The physical mechanism of AdS instability and Holographic Thermalization.

Sang-Jin Sin

Department of Physics, Hanyang Univ. Seoul 133-791, Korea
E-mail: sjsin@hanyang.ac.kr

ABSTRACT: Gravitational falling in AdS has two characteristic properties [1]: i) A thick shell becomes a thin shell. ii) Any shape become spherical. Such focusing character of AdS, for the collapse of dusts, leads to the rapid thermalization mechanism in strongly interacting system. For the collapse of a wave, it explains the cascade of energy to UV through repeated bounces, which has been extensively discussed in recent numerical works. Therefore the focusing is the physical mechanism of instability of AdS. Such sharp contrast between the dust and wave in collapse, together with the experimental observation of rapid thermalization, suggest that the initial condition of created particles in RHIC is in a state with random character rather than a coherent one. Two time scales, one for thermalization and the other for hydro-nization are defined and calculated in terms of the total mass density and energy distribution of the initial particles. We find $t_{th} \sim (1 - c_1/E^2)^{1/2}/T$ so that softer modes thermalize earlier. However, for hydro-nization, $t_{hyd} \sim 1/E^{2/3}T^{1/3}$ therefore harder modes come earlier. We also show that near horizon limit of Dp brane solutions have similar focusing effect which is enough to guarantee the early thermalization.
1 Introduction

The thermodynamics is extremely useful to describe the nature, although a true equilibrated state is very rare. It can be attributed to the fact that thermalization is a very rapid process. Nature seems to know how to arrive at the equilibrium without overshooting, which is more effective if the system is strongly interacting. One example of extremely rapid thermalization\cite{2} is given by Relativistic Heavy Ion Collision (RHIC) experiment: the fireball seems to reach equilibrium in 1fm/c, which is a time for gold ions to pass each other. Certainly this is due to the strongly interacting nature of the quark gluon plasma although the precise mechanism has not been understood, yet. Since understanding this phenomena is beyond perturbative field theory, it is natural to ask if a dual formulation can shed any light on it.

According to the gauge/gravity duality \cite{3}, thermalized state is dual to black hole geometry, and therefore the thermalization process is dual to the black hole formation in the dual picture. Some time ago, in \cite{4}, mapping the entire process of RHIC experiment to the dual gravity language has been tried. Since expansion of the RHIC fireball should be dual to the falling of the matter in the dual gravity, black hole formation from the the collapsing matter seems to be a natural candidate as a dual process of fireball equilibration.

Since there have been many works on this issue, we briefly mention difficulties of existing models. With spherical collapse of scalar wave in global anti-de Ditter space (AdS) \cite{5, 6}, or \cite{7} with homogeneous null hyperplane source in POincare patch, although both of them have mathematical success due to their simplicity, the initial conditions assumed there is too fine tuned to be connected to a real process of thermalization: to an observer outside the shell, the spherical initial configuration is equivalent to preparing an already equilibrated system by the Birkoff theorem. Another useful work is the colliding shock wave model\cite{8}. Its problem is that the initial condition is set BEFORE the collision where
gravity dual is not relevant. The rapid particle creation is the key reason why the fire-ball of RHIC acts as a strongly interacting system: it converts almost all initial kinetic energy into the mass of the created particles such that only 0.1% of kinetic energy play the role of the temperature. The gravity dual is responsible only in this strongly interacting regime which is realized only AFTER the collision. Late time expansion of hydronized QGP was obtained by using the falling horizon model in [9]. There, the goal was the embedding the adiabatic cooling into gravity formalism rather than the mechanism to achieve such quasi-equilibrium configuration. So it is still not clear whether a black hole will be formed starting with generic initial configurations.

In flat space, matter without dissipation mechanism can not make a black hole. Moreover, recent studies on scalar field collapse [10–14] show that even for the spherical shell in AdS space, black hole is formed only after many repeated reflections from the boundary, which is certainly not the dual of ‘thermalization in one passing time’. The natural explanation for the experiment should be such that the black hole is formed in one falling time without any oscillation for any non-spherical/non-homogenous initial configuration in the global-AdS/Poincare-patch.

