A UNIVERSAL PROFILE OF THE DARK MATTER HALO AND THE TWO-POINT CORRELATION FUNCTION

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Received 1999 June 4; accepted 2000 March 27

ABSTRACT

We have found the relation between the two-point spatial correlation function and the density profile of dark matter halos in the strongly nonlinear regime. It is well known that when the density fluctuations grow into dark matter halos whose density profile is \( \rho \propto r^{-\gamma} \) on almost all mass scales, the two-point spatial correlation function obeys a power law with the power index \( \gamma = 2e - 3 \) in the strongly nonlinear regime. We find that the two-point spatial correlation function does not obey the power law when the power index \( e \) is smaller than \( \frac{3}{2} \), such as the density profile \( \rho \propto r^{-\gamma} \) around the center of the halo, which was proposed by Navarro, Frenk, & White in 1996 and 1997. By using the BBGKY equation in the strongly nonlinear regime, it is also found that the velocity parameter \( h = -\langle v \rangle / \delta x \) is not a constant even in the strongly nonlinear regime \( (x = x_{nl} \to 0) \), although it is a constant when \( e > 3/2 \), and then the two-point spatial correlation function can be regarded as the power law. The velocity parameter \( h \) becomes zero at the nonlinear limit of \( x \to 0 \), that is, the stable clustering hypothesis cannot be satisfied when \( e < 3/2 \).

Subject headings: cosmology: theory — dark matter — galaxies: halos —
large-scale structure of universe

1. INTRODUCTION

Formation of the large-scale structures in the expanding universe is one of the most important and interesting problems of cosmology. It is generally believed that these structures have been formed as a result of gravitational instability. Hence, it is very important to clarify evolutions of density fluctuations by the gravitational instability. Here we consider density fluctuations of collisionless cold particles such as cold dark matter, our interest being mainly concentrated on effects of the self-gravity.

In the hierarchical clustering picture, small fluctuations grow and form small clusters of dark matter (dark halos) at first. Then these small dark halos merge into a larger halo or accrete surrounding matter and produce a larger dark matter halo. Evolutions of density fluctuations in the nonlinear regime are especially important to understand the formation of the dark matter halo.

We consider a case in which the matter in almost all regions of high density is subject to the halos, and then a density profile of the halo mainly contributes to the two-point spatial correlation function of the density fluctuations of the universe in the nonlinear regime. When the density profile of the dark halo is given by \( \rho \propto r^{-\gamma} \), then the two-point spatial correlation function in the strongly nonlinear regime \( (r \to 0) \) is determined by the density profile of the dark halo and written in the following way (McClelland & Silk 1977; Sheth & Jain 1997; Padmanabhan & Engineer 1998):

\[
\xi = c_2 \left( \frac{r}{r_{nl}} \right)^{\gamma'}, \quad \gamma' = 2e - 3, \tag{1}
\]

where \( 3/2 < e < 3 \) in order to have the finite value of \( c_2 \) (see eq. [8]) and \( r_{nl} \) is the nonlinear scale.

By the way, a central region of the dark halo was investigated by using \( N \)-body simulations (Frenk, White, & Davis 1988; Navarro, Frenk, & White 1996, 1997, hereafter NFW96, NFW97). Hernquist (1990) investigated an elliptical galaxy analytically. NFW96, NFW97, and Hernquist (1990) claimed that dark halos are not well approximated by isothermal spheres. Instead, the density profile is well approximated by the following form:

\[
\rho \propto \frac{1}{(r/r_s)(1 + r/r_s)^{\mu - 3}}, \tag{2}
\]

where \( r_s \) is a characteristic scale of a dark halo. The parameters \( e \) and \( \mu \) are equal to 1 and 4, respectively, in the Hernquist (1990) case, and \( e = 1 \), \( \mu = 3 \) in the NFW96-NFW97 case. In any case, the density profile of the central region is shallower than the isothermal sphere.

When the density profile is \( \rho = (r/r_{nl})^{-\epsilon} \) with \( e < 3/2 \) around a center of the dark halo, an index of the profile in the outer region should be \( \rho \propto r^{-\epsilon} \) with \( \mu > 3/2 \) in order to have the finite correlation function. In this case, what is the form of the two-point spatial correlation function? The purpose of this paper is to obtain the two-point spatial correlation function in the strongly nonlinear regime when the index of the density profile of the dark halo is shallower than \( 3/2 \) in the central region and the self-similar evolution of the dark halos is satisfied. We also investigate what the form of the velocity parameter \( h \) is, that is, the mean relative peculiar velocity, by using the BBGKY equation.

In § 2, we will briefly review the power-law solutions of the two-point spatial correlation function described in Yano & Gouda (1997, hereafter YG97). We show in § 3 the solution of the two-point spatial correlation function and the mean relative peculiar velocity when there exists the dark halo whose density profile has two powers as in equation (9). We will devote § 4 to the conclusions and discussions.

