Sterile Neutrinos in $E_6$ and a Natural Understanding of Vacuum Oscillation Solution to the Solar Neutrino Puzzle

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(May, 1999)

Abstract

If Nature has chosen the vacuum oscillation solution to theSolar neutrino puzzle, a key theoretical challenge is to understand the extreme smallness of the $\Delta m^2_{\nu_e-\nu_X} (\sim 10^{-10} \text{eV}^2)$ required for the purpose. We find that in a class of models such as $[SU(3)]^3$ or its parent group $E_6$, which contain one sterile neutrino, $\nu_{is}$ for each family, the $\Delta m^2_{\nu_i-\nu_{is}}$ is proportional to the cube of the lepton Yukawa coupling. Therefore fitting the atmospheric neutrino data then predicts the $\nu_e-\nu_{es}$ mass difference square to be $\sim \left(\frac{m_e}{m_{\mu}}\right)^3 \Delta m^2_{\text{atmos}}$, where the atmospheric neutrino data is assumed to be solved via the $\nu_{\mu}-\nu_{\mu s}$ oscillation. This provides a natural explanation of the vacuum oscillation solution to the solar neutrino problem.

I. INTRODUCTION

As is widely known by now, the Super-Kamiokande data has provided conclusive evidence for the existence of oscillations of the muon neutrinos from cosmic rays [1]. While it is yet to be determined what final state the cosmic ray $\nu_{\mu s}$ oscillate to ($\nu_{\tau}$ or $\nu_{\mu s}$), it is known that the mixing angle is near maximal and the $\Delta m^2_{\text{atmos}} \sim 10^{-3} \text{eV}^2$. Similarly the solar neutrino data from Super-Kamiokande and other experiments [2] are also making a very convincing case for oscillations of the electron neutrinos emitted by the Sun in order to understand the observed deficit of the solar neutrinos [4]. Again, in this case also, it is not clear what final state the $\nu_e$ oscillates to on its way from the Sun to Earth. It could either be $\nu_{\mu}$ or $\nu_{es}$. There are however several mixing angle and mass difference possibilities in the solar case [3]. One of the possibilities is the vacuum oscillation of $\nu_e - \nu_{\mu}$ or $\nu_e - \nu_{es}$ type. In order to explain the observations one needs in this case that the $\Delta m^2_{\nu_e-\nu_X} \sim 10^{-10} \text{eV}^2$ and a maximal mixing like in the atmospheric neutrino case. The recent indications of a seasonal dependence of the solar neutrino events in the 708 day Super-Kamiokande data [4] would seem to support this explanation although it is by no means the only way to understand it [5]. If the vacuum oscillation explanation finally wins, then a serious theoretical challenge is to understand the unusually small mass difference squared between the neutrinos needed for the purpose. It is the goal of this letter to propose a way to answer this challenge within a gauge theory framework.
The first observation that motivates our final scenario is the symmetry between the solution to the atmospheric neutrino data and the vacuum oscillation solution to the solar neutrino data in that the mixing angles are maximal. This might suggest a generation independence of the neutrino mixings patterns. An implementation of such an idea would naturally require that in each case i.e. solar as well as atmospheric the active neutrinos (i.e. $\nu_e$ and $\nu_\mu$) oscillate into the sterile neutrinos $\nu_{es}$ to be denoted by $\nu_{es}$ and $\nu_{\mu s}$ respectively. The complete three family picture would then require that there be one sterile neutrino per family. One class of models that lead to such a scenario is the mirror universe picture where the particles and the forces in the standard model are duplicated in a mirror symmetric manner. There is no simple way to understand the ultra small $\Delta m^2$ needed for the vacuum oscillation solution in this case. In this letter we focus on an alternative scheme based on the grand unification group $[SU(3)]^3$ or its parent group $E_6$.

We find it convenient to use the $E_6$ notation. As is well-known under the $SO(10)$ group, the 27-dimensional representation of $E_6$ decomposes to $[16 + 10 - 2 + 1]_3$ where the subscripts represent the $U(1)$ charges. The 16 is well known to contain the left and the right handed neutrinos (to be denoted by us as $\nu_i$ and $\nu_{ci}$, i being the family index). The 10 contains two neutral colorless fermions which behave like neutrinos but are $SU(2)_L$ doublets and the last neutral colorless fermion in the 27, which we identify as the sterile neutrino is the one contained in 1 (denoted by $\nu_{is}$). In general in this model, we will have for each generation a $5 \times 5$ “neutrino” mass matrix and we will show how the small masses for the sterile neutrino and the known neutrino come out as a consequence of a generalized seesaw mechanism. Furthermore, we will see how as a consequence of the smallness of the Yukawa couplings of the standard model, we will not only get maximal mixing between the active and the sterile neutrinos of each generation but also the necessary ultra-small $\Delta m^2$ needed in the vacuum oscillation solution without fine tuning of parameters. The way this comes about in our model is that to the lowest order in the Yukawa couplings, the $\nu_i$ and $\nu_{is}$ form a Dirac neutrino with a mass proportional to the generational Yukawa coupling $\lambda_i$ of the corresponding generation. However they become pseudo-Dirac to order $\lambda_i \ll 1$ leading to nearly degenerate neutrinos with a mass splitting $\Delta m^2 \approx \lambda_i^3$. Therefore fixing the $\Delta m^2_{atmos}$ gives the right value for the $\Delta m^2$ needed for the vacuum oscillation solution.

