EFFECT OF INTERACTING RAREFACTION WAVES ON RELATIVISTICALLY HOT JETS

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ABSTRACT

The effect of rarefaction acceleration on the propagation dynamics and structure of relativistically hot jets is studied through relativistic hydrodynamic simulations. We emphasize the nonlinear interaction of rarefaction waves excited at the interface between a cylindrical jet and the surrounding medium. From simplified one-dimensional (1D) models with radial jet structure, we find that a decrease in the relativistic pressure due to the interacting rarefaction waves in the central zone of the jet transiently yields a more powerful boost of the bulk jet than that expected from single rarefaction acceleration. This leads to a cyclic in situ energy conversion between thermal and bulk kinetic energies, which induces radial oscillating motion of the jet. The oscillation timescale is characterized by the initial pressure ratio of the jet to the ambient medium and follows a simple scaling relation, \(\tau_{\text{oscillation}} \propto (P_{\text{jet,0}}/P_{\text{amb,0}})^{1/2}\). Extended two-dimensional simulations confirm that this radial oscillating motion in the 1D system manifests as modulation of the structure of the jet in a more realistic situation where a relativistically hot jet propagates through an ambient medium. We find that when the ambient medium has a power-law pressure distribution, the size of the reconfinement region along the propagation direction of the jet in the modulation structure \(\lambda\) evolves according to a self-similar relation \(\lambda \propto t^{\alpha/2}\), where \(\alpha\) is the power-law index of the pressure distribution.

Key words: galaxies: jets – hydrodynamics – methods: numerical – relativistic processes – shock waves

Online-only material: color figures

1. INTRODUCTION

Relativistic jets are collimated bipolar outflows that have a velocity almost equal to light speed. They are ubiquitous among astrophysical systems consisting of a compact object surrounded by an accretion disk, e.g., active galactic nuclei (AGNs; Ferrari 1998), microquasars (Mirabel & Rodríguez 1999), and the central engine of gamma-ray bursts (GRBs; Piran 2004; Mészáros 2006). Although there are many works that try to determine the mechanism by which relativistic jets are accelerated and collimated, the process is still not understood.

Aloy & Rezzolla (2006) recently reported that relativistic jets can be powerfully boosted along the interface between the jet and the ambient medium if the jet has sufficiently large velocity and specific enthalpy and is overpressured (see also Aloy et al. 2003, 2005). A rarefaction wave excited at the interface converts the relativistic thermal energy of the plasma into kinetic energy and yields an amplification of the Lorentz factor in the jet–ambient medium interface. This type of boost, which we label rarefaction acceleration, is not possible in Newtonian dynamics but is an inherent process in relativistic hydrodynamics (RHDs).

The relativistic magnetohydrodynamic effects on the rarefaction acceleration were studied by Mizuno et al. (2008) and Aloy & Mimica (2008). Mizuno et al. (2008) showed that, especially in the case of a magnetic field perpendicular to the jet direction, the boost becomes more powerful than that expected from pure hydrodynamic models. Aloy & Mimica (2008) studied the radiative output of magnetized jets bounded by anomalous shear layers in which jets are boosted due to the rarefaction wave. It was indicated theoretically that the boosted Lorentz factor is well described as a function of the initial pressure ratio between the jet and the ambient medium by a simple scaling law derived by Zenitani et al. (2010; see also Komissarov et al. 2010).

Rarefaction acceleration of jets is a process that is expected to occur under realistic conditions. Since a large amount of energy is stored in the jet at its launch site, the jet initially would be overpressured. Actually, the jets from AGNs often appear to be overpressured with relativistic velocity even as they propagate through the interstellar medium (Bicknell & Begelman 1996). In some cases a jet has two components: a faster, lighter component surrounded by a slower, denser component (Giroletti et al. 2004; Meliani & Keppens 2009). The jet that emerges from the central engine of a GRB is also expected to be hot and overpressured when it propagates inside the progenitor star. Furthermore, the fluid of the boundary layer of the jet would be boosted by rarefaction acceleration when the relativistic jet breaks out on the surface of the progenitor star and the external pressure support drops (Tchekhovskoy et al. 2010). The Lorentz factor of the jet can be boosted when it enters the low pressure region outside of the thick accretion torus, which is a possible candidate for the central engine of short GRBs (Aloy et al. 2005).

In this paper, we study in detail how the rarefaction acceleration affects the propagation dynamics of relativistically hot jets through one-dimensional (1D) and two-dimensional (2D) RHD simulations. We focus particularly on the nonlinear interaction of rarefaction waves excited at the interface between the jet and the ambient medium because it might have a potential impact on the boosting process and even alter the dynamics and structure of the jet. This paper is organized as follows: In Section 2, the numerical results of our 1D RHD simulations are presented. We perform 2D simulations in Section 3. Finally, we discuss and summarize our findings in Sections 4 and 5.
2. ONE-DIMENSIONAL SIMULATIONS

2.1. Numeric Models and Setup

In order to investigate the interaction of rarefaction waves excited at the jet–ambient medium interface, we initially set a jet surrounded by ambient gas in the calculation domain, as is schematically shown in Figure 1. We solve the special RHD equations in a 1D axisymmetric cylindrical coordinate system. The fluid velocity has two components: the normal velocity $v_z$ and the tangential velocity $v_r$, to the interface, which separates the jet and the ambient medium. Derivatives of the physical variables in the $z$-direction are assumed to be zero. As the equation of state for the relativistic gas, we adopt the ideal gas equation of state for the relativistic gas, we adopt the ideal gas

\[ \rho = \frac{1}{\gamma c^2} \frac{P}{(\gamma - 1) \rho} \]

where $\gamma \equiv 1/\sqrt{1 - (v_z/c)^2 - (v_r/c)^2}$ is the Lorentz factor and $h = c^2 + \gamma P/(\gamma - 1) \rho$ is the specific enthalpy. The other symbols have their usual meanings.

