Energy gap of ferromagnet-superconductor bilayers

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Abstract

The excitation spectrum of clean ferromagnet-superconductor bilayers is calculated within the framework of the self-consistent Bogoliubov-de Gennes theory. Because of the proximity effect, the superconductor induces a gap in the ferromagnet spectrum, for thin ferromagnetic layers. The effect depends strongly on the exchange field in the ferromagnet. We find that as the thickness of the ferromagnetic layer increases, the gap disappears, and that its destruction arises from those quasiparticle excitations with wavevectors mainly along the interface. We discuss the influence that the interface quality and Fermi energy mismatch between the ferromagnet and superconductor have on the calculated energy gap. We also evaluate the density of states in the ferromagnet, and we find it in all cases consistent with the gap results.

Key words: Energy gap, Proximity effect, Bilayers, Superconductivity

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1 Introduction

Through the mechanism of Andreev reflection[1], a normal (non-superconducting) material in contact with a superconductor acquires superconducting phase co-
herence between its particle and hole wave functions, even though the pairing interactions vanish in this normal material. The associated superconducting correlations give rise to a finite value of the pair amplitude, \( F(\mathbf{r}) = \langle \psi^+_\uparrow(\mathbf{r}) \psi^+\downarrow(\mathbf{r}) \rangle \). This is known as the proximity effect.

The proximity effect occurs also, in a modified way, if the non-superconductor is a ferromagnet. In this case, \( F(\mathbf{r}) \) exhibits spatial oscillations. On the other hand, the pair potential itself, \( \Delta(\mathbf{r}) \) vanishes in the ferromagnet in the absence of attractive coupling. For conventional isotropic superconductors, \( \Delta \) is spatially independent, and corresponds to the minimum excitation energy in the spectrum, or the energy gap, \( E_g \). For ferromagnet-superconductor systems, as in other inhomogeneous cases, \( \Delta(\mathbf{r}) \) depends on position, while the excitation gap for the whole system, \( E_g \), obviously does not. The proximity effect can therefore result in \( E_g \) and \( \Delta \) possessing a rather nontrivial relationship. In this paper we study this question for ferromagnet-superconductor (F/S) bilayers.

Several works involving nonmagnetic normal metal-superconductor heterostructures have discussed in various contexts, the presence or absence of an energy gap in the excitation spectrum.[2,3,4,5,6] For a normal metal layer of width greater than the coherence length in the superconductor, \( \xi_0 \), but smaller than the phase coherence length \( L_\phi \), the appearance of a minigap of order of the Thouless energy has been reported, while if the normal layer is smaller than \( \xi_0 \), the induced gap is of order \( \Delta \). It was found that while interface roughness plays a role, the minigap persistence depends chiefly upon interface quality.[7] These conclusions draw upon the results of calculations carried out within the quasiclassical formalism, which is applicable when certain conditions are satisfied. The main results are modified if the normal metal region is modeled as a non-Fermi liquid, where increased electron correlations may inhibit or completely destroy the minigap altogether.[8]

For F/S structures, the magnetic exchange field in the ferromagnet results in quasiparticle spin splitting, and consequently the pair amplitude rapidly decays and oscillates over a characteristic length scale \( \xi_F \), in which electron-hole pairs remain coherent. This modulation in phase of the pair amplitude has been shown to induce corresponding oscillations in the local density of states (DOS).[9,10,11,12] Impurities in the magnet were shown to play a significant role in the determination of thermodynamic quantities.[9,11] For ballistic F/S structures, the tunneling DOS for clean ferromagnet layers has been calculated for a wide range of exchange fields,[12] and the local DOS has been found to exhibit a gapless state for all positions in the magnet.[10,13] Configurations consisting of weak or thin ferromagnetic layers where \( d_F/\xi_F \ll 1 \), may lead to significantly different results,[14] as we shall see below. The finite geometry can result in the quasiparticle amplitudes undergoing coherent Andreev and normal reflection (at the ferromagnet-insulator interface) in such a way that the minimum value of the excitation spectrum is nonzero due to the relatively
slow characteristic decay of the pair amplitude. Thus, a gap develops. Associated with this, when the dimensions of the ferromagnet are reduced to a scale comparable to $\xi_F$, the greater confinement of the pairing correlations in the ferromagnet restricts the pair amplitude so that it no longer changes sign, and the DOS does not exhibit the corresponding oscillations.

