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Using a sample of $448.1 \times 10^6 \psi(3686)$ events collected with the BESIII detector, a search for the isospin violating decay $\eta_c \to \pi^+ \pi^- \pi^0$ via $\psi(3686) \to \gamma \eta_c$ is presented. No signal is observed, and the upper limit on $B(\psi(3686) \to \eta_c \gamma \eta_c) \times B(\eta_c \to \pi^+ \pi^- \pi^0)$ is determined to be $1.6 \times 10^{-6}$ at the 90% confidence level. In addition, a search for $\eta(1405) \to f_0(980)\pi^0$ in $\psi(3686)$ radiative decays is performed. No signal is observed, and the branching fraction $B(\psi(3686) \to \gamma \eta(1405)) \times B(\eta(1405) \to f_0(980)\pi^0) \times B(f_0(980) \to \pi^+ \pi^-)$ is calculated to be less than $5.0 \times 10^{-7}$ at the 90% confidence level.

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I. INTRODUCTION

As the lowest-lying $c\bar{c}$ state, the pseudoscalar meson $\eta_c$ has attracted considerable theoretical and experimental attention since it was discovered three decades ago [1]. To the lowest order in perturbation theory, the $\eta_c$ decays through $c\bar{c}$ annihilation into two gluons. The $\eta_c$ is then expected to have numerous hadronic decay modes into two- or three-body...
hadronic final states, and many of them have been measured [2]. However, the three-pion decay mode has not yet been studied, but its measurement is important to test isospin symmetry [3–5].

Charmonium radiative decays, especially those of J/ψ and ψ(3686), provide an excellent laboratory for the study of neutral pseudoscalar meson decays. For example, the BESIII experiment using J/ψ radiative decays has performed a series of analyses on three pion decays [6–11], and for the first time reported the observation of the isospin violating decay η(1405) → 3π [12]. Of particular interest in η(1405) → 3π decay is a narrow structure around 0.98 GeV/c² in the ππ mass spectrum, identified with the f₀(980), which can be interpreted under the triangle singularity scheme [13–15].

In this analysis, we perform a search for the isospin violating decay ηc → π⁺π⁻π⁰ using a sample of 448.1 × 10⁶ ψ(3686) events [16] collected with the BESIII [17] detector operating at the BEPCII [18] storage ring. We also perform a search for η(1405) → f₀(980)π⁰ in the ψ(3686) radiative decays to test the “12% rule” [19–21], in which perturbative QCD predicts the ratio of the branching fractions of ψ(3686) and J/ψ into the same final hadronic state is given

\[ Q = \frac{B_{\psi(3686) \rightarrow \eta^0}}{B_{J/\psi \rightarrow \eta^0}} = \frac{B_{\psi(3686) \rightarrow \eta^0\pi^0\pi^0}}{B_{J/\psi \rightarrow \eta^0\pi^0\pi^0}} \approx (12.4 \pm 0.4)\%. \]

The rule is expected to also hold for radiative decays to the same final hadronic state.

II. DETECTOR AND MONTE CARLO SIMULATION

BEPCII is a double-ring e⁺e⁻ collider providing a peak luminosity of 10³³ cm⁻²s⁻¹ at a beam energy of 1.89 GeV. The BESIII detector [17] consists of a helium-based main drift chamber (MDC), a plastic scintillator time-of-flight (TOF) system, a CsI(Tl) electromagnetic calorimeter (EMC), and a multi-layer resistive plate chamber muon counter system. With a geometrical acceptance of 93% of 4π, the BESIII detector operates in a magnetic field of 1.0 T provided by a superconducting solenoidal magnet.

