Traffic Management for Heterogeneous Networks with Opportunistic Unlicensed Spectrum Sharing

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Abstract

This paper studies how to maximize the per-user-based throughput in an $M$-tier heterogeneous wireless network (HetNet) by optimally managing traffic flows between the access points (APs) in the HetNet. The APs in the first $M-1$ tiers can use the licensed spectrum at the same time whereas they share the unlicensed spectrum with the APs in the $M$th tier by the proposed opportunistic carrier sense multiple access with collision avoidance (CSMA/CA) protocol. The APs that access the licensed and unlicensed spectra simultaneously are able to integrate their spectrum resources by the carrier aggregation technique. We first characterize the distribution of the cell load and the channel access probability of each AP using a generalized AP association scheme. For an AP in each tier, the tight lower bounds on its mean spectrum efficiencies in the licensed and unlicensed spectra are derived for the general random models of the channel gain and AP association weights. We define the per-user link throughput and per-user network throughput based on the derived the mean spectrum efficiencies and maximize them by proposing the decentralized and centralized traffic management schemes for the APs in the first $M-1$ tiers under the constraint that the per-user link throughput of the tier-$M$ APs must be above some minimum required value. Finally, a numerical example of coexisting LTE and WiFi networks is provided to validate our derived results and findings.

Index Terms

Heterogeneous networks, unlicensed spectrum sharing, traffic management, stochastic geometry.

I. INTRODUCTION

As more and more versatile services are offered over wireless networks and new generations of wireless smart handsets get wider and wider adoption, considerable data traffic flowing over spectrum-limited cellular networks is an inevitable phenomenon the network operators have
to seriously face. To alleviate the spectrum crunch crisis of a cellular network, an effective means is to make a traditional cellular network migrate to a heterogeneous cellular network in which many different kinds of base stations (BSs), such as macro, micro and small cell BSs, are densely deployed. Although heterogeneous cellular networks have a much higher network capacity compared to their traditional counterpart, their licensed spectrum is still very limited and their per-user link throughput may not be efficiently improved if the network has a huge user population. Accordingly, exploiting more available spectrum for heterogeneous cellular networks is the right track that should be followed, which incubates the idea of extending the service of the cellular BSs to the unlicensed spectrum.

If cellular BSs can access the licensed and unlicensed spectra at the same time, they can integrate all available spectrum resources by using the carrier aggregation technique [1]–[3]. However, extending cellular services to the unlicensed spectrum could severely impact the throughput performance of the existing access points (APs) using the unlicensed spectrum, such as WiFi APs. Hence, how to make different kinds of BSs and APs properly share the unlicensed spectrum and improve their total throughput is an important problem that needs to be investigated thoroughly. To generally and tractably analyze the throughput performance of a heterogeneous wireless network (HetNet) with unlicensed spectrum sharing, the HetNet considered in this paper has $M$ tiers. All APs in the same tier of the HetNet are of the same type and performance and they form an independent Poisson point process (PPP) with certain intensity. Specifically, the APs in the $M$th tier only access the unlicensed spectrum whereas the APs in the first $M-1$ tiers can simultaneously access the licensed spectrum as well as the unlicensed spectrum if they have a chance. All APs use the proposed (slotted nonpersistent) opportunistic CSMA/CA (Carrier sense multiple access with collision avoidance) with random backoff time protocol to contend the channel in the unlicensed spectrum. Such a HetNet model characterizes the coexisting impacts between different kinds of APs in the unlicensed spectrum while the CSMA/CA protocol is adopted.

A. Prior Works on Unlicensed Spectrum Sharing

Earlier prior works on the coexisting interference problem in the unlicensed spectrum focus on how to make APs in different overlaid wireless networks share the unlicensed spectrum with certain fairness. For example, reference [4] proposed a game-theoretical approach to fairly sharing the unlicensed spectrum in multiple coexisting and interfering networks. The interference
modeling and mitigation problems in the unlicensed spectrum are investigated in references [5]–[7]. These works use a small-scale and deterministic network model to formulate their problems and they do not study some coexisting performance metrics, such as coverage and network throughput. Although the authors in [8] developed a more accurate interference analysis for large-scale networks based on the continuum field approximation; however, they do not investigate if the interference in the unlicensed spectrum can be effectively mitigated by using channel access protocols.

A more complicate coexistence problem in the unlicensed spectrum that recently attracts a lot of attentions is how to let the base stations (BSs) or APs originally using the licensed spectrum also be able to access the unlicensed spectrum and use the carrier aggregation technique to boost their overall throughput [2], [3]. A few recent works have already shown that LTE and WiFi networks coexisting in the unlicensed spectrum can significantly improve their entire network throughput [2], [3], [9]–[11]. However, how to manage the traffic flows between LTE BSs and WiFi APs to maximize the total or per-user link throughput in the licensed and unlicensed spectra is not addressed in these works. Although a recent work in [12] studied when to offload the traffic for the LTE network to the WiFi network and share the unlicensed spectrum in order to maximize the per-user link throughput, their network, interference and spectrum sharing are modeled in a deterministic manner so that their observations may not be completely applicable to a large-scale stochastic cellular network.

Offloading traffic from an LTE network to another WiFi network may not increase the per-user throughput since the offloaded users lose their licensed throughput. Hence, as long as the LTE and WiFi networks can coexist without causing severe interference, having them sharing the unlicensed spectrum is a better policy. In [13], [14], small cell BSs are shown to achieve a notable throughput gain if they can cleverly and properly access the unlicensed spectrum without causing much interference to the WiFi APs. A stochastic-geometry-based framework in [10], [15]–[17] is applied to analyze the coexistence performance of large-scale LTE and WiFi networks, but the exact analyses of the coexisting throughput of the APs in the licensed and unlicensed spectra are not studied at all. All the aforementioned prior works do not tractably analyze the achievable ergodic link throughput of each AP in a general random network model while the unlicensed spectrum is shared. As a result, they are unable to reveal some insight about how to maximize the per-user link/network throughput by managing the traffic flows between different APs.
B. Contributions

In this paper, our first contribution is to propose a general $M$-tier HetNet architecture that generally characterizes the licensed and unlicensed spectrum sharing problem of different kinds of APs using the opportunistic CSMA/CA protocol. A general AP association scheme that can cover several pathloss-based AP association schemes is adopted in the HetNet. Under this HetNet model, the coexisting interference models in the licensed and unlicensed spectra can be easily proposed. Our second contribution is to derive the accurate distribution of the cell load of the APs in each tier. This distribution plays a pivotal role in finding the void probability and the association probability of an AP in each tier. The void probability of densely deployed APs is in general not small and thus cannot be ignored. However, almost all previous works overlook this important fact and thus their analytical results could be very inaccurate.

Our third contribution is to derive the identity of the Shannon transformation that leads to the tractable derivations of the mean spectrum efficiency. Due to performing AP association and CSMA to access the unlicensed channel, the resulting transmitting APs in each tier no longer form a PPP so that only the tight lower bounds on the mean spectrum efficiencies of the APs can be found. However, these tight lower bounds are explicitly derived without assuming any specific channel gain and AP association weight models so that they can give us some intuitions regarding how they are affected by channel gain impairments, AP association weights as well as void AP probabilities. These lower bounds are absolutely important since they help us easily judge whether the mean spectrum efficiency of the APs in a particular tier increases or decreases due to traffic loading or offloading of the APs in other tiers.

The traffic management strategy in this paper is to make the APs in the first $M-1$ tiers that can access the licensed and unlicensed spectra be able to achieve higher per-user link throughput whereas the APs in the $M$th tier that only access the unlicensed spectrum just need to have their per-user link throughput above some minimum required value. The per-user link throughput is defined based on the spectrum efficiencies of the APs in the licensed and unlicensed spectra. We propose a decentralized traffic management scheme that maximizes the per-user throughput of the APs in the first $M-1$ tiers while the per-user link throughput requirement of the tier-$M$ APs is satisfied. We also propose a centralized traffic management scheme that can be performed by the backhaul core network to optimally manage traffic offloading or loading of all APs in order to maximize the defined per-user network throughput. This is our fourth contribution. In addition, a numerical example of coexisting LTE BSs and WiFi APs is provided to verify our
II. System Model and Preliminaries

Consider a large-scale interference-limited heterogeneous wireless network consisting of $M$ tiers of access points (APs). All the APs in the same tier are of the same type and performance. Specifically, the APs in the $m$th tier, denoted by set $X_m$, follow an independent marked Poisson point process (PPP) of intensity $\lambda_m$ defined as follows

$$X_m \triangleq \{(X_{mi}, P_m, V_{mi}) : X_{mi} \in \mathbb{R}^2, P_m \in \mathbb{R}_{++}, V_{mi} \in \{0, 1\}, i \in \mathbb{N}_+\}, m \in \mathcal{M},$$

(1)

where $\mathcal{M} \triangleq \{1, 2, \ldots, M\}$, $X_{mi}$ denotes AP $i$ in the $m$th tier and its location, $P_m$ is the transmit power used by the APs in the $m$th tier, and $V_{mi}$ is a Bernoulli random variable indicating whether AP $X_{mi}$ is void or not: if AP $X_{mi}$ is associated at least one user (i.e., it is not void), then $V_{mi} = 1$ and zero otherwise. Without loss of generality, we assume the APs in the $M$th tier only use the unlicensed spectrum to deliver data, and all other APs in the first $M - 1$ tiers primarily use the licensed spectrum and opportunistically use the unlicensed spectrum by carrier aggregation to transmit data if they have a chance to access the unlicensed spectrum. This network model with unlicensed spectrum sharing has a practical application context. For example, in a heterogeneous cellular network, LTE-U macro and small cell base stations (BSs) that consist of the APs in the first $M - 1$ tiers are able to coexist and share the unlicensed spectrum with WiFi APs in the $M$th tier if the LTE-U BSs can use the carrier aggregation technique to integrate the licensed and unlicensed available spectra [2], [3].

