Two-dimensional Configuration and Temporal Evolution of Spark Discharges in Pulsars

Rahul Basu1, George I. Melikidze1,2, and Dipanjan Mitra1,3

1 Janusz Gil Institute of Astronomy, University of Zielona Góra, ul. Szafińska 2, 65-516 Zielona Góra, Poland; rahulbasu.astro@gmail.com
2 Evgeni Kharadze Georgian National Astrophysical Observatory, 0301 Abastumani, Georgia
3 National Centre for Radio Astrophysics, Tata Institute of Fundamental Research, Pune 411007, India

Received 2022 May 20; revised 2022 July 25; accepted 2022 July 25; published 2022 August 29

Abstract

We report on our investigation of the evolution of a system of spark discharges in the inner acceleration region (IAR) above the pulsar polar cap. The surface of the polar cap is heated to temperatures of around $10^6$ K and forms a partially screened gap (PSG), due to thermionic emission of positively charged ions from the stellar surface. The spark lags behind corotation speed during their lifetimes due to variable $E \times B$ drift. In a PSG, spark discharges arise in locations where the surface temperatures go below the critical level ($T_c$) for ions to freely flow from the surface. The spark commences due to the large drop in potential developing along the magnetic field lines in these lower temperature regions and subsequently back-streaming particles heat the surface to $T_c$. Regulation of the temperature requires the polar cap to be tightly filled with sparks and a continuous presence of sparks is required around its boundary since no heating is possible from the closed field line region. We estimate the time evolution of the spark system in the IAR, which shows a gradual shift in the spark formation along two distinct directions resembling clockwise and anticlockwise motions in two halves of the polar cap. Due to the differential shift of the spark pattern in the two halves, a central spark develops representing the core emission. The temporal evolution of the spark process was simulated for different orientations of a non-dipolar polar cap and reproduced the diverse observational features associated with subpulse drifting.

Unified Astronomy Thesaurus concepts: Radio pulsars (1353); Pulsars (1306)

Supporting material: animations

1. Introduction

The pulsar magnetosphere is characterized by a steady outflow of relativistic plasma along the open magnetic field lines. The majority of the electromagnetic radiation from pulsars arises within this outflowing plasma, including the radio emission that results from nonlinear plasma instabilities (Melikidze et al. 2000; Gil et al. 2004; Lakoba et al. 2018; Rahaman et al. 2020). The outflowing plasma is also the source of the dense pulsar wind responsible for the pulsar wind nebulae (de Jager 2007; Kargaltsev et al. 2015). Detailed estimates have constrained the radio emission to originate from heights of less than 10% of the light cylinder radius (von Hoensbroech & Xilouris 1997; Kijak & Gil 1998; Mitra & Rankin 2002, Mitra & Li 2004; Weltevrede & Johnston 2008; Krzeszowski et al. 2009; Mitra 2017). Thus, the location of the plasma generation region is well inside the inner magnetosphere. The prototype for the plasma generation region is the inner vacuum gap (IVG) above the polar caps, extending to heights of $\sim10–100$ m from the stellar surface with a density in potential of $10^{12}$ cm$^{-3}$ across it (Sturrock 1971; Ruderman & Sutherland 1975). The outflowing plasma is generated as a series of spark discharges due to the creation of $e^-e^+$ pairs in the presence of high magnetic fields and is accelerated to relativistic speeds in opposite directions by the large drop in potential, setting up a cascading effect. The charges within each spark screen the electric potential until they leave the gap due to inertial motion and the subsequent sparks are formed resulting in a non-stationary plasma flow.

There are certain limitations to the vacuum gap model of the inner acceleration region (IAR) above the polar cap. The backflowing relativistic charges in the sparks heat the surface to temperatures of $\sim10^6$ K which is close to the critical level ($T_c$) for ions free flow from the surface (Cheng & Ruderman 1980; Jones 1986). As a result, the IAR is not a complete vacuum but forms a partially screened gap (PSG) with a steady density of positively charged ions. It was shown by Gil et al. (2003) that in a PSG, where the ion density ($\rho_i$) is as high as 90% of the Goldreich–Julian corotational density ($\rho_{GJ}$), Goldreich & Julian (1969), spark discharges can develop in the IAR. The formation of a spark is governed by the surface temperature ($T_s$) with thermostatic self-regulation of the potential difference in the IAR. In a purely vacuum gap, there is the presence of unscreened potentials between sparks and hence the discharges cannot be confined at any location on the surface, but are expected to scatter in the direction opposite to the principal normal of the curvature of the local magnetic field lines (Cheng & Ruderman 1980). The thermostatic self-regulation in a PSG ensures that the spark is limited to a finite size with a typical length scale.

Other constraints on the physical properties of the IAR have been obtained from several detailed studies. The surface magnetic field above the polar cap is expected to be highly non-dipolar in nature. The efficiency of the pair cascade leading to the spark discharge as well as the high multiplicity ($10^5$) of the outflowing plasma requires magnetic field lines with a radius of curvature of $\sim10^{-2}-10^8$ cm (Timokhin & Harding 2019), in contrast to $\sim10^8$ cm for purely dipolar fields. The non-dipolar nature of the polar cap has also been supported by measurements of the thermal X-ray...
emission from the hot polar cap surface as well as simultaneous radio/X-ray studies of pulsars (Gil et al. 2008; Hermsen et al. 2013; Geppert 2017; Szary et al. 2017; Hermsen et al. 2018; Arumugasamy & Mitra 2019; Pétri & Mitra 2020; Szajner & Geppert 2020). The sparks are expected to undergo variable \( E \times B \) drift in the IVG, which results in the well-known observational behavior of subpulse drifting (Weltz et al. 2006; Basu et al. 2016) in the pulsed radio emission. It has been suggested that the sparks in the IAR rotate around the magnetic axis (Ruderman & Sutherland 1975; Szary & van Leeuwen 2017), however, in our previous work (Basu et al. 2020b, hereafter Paper I) it was shown that in the absence of any external electric field, the sparks in the IAR would lag behind corotation speed during their lifetimes. The phase behavior associated with drifting periodicity reflects the dynamics of the spark process in the IAR (Basu & Mitra 2018; Basu et al. 2019a). In Paper I the lagging-behind motion of the sparks along with the non-dipolar nature of the surface fields was used to simulate different categories of phase behavior associated with drifting (Basu et al. 2019a).

One important observational feature of the drifting behavior is the absence of drifting in the central core region of the profile window (Rankin 1986; Basu et al. 2019a), and cannot be explained by the lagging-behind motion of the sparks throughout the polar cap. It was recognized in Paper I that the lagging-behind scenario is restricted by the presence of polar cap boundaries beyond which no spark can be formed in the closed field line region. The thermal regulation of the hot polar cap surface requires a continual presence of spark discharges along the boundary since no additional heating is possible from the closed field lines on the other side. In this work, we expanded on the ideas presented in Paper I by including the effects of the polar cap boundary on the configuration of sparks. A two-dimensional model of sparks in a PSG is presented and we explore the evolution of their distribution with time. The constraints from the polar cap boundary coupled with the lagging-behind corotation motion of the sparks during their lifetimes introduce two distinct trajectories for the temporal evolution of the distribution of sparks. One half exhibits a clockwise evolution while the other half shows an anticlockwise behavior. The differential shift in the spark pattern in the two halves as well as the tightly packed nature of the distribution of sparks implies that the heating location at the center is stationary in most configurations of non-dipolar polar caps and accounts for the absence of drifting in the core component.

