Primordial black holes (PBH), which can be naturally produced in the early universe, remain a promising dark matter candidate. They can merge with a supermassive black hole (SMBH) in the center of a galaxy and generate a gravitational wave (GW) signal in the favored frequency region of LISA-like experiments. In this work, we initiate the study of the event rate calculation for such extreme mass ratio inspirals (EMRI). Including the sensitivities of various proposed GW detectors, we find that such experiments offer a novel and outstanding tool to test the scenario where PBHs constitute (fraction of) dark matter. The PBH energy density fraction of DM \( f_{\text{PBH}} \) could potentially be explored for values as small as \( 10^{-3} \sim 10^{-4} \). Further, LISA has the capability to search for PBH masses up to \( 10^{-2} \sim 10^{-3} M_\odot \). Other proposed GW experiments can probe lower PBH mass regimes.

### Introduction.

Dark matter (DM) comprises about 27% of the energy density in our current universe \(^1\). However, the identity of DM remains a mystery. It may be particles beyond the Standard Model, where popular choices are Weakly Interacting Massive Particle and axion. Primordial black holes (PBH) are also a promising candidate with a wide allowed mass range (for a PBH review, please see e.g. \(^2\)). There have been a lot of efforts to study the fraction of DM as PBH, e.g. using gravitational lensing \(^3\sim11\), the CMB temperature anisotropies and polarizations \(^12\sim13\), etc. The validity as well as astrophysical uncertainties of these constraints are still under debate, \(^14\sim15\) and thus it is interesting to explore this possibility through new and independent measurements.

The detection of the gravitational wave (GW) events from black hole binaries by the LIGO and Virgo collaborations \(^16\sim18\) has begun the era of GW astronomy. GW observations provide a novel method to study the universe. Many GW detectors have been proposed (see Ref \(^19\) for a review). In particular, Laser Interferometer Space Antenna (LISA), which aims for a much lower frequency regime than that of LIGO-like ground-based detectors, has been approved recently \(^20\). One major scientific goal of LISA is to measure the GW produced by the merger of a SMBH and a compact object (CO), such as a neutron star, white dwarf or stellar BH. In such EMRIs \(^21\), GW frequencies typically range from \( 10^{-4} \) to \( 1 \) Hz for SMBH masses between \( 10^4 M_\odot \) and \( 10^7 M_\odot \). Once such events are observed, the intrinsic parameters of the binary system can be measured in high precision \(^22\) due to the long-lasting inspiral process before merging.

Aside from their significant impacts for astronomy, the observation of GWs may also open a new avenue to study the possibility of PBHs playing the role of DM. Especially, Ref. \(^23\sim29\) study the interesting question of whether the BHs detected by LIGO can be PBHs which form a non-trivial fraction of DM. Using LIGO and LISA to probe extremely small mass PBH is studied in \(^30\). For PBHs with mass of \( O(10) M_\odot \), it is hard to distinguish them from stellar BHs. However, LIGO is not ideal to probe other PBH mass ranges, either due to the shifted frequency region or reduced magnitude of GW radiation. On the other hand, the mergers between PBHs and SMBHs produce GWs in the favored frequency regions of LISA-like experiments. Such frequencies are mainly determined by SMBH mass and are independent of PBH mass. This indicates that, unlike LIGO, we potentially have the access to a vast mass range of PBHs, which lies outside the mass window of astrophysical COs. Therefore, observation of these events may be used to claim the discovery of PBHs. Moreover, the DM profile peaks at the center of a galaxy, indicating the possibility of a large number density of PBHs in the neighborhood of a SMBH. This may induce a significant EMRI rate caused by PBH-SMBH mergers.

In this letter, we carry out the first study of the event rate estimation for PBH-SMBH mergers, taking into considerations the sensitivities of different experiments. In the next section, we outline the essential ingredients for the calculation. Then we calculate each of them in the later sections. After that, we put everything together and interpret the observable event rate for different experiments as their capabilities to probe PBH-as-DM scenarios. We find these experiments provide us a powerful tool to study a large unexplored parameter space. Not only could the sensitivity to \( f_{\text{PBH}} \) be as good as \( 10^{-3} \sim 10^{-4} \), but also the lower limit of PBH masses that can be probed is potentially far from the astrophysical CO mass region. This could be used to discover PBH
from these GW experiments.

