Split-ring polariton condensate as a macroscopic two-level quantum system

Yan Xue,1,2 Igor Chestnov,1,3 Evgeny Sedov,1,3 Stefan Schumacher,4 Xuekai Ma,4∗ and Alexey Kavokin1 †

1 Westlake University, 18 Shilongshan Road, Hangzhou 310024, Zhejiang Province, China
2 College of Physics, Jilin University, Changchun 130012, China
3 Vladimir State University, Gorkii St. 87, 600000, Vladimir, Russia
4 Department of Physics and Center for Optoelectronics and Photonics Paderborn (CeOPP), Universität Paderborn, Warburger Strasse 100, 33098 Paderborn, Germany

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Superposition states of circular currents of exciton-polaritons1 mimic the superconducting flux qubits.2 The current states are formed by a macroscopic number of bosonic quasiparticles that compose a single quantum state of a many-body condensate.3 The essential difference between a polariton fluid and a supercurrent comes from the fact that polaritons are electrically neutral, and the magnetic field would not have a significant effect on a polariton current.4 Nevertheless, the phase of a polariton condensate must change by an integer number of 2π, when going around the ring.5 If one introduces a π-phase delay line in the ring, the system is obliged to propagate a clockwise or anticlockwise circular current to reduce the total phase gained over one round-trip to zero or to build it up to 2π. We show that such a π-delay line can be provided by a dark soliton6 embedded into a ring condensate and pinned to a potential well created by the C-shape non-resonant pump-spot. The physics of resulting split-ring polariton condensates is essentially similar to the physics of flux qubits. In particular, they exhibit pronounced coherent oscillations7 passing periodically through clockwise and anticlockwise current states. We predict that these oscillations should persist far beyond the coherence time of the polariton condensates. As a consequence the qubits based on split-ring polariton condensates are expected to be characterized by a very high figure of merit8,9 that makes them a valuable alternative to superconducting qubits.

Introduction. While a tremendous progress in the development of quantum technologies is apparent10, it is still unclear which material platform is the most suitable for the realization of future quantum computers and simulators.11 Among the leaders of the quest are superconducting circuits with Josephson junctions,12 cold atoms in optical traps,13 ions,14 purely photonic systems.15 The semiconductor platform legs slightly behind so far, while a lot of interesting fundamental works on two-dimensional quantum systems based on semiconductor quantum dots,17 ring,18 as well as on the topologically protected electronic systems based on semiconductor nanostructures have been published. Recently, a series of papers demonstrated a high potentiality of semiconductor microcavities in the strong light-matter coupling regime for hosting ensembles of phase-locked bosonic condensates of half-light–half-matter quasiparticles: exciton polaritons19,20. It has been argued that the phase locking process in an array of polariton condensates may be used for the minimization of a classical many-body XY-Hamiltonian.21 Exciton-polariton condensates may be formed at elevated temperatures, optically controlled and mutually phase-locked on a picosecond time scale, that constitutes their potential advantages over other material platforms for realisation of quantum simulators. On the other hand, a polariton qubit has never been convincingly demonstrated till now, and it has been argued that the dissipative nature of exciton-polaritons characterised by ultrashort radiative lifetimes would prevent their use for implementations of the most important quantum algorithms.22

Here we argue that a strong fundamental similarity of superfluid polariton flows and superconducting electric currents may be exploited to build a polariton analogue of the superconducting flux qubit. Superconducting flux qubits are based on a superposition of clock-wise and anti-clockwise currents formed by millions of Cooper pairs, see Fig. 1a and 1b. In order to excite the system in a superposition state, the half-quantum flux of magnetic field is passed through the superconducting circuit containing one or several Josephson junctions. The system is forced to generate a circular current to either reduce the magnetic flux to zero or to build it up to a full-quantum flux.

