Progress in Multibaryon spectroscopy

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Confinement XIII
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Lüscher Formalism

\[ \text{det} \left[ (M^\infty)^{-1} + \delta G^V \right] = 0 \]

Infinite volume scattering amplitudes

finite volume spectrum

+ boundary conditions

Physics of interest ← Lattice calculation

+ many others since

Lüscher Commun. Math. Phys. 104 and 105 (1986)
Lüscher Nucl. Phys. B354 (1991) 531-578
Wiese, Nucl. Phys. B Proceedings Supplements 9, 609 (1989)
Lüscher Nucl. Phys. B 354 (1991) 531; Nucl. Phys B 364 (1991) 237
Rummukainen and Gottlieb Nucl. Phys. B 450 (1995) 397-436

PoS(LATTICE2015)008 Hansen
PoS(LATTICE2016)016 Wilson
EPJ Web Conf. 175 (2018) 01016 Briceño
EPJ Web Conf. 175 (2018) 01022 Davoudi
EPJ Web Conf. 175 (2018) 05005 Morningstar/Bulava (Brett et al.)
Morningstar et al. Nucl. Phys. B924 (2017) 477-507
https://github.com/cjmorningstar10/TwoHadronsInBox/
A large and growing literature

The HALQCD Potential Method
  PRL 99 (2007) 022001
  Prog.Theor.Phys 123 (2010) 89-128
  Prog.Theor.Phys 124 (2010) 591-603
  Prog.Theor.Phys 125 (2011) 1225-1240
  PTEP 2012 01A105
  PLB 712 (2012) 437-441
  arXiv:1711.09344 (to appear in PRD)

Other/coupled channels:
  PLB673 (2009) 136-141
  Nucl. Phys. A928 (2014) 89-98
  PTEP 2015 071B01
  PRL 120 (2018) 212001
  Nucl. Phys. A 971 (2018) 113

Theory
  Proc.Jon.Acad.Ser. 87 (2011) 509-517
  PRD 87 (2013) 034512
  PRD 88 (2013) 014036

ππ
  Kurth et al. JHEP 1312 (2013) 015
  PTEP 2018 043B04

Spin-Orbit / Derivative Expansion
  PLB 735 (2014) 19-24
  arXiv:1805.02365

More Than 2 Baryons
  Prog.Theor.Phys. 127 (2012) 723-738

Applications
  PRL 111 (2013) 112503
  PRC 91 (2015) 0110001(R)

Tetraquarks
  PLB 729 (2014) 85-90
  PRL 117 (2016) 242001

and many others!
A large and growing literature

CalLat PLB 765:285-292 (2017)

$\pi$ mass

- $\sim 800$ MeV
  - NPLQCD PRD 87 (2013) 034506
  - Yamazaki et al. PRD 84 (2011) 054506
  - NPLQCD PRD 96 (2017) 114510

- $\sim 510$ MeV
  - Yamazaki et al. PRD 86 (2012) 074514

- $\sim 450$ MeV
  - NPLQCD PRD 92 (2015) 114512

- $\sim 300$ MeV
  - Yamazaki et al. PRD 92 (2015) 014501

H Dibaryon

- NPLQCD Mod.Phys.Lett. A26 (2011) 2587-2595
- NPLQCD Phys.Rev.Lett. 106 (2011) 162001
- HALQCD PRL 106 (2011) 162002
- HALQCD Nucl.Phys. A881 (2012) 28-43
- Green et al. PoS(LATTICE2014)107
- Junnarkar et al. PoS(LATTICE2015)082
- Francis et al. 1805.03966

method of baryon blocks

- Doi + Endres Comput. Phys. Commun 184 (2013) 117
- Detmold + Orginos PRD 87 (2013) 114512

matrix elements + transitions

- NPLQCD PRL 119 (2017) 062002
- NPLQCD PRL 119 (2017) 062003
- NPLQCD PRD 96 (2017) 054505
- NPLQCD PRL 120 (2018) 152002

signal-to-noise

- Parisi Phys. Rept. 103 203 (1984)
- Lepage, Boulder ASI 1989:97-120 (1989)
- NPLQCD PRD 79 (2009) 114502
- NPLQCD PRD 80 (2009) 074501
- NPLQCD PRD 81 (2010) 054505
- NPLQCD Prog.Part.Nucl.Phys. 66 (2011) 1-40
- NPLQCD PRD 96 (2017) 114508

