Detecting and isolating false data injection attacks on electric vehicles of smart grids using distributed functional observers

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Abstract
This paper considers the problem of false data injection attacks (FDIAs) on load frequency control of interconnected smart grids (ISGs) with delayed electric vehicles (EVs) and renewable energies. By intruding incorrect information, unauthorised users can corrupt the system information leading to degradation in the performance and disruptions of ISGs. In this paper, a model of ISGs subject to FDIAs in aggregator of EVs and power plants is first presented. This mathematical representation comprises dynamic interactions of power plants, delayed EVs, renewable energies and FDIAs on both system states and outputs. Based on recent advanced techniques on functional observers and matrix inequalities for time-delay systems, then a new distributed functional observers based scheme is developed to realise the tasks of detecting and isolating FDIAs. Also, an effective procedure presented in tractable linear matrix inequalities is build with an optimisation process for the synthesis of the detector. The proposed detector is distributed, of reduced order, avoids the risk of centralised malicious incidents, therefore easy for implementation and monitoring tasks. The stability of ISGs and contribution of EVs subject to FDIAs are also discussed. Comprehensive simulations are given to demonstrate the effectiveness of our proposed method by using three-area ISGs.

1 | INTRODUCTION

1.1 | Background and motivations

Load frequency control (LFC) is essential in the effective operation of ISGs [1]. Because of variations in load demands, the system frequencies and interchange powers fluctuate from their planned operating points [2]. By taking appropriate control measures, LFC can rebuild the stability of ISGs and preserve the frequencies and interchange powers at the preferred values [3]. In recent years, significant research attention has been focused on the efficient operation of ISGs subject to cyber-attacks (please refer to survey papers on cyber security of power grids [4], smart grids [5], security control of industrial cyber physical systems [6] and false data injection attack (FDIAs) against cyber-physical power systems [7]. Due to the unexpected actions of unauthorised users that violate or illegally acquire, modify and disrupt information of communication networks [5], ISGs are vulnerable to catastrophic disruptions, disclosure of sensitive information and frauds [8]. In [9], the control action provided by distributed energy resources (DERs) was corrupted by attacks. The authors in [10] considered a microgrid with deception attacks. In [11], time delay attacks on LFC of ISGs were addressed.

As a communication network-based component of ISGs, LFC of ISGs is managed by supervision control and data acquisition systems [2]. Critical information for control is transferred within the power network through open communication systems [12]. LFC is the interface between intelligent cyberspace and physical facilities [13]. Because it is highly dependent on the wide utilization of communication networks, LFC is inevitably facing threats created by attackers aiming to cause service outages and infrastructural damages [14]. The approach is to intrude security threats (FDIAs) to interrupt the information transferring within the ISGs, modify the system data and results leading to serious degradations in system stability and performance of LFC [15]. LFC requires reliable information of the system frequencies, interchange power deviations and their area control error to determine the correct amount of power flow within the ISGs. In [16], the authors showed that hackers potentially can destroy the system’s stability by injecting false
and disruptive information into the system. This will lead to incorrect control input signals and hence degrading the system performance and stability. Therefore, it is important to consider the problem of FDIAs in the application of LFC of ISGs and develop efficient regimes to detect and isolate the attacks happen within the systems.

With the purpose of lowering greenhouse emission and noise pollution, integration of REs and EVs have received great research attention in recent literature [3]. REs can provide additional powers without using natural fuels whereas in [17], the deployment of EVs can improve the reliability and flexibility of ISGs. In [18], reserved powers of EVs also support the power plants in the frequency regulation effectively. Due to the powers of REs depend on natural features such as weather and wind speeds, it is unexpectedly intermittent leading to high fluctuation of system frequencies, hence, a bounded control technique was developed for ISGs with REs and energy storages in [19]. To achieve an effective LFC operation of ISGs, the adverse impact of REs intermittent needs to be eliminated for any control and monitoring tasks such fault detection and isolation. With respect to the use of widespread EVs, in order to participate into LFC, an aggregator (master control) and networked or wide-area communication systems comprising power line communications, general packet radio services, internet, Bluetooth and wireless connections is required [3]. Via network communications, the aggregator collects real time information and re-allocate requested power commands to determine charged/discharged power of connected EVs [20]. One important issue of EVs integration is the existence of time delay due to sudden congestion of communication channels, drop-out and disordering of data packets. In [17, 18, 20], networked time delays related to the integration of EVs were considered in the LFC of smart grids. In fact, communication delays can downgrade or even cause to instability of system dynamic performance, hence robust control laws were introduced for ISGs [12], isolated SGs with EVs [17], ISGs with energy storages [19], ISGs with EVs [20], and microgrids [21]. In these works, stability conditions were derived according to Lyapunov theory and techniques based on matrix inequalities to ensure the stability of the closed-loop systems. By employing high complex communication infrastructures, the operation of EVs aggregator faces to the risks of being vulnerable to FDIAs. Indeed, this issue is similar to the problem happened in LFC central facility, deceptive information is potentially and illegally imposed into the transaction between the aggregator and connected EVs. Thus, unauthorized attackers can deteriorate the performance and stability of ISGs and the EVs system.

One of the key functions that smart grid has pledged to perform is to offer a power to fulfil electricity needs with friendly environmentally source of energy while retaining a satisfactory level of adequacy and security that conventional power systems promise [22]. It is therefore important to develop an adequate approach to detect FDIAs in the LFC of the ISGs incorporated friendly sources such REs and EVs and distinguish the FDIAs happened into LFC and the one links to EVs. This is the main motivation of this paper.

1.2 Related works

A novel detection method for LFC based on multilayer perceptron (MLP) classifier was proposed in [14]. For detection purpose, samples of frequency or area control error (ACE) were collected under both normal and compromised circumstances. MLP classifier was applied to map an optimal function between normal and attacked signals. In this method, quantities of training samples are important to the training, testing of MLP based detector. In [15], the authors introduced a model-based fault detection method using real-time load forecast and simulated measurements obtained from equations that govern the functioning of underlying physical systems. In [16], an online attacks detection framework was presented. In this framework, a dynamic watermarking technique was used as the core algorithm to detect tampered information. A detection scheme based on recurrent neural networks for cyber-attacks on DC microgrid was proposed in [23]. In [24], the authors used a full-order observer (FUO)-based method for a delay-free power grid with FDIAs. In [25], a robust detection filter was derived to identify faults on LFC of delay-free power systems. In this work, a FUO was used to estimate the state vector and a residual generator was obtained based on the error between the system state vector and the estimation to determine the existence of faults. In [26], an unknown input FUO-based detection method was deployed for the LFC of delay-free power systems consisting of thermal plants and load demand changes. In [27], the authors used a stochastic estimator based on the observer proposed in [26] to build a fault detector. In [26, 27], residual generators were designed based on the error between the system state vector and its estimated state vector to trigger an alarm. These proposed techniques [25–27] are interesting and effective for detecting FDIAs. However, the structure of these observers used to build the detectors requires current or last updated information of load demand changes. In practice, it is challenging for ISGs to store and update information of REs for monitoring assignments frequently. In [28], a robust detection technique based on unknown input FUO was derived to detect controller and sensor faults of power plant separately without any need of accessing information of load disturbances. In [29], a centralised FUO based detector for actuator and sensing faults were derived. However, the main problem is that the residual generators in the above references was fundamentally founded by the establishment of FUOs [25], unknown input FUOs [24, 26, 28, 29], stochastic estimator based FUOs [27] with centralised architecture, thus resulted in large size of detector, high cost of computation and complexity of implementation. It is highlighted that these detection methods were proposed for delay-free interconnected power systems only. Moreover, the integration of delayed EVs and the issue of FDIAs on EVs were not considered in any of the above-mentioned works. On the other hand, in the previous studies for EVs (see [17, 18, 20] and the references therein), the problem of isolating and detecting of faults was not considered.

In the light of observer developments, FOs have much more advantages than state observers considered in [24–29]. Indeed,
Contributions and organisation of the paper

The contributions of this paper are underlined as follows:

i In this paper, we emphasis on the problem of FDIAs for the LFC operation of ISGs and a proposal of deriving a new method to detect and differentiate the occurrence of each attack. To achieve our objective, we first propose a new model of ISGs incorporating FDIAs, aggregated delay EVs and REs.

ii A new state space model of ISGs encompassing the dynamic interactions of multiple FDIAs on both system states and outputs, communication time delays, EVs and REs.

iii In the next stage, we introduce a new DFO-based detection and isolation schemes to detect and isolate each FDDIA. The proposed scheme is derived based on some advanced developments of FOs, novel of Lyapunov stability for time-delay systems and residual generators-based fault detection.

iv The proposed DFOs detector has the advantages of being less order (small size) comparing to conventional state observer-based detectors. It also handles the existence of time delays, be insensitive to neighbouring FDIAs and the excursion of REs. In addition, the detectors are made of distributed structure, hence, prevented from malicious incidents happened in facility of centralised architecture.

v To synthesize the detector’s gains, we derive an effective procedure in tractable LMI, linear computations and an optimisation process, which can be easily solved by efficient computational robust control tool in MATLAB with flexible programming codes.

Finally, for demonstrating the effectiveness of our method, simulations are conducted with a three-area ISGs comprising reheated thermal plants, delayed EVs and REs. We also consider the stability of time delay ISGs and discuss on EVs contribution into frequency services under abnormal performance caused by FDIAs.

The remaining of this paper is outlined as follows: Section 2 introduces a state space model of an ISG with FDIAs, EVs and REs. The DFO-based detection and isolation method with schematic of implementation and a procedure to obtain the detector’s parameters are derived in Sections 3 and 4. The effectiveness of our proposed methodology is validated in Section 5. Finally, Section 6 concludes the paper.

2 SYSTEM DESCRIPTION AND PROBLEM STATEMENT

Figure 1 represents a block diagram of transfer functions of an ISG with EVs. The system frequencies and interchange powers deviate far from the desirable operating points. To tackle this issue, the local control centre sends a power request command, $P_{ri}(t)$, to adjust the output power of thermal power plant. The description of reheated power plant at local power Area-$i$ depicted in Figure 1 is [20]

\[
P_{gi}(t) = -\frac{1}{T_{gi}} P_{gi}(t) + \frac{K_{gi} K_{ni}}{T_{gi} T_{ni}} X_{gi}(t) + \frac{T_{gi} - K_{gi}}{T_{gi} T_{ni}} P_{ni}(t),
\]

\[
P_{ni}(t) = -\frac{1}{T_{ni}} P_{ni}(t) + \frac{K_{ni}}{T_{ni}} X_{ni}(t),
\]

\[
X_{gi}(t) = -\frac{1}{T_{gi}} X_{gi}(t) + \frac{K_{gi}}{T_{gi}} (P_{gi}(t) - \frac{1}{K_{gi}} f_{i}(t)),
\]

\[
\dot{f}_{i}(t) = -\frac{D_{i}}{M_{i}} f_{i}(t) + \frac{1}{M_{i}} (P_{gi}(t) + P_{ni}(t) - P_{w_{i}}(t)) - \frac{1}{M_{i}} (P_{ri}(t) + P_{w_{i}}(t)).
\]
The operation of LFC requires the knowledge of area control errors, $ACE_i(t)$, $i = 1, \ldots, N$, to ensure small steady-state values of system frequency deviation, $f_i(t)$, and interchange power (power tie-line) deviation, $P_{tie,i}(t)$, when the system subjects to power disturbances. These values are computed according to $f_i(t)$ and $P_{tie,i}(t)$ with a frequency bias constant, $b_i$, as follows \[ ACE_i(t) = P_{tie,i}(t) + f_i(t), \quad i = 1, \ldots, N \] (2)

By using area control error, $ACE_i(t)$, and its integral value, $\vartheta_i(t) = \int ACE_i(t)dt$, the control facility of power Area-$i^{th}$ computes the local power request command, $P_i(t)$, as follows \[ P_i(t) = K_{fi}ACE_i(t) + K_{fi}\vartheta_i(t), \quad i = 1, \ldots, N \] (3)

where $K_{fi}$ and $K_{fi}$ are the controller's parameters.

