Nuclear dependences of azimuthal asymmetries in the Drell-Yan process

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We study nuclear dependences of azimuthal asymmetries in the Drell-Yan lepton pair production in nucleon-nucleus collisions with polarized nucleons. We use the “maximal two-gluon correlation approximation”, so that we can relate the transverse momentum dependent quark distribution in a nucleus to that in a nucleon by a convolution with a Gaussian broadening. We use the Gaussian ansatz for the transverse momentum dependence of such quark distribution functions, and obtain the numerical results for the nuclear dependences. These results show that the averaged azimuthal asymmetries are suppressed.

I. INTRODUCTION

Both the deep inelastic scattering (DIS) off hadrons and the Drell-Yan (DY) process in hadron-hadron collision have been playing very important roles in studying
the structure of hadron and the dynamics of Quantum Chromodynamics (QCD). Correspondingly, the DIS off nuclei and the DY process in hadron-nucleus collision are also very important in studying the nuclear structure and the property of the cold nuclear matter. By studying the corresponding semi-inclusive processes, we can study not only the longitudinal but also the transverse momentum dependence of the parton distribution functions. In this connection, the semi-inclusive DY process is even more suitable to study the structure of hadron or that of nucleus since no fragmentation function is involved. Azimuthal asymmetries are often sensitive physical variables for such studies and thus have attracted much attention. [1–11]

When a parton transmits through a nuclear matter, the multiple gluon scattering with the nuclear matter leads to the energy loss and transverse momentum broadening. [12–24] The multiple parton interaction results in also nuclear dependences of azimuthal asymmetries. This provides a good alternative probe of properties of the nuclear matter. The nuclear dependence of the azimuthal asymmetry in semi-inclusive deep inelastic scattering (SIDIS) have been studied recently [25, 27, 28]. We extend the study [25, 27] to the DY process in nucleon-nucleus collisions in this paper. In Sec. II, we review the result of the differential cross section in DY with polarized nucleon beam in terms of the TMD parton distributions up to twist-2 level. In Sec. III, we study the nuclear dependence of the angular distribution of the DY lepton pair by relating the transverse momentum dependent quark distributions in a nucleus to that in a nucleon. We also illustrate the numerical results with an ansatz of the TMD parton distributions in a Gaussian form. We give a brief summary in Sec. IV.
II. DIFFERENTIAL CROSS SECTION AND AZIMUTHAL ASYMMETRIES

We consider the semi-inclusive DY process in nucleon-nucleus collisions with the transversely or longitudinally polarized nucleon beam,

\[ N(p_1, s) + A(p_2) \rightarrow \gamma^*(q) + X \rightarrow l^+(l) + l^-(l') + X \]  

(2.1)

where \( p_1, p_2, q, l \) and \( l' \) are the four-momenta of the beam nucleon, one nucleon in the nucleus target, the virtual photon, the anti-lepton and the lepton, respectively; and \( s \) denotes the polarization vector of the incident nucleon. We use the light cone coordinate by introducing two light-like vectors \( n_+ = [1, 0, \vec{0}_\perp] \), \( n_- = [0, 1, \vec{0}_\perp] \) and express the momenta \( p_1 \) and \( p_2 \) as,

\[ p_1^\mu = p_1^+ n_+^\mu + \frac{M^2}{2p_1} n_-^\mu, \]  

(2.2)

\[ p_2^\mu = \frac{M^2}{2p_2} n_+^\mu + p_- n_-^\mu \]  

(2.3)

where \( p^+ = p \cdot n_+ = p \cdot n_- \), and \( M \) denotes the mass of the nucleon. We restrict our study to the kinematic region where the transverse momentum \( q_T \) of DY pair is much less than its invariant mass \( Q = \sqrt{q^2} \). In this case, the differential cross section for the semi-inclusive DY process can be calculated in the framework of the transverse momentum dependent (TMD) factorization theorem [31, 32]. Such calculations have been carried out for hadron-hadron collisions, the results can be found in e.g. Refs. [33–35]. We note that such calculations can be extended to nucleon-nucleus in a straigt forward way and, at the twist-2 level, the differential cross section is given by,

\[
\frac{d\sigma}{d^2\Omega d^2q_T dx_1 dx_2} = \frac{\alpha_{em}^2}{4Q^2} \left\{ (1 + \cos^2 \theta) F_0[f_1, f_1] + \sin^2 \theta \cos 2\phi F_1[h_1^+, h_1^+] \\
+ \lambda_s \sin^2 \theta \sin 2\phi F_3[h_{1L}, h_{1T}] - |s_T|(1 + \cos^2 \theta) \sin \phi_s F_2[f_{1T}, f_1] \\
+ |s_T| \sin^2 \theta \sin(2\phi - \phi_s) F_3[h_1, h_1^+] + |s_T| \sin^2 \theta \sin(2\phi + \phi_s) F_4[h_{1T}, h_1^+] \right\}, \tag{2.4}
\]
where \( \lambda \) and \( \vec{s}_T \) are respectively the helicity and the transverse polarization vector of the nucleon; \( \theta, \phi \) and \( \phi_s \) are respectively polar and azimuthal angles of the lepton pair, azimuthal angle of the polarization vector of the nucleon with respect to the transverse vector \( \vec{q}_T \) in the Collins-Soper frame. The \( \mathcal{F}_j[f, h] \)'s \((j = 0 \text{ through } 4)\) are functionals of \( f(x, \vec{k}_T) \) and \( h(x, \vec{k}_T) \) that are defined as convolutions weighted by \( \chi_j(\vec{q}_T, \vec{k}_{1T}, \vec{k}_{2T}) \),

