Bianchi type $V I_h$ Bulk-Viscous String Cosmological Model in $f(R)$ Gravity

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Abstract. In this paper, we derive $f(R)$ gravity field equations with the help of a spatially homogeneous and anisotropic Bianchi type-$V I_h$ space-time, in the presence of a bulk-viscous fluid, containing one-dimensional cosmic strings. Here we obtained the solutions of the field equations, under some specific plausible physical conditions. In particular, cosmological model with bulk-viscous strings in $f(R)$ theory of gravity is obtained by using the special law of variation for Hubble’s parameter proposed by Berman (Nuovo Cimento, B 74, 182 (1983)). Some physical and kinematical properties of the models are also discussed.

1. Introduction
Recent astrophysical data indicate that our Universe is currently in a phase of accelerating expansion, which is indicated by the observational data from the cosmic microwave background (CMB) [1] and observations of type Ia supernovae (SNe) [2] and large scale structure (LSS) [3]. It is believed that the reason for this exotic type of unknown force with huge negative pressure dubbed dark energy (DE). However, the nature and behaviour of DE is still a mystery. Currently, there are two main approaches explaining this accelerated expansion. One way is to construct different dark energy candidates and the other way is modification of Einstein’s theory of gravitation. Among various modified theories of gravitation, $f(R)$ theory of gravity is significant in which a general function of Ricci scalar, $f(R)$, replaces $R$ in the standard Einstein-Hilbert Lagrangian. The $f(R)$ theory of gravity is considered most suitable due to cosmologically important $f(R)$ models. These models consist of higher order curvature invariants as functions of the Ricci scalar. Nojiri and Odintsov [4] have discussed $f(R)$ gravity, provides a natural unification of the early-time inflation and late-time cosmic acceleration. Earlier interest on $f(R)$ theories was motivated by inflationary scenarios as for instance, in the Starobinsky model [5]. The constant curvature solutions in $f(R)$ gravity have been invistigated by Cognola et al. [6]. The theory also gives a natural gravitational alternative to dark energy. Nojiri and Odintsov [7], Capozziello and Laurentis [8] studied many aspects of $f(R)$ gravity. Friedmann-Robertson-Walker (FRW) models, being spatially homogeneous and isotropic in nature, are best for the representation of the large scale structure of the present Universe. However, it is believed that the early uniform. It is also found that some large-angle anomalies [9] appear in cosmic microwave background radiation, which violates the statistical isotropy of the Universe. Bianchi type models, which are homogeneous but not necessarily isotropic, seem to be the most promising explanation of these anomalies. Jaffe et al. [10] found that removing a Bianchi component from...
the Wilkinson microwave anisotropic background are the most suitable to describe the early stages of the Universe. Bianchi type models are among the simplest model with anisotropic background.

Sharif and Shamir [11] have found exact solutions of the Bianchi type-I and V space times using the non-vacuum field equations. These solutions correspond to two models of the Universe, i.e., a singular model and non-singular model. Shamir [12] has studied the exact vacuum solutions of Bianchi type-I, III and Kantowski Sachs space times in the metric version of f(R) gravity. Sharif and Kausar [13] have studies the exact solutions of the Bianchi type-VI_o universe in the metric f(R) gravity and discuss the recent cosmic acceleration. Two types of non-vacuum solutions are found corresponding to isotropic and anisotropic fluids. Adhav [14] studied Bianchi-type-III cosmic string cosmological model in f(R) gravity. Katore [15] have studied the solutions of the Bianchi-type II, VII, IX models string in f(R)gravity. Aditya et al. [16] have studied the Birkhoff’s theorem in f(R) theory of gravity. Recently Aditya and Reddy [17] studied locally rotationally symmetric Bianchi type-I string cosmological models in f(R) theory of gravity.

