu^{\text{diag}} = \text{diag}(\sqrt{m_1^2 + \Delta m_{21}^2}, \sqrt{m_3^2 + \Delta m_{32}^2}, m_3) \) for inverted hierarchy (IH). A clear idea on these mass and mixing observables can be found from the global fit oscillation parameters as presented in Table 2.1.

Using CI parameterisation \( Y_\nu \) can be expressed as the product of a unitary matrix \( U_L \) and some other matrices, but now \( U_L \) is completely identical with the PMNS matrix \( U \). The CI formalism bridge the Dirac Yukawa couplings with the low energy parameters, present in the lepton mixing matrix and by the effective Majorana neutrino mass matrix. Hence, one can expect the low energy neutrino observables being forced to be confined within
Table 1. Standard inputs used in the analysis, taken from [64].

| Parameters     | Normal hierarchy | Best fit (NH) | Inverted hierarchy | Best fit (IH) |
|---------------|------------------|---------------|--------------------|---------------|
| $\sin^2 \theta_{23}$ | 0.433 - 0.609 | 0.582 | 0.436 - 0.610 | 0.582 |
| $\sin^2 \theta_{12}$ | 0.275 - 0.350 | 0.310 | 0.275 - 0.350 | 0.310 |
| $\sin^2 \theta_{13}$ | 0.02044 - 0.02435 | 0.0224 | 0.02064 - 0.02457 | 0.02067 |
| $\Delta m^2_{21}/10^{-5} \text{ eV}^2$ | (6.79 - 8.01) | 7.39 | (6.79 - 8.01) | 7.39 |
| $\Delta m^2_{31}/10^{-3} \text{ eV}^2$ | (2.436 - 2.618) | 2.525 | -(2.601 - 2.419) | 2.512 |
| $\delta_{CP}/^\circ$ | 144 - 357 | 217 | 205 - 348 | 280 |

certain region of parameter space allowed by the observed value of the baryon asymmetry of the Universe. From Eqs. 2.2 and 2.3, the Yukawa coupling matrix can be expressed as,

$$Y_{\nu} = \frac{i}{v} U m^{1/2}_{\text{diag}} R M^{1/2}_{\text{diag}},$$

where, in general $R$ is a complex orthogonal matrix, which implies $RR^T = I$. Here we consider two different form of orthogonal matrix: i) $R = O e^{iA}$ and ii) $R = O e^{A}$, where $A$ is an skew symmetric matrix. Here we choose $O$ to be an identity matrix. Both of these choices for the $R$ matrix especially give rise to exponential growth in the Yukawa coupling parameter space which eventually alters the parameter space for leptogenesis when compared to the case where $R$ is simply a complex orthogonal matrix.

In Ref. [40] we can see the matrix elements of of $A$ are related with the RHN mass ($M_R$) scale. There is no as such fixed choice of values for these matrix elements. One can chose larger values of these matrix elements (here, $a$), which in principle can correspond to a relatively smaller domain for the RHN mass as compared to the classical seesaw scale ($10^{15}$ GeV). A relatively small mass scale of the RHN is also motivated by the requirement of hitting a smaller reheat temperature. This leads us to assume that $M_R \geq T_{RH}$, a condition which further prevents the lepton-number violating processes from washing out the lepton asymmetry produced after the RHNs have decayed. It is worth noting here that, tiny values of the SSMEs can correspond to a very high reheat temperature ($T_{RH}$), which can be clearly understood from the relation of the RHN mass and the SSME mentioned in [40]. As we learn that a high reheat temperature will also lead to a potential danger associated with the gravitino overproduction [65, 66]. Therefore a larger values of these matrix elements are also preferred in order to avoid such problems, at the same time offering a smaller regime for the RHN masses as compared to the type-I seesaw scale. However, there has been reported some limits on the choice of the numerical values of these SSMEs (see e.g., [40]) for a given RHN mass scale. This range can further be constrained by different low energy and high energy observations like LFV and leptogenesis respectively for a wider range of RHN masses. In the following subsections we are going to present the constructions of the CI parameterisations under the consideration of these two orthogonal matrices.
2.2 Case-I: when $R = e^{iA}$

With the above choice of $R$ matrix the Yukawa coupling matrix can be written as,

$$Y_\nu = \frac{i}{v} U m_{\text{diag}}^{1/2} e^{iA} M_{\text{diag}}^{1/2}. \quad (2.6)$$

In case of this complex parameterisation the orthogonal matrix $R = e^{iA}$ can be expanded as [40, 58, 67]:

$$e^{iA} = 1 - \frac{\cosh r - 1}{r^2} A^2 + i \frac{\sinh r}{r} A, \quad (2.7)$$

where, $A$ is a real skew symmetric matrix having satisfied the feature of the $R$ matrix to be orthogonal ($RR^T = I$).

\[
\begin{pmatrix}
0 & a & b \\
-a & 0 & c \\
-b & -c & 0
\end{pmatrix}, \quad (2.8)
\]

with, $r = \sqrt{a^2 + b^2 + c^2}$. For reference, we name this case as the complex case and the later (Case-II) as the real case.

