Thermal Radiation Effect on Entropy in Hydromagnetic Peristaltic Flow of a Jeffrey Nanofluid Through an Unsymmetrical Channel

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Abstract: The present work focuses on the thermal radiation effect in Hydro Magnetic peristaltic pumping of Jeffrey nano fluid in an unsymmetrical channel. The present work looks at the radiation effect, magnetic field, Brownian motion, thermophoresis, and buoyancy forces. Well established high wavelength and tiny Reynolds number assumptions are invoked. The exact solution for the velocity, concentration, temperature has been evaluated. It has been observed that temperature is reducing with increasing of Radiation parameter.

Keywords: Brownian motion, Buoyancy forces, Nanofluid, Peristaltic transport, Thermal radiation, Thermophoresis.

Nomenclature

| Symbol | Description |
|--------|-------------|
| $a_1$, $a_2$ | Wave amplitude |
| $\lambda$ | Wavelength |
| $\mu$ | Coefficient of viscosity |
| $C$ | Concentration |
| $C_0$ | Speed of the wave |
| $C_p$ | Specific heat at constant pressure |
| $D_B$ | Brownian motion coefficient |
| $D_T$ | Thermophoretic diffusion coefficient |
| $d_1 + d_2$ | Width of channel |
| $Ec$ | Eckert number |
| $Gm$ | Local nano particle Grashoff number |
| $I$ | Identity tensor |
| $Gm$ | Hartman number |
| $Nt$ | Thermophoresis |
| $P$ | Pressure in wave |
| $P$ | Pressure in fixed frame |
| $Pr$ | Prandtl number |
| $Re$ | Reynolds number |
| $S$ | Extra stress tensor |
| $Sc$ | Schmidt number |
| $S_{ij}$ | Components of the extra tensor |
| $T_m$ | Mean temperature |
| $T$ | Temperature |
| $t$ | Time |
| $U$, $V$ | Velocity components in the laboratory frame (X, Y) |
| $U$, $V$ | Velocity components in the wave frame (x, y) |

Greek Symbols

| Symbol | Description |
|--------|-------------|
| $\sigma$ | Stefan–Boltzmann constant |
| $\gamma$ | Shear rate |
| $\lambda_1$ | Ratio of relaxation to retardation times |
| $\gamma$ | Shear rate |
| $\lambda_2$ | Retardation time |
| $\rho_f$ | Fluid density |
| $\alpha$ | Thermal diffusivity |

I. INTRODUCTION

Peristalsis is a mechanism for the transitory flow of fluids in the channel interior by means of surface deformations. Peristaltic pumping is the basic mechanism for manufacturing some biomedical instruments. Hypothetical and investigational approaches were initiated on peristalsis of a viscous fluid [1-2].

Sud et al.[3] considered the Magneto hydrodynamics principle to study flow of blood, and noticed that, impact of pertinent affecting magnetic field expedites the blood momentum. Abbasi et al [4] looked at peristaltic pumping of hydro magnetic field with fluctuating viscosity. Further, with the magnetic field impact, peristaltic transport of a Casson fluid in an unsymmetrical stream noticed [5] in addition the fluid flow characteristics analyzed with the extended wavelength and small Reynolds numeral presumption [6]. The extended study is on thermo-physical properties of a carbon nano tube caused by peristaltic pump has been carried out [7].

