Effects analysis of the friction on the buckling of horizontal string

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Abstract. This paper takes a slight string that is under the constraint of a horizontal circular tube as the analysis model, and then establishes the differential equations about the nonlinear buckling behavior of the constrained tubing string under the consideration of the friction and boundary constraints. The paper also analyzes the sinusoidal buckling critical load under the friction coefficient ranged from 0 to 0.6 and four different boundary constraints. At the same time, the research simulate these four different boundary constraints: SS-SS, SS-C, C-SS, and C-C. The results show that C-C is suggested as the boundary constraint in the process of drilling operation, so that when the friction increases, the method of inhibiting the drill string buckling can be utilized to prolong the life of the drill string and reduce the drilling cost.

1 Introduction
Horizontal well technology can reach the remaining oil region of the top layer that vertical drilling left, and deploro the remaining oil from the region unswept because of the influence of edge horn, and remaining in series of strata formed from different physical properties and disproportionate withdrawal in the single layers[1]. In addition, the drilling of horizontal well takes lower cost than the vertical one. Therefore, horizontal well technology is extremely broadly applicable, and can be more economic and effective to increase the recovery ratio and increase production.

Comparing with vertical wells, it is difficult to determine the contact point of pipe string and borehole wall in horizontal wells, which contributes to a more complex situation and a further research. Scholars have made certain research in recent years about the buckling problems for horizontal pipe string that bound by the radial constraint. Guo-hua Gao[2] worked out the buckling equation according to deformation geometric equations and the static equilibrium equations in horizontal hole, and discussed the buckling critical load in horizontal hole under different boundary conditions through the parameter perturbation solution, then pointed out that reducing friction coefficient can enhance the critical drilling pressure of drill string in horizontal hole under the hypothesis of the small angular displacement, but the effects of friction on the helical buckling behavior need yet to be further analyzed. By means of the method of finite element incremental weighted iteration, Feng Liu and Xin-wei Wang[3-4] studied the influence on drill string buckling caused by the tangential friction from the borehole wall and drill string with weight in directional well, also established the equilibrium equations for drill string buckling and the directional well, and established the equilibrium equations for drill string buckling and the corresponding functional expression in the horizontal well. The influence of gravity and torque were considered on the buckling in this mechanical model. Fengwu Liu and Deli Gao[5] obtained the relationships between the deformation and the loads of horizontal drilling string in a state of sinusoidal buckling by energy method. So far the literature on the buckling of horizontal well is still insufficient, therefore, further
studies are needed to achieve more precise control about the horizontal drill string buckling problem.

2 The drill string buckling model
In horizontal wells, the slight drill string bounded by radial will be close to the bottom hole under gravity. Along with increasing of the load, the drill string in initial equilibrium state will lost stability, experience sinusoidal and helical buckling in turn due to the hole radial limitation. As shown in Figure 1, the drill string may occur in the drilling operation.

In order to analyze effectively the buckling characteristic of horizontal drill string bounded by the radial, the tubing drill string is regarded as linear elastic beam according to the actual operating mode of the flexible drill string in horizontal well. If the axial drilling pressure and weight on bit do not exceed key value, it is able to keep the ability to restore the original state. The axial pressure exerts on the circular cross section of the horizontal pipe string. In the initial state, pipe string is at the bottom of the horizontal well since its self-gravity, and the influence of drilling fluid is ignored. Setting up the mechanics analysis model of string buckling, as shown in Figure 2. In the model, the string uses $E$ for the elastic modulus and $EI$ for the bending rigidity, $L$ for the length of the string, $q$ for the weight on per unit length, $p$ for the drilling pressure imposed by the drill string at the left, the right for the axial opposing force at the bottom of pipe string $F_b = P - \int_0^L \mu N_d ds$, $T$ for the torque, $N_r$ for the line density of non-uniform contact force between the wall and pipe string, $f = \mu N_r$ for the axial frictional forces on per unit length, $\alpha$ for the hole deviation angle, $\theta$ for the angular displacement that string deviates from the initial position, $r$ for the center distance between the constraining circular tube and the constrained string. Establishing right angle coordinate system and taking cross section of the string for $x, y$ plane, the axis of the drill string as the $z$ axis.

$$x = r \sin \left( \frac{2\pi z}{p} \right)$$
$$y = 0$$
$$z = z$$

(1)

Figure 1: Schematic diagram of buckling of tubular column in a constrained circular tube.

Figure 2: Mechanical analysis of constrained tubular column with horizontal circular tube

According to the sinusoidal and spiral buckling geometry of the string, and describe them as following geometric relationships:
where x and y are the horizontal displacement of pipe string in the coordinate system respectively; Z is the axial displacement; P is the sinusoidal buckling wavelength of the drill string; r is the center distance between constraining circular tube and constrained string.

Supposing the constrained string always keeps contact with the restraint circular tube in the process of buckling deformation. This paper argues that the drill string is a flexible beam and has no malleability. Comparing with the length of bending string, the radial distance between constraints pipe and the constrained string are petty, therefore, the horizontal displacement is also relatively small. So one can know that the spiral angle \( \beta \) of the drill string buckling is relatively tiny, which can be approximately calculated by

\[
\beta \approx \sin \beta = r \frac{d \theta}{ds}
\]

(2)

3 The buckling equilibrium differential equation

3.1 The buckling Differential Equation with Heavy Tube

Considering the gravity of the drill string and ignoring the friction force, the governing differential equations of the drill string constrained by the circular tube are: (Mitchell, 1988):

\[
EI \theta'' + (F \theta') - 6EI (\theta')^2 \theta'' + q_s \sin \theta / r = 0
\]

(3)

where \( F = P + q_s \), \( q_s = q \cos \alpha \), \( q_s = q \sin \alpha \), \( \alpha \) is the inclined angle.

