Goal programming approach to fully fuzzy fractional transportation problem

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ABSTRACT
In this paper, authors devoted to study a fully fuzzy fractional multi-objective transportation problem by using goal programming approach. Also trapezoidal membership functions are applied to each objective function and constraints to describe a each fuzzy goal. A numerical example is provided to illustrate the efficiency of the multi-objective proposed approach.

1. Introduction
Transportation problem is a special type of linear programming problem (LPP), which deals with shipping commodities from sources to destinations. The basic transportation problem was originally developed by Hitchcock [1]. Depending on the nature of the cost function the transportation problem can be categorized into linear and non-linear transportation problems. In real life situations, all the transportation problems are not single objective. A special type of LPP in which constraints are of equality type and all the objectives are conflicting with each other is called multi-objective transportation problem (MOTP). Buckley and Feuring, in [2], considered the fully fuzzified linear programming problem (FFLP) by establishing all the coefficients and variables of a linear programme as being quantities. They transform the fully fuzzified programming problem into a multi-objective deterministic problem which, treated in the general case, is non-linear.

Fractional programming is important to our daily life, because various optimization problems from engineering, social life and economy consider the minimization of a ratio between physical and/or economical functions, for example cost/time, cost/volume, cost/profit, or other quantities that measure the efficiency of a system is minimized. Fractional linear programmes have a richer set of objective functions. In contrast a linear fractional programming is used to achieve the highest ratio of outcome to cost, the ratio representing the highest efficiency.

Transportation problem comprise a special class of linear fractional programming. In a typical problem of this type of trucking company may be interested in finding the least expensive way of transporting each unit of large quantities of a product from a number of warehouses to a number of stores. The main reason for interest in fractional programming from the fact that linear fractional objective functions occur frequently as a measure of performance in a variety of circumstances such as when satisfying objectives under uncertainty.

The general format of linear fractional problem (LFP) may be written as

\[
\max Z = \frac{c^T x + \alpha}{d^T x + \beta}, \quad c, d \in \mathbb{R}^n, \alpha, \beta \in \mathbb{R}
\]

subject to \(Ax = b, \quad x \geq 0, \quad x \in \mathbb{R}^n. \quad (1)

Transportation problem with fractional objective function are widely used as performance measures in many real life situations.

Goal programming (GP) is a branch of multi-objective optimization, which in turn is a branch of multi-criteria decision analysis (MCDA) also known as multi-criteria decision making. This is an optimization programme. It can be thought of as an extension or generalization of linear programming to multiple, normally conflicting objective measures. Each of these measures is given a GP variant used. A form of linear programming that allows for consideration of multiple goals. GP can be used to determine the optimal solution to a multi-objective decision making problem. GP requires the decision maker to set an aspiration level for each goal which can be a very difficult task as there are...
several of uncertainties in nature must be considered. Lee and Moore [3] applied GP to find a solution of MOTP. Anukokila and Radhakrishnan [4] studied a GP approach for solving multi-objective fractional transportation problem by representing the parameters \((y, \delta)\) in terms of interval valued fuzzy numbers.

On the other hand, GP method has the ability to deal with piecewise linear function with appropriate linearization of the constraints. The point that optimizes a single objective GP determines the point that best satisfies the set of goals in the decision problem. GP attempts to minimize the deviations from the goals. Thus, the objective function contains mainly the deviational variables representing each goal or sub-goal. The deviational variable is represented in two dimensions (a positive and a negative deviation from each sub-goal and/or constraint) in the objective function. GP has been widely applied to solve different real world problems which involve multiple objectives [5–8].

Motivated by Pop and Stancu-Minasian [9], in this paper the author’s extended to solve the fully fuzzy fractional transportation problem (FFFTP) with GP approach and there by the primary objective is thus to introduce the concept and a computational method for solving FFFTP, by using Kerre’s method to evaluate a fuzzy constraint. The method of variation change on the under-and-over deviational variables of the membership goals associated with the fuzzy goals of the model is introduced to solve the problem efficiently by using a linear goal programming methodology. Moreover, the achievement of FFFTP, all the parameters and variables are considered here as trapezoidal fuzzy numbers. Lingo [10] software package is used to solve the optimization problem.

The rest of the paper is organized as follows. Section 2 discuss about the review of the proposed problem. Section 3 provides problem formulation of MOTP and GP formulations. Section 4 presents the preliminaries of trapezoidal fuzzy numbers. In Section 5, FFFTP technique are developed to deriving fuzzy goal programming (FGP) approach also Kerre’s method to apply fuzzy inequalities for fuzzy numbers. An illustrative example provided and the optimal solution is compared with the proposed approach in Section 6.

2. Literature review

Fractional programming has been studied by many researchers Chang [11], Pal et al. [7] and Stanojevic and Stancu-Minasian [12]. To come into a conclusion how important this area of research is we further investigated and noticed that there are entire books and chapters devoted to this subject by researchers by Craven [13], Stancu-Minasian [14]. Stancu-Minasian’s text book [15] contains the state of the art theory and practice of fractional programming. FGP approach studied by Mohamed [6] is an important technique in dealing with conflicting objectives of decision makers for satisfying decision for overall benefit of the organization. Pramanik [16] and Gang [5] for solving MOTP with FGP approach. Pop and Stancu-Minasian [17] proposed a method to solve the fully fuzzified linear fractional programming problem, where all the variables and parameters are represented by triangular fuzzy number. Fuzzy set theory was proposed by Zadeh [18] and has been found extensive in various fields. Ammar [19] and Ebrahimnejad [20] studied multi-objective transportation with fuzzy numbers. Amarpreetkaur and Amit Kumar [21] discussed new approach for solving transportation problem using generalized trapezoidal fuzzy number. Recently Kumar Das and Mandal [22] proposed a new approach for solving fully fuzzy linear fractional programming problems by using multi-objective programming problems. Gupta et al. [23] discussed the exact fuzzy optimal solution of unbalanced fully fuzzy MOTPs using Mehar’s method.

3. Problem formulation

The proposed fuzzy mathematical model programming is based on the following assumptions.

Index Set

- \(i\) = source index for all \(i = 1, 2, \ldots, m\).
- \(j\) = destination index for all \(j = 1, 2, \ldots, n\).
- \(k\) = goals index for all \(k = 1, 2, \ldots, K\).

