Effect of ion motion on relativistic electron beam driven wakefield in a cold plasma

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Abstract

Excitation of relativistic electron beam driven wakefield in a cold plasma is studied using 1-D fluid simulation techniques where the effect of ion motion is included. We have excited the wakefield using a ultra-relativistic, homogeneous, rigid electron beam with different beam densities and mass-ratios (ratio of electron’s to ion’s mass). We have shown that the numerically excited wakefield is in a good agreement with the analytical results of Rosenzweig et al. [Physical Review A. 40, 9, (1989)] for several plasma periods. It is shown here that the excited wake wave is equivalent to the corresponding “Khachatryan mode” [Physical Review E. 58, 6, (1998)]. After several plasma periods, it is found that the excited wake wave gradually modifies and finally breaks, exhibiting sharp spikes in density and sawtooth like structure in electric field profile. It is shown here that the excited wake wave breaks much below the Khachatryan’s wave breaking limit due to phase mixing process.
I. INTRODUCTION

Next generation high-energy accelerators like Colliders (Large Hadron Collider (LHC) and the International Linear Collider) are capable of producing several Trillion electron volts (TeV) energy, but their operation is costly and time-consuming [1]. Plasma based particle acceleration schemes offer a much cheaper alternative. Being an ionized medium, plasmas are an attractive medium for future accelerators because they can support electric field higher than several hundred Giga electron volts (GeV) in a meter which is generally several orders of magnitude stronger than conventional RF accelerators [2, 3]. Therefore plasma based acceleration scheme is found to be suitable for the acceleration of charged particles to higher energies; this dramatically reduces the size of the machine and its cost. Plasma acceleration is a technique for accelerating charged particles, using an electric field associated with plasma wave or other high gradient structures (shock and sheath field). These plasma acceleration structures (waves) are generated either using an ultra-short, ultra-intense laser pulse or an ultra-relativistic electron beam propagating through the plasma [4–10]. Typically plasma acceleration process is categorized into two types, Laser Wakefield Acceleration (LWFA) and Plasma Wakefield Acceleration (PWFA). In LWFA, plasma electron wave is excited using ultra-short, intense laser pulse that expels plasma electron and excites a wake wave having phase velocity equal to the group velocity of the laser pulse due to its radiation pressure. On the other hand, in PWFA, an ultra-relativistic electron beam is used to drive the wake wave instead of a laser pulse. This externally injected electron beam repels the plasma electron and generates wake wave (phase velocity is equal to the velocity of the beam) due to space-charge force [11, 12]. Hence a late coming beam of charged particle rides on this excited wave and gets accelerated to high energies. The success of LWFA scheme has been confirmed in a number of experiments by accelerating charged particle to GeV energies in a meter long plasma [13–15]. In 2007, Blumenfeld et. al. [16] have demonstrated the success of PWFA scheme by accelerating electrons from the tail of a driving beam of energy 42 GeV to maximum energy of 85 GeV at SLAC (Stanford Linear Accelerator Center). In 2014, Litos et al. [17] have minimized the energy spread of the beam (∼ 2 percent) using discrete trailing bunches as a driver in their experiment.

