A new aspects of physics of a photon gas

Levan N. Tsintsadze

Venture Business Laboratory, Hiroshima University, Higashi-Hiroshima, Japan

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Abstract

Bose-Einstein condensation (BEC) and evaporation of transverse photons from the Bose condensate is studied in the case when the density of plasma does not change. The generation of the longitudinal photons (photonikos) by the transverse photons (photons) in terms of the Cherenkov type of radiation in a uniform plasma is demonstrated. The Bogoliubov energy spectrum is derived for photonikos. A new physical phenomena of the "Compton" scattering type in nonlinear photon gas is discussed. To this end, a new version of the Pauli equation in the wavevector representation is derived. The formation of Bose-Einstein condensate and evaporation of photons from the condensate is investigated by means of the newly derived Fokker-Planck equation for photonikos. The relevance of this work to recent discovery of black hole X-ray jets is pointed out.

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I. INTRODUCTION

An immense amount of research has been carried out on propagation of relativistically intense electromagnetic (EM) waves into an isotropic plasma demonstrating that the relativistic oscillatory motion of electrons causes a whole set of interesting and salient phenomena [1]-[6], relevant to the study of laser accelerators of electrons, ions and photons, laser fusion, nonlinear optics, etc. Some of them have already been confirmed by experiments due to the recent progress in compact, high-power, short pulse laser technology. In our previous paper [7] we have suggested a new mechanism for ultrahigh gradient electron and ion acceleration. In addition we have shown over $10^7 G$ generated magnetic fields by the nonpotential ponderomotive force.

The above treatments were restricted to the case of monochromatic EM waves. However, the interaction of relativistically intense radiation with a plasma leads to several kinds of instabilities and the initially coherent spectrum may eventually broaden, and naturally for ultrashort pulses the initial bandwidth is increasingly broad. Moreover, in astronomical plasmas there are a variety of sources of radiation, and in this case we speak about the average density of radiation from all the sources and their spectral distribution. Hence, the natural state of the strong radiation of EM field is with a broad spectrum. In order to study the interaction of spectrally broad and relativistically intense EM waves with a plasma, it was necessary to derive a general equation for the EM spectral intensity. Such an investigation was reported recently [8], [9], where the authors derived a general kinetic equation for the photons in a plasma.

It should be emphasized that the radiation can be in two distinct states. Namely, one is when the total number of photons is not conserved. A good example is a black-body radiation. Another situation is when the total number of photons is conserved. Both these states have been studied in several aspects, mostly for the weak radiation.

It is well known that the nature of EM waves in a vacuum is quite different from the one in a medium. In the vacuum EM wave exists only in motion, however the light can be stopped (wavevector $\vec{k} = 0$, $\omega = \omega(0) \neq 0$) in different mediums and wave-guides. As was shown in Ref. [10] the photons acquire the rest mass and become one of the Bosons in plasmas and posses all characteristics of nonzero rest mass, i.e., we may say that the photon is the elementary particle of the optical field.

Besides, a several reviews and books have been published on theory of Bose-Einstein condensation (BEC) in a quantum Bose liquid [11] and in trapped gases [12], [13]. Kompaneets [14] has shown that the establishment of equilibrium between the photons and the electrons is possible through the Compton effect. In his consideration, since the free electron does not absorb and emit, but only scatters the photon, the total number of photons is conserved. Using the kinetic equation of Kompaneets, Zel’dovich and Levich [15] have shown that in the absence of absorption the photons undergo BEC. Such a possibility of the BEC occurs in the case, when the processes of change of energy and momentum in scattering dominates over the processes involving change of the photon number in their emission and absorption.

Very recently it was shown that exists an another new mechanism of the creation of equilibrium state and Bose-Einstein condensation in a nonideal dense photon gas [16]. More importantly a new effect was predicted in the same paper, namely that the inhomogeneous dense photon gas can be found in the intermediate state.
In the present paper, we consider the BEC and evaporation of the transverse photons (photons) from the Bose condensate. In our study we assume that the intensity of radiation (strong and super-strong laser pulse, non-thermal equilibrium cosmic field radiation, etc.) is sufficiently large, so that the photon-photon interaction can become more likely than the photon-electron interaction. We will show that for certain conditions the variation of the plasma density can be neglected in comparison with the variation of the photon density. In such case the elementary excitations represent the longitudinal photons (photonikos), for which we will derive the well known Bogoliubov’s energy spectrum.

The paper is organized as follows. First in Sec.II a basic equations, describing the relativistic photon-plasma interactions, is presented. The problem of stability of the photon flow is discussed in Sec.III. The derivation of the Bogoliubov energy spectrum for the photonikos is given in the same section. Then in Sec.IV, we derive the Pauli kinetic equation from the Wigner-Moyal equation. Section V is devoted to BEC. The Fokker-Planck equation for photonikos, by which we discuss the possibility of the creation of Bose-Einstein condensate and evaporation of the photons from the condensate, is obtained in the same section. Finally, a brief summary and discussion of our results are given in the last section.

II. BASIC EQUATIONS

If the intensity of photons is sufficiently large, then the photon-photon interaction can become more likely than the photon-plasma particle interaction. Under these conditions, which we shall assume to be realized, we may consider the medium to consist of two weakly interacting subsystems: the photon gas and the plasma, which slowly exchange energy between each other. In other words, relaxation in a photon-plasma system is then a two-stage process: firstly, the statistical equilibrium is established in each subsystem independently, with some average energies $E_\gamma = K_B T_\gamma$ and $E_p = K_B T_p$, where $K_B$ is the Boltzmann constant, $T_\gamma$ is the characteristic ”temperature” associated with the average kinetic energy of the photon, and $T_p$ is the plasma temperature. The photon effective ”temperature”, in general, will differ from the plasma temperature. Slower processes of the equalization of the photon and the plasma temperatures will take place afterwards.

In the following we will show that under some conditions the photon-photon interactions dominate the photon-particle interactions. When the photon-photon interaction takes place, the phases of the waves are, in general, random functions of time. We need therefore not be interested in the phases and can average over them. In such a situation the perturbation state of the photon gas can be described in terms of the occupation number $N(\vec{k}, \omega, \vec{r}, t)$ of photons, and one can study how these numbers change due to the processes of interaction of the photons with each other, or with plasma electrons. Note that $N(\vec{k}, \omega, \vec{r}, t)$ is the slowly varying function in space and time.