In the following sections we present the details of the model and simulations of a tightly packed distribution of sparks in the IAR starting with the physical characteristics of sparks in Section 2. Section 3 demonstrates the effect of the polar cap boundary on the location of the subsequent sparks and their shifts to be in opposite directions in two halves of the polar cap. The two-dimensional configuration of spark discharges and their temporal evolution is explored in Section 4. We used the estimated distribution of sparks and its time evolution to simulate the subpulse drifting behavior. The drifting behaviors for different orientations of the surface non-dipolar magnetic fields are shown in Section 5. A short discussion summarizing the implications of this work is presented in Section 6.

## 2. Physical Parameters of Sparks in a PSG

The details of the spark formation in a PSG were first presented in Gil et al. (2003) and have been further explored in several subsequent works (Gil et al. 2006; Szary 2013; Szary et al. 2015; Mitra et al. 2020, Paper I). In this section, we summarize the different physical parameters such as the spark size, the timescales associated with the spark process, etc., that have been estimated in these earlier works and will be used later to understand the configuration of sparks in the IAR.

### 2.1. Size of Spark

The charge density above the polar cap is limited by the corotational density, \( \rho_{\text{CG}} = \Omega B/2\pi c \), which is many orders of magnitude lower than the density of ions on the stellar crust. As a result, a sufficiently heated polar cap can sustain a free flow of ions from the tail end of the surface charge distribution at the critical temperature, \( T_i \approx e_i/k \), here, \( e_i \) is the binding energy of ions and \( k \) is the Boltzmann constant (Cheng & Ruderman 1980). For surface temperatures, \( T_S < T_i \), a charged depleted acceleration region is formed that is populated by ions with density \( \rho_i < \rho_{\text{CG}} \). A spark is set up in this region, which heats the surface until \( T_S = T_i \) is reached and ionic free flow is restored terminating the spark. The effective drop in potential \( \Delta V_{\text{PSG}} \) across each spark is given as

\[
\Delta V_{\text{PSG}} = \frac{4\pi \eta b B_d \cos \alpha_i h_i^2}{P c},
\]

where \( \eta = 1 - \rho_i/\rho_{\text{CG}} = 1 - \exp[30 \left( 1 - T_i/T_S \right)] \), \( b = B_i/B_d \) where \( B_i \) is the non-dipolar surface magnetic field and \( B_d = 2 \times 10^{12} (P_{-5})^{0.5} \) G, the equivalent dipolar field, \( \alpha_i \) is the angle between the local magnetic field and the rotation axis, \( h_i \) is the perpendicular radius of the spark, and \( P \) and \( P_c \) are the pulsar rotation period and period derivative, respectively. The total energy deposited per unit area on the surface of the polar cap by back-streaming electrons can be estimated by multiplying the energy of each electron with the particle flux in the spark \( P_S = (e \Delta V_{\text{PSG}}) \times (\eta \rho_{\text{CG}}) \). Equating the deposited energy per unit area with the energy radiated from the heated surface, \( P_S = \sigma T^4 \), the typical radius of a spark can be estimated as

\[
h_i = 260 \frac{T_i^2}{\eta b \left( \cos \alpha_i \right)^{0.5}} \left( \frac{P}{P_{-5}} \right)^{0.5} \text{cm},
\]

where the surface temperature is \( T_S = T_6 \times 10^6 \) K.

### 2.2. Timescales Associated with Sparks

There are two relevant timescales associated with sparks, the heating time \( t_h \) or spark duration, the time taken by the spark process to heat the surface to the critical temperature, and the cooling time \( t_c \), which is the time elapsed between the cessation of a spark and the surface below it to cool sufficiently for the next spark to commence. These estimates require understanding the physical properties of the neutron star crust where the heating is taking place. At temperatures around a million Kelvin, crust permeated by highly non-dipolar magnetic fields has been shown to exist in a crystalline state and the surface density \( \rho_s \) can be calculated in such cases as (Lai 2001)

\[
\rho_s \approx 561 A Z^{-3/3} b^{-3/3} \text{g cm}^{-3},
\]

where \( A \) is the mass number and \( Z \) is the atomic number of the constituent atoms in the crust. The crust is made up of iron atoms with \( A = 56 \) and \( Z = 26 \) and the density is \( \rho_s = 4.45 \times 10^3 b^{3/3} \text{g cm}^{-3} \). The duration of the heating and cooling of the crust depends on the depth of heat deposition and can be estimated using the radiation length \( y = 14 \text{ g cm}^{-2} \) for iron.
ions. The corresponding depth is given as

\[ L_R = y/l \rho_s \approx 3.15 \times 10^{-3} \, b^{-6/5} \, \text{cm}. \quad (4) \]

The heat can be deposited up to a depth of several times \( L_R \), \( L \approx 10^{-3} \, \text{cm} \) (Gil et al. 2003).

The total energy carried by back-streaming particles is used to heat the crust with specific heat per unit volume \( C_H \) to a depth of \( L \), \( C_p L \, \partial T/\partial t \approx C_p L T / t_s \), assuming uniform heating. The specific heat of the crust has a contribution from the lattice vibrations as well as the free electrons and is given as

\[ C_H \approx 4.4 \times 10^{13} \rho_e (1 + 0.024 \rho_e^{-2/5} T_b) \approx 2 \times 10^{10} b^{-6/5} (1 + 0.89 b^{-4/5} T_b) \, \text{erg K}^{-1} \text{cm}^{-3}. \quad (5) \]

The spark timescales can be estimated from the above expressions as

\[ t_s \approx 30 \left( 1 + 0.89 b^{-4/5} T_b \right) T_b^{-3} \, \mu s. \quad (6) \]

The sparks lag behind corotation velocity and the total distance during their lifetimes is \( h_s = \eta \rho_{e,fi} t_s \). The corotation velocity was estimated in Paper I to be around \( 10^6 \, \text{cm s}^{-1} \) resulting in \( h_s \approx 10 \, \text{cm} \).

Finally, in order to estimate the cooling timescale, the heat transport equation is used

\[ C_H \, \partial T/\partial t = \partial (\kappa \partial T/\partial l), \quad (7) \]

where \( \kappa \) is the thermal conductivity of the crust. The surface layer for heat penetration is very thin and can be considered to have uniform heat conductivity. The partial differential equation can be approximated as \( \partial T/\partial t \approx T/t_c \) and \( \partial^2 T/\partial l^2 \approx T/L^2 \). This gives an order of magnitude estimate of the cooling timescale as

\[ t_c = \left( L^2 C_H / \kappa \right). \quad (8) \]

In the outermost layers of the neutron star the heat transport is primarily dominated by the electronic transfer and for \( b \sim 10 \) the estimated \( \kappa \sim 10^{12} \, \text{erg cm}^{-1} \, \text{s}^{-1} \, \text{K}^{-1} \) (Gil et al. 2003). Hence, the cooling timescale of the polar cap is around 100 ns, which is more than two orders of magnitude shorter than the duration of sparks. It is possible for the subsequent spark process to commence near the previous spark soon after it dies as the surface below it cools almost instantaneously.

