Decomposition approach for the interdependency analysis of integrated power and transportation systems

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**Abstract:** The increasing deployment of fast charging infrastructure is coupling the operation of power and transportation systems. However, how to evaluate the coupling relationship between these two systems is less studied. This study proposes a look-ahead decentralised framework to solve the integrated power-traffic flow problem using the optimality condition decomposition (OCD) technique. By exploring the similarity between the iterative procedure of the algorithm and the interactive decision-making process of the systems, this study adds two original contributions to this literature. First, the authors demonstrate factors that influence the coupling relationship between the power and transportation systems. However, how to evaluate the coupling relationship between these two systems is less studied. This study proposes a look-ahead decentralised framework to solve the integrated power-traffic flow problem using the optimality condition decomposition (OCD) technique. By exploring the similarity between the iterative procedure of the algorithm and the interactive decision-making process of the systems, this study adds two original contributions to this literature. First, the authors demonstrate factors that influence the coupling relationship between the power and transportation systems. Second, they employ a dynamic multiplier-based OCD algorithm to solve the integrated flow problem in a decentralised manner with improved convergence performance. Their proposed algorithm considers the sensitivity of the electricity prices to the charging demand, which reduces the price fluctuations at some congested electrical nodes to enhance the convergence speed. The case study demonstrates factors that influence the coupling relationship between the power and transportation systems.

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**Nomenclature**

**Transportation network**

- \( N \): set of nodes
- \( \delta^t \): set of travel time arcs
- \( \delta^{cs} \): set of arcs representing charging costs
- \( a \in \delta \): indices for arcs with \( \delta = \delta^t \cup \delta^{cs} \)
- \( w \in \mathcal{W} \): indices for O-D pairs
- \( r \in \mathcal{R} \): indices for routes with \( \mathcal{R} := \bigcup_{w \in \mathcal{W}} \mathcal{R}_w \)

- \( v_w \): traffic flow on arcs
- \( v^{cs} \): vector of EV traffic flows through CSs
- \( f_{rw} \): travel flow on route \( r \) connecting O–D pair \( w \)
- \( c_a \): travel cost on arc \( a \), USD
- \( c \): vector of travel cost on arcs, USD
- \( c_{rw} \): travel cost on route \( r \) connecting O–D pair \( w \), USD

- \( \theta \): perception error parameter
- \( p^v \): EV charging power for unit traffic flow, MW
- \( d_w \): travel request between O–D pair \( w \)
- \( \beta \): economic value of time, USD/min
- \( t_{f} \): free-flow travel time on arc \( a \), min
- \( \gamma \): model parameter for cost function of arc \( a \)
- \( Q^{cs} \): capacity of charging cost arc \( a \)
- \( \delta_{ar} = \delta_{aw} = 1 \) if arc \( a \) belongs to route \( r \) and 0 otherwise

**Power network**

- \( \mathcal{A} \): set of buses in the power network
- \( \mathcal{A}^{cs} \): set of buses connected to CSs
- \( \mathcal{B} \): set of branches in the power network
- \( \pi(j) \): set of all children of bus \( j \)
- \( \lambda_j \): locational marginal prices, USD/MW
- \( \lambda^{cs} \): vector of electricity prices at CSs, USD/MW
- \( p_j \): active power generation, MW
- \( q_j \): reactive power generation, MVAR
- \( v_j \): square magnitude of complex voltage at node \( j \), kV
- \( q_j \): active net load at node \( j \), MW
- \( q_{nj} \): reactive net load at node \( j \), MVAR
- \( L_{ij} \): square magnitude of current from buses \( i \) to \( j \), kA
- \( P_{ij} \): active power flowing from buses \( i \) to \( j \), MW
- \( Q_{ij} \): reactive power flowing from buses \( i \) to \( j \), MVAR

- \( b_{ij} \): generation cost coefficient for quadratic terms, USD/MW^2
- \( b_j \): generation cost coefficient for linear terms, USD/MW
- \( R_j \): resistance of branch \( (i, j) \), \( \Omega \)
- \( X_{ij} \): reactance of branch \( (i, j) \), \( \Omega \)
- \( p_l \): base electricity load, MW
- \( L_0 \): contract energy price with the main grid, USD/MWs
- \( \bar{v}_{ij} \): lower/upper limit of \( v_j \), kV
- \( p_{il} / p_{lj} \): lower/upper limit of \( p_{ij} \), MW
- \( q_{il} / q_{lj} \): lower/upper limit of \( q_{ij} \), MVAR

**1 Introduction**

The proliferation of electric vehicles (EVs) is getting a boost from a fast charging technique that enables drivers to quickly recharge and keep going [1]. Fast chargers can recharge EVs within 15–120 min at the rate of 50–350 kW [2]. With such high charging rates, new fast-charging installations would bring significant impacts on the operation of power grids. Moreover, fast-charging systems are designed to serve drivers who travel long distances and lack of home charging access [2], hence affecting the travel patterns of EV drivers. As such, large-scale fast charging of EVs can establish remarkable interdependency between power and transportation systems [3].
Recently, a growing body of literature has emerged to address the interactions and couplings between the power and transportation systems. Researchers have investigated the equilibrium pattern of the coupled networks from the viewpoint of system operators [4–13]. The traffic flow is typically described by a Wardrop equilibrium pattern [7–12] or a social optimal pattern [4–6], while the power flow is depicted by optimal power flow models [4–12]. Studies explore various objectives, such as computing the equilibrium pattern [4, 7, 9, 10], minimising the total cost of the two systems [8, 12], and designing electricity prices and congestion tolls [5, 6, 11]. However, because of the inherent complexity of computing power and traffic flows, it remains as a challenge to design simple and efficient techniques to acquire integrated flow solutions while protecting the privacy and security of the individual systems.