While electrically neutral polaritons are much less sensitive to the external magnetic field than Cooper pairs, the half-integer quantum ring currents of superfluid polaritons can be efficiently controlled by introducing a phase delay line in a polariton ring that results in formation of a many-body two-level quantum system based on a split-ring polariton condensate. One of efficient methods for the realization of such a delay line is by pinning a dark soliton22 that is characterised by a π-jump of the phase of a superfluid to the slot in the polariton ring. We run numerical experiments showing the presence of a robust coherent oscillations of a polariton qubit state on the Bloch sphere.

Interestingly, the dephasing time in such a system non-resonantly pumped by a cw laser source appears to be orders of magnitude longer than the characteristic oscillation period (125 ps) that may result in very high figures
FIG. 1: (a) Sketch of a flux qubit consisted of a semiconductor ring interrupted by a Josephson junction. The persistent currents generated inside the loop tend either to compensate (\(|\psi\rangle\)) an external magnetic flux or build it up (\(|\psi^\prime\rangle\)) to magnetic flux quantum \(\Phi_0\). (b) Energy levels of a flux qubit. The qubit basis is formed by the symmetric and antisymmetric superpositions of the persistent current states. (c) Loop of a quantum liquid with an embedded defect. The defect causes back-scattering of the quantum currents and leads to the linear coupling between them. (d) Energy diagram of the states with different topological charge \(l\) in the presence of the effective magnetic field. The energy of the state is \(E_l = E_1(\theta = 0)\left[\ell - \theta/2\pi\right]^2\). (e) The semiconductor microcavity excited by the nonresonant pump beam having a C-shape profile, where the intensity on the side opposite to the slit is weakened (shown by a bright red color). Here \(w_r = 6\ \mu m\) and \(w_R = 20\ \mu m\) are the inner and outer radii, respectively. The half-width of the slit is \(w_d = 1\ \mu m\). (f)–(h) Properties of the basis states formed in a polariton superfluid ring predicted with 1D model. (f) The pump distribution contains a slot which is able to pin dark soliton manifesting itself in a density depletion and the \(\pi\) phase jump (panel (h)). (g) The density and (h) the phase of the basis states \(|0\rangle\) and \(|1\rangle\).

doing mer of qubits based on split-ring polariton condensates. This is because the phase gradient that governs the current states of polariton condensates in independent of the overall time-dependent phase characterising the condensate as a whole object. The life time of circular polariton currents is much longer than the coherence time of a polariton condensate that sustains these currents. Our analysis shows a high potentiality of semiconductor microcavities as a platform for realisation of quantum information devices.

Results. The model system.

The system under consideration is schematically shown in Fig. 1. We consider an optical microcavity formed by a couple of semiconductor Bragg mirrors containing an ensemble of embedded quantum wells. The strong coupling of cavity photons and quantum-well excitons results in the appearance of new eigenmodes of the structure, the exciton polaritons (hereafter referred to as polaritons for brevity). In the considered model system, the polaritons are created by the non-resonant continuous-wave optical pump of C-shape, see Methods for details of the pump beam cross-section. Obeying the bosonic statistics, polaritons form a Bose-Einstein condensate. In the particular case considered here the condensate of polaritons represents a macroscopic coherent quantum fluid confined under the C-shape pump due to the finite polariton lifetime.

The origin of the two-level quantum system.

In order to qualitatively describe the effects that will be further revealed by numerical modeling, let us consider a close circuit filled with a quantum liquid. The single-valuedness of the many-body wave function \(\psi(t,x)\) requires from the variation of its phase \(\partial_x \varphi\) to obey the following equality: \(\int_D \partial_x \varphi \, dx = 2\pi \ell\), which is the quantization condition for the topological invariant \(\ell \in \mathbb{Z}\) also known as the winding number. \(D\) is the length of the circuit, \(x\) is the coordinate along the circuit. If the circuit is subjected to some effective vector potential \(A\) similar to the magnetic field penetrating superconducting loop of a flux qubit, the quantization condition becomes \(\int_D \partial_x \varphi \, dx - \theta = 2\pi \ell\). The phase slope \(\theta = \Lambda \Phi/h\) induced by the vector field is defined by its flux \(\Phi\) and dimensional constant \(\Lambda\) which defines pulse rescaling rule.
The field lifts the degeneracy between the counterpropagating currents with opposite winding numbers. At the particular value $\theta = \pi$, the states with $l = 0$ and $l = 1$ are degenerate in energy similar to the a superconducting flux qubit.

