Spin–lattice relaxation phenomena in the magnetic state of a suggested Weyl semimetal CeAlGe

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ABSTRACT
In this work, we report the results of DC susceptibility, AC susceptibility and related technique, resistivity, transverse and longitudinal magnetoresistance and heat capacity on polycrystalline magnetic semimetal CeAlGe. This compound undergoes antiferromagnetic type ordering around 5.2 K \( (T_1) \). Under the application of external magnetic fields, parallel alignment of magnetic moments is favoured above 0.5 T. At low field and temperature, frequency and AC field amplitude response of AC susceptibility indicate the presence of spin–lattice relaxation phenomena. The observation of spin–lattice interaction suggests the presence of the Rashba–Dresselhaus spin–orbit interaction which is associated with inversion and time-reversal symmetry breaking. Additionally, the presence of negative and asymmetric longitudinal magnetoresistance indicates anomalous velocity contribution to the magnetoresistance due to the Rashba–Dresselhaus spin–orbit interaction which is further studied by heat capacity.

1. Introduction
Non-magnetic semimetals have attracted significant attention, both theoretically and experimentally among physicists working in the area of topological materials. In a semimetal, conduction and valence bands cross each other in the Brillouin zone. This crossing persists under various perturbations and preserves certain crystalline symmetry. Such systems exhibit novel topological properties like quantum anomalies [1–4]. These anomalies are characterised by the violation of classical symmetry due to the quantum effect in the presence of an external field [5–7]. In this context, the breaking of either inversion or time-reversal symmetry forms the Weyl points at the Fermi surface, and this phenomenon has attracted significant attention in semimetals. In such cases, Fermi surface consists of open arcs connecting the projection of bulk Weyl points onto the surface Brillouin zone, instead of closed loops. It has remarkable non-trivial topological properties like integer momentum space invariance [8,9].
This invariance arises from the singularities in the electronic structure where the different bands touch, as isolated points [10]. The symmetry breaking occurs either by natural or external perturbation in the form of spin–orbit coupling, thereby leading to interesting topological gapped or gapless states [11–13]. In recent years, magnetic semimetal has emerged as an exclusive topic of research due to the interplay of magnetism and topology [14–16]. In this perspective, a variety of compounds like CeSb, Ce₃Bi₄Pd₃, CeSbTe, Co₃Sn₂S₂ and Co₂MnGa which show the existence of Weyl/Dirac nodes in the magnetic ordered phase has been investigated [16–20]. The presence of localised magnetic moment shifts the Weyl/Dirac nodes leading to an unusual response in the magnetisation [19,21]. Generally, Weyl nodes are noted in a non-centrosymmetric crystal structure. In the centrosymmetric crystal structure, breaking of product symmetry (inversion and time reversal) can also allow the Weyl nodes [22]. It is reported that inversion symmetry maps the Weyl nodes with opposite chirality, whereas the time-reversal symmetry maps the Weyl nodes without changing its chirality [22]. In the presence of spin–orbit coupling, Zeeman energy splits the band in spin-up and spin-down state, which is further mixed by the Rashba–Dresselhaus effect [23]. The Rashba spin–orbit interaction arises from the structural inversion asymmetry, whereas the Dresselhaus spin–orbit interaction arises from the bulk inversion asymmetry [23]. The Rashba–Dresselhaus spin–orbit interaction results in the magnetic texture of band electrons. This phenomenon is known as spin momentum locking and it produces a spin torque which may induce a spin relaxation [24,25].

In this context, the compound CeAlGe is interesting. It has been reported that this compound is ferromagnetically ordered [26] and theoretical studies have predicted a Weyl semimetal state due to breaking inversion and time-reversal symmetry [27]. However, recent experimental results of a single crystal CeAlGe report the magnetic semimetal behaviour along with antiferromagnetic ordering [28,29] which contradicts the results of Ref [26] and [27]. In Ref. [30], it has been reported that in this compound there is the presence of some parallel alignment of magnetic moments in an antiferromagnetic spin matrix. Therefore, it would be interesting to identify the intrinsic magnetic state and also to see whether there is some evidence of Weyl state in this polycrystalline compound.

