INFERENCES FROM THE DISTRIBUTIONS OF FAST RADIO BURST PULSE WIDTHS, DISPERSION MEASURES, AND FLUENCES

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ABSTRACT

The widths, dispersion measures (DMs), dispersion indices, and fluences of Fast Radio Bursts (FRBs) impose coupled constraints that all models must satisfy. The non-monotonic dependence of burst widths (after deconvolution of instrumental effects) on DMs excludes the intergalactic medium as the location of scattering that broadens the FRBs in time. Temporal broadening far greater than that of pulsars at similar high Galactic latitudes implies that scattering occurs close to the sources where high densities and strong turbulence or heterogeneity are plausible. FRB energetics are consistent with supergiant pulses from young, fast, high-field pulsars at cosmological distances. The distribution of FRB DMs is: (1) inconsistent with that of expanding clouds (such as SNRs); (2) inconsistent with space-limited source populations (such as the local Supercluster); and (3) consistent with intergalactic dispersion of a homogeneous source population at cosmological distances. Finally, the FRB log \( N \)–log \( S \) relation also indicates a cosmological distribution aside from the anomalously bright Lorimer burst.

Key words: intergalactic medium – radio continuum: general

1. INTRODUCTION

Following an initial report of a Fast Radio Burst (FRB) by Lorimer et al. (2007), Thornton et al. (2013) discovered four additional events whose large dispersion measures (DM > 500 pc cm\(^{-3}\)) and high Galactic latitudes \( |b| > 40^\circ \) indicated that their sources were at cosmological distances. Subsequently, more FRBs were discovered (the present total is 10). Their measured parameters include fluence, dispersion measure (DM), and pulse widths.

Several FRBs have dedispersed and deconvolved pulse widths \( W_{1300} \) at 1300 MHz of several ms, while only upper limits were found for others. Fitting \( W \propto \nu^\beta \) for FRBs with measured \( W \), their scattering indices \( \beta \approx -4 \) are consistent with theoretical values for multipath propagation spreading in a plasma medium (Williamson 1972; Lee & Jokipii 1975; Rickett 1977) and apparently confirm that the burst widths are not instrumental artefacts.1 In homogeneous Kolmogorov turbulence, \( \beta = -4.4 \) is consistent with all values of \( \beta \) measured for FRBs but inconsistent with many pulsar data (Krishnakumar et al. 2015).

This paper explores the implications of the distributions of FRB scattering widths, fluences, and DMs with the assumption that the DMs indicate cosmological distances. Any explanation of these observations must first account for two facts: (1) all FRBs have DMs within a range of a factor of about three (two if the Lorimer burst is not accepted as an FRB), implying a general property, not an unusual circumstance such as a line of sight that happens to intersect a rare dense cloud; and (2) the dispersion index \( \alpha \), defined by the dispersive delay relation \( \Delta t \propto \nu^\alpha \), is very close to \(-2\) and consistent with exactly \(-2\) for every FRB, implying an upper bound on the density of the dispersing plasma and (combined with the measured DM) a lower bound on its size. These facts are readily accounted for if most of the dispersion is intergalactic, in which case the

1 Incomplete dedispersion of a burst might masquerade as broadening with \( \beta = -3 \) because the propagation delay \( \Delta t \propto (\Delta \nu)^{-3} \) over a narrow frequency band \( \Delta \nu \), but \( \beta = -3 \) is outside the measured range of uncertainty of at least one FRB (Thornton et al. 2013).
Table 1
Summary of FRB Data, Ordered by Estimated Extra-Galactic Dispersion Measures DMg; for the Two Low-b (Galactic Latitude) Bursts These May Be Uncertain and Are Italicized

| FRB    | DMg (pc cm\(^{-2}\)) | b      | S (Jy ms) | \(W_{1300}\) (ms) | Reference                  |
|--------|----------------------|--------|-----------|-------------------|----------------------------|
| 010621 | 213                  | −04.0  | 4.3       | <3.0              | Keane et al. (2012)         |
| 010724 | 350                  | −41.8  | 150.0     | 6.2               | Lorimer et al. (2007)       |
| 121102 | 369                  | −90.2  | 1.2       | <0.5              | Spitler et al. (2014)       |
| 120127 | 521                  | −66.2  | 0.8       | <1.1              | Thornton et al. (2013)      |
| 140514 | 528                  | −54.6  | 1.3       | 1.9               | Petroff et al. (2015a)      |
| 110626 | 677                  | −41.7  | 0.9       | <1.4              | Thornton et al. (2013)      |
| 010125 | 680                  | −20.0  | 5.6       | 5.0               | Burke-Spolaor & Bannister (2014) |
| 131104 | 710                  | −22.2  | 2.3       | 4.0               | Ravi et al. (2015)          |
| 110220 | 910                  | −54.7  | 7.3       | 5.6               | Thornton et al. (2013)      |
| 110703 | 1072                 | −59.0  | 1.8       | <4.3              | Thornton et al. (2013)      |

Note. Most data are from references in the table, but fluences S are from the re-evaluation by Keane & Petroff (2015), except for the “Lorimer Burst” (FRB 010724) and FRB 131104. Pulse widths \(W_{1300}\) are from the references after dedispersion and deconvolution of the instrumental response and scaled, if necessary, \(\propto \nu^{-3}\) to 1300 MHz for consistency with Thornton et al. (2013) (scalings are over modest frequency ratios and insensitive to the scaling exponent).

