Research on the Effects of Machining Parameters on the Surface Location Error for a Milling Process

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Abstract. The surface location error is an inevitable phenomenon which is caused by the forced vibration under a stable machining process. The surface location error (SLE) is dependent on the machining parameters. Therefore, this paper proposes a method to study the effect of each machining parameter on the surface location error. An algorithm based on the cutting force dynamic model is proposed to predict the SLE in frequency domain. And then a case study is performed on the milling process of a real vertical machining center. The variation ranges of the spindle speed, axial cutting depth, radial cutting depth and feed rate per tooth are first determined. Eight specific values of each machining parameter is selected, and an orthogonal experiment table is established to arrange the schemes to predict the corresponding SLEs. Then the range analysis is used to analyse the values of the machining parameters and the obtained SLEs. Results show that the spindle speed is the most dominant factor affecting the SLE and an optimal machining parameters combination related to a minimum SLE is determined based on the optimal level of each machining parameter.

1. Introduction Machining Parameter

The surface location error is defined as the difference between the actual and commanded geometric dimensions of the machined surface, affecting the part quality and productivity [1,2]. For a stable milling, the inherent interrupted cutting conditions results in a periodic cutting force, and then lead to the forced vibrations for the tool and workpiece. The cutting force, which is dependent on the machining parameters and tool-workpiece system properties, cause the tool and workpiece to deflect. Thus, this time-dependent displacement depends on the system frequency response functions (FRFs), excitation frequency and machining parameters. Due to the vibration of the tool-workpiece system, the workpiece may be undercut (less material removed than commanded) or overcut (more material removed than commanded), generating the surface location error of the final surface [3-5].

The surface location error (SLE) has been previously researched and applied to reflecting the geometrical precision of the machined parts [6]. Shirase and Altintas [7] provided the method to predict the SLE for variable pitch helical end mills, and they have found that the variable pitch tools change both SLE and stability behavior. Schmitz and Ziegert [8] have studied the effects of the spindle speed and the tool-workpiece system FRFs on SLE based on the numerical simulation in time domain. Later, they carried out a case study to quantify the relative contributions of geometric, thermal, contouring, and cutting force errors to machined part dimensional errors [9]. Schmitz and Mann [10]
proposed two analytical approaches to calculate the SLE. One is the harmonic balance approach, in which the modal parameters are used to describe the dynamics of the milling system. The other is the new frequency domain approach, which utilized the system FRFs directly. On the basis, Kiran et al. [11] considered both the tool and workpiece flexibility and developed a two degree of freedom (DOF) closed-form frequency domain solution for predicting the SLE. However, the researches addressing the effects of each machining parameter on the SLE are little.

In this paper, a study is developed to analyze how each machining parameter effects the surface location error. Henceforth, the remainder of this paper is organized as follows: a description of the milling dynamic system and SLE prediction in frequency-domain is presented in Section 2. This is followed by a case study in Section 3, in which an orthogonal experimental design is introduced to perform the simulations for predicting the SLE, and then the degree of the effects of each machining parameters on the SLE is discussed. In Section 4, conclusions are summarized.

2. SLE Prediction in Frequency Domain

2.1. Dynamic Model of the Milling Process

A two degree of freedom (DOF) dynamic system is generally used to describe the milling process, and the related dynamic model is shown in figure 1. Then the cutting force in tangential and normal directions ($F_t$ and $F_n$) can be described as follow [12]:

$$F_t = \sum_{i=1}^{N_t} K_t a_i h(\phi_i) = K_t a_f f_i \sin(\phi_i)$$

$$F_n = \sum_{i=1}^{N_n} K_n a_i h(\phi_i) = K_n a_f f_i \sin(\phi_i)$$

where $a_i$ is the axial cutting depth, $h(\phi_i)$ is the instantaneous chip thickness , $f_i$ is the feed per tooth, $\phi_i$ is the $i$th tooth angle, $N_t$ is the number tooth of tooth, and $K_t$ and $K_n$ are the tangential and normal cutting force coefficients. As the tangential and radial force components rotate with the cutting tool, they are projected into the fixed $x$ and $y$-directions:

