The structure and interpretation of cosmology: 
Part II - The concept of creation in inflation and 
quantum cosmology

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Abstract
The purpose of the paper, of which this is part II, is to review, clarify, 
and critically analyse modern mathematical cosmology. The emphasis is 
upon mathematical objects and structures, rather than numerical compu-
tations. Part II provides a critical analysis of inflationary cosmology and 
quantum cosmology, with particular attention to the claims made that 
these theories can explain the creation of the universe.

Keywords: Cosmology, Inflation, Quantum, Creation, Space, Time

1 Introduction
Part I of this paper (McCabe 2004) concentrated on general relativistic cos-
mosology, providing both critical analysis, and an exposition of the mathematical 
structures employed, with the purpose of demonstrating the great variety of 
possible universes consistent with empirical data. Part II now provides a re-
view and critical analysis of inflation and quantum cosmology, concentrating on 
the need to clarify concepts and, in particular, to assess the claims made that 
inflation and quantum cosmology can explain the creation\(^1\) of the universe.

2 Inflation
Inflationary cosmology postulates that the universe underwent a period of acceler-
atory expansion in its early history due to the existence of a scalar field \(\phi\) with 
predicted characteristics. The scalar field is postulated to have an equation of 
state \(p = -\rho\),\(^2\) and a particular type of potential energy function \(V(\phi)\). The

\(^1\)Whilst Grnbaum (1991) has suggested substituting ‘originating’ in the place of ‘creation’, 
to avoid conveying any theological connotations, the latter term is in such widespread usage 
that it is employed in this paper, albeit without the intent of conveying any of those 
connotations.

\(^2\)An equation of state is a functional expression which links the energy density \(\rho\) of a field 
with its pressure \(p\).
inflationary scenarios postulate that there was at least some patch of the early universe in which this scalar field did not reside at the minimum of its potential energy function, and in which the energy density of the scalar field is dominated by its potential energy, $\rho = V(\phi)$. Given the equation of state, this value of the scalar field corresponds to a state of negative pressure, in which gravity is effectively repulsive. A region of space in this so-called ‘false vacuum’ state undergoes exponential expansion until the scalar field eventually falls into the minimum of its potential. After a period of inflation, the false vacuum energy is converted into the energy density of more conventional matter and radiation, and the region of space which underwent inflation subsequently expands in accordance with a Friedmann-Robertson-Walker (FRW) model.

In Guth’s original 1981 proposal, the inflation was driven by a scalar field which sat within a local, but not global, minimum of its potential energy function. Whilst, in classical terms, this state would be stable, quantum tunnelling would eventually cause such a state to decay, thereby ending the inflationary expansion. However, calculations indicated that this type of false vacuum decay would cause density inhomogeneities inconsistent with current observations. The new inflationary scenario, proposed both by Linde (1982), and the pairing of Albrecht and Steinhardt (1982), solved this problem by proposing that the scalar field which drives inflation sits atop a gentle plateau in the potential energy function. With such a scalar field, inflation takes place while the field slowly ‘rolls’ into the global minimum surrounding the plateau. This rolling process does not require quantum tunnelling. Linde (1983a and 1983b) then proposed his chaotic inflationary scenario, in which the scalar field potential can have a simple profile, with no plateau or local minima, just a single global minimum at zero. In Linde’s scenario, inflation occurs because the field begins at a very high value, and slowly ‘rolls’ towards the global minimum.

Originally, the inflationary scalar field was identified as the Higgs field from the Grand Unified Theories (GUTs) of particle physics, and inflation was triggered by spontaneous symmetry breaking of the GUT gauge symmetry. Grand Unified Theories hypothesise that at energies of about $10^{14}$GeV, the electroweak and strong forces merge into a single unified force. Such theories also postulate the existence of Higgs fields. The new inflationary scenario postulated that as the universe approached the age of $10^{-35}$s, the matter in the universe was in its Grand Unified phase, with the electroweak and strong forces unified, and with the GUT Higgs fields all set to zero. As the universe expanded, it cooled, and after $10^{-35}$s the temperature of the universe dropped below the level at which the electroweak and strong forces are unified. This sudden change in the state of the matter in the universe is called the GUT ‘phase transition’. If such a phase transition occurred rapidly when the temperature fell to the critical value, there would be no inconsistency with FRW cosmology, (Blau and Guth 1987, p528). However, the new inflationary scenario proposed that the universe underwent supercooling at the GUT phase transition. In other words, it was proposed that the phase transition occurred slowly compared with the rate of cooling. As a result of supercooling, it was hypothesised that the energy density of the universe became dominated by the energy density of the GUT Higgs fields, and
the thermal component of the energy density became negligible in comparison. A region of space in such a false vacuum state would undergo exponential expansion until the Higgs fields fall into a ‘true vacuum state’. In terms of the classical theory, the set of true vacuum states is simply defined by the global minimum of the potential energy function.

Practitioners of inflation now assert that it is not possible to identify the scalar field responsible for inflation with the Higgs field of GUTs, “since the potential of such a scalar field is too steep,” (Brandenberger 2002, p4). Inflation driven by the potentials of GUT Higgs fields purportedly results in excessive density perturbations; the consequent amplitude of the anisotropies in the cosmic microwave background radiation exceed that which is actually observed. The scalar field responsible for inflation is now widely referred to as the ‘inflaton’, and is often considered in abstraction from particle physics.

The period of acceleratory expansion postulated in inflation has the consequence that the presently observable universe came from a region sufficiently small that it would have been able to reach homogeneity and thermal equilibrium by means of causal processes before the onset of inflation. Inflation thereby solves the so-called ‘horizon problem’ of FRW cosmology. The apparent homogeneity of our observable universe has to be built-in to the initial conditions of a FRW model, dictating the choice of a locally isotropic and locally homogeneous 3-dimensional Riemannian manifold to represent the spatial universe. Regions of space on opposite sides of our observable universe have the same average temperature and density, even though, in a FRW model, they have always lain beyond each other’s particle horizons. Under inflation, the observable universe comes from a region which would have been able to reach a homogeneous state despite starting from a possibly heterogeneous initial state.

However, to regard the horizon phenomenon in the FRW models as a ‘problem’ betrays a methodological assumption that one can only explain things with causal processes rather than by initial conditions. The universe could, quite simply, have been homogeneous from the outset.

A defining characteristic of inflation is that the energy density of the inflating region is constant during the period of acceleratory expansion. The energy density is maintained at the false vacuum energy density, \( \rho_f \), throughout the period of inflation.\(^3\) Although the calculated energy density at the onset of inflation was huge, at, say, \( \rho_f \approx 10^{73} \text{g cm}^{-3} \), by integrating it over the very small region which became the observable universe, one gets a relatively small total energy. Thus, Guth and Steinhardt assert that “probably the most revolutionary aspect of the inflationary model is the notion that all the matter and energy in the observable universe may have emerged from almost nothing,” (1989, p54).

One begins with a region of very small volume at the time inflation was triggered. During inflation, the scale factor of this region increases enormously, but the energy density remains constant. The huge increase in the scale factor

\(^3\)This characteristic plays a key role in the ideas for universe creation ‘in a laboratory’.
means a huge increase in the volume of the region. Thus, during inflation, the total (non-gravitational energy) of the region increases. At the completion of inflation, the energy density is the same that it was to begin with, but the region has a much greater volume. Integrating the energy density over a much larger domain, one gets a larger total energy.

The consequence of this, as Guth and Steinhardt explain, is that “essentially all the non-gravitational energy of the [observable] universe is created as the false vacuum undergoes its accelerated expansion. This energy is released when the phase transition takes place, and it eventually evolves to become everything that we see, including the stars, the planets, and even ourselves,” (1989, p54).

It is important to emphasise that inflation only entails the observable universe to have been created from a very small initial amount of energy. Inflation does not entail that the entire universe was created from almost nothing. The entire spatial universe could have either compact or non-compact topology, and could therefore be either of finite volume, or of infinite volume. In contrast, the observable spatial universe is definitely of finite volume. This entails that the total amount of non-gravitational energy within the observable spatial universe must be finite. If the entire spatial universe is compact, the total amount of non-gravitational energy in the spatial universe will also be finite, but if the entire spatial universe is non-compact, the total non-gravitational energy in the universe could be infinite. It is only if the entire spatial universe is compact, and therefore of finite volume, like the observable universe, that the entire universe could have been created from ‘almost nothing’.

Guth and Steinhardt conclude that “the inflationary model offers what is apparently the first plausible scientific explanation for the creation of essentially all the matter and energy in the observable Universe,” (1989, p54). They acknowledge that “it is then tempting to go one step further and speculate that the entire universe evolved from literally nothing. The recent developments in cosmology strongly suggest that the universe may be the ultimate free lunch,” (1989, p54).

This, of course, is where quantum cosmology enters. Blau and Guth claim that in the scenarios proposed by Vilenkin and Linde, “the universe tunnels directly from a state of ‘absolute nothingness’ into the false vacuum,” and that Hartle and Hawking “have proposed a unique wave function for the universe, incorporating dynamics which leads to an inflationary era,” (1987, p556). These latter claims are over-optimistic, and are typical of the way in which quantum cosmology is often invoked as a *deus ex machina* to explain the initial conditions which are necessary for inflation to occur.

Despite this criticism, one can endorse the interpretation of Guth and Steinhardt, that inflation is able to explain how almost all the non-gravitational energy in our observable universe was created. Inflation, however, clearly cannot explain how space and time were created, and it cannot explain how the initial seed of energy was created. Inflation cannot produce physical something from physical nothing. Inflation could, quite conceivably, be a vital cog in a universe creation theory, but it cannot on its own explain why there is physical something rather than physical nothing.
The hypothetical false vacuum state which drives inflation is distinct from the true vacuum, which, in classical terms, is defined by the global minimum of the potential energy function. Whilst it is conventional to set the global minimum in the classical theory to zero, according to quantum theory the true vacuum state of a field does not have zero energy. In the quantum vacuum, it is believed that virtual particle-antiparticle pairs are constantly created and annihilated. It is believed that the virtual pairs are created \textit{ex nihilo}, and physicists speak of the quantum ‘fluctuations’ of the vacuum.

The nature of the quantum vacuum has inspired a number of universe creation scenarios. For example, in 1973 Edward P. Tryon proposed that our universe was created as a spontaneous quantum fluctuation of some pre-existing ‘vacuum’. Tryon conjectured that all conserved quantities have a net value of zero for the universe as a whole. Noting that in Newtonian theory, the gravitational potential energy is negative, he proposed that there might be a sense in which the negative gravitational energy of the universe cancels the positive mass-energy. He calculated that this might/would be the case if the average density of matter matches the critical density (1973, p396), although he also seemed to predict that the universe is closed (1973, p397).

Tryon’s idea still finds favour today. Guth suggests that the energy created during inflation “comes from the gravitational field. The universe did not begin with this energy stored in the gravitational field, but rather the gravitational field can supply the energy because its energy can become negative without bound. As more and more positive energy materializes in the form of an ever-growing region filled with a high-energy scalar field, more and more negative energy materializes in the form of an expanding region filled with a gravitational field. The total energy remains constant at some very small value, and could in fact be exactly zero,” (Guth 2004, p5-6). However, Tryon’s idea runs aground on a fact Guth mentions in a footnote: “In general relativity there is no coordinate-invariant way of expressing the [gravitational] energy in a space that is not asymptotically flat, so many experts prefer to say that the total energy is undefined,” (ibid., p6). As Wald points out, “it has long been recognized that there is no meaningful local notion of gravitational energy density in general relativity,” (Wald 2001, p20).

In 1978, Brout et al adopted the idea of an initial microscopic quantum fluctuation, but added the idea that the initial state of matter was one with a large negative pressure, which resulted in exponential expansion of the initial fluctuation into an open universe. The creation of an open universe featured in a paper by J.R. Gott in 1982, and in the same year, the papers of Atkatz-Pagels, and Vilenkin addressed the creation of closed universes. Subsequently, Tryon argued (1992) that inflation can be combined with the notion of a quantum vacuum fluctuation to explain the creation of a universe.

On the one hand, Tryon believes that the notion of a quantum fluctuation alone is sufficient to explain the creation of our universe, stating that “although quantum fluctuations are typically microscopic in scale, no princi-
ple limits their potential size and duration, provided that conservation laws are respected. Hence, given sufficient time, it seems inevitable that a universe with the size and duration of ours would spontaneously appear as a quantum fluctuation,” (Tryon, 1992, p571). On the other hand, he acknowledges that “large (and long-lived) universes intuitively seem much less likely than smaller ones,” so he then suggests that inflation could transform an initial microscopic fluctuation into a large universe. He asserts that “inflation greatly enhances the plausibility of creation ex nihilo,” and concludes that “quantum uncertainties suggest the instability of nothingness...inflation might have converted a spontaneous, microscopic quantum fluctuation into our Cosmos,” (1992, p571).