\[
\mathcal{F}_j[f, h] \equiv \frac{1}{3} \sum_a e_a^2 \int \frac{d^2k_{1T}}{(2\pi)^2} \frac{d^2k_{2T}}{(2\pi)^2} \delta^2(\vec{k}_{1T} + \vec{k}_{2T} - \vec{q}_T) \chi_j(\vec{q}_T, \vec{k}_{1T}, \vec{k}_{2T}) \\
\times \left[ f^N(x_1, \vec{k}_{1T}; a) h^A(x_2, \vec{k}_{2T}; \bar{a}) + f^N(x_1, \vec{k}_{1T}; \bar{a}) h^A(x_2, \vec{k}_{2T}; a) \right] \tag{2.5}
\]

where \( f \) and \( h \) are TMD distribution and/or correlation functions of quark or anti-quark. The superscript \( N \) or \( A \) denotes whether it is for the nucleon or the nucleus, \( a \) and \( \bar{a} \) in the arguments denote the flavor of the quark and whether it is for the quark or the anti-quark. The weights \( \chi_j \) are given by,

\[
\begin{align*}
\chi_0 & = 1, \\
\chi_1 & = \frac{1}{M^2} \left[ 2 \left( \vec{k}_{1T} \cdot \vec{q}_T \right) \left( \vec{k}_{2T} \cdot \vec{q}_T \right) - \vec{k}_{1T} \cdot \vec{k}_{2T} \right], \\
\chi_2 & = \frac{1}{M} \vec{k}_{1T} \cdot \vec{q}_T, \\
\chi_3 & = \frac{1}{M} \vec{k}_{2T} \cdot \vec{q}_T, \\
\chi_4 & = \frac{1}{2M^2} \left[ 4 \left( \vec{k}_{1T} \cdot \vec{q}_T \right)^2 \left( \vec{k}_{2T} \cdot \vec{q}_T \right) - 2 \left( \vec{k}_{1T} \cdot \vec{q}_T \right) \left( \vec{k}_{2T} \cdot \vec{k}_{1T} \right) - \vec{k}_{1T} \left( \vec{k}_{2T} \cdot \vec{q}_T \right) \right].
\end{align*}
\]

where \( \vec{q}_T \equiv \vec{q}_T / \sqrt{\vec{q}_T^2} \). All the parton distribution and correlation functions, \( f \)'s and \( h \)'s, given in Eq. (2.4) are defined from the twist-2 decomposition of the quark correlation matrix \[36-38\],

\[
\Phi(x, \vec{k}_T, s) = \left\{ f_1^+ \left( \frac{\gamma_5 \gamma_T \psi^+}{2} \right) + f_1^- \left( \frac{\gamma_5 \gamma_T \psi^-}{2} \right) + h_1^+ \left( \frac{\gamma_5 \gamma_T \psi^+}{2} \right) + h_1^- \left( \frac{\gamma_5 \gamma_T \psi^-}{2} \right) \right\}.
\]

\]

\[
\Phi(x, \vec{k}_T, s) = \left\{ f_1^+ \left( \frac{\gamma_5 \gamma_T \psi^+}{2} \right) + f_1^- \left( \frac{\gamma_5 \gamma_T \psi^-}{2} \right) + h_1^+ \left( \frac{\gamma_5 \gamma_T \psi^+}{2} \right) + h_1^- \left( \frac{\gamma_5 \gamma_T \psi^-}{2} \right) \right\}.
\]

\[
\Phi(x, \vec{k}_T, s) = \left\{ f_1^+ \left( \frac{\gamma_5 \gamma_T \psi^+}{2} \right) + f_1^- \left( \frac{\gamma_5 \gamma_T \psi^-}{2} \right) + h_1^+ \left( \frac{\gamma_5 \gamma_T \psi^+}{2} \right) + h_1^- \left( \frac{\gamma_5 \gamma_T \psi^-}{2} \right) \right\}.
\]

\[
\Phi(x, \vec{k}_T, s) = \left\{ f_1^+ \left( \frac{\gamma_5 \gamma_T \psi^+}{2} \right) + f_1^- \left( \frac{\gamma_5 \gamma_T \psi^-}{2} \right) + h_1^+ \left( \frac{\gamma_5 \gamma_T \psi^+}{2} \right) + h_1^- \left( \frac{\gamma_5 \gamma_T \psi^-}{2} \right) \right\}.
\]
where \( \epsilon^\mu_{\nu\rho\sigma} = \epsilon^{\mu\nu\rho\sigma} n_+ n_- \) with the total antisymmetric tensor \( \epsilon^{0123} = +1 \) and the definition of the components of \( \Phi(x, \tilde{k}_T, s) \) are given by,

\[
\Phi_{\alpha\beta}(x, \tilde{k}_T, s) \equiv \int \frac{p^+ dy^- d^2 y_\perp}{2\pi} \frac{1}{(2\pi)^2} e^{i p^+ y^- - i \tilde{k}_T \cdot y_\perp} \langle N, s | \tilde{\psi}_\beta(0) \mathcal{L}(0, y) \psi_\alpha(y) | N, s \rangle,
\]

where \( \mathcal{L}(0, y) \) is the gauge link that is necessary to ensure the gauge invariance of the matrix. In the DY process, the gauge link in covariant gauge is given by

\[
\mathcal{L}(0, y) = \mathcal{L}[y^-, \bar{y}_\perp; -\infty, \bar{y}_\perp] \mathcal{L}^\dagger[0, \bar{0}_\perp; -\infty, \bar{0}_\perp],
\]

where

\[
\mathcal{L}[y^-, \bar{y}_\perp; -\infty, \bar{y}_\perp] \equiv P \exp \left( -ig \int_{-\infty}^{y^-} d\xi^- A^+(\xi^-, \bar{y}_\perp) \right).
\]