It is generally accepted that the quasiclassical and dirty limit equations are satisfactory for studying many aspects of inhomogeneous superconductivity. If however, the ferromagnet has dimensions of order of or smaller than the elastic mean free path $\ell$, so that quasiparticles undergo very few scattering events, the problem is more suitably solved within the ballistic regime. Furthermore, the inherent geometrical effects associated with ferromagnet layers only a few atomic spacings thick require a full microscopic theory that does not coarse-grain over length scales of order of the Fermi wavelength $\lambda_F$. Another constraint imposed upon the quasiclassical approximation is exhibited by the Andreev[1,15] equations, whereby values of the quasiparticle momentum parallel to the interface, and comparable to the Fermi momentum, are crudely approximated. We will see below that in fact these quasiparticle states play a prominent role in the determination of the gap threshold for F/S systems.

Thus, we examine here the electronic spectrum of F/S bilayers and investigate under what conditions an energy gap can exist in such structures. The problem will be solved within the framework of the self-consistent solutions of the Bogoliubov de-Gennes[16] (BdG) equations. We will calculate self-consistently the DOS, and excitation spectra of a thin ferromagnet film adjoining a bulk superconductor. This microscopic approach complements existing quasiclassical theories and serves to round the current knowledge.

In the next section, we introduce the method and geometry used in our model. In Sec. 3, the influence of the interface transparency, the magnitude of the exchange field, and the mismatch in Fermi levels on the energy gap and local DOS, will be presented.

2 Geometry and model

We consider a heterojunction of total length $d$ in the $z$-direction, consisting of a ferromagnet of thickness $d_F$ and a superconductor of thickness $d_S$. The planar interface is at $z = d_F$, and the free surfaces at $z = 0$ and $z = d$ are specularly reflecting. Upon taking into account the translational invariance in the semi-infinite $x - y$ plane, one can immediately write down the BdG equations[16,10] for the spin-up and spin-down quasiparticle and quasihole wave functions $(u_n^\uparrow, v_n^\uparrow)$,
\[
\begin{align*}
\left[-\frac{1}{2m} \frac{\partial^2}{\partial z^2} + \varepsilon_\perp + W(z) - E_F(z)\right] u_n^\uparrow(z) + \Delta(z) v_n^\downarrow(z) &= \epsilon_n u_n^\uparrow(z), \quad (1) \\
-\left[-\frac{1}{2m} \frac{\partial^2}{\partial z^2} + \varepsilon_\perp + W(z) - E_F(z)\right] v_n^\downarrow(z) + \Delta(z) u_n^\uparrow(z) &= \epsilon_n v_n^\downarrow(z), \quad (2)
\end{align*}
\]

where \(\varepsilon_\perp\) is the transverse kinetic energy, \(\epsilon_n\) are the quasiparticle energy eigenvalues, \(h_0(z) = h_0 \Theta(z - d_F)\) is the magnetic exchange energy, and \(\Delta(z)\) is the pair potential. Scattering at the interface is modeled by the potential \(W(z) = W \delta(z - d_F)\), where \(W\) is the barrier strength parameter. We take the quantity \(E_F(z)\) to equal \(E_{FM}\) in the magnetic side, so that \(E_{F \uparrow} \equiv E_{FM} + h_0\), and \(E_{F \downarrow} \equiv E_{FM} - h_0\), while in the superconducting side, \(E_F(z) \equiv E_F\). From the symmetry of the problem, the solutions for the other set of wavefunctions \((u_n^\downarrow, v_n^\uparrow)\) are easily obtained from those of Eqns. (1,2) by allowing for both positive and negative energies. The BdG equations are supplemented by the self consistency condition for the pair potential,

\[
\Delta(z) = \frac{g(z)}{2} \sum_{\epsilon_n \leq \omega_D} \left[ u_n^\uparrow(z) v_n^\downarrow(z) + u_n^\downarrow(z) v_n^\uparrow(z) \right] \tanh(\epsilon_n/2T),
\]

where \(T\) is the temperature, \(g(z)\) is the effective coupling constant, describing the electron-electron interaction, and \(\omega_D\) is the Debye energy.