2.3 Case-II: when $R = e^{A}$

The Yukawa matrix has the following structure with the choice $R = e^A$

$$Y_\nu = \frac{i}{v} U m_{\text{diag}}^{1/2} e^A M_{\text{diag}}^{1/2}. \quad (2.9)$$

In case of this parameterisation the orthogonal matrix $R = e^A$ can be expanded in the following form [56]:

$$e^A = 1 + \frac{1 - \cos r}{r^2} A^2 + \frac{\sin r}{r} A, \quad (2.10)$$

where, the matrix $A$ and $r$ have the similar definitions as in the former case described above. For simplicity we consider here the elements of the matrix $A$ to obey, $a = b = c = a$, and hence one can have $r = \sqrt{3} a$. Such an assumption allows for a minimal number of free parameters. For convenience, in the rest of this article we name this parameter $a$ to be SSME as we mention in the introduction. This kind of equity among these three elements can be found in a very recent work [68]. Derivation of Eqs. 2.7 and 2.10 can be found in the Appendix A.

3 Leptogenesis

In a temperature regime where all the lepton flavors are distinguishable, it is instructive to consider the flavor dependent as well as resonant leptogenesis approach. The inclusion of flavor effects on leptogenesis provides a very essential modification for the calculation of the final baryon asymmetry (please see Ref. [26, 28, 29, 69]), as compared with the
calculation in the unflavored scenario. In such scenario the lepton asymmetry generated from the decay of the lightest RHN can be expressed as [70, 71],

\[
\epsilon_{\ell} = \frac{1}{8\pi} \left( Y^{\dagger}_i Y_i \right) \sum_{j \neq i} \text{Im} \left[ \left( Y^{\dagger}_i Y_i \right)_{ij} \left( Y^{\dagger}_j Y_j \right)_{ij} \right] \left[ f(x_{ij}) + \frac{\sqrt{x_{ij}}(1 - x_{ij})}{(1 - x_{ij})^2 + \frac{1}{64\pi^2} \left( Y^{\dagger}_i Y_i \right)_{jj}^2} \right]
\]

\[
+ \frac{1}{8\pi} \left( Y^{\dagger}_i Y_i \right) \sum_{j \neq i} \frac{(1 - x_{ij})\text{Im} \left[ \left( Y^{\dagger}_i Y_i \right)_{ij} \left( Y^{\dagger}_j Y_j \right)_{ij} \right]}{(1 - x_{ij})^2 + \frac{1}{64\pi^2} \left( Y^{\dagger}_i Y_i \right)_{jj}^2} + \mathcal{O}(Y_\nu^6),
\]

with the following definition for the loop function \( f(x_{ij}) = \sqrt{x_{ij}} \left[ 1 - (1 + x_{ij})\ln \left( \frac{1 - x_{ij}}{x_{ij}} \right) \right] \)

where, \( x_{ij} = \left( \frac{M_i}{M_j} \right)^2 \). One can define \( Y_\nu \) as the complex Dirac Yukawa coupling, derived in a basis where the RHNs are in the diagonal mass basis.

With the above prescription for lepton asymmetry one can write the analytically approximated solution (from the set of Boltzmann equations given by Eq. 5.1) for the baryon to photon ratio [37, 38, 72] as,

\[
\eta_B \simeq -3 \times 10^{-2} \sum_{\ell,i} \kappa_\ell K^{\text{eff}}_\ell \min \left[ x_c, 1.25 \log (25K^{\text{eff}}_\ell) \right],
\]

where \( z_c = \frac{M_c}{T_c} \) and \( T_c \sim 149 \text{ GeV} \), [72] is the critical temperature, below which the sphalerons freeze out [18, 19]. Here, \( K^{\text{eff}}_\ell = \kappa_\ell \sum_i K_i B_{\ell i} \), with \( K_i = \Gamma_i / H \), the wash out factor and \( \Gamma_i = \frac{M_i}{8\pi} (Y_i Y_\nu^\dagger)_{ii} \) as the tree level heavy-neutrino decay width. The Hubble rate of expansion at temperature \( T \sim M_i \) can be expressed as,

\[
H = 1.66 \sqrt{g^*} \frac{M_i^2}{M_{\text{Pl}}} \quad \text{with} \quad g^* \simeq 106.75 \quad \text{and} \quad M_{\text{Pl}} = 1.29 \times 10^{19} \text{ GeV}.
\]

Here, \( B_{\ell i} \)'s are the branching ratios of the \( N_i \) decay to leptons of \( \ell^{th} \) flavor : \( B_{\ell i} = \frac{|Y_{\nu i \ell}|^2}{(Y_\nu Y_\nu^\dagger)_{ii}} \). Including the Real Intermediate State (RIS) subtracted collision terms one can write the factor \( \kappa \) as,

\[
\kappa_\ell = 2 \sum_{i,j \neq i} \text{Re} \left[ (Y_{\nu})_{i\ell} (Y_{\nu})_{j\ell} \right] \left( Y_{\nu} Y_{\nu}^\dagger \right)_{ij} \text{Re}[i(Y_{\nu} Y_{\nu}^\dagger)_{ii} + (Y_{\nu} Y_{\nu}^\dagger)_{jj}] \right] \times \left( 1 - 2i \frac{M_i - M_j}{\Gamma_i + \Gamma_j} \right)^{-1}.
\]

Once we numerically evaluate \( Y_\nu \), they can be further used in the above equations for computing the baryon to photon ratio. In the following section we are going to present the methodology involved for pursuing the parameter space extraction for leptogenesis using the two different CI formalisms discussed in the previous section 2.

