FEKETE-SZEGÖ INEQUALITY FOR ANALYTIC AND BI-UNIVALENT
FUNCTIONS SUBORDINATE TO CHEBYSHEV POLYNOMIALS

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Abstract. In the present paper, a new subclass of analytic and bi-univalent functions
by means of Chebyshev polynomials is introduced. Certain coefficient bounds for func-
tions belong to this subclass are obtained. Furthermore, the Fekete-Szegö problem in
this subclass is solved.

1. Introduction

The classical Chebyshev polynomials of degree \( n \) of the first and second kinds, which are
denoted respectively by \( T_n(t) \) and \( U_n(t) \), have generated a great deal of interest in recent
years. These orthogonal polynomials, in a real variable \( t \) and a complex variable \( z \), have
played an important role in applied mathematics, numerical analysis and approximation
theory. For this reason, Chebyshev polynomials have been studied extensively, see [8, [10,
[16]. In the study of differential equations, the Chebyshev polynomials of the first and
second kinds are the solution to the Chebyshev differential equations

\[
(1 - t^2)y'' - ty' + n^2 y = 0
\]

and

\[
(1 - t^2)y'' - 3ty' + n(n + 2)y = 0,
\]

respectively. The roots of the Chebyshev polynomials of the first kind are used as nodes
in polynomial interpolation and the monic Chebyshev polynomials minimize all norms
among monic polynomials of a given degree. For a brief history of Chebyshev polynomials
of the first and second kinds and their applications, the reader is referred to [19, 22).

A classical result of Fekete and Szegö [13] determines the maximum value of \( |a_3 - \eta a_2^2| \),
as a non-linear functional of the real parameter \( \eta \), for the class of normalized univalent
functions

\[
f(z) = z + a_2 z^2 + a_3 z^3 + \cdots.
\]

There are now several results of this type in the literature, each of them dealing with
\( |a_3 - \eta a_2^2| \) for various classes of functions defined in terms of subordination (see e.g., [11, 20]).
Motivated by the earlier work of Dziok et al. [10], the main focus of this work is to utilize
the Chebyshev polynomials expansions to solve Fekete-Szegö problem for certain subclass
of bi-univalent functions (see, for example, [5, 6, 7, 14]).

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This paper is divided into three sections with this introduction being the first. In Section 2, we define the class of analytic and bi-univalent functions $B_{\Sigma}(\lambda, \mu, t)$ using the generating function for the Chebyshev polynomials of the second kind, and we also discuss some other definitions and results. Section 3 is devoted to solve problems concerning the coefficients of functions in the class $B_{\Sigma}(\lambda, \mu, t)$. Section 4 is the main part of the paper, we find the sharp bounds of functionals of Fekete-Szegö type.

2. Definitions and preliminaries

Let $A$ denote the class of functions of the form:

$$f(z) = z + \sum_{n=2}^{\infty} a_n z^n,$$

(2.1)

which are analytic in the open unit disk $U = \{z \in \mathbb{C} : |z| < 1\}$. Further, by $\mathcal{S}$ we shall denote the class of all functions in $A$ which are univalent in $U$.

Given two functions $f, g \in A$. The function $f(z)$ is said to be subordinate to $g(z)$ in $U$, written $f(z) \prec g(z)$, if there exists a Schwarz function $\omega(z)$, analytic in $U$, with $\omega(0) = 0$ and $|\omega(z)| < 1$ for all $z \in U$, such that $f(z) = g(\omega(z))$ for all $z \in U$. Furthermore, if the function $g$ is univalent in $U$, then we have the following equivalence (see [17] and [23]):

$$f(z) \prec g(z) \Leftrightarrow f(0) = g(0) \text{ and } f(U) \subset g(U).$$

The Koebe one-quarter theorem [9] asserts that the image of $U$ under each univalent function $f$ in $\mathcal{S}$ contains a disk of radius $\frac{1}{4}$. According to this, every function $f \in \mathcal{S}$ has an inverse map $f^{-1}$, defined by

$$f^{-1}(f(z)) = z \quad (z \in U),$$

and

$$f \left( f^{-1}(w) \right) = w \quad (|w| < r_0(f); r_0(f) \geq \frac{1}{4}).$$

