Distribution characteristics of the flow around four flat plates in staggered at different arrangements

P M Guo¹, B Gao¹ and W B Gou¹

¹ School of Energy and Power Engineering, Jiangsu University, Zhenjiang, 212013, China

gaobo@uj.edu.cn.

Abstract. The three-dimensional turbulent flow around plate cascades in staggered is an important factor to induce hydro-mechanical vibration. Taking the upstream three plates in side-by-side configuration and the downstream single plate in tandem configuration to form the overall staggered layout of the cascades as the subject of investigation. Based on the Large-Eddy-Simulations (LES), the forces experienced by the plates, velocity distributions and three-dimensional vortex structures around the plates are analysed in detail. Some experiments were also carried out to validate the accuracy of the numerical results by LDA measurement. The mean velocity distributions from LES results agree fairly well with the experimental results. Results show that the existence of the downstream plate hinders the development of the upstream wake and has a significant inhibitory effect on the forces experienced by the upstream plates; When the gap spacing is smaller, the stream-wise length of the reverse flow region between the upstream and downstream plate becomes larger, and the minimum velocity in this region is lower than other reverse flow regions; The flow around the plate cascades exhibits high three-dimensional characteristics, and the amplitude of the fluctuation of the forces on the downstream plate under the interference of the upstream wakes is increased obviously. The downstream plate is located in the wake of the upstream cascade, and has a steady flow to the upstream wake. Analysis on the flow can provide theoretical support for the flow control in hydraulic machinery.

1. Introduction
Unsteady flow around a bluff body is a classical researching problem, and it involves many fundamental phenomena such as the boundary layer separation, vortex shedding, wake interference and the excitation source identification. The bluff body flow is occurred commonly in different applications and it is of great interest in the engineering field. Analysis and researches on this problem can provide the theoretical support for the engineering designs to ensure the stable and safe operation of the machines.

Many experimental investigations and extensive numerical studies have been done about the flow around the typical single cylinder model by Morkovin[¹], Williamson & Govardhan[²] and Sumer & Fredsøe[³]. Besides, Cylinder-like structures are also presented in groups commonly in the engineering designs such as the bridges cross-sections design, hydrodynamic loading on ship stabilizers, tall buildings and cooling systems in nuclear power fields. The flow structures around the multiple-cylinders are different from the single-cylinder due to the wake interference effects and the vortex streets formation.
Two cylinders are the most common in the researches in multiple-cylinders models. The idealized arrangements of the two cylinders are classified: (i) Tandem configuration; (ii) Side-by-side configuration; (iii) Staggered configuration. Two cylinders in staggered configuration is the most general arrangement. The center-to-center pitch spacing, diameter of the cylinder and the angle of incidence are the three elements influencing the staggered configuration. Gu & Sun[4] researched this topic in first and more intensive experiments were extended by Sumner et al[5]. However, the experiments were limited in the range of $850 \leq Re \leq 1900$. Low subcritical regime flow patterns were classified into: (i) Single bluff body flow region, where the two staggered cylinders behave as if a single bluff body; (ii) Small incidence angle flow patterns, where the shear layer attachment and flow patterns is marked in the larger gap flow; (iii) Large incidence angle flow patterns, where vortex shedding occurs from both cylinders.

Compared with the cylinder, less studied and understood about the flow structures around a rectangular cross-section body. The major differences between the rectangular body and the cylinder are obvious. On the one hand, fixed separation corner exists in the rectangular body. Separated shear layer at the leading edge of the body plays a significant effect on the flow structures around the rectangular body. On the other hand, the vortex formation and separation from the surfaces of the rectangular body influences the forces experienced by the rectangular body. The behavior of the flow around the rectangular body has a great relationship with the ratio $B/D$, where $B$ is the chord length and $D$ is the depth of the body. Tafti & Vanka[6] investigated the flow structures at low Reynolds number. The flow on the upper and lower surface of the flat plate showed a stable state at $Re=150$, 250, 300, while the boundary layer became unstable with the Reynolds number increasing to 1000. The vortex shedding frequency related to the varying $B/D$ and Reynolds number of $70<Re<2\times10^4$ was researched by Okajima[7], who proposed the Strouhal number is 0.13 for $1\times10^4<Re<2\times10^4$ while the ratio $B/D=1$. Summarizing the research finding of the previous scholars, Naudascher & Wang[8] classified the different vortex shedding mechanism into four patterns: (i) leading-edge vortex shedding (LEVS), (ii) impinging leading-edge vortices (ILEV), (iii) trailing-edge vortex shedding (TEVS), (iv) alternate-edge vortex shedding (AEVS).

