An Analysis of Cosmic Neutrinos: 
Flavor Composition at Source and Neutrino Mixing Parameters

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Abstract

We examine the feasibility of deriving neutrino mixing parameters $\delta$ and $\theta_{13}$ from the cosmic neutrino flavor composition under the assumption that the flavor ratios of the cosmic neutrinos at the source were $F_{\nu_e} + F_{\bar{\nu}_e} : F_{\nu_\mu} + F_{\bar{\nu}_\mu} : F_{\nu_\tau} + F_{\bar{\nu}_\tau} = 1 : 2 : 0$. We analyze various uncertainties that enter the derivation of $\delta$ and $\theta_{13}$ from the ratio of the shower-like to $\mu$-tracks events which is the only realistic source of information on the flavor composition at neutrino telescopes such as ICECUBE. We then examine to what extent the deviation of the initial flavor ratio from $1 : 2 : 0$ can be tested by measurement of this ratio at neutrino telescopes taking into account various sources of uncertainty.

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1 Introduction

According to the current models, the astrophysical objects such as sources of Gamma Ray Bursts (GRBs) \[1\], type I b/c supernovae \[2\] and Active Galactic Nuclei (AGNs) \[3\] can emit beams of neutrinos luminous enough to be detectable at the neutrino telescopes that are under construction. The AMANDA experiment \[4\] at the south pole, which came to the end of its mission in 2006, has set the following bound on the diffuse flux of neutrinos

\[ E_\nu^2 \frac{dF_\nu}{dE_\nu} \leq 8.2 \text{ GeV cm}^{-2} \text{ sr}^{-1} \text{ yr}^{-1}. \]  

A km\(^3\)-scale neutrino telescope named ICECUBE is under construction which encompasses AMANDA. If the bound in (1) is saturated, the completed ICECUBE can collect \(\sim 4000\) cosmic neutrino signal each year \[7\]. Notice that this bound is on the sum of the neutrino fluxes from sources at cosmological distances. In principle, core collapse supernova explosions leading to an intense neutrino flux detectable at km\(^3\)-scale neutrino telescopes can also take place in the close-by galaxies located at a distance of \(\lesssim 10\) Mpc \[6, 7\]. If such an explosion is registered during the time that the ICECUBE is in full swing, ICECUBE can record about a few hundred neutrino events from a single explosion \[7\]. In addition to ICECUBE in the south pole, three neutrino telescopes NEMO \[8\], ANTARES \[9\] and NESTOR \[10\] are being constructed in the Mediterranean sea. Moreover, the so-called KM3NET neutrino telescope \[11\] is planned to be constructed in the Mediterranean sea.

In view of this prospect, extensive studies have been performed on the possibility of deriving information on the mixing parameters of neutrinos by studying the flavor ratio of the neutrinos at the detector \[12, 13, 14\]. The method is based on the following argument. Suppose the flavor ratio at the source was \(F_{\nu_e} + F_{\bar{\nu}_e} : F_{\nu_\mu} + F_{\bar{\nu}_\mu} : F_{\nu_\tau} + F_{\bar{\nu}_\tau} = w_0^e : w_0^\mu : w_0^\tau\). After propagating the distance between the source and the detector, the flavor ratio will become

\[ F_{\nu_e} + F_{\bar{\nu}_e} : F_{\nu_\mu} + F_{\bar{\nu}_\mu} : F_{\nu_\tau} + F_{\bar{\nu}_\tau} = \sum_\alpha w_\alpha^0 P_{\alpha e} : \sum_\alpha w_\alpha^0 P_{\alpha \mu} : \sum_\alpha w_\alpha^0 P_{\alpha \tau}, \]  

where \(P_{\alpha \beta}\) is the probability of \(\nu_\alpha \rightarrow \nu_\beta\). Considering the very long distance between the source and the Earth (i.e., \(\Delta m_{ij}^2 L/(2E_\nu) \gg 1\)), the oscillatory terms in \(P_{\alpha \beta}\) average out \[3\].

\[ P_{\alpha \beta} \equiv P(\nu_\alpha \rightarrow \nu_\beta) = P(\bar{\nu}_\alpha \rightarrow \bar{\nu}_\beta) = \sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2, \]  

\[ ^3\text{For a detailed discussion of the loss of coherence, see} \[19\]. \]
where $U_{\alpha i}$ are the elements of the neutrino mixing matrix. Notice that $P_{\alpha\beta}$ is independent of the neutrino energy $E_\nu$, distance $L$ and the mass square differences $\Delta m_{ij}^2$.

In a wide range of models that lead to detectable cosmic neutrino flux, the neutrino production takes place through $\pi^\pm \to \mu^\pm (e^-)_\mu$ and subsequently $\mu \to e\nu_\mu\nu_\mu$. The flavor ratios at the source are therefore predicted to be $w^0_e : w^0_\mu : w^0_\tau = 1 : 2 : 0$. Thus, by measuring the flavor ratio at Earth, one can in principle derive the absolute values of the mixing matrix elements which yield information on the yet-unknown neutrino parameters $\theta_{13}$ and $\delta$ as well as the deviation of $\theta_{23}$ from $\pi/4$ [12, 13, 14, 15]. It is also suggested to employ cosmic neutrinos to discriminate between the standard oscillation scenario and more exotic possibilities [17, 18].

The flavor identification power of ICECUBE and other neutrino telescopes is limited. In fact, in the energy range of interest ($100 \text{ GeV} < E_\nu < 100 \text{ TeV}$), ICECUBE can only distinguish between shower-like events and the $\mu$-track events. Each of these two types of events can receive contributions from different flavors. As discussed in the next section, several input and assumptions go into derivation of the flavor composition from the ratio of the shower-like events to the $\mu$-track events. Lack of knowledge or uncertainty on these input parameters will lead to uncertainty in derivation of $\theta_{13}$ and $\delta$ from the cosmic neutrinos. To derive information on the unknown mixing parameters ($\theta_{13}$ and/or $\delta$) from the ratio, one should also consider the effect of the uncertainty on the known neutrino mixing parameters, $\theta_{23}$ and $\theta_{12}$. A complete treatment of all these effects is missing in the literature. In the first part of this paper, we study the possibility of deriving $\theta_{13}$ and $\delta$ from the ratio of shower-like events to $\mu$-track events. We take into account the uncertainty of the aforementioned inputs as well as the possible uncertainty in the measurement of the ratio itself. In our analysis, we include contributions to the shower-like and $\mu$-track events whose effects can be larger than the effect of $\delta$ and $\theta_{13}$ but their effects have been overlooked in the literature. We discuss how much the precision of various input parameters has to be improved in order to make the measurement of $\theta_{13}$ or $\delta$ by neutrino telescopes a reality.

Various effects can cause a substantial deviation of the flavor ratio from $w^0_e : w^0_\mu : w^0_\tau = 1 : 2 : 0$. In the second part of the paper, considering the realistic uncertainties in the input, we discuss to what extent the ratio at the source can be determined under the assumption that propagation of neutrinos from the source to detector is simply governed by the standard oscillation formula in Eqs. (23).