To overcome this challenge, centralised, decentralised and centralised decentralised approaches are developed for solving the integrated power-traffic flow problems. There exist studies that solve the problem in a centralised manner, including the manifold suboptimisation algorithm [5, 11], direct search [7], and active-set algorithm [4]. However, since the power and transportation infrastructure is managed by independent system operators [6], currently there are no entities that are able to conduct a centralised operation. As a result, decentralised approaches are highly desirable. A few studies are now available on applying the best response decomposition [8, 9], dual decomposition [6], and the alternating direction method of multipliers (ADMMs) [10]. However, optimality and/or convergence analysis for the application of these algorithms is typically not available. Moreover, the majority of the existing approaches require a central coordinator for updating and distributing the Lagrange multipliers, which may not exist in practice.

Another key challenge of the current decentralised approaches is the convergence performance, where the iterative process could easily oscillate or even diverge [6]. Algorithms such as ADMM have superior convergence properties [14]. However, the performance is highly dependent on parameter tuning. An optimality condition decomposition (OCD) technique is employed in [8, 9], where no parameter tuning or central coordinators are required and a minimal amount of information is exchanged [15, 16]. The OCD algorithm is modified from the Lagrangian relaxation method. The convergence speed is dependent on the Lagrange multipliers that are generally kept static within an iteration. To improve the convergence speed, a dynamic multiplier (DM)-based OCD algorithm is proposed in [15] and further exploited in [17, 18] in power system research communities. The dynamic multipliers incorporate a linear approximation of the Lagrange multipliers, thus reducing oscillations at some ‘sensitive’ points and achieving faster convergence speed. However, the DM-based OCD algorithm has not been investigated in the integrated power-traffic flow problem yet.

This work is motivated by recent advances in power system research to develop a look-ahead decentralised coordinated framework between power and transportation systems. Moreover, we discover that there exists a remarkable similarity between the manner OCD algorithms decompose the integrated flow problem and the way two systems are coupled. We show that the decentralised technique can be used to foster a better understanding of the coupling relationship between the power and transportation systems. More specifically, our paper makes the following contributions:

- First, a look-ahead decentralised framework is proposed for the independent power and transportation system operators to compute the integrated flow solutions with faster convergence speed. Moreover, the optimality and convergence properties of the OCD algorithms are presented.
- Second, the convergence analysis motivates a novel viewpoint for examining the interdependency between power and transportation systems, thereby addressing the influence of various parameters on their coupling relationship. This makes it possible, for the first time, to evaluate the interdependency between the two entities based on the numerical experiments of the algorithms.
- Third, the sensitivity analysis for the power and transportation systems is conducted. Both analytical and numerical techniques for obtaining the sensitivity matrices are provided.

The remainder of this paper is organised as follows. Section 2 introduces the basic concepts of decomposition methods. Section 3 describes the application of the OCD algorithms on the integrated power-traffic flow problem. Section 4 presents the convergence analysis of the integrated flow problem. Section 5 discusses the main results for a case study. Finally, Section 6 summarises the key conclusions.

## 2 Review of decomposition methods

In this section, we briefly review the existing decomposition techniques. They are typically categorised into primal decomposition, dual decomposition, and OCD methods [19].

### 2.1 Primal decomposition

Consider a convex optimisation problem with the form

$$\min_{x,c} f(x) + g(z) \quad \text{s.t.} \quad Ax + Bz = c$$

with decision variables $x \in \mathbb{R}^n$ and $z \in \mathbb{R}^m$, where $A \in \mathbb{R}^{p \times n}$, $B \in \mathbb{R}^{r \times m}$, and $c \in \mathbb{R}^p$. We assume that $f$ and $g$ are convex and differentiable.

Clearly, if variable $Ax$ and $Bz$ were fixed, then the problem would decouple. Consequently, we introduce a slack variable $s \in \mathbb{R}^n$ to represent the value assigned to $Ax$, and $c - s$ is assigned to $Bz$. Explicitly, the algorithm is

$$x^{k+1} := \arg \min_x \{ f(x) : Ax = s^k \}$$

$$z^{k+1} := \arg \min_z \{ g(z) : Bz = c - s^k \}$$

$$s^{k+1} := s^k - q_k (Ax - z^k - \lambda^k)$$

where $q_k$ is a proper step size, $\lambda$ and $\lambda_k$ are the dual variables associated with the constraints $Ax = s^k$ and $Bz = c - s^k$, respectively.

