The role of the buildup oscillations on the speed of resonant tunneling diodes

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Abstract

The fastest tunneling response in double barrier resonant structures is investigated by considering explicit analytic solutions of the time dependent Schrödinger equation. For cutoff initial plane waves, we find that the earliest tunneling events consist on the emission of a series of propagating pulses of the probability density governed by the buildup oscillations in the quantum well. We show that the fastest tunneling response comes from the contribution of incident carriers at energies different from resonance, and that its relevant time scale is given by \( \tau_r = \pi \hbar/|E - \varepsilon| \), where \( \varepsilon \) is the resonance energy and \( E \) is the incidence energy.

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The transport mechanism in resonant tunneling diodes (RTD) and the relevant time scale for its fastest tunneling response have been the subject of intense investigation. In particular, the results of the high-frequency tunneling experiments demonstrated the possibility that the charge transport may occur at time scales even shorter than the lifetime of the quasibound state of the system. Most of the theoretical efforts to estimate the relevant time scales for the tunneling process have been based on stationary approaches; however, it has been widely recognized that the analytic solution of the time-dependent Schrödinger equation (TDSE) provides the most reliable way to tackle this fundamental issue. Although there is in the literature a consensus that the buildup dynamics determines the ultimate speed of RTD’s, the way in which the buildup governs the emission of transmitted particles still needs elucidation.

In this letter we investigate the earliest tunneling events in double barrier resonant structures and its relation to the dynamics of the buildup process in the quantum well, based on the analytic solution of the TDSE. Starting from the formal solution, we show that the earliest evidence of transmitted carriers to the right edge of the structure \((x \geq L)\) consists on the emission of a series of propagating pulses, which are governed by the buildup oscillations in the quantum well.

The initial condition used here is inspired on experimental situations in which the tunneling process begins almost abruptly. It is represented by a cutoff planewave \(\Psi(x, k; t = 0) = \Theta(-x)(e^{ikx} - e^{-ikx})\) impinging on a shutter placed at \(x = 0\), just at the left edge of a DB structure. The tunneling process begins with the instantaneous opening of the shutter at \(t = 0\), enabling the incoming wave to interact with the potential, giving rise to both the buildup in the quantum well and the electronic transport to the right of the DB. For the above initial condition, the solutions \(\Psi^i(x, k; t)\) for the internal region \((0 \leq x \leq L)\), and \(\Psi^e(x, k; t)\) for the external region \((x > L)\), are respectively:

\[
\Psi^i = \phi_k M(y^i_k) - \phi_{-k} M(y^i_{-k}) - i \sum_{n=\infty}^{\infty} \phi_n M(y^i_{kn}),
\]  

(1)
\[
\Psi^e = T_k M(y_k^e) - T_{-k} M(y_{-k}^e) - i \sum_{n=-\infty}^{\infty} T_n M(y_{y_k}^e).
\] (2)

Here \( \phi_k \equiv \phi(x,k) \) is the stationary wave function, \( T_k \equiv t(k) \) is the transmission amplitude, and the factors \( \phi_n(x,k) \equiv 2k u_n(0) u_n(x)/(k^2-k_n^2) \) and \( T_n = \phi_n(L,k) \exp(-ik_nL) \) are given in terms of the resonant eigenfunctions \( u_n(x) \) with complex eigenvalues \( k_n = a_n - ib_n \) \( (a_n, b_n > 0) \). The index \( n \) runs over the complex poles \( k_n \) distributed in the third and fourth quadrants in the complex \( k \)-plane. The \( M' \)'s are the Moshinsky functions with arguments \( y_q(x,t) = e^{-i\pi/4(m/2ht)^{1/2}[x-hqt/m]}, \) and \( y_q^i = y_q^i(0,t), \) where \( q \) stands either for \( \pm k \) or \( k_{\pm n} \). For the particular case of DB systems with isolated resonances i.e. \( |\varepsilon_{n+1} - \varepsilon_n| \ll \Gamma_n \), one term is sufficient; most of the DB diodes with typical potential parameters fall into this category.

The emission of the earliest transmitted particles is described by the external probability density; this is illustrated in Fig. in which we plot the normalized probability density \( |\Psi^e(x,k;t)/T_k|^2 \) as a function of \( x \) for fixed values of time chosen in this example as \( t = 2 \) and 10 ps. For off-resonance incidence (solid lines), the propagation of an oscillatory structure with a sharp defined wavefront is clearly appreciated; to illustrate that this wavefront travels with approximately the classical speed \( v = (2E/m)^{1/2} \), the arrow indicates the position \( x = vt \), for \( t = 10 \) ps. Note that \( |\Psi^e|^2 \) oscillates around the transmission coefficient \( |T_k|^2 \), which is the expected asymptotic value as \( t \to \infty \) at \( x = L \).

