Collective Flow Distributions and Nuclear Stopping
in Heavy-ion Collisions at AGS, SPS and RHIC

Shengqin Feng, Xianbao Yuan and Yafei Shi

Dept of Physics, Science College, China Three Gorges University, Yichang, 443002, China

We study the production of proton, antiproton and net-proton at AGS, SPS and RHIC within the framework non-uniform flow model (NUFM) in this paper. It is found that the system of RHIC has stronger longitudinally non-uniform feature than AGS and SPS, which means that nuclei at RHIC energy region is much more transparent. The NUFM model provides a very good description of all proton rapidity at whole AGS, SPS and RHIC. It is shown that our analysis relates closely to the study of nuclear stopping and longitudinally non-uniform flow distribution of experiment. This comparison with AGS and SPS help us to understand the feature of particle stopping of thermal freeze-out at RHIC experiment.

PACS number(s): 25.75.-q
I. Introduction

Nuclear matter is believed to be compressed to high baryon density during central collisions [1, 2, 3]. In the interaction region of the colliding nuclei, nucleons undergo collisions which reduce their original longitudinal momentum. This loss of rapidity is an important characteristic of the reaction mechanism and is often referred to as stopping power [4]. Relativistic heavy-ion collisions are unique in the sense that secondary collisions of excited baryons are expected to contribute to rapidity loss leading to simultaneous stopping of many nucleons within the interaction volume.

Proton and antiproton provide an experimental tool for measuring baryon production and allow us to explore baryon transport from beam rapidity toward mid-rapidity [5, 6]. The global thermodynamic properties and collective motion of the system at the kinetic freeze-out point can be deduced, albeit in a model-dependent way. BRAHMS collaboration [5] working at RHIC has published rapidity distributions of net proton at central collisions. These results will help us to study the dynamic feature of RHIC and collective movement (flow).

The study of collective flow in high energy nuclear collisions has attracted increasing attentions from both experimental and theoretical point of views. The rich physics of longitudinal and transverse flows is due to their system evolution at early time and nuclear stopping. In general, the collective evolution of the hot and dense matter leaves a distinct imprint on the phase space distribution of the fireball at freeze-out. To disentangle such information from features generated during freeze-out a refined understanding of the decoupling process is needed.

Here we should mention three kinds of models of thermal and collective flow, the first one is the spherically-expanding source may be expected to approximate the fireball created in lower-energy collisions.

At higher energies stronger longitudinal may lead to a cylindrical geometry according to the second kind model postulated by Schnedermann, Sollfrank and Heinz [7]. It accounted for the anisotropy of longitudinal and transverse direction by adding the contribution from a set of fireballs with centers located uniformly in the rapidity region in the longitudinal direction which sketched schematically in Fig.1 and 2 of Ref.[8]. It can account for the wider rapidity distribution when comparing to the prediction of pure thermal isotropic model.

The third kind of model is postulated by Bjorken [9] which has been formulated for asymptotically high energies, where the rapidity distribution of produced particles establishes a plateau at mid-rapidity. As mentioned before [5-10], collisions at available heavy-ion energy regions of AGS, SPS and RHIC are neither fully stopped nor fully transparent, although a significant degree of transparency are observed. Consequently the overall $dN/dy$ distribution of baryons is expected to consist of the sum of the
particles produced in the boost-invariant central zone and the particles produced by the excited fragments. The fact that the observed distributions are flatter at mid-rapidity and wider than those predicted by the thermal isotropic model might point in this direction. Especially at the SPS and RHIC energy region, the central dip of baryon rapidity distribution is clearly shown by the experiment. A nonuniform longitudinal flow model was proposed [8, 10], which assumed that the centers of fireballs were distributed non-uniformly in the longitudinal phase space.

This paper is organized as follows. In Sec.II we give a qualitative description with a geometrical parametrization picture and briefly review the longitudinal Non-Uniform Flow Model (NUFM) [8]. The comparison and analysis of baryon distribution of AGS, SPS, and RHIC with the calculation results of the model are given in Sec.III. Sec.IV contains a summary and conclusions.