Recently, a new version of kinetic equation for the occupation number $N(\vec{k}, \omega, \vec{r}, t)$ of photons, for modes propagating with the wavevector $\vec{k}$ and the frequency $\omega$, at the position $\vec{r}$ and time $t$, was derived in Refs. [8], [9], [17]. It should be emphasized that the previous derivations of kinetic equation were based on envelope equations, were restricted to nonrelativistic plasmas and neglected the time variation of the photon (or the wavepacket) mean frequency. In contrast, our derivation avoids the envelope approximation, is valid for
a fully relativistic plasma, and includes space and time correlations. Since this equation has appeared previously in the literature \cite{8}, \cite{9}, \cite{17}, it is presented here without derivation

$$\frac{\partial}{\partial t} N(k, \omega, \vec{r}, t) + \frac{c^2}{\omega}(\vec{k} \cdot \nabla) N(k, \omega, \vec{r}, t) - \omega_p^2 \sin \frac{1}{2} \left( \nabla_\vec{r} \cdot \nabla_\vec{k} - \frac{\partial}{\partial t} \frac{\partial}{\partial \omega} \right) \cdot \rho \frac{N(k, \omega, \vec{r}, t)}{\omega} = 0 , \quad (1)$$

where $\omega_p = \sqrt{\frac{4 \pi e^2 n_0}{m_0 e}}$, $\rho = \frac{n_e}{n_0} \gamma$, $m_0$ is the electron rest mass, $n_e$ and $n_0$ are the non-equilibrium and equilibrium densities of the electrons, respectively, and $\gamma$ is the relativistic gamma factor of the electrons, which can be expressed as

$$\gamma = \sqrt{1 + Q} = \sqrt{1 + \beta \int d^3 k \int d\omega \frac{N(k, \omega, \vec{r}, t)}{2\pi \omega}} , \quad (2)$$

where $\beta = \frac{2 \hbar \omega_p^2}{m_0 n_0 c^2}$ and $\hbar$ is the Planck constant divided by $2\pi$.

Equation (1) is the generalization of the Wigner-Moyal equation for the classical electromagnetic (EM) field. We specifically note that from this equation follows conservation of the total number of photons, but not the momentum and energy of photons, i.e.,

$$N = 2 \int d\vec{r} \int \frac{d\vec{k}}{(2\pi)^3} \int \frac{d\omega}{2\pi} \frac{N(\vec{k}, \omega, \vec{r}, t)}{\omega} = \text{const.} , \quad (3)$$

where coefficient 2 denotes two possible polarization of the photons. Hence, the chemical potential, $\mu_{\gamma}$, of the photon gas is not zero.

If the local frequency $\omega$ and wavevector $\vec{k}$ are related by the dispersion equation, $\omega = \omega(k)$, then the occupation number is represented as $N(\vec{k}, \omega, \vec{r}, t) = 2\pi N(\vec{k}, \vec{r}, t) \delta(\omega - \omega(k))$ and Eq.(1) reduces to

$$\frac{\partial}{\partial t} N(\vec{k}, \vec{r}, t) + \frac{c^2}{\omega}(\vec{k} \cdot \nabla) N(k, \omega, \vec{r}, t) - \omega_p^2 \sin \frac{1}{2} \left( \nabla_\vec{r} \cdot \nabla_\vec{k} - \frac{\partial}{\partial t} \frac{\partial}{\partial \omega} \right) \cdot \rho \frac{N(k, \vec{r}, t)}{\omega} = 0 . \quad (4)$$

In the geometric optics approximation ($\sin x \approx x$), Eq.(1) reduces to the one-particle Liouville-Vlasov equation with an additional term

$$\frac{\partial}{\partial t} N(\vec{k}, \omega, \vec{r}, t) + \frac{c^2}{\omega}(\vec{k} \cdot \nabla) N(k, \omega, \vec{r}, t) - \frac{\omega_p^2}{2} \left( \nabla_\vec{r} \rho \cdot \nabla_\vec{k} - \frac{\partial}{\partial t} \frac{\partial}{\partial \omega} \right) \cdot \rho \frac{N(k, \vec{r}, t)}{\omega} = 0 . \quad (5)$$

From this kinetic equation we can obtain a set of fluid equations. To this end, we introduce the definitions of mean values of the density, velocity and $Q = \gamma^2 - 1$ of photons:

$$n_{\gamma}(\vec{r}, t) = 2 \int \frac{d^3 k}{(2\pi)^3} \int \frac{d\omega}{2\pi} N(\vec{k}, \omega, \vec{r}, t) . \quad (6)$$

The photon mean velocity is defined by

$$\vec{u}_{\gamma}(\vec{r}, t) = \frac{2}{n_{\gamma}} \frac{d^3 k}{(2\pi)^3} \frac{d\omega}{2\pi} \frac{\vec{k}^2}{\omega} N(\vec{k}, \omega, \vec{r}, t) , \quad (7)$$

and
Having these definitions, we shall construct the transport equations by the usual way. Namely, multiplying Eq.(5) by \( \bar{\hbar} \omega p \), \( \bar{\hbar} \omega \) and integrating over the entire spectral range, in \( \vec{k} \) and \( \omega \), we obtain equations of continuity, motion and \( Q \) of the photon gas:

\[
\frac{\partial n_\gamma}{\partial t} + \text{div} n_\gamma \vec{u}_\gamma = 0 ,
\]

\[
\frac{\partial \vec{u}_\gamma}{\partial t} + (\vec{u}_\gamma \cdot \vec{\nabla}) \vec{u}_\gamma = -\frac{1}{n_\gamma} \vec{\nabla} P_\gamma - \frac{U}{2n_\gamma} \left( \vec{\nabla} \rho + \frac{\vec{u}}{c^2} \cdot \frac{\partial \rho}{\partial t} \right) ,
\]

\[
\frac{\partial Q}{\partial t} + \text{div} Q \vec{u}_\gamma = G \frac{\partial \rho}{\partial t} ,
\]

where we have introduced the following notations in dimensional units

\[
U = \omega_p^2 c^2 \int \frac{d^3 k}{(2\pi)^3} \int \frac{d\omega}{2\pi} \frac{N(\vec{k}, \omega, \vec{r}, t)}{\omega^2} ,
\]

\[
P_\gamma = \frac{2}{3} \int \frac{d^3 k}{(2\pi)^3} \int \frac{d\omega}{2\pi} \left( \frac{\vec{k}c^2}{\omega} - \vec{u}_\gamma \right)^2 N(\vec{k}, \omega, \vec{r}, t) = n_\gamma T_\gamma .
\]