## 3 Results

In the following, we consider a thin ferromagnet layer adjacent to a bulk limit \((d_S \gg \xi_0)\) superconductor. To focus on the parameter range which may yield a gap in the energy spectrum, the thickness of the ferromagnet considered will not exceed the coherence length in the superconductor, i.e., \(d_F < \xi_0\). We introduce the dimensionless parameter \(Z\) to characterize the barrier strength, with \(Z \equiv mW/k_F\), and the measure of mismatch between the Fermi levels is given by the ratio \(\Lambda \equiv E_{FM}/E_F\). The requirement of self-consistency in the problem excludes the possibility of an analytical solution. The BdG equations (1,2) are therefore solved numerically using the approach[10,17] outlined in the Appendix. We assume the low temperature regime (we take \(T/T_c = 0.02\), and a superconducting coherence length \(\xi_0\) of \(k_F\xi_0 = 50\), where \(k_F\) is the Fermi wavevector of the superconductor.

The energy gap, \(E_g\), is the minimum binding energy of a Cooper pair, and its existence will influence various thermodynamic measurements, including heat capacity and thermal conductivity. For other configurations, such as long superconductor-normal metal-superconductor junctions, the gap in the metal and the bulk gap in the superconductor can further inhibit thermal transport. The process of finding \(E_g\) is computationally demanding, as it involves cal-
Fig. 1. Variation of the normalized energy gap in the ferromagnet with the dimensionless ferromagnet thickness $k_F d_F$. The coherence length is given by $k_F \xi_0 = 50$. The different curves correspond to the four values of the magnetic exchange energy labeled in the figure.

calculating self-consistently the entire eigenvalue spectrum and then finding its minimum, for each value of $d_F$. In Fig.1 we illustrate our results for the dependence of $E_g$ on the thickness of the ferromagnet layer. These results are for $Z = 0$ and $\Lambda = 1$. Results for four different exchange fields in the ferromagnet are presented. The top dashed curve corresponds to $h_0/\Delta_0 = 0$, while the subsequent lower curves are for finite values of the exchange field as labeled in the figure. All curves start at $E_g = \Delta_0$ for $d_F = 0$, corresponding to the expected result for a single superconductor in the bulk limit. As the ferromagnet layer grows, $E_g$ decays monotonically towards a gapless superconducting state, at a rate that depends strongly on the strength of $h_0$. In the limit of zero exchange field (normal metal), the gap ceases to exist when $d_F \approx \xi_0$. This thickness at which $E_g$ is destroyed is much smaller than the proximity length characterizing the usual very slow decay of the pair amplitude in the metal.[18] This trend continues for finite $h_0$, with $E_g$ vanishing much more rapidly than the characteristic decay length of the pair amplitude in the magnet. To illustrate this, recall[17,19] the approximate expression for the characteristic length of decay of the pair amplitude in the ferromagnet: $\xi_F \approx (k_F^+ - k_F^-)^{-1}$, which
Fig. 2. Part of the normalized self consistent quasiparticle spectrum as obtained for given values of the normalized transverse energy $\varepsilon_\perp/E_F$. The parameter $h_0/\Delta_0$ is chosen to be 0.8, and $k_F d_F = 15$. The region $|\varepsilon_n/\Delta_0| > 1$ is dominated by the continuum states, these are suppressed for clarity.

can be rewritten as,

$$k_{FM}\xi_F \approx \frac{\pi}{2} \Lambda^{3/2} \left( \frac{\Delta_0}{h_0} \right) k_F \xi_0.$$  \hspace{1cm} (4)

Thus for example, the $h_0/\Delta_0 = 4.0$ curve in Fig. 1 shows the energy gap vanishing at $k_{FM} d_F \approx 7.5$, while the pair amplitude decays over the larger length scale $k_{FM} \xi_F \approx 20$. The discrepancy is even larger if one recalls that the expected decay length might be thought to be a factor of $2\pi$ larger. For the largest field shown ($h_0/\Delta_0 = 8$), the superconducting correlations are further inhibited, failing to generate a gap in the energy spectrum except for ferromagnet nanostructures satisfying $k_F d_F \lesssim 5$.