All users also form an independent PPP $U$ of intensity $\mu$ given by

$$U \triangleq \{U_j : U_j \in \mathbb{R}^2, \forall j \in \mathbb{N}_+\}$$

(2)

and we assume there is a typical user $U_0$ located at the origin without loss of generality. Our following location-dependent analyses will be based on typical user $U_0$ for simplicity since the analytical results do not depend on where the typical user is located due to Slivnyak’s theorem [18]. We consider a downlink transmission scenario in this paper and each user selects its serving AP $X_o$ by adopting the following AP association scheme

$$X_o \triangleq \arg \sup_{X_{mi} \in \bigcup_{m=1}^{M} X_m} W_{mi} \|X_{mi}\|^{-\alpha} = \arg \inf_{X_{mi} \in \bigcup_{m=1}^{M} X_m} W_{mi}^{\frac{1}{\alpha}} \|X_{mi}\|,$$

(3)

where $W_{mi}$ is the random AP association weight with mean $\mathbb{E}[W_{mi}] = \tilde{w}_m$ for AP $X_{mi}$, $\|X_i - X_j\|$ denotes the distance between nodes $X_i$ and $X_j$ for $i \neq j$, and $\|X_{mi}\|^{-\alpha}$ is called the pathloss of
AP $X_m$ with pathloss exponent $\alpha > 2$. Furthermore, we assume that all $W_m$'s are independent and i.i.d. for the same $m$, and the $a$-fractional moment of $W_m$ always exists$^1$ for all $i \in \mathbb{N}_+$ and $m \in \mathcal{M}$, i.e., $\mathbb{E}[W_m^a] < \infty$ for all $a \in (0, 1)$. Note that the scheme in (3) makes users associate an AP in any tier no matter which spectrum the AP primarily/only uses, and it can cover several different pathloss-based AP association schemes by changing the design of the AP association weights. For instance, scheme (3) becomes the biased nearest AP association (BNA) scheme if $W_m$ is a constant, or it becomes the biased mean strongest AP association (BMSA) scheme if $W_m \equiv b_m P_m H_m(s)$ for all $m \in \mathcal{M}$ where $b_m > 0$ is a constant bias and $H_m(s)$ characterizes the large-scale channel shadowing gain of the tier-$m$ APs, and other schemes, etc. [19] [20].

A. AP Association Probability and Cell Load Statistics

The AP association scheme in (3) can be reformulated to statistically represent the weighted pathloss of AP $X_o$ given by

$$W_o \|X_o\|^{-\alpha} \overset{d}{=} \sup_{X_m \in \bigcup_{m=1}^{M} X_m} W_m \|X_m\|^{-\alpha},$$

where $W_o \in \{W_m, m \in \mathcal{M}, i \in \mathbb{N}_+\}$ is the AP association weight used by AP $X_o$ and $\overset{d}{=}$ means the statistical equivalence in distribution. Let $\|\tilde{X}_o\|^{-\alpha} = W_o \|X_o\|^{-\alpha}$ and the cumulative distribution function (CDF) of $\|\tilde{X}_o\|^{-\alpha}$ can be shown as [11], [20]

$$F_{\|\tilde{X}_o\|}(x) = 1 - \exp\left(-\pi x^2 \tilde{\lambda}\right),$$

where $\tilde{\lambda} \triangleq \sum_{m=1}^{M} \tilde{\lambda}_m$ and $\tilde{\lambda}_m \triangleq \lambda_m \mathbb{E}\left[W_m^2\right]$. Thus, the CDF of $\|\tilde{X}_o\|$ essentially indicates that $\|\tilde{X}_o\|$ is the shortest distance between the PPP of intensity $\tilde{\lambda}$ and typical user $U_0$, and the probability that users associate with a tier-$m$ AP is given by

$$\vartheta_m = \frac{\lambda_m}{\tilde{\lambda}} = \frac{\lambda_m \mathbb{E}\left[W_m^2\right]}{\sum_{k=1}^{M} \lambda_k \mathbb{E}\left[W_k^2\right]},$$

which can be used to characterize the distribution of the number of users associating an AP in a particular tier as shown in the following lemma.

**Lemma 1:** Let $A_m$ denote the cell area where all users associate AP $X_{m_i}$ by using the AP association scheme (3). If the cell load of a tier-$m$ AP, denoted as $\mathcal{X}_m(A_m)$, is defined as the

$^1$Throughout this paper, we define the $a$-fractional moment of random variable $Z$ as $\mathbb{E}[Z^a]$ for all $a \in (0, 1)$ and all $m \in \mathcal{M}$.
number of users associating a tier-\(m\) AP and \(\mathbb{E}[W_m^{2/\alpha}]\mathbb{E}[W_m^{-2/\alpha}] < \infty\), then its probability mass function (pmf) is given by

\[
\mathbb{P}[X_m(A_m) = n] = \frac{\Gamma(n + \zeta_m)}{n! \Gamma(\zeta_m)} \left( \frac{\mu \vartheta_m}{\zeta_m \lambda_m} \right)^n \left( \frac{\zeta_m \lambda_m}{\zeta_m \lambda_m + \mu \vartheta_m} \right)^{n+\zeta_m}, \quad n \in \mathbb{N}, \tag{7}
\]

where \(\Gamma(x) = \int_0^{\infty} t^{x-1}e^{-t}dt\) is the Gamma function, \(\zeta_m \triangleq \frac{7}{2} \mathbb{E}[W_m^{2/\alpha}]\mathbb{E}[W_m^{-2/\alpha}]\).

**Proof:** Let us define \(\tilde{X}_m \triangleq \{\tilde{X}_m : \tilde{X}_m = W_m^{-\frac{1}{2}} X_m, X_m \in X_m\}\) so that the intensity of \(\tilde{X}_m\) is \(\tilde{\lambda}_m\) based on (5). Since users adopt the scheme in (3) to select their serving AP and \(\tilde{X}_o\) can be viewed as the nearest point from \(\bigcup_{m=1}^{M} \tilde{X}_m\) to typical user \(U_0\), cell load \(X_m(A_m)\) has the same distribution as \(\tilde{X}_m(A_m)\) where \(A_m\) is the cell area of AP \(\tilde{X}_m \in \tilde{X}_m\) and it is Voronoi-tessellated. Hence, it follows that

\[
\mathbb{P}[X_m(A_m) = n] = \mathbb{P}[\tilde{X}_m(A_m) = n] = \mathbb{E}_{A_m} \left[ \left( \frac{\mu \tilde{\lambda}_m \tilde{A}_m / \tilde{\lambda}}{n!} \right)^n e^{-\mu \tilde{\lambda}_m \tilde{A}_m / \tilde{\lambda}} \right].
\]

Although the exact probability density function (pdf) of a Voronoi-tessellated area is still an open problem, its substitute expression, accurately approximated by a Gamma random variable, is found as [19], [20]

\[
f_{\tilde{A}_m}(x) = \frac{(\zeta_m \lambda_m x)^{\zeta_m}}{x \Gamma(\zeta_m)} e^{-\zeta_m \lambda_m x}.
\]

Thus,

\[
\mathbb{P}[X_m(A_m) = n] = \frac{1}{n!} \left( \frac{\mu \tilde{\lambda}_m \tilde{\lambda}}{\lambda} \right)^n \frac{(\zeta_m \lambda_m)^{\zeta_m}}{\Gamma(\zeta_m)} \int_0^{\infty} x^{n+\zeta_m-1} e^{-\mu \tilde{\lambda}_m \tilde{A}_m / \tilde{\lambda} + \zeta_m \lambda_m x} dx,
\]

and substituting (6) into \(\mathbb{P}[X_m(A_m) = n]\) in above yields the result in (7).

The pmf of the cell load in Lemma 1 reveals that the probability of no users associating a tier-\(m\) AP is

\[
\mathbb{P}[V_m = 0] = \mathbb{P}[X_m(A_m) = 0] = \left( 1 + \frac{\mu \vartheta_m}{\zeta_m \lambda_m} \right)^{-\zeta_m}, \tag{8}
\]

which is the void probability of a tier-\(m\) AP. When \(\zeta_m\) (or \(\mathbb{E}[W_m^{2/\alpha}]\)) goes to infinity, \(\mathbb{P}[V_m = 0]\) reduces to its minimum \(\lim_{\zeta_m \to \infty} \mathbb{P}[V_m = 0] = \exp \left( -\frac{\mu}{\lambda_m} \right)\). Thus, the void probabilities of the APs cannot be ignored if the ratios of the user intensity to the AP intensities are not large, especially for the network with densely deployed APs. However, almost all prior works on the modeling of the multi-tier PPP-based HetNets overlook this important void AP issue and overlooking this issue could lead to inaccurately and excessively modeling the network.
interference $[9], [19]$. Also, the mean cell load of a tier-$m$ AP is given by

$$\mathbb{E} [\mathcal{X}_m(A_m)] = \frac{\bar{\lambda}_m}{\lambda} \mu \times \frac{1}{\bar{\lambda}_m} = \frac{\mu \mathbb{E} [W_m^{2 \alpha}]}{\sum_{k=1}^{M} \lambda_k \mathbb{E} [W_k^{2 \alpha}]}.$$  \hspace{1cm} (9)

To completely balance the mean load between all tiers, all $W_m$’s must be i.i.d. for all $m \in \mathcal{M}$. For example, the (unbiased) nearest AP association scheme that makes user associate their nearest AP can achieve a completely balanced and same mean cell load for all APs in different tiers since all $W_m$’s in (3) are the same constant.

B. Channel Access Protocols

In this paper, all APs are assumed to always have data to transmit to their tagged users. The channel access protocols for the licensed spectrum and unlicensed spectrum are quite different. All APs in the first $M-1$ tiers share the entire licensed spectrum at the same time and they are synchronized when accessing the licensed channel$^2$. Note that the APs in the $M$th tier cannot access the licensed channel and they are only allowed to access the channel in the unlicensed spectrum. All APs have to use the (slotted non-persistent) opportunistic CSMA/CA protocol to access the unlicensed channel$^3$. By adopting such an opportunistic CSMA/CA protocol, the APs that have their channel gain greater than some threshold are qualified and synchronized to contend the unlicensed channel in the predesignated time slots. This opportunistic CSMA/CA protocol is able to make the unlicensed spectrum effectively be utilized by the APs with good channel conditions so as to improve the spectrum sharing efficiency and throughput. Each AP in the $m$th tier that performs the opportunistic CSMA/CA protocol has a sensing region $S_m$ in which all unlicensed channel accessing activities can be detected by the AP. The channel access probability of the opportunistic CSMA/CA protocol is already derived in our previous works $[9] [11]$, and it can be modified for the AP association scheme in (3), as shown in the following lemma.