Obviously, \( P_\gamma \) is the pressure of the photon gas in unit mass. For \( G \) we have the following expression

\[
G = \frac{\hbar \omega_p \omega_p^2}{m_0 c^2 n_\text{e}} \cdot 2 \int \frac{d^3 k}{(2\pi)^3} \int \frac{d\omega}{2\pi} \frac{N(\vec{k}, \omega, \vec{r}, t)}{\omega^3} \approx \frac{\hbar \omega_p}{m_0 c^2} \frac{\omega_p^2}{\omega^3} \cdot \frac{n_\gamma}{n_\text{e}} .
\]

From Eq.(11) follows that the term on the right-hand side (RHS) is much less than the first term on the left-hand side (LFS), if \( G/\gamma^3 \ll 1 \). This inequality allows us to neglect the term on RHS in Eq.(11). With this assumption from Eqs.(9) and (11) follows the frozen-in condition

\[
\frac{Q}{n_\gamma} = \text{const} .
\]

It is important to emphasize that in Eqs.(1), (4), (5) and (10) there are two forces of distinct nature, which can change the occupation number of photons. One force appears due to the redistribution of electrons in space, \( \vec{\nabla} n_e \), and time, \( \frac{\partial n_e}{\partial t} \). The other force arises by the variation of the shape of wavepacket. In other words, this force originates from the alteration of the average kinetic energy of the electron oscillating in a rapidly varying field of the EM waves, and is proportional to \( \vec{\nabla} \gamma \) and \( \frac{\partial Q}{\partial t} \).
III. STABILITY OF THE PHOTON FLOW

In order to study the problem of stability of a photon flow, we consider the propagation of small perturbations in a homogeneous photon flux-plasma medium. First, we will derive a relation between the variation of photon and plasma densities, from which we will establish the condition that allows us to neglect the variation of the plasma density in comparison with the variation of the photon density.

To show this, we suppose the temperature of electrons to be nonrelativistic, i.e., \( T_e \ll m_0e^2c^2\gamma \). In this case, the equations describing a state of the electrons are as follows

\[
\frac{\partial \vec{p}_e}{\partial t} + (\vec{v}_e \cdot \vec{\nabla})\vec{p}_e = -e\vec{E} - m_0e^2c^2\nabla\gamma, \tag{16}
\]

\[
\frac{\partial n_e}{\partial t} + \text{div} \frac{\vec{p}_e}{m_0e}\gamma n_e = 0. \tag{17}
\]

We linearize Eqs. (16) and (17) with respect to perturbations, which are represented as \( \vec{p}_e = \delta\vec{p}_e, n_e = n_{0e} + \delta n_e \) and \( \gamma = \gamma_0 + \delta\gamma = \gamma_0 + \frac{\delta Q}{2\gamma_0} \), where the suffix 0 denotes the constant equilibrium value, and \( \delta\vec{p}_e, \delta n_e, \) and \( \delta Q \) are small variations in the wave. Taking into account the frozen-in condition (15), \( \delta Q \) can be expressed by the variation of the photon density as \( \delta Q = Q_0\frac{\delta n_e}{n_{0e}} = \frac{\gamma_0^2 - 1}{\gamma_0^2 n_{0e}} \delta n_\gamma \). After linearization, we will seek plane wave solutions proportional to \( \exp(i(q \cdot r - \Omega t)) \).

Here we consider the range of low frequencies for which the inequalities \( \Omega < qe < \omega_{pe} \) are fulfilled. In this case the photon flow can no longer excite the Langmuir plasma waves, and the contribution of perturbation of the electron density is rather small in comparison to perturbation of the photon density as

\[
\frac{\delta n_e}{n_{0e}} = -\frac{\gamma_0^2 - 1}{2\gamma_0^2} \frac{\omega_{pe}^2}{\omega_{pe} n_{0e}} \delta n_\gamma. \tag{18}
\]

Therefore, in our consideration the density of the plasma particles remain constant. In this case Eqs. (1) and (4) contain the variation of the relativistic \( \gamma \) factor alone.

In the following we shall demonstrate a new phenomena in the photon gas, which originates from the variation of shape of the wavepacket. Obviously, any waves, which can arise in the photon gas-plasma medium in this case, are the proper waves with the weak decrement of the photon gas. It means that these waves are associated with the photon gas, but not with the plasma.

To this end, we linearize Eq.(4) with respect to the perturbation, which is represented as

\[
N(\vec{k}, \omega(\vec{k}), \vec{r}, t) = N_0(\vec{k}, \omega(\vec{k})) + \delta N(\vec{k}, \omega(\vec{k}), \vec{r}, t). \tag{19}
\]

The result is

\[
\left(\frac{\vec{q} \cdot \vec{k}^2}{\omega} - \Omega\right) \delta N = -\omega_p^2 \frac{\gamma_0^2}{\gamma_0^2} \sum_{l=0}^{\infty} \frac{(\vec{q} \cdot \nabla k)^{2l+1}}{(2l + 1)!(2l+1)} \cdot \frac{N_0(k, \omega(k))}{\omega(k)}, \tag{20}
\]
or after summation we obtain

\[
\left( \frac{\vec{q}k_c^2}{\omega} - \Omega \right) \delta N = -\omega^2_p \frac{\delta \gamma}{\gamma_0^2} \left\{ \frac{N_0^+ (\vec{k} + \vec{q}/2)}{\omega(\vec{k} + \vec{q}/2)} - \frac{N_0^- (\vec{k} - \vec{q}/2)}{\omega(\vec{k} - \vec{q}/2)} \right\},
\]

(21)

where \( \delta \gamma = \frac{1}{2\gamma_0} \delta Q = \frac{\beta}{2\gamma_0} \int \frac{d^3k}{(2\pi)^3} \frac{\delta N(k,\omega,q,\Omega)}{\omega(k)}. \)

After integration over all wavevectors \( \vec{k} \), from Eq.(21) we get the dispersion relation due to the relativistic selfmodulation

\[
1 + \frac{\omega^2_p}{2\gamma_0^3} \beta \int d^3k \left\{ \frac{N_0^+ (\vec{k} + \vec{q}/2)}{\omega(\vec{k} + \vec{q}/2)} - \frac{N_0^- (\vec{k} - \vec{q}/2)}{\omega(\vec{k} - \vec{q}/2)} \right\}/\omega(k) (\frac{\vec{q}k_c^2}{\omega(k)} - \Omega) = 0,
\]

(22)

which has, in general, complex roots.

This equation has been derived and predicted by us first for the monochromatic wave in Ref. [18], and then for the broad spectrum in Ref. [8].

In Eq.(22) \( N_0(k) \) is the occupation number in the equilibrium state and is represented as

\[
N_0(k) = n_0 (2\pi \sigma_k^2)^{-3/2} \exp \left( -\frac{(\vec{k} - \vec{k}_0)^2}{2\sigma_k^2} \right).
\]

(23)

This is a spectral Gaussian distribution, with the average wavevector \( \vec{k}_0 \) and the spectral width \( \sigma_k \).

