On $\gamma$-Regular-Open Sets and $\gamma$-Closed Spaces

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Abstract. The purpose of this paper is to continue studying the properties of $\gamma$-regular open sets introduced and explored in [6]. The concept of $\gamma$-closed spaces have also been defined and discussed.

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1 Introduction

The concept of operation $\gamma$ was initiated by S. Kasahara [7]. He also introduced $\gamma$-closed graph of a function. Using this operation, H. Ogata [8] introduced the concept of $\gamma$-open sets and investigated the related topological properties of the associated topology $\tau_\gamma$ and $\tau$. He further investigated general operator approaches of close graph of mappings.

Further S. Hussain and B. Ahmad [1-6] continued studying the properties of $\gamma$-open(closed) sets and generalized many classical notions in their work. The purpose of this paper is to continue studying the properties of $\gamma$-regular open sets introduced and explored in [6]. The concept of $\gamma$-closed spaces have also been defined and discussed.

First, we recall some definitions and results used in this paper. Hereafter, we shall write a space
in place of a topological space.

2 Preliminaries

Throughout the present paper, X denotes topological spaces.

Definition [7]. An operation \( \gamma : \tau \to P(X) \) is a function from \( \tau \) to the power set of \( X \) such that \( V \subseteq V^\gamma \), for each \( V \in \tau \), where \( V^\gamma \) denotes the value of \( \gamma \) at \( V \). The operations defined by \( \gamma(G) = G \), \( \gamma(G) = \text{cl}(G) \) and \( \gamma(G) = \text{intcl}(G) \) are examples of operation \( \gamma \).

Definition [7]. Let \( A \subseteq X \). A point \( x \in A \) is said to be \( \gamma \)-interior point of \( A \), if there exists an open nbd \( N \) of \( x \) such that \( N^\gamma \subseteq A \) and we denote the set of all such points by \( \text{int}_\gamma(A) \). Thus

\[
\text{int}_\gamma(A) = \{ x \in A : x \in N \in \tau \text{ and } N^\gamma \subseteq A \} \subseteq A.
\]

Note that \( A \) is \( \gamma \)-open [8] iff \( A = \text{int}_\gamma(A) \). A set \( A \) is called \( \gamma \)-closed [1] iff \( X-A \) is \( \gamma \)-open.

Definition [1]. A point \( x \in X \) is called a \( \gamma \)-closure point of \( A \subseteq X \), if \( U^\gamma \cap A \neq \emptyset \), for each open nbd \( U \) of \( x \). The set of all \( \gamma \)-closure points of \( A \) is called \( \gamma \)-closure of \( A \) and is denoted by \( \text{cl}_\gamma(A) \). A subset \( A \) of \( X \) is called \( \gamma \)-closed, if \( \text{cl}_\gamma(A) \subseteq A \). Note that \( \text{cl}_\gamma(A) \) is contained in every \( \gamma \)-closed superset of \( A \).

Definition [7]. An operation \( \gamma \) on \( \tau \) is said to be regular, if for any open nbds \( U, V \) of \( x \in X \), there exists an open nbd \( W \) of \( x \) such that \( U^\gamma \cap V^\gamma \supseteq W^\gamma \).

Definition [8]. An operation \( \gamma \) on \( \tau \) is said to be open, if for any open nbd \( U \) of each \( x \in X \), there exists \( \gamma \)-open set \( B \) such that \( x \in B \) and \( U^\gamma \supseteq B \).

3 \( \gamma \)-Regular-Open Sets

Definition 3.1 [6]. A subset \( A \) of \( X \) is said to be \( \gamma \)-regular-open (resp. \( \gamma \)-regular-closed), if

\[
A = \text{int}_\gamma(\text{cl}_\gamma(A)) \quad \text{(resp. } A = \text{cl}_\gamma(\text{int}_\gamma(A))\text{)}.
\]

It is clear that \( \text{RO}_\gamma(X, \tau) \subseteq \tau_\gamma \subseteq \tau \) [6].