II. The main idea of Non-Uniform Flow Model (NUFM)

The NUFM model we considered [8] contains three distinct assumptions some of which are rather different from those usually contained in other fireball models:

1. We argue that the transparency/stopping of relativistic heavy ion collisions should be taken into account more carefully. A more reasonable assumption is that the fireballs keep some memory on the motion of the incident nuclei, and therefore the distribution of fireballs, instead of being uniform in the longitudinal direction, is more concentrated in the direction of motion of the incident nuclei, i.e., more dense at large absolute value of rapidity. It will not only lead to anisotropy in longitudinal-transverse directions, but also render the fireballs (especially for those baryons) distribute non-uniformly in the longitudinal direction shown in Fig.1.

2. We assume that whether it is at higher RHIC energy region or lower AGS energy region, the freeze-out temperatures are nearly same, around 120 MeV. Since the temperature at freeze-out exceeds 100MeV, the Boltzmann approximation seems reasonable to study RHIC energy region at freeze-out.

3. An ellipticity parameter $e$ is introduced through a geometrical parametrization which can express the non-uniformity of flow in the longitudinal direction. For the central collisions, the nuclear stopping can be studied by the range of rapidity of emission source ($-y_{e0} < y < y_{e0}$) in the center-of-mass system ($y = y_{cm}$).

A parametrization for such a non-uniform distribution can be obtained by using an ellipse like picture on emission angle distribution, as shown in Fig.2, in this scenario, the emission angle is:

$$\vartheta = \tan^{-1}(e \tan \Theta)$$ (1)
Here, the induced parameter $e(0 \leq e \leq 1)$ represents the ellipticity of the introduced ellipse which describes the non-uniform of fire-ball distribution in the longitudinal distribution. The detailed discussions of the NUFM was given at the Ref.[8]. The rapidity distribution of NUFM is:

$$\frac{dn_{NUFM}}{dy} = eKm^2T \int_{\kappa_{\min}}^{\kappa_{\max}} \frac{Q(\vartheta)d\vartheta}{\sin(\vartheta)} \left(1 + 2\Gamma + 2\Gamma^2\right)e^{-1/\Gamma}$$

(2)

Here $K$ is a constant, $\kappa_{\min} = 2\tan^{-1}(e^{-y_e0})$, $\kappa_{\max} = 2\tan^{-1}(e^{y_e0})$, $y_{e0}$ is the rapidity limit which confines the rapidity interval of longitudinal flow. In Eq.2,

$$\Gamma = \frac{T}{m \cosh(y - y_e)}$$

(3)

$$Q(\vartheta) = \frac{1}{\sqrt{1 + \tan^2\kappa} \left| \cos \vartheta \right|}$$

(4)

Here $\kappa = 2\tan^{-1}(e^{-y_e})$, $y_e$ is the rapidity of collective flow, $m$ is the mass of produced particle, $T$ is the temperature parameter. The physical meanings of parameters $e$ and $y_{e0}$ will be discussed in the paper. Changing the integration variable in Eq.2 back to $y_e$, the rapidity distribution can be rewritten as follows:

$$\frac{dn_{NUFM}}{dy} = eKm^2T \int_{y_{e0}}^{y_{e0}} \rho(y_e)dy_e(1 + 2\Gamma + 2\Gamma^2)e^{-1/\Gamma}$$

(5)

Here $\rho(y_e) = \sqrt{\frac{1 + \sinh^2(y_e)}{1 + e^2\sinh^2(y_e)}}$ is the flow distribution of longitudinal flow, $e$ is the parameter $(0 \leq e \leq 1)$ represents the ellipticity of the introduced ellipse which describes the non-uniform of fire-ball distribution in the longitudinal direction, as sketched in Fig.2. It can be seen from Fig.3 that the larger is the parameter $e$, the flatter is the distribution of $\rho(y_e)$ and the more uniform is the longitudinal flow distribution. When $e \to 1$, the longitudinal flow distribution is completely uniform $\rho(y_e) \to 1$.

III Comparison with experiments

A. Rapidity distributions of protons at AGS

In this part, we will discuss the mid-rapidity baryon productions at AGS. This is because rapidity distributions are strongly affected by “memories of the pre-collision state”: Whereas all transverse momentum are generated by the collision itself, a largely unknown fraction of the beam-component of the momenta of the produced hadrons is due to the initial longitudinal motion of the colliding nuclei. In hydrodynamics one
finds that final rapidity distributions are very sensitive to the initialization along the beam direction, and that hydrodynamics evolution is not very efficient in changing the initial distributions [11]. The best way to isolate oneself longitudinally from remnants of the initially colliding nuclei is by going as far away as possible from the projectile and target rapidities, i.e. by studying mid-rapidity hadron production.