In fact, the inverse function is given by

$$f^{-1}(w) = w - a_2 w^2 + (2a_2^2 - a_3)w^3 - (5a_2^3 - 5a_2a_3 + a_4)w^4 + \cdots.$$  

(2.2)

A function $f \in A$ is said to be bi-univalent in $U$ if both $f(z)$ and $f^{-1}(w)$ are univalent in $U$. Let $\Sigma$ denote the class of bi-univalent functions in $U$ given by (2.1). For a brief history and some intriguing examples of functions and characterization of the class $\Sigma$, see Srivastava et al. [21] and Frasin and Aouf [11], see also [2, 3, 4, 12, 15, 18].

The Chebyshev polynomials of the first and second kinds are orthogonal for $t \in [-1, 1]$ and defined as follows:
Definition 2.1. The Chebyshev polynomials of the first kind are defined by the following three-terms recurrence relation:

\[ T_0(t) = 1, \]
\[ T_1(t) = t, \]
\[ T_{n+1}(t) := 2tT_n(t) - T_{n-1}(t). \]

The first few of the Chebyshev polynomials of the first kind are
\[ T_2(t) = 2t^2 - 1, \quad T_3(t) = 4t^3 - 3t, \quad T_4(t) = 8t^4 - 8t^2 + 1, \ldots \] (2.3)

The generating function for the Chebyshev polynomials of the first kind, \( T_n(t) \), is given by:

\[ F(z, t) = \frac{1 - tz}{1 - 2tz + z^2} = \sum_{n=0}^{\infty} T_n(t)z^n \quad (z \in \mathbb{U}). \]

Definition 2.2. The Chebyshev polynomials of the second kind are defined by the following three-terms recurrence relation:

\[ U_0(t) = 1, \]
\[ U_1(t) = 2t, \]
\[ U_{n+1}(t) := 2tU_n(t) - U_{n-1}(t). \]

The first few of the Chebyshev polynomials of the second kind are
\[ U_2(t) = 4t^2 - 1, \quad U_3(t) = 8t^3 - 4t, \quad U_4(t) = 16t^4 - 12t^2 + 1, \ldots \] (2.4)

The generating function for the Chebyshev polynomials of the second kind, \( U_n(t) \), is given by:

\[ H(z, t) = \frac{1}{1 - 2tz + z^2} = \sum_{n=0}^{\infty} U_n(t)z^n \quad (z \in \mathbb{U}). \]

The Chebyshev polynomials of the first and second kinds are connected by the following relations:

\[ \frac{dT_n(t)}{dt} = nU_{n-1}(t); \quad T_n(t) = U_n(t) - tU_{n-1}(t); \quad 2T_n(t) = U_n(t) - U_{n-2}(t). \]

Definition 2.3. For \( \lambda \geq 1, \mu \geq 0 \) and \( t \in (1/2, 1) \), a function \( f \in \Sigma \) given by (2.1) is said to be in the class \( \mathcal{B}_\Sigma(\lambda, \mu, t) \) if the following subordinations hold for all \( z, w \in \mathbb{U} \):

\[ (1 - \lambda) \frac{f(z)}{z} + \lambda f'(z) + \mu zf''(z) \prec H(z, t) := \frac{1}{1 - 2tz + z^2} \] (2.5)

and

\[ (1 - \lambda) \frac{g(w)}{w} + \lambda g'(w) + \mu wg''(w) \prec H(w, t) := \frac{1}{1 - 2tw + w^2}, \] (2.6)

where the function \( g(w) = f^{-1}(w) \) is defined by (2.2).
Remark 2.4. (1) For $\lambda = 1$ and $\mu = 0$, we have the class $B_\Sigma(1, 0, t) := B_\Sigma(t)$ of functions $f \in \Sigma$ given by (2.1) and satisfying the following subordination conditions for all $z, w \in U$:

$$f'(z) < H(z, t) = \frac{1}{1 - 2tz + z^2}$$

and

$$g'(w) < H(w, t) = \frac{1}{1 - 2tw + w^2}.$$ 

This class of functions have been introduced and studied by Altinkaya and Yalçın [5].