We approximate the burst propagation as that in a flat static (Euclidean) universe. For the estimated redshifts, \(z \leq 0.96\) is inferred from the DMs by attributing the dispersion to intergalactic plasma with conventional cosmological parameters (the overwhelming majority of the baryon density distributed homogeneously in intergalactic space (Macquart et al. 2015). Possible inhomogeneity is discussed by Zhou et al. (2014); this only introduces an error of a factor \(\mathcal{O}(1)\), which is less than other uncertainties.

Following the classic model of Williamson (1972), Lee & Jokipii (1975), and Rickett (1977), we approximate the propagation paths as produced by a single scattering at a distance \(aD\) from us and \((1-a)D\) from the source. If the scattering angle is \(\Delta \theta \ll 1\), then the angles are \(\phi \approx (1-a)\Delta \theta\) and \(\chi \approx a\Delta \theta\); the geometry is shown in Figure 1.

We assume that the origin of the pulse width \(W\) is dispersion in propagation path lengths. The incremental delay attributable to scattering by an angle \(\Delta \theta\) (Williamson 1972; Kulkarni et al. 2014) is

\[
W \approx \frac{D}{2c} (\Delta \theta)^2 a (1-a).
\]

Then

\[
\Delta \theta \approx \sqrt{\frac{8cW}{Da(1-a)}} \geq \sqrt{\frac{8cW}{D}} \approx 5 \times 10^{-10},
\]

for all FRBs with measured \(W\). The minimum value is obtained for \(a = 1/2\). The angular width of the received radiation

\[
\phi \approx (1-a)\Delta \theta = \sqrt{\frac{2cW}{D} \frac{(1-a)}{a}}.
\]

Because of its low density, the intergalactic medium is unlikely to be the location of scattering. Table 1 shows that the pulse widths \(W_{1300}\) do not vary monotonically with DMs, in contrast to expectations for intergalactic scattering (Macquart & Koay 2013). In addition, Luan & Goldreich (2014) have shown that Kolmogorov turbulence in the intergalactic medium cannot explain the pulse broadening because the implied dissipation of turbulent energy would be excessive and the turbulence would decay rapidly (this argument does not apply to quasi-static isobaric heterogeneities such as the multi-phase structure of the interstellar medium, and its possible intergalactic analogues).

If a scattering screen is close to the source \((1-a \ll 1)\) or to the observer \((a \ll 1)\), then \(W \propto D \min(a, 1-a)\). The pulse broadening produced by Galactic scattering \((a \ll 1)\) is comparable to that of a Galactic pulsar in the same direction
because the screen’s distance $aD$ is similar, even though $D$ may be seven orders of magnitude greater for the FRB, while $1 - a = O(1)$ for both the Galactic pulsar and the distant FRB.

Galactic pulsars at high $b$ show orders of magnitude less pulse broadening than FRBs (Krishnakumar et al. 2015). Even at low $b$, the nanoshots of the Crab pulsar (Hankins & Eilek 2007) imply, assuming $\nu^{-2}$ scaling from 9.25 GHz, broadening $< 1.0 \mu s$ at 1300 MHz and those of PSR B1937 + 21, scaling only from 1.65 GHz (hence insensitive to the assumed scaling relation), broadening $< 40$ ns (Soglasnov et al. 2004). Unless these are the consequence of focusing, with the same propagation time for all focused paths by Fermat’s principle, they indicate that Galactic scattering cannot contribute significantly to the pulse widths of FRB.

Excluding intergalactic and Galactic scattering as the origin of the pulse broadening implies it takes place near the FRB sources. $\Delta \theta$ must be orders of magnitude larger than the numerical value in Equation (2) because $1 - a \ll 1$. We know nothing of the host galaxies of FRBs, but galaxies at $z \leq 1$ roughly resemble local galaxies. By the preceding argument, their general interstellar media cannot contribute a major fraction of the pulse broadening (or of the DM) unless all the five (four if the Lorimer burst is excluded) FRBs with reported pulse broadening are found in spirals viewed edge-on, a statistical improbability.

The immediate environment of the FRB must be much more strongly scattering than ordinary interstellar media, indicating a causal relation with the FRB itself. FRBs are likely to be young objects because compact, strongly scattering gas clouds would quickly dissipate (Section 4.1) if unbound.

3. PULSAR SUPER-PULSES?

There is an obvious resemblance between FRBs and the giant pulses of some radio pulsars. Like radio pulsars, FRBs are coherent emitters (Lorimer et al. 2007; Thornton et al. 2013; Katz 2014a) with very high brightness temperatures. Could FRBs be the same phenomenon on a more energetic scale, perhaps triggered by collapse of the neutron star (Falcke & Rezzolla 2014)?

The high DMs (and high Galactic latitudes) of FRBs indicate cosmological distances and much greater powers than inferred for Galactic pulsars; Thornton et al. (2013) estimated, assuming isotropic emission, $P \gtrsim 10^{42} \text{erg s}^{-1}$ for FRB 110220, where the lower limit assumes that the observed pulse width is partly intrinsic and not entirely the result of scattering. This should be compared to the classic result for a rotation-powered pulsar’s spindown power (converted to radio emission with an efficiency that is $\ll 1$ for all known radio pulsars)

$$P_{\text{spindown}} \approx \frac{2\mu^2 \Omega^2}{3c^3} \approx 2 \times 10^{44} \mu_{30}^2 \Omega_4^2 \text{ erg s}^{-1},$$

where $\mu_{30} \equiv \mu/10^{30} \text{ G cm}^3$ is the scaled magnetic dipole moment and $\Omega_4 \equiv \Omega/10^4 \text{ s}^{-1}$ is the scaled rotation frequency. This relation is exact for an oblique ($90^\circ$) rotator in a vacuum and is believed to be approximately correct for any angle between magnetic and rotational axes. The angular distribution of this power is unknown; dipole emission and winds from spindown of aligned rotors are roughly isotropic, but how this power is converted to coherent GHz radio emission and its resulting angular distribution is not understood.

