Modeling of deformation processes of structural steels along circular paths

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Abstract. To verify the mathematical model of the theory of processes, numerical calculations of the complex elastoplastic deformation of steel along a flat curvilinear trajectory with simultaneous tension-compression with torsion were carried out. Comparison results are compared with the data of the physical experiment. It has been shown that the mathematical model used qualitatively and quantitatively satisfactorily describes the basic effects of complex plastic deformation for a given class of paths.

1. Introduction
Conducting experimental studies with complex loading beyond the elastic limit is an important part of the formulation of new mathematical models of the processes of elastic-plastic deformation of the material and their verification. To check the physical reliability of the constitutive relations of the theory of plasticity, it is necessary to compare the model calculation data with the experimental data bank available in the literature for various classes of deformation trajectories. Therefore, the study of the laws of complex elastic-plastic deformation of structural materials is important for the development of the fundamental principles of the theory of plasticity. The ideas and results of the construction of the defining relations of the theory of plasticity and the conduct of basic experimental studies are presented in the works [1–9].

This paper presents experimental data on a complex deformation trajectory (tension with torsion – P-M experiments), representing a flat curve of five sections. The realized trajectory is important for demonstrating the very nontrivial connection between stresses and strains during elastic-plastic deformation of the material. To verify the proposed mathematical model, the results of theoretical calculations are compared with experimental data. The calculated and experimental results are given in the vector representation of stresses and strains by A.A. Ilyushin [2–4]. Previously, the variants of the used model of the theory of processes were used to describe the processes of deformation of broken straight-line trajectories [10–12], as well as trajectories with curvilinear sections [13–16].

2. Constitutive equation
In theory processes for numerical modeling of elastic-plastic deformation of materials in Ilyushin's five-dimensional deviatoric vector space $E_5$ at complex flat trajectories with the analytical curvilinear sections used by the constitutive relations [1], considering scalar and vector properties of materials

$$
\frac{d\bar{\sigma}}{ds} = M_i \frac{d\bar{\sigma}}{ds} + \left( \frac{d\sigma}{ds} - M_i \cos \delta_i \right) \frac{\bar{\sigma}}{\sigma}, \quad \frac{d\bar{\kappa}}{ds} + \kappa_i = -\frac{M_i}{\sigma} \sin \delta_i, \quad (1)
$$
where $\mathbf{\sigma}, \mathbf{\varepsilon}$ – stress and strain vectors respectively; $s$ – is the arc length of the trajectory deformation; $\mathbf{\Omega} = \mathbf{\Omega}(s, \kappa_i, \mathbf{\Theta}_0)$ – the angle of delay, which is a functional of the process vector material properties characterizing each point of the trajectory of deformation in the direction of the vector $\mathbf{\sigma}$; $\kappa_i$ – the curvature of the trajectory; $\mathbf{\Theta}_0$ – the break angle at the starting point of the analytical trajectory; $\mathbf{\Theta} = \mathbf{\Theta}(s, \kappa_i, \mathbf{\Theta}_0)$ – the functionality of the process of scalar properties of the material; $M_i$ and $\frac{d\mathbf{\sigma}}{ds}$ – functionals of the deformation process, depending on the parameters of a complex loading $s, \kappa_i, \mathbf{\Theta}_0$.

The constitutive equations of the mathematical model theory in plane problems are equations (1) and universal approximation of functionals depending on all the above parameters is a complex loading for a flat curvilinear trajectories

$$\mathbf{\sigma}(s) = \Phi(s, \mathbf{\Theta}_0, \kappa_i) = \Phi(s) + A\mathbf{\Phi}\mathbf{\Omega} - B\Delta\kappa - \Delta\mathbf{\sigma},$$

$$\frac{d\mathbf{\sigma}}{ds} = \frac{d\Phi}{ds} + A\mathbf{\Phi} \frac{d\mathbf{\Omega}}{ds} - B \frac{d}{ds}(\Delta\kappa_i),$$

$$M_i = 2G_p + (2G - 2G_p)\mathbf{f}^q,$$  \hspace{1cm} (2)

where $\Delta s = s - s^\prime_k$ – the increment of the arc of the trajectory deformation; $s^\prime_k$ – the length of the arc at the break point of the trajectory, or change its curvature; $\Phi(s)$ – a universal hardening function of Odquist-Ilyushin for processes close to simple loading, without regard to their history; $\Delta\mathbf{\sigma} = \Phi(s^\prime_k) - \mathbf{\Theta}_k$ – the difference between the values of the universal functions of Odquist-Ilyushin and the real value of the stress vector module at the point of change sections of the trajectory deformation;

$$\Omega = \left(-\gamma \Delta s \exp[-\gamma \Delta s] + b \left(1 - \exp[-\gamma \Delta s]\right)\right)$$

– is a function of a complex loading, describing scalar «dive» stresses in complex unloading and subsequent plastic deformation;

$$f = f(\mathbf{\Theta}) = \frac{1 - \cos \mathbf{\Theta}}{2}, \quad f_0 = f(\mathbf{\Theta}_0) = \frac{1 - \cos \mathbf{\Theta}_0}{2}$$