4 Results and analysis

As mentioned earlier one of the aims of choosing these two \( R \) matrices is to determine the allowed ranges of the skew symmetric matrix element (a) from the viable parameter space
of leptogenesis in type-I seesaw. Depending on the seesaw scale the allowed range will vary. On the other hand, the significant purpose of doing this analysis lies in the fact that the range of the matrix elements can be indirectly probed via a low energy phenomenon like lepton flavor violation [56]. Note that this range can be very specific for each variety of the $R$ matrix discussed above. However, for both the cases we take a nearly degenerate spectra for the first two RHN masses, $M_1 \approx M_2$, and the third $M_3$ being relatively more massive. This assumption can lead us to have $M_1, M_2, M_3 = 10^8, 10^8 + \Delta M, 10^9$ GeV. We choose a range of this mass splitting $\Delta M$, for instance, to be varied from $(0.001 – 0.1)$ GeV for numerical analysis. This small mass splitting among the RHNs is not fine tuned and can be a natural choice in particular when these special kinds of $R$ matrices are considered.

In the low energy sector apart the phase $\delta$, the values of the three mixing angles have been fixed to their best fit points. While doing this analysis, care has been taken to ensure the perturbativity limit on the Yukawa couplings $(Y_\nu)$ which is generally realized by $|(Y_\nu)_{ij}| \leq \sqrt{4\pi}$.

We present here i) the predictions on low energy parameters namely, the lightest neutrino mass and the Dirac CP phase, ii) the connection between the low energy phase $\delta$ and the high energy CP violation, iii) the role of the Dirac CP phase in different flavored asymmetries, for the above choices of $R$ matrix. We present here the relevant phenomenologies in two different subsections for each individual cases involving the different structures of $R$ matrix. The total number of parameters of interest are respectively, the lightest neutrino mass, Dirac CP phase, the skew symmetric matrix element and the heavy Majorana neutrino mass splitting which can be presented below as,

$$\{m_1, \delta, a, \Delta M\}$$

For simplicity, we have chosen the best fit central values of the neutrino observables namely three mixing angles and two mass squared differences. Also we restrict this analysis for normal hierarchy (NH) of neutrino masses keeping in mind the recent preference for NH [73].

As mentioned earlier we focus on the assumption that the source of CP-violation arises from the low energy phase $\delta$ and hence to ensure this we take the Majorana phases $\alpha$ and $\beta$ to be zero.

In the following subsections we present the various roles of these two parameterisations in bringing a correct order of baryon asymmetry. We divide this analysis into two more parts, one focusing on the low energy predictions and the other for constraining the SSME $a$. These two different kinds of orthogonal matrix $R$ impose constraints on the low energy neutrino oscillation parameters very differently. Interestingly it leads to constrain the lightest neutrino mass and the Dirac CP phase from the requirement of satisfying the baryon to photon ratio.

4.1 Predictions on parameters: when $R = e^{iA}$

Among the four parameters of interest $m_1$ and $\delta$ are in principle low energy parameters as they can be probed in the low energy experiments. Whereas the remaining two $a$ and $\Delta M$ can be considered as high energy parameters as they are assumed to be involved in the
neutrino mass generation mechanism at high energies through the construction of the Yukawa matrix (Eqs. 2.6 and 2.9). We examine the high energy parameters ($a$ and $\Delta M$) by fixing a set of different values of low energy parameters namely the lightest neutrino mass and the Dirac CP phase and vice-versa. As mentioned earlier we focus on the heavy RHN mass regime to be of the $O(10^8)$ GeV, where the three lepton flavors ($e, \mu$ and $\tau$) are completely distinguishable, implying a fully flavored regime.

