UTxO- vs account-based smart contract 
blockchain programming paradigms

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Abstract. We implement two versions of a simple but paradigmatic smart contract: one in Solidity on the Ethereum blockchain platform, and one in Plutus on the Cardano platform, giving annotated code excerpts, with full source code also attached. We get a clearer view of the Cardano programming model in particular by introducing a simple but novel mathematical abstraction which we call idealised Cardano. For each version of the contract, we trace how the architectures of the underlying platforms and their mathematics affects the natural programming styles and natural classes of errors. We prove some simple but novel results about alpha-conversion and observational equivalence for the Cardano platform, and explain why Ethereum does not have them. We conclude with a wide-ranging and detailed discussion in the light of the examples, mathematical model, and mathematical results so far.

1 Introduction

In the context of blockchain and cryptocurrencies, smart contracts are a way to make the blockchain programmable. That is: a smart contract is a program that runs on the blockchain to extend its capabilities.

For the smart contract, the blockchain is just an abstract machine (database, if we prefer) with which it programmatically interacts. Basic design choices in the blockchain’s design can affect the the smart contract programming paradigm which it naturally supports, and this can have far-reaching consequences: different programming paradigms are susceptible to different programming styles, and different kinds of program errors.

Thus, a decision in the blockchain’s design can have lasting, unavoidable, and critical effects on its programmability. It is worth being very aware of how this can play out, not least because almost by definition, applications of smart-contracts-the-programming-paradigm tend to be safety-critical.

In this paper we will consider a simple but paradigmatic example of a smart contract: a fungible tradable token issued by an issuer who creates an initial supply and then retains control of its price—imitating a government-issued fiat currency, but run from a blockchain instead of a central bank (and, just for simplicity, we permit only one initial minting of the token).

We will put this example in the context of two major smart contract languages: Solidity, which runs on the Ethereum blockchain, whose native token
is *ether*; and Plutus, which runs on the Cardano blockchain, whose native token is *ada*.³ We compare and contrast the blockchains’ structure in detail and exhibit their respective smart contracts. Both contracts run, but their constructions are different, in ways that illuminate the essential natures of the respective underlying blockchains and the programming styles that they support.

We will also see that the Ethereum smart contract is arguably buggy, in a way that flows naturally and directly from the underlying programming paradigm of Solidity/Ethereum. So even in a simple example, the essential natures of the underlying systems are felt, and with a mission-critical impact.

**Definition 1.1.** We will use Solidity and Plutus to code a system as follows:

1. An issuer **Issuer** creates some initial supply of a tradable token, located on the blockchain at some location which the **Issuer** controls. Call this location the issuer’s *official portal*.
2. Other parties buy the token from the official portal at a per-token *ether/ada official price*, controlled by the issuer.
   
   Once other parties get some token, they can trade it amongst themselves (e.g. for *ether/ada*), independently of the official portal and on whatever terms and at whatever price they mutually agree.
3. The issuer can update the *ether/ada official price* of the token on the official portal, at any time.
4. For simplicity, the initial supply of tokens is fixed, though it can be redistributed as just described.

2 **Idealised Cardano**

2.1 **The structure of an idealised blockchain**

We start with a novel presentation of a mathematical idealisation of Cardano. Detailed lower-level descriptions are in [1, 2].

**Notation 2.1.** Suppose $X$ and $Y$ are sets. Then:

- Write $\mathbb{N} = \{0, 1, 2, \ldots\}$ and $\mathbb{N}_{>0} = \{1, 2, 3, \ldots\}$.
- Write $\text{fin}(X)$ for the finite powerset of $X$ and $\text{fin}_x(X)$ for the pointed finite powerset (the set of pairs of a $X' \subseteq \text{fin} X$ and some $x \in X'$).
- Write $\text{pow}(X)$ for the powerset of $X$, and $X \xrightarrow{\text{fin}} Y$ for finite maps from $X$ to $Y$ (finite partial functions).

**Definition 2.2.** Let the types of **Idealised Cardano** be a solution to the type equations in Figure 1.

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³ Plutus and Cardano are IOHK designs. The CEO and co-founder of IOHK, Charles Hoskinson, was also one of the co-founders of Ethereum.

⁴ Think: central bank, manufacturer’s price, official exchange rate, etc.