The rapidity distributions of proton for beam kinetic energies of 6, 8, and 10.8 GeV/n Au + Au collisions by E917 [3,12,13] are given in Fig.4. The dotted and solid lines correspond to the results from isotropic thermal model and nonuniform longitudinal flow model (NUFM), respectively. The rapidity limit $y_{e0}$ and ellipticity $e$ at different collision energies used in Table I for comparison. The parameter $T$ is chosen to be 0.12 GeV.

Table I  The value of model-parameters at AGS

| Parameter | Au-Au Collisions energy GeV/n |
|-----------|-------------------------------|
|           | 6 | 8  | 10.8 |
| $e$       | 0.901 | 0.852 | 0.802 |
| $y_{e0}$  | 1.213 | 1.263 | 1.312 |

Fig.4 shows the $dN/dy$ distributions for the most central event classes at all three energies plotted as open circle points of experimental data. The dotted curves show the expected distribution for completed stopping i.e., isotropic emission protons with a Boltzmann energy distribution. It completely fails for the $dN/dy$ distributions. The measured rapidity distributions at all three beam energies suggested that the degree of stopping in central collisions is not completely stopping at the AGS energy region. And the non-uniformity parameter $e < 1$ also suggests that the collective flow in the longitudinal direction at AGS are not completely uniform.

In Ref.8, we compared the parameter values for Si+Al (smaller colliding nuclei) and Au+Au (larger colliding nuclei) collisions listed in Table I of Ref.8. It shown that there was stronger nuclear stopping in the collision of larger nuclei. Here we compare with different collision energies with same collision system. It can be shown from Table.I that the rapidity limit $y_{e0}$ is bigger and the ellipticity $e$ for is smaller for higher collision energy at AGS, which means that decreasing nuclear stopping and uniformity in the longitudinal direction with increasing incident energy.

B. Rapidity distributions of net-proton at different regions

Fig.5 shows net-proton $dN/dy$ measured at AGS[3,14,15], SPS [16] and RHIC [5] from the top 5% central collisions. The real triangle, open square and open circle points are
for the AGS, SPS and RHIC experiment results, respectively. The solid lines are our NUFM calculation results. It can be seen from the Fig.5 that NUFM model reproduces the central dip of the rapidity distribution of the proton at SPS and RHIC in agreement with the experimental findings. Note that the appearance or disappearance of central dip is insensitive to the rapidity limit $y_{eo}$, but depends strongly on the magnitude of the ellipticity $e$ for proton distribution. From that we can see that the central dip strongly relates with the non-uniformity of longitudinal flow distributions. On the other hand, The second parameter in NUFM $y_{eo}$ determines the width of the rapidity distributions. The larger of the distribution limit of emission source $y_{eo}$, the wider of the rapidity distributions. For SPS and RHIC $y_{eo}$ are approximately equal to the half width of two peak distribution. In the sense, the parameter $y_{eo}$ can represent the nuclear stopping power. The parameter of $T$ is chosen to be 0.12 GeV.

The distributions show a strong energy dependence: at AGS, the net protons distribution has a peak at mid-rapidity, the distribution at AGS is narrower than the other two energies, and $e = 0.7327$ which shows no dip at the central rapidity, the collective flow is approximately uniform and the particle distribution shows a peak at the central of rapidity. While at SPS a dip is observed in the middle of rapidity of the distribution. As comparison with AGS, as the decreasing of $e$, the collective flow at $y \sim 0$ is diluted at SPS. At RHIC a broad minimum (dip) has developed spanning several units of rapidity, indicating that at RHIC energies collisions are quite transparent. According to the study of ellipticity $e$, $e = 0.16$ at RHIC gives a maximum non-uniform flow distribution.

