Weak Guidance, Multi-index and Exact Mode Eigenvalue Calculation for Mode Intensity Profiles in Tapers

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Abstract. The fundamental mode intensity profile is calculated for different points along the transition of an optical fibre taper from standard size (125 µm external cladding diameter and 8 µm core diameter) down to micro (1 µm external fibre diameter) and submicro size (down to 440 nm fibre diameter) at 1550 nm operational wavelength. The first section of the taper was evaluated using weak guidance approximation, the second section was treated as a three-index layer structure (double-clad) and evaluated with eigenvalue equations for three refractive indices. An external medium being a liquid with a refractive index close to silica is being considered. The third and thinnest section of the taper was evaluated with the exact mode eigenvalue equation. The results show that the fundamental mode intensity for the third section show a discontinuity just at the fibre edge with a peak amplitude larger than the optical field at the centre of the fibre. The taper shape that complies with the adiabaticity criterion and how the mode intensity profile and the spot size (Petermann 1 definition) of the fundamental mode change along their position on the taper were determined.

1. Introduction

After almost five decades, tapered fibres continue to be identified as a significant topic of research [1][2]. Tapers are still under investigation because of their useful applications as optical fibre sensor and multiplexing devices. Examples of these are evanescent wave biosensors [3][4], functionalization [5][6], hydrogen detection, [7] and Astro-photonic devices [8][9]. The analysis of fibre tapers including the adiabatic concept, has been available since 1983 in a textbook [10] and paper [11]. The concept of adiabaticity means that it is possible to fabricate low loss tapers provided that their geometrical shape varies slowly so it does not allow energy conversion from the fundamental mode (LP_{01} or HE_{11}) to the next circularly symmetric mode (LP_{02} or HE_{12}). This can be implemented based on single mode fibre tapers.

In 2003, a new development was reported: that the fibre taper waist diameter can be downsized to 1 µm (micro optical fibre) or even less (approximately down to 200 nm) in what was called “sub-wavelength diameter silica wires” [12]. Some authors refer to these ultrathin optical fibres as “nano optical fibres”; in this work, the term “sub-micro”, which we believe is more accurate, is used. The
significance of this result was that these fibre devices can have new optical properties and new applications as depicted in the “Tong Tree” [12]. This also has been called “The second life of optical fibre tapers” [13] and the technology entered a very productive phase. However, in all these years little detailed attention has been paid to the evolution of the fundamental mode intensity and its exact shape distribution along all the fibre structure, from the standard size (125 µm) to the micro size (1 µm) and sub-micro size (200 nm) including the transition regions. The core diameter at the standard size is approximately 8 µm for an SMF-28 Corning optical fibre, 64 nm at 1 µm external diameter at 440 nm external diameter. If we consider that the guiding limit is around 200 nm [14] and the original core completely vanishes at microscopic and sub-microscopic sizes, we effectively have a new optical waveguide formed at the cladding and the surrounding medium. The analysis implies the evaluation of the eigenvalue equation at many successive points along the taper and its corresponding mode field and mode intensity profile; the general case is a three-layer refractive index profile waveguide. Also, it is necessary to adjust the central peak intensity to show that as the mode intensity profile extend, the peak amplitude should change to preserve the total amount of optical power in the fundamental mode. We believe that it is important to observe how the mode intensity changes along the tapered fibre. The results show unexpected variations in the mode intensity profile and spot size along the taper length. Furthermore, the application of the adiabatic criterion also yields relatively long tapers.

The shape of fibre tapers has been already studied [15]. We start our analysis assuming an initial hypothetical shape based on an experimental taper reported by Linslal [16]. We propose the theoretical taper shape shown in figure 1, which is actually very close to some practical shapes obtained by other authors. It has been modified to include a section which is “micro”, i.e. 1 µm waist diameter and another section which is “sub-micro” or “sub-wavelength” with a minimum waist diameter of 440 nm. A diameter smaller than 200 nm reaches the guiding limit and the waveguide effect ceases to exist at an operational wavelength of 633 nm. The taper has been assumed to be a symmetrically biconical shape with 19 equidistant points (marked by the letters A-S, 2 mm apart) along the taper from the initial standard size A (125 µm diameter) to the smallest waist diameter of S (440 nm). As the taper is symmetrical, the waveguide parameters in the tapering down section (A-S) are mirrored in the tapering up section (S-A). Note that it will later be found that this taper is not adiabatic, meaning that there is mode conversion and a consequent power loss. However, this taper shape can be modified to meet the adiabatic criterion. The shape of the taper was divided into three sections in order to be able to identify them in the points that the evaluation takes place, within the corresponding eigenvalue equations: section 1 (from point A to F), section 2 (from point G to N) and section 3 (from point O to S).

![Figure 1. Initial theoretical taper shape. Length = 72 mm, Initial external diameter (A) = 125 µm, Taper waist (S) = 0.44 µm.](image)

The refractive index profile of the tapered fibre changes along the taper sections as shown in figure 1. The first section of the taper region has two refractive indices corresponding to a single mode with an initial cladding diameter of 125 µm, initial core radius a (0) = 4 µm, and an operation wavelength of 1550 nm. The refractive indices of the core and cladding are n₁ and n₂ respectively, and the values
for these indices are: \( n_1 = 1.46125 \), \( n_2 = 1.45625 \). The second section of the taper, that is, the transition section, has three refractive indices \( n_1, n_2, n_3 \) and a core radius “a” and a cladding radius “b”. Here \( n_1 \) and \( n_2 \) are the same as the first section, but for \( n_3 \) we have \( n_3 = 1.00029 \) (air) [17]. In the third section the core has become so small that it has effectively disappeared and only two refractive indices \( n_2=1.45125 \) and \( n_3=1.00029 \) remain. The fibre diameter goes from 1.22 µm (O point) to 0.44 µm (S point).