Parameters

- \(x_{ij}\) = number of units transported from source \(i\) to destination \(j\).
- \([C^{ij}_k, D^{ij}_k]\) = per unit cost of transporting from source \(i\) to destination \(j\).
- \(G_k\) = aspiration level of the \(k\)th goal.
- \([\alpha_k, \beta_k]\) = Confidence level of the \(k\)th goal.
- \(U_k\) = Upper tolerance limit for the \(k\)th fuzzy goal.
- \(L_k\) = Lower tolerance limit for the \(k\)th fuzzy goal.

Variables

- \(D^+_k\) = positive deviational variable.
- \(D^-_k\) = negative deviational variable.

Objective function

\[
\text{minimize } F^k(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} C^{ij}_k x_{ij}, \quad k = 1, 2, \ldots, K.
\]
Multi-objective fractional fuzzy transportation problem is
\[
\text{minimize } F_k(x) = \alpha_k + \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}^k x_{ij} = N(x), \quad k = 1, 2, \ldots, K.
\]
Goal programming objective transportation problem is
\[
\text{minimize } F_k(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}^k x_{ij} = \frac{D(x)}{G_k}, \quad k = 1, 2, \ldots, K.
\]

Constraints on supply available for each source \(i\):
\[
\sum_{j=1}^{n} x_{ij} \leq a_i, \quad \forall i.
\]

Constraints on demand for each destination \(j\):
\[
\sum_{i=1}^{m} x_{ij} \geq b_j, \quad \forall j.
\]

Constraints on GP transportation problem:
\[
C_{ij}^k x_{ij} - G_k = D_k^+ - D_k^-, \quad \forall k = 1, 2, \ldots, K
\]
\[
D_k^+ - D_k^- \geq 0, \quad \forall k = 1, 2, \ldots, K
\]
\[
\sum_{j=1}^{n} x_{ij} = A_i, \quad i = 1, 2, \ldots, m
\]
\[
\sum_{i=1}^{m} x_{ij} = B_j, \quad j = 1, 2, \ldots, n
\]
\[
x_{ij} \geq 0, \quad \forall i,j.
\]

Constraints on weighted FGP:
\[
\frac{F_k(x) - l_k}{g_k - l_k} + D_k^- - D_k^+ = 1,
\]
\[
\frac{u_k - F_k(x)}{u_k - g_k} + D_k^- - D_k^+ = 1,
\]
\[
\lambda + D_k^- - D_k^+ \leq 1,
\]
\[
\lambda \geq 0.
\]

Non-negative constraints on decision variables:
\[
x_{ij} \geq 0, \quad \forall i,j.
\]

4. Trapezoidal fuzzy numbers

**Definition 4.1:** A trapezoidal fuzzy number \(\bar{Z}\) is a structure \((z^1, z^2, z^3, z^4) \in \mathbb{R}^4\). The membership function of the trapezoidal number \(\bar{Z}\) is defined in terms of the real numbers \((z^1, z^2, z^3, z^4)\) as follows:
\[
\bar{Z}(y) = \begin{cases} 
0, & y \in (\infty, z^1), \\
\frac{y - z^1}{z^2 - z^1}, & y \in [z^1, z^2], \\
1, & y \in [z^2, z^3], \\
\frac{y - z^3}{z^4 - z^3}, & y \in (z^3, z^4], \\
0, & y \in (z^4, \infty).
\end{cases}
\]

**Definition 4.2 (The principle of extension):** Let us consider the function \(f : X_1 \times X_2 \times \cdots \times X_n \to Y\) defined on a Cartesian product of non-fuzzy sets by values in a non-fuzzy set. Considering the fuzzy sets \(\mathcal{A}_i \subseteq X_i, i = 1, 2, \ldots, n\), the result of applying function \(f\) to the fuzzy sets \(\mathcal{A}_i, i = 1, 2, \ldots, n\), is a fuzzy set \(B\) in \(Y\) described by the membership function \(\mu_B : Y \to [0, 1]\),
\[
\mu_B(y) = \max\{\min\{\mu_{A_i}(x_i) / i = 1, 2, 3, \ldots, n\} / f(x_1, x_2, \ldots, x_n) = y\},
\]
where \(\subseteq\) means the fuzzy inclusion relation.

The principle of extension was formulated by Zadeh [18] in order to extend the known models implying fuzzy elements to the case of fuzzy entities. Applying this principle of extension, the following definitions of the operations with trapezoidal fuzzy number follow.

Given two trapezoidal fuzzy numbers \(\bar{A} = (a^1, a^2, a^3, a^4), \bar{B} = (b^1, b^2, b^3, b^4)\), \(a^1, a^2, a^3, a^4, b^1, b^2, b^3, b^4 > 0\), we have:

(i) the addition of two trapezoidal fuzzy numbers
\[
\bar{A} + \bar{B} = (a^1 + b^1, a^2 + b^2, a^3 + b^3, a^4 + b^4);
\]

(ii) the symmetry of a trapezoidal fuzzy number \(-\bar{A} = (-a^1, -a^2, -a^3, -a^4)\);

(iii) the multiplication of two trapezoidal fuzzy numbers
\[
\bar{A} \cdot \bar{B} = (a^1 b^1, a^2 b^2, a^3 b^3, a^4 b^4);
\]

(iv) the division of two trapezoidal fuzzy numbers
\[
\frac{\bar{A}}{\bar{B}} = (a^1/b^1, a^2/b^2, a^3/b^3, a^4/b^4);
\]

(iv) the inverse of trapezoidal fuzzy number
\[
\bar{1}/\bar{B} = \left(\frac{1}{b^2}, \frac{1}{b^3}, \frac{1}{b^4}\right) \text{ where } \bar{1} = (1, 1, 1, 1).
5. Fully fuzzified fractional transportation problem

Let us consider the fully fuzzified fractional transportation problem

\[
\begin{align*}
\text{minimize } & \quad F^k(x) = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} x_{ij} + \alpha_k \left/ \sum_{i=1}^{m} \sum_{j=1}^{n} D_{ij} x_{ij} + \beta_k \right. \quad = N(x) \\
\text{subject to } & \quad \sum_{i=1}^{m} x_{ij} = A_i^k = 0, \quad j = 1, 2, \ldots, n, \quad \sum_{j=1}^{n} x_{ij} = B_j^k = 0, \quad i = 1, 2, \ldots, m, \quad \sum_{i=1}^{m} A_i^k = \sum_{j=1}^{n} B_j^k, \\
& \quad x_{ij} \geq 0, \quad (2)
\end{align*}
\]

where \(C_{ij}, \alpha_k\) and \(D_{ij}, \beta_k\) represent the coefficients of the fractional transportation objective function. \(A_i^k, B_j^k\) represents the right-hand side of the linear constraints, respectively. The most often used fuzzy numbers are so-called trapezoidal fuzzy numbers, we denote by \([A_i^k, B_j^k]\), where \(k = 1, 2, 3, 4\).