It should be noted that the distribution function \( h_1 \) in Eq. (2.4) is defined as the mixture of \( h_{1T} \) and \( h_{1T}^\perp, \)

\[
h_1(x, \tilde{k}_T) \equiv h_{1T}(x, \tilde{k}_T) + \frac{\vec{k}_T^2}{2M^2} h_{1T}^\perp(x, \tilde{k}_T)
\]

We see that the differential cross section is determined by six TMD quark and anti-quark distributions and correlation functions \( f_1(x, \tilde{k}_T), f_{1T}^\perp(x, \tilde{k}_T), h_{1T}^\perp(x, \tilde{k}_T), \)

\( h_1(x, \tilde{k}_T), h_{1T}(x, \tilde{k}_T) \) and \( h_{1T}^\perp(x, \tilde{k}_T). \) Each of them represents a given aspect of the parton structure of the nucleon. While the physical meaning of some of them are not obvious, some of them are clear and well known. In particular, we know that \( f_{1T}^\perp(x, \tilde{k}_T) \) is the Sivers function\(^{[39, 40]} \) which describes the correlation between the transverse momentum distribution and the transverse polarization of the nucleon, and \( h_{1T}^\perp(x, \tilde{k}_T) \) is the Boer-Mulders function\(^{[41]} \) which describes the correlation between the transverse quark momentum distribution and the transverse quark polarization in an unpolarized nucleon.

We integrate over the polar angle \( \theta \) in Eq. (2.4) and obtain,

\[
\frac{d\sigma}{d\phi d^2 q_T dx_1 dx_2} = \frac{\alpha_{em}^2}{3Q^2} \left\{ 2 \mathcal{F}_0 [f_1, f_1] + \cos 2\phi \mathcal{F}_1 [h_{1T}^\perp, h_{1T}^\perp] \right. \\
+ \lambda_s \sin 2\phi \mathcal{F}_1 [h_{1T}^T, h_{1T}^+T] - 2|s_T| \sin \phi_s \mathcal{F}_2 [f_{1T}^\perp, f_1] \\
+ |s_T| \sin (2\phi - \phi_s) \mathcal{F}_3 [h_1, h_{1T}^\perp] + |s_T| \sin (2\phi + \phi_s) \mathcal{F}_4 [h_{1T}^\perp, h_{1T}^\perp] \left\}, \quad (2.11)
\]
We see that there are five kinds of different azimuthal asymmetries that are given by the average of \( \cos 2\phi \), \( \sin 2\phi \), \( \sin(2\phi - \phi_s) \), \( \sin(2\phi + \phi_s) \) and \( \sin(2\phi + \phi_s) \) respectively. In terms of parton distribution and correlation functions, they are given by,

\[
A_{NA}^{\cos 2\phi} = \frac{\mathcal{F}_1[h_{1\perp}^+, h_{1\perp}^+]}{4F_0[f_1, f_1]},
\]

(2.12)

\[
A_{NA}^{\sin 2\phi} = \frac{\lambda_s \mathcal{F}_1[h_{1L}^+, h_{1\perp}^+]}{4F_0[f_1, f_1]},
\]

(2.13)

\[
A_{NA}^{\sin \phi_s} = -|\vec{s}_T| \frac{\mathcal{F}_2[f_{1T}^+, f_1]}{2F_0[f_1, f_1]},
\]

(2.14)

\[
A_{NA}^{\sin(2\phi - \phi_s)} = |\vec{s}_T| \frac{\mathcal{F}_3[h_1, h_{1\perp}^+]}{4F_0[f_1, f_1]},
\]

(2.15)

\[
A_{NA}^{\sin(2\phi + \phi_s)} = |\vec{s}_T| \frac{\mathcal{F}_4[h_{1T}^+, h_{1\perp}^+]}{4F_0[f_1, f_1]}.
\]

(2.16)

where the subscript \( NA \) denotes the nucleon-nucleus collision. They differ from those for nucleon-nucleon collisions only by the quark distribution and/or correlation functions as given by Eq.(2.7).

We also note that these asymmetries exist in collisions with the unpolarized, longitudinally polarized and transversely polarized nucleon beam respectively. In the unpolarized case, only \( \cos 2\phi \) exists and is determined by the Boer-Mulders functions \( h_{1\perp}^+ \). There is one single spin asymmetry (SSA) \( \sin 2\phi \) in collisions with longitudinally polarized beam, and it is determined by the longitudinal transversity \( h_{1L}^+ \) and the Boer-Mulders function \( h_{1\perp}^+ \). There are three SSA’s in collisions with transversely polarized beam. They are represented by \( \sin \phi_s \), \( \sin(2\phi - \phi_s) \) and \( \sin(2\phi + \phi_s) \). The well known SSA \( \sin \phi_s \) is determined by the Sivers function \( f_{1T}^+ \), while \( \sin(2\phi + \phi_s) \) and \( \sin(2\phi - \phi_s) \) are determined by the Boer-Mulders function \( h_{1\perp}^+ \) together with the pretzelosity \( h_{1T}^+ \) or \( h_1 \) mixed from \( h_{1T} \) and \( h_{1T}^+ \) respectively. Although we still know not much about them, these functions have been studied in semi-inclusive DIS \cite{42-45} and some rough parameterizations have already been made \cite{46-51}.
III. NUCLEAR DEPENDENCE

It has been shown \cite{21, 25} that multiple gluon scattering represented by the
gauge link given by Eq. (2.8) leads to a strong nuclear dependence of TMD parton
distributions and/or correlation functions. Since all the asymmetries presented
above are functionals of these parton correlation functions, we expect strong nuclear
dependence of these asymmetries. We discuss them in the following.