**Lemma 2:** Let $q_{m,0} \triangleq \mathbb{P}[V_m \neq 0]$ denote the non-void probability of a tier-$m$ AP. If the random backoff time of a tier-$m$ AP is uniformly distributed in $[0, \tau_m]$, then its channel access

$^2$Such a licensed channel access protocol is widely used in the cellular networks. In addition, we assume there is only one channel in the licensed spectrum for the ease of analysis.

$^3$Like the case in the licensed spectrum, we assume there is only one available channel in the unlicensed spectrum in order to simplify our following analysis.
probability is

\[
\rho_m = 1 - e^{-\frac{\tau_m}{\tilde{w}_m}} \sum_{k=1}^{M} A_{m,k} \lambda_k^{\tau_k} + \sum_{j=m}^{M} \left( e^{(\tau_j - \tau_{j+1})} \sum_{k=1}^{j} A_{m,k} \left( \frac{\tilde{w}_k}{\tilde{w}_m} \right)^{\frac{2}{\alpha}} \lambda_k^{\tau_{j+1}} \right),
\]

where \( \tau_1 \geq \tau_2 \geq \cdots \geq \tau_M \geq 0, \tau_{M+1} = 0 \), \( A_{m,k} \) is the mean area of region \( S_m \) where the tier-\( k \) APs are distributed, \( \lambda_k^{\tau_k} = \xi_k q_{k,0} \lambda_k \left( \frac{\tau_k}{\tau_k} \right) \), \( \xi_k \in [0, 1] \) is the probability that the unlicensed channel (power) gain from a tier-\( k \) AP to its servicing user is greater than threshold \( \delta > 0 \). If all \( \tau_m \)'s are the same and equal to \( \tau \), (10) reduces to

\[
\rho_m = 1 - \exp \left( -\tau \sum_{k=1}^{M} A_{m,k} \xi_k q_{k,0} \lambda_k \left( \frac{\tilde{w}_k}{\tilde{w}_m} \right)^{\frac{2}{\alpha}} \right).
\]

The channel access probability in (10) indicates not only how much chance a tier-\( m \) AP can successfully access the unlicensed channel in a particular timeslot but also the fraction of time it can access the unlicensed channel in the long-term sense. Adjusting the random backoff time limit \( \tau_m \) can make tier-\( m \) APs have more/less priority or time fraction to access the unlicensed spectrum. For example, if the tier-\( M \) APs represent the WiFi APs, we can make its backoff time limit \( \tau_M \) much shorter than those of the APs in the first \( M - 1 \) tiers so that the throughput of the WiFi APs is able to remain at some level and not significantly reduced when the unlicensed spectrum is shared by many APs in the first \( M - 1 \) tiers at the same time. This is similarly implementing the ideas of the Listen-Before-Talk (LBT) with Carrier Sensing Adaptive Transmission (CSAT) and Licensed-Assisted Access (LAA) protocols proposed in the LTE-U [2], [3]. Another two characteristics of the result in (10) are to consider the void probability of APs as well as the probability of opportunistically having a good channel state. They make the channel access probability more accurate and higher for the APs with good channels. In the following section, we will see how the channel access probability plays a pivotal role in analyzing the throughput of users.

III. SHANNON TRANSFORM, MEAN SPECTRUM EFFICIENCY AND PER-USER THROUGHPUT

In this section, the mean spectrum efficiencies (or called ergodic link capacities per unit bandwidth) of a user in the unlicensed and licensed spectra are studied and they will be used for designing the traffic management schemes in the next section. Finding the explicit expressions of these mean spectrum efficiencies is an important task since they give us insight into achieving the throughput optimality by managing traffic flows between tiers. The prior approach to deriving the mean spectrum efficiency in the literature is based on integrating the function of the coverage
(success) probability under the Rayleigh fading channel model [21]–[23]. This approach does not work for the mean spectrum efficiencies with non-Rayleigh fading channels. In the following analysis, we will show how to derive the mean spectrum efficiency in a tractable form for any random models of the channel gains and AP association weights. First, we need to introduce the identity of the Shannon transform of a nonnegative random variable since it is the key to obtaining the analytically tractable expression of the mean spectrum efficiency without making any specific modeling assumptions on the channel gains and AP association weights.

A. The Shannon Transform and Its Identity

The Shannon transform of a nonnegative random variable \( \Psi \) for any nonnegative \( \eta \in \mathbb{R}_+ \) is defined as [24]

\[
S_\Psi(\eta) = \mathbb{E}[\ln(1 + \eta \Psi)].
\]  

(12)

The Shannon transform in (12) has an identity as shown in the following theorem.

**Theorem 1 (The identity of the Shannon transform):** Suppose the Laplace transform of the reciprocal of nonnegative random variable \( \Psi \) always exists, i.e., \( \mathcal{L}_{\Psi^{-1}}(s) \triangleq \mathbb{E}[e^{-s\Psi^{-1}}] < \infty \). If \( S_\Psi(\eta) \) in (12) exists for any \( \eta \in \mathbb{R}_+ \), then it has the following identity

\[
S_\Psi(\eta) = \int_{0^+}^{\infty} \frac{(1 - e^{-\eta s})}{s} \mathcal{L}_{\Psi^{-1}}(s) ds,
\]  

(13)

which always holds. Furthermore, we can have

\[
\mathbb{E}[S_\Psi(\eta)] = \int_{0^+}^{\infty} \frac{[1 - \mathcal{L}_\eta(s)]}{s} \mathcal{L}_{\Psi^{-1}}(s) ds
\]  

(14)

if \( \eta \) is a nonnegative random variable and its Laplace transform exists.

**Proof:** The Shannon transformation of random variable \( Z \) can be rewritten as

\[
S_Z(\eta) = \int_0^1 \mathbb{E}\left[\frac{\eta Z}{1 + \eta y Z}\right] dy = \int_0^1 \mathbb{E}\left[\frac{1}{1/\eta Z + y}\right] dy.
\]

Since \( \mathcal{L}_{Z^{-1}}(s) \) always exists, for any \( y \in [0, 1] \) we have

\[
\mathbb{E}\left[\frac{1}{1/\eta Z + y}\right] = \int_0^\infty e^{-uy} \mathbb{E}\left[e^{-u/\eta Z}\right] du = \int_0^\infty e^{-uy} \mathcal{L}_{Z^{-1}}(u/\eta) du = \int_0^\infty e^{-\eta y u} \mathcal{L}_{Z^{-1}}(s) \eta ds
\]

and then substituting this result into \( S_Z(\eta) \) yields

\[
S_Z(\eta) = \eta \int_0^\infty \int_0^1 e^{-\eta y u} \mathcal{L}_{Z^{-1}}(s) dy ds = \int_0^\infty \int_0^{1/\eta s} (1 - e^{-\eta y}) s e^{\frac{x}{s}} f_Z(z) dz ds = \int_0^\infty \frac{(1 - e^{-\eta s})}{s} \mathcal{L}_{Z^{-1}}(s) ds,
\]
which is exactly the result in (13). The result in (14) readily follows from (13) and the definition
of the Laplace transform of a nonnegative random variable.

The identity of the Shannon transformation in Theorem[1] provides a tractable means to find the
mean spectrum efficiency of a link if $\eta \Psi$ is the SIR of the link and the Laplace transform of
their reciprocal exists and can be explicitly found. As we will show in the following subsections,
Theorem[1] facilitates the derivations of the mean spectrum efficiencies of an AP in the unlicensed
and licensed spectra for any general random channel gain and AP association weight models.

B. Mean Spectrum Efficiency of the APs in the Unlicensed Spectrum

Consider the scenario that typical user $U_0$ associates a tier-$m$ AP by scheme (3) and is receiving
data over the unlicensed channel. The mean spectrum efficiency (bps/Hz) the tier-$m$ AP offers
to the user in the unlicensed spectrum is defined as

$$ R_{U_m} \triangleq \mathbb{E} \left[ \log_2 \left( 1 + \frac{P_m H_m}{I_{U_m} \| X_o \|^{-\alpha} T_m \Xi_m} \right) \right], \quad m \in \mathcal{M}, \quad (15) $$

where $P_m$ is the transmit power used by AP $X_o \in X_m$, $H_m$ is the random channel (power)
gain with mean $\bar{h}_m$ from AP $X_o$ to typical user $U_0$, and $I_{U_m}$ is the interference as shown in the
following

$$ I_{U_m} \triangleq \sum_{X_{m_i} \in \bigcup_{m=1}^{M} X_m \setminus X_o} P_m H_m V_{m_i} T_{m_i} \Xi_{m_i} \| X_{m_i} \|^{-\alpha} $$
in which $H_{m_i}$ denotes the random channel gain with mean $\bar{h}_m$ from AP $X_{m_i}$ to typical user
$U_0$, $\Xi_{m_i} \in \{0, 1\}$ is a Bernoulli random variable that equals to one if AP $\tilde{X}_{m_i}$’s channel gain is
greater than threshold $\delta$ and zero otherwise, $T_{m_i} \in \{0, 1\}$ is also a Bernoulli random variable
that equals to one if AP $X_{k_i}$ can access the unlicensed channel and zero otherwise, $T_m \in \{T_{m_i}\}$
indicates whether AP $X_o$ can access the unlicensed channel, and $\Xi_m \in \{\Xi_{m_i}\}$ indicates whether
the channel gain of AP $X_o$ in the unlicensed spectrum is greater than threshold $\delta$. For the ease
of analysis, we assume that all $H_{m_i}/\bar{h}_m$’s are i.i.d. for all $i \in \mathbb{N}_+$ and $m \in \mathcal{M}$.

