Optimized VAA Based Synthesis of Elliptical Cylindrical Antenna Array for SLL Reduction and Beam Thinning Using Minimum Number of Elements

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ABSTRACT In this paper, new virtual antenna array (VAA) based synthesis techniques are introduced for side lobe level (SLL) reduction, beam thinning, and number of elements minimization for elliptical cylindrical antenna arrays (ECAA) of radar systems. Thereby, significant improvements in the array gain and directivity are achieved, which enhance the detection range and angular resolution of the radar. Furthermore, the overall implementation cost of the system is highly reduced by saving the number of elements and the corresponding RF chains and simplifying the feeding network. Firstly, the proposed technique decomposes the single transmit/receive ECAA into a separate transmit linear antenna array (LAA) and receive elliptical antenna array (EAA). Secondly, the number of antenna elements, element spacing, and excitations of the created LAA and EAA are optimized using particle swarm optimization (PSO) to produce efficient beamformed patterns. Finally, the Kronker product of the optimized LAA and EAA patterns is performed to form the optimized virtual ECAA (V-ECAA) pattern. We also introduced both the uniform feeding based V-ECAA technique and the non-uniform feeding based V-ECAA synthesis technique for more flexibility and better productivity in antenna arrays design. The simulation results revealed that the uniform feeding based V-ECAA provides an identical pattern to that of the traditional uniform feeding ECAA while saves the number of elements by 66.6%. While in the case of non-uniform feeding based V-ECAA, it provides much lower SLL and narrower HPBW than those of the ECAA while saving the number of elements by 63.8%. Furthermore, the HB is applied to provide additional beam thinning and SLL reduction of the proposed non-uniform V-ECAA that is denoted as (HBV-ECAA). The possibility of practical validations of the synthesized V-ECAAs is verified using the computer simulation technology (CST) microwave studio package, which gives users an integrated design environment and achieves realizable and robust designs.

INDEX TERMS Elliptical cylindrical antenna array (ECAA), hyper beamforming (HB), virtual antenna array (VAA), particle swarm optimization (PSO), radar, side lobe level (SLL).

I. INTRODUCTION Radar systems play an important role in every modern system. The main role of radar is to detect, measure the range, speed, and the direction of the surrounding objects. The radar requires a high directivity and high gain of antenna array system to be able to distinguish between nearby objects. This can be achieved by reducing the half power beamwidth (HPBW) and side lobe level (SLL) of its array radiation pattern. Many algorithms were introduced for linear antenna arrays (LAAs) synthesis such as are particle swarm optimization (PSO), BAT, real-value Genetic Algorithm (RGA), and method of moments/ genetic algorithm (MoM/GA) introduced in [1]–[4].

In [1]–[3], PSO and BAT optimization algorithms are used to improve the performance of LAAs in terms of reducing SLL, reducing HPBW, and directing nulls into particular directions by estimating the optimum values of the LAA excitation coefficients and element spacing. In [4], the hybrid MoM/GA array synthesis technique has been
used to synthesize the arbitrary shaped pattern LAAs with a minimum number of antenna elements. The analytical MoM technique is used to calculate the array excitation coefficients while the GA optimization technique is used to optimize the inter-element spacing of the array. This hybrid technique has proved superior performance compared to only optimization-based synthesis techniques.

For additional performance enhancement in terms of SLL and HPBW, the hyper beamforming (HB) is introduced in [5]–[7]. The HB mainly depends on the sum and difference beam patterns of the array that are raised to the hyper beam exponent to control the SLL and the HPBW. The HB has been applied on the uniform feeding LAA for SLL minimization where the excitation coefficients and the interelement spacing are optimized using the Firefly algorithm (FFA) [5], Seeker Optimization Algorithm (SOA) [6], and collective animal behavior (CAB) algorithm [7]. The simulation results revealed that the SOA provided the lowest SLL compared to that of FFA, RGA, and PSO. While the CAB provided better results in terms of SLL compared to that of RGA and PSO.