A. Nuclear dependence of the TMD parton correlation function

In Ref. \cite{21}, with the assumption that the nucleus is large and weakly bound,
the multiple-nucleon correlation can be neglected, the nuclear effect can only arise
from the final state interaction in the form of multiple gluon scattering that is
encoded into the gauge link in the definition of the TMD parton distributions.
The important trick for the derivations is that the TMD quark distributions in
nucleons or nuclei can be rewritten as a sum of higher-twist collinear parton matrix
elements,

\[
f_1^A(x, \vec{k}_T) = \int \frac{dy^-}{2\pi} e^{i x p^+ y^-} \langle A | \bar{\psi}(0) \gamma^+ \frac{1}{2} e^{W_T(y^-) \cdot \vec{\nabla} k_T} \psi(y^-) | A \rangle \delta^{(2)}(\vec{k}_T),
\]

where \( W_T(y^-) \) is the parton transport operator and given by

\[
W_T(y^-) = i \vec{D}_T(y^-) + g \int_{-\infty}^{y^-} d\xi^- \vec{F}_{+T}(\xi^-),
\]

with \( \vec{D}_T(y^-) \) being the covariant derivative. For simplicity, we have chosen the
light-cone gauge in which the collinear gauge link disappears in the above. The
nuclear effect arises when the the parton transport operator acts on the differ-
ent nucleons. Under the “maximum two-gluon correlation approximation” \cite{21},
the nuclear TMD parton distribution has been expressed in terms of a Gaussian
convolution of the same TMD distribution in a nucleon, i.e.,

\[
f_1^A(x, \vec{k}_T) \approx \frac{A}{\pi \Delta_{2F}} \int d^2 l_T e^{-\frac{(\vec{k}_T - \vec{l}_T)^2}{2 \Delta_{2F}^2}} f_1^N(x, \vec{l}_T),
\]
where $\Delta_{2F}$ denotes the total average squared transverse momentum broadening. Furthermore, it has been shown in Ref. [25] that the relation can be extended to a much more general case so that,

$$\Phi^A(x, \vec{k}_T) \approx A \exp \left[ \frac{\Delta_{2F} \nabla^2_{k_T}}{4} \right] \Phi^N(x, \vec{k}_T)$$

$$= \frac{A}{\pi \Delta_{2F}} \int d^2 l_T e^{-(\vec{k}_T - \vec{l}_T)^2/\Delta_{2F}} \Phi^N(x, \vec{l}_T). \quad (3.4)$$

where the components of the matrix $\Phi^A$ is defined in Eq. (2.7). Recently it has been shown [26] that such simple Gaussian convolution will be broken by the process dependent gauge links in cold nuclear matter.

For the DY azimuthal asymmetries presented in last section for nucleon-nucleus collisions, besides the TMD quark distribution $f^A_1$, the Boer-Mulders distribution $h^\perp_1$ in nucleus is also involved. From the decomposition in (2.7), we can express the Boer-Mulders distribution as the following

$$h^{N/A}_1(x, \vec{k}_T) = M^2 \frac{1}{2k^2_T} \text{Tr} \left[ i \vec{k}_T \cdot \Phi^N \right] \quad (3.5)$$

From the relation (3.4), we can show that Boer-Mulders function in the nucleus is related to that in the nucleon in the exactly same way as the twist-3 distribution in Ref. [25], i.e.,

$$h^{N/A}_1(x, \vec{k}_T) = \frac{A}{\pi \Delta_{2F}} \int d^2 l_T \frac{(\vec{k}_T \cdot \vec{l}_T)}{k^2_T} e^{-(\vec{k}_T - \vec{l}_T)^2/\Delta_{2F}} h^{N/A}_1(x, \vec{l}_T) \quad (3.6)$$

If we take the Gaussian ansatz for the transverse momentum dependence, i.e.,

$$f^N_1(x, \vec{k}_T) = \frac{1}{\pi \alpha} f^N_1(x) e^{-\frac{\vec{k}_T^2}{\alpha}}, \quad (3.7)$$

$$h^{N}_1(x, \vec{k}_T) = \frac{1}{\pi \beta} h^N_1(x) e^{-\frac{\vec{k}_T^2}{\beta}}, \quad (3.8)$$

where we have assumed different flavors have the same Gaussian widths for the same type TMD distributions and suppressed the flavor index. We obtain from
Eqs. (3.3) and (3.6) that

\[ f^{A_1}(x, \vec{k}_T) = \frac{A}{\pi(\alpha + \Delta_{2F})} f^N_1(x) e^{-\vec{F}_T^2}, \quad (3.9) \]

\[ h^{A_\perp 1}(x, \vec{k}_T) = \frac{A\beta}{\pi(\beta + \Delta_{2F})^2} h^N_\perp 1(x) e^{-\vec{F}_T^2}, \quad (3.10) \]