The power request, $P_i(t)$, can be expressed in the following static output feedback control structure \[ P_i(t) = K_{i1}f_i(t) + K_{i2}P_{tie,i}(t) + K_{i3}\vartheta_i(t), \] (4)

where $K_{i1}$, $K_{i2}$, $K_{i3}$ are the controller's parameters.

In this paper, we consider FDIA at the local control facility. Figure 2 shows how FDIA are implanted into LFC of ISGs. As mentioned previously, the area control error, $ACE_i(t)$, is necessary in the construction of power request command, $P_i(t)$, in order to restore the stability of the closed loop system fluctuated by disturbances. $ACE_i(t)$ is computed based on information of frequency, $f_i(t)$, and interchange power, $P_{tie,i}(t)$, hence, the hackers can degrade the LFC performance of the power grid \[ 14, 15].\] Their main strategy is to inject incorrect information, $g_{0i}(t)$ during the process of computing $ACE_i(t)$. As a result, the counterfeit computed values of $ACE_i(t)$ and its integral value are generated \[ ACE_i^*(t) = P_{tie,i}(t) + f_i(t) + g_{0i}(t) = ACE_i(t) + g_{0i}(t), \]

\[ \vartheta_i^*(t) = \int ACE_i^*(t)dt = \vartheta_i(t) + g_{0i}(t), \]

By which the computed control signal Equation (4) becomes \[ P_i(t) = K_{i1}f_i(t) + K_{i2}P_{tie,i}(t) + K_{i3}\vartheta_i^*(t). \] (6)

In this paper, we consider the operation of aggregated EVs. We assume that $M$ of EVs are connected to power Area-$i^{th}$ to participate into LFC without state of charge control (SOC) \[ 17].\] An EV participates into LFC without SOC means that the output power of the EV, $P_{emi}(t)$, is determined by a constant EV gain, $K_{emi}$, as follows \[ P_{emi}(t) = -\frac{1}{T_{emi}}P_{emi}(t) + K_{emi}\varepsilon_{emi}(t) \] (7)

where $\varepsilon_{emi}(t) = \varepsilon_{emi}(t) / M, \varepsilon_{emi}(t) = -\rho_{emi}f_i(t - b_i).$ $K_{emi}$ and $T_{emi}$ are gain and time constants of the $m^{th}$ EV. Here we consider that all EVs have similar time constant, $T_{emi} = T_{emi}$.

The integration of an aggregated EVs requires a networked communication infrastructure with an aggregator (see Figure 3). This aggregator plays the role of a master to obtain the information of all individual EVs and allocate the individual power request command to them. Due to the existence of hacker actions, the request power signal for an individual EV, $\varepsilon_{emi}(t)$, is changed by the injection of an incorrect information, $g_{emi}(t)$.

Hence, a counterfeit command, $\varepsilon_{emi}^*(t) = \varepsilon_{emi}(t) + g_{emi}(t)$, is used for the EV instead of the true information, $\varepsilon_{emi}(t)$. Therefore, the description of the EV taking FDIA into account is presented as follows \[ \varepsilon_{emi}^*(t) = -\frac{1}{M}\rho_{emi}(t - b_i) + g_{emi}(t), \]

\[ P_{emi}^*(t) = -\frac{1}{T_{emi}}P_{emi}^*(t) + K_{emi}\varepsilon_{emi}^*(t) \] (8)

As a result, the output power of aggregated EVs, $P_{emi}^*(t) = \sum_{m=1}^{M} P_{emi}^*(t)$ becomes \[ P_{emi}^*(t) = -\sum_{m=1}^{M} \frac{P_{emi}^*(t)}{T_{emi}} + \sum_{m=1}^{M} \frac{K_{emi}\varepsilon_{emi}^*(t)}{T_{emi}} + \sum_{m=1}^{M} \frac{K_{emi}g_{emi}(t)}{T_{emi}} \] (9)
where $K_a = \sum_{m=1}^{M} K_{am}/M$ is the aggregated EVs gain. $g_2(t) = \sum_{m=1}^{M} (K_{am}g_{am}(t))/K_a$ is considered as FDIAs on the aggregated EVs.

Accordingly, the equation of $f_i(t)$ in Equation (1) becomes

$$j_i(t) = -\frac{D_i}{M_i} f_i(t) + \frac{1}{M_i} \left( P_{gi}^e(t) + P_{gi}^p(t) - P_{mi}(t) \right)$$

$$- \frac{1}{M_i} \left( P_{gi}^e(t) + P_{mi}(t) \right).$$

In order to investigate the abnormal LFC operation of ISGs subject to multiple FDIAs, we develop the following state-space model of the studied system which encompasses the interactions of multiple communication delays, disturbances and FDIAs. First, we denote local state vector, $x_i(t) \in \mathbb{R}^n$, control input vector, $u_i(t) \in \mathbb{R}$, disturbance vector, $d_i(t) \in \mathbb{R}$ and output vector, $y_i(t) \in \mathbb{R}^d$. In the local power Area-$i$ as follows $x_i(t) = [x_{i}(t) X_{i}(t) P_{gi}(t) P_{gi}(t) \tilde{\phi}_i(t)]^T$, $u_i(t) = P_{mi}(t)$, $d_i(t) = P_{gi}^e(t) + P_{mi}(t), y_i(t) = [f_i(t) P_{gi}(t) P_{gi}(t) \tilde{\phi}_i(t)]^T$.

A state-space representation of the local power Area-$i$ is

$$\dot{x}_i(t) = A_{ii} x_i(t) + A_{ii} u_i(t) + \sum_{j=1,j\neq i}^{N} A_{ij} x_j(t) + \Gamma_d d_i(t) + B_i u_i(t)$$

$$y_i(t) = C_{ii} x_i(t) + F_{ii} g_i(t),$$

where $g_i(t) = [g_{i1}(t) g_{i2}(t)]^T \in \mathbb{R}^2$, $S_i = [S_{i1} S_{i2}], F_i = [F_{i1} F_{i2}]$.

System matrices $A_i, A_{ii} \in \mathbb{R}^{n \times n}, A_{ij} \in \mathbb{R}^{n \times n}$, $B_i, \Gamma_i, S_{i1}, S_{i2} \in \mathbb{R}^{n \times 1}, C_i \in \mathbb{R}^{d \times n}$, $F_i, F_{ii} \in \mathbb{R}^{d \times 1}$ are given in the Appendix A.

Accordingly, a state-space model of N-area ISGs is obtained

$$\dot{x}(t) = A x(t) + \sum_{j=1,j\neq i}^{N} A_{ai} x_i(t) - b_i + \Gamma d_i(t) + B a(t) + S g_i(t),$$

$$y(t) = C x(t) + F g(t),$$

where $x(t) = [x_1(t) \cdots x_N(t)]^T \in \mathbb{R}^n$, $d(t) = [d_1(t) \cdots d_N(t)]^T \in \mathbb{R}^N$, $u(t) = [u_1(t) \cdots u_N(t)]^T \in \mathbb{R}^N$, $y(t) = [y_1(t) \cdots y_N(t)]^T \in \mathbb{R}^d$, $g(t) = [g_1(t) \cdots g_N(t)]^T \in \mathbb{R}^d$. Global state, disturbance, control input, output and FDIAs vectors. Matrices $A, A_{ii} \in \mathbb{R}^{n \times n}, A_{ij} \in \mathbb{R}^{n \times n}, B, \Gamma \in \mathbb{R}^{n \times 1}, S \in \mathbb{R}^{n \times 2}, C \in \mathbb{R}^{d \times n}, F \in \mathbb{R}^{d \times 2}, \bar{S} \in \mathbb{R}^{d \times 2}$ are $A = \begin{bmatrix} A_{i1} & \cdots & A_{iN} \\ \vdots & \ddots & \vdots \\ A_{N1} & \cdots & A_{NN} \end{bmatrix}$, $A_{ii} = \text{Diag}(0, \ldots, A_{ii}, \ldots, 0)$, $B = \text{Diag}(B_1, \ldots, B_N)$, $\Gamma = \text{Diag}(\Gamma_1, \ldots, \Gamma_N), C = \text{Diag}(C_1, \ldots, C_N), S = \text{Diag}(S_1, \ldots, S_N), F = \text{Diag}(F_1, \ldots, F_N)$.

**Remark 1.** In this paper, FDIAs in LFC take various forms: (i) counterfeit information of EVs power request, $e_{gi}(t)$ and (ii) injection of incorrect data into the process of computing the area error control signal, $ACE_i(t)$ and its integral value, $\tilde{\phi}_i(t) = \int ACE_i(t) dt$. In the abnormal operation (ISGs subject to FDIAs), instead of reliable signals, $P_{mi}(t)$ and $\tilde{\phi}_i(t)$, counterfeited information of $P_{mi}(t)$ and $\tilde{\phi}_i(t)$ are used for computing control input signals and other local monitoring functions. Therefore, system Equation (11) encompasses behaviours and interactions of a FDIAs, $g_{i1}(t)$, in the output vector $y_i(t)$, and another FDIAs, $g_{i2}(t)$, in dynamical equation of the system state vector, $\dot{x}_i(t), x_i(t)$. For the first time, a mathematical representation of ISGs considering multiple FDIAs, RES and time delays are derived in this paper. Hence, the state-space model Equation (11) is different to those previous works [14–16, 26]. In [14, 15], state space models of ISGs were not derived. In [16], the variations of RES and load demands were not considered in all of the previous works [14–16, 26].

When the FDIAs, $g_{i1}(t)$ and time delays are ignored, the state space model Equation (11) is converted to the model considered in [26]

$$\dot{x}_i(t) = \bar{A}_i x_i(t) + \sum_{j=1,j\neq i}^{N} A_{ij} x_j(t) + \Gamma_d d_i(t) + B_i u_i(t)$$

$$+ S_{i2} g_{i2}(t); y_i(t) = C_{ii} x_i(t),$$

where $\bar{A}_i = A_{ii} + A_{ii}$.

The main objective of this paper is to derive an effective detection method to detect and isolate FDIAs within ISGs. Each local detector is installed at the local power area to implement the tasks of detecting and isolating the local FDIAs only.