We can rewrite Eq.(22) in another form, taking into account a pole in the integral

\[
\Omega - \vec{q} \vec{u} = 0,
\]

(24)

where \( \vec{u} = \frac{\vec{k}_c^2}{\omega(\vec{k})} < c. \)

Using the well known relation

\[
\lim_{\varepsilon \to 0} \frac{1}{x + \varepsilon} = \frac{1}{x} - i\pi \delta(x),
\]

(25)

where \( \phi \) denotes the prescription that at the singularity \( x = 0 \) the principal value is to be taken, Eq.(22) is rewritten in the form

\[
1 + \frac{\omega^2_p}{2\gamma_0^3} \beta \int d^3k \left\{ \frac{N_0^+ (\vec{k} + \vec{q}/2)}{\omega(\vec{k} + \vec{q}/2)} - \frac{N_0^- (\vec{k} - \vec{q}/2)}{\omega(\vec{k} - \vec{q}/2)} \right\}/(\vec{q} \vec{u} - \Omega)
\]

\[
+ \frac{i\pi \omega^2_p}{2\gamma_0^3} \int d^3k \left\{ \frac{N_0^+ (\vec{k} + \vec{q}/2)}{\omega(\vec{k} + \vec{q}/2)} - \frac{N_0^- (\vec{k} - \vec{q}/2)}{\omega(\vec{k} - \vec{q}/2)} \right\} \delta(\Omega - \vec{q} \vec{u}) = 0.
\]

(26)

In the first and the third integrals we now replace the wavevector \( \vec{k} + \vec{q}/2 \) by \( \vec{k} \), and in the second and the forth integrals \( \vec{k} - \vec{q}/2 \to \vec{k} \). Assuming that \( \vec{k} = \vec{k}_0 + \vec{x}, |q|, |\vec{x}| \ll |\vec{k}_0|, \) after integration we obtain the dispersion relation

\[
1 + \frac{\Omega^2 V_E^2}{(\Omega - \vec{q} \vec{u})^2} - q^2 V_s^2 - \alpha^2 q^4 + \frac{i\pi \omega^2_p}{2\gamma_0^3 c^2 \omega(\vec{k}_0)} \left( \partial N_0(\chi_z) \right) \left|_{\chi_z = \frac{\omega(\vec{k}_0)}{\sigma_k^2} (\Omega - \vec{q} \vec{u})} \right. = 0,
\]

(27)
where \( \vec{u}_g = \frac{\vec{k}_0 c^2}{\omega(k_0)} \), \( \vec{q}\vec{u}_g = qu_g \cos \Theta \), \( N_0(\chi_z) = \int d\chi_x \int d\chi_y N_0(\chi) \), \( V_E = c \frac{\omega_p}{\omega(k_0)} \frac{1}{\sqrt{2\gamma_0}} \sqrt{\gamma_0^2 - 1} \gamma_0 \), \( \alpha = \frac{c^2}{2\omega(k_0)} \), and the velocity \( V_s = c \sqrt{3} \sqrt{\gamma_0^2 - 1} \frac{\sigma_k c}{\omega(k_0)} \) can be treated as the sound velocity of longitudinal photons, similar to phonons in the quantum liquid at almost zero temperature [11].

We now first neglect the small imaginary term in Eq.(27), and examine in great detail the following equation

\[
(\Omega - \vec{q}\vec{u}_g)^2 = q^2 \left\{ V_s^2 + \alpha^2 q^2 - V_E^2 \right\}.
\]

This equation has a complex solutions, when the inequality

\[
V_E^2 > V_s^2 + \alpha^2 q^2,
\]

for \( q < \frac{2\omega(k)}{c^2} \sqrt{V_E^2 - V_s^2} \), holds. The unstable solution can be written as \( \Omega = \Omega' + i\Omega'' \), where

\[
\Omega' = qu_g \cos \Theta
\]

and the growth rate for the unstable modes with \( q < \frac{2\omega(k)}{c^2} \sqrt{V_E^2 - V_s^2} \) is

\[
\Omega'' = q\sqrt{V_E^2 - V_s^2 - \alpha^2 q^2}.
\]

This instability for the monochromatic relativistic EM waves was predicted in Ref. [18], and for the broad spectrum at \( V_s = \alpha = 0 \) was disclosed in Ref. [9].

If \( \Omega'' > |\Omega''| \), then the solution (30) clearly describes the emission of the longitudinal photons (photonikos) by the bunch of the transverse photons (photons) inside a resonance cone (\( \cos \Theta = \Omega' / qu_g \)), similar to the well-known Cherenkov emission of EM waves by a charged particles moving in uniform medium with a velocity larger than the phase velocities of emitted waves. Also, here is an analogy with the Landau criterion about the creation of elementary excitations.

From Eq.(31) follows that there exists two range of values of the wavevector \( q \). Namely, for all \( q \) less than \( \frac{2\omega(k)}{c^2} \sqrt{V_E^2 - V_s^2} \), the Cherenkov type mechanism leads to the excitation of photonikos (large scale waves). For the other, \( q \) more than \( \frac{2\omega(k)}{c^2} \sqrt{V_E^2 - V_s^2} \), the frequency \( \Omega = \Omega' + \Omega'' \) is real.

We now consider the case, when \( V_s^2 > V_E^2 \) or \( \omega_p^2 < \gamma_0 \sigma_k^2 c^2 \). If we write Eq.(28) in the moving frame (\( \Omega - \vec{q}\vec{u} \rightarrow \Omega \)), and multiply both sides by the Planck constant \( \hbar \), then we obtain the well known Bogoliubov energy spectrum

\[
\varepsilon(p) = \sqrt{V_s^2 p^2 + \left( \frac{p^2}{2m_{eff}} \right)^2},
\]

where \( \varepsilon = \hbar \Omega, \ p = \hbar q \), and \( m_{eff} = \hbar \omega / c^2 \)

Equation (32) has the same form as the energy of the elementary excitations in the quantum liquid at zero temperature, which was derived first by Bogoliubov.

For small momentum \( (p < m_{eff} V = m_{eff} V_s^2 / V_E^2) \) from Eq.(32) follows \( \varepsilon = pV \approx V_s p \).

This expression show us that the coefficient \( V \) is the velocity of ”sound” in the photon gas,
similar to phonons in the quantum medium. For the large momentum \( p \gg m_{\text{eff}} V \), the photoniko energy (32) tends to \( \frac{p^2}{2m_{\text{eff}}} \), i.e., the kinetic energy of an individual photoniko.