2. POWER-LAW SOLUTIONS

We will briefly review the power-law solutions of the two-point spatial correlation function \( \xi \) and the mean peculiar
velocity \langle v \rangle described in YG97. We use the second BBGKY 
zeroth-moment equation:
\[
\frac{\partial \xi}{\partial t} + \frac{1}{ax} \frac{\partial}{\partial x} [x^2(1 + \xi) \langle v \rangle] = 0 ,
\] (3)

where \(a\) is the scale factor and \(x\) is the comoving coordinate. 
The mean relative peculiar velocity \(\langle v \rangle\) is defined as follows:
\[
\langle v \rangle = \frac{\langle f(x/\xi)(v_x - v_n) f(1) f(2) d^3 v_1 d^3 v_2 \rangle}{\langle f(1) f(2) d^3 v_1 d^3 v_2 \rangle} .
\] (4)

Here the angle brackets show an ensemble mean for any pair 
with the fixed distance \(x \equiv |x_2 - x_1|\), and \(f(i) \equiv f(x_i, v_i) (i = 1, 2)\) is a phase space density 
at a point of the phase space \((x_i, v_i)\).

We assume that the two-point spatial correlation function \(\xi\) 
given by the following power-law form in the strongly nonlinear 
regime:
\[
\xi = \xi_0 a^3 x^{-\gamma} .
\] (5)

Then we obtain from the dimensional analysis of equation (3)
\[
\langle v \rangle = -h \dot{x} a , \quad \beta = (3 - \gamma) h ,
\] (6)

where \(h\) is a constant. We call this parameter \(h\) the velocity 
parameter. This parameter is a value of order 1 (0 \(\leq h \leq 1\)) 
(YG97) when we consider the collisionless cold dark matter 
in the strongly nonlinear regime. We notice that the stability 
condition proposed by Davis & Peebles (1977) corresponds to 
\(h = 1\) (stable clustering). In this case, the collapsed 
object cannot be broken and clustered together to form a larger cluster. Therefore, the relative physical velocity 
of two particles \(\dot{\mathbf{r}} = \langle v \rangle + \dot{x} \mathbf{a}\) is equal to zero \(\langle v \rangle = -\dot{x} a\) in the nonlinear regime. We can consider the other extreme case about the clustering picture. In this case, 
smaller objects have clustered and merged together, and a 
completely virialized object is newly formed. In this case, a 
separation of the particles is expanding with the Hubble velocity, 
and \(h = 0\) (clustering moving).

Here we consider that the parameter \(\epsilon\) of the halo density 
profile has the values 3/2 \(< \epsilon < 3\). We consider the correlation 
function in the nonlinear regime \((\tilde{x} \equiv x/x_{nl} \rightarrow 0\), 
where \(x_{nl}\) is the nonlinear scale written in the comoving coordinate). Then, almost all pairs of the two points with 
a distance \(\tilde{x} < 1\) are included in one halo. Therefore, it is enough 
consider the two-point spatial correlation function within only one halo 
in order to obtain the correlation function at \(\tilde{x} \ll 1\). In this case, 
the relation between the two-point spatial correlation function and 
the density profile is
\[
\xi(r) \propto \int d^3 s \rho(s) \rho(|r + s|)
\] (7)
\[
= c_2 \left( \frac{r}{r_{nl}} \right) ^{-2\epsilon+3} 
\]

(see McClelland & Silk 1977; Sheth & Jain 1997; Padmanabhan & Engineer 1998), where
\[
c_2 = \sum_{m=0}^{\infty} \frac{1}{3 - \epsilon + 2m - \frac{1}{3 - 2\epsilon - 2m}} \times \frac{1}{(2 - \epsilon)(1 - \epsilon) \cdots (2 - \epsilon - 2m)} \frac{1}{(2m + 1)!} .
\] (8)

and \(r_{nl}\) is the nonlinear scale. The relation between both the 
power indices of the halo profile \(\epsilon\) and the two-point spatial 
correlation function \(\gamma\) is given by equation (1). We can see from 
3/2 \(< \epsilon < 3\) and equation (1) that the index of the 
two-point spatial correlation function \(\gamma\) has values between 
0 and 3. When we assume the density profile \(\rho \propto r^{-\epsilon}\) 
with \(\epsilon < 3/2\) around the center of the dark halo, what is the form 
of the two-point spatial correlation function? We consider 
the relation between the two-point spatial correlation function 
and this density profile in the next section.

3. UNIVERSAL DENSITY PROFILE

We consider a form of the two-point spatial correlation function 
when the density fluctuations grow into the dark halos which have the shallower density profile \(\rho \propto r^{-\epsilon}\) with 
\(\epsilon < 3/2\) around the center. In this case, the density profile 
cannot have the single power law with \(\epsilon < 3/2\) in order to 
have the finite two-point spatial correlation function.