A major difficulty of solving fractional programming is that it is a highly non-linear fractional programming problem. In order to solve the problem, the membership function \(\mu_k\) for the \(k\)th fuzzy goal \(F^k(x) \geq G_k\), can be expressed as follows:

\[
\mu_k(F^k(x)) = \begin{cases} 
1 & \text{if } F^k(x) \geq G_k, \\
\frac{F^k(x) - L_k}{G_k - L_k} & \text{if } L_k \leq F^k(x) \leq G_k, \\
0 & \text{if } F^k(x) \leq L_k,
\end{cases}
\]

where \(L_k\) is the lower tolerance limit for the \(k\)th fuzzy goal \((G_k - L_k)\) is the tolerance which is subjectively chosen. Again the membership function \(\mu_k\) for the \(k\)th fuzzy goal \(F^k(x) \leq G_k\), can be expressed as follows:

\[
\mu_k(F^k(x)) = \begin{cases} 
1 & \text{if } F^k(x) \leq G_k, \\
\frac{U_k - F^k(x)}{U_k - G_k} & \text{if } G_k \leq F^k(x) \leq U_k, \\
0 & \text{if } F^k(x) \geq U_k,
\end{cases}
\]

where \(U_k\) is the upper tolerance limit for the fuzzy goal and \((U_k - G_k)\) is the tolerance which is subjectively chosen. In fuzzy programming technique

\[
\begin{align*}
\frac{F^k(x) - L_k}{G_k - L_k} + D_k^- - D_k^+ = 1, \\
\frac{U_k - F^k(x)}{U_k - G_k} + D_k^- - D_k^+ = 1,
\end{align*}
\]

where \(D_k^- (\geq 0)\) and \(D_k^+ (\geq 0)\) with \(D_k^- D_k^+ = 0\) represent the under-and-over deviations, respectively, from the aspired level. In this approach, only the under-deviational variable \(D_k^-\) is required to be minimized to achieve the aspired levels of the fuzzy goals.

In order to solve the problem, we assume that \(L_k \leq F^k(x) \leq U_k\), where \(L_k\) and \(U_k\) are upper and lower bounded of \(F^k(x)\), respectively. Introducing FGP technique the achievement of highest membership value of goal can be represented as follows:

\[
\mu_k(F^k(x)) = \begin{cases} 
1 & \text{if } F^k(x) \geq U_k, \\
\frac{F^k(x) - L_k}{U_k - L_k} & \text{if } L_k \leq F^k(x) \leq U_k, \\
0 & \text{if } F^k(x) \leq L_k.
\end{cases}
\]

Equation (3) can be expressed as the following FGP problem:

\[
\begin{align*}
\text{minimize } & \quad D_k^- - D_k^+ \\
\text{subject to } & \quad \frac{F^k(x) - L_k}{U_k - L_k} + D_k^- - D_k^+ = 1, \\
& \quad \sum_{i=1}^{m} x_{ij} = A_i^k, \quad \sum_{j=1}^{n} x_{ij} = B_j^k, \quad x_{ij} \geq 0, \quad (4)
\end{align*}
\]

where \(D_k^-\) and \(D_k^+\) are respectively. In conventional programming, the under and/or over deviational variables are included in the achievement function minimizing then and that depend upon the type of the objective functions to be optimized.

5.1. Kerre’s method for fuzzy numbers

Kerre’s method [24] defines a fuzzy-max which is problem dependent. Kerre’s provides the result \(M_1 > M_2\). Kerre’s method would favour a fuzzy set with smaller area measurement, regardless of its relative location on the x-axis. It gives a ranking order \(M_3 > M_2 > M_1\). Kerre’s method seems better than Yager’s method and this method not logically sound either. Kerre’s method would result in a smaller Hamming distance between \(M_1\) and the fuzzy-max. Therefore \(M_1 > M_2\), which is against the obvious fact that \(M_2 > M_1\).

**Definition 5.1:** For any two fuzzy numbers \(\tilde{M}\) and \(\tilde{N}\) having their membership functions \(\tilde{M}(x)\) and \(\tilde{N}(y)\) respectively, \(\tilde{M}(x) \leq \tilde{N}(y)\) if and only if

\[
d(\tilde{N}, \max(\tilde{M}, \tilde{N})) \leq d(\tilde{M}, \max(\tilde{M}, \tilde{N})),
\]

where \(d\) represents the Hamming distance between the expressions of the membership functions, which is

\[
d(\tilde{M}, \tilde{N}) = \int_{-\infty}^{+\infty} |(\tilde{M}(x) - \tilde{N}(y))| \, dx \quad \text{and} \quad \max(\tilde{M}, \tilde{N})(z) = \max(\min(\tilde{M}(x), \tilde{N}(y))),
\]

according to Zadeh’s extension principle.
The main concept of comparison of fuzzy numbers is based on the comparison of areas determined by membership functions. We shall operate a system of constraints which has to be satisfied by the components of a trapezoidal fuzzy numbers in order to be considered negative (in the fuzzy Kerre meaning). We apply this way of defining an inequality between fuzzy numbers to be trapezoidal fuzzy numbers \( M \) and \( 0 \). Now we describe the inequality of trapezoidal fuzzy numbers

\[
(m_1, m_2, m_3, m_4) \leq (0, 0, 0, 0) \tag{5}
\]

through a system of deterministic disjunctive constraints. Applying Kerre’s method to (2), the given problem is reduced to the following deterministic multi-objective fractional transportation problem with disjunctive constraints.

**Proposition 5.2:** A trapezoidal fuzzy number \( M = (m_1, m_2, m_3, m_4) \) relation (5) holds if and only if the following system of disjunctive constraints is satisfied:

\[
m_4 \leq 0
\]

(or)

\[
m_1 \leq 0 \leq m_2
\]

\[
m_1(m_1 + m_2 + m_3 + m_4) \leq m_2 m_3 m_4
\]

(or)

\[
m_2 \leq 0 \leq m_3
\]

\[
m_3(m_1 + m_2 + m_3 + m_4) \leq m_1 m_2 m_4
\]

(or)

\[
m_3 \leq 0 \leq m_4
\]

\[
m_4(m_1 + m_2 + m_3 + m_4) \leq m_1 m_2 m_3.
\]

**Proposition 5.3:** The inequality \( (m_1, m_2, m_3, m_4) \leq (0, 0, 0, 0) \) holds if and only if the following system of disjunctive deterministic constraints:

\[
m_4 \leq 0 \quad \text{(or)}
\]

\[
(m_3 \leq 0 \leq m_4) \cap ((m_1 + m_2)(m_3 - m_4)) \leq (m_3^2 + m_4^2)
\]

\[
(m_2 \leq 0 \leq m_3) \cap (m_3 m_4 \leq m_1 m_2)
\]

\[
(m_1 \leq 0 \leq m_2) \cap ((m_2 - m_1)(m_3 + m_4)) \leq (m_2^2 + m_2^2).
\]

In the next section, we use the above considerations to solve the fully fuzzified fractional transportation problems.

