Practicality test of student worksheet (SWS) based on: Action, Process, Object, Schema (APOS model) assisted on Geogebra the subject of Riemann sum

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Abstract. The APOS Model is a Mathematical Learning Model Based on Theory: Action, Process, Object, and Schema (APOS). The APOS Model is a refinement of the Calculus Learning Model Based on APOS Theory. The APOS Model has syntax with phases: Orientation, Practicum, Small Group Discussion, Class Discussion, Exercise or Evaluation. Development of SWS based on APOS Model assisted on Geogebra in the subject of Riemann Sum developed using Plomp design which consists of three stages: 1) preliminary research, 2) prototyping phase, and 3) assessment phase. The purpose of this research was to find out how practicality of SWS was being developed, and how the criteria of practicing SWS based on the APOS Model assisted by Geogebra in the Riemann Sum discussion topic. The instrument used to collect data was the SWS practicality instrument in the form of a Likert scale. Research subjects were: one-to-one test conducted by a lecturer, a teacher, 3 senior students. Small group test by 3 groups of senior students with 3 students in each group. Large group test by 30 students in class B on the 3rd semester of Program Studi Pendidikan Matematika Universitas Bengkulu. After processing the data, the average score of practicality test of SWS was: one to one trial = 82.5%; according to the small group test = 77.63%; according to the large group test = 79.0%. The average value of practicality of SWS by users was 79.71 and belongs to the practical category.

1. Introduction
Definite Integral as a limit to the Riemann Sum was constructed as a general concept of area. The condition did not need an f continues function and was not negative in the closed hose [a, b] [1-4] Suppose the function f was defined in the closed hose [a, b].

1. Make partition of P for [a,b] with the division points $a = x_0 < x_1 < x_2 < .... < x_{i-1} < x_i < .... < x_n = b$. The i-th interval of the partition is $[x_{i-1}, x_i], i = 1, 2,..., n$, the length of the hose $\Delta x_i = x_i - x_{i-1}$. The length of partition P, written $\|P\|$, is defined as $\|P\| = \max \Delta x_i \leq 1 \leq n$

2. Choose $c_i \in [x_{i-1}, x_i], i = 1, 2,..., n$ then form the sum $\sum_{i=1}^{n} f(c_i) \Delta x_i$, that called Riemann Sum of f function in [a, b].

3. Consider the Limit of Riemann Sum $\|P\| \to 0$, is $\lim_{\|P\| \to 0} \sum_{i=1}^{n} f(c_i) \Delta x_i$. 
If the limit exists, the $f$ function is integrated by Riemann at interval $[a, b]$, and written:

$$\lim_{\|P\| \to 0} \sum_{i=1}^{n} f(c_i) \Delta x_i.$$ 

If the limit does not exists, the $f$ function is not integrated by Riemann Sum at $[a, b]$. Example of Riemann Sum Graph:

Figure 1. Left Riemann sum of $x^3$ over $[0, 2]$ using 4 subdivisions [5].

Figure 2. Right Riemann sum of $x^3$ over $[0, 2]$ using 4 subdivisions [5].

Figure 3. Midpoint Rule Riemann sum of $x^3$ over $[0, 2]$ using 4 subdivisions [5].

For the left Riemann sum, approximating the function by its value at the left-end point gives multiple rectangles with base $\Delta x$ and height $f(a + i \Delta x)$. Doing this for $i = 0, 1, ..., n - 1$, and adding up the resulting areas gives. For the right Riemann sum, $f$ is here approximated by the value at the right endpoint. This gives multiple rectangles with base $\Delta x$ and height $f(a + i \Delta x)$. Doing this for $i = 1, ..., n$, and adding up the resulting areas produces. For the midpoint Riemann sum, approximating $f$ at the midpoint of intervals gives $f(a + \Delta x/2)$ for the first interval, for the next one $f(a + 3\Delta x/2)$, and so on until $f(b - \Delta x/2)$. Summing up the areas gives. Examples of manually counting were structured and easy to understand about number of Riemann [6].

