Thermal Bremsstrahlung Radiation in a Two-Temperature Plasma

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Received 2003 month day; accepted 2003 month day

Abstract In the normal one-temperature plasma the motion of ions is usually neglected when calculating the Bremsstrahlung radiation of the plasma. Here we calculate the Bremsstrahlung radiation of a two-temperature plasma by taking into account of the motion of ions. Our results show that the total radiation power is always lower if the motion of ions is considered. We also apply the two-temperature Bremsstrahlung radiation mechanism for an analytical Advection-Dominated Accretion Flow (ADAF) model; we find the two-temperature correction to the total Bremsstrahlung radiation for ADAF is negligible.

Key words: plasmas — radiation mechanisms: thermal

1 INTRODUCTION

In a plasma, electrons are constantly accelerated during their collisions with ions, leading to Bremsstrahlung radiation. Usually, when calculating the radiation power of a plasma, the motion of ions is neglected, because the ion’s mass is much higher than the electron mass. However the motion of ions may not be neglected if the ion temperature is much higher than that of electrons, such as in the ADAF model, in which the temperature of ions $\sim 10^{11}$ k may be much higher than that of electrons ($10^8 \sim 10^9$ k) (Narayan & Yi, 1995). Under such condition, the velocities of ions are comparable to or even higher than that of electrons. Therefore the motion of ions must be taken into account for calculating the thermal Bremsstrahlung radiation of a two-temperature plasma.

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Our calculations show that the radiation power is reduced significantly if the ion temperature is much higher than the electron temperature. However when applying the two-temperature Bremsstrahlung radiation to the ADAF model, we find the total Bremsstrahlung radiation emissivity is not significantly different from the one-temperature Bremsstrahlung radiation, because most of the Bremsstrahlung radiation is produced at large radii where the ion’s temperature is not significantly different from the electron temperature.

2 THERMAL BREMSSTRAHLUNG RADIATION

We begin by considering an individual scattering event between an electron and an ion in which an electron with a velocity $v$ and an impact parameter $b$ is scattered by an ion; we assume the electron’s motion is non-relativistic. Then the radiation power emitted at a specific frequency by this electron is given by (Padmanabhan, 2000, p. 295)

$$W_1(\omega) = \frac{8Z^2e^6}{3\pi\epsilon^3m_e^2v^2b^2}.$$  \hspace{1cm} (1)

Integrating over $b$ with the limit $b_{\text{min}} = Ze^2/m_e v^2$ and $b_{\text{max}} = v/\omega$ (YOU, 1998, p. 282), we get

$$P_1(\nu) = 2\pi P_1(\omega) = 4\pi^2 N_e v \int_{b_{\text{min}}}^{b_{\text{max}}} W_1(\omega)b\,db = \frac{32\pi N_Z Z^2e^6}{3\epsilon^3m_e^2v} \ln \frac{m_e v^3}{2\pi Z e^3 \nu},$$  \hspace{1cm} (2)

where $N_Z$ is the number density of ions. Integrating it over all electrons and assuming the electrons follow the non-relativistic Maxwellian velocity distribution, we get the specific emissivity,

$$j_1(\nu) = \frac{128\pi^2 Z^2 e^6}{3\epsilon^3m_e^2} N_Z N_e \left(\frac{m_e}{2\pi kT}\right)^{3/2} \int_0^\infty \exp\left(-\frac{m_e v^2}{2kT}\right)v \ln \frac{m_e v^3}{2\pi Z e^3 \nu} v\,dv,$$ \hspace{1cm} (3)

where $N_e$ is the number density of electrons.

3 TWO-TEMPERATURE BREMSSTRAHLUNG RADIATION

Assuming that the temperature of ions and electrons are $T_Z$ and $T_e$ respectively, we still begin by considering an individual scattering event. The radiation power of an electron in the rest-frame of an ion is similar to Eq. (1),

$$W_2(\omega) = \frac{8Z^2e^6}{3\pi\epsilon^3m_e^2v^2b^2}.$$ \hspace{1cm} (4)

except that $v$ is the relative velocity between the ion and the electron

$$v = (v_e^2 + v_Z^2 - 2v_e v_Z \cos \theta)^{1/2}.$$ \hspace{1cm} (5)

Integrating over $b$, we get

$$P_2(\nu) = 2\pi P_2(\omega) = 4\pi^2 \int_0^\infty d\omega Z \int_{b_{\text{min}}}^{b_{\text{max}}} W_2(\omega)N(\omega)v\,b\,db.$$ \hspace{1cm} (5)
Assuming that the ions and electrons all follow the non-relativistic Maxwellian velocity distribution, and integrating $P_2(\omega)$ over all the electrons, we get the specific emissivity,

$$j_2(\nu) = \frac{256\pi^3 Z^2 e^6}{3c^3 m_e^2} N_Z N_e \left( \frac{m_e}{2\pi k T_e} \right)^{3/2} \left( \frac{m_Z}{2\pi k T_Z} \right)^{3/2} \int_0^\infty \exp\left(-\frac{m_e v_e^2}{2k T_e}\right) v_e^2 dv_e \int_0^\infty \exp\left(-\frac{m_Z v_Z^2}{2k T_Z}\right) v_Z^2 dv_Z \int_0^{\pi} \ln \frac{m_e v}{2\pi Z^2 e^2} \sin\theta d\theta. \quad (6)$$

Because we are only considering the non-relativistic regime, the radiation power observed in the rest-frame of an ion is the same as that in the laboratory frame.

