Rear-Wheel Steering Control for Enhanced Steady-State and Transient Vehicle Handling Characteristics

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ABSTRACT This paper presents a rear-wheel steering (RWS) control algorithm to enhance the vehicle handling performance without prior knowledge of tire characteristics. A RWS system is a chassis control module that can effectively improve vehicle maneuverability and stability. Since the tire-road friction coefficient is difficult to obtain in real world application, the proposed RWS control algorithm is designed so that it can be implemented without any tire-road information. The proposed RWS control algorithm consists of steady-state and transient control inputs. The steady-state control input is proportional to the driver’s steering input for achieving the desired yaw rate gain. The desired yaw rate gain is obtained through an offline optimization that is aimed to minimize the vehicle lateral velocity. The transient control input consists of feedforward and feedback control inputs. The feedforward input is designed to improve transient responses of the yaw rate. Computer simulation studies have shown that a trade-off relationship between overshoot and response time exists when the RWS control input is a sum of the steady-state and feedforward inputs. To compromise this conflict, a feedback input has been designed. The overshoot can be significantly reduced while the response time is slightly changed via the feedback input. The proposed algorithm has been investigated via computer simulations. The simulation has been conducted for step steer and sine with dwell scenarios under various road friction conditions. The performance of a RWS vehicle was evaluated using objective indices. Simulation results show that the proposed algorithm enhances vehicle handling performance.

INDEX TERMS Rear-wheel steering control, vehicle handling, maneuverability, lateral stability, lateral transient response, performance evaluation.

NOMENCLATURE

| Symbol | Description |
|--------|-------------|
| αf/αr | Front/rear tire slip angle [rad] |
| δf/δr | Front/rear wheel steer angle [rad] |
| β | Vehicle side slip angle [rad] |
| η | A new design parameter in feedforward of transient control input |
| γ | Vehicle yaw rate [rad/s] |
| μ | Road friction [] |
| ωd | Damped natural frequency of system [Hz] |
| ωn | Natural frequency of system [Hz] |
| τ | Time constant of 1st order delay [s] |
| ζ | Damping ratio of system [] |
| ay | Vehicle lateral acceleration [m/s²] |
| aij | Element of matrix A at the i-th row and j-th column |
| bji/cij | Element of matrix B/C at the i-th row |
| C_f/C_r | Front/rear tire cornering stiffness [N/rad] |
| F_yf | Lateral force of front tire [N] |
| F_yr | Lateral force of rear tire [N] |
| G_ss | Steady-state yaw rate gain [] |
| I_z | Yaw moment inertia of vehicle [kg · m²] |
| kδ | Gain of rear-wheel steering in steady-state [ ] |
| K_fb | Feedback gain [] |
| K_us | Understeer gradient [rad · s²/m] |
| L | Distance from the front axle to the rear axle [m] |

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I. INTRODUCTION

Rear-wheel steering (RWS) control has been developed for decades with the aim of improving vehicle handling performance and stability. Steering the rear wheels offers control of rear lateral tire forces, and RWS systems offer great advantages in urban driving situations and on the highway.

In the urban driving situations such as cornering in a narrow alley, the driver's burden to exert steering wheel angle can be reduced when the curvature changes suddenly because the RWS system increases the vehicle’s yaw rate gain by steering the rear wheels in the opposite direction to the front wheels. In the highway driving situations, the vehicle’s lateral stability can be improved throughout the driving range from mild handling to limit handling maneuvers because the RWS system decreases yaw rate gain by rear wheels in the same direction as the front wheels.

The purposes of RWS control broadly fall into two categories: 1) minimize vehicle side slip angle, and 2) track the desired yaw rate. Early rear-wheel steering control methods have been used to enhance vehicle stability and maneuverability by regulating the side slip angle [1]–[4]. Nagai et al. [2] designed a state feedback controller to maintain the zero-side slip angle by constructing the model-following RWS control. Lee et al. [3] proposed a control strategy, i.e., ‘four-wheel independent steering.’ This control strategy aimed to reduce not only the side slip angle but also the actuating power. Eguchi et al. [4] considered both vehicle lateral dynamics and suspension dynamics such as roll steer and compliance steer to make the side slip angle equal to zero.

A number of studies have been proposed to enhance the vehicle’s stability and maneuverability by tracking the output of the reference vehicle model. Ly et al. [5] proposed a yaw rate tracking four-wheel steering (4WS) by means of multi-objective H optimal control. This proposed algorithm accomplishes desired handling characteristics with fewer state variables than conventional model-following control methods. Wagner et al. [6] performed and compared the performance of active steering controllers through the optimization. Concretely, Wagner et al. evaluated front-wheel steering (FWS), rear-wheel steering (RWS), and all-wheel steering (AWS) for the tracking performance of the desired vehicle yaw rate and lateral speed. They concluded that RWS control shows the best performance in terms of actuator costs and vehicle lateral behavior. However, these model-based methods need to pinpoint parameters between the tire and road surface as well as the vehicle parameters.

Most model-based RWS control methods utilize a model that consists of vehicle lateral dynamics and tire dynamics. Based on such models, the RWS or 4WS control inputs are obtained by assuming that the correct vehicle model and parameters are known. Therefore, the performance of control methods can deteriorate when the uncertainties of the vehicle modeling and parameters are presented. For instance, when the driver negotiates a corner with high lateral acceleration conditions, the tire characteristics enter the nonlinear region. In this case, there are differences between nominal and actual parameters, and such parameter errors deteriorate the algorithm’s performance [7]–[10]. Therefore, practitioners often have laborious tasks such as adjusting control gains and parameters. To resolve this inconvenience, there are some methods to design a robust controller in consideration of uncertainties. Russell and Gerdes [11] proposed a state feedback controller to track a reference model and demonstrated stability and robustness to the model uncertainties. Akar [12] proposed a sliding mode controller to track both zero-side slip angle and reference yaw rate, which showed robustness against parameter variations.

In contrast to the model-based control methods that are widely researched, many car manufacturers have adopted a simple proportional RWS control algorithm for application in mass production process. In the proportional RWS control, the proportional gain is the ratio of the rear-wheel steering angle to the front wheel steering angle as a function of vehicle speed. This gain is designed to minimize the steady-state side slip angle. At low speeds, RWS is controlled in the opposite direction (i.e. reverse-phase) to the front wheels for increasing the yaw rate gain while at high speeds RWS is controlled in the same direction (i.e. in-phase) for enhancing the vehicle stability. However, such simple proportional RWS control has some problems via vehicle tests [13]. Brethedauer and Lynch [13] investigated the simple proportional RWS control with respect to various tire types such as winter tires and racing tires. Many test drivers suggest that unpleasant vehicle behavior could occur with respect to quick steering inputs and rear-wheel steering calibrations for tires. This is because the simple proportional RWS control does not consider the vehicle’s transient response. There are more advanced control methods that consider the transient response in RWS control input to resolve this unnatural vehicle lateral behavior. Cho and Kim [14] designed a delayed RWS whereby the time delay between the front-wheel and rear-wheel is extracted from the responses of the optimal 4WS control. Nissan’s phase reversal control [4] considers the suspension characteristics and demonstrated significantly improved vehicle response at high speeds compared to proportional control and first-order delay control [15], [16]. However, Nissan’s control logic requires tire and suspension parameters, and errors can degrade the performance of the control algorithm.

