Supplementary Materials for

Tracking the vector acceleration with a hybrid quantum accelerometer triad

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Real-time phase and frequency control

To address the issues of vibration noise and Doppler shift compensation on each axis of the interferometer, we developed a real-time (RT) phase and frequency compensation system inspired by Ref. (36). This system is based on a Xilinx Artix-7 field-programmable gate array (FPGA) that acquires the three classical accelerometer signals, computes the appropriate frequencies and phases, and communicates them to a direct digital synthesizer (DDS) within our radio-frequency (RF) chain. This RF chain derives an ultra-low phase noise signal at 6.834
GHz that is sent to the optical IQ modulator that generates the two optical Raman frequencies. Through the DDS, we directly control the Raman phase and frequency difference at the microsecond timescale. Using high-speed serial communication with the DDS, the FPGA allows us to adjust these quantities at each Raman pulse.

For each axis of the QuAT, the FPGA performs the following operations. Between each Raman pulse (which occur at times $t_1 = t_0$, $t_2 = t_0 + T$, $t_3 = t_0 + 2T$, and $t_4 = t_0 + 2T + 4\tau_\mu$, where $t_0$ is the initial time-of-flight), the RT system calculates the Doppler shift from the time integral of the acceleration $a_{\mu}^{cl}(t)$ measured by the classical accelerometer:

$$\omega^{\text{RT}}_{\mu,j} = k_\mu \int_{t_{j-1}}^{t_j} a_{\mu}^{cl}(t) \, dt \simeq \begin{cases} 0 & \text{for pulse } j = 1, \\ k_\mu (\bar{a}_\mu + b_\mu)(T + 2\tau_\mu) & \text{for pulse } j = 2, \\ k_\mu (\bar{a}_\mu + b_\mu)(T + 2\tau_\mu) & \text{for pulse } j = 3. \end{cases}$$

(1)

Here, $\bar{a}_\mu$ is the average acceleration during the atom interferometer and $b_\mu$ is the bias of the classical accelerometer projected along axis $\mu$. Just before each pulse, this frequency is added to the previous DDS frequency to compensate the Doppler shift. The Raman frequency then has a step-like temporal profile given by:

$$\Delta \omega^R_{\mu}(t) = \omega_{\text{HF}} + \omega_{\text{rec}} + \sum_j \omega^{\text{RT}}_{\mu,j} H(t - t_j),$$

(2)

where $H(t)$ is the Heaviside step function. This profile can be either phase-continuous or phase-discontinuous (i.e. the phase is reset to zero on each update) depending on the desired mode of the RT system, as we describe below. In addition to the frequency, the FPGA computes the following phase from the classical accelerometer signal:

$$\phi^\text{RT}_\mu = k_\mu \int_0^{t_4} f_\mu(t)a_{\mu}^{cl}(t) \, dt = k_\mu (\bar{a}_\mu + b_\mu)(T + 2\tau_\mu)^2 + \phi^\text{vib}_\mu.$$

(3)

Here, $f(t)$ is the interferometer response function (29, 30), and $\phi^\text{vib}_\mu$ is a phase due to vibration noise that varies randomly from shot-to-shot. For practical reasons, we use a triangular-shaped
response function and ignore the curvature of \( f_\mu(t) \) during the AI pulses:

\[
f_\mu(t) = \begin{cases} 
0 & \text{for } t \leq 0, t > 2T + 2\tau_\mu, \\
 t - (1 - 2/\pi)\tau_\mu & \text{for } 0 < t \leq T + 2\tau_\mu, \\
 2T + (3 + 2/\pi)\tau_\mu - t & \text{for } T + 2\tau_\mu < t \leq 2T + 4\tau_\mu. 
\end{cases}
\] (4)

This accounts for the pulse lengths only to first order. The phase \( \phi^\text{RT}_\mu \) is applied just before the final \( \pi/2 \)-pulse to cancel the kinetic phase due to atomic motion and vibration noise due to mirror motion. We emphasize that these phase and frequency updates must be triggered before each pulse, however to avoid a phase shift that depends on the pretrigger time (20 \( \mu \)s in our case), the updates must occur symmetrically with respect to the temporal center of the interferometer at \( t_2 = t_0 + T + 2\tau_\mu \) (see Fig. 11 in Ref. (40)).

Since the real-time system effectively suppresses mirror vibrations (the primary source of phase noise in the atom interferometers), it allows us to maintain the interferometer at any location on the fringe. To scan the phase of interferometer using the laser, we simply add a controlled laser phase to Eq. (3). This also enables the use of phase modulation schemes similar to those for atomic clocks to lock the interferometer to the central fringe—maximizing the sensitivity of each quantum accelerometer.