The bulk viscosity plays a significant role in cosmology in getting accelerated expansion phase of the Universe popularly known as the inflationary phase. The magnitude of the viscous stress relative to the expansion is determined by the bulk viscosity coefficient. Neutrino viscosity acting in the early era might have considerably reduced the present anisotropy of the blackbody radiation during the process of evolution had been suggested by Misner [18]. Some general characteristics of anisotropic cosmological models in the presence of viscosity are investigated by Belinskii and Khalatnikov [19]. Murphy [20] derived a homogeneous isotropic model by introducing the second viscosity coefficient in the energy-momentum tensor of the fluid context. The coefficient of viscosity decreases as universe expands. Bali and Dave [21], Tripathy et al. [22], Bali and Pradhan [23] and Katore and Shaik [24] have studied various string cosmological models in presence of bulk viscosity. Rao et al [25] obtained anisotropic Universe with cosmic strings and bulk viscosity fluid in a scalar-tensor theory of gravitation. Sagar et al. [26] explored Bianchi type-III bulk viscosity cosmic string models in Brans-Dicke theory of gravitation. Most recently, Santhi et al. [27] studied bulk-viscous string cosmological model in f(R) gravity. Debika et al. [28] have discussed anisotropic Bianchi type-III model in f(R) gravity. Shaikh et al. [29] have studied Bianchi type VI_o cosmological model with bulk-viscosity in f(R) theory. There are extensively works on f(R) gravity in the recent years [31]-[41].

Motivated by the above investigations, in this paper, we focus our attention on the Bianchi type−VI_h bulk-viscous string model in metric f(R) theory of gravity. The plan of the paper is follows: Sect.2, we derive f(R) field equations for Bianchi type-VI_h metric in the presence of bulk-viscous fluid with one-dimensional string. In Section 3.1 is bulk viscous string models are obtained by using Berman’s law [30] for Hubble’s parameter. Section 3.2 is devoted to the bulk-viscous model in f(R) gravity. Some physical parameters are also evaluated for our models. Summary and conclusions are presented in the last section.

Basic formation of f(R) Gravity:
The metric tensor plays an important role in General Relativity. The dependence of Levi-Civita connection on the metric tensor is one of the main properties of GR. However, if we allow torsion in theory, then the connection no longer remains the Levi-Civita connection on the metric tensor vanishes. This is the main idea behind different approaches of f(R) theories of gravity. The action for f(R) gravity is given by

\[ S = \int \sqrt{-g} \left[ \frac{f(R)}{16\pi G} + L_m \right] d^4x, \]

(1)

where f(R) is a general function of the Ricci scalar and L_m is the matter Lagrangian. It is noted that this action is obtained just by replacing R by f(R) in the standard Einstein-Hilbert
The corresponding field equations are found by varying the action with respect to the metric $g_{ij}$

$$F(R) R_{ij} - \frac{1}{2} f(R) g_{ij} - \nabla_i \nabla_j F(R) + g_{ij} \Box F(R) = k T_{ij}, \quad (2)$$

where $f(R) = \frac{df(R)}{dR}$, $\Box = \nabla^i \nabla_i$ where $\nabla_i$ is the covariant derivative and $T_{ij}$ is the standard matter energy-momentum tensor derived from the Lagrangian $L_m$. Now, contracting the field equations, it follows that

$$F(R) R - 2 f(R) + 3 \Box F(R) = k T \quad (3)$$

Using (3) in (2), the field equations take the form

$$F(R) R_{ij} - \nabla_i \nabla_j F(R) - k T_{ij} = g_{ij} \left( \frac{F(R) R - F(R) - k T}{4} \right) \quad (4)$$

Equation (4) is an important relationship between $f(R)$ and $F(R)$, which will be used to simplify the field equations and to evaluate $f(R)$.

2. The Metric and Field Equations

We consider the spatially homogeneous and anisotropic Bianchi type-$VI_h$ space time as

$$ds^2 = dt^2 - A^2(t) dx^2 - B^2(t) dy^2 - C^2(t) e^{2x} dz^2, \quad (5)$$

where $A$, $B$ and $C$ are functions of cosmic time $t$ only and $h$ is an arbitrary constant.