For our initial conditions, we assume a relativistically hot jet with greater pressure and lower rest-mass density than the ambient medium. The initial density and pressure in the jet are chosen as $\rho_{\text{jet}} = 0.1$ and $P_{\text{jet}} = 1$, respectively. Those of the ambient medium are $\rho_{\text{amb}} = 1$ and $P_{\text{amb}} = 0.1$. In addition, the jet velocity in the $z$-direction is relativistic $v_{\text{jet}} = 0.99c$, with a Lorentz factor of $\gamma_{\text{jet}} \sim 7$. The ambient medium does not move, and the normal velocity $v_z$ is set initially to zero in the calculation domain. In this paper, we set a simple model for the jet–ambient medium system in order to investigate the basic physics of the interaction of rarefaction waves excited at the jet–ambient medium interface.

The normalization units in length, velocity, time, and energy density are chosen as the initial jet width $W_{\text{jet}}$, light speed $c$, and rest mass energy density in the ambient medium $\rho_{\text{amb}} c^2$. The computational domain spans $0 < r < 2$. A uniform grid with a grid size $\Delta r = 10^{-3}$ is adopted for our calculations. We use a reflecting boundary condition on the axis $r = 0$. The outer boundary of the grid uses the outflow (zero gradient) boundary condition.

We use a relativistic HLLC scheme for hydrodynamics to solve the RHD equations (Mignone & Bodo 2005). The primitive variables are calculated from the conservative variables following the method of Mignone & McKinney (2007). We use an MUSCL-type interpolation method to attain second-order accuracy in space, while the temporal accuracy obtains second-order using the Runge–Kutta time integration.

2.2. Results of One-Dimensional Simulations

2.2.1. The Jet Boosting Mechanism of Aloy & Rezzolla (2006)

Since the jet is initially hotter and has a higher pressure than the denser, colder ambient medium, three types of hydrodynamic waves are excited at the jet–ambient medium interface. A shock wave propagates outward through the ambient medium, and a rarefaction wave starts to travel toward the center of the jet. Behind the shock wave exists a contact discontinuity, that is, an entropy wave, corresponding to the edge of the jet. The pressure decrease in the jet due to the rarefaction wave accelerates the gas in the jet–ambient medium interface in the tangential direction as a natural outcome of RHDs, as was reported by Aloy & Rezzolla (2006).

Following the analysis of Zenitani et al. (2010), by combining Equations (3) and (4) we can obtain

\[ \gamma^2 \rho h \left( \frac{\partial}{\partial t} + v_z \frac{\partial}{\partial r} \right) v_z = -\frac{v_z}{c} \frac{\partial P}{\partial t} \sim -\frac{\partial P}{\partial t} (\because v_z \sim c). \]

This indicates that a time-decreasing pressure is responsible for the acceleration of the gas in RHDs, unlike the non-relativistic case. The conversion from the thermal energy to bulk kinetic energy of the jet is then constrained by the relativistic Bernoulli equation, which provides enthalpy conservation,

\[ \gamma h \sim \text{const}. \]

The relativistic jet is accelerated by this mechanism, found by Aloy & Rezzolla (2006), in our numeric model.

2.2.2. Temporal Evolution of the System

Figure 2 shows the temporal evolution of our jet–ambient medium system. The color contour represents the spatial distribution of (1) the density $\rho$, (2) the pressure $P$, (3) the radial velocity $v_r$, (4) the tangential velocity $v_z$, and (5) the Lorentz factor $\gamma$. Snapshots of the spatial distribution of the hydrodynamics variables at the time $t = 5, 27, 43, 57$ are illustrated in Figures 3, 4, 5, and 6, respectively. The dashed lines in these figures denote the profiles of hydrodynamic variables at the time.
Figure 2. Temporal evolution of the jet–ambient medium system: (a) the density, (b) the pressure, (c) the radial velocity, (d) the tangential velocity, and (e) the Lorentz factor. In panel (c), the rarefaction region is enclosed by dashed lines. The tangential velocity of the ambient medium is zero in panel (d).

(A color version of this figure is available in the online journal.)

$t = 0, 17, 33,$ and $50$ for better demonstration of the propagation of the rarefaction and shock waves. In the early evolutionary stage ($0 < t < 5$; phase (i)) described in Section 2.2.1, the single rarefaction acceleration of the gas is observed in the jet–ambient medium interface. Figure 3 gives the spatial distribution of the jet–ambient medium system at $t = 0, 1,$ and $5$. The Lorentz factor of the fluid at the jet–ambient interface is boosted to $12$ due to the rarefaction acceleration when $t = 1$ and $5$. One can find that the contact discontinuity moves outward, that is, the jet expands since the contact discontinuity corresponds to the edge of the jet. Note that the head of the rarefaction wave intersects the jet axis, while the Lorentz factor of the fluid inside the jet reaches its maximum value at the tail of the rarefaction wave when $t = 5$.