The paper is organized as follows. In Sect. 2 the mechanism for flavor identification
at neutrino telescopes is described. In Sect. 3 the features of the cosmic neutrino fluxes predicted by the mainstream models are discussed. In Sect. 4 assuming CP conservation, the effects of possible sources of uncertainties on the derivation of $s_{13}$ are studied. An analysis of derivation of $\delta$ in the presence of uncertainties is performed in Sect. 5. Sect. 6 is devoted to discussing various possible mechanisms through which the initial ratios can deviate from $1:2:0$. We discuss whether by measuring the flavor ratio on Earth, it will be possible to differentiate between models. A summary of the conclusions is provided in Sect. 7.

2 Flavor identification

ICECUBE and its Mediterranean counterparts can basically distinguish only two types of events: 1) shower-like events; 2) $\mu$-track events. As shown in the seminal work by Beacom et al. [20], one can derive information on the flavor composition by studying the ratio of the $\mu$-track events to shower-like events. Let us define

$$R = \frac{\text{Number of Muon-track events}}{\text{Number of Shower-like events}}.$$  

There is a threshold energy, $E_{\text{th}}$ below which the neutrino cannot be detected by a neutrino telescope. The value of $E_{\text{th}}$ depends on the structure of the detector and the type of the event (shower-like versus $\mu$-track). Since the detection of neutrinos coming from above suffers from the large background from cosmic rays, the neutrino telescopes mainly focus on upward going neutrinos; i.e., neutrinos that pass through the Earth before reaching the detector (see however, [21]). The Earth is opaque for neutrinos with energies higher than $\sim 100$ TeV [22]. In sum, the neutrino telescopes mainly study the muon neutrinos in the range $E_{\text{th}} \sim 100$ GeV and $E_{\nu}^{\text{cut}} \sim 100$ TeV. In calculating the fluxes we set the upper limit of the integration equal to $E_{\nu}^{\text{cut}} = 100$ TeV. By using this value for the upper limit, we can neglect the attenuation of the neutrino flux crossing the Earth which depends on the direction of the neutrino [23].

Two sources contribute to the $\mu$-track events: (i) Charged Current (CC) interaction of $\nu_\mu$ or $\bar{\nu}_\mu$ producing $\mu$ or $\bar{\mu}$; (ii) CC interaction of $\nu_\tau$ and $\bar{\nu}_\tau$ producing $\tau$ or $\bar{\tau}$ and the subsequent decay of $\tau$ and $\bar{\tau}$ into $\mu$ and $\bar{\mu}$. In the literature, the contribution of $\nu_\tau$ (via $\nu_\tau \rightarrow \tau \rightarrow \mu$) to $\mu$-track events has been overlooked but to study the effect of $\theta_{13}$, one should take into account such sub-dominant effects.
The contribution of $\nu_\mu$ and $\bar{\nu}_\mu$ to the $\mu$-track events can be estimated as

$$\rho AN_A \int \int R_{\mu}(E_\mu, E_{\mu,\text{th}}) \frac{dF_{\nu_\mu}}{dE_{\nu_\mu}} \frac{d\sigma^{CC}}{dE_\mu} dE_\mu dE_{\nu_\mu} + [\text{particle } \rightarrow \text{ antiparticle}],$$  \hspace{1cm} (5)

where $\rho$, $A$ and $N_A$ are respectively the density of the medium (ice/water), the effective area of the detector and the Avogadro number. $R_{\mu}(E_1, E_2)$ is the muon range which is the distance traveled in the medium by a muon with energy $E_1$ before its energy drops below $E_2$. The muon range in ice is given by [24]

$$R_{\mu}(E_1, E_2) = (2.6 \text{ Km}) \ln \left[ \frac{2 + 4.2 \times 10^{-3} E_1}{2 + 4.2 \times 10^{-3} E_2} \right],$$  \hspace{1cm} (6)

where both $E_1$ and $E_2$ are in GeV. Finally, $dF_{\nu_\mu}/dE_{\nu_\mu}$ and $\sigma^{CC}$ are respectively the neutrino flux spectral function at the detector and the cross section of the CC interactions of $\nu_\mu$.

The contributions from $\nu_\tau$ and $\bar{\nu}_\tau$ to $\mu$-track events can be estimated as

$$B\rho AN_A \int_{E_{\text{cut}}}^{E_{\mu,\text{th}}} \int \frac{dF_{\nu_\tau}}{dE_{\nu_\tau}} \frac{d\sigma^{CC}}{dE_{\tau}} f(E_\tau, E_{\mu}) R_{\mu}(E_\mu, E_{\mu,\text{th}}) dE_\mu dE_{\tau} dE_{\nu_\tau} + (\nu_\tau \rightarrow \bar{\nu}_\tau),$$  \hspace{1cm} (7)

where $B \equiv \text{Br}(\tau \rightarrow \mu \nu_\mu \bar{\nu}_\tau) = 17.8\%$. The function $f(E_\tau, E_{\mu})$ in the above equation is the probability density of the production of a muon with energy $E_\mu$ in the decay of a $\tau$ lepton with energy $E_\tau$. That is

$$f(E_\tau, E_{\mu}) \equiv \frac{1}{\Gamma} \frac{d\Gamma(\tau(E_\tau) \rightarrow \mu(E_{\mu})\bar{\nu}_\mu \nu_\tau)}{dE_{\mu}}.$$

(8)

The details of the calculation of $f(E_\tau, E_{\mu})$ can be found in Appendix A.

Three types of events appear as shower: i) the Neutral Current (NC) interactions of all kinds of neutrinos; ii) the CC interactions of $\nu_e$ and $\bar{\nu}_e$; iii) the CC interactions of $\nu_\tau$ ($\bar{\nu}_\tau$) and the subsequent hadronic decay of $\tau$ ($\bar{\tau}$). Showers from NC interaction of all three neutrino flavors can be estimated as

$$\sum_{l=e,\mu,\tau} \rho ALN_A \left[ \int_{E_{\text{cut}}}^{E_{\mu,\text{th}}} \frac{dF_{\nu_l}}{dE_{\nu_l}} \sigma^{NC} dE_{\nu_l} + \int_{E_{\text{cut}}}^{E_{\bar{\nu}_l}} \frac{dF_{\bar{\nu}_l}}{dE_{\bar{\nu}_l}} \sigma^{NC} dE_{\bar{\nu}_l} \right],$$  \hspace{1cm} (9)

where $L$ is the length of the detector. The rate of the electromagnetic showers from the CC interactions of $\nu_e$ and $\bar{\nu}_e$ is

$$\rho ALN_A \left[ \int_{E_{\text{cut}}}^{E_{\text{cut}}} \frac{dF_{\nu_e}}{dE_{\nu_e}} \sigma^{CC} dE_{\nu_e} + \int_{E_{\text{cut}}}^{E_{\text{cut}}} \frac{dF_{\bar{\nu}_e}}{dE_{\bar{\nu}_e}} \sigma^{CC} dE_{\bar{\nu}_e} \right].$$  \hspace{1cm} (10)
The rate of showers originated from the CC interaction of $\nu_\tau$ with the subsequent hadronic decay of the $\tau$ lepton is

$$(1 - B) \rho ALN_A \left[ \int^{E_{\text{cut}}} \frac{dF_{\nu_\tau}}{dE_{\nu_\tau}} \sigma_{\text{CC}}^{\nu_\tau} dE_{\nu_\tau} + \int^{E_{\text{cut}}} \frac{dF_{\bar{\nu}_\tau}}{dE_{\bar{\nu}_\tau}} \sigma_{\text{CC}}^{\bar{\nu}_\tau} dE_{\bar{\nu}_\tau} \right]. \quad (11)$$

Notice that while the shower-like events are given by the length of the detector, the $\mu$-track events are given by the muon range [see Eqs. (5,7)]. This is because muons can emit Cherenkov light and trigger the detector even if they are produced outside the detector but within the range $R_\mu$. In other words, for the muon detection, the effective volume is larger than the geometrical volume.

