A Rely-Guarantee Specification of Mixed-Criticality Scheduling

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Abstract. The application considered here is mixed-criticality scheduling. The core formal approaches used are Rely-Guarantee conditions and the Timeband framework; these are applied to give a layered description of job scheduling which covers resilience to jobs overrunning their expected execution time. A novel formal modelling idea is proposed to handle the relationship between actual time and its approximation in hardware clocks.

Note
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1 Introduction

The objective of the research described in this paper is to develop a framework based on time bands and rely-guarantee conditions for formally specifying and reasoning about properties of mixed-criticality scheduling (MCS); the key correctness issues revolve around timing.

Section 2 outlines the application area while Section 3 briefly sketches published ideas on the timeband framework and rely-guarantee thinking together with the application of both of these formalisms to the specification of cyber physical systems. Section 4 tackles the thorny issue of the passage of time. These ideas are brought together in Section 5 to write layered descriptions of MCS covering both optimistic and resilient modes. The customary summary and statement of future work are given in Section 6.
2 Mixed-criticality scheduling

The implementation of any real-time system requires a run-time scheduler that will follow the rules that define the required behaviour of the scheduling approach that has been chosen to deliver the temporal properties of the application. A human scheduling specialist will choose the basic approach, for example a fixed priority scheme with priorities assigned via the deadline monotonic algorithm, or the Earliest Deadline First protocol. Appropriate scheduling analysis will then be applied to a specification of the application to determine whether all deadlines can be met at run-time by the chosen scheduling approach.

Typical characteristics of an application are the number of jobs or tasks involved, the worst-case execution times of these entries, their deadlines and possible constraints on their arrival patterns. The dynamic run-time scheduler will rely on the validity of these parameters and will guarantee to manage the order of execution in accordance with the rules of the chosen scheduling protocol.

An increasingly important trend in the design of real-time systems is the integration of components with different levels of criticality on a common hardware platform. Criticality is a designation of the level of assurance against failure needed for a system component. A mixed-criticality system is one that has two or more distinct levels (for example safety critical, mission critical and low-criticality). Perhaps up to five levels may be identified (see, for example, the IEC 61508, DO-178B and DO-178C, DO-254 and ISO 26262 standards). Typical names for the levels are ASILs (Automotive Safety and Integrity Levels), DALs (Design Assurance Levels or Development Assurance Levels) and SILs (Safety Integrity Levels).

A key aspect of MCS is that system parameters, such as estimates of the worst-case execution time (WCET) of a job, become dependent on the criticality level of the job. So the same code will have higher WCET estimates for jobs defined to be safety-critical — this is necessary as a higher level of assurance is required than it would be if it is just considered to be mission critical or indeed non-critical. Criticality also has a role when the system becomes overloaded; the jobs of a lower level of criticality may have to be abandoned to provide resources to the safety-critical (high integrity) jobs.

A mixed criticality (MC-) scheduler is one that manages a set of mixed criticality jobs or tasks (the current paper addresses only jobs — extensions to address tasks are mentioned in Section 6.2) so that all deadlines are met if all jobs execute for no more than a lower bound on their execution time. In addition, the MC-scheduler must ensure that all high criticality jobs meet their deadlines if any job executes for more than its lower bound (but less than a conservative upper bound defined for each job).

The following paragraphs relate this objective to the wider topic of Cyber Physical Systems (CPS).

Correctness in safety-critical CPS can be considered from two perspectives: (i) (pre-run-time) verification, and (ii) survivability. Pre-run-time verification of a safety-critical system is the process of ensuring, prior to deployment, that the run-time behaviour of the system will be consistent with expectations. Verifi-
cation assumptions are made regarding the kinds of circumstances that will be encountered by the system during run-time and guarantees are used to specify the required runtime behaviour of the system (provided that the assumptions hold).

In contrast, survivability addresses expectations of system behaviour in the event that the assumptions fail to hold in full (in which case a fault or error is said to have occurred during run-time). Survivability may further be considered to comprise two notions: robustness and resilience [BDBH18]. Informally, the robustness of a system is a measure of the severity and number of faults it can tolerate without compromising the quality of service it offers while resilience refers to the degree of fault for which it can provide degraded yet acceptable quality of service.

