Per-Tone model for Common Mode sensor based alien noise cancellation for Downstream xDSL

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Abstract—For xDSL systems, alien noise cancellation using an additional common mode sensor at the downstream receiver can be thought of as interference cancellation in a Single Input Dual Output (SIDO) system. The coupling between the common mode and differential mode can be modelled as an LTI system with a long impulse response, resulting in high complexity for cancellation. Frequency domain per-tone cancellation offers a low complexity approach to the problem besides having other advantages like faster training, but suffers from loss in cancellation performance due to approximations in the per-tone model. We analyze this loss and show that it is possible to minimize it by a convenient post-training “delay” adjustment. We also show via measurements that the loss of cancellation performance due to the per-tone model is not very large for real scenarios.

I. INTRODUCTION

Cancellation of alien noise in xDSL receivers has attracted significant interest recently [1] [2] [3] [4]. Sources of electromagnetically coupled alien noises include PLC modems, appliances (e.g. treadmill) and switching power supplies [5] [2]. Mitigating the impact of these noises by cancellation can be done using an additional sensor, e.g. a common mode (CM) sensor or an unused twisted pair [6] [7]. In xDSL, while differential mode (DM) is used for transmitting the data signal over the unshielded twisted pair, electro-magnetically coupled noises appear as CM signals. Ideally the CM and DM transmission modes are isolated but in practice, due to cable imbalances, there exist significant leakages between the CM and DM which is how alien/impulse noise leaks into the differential mode signal. The leakage couplings between CM and DM can be modelled as linear time-invariant (LTI) systems [7] with very long impulse responses [1] [8].

Alien noise cancellation can be effected by spatial whitening via optimum linear combination of the CM and DM signals. However these involve high complexity due to the long impulse response if the couplings are modelled as finite impulse response (FIR) systems. Moreover we need to estimate the coupling between CM and DM to derive the linear canceller and this coupling usually needs to be estimated in the presence of a strong useful data signal in DM. This is because estimating the cross-correlation requires that CM be excited by the alien/impulse noise events and noise events may start only after the modem has trained up.

The frequency domain per-tone model of cancellation alluded to in [4] [9] [2], has low-complexity, easily incorporates decision-directed estimation and hence faster training [8] and implementation convenience. In the per-tone model, it is assumed that the alien noise in CM undergoes a circular convolution with the CM-DM coupling function, while in reality the convolution is linear since the alien noise does not have a cyclic structure. This approximation introduces a penalty on the cancellation performance and per-tone cancellation residual noise can be potentially inferior to the Cramer-Rao lower bound (CRLB) for the residual noise.

Following are the main contributions of the paper:

- We provide analytical treatment of the impact of non-cyclic structure of the alien noise signal and quantify the expected loss of performance.
- We outline a method to optimize this loss by a post-training adjustment and we also suggest a method to derive a near-optimal time-domain canceller using the per-tone canceller coefficients.

Notations: Time domain signals, impulse response etc. are denoted by lower-case letters, e.g. \( y(n) \). Frequency domain signals are denoted by upper-case letters e.g. \( Y_d(q) \), \( Y_c \). Matrices are denoted by bold letters while vectors are denoted by bold italicized letters. Superscript * denotes a conjugate.

II. SYSTEM MODEL AND PROPOSED CANCELLER

A. System Model

We assume that the CM-DM coupling modelled as an LTI time domain FIR filter [1] [2] denoted by \( h(n) \) is \( L \) taps long. Let \( x(n) \) be the cyclically extended time domain DMT modulated data signal and \( h_d(n) \) be the \( M \) tap long impulse response of the DM direct channel between the CO transmitter and the CPE receiver. The time domain alien noise signals in the CM and DM are denoted by \( z_c(n) \) and \( z_d(n) \) while \( v_c(n) \) and \( v_d(n) \) constitute the background AWGN noise at the two sensors. There also exists a reverse DM to CM coupling which results in the DM data signal \( x(n) \) leaking into the CM but is ignored in our formulation for simplicity. The received signal

\[ x(n) = y_c(n) + z_c(n) + v_c(n) + \sum_{l=0}^{L-1} h_l \cdot y_d(l) \]