3. Spark Formation and Effect of Polar Cap Boundary

3.1. Sparks in the IAR

In the PSG model the polar cap is positively charged (\( \Omega B < 0 \)) with the surface temperatures maintained around the critical level for free flow of ions (\( T_s \)) due to thermostatic regulation from sparks discharges. When the surface temperature (\( T_s \approx 10^6 \, \text{K} \)) is above \( T_s \), there is a steady outflow of positively charged ions from the stellar surface with density \( \rho_{e,fi} \), screening the potential difference along the IAR. If the temperature becomes lower than \( T_s \), the ion density drops below \( \rho_{e,fi} \) and a large potential difference appears along the IAR. As a result, sparks with cascading e\(^-\)e\(^+\) pairs are set up, with the pairs separated by the large potential difference in the IART the electrons are accelerated downward and heat the surface while the positrons are accelerated away from the surface and give rise to the outflowing plasma. The sparks grow in size until sufficient pairs are produced and the surface is heated to \( T_s \gtrsim T_s \), thereby screening the potential difference across the IAR and terminating the spark. The typical lifetime of a spark is around 30 \( \mu s \) (Equation (6)), during which the spark grows both vertically and laterally (\( h_s \), Equation (2)) forming a cylindrical plasma column. The point of maximum heating lies at the center of the spark and falls off gradually toward the edge with no sharp boundaries in the temperature distribution below the spark. It was shown in Paper I that the spark plasma column lags behind the corotation motion of the pulsar during their lifetimes. As a result, the maximally heated point at the termination of the spark is shifted by a small distance \( h_s \), opposite the corotation direction. After the termination of the spark, the dense plasma column leaves the IAR in about \( \sim 300 \, \text{ns} \), which is comparable to the cooling time of the surface (Equation (8)) and is much shorter than the duration of the sparks. In the absence of any other constraints the subsequent spark forms immediately at the location of maximum heating (hence fastest cooling), shifted by a distance \( h_s \), resulting in an apparent drift motion.

If the size of the polar cap is larger than the lateral size of a spark, then thermal regulation of the surface requires the presence of a system of tightly packed sparks in the IAR. There are no hard borders expected between adjacent sparks, and the region near their edges is heated by particles from all of the surrounding sparks. The polar cap has a well-defined boundary separating the open and closed magnetic field line regions. The closed field line region has constant \( \rho_{e,fi} \) charge density and hence it is not possible for a spark to occur in the closed field line region with the boundary cutting across a spark, i.e., no spark can be formed straddling the boundary between the open and closed field line regions. The continuous heating of the surface around the boundary region requires an annular band of tightly packed sparks to be formed closely bordering the boundary. In order to ensure the continuation of effective heating of the surface, no gaps can appear in the normal direction away from the boundary to the nearest spark during the lagging-behind evolution of the spark process. As a result, the lagging-behind motion of the spark pattern is constrained by the polar cap boundary and subsequent sparks can only evolve around the annular ring either in the clockwise or counterclockwise direction (see Section 3.2). If there is space for more than one ring of sparks in the polar cap, the presence of the outer band constrains the inner sparks to also be arranged in a nested inner concentric ring touching the outer spark band in a closely packed setup. The effective heating of the surface requires the sparks in the inner ring to not have any gaps with the outer ring in the normal direction, thereby constraining the subsequent spark pattern to evolve around the inner ring. Depending on the availability of space, multiple contiguous rings of increasingly smaller radii will be formed in this setup with sparks constrained to evolve around each ring, until there is space for only a single spark at the center. There is no space for the evolution of a spark at the center and after the previous spark heats the surface above \( T_s \) and is extinguished, the next spark is formed around the same location after the surface cools down. The size of the central spark depends on the available space at the center and can be smaller than a typical spark. The arrangement of sparks in concentric rings around a central spark is consistent with the core-cone nature of the average...
shows a schematic of the elliptical polar cap with major axis \(b\) and minor axis \(a\) the boundary of the polar cap at a point specified by the polar angle \(\theta_{PC}\) as
\[
\theta_{sl} = h_D \left/ \sqrt{a_{cap}^2 \sin^2 \theta_{PC} + b_{cap}^2 \cos^2 \theta_{PC}} \right.,
\]
where \(a_{cap}\) and \(b_{cap}\) are the major and minor axes of the elliptical polar cap with \(a_{cap} = b_{cap} = R_{cap}\) for the circular case. The shifts in the spark pattern along the entire polar cap boundary are shown in Figure 2 corresponding to (a) a circular polar cap with the sparks lagging-behind corotation along the \(x\)-axis, (b) a circular polar cap with the corotation direction making a 45° angle in the \(x-y\) plane, (c) elliptical polar cap with eccentricity 0.6 and major axis along the \(x\)-axis, and (d) elliptical polar cap with eccentricity 0.6, and major axis along the \(y\)-axis. The shift is maximum when the direction of corotation is tangential to the curvature of the boundary and goes to zero when the corotation is normal to the boundary. In all cases, there are two distinct regions of spark evolution bounded by the points where the corotation direction is normal to the elliptical/circular boundary of the polar cap. In one half, the shifts are in the negative direction, which signifies a clockwise shift of the spark pattern, while in the other half the shifts are positive resembling an anticlockwise shift. In the elliptical polar caps, the maximum shifts are flatter or steeper compared to the circular case depending on the orientation of the major axis. In realistic situations the differential shifts will be averaged out due to the finite size of the sparks, as the next spark can only form in the available space between two sparks and an average shift will be seen over time. We expect two distinct evolution patterns of the spark discharges along the rim of the polar cap showing clockwise behavior in one half and anticlockwise evolution in the other half. We used this insight into the evolution pattern to understand the two-dimensional configuration of the spark system.

4. Configuration of Sparks and their Temporal Evolution

The distribution of sparks on the polar cap surface in the presence of a PSG has the following constraints:

1. A continuous presence of sparks are required bordering the boundary of the polar cap for effective thermal regulation.
2. The sparks are as tightly packed as possible within the IAR.
3. There are two distinct directions of spark evolution with one half showing a clockwise shift in the pattern and the other half an anticlockwise shift.

The first condition requires the presence of an annulus of sparks bordering the boundary of the polar cap in a tightly packed configuration. The second condition ensures that in the interior of the polar cap the sparks are also formed in concentric radio emission beam (Rankin 1990, 1993; Gil et al. 1993), where the pulsar emission is arranged in concentric rings of coral emission around a central stationary core. The lifetimes of individual sparks are much shorter than the apparent drift motion of the spark pattern, which evolves over several seconds. Hence, the observed drifting behavior reflects the change in the spark pattern and is not associated with the motion of individual sparks.

3.2. Effect of Polar Cap Boundary on Sparks

Figure 1(a) shows a schematic of the evolution of the spark process along the boundary. Initially, the sparks (red outline) are formed close to each other in a tightly packed configuration around a point (red point) on the boundary that is maximally heated by particles from both sparks. During their lifetimes the sparks shift by an average distance \(h_s\) opposite to the corotation direction (dotted black outline). At the end of the life cycle, the location of the maximally heated point (open black circle) shifts around the boundary due to the drift motion. After the surface cools down the next sparks are formed around this shifted point in the boundary resulting in an effective shift of the spark pattern by a distance \(h_D\) along the boundary of the polar cap.

The angular shift \((\theta_{sl})\) in the drift direction of the spark pattern at different points on the boundary is related to the spatial shift \((h_D)\) at a point specified by the polar angle \(\theta_{PC}\) as

\[
\theta_{sl} = h_D / \sqrt{a_{cap}^2 \sin^2 \theta_{PC} + b_{cap}^2 \cos^2 \theta_{PC}},
\]

where \(a_{cap}\) and \(b_{cap}\) are the major and minor axes of the elliptical polar cap with \(a_{cap} = b_{cap} = R_{cap}\) for the circular case. The shifts in the spark pattern along the entire polar cap boundary are shown in Figure 2 corresponding to (a) a circular polar cap with the sparks lagging-behind corotation along the \(x\)-axis, (b) a circular polar cap with the corotation direction making a 45° angle in the \(x-y\) plane, (c) elliptical polar cap with eccentricity 0.6 and major axis along the \(x\)-axis, and (d) elliptical polar cap with eccentricity 0.6, and major axis along the \(y\)-axis. The shift is maximum when the direction of corotation is tangential to the curvature of the boundary and goes to zero when the corotation is normal to the boundary. In all cases, there are two distinct regions of spark evolution bounded by the points where the corotation direction is normal to the elliptical/circular boundary of the polar cap. In one half, the shifts are in the negative direction, which signifies a clockwise shift of the spark pattern, while in the other half the shifts are positive resembling an anticlockwise shift. In the elliptical polar caps, the maximum shifts are flatter or steeper compared to the circular case depending on the orientation of the major axis. In realistic situations the differential shifts will be averaged out due to the finite size of the sparks, as the next spark can only form in the available space between two sparks and an average shift will be seen over time. We expect two distinct evolution patterns of the spark discharges along the rim of the polar cap showing clockwise behavior in one half and anticlockwise evolution in the other half. We used this insight into the evolution pattern to understand the two-dimensional configuration of the spark system.