### 5.2. Solving method for FFTP

First we transform problem (2) into a fully fuzzified fractional transportation problem using the Charnes-Cooper [25] transformation \( (\sum_{i=1}^m \sum_{j=1}^n D_{ij}^k x_{ij} + \beta_k = 1/T_k \) and \( x_{ij}^k = y_{ij}^k \)) and obtain the following problem:

\[
\text{minimize } \left( \frac{\sum_{i=1}^m \sum_{j=1}^n c_{ij}^k y_{ij}^k + \alpha_k T_k}{\sum_{i=1}^m \sum_{j=1}^n c_{ij}^k y_{ij}^k + \alpha_k T_k} \right)
\]

\[
\text{subject to } \sum_{j=1}^n y_{ij}^k - A_{ij}^k T_k \geq 0,
\]

\[
\sum_{i=1}^m y_{ij}^k - B_{ij}^k T_k \leq 0,
\]

\[
\sum_{i=1}^m \sum_{j=1}^n D_{ij}^k y_{ij}^k + \beta_k T_k = 1,
\]

\[
T_k \geq 0.
\]

After aggregating the fuzzy quantities we change the objective function, which is described by a trapezoidal fuzzy number, and obtain the following deterministic multi-objective fractional transportation problem with fuzzy constraints:

\[
\text{minimize } \left( \sum_{i=1}^m \sum_{j=1}^n c_{ij}^k y_{ij}^k + \alpha_1 t^1, \sum_{i=1}^m \sum_{j=1}^n c_{ij}^k y_{ij}^2 + \alpha_2 t^2, \sum_{i=1}^m \sum_{j=1}^n c_{ij}^3 y_{ij}^3 + \alpha_3 t^3, \sum_{i=1}^m \sum_{j=1}^n c_{ij}^4 y_{ij}^4 + \alpha_4 t^4 \right)
\]

\[
\text{subject to } \sum_{j=1}^n y_{ij}^1 - a_1^1 t^1, \sum_{j=1}^n y_{ij}^2 - a_1^2 t^2, \sum_{j=1}^n y_{ij}^3 - a_1^3 t^3, \sum_{j=1}^n y_{ij}^4 - a_1^4 t^4 = 0, \quad j = 1, 2, \ldots, n
\]

\[
\sum_{i=1}^m y_{ij}^1 - b_1^1 t^1, \sum_{i=1}^m y_{ij}^2 - b_1^2 t^2, \sum_{i=1}^m y_{ij}^3 - b_1^3 t^3, \sum_{i=1}^m y_{ij}^4 - b_1^4 t^4 = 0, \quad j = 1, 2, \ldots, n
\]

\[
\left( \sum_{i=1}^m \sum_{j=1}^n d_{ij}^1 y_{ij}^1 + \beta_1 t^1, \sum_{i=1}^m \sum_{j=1}^n d_{ij}^2 y_{ij}^2 + \beta_2 t^2, \sum_{i=1}^m \sum_{j=1}^n d_{ij}^3 y_{ij}^3 + \beta_3 t^3, \sum_{i=1}^m \sum_{j=1}^n d_{ij}^4 y_{ij}^4 + \beta_4 t^4 \right)
\]

\[= 1,
\]

\[0 \leq t^1 \leq t^2 \leq t^3 \leq t^4, \quad 0 \leq y_{ij}^1 \leq y_{ij}^2 \leq y_{ij}^3 \leq y_{ij}^4, \quad i,j = 1, 2, \ldots, m,n. \tag{7}
\]
Solutions \((y^1_j, y^2_j, y^3_j, y^4_j)_{ij=1,2,\ldots,m,n}\) and \((t^1, t^2, t^3, t^4)\) are obtained, namely the trapezoidal fuzzy number \((Y^k_{ij})_{ij=1,2,\ldots,m,n}\) and \(T^k\). Then the optimal solution of problem (6) is \((x^k = \frac{y^k_j}{t^k}, i,j = 1,2,\ldots,m,n)\). Using a symmetrical definition for trapezoidal fuzzy numbers \(M = (m - 1, m, m + 1, m + 2)\).

By applying Proposition 5.2 to the inequality of index \(i\) in Equation (7), we obtain the following disjunctive system of constraints

\[
R_{ij} = R^1_{ij} \cup R^2_{ij} \cup R^3_{ij} \cup R^4_{ij},
\]

where

\[
R^1_{ij} = \begin{cases}
\sum_{i=1}^{m} \sum_{j=1}^{n} y^4_{ij} - a_i^1 t^1 \leq 0, \\
\sum_{i=1}^{m} \sum_{j=1}^{n} y^1_{ij} - b_i^1 t^1 \leq 0, \\
0 \leq t^1 \leq t^2 \leq t^3 \leq t^4, 0 \leq y^1_{ij} \leq y^2_{ij} \leq y^3_{ij} \leq y^4_{ij}.
\end{cases}
\]

\[
R^2_{ij} = \begin{cases}
\sum_{i=1}^{m} \sum_{j=1}^{n} y^1_{ij} - a_i^4 t^4 \leq 0 \leq \sum_{i=1}^{m} \sum_{j=1}^{n} y^2_{ij} - a_i^3 t^3, \\
\sum_{i=1}^{m} \sum_{j=1}^{n} y^3_{ij} - b_i^4 t^4 \leq 0 \leq \sum_{i=1}^{m} \sum_{j=1}^{n} y^4_{ij} - a_i^2 t^2, \\
\sum_{i=1}^{m} \sum_{j=1}^{n} [y^1_{ij} - (a_i^4 t^4 + b_i^4 t^4)] - (a_i^3 t^3 + b_i^3 t^3) + \sum_{i=1}^{m} \sum_{j=1}^{n} y^2_{ij} - (a_i^2 t^2 + b_i^2 t^2) \\
+ \sum_{i=1}^{m} \sum_{j=1}^{n} y^3_{ij} - (a_i^1 t^1 + b_i^1 t^1), \\
i,j = 1,2,\ldots,m,n
\end{cases}
\]

