A PROMETHEE II Method Based on Regret Theory Under the Probabilistic Linguistic Environment

XIANG JIA AND XINFAN WANG
School of Science, Hunan University of Technology, Zhuzhou 412007, China
Corresponding author: Xinfan Wang (zzwxfydm@126.com)

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ABSTRACT The PROMETHEE II method is useful for the decision-making problems to order the alternatives. This paper proposes a PROMETHEE II method based on regret theory under the probabilistic linguistic environment. The probabilistic linguistic evaluations are obtained by applying the basic unit-interval monotonic function-based method. At first, the obtained probabilistic linguistic evaluations are normalized and further transformed to the benefit types. Then, considering the psychological behavior of the decision-makers, the preference value of an alternative over others under each criterion is calculated by combining the regret theory. Hereafter, a group of preference values of an alternative over others under all criteria are weighted aggregated to the preference index. Furthermore, the net-flow is obtained by operating the outgoing flow minus the incoming flow. The alternatives are ordered according to their net-flows. A problem for selecting the cloud serving for a college is resolved to verify the feasibility and the effectiveness of the proposed method.

INDEX TERMS Decision-making, PROMETHEE II method, probabilistic linguistic term sets, regret theory.

I. INTRODUCTION Decision-making is common in our daily life. The main task of the decision-maker (DM) is to order the alternatives according to their associated evaluations. There are several decision-making methods, such as the Technique for Order Preference by Similarity to an Ideal Solution (TOPSIS) [1]–[4], Visekriterijumska Optimizacija I Kompromiso Resenje (VIKOR) [5], [6], aggregation operator-based decision-making method [7]–[12]. In order to propose a simple decision-making method and make it easily understood by the DMs, Brans and Vincke [13] proposed a preference ranking organization method for enrichment evaluations (PROMETHEE) which is divided into the PROMETHEE I and the PROMETHEE II methods. The core of the PROMETHEE is the definition of the preference functions. The preference functions are defined in six conditions, where the criteria are separated into six kinds: usual criteria, Quasi-criteria, criteria with linear preference, level-criteria, criteria with linear preference and indifference area, and Gaussian criteria. The PROMETHEE I method is used to identify the relationships among alternatives, however, there exists incomparability. To avoid the incomparability of the alternatives, Brans and Vincke [13] advanced the PROMETHEE I method to the PROMETHEE II method to express a complete ranking. Since the simplicity and the effectiveness of the PROMETHEE II method, there are several studies aiming to theoretically advance it. Eppe and Smet [14] defined the piecewise linear value functions to approximate the PROMETHEE II’s net flow scores. Dağdeviren [15] integrated the analytic hierarchy process (AHP) and PROMETHEE method to propose a decision-making method. Govindan et al. [16] combined the decision-making trial and evaluation laboratory (DEMATEL)-based DEMATEL-analytic network process (DANP) with PROMETHEE and introduced an evaluation approach. The modified PROMETHEE are utilized to tackle the real-life decision-making problems. Vujosevic and...
Popovic [17] utilized the PROMETHEE method to compare the energy performance of hotel buildings. Taha and Rostam [18] presented the hybrid fuzzy AHP-PROMETHEE decision support system to select the machine tool. Vinodh and Girubha [19] used the PROMETHEE to select the sustainable concepts for manufacturing organization. By considering the reality that DMs prefer to provide the linguistic evaluations, Chen et al. [20] applied the PROMETHEE in investment portfolio decision-making, where the evaluations are 2-tuple linguistic terms. Tian et al. [21] introduced the hesitant fuzzy linguistic information to the PROMETHEE method to allow the DMs to provide several linguistic evaluations. Recently, Liu and Li [22] proposed the PROMETHEE II method under the probabilistic linguistic environment, where the evaluations are probabilistic linguistic term sets (PLTSs).

The PLTS is defined by Pang et al. [2] to overcome the drawback of the hesitant fuzzy linguistic term sets that the probabilities of the involved linguistic terms (LTs) are equal. Exhaustive decision-making methods have been proposed on the basis of the PLTSs. Since the PLTSs are fuzzy sets, to straightforwardly operate them may cause information distortion. To overcome this drawback, the comparison method, operational laws and information measures for the PLTSs are defined. Bai et al. [23] defined the comparisons of the PLTSs, where the comparison is expressed by diagram method. Liao et al. [24] defined the novel operations for the PLTSs. Wang et al. [25] defined the distances for the PLTSs and further applied them to decision-making. Liu et al. [26] defined the entropy measures of the PLTSs including fuzzy entropy, hesitant entropy, and total entropy. The PLTSs are also introduced to several classical decision-making methods. Liao et al. [27] proposed a linear programing method for the probabilistic linguistic information based on the defined inconsistency and the consistency indexes. Wu et al. [28] extracted the probabilistic linguistic MULTIMOORA based on the probabilistic linguistic expectation function and the improved Borda rule. Feng [29] defined the possibility degree for the PLTSs, on the basis of this, a probabilistic linguistic QUALIFLEX method is proposed. Cheng [30] developed the venture capital group decision-making method with interaction, where the evaluations are PLTSs. For more details, please see reference [31]. Hence, the PLTS is a useful tool for describing the evaluations for decision-making methods.

