Multi-Agent Based Load Balancing Dispatch for Power Systems with Renewable Energy

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1 Introduction

Since it is crucial for our society to maintain stable power supply, we need appropriate operation plans in order to avoid malfunctions or black out of the power systems. A power dispatch with load balancing provide such a plan in terms of the quality of service (QoS) [1]. It is formulated as a problem to find a power generation plan such that every load ratio is equal to the same value for all agents and the sum of the power output to be generated is equal to a given demand [2, 3]. Let \( \mathcal{V} = \{1, 2, \ldots, N\} \) be the set of agents and \( N \) be the number of agents. We formulate the power dispatch as a decision problem:

Find \( x_1, x_2, \ldots, x_N \)

s.t. \( \frac{x_1}{\psi_1} = \frac{x_2}{\psi_2} = \ldots = \frac{x_N}{\psi_N} \), \( \sum_{i=1}^{N} x_i = l \) \tag{1}

where \( x_i \in \mathbb{R} \) is a power output plan to be generated at agent \( i \in \mathcal{V} \), \( \psi_i \in \mathbb{R} \) is a positive capacity factor of agent \( i \), and \( l \in \mathbb{R} \) is a given positive constant demand. Note that the desired load ratio \( \beta = x_i/\psi_i \) \( (i \in \mathcal{V}) \) satisfies \( \beta = l/(\psi_1 + \psi_2 + \ldots + \psi_N) \).

In this paper, we suppose that agents generate power output \( \hat{x}_i \in \mathbb{R} \) according to the power generation plan \( x_i \) via renewable energy resources such as solar, wind, or geothermal. Hence, the actual power output \( \hat{x}_i \) generated by agent \( i \) can be modeled as uncertain amount \( \hat{x}_i = x_i + w_i \), where \( \mathbb{E}[w_i] = 0 \), and \( \mathbb{V}[w_i] \leq \sigma_i^2 < \infty \) for all \( i \in \mathcal{V} \). Here \( \mathbb{E}[\cdot] \) and \( \mathbb{V}[\cdot] \) denote the expectation and the variance of a random variable \( \cdot \).

This paper presents the multi-agent based load balancing algorithm for power systems with renewable energy. The aim of this distributed algorithm is to achieve the power generation sequence such that the load ratio asymptotically converges to the same value, while the sum of the power output to be generated is equal to a given demand at each time step.

2 Load Balancing Dispatch with Renewable Energy

We consider a power dispatch with load balancing which is a problem to find a power generation plan such that every load ratio is equal to the same value for all agents and the sum of the power output to be generated is equal to a given demand [2, 3]. Let \( \mathcal{V} = \{1, 2, \ldots, N\} \) be the set of agents and \( N \) be the number of agents. We formulate the power dispatch as a decision problem:

3 Multi-agent System

Since we deal with a multi-agent based algorithm, we introduce a communication graph. Let \( \mathcal{G} = (\mathcal{V}, \mathcal{E}) \) be a digraph which represents a network, where \( \mathcal{E} \subseteq \mathcal{V} \times \mathcal{V} \) is the set of directed edges. We assume that the digraph
\( G \) is weakly connected with no self loop. Then, we consider the following type of distributed power dispatch algorithm:

\[
x_i[k+1] = x_i[k] + u_i[k],
\]

\[
u_i[k] = - \sum_{(i,j) \in \mathcal{E}} p_{ij}[k] + \sum_{(j,i) \in \mathcal{E}} p_{ji}[k] + G^l
\]

where \( k \in \mathbb{N} \) is discrete time, \( x[k] \in \mathbb{R}^n \) is a power generation plan for agent \( i \), \( u_i[k] \) is the amount of increasing/decreasing power output to be generated at agent \( i \), and \( p_{ij}[k] \in \mathbb{R} \) is the amount of power output to be adjusted between agents \( i \) and \( j \). We suppose that the amount of adjustments \( p_{ij}[k] \) is a control variable only for agent \( i \), not for agent \( j \) in the equation (2). On the other hand, \( p_{ji}[k] \) in (2) is determined by in-degree neighbors of agent \( i \), that is, agent \( j \) decides the amount of adjustments \( p_{ij}[k] \). Note that the above update rule preserves sum of \( x_i[k] \), that is, \( \sum_{i=1}^N x_i[k+1] = \sum_{i=1}^N x_i[k] \) for any \( k \), which follows \( \sum_{i=1}^N u_i[k] = 0 \). Thus, if \( \sum_{i=1}^N x_i[k] = l \) is satisfied, the constraint for demands (1) always holds for any time \( k \).