It must be emphasised that the internal details of MC-scheduler, and the theory used to define the associated schedulability analysis (for example the EDF-VD or AMC protocols) are not the emphasis in this paper — there is plenty of prior research on that area [BD17]. Nor is there an explanation of how previously-proposed MC-scheduling algorithms can be shown to satisfy particular sets of rely-guarantee (R/G) specifications — that is (important) future work. This paper only seeks to provide a clear and intuitive specification of the components and thus motivate the formalism. The history of formal methods (such as Hoare Logic) prompts the belief that methods can be developed for showing that specific MC-scheduling algorithms satisfy (or fail to so do) particular R/G specifications.

3 Background approaches

This section describes previously published ideas on which the developments in this paper are based.

3.1 Rely-guarantee reasoning for concurrency

Pre and post conditions are used to document the intended behaviour of sequential programs. Such specifications can be said to document the “Why” rather than the “How” of a component. Furthermore, developments of Tony Hoare’s “axiomatic approach” [Hoa69,Hoa71] provide ways of evolving verified implementations in a top-down style. (Even if the development is not actually undertaken in this way, such a structure provides understandable documentation.) Key to layering such a description is a property that is often referred to as “compositionality”: the specification of a component describes all that need be achieved by its implementation; such specifications insulate a component from considerations about its environment and facilitate the verification of one design step before proceeding to further stages of development.

Finding compositional development methods for concurrent software proved challenging with many initial methods (e.g. AM71,Ash75,Owi75,OG76) needing to reason jointly about the combination of one thread with its sibling threads
and/or environment. This frustrates the ability to achieve top-down design. The Rely-Guarantee approach [Jon81,Jon83a,Jon83b] offers compositional specifications for a class of shared-variable concurrent programs. Just as pre conditions record assumptions about the context in which a component can be deployed, a rely condition indicates what interference a component must tolerate. Thus pre and rely conditions are information to the developer and warnings to anyone who wishes to deploy the specified component. A similar comparison can be made between the two conditions that record properties that the developed code must satisfy: post conditions describe the relation required of starting and finishing states whereas guarantee conditions indicate an upper bound on the impact that steps of the component can have on its environment. A picture of these components of a specification is given in Figure 1.

\[
\begin{array}{cccccc}
\sigma_0 \quad \cdots \quad \sigma_i \quad \sigma_{i+1} \quad \cdots \quad \sigma_j \\
\text{pre} \quad \text{rely} \\
\sigma_f \\
\text{guar} \\
\sigma_j \quad \sigma_{j+1} \\
\text{post} \\
\end{array}
\]

*pre/rely* are assumptions the developer can make
*guar/post* are commitments that the code must achieve

**Fig. 1.** Picturing the parts of a Rely-Guarantee specification

In [Hoa69], Hoare offered proof rules that justify development using the main sequential programming constructs. It is not surprising that the proof rules which justify steps of development employing parallelism are more complicated: they essentially need to show the compatibility of the rely and guarantee conditions. The current paper does not go into these details because description rather than proof is the objective here.

One observation that does carry over from development methods for sequential programs to many applications of the Rely-Guarantee approach is the importance of data abstraction and reification [Jon07]. Although predicates over states provide an element of procedural abstraction, specifications of significant systems can only be made brief and perspicuous by using abstract data types that match the problem. Subsequent development steps must show that representations of the abstractions preserve the properties of the specification. Development and justification of more concrete representations is variously referred to as “refinement” or “reification”.

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3 Pre and post conditions are as in [Hoa69] except that in VDM [Jon80] post conditions are relations over initial and final states.

4 A significant reworking [HJC14,HJC15,HJ18] presents the original rely-guarantee ideas in a more algebraic style.
3.2 Time bands

The motivation for the timeband framework comes from a number of observations about complex time-sensitive systems. Of relevance to this paper are the following:

- systems can be best understood by distinguishing different granularities (of time), i.e. there are different abstract views of the dynamics of the system
- it is useful to view certain actions (events) as atomic and “instantaneous” in one time band, while allowing them to have internal state and behaviour that takes time at a more detailed level of description
- the durations of certain actions are important, but the measuring of time must not be made artificially precise and must allow for tolerance in the temporal domain.

