Jet formation at interaction of a vibrating plate with liquid

V A Aleksandrov, S P Kopysov, L Y Tonkov
Institute of Mechanics, Ural Branch of the Russian Academy of Sciences
34 ul. T. Baramzinoy, Izhevsk, Russia 426067
E-mail: ava@udman.ru

Abstract. In this work, we experimentally investigate the mechanism of jet formation on liquid surface near the edge of a vibrating plate that is partially immersed in the liquid. Under the conditions of resonant bending vibrations, the vibrating plate excites capillary waves in the form of Faraday ripples on the surface of the liquid layer wetting the plate surface. Two-dimensional capillary waves also appear on the curved surface of the liquid near the edges of the vibrating plate. The vibrations of the plate area generate hydrodynamic pressure on the liquid surface, which initiates surface eddy flows. At a certain position of the vibrating plate in the liquid, capillary oscillations in the form of standing waves appear along the boundary of the wetting layer of the liquid, directly under the free edge of the plate. The vibrations of the plate edge modulate the standing waves in the transverse direction, which results in the periodic variation of the surface curvature of the wetting liquid layer, from negative to positive values. The inertia forces, periodically varying with the frequency of the plate vibrations, combined with the Laplace pressure, in the excited standing capillary waves on the surface flow under the plate edge, initiates the periodic ejection of particles of the liquid forming a jet.

1. Introduction
In our previous work, we experimented with a bended rod with a wedge-shaped end surface and a bended plate with free edges, which are partially immersed in liquid at an angle. We showed that the high-frequency vibrations excited by a piezoelectric transducer can atomize the liquid at the wetting layer in the areas with the antinode of vibrations and therefore generate a liquid jet from under the end surface of the rod and the plate edge [1]. In the case of a rectangular plate fixed to the source of vibrations by one edge, a jet can be generated by the areas with the antinode of bending vibrations on the lateral edges of the plate. The conditions to generate a steady jet are the following: the large amplitude of vibrations of a rod or plate (i), and the retention of a certain position of the rod end and the plate edge relative to the free surface of the liquid (ii). In [1], we suggested the mechanism of generation of a liquid jet by a vibrating rod. It can be a high-frequency cumulative effect associated with periodic entry of the flat area of the rod end surface into the liquid at an angle. In this case, the amplitude of the velocity of the liquid wetting layer free surface should be larger than the amplitude of the vibrational speed of the rod surface.

The investigation of plate vibrations in liquid demonstrated that, when a jet is generated by the plate and a liquid wetting layer is present on other areas of the plate surface, with the antinode of bending vibrations, there will be standing capillary waves on the wetting layer...
surface, in the form of Faraday ripples [2]. A jet has not been generated when the free edge of the plate is completely immersed in the liquid or there is no liquid wetting layer on the plate edge. The behavior of the liquid near the free edge of the partially immersed vibrating plate was out of the scope of that work.

Bending vibrations of a plate are normal vibrations with distributed amplitude. The vibrations of the plate surface are maximal in the antinodes and they are absent in the nodes of vibrations. The excitation of the bending vibrations of the plate partially immersed in liquid leads to the vibrations of the liquid layer on the interphase boundary. Depending on the geometry of the interaction of the plate and liquid, the thickness and area of the free surface of the layer wetting the plate surface can vary. The liquid layer on the area of the vibrating plate surface is the volume of liquid bounded by the free surface and the surface of the contact with the plate. The appearance of Faraday ripples on the surface of the liquid layer is due to the parametric excitation of capillary waves, which is described in [3] in more detail.

The production of sprays from a vibrating liquid film is used in ultrasonic atomization [4, 5, 6]. In related works, the liquid contact has a solid support. The goal of our work is to investigate the mechanism of jet formation at the interaction of liquid and a vibrating deformable plate excited by a piezoelectric transducer.