The Astrophysical Journal, 936:35 (13pp), 2022 September 1
Basu, Melikidze, & Mitra
layers with a single spark at the center. The central spark may differ in size depending on the available space. Finally, the third condition associated with subpulse drifting requires the spark pattern in each concentric layer to shift in two different directions in two halves resembling a clockwise and anticlockwise shift. The bounding points of the two layers are locations where the lagging-behind corotation direction is normal to the concentric ring, the starting point being the inward normal and the ending point the outward normal. As the spark pattern shifts away/toward the bounding points smaller empty spaces may open up in these regions. A spark appears in these locations due to the temperature falling below the critical level, and as a consequence, potential difference develops to start the spark process. But there is heating in the surrounding regions from the sparks on either side of this location and hence the difference in potential across it is lower, resulting in smaller sparks. Due to differential motion around it, the central spark is not expected to show any shift but reappears around the same location depending on the surface heating requirements.

In a purely dipolar magnetic field configuration, the polar cap boundary can be approximated to be circular in shape. However, as discussed in the introduction a number of detailed observations have revealed the polar cap to be non-dipolar in nature. In this work, the non-dipolar nature of the surface field is modeled by considering a combination of magnetic dipoles of different strengths (see Section 5). A large star-centered dipole is used to reproduce the large-scale magnetic field away from the stellar surface, while a weaker dipole is placed close to the polar cap to match the surface field strength as well as the estimated polar cap size (Gil et al. 2002; Basu et al. 2020b). In this configuration, the boundary of the non-dipolar polar cap is elliptical in shape with the direction of elongation depending on the location of the surface dipole. Although the magnetic field configuration considered in this work provides a convenient setup for estimating the non-dipolar surface magnetic field, it is not a unique solution. But the spark evolution presented here should be applicable to polar caps with well-defined continuous boundaries. An elliptical polar cap is defined by a major axis ($a_{\text{cap}}$), a minor axis ($b_{\text{cap}}$), and an angle of inclination of the major axis in the $x$–$y$ plane ($\theta_{\text{cap}}$). The drop in potential across the sparks is expected to vary along the different axes in elliptical polar caps and we approximate the spark shape to resemble the polar cap with major axis $a_{\text{spark}}$, minor axis $b_{\text{spark}} = a_{\text{spark}} b_{\text{cap}} / a_{\text{cap}}$, and inclination angle $\theta_{\text{cap}}$, such that $\sqrt{a_{\text{spark}} b_{\text{spark}}} \approx h_i$. The sparks are unlikely to have sharp boundaries but are expected to have a peak density and spread out until the surface is heated by an
adjacent spark. The number of tracks of the concentric spark trajectories in addition to the central core is $N_{\text{trk}} = \text{Int}(a_{\text{cap}}/a_{\text{spark}})$. The maximum number of sparks that can be accommodated within any concentric annulus within the polar cap is given as

$$N_{\text{spark}}^i = \text{Int}(F(a_{\text{out}}^i b_{\text{out}}^i - a_{\text{in}}^i b_{\text{in}}^i)/(a_{\text{spark}} b_{\text{spark}})),$$

$$a_{\text{out}}^i = a_{\text{cap}} - 2(i-1)a_{\text{spark}}, \quad a_{\text{in}}^i = a_{\text{out}}^i - 2a_{\text{spark}},$$

$$b_{\text{out}}^i = b_{\text{cap}} - 2(i-1)b_{\text{spark}}, \quad b_{\text{in}}^i = b_{\text{out}}^i - 2b_{\text{spark}}.$$  \hspace{1cm} (10)

Here, $i = 1, 2, ..., N_{\text{trk}}$ and $F$ is a scaling factor for maximum packing, which we find to be around 0.75. A series of sparks are set up along each of these concentric regions whose angular size is $\theta_{\text{spark}}^i = 2\pi/N_{\text{park}}$, and their centers lie on the ellipse specified by $a_{\text{trk}}^i = (a_{\text{out}}^i + a_{\text{in}}^i)/2$ and $b_{\text{trk}}^i = (b_{\text{out}}^i + b_{\text{in}}^i)/2$. As specified before the dynamics of the evolution of the spark pattern follows two different directions in two halves. In one half, the next sparks are formed and shifted by an angle, $\theta_{\text{u}}^i = -h_D/\sqrt{a_{\text{trk}}^i b_{\text{trk}}^i}$ and in the other half by the angle $\theta_{\text{d}}^i = +h_D/\sqrt{a_{\text{trk}}^i b_{\text{trk}}^i}$, here $h_D$ being the average shift of the spark pattern, which is constant for all $i$. Finally, smaller sparks are set up at either end whose size is variable depending on the available space between the two trajectories.

The two-dimensional spark configuration for a circular polar cap, where $a_{\text{cap}} = b_{\text{cap}}$, with lagging behind the corotation direction along the $x$-axis is shown in Figure 3. The spark pattern evolves to resemble a clockwise shift in the upper half and an anticlockwise shift in the lower half. The two regions are separated at the points where the corotation direction is normal to the circle, i.e., at $\theta_{\text{PC}} = 0^\circ, 180^\circ$ where $\theta_{\text{PC}}$ represents the polar angle. As the spark pattern from either side shifts away from $\theta_{\text{PC}} = 180^\circ$, a gap opens up, which in certain cases, is smaller than the size of a fully formed spark, and smaller size sparks are formed. Similarly, at $\theta_{\text{PC}} = 0^\circ$, the available space reduces due to the encroachment from either side leading to smaller sparks. We have assumed that no phase difference exists between the shifts in the upper and lower halves. However, the sparks in these two regions evolve independently and can have any arbitrary phase difference between them.

**5. Modeling Subpulse Drifting**

The evolution of the distribution of sparks in the IAR is seen as the phenomenon of subpulse drifting in the pulsar radio emission. We used the model of the two-dimensional distribution of sparks presented in the previous section to simulate the different drifting behavior observed in pulsars. The process of generating a single pulse sequence from a pulsar with a non-dipolar polar cap has been presented in Paper I and is summarized below.

