A Method for Multi-objective Optimization and Application in Automobile Impact

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Abstract. A method is proposed for constrained multi-objective optimization problems, which is based on Newton method and the linear weighted sum method. The automobile impact problem is studied in this paper. The multi-objective optimization model of the impact problem would be established. Then, an improved design of front rail is obtained by the algorithm proposed.

Keyword: multi-objective optimization, automobile impact

1. Introduction
Researchers have proposed various methods for multi-objective optimization problems. Generally, these methods can be divided into scalar algorithms and evolutionary algorithms by way of searching the optimal solutions. Typical scalar methods include traditional weighted sum method, constraint method, NBI and so on [1-2]. Evolutionary methods have developed rapidly in multi-objective optimization, such as multi-objective genetic algorithm, NSGA-II [3].

Usually, the engineering optimization problems can't be solved directly. They can be described approximately by surrogate models and then be optimized. The surrogate models are always constructed by polynomial response surface method, radial basis function method, Kriging method and so on, which are generally expressed as secondary or higher order polynomial functions. Based on Newton method and the linear weighted sum method, this paper takes Frisch's method to deal with the constraints.

During the frontal crash of cars, the front rails play an important role in occupant protection for providing rigid support and absorbing impact energy. In order to provide better protection, the collision energy should be absorbed as soon as possible, but the produced peak impact force during the frontal crash is considered the smaller the better. In reality, the two properties can not attain optimum in the meantime. so the design of automobile's front rail becomes a problem of complicated multi-objective optimization. In this paper, the multi-objective optimization problem for automobile impact has been studied well.
2. Theoretical basis of the method

2.1. Theoretical basics

2.1.1. The process of the method. As we all know, impact resistance is the basic performance for structure, especially for vehicle, the performance includes more than one factor. The multi-objective function can be expressed,

\[
\min_{x \in \mathbb{R}^n} F(x) = \left[ F_1(x), F_2(x), \ldots, F_m(x) \right]
\]  

(1)

Where, the design variable \( x = (x_1, x_2, \ldots, x_n) \in \mathbb{R}^n \) is an \( n \) dimensional vector in the optimization problem. \( X \) is the feasible region, so \( x \in X \). \( F(x) \) is the set of functions composed by \( m \) objective functions, which is denoted by \( F_i(x), i = 1, 2, \ldots, m \).

The iterative direction at any point

\[
d = -\left[ \sum_{j=1}^{m} \lambda_j \nabla^2 F_j(x) \right]^{-1} \sum_{j=1}^{m} \lambda_j \nabla F_j(x)
\]

(2)

Denoted by \( \{x_t\} \) a sequence generated in iteration converges to some Pareto optimal points \( x^* \in X \) super-linearly, and the values of \( F(x^*) \) with different weight factors form the Pareto front.

2.2. The process of the method

The flow chart of the completed process for multi-criteria optimization problems by the method is shown in Fig.1.

![Figure 1. The flow chart of the method.](image-url)
3. The multi-objective optimization model of the automobile impact problem and solution

The automobile impact problem is studied in this section, and an improved design of front rail is obtained by the algorithm proposed. The reasonable results and short computing time prove the practicability and high-efficiency of the method. During the frontal crash of cars, the front rails play an important role in occupant protection for providing rigid support and absorbing impact energy. In order to provide better protection, the collision energy should be absorbed as soon as possible, but the produced peak impact force during the frontal crash is considered the smaller the better. In reality, the two properties can not attain optimum in the meantime, so the design of automobile's front rail becomes a problem of complicated multi-objective optimization. As shown in Fig.2, the front rail of a car can be regarded as a crooked channel steel, whose cross-sectional dimensions will be chosen as the design variables to optimize.

![Figure 2. Three-dimensional model of the considered part of a front rail.](image)

3.1. The model by response surface method

For the difficulty in establishing the analytical model of the front rail in frontal crash, the response surface method is employed in this paper. Through central composite design, 20 simulated impact tests within first ten milliseconds with different cross-sectional dimensions are finished by LS-DYNA.

The response surface models of energy absorption \( E_{in} \) and peak crushing force \( F_{\text{max}} \) based on the width \( x_1 \), height \( x_2 \), and thickness \( x_3 \) of the front rail's cross-section are built as follows.

\[
E_{in} = -760064 + 19975.56423 x_1 + 3272.16658 x_2 + 285601 x_3 - 40.95663 x_1 x_2
\]
\[
- 9022.42534 x_1 x_3 + 62.38542 x_2 x_3 - 127.60314 x_1^2 - 13.54378 x_2^2
\]
\[
- 1519.22426 x_3^2 - 0.14222 x_1^2 x_2 + 73.06253 x_1^3 x_3 + 0.20903 x_1 x_2^2
\]

\[
F_{\text{max}} = 11850.90812 - 95.22674 x_1 - 249.22107 x_2 + 2862.91393 x_3
\]
\[
+ 3.94198 x_1 x_2 - 92.53961 x_1 x_3 - 1.44785 x_1^2 + 0.89647 x_2^2
\]
\[
+ 28.61511 x_3^2 + 0.75158 x_1^3 x_3 - 0.01411 x_1 x_2^2
\]

Then, the standard multi-objective optimization model should be established. After converting the optimal problems to minimization, the average value of energy absorption and peak crushing force in the 20 tests are obtained to normalize the model. According to the constraints of sizes of the front rail:

\[
x_1 \in [58, 66], \quad x_2 \in [134, 146], \quad x_3 \in [1.8, 2.2],
\]

the model is:
3.2. The solution of model

In order to solve the above problem, the Frisch’s method is employed to convert the problem to unconstrained multi-objective optimization one. Then, a set of Pareto optimal solutions as well as the Pareto front can be obtained rapidly by the algorithm. The obtained solutions can be abundant reference for improving the design. The absorbed energy and peak crushing force of initial front rail are acquired by the simulation calculation. A comparison of the original design and the obtained Pareto optimal designs can be seen in Fig.3. Obviously, some Pareto optimal designs have better ability of energy absorption and generate smaller peak crushing force during the collision.

![Comparison of the original design and POF](image)

**Figure 3.** Comparison of the original design and the obtained Pareto optimal designs

Based on the better Pareto optimal designs, the original design is improved and the new front rail is check by the simulation calculation.

4. Conclusion

In detail, the initial as well as the improved structure parameters and their property values are listed in Table 1. By 2.77% increased energy absorption and 2.07% reduction in peak crushing force, the improved front rail has better performance in both.

| width / mm | height / mm | thickness / mm | F / kN  | E / J    |
|------------|-------------|----------------|--------|---------|
| Original design | 61          | 141            | 2.0    | 278.452 | 27748  |
| Improved design | 63.4        | 146.0          | 1.92   | 272.65  | 28641.3|
The performances of the two designs are calculated by LS-DYNA. The simulating results of the crushing force and absorbed energy within first ten milliseconds are recorded. Comparison of results between original design and improved one are shown in the in Figure 4, and the partial enlarged details below. The obtained results confirm the superiority of the improved design.

![Comparison of the two designs in force](image)

![Comparison of the designs in Energy](image)

**Figure 4.** Comparison of the simulative results between original and improved designs in properties

**References**

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