Effects of heavy bosonic excitations on QED vacuum

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We discuss the contribution of axion-like excitations (ALE) to the vacuum birrefringence in the limit $m_A \sim \omega$, where $m_A$ is the mass of the excitation and $\omega$ the energy of test photons interacting with an external (intense) magnetic field. The relevance of this term with respect to the QED contribution depends on the ratio $g_A/m_A$ and, from present bounds on the mass and the coupling constant $g_A$, we find that in the present low energy regime, it ranges from $10^{-14}$ to $10^2$ suggesting an interesting alternative to explore.

I. INTRODUCTION

It is well known that, as a consequence of vacuum polarization effects in conventional QED [1, 2, 3, 4], the electromagnetic vacuum presents a birefringent behavior [5, 6, 7, 8]. For example, in the presence of an intense static background magnetic field, there are two different refraction indices depending on the polarization of the incident electromagnetic wave. This is in fact a very tiny effect which could be measured in the next few years [9]. The present bounds for this anisotropy, followed from several experiments [10, 11, 12, 13], are still far from the expected value from QED.

According to the original calculations [6, 7, 8, 14, 15, 16, 17, 18], conventional QED yields for the difference of the refractive indices

$$\Delta n = n_\parallel - n_\perp$$

$$\Delta n = \frac{\alpha}{45 \pi} \left( \frac{B_0}{E_c} \right)^2,$$

where the critical field $E_c = m_e^3 c / \hbar e$, $B_0$ is the external (constant and uniform) magnetic field and $\parallel$ and $\perp$ refer to the polarization of a test wave with respect to the plane determined by the wave vector and the external magnetic field.

Experiments designed to measure such difference are in operation presently, with external magnetic fields ranging between 2 and 5 T (see for instance [10, 11, 12]). Then, one would expect to measure for the difference of refractive indices

$$\Delta n \approx 10^{-23} - 10^{-22}.$$  

As previously mentioned, this anisotropy has not yet been measured, but the experimental results reported in [10, 11, 12] have set the bound

$$\Delta n \leq 10^{-19}.$$  

News are expected in the near future since interesting experiments on vacuum birefringence have been recently proposed. See e.g. [8, 19].
On the other hand, if the interaction of photons with the magnetic field is also mediated by additional axion-like excitations (ALE), new terms will contribute to the anisotropy through axion-photon conversion [15] and these experiments can be used, for instance, to detect axions [20, 21].

In the present paper we will consider this kind of interaction – an effective manifestation of more fundamental degrees of freedom – in the low energy regime as described by a non-renormalizable Lagrangian.

More precisely, for \( \varphi \) and \( \chi \) a pseudoscalar and a scalar field which respectively couple to the photon field invariants \( G \) and \( F \) given by

\[
F := -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = \frac{1}{2} (E^2 - B^2),
\]

\[
G := -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} = E \cdot B,
\]

where \( \tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon_{\mu
u\rho\lambda} F^{\rho\lambda} \), the conventional QED Lagrangian is replaced by the total Lagrangian

\[
\mathcal{L} = \mathcal{L} + \bar{\psi} \left[ i \gamma^{\mu} (i \partial_{\mu} - e A_\mu) - m_e \right] \psi + \mathcal{L}_A + \mathcal{L}_S,
\]

where

\[
\mathcal{L}_A = \frac{1}{2} \partial_{\mu} \varphi \partial^{\mu} \varphi - \frac{1}{2} m_A^2 \varphi^2 - g_A \varphi G,
\]

\[
\mathcal{L}_S = \frac{1}{2} \partial_{\mu} \chi \partial^{\mu} \chi - \frac{1}{2} m_S^2 \chi^2 - g_S \chi F.
\]

The experiments should elucidate which kind of additional effective bosonic degrees of freedom are present in this situation, if any. The purpose of the present paper is to discuss the modifications of (1) in the low energy limit \( \omega << m_a \), where \( \omega \) is the photon energy and \( m_a \), the mass of the ALE.

