A Study on Dynamic Characteristics of Satellite Antenna System considering 3D Revolute Clearance Joint

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Clearances in the joints of real mechanisms are unavoidable due to assemblage, manufacturing errors, and wear. The dual-axis driving and positioning mechanism is one kind of space actuating mechanism for satellite antenna to implement precise guidance and positioning. However, in dynamics analysis and control of the satellite antenna system, it is usually assumed that the revolute joint in the satellite antenna system is perfect without clearances or imperfect with planar radial clearance. However, the axial clearance in an imperfect revolute joint is always ignored. In this work, the revolute joint is considered as a 3D spatial clearance joint with both the radial and axial clearances. A methodology for modeling the 3D revolute joint with clearances and its application in satellite antenna system is presented. The dynamics modeling and analysis of the satellite antenna system are investigated considering the 3D revolute clearance joint. Firstly, the mathematical model of the 3D revolute clearance joint is established, and the definitions of the radial and axial clearance are presented. Then, the potential contact modes, contact conditions, and contact detection of the 3D revolute clearance joint are analyzed. Further, the normal and tangential contact force models are established to describe the contact phenomenon and determine the contact forces in the 3D revolute clearance joint. Finally, a satellite antenna system considering the 3D revolute clearance joint with spatial motion is presented as the application example. Different case studies are presented to discuss the effects of the 3D revolute clearance joint. The results indicate that the 3D revolute clearance joint will lead to more severe effects on the dynamic characteristics of the satellite antenna system. Therefore, the effects of axial clearance on the satellite antenna system cannot be ignored in dynamics analysis and design of the satellite antenna system.

1. Introduction

The dual-axis driving and positioning mechanism is one kind of actuating mechanism for a satellite antenna to meet precise pointing requirements. Generally, in dynamics analysis and control of the satellite antenna system, it is usually assumed that the revolute joint in the satellite antenna system is perfect without clearances or imperfect with planar radial clearances. The axial clearance in the revolute joint is always ignored [1–5]. However, clearances always exist in a real joint due to due to assemblage, manufacturing errors, and wear. Clearances will induce contact and impact in joints, which cause vibration and affect the dynamic performances of the real mechanical systems [6–13]. The effects of clearances on dynamic responses of mechanisms have been studied by many researchers [14–22].

Flores [23] presented a dynamics analysis of a planar linkage mechanism with clearances considering different parameters, such as clearance size and driving speed. Zhao and Bai [24] studied the dynamic characteristics of a space robot manipulator with one planar revolute clearance joint. The nonlinear equivalent spring-damper model is established for the normal contact model in joint clearance. The friction effect is considered using the Coulomb friction model. Erkaya and Uzmay [25, 26] presented a study on decreasing the deviations arising from a clearance joint in planar linkage mechanisms by a neural network-genetic algorithm procedure. Muvengei et al. [27] investigated the
dynamics and motion modes of a slider-crank mechanism with two planar revolute clearance joints. Bai and Sun [28] studied the dynamic responses of a planar mechanism system including three planar revolute clearance joints. Different contact modes caused by the multiclearance joints were investigated. Wang et al. [29] presented a nonpenetration approach of frictional contact analysis for modeling revolute clearance joints of planar rigid multibody systems. Zhang and Zhang [30] proposed a method to minimize the influence of revolute joint clearance of a redundantly actuated mechanism. A planar 3 DOF redundantly actuated 4RRR mechanism with 8 clearance joints was applied as an illustration. Salahshoor et al. [31] studied the effect of joint stiffness on the vibration behavior of a typical slider-crank mechanism with a flexible component and joint clearances. Wang et al. [32] studied the dynamic responses of planar multibody systems with dry revolute clearance joint considering the radial clearance via numerical and experimental approaches. Tan et al. [33] investigated the effects of friction on the dynamic behavior of a planar crank-slider mechanism considering a revolute joint with radial clearance using the LuGre friction model. Chen et al. [34] analyzed the effect of multiple clearances and different friction models on the dynamic behavior of a planar multi-DOF mechanism. The 2 DOF nine-bar planar mechanism was used as the application example. Li et al. [35] investigated the dynamic behavior of a planar rigid-flexible coupling solar array system considering joint clearance numerically. A typical solar array model was used as the application example. Wang and Wang [36] studied the dynamics model of a 4-SPS/PS parallel mechanism with a flexible actuated rod and clearance spherical joint. Pi and Zhang [37] presented a study of dynamics analysis of the classical planar slider-crank mechanism with multiple revolute clearance joints. A general multiple patch-based revolute clearance joint model was proposed. Amiri et al. [38] proposed a control scheme to restrain the clearance and maintain a more stable behavior of planar mechanisms. The approach was based on using a tuned mass damper to reduce the effects of clearances in mechanisms for the passive control purpose. Zan et al. [39] presented a unified motion reliability analysis method for general planar parallel manipulators with interval clearance variables of revolute and prismatic joints. Two typical types of PPMs, the 3RRR PPM and 3PRR PPM, were analyzed as examples. Guo et al. [40] presented a dynamics model to investigate the position secondary motion considering all associated joint clearances. Chen et al. [41] proposed a general methodology for modeling and evaluating the dynamic characteristics of planar multibody systems considering the revolute clearance joints and performed the relevant experimental verification. Erkaya [42] studied the effects of planar clearance on motion accuracy of a six DOF robot system. Different scenarios for clearance values and working periods were performed to fulfill the required motion task of the robot.