There is another trend for synthesizing antenna arrays using a fewer number of elements by using virtual antenna array (VAA) beamforming. The VAA concept is widely used in antenna array-based applications such as radar systems, multiple-input multiple-output (MIMO) systems, direction of arrival (DoA) estimation, and null broadening as introduced in [8]–[14]. In [8], the VAA has been used for MIMO synthetic aperture radar (SAR). Where, the VAA concept is applied on one-dimensional LAA for providing satisfied performance for remote sensing. While in [9], the combination of VAA and MoM/GA beamforming techniques has been introduced for the synthesis of medium-range radar (MRR) and long-range radar (LRR) PAA systems. The VAA is applied on a two-dimensional planar antenna array (PAA) by constructing two orthogonal LAAs. Where, the MoM/GA is used to estimate the excitation coefficients and the interelement spacing of the two orthogonal LAAs to provide the same radiation pattern as the original PAA.

In [10], [11], the modified virtual singular value decomposition (MV-SVD) algorithm and virtual array extension/matrix pencil method/genetic algorithm (VAE/MPM/GA) have been introduced, respectively. They are based on the VAA extension of small size antenna arrays to provide high DoA estimation accuracy. They extend the original antenna array size from $M$ elements to virtual $(2M - 1)$ elements, which allows the system to identify more sources and provide higher resolution. In [12], a VAA-based DoA estimation technique has been introduced where the non-uniform feeding PAA is transformed into a uniform feeding virtual PAA consisting of a fewer number of elements. In addition, the VAA concept has been used for null broadening of antenna arrays as introduced in [13]. In [14], the VAA has been used to implement a hexagonal array configuration for MIMO radar. The hexagonal VAA based MIMO provided better results compared to perpendicular MIMO in terms of SLL, as it reduced the SLL by 3 dB.

It is worth mentioning that the elliptical cylindrical antenna array (ECAA) is one of the most commonly used antenna arrays in radar applications as introduced in [15]. The ECAA is a combination of a set of elliptical antenna arrays (EAs) and LAAs. In [15], three ECAs have been introduced including the uniform feeding ECAA, the non-uniform feeding ECAA, and the hyper beamforming (HB) based uniform and non-uniform feeding ECAs. However, these array configurations have many drawbacks such as the utilization of a large number of antenna elements and RF front-end chains, increased complexity of the feeding networks, increased overall system cost, and moderate SLL and HPBW which limit the detection range and resolution of the radar system.

In this paper, the performances of the three aforementioned ECAs introduced in [15] are enhanced by using a hybrid combination between the VAA, PSO, and HB techniques. The VAA is used to decompose the single transmit/receive ECAA into a separate transmit LAA and a separate receive EAA for initial minimization of the number of antenna elements of the synthesized virtual ECAA (V-ECAA). In the case of uniform feeding V-ECAA, we perfectly implemented the desired uniform feeding ECAA pattern using a much fewer number of antenna elements. While in the case of non-uniform feeding V-ECAA, the PSO is used to optimize the excitation coefficients and element spacing of the constructed transmit LAA and receive EAA to synthesize high-performance V-ECAAs in terms of SLL and HPBW compared to the non-uniform feeding ECAA using minimum number of antenna elements. Furthermore, for more SLL reduction and beam thinning, the HB is applied to the proposed V-ECAA to introduce the HBV-ECAA synthesis technique, which achieved superior performance compared to hyper beamformed ECAA.

The key contributions of this paper are summarized as follows:

I Although the VAA principle is outdated for use in radar systems, the application of the PSO to the separate transmitter and receiver arrays which are the key components of the VAA prior to the Kronker product of their patterns is considered innovative. This is because the use of PSO guarantees optimal allocation and alignment of transmitter and receiver arrays, minimizes the required number of antenna elements, and optimizes both element spacing and excitation prior to the implementation of the VAA principle, which significantly enhances overall array system performance.

II This combination VAA/PSO allows the synthesis of the desired ECAA pattern with minimized number of antenna elements, which reduces the required number of RF front-end chains, simplifies the design of the feeding network, and reduces the overall system cost.

III In case of uniform feeding V-ECAA, the desired ECAA pattern is perfectly implemented while saving the number of elements by a ratio of 66.66%.

IV In case of non-uniform feeding V-ECAA, significant SLL reduction and beam thinning are achieved.
V. The proposed HBV-ECAA is able to reduce the SLL and narrow the HPBW compared to the hyper beamformed ECAA at different hyper beam exponent values.

VI In case of uniform feeding, the HBV-ECAA reduces the SLL by 345.32% and 140.99% compared to that of the hyper beamformed uniform ECAA at \( k = 0.1 \) and 0.3, respectively.

VII In case of non-uniform feeding, the HBV-ECAA reduces the SLL by 18 dB, 29 dB, and 131 dB compared to that of the hyper beamformed non-uniform ECAA at \( k = 0.5, 0.3, \) and 0.1, respectively.