We compute the contribution to the electromagnetic field effective Lagrangian coming from the pseudoscalar field in a gradient approximation, which adds a term to the well-known Heisenberg-Euler effective Lagrangian [1, 2, 4]. Then, we analyze different scenarios for this extra term, considering present bounds for \( g_A \) and \( m_A \). We also show that the phase shift produced by the vacuum birefringence on a test electromagnetic due to a scalar ALP differs in sign with respect to the contribution of a pseudo-scalar ALE (in agreement with [24]).

The paper is organized as follows: in Section II the effective Lagrangian piece coming from an ALE coupled to the electromagnetic field is computed in a gradient expansion. In the next section we calculate the refractive indices modified by the presence of a background magnetic field, in this low photon energy approximation, and discuss its relevance for different values of \( m_A \) and \( g_A \). Finally, in Section IV we give our conclusions and outlook.

II. THE EFFECTIVE LAGRANGIAN

In this section we compute the analogous of the Heisenberg-Euler effective Lagrangian for the system involving the electromagnetic, the fermionic and a pseudoscalar fields. In a second part of this section we will discuss the contribution of a scalar ALE.

As is well known, (for a slowly varying electromagnetic field) the functional integral over the fermionic field leads to the standard Heisenberg-Euler effective Lagrangian, [1, 2, 4, 22], i.e

\[
\mathcal{L}_{HE} = \frac{e^4}{360 \pi^2 m_e^4} \left\{ 4 F^2 + 7 G^2 \right\} + \cdots,
\]

where the dots represent higher order terms in \( F/E_c^2 \) and \( G/E_c^2 \).

Since the pseudoscalar is linearly coupled to \( G \), the functional integral for this field can also be explicitly evaluated to get

\[
\int \mathcal{D} \varphi \exp \left\{ - \frac{i}{2} \int d^4 x \left[ \varphi \left( \partial^2 + m_A^2 \right) \varphi + 2 g_A \varphi G \right] \right\} =
\]

\[
= \left[ \text{Det} \left( \partial^2 + m_A^2 \right) \right]^{-1/2} \left[ \int \frac{\partial^2}{2 g_A} \int d^4 x \int d^4 y \mathcal{G}(x) K(x, y) \mathcal{G}(y) \right],
\]

where \( K(x, y) = (\partial^2 + m_A^2)^{-1} (x, y) \).
The exponential in the right hand side represents a non-local term in the effective action which, in the infrared limit we will be interested in, admits the asymptotic gradient expansion

\[
\frac{i}{2} g^2 \int d^4 x \int d^4 y \, G(x) K(x, y) G(y) = \frac{ig_A^2}{2m_A^2} \sum_{n=0}^{\infty} \left( \frac{-1}{m_A^2} \right)^n \int d^4 x \, G(x) \partial^{2n} G(x),
\]

which is justified for \(m_A \gg \omega\), where \(\omega\) is an energy scale characteristic of the experiment under consideration (see Appendix A). Therefore, in this approximation, the effective Lagrangian for the electromagnetic field can be written as the (formally local) expression

\[
L_{\text{eff}} = F(x) + \frac{e^4}{360 \pi^2 m_e^4} \left\{ 4F(x)^2 + 7G(x)^2 \right\} + \frac{g_A^2}{2m_A^2} \sum_{n=0}^{\infty} \frac{(-1)^n}{m_A^2} \int d^4 x \, \partial^{2n} G(x).
\]

Let us remark that integrating out the pseudo-scalar field is equivalent to eliminating it by means of the Euler-Lagrange equation of motion derived from the quadratic Lagrangian in Eq. (6). In this respect, one should notice that our calculation differs from that of Raffelt and Stodolsky [21] in the range of validity of the respective approximations. Indeed, Ref. [21] considers the relativistic limit for the pseudo-scalar, where \(m_A \ll \omega\), obtaining non-local contributions to the equation of motion of one of the photon propagation modes (see Eqs. (4)-(5) in [21]).