Tryon fails to establish a clear distinction between the possible creation of the material universe from a pre-existing ‘empty’ space-time, and the possible creation of space, time, and matter from physical nothing, the empty set $\emptyset$. In 1973, Tryon imagined our universe as “a fluctuation of the vacuum, the vacuum of some larger space in which our Universe is imbedded,” (Tryon, 1973, p397). This statement seems to indicate that Tryon was thinking of creation from a pre-existing, empty space-time. It seems to indicate that the ‘vacuum’ Tryon refers to is the matter field vacuum of a pre-existing empty space-time. Subsequently, Tryon stated his proposal more carefully, asserting that “the universe was created from nothing as a spontaneous quantum fluctuation of some pre-existing vacuum or state of nothingness,” (1992, p570). From the latter statement, it seems that Tryon now contemplates creation from either a pre-existing empty space-time, or from literally nothing, the empty set.

Even then, however, Tryon argues that “given sufficient time” (1992, p571) quantum fluctuations will yield a universe. This echoes the 1973 proposal that our universe “is simply one of those things which happen from time to time.” Both comments indicate that Tryon considers time to exist before the hypothetical creation of our universe as a vacuum fluctuation. This is consistent with the idea that a universe is created as a quantum fluctuation in a pre-existing space-time. It is inconsistent with the idea that a universe is created from physical nothing, the empty set.

Whilst inflation on its own could only explain the existence of almost all the matter and non-gravitational energy in our universe, by combining inflation with the idea of a quantum fluctuation in a pre-existing space-time, one might be able to explain the existence of all the matter and non-gravitational energy in our universe. One might suggest that there existed an initial space-time in which the matter fields were in their true vacuum states. One might then imagine that some fluctuation of this quantum vacuum created a small region of space in which the inflaton scalar field possesses the necessary initial state for inflation to ensue. The small initial quantum fluctuation would be transformed into a fully-fledged universe. Inflation would transform the small initial amount of non-gravitational energy into enough matter and non-gravitational energy for a universe larger than our own observable universe.

It is important to note that two distinct types of vacuum are at work in such a scenario. Quantum fluctuations of the true vacuum would create a small amount of non-gravitational energy, ‘almost nothing’, and then the properties of the false
vacuum would create, from ‘almost nothing’, sufficient non-gravitational energy for a universe replete with galaxies.

Obviously, such a scenario would only explain the creation of the material universe. It would not be creation from physical nothing, because there would be a pre-existing space-time. Tryon’s idea would not incorporate inflation into a theory which explained why there is physical something rather than physical nothing. Tryon’s idea would, at best, incorporate inflation into a theory which explained why there is some matter and energy, rather than no matter and energy.

Given the current notion of physical space and the current notion of the quantum vacuum, the existence of the quantum vacuum is contingent. It is not a contradiction to imagine the existence of space and the non-existence of the quantum vacuum. It is false to claim that truly empty space, with zero energy, is impossible. There is nothing in the current notion of physical space that entails the presence of the quantum vacuum. So long as space is represented by a differential manifold, and mass-energy is represented by fields on a manifold, it will be possible to imagine empty space. It may well be true that there is no operational procedure which can make a region of space completely empty, but this does not mean that it is impossible for space to be empty. It might also be operationally impossible to change the dimension of physical space, but that does not mean that it is impossible for physical space to be other than 3-dimensional.

Some theory in the future may represent the universe in a way that makes space-time and mass-energy conceptually inseparable, and it may then follow from the nature of space-time that the quantum vacuum exists. However, if this were to be the case, there would no longer be the twofold question of how a material universe could have been created from empty space, and how empty space could have been created from physical nothing. One would have the single question of how the physical universe could have been created from physical nothing. Hence, the notion of the quantum vacuum cannot entail the existence of the material universe. If space-time and mass-energy are conceptually separable, then the presence of the quantum vacuum is merely contingent, hence it cannot entail that empty space must create a material universe. Alternatively, if space-time and mass-energy are conceptually inseparable, then an explanation for the existence of the material universe requires an explanation of how the material universe was created from physical nothing, and the quantum vacuum cannot achieve this.

3 Quantum cosmology

In canonical general relativity, expressed in terms of the ‘traditional’ variables, a configuration of the spatial universe is given by a 3-dimensional manifold $\Sigma$, equipped with a Riemannian metric tensor field $\gamma$, and a matter field configuration $\phi$. The full configuration space of general relativity would be the set of
all possible pairs \((\gamma, \phi)\) on all possible 3-manifolds \(\Sigma\). In canonical quantum gravity, the main object of interest is a state vector \(\Psi\), a function upon the configuration space which satisfies the Wheeler-DeWitt equation.

In path-integral quantum gravity, expressed in terms of the traditional variables, the main object of interest is a transition from an initial configuration \((\Sigma_i, \gamma_i, \phi_i)\) to a final configuration \((\Sigma_f, \gamma_f, \phi_f)\). The interest lies in defining and calculating a propagator \(K(\Sigma_i, \gamma_i, \phi_i; \Sigma_f, \gamma_f, \phi_f)\). In contrast with path-integral non-relativistic quantum mechanics, there are no overt time labels associated with either the initial or final configuration.

To calculate the propagator \(K(\Sigma_i, \gamma_i, \phi_i; \Sigma_f, \gamma_f, \phi_f)\), one might expect to introduce the set \(\mathcal{P}_L(\Sigma_i, \gamma_i, \phi_i; \Sigma_f, \gamma_f, \phi_f)\), of all 4-dimensional Lorentzian space-times which interpolate between \((\Sigma_i, \gamma_i, \phi_i)\) and \((\Sigma_f, \gamma_f, \phi_f)\). Whilst classical general relativity requires that a space-time satisfy the classical dynamical equations, the Einstein field equations, quantum gravity introduces the set of all kinematically possible interpolating space-times, irrespective of whether they satisfy the Einstein field equations.

Each interpolating space-time history is a 4-dimensional Lorentzian manifold-with-boundary \((\mathcal{M}, g)\). The boundary of each \(\mathcal{M}\) must consist of the disjoint union of \(\Sigma_i\) and \(\Sigma_f\). In addition, the restriction of the Lorentzian metric \(g\) to the boundary components must be such that \(g|_{\Sigma_i} = \gamma_i\) and \(g|_{\Sigma_f} = \gamma_f\). Each interpolating space-time must be equipped with a smooth matter field history \(\Phi\), which satisfies the conditions \(\Phi|_{\Sigma_i} = \phi_i\) and \(\Phi|_{\Sigma_f} = \phi_f\).

The initial 3-manifold \(\Sigma_i\) need not be homeomorphic with the final 3-manifold \(\Sigma_f\). Hence, the transition from an initial configuration \((\Sigma_i, \gamma_i, \phi_i)\) to a final configuration \((\Sigma_f, \gamma_f, \phi_f)\) could be a topology changing transition.

The notion of topology change is closely linked with the concept of cobordism. When a pair of \(n\)-manifolds, \(\Sigma_1\) and \(\Sigma_2\), constitute disjoint boundary components of an \(n+1\) dimensional manifold, \(\Sigma_1\) and \(\Sigma_2\) are said to be cobordant. It is a valuable fact for path-integral quantum gravity that any pair of compact 3-manifolds are cobordant, (Lickorish 1963). Not only that, but any pair of compact Riemannian 3-manifolds, \((\Sigma_1, \gamma_1)\) and \((\Sigma_2, \gamma_2)\), are ‘Lorentz cobordant’, (Reinhart 1963). i.e. There exists a compact 4-dimensional Lorentzian manifold \((\mathcal{M}, g)\), with a boundary \(\partial \mathcal{M}\) which is the disjoint union of \(\Sigma_1\) and \(\Sigma_2\), and with a Lorentzian metric \(g\) that induces \(\gamma_1\) on \(\Sigma_1\), and \(\gamma_2\) on \(\Sigma_2\).

This cobordism result is vital because it confirns that topology change is possible. Even when \((\Sigma_1, \gamma_1)\) and \((\Sigma_2, \gamma_2)\) are compact Riemannian 3-manifolds with different topologies, there exists an interpolating space-time.

With each kinematically possible interpolating history, one can associate a real number, the action \(A\)

\[
A = \frac{1}{16} \pi G \int_{\mathcal{M}} S \sqrt{-g} \sqrt{\det g} d^4x + \frac{1}{8} \pi G \int_{\partial \mathcal{M}} Tr K \sqrt{\gamma} d^3x + C + \int_{\mathcal{M}} L_{\text{matter}} \sqrt{-g} d^4x.
\]

\(^4\)In terms of Ashtekar’s ‘new variables’, the geometrical configuration space is not the space of metrics on \(\Sigma\), but the space of connections upon an \(SU(2)\)-principal fibre bundle over \(\Sigma\), (Baez 1995).
S is the scalar curvature, \( K \) is the extrinsic curvature tensor, and \( L_m \) is the matter field Lagrangian density.

One ‘weights’ each possible space-time history with a unimodular complex number \( \exp(iA/\hbar) \). Whilst \( A : \mathcal{P}_L \to \mathbb{R}^1 \) is an unbounded function on the path-space \( \mathcal{P}_L \), the mapping \( \exp(iA/\hbar) : \mathcal{P}_L \to S^1 \subset \mathbb{C}^1 \) is a bounded function.

One could then define the propagator of quantum gravity as:

\[
K(\Sigma_i, \gamma_i, \phi_i; \Sigma_f, \gamma_f, \phi_f) = \int_{\mathcal{P}_L} \exp(iA/\hbar) d\mu,
\]

It has been claimed that in quantum gravity, the creation of a universe from nothing would simply correspond to the special case where \((\Sigma_i, \gamma_i, \phi_i) = \emptyset\). If this were so, then the probability amplitude or probability of a transition from nothing to a spatial configuration \((\Sigma_f, \gamma_f, \phi_f)\) would be given by

\[
K(\emptyset; \Sigma_f, \gamma_f, \phi_f) = \int_{\mathcal{P}_L} \exp(iA/\hbar) d\mu,
\]

where \( \mathcal{P}_L \) is an abbreviation here for \( \mathcal{P}_L(\emptyset; \Sigma_f, \gamma_f, \phi_f) \), the set of all Lorentzian 4-manifolds \((M, g)\) and matter field histories \( \Phi \) which have a single boundary component \( \partial M = \Sigma_f \) on which \( g \) induces \( \gamma_f \), and \( \Phi \) induces \( \phi_f \).

Unfortunately, there are serious technical problems with the definition of the propagator by a Lorentzian path-integral. Firstly, if one permits \( \mathcal{P}_L \) to include non-compact space-times, then the action integral can diverge for some of these space-times. For example, if a non-compact space-time is homogeneous, then the action integral diverges. Because a non-compact homogeneous space-time has no well-defined action \( A \), it cannot be assigned a weight \( \exp(iA/\hbar) \). An asymptotically flat space-time is a notable case of a non-compact space-time for which the action integral is finite, but asymptotically flat geometry is a special case, and is of no cosmological relevance.

Secondly, \( \mathcal{P}_L \) is not finite-dimensional, and no satisfactory measure has been found on \( \mathcal{P}_L \). In the absence of a satisfactory measure on \( \mathcal{P}_L \), integration over \( \mathcal{P}_L \) is not well-defined. Although the integrand \( \exp(iA/\hbar) \) is a bounded function, when it is expanded into its real-imaginary form, \( \exp(iA/\hbar) = \cos A/\hbar + i \sin A/\hbar \), it is clearly oscillatory. Thus, even if one attempted to approximate the propagator by an integral over a finite-dimensional subset of \( \mathcal{P}_L \), the integral would not be finite unless one integrated over a compact subset of \( \mathcal{P}_L \). One attempt to avoid these difficulties is the so-called ‘Euclidean’ path-integral approach to quantum gravity. In this approach, the propagator \( K(\Sigma_i, \gamma_i, \phi_i; \Sigma_f, \gamma_f, \phi_f) \) is defined to be an integral over \( \mathcal{P}_R(\Sigma_i, \gamma_i, \phi_i; \Sigma_f, \gamma_f, \phi_f) \), the set of all compact Riemannian 4-manifolds and matter field histories which interpolate between \((\Sigma_i, \gamma_i, \phi_i)\) and \((\Sigma_f, \gamma_f, \phi_f)\). It would clearly be more appropriate to refer to this approach as the Riemannian path-integral approach to quantum gravity.

A ‘Euclidean’ action \( A_E \) is associated with each interpolating history, and one assigns a weight of \( \exp(-A_E/\hbar) \) to each such interpolating history. The propagator is then defined to be
\begin{equation}
K(\Sigma_i, \gamma_i, \phi_i; \Sigma_f, \gamma_f, \phi_f) = \int_{\mathcal{P}_R} \exp(-AE/\hbar) d\mu.
\end{equation}

A Riemannian manifold \((\mathcal{M}, g)\) has the helpful property that if either i) \(\mathcal{M}\) is compact, or ii) \((\mathcal{M}, g)\) is homogeneous, then \((\mathcal{M}, g)\) must be geodesically complete. Hence, by integrating over compact Riemannian 4-geometries, one would be integrating over geodesically complete geometries; one would be integrating over ‘non-singular’ 4-geometries. For this reason, advocates of Euclidean path-integrals tend to claim that their approach avoids the singularities of classical cosmology. Note, however, that quantum cosmology replaces an individual space-time manifold with objects such as wave-functions and propagators, so the issue of a singularity in the geometry is no longer so pertinent.