(2) For $\mu = 0$, we have the class $B_\Sigma(\lambda, 0, t) := B_\Sigma(\lambda, t)$ of functions $f \in \Sigma$ given by (2.1) and satisfying the following subordination conditions for all $z, w \in U$:

$$(1 - \lambda) \frac{f(z)}{z} + \lambda f'(z) < H(z, t) = \frac{1}{1 - 2tz + z^2}$$

and

$$(1 - \lambda) \frac{g(w)}{w} + \lambda g'(w) < H(w, t) = \frac{1}{1 - 2tw + w^2}.$$ 

This class of functions have been introduced and studied by Bulut et al. [7].

3. Coefficient bounds for the function class $B_\Sigma(\lambda, \mu, t)$

We begin with the following result involving initial coefficient bounds for the function class $B_\Sigma(\lambda, \mu, t)$.

**Theorem 3.1.** Let the function $f(z)$ given by (2.1) be in the class $B_\Sigma(\lambda, \mu, t)$. Then

$$|a_2| \leq \frac{2t \sqrt{2t}}{\sqrt{|(1 + \lambda + 2\mu)^2 - 4t^2[(\lambda + 2\mu)^2 - 2\mu]|}} \quad (3.1)$$

and

$$|a_3| \leq \frac{4t^2}{(1 + \lambda + 2\mu)^2} + \frac{2t}{1 + 2\lambda + 6\mu}. \quad (3.2)$$

**Proof.** Let $f \in B_\Sigma(\lambda, \mu, t)$. From (2.5) and (2.6), we have

$$(1 - \lambda) \frac{f(z)}{z} + \lambda f'(z) + \mu z f''(z) = 1 + U_1(t)w(z) + U_2(t)w^2(z) + \cdots \quad (3.3)$$

and

$$(1 - \lambda) \frac{g(w)}{w} + \lambda g'(w) + \mu wg''(w) = 1 + U_1(t)v(w) + U_2(t)v^2(w) + \cdots, \quad (3.4)$$

for some analytic functions

$$w(z) = c_1z + c_2z^2 + c_3z^3 + \cdots \quad (z \in U),$$
and
\[ v(w) = d_1 w + d_2 w^2 + d_3 w^3 + \cdots \quad (w \in \mathbb{U}), \]
such that \( w(0) = v(0) = 0, |w(z)| < 1 \ (z \in \mathbb{U}) \) and \(|v(w)| < 1 \ (w \in \mathbb{U})\).

It follows from (3.3) and (3.4) that
\[
(1 - \lambda) \frac{f(z)}{z} + \lambda f'(z) + \mu zf''(z) = 1 + U_1(t)c_1 z + \left[ U_1(t)c_2 + U_2(t)c_1^2 \right] z^2 + \cdots
\]
and
\[
(1 - \lambda) \frac{g(w)}{w} + \lambda g'(w) + \mu wg''(w) = 1 + U_1(t)d_1 w + \left[ U_1(t)d_2 + U_2(t)d_1^2 \right] w^2 + \cdots.
\]

A short calculation shows that
\[
(1 + \lambda + 2\mu) a_2 = U_1(t)c_1,
\]
\[
(1 + 2\lambda + 6\mu) a_3 = U_1(t)c_2 + U_2(t)c_1^2,
\]
and
\[
- (1 + \lambda + 2\mu) a_2 = U_1(t)d_1,
\]
\[
(1 + 2\lambda + 6\mu) (2a_2^2 - a_3) = U_1(t)d_2 + U_2(t)d_1^2.
\]

From (3.5) and (3.7), we have
\[
c_1 = -d_1,
\]
and
\[
2 (1 + \lambda + 2\mu)^2 a_2^2 = U_1^2(t) \left( c_1^2 + d_1^2 \right).
\]

By adding (3.6) to (3.8), we get
\[
2 (1 + 2\lambda + 6\mu) a_2^2 = U_1(t) (c_2 + d_2) + U_2(t) (c_1^2 + d_1^2).
\]