In order to analyze the tunneling mechanism at its earliest stages, we present in Fig. a series of snapshots of this process at early times and their correlation with the periodic buildup oscillations observed in the quantum well. In part (a) we illustrate the buildup during this transient at the particular times in which the buildup in the well reaches its maxima and minima, as it oscillates around the stationary value (dotted line). Panel (b) exhibits the birth of the first pulses for the same sequence of times chosen in (a). The correspondence between the buildup oscillations and the successive emission of pulses is evident from the figure. We see that upon completion of one buildup cycle, a pulse is fired; in this way for example, at 2.4 ps, three cycles have been completed and up to this stage three pulses have already been emitted.
In order to find analytically how the buildup governs the fastest tunnelling response and its relevant time scales, we shall exploit the analytical properties of the solutions. As shown in a recent paper, the probability density at any position of the internal region, may be described by the simple formula

$$\left| \Psi^i / \phi \right|^2 = 1 + e^{-\Gamma_n t/\hbar} - 2e^{-\Gamma_n t/2\hbar} \cos [\omega_n t]$$

where $\omega_n = |E - \epsilon_n|/\hbar$. The oscillatory function of the above formula gives the periodicity of the buildup oscillations depicted in Fig. 2 (a). As we shall show below this dynamical behavior also manifests itself outside the structure, and is the key mechanism behind the observed synchronization between the buildup oscillations and the pulse emission. Although the external solution, Eq. (2), is more complex than $\Psi^i$ since the $M$ functions involved in $\Psi^e$ depend on both position and time, a simple expression for $|\Psi^e / T_k|^2$ can be obtained. We start by considering the one-level formula for the external solution,

$$\Psi^e = T_k M(y^e_k) - T_{-k} M(y^e_{-k}) - iT_n M(y^e_{k_n}) - iT_{-n} M(y^e_{k_{-n}}).$$

(4)

From the one level expression for $\phi(L, k)$, the transmission amplitude $T_k$ can be written as $T_k = 2ik \exp(-ikL)u_n(0)u_n(L)/(k^2 - k_n^2)$; thus the factors $iT_n$ and $iT_{-n}$ in the above expression can be readily identified as $iT_n = T_k \exp[i(k - k_n)L]$ and $iT_{-n} = -T_k^* \exp[-i(k - k_n)L]$. Using the symmetry relation for the $M$ functions, $M(y^e_q) = \exp(y^e_q^2) - M(-y^e_q)$, in $M(y^e_k)$ and $M(y^e_{k_n})$, we obtain a suitable representation for $\Psi^e$ consisting on exponential terms and $M$ functions with arguments of the type $y^e_q$ with $q = -k, -k_n, -k_n^*$, which have vanishingly small contributions to the solution. In fact, for a fixed value of the position $x_f$, there exists a time interval starting from $t \gtrsim (mx_f/\hbar k)$, governed exclusively by the exponential terms mentioned above. From these considerations, a simple formula for $|\Psi^e / T_k|^2$ can be obtained, namely,

$$|\Psi^e / T_k|^2 = 1 + e^{-\Gamma_n t/\hbar}e^{2b_n(x_f - L)} - 2e^{b_n(x_f - L)}e^{-\Gamma_n t/2\hbar} \times \cos [(a_n - k)(x_f - L) + \omega_n t]$$

(5)
valid for \( t \gtrsim (mx_f/\hbar) \). By comparison with Eq. (3), we note that \( |\Psi^e/T_k|^2 \) and \( |\Psi^i/\phi|^2 \) have similar time dependence. In fact, at \( x_f = L \), (4) reduces to

\[
|\Psi^e/T_k|^2 = 1 + e^{-\Gamma_n t/\hbar} - 2e^{-\Gamma_n t/2\hbar} \cos [\omega_n t],
\]

which coincides exactly with Eq. (3). This analytical result confirms the qualitative discussion of Fig. 2, that the buildup oscillations in the quantum well govern the emission of the propagating pulses at \( x = L \). Therefore, both the buildup oscillations and the emitted pulses are characterized by the same time scales. It is straightforward to see from Eq. (3) that the maxima of the buildup oscillations occur approximately at the time scales \( \tau_m \) given by