The following example shows that the converse of above inclusion is not true in general.

Example 3.2. Let \( X = \{a, b, c\} \), \( \tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\} \). For \( b \in X \), define an operation \( \gamma : \tau \to P(X) \) by
\[
\gamma(A) = \begin{cases} 
    A, & \text{if } b \in A \\
    cl(A), & \text{if } b \notin A 
\end{cases}
\]

Calculations shows that \(\{a, b\}, \{a, c\}, \{b\}, X, \phi\) are \(\gamma\)-open sets and \(\{a, c\}, \{b\}, X, \phi\) are \(\gamma\)-regular-open sets. Here set \(\{a, b\}\) is \(\gamma\)-open but not \(\gamma\)-regular-open.

**Definition 3.3[7].** A space \(X\) is called \(\gamma\)-extremally disconnected, if for all \(\gamma\)-open subset \(U\) of \(X\), \(cl_\gamma(U)\) is a \(\gamma\)-open subset of \(X\).

**Proposition 3.4.** If \(A\) is a \(\gamma\)-clopan set in \(X\), then \(A\) is a \(\gamma\)-regular-open set. Moreover, if \(X\) is \(\gamma\)-extremally disconnected then the converse holds.

**Proof.** If \(A\) is a \(\gamma\)-clopan set, then \(A = cl_\gamma(A)\) and \(A = int_\gamma(cl_\gamma(A))\), and so we have \(A = int_\gamma(cl_\gamma(A))\). Hence \(A\) is \(\gamma\)-regular-open.

Suppose that \(X\) is a \(\gamma\)-extremally disconnected space and \(A\) is a \(\gamma\)-regular-open set in \(X\). Then \(A\) is \(\gamma\)-open and so \(cl_\gamma(A)\) is a \(\gamma\)-open set. Hence \(A = int_\gamma(cl_\gamma(A)) = cl_\gamma(A)\) and hence \(A\) is \(\gamma\)-closed set. This completes the proof.

The following example shows that space \(X\) to be \(\gamma\)-extremally disconnected is necessary in the converse of above Proposition.

**Example 3.5** Let \(X= \{a, b, c\}\), \(\tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}\}\). Define an operation \(\gamma : \tau \to P(X)\) by \(\gamma(B) = int(cl(B))\). Clearly \(X\) is not \(\gamma\)-extremally disconnected space. Calculations shows that \(\{a\}, \{a, b\}, \{b\}, X, \phi\) are \(\gamma\)-open as well as \(\gamma\)-regular-open sets. Here \(\{a\}\) is a \(\gamma\)-regular-open set but not \(\gamma\)-clopan set.

**Theorem 3.6.** Let \(A \subseteq X\), then (a) \(\Rightarrow\) (b) \(\Rightarrow\) (c), where :

(a) \(A\) is \(\gamma\)-clopan.

(b) \(A = cl_\gamma(int_\gamma(A))\).

(c) \(X - A\) is \(\gamma\)-regular-open.

**Proof.** (a) \(\Rightarrow\) (b). This is obvious.

(b) \(\Rightarrow\) (c). Let \(A = cl_\gamma(int_\gamma(A))\). Then \(X - A = X - cl_\gamma(int_\gamma(A)) = int_\gamma(X - int_\gamma(A)) = int_\gamma(cl_\gamma(X - A))\), and hence \(X - A\) is \(\gamma\)-regular-open set. Hence the proof.
Using Proposition 3.4, we have the following Theorem:

**Theorem 3.7.** If $X$ is a $\gamma$-extremally disconnected space. Then $(a) \Rightarrow (b) \Rightarrow (c)$, where :

(a) $X - A$ is $\gamma$-regular-open.

(b) $A$ is $\gamma$-regular-open.

(c) A is $\gamma$-clopan.

