Collapse of Primordial Filamentary Clouds under Far-Ultraviolet Radiation

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Abstract

The collapse and fragmentation of primordial filamentary clouds under isotropic dissociation radiation was investigated with one-dimensional hydrodynamical calculations. We investigate the effect of the dissociation photon on the filamentary clouds by calculating non-equilibrium chemical reactions. With the external radiation assumed to turn on when the filamentary cloud forms, a filamentary cloud with low initial density \( (n_0 \leq 10^2 \, \text{cm}^{-3}) \) suffers a photodissociation of hydrogen molecules. In such a case, since the main coolant is lost, the temperature increases adiabatically enough to suppress the collapse. As a result, the filamentary cloud fragments into very massive clouds \( (\sim 10^5 \, M_\odot) \). On the other hand, the evolution of the filamentary clouds with high initial density \( (n_0 > 10^2 \, \text{cm}^{-3}) \) is hardly affected by the external radiation. This is because the filamentary cloud with high initial density shields itself from the external radiation. It is found that the fragment mass increases owing to the external radiation. This result is consistent with previous results with one-zone models. It is also found that the fragment mass decreases owing to the external dissociation radiation in the case with a sufficiently large line mass.

Key words: External UV Radiation — Fragmentation: Filamentary Clouds

1. Introduction

It is accepted that the density perturbations collapse and cool due to hydrogen molecules \( (H_2) \) to form so-called population III (popIII) stars (Bromm et al. 1999, 2002; Abel et al. 2000, 2002; Yoshida et al. 2008). PopIII is expected to form in halos with \( \geq 10^6 \, M_\odot \) (Tegmark et al. 1997). If PopIII is a massive star, it is expected to affect neighbor clouds via radiative feedbacks. Radiative feedbacks cause ionization and dissociation. Although the ionization photon tends to be prevented from spreading out of halos because of a large opacity of hydrogen atoms, the dissociation photon tends to spread out of halos (Kitayama et al. 2004). Thus, some regions are expected not to be ionized, but to be photodissociated. We consider the filamentary clouds in such a region.

Filamentary clouds are a possible origin of stars. In general, a non-spherical gas cloud tends to become a sheet-like cloud, and a sheet-like cloud tends to fragment into filamentary clouds (Miyama et al. 1987). In numerical cosmological simulations of first star formation, a filamentary structure is frequently seen (e.g., Abel et al. 1998; Bromm et al. 1999; Greif et al. 2008). Recently, many filamentary structures have been found through the Herschel Gould Belt Survey (André et al. 2010). A filamentary cloud is possible to fragment into many quasi-spherical clouds (Nagasawa 1987; Inutsuka & Miyama 1997). These spherical clouds are expected to become stars, or other astronomical objects. We investigate how this process proceeds when first stars form.

There have been previous studies about the fragmentation of filamentary clouds (Uehara et al. 1996; Nakamura and Umemura 1999, 2001, 2002; Flower 2002; Omukai & Yoshii 2003). Among these, Nakamura and Umemura (1999, 2001, 2002) used one-dimensional hydrodynamical calculations and two-dimensional hydrodynamical calculations. The authors considered many cases with various initial density, temperature, line mass, and initial fraction of \( H_2 \). It is found that the fragment mass is \( 1-500 \, M_\odot \), and has a bimodal distribution when the initial \( H_2 \) fraction is \( 10^{-3} \). However, the effect of external radiation was not considered. Hence, these studies are applicable only to first star formation. Among the studies mentioned above, Omukai and Yoshii (2003) considered the external dissociation radiation. Their work is applicable to the formation of second-generation stars. They calculated the thermal evolution of the filamentary cloud under isotropic external radiation with a one-zone model assuming free-fall. They assumed that the filamentary cloud fragments when its density becomes 100-times higher than the loitering point. Under this assumption, Omukai and Yoshii (2003) concluded that the effect of the external dissociation radiation decreases the fragment mass.

In Bessho and Tsuribe (2012) (hereafter Paper I), we investigated the collapse and fragmentation of filamentary clouds under the isotropic external radiation using one-zone models. We assumed that the external radiation turns on when the filamentary cloud forms. By taking into account the pressure effect explicitly, we found that the filamentary clouds with low initial density \( (n_0 \leq 10^2 \, \text{cm}^{-3}) \) suffer photodissociation, and fragment into very massive clouds \( (\sim 10^4-5 \, M_\odot) \). It was found that the effect of the external radiation increases the fragment mass. The evolution of the filamentary clouds with a high initial density \( (n_0 > 10^2 \, \text{cm}^{-3}) \), or with a sufficiently large line mass, is not affected by the external radiation owing to self-shielding. In Paper I, at first, we assumed uniform filamentary clouds undergoing homologous collapse. However, in realistic situations, filamentary clouds are expected to collapse in runaway fashion. Hence, in Paper I we also introduced a "rarefied filament model" as an improved model that partly includes the effect of run-away collapse. In the rarefied filament model, the
line mass of a collapsing core decreases as the rarefaction wave propagates from the cloud surface. As a result of the rarefied filament model, we found that the effect of external radiation increases the fragment mass. However, this result is apparently inconsistent with Omukai and Yoshii (2003).

The purpose of the present paper is to investigate whether or not the fragment mass increases owing to the effect of external radiation, and to clarify the reason why our result is inconsistent with that of Omukai and Yoshii (2003). For this purpose, we extended our previous investigation using a one-dimensional model that includes the full characteristics of run-away collapse, and obtained more realistic results. Furthermore, in this paper we also consider the further evolution of each fragment.

We assume that the external radiation is isotropic. The intensity of the external dissociation radiation is set according to the distribution of the intensity of the dissociation radiation at a redshift of \( z \approx 10 \) (Dijkstra et al. 2008). The intensity that we consider is moderate (subsection 2.2), and we consider the situation where the dissociation radiation originates from halos out of a halo including the filamentary cloud.