The flow equations are governed, in concern of the viscous dissipation and Joule heating property. In this Buongiorno model [8] thermophoresis and Brownian diffusion are also examined and came to a conclusion that these two play vital role in contributing to heat transfer augmentation. Also in contemporary drug release and cancer therapy [9] role of magnetic field in nano fluids is noteworthy.
Utilizing iron-based nanoparticles Cancer patients undergo radiation and nano fluids bind to tumor cells due to magnetic properties without harming healthy cells. In this regard, few representative studies were made to the conjecture of nano fluids taking thermophoresis and Brownian motion into account [10–12]. Effects of Hartmann number and heat transfer characteristics on CuO–water nano fluid flow have been examined [13]. Contributions were made to describe slanted magnetic field effect on mixed convective flows, considering non-Newtonian fluids in two forms with Joule heating [14]. Impacts of magnetic field, chemical reaction and elastic properties on the ramparts of the blood vessel to understand the dynamics of blood flow is investigated [15]. The investigative solutions for concentration, energy and undimensioned velocity elucidate that for greater values of Hartmann number the trapping bolus size enhances. Magnetic field, thermophoresis, Buoyancy forces and Brownian motion effects are pooled to revise the peristaltic transport of an incompressible Jeffrey nano fluid [16]. Due to radiation parameter, peristaltic move of Jeffrey nano fluid in a channel with Joule heating and viscous dissipation has been addressed here in [17]. A study of unaccustomed thermal radiation, convective mass state aspects in peristaltic model flow is analyzed [18]. Due to it is crucial to study of the blood flow concepts, peristaltic pump of a Jeffrey nano model exposed to the inclined magnetic field is considered in this work. Velocity, concentration, thermal radiation and slip conditions exhibited by the walls and results are obtained for the same. The simplified system of coupled equations is solved numerically using relevant boundary conditions underneath extended wave length and tiny Reynolds number presumptions.

II. MATHEMATICAL FORMULATION

An incompressible Jeffrey nano fluid of non-uniform thickness is considered in two-dimensional unsymmetrical channel. Flow is considered to be free from the electric field. The fluid movement is generated in response to the propagating infinite wave train with invariable pace c0 with wavelength λ along the stretchy channel walls. y = η1, y = η2 are considered to be upper and lower walls. The effect of applied constant magnetic field B0 is insignificant for tiny magnetic Reynolds number. The wave shape is represented by the following equation

\[ H_1 = \eta_1(x, t) = d_1 + (X - c_0 t) \tan \alpha + a_1 \cos \frac{2\pi}{\lambda}(X - c_0 t) \]  
(1)

\[ H_2 = \eta_2(x, t) = -d_2 - (X - c_0 t) \tan \alpha - a_2 \cos \left[ \frac{2\pi}{\lambda}(X - c_0 t) + \phi \right] \]  
(2)

Where the inherent relation for Cauchy stress tensor (t) in an incompressible Jeffrey model is

\[ \tau = -p I + S = \frac{\mu}{1 + \lambda_1} \left( \gamma + \lambda_2 \frac{dy}{dt} \right) \]  
(3)

The geometry of the problem

For incompressible nanofluid the equations leading in the flow are given as

\[ \frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 0 \]  
(4)

\[ \frac{\partial u}{\partial t} + U \frac{\partial u}{\partial x} + V \frac{\partial u}{\partial y} = - \frac{1}{\rho f} \frac{\partial p}{\partial x} + \frac{1}{\rho f} \left[ \frac{\partial}{\partial x} (S_{xx}) + \frac{\partial y}{\partial x} \right] \]  
(5)

\[ \frac{\partial T}{\partial t} + U \frac{\partial T}{\partial x} + V \frac{\partial T}{\partial y} = \alpha \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + \frac{1}{\rho f} \frac{\partial}{\partial x} \left[ X \frac{\partial u}{\partial x} + SXY \frac{\partial u}{\partial y} + D \frac{\partial^2 u}{\partial y^2} \right] \]  
(6)

\[ \frac{\partial C}{\partial t} + U \frac{\partial C}{\partial x} + V \frac{\partial C}{\partial y} = D_{f} \left[ \frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2} \right] + \frac{D_{f}}{\tau} \left[ \frac{\partial u}{\partial x} \frac{\partial C}{\partial y} + \frac{\partial u}{\partial y} \frac{\partial C}{\partial x} \right] \]  
(7)

Radiative heat flux q_r is specified to be

\[ q_r = \frac{-4 \sigma \frac{\partial T^4}{\partial y}}{3k} \]  
(9)

The wave and laboratory frame transformation is given by

\[ x = X - ct, y = Y, u = U - c, v = V, p(x) = P(X, t) \]  
(10)

