Methods of field theory do not straightforwardly apply to systems at finite temperature. Naive perturbation theory fails in the infrared region of energy at high temperature but can be fixed by the partial HTL resummation \[1\]. This approach was generalized in \[2\], and it was shown that fields without dispersion relation \[3\] are necessary for consistent formulation of the theory.

The particle distribution over momenta, \( f(p) \), acquires non-Maxwellian tail in thermodynamic equilibrium \[4\] due to quantum effects, while the particle distribution over energies, \( f(\omega) \), remains Maxwellian. An interesting suggestion was made in Ref. \[5\], that the rates of various processes, including nuclear reactions, may be significantly increased by the quantum tail of the momentum distribution. Non-linear integral equation for the rate of inelastic processes as well as a final formula for the ionization rate was obtained in \[5\]. The formula for the nuclear reaction rates was found in \[6\] under assumption, that it is correct to substitute the particle momentum in the form \( \epsilon_p = p^2/2m \) into the known quasiclassical cross-section \( \sigma(\omega) \) and then average it over the particle distribution over momenta \( f(p) \) rather than the distribution over energies \( f(\omega) \). This procedure becomes even more unclear, if we take into account the fact, that the momentum, \( p \), and energy, \( \omega \), of a colliding particle are independent variables not connected by the usual dispersion relation \( \delta(\omega - p^2/2m) \) \[4\].

Note, that the averaging of \( \sigma(\omega) \) over the distribution over energies \( f(\omega) \), which is Maxwellian, would give the usual reaction rates \[8\] rather than the rates found in \[6\], which are accelerated by many orders of magnitude in certain regimes. One can argue \[3\] that averaging over momentum may not be completely wrong, because the dependence of the scattering amplitude on energy can be neglected in the gaseous approximation \[9\]. However, the connection between the scattering amplitude and the rate of nuclear reactions is not clear. Neither is it obvious that this argument can straightforwardly lead to correct answer in the case of essentially non-linear processes of nuclear reactions. It was shown that non-linearities can be very subtle, subject to various non-trivial cancellations \[11\]. It is therefore important to find the nuclear reaction rates from the first principles.

In this paper we rigorously solve the quasiclassical problem of tunneling through the potential barrier. Our result demonstrates that the cancellations of the type found in \[11\] do not play a role in the case under consideration. We discuss this question in more details in the future publication \[12\].

The tunneling particles undergo simultaneous collisions with other particles of the plasma maintained in thermodynamic equilibrium. The plasma is assumed to be fully ionized. We use the Green–function technique, introduced by Keldysh and Korenman \[13\], and do not rely on any ad hoc assumptions \[6\] about any averaging procedure. Our final result for the nuclear reaction rate \[15\] agrees with that postulated in \[6\], apart from the coefficient depending on the masses of colliding and reacting particles.

A number of different off-shell processes in plasmas were investigated within a framework of thermal field theory. Emission rate of soft photons from hot plasmas was calculated in \[16\]. Using full spectral functions allowed to eliminate unphysical divergences and extend the range of validity of the results obtained by HTL resummation by taking into account off-shell particle propagation. Landau damping was shown to lead to anomalous relaxation of the particle distribution function \[17\]. Off-shell effects
due to particle collisions were treated within 
Born approximation in \[13\]. The relaxation rate, 
calculated with the help of full spectral functions, is slower approximately by a factor of 2 as 
compared to the rates calculated by solving the 
Boltzman equation \[18\]. This relaxation rate is, 
however, an integral quantity, insensitive to the 
details of changes of the kinetic Green function 
due to inclusion of off-shell effects. 

In contrast to the relaxation rate, inherent 
non-linearity of the tunneling problem makes 
the average nuclear reaction rate very sensitive 
to the behavior of this function at large mo-
menta. This explains why we should expect 
off-shell effects to lead to significant corrections 
to the nuclear reaction rates. Indeed, we find 
that certain reactions are accelerated by many 
orders of magnitude as compared to the cal-
culations, performed using spectral functions with 
zero width \[11\]. We solve the tunneling problem, 
neglecting Salpeter corrections due to screening, 
and assume that, the nuclear transformations are 
unaffected by the medium.

Kinetic properties of the particles are de-
scribed by the Green function \( G^{-+}(X_1, X_2) \), 
where \( X = (t, \mathbf{x}) \) \[14\]. Equations for \( G^{-+} \) 
include the interaction between the particles as 
well as their interaction with the external field.
Therefore, the rate of change of this function, 
evaluated at any point after the barrier, will give 
the tunneling rate \( K(x, t) \) for the particles col-
liding simultaneously with other particles \[15\]:

\[
K(x, t) = \left[ (\partial_t - \partial_x)G^{-+}(t_1 \mathbf{x}_1; t_2 \mathbf{x}_2) \right]_{X_1 \rightarrow X_2 = x}
\] (1)