In Paper I, we concentrated on the case where the external radiation turns on when the filamentary cloud forms \( (n \gapprox 10 \text{ cm}^{-3}) \). To clarify the reason for the apparent difference of the conclusions between us and Omukai and Yoshii (2003), we also consider the case where the external radiation turns on at lower density \( (n = 0.1 \text{ cm}^{-3}) \), as in Omukai and Yoshii (2003).

In section 2, we describe our model for the filamentary clouds. We present numerical results in section 3. In section 4, we investigate the reason for the difference between us and Omukai and Yoshii (2003). Section 5 is devoted to conclusions and a discussion.

2. Model for the Filamentary Clouds

2.1. Basic Equations

We assume an axisymmetric filamentary cloud. We do not consider dark matter for simplicity. This simplicity gives us a good approximation in the case with high initial density. In the case with low initial density, the effect of dark-matter gravity is underestimated. The hydrodynamical equation of motion in Lagrangian form is given by

\[
\frac{Dv}{Dt} = -\frac{2Gl}{r} - 2\pi r \frac{dP}{dr},
\]

where \( v \) is velocity in the cylindrical radial direction, \( G \) is the gravitational constant, \( l \) is the line mass within cylindrical radius \( r \), and \( P \) is pressure for an ideal gas, given by

\[
P = nk_BT,
\]

with number density \( n \), temperature \( T \), and Boltzmann constant \( k_B \).

We also solve the energy equation given by

\[
\frac{du}{dt} = -P \frac{1}{\rho} \frac{d}{dt} \rho - \frac{k_BT}{\rho} - \frac{\Lambda_{\text{net}}}{\rho},
\]

where \( \rho \) is the density and \( u \) is the thermal energy per unit mass.

\[
u = \frac{1}{\gamma_{\text{ad}} - 1} \frac{k_BT}{\mu_m},
\]

with adiabatic index \( \gamma_{\text{ad}} \), mean molecular weight \( \mu \), and mass of a hydrogen atom \( m_H \). The symbol \( \Lambda_{\text{net}} \) in equation (3) is the cooling rate per unit volume including lines of \( H \), lines of \( H_2 \), lines of HD, and chemical heating/cooling. Since the continuum processes hardly change the evolution of the filamentary clouds (Paper I), we neglect them. As for lines, we estimate the cooling rate from a detailed balance of the population of energy levels. The escape probability for emission by the transition between levels \( i \) and \( j \) is given by

\[
\beta_{ij} = \frac{1 - e^{-\tau_{ij}}}{\tau_{ij}},
\]

while assuming the velocity profile \( v_i(r) \propto r \) (Castor 1970). The cooling rate is multiplied by this escape probability. Optical depth, \( \tau_{ij} \), is given by

\[
\tau_{ij} = \int_{r_i}^{r_o} \kappa_{ij}(r') dr',
\]

where \( R_o \) is the radius of the outer boundary, \( \kappa_{ij} \) is the opacity for lines, \( h\nu_{ij} \) is the energy difference between levels \( i \) and \( j \), \( n_i \) \( (n_j) \) is the level population at level \( i \) \( (j) \), \( B_{ij} \) and \( B_{ji} \) are the Einstein \( B \)-coefficients, and \( \Delta \nu_{ij} = v_{ij}/c \sqrt{2kB(T)/\mu m_H} \) is the thermal Doppler width of the transition line \( i \rightarrow j \).

We consider non-equilibrium chemical reactions by solving the following equation for each fluid element:

\[
\frac{dX_i}{dt} = \sum_{j,k} k_{ijk} f_i f_j n + \sum_j k_{ij} f_j,
\]

where \( k_{ijk} \) and \( k_{ij} \) are the reaction rates for the formation and destruction of species \( i \), and \( f_i \) is the fraction of species \( i \). We consider fourteen species: \( H, H^+, H^-, H_2, H_2^+, He, He^+, He^{++}, D, D^+, D^-, HD, HD^+, \) and \( e^- \). We consider 26 chemical reactions concerned with \( H \) and \( He \) taken from Nakamura and Umemura (2001), the photodissociation of \( H_2 \) [equation (10)], and 18 chemical reactions concerned with \( H \) and \( D \) taken from Nakamura and Umemura (2002). We solved equation (7) with an implicit integrator.

We solved equations (1)–(7) using 200 spatial meshes in the cylindrical radial direction. As for the initial interval of meshes, we set \( \Delta r_{i+1} = 1.01 \Delta r_i \), where \( \Delta r_i \equiv r_{i+1} - r_i \). Hence, spatial resolution is much better in the central region. The mesh size, \( \Delta r_i \), was checked to be shorter than 1/4 of the local Jeans length\(^1\) at all times (Truelove et al. 1997). We set the outer boundary to be 10 \( R_0 \) as a sufficiently large value, where \( R_0 \) is the effective radius, given by

\[
R_0 = \sqrt{\frac{2f k_BT_0}{\pi G \mu^2 m_H n_0}},
\]

with line mass parameter \( f \) [equation (17)], initial temperature \( T_0 \), and initial number density at the center, \( n_0 \). As for the

\[^1\] The exact definition of Jeans length is given by \( \lambda_J = \left( \frac{\pi}{940 \mu m_H} \right)^{1/2} c_s \).
outer boundary condition, we assume that the external pressure is zero.

We solved equation (1) in the second-order-accurate finite-difference scheme with the artificial viscosity (Richtmyer & Morton 1967). We calculated in the same way as Thoul and Weinberg (1995), except for cylindrical geometry. To check the accuracy of our code, first, we calculated the density distribution of the isothermal equilibrium filamentary cloud \((T = 300 \text{ K})\) with 100 and 200 meshes, and a fixed time step. A drag term \(-2n(r)/dt\) was added to the equation of motion, and was eliminated after the step number reached 125 for the case with 100 meshes and 250 for case with 200 meshes. After 250 steps (100 meshes) and 500 steps (200 meshes), we had the error of the density distribution shown in figure 1. The error for the case with 100 meshes was 4-times of the error for the case with 200 meshes. Hence, our code was second-order-accurate in space.

