Negative transverse magnetoresistance of the two-dimensional electron gas in quantum well with parabolic confinement potential under an in-plane magnetic field

M A Nizametdinova1, F M Hashimzade2, Kh A Hasanov2, M M Babayev2 and B H Mehdiyev2
1Azerbaijan Architecture and Construction University, Baku, Azerbaijan
2Institute of Physics, National Academy of Sciences of Azerbaijan, AZ 1143, Baku, Azerbaijan

E-mail: firudin_hashimzade@yahoo.com

Abstract. We developed a quantitative theory of transverse magnetoresistance for the two-dimensional electron gas (2DEG) in a quantum well with parabolic confinement potential in the magnetic field in the plane of the free movement of electrons. Our theory is based on the solution of Boltzmann kinetic equation for the components of the tensors of kinetic coefficients, when the perturbing force is directed along the plane of the free movement of electrons, and Titeike’s formula for the diagonal component of the conductivity along the direction of confinement, based on the idea of the drift of the centres of cyclotron orbits in the crossed electric and magnetic fields. Numerical calculations have been performed for the 2D electron gas in GaAs/AlGaAs QWs. It is shown, that the magnetoresistance is negative and equal about 50% in the weak magnetic field case, $\omega_c << \omega_0$ if the quantum limit $\hbar \omega_0 > k_0 T$ is fulfilled.

1. Introduction
Much interest has been aroused recently in the behavior of two-dimensional electron systems in the presence of magnetic field parallel to the 2DEG layer. The electrical characteristics of these systems are of interest both for fundamental and applied physics. The study of magnetotransport in these structures makes it possible to understand better the mechanisms and parameters of electron scattering. The theory of the quantum galvanomagnetic effects in size-quantized systems was studied in [1-4]. In [1,2] the case of a strongly degenerate electronic gas was considered with the focus was on oscillation phenomena. In [3] the conductivity in QW structures has been calculated, and the size dependence of conductivity in the quantum limit for different mechanisms of electronic scattering has been considered. The authors used the kinetic equation method and the density matrix approach. In the latter case, the scattering was entered into the equation of motion for the density matrix through the lifetime of a quantum state. In [5] the magnetoresistance was studied for the case of electron scattering on a deformation potential of a three-dimensional crystal in a transverse magnetic field. It has been shown that for semiconductors with low mobility the magnetoresistance is negative in weak transverse magnetic fields. In the same paper it was noted that the negative magnetoresistance is not directly associated with weak localization.
In this paper we have calculated the conductivity tensor components for the current density in a QW for degenerate and non-degenerate electron gas. We considered electron scattering by point defects, with the magnetic field across the direction of confinement, i.e in the plane of a 2DEG. Thus, two cases for the relative arrangement of the current direction and the confinement direction are possible. In the case where the current is located in a plane of a 2DEG it is sufficient to consider the relaxation time approximation and to use the kinetic equation. In the case when the current is along the direction of confinement it is necessary to use the density matrix approach obtained in [6-7] for calculation of the diagonal conduction tensor components.

2. Theory

For practical purpose it is convenient to use the harmonic potential model which enables us to calculate various quantities analytically. We consider a quantum well, in which a 2DEG is confined in the x-direction and a homogenous static magnetic field $\vec{B}=(0,0,B)$, with the vector potential $\vec{A}=(0,xB,0)$ lies in the y-z plane of the layer of a 2DEG parallel to the z-axis. The confining potential in the x-direction is characterized by the parabolic potential of frequency $\omega_0$, $U(x)=m^*\omega_0^2x^2/2$, where $m^*$ is the effective mass of a conduction electron.