Table 1. Riemann Sum for $x^2$ [7].

| $N$ | $R_n \sum_{j=1}^{n} j^2$ | $R_n$ | Decimal for $R_n$ |
|-----|--------------------------|------|------------------|
| 2   | $\frac{2}{4}$           | $\frac{5}{4}$ | 1.25             |
| 4   | $\frac{4}{16}$          | $\frac{45}{32}$ | 1.40625         |
| 8   | $\frac{8}{64}$          | $\frac{51}{32}$ | 1.59375         |
| 16  | $\frac{16}{256}$        | $\frac{935}{512}$ | 1.826171875     |
| 32  | $\frac{32}{1024}$       | $\frac{68640}{32768}$ | 2.0947265625   |
| 64  | $\frac{64}{4096}$       | $\frac{626080}{262144}$ | 2.3883056641   |

- The formula for the sum is $\sum_{j=1}^{n} j^2 = \frac{1}{3} n^3 + \frac{1}{2} n^2 + \frac{1}{6} n$.
- $R_n = \frac{8}{n^3} \sum_{j=1}^{n} j^2 = \frac{8}{n^3} \left(\frac{1}{3} n^3 + \frac{1}{2} n^2 + \frac{1}{6} n\right)$

The formula for the sum is $\sum_{j=2}^{n} j^2 = \frac{1}{3} n^3 + \frac{1}{2} n^2 + \frac{1}{6} n$ So
- $R_n = \frac{8}{n^3} \sum_{j=2}^{n} j^2 = \frac{8}{3} + \frac{8}{n} + \frac{4}{3n^2}$ and $\lim_{n \to \infty} R_n = \frac{8}{3}$
• We have just figured out that the area under the curve $y = x^2$, $0 \leq x \leq 2$, is $A = \int_0^2 x^2 \, dx = \lim_{n \to \infty} R_n = \frac{8}{3}$.

To facilitate the calculation or depiction of graphs like the example above, computer assistance was used. Utilization of computer application programs for mathematics was ideal for using in calculations that required high accuracy, or complete graphics in an appropriate, fast, and accurate manner. One of the computer application programs that could be used as a mathematical learning media was the Geogebra application program. Geogebra was a computer application program for learning mathematics especially for geometry and algebra. According to Geogebra was very useful as a medium of learning mathematics with a variety of activities as follows [8]: (1) As a medium for demonstration and visualization, in this case, in traditional learning, teachers utilized Geogebra to demonstrate and visualize certain mathematical concepts; (2) As a construction assisted, in this case Geogebra was used to visualize the construction of certain mathematical concepts, for example constructing inner circles or circles outside of triangles, or tangents; (3) As a tool for the discovery process, in this case Geogebra was used as a tool for students to find a mathematical concept, for example the location of the points or the characteristics of a satellite dish [9].

Computer algebra systems, dynamic geometry software, and spreadsheets are the main types of educational software currently used for mathematics teaching and learning. Each of the programs has its advantages and is especially useful for treating a certain selection of mathematical topics or supports certain instructional approaches [10]. Geogebra software was a product of technological advances that were currently widely used in learning mathematics. With its various advantages, Geogebra is now widely used as a tool to construct, demonstrate or visualize abstract concepts that exist in mathematics. Geogebra software was very easy to get because it could be downloaded for free. In addition, the Geogebra software was also very easy to operate because it used a very simple syntax or command.

Learning in the context of 2013 curriculum was oriented to produce Indonesian people who are productive, creative, innovative and affective through strengthening attitudes (know why), skills (know how), and integrated knowledge (know what). This orientation was based on the awareness that the development of life and science in the 21st century, there has been a shift in character compared to the previous century. A number of features of the 21st century were that the 21st century was a century of information, computing, automation, and communication [11].