\[
R^3_{ij} = \begin{cases}
\sum_{i=1}^{m} \sum_{j=1}^{n} [y^1_{ij} - (a_i^4 t^4 + b_i^4 t^4)] - (a_i^3 t^3 + b_i^3 t^3) + \sum_{i=1}^{m} \sum_{j=1}^{n} y^2_{ij} - (a_i^2 t^2 + b_i^2 t^2) \\
+ \sum_{i=1}^{m} \sum_{j=1}^{n} [y^3_{ij} - (a_i^1 t^1 + b_i^1 t^1)], \\
i,j = 1,2,\ldots,m,n
\end{cases}
\]

\[
R^4_{ij} = \begin{cases}
\sum_{i=1}^{m} \sum_{j=1}^{n} [y^2_{ij} - (a_i^3 t^3 + b_i^3 t^3)] - (a_i^2 t^2 + b_i^2 t^2) + \sum_{i=1}^{m} \sum_{j=1}^{n} y^3_{ij} - (a_i^1 t^1 + b_i^1 t^1) \\
\sum_{i=1}^{m} \sum_{j=1}^{n} [y^3_{ij} - (a_i^3 t^3 + b_i^3 t^3)] - (a_i^2 t^2 + b_i^2 t^2) + \sum_{i=1}^{m} \sum_{j=1}^{n} y^4_{ij} - (a_i^1 t^1 + b_i^1 t^1), \\
i,j = 1,2,\ldots,m,n
\end{cases}
\]
By applying Proposition 5.3 to the equality in (8), we obtain the disjunctive system of constraints

\[ R_0 = R_0^1 \cup R_0^2 \cup R_0^3 \cup R_0^4, \]

where

\[
R_0^1 = \left\{ \begin{array}{l}
\sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} y_{ij} + \beta_1 t^1 = 1, \\
\sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} y_{ij} + \beta_2 t^2 = 1, \\
\sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} y_{ij} + \beta_3 t^3 = 1, \\
\sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} y_{ij} + \beta_4 t^4 = 1, \\
0 \leq t^1 \leq t^2 \leq t^3 \leq t^4, 0 \leq y_{ij}^0 \\
y_{ij}^0 \leq y_{ij}^1 \leq y_{ij}^2. 
\end{array} \right. 
\]

\[
R_0^2 = \left\{ \begin{array}{l}
\sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} y_{ij} + \beta_0 t^1 \leq 1, \\
\sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} y_{ij} + \beta_0 t^2 \geq 1, \\
\sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} y_{ij} + \beta_0 t^3 \leq 1, \\
\left( \sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} y_{ij} + \beta_0 t^1 \right) \left( \sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} y_{ij} \\
+ d_{ij} y_{ij}^3 + d_{ij} y_{ij}^4 + d_{ij} y_{ij}^5 + \beta_0 t^1 + \beta_0 t^2 \\
+ \beta_0 t^3 + \beta_0 t^4 - 4 \right) \\
= \left( \sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} y_{ij} + \beta_0 t^4 - 1 \right) \\
\sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} y_{ij}^3 + \beta_0 t^3 \\
\sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} y_{ij} + \beta_0 t^2 \\
0 \leq t^1 \leq t^2 \leq t^3 \leq t^4, 0 \leq y_{ij}^0 \leq y_{ij}^2 \\
y_{ij}^0 \leq y_{ij}^1 \leq y_{ij}^0, \quad i,j = 1,2,\ldots,m,n. 
\end{array} \right. 
\]

\[
R_0^3 = \left\{ \begin{array}{l}
\sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} y_{ij} + \beta_0 t^1 \leq 1, \\
\sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} y_{ij} + \beta_0 t^2 \geq 1, \\
\sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} y_{ij} + \beta_0 t^3 \leq 1, \\
\left( \sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} y_{ij} + \beta_0 t^1 \right) \left( \sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} y_{ij} \\
+ d_{ij} y_{ij}^3 + d_{ij} y_{ij}^4 + d_{ij} y_{ij}^5 + \beta_0 t^1 + \beta_0 t^2 \\
+ \beta_0 t^3 + \beta_0 t^4 - 4 \right) \\
= \left( \sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} y_{ij} + \beta_0 t^4 - 1 \right) \\
\sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} y_{ij}^3 + \beta_0 t^3 \\
\sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} y_{ij} + \beta_0 t^2 \\
0 \leq t^1 \leq t^2 \leq t^3 \leq t^4, 0 \leq y_{ij}^0 \leq y_{ij}^2 \\
y_{ij}^0 \leq y_{ij}^1 \leq y_{ij}^0, \quad i,j = 1,2,\ldots,m,n. 
\end{array} \right. 
\]

\[
R_0^4 = \left\{ \begin{array}{l}
\sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} y_{ij} + \beta_0 t^1 \leq 1, \\
\sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} y_{ij} + \beta_0 t^2 \geq 1, \\
\sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} y_{ij} + \beta_0 t^3 \leq 1, \\
\left( \sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} y_{ij} + \beta_0 t^1 \right) \left( \sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} y_{ij} \\
+ d_{ij} y_{ij}^3 + d_{ij} y_{ij}^4 + d_{ij} y_{ij}^5 + \beta_0 t^1 + \beta_0 t^2 \\
+ \beta_0 t^3 + \beta_0 t^4 - 4 \right) \\
= \left( \sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} y_{ij} + \beta_0 t^4 - 1 \right) \\
\sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} y_{ij}^3 + \beta_0 t^3 \\
\sum_{i=1}^{m} \sum_{j=1}^{n} d_{ij} y_{ij} + \beta_0 t^2 \\
0 \leq t^1 \leq t^2 \leq t^3 \leq t^4, 0 \leq y_{ij}^0 \leq y_{ij}^2 \\
y_{ij}^0 \leq y_{ij}^1 \leq y_{ij}^0, \quad i,j = 1,2,\ldots,m,n. 
\end{array} \right. 
\]
Consequently, problem (2) is reduced to the following deterministic multiple objective fractional transportation problem, subject to a conjunctive system of disjunctive non-linear constraints:

\[
\text{minimize } \left( \sum_{j=1}^{m} \sum_{i=1}^{n} c_{ij}^{1} y_{ij} + \alpha_1^{1} t^{1}, \right. \\
\left. \sum_{j=1}^{m} \sum_{i=1}^{n} c_{ij}^{2} y_{ij} + \alpha_2^{2} t^{2}, \right. \\
\left. \sum_{j=1}^{m} \sum_{i=1}^{n} c_{ij}^{3} y_{ij} + \alpha_3^{3} t^{3}, \right. \\
\left. \sum_{j=1}^{m} \sum_{i=1}^{n} c_{ij}^{4} y_{ij} + \alpha_4^{4} t^{4}, \right) \\
\text{subject to } \left( \bigcap_{i=1}^{n} \left( R_{ij}^{1} \cup R_{ij}^{2} \cup R_{ij}^{3} \cup R_{ij}^{4} \right) \right) \\
R_{ij}^{1} \left( y_{ij}^{0} \right) \leq \left( 1 - \delta_{ij}^{1} \right) M_{ij}^{1}, \\
R_{ij}^{2} \left( y_{ij}^{0} \right) \leq \left( 1 - \delta_{ij}^{2} \right) M_{ij}^{2}, \\
R_{ij}^{3} \left( y_{ij}^{0} \right) \leq \left( 1 - \delta_{ij}^{3} \right) M_{ij}^{3}, \\
R_{ij}^{4} \left( y_{ij}^{0} \right) \leq \left( 1 - \delta_{ij}^{4} \right) M_{ij}^{4}, \\
\left( 1 - \delta_{ij}^{1} \right) M_{ij}^{1} + \delta_{ij}^{2} + \delta_{ij}^{3} + \delta_{ij}^{4} \geq 1 \\
\delta_{ij}^{1}, \delta_{ij}^{2}, \delta_{ij}^{3}, \delta_{ij}^{4} \in \{0, 1\}, \\
\text{for each } i = 1, 2, \ldots, m \text{ and } 0 \leq y_{ij} \leq y_{ij}^{0} \leq y_{ij}^{0} \\
\leq y_{ij}^{0}, j = 1, 2, \ldots, n. \\
\tag{9}
\]

According to method described in Patkar and Stancu-Minasian [26], we shall consider the variable \((\delta_{ij}^{1})_{j=1,2,\ldots,m}\) to eliminate the disjunctive and to obtain (9) as system of conjunctive constraints. Solving problem (7)–(9) will allow us to obtain solution \((y_{ij}^{1}, y_{ij}^{2}, y_{ij}^{3}, y_{ij}^{4})_{j=1,2,\ldots,m, (t^{1}, t^{2}, t^{3}, t^{4})}, (\delta_{ij}^{1}, \delta_{ij}^{2}, \delta_{ij}^{3}, \delta_{ij}^{4})_{j=1,2,\ldots,m}\) namely the trapezoidal fuzzy numbers \((y_{ij}^{0})_{j=1,2,\ldots,m}\) and \((t^{1}, t^{2}, t^{3}, t^{4})\), which represents the solution of problem (6).

5.3. Solving FFFTP using GP approach

Applying the minmax form of GP to the fuzzy model of MOTP with the membership function leads the following model:

\[
\text{minimize } \phi \\
\text{subject to } \\
\mu_{k} F^{k}(x) + D_{k}^{0} - D_{k}^{+} = 1, \\
\sum_{j=1}^{m} y_{ij}^{k} - A_{ij}^{k} T^{k} \leq 0, j = 1, 2, \ldots, n, \\
\sum_{i=1}^{n} y_{ij}^{k} - B_{ij}^{k} T^{k} \leq 0, i = 1, 2, \ldots, m, \\
\sum_{i=1}^{m} \sum_{j=1}^{n} (D_{ij}^{k} y_{ij}^{k}) + (\beta_{k} T^{k}) + D_{k}^{0} - D_{k}^{+} = 4, \\
i, j = 1, 2, \ldots, m, n, \\
T^{k} \geq 0, Y_{k} \geq 0, \quad k = 1, 2, \ldots, K. \\
\tag{10}
\]

Now, let the tolerance limits of the two fuzzy objective goals be \((-1, -2\). The membership functions of the goals are obtained as

\[
\mu_{k}(F^{1}(x)) = \frac{\sum_{j=1}^{n} \sum_{i=1}^{m} c_{ij}^{1} y_{ij}^{k} + 1}{3}, \\
\mu_{k}(F^{2}(x)) = \frac{\sum_{j=1}^{n} \sum_{i=1}^{m} c_{ij}^{2} y_{ij}^{k} + 2}{6}. \\
\tag{11}
\]

Then the membership goals can be expressed as

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} \left( c_{ij}^{1} y_{ij}^{k} + 1 \right) + D_{i}^{0} - D_{i}^{+} = 1, \\
\sum_{i=1}^{m} \sum_{j=1}^{n} \left( c_{ij}^{2} y_{ij}^{k} + 2 \right) + D_{i}^{0} - D_{i}^{+} = 1, \\
\tag{12}
\]

where \(D_{i}^{0}, D_{i}^{+} \geq 0\) with \(D_{i}^{0} D_{i}^{+} = 0, i = 1, 2, 3, 4\).

6. Example

To illustrate the FGP approach, consider the following fractional MOTP as

\[
\text{minimize } F^{1}(x) = \frac{\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^{1} y_{ij}^{k} + \alpha_{1}}{\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^{1} y_{ij}^{k} + \beta_{1}}, \\
\text{minimize } F^{2}(x) = \frac{\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^{2} y_{ij}^{k} + \alpha_{2}}{\sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^{2} y_{ij}^{k} + \beta_{2}}, \\
\text{subject to } \\
\sum_{j=1}^{m} y_{ij}^{k} - A_{ij}^{k} T^{k} = 0, \quad j = 1, 2, 3, 4, \\
\sum_{j=1}^{m} y_{ij}^{k} - B_{ij}^{k} T^{k} = 0, \quad i = 1, 2, 3, 4, \\
T^{k} \geq 0, 0 \leq y_{ij}^{k} \leq y_{ij}^{0}, \quad k = 1, 2. \\
\tag{13}
\]
where
\[
\begin{bmatrix}
3 & 4 & 8 & 6 \\
10 & 11 & 12 & 13 \\
11 & 12 & 13 & 14 \\
4 & 5 & 6 & 7 \\
\end{bmatrix},
\begin{bmatrix}
5 & 6 & 7 & 8 \\
6 & 7 & 8 & 9 \\
8 & 9 & 10 & 11 \\
2 & 3 & 4 & 5 \\
\end{bmatrix},
\] (14)
and
\[
\begin{bmatrix}
-3 & -2 & -1 & 0 \\
7 & 8 & 9 & 10 \\
-6 & -5 & -4 & -3 \\
4 & 5 & 6 & 7 \\
0 & 1 & 2 & 3 \\
-4 & -3 & -2 & -1 \\
0 & 1 & 2 & 3 \\
\end{bmatrix},
\begin{bmatrix}
34 \\
34 \\
34 \\
34 \\
34 \\
34 \\
34 \\
\end{bmatrix}
\]
\[= \begin{bmatrix}
31 \\
31 \\
31 \\
31 \\
31 \\
31 \\
31 \\
\end{bmatrix},
\begin{bmatrix}
0.3041 \\
0.3041 \\
0.3041 \\
0.3041 \\
0.3041 \\
0.3041 \\
0.3041 \\
\end{bmatrix}, (15)
\]