In the previous paper [1], we addressed this issue using the special property of the AdS space: any geodesic has the same period and therefore particles arrive at the center simultaneously regardless of their initial position if they start from zero velocity. As a result, a shell of dust particles with arbitrary shape becomes spherical as it falls and it forms black hole when the last particle pass the apparent horizon. It takes a time less than one falling time. Another consequence is that thickness of the initial shell becomes thinner as it falls. See figure 1.

In a Poincare patch, focusing phenomena are robust even in the case we include inter-particle interactions and initial velocity in any non-radial direction. At the moment of creation, zero radial velocities should be assumed for the holographic images of 5000 created particles since there is no reason why they should be moving along holographic direction at the moment of creation. We call this as focusing mechanism of dual gravities.

Here, we will first discuss the wave collapse and suggest that focusing is the physical mechanism of instability of AdS. Then, we will discuss more quantitative prediction of the focusing mechanism: we will conceptually distinguish thermalization time and hydronization time and then and calculate them. We will also show that near horizon limit of Dp brane solutions have similar focusing effect, indicating that the focusing mechanism is the universal feature of any gravity dual. At the end, we will summarize by listing about 15 reasons why focusing is the mechanism of the effective thermalization in gravity dual picture.

2 Mechanism of energy cascade to UV in wave collapse.

What will happen to the collapse of wave rather than dust particles? First, one should be notice that it costs energy to localize a wave packet in a small region, which is the origin of the uncertainty principle of wave mechanics. Such dispersive nature is also responsible for the stability of our material world not collapsing down to a neutron star and also for the
stability of boson star [16]. Gravity can confine a wave packet within the Schwartzshild radius $r_s$ only if the initial configuration is thin and spherical enough, equivalently only if $r_s$ is large enough. For generic configuration gravitational potential energy is not enough to provide such localization. Therefore generic initial wave configuration should bounce back after initial collapse and then fall again. Since nothing can escape from the AdS, which is like a box, the same process repeats again and again.

Why a black hole is formed eventually after enough number of bounces? We can understand this if we visualize a shell-like wave configuration as a collection of infinite number of particles connected by springs. Each time the shell falls, the thickness of the shell decreases due to the focusing mechanism of the AdS. When it bounces back, the decreased thickness does not go back to its initial state due to the attractive inter-particle interaction. After enough number of bounces, the shell becomes thin and spherical enough so that entire wave configuration can be inside its Schwarzshild radius.

In momentum space, these geometric progress has an interesting interpretation. Whatever is the initial configuration of the wave packet, as times goes on, the angular distribution will be shifted towards $l = 0$ to become spherical while the radial distribution will be shifted to larger momentum region to make the shell thinner. That is, cascade of amplitudes to UV regime is derived by the focusing mechanism of AdS space. We believe that this is what is happening in recent numerical works [12–14] with initial spherical shell. If one starts with initially non-spherical shell, one would also observe a cascade of the angular distribution towards zero angular momentum as well as the cascade to UV.

Summarizing, in global AdS, the collapse of arbitrary shape of dust shell forms a black
hole at once, while that of wave shell makes black hole after enough number of oscillations. In both cases, focusing mechanism is the underlying mechanism for the eventual black hole formation. We believe that the focusing is the physical mechanism of the instability of AdS \[10, 11\]: If we dump anything to AdS, it is unstable to become a black hole, although the time depends on the nature of the matter.

Since what is observed in heavy ion collision is ‘thermalization at once’, the initial state of fireball created in heavy ion collision should be considered as an incoherent dust-like state rather than a classical wave configuration, which is dual to particles in a condensed state.