Subsequently, at $t \simeq 5$ the rarefaction waves converge on the central region of the jet and an incident shock wave is excited at the tail of the rarefaction wave in the cylindrical jet, bringing a substantial change in the dynamics (phase (ii) in
The shock wave is excited at the tail of the rarefaction wave as a natural result of the cylindrical shock tube problem in which the characteristic lines intersect. The shock wave is located at \( r = 1 \) when \( t = 17 \), as shown in Figure 4. The gas pressure in the interacting region of rarefaction waves, confined by the surface of the shock wave, is then further reduced and becomes lower than that of the ambient gas (see Figure 4). Since, according to Equation (5), the thermal energy is converted to the bulk kinetic energy of the gas, the Lorentz factor of the gas in the interacting region is further boosted. The peak Lorentz factor of the gas inside the rarefaction region reaches \( \gamma \approx 60 \) at \( t = 44 \) (see Figure 5), which is a factor \( \approx 5 \) higher than that due to the single rarefaction acceleration at the jet–ambient medium interface. The boosted Lorentz factor of the fluid at the jet–ambient medium interface due to the single rarefaction acceleration is roughly 12, as shown in Figures 3–6.

The interaction of rarefaction waves generates a strong inward pressure gradient behind the jet–ambient medium interface, which acts to decelerate the radial expansion of the jet, turning expansion of the jet into contraction around time \( t = 27 \) (see Figures 2(a) and (c)). The contraction of the contact discontinuity results in converging flows inside the jet in phase (iii).

When \( t = 44 \), the inward shock waves preceding the converging flow collide with each other at the center of the jet and propagate outward. The gas bounded by the shock is compressed and heated. Since, according to Equation (5), a time-increasing pressure decelerates the tangential velocity of the jet, the Lorentz factor of the jet reduces and the specific enthalpy increases (see Equation (6)). Thus, in phase (iv), the Lorentz factor of the shock heated gas drops. The spatial distribution of the jet–ambient medium system at the end of phase (iv) \( (t = 57) \) is demonstrated in Figure 6.

When the shock wave encounters the contact discontinuity around \( t = 58 \), the system has almost returned to its initial state. The jet still has sufficiently higher tangential velocity and specific enthalpy than the ambient medium, but the gas pressure in the jet has become smaller than that of the initial state. This is the result of the energy conversion from thermal energy to bulk kinetic energy of the jet. Since the system is restored to a state that is almost the same as the initial conditions, the three types of hydrodynamic waves, an outward propagating shock wave,
a contact discontinuity (the edge of the jet), and a converging rarefaction wave, appear at the jet–ambient medium interface. Therefore, the contracting radial motion of the jet becomes an expanding motion.

2.2.3. Oscillation of the Relativistic Jet

After phase (iv), the system has returned to conditions similar to the initial conditions, and the system repeats the cycle of phases (i)–(iv) until the pressure of the jet becomes equal to that of the ambient gas. During the cycle, the radial motion of the jet oscillates between expansion and contraction, and the system undergoes successive exchanges between the thermal energy and the kinetic energy. Figure 7 shows the temporal evolution of (1) the jet width, (2) the maximum and average tangential velocity, (3) the maximum and average Lorentz factor, and (4) the average of the specific enthalpy in the jet. The oscillations of the Lorentz factor and the averaged specific enthalpy are in anti-phase in accordance with the relativistic Bernoulli relation (6). When the pressure inside the jet becomes almost equal to the ambient gas around time $t = 1000$, the oscillation of the jet ends. The spatial distributions of the density, pressure, radial and tangential velocity, and Lorentz factor at $t = 2000$ are depicted in Figure 8. At the quasi-steady state, there exists an accelerated region localized to the boundary layer of the jet, as shown in Figures 8(d) and (e). The physical parameters in the interface of the jet are comparable to those expected from single rarefaction acceleration (Aloy & Rezzolla 2006). This is because the oscillating motion during the relaxation stage, shown in Figure 7, has little influence on the physical parameters in the boundary layer. At the final quasi-steady state, the pressure in the jet is almost the same as that of the ambient medium. The initial relativistic thermal energy of the jet is converted into the bulk kinetic energy of the jet.

2.3. Scaling Law for the Oscillation Timescale

As shown in Section 2.2.3, the initial non-equilibrium system evolves, through a transition stage, toward a quasi-steady state in which a hydrostatic balance is established in the radial direction. During the transition stage, the oscillation timescale is almost constant, while the oscillation amplitude gradually decreases. In this section, we derive a scaling relation that can reproduce the oscillation timescale of our jet–ambient medium system.
Figure 5. Same as Figure 3, but at $t = 33$ (dashed lines) and 43 (solid lines).

Figure 7(a) indicates that, during the transition stage, the radial size of the jet oscillates around the jet width of the final quasi-steady state. We then approximate the typical oscillating width of the jet by the jet width of the saturated state illustrated in Figure 8. The typical oscillation time of the jet would be determined by the propagation time of the sound waves over the typical oscillating width of the jet. When using the physical parameters at the quasi-steady state, the oscillating time is evaluated as

$$\tau = \bar{\gamma}_{\text{jet}} W_{\text{jet},s} / C_s$$

in the laboratory frame. Here, the subscript $s$ stands for the physical values in the final steady state. Since the thermal energy inside the jet is relativistic, the sound speed is well approximated by $C_s \simeq c/\sqrt{3}$.

