Experimental Consequences of Time Variations of the Fundamental Constants

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Abstract

We discuss the experimental consequences of hypothetical time variations of the fundamental constants. We emphasize that from a purely phenomenological point of view, only dimensionless fundamental constants have significance. Two classes of experiments are identified that give results that are essentially independent of the values of all constants. Finally, we show that experiments that are generally interpreted in terms of time variations of the dimensioned gravitational constant \( G \) are better interpreted as giving limits on the variation of the dimensionless constant \( \alpha G = Gm_c^2/\hbar c \).

Evidence was recently reported by Webb et al. \cite{1} for a time variation of the fine-structure constant \( \alpha = e^2/4\pi\varepsilon_0\hbar c \). The group compared the fine-structure splittings of atomic absorption lines produced by high redshift intergalactic clouds with the same splittings produced by terrestrial atoms. They found a slight difference in the splittings that suggests that \( \alpha \) was lower in the past: \( \Delta\alpha/\alpha = -0.72 \pm 0.18 \times 10^{-5} \). By “past” we mean of order \( 10^{10} \) years ago when the light passed through the clouds. This result is not confirmed by limits in variations of \( \alpha \) over shorter time scales \cite{2} but this could simply indicate a non-linear time variation.

The reported time variations of \( \alpha \) have inspired a variety of theoretical speculations, among them being theories where the speed of light, \( c \) varies with time, thus inducing a variation of \( \alpha \propto c^{-1} \) \cite{3,4}. This has generated a polemic because it is often stated that only dimensionless constants like \( \alpha \) have “physical” significance \cite{4,3,5}. It is perhaps not
sufficiently emphasized that this is due to the experimental nature of physics. Generally speaking, experiments either count events or compare similarly dimensioned quantities. For example, when a length is measured, one really measures the ratio between the length in question and the length of a standard ruler. When an angle is measured, the ratio between the lengths two sides of a triangle is converted to the angle through trigonometry. A velocity is measured by counting the number of “ticks” of a clock as an object moves through a standard distance. A reaction rate is given by counting the number of events during the time that a standard clock gives a standard number of ticks. In all these experiments, only dimensionless numbers are measured. As such, experimental results can only be sensitive to dimensionless combinations of fundamental constants [8].

It is the purpose of this paper, to give a few illustrations of this principle. Following [9], the strategy will be to estimate the complete dependence of an experimental result on the fundamental constants by including their effect on the structure of the experimental apparatus. Once this is done, we can see how the results of an experiment will change with time if the fundamental constants change with time. One of the surprising results will be that there are two classes of experiments that yield results that are largely independent of the values of the fundamental constants or to their time variation.

It is most interesting to start with an experiment that would be naively expected to be sensitive to time variations of $\hbar$. One role of this constant is to relate “particle” properties like momentum to “wave” properties like wavelength. We therefore consider the double slit (Young) interference experiment performed with non-relativistic electrons of momentum $p$. The experiment is performed as in Fig. 1 with, electrons impinging upon a wall with two narrow slits separated by a distance $d$. Beyond the wall, one observes an interference pattern with the angle between interference maxima, $\theta$, determined by $p$, $d$ and $\hbar$:

$$\theta = \frac{\lambda}{d} = \frac{2\pi \hbar}{pd} \tag{1}$$

One is tempted to say that the angle depends on $\hbar$ through the numerator of (1) and that a time variation of $\hbar$ would lead to a time variation of the measured diffraction angle. This is only part of the story since the distance $d$ is determined by the material of the wall whose structure depends on the fundamental constants. Interatomic spacings for
solid materials are generally of order the Bohr radius, \( a_0 = \frac{4\pi \epsilon_0 \hbar^2}{m_e e^2} \) where \( m_e \) and \( e \) are the mass and charge of the electron. This is due to the fact that \( a_0 \) is the only length that can be formed from the three fundamental constants that determine atomic structure: \( \hbar, m_e \) and \( e^2/4\pi \epsilon_0 \). If the fundamental constants change, we can expect that the ratio \( d/a_0 \) is relatively constant. (For crystalline materials, the ratio is basically the number of atomic sites along the distance \( d \)). We therefore write the diffraction angle in the form

\[
\theta = \frac{2\pi \hbar}{pa_0} \frac{1}{d/a_0} = \frac{1}{p} \frac{m_e e^2}{2\epsilon_0 \hbar} \frac{1}{d/a_0} \quad (2)
\]