The energy momentum tensor for a bulk-viscous fluid containing one-dimensional cosmic strings is given by

$$T_{ij} = (\rho + p) u_i u_j - p g_{ij} - \lambda x_i x_j, \quad (6)$$

$$\bar{p} = p - 3 \xi H (= \omega \rho), \quad (7)$$

where $p = \omega_0 \rho$ ($0 < \omega_0 < 1$). Here $\bar{p}$ is the total pressure, which includes the proper pressure $p$, $\lambda$ is the string tension density, $\rho$ is the rest energy density of the system, $\xi(t)$ is the coefficient of bulk viscosity, $3 \xi H$ is generally known as bulk viscous pressure, $H$ is the Hubble parameter of the model and $\omega = \omega_0 - \xi$ (where $\omega_0$ and $\xi$ are constants). We consider $\rho$, $\bar{p}$ and $\lambda$ as functions of time $t$ only.

Also, $u_i$ is the four velocity vector, $x_i$ is a space-like vector, which represents the anisotropic directions of the string and they satisfy

$$g^{ij} u_i u_j = -x^i x_j = 1 \quad u^i x_i = 0. \quad (8)$$

We assume the string to be lying along the Z-axis. The one-dimensional strings are assumed to be loaded with particle and energy density as $\rho = \rho_p + \lambda$ is the rest energy density for a cloud of strings with particles attached to them. $\rho_p$ is density of particles, $\lambda$ is cloud strings tension density, $u^i$ is the four velocity vector and $x^i$ is the direction of strings.

In the co-moving co-ordinate system, from (6) and (8), we have

$$T_1^1 = T_2^2 = -\bar{p}, \quad T_3^3 = \lambda - \bar{p}, \quad T_4^4 = \rho. \quad (9)$$

The field equations (2) for the metric (5) yield the following equations:

$$\left( \frac{\ddot{A}}{A} + \frac{\dot{A} \dot{B}}{AB} + \frac{\dot{A} \dot{C}}{AC} - (1 + h^2) \frac{1}{A^2} \right) F - \frac{1}{2} f(R) + \left( \frac{\dot{B}}{B} + \frac{\dot{C}}{C} \right) \dot{F} + \ddot{F} = -k \bar{p} \quad (10)$$

$$\left( \frac{\ddot{B}}{B} + \frac{\dot{A} \dot{B}}{AB} + \frac{\dot{B} \dot{C}}{BC} - (1 + h) \frac{1}{A^2} \right) F - \frac{1}{2} f(R) + \left( \frac{\dot{A}}{A} + \frac{\dot{C}}{C} \right) \dot{F} + \ddot{F} = -k \bar{p} \quad (11)$$
\[
\left( \frac{\dot{C}}{C} + \frac{\dot{B}C}{BC} + \frac{\dot{A}C}{AC} - h(1 + h)\frac{1}{A^2}\right)F - \frac{1}{2}f(R) + \left( \frac{\dot{A}}{A} + \frac{\dot{B}}{B}\right)\dot{F} + \ddot{F} = k(\lambda - \bar{p}) \quad (12)
\]

\[
\left( \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)F - \frac{1}{2}f(R) + \left( \frac{\dot{A}}{A} + \frac{\dot{B}}{B} + \frac{\dot{C}}{C}\right)\dot{F} = kp. \quad (13)
\]

\[
(1 + h)\frac{\dot{A}}{A} - \frac{\dot{B}}{B} - h\frac{\dot{C}}{C} = 0 \quad (14)
\]

where overhead dot denote differentiation with respect to cosmic time \(t\).

3. Solution of the field equations

Equation (14) yields \(A^{1+h} = kBC^h\), where \(k\) is an integration constant. Without loss of generality, we assume that \(k = 1\), so that we have

\[
A^{1+h} = BC^h. \quad (15)
\]

3.1. Bulk viscous string model (\(\lambda \neq 0\))

Now, using equation (15), the set of field equations (10)-(13) reduce to four non-linear independent equations with six unknowns \(B, C, f(R), \bar{p}, \rho\) and \(\lambda\). Hence, to find a determinate solution of these highly non-linear differential equations, we use the following physically viable conditions:

- The shear scalar \((\sigma^2)\) is proportional to scalar expansion \((\theta)\), which leads to a relationship between the metric potentials, so that we can take
  \[
  B = C^m, \quad \text{where } m \neq 0 \text{ is a constant.} \quad (16)
  \]