If the creation of a universe from nothing corresponds to the special case in which \((\Sigma_i, \gamma_i, \phi_i) = \emptyset\), then in the Euclidean approach the probability amplitude or probability of a transition from nothing to a spatial configuration \((\Sigma_f, \gamma_f, \phi_f)\) would be given by integrating only over the compact Riemannian 4-manifolds and matter field histories \(\mathcal{P}_R = \mathcal{P}_R(\emptyset; \Sigma_f, \gamma_f, \phi_f)\):

\begin{equation}
K(\emptyset; \Sigma_f, \gamma_f, \phi_f) = \int_{\mathcal{P}_R} \exp(-AE/\hbar) d\mu.
\end{equation}

Unfortunately, the Euclidean action \(A_E\) is not positive definite; \(A_E\) can be negative. Moreover, there is no lower bound on the value that the Euclidean action can take. Thus, the integrand in the path integral, \(\exp(-A_E/\hbar) = 1/\exp(A_E/\hbar)\) can ‘blow up exponentially’. This means that the integrand in a Riemannian path-integral can be an unbounded function. If one attempted to approximate the propagator by integrating \(\exp(-A_E/\hbar)\) over a finite-dimensional subset of \(\mathcal{P}_R\), then the integral would not be finite unless one used a special measure. In the Euclidean approach it has been suggested that the transition amplitudes \(K(\emptyset; \Sigma, \gamma, \phi)\) can be approximated by summation over compact Riemannian 4-geometries which are saddle points of the action \(A_E\). However, even if there is a way to approximately calculate the transition amplitudes of quantum gravity, it is highly debatable whether the transition amplitudes \(K(\emptyset; \Sigma, \gamma, \phi)\) could be interpreted as creation \textit{ex nihilo} amplitudes.

In the case of the Lorentzian approach, the first problem is that compact Lorentzian space-times with only a single compact boundary component, are time non-orientable. This means that the single compact boundary cannot be treated as a final boundary, at which the region of space-time ends. It is equally legitimate to treat it as a boundary at which the region of space-time begins.

Suppose instead that one uses a collection of non-compact, time-orientable Lorentzian space-times which end at \((\Sigma, \gamma, \phi)\), and which have no past boundary. Each one of these space-times ‘creates’ \((\Sigma, \gamma, \phi)\) from a prior region of space-time. Thus, all the space-times which determine the purported creation \textit{ex nihilo} probability of \((\Sigma, \gamma, \phi)\), ‘create’ \((\Sigma, \gamma, \phi)\) from a prior region of space-time; they do not individually create \((\Sigma, \gamma, \phi)\) from nothing \(\emptyset\). Indeed, some space-times which terminate with \((\Sigma, \gamma, \phi)\) are past-infinite. Thus, space-times which exist
for an infinite time before reaching \((\Sigma, \gamma, \phi)\) would contribute to the probability of creating \((\Sigma, \gamma, \phi)\) from nothing!

Similarly, in the Euclidean approach, all the Riemannian 4-geometries which determine the purported creation \textit{ex nihilo} probability of \((\Sigma, \gamma, \phi)\), ‘create’ \((\Sigma, \gamma, \phi)\) from a region of 4-dimensional space; they do not individually create \((\Sigma, \gamma, \phi)\) from nothing \(\emptyset\).

These are strong reasons to doubt that \(K(\emptyset; \Sigma, \gamma, \phi)\) could be interpretable as a creation \textit{ex nihilo} probability amplitude in either the Lorentzian or the Euclidean approach. In the Lorentzian approach, when speaking of space-times with no past boundary, it is syntactically acceptable to say that the past boundary component is empty, \(\emptyset\), but one should not think of \(\emptyset\) as a special type of past boundary; it is no past boundary at all. In the Euclidean approach, when speaking of 4-dimensional spaces with no second boundary component, it is syntactically acceptable to say that the second boundary component is empty, \(\emptyset\), but again one should not think of \(\emptyset\) as a special type of second boundary; rather, it is no second boundary at all. A boundary of a manifold must be a topological space, and amongst other things, a topological space must be a non-empty set. \(\emptyset\) is the empty set, hence \(\emptyset\) cannot be a topological space, which entails that \(\emptyset\) cannot be the boundary of a manifold. Cobordism is an equivalence relation between manifolds, hence it is not possible for any manifold to be cobordant with the empty set \(\emptyset\).

Space-times which have no past boundary are not space-times which begin with the empty set \(\emptyset\). As Grnbaum complains, “What...is temporally ‘initial’ about an empty set...? Apparently, the empty set in question is verbally labelled to be ‘initial’ by mere definitional fiat,” (Grnbaum 1991, Section C).

An integration or summation over space-times with no past boundary, can only be interpreted as the probability of \((\Sigma_f, \gamma_f, \phi_f)\) arising from anything, not the probability of \((\Sigma_f, \gamma_f, \phi_f)\) arising from nothing. The absence of a past boundary merely signals the absence of a restriction upon the ways in which \((\Sigma_f, \gamma_f, \phi_f)\) can come about. Every space-time in \(\mathcal{P}_L(\Sigma_i, \gamma_i, \phi_i; \Sigma_f, \gamma_f, \phi_f)\), for each \((\Sigma_i, \gamma_i, \phi_i)\), is a subset of at least one space-time in \(\mathcal{P}_L(\emptyset; \Sigma_f, \gamma_f, \phi_f)\). Every space-time in \(\mathcal{P}_L(\Sigma_i, \gamma_i, \phi_i; \Sigma_f, \gamma_f, \phi_f)\) is part of at least one space-time in \(\mathcal{P}_L(\emptyset; \Sigma_f, \gamma_f, \phi_f)\) which extends further into the past, beyond \((\Sigma_i, \gamma_i, \phi_i)\). It is in this sense that the absence of a past boundary merely signals the absence of a restriction upon the ways in which \((\Sigma_f, \gamma_f, \phi_f)\) can come about. The set of Lorentzian space-times \(\mathcal{P}_L(\emptyset; \Sigma_f, \gamma_f, \phi_f)\) contains all the possible past histories that lead up to \((\Sigma_f, \gamma_f, \phi_f)\), whereas \(\mathcal{P}_L(\Sigma_i, \gamma_i, \phi_i; \Sigma_f, \gamma_f, \phi_f)\) contains the past histories which are truncated at the spatial configuration \((\Sigma_i, \gamma_i, \phi_i)\).

An integration or summation over space-times with no past boundary cannot be interpreted as the probability of a transition from \(\emptyset\) to \((\Sigma_f, \gamma_f, \phi_f)\).

Similarly, in the Euclidean approach, an integration or summation over 4-dimensional spaces in which \((\Sigma_f, \gamma_f, \phi_f)\) is the only boundary component, cannot be interpreted as the probability of a transition from \(\emptyset\) to \((\Sigma_f, \gamma_f, \phi_f)\). The absence of another boundary component merely signals the absence of a restriction upon the 4-dimensional spaces which possess \((\Sigma_f, \gamma_f, \phi_f)\) as a boundary. Every Riemannian 4-geometry in \(\mathcal{P}_R(\Sigma_i, \gamma_i, \phi_i; \Sigma_f, \gamma_f, \phi_f)\), for each \((\Sigma_i, \gamma_i, \phi_i)\),
is a subset of at least one Riemannian 4-geometry in $P_R(\emptyset; \Sigma_f, \gamma_f, \phi_f)$. Every Riemannian 4-geometry in $P_R(\emptyset; \Sigma_f, \gamma_f, \phi_f)$ is part of at least one Riemannian 4-geometry in $P_R(\emptyset; \Sigma_i, \gamma_i, \phi_i)$ which extends to a greater volume, beyond $(\Sigma_i, \gamma_i, \phi_i)$. The set of Riemannian 4-geometries $P_R(\emptyset; \Sigma_f, \gamma_f, \phi_f)$ contains all the possible 4-dimensional spaces which possess $(\Sigma_f, \gamma_f, \phi_f)$ as a boundary, whereas $P_R(\Sigma_i, \gamma_i, \phi_i; \Sigma_f, \gamma_f, \phi_f)$ contains all those that are truncated at the spatial configuration $(\Sigma_i, \gamma_i, \phi_i)$.

To counter these arguments, one could argue that the space-times or 4-dimensional spaces being used would only play a part in the theoretical calculation of the transition probabilities, and would not play a part in any actual physical process. One could argue that the transition from $\emptyset$ to some $(\Sigma, \gamma, \phi)$ only takes place at the quantum level, not at the level of the individual classical space-times or 4-dimensional spaces which are used to calculate the probability of the quantum event. One could argue that the only thing which happens physically is a transition from $\emptyset$ to some $(\Sigma, \gamma, \phi)$. The fact that the space-times used in the Lorentzian approach cannot be said to begin with the empty set, and the fact that they individually create $(\Sigma, \gamma, \phi)$ from a prior region of space-time, does not entail that they cannot be used to calculate the probability of a transition from $\emptyset$ to $(\Sigma, \gamma, \phi)$. The fact that the 4-dimensional spaces used in the Euclidean approach cannot be said to individually create $(\Sigma, \gamma, \phi)$ from the empty set, and the fact that they individually ‘create’ $(\Sigma, \gamma, \phi)$ from a 4-dimensional space, does not entail that they cannot be used to calculate the probability of a transition from $\emptyset$ to $(\Sigma, \gamma, \phi)$.

This counter-argument is inconsistent with the principle that the probability of a transition between two configurations is determined by all the kinematically possible classical histories that can inter-elect between those configurations. Quantum ‘tunnelling’ occurs in non-relativistic quantum theory if there is a transition which is not dynamically possible according to the classical dynamical equations. However, quantum tunnelling in non-relativistic quantum theory can only take place between two configurations, $q_1$ and $q_2$, if there is a kinematically possible classical history that interpolates between them. If there is no such kinematically possible history, then even in quantum theory, a transition between the two configurations is not possible. For example, if $q_1$ and $q_2$ are points that belong to disconnected regions of space, then a transition between $q_1$ and $q_2$ is impossible. Because no manifold $\Sigma$ can be cobordant with the empty set, there are no kinematically possible classical histories which interpolate between $\emptyset$ and $(\Sigma, \gamma, \phi)$. Hence, there cannot be a quantum transition between $\emptyset$ and $(\Sigma, \gamma, \phi)$. In other words, quantum tunnelling between $\emptyset$ and $(\Sigma, \gamma, \phi)$ is impossible.

### 3.1 The Hartle-Hawking Ansatz

The most notorious application of Euclidean path-integral quantum gravity to quantum cosmology is the paper of Hartle and Hawking (1983). It is suggested here that the wave-function of the universe $\Psi_0$ can be specified by Euclidean path-integration, hence the Hartle-Hawking approach provides a meeting point
between the canonical approach and path-integral approach to quantum gravity. The zero subscript indicates that Hartle-Hawking consider this wave-function to be a type of ‘ground state’, which normally means a quantum state of minimum energy. Of all the possible solutions to the Wheeler-DeWitt equation, it is suggested that the ‘Euclidean’ creation \( \text{ex nihilo} \) path-integral generates the correct solution. In elementary quantum theory, a time-dependent quantum state-function, which satisfies the Schrödinger equation, can be generated by a path-integral; here, it is suggested that the time-independent state-function of quantum gravity, which satisfies the Wheeler-DeWitt equation, can also be generated by a path-integral. Hartle-Hawking include the 3-geometries \( \gamma \) and the matter fields \( \phi \) in the domain of the wave-function, \( \Psi_0(\gamma, \phi) \). One could also include the 3-topology \( \Sigma \), although Hartle-Hawking restrict their proposal to compact 3-manifolds.

The Hartle-Hawking Ansatz can be analysed into three separate propositions.\(^5\) The first proposition is that the probability amplitudes \( K(\emptyset; \Sigma_f, \gamma_f, \phi_f) \) are the probability amplitudes of creation \( \text{ex nihilo} \). The second proposition is that these probability amplitudes provide the wave-function of the universe \( \Psi_0(\Sigma_f, \gamma_f, \phi_f) \). i.e.

\[
\Psi_0(\Sigma_f, \gamma_f, \phi_f) = K(\emptyset; \Sigma_f, \gamma_f, \phi_f).
\]

The third proposition is that \( K(\emptyset; \Sigma_f, \gamma_f, \phi_f) \) is specified by path-integration over compact Riemannian 4-geometries. Given the intractability of the full path-integral, a weaker but more plausible proposition can be substituted here: \( K(\emptyset; \Sigma_f, \gamma_f, \phi_f) \) is specified approximately by a summation over select compact Riemannian 4-geometries. There are \( 2^3 = 8 \) possible combinations for accepting or rejecting these propositions. For example, one could agree that the probability amplitudes \( K(\emptyset; \Sigma_f, \gamma_f, \phi_f) \) are equivalent with the wave-function of the universe, but one could reject the proposal that these probability amplitudes are generated by summation over compact Riemannian 4-geometries. One might attempt to use non-compact geometries and Lorentzian geometries instead.

Conversely, one could agree that the probability amplitudes \( K(\emptyset; \Sigma_f, \gamma_f, \phi_f) \) are generated by summation over compact Riemannian 4-geometries, but one need not believe that these amplitudes are equivalent with the wave-function of the universe. Given that the wave-function of the universe is a concept drawn from canonical quantum gravity, one could refuse to grant that it has any connection with path-integral quantum gravity.

