Consistency of ground state and spectroscopic measurements on flux qubits

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We compare the results of ground state and spectroscopic measurements carried out on superconducting flux qubits which are effective two-level quantum systems. For a single qubit and for two coupled qubits we show excellent agreement between the parameters of the pseudospin Hamiltonian found using both methods. We argue, that by making use of the ground state measurements the Hamiltonian of \( N \) coupled flux qubits can be reconstructed as well at temperatures smaller than the energy level separation. Such a reconstruction of a many-qubit Hamiltonian can be useful for future quantum information processing devices.

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Quantum systems are generally characterized by spectroscopic measurements: the system is excited by electromagnetic radiation, with a frequency which matches the level spacing, and the response of this excitation is detected. On the other hand, quantum theory predicts that the Hamiltonian of some quantum-mechanical systems can be completely reconstructed from their ground-state properties. For instance, quantum mechanical treatment of the ammonia molecule in a two-level approximation, shows that its ground state contains information about time-independent Hamiltonian parameters. Superconducting qubits are also described by a similar Hamiltonian. They are micrometer-size quantum systems which can be easily accessed by a macroscopic measuring device. For example, the Hamiltonian parameters of a superconducting flux qubit can be determined from the measurement of its magnetic susceptibility in the ground state. In this Letter we will demonstrate that for a single and two coupled flux qubits the ground-state and the spectroscopic measurements give the same results.

The persistent current, or flux, qubit is a small superconducting loop with three submicron Josephson junctions. Due to the flux quantization only two phases are independent. Thus, the circuit is characterized by a two dimensional potential \( U(\phi_1, \phi_2) \) which, for suitable qubit parameters, exhibits two minima. In the classical case these minima correspond to clockwise and anticlockwise supercurrents in the loop. If the applied magnetic flux equals half a flux quantum, \( \Phi_x = \Phi_0/2 \) (\( \Phi_0 = h/2e \)), both minima have the same potential, leading to a degenerate ground state. According to the quantum mechanics the degeneracy is lifted close to this point and the flux qubit can be described by the Hamiltonian:

\[
H(t) = -\Delta \sigma_x - \varepsilon \sigma_z + A \cos(\omega t) \sigma_z ,
\]

where \( \sigma_x, \sigma_z \) are the Pauli matrices for the spin basis and \( \Delta \) is the tunneling amplitude. The qubit bias is given by \( \varepsilon = I_p (\Phi_x - \Phi_0/2) \), where \( I_p \) is the magnitude of the qubit persistent current. The last term describes the microwave irradiation necessary for the spectroscopy which aims to probe the stationary energy levels represented by the first two, time independent, terms of this Hamiltonian. The eigenvalues of the Hamiltonian depend on the flux bias \( \Phi = \Phi_x - \Phi_0/2 \):

\[
E_{\pm}(\Phi) = \pm \sqrt{(I_p, \Phi)^2 + \Delta^2}.
\]

Spectroscopic measurements detect the excitation of a qubit close to the point where the microwave irradiation matches the level spacing: \( \hbar \omega = \Delta E(\Phi) \equiv E_+(\Phi) - E_-(\Phi) \). By measuring \( \Delta E \) as a function of \( \Phi \) the qubit parameters \( \Delta \) and \( I_p \) can be obtained as has been shown by van der Wal et al. [6].
Alternatively, the same information can be obtained from ground-state measurements. Indeed, let us consider a flux qubit weakly coupled to a classical oscillator consisting of an inductor $L_T$ and a capacitor $C_T$ forming a tank circuit. Due to the mutual inductance $M$ the tank biases the qubit resulting in $\Phi = \Phi_{dc} + \Phi_{rf}$. Provided that the resonant frequency of the tank is small, $\omega_T = 1/\sqrt{L_TC_T} \ll \Delta/h$, and the temperature is low enough, $k_B T \ll 2\Delta$ ($k_B$ is Boltzmann’s constant), the qubit will reside in its ground state $E_-$. The dynamic behavior of the tank-qubit arrangement can be described by the Lagrangian

$$\mathcal{L} = T - U = \frac{1}{2} L_T \dot{q}^2 + E_- (\Phi_{dc} + M \dot{q}) - \frac{1}{2} \frac{q^2}{C_T},$$

(3)

where $q$ is the charge on the tank capacitor and $\dot{q}$ is the circulating current in the tank. Such Lagrangian would lead to the nonlinear equation of motion:

$$0 = \frac{d^2}{dt^2} \frac{\partial \mathcal{L}}{\partial q} - \frac{\partial \mathcal{L}}{\partial \dot{q}} = \left( L_T + M^2 \frac{d^2}{dt^2} E_- (\Phi_{dc} + M \dot{q}) \right) \dot{q} + q \frac{\dot{q}}{C_T},$$