Besides, many studies also have been focused on the topics of mechanism with clearance joints considering parameter uncertainty [43–47], joint wear [48, 49], design optimization [50], joint lubrication [51, 52], flexible body [53, 54], nonlinear analysis [55], joint friction [56, 57], and other topics [58–60]. All the researches indicated that clearance leads to significant effects on dynamic response of mechanism systems. Further, based on the studies of mechanisms with a planar revolute clearance joint, recently, researchers are focused on 3D revolute clearance joints [61–65]. Flores et al. [61] presented a technique for assessing the influences of radial clearance of spatial revolute joints on the kinematics and dynamics of multibody systems. Four different possible motion scenarios of the journal relative to the bearing were considered. However, the axial clearance was not considered in the revolute joint. Bruttì et al. [62] presented a study of the mechanism with one 3D revolute joint considering 4 contact configurations. More recently, Yan et al. [63], Marques et al. [64], and Isaac et al. [65] also presented studies of mechanisms with a 3D revolute clearance joint. These researches indicated that there will be more contact modes in a 3D revolute clearance joint.

In fact, due to the manufacturing and assembling errors, axial clearances always exist in revolute joints, which have been less considered. Most of the previous studies focused on the dynamic responses of mechanisms with planar radial clearances, while axial clearances will result in relative motion along the axial direction between the journal and the bearing of a 3D revolute clearance joint. Correspondingly, it will lead to a more complex contact phenomenon when both the radial and axial clearances are considered. Also, for a highly precise space mechanism with spatial motion, the effects of axial clearance in a revolute joint cannot be ignored.

The dynamics model and simulation of mechanisms considering only radial clearance were widely presented in the literature. Thus, the objective of this work is to study the dynamic characteristics of the space mechanism system considering both the radial and axial clearances in a 3D revolute clearance joint. A methodology for modeling the 3D revolute joint with clearances and its application in satellite antenna system is presented. The contribution of this paper can be summarized as follows: (1) The revolute joint considered in this paper is a spatial joint with both the radial and axial clearances. The mathematic model of the 3D revolute clearance joint is established, and different contact scenarios are presented and discussed. (2) The 3D revolute clearance joint model presented in this work is applied to a high-precision mechanism with spatial motion on spacecraft. The effect of the 3D clearance joint on dynamic performances of a satellite antenna is investigated. Different case studies are presented to discuss the effects of axial clearance. (3) This work develops the model of the revolute clearance joint, and the 3D revolute clearance is more in line with the actual situation of the real revolute joint. Therefore, the three-dimensional methodology of the clearance revolute joint proposed in this paper improves the model of the revolute clearance joint and leads to a more accurate result.

To this end, different case studies are presented to discuss the effects of the 3D revolute clearance joint. Firstly, the mathematic model of the 3D revolute clearance joint is established considering the radial and axial clearances. The definitions of the radial and axial clearances are presented. Then, the potential contact scenarios, contact conditions,
and contact detection of the 3D revolute clearance joint are analyzed. The potential contact scenarios can reveal the different contact phenomena in the 3D revolute clearance joint. The minimum distances between the journal and the bearing in both the axial and radial directions are obtained to determine the contact conditions and detections. Further, the normal and tangential contact force models are established to describe the contact phenomenon and determine the contact forces in the revolute clearance joint. Finally, a satellite antenna system with spatial motion is presented as the demonstrative application example. The dynamic characteristics of the satellite antenna system are investigated considering both the radial and axial clearances in a 3D revolute clearance joint.