Alternatively again, one could accept that the probability amplitudes \( K(\emptyset; \Sigma_f, \gamma_f, \phi_f) \) are equivalent with the wave-function of the universe, and one could accept that these amplitudes are determined by summation over compact Riemannian 4-geometries, but one could deny that these probability amplitudes should be interpreted as creation \( \text{ex nihilo} \) probability amplitudes.

\(^5\)It must be emphasised that Hartle and Hawking made no such threefold distinction themselves.
There seems to be a degree of conceptual confusion in the original expression of the Hartle-Hawking Ansatz. For example, consider the following passage: “our proposal is that the sum should be over compact geometries. This means that the Universe does not have any boundaries in space or time (at least in the Euclidean regime). There is thus no problem of boundary conditions. One can interpret the functional integral over all compact four-geometries bounded by a given three-geometry as giving the amplitude for that three-geometry to arise from a zero three-geometry, i.e. a single point. In other words, the ground state is the amplitude for the Universe to appear from nothing,” (Hartle and Hawking 1983, p2961).

This statement is open to a number of criticisms. Firstly, it is entirely conventional in general relativistic cosmology to represent the universe by a boundaryless differential manifold. It is far from radical to suggest that the universe has no boundary in space or time. Secondly, the concept of a compact 4-manifold is distinct from the concept of a boundaryless 4-manifold. A compact manifold may or may not possess a boundary. A boundaryless manifold may be compact or non-compact. A manifold with boundary may be compact or non-compact. By summing over compact 4-manifolds, one would exclude non-compact 4-manifolds from one’s purview, but one would not exclude compact manifolds which possess a boundary; the Hartle-Hawking proposal is to sum over compact 4-manifolds which possess a single 3-dimensional boundary component.

Thirdly, by moving from classical general relativity to path-integral quantum gravity, one ceases to represent the universe by a single space-time. It is, therefore, difficult to understand in what sense it is ‘the Universe’ which could be bereft of boundary. In quantum cosmology, the universe is represented by a wave-function, not a manifold.

Fourthly, the so-called ‘Euclidean regime’ is a distinct concept from summation over compact 4-geometries. One could propose summation over compact 4-geometries without proposing that the 4-geometries must be Riemannian (‘Euclidean’).

There appear to be two types of boundary conditions at work in the Hartle-Hawking Ansatz. There are boundary conditions on the hypothetical wave-function of the universe, and there are boundary conditions on the individual 4-geometries in the summation. The claim in the above excerpt that there is no problem with boundary conditions, implies that the boundary conditions referred to at this juncture are boundary conditions on the 4-geometries in the summation, not boundary conditions on the wave-function. It is only for compact 4-geometries that the action is guaranteed to be finite. If one were to permit non-compact 4-geometries, one would have to impose boundary conditions to ensure that the action integral of such 4-geometries did not diverge. Hartle-Hawking propose that the wave-function be obtained by summation over compact 4-geometries, which need no spatial boundary conditions. This is the proposed boundary condition on the wave-function.

The confusion created by the Hartle-Hawking Ansatz, and by the decision to name it the ‘no-boundary’ boundary condition, is typified by the account given by Kolb and Turner: “because a compact manifold has no boundaries,
this proposal is referred to as the ‘no-boundary’ boundary condition,” (Kolb and Turner 1990, p462). To reiterate, a compact manifold can have a boundary, and a non-compact manifold need not have a boundary.

The ambiguity of the phrase ‘boundary conditions’, is used by Hawking in his well-known dictum that “the boundary conditions of the Universe are that it has no boundary.” Hawking has stated that “if spacetime is indeed finite but without boundary or edge...it would mean that we could describe the Universe by a mathematical model which was determined completely by the laws of science alone; they would not have to be supplemented by boundary conditions,” (Hawking 1989, p69). A statement like this suppresses the fact that in path-integral quantum gravity, one no longer represents the universe by an individual space-time; one deals with summation over multiple space-times.

The assertion that the laws of science would “not have to be supplemented by boundary conditions” is even more unfathomable because Hawking freely admits that the Euclidean no-boundary proposal “is simply a proposal for the boundary conditions of the Universe,” (Hawking 1989, p68). Hawking must know that the Wheeler-DeWitt equation is a proposed ‘law of science’ which has many possible solutions, and to select a particular solution, one needs to specify boundary conditions. Hawking’s misleading claims for the ‘no-boundary’ proposal have been widely disseminated. Barrow, for example, claims that the Hartle-Hawking Ansatz “removes the conventional dualism between laws and initial conditions,” (Barrow 1991, p67).

Returning to the excerpt from the 1983 paper, Hartle-Hawking interpret their ground state wave-function as giving the amplitude for any 3-geometry “to arise from a zero three-geometry, i.e. a single point. In other words, the ground state is the amplitude for the Universe to appear from nothing.” Hartle-Hawking introduce three distinct concepts here, and treat them as if they are equivalent. First of all they refer to a “zero three-geometry”, then they refer to a “single point”, then they refer to “nothing”. The number zero is a bona fide element of the set of real numbers, and is quite distinct from nothing, the empty set $\emptyset$. Moreover, it is not clear what Hartle-Hawking mean by a “zero three-geometry”. A single point is sometimes considered by mathematicians to be a zero-dimensional manifold, but such an object cannot have any geometry, never mind a “zero three-geometry”. Furthermore, single points never appear in the summations under consideration. The proposed summations are over manifolds which have no initial boundary, so one is dealing with the empty set $(\Sigma_i, \gamma_i, \phi_i) = \emptyset$, not a single point, and not some mythical “zero three-geometry”.

3.2 The WKB and steepest-descent approximations

In papers such as Halliwell (1991), Halliwell and Hartle (1990), Gibbons and Hartle (1990), the ‘Euclidean’ creation ex nihilo proposal, (the Hartle-Hawking Ansatz), developed into the following ‘sum-over-histories’:\footnote{We shall use square brackets hereafter to enclose arguments which are functions or fields on manifolds.}

$$[\Sigma_i, \gamma_i, \phi_i] = \emptyset,$$
\[ \Psi_0[\Sigma, \gamma, \phi] = \sum_{\mathcal{M}} \nu(\mathcal{M}) \int e^{-A_E[\mathcal{M}, g, \Phi]/\hbar} \, d\mu[g, \Phi]. \]

The sum here is understood to be over 4-manifolds \( \mathcal{M} \) which are bounded by \( \Sigma \). \( \nu(\mathcal{M}) \) is a weight that one assigns to each such 4-manifold. For each 4-manifold \( \mathcal{M} \) bounded by \( \Sigma \), the integral is over compact Riemannian 4-geometries and matter field histories \((g, \Phi)\) on \( \mathcal{M} \), which induce \((\gamma, \phi)\) on \( \Sigma \). By summing over 4-manifolds, it is tacitly assumed that there are only a countable number of 4-manifolds bounded by \( \Sigma \). As usual, \( \Sigma \) is only considered to be a compact 3-manifold.

The approach taken in papers such as those listed above is to reject the assumption that the ‘sum-over-histories’ should be taken over all the histories \((\mathcal{M}, g, \Phi)\) which are bounded by \((\Sigma, \gamma, \phi)\). Instead, the manifolds that one should sum over are to be a matter of debate; the weight of each such 4-manifold is to be determined; one restricts the domain of integration to a ‘contour’ of integration, and the particular contour chosen is to be a matter of debate. The restriction of the domain of integration is intended to find a way of making the path-integral convergent, and the measure upon the domain of integration is also considered to be a matter of debate. Only some combinations of these choices, it is argued, will lead to a convergent ‘sum-over-histories’. Moreover, different combinations of these choices will lead to different wave-functions.

In practice, the ‘Euclidean’ creation \textit{ex nihilo} proposal for the wave-function, has only been applied to mini-superspace models, and even then, the ‘Euclidean’ path-integrals have not been calculated. Instead, the so-called ‘steepest-descent’ approximation to the path-integral has been used to obtain a WKB approximation to the wave-function, or a sum of such WKB wave-functions.

The phase of a WKB wave-function approximately satisfies the classical Hamilton-Jacobi equation, hence the WKB approximation is often referred to as the ‘semi-classical’ approximation. In quantum cosmology, however, this can confuse matters because, as we shall see, there is another, more specific sense in which the term ‘semi-classical’ is used.

The difference between ‘oscillatory’ and ‘exponential’ WKB wave-functions has interpretational significance in quantum cosmology, hence a digression to explain the difference is worthwhile. If we abstract momentarily from the context of quantum cosmology, the WKB approximation is used to obtain an approximate wave-function \( \Psi(x) \) under the conditions where the de Broglie wavelength function \( \lambda(x) \) does not change significantly over the distance of one wavelength. For a system of energy \( E \), with a potential \( V(x) \),

\[ \lambda(x) = \frac{2\pi\hbar}{k(x)} = \frac{2\pi\hbar}{\sqrt{2m[E - V(x)]}}. \]

Given that \( \lambda(x) \) only changes by virtue of a change in the potential \( V(x) \), the WKB approximation is valid wherever the potential does not change significantly over the distance of one wavelength.
The domain of a WKB wave-function can be divided into a classically-permitted region, where \( E > V(x) \), and a classically-forbidden region, where \( E < V(x) \). For the first order WKB approximation, in the classically permitted region the wave-function will be ‘oscillatory’:

\[
\Psi(x) = \frac{a_1}{\sqrt{k(x)}} \exp \left( \frac{i}{\hbar} \int_{x_0}^{x} k(x')dx' \right) + \frac{a_2}{\sqrt{|k(x)|}} \exp \left( -\frac{1}{\hbar} \int_{x_0}^{x} |k(x')|dx' \right).
\]

In the classically forbidden region the wave-function will be exponential:

\[
\Psi(x) = \frac{a_1}{\sqrt{|k(x)|}} \exp \left( \frac{1}{\hbar} \int_{x_0}^{x} |k(x')|dx' \right) + \frac{a_2}{\sqrt{|k(x)|}} \exp \left( -\frac{1}{\hbar} \int_{x_0}^{x} |k(x')|dx' \right).
\]

In the classically forbidden region, \( k(x) = \sqrt{2m|E - V(x)|} \) is an imaginary number, hence the difference in the expressions. Given that \( \lambda(x) = (2\pi\hbar/k(x)) \), the de Broglie wavelength will also be imaginary in the forbidden region.

The classical turning points are the regions in which \( |E - V(x)| \) is small. In these regions, \( k(x) \) becomes very small and \( \lambda(x) \) becomes very large. Hence, in the regions near the classical turning points, the change in \( \lambda(x) \) can be significant over the distance of a wavelength. i.e. \( V(x) \) can vary significantly over the distance of a wavelength. Hence, the WKB approximation is not valid in the regions near the classical turning points.

The WKB approximation is often defined to be valid for a wave-function \( \Psi(x) = C(x)e^{iS(x)} \) wherever the phase \( S(x) \) is rapidly varying relative to the modulus \( C(x) \). At first sight, this might seem to suggest that the WKB approximation is invalid in the classically forbidden, exponential regions, where the wave-function is real-valued, and therefore of constant (zero) phase but varying modulus. However, even though \( \Psi(x) \) is real-valued in the forbidden region, one can express the real-valued exponential \( \exp[(\pm i/\hbar) \int_{x_0}^{x} |k(x')|dx'] \) in the form

\[
\exp[\pm iS(x)] = \exp \left( \pm i/\hbar \int_{x_0}^{x} k(x')dx' \right),
\]

and it is this imaginary-valued phase \( S(x) = (\pm i/\hbar) \int_{x_0}^{x} k(x')dx' \) which is rapidly-varying. The phase-change per unit length in the oscillatory and exponential regions is simply \( k(x)/\hbar \), hence small values for \( \lambda(x) \) mean large values for \( k(x) \), and therefore a rapidly-varying phase. The only difference in the forbidden region is that \( k(x) \) is imaginary.

A WKB wave-function is defined analogously in quantum cosmology. The regions of the configuration space in which the wave-function can be given a WKB approximation are those regions in which the phase is rapidly varying with respect to the modulus, entailing that the phase \( \text{approximately} \) satisfies the classical time-independent Hamilton-Jacobi equation of canonical general relativity, (Isham 1992b, p79).

Now, the steepest-descent approximation to a path-integral in quantum theory is the proposition that in some regions of configuration space there is no
need to calculate the entire path-integral. Instead, one need only consider the actions of paths which are classical solutions. In terms of quantum cosmology, if a spatial configuration \((\Sigma, \gamma, \phi)\) lies in a region where the steepest-descent approximation to the path-integral is valid, then one need not consider all 4-dimensional Riemannian histories bounded by \((\Sigma, \gamma, \phi)\). Instead, one need only consider those Riemannian 4-geometries which are saddle points of the action.

A solution of the classical Einstein field equations is a stationary point of the action, and a saddle point is a special kind of stationary point,\(^7\) hence a saddle point is a special type of classical solution. Of all the Riemannian 4-geometries bounded by \((\Sigma, \gamma, \phi)\), the claim is that one need only consider those which are saddle point solutions of the classical Einstein field equations. It is assumed, or reasoned, that the contributions from other Riemannian 4-geometries are either negligible, or cancel out.