The energy gap between basis states appears in the presence of a defect embedded in a circuit, see Fig. 1a–d. The defect causes backscattering of the currents mixing them and giving birth to the new system eigenstates which akin to the linear superposition states of the flux qubit.

For electrically neutral particles the effective vector field can be engineered by rotation of either defect or sampled, or exploiting spin-orbit coupling in the presence of real magnetic fields. Much easier approach implies using a phase delay line embedded in a loop. A properly designed defect is able to pin dark soliton into the slot, which guarantees local variation of the phase on $\pi$. Thus the current states with the phase changing by $\pi$ and $-\pi$ over the remaining part of the circuit superpose forming two states which we will consider as a basis of a polariton split-ring qubit. The angular dependencies of the shape and the phase of these states obtained in a narrow ring condensate are shown in Fig. 1f–h (see Methods for the details).

The dynamics of a split-ring condensate.

In a one-dimensional (1D) semiconductor microcavity modulation of the shape of the nonresonant pump may lead to the appearance of a dark soliton in a polariton condensate. The same happens in the 2D case: a planar microcavity nonresonantly pumped with a C-shape profile laser beam as shown in Fig. 1b. The density of the polariton circular currents, we perform a numerical modeling of the generalized Gross-Pitaevskii equation for the polariton wave function $\psi(r,t)$ coupled to the rate equation for the density of the reservoir of incoherent excitons $n_B(r,t)$. The details of the model and the chosen pump profile are described in Methods.

Figure 2 shows the oscillatory state of the polariton condensate. For the further analysis it will be convenient to characterize non-stationary (circular current) states of the condensate by the normalized average angular momentum introduced as $m(t) = L_z(t)/N(t)$, where $L_z(t) = \int \psi^*(r,t)(x\partial_y - y\partial_x)|\psi(r,t)|^2 dr$ is the actual average angular momentum and $N(t) = \int |\psi(r,t)|^2 dr$ is the number of polaritons in the condensate. In contrast to the winding number $l$, the average angular momentum $m(t)$ continuously varies in the course of the evolution of the condensate. The oscillations of the polariton state in Fig. 2a occur between the states with the average angular momenta of $m \simeq 0.4$ and $m \simeq -0.4$. The panels (b) and (d) show the intensity distribution (left), the phase distribution (middle) in the cavity plane and the angular phase distribution (right) of the polariton states with $m \simeq 0.4$ and $m \simeq -0.4$, respectively.

By slightly changing the pump power and keeping other parameters of the system unchanged, one can switch from the oscillating state to the stationary solution characterized by either $m \simeq 0.4$ or $m \simeq -0.4$, Fig. 2b or 2d, depending on the initial conditions. The system is multi-stable in this regime. Figure 2 illustrates the intermediate state of $m = 0$ visited by the system in the course of the oscillations. We note that, with $r_0$ relative to the radii of the circle marked with red color in Fig. 2 the current direction at the position of...
the slit is different for inner part \((r < r_0)\) and outer part \((r > r_0)\) of the polariton condensate, as shown in the right panels of Fig. 2(b-d). Fig. 2(a) shows the angular momentum for the outer part of the polariton condensate that manifests a pronounced superfluid phase current. The full dynamics of the oscillations illustrated by Fig. 2 is summarized in the Supplementary video.