Thus, we may conclude that the elementary excitations in our case correspond to photonikos.

The decrement, \( \text{Im}\Omega = \Omega' \), of photoniko is derived from Eq.(27), and the result is

\[
\text{Im}\Omega = -\frac{\omega_p}{\gamma_0} F \left( \frac{\gamma_0^2 - 1}{\gamma_0} \right)^2 \exp \left\{ -\frac{(\Omega - \vec{q}\vec{u})^2}{2q^2v_0^2} \right\},
\]

where \( v_0^2 = c^2 \sigma^2 \kappa c^2 \omega^2(k_0) \), \( F = \sqrt{\frac{\pi}{32}} \left( \frac{\omega_p}{\omega(k_0)} \right)^2 \frac{\sigma}{\gamma_k} \).

Clearly the expression (33) is valid for the weakly damped oscillations, that is \( \text{Re}\Omega \gg \text{Im}\Omega \). We note that the decrement has a maximum near the frequency of the Cherenkov resonance \( \Omega \simeq qu \cos \Theta \).

IV. PAULI EQUATION FOR THE PHOTON GAS

In the previous section we have derived the condition (18) allowing the situation, in which the density of plasma almost does not change, to exist. In this case from Eq.(1) follows that photons with different frequencies and wavevectors scatter on the photon bunch (wavepacket), and the equilibrium state may be established. This is a new phenomena of the ”Compton” scattering type, i.e., the statistical equilibrium between the photons and the photon bunch will establish itself as a result of the scattering processes.

In order to better exhibit this, we shall derive the Pauli equation in the wavevector representation. Note that the Wigner-Moyal-Tsintsadze equation (1) is convenient for a study of non-linear processes in the photon gas in the case of so-called weak turbulence states. Such states of the photon gas-plasma means that the energy of the quasi-particle (photoniko) is small compared to the photon energies.

Usually the weak-turbulence states arise in a medium, when a small perturbation grows with the growth rate smaller than the frequency, \( \text{Im}\Omega \ll \text{Re}\Omega \). In this case we can use a series expansions in powers of amplitude of the perturbations.

In the case of weak-turbulence, we can define the distribution function \( N(k, \omega, \vec{r}, t) \) in any approximation for the amplitude of perturbation by iterating the Wigner-Moyal equation. From Eq.(1), we now derive an equation in wavevector space in the limit of the spatial homogeneity, which may be identified with the Pauli equation. To this end, we take the Fourier transform of Eq.(1) to obtain

\[
\frac{\partial}{\partial t} N(k, \omega, \vec{q}, \Omega, t) - i(\Omega - \frac{\vec{q}kc^2}{\omega}) N(k, \omega, \vec{q}, \Omega, t) = \omega_p^2 \int \frac{d^3q'}{(2\pi)^3} \int \frac{d\Omega'}{2\pi} \delta\rho(\vec{q}', \Omega') \sin \frac{1}{2} (\vec{q}' \cdot \nabla_k + \Omega' \frac{\partial}{\partial \omega}) N(k, \omega, \vec{q}' - \vec{q}, \Omega' - \Omega, t),
\]

where \( N(k, \omega, \vec{q}, \Omega, t) = \int d\vec{r} \int dt N(k, \omega, \vec{r}, t) \exp(-i(\vec{q} \cdot \vec{r} - \Omega t)) \), and \( \delta\rho(\vec{q}, \Omega, t) = \int d\vec{r} \int dt \delta\rho(\vec{r}, t) \exp(-i(\vec{q} \cdot \vec{r} - \Omega t)) \) are the slowly varying functions in time.

Let us consider two cases. First, in the case of \( \vec{q} = 0 \) and \( \Omega = 0 \), after changing variables \( -\vec{q} \rightarrow \vec{q} \) and \( -\Omega \rightarrow \Omega \), we rewrite Eq.(34) in the form
\[
\frac{\partial N(\vec{k}, \omega, t)}{\partial t} = -\omega_p^2 \int \frac{d^3q}{(2\pi)^3} \int \frac{d\Omega}{2\pi} \delta \rho(-\vec{q}, -\Omega, t) \hat{L} \frac{N(\vec{k}, \omega, \vec{q}, \Omega, t)}{\omega(k)} .
\]

(35)

Here we have used the identity
\[
\sin \frac{i}{2} (\vec{q} \cdot \nabla_{\vec{k}} + \Omega \frac{\partial}{\partial \omega}) = i \sinh \frac{1}{2} (\vec{q} \cdot \nabla_{\vec{k}} + \Omega \frac{\partial}{\partial \omega}) = i \hat{L} .
\]

(36)

For the second case, when \( \vec{q} \neq 0 \) and \( \Omega \neq 0 \) (in this case \( \partial \ln N/\partial t \ll \Omega \)), Eq.(34) can be written as
\[
(\Omega - \frac{\vec{q}k^2}{\omega}) N(\vec{k}, \omega, \vec{q}, \Omega) = -\omega_p^2 \int \frac{d^3q}{(2\pi)^3} \int \frac{d\Omega}{2\pi} \delta \rho(-\vec{q}, -\Omega) \hat{L} \frac{N(\vec{k}, \omega, \vec{q} - \vec{q}', \Omega - \Omega')}{\omega} .
\]

(37)

By iterating this equation in the case of weak-turbulence we can write an equation for the occupation number \( N \) in any approximation in the amplitude of perturbations. Expanding the integrand about \( \vec{q} = \vec{q}' \), \( \Omega = \Omega' \) and keeping the first term with
\[
N(\vec{k}, \omega, \vec{q} - \vec{q}', \Omega - \Omega') = (2\pi)^4 \delta(\vec{q}' - \vec{q}) \delta(\Omega' - \Omega) N(\vec{k}, \omega) ,
\]

(38)

Substituting Eq.(38) into Eq.(35) we get
\[
\frac{\partial N(\vec{k}, \omega)}{\partial t} = \omega_p^4 \int \frac{d^3q}{(2\pi)^3} \int \frac{d\Omega}{2\pi} \hat{L} \left| \frac{\delta \rho(\vec{q}, \Omega)}{\delta(\vec{q} - \vec{q}') \omega(\Omega - \vec{q} \cdot \vec{u})} \right|^2 \frac{\Omega N(\vec{k}, \omega)}{\omega} .
\]

(39)

Since we are considering the weak-turbulence, for the perturbation the dispersion relation \( \Omega = \Omega(q) \) is valid, i.e.,
\[
\delta \rho(\vec{q}, \Omega) = 2\pi \delta(\Omega - \Omega(q)) \delta \rho(\vec{q}) .
\]

