Muon \((g-2)\): Past, Present and Future

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The muon \((g-2)\) experiment E821 at the Brookhaven National Laboratory has achieved a relative precision of ±0.5 parts per million. A new experiment, E969, with scientific approval but not yet funded, aims to improve this to ±0.2 ppm. The technique and results from E821 will be described, and the proposed improvements for E969 will be discussed.

1. Introduction

A charged particle with spin \(\mathbf{s}\) has a magnetic moment \(\mathbf{\mu} = g_s (e/2m)\mathbf{s}\); an anomaly \(a = (g_s - 2)/2\); and \(\mu = (1+a)e\hbar/2m\); where \(g_s\) is the gyromagnetic ratio, and the latter expression is what one finds in the Particle Data Tables.[1] The \(g\)-value is exactly 2 for a point-like fermion in the Dirac equation, but radiative corrections give rise to a non-zero value for the anomaly \(a\). The lowest order (QED) correction gives \(a = \alpha/2\pi\). For the muon, radiative corrections from QED, virtual hadrons (quarks), and weak gauge bosons are important at the level of measurement.[2] While the QED and weak contributions can be calculated to the necessary accuracy to compare with experiment, the hadronic contribution needs to be obtained using data from \(e^+e^-\) → hadrons.

In a series of three experiments at CERN the muon anomaly was measured to a relative precision of 7.3 parts per million (ppm).[3] Experiment E821 at the Brookhaven Alternating Gradient Synchrotron improved on this by a factor of 14,[4,5,6,7,8] to a relative precision of 0.5 ppm.

While the value of the hadronic contribution has changed over time, the other contributions have remained quite steady.[2] During all of this, there has remained a discrepancy between theory and experiment of between two and three standard deviations when the hadronic contribution is taken from \(e^+e^-\) data. When hadronic \(\tau\)-decay and CVC theory is used to determine the hadronic contribution, the discrepancy is smaller, but there appear to be as yet not understood isospin violation corrections which make it difficult to compare the two methods.[2] Theoretically, the \(e^+e^-\) cross-section is what enters into the dispersion integral. Motivated by this potential discrepancy, a new experiment, E969 has been proposed at Brookhaven to improve the precision to 0.2 ppm.

2. Measurement of the muon anomaly

The method used in the third CERN experiment and the BNL experiment are very similar, save the use of superconducting magnets for the storage ring and inflector, as well as direct muon injection into the storage ring. These experiments are based on the fact that for \(a_\mu > 0\) the spin precesses faster than the momentum vector when a muon travels transversely to a magnetic field. The difference frequency between the cyclotron frequency and the muon spin precession frequency, \(\omega_a = \omega_S - \omega_C = ((g-2)/2)(eB/mc)\) is the frequency with which the spin precesses relative to the momentum, and is proportional to the anomaly, rather than to \(g\). With both an electric and a magnetic field, the spin difference frequency is given by

\[
\tilde{\omega}_a = -\frac{e}{mc} \left[ a_\mu \mathbf{B} - \left( a_\mu - \frac{1}{\gamma^2 - 1} \right) \mathbf{\beta} \times \mathbf{E} \right],
\]

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which reduces to the simpler equation above in the absence of an electric field. Electric quadrupoles were used for vertical focusing, taking advantage of the “magic” $\gamma = 29.3$ at which an electric field does not contribute to the spin motion relative to the momentum.

A precision measurement of $a_\mu$ requires precision measurements of the muon spin precession frequency $\omega_a$, and the magnetic field, which is expressed as the free-proton precession frequency $\omega_p$ in the storage ring magnetic field. These two (average) frequencies plus the fundamental constant $\lambda = \mu_p/\mu_p$ give the anomaly:

$$a_\mu = \frac{\omega_a}{\omega_p} - \frac{\omega_a}{\omega_p}. \quad (2)$$

The experimental signal is the $e^\pm$ from $\mu^\pm$ decay, detected by lead-scintillating fiber calorimeters. Since the highest energy $e^\pm$ are correlated with the muon spin, if one counts high-energy $e^\pm$ as a function of time, one gets an exponential spectrum. The expected form for the positron time distribution is $f(t) = N_0 e^{-t/\tau} \left[1 + A \cos(\omega_a t + \phi)\right]$, however in analyzing the data it is necessary to take a number of small effects into account\[7,8\].