VIII The possibility of practical validations of designed V-EEAs is verified using the well-known computer simulation technology (CST) microwave studio software package. The CST technology offers a more efficient approach to simulating both individual components and full systems, which gives users an integrated design environment. The designers can identify and resolve potential issues such as detuning, signal integrity (SI) and power integrity (PI) problems, and electromagnetic interference (EMI) at an early stage and thus obtain more robust and realizable designs.

The rest of the paper is organized as follows; in Section II, the problem formulation is presented in details, the PSO is introduced in Section III, the proposed VAA/PSO based synthesis techniques are presented in section IV, the hyper beamforming of the proposed V-EEA is introduced in Section V, the simulation results and discussions are introduced in Section VI, and the conclusion is given in Section VII.

II. PROBLEM FORMULATION: TRADITIONAL ECAA

The ECAA is widely used in full duplex single antenna array based radar system. As shown in Fig. 1, the ECAA consists of \( M \) ellipses located in the X-Y plane and encircled around the Z-axis with vertical spacing \( d \). All ellipses have the same major axis \( a \) and the same minor axis \( b \). The \( N \) elements placed in a transversal plane constitute an elliptical antenna array (EAA). While the \( M \) elements aligned along the vertical line on the cylinder surface form a linear antenna array (LAA).

Thereby, the ECAA is a combination of \( N \) number of LAAs and \( M \) number of EAAAs. The EAA pattern is expressed as [15]:

\[
AF_{EAA}(\theta, \phi) = \sum_{n=1}^{N} u_n e^{j\beta \sin \theta (a \cos \phi \cos (\phi_n) + b \sin \phi \sin (\phi_n)) + P_n}
\]

(1)

where \( a \) and \( b \) are the major and minor axes of the ellipse, respectively, \( N \) is the number of the EAA elements, \( u_n \) is the excitation coefficient of the \( n^{th} \) element, \( \beta \) is the wave number, \( \beta = 2\pi / \lambda \), and \( \lambda \) is the wavelength. \( \theta \) is the elevation angle, \( \phi \) is the azimuth angle, and \( \phi_n \) is the angular position of the \( n^{th} \) element that is defined as:

\[
\phi_n = 2\pi (n - 1) / N
\]

(2)

\( P_n \) is a parameter used to steer the main beam towards the desired direction as follows [15]:

\[
P_n = -\beta \sin \theta_0 \cdot (a \cdot \cos(\phi_0)\cos(\phi_n) + b \cdot \sin(\phi_0) \sin(\phi_n))
\]

(3)

where \( \theta_0 \) and \( \phi_0 \) are the elevation angle and azimuth angle of the main beam, respectively.

On the other hand, the LAA pattern can be expressed as [15]:

\[
AF_{LAA}(\theta) = \sum_{m=1}^{M} v_m e^{j\beta (m-1) d (\cos \theta - \cos \theta_0)}
\]

(4)

where \( v_m \) is the excitation coefficient of the \( m^{th} \) antenna element.

The total ECAA pattern is obtained by combining the \( M \) number of EAA patterns as follows:

\[
AF_{ECAA}(\theta, \phi) = \sum_{m=1}^{M} \sum_{n=1}^{N} I_{mn} e^{j\beta \sin \theta (a \cdot \cos \phi \cos (\phi_n) + b \cdot \sin \phi \sin (\phi_n)) + P_n}
\]

(5)

where \( I_{mn} \) is determined by:

\[
I_{mn} = v_m e^{j\beta (m-1) d (\cos \theta - \cos \theta_0) \cdot u_n}
\]

(6)

For uniform feeding ECAA, \( u_n \) and \( v_m \) are set to 1, where \( n = 1, 2, \ldots, N \) and \( m = 1, 2, \ldots, M \). However, the performance of the ECAA can be improved in terms of SLL and HPBW by using non-uniform feeding ECAA. In non-uniform feeding ECAA, \( v_m \) of the elements of the LAAs are set to 1, while \( u_n \) of the EAAAs elements are changed.