This paper proposes a new RWS control design framework that can modify the vehicle handling characteristics using

\[ \frac{l_f}{l_r} \] Distance from the center of gravity to the front/rear axle [m]  
\[ m \] Total mass of vehicle [kg]  
\[ M_c \] Yaw moment [N·m]  
\[ N_r \] Steering gear ratio []  
\[ R \] Radius of curvature [m]  
\[ s \] Laplace operator  
\[ t \] Time [s]  
\[ V_s \] Vehicle longitudinal velocity [m/s]  
\[ V_y \] Vehicle lateral velocity [m/s]
measurable vehicle signals, and, more importantly, without any information from the tire. The proposed RWS control law is a sum of steady-state and transient control inputs. The steady-state part of the proposed algorithm is designed to be proportional to the driver’s steering wheel angle input. This steady-state input modifies steady-state handling characteristics according to vehicle speeds. At low speeds, RWS is controlled in the opposite direction (reversed-phase) to the front-wheels for increasing the vehicle agility (increase yaw rate gain). At high speeds, RWS is controlled in the same direction (in-phase) to the front-wheels for improved vehicle stability (reduced yaw rate gain). The transient control input adjusts the vehicle’s transient response without any information from the tire. To design such transient control input, new vehicle dynamic models and new design parameters are proposed to exclude such tire parameters. Transient responses such as overshoot, rise time, and peak response time are a function of vehicle and tire parameters, and the transient control input can be designed without tire parameters via the proposed control design framework.

The main contributions of the proposed RWS control design framework are as follows:

1) The proposed control algorithm improves vehicle handling performance using measurable sensor signals (such as yaw rate, lateral acceleration, and steering wheel angle) without any information on tire and road surface.
2) This paper proposes a new chassis control design flow: (1) offline optimization; (2) parameterization of optimal solutions; (3) control design.
3) This paper proposes a new lateral dynamic model to design control law.
4) This paper proposes a transient control input without any tire and road friction information based on the proposed lateral dynamic model.

II. LATERAL VEHICLE DYNAMICS OF 4WS VEHICLE

A single-track (bicycle) vehicle model (Figure 1) has been used to describe the vehicle lateral dynamics with RWS control input [7], [17]. The equation of motion can be derived from the bicycle model in Figure 1:

$$\Sigma F_y = F_{sy} + F_{ry} = MV_x (\dot{\beta} + \gamma)$$

$$\Sigma M_z = F_{sy}l_y - F_{ry}l_r = I_z \ddot{\gamma}$$

(1)

The state equation is organized with the two vehicle state variables: 1) side slip angle $\beta$; and 2) yaw rate $\gamma$.

Assuming that linear behavior of lateral tire forces the tire slip angles, the tire forces can be described as:

$$F_{sy} = C_r \cdot \left( \frac{\delta_r - \beta + \frac{l_r \gamma}{V_x}}{\alpha_r} \right)$$

$\frac{F_{ry}}{l_f}$

$$F_{ry} = C_r \cdot \left( \frac{\delta_r - \beta + \frac{l_r \gamma}{V_x}}{\alpha_r} \right)$$

(2)

where $\alpha_f$ and $\alpha_r$ are the slip angles of the front and rear tire, respectively. The tire slip angles are defined by the kinematic relationship with steer angles and vehicle states. The tire lateral forces are actually calculated as $F_{y(f/r)}(k) = C_{y(f/r)}(k) \cdot \alpha_{y(f/r)}(k)$, and the tire cornering stiffness $C_f$ and $C_r$ are the nonlinear coefficients that well represent the vehicle lateral behavior at k step.

From (1) and (2), the state equation of the bicycle model with RWS can be written as

$$\dot{x}(t) = A \cdot x(t) + B \cdot \delta(t) + C \cdot \delta_r(t)$$

$$A = \begin{bmatrix} -\frac{(C_f + C_r)}{mV_x} & -\frac{(C_f l_f - C_r l_r)}{mV_x^2} & 1 \\ -\frac{(C_f l_f - C_r l_r)}{I_z} & \frac{(C_f l_f^2 + C_r l_r^2)}{I_z V_x} & 0 \\ \frac{C_f}{mV_x} & \frac{C_r}{mV_x} & \frac{C_f l_r}{I_z} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{C_f}{I_z} \\ \frac{C_r}{I_z} \\ 0 \end{bmatrix}$$

$$C = \begin{bmatrix} \frac{C_r}{I_z} \\ \frac{C_r}{I_z} \\ 0 \end{bmatrix}$$

(3)

where, the vehicle state $x(t)$ is $[\beta \gamma]^T$.

At steady-state, the derivative terms in (3) are zero ($\dot{\beta} = 0$, $\dot{\gamma} = 0$), and the vehicle dynamics in (3) shrinks to the vehicle cornering kinematics as follows:

$$\delta_f - \delta_r = \frac{L}{V_x} \gamma_{ss} + \frac{m}{L} \left( \frac{l_r}{C_f} - \frac{l_f}{C_r} \right) \cdot V_x \gamma_{ss}$$

(4)

where $\gamma_{ss}$ is the yaw rate in the steady-state, and $K_{ss}$ is the understeer gradient that shows the steering characteristic of the vehicle. The rear-wheel steering angle $\delta_r$ affects vehicle cornering kinematics.
For conventional vehicles without the rear-wheel steering system ($\delta_r = 0$) case, the cornering kinematics and the steady-state yaw rate gain of FWS vehicle $G'_{\gamma,\text{FWS}}$ are derived as:

$$\delta_f = \left( L + K_{us} V_x \right) \cdot \gamma_{ss} \quad (5)$$

$$G'_{\gamma,\text{FWS}} = \frac{\gamma_{ss}}{\delta_f} = \frac{V_x}{L + K_{us} V_x^2} \quad (6)$$

For vehicles with the rear wheel system ($\delta_r \neq 0$) case, the steady-state yaw rate gain for the RWS vehicle $G'_{\gamma,\text{RWS}}$ can be written as follows:

$$G'_{\gamma,\text{RWS}} = \frac{\gamma_{ss,\text{RWS}}}{\delta_f} = \frac{V_x}{L + K_{us} V_x^2} \cdot \left( 1 - \frac{\delta_{r,ss}}{\delta_f} \right)$$

$$= G'_{\gamma,\text{FWS}} \cdot \left( 1 - \frac{\delta_{r,ss}}{\delta_f} \right) \quad (7)$$

where $\gamma_{ss,\text{RWS}}$ is the steady-state yaw rate of RWS vehicle, and $\delta_{r,ss}$ is the rear-wheel steering input in the steady-state. The steady-state yaw rate gain for RWS vehicle is proportional to that for FWS vehicle as described in (6) and (7). Moreover, the ratio between two steady-state gains is a function of front and rear steering wheel angles. Therefore, the steady-state cornering characteristics can be modified by controlling the rear-wheel steering angle proportional to the front-wheel steering angle (the driver’s input). The details of modifying the steady-state cornering characteristics are elucidated in the next chapter.

This paper uses values the vehicle parameters from Table 1. These vehicle parameters were tuned to show similar dynamic characteristics of the given test vehicle—an F-segment vehicle with Rear-Wheel Drive (RWD).

### TABLE 1. Nominal values of vehicle parameter.

| Parameter (Notation) | Value (Units) |
|----------------------|---------------|
| Total mass (m)       | 2055.14 [kg]  |
| Yaw moment inertia ($I_y$) | 4551 [kg·m²] |
| Wheel base (L)       | 3.009 [m]     |
| Distance from C.G to front wheels ($l_f$) | 1.477 [m] |
| Distance from C.G to rear wheels ($l_r$)  | 1.532 [m] |
| Steering gear ratio ($N_s$) | 15.221 |
| Understeer Gradient ($K_{us}$) | 0.0063 [rad · s²/m] |

III. CONTROLLER DESIGN

This section details a new rear-wheel steering (RWS) control algorithm. The proposed algorithm is a combination of the steady-state and transient control inputs as shown in Figure 2.