- The \(f(R)\) theory of gravity has been shown equivalent to scalar tensor theory of gravity, which is incompatible with solar system tests of general relativity, as long as the scalar field propagates over solar system scales [42]. The power law relation between scalar field and average scale factor has already been used by Johri and Sudharsan [43] in the context of FRW Brans-Dicke models with bulk-viscous. However, Uddin et al. [44] have established a result in the context of \(f(R)\) theory of gravity which shows that
  \[
  F(R) \propto a(t)^n, \quad (17)
  \]
  where \(n\) is an arbitrary constant. Thus using power-law relation between \(F\) and \(a\) we have
  \[
  F(R) = F_0(a(t))^n, \quad (18)
  \]
  where \(F_0\) is proportionality constant.

The mean Hubble’s parameter \((H)\) proposed by Berman [30] defined as

\[
H = \beta(a(t))^{-\alpha} \quad (19)
\]

where \(\beta \geq 0, \alpha \geq 0\) are constants and \(a(t)\) is average scale factor. This relation gives a constant value of deceleration parameter. By solving (19), we obtain

\[
a(t) = (\alpha \beta t + c_2)^\frac{1}{\alpha}, \quad \alpha \neq 0 \quad (20)
\]

\[
a(t) = c_3e^{\beta t}, \quad \alpha = 0 \quad (21)
\]

thus the above two values of average scale factor corresponding to two different models of the Universe.
3.1.1. Model with power law expansion ($\alpha \neq 0$): From equations (16), (18) and (19), we obtain the metric coefficients for power-law expansion model of the Universe as

$$A = c_2^{(1+h)(m+h)}\left(\alpha \beta t + c_2\right)^{\frac{3(1+h)(m+h)}{(2m+h^2+mh+h+1)}}$$  (22)

$$B = c_2^{n} \left(\alpha \beta t + c_2\right)^{\frac{3m}{(2m+h^2+mh+h+1)}}$$  (23)

$$C = c_2 \left(\alpha \beta t + c_2\right)^{\frac{3}{(2m+h^2+mh+h+1)}}$$  (24)

$$F = F_0 (\alpha \beta t + c_2)^2$$  (25)

Now the metric (5) can be written as

$$ds^2 = dt^2 - c_2^{2(1+h)(m+h)}\left(\alpha \beta t + c_2\right)^{\frac{6(1+h)(m+h)}{(2m+h^2+mh+h+1)}} dx^2 - c_2^{2m} \left(\alpha \beta t + c_2\right)^{\frac{6m}{(2m+h^2+mh+h+1)}} e^{2\alpha t} dy^2$$

$$- c_2^{2} \left(\alpha \beta t + c_2\right)^{\frac{6}{(2m+h^2+mh+h+1)}} e^{2\beta t} dz^2$$  (26)

Hence, the metric (26) represents the Bianchi type-$V_{I_0}$ Bulk-viscous string cosmological model in the $f(R)$ theory of gravitation.

From equations (10)-(13), we get the string tension density is

$$\lambda = \frac{F_0 (\alpha \beta t + c_2)^2}{k} \left[k_2 + (h-1)c_0^{-2(1+h)(m+h)}(\alpha \beta t + c_2)^{-\frac{6(1+h)(m+h)}{2m+h^2+mh+h+1}}\right], \text{ here } k_2 \text{ is arbitrary constant}$$  (27)

The energy density is

$$\rho = \frac{F_0 (\alpha \beta t + c_2)^2}{k(1+\omega)} \left[k_3(h+1)c^{-2(1+h)(m+h)}(\alpha \beta t + c_2)^{-\frac{6(1+h)(m+h)}{2m+h^2+mh+h+1}}\right], \text{ here } k_3 \text{ is arbitrary constant}$$  (28)

The proper pressure is

$$p = \frac{F_0 (\alpha \beta t + c_2)^2}{k(1+\omega)} \left[k_3(h+1)c^{-2(1+h)(m+h)}(\alpha \beta t + c_2)^{-\frac{6(1+h)(m+h)}{2m+h^2+mh+h+1}}\right]$$  (29)