⁵ This use of ‘pointed’ is unrelated to the ‘points to’ of Notation 2.4.
Fig. 1. Types for Idealised Cardano

REMARK 2.3. 1. Think of \( r \in \text{Redeemer} \) as a key, required as a necessary condition by a validator (below) to permit computation. \( \text{Datum} \) is any data; we set it to be \( \mathbb{N} \) for simplicity. \( \text{Context} \) is a pointed transaction (below), meaning a transaction-viewed-from-a-particular-input.

2. Mathematically a validator \( V \in \text{Validator} \) is the set of \( \text{Redeemer} \times \text{Datum} \times \text{Context} \) tuples it validates but in the implementation we intend that \( V \) is represented by code \( V \) such that
   - from \( V \) we cannot efficiently compute a tuple \( t \) such that \( V(t) = \text{True} \), and
   - from \( V \) and \( t \), we can efficiently check if \( V(t) = \text{True} \).
   If \( t \in V \) then we say \( V \) validates \( t \).

3. A chip \( c = (d, n) \) is intuitively a currency unit (\( \£, \$, \ldots \)), where \( d \in \text{CurrencySymbol} \) is assumed to Gödel encode some predicate defining a monetary policy (more on this in Remark 2.7(2)) and \( n \in \text{TokenName} \) is just a symbol.

4. A value \( v \in \text{Value} \) is intuitively a bag of chips; \( v_c \) is the number of \( c \)s in \( v \).
   - We may abuse notation and define \( v_c = 0 \) if \( c \notin \text{dom}(v) \).
   - If \( \text{dom}(v) = \{c\} \) then we may call \( v \) a singleton value. Note that \( v_c \) need not equal 1; this is a multiset singleton. See line 10 of Figure 2 for the corresponding code.

5. A transaction is a set of inputs and a set of outputs. In a blockchain these are subject to consistency conditions (Definition 2.5): for now we just read a transaction left-to-right such that its inputs ‘consume’ some previous outputs, and ‘generate’ new outputs.

We need a little more notation:

NOTATION 2.4. 1. If \( tx = (I, O) \in \text{Transaction} \) and \( o \in \text{Output} \), say \( o \) appears in \( tx \) and write \( o \in tx \) when \( o \in O \); similarly for an input \( i \in \text{Input} \). We may silently extend this notation to larger data structures, writing for example \( o \in \text{Txs} \) (Definition 2.8(1)).

2. If \( i \) and \( o \) have the same position then say that \( i \) points to \( o \).

3. If \( tx = (I, O) \in \text{Transaction} \) and \( i \in I \) then write \( tx@i \) for the context \(((I, i), O)\) obtained by pointing \( I \) at \( i \in O \).

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6 The ‘crypto’ in ‘cryptocurrency’ lives here.

7 Ada has special status and is encoded as \((0, 0)\).
Definition 2.5. A valid blockchain (or just blockchain) of idealised Cardano is a finite sequence of transactions such that:

1. Distinct outputs appearing in $tx$ have distinct positions.
2. Every input $i$ appearing in $tx$ points to an output in some earlier transaction. Write this unique output $tx(i)$.
3. If $i = (p, k)$ appears in $tx$ and points to an earlier output $tx(i) = (p, V, s, v)$, then $(k, s, tx@i) \in V$.

Notation 2.6. $Txs$ will range over finite sequences of transactions, and $B$ will range over blockchains. We write $\text{valid}(Txs)$ for the assertion “$Txs$ is a blockchain”.

Remark 2.7. In words: A sequence of transactions is a blockchain when every input points to an earlier validating output. We call Figure 1 and Definition 2.5 Idealised Cardano because:

1. The implementation has bells and whistles which we omit for simplicity:
   (a) There is a second type of output for making payments to people.
   (b) Values represent ‘real money’ and so are preserved—the sum of input values must be equal to the sum of output values—unless they are not preserved, e.g. due to fees (which reduce the sum of outputs) or forging (creating) tokens (which increase it).
   (c) Transactions can be made time-sensitive using slot ranges (Remark 2.17); a transaction can only be accepted into a block whose slot is inside the transaction’s slot range.
   All of the above is important for a working blockchain but for our purposes it would just add complexity.
2. We continue the discussion in Remark 2.3(3). If we consider a chip $c = (d, n)$, the currency symbol $d$ is not arbitrary: it is (a Gödel encoding of) a monetary policy predicate. In the implementation, only transactions which satisfy all pertinent monetary policies are admissible.
   This is relevant to our example in Section 3 because we will assume a state chip with a monetary policy which enforces that it is affine (zero or one chips on the blockchain; see Remark 3.3). Explaining the detailed mechanics of how this works would be outside the scope of this paper; here we merely note that it exists.
3. In the implementation, a transaction is a pair of a set of inputs and a list of outputs. This is because an implementation concerns a run on a particular concrete blockchain $B$, and we want to assign concrete positions for outputs in $B$; so with a list the output located at the $j$th output of $i$th transaction could get position $2^j i^3 j$. However, we care here about the theory of blockchains, in the plural. It is better for us to use sets and to leave positions abstract since if positions are fixed by their location in a blockchain then in Theorem 2.16 and the lemmas leading up to it, when we rearrange transactions in a blockchain (e.g. proving an observational equivalence) we could have to track an explicit and messy reindexing of positions in the statement of the results. More on this in Subsection 2.3.
2.2 UTxOs and observational equivalence

We will be most interested in Definition 2.8 when $Txs$ and $Txs'$ are blockchains $B$ and $B'$; however, it is convenient for Lemma 2.14(1) if we consider the more general case of any finite sequences of transactions $Txs$ and $Txs'$:

**Definition 2.8.**
1. Call an output $o \in Txs$ spent (in $Txs$) when a later input points to it, and otherwise call $o$ unspent (in $Txs$).
2. Write $UTxO(Txs)$ for the set of unspent outputs in $Txs$.
3. If $UTxO(Txs) = UTxO(Txs')$ then write $Txs \approx Txs'$ and call $Txs$ and $Txs'$ observationally equivalent.

**Notation 2.9.** Given $Txs$ and a $tx$, write $Txs; tx$ for the sequence of transactions obtained by appending $tx$ to $Txs$. We will mostly care about this when $Txs$ and $Txs; tx$ are blockchains, and if so this will be clear from context.

**Lemma 2.10.** Validity (Definition 2.5) is closed under initial subsequences, but not necessarily under final subsequences:

1. $\text{valid}(Txs; tx)$ implies $\text{valid}(Txs)$.
2. $\text{valid}(tx; Txs)$ does not necessarily imply $\text{valid}(Txs)$.

**Proof.** For part 1: removing $tx$ converts spent outputs into unspent outputs, but this cannot invalidate the conditions in Definition 2.5. For part 2: it may be that an input in $Txs$ points to an output in $tx$; if we then remove $tx$ we would violate condition 2 of Definition 2.5.

A special case of interest is when two transactions operate on non-overlapping parts of the preceding blockchain:

**Notation 2.11.** Suppose $tx$ and $tx'$ are transactions. Write $tx \# tx'$ when the positions mentioned in the inputs and outputs of $tx$, are disjoint from those mentioned in $tx'$, and in this case call $tx$ and $tx'$ apart. Similarly for $tx \# Txs$.

**Lemma 2.12.** $tx \# tx' \iff tx' \# tx$.

**Proof.** An easy structural fact.

**Remark 2.13.** When reading Lemma 2.14 and Theorem 2.16 below, note that $\text{valid}(B; tx)$ (Notations 2.6 and 2.9) can be read as the assertion “it is valid to append the transaction $tx$ to the blockchain $B$”. Similarly, $\text{valid}(B; Txs)$ can be read as “it is valid to extend $B$ with $Txs$”.

**Lemma 2.14.**

1. If $tx \# tx'$ then $\text{valid}(B; tx; tx') \iff \text{valid}(B; tx'; tx) \text{ and } B; tx ; tx' \approx B; tx'; tx.$

2. If $\text{valid}(B; tx'; tx)$ then $\text{valid}(B; tx) \iff tx \# tx'$. (Some real work happens in this technical result, and we use this work to prove Theorem 2.16.)