Let us now present the basic idea of the model for one generation of neutrinos consisting of $\nu_i, \nu_{is}$. Suppose that their mass matrix is given by the following $2 \times 2$ matrix:

$$M_i = m_{0i} \left( \begin{array}{cc} \lambda_i^2 & \lambda_i f_i \\ \lambda_i \bar{f}_i & \lambda_i^2 \bar{\epsilon}_i \end{array} \right)$$

Since $\lambda_i \ll 1$, it is clear that the two neutrinos are maximally mixed with a mass $m_i \simeq \lambda_i f_i m_{0i}$ and $\Delta m^2_i \simeq \lambda_i^3 f_i m_{0i}^2$ provided $\bar{f}_i \simeq 1$. These relations are true generation by generation. We shall show in the next section that a mass matrix of this form emerges naturally from $E_6$ and its subgroup $[SU(3)]^3$ with $\bar{f}_i \approx 1$. The main difference between

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1 An $E_6$ model for the neutrino puzzles was first discussed by Ma [10], where his goal was to understand the smallness of the sterile neutrino masses. Our model is different in many respects and addresses the question of maximal mixing, small $\Delta m^2$'s as well as the small neutrino masses. Our picture also differs from other recently proposed models [11].
these two groups arises from the fact that for the simplest models based on $E_6$ the Yukawa coupling $\lambda_i$ is necessarily related to the Yukawa coupling of the corresponding up type quark. This is not true for $[SU(3)]^\text{3}$, for which $\lambda_i$ is expected to be related to the Yukawa coupling of the corresponding lepton.

Now let us look at the atmospheric neutrino data. Since $\Delta m_2^2 \approx 2 - 5 \times 10^{-3} \text{ eV}^2$, if we choose the $\nu_\mu$ mass to be of order $0.2 - 0.5 \text{ eV}$ (anticipating that we want to accomodate the LSND data [12]), then we get $\lambda_2 \approx 10^{-2}$, which is a typical second generation Yukawa coupling. Note that this is a plausible value even for $[SU(3)]^\text{3}$ since in supersymmetric models $m_\mu \approx \lambda_2 v_d$ and $v_d$ can be considerably less ( e.g 10 GeV for a $\tan\beta \approx 24$) than the standard model value of 246 GeV for the symmetry breaking parameter. Since the same formula applies to the $\nu_e$ and $\nu_{\text{es}}$ sector, assuming no large flavour dependence in the coefficients $f_i$ and $\epsilon_i$, all we need in order to predict their masses and mass differences is the value for $\lambda_1/\lambda_2$. Irrespective of whether we consider $E_6$ or $[SU(3)]^\text{3}$ we find $\lambda_1/\lambda_2 \approx 5 \times 10^{-3}$. This leads to a value for the $\Delta m_1^2 \approx 2 \times 10^{-10} \text{ eV}^2$, which is clearly of the right order of magnitude. Our main point is not to insist on precise numbers but rather to illustrate the idea that a cubic dependence of the neutrino mass difference squares on the generational Yukawa couplings to leptons of the standard model can lead to an understanding of the extreme smallness of $\Delta m^2$ value needed in the vacuum oscillation solution to the solar neutrino puzzle [13].

Extending our idea to the third generation, we find that value of the $\nu_\tau$ mass is $(\lambda_3/\lambda_2)^2 0.2 \text{ eV}$. This implies $m_{\nu_\tau} \approx 2 - 3 \text{ eV}$ for $[SU(3)]^\text{3}$ which is interesting for cosmology since this would mean that about 10-15% of the mass of universe could come from neutrinos. This expectation can eventually be tested when the finer measurements of the angular power spectrum is carried out in the MAP and PLANCK experiments in the next few years. However for minimal versions of $E_6$, $\lambda_3/\lambda_2 \approx m_t/m_c$ yielding a value $m_{\nu_\tau} \approx 20 - 30 \text{ eV}$ which is unacceptable for a realistic cosmology. This means that the simplest $E_6$ model that can accomodate our scenario is one where the quark lepton symmetry is broken. The other possibility is to have some flavour dependence on the coefficients $\tilde{f}_i$ and $m_0 i$. The extent of required flavour dependence is certainly not extreme and we consider models based on both groups as realistic candidates for a complete theory of neutrino masses.