Second, we checked the temporal accuracy of our code in time. We calculated the collapse of the pressure-less uniform filamentary cloud in the free-fall state. The analytic solution is given by

\[
\int_0^{\log F(t)} e^{-x^2} dx = \sqrt{\pi} G\rho(0)t, \tag{9}
\]

where \(F(t) = r(t)/r(0)\), \(\rho(0)\) is the initial density. The time step was set to be \(\Delta t = 10^{-4} t_f(t = 0) = 2 \Delta t\). The calculation was continued until the density became 100-times the initial density. In the case with \(\Delta t\), \(F\) at the end of calculation was 0.0913290. In the case with 2 \(\Delta t\), \(F\) at the end of the calculation was 0.09132307. The analytic solution predicts \(F = 0.9132284\). The error with 2 \(\Delta t\) was 4-times larger than \(\Delta t\). Hence, our code has second-order-accuracy in time.

### 2.2. External Radiation

We assumed that the external dissociation radiation is isotropic. The intensity of the external radiation was set according to Dijkstra et al. (2008). Dijkstra et al. (2008) calculated the probability distribution of the mean intensity of the dissociation radiation at redshift \(z \sim 10\) by estimating the mean intensity of the dissociation photon from the surrounding halos to a single halo. In the present work, we used the mean intensities, whose probability was 0.4 and 0.06, as in Paper I. The external radiation was assumed to be thermal radiation of 120 \(M_\odot\) star and we determined the surface temperature \((T_{\text{sur}} = 95719 \text{ K})\) according to Schaefer (2002). We assumed that the external radiation turns on when the filamentary clouds forms as in section 3. In section 4, we change the density when the external radiation turns on.

We calculate the photodissociation reaction of \(\text{H}_2\),

\[
\text{H}_2 + \gamma \rightarrow \text{H}_2^* \rightarrow 2\text{H}, \tag{10}
\]

(solomon process) where \(\gamma\) is photon with 12.4 eV and \(\text{H}_2^*\) is the excited state of \(\text{H}_2\). The reaction rate is given by

\[
k_{\text{2step}} = 1.4 \times 10^7 f_{\text{sh}} J_{\text{ext}} \text{ s}^{-1}, \tag{11}
\]

where \(J_{\text{ext}}\) is the mean intensity of the external radiation at the surface of the filamentary cloud and \(f_{\text{sh}}\) is the self-shielding function,

\[
f_{\text{sh}} = \min \left[ 1, \left( \frac{N_{\text{H}_2}}{10^{14} \text{ cm}^{-2}} \right)^{-3/4} \right]. \tag{12}
\]

where \(N_{\text{H}_2}\) is the column density of \(\text{H}_2\) (Draine & Bertoldi 1996), estimated as

\[
N_{\text{H}_2}(r) = \int_r^{R_{\text{out}}} n_{\text{H}_2}(r') dr'. \tag{13}
\]

### 2.3. Fragmentation of the Filamentary Cloud

There are two important timescales during the collapse of filamentary clouds. One is the timescale for density evolution, \(t_{\text{dyn}} \equiv \rho_c/\rho_c\), and the other is the timescale for fragmentation, \(t_{\text{frag}} \equiv 5.17/\sqrt{2\pi G}\) (Nagasawa 1987). The latter is the timescale during which the fastest growing mode of perturbation grows to be non-linear. If the fastest growing mode has sufficient time to grow to be non-linear before the fastest growing mode changes, the filamentary cloud is expected to fragment. Thus, we assume that the filamentary clouds fragment when \(t_{\text{dyn}} > t_{\text{frag}}\). After the condition for fragmentation is satisfied, it has been satisfied after that.

We estimate the fragment mass by integrating the region with the density higher than 10% of the central density, \(n_c\),

\[
M_{\text{frag}} = \lambda_{\text{frag}} \int_0^r \rho(r) 2\pi r'dr', \tag{14}
\]

where

\[
\lambda_{\text{frag}} = \frac{2\pi}{0.288} \frac{c_s}{\sqrt{4\pi G\mu m_{\text{H}} n_c}} \tag{15}
\]

is the wavelength of the fastest growing mode for equilibrium filamentary clouds (Nagasawa 1987) and \(c_s\) is the sound speed. Since \(\lambda_{\text{frag}} \propto \rho^{-1/2}\), the fragment mass is smaller when the filamentary cloud reaches higher density before fragmentation. Since the interval of integration in equation (14) is approximately the Jeans length, the Jeans mass at fragmentation is close to the fragment mass (see subsection 3.3).
2.4. Parameters and Initial Conditions

In this paper, we treat three physical quantities as parameters. First is the initial density, \( n_0 \), second is the normalized mean intensity of the external radiation,

\[
J_{21} = \frac{J_{hv=13.6eV,ext}}{10^{-21} \text{erg cm}^{-2} \text{s}^{-1} \text{Hz}^{-1} \text{sr}^{-1}},
\]

(16)

and third is line mass parameter \( f \),

\[
f \equiv \frac{\pi G \mu^2 n_0^2 R_0^2}{2k_B T_0}.
\]

(17)

The line mass parameter is important from the viewpoint of dynamical evolution. The initial density, \( n_0 \), is important for thermal evolution. With respect to photodissociation, the mean intensity, \( J_{21} \), and the initial density, \( n_0 \), are important.

We consider cases with \( \log_{10} n_0 = 1, 1.5, 2, 2.5, 3, 3.5, 4, 4.5, 5, 5.5, \) and \( 6 \) for \( n_0 \) and \( f = 1, 1.5, 2, 2.5, 3, 3.5, 4, 5.5, \) and \( 6 \) for \( f \). For \( J_{21} \), we consider \( J_{21} = 0, 1, 6.5, \) and 10. According to Dijkstra et al. (2008), the case with \( J_{21} = 1 \) represents the weak external radiation case. \( J_{21} = 6.5 \) is the mean intensity with the highest probability (0.4), and \( J_{21} = 10 \) represents the strong radiation case, whose probability is 0.06.