The steady-state RWS control input is proportional to the driver’s front-wheel steering. The proportional gain is obtained through offline numerical optimization. The transient control input improves the transient handling response such as overshoot, rise time, and peak response. The vehicle transient response can be deteriorated with sole steady-state RWS control (the proportional control). For example, the lateral acceleration response of the proportional control is unnatural compared to that of the base vehicle (Figure 3). Figure 3 (a) shows the rear-wheel steering input proportional to the front-wheels, and Figure 3 (b) shows lateral acceleration responses. The base is the FWS vehicle with the specification of Table 1. The RWS is a vehicle with the proportional controller. The computed value is the multiplication of the yaw rate and vehicle speed, which means the steady-state lateral acceleration of the RWS vehicle.

The previous research [13] showed experimentally that this unnatural lateral jerk makes the driver feel unpleasant. This unnatural lateral jerk results from transient handling characteristics—especially the first-time derivative of the side slip angle. There is a difference between the lateral acceleration and the multiplication of the vehicle speed and yaw rate (Figure 3 (b)). Since this difference denotes the first-time derivative of the side slip angle, we conclude that this term is the cause of this unnatural lateral jerk.

To improve this issue, previous studies [14]–[16] added a 1st-order delay term to the rear wheel steering control algorithm. Adding such a delay term improves the lateral transient response by reducing the phase delay between the yaw rate and the lateral acceleration. Here, the previous approach to add a 1st order delay is expanded. The transient RWS control input consists of two parts: (1) feedforward input to control the delay of the yaw rate response, and (2) feedback input to control the first-time derivative of the side slip angle.
where, $\delta_{149286}$

The steady-state yaw rate response generated by (8) can be re-arranged using the yaw rate gains of (9):

$$k_3(V_x) = 1 - \frac{G_{ss,RWS}^r(V_x)}{G_{ss,FWS}^r(V_x)}$$

$$\delta_{r,ss}(t) = k_3(V_x) \cdot \frac{G_{ss,FWS}^r(V_x)}{G_{ss,RWS}^r(V_x)}$$

As described in (9), the steady-state yaw rate gain of RWS vehicles is a multiple of that of the FWS vehicles, and the proportional gain $k_3$ is a design variable. As this gain increases (or decreases), the steady-state yaw rate gain of RWS vehicles decreases (or increases) relative to that of FWS vehicles.

Designing the proportional gain $k_3$ is important because the proportional gain $k_3$ is the sole design variable to modify the steady-state response of vehicles. In this paper, a steady-state gain is designed through offline numerical optimization results. The control objective of the optimization is selected based on [6]. Wagner et al. conducted a performance comparison by configuring various active steering controls to track the reference trajectory. The active steering configurations are the passive vehicle (Base), single-actuation configurations for reference yaw rate tracking ($\text{FWS}^y$, $\text{RWS}^y$), and for lateral velocity minimization ($\text{FWS}^v$, $\text{RWS}^v$), and all-wheel steering (AWS) for tracking both references. Wagner et al. concluded that $\text{RWS}^y$ shows the best performance when comparing the actuator cost and objective assessments with the various criteria. Therefore, in this paper, $\text{RWS}^y$ control based on the optimization plant in [6] has been adopted for the offline numerical optimization.

The optimization results are presented in Figure 4. Figure 4 (a) represents the ratio of RWS yaw rate gain to FWS gain ($G_{ss,RWS}^r/G_{ss,FWS}^r$), and (b) shows the proportional gain ($k_3$) of steady-state control input. The proportional gain becomes smaller than zero at low speeds to increase the yaw rate gain; as a result, steady-state RWS input is controlled in the reverse-phase as the front-wheel steering. Conversely, the steady-state RWS input at high speeds reduces the yaw rate gain to improve vehicle stability by steering in the in-phase as the front-wheels. Correspondingly, the proportional gain is bigger than zero, and the control input is set to in-phase to the front-wheel steering. The speed is about 56kph when the steady-state RWS gain is zero. The results are consistent with how practitioners design the RWS control. Typically, RWS is controlled in the reverse-phase to enhance the yaw rate response at low speed while it is controlled in the in-phase to enhance the vehicle stability at high speeds.

In conclusion, the proportional gain $k_3$ from the optimization results and the steady-state control input $\delta_{r,ss}$ of (8) are re-arranged using the yaw rate gains of (9):

$$\delta_{r,ss}(t) = k_3(V_x) \cdot \frac{G_{ss,FWS}^r(V_x)}{G_{ss,RWS}^r(V_x)} \cdot \frac{G_{ss,FWS}^y}{G_{ss,RWS}^y}$$

However, the control input in (10) and (11) results in an unnatural lateral jerk as illustrated in Figure 3. Therefore, in this paper, the transient control input is designed to compensate for the unnatural lateral jerk.

**B. REAR-WHEEL STEERING CONTROL FOR ENHANCED TRANSIENT HANDLING – FEEDFORWARD CONTROLLER**

The transient RWS control input consists of model-based feedforward and feedback parts. In this subsection,
input and the feedforward part of the transient input is outcome. Analyzed to investigate how such a control input affects the first; subsequently, the closed-loop dynamics will be analyzed to investigate how such a control input affects the outcome. In this subsection, the proposed feedforward transient control input is presented to investigate how such a control input affects the outcome. In this subsection, the proposed feedforward transient control input is presented to investigate how such a control input affects the outcome. In this subsection, the proposed feedforward transient control input is presented to investigate how such a control input affects the outcome. In this subsection, the proposed feedforward transient control input is presented to investigate how such a control input affects the outcome. In this subsection, the proposed feedforward transient control input is presented to investigate how such a control input affects the outcome.

The proposed RWS control input including the steady-state control input for modifying the steady-state yaw rate gain and the feedforward part for enhancing the vehicle's lateral transient response is represented as follows:

$$\delta_f(t) = \delta_{r,ss}(t) + \delta_{r,tf,ff}(t)$$

$$= k_{\delta} \cdot \delta_f(t)$$

$$= \begin{cases} 1 - \eta \cdot \delta_f(t) + K_{us} \cdot a_{r}(t) + \frac{L}{V_s(t)} \cdot \gamma(t) & \text{Transient} \\ \text{Steady-state} \end{cases}$$

where, $\delta_{r,tf,ff}$ is the feedforward control to modify transient characteristics of the rear-wheel steering vehicle.

The control law in (12) consists of two terms: the steady-state control input for modifying the steady-state yaw rate gain and the feedforward part for enhancing the vehicle's transient response. Note that the control law in (12) is the generalized control input of (11). This is because the control input in (12) becomes equal to the steady-state RWS of (11) when the design parameter becomes 1.

The effects of the control input (12) are analyzed. To analyze the effects of the control input in (12), the bicycle model in (3) is newly formulated in the form of the transfer function as follows:

$$\frac{\gamma(s)}{\delta_f(s)} = \frac{1}{\tau s + 1} \cdot \frac{V_s}{L} \cdot \left[ 1 - \frac{k_{\delta}}{\tau s + 1} - K_{us} \cdot a_r(s) \right]$$

(13)

$$\tau = \frac{a_{11}}{a_{21} - |A|} = \frac{L/V_s \cdot (C_f + C_r)}{C_f C_r L^2}$$

(14)

Here, $a_{ij}$ is the element of matrix A at the i-th row and the j-th column, and |X| is the determinant of matrix X. The derivation of (13) and (14) are attached in Appendix A.