This leakage signal is extremely small in comparison to the received DM useful data signal (appx 50 dB attenuation) [1] (also confirmed by lab measurements) and will not lead to any significant performance gain if combined with the DM data signal. The formulation can be modified to include this if needed but is skipped in interest of simplicity.
define the following terms for each tone: because
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corresponding complex valued hermitian-symmetric frequency
domain signals in CM and DM,
yc(n) and yd(n) are given by:
yc(n) = zc(n) + vc(n)
yd(n) = \sum_{k=0}^{L-1} h_d(k) x(n - k) + z_d(n) + vd(n)
zd(n) = \sum_{k=0}^{L-1} h_{cd}(k) z_c(n - k)
(3)
A discrete Fourier transform (DFT) of length \( P \) is applied to
both the CM and DM real-valued time domain signal blocks ( VDSL transmission is baseband )
yc(n) and yd(n), resulting in corresponding complex valued hermitian-symmetric frequency
domain signals. The corresponding frequency domain repre-
sentations for the \( q^{th} \) tone are given by :
yc(q) = Zc(q) + Vc(q)
yd(q) = H_d(q) X(q) + Zd(q) + Vd(q)
where, \( H_d(q) \) is the DM channel coefficient and \( X(q) \) is the
transmitted data symbol. We will now omit the tone index \( q \)
for rest of the paper wherever it is unambiguous to do so. The
alien noise component on any tone in DM, i.e. \( Z_d \) can be split
up as:
\[
Z_d(q) = H_{pertone}(q) Z_c(q) + U(q)
\]
(6)
where \( U \) and \( Z_c \) are uncorrelated and are both independent of
\( V_d \). The uncorrelated term \( U \) consisting of contributions
from neighbouring tones as well as from outside the signal window
used for DFT of \( z_c(n) \) represents the uncancellable (in the
per-tone model) portion of the alien noise signal and arises
because \( z_d(n) = z_c(n) + h(n) \) and \( z_c(n) \) is not cyclic. We
define the following terms for each tone:
\[
I_{cm} = E\{|Z_c|^2\}, \quad I_{dm} = E\{|Z_d|^2\}, \quad I_U = E\{|U|^2\}
\]
\[
\sigma^2_X = E\{|X|^2\}, \quad \sigma^2_{Vc} = E\{|V_c|^2\}, \quad \sigma^2_{Vd} = E\{|V_d|^2\}, \quad \delta = \frac{\sigma^2_{U}}{I_{cm}}
\]
B. Proposed Per-Tone Canceller
The proposed per-tone canceller consists of linearly com-
bining the received frequency domain CM and DM signals
tone-wise with a different cancellation coefficient \( \beta(q) \) used
for each tone.
\[
\hat{Y}_d = Y_d - \beta Y_c.
\]
The optimal cancellation coefficient obtained by minimising
the residual error variance after cancellation is given by:
\[
\beta_{opt} = \frac{E\{|Y_d|Y_c^*\}}{E\{|Y_c|^2\}} = \frac{H_{pertone}}{1 + \delta}
\]
(8)
The variance of the residual noise on any tone after cancella-
tion by the optimum linear canceller designated the Per-Tone
Lower Bound (PTLB) is given by:
\[
\Omega_{PTLB} = I_{dm} \left( \frac{\delta}{1 + \delta} \right)^2 + I_U + \sigma^2_{Vd} + |H_{pertone}|^2 \frac{\delta}{(1 + \delta)^2} \sigma^2_{Vc}
\]
(9)
The impact of uncancellable noise on the cancellation perfor-
mance(vis a vis linear time domain cancellation) is small if the
folded in CM background noise dominates the residual noise
i.e. \( |H_{pertone}|^2 \sigma^2_{Vc} >> I_U \) which happens if the coupling \( H_{cd} \)
is large or if the CM background noise power is very high.
III. ANALYSIS OF PER-TONE CANCELLATION MODEL
We now derive an analytical expression for the coefficient
\( H_{pertone}(q) \), which should be equal to the DFT of \( h_{cd}(n) \)
in case \( z_c(n) \) is cyclic. Figure I shows the CM and DM time
domain signal blocks over which the DFT is performed and
the two blocks are misaligned by \( T \) samples. Note that the
DM alien noise signal \( z_d(n) \) which is modelled as a convolu-
tion of \( z_c(n) \) and \( h_{cd}(n) \) includes portions of \( z_c(n) \) which lie
outside the CM signal block used for the DFT due to the delay spread
of \( h_{cd}(n) \) and also due to the misalignment \( T \).
Consider a decomposition of \( z_c(n) \) , \( z_c(n) = z_{c-cyc}(n) + z_{c1}(n) + z_{c2}(n) \) as shown in figure I where \( z_{c-cyc}(n) \) is a
cyclically extended version of the CM DFT block i.e.
\( z_{c-cyc}(n) = z_c(n) \) for \( -T \leq n \leq P - T - 1 \) while
\( z_{c1}(n) = -z_{c-cyc}(n) \) for \( n < -T \) and \( n > P - T - 1 \) and
\( z_{c2}(n) = 0 \) otherwise. Similarly \( z_{c2}(n) = z_c(n) \) for
\( n > P - T - 1 \) and \( z_{c2}(n) = 0 \) otherwise. The portion of \( z_{c1}(n) \) and \( z_{c2}(n) \) which folds in into the DFT block
for \( z_d(n) \) due to delay spread of \( h_{cd}(n) \) and misalignment
\( T \) comprises the deviation from the cyclic assumption. Note
that this fold-in happens both at the start and end of the
\( P \) sample DFT window. Using (3) and the decomposition
\( z_c(n) = z_{c-cyc}(n) + z_{c1}(n) + z_{c2}(n) \) we can see that:
\[
z_d(n) = \sum_{k=0}^{L-1} (z_{c-cyc}(n-k) h_{cd}(k)) + d(n), 0 \leq n \leq P - 1
\]
(10)
\[
d(n) = \sum_{k=T+1}^{L-1-n} (z_c(-k) - z_c(P - k)) h_{cd}(k + n)
\]
(11)
\[ d(n) = \sum_{k=0}^{n-(P-T)} (z_c(n-k) - z_c(n-k-P)) h_{cd}(k) \]
\[ P - T \leq n \leq P - 1 \]
\[ d(n) = 0, \text{ otherwise} \]

