Universal Constituent-Quark Model for Baryons

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Abstract

We present a relativistic constituent-quark model that covers all known baryons from the nucleon up to $\Omega_{bb\bar{b}}$. The corresponding invariant mass operator includes a linear confinement and a hyperfine interaction based on effective degrees of freedom. The model provides for a unified description of practically all baryon spectra in good agreement with present phenomenology and it can tentatively be employed for the relativistic treatment of all kinds of baryon reactions. Predictions of states still missing in the phenomenological data base, especially in the lesser explored heavy-flavor sectors of charm and bottom baryons, should be important especially for future experiments in these areas.

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Quantum chromodynamics (QCD) is generally considered as the fundamental theory of strong interactions. While it has been well established in the perturbative regime at high energies, QCD still lacks a comprehensive solution at low and intermediate energies, even 40 years after its invention. In order to deal with the wealth of non-perturbative phenomena, various approaches are followed with limited validity and applicability. This is especially also true for lattice QCD, various functional methods, or chiral perturbation theory, to name only a few. In neither one of these approaches the full dynamical content of QCD can yet be included. Basically, the difficulties are associated with a relativistically covariant treatment of confinement and the spontaneous breaking of chiral symmetry (SB\(\chi\)S), the latter being a well-established property of QCD at low and intermediate energies. As a result, most hadron reactions, like resonance excitations, strong and electroweak decays etc., are nowadays only amenable to models of QCD. Most famous is the constituent-quark model (CQM), which essentially relies on a limited number of effective degrees of freedom with the aim of encoding the essential features of low- and intermediate-energy QCD.

The CQM has a long history, and it has made important contributions to the understanding of many hadron properties, think only of the fact that the systematization of hadrons in the standard particle-data base \([1]\) follows the valence-quark picture. Over the decades the CQM has ripened into a stage where its formulation and solution are well based on a relativistic (or more generally Poincaré-invariant) quantum theory. Relativistic constituent-quark models (RCQM) have been developed by several groups, however, with limited domains of validity. Of course, it is desirable to have a framework as universal as possible for the description of all kinds of hadron processes in the low- and intermediate-energy regions. This is especially true in view of the advent of ever more data on heavy-baryon spectroscopy from present and future experimental facilities.

Here, we present a RCQM that comprises all known baryons with flavors \(u, d, s, c,\) and \(b\) within a single framework. There have been only a few efforts so far to extend a CQM from light- to heavy-flavor baryons. We may mention, for example, the approach by the Bonn group who have developed a RCQM, based on the ’t Hooft instanton interaction, along a microscopic theory solving the Salpeter equation \([2]\) and extended their model to charmed baryons \([3]\), still not yet covering bottom baryons. A further quark-model attempt has been undertaken by the Mons-Liège group relying on the large-\(N_c\) expansion \([4, 5]\), partially extended to heavy-flavor baryons \([6]\). Similarly, efforts are invested to expand
other approaches to heavy baryons, such as the employment of Dyson-Schwinger equations together with either quark-diquark or three-quark calculations [7, 8]. Also an increased amount of more refined lattice-QCD results has by now become available on heavy-baryon spectra (see, e.g., the recent work by Liu et al. [9] and references cited therein).

Our RCQM is based on the invariant mass operator

\[ \hat{M} = \hat{M}_{\text{free}} + \hat{M}_{\text{int}}, \]  

(1)

where the free part corresponds to the total kinetic energy of the three-quark system and the interaction part contains the dynamics of the constituent quarks \( Q \). In the rest frame of the baryon, where its three-momentum \( \vec{P} = \sum_{i=1}^{3} \vec{k}_i = 0 \), we may express the terms as

\[ \hat{M}_{\text{free}} = \sum_{i=1}^{3} \sqrt{\hat{m}_i^2 + \hat{k}_i^2}, \]  

(2)

\[ \hat{M}_{\text{int}} = \sum_{i<j}^{3} \hat{V}_{ij} = \sum_{i<j}^{3} \left( \hat{V}_{ij}^{\text{conf}} + \hat{V}_{ij}^{\text{hf}} \right). \]  

(3)

