Regular black holes in Rainbow Gravity

Ednaldo L.B. Junior a, Manuel E. Rodrigues b,c,*, Marcos V. de S. Silva c

a Faculdade de Engenharia da Computação, Universidade Federal do Pará, Campus Universitário de Tucuruí, CEP: 68464-000, Tucuruí, Pará, Brazil
b Faculdade de Ciências Exatas e Tecnologia, Universidade Federal do Pará, Campus Universitário de Abaetetuba, CEP 68440-000, Abaetetuba, Pará, Brazil
c Faculdade de Física, PPGF, Universidade Federal do Pará, 66075-110, Belém, Pará, Brazil

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Abstract

In this work, we consider that in energy scales near to the Planck energy, the geometry, mass, speed of light, the Newtonian constant of gravitation and matter fields will depend on the energy. This type of theory is known as Rainbow Gravity. We coupled the nonlinear electrodynamics to the Rainbow Gravity, defining a new mass function \( M(\sqrt{r}, \epsilon) \), such that we may formulate new classes of spherically symmetric regular black hole solutions, where the curvature invariants are well-behaved in all spacetime. The main differences between the General Relativity and our results in the Rainbow gravity are: a) The intensity of the electric field is inversely proportional to the energy scale. The higher the energy scale, the lower the electric field intensity; b) the region where the strong energy condition (SEC) is violated decreases as the energy scale increases.

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1. Introduction

One of the most interesting predictions of Einstein theory are black hole solutions. These objects have been challenging physicists since Karl Schwarzschild, in 1915, first predicted their
existence by solving the Einstein equations for the vacuum. The Schwarzschild solution, although elegant, had the characteristic that brought out the name “black hole”, which is the existence of a region of spacetime where it is only possible to enter, not to escape from, delimited by the so-called event horizon. This region has a singularity in the origin of the radial coordinate. Since that, other solutions arose in the literature, such as the solution from the first coupling with matter, the Reissner [1] and Nordstrom solution [2], where the coupling with matter was done through Maxwell electromagnetism, resulting in a spherically symmetric, static and charged solution, known as the Reissner-Nordstrom black hole. Another well-known solution of Einstein equations, which can be understood as a generalization of the Schwarzschild metric, is the Kerr solution [3], where rotation effects are considered in a spacetime without matter sources. Since then, numerous works have been done in search of new solutions to Einstein equations. However, these structures remained hidden from the eyes of the scientific community, until April 2019, the Event Horizon Telescope collaboration, led by John Wardle, revealed to the world the first image of a black hole [4], further contributing to the interest in these objects.

One of the great challenges of theoretical physics is the search for a unification of gravitation with quantum theory. Many attempts at unification have been, and are being proposed [5], however, none have the consistency necessary for a complete description of the theory. One of the main motivations for unifying gravity with quantum theory is the violation of Lorentz invariance, which we can consider as an essential requirement for formulating a quantum theory of gravity. An alternative to Lorentz symmetry violation is the modification of the standard energy-momentum dispersion ratio at the ultraviolet limit, this has already been observed in some theories like Horava-Lifshitz [6], loop quantum gravity [7], discreteness spacetime [8] and doubly special relativity [9]. To the doubly special relativity, the dispersion relation may be written as $E^2 f(\epsilon)^2 - p^2 g(\epsilon)^2 = m^2$ where $f(\epsilon)$ and $g(\epsilon)$ are functions of $\epsilon = E/E_P$ with $E$ being the energy of the particle used to analyze the spacetime and $E_P$ is the Planck energy. The functions $f(\epsilon)$ and $g(\epsilon)$ are known as rainbow functions and have phenomenological motivations [10]. The standard energy dispersion relation is recovered in the limit $\lim_{\epsilon \to 0} f(\epsilon) = \lim_{\epsilon \to 0} g(\epsilon) = 1$, known as infrared limit. The linear Lorentz transformations break the invariance of the theory, however, nonlinear Lorentz transformations are those that hold the theory invariant [11], in addition, not only the speed of light is a constant, but also Planck's energy, and it is impossible for a particle to reach energies greater than this limit.

Magueijo and Smolin [12] proposed a generalization from the doubly special relativity to what they called Rainbow Gravity, where the spacetime is represented by a family of parameters in the metric that is parameterized by $\epsilon$, causing the spacetime geometry to depend on the energy of the particle that is being used for test it, thus creating a rainbow of metric. Since then, some researchers have considered the Magueijo and Smolin theory in the study of black holes [13–27] and in generalized theories of gravity [28]-[29]. In the context of nonlinear electrodynamics (NED), we can highlight the M. Momennia et al. work [30], which shows that there is an essential singularity covered by an event horizon and verified the validity of the first law of thermodynamics in the presence of rainbow functions. In [31], the authors obtained exact black hole solutions in the Born-Infeld-dilaton gravity with a energy dependent Liouville-type potential, as well as the thermodynamic quantities. In [32], it has been shown that the inclusion of Gauss-Bonnet gravity in the context of Rainbow Gravity can significantly alter the constraints needed to obtain non-singular universes. One application of Rainbow Gravity is the black hole thermodynamics. Due to the energy scale dependence on the spacetime metric, entropy and temperature also have the same dependence [33]. Thus, the thermodynamics of these new solutions are altered showing the influence of the energy scale on the thermal system. In the NED context,
Dehghani showed the thermal fluctuations of AdS black holes in three-dimensional Rainbow Gravity [34].