2. Experimental devices and method

In the experimental facility, the schematic view of which is shown in Figure 1, rectangular plate 1 was made from transparent polyethylene terephthalate (PET). One edge of the plate is glued to one side of piezoelectric disk transducer 2 which is the source of bending vibrations. The other side of the piezoelectric transducer is soldered to metal holder 3 fixed to the element containing vertically positioned shaft 4 and cylindrical guide 5 with a slide, which allows the guide position adjustment with a screw.

The movement of the plate with the piezoelectric transducer together with the guide was monitored by a micrometer. The plate was bent nearby the free edge. For the excitation of bending vibrations of the plate, on the piezoelectric transducer electrodes the alternating voltage with amplitude about 20 V was supplied by amplifier 6 of signals generated by audio-frequency generator 7 GZ-35. The plate was brought close to the surface of liquid 8 so that the concave area was wetted from above and from below, and at the plate partial immersion in the liquid the free edge of the plate was above the liquid surface. Hydrodynamic processes on the liquid surface nearby the plate and on the surface of the liquid layer wetting the plate were recorded by digital camera 9 Canon EOS 650D or microscope camera OMAX A35140U3. After the excitation of the plate bending vibrations, the frequency of the signals of the generator was adjust to the magnitude at which the formation of a jet outgoing from under the plate edge.
Figure 2. The generation of an alcohol jet by the vibrating PET plate excited by the piezoelectric transducer.

Figure 3. Faraday ripples, cavities with air bubbles under the plate.

Figure 4. Liquid atomization in the areas of the antinodes of bending vibrations.

was observed. The voltage amplitude and the signal frequency were monitored with the use of an oscillograph C1-55 and frequency meter ChZ-34. When it was necessary, the trimming of the plate length was conducted for obtaining a jet at the minimal value of the amplitude of the vibrations. A piezo buzzer FML-20T-4.5A1-100 was used as a piezoelectric transducer. The plate had the size of $48.80 \times 4.60 \times 0.36$ mm. The length of curved part which immersed in the liquid is 18 mm, the radius of curvature of the plate is 15 mm, the depth of the plate dipping is 2 mm. The frequency of the resonance vibrations of the facility, when the interaction of the plate with the liquid led to the jet generation, was 4.7 kHz. The vibrations of the plate were excited at the 10th mode of bending vibrations. The amplitude of the vibrations of the plate free edge in the air reached 60 $\mu$m. Distilled water and ethyl alcohol were used as liquid.

3. Results and discussion

The jet formation by the vibrating polymer plate at the interaction with liquid is shown in Figure 2. Here the amplitude of the vibrations of the plate edge is $30 \div 40$ $\mu$m; at the frequency of 4.7 kHz the maximal value of the instantaneous velocity of the plate edge is $0.9 \div 1.2$ m/s. The velocity of droplets of water and alcohol in the jet is $1.0 \div 1.6$ m/s and $1.4 \div 1.7$ m/s respectively, which is slightly higher than the vibrational speed of the plate edge.
Simultaneously with the jet generation, the liquid atomization from the layer is observed in the antinode of bending vibrations near the interphase boundary on the side of piezoelectric transducer (see Figure3, 4). On the liquid layer wetting the upper side of the plate and on the areas of the plate surface with antinodes of vibrations, two-dimensional capillary waves in the form of Faraday ripples also appear. On the underside of the plate in the centre of the antinodes of vibrations, cavities with microscopic air bubbles are formed as the result of artificial hydrodynamic cavitation.