B. Nuclear dependence of the azimuthal asymmetry

It follows that azimuthal asymmetry \( \cos 2\phi \) in nucleon-nucleon and nucleon-nucleus collisions are given by, respectively,

\[ A^{\cos 2\phi}_{NN} = \frac{1}{4M^2} \frac{\alpha}{4\beta} \frac{S[h_1^+, h_1^+] - \epsilon_a^2 [f^N_1(x_1; a) f^N_1(x_2; \bar{a}) + f^N_1(x_1; \bar{a}) f^N_1(x_2; a)]}{S[f_1, f_1]} \quad (3.11) \]

\[ A^{\cos 2\phi}_{NA} = \frac{1}{4M^2} \frac{\beta^2 (2\alpha + \Delta_{2F})}{(2\beta + \Delta_{2F})^3} \frac{S[h_1^+, h_1^+] - \epsilon_a^2 [f^N_1(x_1; a) f^N_1(x_2; \bar{a}) + f^N_1(x_1; \bar{a}) f^N_1(x_2; a)]}{S[f_1, f_1]} \quad (3.12) \]

where we have defined a shorthand notation

\[ S[f_1, f_1] \equiv \frac{1}{3} \sum_a \epsilon_a^2 [f^N_1(x_1; a) f^N_1(x_2; \bar{a}) + f^N_1(x_1; \bar{a}) f^N_1(x_2; a)] \quad (3.13) \]

It is obvious that we can obtain \( A^{\cos 2\phi}_{NN} \) by simply setting \( \Delta_{2F} = 0 \) in \( A^{\cos 2\phi}_{NA} \). Hence we will only present \( A_{NA} \) in the other azimuthal asymmetries in the following discussion.

The nuclear effect of the azimuthal asymmetry \( \cos 2\phi \) can be measured by the ratio,

\[ R^{\cos 2\phi} \equiv \frac{A^{\cos 2\phi}_{NA}}{A^{\cos 2\phi}_{NN}} = \frac{2\alpha + \Delta_{2F}}{2\alpha} \left( \frac{2\beta}{2\beta + \Delta_{2F}} \right)^3 \exp \left[ \frac{2(\alpha - \beta)(2\alpha + 2\beta + \Delta_{2F})}{2\alpha(2\beta + \Delta_{2F})} \Delta_{2F} \right] \quad (3.14) \]

In the special case where \( \alpha = \beta \), we can obtain a simplified result

\[ R^{\cos 2\phi} = \left( \frac{2\alpha}{2\alpha + \Delta_{2F}} \right)^2 \quad (3.15) \]

which means that the azimuthal asymmetry \( \cos 2\phi \) in DY process in nucleon-nucleus collisions is suppressed compared to that in nucleon-nucleon collisions and
has no dependence on the transverse momentum of the lepton pairs. It is very interesting that the suppression in Eq. (3.15) is in a very similar way with the one of azimuthal asymmetry \( \cos 2\phi \) in SIDIS obtained in [27]. For the general case, the nuclear modification factor can only depend on three independent variable,

\[ \eta \equiv \Delta_2 F / 2\alpha, \quad \hat{q}_T^\alpha \equiv |\hat{q}_T| / \sqrt{2\alpha}, \quad \zeta \equiv \beta / \alpha, \]  

(3.16)

The numerical results are plotted in Fig. (1), with \( \zeta = 2 \) and 0.5, respectively, as functions of \( \eta \), at different scaled transverse momentum \( \hat{q}_T^\alpha \). We can see that they are very similar to the results that have been obtained in [25] and [27]. In the case \( \zeta > 1 \), the azimuthal asymmetry is suppressed and the suppression increases with the transverse momentum \( \hat{q}_T^\alpha \). When \( \zeta < 1 \), the suppression will have opposite dependence on \( \hat{q}_T^\alpha \). Especially, the azimuthal asymmetry could be enhanced, instead of suppression, for large enough transverse momentum \( \hat{q}_T^\alpha \). It means that the nuclear modification of the azimuthal asymmetry and its transverse momentum dependence provides a very sensitive probe to measure the width of the transverse momentum distribution in the TMD quark distribution functions.

We can also calculate the averaged azimuthal asymmetry over \( \vec{q}_L \) which is given by

\[ \bar{A}_{NA}^{\cos 2\phi} = \frac{1}{4M^2} \frac{\beta^2}{(2\beta + \Delta_2 F)} \frac{S \left[ h_1^+ h_1^+ \right]}{S \left[ f_1 f_1 \right]} \]  

(3.17)

Therefore the averaged nuclear modification factor of the azimuthal asymmetry \( \cos 2\phi \) reads,

\[ \bar{R}^{\cos 2\phi} \equiv \frac{\bar{A}_{NA}^{\cos 2\phi}}{\bar{A}_{NN}^{\cos 2\phi}} = \frac{2\beta}{2\beta + \Delta_2 F}. \]  

(3.18)

Now let us turn to the azimuthal asymmetries associated with the polarization of the incident nucleon, we can see four more distribution functions are involved in the Eq. (2.4), \( h_1^L, f_1^T, h_1^T \) and \( h_1^T \). Once more, to illustrate the nuclear dependence of all these azimuthal asymmetries resulted in by these TMD distributions,
we make four more Gussian ansatz assumptions

\[ h_{1L}^{N}(x, \vec{k}_T) = \frac{1}{\pi \sigma_1} h_{1L}^{N} (x) e^{-\frac{\vec{k}_T^2}{\sigma_1^2}}, \quad f_{1T}^{N}(x, \vec{k}_T) = \frac{1}{\pi \sigma_2} f_{1T}^{N} (x) e^{-\frac{\vec{k}_T^2}{\sigma_2^2}}, \quad (3.19) \]

\[ h_{1}^{N}(x, \vec{k}_T) = \frac{1}{\pi \sigma_3} h_{1}^{N} (x) e^{-\frac{\vec{k}_T^2}{\sigma_3^2}}, \quad h_{1T}^{N}(x, \vec{k}_T) = \frac{1}{\pi \sigma_4} h_{1T}^{N} (x) e^{-\frac{\vec{k}_T^2}{\sigma_4^2}}. \quad (3.20) \]
Following the same routine, we can have