Key references include [BHJ20, BH10, WWB12, WOBW10, BBT07, BB06].

The central notion in the framework is that of a time band that is defined by its granularity, $G$, (e.g. 1 millisecond) and its precision, $\rho$, (e.g. 5 microseconds). Granularity defines the unit of time of the band; precision bounds the maximum duration of an event that is deemed to be instantaneous in its band.

Whilst it is the case that system descriptions can be given on a single time axis, inevitably, this has to be a fine granularity and it becomes difficult to “see the wood for the trees.” It is much clearer if the behaviour of a system is given in terms of a finite set of ordered bands. System activities are described in some band $B$ if they engage in significant events at the time scale represented by $B$, i.e. they have dynamics that give rise to changes that are observable or meaningful in the granularity of band $B$.

A complete system specification must address all dynamic behaviours. At the lowest level, circuits (e.g. gates) have propagation delays measured in nanoseconds or even picoseconds; at intermediate levels, tasks/threads have rates and deadlines that are usually expressed in tens of milliseconds; at yet-higher levels, missions can change every hour; and maintenance may need to be undertaken every month.

Understanding the behaviour of circuits allows the worst-case execution time of tasks to be predicted. At a higher band this allows deadlines to be checked and the schedulability of whole missions to be verified.

In this paper, behaviour of the application jobs and the scheduler is placed in a band that reflects the deadlines of the jobs; this might be a band with a granularity of one millisecond, or a finer granularity if that is required. The precision in this band will be sufficiently short so that the duration of certain actions, for example a context switch, can be ignored.

Precision is employed, in Section 4.1, to constrain the difference between external ‘real’ time and any interval interpretation of time as delivered by a hardware clock.
3.3 Specifying resilient CPS

As indicated in Section 3.1, the Rely-Guarantee approach was originally conceived for developments where a specified system was to be decomposed into concurrent processes. The general idea has however been shown to be applicable to contexts where a component is being specified which will execute in an environment that evolves in parallel with the specified component. Examples in [HJJ03, JHJ07] indicate how the specification of the control component can be derived from a specification of an overall cyber-physical system by recording assumptions (rely conditions) about the physical components. These early papers suffered from the fact that time was treated on a single (i.e. the finest) band; in [BHJ20], the timeband approach was used to make the rely and guarantee conditions more intelligible. This indicated that combining the timeband and rely-guarantee approaches can be used to specify CPS. A particular issue with CPS is that they need to be resilient in the sense that they are likely to have layers of required behaviour:

- optimistic (or optimal) behaviour is required when everything is performing in accord with the strongest rely conditions — the control system is required to meet its strongest guarantee condition;
- when some rely conditions are not satisfied, something in the environment is not behaving in an optimal way (this can be caused by a timing problem) — a weaker rely condition can describe a less desirable environment assumption under which the control system can only achieve a weaker guarantee condition;
- such layering of rely and guarantee conditions can be repeated over as many levels as required.

Such nested conditions (combined with time bands) are illustrated in [BHJ20, §4]. Another idea that appears to be useful in specifying CPS is specifying “may/must” constraints: [BHJ20, §4.3] contains an example where a short period of aberrant behaviour can be flagged but the control system is required to report a longer period of misbehaviour. This pattern of specification appears to be useful in a number of situations (again often linked to time).

4 Handling time

Specifying the sort of scheduling problem described in Section 2 presents additional challenges not faced in, for example, [BHJ20]:

- internal machine clocks must be linked to the passage of actual time
- state variables are needed that record the amount of time used by a job.

What follows is a novel approach to these two problems.
4.1 Relating \textit{ClockValue} to \textit{Time}

The first step is to distinguish an abstract notion of \textit{Time} from what clocks record in a computer (a clock will contain a \textit{ClockValue}). Consider a collection of \textit{States} indexed by the abstract notion of \textit{Time}:

\[ \Sigma = \text{Time} \rightarrow \text{State} \]

is a function; \textit{Time} should be “dense” like the real numbers (so \(\Sigma\) cannot be modelled as a list) — but fortunately it transpires that little need be said about \textit{Time} because specifications of operations (e.g. the scheduler and the jobs that it controls) are written with respect to the \(t\) component of \textit{State}:

\[ \text{State} :: t : \text{ClockValue} \]

For a given \(\sigma: \Sigma\) at time \(\alpha\) (because the identifier \(t\) is used for a component of \textit{State}, \(\alpha \in \text{Time}\) is used here), its \textit{ClockValue}\(\sigma(\alpha).t\) can differ from \(\alpha\). This is an issue for a fine time band and time bands need to be chosen such that the allowable difference is within the precision of the coarser band so that it can essentially be assumed that the \textit{ClockValue} in the machine is always sufficiently close to \textit{Time}.