The RT system can be configured to operate in one of two modes that we label open-loop and closed-loop. Table S.1 summarizes the resulting phase and frequency shifts for both modes. Open-loop mode operates in a similar manner to vertically-oriented atomic gravimeters (13). Here, the raw signal from the classical accelerometer is digitally high-pass filtered on the FPGA—nullifying both the DC acceleration \( \ddot{a}_\mu \) and the accelerometer bias \( b_\mu \). In this case, the phase computed by the RT system is simply \( \phi^\text{RT}_\mu = \phi^\text{vib}_\mu \) and the Doppler shift compensation is implemented manually using an “effective chirp rate” \( \alpha_\mu \) defined by the user. Importantly, the frequency updates are made in a phase-continuous manner such that the resulting interferometer phase is \( \Phi_\mu \simeq (k_i\ddot{a}_\mu - \alpha_\mu)(T + 2\tau_\mu)^2 \)—allowing us to measure \( \ddot{a}_\mu \) from the central interference fringe.
In closed-loop mode, the digital filter is disabled and the Doppler shift is compensated as described by Eqs. (1) and (2). In this case, the frequency steps are made in a phase-discontinuous manner to nullify their phase contribution to the total interferometer phase. Instead, the kinetic component of the interferometer is compensated by applying the phase given by Eq. (3) just before the final Raman pulse. The resulting interferometer phase is $\Phi_\mu \simeq -k_\mu b_\mu (T + 2\tau_\mu)^2$—providing a direct measure of the accelerometer bias $b_\mu$. This is the basis on which we hybridize the classical and quantum accelerometer triads, as we describe below. We note that the frequency error initially produced by the accelerometer bias is $k_\mu |b_\mu| T \simeq 2\pi \times 800 \text{ Hz}$ for typical bias levels of $3 \text{ mg}$. This is completely negligible compared to the linewidth of the Raman resonance ($\Omega_\mu \simeq 2\pi \times 50 \text{ kHz}$). This error is further suppressed after a few cycles of the hybridization loop.

### Central-fringe lock

To optimize the sensitivity of the QuAT, we implemented a central fringe lock using the phase modulation protocol illustrated in Fig. S.1. Here, the phase of the interferometer is modulated

| Quantity | Open-loop | Closed-loop |
|----------|-----------|-------------|
| $k_\mu \Delta \mu$ | $k_\mu \alpha_\mu (T + 2\tau_\mu)^2 + \phi^{\text{vib}}_\mu$ | $k_\mu (a_\mu + b_\mu)(T + 2\tau_\mu)$ |
| $\omega^{\text{RT}}_{\mu,j}$ | $\alpha_\mu(T + 2\tau_\mu)$ | $k_\mu (a_\mu + b_\mu)(T + 2\tau_\mu)$ |
| $\varphi_{\mu,j}$ | $\frac{1}{2} j (j - 1) \alpha_\mu (T + 2\tau_\mu)^2$ | 0 |
| $\Delta \phi_\mu$ | $\alpha_\mu (T + 2\tau_\mu)^2 + \phi^{\text{vib}}_\mu$ | $k_\mu (a_\mu + b_\mu)(T + 2\tau_\mu)^2 + \phi^{\text{vib}}_\mu$ |
| $\Phi_\mu$ | $(k_\mu a_\mu - \alpha_\mu)(T + 2\tau_\mu)^2$ | $-k_\mu b_\mu(T + 2\tau_\mu)^2$ |

Table S.1: Summary of frequency and phase shifts for the two modes of the RT system. Here, $\Phi_\mu = k_\mu \Delta \mu - \Delta \phi_\mu$ is the total interferometer phase for axis $\mu = x, y, z$, which contains a kinetic component from the atom’s motion $k_\mu \Delta \mu$ and a laser phase component $\Delta \phi_\mu$. Here, $\Delta \mu = \mu(t_1) - 2\mu(t_2) + \mu(t_3)$ is the difference between wavepacket displacements relative to the mirror during the interferometer, $\Delta \phi_\mu = \varphi_{\mu,1} - 2\varphi_{\mu,2} + \varphi_{\mu,3} + \phi^{\text{RT}}_\mu$ is the difference between laser phases imprinted on the atom at each pulse, with $\varphi_{\mu,j} = \int_{t_0}^{t_j} [\omega^{\text{RT}}_{\mu,j} H(t - t_2) + \omega^{\text{RT}}_{\mu,j} H(t - t_3)] \, dt$ in open-loop and $\varphi_{\mu,j} = 0$ in closed loop.
Figure S.1: Central-fringe locking scheme for the QuAT. For each axis of the triad, the interferometer phase is modulated between $\pm \pi/2$ with alternating momentum transfer directions (labeled as $k_\uparrow$ and $k_\downarrow$, respectively). To initialize the feedback loop, one full cycle of 12 phase measurements is made to determine the full acceleration vector. After this, acceleration components are updated one at a time, cycling from $a_x, a_y, a_z, a_x, a_y, a_z, \ldots$, once every two measurements.

between opposite mid-fringe positions ($\pm \pi/2$), where the sensitivity to phase fluctuations is largest. This scheme has the advantage of being insensitive to fringe offset and contrast variations. We also alternated between $\pm k_\mu$ and combined measurements such that path-independent systematic effects are rejected on the fly (16, 29). In both open- and closed-loop mode, the total interferometer phase is steered toward zero using a simple proportional-integrator (PI) controller with an integrator time constant of $\sim 60T_{cyc} = 96$ s. In open-loop mode, this is achieved by acting on the effective chirp rate $\alpha_\mu$. In closed-loop mode, the lock is controlled through the bias $b_\mu$, fed back to the classical accelerometer stream. The effect of the central fringe lock is clear in the Allan deviation shown in Fig. 4 of the main text. The PI engages after approximately 10 s—initially creating a kink in the Allan deviation. For integration times greater than the time constant of the PI controller, the measurement sensitivity integrates as $1/\sqrt{\tau}$, as expected for white Gaussian phase noise.