The partial immersion of the plate bent at an angle in the liquid and the wetting of the plate surface with the liquid leads to a liquid capillary rise over the plate surface on the phase boundary (Figure 5). The height of the liquid capillary rise increases at the interphase boundary when the plate bending vibrations are excited by the piezoelectric transducer (Figure 6). The additional capillary rise of the liquid layer over the vibrating plate surface indicates that in this liquid layer the pressure is smaller than that in the liquid layer wetting the surface of the nonexcited plate. This is due to the fact that the liquid layer close to the vibrating plate surface obtains the vibrational speed and energy which results in the appearance of the dynamic pressure equal to the liquid specific kinetic energy \( p = \rho v^2 / 2 \), where \( \rho \) is the liquid density, \( v \) is the velocity of the plate surface area.

\[ s = s_0 \cos(k_{bx}x) \cos(\omega t), \]  
(1)

\[ v = \omega s_0 \cos(k_{bx}x) \sin(\omega t). \]  
(2)

The particles of the liquid wetting the plate surface have the same velocity and therefore the specific kinetic energy \( W_l \) of the liquid is

\[ W_l = \frac{\rho}{2} (\omega s_0 \cos(k_{bx}x) \sin(\omega t))^2, \]  
(3)

or

\[ W_l = \frac{\rho}{4} \omega^2 (s_0)^2 \cos^2(k_{bx}x) - \frac{\rho}{4} \omega^2 (s_0)^2 \cos^2(k_{bx}x) \cos(2\omega t). \]  
(4)

The second summand on the right side of equation (4) changes with the doubled frequency of the plate vibrations; therefore, the specific kinetic energy of the liquid layer on the vibrating plate
surface and the hydrodynamic pressure in the layer pulsate. The value of the above summand for the period of vibrations is zero, and the specific energy of the liquid layer on the vibrating plate surface is on average

$$W_l = \frac{\rho}{4} \omega^2 (s_0)^2 \cos^2(k_{bx}x).$$  \hspace{1cm} (5)

The value of the specific energy of the liquid layer vibrating together with the area of the plate surface depends on the coordinate and is maximal at the areas of the plate with the antinode of bending vibrations. The phenomenon of the shift of the liquid layer over the vibrating plate surface towards the antinodes of vibrations indicates the presence of the hydrodynamic pressure gradient

$$\nabla p = -\frac{\rho}{4} \omega^2 (s_0)^2 \sin^2(k_{bx}x).$$  \hspace{1cm} (6)

The maximal value of the pressure gradient is on the layer wetting the areas of the plate surface in the center between the antinodes and nodes of vibrations. The increase of the amplitude of the plate vibrations leads to the appearance of two-dimensional capillary waves in the form of Faraday ripples on the surface of the liquid layer wetting the plate surface in the antinodes of vibrations. When the concave part of the plate is immersed in the liquid, two boundaries of the wetting liquid layer are formed, which envelope the plate surface from above and from below; one of the boundaries is formed near the free edge of the plate. The vertical shift of the plate makes it possible to change the distance from the boundary of the wetting liquid layer to the border of the plate free edge and to select the plate position so that the boundary of the liquid layer will be near the plate edge. When the plate bending vibrations are excited, the amplitude on the area of the free edge surface is maximal, and the same vibrations are passed to the liquid layer on the wetting boundary. It has experimentally been established that for the plate vibrations in liquid, on the phase boundary the parametric excitation of capillary waves takes place, the ridges of which are directed perpendicularly to the boundary of the wetting liquid layer and the plate edge (Figure 7). The width of the steel plate in Figure 7 is 8.8 mm, and that of the PET plate in Figure 8 is 4.6 mm. The calculations made for different frequencies of the excitation of capillary waves both by the PET plate and by the steel plate show that the distance between the ridges of waves is half of the length of a capillary wave, excited by the vibrations with a frequency two times lower than the frequency of the plate vibrations. In addition, an increase in the amplitude of the plate vibrations leads to the increase of the frequency range of the vibrations at which the standing capillary wave remains, which is characteristic of the excitation of parametric vibrations [7]. It has been noticed that the length of the two-dimensional standing capillary waves on the surface of the thin liquid layer on the plate surface on the area with the antinodes of vibrations is a little shorter than the length of the waves on the curved surface of the liquid nearby the plate edge.