NF96-NFW97 and Hernquist (1990) claimed that the 
halo density profile has the form shown in equation (2). The 
characteristic scale \(r_{nl}\) can be rewritten by \(x_{nl}\), where \(\lambda\) is a constant. 
For easy treatment, we assume that the density 
profile is written in the following way:
\[
\rho = \lambda^{-\mu} \left( \frac{r}{r_{nl}} \right) ^{-\kappa(r)} \kappa(r) = \begin{cases} 
\epsilon & (r < r_{nl} \epsilon < 3/2) \\
\mu & (r > r_{nl} \mu > 3/2) 
\end{cases} .
\] (9)

Here we consider the case of \(\mu < 1\) and normalize the density such that \(\rho = 1\) at \(r = r_{nl}\), although the same results are obtained in general for \(\lambda > 1\). Here we have assumed 
that the halos are formed in the nonlinear regime and that 
their density profiles have the same form independently of 
mass scales, as we show in equation (9). In other words, we 
have assumed that their halos evolve self-similarly because 
many authors, such as Jain & Bertshinger (1996), Colombi, 
Bouchet, & Hernquist (1996), and Yano & Gouda (1998), 
have investigated the self-similarity about the two-point 
spatial correlation function, and then the self-similarity is 
confirmed. In this case, each halo should have the self-similarity 
in the same way as the two-point spatial correlation function in the nonlinear regime. It should be noted that the term “halo” in this paper does not have constant 
proper size and evolve in density with time so that their 
density contrast profiles are a universal function of the 
nonlinear scale. In the same way as with the case of 3/2 \(< \epsilon < 3\), 
it is enough to consider the two-point spatial correlation function 
within only one halo in order to obtain the correlation function at \(\tilde{x} \ll 1\). Then, the two-point spatial correlation function in the nonlinear regime can be derived as follows:
\[
\xi(r) \propto \int d^3 s \rho(s) \rho(|r + s|)
\]
\[
\propto \lambda^{-2\mu} \int_{x_{nl}} ds d \cos \theta \ s^2 \left( \frac{s}{r_{nl}} \right) ^{-\kappa(r)} \left( \frac{y}{r_{nl}} \right) ^{-\kappa(\mu)}
\]
\[
\propto c_2 \left( \frac{r}{r_{nl}} \right) ^{-2\epsilon+3} + \frac{1}{3 - 2\epsilon} \lambda^{3-2\epsilon}
\]
\[
+ \frac{1}{2\mu-3} \lambda^{3-2\epsilon} + O\left( \left( \frac{r}{r_{nl}} \right)^2 \right) 
\]
\[
\equiv c_1 + c_2 \tilde{x}^{-\gamma} + O(\tilde{x}^2) ,
\] (10)
stants, and \(c_1 = \lambda^3 - 2\varepsilon / (3 - 2\varepsilon) + \lambda^3 - 2\mu / (2\mu - 3)\) is a positive value because we consider the case that \(\epsilon < 3/2\) and \(\mu > 3/2\). The index \(\gamma\) satisfies \(-\gamma = 3 - 2\varepsilon\) (see eq. [10]). As a result, we can see that in the case of \(\epsilon < 3/2\) the two-point spatial correlation function cannot obey the power law in the nonlinear regime. When the density is continuous at \(r = r_s\), as shown in equation (9), the term of \(O(\bar{x})\) is eliminated. On the other hand, the term of \(O(\bar{x}^2)\) remains. However, there is the possibility that the term of \(\epsilon\) or the higher terms are eliminated when the density profile is smoothly continuous at \(r = r_s\). In the following, we investigate the parameter \(h\) when the two-point spatial correlation function can be expressed by equation (10).

The velocity parameter \(h\) is a constant in the nonlinear limit (see eq. [6]). Here we consider the next order of the parameter \(h\). Therefore, we define the velocity parameter \(h\) in the following way:

\[
h = d_1 + d_2 \bar{x}^\varepsilon ,
\]

where \(d_1\) and \(d_2\) are constants. We should notice that \(\bar{x}\) depends on the scale factor \(a\) because \(\bar{x}\) includes the nonlinear scale \(x_{\text{nl}}\) and that it has a dependence such that \(x_{\text{nl}} \propto a^z\). When the self-similarity of the two-point spatial correlation function is satisfied, we can express that \(a\) is equal to \(2/(n + 3)\) (Davis & Peebles 1977; YG97). Then the BBGKY equation (3) reduces to

\[
\frac{\partial}{\partial \bar{x}} [x^\varepsilon (c_1 + c_2 x^{-\gamma})(d_1 + d_2 \bar{x}^\varepsilon)] ,
\]

and then this is rewritten as follows:

\[
\gamma x c_2 x^{-\gamma} = 3c_1 d_1 + (v + 3)c_1 d_2 \bar{x}^\varepsilon
+ (-\gamma + 3)d_1 c_2 \bar{x}^{-\gamma}
+ (-\gamma + v + 3)c_2 d_2 \bar{x}^{-\gamma + v} .
\]