A large class of regular black hole solutions found in the literature follows the idea that singularity is replaced by a regular distribution of matter [35]. In usual General Relativity, NED produces black holes where singularity is eliminated by a regular field distribution that covers the central core of the black hole. We must consider an important feature of the regular solutions that is linked to so-called energy conditions, all regular black hole solution with spherical symmetry violate, in some region, the strong energy condition (SEC) [36], but not necessarily the dominant (DEC) and weak (WEC) energy conditions.

Our focus in this work is to get black hole solutions that are regular in origin and investigate how this can be changed by the $\epsilon$ power scale parameter. In addition, we are interested in how these new solutions behave in the energy conditions. We consider the natural units $c = G_N = 1$.

### 2. Regular black holes

In order to study the Rainbow Gravity, General Relativity must be reformulated considering a dependency on the energy scale and now should be described by a set of parameters in the connection, $R^{\mu\nu}_{ab}(\epsilon)$, Riemann tensor, $R(\epsilon)_{\mu\nu\lambda\kappa}$, that are constructed using a rainbow metric, and in the stress-energy tensor $T(\epsilon)_{\mu\nu}$. The Greeks index assume the values $(\alpha, \beta, \mu, \nu,... = 0, 1, 2, 3)$. The modified Einstein equations to the Rainbow gravity are given by

$$G(\epsilon)_{\mu\nu} = \kappa(\epsilon)^2 T(\epsilon)_{\mu\nu},$$

where $\kappa(\epsilon)^2 = 8\pi G(\epsilon)/c^4(\epsilon)$ is a coupled constant and $G(\epsilon) = h(\epsilon) G_N$, with $G_N = 1$ being the Newtonian gravitational constant, where $h(\epsilon) = 1/[1 + (1/137)e^2]$ and $c(\epsilon) = g(\epsilon)/f(\epsilon)$ [12,37]. In order to guarantee causality, we need $g(\epsilon) \leq f(\epsilon)$, so that $c(\epsilon) \leq 1$. The general line element, that describes spherically symmetric configurations and depends on $\epsilon$, is written as

$$ds^2 = \frac{e^{a(r)}}{f(\epsilon)^2}dt^2 - \frac{e^{b(r)}}{g(\epsilon)^2}dr^2 - \frac{r^2}{g(\epsilon)^2}d\theta^2 + \sin^2\theta d\phi^2,$$

where $e^{a(r)}$ and $e^{b(r)}$ are general functions of the radial coordinate and the dispersion relations proposed in [38]. $f(\epsilon)$ and $g(\epsilon)$, are $f(\epsilon) = g(\epsilon) = 1/(1 + \lambda \epsilon)$, where $\lambda$ is a constant parameter (in this case $c(\epsilon)$ is a constant). The curvature scalar is given by

$$R(\epsilon) = e^{-b(\epsilon)}g(\epsilon)^2\left[a'' + \left(\frac{a'}{2} - \frac{2}{r}\right)(a' - b') + \frac{2}{r^2}\right] - \frac{2g(\epsilon)^2}{r^2}.$$  \hspace{1cm} (3)

The non-null and independents components of the Riemann tensor are

$$R^{01}_{01} = \frac{1}{4}e^{-b(\epsilon)}g(\epsilon)^2\left(2a'' - a'b' + a'^2\right), \hspace{0.5cm} R^{02}_{02} = R^{03}_{03} = -\frac{e^{-b(\epsilon)}g(\epsilon)^2a'}{2r},$$

$$R^{12}_{12} = R^{13}_{13} = \frac{e^{-b(\epsilon)}g(\epsilon)^2b'}{2r},$$

$$R^{23}_{23} = -\frac{(1 - e^{-b(\epsilon)})g(\epsilon)^2}{r^2}. \hspace{1cm} (4)$$

The Kretschmann scalar, $K = R^{\mu\nu\alpha\beta}R_{\mu\nu\alpha\beta}$, to a static and spherically symmetric spacetime may be written as [39]
\[ K = 4 \left( R_{01}^{01} \right)^2 + 4 \left( R_{02}^{02} \right)^2 + 4 \left( R_{03}^{03} \right)^2 + 4 \left( R_{12}^{12} \right)^2 + 4 \left( R_{13}^{13} \right)^2 + 4 \left( R_{23}^{23} \right)^2, \]
\[ = 4 \left( R_{01}^{01} \right)^2 + 8 \left( R_{02}^{02} \right)^2 + 8 \left( R_{12}^{12} \right)^2 + 4 \left( R_{23}^{23} \right)^2. \] (5)

So, if one of the components of the Riemann tensor is singular, the Kretschmann scalar is also singular. To the line element (2), we get
\[ K(\epsilon) = \frac{1}{4} g(\epsilon)^4 \left( \frac{8 e^{-2b} a'^2}{r^2} + e^{-2b} \left( 2a'' - a'b' + a'^2 \right)^2 + \frac{8 e^{-2b} b'^2}{r^2} + \frac{16 \left( e^{-b} - 1 \right)^2}{r^4} \right). \] (6)

