Model identification and vision-based $H_\infty$ position control of 6-DoF cable-driven parallel robots

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ABSTRACT
This paper presents methodologies for the identification and control of 6-degrees of freedom (6-DoF) cable-driven parallel robots (CDPRs). First a two-step identification methodology is proposed to accurately estimate the kinematic parameters independently and prior to the dynamic parameters of a physics-based model of CDPRs. Second, an original control scheme is developed, including a vision-based position controller tuned with the $H_\infty$ methodology and a cable tension distribution algorithm. The position is controlled in the operational space, making use of the end-effector pose measured by a motion-tracking system. A four-block $H_\infty$ design scheme with adjusted weighting filters ensures good trajectory tracking and disturbance rejection properties for the CDPR system, which is a nonlinear-coupled MIMO system with constrained states. The tension management algorithm generates control signals that maintain the cables under feasible tensions. The paper makes an extensive review of the available methods and presents an extension of one of them. The presented methodologies are evaluated by simulations and experimentally on a redundant 6-DoF INCA 6D CDPR with eight cables, equipped with a motion-tracking system.

1. Introduction
Cable-driven parallel robots (CDPRs) are special parallel robot manipulators where the end-effector or platform is connected to the base via cables only, its movement resulting from the winding and unwinding of the cables around pulleys actuated by motors. This kind of robot manipulator appeared in the late 80s by replacing the segments of the Gough–Stewart platform manipulators by cables in order to solve the major problem of the reduced workspace of rigid parallel robots (Dagalakis, Albus, Wang, Unger, & Lee, 1989).

Compared to serial and parallel manipulator robots actuated by rigid links, cable-actuated robots benefit from interesting features like very large workspace, high-speed motion due to the low moving mass, modular geometry, portability and adaptability to multi-scales. However, their control is a more complex issue as the cables must remain under tension at any time (Oh & Agrawal, 2005). The approaches available in the literature to solve this issue can be classified into two main categories. In the off-line solutions, a path-planning step is used prior to motion in order to design a reference trajectory which guarantees that the cables remain under tension during the predefined motion (Gosselin, 2012; Trevisani, 2012). In the online solutions, an algorithm of tension distribution (also known as force calculation or redundancy resolution) is used to ensure that the control signals maintain the cable tensions inside a predefined feasible workspace during motion (Lafourcade, 2004; Ming & Higuchi, 1994). This is a common solution for redundant manipulators, where the number of cables exceeds the number of degrees of freedom (DoF) of the end-effector, and it is the solution considered in this work.

Dealing with position control of CDPR manipulators, most of the proposed methods rely on joint position measurements. According to the coordinate space chosen to solve this control problem, there are two alternatives. In the first one, the controllers are designed in the joint space. Assuming that a direct position kinematic model (IPKM), the reference end-effector pose is converted into reference joint positions which are then controlled by a feedback loop. Some related works are the joint space PD controller proposed by Kawamura, Kino, and Won (2000) applied to the SEGESTA robot and later to the KNTU robot by Gholami, Aref, and Taghirad (2008), and the joint space PID controller for the redundant suspended ReelAx8 prototype presented by Lamaury, Gouttefarde, Michelin, and Tempier (2012). In the second one, the controllers are designed in the task space. Assuming that a direct position kinematic model
(DPKM) is available, the end-effector pose is calculated from the joint position measurements and a feedback control allows to track a reference pose. Gholami et al. (2008) evaluated such a task space PD controller and compared it to the previous approach. However, for parallel manipulators, the DPKM is difficult to obtain (see, for instance, Carricato and Merlet (2011) for an intensive study on the matter). In the previously mentioned control schemes, the modelling errors and the deformations of the cables result directly in errors on the end-effector pose. One solution for improving the accuracy is, then, to use some exteroceptive sensors in order to obtain a direct measurement of the end-effector pose. Some preliminary works using cameras have been proposed by Dalleg, Gouttefarde, Andreff, Michelin, and Martinet for controlling the redundant suspended ReelAx8 robot (2011) or the large-dimension CoGiRo robot (2012).

CDPRs are nonlinear-coupled MIMO systems with constrained states. For such systems, stability issues arise with linear LTI or model inversion-based controller as soon as the model is not well known or when the parameters evolve during operation. Multi-variable control methodologies that have been developed since the 90s now allow to manage the trade-off between performances and robustness and are then good candidates for CDPRs. Some preliminary results were proposed by Laroche, Chellal, Cuilllon, and Gangloff (2012) for the design of an LTI robust controller that manages both position and tension, including simulation results of a 3-DoF cable-robot with four cables.

The purpose of this paper is twofold. First, a two-step methodology is proposed to accurately estimate the kinematic parameters prior and independently from the dynamic parameters of a physics-based CDPRs model. It requires both the measurement of the motors’ angular positions (measured by optical encoders) and the end-effector pose (provided by a system of cameras). After presenting the available identification methods, experimental results are provided, showing that the identified model fits the system behaviour with good accuracy, and can then be used for control. Second, an original control scheme is developed in two parts: a vision-based position control scheme, for which an $\mathcal{H}_\infty$ methodology is proposed, in addition to a cable tension distribution management algorithm. The position control is achieved in the operational space, making use of an end-effector pose directly measured by a vision-tracking system. A four-blocks $\mathcal{H}_\infty$ design scheme with adjusted weighting filters ensures good trajectory tracking and disturbance rejection for CDPR systems which are nonlinear time invariant (NLTI)-coupled MIMO systems with constrained states. In conjunction with this position control loop, a tension management algorithm aims at ensuring that the control signals maintain the cables under feasible tensions. The paper makes an extensive review of the available methods and presents an algorithm inspired from one of them, to account for the motor dynamics. Results from simulations and experiments are then reported using the redundant 6-DoF CDPR INCA 6D prototype with eight cables manufactured and sold by Haption, equipped with a motion-tracking system Bonita developed by Vicon.

The paper is organised as follows: Section 2 describes the setup composed of the INCA robot and the Bonita motion-capture system. In Section 3, a physics-based model of the 6-DoF CDPRs is developed. In Section 4, the identification methodology is described and implemented. Section 5 is dedicated to the control strategies and includes both simulation and experimental results.

2. System description

2.1 INCA robot

The INCA robot developed by Haption is a haptic device with force feedback that is used in this work as a manipulator to address the control issues raised by CDPRs.

On its 6D version, the INCA 6D has a cubic configuration of 3 m by side, and uses eight driving cables to move the end-effector and eight balancing cables to ensure tension in the driving cables when the motors are non-powered (Figure 1(b)). Each actuator located at one of the eight vertices of the workspace (Figure 1(a)) is composed of a DC motor with a current driver, coupled to both the driving and balancing winches (Figure 1(b)) to store the cables, and is associated with a pulley to guide the cables, thus avoiding shearing. The contact points between the driving cables and the end-effector are called attachment points, and the contact points between the driving cables and the pulleys are called output points.