Considering the DFT of the signal \( z_c(n) \), we get:
\[ Z_d(q) = Z_e(q) \sum_{k=0}^{L-1} h_{cd}(k) W_p^{(k-T)q} + D(q) \]  \hspace{1cm} (13)

\[ D(q) = \sum_{n=0}^{L-T-2} \sum_{k=T}^{1} (z_c(-k) - z_c(P-k)) h_{cd}(k+n) W_p^{nq} \]
\[ + \sum_{n=P-T}^{P-1} \sum_{k=T+1}^{L-1} (z_c(n-k) - z_c(n-k-P)) h_{cd}(k+n) W_p^{nq} \]  \hspace{1cm} (14)

Rearranging the terms in (14), we get the following:
\[ D(q) = \sum_{m=T+1}^{L-1} (z_c(-m) - z_c(P-m)) F_m(q) W_p^{mq} \]
\[ + \sum_{m=0}^{T-1} (z_c(P-T+m) - z_c(-T+m)) G_m W_p^{(P-T+m)q} \]  \hspace{1cm} (15)

where \( F_m(q) = \sum_{i=0}^{L-1-m} h_{cd}(m+i) W_p^{(m+i)q} \) and \( G_m(q) = \sum_{i=0}^{m} h_{cd}(i) W_p^{iq} \) are DFT's of rectangular windowed versions of \( h(n) \). Further rearranging (16), we get:
\[ D(q) = \sum_{m=T+1}^{L-1} R_m(q) h_{cd}(m) W_p^{mq} + \sum_{m=0}^{T-1} S_m(q) h_{cd}(m) W_p^{mq} \]  \hspace{1cm} (17)

where \( R_m(q) = \sum_{k=m}^{L-1} (z_c(-k) - z_c(P-k)) W_p^{(P-k)q} \)
\[ S_m(q) = \sum_{k=0}^{T} (z_c(P-T+k) - z_c(-T+k)) W_p^{(-T+k)q} \]  \hspace{1cm} (18)