\[
A_{NA}^{\sin 2\phi} = \frac{\lambda_s \sigma_1 \beta (2\alpha + \Delta_{2F}) S [h_1^L, h_1^T]}{4M^2 (\sigma_1 + \beta + \Delta_{2F})^2 S [f_1, f_1]} \tilde{q}_T^2 e^{\sigma_1 + \beta - 2\alpha (\sigma_1 + \beta + \Delta_{2F}) \tilde{q}_T^2} (3.21)
\]

\[
A_{NA}^{\sin \phi_\alpha} = -\frac{\tilde{s}_T}{2M} \sigma_2 (2\alpha + \Delta_{2F}) S [f_1^T, f_1^T] \tilde{q}_T^2 e^{\sigma_2 - \alpha \tilde{q}_T^2} (3.22)
\]

\[
A_{NA}^{\sin(2\phi - \phi_\alpha)} = \frac{\tilde{s}_T}{4M (\sigma_3 + \beta + \Delta_{2F})} \sigma_2 (2\alpha + \Delta_{2F}) S [h_1^T, h_1^T] \tilde{q}_T^2 e^{\sigma_2 - \alpha \tilde{q}_T^2} (3.23)
\]

\[
A_{NA}^{\sin(2\phi + \phi_\alpha)} = \frac{\tilde{s}_T}{8M^3 (\sigma_4 + \beta + \Delta_{2F})^4} \sigma_2 (2\alpha + \Delta_{2F}) S [h_1^T, h_1^T] \tilde{q}_T^2 e^{\sigma_2 - \alpha \tilde{q}_T^2} (3.24)
\]

The nuclear modification factors corresponding to the above different azimuthal asymmetries are given by, respectively,

\[
R_{\sin 2\phi} = \frac{A_{NA}^{\sin 2\phi}}{A_{NN}^{\sin 2\phi}} = \frac{(2\alpha + \Delta_{2F})(\sigma_1 + \beta)^3}{2\alpha(\sigma_1 + \beta + \Delta_{2F})^3} e^{\sigma_1 + \beta - 2\alpha (\sigma_1 + \beta + \Delta_{2F}) \tilde{q}_T^2} (3.25)
\]

\[
R_{\sin \phi_\alpha} = A_{NN}^{\sin \phi_\alpha} = \frac{2\alpha(\sigma_2 + \alpha + \Delta_{2F})^2}{2\alpha(\sigma_2 + \alpha + \Delta_{2F})^2} e^{\sigma_2 - \alpha \tilde{q}_T^2} (3.26)
\]

\[
R_{\sin(2\phi - \phi_\alpha)} = \frac{A_{NA}^{\sin(2\phi - \phi_\alpha)}}{A_{NN}^{\sin(2\phi - \phi_\alpha)}} = \frac{(2\alpha + \Delta_{2F})(\sigma_3 + \beta)^2}{2\alpha(\sigma_3 + \beta + \Delta_{2F})^2} e^{\sigma_3 + \beta - 2\alpha (\sigma_3 + \beta + \Delta_{2F}) \tilde{q}_T^2} (3.27)
\]

\[
R_{\sin(2\phi + \phi_\alpha)} = \frac{A_{NA}^{\sin(2\phi + \phi_\alpha)}}{A_{NN}^{\sin(2\phi + \phi_\alpha)}} = \frac{(2\alpha + \Delta_{2F})(\sigma_4 + \beta)^4}{2\alpha(\sigma_4 + \beta + \Delta_{2F})^4} e^{\sigma_4 + \beta - 2\alpha (\sigma_4 + \beta + \Delta_{2F}) \tilde{q}_T^2} (3.28)
\]

In the special case of \(\sigma_1 = \sigma_3 = \sigma_4 = 2\alpha - \beta\) and \(\sigma_2 = \alpha\), we can reduce them into,

\[
R_{\sin 2\phi} = \left(\frac{2\alpha}{2\alpha + \Delta_{2F}}\right)^2, \quad R_{\sin \phi_\alpha} = \frac{2\alpha}{2\alpha + \Delta_{2F}},
\]

\[
R_{\sin(2\phi - \phi_\alpha)} = \frac{2\alpha}{2\alpha + \Delta_{2F}}, \quad R_{\sin(2\phi + \phi_\alpha)} = \left(\frac{2\alpha}{2\alpha + \Delta_{2F}}\right)^3.
\]

For the general cases, we can choose three independent variables for every nuclear modification factor as we did for the azimuthal asymmetry \(\cos 2\phi\). We will choose two same scaled variables \(\eta \equiv \Delta_{2F}/2\alpha, \tilde{q}_T^2 \equiv \tilde{q}_T^2/\sqrt{2\alpha}\) for all the nuclear modification factors, and the rest \(\zeta \equiv (\sigma_1 + \beta)/2\alpha, \sigma_2/\alpha, (\sigma_3 + \beta)/2\alpha\) and \((\sigma_4 + \beta)/2\alpha\), corresponding to the sin \(2\phi\), sin \(\phi_\alpha\), sin \((2\phi - \phi_\alpha)\) and sin \((2\phi + \phi_\alpha)\), respectively. With these variables, it is obvious that \(R_{\cos 2\phi}\) and \(R_{\sin 2\phi}\) are the same functions while
\( R_{\sin \phi_s} \) and \( R_{\sin(2\phi-\phi_s)} \) are the same. It should be noted that \( R_{\sin \phi_s} \) and \( R_{\sin (2\phi-\phi_s)} \) are very similar to the result of azimuthal asymmetry \( \cos \phi \) in SIDIS obtained in Ref. [25] and \( R_{\cos 2\phi} \) and \( R_{\sin 2\phi} \) are very similar to the result of azimuthal asymmetry \( \cos 2\phi \) in SIDIS obtained in Ref. [27]. Besides, in the new scaled variables, the only difference between different azimuthal asymmetry is up to an overall factor \( \zeta / (\zeta + \eta) \) with right power order. Since our calculation is only qualitative, we will not show their numerical results in plots one by one. The shapes and features of these azimuthal asymmetry associated with the polarization are very similar to the unpolarized azimuthal asymmetry \( \cos 2\phi \) shown in Fig. (1). The averaged angular asymmetries over the transverse momentum \( \vec{q}_T \) can be obtained in a very straightforward way,