Equation (4) indicates that a combination of high dipole moment and fast spin would be required to explain FRBs as rotation-powered pulsars. Radio pulsars are known with such high values of $\mu$ and $\omega$, but not in combination. The combination, if it occurs, would lead to a short spin-down time

$$t_{\text{spindown}} = \frac{1}{2} P_{\text{spindown}} = \frac{3}{4} \Omega_4 \approx \frac{10 \text{ year}}{(\mu_{30} \Omega_4)^2} \approx 10^3 \text{ year},$$

where $I \sim 10^{45} \text{ cm}^2$ is the moment of inertia and the last inequality uses Equation (4). For a neutron star spinning near breakup and $P_{\text{spindown}}$ as large as required by a cosmological distance for FRB 110220 (Thornton et al. 2013), $t_{\text{spindown}}$ could be as long as $\sim 10^3$ year, but this would require implausibly efficient emission and optimal choice of parameters.

3.1. Soft Gamma Repeaters (SGRs)?

A fundamental property of rotation-powered pulsar models is that their instantaneous power (including that into a particle wind that is believed to dominate the energetics) is limited by Equation (4); there is no intermediate energy reservoir between the rotation and accelerated particles or radiated fields that could be drawn down in sudden bursts of higher power. Rotation-powered pulsar models are thus distinguished from “magnetar” models (Katz 1982; Thompson & Duncan 1992; Mereghetti 2008) of SGR powered by dissipation of magnetic energy. The upper limit to the rate of energy release in a magnetar model is set by the poorly understood process of magnetic reconnection, and might be as high as $\mathcal{O}(R^{24}B^2/c) \sim 10^{46} \mu_{30}^2 \text{ erg s}^{-1}$, as required to account for the 2004 December 27 outburst of SGR 1806–20, assuming isotropic emission as would be expected for a magnetically trapped pair plasma.

In SGRs this power is thermalized, appearing as roughly blackbody emission of soft gamma-rays or hard X-rays, not as the high brightness radio frequency emission or relativistic particle acceleration of rotation-powered pulsars; thermalization may be inevitable at intensities $\gtrsim 10^{28} \text{ erg cm}^{-2} \text{ s}^{-1}$ (Katz 1996). Perhaps the same object might be both an SGR and an FRB (Kulkarni et al. 2014), though at different times or at different places in its magnetosphere. For example, PSR J1745–2900 is observed both as an anomalous X-ray pulsar (and hence is often termed a magnetar; Mori et al. 2013) and as a radio pulsar (Eatough et al. 2013) emitting high-brighness nonthermal radiation.

A Galactic FRB, if above the horizon, could be detected in the far sidelobes of any radio telescope observing in the L-band (Katz 2014a) with sufficient time resolution to detect a transient. No such detections of the known giant outbursts of Galactic SGRs have been reported, but it is unclear if any radio telescope was operating in a suitable mode, if the SGRs were above the horizon, or if such a signal would have been rejected as interference.

4. DISPERSION LOCAL TO FRBs?

4.1. Distribution of Dispersion Measures in Young SNRs

Some of the difficulties of the pulsar supergiant-pulse hypothesis would be mitigated if the dispersion were produced
in the pulsar’s immediate vicinity so that the DM would not imply cosmological distances, reducing the required energy (Connor et al. 2015; Pen & Connor 2015). Rotation-powered pulsar models of FRBs imply short-lived (hence, young) sources, but the constraints of Section 3 would be mitigated if they are cosmologically local. If so, a surrounding young SNR might be the chief source of dispersion (Connor et al. 2015; Kulkarni et al. 2014), possibly with a major Galactic contribution for the two low-latitude FRBs. The local DM of a source at the center of a spherical cloud of ionized gas of mass \( M \) and radius \( R \) is

\[
\text{DM}_{\text{local}} = 818 \frac{M}{M_\odot} \left( \frac{R}{0.1 \text{ pc}} \right)^2 f \text{ pc cm}^{-3},
\]

(6)

where \( f = 1 \) for a homogeneous sphere and \( f = 1/3 \) for a thin shell, implying \( R \approx 0.1 \text{ pc} \) for an SNR, lost stellar envelope, etc. providing much of the DM of an FRB.

As discussed in Section 2, this cloud must explain most of the pulse broadening, whether or not it contributes most of the DM. If the customary Kolmogorov turbulence model of diffractive scattering (Rickett 1977) is assumed, then

\[
C_{n_e}^2 \approx 0.5 \left( \frac{W}{5 \text{ ms Gpc}} \right)^{5/6} \left( \frac{0.1 \text{ pc}}{R} \right) \text{m}^{-20/3},
\]

(7)

This is comparable to values found for the more distant and highly dispersed Galactic pulsars (Krishnakumar et al. 2015), although the measured pulsar \( C_{n_e}^2 \) range over orders of magnitude. Lewandowski et al. (2013) have shown that the scattering spectral indices of many pulsars are far from the Kolmogorov model prediction of \(-4.4\), and there is no a priori reason to expect either this model or values of \( C_{n_e}^2 \) found for the Galactic interstellar medium to be applicable to the environments of FRB.