To write down the above formulas, several simplifications have been made:

- Obviously, neutrinos entering the detector through different zenith angles have propagated different lengths inside the earth so the amount of attenuation is different for them. In other words, to be precise, the zenith angle dependence of $E_{\text{cut}}$ has to be taken into account.

- The energy threshold for detecting the neutrino also depends on the direction. For the vertically propagating muon at ICECUBE, $E_{\text{th}}^\mu$ is about 20 GeV; that is while, for the muons propagating horizontally, $E_{\text{th}}^\mu$ is about 100 GeV.[25].

- For the high energy muons with $E_\mu > 1$ TeV, the muon ranges both in the ice and rock exceed 1 km, (see Eq. (6) and Ref. [24]). The depth of the ice at the site of ICECUBE is about 2810 m; so a considerable number of muons reaching the ICECUBE would be produced in the rock beneath the ice where the density is quite different.

- As mentioned before, for neutrinos with energies higher than 100 TeV the Earth is opaque. For very high energies, $\nu_\tau$ can however be regenerated through $\nu_\tau \rightarrow \tau \rightarrow \nu_\tau \rightarrow \cdots \rightarrow \nu_\tau$. As a result, $\nu_\tau$ and $\bar{\nu}_\tau$ with $E_\nu \gg 100$ TeV can give a contribution to the upward-going neutrino flux with $E < 100$ TeV. The neutrino flux at high energies is expected to be suppressed. Thus, such a contribution is expected to be negligible[26]. This assumption can in principle be tested by measuring the downward-going shower-like events (which have not traversed the Earth).

Throughout the present analysis, we use the approximate formulae [5,7,9,10,11]. We determine how much the uncertainties in various inputs have to be improved in order not to be an
obstacle for determination of $\delta$ and $s_{13}$. As we shall see, our conclusion is that even without the above subtleties, the required precision in certain input parameters is so fine that seems beyond reach in the foreseeable future. Taking into account the above uncertainties not only will not change our conclusion but will further confirm it.

3 The standard picture

To evaluate $R$, several input parameters have to be known: i) the energy spectrum of the incoming neutrinos; ii) the ratio of the neutrino flux to the anti-neutrino flux; iii) the initial ratio $w^0_e : w^0_\mu : w^0_\tau$. We rely on the predictions of the models for such input parameters. Although the models differ in details, they share some common features. From now on, we call these features the “standard picture.” The features of the standard picture are enumerated below.

- In the standard picture neutrinos are produced in the following chain of processes. First, the energetic protons in jets collide on $\gamma$ or on the background protons and produce $\pi^+$ and $\pi^-$. Then,

\[
\begin{align*}
\pi^+ &\rightarrow \mu^+\nu_\mu, & \pi^- &\rightarrow \mu^-\bar{\nu}_\mu, \\
\mu^+ &\rightarrow e^+\bar{\nu}_\mu\nu_e, & \mu^- &\rightarrow e^-\nu_\mu\bar{\nu}_e.
\end{align*}
\]  

(12)

Thus, $w^0_e : w^0_\mu : w^0_\tau = 1 : 2 : 0$.

- The energy spectra of the neutrinos follow power law distributions:

\[
\frac{dF_{\nu_\beta}}{dE_{\nu_\beta}} = N_\beta E_{\nu_\beta}^{-\alpha}
\]  

(13)

where $N_\beta$ is the normalization factor for each neutrino and anti-neutrino flavor. $\alpha$ is the spectral index. In the standard picture where the initial protons are accelerated to high energies via Fermi acceleration mechanism, the spectral index is expected to be equal to 2 [27].

- Regardless of the relative amount of $\pi^+$ and $\pi^-$, we expect $N_{\nu_\mu} = N_{\bar{\nu}_\mu}$. However, $N_{\bar{\nu}_e}/N_{\nu_e}$ depends on the initial composition of $\pi^+$ to $\pi^-$ which in turn depends on the details of the model.
4 Uncertainties and their impact on $\theta_{13}$ measurement

Our knowledge of the sources of the cosmic neutrinos is quite limited and mostly speculative. A myriad of known and un-known effects can cause deviation of the initial flux from the standard picture that was described in the previous section. In this section, we compare the effect of a deviation from the standard picture on $R$ with the effect of a nonzero $s_{13}$. Here, we assume that the neutrino mass matrix conserves CP. A discussion of CP-violation is given in sect. 5.

Let us fix our convention for the mixing angles. Here, we use the standard parametrization of PDG [28] for the neutrino mixing matrix with [29]

$$0 \leq \theta_{13} < 0.2 < \frac{\pi}{2} \text{ and } 0 \leq \delta < 2\pi.$$  

The sensitivity of $P_{\alpha\beta}$ on the phase $\delta$ is through $\sin \theta_{13} \cos \delta$; so the CP-conserving cases with $\delta = 0$ and $\delta = \pi$ will have distinct effects. We will consider both cases. Throughout this section we take $w^0_e : w^0_\mu : w^0_\tau = 1 : 2 : 0$.

In sect. 4.1, we discuss the effect of the variation of the spectrum on ratio $R$. In sect. 4.2, we discuss the dependence of $R$ on $N_{\bar{\nu}_e}/N_{\nu_e}$. Sect. 4.3 gives a brief discussion of the neutrino nucleon uncertainty and its effects.

4.1 Energy spectrum of incoming flux

As mentioned before, in the standard picture, the neutrino flux follows a power-law spectrum of form Eq. (13) with $\alpha = 2$. However, more careful considerations of the details of the Fermi acceleration and the properties of the target particles show that $\alpha$ can deviate from 2 and take any value in the range (1,3) [16, 30, 31].

The energies of the muons and showers entering a neutrino telescope can be measured. However, extracting the energy of the incoming neutrinos that induce such events is not straightforward. In the case of $\mu$-track events, the muon can lose a substantial part of its energy before entering the detector. On the other hand, limiting the analysis to the muons produced inside the detector will reduce the statistics. In the case of the shower-like events originating from the NC interaction of neutrinos, the energy of the shower does not give the energy of the initial neutrino because a part of the energy is carried away by the final neutrino which escapes detection. Nevertheless, it is shown in [20] that for $E_\nu^2 dF_\nu/dE_\nu = 0.25$ GeV cm$^{-2}$ sr$^{-1}$ yr$^{-1}$ after one year of data-taking, $\alpha$ can be determined with 10 % uncertainty.
Figure 1: The dependence of $R$ on $\sin^2 \theta_{13}$ for different values of the spectral index, $\alpha$. The thicker lines correspond to $\delta = \pi$ and the thinner ones correspond to $\delta = 0$. We have used the central values for the neutrino-nucleon cross section [22] and have set $N_{\bar{\nu}_e}/N_{\nu_e} = 0.5$ and $(N_{\bar{\nu}_\mu} + N_{\nu_\mu})/(N_{\bar{\nu}_e} + N_{\nu_e}) = 2$. The input for $\theta_{12}$ and $\theta_{23}$ are set equal to the best fit in [29]. The vertical line at 0.041 shows the present bound at 3$\sigma$ [29].