To provide additional perspective on the spectral features of the proximity effect, we show in Fig. 2 a relevant portion ($|\varepsilon_n/\Delta_0| \leq 1$) of the calculated eigenvalues as obtained (under the same conditions as in the previous Figure) for given transverse energies, $\varepsilon_\perp$. For a homogeneous superconductor, a gap separates the continuum states that exist for $|\varepsilon_n/\Delta_0| > 1$. With the inclusion of a ferromagnet, in addition to the scattering states there exists a discrete quasiparticle excitation spectrum in the range $|\varepsilon_n/\Delta_0| < 1$ (see Fig. 2). The bound states are visibly asymmetric about the Fermi level due to the effects
the exchange field has on the particle-hole amplitudes. The existence of a finite number of bound-state branches of localized quasiparticles is also characteristic of antisymmetric domain walls.[20] In Fig. 2 we can further see that the minimum in $|\epsilon_n|$, which corresponds to the energy gap seen in Fig. 1 for $k_Fd_F = 15$, where $E_g \approx 0.09\Delta_0$, is sharp, and that it occurs for quasiparticles whose in-plane momentum is close to the Fermi level ($\epsilon_\perp/E_F \approx 1$). This is due in part to those quasiparticles not coupling to those states responsible for superconductivity. This result is typical of what we find for other exchange fields, the only difference being the value of $E_g$ [see Fig. 1]. It is therefore those excitations with a dominant momentum component parallel to the interface, that are significant in contributing to the filling in of the gap. In the Andreev or quasi-classical approximation scheme, these quasiparticles have “trajectories” which are not treated accurately, and those with $\epsilon_\perp/E_F > 1$ are neglected altogether. This discrepancy has also been revealed[15] in normal metal-superconductor systems, where normal reflection dominates the Andreev reflection process for quasiparticles with large momentum parallel to the interface.

The previous results in Fig. 1 showed the strong influence the exchange energy has on $E_g$. Another quantity of great experimental interest is the local density of one particle excitations in the magnet. Current experimental tools such as the scanning tunneling microscope (STM) have atomic scale resolution, and make this quantity experimentally accessible. When well defined quasiparticles exist, the tunneling current is simply expressed as a convolution of the one-particle spectral function of the STM tip with the spectral function for the F/S system.[21] The resultant tunneling conductance, which is proportional to the density of states (DOS), is then given as a sum of the individual contributions to the DOS from each spin channel.

$$N(z, \epsilon) = N_\uparrow(z, \epsilon) + N_\downarrow(z, \epsilon),$$  

(5)

where the local DOS for each spin state is given by

$$N_\uparrow(z, \epsilon) = -\sum_n \left\{ [u_n^\uparrow(z)]^2 f'(\epsilon - \epsilon_n) + [v_n^\uparrow(z)]^2 f'(\epsilon + \epsilon_n) \right\},$$  

(6)

$$N_\downarrow(z, \epsilon) = -\sum_n \left\{ [u_n^\downarrow(z)]^2 f'(\epsilon - \epsilon_n) + [v_n^\downarrow(z)]^2 f'(\epsilon + \epsilon_n) \right\}. $$  

(7)

