Do Pulsar and Fast Radio Burst dispersion measures obey Benford’s law?

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We check if the first significant digit of the dispersion measure of pulsars and Fast Radio Bursts (using the CHIME catalog) is consistent with the Benford distribution. We find a large disagreement with Benford’s law with $\chi^2$ close to 80 for 8 degrees of freedom for both these aforementioned datasets. This corresponds to a discrepancy of about $7\sigma$. Therefore, we conclude that the dispersion measures of pulsars and FRBs do not obey Benford’s law.

I. INTRODUCTION

Many naturally occurring distributions tend to adhere to a logarithmic distribution as predicted by Benford’s law [1, 2], which is sometimes often referred to as the significant digit law or the law of anomalous numbers. This law states that given a distribution of numbers, the fraction of numbers with leading digit $d$ is given by [3, 4]:

$$P(d) = \log(1 + \frac{1}{d})$$

(1)

This law has been widely applied to an assortment of fields, including biology [5, 6], finance [7, 8], geophysics [9], seismology [10], spectroscopy [11, 12], finance to detect banking frauds [8], Nuclear and Particle Physics [13–16], etc. This law has been derived using a central-limit-like theorem for significant digits [17], as well as using Markov process [18].

Benford’s law has also been widely applied to a variety of astrophysical datasets, such as distances to stars and galaxies [19, 20], masses, orbital periods and semi-major axes of asteroids [21], GAIA-2 parallaxes [22], light curves of cataclysmic variables and other X-ray transient sources [23], etc.

In this work, we check if the dispersion measures of pulsars and Fast radio bursts (FRBs) obey Benford’s law. Pulsars are rotating neutron stars which emit pulsed emission in radio waves, and have been widely used as probes of nearly all branches of Physics and Astronomy [24]. FRBs are short-duration radio bursts located at extragalactic distances [25]. Previously, Benford’s law has also shown to be true for a whole slew of other pulsar properties such as the time derivative of the barycentric period, first and second time derivative of rotation frequency, period derivative, spin down age, proper motions, spin-down luminosity, fluxes, transverse velocity [26]. However, this same work also showed that Benford’s law does not hold true for barycentric period and barycentric rotation frequency. Dispersion measure (DM) is the integrated free electron column density in the ionized interstellar medium. Precise measurements of DM for millisecond pulsars is one of the main goals of various pulsar timing array experiments [27, 28].

II. DATASET AND ANALYSIS

A. Dataset

For this work, we downloaded the DM (in units of $cm^{-3}pc$) for the radio pulsar population, along with the measurement uncertainties from the ATNF online catalogue (version 1.67) [29]. The dataset contains 3319 pulsars. We removed 122 pulsars for which no DM measurements were available. We did this analysis for all the remaining 3197 pulsars with DM measurements as well as a smaller sample of 3143 pulsars for which fractional errors in DM were less than 10%.

For FRBs, we used the DM measured from the CHIME sample [30], which contains 536 FRBs in the same units as the pulsar population. This sample includes 474 one-off events and 62 repeaters. None of the FRBs had fractional errors greater than 10%. So there was no need to cull any objects from the FRB sample for our analysis.

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$^1$http://www.atnf.csiro.au/research/pulsar/psrcat/
The first digit distribution for the complete pulsar sample can be found in Fig. 1, whereas the same distribution after removing the data points in which the fractional errors are less than 10% can be found in Fig. 2. For both the figures, the expected Benford distribution along with associated binomial errors $\sqrt{NP(d)(1 - P(d))}$ are also shown. By eye, we see that the Benford distribution is not obeyed although the relative rank of the frequency of leading digits agrees with Benford’s law. Most of the discrepancy occurs for the first two digits which undershoot (1) and overshoot (2) the expected values respectively.