However, wake patterns, vortex structures and forces characteristics vary greatly about the flow between the single rectangular body and the multiple- rectangular bodies. Shun C. Yen & Jung H. Liu[9] investigated the flow characteristics around two side-by-side identical square cylinders at $2262 \leq Re \leq 28000$. And the flow structures were classified into three modes: (i) single mode, (ii) gap-flow mode, (iii) couple vortex-shedding mode. When the rectangular bodies arranged in tandem, the downstream body was in the wake with a large number of unsteady vortices. Chen & Chiu[10] found the Karmen vortex street was split by the leading edge of the rectangular body, and the “secondary vortex” generated on the surface of the body. What the flow structures will be when the rectangular bodies arranged in staggered. This present investigation aims to show the flow around four flat plates arranged in staggered.

2. Numerical simulation

2.1. Governing equations

In order to study the flow, the incompressible Navier-Stokes equations used in the LES Method are solved:

$$\frac{\partial \bar{u}_i}{\partial x_i} = 0$$  \hspace{1cm} (1)

$$\frac{\partial \bar{u}_i}{\partial t} + \frac{\partial \bar{u}_i \bar{u}_j}{\partial x_j} = -\frac{1}{\rho} \frac{\partial \bar{p}}{\partial x_i} + \nu \left( \frac{\partial^2 \bar{u}_i}{\partial x_j \partial x_j} - \frac{\partial \bar{u}_j}{\partial x_j} \right), \hspace{1cm} (i=1,2,3)$$  \hspace{1cm} (2)

Where $u_i$ is the velocity component in the $i$ direction, $\bar{p}$ is the pressure field; $\nu$ stands for the kinematic viscosity and $\rho$ for the density of the fluid. The influence of the small scales on the large
scales takes place through the SGS stress expressed according to Boussinesq’s assumption in equation (3).

\[ \tau_{ij} = \frac{1}{2} \frac{\partial u_i}{\partial x_j} + \frac{1}{2} \frac{\partial u_j}{\partial x_i} \]

The present study selected the SGS model to close the equations which was proposed by Smagorinsky\(^{11}\), then the \( \tau_{ij} \) is defined in equation (4).

\[ \tau_{ij} - \frac{1}{3} \delta_{ij} \tau_{kk} = -2 \nu_t \bar{S}_{ij} \]

Where \( \delta_{ij} \) is the Kronecker delta, \( \nu_t \) is the SGS kinematic viscosity. And \( \bar{S}_{ij} \) is the strain rate tensor for the resolved scale defined in equation (5).

\[ \bar{S}_{ij} = \frac{1}{2} \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \]

In the Smagorinsky-Lilly model, the sub-grid kinematic viscosity \( \nu_t \) is modeled by

\[ \nu_t = l_i^2 |\bar{S}_{ij}| \]

Where \( l_i \) is the mixing length for the SGS, and \( |\bar{S}| = \sqrt{2S_{ij}S_{ij}} \), \( l_i \) can be computed using

\[ l_i = \min(ky, C_s \Delta^{1/3}) \]

Where \( k \) is the von Karman constant \( (k=0.42) \), \( y \) is the distance to the nearest wall. And the \( \Delta \) is the filter width. The Smagorinsky constant \( C_s=0.1 \) is used in this present work.

2.2. A Computational overview

The computational domain, boundary conditions and arrangements of experimental set-up are presented in figure 1. The computational domain is a rectangular solid with the dimension parameter \( 420 \times 100 \times 20 \text{mm} \). Four plates with round leading edges are arranged in staggered in the computational domain. Three upstream plates are in side-by-side configuration named as plate 1, plate 2 and plate 3 respectively and one downstream plate is in tandem with plate 2 named as plate 4.

Figure 1. Schematic diagram of the computational model and experimental arrangement

The incoming flow is characterized by a \( Re=\frac{uD}{\nu} = 3.0 \times 10^4 \), where \( u \) is the free stream-wise velocity. The outflow boundary condition is used at the outlet boundary. Symmetry boundary conditions are used in the lateral surfaces. And the surfaces of the plates use the no-slip boundary conditions. The momentum equations in the pressure-based solver is set as the second-order accurate central differencing spatial discretization scheme. The time step size is set to \( \Delta t = 5 \times 10^{-5} \text{ s} \) by calculating the equation \( \Delta t = \frac{D}{u} \). This is to guarantee the value of \( \Delta t \) small enough to satisfy the Courant-Friedrichs-Lew (CFL) number in most part of the computational domain.