DWF on MILC

- NPLQCD PRL 97 (2006) 012001

Mirage Plateaus and Sanity Checks

- HALQCD JHEP 1610 (2016) 101
- HALQCD PRD 96 (2017) 034521
- NPLQCD arXiv:1705.09239
| Software | References |
|----------|------------|
| METAQ | Berkowitz arXiv:1702.06122 [github.com/evanberkowitz/metaq]
| | Berkowitz et al. EPJ (LATTICE2017) 175 09007 (2018) |
| chroma | Edwards and Joo (SciDAC, LHPC and UKQCD Collaborations) Nucl. Phys. Proc. Suppl 140, 832 (2005) |
| QDP++ | Clark et al. Comput. Phys. Commun. 181 1517 (2010) |
| | Babich et al. Supercomputing 11, 70 |
| hdf5 in QDP++ | Kurth et al. PoS LATTICE2014 045 (2015) |
| qmp | Chen, Edwards, and Watson et al. 
[https://github.com/usqcd-software/qmp](https://github.com/usqcd-software/qmp) |
| mpi_jm | Berkowitz et al. EPJ (LATTICE2017) 175 09007 (2018) 
McElvain et al. [https://github.com/kenmcelvain/mpi_jm/](https://github.com/kenmcelvain/mpi_jm/) |
Lüscher Formalism

\[ \text{det} \left[ (M^\infty)^{-1} + \delta G^V \right] = 0 \]

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+ many others since

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Morningstar et al. Nucl. Phys. B924 (2017) 477-507
https://github.com/cjmorningstar10/TwoHadronsInBox/
Two-Nucleon Spectrum

• Spectrum given by effective mass of (schematic) NN correlator:

\[ \langle \Omega | \mathcal{O}[J' \ell' S']_{\Lambda' \mu', \text{Im} I} (t) \overline{\mathcal{O}[J \ell S]}_{\Lambda \mu, \text{Im} I} (0) | \Omega \rangle \]

• Box breaks rotational symmetry → spectrum falls into irreps of \( O_H \), not SO(3).

| Isospin 0 | Isospin 1 |
|-----------|-----------|
| Partial wave | Irreps | Partial wave | Irreps |
| \(^1P_1\) | \( T_1^- \) | \(^1S_0\) | \( A_1^+ \) |
| \(^3S_1, \, ^3D_1\) | \( T_1^+ \) | \(^3P_0\) | \( A_1^- \) |
| \(^3D_2\) | \( E^+ \oplus T_2^+ \) | \(^3P_1\) | \( T_1^- \) |
| \(^3D_3\) | \( A_2^+ \oplus T_1^+ \oplus T_2^+ \) | \(^3P_2, \, ^3F_2\) | \( E^- \oplus T_2^- \) |
| \(^1F_3\) | \( A_2^- \oplus T_1^- \oplus T_2^- \) | \(^1D_2\) | \( E^+ \oplus T_2^+ \) |
| \(^3F_3\) | \( A_2^- \oplus T_1^- \oplus T_2^- \) | \(^3F_4\) | \( A_1^- \oplus E^- \oplus T_1^- \oplus T_2^- \) |
Non-interacting States
Luu & Savage 1101.3347 (arXiv version is better!)

• Project to eigenstates of a noninteracting theory in a box.

• Full volume information $\rightarrow$ exactly project to any desired irrep

\[ n^2 = 0 \]
\[ n^2 = 1 \]
\[ n^2 = 2 \]
\[ n^2 = 3 \]
Non-interacting States

Luu & Savage 1101.3347 (arXiv version is better!)

- Project to eigenstates of a noninteracting theory in a box.
- Full volume information $\rightarrow$ exactly project to any desired irrep

\[
\begin{align*}
T_1^- \\
\text{\(n^2=1\)} & \quad \text{\(n^2=3\)} \\
\text{\(n^2=2\)} & \quad \text{\(n^2=4\)}
\end{align*}
\]
Two-Nucleon Spectrum

- Spectrum given by effective mass of (schematic) NN correlator:

\[
\langle \Omega \left| \mathcal{O}_{\lambda' \mu', \text{Im} I}^{J' \ell' S'}(t) \mathcal{O}_{\lambda \mu, \text{Im} I}^{J \ell S}(0) \right| \Omega \rangle
\]
Two-Nucleon Spectrum

- Spectrum given by effective mass of (schematic) NN correlator:

\[
\langle \Omega | \mathcal{O}[J'^\ell'S']_{\Lambda'\mu',\text{Im}I} (t) | \mathcal{O}[J^\ell S]_{\Lambda\mu,\text{Im}I} (0) | \Omega \rangle
\]

\[\delta G^V\]
Two-Nucleon Spectrum

- Spectrum given by effective mass of (schematic) NN correlator:

\[
\langle \Omega | \mathcal{O}_{\Lambda', \mu', \text{Im}_I}^{J', \ell', S'}(t) | \mathcal{O}_{\Lambda, \mu, \text{Im}_I}^{J, \ell, S}(0) | \Omega \rangle
\]

SINK

exact projection

SOURCE

exact projection

Francis et al. 1805.03966
H binding energy = 19\(\pm\)10 MeV
\(m_\pi \sim 960\) MeV SU(3)-symmetric via distillation

Method of baryon blocks:
Doi + Endres Comput. Phys. Commun 184 (2013) 117
Detmold + Orginos PRD 87 (2013) 114512
Two-Nucleon Spectrum

• Spectrum given by effective mass of (schematic) NN correlator:

$$\langle \Omega | \mathcal{O}[J' \ell' S']_{\Lambda' \mu', \text{Im}_I}(t) \mathcal{O}[J \ell S]_{\Lambda \mu, \text{Im}_I}(0) | \Omega \rangle$$