The coupled with NED is made through the stress-energy tensor \( T(\epsilon)_{\mu \nu} \), that is
\[ T(\epsilon)_{\mu \nu} = L(F) \delta_{\mu \nu} - \frac{\partial L(F)}{\partial F} F_{\nu \sigma} F^{\mu \sigma}, \] (7)
where \( L(F) \) is a nonlinear function of the electromagnetic scalar \( F \). We are not considering the dependence on derivative terms as \( \nabla_{\mu} F \) or \( \nabla_{\mu} \nabla_{\nu} F \). The Lagrangian \( L(F) \) will not be fixed yet. We will determine \( L \) and its derivative through the Einstein equations. We define the 4-potential as being a 1-form \( A = A_{\mu} dx^{\mu} \). The Maxwell tensor is a 2-Form defined as \( \hat{F} = (1/2) F_{\mu \nu} dx^{\mu} \wedge dx^{\nu} \), where the components are \( F_{\mu \nu} = \nabla_{\mu} A_{\nu} - \nabla_{\nu} A_{\mu} \). Then, the electromagnetic scalar becomes \( F = (1/4) F^{\mu \nu} F_{\mu \nu} \). As we are considering a spherically symmetric and static spacetime, we have the invariance under both time translation and rotation em relation to the center, and then we have two Killing vectors. These Killing vectors satisfy the equation \( \mathcal{L}_{K} g_{\mu \nu}(r, \theta) = 0 \), where \( \{K^\sigma \} \) are the vector fields and \( \mathcal{L} \) is the Lie derivative [40]. From the same Killing vectors, we have \( \mathcal{L}_{K} F^{\mu \nu}(r, \theta) = 0 \). With that, there are only two non-null and independent components of the Maxwell tensor, \( F^{10}(r) \) and \( F^{23}(r, \theta) \) [41]. If we consider that the black hole is only electrically charged, the only non-zero and independent component of the Maxwell tensor is \( F^{10}(r) \) [42]. The modified Maxwell equations are
\[ \nabla_{\mu} \left[ F^{\mu \nu} \frac{\partial L}{\partial F} \right] = \partial_{\mu} \left[ \sqrt{-g} F^{\mu \nu} \frac{\partial L}{\partial F} \right] = 0, \] (8)
whose the solution, to the line element (2), is given by [47]
\[ F^{10}(r, \epsilon) = f(\epsilon) g(\epsilon) \frac{Q}{r^2} e^{-(a+b)/2} \left( \frac{\partial L}{\partial F} \right)^{-1}, \] (9)
with \( Q \) being the electric charge.

Using (2), (9) and (7), with \( L_F = \partial L / \partial F \), the components of the field equations are
\[ \frac{g(\epsilon)^2}{r^2} \left[ e^{-b} (rb' - 1) + 1 \right] = \kappa(\epsilon)^2 \left( L + g(\epsilon)^4 \frac{Q^2}{r^4} L_F \right)^{-1}, \] (10)
\[ -\frac{g(\epsilon)^2}{r^2} \left[ e^{-b} (ra' + 1) - 1 \right] = \kappa(\epsilon)^2 \left( L + g(\epsilon)^4 \frac{Q^2}{r^4} L_F \right)^{-1}, \] (11)
\[ -\frac{g(\epsilon)^2 e^{b}}{4r} \left[ (2 + a' r) (a' - b') + 2ra'' \right] = \kappa(\epsilon)^2 L. \] (12)
From (7) and (9), we obtain that \( T_{0}^{0} = T_{1}^{1} \). With that condition, from (10) and (11), we obtain we may find \( a'(r) = -b'(r) \). Integrating that expression, and considering the integration constant
equal to zero, we have \( a = -b \). We may identify \( T_0^0 = \rho \), \( T_1^1 = -p_r \) and \( T_2^2 = T_3^3 = -p_t \) where \( \rho \) is the energy density and \( p_r \) and \( p_t \) are, respectively, the radial and tangential pressures. To the line element (2), we get

\[
\rho(r, \epsilon) = e^{-b g(\epsilon)} \left( \frac{rb'}{r^2 8 \pi f(\epsilon) h(\epsilon)} + \frac{e^b - 1}{28 \pi f(\epsilon) h(\epsilon)} \right), \tag{13}
\]

\[
p_r(r, \epsilon) = -\rho(r, \epsilon), \tag{14}
\]

\[
p_t(r, \epsilon) = -\frac{e^{-b g(\epsilon)} 16 \pi f(\epsilon) h(\epsilon)}{r^2} \left[ \frac{rb'^2 - rb'' - 2b'}{2} \right]. \tag{15}
\]

The energy conditions will be given by [43]:

\[
SEC = \rho + p_r + 2p_t \geq 0, \tag{16}
\]

\[
WEC_{1,2} = \rho + p_r, t \geq 0, \tag{17}
\]

\[
WEC_3 = \rho \geq 0, \tag{18}
\]

\[
DEC_{2,3} = \rho - p_r, t \geq 0, \tag{19}
\]

where \( DEC_1 \equiv WEC_3 \) and \( NEC_{1,2} \equiv WEC_{1,2} \).

