Tool for assessing the quality of electricity from renewable energy sources

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Abstract. The article deals with the problem of quality control of electrical energy. A method for increasing the accuracy of measurements at the software level is considered in detail, using the example of a Hall sensor. Determination of the Hall sensor hardware function is based on the assumption of a stationary current density distribution. It is shown that the hardware function of the sensor has a characteristic maximum, which must be taken into account when determining the quality indicators of electrical energy in electrical networks. The results of modeling the potential distribution in the Hall sensor are presented. The influence of the geometric linear dimensions of the sensor on the instrumental function is analyzed.

1. Introduction

In the Altai Territory, there are two problems: a shortage of its own electricity and, as a result, a high tariff for electricity [1]. Altai Territory has a high potential for the construction of power plants operating on renewable energy sources (sun, wind). This requires investment and unoccupied land.

Within the framework of the Agreement between Altayenergosbyt OJSC, Altai Territory Administration, IDGC of Siberia OJSC and Vent Rus LLC dated 04.05.2010 on cooperation in the development of wind energy in the Altai Territory, work is underway to implement investment projects for the construction of wind power plants.

The hydropower potential of the rivers of the Altai Territory is able to significantly reduce the power supply deficit in rural areas and villages remote from the existing power system, as well as areas with single-circuit and radial physically worn out power lines. LLC EC Energia plans to implement the project "Construction of small and micro-hydroelectric power plants in the Altai Territory with a total installed capacity of 31.6 MW", which includes the following small hydroelectric power plants: Gilevskaya Micro-HPP with a capacity of 2.4 MW at the Gilevsky reservoir in the Loktevsky district; Soloneshenskaya SHPP with a capacity of 1.2 MW on the Anuy River in the Soloneshensky District.

The generated electrical energy must comply with the regulations [2, 3]. In particular, indicators of the quality of electrical energy are determined by the state standard GOST 32144–2013 “Electrical Energy. Electromagnetic compatibility of technical means. Electricity quality standards in general-purpose power supply systems.

Electricity quality indicators require continuous monitoring [4]. To process the measurement results, modern algorithms are used, including those based on neural networks [5]. Evaluation of the parameters of the quality of electrical energy is also necessary for the management of urban electrical networks within the Smart Grid [6].

2. Mathematical model

Evaluation of indicators of the quality of electrical energy is possible based on the analysis of the parameters of the electromagnetic field. To determine the characteristics of the magnetic field, it is proposed to use a Hall sensor [7].

If the measured magnetic field is small enough, as well as the sensor current, the Hall sensor can be considered linear. In this case, the Hall potential difference and the measured value of the magnetic field have a linear relationship, which can be represented as a convolution integral:
\[ u_H(x, y) = \frac{IK_H}{S} \int_S r(x', y')B(x - x', y - y') \, dx' \, dy' \]  

(1)

where \( I \) is the current flowing in the Hall sensor through its contacts; \( S \) is the cross-sectional area of the Hall sensor; \( K_H \) is the coefficient (constant) of the Hall sensor; \( r(x, y) \) is the hardware function \( r(x, y) \); \( B(x, y) \) is the magnetic field distribution inside the sensor volume.

Relation (1) shows that the potential difference at the potential contacts of the Hall sensor is determined by the hardware function \( r(x, y) \), which is invariant to the field distribution and the current flowing through the sensor.

Obviously, when \( r(x, y) = 1 \), the minimum change in the measured value that the sensor can detect is primarily determined by its dimensions, ideally \( r(x, y) = S_0(x, y) \). If the sensor is placed in a uniform magnetic field, it is necessary to enter the normalization

\[ \int_S r(x, y) \, dx \, dy = S \]  

(2)

Let us assume that the general view of the apparatus function \( r(x, y) \) depends on the geometric dimensions of the sensor and its contacts and the current density.

To substantiate (1) and determine the hardware function of the Hall sensor, we will assume that a direct current \( I \) flows through its contacts. The \( x \) axis passes through the current contacts located on the left and right edges of the crystal. The \( y \) axis passes through potential contacts located on the upper and lower faces of the crystal. In the absence of a magnetic field, a stationary current density distribution \( j_0(x, y) \) is established inside the sensor, which is potential [8]

\[ \begin{align*}
rot j_0(x, y) &= 0 \\
\text{div} j_0(x, y) &= 0
\end{align*} \]  

(3)  

(4)

According to [9], in this case there should be a scalar potential function \( \phi_0 \), which determines the distribution of the electric field inside the sensor

\[ E = -\text{grad}(\phi_0) \]  

(5)

The current density is calculated according to Ohm's law, written in differential form

\[ j_0 = \sigma E = -\sigma \text{grad} \phi_0 \]  

(6)

Substituting (6) into equation (4), we obtain the Laplace equation for the potential distribution \( \phi_0(x, y) \),

\[ \Delta \phi_0(x, y) = 0 \]  

(7)

3. Results and Discussion

Let us assume that the conductivity of the material of the sensor contacts, both current and potential, is much higher than the conductivity of the Hall sensor crystal, in this case only the normal component of the electric field strength will be present at the border of the sensor and the contact.