– is a function of a complex loading taking into account the orientation of the stress vector in the process of deformation and its value at the break point of the trajectory; $A, B, b, \gamma, p, q$ – parameters approximations. To approximate a generic function hardening of Odquist-Ilyushin in proportional loading is used

$$\mathbf{\Theta} = \Phi(s) = \left\{ \begin{array}{ll}
\frac{2G}{\gamma} \left(1 - \exp[-\beta s]\right), & \text{if } 0 \leq s \leq s^\prime, \\
\sigma^\prime + 2G_s(s - s^\prime) + \sigma_t \left(1 - \exp[-\beta(s - s^\prime)]\right), & \text{if } s > s^\prime;
\end{array} \right. $$

$$\sigma^\prime = \sqrt{\frac{2}{3}} s^\prime; \quad \sigma^\prime, \sigma^\prime – \text{yield strength tensile; } s^\prime – \text{border sections of the deformation chart that separates the elastic part of the chart and the area yield } \left(0 \leq s \leq s^\prime\right) \text{ from the plot of self-hardening of material } \left(s > s^\prime\right); G – \text{shear modulus; } \sigma_t, G_s, \alpha, \beta \text{ is the experimentally determined material parameters from experiments on simple loading.}$$

Under given initial conditions for the components $e_k (k = 1, 3)$ of the strain vector $\mathbf{\varepsilon}$ and angle $\mathbf{\Theta}_0$, the basic equations of the model are given to the Koshi problem for the numerical solution of which
determine the integration component $S_k (k = 1, 3)$ of the stress vector $\bar{\sigma}$ and the angle of delay used Runge-Kutta method of fourth order accuracy in the linear algebra package MathWorks MATLAB.

3. **Comparison of numerical results with the experimental ones**

Experimental research was performed on an automated calculation-experimental complex of SN-EVM computer that implements the three-parameter impact on the model (axial tension-compression, torsion and internal pressure). As the sample used thin-walled tubular specimens made of St3 steel in the delivery state, with the working part of: length $l = 110$ mm, thickness $h = 1$ mm, and the diameter of the middle surface $d = 31$ mm. The Initial isotropy of the material specimens with a sufficient degree of accuracy was confirmed in experiments under simple loading conditions (tension, compression and torsion), where after processing, these experimental chart were taken the following values of the material parameters for steel St3 in approximation (6): $\sigma^* = 220$ MPa, $s^* = 10^{-2}$, $2G = 1.57 \cdot 10^8$ MPa, $\beta = 70$, $\alpha = 1500$, $\sigma_r = 54.9$ MPa, $2G_\tau = 1181.5$ MPa.

The program of the experiment at kinematic loading represents in the deviatoric space of deformations $e_i - e_i$ a flat curve according to the type «displaced fan», consisting of five sections (Figure 1).

![Figure 1. Trajectory of deformation](image)

The first straight section is implemented is proportional to the axial tension component $e_i$ to the value $e_i^* = 2\%$ in the second section without break was implemented in the trajectory of constant curvature (tension-compression and torsion) in the form of a circle of radius $R = 0.25\%$ and curvature $\kappa_1 = 400$; for the circle one complete revolution. At the third section, made a full rotation of a circle of radius $R = 0.5\%$ and curvature $\kappa_1 = 200$, and the fourth – a full rotation of a circle of radius $R = 0.75\%$ and curvature $\kappa_1 = 133.33$. On the last fifth section was stretching to the value $e_i^* \approx 3.9\%$ under constant $e_3$.

In the terminology of A. A. Ilyushin [2] implemented trajectory as flat curve is smooth, as in places of transition sections are missing a break points, that is the functions $\bar{J} = \bar{J}(x)$ describing the trajectory in a linear space, the first derivatives are continuous, and the second to suffer a rupture. Experiments show [16] that on smooth trajectories of change of curvature at the junction of the sections is equivalent to a break the trajectory with the formation of stress «dives».
However, as shown presented in this experiment, the effect of a complex loading is not immediately in the transition to the curved area, and approximately a quarter turn of a circle. This circumstance was taken into account in the calculation of the split sections of the circle into two parts (with lengths equal to a quarter and three quarters of a circle) with the introduction of them fictitious break angle to describe a scalar dives stresses using formulas (3), (5). As shown by experiments with two-link broken [1, 10], the influence of complex loading begins to significantly occur at the break angle $\theta^0 \geq 90^\circ$. So for the first parts of sections of circles, where the influence of complex loading slightly, was adopted $\theta^0 = 45^\circ$. For the second part of the circles, where the influence of complex loading significantly on it $\theta^0 = 90^\circ$. Figure 3 show chart $\sigma - s$ characterizing scalar properties of the materials, figure 4 show chart $\delta - \Delta s$ characterizing vector properties of the materials. Figure 5 and figure 6 shows the local strain chart $S_i - e_i$ and $S_j - e_j$ respectively. The numbers 1, 2, 3, 4, 5 in Figures 1–6 are designated the start point of the corresponding sections of the realized trajectory. Experimental data in the figures are indicated by dots, and the model estimated data – the solid line.

Figure 2. The response $S_i - S_3$.

Figure 3. Charts of deformation $\sigma - s$. 
Figure 4. Chart $\theta_1 - \Delta s$.

Figure 5. Chart $S_1 - \Theta_1$

Figure 6. Chart $S_3 - \Theta_3$
It can be seen qualitative and acceptable quantitative coincidence of experimental and accounting data about proposed mathematical model for the realized trajectory of the deformation in the form of a flat curve type «displaced fan» circle trajectories, which indicates the correctness of modeling the process of the complex elastic-plastic deformation of the material.

4. Conclusion
Verification of the mathematical model of the process theory by comparing the numerical results with the experimental data for elastic-plastic deformation on a flat curved path shows the correctness of the modeling of complex elastic-plastic deformation of structural steel for this class of trajectories.

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