Jet broadening in unstable non-Abelian plasmas

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We perform numerical simulations of the SU(2) Boltzmann-Vlasov equation including both hard elastic particle collisions and soft interactions mediated by classical Yang-Mills fields. Using this technique we calculate the momentum-space broadening of high-energy jets in real-time for both locally isotropic and anisotropic plasmas. In both cases we introduce a separation scale which separates hard and soft interactions and demonstrate that our results for jet broadening are independent of the precise separation scale chosen. For an isotropic plasma this allows us to calculate the jet transport coefficient \( \hat{q} \) including hard and soft non-equilibrium dynamics. For an anisotropic plasma the jet transport coefficient becomes a tensor with \( \hat{q}_{\perp} \neq \hat{q}_{\parallel} \). We find that for weakly-coupled anisotropic plasmas the fields develop unstable modes, forming configurations where \( B_\perp > E_\perp \) and \( E_z > B_z \) which lead to \( \hat{q}_{\perp} > \hat{q}_{\parallel} \). We study whether the effect is strong enough to explain the experimental observation that high-energy jets traversing the plasma perpendicular to the beam axis experience much stronger broadening in rapidity, \( \Delta \eta \), than in azimuth, \( \Delta \phi \).

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INTRODUCTION

High transverse momentum jets produced in heavy-ion collisions represent a valuable tool for studies of the properties of the hot parton plasma produced in the central rapidity region \([1]\). This is due to the fact that jets couple to the plasma causing the jet to broaden in momentum space and to lose energy. The magnitude of momentum-space broadening and energy loss experienced by a parton depends on whether or not one assumes the matter to be hadronic or partonic in nature. Hence, it is one of the primary observables to ascertain experimentally whether or not the plasma has been produced. At very high energies it is expected that hard bremsstrahlung processes dominate the light quark or gluon energy loss \([2]\); however, at intermediate energies the inclusion of both collisional and radiative processes is necessary in order to make phenomenological predictions. Here we present first results from real-time solution of the SU(2) Boltzmann-Vlasov equation for locally isotropic and anisotropic plasmas which include both hard (scattering) and soft (classical field) processes.

We first demonstrate that in a locally isotropic plasma, one can obtain a cutoff-independent transport coefficient \( \hat{q} \), which measures the square of the transverse momentum transfer per mean free path. This measurement lays the groundwork for determining other jet transport properties like the energy-loss spectrum from real-time simulations. We then perform a similar measurement in a locally anisotropic plasma and demonstrate that for a non-Abelian plasma unstable modes can cause asymmetric broadening of jets. This may be relevant for recent measurements of di-hadron correlations which provide evidence for an asymmetric broadening of jet profiles in the plane of pseudorapidity (\( \eta \)) and azimuthal angle (\( \phi \)), with \( \Delta \eta > \Delta \phi \), which has been called “the ridge” by experimentalists \([1,3]\). Here we will show that this asymmetry could partly be caused by unstable plasma modes which are induced by the longitudinal expansion of the plasma.

At the earliest times after an ultrarelativistic heavy-ion collision, before thermalization and hydrodynamic expansion, the plasma undergoes rapid longitudinal expansion. This expansion can lead to an oblate anisotropic \( \langle p_T^2 \rangle \ll \langle p_z^2 \rangle \) momentum distribution in the local rest frame \([4,5]\). It has been shown that instabilities develop \([6]\) in such anisotropic plasmas which lead to the formation of long-wavelength chromo-magnetic and chromo-electric fields. These fields can then affect the propagation of hard jets and their induced hard radiation field. In the Abelian case soft transverse magnetic fields, \( B_\perp \), dominate all other field components and therefore in this case one expects the longitudinal pressure induced by unstables modes to be larger than the transverse pressure \([6,8]\). This pressure asymmetry causes asymmetric broadening of jets with larger broadening along the longitudinal or \( \eta \) direction.

The situation is more complicated in non-Abelian plasmas since then, in addition to generating large coherent \( B_\perp \) and \( E_\perp \) domains, one also generates large-amplitude longitudinal fields \( B_z \) and \( E_z \). It is therefore not obvious a priori that the pressure generated by other field components will result in an asymmetric broadening of the jet. Through our numerical simulations we find that for oblate parton momentum distributions, that at different times either \( E_z > B_z \) or \( B_\perp > E_\perp \) with the net effect being a factor of 1.5 stronger longitudinal than transverse
where $\gamma$-Mills equation for the soft gluon fields, with generators \(v\). Parametrically small, \(q\) for the appropriate for the SU(2) gauge group. All particles, which leads to Wong’s equations \(9\). The value is fixed by the length of the lattice \(L\), given by \(\sqrt{k^*} a\). Since \(g\sim 1\), in practice, we choose \(k^* = \sqrt{3}\pi/a\) to be on the order of the temperature for isotropic systems, and on the order of the hard transverse momentum scale for anisotropic plasmas. Independently, one should have \(m_\infty L \gg 1\) and \(m_\infty a \ll 1\): the first condition ensures that the relevant soft modes actually fit on the lattice while the latter corresponds to the continuum limit. Here, \(m_\infty\) denotes the soft scale and is given by \(m_\infty = g a N_c \int \frac{d^3p}{(2\pi)^3} \frac{f(p)}{|p|} \sim g^2 N_c n_2 p_h\).
As we have argued above, we shall choose the inverse lattice spacing to be on the order of the temperature of the medium. Thus, with \( m_{\infty}a \ll 1 \) roughly translates into