Figure 3 shows the dynamics of the split-ring polariton condensate on the Bloch sphere (see details of the mapping in Methods). Retaining only the classical fluctuations of the initial polariton field by setting the stochastic processes to zero, \(dW = 0\) (Fig. 3 and Fig. 5), we obtain the stable oscillations that persist during over 10 ns (truncation time of this numerical simulation) and correspond to a circular trajectory close to the equator of the Bloch sphere. When including the quantum fluctuations described by Eq. (3) in Methods, we find that the uncertainty in the mapping procedure to the Bloch sphere becomes larger while it does not affect the oscillation stability. Fig. 3 shows the case of a 10% fluctuation. We underline that the coherent oscillations predicted by this simulation persist over tens of nanoseconds or even longer. This is much beyond the single polariton lifetime (as short as 6 ps in our case) and the coherence time of the condensate as a whole (about 100 ps). This is because the circular current states in split-ring condensates are governed by the spatial distribution of the phase and the density of the condensate. The localisation radius of the condensate \((\mu R = 20 \mu m \text{ in our case})\) is much smaller than the coherence length in system (over 100 \(\mu m\)) which is why the coherence of superfluid polariton currents is well preserved. This is consistent with the recent experimental results. The coherence time of the condensate as a whole which characterises the time-dependence of the overall phase of the ring condensate does not impose limitations on the life-time of polariton currents.

The stable persistent oscillations of the considered quantum system in the vicinity of the equator of the Bloch sphere are observed if the parameter \(P_1\) characterising the step in the pump power distribution (see Methods) is in the range of \([0.1 : 0.3] \times P_0\). With the decrease of \(P_1\) down to \(0.09 \times P_0\), the stability of the oscillations is reduced and the system shows a fast decay. Fig. 4 shows the variation of the dynamics of the system resulting from the variation of the value of \(P_1\): while \(t\) is shorter than 3000 ps that corresponds to the point \(B\), the system exhibits the same stable oscillations as those shown in Fig. 2 and 3. Next, as a result of the decrease of \(P_1\) from \(0.1 \times P_0\) to \(0.09 \times P_0\) at the point \(B\), the fast decay of the oscillations is observed, so that, eventually, the system relaxes to one of the basis states, in particular, to the state \(|0\rangle\) shown in Fig. 5a. Note that the trajectory on the Bloch sphere that describes the decay of the oscillations has been smoothed with the Bezier function for clarity. We emphasize that the decay time of oscillations is still independent of the coherence time of the condensate in this regime. It is fully governed by the magnitude of the step potential that controls the oscillations given by \(P_1\).

It is clearly seen in Fig. 5a that the wave function of the split-ring condensate is anti-symmetric either with respect to the vertical axis \((y = 0)\) or with respect to the center of the slit. It looks like two counter-rotating vortices encounter at the center of the slit and show the destructive interference pattern that is pinned to the slit: a \(\pi\) phase jump appears at \(y = 0\). With the further decrease of \(P_1\) down to \(0.03 \times P_0\), the oscillations relaxes to the basis state \(|1\rangle\) shown in Fig. 5a. This state is similar to the one shown in Fig. 2g. It represents the symmetric pattern corresponding to the constructive interference with a pair of \(\pi\) phase jumps close to the vertical axis \((y = 0)\). Note that both basis states are charac-
FIG. 5: Two stable orthogonal states of the split-ring polariton condensate corresponding to the two poles of the Bloch sphere: $|0\rangle$ and $|1\rangle$, respectively. (a) $P_1 = 0.09 \times P_0$ and (b) $P_1 = 0.03 \times P_0$. The density and phase profile in the 2D case are given in left and middle panels of the figure, respectively, the angular dependencies of the phase corresponding to different fixed radii are shown with different colors in the right panels ($r = 8 \mu m$, $13 \mu m$ and $18 \mu m$ are shown with red, green and black colors, respectively).

...terised by zero average current flow and 0 mean angular momentum. Besides, they are nearly perfectly orthogonal. Their orthogonality is essential for mapping the system to the Bloch sphere. The energy splitting estimated from their chemical potentials is much larger than one deduced from the oscillatory dynamics, because of the difference of the blue shifts of the condensate energy for different pump power intensities.