Seen from the above decision-making methods, there are still two issues: (1) The probabilistic linguistic evaluations are straightforwardly obtained and do not consider the influence of multiple periods. (2) The probabilistic linguistic decision-making methods do not take into account the regret of the DMs.

Xu [32] defined the weight vector of multiple periods and pointed out the importance of the dynamic decision-making. Simultaneously, the probabilistic linguistic evaluations cannot be straightforwardly obtained in the condition that the linguistic evaluations are given in multiple periods. Thus, the determination of the weight vector of periods and the method for obtaining the probabilistic linguistic evaluations should be investigated. Tian et al. [33] modified the TODIM method by considering the psychological state of the DMs. Liu and You [34], Liu and Teng [35], and Zhang et al. [36] introduced the PLTSs to the TODIM method. They further pointed out that the psychological behaviors play an important role in decision-making. It should be highlighted that Tian et al. [21] combined the PROMETHEE with the prospect theory and further presented group decision-making models for venture capitalists, where the hesitant fuzzy linguistic term sets are introduced to portray the evaluations. The models they presented are sufficient, however, they failed to deeply investigate the fuzziness of the evaluations. Due to the complexity of the decision-making problems, the DMs not only provide several linguistic evaluations or an uncertain linguistic evaluation, but also hold their tendency or confidence degree for each involved linguistic evaluation. The latter constructs the probabilistic linguistic information. Tian et al. [21] described the bounded rationality instead of the complete rationality in the light of the prospect theory. They emphasized that the DMs tend to be risk averse for gains and risk seeking for losses, however, they failed to consider another common psychological behavior, the regret. The prospect theory measures the attitude of the DMs over the alternatives when difference exists between their evaluations and their reference points, it cannot be utilized when the DMs straightforwardly compare the pairwise alternatives. In such case, the DMs feel joyful when choosing the alternative that is superior to others, while they feel regret when choosing the alternative that is inferior to others. The most important is, the cores of the regret theory and the PROMETHEE are the same, i.e., the comparison. Hence, to combine the regret with the PROMETHEE is to enhance the cores of them and ensure the feasibility at the same time. On the basis of the above analyses, we propose a PROMETHEE II method based on the regret theory under the probabilistic linguistic environment, where the evaluations are obtained from multiple periods.

The main contributions are summarized as follows: (1) The PLTSs are constructed to portray the evaluations obtained from multiple periods. (2) The regret avoidance of the DMs are fully considered in the proposed method. (3) The regret theory is firstly introduced to the PROMETHEE II method. (4) The detailed decision-making steps of the proposed method are presented.

The rest of this paper is organized as follows: Section 2 reviews several concepts of the PLTSs. Section 3 proposes a PROMETHEE II method in details. Section 4 uses the proposed method to help a college to order four cloud serving. Section 5 conducts sensitivity analyses and comparative analysis for the proposed method. The conclusion and future works are included in the last section.

II. PRELIMINARIES

In this section, we review several concepts with respect to the PLTSs.

For a subscript-symmetric LTS $S = \{s_\xi | \xi = -\tau, \cdots, -1, 0, 1, \cdots, \tau\}$, where $\tau$ is a positive integer, the mid LT $s_0$
represents an assessment of “indifference”, and with the rest of the LTs being placed symmetrically around it, Xu [37] defined:

1. \( s_{\xi_1}, s_{\xi_2} \in S \), if \( \xi_1 > \xi_2 \), then \( s_{\xi_1} > s_{\xi_2} \);
2. The negation operator is \( \text{neg} (s_{\xi_1}) = s_{-\xi_1} \), especially, \( \text{neg} (s_0) = s_0 \).

The operational laws are also defined.

Definition 1 [37]: Let \( s_{\xi_1}, s_{\xi_2} \in S \) and \( \lambda \in [0, 1] \), then

1. \( s_{\xi_1} \oplus s_{\xi_2} = s_{\lambda \xi_1 + \xi_2} \);
2. \( \lambda s_{\xi_1} = s_{\lambda \xi_1} \).

Definition 2 [2]: Let \( S = \{ s_{\xi} \mid \xi = -\tau, \cdots, -1, 0, 1, \cdots, \tau \} \) be a LTs, \( \tau \) is a positive integer, a PLTS can be defined as:

\[
L(p) = \left\{ \left( L^{(l)} \left( p^{(l)} \right) \right) \mid L^{(l)} \in S, p^{(l)} \geq 0, l = 1, 2, \cdots, \#L(p) \right\},
\]

where \( L^{(l)} \left( p^{(l)} \right) \) is the LT \( L^{(l)} \) associated with the probability \( p^{(l)} \), and \( \#L(p) \) is the number of all different LTs in \( L(p) \).