In the absence of a magnetic field inside the sensor, there is no current through the upper potential faces, therefore, from the side of these contacts \( j_{0y} \left( y = \pm \frac{b}{2} \right) = 0 \), then, taking into account (7), we obtain the first boundary condition

\[ \frac{\partial \phi_0}{\partial y} (x, y = \pm \frac{b}{2}) = 0 \]  

(8)

From the side of the current contact \( (x = \pm \frac{a}{2}, |y| \leq \frac{\varepsilon}{2}) \), taking into account the constancy of the current and its uniform distribution over the entire current contact, only the \( x \)-component of the electric field strength will exist, therefore the current density

\[ j_{0x} = (x = \pm \frac{a}{2}, |y| \leq \frac{\varepsilon}{2}) = \frac{l}{d} \]  

(9)

where \( l \) is the total current flowing through the contact; \( d \) is the Hall sensor thickness; \( \varepsilon \) is the current contacts width.

From the side of the Hall sensor at the boundary \( (x = \pm \frac{a}{2}, |y| \leq \frac{\varepsilon}{2}) \), the potential \( \phi_0 \) and the current density \( j_{0x} \) are related by relation (7), therefore the second boundary condition for the potential \( \phi_0 \)

\[ \frac{\partial \phi_0}{\partial x} (x = \pm \frac{a}{2}, |y| \leq \frac{\varepsilon}{2}) = -\frac{l}{d \varepsilon \sigma} \]  

(10)

where \( \sigma \) is the conductivity of the Hall sensor.
For current density outside the current contacts
\[ \text{div} j_0(x, y) = 0 \quad \frac{\partial \varphi_0}{\partial x}(x = \pm \frac{a}{2}, |y| \leq \frac{\epsilon}{2}) = -\frac{l}{\varepsilon d} = 0 \quad (11) \]

Expressions (9) - (11) represent the well-known Neumann boundary value problem [10] for the potential distribution \( \varphi_0 \).

If the width of the current and potential contacts of the sensor is much less than the linear dimensions of the sensor, the conductivity of the contact material is multiples of the conductivity of the crystal, the sensor is placed in a uniform magnetic field directed along the \( z \) axis (there is no tangential component of the magnetic induction vector), then the potential distribution \( \varphi_0 \) along the \( z \) axis can be considered uniform and one can pass from the three-dimensional model of the Hall sensor to its two-dimensional approximation. In the case of a rectangular sensor with dimensions \( a \times b \), the potential distribution is found by separation of variables.

The potential \( \varphi_0(x, y) \) must satisfy the symmetry conditions. In particular, the potential along the \( y \)-axis must be even, and odd along the \( x \)-axis. Then, taking these requirements into account, we obtain the potential distribution
\[ \varphi_0(x, y) = -\frac{l}{bd} x - \frac{l}{\varepsilon d} b \sum_{m=1}^{\infty} \frac{\sin(\frac{2\pi m}{b} x)}{(\pi m)^2 \sinh(\frac{\pi m c}{b})} \frac{2\pi m x}{b} \cos \frac{2\pi m y}{b} \quad (12) \]

This distribution can be obtained alternatively by numerically integrating the Laplace equation (7), taking into account the boundary conditions (9-11).

Let us transform the integral in expression (1) into a multiple integral. To do this, replace the variables of integration \( x, y \) with \( x', y' \) taking into account that the center of the Hall sensor is at the point with coordinates \( x, y \), the parity of the functions \( \varphi \), \( j_0(x, y) \):
\[ u_H(x, y) = \frac{1}{ane} \int_{-\frac{b}{2}}^{\frac{b}{2}} \int_{-\frac{a}{2}}^{\frac{a}{2}} v(y') j_{ox}(x', y') B(x - x', y - y') dx' dy' \quad (13) \]

Comparison of (13) with (1) gives an expression for the apparatus function of the Hall sensor
\[ r(x, y) = \frac{\nu(y)S_{ane1K_H}}{j_{ox}(x, y)} \quad (14) \]

Integrating (14) over the entire surface of the sensor and taking into account the normalization condition (2), we obtain
\[ K_H = \frac{1}{ane} \int_{-\frac{b}{2}}^{\frac{b}{2}} v(y) \left[ j_{ox}(x, y) dx \right] dy \quad (15) \]

The distribution of \( x \) component of the current density \( j_0(x, y) \) which is included in (15), can be determined from the distribution (12) of the potential \( \varphi_0(x, y) \):
\[ j_{ox}(x, y) = -\sigma \frac{\partial \varphi_0}{\partial x} = \frac{l}{bd} + \frac{l}{\varepsilon d} \sum_{m=1}^{\infty} \frac{2\sin(\frac{2\pi m}{b} x)}{\pi m \sinh(\frac{\pi m c}{b})} \frac{2\pi m x}{b} \cos \frac{2\pi m y}{b} \quad (16) \]

To find the current density distribution \( j_e(x, y) = -\varepsilon \text{grad} \varphi_H \), we first determine the potential distribution \( \varphi_H(x, y) \) caused by the hollowing of the potential difference \( u_H \) on the potential contacts of the sensor.