2. Model of Revolute Joint with Radial and Axial Clearances

2.1. Definition of Ideal 3D Revolute Joint without Clearances. The 3D revolute joint without clearance between bodies \( i \) and \( j \) is shown in Figure 1. Any point on the axis of the revolute joint has constant coordinates in both local coordinate systems. \( P_0 \) is an arbitrary point on the joint axis. Two other points, \( P_i \) on body \( i \) and \( P_j \) on body \( j \), are also chosen as arbitrary points on the joint axis. It is clear that vectors \( s_i \) and \( s_j \) must remain parallel. Therefore, there are five constraint equations for a 3D revolute joint without clearance:

\[
\begin{align*}
\mathbf{r}_i + A_i s_i - s_j - A_j s_j &= 0, \\
\mathbf{s}_i \times \mathbf{s}_j &= \mathbf{s}_i, \\
\end{align*}
\]

where \( A_i \) and \( A_j \) are the transformation matrices that define the orientation of bodies \( i \) and \( j \), respectively. \( \mathbf{s}_i \) is the skew-symmetric matrix associated with the vector \( \mathbf{s}_i \).

Thus, the revolute joint without clearances is described by a set of holonomic algebraic constraints. There is only one relative degree of freedom (DOF) between two bodies connected by a 3D ideal revolute joint without clearance.

2.2. Definition of 3D Revolute Joint with Clearances. Figure 2 shows the configuration of a 3D revolute joint with both the radial and axial clearances. Comparing with the ideal 3D revolute joint without clearances, although a revolute joint with clearances does not restrict any degrees of freedom, it limits the journal to move within the bearing. When both the radial and axial clearances are presented in a 3D revolute joint, the dynamics of the clearance joint are controlled by contact forces applied on the journal and bearing. Therefore, in this work, the mechanical bodies connected by the 3D revolute clearance joint are modelled as contact rigid bodies. And consequently, contact-impact forces control the dynamics of the clearance joint. Therefore, the constraints of the revolute clearance joint are modelled as contact force constraints.

The components of the 3D revolute clearance joint are idealized as two cylinders, where the journal is considered as a cylinder with an end flange. According to the geometric description, as shown in Figure 1(a), the axial clearance \( C_a \) and the radial clearance \( C_r \) between the journal and the bearing are defined as follows:

\[
\begin{align*}
C_a &= \frac{L_j - L_i}{2}, \\
C_r &= R_i - R_j,
\end{align*}
\]

where \( L_j \) and \( R_j \) are the length and the radius of the journal, respectively. \( L_i \) and \( R_i \) are the length and the radius of the bearing, respectively.

In Figure 2(b), two local coordinate systems \( O_i-x_iz_i \) and \( O_j-x_jy_jz_j \) are attached to the journal and bearing, respectively. Axis \( z_i \) and axis \( z_j \) are along to the central axes of the journal and bearing, respectively. \( O_i \) is the geometric center of the journal, and \( O_j \) is the center of the end face circle of the bearing. Additionally, a global coordinate system \( O-XYZ \) is fixed to the ground. Initially, the three coordinate systems keep the same orientation. Therefore, the relative motion between the journal and the bearing is equivalent to the relative motion between the two local coordinate systems \( O_i-x_iz_i \) and \( O_j-x_jy_jz_j \), which are fixed to the journal and bearing, respectively.

2.3. Contact Scenarios in 3D Revolute Clearance Joint. Considering that both the axial and radial clearances exist in such a 3D revolute joint, the contacts of the journal and bearing in the axial and radial directions are independent. Therefore, it may contact in the axial direction, radial direction, or both directions. In terms merely of the radial clearance condition, four potential contact scenarios in the radial direction can be recognized, that is, no contact, one-point contact, two-point contact, and line contact. Similarly, when considering the axial clearance separately, there are also four potential contact scenarios in the axial direction, that is, no contact, one-point contact, two-point contact, and surface contact.
Furthermore, 13 potential contact scenarios, as shown in Figure 3, can be found when both the axial and radial clearances are considered in a 3D revolute clearance joint. It should be noted that the joint’s behavior is the function of these 13 scenarios which depends on joint geometry and the dynamics configuration of the system. Thus, some of these potential cases can be eliminated and not further happen.