The eigenvalues and eigenfunctions of the Schrodinger equation are given by

$$\varepsilon_n = \left( \frac{1}{2} + N \right) \omega + \left( \frac{\omega_n}{\omega^*} \right)^2 \frac{\hbar^2 k_n^2}{2m^*} + \frac{\hbar^2 k_z^2}{2m^*},$$

$$\phi_n(x,y,z) = \frac{1}{\sqrt{L_x L_y \sqrt{\pi \sqrt{2^N N!}}}} \exp \left\{-\left( \frac{x-x_n}{\sqrt{2R}} \right)^2 \right\} H_N \left( \frac{x-x_n}{R} \right) e^{i(k_n y + k_z z)}$$

where $\omega = \sqrt{\omega_0^2 + \omega^2}$ is the “hybrid” frequency, $\omega_c = eB/m^*c$ is the cyclotron frequency of electrons and $N$ is the oscillation quantum number, $x_n = -\omega R^2 k_n / \omega$ is the oscillator center, and $R = \sqrt{h/m\omega}$, $H_N(\frac{\xi}{R})$ is the Hermite polynomial, $\alpha = (N, k_n, k_z)$ is a set of quantum numbers that determine the electron states in a magnetic field.

The transverse component of the magnetoresistance in a magnetic field is defined by

$$\rho_{yx} = \frac{\sigma_{yx}}{\sigma_{xx} \sigma_{yy} - \sigma_{yx}^2 \sigma_{xx}}$$

For the calculation of kinetic coefficients $\rho_{yx}$, it is necessary to calculate both diagonal and non-diagonal components of conductivity tensor $\sigma_{ik}$. Note that in bulk semiconductors $\sigma_{yx} \gg \sigma_{xx}$. It is related to the fact that a decrease in scattering potential results in the diagonal electric conductivity tensor components tending to zero, while the non-diagonal components stay finite [7, 8]. In QW, in contrast to the bulk crystal, the diagonal components of the conductivity tensor can exceed the non-diagonal ones [4].

The expression for non-diagonal components, which was obtained using the density matrix approach, is given in [9] as

$$\sigma_{yx} = -\sigma_{xy} = \frac{\omega_e^2 n}{m^* \omega^*}$$

where

$$n = \frac{k_B T m^*}{\pi \hbar^2} \frac{\omega}{\omega_0} \sum_N \ln \left( 1 + e^{\gamma - \zeta_N} \right)$$

is the density of the two-dimensional electron gas, and
Here $\zeta$ is the chemical potential of the electrons, and $k_0$ is the Boltzmann constant.

In the limit of strong magnetic fields, $\omega_0 << \omega_c$, or equivalently, in the bulk case, when $\omega_0 \to 0$, the energy spectrum (1) equals that of an electron in a magnetic field. In this case the expression for $\sigma_{yx}$ in (5) coincides with that for the non-diagonal component of the conductivity tensor of the bulk semiconductor material.

For the calculation of the diagonal components of conductivity tensor $\sigma_{xx}$ when the electric field is perpendicular to the plane of two-dimensional electron gas we will make use of the expressions obtained in [6] and [8], modified so as to allow for 2D character of the electron gas.

The scattering mechanisms considered in the present work are the elastic electron scattering on short-range potential.

Further we will focus on the extreme situation, namely, the quantum limit in which the scattering of electrons is confined within the $N = N' = 0$ level. For the quantum well in a magnetic field the quantum limit criterion is $\hbar \omega \gg k_0 T$. This case is the most interesting in practice for the transport phenomena.

The expression for $\sigma_{xx}$ is given by

$$\sigma_{xx} = \frac{n e^2 \omega_0^2}{m \tau_0 \omega_0} A_1$$

(7)

$$A_1 = \frac{1}{\log(1 + e^\eta)} \int_0^\eta \left( -\frac{\partial f_0}{\partial x} \right) e^{-2ax} I_0(ax)^2 \left(1 - \frac{I_1(ax)}{I_0(ax)}\right) dx$$

(8)

where

$$\tau_0 = \frac{L_0 L_z}{N_d} \frac{\pi R^2 \hbar^2}{m V_0^2}, \quad a = \frac{k_0 T \omega_0^2}{\hbar \omega \omega_0^2}, \quad f_0 = \left(1 + \exp\left(x - \tilde{\eta}\right)\right)^{-1}, \quad \tilde{\eta} = \eta - x_0$$

(9)

Here $I(\xi)$, $N_d$ and $V_0$ are the modified Bessel function of the first kind, two-dimensional concentration, and the strength of point defects, respectively.

