Calculation of Shear Strength of Reinforced Concrete Members with Circular Section using the Extended Modified Compression Field Theory

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Abstract
In this paper, the shear strength of circular reinforced concrete (RC) columns was obtained using numerical models based on a modified compression field theory that enabled the sectional analysis of the beams. By comparing the computational results with previous experimental results, we were able to assess the suitability of this numerical model for estimating the shear strength of circular RC columns. Additionally, the validity range of the shear strengths was evaluated by substituting rectangular column sections for circular column sections. The relationship between the shear strength of the circular RC columns and the shear span-depth ratio were also discussed. On a final note, we examined by calculating the shear strength of the numerical models by using equations to estimate the concrete strength of confined concrete.

Keywords: modified compression field theory; reinforced concrete column; circular section; shear strength

1. Introduction
The theory regarding the shear strength of reinforced concrete members has developed remarkably in Japan. An equation for ultimate shear strength has been proposed in the 'Design guidelines for earthquake resistant RC buildings based on the ultimate strength concept' (hereafter referred to as the seismic design guidelines) issued by the Architectural Institute of Japan (AIJ, 1990) and is based on the plasticity theory. (AIJ, 1990)1) It has since been used as the equation for shear designs in place of previously used experimental equations.

There is, however, no clear description of the criteria and guidelines for the ultimate shear strength of members that have circular sections in their superstructure; in the seismic design guidelines, the sectional replacement method, which uses the equation for the ultimate shear strength of a rectangular cross-section, is tentatively recommended. The AIJ sectional replacement method is described as follows: 'shear design for members with a circular cross-section is performed by replacing the circular cross-section with a square cross-section that has the same area'.

The objective of this study is to obtain the shear strength of circular RC columns; to do this, we will use a numerical analytical model based on a modified compression field theory that has been extended to apply to sectional analysis for beams. The suitability of the analytical model is examined by comparing it to past test results for flexural shear strength. Subsequently, we will determine how effective the numerical model is for replacing the AIJ sectional replacement model by comparing the results of the two. In addition, the relationship between shear span-depth ratio and the shear strength is examined. This ratio is considered to be a main factor that affects the shear strength of an RC column. Finally, since the transverse constraint effect of circular RC columns is considered to be larger than that of square columns, the concrete strength obtained from an estimation equation for confined concrete strength has been included in the proposed numerical analytical model. The relationship between the confined concrete strength and shear strength is also examined through a comparison of the shear strengths obtained from the computational results of the model and from earlier experimental results.

2. Analytical Model
2.1 Extended Modified Compression Field Theory
Modified compression field theory2 (Vecchio et al., 1986) is applicable to the prediction of the elastoplastic behavior of RC elements that have been subjected to uniform shear stress and axial stress; however, this theory cannot be applied for conditions where
the stress or strain gradient is generated at the cross-section of a column. Therefore, we propose dividing the cross-section of a RC column into thin layers, so that it can be analysed; an example of such a division is shown in Fig.1. The modified compression field theory can then be applied to cases where the RC element in each divided layer is subjected to both uniform shear stress and normal stress. Using this method, it becomes possible to perform a sectional analysis (Vecchio et al., 1986; Nakamura et al., 1994-1995) of RC beams that have been subjected to bending moments, shear forces and axial forces.

2.2 Concrete Stress-strain Relationships

With regards to concrete stress-strain relationships, the principal compressive stress can be calculated using Equation (1) of the parabola formula in order to determine the principal tensile strain's relationship using \( \beta \).

The principal tensile stress, which is the result of the tension of the concrete stiffening after cracking, takes into account Equation (2).

Furthermore, the stress-strain relationships of reinforcing bars can be obtained from a bi-linear equation.

Principal compressive stress:

\[
f_{c2} = f_c + \beta \frac{f_{cr}}{f_c} \left( \frac{\varepsilon_2}{\varepsilon_c} \right)^2
\]

Principal tensile stress:

\[
f_{e1} = \frac{f_{cr} \left( 2 + \frac{f_{cr}}{f_c} \frac{\varepsilon_2}{\varepsilon_c} \right)}{1 + \sqrt{200 \cdot \varepsilon_t}}
\]

\[
\beta = \frac{1 - 0.8 \cdot 0.34 \cdot \frac{f_r}{f_c}}{1 + \sqrt{200 \cdot \varepsilon_t}} = 0.33 \frac{f_r}{f_c}
\]

\( f_c \) : is the concrete cylinder strength
\( f_{cr} \) : is the concrete tensile cracking stress
\( f_{e1} \) : is the principal tensile stress
\( f_{e2} \) : is the principal compressive stress
\( \varepsilon_c \) : is the principal tensile strain
\( \varepsilon_c \) : is the principal compressive strain

2.3 Strength of Confined Concrete

For strength of confined concrete, the equation used was determined by Sakino's formula shown in equation (3).