The non-dipolar magnetic field in the polar cap is modeled using a large star-centered dipole, which dominates the magnetic field away from the star surface along with one or more surface dipoles that determine the magnetic field structure at the polar cap (Gil et al. 2002). We use a spherical coordinate system with the origin located at the center of the neutron star and the rotation axis along $\theta = 0^\circ$. The star-centered dipole located at the origin is specified as $d = (d, \theta_{\text{d}}, 0^\circ)$, where $d$ represents the dipole moment and $\theta_{\text{d}}$ the declination angle between the rotation and magnetic axis of the star. The surface dipoles are represented as $m_i = (m_i^r, \theta_{m_i}^r, \phi_{m_i}^r)$, where $i = 1, 2, ..., N$ in case more than one is used, with dipole moments $m_i^r = 0.001$–0.05$d$ much smaller than the star-centered dipole, and the orientation of each dipole specified by the angles $\theta_{m_i}^r$ and $\phi_{m_i}^r$. The surface dipoles are located at $r_i = (r_i^c, \theta_{m_i}^r, \phi_{m_i}^r)$ close to the surface with $r_i^c = 0.95R_S$, $R_S = 10\text{ km}$ the neutron star radius, while the angles $\theta_{m_i}^r$ and $\phi_{m_i}^r$ can be varied to shift the location away from the purely dipolar polar cap. At a distance...
of $\sim 30R_\odot$ the contribution of surface dipole moments becomes negligible and the magnetic field is largely dipolar with contribution from $d$. The opening angle corresponding to the last open dipolar field line at $30R_\odot$ is used to estimate the outline of the modeled polar cap surface by numerically solving the magnetic field line equations (see Paper I for details). The elliptical outline of the polar cap is estimated using nonlinear least squares fits (Press et al. 1992) to obtain $a'_\text{cap}$, $b'_\text{cap}$, and $\theta'_\text{cap}$, and the location of the center is defined by $\theta'_\text{cap}$ and $\phi'_\text{cap}$.

As the pulsar rotates the pulsed emission is seen when the line of sight (LOS) passes through the open field line across the radio emission region. The pulsar profile shape and the drifting behavior are dependent on the LOS cut across this emission beam. The LOS is defined by the angle $\beta$, which is the minimum angle between the axis of the star-centered dipole and LOS. In the spherical coordinate system defined at the center of the neutron star, with the rotation axis aligned along the $z$-axis, the track of the LOS and consequently a train of pulsed emission is obtained by a continuous change in the coordinate $\phi = 2\pi/P$, for a fixed azimuth angle $\theta = \theta_d + \beta$. The non-dipolar polar cap is usually shifted from the dipolar case with the magnetic field lines twisting as they connect with the dipolar emission region around 10–100$R_\odot$. As a result, the outline of the LOS gets modified as it traverses the distribution of sparks resulting in different drifting behavior for different surface magnetic field configurations. Once the LOS enters the open field line region the corresponding field line at the surface is estimated by numerically solving the magnetic field line equations. The spark intensity at that point on the surface is estimated and recorded. It is assumed that the intensity of the radio emission follows that of the spark, but in realistic cases, the radio emission in the subpulses is a result of nonlinear plasma processes (Melikidze et al. 2000), which introduces additional features. As the LOS traverses the extent of the pulse window an intensity pattern due to the distribution of sparks is obtained and forms the single pulse. In subsequent rotations the distribution of sparks evolves and gives different intensity patterns, reflecting the drifting behavior.

In order to estimate the two-dimensional distribution of sparks a Cartesian $x'y'$-plane\(^5\) is defined to contain the elliptical polar cap with the origin at the center of the ellipse. The boundary of the upper and lower halves of the polar cap signifying clockwise and anticlockwise evolution of the spark pattern is specified by the angles $\theta'_s = 3\pi/2 - \phi'_\text{cap}$ and $\theta'_s = \pi/2 - \phi'_\text{cap}$, where $\theta'$ is the polar angle in the $x'y'$-plane.

The spark pattern diverges away from $\theta'_s$ and converges toward $\theta'_c$. We have assumed a Gaussian distribution of intensity for each spark with elliptical symmetry:

$$I_{\text{spark}}(x', y') = I_0 \exp \left( -\frac{1}{2} \left[ (x' - x'_c)^2/a_{\text{spark}}^2 + (y' - y'_c)^2/b_{\text{spark}}^2 \right] \right). \quad (11)$$

where $I_0$ is the peak intensity of the spark and $x'_c$, $y'_c$ corresponds to the center of the spark. The sparks do not have sharp borders and the intensity pattern is drawn primarily as guidelines to show the distinction between adjacent sparks. The location of the sparks within the elliptical polar cap and the evolution of their distribution with time is described in the previous section. Similarly to in Paper I, we have considered three different magnetic field configurations whose behavior represents the

---

**Table 1**

| Physical Parameters of the Elliptical Polar Cap |
|-----------------------------------------------|
| $a'_\text{cap}$ (m) | $b'_\text{cap}$ (m) | $\theta'_\text{cap}$ (°) | $\phi'_\text{cap}$ (°) | $b$ | $\cos \alpha_l$ | $\eta$ |
|-----------------------------------------------|
| Coherent | 127.6 | 80.4 | 45.9 | 16.2 | 5.3 | ~4 | ~0.85 | 0.038 |
| Phase-stat | 75.3 | 21.6 | 52.4 | 51.2 | 17.9 | ~19 | ~0.8 | 0.021 |
| Bi-drift | 62.2 | 36.1 | 148.5 | 3.2 | 100.1 | ~26 | ~0.79 | 0.013 |

---

\(^5\) The primed coordinates are used to distinguish from the star-centered coordinate system used for estimating the LOS traverse.

---

**Figure 4.** Two-dimensional distribution of the spark pattern for an elliptical polar cap with a major axis of 127.6 m, minor axis of 80.35 m, and tilted by an angle of 45° in the plane of the ellipse. The sparks are arranged in two concentric elliptical annuli around a central spark in a tightly packed configuration. The spark pattern evolves with time to show a clockwise shift in the left half and an anticlockwise shift in the right half bounded by the points $\theta'_\text{cap} = 264.7°$ and $\theta'_\text{cap} = 84.7°$, where the pattern shifts away from $\theta'_\text{cap}$ and converges toward $\theta'_\text{cap}$. The LOS traverse across the emission beam at an angle $\beta = 3°$ from the center and its imprint on the polar cap is also shown. The dynamical evolution of the distribution of sparks across the LOS results in coherent subpulse drifting. An animation showing the evolution of the spark configuration with time is available.

(An animation of this figure is available.)
three distinct observational classes of subpulse drifting, viz., coherent phase-modulated drifting, phase stationary drifting, and bi-drifting with opposite drift directions in different parts of the emission window (see Basu et al. 2019a). We present below the physical characteristics of the PSG for each magnetic field configuration, the temporal evolution of the two-dimensional spark configuration, and the simulated single pulses showing subpulse drifting.

5.1. Coherent Phase-modulated Drifting

Coherent phase-modulated drifting is the prototype of the subpulse drifting behavior with the subpulses showing a systematic shift throughout the entire pulse window. These are mostly associated with LOS traverses toward the edge of the emission beam and are seen in pulsar average profiles with single or double conal components (Rankin 1986; Basu et al. 2019a). In Paper I, the magnetic field configuration used to demonstrate this behavior consisted of a star-centered dipole $d = (d, 15^\circ, 0^\circ)$ and one surface dipole $m = (0.001d, 0^\circ, 0^\circ)$ located at $r = (0.95R_S, 18^\circ 86, 10^\circ 99)$. The dipolar magnetic axis is tilted at an angle of $15^\circ$ with the rotation axis while the non-dipolar polar cap is located around $5^\circ$ away in the $\phi$-axis from the dipolar polar cap.