$$F_x = \sum_{i=1}^{N_t} F_t \cos(\phi_i) + F_n \sin(\phi_i)$$

$$F_y = \sum_{i=1}^{N_t} F_t \sin(\phi_i) - K_n \cos(\phi_i)$$

Considering not all teeth engaged in cutting at the same moment, the final expression of the cutting force in two directions is:
\[
F_x(\phi) = \frac{a_x}{2} K_x f_n \sum_{n=1}^{\infty} g(n\phi) \left( \sin(2n\phi) + \frac{K_n}{K_x} \sin(n\phi) \right)
\]
\[
F_y(\phi) = \frac{a_y}{2} K_y f_n \sum_{n=1}^{\infty} g(n\phi) \left( -2\sin^2(n\phi) + \frac{K_n}{K_y} \sin(2n\phi) \right)
\]

(3)

where \( g(\phi) \) is the function defining whether the current tooth engages in cutting. If the angle is between the cut entry and exit angles, \( g(\phi) \) equals 1; otherwise, \( g(\phi) \) equals 0. \( \phi_i \) is the \( i \)th tooth angle: 
\[
\phi_i = \omega t + (2\pi/N)(i - 1) \text{(rad)},
\]
where \( \omega \) (rad/s) is the rotating frequency of the spindle.

A Fourier series is applied to representing the periodic cutting force components:

\[
F_x(\phi) = \sum_{n=-\infty}^{\infty} a_n + \sum_{n=1}^{\infty} \left( a_n \cos(n\phi) + b_n \sin(n\phi) \right)
\]
\[
a_n = \frac{1}{\pi} \int_{0}^{\pi} F_x(\phi) \cos(n\phi) d\phi
\]
\[
b_n = \frac{1}{\pi} \int_{0}^{\pi} F_x(\phi) \sin(n\phi) d\phi
\]

(4)

where \( a_n, a_n, \) and \( b_n \) are the Fourier coefficients, and the expressions of \( F_x(\phi) \) and \( F_y(\phi) \) are the same.

2.2. The SLE Prediction Algorithm

A visual description of SLE phenomenon under the overcut case is represented in figure 2. Four steps are summarized to realize the SLE prediction:

- Obtain the cutting forces in \( x \) and \( y \) directions \( F_x(t) \) and \( F_y(t) \) in time domain based on the milling conditions, and then apply the discrete Fourier transform to obtain the cutting force \( F_x(\omega) \) and \( F_y(\omega) \) in frequency domain;
- Calculate the displacements in \( x \) and \( y \) directions in frequency domain: \( X(\omega) = FRF_x(\omega) F_x(\omega) \) and \( Y(\omega) = FRF_y(\omega) F_y(\omega) \), where \( FRF_x(\omega) \) and \( FRF_y(\omega) \) are the frequency response functions of the tool tip in two directions;
- Compute the inverse Fourier transform of \( X(\omega) \) and \( Y(\omega) \) to get the displacements in \( x \) and \( y \) directions \( X(t) \) and \( Y(t) \);
- Using a trimming algorithm to isolate the machined surface from the entire final tool path, and then predict the SLE according to the machined surface geometry.

First, the points with the coordinate values meeting the following condition are selected:

\[
S_y < -Q_c \cdot \frac{D}{2} \quad \text{down milling}
\]
\[
S_y > Q_c \cdot \frac{D}{2} \quad \text{up milling}
\]

(5)

where \( S_y \) is the location coordinate in \( y \) direction, \( D \) is the tool diameter, and \( Q_c \) is the appropriately defined coefficient, such as 0.95. The selected points are arranged on the tool path in the ascending \( x \)-
feed direction. Values of these points are compared repeatedly to keep the lower ones for down milling or the higher ones for up milling, determining the final machined surface geometry. Then the surface location error is calculated using the mean value of the final selected points ($S_{\text{average}}$) and the tool diameter $D$:

$$S_{\text{LE down milling}} = \left( \frac{D}{2} + S_{\text{average}} \right)$$
$$S_{\text{LE up milling}} = \left( \frac{D}{2} - S_{\text{average}} \right)$$

(6)

3. A Case Study and Results Discussion

To illustrate the application of the surface location error prediction in real milling process and further discuss the effects of all machining parameters on the surface location error, a case study is performed on a real vertical machining center.