2.4. Contact Conditions. The potential contact points on the journal and bearing are shown in Figures 4 and 5, which are simplified to calculate the minimum radial and axial distances, respectively [63].

From Figure 4, it can be observed that the potential contact points in the radial direction must be on the two circles of the bearing end face. The centers of the end faces of the journal are point \( O_i \) and point \( O'_i \), respectively. The vector \( \mathbf{d}'_i \) with orientation from \( O_j \) to \( O_i \) connects the origins of the local coordinate systems \( O_j x'_j y'_j z'_j \) and \( O_i x y z \). \( \mathbf{d}'_i \) denotes the projection of \( \mathbf{d}'_i \) on the cross section of the journal. The coordinates of points \( O_i, O'_i, O_j, M \), and \( N \) at the global coordinate systems at time \( t \) are defined as \( O_i(t) = (x_i, y_i, z_i)^T, O'_i(t) = (x'_i, y'_i, z'_i)^T, O_j(t) = (x_j, y_j, z_j)^T, M(t) = (x_M, y_M, z_M)^T, \) and \( N(t) = (x_N, y_N, z_N)^T. \) Further, through the center of the section \( O_i \), a vector \( \mathbf{d}'_i \) perpendicular to the vector \( \mathbf{d}'_i \) is made. Then, the tangent plane \( \tau_i \) of the journal flank is acquired, which is parallel to the vector \( \mathbf{d}'_i \). The tangent plane \( \tau_i \) connects the origins of the local coordinate systems \( O_j x'_j y'_j z'_j \) and \( O_i x y z \). The intersection point is defined as point \( P \). The tangent plane \( \tau_2 \) of the end face circle of the bearing is parallel to the vector \( \mathbf{d}'_i \) and the tangent point is defined as \( Q \). Therefore, the tangent plane \( \tau_2 \) is parallel to the tangent plane \( \tau_2 \). Further, it can be found that point \( Q \) is the potential contact point. Obviously, the radial minimum distance between the journal and the bearing at the left side equals the distance between the planes \( \tau_1 \) and \( \tau_2 \) [63, 66].

Therefore, the calculation of the minimum distance in the radial direction can be expressed as follows:

\[
\delta_r = R_i \cdot \cos \theta - R_j - |\mathbf{d}'_i|,
\]

where \( \delta_r \) represents the minimum distance between the journal and the bearing in the radial direction. \( \theta \) is the angle between lines \( O_iQ \) and \( O_jP \), as shown in Figure 4.

\[
\mathbf{d}'_i = O_i - O'_i,
\]

\[
\mathbf{d}'_j = O_j - O',
\]

\[
|\mathbf{d}'_i| = \sqrt{\left(\mathbf{d}'_i^T\right)^2}.
\]

In particular, in the case where the axes of the journal and bearing are parallel to each other, the angle \( \theta \) becomes zero. Then, (3) can be expressed as

\[
\delta_r = R_i - R_j - |\mathbf{d}'_i|.
\]

Further, the contact condition of the journal and bearing in the radial direction can be expressed as

\[
\begin{align*}
\delta_r &> 0, & \text{free motion in radial direction,} \\
\delta_r &\leq 0, & \text{contact and impact in radial direction.}
\end{align*}
\]

Similarly, the potential contact point on the journal and bearing along the axial direction is depicted in Figure 5, which is employed to calculate the minimum axial distance between the journal and bearing. It can be seen that axial contact occurs between the end flange of the journal and the end face of the bearing. It should be noted that the wall thickness of the actual bearing should be taken into account.
In order to calculate the minimum radial distance, plane \( \tau_2 \) is tangent to the left end face circle of the outer surface of the bearing at point \( Q' \), which is the potential contact point for calculating the minimum axial distance. Further, it can be found that the distance between the point \( Q' \) and the end flange is the minimum distance along the axial direction, which can be expressed as

\[
\delta_a = L_{ij} - a - b - \left| d''_z \right| - R_{m} \cdot \sin \theta,
\]  

where \( \delta_a \) represents the minimum distance between the journal and bearing in the axial direction. \( a \) is the norm of vector \( d''_z \). \( b \) is the projection of \( R_{m} \) on the journal axis, as shown in Figure 5.