Hence, with this aim, in this manuscript, we report the results of DC susceptibility, AC susceptibility and related technique, resistivity, transverse and longitudinal magnetoresistance and heat capacity on the polycrystalline magnetic semimetal CeAlGe. This compound undergoes an antiferromagnetic type ordering around 5.2 K at low applied magnetic fields. As the magnitude of the applied field is increased, the parallel alignment of magnetic moments is favoured above 0.5 T. A detailed investigation through AC susceptibility reveals the presence of spin–lattice relaxation in this compound. The observed spin–lattice relaxation is a signature of the presence of the Rashba–Dresselhaus spin–orbit interaction. Furthermore, the existence of negative and asymmetric longitudinal
magnetoresistance provides evidence of the anomalous velocity contribution to the magnetoresistance due to the Rashba–Dresselhaus spin–orbit interaction.

2. Experimental details

The compound CeAlGe is prepared by arc melting stoichiometric amounts of respective high purity (>99.9%) elements in an atmosphere of argon. For better homogeneity, an ingot is re-melted a number of times by turning over each time. The weight loss after the final melting is <1%. After melting, the ingot is wrapped in a tantalum foil and sealed in an evacuated quartz tube. The compound is annealed at 650°C for 48 h and quenched in ice water. The compound thus obtained is characterised by X-ray diffraction (XRD) using Rigaku smartlab diffractometer using monochromatised Cu Kα1 radiation at room temperature. Figure 1 shows the Rietveld refined (using Fullprof suite) powder XRD pattern of this compound at room temperature. The compound crystallises in the tetragonal structure and is in a single phase. The structure can be refined by either of two space groups: I41/amd (centrosymmetric) or I41/mmd (non-centrosymmetric). We tried to analyse the pattern using both space groups. It is observed that R-factor and goodness of fit are smaller for I41/mmd space group which points to the presence of non-centrosymmetric structure. In order to get an idea about the stoichiometry, energy-dispersive X-ray spectroscopy measurement is carried out. The average atomic stoichiometry of the compound is in accordance with the expected values. From scanning electron microscope images, it is revealed that the average crystallite size is of the order of a micrometre.

Temperature (T)- and magnetic field (H)-dependent

![Figure 1. Rietveld refined powder XRD pattern of CeAlGe. The difference between the observed and experimental pattern and the Bragg position is also shown. Lattice parameters obtained from the fitting are also represented.](image-url)
magnetisation \((M)\) studies are performed using Magnetic Property Measurement System (MPMS), while temperature- and magnetic field-dependent heat capacity and resistivity are performed using Physical Property Measurement System (PPMS), by Quantum design, USA. Resistivity, transverse magnetoresistance (magnetic field is perpendicular to the current direction) and longitudinal magnetoresistance (magnetic field is parallel to the current direction) measurements are carried out on the pellets of specific shapes. In these measurements, current inhomogeneity is minimised by the procedure mentioned in the supplement information of Ref. [31].