Schemes to avoid all-to-all source displacements single-baryon improvement
Spatially Displaced Two-Nucleon Operators
Source Overlap

• Exact projection source-side costs \((\text{volume})^2\)

• Pick displacements

| name    | \(\Delta x \sim\) | # solves |
|---------|-------------------|----------|
| local   | (0, 0, 0)        | 1        |
| face    | (0, 0, 1)        | 6        |
| edge    | (0, 1, 1)        | 12       |
| corner  | (1, 1, 1)        | 8        |
|         | (0, 1, 2)        | 24       |
|         | (1, 1, 2)        | 24       |
|         | (1, 2, 3)        | 48       |
Source Overlap
Luu & Savage 1101.3347 (arXiv version is better!)

Project Luu & Savage momentum sources to corner as a function of $\pi \Delta x/L$
Source Overlap
Luu & Savage 1101.3347 (arXiv version is better!)

Project Luu & Savage momentum sources to corner as a function of $\pi\Delta x/L$

Could get away with just 1 solve at the origin

\[ S \]
\[ P \]
\[ D \]
\[ F \]

$m_L=0$  $m_L=1$  $m_L=2$  $m_L=3$
Source Overlap
Luu & Savage 1101.3347 (arXiv version is better!)

Project Luu & Savage momentum sources to corner as a function of $\pi \Delta x / L$

Could get away with just 1 solve at the origin but would sacrifice higher partial waves.

$S$
$P$
$D$
$F$

$m_L = 0$  $m_L = 1$  $m_L = 2$  $m_L = 3$
Could get away with just 1 solve at the origin but would sacrifice higher partial waves.
Project Luu & Savage momentum sources to corner as a function of $\pi \Delta x / L$

2 solves: origin and $\frac{(L,L,L)}{2}$

higher partial waves still inaccessible.

$m_L=0$  $m_L=1$  $m_L=2$  $m_L=3$
\( I=1 \) P-wave

CalLat PLB 765:285-292 (2017)

\[ q^3 \cot \delta P_2^3/m_{\pi}^3 \]

\[ (q/m_{\pi})^2 \]

\[ \delta P_2^3 \]

\[ (q/m_{\pi})^2 \]

WP/JLab cfigs.

\[ m_{\pi} \sim 800 \text{ MeV} \]

\[ b \sim 0.145 \text{ fm} \]
Solid Geometry on the 3-Torus for Fun and Profit: Propagator Reuse

See also: EPJ Web Conf. 175 (2018) 05024 Wu et al.
Solid Geometry on the 3-Torus for Fun and Profit: Propagator Reuse

\[ O = (0,0,0) \]
Solid Geometry on the 3-Torus for Fun and Profit: Propagator Reuse

$O = (0,0,0)$

$A = (L,L,L)/2$
Solid Geometry on the 3-Torus for Fun and Profit: Propagator Reuse

O = (0,0,0)
A = (L,L,L)/2

See also: EPJ Web Conf. 175 (2018) 05024 Wu et al.
Solid Geometry on the 3-Torus for Fun and Profit: Propagator Reuse

O = (0,0,0)
A = (L,L,L)/2
C = (±1,±1,±1) Δx

See also: EPJ Web Conf. 175 (2018) 05024 Wu et al.
Solid Geometry on the 3-Torus for Fun and Profit: Propagator Reuse

$O = (0,0,0)$

$A = (L,L,L)/2$

$C = (±1,±1,±1) \Delta x$

10 local sources

See also: EPJ Web Conf. 175 (2018) 05024 Wu et al.
Solid Geometry on the 3-Torus for Fun and Profit: Propagator Reuse

O = (0,0,0)
A = (L,L,L)/2
C = (±1,±1,±1) Δx

10 local sources
OA maximally displaced

See also: EPJ Web Conf. 175 (2018) 05024 Wu et al.
Solid Geometry on the 3-Torus for Fun and Profit: Propagator Reuse

\[ O = (0,0,0) \]
\[ A = (L,L,L)/2 \]
\[ C = (±1,±1,±1) \Delta x \]

10 local sources
\( OA \) maximally displaced
\( OC \) corner(\( \Delta x \)) around \( O \)

See also: EPJ Web Conf. 175 (2018) 05024 Wu et al.
Solid Geometry on the 3-Torus for Fun and Profit: Propagator Reuse

\[ O = (0,0,0) \]
\[ A = (L,L,L)/2 \]
\[ C = (\pm1,\pm1,\pm1) \Delta x \]

10 local sources
OA maximally displaced
OC corner(\(\Delta x\)) around O
AC corner(\(L/2-\Delta x\)) around A

See also: EPJ Web Conf. 175 (2018) 05024 Wu et al.
Solid Geometry on the 3-Torus for Fun and Profit: Propagator Reuse

\[ O = (0,0,0) \]
\[ A = (L,L,L)/2 \]
\[ C = (\pm 1, \pm 1, \pm 1) \Delta x \]