The solution to this problem, similarly to (12) Respectively
\[
\varphi_H(x, y) = -\frac{u_H}{a \sigma R} y - \frac{u_H}{\epsilon \sigma R} \sum_{m=1}^{\infty} \frac{\sin\left(\frac{2\pi m \epsilon}{a}\right)}{(\pi m)^2 \text{ch}\left(\frac{\pi \epsilon b}{a}\right)} \frac{2\pi m y}{a} \cosh\left(\frac{2\pi \epsilon y}{a}\right) \frac{2\pi m x}{a}
\]

(17)

where \( R \) is the output resistance of the sensor.

Respectively

\[
\begin{aligned}
\psi(x, y) &= -\sigma \frac{\partial \varphi_H}{\partial y} = \frac{u_H}{a \sigma R} + \frac{u_H}{\epsilon \sigma R} \sum_{m=1}^{\infty} \frac{\sin\left(\frac{2\pi m \epsilon}{a}\right)}{(\pi m)^2 \text{ch}\left(\frac{\pi \epsilon b}{a}\right)} \frac{2\pi m y}{a} \cos\left(\frac{2\pi \epsilon y}{a}\right) \\
&\quad \times \frac{2\pi m x}{a}
\end{aligned}
\]

(18)

After making the transformations, we get the expression for the auxiliary function \( v(y) \)

\[
v(y) = \left[ \frac{b^2}{2} + \frac{a^2}{\epsilon} \right] \sum_{m=1}^{\infty} \frac{\sin\left(\frac{2\pi m \epsilon}{a}\right)}{(\pi m)^2 \text{ch}\left(\frac{\pi \epsilon b}{a}\right)} \int_{-\frac{b}{2}}^{\frac{b}{2}} \int_{-\frac{a}{2}}^{\frac{a}{2}} \sin\left(\frac{2\pi m \epsilon}{a}\right) \frac{2\pi m y}{a} \cos\left(\frac{2\pi m x}{a}\right) dx \\
&\quad \times \frac{2\pi m x}{a} \cos\left(\frac{2\pi m y}{a}\right) \frac{2\pi m x}{a}
\]

(19)

Integral (19) in the general case is found numerically, but for the particular case \((a = b)\) it is greatly simplified and gives \( v(y) = 1 \). Then

\[
K_H = \frac{1}{\alpha e l} \int_{-\frac{a}{2}}^{\frac{a}{2}} \int_{-\frac{b}{2}}^{\frac{b}{2}} j(x, y) dy dx = \frac{1}{\alpha e l}
\]

(20)

\[
r(x, y) = 1 + \frac{2a}{\epsilon} \sum_{m=1}^{\infty} \frac{1}{\pi m \text{ch}(\pi m)} \sin\left(\frac{2\pi m \epsilon}{a}\right) \frac{2\pi m x}{a} \cosh\left(\frac{2\pi \epsilon y}{a}\right) \frac{2\pi m x}{a} \cos\left(\frac{2\pi m y}{a}\right) \frac{2\pi m x}{a}
\]

Let us present the results of mathematical modeling of the apparatus function of the Hall sensor of the SS495A type with a crystal \(5 \times 5 \text{ mm}\) in size, \(1 \text{ mm}\) thick and with a width of potential and current contacts \(mm\). For modeling and calculating the instrumental function, as well as for modeling the potential distribution, the first thirty terms of the series were used, while the calculation showed that the error does not exceed 0.1\%. The total number of calculated grid points was 34x35. Figure 1 shows the surface of the hardware function; it has pronounced extreme values at the location of the current contacts.
4. Conclusion

For an ideal point sensor, there is a single and narrow extremum of the apparatus function \( r(x, y) \) of the form of a delta function. Physically, this is possible if the linear dimensions of one of the current contacts are multiples of the linear dimensions of the other.

Thus, the use of Hall sensors is possible and advisable for continuous monitoring of indicators of the quality of electrical energy. Hall sensors of the following types SS49E can be used for these purposes. SS49B, SS495A1, SS495A2, SS496A. Their Hall potential difference and the measured value of the magnetic field have a linear relationship. Since these types of sensors have the same crystal size, the results of mathematical modeling of the instrumental function can be used for them.

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