\[
\begin{align*}
\bar{A}_{N_{A}}^{\sin 2\phi} &= \frac{\lambda_{s}}{4M^2 (\sigma_1 + \beta + \Delta_{2F})} \frac{\sigma_1 \beta}{S[f_1^+, f_1]} S[h_1^+, h_1^+] \tag{3.31} \\
\bar{A}_{N_{A}}^{\sin \phi_s} &= \left| \vec{s}_T \right| \frac{\sqrt{\pi} \sigma_2}{2M 2(\sigma_2 + \alpha + \Delta_{2F})} \frac{\sigma}{S[f_1^+, f_1]} S[f_1^+, f_1] \tag{3.32} \\
\bar{A}_{N_{A}}^{\sin(2\phi-\phi_s)} &= \left| \vec{s}_T \right| \frac{\sqrt{\pi} \beta}{4M 2(\sigma_3 + \beta + \Delta_{2F})} \frac{\sigma}{S[f_1^+, f_1]} S[h_1^+, h_1^+] \tag{3.33} \\
\bar{A}_{N_{A}}^{\sin(2\phi+\phi_s)} &= \left| \vec{s}_T \right| \frac{3\sqrt{\pi} \sigma_4^2 \beta}{8M^3 4(\sigma_4 + \beta + \Delta_{2F})^3} \frac{\sigma}{S[f_1^+, f_1]} S[f_1^+, f_1] \tag{3.34}
\end{align*}
\]

It follows that the averaged nuclear modifications of the single spin azimuthal asymmetries read

\[
\begin{align*}
\bar{R}_{\sin 2\phi} &= \frac{\sigma_1 + \beta}{\sigma_1 + \beta + \Delta_{2F}}, & \bar{R}_{\sin \phi_s} &= \left( \frac{\sigma_2 + \alpha}{\sigma_2 + \alpha + \Delta_{2F}} \right)^{1/2} \tag{3.35} \\
\bar{R}_{\sin(2\phi-\phi_s)} &= \left( \frac{\sigma_3 + \beta}{\sigma_3 + \beta + \Delta_{2F}} \right)^{1/2}, & \bar{R}_{\sin(2\phi+\phi_s)} &= \left( \frac{\sigma_4 + \beta}{\sigma_4 + \beta + \Delta_{2F}} \right)^{3/2} \tag{3.36}
\end{align*}
\]

### IV. SUMMARY

Within the framework of the TMD factorization, the nuclear dependence of the azimuthal asymmetry in polarized DY process has been studied. We find the
nuclear modifications of the azimuthal asymmetry $\cos 2\phi$ and $\sin 2\phi$ are the same and in a very similar way with the azimuthal asymmetry $\cos 2\phi$ in SIDIS obtained in [27]. The nuclear effects of the azimuthal asymmetry $\sin \phi_s$ and $\sin(2\phi - \phi_s)$ are the same and similar to the azimuthal asymmetry $\cos \phi$ in SIDIS obtained in [25]. Among all the azimuthal asymmetries we have considered, the nuclear dependence of the azimuthal asymmetry $\sin(2\phi + \phi_s)$ is most suppressed. The nuclear modification of the azimuthal asymmetry and its nontrivial transverse momentum dependence provides a very sensitive probe to measure the width of the transverse momentum distribution in the TMD quark distribution functions.