3. Result and discussion

3.1. DC susceptibility study

Figure 2(a) shows the temperature-dependent DC magnetisation divided by the magnetic field \((M/H)\) under zero-field cooling (ZFC) and field cooling (FC) conditions in the field range 0.01–0.3 T. It is observed that \(M/H\) increases rapidly and shows a peak around 5.2 K \((T^1)\), followed by a hump around 3.0 K (possibly due to some spin rearrangements) at 0.01 T. The bifurcation between ZFC and FC curves starts from \(T^1\). As the magnitude of the applied field increases the bifurcation between the curves reduces and is suppressed around 0.3 T. Above 0.3 T, \(M/H\) curves saturate below 5.2 K (shown in Figure 2(b)). The inverse magnetic susceptibility of the compound is fitted with the Curie–Weiss law in the temperature range 50–300 K at 0.1 T (inset of Figure 2(a)). Below this temperature, a non-linear deviation is observed, which could be due to the crystalline electric field effect [32]. The obtained effective moment \((\mu_{\text{eff}})\) and Curie–Weiss temperature \((\theta_p)\) are 2.8 \(\mu_B\) and \(-32\) K, respectively. The negative value of \(\theta_p\) indicates the dominance of antiferromagnetic interactions. It is also noted that \(T^1\) decreases on increasing the magnitude of the applied field. Hence, the peak around \(T^d\) arises due to the antiferromagnetic ordering which is in accordance with Refs [28,29]. Above 0.5 T, some partial parallel alignments of magnetic moments are present [28,30]. On further increasing magnetic field, this alignment of magnetic moments increases which is responsible for the increase of saturation temperature of \(M/H\) curve to a higher temperature (as shown in the inset of Figure 2(b)). In order to shed more light, the magnetic field response of isothermal magnetisation \(M\) at different temperatures is measured (shown in Figure 2(c)). The curve shows the weak hysteresis at temperatures below \(T^1\) (shown in the inset of Figure 2(c)). However, at the high field, the magnetisation tends to saturate due to parallel alignment among the magnetic moments. From the above studies, it can be concluded that at low field, below \(T^1\), there is some parallel alignment of magnetic moments in an antiferromagnetic spin matrix. This gives rise to the Zeeman energy which increases as the magnetic field is increased because high field favours the parallel alignment among the magnetic moments.
Figure 2. (a) Temperature-dependent DC magnetisation divided by the magnetic field ($M/H$) under ZFC and FC condition in the field range 0.01–0.3. Inset: Temperature-dependent $H/M$ at 0.1 T. The red line shows the Curie–Weiss law fitting. (b) Temperature-dependent $M/H$ under ZFC condition in the field range 0.4–7 T. Inset: Temperature-dependent derivative of $M/H$ in the field range 2–7 T. (c) Isothermal magnetisation plotted as a function of the magnetic field at 1.8, 4 and 7 K. Inset: The expanded form of the same curves at low fields.
3.2. AC susceptibility: spin–lattice relaxation phenomena

To further explore the complex magnetic state present in this compound, AC susceptibility study is done. Figure 3(a,b) shows the temperature response real part ($\chi'_\text{ac}$) and imaginary part ($\chi''_\text{ac}$) of AC susceptibility measured at 3 (10$^{-4}$) T AC field along with superimposed DC fields of different magnitudes. At 0 T DC field, a sharp peak is noted around $T^i$. With increasing DC field, $T^i$ shifts to lower temperature. Above 0.5 T, the signal of AC susceptibility is suppressed. The above observation is because the exchange energy accountable for the development of the antiferromagnetic state is suppressed above 0.5 T. The signature of the peak is also noted in the $\chi''_\text{ac}$ due to the dissipation of magnetic energy below 0.5 T. The upper and lower inset of Figure 3(b) shows the temperature response of $\chi'_\text{ac}$ and $\chi''_\text{ac}$ measured in 3 (10$^{-4}$) T AC field and 0 T DC field under different frequencies. It is observed that with increasing frequency, $T^i$ remains unchanged. This observation rules out the presence of a spin freezing mechanism in this compound [33]. Figure 3(c) shows the DC magnetic field-dependent $\chi'_\text{ac}$ at 7 Hz and AC field of 3 (10$^{-4}$) T. In paramagnetic region (above $T^i$), $\chi'_\text{ac}$ decreases with increasing magnetic field in both directions. Below $T^i$ (at 2.5 K), $\chi'_\text{ac}$ decreases but a peak is observed near 0.39 T in both directions of fields. These peaks shift to the higher fields on decreasing temperature (shown in the inset of Figure 3(c)). This observed peak gives an indication of the presence of spin dynamic under the application of an external magnetic field. Hence, our observation indicates the existence of an unusual magnetic state in the low field and temperature region in this compound.