In Fig. 1 we fix $m_1$, and $\delta$ at different set of values and varied the high energy parameter $a$ from $1 - 10$ along with the mass splitting $\Delta M$ from $(0.001 - 0.1)$ GeV. We perform this analysis for some benchmark values of the Dirac CP phase $\delta = 3\pi/2, \pi, \pi/2$, and for a particular value of $m_1 = 0.001$ eV. Here we notice that for decreasing value of $\delta$ the upper bound on $a$ gets shifted towards higher values of $a$ to meet the observed $\eta_B$. A similar kind of observation we have when we repeat this analysis for some benchmark values of the lightest neutrino mass. In the right panel of Fig.1 we fixed $m_1$ at different values setting $\delta$ at $\pi/2$ to see the constraint on $a$ coming from the observed $\eta_B$. We see that the allowed parameter space for the observed $\eta_B$ (the darkened regions in each plot) forces $a$ to have a narrow region. For an RHN mass regime $O(10^8)$ GeV, we report a maximum value of $a$ to be close to 2.42. From the left panel of this figure it is clear that the highest $a$ value one can have in this set up is for a smaller value of $\delta$, which is $\pi/2$. In the right panel one can see that, for the lightest neutrino mass $m_1$ to be 0.001 eV, the allowed $a$ value is largest when compared to the $a$ values at the other two choices of $m_1$. Note that a particular allowed value of $a$ corresponds to a specific value for the heavy RHN mass as also mentioned in [40]. Although, we do not see such restrictions on the Majorana mass splitting $\Delta M$ in order to satisfy the $\eta_B$ constraint, as long as the chosen range of $\Delta M$ is provided as mentioned before.

We also observe that choosing some benchmark values of $a$ from this range can constrain the Dirac CP phase and the range of the lightest neutrino mass to achieve the observed $\eta_B$ as evident from Fig. 2. In this figure we fix $a$ and $\Delta M$ and vary the low energy parameters $\delta$ and $m_1$ from (0-2$\pi$) and (0.001-0.009) eV respectively. Here also we
In the left (right) panel we show the variation of $\eta_B$ as a function of the Dirac CP phase $\delta$ ($m_1$ for NH) for different benchmark values of the parameter $a$. The darkened points exhibit the constrained values of Dirac CP phase and the lightest neutrino mass for the chosen hierarchy.

We fix $a$ at some benchmark values such as $a=1, 2, 3$ with $\Delta M = 0.001$ GeV. In the left side of this figure one can see that for $\delta = \pi/2$ one obtains a maximum $\eta_B$ for any choice of $a$. We see that larger values of $a$ decreases $\eta_B$ from it’s observed value (say for $a=3$). This might be due to the reason that, the larger value of $a$ enhances the order of Yukawa couplings (having a hyperbolic kind of dependency) which, further increases the amount of wash out ($K$) well enough to suppress the order of a produced asymmetry. One can notice here that for $a=2$, $\delta$ prefers the range from $(0 - 2\pi)$ except those points which are around $\pi/2$. In the right panel of Fig. 2 for $a=2$, $m_1$ agrees with the observed BAU when it is more than or equal to 0.0018 eV. Thus it is understood that, for $a = 2$, one should have two ranges for $\delta$ one from $0 - 0.8$ (rad) and other from $2.3 - 6$ (rad) as allowed by the observed $\eta_B$. Similarly, the right panel of this figure shows the restriction on the lightest neutrino mass which has been varied from 0.001 – 0.009 (eV). The baryon to photon ratio sets a lower bound on this mass parameter as 0.0018 eV, for $a = 2$.

### 4.2 Predictions on parameters: when $R = e^A$

For numerically evaluating the baryon asymmetry while making use of the above $R$ matrix, we vary $a$ from 0.01 – 10 and $\Delta M$ from 0.001 – 0.1 GeV. Here we fix $m_1$ at 0.008 eV. We examine the relevant parameter space for $\eta_B$ considering both CP conserving and violating values of $\delta = (0 - 2\pi)$. For this real $R$ matrix, as expected from the Eq. 2.10 the behaviour of the parameterisation is sinusoidal. As the value of $a$ vary over the range from 0.001 – 10, we see a periodic behaviour as evident in the left of Fig. 3 from 0.01 – 3.65 and then from 3.65 – 7.32 and so on. We consider the range of $a$ to be 0.01 – 3.65 for further analysis, as it is clear that the other range of $a$ is not going to alter the phenomenology.

For both the choices of $\delta = \pi/2$ and $3\pi/2$ along with for $m_1 = 0.008$, we see a similar behaviour of $\eta_B$ when plotted with respect to $a$. We observe an over estimation of $\eta_B$ for all $a$ values, except for the cases when $a$ is equal to 0.01, 0.35, 1.12 and 3.65, where they give rise to the observed $\eta_B$. We also show the estimated $\eta_B$ as a function of $\Delta M$ as evinced in the right of Fig. 3. We see that the observed baryon asymmetry allows for the entire range of $\Delta M$ although accommodating more $\Delta M$ at it’s smaller values. We see...
Figure 3. $\eta_B$ as a function of the parameter $a$ (left) and $\Delta M$ (right) fixing $m_1 = 0.008\text{eV}$ along with $\delta = \pi/2$. An odd multiple of $\pi/2$ for $\delta$ does not bring any major change in the analysis for this real $R$ case. Thus we focus here to show the variation of $\eta_B$ only for $\delta = \pi/2$. The dark color points satisfy the respective parameter spaces which are allowed by the bound on the observed $\eta_B$.

Figure 4. Variation of $\eta_B$ w.r.t. Dirac CP phase (left) and the lightest neutrino mass (right) for different benchmark values of $a$. For explanation please refer to the text.

that for $\delta = 0, \pi$ and $2\pi$ values the CP-asymmetry exactly vanishes. However, when there is slight deviation from these CP conserving values, $\delta$ has a wide range of allowed values, which satisfy the $\eta_B$ constraint.