4 RESULTS

We make numerical calculations of the two-temperature plasma radiation; all ions are assumed to be protons. The results are shown in Figs. 1–3. We can see that in all cases the two-temperature Bremsstrahlung radiation emissivity is lower than the one-temperature case, and the difference is greater for higher electron and/or ion temperatures.

We then calculate the total luminosity following an analytical ADAF model (Mahadevan, 1997), assuming spherical accretion and with all the self-similar equations as showed in Mahadevan (1997). The electron temperature $T_e$ is around $10^9$ k, and is
assumed to be constant for \( r < 10^3 \), where \( r \) is the dimensionless radius of the accretion disk, defined in \( R = rR_{Schw} = r\frac{2GM}{c^2} \). The ion temperature given by Mahadevan (1997) is approximated to

\[
T_i = 9.99 \times 10^{11} r^{-1} \text{k} \tag{7}
\]

The temperature profile is shown in Fig. 4. We simply assume an optically thin accretion disk model and integrate the specific emissivity over all radii to get the total luminosity. The ratio between the luminosity of two-temperature and one-temperature Bremsstrahlung radiation \( L_2/L_1 \) is about 0.964 for \( T_e = 10^9 \text{k} \) and 0.950 for \( T_e = 10^8 \text{k} \), respectively. The small difference between the two cases is due to the small difference between the ion temperature and the electron temperature in most of the accretion flow volume; only at small radii could \( T_Z/T_e \) exceed 1000 where the two types of Bremsstrahlung radiation become significantly different. We therefore conclude that in the ADAF model the correction due to the two-temperature Bremsstrahlung is negligible.

![Fig. 3](image1.png) The ratio between the two types of radiation emissivity \((j_2/j_1)\) as a function of the ratio between the ion temperature to the electron temperature \((T_Z/T_e)\). If \( T_Z/T_e \) is less than 100, \( j_2/j_1 \) is very close to unity. When \( T_Z/T_e \) is more than 1000, the difference between \( j_2 \) and \( j_1 \) becomes significant.

![Fig. 4](image2.png) The ion temperature profile. The horizontal axis denotes the dimensionless radius \( r (R = rR_{Schw} = r\frac{2GM}{c^2}) \) of the accretion disk, ranging from 3 to 1000.

5 DISCUSSION

In a plasma with a high electron temperature, the bremsstrahlung from electron-positron \((e^+e^-)\), electron-electron \((ee)\), positron-positron \((e^+e^+)\) collisions may become important (Svensson, 1982). In our non-relativistic case, \( e^+e^- \) pair creation and annihilation can be neglected, then we only need to consider the \( ee \) bremsstrahlung, in addition to
the $e$-proton bremsstrahlung we have calculated above. For $T_e < 10^9$ k, we calculate the cooling rates of electron-ion bremsstrahlung ($q_{ei}$) and $ee$ bremsstrahlung ($q_{ee}$) according to Svensson (1982) and Narayan & Yi (1995), and get that $q_{ee}/q_{ei} < 0.3$. Therefore the electron-ion bremsstrahlung dominates the radiation power and the $ee$ bremsstrahlung can also be neglected in non-relativistic cases.

In summary, our results show that the two-temperature Bremsstrahlung radiation power is significantly lower than the one-temperature Bremsstrahlung radiation if the ion temperature is more than $1000T_e$. Although the temperature difference in the ADAF model could exceed this critical value, the luminosity correction due to this effect is still negligible due to the rapid decrease of the ion temperature at large radii. However if in some more extreme astrophysical environment the ion temperature is significantly higher than the electron temperature, the two-temperature Bremsstrahlung radiation calculated in this work should be taken into account.

**Acknowledgements** We thank the anonymous referees for valuable suggestions and comments which have improved the manuscript significantly. This study is supported in part by the Special Funds for Major State Basic Research Projects (10233010) and by the National Natural Science Foundation of China.

**References**

Junhan YOU, 1998, Radiation Mechanisms in Astrophysics, 2nd ed., Beijing: Science Press
Mahadevan R., 1997, ApJ, 447, 585
Narayan R., Yi, I., 1995, ApJ, 452, 710
Svensson R., 1982, ApJ, 258, 335
Padmanabhan T., 2000, Theoretical Astrophysics, Cambridge: Cambridge University Press