Assume that the precision of the band is \(\rho\), equality \(=\rho\) is with respect to that precision. Then the relationship between \(t\) and \(\alpha\) is defined by the following predicate:

\[ P_t(\sigma) \overset{\text{def}}{=} \forall \alpha \in \text{Time} : \sigma(\alpha).t =_{\rho} \alpha \]

4.2 Tracking execution time

The set \textit{State} can for the most part be viewed as in any model-oriented specification in say VDM or Event-B. Specified software operations cause changes from one state to another value in \textit{State}.

\[ \text{State} :: t : \text{ClockValue} \]

\[ \text{active} : I \overset{m}{\rightarrow} \text{Job} \]

\[ \text{Job} :: e : \text{Duration} \]

\[ \text{run} : \mathbb{B} \]

\[ \text{info} : \text{JobInfo} \]

Information about any \textit{Job} that has started and not finished is stored in the \textit{active} map. If the job is running at \textit{ClockValue} (time) \(t\) then the Boolean flag \textit{run} is true (otherwise it is false). The Scheduler controls when a job is running by flipping the \textit{run} flag (\textit{JobInfo} is discussed below.)

\[^5\] At the finer time band, issues such as clock drift could also be formalised. In a distributed system, there would be a local \textit{State} for each processor and their clocks could also differ within the appropriate precision.

\[^6\] To model multi-processors, this can be the case with more than one \textit{Job}.
The predicate $P_t$ above defines how $t$ values relate to $Time$ but the specifications of jobs and Scheduler also need to refer to the execution time of a job: the $e$ field of a Job records the summation of its execution time up to $ClockValue$ $t$.

Neither the scheduler nor any job can change $t$ or $e$ which are instead linked to the autonomous (i.e. not under the control of software) progress of time: any job that is running over a period of time from $\alpha_1$ to $\alpha_2$ will have its $e$ field advanced by as much as the difference in the $Time$ i.e. $e_2 - e_1$ must be equal, within the precision of the time band, to $\alpha_2 - \alpha_1$; furthermore, when job $i$ is not running, its $e$ field remains unchanged. Thus the link between $Time$ and $e$ is defined by a predicate over $\Sigma$:

$$P_e(\sigma) \overset{\text{def}}{=} \forall \alpha_1, \alpha_2 \in Time, i \in Index \cdot$$

$$(\forall \alpha \mid \alpha_1 \leq \alpha \leq \alpha_2 \cdot (\sigma(\alpha).active)(i).run) \Rightarrow$

$$\sigma(\alpha_2).active(i).e - \sigma(\alpha_1).active(i).e = \rho_\alpha \alpha_2 - \alpha_1) \land$$

$$(\forall \alpha \mid \alpha_1 \leq \alpha \leq \alpha_2 \cdot \neg(\sigma(\alpha).active)(i).run) \Rightarrow$$

$$\sigma(\alpha_2).active(i).e = \sigma(\alpha_1).active(i).e$$

Although $Time$ is dense and progresses outside the influence of the software ($Scheduler$ or $Jobs$), it is precisely that software that brings about discrete changes to $State$. The specifications of the software components are written with respect to $State$ and thus relate to $t$: $ClockValue$ but that $t$ component is changed by the progress of $Time$. Essentially, real time is about the $\Sigma$ function whereas programs actually bring about discrete changes in $States$ (except, of course, neither $Scheduler$ nor $Jobs$ can write to $t$ or $e$ whose values are constrained by $P_t/P_e$ above).