## 3 | DFO BASED DETECTION SCHEME FOR FDIAs

In this section, we develop a FDIAs detection method for ISGs. In this method, a local detector, $D_i$, is placed at local Area-$i$th to detect the existence of local FDIAs, $g_{i1}(t)$ and $g_{i2}(t)$, injected by hackers. At first, we propose a local residual generator, $r_i(t)$, which is used to construct the local detector at local power Area-$i$th, $i = 1, \ldots, N$.

$$r_i(t) = T_{i} \zeta_i(t) + E_{i} y_i(t).$$

In Equation (14), $r_i(t) \in \mathbb{R}$ is a real function. Matrices $T_i \in \mathbb{R}^{1 \times q_i}, E_i \in \mathbb{R}^{1 \times p_i}$ are residual generator matrices that will be designed later. $\zeta_i(t) \in \mathbb{R}^{q_i}$ is computed based on the following DFO

$$\dot{\zeta}_i(t) = N_{i} \zeta_i(t) + N_{i} \zeta_i(t) - b_i + J_f y_i(t)$$

$$+ J_{i} y_i(t) - b_i + H_i u_i(t) + \sum_{j=1,j\neq i}^{N} J_{ij} y_i(t),$$

where $N_i, N_{ii} \in \mathbb{R}^{q_i \times q_i}, J_{ij} J_{ij} \in \mathbb{R}^{q_i \times p_i}, J_{ij} \in \mathbb{R}^{q_i \times p_i}, H_i \in \mathbb{R}^{d \times 1}$ are the observer gains and be determined such that $\zeta_i(t)$
asymptotically converges to a linear function of the local state vector, \( \dot{z}_i = L_i x_i(t) \) when there are no FDIAs within the power grid, \( g_i(t) = 0 \). In Equation (15), \( \dot{z}_i(t) = C_{ji} x_j(t) + F_{ji} g_j(t) \) is a vector of the augmented outputs from neighbouring power areas, where \( C_{ji} \in \mathbb{R}^{n_i \times n_j} \), \( F_{ji} \in \mathbb{R}^{n_i \times 2} \).

For FDIAs detection purpose, \( T_{ni} E_{ni} L_i \) and DFO matrices should achieve the following detection requirements: (i) (safe case) \( \lim_{t \to \infty} r_i(t) = 0 \) if \( g_i(t) = 0 \); (ii) (unsafe case) \( \lim_{t \to \infty} r_i(t) \neq 0 \) if \( g_i(t) \neq 0 \). We define \( \theta_i(t) = z_i(t) - L_i x_i(t) \) as the error between \( z_i(t) \) and \( \dot{z}_i(t) \).

**Theorem 1.** For any given control signal, \( u_i(t) \), disturbance, \( d_i(t) \) and with no FDIAs, \( z_i(t) \) will asymptotically converge to \( \dot{z}_i = L_i x_i(t) \) if there exist matrices \( N_i, N_{ib} J_i, J_{ib}, J_{ij}, H_i, L_i \) with appropriate dimensions satisfying

\[
\dot{\theta}_i(t) = N_i \dot{\theta}_i(t) + N_{ib} \theta_i(t - h_i) \quad \text{is asymptotically stable,} \tag{16}
\]

\[
\left[ \Omega_{i1} \Omega_{i2} \Omega_{i3} \Omega_{i4} \Omega_{i5} \Omega_{ib} \right] = 0, \tag{17}
\]

where

\[
\begin{align*}
\Omega_{i1} &= -L_i \Gamma_i, \\
\Omega_{i2} &= N_i L_i + J_i C_i - L_i A_{ib}, \Omega_{i6} = \left[ \Omega_{i6}^j \right]_j \neq i, \\
\Omega_{i3} &= N_{ib} L_i + J_{ib} C_i - L_i A_{ib}, \Omega_{i4} = H_i - L_i B_i, \\
\Omega_{i5} &= \left[ \Omega_{i5}^j \right]_j \neq i, \Omega_{i5}^i = J_{ij} C_{ij} - L_i A_{ij}, \Omega_{i6} = J_{ij} F_{ij}.
\end{align*}
\]

**Proof of Theorem 1.** Let us define \( Y_i, Y_{i1}, Y_{i2} \) and \( \Omega_i \) which will be used later as \( Y_i = T_i L_i + E_i C_i, \ Y_{i1} = J_i F_i - L_i S_i, \ Y_{i2} = J_{ib} F_{ib}, \ Y_{i3} = J_i F_i + E_i, \ \Omega_i = (J_i F_i - L_i S_i) g_i(t) + J_{ib} F_{ib}(t - h_i) = Y_{i1} g_i(t) + Y_{i2} g_i(t - h_i) \).

Taking the derivative of \( \dot{\theta}_i(t) \), we obtain

\[
\dot{\theta}_i(t) = N_i \dot{\theta}_i(t) + N_{ib} \theta_i(t - h_i) - L_i \Gamma_i d_i(t) + \sum_{j=1, j \neq i}^{N} (J_{ij} C_{ij} - L_i A_{ij} x_j(t)) \tag{18}
\]

\[
+ \sum_{j=1, j \neq i}^{N} (J_{ib} C_{ij} - L_i A_{ib} x_j(t)) + \sum_{j=1, j \neq i}^{N} (J_{ij} F_{ij} \dot{g}_j(t)) \tag{19}
\]

\[
\dot{\theta}_i(t) = N_i \dot{\theta}_i(t) + N_{ib} \theta_i(t - h_i) + \Omega_{i1} d_i(t) + \Omega_{i2} x_i(t) + \Omega_{i3} x_i(t - h_i) + \Omega_{i4} \theta_i(t) + \Omega_{i5} \theta_i(t - h_i) \tag{20}
\]

\[
+ \sum_{j=1, j \neq i}^{N} \Omega_{i6}^j x_j(t) + \sum_{j=1, j \neq i}^{N} \Omega_{i6}^j \theta_j(t) \tag{21}
\]

For \( N_i, N_{ib}, J_i, J_{ib}, J_{ij} \) satisfying the conditions Equations (16) and (17) and with no FDIAs, \( g_i(t) = 0, \Omega_i = 0 \), we have

\[
\dot{\theta}_i(t) = N_i \dot{\theta}_i(t) + N_{ib} \theta_i(t - h_i). \tag{22}
\]

As can be seen that Equation (20) is a time-delay system and \( \dot{\theta}_i(t) \) will asymptotically converge to zero, \( (\dot{\theta}_i(t) \to 0) \), if the stability of Equation (20) is guaranteed (i.e. the condition Equations (16) and (17) are satisfied). This completes the proof of Theorem 1.

It is noted that \( \dot{\theta}_i(t) = z_i(t) - L_i x_i(t) \). From Equation (14), the local residual generator, \( r_i(t) \), can be rewritten in the following form

\[
r_i(t) = T_i (\dot{\theta}_i(t) + L_i x_i(t)) + E_i (C_i x_i(t) + F_{ib} g_i(t)) \tag{23}
\]

\[
= T_i \dot{\theta}_i(t) + (T_i L_i + E_i C_i) x_i(t) + E_i F_{ib} g_i(t) \tag{24}
\]

\[
= T_i \dot{\theta}_i(t) + Y_i x_i(t) + Y_{i3} g_i(t). \tag{25}
\]

When \( Y_i = 0 \), Equation (21) becomes

\[
r_i(t) = T_i \dot{\theta}_i(t) + Y_{i3} g_i(t). \tag{26}
\]

In the following part, according to Theorem 1, Equations (19) and (26) we propose a method using \( r_i(t) \) to detect local FDIAs.

i When no FIA exist within the systems (\( \Omega_i = 0 \) and \( Y_{i3} g_i(t) = 0 \)) and Theorem 1 is satisfied, residual generator, \( r_i(t) \to 0 \) as \( t \to \infty \).

ii In contrast, when FDIAs exist in the system, \( \Omega_i \neq 0 \) or \( Y_{i3} g_i(t) \neq 0 \), and Theorem 1 is satisfied, \( r_i(t) \) will not asymptotically converge to 0, \( (r_i(t) \not\to 0) \).

Therefore, residual generator, \( r_i(t) \), can be used as an indicator for FDIAs detection purpose. The schematic implementation of DFO based FDIAs detection method is shown on Figure 4. In this scheme, a detector (a residual generator based observer) is located at the local power Area-\( i \)-th to detect its FDIAs. The residual value is computed according to matrices \( T_i, E_i \) together with a DFO.

In the following, we develop corollary 1 such that \( r_i(t) \) reacts to FDIAs \( (r_i(t) \to 0) \).

**Corollary 1.** Local residual generator, \( r_i(t) \to 0 \) as \( t \to \infty \) for any local FDIAs, \( g_{i1}(t), g_{i2}(t) \), if the following conditions are satisfied

\[
Y_i = 0, \quad \text{Diag} \left( Y_{i1}^1, Y_{i2}^1, Y_{i3}^1 \right) \neq 0, \tag{27}
\]

\[
\text{Diag} \left( Y_{i1}^2, Y_{i2}^2, Y_{i3}^2 \right) \neq 0. \tag{28}
\]
where \( Y_{i1} = \{ Y_{i1}^1 \, Y_{i1}^2 \}, Y_{i2} = J_i F_i - L_i S_i, Y_{i3} = J_i F_i - L_i S_i \), \( Y_{j1} = \{ Y_{j1}^1 \, Y_{j1}^2 \}, Y_{j2} = J_j F_j, Y_{j3} = J_j F_j \), \( Y_{j3} = E_i F_i, Y_{j3} = E_j F_j \).

**Proof of Corollary 1.** Let us consider \( Y_{i1}, Y_{i2}, Y_{i3}, Y_{j1}, Y_{j2}, Y_{j3} \) satisfying Corollary 1. We assume that only \( g_{i1}(t) \) exists at the local power.\( \hat{\xi}_i(t) = 0 \), \( g_{j2}(t) = 0 \). According to Corollary 1, we have one of the terms \( Y_{i1}^1 g_{i1}(t) \), \( Y_{i1}^2 g_{i1}(t) \) and \( Y_{i3}^3 g_{i1}(t) \) is not zero. Therefore, \( \Omega_i \neq 0 \) or \( Y_{i3}^3 g_{i1}(t) \neq 0 \) leading to \( r_i(t) \rightarrow 0 \) as \( t \rightarrow \infty \). The proof can be applied to \( g_{j2}(t) \). This completes the proof of Corollary 1. \( \square \)

**Remark 2.** We consider model Equation (11) where \( g_{i1}(t) = 0 \), \( b_i = 0 \). By using a similar structure of DFO and residual generator form in Equations (14)-(15), a detector for Equation (11) can be developed.

\[
\tilde{z}_i(t) = N_i \tilde{z}_i(t) + J_i \tilde{y}_i(t) + H_i m_i(t) + \sum_{j=1, j \neq i}^{N_i} J_{ij} \tilde{y}_j(t). \tag{24}
\]

The derivative of \( \tilde{z}_i(t) \) is \( \dot{\tilde{z}}_i(t) = N_i \tilde{z}_i(t) - L_i \tilde{y}_i(t) \). The local residual generator is \( r_i(t) = J_i \tilde{z}_i(t) + Y_{i3} \tilde{z}_i(t) \). Therefore, \( r_i(t) \) reacts to \( g_{j2}(t) \), if the Theorem 1 holds, \( Y_{j1} = 0 \) and the following condition is satisfied \( L_i \tilde{y}_i \neq 0 \). This completes Remark 2.

**Corollary 2.** When Theorem 1 holds and \( Y_{j1} = 0 \), \( r_i(t) \) is insensitive to a local FDLA, \( g_{j2}(t) \), if there exist matrices \( Y_{i1}^k, Y_{i2}^k, Y_{i3}^k \) such that the following conditions are satisfied

\[
\text{Diag} \left( \begin{array}{ccc} Y_{i1}^k & Y_{i2}^k & Y_{i3}^k \end{array} \right) = 0. \tag{25}
\]

where \( Y_{i1}^k, Y_{i2}^k, Y_{i3}^k \) are given in Corollary 1.