The simplest version of the steepest-descent approximation proposes that to each \((\Sigma, \gamma, \phi)\), there is some Riemannian 4-geometry \((\mathcal{M}, g, \Phi)\), with boundary \((\Sigma, \gamma, \phi)\), which is a dominant saddle point of the action functional \(A_E\). Where this approximation is valid, the wave-function then has the form\(^8\)

\[
\Psi_0 \sim \nu(\mathcal{M}) \Delta_{WKB}[\Sigma, \gamma, \phi; \mathcal{M}, g, \Phi] e^{-A_E[\Sigma, \gamma, \phi; \mathcal{M}, g, \Phi]}. \]

Notice the presence of the so-called WKB pre-factor \(\Delta_{WKB}\).

A less simple version of the approximation proposes that amongst the 4-geometries with boundary \((\Sigma, \gamma, \phi)\), there can be multiple saddle points of the action, \(\{(\mathcal{M}_i, g_i, \Phi_i) : i = 1, 2, \ldots\}\), with no single dominant contribution. The wave-function then has the form

\[
\Psi_0 \sim \sum_i \nu(\mathcal{M}_i) \Delta_{WKB}[\Sigma, \gamma, \phi; \mathcal{M}_i, g_i, \Phi_i] e^{-A_E[\Sigma, \gamma, \phi; \mathcal{M}_i, g_i, \Phi_i]}. \]

The proponents of the steepest-descent approximation in quantum cosmology claim that dominant contributions to the path-integral come from complex 4-geometries which are saddle points of the action. Halliwell states that “one generally finds that the dominating saddle-points are four-metrics that are not real Euclidean, or real Lorentzian, but complex, with complex action,” (Halliwell 1991, p185). He also asserts that “it appears to be most commonly the case for generic boundary data that no real Euclidean solution exists, and the only solutions are complex,” (Halliwell 1991, p184).

Halliwell and Hartle state that “a semi-classical approximation to \(\Psi_0\) . . . arises when, in the steepest descent approximation to the functional integral, the dominating saddle points are complex,” (1990, p1817). This is the second sense in which the wave-function can be ‘semi-classical’. Used in this context, the term is not meant to be synonymous with the WKB approximation, but to indicate that there is a sense in which classical space-times can be recovered from the wave-function, as we shall see below.

\(^7\)A saddle point is a stationary point which is not an extremum.

\(^8\)\(\hbar\) is omitted hereafter to avoid unnecessary clutter.
Halliwell points out that the ‘Euclidean’ gravitational action \( A_E \) of real Riemannian (‘Euclidean’) 4-geometries is unbounded from below. “This means that the path-integral will not converge if one integrates over real Euclidean metrics,” he asserts. “Convergence is achieved only by integrating along a complex contour in the space of complex four-metrics,” (Halliwell 1991, p 172). For a steepest-descent approximation to a path-integral to be valid, there must be a contour of integration which passes through the relevant stationary points, and whose contributions decline rapidly away from those stationary points, i.e. there must be a steepest-descent contour, (Butterfield and Isham 1999, p55).

The practitioners of quantum cosmology claim that complex geometries are vital to recovering the notion of classical Lorentzian space-time. To reiterate, in those regions of configuration space where the wave-function can be given a WKB approximation, the wave-function can be either exponential \( C \exp(-S) \), or oscillatory \( C \exp(iS) \), (Halliwell 1991, p181). It is in the regions where the wave-function is oscillatory that the notion of a classical Lorentzian space-time can be recovered. Consider again the steepest-descent approximation for a single dominant saddle point:

\[
\Psi_0 \sim \nu(M) \Delta_{WKB}[\Sigma, \gamma, \phi; M, g, \Phi] e^{-A_E[\Sigma, \gamma, \phi; M, g, \Phi]}.
\]

If \( A_E \) is real, then the wave-function is clearly exponential \( C \exp(-S) \). If, however, \( A_E \) is complex, then the wave-function will be oscillatory. If \( A_E \) is a complex number, as it is for a complex saddle point, then \( A_E = \text{Re}(A_E) + i \text{Im}(A_E) \), and one can factorize \( \exp(-A_E) \) as follows:

\[
e^{-A_E} = e^{-\text{Re}(A_E)} e^{-i \text{Im}(A_E)}.
\]

In this event the wave-function \( \Psi_0[\Sigma, \gamma, \phi] \) can be written as

\[
\Psi_0 \sim \nu(M) \Delta_{WKB}[\Sigma, \gamma, \phi; M, g, \Phi] e^{-\text{Re}(A_E[\Sigma, \gamma, \phi; M, g, \Phi])} e^{-i \text{Im}(A_E[\Sigma, \gamma, \phi; M, g, \Phi])}.
\]

This wave-function clearly has the oscillatory form \( \Psi_0[\Sigma, \gamma, \phi] = C[\Sigma, \gamma, \phi] e^{iS[\Sigma, \gamma, \phi]} \). The modulus \( C[\Sigma, \gamma, \phi] \) is given by

\[
C[\Sigma, \gamma, \phi] = \nu(M) \Delta_{WKB}[\Sigma, \gamma, \phi; M, g, \Phi] e^{-\text{Re}(A_E[\Sigma, \gamma, \phi; M, g, \Phi])}.
\]

In this case, the phase of the wave-function is determined by the imaginary part \(-i \text{Im}(A_E[\Sigma, \gamma, \phi; M, g, \Phi])\) of the action, and the real part of the action contributes the factor

\[
e^{-\text{Re}(A_E[\Sigma, \gamma, \phi; M, g, \Phi])}
\]

to the modulus. Being exponential, this factor can dominate the modulus, hence the square modulus \( \exp(-2 \text{Re}(A_E)) \) is often taken to provide a probability distribution. The smaller the real part of the action, the greater the contribution.
According to Halliwell, an oscillatory WKB wave-function is peaked about a set of Lorentzian solutions of the classical equations. The ‘classical trajectories’ are defined to be the integral curves of the vector field $\nabla S$, the gradient of the phase of the wave-function. The wave-function is claimed to be peaked, not about a single classical solution, but about a set of classical solutions. The integral curves of $\nabla S$ constitute a congruence of the subset of the configuration space in which the wave-function can be approximated by an oscillatory WKB wave-function.

Halliwell claims that the squared-modulus $|C|^2$ is constant along each classical trajectory, and therefore provides a probability measure on the classical trajectories.

3.3 Signature change space-times and intrinsic time

To illustrate the emergence of classical Lorentzian paths in those regions of the configuration space in which an oscillatory WKB wave-function is valid, let us consider the case of signature change space-times. Those signature-change space-times $\mathcal{M}$ relevant to the Hartle-Hawking Ansatz consist of a 4-dimensional region of compact Riemannian geometry $\mathcal{M}_R$, in which there is no time, an adjoining 4-dimensional region of Lorentzian space-time $\mathcal{M}_L$, and a compact 3-dimensional signature-changing hypersurface $\Sigma$, which separates the two regions, so that (Gibbons and Hartle 1990, p2460)

$$\mathcal{M} = \mathcal{M}_L \cup \mathcal{M}_R$$

$$\partial \mathcal{M}_L = \Sigma = \partial \mathcal{M}_R.$$

One considers a Riemannian metric on $\mathcal{M}_R$ and a Lorentzian metric on $\mathcal{M}_L$, which are such that they induce the same spacelike geometry on the 3-manifold $\Sigma$.

Although such a signature-change space-time is not a complex 4-geometry, it does define a complex action, with the action of the Riemannian region providing the real component, and the action of the Lorentzian region providing the imaginary component, (Gibbons and Hartle 1990, p2460).

Suppose that one has a signature-change space-time which is a saddle point of the complex action, and suppose that the Lorentzian region can be foliated by a one-parameter family of spacelike hypersurfaces. Each space-like slice $(\Sigma, \gamma, \phi)$ can be treated as the boundary of a signature-change space-time if one removes the Lorentzian region to the future of that slice. The signature-change space-time consists of the prior region of Lorentzian space-time, and the entire region of Riemannian space. Assuming that the complex action $A_E$ of this truncated signature-change space-time provides the dominant saddle-point contribution to the wave-function value $\Psi_0[\Sigma, \gamma, \phi]$ in the steepest-descent approximation to the Hartle-Hawking path-integral, one can set $\Psi_0 \sim e^{-A_E}$. The actions of the 4-geometries bounded by the slices $(\Sigma, \gamma, \phi)$ in the Lorentzian region differ only by the value of their imaginary component. The real component is determined by the action of the Riemannian region, and this is common to all the slices.
in the Lorentzian region, hence the real component does not vary. The real component determines the modulus $C[\Sigma, \gamma, \phi]$ of the wave-function, hence the modulus of the wave-function does not vary for the slices in the Lorentzian region. The imaginary component determines the phase of the wave-function, hence the phase of the wave-function varies amongst the slices in the Lorentzian region. The gradient of the phase, $\nabla S$, therefore gives the classical Lorentzian paths in that part of the configuration space in which the wave-function is determined by dominant signature-change saddle points, part of the region in which the wave-function is said to be oscillatory.

In those regions of the configuration space where the wave-function is exponential, one does not have a congruence of classical ‘Euclidean’ paths, (Halliwell 1991, p182). Suppose that one has a real 4-dimensional compact Riemannian geometry, which is a saddle-point solution of the classical field equations. Suppose that this geometry can be foliated by a one-parameter family of hypersurfaces which are, of necessity, spacelike themselves, and suppose that one chooses an ordering for these slices. Each slice $(\Sigma, \gamma, \phi)$ can be treated as the boundary of a compact Riemannian 4-geometry if one removes the region which is ordered to be ‘greater than’ $(\Sigma, \gamma, \phi)$. Moreover, such an operation renders $(\Sigma, \gamma, \phi)$ as the boundary of a saddle-point solution of the classical equations. Assuming the steepest-descent approximation to the Hartle-Hawking path-integral gives the wave-function value $\Psi_0[\Sigma, \gamma, \phi]$, and assuming that the action $A_E$ of this truncated Riemannian region provides the dominant saddle-point contribution, one can set $\Psi_0 \sim e^{-A_E}$. For successive slices $(\Sigma, \gamma, \phi)$ in the Riemannian 4-geometry, the size of the truncated region grows, and the real-valued action and wave-function vary accordingly. Hence, the path in configuration space consisting of successive slices of the Riemannian 4-geometry is assigned complex numbers of constant (zero) phase and varying modulus. The integral curves of $\nabla S$ do not correspond to these paths, and the wave-function is not peaked over such paths in configuration space.

This nice clean distinction between regions of the configuration space breaks down if one considers configurations which can be embedded in both a foliation of the Lorentzian region in a signature-change saddle-point space-time, and in a foliation of a saddle-point Riemannian 4-geometry.

It is often said that classical space-time must be a prediction of quantum cosmology in the ‘late’ universe. This means that the oscillatory WKB approximation must be valid in the part of the configuration space which contains the large 3-geometries (i.e. large scale-factors), because large geometries correspond to the universe that we presently inhabit. Not only is the concept of Lorentzian space-time lost in those regions of the configuration space in which the oscillatory WKB approximation is invalid, but the notion of time itself seems to be lost in those regions.

In quantum mechanics in general, wherever the oscillatory WKB approximation is valid, one can interpret the wave-function to describe a classical statistical ensemble. In this sense, wherever the wave-function of quantum gravity is
given by an oscillatory WKB approximation, one does indeed recover a family of Lorentzian space-times. However, in the particular case of quantum cosmology, this poses an interpretational problem because it implies a statistical ensemble of universes. It appears that even where the wave-function is ‘semi-classical’, its interpretation requires one to accept that there are many universes, each of which follows one of the classical trajectories.

The recovery of classical space-times faces a problem when there is more than one stationary point in the steepest-descent approximation to the wave-function. The wave-function then has the form

$$\Psi_0[\Sigma, \gamma, \phi] \sim \sum_i C_i[\Sigma, \gamma, \phi] e^{iS_i[\Sigma, \gamma, \phi]}$$

As a consequence, there is an entire family of different congruences, each determined by $\nabla S_i$. One no longer has a unique family of classical trajectories. The interpretational difficulties are therefore magnified. Even if one postulates the existence of many universe, the wave-function in this case would seem to describe those universes to be in a state of quantum superposition.9

Kossowski and Kriele (1994a, p115 and 1994b, p297) suggest that the ‘Euclidean’ creation ex nihilo proposal reduces in the classical limit to a signature-changing space-time. They state that “our point of view is that the path-integral argument of Hartle and Hawking gives initial conditions for the classical Einstein equation at the [signature change hypersurface],” (Kossowski and Kriele, 1994a, p116). On the previous page, (p115), they assert that Hartle and Hawking are able to “calculate rather than assume the initial state for the Lorentzian part of the universe . . . they obtain an initial state at [the signature change hypersurface] by path integration over all Riemannian metrics (defined on the Riemannian region).” If the oscillatory region of the configuration space can be foliated by the congruence $\nabla S$ of classical Lorentzian space-times, and the modulus is constant along each such path, then the wave-function on the boundary between the oscillatory region and the exponential region would effectively determine a probability distribution across a family of initial configurations for classical Lorentzian space-times.