III. PARTICLE SWARM OPTIMIZATION (PSO)

Particle Swarm Optimization (PSO) is one of the meta-heuristic optimization paradigms that is widely used in the last two decades. The PSO algorithm uses a swarm of particles, which traverse a multidimensional search space to reach to...
the optima. It has the ability to solve the unsupervised applications and complex multidimensional problems. Also, it distinguishes by the low computational complexity, low required memory, and high accuracy features. In addition, PSO has many features over heuristic optimization algorithms such as GA: (i) PSO has two populations (best position and current position), thereby, it provides more diversity and exploration over a single population. (ii) It provides faster convergence and more variety in search trajectories. (iii) The efficiency of heuristic algorithms is highly dependent on the algorithm parameters and the number of variables [16].

In PSO, each state is considered as a unique particle with a position \( \mathbf{x}_i \) and velocity \( \mathbf{v}_i \) at each iteration \( t \). For each particle, the better position is realized by immediately updating its velocity and position considering its previous position and the previous best positions of the other particles. The velocity of the \( j \)th particle at the \( i \)th iteration is updated as follows [17]:

\[
\mathbf{v}_i^{j+1} = \left[ \sigma \times \mathbf{v}_i^j \right] + \left[ u_1 \times \text{rand} \left( 1, 1 \right) \times \left( \mathbf{P}_i - \mathbf{g}_i \right) \right] + \left[ u_2 \times \text{rand} \left( 1, 1 \right) \times \left( \mathbf{P}_g - \mathbf{g}_i \right) \right]
\]

while the position of the \( j \)th particle at the \( i \)th iteration is updated as follows:

\[
\mathbf{x}_i^{j+1} = \mathbf{x}_i^j + \mathbf{v}_i^{j+1}
\]

where \( u_1 \) is the cognitive parameter, \( u_2 \) is the social parameter, \( \sigma \) is the inertia weight index, and \( \text{rand} \left( 1, 1 \right) \) is a MATLAB command that generates a random value within the range \([0, 1]\). \( \sigma \) organizes the influence of the previous velocity on the new velocity during the adjusting process. \( \mathbf{P}_i \) is a best position of the particles in iteration \( i \) and \( \mathbf{P}_g \) is the global best position, which is considered as the best \( \mathbf{P}_g \) through all iterations.

**IV. PROPOSED VAA/PSO BASED SYNTHESIS TECHNIQUES OF ECAA**

In this section, a VAA/PSO based synthesis technique denoted as V-ECAA is introduced for ECAAs synthesis using minimum number of antenna elements. The VAA concept is widely used in radar systems having separate transmit (TX) and receive (RX) antenna arrays. The resultant VAA pattern is the multiplication of the TX and RX arrays patterns [8], [9]. Thereby, the ECAA shown in Fig. 1 can be synthesized using the VAA concept as shown in Fig. 2 where the TX antenna array is formed as a LAA, while the RX antenna array is formed as an EAA. The transmit LAA has \( M_{TX} \) elements which are placed on the Z-axis with uniform element spacing, \( d_s \).

The transmit LAA pattern is given by:

\[
AF_{TX}(\theta, \phi) = \sum_{m=1}^{M_{TX}} q_m e^{j(b(m-1)d_s*cos\theta - \phi)}
\]

where \( q_m \) is the excitation coefficient of the \( m \)th antenna element, \( m = 1, 2, \ldots, M_{TX} \). While, the receive antenna array is designed as a single EAA consisting of \( N_{RX} \) elements and located in X-Y plane. The receive EAA pattern is given by:

\[
AF_{RX}(\theta, \phi) = \sum_{n=1}^{N_{RX}} p_n e^{j(\beta(\frac{n-1}{2}d_s*sin\theta - \phi))}
\]

While the transmit LAA pattern is given by:

\[
AF_{TX}(\theta, \phi) = \sum_{m=1}^{M_{TX}} q_m e^{j(b(m-1)d_s*cos\theta - \phi)}
\]

Where \( p_n \) is the excitation coefficient of the \( n \)th antenna element, \( n = 1, 2, \ldots, N_{RX} \). The receive EAA pattern can be expressed as [8], [9]:

\[
AF_{RX}(\theta, \phi) = AF_{TX} \odot AF_{RX}
\]

**FIGURE 2. The geometry of the proposed V-ECAA.**

where \( \odot \) is the Kronner product. In the following sections, both the uniform feeding V-ECAA (UV-ECAA) and the non-uniform feeding V-ECAA (NUV-ECAA) arrays are introduced.