Table II  The comparison of value of model-parameters of net proton distributions at AGS, SPS and RHIC

| energy region | Parameter | $e$     | $y_{eo}$ | $<\delta y>$ | $\chi^2/N$ |
|---------------|-----------|---------|----------|-------------|-------------|
| AGS           | $e$       | 0.7237  | 1.104    | 1.0         | 5.469       |
| SPS           | $y_{eo}$  | 0.6535  | 2.105    | 2.0         | 3.003       |
| RHIC          | $y_{eo}$  | 0.16    | 2.458    | 2.2 ± 0.4   | 4.134       |

Therefore, the two parameters of $e$ and $y_{eo}$ introduced by NUFM have their own physical meanings. $e$ is a physical quantum that represents the non-uniformity of longitudinally collective flow. From Table II, we know that the ellipticity $e$ decreases with increasing energy from AGS to RHIC. $y_{eo}$ represents the rapidity limit of emission source which correspond to the nuclear stopping power, the larger of the value of $y_{eo}$ the smaller of the nuclear stopping power. As shown in Fig.5 and Table II, we can understand that at RHIC energies collisions are quite transparent and the flow distribution at longitudinal direction is more nonuniform.
C. Rapidity distributions of proton, anti-proton and net-proton at RHIC

The energy loss of colliding nuclei is a fundamental quantity determining the energy available for particle production (excitation) in heavy-ion collisions. Because the baryon number is conserved, the rapidity distributions are only slightly affected by re-scattering in later stage of the collision, the measured net-baryon \((B - \overline{B})\) distribution retains information about the energy loss and allows the degree of nuclear stopping to be determined.

It can be seen from the Fig.7 that the NUFM model reproduces the central dip of the rapidity distribution of net-protons, given by BRAHMS Collaboration [5] working at RHIC.

Fig.7 shows the resulting rapidity densities \(dN/dy\) as a function of rapidity. The most prominent feature of the distributions is that proton and anti-proton show platter distributions, and decrease at rapidities away from mid-rapidity. But the net-proton distribution is absolutely different from that of the \(p\) and \(\overline{p}\), it shows a dip at the central of rapidity \(y \approx 0\). From the distribution of \(\overline{p}\), we can study the pair production of \(p - \overline{p}\), which distributes nearly uniformly in the longitudinal direction.

IV. Summary and Conclusions

In high energy heavy ion collisions, there will have a large amount energy lose in relativistic heavy ion collisions, and deposit at region of central rapidity. Soft hadronic observances measure directly the final "freeze-out" stage of the collision, when hadrons decouple from the bulk and free-stream to the detectors. Freeze-out may correspond to a complex configuration in the combined coordinate-momentum space, with collective components (called flow) generating space-momentum correlations, as well as geometrical and dynamical anisotropic. A detailed experimental experimental-driven understanding of the freeze-out configuration is the crucial first step in understanding the system’s prior evolution and the physics of hot colored matter. The isotropic thermal model and cylindrical-symmetric longitudinal flow model have discussed the collective flow feature, respectively. But they cannot exhibit the central dip distribution especially at higher collision energy when the nuclear have more transparency. Non-uniform Flow Model (NUFM) can not only discuss the collective flow distribution feature, but also express the dip distribution.

In the paper we argue that the transparency/stopping of relativistic heavy-ion collisions should be taken into account more carefully. we assumes that the centers of the fireballs are distributed non-uniformly in the longitudinal phase space. A ellipticity \(e\) is introduced through a geometrical parametrization which can express the...
non-uniformity of flow in the longitudinal direction, i.e., the centers of fireballs of proton and net-proton prefer to accumulate in the two extreme rapidity regions \( y \approx \pm y_{e0} \) in the c.m.s. frame of relativistic heavy ion collisions, and accordingly the distribution is diluted in the central rapidity region \( (y \approx 0) \). It is found that the depth of the central dip depends on the magnitude of the parameter \( e \), i.e., the non-uniformity of the longitudinal flow is describes the depth of the central dip for proton, anti-proton and net-proton.

Comparing with our early work [8], we analyze and study the features of collective flow and nuclear stopping at RHIC, SPS and AGS energy regions are given in this paper, respectively. Through comparing the feature of collision energy regions, it is found that the uniformity of collective flow in the longitudinal decrease with increasing collision energy, and much more transparent with increasing incident energy. The feature of net-proton distribution at RHIC is completely different from that of at AGS. Qualitatively speaking, the collision of high-energy heavy-ions can be divided into two different energy regions: the "baryon-free quark-gluon plasma region" (or the pure quark-gluon plasma region) with \( \sqrt{s} \geq 100 \text{ GeV per nucleon} \), and the baryon rich plasma region (or the stopping region) with \( \sqrt{s} \approx 5 \sim 10 \text{ GeV per nucleon} \), which corresponds to about many tens of GeV per projectile nucleon in the laboratory system. In the baryon-free quark gluon plasma region, we need to know the nuclear stopping power to determine whether the beam baryons and the target baryons will recede away from the center of mass without being completely stopped, leaving behind quark-gluon plasma with very little baryon content. In the baryon-rich region or the stopping region, the nuclear stopping power determines whether the colliding baryons will be stopped in the center-of-mass system and pile up to form a quark-gluon plasma with a large baryon density.