The polar cap is elliptical in shape with the fitting parameters shown in Table 1. The surface magnetic field strength is characterized by average $b \sim 4$ (see Figure 17 in Paper I). We are primarily interested in the polar caps of long-period older pulsars that show subpulse drifting. A number of studies have shown that the average emission beam in such cases comprises a central core surrounded by two concentric rings of conal emission (Rankin 1990, 1993; Mitra & Deshpande 1999; Mitra & Rankin 2002). The presence of two conal rings in such pulsars is also expected from the spark sizes obtained from the PSG model (Mitra et al. 2020). Hence, using $N_{\text{spark}} = 2$, the typical spark size can be estimated as $a_{\text{spark}} \sim 23.6$ m, $b_{\text{spark}} \sim 14.9$ m, and $h_{\text{spark}} \sim 18.8$ m. The screening factor for the PSG is obtained from Equation (2) as $\eta \sim 0.038$. The drifting periodicity ($P_3$) in a PSG

---

**Table 2**

| $l$ | $a_{\text{out}}$ (m) | $a_{\text{in}}$ (m) | $b_{\text{out}}$ (m) | $b_{\text{in}}$ (m) | $N_{\text{spark}}$ | $\theta_{\text{spark}}$ (°) | $a_{\text{tik}}$ (m) | $b_{\text{tik}}$ (m) | $\omega_{u,d}$ (deg s$^{-1}$) |
|-----|----------------------|----------------------|----------------------|----------------------|--------------------|------------------------|----------------------|----------------------|-------------------------|
| Outer | 1 | 127.6 | 80.4 | 80.4 | 50.6 | 13 | 27.7 | 104.0 | 65.5 | $\mp 5.6$ |
| Inner | 2 | 80.4 | 33.2 | 50.6 | 20.8 | 7 | 51.4 | 56.8 | 35.7 | $\mp 10.43$ |

---

Figure 5. Single pulse simulations demonstrating the coherent subpulse drifting. (a) Pulse stack with 128 simulated single pulses and (b) LRFS across the pulse window. The drifting periodicity, $P_3 = 4.9P$, is seen as a peak frequency, $f_p \sim 0.2$ cycles/$P$. The time evolution of the spark pattern is reflected in the phase behavior across the profile (top window).

---

The Astrophysical Journal, 936:35 (13pp), 2022 September 1

Basu, Melikidze, & Mitra
Table 3
Details of the Distribution of Sparks in the Polar Cap Exhibiting Phase Stationary Drifting

| i     | a_{out} (m) | a_{in} (m) | b_{out} (m) | b_{in} (m) | N_{spark} | \theta_{spark} (°) | a_{pk} (m) | b_{pk} (m) | \omega_{\delta,d} (deg s^{-1}) |
|-------|-------------|-------------|-------------|-------------|-----------|-------------------|-------------|-------------|-----------------------------|
| Outer | 1           | 75.3        | 47.5        | 21.6        | 13        | 27.7              | 61.4        | 17.6        | ±2.9                        |
| Inner | 2           | 47.5        | 19.7        | 13.6        | 7         | 51.4              | 33.6        | 9.6         | ±5.3                        |

Figure 6. Two-dimensional distribution of the spark pattern for an elliptical polar cap with a major axis of 75.3 m, minor axis of 21.6 m, and tilted by an angle of 52°4 in the plane of the ellipse. The sparks are arranged in two concentric elliptical annuli around a central spark in a tightly packed configuration. The spark pattern evolves with time to show a clockwise shift in the left half and an anticlockwise shift in the right half bounded by the points \( \theta_{l} = 252°1 \) and \( \theta_{r} = 72°1 \), where the pattern shifts away from \( \theta_{l} \) and converges toward \( \theta_{r} \). The LOS traverse across the emission beam centrally at an angle \( \beta = -0°2 \) and its imprint on the polar cap is also shown. The dynamical evolution of the distribution of sparks across the LOS results in phase stationary subpulse drifting in the inner and outer concentric rings, but no periodic behavior in the central component. An animation showing the evolution of the spark configuration with time is available.

The rate of shifting of the patterns in the two halves of each ring is estimated as \( \omega_{\delta,d} = \pm \theta_{\text{spark}} / P_{Y} \). The average shift of the spark pattern during a rotation period \( P = 1 \text{ s} \) is \( h_{P} = |\omega_{\delta,d}| \sqrt{a_{pk} b_{pk}} \approx 8.1 \text{ m} \) (see Section 4).

We assume the emission to originate from an average height of 30R_{S} where the opening angle of the open field line region is \( \rho = 4°55 \). In order to simulate the single pulses exhibiting coherent drifting an outer LOS traverse is considered with \( \beta = 3° \), such that \( \beta / \rho = 0.66 \). We simulated 128 single pulses using the above setup and the corresponding pulse stack is shown in Figure 5(a). The average profile shows a barely resolved double-peak structure resulting from the shifted non-dipolar polar cap and the corresponding shift in the LOS cut across it (see Figure 4). The single pulses show prominent drift bands with systematic variations across the entire window as expected for coherent drifting behavior. The drifting behavior is characterized using longitude resolved fluctuation spectra (LRFS; Backer 1973) as shown in Figure 5(b). The LRFS is estimated by determining the fast Fourier transform across the 128 pulses along each pulsar longitude within the emission window. Drifting periodicity is seen as the peak amplitude in frequency, \( f_{p} = 0.2 \text{ cycles} / P \) (left window) while the evolution of the spark pattern across the LOS is reflected in the large phase variations seen across the emission window (top window). The actual phase variations show a continuous change from the leading to the trailing edge of the profile, but have been wrapped around \( \pm 180° \) in the figure for convenience of plotting.

5.2. Phase Stationary Drifting

Phase stationary drifting corresponds to the cases where the subpulses do not show a significant shift in position across the emission window but periodically change in intensity. As a result, the phases associated with the drifting periodicity are relatively flat. Phase stationary drifting is usually seen in pulsars with multiple components having core-cone profiles where the central core component does not exhibit any periodicity (Rankin 1986; Basu et al. 2019a, 2020a). Such profiles correspond to the central LOS traverse of the emission beam with small \( \beta \). We use the magnetic field configuration of Paper I to study this drifting behavior where the star-centered dipole is specified as \( d = (d_{x}, 45°, 0°) \) and there is one surface dipole \( m = (0.05d_{x}, 0°, 0°) \) located at \( r = 0.95R_{S}, 57°08, 20°66° \). The dipolar magnetic axis is tilted by an angle of 45° with the rotation axis while the non-dipolar polar cap is located around 20° away from the dipolar polar cap.

The polar cap is highly elliptical in shape and much smaller in size compared to the dipolar case; the fitting parameters are

Dynamical evolution of the spark pattern (evolution of the spark configuration with time) is available.

(An animation of this figure is available.)

is given by \( P_{3} = 1/2 \pi \eta \cos \alpha_{l} \), where \( \alpha_{l} \) is the angle of the local non-dipolar magnetic field with the rotation axis (Mitra et al. 2020). The average \( \cos \alpha_{l} \) is 0.85 for the magnetic field configuration (Figure 17 in Paper I) and we can estimate \( P_{3} \) to be 4.9P.
shown in Table 1. The surface magnetic field strength is characterized by average $b \sim 19$ (see Figure 18 in Paper I). Assuming $N_{\text{nth}} = 2$, the typical spark size can be estimated as $a_{\text{spark}} \sim 13.9$ m, $b_{\text{spark}} \sim 4.0$ m, and $h \sim 7.5$ m. The screening factor for the PSG is obtained from Equation (2) as $\eta \sim 0.021$. The average $\cos \alpha_{\text{e}}$ is 0.8 for the magnetic field configuration (Figure 18 in Paper I) and we can estimate $P_3$ to be $9.7P$. The two-dimensional distribution of the sparks and their evolution with time is shown in Figure 6. The sparks are distributed around two elliptical rings surrounding a central spark in a tightly packed configuration. The details of the distribution of sparks for each track are reported in Table 3 and show the different major $(a)$ and minor $(b)$ axes describing each ring, the maximum number of fully formed sparks that can be accommodated along these rings ($N_{\text{spark}}$), the angular size of the sparks ($\theta_{\text{spark}}$, as explained in Section 4 and the rate of shifting of the patterns $\omega_{\text{dip}}$. The two bounding points around the distribution of sparks show evolution in opposite directions are $\theta_1 = 252^\circ, 7^\circ$ and $\theta_2 = 72^\circ, 1$, where the pattern shifts away from $\theta_1$ and converges toward $\theta_2$. The average shift of the spark pattern during a rotation period ($P = 1$ s) is $h_0 \sim 1.7$ m.