APOS was a learning theory that was specific to mathematics learning at the college level, which integrated using of computers, learning in small groups and paying attention to mental constructions carried out by students in understanding a mathematical concept. The constructions of mental mentioned were: action, process, object and schema which was abbreviated by APOS [12-14]. APOS Model was a model based on student-centered learning (SCL) and computer assisted. The use of computers was as a tool to construct material by students [13]. The basic idea of student-centeredness was that students might not only choose what to study, but how and why that topic might be an interesting one to study [15]. SCL was a learning strategy that places students as active and independent subjects / students with psychological conditions as adult learners, was fully responsible for their learning, and was able to learn beyond the classroom. With these principles students were expected to have and live a life-long learner spirit and master hard skills and soft skills that support one another. On the other hand, the lecturers switch functions to become facilitators, including as learning partners, no longer as the main source of knowledge [15].

Calculus Integral was a compulsory course that offered in the Department / Mathematics Education Study Program in all University in Indonesia. In the Bachelor’s Degree of Program Studi Pendidikan Matematika Universitas Bengkulu, Calculus Integral was given a weight of 4 (3-1) Credit Semester. The learning model applied to Integral Calculus was a Mathematical Learning Model Based on Theory of Action, Process, Object, Schema (APOS Model) which has syntax with phases: Orientation, Practicum, Small Group Discussion, Class Discussion, Exercise and Evaluation [16]. Each phase was given the following time: orientation phase 20 ', practicum 50 ', small group
discussion 50', class discussion 50', and 30 minutes of training / evaluation. All activities were carried out in class, students were divided into heterogeneous small groups consisting of 3-4 student each group. For practicum students brought a laptop. APOS Model, was a refinement of the Calculus Learning Model Based on APOS Theory (MPK-APOS) which has been declared valid, practical and effective [17]. To support the implementation of the APOS Model, an APOS Model Based Worksheet was needed.

In academic year 2017/2018 and 2018/2019 the APOS Model was applied to the Integral Calculus course supported by worksheet based on APOS Model with Maple. The problem that occurred at that time was that some students had difficulty painting graphics manually [18,19]. At the time of Micro Teaching in the VI C class Mathematics Education Study Program Faculty of Teacher Training and Education on Academic Year 2018 / 2019 there were students who made student worksheet based on APOS with Geogebra about Riemann Sum. Geogebra that was more dynamic, and easier to execute than Maple has caught the attention of the writer.

Based on Figure 4, it could be seen that there were many commands that must be typed by the user, Maple which was sensitive to distinguish between uppercase and lowercase letters made the program not executable. The following was an example of Geogebra's command regarding Riemann Sum.

![Figure 4. Example of Maple’s command for Riemann Sum [20.]](image)

According to figure 5 and 6, it could be seen that with less command, Geogebra could showed the area and Riemann sum fast. The writer follows up to be developed for student used. Having adapted to the needs of students, carried out the process of validity test. The result of validity test stated that student worksheet developing was valid and could be continued by conducting the student worksheet practicality trials [22]. This article deals specifically with practicality testing. The purpose of the practicality test was to find out whether the student worksheet being developed were practical. As
shown in example from Table 1 above, if the value of n greater, if the calculation was done manually it would took a long time and with a high degree of accuracy to find Riemann Sum. For this reason, computer assisted were needed to calculate it. In the study of Integral Calculus with the topic of Riemann Sum, the APOS Model was assisted by Geogebra and supported by student worksheet. The following were examples of student worksheet content in, Practicum and Small Group Discussion phases.