Letting $\Psi_{U_m} \triangleq P_m H_m T_m \Xi_m / I_{U_m} \| X_o \|^{-\alpha}$ and using the definition of the Shannon transform in
the previous subsection, $R_{U_m}$ can be further expressed as

$$ R_{U_m} = \frac{S_{\Psi_{U_m}}(1)}{\ln(2)} = \frac{\rho_m \xi_m}{\ln(2)} \int_0^{\infty} \frac{1 - L_{H_m \Xi_m \| X_o \|^{-\alpha}}(s)}{s} \mathcal{L}_{F_{m_i} T_{m_i} \Xi_m}(s) ds, \quad (16) $$

where $\xi_m \triangleq \mathbb{P}[\Xi_m = 1]$ and $\rho_m \triangleq \mathbb{P}[T_m = 1]$ for all $m \in \mathcal{M}$. The explicit expression of $R_{U_m}$ is
derived as shown in the following theorem.
**Theorem 2:** Suppose users adopt the AP association scheme in (3). If all non-void APs that use the opportunistic CSMA/CA protocol to access the unlicensed channel, the mean spectrum efficiency of the user in the unlicensed spectrum in (16) can be explicitly lower bounded by

\[
R_{U_m} \geq \frac{\rho_m \xi_m}{\ln(2)} \int_0^\infty \frac{[1 - \mathcal{L}_{\tilde{H}}(u)]}{u \left( \sum_{k=1}^{M} q_k \xi_k \rho_k \tilde{\vartheta}_k \ell_H(\frac{\tilde{w}_m \tilde{P}_k}{\tilde{w}_m h_m P_m u, \frac{2}{\alpha}}) + 1 \right)} \, du,
\]

where \( \tilde{H} \triangleq \frac{H_m \tilde{w}_m}{W_m h_m} \) is a random variable with unit mean and \( \ell_Z(x, y) \) for \( y \in (0, 1) \) is defined as

\[
\ell_Z(x, y) \triangleq x^y \Gamma(1 - y) \mathbb{E}[Z^y] + \int_0^1 \mathcal{L}_Z \left( x t^{-\frac{1}{y}} \right) \, dt - 1.
\]

If all \( W_m \)'s are deterministic and equal to constant \( \tilde{w}_m \) and all \( H_{m_i} \)'s are i.i.d. random variables with unit mean for all \( m \in \mathcal{M} \), then (17) reduces to

\[
R_{U_m} \geq \frac{\rho_m \xi_m}{\ln(2)} \int_{0+}^\infty \frac{[1 - \mathcal{L}_H(u)]}{u \left( \sum_{k=1}^{M} q_k \xi_k \rho_k \tilde{\vartheta}_k \ell_H(\frac{\tilde{w}_m \tilde{P}_k}{\tilde{w}_m h_m P_m u, \frac{2}{\alpha}}) + 1 \right)} \, du,
\]

where random variable \( H \) with unit mean has the same distribution as all \( H_{m_i} \)'s.

**Proof:** See Appendix A.

It is worth mentioning a few features of the lower bounds in (17) and (19). First of all, the lower bound is in general fairly tight since the location correlations between the non-void APs due to AP association and opportunistic CSMA/CA are usually very weak [19], [20]. When either the user intensity or the channel gain threshold for opportunistic CSMA/CA increases, these location correlations due to AP association is weakened so that \( R_{U_m} \) is very close to its lower bound. The tightness of the lower bound in (17) will be verified by the numerical results presented in Section V-B. Second, the lower bound in (17) is valid for all random models of channel gains and AP association weights as long as the Laplace transforms of the channel gains and the AP association weights exist, which is never derived in the prior works. This is a very important feature since we are able to realize how different channel and AP association models affect \( R_{U_m} \) and gain some insights about how to improve \( R_{U_m} \) by appropriately designing the AP association weights in order to manage the traffic flows between different tiers of the APs. Third, according to Jensen’s inequality, we know \( \mathcal{L}_Z(u) \geq \exp(-u\mathbb{E}[Z]) \), which means the lower bound in (17) will increase (i.e., \( R_{U_m} \) will increase) if \( H_m \) and/or \( W_m \) are not random. For example, in the special case that there are no random channel gain impairments and users adopt the BNA scheme (i.e., \( H_m \equiv 1 \) and all \( W_{m_i} \)'s are equal to constant \( w_m \) for all \( i \in \mathbb{N}_+ \)) or in the special case of the AP association scheme with \( W_{m_i} = b_m P_m H_{m_i} \) for all \( m \in \mathcal{M} \), the
lower bound in (19) becomes

$$R_u \geq \frac{\rho_m \xi_m}{\ln(2)} \int_{0^+}^{\infty} \frac{(1 - e^{-u})}{u} \left( \ell(u, \frac{2}{\alpha}) \sum_{k=1}^{M} q_{k,0} \xi_k \rho_k \vartheta_k + 1 \right) \, du,$$

(20)

where \( \ell(x, y) \triangleq x^y \Gamma(1 - y) + \int_{0}^{1} e^{-xt - \frac{1}{2}y} \, dt - 1 \), and this is the maximum lower bound achieved by the biased BNA scheme.

C. Mean Spectrum Efficiency of the APs in the Licensed Spectrum

For the mean spectrum efficiency of a user associating a tier-\( m \) AP in the licensed spectrum, its formal definition can be written as

$$R_{L_m} = \mathbb{E} \left[ \log_2 \left( 1 + \frac{P_m H_m}{I_{L_m} \|X_o\|^\alpha} \right) \right], \quad m \in \mathcal{M} \setminus M,$$

(21)

where \( X_o \in \mathcal{X}_m \) and \( I_{L_m} \) is given by

$$I_{L_m} \triangleq \sum_{X_{m_i} \in \bigcup_{m=1}^{M-1} \mathcal{X}_m \setminus X_o} P_m H_{m_i} V_{m_i} \|X_{m_i}\|^{-\alpha}.$$

(22)

Note that all channel gains in (21) are evaluated in the licensed spectrum. The explicit result of \( R_{L} \) is shown in the following theorem.

**Theorem 3:** If users adopt the AP association scheme in (3), then the mean spectrum efficiency of a user in the licensed spectrum defined in (21) has a lower bound given by

$$R_{L_m} \geq \frac{1}{\ln(2)} \int_{0^+}^{\infty} \frac{[1 - \mathcal{L}_{\tilde{H}}(u)]}{u} \, du \left( \sum_{k=1}^{M-1} q_{k,0} \vartheta_k \ell_{\tilde{H}} \left( \frac{\bar{w}_m \bar{h}_k P_k}{\bar{w}_k \bar{h}_m P_m} \, u, \frac{2}{\alpha} \right) + 1 \right), \quad m \in \mathcal{M} \setminus M.$$

(23)

If all \( W_m \)'s are deterministic and equal to \( \bar{w}_m \) and all \( H_{m_i} \)'s are i.i.d. random variables with unit mean for all \( m \in \mathcal{M} \), (23) reduces to

$$R_{L_m} \geq \frac{1}{\ln(2)} \int_{0^+}^{\infty} \frac{[1 - \mathcal{L}_{H}(u)]}{u} \, du \left( \sum_{k=1}^{M-1} q_{k,0} \vartheta_k \ell_{H} \left( \frac{\bar{w}_m \bar{h}_k P_k}{\bar{w}_k \bar{h}_m P_m} \, u, \frac{2}{\alpha} \right) + 1 \right).$$

(24)

**Proof:** First, consider the case that all \( W_{m_i} \)'s are random variables. Since all APs in the first \( M - 1 \) tiers access the licensed spectrum without using using the opportunistic CSMA/CA protocol, the mean spectrum efficiency for a user associating a tier-\( m \) AP can be explicitly obtained as

$$\mathbb{E} \left[ \log_2 \left( 1 + \frac{P_m H_m}{I_{L_m} \|X_o\|^\alpha} \right) \right] = \frac{1}{\ln(2)} \int_{0^+}^{\infty} \frac{[1 - \mathcal{L}_{\tilde{H}_m}(u)]}{u} \, du \left( \sum_{k=1}^{M-1} q_{k,0} \vartheta_k \ell_{\tilde{H}_m} \left( \frac{\bar{w}_m \bar{h}_k P_k}{\bar{w}_k \bar{h}_m P_m} \, u, \frac{2}{\alpha} \right) + 1 \right)$$

(25)
by applying the result of $R_{U_m}$ for the mean spectrum efficiency for $m$th tier with $\rho_k = \xi_k = 1$ for all $k \in \mathcal{M}$. Substituting (25) into (21) results in (23). The result in (24) can be readily inferred from (23) by letting $\hat{H}_m = H$ for all $m \in \mathcal{M}$.

The lower-bound results in Theorem 3 are also valid for any random models of the channel gains and AP association weights. Note that prior works on the spectrum efficiency in the licensed spectrum are only derived for Rayleigh fading channels and do not consider the impact of the void APs. The lower bound on $R_{L_m}$ in (23) is obtained by assuming the non-void correlated APs form $M$ independent thinning homogeneous PPPs and in general it is also very tight, like the lower bounds on $R_{U_m}$ in Theorem 2, since the location correlations of the non-void APs are fairly weak. Thus, as the user intensity goes to infinity, $q_{k,0}$ converges to one so that $R_{L_m}$ exactly reduces to the lower bound given by

$$\lim_{\mu \to \infty} R_{L_m} = \frac{1}{\ln(2)} \int_{0^+}^{\infty} \frac{[1 - \mathcal{L}_{\hat{H}_m}(u)] \, du}{u \left[ \sum_{k=1}^{M-1} \varrho_k \ell(u, \frac{2}{\alpha}) \sum_{k=1}^{M-1} q_{k,0} \varrho_k + 1 \right]} \cdot (26)$$

which is the lowest mean spectrum efficiency of the tier-$m$ APs in the licensed spectrum. This indicates that the mean spectrum efficiency is significantly underestimated in a dense network if the void AP impact is not considered in the interference model. Also, if the AP association scheme in (3) with $W_{m_i} = b_m P_m H_m$ can characterize the channel gains, the lower bound in (23) becomes

$$R_{L_m} \geq \frac{1}{\ln(2)} \int_{0^+}^{\infty} \frac{(1 - e^{-u}) \, du}{u \left[ \ell(u, \frac{2}{\alpha}) \sum_{k=1}^{M-1} q_{k,0} \varrho_k + 1 \right]} \cdot (27)$$

which is the maximum lower bound achieved by AP association and it is also the lower bound for the scenario that channels do not suffer random impairments and users select their APs by using the BNA scheme.