We define the control variable \( p_{ij}[k] \) as

\[
p_{ij}[k] = \mu[k] a_{ij} \left( \frac{x_i[k]}{\psi_i} - \frac{x_j[k]}{\psi_j} \right),
\]

\[\hat{x}_i[k] = x_i[k] + w_i[k],\]

where \( \mu[k] \in \mathbb{R} \) is a communication gain to be determined later, \( a_{ij} \in \mathbb{R} \) is a positive static weight corresponding to the edge \( (i, j) \in \mathcal{E} \), \( a_{ij} = 0 \) if \( (i, j) \notin \mathcal{E} \), and \( \hat{x}_i[k] \in \mathbb{R} \) is an actual power output generated by agent \( i \) at time \( k \). Note that the actual power output \( \hat{x}_i[k] \) would be generated by renewable energy resources according to the power generation plan \( x_i[k] \), that is, the disturbance \( w_i[k] \) represents the fluctuations of power output caused by renewable energy resources. We assume that \( \mathbb{E}[w_i[k]] = 0 \), \( \text{Var}[w_i[k]] \leq \psi_i^2 < \infty \), and the disturbance \( w_i[k] \) follows an independent and identically distributed with respect to \( \iota \) and \( k \).

In order to represent the distributed dispatch algorithm (2) in a compact form, we define

\[
L = \text{diag}(A + A^T) \mathbf{1}_N - (A + A^T),
\]

where \( A \) is an adjacency matrix whose \( i,j \)-th element is \( a_{ij} \) if \( (i,j) \in \mathcal{E} \) or 0 otherwise, \( \mathbf{1}_N \) is the \( N \)-dimensional vector whose elements are all 1, and \( \text{diag}(a) \) denotes the diagonal matrix whose diagonal elements correspond to the column vector \( a \). By using the matrix \( L \), the consensus algorithm (2) can be rewritten as

\[
x[k+1] = (I_N - \mu[k] L \psi \Psi) x[k] - \mu[k] L \psi w[k],
\]

where

\[
x[k] = [ x_1[k] \ x_2[k] \cdots \ x_N[k] ]^T \in \mathbb{R}^N,
\]

\[
\psi = \text{diag}([ 1/\psi_1 \ 1/\psi_2 \cdots \ 1/\psi_N ]^T) \in \mathbb{R}^{N \times N}
\]

\[
w[k] = [ w_1[k] \ w_2[k] \cdots \ w_N[k] ]^T \in \mathbb{R}^N.
\]

Let a deviation \( \tilde{x}[k] \in \mathbb{R} \) be

\[
\tilde{x}[k] = x[k] - \beta \psi^{-1} \mathbf{1}_N = x[k] - \frac{\beta \psi^{-1} \mathbf{1}_N}{\| \psi^{-1} \mathbf{1}_N \|^2},
\]

where \( \| a \| \) is the Euclidean norm. We then employ a state coordinate transformation

\[
\begin{bmatrix}
\xi_1[k] \\
\xi_2[k]
\end{bmatrix}
= \begin{bmatrix}
S^T \\
\gamma^T
\end{bmatrix} \psi^{1/2} \tilde{x}[k],
\]

\[
x[k] = \psi^{-1/2} \begin{bmatrix}
S & \gamma
\end{bmatrix} \begin{bmatrix}
\xi_1[k] \\
\xi_2[k]
\end{bmatrix},
\]

where \( \gamma = \psi^{-1/2} \mathbf{1}_N / \| \psi^{-1/2} \mathbf{1}_N \| \) and \( S \in \mathbb{R}^{N \times (N-1)} \) is the orthonormal complement of \( \gamma \). Applying these transformations to the compact form (3), we have

\[
\begin{align*}
\xi_1[k+1] &= \left( I_{N-1} - \mu[k] S^T \psi^{1/2} L \psi^{1/2} S \right) \xi_1[k] - \mu[k] S^T \psi^{1/2} \psi L \psi w[k], \\
\xi_2[k+1] &= \xi_2[k] = \xi_2[k] = \frac{l}{\| \psi^{-1/2} \mathbf{1}_N \|},
\end{align*}
\]

\[
\tilde{\xi}_1[k] = S^T \psi^{1/2} \tilde{x}[k] = \xi_1[k],
\]

\[
\tilde{\xi}_2[k] = \gamma^T \psi^{1/2} \tilde{x}[k] = \xi_2[k] - \frac{l}{\| \psi^{-1/2} \mathbf{1}_N \|} = 0.
\]

Since

\[
\| \psi^{1/2} \tilde{x}[k] \|^2 = (\tilde{x}[k]^T \psi^{1/2} (SS^T + \gamma \gamma^T) \psi^{1/2} \tilde{x}[k])
\]

\[
= \| \tilde{\xi}_1[k] \|^2 + \| \tilde{\xi}_2[k] \|^2
\]

\[
= \| \tilde{\xi}_1[k] \|^2 = \| \xi_1[k] \|^2,
\]

we can evaluate the convergence of \( \tilde{x}[k] \) as that of \( \xi_1[k] \).