As described in (13), the vehicle’s yaw rate (or driver’s steering angle) is regarded as the sole output (or input) of the system. The rear-wheel steering angle and the lateral acceleration is set as the external input of the system. Note that this system is a first-order delay system with the gain.

In the case of vehicles with the steady-state RWS control input only ($\delta_r = \delta_{r,ss}$), the lateral response can be expressed as follows:

$$\frac{\gamma(s)}{\delta_f(s)} = \frac{1}{\tau s + 1} \cdot \frac{V_s}{L} \cdot \left[ 1 - k_{\delta} - K_{us} \cdot a_r(s) \right]$$

(15)

As mentioned above, the goal of the feedforward transient control input is to enhance the transient response in (13). By substituting (12) into (13), the closed-loop dynamics with the control law in (12) can be derived as:

$$\frac{\gamma(s)}{\delta_f(s)} = \frac{1}{\eta \tau s + 1} \cdot \frac{V_s}{L} \cdot \left[ 1 - k_{\delta} - K_{us} \cdot a_r(s) \right]$$

(16)

As described in (16), by changing the design parameter $\eta$, it is possible to modify the transient response of the vehicle lateral behavior. Since the parameter $\eta$ is a coefficient of the time constant, the system becomes sluggish or responsive according to the value of such a parameter. We note that the model-based feedforward control input is described. The goal of the transient RWS control is to enhance lateral transient response compared to the transient response when only steady-state control in (11) is exerted to the vehicle. For example, the goal of the transient control is to minimize yaw rate overshoot and unnatural lateral acceleration response. In this subsection, the proposed feedforward part of the transient control input will be presented first; subsequently, the closed-loop dynamics will be analyzed to investigate how such a control input affects the outcome.

The proposed RWS control input including the steady-state input and the feedforward part of the transient input is illustrated in Figure 4. The ratio of steady-state RWS gain obtained by numerical optimization is shown in Figure 4 (a). The ratio of yaw rate gain as the ratio to the base vehicle is reduced vehicle speed increases. (b) Proportional gain $k_{\delta}$: based on about 56 kph, RWS is controlled to be in-phase at high speed and reverse-phase at low speed.

**FIGURE 4.** Illustration of the ratio of steady-state RWS gain obtained by numerical optimization. (a) Ratio of yaw rate gain $G_{r,ss}/G_{r,ss}$ as the ratio to the base vehicle is reduced vehicle speed increases. (b) Proportional gain $k_{\delta}$: based on about 56 kph, RWS is controlled to be in-phase at high speed and reverse-phase at low speed.
the control input in (12) can modify the transient response of the vehicle without any information on tire and vehicle parameters.

Analysis of the design parameter \( \eta \) has been conducted via computer simulations. The trade-off relationship between the rise time and the overshoot was discovered via computer simulations. The conceptual figure and the simulation results are presented in Figure 5. Figure 5 (a) is a schematic diagram of the concept of the design parameter \( \eta \). When \( \eta \) is bigger than 1, the system becomes sluggish: the rise time of yaw rate response is increased, and overshoot is decreased. On the other hand, the system becomes responsive when \( \eta \) becomes smaller than 1: The rise time in the transient region is improved. Moreover, the notable point in this case is the undershoot response of the rear wheel steering angle command. Figure 5 (b) shows the simulation results of changing the design parameter \( \eta \) revealing similarities with the conceptual diagram. The changes of \( \eta \) also affect the transient response of the lateral acceleration. The design of \( \eta \) is also aimed at modifying such a nonlinear transient response as shown in Figure 3. Based on these characteristics, \( \eta \) is tuned with reference to the optimal results in the step steer scenario such as the yaw rate’s response time and the lateral acceleration’s transient response.

Figure 6 shows the tuning results using (12) for mimicking the response time of the optimal RWS control’s yaw rate. In the case of 30 kph presented in Figure 6 (a), the proposed control law (12) can imitate not only the response time but also the overshoot of the optimal yaw rate, by setting the design parameter \( \eta = 1.3 \). In the case of 110 kph (Figure 6 (b)), the control law (12) can imitate the response time by setting the design parameter \( \eta = 0.6 \). However, the overshoot increases noticeably. The overshoot must be reduced since the overshoot of the yaw rate response is related to the lateral instability. Therefore, under high-speed driving, an additional control input is required to minimize the overshoot. The algorithm will be mentioned in detail in section 3.3.
C. REAR-WHEEL STEERING CONTROL FOR ENHANCED TRANSIENT HANDLING – FEEDBACK CONTROLLER

The feedback transient control input is proposed to compensate for the trade-off between the overshoot and response time that the feedforward transient control input has illustrated in Figure 5 and 6. Since the excessive overshoot of the yaw rate response in Figure 6 results from the side slip rate (the first time derivative of the side slip angle), the proposed feedback control is formulated as follows:

\[
\delta_{r, fb}(t) = -K_{fb} \cdot V_s(t) \cdot (\dot{\beta}(t) - \dot{\beta}_{des}(t)) \\
= -K_{fb} \cdot (a_x(t) - V_s(t) \cdot \gamma(t)) \tag{17}
\]

where \(\delta_{r, fb}\) is the feedback control to modify transient characteristics of the rear-wheel steering vehicle, and \(K_{fb}\) is the feedback gain that is a positive number. \(\dot{\beta}\) is the time-derivative of the side slip angle, and \(\dot{\beta}_{des}\) is the desired \(\dot{\beta}\) that is zero in this paper.

The final form of the proposed control algorithm is obtained by adding (17) to (12) as follows:

\[
\delta_{r, proposed}(t) = \delta_{r, ss}(t) + \delta_{r, ff}(t) + \delta_{r, fb}(t) \\
= k_2 \cdot \delta(t) + \left\{ \begin{array}{l}
\text{Steady-state} \\
\quad + \left( \frac{1}{\eta} - 1 \right) \left\{ (k_4 - 1) \cdot \delta(t) + K_{us} \cdot a_x(t) + \frac{L}{V_s(t)} \cdot \gamma(t) \right\} \\
\text{Feedforward for Transient} \\
\quad - K_{fb} \cdot (a_x(t) - V_s(t) \cdot \gamma(t)) \end{array} \right. \tag{18}
\]

The proposed RWS algorithm determines the steady-state response using the yaw rate gain ratio, and modifies the transient response such as overshoot and peak response time by tuning \(\eta\) and \(K_{fb}\). Yaw rate, lateral acceleration, and steering wheel angle in control law (18) are measured from sensors mounted on the vehicle. The vehicle longitudinal speed is estimated using the wheel speed and longitudinal acceleration [18].

Verification is required to ensure that the feedback control reduces the overshoot of the yaw rate without changing the steady-state yaw rate gain. The verification is processed in two parts: (1) the feedback control input does not change the steady-state yaw rate gain, and (2) the feedback control input reduces the overshoot.