Since the elements in PLTSs can be ordered in any permutations, which causes the distortion of information, an order method for the PLTSs is defined.

Definition 3 [2]: Given a PLTS \( L(p) = \{ L^{(l)} \left( p^{(l)} \right) \mid l = 1, 2, \cdots, \#L(p) \} \) and \( r^{(l)} \) is the subscript of LT \( L^{(l)} \). \( L(p) \) is called an ordered PLTS, if the LTs \( L^{(l)} \left( p^{(l)} \right) \) are arranged according to the values of \( r^{(l)} p^{(l)} \) (\( l = 1, 2, \cdots, \#L(p) \)) in descending order.

For the PLTSs where the involved probabilities are incomplete, normalization method is defined.

Definition 4 [2]: Given a PLTS \( L(p) \) with \( \#L(p) \leq 1 \), then the associated PLTS \( \bar{L}(p) \) is defined by

\[
\bar{L}(p) = \left\{ \left( L^{(l)} \left( p^{(l)} \right) \right) \mid l = 1, 2, \cdots, \#L(p) \right\},
\]

where \( p^{(l)} = \frac{p^{(l)}}{\sum_{l=1}^{\#L(p)} p^{(l)}} \), for all \( l = 1, 2, \cdots, \#L(p) \).

Different PLTSs may have different numbers of the involved LTs which causes great trouble to operate. For two PLTSs

\[
L_1(p_1) = \left\{ L_1^{(l)} \left( p_1^{(l)} \right) \mid l = 1, 2, \cdots, \#L_1(p_1) \right\}
\]

and

\[
L_2(p_2) = \left\{ L_2^{(l)} \left( p_2^{(l)} \right) \mid l = 1, 2, \cdots, \#L_2(p_2) \right\},
\]

where \( \#L_1(p_1) \) and \( \#L_2(p_2) \) are the numbers of LTs in \( L_1(p_1) \) and \( L_2(p_2) \) respectively, Pang et al. [2] pointed out that if \( \#L_1(p_1) > \#L_2(p_2) \), then \( \#L_1(p_1) - \#L_2(p_2) \) the smallest LTs in \( L_2(p_2) \) should be added to \( L_2(p_2) \) and their associated probabilities are zero.

The comparison between the PLTSs is also significant. Pang et al. [2] defined a method to compare the PLTSs based on the mean-variance rule.

Definition 5 [2]: Let

\[
L_1(p_1) = \left\{ L_1^{(l)} \left( p_1^{(l)} \right) \mid l = 1, 2, \cdots, \#L_1(p_1) \right\}
\]

and

\[
L_2(p_2) = \left\{ L_2^{(l)} \left( p_2^{(l)} \right) \mid l = 1, 2, \cdots, \#L_2(p_2) \right\}
\]

be two PLTSs, \( r_1^{(l)} \) and \( r_2^{(l)} \) be the subscripts of \( L_1^{(l)} \) and \( L_2^{(l)} \) respectively, then

1. if \( E(L_1(p_1)) > E(L_2(p_2)) \), then \( L_1(p_1) > L_2(p_2) \);
2. if \( E(L_1(p_1)) = E(L_2(p_2)) \), then
   a. if \( \sigma(L_1(p_1)) < \sigma(L_2(p_2)) \), then \( L_1(p_1) > L_2(p_2) \);
   b. if \( \sigma(L_1(p_1)) = \sigma(L_2(p_2)) \), then \( L_1(p_1) = L_2(p_2) \),

where \( E(L_1(p_1)) = s_{\tau_1} = \frac{\sum_{l=1}^{\#L_1(p_1)} r_1^{(l)} p_1^{(l)}}{\sum_{l=1}^{\#L_1(p_1)} p_1^{(l)}} \) and \( \sigma(L_1(p_1)) = \left( \sum_{l=1}^{\#L_1(p_1)} \left( p_1^{(l)} (r_1^{(l)} - \bar{r}_1)^2 \right) \right)^{1/2} \)

are the deviation degrees of \( L_1(p_1) \) and \( L_2(p_2) \) respectively.

In order to measure the difference between PLTSs, Lin and Xu [38] defined the normalized Hamming distance (NHD).