**Ingredients for EMRI Rate Calculation.** EMRI has been carefully studied in the context of astrophysics. In particular, the merger rate between SMBH and astrophysical COs has been calculated. Let us first summarize the key ingredients in this calculation.

The event rate observed by a GW detector can be written as,

\[
\Gamma = \int R(M,\mu) \left( \frac{dn(M,z)}{dM} \right) (p(s,z)ds) \left( \frac{dV_r}{dz} dz \right),
\]

where \(R(M,\mu)\) is the intrinsic EMRI rate in a galaxy hosting a SMBH with mass \(M\). The mass of the CO is \(\mu\). The \(dn(M,z)/dM\) and \(p(s,z)\) are the mass spectrum and spin, \(s\), distribution of SMBHs. They are functions of redshift \(z\) due to the evolution of galaxies. If one only focuses on late times, \(z\)-dependence may be approximately removed. From the popIII model \(31\), most of the SMBHs within the LISA range, i.e. with mass comparable or smaller than \(10^7 M_\odot\), are expected to have near maximal spins \(32\). Further, EMRI rates are calculated with various spin distributions, and the difference appears to be less than 10%. Thus in the following discussion, we fix \(s = 0.999\).

In addition \(dV_r/dz\) is the comoving volume integral as a function of \(z\). Since the GW strength decreases when distance increases, not all EMRI events are detectable by a GW detector. Thus the sensitivity of an experiment imposes a maximum \(z, z_{\text{max}}\), as a function of \((M, s, \mu)\), the details of which we will discuss in later sections.

Among these ingredients, the most non-trivial is \(R(M,\mu)\). The intrinsic EMRI rate can be calculated by solving the Fokker-Planck equation, which describes the diffusion of the CO distribution functions. The result is a function of the mass and density of the CO. Although the precise result has yet to be obtained by numerical calculations, qualitative estimation is possible and agrees well with numerics \(33\).

As far as is known, the detailed numerical calculation on \(R\) is only done assuming COs are white dwarfs, neutron stars and stellar BHs. It is important to derive a reasonable estimation on intrinsic EMRI rate for PBHs whose mass and number density are dramatically different from those of astrophysical COs. We will follow the analysis in \(33\) and present an analytical formula to scale \(R\) for stellar BHs as a function of the PBH’s properties.

In the next few sections, we prepare the ingredients for the calculation of Eq. (1). We first discuss the DM profile, which determines the number density of PBHs near a SMBH. Astrophysical empirical equations are applied to relate DM profiles to SMBH masses. Then we review the calculation of the GW strain from EMRIs. We show sensitivities of various GW detectors and discuss the calculation of signal-to-noise-ratio (SNR). We also consider the subtlety of how detector operation time affects the SNR estimation. After that, we present a detailed analysis of how the intrinsic EMRI rate scales as a function of PBH number density and mass. Last, we put everything together to study the event rate for various GW detectors.

**Dark Matter Halo Profile.** The PBH-SMBH merger rate highly depends on the number density of PBHs around SMBH. EMRIs are mainly produced by COs within the radius of influence of the SMBH \(31\).

\[
r_h = \frac{GM}{\sigma^2} = 2 \text{pc} \left( \frac{M}{3 \times 10^6 M_\odot} \right)^{1/2},
\]

where \(\sigma\) is the velocity dispersion in the bulge, and the following \(M - \sigma\) relation \(35, 37\) is applied:

\[
M = 10^8 M_\odot \left( \frac{\sigma}{200 \text{km/s}} \right)^4.
\]

Since \(r_h\) is \(O(\text{pc})\), the EMRI rate is sensitive to the DM energy density in the innermost region. While collisionless N-body simulations of cold DM indicate a cuspy profile \(38–41\), a cored profile may be obtained if other effects, such as baryonic feedback, are taken into consideration \(42\). On the other hand, assuming adiabatic growth of SMBHs, a spike around the galactic center can be induced \(43, 44\) and is more pronounced for a Kerr SMBH \(45\). Especially, in \(29\), a spike connected to the NFW profile is used to study the PBH-PBH merger rate, which is enhanced as expected. In this letter, we only use the NFW profile \(39, 40\) as an illustration and note that cored (spiky) profiles may lead to smaller (larger) rates.