### 2.2 Dual decomposition

The Lagrangian for (1) is

$$\mathcal{L}(x, z, \lambda) = f(x) + g(z) + \lambda^T (Ax + Bz - c)$$

where $\lambda \in \mathbb{R}^p$ is the Lagrange multiplier corresponding to the constraint $Ax + Bz = c$. Given the dual variable $\lambda$, the Lagrangian is separable with respect to $x$ and $z$. The algorithm consists of the iterations

$$x^{k+1} := \arg \min_x \mathcal{L}(x, z^k, \lambda^k)$$

$$z^{k+1} := \arg \min_z \mathcal{L}(x^k, z, \lambda^k)$$

$$s^{k+1} := \lambda^k - q_k (Ax^{k+1} + Bz^{k+1} - c)$$

Other dual-based decomposition methods, such as the Lagrangian relaxation and ADMM, can be viewed as an extended version of the dual decomposition. Interested readers may refer to [14] for further details on these algorithms.
The OCD technique is motivated by natural decomposition of the optimality conditions of the original problem [16]. The OCD algorithm consists of iterating the updates

\[ \lambda^{k+1} := \arg \min_{x} \{ f(x); Ax + Bz^{k} = c \} \]  \hspace{1cm} (9)

\[ z^{k+1} := \arg \min_{z} \mathcal{L}(x^{k+1}, z, \lambda^{k+1}) \]  \hspace{1cm} (10)

where \( \lambda^{k+1} \) is the Lagrange multiplier associated with the constraint \( Ax + Bz^{k} = c \) in problem (9). Notice that the solution of problem (10) does not depend on the value of \( x^{k+1} \) in subproblem (10). The variables exchanged between (9) and (10) are in fact \( x^{k+1} \) and \( z^{k+1} \), which are primal and dual variables, respectively.

The optimality and convergence properties of the OCD algorithm are given as follows. If we combine the first-order Karush–Kuhn–Tucker (KKT) optimality conditions of the subproblems (9) and (10), it can be observed that they are identical to those of the centralised problem (1). Hence, the optimality of the OCD algorithm is guaranteed.

The convergence properties of the OCD algorithm are demonstrated by constructing a fixed point mapping [20]. With this aim, let us denote by \( u: \mathbb{R}^{m} \to \mathbb{R}^{p} \) the mapping of problem (9) as

\[ \lambda^{k+1} := u(z^{k}) \]  \hspace{1cm} (11)

and denote by \( v: \mathbb{R}^{p} \to \mathbb{R}^{m} \) the mapping of problem (10) as

\[ z^{k+1} := v(\lambda^{k+1}) \]  \hspace{1cm} (12)

The OCD algorithm then reads as finding a vector \( z \in \mathbb{R}^{m} \), that is a fixed point of the composite mapping from (11) and (12), i.e.

\[ z^{k+1} = (v \circ u)(z^{k}) := u(z^{k}) \]  \hspace{1cm} (13)

The convergence of the proposed algorithm is formalised in the following statement.

**Proposition 1: (existence and uniqueness of the fixed point):**

The OCD algorithm converges to a unique fixed point \( z^{*} \in \mathbb{R}^{m} \) of (13) provided that for a given matrix norm, the following condition is valid [21]:

\[ \| u'(z^{*}) \| < 1 \]  \hspace{1cm} (14)

where \( u'(z^{*}) \) is the Jacobian matrix at \( z^{*} \). Then choosing an initial iterate \( \tilde{z} \) sufficiently close to \( z^{*} \), the iterates \( z^{k} \) will converge to the optimal solution \( z^{*} \) as \( k \to \infty \).

Since the spectral radius of a matrix is bounded by its matrix norm [21, Theorem 7.16], we have the following result.

**Remark 1:** A less strict condition for convergence of the OCD algorithm is

\[ \rho(u'(z^{*})) < 1 \]  \hspace{1cm} (15)

where \( \rho(\cdot) \) is the spectral radius of a matrix, which is defined as the maximal modulus of all the eigenvalues.

A complete proof for optimality and convergence is given in Section 9.1 of the Appendix. Since the Lagrange multiplier \( \lambda^{k+1} \) is regarded as a constant in the subproblem (10), this method is referred to as the static multiplier (SM)-based OCD algorithm to distinguish it with the method introduced in the following subsection [15].

### 2.4 DM-based OCD algorithm

The SM-based OCD algorithm has a linear rate of convergence provided that \( u'(z^{k}) \neq 0 \) [22]. The magnitude of \( \| u'(z^{k}) \| \) determines the rate of convergence, and the closer it approaches 1, the slower the iterations terminate [21]. This could lead to numerical oscillations or even failure of convergence. Observe that at iteration \( k + 1 \), if the value of \( \lambda^{k+1} \) is anticipated before solving (10), the divergence can be avoided in advance [15, 18]. To this end, dynamic multipliers are incorporated into the subproblem formulation, in which a linear approximation of the Lagrange multipliers is introduced

\[ z^{k+1} := \lambda^{k} + S^{k}(Bz^{k} - Bz^{k-1}) \]  \hspace{1cm} (16)

where \( S^{k} \in \mathbb{R}^{p \times p} \) is the sensitivity matrix of the Lagrange multiplier \( \lambda^{k} \) with respect to \( Bz^{k} \).

By substituting (16) into (10), we then revise the objective of subproblem (10) as follows:

\[ \mathcal{L}(z, z, \lambda, S) = g(z) + (\lambda - SBz)^{T}Bz + \frac{1}{2}(Bz)^{T}S(Bz) \]  \hspace{1cm} (17)

where \( \tilde{z} \) is fixed to the last computed value of \( z \). The coefficient 2 is introduced for satisfying the optimality conditions [18] (see Section 9.2 of the Appendix). Since the solution of the subproblem for \( z^{k+1} \) does not directly depend on \( x^{k+1} \), for ease of presentation, we omit the terms related to \( x^{k+1} \) in the expression of \( \mathcal{L} \) in (17). The algorithm then becomes

\[ x^{k+1} := \arg \min_{x} \{ f(x); Ax + Bz^{k} = c \} \]  \hspace{1cm} (18)

\[ z^{k+1} := \arg \min_{z} \mathcal{L}(z, z^{k}, \lambda^{k+1}, S^{k}) \]  \hspace{1cm} (19)

A proof of optimality and improved convergence analysis for the DM-based OCD algorithm is provided in Section 9.2 of the Appendix.