Definition 6 [38]: Let

\[
L_1(p_1) = \left\{ L_1^{(l)} \left( p_1^{(l)} \right) \mid l = 1, 2, \cdots, \#L_1(p_1) \right\}
\]

and

\[
L_2(p_2) = \left\{ L_2^{(l)} \left( p_2^{(l)} \right) \mid l = 1, 2, \cdots, \#L_2(p_2) \right\}
\]

be two PLTSs, \( \#L_1(p_1) = \#L_2(p_2) \), \( r_1^{(l)} \) and \( r_2^{(l)} \) be the subscripts of \( L_1^{(l)} \) and \( L_2^{(l)} \) respectively, the NHD between them is defined as

\[
d(L_1(p_1), L_2(p_2)) = \frac{1}{\#L_1(p_1)} \sum_{l=1}^{\#L_1(p_1)} \frac{r_1^{(l)}}{2\tau} - \frac{r_2^{(l)}}{2\tau}.
\]

III. DECISION-MAKING METHOD

In this section, we describe the decision-making problems and then propose a PROMETHEE II method based on regret theory under the probabilistic linguistic environment. A probabilistic linguistic evaluation obtaining method is firstly proposed, subsequently, the decision-making method combining the obtained probabilistic linguistic evaluations and the regret theory-based PROMETHEE II method is proposed.
A. DESCRIPTION OF DECISION-MAKING PROBLEMS

Let $A = \{ A_i | i \in M \}$ be an alternative set, $M = \{1, 2, \ldots, m\}$, $m \geq 2$, $C = \{ C_j | j \in N \}$ be the criterion set with respect to the alternatives, $N = \{1, 2, \ldots, n\}$, $n \geq 2$. The weight vector of $C$ is $\omega = \left(\omega_1, \omega_2, \ldots, \omega_n\right)^T, 0 \leq \omega_j \leq 1, \sum_{j=1}^{n} \omega_j = 1$. The probabilistic linguistic evaluation of alternative $A_i$ under the criterion $C_j$ given by the DM is expressed as $e_{ij}$. The task of the DM is to give the evaluation with respect to the alternatives under each criterion and further order the alternatives.

B. OBTAINING PROBABILISTIC LINGUISTIC EVALUATIONS

Xu [39] pointed out the dynamic decision-making (or multi-periods decision-making) plays an important role in modern decision science. Actually, the evaluations with respect to multiple periods enable the DM to make a more scientific decision. Let $t_1, t_2, \ldots, t_k, \ldots, t_q$ be $q$ periods, $q \geq 2$, and $(e_{ij})^k$ be the linguistic evaluation of alternative $A_i$ under the criterion $C_j$ in the period $t_k$. Xu [37] indicated that each period has a weight and further defined the weight vector of $q$ periods as $w = (w_1, w_2, \ldots, w_q)^T, 0 \leq w_k \leq 1, \sum_{k=1}^{q} w_k = 1$.

In order to obtain the weight vector of periods, Yager [40] proposed the basic unit-interval monotonic (BUM) function based method: Let $Q : [0, 1] \rightarrow [0, 1]$ be a function having the following properties:

(i) $Q(0) = 0$,
(ii) $Q(1) = 1$,
(iii) $Q(x) \geq Q(y)$ if $x \geq y$.

Then $Q$ is a BUM function. By using the BUM function, the weight vector $w$ can be obtained as:

$$w_k = Q\left(\frac{k}{q}\right) - Q\left(\frac{k-1}{q}\right), \quad k = 1, 2, \ldots, q. \quad (2)$$

Let $Q(x) = x^\alpha, \alpha > 0$, then

$$w_k = \left(\frac{k}{q}\right)^\alpha - \left(\frac{k-1}{q}\right)^\alpha, \quad k = 1, 2, \ldots, q. \quad (3)$$

The linguistic evaluations and the associated weights of periods can be integrated to the probabilistic linguistic evaluation

$$e_{ij} = \left\{ (e_{ij}^{(1)} (w_1), e_{ij}^{(2)} (w_2), \ldots, e_{ij}^{(k)} (w_k), \ldots, e_{ij}^{(q)} (w_q)) \right\}. \quad (3)$$

Note that the term $w_k$ in $e_{ij}$ is the weight of period rather than the probability. Since Pang et al. [2] pointed out that term $p$ in the PLTSs can be interpreted as the importance, $e_{ij}$ is also a PLTS. To deal with the information obtained from multiple periods, existing techniques mainly focus on aggregating them, such as aggregation operator [12], [32], [39]. The key should be pointed out is that, although aggregation operator simplifies the calculation, the aggregated result cannot represent the individual characteristics. Especially for the condition that the evaluations are obtained from several periods rather than numerous periods, it is property to integrate them to the PLTSs rather than straightforwardly aggregate them.

C. DECISION-MAKING PROCESS

To ensure the probabilistic linguistic evaluations can be straightforwardly operated, they should be ordered, normalized and added the smallest LT. For convenience, we call a probabilistic linguistic evaluation which has been ordered, normalized and added the smallest LT the normalized evaluation. There are two kinds of criteria: benefit criteria and cost criteria. Benefit criteria express that a higher evaluation is better, while cost criteria express that a lower one is better. For consistency, the evaluations under the cost criteria should be transformed to the benefit types. Tian et al. [21] defined an approach for transforming the hesitant fuzzy linguistic information under the cost criterion to the benefit type, where the subscripts of the involved LTs are transformed to their opposite numbers. On the basis of this, we extend this approach to accommodate the probabilistic linguistic environment. If $e_{ij}$ is a normalized evaluation under the cost criterion, the benefit type of which is

$$-e_{ij} = \left\{ (-e_{ij}^{(1)} (w_1), (-e_{ij}^{(2)} (w_2), \ldots, (-e_{ij}^{(q)} (w_q)) \right\}. \quad (4)$$