In 1979, Tajima and Dawson first proposed the basic concepts of plasma acceleration and
its possibilities \cite{18}. In 1984, a group of scientists from UCLA (University of California, Los Angeles) designed the first experimental device for wakefield acceleration and produced an accelerating gradient several orders of magnitude higher than conventional RF accelerators \cite{19}. Plasma wakefield Acceleration (PWFA) scheme was first proposed by Chen, Huff and Dawson in 1984 as a means of coupling the relativistic electron beam to the plasma electron wave \cite{20}. As stated above, in PWFA, an ultra-relativistic electron beam propagates through plasma which expels the nearby plasma electrons. Ions do not respond because of their heavy mass and they only provide a neutralizing background. These repelled plasma electrons are then attracted by the massive ions which are left behind and they overshoot their corresponding initial positions because of their inertia. Hence a plasma wave (wake wave) is excited just behind the beam that has phase velocity equal to the velocity of the beam \cite{21–23}. The structure of this relativistic electron beam driven wakefield has already been studied extensively in 1-D as a function of beam density ($n_b$) and beam velocity ($v_b$) by several authors \cite{24–27}, where the ion motion was completely neglected because of their high inertia. In 2015, excluding the effect of ion motion, Ratan et al. \cite{27} presented a fully generalized analytical treatment for arbitrary beam density and verified the analytically excited structures by fluid simulations. They also showed that the beam can be considered to be rigid for several plasma periods if the velocity of the beam ($v_b$) is greater than $0.99c$ i.e. $\gamma_b \gg 1$; where $c$ is the speed of light and $\gamma_b = (1 - v^2_b/c^2)^{-1/2}$ is the Lorentz factor associated with the beam velocity ($v_b$). For lower values of $v_b$, the beam was found to be compressed for $l_b < \lambda_p$ and split into different beamlets for $l_b > \lambda_p$; where $l_b$ and $\lambda_p$ are the beam length and plasma wavelength respectively. Recently, in their another report \cite{28}, it was shown that the excited wake wave gradually modifies with time and finally breaks, manifested by the appearance of spikes in the density profile, via phase mixing process. It was found that the excited wake wave is a “Akhiezer-Polovin” (AP) mode, excited using the same parameter values of the wake wave. The underlying mechanism behind the wake-wave breaking was understood in terms of phase-mixing process of AP mode. They also showed that the numerical wake wave breaking time (minimum time needed to break the wave) matches with the analytically estimated value.

In 1998, Khachatryan et al. \cite{29} reported in the study of strong plasma waves (i.e. $\gamma \gg 1$) that plasma ions (even heavy ions) make an essential contribution to the process of charge
separation under the influence of such a strong field where maximum relativistic wavelength and amplitude of the wave grow in proportion to $\gamma^{3/2}$. Here, $\gamma = (1 - v^2/c^2)^{-1/2}$ is the Lorentz factor associated with the velocity of the electrons. The study of relativistic plasma waves, including ion motion, is also important for some astrophysical phenomenon. In the polar region of the pulsars, it is considered to be filled with electron-positron plasma and energetic charged particles are being generated from the plasma waves. As stated above, in PWFA, strong plasma waves are excited in plasma, using the relativistic electron beam. Therefore it is important to incorporate the dynamics of ion for the study replicating the structure of the wakefield in such applications. In this paper, with the help of the fluid simulation, we have studied the structure of the relativistic electron beam driven wakefield where the effect of the ion motion is included. Including ion motion, Rosenzweig et al. [30] presented a semi-analytical form of the electron beam driven wakefield and estimated the approximate value of transformer ratio (for mass ratio $\mu = \frac{m_e}{m_i} \ll 1$) only for beam density equal to the half of plasma density using multiple-fluid (ion and electron fluid) model. For further extension of Rosenweig’s work [30], we have performed our simulation for arbitrary mass ratio ($\mu$) and beam density ($n_b$) using a rigid beam. We have used the velocity of the beam larger than $0.99c$ throughout our simulation to avoid the deformation in beam density. It is shown that simulation results match with the semi-analytical results given by Rosenzweig et al. [30] for different beam density and mass ratio. The transformer ratio ($R$) which determines the efficiency in the acceleration process is also studied as a function of mass ratio and beam density ($n_b$). We have shown that the excited wake wave is identical to corresponding Khachtryan’s mode [29], excited using the same parameter values of the wakefield. We have observed in our simulation that the density of the excited wake wave also gradually modifies and becomes spiky after several plasma periods. The corresponding electric field profile turns into sawtooth form which is a clear signature of wave breaking [31, 32]. This particular feature observed in the present simulation has been found to be absent in the analytical calculations given in ref. [30]. The physical mechanism behind the wave breaking has been understood in terms of phase mixing process of the wake wave. It is seen here that the numerically obtained wave breaking limit lies much below the analytically estimated value given by [29].

In next section (Section -II), we present the basic equations governing the excitation of
relativistic electron beam driven wakefield. We have discussed our numerical techniques for this study in section-III. Our numerical observations and the detail discussion of the results has been covered in section-IV followed by a summary in section V.