### 3 Integrated power-traffic flow problem

In this section, we investigate the properties and performance of the OCD algorithms on an integrated power-traffic flow problem, as shown in Fig. 1. We detail the formulation of the integrated flow problem in a centralised optimisation problem, which is then decomposed into the power and transportation subproblems based on the OCD technique. Interested readers may refer to our previous work [23] for further details. Please reference the Nomenclature section for definitions.

#### 3.1 Centralised formulation of the integrated power-traffic flow problem

We now detail the mathematical formulation of the integrated flow problem as follows:

[Fig. 1 Integrated power-traffic flow problem coupled by fast charging of EVs]
\[
\begin{align*}
\min & \quad \left[ \sum_{a \in \mathcal{A}} \int_{a}^{\infty} c_a(\omega, \lambda_a) \, d\omega - \sum_{a \in \mathcal{A}} \lambda_a P_{a0} v_a \right. \\
& \quad + \frac{1}{g} \sum_{a \in \mathcal{A}} \sum_{r \in \mathcal{R}} f_{ru}(\ln f_{ru} - 1) \\
& \quad + \left. \left[ \sum_{a \in \mathcal{A}} b_i(p_j^f) + \sum_{k \in \mathcal{M}} b_k p_j^f + \lambda_0 \sum_{k \in \mathcal{M}} P_{ak} \right] \right] \\
\text{over} & \quad v_a, \forall a \in \mathcal{A} \quad f_{ru}, \forall r \in \mathcal{R}, w \in \mathcal{W} \quad p_j^f, q_j^f, v_j, \forall j \in \mathcal{J} \quad P_{ji}, Q_{ji}, \forall (i, j) \in \mathcal{B} \quad \nu \in \mathcal{W} \\
\text{given} & \quad R_{ji}, X_{ji} \quad \forall (i, j) \in \mathcal{B} \quad d_{a0}, d_a, \forall a \in \mathcal{A} \quad \mathcal{V} \in \mathcal{W} \\
\text{s.t.} & \quad \sum_{a \in \mathcal{A}} f_{ru} d_{ru} = v_a, \quad \forall a \in \mathcal{A} \\
& \quad f_{ru} \geq 0, \quad \forall r \in \mathcal{R} \\
& \quad p_j = P_j - R_j L_j = \sum_{k \in \mathcal{M}} P_{kj}, \quad \forall j \in \mathcal{J} \\
& \quad q_j = Q_j - X_j L_j = \sum_{k \in \mathcal{M}} Q_{kj}, \quad \forall j \in \mathcal{J} \\
& \quad \nu_j = \nu_i - 2(R_i P_{ij} + X_i Q_{ij}) + (R_j + X_j) L_{ij}, \quad \forall (i, j) \in \mathcal{B} \\
& \quad \frac{p_j^f + Q_j^f}{l_{ij}} \leq L_{ij}, \quad \forall (i, j) \in \mathcal{B} \\
& \quad R_i L_{ij} - P_j \leq 0, \quad \forall (i, j) \in \mathcal{B} \\
& \quad X_i L_{ij} - Q_j \leq 0, \quad \forall (i, j) \in \mathcal{B} \\
& \quad p_j^f \leq p_j^f \leq \bar{p}_j, \quad q_j^f \leq q_j^f \leq \bar{q}_j, \quad \forall j \in \mathcal{J} \\
& \quad \nu_j \leq \nu_j \leq \bar{\nu}_j, \quad \forall j \in \mathcal{J} \\
\end{align*}
\]

The objective function (20) consists of two types of costs. The first three terms are the integrals of the total perceived travel costs excluding the charging expense. Note that \( \theta \) is a positive parameter in discrete choice models, describing the level of stochasticity in drivers’ choice behaviour [24]. A larger value of \( \theta \) represents smaller perception errors, resulting in a more deterministic choice behaviour (see Section 9.3 of the Appendix for details on discrete choice models). The remaining terms of (20) represent the total power generation cost. Specifically

\[
\sum_{a \in \mathcal{A}} \left[ b_i(p_j^f) + b_j p_j^f \right]
\]

represents the power production cost of the local generators, and

\[
\lambda_0 \sum_{k \in \mathcal{M}} P_{ak}
\]

represents the energy purchase cost from the main grid, where \( P_{ak} \) denotes the active power transmitted through the branches directly connected to the slack bus, and \( \lambda_0 \) is the contract energy price with the main grid.

The proposed model has two types of constraints. The first, consisting of constraints (21)–(23), imposes feasible traffic flow restrictions. Constraint (21) enforces the flow conservation law, which states that the set of routes carrying the flow must meet the demand. Constraint (22) defines the route–arc incidence relationship, requiring that the flow on an arc \( a \) is the amount of traffic using routes that include arc \( a \). Constraint (23) restricts the route flow to be non-negative.