3 Focusing in other dual gravities.

Now we come to the one of the main question: Is the focusing mechanism is special and working only for AdS space, or generic to a large class of gravities? If the answer is the former, focusing is not very useful as a mechanism of thermalization, because we do not know whether the gravity dual of the given system is precisely the AdS. Here, we will show that the field theory limit of all Dp branes solutions have focusing mechanism if \( p \leq 5 \).

Particles interacting via gluon exchange are mapped into non-interacting particles moving in the background AdS space. Only the residual non-gluonic interaction should be handled by inter-particle interaction. Therefore we consider only non-interacting particles, moving in a gravitational background given by

\[
\text{d}s^2 = -g_{tt}\text{d}t^2 + g_{rr}\text{d}r^2 + g_{ii}\text{d}x^i\text{d}x^i. \tag{3.1}
\]

The equation of motion is given by the action

\[
S = -m\int \sqrt{-g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu}\text{d}t, \quad \text{with } \dot{x} = \frac{dx}{dt}. \tag{3.2}
\]

We set \( R = 1, c = 1 \). The radial motion can be first integrated to give the energy conservation:

\[
\frac{mg_{tt}}{\sqrt{g_{tt} - g_{rr}\dot{r}^2}} = E. \tag{3.3}
\]

If the motion starts with zero radial velocity from the initial radial position \( r_0 \), then

\[
E = m\sqrt{g_{tt}(r_0)}, \tag{3.4}
\]

which can be used as a dictionary between the conserved energy \( E \) and the initial radial position. Namely we can assign the initial position \( r_0 \) for a particle whose energy is \( E \) such that they are related by eq.(3.4).

Introducing \( v_c = \sqrt{g_{tt}(r_0)/(1 + g_{tt}(r_0))} \), we have \( E = m/\sqrt{1 - v_c^2} \). Therefore \( v_c \) can be interpreted as radial velocity when it arrives at the center. The equation of motion can be formally integrated:

\[
t = \int_r^{r_0} \frac{dr}{\sqrt{g_{tt}(r)/g_{rr}(r)} \cdot (1 - g_{tt}(r)/\epsilon^2)}, \tag{3.5}
\]

where the \( \epsilon = E/m \) is the energy per unit mass.
For the global AdS metric, the period of the motion is $T_{\text{fall}} = \frac{\pi R}{2 c}$, which is independent of the initial position. This is in fact a unique property of AdS, which is not shared by any other metric known to us. If thermalization is really depends on this exact synchronization, the focusing mechanism would not be a universal one and we have a less chance to explain the RHIC experiment in terms of the focusing mechanism.

Fortunately, it turns out that the exact synchronization in finite time is not really necessary for thermalization. For the Poincare patch of the AdS, $g_{tt} = \frac{r^2}{1 - g_{rr}}$ and the falling time is infinite. However, as we can see from the expansion [4],

$$r = \frac{\epsilon}{\sqrt{1 + (\epsilon t)^2}} = \frac{1}{t} - \frac{1}{2 \epsilon^2 t^2} + O(1/t^4).$$

The leading term is independent of initial condition and the subleading terms rapidly vanish as time goes on. Therefore all the particles will be inside the apparent horizon within finite time and we conclude that focusing mechanism still works for the AdS with flat boundary.

This mechanism does not request arrival at the center simultaneously for many particles. Infinite falling time with bounding gravitation potential together with the finite would-be-horizon radius seem to be all we need. To support this conjecture, we now show that above mentioned property holds for near horizon geometry of other $D_p$ branes. Let’s start with $D_4$ case where $g_{tt} = \frac{r^3}{2} = \frac{1}{g_{rr}}$. We can show that

$$r(t) = 4/t^2 - 8a/(\epsilon t)^3 + \cdots,$$

where $a = 2\sqrt{\pi} \Gamma(2/3) / \Gamma(1/6) \simeq .86237$. So, only after $t \to \infty r(t) \to 0$ meaning that falling time is infinite and the leading term is independent of the initial condition. Therefore falling in the $D_4$ is qualitatively the same as that in the Poincare patch of AdS.