Monte Carlo (MC) simulations are used to determine detector efficiency, optimize event selection and estimate backgrounds. The BESIII detector is modeled with GEANT4 [22]. For the inclusive MC, the production of the ψ(3686) resonance is simulated by the MC event generator KKMC [23, 24], and the decays are generated by EVTGEN [25, 26] for known decay modes with branching fractions being set to Particle Data Group (PDG) [2] world average values, while the remaining unknown decays are generated by LUNDCHARM [27]. For ψ(3686) → γηc, ηc → π⁺π⁻π⁰ decays, the line shape of the ηc meson is described by \( E_γ^2 \times |BW(m)|^2 \times D(E_γ) \), where m is the π⁺π⁻π⁰ invariant mass, \( E_γ = \frac{M^2_{\psi(3686)} - m^2}{2M_{\psi(3686)}} \) is the energy of the transition photon in the rest frame of ψ(3686), \( BW(m) = \frac{M_{\eta c} \Gamma_{\eta c}}{m^2 - M_{\eta c}^2 + iM_{\eta c} \Gamma_{\eta c}} \) is a relativistic Breit-Wigner function, \( M_{\eta c} \) and \( \Gamma_{\eta c} \) are the mass and width of ηc, \( D(E_γ) = \frac{E_γ^2}{E_0(E_γ + (E_0 - E_γ)^2)} \) is a function introduced by the KEDR collaboration [28], which damps the low-mass divergent tail, where \( E_0 = \frac{M^2_{\psi(3686)} - M_{\eta c}^2}{2M_{\psi(3686)}} \) is the peak energy of the transition photon. In the MC simulation, ηc → π⁺π⁻π⁰ events are generated according to a phase space distribution.

III. DATA ANALYSIS

A. ψ(3686) → γηc, ηc → π⁺π⁻π⁰

For ψ(3686) → γηc with ηc subsequently decaying into π⁺π⁻π⁰, the final state in this analysis is π⁺π⁻γγγ. Charged tracks must be in the active region of the MDC, corresponding to \( |\cos \theta| < 0.93 \), where θ is the polar angle of the charged track with respect to the beam direction, and are required to pass within ±10 cm of the interaction point in the beam direction and 1 cm of the beam line in the plane perpendicular to the beam. Photon candidates must have minimum energies of 25 MeV in the EMC barrel (\( |\cos \theta| < 0.8 \)) or 50 MeV in the EMC end-caps (0.86 < \( |\cos \theta| < 0.92 \)). To eliminate photons radiated from charged particles, each photon must be separated by at least 10° from any charged track. A requirement on the photon time, TDC, in the EMC, 0 ≤ TDC ≤ 14 (50 ns/count), is used to suppress noise and energy deposits unrelated to the event. Events with at least three photons are selected for further analysis. The two charged tracks are required to be identified as pions using the combined information of dE/dx from the MDC and the flight time from the TOF.

A four-constraint (4C) kinematic fit imposing energy-momentum conservation is performed under the γγγπ⁺π⁻ hypothesis, and the fit results are used for the kinematic quantities below. If there are more than three photon candidates in an event, the combination with the smallest \( \chi^2_{4C} \) is retained, and \( \chi^2_{4C} \) is required to be less than 20. To sup-
press the background events with two or four photons in the final states, 4C kinematic fits are also performed under the $\gamma\gamma\pi^+\pi^-$ and $\gamma\gamma\gamma\pi^+\pi^-$ hypotheses, and $\chi^2_{4C}$ is required to be less than the $\chi^2$ values of the $\gamma\gamma\pi^+\pi^-$ and $\gamma\gamma\gamma\pi^+\pi^-$ hypotheses. To select $\pi^0$ candidates, the invariant mass of two photons, $M_{\gamma\gamma}$, must satisfy $|M_{\gamma\gamma} - m_{\pi^0}| < 0.015$ GeV/$c^2$, where $m_{\pi^0}$ is the nominal $\pi^0$ mass [2]. If more than one $\gamma\gamma$ combination satisfies this requirement, the one with $M_{\gamma\gamma}$ closest to $m_{\pi^0}$ is selected. To reject background events with an $\eta$ in the final state, we require that the invariant masses of the other two possible photon pairs are not within the $\eta$ mass region, $|M_{\gamma\gamma} - m_{\eta}| > 0.02$ GeV/$c^2$, where $m_{\eta}$ is the nominal $\eta$ mass [2]. In order to reduce the $\omega \rightarrow \gamma\pi^0$ background, $|M_{\pi\pi\pi\eta} - m_{\omega}| > 0.05$ GeV/$c^2$ is required, where $M_{\gamma\gamma\pi\pi}$ and $m_{\omega}$ are the $\gamma\pi^0$ invariant mass and nominal $\omega$ mass [2], respectively. Events with a $\gamma\pi^+\pi^-\pi^0$ invariant mass in the vicinity of the $J/\psi$ peak in Fig. 1 are dominated from $J/\psi \rightarrow \pi^0 J/\psi$, $J/\psi \rightarrow \pi^+\pi^-\pi^0$ and $J/\psi \rightarrow \gamma \chi_{cJ}$, $\chi_{cJ} \rightarrow \gamma J/\psi$, $J/\psi \rightarrow \pi^+\pi^-\pi^0$, while the other background events, mainly from $\psi(3686) \rightarrow \rho\pi\pi$, contribute a smooth shape in the $\eta_c$ mass region. Using the off-resonance continuum data sample taken at a center-of-mass energy of 3.65 GeV, corresponding an integrated luminosity of 44 pb$^{-1}$ [29], we also investigate the background events from QED processes. There are no peaking contributions except for a small $J/\psi$ peak due to the initial state radiation process $e^+e^- \rightarrow \gamma_{ISR} J/\psi$.