We assume the initial density distribution to be

\[
n(r) = n_0 \left(1 + \frac{r^2}{R_0^2}\right)^{-\frac{3}{2}},
\]

(18)

with \( R_0 \) given in equation (8). With \( f = 1 \), equation (18) represents the equilibrium density distribution for an isothermal filamentary cloud (Ostriker 1964). In this paper, we concentrate on a collapsing filamentary cloud with \( f > 1 \). The initial velocity distribution is assumed to be

\[
v(r) = \frac{c_s}{R_0 + \sqrt{R_0^2 + r^2}} r.
\]

(19)

Equation (19) indicates that infall velocity is proportional to the radius at \( r \to 0 \), and is constant \((\sim c_s)\) at \( r \to \infty \). Although the actual initial velocity may be different from equation (19), depending on the details of the dynamical evolution of the filamentary cloud formation, the results in this paper will not change qualitatively unless the initial velocity is much faster than a few times of equation (19).

As the initial temperature, we adopt \( T_0 = 300 \text{K} \), assuming that the filamentary clouds form after the cloud undergoes \( \text{H}_2 \) cooling. We also adopt \( f_{\text{H}_2} = 10^{-4} \) and \( f_{\text{e}} = 10^{-4} \). As for the value of \( f_{\text{H}_2} \), we refer to the result given in Paper I. The fraction of the electron is set in order not to change \( f_{\text{H}_2} \) artificially via the \( \text{H}^- \) channel. The initial fraction of the proton is set to be \( f_p = 10^{-4} \) for charge conservation. We assume \( [\text{H}] / [\text{D}] = 4 \times 10^{-3} \), which is consistent with observations of the deuterium Lyra feature (e.g., O’Meara et al. 2001). The initial fraction of the others is set to be zero.

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If a filamentary cloud forms as a result of fragmentation of the fastest growing mode in a sheet-like cloud, the typical value of \( f \) is 2 (Miyama et al. 1987).

3. Results of One-Dimensional Hydrodynamical Calculations

3.1. Cases without External Radiation

To investigate the effect of external radiation, at first we show the results for the cases without the external radiation.

3.1.1. Low density filamentary cloud with small line mass

First, we show the results for the case with a low initial density and a small line mass, \((f, n_0, J_{21}) = (1.5, 10 \text{cm}^{-3}, 0)\) (figure 2). In the early stage of collapse, \( \text{H}_2 \) cooling dominates adiabatic heating a little, and the temperature decreases. After the density reaches \( n \sim 10^3 \text{cm}^{-3} \), adiabatic heating dominates \( \text{H}_2 \) cooling. For \( n > n_{\text{crit}} \sim 10^4 \text{cm}^{-3} \), the \( \text{H}_2 \) cooling rate is proportional to \( n \), while it is proportional to \( n^2 \) when \( n < n_{\text{crit}} \). Hence, since the cooling time becomes constant for \( n > n_{\text{crit}} \), the dynamical time becomes constant and longer than the fragmentation time. When \( n \) reaches \( \sim 2 \times 10^5 \text{cm}^{-3} \), the condition for fragmentation is satisfied with \( M_{\text{frag}} \sim 1220 M_\odot \). To ensure that fragmentation occurs, we continue to calculate the evolution until the free-fall time has past after the condition for fragmentation is first satisfied. After the condition for fragmentation is satisfied, it has been satisfied.

3.1.2. Low-density filamentary cloud with a large line mass

Next, we show the result for the case with a low initial density and a large line mass, \((f, n_0, J_{21}) = (6, 10 \text{cm}^{-3}, 0)\) (figure 3). Owing to the larger line mass, the filamentary cloud collapses to a higher density than in figure 2. The collapse continues up to high density \((n \sim 10^6 \text{cm}^{-3}) \), and the
filamentary cloud fragments. The fragment mass is \( \sim 490 \, M_\odot \). Until fragmentation, adiabatic heating and \( \text{H}_2 \) cooling balance, and the temperature is approximately constant (\( T \sim 350 \, \text{K} \)).

### 3.1.3 High-density filamentary cloud with a small line mass

As the final example, we show the result for the case with a high initial density and a small line mass, \( (f, n_0, J_{21}) = (1.5, 10^6 \, \text{cm}^{-3}, 0) \) (figure 4). In the early stage of collapse, adiabatic heating dominates \( \text{H}_2 \) cooling, and the temperature increases. During collapse, the \( \text{H}_2 \) cooling rate increases, and approximately balances with the adiabatic heating rate at \( n \sim 10^7 \, \text{cm}^{-3} \). After the temperature decreases a little, the fragmentation condition is satisfied at \( n \sim 3 \times 10^5 \, \text{cm}^{-3} \), since the collapse is suppressed owing to high temperature. The fragment mass is \( \sim 370 \, M_\odot \).

In summary, in cases without the external radiation, the filamentary cloud undergoes approximately isothermal states, and fragments. This feature comes from the fact that the \( \text{H}_2 \) cooling and adiabatic heating compete with each other. The results given in this subsection are similar to that of previous works (Nakamura & Umemura 2001, 2002). For a parameter set the same as that in figure 2, in Paper I, the fragment mass was \( 23 \, M_\odot \) in the uniform model and \( 3500 \, M_\odot \) in the rarefied filament model. A fragment mass of \( 1220 \, M_\odot \) in the one-dimensional model is close to the results of the rarefied filament model. This result indicates that the effect of run-away collapse is important to estimate the fragment mass.

### 3.2 Cases with the External Radiation

In this subsection, using the same parameters as for figures 2, 3, and 4, we consider how the external radiation changes the thermal evolution and the fragment mass of the filamentary cloud.

#### 3.2.1 Low-density filamentary cloud with a small line mass and strong radiation

First, we show the result for the case with a low initial density, small line mass, and strong external radiation, \( (f, n_0, J_{21}) = (1.5, 10^6 \, \text{cm}^{-3}, 10) \) (figure 5), where the external radiation is added to the case of figure 2. This case would be affected by the external radiation because of low density. Most of \( \text{H}_2 \) is photodissociated in the early stage of collapse, and the temperature increases adiabatically. The filamentary cloud fragments into very massive fragments \( \sim 2.4 \times 10^5 \, M_\odot \) at \( n \sim 30 \, \text{cm}^{-3} \). This result demonstrates that in the case with low initial density and small line mass the effect of the external radiation increases the fragment mass, and changes the thermal evolution.