II. GOVERNING EQUATIONS

The basic equations governing the excitation of 1-D relativistic electron beam driven wakefield in a cold plasma are the relativistic fluid-Maxwell equations. These equations contain the continuity and momentum equations for electron beam, plasma electrons and plasma ions. We have used Poisson’s equation for calculating the electric field considering that the electron beam is moving along $z$-direction in an infinite, homogeneous plasma channel, since we focus on exciting a relativistic electron beam driven wakefield only in the longitudinal direction (along the beam propagation). Therefore, neglecting the variation of plasma parameters (density, velocity and electric field for both the electrons and ions) in transverse directions (transverse to the beam propagation), the basic normalized governing equations in 1-D are,

\[
\frac{\partial n}{\partial t} + \frac{\partial (n v)}{\partial z} = 0 \quad (1)
\]

\[
\frac{\partial p}{\partial t} + v \frac{\partial p}{\partial z} = -E \quad (2)
\]

\[
\frac{\partial n_i}{\partial t} + \frac{\partial (n_i v_i)}{\partial z} = 0 \quad (3)
\]

\[
\frac{\partial p_i}{\partial t} + v_i \frac{\partial p_i}{\partial z} = -\mu E \quad (4)
\]

\[
\frac{\partial n_b}{\partial t} + \frac{\partial (n_b v_b)}{\partial z} = 0 \quad (5)
\]

\[
\frac{\partial p_b}{\partial t} + v_b \frac{\partial p_b}{\partial z} = -E \quad (6)
\]

\[
\frac{\partial E}{\partial z} = (n_i - n - n_b) \quad (7)
\]

where $p = \gamma v$, $p_i = \gamma_i v_i$ and $p_b = \gamma_b v_b$ are the $z$-components of momentum of plasma electron, plasma ion and beam electron having $z$-component of velocity $v$, $v_i$ and $v_b$ respectively. Here, $\gamma = (1 - v^2)^{-1/2}$, $\gamma_i = (1 - v_i^2)^{-1/2}$ and $\gamma_b = (1 - v_b^2)^{-1/2}$ are the relativistic
factors associated with plasma electron, plasma ion and beam electron respectively. In the
above equations, \( n, n_i \) and \( n_b \) represents the density of plasma electron, plasma ion and elec-
tron beam respectively. \( E \) and \( \mu \) represents the \( z \)-component of the electric field and mass
ratio (ratio of electron to ion mass) respectively. We have used the normalization factors
as, \( t \rightarrow \omega_{pe} t, z \rightarrow \omega_{pe} z, E \rightarrow \frac{eE}{m_e\omega_{pe}}, v \rightarrow \frac{v}{c}, v_i \rightarrow \frac{v_i}{c}, v_b \rightarrow \frac{v_b}{c}, p \rightarrow \frac{p}{m_e c}, p_i \rightarrow \frac{p_i}{m_i c}, p_b \rightarrow \frac{p_b}{m_b c}, \)
\( n \rightarrow \frac{n}{n_0}, n_i \rightarrow \frac{n_i}{n_0} \) and \( n_b \rightarrow \frac{n_b}{n_0} \). Equations (1-7) are the key equations needed to examine
the excitation of 1-D electrostatic relativistic electron beam driven wakefield excitation in a
cold plasma.