The computational domain is divided into several blocks to control the grid quality in figure 2. In the near plate surface regions, refined grid solution is adopted to resolve the viscous sub-layer. The
distance of the first node away from the plate surface is $2.5 \times 10^{-3}D$, and the mesh spacing is properly increased at a ratio of 1.04 away from the cylinder surface, which is smaller than 1.1 times used in the literature $^{[12-14]}$. The total grid number is $6.0 \times 10^6$, and the averaged $y^+$ value reaches the range of 0.211~2.14, which meets the LES calculation requirements.

![Computational grid distributions](image)

**Figure 2.** Computational grid distributions

### 3. Experimental verification

Experimental measurements are performed to validate the accuracy of the numerical results. The experiment is carried out in a closed-circuit water tunnel, and the working section is 100×20mm in cross-section with 250mm in length. The Flow Explorer two-dimensional LDA system is used as the measuring equipment in this experiment. Mean stream-wise velocity distributions along the wake centerline and mean stream-wise velocity distributions at different $z$-axis in the near wake are captured to compare with the results from LES.

In experiment, the measuring system is installed in a three-dimensional, computer-controlled traversing mechanism with the 0.01mm movement resolution per step. And hollow glass spheres produced by DANTEC Dynamics are used as tracer particles seeded in the water flow. The maximum number of samples to acquire is 50000, and maximum the acquisition time is set to 10s. This is in order to ensure the lowest data rate was maintained at 1000Hz and the validation was kept over 90%.

The mean stream-wise velocity distributions in the wake are shown in figure 3. The length and velocity are normalized by $D$ and the incoming flow velocity $U_\infty$. Figure 3(a) shows the stream-wise velocity distributions along the wake centerline of plate2. Two reverse flow regions can be observed both by the LES and LDA. The velocity distributions from LES in the second reverse flow region agree well with the experimental results. Although there are some differences of the results from LES and LDA in the first reverse flow region, the lowest velocity in the region is consistent well.

Figure 3(b) shows the stream-wise velocity distributions at a $z$-direction in the near wake of the plate 4. Due the special arrangement of plates, three reverse flow regions existed in the wake. The stream-wise length of these reverse flow regions and the velocity distributions in the three reverse flow regions coincide well between the results from LES and LDA. In conclusion, the current adopted numerical methods are appreciate and have enough accuracy to predict the flow structures around the plates.
4. Results and discussions

4.1. Drag and lift forces experienced by the plates at different gap spacing $n$

The time-domain signals of the drag and lift forces experienced by plates at different gaps are presented in figure 4.

The mean coefficient of the drag and lift force on the plates are defined as:

$$C_d = \frac{F_x}{0.5 \rho U_\infty^2 A}$$  \hfill (8)

$$C_l = \frac{F_y}{0.5 \rho U_\infty^2 A}$$  \hfill (9)

Where $F_x$ is the force acting on the plate in the stream-wise direction. Where $F_y$ is the force acting on the plate in the cross-stream direction.

In view of the initial conditions, the first 50 non-dimensional time units are discarded. Due to the different positions of plates in the flow field, the fluctuations of the force coefficients experienced by different plates are quite different.

The lift coefficient of all the plates exhibits periodicity at the gap spacing equals to 20mm. It is clearly to observe that the mean lift coefficient for plate 2 and plate 4 is 0, while the value for plate 1 is negative and for plate 3 is positive.

The differences of the lift coefficient between the plates also can be observed by comparing the lift and drag coefficient on the same plate. For plate 1 and plate 2, the mean drag coefficient is significantly larger than lift coefficient, while is less than for plate 3. And the time signals curves of the lift coefficient and the drag coefficient for plate 4 are interweaved. This may be related to the location of the plate in the flow field, and the flow around the plates will be discussed in the following.

When the gap spacing increases to 40mm, the amplitude of the fluctuation of the force for all the plates is obviously increased. And the maximum fluctuation of lift coefficient is plate 4. This is due to the plate 4 is located on the wake of upstream plates, and with the gap spacing increasing, the vortex structures shedding from the upstream plates developed fully.