In order to derive the typical oscillation time of the jet–ambient medium system, we need to estimate the average Lorentz factor in the jet $\bar{\gamma}_{\text{jet}}$ in the quasi-steady state. Since the total energy in the jet is almost conserved during the transition phase, we can obtain the following relation neglecting the rest mass energy, which is lower than the relativistic thermal energy in the jet,

$$W_{\text{jet},s}^2 \gamma_{\text{jet}}^2 P_{\text{amb},0} = W_{\text{jet},0}^2 \gamma_{\text{jet},0}^2 P_{\text{jet},0}. \quad (8)$$

Note that we replace the pressure in the jet in the steady state $P_{\text{jet},s}$ with that of the initial ambient medium $P_{\text{amb},0}$ because a pressure balance is established inside and outside the jet in the final quasi-steady state.

From Equations (7) and (8), we can give the scaling law for the oscillation time

$$\tau = \sqrt{3} \gamma_{\text{jet},0} \left( \frac{W_{\text{jet},0}}{c} \right) \left( \frac{P_{\text{jet},0}}{P_{\text{amb},0}} \right)^{1/2}. \quad (9)$$

In Figure 9, we plot the oscillation time averaged over 10 cycles for numeric runs with different initial pressure ratios. The solid

5 Our models indicate that the loss of the total energy is less than 0.4% in the transition phase.
line represents the analytic scaling we derived. This indicates that our numeric results are well captured by our simple scaling law shown in Equation (9).

Magnetic fields, which are expected to exist in relativistic jets from AGNs and GRBs (Blandford & Payne 1982; Uchida & Shibata 1985; Shibata & Uchida 1986; Blandford 2000; Lyutikov & Blandford 2003), may alter the oscillation mechanism we have presented. In the Poynting flux jet, the magnetic pressure is dominant over the gas pressure. Since the sum of the magnetic pressure and the gas pressure contributes to the rarification acceleration (Mizuno et al. 2008; Aloy & Mimica 2008; Zenitani et al. 2010; Tchekhovskoy et al. 2010; Komissarov et al. 2010), it is expected that the magnetic pressure would play much the same role as the relativistic thermal pressure in the jet oscillation.

3. TWO-DIMENSIONAL SIMULATIONS

3.1. Numeric Models and Setup

We investigate how the oscillating motion found in the 1D model affects the propagation dynamics and the structure of the jet in a more realistic 2D axisymmetric system. Our axisymmetric simulation of the jet propagation has been carried out in cylindrical coordinates \((r, z)\), where the \(z\)-axis coincides with the symmetric axis (see Figure 1). Relativistically hot flow is continuously injected into the ambient medium from the lower boundary of the computational domain, \(z = z_{\text{low}}\). The governing equations we solved are

\[
\frac{\partial}{\partial t}(\gamma \rho) + \frac{1}{r} \frac{\partial}{\partial r}(r \gamma \rho v_r) + \frac{\partial}{\partial z}(\gamma \rho v_z) = 0, \tag{10}
\]

\[
\frac{\partial}{\partial t}(\gamma^2 \rho v_r) + \frac{1}{r} \frac{\partial}{\partial r}\left[r(\gamma^2 \rho v_r^2 + P c^2)\right] + \frac{\partial}{\partial z}\left[\gamma^2 \rho v_r v_z\right] = \frac{P c^2}{r}, \tag{11}
\]

\[
\frac{\partial}{\partial t}(\gamma^2 \rho v_z) + \frac{1}{r} \frac{\partial}{\partial r}\left[r(\gamma^2 \rho v_z^2 + P c^2)\right] + \frac{\partial}{\partial z}\left[\gamma^2 \rho v_z \right] = 0, \tag{12}
\]
Figure 7. Temporal evolution of (a) the jet width, (b) the maximum and average of the tangential velocity \((v_{\text{z,max}} / c)\), (c) the maximum and average of the Lorentz factor \((\gamma_{\text{max}} \text{ and } \bar{\gamma})\), and (d) the average of the specific enthalpy in the jet. The initial Lorentz factor inside the jet is roughly seven and is shown by the dotted line in panel (c). However, the initial specific Lorentz factor of the jet is 41 and is presented by a dashed line in panel (d).

\[
\frac{\partial}{\partial t}(\gamma^2 \rho h - P) + \frac{1}{r} \frac{\partial}{\partial r} \left( r (\gamma^2 \rho h v_r) \right) + \frac{\partial}{\partial z} \left[ \gamma^2 \rho h v_z \right] = 0, \tag{13}
\]

where we use the normalization units and symbols defined in Section 2 in order to compare the results of the 1D and 2D calculations. The injection flow has the same hydrodynamic parameters as the 1D model, that is, \(\rho_{\text{jet},0} = 0.1\), \(P_{\text{jet},0} = 1.0\), \(v_r,0 = 0\), \(v_z,0 = 0.99c\), and \(\gamma_{\text{jet},0} \sim 7\).

For the simulation presented here, we use simple power-law models for the ambient medium. When assuming a polytropic atmosphere, the ambient medium has pressure and density distributions

\[
P_{\text{amb}} = P_{\text{amb},0} \left( \hat{r} / W_{\text{jet},0} \right)^{-\alpha}, \tag{14}
\]

\[
\rho_{\text{amb}} = \rho_{\text{amb},0} \left( \hat{r} / W_{\text{jet},0} \right)^{-3\alpha / 4}, \tag{15}
\]

where \(\hat{r} = \sqrt{r^2 + z^2}\) is the spherical radius in the cylindrical coordinate system and \(P_{\text{amb},0} = 0.1\) and \(\rho_{\text{amb},0} = 1\). Note that the model with \(\alpha = 0\) corresponds to a uniform ambient medium model. For simplicity, we neglect the effect of the gravity, and the ambient medium initially does not move in the calculation domain. We set the power-law index of the ambient medium \(\alpha\) as 0.0, 0.4, and 0.8 in the following simulations.