As shown in Preliminaries, the involved LTs are transformed in the light of the negation operator. Suppose $\{(s_{\xi_1}) (w_1), (s_{\xi_2}) (w_2), (s_{\xi_3}) (w_3)\}$ is a probabilistic linguistic evaluation under the cost criterion, the benefit type of which is $\{(s_{\xi_1}) (w_1), (s_{\xi_2}) (w_2), (s_{\xi_3}) (w_3)\}$. For $\xi_k \in \{\xi_1, \xi_2, \xi_3\}$, there are possibly three conditions: $\xi_k = 0$, $\xi_k = 0$, $\xi_k < 0$. If $\xi_k > 0$, the linguistic evaluation is relatively high. Since cost criteria express that a lower one is better, a higher one is actually dissatisfied. $s_{\xi_1}$ is the symmetrical element of $s_{\xi_3}$ in the LTSs. The satisfied degree of $s_{\xi_3}$ under the benefit criterion and that of $s_{\xi_3}$ under the cost criterion are identical. On the contrast, this also holds if $\xi_k < 0$. If $\xi_k = 0$, the meaning of $s_{\xi_3}$ and $s_{\xi_1}$ are “indifference” no matter under the cost criteria or the benefit criteria. By doing this, the type of the probabilistic linguistic evaluations can be flexibly transformed.

Regret is a common psychological behavior. Since regret is qualitative, it can be hardly described. Loomes and Sugden [41] introduced the regret theory to express the regret by specific function. The key point of the regret theory is the comparison. We take alternatives $A_i$ and $A_h$ as an example here. If $e_{ij}$ is larger than $e_{hj}$, the DM is joyful to prefer $A_i$ under the criterion $C_j$. If $e_{ij}$ is smaller than $e_{hj}$, the DM is regret to prefer $A_i$ under the criterion $C_j$. Since the degree of preference can be measured, DMs’ regret can be quantitatively expressed by the regret theory. Also, the DM whose psychological behavior is regret prefer to make a complete comparison such that $e_{ij} \geq e_{hj}$ or $e_{ij} < e_{hj}$. Hence, we consider the usual criteria in decision-making problems. By combining the regret theory and the PROMETHEE II method, the preference function can be expressed as

$$\Delta_j (A_i, A_h) = \begin{cases} 1 - e^{-bd(e_{ij}, e_{hj})}, & \text{if } e_{ij} \geq e_{hj}, \\ 1 - e^{bd(e_{ij}, e_{hj})}, & \text{if } e_{ij} < e_{hj}. \end{cases} \quad (4)$$
where $\delta$ is the index of regret avoidance, $\delta > 0$, $d(e_{ij}, e_{ij})$ is the NHD between $e_{ij}$ and $e_{ij}$, $\Delta_j(A_i, A_h)$ is the preference value of $A_i$ over $A_h$ under the criterion $C_j$.

It is obvious that the larger value of $\delta$ is, the higher absolute value of the preference value will be. If $e_{ij}$ is equal or larger than $e_{ij}$, then the value of $\Delta_j(A_i, A_h)$ is equal or larger than zero, which means $A_i$ is preferred to $A_h$ under the criterion $C_j$. If $e_{ij}$ is smaller than $e_{ij}$, then the value of $\Delta_j(A_i, A_h)$ is smaller than zero, which means $A_h$ is preferred to $A_i$ under the criterion $C_j$.

The preference index can be obtained as

$$\pi(A_i, A_h) = \sum_{j=1}^{n} \omega_j \Delta_j(A_i, A_h), \quad (5)$$

where $\omega_j$ is the weight of criterion $C_j$, $\Delta_j(A_i, A_h)$ is the preference value of $A_i$ over $A_h$ under the criterion $C_j$, $\pi(A_i, A_h)$ is the preference index of $A_i$ over $A_h$.

The outgoing flow

$$\phi^+(A_i) = \sum_{h \neq i} \pi(A_i, A_h) \quad (6)$$

and the incoming flow

$$\phi^-(A_i) = \sum_{h \neq i} \pi(A_h, A_i) \quad (7)$$

can be defined.

Subsequently, the net-flow can be expressed as

$$\phi(A_i) = \phi^+(A_i) - \phi^-(A_i). \quad (8)$$

The net-flow can be used to rank the alternatives: $A_i$ outranks $A_h$, if and only if $\phi(A_i) > \phi(A_h)$. $A_i$ is indifference to $A_h$, if and only if $\phi(A_i) = \phi(A_h)$.

### D. Decision-Making Steps

We summarize the decision-making steps as follows:

**Step 1:** Obtain the probabilistic linguistic evaluations.

**Step 2:** Normalize the obtained evaluations and then transform the normalized evaluations under the cost criteria to the benefit types.