\[
g^2 N_c \frac{n_g}{T^3} \ll 1 .
\]  \( \text{(11)} \)

In order to satisfy this relation with \( g \sim 1 \), in our numerical simulations below we shall assume an extremely hot medium, \( T^3 \gg n_g \). However, this should be viewed simply as a numerical procedure which ensures that the simulations are carried out near the continuum (or weak-coupling) limit. We shall verify below that transverse momentum broadening of a high-energy jet passing through a thermal medium is independent of \( T \) if the density and the ratio of jet momentum to temperature is fixed; compare to Eq. \( \text{(13)} \) below. One may therefore obtain a useful “weak-coupling” estimate of \( \langle p^2 \rangle \) (resp. for the related transport coefficient \( \hat{q} \)) by extrapolating our measurements down to realistic temperatures.

**JET BROADENING IN AN ISOTROPIC PLASMA**

We first consider a heat-bath of particles with a density of \( n_g = 10/\text{fm}^3 \) and an average particle momentum of \( 3T = 12 \text{ GeV} \). The rather extreme “temperature” is chosen to satisfy the above conditions on \( N_c = 32 \cdots 128 \) lattices, assuming \( L = 15 \text{ fm} \). For a given lattice (resp. \( k^* \)) we take the initial energy density of the thermalized fields to be

\[
\int \frac{d^3 k}{(2\pi)^3} k^* f_{\text{Bose}}(k^*) \Theta(k^* - k) ,
\]  \( \text{(12)} \)

where \( f_{\text{Bose}}(k) = n_g/(2T^3\zeta(3))/(e^{k/T} - 1) \) is a Bose distribution normalized to the assumed particle density \( n_g \), and \( \zeta \) is the Riemann zeta function. This is equivalent to the energy density of Bose-distributed particles with momenta below the separation momentum \( k^* \). The initial spectrum is fixed to Coulomb gauge and \( A_t \sim 1/k \) (in continuum notation); also, for simplicity we set \( E_i = 0 \) at the initial time but electric fields build up quickly within just a few time steps.

We then measure the momentum broadening \( \langle p^2 \rangle(t) \) of high-energy test particles \( \langle p/3T \approx 5 \rangle \) passing through this medium. Fig. \( \text{1} \) shows that in the collisionless case, \( C = 0 \), the broadening is stronger on larger lattices, which accommodate harder field modes. However, Fig. \( \text{2} \) demonstrates that collisions with momentum exchange larger than \( k^*(a) \) compensate for this growth and lead to approximately lattice-spacing independent results even when \( k^* \) varies by a factor of four.

Figs. \( \text{1} \) and \( \text{2} \) show that the relative contributions to \( \langle p^2 \rangle \) from soft and hard exchanges can depend significantly on \( k^* \), even for \( p/k^* = O(10) \). It is clear, therefore, that transport coefficients obtained in the leading logarithmic (LL) approximation from the pure Boltzmann approach (without soft fields) will be rather sensitive to the infrared cutoff \( k^* \). Fitting the difference of Fig. \( \text{2} \) and Fig. \( \text{1} \) (i.e., the hard contribution) to the LL formula

\[
\frac{d\langle p^2 \rangle}{dt} = \frac{C_A g^4}{C_F 8\pi} n_g \log \left( \frac{C^2 p^2}{k^* g^2} \right) ,
\]  \( \text{(13)} \)

gives \( C \approx 0.43, 0.41, 0.31 \) for \( k^*/T = 2\sqrt{3}, \sqrt{3}, 0.5\sqrt{3} \), respectively. For the full calculation \( C \approx 0.61 k^*/(\sqrt{3}T) \).

A related and frequently used transport coefficient is \( \hat{q} \). It is the typical momentum transfer (squared) per collision divided by the mean-free path, which is nothing but \( \langle p^2 \rangle(t)/t \). From Fig. \( \text{2} \) \( \hat{q} \approx 2.2 \text{ GeV}^2/\text{fm} \) for \( N_c = 2 \), \( n_g = 10/\text{fm}^3 \) and \( p/(3T) \approx 5 \). Our cut-off independent value for \( \hat{q} \) is in the range extracted from phenomenological analyses of jet-quenching data from RHIC \( \text{[13]} \).