Fig. 1 shows $R$ versus $\sin^2 \theta_{13}$ for $\cos \delta = \pm 1$ and various values of $\alpha$. As seen from the figure when $\delta = 0$, the sensitivity of $R$ to $s_{13}^2$ is very mild and less than 2%. That is while for $\cos \delta = -1$, the sensitivity to $s_{13}^2$ is about 10%. The disparity between $\cos \delta = +1$ and $\cos \delta = -1$ means that for $s_{13}^2 \sim 0.04$, the contributions from $s_{13} \cos \delta$ and $s_{13}^2$ are comparable. In fact, expanding $R$ in powers of $s_{13}$ confirms this claim:

$$R \simeq r_1 + r_2 s_{13} \cos \delta + r_3 s_{13}^2 \cos^2 \delta,$$

where for the central curve with $\alpha = 2.0$, $r_1 \simeq 2.55$, $r_2 \simeq -0.66$ and $r_3 \simeq 2.65$. If $\theta_{23}$ deviates from $\pi/4$, in addition to the $s_{13}^2 \cos^2 \delta$ term, a term proportional to $s_{13} \cos \delta$ has to be added to Eq. (14).

As seen from the figure, even for $\cos \delta = -1$, the sensitivity to $s_{13}^2$ can be obscured by the 10% uncertainty in $\alpha$. However, for $s_{13}^2 > 0.02$, the bands between $\alpha = 2.2$ and 1.8 for $\cos \delta = 1$ and $\cos \delta = -1$ have no overlap. This means that for $s_{13}^2 > 0.02$, 10% precision in $\alpha$ is enough to distinguish $\cos \delta = 1$ from $\cos \delta = -1$.
Notice that the curve with $\alpha = 2$ is closer to that with $\alpha = 2.2$ than that with $\alpha = 1.8$. This means that the effect of the uncertainty decreases by increasing the value of $\alpha$.

![Figure 2: The dependence of $R$ on $\sin^2 \theta_{13}$ for different values of the $p$ parameter defined in Eq. (15). We have set $\delta = \pi$. The rest of the input parameters are the same as in Fig. 1. The vertical line at 0.041 shows the present bound at 3$\sigma$ [29].](image)

Considering the unknown nature of the production mechanism, it is not dismissed that the energy spectrum of neutrinos does not follow a simple power-law form. For example the spectrum can be a sum of two power-law functions each originating from a separate mechanism (i.e., $aE^{-\alpha_1} + bE^{-\alpha_2}$). We study such a possibility in Fig. 2, where we have taken the shape of the spectrum to be of the form:

$$E^{-2} + p \left( \frac{E^{-1}}{100 \text{ TeV}} \right),$$

where $p$ is a dimensionless parameter that determines the magnitude of the second term. Both terms can originate from the Fermi acceleration mechanism [32]. Notice that the second term is subdominant. The curves from up to down respectively correspond to $p = 1, 0.5, 0.3, 0.1$ and 0. As seen from the figure, the uncertainty on $p$ obscures the extraction of the mixing angle $\theta_{13}$.
4.2 Uncertainty in $N_{\bar{\nu}_e}/N_{\nu_e}$

Since the source is made of matter rather than anti-matter, we in general expect $\pi^+$ to dominate over $\pi^-$ and therefore $0 < N_{\bar{\nu}_e}/N_{\nu_e} < 1$. There is not any established or proposed method for determining $N_{\bar{\nu}_e}/N_{\nu_e}$ in the neutrino telescopes in the energy interval (100 GeV, 100 TeV). As a result, this ratio appears as a source of uncertainty in determination of $R$. Curves in the Fig. 3 show the dependence of $R$ on $\sin^2 \theta_{13}$ for two extreme values $\lambda \equiv N_{\bar{\nu}_e}/N_{\nu_e} = 0$ and $\lambda = 1$. For $\delta = \pi$, the variation of $N_{\bar{\nu}_e}/N_{\nu_e}$ in the interval $[0, 1]$ causes a change in $R$ of about 5 % which can obscure the determination of $s_{13}$. Notice that for $s_{13}^2 > 0.005$ the bands between $\lambda = 1$ and $\lambda = 0$ for $\delta = 0$ and $\delta = \pi$ are separate, so the uncertainty in $\lambda$ will not cause a problem for discriminating between $\cos \delta = \pm 1$.

![Figure 3](image_url)  

Figure 3: The dependence of $R$ on $\sin^2 \theta_{13}$ for different values of the parameters $\lambda \equiv N_{\bar{\nu}_e}/N_{\nu_e}$. The thicker lines correspond to $\delta = \pi$ and the thinner ones correspond to $\delta = 0$. The spectral index has been set equal to 2. The rest of the input parameters are the same as in Fig. (1).

4.3 Uncertainties in cross sections

To calculate the cross section, information on the Parton Distribution Functions (PDFs) of the nucleon is needed. The center of mass energy of a system composed of a neutrino with energy $E_{\nu} \sim 100$ TeV incident on a proton at rest is $(2E_{\nu}m_p)^{1/2} = 450$ GeV. The center
of mass energy of the $e - p$ HERA collider is about 320 GeV. Thus, to calculate $\sigma_{\nu N}$ in the energy range relevant for this study (i.e., $100 \text{ GeV} \lesssim E_\nu \lesssim 100 \text{ TeV}$) the results of the HERA experiment can be employed. The current uncertainty on the PDFs is about 3 % \cite{33}. LHC can further improve the precision of the PDFs.

The uncertainty in the cross section of all types of neutrinos (each flavor of neutrino and anti-neutrino) originates from the same uncertainties in the PDFs. As a result, the resultant uncertainty in the numerator and denominator of $R$ cancel each other so the certainty in the cross section will not be a limiting factor for determining $\theta_{13}$ (or $\delta$) from the cosmic neutrino flavor composition.

5 Determination of $\delta$

In the literature it is suggested to derive the Dirac CP-violating phase ($\delta$) by studying the flavor composition of the cosmic neutrinos \cite{12, 13, 14}. Considering the expenses and challenges before measuring this phase through the more conventional proposals (i.e., neutrino factory or superbeam methods), it is worth giving this possibility a thorough consideration. However, in the literature the flavor identification power of the detector has not been realistically treated. To be specific, the quantities that have been previously analyzed in the context of deriving $\delta$ are ratios such as $R' \equiv F_{\nu_i}/(F_{\nu_e} + F_{\nu_\tau})$ \cite{12, 13, 14} which cannot be directly derived at neutrino telescopes. In this section, we assess the possibility of measuring $\delta$ considering realistic flavor identification power of neutrino telescopes (i.e., studying ratio $R$) and taking into account various sources of uncertainty for the first time.