\[A^k = [5 \ 6 \ 7 \ 8] \text{ and } B^k = [-3 \ -2 \ -1 \ 0].\]

\[\begin{bmatrix}
3y_{11}^1 + 10y_{12}^1 + 11y_{13}^1 + 4y_{14}^1 + 4y_{21}^1 \\
+11y_{22}^1 + 12y_{23}^1 + 5y_{24}^1 + 8y_{31}^1 \\
+13y_{32}^1 + 13y_{33}^1 + 6y_{34}^1 + 6y_{41}^1 \\
+13y_{42}^1 + 14y_{43}^1 + 7y_{44}^1 + 41y_{41}^1 \\
\end{bmatrix}
\] (3y_{11}^1 + 7y_{12}^1 - 6y_{14}^1 - 2y_{21}^1
+8y_{22}^1 - y_{23}^1 - 5y_{24}^1
-y_{31}^1 + 9y_{32}^1 - 2y_{33}^1 - 4y_{34}^1
+10y_{42}^1 - 3y_{43}^1 - 3y_{44}^1 + 41y_{41}^1)
\]

\[\begin{bmatrix}
\begin{bmatrix}
-31 & -31 & -31 & -31 \\
31 & 31 & 31 & 31 \\
31 & 31 & 31 & 31 \\
31 & 31 & 31 & 31 \\
\end{bmatrix}
\begin{bmatrix}
0.3041 \\
0.3041 \\
0.3041 \\
0.3041 \\
\end{bmatrix}, (15)
\]

\[\begin{bmatrix}
10y_{11}^1 + 10y_{12}^1 + 11y_{13}^1 + 4y_{14}^1 + 4y_{21}^1 \\
+11y_{22}^1 + 12y_{23}^1 + 5y_{24}^1 + 8y_{31}^1 \\
+13y_{32}^1 + 13y_{33}^1 + 6y_{34}^1 + 6y_{41}^1 \\
+13y_{42}^1 + 14y_{43}^1 + 7y_{44}^1 + 41y_{41}^1 \\
\end{bmatrix}
\] (3y_{11}^1 + 7y_{12}^1 - 6y_{14}^1 - 2y_{21}^1
+8y_{22}^1 - y_{23}^1 - 5y_{24}^1
-y_{31}^1 + 9y_{32}^1 - 2y_{33}^1 - 4y_{34}^1
+10y_{42}^1 - 3y_{43}^1 - 3y_{44}^1 + 41y_{41}^1)

Using fuzzy programming technique the optimal solution of the problem is obtained by using the following steps.

**Step 1:**
\[\begin{align*}
y_{11}^1 &= 0.1041, \quad y_{12}^1 = 0.2, \quad y_{13}^1 = 0.1719, \\
y_{21}^1 &= 0.1057, \\
y_{23}^1 &= 0.2347, \quad y_{24}^1 = 0.1735, \quad y_{31}^1 = 0.1619, \\
y_{33}^1 &= 0.2694, \\
y_{34}^1 &= 0.1752, \quad y_{41}^1 = 0.1090, \quad y_{43}^1 = 0.3041, \\
y_{44}^1 &= 0.1768.
\end{align*}\]

**Step 2:**
\[F^1(x) = 0.9090, \quad F^2(x) = 0.9298 \text{ (substituting } k = 2 \text{ in Equation (2)).}\]

We attach now to this problem to a fully fuzzy fractional transportation problem. Consider its real number coefficient \(m\) as being symmetric trapezoidal fuzzy number \(\mathcal{M}\) of spread 2, having the following form:
\[
\mathcal{M} = (m^1, m^2, m^3, m^4), \quad m^1 = m - 2, \quad m^2 = m - 1, \quad m^3 = m, \quad m^4 = m + 1.
\]

Using (2), we get the fully fuzzed fractional transportation problem as
\[
\begin{align*}
\text{minimize} & \quad F(x) \\
\text{subject to} & \quad \begin{align*}
& x_{11} \leq A^k, \quad x_{12} \leq A^k, \quad x_{13} \leq A^k, \quad x_{14} \leq A^k, \\
& x_{21} \leq A^k, \quad x_{22} \leq A^k, \quad x_{23} \leq A^k, \quad x_{24} \leq A^k, \\
& x_{31} \leq A^k, \quad x_{32} \leq A^k, \quad x_{33} \leq A^k, \quad x_{34} \leq A^k, \\
& x_{41} \leq A^k, \quad x_{42} \leq A^k, \quad x_{43} \leq A^k, \quad x_{44} \leq A^k,
\end{align*}
\end{align*}
\]

According to the method described in Section 5, in order to obtain the solution of the problem we solve the following multiple objective transportation problem (8)
still having fuzzy constraints using (14) & (15) we get

\[
\min F^1(x) = 3y_{11}^1 + 10y_{12}^1 + 11y_{13}^1 + 4y_{14}^1 + 4y_{21}^1 \\
+ 11y_{22}^1 + 12y_{23}^1 + 5y_{24}^1 + 8y_{31}^1 + 12y_{32}^1 \\
+ 13y_{33}^1 + 6y_{34}^1 + 6y_{41}^1 + 13y_{42}^1 \\
+ 14y_{43}^1 + 7y_{44}^1 + \alpha_1,
\]