3.1. The Impact Testing to Obtain the Tool Tip FRFs

According to the SLE algorithm in section 2.2, the dynamics of the milling system which are commonly presented by the tool tip frequency response functions are the prerequisite. Therefore, a impact testing has been performed at the tool tip to obtain the FRFs in $x$ and $y$ directions. As shown in figure 3, an impact hammer is used to excite at tool tip, and the responses are picked up by an accelerometer pasted on the opposite position. Then the force signal and acceleration signal are processed by the modal analysis software to obtain the FRFs in two directions show in figure 3.

![Figure 3. The impact testing and an example of the FRFs](image)

3.2. Orthogonal Experimental Design

The milling conditions are determined as: down milling, the diameter of the end mill with a helical angle of 45 degree is 20 mm, the tooth number is 4, the workpiece is the commonly used 45 steel, the tangential and normal force coefficients are 1740 and 796 Mpa, and the variation ranges of the spindle speed $\Omega$, axial cutting depth $a_p$, radial cutting depth $a_e$ and feed rate per tooth $f_z$ are: [40, 10000] (rpm), [0.02, 10] (mm), [0.02, 20] (mm) and [0.02, 0.2] (mm/per tooth).

In order to study the effect of each machining parameter on the SLE, an orthogonal experimental design is introduced to arrange the SLE prediction. Each machining parameters is determined to have 8 specific values, and 81 schemes are finally arranged as shown in Table 1. For each scheme, the axial cutting depth are recalculate based on the milling stability analysis to guarantee it smaller than the limiting axial cutting depth, and all the simulations are performed in the Matlab software.

To further analyze the effect degree of each machining parameter on the SLE, the range analysis is used to analyze the values of the machining parameters and calculated SLEs in Table 1. A larger range value $R_i$ of one machining parameter means that it affects SLE more significantly. And then the optimal level of each machining parameter is determined to obtain the optimal combination of machining parameters to have a minimum SLE.

$$R_i = \max(k_j) - \min(k_j)$$

(7)

where $i$ and $j$ mean the factor number and level number of the orthogonal experimental design respectively. $k_j$ is the sum of the SLEs corresponding to the $j_{th}$ level of the $i_{th}$ factor in Table 1.
As the calculated range values for the \( \Omega, a_p, a_e \) and \( f_z \) are 1053, 27, 390 and 469 respectively, the effect order of the four machining parameter on SLE is: \( \Omega > f_z > a_e > a_p \). The optimal combination with the minimum SLE is \( \Omega(10), a_p(2), a_e(2), f_z(2) \).