In this section, we study the frequency response $\chi'_\text{ac}$ and $\chi''_\text{ac}$ at a fixed temperature, DC and AC field. Figure 4(a) and inset shows the frequency-dependent normalised (with respect to the value at 500 Hz) $\chi'_\text{ac}$ and $\chi''_\text{ac}$ at 7 Hz, 0 T DC and 3 (10$^{-4}$) AC field at different temperature. It is noted that with decreasing frequency (at 1.8 K), $\chi'_\text{ac}$ increases and a maximum and minimum occur around 125 and 55 Hz, respectively. Below 55 Hz, $\chi'_\text{ac}$ increases. A similar feature is also noted at other temperatures. For $\chi''_\text{ac}$, a minimum and a broad maximum are observed around 76 and 4 Hz, respectively and a similar feature is observed for other measurement temperatures. This behaviour of AC susceptibility can be understood on the basis of modified Cole–Cole formalism [21]

$$\chi_\text{ac}(\omega) = \chi_\text{ac}(\infty) + [\chi_\text{ac}(0)-\chi_\text{ac}(\infty)]/1 + (i\omega\tau_0)^{1-\alpha}$$ (1)

where $\chi_\text{ac}(0)$ is the isothermal susceptibility extrapolated to zero frequency where spin–lattice relaxation is active and $\chi_\text{ac}(\infty)$ is the adiabatic susceptibility extrapolated to infinite frequency where spin–spin relaxation is active; $\omega = 2\pi f$ is the angular frequency; $\tau_0 = 1/2\pi f_0$ is the characteristic relaxation time and $\alpha$ is the width of frequency distribution. For an infinite broad distribution, $\alpha$ is...
Figure 3. (a) Temperature-dependent $\chi'_{ac}$ at different superimposed DC fields at 7 Hz in the presence of $3 \times 10^{-4}$ T AC field. (b) Temperature-dependent $\chi''_{ac}$ under similar conditions. Upper inset: Temperature-dependent $\chi'_{ac}$ at different frequencies under similar conditions. Lower inset: Temperature-dependent $\chi''_{ac}$ at similar frequencies under similar conditions. (c) DC magnetic field (3 to $-3$ T) dependent $\chi'_{ac}$ at 7 Hz in the presence of $3 \times 10^{-4}$ T AC field in the temperature range 1.8–7 K. Inset: Magnified plot of the same figure at low fields.
1 and it is 0 for a single relaxation process. Equation (1) can be decomposed in terms of the real and imaginary part as

\[
\chi'_{ac}(v) = \chi_{ac}(0) + \left[ A_0 \left\{ 1 + (\omega \tau_0)^{-\alpha} \sin(\pi \alpha / 2) \right\} / \left\{ 1 + 2(\omega \tau_0)^{-\alpha} \sin(\pi \alpha / 2) + (\omega \tau_0)^{2(1-\alpha)} \right\} \right]
\]  

(2)

\[
\chi''_{ac}(v) = A_0 \left\{ (\omega \tau_0)^{-\alpha} \cos(\pi \alpha / 2) \right\} / \left\{ 1 + 2(\omega \tau_0)^{-\alpha} \sin(\pi \alpha / 2) + (\omega \tau_0)^{2(1-\alpha)} \right\}
\]  

(3)

where \( A_0 = \chi_{ac}(0) - \chi_{ac}(\infty) \) and positive \( A_0 \) indicate the dominance of spin–lattice relaxation behaviour [21]. We tried to fit the curves in Figure 4(a) and inset with Equations (2) and (3), respectively. Due to weak frequency

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dependence, good fitting of the $\chi_{ac}'(\omega)$ curve is not obtained with Equation (3). However, a worthy fitting is obtained for $\chi_{ac}'(\omega)$ in the low-frequency region (below 76 Hz) for 1.8, 3.6 and 5.4 K (shown in Figure 4(a)). A positive $A_0$ is obtained implying that $\chi_{ac}(0)$ is greater than $\chi_{ac}(\infty)$. Hence, it can be said that spin–lattice interaction is dominant below 76 Hz in this compound. The parameters $\alpha$ and $\tau_0$ roughly obtained from fitting are summarised in Table 1. These parameters indicate a broad relaxation process in spin–lattice interaction. In the high-frequency region (above 76 Hz), the change of slope of AC susceptibility curve indicates competition between spin–lattice relaxation and antiferromagnetic interactions. Here, we would like to mention that the effect of superimposed DC fields (above 0.5 T) on these curves is negligible and the features in AC susceptibility are suppressed.