III. FLUID SIMULATION OF THE RELATIVISTIC ELECTRON BEAM DRIVEN
WAKEFIELD

In this section, we present numerical techniques used to study the relativistic electron
beam driven wakefield excitation in a cold plasma. We have constructed a 1-D fluid code
using LCPFCT routines, which is based on flux-corrected transport scheme [33]. The basic
principle of this scheme is based on the generalization of two-step Lax-Wendroff method
[34]. Ratan et al. [27] showed that beam can be considered to be rigid only if \( \gamma_b \gg 1 \). In this limit, beam evolution equations (5) and (6) can be neglected. In our present
simulation, we have also considered \( \gamma_b \gg 1 \) in all cases. Therefore, in simulation, the
beam propagates along \( z \)- direction with a speed close to the speed of light inside the
plasma without any deformation in its shape. Using LCPFCT routine, we have solved
the equations ((1),(2), (3),(4) and (7)) with non-periodic boundary conditions along
\( z \)-direction. Here the driver beam is allowed to propagate from one end of the simulation
window to its other end. Electron beam itself perturbs the system and excites the wake
wave. We have recorded the profile of electron density (\( n \)) and electric field (\( E \)) with time.
The simulations have been carried out using the spatial resolution \( \Delta z = 0.05 \).
The temporal resolution \( i.e. \) time step (\( \Delta t \)) is then calculated from Courant-
Friedrichs-Lewy (CFL) condition \( \Delta t = C_n \Delta z/u_{max} \), where \( u_{max} = 1 \) and \( C_n = 0.2 \),
known as CFL number [33]. We have observed that our simulation results (inside
and outside the beam) remain unchanged by doubling the number of grid points
i.e. \( \Delta z = 0.025 \). Regardless of its resolution, however, the simulation results in
both cases (\( \Delta z = 0.05 \) and \( 0.025 \)) converge to the same wake field solution. We
have also tried different initialization methods viz. when the beam and the wake field is initialized in a self-consistent way and also when the beam is “simply put” inside the plasma in the simulation. We find that our simulation results are independent of the initialization method; the solution always converges to the same wake field profile in both the cases. This fact, i.e. independence of wake field profile to initialization method, is also discussed in one of our earlier work [27].

IV. NUMERICAL OBSERVATIONS AND DISCUSSION

Here we present the numerically obtained profiles of perturbed electron density \( n_1 \) and electric field \( E \) profile with time for different beam density and mass ratio. In all our simulations, we have used beam velocity \( v_b = 0.99 \) and beam length \( l_b = 4 \). The numerical perturbed density \( n_1 \) and electric field \( E \) profiles of the excited wake wave are shown in figures (1) and (2) respectively at different times for \( n_b = 0.1 \) and \( \mu = 1 \). The numerical perturbed density \( n_1 \) and electric field \( E \) profiles are plotted in figures (3) and (4) respectively at different times for \( n_b = 0.2 \) and \( \mu = 1 \). We have obtained the corresponding analytical profiles of wakefield excited for the same parameters used in our simulation from the semi-analytical calculation given in ref. [30]. These analytical profiles (in blue lines) are shown in figures (1-4). It is clear from figures (1-4) that the numerical profiles match well with the analytical profiles for different beam densities and mass-ratios. In figure (5), we have shown the plot of perturbed electron density profile \( n_1 \) obtained for \( \mu = 1/1836, n_b = 0.2 \) at \( \omega_{pe} t = 50 \) along with the profile, where the effect of ion motion is completely neglected [27]. As expected, it is clear from figure (5) that ion motion may be neglected for small values of \( \mu \). In figure (6), we have plotted analytical values of transformer ratio \( R = \frac{E_+}{E_-} \) obtained from ref. [30] along with numerical values as a function of \( \mu \) for two different values of \( n_b = 0.1 \) and 0.5; where \( E_+ \) and \( E_- \) are the maximum value of accelerating electric field behind the beam and maximum decelerating electric field inside the beam respectively. We have found that numerically obtained transformer ratio matches well with the analytical values. In figure (7), we have plotted the transformer ratio (shown in squares) obtained from fluid simulation on the top of its analytical
values (solid red line) obtained from semi-analytical theory \(^{30}\) for \(\mu = 1/1836\) as a function of beam density \((n_b)\). In addition, we have also plotted the analytical result for the transformer ratio \(R\) vs. beam density \(n_b\), given in ref. \(^{27}\) for \(\mu = 0\) on top of the \(\mu = 1/1836\) curve. As expected the curve for \(\mu = 0\) closely matches the curve for \(\mu = 1/1836\). Further, we have also plotted the semi-analytical values (shown in dotted line) obtained for \(\mu = 1\) from ref \(^{30}\) along with some values obtained from simulation. We find a good match between theory and simulation. It is observed that in both cases transformer ratio \(R\) settles down to unity for large values of \(n_b\). The transformer ratio determines the energy gain of the acceleration along the dephasing length, which is the maximum length over which electrons are accelerated. Typically, higher the value of transformer ratio \((R)\) larger the energy gain in the process of acceleration.