D. Per-User Throughput Analysis

We have characterized the mean spectrum efficiencies of the licensed and unlicensed spectra in the previous subsections, which can be used to characterize the total (licensed and unlicensed) link throughput of a tier-$m$ AP. Assume the bandwidths of the licensed spectrum and unlicensed spectrum are denoted by $B_L$ and $B_U$, respectively. Accordingly, the total link throughput of a tier-$m$ AP can be expressed as

$$C_m = B_L R_{L_m} \mathbb{1}(m \neq M) + B_U R_{U_m}, \ m \in \mathcal{M}, \ (28)$$
where \( \mathbb{I}(E) \) is an indicator function which is one if event \( E \) is true and zero otherwise. Note that the link throughput of a tier-\( M \) AP, only in the unlicensed spectrum, is \( C_M = B_U R_{U_M} \). The total link throughput of each AP highly depends on how the AP association weights in (3) are designated. For instance, when \( \mathbb{E}[W_m^{2/\alpha}] \) becomes larger, more users associate the tier-\( m \) APs (i.e., more traffic is offloaded to the tier-\( m \) APs) so that \( R_{L_m} \) and \( R_{U_m} \) would change very likely due to interference variations since the void probability of the tier-\( m \) APs reduces and the void probabilities of the APs in other tiers increase correspondingly. In this case, whether \( C_m \) increases (or decreases) is dependent upon whether the interferences in the licensed and unlicensed spectra decrease (or increase) and the channel access probability of the APs in the unlicensed spectrum decreases (or increases) due to offloading traffic to the tier-\( m \) APs.

Recall that the mean cell load of a tier-\( m \) AP given in (9), denoted by \( \mathbb{E}[X_m(A_m)] \), is the mean number of users associating a tier-\( m \) AP. By assuming all users equally share the spectrum bandwidths, the per-user link throughput of the tier-\( m \) APs is defined as

\[
c_m \triangleq \frac{C_m}{\mathbb{E}[X_m(A_m)|V_m = 1]} = \frac{q_{m,0}\lambda_m}{\mu \vartheta_m} \left[ B_L R_{L_m} \mathbb{I}(m \neq M) + B_U R_{U_m} \right].
\]  

(29)

In general, \( c_m \) decreases as the traffic offloaded to the tier-\( m \) APs increases in that \( \mathbb{E}[X_m(A_m)] \) increases but \( C_m \) may not increase as explained in above. We can call \( \sum_{m=1}^M c_m \vartheta_m \) “average” per-user throughput since \( \vartheta_m \) is the probability that a user associates a tier-\( m \) AP, and also \( \sum_{m=1}^M c_m \vartheta_m \) can be interpreted as the per-user network throughput because it can be explicitly written as

\[
\sum_{m=1}^M c_m \vartheta_m = \frac{B_L}{\mu} \sum_{m=1}^M q_{m,0}\lambda_m \left[ R_{L_m} \mathbb{I}(m \neq M) + \frac{B_U}{B_L} R_{U_m} \right] = \frac{1}{\mu} \sum_{m=1}^M q_{m,0}\lambda_m C_m,
\]

(30)

where \( \frac{1}{\mu} \sum_{m=1}^M q_{m,0}\lambda_m C_m \) characterizes how much network throughput (per unit area) a user can obtain when the void cell issue is considered and the unlicensed spectrum is opportunistically shared by all APs. In the following section, we will study how to maximize the per-user link throughput and per-user network throughput by the proposed traffic management methods.

IV. TRAFFIC MANAGEMENT FOR COEXISTING LICENSED AND UNLICENSED APs

The underlying HetNet considered in this paper models a realistic scenario in which unlicensed WiFi APs in the \( M \)th tier share their unlicensed spectrum with licensed LTE APs in the first \( M-1 \) tiers. The traffic management in this network focuses on how to offload or load traffic between two orthogonal spectrum domains so that the per-user-based throughput increases. Since the APs...
in the first $M-1$ tiers could simultaneously assess the unlicensed channel, their transmitting behaviors in the unlicensed spectrum definitely affect the throughput of the APs in the $M$th tier. Accordingly, the primary premise of managing the traffic in different tiers is to make the per-user link throughput of the APs in the $M$th tier greater than or equal to the minimum guaranteed throughput that is offered to the unlicensed users. Namely, assuming the per-user link throughput of the tier-$M$ APs must be at least greater than some minimum value $c_{\min}$, i.e., $c_M \geq c_{\min}$, which can be explicitly written by using $c_M$ in (29) as

$$\frac{q_{M,0}B_U}{c_{\min} \mu} \left( \sum_{m=1}^{M} \lambda_m \mathbb{E}[W_m^2] \right) \geq \frac{\mathbb{E}[W_M^2]}{R_{U,M}}.$$  (31)

Using $R_{L,m}$, $R_{U,m}$ and $c_m$ found in Section III and the constraint in (31), we are able to investigate how to maximize the per-user link throughput of the APs in any particular tier and the per-user network throughput by optimizing $\mathbb{E}[W_m^{2/\alpha}]$ for all $m \in \mathcal{M}$ in the following subsections. We first study the scheme of decentralized traffic management that makes APs locally decide to offload/load their traffic in order to maximize their per-user link throughput. Then we study the method of centralized traffic management that optimizes the per-user network throughput.

### A. Decentralized Traffic Management

According to Theorems 2, 3 and the per-user link throughput of the tier-$m$ APs given in (29), $c_m$ is significantly affected by all $\{\mathbb{E}[W_m^{2/\alpha}]\}$. Now our focus here is to study when an AP in a particular tier should independently offload its traffic to APs in other tiers (i.e., reduce its AP association weight) or load traffic from them (i.e., increase its AP association weight) so that its per-user throughput increases. This is essentially a “decentralized” traffic management problem since the traffic loading or offloading decision is independently made by each AP using its local information. Specifically, this decentralized traffic management problem is how to increase or even maximize $c_m$ by unilaterally optimizing $\mathbb{E}[W_m^{2/\alpha}]$ of a tier-$m$ AP under the constraint (31). Namely, we would like to solve the following optimization problem of $\omega_m \triangleq \mathbb{E}[W_m^{2/\alpha}]$ given by

$$\begin{cases} \max_{\omega_m \geq 0} c_m(\omega_m) \\ \text{s.t.} \quad \left( \sum_{k=1}^{M} \lambda_k \omega_k \right) \frac{R_{U,M}}{\omega_M} \geq \frac{c_{\min} \mu}{q_{M,0}B_U} \end{cases}.$$  (32)

The solution of this optimization problem exists as shown in the following lemma.

**Lemma 3:** Let $\Omega_m$ be the feasible set of $\omega_m$ with the constraint (31), i.e., it is

$$\Omega_m \triangleq \left\{ \omega_m \in \mathbb{R}^+: \left( \sum_{k=1}^{M} \lambda_k \omega_k \right) \frac{R_{U,M}}{\omega_M} \geq \frac{c_{\min} \mu}{q_{M,0}B_U}, \omega_k > 0, k \in \mathcal{M} \setminus m \right\}.$$  (33)
If $\Omega_m$ is nonempty, there exists a maximizer $\omega^*_m$ of $c_m$ over $\Omega_m$ for all $m \in \{1, 2, \ldots, M - 1\}$.

**Proof:** First, we would like to show that $\Omega_m$ is a compact set if it is nonempty. According to (10) and (17), we can infer the following facts

$$\lim_{\omega_m \to 0} \left( \sum_{k=1}^{M} \lambda_k \omega_k \right) \frac{R_{U_M}}{\omega_M} < \infty \text{ and } \lim_{\omega_m \to \infty} \left( \sum_{k=1}^{M} \lambda_k \omega_k \right) \frac{R_{U_M}}{\omega_M} = 0.$$

Since $\left( \sum_{k=1}^{M} \lambda_k \omega_k \right) \frac{R_{U_M}}{\omega_M}$ is bounded, there must exist a positive number $R^\dagger_{U_M}$ at $\omega^\dagger_m$ such that

$$R^\dagger_{U_M} \left( \omega^\dagger_m \right) = \sup_{\omega_m > 0} \left\{ \left( \sum_{k \in M} \lambda_k \omega_k \right) \frac{R_{U_M}}{\omega_M} \right\} \geq \frac{c_{\min}}{q_{M,0}B_U}$$

if $\Omega_m$ is nonempty. For $\omega_m \in (\omega^\dagger_m, \infty)$, $\left( \sum_{k=1}^{M} \lambda_k \omega_k \right) \frac{R_{U_M}}{\omega_M}$ is a monotonically decreasing function of $\omega_m$. Accordingly, there must exist a $\omega^\dagger_m$ such that $\left( \sum_{k=1}^{M} \lambda_k \omega_k \right) \frac{R_{U_M}}{\omega_M} \geq \frac{c_{\min}}{q_{M,0}B_U}$ for $\omega_m \in (0, \omega^\dagger_m]$. Hence $\Omega_m = (0, \omega^\dagger_m]$ is closed and bounded. Furthermore, $c_m$ is a continuous function of $\omega_m$ over $\Omega_m$. According to the Weierstrass theorem [25], $\omega^*_m = \arg \max_{\omega_m \in \Omega_m} c_m(\omega_m)$ must exist.

Lemma 3 reveals that $\omega^*_m \in \{ \arg \max_{\omega_m \in \Omega_m} c_m(\omega_m) \}$ and $\frac{\partial c_m}{\partial \omega_m} \big|_{\omega_m = \omega^*_m} = 0$. However, finding $\omega^*_m$ needs the values of other $\omega_k$’s which are in general unknown for the APs in the $m$th tier in the decentralized context. Hence, we devise a recursive scheme of finding $\omega^*_m$ as follows.

The two fundamental traffic management rules for a tier-$m$ AP can be easily realized as

$$\frac{\partial c_m}{\partial \omega_m} < 0 \Leftrightarrow \begin{cases} \text{loading traffic reduces } c_m \\ \text{offloading traffic increases } c_m \end{cases}$$

and

$$\frac{\partial c_m}{\partial \omega_m} > 0 \Leftrightarrow \begin{cases} \text{loading traffic increases } c_m \\ \text{offloading traffic reduces } c_m \end{cases}.$$ (34)

These two rules manifest that the APs in the first $M - 1$ tiers need to offload traffic if $\frac{\partial c_m}{\partial \omega_m} < 0$ and load traffic if $\frac{\partial c_m}{\partial \omega_m} > 0$ under the constraint that the APs in the $M$th tier need to maintain their per-user throughput above the threshold value $c_{\min}$. According to the facts in (34) and (35), we develop a decentralized traffic management scheme for the APs in each tier as shown in the following theorem.