This work was supported in part by the Major Science Foundation of Education Department of Hubei Province of China Grant No. 2003Z002 and China Three Gorges University for Key Subjects Grant No. 2003C02.

References

[1] L.Ahle et al., Nucl.Phys. A590, 249c(1995).

[2] L.Ahle et al., Nucl.Phys. A610, 139c(1996).

[3] B.B.Back et al., Phys. Rev. Lett. 86, 1970(2001).

[4] W.Busza and A.S.Goldhaber, Phys. Lett. B 139, 235(1984).
[5] I.G. Bearden et al. (BRAHMS Collaboration), Phys. Rev. Lett. 93, 102301 (2004).

[6] S. A. Bass, B Müller, and D. K. Srivastava, Phys. Rev. Lett. 91, 052302 (2003).

[7] E. Schnedermann and U. Heinz, Phys. Rev. C 50, 1675 (1994); P. Braun-Munzinger, J. Stachel, J. P. Wessels, and N. Xu, Phys. Lett. B 344, 43 (1995).

[8] S. Q. Feng, L. Feng, and L. S. Liu, Phys. Rev. C 63, 014901 (2000).

[9] J. D. Bjorken, Phys. Rev. D 27, 140 (1983).

[10] X. Cai, S. Q. Feng, Y. D. Li, C. B. Yang, and D. C. Zhou, Phys. Rev. C 51, 3336 (1995).

[11] K. J. Eskola, K. Kajantie, and P. V. Ruuskanen, Euro. Phys. J. C 1, 627 (1998).

[12] L. Ahle et al., Phys. Rev. C 58, 3523 (1998).

[13] T. Abbott et al., Nucl. Instrum. Methods Phys. Res. Sect. A 290, 41 (1990).

[14] L. Ahle et al., Phys. Rev. C 60, 064901 (1999).

[15] J. Barrette et al., Phys. Rev. C 62, 024901 (2000).

[16] H. Appelshauser et al., Phys. Rev. Lett. 82, 2471 (1999).
Figure captions

Fig.1 Schematic sketch of the distribution of fire-balls in the non-uniform flow model (NUFM).

Fig.2 Schematic sketch of the emission angle $\vartheta$ in the nonuniform flow model (NUFM).

Fig.3 The distributions $\rho(y_e)$ of the center of fire-balls as a function of $y_e$. When $e \to 1$, the non-uniform distribution turns to the uniform distribution $\rho(y_e) \to 1$.

Fig. 4(a, b, c) are the rapidity distributions for central 10.8, 8 and 6 GeV/n Au+Au collisions at AGS, respectively. Open circles are the experimental data for Au+Au collisions taken from Ref. [3]. dotted and solid lines are the distributions from the isotropic thermal model, and non-uniform longitudinal flow model (NUFM) respectively. The temperature $T = 0.12$ GeV.

Fig.5 Rapidity distributions of net-proton in central collisions at AGS, SPS and RHIC. The experimental data are taken from Ref.[5]. The solid line is our calculation using the NUFM model. The temperature $T = 0.12$ GeV.

Fig.6 The fire-ball distribution functions $\rho(y_e)$ verus rapidity $y_e$ with the non-uniform flow model (NUFM) at AGS , SPS and RHIC three different energy regions.

Fig.7 Rapidity distributions of proton, anti-proton and net-proton for central GeV/n Au+Au collisions at RHIC. dotted line, dashed line and solid line are the calculation results using NUFM for proton, anti-proton and net-proton distributions, respectively. The data are taken from Ref.[5]. (here, for proton: $e = 0.9951$, $y_{eo} = 3.2$; for anti-proton: $e = 0.9802$, $y_{eo} = 2.4$; for net-proton: $e = 0.16$, $y_{eo} = 2.458$.)
Fig. 3
Fig. 4
Fig. 5