The dependence of the length $\lambda$ of the capillary waves on the frequency $f$ of the excited vibrations can be obtained from the expression for the dependence of the circular frequency $\omega$ on the wave vector $k$ for capillary-gravity waves on the liquid surface [8]

$$\omega^2 = (gk + \sigma k^3/\rho) \tanh(kH),$$  \hspace{1cm} (7)

where $g$ is the acceleration due to gravity, $\sigma$ is the surface tension, $\rho$ is the density and $H$ is the liquid depth; the multiplier $\tanh(kH)$ is the hyperbolic tangent of the product of the wave vector and the depth.

For purely capillary waves excited by the vibrating plate, the first term on the right-hand side of the equation can be ignored and we have

$$\omega^2 = \frac{\sigma k^3}{\rho} \tanh(kH).$$  \hspace{1cm} (8)
Figure 7. Capillary waves on the water surface under the free edge of the steel plate at the vibrations with a frequency of 5.4 kHz.

Figure 8. Capillary waves on the water surface under the PET plate edge at the vibrations with a frequency of 4.7 kHz.

Taking into account that \( \omega = 2\pi f \) and \( k = 2\pi \lambda \), we obtain the following dependence of the capillary wave length on the frequency

\[
\lambda^3 = \frac{2\pi \sigma}{\rho f^2} \tanh(2\pi H/\lambda).
\] (9)

Equation 9 can be used for a thin liquid layer with the thickness \( H < \lambda/2 \), when the value \( \tanh(2\pi H/\lambda) < 1 \). In this case there is the necessity of the experimental determination of the thickness of the liquid layer, on the surface of which capillary waves are excited.

For capillary waves in deep liquid and in the layer of liquid with the thickness \( H \geq \lambda/2 \), the value \( \tanh(2\pi H/\lambda) = 1 \) and the relation

\[
\lambda^3 = \frac{2\pi \sigma}{\rho f^2}
\] (10)

is fulfilled. This equation is applicable to capillary waves on the liquid surface near the edges of the vibrating plate. The surface tension of water \( \sigma = 72.310^{-3} \) N/m and its density \( \rho = 1000 \) kg/m\(^3\), and alcohol has \( \sigma = 22.310^{-3} \) N/m and \( \rho = 790 \) kg/m\(^3\). Using these values we can obtain convenient analysis expressions for the length of a capillary wave in millimeters on the surface of water and alcohol and, respectively, \( \lambda = 0.77 f^{-2/3} \), \( \lambda = 0.56 f^{-2/3} \) where the frequency is given in kilohertz.

The parametric excitation of capillary waves in the liquid layer which is in contact with the vibrating plate surface is due to that at the vibrations of the plate the particles in the liquid get both impulse and acceleration

\[
\lambda^3 = \frac{2\pi \sigma}{\rho f^2}.
\] (11)

The volume of the liquid layer with the thickness \( H \) and the surface area \( \Delta S \) has the mass \( m = \rho \rho \Delta S H \). At the accelerated movement this mass of the liquid layer acts on the plate with the force equal to the force of inertia

\[
F = m \frac{d^2 s}{dt^2} = \rho \Delta S H \omega^2 s_0 \cos(k_{bx}x) \cos(\omega t).
\] (12)

Correspondingly, in the liquid layer at vibrations together with the plate, the pressure \( p_i \) of the forces of inertia appears, which in the antinode of the plate vibrations is

\[
p_i = F/\Delta S = \rho H \omega^2 s_0 \cos(\omega t).
\] (13)
The inertial force pressure in the liquid layer depends on the thickness in the direction of the vibrations and it periodically changes with the frequency of the plate area vibrations. For each period of vibrations with the beginning of the plate motion towards the liquid layer the pressure has a maximal positive value and at the moment, when the direction of the motion is from the liquid layer, the pressure has a maximal negative value. The inertial force pressure also produces an effect on the curved surface areas on the phase boundary; at the periodic variation of the inertial force pressure, the vibrations of the wetting layer boundaries are excited. Such process can be considered as a periodic variation of the area of the liquid free surface as the result of the action of additional periodically changing surface forces due to the inertial forces. This leads to the parametric excitation of capillary vibrations in the form of standing waves on the boundary of the liquid layer wetting the plate surface.