The NFW profile can be parametrized as

\[
\rho(r) = \frac{\rho_s}{r_c(1 + r/r_c)^2},
\]

where \(\rho_s\) and \(R_c\) are the characteristic density and scale radius, respectively. The enclosed mass within a radius \(R\) (equivalently, the dimensionless radius \(c = R/R_s\)) is

\[
m_{\text{Halo}} = \int_0^{R_{\text{max}}} 4\pi r^2 \rho(r)dr = 4\pi \rho_s R_s^3 g(c_{\text{max}}),
\]

where the function \(g(x) = \ln(1 + x) - x/(1 + x)\) is defined for later convenience. Since \(n_{\text{Halo}}\) diverges, a cutoff radius is conventionally defined such that the enclosed average DM energy density is 200 times the critical density of the universe \(\rho_c\). The DM halo profile can then be specified by the two parameters \(c_{200}\) and \(M_{200}\), where \(M_{200}\) is the enclosed DM halo mass, and \(c_{200}\) is the corresponding radius in units of \(R_s\):

\[
\rho_s = \frac{200}{3} \frac{c_{200}^3}{g(c_{200})} \rho_c; \quad R_s = \left[ \frac{M_{200}}{4\pi \rho_s g(c_{200})} \right]^{1/3}.
\]

Further, at late times in the universe, i.e. at small redshift, \(c_{200}\) and \(M_{200}\) can be related through the
concentration-mass relation [46],

\[ c_{200} = 10^{0.905} \left( \frac{M_{200}}{10^{12} h^{-1} M_{\odot}} \right)^{-0.101}. \]  

(7)

Here \( h = 0.673 \) is the Hubble parameter at present time. The DM halo can then be specified by a single parameter, chosen here as \( M_{200} \). Since Eq. (7) only holds at small \( z \), we truncate the spatial integral in the rate calculation at a maximal distance. More explicitly, we take \( z \leq 1 \) (\( r_0 \leq 3.5 \text{Gpc} \)).

Last, we need the connection between the halo mass \( M_{200} \) and the SMBH mass \( M \). This is given in [17],

\[ \frac{M}{3 \times 10^9 M_{\odot}} \approx 3.3 \left( \frac{M_{200}}{10^{12} M_{\odot}} \right)^{1.65}. \]  

(8)

Therefore, the DM halo profile can be expressed as a simple function of the SMBH mass. We note that the total DM mass within \( r_h \), according to the above NFW profile, is \( \sim 10^{-2} \) of the SMBH mass. Thus the existence of DM can be treated as small perturbation.

**Gravitational Wave Strain and SNR.** Modeling GW emission from an EMRI system is non-trivial. Several formalisms have been studied. For example, the numerical-kludge model [38, 49] is more accurate but computationally expensive. The analytic kludge model (AK) [22, 50], on the other hand, is cheaper but at the price of accuracy. Within AK formalism, the two ways to truncate the calculation are labeled as AKK and AKS, which tend to give optimistic/conservative estimates of SNR. These two choices characterize the uncertainties of the calculation. Last, gravitational wave emission can also be approximately calculated for circular and equatorial EMRIs by solving the Teukolsky equation [51–53]. This method is also used in [54] to estimate the EMRI rate for LISA. Although the orbits of EMRIs generically have moderate eccentricity and are inclined, the result consistently falls between those from AKK and AKS, as shown in [32].