#### 3.2.2 Low-density filamentary cloud with a large line mass and strong radiation

Next, we show the result for the case with a low initial density, large line mass, and strong external radiation, \( (f, n_0, J_{21}) = (6, 10^6 \, \text{cm}^{-3}, 10) \) (figure 6), where the external radiation is added to the case of figure 3. This case is also excepted to be affected by the external radiation because of low density. However, the filamentary cloud may collapse up to a higher density than in figure 5, because of the large line mass. In figure 6, it can be seen that most of the \( \text{H}_2 \) is photodissociated in the early stage of collapse, and temperature increases adiabatically, as in figure 5. However, since the filamentary cloud is more massive than in figure 5, stronger gravity and a large inertia help the collapse. Fragmentation does not occur during the early adiabatic phase, and collapse continues until the density becomes higher than in figure 5. At \( n \sim 10^7 \, \text{cm}^{-3} \), \( \text{H}_2 \) starts to form, and shields itself from the external radiation. The filamentary cloud starts to cool owing to \( \text{H}_2 \) cooling. After \( n \sim 10^7 \, \text{cm}^{-3} \), since \( \text{H}_2 \) cooling balances with adiabatic...
heating, the temperature becomes nearly constant \( T \approx 400 \) K. The filamentary cloud fragments into clouds with \( n \approx 590 \, M_\odot \) at \( n \approx 10^6 \) cm\(^{-3}\). The fragment mass and density at fragmentation are similar to the case without the external radiation (figure 3).

In the case with a large line mass \((f = 6)\), it is found that the fragment mass is hardly affected by the external radiation, although the evolution of the temperature is affected by the external radiation in the early stage of collapse.

3.2.3. High-density filamentary cloud with a small line mass and strong radiation

Finally, we show the result for the case with a high initial density, small line mass, and strong external radiation, \((f, n_0, J_{21}) = (1.5, 10^6 \, \text{cm}^{-3}, 10)\) (figure 7), where the external radiation is added to the case of figure 4. This case may not be affected by the external radiation because of the high density. Since the initial density is high enough to shield the filamentary cloud from the external radiation, \(H_2\) near the center of the cloud is not photodissociated. The evolution of the temperature at the center is hardly affected by the external radiation, and is similar to figure 4. The filamentary cloud fragments into clouds with \( n \approx 10^7 \) cm\(^{-3}\). It is found that the effect of the external radiation is not important in the case with a high initial density \((n_0 = 10^6 \, \text{cm}^{-3})\).

In summary, in the case with a low initial density \((n_0 < 10^2 \, \text{cm}^{-3})\), the filamentary cloud suffers photodissociation in the early stage of collapse. In such a case, the temperature increases adiabatically. The filamentary cloud with a small line mass \((f = 1.5)\) fragments during the adiabatic phase. On the other hand, the filamentary cloud with a large line mass \((f = 6)\) does not fragment during the adiabatic phase, and collapses while shielding itself from the external radiation. In this case, the fragment mass is hardly affected by the external dissociation radiation.

The one-zone model predicts a fragment mass different from \(2.4 \times 10^5 \, M_\odot\), which the one-dimensional model predicts. For example, for the parameter set to be the same as in figure 5,
of Paper I, the uniform model predicted $1.5 \times 10^5 \, M_\odot$, and the rarefied model predicted $2.7 \times 10^4 \, M_\odot$. The difference between one-zone models and the one-dimensional model originates from the difference in the dynamical equation (virial equation in the one-zone model and the hydrodynamical equation of motion in the one-dimensional model). In the one-dimensional model, the collapse is a run-away collapse, and the fragmentation condition is satisfied at a lower density, since the free-fall time balances with the sound crossing time in the central dense region. Hence, in the one-dimensional model, the fragment mass becomes larger than that in the one-zone model.

### 3.3. Property of the Filamentary Cloud at Fragmentation

In this subsection, we show the profile of the physical quantities (density, temperature, infall velocity, and ratio of pressure gradient to gravitational force) at fragmentation. We focus on the density profile, and investigate whether or not a universal profile at fragmentation exists. Moreover, we compare the fragment mass [equation (14)] with the Jeans mass estimated with the central density and temperature.

#### 3.3.1. Case without external radiation

We show the profiles of the density, temperature, infall velocity, and ratio of pressure gradient to the gravitational force at fragmentation in the case with $(f, n_0, J_{21}) = (1.5, 10 \, \text{cm}^{-3}, 0)$ (figure 8). In diagram (a) of figure 8, it is can be seen that the dense central region within the Jeans length ($\lambda_J \sim 2.3 \times 10^{18} \, \text{cm}$) has a uniform density, and the density profile in the outer envelope is proportional to $r^{-4}$. This density profile is similar to that of the equilibrium solution for the isothermal filamentary cloud (Ostriker 1967). However, between $r = 10^{19} \, \text{cm}$ and $r = 10^{20} \, \text{cm}$, slope of the density profile is sallower than $r^{-4}$. The temperature is highest outside $r_{\text{cool}} \sim 2 \times 10^{18} \, \text{cm}$, where $t_{\text{cool}} = t_{\text{ff}}$, and the pressure gradient force is stronger than the gravity force outside $r_{\text{cool}}$. Hence, matter is pushed outward. The velocity profile is in proportion to the radius in the central dense region, and is constantly larger than the sound speed in the outer envelope. The ratio of the pressure gradient to gravity inside $r_{\text{cool}}$ is nearly 1 ($\sim 1.01$) inside $r_{\text{cool}}$. In diagram (b), a drop of temperature at the surface can be seen. Since we assume that the external pressure is zero, adiabatic cooling occurs at several meshes of the surface. Moreover, these meshes are pushed by inner meshes with higher pressure, and fall more slowly than the inner meshes. However, these effects do not affect the central region. The fragment mass is $1220 \, M_\odot$ which is close to the Jeans mass ($1140 \, M_\odot$), estimated with the central density and the temperature. Since $t_{\text{dyn}}$ is about 6-times of $t_{\text{ff}}$ at the center when the filamentary cloud fragments, the pressure gradient force is important to calculate any further evolution of the fragments. Further evolution of each fragment is shown in section 4.
3.3.2. Case with the external radiation