Using the latter, shear strength including the constraint effect was required.

\[
f_{cc} = f_r + 4.18 \cdot f_{cc}
\]

\[
= 0.8 \cdot f_c + 4.18 \left( \frac{1}{2} \rho h \cdot f_{cr} \left( 1 - \frac{S}{2 \cdot D_c} \cdot \frac{f_{cc}}{f_{cr}} \right) \right)
\]

\( f_{cc} \) : is the Strength of confined concrete
\( f_r \) : is the Coefficient of lateral stress
\( f_{cr} \) : is the Yield strength of transverse reinforcement
\( f_p \) : is the Compressive strength of concrete cylinder
\( f_c \) : is the Concrete cylinder strength
\( \rho h \) : is the Volumetric ratio of rectilinear transverse reinforcement
\( D_c \) : is the Distance between centroids of perimeter transverse reinforcement
\( S \) : is the Spacing of transverse reinforcement

3. Numerical Analysis

3.1 Failure Mode in the Analysis Model

The failure mode in this study is classified as being either a shear failure or a flexural failure. The conditions for such failures are as follows:

Shear failure mode: Beyond the yield strain, the yield region of the strain of the transverse reinforcement expands in a circumferential direction in the vicinity of the central part of the depth of the cross-section. Along with the expansion, the calculation becomes harder to converge as the increase in the strain of the transverse reinforcement in an axial direction on the compression side increases beyond the yield strain and further increases in the region of the compressive yield concrete layer.

Flexural failure mode: The calculation does not converge either due to the increase in the axial strain of the main reinforcement on the tension side beyond the yield strain nor due to the increase in the region of the concrete layer in order to reach the compressive strength on the compressive side followed by a gradual expansion of the region toward the central region of the depth of the cross-section.

3.2 Comparison with Previous Experimental Results

We have examined whether a circular RC column could be analyzed for shear strength by our model.

Fig.2. shows a comparison of the computational results of the study with those of previous bending shear experiments. The specimens used in the comparison were 105 circular section column pieces that had previously been described in manuscripts (Dongbum et al., 2002; Yamada et al., 2003; Korenaga et al., 1998; Sako et al., 1999; Nagae et al., 1999; Arai et al., 2000; Sako et al., 2000; Sako et al., 2001; Yamamoto et al., Honjou, 2001; Yano et al., 2002; Yamamoto et al., 2002; Suzuki et al., 1988; Itoh et al., 1986-1988).

The average value and
The coefficient of variation of the force ratio were 1.14 and 0.13, respectively. The figure shows that the analytical values were in good agreement with the result from the experiments.

Fig. 3. shows a comparison between the shear failure of 74 specimens as determined by both experiments and the analytical model. The average value was 1.11, which indicates that the analytical values were close to the experimental ones.

3.3 Review of the AIJ Sectional Replacement Method

We also examined whether the numerical analysis model could be used to replace the AIJ sectional replacement model. Various factors of the RC column specimens used in experiments conducted by Kim et al. (Dongbum et al., 2002) and Yamada et al. (Yamada et al., 2003) were used to examine the suitability of our analytical model.

![Fig. 2. Correlations between the Experimental Shear Force and the Calculated Shear Force for 105 Circular Section Column Pieces](image)

![Fig. 3. Correlations between the Experimental Shear Force and the Calculated Shear Force for the Shear Failure of 74 Specimens](image)

![Fig. 4. Comparison between the Circular and Rectangular Sections, as Obtained from Experiments](image)

![Fig. 5. Comparison between the Circular and Rectangular Cross-sections Used in the Analysis](image)
with a coefficient of variation of 0.09. As such, it is possible to use the AIJ sectional replacement model instead. Fig.5, also shows a comparison between the shear strength of a circular RC column, as calculated by the circular section, and the shear strength as calculated by replacing the circular section with equivalent rectangular sections. The □ and △ symbols denote the results when the sections were replaced by squares and rectangles, respectively.