Here, the \( \hat{\vec{k}}_i \) correspond to the three-momentum operators of the individual quarks with rest masses \( m_i \) and the \( Q-Q \) potentials \( \hat{V}_{ij} \) are composed of confinement and hyperfine interactions. By employing such a mass operator \( \hat{M}^2 = \hat{P}_\mu \hat{P}_\mu \), with baryon four-momentum \( \hat{P}_\mu = (\hat{H}, \hat{P}_1, \hat{P}_2, \hat{P}_3) \), the Poincaré algebra involving all ten generators \( \{\hat{H}, \hat{P}_i, \hat{J}_i, \hat{K}_i\} \) \( (i = 1, 2, 3) \), or equivalently \( \{\hat{P}_\mu, \hat{J}_{\mu\nu}\} \) \( (\mu, \nu = 0, 1, 2, 3) \), of time and space translations, spatial rotations as well as Lorentz boosts, can be guaranteed. The solution of the eigenvalue problem of the mass operator \( \hat{M} \) yields the relativistically invariant mass spectra as well as the baryon eigenstates (the latter, of course, initially in the standard rest frame).

We adopt the confinement depending linearly on the \( Q-Q \) distance \( r_{ij} \)

\[ V_{ij}^{\text{conf}}(\vec{r}_{ij}) = V_0 + Cr_{ij} \]  

(4)

with the strength \( C = 2.33 \text{ fm}^{-2} \), corresponding to the string tension of QCD. The parameter \( V_0 = -402 \text{ MeV} \) is only necessary to set the ground state of the whole baryon spectrum, i.e., the proton mass; it is irrelevant, if one considers only level spacings.

The hyperfine interaction is most essential to describe all of the baryon excitation spectra. In a unified model the hyperfine potential must be explicitly flavor-dependent. Otherwise, e.g., the \( N \) and \( \Lambda \) spectra with their distinct level orderings could not be reproduced simultaneously. At least for baryons with flavors \( u, d, \) and \( s \) the type of hyperfine interaction
taking into account SBχS has been most successful over the past years. Obviously, it grabs the essential degrees of freedom governing the behavior of low-energy baryons [10–12]. The RCQM constructed along this dynamical concept, i.e., on Goldstone-boson exchange (GBE), has provided a comprehensive description of all light and strange baryons [13, 14]. This is not only true with regard to the spectroscopy but to a large extent also for other baryon properties, like electromagnetic and axial form factors [15] and a number of other reaction observables (for a concise summary see ref. [16]). It has been tempting to extend this successful concept even to the heavier flavors $c$ and $b$. By such studies one should in addition learn about the proper light-heavy and heavy-heavy hyperfine $Q$-$Q$ interactions. Some exploratory work in this direction had already been undertaken some time ago in ref. [17], hinting to promising results also for charm and bottom baryons.

Therefore we have advocated for the hyperfine interaction of our universal RCQM the $SU(5)_F$ GBE potential

$$V_{\text{id}}(\vec{r}_{ij}) = \left[ V_{24}(\vec{r}_{ij}) \sum_{a=1}^{24} \lambda_i^a \lambda_j^a + V_0(\vec{r}_{ij}) \lambda_i^0 \lambda_j^0 \right] \vec{\sigma}_i \cdot \vec{\sigma}_j. \quad (5)$$

Here, we take into account only its spin-spin component, which produces the most important hyperfine forces for the baryon spectra. While $\vec{\sigma}_i$ represent the Pauli spin matrices of $SU(2)_S$, the $\lambda_i^a$ are the generalized Gell-Mann flavor matrices of $SU(5)_F$ for quark $i$. In addition to the exchange of the pseudoscalar 24-plet also the flavor-singlet is included because of the $U(1)$ anomaly. The radial form of the GBE potential resembles the one of the pseudoscalar meson exchange

$$V_{\beta}(\vec{r}_{ij}) = \frac{g_{\beta}^2}{4\pi} \frac{1}{12m_im_j} \left[ \mu_{\beta}^2 e^{-\mu_{\beta} r_{ij}} r_{ij} - 4\pi \delta(\vec{r}_{ij}) \right], \quad (6)$$

for $\beta = 24$ and $\beta = 0$. Herein the $\delta$-function must be smeared out leading to [14] [18]

$$V_{\beta}(\vec{r}_{ij}) = \frac{g_{\beta}^2}{4\pi} \frac{1}{12m_im_j} \left[ \mu_{\beta}^2 e^{-\mu_{\beta} r_{ij}} r_{ij} - \Lambda_{\beta}^2 e^{-\Lambda_{\beta} r_{ij}} \right]. \quad (7)$$

Contrary to the earlier GBE RCQM [13], which uses several different exchange masses $\mu_\gamma$ and different cut-offs $\Lambda_\gamma$, corresponding to $\gamma = \pi$, $K$, and $\eta=\eta_8$ mesons, we here managed to get along with a universal GBE mass $\mu_{24}$ and a single cut-off $\Lambda_{24}$ for the 24-plet of $SU(5)_F$. Only the singlet exchange comes with another mass $\mu_0$ and another cut-off $\Lambda_0$ with a separate coupling constant $g_0$. Consequently the number of open parameters in the hyperfine interaction could be kept as low as only three (see Tab. [I]).
TABLE I. Free parameters of the present GBE RCQM determined by a best fit to the baryon spectra.