The increase of the amplitude of the plate vibrations causes the space modulation of capillary waves in the crosswise direction with the result that the capillary waves become two-dimensional (Figure 8). Moreover, if in the process of vibrations, the liquid layer boundary reaches the border of the plate edge, the curvature of the free surface of the liquid layer along the plate edge periodically changes and, as a result, the ridges of two-dimensional capillary waves are stretched up to the detachment of particles of the liquid. In general, this process leads to the atomization of the liquid in the layer wetting the surface of the vibrating plate. It is established that on the curved free surface under the edge of the vibrating plate, surface eddy flows appear on either side of the plate edge. The liquid surface in these flows moves in the opposite directions so that between them a rising stream appears moving under the edge of the plate in the direction perpendicular to this edge (Figure 9). The observed motion of the liquid can be explained by that the pressure of the liquid under its free surface near the vibrating plate is smaller than that under the undisturbed surface far from the plate. As has been mentioned above, the pressure difference is determined by the liquid specific energy got from the vibrating plate area.

![Figure 9](image1.png) **Figure 9.** The surface flow and capillary vibrations in the liquid under the vibrating plate edge.

![Figure 10](image2.png) **Figure 10.** The mechanism of the jet formation under the free edge of the vibrating plate.

The further increase of the amplitude of the plate bending vibrations leads to the periodic inertial detachment of relatively large droplets of the liquid from under the vibrating plate edge creating a jet. The formation of the liquid jet by the vibrating plate takes place as the result of the appearance of the surface flow on the curved surface of the liquid and the excitation of two-dimensional capillary waves on the moving liquid surface (Figure 10) at the same time.

In general, the discussed interactions of liquid and a vibrating plate excited with piezoelectric transducer are threshold. If the excitation of capillary waves on the piezoelectric transducer plate electrodes enough to apply a voltage of about 1 V, the generation of the jet occurs at an amplitude voltage of at least $15 \div 20\text{V}$. Figure 11 shows the regions of amplitude and frequency of voltage on the electrodes of the piezoelectric transducer, within which vibrating plate sizes specified excites capillary waves, produces spraying and forms a jet.
Figure 11. Regions of amplitudes and frequencies of electrode voltage of the piezoelectric transducer FML-20T-6.0A1-100 in case of excitation in water of capillary waves — 1, liquid atomization — 2 and generations of a jet — 3 by steel curved plate 40.0 × 8.6 × 0.1 mm.

4. Conclusion
The vibrations of a plate partially immersed in liquid excite capillary waves not only on the surface of the liquid layer wetting the plate surface, but also on the curved surface of the liquid near the plate edges. Since the capillary waves formed on the boundary of the liquid layer that wets the surface of the plate near its free edge are modulated by the plate vibrations, two-dimensional capillary waves appear. The capillary oscillations with the frequency of the plate vibrations periodically change the surface curvature of the wetting liquid layer near the plate edge from negative to positive. In addition, the plate vibrations cause the appearance of the hydrodynamic pressure in the vibrating liquid. As a result, the surface eddy flows appear in the liquid near the plate. The liquid flow under the free edge of the vibrating plate is directed up to the free edge. The overlapping of the pressure of the inertial forces and the Laplace pressure of the two-dimensional capillary waves on the surface flow generated by the plate vibrations results in periodic inertial detachment of the liquid particles from under the plate edge, and thus forms the jet.

4.1. Acknowledgments
The work is supported by the Russian Foundation for Basic Research (grant 16-41-180276_а).