Let us suppose that we have fixed a 4-dimensional manifold $\mathcal{M} = \mathcal{M}_L \cup \mathcal{M}_R$ as above. Presumably, Kossowski and Kriele would wish to consider, for each possible pair $(\gamma, \phi)$ on $\Sigma$, a path-integral over all possible Riemannian 4-geometries and matter field histories $(g, \Phi)$ on $\mathcal{M}_R$ which induce $(\gamma, \phi)$ on $\Sigma$. According to the Hartle-Hawking proposal, this would yield a quantum state-function $\Psi_0[\gamma, \phi]$, whose domain is the set of all possible 3-metrics and matter fields on the hypersurface $\Sigma$. Kossowski and Kriele propose that this state function should be considered as “the initial state for the Lorentzian part

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9The reader should be aware that physicists are fond of something called the decoherent histories interpretation, which purportedly explains how this is consistent with our observation of an individual classical space-time. See Halliwell (1989).
of the universe.” Unfortunately, the quantum state-function $\Psi_0[\gamma, \phi]$ does not constitute initial conditions for the classical Einstein equation, and $|\Psi_0[\gamma, \phi]|^2$ does not even constitute a probability distribution over the initial conditions for the Einstein equation. Recall that initial conditions in a classical theory do not merely consist of a configuration, but also a rate of change of configuration. Initial conditions for the Einstein equation on a hypersurface $\Sigma$ consist of a 3-metric, the extrinsic curvature tensor or conjugate momentum tensor field, the matter fields, and the first order matter field time derivatives. At best, the Hartle-Hawking wave-function could provide quantum initial conditions, rather than the classical initial conditions suggested by Kossowski and Kriele. To reconcile this with the apparent time-independence of the wave-function, it is necessary to invoke the notion of ‘intrinsic’ time.

The idea here is that time can be found in the domain of the wave-function. A genuine configuration space in canonical quantum gravity is infinite-dimensional; there will be an infinite number of degrees of freedom. Intrinsic time advocates suggest that one can split the degrees of freedom into those which are ‘physical’, and those which are ‘non-physical’. The physical degrees of freedom are sufficient to pin down the configuration, whilst the non-physical are redundant degrees of freedom, which purportedly contain information about intrinsic time.

Isham (1988, p396) argues that since the degrees of freedom include an internal definition of time, it would be incorrect to add an external time label to the state function $\Psi$. Instead, the internal time is treated as a function $T[\Sigma, \gamma, \phi]$ of the configuration, and $\Psi[\Sigma, \gamma, \phi]$ gives the probability amplitude of the physical configuration $(\Sigma, \gamma, \phi)_{\text{phys}}$ at the internal time $T[\Sigma, \gamma, \phi]$.

One could presumably fix the physical degrees of freedom, but allow the internal time to vary; the probability amplitude of a physical configuration would vary with internal time. One could also, presumably, fix the value of the internal time, and consider all the possible physical configurations at that value of the internal time. The square-modulus of the wave-function would then provide a probability distribution over all the possible physical configurations at that internal time. By allowing the internal time to vary, one would have a varying probability distribution over the possible physical configurations. One could write the wave-function as

$$\Psi[\Sigma, \gamma, \phi] = \Psi[(\Sigma, \gamma, \phi)_{\text{phys}}, T] = \Psi_T[\Sigma, \gamma, \phi]_{\text{phys}}.$$ 

There are supposedly many different choices of internal time. The Wheeler-DeWitt equation purportedly governs the time-dependence of the wave-function for any choice of internal time. If one were to specify $\Psi_0[\Sigma, \gamma, \phi]_{\text{phys}}$ at some internal time $T = 0$, then the Wheeler-DeWitt equation would purportedly determine $\Psi_T[\Sigma, \gamma, \phi]_{\text{phys}}$ at any other value $T$ of internal time.

The notion of intrinsic time can also be applied to path-integral quantum gravity. One interprets the transition amplitude

$$K(\Sigma_i, \gamma_i, \phi_i; \Sigma_f, \gamma_f, \phi_f),$$
as the amplitude of a transition from the physical configuration \((\Sigma_i, \gamma_i, \phi_i)_{\text{phys}}\) at the internal time \(T[\Sigma_i, \gamma_i, \phi_i]\), to the physical configuration \((\Sigma_f, \gamma_f, \phi_f)_{\text{phys}}\) at the internal time \(T[\Sigma_f, \gamma_f, \phi_f]\).

The idea of intrinsic time is, however, difficult to implement in practice, and existing attempts use mini-superspace models. The concept of (internal) time may have a limited domain of validity.

G.F.R. Ellis asserts that the Hartle-Hawking Ansatz is “a scheme whereby the origin of the Universe is separated from the issue of the origin of time,” (Ellis 1995, p326). This is a dubious interpretation. Recall that part two of the Hartle-Hawking Ansatz is that the probability amplitude of \((\Sigma, \gamma, \phi)\) being created from nothing, is given by a path-integral over compact Riemannian 4-geometries which are bounded by \((\Sigma, \gamma, \phi)\). It could then be suggested that once a universe has been created \textit{ex nihilo}, it evolves as a Lorentzian space-time thereafter. This is a distinct proposal, and not one made by Hartle and Hawking, but let us consider it for the sake of argument. Even then, one need not accept Ellis’ interpretation that the origin of time is separate from the origin of the universe. One could suggest that the Riemannian 4-manifolds only have a part to play in the creation \textit{ex nihilo} calculations, not in any actual processes, hence there would be no actual signature change process. One would merely integrate over Riemannian 4-manifolds to find the creation \textit{ex nihilo} probabilities. There would be creation from nothing, and Lorentzian space-time thereafter, with no intermediate Riemannian geometry. In this case, the creation of a universe would coincide with the creation of time, contrary to Ellis’ suggestion.

Isham recognizes that one need not ascribe physical status to the Riemannian geometries used in the Hartle-Hawking definition of the wave-function \(\Psi_0\). He recognizes that a “‘phenomenological’ four-dimensional (Lorentzian) space-time that is reconstructed from the canonical state \(\Psi_0\) is not necessarily related, either metrically or topologically, to any four-dimensional manifold that happens to be used in the construction of the state. Indeed, if the concept of ‘time’ is only semi-classical, it is incorrect to talk at all about a four-dimensional manifold at the quantum level,” (Isham 1991b, p356).

It is the use of signature-change saddle points in the steepest-descent approximation to the wave-function \(\Psi_0[\Sigma, \gamma, \phi]\), rather than the use of the genuine ‘Euclidean’ path-integral, which has inspired some authors to draw a line between the creation of a universe, and the origin of time. Gibbons and Hartle (1990, p2459), for example, consider a signature change solution of the classical Einstein equation, to be a “tunneling solution.” They state that such “tunneling solutions describe the universe ‘tunneling from nothing’, and are the dominant contributors to the semiclassical approximations to the ‘no-boundary’ proposal,” (1990, p2460).

To describe a signature change solution of the classical Einstein equation as a tunnelling solution, brings tunnelling down to the level of classical theory, when it should be exclusively a quantum phenomenon. Signature change should not, in itself, be considered as an occurrence of tunnelling.

Another difficulty with the introduction of signature change space-times, is that the interpretation of the probability amplitude \(\Psi_0[\Sigma, \gamma, \phi]\) assigned to
a triple \((\Sigma, \gamma, \phi)\) becomes ambiguous. Is \(\Psi_0[\Sigma, \gamma, \phi]\) the probability amplitude that \((\Sigma, \gamma, \phi)\) be created from nothing, or is it the probability amplitude of \((\Sigma, \gamma, \phi)\) being the initial configuration of the Lorentzian region of the universe? Could it even be both? Prima facie, the creation of \((\Sigma, \gamma, \phi)\) from nothing would seem to require a direct transition from \(\emptyset\) to \((\Sigma, \gamma, \phi)\). When \((\Sigma, \gamma, \phi)\) is the boundary of a 4-dimensional Riemannian region, it is difficult to interpret it to have been created from nothing. Only if one interprets the Riemannian 4-geometries bounded by \((\Sigma, \gamma, \phi)\) as calculational fictions, could one maintain the creation \textit{ex nihilo} interpretation.

One could argue that an individual signature-changing space-time should be construed merely as a classical version of the quantum tunnelling of quantum cosmology. But this then contradicts the idea that the Riemannian region is classically forbidden, as seen in the Hartle-Hawking de Sitter mini-superspace model that we will encounter in Section 4. Is signature change part of quantization, or is it a preparation for quantization?

If the wave-function of the universe is interpreted epistemologically, so that it is thought to provide merely an incomplete description, then one can interpret the wave-function to provide a statistical description of an ensemble of universes. If one interprets the probabilities of the wave-function \(\Psi_0\) epistemologically, then one could conceivably assert that individual signature change space-times exist in the statistical ensemble. If, however, one interprets the wave-function and its probabilities ontologically, then what actually exists would be the wave-function \(\Psi_0\). Individual signature-change space-times would not exist.

4 Mini-superspace quantum cosmology

In an effort to make the process of finding solutions to the Wheeler-DeWitt more tractable, ‘mini-superspace’ models were employed in quantum cosmology. Such models fix all but a finite number of degrees of freedom before quantization. The intention in this section is to review, clarify, and critically analyse mini-superspace quantum cosmology. In particular, the claim that Vilenkin’s ‘tunnelling boundary condition’ provides the probability of creating a universe from nothing, will be subjected to critical scrutiny.

To reiterate, in canonical general relativity, expressed in terms of the ‘traditional’ variables, the set of all possible geometrical configurations of the spatial universe corresponds to the set of all 3-dimensional Riemannian manifolds \((\Sigma, \gamma)\). This set of all 3-dimensional Riemannian geometries is a disconnected topological space. Each component of the disconnected space corresponds to the set \(\mathcal{C}(\Sigma)\) of all Riemannian metric tensors upon a fixed 3-manifold \(\Sigma\).

Although \(\mathcal{C}(\Sigma)\) is referred to as a configuration space, there exist distinct elements of \(\mathcal{C}(\Sigma)\) which are isometric. This can be understood by the action of \(Diff(\Sigma)\), the diffeomorphism group of \(\Sigma\), upon the space of metrics \(\mathcal{C}(\Sigma)\). A diffeomorphism \(\phi: \Sigma \rightarrow \Sigma\) maps a metric \(h \in \mathcal{C}(\Sigma)\) to another metric \(h'\) by pullback, \(h' = \phi^* h\). That is, \(h'_p(v, w) = h_{\phi(p)}(\phi_*(v), \phi_*(w))\) at each point \(p \in \Sigma\), and for each pair of vectors \(v, w \in T_p \Sigma\).
The orbits of the action of $Diff(\Sigma)$ are the isometry equivalence classes of Riemannian metric tensor fields on $\Sigma$. Hence, one considers the quotient $S(\Sigma) = \mathcal{C}(\Sigma)/Diff(\Sigma)$ to be the set of all possible intrinsic Riemannian geometries of $\Sigma$. $S(\Sigma)$ is known as the superspace of the 3-manifold $\Sigma$.

Whilst a wave-function $\Psi$ on the configuration space $\mathcal{C}(\Sigma)$ must satisfy both the Wheeler-DeWitt equation, and then additional constraint equations to ensure it is invariant under diffeomorphisms of $\Sigma$, a wave-function on superspace $S(\Sigma)$ need merely satisfy the Wheeler-DeWitt equation.

In the absence of matter, a true wave-function $\Psi[\gamma]$ of the universe, in the traditional variables configuration representation, would be a complex-valued functional upon the entire disconnected space of possible 3-geometries. However, in most of the existing literature on quantum cosmology, it is conventional to tacitly restrict the topological degrees of freedom; in particular, a compact and orientable 3-manifold $\Sigma$ is fixed from the outset. Typically, the three-sphere $S^3$ is chosen.

Even by fixing the topological degrees of freedom, however, the set of all Riemannian 3-geometries and matter field configurations on $\Sigma$ is still infinite-dimensional. Thus, even by omitting the 3-topology as an argument of the wave-function, the latter will still be a function $\Psi[\gamma, \phi]$ on an infinite-dimensional manifold. In general, it is difficult to solve differential equations on an infinite-dimensional manifold, hence it is very difficult to find any solutions of the Wheeler-DeWitt equation, and even more difficult to select a unique solution which satisfies some ‘boundary’ conditions.

Thus, in an effort to make things more tractable, mini-superspace models were employed in quantum cosmology. In these models, symmetries such as homogeneity and isotropy were imposed, and all but a finite number of degrees of freedom were frozen before quantization. Hence, the superspace of such models is a finite-dimensional submanifold of the full superspace, and efforts can be made to find solutions of the Wheeler-DeWitt equation restricted to such finite-dimensional domains.\(^{10}\)

To demonstrate the mini-superspace technique in the traditional variables, we shall begin by considering a well-known model with only one degree of freedom, (Kolb and Turner 1990, p458-464). This particular model will also enable us to discuss one of the ‘creation from nothing’ claims made for mini-superspace quantum cosmology.