By the time a statistically significant number of cosmic neutrino events is collected, we expect noticeable improvement in determination of the input parameters. In particular, in the case that the parameters are in favorable range, we expect progress in the following measurements:

- If $\sin^2 \theta_{13}$ is close to the present bound, the forthcoming experiments can measure its value with a precision of $\Delta \sin^2 \theta_{13}$:

$$\sin^2 \theta_{13} = \sin^2 \bar{\theta}_{13}(1 \pm \Delta \sin^2 \theta_{13}/\sin^2 \bar{\theta}_{13}).$$

In fact, for relatively large values of $\sin^2 \theta_{13}$, the uncertainty $\Delta \sin^2 \theta_{13}/\sin^2 \bar{\theta}_{13}$ can be reduced to as small as 5 % \cite{34, 35}.
• In case that statistically significant number of cosmic neutrinos are collected, $R$ can be measured with an uncertainty of $\Delta R$:

$$R = \bar{R} (1 \pm \Delta R / \bar{R}) .$$

As shown in [20], a precision of $\Delta R / \bar{R} \simeq 7\%$ can be obtained provided that the number of events exceeds $\sim 300$. This would be achieved with a neutrino flux of $E^2 \frac{dF_{\nu}}{dE_\nu} = 0.25$ GeV cm$^{-2}$ sr$^{-1}$ yr$^{-1}$ after a couple of years of data-taking.

• The forthcoming long-baseline [36] and reactor neutrino [37] experiments can respectively measure the solar and atmospheric mixing angles by a precision of $\sim 6\%$. That is

$$\sin^2 \theta_{12} = \sin^2 \bar{\theta}_{12} (1 \pm 6\%),$$

$$\sin^2 \theta_{23} = \sin^2 \bar{\theta}_{23} (1 \pm 6\%).$$

The present best-fit values are $\sin^2 \bar{\theta}_{12} = 0.32$ and $\sin^2 \bar{\theta}_{23} = 0.5$.

Remember that the ratio $N_{\bar{\nu}_e} / N_{\nu_e}$ cannot be measured. Since the initial jets creating the charged pions (and subsequently the neutrinos) are mainly made of protons rather than anti-protons, we expect $N_{\bar{\nu}_e} / N_{\nu_e} \leq 1$. Considering all these uncertainties, the question is whether it will be possible to extract the value of $\delta$.

Fig. 4 addresses this question. Drawing the plot, we have assumed that $\bar{R}$ will be found to have a typical value of 2.53 with an uncertainty of $\Delta R / \bar{R}$. This value of $\bar{R}$ can be obtained by taking maximal CP-violation ($\delta = \pi/2$), $\sin^2 \theta_{13} = 0.03$, $w_e^0 : w_\mu^0 : w_\tau^0 = 1 : 2 : 0$, $\alpha = 2$ and $N_{\bar{\nu}_e} / N_{\nu_e} = 0.5$. We have looked for solutions in the $\delta - \alpha$ plane for which $R = 2.53 (1 \pm \Delta R / \bar{R})$, varying the rest of the relevant parameters in the ranges indicated in the caption of Fig. 4. The regions covered with dots, little triangles and crosses respectively correspond to 7%, 1.5% and 1% precision in the measurement of $R$. Notice that the figure is symmetric under $\delta \to 2\pi - \delta$. The symmetry originates from the fact that the dependence of $R$ on $\delta$ is through $\cos \delta$. As mentioned earlier, $\alpha$ can be independently determined by the measurement of the energy spectrum with about 10% precision. (For $\alpha = 2$, the direct measurement of the energy spectrum can restrict the value of $\alpha$ to the region between the vertical lines at $\alpha = 1.8$ and 2.2.) As seen from the figure, with $\Delta R / \bar{R} = 7\%$, $\delta$ cannot be constrained. In fact, any point between the vertical lines can be a solution. The figure shows that reducing $\Delta R / \bar{R}$ to 1% (but keeping the rest of the uncertainties as before), some parts of the solutions can be excluded. In particular, the region around $\delta = \pi$ will not be a
solution anymore. Notice that along with \( \delta = \pi/2, \delta = 0 \) is also a solution. This means that despite maximal CP-violation, the CP-violation cannot still be established. We examined the robustness of this result. We found that reducing the uncertainties in the mixing angles (even in \( \theta_{13} \)) will not noticeably change the overall conclusion. However, the sensitivity to \( \Delta R/\bar{R} \) seems to be high. Notice that with a precision of \( \Delta R/\bar{R} = 1.5\% \), there are some regions of solutions (covered by the triangles) that can be excluded if the uncertainty is reduced to 1\% (\( i.e. \), they are not covered with crosses). As we will see below, the sensitivity to power index is also high.

Drawing Figs. (5-a,5-b), we have respectively taken \( \bar{R} = 3.3 \) (corresponding to \( \alpha = 1 \)) and \( \bar{R} = 3.59 \) (corresponding to \( \alpha = 3 \)). The rest of the input is the same as Fig. (4). From Fig. (5-b) we observe that for \( \alpha = 3 \), the measurement of \( R \) with 7\% uncertainty determines \( \alpha \) with better than 6 \% precision which will probably be more accurate than the direct determination of \( \alpha \) from the energy spectrum measurement. Notice that when \( \alpha = 1 \) or 3, even a precision of \( \Delta R/\bar{R} = 1\% \) will not be enough to constrain \( \delta \). However, from Fig. (5-a) we observe that in case of \( \alpha = 1 \) and \( \delta = \pi/2 \), if the error in direct measurement

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**Figure 4**: Points in the \((\alpha, \delta)\) space consistent with \( R = 2.53 \pm \Delta R \). True values of the \((\alpha, \delta)\) pair are \((2, \pi/2)\). Points displayed by dots, triangles and crosses respectively correspond to \( \Delta R/\bar{R} = 7\% \), \( \Delta R/\bar{R} = 1.5\% \) and \( \Delta R/\bar{R} = 1\% \). To draw this figure we have varied \( \sin^2 \theta_{13} \in (0.028, 0.032) \), \( \sin^2 \theta_{12} \in (0.30, 0.34) \), \( \sin^2 \theta_{23} \in (0.47, 0.53) \) and \( N_{\nu_e}/N_{\nu_e} \in (0, 1) \).
of $\alpha$ is reduced to 10% (i.e., if $\alpha$ is constrained to the region between the dashed vertical lines) and if $R$ is measured with 1% precision, one can exclude solutions around $\delta = \pi$ but CP-violation cannot still be established.

As emphasized in [13], the sensitivity of $R$ to the variation of $\theta_{23}$ is significant. In fact, the present uncertainty (i.e., $\sin^2 \theta_{23} = 0.50^{+0.18}_{-0.16}$ at 3$\sigma$ c.l. [29]) leads to a sizeable uncertainty of $\sim 20\%$ in $R$. As mentioned earlier the forthcoming experiments [37] can improve the precision of $\sin^2 \theta_{23}$ to 6 %. A variation of 6% in $\sin^2 \theta_{23}$ leads to a $\sim 4\%$ change in $R$ which is comparable to the effect of $\cos \delta$ for $\sin^2 \theta_{13} = 0.03$. To pinpoint the effect of the uncertainty of $\theta_{23}$ on $R$, we have presented Fig. 6. The input of Fig. 6 is similar to Fig. 4 but to study the effect of $\theta_{23}$, we have fixed $N_{\nu_e}/N_{\nu_x}$, $\theta_{12}$ and $\theta_{13}$ to their central values. Comparing Figs. 4 and 6 we find that improving the precision of $\sin^2 \theta_{23}$ from 6% to 1% can help us to remove a substantial part of the spurious solutions. Especially for $\Delta R/\bar{R} = 1\%$ solutions with $3\pi/4 < \delta < 5\pi/4$ can be removed by reducing the uncertainty of $\sin^2 \theta_{23}$ to 1 %.