As can be seen from Fig.5, the average value of the force ratio between the circular and AIJ sectional replacement sections and the coefficient of variation are 0.95 and 0.09 respectively, this indicates the suitability of replacing the circular sections with the AIJ sectional replacement model. In addition, we also found that the shear strength in the AIJ sectional replacement sections tended to be larger than those in the circular sections judging from the average force ratio value of 0.95.

### 3.4 Examination Based on the Shear Span-depth Ratio

Fig.6 shows the relationship between the shear force, Q, and the bending moment, M, of two specimens at failure in a shear strength analysis, which used M/QD as a parameter, i.e. a circular section has (M/QD = 1.0), while a square section has (M/QD = 1.12) (Ohmiya et al., 2003). In this case, we replaced various factors associated with the circular section specimen with equivalent factors for the square section. The relationship between the diameter D of the circular section and the depth of the replacement square cross-section, D, is expressed by \( D' = 0.89D \).

We found that the M – Q relationship at the time of a shear or flexural failure tended to show interaction curves in both the circular and square sections. Because a border of shear failure and flexural failure (circle: 1.5 – 2.0, square: 2.25 – 2.81) existed on the M – Q curve, it may be possible to obtain this border area analytically. Furthermore, we found that, in the region of shear failure, the shear strength in the square section was larger than that in the circular section.

This may be due to the impact of the way in which the reinforcement effect of the transverse reinforcement of the square section is described in the literature 20) - 21). (Ohmiya et al., 2003; Sakino et al., 1993)

### 3.5 Review of the Constraint Effect

In using the numerical analysis to calculate the shear strength, we reviewed how the consideration of the confinement effect of the concrete affected the computational results. An equation proposed by Sakino et al., 22) estimates concrete strength was used to calculate the strength of the confined concrete. For the 105 specimens of confined concrete mentioned in section 3.2, the ratio between the concrete strength, \( f'c \), and the cylinder strength, \( fc \), i.e. \( f'c/fc \) (here after referred to as the ratio of strength increase) was obtained by this equation. The proportion of specimens whose ratio of strength increase was less than 1.0, between 1.0 – 1.1, between 1.1 – 1.2 and more than 1.2 was 56%, 17%, 12% and 15% respectively.

Since the increase in strength due to factors such as the volume ratio of transverse reinforcement and yield strength was small for specimens analyzed by this study, it suggests that the differences between the values obtained by the estimation equation of the concrete strength and those of the cylinder strength were small.

Fig.7 shows a comparison between the shear strength of the experimental and calculated values based on a consideration of the confinement effect exploiting the estimated strength of the confined concrete. Thirteen of the specimens used for the comparison were based on the literature; 14 had a larger transverse reinforcement ratio and a yield strength showing an average value of 1.26 for the ratio of strength increase, while six of them were higher than 1.2. These specimens were selected based on the assumption that they might be significantly affected by the confinement effect. In addition, the computational results of the cylinder strength are also shown in the figure for the purposes of comparison.
The force ratio of the experimental and calculated values based on the estimated strength of the confined concrete was 1.00 on average with a coefficient of variation of 0.10 (denoted by $\circ$ in the figure). Furthermore, the force ratio of the experimental and calculated values based on the cylinder strength was 1.17 on average with a coefficient of variation of 0.13 (denoted by $\square$ in the figure). We found that the calculated results of the RC column with the larger reinforcement ratio, or the high-strength reinforcing bar, agreed well with the experiment results when we took into account the confinement effect of concrete.

4. Summary

The results obtained by this study are summarized below:

1) Even though there was a slight difference in the consideration of the force ratio of past experimental results and the computational results obtained by this study, which was 1.14 on average and had a coefficient of variation of 0.13, the numerical analytical method proposed by this study may be able to predict the shear strength of RC columns that have circular sections in the shear failure region.

2) The analytical method proposed by this study may be suitable for the replacement of the AIJ sectional replacement method.

3) It is possible to find the border of shear failure and flexural failure by using $M/QD$ as a parameter by using the proposed numerical analytical model.

4) The computational results of RC columns with larger yield strengths and reinforcement ratios of transverse reinforcement were closer to the experimental values when the model used an estimated strength of the confined concrete.

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