Melting temperature for scalar hadrons in AdS / QCD models

Adolfo Ibañez and Alfredo Vega
Instituto de Física y Astronomía, Universidad de Valparaíso, A. Gran Bretaña 1111, Valparaíso, Chile.
E-mail: alfredo.vega@uv.cl

Abstract.
We consider an analysis to potentials related to Schrödinger like equations for scalar fields in a 5D AdS black hole background with dilaton in order to get melting temperatures for different hadrons in a thermal bath. In approach considered it is not necessary to calculate spectral functions and it is easy to get results for different kind of hadrons. Here we present results for scalar mesons, glueballs, hybrid mesons and tetraquarks.

1. Introduction
Actually in several places are realizing experiments to understand how temperature affect hadron properties, and this topic has attracted interest of theoreticians considering several approaches to describe hadrons under these conditions (e.g see [1, 2]). The main theoretical tools used are lattice QCD and sum rules, and in last 15 years was added techniques based on Gauge / Gravity dualities (e.g see [3, 4, 5, 6, 7]). In the last kind of models, temperature is included considering black hole backgrounds in AdS in $d + 1$ dimensions.

With models based on holographic correspondence it is possible to calculate several properties that vary with temperatures as masses and melting temperatures using the spectral function calculated in this approach [5, 6, 7]).

In this work we consider a procedure to calculate melting temperatures for hadrons in a thermal bath without calculate the spectral function. We use a holographic model, and melting temperature it is obtained since an analysis of potential associated to Schrödinger like equations that describe hadrons in AdS side [9, 10]. Here we restrict us to study scalar hadrons, which according to AdS dictionary are related to scalar modes [8], and are shown results for melting temperatures for mesons, glueballs, hybrid mesons and tetraquarks in a thermal bath.

This paper consist of fourth sections and an appendix. The introduction is followed by section 2 where we describe the procedure used to calculate melting temperatures considering an analysis of potentials of equation of motions in AdS side. In section 3 are presented and discussed the results. Section 4 considers conclusions and future perspectives. Additionally we consider an appendix with Liouville substitution, which is the procedure used to get Schrödinger like equations and potentials which are analysed in this work to get melting temperatures.
2. The model

In AdS / QCD models, scalar hadrons in AdS side are modelled by scalar models in a 5D curved space with dilaton with an action [11, 12, 13, 14]

$$S = \frac{1}{2K} \int d^5 x \sqrt{-g} e^{-\phi(z)} \left[ g^{MN} \partial_M X(x,z) \partial_N X(x,z) + m_5^2 X^2(x,z) \right]$$  \hspace{1cm} (1)

In order to introduce temperature in this description it is necessary to consider a 5D AdS black hole metric [15].

$$ds^2 = e^{2A(z)} \left[ -f(z) dt^2 + \sum_{i=1}^3 (dx_i)^2 + \frac{1}{f(z)} dz^2 \right],$$ \hspace{1cm} (2)

or

$$g_{MN} = e^{2A(z)} \text{diag} \left( -f(z), 1, 1, 1, \frac{1}{f(z)} \right).$$ \hspace{1cm} (3)

In this case the equation of motion is

$$e^{B(z)} f(z) \partial_z \left[ e^{-B(z)} f(z) \partial_z \psi \right] - f(z) e^{2A(z)} m_5^2 \psi + \omega^2 \psi - f(z) q^2 \psi = 0,$$ \hspace{1cm} (4)

where $B(z) = \phi(z) - 3A(z)$.

For particles at rest ($\vec{q} = \vec{0}$), last equation change to

$$\partial_z \left[ e^{-B(z)} f(z) \partial_z \psi \right] + \left[ \frac{\omega^2}{e^{B(z)} f(z)} - e^{-\phi(z) + 5A(z)} m_5^2 \right] \psi = 0.$$ \hspace{1cm} (5)

By using the Liouville substitution (see appendix), it is possible obtain a Schrödinger like equation which is related to the previous one, which potential is,

$$V \left( z(\xi) \right) = e^{B(z(\xi))} f \left( z(\xi) \right) e^{-\phi(z(\xi)) + 5A(z(\xi))} m_5^2 - \left[ e^{-2B(z(\xi))} \right]^{-\frac{1}{4}} \frac{d^2}{d\xi^2} \left( e^{-2B(z(\xi))} \right)^{\frac{1}{4}}.$$ \hspace{1cm} (6)

This potential at zero temperature ($f(z) = 1$) has normalizable modes ([11, 12, 13, 14]), but at the finite temperature it has the form of quasi bound states, and depth of the well it is reduced when temperature is increased, and at some temperature the well disappear. This temperature it is interpreted as melting temperature in thermal bath [9, 10]. In literature it is possible to find a different way to obtain a potential to extract the melting temperature, but it is necessary to do supposition about how mass vary with temperature [16].