\( R_m(q) \) and \( S_m(q) \) are DFTs of rectangular windowed versions of the alien noise signal and can alternately be written as a convolution the DFTs of \( z_c(n) \) and the rectangular window. This leads to the following:
\[ R_m(q) = \frac{1}{P} \sum_{k=0}^{P-1} Z_{c}^{\text{start-res}}(q-k) B_m(k) \]  \hspace{1cm} (21)
\[ S_m(q) = \frac{1}{P} \sum_{k=0}^{P-1} Z_{c}^{\text{end-res}}(q-k) C_m(k) \]  \hspace{1cm} (22)

where:
\[ Z_{c}^{\text{start-res}}(q) = \sum_{k=-T}^{k=P-T-1} (z_c(k-P) - z_c(k)) W_p^{kq} \]  \hspace{1cm} (23)
\[ Z_{c}^{\text{end-res}}(q) = \sum_{k=-T}^{k=P-T-1} (z_c(k+P) - z_c(k)) W_p^{kq} \]  \hspace{1cm} (24)

and \( B_m(k) \) and \( C_m(k) \) correspond to DFTs of rectangular windows and are given by:
\[ B_m(q) = \sum_{k=-T}^{k=P-T-1} W_p^{kq}, T + 1 \leq m \leq L - 1 \]  \hspace{1cm} (25)
\[ C_m(q) = \sum_{k=-T}^{k=P-T-1} W_p^{kq}, 0 \leq m \leq T - 1 \]  \hspace{1cm} (26)

The coefficient \( H_{\text{pertone}(q)}(q) \) is given by
\[ H_{\text{pertone}(q)}(q) = \frac{E\{Z_c(q)Z^*_e(q)\}}{E\{Z_c(q)Z^*_c(q)\}} \]  \hspace{1cm} Using (13), we get
\[ H_{\text{pertone}(q)}(q) = \sum_{k=0}^{L-1} h_{cd}(k) W_p^{(k-T)q} + \frac{E\{Z_c(q)Z^*_c(q)\}}{E\{Z_c(q)Z^*_e(q)\}} \]  \hspace{1cm} (27)

Assuming \( z_c(n) \) is wide-sense stationary and white, using (21), (22), (23), (24) and noting that \( B_m(0) = m - T, C_m(0) = T + m \), (27) reduces to:
\[ H_{\text{pertone}(q)}(q) = \sum_{k=0}^{T-1} h_{cd}(k) \left(1 + \frac{k - T}{P}\right) W_p^{(k-T)q} \]  \hspace{1cm} (28)
\[ + \sum_{k=T}^{L-1} h_{cd}(k) \left(1 - \frac{k - T}{P}\right) W_p^{(k-T)q} \]

From (28) it can be seen that the inverse DFT of per-tone coefficients \( H_{\text{pertone}(q)}(q) \) yields a cyclically shifted (by \( T \) samples) time domain signal \( h_{\text{pertone}(n)}(n) \) which is related to the true impulse response \( h_{cd}(n) \) cyclically shifted by \( T \) samples. Therefore we have proven the following result:

**Lemma 3.1:** If \( z_c(n) \) is assumed to be white and wide-sense stationary and \( T < L \) and \( T \geq 0 \) (i.e. CM DFT window is ahead in time w.r.t DM DFT window), \( h_{\text{pertone}(n)}(n) \) derived from the inverse DFT of the per-tone coefficients \( H_{\text{pertone}(q)}(q) \) is related to the true impulse response \( h_{cd}(n) \) as:
\[ h_{\text{pertone}(i)}(i) = h_{cd}^{(T)}(i) \left(1 - \frac{i}{P}\right), \quad i = 0, 1, \ldots L - T \]  \hspace{1cm} (29)
\[ h_{\text{pertone}(P + i)} = h_{cd}^{(T)}(P + i) \left(1 + \frac{i}{P}\right), \quad i = -1, \ldots, -T \]  \hspace{1cm} (30)
where \( h_{cd}^{cyc-T} \) is the cyclically shifted version of \( h_{cd}(n) \). Corresponding results can also be derived for the scenarios where \( z_c(n) \) is coloured or \( T > L \) or \( T < 0 \) (i.e. CM DFT window lags the DM DFT window) but discussing all these scenarios is beyond the scope of this paper.