Therefore, the contact condition in the axial direction can be expressed as

\[
\delta_a > 0, \quad \text{free motion in axial direction},
\]

\[
\delta_a \leq 0, \quad \text{contact and impact in axial direction}.
\]
2.5. Contact Detection. Contact detection is one of the most important aspects in contact analysis. Since the contact-impact phenomena in the radial and axial directions are independent in the 3D revolute clearance joint, a separate condition is employed to deal with the contact detection in each direction. Based on the minimum radial and axial distances, the contact detection between the journal and bearing can be judged according to the positive and negative values of the distances. Theoretically, the radial or axial distance is equal to zero at the switching instantaneous moment, but it cannot be accurately found in the numerical simulation. Therefore, in numerical simulation, the sign of the minimum distance could reveal the change of contact status. When the minimum distance changes sign from time $t$ to $t + \Delta t$, contact must occur and the contact can be detected by

\[ \delta_n(t) \cdot \delta_n(t + \Delta t) \leq 0, \quad (9) \]
\[ \delta_r(t) \cdot \delta_r(t + \Delta t) \leq 0. \quad (10) \]

When at least one of the equations is satisfied, contact will occur during $t$ to $t + \Delta t$. It should be noted that the contact could occur in the radial and axial directions simultaneously.

Therefore, contact is detected when at least one of Equations (9) and (10) is verified. An alternative way to determine the presence of contact for multiple time-step size integrators is using the characteristics of the integration algorithm selected. The step size is proposed to reach smaller values to keep the integration tolerance error. Thus, the impact velocity and the direction of the contact can be predicted in the overall system motion.

3. Contact Force Models

Usually, there will always be contact/impact in a real joint with clearances in mechanisms, which includes normal contact forces and tangential friction forces simultaneously. Tian et al. [67] presented a comprehensive survey of dynamics of multibody mechanical systems with clearance or imperfect joints, in which the contact force model used for modeling the clearance joint is discussed. Therefore, to evaluate the contact forces efficiently between the bearing and the journal for a revolute joint with clearance, special attention must be given to the numerical description of the contact force model [68–73]. There have been several normal contact force models used to determine the contact forces in a clearance joint.

In this work, the Lankarani and Nikravesh [72] contact force model is used to describe the normal contact of clearance joint, and it is expressed as

\[ F_n = K\delta^n \left[ 1 + \frac{3(1 - c_e^2)\dot{\delta}}{4\delta^{(-)}_e} \right], \quad (11) \]

where $\delta$ is the deformation, $\dot{\delta}$ is the relative deformation velocity, $c_e$ is the coefficient of restitution, and $\delta^{(-)}_e$ is the initial normal relative velocity of the impact point. $K$ is the contact stiffness coefficient and it is obtained from

\[ K = \frac{4}{3\pi(\sigma_i + \sigma_j)} \left[ \frac{R_i R_j}{R_i - R_j} \right]^{1/2}, \]
\[ \sigma_i = \frac{1 - \nu_i^2}{\pi E_i}, \]
\[ \sigma_j = \frac{1 - \nu_j^2}{\pi E_j}, \quad (12) \]

where $\nu$ and $E$ are the Poisson ratio and Young modulus, respectively. $R_i$ and $R_j$ are radii of the joint elements.

The tangential contact of the clearance joint is represented using the friction force model. Here, a modified Coulomb friction model with dynamic friction coefficient is used for the tangential contact of the clearance joint, which can avoid numerical difficulties [5, 9, 14]. The expression of the modified Coulomb friction model is shown as
\[ F_t = -\mu(v_t) F_n \frac{V_t}{|V_t|}, \] (13)

where the dynamic friction coefficient \( \mu(v_t) \) is a function of tangential velocity and it is given as

\[
\mu(v_t) = \begin{cases} 
-\mu_d \text{ sign } (v_t), & \text{for } |v_t| > v_{cr}, \\
\mu_s + \mu_d \left( \frac{|v_t| - v_{cr}}{v_{cr} - v_s} \right)^2 - \left( 3 - \frac{|v_t| - v_{cr}}{v_{cr} - v_s} \right) \text{ sign } (v_t), & \text{for } v_s \leq |v_t| \leq v_{cr}, \\
\mu_s - 2\mu_c \left( \frac{|v_t| + v_s}{2v_s} \right)^2 \left( 3 - \frac{|v_t| + v_s}{v_{cr}} \right), & \text{for } |v_t| < v_s,
\end{cases}
\] (14)

where \( v_t \) is the relative sliding velocity. \( \mu_d \) and \( \mu_s \) are dynamic and static friction coefficients, respectively. \( v_s \) and \( v_{cr} \) are the given bounds for the tangential critical velocity.