**Proof of Corollary 2.** The proof can be obtained by using similar lines as in the proof of Corollary 1. Hence, we omit it here.

By using Corollary 2, we can build extra detectors-D\( \hat{D}_k \), \( k = 1, 2 \), where \( D\hat{D}_k \) includes a residual generator, \( r\hat{D}_k(t), k = 1, 2 \), which is insensitive to the FIDA of \( g_{j2}(t) \) while it is sensitive to the remaining local FDLA. By using the extra detectors together with the main detector \( D_j \) (residual \( r_j(t) \)), FDLAs can be isolated.

This completes Remark 2. \( \square \)

**Remark 4.** In this Remark, we develop a CFO based FDLAs detection method for ISGs. In this design, a centralised global detector, \( D \), will detect the happening of all FDLAs. A global residual generator, \( r(t) \in \mathbb{R} \), which is used to form the global detector.

\[
r(t) = T\tilde{z}(t) + E\tilde{y}(t). \tag{26}
\]

In Equation (26), \( T \in \mathbb{R}^{1 \times d}, E \in \mathbb{R}^{1 \times p} \) are residual generator's matrix gains. \( \tilde{z}(t) \in \mathbb{R}^d \) is computed based on structure of CFO

\[
\hat{z}_i(t) = N_i \tilde{z}_i(t) + \sum_{j=1}^{N_i} N_{ij} \tilde{z}(t - b_j) \tag{27}
\]

\[
E \tilde{y}(t) + H m_i(t),
\]

where \( \tilde{N}, N_{ij} \in \mathbb{R}^{d \times q}, J_{ij} \in \mathbb{R}^{d \times l}, H \in \mathbb{R}^{d \times w} \) are the CFO gains and be determined such that \( \tilde{z}(t) \) asymptotically converges to a linear functional of the local state vector, \( \tilde{z}(t) = L x(t) \) when there are no FDIAs within the ISG, \( g_{j2}(t) = 0 \).

For the detection purpose, \( T, E, L \) and CFO matrices should achieve the following detection requirements: (i) (safe case) \( \lim_{t \to \infty} r(t) = 0 \) if \( g_{j2}(t) = 0 \); (ii) (unsafe case) \( \lim_{t \to \infty} r(t) \neq 0 \) if \( g_{j2}(t) \neq 0 \).

**Corollary 3.** For any given control signal, \( u(t) \), disturbance, \( d(t) \) and with no FDIAs, \( \tilde{z}(t) \) asymptotically converges to \( \tilde{z}(t) = L x(t) \) if there exist matrices \( \tilde{N}, N_{ij}, J_{ij}, J_{ij} \), \( L \) with appropriate dimensions satisfying

\[
\hat{\theta}(t) = \tilde{N} \theta(t) + \sum_{i=1}^{N_i} N_{ij} \theta(t - b_j) \text{ is asymptotically stable}, \tag{28}
\]

\[
[ \Omega_1 \quad \Omega_2 \quad \Omega_3 \quad \Omega_4 ] = 0,
\]

where \( \Omega_1 = - L \Gamma, \Omega_2 = \tilde{N} L + J C - L A, \Omega_3 = \Omega_4 \times \Omega_3, \Omega_4 = N_{ij} L + J_{ij} C - L A_{ij}, \Omega_4 = H - L B \).

**Proof of Corollary 3.** We define \( Y = \tilde{N} + E C, Y_1 = JF - LS, Y_2 = J_{ij} F, Y_3 = E F, \Omega = Y_{i3} \tilde{z}(t) + \sum_{i=1}^{N_i} Y_{i2} \tilde{z}(t - b_j) \).

We have \( \hat{\theta}(t) = \tilde{N} \theta(t) + \sum_{i=1}^{N_i} N_{ij} \theta(t - b_j) + \sum_{i=1}^{N_i} Y_{i2} \tilde{z}(t - b_j) + \Omega_1 d(t) + \Omega_2 \tilde{z}(t) + \Omega_4 \tilde{z}(t) + \Omega_4 \).

For \( \tilde{N}, N_{ij}, J_{ij} \) satisfying Equation (28) and with no FDIAs, \( g_{j2}(t) = 0 \), \( \Omega = 0 \), we obtain \( \hat{\theta}(t) = \tilde{N} \theta(t) + \sum_{i=1}^{N_i} N_{ij} \theta(t - b_j) \). This is a linear system with multiple time delays and \( \theta(t) \) will asymptotically converge to zero, \( \theta(t) \rightarrow 0 \), if the stability is guaranteed. This completes the proof of Corollary 3. \( \square \)

The residual generator is \( r(t) = T \tilde{z}(t) + Y \tilde{z}(t) + Y_{i3} \tilde{z}(t) \). When \( Y = 0 \), \( r(t) \) becomes \( r(t) = T \tilde{z}(t) + Y_{i3} \tilde{z}(t) \). Now, according to Corollary 3, we propose a method using \( r(t) \) to detect FDLAs. When no FDIAs exist within the systems \( \Omega = 0 \) and \( Y_{i3} \tilde{z}(t) = 0 \) and Corollary 3 is satisfied, residual generator, \( r(t) \rightarrow 0 \) as \( t \to \infty \). In contrast, when FDLAs exist in the system, \( \Omega \neq 0 \) or \( Y_{i3} \tilde{z}(t) \neq 0 \), and Corollary 3 is satisfied, \( r(t) \) will not asymptotically converge to \( 0, r(t) \rightarrow 0 \).
Corollary 4. Residual generator, \( r(t) \rightarrow 0 \) as \( t \rightarrow \infty \) for any FDLa, if the following conditions are satisfied
\[
Y = 0, \quad \text{Diag}(Y_1^k, Y_2^k, Y_3^k) \neq 0, \quad k = 1, \ldots, 2N_i
\]
where
\[
Y_1 = [Y_1^1 \ldots Y_{2N_i}^1], \quad Y_1^k = iF_k - Ls^k, \quad Y_2 = [Y_2^1 \ldots Y_{2N_i}^1], \quad Y_2^k = Js^k, \quad Y_3 = [Y_3^1 \ldots Y_{2N_i}^1], \quad Y_3^k = EF_k, \quad s^k
\]
\( F_k \) is \( k \times k \) columns of \( S \) and \( F \).

Proof of Corollary 4. The proof for this Corollary can be obtained by using a similar technique in Corollary 1, hence we omit here. This completes Corollary 4 and Remark 4.

4 | DFO BASED DETECTOR SYNTHESIS

We now introduce the next stages of detector synthesis to find the unknown parameters of detector-\( D_2 \), \( T_1 \), \( E_i \), \( N_i \), \( N_s \), \( J_{ib} \), \( J_{ij} \) and \( H_i \) in Equations (14)-(15). Let we introduce following Lemmas 1 and 2 which are necessary for the synthesis procedure of \( D_2 \).

Lemma 1 ([34]). We consider a linear equation \( X \Omega_i = \Omega_2 \), \( X \in \mathbb{R}^{m \times p} \), \( \Omega_1 \in \mathbb{R}^{m \times m} \), \( \Omega_2 \in \mathbb{R}^{m \times m} \). There exists a solution \( X \) if and only if
\[
\text{rank} \left( \begin{bmatrix} \Omega_2 \\ \Omega_1 \end{bmatrix} \right) = \text{rank}(\Omega_1), \quad (30)
\]
and \( X \) can be obtained as \( X = \Omega_2 \Omega_1^+ + Z(l_p - \Omega_1 \Omega_1^+) \) where \( \Omega_1^+ \) is the Moore-Penrose inverse of \( \Omega_1 \), \( Z \in \mathbb{R}^{m \times m} \) is an arbitrary matrix.

Lemma 2 ([33]). We consider the homogenous equation
\[
Y \Omega = 0, \quad (31)
\]
where \( Y \in \mathbb{R}^{m \times m} \) is an unknown matrix, \( \Omega \in \mathbb{R}^{m \times m} \) is a known matrix.

Theorem 2. For given scalars \( b, \delta \), and matrices \( N_i, N_{ib} \in \mathbb{R}^{p \times q}, \) system Equation (20) is asymptotically stable if there exist positive definite matrices \( P, Q, R, \in \mathbb{R}^{p \times p} \), and a matrix \( X_i \in \mathbb{R}^{p \times q} \) satisfying the following matrix inequality
\[
\Lambda_1 = \Lambda_1 + \Lambda_2 + \Lambda_3 + \Lambda_4 < 0, \quad (32)
\]
where \( \Lambda_1 = \epsilon_1^T P_1 r_1 + \epsilon_2^T P_1 r_2, \quad \Lambda_2 = \epsilon_1^T Q_1 r_1 - \epsilon_2^T Q_1 r_2 + \epsilon_2^T (b_i R_i) r_2, \quad \Lambda_3 = -F_i^T R_i F_i, \quad \Lambda_4 = M_i N_i + N_i^T M_i^T, \quad \mathcal{M}_i = (\epsilon_1 + \delta_i)^T P_i, \quad N_i = -X_i N_i r_1 + X_i N_i r_2, \quad \epsilon_2 = [0_{q \times N_i} I_{0_{q \times N_i}}]^T, \quad 0 \leq l_i \leq 4, \quad P_i = [P_i^T P_i], \quad F_i = r_1 - r_3, \quad \mathcal{R}_i = \text{Diag}(R_i, 3R_i, 5R_i), \quad F_i = r_1 - r_3, \quad \mathcal{R}_i = r_1 - r_3 + 6r_4 - 6r_5.

Proof of Theorem 2. We denote some following notations
\[
\chi(t) = [\Omega_i(t) \Omega_i(t) \Omega_i(t)]^T, \quad X_i(t) = (\Omega_i(t) - X_i(t) - b_i)^T \Omega_i(t) \Omega_i(t) R_i \Omega_i(t), \quad \mathcal{X}_i(t) = (\Omega_i(t) - X_i(t) - b_i)^T \Omega_i(t) \Omega_i(t) R_i \Omega_i(t). \quad (33)
\]
By taking the derivative of \( V_i(\chi_i(t), t) \), we obtain
\[
V_i'(\chi_i(t), t) \leq \chi_i(t) \mathcal{X}_i(t) \chi_i(t), \quad (34)
\]
and this completes the proof of Theorem 2.

Now we are in the main stage of this section. In this step, we solve the condition presented in Theorem 1 and corollary 1 for the exits of proposed detector-\( D_2 \). By solving these conditions, the detector’s parameters \( (N_i, N_{ib}, J_{ib}, J_{ij}, T_i, H_i) \) can be obtained. To solve these conditions, above Lemma 1, Lemma 2 and Theorem 2 will be used. For the ease of presentation, we present the process of solving Equations (16), (17) and (32) in a procedure of four main stages which can be implemented by using various computing tools.