### 4 Convergence Analysis

Let us define the transition matrix \( \Phi(k,m) \) as

\[
\Phi(k,m) = \begin{cases}
(I_{N-1} - \mu[k-1] S^T \psi^{1/2} L \psi^{1/2} S) \\
\cdot \\
\cdot \\
\cdot \\
(I_{N-1} - \mu[k-2] S^T \psi^{1/2} L \psi^{1/2} S) \\
\cdots \\
(I_{N-1} - \mu[k-m] S^T \psi^{1/2} L \psi^{1/2} S)
\end{cases}
\]

if \( k > m \)

\[
I_{N-1}
\]

otherwise.

Then, the solution of \( \xi_1[k] \) can be written as

\[
\xi_1[k] = \Phi(k,1) \xi_1[1] - \sum_{m=1}^{k-1} \mu[m] \Phi(k,m+1) S^T \psi^{1/2} L \psi w[m],
\]

Since \( \mathbb{E}[w[k]] = 0 \) for any \( k \), the expectation of \( \| \xi_1[k] \|^2 \) can be evaluated with a deterministic term and a stochastic term, i.e.,

\[
\mathbb{E} \left[ \| \xi_1[k] \|^2 \right]
\]
\[= \|\Phi(k, 1)\xi_1[1]\|^2 + \mathbb{E} \left[ \left( \sum_{m=1}^{k-1} \mu_m \Phi(k, m + 1) S^T \Psi^{1/2} L \Psi w[m] \right)^2 \right] \]. \tag{5}

Let \( \lambda_2 \in \mathbb{R} \) be the second smallest eigenvalue of the matrix \( L \Psi \) and \( \lambda_N \in \mathbb{R} \) be the largest eigenvalue of the matrix \( L \Psi \). This means that \( \|\Psi^{1/2} L \Psi^{1/2}\| = \lambda_N \). To prove main results, we prepare the following lemma \[3\].

**Lemma 1.** The matrix \( L \Psi \) satisfies
\[\|I_{N-1} - \frac{1}{\lambda_N} S^T \Psi^{1/2} L \Psi^{1/2} S\| \leq 1 - \frac{\lambda_2}{\lambda_N}.\]

**4.1 Diminishing Gain**

Let us select the diminishing gain \( \mu_d[k] \) as
\[\mu_d[k] = \frac{1}{\lambda_2 (k_0 + k)}, \quad k_0 \geq \frac{\lambda_N}{\lambda_2} - 1,\]
where \( k_0 \in \mathbb{N} \). Then we have the first main result.

**Theorem 1.** Let the communication gain \( \mu_d[k] \) be \( 1/(\lambda_2 (k_0 + k)) \), where \( k_0 \geq \lambda_N/\lambda_2 - 1 \). Then, for any initial state \( x[1] \), the deviation \( \tilde{x}[k] \) satisfies
\[\lim_{k \to \infty} \mathbb{E} [\|\tilde{x}[k]\|] = 0.\]

**Proof.** From Lemma 1, we obtain
\[\left\| I_{N-1} - \frac{1}{\lambda_N} S^T \Psi^{1/2} L \Psi^{1/2} S \right\| \leq 1 - \frac{1}{k_0 + k},\]
which means that the norm of \( \Phi(k, m) \) satisfies
\[\|\Phi(k, m)\| \leq \frac{k_0 + m - 1}{k_0 + k - 1}.\]
Then we have
\[\|\Phi(k, 1)\xi_1[1]\| \leq \frac{k_0}{k_0 + k - 1} \|\xi_1[1]\|\]
and thus we can evaluate the deterministic term as
\[\lim_{k \to \infty} \|\Phi(k, 1)\xi_1[1]\|^2 \leq \lim_{k \to \infty} \frac{k_0^2}{(k_0 + k - 1)^2} \|\xi_1[1]\|^2 = 0.\] \tag{6}

We can also compute the stochastic term as
\[\mathbb{E} \left[ \left( \sum_{m=1}^{k-1} \mu_d[m] \Phi(k, m + 1) S^T \Psi^{1/2} S \Psi w[m] \right)^2 \right] \leq \mathbb{E} \left[ \left( \sum_{m=1}^{k-1} \mu_d[m] \Phi(k, m + 1) S^T \Psi^{1/2} S \Psi V \right)^2 \right].\]

**4.2 Constant Gain**

Let us choose the constant gain
\[\mu_c[k] = \frac{1}{\lambda_N}.\]
Then we have the second main result.