Furthermore, AC field ($h_{ac}$)-dependent studies of $\chi_{ac}'$ and $\chi_{ac}''$ at 7 Hz, 0 T DC field is carried out at different temperatures. Figure 4(b) and inset shows the normalised (with respect to the value at 9 ($10^{-4}$) T AC field) $\chi_{ac}'$ and $\chi_{ac}''$ curves at different temperatures. As noted from the figure, above the ordering temperature, $\chi_{ac}'$ varies insignificantly with $h_{ac}$. The below ordering temperature, $\chi_{ac}'$ increases with increasing $h_{ac}$. A change of slope is noted around 5.2 K and a broad maximum is noted around 5.4 K which is centred around 4 ($10^{-4}$) T. $\chi_{ac}''$ also increases with increasing $h_{ac}$ for all temperatures, except in the temperature range 5.2–5.6 K. In this temperature region, a broad maximum is noted. These observations may be arising due to the presence of spin–orbit coupling. It is well known that orbital magnetic moment diminishes the magnetisation, while spin magnetic moment favours it in the presence of the external magnetic field. Equal contribution of both of these moments might be responsible for this observed maximum near the magnetic phase boundary. This observation suggests that the magnetic moments interact with lattice via the spin–orbit coupling as it is the key phenomenon responsible for the coupling of spin to the lattice [34].

The presence of Zeeman energy splits the bands into spin-up and spin-down states in the presence of spin–orbit coupling. As a result, these states are further interacted by the Rashba–Dresselhaus effect and lead to the breaking of time-reversal symmetry [27]. The Rashba effect arises due to structural inversion symmetry breaking and splits the spin sub-bands, while the Dresselhaus effect arises due to an additional symmetry breaking, resulting in strain-induced Dresselhaus spin–orbit interaction. Thus, both Rashba and Dresselhaus effect adds an additional

### Table 1. Parameters ($\alpha$ and $\tau_0$) obtained from the AC susceptibility curves fitting with Equation (2) in the low-frequency region at 0 DC field.

| $T$ (K) | $\alpha$ | $\tau_0$ |
|-------|----------|----------|
| 1.8   | 0.380 ± 0.018 | 0.027 ± 0.017 |
| 3.6   | 0.390 ± 0.022 | 0.040 ± 0.026 |
| 5.4   | 0.398 ± 0.015 | 0.018 ± 0.005 |
interaction in spin–orbit coupling. An inequality between Rashba and Dresselhaus spin–orbit interaction leads to a non-equilibrium condition in the magnetic state [25]. As a result, exchange energy associated with the conduction electrons behaves like a torque [35]. This torque acting on magnetic moments results in a spin–lattice relaxation behaviour in the magnetic state of this compound.

3.3. Resistivity, transverse and longitudinal magnetoresistance: evidence of Rashba and Dresselhaus spin–orbit interactions