(40)

Noting the relation
\[
\sinh(y \frac{\partial}{\partial x}) f(x) = \frac{1}{2} \left\{ f(x + y) - f(x - y) \right\}
\]

(41)

and \( \delta \rho(-\vec{q}, -\Omega) = \delta \rho^*(\vec{q}, \Omega) \) (where the asterisk * depicts the Hermitian conjugate), we obtain from Eq.(39)
\[
\frac{\partial N(\vec{k}, \omega)}{\partial t} = i \omega_p^4 \int \frac{d^3q}{(2\pi)^3} \left| \frac{\delta \rho(\vec{q})}{\delta(\vec{q} - \vec{q}') \omega(\vec{k} + \vec{q}/2)} \right|^2 \left\{ \frac{N(\vec{k} + \vec{q}, \omega + \Omega)}{\omega(\vec{k} + \vec{q})} - \frac{N(\vec{k}, \omega)}{\omega(\vec{k})} \right\} \times \left[ \frac{1}{\Omega - \vec{q} \cdot \vec{u}} + \frac{1}{\Omega + \vec{q} \cdot \vec{u}} \right] ,
\]

(42)

where \( \vec{u} = (\vec{k} + \vec{q}/2) \omega^2 / \omega(\vec{k} + \vec{q}/2)^3 \) and use was made of dispersion relation \( \omega = \omega(k) \).

Use of Eq.(25) in Eq.(42) yields the following relation
\[ \frac{\partial N(\vec{k})}{\partial t} = \sum_{\pm} \int \frac{d^3q}{(2\pi)^3} \left[ W_{\pm}(\vec{k} + \vec{q}, \vec{k}) N(\vec{k} + \vec{q}) - W_{\pm}(\vec{k}, \vec{k} + \vec{q}) N(\vec{k}) \right]. \] (43)

Here we have introduced the scattering rate
\[ W_{\pm} = \frac{\pi}{2} \frac{\omega_{p}^4 |\delta \rho(q)|^2}{\omega(k + q/2)\omega(k)} \delta(\Omega \pm \vec{q} \cdot \vec{u}). \] (44)

Equation (43) describes the three wave interaction, as illustrated in Fig.1. First (a) diagram exhibits the absorption of the photoniko by the photon, whereas the second (b) diagram shows the emission of the photoniko by the photon. In other words, the photon passing through the photon bunch (wavepacket) absorbs and emits photonikos, with frequencies \( \Omega = \mp(\omega - \omega') \) and wavevectors \( \vec{q} = \mp(\vec{k} - \vec{k}') \).

Integral in Eq.(43) is the elastic collision integral and describes the photon scattering process on the variation of shape of the photon bunch.

It should be emphasized that the kinetic equation (43) is a new version of the Pauli equation in the wavevector representation [19]. Note that the equation type of (43) has been obtained by Pauli for a quantum system, whereas Eq.(43), derived for the dense photon gas, is pure classical. This equation indicates that the equilibrium of the photon gas is triggered by the perturbation \( \delta \rho = \delta(n/n_o \gamma) \), in particular by the perturbation in the wavepacket.

V. BOSE-EINSTEIN CONDENSATION IN THE PHOTON GAS

As we have mentioned in the Introduction, Zel’dovich and Levich have shown that the usual Compton scattering leads to the BEC in the nonrelativistic photon gas-plasma medium.

In this section, we will show that exists another new mechanism of BEC in a non-ideal dense photon gas. As was mentioned above, if the intensity of radiation is sufficiently large, then the photon-photon interaction can become more likely than the photon-electron interaction.

It is well known from plasma physics that the transverse photon can decay into the transverse photon and a plasmon (electron Langmuir and ion sound plasmons). For this process to take place it is necessary to have a fluctuation of the plasma density.

In our consideration, as was shown in Sec.III, the density of plasma remains constant. So that only one mechanism (relativistic effect), which can cause BEC, is the decay of the transverse photon into the transverse and the longitudinal (photoniko) photons. That is, in the weak-turbulence limit this mechanism is realized by three wave interaction, as we have discussed in Sec.IV. Namely, the photon \( (\omega, \vec{k}) \) generates photoniko and scatters on latter, with the frequency \( \omega' \) and the wavevector \( \vec{k}' \). In this case, the wave number and frequency matching conditions are satisfied, i.e., \( \vec{k} = \vec{k}' + \vec{q} \), and \( \omega = \omega' + \Omega \). These processes continue as a cascade \( (\omega' = \omega'' + \Omega', \vec{k}' = \vec{k}'' + \vec{q}', \text{etc.}) \) till the wavevector of the photon becomes zero and the frequency \( \omega = \omega_p/\gamma^{1/2} \). This cascade leads to the BEC of the photons and the creation of the photoniko gas. When the density of latter is sufficiently small, the photonikos may be regarded as non-interacting with each other, however they can exchange energies and momentum with a bunch of the photons by scattering mechanism, and finally such process can lead to the equilibrium state of the photoniko gas, with the Bose distribution.
\[ n_L = \frac{1}{\exp\left(\frac{\hbar^2 \overrightarrow{p} \cdot \overrightarrow{u}}{2k_B T} \right) - 1}, \]  

(45)

where \( \Omega \) is the Bogoliubov spectrum (32).

We note here that since the total number of photonikos does not conserve, the chemical potential of this gas is zero.

The result of BEC is that the ground state is filled by photons with the total rest energy \( E_0 = N_0 \frac{\hbar \omega_p}{\gamma^{1/2}} \), where \( N_0 \) is the total number of photons in the ground state.

We now express explicitly the energy of the ground state, \( E_0 \), through the total number, \( N_0 \), and the volume of the photon gas in two limits. First, we consider ultrarelativistic case, when \( \gamma = \sqrt{1 + Q} \approx Q^{1/2} = (\hbar \bar{\omega}_p/m_0 c^2)^{2/3}(N_0/N_{0e})^{2/3}(1/V^{1/3}) \), where \( \bar{\omega}_p = (4\pi e^2 N_{0e}/m_0 e^2)^{1/2} \), \( N_{0e} \) is the total number of electrons.

The chemical potential of the photon gas is

\[ \mu = \frac{\partial E_0}{\partial N_0} = 2 \left( \frac{N_{0e}}{N_0} \right)^{1/3} (\hbar \bar{\omega}_p)^{2/3} (m_0 c^2)^{1/3}. \]  

(46)

Note that in quantum Bose liquid at \( T = 0 \), the chemical potential increases with increase of \( N_0 \), whereas in the present case \( \mu \) decreases as shows Eq.(46).