2. Eigenvalue equations for the fundamental mode propagation
To determine the mode intensity profile evolution along the taper we require the solution of three different sets of eigenvalue equations. In order to evaluate mode intensity profile of the fundamental mode for points A to E, we use the weakly guiding mode equation [18]. The reason is that in this section we have two refractive indices with very similar values.

\[
\frac{u J_1(u)}{J_0(u)} = \frac{K_1(w)}{K_0(w)}
\]  

Where “u” and “w” are the modal parameters as defined as: 
\( u = a(k_0^2 n_1^2 - \beta^2)^{1/2} \), 
\( w = a(\beta^2 - k_0^2 n_2^2)^{1/2} \), 
\( a = \) core radius, 
\( k_0 = \frac{2\pi}{\lambda} \), 
\( \beta \) is the mode propagation constant. 
\( J \) is the Bessel function of the first kind of order \( \nu \), 
\( K \) is the modified Bessel function of the second kind of order \( \nu \).

For point F to N, we use the Monerie equation [19].

For the last section (third part) we use the exact mode equation:

\[
\left( \frac{J_q(u)}{uJ_q(u)} + \frac{K_q(w)}{wK_q(w)} \right) \left( \frac{J_q(u)}{uJ_q(u)} + \frac{n_3^2 K_q(w)}{n_2^2 wK_q(w)} \right) = \left( \frac{q\beta}{kn_2} \right)^2 \left( \frac{V}{UW} \right)^4
\]  

3. Results and discussion
We can observe the mode intensity profiles in figure 2 for points A to F. In this case, we attend only at the shape of the mode intensity profile and we can see how it expands and changes its shape as the fibre is tapered down.

![Figure 2](image_url)  

**Figure 2.** Fundamental mode intensity from points A to F. Colour blue is the field intensity part propagated into the core and colour black is propagated into the cladding.
For the second section our results are as shown in figure 3:

![Figure 3. Fundamental mode intensity from points G to N.](image)

And for the last section the results are shown in figure 4 below:

![Figure 4. Fundamental mode intensity from points O to S.](image)

4. Adiabaticity condition and spot size evolution
It is key to find out if the optical fibre taper is adiabatic, that is the taper has very low loss. The adiabatic criterion for optical fibre tapers it is well known [9]. figure 5 shows the adiabatic limit slope (3 and 4) for the proposed shape and b) for adjusted shape.
\[
\frac{da(z)}{dz} \ll \left| \frac{a(z)}{z_b} \right|
\]

\[
z_b = \frac{2\pi}{\beta_1 - \beta_2^2}
\]

Where \(a(z)\) is the effective core radius of the waveguide (in some cases is \(b(z)\)), \(z_b\) is the beat length between the LP\(_{01}\) and LP\(_{02}\) modes (or HE\(_{11}\) and HE\(_{12}\) modes), \(\beta_1\) is the propagation constant of LP\(_{01}\) (or HE\(_{11}\) mode) and \(\beta_2\) is the propagation constant of LP\(_{02}\) (or HE\(_{12}\) mode). The rate of change of the taper (taper slope) should be lower than the rate of change given by the theoretical limit slope which is a function of the effective core radius of the fibre divided by the beat length of the first circular symmetric modes. It should be noted that when the taper is single mode, \(\beta_2\) corresponds to the cut-off condition of the LP\(_{02}\) mode equal to \(n_2k_0\) or \(n_3k_0\).

**Figure 5.** a) Limit slope by the adiabatic criterion and b) adjusted taper slope.

In order to show how the spot size radius \(\omega_0\) or alternatively the mode field diameter (MFD=2\(\omega_0\)) changes along the taper, we use the first Petermann definition as given by equation (5):

\[
\omega_0 = \sqrt{2} \left[ \int_{\omega_0}^{\infty} \varphi^2(r) r^3 \, dr \right]^{1/2}
\]

The spot size is a measure of the expansion or contraction of the mode field, and therefore the field intensity profile. In this work, the spot size definition for the Gaussian approximation for the mode field is not used as it does not apply for most of the points along the taper. **Figure 6** shows the spot size evolution along the position of the taper.

**Figure 6.** Spot size evolution along the tapered in a fibre.
The spot size radius increases in the first section of the taper, reaching a maximum (21.20 µm) at point F with 23.13 µm of fibre external radius, however the spot size evolution behaves now very differently with a minimum (0.76 µm) at point P with 0.81 µm of fibre external radius. The spot size radius increases to 4.84 µm with a 220 nm of fibre external radius.

5. Conclusions
This work has presented a realistic simulation of the evolution of the mode intensity profile along a tapered fibre, from standard size (125 µm external diameter) to micron and sub-micron size external diameter, with 440 nm being the smallest waist size operating at 1550 nm wavelength for the case of a taper in air. Optimum fibre external diameter values for smallest spot sizes and largest mode field intensities were found. It is expected that this analysis will be useful for researchers in the area of evanescent submicron optical fibre sensors. A reduction in outer diameter size (440 nm) still guides the fundamental mode with an expanded evanescent field was obtained.

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