In this letter, we adopt the result from [53] where the GW strain is organized into a set of harmonics \( h_{c,m}(f) \) with \( m \) the harmonic number,

\[ h_{c,1} = \frac{5}{\sqrt{672\pi}} \frac{\eta^{1/2} M}{r_o} \Omega^{1/6} h_{c,1}, \]

\[ h_{c,m} = \frac{\sqrt{5(m+1)(m+2)(2m+1)!m^{2m-1}}}{12\pi(2m-1)!m!(2m+1)!} \frac{\eta^{1/2} M}{r_o} \times \tilde{\Omega}^{(2m-5)/6} h_{c,m}, \quad m \geq 2 . \]  

(9)

The equations are in geometrized units (\( G = 1 \) and \( c = 1 \)). Here \( \eta \) is the ratio of the inspiraling object mass \( \mu \) and SMBH mass \( M \), i.e. \( \eta = \mu / M \). \( r_o \) is the distance from the merger to us. A dimensionless orbital angular velocity \( \Omega \) is defined as \( \Omega = M \Omega = 1/(\tilde{\tau}^{3/2} + s) \) where \( \tilde{\tau} \equiv \tau / M \) with \( \tau \) being the Boyer-Lindquist radial coordinates of the orbit. \( h_{c,m} \) is the relativistic correction and is provided in [53] with various choices of \( s \) and \( r \).

The maximal frequency of GW radiation \( f_{\text{max}} \) occurs at the innermost stable circular orbit (ISCO) at radius \( r_{\text{ISCO}} \), which is a function of \( M \) and \( a \). In Fig. 1 we show \( h_{c,2} \) with different choices of \( \mu \). The experimental sensitivity is quantified by \( h_{n}(f_m) \equiv \sqrt{T S_n(f_m)} \), where \( S_n(f_m) \) is the one-sided noise power spectral density [19]. Optimistic and pessimistic LISA configurations N2A5M5L6 (C1) and N1A1M2L4 (C4) [51] are presented [72]. We also include several other proposed experiments, i.e. Taiji GW project, Big Bang Observer (BBO), DECI-hertz Interferometer Gravitational wave Observatory (DECIGO) [19], and Ultimate-DECIGO (UDECIGO) [50].

It is instructive to make a qualitative comparison between LISA and LIGO at this point. While LISA and LIGO have their best sensitivities at different frequency regimes, \( h_n \) of LISA and LIGO are at a similar order of magnitude. Around \( r_{\text{ISCO}} \), \( h_c \) scales as \( \sqrt{\mu / M} \). The events observed by LIGO have masses as \( O(10) M_{\odot} \). At the same distance, a similar order of magnitude of \( h_c \) can be achieved if \( \mu \sim 10^{-3} M_{\odot} \) when \( M \sim 10^6 M_{\odot} \). This indicates the possibility for LISA-like GW detectors to probe light PBHs.

A GW signal can be detected only if the SNR is above a certain threshold. The SNR can be calculated as

\[ \text{SNR}^2 = S^2 / N^2 = \sum_m \int [h_{c,m}(f_m)]^2 d \ln f_m, \]  

where \( S \) and \( N \) are the signal and noise obtained with matched-filtering [19]. A widely adopted choice of threshold is \( \text{SNR} \geq 15 \).

One subtlety appears when calculating SNR. While the slow inspirals may last for a very long time, e.g. \( O(\text{Gyr}) \),
LISA-like GW detectors can only operate at timescales $\mathcal{O}(\text{yr})$. The GW frequency increases during inspiral and achieves its maximal value $f_{\text{max}}$ when $r \sim r_{\text{ISCO}}$, after which the inspiral stops and the plunge occurs. Only a finite frequency window near the maximal frequency can be recorded during the operation time of an experiment. A truncation needs to be imposed accordingly for the integration range in Eq. (10). This can be calculated by the total time remaining before the plunge \cite{53,57}

$$T = \frac{5}{256} \frac{1}{\mu} \frac{M^2}{\Omega R^3/3},$$

where $\Omega$ is the general relativistic correction with details listed in \cite{53}. Since we are focused on the merger events, setting $T$ to the operation time gives the lower bound of the frequency integral $f_{\text{min}}$. Note for smaller PBH masses, the integration range can be very small since GW radiation power is lower for a lighter CO. Thus light PBHs linger around ISCO for a longer time and the frequency variation is tiny on timescales $\mathcal{O}(\text{yr})$. For light PBHs, the variation of frequency during $\mathcal{O}(\text{yr})$ is small.