Detector synthesis procedure:

Step 1: Solve \( \Omega_1 = 0, \Omega_4 = 0, \Omega_3 = 0 \) in Theorem 1 to obtain \( L_{ij}, f_{ij}, j = 1, \ldots, N_i, j \neq i, \text{then } H_i \). Let we denote \( \Xi_i = \Xi_{i1} = \Xi_{i2} = 0_{N_i}, \) where \( \Xi_{i1} = \text{Diag}(C_{1i}, \ldots, C_{(i-1)}, C_{(i+1)}, \ldots, C_{N_i}), \) \( C_{ij} = [A_{ij} \ldots A_{ij}, A_{ij}], \) \( A_{ij} \in \mathbb{R}^{p \times m}, \) \( \Xi_{i2} = [I_{1i} \ldots I_{(i-1)}, I_{(i+1)} \ldots I_{N_i}], \) \( \Psi_i = [0_{N_i \times N_i}], \) \( p_i = [N_i \times N_i], \)
From \( \Omega_1 = 0 \) and \( \Omega_3 < 0, j \neq i \) in Equations (16)-(17), we obtain the following equality
\[
\Psi_i \Xi_i = 0, \quad (34)
\]
Equation (34) has solution if \( \Psi_i \), if \( \Xi_i \) satisfies Lemma 2. To do that, we use Matlab software with “rank” command to check
\[
\text{rank}(\Xi_i) < (p_i + n_i), \text{then } \Psi_i \text{ can be obtained by taking some rows of } \Psi_i = \mathcal{N}(\Xi_i). \text{ Here the DFO parameter, } f_{ij} \text{ and matrix}
\]
\[
\Psi_i \Xi_i = 0, \quad (34)
\]
$L_\tau$ can be obtained from $\Psi_\tau$. Finally, $H_i$ is obtained from $\Omega_{ih} = 0$ in Equations (16) and (17), $H_i = L_\tau B_i$.

**Step 2:** To solve conditions of $\Omega_{i2}$ and $\Omega_{i3}$ in Theorem 1 for the existence of $N_i$, $N_{ih}, J_i, J_{ih}$. We denote $\Phi_{i2} = [I_{A_i} L_{\tau} A_i]$ and $\Phi_{i3} = \text{diag}(E_i, \Xi_i)$, where $E_i = [T_i, C_i]^T$. From $\Omega_{i2} = 0$ and $\Omega_{i3} = 0$ in Equations (16)-(17), we obtain the following results

$$[N_i \ J_i \ N_{ih} \ J_{ih}] \Phi_{i1} = \Phi_{i2}. \quad (35)$$

According to Lemma 1, Equation (35) has solution if it satisfies the following condition of matrix rank. $\text{rank}([\Phi_{i2}^T \ \Phi_{i3}^T]^T) = \text{rank}(\Phi_{i1})$. If this condition of rank $\Phi_{i2}, \Phi_{i3}$ are satisfied, detector’s matrices $N_i, N_{ih}, J_i, J_{ih}$ can be rewritten in the following structure

$$= \Phi_{i2} \Phi_{i3}^T + Z_i(I_{2(p+q_i)} - \Phi_{i1} \Phi_{i3}^T), \quad (36)$$

where $N_i = N_{i1} + Z_i N_{i2}, \ N_{ih} = N_{ih1} + Z_i N_{ih2}, \ J_i = J_{i01} + Z_i J_{i02}, \ J_{ih} = J_{ih1} + Z_i J_{ih2}, \ \Phi_{i3}^+ = \text{Moore–Penrose pseudo-inverse of } \Phi_{i3}, Z_i$ will be obtained later and $N_{i1}, N_{i2}, N_{ih1}, N_{ih2}, J_{i01}, J_{i02}, J_{ih1}, J_{ih2}$ are

$$N_{i1} = \Phi_{i2} \Phi_{i3}^+ \Lambda_i, \ N_{i2} = (I_{2(p+q_i)} - \Phi_{i1} \Phi_{i3}^+) \Lambda_i, \ J_{i01} = \Phi_{i2} \Phi_{i3}^+ \Lambda_{i2}, \ J_{i02} = (I_{2(p+q_i)} - \Phi_{i1} \Phi_{i3}^+) \Lambda_{i2}, \ N_{ih1} = \Phi_{i2} \Phi_{i3}^+ \Lambda_{i3}, \ N_{ih2} = (I_{2(p+q_i)} - \Phi_{i1} \Phi_{i3}^+) \Lambda_{i3}, \ J_{ih1} = \Phi_{i2} \Phi_{i3}^+ \Lambda_{i4}, \ J_{ih2} = (I_{2(p+q_i)} - \Phi_{i1} \Phi_{i3}^+) \Lambda_{i4}, \ \Lambda_i = [I_{q_i}, \ 0_{p \times (q_i + 2p_i)}]^T, \ \Lambda_{i2} = [0_{p \times q_i}, \ I_{p_i}, \ 0_{p \times (q_i + p_i)}]^T, \ \Lambda_{i3} = [0_{p \times (q_i + p_i)}], \ I_{p}, \ 0_{p \times (q_i + p_i)}]^T, \ \Lambda_{i4} = [0_{p \times (q_i + p_i)}, \ I_{p_i}]^T.$$

As can be seen that, after some calculations, we have presented $N_i, N_{ih}, J_i, J_{ih}$ by some matrices where $N_{i1}, N_{i2}, N_{ih1}, N_{ih2}$ are known and $Z_i$ need to be calculated. Then, the equation for DFO error, $\tilde{\theta}_i(t)$ in Equation (20) becomes the following construction

$$\tilde{\theta}_i(t) = (N_{i1} + Z_i N_{i2}) \tilde{\theta}_i(t) + (N_{ih1} + Z_i N_{ih2}) \tilde{\theta}_i(t - b_i). \quad (37)$$

**Step 3:** We derive $Z_i$ such Equation (37) is asymptotically stable, then we obtain $N_i, N_{ih, J_i, J_{ih}}$ from Equation (37). To do that, we develop the following Corollary 5 for synthesising $Z_i$. The corollary 5 is derived according to Theorem 2. Roughly speaking, corollary 5 is a presentation of Theorem 2 in a tractable LMI form, therefore, which can be solved automatically by LMI solver in MATLAB.

**Corollary 5.** For given positive scalars $b_i$ and $\delta_i$, system Equation (37) is asymptotically stable if there exist positive definite matrices $P_i, Q_i, R_i \in \mathbb{R}^{p \times p}$ and matrices $X_i, Y_i$ with appropriate dimension satisfying the following LMI

$$X_i^T = X_i + P_i X_i, \ Y_i = [P_i, Q_i, \ P_i], \ \text{Rank}(P_i) < b_i + q_i \ \text{and matrices } X_i, Y_i \text{ with appropriate dimension satisfying the following LMI}$$

$$X_i = \Lambda \ 4 < 0, \quad (38)$$

**Proof of Corollary 5.** Let we define $Y_i = X_i Z_i$, the apply $N_i = N_1 + Z_i N_{i2}, \ N_{ih} = N_{ih1} + Z_i N_{ih2}, \ we$ can obtain Equation (32). It shows that LMI Equation (32) hold if LMI Equation (38) holds. This completes proof of Corollary 5.

By using robust control toolbox in Matlab to solve the LMI Equation (38), $Z_i$ and detector gains $N_i, N_{ih}, J_i, J_{ih}$ can be obtained as

$$Z_i = X_i^{-1} Y_i, \ N_i = N_1 + Z_i N_{i2}, \ N_{ih} = N_{ih1} + Z_i N_{ih2}, \ J_i = J_{i01} + Z_i J_{i02}, \ J_{ih} = J_{ih1} + Z_i J_{ih2}. \quad (39)$$

This completes Step 3 of the procedure.

**Step 4:** We solve condition of $Y_i = 0$ in Corollary 1 to obtain residual generator gains $T_i$ and $E_i$. We denote $\Sigma_i = [T_i, E_i]$. From $\hat{\Sigma}$ derived in Step 2 of this procedure and the condition of $\Sigma_i = 0$ in Equation (23), we obtain the following result

$$\Sigma_i \Xi_i = 0. \quad (40)$$

From Lemma 2, we use MATLAB software test $\text{rank}(\Xi_i) < q_i + p_i$. Thus $T_i, E_i$ are taken from rows of $\Sigma_i = \mathcal{N}(\Xi_i)$. Finally, we test the conditions $\text{Diag}(Y_{i1}, Y_{i2}, Y_{i3}) \neq 0, \ \text{Diag}(Y_{i1}', Y_{i2}', Y_{i3}') \neq 0$ in Equation (23). If all two conditions are not satisfied, the residual generator is insensitive to both FDAs whereas if one of them is satisfied, the residual generator is sensitive to one FDA and insensitive to other, go back to choose another value of $L_\tau$ in Step 1. Otherwise, this completes the detector synthesis procedure. The procedure is demonstrated in Figure 5.
RESULTS AND DISCUSSION

LFC of time delay ISGs

Stability of LFC for ISGs

In this section, a three-area ISG with reheated thermal power plants, delayed EVs and RES are used for demonstrating our detection method. The intermittent of RES and demand changes are formulated as following time-varying functions $d_i(t) = d_{oi} + d_{1i} \sin(\phi_{1i} t) + d_{2i} \cos(\phi_{2i} t)$ puMW, where $d_{oi}$, $d_{1i}$, $d_{2i}$, $\phi_{1i}$, $\phi_{2i}$, $i = 1, 2, 3$ are the magnitudes of load demand, RES and the frequencies of the functions as follows $d_{0i} = 0.08$, $d_{1i} = 0.01$, $d_{2i} = 0.05$, $d_{2i} = 0.005$, $d_{2i} = 0.01$, $\phi_{1i} = \sqrt{0.002}$, $\phi_{2i} = \sqrt{0.007}$, $d_{0i} = 0.05$, $d_{1i} = 0.01$, $d_{2i} = 0.005$, $d_{2i} = 0.01$, $\phi_{1i} = \sqrt{0.003}$, $\phi_{2i} = \sqrt{0.006}$, $d_{0i} = 0.13$, $d_{1i} = 0.01$, $d_{2i} = 0.005$, $\phi_{1i} = \sqrt{0.005}$, $\phi_{2i} = \sqrt{0.004}$. The demonstration of $d_1(t)$ and $d_2(t)$ are provided in Figure 6. Other parameters of studied systems can be gathered from Appendix. The system control input signals, $u_i(t)$, $i = 1, 2, 3$ can be obtained by following computation $u_i(t) = K_{pi} f_i(t) + K_{ti} P_{tie}^i(t) + K_{si} \dot{\phi}_i(t)$, where $K_2 = 0.1$, $K_{si} = 0.0425$, $K_{ti} = -0.1$ are LFC controller gains. Time delayed $b, b = 0.1$s.

We undertook Scenario I to show that above controller is adequate to achieve the main objectives of LFC in normal operation of the ISG (without FDIAs, $g(t) = 0$). Figures 8 and 9 illustrate the responses of frequency deviations, $f_i(t)$, and interchange power deviations, $P_{tie}^i(t)$, $i = 1, 2, 3$ of closed-loop systems for the duration of 180 seconds (from the second of 140 to 320) of simulation. These simulation results indicate that $f_i(t)$ and $P_{tie}^i(t)$ are brought back in very small bounds of desirable value, $f_i(t) \in [-0.0062, 0.0039]$ Hz, $f_2(t) \in [-0.0077, 0.0055]$ Hz, $f_3(t) \in [-0.0066, 0.0029]$ Hz, $P_{tie}^i(t), i = 1, 2, 3$ belongs to a range of $[-0.01, 0.01]$ puMW which is corresponding to 10% of fluctuations on RES and demand.