For general $D_p$ brane case,

$$r(t) = (\beta/t)^{\beta} + \cdots, \quad \text{with } \beta = \frac{2}{5 - p}, \quad \text{for } p < 5.$$

The difference in the initial condition will decay away with power law. Using the hypergeometric function, we can express the exact solution:

$$t = \frac{1}{\alpha - 1} \left[ \frac{1}{\Gamma(\frac{1}{\alpha - 1})} \sum_{n=0}^{\infty} \frac{1}{(\alpha - 1) n!} (1 - \frac{1}{\alpha} - \frac{1}{\alpha} r_0^\alpha) - \Gamma(\frac{1}{\alpha}) \Gamma(\frac{1}{2}) \frac{1}{\Gamma(\frac{1}{2} - \frac{1}{\alpha})} r_0^{\alpha - 1} \right],$$

where $\alpha = \frac{7 - p}{2}$ and $r_0^\alpha = \epsilon^2$. For $p=5$, we can find a simple solution

$$r(t) = r_0 / \sinh^2(t/2),$$

so that the difference in the initial positions will be exponentially washed out: $\Delta r \sim 4\Delta r_0 \cdot e^{-t}$ for late time. Notice that $r_0 = \epsilon^2$ for $p=5$ with eq.(3.4).

Technically, the origin of the focusing mechanism in these background is the behavior of metric $g_{tt}/g_{rr} \sim r^\alpha$ with $\alpha \geq 1$, which in turn can be attributed to taking the Maldacena limit from the D$p$ geometry: Had we keep the 1 in $H_p = 1 + c/r^{7-p}$, we would not have focusing mechanism. So the focusing mechanism is the property of neck region of original $D_p$ geometry.
4 Conserved Energy v.s Radial position : AdS/CFT dictionary

So far we mainly discussed the radial motion. In the presence of the velocity in the boundary direction, we have difficulty in the global AdS since that will create angular momentum and therefore build up centrifugal barrier, which will forbid falling to form the black hole formation. To avoid this, we examine the effect of the initial horizontal velocity in the Poincare patch where the equation of motion is given by

\[ \frac{mr^2}{\sqrt{r^2(1 - x^2)}} - \frac{\dot{r}^2}{r^2} = E, \quad \frac{m\dot{r}^2}{\sqrt{r^2(1 - x^2)}} - \frac{\dot{r}^2}{r^2} = p, \quad (4.1) \]

At the moment of creation \( \dot{r} = 0 \) although \( \ddot{r} \neq 0 \), so that

\[ r_0 \cdot \frac{m}{\sqrt{1 - v^2}} = E, \quad r_0 \cdot \frac{mv}{\sqrt{1 - v^2}} = p \quad (4.2) \]

The right hand sides of above equations are conserved energy and momentum while the left hand sides (LHS) are product of two: initial bulk height times bulk energy/momentum. If we identify the the conserved quantities as those at the the boundary and the energy and momentum in the LHS as bulk quantity, it is consistent with the prescription of Polchinski and Strassler [17]. According to above relation, we can attribute the boundary energy partly to the bulk energy and partially as bulk initial height such that eq.(4.2) holds. Since the bulk energy \( m/\sqrt{1 - v^2} \) is always bigger than the mass \( m \), \( r_0 \) has maximum value \( E/m := \epsilon \). The question of how much bulk kinetic energy is assigned is matter of choice and in this sense there is a holographic gauge choice. We choose the gauge where \( r_0 \) is maximum and \( v = 0 \). Therefore in our gauge, all the energy of a created particle is attributed to the height of its bulk-image. This is a justification postulated in the previous work [1].