Intrinsic EMRI Rate for PBH-SMBH. A CO can change its orbit in two ways: i) gravitationally scattering with another CO object, or ii) losing energy by GW radiation. If gravitational scattering brings a CO to an orbit direct falling into a SMBH, this plunge will not produce a GW observable by LISA-like detectors. On the other hand, if a SMBH-CO merger is induced by GW radiation after many orbits, this results in a slow inspiral which can be potentially detected. This will be our focus. \cite{73}

The intrinsic EMRI rate induced by SMBH-stellar BH slow inspiral has been calculated using the Fokker-Plank equation in \cite{58,60}. The stellar BH mass is set to be $10 M_\odot$, and the number density is taken to be 0.1% of the total number density of astrophysical objects within $r_h$. It can be explicitly written as \cite{60}

$$n_{\text{BH}} = 40 \text{ pc}^{-3} \left(\frac{M}{3 \times 10^6 M_\odot}\right)^{-1/2}.$$  \hspace{1cm} (12)

The intrinsic EMRI rate of such system scales with $M$ as \cite{58,60}

$$\mathcal{R}_{\text{astro}}(M) = 400 G \text{yr}^{-1} \left(\frac{M}{3 \times 10^6 M_\odot}\right)^{-0.15}.$$  \hspace{1cm} (13)

Now we study how Eq. (13) scales as a function of PBH number density and mass.

First, we rescale the number density of PBHs with respect to that of stellar BHs in Eq. (12),

$$\mathcal{G}(M,\mu) = f_{\text{PBH}} \frac{\rho_{\text{NFW}}(M, r_h(M))/\mu}{n_{\text{BH}}(M)}. \hspace{1cm} (14)$$

For example, when $\mu = 10 M_\odot$ and $M = 10^6 M_\odot$, $\mathcal{G}$ is $\mathcal{O}(1)$.

The timescale that brings a PBH to an orbit of slow inspiral can be written as a function of relaxation time $t_h$ at $r_h$. According to \cite{61}, for generic astrophysical objects, the relaxation time is determined by the species with largest $m_i^2 n_i$ where $m_i$ and $n_i$ are the mass and number density of each species. Using the NFW profile, the total mass of the PBH within $r_h$ is only a small fraction. Given the parameter choice in \cite{60}, the relaxation of PBHs is mainly controlled by their scattering with main-sequence stars (MS). Accordingly we expect $t_h$ is approximately independent of PBH mass.

The angular momentum relaxation time can be written as

$$t_J(J, a) = t_h \left[ J \left( \frac{J_m(a)}{J_m(\infty)} \right) \right]^2 \left( \frac{a}{r_h} \right)^p.$$  \hspace{1cm} (15)

Here $a$ is the semi-major axis of an orbit, and $J_m(a) = \sqrt{Ma}$ is the maximal (circular) angular momentum for a specific energy. $p$ is related to the spatial profile of the astrophysical objects which dominate the relaxation process of PBHs, i.e. $n_{\text{MS}} \sim r^{-3/2-p}$.

Now let us estimate the timescale of a slow inspiral. This process lasts a long time, much longer than the period of the orbit. The energy carried away by gravitational radiation per period is $\mathcal{E}_1$ \cite{58,60}:

$$\Delta E = E_1 \left( \frac{J}{J_{\text{ic}}} \right)^{-7}$$

with

$$E_1 = \frac{85}{3} \frac{\pi}{2^{13}} \frac{\mu}{M}; \hspace{0.5cm} J_{\text{ic}} = 4M.$$  \hspace{1cm} (17)

Note the energy and angular momentum are defined in units of PBH mass $\mu$.