10 local sources
\[ OA \text{ maximally displaced} \]
\[ OC \text{ corner}(\Delta x) \text{ around } O \]
\[ AC \text{ corner}(L/2-\Delta x) \text{ around } A \]

+ additional combinations of just C sources

See also: EPJ Web Conf. 175 (2018) 05024 Wu et al.
Solid Geometry on the 3-Torus for Fun and Profit: Propagator Reuse

\[ O = (0,0,0) \]
\[ A = (L,L,L)/2 \]
\[ C = (\pm 1, \pm 1, \pm 1) \Delta x \]

10 local sources
OA maximally displaced
OC corner(\(\Delta x\)) around O
AC corner(\(L/2-\Delta x\)) around A

+ additional combinations of just C sources

See also: EPJ Web Conf. 175 (2018) 05024 Wu et al.
Single-Nucleon Improvements
Calm Baryons via Matrix Prony

$y(t) = \begin{pmatrix} y_{PS}(t) \\ y_{SS}(t) \end{pmatrix}$

$am_{\text{eff}}(t)$

t/a

1.300
1.275
1.250
1.225
1.200

4 6 8 10 12 14

References:
- Doi + Endres Comput. Phys. Commun 184 (2013) 117
- Detmold + Orginos PRD 87 (2013) 114512
Calm Baryons via Matrix Prony

Assume a transfer operator

\[ y(t + \tau) = \hat{T}(\tau)y(t) \]

\[ y(t) = \begin{pmatrix} y_{PS}(t) \\ y_{SS}(t) \end{pmatrix} \]
Calm Baryons via Matrix Prony

EPJ Web Conf. 175 (2018) 01016 EB et al.

\[ y(t) = \begin{pmatrix} y_{PS}(t) \\ y_{SS}(t) \end{pmatrix} \]

Assume a transfer operator
\[ y(t + \tau) = \hat{T}(\tau)y(t) \]

Ansatz: 2 states meaningfully contribute from \( t_i \) to \( t_f \)
Calm Baryons via Matrix Prony

EPJ Web Conf. 175 (2018) 01016 EB et al.

\[
\begin{align*}
\text{am} & \text{eff}(t) \\
& \text{at } t/a \\
& \text{PS} \\
& \text{SS} \\
& \text{MP Result}
\end{align*}
\]
Calm Baryons via Matrix Prony

EPJ Web Conf. 175 (2018) 01016 EB et al.

+ another noisy state at ~ 1.9 ↑
NN with Calm Baryons

EPJ Web Conf. 175 (2018) 01016 EB et al.

$NN : A_1^+ : ^1S_0$

$\begin{align*}
\alpha_{NN}(t) & \equiv f_{NN}(t) \\
& \equiv f_{NN}(t)
\end{align*}$

WM/JLab cfgs.

$m_\pi \sim 800$ MeV

$b \sim 0.145$ fm

$n^2 = 0$

$n^2 = 1$

$n^2 = 2$
NN with Calm Baryons

EPJ Web Conf. 175 (2018) 01016 EB et al.

$NN : T_1^+ : {}^3S_1$

$\begin{align*}
&n^2 = 0 \\
&n^2 = 1 \\
&n^2 = 2
\end{align*}$

WM/JLab cfgs.
$m_\pi \sim 800 \text{ MeV}$
$b \sim 0.145 \text{ fm}$
Preliminary NN Results at $m_\pi \sim 350$ MeV
Möbius Domain Wall on HISQ
CallLat PRD96 (2017) 054513

- For $g_A$ and $\pi^- \rightarrow \pi^+ 0\nu\beta\beta$ we used MILC ensembles of $N_f=2+1+1$ HISQ
  - Follana et al. PRD 75 (2007) 054502
  - Bazavov et al. PRD82 (2010) 074501, PRD87 (2013) 054505

- DWF on asqtad quite successful
  - Renner at al. [LHPC] NPPS 140 (2005) 255-260
  - LHPC; NPLQCD; Aubin, Laiho, Van de Water; …

- Well-developed mixed-action EFT
  - Bar, Bernard, Rupak, Shoresh; Tiburzi; Chen, O'Connell, Van de Water, Walker-Loud; …

- We generated 10 000 thermalized $24^3 \times 64$ $m_\pi \sim 350$ MeV, $a \sim 0.12$ fm HISQ configurations ($m_\pi L = 5.1$)
  - milc

- Fantastic GPU MDWF solver in QUDA
  - Kim and Izubuchi PoS(LATTICE2013)033
  - and substantial subsequent enhancements

- (So far) one measurement each.
  - This means 10 solves on a single time slice!