Figure 5(a) shows the temperature-dependent resistivity ($\rho$) at 0 T up to 300 K. With decreasing temperature, resistivity decreases and an anomaly is noted around $T^I$ (the upper inset of Figure 5(a)). The lower inset of Figure 5(a) shows the temperature-dependent resistivity under different applied fields up to 15 K. It is observed that anomaly near $T^I$ shifts to the right in temperature above 0.5 T, which is an analogy with the DC susceptibility results. Figure 5(b) displays the transverse magnetoresistance (MR) [= ($\rho (H) - \rho (0)$)/$\rho (0)$] at different temperatures. From the figure, it is noted that MR increases with the increasing magnetic field in both directions at 20 and 15 K. The positive MR is likely to be arising from the orbital contribution [36]. Below the 15 K, MR decreases with the increasing magnetic field and it is noted that the slope of the curve changes below $T^I$. Figure 5(c) shows the longitudinal MR measured at different temperatures. Similar to transverse MR, longitudinal MR is positive above 15 K and negative below it. It is also noted that the slope changes of MR are insignificant below $T^I$. However, interestingly, it is observed that the longitudinal MR is asymmetric (asymmetric behaviour is insignificant in transverse MR). The asymmetric MR is calculated using the following formula [37,38]:

$$\text{Asym MR} = \frac{[\text{MR (H+) - MR (H-)]}}{2}$$

(4)

where MR (H+) and MR (H−) are the magnetoresistance for the positive and negative magnetic field, respectively. The inset of Figure 5(c) shows the longitudinal asym MR at the selected temperature. It is observed that asym MR increases with an increasing magnetic field. The observed asymmetry in MR may arise due to the current jetting effect. The current jetting effect arises due to the presence of inhomogeneous conductivity. This can cause a distortion in the current path due to the rise in the perpendicular component, which flows normal to the major current direction. This component decreases the potential drop between voltage electrodes resulting in the negative and possible asymmetric MR [37]. However, the presence of positive MR and insignificant asymmetry in the transverse MR suggest that current jetting effect is minimal. In fact, this mechanism is understood on the basis of anomalous velocity term associated with non-zero Berry curvature [39,40]. Generally, electron velocity, $v$, can
Figure 5. (a) Temperature-dependent resistivity at 0 T. Upper inset: Magnified resistivity curve in the low-temperature region. Lower inset: Temperature-dependent resistivity under different fields. (b) Magnetic field response of transverse MR at different temperatures. (c) Magnetic field response of longitudinal MR at different temperatures. Inset: Longitudinal Asymmetric MR (Asym MR) plotted as a function of the magnetic field at selected temperatures.
be expressed as \[37,41\]
\[
\vec{v} = \nabla_k \varepsilon(k) + e\vec{E} \times \vec{\Omega}(\vec{H}, \vec{k})
\] (5)

where \(\varepsilon(k)\) is the electron energy, \(\vec{E}\) is the electric field, \(\vec{k}\) is the wave vector of electron and \(\vec{\Omega}(\vec{H}, \vec{k})\) is the Berry curvature. The total current \(\vec{j}\) can be expressed as in a diffusive transport \[37,38\]
\[
\vec{j} = \frac{d^3\vec{k}}{(2\pi)^3} \left[ \vec{v} + e\vec{E} \times \vec{\Omega}(\vec{H}, \vec{k}) + \frac{e}{c} (\vec{\Omega}(\vec{H}, \vec{k}) \cdot \vec{v}) \vec{H} \right] n_{\vec{k}}
\] (6)

where \(n_{\vec{k}} = f^0_{\vec{k}} + g_{\vec{k}}(\vec{H}, \vec{E})\) is the electron distribution function, \(f^0_{\vec{k}}\) and \(g_{\vec{k}}(\vec{H}, \vec{E})\) are equilibrium and non-equilibrium distribution function, respectively. In the presence of a magnetic field \((\vec{H})\), an induced term \(g_{\vec{k}}(\vec{H}, \vec{E})\) is added which results in non-vanishing anomalous velocity contribution to the total current. It can be said that the presence of the Rashba–Dresselhaus spin–orbit interaction (which is associated with the non-zero Berry curvature) is responsible for the non-trivial dependence of \(g_{\vec{k}}(\vec{H}, \vec{E})\), which depends on the orientation of \(\vec{H}\) and \(\vec{E}\) resulting in the anomalous velocity contribution to the MR \[42–45\]. The above statement can be understood on the basis that with an increase in the magnetic field, parallel alignment among magnetic moments increases and it leads to the enhancement of Zeeman energy. This energy associated with the Rashba–Dresselhaus spin–orbit interaction leads to an asymmetry in longitudinal MR, which gives a signature of the time-reversal symmetry breaking. As stated before, asymmetry in MR is observed only in the current direction parallel to the magnetic field, however, a considerable change of slope below \(T^t\) is noted only for transverse MR. Hence, it can be said that magnetic ordering strength is stronger in the transverse direction and it might dominate over the anomalous velocity contribution to the MR leading to the observation of insignificant asymmetric MR when the direction of current is perpendicular to the magnetic field.