**Theorem 4:** For the APs in the $m$th tier and $m \in \{1, 2, \ldots, M - 1\}$, the following decentralized traffic management scheme can maximize their per-user throughput under the constraint
\[ \omega_m(n + 1) = \frac{c^*(n)N_m(n)\omega_m(n)}{C_m(n)}, \quad \omega_m(0) > 0, \quad n \in \mathbb{N}, \]  \hspace{1cm} (36)

where \( N_m(n) = \frac{1}{n} \sum_{i=0}^{n-1} N_m(i) \) denotes the average number of the users tagged a tier-\( m \) AP at time \( n \) and \( c^*(n) \triangleq \max\{c_{\text{min}}, c^*(n-1), C_m(n)/N_m(n)\} \). In addition, as \( n \) goes to infinity this scheme makes \( \omega_m \) converge to \( \omega^*_m \) that is the fixed point of the function \( \Upsilon_m(x) \) given by

\[ \Upsilon_m(x) = \frac{q_m,0(x)C_m(x)}{\mu c_{\text{min}}} \left( \sum_{k \in \mathcal{M} \setminus m} \lambda_k \omega_k + \lambda_m x \right). \]  \hspace{1cm} (37)

**Proof:** See Appendix B. 

Since a tier-\( m \) AP is able to estimate \( R_{L_m}(n) \) and \( R_{U_m}(n) \) and obtain other parameters in (36) locally, the scheme in (36) can be performed by the tier-\( m \) AP. Function \( \Upsilon_m(x) \) in (37) can assist us to properly determine the initial value \( \omega_m(0) \) of \( \omega_m(n) \) provided that each tier-\( m \) AP initially knows all other \( \omega_k \)'s and this would make \( \omega_m(n) \) quickly converge to \( \omega^*_m \). Note that in general the per-user link throughput achieved by the scheme in (36) in the steady state is just a suboptimal result because other \( M-1 \) parameters \( \{\omega_k, k \in \mathcal{M} \setminus m\} \) are not jointly optimized.

In the following subsection, a centralized traffic management scheme to maximize the per-user network throughput is proposed.

**B. Centralized Traffic Management**

If all the APs in the HetNet can send their information to their backhaul core network, the centralized traffic management is implementable. Under this circumstance, we can maximize the per-user network throughput in (30) under the constraint (31) by optimizing all \( \{\omega_m\} \) at the same time. That is, for the centralized traffic management we need to solve the following optimization problem of the per-user network throughput:

\[
\begin{align*}
\max_{\{\omega_m\} > 0} & \quad \sum_{m=1}^{M} q_m,0 \lambda_m \left[ R_{L_m} I(m \neq M) + \frac{B_L}{E_L} R_{U_m} \right] \\
\text{s.t.} & \quad \left( \sum_{m=1}^{M} \lambda_m \omega_m \right) \frac{R_{U_M}}{\omega_M} \geq \frac{c_{\text{min}} \mu}{q_m,0 B_L}.
\end{align*}
\]  \hspace{1cm} (38)

Note that the objective function of the optimization problem in (38) is the per-user network throughput normalized by constant \( \frac{\mu}{E_L} \) which does not affect the optimization solutions. The solution of this optimization problem exists as shown in the following lemma.
Lemma 4: Let $\Omega$ be the feasible set of the $M$-tuple vector $(\omega_1, \omega_2, \ldots, \omega_M)$ with the constraint (31) and it is expressed as

$$
\Omega \triangleq \left\{ (\omega_1, \omega_2, \ldots, \omega_M) \in \mathbb{R}^M_+ : \left( \sum_{m=1}^{M} \lambda_m \omega_m \right) \frac{RU_M}{\omega_M} \geq \frac{c_{\min \mu U_M}}{q_{M,0} B_U} \right\}.
$$

If $\Omega$ is nonempty, there exists an optimal $M$-tuple vector $(\omega_1^*, \omega_2^*, \ldots, \omega_M^*)$ that maximizes the per-user network throughput $\sum_{m=1}^{M} c_m \vartheta_m$.

**Proof:** According to Lemma 3, we know there exists an $M$-tuple vector $(\omega_1, \omega_2^*, \ldots, \omega_M)$ in set $\Omega_m$ if $\Omega_m$ is nonempty. In other words, there must exist a vector $(\omega_1^*, \omega_2^*, \ldots, \omega_M)$ in $\Omega_1 \times \Omega_2 \times \cdots \times \Omega_M \triangleq \prod_{m=1}^{M} \Omega_m$ if $\prod_{m=1}^{M} \Omega_m$ is nonempty since all $\Omega_m$’s are compact. Let $\Omega$ be an $M$-dimensional closed ball that encloses $\prod_{m=1}^{M} \Omega_m$, i.e., $\prod_{m=1}^{M} \Omega_m \subseteq \Omega$. Hence, $\Omega$ is compact as well as nonempty if $\prod_{m=1}^{M} \Omega_m$ is nonempty. Also, $\sum_{m=1}^{M} c_m \vartheta_m$ is continuous over $\prod_{m=1}^{M} \Omega_m$ in that $c_m$ is continuous over $\Omega_m$ for all $m \in \mathcal{M}$ and $\sum_{m=1}^{M} c_m \vartheta_m$ is a linear combination of all $c_m$’s, which follows that $\sum_{m=1}^{M} c_m \vartheta_m$ is also continuous over $\Omega$. Since $\sum_{m=1}^{M} c_m \vartheta_m$ is continuous over $\Omega$ and $\Omega$ is compact, there must exist an optimal $M$-tuple vector $(\omega_1^*, \omega_2^*, \ldots, \omega_M^*)$ that maximizes $\sum_{m=1}^{M} c_m \vartheta_m$ due to the Weierstrass theorem.

In general, the optimal solution of the optimization problem in (38) is hardly to be found in closed-form. Nonetheless, numerical techniques can be applied to acquire it in practice. In addition, since the per-user network throughput can also be interpreted as the “average” of the per-user link throughput as aforementioned, the solution of the optimization in (38) essentially also maximizes the per-user link throughput in an average sense.

V. Numerical Example for Coexisting LTE and WiFi Networks

In this section, we provide some numerical results by simulating a scenario that there are four tiers in the HetNet consisting of LTE BSs and WiFi APs. The first tier of the HetNet consists of the macro BSs that do not access the unlicensed spectrum, the second and third tiers consist of picocell and femtocell BSs, and the fourth tier consists of the WiFi APs. The BSs and APs in the last three tiers use the opportunistic CSMA/CA protocol to access the unlicensed spectrum. All channels undergo Rayleigh fading and log-normal shadowing and all users adopt the BMSA scheme defined in Section II to associate their serving BSs or APs. Specifically, the AP association weight in (3) for the BSs in the first three tiers is designated as $W_{m_i} = b P_m H_{m_i}^{(s)}$ for $m \in \{1, 2, 3\}$ in which $b > 0$ is a constant bias and $H_{m_i}^{(s)}$ characterizes...
TABLE I
NETWORK PARAMETERS FOR SIMULATION

| Parameter               | Macrocell (1) | Picocell (2) | Femtocell (3) | WiFi (4) |
|-------------------------|---------------|--------------|---------------|----------|
| Power $P_m$ (W)         | 20            | 1            | 0.2           | 0.1      |
| Intensity $\lambda_m$ (APs/m$^2$) | $5 \times 10^{-6}$ | $5 \times 10^{-5}$ | $2.5 \times 10^{-4}$ | $5 \times 10^{-4}$ |
| Maximum Backoff Time $\tau_m$ | $\infty$      | 2            | 1             |          |
| Sensing Area $S_m$ (m$^2$) | N/A          | 900$\pi$    |               |          |
| CSMA Threshold $\delta$ | N/A          | 4.481        |               |          |
| $H_{m_i} = H_{m_i}^{(f)} \times H_{m_i}^{(s)}$ | $\sim \exp(1,1) \times \ln \mathcal{N}(0,3(\text{dB}))$ |               |          |
| Unlicensed Bandwidth $B_U$ | 160 MHz      |              |               |          |
| Licensed Bandwidth $B_L$ | 100 MHz      |              |               |          |
| Pathloss Exponent $\alpha$ | 4           |              |               |          |
| User Intensity $\mu$ (users/Km$^2$) | 500        |              |               |          |

the channel gain due to log-normal shadowing\(^4\) whereas the AP association weight for the WiFi APs in the fourth tier is $W_4 = P_4 H_4^{(s)}$, i.e., adopting the unbiased MSA scheme. Note that we have $\omega_m = (bP_m)^{\frac{1}{\alpha}} \mathbb{E}[H_{m_i}^{(s)}]^{\frac{2}{\alpha}}$ for $m \in \{1, 2, 3\}$ and $\omega_4 = P_4^{\frac{1}{\alpha}} \mathbb{E}[H_4^{(s)}]^{\frac{2}{\alpha}}$. All the network parameters for simulation are listed in Table I. In the following, we first provide the simulation results of the void BS/AP probability and the channel access probability for the proposed opportunistic CSMA/CA protocol. Next, the simulation results of the mean spectrum efficiencies in the licensed and unlicensed spectra are given to demonstrate the tightness of the lower bounds on the mean spectrum efficiencies derived in Section III. Finally, we show the simulation results of traffic offloading from the LTE network to the WiFi network in order to validate the previous discussions on the per-user link throughput and per-user network throughput with the decentralized and centralized traffic management schemes.

A. Simulation Results of the Void BS/AP Probability and the Channel Access Probability

The void probability of an AP in a particular tier is already derived in (8). To verify the accuracy of (8) and illustrate this void AP issue that cannot be overlooked in our HetNet setting here, the simulation results of the void probabilities of the BSs and APs in all tiers are presented in Fig. I. As shown in Fig. I, the theoretical and simulated void probabilities are almost the same for

\(^4\)In this section, the channel gain $H_{m_i}$ is equal to $H_{m_i}^{(f)} \times H_{m_i}^{(s)}$ where $H_{m_i}^{(f)}$ characterizes the channel gain due to Rayleigh fading and $H_{m_i}^{(s)}$, as already specified, characterizes the channel gain due to log-normal shadowing.
Fig. 1. Simulation results of the void probabilities of the BSs and APs in the four different tiers.