A measurement of the motor positions and currents are, respectively, achieved by incremental optical encoders and current sensors.

2.2 Bonita motion-capture system

The Bonita motion-capture system used to measure the pose of the INCA end-effector is composed of six infrared (IR) cameras (Figure 1(a)) and a tracker software running on a Windows PC, both from Vicon company. Each camera has its own emitting source and delivers a greyscale image with VGA resolution up to a 250 Hz frame rate. Assuming that this stereo system has been previously calibrated, the pose of the INCA end-effector fitted with five
retro-reflective fixed markers can be tracked by the software.

The temporal and spatial performances of the pose reconstruction are critical for the robot control and have been evaluated as follows:

- the delay between the start of the image acquisition and the availability of the pose measurement has been evaluated at 10.7 ± 0.7 ms with a 200 Hz camera frame rate. This latency of roughly twice the acquisition period is the sum of one period of image acquisition and one period for the pose reconstruction.
- the accuracy of the pose is of 1.7 ± 0.4 mm. It has been estimated by the RMS error in the IR camera images between the current position of the visible markers and their expected positions given by the reconstructed pose of the end-effector.

### 2.3 Real-time control architecture

To allow the controller to operate at a higher frequency than the frequency of the Bonita motion-capture system, the control architecture summed up in Figure 2 is designed as the following two asynchronous real-time tasks:

- the main periodic task is run by a PC under RT Linux at 1 kHz. It consists in reading the motor currents and the motor positions, checking the consistency of the robot state and finally computing the reference for the motor currents.
- a secondary task is run by a PC under Windows asynchronously at 200 Hz. It waits for the availability of the end-effector pose provided by the vision-tracking system.

### 3. Modelling of the 6-DoF CDPRs

The physics-based model considered in this work is derived from the general model for the m-DoF CDPR manipulators with n cables, which is augmented with the pretension system (balancing cables, winches and springs) of the INCA prototype. It is assumed that the cables are of negligible mass (straight, no sagging) and of infinite stiffness (inextensible).

#### 3.1 Kinematics modelling

##### 3.1.1 Position kinematics

Denoting $\mathcal{R}_o$, the base reference frame and $\mathcal{R}_e$ the end-effector frame centred at its centre of mass, the pose of the end-effector can be represented by the vector $
abla X_e = [P_e^T \quad \Phi_e^T]^T$, with $P_e$ the position of the origin and $\Phi_e$ a representation of the orientation of $\mathcal{R}_e$ with respect to the base reference frame $\mathcal{R}_o$ (Figure 3(a)). Considering $\Phi_e =$
[φ_r φ_p φ_y]^T, the rotation matrix \( \alpha_R \) from \( \mathcal{R}_o \) to \( \mathcal{R}_e \) has been chosen as a composition of three successive rotations roll–pitch–yaw (Khalil & Dombre, 1999) of angles (φ_r, φ_p, φ_y), respectively, around the principal axes (X_o, Y_o, Z_o) of \( \mathcal{R}_o \):

\[
\alpha_R(\Phi_e) = \begin{bmatrix}
c_y c_p & c_y s_p s_r - s_y c_r & c_y s_p c_r + s_y s_r \\
s_y c_p & s_y s_p s_r + c_y c_r & s_y s_p c_r - c_y s_r \\
-s_p & c_p s_r & c_p c_r
\end{bmatrix}
\] (1)

with \( s_k = \sin \phi_k \) and \( c_k = \cos \phi_k \), \( k \) standing for \( r, p \) or \( y \).

The attachment point \( E_i \) with coordinates \( e_{ri} \) fixed in \( \mathcal{R}_o \) is located at the position \( P_0 = P + \alpha_R(\Phi_e) e_{ri} \) in \( \mathcal{R}_o \). With \( P_0 \) denoting the position of an output point \( O_i \) fixed in \( \mathcal{R}_o \), the length of cable \#i can be written \( L_i(X_e) = \| l_i(X_e) \|_2 \), where \( l_i(X_e) = P + \alpha_R(\Phi_e) e_{ri} - P_0 \). We then stack the lengths of the cables in \( L = [L_1 ... L_n]^T \).

By convention, the angular position is considered as increasing during winding. With a null reference of the motor positions \( \theta_o = \Theta_{\mathcal{R}_e \mathcal{R}_o} \) for the initial pose of the end-effector \( X_{eo} = [P_{eo}^T \Phi_{eo}]^T \), the IPKM is then given by

\[
\theta(X_e, \alpha_K) = -R_{pm}^{-1}(L(X_e, \alpha_K) - L(X_{eo}, \alpha_K)) \] (2)

where \( \alpha_K \) is the vector of the parameters involved in the kinematic model and \( R_{pm} = \text{diag}(r_{pm1}, ..., r_{pmn}) \) contains the radii of the driving winches.

### 3.1.2 Velocity kinematics

Differentiating Equation (2) with respect to time yields the inverse velocity kinematic model (IVKM):

\[
\dot{\theta} = -R_{pm}^{-1} J(X_e) V_e \] (3)

where the \( i \)th row of the inverse kinematics Jacobian matrix \( J \) is the same as for a rigid parallel manipulator (Merlet, 1997) and is given by \( J_i(X_e) = [u_i^T(X_e) r_i(\Phi_e) \times u_i(X_e)]^T \), in which \( u_i(X_e) = l_i(X_e) / L_i(X_e) \) is the unit direction vector of the \( i \)th driving cable (Figure 3(a)). The end-effector velocity \( V_e = [v_e^T w_e^T]^T \) (including the linear \( v_e \) and angular \( w_e \) velocities) can then be converted into the time derivative of end-effector pose \( X_e \) by

\[
V_e = A_o(\Phi_e) \dot{X}_e \] (4)

with \( A_o(\Phi_e) = \text{diag}(I_{3 \times 3}, J_{rpy}(\Phi_e)) \), in which the matrix \( J_{rpy} \) maps the angular velocity \( w_e \) to the time derivative of the chosen orientation representation \( \Phi_e \), such as \( w_e = J_{rpy}(\Phi_e) \dot{\Phi}_e \). For a roll–pitch–yaw representation of the rotation, \( J_{rpy} \) has the following form:

\[
J_{rpy}(\Phi_e) = \begin{bmatrix}
c_y c_p & -s_y & 0 \\
s_y c_p & s_y c_p & 0 \\
-s_p & 0 & 1
\end{bmatrix}
\] (5)

Denoting \( \vec{J}(X_e) = R_{pm}^{-1} J(X_e) A_o(\Phi_e) \) and recalling the dependance with respect to the kinematic parameter vector \( \alpha_K \), the IVKM is finally rewritten as

\[
\dot{\theta} = -\vec{J}(X_e, \alpha_K) \dot{X}_e \] (6)