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[1] H. Georgi and H. Politzer, Phys. Rev. Lett. 40, 3 (1978).
[2] R. N. Cahn, Phys. Lett. B 78, 269 (1978).
[3] E. L. Berger, Phys. Lett. B 89, 241 (1980).
[4] K. A. Oganesian, H. R. Avakian, N. Bianchi and P. Di Nezza, Eur. Phys. J. C 5, 681 (1998).
[5] J. Chay and S. M. Kim, Phys. Rev. D 57, 224 (1998).
[6] Z. T. Liang and X. N. Wang, Phys. Rev. D 75, 094002 (2007).
[7] J. C. Collins and D. E. Soper, Phys. Rev. D 16, 2219 (1977).
[8] J. C. Collins, Phys. Rev. Lett. 42, 291 (1979).
[9] C. S. Lam and W. K. Tung, Phys. Rev. D 18, 2447 (1978).
[10] C. S. Lam and W. -K. Tung, Phys. Lett. B 80, 228 (1979).
[11] C. S. Lam and W. -K. Tung, Phys. Rev. D 21, 2712 (1980).
[12] R. Baier, Y. L. Dokshitzer, A. H. Mueller, S. Peigne and D. Schiff, Nucl. Phys. B 484, 265 (1997).
[13] G. T. Bodwin, S. J. Brodsky and G. P. Lepage, Phys. Rev. D 39, 3287 (1989).
[14] M. Luo, J. -w. Qiu, G. F. Sterman, Phys. Lett. B279, 377-383 (1992); Phys. Rev. D49, 4493-4502 (1994); Phys. Rev. D50, 1951-1971 (1994).
[15] X. F. Guo, Phys. Rev. D 58, 114033 (1998).
[16] U. A. Wiedemann, Nucl. Phys. B 588, 303 (2000)
[17] X. -f. Guo and X. -N. Wang, Phys. Rev. Lett. 85, 3591 (2000) [hep-ph/0005044].
[18] X. -N. Wang and X. -f. Guo, Nucl. Phys. A 696, 788 (2001) [hep-ph/0102230].
[19] R. J. Fries, Phys. Rev. D 68, 074013 (2003).
[20] A. Majumder, B. Muller, Phys. Rev. C77, 054903 (2008).
[21] Z. T. Liang, X. N. Wang and J. Zhou, Phys. Rev. D 77, 125010 (2008)
[22] F. D’Eramo, H. Liu and K. Rajagopal, Phys. Rev. D 84, 065015 (2011)
[23] F. D’Eramo, H. Liu, K. Rajagopal, Nucl. Phys. A855, 457-460 (2011).
[24] F. D’Eramo, H. Liu and K. Rajagopal, J. Phys. G 38, 124162 (2011).
[25] J. -H. Gao, Z. -t. Liang, X. -N. Wang, Phys. Rev. C81, 065211 (2010).
[26] A. Schafer and J. Zhou, arXiv:1305.5042 [hep-ph].
[27] Y. -k. Song, J. -h. Gao, Z. -t. Liang, X. -N. Wang, Phys. Rev. D83, 054010 (2011).
[28] J. -H. Gao, A. Schafer and J. Zhou, Phys. Rev. D 85, 074027 (2012).
[29] R. J. Fries, B. Muller, A. Schafer, E. Stein, Phys. Rev. Lett. 83, 4261-4264 (1999).
R. J. Fries, A. Schafer, E. Stein, B. Muller, Nucl. Phys. B582, 537-570 (2000).
[30] F. Gelis, J. Jalilian-Marian, Phys. Rev. D76, 074015 (2007).
[31] J. C. Collins and D. E. Soper, Nucl. Phys. B 193, 381 (1981) [Erratum-ibid. B 213, 545 (1983)] [Nucl. Phys. B 213, 545 (1983)].
[32] X. -d. Ji, J. -P. Ma and F. Yuan, Phys. Lett. B 597, 299 (2004) [hep-ph/0405085].
[33] D. Boer, Phys. Rev. D 60, 014012 (1999) [hep-ph/9902255].
[34] S. Arnold, A. Metz and M. Schlegel, Phys. Rev. D 79, 034005 (2009) [arXiv:0809.2262 [hep-ph]].
[35] Z. Lu, B.-Q. Ma and J. Zhu, Phys. Rev. D 84, 074036 (2011) [arXiv:1108.4974 [hep-ph]].
[36] P. J. Mulders and R. D. Tangerman, Nucl. Phys. B 461, 197 (1996) [Erratum-ibid. B 484, 538 (1997)] [hep-ph/9510301].
[37] K. Goeke, A. Metz and M. Schlegel, Phys. Lett. B 618, 90 (2005) [hep-ph/0504130].
[38] A. Bacchetta, M. Diehl, K. Goeke, A. Metz, P. J. Mulders and M. Schlegel, JHEP 0702, 093 (2007) [hep-ph/0611265].
[39] D. W. Sivers, Phys. Rev. D 41, 83 (1990).
[40] D. W. Sivers, Phys. Rev. D 43, 261 (1991).
[41] D. Boer and P. J. Mulders, Phys. Rev. D 57, 5780 (1998) [hep-ph/9711485].
[42] H. Mkrtchyan, P. E. Bosted, G. S. Adams, A. Ahmidouch, T. Angelescu, J. Ar- rington, R. Asaturyan and O. K. Baker et al., Phys. Lett. B 665, 20 (2008) [arXiv:0709.3020 [hep-ph]].
[43] A. Airapetian et al. [HERMES Collaboration], Phys. Rev. Lett. 103, 152002 (2009) [arXiv:0906.3918 [hep-ex]].
[44] H. Avakian et al. [CLAS Collaboration], Phys. Rev. Lett. 105, 262002 (2010) [arXiv:1003.4549 [hep-ex]].
[45] M. G. Alekseev et al. [COMPASS Collaboration], Phys. Lett. B 692, 240 (2010) [arXiv:1005.5609 [hep-ex]].
[46] M. Anselmino, M. Boglione, U. D’Alesio, A. Kotzinian, F. Murgia and A. Prokudin, Phys. Rev. D 71, 074006 (2005) [hep-ph/0501196].
[47] J. C. Collins, A. V. Efremov, K. Goeke, M. Grosse Perdekamp, S. Menzel, B. Meredith, A. Metz and P. Schweitzer, Phys. Rev. D 73, 094023 (2006) [hep-ph/0511272].
[48] W. Vogelsang and F. Yuan, Phys. Rev. D 72, 054028 (2005) [hep-ph/0507266].
[49] M. Anselmino, M. Boglione, U. D’Alesio, A. Kotzinian, S. Melis, F. Murgia, A. Prokudin and C. Turk, Eur. Phys. J. A 39, 89 (2009) [arXiv:0805.2677 [hep-ph]].

[50] M. Anselmino, M. Boglione, U. D’Alesio, A. Kotzinian, F. Murgia, A. Prokudin and C. Turk, Phys. Rev. D 75, 054032 (2007) [hep-ph/0701006].

[51] M. Anselmino, M. Boglione, U. D’Alesio, A. Kotzinian, F. Murgia, A. Prokudin and S. Melis, Nucl. Phys. Proc. Suppl. 191, 98 (2009) [arXiv:0812.4366 [hep-ph]].