**Step 3:** Calculate the preference value of $A_i$ over other alternatives under each criterion by using equation (4).

**Step 4:** Calculate the preference index of $A_i$ over other alternatives by using equation (5).

**Step 5:** Calculate the outgoing flow and the incoming flow of $A_i$ by using equations (6) and (7) respectively.

**Step 6:** Calculate the net-flow of $A_i$ by using equation (8) and then rank the alternatives.

### IV. Illustrative Example

A college in Zhuzhou is seeking the cloud serving. Cloud servings are the increase, use, and interactive modes of internet-based related services, which usually involve the dynamic, easily expandable and often virtualized resources through the internet. There are four alternatives, Ali cloud ($A_1$), Tencent cloud ($A_2$), Jinshan cloud ($A_3$), and Hecai cloud ($A_4$) are to be ordered. The criteria with respect to the alternatives are efficiency of cloud computing ($C_1$), space of cloud storage ($C_2$), and safety ($C_3$). The weight vector of criteria is given by the DM as $(0.4, 0.3, 0.3)^T$. The basic LTS is $S = \{-3, -2, -1, 0, 1, 2, 3\}$ i.e. [none, very low, low, medium, high, very high, perfect]. The evaluations of alternatives are previously given by the DM in three periods, which are shown as Table 1.

**Step 1:** Let $\alpha = \frac{1}{2}$, the weight vector of period can be obtained by using equation (3) as $(0.5776, 0.2389, 0.1835)^T$. The evaluations can be further integrated to the probabilistic linguistic evaluations as shown as Table 2.

**Step 2:** Normalize the probabilistic linguistic evaluations. Since the criteria are all benefit, there is no need to transform. The normalized evaluations are shown as Table 3.

**Step 3:** Calculate the preference value. We take the calculation of $\Delta_1(A_1, A_2)$ as an example. According to the reference [42], we let $\delta = 0.88$. Since the NHD between $e_{11}$ and $e_{21}$ is 0.0556 and $e_{11} > e_{21}$ according to the score values, $\Delta_1(A_1, A_2) = 1 - e^{-0.88 \times 0.0556} = 0.0477$.

The preference values under the criterion $C_1, C_2$, and $C_3$ are integrated to Table 4-Table 6. In Table 4, the element in row $i$, column $h$ represents $\Delta_1(A_i, A_h)$. In Table 5, the element in row $i$, column $h$ represents $\Delta_2(A_i, A_h)$. In Table 6, the element in row $i$, column $h$ represents $\Delta_3(A_i, A_h)$.
TABLE 3. Normalized evaluations.

|      | $C_1$                      | $C_2$                      | $C_3$                      |
|------|---------------------------|---------------------------|---------------------------|
| $A_1$| $(s_1(0.7611), s_2(0.5776), s_3(0.4224), s_4(0.5776), s_5(0.4224))$ | $(s_3(0.5776), s_4(0.5776), s_5(0.5776))$ | $(s_1(0.5776), s_3(0.5776), s_5(0.5776))$ |
| $A_2$| $(s_1(0.2389), s_2(0.1835), s_3(0.2389), s_4(0.2389), s_5(0.2389))$ | $(s_3(0.2389), s_5(0.2389))$ | $(s_1(0.2389), s_3(0.2389), s_5(0.2389))$ |
| $A_3$| $(s_1(0.5776), s_2(0.1835), s_3(0.5776), s_5(0.5776))$ | $(s_3(0.5776), s_5(0.5776))$ | $(s_1(0.5776), s_3(0.5776), s_5(0.5776))$ |
| $A_4$| $(s_1(0.4224), s_2(0.1835), s_3(0.5776), s_5(0.5776))$ | $(s_3(0.5776), s_5(0.5776))$ | $(s_1(0.5776), s_3(0.5776), s_5(0.5776))$ |

TABLE 4. Preference value under criterion $C_1$.

|      | $A_1$ | $A_2$ | $A_3$ | $A_4$ |
|------|-------|-------|-------|-------|
| $A_1$| 0     | 0.0477| 0.0352| 0.0724|
| $A_2$| -0.0501| 0     | -0.0314| 0.0391|
| $A_3$| -0.0365| 0.0305| 0     | 0.0557|
| $A_4$| -0.0781| -0.0407| -0.0589| 0     |

TABLE 5. Preference value under criterion $C_2$.

|      | $A_1$ | $A_2$ | $A_3$ | $A_4$ |
|------|-------|-------|-------|-------|
| $A_1$| 0     | 0.0305| 0.0490| 0.0089|
| $A_2$| -0.0314| 0     | 0.0191| -0.0351|
| $A_3$| -0.0515| -0.0195| 0     | -0.0421|
| $A_4$| -0.0090| 0.0339| 0.0404| 0     |

TABLE 6. Preference value under criterion $C_3$.

|      | $A_1$ | $A_2$ | $A_3$ | $A_4$ |
|------|-------|-------|-------|-------|
| $A_1$| 0     | 0.0418| -0.0538| -0.0566|
| $A_2$| -0.0436| 0     | -0.0997| -0.1027|
| $A_3$| 0.0510| 0.0907| 0     | -0.1027|
| $A_4$| 0.0536| 0.0931| 0.0081| 0     |

Step 4: Calculate the preference index by using equation (5). The preference index are integrated to Table 7, where the element in row $i$, column $h$ represents $\pi(A_i, A_h)$.