Fig. 2. Simulation results of the channel access probabilities of the BSs and APs in the unlicensed spectrum when the opportunistic CSMA/CA is adopted.
the BSs or APs in any particular tiers, which verifies the correctness and accuracy of the derived pmf in (7) as well as the void AP probability in (8). In addition to the void probability of macro BSs, all the void probabilities of the BSs and APs in the last three tiers are actually not small at all so that the voidness issue of the dense-deployed BSs or APs should be definitely considered in modeling and analyzing the interference. Also, we can see that all the void probabilities of the BSs in the first three tiers increase whereas the void probability of the WiFi APs decreases while offloading traffic from the LTE network to the WiFi network. Thus, offloading or loading traffic also significantly affects the void probabilities especially for those of BSs and APs with high intensity, and this gives rise to strong impacts on the mean spectrum efficiency and per-user link/network throughput as shown in the following subsections. The simulation results of the channel access probabilities of the BSs and APs in all tiers are shown in Fig. 2. They not only verify the correctness of the channel access probability given in (10) but also indicate that overlooking the void cells while modeling the contending channel behaviors among the BSs and APs can also make the channel access probabilities be seriously underestimated. Furthermore, Fig. 2 also shows that all channel access probabilities almost remain unchanged as $b^{-1}$ increases since all small BSs originally can access the unlicensed channel and the intensity of the macro BSs is small so that offloading traffic does not make the total intensity of BSs and APs contending the unlicensed channel alter much.

B. Simulation Results of the Mean Spectrum Efficiency

The simulation results of the mean spectrum efficiencies in the licensed and unlicensed spectra are shown in Fig. 3. Since all BSs in the first three tiers use the BMSA scheme with the same bias, their mean spectrum efficiencies in the licensed spectrum are the same, i.e., $R_{L1} = R_{L2} = R_{L3} = R_L$. Also, note that $R_{U1} = 0$ since macro BSs do not access the unlicensed spectrum, and the BSs in the second and third tiers have the same spectrum efficiency in the unlicensed spectrum, i.e., $R_{U2} = R_{U3} = R_U$, because they have the same channel access probability in the unlicensed spectrum. As a result, the BSs in the second and third tiers have the same sum of the mean spectrum efficiencies in the licensed and unlicensed spectra, i.e., $R_L + R_U$. From Fig. 3, we can gain a few important observations. First, the theoretical lower bound on $R_L + R_U$ is very tight to the simulated result of $R_L + R_U$, and the lower bound on $R_{U1}$ is also very close to the simulated result of $R_{U1}$. Thus, the derived lower bounds in (17) and (23) are fairly tight, as we already emphasized this point in the previous section. Second, when LTE
BSs offload their traffic, $R_L$ significantly increases and $R_U$ slightly increases so that $R_L + R_U$ significantly increases, as expected, whereas the mean spectrum efficiency $R_{U_4}$ of the WiFi APs just slightly reduces. Hence, letting LTE small cell BSs and WiFi APs coexist and share the unlicensed spectrum indeed improves their total mean spectrum efficiency in the unlicensed spectrum. Third, since the mean spectrum efficiency of the WiFi APs just slightly reduces as more traffic is offloaded from the LTE network to the WiFi network, offloading the traffic from the LTE network to the WiFi network is the best traffic management strategy for the BSs in this network setting. In Fig. 4 we show the simulation results of the mean spectrum efficiencies with and without considering the void AP/BS phenomenon in the interference model. As can be observed, $R_L$ is significantly underestimated when the void BSs are not modeled in the interference. This validates our previous claim that the void BSs and APs should be considered in a densely deployed HetNet model. Similarly, in the unlicensed spectrum the mean spectrum efficiencies of the LTE BSs and the WiFi APs are also underestimated when void BSs and APs are not considered. As the traffic is offloaded from the LTE network to the WiFi network, the inaccuracy of the mean spectrum efficiencies of the LTE BSs without considering void BSs is exacerbated in that the void cell probabilities of the LTE BSs increase. On the contrary, the voidness impact on the mean spectrum efficiency of the WiFi network is alleviated since the
offloaded traffic helps the WiFi APs reduce their void probability.

C. Simulation Results of the Per-User Link Throughput and Per-User Network Throughput

The simulation results of the per-user link throughput of the APs in the four different tiers are shown in Fig. 5 when the traffic is offloaded from the LTE network to the WiFi Network. The minimum required per-user link throughput of an WiFi AP is $c_{\min} = 120$ Mbps. Initially, the unlicensed per-user link throughput of the WiFi APs is much higher than $c_{\min}$ so that all BSs start to offload their traffic, i.e., $b^{-1}$ increases. As can be seen, offloading traffic from the LTE network to the WiFi network largely improves the per-user link throughput of the LTE BSs since \( \frac{\partial c_m}{\partial \omega_m} < 0 \) for \( m \in \{1, 2, \ldots, M - 1\} \) holds in this context. Although the per-user link throughput of the WiFi APs also reduces, the throughput loss of the WiFi APs is actually not much. Accordingly, offloading the traffic from LTE to WiFi as much as possible can significantly benefit the per-user link throughput of the LTE BSs as long as \( \frac{\partial c_m}{\partial \omega_m} < 0 \) holds and the required per-user link throughput of the WiFi APs is maintained, as expected. If the decentralized traffic management scheme in Theorem 4 is performed, the optimal value of $\omega_m$ is approximately found as $\omega_m^* \approx 0.588 \sqrt{P_m} = 1.284 \times \sqrt{0.21P_m}$ since $\mathbb{E}[(H_m^{(s)})^{1/2}] = \exp(2 \times 2/4^2) \approx 1.284$ for all $m \in \{1, 2, \ldots, M - 1\}$ so that $b \approx 0.21$ (or $b^{-1} \approx 4.75$) which can be verified by the value

Fig. 4. Simulation results of the mean spectrum efficiencies with and without considering the void cell phenomenon in the interference model of the licensed and unlicensed spectra.
Fig. 5. Simulation results of the per-user link throughput of the BSs and APs in the four different tiers. The required per-user link throughput for the WiFi users is set as $c_{\min} = 120$ Mbps.

Fig. 6. Simulation results of the per-user network throughput of the BSs and APs in the four different tiers. The required per-user link throughput for the WiFi users is set as $c_{\min} = 120$ Mbps.

of $b^{-1}$ in Fig. 5 as the (red) curve of $c_4$ reduces to 120. As $b^{-1} > 4.75$, traffic offloading from the LTE network to the WiFi network stops. Furthermore, Fig. 5 also illustrates that the derived
lower bound on $c_m$ is a very tight to the simulated $c_m$ and its accuracy improves as the intensity of the APs is low (see curve $c_4$ for the macro BSs in Fig. 5). Finally, the simulation results of the per-user network throughput are shown in Fig. 6 while offloading traffic from the LTE network to the WiFi network. Like the case of the per-user link throughput, offloading traffic from the LTE network to the WiFi network initially improves the per-user network throughput, but it eventually leads to the reduction in the per-user network throughput due to too much offloading. When the centralized traffic management is performed, the optimal value of $\omega_m$ is approximately found as $\omega_m^* \approx 0.6326 \sqrt{P_m} = 1.284 \times \sqrt{0.2427 P_m}$ so that $b \approx 0.2427$ (or $b^{-1} \approx 4.12$), which can be verified in Fig. 6 where the simulated per-user network throughput achieves its maximum at $b^{-1} \approx 4$. Hence, there indeed exists an optimal 4-tuple vector $(\omega_1^*, \omega_2^*, \omega_3^*, \omega_4^*)$ that maximizes the per-user network throughput as shown in Lemma 3.

VI. CONCLUSION

In this paper, we consider an $M$-tier HetHet in which all APs in any particular tier form an independent PPP and the APs in the first $M-1$ tiers can simultaneously access the licensed spectrum and use the opportunistic CSMA/CA protocol to share the unlicensed spectrum with the APs in the $M$th tier. The distribution of the cell load is studied first since we need it to find the per-user link and network throughputs. Most importantly, it characterizes the void probability of an AP in each tier that impacts the channel access probability of an AP in the unlicensed spectrum as well as the interference model especially in a densely deployed HetNet. A novel approach is devised to find the tight lower bounds on the mean spectrum efficiencies of an AP in the licensed and unlicensed spectra for any random models of the channel gain and AP association weights. The per-user link throughput and network throughput are proposed, and they are used to develop the decentralized and centralized traffic management schemes, respectively. These two traffic management schemes are shown to be able to maximize the per-user link throughput and network throughput under the constraint posed on the per-user link throughput of the tier-$M$ APs.

APPENDIX

A. Proof of Theorem 2

We first show the case that all $W_m$’s are random. Let $\Delta_{U_m} = P_m H_m / I_{U_m} \| X_o \|^\alpha$ be the signal-to-interference ratio (SIR) of the typical user when AP $X_o$ is from the $m$th tier. Thus,
\[ R_{U_m} = \mathbb{E}[\log_2(1 + T_m \Xi_m \Delta_{U_m})] = \xi_m \rho_m \mathbb{E}[\log_2(1 + \Delta_{U_m})] \] and by using the identity of the Shannon transform in Theorem [13] it can be expressed as

\[ R_{U_m} = \frac{\xi_m \rho_m}{\ln(2)} \mathbb{E}[\ln(1 + \Delta_{U_m})] = \frac{\xi_m \rho_m}{\ln(2)} S_{\Delta_{U_m}}(1) = \frac{\xi_m \rho_m}{\ln(2)} \int_0^\infty \int_0^1 e^{-sy} \mathcal{L}_{\Delta_{U_m}}^{-1}(s) dy ds \quad (40) \]

The Laplace transform of \( \Delta_{U_m}^{-1} \) can be further explicitly expressed as

\[ \mathcal{L}_{\Delta_{U_m}}^{-1}(s) = \mathbb{E} \left[ \exp \left( - \sum_{x_k \in \bigcup_{m=1}^M x_k \setminus x_o} \frac{P_k H_k V_k T_k \Xi_k \| X_o \|^\alpha}{P_m H_m \| X_k \|^\alpha} \right) \right] = \mathbb{E} \left[ \exp \left( - \sum_{x_k \in \bigcup_{m=1}^M x_k \setminus x_o} \frac{P_k H_k V_k T_k \Xi_k W_m \| X_o \|^\alpha}{P_m H_m W_k \| X_k \|^\alpha} \right) \right] \]

where \( \tilde{X}_o, \tilde{X}_{m_i} \) and \( \tilde{X}_m \) are all already defined in the Proof of Lemma [1]. (a) follows from (7) and (b) is obtained by letting \( S_{m,k} \triangleq \frac{s P_k H_k W_m}{P_m H_m W_k} V_k T_k \Xi_k \). Now be aware that all \( T_k \)'s are independent since we assume all channel gains are independent, all \( V_k \)'s are not completely and mutually independent since cell association could induce the location correlations between the non-void APs [19] [20], all \( \Xi_k \)'s are not completely and mutually independent as well since all non-void APs use CSMA/CA to access the unlicensed channel and the resulting APs accessing the channel is not a PPP any more and instead they become a Martén hard-core point process (MHPP) [9], [26].