Using the line element (2), we may choose different models of mass functions to construct regular black holes solutions. To do that, we introduce the mass function through the coefficient of the metric \( g_{00} \) as

\[
e^{a(r)} = 1 - 2 \frac{G(\epsilon) M(r, \epsilon) g(\epsilon)}{c^2(\epsilon) r}, \tag{20}
\]

where, to formulate that, we follow [12], that consider \( r \to r(\epsilon) = r/g(\epsilon) \). If the solution is regular in \( r = 0 \), \( M(r, \epsilon) \) must satisfy the condition \( \lim_{r \to 0} [M(r, \epsilon)/r] \to 0 \).

From the equations (10)-(12), \( L \) and \( LF \), in terms of \( M(r, \epsilon) \), are

\[
L = f^2(\epsilon) g(\epsilon) h(\epsilon) \frac{M''(r, \epsilon)}{r \kappa^2(\epsilon)}, \tag{21}
\]

\[
LF = \frac{q(\epsilon)^2 g^2(\epsilon) \kappa^2(\epsilon)}{r^2 f^2(\epsilon) h(\epsilon)} \left( -2M'(r, \epsilon) + M''(r, \epsilon) \right). \tag{22}
\]

The Lagrangian density \( L \) and its derivative \( LF \) must satisfy the relation

\[
LF = \frac{\partial L}{\partial r} \left( \frac{\partial F}{\partial r} \right)^{-1}. \tag{23}
\]

In order to do that, we need the electric field, that is

\[
F^{10}(r, \epsilon) = f^3(\epsilon) h(\epsilon) \frac{2M'(r, \epsilon) - r M''(r, \epsilon)}{Q g(\epsilon) \kappa^2(\epsilon)}. \tag{24}
\]

Using the relation \( F = -e^{a+b} [F^{10}(r, \epsilon)]^2 (2 f^2(\epsilon) g^2(\epsilon))^{-1} \), it’s possible to show that (23) is satisfied.

We will consider some model to the mass function in order to obtain regular solutions that depend on \( \epsilon \).
Fig. 1. Graphical representation of the functions $R(r, \epsilon)$ and $K(r, \epsilon)$, to $m = 10Q$, $Q = 0.8$ and $\lambda = 1$ to different values of energy with the result from GR. (For interpretation of the colors in the figure(s), the reader is referred to the web version of this article.)

2.1. Culetu-type solution

The solution proposed by Culetu in [45] is described by the line element

$$ds^2 = \left( 1 - \frac{2m}{r} e^{-p_0/r} \right) dt^2 - \frac{1}{1 - \frac{2m}{r} e^{-p_0/r}} dr^2 - r^2 \left( d\theta^2 + \sin^2 \theta d\phi^2 \right),$$

(25)

where $p_0$ is a parameter associated with the charge $Q$ and mass $m$, such that $p_0 = Q^2/2m$ generates regular solution in GR, which is asymptotically Reissner-Nordstrom. Let’s extend the Culetu solution to the Rainbow Gravity using the line element (2) with (20) and the condition $r \to r(\epsilon) = r/g(\epsilon)$. The mass function of the Culetu solution to the Rainbow gravity is

$$M(r, \epsilon) = me^{-Q^2g(\epsilon)/(2mr)},$$

(26)

with $m$ being the black hole mass. This model was considered and analyzed in [45] to the case $g(\epsilon) = 1$.

Using (26) and (20) in (2), we may get the curvature scalar (3) and the Kretschmann scalar (6) to this model, that are

$$R(r, \epsilon) = -\frac{e^{-Q^2g(\epsilon)/mr} Q^4 f^2(\epsilon) g^3(\epsilon) h(\epsilon)}{2mr^5}.$$  

(27)

$$K(r, \epsilon) = \frac{e^{-Q^2g(\epsilon)/mr}}{4m^2r^{10}} f(\epsilon)^4 g(\epsilon)^2 h(\epsilon)^2 \left[ 192m^4 r^4 + Q^2 g(\epsilon)(Q^2 g(\epsilon) - 4mr) \right. \right. \left. \left. \times \left( 48m^2 r^2 + Q^2 g(\epsilon)(Q^2 g(\epsilon) - 12mr) \right) \right].$$  

(28)

The scalars (27) and (28) are regular in all spacetime to $m > 0$, $f(\epsilon) > 0$, $g(\epsilon) > 0$ and $h(\epsilon) > 0$, once in the limits $r \to 0$ and $r \to \infty$, both $R(r, \epsilon)$ and $K(r, \epsilon)$ do not present divergences. In Fig. 1, the behavior of the scalars is exhibit and we compare to different values of $\epsilon$.