The total pressure

$$\bar{p} = \omega \rho = \frac{\omega}{k(1+\omega)} \left[k_3(\alpha \beta t + c_2)^{-2(1+h)(m+h)}(\alpha \beta t + c_2)^{-\frac{6(1+h)(m+h)}{2m+h^2+mh+h+1}}\right] F_0 (\alpha \beta t + c_2)^{-\frac{n}{n}}$$  (30)

The coefficient of bulk-viscousity is

$$\xi = \frac{p - \bar{p}}{3H} = \frac{(\omega_0 - \omega)}{3k\alpha \beta (1+\omega)} \left[k_3(\alpha \beta t + c_2)^{-2(1+h)(m+h)}(\alpha \beta t + c_2)^{-\frac{6(1+h)(m+h)}{2m+h^2+mh+h+1}}\right] F_0 (\alpha \beta t + c_2)^{-\frac{n}{n} + 1}$$  (31)

The Ricci scalar function is

$$f(R) = \left[\frac{k_3}{(\alpha \beta t + c_2)^2} + 2 \frac{(1+h)(m+h)}{(1+\omega)} c_2^{-2(1+h)(m+h)}(\alpha \beta t + c_2)^{-\frac{6(1+h)(m+h)}{2m+h^2+mh+h+1}}\right] F_0 (\alpha \beta t + c_2)^{\frac{n}{n}}$$  (32)
**Physical parameters:** The physical parameters are important in discussion of cosmology of obtained anisotropic bulk viscous string model.

Spatial volume ($V$) of the model

$$V = ABC = C_1(\alpha \beta t + c_2)^{\frac{3}{2}}$$

(33)

The average scale parameter ($a(t)$) is

$$a(t) = (\alpha \beta t + c_2)^{\frac{1}{2}}$$

(34)

The directional Hubble’s parameters are

$$H_1 = \frac{3\beta(1+h)(m+h)}{(2m+h^2+mh+h+1)(\alpha \beta t + c_2)}$$

$$H_2 = \frac{3m\beta}{(2m+h^2+mh+h+1)(\alpha \beta t + c_2)}$$

$$H_3 = \frac{3\beta}{(2m+h^2+mh+h+1)(\alpha \beta t + c_2)}$$

(35)

The mean Hubble’s parameter (H) is

$$H = \frac{\alpha \beta}{(\alpha \beta t + c_2)}$$

(36)

The scalar expansion ($\theta$)

$$\theta = \frac{3\beta}{(\alpha \beta t + c_2)}$$

(37)

The shear scalar ($\sigma^2$) in the model is

$$\sigma^2 = \frac{9\beta^2(1+h)^2(m+h)^2 + m^2 + 1}{(2m+h^2+mh+h+1)(\alpha \beta t + c_2)^2}$$

(38)

The deceleration parameter ($q$) is

$$q = 0$$

(39)

3.1.2. **Model with exponential expansion** ($\alpha = 0$): In this case, we get the average scale factor as

$$a(t) = c_2 e^{\beta t}$$

(40)

by substituting average scale parameter, we get

$$A = c_0^{(1+h)(m+h)} e^{\frac{3\beta(1+h)(m+h)t}{2m+h^2+mh+h+1}}$$

(41)

$$B = c_2^{3m\beta(1+h)(m+h)} e^{\frac{3m\beta(1+h)(m+h)t}{2m+h^2+mh+h+1}}$$

(42)

$$C = c_2^{\frac{3}{2}} e^{\frac{3\beta(1+h)(m+h)t}{2m+h^2+mh+h+1}}$$

(43)

$$F = F_0 c_2^\beta e^{\beta t}$$

(44)

Now metric (4) can be written as

$$ds^2 = dt^2 - c_2^{2(1+h)(m+h)} e^{\frac{6\beta(1+h)(m+h)t}{2m+h^2+mh+h+1}} dx^2 - c_2^{2m} e^{\frac{6m\beta(1+h)(m+h)t}{2m+h^2+mh+h+1}} e^{2\beta t} dy^2 - c_2^{\frac{3}{2}} e^{\frac{3\beta(1+h)(m+h)t}{2m+h^2+mh+h+1}} e^{2h x} dz^2$$

(45)
Hence, the metric (45) represents the Bianchi type-V $I_h$ Bulk-viscous string cosmological model in the $f(R)$ theory of gravitation.