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*We don’t necessarily know $B; tx; tx'$ is a blockchain, which is why we stated Definition 2.8 for sequences of transactions.*
Proof. 1. By routine checking Definition 2.5.
2. Suppose \( \neg(\text{tx}\#\text{tx}') \), so some position \( p \) is mentioned by \( \text{tx} \) and \( \text{tx}' \). If \( p \) is in an input in both \( \text{tx} \) and \( \text{tx}' \), or an output in both, then \( \text{valid}(B;\text{tx}'\text{;}\text{tx}) \) is impossible because each input must point to a unique earlier output, and each output must have a unique position. If \( p \) is in an input in \( \text{tx} \) and an output in \( \text{tx}' \) then \( \neg\text{valid}(B;\text{tx}) \), because now this input points to a nonexistent output. If \( p \) is in an output in \( \text{tx} \) and an input in \( \text{tx}' \) then \( \text{valid}(B;\text{tx}'\text{;}\text{tx}) \) is impossible, because each input must point to an earlier output.

Conversely suppose \( \text{tx}\#\text{tx}' \). Then \( \text{tx}' \) must point only to outputs in \( B \) and removing \( \text{tx} \) cannot disconnect them and so cannot invalidate the conditions in Definition 2.5.

Remark 2.15. Theorem 2.16 below gives a sense in which UTxO-based accounting is ‘stateless’. With the machinery we now have the proof looks simple, but this belies its significance, which we now unpack in English:

Suppose we submit \( \text{tx} \) to some blockchain \( B \). Then either the submission of \( \text{tx} \) fails and is rejected (e.g. if some validator objects to it)—or it succeeds and \( \text{tx} \) is appended to \( B \).

If it succeeds, then even if other transactions \( \text{Txs} \) get appended first—e.g. they were submitted by other actors whose transactions arrived first, so that in-between us creating \( \text{tx} \) and the arrival of \( \text{tx} \) at the main blockchain, it grew from \( B \) to \( B;\text{Txs} \)—then the result \( B;\text{Txs};\text{tx} \) is up to observational equivalence equivalent to \( B;\text{tx};\text{Txs} \), which is what would have happened if our \( \text{tx} \) had arrived at \( B \) instantly and before the competing transactions \( \text{Txs} \).

In other words: if we submit \( \text{tx} \) to the blockchain, then at worst, other actors’ actions might prevent \( \text{tx} \) from getting appended, however, if our transaction gets onto the blockchain somewhere, then we obtain our originally intended result up to observational equivalence.

Theorem 2.16. Suppose \( \text{valid}(B;\text{Txs};\text{tx}) \) and \( \text{valid}(B;\text{tx}) \). Then

1. \( \text{valid}(B;\text{tx};\text{Txs}) \) and
2. \( B;\text{tx};\text{Txs} \cong B;\text{Txs};\text{tx} \).

Proof. Using Lemma 2.14(2) \( \text{tx}\#\text{Txs} \). The rest follows using Lemma 2.14(1).

Remark 2.17. The Cardano implementation has slot ranges (Remark 2.7(1c)). These introduce a notion of time-sensitivity to transactions which breaks Theorem 2.16(1), because \( \text{Txs} \) might be time-sensitive. A milder form holds which we sketch as Proposition 2.18 below. It is still reasonable and useful to consider a ‘pure’ idealised Cardano without slot ranges, and use this idealisation to observe that the system has certain purity properties provided we avoid stateful constructs, like timing.

Proposition 2.18. If we extend our notion of blockchain with slot ranges, which restrict validity of transactions to defined time intervals, then Theorem 2.16 weakens to:
If valid($B; tx; Txs$) and valid($B; Txs; tx$) then $B; tx; Txs \cong B; Txs; tx$.

Proof. As for Theorem 2.16, noting that slot ranges are orthogonal to UTxOs, provided the transactions concerned can be appended.

2.3 $\alpha$-equivalence and more on observational equivalence

Our syntax for $B$ in Definition 2.5 is name-carrying; outputs are identified by unique markers which—while taken from $\mathbb{N}$; see “Position=$\mathbb{N}$” in Figure 1—are clearly used as atoms or names to identify binding points on the blockchain, to which at most one later input may bind. Once bound this name can be thought of graphically as an edge from an output to the input that spends it, so clearly the choice of name/position—once it is bound—is irrelevant up to permuting our choices of names. This is familiar from $\alpha$-equivalence in syntax, where e.g. $\lambda a.\lambda b.ab$ is equivalent to $\lambda b.\lambda a.ba$. We define:

**Definition 2.19.**
1. Write $B =_{\alpha} B'$ and call $B$ and $B'$ $\alpha$-equivalent when they differ only in their choice of positions of spent output-input pairs (Definition 2.8(1)).