### 3.2 Dynamics modelling

#### 3.2.1 End-effector dynamics

The Newton–Euler equations (Khalil & Dombre, 1999) applied to the end-effector rigid body of mass \( M_e \) and moment of inertia tensor \( I_e \), written at its centre of mass...
Combining (7)–(9) with the use of Equations (2), (4), and (6) and their time derivative, the direct dynamic model (DDM) of the system written in the operational space is given by

\[ M_e(X_e) \ddot{X}_e + C_e(X_e, \dot{X}_e) + K_e(X_e) + G_e = A_e^T(\Phi_e) F_{eo} \]

under the \( n \) constraints expressing that the cables are kept inside a feasible tensions workspace \([T_{min} \ T_{max}]\):

\[ T_{min} \leq T(I_m, \theta, \dot{\theta}, \ddot{\theta}) \leq T_{max} \]

The matrices involved in the model can be written as

\[ \begin{bmatrix} M_e(X_e) & A_e^T(\Phi_e) A_e(X_e) A_e(\Phi_e) + \ddot{J}^T(X_e) J_{eq} \dot{J}(X_e) \\ K_e(X_e) & \ddot{J}^T(X_e) K_{eq} R_{pm}^{-1} L(X_e) \\ G_e & -A_e^T(\Phi_e) G = -G \\ C_e(X_e, \dot{X}_e) & [A_e^T(\Phi_e) A_e(X_e) A_e(\Phi_e) + \ddot{J}^T(X_e) F_{eq} \dot{J}(X_e) + \ddot{J}^T(X_e) J_{eq} \dot{J}(X_e, \dot{X}_e)] \\ & \times \dot{X}_e + A_e^T(\Phi_e) B_0 (\Phi_e, \dot{\Phi}_e) \\ & + \ddot{J}^T(X_e) f_{eq} \text{sign}(\dot{J}(X_e) \dot{X}_e) \end{bmatrix} \]

and the wrench \( F_{eo} \) resulting from the motor-current vector \( I_m \) is

\[ F_{eo} = W_f(X_e) \ I_m \]

where \( W_f(X_e) = W(X_e) \ R_{pm}^{-1} K_{em} \). This model relies on the kinematic parameters stacked in \( \alpha_k \) and also on the dynamic parameters (inertias, torque constant, friction ratios...) stacked in \( \alpha_D \).

### 3.2.4 Nominal linear model

As can be seen in the system dynamics given by the ODEs system (10), CDPRs are NLTI-coupled MIMO systems. So, the linearisation of (10) is required for the use of the \( H_{\infty} \) methodology presented in Section 5. It leads to the linear-coupled ODEs system:

\[ M_o \delta \ddot{X}_c + C_o \delta \dot{X}_c + K_o \delta X_c = A_o^T(\Phi_e) \delta F_{eo} \]

where the linearised dynamic matrices could be evaluated analytically by differentiating \( F_{eo} \) from (10) at the operating point \( op = (X_{eo}, \dot{X}_{eo}, \ddot{X}_{eo}) \):

\[ M_o = \frac{\partial(A_o^T F_{eo})}{\partial X_c} \bigg|_{op} \quad C_o = \frac{\partial(A_o^T F_{eo})}{\partial \dot{X}_c} \bigg|_{op} \quad K_o = \frac{\partial(A_o^T F_{eo})}{\partial \ddot{X}_c} \bigg|_{op} \]
and are expressed at the operating or equilibrium point $op = (X_e, 0, 0)$ by

$$
\begin{align*}
M_o &= A_T^T (\Phi_{o_e}) A_e (X_e, 0, 0) + J^T (X_e) J_{eq} J (X_e) \\
C_o &= J^T (X_e) F_{o_e} J (X_e) \\
K_o &= J^T (X_e) K_{eq} J (X_e) + \left[ \frac{\partial J^T}{\partial X_e} \right] K_{eq} R_{pm}^{-1} L (X_e) \\
\ldots & & \left[ \frac{\partial J}{\partial \phi} \right] K_{eq} R_{pm}^{-1} L (X_e)
\end{align*}
$$

Coulomb frictions are naturally removed in the linear model. They can be considered as disturbances acting on the wrench that will be rejected by the controller.

Hence, the state-space representation of the linearised system based on state $x = [\delta X^T \delta \dot{X}^T]^T \in \mathbb{R}^{12}$, control $u = \delta F_e \in \mathbb{R}^6$ and measurement $y = \delta X_e \in \mathbb{R}^6$ vectors can be written as

$$
\begin{align*}
\dot{x} &= A_o x + B_o u \\
y &= C_o x + D_o u
\end{align*}
$$

with the state-space matrices expressed as $A_o = \begin{bmatrix} 0_{6 \times 6} & I_{6 \times 6} \\ -M_c^{-1} K_c - M_c^{-1} C_o \end{bmatrix}$, $B_o = \begin{bmatrix} 0_{6 \times 6} \\ M_c^{-1} A_c^T (\phi_e) \end{bmatrix}$, $C_o = \begin{bmatrix} I_{6 \times 6} & 0_{6 \times 6} \end{bmatrix}$ and $D_o = 0_{6 \times 6}$.

4. Model identification

A number of approaches are available in the literature for identification of robot models. Yu, Chen, and Li (2011) have considered the estimation of the kinematic parameters of a rigid parallel manipulator. For estimation of dynamic parameters, the reader can refer to Gautier and Poignet (2002) and Gautier, Janot, and Vandanjon (2013) for the electromechanics systems in general and for the serial robots particularly and to Poignet, Ramdani, and Vivas (2003) for the case of parallel robots.

4.1 Identification methodology

Following Renaud et al. (2006), we propose to estimate the parameters sequentially in two steps, thus reducing the number of parameters to be considered at one time:

Step 1: The kinematic parameters are first estimated using the IPKM (see Vischer & Clavel, 1998, for an example of determination of the kinematic parameters of a parallel robot).

Step 2: The dynamic parameters are then estimated from the DDM, using the identified kinematic parameters.

4.1.1 Step 1: Kinematic parameters identification

Considering a set of $N_s$ measurements, the estimate $\hat{\alpha}_K$ of the vector $\alpha_K$ of the kinematic parameters can be determined by minimising the following criterion $E(\alpha_K)$ on the motor positions (see Figure 4(a)):

$$
E(\alpha_K) = \sum_{i=1}^{n} \sum_{k=1}^{N_s} (\theta_{ik} - \hat{\theta}_{ik}(\alpha_K))^2
$$

with

$$
\frac{\partial \theta_{ik}}{\partial \alpha_K} = \hat{\theta}_{ik}(t = kT_s) \quad \text{motor position measurements.}
$$

$$
\frac{\partial \hat{\theta}_{ik}(\alpha_K)}{\partial \alpha_K} = \hat{\theta}_{ik}(t = kT_s, \alpha_K) \quad \text{motor position estimations using the IPKM.}
$$

Minimising $E(\alpha_K)$ is a nonlinear least squares optimisation problem that can be solved iteratively using numerical algorithms such as gradient-descendant, Gauss–Newton or Levenberg–Marquardt. Denoting $\sigma_\theta^2$ the variance of the measurement error on the joint positions and $A$ the sensitivity matrix of the model of

**Figure 4.** Parameter estimation schemes. (a) Kinematic parameter estimation scheme. (b) Dynamic parameter estimation scheme.
dimension $nN_i \times n\alpha_k$

$$A = \begin{bmatrix}
\frac{d\hat{\theta}_1}{d\alpha_k} \\
\vdots \\
\frac{d\hat{\theta}_{N_i}}{d\alpha_k}
\end{bmatrix}$$ (19)

the covariance matrix of the error on the estimates can be given as $\Sigma_{\alpha_k}^2 = \sigma_\theta^2 (A^T A)^{-T}$ (Walter & Pronzato, 1997).