The calculation domain spans \((100 W_{\text{jet},0} \times 1500 W_{\text{jet},0})\) in the \((r \times z)\)-plane, which corresponds to a \(1600 \times 24,000\) grid. A uniform resolution of eight numeric cells over the radius of the injection jet is used. At the lower boundary \(z_{\text{low}} = 1\) hydrodynamic variables are fixed inside the jet injection region \((0 < r < 0.5)\), while the boundary conditions are reflective outside the jet injection region. An outflow boundary condition is imposed on the outer boundaries of the grid, and the symmetry axis is reflecting.

3.2. Analytic Estimation: The Size of Cusp-Shaped Boosted Region

Our 1D models suggest that the relativistically hot jet alternates between acceleration and deceleration phases with the oscillation period derived in Equation (9) when it propagates through a uniform ambient medium. This is a consequence of the in situ conversion between thermal and bulk kinetic energies inside the jet. When the relativistically hot jet is continuously injected into the ambient medium in more realistic multi-dimensional situations, we can expect that the radial oscillating motion that appeared in the 1D system manifests as the periodically modulated structure of the jet along the jet propagation direction. From the scaling law for the oscillation period of the jet obtained in Section 2.3, we estimate here the typical size of the region where the relativistic jet is boosted.

The space–time diagram of Figure 2 shows that the boosted region of the jet has a cusp shape, which corresponds to the interacting region of rarefaction waves. The multi-dimensional extension of this jet oscillation in the 1D system would provide the periodic formation of the cusp-shaped boosted region confined by oblique shocks (Norman et al. 1982; Sanders 1983) along the propagation direction of the jet. In previous numeric works, shock waves confining the flow to a narrow region were observed inside relativistic jets; this is called the reconfinement shock (e.g., Marti et al. 1997; Gomez et al. 1997; Komissarov & Falle 1997, 1998; Aloy et al. 2000a, 2000b; Agudo et al. 2001; Zhang et al. 2003, 2004; Mizuta et al. 2006; Mizuta & Aloy 2009; Perucho & Marti 2007; Morsony et al. 2007, 2010; Lazzati et al. 2009; Mimica et al. 2009; Nagakura et al. 2011). The
Figure 8. Spatial distribution of the initial non-equilibrium and final quasi-steady states of the jet: (a) density, (b) pressure, (c) radial velocity, (d) tangential velocity, and (e) Lorentz factor. The dashed and solid lines represent the initial \((t = 0)\) and final \((t = 2000)\) states, respectively.

Figure 9. Relation between the oscillation timescale of the system and initial pressure ratio of the jet to the ambient gas (Equation (9)). Diamonds denote the oscillation time averaged over 10 cycles for each parameter.

The cusp-shaped boosted region, which results from the jet oscillation due to the interacting rarefaction waves, would appear in the multi-dimensional system as the reconfinement region found in these previous works.

We can approximate the size of the cusp-shaped boosted region, that is, the reconfinement region, by the propagation distance of the jet fluid during the typical oscillation time \(\tau\) written in Equation (9). Since the fluid velocity of the relativistic jet is almost equal to the speed of light, the propagation distance \(\lambda\) is given by

\[
\lambda = c\tau = \sqrt{3} \gamma_{jet,0} W_{jet,0} \left( \frac{P_{jet,0}}{P_{amb,0}} \right)^{1/2}.
\] (16)

The size of the reconfinement region is proportional to the Lorentz factor of the injected jet and the square root of the initial pressure ratio between the jet and the ambient medium. When the relativistically hot jet is injected continuously into the uniform ambient medium, the cusp-shaped reconfinement region with size \(\lambda\) will be formed periodically inside the jet. The scaling law that relates the typical size of the reconfinement region to the pressure of the ambient medium was also
analytically derived by Daly & Marscher (1988). We compare our scaling to it later in Section 4.

In a gravitationally bounded atmosphere, which is plausible in the central engine of relativistic jets, the ambient gas should be stratified and have a pressure distribution along the propagation direction of the jet. Since the radial oscillation of the relativistic jet is controlled by the pressure ratio between the jet and the ambient medium, the size of the reconfinement region $\lambda$ evolves with time when the jet propagates through the stratified medium. Adopting the pressure profile for the stratified ambient medium
described by Equation (14), the size of the reconfinement region along the \( z \)-axis is evaluated as
\[
\lambda = \sqrt{3} \gamma_{\text{jet},0} W_{\text{jet},0} \left( \frac{P_{\text{jet},0}}{P_{\text{amb},0}} \right)^{1/2} \frac{z}{W_{\text{jet},0}} \alpha/2 .
\]
Since the fluid velocity inside the reconfinement region should be relativistic, we can provide the following relation by replacing \( z \) in Equation (17) with \( ct \),
\[
\lambda \propto t^{\alpha/2} .
\]
This implies that the cusp-shaped reconfinement region evolves self-similarly in the multi-dimensional system (see also Komissarov & Falle 1998; Bromberg & Levinson 2007; Nalewajko & Sikora 2009; Kohler et al. 2012).