Figure 5: The same as Fig. 4 except that in figure (a), we have taken $\bar{R} = 3.30$ (corresponding to $\alpha = 1$) and in figure (b), we have taken $\bar{R} = 3.59$ (corresponding to $\alpha = 3$). Points displayed by dots and triangles respectively correspond to $\Delta R/\bar{R} = 7\%$ and $\Delta R/\bar{R} = 1\%$. 
Figure 6: Points in the \((\alpha, \delta)\) space consistent with \(R = 2.53 \pm \Delta R\). As in Fig. 4, the true values of the \((\alpha, \delta)\) pair are \((2, \pi/2)\). Points displayed by dots and triangles respectively correspond to \(\Delta R/\bar{R} = 7\%\) and \(\Delta R/\bar{R} = 1\%\). To draw this figure we have fixed \(\sin^2 \theta_{13} = 0.0301\), \(\sin^2 \theta_{12} = 0.32\), \(N_{\bar{\nu}_e}/N_{\nu_e} = 0.5\) and varied \(\sin^2 \theta_{23}\) in 0.5 \((1 \pm 1\%)\).

6 Initial flavor composition

In the previous sections, in accordance with the “standard picture”, we had assumed that \(w_0^e : w_0^\mu : w_0^\tau = 1 : 2 : 0\). Various mechanisms can intervene to cause a deviation from this simple prediction. For example, in the so-called stopped muon scenario, the muon production takes place inside a high magnetic field so the muons come to rest before decay. As a result, the neutrinos produced from the decay of the muon would be below the detection energy threshold. This effectively leads to \(w_0^e : w_0^\mu : w_0^\tau = 0 : 1 : 0\) \([30]\). On the other hand, a contribution from neutron decay, \(n \rightarrow \bar{\nu}_e p e^-\), would increase \(w_0^e/w_0^\mu\). Even in the standard picture, along with the production of \(\pi^\pm\), charged Kaons can also be produced in the jets. Like the case of the charged pions, the main decay mode of charged Kaon is \(K \rightarrow \mu \nu_\mu\); so neglecting the subdominant modes \(\text{Br}(K \rightarrow \pi^0 \mu \nu_\mu) = 3.3\%\) and \(\text{Br}(K \rightarrow \pi^0 e \nu_e) = 5\%\) \([28]\), we would naively expect that flavor composition of the neutrinos from the Kaon chain also follow the standard picture (i.e., \(w_0^e : w_0^\mu : w_0^\tau \approx 1 : 2 : 0\)) but there is a subtlety here. In the chain process, \(\pi \rightarrow \mu \nu_\mu, \mu \rightarrow \nu_e \nu_\mu e\), the average energy of each of the produced neutrinos in
the rest frame of the pion is about \( m_\pi/4 \). That is while in the case of Kaon decay, the average energies of the neutrinos are different: The energy of \( \nu_\mu \) produced directly from the Kaon decay in the Kaon rest frame is \( (m_K^2 - m_\mu^2)/(2m_K) \approx 236 \text{ MeV} \); that is while, the average energies of the neutrinos from the secondary muon are \( [(m_K^2 + m_\mu^2)/(2m_K)]/3 \approx 86 \text{ MeV} \).

As a result, limiting the detection to an energy range (e.g., \((100 \text{ GeV}, 100 \text{ TeV})\)) the ratio \( w_0^e : w_0^\mu \) (i.e., \( \int^{E_{\text{cut}}}_{E_{\text{th}}} (dF_0^{\nu_e}/dE + dF_0^{\bar{\nu}_e}/dE)dE : \int^{E_{\text{cut}}}_{E_{\text{th}}} (dF_0^{\nu_\mu}/dE + dF_0^{\bar{\nu}_\mu}/dE)dE \)) will deviate from 1:2. In fact, the ratio \( w_0^e : w_0^\mu \) would depend on the energy spectrum of the initial charged Kaon.

The mechanisms that we pointed out above are all processes that expected to exist and play at least a subdominant role within the framework of the mainstream models. None of these mechanisms creates \( \tau \) neutrino or anti-neutrino at the sources (i.e., they all yield \( w_0^e : w_0^\mu : w_0^\tau = 0 \)). However, more exotic mechanisms can be at work to create \( \nu_\tau \) or \( \bar{\nu}_\tau \) at source: In principle, the collision of \( pp \) or \( p\gamma \) at the source can create \( D \) meson whose decay can produce \( \nu_\tau \) and \( \bar{\nu}_\tau \). According to [38], the contribution of the \( D \) meson to the neutrino flux becomes important only for \( E_\nu > 10^5 \text{ TeV} \). For lower values of energy in which we are interested in, the contribution is about three orders of magnitude below the present bound. To our best knowledge, within the Standard Model (SM) of particles, in the energy range \( 100 \text{ GeV}-100 \text{ TeV} \), \( w_0^\tau \) remains much smaller than \( w_0^\mu \) and \( w_0^e \). However, if neutrinos have some yet unexplored properties beyond SM, tau neutrinos or anti-neutrinos can be produced at the source. For example, suppose neutrinos posses tiny transition magnetic moments of form

\[ \mu_{\tau e} F_{\alpha\beta} \nu^T_e C \sigma^{\alpha\beta} \nu_\tau \quad \text{and/or} \quad \mu_{\tau \mu} F_{\alpha\beta} \nu^T_\mu C \sigma^{\alpha\beta} \nu_\tau . \]  

There are strong bounds on the transition moments, |\( \mu_{\alpha\beta} \)|, from different terrestrial experiments and astrophysical observations [28, 39]:

\[ \mu_{\alpha\beta} < 3 \times 10^{-12} \mu_B, \]  

where \( \mu_B \) is the Bohr magneton. The magnetic field inside the source can be so large that even a transition moment as tiny as \( 10^{-13} \mu_B \) can lead to a sizeable production of tau neutrino and anti-neutrino at the source [40]. In fact according to the models [41], it is possible to have

\[ \left( \frac{B}{10^9 \text{ Gauss}} \right) \left( \frac{L}{10^8 \text{ cm}} \right) \left( \frac{\mu_{\alpha\beta}}{10^{-13} \mu_B} \right) \gtrsim 1 \]  

where \( B \) is the magnetic field and \( L \) is the linear size of the volume in which the magnetic field is as large as \( B \). If condition (18) is fulfilled, the flavor composition at source will
considerably deviate from the standard $1:2:0$. Let us normalize the initial flavor ratio such that $w_\mu^0 = 1$:

$$w_e^0 : w_\mu^0 : w_\tau^0 = w_e^0 : 1 : w_\tau^0.$$ 

In this section, we discuss whether by merely studying $R$ and without theoretical prejudice, one can extract $w_e^0$ and $w_\tau^0$. Actually, the possibility of deriving information on $w_e^0$ and $w_\tau^0$ from the measurement of the flavor ratio at the detector has been discussed in the literature [42]. Like the case of measurement of neutrino parameters, the flavor identification power of the neutrino telescopes has not been realistically treated in the previous studies. Here we investigate this possibility considering realistic flavor identification power of neutrino telescopes (i.e., studying ratio $R$) and taking into account various sources of uncertainty for the first time.