2. If $\Phi$ is an assertion about blockchains, write “up to $\alpha$-equivalence, $\Phi$” for the assertion “there exist $\alpha$-equivalent forms of the arguments of $\Phi$ such that $\Phi$ is true of those arguments”.

Lemma 2.20 checks that observational equivalence interacts well with being a valid blockchain and appending transactions. We sketch its statement and its proof, which is by simple checking. In words it says: extensionally, a blockchain up to $\alpha$-equivalence is just its UTxOs:

**Lemma 2.20.**
1. If $B \cong B'$ and valid($B; tx$) and valid($B'; tx$) then $B; tx \cong B'; tx$.

2. If $B =_{\alpha} B'$ then $B \cong B'$.

3. Up to $\alpha$-equivalence, if $B \cong B'$ then valid($B; tx$) $\iff$ valid($B'; tx$).

4. Up to $\alpha$-equivalence, if $B \cong B' \land$ valid($B; tx$) then $B; tx \cong B'; tx$.

**Remark 2.21.** We need $\alpha$-conversion in cases 3 and 4 of Lemma 2.20 because valid($B; tx$) might fail only because of an accidental name-clash between the position assigned to a spent output in $B$, and a position in an unspent output of $tx$; in this case, we need $\alpha$-conversion to rename the bound position and avoid this name-clash.

This phenomenon is familiar from syntax, e.g. we know to $\alpha$-convert $a$ in $(\lambda a.b)[b:=a]$ to obtain (up to $\alpha$-equivalence) $\lambda a'.a$.

There are many approaches to working up to $\alpha$-conversion; graphs, de Bruijn indexes [4], name-carrying syntax with an explicit equivalence relation as required (which we use in this paper), the nominal abstract syntax developed by
the first author with others [5], and [6]. Studying what works best for a structural theory of blockchains is future research.

In this paper we have used raw name-carrying syntax, possibly quotiented by equivalence as above; a more sophisticated development might require more. Note that the implementation solution discussed in Remark 2.7(3) corresponds to a de Bruijn indexe approach, and that in the context of our needs for this paper, this does not solve all problems, as discussed in that Remark (see ‘messy reindexing’).

3 The Plutus smart contract

Definition 3.1. Relevant parts of the Plutus code to implement Definition 1.1 are in Figure 2. Full source is at https://arxiv.org/src/2003.14271v2/anc.

Remark 3.2. 1. Chip corresponds to Chip from Figure 1.
2. Config stores necessary configuration: the issuer (represented as a hash of their public key), the chip to be traded, and the state chip (discussed below in Remark 3.3).
3. tradedChip packs up \( n \) of cTradedChip in a value (think: a roll of quarters).
4. Action is a datatype which stores labels of a transition system, which in our case are either ‘buy’ or ‘set price’.
5. State Integer corresponds to Datum in Figure 1. stateData on line 29 retrieves this datum and uses it as the price of the token.
6. value’ (lines 26 and 29) is the amount of ada paid to the Issuer. It is so called (with a dash) because value is an existing function from the Plutus Ledger library.

Remark 3.3. In Remark 2.15 we described in what sense Idealised Cardano (Figure 1) is stateless. Yet Definition 1.1(3) specifies that Issuer can set a price for the traded chip. This seems stateful. How to reconcile this?

We create a state chip cStateChip, whose monetary policy (Remark 2.7(2)) enforces that it is affine and monotone increasing, and thus linear once created. The Issuer issues an initial transaction to the blockchain which sets up our trading system, and creates a unique UTxO that contains this state chip in its value, with the price in its Datum field.

The UTxO with the state chip corresponds to the official portal from Definition 1.1, and its state datum corresponds to the official price.

Monetary policy ensures this is now an invariant of the blockchain as it develops, and anybody can check the current price by looking up the unique UTxO distinguished by carrying precisely on cStateChip, and looking at the value in its Datum field. The interested reader can consult the source code.
data Chip = MkChip
  { cSymbol :: !CurrencySymbol
    , cName :: !TokenName
  }

data Config = MkConfig
  { cIssuer :: !PubKeyHash
    , cTradedChip, cStateChip :: !Chip
  }

tradedChip :: Config → Integer → Value
  tradedChip MkConfig{..} n = singletonValue cTradedChip n

data Action =
  SetPrice !Integer
  | Buy !Integer

transition :: Config → State Integer → Action
  → Maybe (TxConstraints Void Void, State Integer)

transition c s (SetPrice p)
  - ACTION: set price to p
  | p < 0      = Nothing
  | otherwise = Just

    . mustBeSignedBy (cIssuer c)
    . s{stateData = p}

  | p ≤ 0       = Nothing
  | otherwise = Just

    . mustPayToPubKey (cIssuer c) value'

    . seller been paid?