(4)

however for small amplitude of $\dot{q}$ the Lagrangian can be linearized by replacing $E_- (\dot{q})$ by its second order Taylor expansion around $\Phi_{dc}$. Consequently, we obtain the simple Lagrangian of a particle in a parabolic potential well:

$$\mathcal{L} = \frac{1}{2} m^* \dot{q}^2 - \frac{1}{2} k^* q^2. $$

(5)

The equation of motion which can be obtained from this Lagrangian is just the simple equation for a particle in a parabolic potential, $m^* \dot{q} = k^* q$, where

$$m^* = \left( L_T + M^2 \frac{d^2}{dt^2} E_- (\Phi_{dc}) \right),$$

(6)

is the effective mass and $k^* = 1/C_T$ is the curvature of the parabolic potential well. Thus, the resonant frequency of the tank-qubit arrangement

$$\omega_0 = \sqrt{\frac{k^*}{m^*}} \approx \omega_T \left( 1 - \frac{M^2}{2L_T} \frac{d^2}{dt^2} E_- (\Phi_{dc}) \right),$$

(7)

contains information on the curvature of the ground state of the qubit. Differentiating Eq. (2) results in:

$$\frac{d^2}{d\Phi_{dc}^2} E_- (\Phi_{dc}) = -\frac{(I_p \Delta)^2}{(\varepsilon^2 (\Phi_{dc}) + \Delta^2)^{3/2}},$$

(8)

showing that $\Delta$ and $I_p$ can be determined from the dependence of the resonance frequency of the tank circuit on the applied flux.

In order to compare both methods we fabricated a two qubit sample like the one shown in (Fig. 1). As either one of the qubits can be biased far away from degeneracy, the single qubit properties can be studied as well. This can be understood if we consider the Hamiltonian of two coupled flux qubits:

$$H_{2qbs} = -\Delta_a \sigma_z^{(a)} - \Delta_b \sigma_z^{(b)} - \varepsilon_a \sigma_x^{(a)} - \varepsilon_b \sigma_x^{(b)} + J \sigma_z^{(a)} \sigma_z^{(b)},$$

(9)

where $J$ is the Josephson coupling energy provided by the large connecting Josephson junction. Suppose qubit $a$ is the one biased far from its degeneracy point in such a way that $\varepsilon_a$ is large in comparison with the other energy variables. Then, qubit $a$ has a well defined ground state with averaged spin variables $\langle \sigma_z^{(a)} \rangle = 1$ and $\langle \sigma_x^{(a)} \rangle = 0$ which can be av-

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**FIG. 2:** Comparison of the ground state and spectroscopic measurements for qubit $b$. (a) Ground state measurements. Presented is the dependence of the phase shift between the tank circuit voltage and bias current on the flux bias. The solid lines are experimental data fitted by the theoretical curves (dashed curves) for qubit parameters $I_p = 225$ nA and $\Delta/h = 1.75$ GHz. The curves correspond to various values of the $rf$-bias current on the tank circuit resulting in $rf$ voltage amplitudes, from top to bottom, $V_T = 4.3, 2.9, 0.5$, and $0.3 \mu$V. (b) Amplitude of the tank voltage as a function of the normalized magnetic flux in the qubit at the driving frequencies, from top to bottom, $\omega/2\pi = 18, 5$, and $3.5$ GHz. The curves have been shifted for clarity. The resonant excitation in the flux qubit results in the peak-and-dip at the positions defined by the condition $\Delta E (\Phi_{dc}) = \hbar \omega$. (c) Energy gap $\Delta E$ between the qubit energy levels determined from the positions of the mid points of the peak-and-dip structures (solid squares). The solid line is the theoretical curve calculated from Eq. (2) using the parameters $I_p = 225$ nA and $\Delta/h = 1.75$ GHz obtained from the ground-state measurements. The effective temperature $T \approx 70$ mK $\approx 1.4$ GHz $\cdot h/k_B$ is smaller than the minimal energy level separation $2\Delta$. 