It is amusing to note that whereas equation (1) gives \( \theta \propto \hbar \), equation (2) gives \( \theta \propto \hbar^{-1} \). However, we are not finished since nature does not usually provide us with electrons of momentum \( p \). Rather, they must be prepared which can be done by passing electrons initially at rest through a potential difference, \( \phi \), so that \( p^2/2m_e = e\phi \). The potential difference can be maintained by placing a charge, \( \pm q \), on each of two square plates of area, \( A = x^2 \), separated by a distance \( z \). This configuration gives \( \phi = N_q e z/\epsilon_0 x^2 \) where...
\( N_q = q/e \) is the number of fundamental charges per plate. As with the interslit distance, \( x/a_0 \) and \( z/a_0 \) should be insensitive to changes in the fundamental constants so we write the momentum in the form \( p^2/2m_e = N_q(e^2/\epsilon_0a_0)(za_0)/x^2 \). Substituting this into \( \theta \) we get the final result:

\[
\theta = \sqrt{\frac{\pi}{2}} \frac{x/a_0}{d/a_0\sqrt{N_qz/a_0}}
\]

To the extent that \( x/a_0, d/a_0 \) and \( z/a_0 \) do not depend on the fundamental constants, the interference angle does not depend on any fundamental constant but only on the number and type of the particles used to construct the experimental apparatus.

This surprising result is, in fact, obvious since an angle is a dimensionless quantity and there is no dimensionless combination of \( \hbar, e^2/4\pi\epsilon_0 \), and \( m_e \). Measurements of angles using non-relativistic electrons and performed with apparatuses whose size depend only on atomic structure are therefore “constant-free”. Among such experiments are the Young experiment discussed here and the Davisson-Thomson experiment on electron diffraction by crystals. These quintessential quantum experiments give results that are largely independent of the parameters of the theory (e.g. of \( \hbar \)) and depend only on the fact that the dynamics is governed by quantum mechanics. In the distant future when variations of the constants are routinely monitored at the N.I.S.T., these experiments could serve as fiducial experiments to check systematics.

Of course relativistic (fine-structure) corrections to interatomic spacings would lead to a small dependence of the diffraction angles on \( \alpha \). We also note that the time variation of a constant, say of \( e^2 \), would introduce a new constant \( \tau_e = e/\dot{e} \) that gives the time scale over which \( e^2 \) changes be a significant amount. With this new constant we can now form the dimensionless quantity \( (e^2/\hbar)a_0/\tau_e \) which is just the fractional change in \( e \) during one Bohr revolution. Depending on the dynamics that drives the variation of the constants, the interference angle could conceivably depend on some power of this quantity. On the other hand, if \( \tau_e \) is of a cosmological scale, the parameter is tiny and we might expect that the diffraction angle is practically unaffected.

Our experiment to search for time variations of \( \hbar \) can be criticized because the size of the apparatus depends on \( \hbar \). We can avoid this criticism by using the classical electron radius, \( r_e = e^2/(4\pi\epsilon_0m_ec^2) = \alpha^2a_0 \), to define the dimensions of the experimental apparatus.
We then write (3) as
\[ \theta = \sqrt{\frac{\pi}{2}} \alpha^{-1} \frac{x/r_e}{d/r_e\sqrt{N_qz/r_e}} \] (4)
The electron diffraction angle now depends on \( \alpha \) if the distances are maintained as multiples of \( r_e \). Experimentally, this is difficult but can be done by filling a small cube of volume \( l^3 \) with a known number \( N \) of electrons. If the cube is uniformly irradiated by photons of energy \( E_\gamma \ll m_e c^2 \), the probability that a photon is scattered is \( P = N\sigma_T/l^2 \) where \( \sigma_T = 8\pi r_e^2/3 \) is the Thompson scattering cross section. (We take \( N \) small enough so that \( P \ll 1 \).) The probability \( P \) can be measured so the length \( l \) can be defined as a multiple of \( r_e \). This length can then be compared with the lengths in an experimental apparatus and an appropriate expansion-contraction scheme can insure that all lengths remain a fixed multiple of \( r_e \) even if the constants vary.

We now come back to the original, more practical, experimental arrangement using solid materials to define the apparatus. The diffraction angle cannot depend on the constants because there is no dimensionless combination of the relevant constants. We can make \( \theta \) depend on fundamental constants by accelerating protons instead of electrons. In this case the angle is given by (3) multiplied by \( \sqrt{m_p/m_e} \) where \( m_p \) is the proton mass. The presence of two constants with dimensions of mass leads to a new dimensionless quantity \( m_p/m_e \) upon which the angle can depend.