sub to

\[
\begin{align*}
(y_{11}^1 - 4t_1^1, y_{12}^1 - 3t_2^1, y_{13}^1 - 5t_3^1, y_{14}^1 - 6t_4^1) & \geq 0, \\
(y_{21}^1 - 0, y_{22}^1 - 2t_2^1, y_{23}^1 - t_3^1, y_{24}^1 - 3t_4^1) & \geq 0, \\
(y_{31}^1 - t_1^1, y_{32}^1 - 2t_2^1, y_{33}^1 - 3t_3^1, y_{34}^1 - 4t_4^1) & \geq 0, \\
(y_{41}^1 - 0, y_{42}^1 - t_2^1, y_{43}^1 - 2t_3^1, y_{44}^1 - 4t_4^1) & \geq 0, \\
(y_{21}^1 - 0, y_{22}^1 - 2t_2^1, y_{23}^1 - t_3^1, y_{24}^1 - 4t_4^1) & \geq 0, \\
(y_{31}^1 - t_1^1, y_{32}^1 - 2t_2^1, y_{33}^1 - 3t_3^1, y_{34}^1 - 4t_4^1) & \geq 0, \\
(-3y_{11}^1 + 7y_{12}^1 - 6y_{14}^1 - 2y_{21}^1 + 8y_{22}^1 - y_{23}^1 \\
- 5y_{24}^1 - y_{31}^1 + 9y_{32}^1 - 2y_{33}^1 \\
- 4y_{34}^1 + 10y_{42}^1 - 3y_{43}^1 - 3y_{44}^1 + \beta_1) & \geq 0, \\
0, \quad i, j = 1, 2, 3, 4.
\end{align*}
\]

Similarly we solve \( F^2(x) \) by the same procedure. By evaluating the fuzzy constraints with Kerre’s method, described in Section 5, we obtain the following equivalent system of disjunctive constraints:

\[
\begin{align*}
[R_1 \cup (R_2 \cap R_3) \cup (R_4 \cap R_5 \cap R_6) \cup (R_7 \cap R_8 \cap R_9 \cap R_{10})] & \\
\cap [R_{11} \cup (R_{12} \cap R_{13} \cap R_{14} \cap R_{15})] & \\
\cap [(R_{16} \cap R_{17} \cap R_{18}) \cup (R_{19} \cap R_{20})] & \\
\cap [(R_{21} \cap R_{22} \cap R_{23} \cap R_{24}) & \\
\cup ((R_{25} \cap R_{26} \cap R_{27} \cap R_{28}) \cup (R_{29} \cap R_{30} \cap R_{31} \cap R_{32})] & \\
\cap [R_{33} \cap R_{34} \cap R_{35} \cap R_{36} \cap R_{37} \cap R_{38} & \\
\cap R_{39} \cap R_{40} \cap R_{41} \cap R_{42} \cap R_{43} \cap R_{44}]. &
\end{align*}
\]

In order to obtain a synthesis function of the form objective function from (16) and applying to it the results presented in Stancu-Manasian [27] we use the importance coefficients \( \pi_1 = 0.1, \pi_2 = 0.3 \) respectively.

Now obtain the optimal values

\[
F^1(x) = 0.8333; \quad F^2(x) = 0.5285
\]

with

\[
y_{11} = 1, \quad y_{12} = 2.6667, \quad y_{14} = 1.6667 \quad \text{and} \quad y_{21} = 0.4285, \quad y_{23} = 0.2857, \quad y_{24} = 1.8571,
\]

\[
y_{31} = 1.7142, \quad y_{34} = 1.5714.
\]

The optimum of the synthesis function \( (\pi_1 F^1, \pi_2 F^2) \) is reached in \((0.0833, 0.1585)\). Therefore the fuzzy number \( F^1(x) = 0.0833, F^2(x) = 0.1585 \).

Step 3. Using (9), the membership goal can be restated as

\[
\minimize \phi
\]

subject to

\[
\begin{align*}
3y_{11} + 10y_{12} + 11y_{13} + 4y_{14} + 4y_{21} \\
+ 11y_{22} + 12y_{23} + 5y_{24} + 8y_{31} + 12y_{32} \\
+ 13y_{33} + 6y_{34} + 6y_{41} + 13y_{42} \\
+ 14y_{43} + 7y_{44} + \alpha_1
\end{align*}
\]

subject to

\[
\begin{align*}
(y_{11} - 4t_1, y_{12} - 3t_2, y_{13} - 5t_3, y_{14} - 6t_4) & \geq 0, \\
(y_{21} - 0, y_{22} - 2t_2, y_{23} - t_3, y_{24} - 3t_4) & \geq 0, \\
(y_{31} - t_1, y_{32} - 2t_2, y_{33} - 3t_3, y_{34} - 4t_4) & \geq 0, \\
(y_{41} - 0, y_{42} - t_2, y_{43} - 2t_3, y_{44} - 4t_4) & \geq 0, \\
(-3y_{11} + 7y_{12} - 6y_{14} - 2y_{21} + 8y_{22} - y_{23} \\
- 5y_{24} - y_{31} + 9y_{32} - 2y_{33} \\
- 4y_{34} + 10y_{42} - 3y_{43} - 3y_{44} + \beta_1) & \geq 0, \\
0, \quad i, j = 1, 2, 3, 4.
\end{align*}
\]

The problem was solved by the linear interactive global optimization (LINGO) software the compromise solution is presented as follows:

\[
\mu_1(F^1(x)) = 1.0667,
\]

\[
\mu_2(F^2(x)) = 0.0952 \quad \text{(substitute} \quad k = 2 \text{in step 3)}
\]

with \( y_{11} = 2.3, \quad y_{12} = 0.3, \quad y_{13} = 3.667, \)

\[
y_{22} = 0.2, \quad \lambda = 1, \quad D_k = 0, \quad \phi = 1,
\]

\[
y_{32} = 0.1, \quad y_{41} = 2. \quad
\]

\[
y_{21} = 1, \quad y_{23} = 1.333, \quad y_{24} = 1.09,
\]

\[
y_{22} = 0.97, \quad y_{23} = 2.07, \quad y_{24} = 0.3, \quad y_{31} = 2.3095,
\]

\[
y_{32} = 0.9523, \quad y_{33} = 2.047, \quad y_{34} = 0.2857.
\]

Thus the trapezoidal fuzzy numbers are very closely to the real numbers (i.e. unknowns) 0 and 1 approximately.
7. Conclusion

In fuzzy programming technique, decision maker can obtain a satisfying solution. The proposed approach allows using of the fully fuzzy fractional transportation problem and Kerre’s method in the process of obtaining the preferred solution with the decision making process. The main contribution of this proposed paper is the GP approach for solving multi-objective fractional transportation problem with trapezoidal fuzzy number. The FGP solution procedure provides the most satisfactory solution for all the decision makers at both the levels by reaching the aspired levels of the membership goals. The method can be easily implemented to solve any non-linear multi-objective programming problem.

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