5.1.2 Stability and stabilisation for time-delay ISGs

We consider the stability of time-delay ISGs embedded LFC in this subsection. Firstly, the global control input signal, $u(t)$ can be rewritten in the form of $u(t) = KC\xi(t)$ and the state space model of ISGs without FDIAs is

$$\dot{x}(t) = \Lambda x(t) + \Lambda_1 x(t - b) + \Gamma d(t).$$  \hspace{1cm} (41)

In system Equation (41), $\Lambda_1 = \sum_{i=1}^{N} A_{hi} \Lambda_1 \Lambda = A + BKC$ where $\Lambda, \Lambda_1, B, \Gamma$ are defined in Equation (12). For ease of presentation, we consider $b = b$ for all local SGs. We denote $\tilde{z}(t) = E\tilde{x}(t)$, $\tilde{x}(t) \in \mathbb{R}^n$, $E \in \mathbb{R}^{n \times n}$ is the observation vector. This vector is for the evaluation of system robust performance, $\gamma$, presented in the relationship between the observation vector, $\tilde{z}(t)$ and $d(t)$ (the reader can refer to [20], [17] for the theory of robust stabilisation). For a given controller gain $K$, the following Theorem 3 presents a sufficient condition under which system Equation (41) is asymptotically stable with a pre-selected robust performance index (RPI), $\gamma$.

Theorem 3. For given positive scalars $b, \gamma$, and $\Lambda$, system Equation (41) is asymptotically stable with a given RPI, $\gamma$, if there exist symmetric positive definite matrices $P, Q, R \in \mathbb{R}^{n \times n}$, and $X \in \mathbb{R}^{n \times n}$ satisfying the following matrix inequality

$$\Lambda = \begin{bmatrix} \Lambda_{11} & \Lambda_{12} & \Lambda_{13} \\ \Lambda_{21} & \Lambda_{22} & 0 \\ \Lambda_{31} & 0 & \Lambda_{33} \end{bmatrix} < 0,$$  \hspace{1cm} (42)
where \( \tilde{A}_{11} = \sum_{j=0}^{3} \Theta_j, \tilde{A}_{12} = \tilde{U} T, \tilde{A}_{13} = \tilde{R}_1 E^T, \tilde{R} = h^2 \tilde{R}, \Theta_0 = \tilde{E}_0 \tilde{P}_0 + \tilde{E}_0 \tilde{P}_0 T, \tilde{R} = \text{diag}(\tilde{R}, \tilde{R}, \tilde{R}), \tilde{A}_{22} = - \gamma_I T, \Theta_1 = \tilde{E}_0 \tilde{P}_1 - \tilde{E}_0 \tilde{P}_0 T, \tilde{A}_2 = - F^T \tilde{R} \tilde{F}, \tilde{A}_{13} = - I, \Theta_1 = \tilde{U} \tilde{V} + \tilde{V}^T \tilde{U} f, \tilde{U} = (\tilde{E}_0 + \tilde{A}_{12} E^T)^T, \tilde{X}(t) = [x^T(t) \tilde{x}^T(t) x^T(t) - b \tilde{X}_2^T(t) \tilde{R}_2^T(t) \tilde{X}_2^T(t), \tilde{V} = - \tilde{Z}_1 + \tilde{Z}_1 + \tilde{A}_3 \tilde{Z}_2, \tilde{X}_1(t) = \frac{1}{2} \int_{t-j}^{t} x(i) d \tau, \tilde{Z}_2(t) = \frac{2}{h} \int_{t-j}^{t} x(u) d \tau.

Proof of Theorem 3. Let \( \tilde{P}, \tilde{Q}, \tilde{R} \) satisfy Equation (42). We construct the following Lyapunov–Krasovskii functional candidate \( \tilde{V} = \tilde{V}_1(t) + \tilde{V}_2(t) + \tilde{V}_3(t), \tilde{V}_1(t) = x^T(t) \tilde{P} x(t), \tilde{V}_2(t) = \int_{t-j}^{t} x^T(i) \tilde{Q} x(i) d \tau, \tilde{V}_3(t) = h \int_{t-j}^{t} \tilde{x}^T(u) \tilde{R} \tilde{x}(u) d \tau.

Then, we obtain \( \tilde{V} = \tilde{X}^T(t) \tilde{P} \tilde{X}(t) + \tilde{X}_1(t) + \tilde{X}_2(t) = \frac{2}{h} \int_{t-j}^{t} x(u) d \tau \), which implies that \( \tilde{V} = \tilde{X}^T(t) \tilde{P} \tilde{X}(t) + \tilde{X}_1(t) + \tilde{X}_2(t) = \frac{2}{h} \int_{t-j}^{t} x(u) d \tau \) is a candidate Lyapunov–Krasovskii functional candidate.

Adding the left-hand side of the above equation to \( \tilde{V} \), we have \( \tilde{V} = \tilde{X}^T(t) \tilde{P} \tilde{X}(t) + \tilde{X}_1(t) + \tilde{X}_2(t) = \frac{2}{h} \int_{t-j}^{t} x(u) d \tau \), which implies that \( \tilde{V} = \tilde{X}^T(t) \tilde{P} \tilde{X}(t) + \tilde{X}_1(t) + \tilde{X}_2(t) = \frac{2}{h} \int_{t-j}^{t} x(u) d \tau \) is a candidate Lyapunov–Krasovskii functional candidate.

Now, we are in the stage to prove that system Equation (41) is asymptotically stable with a RPI \( \gamma \). For \( d(t) = 0 \), Equation (43) implies that \( \tilde{V} \) is a negative definite, and thus system Equation (41) is asymptotically stable. For \( d(t) \neq 0 \), integrating both sides of Equation (43) from zero to \( t_f > 0 \), we obtain \( \tilde{V}(t_f) - \tilde{V}(0) + \int_{0}^{t_f} \tilde{x}^T(i) \tilde{Z}_2(i) \tilde{x}(i) d \tau \leq \gamma^2 \int_{0}^{t_f} \tilde{d}^T(t) \tilde{d}(t) d \tau \). Therefore, if Equation (42) holds, the linear time-delay system with disturbance in Equation (41) will be asymptotically stable with a given RPI, \( \gamma \). The proof of Theorem 3 is completed.

As can be seen that, by using the above condition Equation (42), the stability of a closed-loop ISGs Equation (41) with pre-determined controller can be considered. This stability condition Equation (42) can be extended to cater for the design of a robust controller to stabilize ISGs asymptotically with a given RPI, \( \gamma \).

5.1.3 Stability of ISGs subjects to FDIA

In this part, we will consider the stability of ISGs with FDIA.

Let us reform the state space model of IGs from Equations (11) and (41) with the use of a static output feedback controller, 

\[ \dot{x}(t) = \sum_{j=0}^{3} \Theta_j x(t), \quad \dot{x}(t) = \tilde{U} T x(t), \quad \dot{x}(t) = \tilde{R}_1 E^T x(t), \quad \dot{R} = h^2 \tilde{R}, \Theta_0 = \tilde{E}_0 \tilde{P}_0 + \tilde{E}_0 \tilde{P}_0 T, \tilde{R} = \text{diag}(\tilde{R}, \tilde{R}, \tilde{R}), \tilde{A}_{22} = - \gamma_I T, \Theta_1 = \tilde{E}_0 \tilde{P}_1 - \tilde{E}_0 \tilde{P}_0 T, \tilde{A}_2 = - F^T \tilde{R} \tilde{F}, \tilde{A}_{13} = - I, \Theta_1 = \tilde{U} \tilde{V} + \tilde{V}^T \tilde{U} f, \tilde{U} = (\tilde{E}_0 + \tilde{A}_{12} E^T)^T, \tilde{X}(t) = [x^T(t) \tilde{x}^T(t) x^T(t) - b \tilde{X}_2^T(t) \tilde{R}_2^T(t) \tilde{X}_2^T(t), \tilde{V} = - \tilde{Z}_1 + \tilde{Z}_1 + \tilde{A}_3 \tilde{Z}_2, \tilde{X}_1(t) = \frac{1}{2} \int_{t-j}^{t} x(i) d \tau, \tilde{Z}_2(t) = \frac{2}{h} \int_{t-j}^{t} x(u) d \tau.

5.2 Influences of FDIA on LFC performance of ISGs

We take Scenario II of simulation to show the unfavourable impacts of FDIA, \( g_{ij}(t) \), \( g_{ij}(t) \), \( i = 1, 2, 3 \), on the performance of closed-loop ISG. For demonstrative goal, FDIA are considered as time-varying functions of \( g_{ij}(t) = a_{ij} \sin(\sqrt{b_{ij}} t) + \tilde{d}_{ij}(\cos(\sqrt{b_{ij}} t)), \) where \( a_{ij}, b_{ij}, \sqrt{b_{ij}}, \sqrt{b_{ij}} \) are magnitudes and frequencies of sin and cos components. The FDIA and their parameters are shown in Figures 10 and 11 and be tabulated in Table 1.

In this Scenario II, FDIA happen in durations of 100 seconds for the six cases (Scenario II to Scenario H6). The results of analysis are obtained by using MATLAB software version 2019b with the E15 Thinkpad Lenovo computer. The model of three-area ISGs is built by MALAB programming in m-file.
which is convenient to observe the simulated signals for class of linear time-delay systems. The simulation step time is 0.01 s.

Figure 12 illustrates the responses of $f_1(t)$ in abnormal operation of the ISG, where each FDIAs, $g_{1i}(t), i = 1, 2, 3$ (FDIAs on computation of ACEs) are considered. It can be concluded that an individual FDIAs $g_{1i}(t)$ at a local power Area $i^{th}$ can degrade not only the performance of local grid but also the neighbouring power areas. All three FDIAs, $g_{1i}(t), i = 1, 2, 3$ on computation of ACE, $f_i(t)$ (Scenarios II, III and IV) lead to unfavourable oscillations of $f_1(t)$ during these times. Furthermore, after the FDIAs have left, $f_1(t)$, was still oscillated from some seconds. By $g_{1i}(t)$, the frequency deviation, $f_1(t) \in [-0.058, 0.043]$ Hz, the deviation has increased ten to fourteen times compared to Scenario I where $f_1(t) \in [-0.0062, 0.0039]$ Hz. For neighbouring power areas, the FDIAs of $g_{21i}(t)$, $g_{31i}(t)$ lead to the magnitude of fluctuation in frequency rises three time, $f_1(t) \in [-0.011, 0.011]$ Hz, to six times, $f_1(t) \in [-0.015, 0.08]$ Hz, than $f_1(t)$ obtained in Scenario I (normal operation of the ISG).

Regarding to EVs, Figure 13 shows LFC performance of the closed-loop ISG in abnormal operations where each FDIAs, $g_{2i}(t), i = 1, 2, 3$ (the FDIAs on aggregators of EVs) are considered. It can be seen from (Scenarios II, III and IV) that each FDIAs has made adverse impacts and leads to the low-quality in performance of $f_1(t)$. For example, in Scenario II, the injection of $g_{21i}(t)$ FDIAs leads to $f_1(t)$ fluctuates six times more than Scenario I. After the FDIAs finished, the frequency is still oscillated. For FDIAs from neighbouring areas, the occurrence of $g_{22i}(t)$, $g_{23i}(t)$ FDIAs lead to the magnitudes of $f_1(t)$ changes four to seven times than the result obtained in Scenario I. According to these simulations results, it is clear that the adverse impacts of FDIAs happen on both ACEs and aggregators of EVs is important and need to be considered for the efficient LFC operation of ISGs. In the next section of this paper, we will synthesize our proposed DFO based detectors to identify, isolate FDIAs. The co-operation of DFO detectors and isolators not only differentiate the FDIAs for between sub-systems but also differentiate FDIAs of ACE computations and EVs aggregators within a local SG.