**Proof.** $(a) \Rightarrow (b)$. Suppose $X$ is $\gamma$-extremally disconnected space. From Proposition 3.4, $X - A$ is a $\gamma$-open and $\gamma$-closed set, and hence $A$ is a $\gamma$-open and $\gamma$-closed set. Thus $A = int_\gamma(cl_\gamma(A))$ implies $A$ is $\gamma$-regular-open set.

$(b) \Rightarrow (c)$. This directory follows from Proposition 3.4. This completes as required.

Combining Theorems 3.6 and 3.7, we have the following:

**Theorem 3.8.** If $X$ is a $\gamma$-extremally disconnected space. Then the following statements are equivalent:

(a) $A$ is $\gamma$-clopan.

(b) $A = cl_\gamma(int_\gamma(A))$.

(c) $X - A$ is $\gamma$-regular-open.

(d) $A$ is $\gamma$-regular-open.

**Theorem 3.9.** Let $A \subseteq X$ and $\gamma$ be an open operation. If $cl_\gamma(A)$ is a $\gamma$-regular-open set. Then $A$ is a $\gamma$-open set in $X$. Moreover, if $X$ is extremally $\gamma$-disconnected then the converse holds.

**Proof.** Suppose that $cl_\gamma(A)$ is a $\gamma$-regular-open sets. Since $\gamma$ is open, we have $A \subseteq cl_\gamma(A) \subseteq int_\gamma(cl_\gamma(cl_\gamma(A))) = int_\gamma(cl_\gamma(A)) = int_\gamma(A)$. This implies that $A$ is $\gamma$-open set.

Suppose that $X$ is $\gamma$-extremally disconnected and $A$ is $\gamma$-open set. Then $cl_\gamma(A)$ is a $\gamma$-open set, and hence $\gamma$-clopan set. Thus by Theorem 3.8, $cl_\gamma(A)$ is a $\gamma$-regular-open set. This completes the proof.

**Corollary 3.10.** Let $X$ be a $\gamma$-extremally disconnected space. Then for each subset $A$ of $X$, the set $cl_\gamma(int_\gamma(A))$ is $\gamma$-regular-open sets.
Definition 3.11. A point \( x \in X \) is said to be a \( \gamma\theta \)-cluster point of a subset \( A \) of \( X \), if \( \text{cl}_\gamma(U) \cap A \neq \emptyset \) for every \( \gamma \)-open set \( U \) containing \( x \). The set of all \( \gamma\theta \)-cluster points of \( A \) is called the \( \gamma\theta \)-closure of \( A \) and is denoted by \( \gamma\text{cl}_\theta(A) \).

Definition 3.12. A subset \( A \) of \( X \) is said to be \( \gamma\theta \)-closed, if \( \gamma\text{cl}_\theta(A) = A \). The complement of \( \gamma\theta \)-closed set is called \( \gamma\theta \)-open sets. Clearly a \( \gamma\theta \)-closed (\( \gamma\theta \)-open) is \( \gamma \)-closed (\( \gamma \)-open) set.

Proposition 3.13. Let \( A \) and \( B \) be subsets of a space \( X \). Then the following properties hold:

(1) If \( A \subseteq B \), then \( \gamma\text{cl}_\theta(A) \subseteq \gamma\text{cl}_\theta(B) \).

(2) If \( A_i \) is \( \gamma\theta \)-closed in \( X \), for each \( i \in I \), then \( \bigcap_{i \in I} A_i \) is \( \gamma\theta \)-closed in \( X \).

Proof. (1). This is obvious.

(2). Let \( A_i \) be a \( \gamma\theta \)-closed in \( X \) for each \( i \in I \). Then \( A_i = \gamma\text{cl}_\theta(A_i) \) for each \( i \in I \). Thus we have \( \gamma\text{cl}_\theta(\bigcap_{i \in I} A_i) \subseteq \bigcap_{i \in I} \gamma\text{cl}_\theta(A_i) = \bigcap_{i \in I} A_i \subseteq \gamma\text{cl}_\theta(\bigcap_{i \in I} A_i) \).