The scalar $R(r, \epsilon)$ and $K(r, \epsilon)$, that depend on the energy $\epsilon$, are regular in all spacetime and zero in the infinity, which characterize a regular and asymptotically flat solution.
Substituting (26) in (24), the electric field becomes

$$F^{10}(r, \epsilon) = -e^{-\frac{Q^2 g(\epsilon)}{2m r}} \frac{Q (8mr - Q^2 g(\epsilon))}{4m r^3 \kappa(\epsilon)^2} f^3(\epsilon) g^2(\epsilon) h(\epsilon).$$  \hspace{1cm} (29)

The electric field behavior is exhibit in Fig. 2 to different values of $\epsilon$ and we also compare to the GR result. From Fig. 2, we see that the intensity of the electric field is always regular and presents a minimum and a maximum. We may also see from Fig. 2 that the intensity of the electric field (29) decreases as $\epsilon$ increases. Since we have $F^{10}$, we may evaluate $F(r, \epsilon)$ and the Lagrangian density $L(r, \epsilon)$. Using (29), $F$ is given by

$$F(r, \epsilon) = -e^{-\frac{Q^2 g(\epsilon)}{2m r}} \frac{Q^2 (Q^2 g(\epsilon) - 8mr)^2 h(\epsilon)^2 f(\epsilon)^4}{32m^2 r^6 \kappa(\epsilon)^4},$$  \hspace{1cm} (30)

and from (21) and (26), the Lagrangian density to the nonlinear electrodynamics is

$$L(r, \epsilon) = -e^{-\frac{Q^2 g(\epsilon)}{2m r}} \frac{Q^2 (Q^2 g(\epsilon) - 4mr)^2 f(\epsilon)^2 h(\epsilon)}{4m r^5 \kappa(\epsilon)^2}.$$  \hspace{1cm} (31)

The behavior of (30) (right) and (31) (left), in terms of the radial coordinate are analyzed in Fig. 3. It’s clear the nonlinearity of the electromagnetic theory and the regularity of these functions. The form of $L(F)$ does not depend on the energy scale.
Using the function $M(r, \epsilon)$ to the model that we are considering in (16)-(19) with (13), (14) and (15), we get

$$SEC = e^{-\frac{Q^2 g(\epsilon)^2}{4mr} \left( 4mr - Q^2 g(\epsilon) \right) g(\epsilon)} \frac{Q^2 g^6(\epsilon)}{16 m \pi r^3 f^2(\epsilon)},$$

$$WEC_1 = 0,$$

$$WEC_2 = e^{-\frac{Q^2 g(\epsilon)^2}{4mr} \left( 8mr - Q^2 g(\epsilon) \right) g(\epsilon)} \frac{Q^2 g^6(\epsilon)}{32 m \pi r^5 f^2(\epsilon)},$$

$$WEC_3 = e^{-\frac{Q^2 g(\epsilon)^2}{8\pi r^4 f^2(\epsilon)}},$$

$$DEC_2 = e^{-\frac{Q^2 g(\epsilon)^2}{4\pi r^4 f^2(\epsilon)}},$$

$$DEC_3 = e^{-\frac{Q^2 g(\epsilon)^2}{32 m \pi r^5 f^2(\epsilon)}}.$$

The energy conditions are represented in Fig. 4. All conditions are satisfied to $r \geq Q^2 g(\epsilon)/4m$, and inside the event horizon, SEC and WEC are violated. The region where these functions are violated is attenuated as $\epsilon$ increases. The region where WEC is violated is smaller than the SEC, since we need $r \geq Q^2 g(\epsilon)/8m$ to satisfy the first one.

2.2. Balart-Vagenas-type solution

Let’s consider the mass function proposed in [44] in GR, where we make $M(r) \rightarrow M(r, \epsilon)$ with $r \rightarrow r(\epsilon) = r/g(\epsilon)$.

$$M(r, \epsilon) = m \left( 1 + \frac{Q^2 g(\epsilon)}{6mr} \right)^{-3}.\quad (38)$$

To $g(\epsilon) = 1$, we recover the GR case. As we did with the solution before,

$$e^{a(r)} = e^{-b(r)} = 1 - \frac{432 m^4 r^2 f(\epsilon)^2 h(\epsilon)}{g(\epsilon)(6mr + Q^2 g(\epsilon))^3},\quad (39)$$

$$F^{10}(r, \epsilon) = \frac{243 m^5 Q r^3 f(\epsilon)^3 g(\epsilon)}{\pi^2 h(\epsilon) (6mr + Q^2 g(\epsilon))^5},\quad (40)$$

and the curvature invariants $R(r, \epsilon)$ and $K(r, \epsilon)$ are

$$R(r, \epsilon) = -\frac{5184 m^4 Q^4 f(\epsilon)^2 g(\epsilon)^3 h(\epsilon)}{(6mr + Q^2 g(\epsilon))^5},\quad (41)$$

$$K(r, \epsilon) = \frac{4478976 m^8 f(\epsilon)^4 g(\epsilon)^2 h(\epsilon)^2}{(6mr + Q^2 g(\epsilon))^{10}} \times \left[ 648m^4 r^4 + Q^2 g(\epsilon) \left( 126m^2 Q^2 r^2 g(\epsilon) + Q^6 g(\epsilon)^3 - 216m^3 r^3 \right) \right],\quad (42)$$

that are always regular. In the black hole center we find
Fig. 4. Graphical representation of the energy conditions $SEC$, $WEC_2$, $WEC_3$, $DEC_2$ e $DEC_3$ with $m = 10Q$, $Q = 0.8$ and $\lambda = 1$ for some values of the energy $\epsilon$. We also see the GR case.

\[
\lim_{r \to 0} \{R(r, \epsilon), K(r, \epsilon)\} = \left\{ -5184m^4 f(\epsilon)^2 h(\epsilon) / Q^6 g(\epsilon)^2, \\
4478976m^8 f(\epsilon)^4 h(\epsilon)^2 / Q^{12} g(\epsilon)^4 \right\}, \tag{43}
\]

which guarantees the regularity. We can also see the regularity in Fig. 5 since we have no divergence in all spacetime. In the infinity of the radial coordinate the invariants tend to zero, so that, the solution is asymptotically flat. In Fig. 6 we see how behaves the intensity of the electric field. This function has only one maximum and it’s clear the regularity, once we have it’s finite everywhere. We may also see in Fig. 6 that the intensity decreases as $\epsilon$ increases, so that, the GR case has the greater value to the electric field.