4) Untuk mencari luas poligon luar, pada menu input. Tuliskan rumus: **UpperSum(f, x(A), x(B), n)**
5) Untuk mencari luas poligon dalam, pada menu input. Tuliskan rumus: **LowerSum(f, x(A), x(B), n)**
6) Untuk mencari integral (luas bangun di bawah kurva), pada menu input. Tuliskan rumus: **e=Integral(f, x(A), x(B)).**

Figure 7. Example of Practicum phase content on Student Worksheet

According to figure 7, it could be seen that Geogebra’s command was simple and easy to input it.

| No | Jumlah n | Luas Poligon Dalam | Luas Poligon Luar |
|----|----------|-------------------|------------------|
| ... |          |                   |                  |
| 3  | n = 5    | A₅ = f(x₁) Δx + f(x₂) Δx + f(x₃)Δx + f(x₄)Δx + f(x₅)Δx = ....... | A₅ = f(x₁) Δx + f(x₂) Δx + f(x₃)Δx + f(x₄)Δx + f(x₅)Δx = ....... |
| 4  | N=10     | A₁₀ = ....         | A₁₀ = ....       |
| ... |          |                   |                  |

Figure 8. Example content SWS on Discussion phase of small group.

2. Methods

Development of SWS based on APOS Model assisted on Geogebra in the subject of Riemann Sum developed using Plomp design which consists of three stages: 1) preliminary research, 2) prototyping phase, and 3) assessment phase [23]. The purpose of this research was to find out how practicality of SWS was being developed, and how the criteria of practicing SWS based on the APOS Model assisted by Geogebra in the Riemann Sum discussion topic. The instrument used to collect data was the SWS practicality instrument in the form of a Likert scale. Research subjects were: one-to-one test conducted by a lecturer, a teacher, 3 senior students. Small group test by 3 groups of senior students with 3 students in each group. Large group test by 30 students in class B on the 3rd semester of Program Studi Pendidikan Matematika Universitas Bengkulu.

The practicality aspects could only be fulfilled if: (1) experts and practitioners stated that the development could be applied, and (2) reality showed that what was developed could be applied. To measure the level of practicality associated with developing instruments in the form of learning material, Nieyen [24] argued that to measure practicality by seeing whether lecturers (and other experts) consider that the material was easy and could be used by lecturers and students. SWS development that was developed in the development research, the SWS said to be practical if experts and practitioners state that theoretically that SWS could be applied in the field and the level of implementation of the SWS was included in the "good" category.
Practicality Data Analysis of SWS consisted of data from the practicality of the Worksheet based on the APOS Model by lecturers, teachers and students. For data in the form of a questionnaire using a Likert scale, the data analysis technique obtained the same as the validity data i.e. the data collected then tabulated. The results of tabulations for each bill searched for by the formula.

\[
Practicality score (P) = \frac{\text{score}}{\text{maximum score}} \times 100\%
\]

Category of SWS practicality based on APOS Model assisted on Geogebra was explained on table 2 [25].

**Table 2.** Category of SWS practicality based on APOS Model assisted on Geogebra.

| No | Level of Achievement (%) | Category       |
|----|---------------------------|----------------|
| 1  | 0 – 20                    | Not Practical  |
| 2  | 21 – 40                   | Less Practical |
| 3  | 41 – 60                   | Enough Practical|
| 4  | 61 – 80                   | Practical       |
| 5  | 81 – 100                  | Very Practical  |

3. Results and discussion

After processing the data, the average score of practicality test of SWS was: one to one trial = 82.5%; according to the small group test = 77.63%; according to the large group test = 79.0%. The average value of practicality of SWS by users was 79.71 and belongs to the practical category. The following were the results of the student worksheet practicality test on graph form.

![results of the student worksheet practicality test](image)

It could be seen in Figure 9 that the one-on-one test scores higher than the classical test. The following were the results of practicality tests for each phase of the student worksheet based on APOS Model. If looking back at Figure 4, it was seen that the average value of the one-on-one test was higher than the value of the small group test, or the large group test. One on one test carried out by a lecturer, a teacher, and 2 senior students who had conducted Integral Calculus learning-based model APOS assisted on Maple 11. The user who performed one-on-one test to give high marks for working with Geogebra much easier than working with Maple, which was very sensitive, which was to distinguish between uppercase and lowercase letters.