The remaining constraints (24)–(31) impose the feasible power flow restrictions [25]. Constraint (24) defines the active power balance at each bus. The left-hand side of each equality is the active power injection, which should include charging demand if it is connected to a charging station (CS). Let \( \mathcal{M}^{\text{cs}} \) be the set of coupled buses in the power network, which indicates that bus \( j \in \mathcal{M}^{\text{cs}} \) provides electrical energy for the nearest CS on arc \( a \in \mathcal{A} \). Thus, we have the following relationship:

\[
p_j := -p_j^f + p_j^f + P_{a0} v_a, \quad \forall j \in \mathcal{M}^{\text{cs}}, a \in \mathcal{A} \]

where \( p_j^f, p_j^f, \) and \( P_{a0} v_a \) are the active power generation, base electricity load, and EV charging demand at bus \( f \), respectively, \( v_a \) is the traffic flow on charging arc \( a \), on which the EVs consume energy from bus \( j \in \mathcal{M}^{\text{cs}} \), and \( p^o \) is the average charging demand for unit traffic flow. Constraint (25) enforces the nodal reactive power balance. Constraint (26) describes the forward voltage drop on each branch. Constraint (27) gives a comic relaxation of branch power definition. Constraint (28) prevents reverse active and reactive power flows on both sides of a branch. Constraints (30) and (31) impose the security constraints on active power generation, reactive power generation, and nodal voltage, respectively.

3.2 Decomposed power and transportation subproblems

Using the OCD algorithms presented in the previous section, the convex optimisation problem in (20)–(31) can be decomposed into the power and transportation subproblems.

First, the influence of traffic flow on the power network is the injection of the charging load. Clearly, if the charging demand is given, i.e. \( v_a = v_a^{(0)} \), then we have the subproblem of economic dispatch for the power network

\[
\begin{align*}
\min & \quad \sum_{j \in \mathcal{J}} \left[ b_i(p_j^f) + b_j p_j^f \right] + \lambda_0 \sum_{k \in \mathcal{M}} P_{ak} \\
\text{over} & \quad p_j^f, q_j^f, v_j, \forall j \in \mathcal{J} \quad P_{ji}, Q_{ji}, \forall (i, j) \in \mathcal{B} \quad \nu \in \mathcal{W} \\
\text{given} & \quad v_a^{(0)}, v_a \in \mathcal{A} \quad f_{ru}, \forall r \in \mathcal{R}, w \in \mathcal{W} \\
\text{s.t.} & \quad \lambda_j \nu_j \leq \nu_j \leq \bar{\nu}_j, \quad \forall j \in \mathcal{J} \\
\end{align*}
\]

(24) – (31)

where we obtain the updated Lagrange multiplier \( \lambda_j \) associated with (36), namely, the locational marginal price (LMP), which is taken as the electricity price [26].

For the transportation network, if the electricity prices are fixed, the distribution of traffic flows is determined. Then we have the subproblem of traffic assignment for the transportation network given \( \lambda_j = \lambda_j^{(k+1)} \)

\[
\begin{align*}
\min & \quad \sum_{a \in \mathcal{A}} \int_{a}^{\infty} c_a(\omega, \lambda_a^{(k+1)}) \, d\omega \\
& \quad + \frac{1}{g} \sum_{a \in \mathcal{A}} \sum_{r \in \mathcal{R}} f_{ru}(\ln f_{ru} - 1) \\
\text{over} & \quad v_a, v_a \in \mathcal{A} \quad f_{ru}, \forall r \in \mathcal{R}, w \in \mathcal{W} \\
\text{given} & \quad \lambda_j^{(k+1)}, \forall j \in \mathcal{J} \quad d_a, \forall w \in \mathcal{W} \\
\text{s.t.} & \quad (21) - (23)
\end{align*}
\]

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### 3.3 Subproblem formulation using DM-based OCD

As noted before, the SM-based OCD algorithm may have slow convergence or numerical oscillations when condition (14) is narrowly met. This could occur when large-scale EV charging demand is integrated into the power grid, as demonstrated in Section 5. To this end, the DM-based OCD algorithm is utilised in the integrated flow problem.

Let us define the electricity prices at CSs as \( \lambda^{(k)}_j = [\lambda_{j,a}]_{a \in \mathcal{A}} \) and EV traffic flows through CSs as \( \tau^{(k)} = [\tau_{j,a}]_{a \in \mathcal{A}} \). The sensitivities of the electricity prices with respect to the EV flows are given in the matrix form as

\[
S = \begin{bmatrix}
\frac{\partial \lambda_{i}}{\partial v_{j}} & \cdots & \frac{\partial \lambda_{i}}{\partial v_{j}^{(k-1)}} \\
\vdots & \ddots & \vdots \\
\frac{\partial \lambda_{i}}{\partial v_{j}} & \cdots & \frac{\partial \lambda_{i}}{\partial v_{j}^{(k-1)}}
\end{bmatrix}
\]

(40)

Hence, the electricity price can be approximated as a linear function of EV traffic flows as

\[
\hat{\lambda}^{k+1}_j = \lambda^{(k)}_j + S^{(k)}_{jk}(\hat{v}_a^{(k)} - v_a^{(k-1)})
\]

(41)

where \( S^{(k)}_{jk} \) is the \((j, a)\)th entry of the sensitivity matrix \( S \) at iteration \( k \).