The interference angle can also depend on the fundamental constants if we use photons, in which case we introduce the speed of light, \( c \), into the problem. For example, if we use photons from the \( n = 2 \) to \( n = 1 \) transition of atomic hydrogen, we have a photon momentum of \( p_\gamma = (3/8)\alpha^2 m_e c \) giving
\[ \theta = \frac{16\pi}{3} \alpha^{-1} \frac{1}{d/a_0} \] (5)
A time variation of the fine-structure constant would then yield a time dependence of the diffraction angle.

Having shown that a quintessential quantum experiment gives results that are independent of \( \hbar \), we will now consider the quintessential relativistic effect, the twin paradox. Two identical clocks are needed, one in free fall, and the second departing from the first with velocity \( v \), stopping, and then returning to the first with the same velocity \( v \). The
number of ticks on the two clocks, $N_1$ and $N_2$, counted during the time interval are related by

$$\frac{N_2}{N_1} = \sqrt{1 - \frac{v^2}{c^2}}.$$  \hfill (6)

We can now ask how this ratio will change if the fundamental constants vary while maintaining the validity of the basic formula. The problem is to give a prescription for determining the velocity $v$. If $v$ is determined by comparing the velocity of the clock with $c$, we will clearly never detect any effect since clocks of a given $v/c$ will always give the same $N_2/N_1$. To detect an effect, we must define the velocity differently. For instance, we can use clocks with the same velocity as that of the electrons produced in Figure 1

$$\frac{v^2}{c^2} = 8\pi N_q \alpha^2 \frac{z a_0}{x^2}$$  \hfill (7)

This shows that if the experiment is built from solid objects so that $z a_0/x^2$ is insensitive to changes in the fundamental constants, then a time variation of $N_2/N_1$ should be interpreted as a time variation of $\alpha^2$. On the other hand, if we design the experiment so that $z$ and $x$ are fixed multiples of the classical electron radius $r_e = \alpha^2 a_0$, then the ratio $N_2/N_1$ is constant-free. This is because there is no dimensionless combination of the relevant constants, $e^2/4\pi \varepsilon_0$, $m_e$ and $c$. Just as non-relativistic quantum experiments give results that are $\hbar$ independent, non-quantum relativistic experiments can give results independent of $c$.

Since the twin paradox experiment cannot give unambiguous information on the time variation of $c$, we will now consider experiments that directly measure $c$. One might hope that a time variation of $c$ would lead to a time variation of the ratio between the flight time $x/c$ over a distance $x$ and the period $T$ of a standard clock. This amounts to counting the number of ticks $= (x/c)/T$ that the clock makes during the time the photon travels the distance $x$.

If we define the distance $x$ by photon time-of-flight as is now done in SI units, we will obviously not be able to detect a change in $c$. We will then go back to the old procedure of defining lengths in terms of physical rods. We therefore write

$$\frac{x/c}{T} = \frac{a_0/c}{T} \frac{x}{a_0} = \frac{\hbar}{\alpha m_e c^2 T} \frac{x}{a_0}$$  \hfill (8)
and suppose that the ratio $x/a_0$ is insensitive to changes in the constants.

The ratio $x/cT$ will now depend only on the clock that we choose to use. Atomic clocks based on hyperfine splittings of atomic lines have periods of order

$$T_{hf} \propto \frac{\hbar}{\alpha^4 g (m_e/m_p) m_e c^2}$$

where $g$ is the nuclear gyromagnetic ratio. This gives

$$\frac{x/c}{T} = g \alpha^3 \left( \frac{m_e}{m_p} \right) \frac{x}{a_0}$$

If $x/a_0$ is constant-free, this attempt to detect a change in $c$ yields the time variation of $g \alpha^3 m_e/m_p$. It is basically the technique of Turneaure and Stein [3] who, by looking for the desychronization of a superconducting cavity oscillator and an atomic clock, set an upper limit of $4.1 \times 10^{-24} \text{yr}^{-1}$ on the logarithmic derivative of $g \alpha^3 m_e/m_p$.

The use of a clock based on hyperfine splittings to measure variations of $c$ can be criticized because the clock period depends on a relativistic effect and therefore explicitly on $c$. It is possible to find quantum processes that have periods that are $c$-independent. The first type gives periods that are multiples of the rotational period of an electron in a Bohr orbit:

$$T_e = \frac{a_0}{e^2/4\pi \epsilon_0 \hbar} = \frac{\hbar}{\alpha^2 m_e c^2}$$

An example of such a clock is a mechanical vibrator. The sound speed of a crystal consisting of nuclei of mass $\sim Am_p$ is of order $(a_0/T_e)(m_e/Am_p)^{1/2}$ so a rod of length $L$ has a fundamental period $\propto (m_p/m_e)^{1/2}(L/a_0)T_e$. The use of such a clock would give

$$\frac{x/c}{T} = \alpha (m_e/m_p)^{1/2} \frac{x}{L}$$

Since $x/L$ is constant-free for material objects, a variation of the measured speed of light would then be interpreted as a variation of the quantity $\alpha (m_e/m_p)^{1/2}$.

