Complementarity of muon-conversion and linear collider-based experiments for lepton-flavor violating $U(1)$ gauge bosons

Brandon Murakami and James D. Wells

University of California, Davis, California 95616

(Dated: November 28, 2001)

In general, attempts to extend the Standard Model will include extra gauge structure. We parameterize string and technicolor models for a $Z'$ boson with primitive lepton flavor violating interactions. Calculations for its muon conversion rate $(\mu^- N \rightarrow e^- N)$ on Titanium are made and used to show the potential of forthcoming experiments MECO at Brookhaven National Laboratory and PRIME at the Japan Hadron Facility (to be renamed). For reasonable choices of parameters, such $U(1)$ bosons with masses the order 10 TeV and $\mu e Z'$ charge as low as $\sim 10^{-5}$ are demonstrated to be accessible for MECO and PRIME. Also, a demonstration of the complementarity of parameter space coverage for future colliders and muon conversion experiments is given.

In the grander endeavors to unify the forces, explain the spectrum of fermion masses, and understand the hierarchy problem, additional gauge and vector bosons generally arise from string models, GUTs, $N \geq 2$ supersymmetry, technicolor models, and models with extra dimensions. Models with extended gauge groups, such as string models [1, 2, 3] and technicolor [4, 5, 6], have potential to produce additional $U(1)'$ (or $Z'$) gauge bosons with flavor-dependent couplings to the fermions. Lepton flavor violation (LFV) is allowed in many extensions of the standard model [7]. LFV may occur at tree-level bosonic exchange by either flavor off-diagonal $Z'$ charges or indirectly through the standard model $Z$ boson mixing with the $Z'$. The additional fermions that arise from extended gauge models may further promote LFV currents by mixing with the standard model (SM) leptons [8]. Such a boson's mass may be on the order of 1 TeV in models such as those of the supersymmetric model [14, 15] or model with singlet neutrinos propagating in extra dimensions [16], neutrino mixing would not be sufficient to explain a muon conversion signal at MECO or PRIME [13]. However, in a model with a $+2$ electrically charged Higgs boson also exhibit muon conversion potential through loop diagrams with charged leptons provided a $y_{ij} \phi^+ \bar{\phi} e_{Ri} e_{Rj}$ interaction exists [17]. Other exotic scenarios are reviewed in Ref. [18]. In our analysis, we assume muon conversion is due entirely to LFV $Z'$ bosons.

Any model with a $Z'$ that has flavor couplings not proportional to the identity matrix in which the fermions are in their “natural basis” (before Yukawa diagonalization) will have a non-zero $\mu e Z'$ vertex in the physical basis. Let us call this a flavor-dependent vector-boson $G_\mu$. These models will provide the coupling constant $y_G$ and charges that form six matrices $q_{ij}^\psi$ for the fermions $\psi$—the left- and right-handed lepton, up-type quarks, and down-type quarks. The $Z - G$ mixing angle $\theta$ and the mass $m_G$ are free parameters. In our analysis, other parameters are the six unitary matrices $U_\psi$ that perform the Yukawa rotations with the constraint $V_{CKM} = U_{uL}^\dagger U_{dL}$. The charges in the physical basis are denoted $Q_{ij}^\psi \equiv U_{ik}^\psi q_{kl}^\psi U_{lj}^\dagger$. We express the $G$-fermion

\[ m_{Z'} \equiv U_{eR}^\dagger U_{\mu R} \phi^+ \bar{\phi} e_{Ri} e_{Rj} \]

\[ m_{Z'} \equiv U_{eR}^\dagger U_{\mu R} \phi^+ \bar{\phi} e_{Ri} e_{Rj} \]
interactions as
\[ \mathcal{L} \supset \frac{g_G}{\sin \theta_W} \sum_\psi \bar{\psi} \gamma^\mu (q_{ij}^\psi P_L + q_{ij}^\psi R P_R) \psi_j G_\mu \]  
where \( \psi' \) denotes the fermions in the natural basis, and
\[ \mathcal{L} \supset \frac{g_G}{\sin \theta_W} \sum_\psi \bar{\psi} \gamma^\mu (Q_{ij}^\psi P_L + Q_{ij}^\psi R P_R) \psi_j G_\mu \]
where the fermions are in the physical basis.