\[
\tau_m = \frac{(2m - 1)\pi\hbar}{\Delta E_n}, \quad m = 1, 2, 3, ...
\]

where \( \Delta E_n \equiv |E - \epsilon_n| \) measures the deviation of the incidence energy \( E \) from resonance \( \epsilon_n \). Note that the same result is obtained for incidence above and below resonance, \( E = \epsilon_n \pm \Delta E_n \), in view that Eq. (3) is irrespective to this choice. According to Eq. (3), the intensity of the pulses reaches maximum values at \( x = L \) also at these time scales. The formation and emission of the first of such pulses constitutes the earliest evidence of carrier presence at the transmitted region; thus, the relevant time scale for the fastest tunneling response corresponds to \( m = 1 \), and is simply given by

\[
\tau_r = \frac{\hbar \pi}{\Delta E}.
\]

With regard to the times scales involved in the tunneling process, it is important to emphasize the differences between the response time \( \tau_r \) and other relevant time scales, such as the lifetime, \( \tau_l = \hbar/\Gamma \), and the buildup time, \( \tau_b \). In what follows we shall discuss these differences and their implications.

The expressions for \( \tau_r \) and \( \tau_l \) are similar since they are both inversely proportional to a certain energy width. However despite of this resemblance, there is a fundamental difference: on the one hand the lifetime is an \textit{intrinsic} property of the resonant structure since it depends
exclusively on the system parameters through the resonance width $\Gamma$; on the other hand, the
time scale $\tau_r$ is not an intrinsic property of the system since it takes into account external
information, namely, the energy of the incident carriers through the “off-resonance width”,
$\Delta E$. This difference is crucial for understanding the ultrafast response in DB diodes, and
an important consequence is that depending on $\Delta E$, the response time $\tau_r$ may be greater
or even shorter than $\tau_l$; in fact, it is easy to see from Eq. (8) that the condition for $\tau_r < \tau_l$
is simply $\Delta E > \pi \Gamma$. This is illustrated in Fig. 3, in which we compare the positions of the
main peak of the plots of $|\Psi^e/T_k|^2$ as a function of time and fixed position at $x = L$, using
two different values of $\Delta E$.

The buildup time $\tau_b$ is the duration of the transient regime in which the time-dependent
probability density $|\Psi^i|^2$ reaches its level-off given by the stationary probability density, $|\phi|^2$.
As shown in a recent work such a transient is of approximately ten lifetimes, for any DB
structure with isolated resonances; see Fig. 2 (b) of Ref. 6. As shown in the previous
paragraph, $\tau_r$ can be shorter than $\tau_l$ and consequently than $\tau_b$, which means that several
buildup oscillations (and emitted pulses) may occur before $\tau_b$. This is clearly illustrated
in Fig. 2 where for example at $t = 2.4$ ps ($\approx 0.38\tau_b$) significant evidence of transmitted
carriers at the right of the structure is appreciated; in fact, up to this time the head of the
transmitted wave has already traveled more than $10^3$ nm, despite the fact that the buildup
in the quantum well has not been fully established.

The above results emphasize on the importance of the off-resonant carriers in ultrafast
tunneling, which play an important role in typical tunneling experiment with RTD’s where
the energies of the available carriers are distributed within a finite interval $0 < E < E_F$,
where $E_F$ is the Fermi energy of the emitter. In fact, although off-resonance carriers have a
smaller transmission coefficient, the main contribution to the tunneling current comes from
energies near resonance rather than from the resonance alone (i.e. the Tsu-Esaki formula
involves an integration over the whole Fermi interval, $0 < E < E_F$). As an implication of
our analysis, when the resonance $\varepsilon$ is immersed into the Fermi sea and the resonance width
$\Gamma$ is such that $E_F > \pi \Gamma$, there will be carriers of the Fermi interval fulfilling $\Delta E > \pi \Gamma$ and
hence contributing with tunneling responses faster than the lifetime. The above situation should be present in tunneling experiments on DB systems in which $E_F \gg \pi \Gamma$.

We conclude our discussion with the following remarks: (i) Since the tunneling dynamics critically depends on the shape of the incoming wave, whenever one deals with the problem of the response time, one must clearly specify to what initial condition it corresponds. In this respect, we stress that the response time derived here corresponds to cutoff initial plane waves. (ii) In view that plane waves are the “building blocks” of wavefunctions with more general shapes, the results obtained here may also give insight into the more intricate behavior expected for the case of incoming wavepackets.\textsuperscript{12} (iii) The emphasis of the present study is the exploration of genuine quantum dynamical effects occurring in the transient regime; in particular, we have shown that the earliest tunneling events in a DB structure consist on the firing of a series of propagating pulses whose periodic emission at the right edge of the structure is governed by the buildup oscillations in the quantum well. (iv) The importance of the off-resonance incident carriers on the response time is demonstrated within a purely coherent picture, and a closed formula (valid near resonance) for the corresponding time scale has been derived from the analytic solution.