A second class of $c$-independent periods can be found by replacing $e^2/4\pi \epsilon_0$ with $Gm_p^2$ and $m_e$ by $m_p$:

$$T_G = \frac{\hbar^3}{G^2 m_p^5} = \frac{\hbar}{\alpha_G^2 m_p c^2}$$

where $\alpha_G = Gm_p^2/\hbar c$ is the gravitational equivalent of the fine-structure constant. An example of a clock whose period is a multiple of $T_G$ uses a particle revolving near the surface of a totally degenerate (white dwarf) star consisting of $N_e$ electrons and $N_p$ nucleons.
The radius of such a star is \( R \sim \bar{h}^2 N_e^{5/3} / (Gm_e m_p^2 N_p^2) \) and the particle has a revolution period of

\[
T \sim N_p^{-1} \left( \frac{m_p}{m_e} \right)^{3/2} \left( \frac{N_p}{N_e} \right)^{5/2} T_G \tag{14}
\]

Substituting this into (8) gives

\[
\frac{x}{cT} \sim \alpha G \left( \frac{m_e}{m_p} \right)^{1/2} N_p \left( \frac{N_p}{N_e} \right)^{5/2} (x/a_0) \tag{15}
\]

A time variation of the measured speed of light using this clock would be interpreted as a variation of \((m_e/m_p)^{1/2} \alpha G/\alpha\).

We end with a comment on searches for time variations of the dimensional gravitational constant \( G \). One way to do this is to search for anomalies in the movement of objects in the Earth’s gravitational field [2]. The period \( P \) of an object in a circular orbit of radius \( r \) is given by

\[
P^2 = \frac{4\pi^2 r^3}{GM_\odot} = \frac{4\pi^2 r^3}{GN_\odot m_p} \tag{16}
\]

where in the second form we approximate the solar mass by \( N_\odot m_p \) where \( N_\odot \) is the number of nucleons in the Sun. If \( G \) varies in time we can expect \( P \) to vary in time. Of course, there are two problems here. First, what one would actually observe is time variation of the ratio of \( P \) and the period \( T \) of a standard clock. Second, a time variation of \( G \) might be expected to yield a time variation of the orbital radius. If the radius changes, one must extrapolate the period back to the original radius using (16). To do this, we need to measure \( r \), which can be done with radar, \( r = ct \), where \( t \) is the time for a photon to travel the distance \( r \). (We ignore small general relativistic corrections.) The time \( t \) can then be fixed to be a given number \( N \) of periods of the standard clock, \( r = cNT \). The ratio of the orbital period (fixed \( r \)) to the clock period is now given by

\[
\frac{P^2}{T^2} = \frac{4\pi^2 N_\odot^3}{c^3 T} \left( \frac{c^3}{G m_p} \right) = \frac{4\pi^2 N_\odot^3}{\alpha G \alpha^4 g(m_e/m_p)^2} \tag{17}
\]

where in the second form we use \( T = T_{hf} \) appropriate for atomic clocks. An anomalous variation of an orbital period can then be interpreted as a variation of \( \alpha G \alpha^4 g(m_e/m_p)^2 \).

While not claiming to have analysed all experiments used to limit the variations of \( G \), this suggests that these limits can all be interpreted as limits on the variation of \( \alpha G \) and some combination of \( \alpha \) and various mass ratios and gyromagnetic ratios.
Since non-gravitational methods yield limits on $\alpha$, $m_e/m_p$ and $g$ [9, 2], that are considerably stricter than the limits on time variations based on orbital anomalies, an observed variation of an orbital period can be safely interpreted simply as a variation of $\alpha G$. Traditionally, limits on orbital anomalies have been interpreted as limits on the time variation of $G$ with the caveat “assuming all other constants non-varying.”

In summary, we see that only dimensionless parameters have phenomenological significance. It should however be emphasized that theories are generally expressed in ways that use dimensioned parameters. If nature is such that the dimensionless parameters objectively vary with time, it is conceivable that the underlying dynamics is most simply expressed as a time variation of one or more dimensioned parameters.

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