**A. Proposed UV-ECAA**

In this section, the UV-ECAA is introduced to synthesize the same pattern as that of the uniform feeding ECAA. In this case, the excitation coefficients of transmit LAA and receive EAA (\( q_m \) and \( p_n \)) are set to 1 in (9) and (10). While the number of elements \( N_{RX} \), \( M_{TX} \) and the inter element spacing \( d_s \) are optimized using PSO to minimize the designed cost function \( \mathcal{C}_{err} \) under the constraint that both the synthesized pattern \( AF_{UV-ECAA}(\theta, \phi) \) and the original pattern \( AF_{ECAA}(\theta, \phi) \) have the same HPBW as follows:

\[
\mathcal{C}_{err} = \frac{1}{L} \sum_{l=1}^{L} \left[ \left\| AF_{UV-ECAA}(l) - AF_{ECAA}(l) \right\|^2 \right]_{\text{HPBW}_{UV-ECAA}} = \text{HPBW}_{ECAA}
\]

where \( L \) is the number of samples used to represent the array pattern, \( \text{HPBW}_{UV-ECAA} \) and \( \text{HPBW}_{ECAA} \) are the HPBWs of the synthesized UV-ECAA and the uniform feeding ECAA, respectively. It is worth mentioning that the minimization of the cost function corresponds to the minimization of the mean square error between \( AF_{UV-ECAA}(\theta, \phi) \) and \( AF_{ECAA}(\theta, \phi) \).
B. PROPOSED NUV-ECAA

In this section, the NUV-ECAA is introduced to improve the performance of the non-uniform feeding ECAA in terms of SLL and HPBW. In this case, both the excitation coefficients \( q_m \) and \( p_n \), the number of elements \( N_{RX} \) and \( M_{TX} \), and the inter element spacing \( d_s \) are optimized using PSO to minimize the SLL of the AF \( ECAA(\theta, \phi) \). The PSO determines the optimum values of \( n \), \( m \), \( N_{RX} \), and \( M_{TX} \) to minimize the designed cost function, \( CF_{NUV} \).

\[
CF_{NUV} = 20\log_{10}\left[ \frac{\text{max}(AF_{NUV-ECAA}(\theta_{SL}))}{\text{AF}_{NUV-ECAA}(\theta_{0})} \right] \quad \text{HPBW}_{NUV-ECAA} < \text{HPBW}_{ECAA}
\]  

where \( \theta_{SL} \) is the angle of the side lobe of the synthesized pattern \( AF_{NUV-ECAA}(\theta, \phi) \). The amplitude of the main beam at the main beam direction \( \theta_0 \) is \( AF_{NUV-ECAA}(\theta_0) \) and HPBW \( _{NUV-ECAA} \) is the HPBW of the synthesized pattern \( AF_{NUV-ECAA}(\theta, \phi) \). Fig. 3 shows the flow chart, which describes the proposed UV-ECAA and NUV-ECAA synthesis techniques. To minimize the cost function \( CF_{NUV} \), the amplitude of the main beam should be maximum and the amplitude of the maximum side lobe should be minimum under the constraint that the synthesized HPBW \( _{NUV-ECAA} \) be less than that of the original non-uniform ECAA.

V. HYPER BEAMFORMING OF THE PROPOSED V-ECAA

In this section, the HB is applied to the proposed V-ECAA (HBV-ECAA) to provide additional SLL reduction and beam thinning. The HB concept is based on the generation of sum and difference patterns, which are raised to the power of a hyper beam exponent parameter. The sum pattern is obtained by taking the summation of the absolute values of left and right patterns. While, the difference is the absolute value of the difference between the left and right patterns. The right and left patterns are originated from dividing the antenna pattern crossing the x-axis. Starting from 0° going to the –ve x-axis results in left pattern. While, the right pattern is obtained starting from 0° going to the +ve x-axis [5]–[7]. The HB is applied only on the receive EAA, while the transmit LAA is kept unchanged.