The characteristic lifetime \( T \) of an expanding cloud

\[
T \approx \frac{R}{V} \approx 30 \frac{R}{0.1 \text{ pc}} \frac{3000 \text{ km s}^{-1}}{V} \text{ year}
\]

\[
\approx 30 \left( \frac{f}{\text{DM}_{1000}} \frac{M}{M_\odot} \right) \frac{3000 \text{ km s}^{-1}}{V} \text{ year},
\]

(8)

where \( V \) is the expansion velocity and \( \text{DM}_{1000} \equiv \text{DM}_{\text{local}}/(1000 \text{ pc cm}^{-3}) \). At \( R = 0.1 \text{ pc} \) only \( \approx 10^{-3} n_{\text{ISM}} M_\odot \) of interstellar material will have been swept up for an interstellar density of \( n_{\text{ISM}} \text{ atoms cm}^{-3} \), so \( V \) is nearly the initial explosion velocity. If \( V \) is within the range of \( 3000-30,000 \text{ km s}^{-1} \) of SN ejecta, then the age of the cloud is \( t \approx T \lesssim 30 \text{ years} \). If FRBs are found within such clouds and if repetitive bursts are observed, their DMs will decrease monotonically and smoothly according to (6) with \( R = Vt \).

The absence of known SNRs within the last \( \sim 30 \text{ years} \) at the high Galactic latitude positions of most FRBs sets lower bounds on their distances.

The hypothesis that most of the dispersion is produced in an expanding cloud around the FRBs also predicts the distribution of DMs, subject to the unknown event rate (for example, bursts may be more frequent in younger objects or may not occur until after a latency period from their birth). If we assume an age-independent event rate (at least over the period at which the SNR provides much of the observed DM), then from (6) and (8) we find

\[
\frac{dN}{d\text{DM}} \propto \text{DM}^{-3/2}.
\]

(9)

Figure 2. Cumulative distribution of dispersion measures of high latitude (\( \theta_0 > 20^\circ \)) FRBs compared to a fitted distribution of dispersion measures of expanding SNR shells. The predicted divergence at small dispersion measures is absent, requiring an early cutoff to FRB activity. The dotted lines indicate the cutoff ages required to avoid a (not observed) divergence of the population at low DM; these ages are given for a \( 1 M_\odot \) shell with expansion speed \( V = 3000 \text{ km s}^{-1} \) and scale \( \propto M^{1/2}/V \). The observed FRB distribution also disagrees with the prediction of the SNR hypothesis for DM \( \gtrsim 700 \text{ pc cm}^{-3} \).

In Figure 2 we show the cumulative distribution of DMs of the high latitude FRBs shown in the Table 1, and compare it to the relation \( N \propto \text{DM}^{-1/2} \) predicted by Equation (9). Even though only eight such FRBs are known, their cumulative distribution is robust. The fit is not good.

The observed distribution of DMs does not display the long “tail” of high DM bursts predicted by cosmologically local models in which much of the dispersion is provided by an expanding gas cloud such as (but not necessarily) an SNR. Nor does it show the predicted divergence at low DM. Even if the absence of this low-DM divergence were explained by short active lifetimes of the FRB sources as required by Equation (5) and consistent with Equation (8), the deficiency of high DM FRBs argues against the hypothesis that most of the dispersion is provided by expanding SNRs (Connor et al. 2015; Kulkarni et al. 2014). It therefore argues in favor of the cosmological distances inferred by attributing the DMs to the intergalactic medium. Then a deficiency of high DM FRBs may be attributed to the lesser fluence (greater luminosity distance) of more distant FRBs.

This does not require rejection of the hypothesis that FRBs are produced by high field fast rotation powered pulsars. Attributing most of the dispersion to the intergalactic medium...
and accepting cosmological distances would resolve the difficulty (Equation (8)) that if an SNR is to provide the dispersion, its rapid expansion (especially if \( V > 3000 \text{ km s}^{-1} \), as found for most SNe) would require FRB activity to occur very soon after the pulsar’s birth. However, the constraint \( t < t_{\text{spindown}} \) would remain.

### 4.2. Quasi-static Clouds

If the dispersion occurs in a stable quasi-static plasma cloud, then the condition that FRB activity occurs within a time given by Equation (8) would no longer apply. Such clouds need not be associated with the compact source of the FRB, but might be, for example, dense formerly molecular clouds ionized by a starburst or the ionized interstellar medium of an irregular galaxy with rapid star formation. Specific examples in regions that may be plausible sites of FRBs include dense ionized star-forming structures (Kulkarni et al. 2014) and circum-galactic nuclear gas (Pen & Connor 2015).