Although the focus in Section 5 is on specifications for the scheduler and the jobs to which it is allocating time, the progress of $Time$ is really a third concurrent process. What could be thought of as a guarantee condition of this enigmatic process (the conjunction of $P_e$ and $P_t$) can be used as an assumption in any reasoning about the components ($Scheduler$ and $Jobs$) that are to be programmed.

5 Job-Based Scheduling

As outlined in Section 2, scheduling work can be divided into two parts. A human scheduler performs “schedulability analysis” which considers the likely arrival pattern –and estimates of the worst case execution time– of jobs and chooses a scheduling algorithm for the run-time scheduling software (which comprises the second part). The aim of the Scheduler software component is to ensure that jobs complete execution by their respective deadlines. The rely conditions of the combined scheduling activities would detail the inputs to the static part

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7 The difference between two values of type $Time$ is a $Duration$ as is the difference between two values of type $Duration$. $Duration$s are non-negative.

8 In practical scheduling applications, there is likely to be a $task$ $level$ above jobs but this is not addressed in the current paper.
of scheduling. In this section, the focus is on specifying the run-time scheduling software using rely and guarantee conditions.

In safety-critical situations, resilience is crucial and minor deviation from the estimated run times must not be allowed to cause highly-critical (hi-crit) jobs to miss their deadlines. The categorisation of jobs as hi/lo-crit is part of the off-line schedulability analysis. Here, nested rely and guarantee conditions are used to specify an optimistic mode in which all jobs can meet their deadlines and a fault-tolerant mode in which only hi-crit jobs are guaranteed to meet their deadlines.

The overall run-time system can be considered to consist of:

\[ Passage-of-Time || SCHEDULER || ||, JOB(i) \]

But it is important to understand the “frames” of these components. Starting with the enigmatic Passage-of-Time this is the only component that relates to Σ and it guarantees both \( P_t \) and \( P_e \) (recall that \( P_t \) links \( α \in Time \) with \( σ(α).t) \).

The SCHEDULER has a frame of State because it has purview over the whole collection of JOBS. An individual JOB(\( i \)) only has access to its own Job information.

The run-time Scheduler manages the execution of Jobs (indexed by \( i ∈ I \) — the set \( I \) is an arbitrary index set). As indicated in Section 4.2 active contains information about those jobs that have started and not yet finished.

As defined above, the local state for each job is as follows:

\[
Job :: e : Duration \\
run : B \\
info : JobInfo
\]

As mentioned in Section 4.1, the time (\( e \)) that a job has been executing cannot be changed directly by the scheduler and is updated in accordance with \( P_e \).

Static information about a Job is contained in:

\[
JobInfo :: d : ClockValue \\
C : Duration \\
crit : Lo | Hi
\]

The deadline by which a job should complete (\( d \)) is set when a job starts (see below) — it is shown to be of type ClockValue because, although deadlines relate to the external world, software can only access the internal timers (thus \( t \)) — but \( P_t \) ensures that software can use \( t:ClockValue \) as an acceptable surrogate. The \( C \) field contains the expected maximum execution time of the job. Finally criticality can either be a simple flag (Lo) or, for high criticality jobs, can store an extra allocation that can be used if they overrun:

\[
Hi :: X : Duration
\]

5.1 Starting and ending Jobs

The “life cycle” of a JOB is:

\footnote{In reality, there could be many levels of criticality but the approach can be illustrated with just two.}
– A JOB appears (not controlled by the SCHEDULER) when it enters active; this happens at $\alpha \in \text{Time}$; this time is registered internally as $\text{ClockValue}$, $t$ (with $t = \rho \alpha$). The execution time $(e)$ at the start of a job is set to zero; and its absolute deadline $d$ is set to $t + D$ ($D$ being a relative deadline); The Job is not initially in run mode (but if it has high priority, this might happen almost immediately).

– In general, a JOB will go through a number of cycles out/in of run mode;
– $P_e$ forces recording in its $e$ the sum of time it is executing.
– When a JOB finishes, it is removed from active.

Notice that starting a Job does not make it run — that is the role of the Scheduler. Thus, jobs start and are added to active; the scheduler sets and resets their run field.

5.2 Dynamic Scheduler

The run field of a Job records whether it is actively executing. In fact, the only way the Scheduler can allow a job to progress, is change its run filed and this makes the way the scheduler achieves its specification rather indirect in that it relies on $P_e$.