First, (13) is reformulated by substituting (18) to identify whether the feedback gain changes the steady-state yaw rate gain.

\[
\frac{\gamma(s)}{\delta_f(s)} = \frac{1}{\eta f \tau_s + 1} \cdot \eta f \cdot \frac{\eta}{\eta} \\
\quad \times \frac{V_s}{L} \left[ 1 - k_4 \left( K_{us} - \eta K_{fb} \right) \right] \frac{a_x(s)}{\delta_f(s)} \\
\eta_f = \frac{\eta}{1 + \eta K_{fb} V_s^2 / L} \tag{19}
\]

Here, \(\eta_f\) is a function of \(\eta\) and \(K_{fb}\) that can express the yaw rate in the same way of (16). The derivation of (19) is attached in Appendix B. To obtain the steady-state yaw rate gain, (19) can be re-written as follows by applying the steady-state yaw rate gain, \(\gamma_{ss}\) and the final value theorem:

\[
\frac{\gamma_{ss}}{\delta_f} = \frac{V_s}{L + K_{us} \cdot V_s^2} \cdot (1 - k_3) = G_{ss, RWS}^f \tag{20}
\]

The results in (20) show that the steady-state yaw rate gain does not change even if the feedback control is added.

Second, this data verified that the feedback control in (17) reduces the overshoot. The transfer function of the closed-loop dynamics is derived by substituting (18) into (3) as follows:

\[
\frac{\gamma(s)}{\delta_f(s)} = \frac{ps + q}{s^2 + 2\xi \omega_n s + \omega_n^2} \tag{21}
\]

where, \(p = \frac{(k_3 - 1) \left( \frac{1}{\eta} - 1 \right) c_2}{1 - c_1 \left( \frac{1}{\eta} - 1 \right) K_{us} V_x + c_1 V_x K_{fb}}, \quad q = \frac{(k_3 - 1) \left( \frac{1}{\eta} - 1 \right) |A|}{1 - c_1 \left( \frac{1}{\eta} - 1 \right) K_{us} V_x + c_1 V_x K_{fb}} \),

\[
\omega_n^2 = \frac{1}{1 - c_1 \left( \frac{1}{\eta} - 1 \right) K_{us} V_x + c_1 V_x K_{fb}} \tag{21}
\]

Here, \(\zeta\) is the damping ratio of the system, and \(\omega_n\) is the natural frequency of the system. Term \(c_1\) is the elements of the matrix \(C\), and \(A_i\) is the i-th column vector of matrix A. \(|X|\) is the determinant of matrix X, and \(tr(X)\) is the trace of matrix X.

To verify that the overshoot in the step steer scenario is reduced, the yaw rate response in the step-steer scenario is derived by substituting \(\delta_f(s) = k/s\) into (21). Term \(k\) is the step amplitude of steering command, and the value of \(k\) is SWA (45 deg) in this analysis.

\[
\gamma(s) = \frac{kps + kq}{s \left( s^2 + 2\xi \omega_n s + \omega_n^2 \right)} \tag{22}
\]
Here, \( \gamma_{ss} \) is the steady-state yaw rate \((k \cdot G_{ss,RWS}^y)\) in the step steer scenario, and \( \omega_d \) is the damped natural frequency of the system \((\omega_n \cdot \sqrt{1 - \zeta^2})\). Expression (23) is obtained by transforming (22) from the s-domain to the t-domain.

For a given step steering input, the overshoot of the yaw rate is calculated as \( \text{Overshoot} = \frac{\gamma_{ss} - \gamma_{ss,\text{RWS}}}{\gamma_{ss}} \times 100[\%] \), and the peak response time is calculated as \( t_{\gamma,\text{resp}} = t_{\gamma,\text{max}} - t_{\text{step steer}} \). Term \( t_{\text{step steer}} \) denotes the time required for the steering wheel angle rise from 0% to 50%. Reducing overshoot is equivalent to reducing the peak value \((\text{i.e. maximum value, } \gamma_{\text{max}})\) of the yaw rate. Likewise, since \( t_{\text{step steer}} \) is fixed, reducing the peak response time \( t_{\gamma,\text{resp}} \) is equivalent to reducing the peak time \( t_{\gamma,\text{max}} \).

\[
\frac{d\gamma}{dt}(t_{\gamma,\text{max}}) = 0 \quad \gamma(t_{\gamma,\text{max}}) = \gamma_{\text{max}}
\]

The relationship between the peak time/yaw rate and the feedback gain is numerically analyzed. The peak time and the peak yaw rate at 110 kph of vehicle speed were calculated from (25). The peak response time and overshoot are shown in Figure 7. The yaw rate overshoot monotonically decreases as the feedback gain \( K_{fb} \) increases. However, the relationship between \( t_{\gamma,\text{resp}} \) and the feedback gain is a convex function, and \((K_{fb}, t_{\gamma,\text{resp}}) = (0.016, 0.17)\) is a local minimum point. Since minimizing both the overshoot and the peak response time is the goal of the control design, the feedback gain \( K_{fb} \) is set to 0.016 in the step steer scenario at the 110 kph vehicle speed. The ‘X’ mark indicates the proposed feedback gain in this case.

Figure 8 represents the locus of the closed-loop poles of the transfer function (21) and indicates the yaw rate response to the driver’s steering input. The feedback gain \( K_{fb} \) varies from 0 to 0.05 under the condition that is dry asphalt \((\mu = 1)\) at high-speed (110 kph). Figure 8 (a) shows the root-locus plot [19]–[21] of the proposed RWS algorithm. Figure 8 (b) shows the change of the poles with respect to the change of the feedback gain in the log scale x-axis. As the feedback gain \( K_{fb} \) increases, the pole first moves towards the left-hand plane (LHP; Figure 8 (a)) but the poles bifurcate around \( K_{fb} = 0.032 \) as illustrated in Figure 8 (a) and (b).

The feedback gain of the proposed algorithm is determined by the process shown in Figure 7 and places the poles before the bifurcation [22], [23]. Figure 8 (a) shows that the feedback control increases the damping ratio \( \zeta \) from 0.16 to 0.6. This means that the feedback control is effective for overshoot reduction. In Figure 8 (b), the feedback control places the real part of the poles from -0.1 to -0.3. This means that the feedback control improves the stability of the RWS control system.

The blue solid line in Figure 8. (b) indicates the pole changes when the tire cornering stiffnesses are
\( C_f = 20000 \text{ N/rad} \) and \( C_r = 26800 \text{ N/rad} \), which are the linear cornering stiffnesses of the test vehicle. The linear cornering stiffnesses nicely represent the vehicle’s lateral dynamics in mild driving region. However, the tire cornering stiffness can be changed during driving according to vehicle states, tire states, road friction, etc. The closed-loop poles’ change under such a condition was investigated to analyze the control performance under varying tire stiffnesses conditions.

Figure 9 shows a closed-loop pole change regarding the tire cornering stiffness variation based on (21). As illustrated in Figure 9, the value of the closed-loop poles increases as the tire cornering stiffnesses increase. This is because increased tire stiffness increases the natural frequency of the closed-loop system. Moreover, the data suggest that the poles are still present on the left-hand plane (stable closed-loop poles) regardless of the tire cornering stiffness.

The best feedback gain is changed regarding vehicle speed as illustrated in Figure 10. Figure 10 (a) shows the feedback gain with vehicle speeds obtained via the same process as in Figure 7. At low speeds (below 56 kph), the feedback gain is set to 0 because additional feedback control increases the peak yaw rate response time. The detailed process of determining the feedback gain at low speeds can be found in Appendix C. The feedback gain is increased as vehicle speed increases at high speeds (over 56 kph).