PoS LATTICE2016 (2016) 017 Nicholson
Calm Nucleon

MDWF on HISQ
a12m350
$24^3 \times 64$
$m_n L = 5.1$
Source Study
$I=1 \ A_1^+ \ n^2=0 \ (^{1}S_0 \ \text{Dineutron})$

$\mathbf{O} \sim 3L/8 \ (\pm 1,\pm 1,\pm 1)$
$\mathbf{A} \sim 1L/8 \ (\pm 1,\pm 1,\pm 1)$

$\mathbf{OO} \ n^2 = 0$
Source Study

$I=1$ $A_{1+} n^2=0$ ($^1S_0$ Dineutron)

MDWF on HISQ
$a_{12m350}
24^3 \times 64$
$m_nL = 5.1$

$\mathbf{O} \sim 3L/8 (\pm 1, \pm 1, \pm 1)$
$\mathbf{AC} \sim 1L/8 (\pm 1, \pm 1, \pm 1)$
Source Study
$I=0 \ T_1^+ \ n^2=0 \ (^3S_1 \ \text{Deuteron})$

MDWF on HISQ
a12m350
$24^3 \times 64$
m$_\pi$L = 5.1

$\mathbf{O}$ $\mathbf{C}$ $\mathbf{A}$

$OC \sim 3L/8 \ (\pm 1, \pm 1, \pm 1)$
$AC \sim 1L/8 \ (\pm 1, \pm 1, \pm 1)$
Source Study

$I=0 \ T_1^+ \ n^2=0 \ (^3S_1 \ Deuteron)$

\[ \mathbf{I}_0 \ T_1^+ \ n^2=0 \ (^3S_1 \ Deuteron) \]

\[ \mathbf{O}_C \sim \frac{3L}{8} \ (\pm 1, \pm 1, \pm 1) \]

\[ \mathbf{A}_C \sim \frac{1L}{8} \ (\pm 1, \pm 1, \pm 1) \]
I=1 $^1S_0$ Dineutron

Dineutron binding energies at nearby $m_\pi$:
Yamazaki et al. $m_\pi \sim 300$ MeV $\quad 8.5(0.7)(^{+1.6}_{-0.5})$ MeV
NPLQCD $m_\pi \sim 450$ MeV $\quad 12.5(^{+3.0}_{-5.0})$ MeV

MDWF on HISQ
a12m350
$24^3 \times 64$
$m_\pi L = 5.1$
I=0 $^3S_1$ Deuteron

Deuteron binding energies at nearby $m_{\pi}$:
Yamazaki et al. $m_{\pi}\sim 300$ MeV $14.5(0.7)(^{+2.4}_{-0.8})$ MeV
NPLQCD $m_{\pi}\sim 450$ MeV $14.4(^{+3.2}_{-2.6})$ MeV
Outlook
Three Neutrons In A Box
Jan-Lukas Wynen, EB, Tom Luu, Andrea Schindler, John Bulava

$\pi \sim 368$ MeV
$a \sim 0.09$ fm
$48^3 \times 96$
175 measurements

CLS H107
RQCD PRD 94 (2016) 074501
Future Methods

Variational Methods

Distillation
Peardon et al. PRD 80 (2009) 054506

Francis et al. 1805.03966
H binding energy = 19±10 MeV
m_π ~ 960 MeV SU(3)-symmetric

Stochastic LapH
Morningstar et al. PRD 83 (2011) 114505

Harmonic-Oscillator Basis Effective Theory
McElvain and Haxton 1607.06863
McElvain APS April 2017 62 1 BAPS.2017.APR.C13.5

Finite Volume Matching

nEFT in Finite Volume
Barnea Few Body 22 (2018)
EFT for Lattice Nuclei
Barnea, Eliyahu, Bazak (forthcoming)