Equation (8) is then replaced by the Jeans condition that the cloud is stable against gravitational collapse. This would constrain its parameters:

\[
\sqrt{\frac{GM}{R}} \lesssim c_s = \sqrt{\frac{5k_B T (1 + \mu_e)}{3m_p}},
\]

(10)

where \( c_s \) is the sound speed and \( \mu_e \approx 0.85 \) is the number of electrons per baryon. Substituting \( M \approx R^3 \rho_n n_e/\mu \) and DM \( \approx n_e R \) (attributing the dispersion to the source’s plasma cloud, not the intervening line of sight), we find

\[
R \lesssim \frac{5(1 + \mu_e) \mu_e k_B T}{3GDM \rho_n^2} \approx 5 \times 10^{21} \frac{T_{\text{8000}}}{\text{DM}_{1000}} \text{ cm}
\]

(11)

and

\[
n_e \sim \frac{\text{DM}}{R} \gtrsim 0.6 \text{ DM}^2_{1000} T_{\text{8000}}^{-1} \text{ cm}^{-3},
\]

(12)

where we normalize the temperature \( T_{\text{8000}} \equiv T/8000^\circ \text{K} \) (following Kulkarni et al. 2014) and the dispersion measure \( \text{DM}_{1000} \equiv \text{DM}/1000 \text{ pc cm}^{-3} \), and assume complete ionization and cosmic abundances. The corresponding mass is

\[
M \lesssim \frac{25k_B T (1 + \mu_e)^2 \mu_e}{9G^2 \rho_n^2 \text{DM}} \approx 8 \times 10^7 \frac{T_{\text{8000}}}{\text{DM}_{1000}} \text{ M}_\odot.
\]

(13)

The hydrodynamic time

\[
T_J \sim \frac{R}{c_s} \approx \sqrt{\frac{5k_B T (1 + \mu_e)}{3m_p}} \frac{\mu_e}{G \rho_n \text{DM}} \approx 10^8 \frac{T_{\text{8000}}^{1/2}}{\text{DM}_{1000}} \text{ year}
\]

(14)

has no explicit dependence on the unknown parameters \( n_e, R, \) and \( M, T_J \) is long enough to avoid the statistical problems (Section 4.1) posed by attributing the DMs to young SNRs, whose youth implies that only a very few are active with the observed DMs at any time. The dispersive cloud could be more compact and dense than the bounds (11) and (12), perhaps by a large factor.

These bounds are consistent with dense quasi-static compact clouds in the FRB neighborhood (Pen & Connor 2015) while avoiding the rapid expansion and short lifetime implied by attributing dispersion to young SNRs. Much smaller \( R \) and \( M \) and larger \( n_e \) than the bounds are possible. The bounds also admit a protogalaxy or starburst ionized by an initial generation of hot luminous stars, providing the observed DMs. Such sites may be plausible locales for FRBs, but give no clues to the origin of the FRBs themselves beyond indicating a relation with massive stars and high rates of star formation and death.

### 5. HOW MANY FRB SOURCES?

Models of FRBs may be divided into two general classes: those in which they are the product of catastrophic events (e.g., SN, neutron star mergers, neutron star accretion by black holes, and GRBs) that destroy their participants and cannot repeat, or those in which they are the product of non-catastrophic events (e.g., giant pulsar pulses and SGR outbursts) that can repeat. In the former case, the number of sources will equal the number of observed FRBs, but in the latter case repetitions may be observed. The confirmed observation of a single repetition would establish their origin in non-catastrophic events. Here we consider the constraints that can be placed on \( N_{\text{sources}} \) non-catastrophic sources if, as at present, no repetitions have been observed.

There are two constraints on the number of presently active detectable FRB sources \( N_{\text{sources}} \equiv B T \), where \( B \) is their birth rate within the volume from which FRBs may be detected and \( T \) is their active lifetime (during which they have the defining properties of FRBs, including outbursts and DMs). If the bursts occur stochastically without any latency period following a burst, then the absence of coincidences among \( N_{\text{FRB}} \) observed FRBs implies

\[
N_{\text{sources}} \gtrsim N_{\text{FRB}}^2 = 100.
\]

(15)

As more FRBs are discovered, either coincidences (repetitions) will be observed, implying

\[
N_{\text{sources}} \sim \frac{N_{\text{FRB}}^2}{N_{\text{coincidences}}},
\]

(16)

or the lower bound of Equation (15) will increase.

The absence of repetitions of any individual FRB implies

\[
N_{\text{sources}} \gtrsim \Omega_{\text{FRB}} \tau_{\text{min}} \sim 10^3,
\]

(17)

where \( \Omega_{\text{FRB}} \) is the all-sky FRB rate and \( \tau_{\text{min}} \) is the empirical lower bound on the repetition time of an individual source. Keane & Petroff (2015) and Ravi et al. (2015) estimate \( \Omega_{\text{FRB}} \sim 0.03 \text{ s}^{-1} \) with an uncertainty as large as a factor of three and \( \tau_{\text{min}} \sim 10 \text{ hr} \).

If the bursts are stochastic, then \( \tau_{\text{min}} \lesssim \tau_{\text{total}} \), the total time beams pointed in the known directions to FRBs, summed over all FRBs, without observing a repetition. Law et al. (2014) found no recurrences in \( 1.1 \times 10^5 \) s of observations of a single FRB, implying a 95% confidence bound of \( \tau_{\text{min}} > 2.7 \times 10^8 \), giving the numerical estimate in (17). On the other hand, if there is a latency period between FRBs from a single source, then, depending on how the observing time was distributed, \( \tau_{\text{min}} \) may be as short as \( \tau_{\text{cont}} \), the longest duration of continuous observation of an individual FRB location without a repetition.\(^2\)

\(^2\) It is not necessary that a beam is pointed to a single FRB for this time because if they all have the same properties, staring in all directions in which an FRB has been observed is equivalent. It is also assumed that localization is good enough that the chance of misidentifying a new source as a repetition of a previously observed source is negligible; for 15’ localization and \( N_{\text{FRB}} \sim 10 \) this chance is \( \sim 2 \times 10^{-5} \).
repetition. The conditions (15)–(17) may be used to test models of \( N_{\text{sources}} \) against the empirical parameters of \( N_{\text{FRB}}, \tau_{\text{min}} \) and \( \Omega_{\text{FRB}} \), and thereby to constrain models of the sources, of their astronomical environments and of their distances.