We perform an unbinned maximum likelihood fit to the $M_{\pi^+\pi^-\pi^0}$ distribution in the range of [2.80, 3.15] GeV/$c^2$. In the fit, the $\eta_c$ signal shape is obtained from exclusive MC samples, the $J/\psi$ background shape is described by a Breit-Wigner function convolved with a Gaussian function, and the smooth background is described by a 2$^{nd}$-order Chebychev polynomial function, where all the parameters are free. The fit, shown in Fig. 2, yields $N = 15 \pm 44$ $\eta_c$-candidate events, consistent with zero. To obtain an upper limit on the signal yield, a series of unbinned maximum likelihood fits to the $M_{\pi^+\pi^-\pi^0}$ distribution are performed for different values $N$ of the $\eta_c$ signal yield. The upper limit on $N$ at the 90% confidence level (C.L.), $N_{UL}^{\eta_c}$, is the value of $N$ yielding 90% of the integral of the likelihood over all non-negative values of $N$. The fit-related uncertainties on $N_{UL}^{\eta_c}$ are considered by varying fit ranges, changing the order of the Chebychev polynomial function for the background shape and changing the mass and width of the $\eta_c$ within one standard deviation from the central values for the signal shape. The maximum upper limit amongst the variations, $N_{UL}^{\eta_c} = 121$, is used to calculate the upper limit on the branching fraction.

Figure 1. The distributions of $M_{\pi^+\pi^-\pi^0}$ in the vicinity of the $\eta_c$ signal, dots with error bars are data, the solid line is the $\eta_c$ line shape from the exclusive MC simulation, and the dashed line are the backgrounds estimated from inclusive MC sample and initial state radiation process $e^+e^- \rightarrow \gamma_{ISR} J/\psi$.

Figure 2. The result of the fit on the $\pi^+\pi^-\pi^0$ mass spectrum in the $\eta_c$ region. Dots with error bars are data, the solid curve shows the result of unbinned maximum likelihood fit, the dotted curve is the $\eta_c$ signal, the long-dashed curve is the $J/\psi$ background, and the short-dashed curve is the main background.
B. $\psi(3686) \rightarrow \gamma \eta(1405)$, $\eta(1405) \rightarrow f_0(980)\pi^0$