### 3.4. Heat capacity study

Figure 6(a) shows the temperature-dependent heat capacity divided by temperature \((C/T)\) in different fields. It is noted that a transition appears near \(T^t\). With increasing magnetic field, \(T^t\) is shifted to the lower temperature. Above 0.5 T, with an increase in the magnetic field, the peak broadens and \(T^t\) shifts toward higher temperature. In order to extract the electronic \((\gamma)\) and phonon \((\beta)\) contribution, the following equation is fitted to the 0 T curve above \(T^t\) (the upper
The obtained parameters $\gamma$ and $\beta$ are 22 mJ mol$^{-1}$ K$^{-2}$ and 0.26 mJ mol$^{-1}$ K$^{-4}$, respectively. The value of $\gamma$ indicates the insignificant hybridisation of localised magnetic moment with the conduction electrons [46,47]. Above 0.5 T, $C$ varies as $T^3$ in the low-temperature region (where the phonon contribution is negligible). The curves above 0.5 T are fitted with the following equation:

$$C/T = \gamma + \beta T^2$$

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$$C/T = \gamma + \beta T^2$$

**Figure 6.** (a) Temperature response of $C/T$ at selected magnetic fields. Upper inset: $T^2$ dependence of $C/T$ at 0 T. The solid red line is the fit of Equation (7). (b) $T^2$ dependence of $C/T$ at high fields. Solid red lines are the fit of Equation (8). Inset: Effective electron velocity ($v^*$) (left axis) and $\gamma$ (right axis) plotted as a function of the magnetic field.
\[ C = \gamma T + \left( k_B T / v^* \right)^3 k_B \]  

(8)

where \( k_B \) is the Boltzmann constant and \( v^* \) is the effective Fermi velocity. This relation is proposed for a linear dispersion \( \varepsilon (k) = \hbar v^* k \). Due to the presence of very small Debye temperature (estimated from \( \beta \)), it is expected that \( \varepsilon (k) \) cannot be due to acoustic phonons. It can be understood on the basis of the Rashba–Dresselhaus spin–orbit interaction [24]. Figure 6(b) shows \( T^2 \)-dependent \( C/T \) at selected fields and linear fitting of Equation (8) gives the value \( \gamma \) and \( v^* \). It is observed that \( \gamma \) decreases, while \( v^* \) increases with an increasing magnetic field (inset of Figure 6(b)) and this feature is similar to that reported in Ref. [48]. Hence, it can be said that the Rashba–Dresselhaus spin–orbit effect and linear dispersion suggests the presence of symmetry (inversion and time reversal) breaking. As a result, this might be responsible for the spin–lattice relaxation phenomena in magnetic state and such behaviour is expected in the vicinity of Weyl nodes [49]. In our case, the chiral anomaly is not seen clearly, as it may be hidden due to the interplay of magnetic interaction in this compound. For unambiguous identification of this hidden signature, ARPES, NMR, X-ray scattering, field-dependent neutron diffraction studies, etc. are necessary.

4. Conclusion

Our study on CeAlGe reveals that, above 0.5 T and below \( T^1 \), antiferromagnetic ordering is suppressed and parallel alignment of magnetic moments is favoured, while at low field and temperature, spin–lattice relaxation phenomena are noted. The spin–lattice interacts via the Rashba–Dresselhaus spin–orbit interaction due to breaking of symmetry and it results in anomalous velocity contribution to the MR.

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