6. DISTANCES FROM COMPARISON TO SN RATE

Here we consider the consequences of the plausible but unproven assumption that FRBs are associated with SNe. The number of SNRs of ages \( t < T \) (Equation (8)) associated with our Galaxy (out to distances \(~1 \text{ Mpc}) with sufficient column density to provide the observed FRB dispersions is inferred from a Galactic SN rate of \( 0.03 \text{ yr}^{-1} \) to be \( N_{\text{SNR}} \sim O(1) \), inconsistent with the observation of 10 FRBs. Equation (15) provides an even stronger argument against such close SN-associated sources. Further, the all-sky FRB rate \( \Omega_{\text{FRB}} \sim 0.03 \text{ s}^{-1} \) would imply a repetition time of an individual source \( \tau \sim N_{\text{sources}}/\Omega_{\text{FRB}} = N_{\text{SNR}} \sim 30 \text{ s} \). The hypothesis of such rapid repetitions of FRBs is excluded empirically by orders of magnitude (Law et al. 2014; Petroff et al. 2015a).

If FRBs are associated with SNe at a rate of order one-to-one (the FRBs do not repeat), comparison of the rates of the two classes of events shows that their distances must be cosmological: The SN rate is estimated (Sharon et al. 2007) to be \( \Omega_{\text{SN}} \approx 0.098 \times 10^{-12} M_{\odot} \text{ yr}^{-1} \). Standard cosmological parameters indicate a local baryon density of \( \rho_{\text{baryon}} = 1.9 \times 10^{-77} \text{ cm}^{-3} \) and an SN rate of \( \Omega_{\text{SN}} \rho_{\text{baryon}} \approx 1.9 \times 10^{-77} \text{ cm}^{-3} \text{ yr}^{-1} \). Comparison to the all-sky FRB rate of \( \Omega_{\text{FRB}} \approx 0.03 \text{ s}^{-1} \) indicates

\[
D \sim \left( \frac{3}{4\pi \Omega_{\text{SN}} \rho_{\text{baryon}}} \right)^{1/3} \sim 1 \text{ Gpc.} \tag{18}
\]

With these assumptions, FRBs must originate at cosmological distances even if much of their DMs are local to their sources. Local matter might be the source of dispersion if FRBs are giant pulsar pulses or SGR outbursts (Kulkarni et al. 2014), but neither of these can explain the distribution of DMs (Section 8).

If, on the other hand, many FRBs are associated with each SN (as is plausible if they are supergiant pulsar pulses), we can still set a lower bound on the distance out to which FRBs are observed by requiring that the number of FRB sources be at least the number of SNe in the active lifetime \( T_{\text{FRB}} \) of the product of an SN:

\[
D \gtrsim \left( \frac{3N_{\text{sources}}}{4\pi \Omega_{\text{SN}} \rho_{\text{baryon}} T_{\text{FRB}}} \right)^{1/3} \sim 10 \text{ Mpc}; \tag{19}
\]

the numerical value assumes \( T_{\text{FRB}} \sim 3000 \text{ year} \), the estimated active lifetime of a SGR. If, instead, \( T_{\text{FRB}} \) is taken to be \(~30 \text{ years}, the duration over which an SNR can provide the observed DM (8), then \( D \gtrsim 50 \text{ Mpc} \). Finally, the absence of an obvious correlation with a cosmologically local structure (clusters and superclusters of galaxies) suggests \( D \gtrsim O(100) \text{ Mpc} \).

7. LOG N—LOG S

The distribution of fluxes or fluences for a population of objects is a classic tool for determining their distribution in space. It was used in the early days of radio astronomy to exclude steady-state cosmology and in gamma-ray burst astronomy it indicated their origin at cosmological distances. Because \( N(S) \) is the cumulative number of sources with flux or fluence greater than some threshold \( S \), meaningful conclusions can be drawn from remarkably small values of \( N(S) \). For FRB \( S \) must be the fluence. In flat static (Euclidean) geometry an inverse square law holds and \( N(S) \sim S^{-2/3} \). Results for real cosmological models cannot be disentangled from the evolution of the FRB event rate, which is completely unknown; therefore, we compare to the Euclidean result, which is approximately valid for \( z \lesssim 1 \).

The data are shown in Figure 3. If allowance is made for the likely incompleteness of the sample for \( S \sim 1 \text{ Jy ms} \), the distribution is consistent with homogeneous Euclidean space as is also suggested by its isotropy, except for the anomalous Lorimer burst. Keane & Petroff (2015) give only a lower bound on its fluence of 31.5 Jy ms rather than the specific (but uncertain) value of 150 Jy ms cited by Lorimer et al. (2007) and used in the figure; however, even if this minimum is its actual value, its deviation from the power law would still be problematic.

These results do not determine a distance scale, but do indicate that FRBs are not limited to a bounded region. Provided the deviation from the power law at low \( S \) is attributed to artefacts (for example, different research groups or instruments having differing calibrations, or thresholds for detection or acceptance of transients as FRB), they argue against models in which FRBs are associated with bounded structures (the Solar System, Galaxy, local group, Local
8. DISTRIBUTION OF DISPERSION MEASURES—COSMOLOGICAL

It is possible to use the distribution of FRB DMs to constrain their spatial distribution if the dispersion is attributed to a uniform intergalactic medium. This is complementary to the use of the $\log N$–$\log S$ distribution. Unlike $\log N$–$\log S$, this method is specific to FRBs.