Here thermal broadening is accounted for by the derivative of the Fermi function, $f'(\epsilon) = \partial f/\partial \epsilon$. To investigate further how the previous results interrelate with the local DOS (Eq. 5), we show in Fig. 3 the local DOS $N(\epsilon)$, defined as $N(z, \epsilon)$ integrated over the ferromagnetic layer. This quantity is shown for two exchange fields, differing by a factor of ten. The ferromagnet has a thickness of $d_F = 9k_F^{-1}$: from Fig. 1 this corresponds to an energy gap of $E_g \approx 0.5$ for $h_0/\Delta_0 = 0.8$, and an absence of an energy gap for $h_0/\Delta_0 = 8$. This is
consistent with the DOS calculations, as Fig. 3 illustrates: for the smaller exchange field, there is a complete absence of states for $\epsilon/\Delta_0 \lesssim 0.5$, while the DOS is gapless at all energies for $h_0/\Delta_0 = 8$. The observed energy gap for $h_0/\Delta_0 = 0.8$ is a result of the underlying competition between quantum size effects and constructive interference between the spin-split quasiparticle amplitudes. As expected, the particle-hole asymmetry is more prevalent in the stronger magnet (within the subgap region), as the two uppermost peaks are clearly shifted relative to one another.

Next we consider the effect that an interface barrier or Fermi energy mismatch has on the electronic structure. Experimentally, some degree of Fermi energy mismatch is unavoidable, particularly with alloys, and the transparency of the interface can be degraded due to a thin oxide layer. In Fig. 4 we illustrate the normalized energy gap $E_g$ as a function of $k_{FM}d_F$ (where $k_{FM}$ is the wavevector corresponding to $E_{FM}$), for two $\Lambda$ and $Z$ values that each differ by a factor of three. We fix the exchange field at $h_0 = 0.8\Delta_0$, keeping in mind that higher fields obey similar trends so that generality is not lost. Also, in
Fig. 4. Influence of interface scattering and Fermi energy mismatch on the energy gap as a function of the ferromagnet thickness. Both the effects of increasing the mismatch (decreasing $\Lambda$ from unity, see text) and increasing scattering strength (increasing $Z$) serve to destroy the energy gap $E_g$ at an increased rate.

In order to isolate each of the effects, whenever $\Lambda$ is different from unity, $Z$ is zero, and vice versa. The figure shows that increased mismatch (smaller $\Lambda$) or increased interface scattering (larger $Z$), induces the same trend of reducing $E_g$ as a function of $k_{FM}d_F$. The functional form of the graphs are distinct however: for $\Lambda < 1$, the tail-end decay is slower than the near linear decay in $E_g$ exhibited in the case of finite $Z$. Further differences in the effects of the two parameters are even more evident in the DOS spectra. To this end, Fig. 5 shows the DOS in the ferromagnet for two different values of the mismatch parameter $\Lambda$, and one value of $Z$. Upon comparison with the previous Fig. 4, we again find consistency between the DOS and $E_g$ calculations: the energy gap is reduced at $k_{FM}d_F = 10$ for both $\Lambda = 0.2$, and $Z = 0.2$. This is reflected in the DOS, where Fig. 5 illustrates a small but finite number of states centered about the Fermi level. Although the quasiparticle spectrum has just turned gapless for $\Lambda = 0.2$, and $Z = 0.2$, the DOS signature is substantially different, the most prominent feature being the larger number of subgap bound states for $\Lambda = 0.2$, which can be attributed to the larger Fermi energy mismatch, and subsequently, increased normal reflections.
4 Conclusions

We have self-consistently calculated the electronic structure for a F/S heterostructure consisting of a thin ferromagnet layer adjoining a bulk superconductor. Our fully microscopic method revealed the dependence of the transition to a gapless superconducting state on the relevant physical parameters such as exchange field energy, interface transparency, and Fermi energy mismatch. The energy gap decreases with the width of the F layer at a rate that depends upon the dimensionless ferromagnet width \( k_{FM}d_F \). The characteristic length for gap suppression is much smaller than the length scale describing the decay of the pair amplitude in the ferromagnet: thus \( E_g \) is suppressed more rapidly than the pair amplitude. Fermi level mismatch and interface barrier both serve to reduce the gap further, however the functional forms of the respective reductions are different. The local DOS in the ferromagnet is in all cases consistent with the above observations. The self-consistent quasiparticle spectra reveals that the gap onset is due to quasiparticle states with a large momentum component parallel to the interface. These states however, occupy
a relatively small region of momentum space, and when calculating integral quantities over the full solid angle, good agreement between our exact theory and quasiclassical theory might be possible.