At the boundary, the thermalization is complicated process of strongly interacting particles. But in the bulk, the particles ‘interact’ only with the background metric, namely they are just free falling many-body system. This is the simplicity obtained from the dual gravity formalism:

5 Time for Thermalization and Hydro-nization

How long does it take to thermalize? A natural definition for the thermalization is the time at which the last particle in the process passes the apparent horizon \( r_H \), which is determined by the total energy inside the system:

\[ r_H^4 = \frac{c_2}{V} \sum_i E_i, \quad (5.1) \]

where \( c_2 \) is a constant proportional to the Newton’s constant \( G_N \), \( V \) is the volume of the 3-space and \( E_i \) is the energy of \( i \)-th particle. If the system has two well separated parts in energy distribution, we can model it as two well separated shells. It is easy to prove that radial motion of particle with higher initial position will catch up the lower particle
but never overtakes. Therefore, we can conclude that soft modes thermalize first and then hard modes thermalize on top of the former.

The thermalization time is simply given by:

\[ t_{TH} = \sqrt{\frac{1}{r_H^2} - \frac{1}{r_0^2}} = \frac{1}{\pi T} \cdot \sqrt{1 - \left(\frac{\pi T m}{E^2}\right)^2}. \]  

(5.2)

where \( r_H \), and \( T \) are the radius of the ‘would-be-horizon’ and its corresponding temperature which will be reached by the system after thermalization. \( r_0 \) is the initial height of the particle with highest energy. If we consider the initial energy distribution of particles in the system such that it is not concentrated on specific energy scale, then the highest energy scale is much bigger than the temperature. If \( r_H << r_0 \) the thermalization time is given by the inverse temperature,

\[ t_{Th} \simeq \frac{1}{r_H} = \frac{1}{\pi T}, \]  

(5.3)

Otherwise, thermalization time should be less than this.

Now look at the falling of two particles whose initial heights are order of magnitude different. Although there is no passing, particles with higher radial position almost catch up the lower particles within half of the thermalization time. Such phenomena strongly suggests that approximate isotropization happens much before the actual thermalization. See figure 2.

To an external observer, after rapid process of isotropization, not much change will happen although the shell is still falling. Only residual process of isotropization is under progress. We can identify such stage as hydronized state. This is the regime where the metric is still time dependent but hydrodynamics works effectively. One can quantify the hydro-nization time \( t_{hy} \) as follows: Take a shell of thickness \( \Delta r_0 := r_{0,\text{max}} - r_{0,\text{min}} \) which is, say, 10% of its average height \( r_0 \). This amounts to taking the particles whose initial energy distribution width is \( \Delta E/E = 10\% \) around its average value \( E \). We can say that the
system is hydro-nized after time \( t_{hy} \), if the maximum difference of radial positions in the shell is within 10% of the horizon radius, that is if \( \Delta r(t_{hy})/r_H \leq 0.1 \) which is equivalent to \( \Delta E(t_{hy})/T \leq 0.1 \), which can be inverted to give

\[
t_{hy} = \frac{1}{r_0} \left( \frac{r_0}{r_H} \right)^{2/3} - 1 = \frac{m}{E} \sqrt{\frac{E}{\pi T m}}^{2/3} - 1.
\]  

(5.4)

When initial energy per mass of the shell is much bigger than the final temperature, we can get

\[
t_{hy} = \left( \frac{m}{\pi^2} \right)^{1/3} \frac{E}{E^{1/3} T^{2/3}}.
\]  

(5.5)

It says that the hydro-nization is faster for higher initial energy. Therefore hard modes hydro-nize first and then soft modes follows.

6 Summary and Conclusion

In this paper we answer to the question what is the physical mechanism for the instability of the AdS, namely why any thing dumped into AdS cause black hole formation. We also demonstrated how to use it to characterize the thermalization and hydro-nization. Falling particles arrive at the center simultaneously independent of the initial height, which causes two remarkable effects: It makes a shell of arbitrary shape spherical and makes thick shell thin. We argued that these effects are not only the mechanism for the formation of the black hole in one falling time when applied to the dust particles, but also are the physical mechanism of the cascade to UV modes and cascade to lower angular momentum mode. Focusing in AdS is the underlying mechanism of the instability of AdS. Two distinguished time scales for hydro-nization and thermalization are defined and calculated. The former is the time for black hole formation and the latter is time for isotropization.