Technicolor bosons may have similar parameterization. Ref. [20] reviews such models that replace a \( U(1)_Y \) or \( SU(2)_L \) of the standard model with a pair of gauge groups \( U(1)_h \times U(1)_h \) or \( SU(2)_h \times SU(2)_h \). The gauge group with the “h” ("light") subscript is assigned SM-like couplings to the first two fermion families, and the other (“h” for “heavy”) receives SM-like couplings to the third generation. The \( U(1)_h \times U(1)_h \) symmetry is broken by a new Higgs boson at some scale higher than the weak scale to form a \( U(1)_Y \) representation. In either case, after properly enforcing electroweak symmetry breaking there will be an additional \( U(1)' \) boson with generation-diagonal couplings such that the first two generations have equal \( U(1)' \) charge but different than the third. The relative strength of these charges are determined completely by the mixing angle of the \( Z \) and \( Z' \). If these gauge groups are considered to act on the fermions in the natural basis, there will, in general, be generation-off-diagonal LFV \( G \) couplings in the physical basis.

With the objective of choosing the minimum number of parameters while maintaining arbitrary LFV couplings and boson mass, we define our model as follows. Additional fermions are necessary to cancel the anomalies that arise with an additional gauge boson; their effects are ignored. For the purposes of illustrating the \( G \) boson effects on LFV interactions in the most conservative way, the \( Z - G \) mixing angle \( \theta \) is considered negligible, and the mixing of the kinetic gauge terms, i.e. \( \frac{1}{2} Z^\mu \nu G_{\mu \nu} \), is also omitted. Upper limits on phenomenological constraints on \( |\chi|^2 \) are on the orders of \( 10^{-5} \) for gauge mediated SUSY breaking and \( 10^{-16} \) for gravity mediated SUSY breaking [20]. Relative to the amplitude for tree-level \( G \)-exchange in \( t \)-channel muon conversion, the amplitude for kinetic mixing would include \( |\chi|^2 \) suppression, and so is generally negligible anyway.

In further efforts to be conservative on the effects of generation-dependent \( Z' \)'s, we set the first two generations of right-handed lepton charges equal and the charge matrix diagonal, \( q_i^R = \text{diag}(q_{11}^R, q_{12}^R, q_{13}^R) \) (in the natural basis). These assignments will allow the \( \mu G \) charge \( Q_{12}^\mu \) to be non-zero and have an arbitrary value. We set the left-handed lepton charges to zero \( q_i^L = 0 \). Other gauge interaction choices (i.e. purely axial vector, vector and axial vector, mixed helicities, etc.) would have no impact on our study. We choose the \( G \)-quark interactions to be \( Q_{11}^u = Q_{12}^d = Q_{11}^d = 1 \), and \( Q_{11}^d = 1 \). All charges not noted have no direct relevance for our calculations.

For the Yukawa unitary rotation matrices, we choose as many possible to be trivial, \( U^{1L} = U^{uR} = U^{dL} = U^{eR} = 1 \). \( U^{uL} = V_{CKM} \) is necessary to meet the definition of the CKM matrix. The meaningful rotation is assigned to \( U^{dR} \) here, and has the parameterized form of the CKM matrix,
\[ U^{dR} = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} \\ -s_{12} c_{23} - c_{12} s_{23} s_{13} & c_{12} c_{23} - s_{12} s_{23} s_{13} & c_{23} c_{13} \\ s_{12} s_{23} - c_{12} c_{23} s_{13} & c_{12} s_{23} - s_{12} c_{23} s_{13} & c_{23} c_{13} \end{pmatrix}. \]  
(3)
The notation \( s_{ij} \) and \( c_{ij} \) means sines and cosines of parameters \( \theta_{12}, \theta_{23}, \theta_{13} \) which need not be the CKM values. We have ignored the allowed complex phase for simplicity.