All other parameters entering the model have judiciously been predetermined by existing phenomenological insights. In this way the constituent quark masses have been set to the values as given in Tab. II. The 24-plet Goldstone-boson (GB) mass has been assumed as the value of the \( \pi \) mass and similarly the singlet mass as the one of the \( \eta' \). The universal coupling constant of the 24-plet has been chosen according to the value derived from the \( \pi-N \) coupling constant via the Goldberger-Treiman relation.

TABLE II. Fixed parameters of the present GBE RCQM predetermined from phenomenology and not varied in the fitting procedure.

We have calculated the baryon spectra of the relativistically invariant mass operator \( \hat{M} \) to a high accuracy both by the stochastic variational method [19] as well as the modified Faddeev integral equations [20, 21]. The present universal GBE RCQM produces the spectra in the light and strange sectors with similar or even better quality than the previous GBE RCQM [13] (see Figs. 1 and 2). Most importantly, the right level orderings specifically in the \( N, \Delta \) and \( \Lambda \) spectra as well as all other \( SU(3)_F \) ground and excited states are reproduced.
in accordance with phenomenology. The reasons are exactly the same as for the previous GBE RCQM, which has already been extensively discussed in the literature [11, 13, 14]. Unfortunately, the case of the Λ(1405) excitation could still not be resolved. It remains as an intriguing problem for all three-quark CQMs.

What is most interesting in the context of the present work are the very properties of the light-heavy and heavy-heavy $Q$-$Q$ hyperfine interactions. Can the GBE dynamics reasonably account for them? In Figs. 3 and 4 we show the spectra of all charm and bottom baryons that experimental data with at least three- or four-star status by the PDG [1] are available for [?]. As is clearly seen, our universal GBE RCQM can reproduce all levels with respectable accuracy. In the Λ_{c} and Σ_{c} spectra some experimental levels are not known with regard to their spin and parity $J^{P}$. They are shown in the right-most columns of Fig. 3. Obviously they could easily be accommodated in accordance with the theoretical spectra, once their $J^{P}$’s are determined. Furthermore the model predicts some additional excited states for charm and bottom baryons that are presently missing in the phenomenological data base.

Of course, the presently available data base on charm and bottom baryon states is not yet very rich and thus not particularly selective for tests of effective $Q$-$Q$ hyperfine forces. The situation will certainly improve with the advent of further data from ongoing and planned experiments. Beyond the comparison to experimental data, we note that the theoretical spectra produced by our present GBE RCQM are also in good agreement with existing
lattice-QCD results for heavy-flavor baryons. This is especially true for the charm baryons vis-à-vis the recent work by Liu et al. [9]. Further comparisons with results from lattice QCD and alternative methods will be given in a forthcoming more detailed article [22], where also a number of additional theoretical spectra up to Ω_{bbb} will be presented (for which, however, no phenomenological data exist so far).

We emphasize that the most important ingredients into the present RCQM are relativity, or more generally Poincaré invariance, and a hyperfine interaction that is derived from an interaction Lagrangian built from effective fermion (constituent quark) and boson (Goldstone boson) fields connected by a pseudoscalar coupling [11]. It appears that such kind of dynamics is quite appropriate for constituent quarks of any flavor. The effects of the hyperfine forces do not at all become tentatively small for baryons with charm and bottom flavors. In some cases at least the heavy-light interactions are of the same importance for the
level spacings as the light-light interactions. This has already been seen for charm baryons in the work by the Bonn group [3] and is also true for our universal GBE RCQM (as will be detailed in ref. [22] too). It is furthermore in line with findings from earlier lattice-QCD calculations [23].

As a result we have demonstrated by the proposed GBE RCQM that a universal description of all known baryons is possible in a single model. Here, we have considered only the baryon masses (eigenvalues of the invariant mass operator $\hat{M}$). Beyond spectroscopy the present model will be subject to further tests with regard to the baryon eigenstates, which are simultaneously obtained from the solution of the eigenvalue problem of $\hat{M}$. They must prove reasonable in order to make the model a useful tool for the treatment of all kinds of baryons reactions within a universal framework.
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