Similarly, the coupling with a scalar field \(\chi(x)\) as in Eq. (7) adds to the gradient expansion of the effective Lagrangian in Eq. (11) the piece

\[
\Delta L_{\text{eff}} = gS^2 \frac{2m_S^2}{2m_S^2} \sum_{n=0}^{\infty} \frac{(-1)^n}{m_S^2} \int d^4 x \, \partial^{2n} F(x).
\]

Therefore, we get

\[
L_{\text{eff}} = F(x) + \frac{1}{2} \left\{ \left[ 4\varrho F(x)^2 + S \sum_{n=0}^{\infty} \frac{(-1)^n}{m_S^2} \int d^4 x \, \partial^{2n} F(x) \right] + \left[ 7\varrho G(x)^2 + A \sum_{n=0}^{\infty} \frac{(-1)^n}{m_A^2} \int d^4 x \, \partial^{2n} G(x) \right] \right\},
\]

where the parameters \(\varrho\), \(A\) and \(S\) are defined as

\[
\varrho := \frac{e^4}{180 \pi^2 m_e^4} = \frac{4 \alpha^2}{45 m_e^4},
\]

\[
A := \frac{g_A^2}{m_A^2},
\]

\[
S := \frac{g_S^2}{m_S^2}.
\]

In the next section we will calculate the contribution of these terms to the refractive indices and discuss possible physical implications.

### III. POLARIZATION PHENOMENA IN ELECTROMAGNETIC BACKGROUNDS

The polarization of the fermionic vacuum in the presence of an electromagnetic background makes it to act like a birefringent medium with two different indices of refraction, depending on the direction and polarization of the propagating wave [6, 7]. In this Section we evaluate the contributions to the refractive indices due to the additional coupling with the pseudoscalar and scalar field.

Following [8], we will consider the expression in Eq. (13) as the effective Lagrangian for the total electromagnetic field, consisting in the sum of an intense constant uniform background field, \(F_{\mu\nu}\), and a test wave of low frequency (large wavelength), \(f_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu\), and look for the Euler-Lagrange equations of motion for the last one.
In so doing, it is sufficient to retain the piece quadratic in $f_{\mu\nu}$, since the linear term is a total divergence for a constant background, and the higher order terms (cubic and quartic in $f_{\mu\nu}$) are suppressed by factors of the order of the ratio between the intensity of the fluctuation and that of the background.

We get for that piece

$$\mathcal{L}^{(2)} = -\frac{1}{4} f_{\mu\nu} M^{\nu\alpha\beta} f_{\alpha\beta}$$

(15)

where the operator

$$M^{\nu\alpha\beta} := \frac{1}{2} \left[ 1 + (4\theta + S) \mathcal{F} \right] (g^{\mu\alpha} g^{\nu\beta} - g^{\mu\beta} g^{\nu\alpha}) + \frac{1}{2} (7\theta + A) \mathcal{G} \varepsilon^{\nu\alpha\beta} -$$

$$- \frac{1}{2} F^{\nu\mu} F^{\alpha\beta} \left[ 4\theta + S \sum_{n=0}^{\infty} \left( \frac{k^2}{m^2} \right)^n \right] - \frac{1}{2} \tilde{F}^{\nu\mu} F^{\alpha\beta} \left[ 7\theta + A \sum_{n=0}^{\infty} \left( \frac{k^2}{m^2_\Lambda} \right)^n \right]$$

(16)

satisfies $M^{\nu\alpha\beta} = M^{\alpha\beta\nu} = -M^{\nu\alpha\beta}$ and commutes with the derivatives $\partial_\lambda$ (for a constant background field $F_{\mu\nu}$). The corresponding equation of motion for the fluctuation writes simply as $M^{\nu\alpha\beta} \partial_\nu \partial_\alpha a_\beta = 0$.