Explicitly, the right-handed leptons have charges
\[ Q_{12}^\mu = U^{dR \dagger} q_i^R U^{dR}. \]  
(4)
Note, if \( U^{dR} \approx V_{CKM} \) we would expect an estimated value of \( |Q_{12}^\mu| \approx 10^{-4} \). If it is expected that the lepton generation-transitions should follow in a similar manner as the neutrino mixing, the large mixing angle (LMA) solution and “low probability, low mass” (LOW) solution both yield roughly \( |Q_{12}^\mu| \approx 0.1 \) [21].

The muon conversion rate is [22]
\[ B = \frac{G_F^2 \alpha^3 m_\mu^5}{2 \pi^4} Z_{\text{eff}}^4 \left| F_P \right|^2 \left( |Q_{12}^L|^2 + |Q_{12}^R|^2 \right) \times \left| \frac{g_G}{g_Y} \sin \theta \cos \theta \left( 1 - \frac{m_W^2}{m_\mu^2 \cos^2 \theta_W} \right) \right| \left[ \frac{1}{2} (Z - N) - 2 Z \sin^2 \theta_W \right]. \]
FIG. 1: LFV boson mass $m_G$ (GeV) vs. the muon conversion rate $B \equiv \Gamma(\mu^- N \rightarrow e^- N)/\Gamma(\mu^- N \rightarrow \nu_e N)$. The potential for muon conversion experiments to access our model’s $G$ boson is demonstrated here. The solid diagonal lines represent different values of the $\mu eG$ vertex couplings $Q_{12}^{\mu e}$. The horizontal dotted lines represent the limits set or to be set by muon conversion experiments. The information here is best appreciated by noting the surprisingly large boson mass and small couplings possibly accessible by experiment.

\[ + g_G^2 \left( \sin^2 \theta + \frac{m_W^2}{m_G^2 \cos^2 \theta} \cos^2 \theta \right) \left[ (2Z + N) (|Q_{11}^{u_L}|^2 + |Q_{11}^{u_R}|^2) + (Z + 2N) (|Q_{11}^{d_L}|^2 + |Q_{11}^{d_R}|^2) \right]^2, \]

and for $^{48}$Ti, nuclear form factor $F_P = 0.54$, $Z_{\text{eff}} = 17.6$, and the muon capture rate $\Gamma_{\text{capture}} = 2.6 \times 10^{-6}$ s$^{-1}$. For this model’s parameterization, the muon conversion rate is

\[ B = 0.157 \left( \frac{g_G}{g_Y} \right)^4 |Q_{12}^{\mu e}|^2 \left( \frac{1\text{TeV}}{m_G} \right)^4. \]

Fig. 1 shows the potential of muon conversion experiments. The reach for extremely heavy bosons or moderately massive bosons with extremely small LFV couplings is intriguing. For example, a 10 TeV $G$ boson with off-diagonal couplings $g_G g_Y Q_{12}^{\mu e}$ as small $10^{-5}$ may be implied by a signal at MECO or PRIME. Furthermore, these experiments have potential to imply $G$ bosons of astonishing high masses: roughly from $\mathcal{O}(10 \text{ TeV})$ to $\mathcal{O}(1,000 \text{ TeV})$ for reasonable choices of off-diagonal couplings. With the promise of strict bounds or detection, this exemplifies the importance of muon conversion experiments.