The number of FRBs with origins between distances of $r$ and $r + dr$

$$
dN = K(r) r^2 dr,
$$

where $K(r)$ should include the effects of the geometry of the universe, the redshift of the event rate, the evolution and varying density of the source population, and the $K$-correction (redshift of the spectrum). The source evolution and $K$-correction are unknown. For low redshift $z < 1$ $K(r) \propto K_1$, a constant. More generally, $K(r)$ contains three powers of $1 + z$ from the variation of the source density minus one power from the redshift of the event rate, and minus one power from the redshift of the photon energy (because fluence rather than flux is measured, the rate of photon arrival does not contribute another factor). If the source spectrum is a power law $S \propto \nu^{-\gamma}$ then $K(r) \propto (1 + z)^{\gamma - 1}$. With $\gamma$ and the source evolution unknown, treating $K(r)$ as a constant may not be far wrong for $z \lesssim 1$.

In the same spirit of approximation

$$
\frac{dz}{H_0 (1 + z)} = \frac{c}{H_0} dr
$$

$$
r = \frac{c}{H_0} \ln(1 + z)
$$

$$
n_e = n_0 (1 + z)^3 = 1.6 \times 10^{-7} (1 + z)^3 \text{ cm}^{-3}
$$

$$
d\text{DM}_{\text{eff}} = n_e dr = \frac{n_0 c}{H_0} (1 + z) d \ln(1 + z)
$$

$$
\text{DM}_{\text{eff}} = \frac{n_0 c}{H_0} [(1 + z) - 1]
$$

where the baryons are assumed to be homogeneously distributed and ionized with the present-day density of $n_0 = 1.6 \times 10^{-7} \text{ cm}^{-3}$ and $\text{DM}_{\text{eff}}$ is the measured DM allowing for the higher frequency of the observed radiation in the distant universe. Then

$$
\frac{dN}{d\text{DM}} = \frac{K c^2}{n_0 H_0^2} \left[\ln \left(1 + \frac{H_0 \text{DM}}{n_0 c}\right)\right]^2.
$$

This is readily integrated

$$
N = \frac{K c^3}{3 H_0^3} \left[\ln \left(1 + \frac{H_0 \text{DM}}{n_0 c}\right)\right]^3 \propto \left[\ln \left(1 + \frac{\text{DM}}{682 \text{ pc cm}^{-3}}\right)\right]^3.
$$

Figure 4 shows the cumulative distribution of $N$ as a function of DM and a curve of the form of Equation (28) with one fitted parameter ($K$). The fit is good. The most striking feature is the absence, in both the data and the theoretical curve, of FRBs with low DMs. This is explained by the small volume $\propto z^3$ of space with low $z$; Euclidean geometry and the absence of cosmic evolution are good approximations at small redshifts. This supports the interpretation of the DMs of FRB as resulting from propagation through intergalactic space, independent of any specific model of their origin. In contrast, models (Kulkarni et al. 2014; Pen & Connor 2015) that attribute FRB DMs to quasi-static clouds (Section 4.2) in their vicinities make no specific predictions of the distribution of DMs, but do not naturally explain the data shown in Figure 4.

This result is robust against changes in the assumptions. For example, ignoring the $(1 + z)^{-2}$ factor in $d\text{DM}_{\text{eff}}$ that results from the $z$ dependence of the frequency of radiation along its path. The low fluence deviation cannot be explained by the variation of sensitivity across a beam. The reported fluences are nominal values that apply if the FRB is centered in the beam and are not corrected for the beam pattern (a correction that cannot be made because the location of the FRB within the beam is not known, except possibly for the Lorimer burst that was detected in the sidelobes of two beams in addition to its principal detection). Each element of beam solid angle contributes a distribution of $\delta N(S) \propto S^{-3/2}$, and their sum is $N(S) \propto S^{-1/2}$. 

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[^3]: The low fluence deviation cannot be explained by the variation of sensitivity across a beam. The reported fluences are nominal values that apply if the FRB is centered in the beam and are not corrected for the beam pattern (a correction that cannot be made because the location of the FRB within the beam is not known, except possibly for the Lorimer burst that was detected in the sidelobes of two beams in addition to its principal detection). Each element of beam solid angle contributes a distribution of $\delta N(S) \propto S^{-3/2}$, and their sum is $N(S) \propto S^{-1/2}$. 

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![Figure 4. Cumulative distribution of FRBs (the eight at high Galactic latitudes for which the Galactic contributions to their dispersion measures are small and can be reliably subtracted) vs. the extragalactic part of their dispersion measures. The “Intergalactic” curve (Equation (28)) describes a simple model of dispersion attributable to the intergalactic medium. Agreement is good.](image-url)
path leads to a fit similar to that of Figure 4 (but with a different $K$). This robustness is not surprising because the model is dominated by the $\propto z^3$ dependence of the volume and $N$ in the local (Euclidean) limit.

9. DISCUSSION

Understanding the high brightness emission of FRBs may be as elusive as understanding the emission of radio pulsars has been, nearly 50 years after their discovery. Radio pulsars were very quickly identified on the basis of their periods and slowing with rotating neutron stars. No such clues exist for FRBs.