Here we analyse this potential for a specific metric and dilaton. And we consider different values for $m_5$, which according at AdS / CFT dictionary is related to dimension of operators that create scalar hadrons. Expression that relate AdS mass with operator dimension is [8, 11]

$$m_5^2 R^2 = \Delta (\Delta - 4),$$ \hspace{1cm} (7)

where $R$ is the AdS radii (without loss of generality can be considered as one) and $\Delta$ is the dimension of operator that create hadrons. In this work are considered mesones ($\Delta = 3$; $m_5^2 = -3$), glueballs ($\Delta = 4$; $m_5^2 = 0$), hybrid mesons ($\Delta = 5$; $m_5^2 = 5$) and tetraquarks ($\Delta = 6$; $m_5^2 = 12$) [11].

In this work was used

$$f(z) = 1 - \frac{z^4}{z_h^4}, \hspace{1cm} 0 < z < z_h,$$ \hspace{1cm} (8)

so fare,
Figure 1. Plots shows the change in potential shapes for different temperatures for several scalar hadrons with a different number of constituents. Continuous lines correspond to cases considering the melting temperature for each hadron. Without lost of generality we use $R = 1$. All temperatures are in GeV.

\[
\xi = \int_0^z \frac{1}{1 - \frac{t^2}{z_h^2}} \, dt = \frac{z_h}{2} \left( -\arctan \left( \frac{z}{z_h} \right) + \frac{1}{2} \log \left( \frac{z - z_h}{z + z_h} \right) \right)
\]

where $z_h$ is localization of event horizon, which is related with temperature by

\[
z_h = \frac{1}{\pi T}
\]

3. Results

At zero temperature we got the traditional soft wall model, and as it is possible to see in plots, increasing temperature the potential shape vary and at some characteristic temperature wells in potential disappear [9, 10, 16]. This is the melting temperature for each hadron in a thermal bath. In plots, continuous line shows potential at melting temperature, and as you can see, mesons are the more resistant hadrons to be melted in a thermal bath.
4. Conclusions
We consider an approach based on Gauge / Gravity ideas to calculate hadron temperature melting without to do calculations of spectral functions. Calculations consider in a simple way hadrons with arbitrary number of constituents (here we restrict us to mesons, glueballs, hybrid mesons and tetraquarks), and it shows that for scalars case, mesons are most resistant hadrons to be melted on a thermal bath.

This procedure can be applied to study another kind of hadrons (vectors, dirac, etc) and with small changes could be used to study properties of hadrons in dense media.

Appendix A. Liouville substitution

Lets consider the equation (e.g see [17])

\[
\frac{d}{dz}\left[p(z)\frac{d}{dz}\psi(z)\right] + \left[\omega^2g(z) - q(z)\right]\psi(z) = 0. \tag{A.1}
\]

The Liouville substitution

\[
\psi(z) = v(\xi)[p(z)g(z)]^{-\frac{1}{4}}\xi = \int_0^z \left[\frac{g(t)}{p(t)}\right]^\frac{1}{4} dt \tag{A.2}
\]

allows us to get a Schrödinger like equation as,

\[
\frac{d^2 v(\xi)}{d\xi^2} + \left[\omega^2 - Q(\xi)\right]v(\xi) = 0 \tag{A.3}
\]

where

\[
Q(\xi) = \frac{q(z(\xi))}{g\left(z(\xi)\right)} + \left[p(z(\xi))\frac{g\left(z(\xi)\right)}{g\left(z(\xi)\right)}\right]^{-\frac{1}{4}}\frac{d^2}{d\xi^2}\left[p(z(\xi))\frac{g\left(z(\xi)\right)}{g\left(z(\xi)\right)}\right]^\frac{1}{4} \tag{A.4}
\]

Acknowledgments
This work was supported by FONDECYT (Chile) under Grant No. 1141280.

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