Substituting (41) into (38), the objective function for the transportation subproblem becomes

\[
\sum_{a \in \mathcal{A}} \int_{\mathcal{D}} c_{ja}(\alpha, \lambda^{(k+1)}_j) \, d\alpha + \frac{1}{g} \sum_{e \in \mathcal{E}} \sum_{r \in \mathcal{R}_e} f_{re}(\ln f_{re} - 1)
\]

Table 1 summarises the features of the algorithms introduced in Section 2 and their application on the integrated power-traffic flow problem [27]. In the next section, we give the convergence analysis of the OCD algorithms on the integrated flow problem, where we find that the convergence speed and the interdependency between the coupled systems are closely related.

### 4 Convergence analysis

This section presents the convergence analysis for the decentralised solutions of the integrated power-traffic flow problem. Based on (14), the iterative procedure described in Section 3.2 converges to the centralised solution if

\[
\| S^{(k+1)} \hat{S} \| < 1
\]

(42)

where \( S \) is defined in (40) and \( \hat{S} \) is the sensitivity matrix for the transportation network, containing the sensitivities of the EV traffic flows with respect to the electricity prices at CSs, i.e.

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**Table 1**: Comparison of the decomposition methods

| Algorithms          | Decomposed by          | Category      | Shared information | Suitable for                  | Results for the problem (20)–(31) |
|---------------------|------------------------|---------------|-------------------|-------------------------------|----------------------------------|
| SM-based OCD        | optimality conditions  | decentralised | primal and dual   | several coupling             | diverge for large \( p^{\infty} \) and \( \theta \) |
| DM-based OCD        | augmented Lagrangian   | decentralised | primal and dual   | few coupling constraints      | improved convergence             |
| ADMM                | Lagrangian             | decentralised | primal and dual   | few coupling constraints      | converge for scaled form ADMM    |
| dual decomposition  | Lagrangian             | distributed   | dual variables    | few coupling variables        | failure since (38) and (39)      |
| primal decomposition| primal problem         | distributed   | primal variables  |                              |                                  |

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Fig. 2: Decomposition of the integrated power-traffic flow problem by the SM-based and DM-based OCD algorithms (ED: economic dispatch; TAP: traffic assignment problem)

 Clearly, from (42), the convergence properties of the integrated flow problem are determined by the sensitivity matrices \( S \) and \( \hat{S} \). Moreover, the problem is decomposed in a way that is consistent with the physical connections (i.e. prices and energy) between the two networks, as depicted in Fig. 2. Hence, these sensitivity matrices manifest the factors that influence the interdependency between the networks. In addition, when the systems become more interdependent, that is when they are more ‘sensitive’ to the inputs of the other, the slower the algorithm converges to optimal solution [21].

To acquire insight into the convergence properties and the interdependency between the power and transportation systems, we describe the sensitivity analysis procedure reported in [28, 29] for the power and transportation networks in further detail.

### 4.1 Sensitivity analysis for the power system

The sensitivity of LMPs with respect to electrical demand can be obtained by the perturbation technique reported in [17, 28]. The problems (35)–(37) are expressed compactly as

\[
\min_x \varphi(x, \epsilon) \quad (44)
\]

s.t. \( g(x, \epsilon) = 0: \kappa \)

(45)

\( h(x, \epsilon) \leq 0: \mu \)

(46)

with parameter vector \( \epsilon \) and decision variable vector \( x := (p_f, q_m, v_i, \forall i \in \mathcal{I}, P, Q, L_i, \forall (i, j) \in \mathcal{Z}) \), where the objective function (44) refers to (35), the equality constraints \( g(x, \epsilon) = (g(x, \epsilon), \ldots, g_n(x, \epsilon))^T \) consist of (24)–(26), the inequality constraints \( h(x, \epsilon) = (h_1(x, \epsilon), \ldots, h_n(x, \epsilon))^T \) include (27)–(31), and \( \kappa \) and \( \mu \) are the vectors of Lagrange multipliers associated with (45) and (46), respectively. Assume that the optimal solution is regular and non-degenerate.
Moreover, the cost function for a charging cost arc $a \in A^{ev}$ is

$$c_a(\omega_a, \epsilon_a) = p^{ev}(\lambda_j + \epsilon_a) + \beta_{\omega_a} \left[ 1 + \gamma_a \left( \frac{v_a}{Q_c} \right)^i \right] \tag{54}$$

By the implicit function theorem \[31\], $v_a$ can be expressed as a function of a free variable $\epsilon$ and an intermediate variable $c_a$ that depends on $\epsilon$ \[29\], i.e.

$$v_a = \psi_a(c_a(\epsilon), \epsilon) \tag{55}$$

Using Leibniz’s rule, one obtains the sensitivities of the EV flows with respect to the parameter $\epsilon$

$$\frac{\partial v_a}{\partial \epsilon} = -\frac{p^{ev}'(Q_c)^i}{4p^{ev}_b \epsilon(\alpha_{v_a})} \frac{\partial \psi_a}{\partial c_a} - \frac{\partial \psi_a}{\partial v_a} \frac{\partial v_a}{\partial \epsilon} \tag{56}$$

where $\delta_{ab}$ is an indicator parameter, that is 1, if $a = b$ and 0 otherwise, and by (54), one obtains

$$\frac{\partial \psi_a}{\partial v_a} = \frac{(Q_c)^i}{4p^{ev}_b \epsilon(\alpha_{v_a})} \tag{57}$$

Substituting (53) into (56) yields the sensitivity matrix $\dot{S}$.