It is important to note that, in a general case, the ring condensate is not expected to relax to the lowest energy state corresponding to the pole of the Bloch sphere. This is a characteristic feature of polariton lasers: out of all quantum states the system chooses one that maximises the occupation number of the condensate, not one that is characterised by the lowest energy $|0\rangle$ and $|1\rangle$. This is why incoherent processes of acoustic-phonon assistant energy relaxation are not expected to affect the dynamics of qubits based on split-ring polariton condensates.

C-shape potentials.

... Till now we were considering the polariton condensates imprinted in a planar microcavity by means of the optical pumping. Their spatial localisation was imposed by the shape of the non-resonant optical pump beams used for their excitation. An alternative way to realize split-ring polariton condensates is by using laterally confined C-shape potentials produced by chemical etching of planar cavities. In this case, we expect a stronger confinement of polaritons and more control tools for shaping the condensates. The drawback of this system as compared to fully optically induced split-ring condensates is in its rigidity: each time to change the geometry of an array of polariton condensates one would need to grow a new sample. We also consider the combined method of lateral confinement by use of the etched micropillars where ring condensates are formed due to the repulsion of polaritons from the exciton reservoir formed in the center of the pillar by a non-resonant optical pumping. Persistent superfluid currents of exciton-polaritons were recently observed in such structure. Shifting the pump spot from the center of the pillar one should be able to realize half-moon condensates suitable for the generation of circular currents with half-integer angular momenta. Numerical simulations of the coherent oscillations in polariton condensates confined to C-shape potentials are given in the Supplementary material.

Discussion. The simulations described above demonstrate that at certain conditions split-ring polariton condensates behave as coherent two level quantum systems demonstrating long-standing coherent oscillations. Considering this system as a qubit, one should be able to estimate its figure of merit given by the ratio of the decoherence time to the characteristic single logic operation time. For the set of parameters used in our simulations that corresponds to a conventional GaAs-based microcavity, the decoherence time appears to be many orders of magnitude longer than the single polariton lifetime and than the coherence time of the condensate as a whole. Even accounting for quantum fluctuations, we do not observe any sizable decay of oscillations over a time-scale of ten nanoseconds for a certain range of the control pump power $P_1$. Estimating the single logic operation time by the period of the oscillations on the Bloch sphere that is of the order of 125 ps in our case, we end up with a figure of merit of more than 100, that matches those of best superconducting qubits.

This high figure of merit can be achieved in a split-ring condensate because it is localised in a spot that is much smaller than the coherence length in the system and because the overall phase of the condensate that is subject to a fast decoherence is fully decoupled from the superfluid phase current dynamics which defines the trajectory of the considered quantum system on a Bloch sphere. It is also important that at the cw pumping the energy relaxation of the condensate as a whole does not occur. The fluctuations of the number of particles in the condensate do not have any significant effect on the energy splitting between $|0\rangle$ and $|1\rangle$ states in the Bloch sphere that controls the frequency of oscillations. The system is efficiently controlled by the optical pump parameter $P_1$.

Implementation of quantum algorithms

In order to fully characterize the applicability of polariton qubits for quantum information processing one should address the questions of setting the initial quantum state of a polariton qubit, coupling between different qubits, elementary logic operations and read-out of the information from a set of polariton qubits. While discussing these issues in their integrity is out of the scope of this work, we would like to briefly express our vision on the concept of quantum information processing with use of polariton qubits.
Setting a split-ring condensate into a given quantum state can be achieved with use of a resonant femtosecond pump pulse focused on the specific spot on the ring. A similar technique has been employed for setting the phase of polariton Rabi oscillations. The read-out of the quantum state of the qubit can be done combining the time- and spatially-resolved photoluminescence and interferometry measurements. Note that this is a “weak measurement” method that does not fully destroy the measured quantum state, while it perturbs it to some extent. Conceptually, in a similar way, a SQUID-based read-out perturbs but does not fully destroy the quantum state of a superconducting flux qubit. To our knowledge, the proposed optical read-out technique is currently being used for studies of XY-simulators based on an array of exciton-polariton condensates. Finally, the coherent coupling between qubits can be achieved by means of the exchange by exciton-polaritons ballistically propagating in the plane of a microcavity between the condensates, in a full similarity to the coupling mechanism employed in XY-simulators. This simple coupling scheme proved efficient for pairing of nearest neighbours in an array of polariton condensates. In order to achieve coupling of distant condensates one can use one-dimensional optical waveguides imprinted lithographically in the plane of a semiconductor microcavity.