Therefore, we have \( \gamma\text{cl}_\theta(\bigcap_{i \in I} A_i) = \bigcap_{i \in I} A_i \) and hence \( \bigcap_{i \in I} A_i \) is \( \gamma\theta \)-closed. Hence the proof.

Theorem 3.14. If \( \gamma \) is an open operation. Then for any subset \( A \) of \( \gamma \)-extremally disconnected space \( X \), the following hold:

\[
\gamma\text{cl}_\theta(A) = \bigcap \{ V : A \subseteq V \text{ and } V \text{ is } \gamma\theta\text{-closed} \}
\]

\[
= \bigcap \{ V : A \subseteq V \text{ and } V \text{ is } \gamma\text{-regular-open} \}
\]

Proof. Let \( x \notin \gamma\text{cl}_\theta(A) \). Then there is a \( \gamma \)-open set \( V \) with \( x \in V \) such that \( cl_\gamma(V) \cap A = \emptyset \). By Theorem 3.9, \( X - cl_\gamma(V) \) is \( \gamma\theta\)-regular-open and hence \( X - cl_\gamma(V) \) is a \( \gamma\theta \)-closed set containing \( A \) and \( x \notin X - \gamma\text{cl}_\theta(V) \). Thus we have \( x \notin \bigcap \{ V : A \subseteq V \text{ and } V \text{ is } \gamma\theta\text{-closed} \} \).

Conversely, suppose that \( x \notin \bigcap \{ V : A \subseteq V \text{ and } V \text{ is } \gamma\theta\text{-closed} \} \). Then there exists a \( \gamma\theta \)-closed set \( V \) such that \( A \subseteq V \) and \( x \notin V \), and so there exists a \( \gamma \)-open set \( U \) with \( x \in U \) such that \( U \subseteq cl_\gamma(U) \subseteq X - V \). Thus we have \( cl_\gamma(U) \cap A \subseteq cl_\gamma(U) \cap V = \emptyset \) implies \( x \notin \gamma\text{cl}_\theta(A) \). The proof of the second equation follows similarly. This completes the proof.

Theorem 3.15. Let \( \gamma \) be an open operation. If \( X \) is a \( \gamma \)-extremally disconnected space and \( A \subseteq X \). Then the followings hold:

(a) \( x \in \gamma\text{cl}_\theta(A) \) if and only if \( V \cap A \neq \emptyset \), for each \( \gamma\theta\text{-regular-open} \) set \( V \) with \( x \in V \).
(b) A is γ-θ-open if and only if for each \( x \in A \) there exists a γ-regular-open set \( V \) with \( x \in V \) such that \( V \subseteq A \).

(c) A is a γ-regular-open set if and only if A is γ-θ-clopan.

**Proof.** (a) and (b) follows directly from Theorems 3.8 and 3.9.

(c) Let A be a γ-regular-open set. Then A is a γ-open set and so \( A = cl_\gamma(A) = cl_\theta(A) \) and hence A is γ-θ-closed. Since \( X - A \) is a γ-regular-open set, by the argument above, \( X - A \) is γ-θ-closed and A is γ-θ-open. The converse is obvious. Hence the proof.

It is obvious that γ-regular-open ⇒ γ-θ-open ⇒ γ-open. But the converses are not necessarily true as the following examples show.

**Example 3.16.** Let \( X = \{a, b, c\}, \tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\} \). For \( b \in X \), define an operation \( \gamma : \tau \to P(X) \) by

\[
\gamma(A) = \begin{cases} 
  A, & \text{if } b \in A \\
  cl(A), & \text{if } b \notin A 
\end{cases}
\]

Calculations shows that \( \{a, b\}, \{a, c\}, \{b\}, X, \phi \) are γ-open sets as well as γ-θ-open sets and γ-regular-open sets are \( \{a, c\}, \{b\}, X, \phi \). Then the subset \( \{a, b\} \) is γ-θ-open but not γ-regular-open.