In the paper, the research object is horizontal drill string, so the deviation angle in the formula is \( \alpha = 90^\circ \), then, \( q_s = q_\theta, q_s = 0 \).

Eq. (3) and Eq. (4) can be simplified as:

\[
EI \theta'' + (F \theta') - 6EI (\theta')^2 \theta'' + q \sin \theta / r = 0
\]

(5)

\[
N_i = rF (\theta')^2 + q_s \cos \theta - EI r (\theta')^4
\]

(6)

3.2 The buckling differential equations considering friction

Under the self-weight, the tube column, which is constrained by a horizontal circular tube, will tightly close to the bottom of the circular pipe, and maintain a straight line balance. Considering the axial friction, the horizontal drill string column to the differential unit, the following approximation is employed:

\[
dF \approx dF \mu = -\mu N_i
\]

(7)

where \( F \) is the axial force, and \( s \) is the deformation path column central line tube, and \( \mu \) is the friction coefficient between the columns with circular tubes. It is considered that the direction of axial friction is opposite to the direction of axial load, so the simplify problem can be done.

By Eq. (7), combining with the boundary condition, the Eq. (8) can be obtained as:

\[
F \approx P - \mu \int_0^s N_i ds
\]

(8)

where \( P \) is the axial force acting at the left side (s=0) of the string.

Substituting Eq. (8) into Eq. (5) and Eq. (6), get the equations:

\[
EI \theta'' + \left( P - \mu \int_0^s N_i ds \right) \theta'' - \mu N_i \theta' - 6EI (\theta')^2 \theta'' + q \sin \theta / r = 0
\]

(9)
\[ N_i(s) = \left( P - \mu \int_0^s N_i ds \right) r(\theta)^2 + q \cos \theta - EI r(\theta)^4 \]  

Eq. (9) and Eq. (10) are nonlinear differential equations of flexible tubular columns confined by horizontal circular tubes. Eq. (9) and Eq. (10) are coupled, which are different from the Eq. (3) and Eq. (4).

When the solution is obtained by the value of a and B, the deformation path of the center line of the pipe string can be calculated by the following formula:

\[
\begin{bmatrix}
  u \\
  v \\
  w
\end{bmatrix} = \begin{bmatrix}
  r \cos \theta \\
  r \sin \theta \\
  0
\end{bmatrix} + \int_0^s \begin{bmatrix}
  -\frac{r}{ds} \sin \theta \\
  \frac{r}{ds} \cos \theta \\
  1 - \frac{1}{2} \frac{r^2}{ds} \frac{d\theta}{ds}
\end{bmatrix} ds
\]

where \( u \), \( v \), \( w \) respectively on behalf of the position vector of the deformed rod centerline.

### 4 Simulation and analysis

Based on the established mathematical model, the simulate and analyze be performed. The main calculation parameters are shown in Table 1, the variable parameters are not defined and assigned. In addition to the fixed system parameters, the variable parameters used in the simulation are the friction coefficient between the string and borehole wall and boundary conditions. The four kinds of different boundary constraints will be considered as follows: simply supported-simply supported (SS-SS); simply supported-clamped (SS-C); clamped-simply supported (C-SS); clamped-clamped (C-C).

The curves about relationships between the horizontal drill string sinusoidal buckling critical loads and the friction coefficients under the SS-SS and SS-C, C-SS, C-C boundary constraints are shown as in Figure 3. We find the relationships between horizontal string sinusoidal buckling critical load and the friction coefficient should also be determined to the boundary conditions. The drill string buckling critical load increases along with the increasing of the friction coefficient. In C-SS and SS-SS constraints, the buckling critical load increases along with the increasing of the friction coefficient, but the range is relatively small. In the C-C and S-C constraints, the critical buckling load decreases along with the increasing of the friction coefficient, and the magnitude is also minor. When the friction coefficients is same, and among the four kinds of boundary conditions, the buckling critical load has the minimum value of only 0.80044N under the SS-SS restriction, while the maximum value of 16.22N under the restriction of C-C. Therefore, the clamped-clamped (C-C) is suggested as the boundary constraint in the process of drilling operation, so when the friction increasing, the method of inhibiting the drill string buckling can be utilized to prolong the life of the drill string and reduce the drilling cost.

| Parameters                  | Symbol | Value |
|-----------------------------|--------|-------|
| Length/（m）                 | L      | 10    |
| Diameter of Mineshaft/（mm） | D      | 32    |
| External Diameter/（mm）     | d₁     | 14    |
| Poisson Ratio               | ν      | 0.3   |
| Elasticity Modulus/（GPa）   | E      | 127   |
| Density/（kg/m³）             | ρ      | 8900  |
| Hole Deviation Angle/（°）   | α      | 90    |
5 Conclusions
Considering to the characteristics of mutual influence of the gravity on the drill string mathematical model was established, in which the axial force, the contact force, and the friction in the actual drilling operation process are considered. The research result show when the friction coefficient is the same, the critical buckling load under the SS-SS constraint is the smallest and the C-C constraint is the largest in the four boundary conditions. Therefore, in the actual process of the drilling, the C-C used as the boundary constraint method is recommended. When the friction resistance increasing, the drill string buckling can be restrained, which prolonged the life of the drill string and reduced the cost of the drilling.

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