Andersen, Bulava, Hörz,
Morningstar, CalLat
Backup Slides
| abbr.     | $N_{\text{cfg}}$ | volume           | $\sim a$ [fm] | $m_t/m_s$ [MeV] | $\sim m_{\pi_5}L$ | $N_{\text{src}}$ | $L_5/a$ | $aM_5$ | $b_5$ | $c_5$ | $am_t^{\text{val.}}$ | $\sigma_{\text{smr}}$ | $N_{\text{smr}}$ |
|-----------|------------------|------------------|--------------|-----------------|-------------------|-----------------|--------|--------|------|------|----------------------|------------------|----------|
| a15m400   | 1000             | $16^3 \times 48$ | 0.15         | 0.334           | 400               | 8               | 12     | 1.3    | 1.5  | 0.5  | 0.0278               | 3.0              | 30       |
| a15m350   | 1000             | $16^3 \times 48$ | 0.15         | 0.255           | 350               | 16              | 12     | 1.3    | 1.5  | 0.5  | 0.0206               | 3.0              | 30       |
| a15m310   | 1960             | $16^3 \times 48$ | 0.15         | 0.2             | 310               | 24              | 12     | 1.3    | 1.5  | 0.5  | 0.01580              | 4.2              | 60       |
| a15m220   | 1000             | $24^3 \times 48$ | 0.15         | 0.1             | 220               | 12              | 16     | 1.3    | 1.75 | 0.75 | 0.00712              | 4.5              | 60       |
| a15m130   | 1000             | $32^3 \times 48$ | 0.15         | 0.036           | 130               | 5               | 24     | 1.3    | 2.25 | 1.25 | 0.00216              | 4.5              | 60       |
| a12m400   | 1000             | $24^3 \times 64$ | 0.12         | 0.334           | 400               | 8               | 8      | 1.2    | 1.25 | 0.25 | 0.02190              | 3.0              | 30       |
| a12m350   | 1000             | $24^3 \times 64$ | 0.12         | 0.255           | 350               | 8               | 8      | 1.2    | 1.25 | 0.25 | 0.01660              | 3.0              | 30       |
| a12m310   | 1053             | $24^3 \times 64$ | 0.12         | 0.2             | 310               | 8               | 8      | 1.2    | 1.25 | 0.25 | 0.01260              | 3.0              | 30       |
| a12m220S  | 1000             | $24^3 \times 64$ | 0.12         | 0.1             | 220               | 4               | 12     | 1.2    | 1.5  | 0.5  | 0.00600              | 6.0              | 90       |
| a12m220   | 1000             | $32^3 \times 64$ | 0.12         | 0.1             | 220               | 4               | 12     | 1.2    | 1.5  | 0.5  | 0.00600              | 6.0              | 90       |
| a12m220L  | 1000             | $40^3 \times 64$ | 0.12         | 0.1             | 220               | 4               | 12     | 1.2    | 1.5  | 0.5  | 0.00600              | 6.0              | 90       |
| a12m130   | 1000             | $48^3 \times 64$ | 0.12         | 0.036           | 130               | 3               | 20     | 1.2    | 2.0  | 1.0  | 0.00195              | 7.0              | 150      |
| a09m400   | 1201             | $32^3 \times 64$ | 0.09         | 0.335           | 400               | 8               | 6      | 1.1    | 1.25 | 0.25 | 0.0160               | 3.5              | 45       |
| a09m350   | 1201             | $32^3 \times 64$ | 0.09         | 0.255           | 350               | 8               | 6      | 1.1    | 1.25 | 0.25 | 0.0121               | 3.5              | 45       |
| a09m310   | 784              | $32^3 \times 96$ | 0.09         | 0.2             | 310               | 8               | 6      | 1.1    | 1.25 | 0.25 | 0.00951              | 7.5              | 167      |
| a09m220   | 1001             | $48^3 \times 96$ | 0.09         | 0.1             | 220               | 6               | 8      | 1.1    | 1.25 | 0.25 | 0.00449              | 8.0              | 150      |
Our Lattice QCD Action
Möbius Domain Wall Fermion on HISQ sea
CalLat PRD96 (2017) 054513

| HIGQ gauge configuration parameters | valence parameters |
|-------------------------------------|-------------------|
| abbr.     | N_{cfg} | volume  | \sim a [fm] | m_{t}/m_{s} | \sim m_{\pi}L | N_{src} | L_{5}/a | aM_{5} | b_{5} | c_{5} | a m_{t}^{val.} | \sigma_{smr} | N_{smr} |
| a15m400   | 1000    | 16^3 \times 48 | 0.15   | 0.334 | 400 | 4.8 | 8 | 12 | 1.3 | 1.5 | 0.5 | 0.0278 | 3.0 | 30 |
| a15m350   | 1000    | 16^3 \times 48 | 0.15   | 0.255 | 350 | 4.2 | 8 | 12 | 1.3 | 1.5 | 0.5 | 0.0206 | 3.0 | 30 |
| a15m310   | 1960    | 16^3 \times 48 | 0.15   | 0.2   | 310 | 3.8 | 24 | 12 | 1.3 | 1.5 | 0.5 | 0.01580 | 4.2 | 60 |
| a15m220   | 1000    | 24^3 \times 48 | 0.15   | 0.1  | 220 | 4.0 | 12 | 16 | 1.3 | 1.75 | 0.75 | 0.00712 | 4.5 | 60 |
| a15m130   | 1000    | 32^3 \times 48 | 0.15   | 0.036 | 130 | 3.2 | 5 | 24 | 1.3 | 2.25 | 1.25 | 0.00216 | 4.5 | 60 |
| a12m400   | 1000    | 24^3 \times 64 | 0.12   | 0.334 | 400 | 5.8 | 8 | 8 | 1.2 | 1.25 | 0.25 | 0.02190 | 3.0 | 30 |
| a12m350   | 1000    | 24^3 \times 64 | 0.12   | 0.255 | 350 | 5.1 | 8 | 8 | 1.2 | 1.25 | 0.25 | 0.01660 | 3.0 | 30 |
| a12m310   | 1053    | 24^3 \times 64 | 0.12   | 0.2   | 310 | 4.5 | 8 | 8 | 1.2 | 1.25 | 0.25 | 0.01260 | 3.0 | 30 |
| a12m220S  | 1000    | 24^3 \times 64 | 0.12   | 0.1  | 220 | 3.2 | 4 | 12 | 1.2 | 1.5 | 0.5 | 0.00600 | 6.0 | 90 |
| a12m220   | 1000    | 32^3 \times 64 | 0.12   | 0.1  | 220 | 4.3 | 4 | 12 | 1.2 | 1.5 | 0.5 | 0.00600 | 6.0 | 90 |
| a12m220L  | 1000    | 40^3 \times 64 | 0.12   | 0.1  | 220 | 5.4 | 4 | 12 | 1.2 | 1.5 | 0.5 | 0.00600 | 6.0 | 90 |
| a12m130   | 1000    | 48^3 \times 64 | 0.12   | 0.036 | 130 | 3.9 | 3 | 20 | 1.2 | 2.0 | 1.0 | 0.00195 | 7.0 | 150 |
| a09m400   | 1201    | 32^3 \times 64 | 0.09   | 0.335 | 400 | 5.8 | 8 | 6 | 1.1 | 1.25 | 0.25 | 0.0160 | 3.5 | 45 |
| a09m350   | 1201    | 32^3 \times 64 | 0.09   | 0.255 | 350 | 5.1 | 8 | 6 | 1.1 | 1.25 | 0.25 | 0.0121 | 3.5 | 45 |
| a09m310   | 784     | 32^3 \times 96 | 0.09   | 0.2  | 310 | 4.5 | 8 | 6 | 1.1 | 1.25 | 0.25 | 0.00951 | 7.5 | 167 |
| a09m220   | 1001    | 48^3 \times 96 | 0.09   | 0.1  | 220 | 4.7 | 6 | 8 | 1.1 | 1.25 | 0.25 | 0.00449 | 8.0 | 150 |