### 3.3. Self-Similar Evolution of Reconfinement Region

#### 3.3.1. Uniform Ambient Medium Model

Figure 10 shows the temporal evolution of the injected relativistically hot jet for a uniform ambient model (power-law index \( \alpha = 0 \) in Equations (14) and (15)) in the early evolutionary stage. The color contour represents the spatial distribution of the (1) density \( \rho \), (2) pressure \( P \), and (3) Lorentz factor \( \gamma \) when \( t = 100 \) (left), 200 (middle), and 300 (right), respectively. As expected from 1D models, the cusp-shaped rarefaction regions confined by oblique shocks are formed inside the jet. In those regions, the fluid of the jet is accelerated due to the interaction of rarefaction waves and subsequent in situ energy conversion from relativistic thermal energy to bulk kinetic energy (see Section 2.2). Since the rarefaction waves are repeatedly excited behind the jet head, the reconfinement regions are periodically formed along the jet-propagation direction and create the modulated structure of the jet in the early evolutionary stage.

Figure 11 gives the spatial distribution of the (1) density \( \rho \), (2) pressure \( P \), and (3) Lorentz factor \( \gamma \) when \( t = 2000 \). As the jet propagates, the modulated structure of the jet has a loss in coherency except at the region near the injection point. This is because there are vortices that are episodically created by a Kelvin–Helmholtz like instability that develops between the jet and the backflow near the jet head (Mizuta & Aloy 2009). Since the continuously induced vortices make the ambient medium inhomogeneous, the modulation structure of the jet becomes incoherent. Then, the size of the reconfinement region violently fluctuates, except the region near the injection point, where the ambient inhomogeneity remains relatively weak.

The spatial distribution of the Lorentz factor near the jet injection point is shown in Figure 12(b). In order to compare it to the result of the 1D calculation, the temporal evolution of the Lorentz factor in the jet–ambient system of the 1D simulation is shown in Figure 12(a). The size of the cusp-shaped rarefaction

![Figure 11. Snapshots of the spatial distribution of the (a) density, (b) pressure, and (c) Lorentz factor in the uniform ambient medium model when \( t = 2000 \). (A color version of this figure is available in the online journal.)](image)

![Figure 12. Left panel represents the time evolution of the Lorentz factor in the jet–ambient medium system of the 1D simulation. The right panel demonstrates the spatial distribution of the Lorentz factor at \( t = 2000 \) in the uniform ambient model (\( \alpha = 0 \)) of the 2D simulation. (A color version of this figure is available in the online journal.)](image)
region in both cases seems to be almost the same, although the vertical axis in Figure 12(a) is the time. The typical size of the reconfinement region in the jet-propagation direction can be estimated theoretically from relation (16). With using the parameters $\gamma_{\text{jet}, 0} \simeq 7, W_{\text{jet}, 0} = 1,$ and $P_{\text{jet}, 0}/P_{\text{amb}, 0} = 10$, we can obtain $\lambda \simeq 40$, indicating that our scaling relation captures the result of the 2D simulation. The width of the contact layer of the 2D simulation is larger than that of the 1D calculation due to the lower resolution of the simulation.

The pressure on the $z$-axis in the 1D and 2D simulations is shown in Figures 13(a) and (b), respectively. Note that the horizontal axis represents the time $t$ in Figures 13(a) while Figure 13(b) depicts the pressure in a 1D cut along the $z$-axis of Figure 11(b). In 1D models, the oscillation of the jet is a transient phenomena that terminates when the pressure inside the jet falls to the same level as that of the ambient medium (see Section 2.2.3). On the other hand, in the 2D case, the pressure inside the jet continues to vary, and a pressure balance between the jet and the ambient medium is not achieved. The predicted size of the reconfinement region apparently seems to exist only near the injection point.

Figure 14 shows the time–distance diagram for the pressure along the $z$-axis. The reconfinement region near the injection point does not move with time, while the reconfinement region formed behind the jet head propagates along the jet direction. Though the modulation properties of the jet change due to the ambient medium inhomogeneity, the typical size of the reconfinement region is expected to be maintained at a constant value at any time as shown in Figure 14.

The temporal evolution of the Fourier spectrum of the pressure measured along the jet axis is given in Figure 15. Here, the Fourier transformation $P(k)$ is obtained from

$$P(k) = \frac{1}{L_{\text{jet}}} \int_{L_{\text{jet}}} P_{z-\text{axis}}(z)e^{-ikz}dz,$$

where $L_{\text{jet}}$ is the length of the jet in the propagation direction. The solid, dashed, and dotted curves indicate the spectra at $t = 600, 1200,$ and $2000$, respectively. Despite the different evolutionary phases, all the spectra show a peak at around $k/2\pi \simeq 1/40$, which corresponds to the inverse of the typical size of the reconfinement region derived from Equation (16), and we can find that there is a second harmonic in each spectrum. This suggests that the typical size of the reconfinement region inside the jet is essentially determined by the modulation caused by the interaction of rarefaction waves even though the ambient medium inhomogeneity causes fluctuation in jet structure.

### 3.3.2. Power-Law Models

There is one main important difference between the power-law models with $\alpha \neq 0$ and the uniform ambient model ($\alpha = 0$). For models with decreasing pressure profiles with distance from...
the launching point of the jet, the jet expands more than in the uniform ambient model. The jet is then further accelerated because the pressure inside the reconfinement region decreases more drastically and a larger amount of thermal energy is converted into bulk kinetic energy.