We show profiles of the same quantities as figure 8 for the case with \((f, n_0, J_{21}) = (1.5, 10 \text{ cm}^{-3}, 10)\) in figure 9. In figure 9, it is seen that except for temperature, the profiles of the physical quantities are similar to figure 8. Most of the H\(_2\) is photodissociated, and the filamentary cloud loses the ability to cool. Hence, the temperature is higher in the central dense region than in the outer envelope. The ratio of the pressure gradient to gravity is larger than \(1/(\text{CAN}1.2)\). The fragment mass is \(2.4 \times 10^5 M_\odot\), which is close to the Jeans mass (\(3.0 \times 10^5 M_\odot\)) estimated with the central density and the temperature. In the case with external radiation, since \(t_{\text{dyn}}\) is about 5-times of \(t_{\text{ff}}\) at the center when the filamentary cloud fragments, the pressure gradient force is important to calculate any further evolution of the fragments (see section 4). In figure 10, the density profiles in figures 8 and 9 are simultaneously plotted, and each profile is found to be similar to each other. The profiles at \(r/J = 0.5\) are similar to the profile of the isothermal filamentary cloud in the equilibrium state.

3.4. Fragment Mass

We show how much the effect of the external dissociation radiation changes the fragment mass. Figure 11 shows the fragment mass for all of the parameters in the \(n_0-f\) plane using contours. The results for the cases with \(J_{21} = 0, 1, 6.5,\) and 10 are presented in different diagrams. In the case with the external radiation, it can be seen that the filamentary clouds fragment into very massive clouds (> \(10^5 M_\odot\)) in the cases with low initial density \((n_0 \leq 10^2 \text{ cm}^{-3})\). Since very massive fragments are not seen in the case without the external radiation, this can be regarded as a result of the effect of the dissociation photon. This feature is similar to the result of the one-zone...
models in Paper I. Thus, the formation of very massive fragments under external radiation with moderate intensity can be regarded as a robust result, provided that the external radiation turns on when the filamentary cloud forms.

Diagram (a) of figure 11 is similar to figure 6 of Nakamura and Umemura (2001). In the range of 2–100 $M_\odot$, the contours are dense. This is because the filamentary cloud becomes isothermal once $H_2$ cooling becomes effective owing to the three-body reaction, and continues to collapse to high density ($\sim 10^{13}$ cm$^{-3}$). In such a case, the fragment mass is small ($\sim 2–10 M_\odot$). This feature can be seen in Nakamura and Umemura (2001).

Nakamura and Umemura (2002) concluded that there are some parameter sets where HD is the main coolant. However, in our results, HD is found not to be important. Deuterated hydrogen molecules HD mainly forms from $H_2$. Since the initial $H_2$ fraction described in this paper is assumed to be small ($10^{-4}$), even in the case without external radiation, a sufficient amount of HD to cool does not form. This result is consistent with that of Nakamura and Umemura (2002). In the case with the external radiation, since $H_2$ is photodissociated, HD is less important.

We show the effect of the external radiation on the fragment mass quantitatively. Figure 12 shows similar contours, but about the ratio of the fragment mass between the cases with and without the external radiation. In addition to figure 11, figure 12 clearly shows that the filamentary clouds with low initial density ($n_0 \leq 10^2$ cm$^{-3}$) and moderate $f$ ($< 4.5$) fragment into more massive fragments than the case without the external radiation. This feature agrees with the results of the rarefied filament model in Paper I.

It can be seen that the fragment mass for the case with high initial density ($n_0 > 10^2$ cm$^{-3}$) does not increase, owing to the external dissociation radiation. This is because the initial density is high enough for the filamentary cloud to shield itself from the external dissociation radiation. In this case, the evolution is similar to the case without the external radiation.

3.5. Self-Shielding Function of Wolcott-Green et al. (2011)

Wolcott-Green et al. (2011) have recently suggested a self-shielding function, indicated by three-dimensional radiative transfer. In this subsection, we investigate the dependence of the fragment mass on the self-shielding function quantitatively. The new self-shielding function is given by

$$ f_{\text{sh, WG}} = \frac{0.965}{(1 + x/b_5)^{1.1}} + \frac{0.035}{(1 + x)^{0.5}} \exp[-8.5 \times 10^{-4}(1 + x)^{0.5}], $$

(20)

where

$$ f_{\text{sh, WG}} = \frac{0.965}{(1 + x/b_5)^{1.1}} + \frac{0.035}{(1 + x)^{0.5}} \exp[-8.5 \times 10^{-4}(1 + x)^{0.5}], $$

(20)
Fig. 12. Ratio of the fragment mass with the external radiation [(a) $J_{21} = 1$, (b) $J_{21} = 6.5$, and (c) $J_{21} = 10$] to that without the external radiation ($J_{21} = 0$). The number near each solid line is the ratio of the fragment mass.

Fig. 13. Ratio of the fragment mass with $f_{\text{sh, WG}}$ to that with $f_{\text{sh}}$ in the case with $J_{21} = 10$. The number near each solid line is the ratio of the fragment mass.

Since $f_{\text{sh, WG}}$ is 1–10 times larger than the original $f_{\text{sh}}$ [equation (12)], Wolcott-Green et al. (2011) suggested that the effect of photodissociation is actually stronger than the result with $f_{\text{sh}}$. Thus, the results which we have considered are expected to be modified quantitatively.

To see the difference between the shielding function, we calculated the fragment mass using $f_{\text{sh, WG}}$ in the case with $J_{21} = 10$. Figure 13 shows the ratio of the fragment mass between the cases with $f_{\text{sh, WG}}$ and with $f_{\text{sh}}$. When we use $f_{\text{sh, WG}}$, the fragment mass increases compared to the case with $f_{\text{sh}}$, especially for the cases with $n_0 = 10–10^3 \text{cm}^{-3}$ and $f \geq 2$. This feature is consistent with the relation $f_{\text{sh, WG}} \geq f_{\text{sh}}$.