We have focused here on the spectral properties of the ferromagnet, in the clean limit. This regime is most appropriate for layers whose dimensions do not exceed the mean free path \( d_F \lesssim \ell \). Since, we have considered only thin layers, this assumption is consistent with our calculations. For situations where the effects of impurities or inelastic effects may be important, a finite DOS at low energies would likely arise. The general method used here however has found good overall agreement with experiments involving thin nonmagnetic normal metal-superconductor bilayers, and bulk F/S structures.[17] Furthermore, previous calculations[13,22] within the ballistic limit have also found agreement with experiment. Other complexities may arise in actual experimental conditions, such as grain boundaries and domain walls not accounted for here. The fabrication of relatively clean heterostructures,[23,24] and structures with highly transparent interfaces[25] is possible however. Our calculations are therefore realistic, and the results should be reproducible experimentally.

\section{A Numerical method}

We solve Eqns. (1,2) by expanding the quasiparticle amplitudes in terms of a finite subset of a set of orthonormal basis vectors, \( u_n^\uparrow(z) = \sum_q u_{nq}^\uparrow \phi_q(z) \), and \( v_n^\downarrow(z) = \sum_q v_{nq}^\downarrow \phi_q(z) \). We use the complete set of eigenfunctions \( \phi_q(z) = \langle z | q \rangle = \sqrt{2/d} \sin(k_q z) \), where \( k_q = q/\pi d \), and \( q \) is a positive integer. The finite range of the pairing interaction \( \omega_D \) permits the number \( N \) of such basis vectors to be cut off in the usual way.[10] Once this is done, we arrive at the following \( 2N \times 2N \) matrix eigensystem,

\[
\begin{bmatrix}
H^+ & D \\
D & H^-
\end{bmatrix}
\Psi_n = \epsilon_n \Psi_n,
\]

where \( \Psi_n^T = (u_{n1}^\uparrow, \ldots, u_{nN}^\uparrow, v_{n1}^\downarrow, \ldots, v_{nN}^\downarrow) \). The matrix elements \( H_{qq'}^+ \) connecting \( \phi_q \) to \( \phi_{q'} \) are constructed from the term found in brackets in Eq.(1),

\[
H_{qq'}^+ = \left\langle q \left| -\frac{1}{2m} \frac{\partial^2}{\partial z^2} + \varepsilon_\perp + W(z) - E_F(z) \right| q' \right\rangle = \left[ \frac{k_q^2}{2m} + \varepsilon_\perp \right] \delta_{qq'} + \int_0^d dz \phi_q(z) W(z) \phi_{q'}(z) - E_F^+ \int_0^{d_F} dz \phi_q(z) \phi_{q'}(z)
\]
The expression for $H_{qq'}^{-}$ is calculated similarly using Eq.(2). The “off-diagonal” matrix elements $D_{qq'}$ are given as,

$$\begin{align*}
D_{qq'} &= \langle q | \Delta(z) | q' \rangle = \int_{d_F}^{d} dz \phi_q(z) \Delta(z) \phi_{q'}(z).
\end{align*}$$

(A.3)

After performing the integrations, Eq.(A.2) can be expressed as

$$\begin{align*}
H^+_{qq'} &= \frac{2W}{d} \sin(k_q d_F) \sin(k_{q'} d_F) - \frac{E_F}{d} \left[ \sin((k_{q'} - k_q)d_F) - \sin((k_{q'} + k_q)d_F) \right] - \frac{E_F}{d} \left[ \frac{\sin((k_{q'} + k_q)d_F)}{(k_{q'} + k_q)} - \frac{\sin((k_{q'} - k_q)d_F)}{(k_{q'} - k_q)} \right], \quad q \neq q',
\end{align*}$$

(A.4)

while the diagonal matrix elements are written as,

$$\begin{align*}
H^+_{qq'} &= \left[ \frac{k^2}{2m} + \varepsilon_\perp \right] + \frac{W}{d} \left[ 1 - \cos(2k_q d_F) \right] - \frac{E_F}{d} \left[ \frac{\sin(2k_q d_F)}{2k_q} \right] - d_s + \frac{\sin(2k_q d_F)}{2k_q}, \quad q = q'.
\end{align*}$$