This result enables us to derive the true impulse response of the coupling function from the per-tone coefficients, which can be estimated quickly and with low complexity. The true impulse response can derive an optimum time-domain canceller (e.g. based on the MMSE criterion) which does not suffer from the loss in per-tone cancellation or can be used for a post-training “delay” adjustment to optimize the training complexity.

A. Optimizing per-tone cancellation via delay adjustment

The uncancellable residual time domain signal with per-tone cancellation is given by:

\[
\begin{align*}
    r(n) &= d(n) + \sum_{k=0}^{L-1} (h_{cd}(k) - h_{\text{pertone}}(k)) z_{c-cyc}(n-k) \\
    \xi(h, T) &= 2 \left( \sum_{m=1}^{L-T-1} \left( h_m^T R_m h_m - h_m^T P_m h_m \right) + \sum_{t=0}^{T-1} \left( h_t^T R_t h_t + h_t^T P_t h_t \right) \right)
\end{align*}
\]

The total uncancellable noise energy in a DMT symbol given by \( \xi(h, T) \) can be estimated by using Lemma 3.1 (stated without proof).

\[
\begin{align*}
    E(r(n)) &= \sum_{n=0}^{N-1} E[r(n)^2] \\
    \frac{1}{2} \sum_{q=0}^{P-1} I_U(q) &= \sum_{n=0}^{N-1} E[r(n)^2].
\end{align*}
\]

where \( h_{\text{cm}}^i = \begin{bmatrix} h(T+m) \\ \vdots \\ h(L-1) \end{bmatrix} \) and \( h_{\text{cm}}'' = \begin{bmatrix} h(0) \\ \vdots \\ h(t) \end{bmatrix} \) are rectangular windowed versions of the impulse response \( h(n) \) while \( R_m \) and \( P_m \) are \( (L - T - m) \times (L - T - m) \) and \( (t + 1) \times (t + 1) \) autocorrelation matrices of \( z_c(n) \) which is assumed to be wide-sense stationary and

\[
\begin{bmatrix} r(N) & r(N+1) & \cdots & r(N+t) \\ r(N-1) & r(N) & \cdots & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ r(N) & r(N+1) & \cdots & r(N+L+m+T+1) \\ r(N-1) & r(N) & \cdots & \cdots \\ \vdots & \vdots & \ddots & r(N) \end{bmatrix}
\]

and

\[
\begin{bmatrix} r(N) & r(N+1) & \cdots & r(N+L+m+T+1) \\ r(N-1) & r(N) & \cdots & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ r(N) & r(N+1) & \cdots & r(N) \end{bmatrix}
\]

matrices also consist of the autocorrelation terms of \( z_{cm}(n) \).

The above result assumes that \( P >> L \) and the autocorrelation terms \( r(n) = E[z_c(i)z_c(i+n)] \) are independent of the method used for training the per-tone canceller and can be conveniently applied as a post-training adjustment resulting in a modified position for the CM DFT window and new values for the per-tone coefficients.

C. Measurement Results

Figure 3 shows the measured CM-DM coupling function for a 400m 24 AWG loop obtained by injecting white noise using a specially designed CM injection probe and measuring the signals at the CM and DM sensors. It is seen that the delay spread of the impulse response is quite large (\( \approx 700 \) samples). Figure 4 shows the corresponding post-cancellation performance for per-tone (with different values of misalignment) as well as time-domain linear cancellation with alien noise modelled as stationary white noise (Refer table ?? for the simulation parameters). It is seen that the gap between per-tone cancellation and linear cancellation is small for majority of the bins when the misalignment is optimum.
Remark: For REIN sources like home appliances the length of the impulse noise burst may be much smaller than the DMT symbol. For these cases, the uncancelable noise energy can be small given that start and/or end portion of the uncancelable signal $u(n)$ (refer (11),(12)) may be zero due to the shorter length of the noise burst.

IV. Conclusion

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