Figure 16 shows the temporal evolution of the injected relativistically hot jet for the power-law ambient medium model with $\alpha = 0.8$. The meanings of the color contours and the columns are the same as those in Figure 10. Note that the vertical and horizontal scale of Figure 16 is 2.5 times larger than that...
of Figure 10. It is found that, though the inflow flux is the same for each model, the maximum Lorentz factor inside the reconfinement region reaches $\sim 180$ for the power-law model with case $\alpha = 0.8$, while it is $\sim 45$ for the uniform ambient medium model (see Figures 10(c) and 12(b)).

The decreasing ambient pressure results in a longer oscillation timescale as suggested in Equation (9). The size of the reconfinement region in the power-law models thus becomes larger than that in the uniform ambient medium model. Additionally, the results shown in Figure 16 imply that the reconfinement region evolves self-similarly.

In order to confirm the self-similarity, we investigate the time trajectory of the cusp of the reconfinement region on the jet axis, depicted by diamonds in Figure 17, after the cusp-shaped region is formed. Overplotted in Figure 17 is the curve denoting the scaling relation (18), which is obtained from the results of the 1D models (see Section 3.2). Panels (a) and (b) correspond to the cases with $\alpha = 0.4$ and $\alpha = 0.8$. These figures confirm that the reconfinement region evolves self-similarly once the cusp-shaped reconfinement region is formed.

4. DISCUSSION

Aloy & Rezzolla (2006) discussed for the first time a new mechanism in RHDs that can act as a powerful booster in jets; rarefaction acceleration. This mechanism is purely hydrodynamic and operates when the jet–ambient medium satisfies the following physical conditions: The jet should be hot and over pressured and should have a relativistic velocity to the jet–ambient medium interface. The hot jet stores internal energy comparable to or greater than its rest mass energy. When the pressure of the jet is greater than that of the ambient medium, the rarefaction wave is excited at the jet–ambient medium interface, and the relativistic thermal energy of the plasma can be converted into the bulk kinetic energy of the jet near the interface. An expansion of the relativistically hot jet to the under pressured ambient medium is responsible for the rarefaction acceleration of the jet.

The rarefaction acceleration should be commonly operated in the jet that satisfies these conditions. This has been observed in many previous simulations as expected (e.g., Gomez et al. 1997; Aloy et al. 2000b; Scheck et al. 2002; Zhang et al. 2003). The reconfinement region confined by the oblique shock has been also formed in these previous simulations as a natural outcome of the rarefaction acceleration in the multidimensional system. However, the rarefaction acceleration cannot be observed in the cold jet where the internal energy of the fluid is lower than the rest mass energy of the fluid even if it shows the reconfinement shock (e.g., Komissarov & Falle 1997).

Daly & Marscher (1988) explored the dynamics of relativistic, hot, and over pressured jets on the basis of simplified, quasilinear, hydrodynamic equations for adiabatic, steady, cylindrically symmetric, and irrotational flows. In that work, they found the variety of flow structures with oscillating cross section or standing shocks. In addition, they discussed scaling laws that relate the intrinsic properties of the jet to the pressure of the ambient medium.

In this sense, our finding is that there exists a close connection between the rarefaction acceleration mechanism discussed by Aloy & Rezzolla and the flow structure with the reconfinement region and oscillating cross section quantitatively by using 1D and 2D hydrodynamic simulations. It is remarkable that a simple 1D model based on Aloy & Rezzolla (2006)’s mechanism can so well reproduce the complicated 2D structure of jets not only in the rarefaction acceleration region but also in the reconfinement region. Since the 1D model is simple and easy to understand, these findings will be useful for more detailed study of jets.

It should be emphasized that the scaling law we obtained in this work (Equation (16)) is different from that found by Daly & Marscher (1988). This is because the approximations used for deriving the scaling in their work is applicable only to the system with a small pressure difference between the jet and the ambient medium. Our scaling law can capture the flow properties of the jet–ambient medium system in the wider parameter range than that covered by the scaling law of Daly & Marscher (1988).

The simple scaling law for the self-similarity of the reconfinement region (18) would predict the evolution of the reconfinement region in the context of both AGN jets and GRBs. The reconfinement shocks due to the interacting rarefaction waves may be able to account for the confinement of the relativistic jet from AGNs on a sub-parsec scale (Junor et al. 1999; Kovalev et al. 2007; Acciari et al. 2009). Some authors propose that the radio knots in the jet may correspond to reconfinement regions (e.g., Daly & Marscher 1988; Gomez et al. 1995; Bicknell & Begelman 1996; Komissarov & Falle 1997; Stawarz et al. 2006; Mimica et al. 2009). The scaling law (18) predicts that radio knots in the jet from AGNs evolve self-similarly with time depending on the power-law index of the ambient pressure distribution when the relativistically hot and steady jet is formed near the central engine.

Although the numeric work in this paper is based on simulations with a constant engine that powers the jet, the variability injected by the central engine also leads to inhomogeneity in relativistic jets (Gomez et al. 1997; Mimica et al. 2009;
Morsony et al. (2010). This might be a promising mechanism for generating internal shocks in relativistic jets, accounting for the time-variable emission properties of AGNs and GRBs (Takahashi et al. 2000; Piran 2004; Mimica et al. 2005; Mészáros 2006). The relation for inhomogeneity in the jet between the variability injected by the central engine and the interaction of the relativistic jet with the ambient medium also needs to be investigated in a more realistic situation. Actually, for the case of power-law ambient models, we need a larger calculation domain while retaining the same resolution to study the effect of the vortices developed around the jet head on the propagation dynamics of the jet. This is because the jet expands more rapidly and the modulation structure becomes larger as the power-law index $\alpha$ increases. However, these studies go beyond the scope of this paper and will be studied in a separate paper.