    . s{stateValue = stateValue s − sold}

    . sell chips!

where

value' = lovelaceValueOf (m * stateData s)
  - final value buyer pays

sold = tradedChip c m
  - no. chips buyer gets

guarded :: HasNative s ⇒ Config → Integer → Integer
  → Contract s Text ()
guarded c n maxPrice =

void $ withError $ runGuardedStep (client c) (Buy n) $

λₚₚ → if p ≤ maxPrice then Nothing else Just ()

Fig. 2. Plutus implementation of the tradable token
contract Changing {
    address payable public issuer; // issues the token
    uint public price; // current price
    mapping (address => uint) public balances; // tracks who owns how many tokens

    constructor (uint _count, uint _price) public {
        require (_count > 0, "count must be positive");
        require (_price > 0, "price must be positive");
        issuer = msg.sender;
        price = _price;
        balances[msg.sender] = _count;
    }

    function send(address _receiver, uint _amount) public {
        require (_amount ≤ balances[msg.sender], "balance too low");
        balances[msg.sender] = _amount;
        balances[_receiver] += _amount;
    }

    function buy() public payable {
        uint _tokens = msg.value / price;
        require (_tokens ≤ balances[issuer], "not enough tokens");
        issuer.transfer(msg.value);
        balances[issuer] -= _tokens;
        balances[msg.sender] += _tokens;
    }

    function setPrice (uint _newPrice) public {
        require (msg.sender == issuer, "only issuer can set price");
        price = _newPrice;
    }
}

Fig. 3. Solidity implementation of the tradable coin
4 The Solidity smart contract

4.1 Description

Remark 4.1. The Ethereum blockchain is account-based: it can be thought of as a state machine whose transitions modify a global state of contracts containing functions and data.

Thus the contract Changing (line 1 of Figure 3)—once deployed—is located on the Ethereum blockchain, along with its functions and state. Intuitively this contract and its state is ‘in one place’ on the blockchain, and is not spread out across multiple UTxOs as in Cardano. There is no need for the state-chip mechanism from Remark 3.3.

Remark 4.2. We now briefly read through the code:

1. address is an address for the Issuer. It is payable (it can receive ether) and public (its value can be read by any other function on the blockchain).\(^{11}\)
2. constructor is the initialisation subroutine of the contract, which sets it up such that issuer (the agent who triggers the contract) has all of the new token.
3. send is a function to send money to another address. Note that this is not in the Plutus code; this is because we got it ‘for free’ as part of Cardano’s in-built support for currencies (this is also why Idealised Cardano has the type Value in Figure 1).
4. buy on line 20 is analogue to the Buy transition in Figure 2.
5. The rest of the code is elementary.

4.2 Discussion

Remark 4.3. In line 21 of Figure 3 we calculate the tokens purchased using a division of value (the total sum paid) by price.\(^{11}\)

In contrast, in line 29 of Figure 2 we calculate the sum by multiplying the number of tokens by the price of each token.

There is a reason for this: we cannot perform the multiplication in the Solidity buy code because we do not actually know the price per token at the time the function acts to transition the blockchain. We can access a price at the time of invoking buy by querying Changing.price(), but by the time that invocation reaches the Ethereum blockchain and acts on it, the virtual machine might have undergone transitions, and the price might have changed.

Because Ethereum is stateful and has nothing like Remark 2.15 and Theorem 2.16, this cannot be fixed.

\(^{10}\) This technique was developed by the IOHK Plutus team.
\(^{11}\) Any data on the Ethereum blockchain is public in the external sense that it can be read off the binary data of the blockchain as a file on a machine running it, however, not all data is public in the internal sense that it can be accessed from any code running on the Ethereum virtual machine.
Remark 4.4. One might counter that this could indeed be fixed, just by changing \texttt{buy} to include an extra parameter which is the price that the buyer expects to pay, and if this expectation is not met then the function terminates. However, the issuer controls the contract (\texttt{issuer = msg.sender} on line 9 of Figure 3), including the code for \texttt{buy}, so this safeguard can only exist at the issuer’s discretion—and our issuer, whether through thoughtlessness or malice, has not included it.