To get a view on how $\delta$ influences $\eta_B$ for the real $R$ case, we vary $\delta$ from 0 to $2\pi$ and $m_1$ from 0.001 to 0.008 eV. With this second kind of orthogonal matrix, unlike the former one the source of CP violation arises only through the low energy phase $\delta$ to give rise to an adequate amount of lepton asymmetry\(^4\). We present the variation of $\eta_B$ with $\delta$ and $m_1$ in the left and right of Fig. 4, for a fixed $\Delta M$ at 0.001 GeV and considering different values of $a$ as $a = 0.1, 1, 2, 3, 3.6$. As can be observed from the left side of this figure, only small values of $a$ can allow the entire range for $\delta$ in order to satisfy the observed $\eta_B$. On the other hand for all values of $a > 1$, $\delta$ is preferred to be placed around 0, $\pi, 2\pi$. In the right side of Fig. 4 for the same set of $a$ values we show the variation of $\eta_B$ with respect to the lightest neutrino mass $m_1$, which evinces that the entire range of $m_1$ can give rise to the observed $\eta_B$.

\(^4\)as we have no other phase associated with the CI parameterisation involving real $R$. 
4.3 Dependence of the flavored asymmetries on a varying range of the Dirac CP phase

The ideal reason is that realistic leptogenesis is a dynamical process, involving the creation and destruction of the heavy RH Majorana neutrinos, and of a lepton asymmetry that is distributed among distinguishable lepton flavors. Investigation of flavored asymmetries due to the variation in Dirac CP phases have been explored earlier by the authors in Ref. [71] in the background of inverse and linear seesaw. Here, we execute the same in a model independent way in the light of CI parameterisations realized by the presence of these special orthogonal matrices. When flavour effects are accounted for, the final value of the baryon asymmetry is the sum of three contributions from each lepton flavor. Each term is given by the CP asymmetry in a given lepton flavour $\ell$, properly weighted by a wash-out factor induced by the same lepton number violating processes. The wash-out factors also become flavour dependent. We present here the role played by the Dirac CP phase in bringing an appreciating amount of flavored asymmetry in the form of contours of $\eta_B$ for the temperature-function ($z$) variation. In Figs. 5 and 6 we show the contours for complex and real cases respectively. It is also to mention that, the contours have been constructed with the choice of two different values of the lightest neutrino mass ($m_1$ here for NH) and for which one can notice the important changes having been brought into.

For both complex and real cases we obtain the major contribution to final $\eta_B$ from the $\mu$ flavor compared to $e$ and $\tau$ flavors. As observed in Fig. 6, presenting for the real $R$ matrix the major contribution to $\eta_B$ is provided by the entire range of $\delta$ except for when $\delta$ becomes equal to around $0, \pi, 2\pi$. We also observe an increase in the final $\eta_B$ for a smaller $m_1$ irrespective of all the lepton flavors alike the complex case.

As we learn that the connection between the low and high energy CP violation becomes unavoidable once we choose the temperature regime of leptogenesis to be as such where the three lepton flavors act distinguishably. This connection can be well understood on having a subtle look on the variation of the asymmetries associated with each lepton flavor with the low energy CP phase that we considered to be non zero. To realize this, if we look at Fig. 5 for the complex $R$ case it is evident that, for both the choices of the lightest neutrino mass mentioned above the asymmetry due to electron flavor is seemingly found to be insensitive to a particular value of the $\delta$, as there is no sharp rise of the respective asymmetry for a particular $\delta$ value. However, the asymmetry due to the $\mu$ and $\tau$ flavors are quite sensitive to the value of $\delta$ around $\pi/2$. For the real $R$ case one can notice from the Fig. 6 that, the asymmetries due all the lepton flavors are very much sensitive around some specific values of $\delta$ like $\pi/2, 3\pi/2$. For the $m_1 = 0.002$ eV, one can see that $\eta_B$ is over estimated due to maximum contribution by the electron and muon flavor. A similar inference is also visible for muon and electron flavor given $m_1 = 0.008$ eV, however with an expected range of $\eta_B$. In the left hand side for $m_1 = 0.002$ eV, the asymmetry due to electron and $\mu$ and $\tau$-flavor reaches the maxima around $\delta > \pi/2$ and $\delta < 3\pi/2$, however attains a minimum value at $\delta = \pi$. For $m_1 = 0.008$ eV, the electron and muon flavored asymmetries attains maximum value around $\delta = \pi/2$ and $3\pi/2$. Although for the $\tau$- flavored asymmetry a slight shift of $\delta$ from $\delta = \pi/2$ and $3\pi/2$ is noticed as required by the observed $\eta_B$. 