(A.5)

The self-consistency condition, Eq.(3) is now transformed into,

$$\begin{align*}
\Delta(z) &= \frac{\pi \lambda}{k_F d} \sum_{q \neq q'} \sum_q \int d\varepsilon_\perp \left[ u_{np}^\dagger v_{np'}^\dagger + u_{np}^\dagger v_{np'} \right] \sin(k_p z) \sin(k_{p'} z) \tanh(\varepsilon_n/2T),
\end{align*}$$

(A.6)

where $\lambda = g(z) N(0)$, and $N(0)$ is the DOS for both spins of the superconductor in the normal state. The quantum numbers $n$ encompass the continuous transverse energy $\varepsilon_\perp$, and the quantized longitudinal momentum index $q$. The matrix eigensystem Eq. (A.1) and the self-consistency condition (A.6) are then solved numerically, using an iterative scheme developed and described in earlier work[10,17].

References

[1] A. F. Andreev, Zh. Éksp. Teor. Fiz. 46, 1823 (1964) [Sov. Phys. JETP 19, 1228 (1964)].
[2] F. Zhou, P. Charlat, B. Spivak, and B. Pannetier, J. Low Temp. Phys. 110, 841 (1998).
[3] J. A. Melson, P.W. Brouwer, K. M. Frahm, and C. W. J. Beenakker, Europhys. Lett. 35, 7 (1996).
[4] W. Belzig, C. Bruder, and G. Schön, Phys. Rev. B 54, 9443 (1996).
[5] P. M. Ostrovsky, M. A. Skvortsov, and M. V. Feigel’man, JETP Lett. 75, 336 (2002).
[6] D. A. Ivanov, R. Roten, and G. Blatter, Phys. Rev. B 66, 052507 (2002).
[7] S. Pilgram, W. Belzig, and C. Bruder, Phys. Rev. B 62, 12462 (2000).
[8] B. K. Nikolić, J.K. Freericks, and P. Miller, Phys. Rev. Lett. 88, 077002 (2002).
[9] I. Baladié and A. Buzdin, Phys. Rev. B 64, 224514 (2001).
[10] K. Halterman and O.T. Valls, Phys. Rev. B 65, 014509 (2002).
[11] F.S. Bergeret, A.F. Volkov and K.B. Efetov, Phys. Rev. B 65, 134505 (2002).
[12] M. Zareyan, W. Belzig, and Yu. V. Nazarov, Phys. Rev. B 65, 184505 (2002).
[13] G. Sun, D.Y. Xing, J. Dong, and M. Liu, Phys. Rev. B 65, 174508 (2002).
[14] E. Vecino, A. Martín-Rodero and A. L. Yeyati, Phys. Rev. B 64, 184502 (2001).
[15] O. Šipr and B.L. Györffy, J. Low Temp. Phys. 106, 315 (1997).
[16] P.G. de Gennes, Superconductivity of Metals and Alloys (Addison-Wesley, Reading, MA, 1989).
[17] K. Halterman and O.T. Valls, Phys. Rev. B 66 224516 (2002).
[18] D.S. Falk, Phys. Rev. 132, 1576 (1963).
[19] E.A. Demler, G.B. Arnold, and M.R. Beasley, Phys. Rev. B 55, 15174 (1997).
[20] S. M. M. Virtanen, and M. M. Salomaa, J. Phys.: Condens. Matter 12, L147 (2000).
[21] See, e.g., F. Gygi and M. Schluter, Phys. Rev. B 41, 822 (1990).
[22] M. Zareyan, W. Belzig, and Y. V. Nazarov, Phys. Rev. Lett. 86, 308 (2001).
[23] S. K. Upadhyay, A. Palanisami, R. N. Louie, and R. A. Buhrman, Phys. Rev. Lett. 81, 3247 (1998).
[24] R. J. Soulen et al., Science 282, 85 (1998).
[25] P. A. Kraus, A. Bhattacharya, and A. M. Goldman, Phys. Rev. B 64, 220505 (2001).