5.3 FDIAs detector synthesis

In this subsection, we are in the stage of synthesising the DFO detector gains, $N_1, N_{ih}, j_1, j_1, T_i, E_i$ proposed in Sections 2–4. To do that, three local DOF detectors are designed and located at three local SGs to implement the task of detecting local FDIAs and triggering the alarm (or LED). The simulation data, time delays, RES fluctuations, and FDIAs are from Scenarios I–II.

We take Scenario III with the main focus on the design of local power Area-1 st, $D_i$. The main task of the detector, $D_1$, is to detect the local FDIAs, $g_{11i}(t)$ and $g_{21i}(t)$. The ultimate detection goals of $D_1$, are explained as follows: (i) if the alarm/LED is ON, there is at least one FDIAs, $g_{11i}(t)$ or $g_{21i}(t)$, happened; (ii) if the alarm/LED is OFF, there is no FDIAs of $g_{11i}(t)$ or $g_{21i}(t)$ happened at the power Area-1 st. On the contrary, regarding to the existence of attacks: (iii) if any attacks of $g_{11i}(t)$ and $g_{21i}(t)$ happened, the LED is ON; (iv) if no attack of $g_{11i}(t)$ and $g_{21i}(t)$ happened, the LED is OFF. System matrices $A_{1i}, A_{1ih}, A_{12i}, A_{13i}, B_{i1}, C_{i1}, C_{i2}, C_{i3}, \Gamma_i, F_{11}, F_{12}, j_{i1}, j_{i2}$ are given in the Appendix. Time delay $b_1 = 0.1$ s. There are six information comprising four local measurements and two remote signals required from remote power Areas 2 nd and 3 rd are required for obtaining the gains of $D_1$. By employing the synthesis procedure (Section 4), we obtained $I_1, j_{12i}, j_{13}$ in Step 1 and $N_{11}, N_{12}, N_{ih1}, N_{1h2}, j_{101}, j_{102}, j_{103}, j_{12}$ in Step 3 of the algorithm. All of these steps can be easily implemented by using Matlab.
software with basic algebra. In Step 3, we solve Equation (38) to obtain $Z_1$, here (LMI) tool in Matlab robust control toolbox with an optimisation process containing searching parameter of $\delta = 0.6$ are used. Accordingly, $D_1$-detector’s matrix gains, $N_1$, $N_{1b}, J_1,J_{1b}, J_{12}, J_{13}$ can be obtained as

\[
N_1 = \begin{bmatrix} -1.113275 & 0 & 0 \\ 0 & -1.113275 & 0 \\ 0 & 0 & -1.113275 \end{bmatrix},
\]

\[
N_{1b} = \begin{bmatrix} -0.117131 & 0 & 0 \\ 0 & -0.117131 & 0 \\ 0 & 0 & -0.117131 \end{bmatrix},
\]

\[
J_1 = \begin{bmatrix} 0 & 0.4905 & 0 \\ 0.4250 & 0 & 1 \\ 0 & 0 & 1.113275 \end{bmatrix},
\]

\[
J_{1b} = \begin{bmatrix} 0 & 0.117131 & 0 \\ 0 & 0 & 0.117131 \\ 0 & 0 & 0.117131 \end{bmatrix},
\]

\[
J_{12} = \begin{bmatrix} 0 & 0 \\ -0.2725 & 0 \end{bmatrix},
\]

\[
J_{13} = \begin{bmatrix} 0 \\ 0.117131 \\ 0 \end{bmatrix}.
\]

As can be seen that $N_1$ is three rows with full rank, therefore the order of the design DFO detector is third orders, which is smaller size than the order of FUO based detectors. We will discuss the advantage later. Now, we consider building the residual generator, $r_1(t)$ for detector $D_1$. The form of $r_1(t)$ defined as $r_1(t) = T_1 z(t) + E_1 y_1(t)$ from Equation (14), where $z(t)$ is from the designed DFOs and pair matrices ($T_1, E_1$) can be obtained from Step 4 of the synthesis algorithm in Section 4. Theoretically, $r_1(t) \neq 0$ is enough for the FDIAs detection. In practical implementation a comparable value, $r^*_1(t) = \|r_1(t)\|$ should be developed and compared to a pre-determined values of system threshold $\tilde{r}_1(t)$ to trigger the alarm or turn on the LED. By employing Step 4 of the algorithm, we have found that there exist two values of residual generators, $r_{1u}(t)$ and $r_{1d}(t)$, satisfying our conditions with two solution for pairs of ($T_{1u}, E_{1u}$) and ($T_{1d}, E_{1d}$). Therefore, two residual values can be built as $r_{1u}(t) = T_{1u} z(t) + E_{1u} y_1(t)$ and $r_{1d}(t) = T_{1d} z(t) + E_{1d} y_1(t)$.

However, by taking many tests, we recognised that the residual generators, $r_{1u}(t)$ and $r_{1d}(t)$, may clearly reacts to one FDIAs but not strongly reacts to the other ones. In order to tackle the gap between the theoretical calculations and practical observations, we have taken the following consideration. To ensure the detector, $D_1$, can detect all two FDIAs clearly, we develop the comparable value, $r^*_1(t)$ as follows $r^*_1(t) = \sqrt{(r_{1u}(t))^2 + (r_{1d}(t))^2}$ and undertake the comparison between $r^*_1(t)$ and a threshold $\tilde{r}_1 = 10^{-i}$ to trigger the alarm. The schematic implementation of $r^*_1(t)$ is shown in Figure 14.

Figure 15 presents $r^*_1(t)$ and the alarm status when no attacks occurred to the local power Area $1^{st}$, $g_{11}(t) = 0$ and $g_{12}(t) = 0$, (Scenario III1). As can be observed from Figure 15, $r^*_1(t) = 0$ and the Alarm (LED) is “OFF” with status “00”. On the other hand, Figures 16 and 17 illustrate $r^*_1(t)$ and Alarm status when each attack, $g_{11}(t)$ and $g_{12}(t)$, happened at the local power Area 1st (Scenarios III2 to III3). As can be seen from Figures 16 and 17, when $r^*_1(t)$ is clearly larger than threshold, the proposed detector can recognize $g_{11}(t)$, $g_{12}(t)$ clearly. Regarding to the alarm status, the alarm is triggered to On (set to be 1) in Scenarios III2 to III3. For example, $g_{11}(t) \neq 0$ in scenario III2, the alarm/LED is zero before the attack happened and triggered during 150 to 250 s period. These results also indicate that after the FDIAs has left, the alarm status go back to zeros after some seconds of settling time.

### 5.4 FDIAs isolation scheme

The isolation of each local FDSA positions in the local power areas is also very important. The information of each local FDSIs position is used to improve monitoring activities of local power grids (further action to handle the attacks). By using...
### TABLE 2 Isolating each local FDIAs, $g_1(t)$ and $g_2(t)$

| Local attacks | Detector $D_1$ | Detector $D_{11}$ | Detector $D_{12}$ |
|---------------|---------------|-----------------|-----------------|
| No FDLA       | OFF           | OFF             | OFF             |
| $g_1(t)$      | ON            | OFF             | ON              |
| $g_2(t)$      | ON            | ON              | OFF             |

**FIGURE 18** $D_{11}$ responses to FDIAs: (a) $g_1(t)$, (b) $g_2(t)$

distributed detection methods, each local detector can realise
the existence of local FDLA at local power areas and the FDLA
in other power Areas do not impact to the performance of
the local detector. It implies that the local FDLA is isolated
to others from neighbouring areas. In this part, we further
to isolate each local FDLA at a local power area. This means
the isolators differentiates the existences of $g_1(t)$ and $g_2(t)$. The main concept bases on the design
the detector (residual generator-based observer) that insensitively reacts to one FDLA
and sensitively reacts the remaining of FDIAs. To do that,
we develop two more local detectors: (i) $D_{11}$ is insensitive to
$g_2(t)$ and sensitive to $g_1(t)$; (ii) $D_{12}$ is insensitive to $g_1(t)$ and
sensitive to $g_2(t)$. By using detectors, $D_1$, $D_{11}$ and $D_{12}$, the
isolation of two local FDIAs is obtained (see Table 2).

In order to show the effectiveness of the proposed isolation
schemes, we undertook the Scenario IV. At first, by using
apply Remark 3 and detector design in Section 3, the two second
order DFO based detectors, $D_{11}$, $D_{12}$ are obtained to recognize
each local individual FID, $g_1(t)$ and $g_2(t)$. Figures 18 and 19
show the Alarm status of each detector to the existence of local
FDIAs, $g_1(t)$, $g_2(t)$. As can be seen that, the $D_{11}$ is insensitive
to $g_1(t)$ while it reacts to $g_2(t)$ clearly. It is an important
objective of our paper.

#### 5.5 Advantages of DFO scheme

As we have discussed in the introduction, the adverse impacts of
REs intermittent need to be eliminated in the design of detectors. In Theorem 1 (see Equations (16) and (17)), the constraint
of $\Omega_i = 0$ has been added to ensure that the variation in high
frequency of REs will not impact the performance of detector.

**FIGURE 19** $D_{12}$ responses to FDIAs: (a) $g_1(t)$, (b) $g_2(t)$

Indeed, the above section of detector synthesis has shown that
advantage. By employing the constraint of $\Omega_i, \Omega_6$ to guarantee
that the neighbouring FDIAs does not convert into the resid-
ual of generator, therefore, it is clear that our DFO detector is
insensitive to those remote FDIAs. On the other hand, our pro-
sposed DFOs detector scheme is a distributed architecture and
having advantage of minimum orders which are the main pre-
sentation in this subsection.

#### 5.5.1 Distributed implemetation

The main characteristic of our proposed scheme is a distributed
structure. By this, a detector of low order is located at each area
of a SG to detect the local FDIAs only rather than to detect
all FDIAs within the global power grid based on a centralised
detector leading to the convenience of easy implementation. To
verify that, in this subsection, we consider CFO based detector
to recognise all FDIAs within SGs. According to the Remark 3
in Section 2 and a procedure which is similar to Section 4, a
sixth-order CFO detector can be obtained. The schematic of
implementation of CFOs detector are given in Figure 20.

Clearly, CFO detector requires the access of large number of
instant information of control signals and measurements con-
temporarily. In contrast, the proposed DFO detector needs less
information so that less cost of implementation. Furthermore,
CFO detector requires a large central facility which can be sen-
sitive to malicious incidents. Similar to the processing of build-
ning comparable value for DFO detector, to avoid some sensi-
tivities of practical calculation, the global centralised compara-
ble value of $r^*$ is computed based on six residual generators,

$$ r^* = \sqrt{\sum_{t=1}^{6} (r(t))^2}. $$

**FIGURE 20** Schematic implementation of the CFO based detector

**FIGURE 21** CFO Detector responses to $g_1(t)$ (Scenario V): (a) $r^*$, (b) triggered alarm status

**FIGURE 22** CFO Detector responses to $g_2(t)$ (Scenario V): (a) $r^*$, (b) triggered alarm status

However, it is noted that, it has disadvantage as we have discussed above.
In the following part, we conduct Scenario VI to consider the situation that the centralised facility of CFO detector is impacted by malicious incidents. For example, CFO detector’s gains of $N_i, J_i$ are destroyed and considered as null matrix from the seconds of 100 to 400. It is noted that the whole ISGs now have only one detector. The performance of CFO detector under malicious incidents of N and J are respectively shown in Figures 23 and 24. From these results of Scenario VI, it is clear that when the central facility of the CFOs detector operates under the abnormal of malicious incidents, the performance of detector is degraded or further destroyed leading to the incorrect activities in implementing the task of detecting $g_1(t)$, i.e. the alarm operates incorrectly.