additional HISQ ensembles generated @ LLNL
available to interested parties
Gradient Flow smearing of HISQ cffgs more effective at reducing residual chiral symmetry breaking than the HYP smearing used in DWF on asqtad $m_{\text{res}} < 0.1 m_{\text{I}}$ on all ensembles for small-to-moderate $L_5$ and $M_5 \leq 1.3$
Source Geometry
Different Sources Agree

CalLat PLB 765:285-292 (2017)

L=24
$A_1^+ \ n^2=1$

| name   | $\Delta x \sim$ | # solve |
|--------|-----------------|---------|
| local  | (0, 0, 0)       | 1       |
| face   | (0, 0, 1)       | 6       |
| edge   | (0, 1, 1)       | 12      |
| corner | (1, 1, 1)       | 8       |

WM/JLab cgs.
$\sqrt{m_\pi} \sim 800$ MeV
$b \sim 0.145$ fm
Sources

| Solid Name                  | Solid            | Count |
|-----------------------------|------------------|-------|
| (0,0,1)                     | Octahedron       | 6     |
| (1,1,1)                     | Cube             | 8     |
| (0,1,1)                     | Cuboctahedron    | 12    |
| (0,1,2)                     | TruncatedOctahedron | 24   |
| (1,1,2)                     | SmallRhombicuboctahedron | 24 |
| (1,2,3)                     | GreatRhombicuboctahedron | 48 |
| local point                 |                  | 1     |
| face octahedron             |                  | 6     |
| edge cuboctahedron          |                  | 12    |
| corner cube                 |                  | 8     |
| knight’s move truncated octahedron |          | 24    |
| small rhombicuboctahedron   |                  | 24    |
| great rhombicuboctahedron   |                  | 48    |
## States

Courtesy of Amy Nicholson and Raul Briceño

| $n^2$ | $S$ | $I=1$ | $I=0$ |
|-------|-----|-------|-------|
|       | $^1S_0$ | $^1D_2$ | $^1D_2$ | $^1F_3$ | $^1P_1$ | $^1F_3$ |
| 0     | 1    | -     | -     | -     | -     | -     |
| 1     | 1    | -     | -     | -     | e     | 1     |
| 2     | 1    | -     | 1     | e     | -     | 1     |
| 3     | 1    | -     | 1     | -     | 1     | 1     |
| 4     | 1    | -     | e     | -     | 1     | -     |

| $n^2$ | $S$ | $I=1$ | $I=0$ |
|-------|-----|-------|-------|
| 0     | -   | 1     | -     | -     | -     | -     |
| 1     | -   | 2     | 1     | -     | 1     | 1     |
| 2     | 1   | 1     | 3     | 2     | 1     | e     |
| 3     | -   | 1     | 2     | 1     | 1     | -     |
| 4     | -   | 2     | 1     | -     | 1     | 1     |

$^3D_3$ $^3S_1$ $^3D_2$ $^3D_2$ $^3P_0$ $^3P_3$ $^3P_1$ $^3P_2$ $^3P_2$ $^3P_2$

$^3D_1$ $^3D_3$ $^3D_3$ $^3F_4$ $^3F_3$ $^3F_2$ $^3F_2$ $^3F_4$ $^3F_3$ $^3F_4$

$^3D_3$ $^3F_4$ $^3F_3$ $^3F_4$ $^3F_4$ $^3F_4$

### e: edges only
Matrix Prony
Calm Baryons via Matrix Prony

EPJ Web Conf. 175 (2018) 01016 EB et al.

\[ y(t) = \begin{pmatrix} y_{PS}(t) \\ y_{SS}(t) \end{pmatrix} \]
Calm Baryons via Matrix Prony

\[ y(t) = \begin{pmatrix} y_{PS}(t) \\ y_{SS}(t) \end{pmatrix} \]