For the case of the electric field and the temperature gradient directed along the plane of two-dimensional electron gas, we use the solution of the kinetic equation to calculate the diagonal components of tensors $\sigma_{yy}$

$$\sigma_{yy} = \frac{n e^2 \tau_0 \omega_0^4}{m \omega_0^4} A_2$$

(10)

$$A_2 = \frac{1}{\log(1 + e^\eta)} \int_0^\eta \left( -\frac{\partial f_0}{\partial x} \right) e^{2ax} \left(1 + \frac{I_1(ax)}{I_0(ax)}\right) dx$$

(11)

For the relative change of the resistance in transverse magnetic field we obtain

$$\frac{\Delta \rho_{yy}(B)}{\rho(0)} = \left( \frac{\omega}{\omega_0} \right)^4 \frac{A_1}{A_1 A_2 + 1} - 1$$

(12)

3. Results and discussion

We present the numerical calculations for a $GaAs/Al_xGa_{1-x}As$ parabolic quantum well. We use the
following set of physical parameters. The effective mass, \( m' = 0.066 m_0 \), where \( m_0 \) is the free electron mass. The parameter of the parabolic potential is \( \omega_c = 1.4 \times 10^{13} \text{ s}^{-1} \), and for the temperature we assume \( T = 5 \text{ K} \). Notice that in the calculations for the quantum limit criterion the Fermi level must be between the first and second subbands \( \hbar \omega / 2 \leq \zeta < 3 \hbar \omega / 2 \).

In Fig.1 we plot the relative change of magnetoresistance in QW as a function of dimensionless parameter \( \omega_c / \omega_0 \) for different 2DEG concentrations (\( T = 5 \text{ K} \)). The dashed line correspond to the case \( A_i = A_2 = 1 \).

Fig.1 The dependence of the relative change of magnetoresistance as a function of dimensionless parameter \( \omega_c / \omega_0 \) for different 2DEG concentrations (\( T = 5 \text{ K} \)).

The analysis of the results for scattering on point defects shows that when the dependence of the form factor on the magnetic field in terms of the matrix element of scattering is neglected the dependence of the magnetoresistance on the magnetic field does not change significantly. With a weak filling of the conduction band this is quite justified, as it is evident from Fig.1. For the case of DA scattering mechanism on acoustic phonons and alloy disorder scattering, the scattering probability does not depend on the momentum transfer, and in this approximation the dependence of magnetoresistance on the magnetic field takes the form

\[
\frac{\Delta \rho_x (B)}{\rho(0)} = \frac{1}{2} \left( \frac{\omega}{\omega_0} \right)^{7/2} - 1
\]

For other scattering mechanisms such a simple relationship cannot be obtained, but in a weak field, as seen from (7) and (10), when conditions \( \omega_0 >> \omega_c \) and \( \omega_c > 1 \) are satisfied, we have \( \sigma_{xx} \sigma_{yy} = \sigma_{xy}^2 \), and magnetoresistance is negative.

References

[1] Simserides C D, 1999 J. Phys.: Condens. Matter 11, 5131
[2] Blokh M D, 1975 Sov. Phys. Solid State 17, 567  (Engl.Transl).
[3] Kubakaddi S S, Mulimani B G and Jali V M, 1986 Phys. Stat.Sol. (b) 137, 683
[4] Sinyavskii E P, and Khamidullin R A, 2002 Semiconductors 36, 924
[5] Kaminskii V E, 2002 Semiconductors 36, 1276
[6] Adams E N and Holstein T D, 1959 J.Phys. Chem. Sol. 10 , 254
[7] Anselm A I, and Askerov B M 1967 Sov. Phys. Solid State 9, 22
[8] Askerov B M 1994 Electron Transport Phenomena in Semiconductors (Singapore: World Scientific)
[9] Hashimzade F M , Hasanov Kh A, Mehdiyev B H and Cakmak S 2010 Phys. Scr. 81, 015701