### 4.3 Numerical methods for sensitivity analysis

For large network systems, the aforementioned analytical methods to compute the sensitivity matrix may become computationally intensive. A more practical way to obtain the sensitivity matrix is by numerical approaches \[18\].

Take the sensitivity analysis for the power network as an example. The sensitivity matrix can be approximated by the least-squares solution

$$S^{k+1}_{\omega_a} = \frac{\sum_{m=2}^{k+1} \left( v_a^{(m)} - v_a^{(m-1)} \right) j^{(m+1)} - j^{(m)}}{\sum_{m=2}^{k+1} \left( v_a^{(m)} - v_a^{(m-1)} \right)} \tag{58}$$

where the sensitivity matrix at iteration $k + 1$ can be obtained by the historical values of $v_a$ and $j_i$.

In the numerical methods, the power network is treated as a black box and the charging traffic flows and electricity prices are taken as inputs and outputs, respectively, shown in Fig. 3.

### 5 Case study

This section investigates the performance of the SM-based and DM-based OCD algorithms. In our case study, we consider the coupled transportation network of Sioux Falls city (shown in Fig. 4) and a 141-bus radial distribution network (shown in Fig. 5) \[32–34\]. The CCSs located in Fig. 4 are assumed to consume the electrical energy from the nearest buses in Fig. 5, and the coupled arcs and buses are labelled by the same colours in the figures. Here, the charging powers of General Motors Chevrolet Bolt and Tesla supercharger are adopted. The Chevrolet Bolt EV has three charging powers of General Motors Chevrolet Bolt and Tesla supercharger are adopted. The Chevrolet Bolt EV has three charging powers of level one at 1 kW, level two at 7.68 kW, and DC fast charging at 50 kW \[35\], while the Tesla supercharger can charge vehicles up to the rate of 350 kW \[36, 37\]. The base scenario is set at $p^{ev} = 350$ kW and $\theta = 10$. The sensitivity matrix is obtained by numerical methods.

The correlation between the interdependency of the coupled systems and the convergence performance of the OCD algorithm is depicted in Figs. 6 and 7. The bar graph in Fig. 6 (left Y-axis) shows the electricity prices at CSS for varying EV charging power $p^{ev}$. It is clear from the graph that the electricity prices at all CSs increase with $p^{ev}$, indicating that the charging costs play a more critical role in drivers’ route choice. As expected, the number of iterations using the SM-based OCD algorithm also increases with the growing EV charging power. Specifically, the SM-based OCD

\[830\]

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algorithm fails to reach a converged solution when the average charging power reaches $p^{ev} = 350$ kW. To acquire insight on the interactions between the power and transportation systems, the price of electricity at CSs obtained by coordinated operation and shortest path strategies is provided in Fig. 7. The bar graph shows that the electricity price at coordinated operation is always lower than that using the shortest path strategy for varying proportion of EV integration. Moreover, the gap between using the coordinated operation and shortest path strategies becomes wider as the proportion of EV integration grows. This result can be understood by noting that the electric information helps EV drivers to avoid charging at congested stations, thus alleviating the power congestion on the system level. Additionally, the number of iterations of the SM-based OCD algorithm also increases when the EV integration becomes larger. Both Figs. 6 and 7 validate our analysis in Section 4 that when the coupled systems become more closely interacted, the convergence speed of the OCD algorithm gets slower. These observations can be used for system operators to determine when the coordinated operation of the power and transportation systems is imperative. For example, the power and transportation systems can be operated independently when applying level 2 (7.68 kW) charging or when the percentage of EV integration is <40%.

To compare the performance of the SM-based and DM-based OCD algorithms, the iterative evolution of the electricity prices at CSs using the static and dynamic algorithms is reported in Figs. 8 and 9, respectively. Fig. 8 shows that the SM-based algorithm oscillates through the iterative procedure and fails to converge after 50 iterations. In contrast, the DM-based algorithm converges to the optimal electricity prices after only seven iterations, which significantly improves the convergence speed. These two figures indicate that with the prediction ability, the DM-based OCD algorithm can anticipate the potential change of the electricity prices, hence significantly enhancing the convergence performance.

The comparison of the SM-based and DM-based OCD algorithms is further elucidated in Figs. 10 and 11. The iterative procedure of electricity price at CS 6 to the optimal price using the SM-based and DM-based OCD algorithms is portrayed in Fig. 10. While the SM-based algorithm fails to converge after 50 iterations, the DM-based algorithm converges in seven iterations.
Moreover, the convergence properties of the proposed algorithm provide a novel metric for evaluating the interdependency between the power and transportation systems. The numerical simulations demonstrate that a greater amount of charging demand and more deterministic choice behaviour can create stronger interactions between the two entities. Our sensitive analysis and numerical experiments can provide useful suggestions for government agencies to decide whether to apply an integrated transportation strategy. In the future, we will consider more detailed power and transportation system models, investigate the dynamics of the systems, and examine their coupling relationship over certain time periods.

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the optimality condition (59), thus achieving the optimal solution of problem (1). □

Proof: The following definition is required for the proof of convergence [39, Theorem 9.3].

Definition 1: Let \( z^* \) be a fixed point of (13). If the Jacobian matrix \( \| J(z^*) \| < 1 \), then \( u \) is a contraction mapping at \( z^* \).