**Theorem 2.** Let the communication gain \( \mu_c[k] \) be \( 1/\lambda_N \). Then, for any initial state \( x[1] \), the deviation \( \tilde{x}[k] \) satisfies
\[\lim_{k \to \infty} \mathbb{E} [\|\tilde{x}[k]\|] < \infty.\]

**Proof.** Since \( \mu_c[k] = 1/\lambda_N \), we see
\[\Phi(k, m) = \left( I_{N-1} - \frac{1}{\lambda_N} S^T \Psi^{1/2} L \Psi^{1/2} S \right)^{k-m}.\]

From Lemma 1, the norm of \( \Phi(k, m) \) satisfies
\[\|\Phi(k, m)\| \leq \left( 1 - \frac{\lambda_2}{\lambda_N} \right)^{k-m}.\]
Then we have
\[\|\Phi(k, 1)\xi_1[1]\| \leq \left( 1 - \frac{\lambda_2}{\lambda_N} \right)^{k-1} \|\xi_1[1]\|\]
and thus we can evaluate the deterministic term as
\[\lim_{k \to \infty} \|\Phi(k, 1)\xi_1[1]\|^2 \leq \lim_{k \to \infty} \left( 1 - \frac{\lambda_2}{\lambda_N} \right)^{2(k-1)} \|\xi_1[1]\|^2 = 0.\] \tag{7}

We can also compute the stochastic term as
\[\mathbb{E} \left[ \left( \sum_{m=1}^{k-1} \mu_c[m] \Phi(k, m + 1) S^T \Psi^{1/2} L \Psi w[m] \right)^2 \right] \leq \mathbb{E} \left[ \left( \sum_{m=1}^{k-1} \mu_c[m] \Phi(k, m + 1) S^T \Psi^{1/2} S \Psi V \right)^2 \right].\]
\[
\frac{\lambda_2^2}{2\lambda_N\lambda_2 - \lambda_2^2} \text{Tr} (V\Psi),
\]
which is bounded independently of \( k \). Thus we have
\[
\lim_{k \to \infty} \frac{1}{\lambda_2} \left[ \sum_{m=1}^{k-1} \mu_k[m] \Phi(k, m + 1) S^T \psi^{1/2} L\psi w[m] \right]^2 
\leq \frac{\lambda_2^2}{2\lambda_N\lambda_2 - \lambda_2^2} \text{Tr} (V\Psi) < \infty. \tag{9}
\]
With (4), (5), (8), and (9), Theorem 2 holds. \( \square \)

These theorems claim that the proposed distributed algorithm can achieve the desired consensus in mean square or with bounded error variance if we select a suitable communication gain.

Notice also that \( \mu_k[1] = \mu_k[k] \) when we select \( k_0 = \lambda_2/\lambda_N - 1 \) for \( \mu_k[k] \). That is, the diminishing gain \( \mu_k[k] \) is always less than or equal to the constant gain \( \mu_k[k] \).

5 Numerical Examples

In this section, we show a numerical example. We basically followed the settings of [2, 3], where \( N = 4 \). The settings are shown as follows:

\[
\begin{align*}
E &= \{1, 2, 3, 4\}, \\
\Psi &= \text{diag} \left( \begin{bmatrix} 1/120 & 1/150 & 1/200 & 1/280 \end{bmatrix} \right), \\
A &= \begin{bmatrix} a_{12} & a_{21} = 3, & a_{23} = 7, & a_{34} = 4, & a_{43} = 9, \\
x_1[1] &= 100, & x_2[1] &= 120, & x_3[1] &= 89, & x_4[1] &= 230.
\end{bmatrix}
\end{align*}
\]

Note that a desired value of the weighted consensus was
\[
\beta \Psi^{-1} 1 = \begin{bmatrix} 86.24 & 107.80 & 143.73 & 201.23 \end{bmatrix}^T.
\]

The largest eigenvalue \( \lambda_N \) of \( L\Psi \) is 0.2514 and the second smallest eigenvalue \( \lambda_2 \) of \( L\Psi \) is 0.0378. We used the upper bound of the variance for the agents as
\[
v_1^2 = 20, \quad v_2^2 = 40, \quad v_3^2 = 35, \quad v_4^2 = 7.
\]

We selected the communication gain \( \mu_k[k] \) as
\[
\mu_k[k] = \frac{1}{0.0378(6 + k)}.
\]

Figs 1 and 2 depict the behavior of \( x[k] \) and \( \tilde{x}[k] \), respectively. These figures indicate that our proposed algorithm works well.

6 Concluding Remarks

We have considered a multi-agent based load balancing dispatch for power systems with renewable energy. We have derived that a suitable communication gain of the distributed algorithm for the desired consensus in mean square or with bounded error variance.

Acknowledgment: This research was supported by JST CREST Grant Number JPMJCR15K2, Japan.

![Fig. 1: Behavior of each element of the state \( x[k] \)](image1.png)

![Fig. 2: Behavior of each element of the deviation \( \tilde{x}[k] \)](image2.png)

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