The final state for $\psi(3686) \rightarrow \gamma \eta(1405)$, $\eta(1405) \rightarrow f_0(980)\pi^0$ with $f_0(980) \rightarrow \pi^+\pi^-$ is also $\pi^+\pi^-\pi^0$, so we also perform a search for $\eta(1405) \rightarrow f_0(980)\pi^0$ in $\psi(3686)$ radiative decays. The same event selection is used for events with $\pi^+\pi^-\pi^0$ invariant mass within the region of $[1.20, 2.00]$ GeV/$c^2$, and the resulting $\pi^+\pi^-\pi^0$ invariant mass distribution is shown in Fig. 3. A narrow structure around 0.98 GeV/$c^2$ is observed, which is consistent with that observed in $J/\psi \rightarrow \gamma \eta(1405)$, $\eta(1405) \rightarrow f_0(980)\pi^0$ [12]. After requiring the $\pi^+\pi^-$ invariant mass to satisfy $|M_{\pi^+\pi^-}-m_{f_0}| < 0.04$ GeV/$c^2$, where $m_{f_0}$ is the nominal mass of $f_0(980)$ [2], there is no apparent $\eta(1405)$ signal in the $M_{\pi^+\pi^-\pi^0}$ distribution, shown in Fig. 4. The background events are investigated using $\pi^0$ sidebands ($0.100 < M_{\pi\gamma\gamma} < 0.115$ GeV/$c^2$ and $0.155 < M_{\gamma\gamma} < 0.170$ GeV/$c^2$), $f_0(980)$ sidebands ($0.90 < M_{\pi^+\pi^-} < 0.94$ GeV/$c^2$ and 1.04 $< M_{\pi^+\pi^-} < 1.08$ GeV/$c^2$), and the inclusive MC decays, and no obvious peaking structures are observed around 1.4 GeV/$c^2$.

IV. SYSTEMATIC UNCERTAINTIES

The systematic uncertainties in branching fraction measurements mainly come from the tracking, photon detection, and particle identification (PID) efficiencies, the 4C kinematic fit, the $\pi^0$ mass window requirement, the uncertainties of $B(\pi^0 \rightarrow \gamma\gamma)$ and the number of $\psi(3686)$ events, and the fitting related uncertainties.

The MDC tracking efficiency is studied with clean channels of $J/\psi \rightarrow \pi\pi\pi\pi$ and $J/\psi \rightarrow \rho\pi$ [9], and the MC simulation is found to agree with data within 1%. Therefore 2% is taken as the systematic uncertainty for the two charged tracks in the final states.

The photon detection efficiency is studied with the control sample $J/\psi \rightarrow \pi^+\pi^-\pi^0$ [30]. The difference between data and MC is less than 1% per photon. Therefore 3% is assigned as the systematic uncertainty from the three photons.

The $\pi^\pm$ particle identification efficiency is studied using a clean control sample of $J/\psi \rightarrow \rho\pi$ events, and the PID efficiency for data agrees with that of the Monte Carlo simulation within 1%. In this analysis, two charged tracks are identified as pions, so 2% is taken as the systematic uncertainty.

The uncertainty associated with the 4C kinematic fit comes from the inconsistency between data and MC simulation; this difference is reduced by correcting the track helix parameters of the MC simulation,
as described in detail in Ref. [31]. The correction parameters for pions are obtained by using control samples of $\psi(3686) \to \pi^+\pi^-\pi^0$. In this analysis, the efficiency obtained from the corrected MC samples is taken as the nominal value, and we take the differences between the efficiencies with and without correction, 4.5% for $\eta_c \to \pi^+\pi^-\pi^0$, and 3.1% for $\eta(1405) \to f_0(980)\pi^0$, as the systematic uncertainties.

The uncertainty due to the width of $f_0(980)$ is estimated by varying its parameters by $1\sigma$ in the MC simulation, where the parameters are obtained from the fit to data. The relative change of the detection efficiency, 5.4%, is taken as the corresponding systematic uncertainty.

The uncertainty related with the $\pi^0$ mass window requirement is studied with control samples of $\psi(3686) \to \pi^+\pi^-\pi^0$ for both data and MC simulation. We fit the $\gamma\gamma$ invariant mass distribution to determine the $\pi^0$ signal yields, and the $\pi^0$ efficiency is the ratio of the $\pi^0$ yields with and without the $\pi^0$ mass window requirement, where the $\pi^0$ yield is obtained by integrating the fitted signal shape. The difference in efficiencies between data and MC simulation, 0.8%, is assigned as the systematic uncertainty.

The branching fraction uncertainty of $\pi^0 \to \gamma\gamma$ is taken from the PDG [2] and is 0.03%. The uncertainty of the number of $\psi(3686)$ events is 0.65% [16].