To summarize, we have demonstrated that a coherent many-body quantum system represented by a bosonic condensate of exciton-polaritons placed in a split-ring geometry sustains stable and long-living oscillations between two mutually coherent states. The polariton system qualitatively reproduces the behaviour of a superconducting flux qubit. In a remarkable similarity to the flux qubit, in the considered split-ring polariton condensate a two-level quantum system is formed by superposition states of clockwise and anti-clockwise circular currents. The size of the system is much less than the coherence length of a polariton condensate, which is why superfluid polariton currents are well preserved and they are not sensitive neither to the overall coherence time of the condensate nor to the single polariton lifetime. This ensures a high figure of merit for qubits based on split-ring polariton condensates. The present analysis opens way to the realisation of a new semiconductor platform for quantum information processing. The evident advantages of the considered quantum system are in its high scalability, full optical control, high operation temperature, ultrafast logic operations and potential integrability with classical semiconductor based nano-electronic devices.

Methods

The model. To predict the dynamics of a 2D split-ring polariton condensate in a semiconductor microcavity, the driven-dissipative Gross-Pitaevskii (GP) equation coupled with the rate equation for the density of the exciton reservoir are used:

$$d\psi(r, t) = \left[ \frac{i\hbar}{2m_s} \nabla^2 - \frac{i}{\hbar}(g_e|\psi(r, t)|^2 + g_r n_R(r, t)) + \frac{1}{2}(Rn_R(r, t) - \gamma_c) + V(r) \right] \psi(r, t) dt + dW,$$

$$\frac{\partial n_R(r, t)}{\partial t} = P(r) - (\gamma_R + R|\psi(r, t)|^2)n_R(r, t),$$

where $\psi(r, t)$ is the wavefunction of a polariton condensate and the boundary condition of the limited size of material is considered in the numerical simulations. $n_R(r, t)$ is the exciton reservoir density, $m^* = 10^{-4} m_e$ is the effective mass of polaritons on the lower branch ($m_e$ is the free electron mass), $\gamma_c = 1/6 \text{ ps}^{-1}$ and $\gamma_R = 2\gamma_c$ are the polariton and the exciton reservoir decay rates, respectively. $R = 0.01 \text{ ps}^{-1} \mu m^2$ is the rate of the stimulated scattering from the exciton reservoir to the polariton condensate. $g_e = 6 \times 10^{-3} \text{ meV } \mu m^2$ and $g_r = 2g_e$ represent the interaction constants between polaritons and between polaritons and the exciton reservoir, respectively. $V(r)$ is the external potential. The cw nonresonant pump $P(r)$ of C-shape, as shown in Fig. 1, is a super-Gaussian beam with an intensity deep in the center:

$$P(r) = \left\{ 
\begin{array}{ll}
P_0 \left[ 1 - e^{-\left(\frac{x}{w_R}\right)^{20}} \right] & 1 - e^{-\left(\frac{x}{w_R}\right)^{20}} e^{-\left(\frac{y}{w_R}\right)^{20}}, x > 0, \\
(P_0 - P_1) \left[ 1 - e^{-\left(\frac{x}{w_R}\right)^{20}} \right] e^{-\left(\frac{y}{w_R}\right)^{20}}, & x \leq 0,
\end{array}
\right.$$  

where a control factor $P_1$ aimed to weaken the light intensity on the left side of the pump, is introduced to modulate the current flow velocity in the split-ring polariton condensate. $P_1$ is a vital parameter for the realization and control of a polariton qubit. While the classical fluctuations are taken into account in the initial polariton state and the boundary condition of the limited size of material is considered in the numerical simulation, the coherent Wigner approximation (TWA) is aimed to weaken the light intensity on the left side of the pump, is introduced to modulate the current flow velocity in the split-ring polariton condensate.