The string tension density is
\[ \lambda = \frac{F_0 c_2^\beta e^{n \beta t}}{k} \left[ k_1 + (h - 1) c_2^{-2(1+h)(m+h)} e^{-6\beta(1+h)(m+h)t} \right], \text{ here } k_1 \text{ is arbitrary constant (46)} \]

The energy density is
\[ \rho = \frac{F_0 c_2^\beta e^{n \beta t}}{k(1 + \omega)} \left[ k_2 + (h - 1) c_2^{-2(1+h)(m+h)} e^{-6\beta(1+h)(m+h)t} \right], \text{ here } k_2 \text{ is arbitrary constant (47)} \]

The Proper pressure is
\[ p = \frac{\omega_0 F_0 c_2^\beta e^{n \beta t}}{k(1 + \omega)} \left[ k_2 + (h - 1) c_2^{-2(1+h)(m+h)} e^{-6\beta(1+h)(m+h)t} \right] \]
\[ (48) \]

The total pressure is
\[ \bar{p} = \frac{\omega F_0 c_2^\beta e^{n \beta t}}{k(1 + \omega)} \left[ k_2 + (h - 1) c_2^{-2(1+h)(m+h)} e^{-6\beta(1+h)(m+h)t} \right] \]
\[ (49) \]

The coefficient of bulk-viscosity is
\[ \xi = \frac{\beta(\omega_0 - \omega) F_0 c_2^\beta e^{n \beta t}}{3k(1 + \omega)} \left[ k_2 + (h - 1) c_2^{-2(1+h)(m+h)} e^{-6\beta(1+h)(m+h)t} \right] \]
\[ (50) \]

The Ricci scalar function is
\[ f(R) = \left[ k_3 + 2\left( \frac{1 + h}{1 + \omega} \right) c_2^{-2(1+h)(m+h)} e^{-6\beta(1+h)(m+h)t} \right] F_0 c_2^\beta e^{n \beta t} \]
\[ (51) \]

**Physical parameters:**
The physical parameters are important in discussion of cosmology of obtained anisotropic bulk viscous string model ($\alpha = 0$).

Spatial volume of the model obtained as
\[ V = ABC = e^{(1+h)x} e^{3\beta t} \]
\[ (52) \]

The average scale parameter is
\[ a = (V)^\frac{1}{3} = c_2 e^{\beta t} \]
\[ (53) \]

The Hubble’s parameter is
\[ H = \frac{1}{3}[H_1 + H_2 + H_3] = \beta \]
\[ (54) \]

The scalar expansion is
\[ \theta = 3H = 3\beta \]
\[ (55) \]

The shear scalar is
\[ \sigma^2 = \frac{1}{2} \sigma_{ij} \sigma^{ij} = \frac{9\beta^2 \left[ (1+h)^2(m+h)^2 + m^2 + 1 \right]}{(2m + h^2 + mh + h + 1)^2} \]
\[ (56) \]

The deceleration parameter is
\[ q = \frac{d}{dt} \left( \frac{1}{H} \right) - 1 = -1 \]
\[ (57) \]

The deceleration parameter $q = -1$ ($< 0$), which shows that the Universe is accelerating.
3.2. Bulk viscous model ($\lambda = 0$):

In this case field equations (10)-(13) reduce to

\[
\left(\frac{\dot{A}}{A} - \frac{\dot{B}}{B} + \frac{\dot{C}}{AC} - \frac{BC}{BC} + (h-h^2)\frac{1}{A^2}\right)F + (\frac{\dot{B}}{B} - \frac{\dot{A}}{A})F = 0 \tag{58}
\]

\[
\left(\frac{\dot{A}}{A} + \frac{\dot{B}}{B} - \frac{\dot{C}}{AC} - \frac{BC}{BC} + (h+h^2)\frac{1}{A^2}\right)F + \frac{\dot{C}}{C}F = k(\rho + \bar{p}) \tag{59}
\]

Now (58) and (59) are two independent equations in five unknowns $A$, $B$, $\bar{p}$, $f(R)$ and $\rho$. Hence to find a determinate solution we use the physically possible conditions given in Sec.3.