We have observed in our simulation that the numerical profiles of perturbed electron density \((n_1)\) and electric field \((E)\) match with the analytical results, obtained using quasistatic approximation, for several plasma periods (see figures (1-5)). After several plasma periods, subsequently, they start to deviate. The amplitude of the electron density gradually increases and shows spiky behaviour at later times \((\omega_{pe}t = 160)\) shown in figure (8). We note here that, for a rigid beam (which is assumed in our work), and for a “perfectly” noise free system, the analytical results obtained from quasistatic approximation \(^{30}\) should give a valid description of the system. But we observe here that, after several plasma periods, the analytical results deviate from the simulation result which show a complete breakdown of quasistatic approximation. This is because, in a realistic situation (and also in numerical simulations), perturbations to physical variables associated with the wake wave are inevitably present; this is true even in the case of a non- evolving driver beam which itself excites these perturbations along with the wake wave. These perturbations result in a slow variation of physical quantities associated with the wake wave outside the beam and produce a real physical effect which is described by the phase mixing process (discussed later). Inside the beam, such a phenomenon does not occur as the wake is forced to oscillate at a frequency which is decided by the beam density. Thus the analytical results obtained using the quasistatic
approximation, which neglects the slow variation of physical quantities associated with the wake wave, exhibit deviation from the simulation results, outside the beam. This feature, indicating the density bursts, is known as wave breaking \cite{31, 32}. To understand the basics of wave breaking process of the wakefield, we first identify that the wake wave is the corresponding “Khachatryan mode” \cite{29}. It is well known that, including the ion dynamics, the solution of 1-D relativistic fluid-Maxwell equations (equations (1), (2), (3), (4) and without the beam term in Poisson equation (7)) in a cold plasma is a “Khachatryan mode” \cite{29} which is parameterized in terms of $\mu$, $\beta_{ph}$ and $E_{max}$; where $\beta_{ph}$ and $E_{max}$ are the phase velocity and the maximum amplitude of the electric field associated with the wave. Therefore the electron beam driven wakefield (structure behind the beam) which is a solution of (equations (1-4, 7)) with $n_b = 0$, is a corresponding Khachatryan mode excited using the same values of $\mu$, $\beta_{ph} = v_b$ and $E_{max}$ of the wake wave. Using the value of $\mu$, $\beta_{ph}$ and $E_{max}$ from the simulation, we have plotted the corresponding Khachatryan’s mode on top of the wake wave excited for $n_b = 0.1$, $v_b = 0.99$, $l_b = 4$ and $\mu = 1$ in figure (9). It is seen that the structure of the wake wave shows a good match with the corresponding Khachatryan mode. It is already well accepted that the amplitude of a wave sustained in a medium is limited by its wave breaking limit. If the amplitude of the wave exceeds this limit, it breaks resulting in the destruction of coherent motion. In 1998, Khachatryan et al. \cite{29} analytically calculated the wave breaking limit for a relativistically intense plasma wave (including ion motion) in terms of the maximum amplitude of electric field as, $E_{WB} = \sqrt{2} \gamma_{ph} \left[ 1 + \left( 1 - \xi_1^{-\frac{1}{2}} \xi_2^2 \right) \right] / \mu$; where $\xi_1 = 1 + \mu$, $\xi_2 = 1 + \left[ \mu (\gamma_{ph} - 1) / (\gamma_{ph} + 1) \right]$ and $\gamma_{ph} = (1 - \beta_{ph}^2)^{-\frac{1}{2}}$. In our simulation, the wake wave (which is a Khachatryan mode) having phase velocity equal to the velocity of the beam breaks after several plasma periods, exhibiting sharp spikes in density profile. In simulation, at the point of wave breaking, we note the corresponding maximum amplitude of electric field ($E_{WB}$) for different values of $\mu$, where $n_b = 0.2$, $l_b = 4$ and $v_b = 0.99$. In figure (10), we have plotted both numerical and theoretical values of $E_{WB}$ (Khachatryan’s wave breaking limit) as a function of mass ratio ($\mu$). It is seen that the wave-breaking limit of numerically excited wake-wave lies much below the analytically estimated limit. Here the wake wave breaks much below the analytical wave breaking limit. In other words, the wake wave breaks before it reaches to its wave breaking amplitude. In our simulation, the numerically excited wake wave breaks via the phase mixing process, which arises because of relativistic mass variation effects \cite{28, 31, 35-37}. Physically, the
propagation of the non-evolving driver beam inside the cold plasma, excites a wake wave along with perturbations, in our simulations. As shown above, the wake wave excited outside the beam has been identified with the corresponding Khachatryan mode [29]. The perturbations to the Khachatryan mode, causes the frequency of the wake wave to become a function of position. This is because frequency of an oscillating electron fluid element depends on relativistic mass (energy) which in turn becomes a function of position because of the perturbations. For a perturbed Akhiezer–Polovin mode (i.e. with immobile ions), the spatial dependence of frequency has been explicitly shown numerically in refs. [28, 36] and analytically in ref. [37]. The spatial dependence of frequency eventually results in crossing of electron fluid elements resulting in breaking of the wake wave even in a cold plasma [31, 35]. Therefore the wake wave breaks due to phase mixing process before it reaches to its wave breaking limit ($E_{WB}$). Khachatryan et al. [29] calculated the wave breaking limit without considering the contribution of phase mixing process in their theory.