The non-dimensional quantities are defined as in below:
\[
\begin{align*}
& u' = u, v' = v, x' = x, y' = y, t' = t, \eta' = \frac{n}{d}, \\
& p' = \frac{d^2 p}{c d^2}, k' = k, \\
& \theta = \frac{T-T_0}{T_1-T_0}, \phi = \frac{C-C_0}{C_1-C_0}, \\
& Gr = \frac{(T_1-T_0) n p f g \beta d^2 (1-C_0)}{c p u}, \\
& Nb = \frac{\tau \phi (c_1-C_0)}{v}, \\
& Nt = \frac{\tau \phi (T_1-T_0)}{T_0 v}, \\
& Rn = \frac{16 \sigma T_0^3}{3 \nu \mu c_f^2}, \\
& \delta = \frac{d_1}{x}, \lambda_2 = \frac{x_C}{d_1}, M = \frac{n}{\nu} B_0 d_1, \\
& \phi = \frac{\theta p c_0}{\lambda}, \\
& pr = \frac{\theta p c_0}{k}.
\end{align*}
\]

After primes are omitted from equations (4)–(7), the stream function \( \psi \) as follows,
\( u = \psi_y, v = -\psi_x, \)
(12)

After employing extended wavelength and low Reynolds number approximations in the above non-dimensional leading flow equations one has subsequent problems
\[
\begin{align*}
& \left(1 + \frac{1}{1+\lambda_1} \frac{\partial^3 \psi}{\partial y^3} + Gr \frac{\partial \theta}{\partial y} + Gm \frac{\partial \phi}{\partial y} - M^2 \frac{\partial^2 \psi}{\partial y^2} \right) = 0, \\
& (1 + Rn Pr(1 + \theta^3)) \frac{\partial^3 \psi}{\partial y^3} + Nt Pr \frac{\partial \theta}{\partial y} + (3Rn(1+\theta)^2) + \\
& Nt Pr \theta \psi_2 + Br 11 + \lambda_1 \theta \psi_2 + M2 \phi \psi_2 = 0.
\end{align*}
\]
(13)  (14)

The relevant boundary conditions are
\[
\begin{align*}
& \psi = \frac{q}{2}, \frac{\partial \psi}{\partial y} = -1, \theta = 0, \phi = 0, \text{ at } \ y = h_1 = 1 + \tan \alpha + a \cos(2\pi x), \\
& \psi = \frac{q}{2}, \frac{\partial \psi}{\partial y} = -1, \theta = 1, \phi = 1, \text{ at } \ y = h_2 = -d - \tan \alpha - b \cos(2\pi x + \varphi),
\end{align*}
\]
(15)  (16)  (17)

Here flux \( q \) in wave frame and \( a, b, d, \phi \) must satisfy
\[
a^2 + b^2 + 2 ab \cos \phi \leq (1 + d)^2
\]
(18)

The flow speed in fixed frame and wave are connected by
\[
Q = q + 1 + d
\]
(19)

II. A. Second Law Analysis

The volumetric local entropy generation of Jeffrey Nano fluid in the magnetic field existence can be described as
\[
S_{gen} = \left(1 + \frac{4}{3} Rd \right) \left(T_1^2 + T_2^2 \right) + \frac{\mu_n f}{(1+\lambda_1)} \left[ 2 \left(u_x^2 + vy^2 + uw + v^2 + 2\alpha \theta + 2\gamma \theta \right) \right]
\]
(20)

In the convection procedure, the two sources of irreversibility, the heat transfer and fluid friction of give rise to generation of entropy. It is perceptible from the above Eq. (20) that contribution of heat transfer is caused to the first term, the second is the effect of fluid resistance to local entropy generation and significance of the third term is because of the magnetic field effect.

\[
S_{gen} = \frac{k_{nf}}{T_0^2} \left( T_1^2 + T_2^2 \right) + \frac{\mu_n f}{(1+\lambda_1) T_0} \left[ 2 \left(u_x^2 + vy^2 + uw + v^2 + 2\alpha \theta + 2\gamma \theta \right) \right]
\]
(21)

Using the dimensionless quantities introduced in Eq. (10), it can be obtained from the Eq. (21)
\[
N_s = \frac{T_0^2 d_1^2 S_{gen}}{k (T_1^2)} + \frac{4}{3} Rd \left[ \theta_y^2 \right],
\]
(22)

Writing \( N_s = N_h + N_v \), where \( N_h = \left( \theta_y^2 \right) \) and \( N_v = \frac{Br}{\Delta} \left[ \frac{1}{1+\lambda_1} (\psi_{yy})^2 + M^2 \psi_y^2 \right] \)
(23)

Here \( N_h \) corresponds to heat transfer irreversibility and \( N_v \) corresponds to viscous dissipation and the magnetic field effect.