In Cardano the situation is different and the buyer has more control, because by Theorem 2.16 a party issuing a transaction knows (up to observational equivalence) precisely what the inputs and outputs will be; the only question is whether it successfully attaches to the blockchain.

Another subtle error in the Ethereum code is that the `/` on line 21 is integer division, so there may be a rounding error.\footnote{It would be unheard of for such elementary mistakes to slip into production code; and even if it did happen, it is hardly conceivable that such errors would happen repeatedly across a wide variety of programming languages. That was sarcasm, but the point may bear repeating: programmer error and programming language design are two sides of a single coin.}

Remark 4.5. The Ethereum code contains two errors, but the emphasis of this discussion is not that it is possible to write buggy code (which is always true) rather, we draw attention to how and why the underlying accounts-based structure of Ethereum invites and provides cover for certain errors in particular.

On the other hand, the Plutus code is more complex than the Ethereum code. Plutus is arguably conceptually beautiful, but the pragmatics as currently implemented are fiddly and the amount of boilerplate required to get our Plutus contract running much exceeds that required for our Ethereum contract. This may improve with time as Plutus matures and libraries improve—Plutus has been streamlined just in the time this paper was written, and in at least one instance this was because of a suggestion made by the authors while designing this contract\footnote{—the need for \texttt{runGuardedStep}.}—but it seems unlikely that the gap will be totally closed.

So Ethereum is simple, direct and alluring, but can be dangerous, whereas Plutus places higher burdens on the programmer but enjoys some good mathematical properties which can enhance programmability. This is the tradeoff.

Remark 4.6 (Mechanics of contracts on the blockchain). It is convenient to call the code referred to in Figures 2 and 3 contracts, but this is in fact an imprecise use of the term: a contract is an entity on the blockchain, whereas the figures describe schemas or programs which, when invoked with parameters, attempt to push a contract onto the blockchain.

\begin{itemize}
\item For Plutus, each time we invoke the contract schema for some choice of values for \texttt{Config}, the schema tries to creates a transaction on the blockchain which generates a UTxO which carries a state-chip and an instance of the contract. The contract is encoded in the monetary policy of the state-chip (which enforces linearity once created) and in the Validator of that UTxO.
\end{itemize}
The issuer’s address is encoded in the Validator field of the UTxO. A mathematical presentation of the data structures concerned is in Figure 1.

- For Ethereum, Figure 3 also describes a schema which deploys a contract to the blockchain. Each time we invoke its constructor we generate a specific instance of `Changing`. Configuration data, given by the constructor arguments, is simply stored as values of the global state variables of that instance, such as `issuer` (line 2 of Figure 3).

Remark 4.7 (State & possible refinements). In Solidity, state is located in variables in contracts. To query a state we just query a variable in a contract of interest, e.g. `price` or `balances` in contract `Changing` in Figure 3.

In Plutus, state can be more expensive. State is located in unspent outputs, see the `Datum` and `Value` datatype fields in `Output` in Figure 1. In a transaction that queries or modifies state, we consume any UTxOs containing state to which we refer, and produce new UTxOs with new state. Even if our queries are read-only, they destroy the UTxOs read and create new UTxOs.\(^{14}\)

Suppose a UTxO contains some state information that is very popular, so that multiple users submit transactions to the blockchain simultaneously, all referencing the same state-bearing UTxO. Only one transaction can succeed and be appended and the other transactions will not link to the new state UTxO which is generated—even if it contains the same state data as the older version. Instead, the other transactions will fail, because the particular incarnation of the state UTxO to which they happened to link, has been consumed.

This could become a bottleneck. In the specific case of our Plutus contract, it could become a victim of its own success if our token is purchased with high enough frequency, since access to the state UTxO and the tokens on it could become contested and slow and expensive in user’s time. The state UTxO becomes, in effect, constantly locked by competing users—though note the the blockchain would still be behaving correctly in the sense that nobody can lose tokens.

For a more scaleable scenario, some parallel redundancy with a monetary policy for the state chip(s) implementing a consensus or merging algorithm, might be required. This is important future work, but note that whatever mechanism is constructed would be ‘write once, deploy in many contracts’.