Even with only 10 published FRBs and without a specific model of their sources, statistical arguments can constrain their origin, both physical and spatial. Their qualitative resemblance to radio pulsars, in particular to the rare giant pulses of a few pulsars, suggests that they may be analogous events, writ large. Elementary energetic considerations show that this may be possible, but if so, their sources must be very young neutron stars.

This leads to the hypothesis that, surrounded by young, dense, compact SNRs, much of the DMs of FRBs may be attributed to those SNRs (Connor et al. 2015; Kulkarni et al. 2014). If so, the distances to FRBs may be much closer than those indicated by attributing their DMs to the intergalactic medium, relaxing the constraints on their energetics implied by the assumption of cosmological distances. This hypothesis predicts a specific form for the distribution of FRB DMs. Unfortunately, comparison to the data (Figure 2) shows a poor fit at both low and high DMs. The former might be explained by termination of FRB activity as the neutron star spins down (or ages in some other manner), in analogy to the pulsar “death line”, but the absence of FRBs with very high DMs is an argument against this origin of their dispersion.

The classic $\log N - \log S$ test, where $S$ is the FRB fluence rather than the (unmeasured) flux, may also be applied (Figure 3). A fit to the relation $N \propto S^{-3/2}$ applicable to Euclidean geometry (cosmologically local sources) is adequate at intermediate values of $S$. A deficiency of low fluence FRBs may be explained by sample incompleteness. If so, the fit at higher fluences indicates a source population with $z \lesssim 1$, but excludes a spatially bounded distribution such as the Galactic halo.

Similar conclusions may be drawn from the distribution of FRB DMs less than a cutoff (Figure 4). A simple model that includes both cosmologically local and distant sources (but one that ignores the unknown evolution of the source population, and hence must be considered only qualitative for $z \gtrsim 1$) provides a good fit. In contrast to the similar inferences from the $\log N - \log S$ data, this conclusion does not depend on additional explanations of apparent deviations from the model. A robust conclusion independent of any caveats applicable for $z \gtrsim 1$ is that the source population is homogeneously distributed in space at smaller $z$. This is an essentially Euclidean conclusion, but the satisfactory fit for $z \sim 1$ suggests that the distances are those inferred from the assumption that the DMs are intergalactic. This is supported by the failure of the alternative expanding cloud (SNR) models of the dispersing matter (Figure 2). The fact that this cosmological model of the distribution of DMs is consistent with the data implies that nearly all the dispersion is intergalactic, in contrast with models (Kulkarni et al. 2014, 2015) in which a large portion of the DM is attributed to gas local to the FRB.

The extraordinarily intense Lorimer burst is not consistent with $N \propto S^{-3/2}$. The absence of weaker events was noted by Lorimer et al. (2007), and indicates that it comes from a different population. With a fluence $\sim 100$ times the detection threshold, a homogeneous distribution in Euclidean space (its comparatively small DM indicates it must be cosmologically local) would suggest that $\sim 1000$ weaker events should have been detected, in contrast to the 10 actually detected, an anomaly that is significant at the $\approx 99\%$ level (the detection of 10 weaker events implies a $1\%$ probability of detecting one as strong as the Lorimer burst). If this was neither a statistical fluke (because of the need to reject anthropogenic transients, early searches may have had effective detection thresholds higher than their nominal thresholds) nor anthropogenic interference, it implies the Lorimer burst came from a distinct space-limited population whose local density far exceeds its mean cosmological density. Speculative possibilities include classes of intrinsically less luminous bursts in the Galactic halo or the Local Supercluster.

APPENDIX

SPACE RADAR AS A SOURCE OF CHIRPED PULSES?

Since the discovery of FRBs, there has been concern that they might be anthropogenic rather than astronomical phenomena. This concern was exacerbated by the discovery of manifestly anthropogenic perytons by Burke-Spolaor et al. (2011), whose sources were recently identified by Petroff et al. (2015b) as microwave ovens. Could FRBs also be anthropogenic interference, perhaps chirped radar pulses? A ground-based radar might enter the sidelobes of a radio telescope, but it would appear in all 13 Parkes beams, unlike the FRBs. Hence, we consider a radar on a satellite that might pass through a beam.

Radar systems may use chirped emission, compressed upon reception into narrow pulses, to obtain accurate range measurements without requiring excessive peak transmitted powers. The observation of FRBs in a single beam at Parkes in contrast to perytons (Burke-Spolaor et al. 2011), indicates a distance of $\gtrsim 20$ km outside the first Fresnel zone, consistent with a radar satellite. There is no obvious reason for a radar to have a chirp of $\omega \propto t^{-1/2}$ as observed, nor is there an obvious reason not to. However, the observed dispersed pulse durations of several tenths of a second would imply, for monostatic radar, target distances of at least half that light travel time to avoid interference of the transmission with the received scattered radiation. Such distances would be $\sim 10^{10}$ cm, far beyond the range to plausible targets, and the return would be undetectably weak.

In contrast, bistatic radar can use arbitrarily long pulses. The pulse repetition intervals would have to have been longer than the lengths of time the radars were anywhere in the 13 beams of the Parkes Multibeam Pulsar Survey (about 0.3 s for a radar near the zenith in low Earth orbit), yet the pulse durations must have been shorter than the time required to cross a single beam. This explanation would also require at least as many radar satellites, each with a different chirp rate, as FRBs because each FRB had a different DM, or satellites whose chirp rates were variable in some non-obvious manner. This combination
of requirements makes the hypothesis of interference by an orbital chirped source implausible.

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