It is straightforward to see that, when (14) is satisfied, \( u \) is a contraction mapping, thus following the contraction mapping theorem stated in the following [21, Theorem 18.18].

Theorem 1: If \( u: \mathbb{R}^p \rightarrow \mathbb{R}^p \) is a contraction mapping on a closed bounded domain \( \Omega \subset \mathbb{R}^p \), then \( u \) admits a unique fixed point \( z^* \in \Omega \). Moreover, starting with any initial point \( z^0 \in \Omega \), the iterates \( z^{k+1} = u(z^k) \) necessarily converge to the fixed point \( z^* \).

Hence, the SM-based OCD algorithm converges to the optimal solution when \( u \) satisfies (14).

9.2 Proof of optimality and convergence for the DM-based OCD algorithm

Proof: The proof is very similar to that of the SM-based OCD algorithm. Since the subproblems (9) and (18) remain the same, we will show that the optimality conditions of the subproblems (10) and (19) are identical [18]. To see this, we give the optimality condition of (19) as

\[
\mathbf{V}_c(g(z) + B^T(\lambda - SBz + SB\bar{z})) = 0
\]

where at convergence \( z = \bar{z} \), we can eliminate the sensitivity terms \( SBz \) and \( SB\bar{z} \). Hence, the conditions (60) and (59b) are identical, so the proof is complete. □

Proof: Following a similar proof given in [18], we compare the solution improvement between two consecutive iterations. Assume that both algorithms start with \( \lambda^0 \). Within iterations \( k \) and \( k + 1 \), the first-order optimality conditions for the SM-based OCD algorithm are

\[
\mathbf{V}_c(g(z^{k+1}) + B^T\lambda^{k+1}) = 0 \quad \text{(61a)}
\]

\[
\mathbf{V}_c(f(z^{k+1}) + A\lambda^{k+1}) = 0 \quad \text{(61b)}
\]

Hence, the solution improvement for the SM-based algorithm is

\[
\lambda^{k+1} - \lambda^k = -A^T\mathbf{V}_c f(z^{k+1}) + B^T\mathbf{V}_c g(z^{k+1})
\]

Analogously, based on the optimality conditions (60), the solution improvement for the DM-based OCD algorithm is

\[
\lambda^{k+1} - \lambda^k = -A^T\mathbf{V}_c f(z^{k+1}) + B^T\mathbf{V}_c g(z^{k+1}) - S^\lambda B\bar{z}^{k+1} + S^\lambda B\bar{z}^{k+1}
\]

where \( S^\lambda \) is the Lagrange multiplier at iteration \( k + 1 \) for the DM-based OCD algorithm.

Subtracting (62) from (63), one obtains

\[
\lambda^{k+1} - \lambda^k = S^\lambda B(z^{k+1} - z^*)
\]

It has thus been shown that within each iteration, the DM-based OCD algorithm can approach the optimal solution by a linear approximation closer to the optimal value than the SM-based OCD algorithm, which eventually leads to improved convergence of the DM-based OCD algorithm. □

9 Appendix

9.1 Proof of optimality and convergence for the SM-based OCD algorithm

Proof: An underlying assumption for the proof is that the solution is a regular point with no active inequality constraints degenerate [16, 38]. Hence, the first-order KKT optimality conditions are necessary for the solution to be a local minimiser. Assume that problem (1) is convex. The first-order optimality conditions for problem (1) are

\[
\mathbf{V}_c(f(z)) + \lambda^T \mathbf{V}_c(Ax + Bz - c) = 0 \quad \text{(59a)}
\]

\[
\mathbf{V}_c(g(z)) + \lambda^T \mathbf{V}_c(Ax + Bz - c) = 0 \quad \text{(59b)}
\]

\[
Ax + Bz = c \quad \text{(59c)}
\]

It can be observed that when the static OCD algorithm converges, the first-order optimality conditions for problem (9) are (59a) and (59c), and the optimality condition for problem (10) is (59b). Thus, the converged solution of the SM-based OCD algorithm satisfies
9.3 Brief introduction to discrete choice models

An expanded transportation network is established to capture both travel time and charging costs into a graph, as shown in Fig. 12. Pseudo arcs are added to represent the charging costs, shown as the red arcs $e^A = \{5, 6\}$ in Fig. 12b. The perceived utility of travelling route $r$ between O–D pair $w$, $U_{rw}$, is given by [40]

$$U_{rw} = -\theta c_{rw} + \xi_{rw}, \quad \forall r \in \mathcal{R}_w, w \in \mathcal{W}$$

(65)

where $\theta$ is a positive unit scaling parameter, $c_{rw}$ is the actual travel cost, $-\theta c_{rw}$ is the measured utility, and $\xi_{rw}$ is the random error term associated with the route concerned. We utilise the logit-based discrete choice model. The probability of choosing route $r$, $P_{rw}$, is given by [24]

$$P_{rw} = \frac{\exp(-\theta c_{rw})}{\sum_{j \in \mathcal{R}_w} \exp(-\theta c_{wj})}, \quad \forall r \in \mathcal{R}_w, w \in \mathcal{W}$$

(66)

Notice that $\theta > 0$ is a given parameter, measuring the different degrees of travellers’ perception errors on the travel cost. If $\theta \to \infty$, the perception is reliable that the resulting traffic pattern approximate to that of the deterministic traffic flow pattern and vice versa [40].