We may also show the behavior of $F(r, \epsilon)$ and $L(r, \epsilon)$ in terms of the radial coordinate and compare to the GR case, Fig. 7.
Fig. 5. Scalars $R(r, \epsilon)$ and $K(r, \epsilon)$ to $m = 10Q$, $Q = 0.8$ and $\lambda = 1$ for some values of $\epsilon$ and to GR.

Fig. 6. Electric field intensity (40) with $m = 10Q$, $Q = 0.8$, $\lambda = 1$, $G = 1$, $\epsilon = 0.2$, $\epsilon = 0.4$, $\epsilon = 0.6$ and $\epsilon = 0.8$. The red line represents the GR case.

Fig. 7. Graphical representation of $F(r, \epsilon)$ (left) and $L(r, \epsilon)$ (right) to $m = 10Q$, $Q = 0.8$, $\lambda = 1$ and $G = 1$ with $\epsilon = 0.2$, $\epsilon = 0.4$, $\epsilon = 0.6$ and $\epsilon = 0.8$ and GR.

The electromagnetic scalar and Lagrangian density are always regular, and the nonlinearity of $L(F)$ is evident, as it’s expect to this type of solution.

The energy conditions are found through (38) and (16)-(19), so that, we have

$$SEC = \frac{324m^4Q^2g(\epsilon)^6(6mr - Q^2g(\epsilon))}{\pi f(\epsilon)^2(6mr + Q^2g(\epsilon))^5},$$

$$WEC_1 = 0,$$
Fig. 8. Energy conditions \(SEC, WEC_2, WEC_3, DEC_2\) and \(DEC_3\) with \(m = 10Q\), \(Q = 0.8\) and \(\lambda = 1\) for some of values of \(\epsilon\). The red line represents the GR case.

\[
WEC_2 = \frac{1944m^5 Q^2 r g(\epsilon)^6}{\pi f(\epsilon)^2 (6mr + Q^2 g(\epsilon))^3},
\]
\[
WEC_3 = \frac{162m^4 Q^2 g(\epsilon)^6}{\pi f(\epsilon)^2 (6mr + Q^2 g(\epsilon))^4},
\]
\[
DEC_2 = \frac{324m^4 Q^4 g(\epsilon)^6}{\pi f(\epsilon)^2 (6mr + Q^2 g(\epsilon))^4},
\]
\[
DEC_3 = \frac{324m^4 Q^4 g(\epsilon)^7}{\pi f(\epsilon)^2 (6mr + Q^2 g(\epsilon))^5}.
\]

The behavior of (44)-(49), as functions of \(r\), is exhibit in Fig. 8. As in GR, to the model (38) [44], the conditions NEC, WEC and DEC are always satisfied. However, SEC is violated to \(r < \)
$Q^2 g(\epsilon)/6m$, inside the event horizon, but as we increase the $\epsilon$, the region where this condition is violated is attenuated. This result shows explicit that the energy scale may modify the energy conditions.

3. Conclusion

In this work, we proposed regular black solutions in the Rainbow Gravity. To that, we consider the coupled of the gravitational theory with a NED. As the line element depends on the energy scale, both the geometric sector and the matter sector of the Einstein equations are modified in order to also depend on the energy scale. The mainly equations are written in terms of a mass function, which will change for different regular models.

We presented two examples of mass functions, that are modifications from the Culetu and Balart-Vagenas solutions to $M(r, \epsilon)$, through the Rainbow functions. In the Culetu-type solution, we showed that $R(r, \epsilon)$ and $K(r, \epsilon)$ are regular and asymptotically flat to $0 < \epsilon < 1$. As the Kretschmann scalar tends to zero in the center, this solution has a Minkowski core [46]. The electromagnetic Lagrangian density is nonlinear in the scalar $F$ and is regular in all spacetime, as well as the electric field. About the energy conditions, SEC and WEC are violated near to black hole center and, to higher values of energy $\epsilon = E/E_p$, smaller is the region where this happens. DEC is always satisfied. The Balart-Vagenas-type solution, as the example before, is curvature regular and the electromagnetic theory is nonlinear. To this model, only the strong energy condition is violated and the region where it is violated decreases as the energy increases. To both models $\epsilon$ attenuate the intensity of the electric field.

A possible continuation of this work would be the study of black hole thermodynamics associated to regular solutions in Rainbow Gravity, similar to what is found in the literature [33].