We assume the emission to originate from an average height of $30R_3$ where the opening angle of the open field line region is $\rho = 4^\circ 55$. In order to simulate the single pulses exhibiting phase stationary drifting a central LOS traverse is considered with $\beta = -0^\circ 2$, such that $\beta/\rho = 0.04$. We simulated 128 single pulses using the above setup and the corresponding pulse stack is shown in Figure 7(a). The average profile has five components where the central core component corresponds to the central region with a constant presence of a spark. As a result, the core component does not show any drifting, which is an established observational result. The surrounding components show phase stationary drifting with a periodic change in intensity but very little systematic variations across the longitudes. The drifting behavior is characterized using the time evolution of the spark pattern is reflected in the phase behavior across the profile (top window).

5.3. Bi-drifting: Reversals in Drift Directions

The bi-drifting phenomenon is a unique drifting behavior seen in a handful of pulsars (Champion et al. 2005; Weltevrede 2016; Szary & van Leeuwen 2017; Basu & Mitra 2018; Basu et al. 2019b; Szary et al. 2020; Shang et al. 2022) where the drifting direction is opposite in different components of the pulsar profile. These pulsars usually have wider profiles that suggest a small inclination angle $\theta$ between the rotation and the magnetic axis. The magnetic field configuration used for simulating bi-drifting in Paper I consisted of a star-centered dipole specified as $d = (d, 5^\circ, 0^\circ)$ and one surface dipole $m = (0.005d, 0^\circ, 0^\circ)$ located at $r = (0.95R_3, 5^\circ, 120^\circ)$. The dipolar magnetic axis has a
Table 4
Details of the Distribution of Sparks in the Polar Cap Exhibiting Bi-drifting

| $i$ | $a_{out}$ (m) | $a_{in}$ (m) | $b_{out}$ (m) | $b_{in}$ (m) | $N_{spark}$ | $\theta_{spark}$ ($^\circ$) | $a_{trk}$ (m) | $b_{trk}$ (m) | $\omega_{drift}$ (deg s$^{-1}$) |
|-----|---------------|--------------|---------------|--------------|-------------|-------------------------|--------------|--------------|---------------------|
| Outer | 1 | 62.2 | 39.2 | 36.1 | 22.7 | 13 | 27.4 | 50.7 | 29.4 | ±1.8 |
| Inner | 2 | 39.2 | 16.2 | 22.7 | 9.3 | 7 | 51.4 | 27.7 | 16.0 | ±3.3 |

The Astrophysical Journal, 936:35 (13pp), 2022 September 1
Basu, Melikidze, & Mitra

Figure 8. Two-dimensional distribution of the spark pattern for an elliptical polar cap with a major axis of 62.2 m, minor axis of 36.1 m, and tilted by an angle of 148.5° in the plane of the ellipse. The sparks are arranged in two concentric elliptical annuli around a central spark in a tightly packed configuration. The spark pattern evolves with time to show a clockwise shift in the upper half and an anticlockwise shift in the lower half bounded by the points $\theta_1' = 169.5^\circ$ and $\theta_2' = -10.5^\circ$, where the pattern shifts away from $\theta_1'$ and converges toward $\theta_2'$. The LOS traverses across the emission beam between the center and the edge at an angle $\beta = 2^\circ$ and its imprint on the polar cap is also shown. The dynamical evolution of the distribution of sparks across the LOS results in bi-drifting with opposite drift directions in the upper and lower halves. An animation showing the evolution of the spark configuration with time is available.

(A low inclination angle of 5° with the rotation axis, while the non-dipolar polar cap is rotated by a large angle of around 100° away from the dipolar polar cap.

The polar cap is elliptical in shape and is tilted in the opposite direction compared to the previous cases. The parameters of the elliptical fit to the polar cap outline are reported in Table 1. The surface magnetic field strength is characterized by an average $b \sim 26$ (see Figure 19 in Paper I). Assuming $N_{trk} = 2$, the typical spark size can be estimated as $a_{spark} \sim 11.5$ m, $b_{spark} \sim 6.7$ m, and $h_{s} \sim 8.8$ m. The screening factor for the PSG is obtained from Equation (2) as $\eta \sim 0.013$. The average $\cos \alpha_1$ is 0.79 for the magnetic field configuration (Figure 19 in Paper I) and the expected $P_3$ is 15.7P.

The two-dimensional distribution of the sparks and their evolution with time is shown in Figure 8. The sparks are distributed around two elliptical rings surrounding a central spark in a tightly packed configuration. The details of the distribution of sparks for each track are reported in Table 4 and show the different major ($a$) and minor ($b$) axes describing each ring, the maximum number of fully formed sparks that can be accommodated along these rings ($N_{spark}$), the angular size of the sparks ($\theta_{spark}$), as explained in Section 4, and the rate of shifting of the patterns $\omega_{drift}$. Due to the large shift in the location of the non-dipolar polar cap the distribution of sparks evolves along the upper and lower parts of the concentric rings, in contrast with the previous cases where the changes happened on the left and right halves. The two bounding points around which the distribution of sparks show evolution in opposite directions are $\theta_1' = 169.5^\circ$ and $\theta_2' = -10.5^\circ$, where the pattern shifts away from $\theta_1'$ and converges toward $\theta_2'$. The average shift of the spark pattern during a rotation period ($P = 1$ s) is $h_{d} \sim 1.2$ m.

We assume the emission to originate from an average height of $30R_S$ where the opening angle of the open field line region is $\rho = 4.55$. In order to show the full effect of the bi-drifting behavior LOS traverse halfway between the center and the edge is considered with $\beta = 2^\circ$, $\beta/\rho = 0.4$, such that all conal components are visible without being affected by the stationary cone. In contrast to the previous two cases, the LOS is roughly perpendicular to the direction of the shifts in the spark pattern. We simulated 128 single pulses using the above setup and the corresponding pulse stack is shown in Figure 9(a). The average profile has four components with prominent drifting behavior where the leading two components have opposite shifts in their drifting pattern compared to the trailing components.

The drifting behavior is characterized using the LRFS as shown in Figure 9(b). The drifting periodicity is seen as the peak frequency, $f_p \sim 0.06$ cycles/$P$ (left window), while phase changes show opposite slopes in the leading and trailing components (top window).

6. Summary and Conclusion

We have presented a model of the two-dimensional distribution of spark discharges in the IAR above the pulsar polar cap and its evolution with time. The polar cap is dominated by non-dipolar magnetic fields and is smaller in size and usually shifted compared to an equivalent dipolar polar cap. The surface of the polar cap is heated to temperatures of $\sim 10^6$ K, around the critical temperature for ionic free flow, resulting in a PSG. The spark discharges in this system are set up when the temperature goes below the critical level opening up a large difference in potential for pair cascades to commence. The sparks are essentially a mechanism to regulate the surface temperature where the surface acts as a thermostat.

The primary features of the distribution of sparks are summarized as follows:

1. The sparks are formed in a tightly packed configuration governed by the surface temperature. The sparks spread out around the point of origin depending on the available drop in potential across the field lines, with decreasing particle densities away from the peak location. The region lying between the sparks is also heated by the diffusion of particles from surrounding sparks and there are no effective gaps between sparks.
The sparks have typical durations of $\sim 10 - 100$ μs, which is the time taken to reach the critical temperature. The spark is a local event on the surface, governed by thermostatic regulation, and is unaffected by distant sparks. During the spark process, the charges within the spark lag behind the corotation motion due to $E \times B$ drift. The surface cooling timescale of $\sim 100$ ns is much shorter than the spark duration, which ensures that immediately after the cessation of the spark the next spark can be formed at a nearby point.