The basis states shown in Fig. 1 are obtained for a narrow-ring polariton condensate with a 1D model, $\psi(r, t) \rightarrow \psi(\phi, t)$, where $\phi$ corresponds to the angular coordinate. The parameters are the same as shown above. The pump power amplitude is $3.5 \times P_0$.

Mapping the dynamics on the Bloch sphere. The procedure of mapping the dynamics of the split-ring condensate on the Bloch sphere is described here. For a qubit based on a two-level system formed by the states $|0\rangle$ and $|1\rangle$, any state $|\psi\rangle$ on the surface of the Bloch sphere can be represented as a linear combination of two basis states: $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$ imposing the normalization condition of $|\alpha|^2 + |\beta|^2 = 1$. In the basis of circular current states, this expression may be rewritten as: $|\psi\rangle = e^{-i\frac{\pi}{4}} \frac{|0\rangle + |1\rangle}{\sqrt{2}}$. 

$$\langle dW(r)W(r') \rangle = 0,$$

$$\langle dW(r)W^*(r') \rangle = \frac{dt}{2\Delta x dy}(Rn_R + \gamma_c)\delta_{rr'},$$

The basis states shown in Fig. 1 are obtained for a narrow-ring polariton condensate with a 1D model, $\psi(r, t) \rightarrow \psi(\phi, t)$, where $\phi$ corresponds to the angular coordinate. The parameters are the same as shown above. The pump power amplitude is $3.5 \times P_0$.
whose components are defined as: $S = S_x e_x + S_y e_y + S_z e_z$ whose components are defined as: $S = S_x e_x + S_y e_y + S_z e_z$ and $| \pm \rangle = \pm | 0 \rangle$. Here $| \pm \rangle$ describes two states of $m = 0$. For simplicity we associate $|m \approx \pm 0.4 \rangle$ (points B and D in Fig. 2) with $| \langle \rangle \rangle$ and $| \langle \rangle \rangle$ and choose these states as the basis for our numerical fitting procedure. In this way we obtain:

$$| \psi \rangle = \alpha |m = 0.4 \rangle + \beta_0 |m = -0.4 \rangle = \alpha |0 \rangle + \beta |1 \rangle. \quad (4)$$

The quantum state visited by the condensate can be characterized by a pseudospin vector $S = S_x e_x + S_y e_y + S_z e_z$ whose components are defined as: $S_x = \frac{1}{2} (\alpha^* \beta + \alpha \beta^*)$, $S_y = \frac{1}{2} (\alpha^* \beta - \alpha \beta^*)$, $S_z = \frac{1}{2} (|\alpha|^2 - |\beta|^2)$. The mapping of the condensate dynamics to the Bloch sphere is realised using the method of Maximum Inherit Optimization (MIO).

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* Electronic address: xy4610@jlu.edu.cn
† Electronic address: xuekai.ma@gmail.com
‡ Electronic address: a.kavokin@westlake.edu.cn

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Supplemental material: Split-ring polariton condensate as a macroscopic two-level quantum system

Yan Xue, Igor Chestnov, Evgeny Sedov, Stefan Schumacher, Xuekai Ma, Alexey Kavokin

Half-quantum currents trapped in C-shape potentials

FIG. S1: (a) Pump profile with a flat top. (b) Narrow C-shape potential well and the potential depth is $-1\text{ meV}$. (c) Time evolutions of the angular momentum of the condensate for different pump intensities: $P_0 = 1.3P_{th}$ (red line), $P_0 = 3.3P_{th}$ (green line), and $P_0 = 6.6P_{th}$ (blue line). Density (middle row) and phase (bottom row) distributions of condensates at different pump intensities: (d,e) $P_0 = 1.3P_{th}$, (f) $P_0 = 3.3P_{th}$, and (g) $P_0 = 6.6P_{th}$. (d) and (e) correspond to the black points in (c). $P_{th}$ is the condensation threshold. Here, the parameters are: $\gamma_c = 0.3\text{ ps}^{-1}$, $m^* = 2 \times 10^{-3} m_e$, $g_c = 3 \times 10^{-3}\text{ meV}\mu m^2$.