Figure 5. Contour of $\eta_B$ corresponding to the different lepton flavors as a function of the Dirac CP phase $\delta$ and the temperature function $z$ for a fixed value of the parameter $a = 2.42$. In the left (right) panel we fix the $m_1 = 0.001(0.01)$ eV respectively. The relevant explanation is detailed in the text.
Figure 6. Contour plot of $\eta_B$ corresponding to the different lepton flavors as a function of the Dirac CP phase $\delta$ and the temperature-function $z$. In the left (right) panel we fix the parameters $a = 1.11$ with $m_1 = 0.008$ eV ($a = 2$ with $m_1 = 0.002$ eV) respectively. The relevant explanation is detailed in the text.

5 Boltzmann Equation for leptogenesis

We learn that in a temperature regime where the lepton flavors act non identically, one should consider the flavor dependent Boltzmann equations (BEQ) governing the evolution
of RHN population and the lepton charge density, which are given below [38]. In solving the BEQs we considered the initial conditions $\eta_\alpha (z_{in}) = 0$ for the lepton asymmetry and $\eta_N (z_{in}) = \eta^{\text{eq}}$ for the RHN abundance. Given a strong washout scenario naturally obtained from the model parameter space the initial condition for RHN abundance is set. This fact can also be driven by the argument that, as the RHNs are relatively less massive, it facilitates a thermal population of these RHNs which are supposed to be produced much before the asymmetry is yielded.

\[
\frac{d\delta\eta_{N_i}}{dz} = \frac{K_1(z)}{K_2(z)} [1 + (1 - K_i z) \delta\eta_{N_i}] \quad \text{with} \quad i = 1, 2 
\]

\[
\frac{d\eta_\ell}{dz} = z^3 K_1(z) K_i \left( \delta\eta_{N_i} \epsilon_\ell - \frac{2}{3} B_{d\ell} \eta_\ell \right), \quad \text{with} \quad \ell = e, \mu, \tau
\]

with $K_1, \ K_2$ as the modified Bessel function of the second kind. One can define $\delta\eta_N = \frac{N_N}{N_N^{\text{eq}}} - 1$ as the parameter which estimates the deviation of the RHN abundance from the Equilibrium abundance, $K_i = \frac{\Gamma_i}{\xi(3)H}$ as the wash out parameter with the definition of $\Gamma_i$ and $H$ provided in the Section 3. Using the solution of the above BEQs one can estimate the baryon to photon ratio as follows:

\[
\eta_B = -\frac{28}{51} \frac{1}{27} \sum_{\ell} \eta_\ell
\]

where, $\eta_\ell$ is the yield for lepton charge density obtained from the solution of the above BEQs. The factor of $28/51$ arises from the fraction of lepton asymmetry reprocessed into a baryon asymmetry by the electroweak sphalerons while $1/27$ is the dilution factor from photon production until the recombination epoch.

**Figure 7.** In the left (right) panel we show the nature of the evolution of the RHN number density and lepton asymmetry from the RHN decay for the particular cases complex and real respectively. The upper black line presents the evolution of RHN population. Different colours here indicate the lepton asymmetry evolution for different lepton flavors. For this figure we set the $\delta = \pi/2$. However the $a$ value is different such as, for complex $a = 2.42$ and real $a = 1.11$. For relevant explanations please refer to the text.
Fig. 7 illustrates the deviation of RHN abundance from Equilibrium abundance and the evolution of lepton charge density corresponding to each lepton flavors. As followed from the prescriptions of lepton asymmetry and all the wash out parameters, both explicitly depend on the Yukawa couplings ($Y_{\nu}$ being regulated by the $R$ matrices of interest). Therefore it is imperative to study the evolutions of lepton asymmetry resulted for each cases of different $R$ matrices. In the left (right) panel of this figure we present the respective evolution plots for the two different cases discussed above, where we have noticed that the final asymmetry receives major contribution from the muon flavor among the three.

6 Comment on Lepton flavor violation

In the previous sections we mainly discussed about the leptogenesis parameter space in the context of two different choices of the orthogonal matrix $R$. In this section we further investigate, whether the favourable parameter space for leptogenesis can give rise to an expected branching ratio of a LFV decay process. For simplicity we have considered here the branching for $\mu \rightarrow e + \gamma$ decay process, which presently provides the strongest bound. As we have seen, the rates of the LVF processes in the canonical type-I seesaw model with massive neutrinos are so strongly suppressed that these processes are not observable in practice, one has e.g., $\text{BR}(\mu \rightarrow e + \gamma) < 10^{-47}$ [74, 75].

However, the presently planned near future sensitivity of $\text{BR}(\mu \rightarrow e + \gamma) < 6 \times 10^{-14}$, having the present bound to be $< 5.3 \times 10^{-13}$ (taken from Refs. [75–77]). A search for a theoretically well motivated framework to account this large number for $\text{BR}(\mu \rightarrow e + \gamma)$ is hence looked for.

| Parameteraisation     | $\text{Br}(\mu \rightarrow e\gamma)$ | $a$   |
|-----------------------|--------------------------------------|-------|
| Complex case          | $5.67 \times 10^{-13}$               | 9.78  |
|                       | $3.83 \times 10^{-35}$               | 2.42  |
| Real case             | $2.47 \times 10^{-40}$               | 2.45  |

Table 2. Attainable $\text{Br}(\mu \rightarrow e\gamma)$ for the complex case and the real case.