The values obtained for $a_\mu$ by E821 are shown in Fig. 1 along with one theory value using $e^+e^-$ data for the lowest-order hadronic contribution\[9\]. The discrepancy with theory varies between 2.2 and 2.8 standard deviations when using the $e^+e^-$ data for the hadronic contribution, and about one-third of this when using the $\tau$-data. The improvement of the $e^+e^-$ data, and the understanding of the related theoretical issues is under active study worldwide.\[10\]

3. Future Muon $(g - 2)$ Experiments

To increase the sensitivity of the comparison between theory and experiment, a new collaboration was formed to continue the measurement of $(g - 2)$ at BNL. The goal is a total error of 0.2 ppm, a factor of 2.5 better than E821. This increased precision, combined with the expected improvements in the knowledge of the hadronic contribution should give at least a factor of two reduction in the combined experiment-theory uncertainty when comparing theory with experiment. In September 2004 the new experiment E969\[11\] received enthusiastic scientific endorsement by the Laboratory. The funding situation is less clear, and E9669 is expected to be considered by the HEPAP sub-panel P5 in early 2006.

E821 achieved a final uncertainty on the measurement of the muon anomalous magnetic moment $a_\mu$ of 0.54 ppm, which is dominated by the statistical error of 0.46 ppm. For our last data set the systematic uncertainties on the knowledge of $\langle B \rangle$ and $\omega_a$ were 0.17 ppm and 0.21 ppm respectively, for a total systematic of 0.27 ppm.

A further increase in precision is possible if a higher muon storage rate can be obtained, and the systematic uncertainties present in E821 are reduced. The proposed 0.2 ppm uncertainty is derived from a 0.14 ppm statistical error, and equal total systematic uncertainties of 0.1 ppm from the measurements of $\omega_p$ and $\omega_a$. Ten times more events compared to E821 are needed.

An important feature of the upgraded experiment is a new front-end to the beamline. In E821 pions slightly higher than the magic momentum decay in a FODO decay channel, as shown in Fig. 2. Forward decay $\mu$ were accepted at a momentum slit, but because of the large tail on the $\pi$ distribution, the $\pi: \mu$ ratio was about 1:1. In the new experiment backward decays in a $\pi$ beam of
Figure 2. The E821 Beamline. Pions produced at $0^\circ$ are collected in Q1Q2 and the momentum is selected on collimators K1-K2. The pion decay channel is 80 m in length. Forward muons are selected at the collimator K3K4.

5.3 GeV/c will be used to produce muons of 3.1 GeV, thus eliminating the baseline shifts in the detectors that were caused by the pion “flash” at injection. To increase the muon flux, we will double the number of quadrupoles in the decay channel, and open the ends of the inflector magnet. This will gain a factor of four in muon flux.

A number of experimental systems will need to be upgraded for E969. To handle the increased rates new segmented detectors and their downstream electronics will be developed. The magnetic field measurement and control will need to be improved, and the magnet will be shimmed further. Also, the muon kicker will need to be upgraded. All of these issues are presently under study.

While it is possible to improve the experimental value of $a_\mu$ further, to below 0.1 ppm, the motivation for such an improvement would need to be driven by a better understanding of the hadronic contribution. Certainly our upgraded experiment E969, combined with expected improvements in the knowledge of the hadronic contribution, will present the community with a new, more stringent test of the standard model. It is clear that interest in our result, and in our ability to improve upon it, remains high.

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