Najjar and Balachandar\cite{[15]} defined the regime H and regime L to represent the high and low drag regimes, respectively. As shown, the flow patterns switching between the two regimes is also more obvious at large spacing. In conclusion, at a certain spacing, the presence of the downstream plate has an inhibitory effect on the force of the upstream plate, and the effect is closely related to the gap spacing.
In order to analyse the influence of the existence of downstream plate on the vortex shedding frequency, the spectrum analysis of the lift force is carried out by FFT shown in figure 5. The peak in the spectrum corresponds to the vortex shedding frequency.

At the gap spacing is 20mm, plate 1 shares the same frequency as plate 3, which is 152.207Hz. The vortex shedding frequency of plate 2 is slightly lower, and the frequency is the lowest of plate 4. Furthermore, the magnitude of the lift coefficient at the main frequency experienced by the plate 2 is the lowest when compared with others. Therefore, it is concluded that the presence of the downstream plate 4 inhibits the vortex shedding from the upstream plate 2.

When the gap spacing increases to 40mm, the domain frequency in four spectra is the same, which is 141.773Hz marked in the pictures. It shows that the existence of the downstream plate has no influence on the vortex shedding from upstream plates. And the magnitude of the lift coefficient in spectrum experienced by plate 4 is obviously increased due to the full development of upstream vortex structures.

Figure 5. Spectrum analysis of the lift coefficient

4.2. Mean stream-wise velocity distributions
Distributions of the time-averaged stream-wise velocity along the wake centerline of the plates are shown in figure 6.

The velocity distributions in figure 6(a) are the plates arranged at the gap spacing of 20mm. The flow direction is from left to right. It can be clearly captured that there is a reverse flow region behind each plate. The size of the reverse flow region is described as a region in which \( u/U_0 < 0 \) along the wake centerline is defined.

The velocity distribution in the reverse flow region of plate1 and plate 3 is almost the same. While because of the existence of the downstream plate 4, the stream-size length of reverse flow region of plate 2 is obviously larger than that of plate1 and plate 3. And the minimum flow velocity in the reverse flow region is also the lowest of plate 2. On the contrary, the minimum flow velocity is the highest in the reverse flow region of plate 4.
As the gap spacing increase to 40mm in figure 6(b), the velocity distribution curve along the wake centerline is almost coincident with each other. This means when the gap spacing increases to 40mm, the downstream plate has little influence on the velocity distribution of the upstream plate.

![Figure 6. Mean velocity distributions of the flow direction component on plate centreline](image)

In order to analyse the velocity distributions in the reverse flow region, the stream-wise velocity at different locations in the near wake are shown in figure 7. The selection of the location is in the middle of the gap and at the same distance which is 0.5D away from the upstream plate trailing edge and the downstream plate leading edge.

Three reverse flow regions are observed clearly from the velocity curves, and the velocity recovery speed differs a lot in these reverse flow regions. When the gap spacing equals to 20mm, at \(-3 < z/D < -2\) and \(2 < z/D < 3\), the velocity increases gradually away from the edge of the upstream plate. The minimum velocity is negative in the location \(x/D=4.25\) while is positive in the location \(x/D=5.75\). At \(-1 < z/D < 1\), the minimum velocity is negative in the location \(x/D=5.75\) while is almost coincident in the location \(x/D=4.25\) and \(x/D=5\). And the minimum velocity all is negative in these three locations which means the reverse flow region covers the whole gap between the plate 2 and plate 4.

While the gap spacing increases to 40mm, in all three reverse flow regions, the velocity increases gradually away from the edge of the upstream plate. It can be captured clearly that the minimum velocity in the location \(x/D=4.25\) in all three reverse flow regions is negative while is positive in the location \(x/D=8.25\).

![Figure 7. Mean stream-wise velocity distributions in the near wake of the downstream plate at different z locations](image)

4.3. Flow structures around the plates
The three-dimensional vortex structures and two-dimensional instantaneous streamlines around the plates are shown in figure 8. The wake exhibits high three-dimensional characteristics, and there is a high vorticity region in the near wake of plates.

![Figure 8. Flow structures around the plates](image)
At \( Re=3\times10^4 \), the hairpin vortex structures exist on the lower surface of plate 1 and upper surface of plate 3. While due to the upstream three plates arranged in side-by-side, there are no vortex structures existing on the upper and lower surfaces of the plate 2. And the vortex structures also can be captured clearly in the streamlines. This phenomenon is similar to the “single vortex street regime” in circular cylinders arranged in side-by-side. The vortex structures at the gap between the plates are suppressed under the spacing of the cascades is small enough.