We begin by selecting the 3-manifold to be $S^3$, and we only consider metric tensors of the form

$$ds^2 = R^2(d\chi^2 + \sin^2 \chi (d\theta^2 + \sin^2 \theta d\phi^2)).$$

Each metric tensor of this type equips $S^3$ with a homogeneous and isotropic Riemannian geometry. The scale factor $R \in [0, \infty)$ is the only permitted degree of freedom in the spatial geometry. In this particular model, it is also the only

\(^{10}\)In Bojowald’s recent application of loop quantum gravity to quantum cosmology, he quantizes the kinematics of the full theory, and then seeks quantum states which correspond, in some sense, to homogeneous and isotropic space. See Ashtekar (2002).
degree of freedom, geometrical or non-geometrical. The matter field is chosen to be a massive scalar field, fixed at some constant value $\phi$; the value of the field is the same at each point of $S^3$. The selection of a particular massive scalar field includes the selection of a potential energy function $V(\phi)$. Hence, by fixing a particular value $\phi$, one fixes a particular energy density $\rho_{\phi}$. It will be helpful in what follows to define a cosmological constant $\Lambda = 8\pi G \rho_{\phi}$ from the energy density of the scalar field.

With the mini-superspace now defined, it is clear that a wave-function will simply be a function $\Psi(R)$ of the possible values for the scale factor. In general terms, the Wheeler-DeWitt equation will have the form $(\nabla^2 - U(R))\Psi(R) = 0$. With a factor-ordering ambiguity $a$, the Wheeler-DeWitt operator has the form:

$$(R^{-a} \frac{\partial}{\partial R} R^a \frac{\partial}{\partial R} - U(R)).$$

With the potential $U(R)$ defined to be

$$U(R) = \frac{9\pi^2}{4G^2}(R^2 - \frac{\Lambda}{3}R^4),$$

and with $a$ set to $a = 0$, the Wheeler-DeWitt equation takes the form, (Kolb and Turner, p459):

$$\left[ \frac{\partial^2}{\partial R^2} - \frac{9\pi^2}{4G^2}(R^2 - \frac{\Lambda}{3}R^4) \right] \Psi(R) = 0.$$

This equation clearly resembles the time-independent Schrödinger equation $H\Psi = E\Psi$ for a system constrained to move in $[0, \infty)$, with a fixed total energy $E = 0$, and subject to the potential $U(R)$.

This mini-superspace model has a profound relationship with de Sitter space-time, a solution of the classical equations. To see this, one introduces $R_0 = (\Lambda/3)^{-1/2} = (8\pi G \rho_{\phi}/3)^{-1/2}$. One can then split the configuration space $[0, \infty)$ into $0 < R \leq R_0$ and $R \geq R_0$. The potential $U(R)$ is positive in the region $0 < R < R_0$, hence with the total energy fixed at $E = 0$, this region is classically forbidden. $R_0$ is the classical turning point, at which the potential is zero. Hence, at $R = R_0$, the kinetic energy of a classical system would have to be zero. In the region $R > R_0$, the potential is negative, so $R \geq R_0$ is a classically permitted region of the configuration space for the $E = 0$ system. Intriguingly, the potential $U(R)$ is zero at $R = 0$, hence $R = 0$ is also a classically permitted configuration. A classical system at $R = 0$ would have zero kinetic energy and zero potential energy, and would remain at $R = 0$.

To understand the link between this mini-superspace model and de Sitter space-time, recall that de Sitter space-time is $\mathbb{R}^1 \times S^3$ equipped with the metric

$$ds^2 = -dt^2 + R^2(t)(d\Omega_3^2),$$

where $R(t) = R_0 \cosh(R_0^{-1}t)$, and $d\Omega_3^2$ is the standard metric on the 3-sphere.
De Sitter space-time can be treated as a solution of the Einstein Field equations with a cosmological constant $\Lambda = 8\pi G \rho_{\text{vac}}$. The vacuum energy density $\rho_{\text{vac}}$ corresponds to the energy density $\rho_\phi$ of the scalar field in the mini-superspace model.

If one foliates de Sitter space-time by the one-parameter family of homogeneous $t =$ constant spacelike hypersurfaces, then the resulting family of spatial configurations corresponds to a curve in the one-dimensional configuration space $[0, \infty)$ under consideration. One has a classical universe which contracts from the infinite past to a minimum scale factor of $R_0 = (\Lambda/3)^{-1/2}$, and then expands without limit into the infinite future. Thus, the region $[0, R_0)$ of the configuration space is not entered by the classical de Sitter space-time. In the mini-superspace model, this corresponds to the fact that $0 < R < R_0$ is a classically forbidden region. The classical turning point $R_0$ of the mini-superspace model corresponds to the minimum radius of de Sitter space-time. The scale factor of de Sitter space-time only occupies the classically permitted region $R \geq R_0$ of the configuration space.

The transition to quantum theory involves the serious consideration of all kinematically possible paths through a configuration space. De Sitter space-time provides a path in configuration space which is dynamically possible according to the classical theory. By considering all kinematically possible paths, the classically forbidden region $0 < R < R_0$ becomes traversable. There are kinematically possible paths which do enter $0 < R < R_0$. The most startling consequence of this is that a quantum system which begins at $R = 0$, can tunnel through the potential barrier, and reach $R > R_0$. Some quantum cosmologists interpreted this as a prototypical model for the creation of the universe ex nihilo.

To actually calculate the probability of a system tunnelling from $R = 0$ to $R > 0$, it is of course necessary to provide a solution $\Psi(R)$ for the Wheeler-DeWitt equation of this one-dimensional mini-superspace model. Vilenkin, Linde and Hartle-Hawking make competing proposals for this wave-function, (Vilenkin 1998).

For the $R > R_0$ region, the WKB solutions of the Wheeler-DeWitt equation are

$$\Psi_{\pm}(R > R_0) \propto \frac{1}{\sqrt{|k(R)|}} \exp \left( \pm i \int_{R_0}^{R} k(R')dR' \mp i \frac{\pi}{4} \right),$$

where $k(R) = \sqrt{(-U(R))}$ for the $E = 0$ mini-superspace model under consideration.

For the $R < R_0$ region, the WKB solutions are

$$\Psi_{\pm}(R < R_0) \propto \frac{1}{\sqrt{|k(R)|}} \exp \left( \pm \int_{R}^{R_0} |k(R')|dR' \right).$$

Vilenkin claims that the ‘ingoing’ wave $\Psi_{+}(R > R_0)$ corresponds to a contracting universe, and that it is the ‘outgoing’ wave $\Psi_{-}(R > R_0)$, satisfying the condition $i\Psi^{-1}\partial\Psi/\partial R > 0$, which corresponds to an expanding universe. He
claims that the wave-function should be $\Psi_-(R > R_0)$ in the classically permitted region, and a combination of ingoing and outgoing modes in the classically forbidden region. From this, he calculates that the probability for tunnelling through the potential barrier from $R = 0$ should be $\sim \exp(-|A_E|)$. Vilenkin, however, also makes the dubious assertion that this provides the probability of creating an expanding universe from ‘nothing’. Vilenkin equates $R = 0$ with nothing in this context.

Linde proposes that the wave-function in the classically allowed region should be a combination of incoming and outgoing modes, $\frac{1}{2} [\Psi_+(R > R_0) + \Psi_-(R > R_0)]$, and should be $\Psi_+(R < R_0)$ in the classically forbidden region. The Hartle-Hawking proposal is also that the wave-function in the classically allowed region should be a combination of incoming and outgoing modes, $\Psi_+(R > R_0) - \Psi_-(R > R_0)$, and should be $\Psi_-(R < R_0)$ in the classically forbidden region. Hartle-Hawking calculate the probability of a transition from $R = 0$ to $R = R_0$ by means of a signature change scenario. They take the Riemannian four-sphere $S^4$, and they remove one hemisphere, joining the equator to Lorentzian de Sitter space-time at its minimum radius $R_0$. They take the Euclidean action $A_E$ of the compact Riemannian region, and they assert that the probability of a transition from $R = 0$ to $R = R_0$ is $\sim \exp(-A_E)$.

The four-sphere region, as a Riemannian solution of the classical vacuum field equations,

$$R_{\mu\nu} = \Lambda g_{\mu\nu},$$

a so-called ‘gravitational instanton’, is automatically geodesically complete. As pointed out in Section 3, this is a generic outcome of the Euclidean approach, and motivates the suggestion that an initial singularity can be avoided in Euclidean quantum cosmology. Given, however, that mini-superspace quantum cosmology replaces an individual space-time manifold with a wave-function, the issue of a singularity in the geometry has been by-passed; a geometric object has been replaced by an object from a function space. The only sense in which the question of geometric singularities might re-emerge is with respect to the classical paths in configuration space which can, under certain conditions, be derived from the wave-function.

The approaches of Vilenkin, Linde and Hartle-Hawking are each vulnerable to the following objection: In quantum mechanics, there is a probabilistic propensity for a system to make a transition from one side of a potential barrier to the other. If a transition takes place, the common interpretation is that state reduction has taken place, and the probabilistic propensities are replaced by a definite position on the other side of the potential barrier. Given that the universe is a closed system, the concept of state vector reduction is not obviously applicable in quantum cosmology. Hence, unless one subscribes to an interpretation of quantum theory which rejects state vector reduction, the concept of tunnelling through a potential barrier cannot explain the creation of the universe. In general, therefore, quantum cosmologists have been forced to explore and apply various ‘no-collapse’ interpretations of quantum theory. It may be,
of course, that a no-collapse interpretation of quantum theory is the correct one to take, but it is important to emphasise the dependence of quantum cosmology explanations upon non-standard interpretations of quantum theory.

Vilenkin’s attempt to equate ‘nothing’ with $R = 0$ also seems completely incorrect. If one considers $S^3$ with a metric

$$ds^2 = R^2(d\chi^2 + \sin^2 \chi(d\theta^2 + \sin^2 \theta d\phi^2)),$$

then setting $R = 0$ simply removes the geometry from $S^3$. One still has the 3-manifold $S^3$, a 3-dimensional space without any geometry. Thus, Vilenkin’s theory might conceivably show that the universe was created from a 3-dimensional space, bereft of geometry and matter, but it cannot show that the universe was created from nothing. Moreover, in this one-dimensional mini-superspace model, the scalar field is fixed at a constant value $\phi$ on $S^3$. Hence, when $R = 0$, the scalar field still presumably exists on $S^3$. One has a 3-manifold $S^3$, equipped with a matter field; this is far from being nothing.

Prugovecki associates Vilenkin’s scenario with Tryon’s idea for the creation of the universe as a fluctuation of ‘nothing’. Prugovecki argues that “the concept of a wave function, representing a quantum particle, ‘tunneling through’ the potential barrier to which another system of existing quantum particles gives rise, is operationally well-defined, and it makes physical sense; however, what is the possible physical meaning of Nothing tunneling through a potential barrier produced by Nothing, in order to ‘create’ our Universe?” (Prugovecki 1992, p454).

Prugovecki (1992, p481, note 33) asks, with justification, “what is it that is supposedly ‘tunneling’, and through a barrier of what does that purported ‘tunneling’ take place in the Tryon-Vilenkin ‘scenario’?” .

### 4.1 Vilenkin’s tunnelling boundary condition

Vilenkin’s stipulation that $i\Psi^{-1}\partial\Psi/\partial R > 0$ for the one-dimensional mini-superspace model discussed above, is a special case of his ‘tunnelling boundary condition’ on the wave-function of the universe. To understand Vilenkin’s boundary condition in greater generality, we shall now review, clarify, and critically analyse a mini-superspace model with two degrees of freedom. This mini-superspace model is of particular interest because it has been used to address the question of whether quantum cosmology can predict the false vacuum necessary for inflation to take place. The notion that Vilenkin’s tunnelling boundary condition specifies the probability of creating a universe from nothing, will continue to receive attention, and in particular, Vilenkin’s notion of a boundary will be subjected to critical scrutiny.

We again select the 3-manifold $\Sigma$ to be $S^3$, and we again consider only the metrics on $S^3$ of the form

$$ds^2 = R^2(d\chi^2 + \sin^2 \chi(d\theta^2 + \sin^2 \theta d\phi^2)).$$
The scale factor \( R \in [0, \infty) \) is therefore the only contemplated degree of freedom in the spatial geometry.

We select the matter field on \( S^3 \) to be a massive scalar field \( \phi \) of constant value across \( \Sigma \). The value of \( \phi \) is the degree of freedom in the matter field. The potential energy density \( V(\phi) \) is considered to be a function of \( \phi \). Different forms of the function \( V(\phi) \) yield different mini-superspace models. The spatial geometry is fixed to be homogeneous and isotropic, and the matter field is also clearly homogeneous.

A wave-function in this model therefore has the form \( \Psi(R, \phi) \), and the Wheeler-DeWitt equation will be, \((\text{Kolb and Turner, p463)}):

\[
R^{-a} \frac{\partial}{\partial R} R^a \frac{\partial}{\partial R} - \frac{1}{R^2} \frac{3}{4 \pi G} \frac{\partial^2}{\partial \phi^2} - U(R, \phi) \right] \Psi(R, \phi) = 0,
\]

where \( a \) is a factor ordering ambiguity, and where the ‘superpotential’ \( U(R, \phi) \) is given by

\[
U(R, \phi) = \frac{9 \pi^2}{4 G^2} \left( R^2 - R^4 \frac{8 \pi G}{3} V(\phi) \right).
\]

The Wheeler-DeWitt equation here clearly has the form \((\nabla^2 - U) \Psi = 0\), where \( \nabla^2 \) is a Laplacian. Once again the Wheeler-DeWitt equation resembles the time-independent Schrödinger equation \( H \Psi = E \Psi \) for a particle of fixed total energy \( E = 0 \), moving in a potential \( U \).