Fig. 2 demonstrates how a high-energy linear collider (LC) would complement muon conversion experiments. Together, both programs can cover all but the “quadrant” of small coupling $Q_{12}^{\mu e}$ and large mass $m_G$. With precision measurements of $e^+ e^- \rightarrow \mu^+ \mu^-$, measuring a 1% difference in the standard model cross section sets limits on what masses of a $G$ boson may be implied. The contributions from the $t$-channel exchange (which utilize the LFV vertex) are relevant only for the parameter space region of relatively small off-diagonal charge $Q_{11}^{\mu e}$ and low boson mass, which is ruled out by SINDRUM II. Naively, one might expect $e^+ e^- \rightarrow e\mu$ to be a better probe, however the limits from this process would also be ruled out by SINDRUM II. For small charge $Q_{12}^{\mu e}$, the mass limits are approximately independent of the coupling. The Tevatron should be able to continue its search for $Z'$ gauge bosons above 1 TeV, and the LHC will be able to discover most $Z'$ bosons with mass above about 5 TeV. Combining the $\mu - e$ LFV searches with the collider searches provides a much enhanced ability to search for new gauge symmetries.
FIG. 2: LFV boson mass $m_G$ (GeV) vs. the $\mu e G$ vertex charge $Q_{12}^{fl}$. The vertical lines show the limit of a 1% difference in $\Delta \sigma(e^+e^- \rightarrow \mu^+\mu^-)/\sigma_{SM}(e^+e^- \rightarrow \mu^+\mu^-)$ where $\Delta \sigma$ is the difference of the cross section of our extended model and standard model. The diagonal lines show the charge $Q_{12}^{fl}$ necessary for a muon conversion experiment to detect a $G$ boson of a certain mass provided the experiment has the ability to reach the muon conversion rate noted on the plot. These plots show linear collider and muon conversion experiments to be highly complementary: 1) In the upper left quadrant, both muon conversion experiments and an NLC will detect our model’s boson. 2) In the upper right quadrant, only MECO and PRIME have potential. 3) In the lower left quadrant, only an NLC has potential. 4) In the lower right quadrant, neither have potential.

For the $G$ boson model, the rate for the magnetic interaction $\mu \rightarrow e\gamma$ is zero by virtue of all left-handed lepton charges being zero and the chirality flips inherent in magnetic moments. Therefore, $\mu \rightarrow e\gamma$ does not constrain this model. Nevertheless, if we chose a model with non-zero left- and right-handed lepton charges, such as a model with purely vectorial couplings, muon conversion experiments would generally still be more useful in constraining lepton-violating $Z'$ models.

The current $\mu^+ \rightarrow e^+e^-e^-$ limit is $\Gamma(\mu \rightarrow eee)/\Gamma(\mu \rightarrow e\nu\bar{\nu}) = 1.0 \times 10^{-12}$, held by SINDRUM [25]. This limit is competitive with the current SINDRUM II limit for muon conversion. For example, a 10 TeV $G$ boson would be at the verge of being seen in $\mu \rightarrow eee$ if it had charges of $Q_{11}^{fl} = 1$ and $Q_{12}^{fl} = 5.4 \times 10^{-3}$. Next generation $\mu - e$ conversion experiments would then far outpace current constraints from $\mu \rightarrow eee$. We extend this study in Ref. [26] to a more complete study, including discussions of $\mu \rightarrow e\gamma, \mu \rightarrow eee$, the muon anomalous magnetic moment, and other relevant lepton observables.

[1] E. Nardi, Phys. Rev. D 48, 1240 (1993) [hep-ph/9209223].
[2] G. B. Cleaver, A. E. Faraggi, D. V. Nanopoulos and T. ter Veldhuis, hep-ph/0002292.
[3] S. Chaudhuri, S. Chung, G. Hockney and J. Lykken, Nucl. Phys. B 456, 89 (1995) [hep-ph/9501361].
[4] T. Rador, Phys. Rev. D 59, 095012 (1999) [hep-ph/9810255].
[5] D. J. Muller and S. Nandi, Phys. Lett. B 383, 345 (1996) [hep-ph/9602306].
[6] C. Yue, G. Liu and J. Li, Phys. Lett. B 496, 89 (2000) [hep-ph/0011144].
[7] J. L. Feng, hep-ph/0101122.
[8] J. Bernabeu, E. Nardi and D. Tommasini, Nucl. Phys. B 409, 69 (1993) [hep-ph/9306251].
[9] P. Wintz, in Proceedings of the First International Symposium on Lepton and Baryon Number Violation, p. 534 (1998).
[10] W. Molzon, private communication.
Acknowledgments

We thank K. Dienes and S. Mrenna for helpful discussions. This work was supported by the Department of Energy and the Alfred P. Sloan Foundation.