The specifications are expressed at two levels: an optimistic mode in which all jobs can be scheduled successfully and a resilient mode in which lo-crit jobs can be abandoned if necessary to meet the deadlines of hi-crit jobs.

Optimistic mode The overall scheduling objective is to make sure that jobs can finish by their respective deadlines; if all jobs were of equal criticality, this could be expressed by requiring that the following optimistic invariant is maintained:

$inv\text{-}State_O : \text{State} \rightarrow \mathbb{B}$

$inv\text{-}State_O(\text{mk-State}(t, \text{active})) \triangleq \forall \text{mk-Job}(e, \text{mk-JobInfo}(d, C)) \in \text{rng active} \cdot C - e \leq d - t$

This states that all jobs currently in the system must have room to finish by their deadline time ($d$). A corollary of this is that each job would terminate no later than its deadline.\footnote{The condition above is necessary but not in itself sufficient: prior schedulability analysis will have ascertained that the expected job arrival pattern will not be such as to make the sum of remaining execution times exceed capacity (e.g. it will not by possible for two jobs to arrive $n$ units of duration before their deadline and both jobs having worst case execution time of $n$).

Preserving $inv\text{-}State_O$ can be viewed as the key obligation of the scheduler but the only way in which it can achieve this is by allocating time to Jobs in

$\text{rng active}$.
active which entails setting their run field to true ($P_e$ then governs the increase in their $e$ field).\footnote{Readers familiar with Rely-Guarantee literature might wonder if such invariants should be couched as guarantee conditions of the form: $inv-StateO(st) \Rightarrow inv-StateO(st')$}

To facilitate maintenance of this invariant, the rely condition of the scheduler must cover both reliance on all Jobs respecting their $C$ and the JobInfo values of each job are unchanging (remember that jobs can arrive in—and leave from—active).

**SCHEDULER**

ext rd $t$ : ClockValue
rd active : $I \xrightarrow{m} Job$

\[ rely(\forall mk\cdot Job(e', info') \in rng active' \cdot e' \leq info'.C) \wedge \]
\[ \forall i \in (\text{dom active} \cap \text{dom active'}) \cdot active'(i).run = active(i).run \wedge active'(i).info = active(i).info \]

Correspondingly, the specification of each job is defined as follows:

**JOB**($i$)

ext rd job : Job
rely job'.info = job.info
guar job'.e' $\leq$ (job.info).C

The index $i$ locates a Job in the active map of State.

The standard Rely/Guarantee proof obligation for parallel composition requires that the rely conditions of each component follow from the guarantee conditions of their sibling processes.

**Theorem 1.** For JOB:

\[ guar-SCHEDULERO(st, st') \Rightarrow rely-JOB(i)((st.active)(i), st'.active)(i)) \]

Is immediate from their respective frames.

**Theorem 2.** For SCHEDULER:

\[ \land_i guar-JOB(i)((st.active)(i), st'.active)(i)) \Rightarrow rely-SCHEDULERO(st, st') \]

Is also immediate.

Although the guarantee conditions of jobs imply the rely condition of the scheduler, it cannot be implemented unless the developer can also assume that $P_e$ is satisfied.
Notice that the value $C$ has two roles: each job relies on its environment behaving according to whatever model or measuring process was used to derive $C$, but the job also has a contract with the scheduler not to execute for more than $C$. The scheduler is assumed to have used some form of analysis to verify (offline usually) that if all jobs respect their guarantee conditions then it will indeed provide the necessary capacity to each job. Hence the job can rely upon receiving $C$ before its deadline.

It should again be noted that this specification of the Scheduler’s behaviour does not include the actual details of the scheduling algorithm or dispatching policy — it is just a specification of what the Scheduler must achieve (its obligations). For a specific set of jobs it may not be possible to derive a valid scheduler that can meet this specification.

**Resilient mode** Consider what happens if a job overruns its estimated $C$: the guarantee condition of that job no longer holds and this invalidates the rely condition of the Scheduler. If the specifications given above were the only requirement on an implementation, the invalidated rely-$SCHEDULER$ removes the obligation of the developed Scheduler to function according to its specification.