**Example 3.17.** Let \( X = \{a, b, c\}, \tau = \{\phi, X, \{a\}, \{b\}, \{a, b\}, \{a, c\}\} \) be a topology on X. For \( b \in X \), define an operation \( \gamma : \tau \to P(X) \) by

\[
\gamma(A) = A^\gamma = \begin{cases} 
  cl(A), & \text{if } b \in A \\
  A, & \text{if } b \notin A 
\end{cases}
\]

Calculations shows that \( \{\phi, X, \{a\}, \{a, c\}\} \) are γ-open sets and \( \{\phi, X, \{a, c\}\} \) are γ-θ-open sets. The the subset \( \{a\} \) is γ-open but not γ-θ-open.

4 γ-Closed Spaces

**Definition 4.1.** A filterbase \( \Gamma \) in X, γ-R-converges to \( x_0 \in X \), if for each γ-regular-open set \( A \) with \( x_0 \in A \), there exists \( F \in \Gamma \) such that \( F \subseteq A \).

**Definition 4.2.** A filterbase \( \Gamma \) in X γ-R-accumulates to \( x_0 \in X \), if for each γ-regular-open set \( A \)
with \( x_0 \in A \) and each \( F \in \Gamma, F \cap A \neq \phi \).

The following Theorems directly follow from the above definitions.

**Theorem 4.3.** If a filterbase \( \Gamma \) in \( X \), \( \gamma \)-R-converges to \( x_0 \in X \), then \( \Gamma \) \( \gamma \)-R-accumulates to \( x_0 \).

**Theorem 4.4.** If \( \Gamma_1 \) and \( \Gamma_2 \) are filterbases in \( X \) such that \( \Gamma_2 \) subordinate to \( \Gamma_1 \) and \( \Gamma_2 \) \( \gamma \)-R-accumulates to \( x_0 \), then \( \Gamma_1 \) \( \gamma \)-R-accumulates to \( x_0 \).

**Theorem 4.5.** If \( \Gamma \) is a maximal filterbase in \( X \), then \( \Gamma \) \( \gamma \)-R-accumulates to \( x_0 \) if and only if \( \Gamma \) \( \gamma \)-R-converges to \( x_0 \).

**Definition 4.6.** A space \( X \) is said to be \( \gamma \)-closed, if every cover \( \{V_\alpha : \alpha \in I\} \) of \( X \) by \( \gamma \)-open sets has a finite subset \( I_0 \) of \( I \) such that \( X = \bigcup_{\alpha \in I} \text{cl}_\gamma(V_\alpha) \).

**Proposition 4.7.** If \( \gamma \) is an open operation, Then the following are equivalent:

1. \( X \) is \( \gamma \)-closed.
2. For each family \( \{A_\alpha : \alpha \in I\} \) of \( \gamma \)-closed subsets of \( X \) such that \( \bigcap_{\alpha \in I} A_\alpha = \phi \), there exists a finite subset \( I_0 \) of \( I \) such that \( \bigcap_{\alpha \in I_0} \text{int}_\gamma(A_\alpha) = \phi \).
3. For each family \( \{A_\alpha : \alpha \in I\} \) of \( \gamma \)-closed subsets of \( X \), if \( \bigcap_{\alpha \in I_0} \text{int}_\gamma(A_\alpha) \neq \phi \), for every finite subset \( I_0 \) of \( I \), then \( \bigcap_{\alpha \in I} A_\alpha \neq \phi \).
4. Every filterbase \( \Gamma \) in \( X \) \( \gamma \)-R-accumulates to \( x_0 \in X \).
5. Every maximal filterbase \( \Gamma \) in \( X \) \( \gamma \)-R-converges to \( x_0 \in X \).