In order to quantify how well the pulsar dispersion measures adhere to Benford’s law we use Pearson $\chi^2$

$$\chi^2 = \sum_{d=1}^{9} \frac{(N_B(d) - N_O(d))^2}{N_B(d)},$$  

where $N_B(d)$ indicates the expected count of occurrences according to Benford’s law and $N_O(d)$ is the observed number of occurrences for a single digit $d$. For the full sample of pulsars, we obtain a $\chi^2$ value of 78 for 8 degrees of freedom. If we discard the measurements for which the fractional error in the dispersion measure is more than 10%, we obtain a Pearson $\chi^2$ value of 81. Both these measurements correspond to $p$-values of $10^{-13}$ and $10^{-14}$, or in terms of $Z$-score, a discrepancy of 7.3$\sigma$ and 7.6$\sigma$, respectively, using the prescription in [31, 32].

The first digit distribution of FRBs can be found in Fig. 3. As we can see, the first two digits are much smaller
FIG. 2: Distribution of the first digit for the DM of 3143 pulsars downloaded from the ATNF catalog, for which the fractional errors in DM were less than 10%.

| Dataset             | $\chi^2$/dof | p-value  | Discrepancy Significance |
|---------------------|--------------|----------|--------------------------|
| Pulsar (Full)       | 78/8         | $10^{-13}$| 7.3σ                     |
| Pulsar ($\sigma_{DM}/DM < 10\%$) | 81/8       | $10^{-14}$| 7.6σ                     |
| FRB                 | 77/8         | $3 \times 10^{-13}$| 7.2σ                     |

TABLE I: Summary of our results on Benford analysis of first digit of DM of pulsars and FRBs. As we can see, none of the datasets agree with Benford’s law.

than the expectations due to Benford distribution. The $\chi^2$ value we obtain is about 77 for 8 degree of freedom, corresponding to a p-value of $3 \times 10^{-13}$ or 7.2σ discrepancy.

A tabular summary of all our results can be found in Table I. We conclude that both pulsar and FRB DMs do not adhere to the Benford distribution.
III. CONCLUSIONS

In this work the first significant digit of the DM for pulsars and FRBs is examined for adherence to Benford’s distribution. For pulsars we considered the full dataset for which DM measurements were available, as well as those for which the fractional errors in DM were less than 10%. For FRBs, we used the dataset from CHIME catalog.

The distribution of first digit of DM for the aforementioned datasets can be found in Figs. 1, 2, and 3. None of the three datasets agree with Benford distribution. A tabular summary of our results can be found in Table I. All the datasets show a disagreement with Benford’s law with significance between 7.2-7.6σ. Therefore, we conclude that the DM of pulsars and FRBs do not agree with Benford’s law, even though some other properties of pulsars have been previously shown to adhere to Benford’s law [26].

The main reason for this is that given the physical location of pulsars/FRBs, there is not enough dynamic range in the data for them to adequately obey the Benford distribution.

For pulsars, there is a dearth (only 6%) of objects with DM < 20cm⁻³pc. Majority of the pulsars (35%) have DMs between 20 and 100 cm⁻³pc and only about 20% of them have DMs of between 100-200 cm⁻³pc. This is due to the fact that the detection of pulsars is limited by the sensitivity of radio telescopes, and the DM distribution of pulsars is very much direction dependent. Therefore, the number of occurrences of the number one in the first digit of the DM is suppressed, whereas the same is enhanced for the number two compared to the expectations from Benford’s law.

Similarly, since FRBs are located at cosmological distances, only a small fraction of FRBs (8%) have DM <
200 cm$^{-3}$pc, since there are very few Galaxies nearby. Furthermore, there are very few FRBs (15\%) with DM > 1000 cm$^{-3}$pc, since radio telescopes are not sensitive to very high redshift FRBs. Therefore, the full dynamic range of the DMs for FRBs is only about a factor of 20-30, and the number of occurrences of the number one in the first digit of DM is suppressed compared to Benford’s law.

Therefore for the above reasons, the DMs of pulsars and FRBs do not obey Benford’s law, and therefore our results are not unexpected or a surprise.