For an orbit with high eccentricity, periapse approximately remains a constant, and the time for a CO with initial specific energy $\epsilon_0$ to finish the inspiral is

$$t_0 = \int_{\epsilon_0}^{\infty} \frac{d\epsilon}{d\epsilon/dt} \approx \frac{2 \pi \sqrt{Ma}}{\Delta E} \sim \mu^{-1}.$$  \hspace{1cm} (18)

Here we only pay attention to its dependence on $\mu$ since the goal is to estimate the intrinsic EMRI rate by rescaling Eq. (13).

It is important to ensure that the slow inspiral can continue without being disrupted by further scatterings. A critical value of $a$ is defined by the ratio of $t_0$ and $t_J$, i.e.
\( t_0(J_{lc}, a_c)/t_f(J_{lc}, a_c) = 1 \). For an orbit with \( a < a_c \), a CO has a large chance to fall into SMBH without disruptions. This critical value \( a_c \) is given by,

\[
\frac{a_c}{r_h} = \left( \frac{d_c}{r_h} \right)^{3/2-p} \quad \text{and} \quad d_c = \left( \frac{8\sqrt{ME_1 t_h}}{\pi} \right)^{2/3}.
\]

(19)

Using the analytic solution of the Fokker-Planck equation in \([33]\), one obtains an estimation of the intrinsic EMRI rate for PBHs with arbitrary mass,

\[
\mathcal{R}_{PBH}(M, \mu) = \int_0^{a_c} \frac{da}{\ln(J_m(a))/J_{lc}} \left( \frac{r_h}{a} \right)^p \sim \frac{n_{PBH}(r_h)}{t_h \ln(J_m(a))} \left( \frac{a_c}{r_h} \right)^{3/2-2p} \sim G(M, \mu) \mu^{\frac{4p-1}{2}} \mathcal{R}_{astro}(M).
\]

(20)

where \( n_{PBH}(a) \) is the PBH number density at \( a \) \([74]\).

As shown in Eq. (20), the intrinsic EMRI rate is sensitive to the choice of \( p \), which ranges from 0 to 0.25 \([59, 62, 63]\). To show its effects qualitatively, we present the results with different choices of \( p \) in the next section.

**PBH Constraints.** Finally, to estimate event rate, we take the mass spectrum of SMBHs given in Ref. \([31, 32]\),

\[
\frac{dn}{d\ln M} = 0.005 \left( \frac{M}{3 \times 10^8 M_\odot} \right)^{-0.3} \text{Mpc}^{-3},
\]

(21)

with the range of the SMBH masses taken to be \( 10^4 M_\odot \leq M \leq 10^7 M_\odot \). One can convert the expected observable PBH-SMBH EMRI rate into the sensitivity to PBH energy density fraction of DM, \( f_{PBH} \).

Once such EMRI events are observed, the detailed waveform provides an excellent handle to extract information on the system \([22, 32]\), and \( \mu \) can be measured by analyzing the time-dependence of the orbit. The stellar BHs are expected to have masses ranging from 5 to few tens \( M_\odot \) \([65]\). If PBHs are within the same mass regime, e.g. motivated in \([62]\), stellar BHs may behave as a background of the PBH search. Further, mergers between SMBH and other astrophysical COs, such as neutron stars and white dwarfs, may also contribute as PBH-SMBH background. The mass of white dwarfs (neutron stars) is unlikely to be smaller than 0.6 \( M_\odot (1 M_\odot) \). If PBHs are much lighter than those astrophysical COs, the background is free. In that case, one event observed is enough to declare discovery.

In Fig. 2 with various choices of GW detectors, we present the value of \( f_{PBH} \) which generate one PBH-SMBH EMRI with SNR > 15 during a 5-year operation of the experiment. The dark grey region starts at 3 \( M_\odot \) where stellar BHs begin to contribute as background. From 0.3 \( M_\odot \), white dwarfs and neutron stars become important. We stop our calculation at \( \mu = 10^2 M_\odot \) so that EMRI remains a reasonable approximation, especially for galaxies with light SMBHs (\( 10^4 M_\odot \)). The existing constraints on \( f_{PBH} \) are included, and LISA-like GW experiments have good potential to probe the unexplored parameter space.