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eraged out of the two-qubit Hamiltonian [9], reducing it to: $H_{\text{qbs,red}} = -\Delta a_s \sigma_z^{(b)} - (\varepsilon_b - J) \sigma_z^{(b)}$. Apart from the offset in the bias term due to the coupling this is identical to the single qubit Hamiltonian [1]. This offset can be easily compensated and measured allowing the determination of the coupling energy $J$ [9]. The qubit parameters, $\Delta_a$ and $J_p^{(a)}$, are determined from the ground state measurement, as it is described above. Analogously, biasing qubit $b$ far from the degeneracy point the parameters for qubit $a$, $\Delta_a$ and $J_p^{(a)}$, can be determined. In a similar way the parameters of a $N$-qubit Hamiltonian can be completely reconstructed from the ground-state measurements as has already been demonstrated, for instance, for four qubits circuits [10].

The qubit parameters can be probed by either the ground state (adiabatic) measurements or by making use of spectroscopy. However it naturally raises the question of whether the ground-state and the spectroscopy measurements are consistent? While addressing this problem in this Letter we study both approaches in situ for one and two coupled flux qubits.

Experimentally, the shift of the resonance frequency can be obtained by driving the tank circuit with a rf current $I_{rf}$ at a frequency close to the resonant frequency $\omega_T$ and measure the phase shift $\Theta$ between the rf voltage and driving current. For a small qubit inductance $L$, the phase shift $\Theta$ is defined by [5]:

$$\tan \Theta = \frac{M^2 Q \, d^2 E_z}{L_T \, d \Phi_{dc}}. \quad (10)$$

The mutual inductance $M$, tank inductance $L_T$ and quality factor $Q$ can be measured independently giving a value of 23.4 pH for this prefactor. The results of such measurements are shown in Fig. 2(a). Note that the sample was thermally anchored to the mixing chamber of a dilution refrigerator at a temperature $T_{mix} \approx 10$ mK. The effective temperature of the sample $T$ is higher and we estimated from the best theoretical fits that $T \approx 70$ mK [11].

It is important to note that thermal excitations can modify the measured signal, which would result in erroneous qubit parameters. In practice, thermal excitations are not negligible when $k_B T \gtrsim 2\Delta$. Nevertheless, if $k_B T < 2\Delta$ the dispersive measurement provides a correct value of qubit parameters [11]. This statement is also confirmed by a good agreement between both ground state and spectroscopic measurements (see Fig. 2(c)).

In fact the tank circuit can be used as detector for the spectroscopy measurements as well, since the variation in the population of the qubits’ energy levels results in the change of the effective impedance of the tank circuit [12]. The tank circuit is insensitive to the microwave signal itself since $\omega_T \ll \Delta/\hbar$ and $Q \gg 1$. However, if the microwave frequency is close to the qubit level separation, the system damps or amplifies the voltage on the tank, mimicking the Sisyphus mechanism of damping (and heating) of the tank known from quantum optics [14]. This effect generates the peak-dip structure in the $V_T(\Phi_{dc})$ dependence around the resonance (see Fig. 2(b)).

![FIG. 3: (color online). Landau-Zener interferometry for qubit $b$. Dependence of the tank voltage phase shift $\Theta$ on the dc flux bias $\Phi_{dc}$ and the ac flux amplitude $\Phi_{ac}$ (the microwave amplitude). The spots along the $\Phi_{dc}$ axis correspond to the multiphoton resonances at the positions defined by the relation $\Delta E(\Phi_{dc}) \approx n \cdot \hbar \omega$; the numbers from 1 to 7 show the position of the $n$-photon resonances. The changes along the $\Phi_{ac}$ axis are due to the St"uckelberg oscillations in the qubit. The calibration of the driving power of the ac flux can be done either with the distance between these oscillations (black arrow) or from the slope of the interference fringes (white line).](Image 362x629 to 513x739)

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[12 13]. The position of the resonances is the point where a peak changes to a dip. From the positions of the mid points of the peak-dip structures one can determine the energy gap $\Delta E$ between the energy levels. The obtained agreement between the adiabatic and spectroscopic measurement for weak driving regime is excellent (see Fig. 2(c)).

With increasing the microwave power, the Landau-Zener interference pattern of the qubit is clearly visible. The qubit’s response in the strong driving regime is demonstrated in Fig. 3 where the tank voltage phase shift is presented as a function of the microwave amplitude and the dc flux bias. The position of the multiphoton resonances is approximately given by the relation $\Delta E(\Phi_{dc}) \approx n \cdot \hbar \omega$, where the energy gap $\Delta E$ is calculated using the parameters $I_p$ and $\Delta$ obtained from the ground state measurement. Moreover, the Landau-Zener interferometry allows the calibration of the microwave power to the ac flux due to the periodicity of the St"uckelberg oscillations on the parameter $4I_p \Phi_{ac}/\hbar \omega$ with the period $2\pi$ [15 16]. It follows that the distance between the resonances (shown by the black arrow in Fig. 3) is approximately equal to $\delta \Phi_{ac} = \frac{\pi}{2} \hbar \omega/I_p$. Alternatively, the calibration can be made using the slope of the interference fringes (white line in Fig. 3) [17 18].