By using (3.10) in (3.11), we obtain
\[
\left[ 2 (1 + 2\lambda + 6\mu) - \frac{2U_2(t)}{U_1^2(t)} (1 + \lambda + 2\mu)^2 \right] a_2^2 = U_1(t) (c_2 + d_2).
\]

It is fairly well known [9] that if \(|w(z)| < 1\) and \(|v(w)| < 1\), then
\[
|c_j| \leq 1 \text{ and } |d_j| \leq 1 \text{ for all } j \in \mathbb{N}.
\]

By considering (2.4) and (3.13), we get from (3.12) the desired inequality (3.1).

Next, by subtracting (3.8) from (3.6), we have
\[ 2 \left( 1 + 2\lambda + 6\mu \right) a_3 - 2(1 + 2\lambda + 6\mu)a_2^2 = U_1(t) (c_2 - d_2) + U_2(t) \left( c_1^2 - d_1^2 \right). \quad (3.14) \]

Further, in view of (3.9), it follows from (3.14) that
\[ a_3 = a_2^2 + \frac{U_1(t)}{2(1 + 2\lambda + 6\mu)} (c_2 - d_2). \quad (3.15) \]

By considering (3.10) and (3.13), we get from (3.15) the desired inequality (3.2). This completes the proof of Theorem 3.1.

Taking \( \lambda = 1 \) and \( \mu = 0 \) in Theorem 3.1, we get the following corollary.

**Corollary 3.2.** [7] Let the function \( f(z) \) given by (2.1) be in the class \( \mathcal{B}_\Sigma(t) \). Then
\[ |a_2| \leq \frac{t\sqrt{2t}}{\sqrt{1 - t^2}}, \]
and
\[ |a_3| \leq t^2 + \frac{2}{3} t. \]

For Corollary 3.2, it’s worthy to mention that Altinkaya and Yalçın [5] have obtained a remarkable result for the coefficient \( |a_2| \), as shown in the following corollary.

**Corollary 3.3.** Let the function \( f(z) \) given by (2.1) be in the class \( \mathcal{B}_\Sigma(t) \). Then
\[ |a_2| \leq \frac{t\sqrt{2t}}{\sqrt{1 + 2t - t^2}}. \]

Taking \( \mu = 0 \) in Theorem 3.1 we get the following corollary.

**Corollary 3.4.** [7] Let the function \( f(z) \) given by (2.1) be in the class \( \mathcal{B}_\Sigma(\lambda, t) \). Then
\[ |a_2| \leq \frac{2t\sqrt{2t}}{\sqrt{(1 + \lambda)^2 - 4t^2\lambda^2}} \]
and
\[ |a_3| \leq \frac{4t^2}{(1 + \lambda)^2} + \frac{2t}{1 + 2\lambda}. \]

4. **Fekete-Szegö inequality for the function class \( \mathcal{B}_\Sigma(\lambda, \mu, t) \)**

Now, we are ready to find the sharp bounds of Fekete-Szegö functional \( a_3 - \eta a_2^2 \) defined for \( f \in \mathcal{B}_\Sigma(\lambda, \mu, t) \) given by (2.1).
Theorem 4.1. Let the function \( f(z) \) given by (2.1) be in the class \( B_\Sigma (\lambda, \mu, t) \). Then for some \( \eta \in \mathbb{R} \),

\[
|a_3 - \eta a_2^2| \leq \left\{ \begin{array}{ll}
\frac{4t}{1+2\lambda+6\mu}, & |\eta - 1| \leq M \\
\frac{16|\eta - 1|^3}{|1+\lambda+2\mu|^2 - 4(\lambda+2\mu)^2 - 2\mu|}, & |\eta - 1| \geq M
\end{array} \right.
\]

(4.1)

where

\[
M := \frac{|(1 + \lambda + 2\mu)^2 - 4t ((\lambda + 2\mu)^2 - 2\mu)|}{4(1 + 2\lambda + 6\mu)t^2}.
\]