The left pattern of the receive EAA is synthesized using the EAA elements from 1 to \( N_{RX} + 1 \) as follows [5]–[7]:

\[
AF_{RX-left} = \sum_{n=1}^{N_{RX}} p_n e^{j\beta \sin \theta \cdot (a \cdot \cos \phi \cdot \cos (\phi_n) + b \cdot \sin \phi \cdot \sin (\phi_n) + P_n)}
\]  

While, the right pattern of the RX antenna array is synthesized using the remaining elements from \( \frac{N_{RX}}{2} + 1 \) to \( N_{RX} \) as follows [5]–[7]:

\[
AF_{RX-right} = \sum_{n=\frac{N_{RX}}{2} + 1}^{N_{RX}} p_n e^{j\beta \sin \theta \cdot (a \cdot \cos \phi \cdot \cos (\phi_n) + b \cdot \sin \phi \cdot \sin (\phi_n) + P_n)}
\]  

Then, the sum and difference patterns are generated as follows [5]–[7]:

\[
AF_{RX-sum} = \left| AF_{RX-left} \right| + \left| AF_{RX-right} \right|
\]

\[
AF_{RX-diff} = \left| AF_{RX-left} - AF_{RX-right} \right|
\]

Fig. 4 and Fig. 5 show examples for the sum and difference patterns generation from 10-elements EAA.
Consequently, the hyper beam pattern of the receive antenna array can be expressed as [5]–[7]:

$$ AF_{RX-\text{hyper}} = \left( (AF_{RX-\text{sum}})^k - (AF_{RX-\text{diff}})^k \right)^{1/k} $$

(18)

where $k$ is the hyperbeam exponent, which lies within the range (0.1 : 1). If $k$ equals 0.1, the $AF_{RX-\text{hyper}}$ pattern will have large spike depth towards the main beam direction. However, if $k$ exceeds 1, the side lobes of the $AF_{RX-\text{hyper}}$ pattern will be higher than that of $AF_{RX}$. From (11), the synthesized pattern using the HBV-ECAA is written as follows:

$$ AF_{HBV-\text{ECAA}} = AF_{TX} \otimes AF_{RX-\text{hyper}} $$

(19)

When HB applying is applied to the UV-ECAA, the synthesis technique is denoted as HBUV-ECAA. While in case of NUV-ECAA, it is denoted as HBNUV-ECAA. The flow chart of proposed HBUV-ECAA and HBNUV-ECAA techniques is shown in Fig. 6.

VI. SIMULATION RESULTS AND DISCUSSIONS

In this section, several simulations are carried out to verify the superiority of the proposed V-ECAA compared to the traditional uniform feeding ECAA (U-ECAA) and the non-uniform feeding ECAA (NU-ECAA). In addition, the superiority of the proposed HBV-ECAA compared to the hyper beamformed ECAA is verified. In the simulations, the proposed V-ECAA and HBV-ECAA are compared to the traditional U-ECAA and NU-ECAA introduced in [15], respectively at the same specifications listed in Table 1. The number of PSO iterations is denoted as $N_{PSO}$. The simulations results are divided into four sections as follows:

1) **UV-ECAA versus U-ECAA**

2) **NUV-ECAA versus NU-ECAA**

3) **HBV-ECAA versus HB-ECAA**

4) **Implementation of the Synthesized V-ECAAs Using CST Microwave Studio**

A. **UV-ECAA versus U-ECAA**

In this section, the proposed UV-ECAA is compared to the traditional U-ECAA that is generated using $M = 3$ and $N = 12$ elements as introduced in [15]. In this case, the U-ECAA is formed from 36 elements. Applying the proposed UV-ECAA technique, the rectangular plot of the U-ECAA and the UV-ECAA patterns are shown in Fig. 7. While, the PSO optimized values of $N_{RX}$, $M_{TX}$, $d_s$, $N_{PSO}$, and $CF_{UV}$ are listed in Table 2. As shown in Fig. 7, it is clear that the proposed UV-ECAA provides an identical pattern to that of the U-ECAA. They have the same SLL and HPBW, which equal $-8.5$ dB and $17.82^\circ$, respectively. Furthermore, the UV-ECAA reduces the number of elements from 36 to only 12 elements such that; ($N_{RX} = 9$ elements are used to form the receive EAA and $M_{TX} = 3$ elements are used to form the transmit LAA). Accordingly, there is 66.6% saving in the overall number of antenna elements.