There is a subtle point here. If within the lifetime of the ICECUBE (or a more advanced neutrino telescope) neutrino flux from a GRB in a close-by galaxy (a $\sim 5$ Mpc far away galaxy) is detected, the statistics will be high enough to extract information on the flavor composition of the flux from this individual source. For sources located at cosmological distances ($\gtrsim 100$ Mpc), the best can be done is to combine the information from different sources. Each source may be different and emit neutrino flux with a different flavor composition. From the “average” $R$, only “average” values of $w_e^0$ and $w_\tau^0$ over these sources can be derived.

Taking into account the relevant uncertainties in the input parameters, we look for values of $w_e^0 : w_\mu^0 : w_\tau^0$ that are consistent with $R = \bar{R} \pm \Delta R$. To perform this analysis, we take $\theta_{13} = 0$. For any other value of $\theta_{13}$, the same analysis can be repeated. The results are robust against the variation of $\theta_{13}$ within the present bound. If $\theta_{13} = 0$, by the time that enough cosmic neutrinos are collected, the Daya-Bay [43] and Double-Chooz [35] experiments can set the bound $\sin^2 \theta_{13} < 0.003$. We vary $\sin^2 \theta_{13}$ between zero and 0.003. In this case, there is no hope of measuring $\delta$ so we allow $\delta$ to vary between 0 and $2\pi$. We take the energy spectrum to be of form $E^{-2}$ and assume that its power-law behavior will be established and the spectral index will be measured with 10 % precision. Again, we vary $N_{\bar{\nu}_e}/N_{\nu_e}$ within [0,1].

In Fig. (7), we consider two possibilities: (i) the standard case with $w_e^0 : w_\mu^0 : w_\tau^0 = 0.5 : 1 : 0$ leading to $\bar{R} = 2.53$ (see Fig. 7-a); (ii) the case of stopped muons with $w_e^0 : w_\mu^0 : w_\tau^0 = 0 : 1 : 0$ yielding $\bar{R} = 3.20$ (see Fig. 7-b). From these figures we observe that with a precision of $\Delta R/\bar{R} = 7\%$, these two scenarios can be easily discriminated. These two can also be
Figure 7: Points in the $(w^0_e, w^0_\tau)$ plane consistent with $R = \bar{R} \pm \Delta R$. The ratios are normalized such that $w^0_\mu = 1$. The true values of $(w^0_e, w^0_\tau)$ are denoted by ★. Points displayed by dots and triangles respectively correspond to $\Delta R/\bar{R} = 7\%$ and $\Delta R/\bar{R} = 1\%$. In drawing this figure we have varied $\sin^2 \theta_{13} \in (0, 0.003)$, $\delta \in (0, 2\pi)$, $\alpha \in (1.8, 2.2)$, and $N_{\bar{\nu}_e}/N_{\nu_e} \in (0, 1)$. Drawing Fig. (a), we have taken $\bar{R} = 2.53$ which corresponds to the standard picture with $w^0_e = 1/2$ and $w^0_\tau = 0$. In case Fig. (b), we have set $\bar{R} = 3.2$ which corresponds to the stopped muon scenario with $w^0_e = w^0_\tau = 0$.

Figure 8: The same as Fig. 7 except that $\bar{R}$ is set equal to 1.96, which corresponds to $w^0_e : w^0_\mu : w^0_\tau = 4 : 1 : 1$. 
discriminated from the scenario in which the neutrino production mechanism is \( n \rightarrow p e \bar{\nu}_e \) \( (i.e., \ w^0_e : w^0_\mu : w^0_\tau = 1 : 0 : 0) \). When we restrict the analysis to \( w^0_\tau = 0 \) \( (i.e., \ the \ case \ without \ exotic \ neutrino \ properties) \) from these figures we observe that the measurement of \( R \) stringently constrains \( w^0_e : w^0_\tau \) which in turn sheds light on the production mechanism. However, once the assumption of \( w^0_\tau \) is relaxed, a wide range of \( w^0_e : w^0_\mu : w^0_\tau \) can be a solution. For example, the exotic case of \( w^0_e : w^0_\mu : w^0_\tau = 0 : 0 : 1 \) leads to the same value of \( \bar{R} \) as the stopped muon scenario.

The input for Fig. 8 is the same as that for Fig. 7 except that in Fig. 8 \( w^0_e : w^0_\mu : w^0_\tau = 4 : 1 : 1 \). For this flavor ratio, the central value of \( R \) is \( \bar{R} = 1.96 \). Notice that with \( \Delta R/\bar{R} \), this exotic flavor ratio can be discriminated from the two standard cases that we have mentioned. That is \( w^0_e : w^0_\mu : w^0_\tau = 0.5 : 1 : 0 \) or \( w^0_e : w^0_\mu : w^0_\tau = 0 : 1 : 0 \) are not solutions for \( R = 1.96(1 \pm 7\%) \).

7 Conclusions and Discussions

Under the assumption that the initial flavor ratios at the source were \( w^0_e : w^0_\mu : w^0_\tau = 1 : 2 : 0 \), we have studied the possibility of deriving \( \theta_{13} \) and/or \( \delta \) from cosmic neutrinos taking into account various uncertainties.

ICECUBE and other neutrino telescopes that are going to collect neutrino events in the energy range \( 100 \text{ GeV} < E_\nu < 100 \text{ TeV} \) will be sensitive only to two types of events; \( i.e., \) shower-like and \( \mu \)-track events. The ratio of these two, \( R \), yields only one piece of information on the mixing parameters. Under the assumption of CP conservation, \( \cos \delta = \pm 1 \), we have discussed the possibility of extracting \( s_{13} \) from the measurement of \( R \). We have found that for \( \cos \delta = 1 \), the sensitivity of \( R \) to \( s^2_{13} \) is very mild. For \( \cos \delta = 1 \), the derivation of \( s_{13} \) from \( R \) would require measurement of \( R \) with a precision better than 2% which does not seem achievable. In the case of \( \cos \delta = -1 \), as \( s^2_{13} \) varies between zero and the present upper bound, \( R \) changes by 10% which is in principle resolvable by ICECUBE [20].

We have found that a 10% uncertainty in the energy spectrum (to be precise, 10% uncertainty in the spectral index in Eq. (13)) is a major source of error in the derivation of \( s_{13} \). However, for \( s^2_{13} > 0.02 \), the solutions with \( \cos \delta = 1 \) and \( \cos \delta = -1 \) can be distinguished despite a 10% uncertainty in the spectral index. We have also studied the effect of a deviation from the power-law spectrum. Our conclusion is that in order to derive
$s_{13}$ from $R$, a precision better than 5% in the measurement of the energy spectrum is required.

We have also studied the effects of the variation of the neutrino to anti-neutrino ratio on $R$. We have found that when $N_{ar{\nu}_e}/N_{\nu_e}$ (see Eq. (13) for the definition) varies between 0 and 1, $R$ changes by 5% which is comparable to the effect of $s_{13}$. Unfortunately with the current techniques, it is not possible to measure $N_{\bar{\nu}_e}/N_{\nu_e}$ in the energy range $100 \text{ GeV} < E_\nu < 100 \text{ TeV}$ so this source of uncertainty cannot be eliminated by measurement and one should rely on the models to predict the value of $N_{\bar{\nu}_e}/N_{\nu_e}$.

The uncertainty in the neutrino nucleon cross section, $\sigma_{\nu N}$, induces relatively large uncertainty in the evaluation of the shower-like and $\mu$-track events. However, when we take their ratio, the uncertainties cancel each other out so the derivation of the neutrino parameters from $R$ does not suffer from the uncertainty in $\sigma_{\nu N}$.