We now find the pressure in the condensate, \( P_c \), as \( P_c = -\partial E_0/\partial V \), and write the equation of state for given \( N_0 \), the result is

\[ P_c V^{4/3} = \text{const}. \]  

(47)

Next in the nonrelativistic limit, we have \( \gamma \approx 1 + (1/2)(\hbar \bar{\omega}_p/m_0 c^2)(N_0/N_{0e})(1/V^{1/2}) \). Then for the equation of state and the chemical potential we get the following expressions

\[ P_c V^{3/2} = \text{const} \]  

(48)

and

\[ \mu = \hbar \bar{\omega}_p \left( 1 - \frac{N_0}{2N_{0e}} \frac{\hbar \omega_p}{m_0 c^2} \right). \]  

(49)

The second term in Eq.(49) is always less than one.

We note that when the conditions for the formation of BEC arise, then the photon gas in plasmas can be far from the equilibrium. The kinetics of this phenomena is a very important issue not only in the study of the contemporary problems of short pulse laser-matter interaction, but also for understanding of the processes, which take place on cosmic objects in the presence of strong radiation of EM field.

The kinetic equation (43) is the simplest nonequilibrium model in which we can expect BEC. To show this, we introduce the number density of the photons in the condensate as \( n_0 = \lim_{V \to \infty} \frac{N_0}{V} \). Then we can write \( N(\overrightarrow{k}, t) = 4\pi^3 \delta(\overrightarrow{k}) n_0(t) \), which means that the photons with a zero wavevector are in the condensate.

We next discuss another situation in which the photon with the wavevector \( \overrightarrow{k} \) is collided with the photoniko with \( \overrightarrow{q} \). For such processes exists the probability of the condition \( \overrightarrow{k} + \overrightarrow{q} = 0 \) to be satisfied for the photon and photoniko. Thus the photons absorbing the
photoniks will pass to the ground state. Therefore, the occupation number \(N(\vec{k} + \vec{q})\), which is under integral in Eq. (43), can be written as

\[
N(\vec{k} + \vec{q}) = 4\pi^3 \delta(\vec{k} + \vec{q}) n_1(t) + N(\vec{k} + \vec{q})_{\vec{k} \neq -\vec{q}},
\]

where \(n_1(t)\) is the density of pairs, for which the equality \(\vec{k} + \vec{q} = 0\) holds.

Substituting Eq. (50) into Eq. (43), for the Bose-Einstein condensate density \(n_0(t)\) we obtain

\[
\frac{\partial n_0(t)}{\partial t} = \frac{n_1(t) - n_0(t)}{\tau_0} + 2 \int_{k \neq 0} \frac{d^3k}{(2\pi)^3} \int \frac{d^3q}{(2\pi)^3} N(k', t) W(\vec{k}', \vec{k} - \vec{q}),
\]

where \(\tau_0^{-1} = \int \frac{d^3q}{(2\pi)^3} W(q) = \frac{\pi}{2} \omega^2 \int \frac{d^3q}{(2\pi)^3} |\delta \rho(q)|^2 \delta(\Omega - \frac{q^2 c^2 \gamma}{2\omega})^{1/2}\), and in the last term we have replaced \(\vec{k} + \vec{q} \rightarrow \vec{k}'\).

We specifically note here that the first and the third terms in Eq. (51) are the sources of production of the Bose-Einstein condensate, whereas the second term describes the evaporation of photons from the ground state.

As we can see from Eq. (51), the stationary solution may exist, and it is

\[
n_0 = n_1 + < \tau > n_2,
\]

where \(< \tau >^{-1} = \frac{2}{n_2} \int \frac{d^3k}{(2\pi)^3} W(k)\), with \(\tau(k)^{-1} = \int \frac{d^3q}{(2\pi)^3} W(k', k - q)\).

Equation (52) has an obvious physical meaning, which is the following: the departure of photons from the Bose-Einstein condensate should, under steady-state conditions, be completely balanced by their arrival from other modes.

The problem of BEC and evaporation of the Bose-Einstein condensate we can investigate by Fokker-Planck equation, which we shall derive from the Pauli equation. Here we suppose that the wavevector \(\vec{q}\) and the frequency \(\Omega\) of photoniks are small in comparison with \(\vec{k}\) and \(\omega\). This allows us to use the following expansion in the integrand

\[
W(\vec{k} + \vec{q}, \vec{k}) N(\vec{k} + \vec{q}) \approx W(k) N(k) + \vec{q} \frac{\partial}{\partial k} (W N) |_{q=0} + \frac{q_i q_j}{2} \frac{\partial^2 W N}{\partial k_i \partial k_j} |_{q=0} + \ldots.
\]

Substituting this expression into Eq. (43), we obtain the Fokker-Planck equation for photons, which describes the slow change of the occupation number in the wavevector space

\[
\frac{\partial N(k)}{\partial t} = \frac{\partial}{\partial k_i} \left\{ A_i N(k) + \frac{1}{2} \frac{\partial}{\partial k_j} (D_{ij} N(k)) \right\},
\]

where

\[
A_i = \int \frac{d^3q}{(2\pi)^3} q_i W(q, k),
\]

\[
D_{ij} = \int \frac{d^3q}{(2\pi)^3} q_i q_j W(q, k).
\]
The quantity $A_i$ is the dynamic friction coefficient, whereas $D_{ij}$ is the diffusion tensor in the wavevector space.

Introducing the following definition $B_i = A_i + \frac{\partial D_{ij}}{\partial k_j}$, and noting that the expression on RHS in Eq.(54) is divergent in wavevector space, Eq.(54) can be rewritten in the form

$$\frac{\partial N}{\partial t} + \frac{\partial}{\partial k_i} j_i = 0 ,$$

(57)

where $j_i = -B_i N - \frac{D_{ij}}{2} \frac{\partial N}{\partial k_j}$ is the photon flux density in wavevector space. The fact that the flux should be zero allowed us to express $B_i$ and $D_{ij}$ in terms of one another.