Vortices shed at the trailing edge of the upstream plates, and then go down with the main flow, eventually roll up into von Karman vortex street. At the gap spacing equals to 20mm, it can be seen from the instantaneous streamlines that small vortex structures are filled fully in the gap region between plate 2 and plate 4. And no obviously vortex street is observed. While in the wake of plate 1 and plate 3, there has been performed a vortex street.

As the gap spacing increases to 40mm, three vortex streets are preformed behind the upstream three plates, and there are no differences between the wake of the three upstream plates. But for the downstream plate, the downstream plate is located in the interference of the vortex streets of upstream plates.

![Three-dimensional vortex structures and two-dimensional streamlines](image)

**Figure 8.** Three-dimensional vortex structures and two-dimensional streamlines

5. **Conclusions**

The downstream plate at a certain position in the wake will prevent the development of the vortex structures from upstream plates. At a smaller gap, the magnitudes of lift and drag coefficient experienced by upstream plates decrease obviously and the vortex shedding frequency reduces. At a larger gap, flow patterns switching between regime H and regime L is more obvious and the flow around the plates shares the same frequency. The presence of the downstream plates has a steady flow effect to the upstream wake.

The stream-wise length of the reverse flow region of upstream plate is obviously increased due to the existence of the downstream plate at a smaller gap spacing. And the minimum velocity in this region is lower than other reverse flow regions.
The structures of the flow around the plates exhibit a high degree of three-dimensional characteristics. Mature and staggered arrangement of vortex structures develop on the upper and lower surfaces of the plate. At a smaller gap, small vortex structures are filled fully in the gap region, while at a larger gap, the vortex street is formed.

Acknowledgment
The authors gratefully acknowledge the financial support of Natural Science Foundation of China (51576090).

References
[1] Morkovin M V 1964 Flow around circular cylinder-a kaleidoscope of challenging fluid phenomena Asme Symposium on Fully Separated Flows
[2] Williamson C H K and Govardhan R 2004 Vortex-induced vibrations Fluid Mechanics 36 413–455
[3] Sumer B M and Fredsøe J 1997 Hydrodynamics around Cylindrical Structures Coastal Engineering 33 69–69
[4] Gu Z and Sun T 1999 On interference between two circular cylinders in staggered arrangement at high subcritical Reynolds numbers Journal of Wind Engineering and Industrial Aerodynamics 80 287–309
[5] Sumner D, Price S J and Paidoussis M P 2000 Flow-pattern identification for two staggered circular cylinders in cross-flow Journal of Fluid Mechanics 411 263–303
[6] Tafti D K and Vanka S P 1991 A numerical study of flow separation and reattachment on a blunt plate Physics of Fluids A 3 2887-2909
[7] Okajima A 2006 Strouhal numbers of rectangular cylinders Journal of Fluid Mechanics 123 379–398
[8] Naudascher E and Wang Y 1993 Flow-induced vibrations of prismatic bodies and grids of prisms Journal of Fluids and Structures 7 341–373
[9] Yen S C and Liu J H 2011 Wake flow behind two side-by-side square cylinders International Journal of Heat and Fluid Flow 32 41-51
[10] Fang F M, Chen J C and Hong Y T 2001 Experimental and analytical evaluation of flow in a square-to-square wind tunnel contraction Journal of Wind Engineering and Industrial Aerodynamics 89 247-262
[11] Smagorinsky J 1963 General circulation experiments with the primitive equations Monthly Weather Review 91 99–164
[12] Berrone S, Garbero V and Marro M 2011 Numerical simulation of low-Reynolds number flows past rectangular cylinders based on adaptive finite element and finite volume methods Computers and Fluids 40 92–112
[13] Lam K and Lin Y F 2009 Effects of wave length and amplitude of a wavy cylinder incross-flow at low Reynolds numbers Journal of Fluid Mechanics 620 195–220
[14] Lam K, Lin Y F, Zou L and Liu Y 2010 Investigation of turbulent flow past a yawed wavy cylinder Journal of Fluids and Structures 26 1078–1097
[15] Najjar F M 1998 Low-frequency unsteadiness in the wake of anormal flat plate Journal of Fluid Mechanics 370 101–147