We have then studied the possibility of deriving $\delta$ from the cosmic neutrino flavor composition assuming that $s_{13}$ will be measured by other experiments with a reasonable precision. Since the dependence of the oscillation probabilities of the cosmic neutrinos on $\delta$ is through $\cos \delta$, there is a symmetry under $\delta \rightarrow 2\pi - \delta$. On the other hand, the sensitivity of the CP-odd combination $P(\nu_\mu \rightarrow \nu_e) - P(\bar{\nu}_\mu \rightarrow \bar{\nu}_e)$ (which is proposed to be measured by neutrino factory or superbeam facilities) to $\delta$ is through $\sin \delta$ and would therefore suffer from a degeneracy under $\delta \rightarrow \pi - \delta$. To resolve the latter degeneracy, it is suggested to employ various baselines [44] and/or study the energy dependence of the oscillation probability [45]. Derivation of $\cos \delta$ from the cosmic neutrino flavor composition can be considered as an alternative method to solve this degeneracy. We have studied the effects induced by the error in the mixing angles ($\theta_{12}$, $\theta_{23}$ and $\theta_{13}$) and the measurement of $R$ as well as by the uncertainties in neutrino-antineutrino ratio and the energy spectra. We have found that even with a precision of 1% in $R$, CP cannot be established. That is even for the maximal CP-violation ($\delta = \pi/2$), $\delta = 0$ cannot be ruled out. However, in this case, $\delta = \pi$ can be excluded provided that $\Delta R/\bar{R}$ is reduced to less than 1%.

Within the SM of the particles, various mechanisms can deviate $w_\mu^0/w_e^0$ from 2 but within the energy range $100 \text{ GeV} < E_\nu < 100 \text{ TeV}$, $w_\tau^0$ still remains much smaller than $w_e^0$ and $w_\mu^0$. The conditions in the source of cosmic neutrinos are so extreme that the beyond SM properties of neutrinos can play a role to significantly distort the initial flavor ratio of the cosmic neutrinos. In particular, a nonzero $\mu_{\tau e}$ or $\mu_{\tau \mu}$ transition moment close to the present bound can lead to $w_\tau^0 \sim w_\mu^0 \sim w_e^0$. In the second part of the paper, we have relaxed the
assumption $w^0_e : w^0_\mu : w^0_\tau = 1 : 2 : 0$ and have studied the possibility of extracting the initial $w^0_e : w^0_\mu : w^0_\tau$ from the cosmic neutrino data. We have found that with a precision of $\Delta R/\bar{R} = 7\%$, one can discriminate between the standard picture with $w^0_e : w^0_\mu : w^0_\tau = 1 : 2 : 0$ and the stopped muon scenario with $w^0_e : w^0_\mu : w^0_\tau = 0 : 1 : 0$. When based on theoretical prejudice, we restrict the analysis to $w^0_\tau = 0$, we find that $w^0_\mu/w^0_e$ can be constrained with reasonable accuracy but relaxing $w^0_\tau = 0$ will open up the possibility of different solutions.

We have also enumerated a number of other effects that can be comparable to that of nonzero $s_{13}$ but have been overlooked in the literature. Calculating these effects requires the knowledge of details of the detector and the shape of the neutrino spectrum. Uncertainty in this knowledge will lead further uncertainty in the determination of $\delta$ and $\theta_{13}$. As we demonstrated in the present paper, even in the absence of these effects, the uncertainties are too large to allow for the determination of $\delta$ and $\theta_{13}$. Potential uncertainties in these effects will further confirm this conclusion. For the purpose of establishing substantial deviation of $w^0_e : w^0_\mu : w^0_\tau$ from $1 : 2 : 0$, these effects have to be taken into account however the uncertainties in the evaluation of these effects are not expected to be so large to change our positive conclusion in the second part of this paper. Throughout this paper, we have assumed that the propagation of the cosmic neutrinos from the source to the detector is governed by the standard oscillation formula. The effects of a deviation from the standard oscillation probability will be presented elsewhere.

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**A Calculation of $f(E_\tau, E_\mu)$**

In this section we calculate the function $f(E_\tau, E_\mu)$ which is the probability density of the emission of a muon with energy $E_\mu$ in the decay of a $\tau$ lepton ($\tau \rightarrow \mu \nu_\mu \nu_\tau$) with energy $E_\tau$ (see Eq. (8) for the definition). In the rest frame of $\tau$, the partial decay rate of an unpolarized
is given by the following well-known formula (see [46])

\[
\frac{1}{\Gamma'} \frac{d^2\Gamma'}{dE'_\mu d\Omega'} = \frac{12}{\pi m^2} \left( 1 - \frac{4E'_\mu}{3m_\tau} \right) E'^2_\mu, \tag{19}
\]

where the effects of \(m^2_\mu/m^2_\tau \ll 1\) are neglected. Quantities in the rest frame of the decaying \(\tau\) lepton are denoted by a prime. In the rest frame of the \(\tau\) lepton, \(0 < E'_\mu < m_\tau/2\). The number of emitted muons in certain direction within the solid angle \(d\Omega'\) and with energy in the interval \([E'_\mu, E'_\mu + dE'_\mu]\) is a Lorentz invariant quantity:

\[
\frac{1}{\Gamma} \frac{d^2\Gamma}{dE\mu d\Omega} \frac{dE\mu d\Omega}{dE'_\mu d\Omega'} = \frac{1}{\Gamma'} \frac{d^2\Gamma'}{dE'_\mu d\Omega'} dE'_\mu d\Omega'. \tag{20}
\]

From this equality, we obtain

\[
\frac{1}{\Gamma} \frac{d^2\Gamma}{dE\mu d\Omega} dE\mu d\Omega = \frac{12}{\pi m^2} \sum \gamma(1 - \beta \cos \theta) E\mu \gamma(1 - \beta \cos \theta) E^2\mu dE\mu \sin \theta d\theta d\phi, \tag{21}
\]

where \(\gamma = E_\tau/m_\tau\) and \(\beta = \sqrt{1 - 1/\gamma^2}\). The z-axis is taken along the direction of motion of \(\tau\). The quantities \(E_\mu, \theta\) and \(\phi\) take values in the following intervals

\[
0 \leq \phi < 2\pi, \quad 0 < E_\mu < E^2_\tau (1 + \beta), \quad 0 \leq \theta \leq \theta_{max} \tag{22}
\]

where

\[
\theta_{max} = \arccos \left[ \max \left\{ \frac{1}{\beta} \left( 1 - \frac{m_\tau}{2\gamma E_\mu} \right), -1 \right\} \right]. \tag{23}
\]

By integrating over \(\theta\) and \(\phi\) in Eq. (21), we obtain

\[
f(E_\tau, E_\mu) = \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\theta_{max}} \frac{1}{\Gamma} \frac{d^2\Gamma}{dE\mu d\Omega} \sin \theta d\theta d\phi. \tag{24}
\]

In the limit \(\beta \to 1\) (or equivalently, \(\gamma \gg 1\)),

\[
f(E_\tau, E_\mu) \approx \frac{5}{3E_\tau} - \frac{3E^2_\mu}{E^3_\tau} + \frac{4E^3_\mu}{3E^5_\tau}. \tag{25}
\]

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