Proof. Let \( f \in B_\Sigma (\lambda, \mu, t) \). By using (3.12) and (3.15) for some \( \eta \in \mathbb{R} \), we get

\[
a_3 - \eta a_2^2 = (1 - \eta) \left[ \frac{U_3^1(t) (c_2 + d_2)}{2(1 + 2\lambda + 6\mu)U_2^1(t) - 2(1 + \lambda + 2\mu)^2 U_2(t)} + \frac{U_1(t) (c_2 - d_2)}{2(1 + 2\lambda + 6\mu)} \right]
\]

\[
= U_1(t) \left[ \left( h(\eta) + \frac{1}{2(1 + 2\lambda + 6\mu)} \right) c_2 + \left( h(\eta) - \frac{1}{2(1 + 2\lambda + 6\mu)} \right) d_2 \right],
\]

where

\[
h(\eta) = \frac{U_2^1(t) (1 - \eta)}{2 [(1 + 2\lambda + 6\mu)U_2^1(t) - (1 + \lambda + 2\mu)^2 U_2(t)]}.
\]

Then, in view of (2.4), we easily conclude that

\[
|a_3 - \eta a_2^2| \leq \left\{ \begin{array}{ll}
\frac{4t}{1+2\lambda+6\mu}, & |h(\eta)| \leq \frac{1}{2(1+2\lambda+6\mu)} \\
8|h(\eta)|t, & |h(\eta)| \geq \frac{1}{2(1+2\lambda+6\mu)}
\end{array} \right.
\]

This proves Theorem 4.1. \( \square \)

We end this section with some corollaries concerning the sharp bounds of Fekete-Szegö functional \( a_3 - \eta a_2^2 \) defined for \( f \in B_\Sigma (\lambda, \mu, t) \) given by (2.1).

Taking \( \eta = 1 \) in Theorem 4.1, we get the following corollary.

Corollary 4.2. Let the function \( f(z) \) given by (2.1) be in the class \( B_\Sigma (\lambda, \mu, t) \). Then

\[
|a_3 - a_2^2| \leq \frac{4t}{1+2\lambda+6\mu}.
\]

Taking \( \lambda = 1 \) and \( \mu = 0 \) in Theorem 4.1, we get the following corollary.

Corollary 4.3. Let the function \( f(z) \) given by (2.1) be in the class \( B_\Sigma (t) \). Then for some \( \eta \in \mathbb{R} \),

\[
|a_3 - \eta a_2^2| \leq \left\{ \begin{array}{ll}
\frac{4t}{3}, & |\eta - 1| \leq \frac{1-t^2}{3t^2} \\
\frac{4|\eta - 1|^3}{1-t^2}, & |\eta - 1| \geq \frac{1-t^2}{3t^2}
\end{array} \right.
\]

Taking \( \eta = 1 \) in Corollary 4.3, we get the following corollary.
Corollary 4.4. Let the function $f(z)$ given by (2.1) be in the class $B_{\Sigma}(t)$. Then

$$|a_3 - a_2^2| \leq \frac{4}{3}t.$$ 

Taking $\mu = 0$ in Theorem 4.1 we get the following corollary.

Corollary 4.5. Let the function $f(z)$ given by (2.1) be in the class $B_{\Sigma}(\lambda, t)$. Then for some $\eta \in \mathbb{R}$,

$$|a_3 - \eta a_2^2| \leq \begin{cases} \frac{4t}{1 + 2\lambda}, & |\eta - 1| \leq \frac{|(1+\lambda)^2 - 4t^2\lambda^2|}{4(1+2\lambda)t^2} \\ \frac{16|\eta - 1|}{|(1+\lambda)^2 - 4t^2\lambda^2|}, & |\eta - 1| \geq \frac{|(1+\lambda)^2 - 4t^2\lambda^2|}{4(1+2\lambda)t^2} \end{cases}$$

(4.2)

Taking $\eta = 1$ in Corollary 4.5 we get the following corollary.

Corollary 4.6. Let the function $f(z)$ given by (2.1) be in the class $B_{\Sigma}(\lambda, t)$. Then

$$|a_3 - a_2^2| \leq \frac{4t}{1 + 2\lambda}.$$ 

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