Step 5: Calculate the outgoing flows and the incoming flows by using equations (6) and (7) respectively.

Step 6: Calculate the net-flow by using equation (8). The net-flows of $A_1, A_2, A_3$, and $A_4$ are $0.1433$, $-0.2012$, $0.0591$, and $-0.0012$ respectively. Thus, the order of four alternatives is $A_1 > A_3 > A_4 > A_2$.

TABLE 7. Preference index.

|      | $A_1$ | $A_2$ | $A_3$ | $A_4$ |
|------|-------|-------|-------|-------|
| $A_1$| 0     | 0.0408| 0.0127| 0.0147|
| $A_2$| -0.0426| 0     | -0.0368| -0.0257|
| $A_3$| -0.0148| 0.0336| 0     | 0.0072|
| $A_4$| -0.0179| 0.0218| -0.0090| 0     |

TABLE 8. Weight vectors and orders.

| Value of $\alpha$ | Weight vector | Order of alternatives |
|-------------------|---------------|-----------------------|
| $\frac{1}{3}$     | $(0.6934, 0.1802, 0.1264)^T$ | $A_1 > A_3 > A_4 > A_2$ |
| $\frac{1}{2}$     | $(0.5776, 0.2389, 0.1835)^T$ | $A_1 > A_3 > A_4 > A_2$ |
| $1$               | $(\frac{1}{3}, \frac{1}{3}, \frac{1}{3})^T$ | $A_1 > A_3 > A_4 > A_2$ |
| $2$               | $(0.1111, 0.3333, 0.5556)^T$ | $A_1 > A_3 > A_4 > A_2$ |
| $3$               | $(0.0370, 0.2592, 0.7038)^T$ | $A_1 > A_3 > A_4 > A_2$ |

V. ANALYSES

In this section, we conduct two sensitivity analyses and comparative analysis to verify the stability and the progressiveness of the proposed method.

A. SENSITIVITY ANALYSIS FOR $\alpha$

The value of $\alpha$ is decisive for determining the weight vector of periods. If $\alpha$ is changed, the weight vector of periods changes consistently. In other words, the value of $\alpha$ influences the total decision-making process. We let $\alpha = \frac{1}{3}, \frac{1}{2}, 1, 2, 3$ to test the stability of the proposed method. Five results of the order of alternatives in Section 4 are integrated in Table 8.

It can be found that the weight vector of periods changes according to the value of $\alpha$. If $0 < \alpha < 1$, the weight of the former period is larger than that of the later period. If $\alpha = 1$, the weights of all periods are identical. If $\alpha > 1$, the weight of the later period is larger than that of the former period. The evaluations of later periods of $A_2$ under the criteria $C_1$ and $C_2$ are larger than that of $A_4$, the sum of the weights of $C_1$ and $C_2$ are 0.7, which is larger than that of $C_3$, hence $A_2$ outranks $A_4$ when $\alpha = 2$ and $\alpha = 3$. The value of the net-flow of $A_1$ is always the highest among five alternatives, hence the proposed method is stable.

B. SENSITIVITY ANALYSIS FOR $\delta$

Note that the change of the value of $\delta$ can also affect the order result. To figure out the influence to the order caused by $\delta$, we hold $\alpha = \frac{1}{2}$. Since $\delta > 0$, the proposed method is used to compute the order of four alternatives in Section 4 in the conditions where $\delta$ is varied from 0.01 to a sufficiently large value. The orders are summarized in Table 9. The values of net-flow in four intervals are represented in Figure 1, Figure 2, Figure 3, and Figure 4 respectively, where the full line, the dotted line, the line with “-+-”, and the line with "--".
TABLE 9. Orders in different intervals.

| Interval | Order |
|----------|-------|
| \( \delta \in (0, 29.95] \) | \( A_4 > A_1 > A_2 > A_3 \) |
| \( \delta \in [29.96, 45.28) \) | \( A_3 > A_1 > A_2 > A_4 \) |
| \( \delta \in [45.29, 79.00] \) | \( A_3 > A_1 > A_2 > A_4 \) |
| \( \delta \in [79.01, +\infty) \) | \( A_3 > A_1 > A_2 > A_4 \) |

FIGURE 1. Values of net-flow in the first interval.