Figure 10 (b) displays the value of the design parameter \( \eta \). Section 3.2 shows that in the low speed region, the yaw rate response of the optimal solution can be imitated only by adopting the design parameter \( \eta \). Therefore, the design parameter \( \eta \) is adjusted to mimic the optimal solution while the feedback gain is set to zero. However, both the design parameter \( \eta \) and the feedback gain must be tuned to mimic the optimal solution at high speed. At high speeds, \( \eta \) is adjusted to show a similar performance with the optimal solution when the feedback gain is set to Figure 10 (a).

Figure 11 shows the performance of the proposed algorithm in (18). The proposed algorithm in (18) is compared with control inputs in (8) and (12) and the optimal solution. The simulation scenario is a 45 deg (300 deg/s) step steer at 110 kph vehicle speed on the dry asphalt. ‘Feedforward’ in Figure 11 stands for the control input in (12) with \( \eta = 0.6 \). ‘Proposed’ is the control input in (18) with \( (\eta, K_{fb}) = (0.8, 0.016) \). Feedback using \( (\eta, K_{fb}) \) noticeably reduces the overshoot of yaw rate to 13.07%, while feedforward using \( \eta \) changed the results by 40.14%. Moreover, \( \delta_r, \delta_r, K_{fb} \) in (17) acts as a side slip angle feedback control as illustrated in Figure 11, and it helps the
TABLE 2. Performance evaluation indices.

| Scenario   | Performance index | Calculation                                                                 |
|------------|------------------|-----------------------------------------------------------------------------|
| Step steer | Overshoot        | \( \text{Overshoot} = \frac{y_{\text{max}} - y_{ss}}{y_{ss}} \times 100 \% \) |
|            | Peak Response Time| \( t_{p,\text{resp}} = t_{p,\text{max}} - t_{\text{SWEA.50\%}} \) [sec]       |
|            | TB Factor        | \( \text{TB} = t_{p,\text{resp}} \times \beta_{ss} \) [sec \cdot \text{deg}]       |
|            | Yaw rate Gain    | \( \frac{y_{ss}}{\delta_f} \) [1/s]                                      |

![Figure 10](image.png)

**FIGURE 10.** Changes of feedback gain and design parameter \( \eta \) with vehicle speed. (a) Feedback gain \( (K_{fb}) \): the feedback gain is increased as vehicle speed increases. (b) Design parameter \( (\eta) \): the design parameters are reduced as vehicle speed increases.

feedforward control to converge quickly in controlling the steady-state side slip angle to zero. Moreover, the RWS of the proposed algorithm was initially steered to the opposite direction of the front steer angle as shown in Figure 11 (a). This initial undershoot command of the proposed algorithm is identical to the RWS command of the optimal solution.

IV. SIMULATION RESULTS

The simulation results provide analysis and understanding to investigate the performance of the proposed algorithm. Three simulation results were performed in this paper. First, the step steer scenario based-on ISO-7401 was conducted to compare the optimal control results. The comparison results are evaluated whether the proposed control algorithm implements the optimal performance with overshoot, response time, and TB factor as the objective criteria [24]–[27]. Second, the proposed control algorithm is verified in that it performs well even in the sine with dwell scenario. Third, the robustness of the proposed control algorithm is investigated for low friction road conditions. These simulations are compared with three different controllers: 1) Base vehicle, F-segment sedan with Rear-Wheel Drive (RWD); 2) Proportional RWS controller with the gain expressed as a function of vehicle speed in Figure 4; and 3) Optimal RWS controller calculated by offline numerical optimization.

A. OPEN-LOOP STEERING COMMAND – STEP INPUT

The proposed RWS control algorithm is carried out to emulate the lateral behavior of a reference model. The reference model is conducted based on the offline numerical optimization of [6]. The numerical optimization is simulated through open-loop maneuvers that do not involve the driver’s intention. In this section, a step steer scenario is adopted that satisfies the ISO-7401 with a 45 deg steering wheel angle and a 300 deg/s steering rate. The optimal control (i.e. RWS\( ^{v_y} \) control of [6]) is compared with the passive vehicle (only FWS) through objective criteria such as overshoot, response time, and TB factor [23]–[26] of the following Table 2.

Here, \( y_{\text{max}} \) is the maximum value of yaw rate, and \( \beta_{ss} \) is the steady-state value of side slip angle. \( t_{p,\text{max}} \) is the time required for the response to reach the first peak of the overshoot, and \( t_{\text{SWEA.50\%}} \) is the time required for the steering wheel angle rise from 0% to 50%; it serves as a reference point in calculating...
the peak response time $t_{\gamma, \text{resp}}$. The primary concern in this section is to assess how well the proposed control algorithm implements the best performance of the numerical optimization based on objective criteria.

Figures 12 and 14 (a) are simulation results and performance assessment graphs at low speeds. Figure 12 (a) shows “SWA/Gear-ratio (15.221)"; This means the steering command divided by the gear ratio of vehicle, and it represents the front-wheel steering input by the driver. Steady-state control gain and transient control gains are set for emulating the target behavior of the optimal control. These are designated as $(k_\delta, \eta, K_{fb}) = (-0.501, 1.3, 0)$. The performance assessment graph in Figure 14 (a) shows that the RWS control at low speeds improves vehicle maneuverability by increasing the steady-state yaw rate gain from 0.16 [1/s] to 0.25 [1/s]. The rear-wheel steering is controlled in the opposite direction to the front-wheels, which increases the yaw rate gain to make the TB factor zero. Vehicles with RWS control input (proposed, optimal, proportional) reduce the peak response...
FIGURE 13. Result of step steer at 110 kph, 45 deg (300 deg/s), dry asphalt, with $k_\delta = 0.357$, $\eta = 0.8$, $K_{fb} = 0.016$.

The proposed algorithm is slightly insufficient to mimic the optimal control’s overshoot, but greatly reduces overshoot compared to the proportional and base vehicle. On the other hand, the proposed algorithm shows the fastest response (0.17 s) in terms of peak response time.

In terms of overshoot, performance is evaluated as follows: optimal (7.1%) $\leq$ proposed (7.4%) $<\propto$ proportional (18.5%) $<\propto$ base (20.9%). From an overall perspective, the proposed RWS control algorithm at low speeds shows better performance compared to the proportional control and base, and the proposed algorithm emulates the optimal results very well.

FIGURES 13 and 14 (b) illustrate the simulation results and performance assessment at high speed. Steady-state control gain and transient control gains are set to emulate the target behavior of the optimal control. These values are designated as $(k_\delta, \eta, K_{fb}) = (0.357, 0.8, 0.016)$, respectively. The rear-wheel steering is controlled in the same direction to front-wheels, which decreases the yaw rate gain to make the TB factor zero. The proposed algorithm is slightly insufficient to mimic the optimal control’s overshoot, but greatly reduces overshoot compared to the proportional and base vehicle. On the other hand, the proposed algorithm shows the fastest response (0.17 s) in terms of peak response time.

The simulation results can be explained in the physical analysis and control design point of view. From the physical analysis viewpoint, one sees a difference in RWS command. Compared to the optimal control, input-delay due to the initial undershoot is similar, but it shows that the proposed algorithm subsequently has an overshoot-shaped control input. This results in greater lateral forces on the rear-axle creating a faster yaw rate response. From the control design point of view, the feedback control reduces the overshoot and peak response time compared to the feedforward control. Figure 7 illustrates that the proposed algorithm has a faster response than the optimal result by further reducing the peak response time using the feedback control. Furthermore, as shown in Figure 13 (d), the optimal control and the proposed algorithm has a linear transient response of the lateral acceleration while the proportional control has a nonlinear (stair-shaped) transient response. Additionally, the proposed algorithm converges the side slip angle to zero better than the base and the proportional control. In this regard, the proof of stability has been attached in the Appendix D.