Here we summarize by listing the evidences why the FOCUSING and its two consequences are the mechanism of rapid thermalization/hydronization.

1. Falling in bulk is necessary for the fireball to expand at boundary before thermalization. These two are dual to each other.

2. It enforces dust shell to form the black hole within one falling time.

3. For the wave collapse, black hole forms only after enough number of oscillations [10–13]: We can understand the bouncing mechanism from the wave nature: as the field configuration collapses gravitationally, uncertainty principle activates the kinetic term for the localized wave packet, which generates pressure and causes a bounce.

4. The eventual formation of black hole for the collapsing wave can be understood: each time it fall, it becomes thinner, which enforces cascade of energy to UV. When it bounces back, attractive interaction partially preserve two consequences of the falling.
5. Sharp contrast between dust and wave enforces us to decide the nature of the initial condition after collision. The created particles should be to be non-coherent dust like state rather than a wave-like state which is dual to highly correlated many body system.

6. We only need to consider non-interacting particles since gluon exchange is transformed into geometric background.

7. In poincare patch, initial non-radial velocity does not destroy the focusing effect.

8. Precise dictionary exist between bulk radial position and boundary energy such that we need to consider only radial falling.

9. Attractive residual interaction does not destroy the focusing effect in Poincare patch.

10. It allows quantitative discussion for time scales of Thermalization as black hole formation time.

11. It allows quantitative discussion for time scales of hydronization as isotropization time.

12. Focusing generates entropy creation and ir-reversibility by the horizon formation.

13. The mechanism working for global AdS (the exact syncronization) is a unique property of AdS and it is not shared by other geometry.

14. The mechanism working for Poincare patch of AdS is universal to all gravity duals, that is, infinite falling time of near horizon geometry (of Dp branes) allows all known gravity dual to have focusing effect.

15. Assuming a spherical symmetry or setting initial condition before the particle creation can be a serious problem of a model for RHIC collision.

It would be very interesting if the intuition obtained in this paper can be utilized to understand the cascade [10–15] and inverse-cascade [18–21] of the energy of the holographic fluids. While the cascade of 3+1 dimension looks natural from our point of view, for inverse cascade of 2+1 dimension seems to request other idea. Also it is necessary to examine other backgrounds to see the universality of the focus mechanism for dual gravity.

In this paper we restricted ourselves in the extremal backgrounds. It would be interesting to consider the non-extremal backgrounds. For confining backgrounds, the geometry cap off in the IR region and we have a natural IR cut-off. In such case, some particles will bounce from the core boundary before others pass the horizon radius. Therefore naively it is expected to be impossible to reach thermalization at once. However, numerical experiment shows that confining metric for D3 has strong focusing mechanism: The falling time from the initial height to the core boundary is almost the same and saturates to a fixed value as its initial height increases. It may not form a black brane but may give the hydronization very quickly. Other possibility is that if the total mass or mass density
is large enough such that its apparent horizon radius is larger than the core radius \( r_{KK} \), falling particles might form a black brane after enough number of bounces. If the particles are falling in the background of black hole metric, there is no question of thermalization. But it is still interesting if non-passing property still holds in this background. We leave studying these issues to future works.

Acknowledgments

I’d like to thanks Y.Seo, S.Seki, I. Takaaki, Y. Zhou for discussions and H.Bae for drawing figures. I also appreciate questions and comments of S. Das, J.Erdmenger, G. Horowitz, G. Policastro, G. Semenoff and T. Takayanagi. I’d like to thank APCTP, BIRS, MPI, Newton institute and YITP for the hospitality during the workshops on holography. This work was supported by Mid-career Researcher Program through NRF grant No. NRF-2013R1A2A2A05004846. It is also supported by the NRF grant through the SRC program CQUeST with grant number 2005-0049409.