Assume the meaning of \( \Delta_j (A_i, A_h) \rightarrow -\infty \) is the same as dissatisfaction of \( A_i \) compared to \( A_h \), after rewriting the \( \Delta_j (A_i, A_h) \rightarrow -\infty \) to \( \Delta_j (A_i, A_h) = 0 \), it is obvious that the equation (4) reduces to the preference function of the classical PROMETHEE II method. Take the condition of \( \delta \in [79.01, +\infty) \) as an example, \( A_4 \) is superior to other alternatives. However, by observing the original evaluations, the most of \( A_4 \) are notably lower than \( A_1 \) and \( A_3 \). Hence, the order \( A_4 > A_3 > A_1 > A_2 \) is not reasonable, and the value of \( \delta \) should be controlled in a low level. Fortunately, in the low level \((0, 29.95]\), the order remains \( A_1 > A_3 > A_4 > A_2 \), which also indicates the proposed method is stable.

C. COMPARATIVE ANALYSIS

To verify the progressiveness of the proposed method, we compare the proposed method to the PL-PROMETHEE II method proposed by Liu and Li [22]. We use the PL-PROMETHEE II method to cope with the illustrative example in Section 4, the order of the alternatives is \( A_1 > A_3 > A_4 > A_2 \). The comparative analysis can be conducted on four parts:

1. The probabilistic linguistic evaluations in reference [22] are straightforwardly obtained, while they are obtained by using the BUM function based method in the proposed method. We not only consider the hesitancy of the DM, but also consider the weight of the periods. The obtained probabilistic linguistic evaluations can reflect the fuzziness and the weight vector of periods simultaneously. By varying the value of index \( \alpha \), the weight vector of periods can be properly adjusted.

2. The preference function in reference [22] is defined by combining the possibility degree of PLTSs, while it is defined by combining the regret theory in the proposed method. The psychological behavior of the DM is considered in the proposed method, which is a significant point in human nature and inevitably affect the decision-making process.

3. It can be found in Table 4-Table 7 that if \( \Delta (A_i, A_h) > 0 \), then \( \Delta (A_h, A_i) < 0 \). The signal in upper triangle and that in lower triangle both satisfy the condition of the proposed method.

\[ \Delta_j (A_i, A_h) = \begin{cases} e_{ij} - e_{hj} & \text{if } e_{ij} > e_{hj} \\ e_{hj} - e_{ij} & \text{if } e_{ij} \leq e_{hj} \end{cases} \]

\[ \Delta (A_i, A_h) = \Delta_j (A_i, A_h) \]

\[ \Delta_j (A_i, A_h) \rightarrow -\infty \]

\[ \Delta (A_i, A_h) \rightarrow -\infty \]
The sensitivity analyses by varying the parameters in the BUM probability of the proposed PROMETHEE II method is verified by integrated to the probabilistic linguistic evaluations. The stability values of an alternative over others are aggregated to the linguistic evaluations and their associated weights are applied to obtain the weights of periods, later, in multiple periods is considered. The BUM function based method is solved to exhibit the effectiveness and the feasibility of the proposed method.

A PROMETHEE II method is proposed based on regret theory under the probabilistic linguistic environment. The probabilistic linguistic evaluations are obtained by using the BUM function based method, they are firstly normalized and transformed to the benefit types. The preference value of an alternative over others under each criterion is then calculated by considering the regret behavior. Furthermore, the preference values of an alternative over others are aggregated to the preference index by combining the weight vector of criteria. Hereafter, the preference indexes of an alternative over others and that of others over this alternative are aggregated to the outgoing and incoming flow respectively. Afterwards, the net-flows of alternatives are obtained. On the basis of this, the alternatives can be ordered by comparing their net-flows, the higher the net-flow is, the former the alternative ranks. An illustrative example for selecting the cloud serving is solved to exhibit the effectiveness and the feasibility of the proposed method.

The condition that the DM give their linguistic evaluations in multiple periods is considered. The BUM function based method is applied to obtain the weights of periods, later, the linguistic evaluations and their associated weights are integrated to the probabilistic linguistic evaluations. The stability of the proposed PROMETHEE II method is verified by the sensitivity analyses by varying the parameters in the BUM function and the regret theory respectively. The regret of the DM is also considered and further expressed by the regret theory when comparing two alternatives. The comparative analysis exhibits the progressiveness of the proposed method compared to the PL-PROMETHEE II method.

In future work, we will develop the PROMETHEE II method in two conditions: one is to investigate the decision-making problems by combining the weight determining method; the other is to consider the correlation among criteria in decision-making problems.

**DECLARATION OF COMPETING INTEREST**

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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XIANG JIA was born in 1996. He is currently pursuing the master’s degree in science with the Hunan University of Technology, Zhuzhou, Hunan, China. His research interests include fuzzy sets and decision making.

XINFAN WANG received the Ph.D. degree in management science and engineering from Central South University, Changsha, Hunan, China, in 2014. He is a currently a Professor with the School of Science, Hunan University of Technology, Zhuzhou, Hunan, China. His research interests include aggregation operators, decision making, and uncertainty.