Figure 15 shows the performance indices throughout the driving region: from low to high lateral acceleration. These results are obtained by increasing the amplitude of step steering input at 110 kph. Figure 15 (a) shows the changes of the yaw rate overshoot. The base vehicle’s overshoot exceeds 30%, and the proportional controller’s overshoot rises to 27% as the lateral acceleration increases. The proposed algorithm also increases the overshoot by more than 10% in limit handling ($Ay>0.6g$), but it shows good performance in mild driving ($Ay<0.6g$) by maintaining the overshoot less than 10%. Figure 15 (b) represents the peak response time according to the lateral acceleration. Compared to the proportional controller exceeded 0.25 s, the proposed algorithm maintains below 0.2 s from mild driving to limit handling. Figure 15 (c) represents the changes in TB factor. In limit handling, the RWS vehicles minimize the side slip angle to zero while the base vehicle increases a TB factor to more than 1 deg-s.

B. OPEN-LOOP STEERING COMMAND – SINE WITH DWELL

The proposed control algorithm has been simulated to verify whether it performs well for the sine with dwell scenario. The steering input is configured as $0.4 g/0.8 g$ Ay level...
in the first peak. This is based-on ISO-19365 (sine wave of 0.7 Hz frequency, 500 ms delay at the dwelling zone). The performance test is conducted with the previously set $\eta$ and $K_{fb}$. This result comparison was conducted for the following three controlled vehicles: base, proportional, and proposed. The purpose of this section is to identify the effect of RWS intervention and transient controller on vehicle stability.

Figure 16 shows the simulation results for the sine dwell test with $Ay$ 0.4 g level. The vehicle speed is 110 kph, and SWA for a 0.4g level of $Ay$ requires about 25 deg for the base vehicle and about 45 deg for the RWS vehicles. Figure 16 (b) shows that the proposed algorithm has a linearity yaw rate response to the driver’s steering input in the dwelling area (2.5 s – 4 s). The proposed algorithm then shows a quick convergence and a smaller overshoot compared to other vehicles. Comparing the side slip angle in Figure 16 (c), one sees that the proposed algorithm significantly reduces below 0.09 deg while the base vehicle produces a side slip angle of up to 1.2 deg; the proportional controller produces a value up to 0.24 deg. Additionally, the proposed algorithm has the smallest rate of change in the side slip angle. This can be seen in the graphs of yaw rate and lateral acceleration. The proposed algorithm shows that the first peak time of yaw rate and lateral acceleration are almost the same, while the base and proportional control have a time gap. This result is explained by the effect of the feedback control in (17).

Figure 17 represents the simulation results for the sine dwell test at $Ay$ 0.8g level where the base vehicle spins out. The vehicle speed is 110 kph and SWA for 0.8g level of $Ay$ requires about 55 deg for the base vehicle and about 100 deg for RWS vehicles. For a mild maneuver ($Ay$ 0.4g level), the proposed algorithm has a similar performance in limit handling maneuver ($Ay$ 0.8g level). Thus, it can be seen that the proposed algorithm, tuned to emulate the optimal control to minimize side slip angle for step steer, performs the control objective well for other steering inputs.

C. PERFORMANCE COMPARISON ON A LOW FRICTION ROAD
The robustness of the proposed RWS algorithm has been investigated for low tire-road friction cases. In contrast to the
results in section 4.1 where the RWS controlled vehicle performs the optimal performance on high friction roads ($\mu = 1$), this section shows what happens to RWS performance when the road surface condition is changed to low friction roads ($\mu = 0.3$). The performance of the proposed RWS control algorithm has been compared to the base vehicle, the steady-state control input in (11), and the feedforward control without feedback control in (12). The simulation was conducted to investigate the performance of the proposed algorithm for

the following scenarios: step steer (45 deg, 300 deg/s) and sine with dwell (0.3g level of the first peak $A_y$).

Figure 18 illustrates the performance changes of RWS controllers when driving on a low friction road. The base vehicle spins out and has a loss of stability. In the case of the steady-state control and the feedforward control tuned on high friction roads, the yaw rate and side slip angle of the vehicle did not diverge but oscillated over a large period. In the case of the proposed algorithm, the algorithm
FIGURE 18. Step steer at 110 kph, 45deg (300deg/s), icy asphalt, with $k_\delta = 0.357$, $\eta = 0.8$, $K_{fb} = 0.016$.

Figure 18 shows the results of step steer scenario on the icy asphalt driving condition. The base vehicle and the steady-state control tuned on high friction roads lead to spin out and loss of lateral stability. The feedforward and the proposed RWS algorithm do not diverge. Compared to dry asphalt results (Figure 12 - 13), the proposed algorithm has the following features: slightly increased overshoot (7.4% to 10.5%) and peak response time (0.26 s to 0.28 s); this is superior to other controllers. In terms of lateral acceleration, the proposed algorithm maintains the linear shape response in the transition area while the steady-state control has a nonlinear shape.

tuned on dry asphalt has a 0.5 deg offset in side slip angle, but it converges fast. Compared to the dry asphalt results (Figure 12 - 13), the proposed algorithm has the following features: slightly increased overshoot (7.4% to 10.5%) and peak response time (0.26 s to 0.28 s); this is superior to other controllers. In terms of lateral acceleration, the proposed algorithm maintains the linear shape response in the transition area while the steady-state control has a nonlinear shape.

FIGURE 19. Sine with dwell at 110 kph, peak Ay 0.3g, icy asphalt, with $k_\delta = 0.357$, $\eta = 0.8$, $K_{fb} = 0.016$.

Figure 19 shows the results of sine with dwell scenario on the icy asphalt driving condition. The base vehicle and the steady-state control tuned on high friction roads lead to spin out and loss of lateral stability. The feedforward and the proposed RWS algorithm do not diverge. Compared to dry asphalt results (Figure 15 - 16), the RWS input of the proposed algorithm on the low friction road differs remarkably from RWS input of the steady-state control. This difference
results from the feedback controller that improves the vehicles’ lateral stability.

V. CONCLUSION
A new rear-wheel steering control method to enhance the vehicle handling characteristics without any information on tire characteristics parameters has been presented. The proposed algorithm consists of the steady-state and transient control input. The steady-state control input is proportional to the driver’s front-wheel steering. The proportional gain is pre-determined as a function of vehicle speed using the offline optimization results, which is designed to minimize vehicle lateral velocity. The transient control input enhances the lateral transient response of the vehicle’s yaw rate and lateral acceleration, and is designed as a combination of the feedforward and feedback inputs. In the feedforward input, a new feedforward gain is introduced to adjust the transient response of the vehicle’s yaw rate. The feedback input significantly reduces the yaw rate overshoot via the first-time derivative of the vehicle side slip angle.

Computer simulations were used to evaluate the proposed control algorithm and were compared with three different control systems: (1) base (passive) vehicle; (2) proportional RWS system with the pre-determined rear-to-front steering ratio that is a function of the vehicle speed; and (3) optimal RWS system obtained by offline numerical optimization. The simulation results show that the proposed algorithm nicely emulates the optimal handling performance for step steer scenarios without any information on tire parameters. Moreover, compared to the other RWS control algorithms, the proposed control algorithm shows superior performance in the vehicle’s lateral stability and maneuverability even under various steering and road surface conditions.