References

[1] E. Oh and S. -J. Sin, arXiv:1302.1277 [hep-th].
[2] U. W. Heinz and P. F. Kolb, Nucl. Phys. A 702, 269 (2002) [hep-ph/0111075].
[3] J. M. Maldacena, The Large N limit of superconformal field theories and supergravity, Adv. Theor. Math. Phys. 2 (1998) 231–252, [hep-th/9711200].
[4] E. Shuryak, S. -J. Sin and I. Zahed, J. Korean Phys. Soc. 50, 384 (2007) [hep-th/0511199].
[5] U. H. Danielsson, E. Keski-Vakkuri and M. Kruczenski, JHEP 0002, 039 (2000) [hep-th/9912209].
[6] V. Balasubramanian, A. Beramonti, J. de Boer, N. Copland, B. Craps, E. Keski-Vakkuri, B. Muller and A. Schafer et al., Phys. Rev. D 84, 026010 (2011) [arXiv:1103.2683 [hep-th]].
[7] S. Bhattacharyya and S. Minwalla, [arXiv:0904.0464 [hep-th]].
[8] P. M. Chesler and L. G. Yaffe, Phys. Rev. Lett. 99, 152001 (2007) [arXiv:0706.0368 [hep-th]].
[9] R. A. Janik and R. B. Peschanski, Phys. Rev. D 73, 045013 (2006) [hep-th/0512162]. ; M. P. Heller, R. A. Janik and P. Witaszczycy, Phys. Rev. Lett. 108, 201602 (2012) [arXiv:1103.3452 [hep-th]]. ; S. Nakamura and S. -J. Sin, JHEP 0609, 020 (2006) [hep-th/0607123]. ; S. -J. Sin, S. Nakamura and S. P. Kim, JHEP 0612, 075 (2006) [hep-th/0610113].
[10] P. Bizon and A. Rostworowski, Phys. Rev. Lett. 107, 031102 (2011) [arXiv:1104.3702 [gr-qc]].
[11] O. J. C. Dias, G. T. Horowitz and J. E. Santos, Class. Quant. Grav. 29, 194002 (2012) [arXiv:1109.1825 [hep-th]].
[12] D. Garfinkle and L. A. Pando Zayas, Phys. Rev. D 84, 066006 (2011) [arXiv:1106.2339 [hep-th]]; D. Garfinkle, L. A. Pando Zayas and D. Reichmann, JHEP 1202, 119 (2012) [arXiv:1110.5823 [hep-th]].
[13] A. Buchel, L. Lehner and S. L. Liebling, Phys. Rev. D 86, 123011 (2012) [arXiv:1210.0890 [gr-qc]].

[14] H. Bantilan, F. Pretorius and S. S. Gubser, Phys. Rev. D 85, 084038 (2012) [arXiv:1201.2132 [hep-th]].

[15] A. Adams, P. M. Chesler and H. Liu, arXiv:1212.0281 [hep-th].

[16] S. -J. Sin, Phys. Rev. D 50, 3650 (1994) [hep-ph/9205208].

[17] J. Polchinski and M. J. Strassler, Phys. Rev. Lett. 88, 031601 (2002) [hep-th/0109174].

[18] F. Carrasco, L. Lehner, R. C. Myers, O. Reula and A. Singh, Phys. Rev. D 86, 126006 (2012) [arXiv:1210.6702 [hep-th]].

[19] A. Adams, P. M. Chesler and H. Liu, arXiv:1307.7267 [hep-th].

[20] S. R. Green, F. Carrasco and L. Lehner, arXiv:1309.7940 [hep-th].

[21] C.-k. Chan, D. Mitra, and A. Brandenburg. Dynamics of saturated energy condensation in two-dimensional turbulence. Phys. Rev. E, 85(3):036315, March 2012.