For $\eta_c \to \pi^+\pi^-\pi^0$ and $\eta(1405) \to f_0(980)\pi^0$, the uncertainties from the fitting range, background shape, and the signal shape have already been considered, since we select the maximum upper limit from amongst various fits described above.

Table I summarizes all contributions to the systematic uncertainties on the branching fraction measurements. In each case, the total systematic uncertainty is given by the quadratic sum of the individual contributions, assuming all sources to be independent.

V. RESULTS

To be conservative, the upper limit on the branching fraction is determined by

$$B(\psi(3686) \to \gamma X) < \frac{N_{UL}}{N_{\psi(3686)} \times \varepsilon \times B(\pi^0 \to \gamma\gamma) \times (1 - \delta_{syst})},$$

where $X$ stands for $\eta_c (\eta_c \to \pi^+\pi^-\pi^0)$ or $\eta(1405) (\eta(1405) \to f_0(980)\pi^0 \to \pi^+\pi^-\pi^0)$, $\varepsilon$ is the detection efficiency obtained from the MC simulation and $\delta_{syst}$ is the total systematic uncertainty.

The detection efficiencies are 18.4% and 18.5% for $\eta_c \to \pi^+\pi^-\pi^0$ and $\eta(1405) \to f_0(980)\pi^0$, respectively, which are determined with MC simulation by assuming the polar angle of radiative photon follows the distribution $1 + \cos^2 \theta_c$. The upper limits at the 90% C.L. on $B(\psi(3686) \to \gamma\eta_c) \times B(\eta_c \to \pi^+\pi^-\pi^0)$ and $B(\psi(3686) \to \gamma\eta(1405)) \times B(\eta(1405) \to f_0(980)\pi^0) \to \pi^+\pi^-\pi^0$) are calculated to be $1.6 \times 10^{-6}$ and $5.0 \times 10^{-7}$, respectively.

VI. SUMMARY

Using $448.1 \times 10^6 \psi(3686)$ events accumulated with the BESIII detector, the search for $\eta_c \to \pi^+\pi^-\pi^0$ is performed for the first time. No obvious $\eta_c$ signal is seen in the $\pi^+\pi^-\pi^0$ mass spectrum, and the 90% C.L upper limit on $B(\psi(3686) \to \gamma\eta_c) \times B(\eta_c \to \pi^+\pi^-\pi^0)$ is $1.6 \times 10^{-6}$. Using the branching fraction of $\psi(3686) \to \gamma\eta_c$, [3.4 ± 0.5] × 10^{-3}, the upper limit for $B(\eta(1405) \to \pi^+\pi^-\pi^0)$ is calculated to be $5.5 \times 10^{-4}$. We also search for $\psi(3686) \to \gamma\eta(1405)$, $\eta(1405) \to f_0(980)\pi^0$. No obvious structure around the $\eta(1405)$ is observed, and the 90% C.L upper limit on $B(\psi(3686) \to \gamma\eta(1405)) \times B(\eta(1405) \to f_0(980)\pi^0) \times B(f_0(980) \to \pi^+\pi^-\pi^0)$ is calculated to be less than $3.3 \times 10^{-2}$, which indicates that this process also violates the “12% rule”.

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Table I. Summary of systematic uncertainty sources and their contributions (in %).

| Source                        | $\eta_1 \rightarrow \pi^+ \pi^- \pi^0$ | $\eta(1405) \rightarrow f_0(980) \pi^0$ |
|-------------------------------|---------------------------------------|------------------------------------------|
| MDC tracking                  | 2.0                                   | 2.0                                      |
| Photon detection              | 3.0                                   | 3.0                                      |
| Particle ID                   | 2.0                                   | 2.0                                      |
| 4C kinematic fit              | 4.5                                   | 3.1                                      |
| $\pi^0$ mass window           | 0.8                                   | 0.8                                      |
| Width of $f_0(980)$           | ...                                   | 5.4                                      |
| $B(\pi^0 \rightarrow \gamma\gamma)$ | 0.03                        | 0.03                                      |
| Number of $\psi(3686)$        | 0.65                                   | 0.65                                      |
| Total                         | 6.2                                   | 7.5                                      |

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