Resonance Phenomena in the Macroscopic Quantum Tunnelling

S Rombetto$^{1,2}$, V Corato$^{1,3}$, Yu N Ovchinnikov$^4$, B Ruggiero$^1$ and P Silvestrini$^{1,3}$

$^1$Istituto di Cibernetica "E.Caianiello" del CNR, I-80078, Pozzuoli
$^2$Università degli Studi di Napoli "Federico II"
$^3$Dipartimento di Ingegneria dell’Informazione, Seconda Universitá di Napoli
$^4$L D Landau Institute for Theoretical Physics, Academy of Science, Moscow Russia
E-mail: sara@na.infn.it

Abstract. In this paper we present a theoretical approach to describe the quantum behaviour of a macroscopic system interacting with an external field at frequencies close to resonant condition. Moreover we apply our results to simulate resonant phenomena in rf SQUIDs, whose parameters lie in the range typically used in the experiments.

1. Introduction
In order to observe quantum effects in a macroscopic system, the quantum system must be essentially decoupled from the degrees of freedom describing the environment [1]. The interaction with an external thermal bath leads to the appearance of the finite width $\gamma$ of quantum levels [2, 3, 4, 5, 6] as well as to relaxation and decoherence effects. As well known, transitions between levels can occur either by thermal decay inside the same potential well or by tunnelling between resonant levels in different wells. Resonant phenomena can take place only if the energy width $\gamma$ is small compared to the energy difference between levels $\bar{\hbar}w_j$ ($\gamma \ll \bar{\hbar}w_j$), where $w_j$ is of the order of the Josephson plasma frequency.

When the states involved in the transition correspond to macroscopically distinct ones, as for instance different flux states in a rf-SQUID, the interacting energy states can be visualized as belonging to different wells of the double potential well and the transition process is characterized by tunnelling across the potential barrier. In such a case the non-zero width of levels leads to a reduction of coherence during the tunnelling process and so the behaviour of the system strictly depends on the ratio between the two parameters $\gamma$ and $T_N$, where $T_N$ is the tunnelling amplitude [7]. Coherent control of the quantum device state can be achieved by using microwave irradiation through the bias lines to induce transitions from the ground state to higher energy states [8, 9, 10, 11, 12, 13, 14, 15].

We present a theoretical approach to describe the quantum behaviour of a macroscopic system interacting with an external field at frequencies close to resonant condition, whose parameters lie in the range typically used in the experiments. The analysis is done by using density matrix formalism. All the calculations have been performed in the quasiclassical approximation, while the details are reported elsewhere [16]. The hamiltonian describing the system in the presence
of an external field with frequency $\omega$ and intensity $I$ is

$$H_0 = -\frac{1}{2M} \frac{\partial^2}{\partial \varphi^2} + U_0 \left[ \frac{1}{2M} (\varphi - \varphi_x^2) + \beta_L \cos \varphi \right] + \frac{I}{2e} \cos(\omega t) \varphi$$

where $\varphi_x$ is the external magnetic flux, $M$ is the 'mass' of the junction defined as

$$M = \left( \frac{\hbar}{2e} \right)^2 C$$

and $C$ is the junction capacitance. In eq.(1) quantities $U_0$, $\varphi_x$, $\beta_L$ are free parameters of the system and are defined as

$$U_0 = \left[ \frac{\Phi_0^2}{2\pi L} \right]$$

$$\beta_L = \frac{2\pi LI_c}{\Phi_0}$$

The simplest case is when $U_0$ and $\beta_L$ are constant, while $\varphi_x$ changes around a critical point $\varphi_x^{(0)}$. As usual, the dissipation is accounted for by an effective resistance $R$ in the RSJ model for the junction [1], [6].

2. The moderate underdamped regime and the extremely underdamped limit

First we consider the resonant tunnelling assisted by microwave irradiation under the condition $\gamma \gg T_N$, this corresponds to a dissipative system in the moderate underdamped quantum regime. This process can be visualized as a resonant pumping from the ground state to an excited level of the left potential well followed by resonant tunnelling from left to right well.

Moreover the condition $\gamma \gg T_N$ assures that the excited level in resonance with the microwave is not too close to the top of the potential barrier so that thermally induced decay between levels in the same well is the dominant transition with respect to the tunnelling process. The macroscopic tunnelling can really be observed only if the number of levels in the left potential well is not large (say less than 10) [17].

We also analyse the small viscosity limit, corresponding to the condition $\gamma \ll T_N$, referred as the extremely underdamped regime. When the two levels in different wells are in resonance, a
coherent superposition of distinct states occurs and a gap $\Delta$ appears between the energy levels. Then the wave functions spread all over the two wells, generating delocalized states with energies $E_{f_1}$ and $E_{f_2}$ (see Fig.1).

For the numerical simulations [16] we used the following parameters:

- $\beta_L = 1.75$
- $C = 0.1 \text{ pF}$
- $L = 210 \text{ pH}$
- $R = 6 \text{ M}\Omega$

and pumping frequencies ranging from $\nu = \omega/2\pi = 25.1 \text{ GHz}$ to $\nu = 25.8 \text{ GHz}$.

These parameters assure that the excited pumped level in the left potential well is close to some level in right potential well.

The transition probability $W$ from left potential well to the right potential well has been numerically calculated for a rf-SQUID as function of the external flux $\phi_x$ for both regimes. In the moderate underdamped regime the transition probability $W$ shows two peaks (see Fig.2): the first is connected with the resonance tunnelling and the second one is associated with the resonance pumping.

![Figure 2. Transition probability $W$ vs. $\phi_x$ for different values of the pumping frequency. The two peaks are due to the resonant tunnelling between levels in different wells and to the resonant pumping between levels in the same well. Curves are obtained by using the following parameters: $\beta_L=1.75$, $L=210 \text{ pH}$, $C=0.1 \text{ pF}$, and $R=6 \text{ M}\Omega$.](image)

In the extremely underdamped limit it’s possible to observe three peaks in the transition probability distribution [16]. In fact, for a fixed value of the pumping $\omega$, one maximum is connected with resonance tunnelling and two others with the resonant pumping of the two levels close to the barrier top (see Fig.3).

Note that in considered problems friction has a relevant role in both regimes since it leads to a finite width $\gamma$ of levels and destroys the coherence by tunnelling [7]. In both regimes the position of peaks strictly depends on the pumping frequency and on the external flux biasing the rf-SQUID.

3. Conclusion
The distribution function has been calculated by varying $\phi_x$ and for different values of the pumping frequency $\nu$. The predicted phenomena can be observed by escape rate measurements.
Figure 3. Transition probability $W$ vs. $\varphi_x$ for different values of the pumping frequency. $W$ presents three peaks: one is due to the resonance tunnelling and two others are due to the resonant pumping of the two levels close to the barrier top. Curves are obtained by using the following parameters: $\beta_L=1.75$, $L=210$ pH, $C=0.1$ pF, and $R=6$ MΩ.

[9]. Work is in progress to realize new experiments on the resonant phenomena in rf-SQUID-based device in the presence of external microwave irradiation.

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