A tractable lower bound on \( \mathcal{L}_{\Delta_{U_m}}^{-1} \) can be obtained by treating all \( V_k \)'s (\( \Xi_k \)'s) are independent and i.i.d. for same \( k \) so that all non-void APs that have good channels and successfully access to unlicensed channel become a thinning homogeneous PPP which induces larger interference. Accordingly, we have

\[ \mathcal{L}_{\Delta_{U_m}}^{-1}(s) \geq \mathbb{E} \left[ \exp \left\{ - \pi \sum_{k=1}^M \tilde{X}_k \mathbb{E}_{S_{m,k}} \left[ \int_0^\infty \left( 1 - e^{-S_{m,k} \left( 1 + \frac{x}{\| X_o \|^2} \right)^{-\frac{2}{\alpha}}} \right) dx \right] \right\} \right] \]

by following the proof of Proposition 2 in [20] since \( \tilde{X}_o \) is the nearest point in the point process of \( \bigcup_{m=1}^M \tilde{X}_m \) to the typical user. Also, we can show the following identity

\[ \int_0^\infty \left( 1 - e^{-S_{m,k} \left( 1 + \frac{x}{\| X_o \|^2} \right)^{-\frac{2}{\alpha}}} \right) dx = \| \tilde{X}_o \|^2 S_{m,k}^\frac{2}{\alpha} \Gamma \left( 1 - \frac{2}{\alpha} \right) + \int_0^1 e^{-S_{m,k} u^{-\frac{2}{\alpha}}} du - 1 \]

by using the algebraic technique in the proof of of Proposition 2 in [20]. Hence, we have

\[ \mathbb{E}_{S_{m,k}} \left[ \int_0^\infty \left( 1 - e^{-S_{m,k} \left( 1 + \frac{x}{\| X_o \|^2} \right)^{-\frac{2}{\alpha}}} \right) dx \right] = \| \tilde{X}_o \|^2 q_{k,0} \xi_k \rho_k \left\{ \frac{P_k \bar{w}_m \bar{h}_k}{P_m \bar{w}_k \bar{h}_m} \right\}^\frac{2}{\alpha} \mathbb{E} \left[ \left( \frac{H_k / \bar{h}_k}{W_k / \bar{w}_k} \right)^\frac{2}{\alpha} \right] \times \Gamma \left( 1 - \frac{2}{\alpha} \right) + \int_0^1 \mathcal{L}_{\tilde{H}} \left( \frac{s P_k \bar{w}_m \bar{h}_k u^{-2}}{P_m \bar{w}_k \bar{h}_m} \right) du - 1 \right\} = \| \tilde{X}_o \|^2 q_{k,0} \xi_k \rho_k \ell_{\tilde{H}} \left( \frac{P_k \bar{w}_m \bar{h}_k s}{P_m \bar{w}_k \bar{h}_m \bar{H}} \right)^{\frac{2}{\alpha}} \right\} \]
and this follows that

\[ \mathcal{L}_{\Delta_{\omega_m}^{-1}}(s) \geq \mathbb{E}_{\hat{H}} \left[ \exp \left\{ -\pi \| \hat{X}_\omega \|^2 \sum_{k=1}^{M} q_{k,0} \xi_k \rho_k \lambda_k \ell \left( \frac{s P_k \bar{w}_m \bar{h}_k}{P_m \bar{w}_k \bar{h}_m \bar{H}}, \frac{2}{\alpha} \right) \right\} \right] \]

\[ = \mathbb{E}_{\hat{H}} \left[ \int_{0}^{\infty} \pi \lambda \exp \left\{ -\pi x \lambda \left( \sum_{k=1}^{M} q_{k,0} \xi_k \rho_k \theta_k \ell \left( \frac{s P_k \bar{w}_m \bar{h}_k}{P_m \bar{w}_k \bar{h}_m \bar{H}}, \frac{2}{\alpha} \right) + 1 \right) \right\} dx \right] \]

\[ = \mathbb{E}_{\hat{H}} \left[ \left( \sum_{k=1}^{M} q_{k,0} \xi_k \rho_k \theta_k \ell \left( \frac{s P_k \bar{w}_m \bar{h}_k}{P_m \bar{w}_k \bar{h}_m \bar{H}}, \frac{2}{\alpha} \right) + 1 \right)^{-1} \right]. \]

Then we know

\[ \int_{0}^{\infty} \mathcal{L}_{\Delta_{\omega_m}^{-1}}(s) ds \geq \int_{0}^{\infty} \int_{0}^{\infty} e^{-sx} f_{\hat{H}}(x) ds dx \]

\[ = \int_{0}^{\infty} \int_{0}^{\infty} e^{-uxy} f_{\hat{H}}(x) x dx du. \]

According to (40), \( R_{U_m} \) can be lower bounded as

\[ R_{U_m} = \frac{\xi_m \rho_m}{\ln(2)} \int_{0}^{\infty} \int_{0}^{1} e^{-xy} \mathcal{L}_{\Delta_{\omega_m}^{-1}}(s) ds dy \geq \frac{\xi_m \rho_m}{\ln(2)} \int_{0}^{\infty} \int_{0}^{\infty} \int_{0}^{\infty} \int_{0}^{\infty} e^{-uxy} f_{\hat{H}}(x) x dx du, \]

which is exactly the lower bound in (17) by carrying out those inner double integrals for variables \( x \) and \( y \). For the case that all \( W_m \)'s are constant \( \bar{w}_m \) for all \( m \in \mathcal{M} \), letting all \( W_m \) leave in function \( \ell(\cdot, \cdot) \) yields the result in (19) with \( \hat{H} = H_m / \bar{h}_m \) for all \( m \in \mathcal{M} \).

\[ B. \text{ Proof of Theorem 4} \]

First notice that the coefficient of \( \omega_m(n) \) in (36) has the physical meaning that \( c^*(n) \) is normalized by the per-user link throughput of a tier-\( m \) AP in the unlicensed spectrum. The tier-\( m \) APs will offload (load) its traffic in the next time if this coefficient at time \( n \) is smaller (greater) than one. In other words, the per-user link throughput of the tier-\( m \) APs can continuously increase (decrease) under the traffic management scheme in (36) as long as it is smaller (larger) than \( c^*(n) \) for all \( n \in \mathbb{N} \). Now we want to show that \( \omega_m(n) \) converges to a steady-state value \( \omega_m^* \) as \( n \) goes to infinity. Consider a Lyapunov function \( V(n) := (\omega_m(n) - \omega_m^*/2)^2 \) and we can have

\[ V(n+1) - V(n) = (\omega_m(n+1) - \omega_m^*/2)^2 - (\omega_m(n) - \omega_m^*/2)^2 \]

\[ = \left[ \omega_m(n+1) - \omega_m(n) \right] \left[ \omega_m(n+1) + \omega_m(n) - \omega_m^* \right] \]

\[ = \left[ \frac{c^*(n) N_m(n)}{C_m(n)} - 1 \right] \left[ \frac{c^*(n) N_m(n)}{C_m(n)} + 1 - \frac{\omega_n}{\omega_m(n)} \right] \omega_m^2(n) < 0, \]
which yields the following two constraints
\[
1 < \frac{c^*(n)N_m(n)}{C_m(n)} < \frac{\omega_m^*}{\omega_m(n)} - 1 \quad \text{and} \quad \frac{\omega_m^*}{\omega_m(n)} - 1 < \frac{c^*(n)N_m(n)}{C_m(n)} < 1.
\]
As long as \( w_m(n) \) and \( \frac{c^*(n)N_m(n)}{C_m(n)} \) satisfy these two constraints, \( \omega_m(n) \) converges to \( \omega_m^* \) as \( n \) goes to infinity based on the Foster-Lyapunov criterion \[27\]. In other words, as \( n \) goes to infinity, \( c^* \) will converge to \( \sup_n \{C_m(n)/N_m(n)\} \). Also, these two constraints always hold since \( \frac{c^*(n)N_m(n)}{C_m(n)} < 1 \) (i.e., \( C_m/N_m(n) > c^*(n) \)) makes \( \omega_m(n) \) reduce and approach to \( \omega_m^* \) if \( \omega_m(n) > \omega_m^*/2 \) and \( \frac{c^*(n)N_m(n)}{C_m(n)} > 1 \) (i.e., \( C_m/N_m(n) > c^*(n) \)) makes \( \omega_m(n) \) reduce and approach to \( \omega_m^* \) if \( \omega_m(n) < \omega_m^*/2 \). Therefore, the decentralized traffic management scheme in (36) will make \( \omega_m(n) \) converge to \( \omega_m^* \) as time goes to infinity and thus we must have \( 0 < \frac{c^*(n)N_m(n)}{C_m(n)} < 1 \), i.e., \( C_m(\omega_m^*)/N_m(\infty) > c^*(\infty) \) for all \( m \in M \), and \( c_M \geq c_{\min} \) is surely satisfied. In addition, since \( N_m(\infty) = \mu \vartheta_m(\omega_m^*)/\lambda_m \), in the steady state \( \omega_m^* \) must satisfy the following constraint
\[
\frac{\mu c^* \vartheta_m(\omega_m^*)}{\lambda_m C_m(\omega_m^*)} = 1 \Rightarrow \omega_m^* = \frac{C_m(\omega_m^*)}{\mu c^*} \left( \sum_{k \in M \setminus m} \lambda_k \omega_k + \lambda_m \omega_m^* \right),
\]
which indicates \( \omega_m^* \) is the fixed point of \( \Upsilon_m(x) \) in (37).

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