5. SUMMARY

We study the nonlinear evolution of the interacting rarefaction waves excited at the cylindrical jet–ambient medium interface through 1D RHD simulations. We find that an enhanced decrease in the relativistic pressure due to the interaction of rarefaction waves transiently yields a more powerful boost of the Lorentz factor of the bulk jet than that expected from a single rarefaction wave. The cyclic in situ energy conversion between thermal energy and bulk kinetic energy is a natural relativistic outcome of the jet scenario studied and responsible for the radial oscillating motion of the jet. The oscillation timescale is characterized by the initial pressure ratio of the jet to the ambient medium and follows a simple scaling relation $\tau_{\text{oscillation}} \propto (P_{\text{jet},0}/P_{\text{amb},0})^{1/2}$.

We confirm from extended 2D simulations that repeated excitation and convergence of rarefaction waves result in the alignment of the interacting regions of rarefaction waves, confined by oblique shocks, along the propagation direction of the jet when a relativistically hot jet propagates through an ambient medium. The evolution of the reconfinement region in which the fluid of the jet is powerfully boosted due to the interacting rarefaction waves has a self-similar property when the relativistic jet propagates through the ambient medium which has a power-law pressure distribution. The evolution of the size of the reconfinement region along the jet direction follows a simple scaling law $\lambda \propto t^{\alpha/2}$, where $\alpha$ is the power-law index of the pressure distribution. In the uniform ambient medium model ($\alpha = 0$) especially, the typical size of the reconfinement region inside the jet is essentially determined by the modulation caused by the interaction of rarefaction waves.

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APPENDIX

CONVERGENCE TESTS

A.1. One-Dimensional Riemann Problem with Transverse Velocity

The 1D relativistic Riemann problem with large transverse velocity requires high resolution in order to numerically resolve
Figure 19. Solutions of Riemann problem of our jet model in the 1D (x-direction) cartesian coordinate with the resolution of $\Delta x = 2 \times 10^{-5}$. Diamonds and solid lines illustrate the numeric and analytic solutions, respectively. (A color version of this figure is available in the online journal.)

Figure 20. Same as Figure 19 but for the case with $\Delta x = 10^{-3}$. (A color version of this figure is available in the online journal.)
the complicated structure (Zhang & MacFadyen 2006; Mizuta et al. 2006; Mizuno et al. 2008). We have tested our code on a 1D (x-direction) Riemann problem with transverse velocity using uniform resolutions of $\Delta x = 10^{-3}$ and $2 \times 10^{-5}$ when the calculation domain spans $0 < x < 1$. We have used the initial conditions from Zhang & MacFadyen (2006) with adiabatic index $\Gamma = 5/3$ as follows:

Left state ($0 < x < 0.5$): $\rho_L = 1$, $P_L = 10^3$, $v_{x,L} = 0$ and $v_{z,L} = 0.99c$.

Right state ($0.5 < x < 1$): $\rho_R = 1$, $P_R = 10^{-2}$, $v_{x,R} = 0$ and $v_{z,R} = 0.99c$.

Figure 18 denotes numeric and analytic solutions of this problem at $t = 0.6$. Analytic solutions in the Appendix are calculated with the code of Giacomazzo & Rezzolla (2006). The upper and lower panels correspond to the cases with calculated with the code of Giacomazzo & Rezzolla (2006) with adiabatic index $\Gamma = 5/3$ as follows:

Left state ($0 < x < 0.5$): $\rho_L = 1$, $P_L = 10^3$, $v_{x,L} = 0$ and $v_{z,L} = 0.99c$.

Right state ($0.5 < x < 1$): $\rho_R = 1$, $P_R = 10^{-2}$, $v_{x,R} = 0$ and $v_{z,R} = 0.99c$.

Figure 19 shows the numeric and analytic solutions of our jet model at $t = 0.1$ in the case with $\Delta x = 2 \times 10^{-5}$. The numeric solution resolves the hydrodynamic structure of the analytic solution very well. In the case with $\Delta x = 10^{-3}$, the position of the right-propagating shock front is not correctly captured, while the numeric solution is able to capture the hydrodynamic structure of the rarefaction wave and the contact discontinuity as shown in Figure 20. In this paper, we focus on the interaction of rarefaction waves excited at the jet–ambient medium interface. Since the shock wave propagates outward from the jet–ambient medium interface, the interaction of rarefaction waves inside the jet is not affected by the outward-going shock wave.

In our calculations in Section 2, shock waves are excited inside the cylindrical jet. In order to investigate the effect of these shock waves, we have compared results of the calculation we performed in Section 2 to those of the high-resolution calculation, in which a uniform grid with the grid size $\Delta r = 2 \times 10^{-5}$ is adopted in the cylindrical coordinate. Figure 21 illustrates the temporal evolution of (1) the jet width, (2) the maximum, and (3) the average Lorentz factor in the jet in both numeric runs. Note that the horizontal axis corresponds to the time for the duration $0 < t < 100$ demonstrated in Figure 3. The jet width in the case with $\Delta r = 10^{-3}$ is up to a factor 1.07 larger than that in the high-resolution case because of numeric diffusion of the contact discontinuity. This leads to a 2% decrease in the average Lorentz factor inside the jet compared to the high-resolution case. We can find, however, only small differences of the maximum Lorentz factor in the jet between these numeric runs. Therefore, our choice of resolution does not significantly impact our main results.