The Geogebra command in the Practicum phase, for example, could be seen in Figure 3 above. When the Geogebra command was executed a graphic image and integral results would appeared. If the goal was to introduce how the process of the area became the limit of the Riemann Sum and then it was symbolized by a definite integral. Student worksheet users such as lecturers, teachers and senior students agree that student worksheet of Riemann Sum was practical with an average practicality questionnaire = 82.5%;
The results of practicality tests in small groups conducted by 3 senior students who have used Maple when studying Integral Calculus, the average value was 77.63%; Unlike the one-on-one test, the small group test gave a smaller value than the individual test, because in small groups students could discussed the weaknesses and strengths of student worksheet assisted on Geogebra compared to student worksheet assisted on Maple.

Practicality test results by a large group of students (one class), which was students of class A Program Studi Pendidikan Matematika Universitas Bengkulu on academic year 2018/2019. They worked on student worksheet in small heterogeneous groups of 3 student. During the practicum phase, all students looked enthusiastic and happy to learn it. It could be said that the practicum phase was going well. Problems arise when in the Class Discussion phase, where a selected group presented the results of their group discussion in front of the class. For question with n definite, students easily explained how to calculate the results of the Riemann sum. If n went to infinite, student answered directly using the formula definite integral, but they could not solved the problem about Riemann Sum manually.

When the lecturer asked how to answer it manually without the help of Geogebra especially on n went to infinite, students looked confused. To solve this problem, the lecturer immediately took over and explained how to calculate the Riemann Sum if n went to infinite.

The lecturer reminded about sigma and about limits. Then the lecturer asked the students to solve the problem of calculating the Riemann Sum to go to infinite as practice. The lecturer also gave a hard problem to do at home. Before the lecture ended, the lecturer asked students to fill out a questionnaire. After being processed the practicality test results obtained by a large group (Class) = 79.0%. The average value of practicality of SWS by users was 79.71 and belongs to the practical category.

Tasman et al concluded that the research shows that the use of geogebra by providing dynamic visualization builds students understanding on the definition of definite integral. The students need to see or imagine and understand why the definite integral also means the area under the curve. This research confirm the finding of Monika Dockendorff and Horacio Solar about integrating learning with ICT will give good impact on teacher conceptions about teaching and learning mathematics since our students are perspective teachers [26].

It could be seen in Figure 2 that the Maple application program provided instructions to calculate in more detail so that it reflected the completion steps manually. It's just that the order was complicated and Maple was sensitive because it distinguished uppercase and lowercase letters. Once working with Geogebra, it was feeling more dynamic and practical. But unfortunately student worksheet that developed have not been able to choose the Geogebra that was able to guide students to understand how to calculate the number of Riemann manually.

What needed to be done if continued to use Geogebra as a learning resource is to try to create a student worksheet based on the APOS Model with the Scientific approach [19]. The scientific approach could be implemented in the Small Group Discussion phase, so students were expected to understand how to calculate the Riemann sum for n to infinity. If it was still difficult to calculate the Riemanns sum manually, then previously in the Orientation phase the lecturer must gave direction to students about sigma, limits and the Riemann sum.

4. Conclusion
Student worksheet about Riemann sum based on the APOS Model assisted on Geogebra that was developed, after a practicality test, concluded that: one to one trial = 82.5%; according to the small group test = 77.63%; according to the large group test = 79.0%. The average value of practicality of student worksheet by users was 79.71 and belongs to the practical category. To overcome the weaknesses of Geogebra which had not been able to display the results of the calculation of the Riemann sum for n to infinity in detail was to provide assistance classically. The lecturer directed students to calculate Riemann sum step by step for n to infinity manually, by reminding about the
sigma and limits of a function. Another way was using student worksheet based on APOS Model assisted on Geogebra with Scientific approaches.

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