4.1.2 Step 2: Dynamic parameters identification

The identification of the dynamic parameters of a robot is a complex problem, due to the nonlinear nature of both the input–output behaviour and the parameter dependence of these MIMO systems. Identification methodologies for robots can be classified according to the minimisation criterion (Gautier et al., 2013):

- **Inverse dynamic identification model (IDIM):** the inverse dynamic model (IDM) is written in a linear form according to the parameters to be estimated, thus allowing to compute their estimate in one shot by minimising the quadratic error between the measured input and its estimation computed on the same output trajectory (Gautier & Poignet, 2002). This method requires the estimation of the speed and the acceleration, through bandpass filtering of the joint position at high sampling rate.

- **Output error (OE):** the quadratic error between measured outputs and their estimate is minimised, where the output estimate is computed with the DDM, the system being fed with the same input signal. This technique can be implemented in an open-loop (OLOE) or closed-loop (CLOE) situations (Janot, Gautier, Jubien, & Vandanjon, 2014). This very popular method often yields a non-convex problem with possible local minima and is then sensitive to initialisation.

Despite its drawbacks, the OE method has been chosen as it is more realistic (the errors are mainly on the measurement signals) and thus leads to a more accurate model for control synthesis. Beneficing from the stabilisation effect of the balancing springs, the experiments were done in open loop. For a robot without balancing springs, a closed-loop framework would better suit to ensure that the platform remains within a limited workspace. An excitation is first designed, leading to a set of input and output data collected from $N_s$ samples at a sampling rate $f_s$. Then, the estimate $\hat{\alpha}_D$ of the vector $\alpha_D$ of the dynamic parameters can be determined by minimising the following dynamic identification error criterion $E(\hat{\alpha}_K, \alpha_D)$, defined in the operational space in terms of end-effector posture (see Figure 4(b) with $c = 0$):

$$E(\hat{\alpha}_K, \alpha_D) = \sum_{i=1}^{6} \sum_{k=1}^{N_i} (\hat{X}_{cik} - \hat{X}_{cik}(\hat{\alpha}_K, \alpha_D))^2$$ (20)

or also in the joint space in terms of motors’ positioning (see Figure 4(b) with $c = 1$):

$$E(\hat{\alpha}_K, \alpha_D) = \sum_{i=1}^{n} \sum_{k=1}^{N_i} (\hat{\theta}_{ik} - \hat{\theta}_{ik}(\hat{\alpha}_K, \alpha_D))^2$$ (21)

with

- $\hat{X}_{cik} = X_i(t = kT_s)$ end-effector pose measurement,
- $\hat{X}_{cik}(\hat{\alpha}_K, \alpha_D) = \hat{X}_i(t = kT_s, \hat{\alpha}_K, \alpha_D)$ end-effector pose estimation using the DDM,
- $\hat{\theta}_{ik} = \theta_i(t = kT_s)$ motor position measurements,
- $\hat{\theta}_{ik}(\hat{\alpha}_K, \alpha_D) = \hat{\theta}_i(t = kT_s, \hat{\alpha}_K, \alpha_D)$ motor position estimates using the DDM and IPKM.

As for the kinematic parameters, minimising $E(\hat{\alpha}_K, \alpha_D)$ is a nonlinear least-square optimisation problem that can be solved iteratively using the same algorithms mentioned for the kinematic parameters estimation. However, the evaluation of the objective function requires the simulation of the DDM, which is very demanding in computation time.

4.2 Implementation and results

The proposed identification methodology has been implemented on the INCA 6D CDPR. For the $i$th output of a MIMO system of input vector $u$ (dim($u$) = $N_u$) and output vector $y$ (dim($y$) = $N_y$) among a set of $N_s$ samples, the fit is defined by

$$FIT_i(\%) = \left( 1 - \frac{\sum_{i=1}^{N_s} \sum_{k=1}^{N_i} (y_{ik} - \hat{y}_{ik})^2}{\sum_{i=1}^{N_s} \sum_{k=1}^{N_i} (y_{ik} - \bar{y}_i)^2} \right) \times 100$$ (22)

where $\hat{y}$ is the estimation of the output vector $y$ and $\bar{y}_i$ the average of the $i$th output $y_i$.

4.2.1 Initial values of the parameters

The reference frame $R_o$ is located at the centre of the workspace (Figure 5(a)), and the initial position for the frame $R_e$ attached to the end-effector is chosen equal to $R_o$ (Figure 5(b)).

Each motor is controlled in current and the current loop, i.e. the transfer between the current reference $I_{m1}^* = 1 / (1 + \tau_1 s)$ with a time constant $\tau_1$. 

The initial values of the kinematic (Figure 5) and dynamic parameters of the INCA 6D robot, known by design or calculated, are given in Table 1. The different actuators are assumed to have the same parameters.

Coulomb friction parameter $f_{ceq}$ has been evaluated by powering each motor separately with a ramp signal for the reference current slowly varying from $I_{\text{min}} = 0$ A to $I_{\text{max}} = 3$ A with a slope of 1 A/s, whereas the other motors are controlled to 0 A. The Coulomb friction coefficient is then calculated when detecting the minimum value of the current that produces a motion of the end-effector. The worse case over the eight winders has been considered for $f_{ceq}$ given in Table 1.

4.2.2 Estimation of the kinematic parameters

The test consists in moving the effector in eight different directions by applying successively a current reference of 1 A on one motor and 0 A on the other ones. The positions of the motors and the pose of the end-effector are acquired at steady state.

The identification scheme presented in Section 4.1.1 and summarised in Figure 4(a) has been implemented using the Levenberg–Marquardt optimiser. The estimates of the kinematic parameters are given in Table 1. Notice that parameters $a$ and $r_{pe}$ are not involved in the kinematics modelling and then were not involved in the estimation procedure. The comparison between the positions of the motors estimations issued from the IPKM and the experimental identification data is shown in Figure 6. The error on the motor position is centred (less than 0.4°, i.e. about 1 mm on the cable length) and the standard deviation is 45°, corresponding to 13.7 mm on the cable length. The standard deviation of the errors on the estimates provided in Table 1 are computed from the diagonal terms of $\Sigma_{\alpha_e}$. 