Providing resilience requires that a weaker specification should be met in such circumstances. There are then two issues: defining the weaker (less optimal) set of conditions and ensuring a smooth transition.

In resilient mode, the scheduler only guarantees that hi-crit jobs get serviced and lo-crit jobs might be terminated or fail to complete by their deadlines. The specification does not require such terminations, it simply specifies a lower bound on the Scheduler. As indicated at the beginning of Section, the optimistic invariant preserves enough resource to be able to complete any job whether hi or lo-crit. In contrast, $inv-State_{R}$ enshrines a cautious approach of making sure that hi-crit jobs can meet their deadlines even if they all use their extra time allocation:

$$inv-State_{R} : State \rightarrow \mathbb{B}$$

$$inv-State_{R}(mk-State(t, active.)) \triangleq$$

$$\forall mk-Job(e, , mk-JobInfo(d, C, crit)) \in rng active \cdot$$

$$crit \in Hi \Rightarrow C + crit.X - e \leq d - t$$

As in optimistic mode, maintaining $inv-State_{R}$ is a key issue for the Scheduler. Furthermore, the implementation is required to meet both the optimistic behaviour and provide the specified resilient mode. This raises the issue of the transition between modes (i.e. what happens when the Scheduler has to change modes because of an overrunning job).

If it were the case that $inv-State_{O}$ implied $inv-State_{R}$, things would be simple. In that $inv-State_{R}$ is concerned with only hi-crit jobs, it is a relaxation. But these

\[12\] Paradoxically, it would illustrate the use of nested conditions rather better if there were more criticality levels because each failure could cascade to a lower level. The reason for limiting to two levels in this paper is just notational brevity.
hi-crit jobs can now demand more resource so the unguarded implication does not hold. It is necessary to find conditions under which a smooth transition can be achieved.

**Theorem 3.** Under suitable conditions:

\[ \text{conds}(st) \land \text{inv-State}_O(st) \Rightarrow \text{inv-State}_R(st) \]

One case where it holds trivially is if all of the extra (X) allowances are zero.

**Theorem 4.** Another option is for the Scheduler to maintain that degree of reserve for all hi-crit jobs since:

\[ \text{inv-State}_R(st) \land \text{inv-State}_O(st) \Rightarrow \text{inv-State}_R(st) \]

is obviously immediate.

In resilient mode (SCHEDULER\_R), the weaker rely condition concerns only hi-crit jobs and accepts that their execution might need the extra (X) execution time. So the invariant only requires that the scheduler concerns itself with hi-crit jobs.

What actual Schedulers do is, in effect, to retain enough slack to be able to manage transitions that are judged in the scheduling analysis to be important.

Consider the following example:

To see that \( \text{inv-State}_O \) is not strong enough as an invariant to handle mixed criticality, consider the following example which shows that there is an issue when resilience to overrunning hi-crit jobs is included. Suppose there was a hi-crit job \( a \) with: \( e_a = 10, \ d_a = 56, \ C_a = 15 \) and \( X_a = 3 \). There might also be some other lo-crit jobs so \( \text{active} \) would contain:

\[
\{ a \mapsto \text{mk-Job}(10, \text{true}, \text{mk-JobInfo}(56, 15, \text{mk-Hi}(3))), \ b \mapsto \ldots \}
\]

If this situation existed at \( t = 52 \), there would be insufficient time to complete the hi-crit job \( a \) by its deadline. (Presumably some other job with an earlier deadline would have been using the resource.) A scheduler could abandon execution of any lo-crit jobs such as \( b \) and employ the eXtra allowance stored for \( a \) — in the example there is an extra allowance of 3 units of time to be allocated to \( a \) but it is too late to meet the deadline (56) so this does not help. Therefore the situation must not be allowed to arise at \( t = 52 \) — it is clear that the scheduler has not kept enough “fat” to be able to complete \( a \) by its deadline.

Notice that this represents what the scheduler must do — it is at liberty to attempt to schedule more jobs than required including some that are marked as lo-crit.