Since we consider \(\epsilon < 3/2\), the relation \(-\gamma = 3 - 2\varepsilon > 0\) is satisfied. Therefore, the left side of equation (13) is zero in the nonlinear limit \((\bar{x} = x/x_{\text{nl}} \to 0)\). As a result, \(d_1\) must be zero because \(c_1\) is a positive value. Then the first and third terms of the right side of equation (13) vanish. Therefore, in the limit of \(\bar{x} \to 0\), the second term of the right side is the same order of the left side, so \(v\) is equal to \(-\gamma\). Then, the last term of the right side is higher order. Thus, we obtain the following relation:

\[
\gamma x c_2 = (v + 3)c_1 d_2 ,
\]

or, equivalently,

\[
v = \frac{-3c_1 d_2}{c_1 d_2 + \alpha c_2} .
\]

Here \(\alpha\) and \(c_2\) are of order 1. Furthermore, \(\gamma\) and \(v + 3\) are the same order. The constant \(c_1\) is the value of the two-point spatial correlation function in the nonlinear limit \((\bar{x}(0))\). Then \(c_1\) is greater than 1. If we assume \(c_1 \sim 10^2\), then \(d_2 \sim 10^{-2}\).

Finally, we find that the velocity parameter is written in the following form:

\[
h = d_2 \bar{x}^\varepsilon , \quad v = 3 - 2\epsilon \left(\epsilon < \frac{3}{2}\right) , \quad d_2 \ll 1 .
\]

The velocity parameter \(h\) is zero in the nonlinear limit. This means that the stability condition \((h = 1)\) cannot be satisfied at least in the case of \(\epsilon < 3/2\).

We obtain the form of the velocity parameter as shown in equation (16), and this velocity parameter is related to the two-point spatial correlation function through \(v\). However, we cannot obtain the values of \(d_2\) and \(v\) from the second BBGKY zeroth-moment equation. If we would like to do so, we must solve higher moment equations.

4. CONCLUSIONS AND DISCUSSION

We have investigated the relation between the two-point spatial correlation function and the density profile of the dark matter halos in the strongly nonlinear regime. It is found that when the density fluctuations evolve to the dark matter halos whose density profile is \(\rho \propto r^{-\epsilon}\) \((\epsilon < 3/2)\) around the center of the halo, the two-point spatial correlation function cannot obey the power law. Furthermore, we find by using the BBGKY equation that the velocity parameter \(h = -v/\alpha x\) is not a constant in the case of the shallower density profile \((\epsilon < 3/2)\), although \(h\) is a constant in the case of \(\epsilon > 3/2\). The velocity parameter \(h\) becomes 0 at the nonlinear limit, that is, the stable clustering hypothesis cannot be satisfied at least in the case of \(\epsilon < 3/2\).

NFW96-NFW97 proposed that the index of the density profile of the halo is independent of the initial condition. After that, other authors, such as Huss, Jain, & Steinmetz (1999), Moore et al. (1998), Primack & Bullock (1998), etc., also investigated the index of the density profile of the dark halos. These investigations resulted in different values of the index of the density profile, while all results suggest \(\epsilon < 3/2\). For example, Moore et al. (1998) proposed that the value of the index of the density profile is \(\epsilon = 1.5\). However, even if we chose this value, it is found from our results that the index of the two-point spatial correlation function is equal to zero or the velocity parameter \(h = 0\) in the nonlinear limit.

If the self-similarity of the correlation function and the velocity parameter \(h \neq 0\) (which means that the correlation function obeys the power law with the power index \(\gamma \neq 0\)) are satisfied, we could say that the NFW96-NFW97 density profile and/or the profiles which the above authors proposed never appear. On the other hand, if the self-similarity is satisfied and the NFW96-NFW97 density profile and/or the profiles which the above authors proposed appear, we could say that the velocity parameter \(h\) is equal to zero in the nonlinear limit, that is, the index of the two-point spatial correlation function in the nonlinear limit is equal to zero. This means that the stable clustering picture \((h = 1)\) is not satisfied.

We would like to thank Y. Fujita and M. Nagashima for useful discussion. We are also grateful to F. Takahara and M. Sasaki for useful suggestions. This work was supported in part by Research Fellowships of the Japan Society for the Promotion of Science for Young Scientists 4746 and in part by the grant-in-aid for Scientific Research 10640229 from the Ministry of Education, Science, Sports, and Culture of Japan.
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