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**Table 1.** Initial and estimated values of the kinematic and dynamic parameters of the INCA 6D robot (parameters with an upperscript $a$ have been set to their initial value).

| Kinematic parameters | Initial values | Estimated values | STD  |
|----------------------|----------------|-----------------|------|
| $L_x$ (m)            | 2.53           | 2.50            | 6.8  |
| $L_y$ (m)            | 2.8            | 2.75            | 7.0  |
| $L_z$ (m)            | 3              | 3.05            | 5.4  |
| $a$ (mm)             | 41             | 41$^a$          | –    |
| $l$ (mm)             | 153            | 138             | 18.7 |
| $\theta_{xy}$ (°)    | 70             | 63.0            | 0.6  |
| $\theta_{xz}$ (°)    | 24             | 21.6            | 0.3  |
| $r_{pe}$ (mm)        | 6              | 6$^a$           | –    |
| $r_{pm}$ (mm)        | 17.5           | 17.5            | 1.0  |

| Dynamic parameters   | Initial values | Estimated values | STD  |
|----------------------|----------------|-----------------|------|
| $M_e$ (kg)           | 0.157          | 0.289           | 0.3  |
| $e_{f_{exx}}$ (kg m$^2$) | $4.97 \times 10^{-3}$ | $1.49 \times 10^{-4}$ | 10.3 |
| $e_{f_{eyy}}$ (kg m$^2$) | $6.91 \times 10^{-3}$ | $2.07 \times 10^{-4}$ | 13.0 |
| $e_{f_{ezz}}$ (kg m$^2$) | $6.91 \times 10^{-3}$ | $2.07 \times 10^{-4}$ | 13.0 |
| $e_{f_{exy}}$ (kg m$^2$) | 0             | 0$^a$           | –    |
| $e_{f_{exz}}$ (kg m$^2$) | 0             | 0$^a$           | –    |
| $e_{f_{eyz}}$ (kg m$^2$) | 0             | 0$^a$           | –    |
| $e_{f_{exx}}$ (kg m$^2$) | 0             | 0$^a$           | –    |
| $j_{eq}$ (kg m$^2$)   | $2.91 \times 10^{-5}$ | $3.88 \times 10^{-5}$ | 0.4  |
| $f_{eq}$ (N m)/(rad/s) | $3.1 \times 10^{-3}$ | $6.68 \times 10^{-5}$ | 3.8  |
| $f_{eq}$ (N m)/(rad/s) | $1.8 \times 10^{-3}$ | $4.4 \times 10^{-5}$ | 3.3  |
| $k_p$ (N/m)           | 16             | 14.4            | 0.05 |
| $k_{pm}$ (N m/A)      | $60.3 \times 10^{-3}$ | $60.3 \times 10^{-3}$ | –    |
| $\tau$ (ms)           | 1.3            | no id.          | –    |
The corresponding numerical values of the fit (22) on the motors positions are provided in the first column of Table 2. One can notice that the model reproduces accurately the measured positions.

### 4.2.3 Estimation of the dynamic parameters

#### 4.2.3.1 Experiment.

The choice of the excitation trajectories is an important point for the identification procedure. Indeed, the model needs not only to be structurally identifiable, but the excitation trajectory must also be rich enough. The excitation trajectories must contain both slow (for friction and stiffness) and fast (for inertia) dynamics.

For the current case of a CDPR, the trajectories should be also large enough to excite the nonlinear behaviour, the cable tension must be sufficiently high to avoid slackness, and sufficiently slow not to excite the vibrations of the cables.

Motor current signals have been chosen as decoupled pseudo-random binary sequence (PRBS) that are usually used in system identification as they can both excite the different inputs of a multivariable system in a decoupled

### Table 2. Identification results: fit on the motors positions.

| PRBS parameters | Dynamic | Dynamic | Dynamic | Dynamic | Dynamic | Dynamic | Dynamic | Dynamic |
|-----------------|---------|---------|---------|---------|---------|---------|---------|---------|
|                 | Ident   | Val(a)  | Val(b)  | Val(c)  | Val(d)  | Val(e)  | Val(f)  |
| $I_{\text{min}}$ (A) | 0       | 0       | 0       | 0.5     | 0.5     | 0       | 0.5     |
| $I_{\text{max}}$ (A) | 1       | 1       | 1       | 1.5     | 1.5     | 1       | 1.5     |
| $f_{\text{max}}$ (Hz) | 1       | 1       | 2       | 1       | 2       | 5       | 5       |

| Motors          | Kinematic | Dynamic | Dynamic | Dynamic | Dynamic | Dynamic | Dynamic | Dynamic |
|-----------------|-----------|---------|---------|---------|---------|---------|---------|---------|
|                 | Ident     | Val(a)  | Val(b)  | Val(c)  | Val(d)  | Val(e)  | Val(f)  |
| Motor 1         | 88.6      | 91.2    | 92.5    | 84.8    | 86.1    | 90.1    | 83.3    | 82.2    |
| Motor 2         | 88.2      | 85.1    | 88.0    | 84.8    | 81.4    | 86.8    | 69.9    | 69.3    |
| Motor 3         | 85.7      | 88.2    | 89.1    | 82.6    | 81.5    | 82.6    | 80.4    | 77.9    |
| Motor 4         | 92.9      | 93.9    | 93.4    | 88.2    | 85.7    | 89.1    | 79.3    | 74.1    |
| Motor 5         | 88.9      | 86.8    | 84.7    | 77.9    | 82.0    | 82.6    | 79.9    | 79.7    |
| Motor 6         | 84.1      | 93.6    | 92.0    | 87.1    | 85.4    | 89.3    | 81.3    | 75.9    |
| Motor 7         | 83.4      | 90.3    | 90.5    | 83.7    | 86.0    | 88.8    | 83.0    | 82.0    |
| Motor 8         | 95.1      | 85.7    | 87.8    | 82.1    | 84.3    | 87.0    | 70.9    | 72.0    |
| Average         | 88.4      | 89.3    | 89.7    | 83.9    | 84.1    | 87.0    | 78.5    | 76.6    |

*Parameters that have been kept equal to their initial value.*
fashion and are able to excite over a large bandwidth. For the identification trajectory, the low level $I_{\text{min}} = 0$ A and high level $I_{\text{max}} = 1$ A of the signals are adjusted with respect to the constraints on the cable tensions. The upper bound of the frequency band $[0 f_{\text{max}}]$ was set to $f_{\text{max}} = 1$ Hz, during a time interval $[0,5]$ s.