The coefficient of bulk-viscousity is

\[
\xi = \frac{p-\bar{p}}{3H} = \frac{\alpha(1+h)(\alpha \beta + c_2)}{(m+h) + (m+1)(1+h)} \left[\frac{k_1}{(\alpha \beta + c_2)^2} + (h+h^2)(\alpha \beta + c_2)^{-2(m+h)}\right] \tag{68}
\]

The Ricci scalar function is

\[
f(R) = \frac{F_0(\alpha \beta + c_2)^\frac{m}{2}}{k(1+\omega)} \left[\frac{k_2}{(\alpha \beta + c_2)^2} + 2(h+h^2)(\alpha \beta + c_2)^{-2(m+h)}\right] \tag{69}
\]
Physical parameters:
The physical parameters are important in discussion of cosmology of obtained anisotropic bulk-viscous model.

Spatial volume \(V\) of the model

\[
V = ABC = (\alpha \beta t + c_2) \frac{(m+h)+(m+1)(1+h)}{\alpha(1+h)}
\]

The average scale parameter \(a(t)\) is

\[
a(t) = (\alpha \beta t + c_2) \frac{(m+h)+(m+1)(1+h)}{3\alpha(1+h)}
\]

The mean Hubble’s parameter \(H\) is

\[
H = \frac{(m + h) + (1 + h)(m + 1)}{3\alpha(1 + h)(\alpha \beta t + c_2)}
\]

The scalar expansion \(\theta\)

\[
\theta = \frac{(m + h) + (1 + h)(m + 1)}{\alpha(1 + h)(\alpha \beta t + c_2)}
\]

The shear scalar \(\sigma^2\) in the model is

\[
\sigma^2 = \frac{9\beta^2 \left( (1 + h)^2(m + h)^2 + m^2 + 1 \right)}{(2m + h^2 + mh + h + 1)(\alpha \beta t + c_2)^2}
\]

The deceleration parameter \(q\) is

\[
q = \frac{3\alpha(1 + h)\beta}{(m + h) + (m + 1)(1 + h)} - 1
\]

4. Summary and Conclusions:
The main purpose of this paper is to discuss the well-known phenomenon of the Universe is an accelerated expansion in the context of \(f(R)\) gravity. We have investigated solution of the Bianchi type-V field equations. We have obtained cosmological models corresponding to bulk-violous fluid and bulk-violous cosmic strings using some physically viable conditions.

Power law models (26) and (64) of the Universe has a point type singularity at \(t^* = -\frac{c_2}{\alpha \beta}\). Physical parameters of these models (26) and (64) like mean Hubble’s parameter \(H\), expansion scalar \(\theta\) and shear scalar \(\sigma^2\) are all infinite at this point but spatial volume \(V\) of the model vanishes. Energy density \((\rho)\) and string tension density \((\lambda)\), coefficient of bulk-viscosity \((\xi)\), total pressure \((\bar{p})\) and Ricci scalar function \(f(R)\) are also infinite while the metric potentials vanish at the singular point \(t^* = -\frac{c_2}{\alpha \beta}\).

The exponential model (45) is non-singular model of the Universe. In this model observed that the spatial volume of bulk-viscosity is exponentially increasing with time and doesn’t vanish throughout the evolution of the Universe, whereas the expansion scalar \(\theta\), shear scalar \(\sigma^2\) and mean Hubble’s parameter \(H\) are constant without depending on time. Which shows that the volume of the Universe increasing with uniform rate of expansion. Also, the model is free from singularities. The deceleration parameter \(q\) is very important vector for understanding cosmic evolution. If \(q < 0\), the model accelerates and when \(q > 0\), the model decelerates in the standard way. In this work, it is observed that the present day scenario. String densities and pressure become constants when \(t = 0\).
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