The specifications are:

\[
\begin{align*}
SCHEDULER\_R \\
\text{ext rd} \ t & \ : \ ClockValue \\
\text{rd active} & \ : \ I \xrightarrow{m} \text{Job}
\end{align*}
\]
r\(\forall m \cdot \text{Job}(e', \text{info}) \in \text{rng} \text{ active'} \cdot\)
\[
\begin{align*}
\text{info'.crit} \in H_i \Rightarrow e' &\leq C + (\text{info'.crit}).X) \land \\
\forall i \in (\text{dom active} \cap \text{dom active'}) \cdot
\text{active'}(i).\text{run} \geq \text{active}(i).\text{run} \land \text{active'}(i).\text{info} = \text{active}(i).\text{info}
\end{align*}
\]

In resilient mode each hi-crit job promises to stay within its extended execution time; lo-crit jobs make no promises.

\[
\begin{align*}
\text{JOB}(i) \\
\text{ext rd } e & : \text{Duration} \\
\text{rd info} & : \text{JobInfo} \\
\text{rely info'} & \geq \text{info} \\
\text{guar info'.crit} \in H_i \Rightarrow e' &\leq C + (\text{info'.crit}).X
\end{align*}
\]

It is important to realize that the design of the scheduler must create code that satisfies both \textit{SCHEDULER}_O and \textit{SCHEDULER}_R: all jobs will meet their deadlines providing none are greedy and hi-crit jobs will get the extra resources to meet their deadlines — if necessary at the expense of lo-crit jobs being abandoned.

6 Conclusions

The formal methods basis for this paper is the timebands framework and the rely-guarantee approach. These ideas have previously been shown to be applicable to the specification of Cyber Physical Systems (CPS). Time bands have been used to avoid the confusion that arises when coarse-grained concepts are discussed at a fine granularity. Rely-guarantee conditions both help separate documentation of components and – when nested – help distinguish resilient modes of operation from the ideal behaviour of a system.

The targeted application of this paper is Mixed Criticality Scheduling; tackling MCS has required an important extension to the previously used notations.

6.1 Summary

The objective in writing this paper was to take existing ideas on the timeband framework and rely-guarantee approaches and to extend them so that they can be useful in specifying MCS. To achieve this, a novel approach to viewing time as a separate index has been proposed: assumptions about the relationship between actual \textit{Time} and what happens inside a computer are recorded essentially as guarantee conditions on the unstoppable progress of \textit{Time}.

With MCS, the trigger that indicates that ideal behaviour cannot be achieved is related to time: if hi-crit jobs need more resources than their optimistic estimates, a scheduler has to take action such as abandoning lo-crit jobs. Layered rely-guarantee conditions cope well with describing such nested modes.

The important task of relating the passage of \textit{Time} in the real world with what is going on inside the computer has been handled by having a model in
which there is a function from Time to machine states; this function cannot
be affected by programs but programs do bring about discrete changes in the
machine states. Crucially, the relationship between the Time index and values
in the machine states is defined by a predicate that can be thought of as a
guarantee condition. Appropriate definition of the precision of the time band
concerns issues such as clock accuracy and drift.

6.2 Further work

Of course, much work remains to be done. This introductory paper is restricted
to scheduling jobs. Sketches for describing the “task level” using the same formal
ideas exist and show that they appear to suffice. This material will be the subject
of a companion paper that will also say more on the transition between modes.

There is extensive published work on MC scheduling and implementation,
but little on their formal specification. The Rely-Guarantee (R/G) approach
has proved to be a useful formalism for specifying non-real-time safety-critical
systems and the main contribution here is to extend R/G to (i) time and (ii)
multiple criticalities. Such a formalism will prove to be essential since the notion
of mixed criticality has subtle semantics: concepts such as correctness, resilience
and robustness are rarely straightforward or intuitive for such systems.

It is important to remember that the material in the body of this paper
concerns only specifications. These specifications set a necessary condition on
implementations but they are at liberty to achieve more. A typical scheduler
will ensure that jobs with the earliest deadline are executed first (EDF); more
useful schedulers might abandon lo-crit jobs in stages if so doing provides enough
resource for the hi-crit jobs. The usefulness of the specifications in the body of
this paper has to be judged by seeing how easy it is to show that such imple-
mentations satisfy the specifications proposed here.

An ambitious extension that has not yet been worked on would be to specify
probability distributions on assumptions about timing and on the commitment
to timely job completion.

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