CRediT authorship contribution statement

All authors participated fully in the preparation stages.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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References

[1] H. Reissner, Über die Eigengravitation des elektrischen Feldes nach der Einsteinschen Theorie, Ann. Phys. 50 (106–120) (1916), 19163550905.
[2] G. Nordstrom, On the energy of the gravitation field in Einstein’s theory, Proc. K. Ned. Akad. Wet. 20 (1918) 1238–1245, 1918KNAB...20.1238N.
[3] R.P. Kerr, Gravitational field of a spinning mass as an example of algebraically special metrics, Phys. Rev. Lett. 11 (1963) 237, https://doi.org/10.1103/PhysRevLett.11.237.
[4] Event Horizon Telescope Collaboration, First M87 event horizon telescope results. I. The shadow of the supermassive black hole, Astrophys. J. 1 (875) (2019) L1, arXiv:1906.11238.

[5] S.M. Carroll, J.A. Harvey, V.A. Kostelecký, C.D. Lane, T. Okamoto, Noncommutative field theory and Lorentz violation, Phys. Rev. Lett. 87 (2001) 141601, arXiv:hep-th/0105082.

[6] P. Horava, Quantum gravity at a Lifshitz point, Phys. Rev. D 79 (2009) 084008, arXiv:0901.3775;

P. Horava, Spectral dimension of the universe in quantum gravity at a Lifshitz point, Phys. Rev. Lett. 102 (2009) 161301, arXiv:0902.3657.

[7] G. Amelino-Camelia, J.R. Ellis, N.E. Mavromatos, D.V. Nanopoulos, Distance measurement and wave dispersion in a Liouville-string approach to quantum gravity, Int. J. Mod. Phys. A 12 (1997) 607, arXiv:hep-th/9605211;

G. Amelino-Camelia, Quantum spacetime phenomenology, Living Rev. Relativ. 16 (2013) 5, arXiv:0806.0339.

[8] G. ’t Hooft, Quantization of point particles in 2+1 dimensional gravity and space-time discreteness, Class. Quantum Gravity 13 (1996) 1023, arXiv:gr-qc/9601014.

[9] J. Magueijo, L. Smolin, Generalized Lorentz invariance with an invariant energy scale, Phys. Rev. D 67 (2003) 044017, arXiv:gr-qc/0207085.

[10] M. Dehghani, Thermodynamics of novel charged dilaton black holes in gravity’s rainbow, Phys. Lett. B 785 (2018) 274, physletb.2018.08.045.

[11] R. Garattini, E.N. Saridakis, Gravity’s rainbow: a bridge towards Horava-Lifshitz gravity, Eur. Phys. J. C 75 (7) (2015) 343, arXiv:1411.7257.

[12] J. Magueijo, L. Smolin, Gravity’s rainbow, Class. Quantum Gravity 21 (2004) 1725, arXiv:gr-qc/0305055.

[13] Y. Ling, Xiang Li, Hongbao Zhang, Thermodynamics of modified black holes from gravity’s rainbow, Mod. Phys. Lett. A 22 (2007) 2749–2756, arXiv:gr-qc/0512084.

[14] P. Galan, G.A. Mena Marugan, Entropy and temperature of black holes in a gravity’s rainbow, Phys. Rev. D 74 (2006) 044035, arXiv:gr-qc/0608061.

[15] Cheng-Zhou Liu, Jian-Yang Zhu, Hawking radiation and black hole entropy in a gravity’s rainbow, Gen. Relativ. Gravit. 40 (2008) 1899–1911, arXiv:gr-qc/0703055.

[16] C. Leiva, J. Saavedra, J. Villanueva, Geodesic structure of the Schwarzschild black hole in rainbow gravity, Mod. Phys. Lett. A 24 (2009) 1443–1451, arXiv:0808.2601.

[17] A. Farag Ali, Black hole remnant from gravity’s rainbow, Phys. Rev. D 89 (2014) 104040, arXiv:1402.5320.

[18] A. Farag Ali, M. Faizal, B. Majumder, Absence of an effective horizon for black holes in gravity’s rainbow, Europhys. Lett. 109 (2015) 20011, arXiv:1406.1980.

[19] A. Farag Ali, M. Faizal, M.M. Khalil, Absence of black holes at LHC due to gravity’s rainbow, Phys. Lett. B 743 (2015) 295, arXiv:1410.4765.

[20] Y. Gim, W. Kim, Black hole complementarity in gravity’s rainbow, J. Cosmol. Astropart. Phys. 05 (2015) 002, arXiv:1501.04702.

[21] S.H. Hendi, Mir Faizal, B. Eslam Panah, S. Panahiyan, Charged dilatonic black holes in gravity’s rainbow, Eur. Phys. J. C 76 (5) (2016) 296, arXiv:1508.00234.

[22] J.W. Kim, S. Kook Kim, Y-Jai Park, Thermodynamic stability of modified Schwarzschild-AdS black hole in rainbow gravity, Eur. Phys. J. C 76 (10) (2016) 557, arXiv:1607.06185.

[23] Zhong-Wen Feng, Shu-Zheng Yang, Thermodynamic phase transition of a black hole in rainbow gravity, Phys. Lett. B 772 (2017) 737–742, arXiv:1708.06627.

[24] Y. Gim, H. Um, Wontae Kim, Black hole complementarity with the generalized uncertainty principle in gravity’s rainbow, J. Cosmol. Astropart. Phys. 02 (2018) 060, arXiv:1712.04444.

[25] R. Garattini, Gravity’s rainbow and black hole entropy, J. Phys. Conf. Ser. 942 (1) (2017) 012011, arXiv:1712.09729.

