NEW QCD SUM RULES FOR IN-MEDIUM NUCLEONS

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New QCD sum rules for nucleons in nuclear matter are derived from a mixed correlation function of spin-1/2 and spin-3/2 interpolating fields. These sum rules allow a determination of the scalar self-energy of the nucleon independent of the poorly known four-quark condensates. An analysis of these new sum rules in concert with previous nucleon sum rules from spin-1/2 interpolators indicates consistency with the expectations of relativistic mean-field phenomenology. We find $M^* = 0.68 \pm 0.08$ GeV and $\Sigma_v = 0.31 \pm 0.06$ GeV at nuclear matter saturation density.

1 Introduction

Understanding the observed properties of hadrons and nuclei from quantum chromodynamics (QCD) is a principal goal of nuclear theorists. The QCD sum-rule approach is a particularly useful method for connecting the properties of QCD to observed nuclear phenomena. Recent progress in understanding the origin of the large and canceling isoscalar Lorentz-scalar and -vector self-energies for propagating nucleons in nuclear matter has been made via the analysis of QCD sum rules generalized to finite nucleon density. These large self-energies are central to the success of relativistic nuclear phenomenology.

However, the previous sum-rule predictions for the scalar self-energy are sensitive to the density dependence of certain “scalar-scalar” dimension-six four-quark condensates. This density dependence is unknown. There are at least two ways to clarify the situation. One direction is to attempt to better determine the density dependence of the four-quark condensates via modeling. An alternative approach, which is adopted here, is to derive new QCD sum rules for the scalar self-energy that do not depend on the four-quark condensates.

The new sum rules are obtained from a mixed correlator of generalized spin–1/2 and spin–3/2 interpolating fields. The spin–1/2 states remain projected, and one generates additional

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sum rules for the scalar self-energy that are independent of the problematic four-quark
condensates. These new sum rules, along with previous sum rules from spin-1/2 interpolators
that are also independent of four-quark condensates [2], provide an interesting forum for
examining the expectations of phenomenological approaches. In this work we present results
from the first analysis of the new sum rules; more detailed analyses will be given in future
publications [4].

2 New QCD Sum Rules

The finite-density QCD sum-rule approach focuses on a correlation function of interpolating
fields carrying the quantum numbers of the hadron of interest

\[ \Pi_{\mu\nu}^{12}(q) \equiv i \int d^4 x e^{iq\cdot x} \langle \Psi_0 | T \left[ \gamma_\mu(x) \chi^\mu_{\mu}(x) \right] | \Psi_0 \rangle. \]  

(1)

Here, the ground state of nuclear matter |Ψ_0⟩ is characterized by the rest-frame nucleon
density ρN and by the four-velocity u^μ. The nucleon interpolating fields are [5, 6]

χ^1_μ(x) = \gamma_\mu \gamma_5 e^{abc} \left\{ \left[ u^T_a(x) C \gamma_5 d_b(x) \right] u_c + \beta \left[ u^T_a(x) C d_b(x) \right] \gamma_5 u_c(x) \right\},

(2)

χ^2_ν(x) = e^{abc} \left\{ \left[ u^T_a(x) C \sigma_\rho\lambda d_b(x) \right] \sigma^{\rho\lambda} \gamma_\nu u_c(x) - \left[ u^T_a(x) C \gamma_\nu u_b(x) \right] \sigma^{\rho\lambda} \gamma_\nu d_c(x) \right\},

(3)

where T denotes a transpose in Dirac space, C is the charge conjugation matrix, and β is a
parameter allowing for arbitrary mixing of the two independent spin-1/2 interpolators. For
β = −1, the correlator of χ^1_μ and \overline{χ}^1_ν gives the sum rules discussed in Ref. [2].

In the rest frame of the medium, the analytic properties of the invariant functions can
be studied through Lehman representations in energy. The quasi-nucleon excitations are
characterized by the discontinuities of the invariant functions across the real axis, which are
used to identify the on-shell self-energies. A representation of the correlation function can
be obtained by introducing a simple phenomenological Ansatz for these spectral densities.

On the other hand, the correlation function can be evaluated at large space-like momen-
ta using an operator product expansion (OPE). This expansion requires knowledge of
QCD Lagrangian parameters and finite-density quark and gluon matrix elements. Finite-
density QCD sum rules, which relate the nucleon self-energies in the nuclear medium to these
QCD inputs, are obtained by equating the two different representations using appropriately
weighted integrals [2].

The correlator Π_{\mu\nu}^{12}(q) contains nine distinct structures at finite density. Two of these
sum rules are independent of the problematic “scalar-scalar” four-quark condensates and are
dominated by the leading terms of the OPE.

To analyze the sum rules, we follow the techniques introduced in Ref. [3, 4] for deter-
mining the valid Borel region. We limit the continuum model contributions to 50% of the
phenomenology, and maintain the contributions of the highest dimensional operators in the
OPE to less than 10% of the sum of OPE terms. This defines a region in Borel parameter
space where a sum rule should be valid. Uncertainties are estimated via a new Monte-Carlo
error analysis introduced in Ref. [3].

As in previous works on finite-density sum rules, we use the linear density approximation
for estimating the in-medium condensates. Central values and uncertainties of QCD vac-
umuum parameters are taken from Ref. [6]. Finite-density parameters are in accord with the
estimates of Ref. [2]. Since the mixed condensate \( \langle g_i \bar{q} \sigma \cdot G q \rangle_{\rho N} \) is chirally odd, we assume
the same density dependence as for the quark condensate.

The one firm conclusion from previous in-medium studies is that the vector self-energy
is positive and a few hundred MeV. Hence, we begin by fixing \( \Sigma_v = 0.25 \) GeV at saturation
density and searching for a region in which both sides of the QCD sum rules are valid.