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REFERENCES

a) Electronic mail: rromo@faro.ens.uabc.mx.

1 T. C. L. G. Sollner, E. R. Brown, W. D. Goodhue, and H. Q. Le, Appl. Phys. Lett. 50, 332 (1987); D. D. Coon and H. C. Liu, ibid. 49, 94 (1986); M. A. Talebian and W. Pötz, ibid. 69, 1148 (1996); M. N. Feiginov, ibid. 76, 2904 (2000);

2 T. C. L. G. Sollner, W. D. Goodhue, P. E. Tannenwald, C. D Parker and D. D. Peck, Appl. Phys. Lett. 43, 588 (1983).

3 B. Ricco and M. Ya Azbel, Phys. Rev. B 29, 1970 (1984).

4 S. Luryi, Appl. Phys. Lett. 47, 490 (1985).

5 M Tsuchiya, T. Matsusue, and H. Sakaki, Phys. Rev. Lett. 59, 2356 (1987); H. Yoshimura, J. N. Schulman, H. Sakaki, ibid. 64, 2422 (1990).

6 J. Villavicencio and R. Romo, Appl. Phys. Lett. 77, 379 (2000).

7 It is important to emphasize that the shutter is a device that aids to visualize the initial condition and hence is not part of the system i.e. it does not appear as a potential in the Hamiltonian. More general initial conditions can be treated as external perturbation to the potential, see for example: M. Ya. Azbel, Sol. St. Comm. 91 6 439 (1994); M. Kleber, Phys. Rep. 236, (6) 331-393 (1994) (p. 382).

8 The formalism was originally developed by G. García-Calderón by considering an absorbing shutter as the initial condition, G. García-Calderón and A. Rubio, Phys. Rev. A 55, 3361 (1997). Formulas (1) and (2) were obtained by J. Villavicencio for a modified initial condition: the reflecting shutter, J. Villavicencio Ph. D. Thesis (2000).

9 G. García-Calderón and A. Rubio, Phys. Rev. A 55, 3361 (1997).

10 R. Romo and J. Villavicencio, Phys. Rev. B 60, R2142 (1999).
The parameters of the symmetrical DB structure are: barrier heights $V_0 = 0.23$ eV, barrier widths $b_0 = 5$ nm, well width $\omega_0 = 5$ nm, and effective mass of the electron $m = 0.067m_e$; the position and width of the first resonance are respectively, $\varepsilon = 80.11$ meV, and $\Gamma = 1.033$ meV. We have dropped here the subindex $n$ since we are dealing with the first resonance.

J. A. Støvneng and E. H. Hauge, Phys. Rev. B 44 13582 (1991).
FIGURES

FIG. 1. Plot of $|\Psi^e(x, k; t)/t(k)|^2$ as a function of the distance $x$ for two fixed values of time $t = 2, 10$ ps (solid lines). The incidence energy is below resonance $E = \varepsilon - \Delta E = 74.97 \text{ meV}$ ($\Delta E = 5\Gamma$); for comparison to the classical propagation, the arrow indicates the position $x = vt$ for $t = 10$ ps. The special case of incidence at resonance $E = 80.11 \text{ meV}$ is also included for comparison (dashed lines).

FIG. 2. Early times of the tunneling process in a DB structure. Part (a) illustrates the buildup oscillations inside the structure (solid line) around the stationary probability density $|\phi|^2$ (dotted line); the potential barriers are schematically represented by the dashed-dotted lines. For the same sequence of times, part (b) shows the birth and emission of the first pulses which are the earliest tunneling events occurring in the structure. Note the synchronization with the buildup dynamics: for each buildup oscillation, a pulse is emitted.

FIG. 3. Plots of $|\Psi^e(x, k; t)/t(k)|^2$ as a function of time at the fixed position $x = L = 15$ nm, for two values of $\Delta E$. We see that depending on the value of $\Delta E$, the response time $\tau_r$ may be greater or shorter than the lifetime of the quasibound state.
Figure 1  Roberto Romo. Applied Physics Letters.
Figure 2  Roberto Romo. Applied Physics Letters.
Figure 3  Roberto Romo.  Applied Physics Letters