### 4. Case Study and Results

#### 4.1. Properties of the Satellite Antenna System

In this section, a space antenna system with spatial motion is used as the numerical example to investigate the effects of 3D revolute clearance joints on the dynamic responses of the satellite antenna system. Most of the previous studies focused on the dynamic responses of mechanisms with planar radial clearances and ignored the axial clearance. For a highly precise space mechanism with spatial motion, the effects of axial clearance in a revolute joint cannot be ignored. Therefore, first, we considered the case of the 3D revolute joint with radial clearance only. Second, we added the axial clearance to the 3D revolute joint with radial clearance. Then, we compared the simulation results for the case without axial clearance and the case with axial clearance to investigate the effects of axial clearance. Further, the effects of axial clearance size on dynamic characteristics of the space mechanism were discussed.

The main structure of the satellite antenna system consists of the spacecraft body, two shafts, two revolute joints, and a flexible antenna reflector, as shown in Figure 6. The
4.2. Effects of the 3D Revolute Clearance Joints on the Satellite Antenna System

4.2.1. Effects of 3D Revolute Clearance Joint. In this section, three case studies, as listed in Table 3, are presented with different initial configurations to investigate the effects of the 3D revolute clearance joint on the dynamic performances of the satellite antenna.

In the first case study, the revolute joint is considered as an ideal joint, that is, there are no clearances in the revolute joint. In the second case study, there is only radial clearance considered in the 3D revolute clearance joint and there is no axial clearance. In the third case, both the radial and axial clearances exist in the 3D revolute clearance joint. The effects of the axial clearance are investigated.

The simulation results are presented as Figures 7–9, which show the angle displacement, angular velocity, and angular acceleration of the satellite antenna system. Figure 10 presents the contact forces in the revolute clearance joint.

Figure 7 shows that the clearance will lead to deviation of the motion angle compared with that of the ideal case. When the revolute joint is considered as the 3D revolute clearance joint considering both the radial and axial clearances, the deviations in the motion profile of the antenna are increased. Thus, it decreases the motion accuracy of the satellite antenna when the axial clearance is also considered in the 3D revolute clearance joint. Figure 8 shows that the effects of clearances on angular velocity are much more obvious. The angular velocity is shaky and deviates from the ideal case without clearances. When both the radial and axis clearances are considered in the 3D revolute joint, the deviations of the antenna motion are bigger than that of only the radial clearance considered in the 3D revolute joint. Figure 9 shows that, when both the radial and axial clearances are considered in the 3D revolute clearance joint, the shake of the satellite antenna is extraordinarily obvious. The shaky peaks of the

| Cases          | Description                                      | Clearance size (mm)   |
|---------------|--------------------------------------------------|-----------------------|
| Case 1        | Ideal revolute joint without clearance           | $C_r = C_a = c = 0$   |
| Case 2        | 3D revolute joint with radial clearance          | $C_r = 0.2, C_a = 0$  |
| Case 3        | 3D revolute joint with both radial and axial clearances | $C_r = C_a = c = 0.2$ |

Table 3: Different case studies of 3D revolute clearance joint.
The angular acceleration of the satellite antenna increase sharply compared with those of only the radial clearance considered in the revolute joint. The reasons are that clearances in the joint lead to contact and impact forces in the clearance joint for the spatial motion of the satellite antenna, as shown in Figure 10. It clearly shows that clearances lead to impulse type contact forces and the contact forces are high-frequency shaky with high peaks. Comparing with Figures 10(a)–10(c) shows that the existence of the axial clearance in the 3D revolute clearance joint leads to more severe effects, especially in the axial direction of the contact forces. The contact forces in the axial direction are increased sharply with much higher shaky peaks than that of only considering radial clearance. It indicates that the existence of clearances will lead to undesirable vibrations of the satellite antenna and influence the movement stability of the satellite antenna, especially when the axial clearance is also considered in the revolute joint. The 3D revolute clearance joint will lead to more severe effects on the dynamic performances of the satellite antenna when both the radial and axial clearances are considered. Therefore, the effects of axial clearance on the satellite antenna cannot be ignored in the dynamics analysis of the satellite antenna system.
Meanwhile, the simulation results are compared to other studies from previous literatures on planar and spatial revolute clearance joints [9, 10, 23, 61–63]. The previous researches showed that the planar revolute joint with radial clearance had important effects on the dynamic responses, especially the acceleration of multibody mechanical systems. Also, the previous studies indicated that the 3D revolute clearance joint will lead to more significant effects on dynamic responses of mechanism systems. Therefore, the simulation results are validated by other data published on the field of dynamics of mechanism systems with clearance joints.