V. SUMMARY

We have studied relativistic electron beam driven wakefield in a cold plasma using fluid simulation techniques where the effect of ion motion is included. It is shown that simulation results match with the analytical results for different beam density and mass ratio. At later times, the numerical result gradually deviates from the analytical result and finally breaks via phase mixing process. We have shown that the excited wake wave is alike to the corresponding Khachatryan’s wave [29]. We have numerically obtained the wave breaking limit which is found to be much below the analytically estimated values by Khachatryan et al. [29]. The underlying reason for this is understood in terms of phase mixing process which arises here because of relativistic mass variation effects. The contribution of phase mixing to the wave breaking limit by Khachatryan et al. [29] was not considered in the
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FIG. 1. Plot of normalized perturbed electron density ($n_1$) profile at different times for the normalized beam density ($n_b$)=0.1, $l_b = 4$ beam velocity ($v_b$) =0.99 and $\mu = 1$. 
FIG. 2. Plot of normalized electric field ($E$) profile at different times for the normalized beam density ($n_b$)=0.1, beam velocity ($v_b$) =0.99, $l_b = 4$ and $\mu = 1$. 
FIG. 3. Plot of normalized perturbed electron density ($n_1$) profile at different times for the normalized beam density ($n_b$) = 0.2, beam velocity $v_b = 0.99$, $l_b = 4$ and $\mu = 1$. 
FIG. 4. Plot of normalized electric field ($E$) profile at different time for the normalized beam density ($n_b$)=0.2, beam velocity $v_b = 0.99$, $l_b = 4$ and $\mu = 1$. 

\[ \omega_{pe} t = 25 \]

\[ \omega_{pe} t = 50 \]
FIG. 5. Plot of normalized perturbed electron density ($n_1$) profile at different times for the normalized beam density ($n_b$)=0.2, beam velocity $v_b = 0.99$ and $l_b = 4$ and $\mu = 1/1836$.

FIG. 6. Plot of transformer ratio ($R$) vs. mass ratio ($\mu$) for $n_b = 0.5$ and $n_b = 0.1$
FIG. 7. Plot of semi-analytical and numerical values of transformer ratio (R) as a function of beam density ($n_b$) for $\mu = 1$ and $\mu = 1/1836$. The blue circles indicate the values of transformer ratio obtained from the analytical expression given by Ratan et. al. [27] for $\mu = 0$.

FIG. 8. Plot of plasma electron density ($n_1 = n - 1$) at different times $t = 60, 160$ for $\mu = 1$, $n_b = 0.1$, $v_b = 0.99$ and $l_b = 4$. 
FIG. 9. Comparison of numerical electric field profile \((E)\) with the analytical and Khachatryan mode for \(n_b = 0.1, l_b = 4, v_b = 0.99\) and \(\mu = 1\).

FIG. 10. Plot of maximum amplitude of wave breaking electric field \((E_{WB})\) as a function of mass ratio \((\mu)\) for \(n_b = 0.2, l_b = 4\) and \(v_b = 0.99\) or \(\gamma_{ph} = 7.08\).