4. Criterion for an Increase of the Fragment Mass

Omukai and Yoshii (2003) calculated the evolution of the filamentary cloud under external dissociation radiation, assuming free-fall. The authors assumed that fragmentation occurs at a density of 100-times higher than the loitering point, and concluded that the effect of the external dissociation radiation decreases the fragment mass. This conclusion apparently disagrees with our results in section 3. In this section, we consider whether or not the effect of the external radiation increases the fragment mass when the filamentary cloud reaches the loitering point. The initial density of the filamentary cloud is assumed to be very low ($n_0 = 0.1 \text{cm}^{-3}$). As
the timing when the external radiation turns on, we consider various cases. The investigation described in this section provides a systematic study that includes the situation of Omukai and Yoshii (2003).

4.1. Whether the Filament Reaches the Loitering Point

Suppose a filamentary cloud with \( n_0 = 0.1 \text{ cm}^{-3} \) and the external radiation turns on when the density reaches \( n_{UV} \). We considered the cases with \( n_{UV} = 0.1 \text{ cm}^{-3}, 1 \text{ cm}^{-3}, \) and \( 10 \text{ cm}^{-3} \). We also assumed \( T_0 = 300 \text{ K} \) and \( f_{H_2} = 0 \) at \( n_0 \). As for the evolution of the filamentary cloud, we solved the one-dimensional hydrodynamics as described in section 3.

We calculated the evolution of the filamentary cloud with various values of \( f \). In order for the filamentary cloud not to fragment during the adiabatic phase in the case with \( J_{21} = 10 \), it is found that \( f \) is required to be larger than \( 30, 25, 10, \) and \( 5 \) for various values of \( n_{UV} \#:n_{UV} = 0.1 \text{ cm}^{-3}, 1 \text{ cm}^{-3}, 10 \text{ cm}^{-3}, \) and \( \infty \), respectively. Hence, we investigated how massive fragments are in the cases with \( f = 30, 25, 10, \) and \( 5 \). Does the fragment mass increase when the filamentary cloud reaches the loitering point?

First, we show the case with \( f = 30 \), where the filamentary cloud reaches the loitering point for any \( n_{UV} \). Figure 14 shows the thermal evolution of the filamentary cloud with \( f = 30, n_0 = 0.1 \text{ cm}^{-3}, J_{21} = 10 \), and various \( n_{UV} \). Thin lines indicate the thermal evolution of each fragment (see subsection 4.2). The fragment mass is largest (550 \( M_\odot \)) in the case with \( n_{UV} = \infty \). The fragment mass is smaller in the case of \( n_{UV} = 0.1 \text{ cm}^{-3} \) when it does not reach the loitering point. However, each fragment may cause sub-fragmentation, and the mass of the final outcome may be as small as the fragment mass in the case without the external radiation. To clarify the mass of the final outcome, we investigated the further evolution of each fragment.

4.2. Sub-Fragmentation

It is found that the effect of the external radiation increases the fragment mass when the filamentary cloud fragments before the loitering point. However, each fragment may cause sub-fragmentation, and the mass of the final outcome may be as small as the fragment mass in the case without the external radiation. To clarify the mass of the final outcome, we investigated the further evolution of each fragment.

We used the one-zone model for each fragment. We assumed that each fragment is spherical and has \( M_{\text{frac}} \) (see equation (14)), chemical composition same as at the center of the filamentary cloud, radius \( r_{0.1} \) where \( n(r_{0.1}) = 0.1 n_0 \). The infall velocity at fragmentation is proportional to the radius, and we use the infall velocity at \( r_{0.1} \) as the infall velocity of each fragment. The density of the fragment is estimated from...
the mass and radius as
\[ \rho_{\text{frag}} = \frac{M_{\text{frag}}}{4\pi R_{\text{frag}}^3}. \]  

This density, \( \rho_{\text{frag}} \), is different from the density of the filamentary cloud before fragmentation, and corresponds to that at the end of fragmentation. The temperature and chemical fraction of each fragment is approximated by the value when the filamentary cloud fragments. According to the result given in subsection 3.3, from the viewpoint of dynamical evolution, we should consider the effect of the pressure gradient force. Hence, we treat the pressure effect explicitly using the virial equation for a uniform sphere. The virial equation is given by
\[ \frac{dv}{dt} = \frac{10 k_B T}{3 \mu m_H R} \frac{GM}{R^2}, \]
where \( R \) is the radius and \( M \) is the mass of the cloud (see the Appendix). Radiative transfer is treated by the same method as in Omukai (2001), except that we used \( f_{\text{sh},G_W} \) instead of \( f_{\text{sh}} \). The same routine as in section 3 was used for chemical reactions.

In figure 15, it can be seen that if each fragment undergoes sub-fragmentation at the loitering point (\( n \sim 10^3 \text{ cm}^{-3} \)), the mass of the final outcome is \( \sim 10^6 M_\odot \). This is larger than fragment mass in the case without the external radiation. Hence, even if sub-fragmentation occurs, the mass of the final outcome increases, owing to the external radiation when the filamentary cloud fragments before the loitering point. This tendency is found in other cases with \( f = 25 \) and 10.

When the filamentary cloud reaches the loitering point, the temperature of each fragment increases adiabatically after fragmentation (\( n \sim 10^6 \text{ cm}^{-3} \)). This is because gravity dominates the pressure gradient after fragmentation, and the dynamical time becomes shorter than the cooling time.