II. THE MODEL

Let us now proceed to construct the mass matrix in Eq. (1) in the context of an $E_6$ model. As usual, we will assign matter to the $27$ dimensional representation of the group and we have already noted that there are five neutrino-like fields in the model which will mix among each other subsequent to symmetry breaking. It is therefore necessary to describe the symmetry breaking of $E_6$. To implement the symmetry breaking we use three pairs of $27 + \bar{27}$ representations and one $78$-dim. field. The pattern of symmetry breaking is as follows:

1) $<27_1>$ and $<\bar{27}_1>$ have GUT scale vevs in the SO(10) singlet direction.
2) $<27_{16}>$ and $<\bar{27}_{16}>$ have GUT scale vevs in the $\nu$ and $\nu^c$ directions respectively. They break SO(10) down to SU(5).
3) The $<78_{[1,45]>}$ completes the breaking of SU(5) to the standard model gauge group at the GUT scale. We assume the VEVs reside both in the adjoint and in the singlet of SO(10).
4) $< 27_{10} >$ and $< \overline{27}_{10} >$ contain the Higgs doublets of the MSSM. It is assumed that $H_u$ and $H_d$ are both linear combinations arising partially from the $< 27_{10} >$ and partially from the $< \overline{27}_{10} >$.

In addition to the above there is another field labelled by $27'$ whose $\nu^c$ component mixes with a singlet $S$ and one linear combination of this pair (denoted by $S'$) remains light below the GUT scale. As a consequence of radiative symmetry breaking this picks up a VEV at the electroweak scale. We will show later how this can occur. The remaining components of $27'$ have GUT scale mass.

Let us now write down the relevant terms in the superpotential that lead to a $5 \times 5$ “neutrino” mass matrix of the form we desire. To keep matters simple let us ignore generation mixings, which can be incorporated very trivially.

\[
W = \lambda_i \psi_i \psi_i 27_{10} + f_i \psi_i \psi_i 27' + \frac{\alpha_i}{M_{Pl}} \psi_i \psi_i 27_1 78_{[1, 45]} + \frac{\gamma_i}{M_{Pl}} \psi_i \psi_i \overline{27}_{16} 27_{16}
\]  

We have chosen only a subset of allowed terms in the theory and believe that it is reasonable to assume a discrete symmetry (perhaps in the context of a string model) that would allow only this subset. In any case since we are dealing with a supersymmetric theory, radiative corrections will not generate any new terms in the superpotential.

Note that in Eq. (2), since it is the first term that leads to lepton and quark masses of various generations, it carries a generation label and obeys a hierarchical pattern, whereas the $f_i$’s not being connected to known fermion masses need not obey a hierarchical pattern. We will from now on assume that each $f_i \approx 1$, and see where it leads us.

After substituting the VEVs for the Higgs fields in the above equation, we find a $5 \times 5$ mass matrix of the following form for the neutral lepton fields of each generation in the basis ($\nu, \nu_s, \nu^c, E_0^u, E_0^d$):

\[
M = \begin{pmatrix}
0 & 0 & \lambda_i v_u & f_i v' & 0 \\
0 & 0 & 0 & \lambda_i v_d & \lambda_i v_u \\
\lambda_i v_u & 0 & M_{\nu^c,i} & 0 & 0 \\
f_i v' & \lambda_i v_d & 0 & 0 & M_{10,i} \\
0 & \lambda_i v_u & 0 & M_{10,i} & 0
\end{pmatrix}
\]  

Here $M_{\nu^c,i}$ is the mass of the right handed neutrino and $M_{10,i}$ is the mass of the entire 10-plet in the $27$ matter multiplet. Since 10 contains two full SU(5) multiplets, gauge coupling unification will not be effected even though its mass is below the GUT scale.