B. **NUV-ECAA versus NU-ECAA**

In this section, the proposed NUV-ECAA is compared to the traditional NU-ECAA introduced in [15]. Performing the proposed NUV-ECAA, Fig. 8 shows the rectangular plot of the synthesized NUV-ECAA pattern compared to the NU-ECAA pattern. However, the PSO optimized values
of $N_{RX}$, $M_{TX}$, $d_s$, $N_{PSO}$, and $C_{\mathcal{F}_{UV}}$ are listed in Table 3. A comparison between the NUV-ECAA and the NU-ECAA patterns in terms HPBW, SLL, and number of elements is listed in Table 4. The optimum values of $q_m$ and $p_n$ are listed in Table 5. While, the excitation coefficients of the NU-ECAA ($u_n$) are listed in Table 6. By analyzing the results, it is clear that the NUV-ECAA pattern provides better performance than the NU-ECAA pattern in terms of SLL and HPBW. It provides a $3.3^\circ$ narrower HPBW than that of the NU-ECAA pattern. Also, it provides a 7 dB lower SLL than that of the ECAA pattern. Furthermore, it decreases the number of elements from 36 elements to 13 elements. Accordingly, the number of elements is reduced by 63.8%.

### C. HBV-ECAA VERSUS HB-ECAA

In this section, the HB is applied on both the proposed UV-ECAA and NUV-ECAA for comparison with the hyperbeamformed uniform ECAA (HBU-ECAA) and hyper beamformed non-uniform ECAA (HBNU-ECAA), respectively as follows:

1) **HBUV-ECAA VERSUS HBU-ECAA**

   In this section, the HB is applied on the UV-ECAA introduced in section (VI. A) and on the U-ECAA introduced in [15] at $k = 0.1$ and 0.3. Fig. 9 and Fig. 10 show the rectangular plot of synthesized 12-elements HBUV-ECAAs and the HBU-ECAAs at $k = 0.1$ and 0.3. A comparison between the synthesized HBUV-ECAA patterns and the HBU-ECAA patterns is presented in Table 7. By analyzing the results, it is revealed that the HBUV-ECAA and the HBU-ECAA provides the same HPBW at $k = 0.1$ and 0.3. While, the HBUV-ECAA provides much lower SLL than that of the HBU-ECAA, as it reduces the SLL from $-41.74$ dB to $-185.68$ dB and from $-17.22$ dB to $-41.5$ dB at $k = 0.1$ and 0.3, respectively. Accordingly, the SLL is reduced by 345.32% and 140.99%, at $k = 0.1$ and 0.3, respectively.

2) **HBNUV-ECAA VERSUS HBNU-ECAA**

   In this section, the HB is applied on the NUV-ECAA introduced in section (VI. B) and on the NU-ECAA introduced...
The comparisons between the rectangular plots of the synthesized 13-elements HBNUV-ECAAs and the HBUN-ECAAs at \( k = 0.5 \), 0.3, and 0.1, respectively. Also, they decrease the HPBW by 4°, 0.9°, and 0.2° at \( k = 0.5 \), 0.3, and 0.1, respectively. Furthermore, the number of elements, which is used to implement the HBV-ECAAs, is lower than that of the HBUN-ECAAs by 23 elements. Thereby reducing required RF front end channels, and hence reducing the overall system cost. By comparing the results in Table 4 to the hyper beamforming results in Table 8, it is clear that the HB plays an important role in SLL reduction and beam thinning. As, it decreases the SLL by 12.5 dB, 34 dB, and 119 dB at \( k = 0.5 \), 0.3, and 0.1, respectively in case of HBNU-ECAAs. While in case of HBNUV-ECAAs, it decreases the SLL by 23.5 dB, 56 dB, and 243 dB at \( k = 0.5 \), 0.3, and 0.1, respectively.

Finally, the simulation results of the V-ECAA compared to ECAA for uniform feeding, non-uniform feeding, and applying HB on them are summarized in brief in Table 9 which contains the computed SLL, HPBW, and number of elements.

### D. IMPLEMENTATION OF THE SYNTHESIZED V-ECAAs USING CST MICROWAVE STUDIO

In this section, the synthesized V-ECAAs and the traditional ECAAs are implemented using CST Microwave Studio taking into account the effect of mutual coupling between the...
antenna elements. The mutual coupling affects the antenna characteristics drastically in terms of the SLL and main beam properties and therefore degrades the performance of the system [18], [19]. In this section, the V-ECAAs and ECAAs are implemented using a \( \frac{\lambda}{2} \) dipole antenna radiating at resonance frequency \( f_o = 0.305 \) GHz. The dipole structure, H-plane, and E-plane patterns are shown in Fig. 14. While, the scattering parameter \( |S_{11}| \) versus frequency of the dipole antenna is shown in Fig. 15. The patterns of the synthesized V-ECAAs are compared to those of the traditional ECAAs in terms of HPBW and SLL under practical conditions that have been adjusted in CST simulations. The CST simulations are divided into three sections as follows:

1) **CST Implementation of UV-ECAA versus U-ECAA.**
2) **CST Implementation of NUV-ECAA versus NU-ECAA.**
3) **CST Implementation of HBNUV-ECAA versus HBNU-ECAA.**

1) **CST IMPLEMENTATION OF UV-ECAA VERSUS U-ECAA**
In this section, the 12-elements UV-ECAA and 36-elements U-ECAA arrays presented in section (IV.A) are implemented using CST as shown in Fig. 16. The rectangular plots of their...
patterns are shown in Fig. 17. While, a comparison between the synthesized UV-ECAA and the U-ECAA in terms of HPBW, SLL, and number of elements is listed in Table 10. It is evident from the review of the data in Table 10 that the UV-ECAA and U-ECAA patterns have the same HPBW. However, the UV-ECAA has a lower SLL than that of the U-ECAA by 2.41 dB.

2) CST IMPLEMENTATION OF NUV-ECAA VERSUS NU-ECAA

In this section, the synthesized 13-element NUV-ECAA and 36-element NU-ECAA arrays introduced in section (IV.B) are implemented using the CST as shown in Fig. 18. Fig. 19 displays the rectangular patterns of the synthesized NUV-ECAA and NU-ECAA. While the distinction between them in terms of HPBW, SLL, and number of elements is seen in Table 11. The results revealed that the NUV-ECAA has superior performance compared to the NU-ECAA in terms of HPBW and SLL, as it provides narrower HPBW by $1.81\degree$ and much lower SLL by 18.2 dB than the NU-ECAA. At the same moment, the NU-ECAA has a very high SLL equal to $-4.73\text{dB}$.

3) CST IMPLEMENTATION OF HBNUV-ECAA VERSUS HBNU-ECAA

In this section, we applied the HB on the implemented NUV-ECAA and NU-ECAA at $k = 0.5$ as an example. Fig. 20 shows the rectangular plot of the synthesized
In this paper, the combination between VAA and PSO is utilized to improve the properties of the traditional ECAAs in terms of SLL and HPBW, thereby the array directivity, gain, and the receiver sensitivity are significantly improved. The VAA is used to decompose the single transmit/receive ECAA into a separate transmit linear antenna array (LAA) and receive elliptical antenna array (EAA). While PSO is used to optimize the number of antenna elements, element spacing, and excitations of the LAA and EAA. In the case of uniform feeding, the proposed UV-ECAA provides an identical pattern to that of the uniform feeding ECAA using 66.6% fewer number of antenna elements. While in case of non-uniform feeding, the proposed NUV-ECAA provides better results than that of the non-uniform ECAA in terms of SLL and HPBW and saves 63.8% of the antenna elements. In case of applying the hyper beamforming, the simulation results revealed that the proposed HBV-ECAA provides significant reductions in the SLL and HPBW at different values of hyper beam exponent for both uniform and non-uniform feeding. In addition, the CST simulation results revealed that the synthesized V-ECAAs have superior performance compared to the traditional ECAAs in cases of uniform and non-uniform feeding. The implemented V-ECAAs provide much lower SLL and narrower HPBW than those of the implemented ECAAs.

VII. CONCLUSION

In this paper, the combination between VAA and PSO is utilized to improve the properties of the traditional ECAAs in terms of SLL and HPBW, thereby the array directivity, gain, and the receiver sensitivity are significantly improved. The VAA is used to decompose the single transmit/receive ECAA into a separate transmit linear antenna array (LAA) and receive elliptical antenna array (EAA). While PSO is used to optimize the number of antenna elements, element spacing, and excitations of the LAA and EAA. In the case of uniform feeding, the proposed UV-ECAA provides an identical pattern to that of the uniform feeding ECAA using 66.6% fewer number of antenna elements. While in case of non-uniform feeding, the proposed NUV-ECAA provides better results than that of the non-uniform ECAA in terms of SLL and HPBW and saves 63.8% of the antenna elements. In case of applying the hyper beamforming, the simulation results revealed that the proposed HBV-ECAA provides significant reductions in the SLL and HPBW at different values of hyper beam exponent for both uniform and non-uniform feeding. In addition, the CST simulation results revealed that the synthesized V-ECAAs have superior performance compared to the traditional ECAAs in cases of uniform and non-uniform feeding. The implemented V-ECAAs provide much lower SLL and narrower HPBW than those of the implemented ECAAs.

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