Circular superfluid currents with half-quantum angular momenta and Bloch oscillations can also be found in the case of a condensate confined to a C-shape external potential created e.g. by etching of a planar microcavity sample. In order to describe the system in this case we introduce the additional stationary potential $V(r)$ in the right-hand side of Eq. (1) in the main text. We consider the non-resonant excitation of the system by a broad pump as illustrated in Fig. S1. The considered potential contains a narrow barrier, which is different from the optically induced potential distribution shown in Fig. 1(a) in the main text: in the main text the slit in the pump-ring corresponded to a potential well for the polariton condensate. Under the excitation of a non-resonant broad pump with a relatively low intensity ($P_0 = 1.3 \times P_{th}$), an oscillation state with its angular momentum varying between $m = \pm 0.5$ is obtained, as shown in Figs. S1(c) and S1(e). The tunneling of exciton-polaritons through the narrow barrier mimics the Josephson dynamics. It is important to underline that Josephson oscillations between two condensates observed in K. Lagoudakis et al, PRL 105, 120403 (2010) are limited in time by the coherence time of polariton condensates. In contrast, in the ring geometry we work with a single condensate. The fluctuations of its overall phase do not affect the phase difference between its parts situated to the right and left sides of the potential barrier, which is why the oscillations persist on a much longer time-scale in our case. As the pump intensity increases, the system achieved a steady state whose normalised angular momentum is fractional as shown in Fig. S1(f). Note that this state is different from that in Fig. 2 in the main text, since there is no clear $\pi$ phase jump at the potential barrier. Making the pump intensity much stronger, one can see clearly the density of the tunneling under the potential barrier [Fig. S1(g)]. Tunneling preserves the phase in this case, so that the angular momentum of the condensate approaches $m = 1$ [Fig. S1(c)].

We note that the shape of the external potential strongly influences the spatial distribution of the condensate. If the potential width is increased [Fig. S2(a)], the same broad pump in Fig. S2(a) can create a C-shape solution with the angular momentum $m \simeq -0.5$ [Fig. S2(b)], and a clear $\pi$ phase jump is observed at the potential barrier as shown in Fig. S2(c). This solution is very similar to that of Fig. 2 in the main text. As the pump intensity increases, the phase difference between both sides of the potential barrier changes from $\pi$ to $0$ phase, leading to the formation of an integer angular momentum state [Figs. S2(b) and S2(d)].

FIG. S2: (a) C-shape potential well and the potential depth is $-1\text{ meV}$. (b) Density (middle row) and phase (bottom row) distributions of the condensate at different pump intensities: (c) $P_0 = 1.3P_{th}$, (d) $P_0 = 3.3P_{th}$, and (e) $P_0 = 6.6P_{th}$. The parameters are: $\gamma_c = 0.3\text{ ps}^{-1}$, $m^* = 2 \times 10^{-3} m_e$, $g_c = 3 \times 10^{-3}\text{ meV}\mu m^2$.
FIG. S2: (a) Broad C-shape potential well of the depth of $-1$ meV. (b) Time evolutions of the angular momenta for different pump intensities: $P_0 = 2P_{th}$ (green line) and $P_0 = 2.6P_{th}$ (blue line). Density (middle row) and phase (bottom row) distributions of condensates at different pump intensities: (c) $P_0 = 2P_{th}$ and (d) $P_0 = 2.6P_{th}$. 