Note that the $3 \times 3$ mass matrix involving the $(\nu^c, E_0^u, E_0^d)$ have superheavy entries and will therefore decouple at low energies. Their effects on the spectrum of the light neutrinos will be dictated by the seesaw mechanism [14]. The light neutrino mass matrix involving $\nu_i, \nu_{is}$ can be written down as:

\[
M_{light} \approx \frac{1}{M_{\nu^c,i}} \begin{pmatrix}
\lambda_i v_u & f_i v' & 0 \\
0 & \lambda_i v_d & \lambda_i v_u \\
\end{pmatrix} \begin{pmatrix}
1 & 0 & 0 \\
0 & 0 & \epsilon \\
0 & \epsilon & 0
\end{pmatrix} \begin{pmatrix}
\lambda_i v_u & 0 \\
f_i v' & \lambda_i v_d \\
0 & \lambda_i v_u
\end{pmatrix}
\]

$^2$Although the form of this mass matrix is same as in [10], the results of our paper are different.
where $\epsilon_i = M_{10,i}/M_{\nu_c,i}$. Note that $\epsilon_i$ is expected to be of order one. This leads to the $2 \times 2$ mass matrix for the $(\nu, \nu_c)$ fields of each generation which is of the form in Eq. (1),

$$M_i = m_{0i} \left( \begin{array}{cc} \lambda_i^2 & \lambda_i \bar{f}_i \\
\lambda_i \bar{f}_i & \lambda_i^2 \bar{\epsilon}_i \end{array} \right)$$

(5)

Here $m_{0i} = \frac{v^2}{M_{\nu_c,i}}$, $\bar{f}_i = f_i \epsilon / v_u$, and $\bar{\epsilon}_i = 2 \epsilon_i \cot \beta$. Taking $M_{Pl} \sim 10^{19} GeV$, $M_{GUT} \sim 10^{16}$ and reasonable values of the unknown parameters e.g. $\alpha_i \approx 0.1$, $\gamma_i \approx 0.1$, $f_i \approx 1$, $v' \approx v_u$, we get $m_{0i} \approx 20$ eV and $\epsilon \approx 1$ which leads us to the desired pattern of masses and mass differences outlined in the introduction.

A crucial assumption in our analysis is that one of the Higgs fields has a vev along $\nu_c$ direction with a low scale (the $v'$ above). We will now demonstrate what kind of a superpotential can lead to such a situation.

Consider

$$W = M 27' \overline{27'} + S 27' 27_{16}$$

(6)

Since $27_{16}$ has a VEV, this implies that one linear combination of $S$ and the $\nu_c$ component of $27'$ (denoted by $S'$) remains light while everything else in $27'$ and $\overline{27'}$ become heavy. If in addition the superpotential contains the couplings

$$W = S 27_{10} \overline{27}_{10} + S^3$$

(7)

since $27_{10}$ and $\overline{27}_{10}$ have electroweak scale VEVs the light combination of $S$ and $27'$ ($S'$) also picks up an electroweak scale VEV from the trilinear soft supersymmetry breaking terms. Note that this is inevitable as long as electroweak symmetry is broken because such a trilinear term then becomes a linear term in the potential for $S'$ and hence $S'$ must pick up a VEV. We thus see that it is possible to get vev for the singlet field $\nu_c$ in the desired $27$-plet of the order of the electroweak scale.

Let us next address the question of the generation mixing. We will assume that it parallels that in the quark sector i.e. the mixing angles to start with are small. Since the neutrino mixings have an additional contribution coming from their seesaw mechanism, we can easily have them be smaller than the corresponding quark mixings. This is for instance what one would like in order to fit the LSND data. We do not get into the details of this since clearly it does not effect the main point of the paper.

Let us end with a few comments on the phenomenological and cosmological implications of the model. The most severe test of this model will come from the understanding of big bang nucleosynthesis [15]. Our model within the standard assumptions that go into the discussion of BBN would imply $N_{\nu} = 6$ i.e. three extra neutrinos. However, in models with sterile neutrinos, possibilities of large lepton asymmetry at the BBN era has been discussed [16].

The second point that needs emphasizing is that in our model, both the solar and atmospheric neutrinos involve separate sterile neutrinos in the final state. There are well known tests [17] of such models for the atmospheric neutrino oscillations [18] where one looks for neutral pion production. For solar neutrinos, our model is testable by the neutral current measurement planned for the SNO experiment [19].
In conclusion, in this paper we have pointed out a simple way to understand theoretically challenging possibility of a tiny mass difference squared that may arise if the solar neutrino puzzle is to be solved via the vacuum oscillation solution. We exploit an apparent symmetry between the solar and the atmospheric case arising from the maximality of mixing angles to suggest that the ultra small $\Delta m^2_{\text{solar}}$ may be understandable in models of $E_6$ type, which automatically contain a sterile neutrino in each $27$ that also contains other known particles of each generation and where the generational neutrino mass difference squared may be proportional to the cube of the lepton Yukawa couplings. In this model we can also accommodate the indication for neutrino oscillations from LSND.

This work has been supported by the National Science Foundation grant under no. PHY-9802551.
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