[26] S. Hossein Hendi, M. Momennia, AdS charged black holes in Einstein-Yang-Mills gravity’s rainbow: thermal stability and P-V criticality, Phys. Lett. B 777 (2018) 222–234, j.physletb.2017.12.033.

[27] B. Eslam Panah, Effects of energy dependent spacetime on geometrical thermodynamics and heat engine of black holes: gravity’s rainbow, Phys. Lett. B 787 (2018) 45, arXiv:1805.03014.

[28] R. Garattini, Distorting general relativity: gravity’s rainbow and f(R) theories at work, J. Cosmol. Astropart. Phys. 1306 (2013) 017, arXiv:1210.7760.

[29] S.H. Hendi, B. Eslam Panah, S. Panahiyan, F(R) gravity’s rainbow and its Einstein counterpart, Adv. High Energy Phys. 2016 (2016) 9813582, arXiv:1607.03383.

[30] S.H. Hendi, S. Panahiyan, B. Eslam Panah, M. Momennia, Thermodynamic instability of nonlinearly charged black holes in gravity’s rainbow, Eur. Phys. J. C 76 (2016) 150, https://doi.org/10.1140/epjc/s10052-016-3994-z.

[31] S.H. Hendi, B. Eslam Panah, S. Panahiyan, M. Momennia, Dilatonic black holes in gravity’s rainbow with a nonlinear source: the effects of thermal fluctuations, Eur. Phys. J. C 77 (2017) 647, arXiv:1708.06634.
In A. H. L. M. J. M.M. O.B. E. M. M. S. E.L.B. Junior, M.E. Rodrigues and M.V. de S. Silva, Nuclear Physics B 961 (2020) 115244.

[32] S. Hossein Hendi, M. Momennia, B.E. Panah, Mir Faizal, Nonsingular universes in Gauss-Bonnet gravity’s rainbow, Astrophys. J. 827 (2016) 153, arXiv:1703.00480.

[33] M. Dehghanian, Thermodynamics of charged dilatonic BTZ black holes in rainbow gravity, Phys. Lett. B 777 (2018) 351–360, j.physletb.2017.12.048.

[34] M. Dehghanian, Thermal fluctuations of AdS black holes in three-dimensional rainbow gravity, Phys. Lett. B 793 (2019) 234–239, physlett.2019.04.058.

[35] E. Ayón-Beato, A. García, Regular black hole in general relativity coupled to nonlinear electrodynamics, Phys. Rev. Lett. 80 (1998) 5056–5059, arXiv:gr-qc/9911046;
    K.A. Bronnikov, Regular magnetic black holes and monopoles from nonlinear electrodynamics, Phys. Rev. D 63 (2001) 044005, arXiv:gr-qc/0006014;
    I. Dymnikova, Regular electrically charged structures in nonlinear electrodynamics coupled to general relativity, Class. Quantum Gravity 21 (2004) 4417–4429, arXiv:gr-qc/0407072;
    M. Novello, V.A. De Lorenzi, J.M. Salim, R. Klippert, Geometrical aspects of light propagation in nonlinear electrodynamics, Phys. Rev. D 61 (2000) 045001, arXiv:gr-qc/9911085.

[36] O.B. Zaslavskii, Regular black holes and energy conditions, Phys. Lett. B 688 (2010) 278, arXiv:1004.2362.

[37] M.M. Amber, J.F. Donoghue, On the running of the gravitational constant, Phys. Rev. D 85 (2012) 104016, arXiv: 1111.2875 [hep-th].

[38] J. Magueijo, L. Smolin, Lorentz invariance with an invariant energy scale, Phys. Rev. Lett. 88 (2002) 190403, arXiv:hep-th/0112090.

[39] K.A. Bronnikov, S.G. Rubin, Black Holes, Cosmology and Extra Dimensions, World Scientific, Singapore, 2013.

[40] R.M. Wald, General Relativity, The University of Chicago Press, Chicago, 1984.

[41] J. Wainwright, P.E.A. Yaremovicz, Killing vector fields and the Einstein-Maxwell field equations with perfect fluid source, Gen. Relativ. Gravit. 7 (1976) 345–359.

[42] J. Plebanski, A. Krasinski, An Introduction to General Relativity and Cosmology, Cambridge University Press, New York, 2006, ISBN-13: 978-0521856232.

[43] M. Visser, Lorentzian Wormholes: From Einstein to Hawking, United Book Press, Springer-Verlag, New York, 1995, ISBN: 1563966530, 9781563966538.

[44] L. Balart, E.C. Vagenas, Regular black holes with a nonlinear electrodynamics source, Phys. Rev. D 90 (2014) 124045, arXiv:1408.0306.

[45] H. Culetu, On a regular modified Schwarzschild spacetime, arXiv:1305.5964;
    H. Culetu, On a regular charged black hole with a nonlinear electric source, Int. J. Theor. Phys. 54 (2015) 2855, arXiv:1408.3334.

[46] A. Simpson, M. Visser, Regular black holes with asymptotically Minkowski cores, Universe 6 (1) (2019) 8, arXiv:1911.01020.

[47] In [44], the electric field differ from the result obtained here, in the context of General Relativity, since the authors consider that the solution to the modified Maxwell equations is written as $F_{10}^{\delta L/\delta F} = Q/(4\pi r^2)$. 