In Fig. \( \text{3} \) we show that \( \hat{q} \) is indeed largely independent of the separation scale \( k^* \). In these simulations, test parti-
cles were explicitly bunched into colorless jets, such that radiative energy loss does not contribute. This explains why one does not need bremsstrahlung processes in the approach. Here, we investigate their effect on the momentum broadening of jets, including the effect of collisions. The initial momentum distribution for the hard plasma gluons is taken to be

\[
f(p) = n_g \left( \frac{2\pi}{p \delta} \right)^2 \delta(p_x) \exp(-p_L/p_h),
\]

with \( p_L = \sqrt{p_x^2 + p_y^2} \). This represents a quasi-thermal distribution in two dimensions with average momentum = 2 \( p_h \). We initialize small-amplitude fields sampled from a Gaussian distribution and set \( k^* \approx p_h \), for the reasons alluded to above. The band of unstable modes is located below \( k^* \).

We find that binary collisions among hard particles reduce the growth rate of unstable field modes, in agreement with expectations \[10\]. However, for \( p_h = 16 \text{ GeV}, L = 5 \text{ fm}, n_g = 10/\text{fm}^3, g = 2, m_\infty \approx 1/\text{fm} \) and \( k^* \approx 1.7p_h \), the reduction of the growth rate is only approximately 5%, increasing to about 15% when \( k^* \approx 0.9p_h \). This is due to fewer available field modes and more randomizing collisions.

Next, we add additional high momentum particles with \( p_x = 12p_h \) and \( p_x = 6p_h \), respectively, to investigate the broadening in the \( y \) and \( z \) directions via the variances

\[
\hat{q}_L(p_x) := \frac{d}{dt} \langle (\Delta p_L)^2 \rangle, \quad \hat{q}_L(p_x) := \frac{d}{dt} \langle (\Delta p_z)^2 \rangle.
\]

The quantity \( \sqrt{\hat{q}_L/\hat{q}_L} \) can be roughly associated with the ratio of jet correlation widths in azimuth and rapidity: \( \sqrt{\hat{q}_L/\hat{q}_L} \approx (\Delta \eta)/(\Delta \phi) \).

Fig. 5 shows the time evolution of \( \langle p_L^2 \rangle \) and of \( \langle p_z^2 \rangle \). The strong growth of the soft fields sets in at about \( t \approx 10 \text{ fm}^{-1} \) and saturates around \( t \approx 25 \text{ fm}^{-1} \) due to the finite lattice spacing (also see \[12\]). Outside the above time interval the ratio \( \hat{q}_L/\hat{q}_L \approx 1 \). During the period of instability, however,

\[
\hat{q}_L/\hat{q}_L \approx 2.3,
\]

for both jet energies shown in Fig. 5. We find approximately the same ratio for denser plasmas \( (n_g = 20/\text{fm}^3 \) and \( n_g = 40/\text{fm}^3 \)). Reducing the number of lattice sites and scaling \( p_h \) down to 8 GeV gives \( \hat{q}_L/\hat{q}_L \approx 2.1 \). However, these latter runs are rather far from the continuum limit and lattice artifacts are significant \[12\].

The explanation for the larger broadening along the beam axis is as follows. In the Abelian case the insta-
bility generates predominantly transverse magnetic fields which deflect the particles in the z-direction [3].

In the non-Abelian case, however, on three-dimensional lattices transverse magnetic fields are much less dominant (see, e.g. Fig. 5 in [12]) although they do form larger coherent domains in the transverse plane at intermediate times than $E_L$, Fig. 6. Longitudinal fields and locally non-zero Chern-Simons number $\sim \text{tr } E \cdot B$ emerge, also. Nevertheless, Fig. 6 shows that $E_z > B_z$, aside from $B_L > E_L$. Hence, the field configurations are such that particles are deflected preferentially in the longitudinal $z$-direction (to restore isotropy).

A third contribution to $p_z$ broadening in an expanding plasma, not considered explicitly here, is due to a longitudinal collective flow field which “blows” the jet fragments to the side [17]. This mechanism is also available for collision dominated plasmas with (nearly) isotropic momentum distribution. However, rather strong flow gradients seem to be required to reproduce the observed broadening of midrapidity jets (the flow velocity has to vary substantially within the narrow jet cone). In contrast, color fields will naturally deflect particles with lower momentum by larger angles ($\Delta p \sim E, B$): the jet profile broadens even if the induced radiation is exactly collinear. It is therefore important to determine, experimentally, whether the asymmetric broadening is related to the macroscopic collective flow or to an anisotropy of the plasma in the local rest frame. More detailed simulations should account, also, for the fact that small-$x$ gluons are already correlated over large rapidity intervals at the initial time [18].

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