Vilenkin’s ‘tunnelling’ boundary condition, if it could be shown to be meaningful, would be a genuine boundary condition on the wave-function \( \Psi \). Vilenkin conceives that superspace has a boundary, and he attempts to identify a unique wave-function \( \Psi \) by its behaviour on the boundary of superspace. Thus, the boundary of Vilenkin’s boundary condition is a topological boundary of the domain of the wave-function. Attempts to practically implement Vilenkin’s boundary condition have been restricted to mini-superspace models, where the boundary is the boundary of the mini-superspace.

Vilenkin’s notion of the boundary of superspace is rather unclear. He asserts that “the boundary of superspace can be thought of as consisting of singular configurations which have some points or regions with infinite three-curvature or with infinite \( \phi \) or \((\partial \phi)^2\), as well as configurations of infinite three-volume,” (Vilenkin 1988, p889).\(^{11}\) This is a wholly inadequate definition of the boundary of superspace. Suppose that we have fixed a compact, orientable 3-manifold \( \Sigma \), and that we introduce the configuration space \( \mathcal{C}(\Sigma) \) of all Riemannian metric tensor fields on \( \Sigma \). Let \( T^*\Sigma \) denote the cotangent bundle of \( \Sigma \), and let \( \odot^2 T^*\Sigma \) denote the 2-fold symmetric tensor product of the cotangent bundle. \( \mathcal{C}(\Sigma) \) is an open positive cone in the set of smooth cross sections \( C^\infty(\odot^2 T^*\Sigma) \) of the vector bundle \( \odot^2 T^*\Sigma \). One can now ask: Should the boundary of the configuration space be the topological boundary of \( \mathcal{C}(\Sigma) \), considered as an open subset in the topological space \( C^\infty(\odot^2 T^*\Sigma) \)? Should one then take the quotient of this

\(^{11}\)Vilenkin only contemplates compact 3-topology, so infinite volume is considered to be a kind of pathology.
boundary with respect to the $Diff(\Sigma)$ action, to find the boundary of the superspace $\mathcal{C}(\Sigma)/Diff(\Sigma)$?

The boundary of an open subset $\mathcal{X}$ of a topological space $\mathcal{T}$, is the set of points in the closure of $\mathcal{X}$ which do not belong to $\mathcal{X}$. The closure of an open subset $\mathcal{X}$ is the union of $\mathcal{X}$ with the set of accumulation points of $\mathcal{X}$ which do not belong to $\mathcal{X}$. Thus, the boundary of an open subset $\mathcal{X}$ is the set of those accumulation points which do not belong to $\mathcal{X}$. An accumulation point $x$ of a subset $\mathcal{X}$ is a point for which every neighbourhood contains points of $\mathcal{X}$ other than $x$. For first-countable topological spaces, of which manifolds are particular cases, a point $x$ is an accumulation point of a subset $\mathcal{X}$ if and only if there is a sequence of points in $\mathcal{X} - x$ which converges to $x$.

The boundary of $\mathcal{C}(\Sigma)$ will be dependent upon the topological space that $\mathcal{C}(\Sigma)$ is considered to be an open subset of. Even if one fixes the set $\mathcal{T}$ that $\mathcal{C}(\Sigma)$ is considered to be a subset of, one can vary the topology of $\mathcal{T}$. The closure of $\mathcal{C}(\Sigma)$, and therefore its boundary, will be different in different topologies. In general, the coarser the topology of the set in which $\mathcal{C}(\Sigma)$ is considered to be a subspace, the larger the closure will be. Vilenkin discusses none of these questions. Moreover, no matter what the topology of $\mathcal{T}$, the accumulation points of $\mathcal{C}(\Sigma)$ in $\mathcal{T}$ will not correspond to tensor fields which diverge at points or regions of $\Sigma$. Infinity, $\infty$, is not an element in the field of real numbers, and we are dealing with a module $C^\infty(\mathbb{R}^2T^*\Sigma)$ of tensor fields over the ring of real-valued scalar fields on the manifold $\Sigma$.

However, to understand Vilenkin’s notion of a superspace boundary, one might be able to densely embed $\mathcal{C}(\Sigma)$ in an appropriate space constructed from sequences of points in $\mathcal{C}(\Sigma)$. Each $\gamma \in \mathcal{C}(\Sigma)$ could be mapped to an equivalence class of sequences which converge to $\gamma$. Sequences which tend towards infinite volume, infinite curvature, or infinite matter field values, for example, do not converge to points of $\mathcal{C}(\Sigma)$, but might form the boundary of $\mathcal{C}(\Sigma)$ in a suitable dense embedding.

For example, take an arbitrary point $\gamma \in \mathcal{C}(\Sigma)$ in the configuration space, and consider the sequence of points $S = \{P(N) = N^2\gamma : N \in \mathbb{Z}_+\}$. $\mathbb{Z}_+$ is the set of positive integers. The sequence has no limiting point in $\mathcal{C}(\Sigma)$ as $N \to \infty$. It is a sequence of homothetic 3-geometries in which the volume grows without limit. For any integer $M$ between 1 and $\infty$, one can get a subsequence of $S$ by taking only the members of $S$ up to $P(M)$. Each such sequence $S_M$ does converge to a point of $\mathcal{C}(\Sigma)$; it converges to $P(M)$. The sequence of $S_M$-sequences converges to the sequence $S$. That is, $\lim_{M \to \infty} S_M = S$. Thus, by this construction, one can treat $S$ as a boundary point of $\mathcal{C}(\Sigma)$.

When Vilenkin speaks of a configuration of infinite volume, one might interpret this to be a metaphorical way of referring to a sequence of configurations in which the volume increases without limit. It could only be metaphorical because the volume is finite in each member of the sequence. One might treat Vilenkin’s talk of infinite curvature and infinite matter field values similarly.

Vilenkin attempts to divide the boundary of superspace into a ‘singular’ boundary and a ‘regular’ boundary. He defines every regular boundary configuration to be a ‘critical’ slice of a topology-changing 4-geometry. The slicing, or
‘foliation’, is defined here by a Morse function \( f(x) \), and a critical slice contains critical points of the Morse function, points \( x_0 \) at which \( \partial_n f(x_0) = 0 \), (Vilenkin 1994, p2588-2589). The 3-geometry on such a slice is degenerate at the isolated critical points. Vilenkin defines the singular boundary to be composed of those configurations with infinite volume, curvature etc, which cannot be embedded as a critical slice in a 4-geometry.

To express his boundary condition, Vilenkin employs a probability current density

\[
J = i/2(\Psi^* \nabla \Psi - \Psi \nabla \Psi^*),
\]

and he states that “this current can be identified with the probability flux in superspace,” (Vilenkin 1988, p889). The tunnelling boundary condition is that the wave-function of the universe should only include ‘outgoing modes’ at the singular boundary, “carrying flux out of superspace,” (Vilenkin 1988, p889). Vilenkin is proposing that the probability flux vector field \( J \), associated with \( \Psi \), must point out of superspace at the singular boundary. According to Vilenkin, this corresponds to a non-singular beginning to the universe.

Vilenkin states that a WKB wave-function can be written as a superposition

\[
\Psi = \sum_n C_n e^{iS_n},
\]

where the \( S_n \) are rapidly varying functions, each of which satisfies the time-independent Hamilton-Jacobi equation on mini-superspace:

\[
\|\nabla S_n\|^2 + U = 0.
\]

Vilenkin asserts that the current for the nth term is

\[
J_n = -|C_n|^2 \nabla S_n,
\]

and that the tunnelling boundary condition requires that the vector fields \( -\nabla S_n \) should point out of superspace at the singular boundary, (Vilenkin 1988, p890). He states that each function \( S_n \) defines a congruence of ‘classical trajectories’ in the mini-superspace, the integral curves of the vector fields \( -\nabla S_n \). The tunnelling boundary condition means that these classical paths can end at the singular boundary, but not begin there.

Vilenkin recognizes that one need not restrict attention to the (Riemannian) geometries on a fixed 3-manifold: “We can define the extended superspace...including all possible topologies. It can be split into topological sectors, with all metrics in the same sector having the same topology,” (1994, p2588). If one restricts attention to 3-geometries which are non-degenerate at all points, then the superspace of all possible 3-geometries is a disconnected topological space, with each component corresponding to geometries having the same topology. Vilenkin, in contrast, seems to envisage a connected space, with the metrics of different topology occupying disjoint open subsets, separated from each other by boundaries. In line with this, he originally held that “topology
changing transitions...occur through the boundary of the corresponding superspace sectors,” (1994, p2588). The boundaries between different topological sectors are Vilenkin’s regular boundaries, each point of which is the critical slice of a topology-changing 4-geometry, (1994, p2589). One can foliate a topology-changing space-time by a one-parameter family of spacelike hypersurfaces if one permits the metric on some of those spacelike hypersurfaces to be degenerate at isolated points (Borde 1994). If one enlarges the space of 3-geometries to include those which are degenerate at isolated points, then one can represent a topology-changing space-time as a curve in this enlarged configuration space.

Whilst the outgoing-wave boundary condition was imposed on the singular part of the boundary, “the boundary condition on [the] regular boundary was supposed to enforce conservation of probability flux as it flows from one topological sector to another,” (Vilenkin 1994, p2589). Vilenkin held that there is a boundary between the ‘null topological sector’, which contains nothing, and a sector such as that containing all the $S^3$-geometries, (1994, p2588). He believed this boundary to be part of the regular boundary: “the probability flux is injected into superspace through the boundary with the null sector; it then flows between different topological sectors through the regular boundaries, and finally flows out of superspace through the singular boundary,” (1994, p2589).

Vilenkin later held that “topology change does not necessarily occur between configurations at the boundaries of superspace sectors, but generally involves configurations in the interiors of these sectors,” (1994, p2589). Whilst topology change must occur through a critical slice, Vilenkin contends that such critical slices can lie in the interior of superspace sectors. It seems, then, that every regular boundary configuration is a critical slice, but not every critical slice is part of the regular boundary.

In terms of an $S^3$ mini-superspace model with a scale factor $a$ and matter field $\phi$, each point in the mini-superspace is a pair $(a, \phi)$. In terms of an $S^3$ mini-superspace model with a scale factor $a$ mapped to $\alpha = \ln a$, and a matter field $\phi$, each point in this mini-superspace is a pair $(\alpha, \phi)$. The mini-superspace of pairs $(a, \phi)$ is the manifold $(0, \infty) \times (-\infty, +\infty)$, whilst the mini-superspace of pairs $(\alpha, \phi)$ is the manifold $(-\infty, +\infty) \times (-\infty, +\infty)$. Vilenkin asserts (1994, p2588) that “the surface $\alpha = -\infty, |\phi| < \infty$ can be thought of” as the boundary between the ‘null topological sector’ and the sector associated with $S^3$. Thus, using the interpretation of Vilenkin’s boundary-concept suggested above, each sequence of points $(\alpha_n, \phi_n)$ in which $\lim_{n \to \infty} \alpha_n = -\infty$, is a point of the (regular) boundary with the null sector. In other words, a sequence of configurations in which the scale factor $a = \exp(\alpha)$ tends to zero, and the matter field $\phi$ converges to a finite value, is a point of the (regular) boundary with the null sector. Sequences in which $\lim_{n \to \infty} \alpha_n = -\infty$ and $\lim_{n \to \infty} |\phi_n| = \infty$, or sequences in which $\lim_{n \to \infty} \alpha_n = +\infty$, correspond to points in the singular boundary.\footnote{Sequences in which $\lim_{n \to \infty} \alpha_n = +\infty$, are sequences in which the scale factor, and thus the volume of space, increase without limit. Sequences in which $\lim_{n \to \infty} |\phi_n| = \infty$, are sequences in which the matter field either increases without limit, or decreases without limit. It is worth emphasizing once again that each point in any of these sequences would be a point of the mini-superspace, hence the scale factor and matter field would be finite for each point.}
Vilenkin asserts that the probability flux is “injected” through the regular boundary \( \alpha = -\infty, |\phi| < \infty \), the boundary with the ‘null topological sector’, and “flows out of superspace through the remaining boundary (\( \alpha \to -\infty \) with |\phi| \to \infty, or \( \alpha \to +\infty \)).” (ibid.).

However, the ‘null topological sector’ is just the empty set, and there is no reason to think of it as sharing a boundary with a non-empty set of geometries. Hence, the integral curves on the configuration space (or superspace) which result from Vilenkin’s boundary condition on the wave function, should not be interpreted as describing an ensemble of universes which are created from nothing.

Vilenkin’s ideas are intriguing, but it is not established that they can be meaningfully extended to infinite-dimensional superspaces, his notion of the boundary of a superspace is unsatisfactory, and a wave-function satisfying the tunnelling boundary condition cannot be interpreted as describing creation from nothing.

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