4. Conclusions

Table 1. The orthogonal experimental design table for the SLE prediction

| Order | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 |
|-------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|
| \( \Omega \) (rpm) | 300 | 400 | 900 | 900 | 500 | 500 | 700 | 400 | 700 | 500 | 300 | 300 | 300 | 100 |
| \( a_p \) (mm) | 6.2 | 2.5 | 1.2 | 2.5 | 2.5 | 1.2 | 5 | 10 | 6.2 | 7.5 | 3.7 | 5 | 5 | 0.6 |
| \( a_e \) (mm) | 200 | 50 | 25 | 125 | 175 | 90 | 150 | 75 | 50 | 75 | 125 | 9 | 12.5 | 2.5 |
| \( f_z \) (×10^1 mm) | 1.7 | 0.5 | 1.5 | 1.0 | 0.2 | 1.7 | 1.0 | 1.7 | 0.2 | 0.7 | 0.7 | 12 | 0.7 | 12.5 |
| \( SLE \) (μm) | 4.8 | 35. | 3.4 | 3.8 | 0.5 | 9.8 | 5.0 | 8.6 | 6.7 | 0.7 | 12. | 0.3 | 3.2 | 26. |
| Order | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 |
| \( \Omega \) (rpm) | 600 | 100 | 300 | 400 | 700 | 900 | 900 | 500 | 800 | 800 | 800 | 600 | 300 | 500 |
| \( a_p \) (mm) | 6.2 | 0.4 | 0.9 | 1.8 | 1.2 | 0.4 | 0.6 | 0.5 | 0.4 | 0.7 | 2.2 | 2.5 | 0.5 | 0.3 |
| \( a_e \) (mm) | 2.5 | 20 | 5 | 9 | 10 | 17. | 5 | 10 | 15 | 15. | 7.5 | 12. | 7.5 | 10 |
| \( f_z \) (×10^1 mm) | 2 | 2 | 0.7 | 2 | 0.7 | 1.7 | 0.2 | 1.5 | 1 | 1.7 | 0.2 | 1.5 | 0.2 | 0.2 |
| \( SLE \) (μm) | 94 | 7.3 | 0.0 | 204. | 8.2 | 7.0 | 2.0 | 8.6 | 5.3 | 17. | 48. | 172. | 0.4 | 1.3 |
| Order | 29 | 30 | 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 | 41 | 42 |
| \( \Omega \) (rpm) | 800 | 900 | 800 | 800 | 400 | 800 | 800 | 700 | 300 | 400 | 700 | 700 | 600 | 300 |
| \( a_p \) (mm) | 0.3 | 0.6 | 0.5 | 0.6 | 1.2 | 0.9 | 0.9 | 0.3 | 1.3 | 1.3 | 2.7 | 1.1 | 0.6 | 0.4 |
| \( a_e \) (mm) | 20 | 10 | 12. | 5 | 10 | 12. | 5 | 20 | 15 | 9. | 25 | 20 | 9 | 15 |
| \( f_z \) (×10^1 mm) | 0.2 | 0.5 | 0.2 | 1.7 | 1.2 | 2 | 1 | 0.7 | 1.5 | 1 | 1.2 | 0.2 | 2 | 1 |
| \( SLE \) (μm) | 0.9 | 0.1 | 0.2 | 13. | 106 | 5.6 | 6.9 | 2.7 | 65. | 27. | 28. | 104 | 0.0 | 1.3 |
| Order | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 | 51 | 52 | 53 | 54 | 55 | 56 |
| \( \Omega \) (rpm) | 300 | 100 | 900 | 300 | 600 | 900 | 100 | 400 | 600 | 600 | 300 | 900 | 400 |
| \( a_p \) (mm) | 0.4 | 1.3 | 0.4 | 0.5 | 2.8 | 0.6 | 0.6 | 0.8 | 4.3 | 1.2 | 3.1 | 0.7 | 0.9 | 1.8 |
| \( a_e \) (mm) | 17. | 5 | 15 | 10 | 10 | 9 | 15 | 10 | 2.5 | 15 | 9. | 7.5 | 5 | 10 |
| \( f_z \) (×10^1 mm) | 0.5 | 1 | 0.2 | 1.5 | 1 | 1.2 | 1.2 | 1.7 | 0.2 | 0.7 | 0.5 | 1.7 | 2 | 2 | 1.2 |
| \( SLE \) (μm) | 1.8 | 4.1 | 1.4 | 2.3 | 175 | 1.9 | 7.4 | 59. | 14. | 42. | 271 | 2.4 | 7.6 | 129 |

| Order | 2 | 8 | 5 | .2 | 5 | 6 | 36 | 13 | 69 | .4 | 4 | 7.6 | .5 |
An algorithm based on the dynamic model of cutting force is proposed to predict the surface location error in frequency domain, and the SLE is the function of the spindle speed, axial cutting depth, radial cutting depth and feed rate per tooth.

The spindle speed, axial cutting depth, radial cutting depth and feed rate per tooth are defined as the variables, and an orthogonal experimental design method is utilized to study the effects of each machining parameter on the SLE. Then an orthogonal table including 81 schemes is determined to conduct the SLE simulations in Matlab software.

The range analysis is used to analyze the values of the machining parameters and predicted SLEs. Comparing the results, the speed spindle show dominant effects on the SLE, and the axial cutting depth has smaller effect on the SLE; and the optimal level of each machining parameter can be determined to obtain the optimal machining parameters combination to have a minimum SLE.

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