Let us now look for a solution of the form $a_\beta(x) = \varepsilon_\beta(k) e^{-ikx}$ which, replaced in the previous equation, leads to

$$\frac{1}{2} \left[ 1 + (4\theta + S) \mathcal{F} \right] [(k \cdot \varepsilon) k^\mu - k^2 \varepsilon^\mu] +$$

$$+ \frac{1}{2} (k_\nu F^{\nu\mu}) (k_\alpha F^{\alpha\beta} \varepsilon_\beta) \left[ 4\theta + S \sum_{n=0}^{\infty} \left( \frac{k^2}{m^2} \right)^n \right] + \frac{1}{2} \left( k_\nu \tilde{F}^{\nu\mu} \right) \left( k_\alpha \tilde{F}^{\alpha\beta} \varepsilon_\beta \right) \left[ 7\theta + A \sum_{n=0}^{\infty} \left( \frac{k^2}{m^2_\Lambda} \right)^n \right] = 0.$$  

(17)

In the asymptotic large mass expansion we are employing, these series can be summed up to get

$$\frac{1}{2} \left[ 1 + (4\theta + S) \mathcal{F} \right] [(k \cdot \varepsilon) k^\mu - k^2 \varepsilon^\mu] +$$

$$+ \frac{1}{2} (k_\nu F^{\nu\mu}) (k_\alpha F^{\alpha\beta} \varepsilon_\beta) [4\theta + S(k)] + \frac{1}{2} \left( k_\nu \tilde{F}^{\nu\mu} \right) \left( k_\alpha \tilde{F}^{\alpha\beta} \varepsilon_\beta \right) \left[ 7\theta + A(k) \right] = 0,$$

(18)

where we have called

$$A(k) := \frac{A}{1 - k^2/m^2_\Lambda}, \quad S(k) := \frac{S}{1 - k^2/m^2_S}.$$  

(19)

Eq. (18) implies that the polarization vector must have the form

$$\varepsilon'_\nu(k) = \xi_0 k^\nu + \xi_1 k_\mu F^{\mu\nu} + \xi_2 k_\mu \tilde{F}^{\mu\nu}.$$  

(20)

where $\xi_0$ represents a gauge transformation and can not be determined from those (gauge invariant) equations.

Taking into account that

$$k_\mu F_{\mu\nu} \tilde{F}^{\nu\lambda} k^\lambda = k^2 \mathcal{G},$$

$$k_\nu \tilde{F}_{\nu\mu} \tilde{F}^{\mu\nu} k^\lambda = k^2 M_{\mu\nu} F^{\nu\lambda} k^\lambda - 2 \mathcal{F}^2 k^2,$$

(21)

it is straightforward to show that the propagation normal modes satisfy

$$\begin{pmatrix}
[1 + (4\theta + S) \mathcal{F}] k^2 + [4\theta + S(k)] k \cdot F \cdot F \cdot k & [4\theta + S(k)] \mathcal{G} k^2 \\
[7\theta + A(k)] \mathcal{G} k^2 & [1 - (10\theta + 2A(k) - S) \mathcal{F}] k^2 + [7\theta + A(k)] k \cdot F \cdot F \cdot k
\end{pmatrix}
\begin{pmatrix}
\xi_1 \\
\xi_2
\end{pmatrix}
= 0.$$  

(22)

Here

$$k \cdot F \cdot F \cdot k := k^\mu M_{\mu\nu} F^{\nu\lambda} k^\lambda =$$

$$= \omega^2 \mathbf{E}^2 + k^2 \mathbf{B}^2 - 2\omega \mathbf{k} \cdot \mathbf{E} \times \mathbf{B} - (\mathbf{k} \cdot \mathbf{E})^2 - (\mathbf{k} \cdot \mathbf{B})^2,$$

(23)
where \( \omega = k_0 \), and \( \mathbf{E} \) and \( \mathbf{B} \) are the background electric and magnetic fields, respectively.

Nontrivial solutions of Eq. (22) require the determinant of the matrix on the left hand side to vanish. If, for simplicity, we consider the case in which \( G = 0 \) (which corresponds to \( \mathbf{E} \perp \mathbf{B} \), or pure magnetic or pure electric background field), the non-diagonal elements of this matrix vanish, and the normal modes are determined by setting the diagonal elements equal to zero.