After determining the single qubit parameters far away from the degeneracy points, we investigated the two qubit behavior. Firstly, the coupling energy $J$ was determined from the offset of the qubit dips from the $\Phi_{a/b} = 0$ lines, visible in the pure ground state measurements presented in Fig. 4(a). Then the qubits were driven by various ac magnetic fluxes $\Phi_{ac}\sin \omega t$. In Fig. 4(b) a frequency in-between both qubit gaps was used and therefore only the transitions to the first excited state are visible. For higher frequencies, also the sec-
FIG. 4: (color online). Spectroscopy of the system of two coupled flux qubits. The dependence of the tank voltage phase shift $\Theta$ on the flux biases in qubits $a$ and $b$ is presented for measurements without microwave excitation in (a) and for microwave excitation with $\omega/2\pi = 14.125$, 17.625 and 20.75 GHz in (b) till (d) respectively. Inset shows the transition to the third excited level. The blue, magenta and white-dotted lines in the pictures with microwave excitation show the expected positions of the resonant excitations of the qubits to the first, second and third excited levels respectively, calculated from the energy eigenvalues of Hamiltonian $H$ with parameters: $\Delta_a/h = 7.9$ GHz, $\Delta_b/h = 1.75$ GHz, $I_{0a} = 120$ nA, $I_{0b} = 225$ nA, and $J/h = 1.9$ GHz. The trough around $\Phi_b = 0$ is due to the ground state curvature of qubit $a$ and corresponds to the ground state measurements of Fig. (2). The shallow trough around $\Phi_a = 0$ visible in figure (a), is due to qubit $a$.

ond and third excited states become visible as can be seen in subfigures (c) and (d). Here also both types of the measurements (ground-state and spectroscopic) result in the same set of parameters for the system. Finally we would like to note that the theoretical calculations allow us to plot analogous to Figs. 3 and 4 graphs (to be published elsewhere [11]).

In conclusion, the equivalence of the ground-state and spectroscopic approaches for the measurement of the qubit system parameters was demonstrated. We have probed the one- and two- flux qubit systems by using a dispersive measurement technique. It was shown that the ground state measurement gives the same qubit parameters as the spectroscopy in the weak (Figs. 2 and 4) as well as in the strong driving regime (Fig. 3).

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[1] R. P. Feynman, R. B. Leighton, and M. Sands, The Feynman lectures on physics. Quantum mechanics (California Institute of Technology, 1965).
[2] Yu. Makhlin, G. Schön, and A. Shnirman, Rev. Mod. Phys. 73, 357 (2001).
[3] J. Q. You and F. Nori, Phys. Today 58, 42 (2005).
[4] J. E. Mooij et al., Science 285, 1036 (1999).
[5] Ya. S. Greenberg et al., Phys. Rev. B 66, 214525 (2002).
[6] C. H. van der Wal et al., Science 290, 773 (2000).
[7] This arrangement has been already described phenomenologically – oscillator perturbed by two-level system [5]. Here we present a more general approach to this problem.
[8] For finite $I_{rf}$ the response of the tank circuit can be calculated numerically [5].
[9] M. Grajcar et al., Phys. Rev. B 72, 020503 (2005).
[10] M. Grajcar et al., Phys. Rev. Lett. 96, 047006 (2006).
[11] S.N. Shevchenko, S.H.W. van der Ploeg, M. Grajcar, E. Il’ichev, A.N. Omelyanchouk, and H.-G. Meyer, arXiv:0808.1520
[12] M. Grajcar et al., Nat. Phys. 4, 612–616 (2008)
[13] S.N. Shevchenko, Eur. Phys. J. B 61, 187 (2008).
[14] D. J. Wineland, J. Dalibard, and C. Cohen-Tannouji, J. Opt. Soc. B9, 3242 (1992).
[15] A.V. Shytov, D.A. Ivanov, and M.V. Feigel’man, Eur. Phys. J. B 36, 263 (2003).
[16] S.N. Shevchenko and A.N. Omelyanchouk, Low Temp. Phys. 32, 973 (2006).
[17] W. D. Oliver et al., Science 310, 1653 (2005).
[18] M. Sillanpää, T. Lehtinen, A. Païla, Yu. Makhlin, and P. Hakonen, Phys. Rev. Lett. 96, 187002 (2006).

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