The equilibrium distribution function we choose to be Gaussian

$$N(k) = \text{const} \cdot e^{-\frac{k^2}{2\sigma_k^2}} .$$

(58)

Substituting expression (58) into the equation $\vec{j} = 0$, we obtain

$$B_i \sigma_k^2 = \frac{1}{2} k_j D_{ij} .$$

(59)

Finally the transport equation of the photon gas takes the form

$$\frac{\partial N}{\partial t} = \frac{\partial}{\partial k_i} \left\{ \frac{D_{ij}}{2} \left( \frac{k_j N}{\sigma_k^2} + \frac{\partial N}{\partial k_j} \right) \right\} .$$

(60)

In order to solve Eq.(60), we consider a simple model representing the diffusion tensor as $D_{ij} = D_0 \delta_{ij}$, with $D_0 = \text{const}$. Such situation is realized when $u \cdot \cos \Theta \ll qe^2/\omega$. With this assumption Eq.(60) reduces to

$$\frac{\partial N}{\partial t} = a \frac{\partial}{\partial k_i} (\vec{k} N) + \frac{D_0}{2} \nabla_k^2 N ,$$

(61)

where $a = \frac{D_0}{2\sigma_k^2}$.

To discover the physical meaning of terms on RHS of Eq.(61), we consider them separately. First we neglect the diffusion term in Eq.(61), and assuming $k(0, 0, k)$, we get for the new function $f = k N$

$$\frac{\partial f}{\partial t} - ak \frac{\partial f}{\partial k} = 0 .$$

(62)

The general solution of this equation is an arbitrary function of $k_0 = ke^{at}$ (where $k_0$ is the initial value of the wavevector), i.e., $f(k_0) = f(ke^{at})$. This function is constant on the curves $k_0 = ke^{at}$, while at $t \to \infty$ the wavevector goes to zero ($k \to 0$). The meaning of this is that the occupation number would tend to peak toward the origin as

$$N(k, t) = \frac{f}{k} = \text{const} \cdot e^{at} .$$

(63)

Thus we conclude that the friction effect leads to BEC of the photon gas.
It should be emphasized that the exponential decay of the wavevector \( k = k_0 e^{-at} \) is due to the linearity of the Fokker-Planck equation. Note that we have neglected all nonlinear terms in deriving the Pauli equation. The presence of nonlinearity could saturate the exponential growth of \( N(k, t) \). In the above approximation (63) the condensate formation time, \( t_c \), is defined by the relation \( at_c \sim 1 \).

Second, we suppose that \( W(\vec{q}, \vec{k}) \) is the even function with respect to \( \vec{q} \). In this case \( A = 0 \), and we have

\[
\frac{\partial N}{\partial t} = \frac{D_0}{2} \nabla_k^2 N . \tag{64}
\]

Assuming that initially all photons are in the ground state with \( k = 0 \), and the occupation number of the photons is \( N = 4\pi^3 n_0 \delta(\vec{k}) \), then the solution of Eq.(64) reads

\[
N(k, t) = n_0 e^{-k^2 / (2D_0 t)} / (2\pi D_0 t)^{1/2} . \tag{65}
\]

In the course of time, the number of photons with \( k = 0 \) decreases as \( t^{-1/2} \). The number of photons in the surrounding wavevector space rises correspondingly, and initially peaked at the origin is to be flatten out. Let us determine the mean square wavevector from origin at time \( t \). From expression (65) we have

\[
<k^2> = D_0 t . \tag{66}
\]

Thus, \( \sqrt{<k^2>} \) increases as the square root of time.

Eqs. (65) and (66) manifest that the evaporation of the photons from the condensate takes place.

We now derive a relation between the diffusion time, \( t_D \), and the time of the condensation. From Eqs.(63) and (65) we obtain

\[
t_c \sim \frac{2\sigma_k^2}{D_0} \quad \text{and} \quad t_D \sim \frac{k^2}{2D_0} ,
\]

where \( \sigma_k = \frac{1}{2r_0} \), as follows from the Wigner function \( N(\vec{k}, \vec{r}) = N_0 \exp \left( -\frac{r^2}{2r_0^2} - \frac{k^2}{2\sigma_k^2} \right) \). Note that \( r_0 \) is the initial cross size of the photon bunch.

Finally, we arrive at the desired relation

\[
\frac{t_D}{t_c} = k^2 r_0^2 . \tag{67}
\]

From here it is evident that for the condensation and evaporation of photons, it is necessary that the following inequality \( t_D \gg t_c \) is satisfied. This is to be expected, as in realistic astrophysical objects with a strong radiation, as well as in laboratory experiments with a laser radiation, the following condition \( k^2 r_0^2 \gg 1 \) is valid.

\[\text{VI. SUMMARY AND DISCUSSIONS}\]

We have investigated a class of problems involving the interaction of spectrally broad and relativistically intense EM radiation with a plasma in the case when the photon-photon
interaction dominates the photon-particle interactions. We have presented a new concept of the establishment of equilibrium between the photon and the wavepacket of EM field (the dense photon bunch). This is a fully relativistic effect due to the strong EM radiation. We have established the condition under which the variation of the plasma density can be neglected in comparison with the variation of the photon density. In such case the elementary excitations represent the photonikos, for which we have derived the well known Bogoliubov energy spectrum. We have studied the BEC and evaporation of the photons from the Bose condensate in the case when the density of plasma does not change. To this end, from the Wigner-Moyal-Tsintsadze equation [8], [9], [17] we have derived a new version of the Pauli kinetic equation for the photon gas. For the case, when the wavevector and frequency of the photoniko is small in comparison with the wavevector and frequency of the photon, we have derived the Fokker-Planck equation for photonikos. We have presented a simple model, which exhibits the possibility of the creation of Bose-Einstein condensate and evaporation of photons from the condensate. We think that these processes can be detectable in next generation experiments with appropriate instrumentation. In fact, a number of experiments have been carried out in which plasmas are irradiated by laser beams with intensities up to $3 \cdot 10^{20} W/cm^2$. At such intensities the photon density is of the order of $n_\gamma \sim 10^{29} cm^{-3}$. For the plasma densities up to $n_e \sim 10^{19}$, we should expect that the Bose-Einstein condensate becomes observable. The theory developed in this paper should also be the case for astrophysical objects, such as a black hole. In this connection we speculate that the recently observed radiation from the black hole may be attributed to the evaporated photons from the Bose condensate. That is initially all photons fall into Bose-Einstein condensate, and then after a certain time, as discussed in this paper, some photons undergo evaporation from the condensate. These processes may also explain the observable variation in radiation intensity, from being undetectable to one of the brightest sources on the sky. Note that these sources can turn off for decades, and new ones are always being found. In addition Universe is filled with a gas of photonikos. This gas may play the decisive role in the expansion of the Universe. Moreover, it may also be useful for explaining certain processes in supernovae explosion. Finally, the present theory may also find a valuable application in the future space technology, as well as in nonlinear optics.
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FIGURES

FIG. 1. a) Absorption of the photoniko by the photon
b) Emission of the photoniko by the photon
Fig. 1a Tsintsadze
Fig. 1b Tsintsadze