Cross validations have been performed with different PRBS trajectories, with additional data that were not used for training the model. Results are reported in Table 2. First, Val(a) is the continuation of the identification trajectory over $[5–10]$ s, thus obtained with the same tuning parameters. Then, Val(b) is obtained by varying the frequency $f_{\text{max}}$; Val(c) by varying the current levels $I_{\text{min}}$ and $I_{\text{max}}$ and Val(d) by varying both of them. Notice that the average fit obtained with the initial values of the parameters was of 3.5%. It reached 45% when dividing the initial value of $f_{\text{eq}}$ by 10. These figures show that the parameter estimation step is paramount for improving the accuracy of the model.

The validation trajectories Val(e) and Val(f) with much higher frequency $f_{\text{max}}$ allow to evaluate the effects of the cable vibrations that were observed during these experiments and that are not accounted for in the model.

4.2.3.2 Estimated dynamic parameters. Applying the dynamic identification scheme presented previously in Figure 4(b) (considering the joint positions as outputs, i.e. with $c = 1$), the dynamic parameters are estimated iteratively using the Levenberg–Marquardt optimiser for solving the nonlinear least-square optimisation problem of Equation (21).

The estimates of the dynamic parameters are given in Table 1. Parameters $k_{\text{em}}$ and $\tau_i$ have been maintained at their initial values, that have been confirmed by previous experiments. The non-diagonal components of the moment of inertia are set to:

$$I_{\text{exy}} = I_{\text{eyz}} = I_{\text{exz}} = 0 \text{ kg·m}^2,$$

and the diagonal components $I_{\text{eyy}}$ and $I_{\text{ezz}}$ are set as equal ($I_{\text{ezz}} = I_{\text{eyy}}$), because of the symmetry of the end-effector.

An evaluation of the standard deviation of the estimates is given in the last column of Table 1. This evaluation has been done by considering the sum of the parameter variances when considering all the errors on the kinematic parameters. More precisely, the dynamic parameter identification has been run seven more times, considering an error of 1% on each kinematic parameter, thus resulting in a sensitivity matrix expressing the effect of an error on the estimated parameter.
on each kinematic parameter on the dynamic parameter estimate. The variances of all the kinematic parameters, weighted by their sensitivities, are then summed up to produce the variances of the dynamic parameter estimates and finally their relative standard deviations given in Table 1. One can notice that the highest deviations are for the inertias and are of 13%.

In Figure 7 are provided for the first three motors: the reference current (on the top), the estimated and measured motor angular positions (2nd row) and a zoom on the motor position error (lower row).

Table 2 provides the numerical values of the fit (22) on the motor positions. The fit obtained with the identification trajectory (see the column labeled “Dynamic ident”) is complemented with the fit obtained for six different validation trajectories with variable amplitudes and frequencies. One can see that the model reproduces accurately the measurement, even on the validation data that were not used for identification. These results validate the assumptions used for deriving the model such as the negligible mass of the cables and their infinite stiffness.

### 4.2.4 Frequency-domain behaviour

The linearisation has been performed for the end-effector velocity and acceleration equal to zero. The nominal model corresponds to a position of the platform at the centre of the workspace, with the nominal values of the kinematic and dynamic parameters previously estimated.

The frequency behaviour of the nominal linear model $G_p(s)$ is reported in Figure 8. The singular values for both position and orientation models behave as a second order under-damped LTI system with a resonance at the frequencies close to 5 rad/s for position and of 22 rad/s for orientation.

![Singular values of the nominal model $G_p(s)$: position and orientation parts.](image)

5. Position control and tension distribution

The system can be considered as redundant as the number of actuators exceeds the number of DoF of the platform (refer to Kanoun, Lamiraux, & Wieber, 2011 for a more general reflexion on redundant systems). The extra DoF are used to handle the cable tension. The considered control scheme presented in Figure 9 is composed of two parts detailed in this section:

- **Position control** that aims to control the pose of the end-effector at a reference value. It is typically a dynamic position-based visual control scheme (also known as position-based visual servoing (PBVS) or 3D visual servoing (3DVS)).
- **Tension distribution** that aims to maintain the tension of the cables inside a feasible workspace.

### 5.1 Position control

5.1.1 $H_\infty$ synthesis

The $H_\infty$ synthesis provides a framework for tuning dynamic output-feedback MIMO controllers in the frequency domain, allowing to handle easily the robustness versus performance trade-off (Duc & Font, 1999).

In the standard synthesis scheme described in Figure 10, the controller $K(s)$ to be designed closes the loop of the extended plant $G_e(s)$, thus modifying the performance channel from $v$ to $z$. Weighting filters located at the input ($W_i(s)$) and output ($W_o(s)$) complete the so-called augmented scheme.

![Control scheme.](image)
A number of problems can be recast into the following $H_\infty$ synthesis problem that consists in computing $K(s)$ that:

- stabilises the weighted closed-loop system in Figure 10;
- minimises $\gamma := \min_{s \in \mathbb{C}} \| T(s) \|_{\infty}$, i.e. the $H_\infty$ norm of the weighted closed-loop system $T(s) = W_o(s) T_{zn}(s) W_i(s)$.

Assuming that the weighting filters $W_i(s)$ and $W_o(s)$ are diagonal, and denoting $W_i(s) = \text{diag}(W_{i1}(s), \ldots, W_{in}(s))$ and $W_o(s) = \text{diag}(W_{o1}(s), \ldots, W_{om}(s))$, where $n_i$ and $n_o$ being the dimensions of $v$ and $z$, respectively, then any SISO transfer $T_{zn}(s)$ between input $v_k$ and output $z_i$ satisfies:

$$|T_{zn}(j\omega)| \leq \frac{\gamma}{|W_d(j\omega)| \cdot |W_k(j\omega)|} \quad (\forall \omega \in \mathbb{R}^+)$$

(23)

Therefore, $\frac{1}{W_d(s) W_k(s)}$ is a template for the performance channel $T_{zn}(s)$ and the template is satisfied as soon as $\gamma \leq 1$.

The resolution of the standard problem of the $H_\infty$ synthesis can be done by solving the resulted Riccati (Dolye, Glover, Khargonekar, & Francis, 1989) or LMI equations (Gahinet & Apkarian, 1994) resulting in full-order controllers, i.e. controllers with the same order as the augmented plant. Solvers are also available for the structured $H_\infty$ synthesis: let us mention the HIFOO package (Burke, Henrion, Lewis, & Overton, 2006) and hinfsyn available in the robust control toolbox (Apkarian & Noll, 2006).

To summarise, the method consists in the following steps:

- select the performance channels,
- design the weighting functions according to the requirements,
- compute the augmented plant and design the controller,
- iterate the two previous steps until $\gamma$ is close to one.