There are two sum rules obtained from the correlator of \( \chi_i^1 \) and \( \chi_i^2 \) that are independent
of the problematic four-quark condensates. One of these satisfies the validity criteria.

\[
\lambda_i^2 \Sigma_v e^{-(E_i^2-q^2)/M^2} = \frac{5 + 2\beta + 5\beta^2}{48\pi^2} M^4 E_1 \langle \bar{q}^i q \rangle_{\rho N} L^{-4/9} \\
+ \frac{5(5 + 2\beta + 5\beta^2)}{72\pi^2} E_q M^2 E_0 \langle q^i \bar{D}_0 q \rangle_{\rho N} L^{-4/9} \\
- \frac{7 + 10\beta + 7\beta^2}{192\pi^2} M^2 E_0 \langle g^i \bar{q} \sigma \cdot G q \rangle_{\rho N} L^{-4/9} \\
+ \frac{5 + 2\beta + 5\beta^2}{8\pi^2} q^2 \left( \langle q^i \bar{D}_0 \bar{D}_0 q \rangle_{\rho N} + \frac{1}{12} \langle g^i \bar{q} \sigma \cdot G q \rangle_{\rho N} \right) L^{-4/9} \\
+ \frac{5 + 2\beta + 5\beta^2}{12} \bar{E}_q \langle q^i \bar{q} \rangle_{\rho N}^2 L^{-4/9}. \tag{4}
\]

Here \( \lambda_i \) denotes the coupling of \( \chi_i^1 \) to the quasi-nucleon state. We have also defined
\( M_N^* \equiv M_N + \Sigma_s \), \( E_q \equiv \Sigma_v + \sqrt{q^2 + M_N^*} \), \( \bar{E}_q \equiv \Sigma_v - \sqrt{q^2 + M_N^*} \),
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\( M_N^* \equiv M_N + \Sigma_s \), \( E_q \equiv \Sigma_v + \sqrt{q^2 + M_N^*} \), \( \bar{E}_q \equiv \Sigma_v - \sqrt{q^2 + M_N^*} \),
and \( \Sigma_s \) are the scalar and vector self-energies of the nucleon in nuclear matter,
respectively. The anomalous dimensions of various operators have been taken into account
through the factor \( L \equiv \ln(M^2/\Lambda_{QCD}^2)/\ln(\mu^2/\Lambda_{QCD}^2) \) [1]. We have also defined \( E_0 \equiv 1 - e^{-s_0/M^2} \) and \( E_1 \equiv 1 - e^{-s_0/M^2} (s_0/M^2 + 1) \), which account for excited-state contributions [2].

Of the two favorable new sum rules obtained from the overlap of spin-1/2 and spin-3/2
interpolators, only the sum rule at the structure \( \gamma_{\mu} q \bar{q}_\nu \), satisfies the validity criteria.

\[
\lambda_i \lambda_2 \frac{1}{M_N^*} e^{-(E_i^2-q^2)/M^2} = \frac{1}{8\pi^2} \langle \bar{q} q \rangle_{\rho N} M^2 E_0 L^{8/27} - \frac{3 - \beta}{64\pi^2} \langle g_i \bar{q} \sigma \cdot G q \rangle_{\rho N} L^{-2/27} \\
+ \frac{3 + 5\beta}{96} \langle \bar{q} q \rangle_{\rho N} \langle \frac{\alpha_s}{\pi} G^2 \rangle_{\rho N} \frac{1}{M^2} L^{8/27}. \tag{6}
\]

The parameter \( \beta \) is selected to minimize continuum model contributions while maintain-
ing reasonable higher-dimension operator contributions such that the pole may be resolved
from the continuum contributions [4]. The optimal choice for the sum rule of (6) is \( \beta = -0.7 \).

The continuum model contribution of (6) is independent of \( \beta \). Since the second term of (6)
has little overlap with the ground state nucleon [4], we set \( \beta = 0 \).
3 Implications

In vacuum, higher-dimensional condensates are typically evaluated by adopting a factorization prescription. It is generally accepted that factorization fails for the four-quark operator. However, the dimension seven operator appearing in (6) appears to be reasonably well approximated by the factorization prescription in vacuum [6]. As a chirally odd operator its density dependence should be qualitatively similar to that of the quark condensate. Fortunately, this behavior arises naturally in the factorized form where the density dependence of the gluon condensate is estimated to be a 7% effect. Hence the large uncertainties associated with the density dependence of factorized operators are largely eliminated in these new sum rules. We will further examine the sensitivity of these assumptions in a later work [4].

The density dependence of the new sum rule of (6) is predominantly governed by the quark condensate and is common to all terms of the OPE. As such, the effects of increasing density will be to reduce the residue of the pole while the pole position remains largely unchanged. This result is the key feature absent in the former in-medium nucleon analysis [2]. The approximate invariance of \( M^* + \Sigma_v \) is manifest in (6).

Fig. 1a displays the valid Borel regimes for the two sum rules (4) and (6). The distributions for \( M^* \) and \( \Sigma_v \) are illustrated in Fig. 1b. We find \( M^* = 0.68 \pm 0.08 \) GeV and \( \Sigma_v = 0.31 \pm 0.06 \) GeV at nuclear matter saturation density. These results are consistent with the expectations of relativistic phenomenology.

![Figure 1](image_url)

Figure 1: (a) Illustration of the valid Borel regimes for the sum rules of (4) (large dash) and (6) (fine dash). Both continuum model contributions (limited to 50%) and highest dimension operator contributions (limited to 10%) are illustrated. (b) Histogram for \( \Sigma_v \) and \( M^* \) obtained from a Monte Carlo sample of 1000 QCD parameters.
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