For definiteness, let us consider the case of a pure magnetic background field: \( F_{12} = B_0 = -F_{21} \ (B_0 = B_0 \epsilon_3) \) and the other components equal to zero. We get for the two normal modes

\[
\begin{align*}
[1 - \frac{1}{2}(4\rho + S)B_0^2] (\omega_1^2 - k^2) + [4\rho + S(k)] B_0^2 k_{\perp}^2 &= 0, \\
[1 + \frac{1}{2}(10\rho + 2A(k) - S)B_0^2] (\omega_2^2 - k^2) + [7\rho + A(k)] B_0^2 k_{\perp}^2 &= 0,
\end{align*}
\]

(24)

where \( k = (\omega, \mathbf{k}) \) and \( k_{\perp} \) is the component of \( \mathbf{k} \) perpendicular to the magnetic field. This implies the dispersion relations implicitly given by the following equations

\[
\begin{align*}
\omega_1^2 &= k^2 - \left( \frac{(4\rho + S(k)) B_0^2}{1 - \frac{1}{2}(4\rho + S) B_0^2} \right) k_{\perp}^2, \\
\omega_2^2 &= k^2 - \left( \frac{(7\rho + A(k)) B_0^2}{1 + \frac{1}{2}(10\rho + 2A(k) - S) B_0^2} \right) k_{\perp}^2.
\end{align*}
\]

(25)

Notice that, in these equations, \( A(k) \) and \( S(k) \) also depend on the normal frequencies \( \omega_1 \) and \( \omega_2 \).

These equations can be solved in a large mass expansion (consistent with the gradient expansion we have employed) to get, for example,

\[
\begin{align*}
\omega_1^2 &= k^2 \left\{ 1 - (4\rho + S) B_0^2 \sin^2 \theta - \frac{1}{2} (4\rho + S)^2 B_0^4 \sin^2 \theta \right. \\
&\quad + \frac{k^2}{m^2} \left[ (4\rho + S) S B_0^4 \sin^4 \theta \right] + O \left( \frac{k^2}{m^2} \right)^2 \right\}, \\
\omega_2^2 &= k^2 \left\{ 1 - (7\rho + A) B_0^2 \sin^2 \theta + \frac{1}{2} (7\rho + A)(10\rho + 2A - S) B_0^4 \sin^2 \theta \right. \\
&\quad + \frac{k^2}{m^2} \left[ (7\rho + A) A B_0^4 \sin^4 \theta \right] + O \left( \frac{k^2}{m^2} \right)^2 \right\},
\end{align*}
\]

(26)

where, in the right hand sides, we have retained just the first terms in \( \rho B_0^2 \), \( AB_0^2 \), and \( SB_0^2 \), parameters whose values will be consider comparable for the time being. The angle \( \theta \) is defined by \( \cos \theta = \mathbf{k} \cdot \mathbf{B}_0/|\mathbf{B}_0| \).

Let us now introduce the refractive index as \( n(\omega) := |n(\omega)| \), where \( n(\omega) := k/\omega \), and discuss the anisotropy induced by the non-linear terms in the effective Lagrangian. From Eq. (22) we get for the indices and polarization vectors

\[
\begin{align*}
n_1(\omega)^2 &= 1 + (4\rho + S) B_0^2 \sin^2 \theta - \frac{1}{2} (4\rho + S)^2 B_0^4 (\cos 2\theta - 2) \sin^2 \theta - \\
&\quad - \frac{\omega^2}{m^2} \left[ (4\rho + S) S B_0^4 \sin^4 \theta \right] + O \left( \frac{\omega}{m} \right)^4, \\
\varepsilon_1 &= n_1 \times B_0, \\
n_2(\omega)^2 &= 1 + (7\rho + A) B_0^2 \sin^2 \theta - \frac{1}{2} (7\rho + A)(3\theta + A - S + (7\rho + A) \cos 2\theta) B_0^4 \sin^2 \theta - \\
&\quad - \frac{\omega^2}{m^2} \left[ (7\rho + A) A B_0^4 \sin^4 \theta \right] + O \left( \frac{\omega}{m} \right)^4, \\
\varepsilon_2 &= B_0 - n_2 (n_2 \cdot B_0),
\end{align*}
\]

(27)
for the transverse and parallel polarizations respectively, where the terms transverse and parallel refer to the orientation of the polarization vector with respect to the plane determined by the vectors $k$ and $B_0$ (The gauge parameter $\xi_0$ has been chosen in such a way that $\varepsilon_1^0 = 0$). Notice that the right hand sides of Eq. (27) with $A, S \to 0$ coincide with the result in [7]. Moreover, it agrees with the result by Raffelt and Stodolsky in [21] (see also [22, 23]) in the sense that, in this case, it is only the parallel mode which is affected by the presence of the pseudoscalar ALE (even though it is the relativistic small pseudo-scalar mass limit which has been considered in that reference).