### 5. Conclusions and Discussions

In this paper, the collapse and fragmentation of a primordial filamentary cloud was investigated using one-dimensional hydrodynamical calculations with the effect of the external dissociation radiation. Especially, the effect of run-away collapse on the fragment mass is considered by comparing with previous results with one-zone models. The results are summarized as follows:

- Compared with the uniform model in Paper I, the one-dimensional filament model predicts a lower fragmentation density and a larger fragment mass. This is because fragmentation occurs only in the central region with a low virial temperature in the one-dimensional model.
- Compared with the rarefied filament model described in Paper I, the one-dimensional filament model predicts a similar fragment mass. This explains that the discrepancy between the uniform filament model and the one-dimensional filament model mainly comes from run-away collapse, which is partly induced by the pressure effect.
- As long as the external radiation is assumed to turn on when the filamentary clouds form, low initial density (\( n_0 \leq 10^2 \text{ cm}^{-3} \)) filamentary clouds with moderate line mass are expected to fragment into very massive clumps (\( \sim 10^5 M_\odot \)) as a result of the photodissociation of molecular hydrogen. This result, which is originally indicated in Paper I, is confirmed in this paper using one-dimensional hydrodynamical calculations.
- The external dissociation radiation increases the fragment mass when the filamentary cloud fragments during the adiabatic phase after the external radiation turns on. On the other hand, when the filamentary cloud with sufficient line mass reaches the loitering point, the dissociation radiation decreases the fragment mass, which is consistent with Omukai and Yoshii (2003).

As we can be seen in figure 15, the thermal evolution of a filamentary cloud and a spherical cloud is different after each cloud reaches the loitering point. The thermal evolution of the filamentary cloud is isothermal, and the temperature of the spherical cloud increases. This is explained as follows: since the central region of the filamentary cloud is approximately dynamical equilibrium (subsection 3.3), we have

\[ \frac{1}{P} \frac{d}{dt} \frac{1}{\rho} \sim \frac{Gl}{r} \]

\[ T \propto n m_H G l \propto \text{const.} \]

As a spherical cloud, the central region is not in dynamical equilibrium. Hence, we investigated the relation between \( T \) and \( n \) from the balance between adiabatic heating and \( H_2 \) cooling. The adiabatic heating rate is

\[ -P \frac{d}{dt} \frac{1}{\rho} \sim P \frac{1}{t_{\text{ff}}} \propto T n^{1/2}, \]

where we assume that the spherical cloud collapses on a free-fall timescale. When the density is larger than the critical density of \( H_2, \Lambda_{H_2} \propto n T^a \) (\( a \sim 3.8 \) at \( T \sim 300 \text{ K} \) and \( a \sim 4.8 \) at \( T \sim 1000 \text{ K} \), and we have

\[ \frac{\Lambda_{H_2}}{\rho} \propto \frac{n T^a}{n} \propto T^a. \]

From equations (27) and (28), we have \( T \propto n^{1/2(a-1)} \). The temperature depends on the density as,

\[ T \propto \begin{cases} n^{1/2(a-1)} & \text{sphere} \\ \text{const.} & \text{filament} \end{cases} \]

In this paper, we have considered a filamentary cloud with a variety of line mass. Hence, we estimated \( f \) for the filamentary clouds based on cosmological simulations. As an example, we refer to figure 2 of Greif et al. (2008). In this figure, a filamentary cloud with density \( n \sim 10^{-2} \text{ cm}^{-3} \) and radius \( \sim 7 \text{ kpc} \) is shown. The line mass of this filamentary cloud is \( \sim 7.8 \times 10^{16} \text{ g cm}^{-1} \). When the temperature is \( 300 \text{ K} \), the critical line mass is \( I_{\text{crit}} \sim 3.5 \times 10^{17} \text{ g cm}^{-1} \). Hence, \( f \sim 22 \) and according to our results presented in section 4, the fragment mass may increase owing to the external radiation if the external radiation turns on at \( n \leq 1 \text{ cm}^{-3} \).

For simplicity, the model and numerical calculations described in this paper are one-dimensional for the filamentary cloud and one-zone for each fragment. In order to discuss fragmentation, we assumed the condition for fragmentation...
Regarding the first term on the right-hand side of equation (A2),
\[ \int r^2 \, dM = \int 4\pi r^4 \rho \, dr = \frac{4\pi \rho}{5} R^5 = \frac{3}{5} MR^2, \quad (A3) \]
and
\[ \frac{1}{2} D_r^2 \int r^2 \, dM = \frac{1}{2} D_r^2 \left( \frac{3}{5} MR^2 \right) = \frac{3}{5} M \left( \frac{D_r}{Dt} \right)^2 + \frac{3}{5} MR \frac{D^2 R}{Dt^2}. \quad (A4) \]
Regarding the second term,
\[ \int v^2 \, dM = \int 4\pi r^2 \rho v^2 \, dr = \int 4\pi \rho r^2 \left( \frac{D_r}{Dt} \right)^2 \frac{r^2}{R^2} \, dr = \frac{3}{5} M \left( \frac{D_r}{Dt} \right)^2, \quad (A5) \]
where we use
\[ v = \left( \frac{D_r}{Dt} \right) \frac{r}{R}. \quad (A6) \]
since the velocity is in proportion to \( r \), because of the uniform density. Hence, the left-hand side of equation (A2) is
\[ \int 4\pi r^3 \frac{D_v}{Dt} \, dr = \frac{3}{5} MR \frac{D^2}{Dt^2} R. \quad (A7) \]
On the other hand, concerning the right-hand side of the equation of motion, the term for the pressure gradient is
\[ -\int 4\pi r^4 \frac{dP}{dr} \, dr = 3(\gamma_{adi} - 1) \frac{k_B T}{\mu m_H} M. \quad (A8) \]
where \( \gamma_{adi} \) is the adiabatic index. The second term is
\[ -\int 4\pi r^5 \frac{GM}{r^2} \rho \, dr = \int (4\pi \rho)^2 G \frac{r^4}{R^4} \, dr = -\frac{3}{5} GM^2. \quad (A9) \]
Finally, we have the virial equation for a uniform sphere,
\[ \frac{3}{5} MR \frac{D_v}{Dt} = 3(\gamma_{adi} - 1) \frac{k_B T}{\mu m_H} R + \frac{3GM}{5R}. \quad (A10) \]
where we use \( \gamma_{adi} = 5/3 \).

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