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[1] R. S. Pinkham, The Annals of Mathematical Statistics 32, 1223 (1961).
[2] T. P. Hill, American Scientist 86, 358 (1998).
[3] S. Newcomb, American Journal of Mathematics 4, 39 (1881).
[4] F. Benford, Proceedings of the American Philosophical Society 78, 551 (1938).
[5] J. H. Caceres, J. L. P. Garcia, C. M. Ortiz, and L. G. Dominguez, J. Biomed 1, 27 (2008).
[6] S. Docampo, M. del Mar Trigo, M. J. Aira, B. Cabezudo, and A. Flores-Moya, Aerobiologia 25, 275 (2009).
[7] M. J. De Ceuster, G. Dhaene, and T. Schatteman, Journal of Empirical Finance 5, 263 (1998).
[8] C. Durtschi, W. Hillison, and C. Pacini, Journal of forensic accounting 5, 17 (2004).
[9] M. Sambridge, H. Tkalić, and A. Jackson, Geophysical research letters 37 (2010).
[10] G. Sottili, D. M. Palladino, B. Giaccio, and P. Messina, Mathematical Geosciences 44, 619 (2012).
[11] G. Whyman, E. Shulzinger, and E. Bormashenko, Results in Physics 6, 3 (2016).
[12] E. Bormashenko, E. Shulzinger, G. Whyman, and Y. Bormashenko, Physica A: Statistical Mechanics and its Applications 444, 524 (2016).
[13] B. Buck, A. Merchant, and S. Perez, European Journal of Physics 14, 59 (1993).
[14] D.-D. Ni, W. Lai, and R. Zhong-Zhou, Communications in Theoretical Physics 51, 713 (2009).
[15] J. Farkas and G. Gyurky, Acta Phys. Polon. B41, 1213 (2010), 1006.3615.
[16] A. Dantuturi and S. Desai, Physica A 506, 919 (2018), 1709.09823.
[17] T. P. Hill, Proceedings of the American Mathematical Society 123, 358 (1995).
[18] A. Burgos and A. Santos, American Journal of Physics 89, 851 (2021), 2101.12068.
[19] T. Alexopoulos and S. Leontsinis, Journal of Astrophysics and Astronomy 35, 639 (2014).
[20] A. Shukla, A. K. Pandey, and A. Pathak, Journal of Astrophysics and Astronomy 38, 7 (2017), 1606.05678.
[21] M. D. Melita and J. E. Miraglia, New Astronomy 89, 101554 (2021).
[22] J. de Jong, J. de Brujine, and J. De Ridder, Astron. & Astrophys. 642, A205 (2020), 2008.12271.
[23] M. A. Moret, V. de Senna, M. G. Pereira, and G. F. Zebende, International Journal of Modern Physics C 17, 1597 (2006).
[24] D. R. Lorimer and M. Kramer, Handbook of Pulsar Astronomy (2012).
[25] E. Petroff, J. W. T. Hessels, and D. R. Lorimer, Astronomy and Astrophysics Review 30, 2 (2022), 2107.10113.
[26] L. Shao and B.-Q. Ma, Astroparticle Physics 33, 255 (2010), 1005.1762.
[27] M. A. Krishnakumar, P. K. Manoharan, B. C. Joshi, R. Girgaonkar, S. Desai, M. Bagchi, K. Nobleson, L. Dey, A. Susobhanan, S. C. Susarla, et al., Astron. & Astrophys. 651, A5 (2021), 2101.05334.
[28] K. Nobleson, N. Agarwal, R. Girgaonkar, A. Pandian, B. C. Joshi, M. A. Krishnakumar, A. Susobhanan, S. Desai, T. Prabhu, A. Bathula, et al., Mon. Not. R. Astron. Soc. 512, 1234 (2022), 2112.06908.
[29] R. N. Manchester, G. B. Hobbs, A. Teoh, and M. Hobbs, Astron. J. 129, 1993 (2005), astro-ph/0412641.
[30] CHIME/FRB Collaboration, M. Amiri, B. C. Andersen, K. Bandura, S. Berger, M. Bhardwaj, M. M. Boyce, P. J. Boyle, C. Brar, D. Breitman, et al., Astrophys. J. Suppl. Ser. 257, 59 (2021), 2106.04352.
[31] G. Cowan, K. Cranmer, E. Gross, and O. Vitells, European Physical Journal C 71, 1554 (2011), 1007.1727.
[32] S. Ganguly and S. Desai, Astroparticle Physics 94, 17 (2017), 1706.01292.