III. DISCUSSION

The non-linear coupled differential equations are solved in this paper. The physical impacts of pertinent parameters are inspected by interpreting graphical outcomes.

Velocity Profile

To study velocity, various compliant embedded parameters have been considered (Fig. 2(a)–2(d)). Fig. 2(a) discloses that at the middle of the channel \( M \) has a resistive role for the velocity. It witnesses the statement that the resistive nature is due to Lorentz force. It is perceived that velocity is increasing at the lower division of the channel with Jeffrey fluid parameter (Fig. 2(b)). The impact of \( Gr \) and \( Gm \) on velocity is presented in (Fig. 2(c), 2(d)) and it is found that velocity enhances near the lower wall.
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Fig. 2(b) Variations in velocity for $\lambda_1$

Fig. 2(c) Variations in velocity for $Gr$

Fig. 2(d) Variations in velocity for $Gm$

Temperature Profiles

In Fig. 3(a)-3(f) the outline of temperature is represented in response of pertinent parameters. The effects of Radiation (Rn) and Jeffrey fluid parameter ($\lambda_1$) on the temperature is disclosed in Fig. 3(a) and 3(b), it is discerned that temperature retarded for both of the parameters Rn and $\lambda_1$ vary. Fig. 3(c) depicts that, raise in the value of Nb causes enhance in temperature because it makes the motion of nano particles stronger. The variations in temperature distribution of fluid on Nt are plotted in Fig. 3(d). From Fig. 3(d) it can be depicted that the temperature rises with increase in Nt. Similar effects were found in Br number and Magnetic number.

Nano particle concentration profiles

It can be revealed from Fig. 4(a) that concentration abates as Magnetic parameter M enhancing. From Fig. 4(b) it can be noted that nano particle concentration (Gm) increases as Nt increases. From Fig. 4(c) and Fig. 4(d), it can be depicted that the Gm is decreasing with the increasing Nt and Br, however, opposite behavior can be depicted from the effects of Rn and $\lambda_1$.

Entropy

Influence of various governing parameters on entropy can be depicted from the Fig. 5(a)-5(c). Corresponding to the enhancement in parameter M entropy generation reduces. Similar behavior is observed in the case of influence of Nb, Nt and Rn. Entropy has larger near the walls for Br and Gr.
Fig. 3(d) Variations in temperature for $N_t$

Fig. 3(e) Variations in temperature for $B_r$

Fig. 3(f) Variations in temperature for $M$

Fig. 4(a) Variations in concentration for $M$

Fig. 4(b) Variations in concentration for $N_b$

Fig. 4(c) Variations in concentration for $N_t$

Fig. 4(d) Variations in concentration for $B_r$

Fig. 4(e) Variations in concentration for $R_n$
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Fig. 4(f) Variations in concentration for $\lambda_1$

Fig. 5(a) Variations in entropy for $M$

Fig. 5(b) Variations in entropy for $Br$

Fig. 5(c) Variations in entropy for $Nb$

Fig. 5(d) Variations in entropy for $Br$

Fig. 5(e) Variations in entropy for $Rn$
An analysis has been carried out for the effect of thermal radiation on entropy in hydro magnetically driven peristaltic flow of a Jeffrey nanofluid. Expressions for velocity, temperature, and concentration have been derived analytically, and entropy generation is computed. The main conclusions can be drawn as follows:

- At the middle of the channel, the magnetic parameter has a resistive role for the velocity.
- It is discerned that temperature retarded for the Radiation and Jeffrey fluid parameters.
- Concentration abates as Magnetic parameter enhancing, and decreasing with the increasing thermophoresis and Brinkman number.
- Corresponding to the enhancement in magnetic parameter entropy generation reduces. Similar behavior is observed in the case of Brownian motion, thermophoresis, and Radiation parameter.

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