**Proof.** (2) \( \iff \) (3). This is obvious.

(2) \( \Rightarrow \) (1). Let \( \{A_\alpha : \alpha \in I\} \) be a family of \( \gamma \)-open subsets of \( X \) such that \( X = \bigcup_{\alpha \in I} A_\alpha \). Then each \( X - A_\alpha \) is a \( \gamma \)-closed subset of \( X \) and \( \bigcap_{\alpha \in I} (X - A_\alpha) = \phi \), and so there exists a finite subset \( I_0 \) of \( I \) such that \( \bigcap_{\alpha \in I_0} \text{int}_\gamma(X - A_\alpha) = \phi \), and hence \( X = \bigcup_{\alpha \in I_0} (X - \text{int}_\gamma(X - A_\alpha)) = \bigcup_{\alpha \in I_0} \text{cl}_\gamma(A_\alpha) \). Therefore \( X \) is \( \gamma \)-closed, since \( \gamma \) is open.

(4) \( \Rightarrow \) (2). Let \( \{A_\alpha : \alpha \in I\} \) be a family of \( \gamma \)-closed subsets of \( X \) such that \( \bigcap_{\alpha \in I} A_\alpha = \phi \). Suppose that for every finite subfamily \( \{A_{\alpha_i} : i = 1, 2, ..., n\} \), \( \bigcap_{i=1}^n \text{int}_\gamma(A_{\alpha_i}) \neq \phi \). Then \( \bigcap_{i=1}^n (A_{\alpha_i}) \neq \phi \) and \( \Gamma = \{\bigcap_{i=1}^n A_{\alpha_i} : n \in N, \alpha_i \in I\} \) forms a filterbase in \( X \). By (4), \( \Gamma \) \( \gamma \)-R-
Proposition 4.11. Every filterbase $F$ in $X$ determines a net $(\xi_i)_{i \in D}$ in $X$.

Proposition 4.10. Let $(\xi_i)_{i \in D}$ be a net in $X$. For the filterbase $F((\xi_i)_{i \in D}) = \{\{x_i : i \leq j\} : j \in D\}$ in $X$,

(1) $F((\xi_i)_{i \in D})$ $\gamma$-converges to $x$ if and only if $(\xi_i)_{i \in D}$ $\gamma$-converges to $x$.

(2) $F((\xi_i)_{i \in D})$ $\gamma$-accumulates to $x$ if and only if $(\xi_i)_{i \in D}$ $\gamma$-accumulates to $x$.

Proposition 4.11. Every filterbase $F$ in $X$ determines a net $(\xi_i)_{i \in D}$ in $X$ such that

Definition 4.8. A net $(x_i)_{i \in D}$ in a space $X$ is said to be $\gamma$-converges to $x \in X$, if for each $\gamma$-open set $U$ with $x \in U$, there exists $i_0$ such that $x_i \in cl_\gamma(U)$ for all $i \geq i_0$, where $D$ is a directed set.

Definition 4.9. A net $(x_i)_{i \in D}$ in a space $X$ is said to be $\gamma$-accumulates to $x \in X$, if for each $\gamma$-open set $U$ with $x \in U$ and each $i$, $x_i \in cl_\gamma(U)$, where $D$ is a directed set.

The proofs of following Propositions are easy and thus are omitted:

Proposition 4.10. Let $(\xi_i)_{i \in D}$ be a net in $X$. For the filterbase $F((\xi_i)_{i \in D}) = \{\{x_i : i \leq j\} : j \in D\}$ in $X$,
(1) $F_{\gamma}$-R-converges to $x$ if and only if $(x_i)_{i \in D} \gamma$-R-converges to $x$.

(2) $F_{\gamma}$-R-accumulates to $x$ if and only if $(x_i)_{i \in D} \gamma$-R-accumulates to $x$.

From Propositions 4.11 and 4.12, filterbses and nets are equivalent in the sense of $\gamma$-R-converges and $\gamma$-R-accumulates. Thus we have the following Theorem:

**Theorem 4.13.** For a space $X$, the following are equivalent:

1. $X$ is $\gamma$-closed.
2. Each net $(x_i)_{i \in D}$ in $X$ has a $\gamma$-R-accumulation point.
3. Each universal net in $X$ $\gamma$-R-converges.

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