One more small refinement: as things stand, every token that is ever bought must go through the contract at the official price—even if it is the issuer trying to withdraw it. In practice, the issuer might want to code a way to get their tokens out without having to go through `buy`.

Remark 4.8 (Resisting DOS attacks). An aside contrasting how Cardano and Ethereum manage their fees to prevent denial-of-service (DOS) attacks: In Cardano, users add transactions to the blockchain and pay fees of the form \(a + bx\) where \(a\) and \(b\) are constants and \(x\) is the size of the transaction in bytes. This prevents DOS attacks because e.g. if we flood the blockchain with \(n\) transactions containing tiny balances, we pay \(\geq na\) transaction fees. In Ethereum, storage

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\(^{14}\) This is distinct from a user inspecting the contents of UTxOs from outside the blockchain, i.e. by reading state off the hard drive of their node or Cardano wallet.
costs a fee (denominated in gas); if we inflate balances—which is a finite but unbounded mapping from addresses to integers—with tiny balances, we may run out of gas.

### 4.3 Summary of the critical points

1. The errors in the Solidity code are not necessarily obvious.
2. Even if the buyer knows to watch out for them, they cannot be defended against without rewriting the contract, which requires the cooperation of the seller (who controls the contract and who might even be hostile and have planted the bug on purpose).
3. In Plutus, the buyer creates a transaction, and determines its inputs and outputs at the time of its creation. A transaction might be rejected if the available UTxOs change—but if it succeeds then the buyer knows the outcome. This gives the user of a contract an assurance that if they anticipate an error (or attack) then they can guard against it independently of the contract’s designer.

   In Ethereum this is impossible: a buyer can propose a transition to the global virtual machine by calling a function on the Ethereum blockchain, but it is the designer of the contract who has designed that function—and because of concurrency, the buyer cannot know the values of the inputs to this function at the time of its execution on the blockchain.

### 5 Conclusions

We have seen that Ethereum is an accounts-based blockchain system whereas Cardano (like Bitcoin) is UTxO-based, and implemented a specification (Definition 1.1) in Solidity (Section 4) and in Plutus (Section 3), and we have given a mathematical abstraction of Cardano, idealised Cardano (Section 2). These have raised some surprisingly non-trivial points, both quite mathematical (e.g. Subsection 2.3) and more implementational (e.g. Subsection 4.2), which are discussed in the body of the paper.

The accounts-based paradigm lends itself naturally to an imperative programming style: a smart contract is a program that manipulates a global state mapping global variables (called ‘accounts’) to values. The UTxO-based paradigm lends itself naturally to a functional programming style: the smart contract is a function that takes a UTxO-list as its input, and returns a UTxO-list as its output, with no other dependencies. See Theorem 2.16 and the preceding discussion and lemmas in Subsection 2.2 for a mathematically precise rendering of this intuition, and Subsection 2.3 for supplementary results.

Smart contract programming is naturally concurrent; the blockchain must expect to scale to thousands, if not billions, of concurrent users, but the problems inherent to concurrent imperative programming are well-known and predate

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15 How to reach distributed consensus in such an environment is another topic, with its own attack surface. Cardano uses Ouroboros consensus [7].
smart contracts by decades. This is an issue with Ethereum/Solidity and it is visible even in our simple (but paradigmatic) example. Real-life examples, like the infamous DAO hack\textsuperscript{16}, are clearly related to the problem of transactions having unexpected consequences in a stateful, concurrent environment.

Cardano’s UTxO-based structure invites a functional programming paradigm, and this is reflected in Plutus. The price we pay is arguably an increase in conceptual complexity.

Put simply: accounts are easier to think about than UTxOs, and imperative programs are easier to read than functional programs—at least initially—but this ease of use makes us vulnerable to serious classes of hard-to-trace errors, especially in a concurrent safety-critical context. Such trade-offs may be familiar to many readers with a computer science or concurrency background.

We hope that this paper will provide an accessible, yet technically precise, entry point for interested readers, and serve as a guide to some of the design considerations of working in this area.

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\textsuperscript{16} The DAO hack enabled an unknown hacker to steal approximately 70 million USD. Ethereum chose to revert the theft, and to do this a hard fork of the Ethereum blockchain was required [8].