The difference in refractive indices induced by the background field is maximal at $\theta = \pi/2$ ($k \perp B_0$) and vanishing for $\theta = 0$ ($k \parallel B_0$). Moreover, the polarization vector for the parallel mode is not perpendicular to $k$: $n_2 \cdot \varepsilon_2 = (n_2 \cdot B_0)(1 - n_2^2) \neq 0$.

Notice that the first non-vanishing contribution proportional to $\omega^2/m_A^2$ is $O(\varrho^2 B_0^4)$ which, for typical values of $B_0 \sim 1$T, turn out to be of the order $10^{-50}$. Therefore, for practical purposes, it is enough to preserve only the zero order term in the large mass expansion, retaining in it only the first order in $\varrho B_0^2$, $AB_0^2$ or $SB_0^2$.

Notice that, up to this order of approximation, the presence of the pseudoscalar field reflects in the refractive index of the parallel mode, while the scalar field has no effect on it and affects only the refractive index of the transverse mode.

Different values for the refractive indices of the normal modes imply birefringence. Indeed, a linearly polarized electromagnetic wave of wavelength $\lambda = 2\pi/\omega$ which, for example, propagates perpendicularly to the uniform magnetic field $B_0$ along a distance $L$, turns into an elliptically polarized wave with a phase shift between the two rays given by

$$\phi = 2\pi(n_2 - n_1) \frac{L}{\lambda} \approx \pi(3g + A - S)B_0 \frac{L}{\lambda}.$$  \hspace{1cm} (28)

Therefore, the sign of the contribution to the phase shift depends on what kind of bosonic excitation realizes this low-energy effective coupling of fundamental degrees of freedom with the electromagnetic field. In a first approach, we could estimate $S \approx A$. Then, the sign of $\phi$ would allow to discriminate among the presence of a scalar or a pseudoscalar ALE coupled to the electromagnetic field. This is in agreement with the analysis in [24].

Indeed, in terms of the refraction index arising just from QED, $\Delta n_{QED}$ in (4), we have for the difference of refractive indices

$$\Delta n = \Delta n_{QED} \left(1 + \frac{A - S}{3\varrho}\right),$$  \hspace{1cm} (29)

Let us consider the relative weight of the contributions from the fermionic and the pseudoscalar fields. This is simply given by the quotient

$$\frac{A}{3\varrho} = \frac{15 m_e^4 g_A^2}{4 \alpha^2 m_A^2} = 4.4 \times 10^{27}(eV)^4 \times \left(\frac{g_A}{m_A}\right)^2.$$  \hspace{1cm} (30)

Bounds for $g_A$ and $m_A$ for the axion-photon coupling can be found in [26]. For example, from the optical experiments reported in [10, 11, 12, 27], we can take $m_A \sim 10^{-3}$ eV and $g_A \sim 3 \times 10^{-16}$ eV$^{-1}$, leading to

$$\frac{A}{3\varrho} \sim 4 \times 10^2$$

for which $\Delta n$ is two orders of magnitude below the bound [3] and two orders of magnitude greater than $\Delta n_{QED}$. For different bounds as for invisible axions with larger masses [28, 29], $m_a \sim 1$ keV, $g_A \sim 3 \times 10^{-18}$ eV$^{-1}$, and one has

$$\frac{A}{3\varrho} \sim 4 \times 10^{-14}.$$  \hspace{1cm} (31)

The weight of this contribution is, in this case, negligible with respect to that of QED.