The LFV processes and leptogenesis are related by the Yukawa couplings $Y_{\nu}$. As we learn, the matrix $Y_{\nu}$ can be expressed in terms of all the neutrino mass and mixing parameters which are respectively, light neutrino and heavy RHN masses, the neutrino mixing matrix $U$, and the orthogonal matrix $R$. Leptogenesis can take place only if $Y_{\nu}$ is complex. However, this is not a necessary condition for the calculation of the branching ratios associated with LFV decays. The branching for a particular LFV process depends explicitly on $(Y_{\nu}^\dagger Y_{\nu})_{ij}$. Therefore, the predictions on $\text{BR}(l_i \rightarrow l_j\gamma)$ are directly linked to the magnitude of the matrix elements involved in the orthogonal matrix $R$. Using the complex form of the $R$ matrix authors in [40] have reported two findings. One is that it is difficult to get a common viable regime for leptogenesis and branchings for various LFV decays for such high RHN mass scale and the second is using the complex $R$ matrix as discussed in this analysis the branching ratios for different LFV decays $(l_i \rightarrow l_j\gamma)$ are
expected to have an unsuppressed value in comparison to what one obtains considering a real $R$ matrix.

To compute the branching ratio for this decay process we followed prescriptions from \cite{40, 72}. As illustrated in the previous sections, about the role of the orthogonal matrix being very deterministic in the leptogenesis scenario, that also holds good for the calculation of $\text{BR}(\mu \rightarrow e + \gamma)$. This can be understood from the Table 6, where we present the quantitative results for the enhancement factors for the mentioned LFV decay channel. For understanding the variation of the branching ratio with respect to $a$, one may look at the Fig.8.

![Figure 8](image.png)

**Figure 8.** $\text{BR}(l_i \rightarrow l_j \gamma)$ plots as a function of the skew symmetric matrix element for the complex (left) and real (right) cases.

In addition to this, we also report that for a large $a$ value one could achieve a common regime for both baryon asymmetry and an unsuppressed branching for LFV decays, for a much smaller mass regime for the RHNs in case of the complex parameterisations.

Given the existing experimental bound on $\text{BR}(\mu \rightarrow e \gamma)$, our results can be used, in particular, to further constrain the SSME parameter space (here, in particular range of $a$) along with a possible mass window for the RHNs starting from a TeV to that naturally required for type-I seesaw scale. This requires a more detailed numerical analysis which is beyond the scope of the present work.

7 Conclusion and discussion

In this work we widely explore the appealing role of a Casas-Ibarra parameterisation where the orthogonal matrix has a special form (one as $e^{iA}$ another as $e^A$). Here we discuss various attractive aspects of these two cases in the context of having an accessible leptogenesis parameter space. In doing so, the active involvement of these special orthogonal matrices in the extraction of neutrino Yukawa is realized with the help of CI formalism. The same coupling acts indispensably in the generation of light Majorana neutrino mass, leptogenesis and the branching ratios associated with the various LFV decays. The light neutrino masses are assumed to be offered by the canonical type-I seesaw, with a choice of the RHN mass scale to be around $10^8$ GeV along with a quasi degenerate spectrum consisting the three
RHNs. With this mass regime of the RHNs we performed a detailed analysis on thermal leptogenesis in the light of these $R$ matrix formalism mentioned above. The results of this analysis highlight the reliance of high energy CP violation on the leptonic CP violation present in the form of Dirac CP phase in the PMNS matrix. We show that with the given choice of the RHN mass regime, the Dirac CP violation is sufficient to bring out an ample amount of lepton asymmetry. However, this RHN mass window can have a wider range depending on the chosen range for the skew symmetric matrix element. Moreover, we have also studied the time evolution of the RHN abundance and flavored asymmetries by numerically solving the coupled Boltzmann equations. We summaries our main findings below:

- The structural influence provided by the complex $R$ matrix on the neutrino Yukawa couplings give rise to important predictions in low energy parameter space. The complex $R$ matrix impose a restriction on the Dirac CP phase values, and range of the lightest active neutrino mass in order to account for the observed baryon to photon ratio. These predictions can be further tested in the future and ongoing neutrino oscillation experiments. Not only that, we are able to constrain the SSME ($a$) parameter space in this complex $R$ case, which can further be probed through the Branching ratios of various LFV decays. For the complex $R$ matrix, the connection between low energy and high energy CP violation is found to be very prominent, with a preference for the Dirac CP phase ($\delta$) to be around $\pi/2$. The observed value of the baryon to photon ratio allow the SSME to range from $(1.67 - 2.42)$ for the given choice of RHN mass.