Assume a transfer operator

\[ y(t + \tau) = \hat{T}(\tau)y(t) \]
Calm Baryons via Matrix Prony

EPJ Web Conf. 175 (2018) 01016 EB et al.

\[
y(t) = \begin{pmatrix} y_{PS}(t) \\ y_{SS}(t) \end{pmatrix}
\]

\[
y(t + \tau) = \hat{T}(\tau)y(t)
\]

\[
\hat{T} = M^{-1}V
\]

\[
My(t + \tau)y^T(t) = Vy(t)y^T(t)
\]
Calm Baryons via Matrix Prony

EPJ Web Conf. 175 (2018) 01016 EB et al.

$$M = \left[ \sum_{t=t_i}^{t_f} y(t+\tau) y^T(t) \right]^{-1}$$

$$V = \left[ \sum_{t=t_i}^{t_f} y(t) y^T(t) \right]^{-1}$$

$$\hat{T} = M^{-1}V$$

$$My(t+\tau)y^T(t) =Vy(t)y^T(t)$$

Ansatz: 2 states meaningfully contribute from \(t_i\) to \(t_f\)
Calm Baryons via Matrix Prony

EPJ Web Conf. 175 (2018) 01016 EB et al.

Slide 30

\[
M = \left[ \sum_{t=t_i}^{t_f} y(t + \tau) y^T(t) \right]^{-1} \\
V = \left[ \sum_{t=t_i}^{t_f} y(t) y^T(t) \right]^{-1} \\
\hat{T} = M^{-1} V \\
M y(t + \tau) y^T(t) = V y(t) y^T(t)
\]

Ansatz: 2 states meaningfully contribute from \( t_i \) to \( t_f \)
Calm Baryons via Matrix Prony

EPJ Web Conf. 175 (2018) 01016 EB et al.
Calm Baryons via Matrix Prony

EPJ Web Conf. 175 (2018) 01016 EB et al.

+ another noisy state at ~ 1.9 ↑

$am_{\text{eff}}(t)$

t/\alpha

PS
SS
MP Result

R. Prony J. de l’Ecole Polytechnique (1795) 2
$m_\pi \sim 800 \text{ MeV}$
\begin{align*}
q^{2\ell+1} \cot(\delta_{\ell}) &= -\frac{1}{a_{\ell}} + \frac{1}{2} r_{\ell} q^2 + \frac{1}{4!} P_{\ell} q^4 + \cdots \\
1S_0 &= 21.8(\pm 3.2)(\pm 0.8)\text{MeV} \\
3S_1 &= 30.7(\pm 2.4)(\pm 0.5)\text{MeV} \\
3S_1 &= 3.3(\pm 1.0)(\pm 0.6)\text{MeV}
\end{align*}
S wave

\[ q^{2\ell+1} \cot(\delta_\ell) = -\frac{1}{a_\ell} + \frac{1}{2} r_\ell q^2 + \frac{1}{4!} P_\ell q^4 + \cdots \]

\[ q^{2\ell+1} \cot(\delta_\ell) = iq \]

\[ ^1S_0 = 21.8(\pm 3.2)(\pm 0.8)\text{MeV} \]

\[ ^3S_1 = 30.7(\pm 2.4)(\pm 0.5)\text{MeV} \]

\[ ^3S_1 = 3.3(\pm 1.0)(\pm 0.6)\text{MeV} \]
I=0 P-wave

CalLat PLB 765:285-292 (2017)

WM/JLab cfgs.

\[ m_\pi \sim 800 \text{ MeV} \]

\[ b \sim 0.145 \text{ fm} \]
I=1 P-wave

CalLat PLB 765:285-292 (2017)

WM/JLab cfs.

\(m_\pi \sim 800\text{ MeV}\)
\(b \sim 0.145\text{ fm}\)
I=0 D-wave

CalLat PLB 765:285-292 (2017)

😍 No $^3D_1$
without disentangling partial waves

WM/JLab cfgs.
$m_\pi \sim 800$ MeV
$b \sim 0.145$ fm

\[ q^5 \cot \delta_{D_2} m_\pi^5 \]

\[ (q/m_\pi)^2 \]
I=1 D-wave

CalLat PLB 765:285-292 (2017)

WM/JLab cffgs.
m_π ~ 800 MeV
b ~ 0.145 fm
F Waves

CalLat PLB 765:285-292 (2017)

F-waves require disentangling from P-waves

Other

WM/JLab cfrgs.

$\eta \sim 800$ MeV

$b \sim 0.145$ fm

$NPLQCD$ PRD 87 (2013) 034506

$NPLQCD$ PRD 96 (2017) 114510
$m_\pi \sim 350$ MeV
I=1 $^1S_0$ Dineutron

MDWF on HISQ
a12m350
$24^3 \times 64$
m$_\pi$L = 5.1
$I=0 \ ^3S_1$ Deuteron

MDWF on HISQ
a12m350
$24^3 \times 64$
$m_\pi L = 5.1$