But let us stress that, strictly speaking, our results are valid in the large-mass approximation, where $m_A S$ is larger that the photon energy in the probe ray. Since the frequency of the laser employed in the experiments [10, 11, 12, 30] is around 1eV, our results suggest to look for ALE with masses $m_A \gtrsim 1$eV, which corresponds to a much heavier excitation than the conventional axion-like particles considered previously in this context.
IV. DISCUSSION AND CONCLUSIONS

In this paper we have speculated about the role a heavy axion-like excitation can play in the effective electromagnetic phenomena at low energies. In a large mass approximation, we have considered excitations of both pseudoscalar and scalar nature, and studied their contributions to the vacuum birefringence phenomenon.

Even though the effects of a pseudo-scalar axion-like particle on the electromagnetic vacuum birefringence and dichroism properties have been extensively considered in the literature, the novelty of our research resides on the inclusion of heavy bosonic excitations in a gradient expansion of the effective Lagrangian (leading to a simple large-mass approach). The contribution of this term to the standard vacuum birefringence predicted by QED, depends on the ratio $g_A/m_A$, and it can be numerically estimated by using present bounds of both parameters, arising from experiments of axion-photon couplings, for instance. We have found that, according to these bounds, this contribution could be non-negligible. In any case, the bound (3) is not saturated.

But, strictly speaking, since the optical experiments have been done employing lasers of frequencies $\omega \approx 1$ eV, the validity of our approximation requires that $m_A \gtrsim 1$ eV, at least three orders of magnitude grater than the masses of axion-like particles previously considered in this context. This gives for the coupling constant

$$g_A > 10^{-12} (\text{eV})^{-1}. \quad (31)$$

On the other hand, following [30] we can estimate the photon-ALE conversion probability as

$$P_{\gamma \rightarrow \text{ALE}} = \frac{g_A^4 B_0^4}{8 q m_A^4} \log \left( \frac{2 q m_A^4}{g_A^4 B_0^4} \right), \quad (32)$$

where $q$ is the quality factor of the laser source ($q = \Delta/m_A$, with $\Delta$ the laser bandwidth).

Taking, for example, the data from the LULI-BOSONS project [31], $q \sim 10^{-5}$ and $B_0 \sim 2 \times 10^2$ eV $^2$, we get a photon-boson conversion probability of the order of $10^{-34}$, with an extremely small rotation angle of the polarization plane. This is consistent with the non-observation of vacuum dichroism.

Although this probability is similar to the one obtained in the standard calculation in axion physics [32], the philosophy of our approach is quite different and could justify new experimental efforts, eventually beyond the present optical experiments.

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APPENDIX A: LARGE MASS APPROXIMATION

As pointed out in Section III in order to determine the effects on the refractive indices, it is sufficient to retain the piece of the effective Lagrangian quadratic in the fluctuating electromagnetic field.

For the case of interest, of a large constant and uniform background magnetic field plus a (laser) test wave of frequency $\omega$, we have

$$\mathcal{G} = B \cdot E \simeq B_0 \cdot \mathcal{E} + \cdots, \quad (A1)$$

where $\mathcal{E} = \varepsilon e^{-i(\omega t - \mathbf{k} \cdot \mathbf{x})}$ is the electric field of the test wave (with $\varepsilon$ the polarization vector) and the dots stand for a term quadratic in the fluctuating field.

In terms of the Fourier transform of $\mathcal{G}(x)$,

$$\tilde{\mathcal{G}}(q) = \int d^4 x \, e^{i(q-k) \cdot x} B_0 \cdot \varepsilon, \quad (A2)$$

the argument of the exponential factor in the right hand side of Eq. (4) reduces to

$$\frac{1}{2} \int \frac{d^4 q}{(2\pi)^4} \frac{\tilde{\mathcal{G}}^*(q) \tilde{\mathcal{G}}(q)}{-q^2 + m_A^2} \equiv 1 \int d^4 x \, \mathcal{G}(x)^2 \left[ 1 + O \left( \frac{\omega^2}{m_A^2} \right) \right], \quad (A3)$$

showing that our gradient approximation applies when $m_{A,S}$ is grater that the energy of the test (laser) photons.

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