### 5.5.2 Minimum-order detector

Another advantage of our detection scheme is that our detector is reduced order (small size and weight). The order of our DFO detector is determined according to the number of independent rows of $L_y$, further, the rank of matrix $N_i$. Therefore, to minimize the size of detector, number of rows of matrix $L_y$ should be minimized. This purpose can be achieved by considering Step 1 of the procedure. In this step, $L_y$ is selected to satisfy the condition Equation (34) as $\Psi_j \Xi_j = 0$, where $\Psi_j$ contains $L_y$ as $\Psi_j = [\Xi_j, L_y]$, and $\Xi_j$ are given in Step 1. In previous section, by obtaining $\Psi_j$ and further $L_y$ has three rows and third-order DFO detector is designed. By minimizing the number of rows of $\Psi_j$, a second-order DFO detector can be obtained. Therefore, our detection method can achieve a very small size of the detector. This is the important motivation behind our research.

In order to further highlight the advantages of FOSs, we use full-order state observer to design CFUO detector and compare to the CFO detector. The structure of unknown input CFUO has been extensively considered in [30] and the CFOU detector can be designed from chapter 2 in [30] and our procedure in Section 4. Here, the CFOU comparable signal is built as $r^*(t) = [\gamma(t) - C\hat{x}(t)]$ where $\hat{x}(t)$ is the estimation of the state vector $x(t)$. For a three-area ISG, the order of CFUO based detector is always equal to the number of the state variables which is 20. This order is significantly higher than sixth-order of CFO detector (presented in Section 5.5.1) and third-order DFO detector (presented in Section 5.3) and second-order DFO detector (in above discussion). Figures 25 and 26 show the performance of CFOU based detector subjects to FDIAs, $g_1(t), g_1(t)$. As can be seen that, the observer can implement the task of detecting FDIAs, however, as we have presented, the order of this observer is significant high leading to high cost of computation. With regard to eigenvalue, let we consider ISGs without time-delay consideration, the CFOU detector without time-delay consideration can be obtained (we call it as $\hat{D}_1$). Without FDIAs consideration, the observer error is $\hat{\theta}_1(t) = N_1\hat{\theta}_1(t)$. Now we combine state space model of original IGSs with DFOs as follows

$$
\begin{bmatrix}
\dot{x}(t) \\
\dot{\theta}_1(t)
\end{bmatrix} =
\begin{bmatrix}
\bar{A} & 0 \\
0 & N_i
\end{bmatrix}
\begin{bmatrix}
x(t) \\
\theta_1(t)
\end{bmatrix}
+ 
\begin{bmatrix}
0 \\
\Gamma
\end{bmatrix}
d(t)
$$

$$
\bar{A}N_1 x(t) + \Gamma_N d(t),
$$

where $\bar{A} = \bar{A} + \bar{A}_b \bar{\theta}$ and $\bar{A}_b$ are defined in Equation (44).
As can be seen that the eigenvalues $A_N = \begin{bmatrix} \bar{A} & 0 \\ 0 & N_1 \end{bmatrix}$ combines the eigenvalues of $\bar{A}$ and eigenvalues of $N_1$. By using eig function in MATLAB, we obtain the similar results.

### 5.6 | EVs frequency support analysis

In this part, we undertake an analysis on the contribution of EVs to the frequency regulation. In this paper, EVs participate into LFC via the operation of droop loop which mimics the operation of power plant’s primary frequency regulation. The contribution of EVs into the frequency service is presented by EVs droop constant $\rho_{\alpha}$. Throughout of all previous parts, $\rho_{\alpha} = \bar{\rho}_d / R_g$, where $\bar{\rho}_d = 2$, hence EVs contribute $\frac{\bar{\rho}_d + 1/R_g}{2} = 67\%$ into the services. Under the impacts of FDIAs $g_{11}(t)$ and $g_{12}(t)$, the system frequency, $f_1(t)$ belongs to the range of $[-0.058 \text{ Hz}; 0.043 \text{ Hz}]$ and $[-0.061 \text{ Hz}; 0.034 \text{ Hz}]$ respectively. By employing the stability condition presented in subsection 5.1, the maximum value of $\rho_{\alpha}$ while the system still be stable is around $3/R_g$, which presents 75\% of frequency service.

To emphasis on the effect of EVs contribution in abnormal operation of FDIAs, we observe the systems performance under abnormal operations subjects to $g_{11}(t)$ and $g_{12}(t)$ when $\bar{\rho}_d$ changes. By adjusting the values of $\bar{\rho}_d$ from 0.1 to 1, the contribution of EVs is from 9\% to 50\%. Figure 27 shows the responses of $f_1(t)$ in some abnormal operations. As can be seen that, under the impacts of FIDAs, for $\rho_{\alpha}$ smaller than $3/R_g$, better performance of $f_1(t)$ can be achieved with more contributions of EVs.

For the case of $g_{11}(t) \neq 0$, (iii) (Scenario VIII1): $f_1(t) \in [-0.068; 0.049]$ Hz EVs contributes 50\% of the service, iv) (Scenario VIII2): $f_1(t) \in [-0.074; 0.043]$ Hz while EVs contributes 9\%. On the other hand, for the case of $g_{12}(t) \neq 0$, (i) (Scenario VIII3): $f_1(t) \in [-0.074; 0.043]$ Hz while EVs contributes 50\% of the service, (ii) (Scenario VIII4): $f_1(t) \in [-0.088; 0.076]$ Hz while EVs contributes 9\%.

In summary, with extensive analysis and evaluation through Section 5, we have demonstrated the capability of our proposed DFO based detection scheme.

### 6 | CONCLUSION

In this paper, the issue of FDIAs on LFC of ISGs has been considered. A new mathematic representation of ISGs incorporating FDIAs, intermittent of RES, aggregated EVs with communication delays has been proposed. With the goal of detecting and isolating FDIAs within ISG, we have derived a new DFO based detection and isolation schemes based on some developments of FOs, stability of time-delay systems, EVs integrations and residual generator-based fault detection. We have proposed an effective procedure in tractable LMIs and an optimisation process for synthesising the detector’s gains. This procedure can be easily solved by robust control toolbox in MATLAB with together with C Programming. Our DFOs detectors have the advantages of insensitive to neighbouring FDIAs and RES variation, malicious incident of centralised architecture while being able to deal with time delays. Furthermore, the detector has smaller size than the conventional state observer-based detectors. The distributed architecture of detector leads to the reduction in cost of computation and the convenience for implementation monitoring tasks. We have also discussed the stability of time-delay ISGs and the EVs contributions into frequency service under abnormal of FDIAs. Various comprehensive simulations have been carried out with a three-area ISG to validate the effectiveness of the proposed scheme. The studied SGs can be enlarged to include various topologies of ISGs. In the future work, the design method of this paper can be extended to deal with wider classes of systems that include non-linearities. On the other hand, the stabilisation and mitigation problem for ISGs should be further addressed. In this case, the combination of DFOs and decentralised controller needs further development.

**NOMENCLATURE**

- EVs, LFC: Electric vehicles, load frequency control.
- ISG, RES: Interconnected smart grid, renewable energies.
- FDIA, FUO: False data injection attack, full-order observer.
- DFO, CFO: Distributed and centralised functional observer.
- LMI: Linear matrix inequality.
- LKF: Lyapunov Krasovskii functionals.
- $i, j$: Power Area-$i^th$, time delay.
- $\delta, \delta$: Optimisation searching variables in LMI.
- $n_i$: The number of state variables of Area-$i^th$.
- $M_i, D_i$: Inertia constant, load damping coefficient.
- $f_i, b_i$: Frequency deviation, frequency bias constant.
- $R_g, \rho_{\alpha}$: Governor and EVs droop characteristic.
- $K_g, K_e$: Governor and turbine gain constants.
- $T_e$: EVs gain and time constants.
- $T_r$: Governor and turbine time constants.
- $K_{re}, T_{re}$: Reheater gain and time constants.
- $ACE_i, b_i$: Area control error, frequency bias constant.
- $P_{li}, P_{wi}$: Load demand and wind power deviations.
- $X_{gi}, P_{gi}$: Governor position and EVs power deviations.
- $P_{ri}, P_{ri}$: Power and reheater output deviations.
$P_{ci}$ Incremental change in power command.
$g_{i1}, g_{i2}$ FDIAs at power plants and aggregators.
$g_{emi}$ Fidia at aggregator related to the mth EV.
$\eta_1$ Integral value of area error control.
$r_i(t), r_{ij}(t)$ Residual generator and detector comparable value.
$N_{b}, N_{ij}, J_{ij}, H_{ij}, J_{Ai}, L_{Ai}, T_{j}, E_{i}$ are detector’s matrices.

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**APPENDIX A**

Simulation data of the ISG in Figure 1 is given [1, 3, 17] as follows:

\[
K_0 = 1, \ T_0 = 0.3, \ K_{\theta} = 1, \ T_{\theta} = 0.08, \ R_{\theta} = 2.4, \\
K_{\phi} = 5, \ T_{\phi} = 10, \ M_i = 0.1667, b_i = 0.425, D_i = 0.0083, \\
T_{\psi} = 1, \ K_{\psi} = 1, \ R_{\psi} = 2/T_{\phi}, 2\pi T_{12} = 0.2725, \\
2\pi T_{13} = 0.2180, 2\pi T_{23} = 0.1635, \ T_{ji} = T_{ji}, i = 1, 2, 3.
\]
Matrices $A_{ij}, A_{ih} \in \mathbb{R}^{n_i \times n_i}, A_{ij} \in \mathbb{R}^{n_i \times n_j}, C_i \in \mathbb{R}^{p_i \times n_i}, B_i, \Gamma_i,$ $S_1, S_2 \in \mathbb{R}^{n \times 1}, F_{i1}, F_{i2} \in \mathbb{R}^{p \times 1},$ are 

\[
\begin{align*}
A_{ij} &= \begin{bmatrix}
-\frac{2\pi}{T_{ij}} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}, \\
A_{ih} &= \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\end{align*}
\]

\[
B_i = \begin{bmatrix}
0 & \frac{K_i}{T_i} & 0 \\
0 & 0 & 0
\end{bmatrix}, \quad \Gamma_i = \begin{bmatrix}
-\frac{1}{M_i} & 0 \\
0 & 0
\end{bmatrix}, \quad S_2 = \begin{bmatrix}
0 & 0 & 1 \\
0 & 0
\end{bmatrix}, \quad S_1 = \begin{bmatrix}
0 & 0 & 1 \\
0 & 0
\end{bmatrix}.
\]

For the design of $D_1, C_{ij} = \begin{bmatrix} 1 & 0 \end{bmatrix}, F_{i1} = \begin{bmatrix} 0 \end{bmatrix}.$

$E_{1a} = \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}, E_{1b} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}, T_{1a} = \begin{bmatrix} -1 & 0 & 0 \end{bmatrix},$ 

$T_{1b} = \begin{bmatrix} 0 & 0 & -1 \end{bmatrix}.$