There are several important features of this sensitivity curve.

i). When \( \mu \) is not too small, with a sufficiently sensitive GW detector, all EMRIs happening within \( z = 1 \) can be observed. As indicated in Eq. (20), the intrinsic EMRI rate \( \mathcal{R}_{PBH}(M, \mu) \) is independent of \( \mu \) when \( p = 0 \). This explains the flatness of \( f_{PBH} \) curves in the large \( \mu \) regime. When lowering \( \mu \), not all EMRIs exceed the SNR threshold. This produces the turning point which is determined by the detector sensitivity.

ii). As discussed below Eq. (11), for a fixed \( \mu \), smaller \( M \) gives a larger integration range of \( \Delta f/f \) in the calculation of SNR, i.e. \( \Delta f/f \sim 1/M^2 \). Although the gravitational wave strain scales as \( h_n \sim \sqrt{M} \), a better SNR can still be achieved for lighter SMBH assuming \( h_n \) is the same. Given the SMBH mass distribution also increases when \( M \) decreases as shown in Eq. (21), this indicates that a GW experiment may have better sensitivity for lighter PBHs if its best frequency region is higher. This is why the reach of DECIGO is comparable to that of BBO even though its sensitivity is worse in lower frequency.

In Fig. 2, we also study the reach limit with a different choice of \( p \), shown as the dashed curve for LISA(C1).
For $p \neq 0$, the dependence on $\mu$ becomes non-trivial for the intrinsic EMRI rate. When $p$ is positive, the probed region is further extended in the lighter PBH region. As discussed above, $p$ is related to the spatial distribution of the astrophysical objects, presumably MS, and controls the relaxation time. It also affects the EMRI rate of merging SMBHs and ordinary astrophysical COs, the observation of which can help to reduce the uncertainty in our PBH-SMBH rate calculation.

**Discussion.** In this letter, we explore the possibility of using LISA-like GW detectors to look for PBH-SMBH EMRI events. The frequency of the GWs is mainly determined by the mass of SMBH, and a vast range of PBH masses can be probed by such experiments. Especially, a BH much lighter than $0.3 \, M_\odot$ is not expected from astrophysics. The detection of such a SMBH-PBH merger outside the astrophysical CO mass window is potentially enough to declare the discovery of PBHs.

We find that LISA-like GW experiments provide a novel and promising way to test the scenario where PBHs are (a fraction of) DM. The sensitivity to $f_{\text{PBH}}$ in certain mass regimes could be as good as $10^{-3} \sim 10^{-4}$, which is much better than the existing constraints.

Our analysis here initiates the study of PBHs as DM using LISA-like GW detectors which connects astronomy and GW and DM physics. We expect that our current results can be significantly improved with better knowledge from those interdisciplinary areas in the future. For example, we truncate our calculation at $z = 1$ due to the uncertain validity of astrophysical empirical relations, such as Eq. (7) at high redshift. With a better understanding of such a relation, the higher $z$ region could be included, and a much smaller $f_{\text{PBH}}$ may be explored. Furthermore, astrophysical uncertainties, such as mass and spin distributions of SMBHs, would affect the rate estimation. The observation of EMRI events induced by astrophysical COs also provides valuable information. This may have feedback to the PBH calculation and reduce the theoretical uncertainties.

As a final comment, as we discussed above, lighter SMBHs may potentially be more beneficial to search for small mass PBHs, both because of the higher number density from the SMBH mass spectrum as well as the larger integration window on frequency in the SNR calculation. This serves as a guideline for the optimization of a light PBH search in future LISA-like GW experiments.

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[74] For simplicity, we assume PBH and MS share the same power law, i.e.,

\[ p \]

is not difficult to derive a similar formula with different choices on

\[ M \]. It is not difficult to derive a similar formula with different choices on \( p \).