3. The evolution of the tightly packed distribution of sparks in a thermally regulated surface depends on the boundary of the polar cap. No spark discharges can take place in the closed field line region and hence thermal regulation requires the continuous presence of a spark around the boundary. As the sparks on the boundary drift opposite to the corotation direction during their lifetimes, the location of maximally heated points moves around the boundary leading to the time evolution of the spark pattern. The individual sparks are short-lived and do not participate in any long-term periodic behavior. The locus of the heating points on the surface around which the subsequent spark takes place shows a gradual but continuous shift with time.

4. The evolution of the spark pattern on the boundary ensures that similar changes take place in the interior as well, since these sparks also affect the heating on the other side. The sparks are set up around concentric rings where two distinct directions of evolution arise bounded by points where the corotation direction is normal to the curvature of the boundary. In one half, the pattern represents a clockwise shift while in the other half an anticlockwise shift is set up. As the pattern shifts, gaps open up in the two normal points in each ring where smaller sparks arise to aid the thermal regulation. The evolution of the spark pattern described above requires the presence of a polar cap boundary with regular curvature like an ellipse or a circle. In the case of irregular shapes, it is possible that only part of the polar cap shows evolution or the spark pattern is stationary in the case of extremely irregular shapes.

5. Due to the differential shift of spark patterns in two halves, the heating location at the center remains stationary with sparks forming at the same place at regular intervals. This resembles the core component in the pulsar profile, which does not exhibit any drifting behavior.

The evolution of the spark process in the IAR was used to simulate the subpulse drifting behavior in pulsars. We considered three different surface magnetic field configurations, outlined in Paper I, to reproduce the different classes of
drifting observed in the pulsar population. In each case, a detailed characterization of the spark properties on the surface was carried out. These provide the template for constraining the physical characteristics of the IAR in pulsars where subpulse drifting has been measured and will be explored in future works. A complete determination of the IAR properties will likely require additional information from X-ray observations to constrain the polar cap temperature and size.

We thank the referee for comments that helped to improve the paper. D.M. acknowledges the support of the Department of Atomic Energy, Government of India, under project No. 12-R&D-TFR-5.02-0700. D.M. acknowledges funding from the grant “Indo-French Centre for the Promotion of Advanced Research - CEFIPRA” grant IFC/F5904-B/2018. This work was supported by grant 2020/37/B/ST9/02215 from the National Science Center, Poland.

ORCID iDs
Rahul Basu @ https://orcid.org/0000-0003-1824-4487
George I. Melikidze @ https://orcid.org/0000-0003-1879-1659
Dipanjan Mitra @ https://orcid.org/0000-0002-9142-9835

References
Arumugasamy, P., & Mitra, D. 2019, MNRAS, 489, 4589
Backer, D. C. 1973, ApJ, 182, 245
Basu, R., Lewandowski, W., & Kijak, J. 2020a, MNRAS, 499, 906
Basu, R., Mitra, D., & Melikidze, G. I. 2020b, MNRAS, 496, 465
Basu, R., & Mitra, D. 2018, MNRAS, 475, 5098
Basu, R., Mitra, D., Melikidze, G. I., et al. 2016, ApJ, 833, 29
Basu, R., Mitra, D., Melikidze, G. I., & Skrzypczak, A. 2019a, MNRAS, 482, 3757
Basu, R., Paul, A., & Mitra, D. 2019b, MNRAS, 486, 5216
Champion, D. J., Lorimer, D. R., McLaughlin, M. A., et al. 2005, MNRAS, 363, 929
Cheng, A. F., & Ruderman, M. A. 1980, ApJ, 235, 576
de Jager, O. C. 2007, ApJ, 658, 1177
Geppert, U. 2017, JApA, 38, 46
Gil, J., Haberl, F., Melikidze, G., et al. 2008, ApJ, 686, 497
Gil, J., Lyubarsky, Y., & Melikidze, G. I. 2004, ApJ, 600, 872
Gil, J., Melikidze, G., & Zhang, B. 2006, ApJ, 650, 1048
Gil, J., Melikidze, G. I., & Geppert, U. 2003, A&A, 407, 315
Gil, J. A., Kijak, J., & Seiradakis, J. H. 1993, A&A, 272, 268
Gil, J. A., Melikidze, G. I., & Mitra, D. 2002, A&A, 388, 235
Goldreich, P., & Julian, W. H. 1969, ApJ, 157, 869
Hermsen, W., Hessels, J. W. T., Kuiper, L., et al. 2013, Sci, 339, 436
Hermsen, W., Kuiper, L., Basu, R., et al. 2018, MNRAS, 480, 3655
Jones, P. B. 1986, MNRAS, 218, 477
Kargaltsev, O., Cerutti, B., Lyubarsky, Y., & Striani, E. 2015, SSrv, 191, 391
Kijak, J., & Gil, J. 1999, MNRAS, 299, 855
Kraszowski, K., Mitra, D., Gupta, Y., et al. 2009, MNRAS, 393, 1617
Lai, D. 2001, RVMP, 73, 629
Lakoba, T., Mitra, D., & Melikidze, G. 2018, MNRAS, 480, 4526
Melikidze, G. I., Gil, J. A., & Pataraya, A. D. 2000, ApJ, 544, 1081
Mitra, D. 2017, JApA, 38, 52
Mitra, D., Basu, R., Melikidze, G. I., & Arjunwadkar, M. 2020, MNRAS, 492, 2468
Mitra, D., & Deshpande, A. A. 1999, A&A, 346, 906
Mitra, D., & Li, X. H. 2004, A&A, 421, 215
Mitra, D., & Rankin, J. M. 2002, ApJ, 577, 322
Pétri, J., & Mitra, D. 2020, MNRAS, 491, 80
Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. 1992, Numerical Recipes in C: The Art of Scientific Computing (Cambridge: Cambridge Univ. Press)
Rahaman, S. M., Mitra, D., & Melikidze, G. I. 2020, MNRAS, 497, 3953
Rankin, J. M. 1986, ApJ, 301, 901
Rankin, J. M. 1990, ApJ, 352, 247
Rankin, J. M. 1993, ApJ, 405, 285
Ruderman, M. A., & Sutherland, P. G. 1975, ApJ, 196, 51
Shang, L.-H., Bai, J.-T., Ding, S.-J., & Zhi, Q.-J. 2022, RAA, 22, 025018
Sturrock, P. A. 1971, ApJ, 164, 529
Szary, A. 2013, arXiv:1304.4203
Szary, A., Gil, J., Zhang, B., et al. 2017, ApJ, 835, 178
Szary, A., Melikidze, G. I., & Gil, J. 2015, MNRAS, 447, 2295
Szary, A., & van Leeuwen, J. 2017, ApJ, 845, 95
Szary, A., van Leeuwen, J., Weltevrede, P., & Maan, Y. 2020, ApJ, 896, 168
Sznajder, M., & Geppert, U. 2020, MNRAS, 493, 3770
Timokhin, A. N., & Harding, A. K. 2019, ApJ, 871, 12
von Hohensbroech, A., & Xilouris, K. M. 1997, A&AS, 126, 121
Weltevrede, P. 2016, A&A, 590, A109
Weltevrede, P., Edwards, R. T., & Stappers, B. W. 2006, A&A, 445, 243
Weltevrede, P., & Johnston, S. 2008, MNRAS, 391, 1210