### 4.2.2. Effects of Axial Clearance Size

In this section, the effects of the axial clearance size of the 3D revolute clearance

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**Table 4: Different axial clearance sizes of 3D revolute clearance joint.**

| Cases   | Description                          | Clearance size (mm) |
|---------|--------------------------------------|---------------------|
| Case 1  | Ideal revolute joint without clearance | $C_r = C_a = c = 0$ |
| Case 2  | 3D revolute clearance joint           | $C_r = 0.2, C_a = 0$ |
| Case 3  | 3D revolute clearance joint           | $C_r = 0.2, C_a = 0.2$ |
| Case 4  | 3D revolute clearance joint           | $C_r = 0.2, C_a = 0.3$ |
| Case 5  | 3D revolute clearance joint           | $C_r = 0.2, C_a = 0.4$ |

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**Figure 10: Contact forces in 3D revolute clearance joint.**

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**Figure 10**: Contact forces in 3D revolute clearance joint.
joint are investigated. Five case studies are presented, as shown in Table 4. The simulation results are presented in Figures 11–13. It also can be found that the bigger size of the axial clearance will induce more severe shaky responses and much higher shaky peaks. Therefore, a bigger size of the axial clearance will lead to worse performance of the satellite antenna system.

The fact that the existence of both the radial and axial clearances in 3D revolute clearance joints has an important effect on the dynamic performances of the satellite antenna supports the idea that the axial clearance in the revolute joint cannot be ignored. The axial clearance in a 3D revolute clearance joint must be considered in the dynamics analysis and design of the real satellite antenna systems.
5. Conclusions

Generally, axial clearances always exist in revolute joints due to the manufacturing and assembling errors, which have been less considered. However, axial clearances will result in relative motion along the axial direction between the journal and bearing of a 3D revolute clearance joint. Correspondingly, it will influence the dynamic characteristics of mechanisms. In this paper, a methodology for modeling the 3D revolute joint with clearance and its application in satellite antenna system are presented.

The mechanical bodies connected by the 3D revolute clearance joint are modelled as contact rigid bodies, and it is modelled as contact force constraints. Consequently, contact-impact forces control the dynamics of the clearance joint. For a 3D spatial revolute clearance joint, there will be 13 potential contact scenarios, which can reveal the different contact phenomena. A satellite antenna system with spatial motion is presented as the demonstrative application example. Then, the dynamic characteristics of the space antenna system considering both the radial and axial clearances in the 3D revolute clearance joints are investigated. Different case studies are presented and discussed.

The numerical simulation results indicate that (1) the dynamic responses of the antenna system are obviously shaky with much higher peaks when considering both the radial and axial clearance. Therefore, the 3D revolute clearance joint leads to much more severe effects on the dynamic performances of the satellite antenna system. (2) The contact forces exist in both the radial and axial directions. The existence of the axial clearance in the 3D revolute clearance joint leads to more severe effects on the 3D revolute clearance joint.
joint, which will induce undesirable vibrations and affect the movement stability and accuracy of the satellite antenna. (3) The bigger size of the axial clearance will induce much more severe shaky responses and much higher shaky peaks.

Clearances in revolute joints play a crucial role in predicting accurately the dynamic responses of the satellite antenna system. The studies of this paper indicate that the effects of the axial clearance on the satellite antenna system cannot be ignored. The methodology for modeling the 3D revolute joint with clearances is the basis of performance analysis and system design of space mechanisms, which develops the revolute joint to real engineering application.

Data Availability
The data used to support the findings of this study are included within the article.

Conflicts of Interest
The authors declare that there is no conflict of interest regarding the publication of this paper.

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