Since the proposed RWS control algorithm exhibits good performance at the simulation level, the rear-wheel steering control algorithm for the target vehicle could be developed via a real-time software tool. Real-time implementation and vehicle tests for the evaluation of this algorithm are the topics of our future research.

APPENDIX A
DERIVATION PROCESS OF (13)
In (3) representing the vehicle bicycle model, summarizing in the direction of eliminating the side slip angle \( \beta \) is as follows:

\[
a_v \frac{V_x}{V_x} = \dot{\beta} + \gamma = a_{11}\beta + (a_{12} + 1) \gamma + b_1 \delta_f + c_1 \delta_r \quad \text{(A.26)}
\]

\[
\beta = \frac{1}{a_{21}} \cdot (\dot{\gamma} - a_{22} \gamma - b_2 \delta_f - c_2 \delta_r) \quad \text{(A.27)}
\]

where, \( a_{ij} \) is the element of matrix \( A \) at the \( i \)-th row and the \( j \)-th column, and \( b_i, c_i \) are the elements of the matrices \( B \) and \( C \), respectively. Equation (A.26) represents the ratio of the lateral acceleration to the vehicle speed using the first row of (3). Equation (A.27) is the second row of (3) and is used for canceling the side slip angle in (A.26). The lateral dynamics can be expressed with the measurable vehicle signals such as yaw rate and lateral acceleration, as shown in (A.28) below:

\[
a_{11} \ddot{\gamma} + (a_{21} - |A|) \cdot \gamma = a_{21} \cdot \frac{a_v}{V_x} + |A| B \cdot \delta_f + |A| C \cdot \delta_r \quad \text{(A.28)}
\]

where, \( A \) is the \( i \)-th column vector of matrix \( A \), and \( |A| \) is the determinant of matrix \( A \). Equation (A.28) can be transformed from the time domain (real variable \( t \)) to the frequency domain (complex variable \( s \)) by Laplace transformation. The process of expressing the vehicle lateral response is as follows:

\[
a_{11} s + (a_{21} - |A|) \cdot \gamma(s)
\]

\[
= a_{21} \cdot \frac{a_v(s)}{V_x} + |A| B \cdot \frac{\delta_f(s)}{s} + |A| C \cdot \frac{\delta_r(s)}{s} \quad \text{(A.29)}
\]

\[
\frac{\gamma(s)}{\delta_f(s)} = (a_{11} s + (a_{21} - |A|))^{-1} \times |A| B + |A| C \cdot \frac{\delta_f(s)}{s} + a_{21} \cdot \frac{a_v(s)}{V_x} \cdot \frac{\delta_r(s)}{s}
\]

\[
\left(1 - \delta_f(s) - K_{us} \cdot \frac{a_v(s)}{\delta_f(s)} \right) \quad \text{(A.30)}
\]

**APPENDIX A-1**
**LATERAL RESPONSE OF (16)**
By substituting equation (12) into equation (13), vehicle lateral response of the proposed algorithm is derived as follows:

\[
\gamma(s) = \frac{1}{\tau s + 1} \cdot \frac{V_x}{L} \times \left[ 1 - \left( \frac{k_d}{\eta} + \frac{1}{\eta} \right) \cdot \frac{a_v(s)}{\delta_f(s)} \cdot \frac{\delta_f(s)}{s} \right] 
\]

\[
\gamma(s) = \frac{1}{\eta} \cdot \frac{1}{\tau s + 1} \cdot \frac{V_x}{L} \left[ 1 - \left( \frac{k_d}{\eta} - K_{us} \cdot \frac{a_v(s)}{\delta_f(s)} \right) \right] \cdot \frac{\gamma(s)}{\delta_f(s)} \quad \text{(A.31)}
\]

Transpose the yaw rate term to the left, and replace \( k_d \) using (8). And then, equation (A.32) is expressed as follows:

\[
\frac{\gamma(s)}{\delta_f(s)} = \frac{1}{\eta \tau s + 1} \cdot \frac{V_x}{L} \left[ 1 - \left( \frac{\delta_{r,ss}(s)}{s} - K_{us} \cdot \frac{a_v(s)}{\delta_f(s)} \right) \right] \quad \text{(A.32)}
\]

**APPENDIX B**
**DERIVATION PROCESS OF (19)**
By substituting equation (18) into equation (13), vehicle lateral response with proposed algorithm is derived as
The peak time and the peak yaw rate at 30 kph of vehicle speed were calculated from (25). The peak response time and overshoot are shown in Figure 20.

By summarizing equation (B.34), the yaw rate response to the driver’s steering input is expressed as follows:

\[
\frac{\gamma(s)}{\delta_f(s)} = \frac{1}{\eta} \left( \frac{1}{\eta + K_{fb} V_x^2} \right) \cdot \frac{L}{V_x} \cdot \frac{\gamma(s)}{\delta_f(s)} - \left( 1 - k_\delta - (K_{us} - \eta K_{fb}) \cdot \frac{a_v(s)}{\delta_f(s)} V_x \cdot \frac{\gamma(s)}{\delta_f(s)} \right) + K_{us} \cdot \frac{a_v(s)}{\delta_f(s)} \cdot \frac{\gamma(s)}{\delta_f(s)} \right] 
\]

\[
\frac{\gamma(s)}{\delta_f(s)} = \frac{1}{\eta} \left( \frac{1}{\eta + K_{fb} V_x^2} \right) \cdot \frac{L}{V_x} \cdot \frac{\gamma(s)}{\delta_f(s)} - \left( 1 - k_\delta - (K_{us} - \eta K_{fb}) \cdot \frac{a_v(s)}{\delta_f(s)} V_x \cdot \frac{\gamma(s)}{\delta_f(s)} \right) + K_{us} \cdot \frac{a_v(s)}{\delta_f(s)} \cdot \frac{\gamma(s)}{\delta_f(s)} \right] 
\]

\[
\eta_f = \frac{1}{\eta} \left( \frac{1}{\eta + K_{fb} V_x^2} \right) \cdot \frac{L}{V_x} \cdot \frac{\gamma(s)}{\delta_f(s)} - \left( 1 - k_\delta - (K_{us} - \eta K_{fb}) \cdot \frac{a_v(s)}{\delta_f(s)} V_x \cdot \frac{\gamma(s)}{\delta_f(s)} \right) + K_{us} \cdot \frac{a_v(s)}{\delta_f(s)} \cdot \frac{\gamma(s)}{\delta_f(s)} \right] 
\]

\[
\eta_f = \frac{1}{\eta} \left( \frac{1}{\eta + K_{fb} V_x^2} \right) \cdot \frac{L}{V_x} \cdot \frac{\gamma(s)}{\delta_f(s)} - \left( 1 - k_\delta - (K_{us} - \eta K_{fb}) \cdot \frac{a_v(s)}{\delta_f(s)} V_x \cdot \frac{\gamma(s)}{\delta_f(s)} \right) + K_{us} \cdot \frac{a_v(s)}{\delta_f(s)} \cdot \frac{\gamma(s)}{\delta_f(s)} \right] 
\]
on the parameterization afforded by offline numerical optimization. However, there is no guarantee from the stability point of view. Thus, the proposed algorithm has been verified whether the controller can extend the region of attraction on the phase plane.

The results of the phase portrait are analyzed under assumptions of constant steering wheel angle 45deg and vehicle speed 110kph. The region of attraction (RoA) converging to the stable node is represented as green area in Figure 21. It can be shown that the proposed algorithm expands the RoA compared to the uncontrolled vehicle.

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