### 5.1.2 Controller design

Figure 11 depicts the synthesis scheme. The control scheme is deduced by ignoring the weighting filters $W_1(s)$, $W_2(s)$ and $W_3(s)$. It allows to control the measurement vector $y = X_e$ to the reference $r = X_e^*$, while rejecting the disturbance $d$ that is added on the control vector $u$ and considering the measurement noise $b_m$. As $b_m$ and $r$ have the same effects on the signals of interest $e$ and $u$, only $r$ is considered for building the extended plant $G_e(s)$, leading to a four-block design scheme equivalent to the scheme of Figure 10 by selecting the signals $v = [r^T d^T]^T$ and $z = [e^T u^T]^T$, and the weighting filters $W_1(s) = \text{diag}(e_{6 \times 6}, W_3(s))$ and $W_o(s) = \text{diag}(W_1(s), W_2(s))$.

The weighing filters are chosen as following:

- $W_1(s)$ allows to tune the sensitivity function $S(s) = T_{er}(s)$, i.e. the transfer from $r$ to $e$. It is chosen as constant, to impose the modulus margin $\Delta_M$. As $\Delta_M = 1/\|S(s)\|_\infty$, we select $W_1(s) = \Delta_M^s$, where $\Delta_M^s$ denotes the required modulus margin.
- $W_2(s)$ allows to tune $T_{ur}(s)$, the transfer between $r$ to $u$, which is equal to $T_{ubu}(s)$. So, $W_2(s)$ has the charge of reducing the effect of the measurement noise on the control signal. Thus, $W_2(s)$ must amplify in the high frequencies and $1/W_2(s)$ is chosen as a low-pass filter.
- $W_3(s)$ is used to penalise the disturbance in the low frequency, thus enhancing the disturbance rejection properties.

The desired performances include a bandwidth of 5 rad/s, a modulus margin of 6 dB and an accuracy of $10^{-3}$ due to $d$ for both position and orientation, with a reasonable amplitude of the control inputs for both forces and moments. The weighting filters $W_1(s)$, $W_2(s)$ and $W_3(s)$ are chosen under the shape $W_k(s) = \text{diag}(w_{k1}(s) I_{3 \times 3}, w_{k2}(s) I_{3 \times 3})$, where the SISO weighting filters $w_{k1}(s)$ correspond to the positions or forces and $w_{k2}(s)$ corresponds to the orientations or moments. The weighting filters were tuned to achieve the requested closed-loop performances, resulting in

$$
\begin{align*}
\begin{cases}
  w_{11}(s) = 0.5 \\
  w_{21}(s) = \left( \frac{250}{s + 10^{-2}} \right)^2 \\
  w_{31}(s) = \left( \frac{s + 10}{s + 10^4} \right)^2
\end{cases}
\end{align*}
$$


Notice that the more simple two-block scheme, based on weighting filters \( W_1(s) \) and \( W_2(s) \) only, was not sufficient to reject the disturbances induced by the nonlinear behaviour of the system.

The frequency response of the controller (singular values) is given in Figure 12. For both position and orientation, the controller exhibits a high gain in low frequencies that favours the disturbance rejection, then the gain increases in the medium frequencies, bringing phase to allow a high bandwidth and finally decreases in the high frequencies, thus reducing the effects of the measurement noise.

Figure 13 shows the singular values of the closed-loop transfer functions of interest involved in the \( H_\infty \) synthesis and their corresponding templates. The resolution has been done with the Riccati method. The obtained value of \( \gamma = 0.92 \) being slightly smaller than 1, the templates are fully satisfied and the tracking trajectories performances slightly exceed those intended with a bandwidth of 6 rad/s for position and of 2 rad/s for orientation, a modulus margin of 0.57 for position and of 0.61 for orientation and a static error less than 1 % on position and orientation (see Figure 13(a)).

5.2 Tension distribution

5.2.1 Problem statement and state of the art

The wrench \( F_{ce} \) computed by the \( H_\infty \) controller must be converted into motor current vector \( I^*_m \). This requires to solve the system of linear equations (13) under the inequality constraints (11), in order to ensure that the cable tension vector \( T \) remains inside the interval \([T_{min} T_{max}]\). Due to the actuation redundancy, the system of Equations (13) is under-determined and then has an infinity of solutions (assuming that \( W_I \) has full rank \( r = n - m \)). This can be exploited to minimise the distance towards some objective value \( I_{obj} \) corresponding to an objective tension vector \( T_{obj} \). This is done by solving the quadratic optimisation problem of the objective function \( E(I^*_m, \lambda) \) given by

\[
E(I^*_m, \lambda) = \frac{1}{2} (I^*_m - I_{obj})^T (I^*_m - I_{obj}) + (F_{cv} - W_I I^*_m)^T \lambda
\]

where \( \lambda \in \mathbb{R}^m \) is the Lagrange multiplier associated to the equality constraints \( F_{cv} - W_I I^*_m = 0 \). The solution can be written as

\[
I^*_m = I^*_{mp} + I^*_{mn}
\]

where

- \( I^*_{mp} \), the particular solution that minimises the norm of \( I^*_m - I_{obj} \), acts on the end-effector pose:

\[
I^*_{mp} = W^{-1}_I(X_e) F_{cv}
\]

with \( W^+_I = W^T_I (W_I W^T_I)^{-1} \) the Moore–Penrose pseudo-inverse of the wrench matrix \( W_I \).

- \( I^*_{mn} \), the homogeneous solution of \( W_I \) null-space, acts on the tensions, without effect on the end-effector pose:

\[
I^*_{mn} = [I_{n \times n} - W^+_I(X_e) W_I(X_e)] I_{obj}
\]

A number of approaches are available in the literature for the determination of the objective tension vector \( T_{obj} \) that maintains the cable tension vector \( T \) inside a feasible tensions workspace. The available works can be classified into two categories. The first approach opted for iterative algorithms, so that efficient constrained optimisation methods can be used such as linear programming methods (LPM) (Gosselin & Grenier, 2011; Ming & Higuchi, 1994; Oh & Agrawal, 2005), but the cable tension continuity is not guaranteed. Other optimisation methods are also used such as nonlinear programming methods (NLPM) in the particular case of quadratic programming methods (QPM) (Oh & Agrawal, 2005; Vafaei, Aref, & Taghirad, 2010), and the general NLPM with the gradient descent method to solve the problem in a quadratic formulation (Gholami et al., 2008). These quadratic methods guarantee the tension continuity but
Figure 13. Singular values of the closed-loop transfer functions of interest $T_{z_k}(s)$ and their corresponding templates $\gamma/W_k(s)W_k(s)$. (a) Transfer $T_k(s) = S(s)$ with template $\gamma/w_1(s)$. (b) Transfer $T_{w_k}(s) = SG(s)$ with template $\gamma/w_2(s)$. (c) Transfer $T_{\omega_k}(s) = KS(s)$ with template $\gamma/w_2(s)$. (d) Transfer $T_{w_0}(s) = KSG(s)$ with template $\gamma/w_2w_2(s)$.

have a non-predictable runtime, which is a drawback for real-time implementation. The second approach relies on non-iterative algorithms to handle the real-time control constraints. For instance, Mikelsons, Bruckmann, Hiller, and Schramm (2008) proved that the centre of gravity (CoG) of the feasible tension distribution workspace (the set of solutions of Equation (13) satisfying the tension constraint (11)) is a solution that ensures the tensions continuity. Recently, Gouttefarde, Lamaury, Reichert, and Bruckmann (2015) have proposed contributions to improve the CoG method to the case of two degrees of redundancy.