- For the real $R$ matrix we have noticed an unusual behaviour of the Dirac phase in obtaining a non-zero lepton asymmetry. In order to satisfy the leptogenesis constraint we see that, the real $R$ matrix predicts the Dirac CP phase to be around certain CP conserving values which makes us convenient to write $\delta \approx 0, \pi, 2\pi$. One can expect for this parameterisation that, since the orthogonal matrix is chosen to be a real one, the source of CP violation in the Yukawa couplings is solely contributed by the low energy CP phase $\delta$, present in the lepton mixing matrix. Thereby, a large value of $\delta$ can be naturally expected to generate a sufficient amount of lepton asymmetry. However, this is not obtained in the real case unlike the complex case. For this choice of $R$ matrix we obtain the constraint region of $\delta$ very close to the CP conserving values as mentioned above. This result makes the real case very different from the complex case. Unlike the complex $R$ case, the observed $\eta_B$ restricts the SSME to pick certain values which come after certain periodic intervals. This is due to the reason that the Yukawa couplings in this case are guided by sinusoidal functions of $\sqrt{3}a$. Some benchmark points however could be noticed like $a = 0.125, 0.39, 1.11$ etc. for the chosen RHN mass appropriate for flavored leptogenesis. However, in both of the two cases of $R$ matrix the order of the RHN mass degeneracy ($\Delta M$) does not differ much in order to have a successful leptogenesis.

- We have shown the influence of the low energy phase on asymmetries associated with
each lepton flavor in the form of contour plots. For the complex $R$ case we obtain a direct connection between the maximal low energy CP violation (implying $\delta \approx \pi/2$) and an enhanced lepton asymmetry for all the flavors for any choice of the lightest neutrino mass. This scenario is seemingly different in case of the real $R$ matrix. Such connection between the low and high energy CP violation is realized for the real $R$ matrix for a higher value of the lightest neutrino mass as we notice it to be $0.008$ eV. This correlation does not hold true for a smaller value of the lightest neutrino mass as we see for the case of $m_1 = 0.002$ eV.

- We find that regions of successful leptogenesis which in principle rely on the SSME parameter space, do not actually coincide with the current or future reach of LFV experiments for the afore mentioned choice of RHN mass regime and the allowed range of the skew symmetric matrix element. Taking into account the leptogenesis constraints on the relevant parameters we learn that an increase of the $a$ value can drastically enhance the branching of $\mu \to e\gamma$ decay approximately by a factor of $10^{12}$ when one considers the complex $R$ matrix for Yukawa extraction. An increase in the branching requires $a$ value to be around 8.97 for an RHN mass of the order $10^8$ GeV. However, this combination is unable to produce an ample amount of lepton asymmetry we are looking after. On the other hand this kind of enhancement in the branching is not at all obtained in the case of the later choice of $R$ matrix, even after choosing a larger value of $a$. This is due to the hyperbolic kind of escalation in the overall Yukawa coupling matrix elements in the complex $R$ case, which is absent in case of the real $R$ matrix.

To conclude, these two special $R$ matrices lead to potentially interesting phenomenological predictions as we have seen in the present analysis. A detailed investigation is needed to underscore the importance of all the PMNS phases in the leptogenesis process through the consideration of the Casas-Ibarra formalism involving these special choices of the orthogonal matrix, which can be found in an upcoming work. A study in a non-minimalistic scenario considering the presence of the Majorana phases, not only can shed light on the source of high energy CP violation but also can exclude some region of free parameter space associated with the effective neutrino mass $m_{\beta\beta}$ governing the neutrino less double beta decay ($0\nu\beta\beta$) subject to some constraints that might put restrictions on these phases. In addition, the present analysis leaves an imprint for a future study involving various regimes of the RHN mass, which will alter the parameter space associated with the skew symmetric matrix element, that can be further examined in the context of lepton flavor violation related searches.
Appendix

A Expansion of the $R$ matrices:

Following the definition of the skew symmetric matrix $A$ from Eq. 2.8 and the condition of equity among the matrix elements, the characteristic polynomial for $A$ can be written as,

$$|A - \lambda I| = 0 \Rightarrow \lambda^3 + \lambda r^2 = 0,$$

(A.1)

which further leads to, $\lambda^3 = -r^2 \lambda$. Using the Cayley-Hamilton theorem which states that, every square matrix over a commutative ring satisfies its own characteristic equation, one can simply write $A^3 = -r^2 A$. Now on expanding,

$$e^{iA} = 1 + iA - \frac{A^2}{2!} - \frac{iA^3}{3!} + \frac{A^4}{4!} - \frac{iA^5}{5!} + \text{etc.}$$

Using $A^3 = -r^2 A$, and after separating the real and imaginary parts of $e^{iA}$ we can write,

$$\text{Im}(e^{iA}) = iA \left( \frac{1}{r} \left( r + \frac{r^3}{3!} + \text{.....} \right) \right) = \frac{iA \sinh r}{r}$$

$$\text{Re}(e^{iA}) = -\frac{A^2}{r^2} \left( \frac{r^2}{2!} + \frac{r^4}{4!} + \text{.....} \right) = -\frac{A^2}{r^2} (\cosh r - 1)$$

The above formulations lead one to write,

$$R = e^{iA} = 1 - \frac{\cosh r - 1}{r^2} A^2 + \frac{i \sinh r}{r} A.$$

The same procedure will help us to derive the analogous formula for the other parameterisation when, $R = e^A$.

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