5.2.2 Considered algorithm

The proposed approach is inspired from the algorithm proposed by Lafourcade (2004) appropriate to satisfy real-time constraints (currently less than 1 ms). It has been improved to account for the motor dynamics (9) by considering a variable $I_{obj}$ and accounting for the current limits of the motors. It consists in:

1. selecting $T_{obj}$ inside the feasible tension workspace and calculating $I_{obj}$ from Equation (9) by considering $T = T_{obj}$ and solving in $I_{obj} = I_m$;
2. computing the current reference $I_m^*$ that satisfies constraint (13) while minimising the mean-square error on $I_m^* - I_{obj}$ without considering the tension inequality constraint (11) as given by (27);
3. if some inequality constraints on the tensions are violated, then $q$ concerned inequalities selected among them are transformed into equality constraints on the currents (not more than $r$ tension inequality constraints can be saturated simultaneously) and included into the optimisation
problem. All combinations of $1$ to $r$ violated tension inequality constraints are considered until one solution is met that satisfies all the non-saturated tension inequalities constraints. The process is repeated until a feasible solution is met. If no solution is found, the vector $I_{\text{obj}}$ can be scaled by a scalar factor without impact on the trajectory. If the problem has no solution, the reference trajectory must be decelerated.

More precisely, for a combination of $q$ tension constraints, the current vector $I_m^*$ is obtained by solving the new quadratic optimisation problem of the objective function $E_{\text{sat}}(I_m^*, \lambda, \mu)$ given by

$$E_{\text{sat}}(I_m^*, \lambda, \mu) = \frac{1}{2} (I_m^* - I_{\text{obj}})^T (I_m^* - I_{\text{obj}}) + (F_{\text{en}} - W_{\text{f}} I_m^*)^T \lambda + (S^T I_m^* - I_{\text{sat}})^T \mu$$

(29)

where

- the selection matrix $S = [s_1 \ldots s_q] \in \mathbb{R}^{n \times q}$ concatenates the vectors $s_k$ of the canonical basis of $\mathbb{R}^n$ to select the combinations of the violated tension constraints to be saturated,
- the vector $\mu \in \mathbb{R}^q$ is the Lagrange multiplier associated to the current equality constraints $S^T I_m^* - I_{\text{sat}} = 0$, such as the vector $I_{\text{sat}} \in \mathbb{R}^q$ is the current vector that corresponds to the saturated tension vector $T_{\text{sat}}$ and is calculated from Equation (9).

The solution of the optimisation problem (29) can be written as

$$I_m^* = I_{mp}^* + I_{mt}^*$$

(30)
where

\[
I_{mt}^* = I_{mn}^* + \left[ W_t^+(X_e) W_t^s(X_e) - S \right] \\
\times \left[ S^T W_t^+(X_e) W_t^s(X_e) - I_{qx}q \right]^{-1} \Delta_{sat} \quad (31)
\]

in which the resulting saturated wrench matrix is \( W_t^s(X_e) = W_t(X_e) S \), and the vector of the excessive motor currents \( \Delta_{sat} = I_{sat} - S^T (I_{mp} + I_{mn}^*) \) is an image of the excess in cable tensions evaluated from Equation (9).

### 5.3 Experiments

#### 5.3.1 Evaluation tasks

A set of trajectories have been chosen in order to evaluate the performance of the controlled system. Starting from the centre of the workspace identified by a null position and orientation, the trajectory consists in reaching one after the other the vertices of a square centred into the workspace, belonging to the plane \((X_o, Y_o)\) and of width 0.2 m. The feasible tension workspace is defined by the boundaries \( T_{min} = 1.48 \text{ N} \) and \( T_{max} = 18.5 \text{ N} \) (the maximum tension supported by the cables being of 247 N), which have been calculated based on the static model of the actuators, considering the limits of the motor currents \( I_{min} = 0 \text{ A} \) and \( I_{max} = 3 \text{ N} \), and the limits of the unwinding cable lengths \( L_{min} = 0 \text{ m} \) and \( L_{max} = 4.82 \text{ m} \). The objective tension vector is chosen as \( T_{obj} = 10 \text{ N} \).

#### 5.3.2 Results

The experimental results obtained with the square trajectory are reported in Figure 14. One can notice that the position is properly controlled in a decoupled fashion (Figure 14(a)), while the currents (Figure 14(b)) and tensions (Figure 14(c)) remain inside the prescribed workspace and the tensions are close to \( T_{obj} = 10 \text{ N} \). The cables’ tensions have been estimated using the dynamic model of the actuators (9).

Simulation results provided in Figure 14 include Coulomb frictions that were found as a major source of misfit between the simulated and the measured signals. As shown in Figure 15, the prediction provided by the model is much more accurate as soon as Coulomb friction is included.

### 6. Conclusion

This paper is a methodological contribution for the identification and control of 6-DoF CDPRs. First, a two-step identification methodology has been proposed to estimate sequentially the combined kinematic and dynamic parameters of 6-DoF CDPRs. The method has been successfully implemented on the INCA 6D robot, allowing to improve the input–output behaviour of the model, as shown by the good fit on the motor positions. Moreover, the estimated values of both kinematic and dynamic parameters remain close from their initial guessed values, except for the viscous friction term. The validity of the identified model remains satisfying even when the cables are subject to non-negligible vibrations, as could be observed on the validations with faster trajectories. Then, an \( H_\infty \) methodology has been proposed for the vision-based position control of 6-DoF CDPRs, which has been then implemented to the INCA 6D robot. The simulation and experiment results have shown that the \( H_\infty \) position controller combined with the tension distribution algorithm allows good bandwidth, accuracy and disturbance rejection, while maintaining the cables under feasible tensions. Notice that strong disturbance rejection properties had to be conferred to the controller in order to dominate the nonlinear behaviour of the system. One strong limitation of the \( H_\infty \) methodology is that it leads to an LTI controller that is restricted to a neighbourhood of the nominal position. However, the robustness of the designed controller was sufficient for an evolution in a reasonably large domain. For a larger workspace, gain-scheduling techniques would be fruitful to adapt the controller behaviour.

### Note

1. The \( H_\infty \) norm of a linear system is the maximum singular value of its complex gain over frequency. It is also the maximum amplification when considering the \( L_2 \) norm
on signals. For a system $G(s)$ of input $u(t)$ and output $y(t)$, $\|G(s)\|_\infty = \max_{\omega \in \mathbb{R}} \sigma(G(j\omega)) = \max_{u(t)} \frac{\|y(t)\|_\infty}{\|u(t)\|_\infty}$.

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