Now we will proceed with solution of the ki-
etic equations for \( G^{-+}(X_1, X_2) \). They can be 
written as Dyson equations for \( G^{\alpha\beta}_{12} \) \[13\]:

\[
G^{\alpha\beta}_{12} = G^{(0)\alpha\beta}_{12} + \int G^{(0)\alpha\gamma}_{14} \Sigma_{43} G^{\delta\beta}_{32} d^4X_4 d^4X_3,
\] (2)

where we use subscripts 1..4 to denote depen-
dence on \( X_1..X_4 \). Dependence on \( G^{(0)\alpha\beta}_{12} \) can be 
eliminated \[13\] by acting on both sides of the 
Eq. (2) with the operator

\[
\hat{G}^{-1} = i \frac{\partial}{\partial t_1} + \Delta_1 \equiv \frac{\partial}{\partial t_1} + \frac{\Delta_1}{2m} + \hat{L}(z_1),
\]

where \( \Delta_1 = \partial^2_{z_1} + \partial^2_{\mu_1} \). Since we are interested 
in the steady state solution, one can see that, 
the properties of this operator can be fully ac-
counted for if we introduce the function \( g(z, k) \), 
such that

\[
\left[ \frac{1}{2m} \frac{\partial^2}{\partial z^2} - U(z) \right] g(z, k) = -\epsilon_k g(z, k),
\] (3)

where \( \epsilon_k = k^2/2m \). Since the operator \( \hat{L} \) is 
Hermitian, we will use the property of its eigen-
functions \( g(z, k) \):

\[
\int_{-\infty}^{\infty} g(z, k_1)g^*(z, k_2) dz = 2\pi\delta(k_1 - k_2)
\] (4)

Now let us solve for \( G^{(0)-+}_{12} \), which satisfies

\[
\hat{G}^{-1}_{12} G^{(0)-+}_{12} = 0.
\] (5)

It is easier to find \( G^{(0)-+}_{12} \) by using its definition

\[
G^{(0)-+}_{12} = i \langle \psi^+_2 \psi_1 \rangle,
\] (6)

where \( \psi^t, \psi \) are the Heisenberg operators of cre-
ation and annihilation of the particles at a point \( X \), and \( \langle .. \rangle \) means quantum and statistical av-
ergaging. Since \( \psi \) evolves according to \( \hat{G}^{-1} \psi = 0 \), 
one can see, that it is equal to

\[
\psi(X) = \sum_{\mathbf{q}_{\perp} k} \hat{a}_{\mathbf{q}_{\perp} k} g(z, k) e^{-i\mathbf{q}_{\perp} \mathbf{x} + i\mathbf{q}_{\perp} \cdot \mathbf{x}^+}
\] (7)

Here \( \hat{a}_{\mathbf{q}_{\perp} k} \) is the annihilation operator of the 
particle with momentum \( (\mathbf{q}_{\perp}, k) \), and \( \epsilon_{\mathbf{q}_{\perp} k} \equiv
Q^2/2m + k^2/2m \).

We substitute \( \psi(X) \) from (7) into Eq. (8) and 

obtain

\[
G^{(0)-+}_{\omega_{\mathbf{q}_{\perp}}}(z_1, z_2) = 2\pi i \int_{-\infty}^{\infty} \frac{dk}{2\pi} \delta(\omega - \epsilon_{\mathbf{q}_{\perp} k} + \mu) n(\mathbf{q}_{\perp} k) g(z_1, k) g^*(z_2, k),
\] (8)

where we denote \( \langle a_{\mathbf{q}_{\perp} k}^+ a_{\mathbf{q}_{\perp} k} \rangle \equiv n(\mathbf{q}_{\perp} k); \mu \) is the 
chemical potential.

It is easy to see that, in thermodynamic equi-
librium we are considering, the function \( n(\mathbf{q}_{\perp} k) \) 
is equal to Fermi distribution \( n_F(\epsilon_{\mathbf{q}_{\perp} k}) \). To con-
vince ourselves we take \( G^{(0)-+}_{\omega_{\mathbf{q}_{\perp}}}(z_1, z_2) \) at such
values of $z_1, z_2$, when $U(z)$ is negligible and $g(z, k) \Rightarrow \exp(ikz)$. One more Fourier transform
\[ f \exp[-i\varphi(z_1 - z_2)]d(z_1 - z_2) \] will give $G^{(0)-+}_{\omega k}$, which has to coincide with the known expression
\[ \frac{1}{G^{(0)-+}_{\omega k}} = 2\pi i \delta(\omega - \epsilon_k + \mu)n_F(\epsilon_k). \]

Before proceeding further with solving for $G^{(0)-+}$ we first find $K^{(0)}$, which is the tunneling rate obtained by using $G^{(0)-+}_{\omega k}$ in Eq. (11). Since we are interested in $K^{(0)}$ behind the barrier, where $U(z) = 0$, we can exchange $\partial_{t_1}$ for $i\Delta/2m$ as is clear from the form of the operator
\[ \hat{G}^{-1}_{1} \] and Eq. (3). Then the formula for the rate $K^{(0)}$ takes the form:
\[ K^{(0)}(z) = \frac{i}{2m} \int \frac{d\omega}{2\pi} \int \frac{d^2 q}{(2\pi)^2} \left[ \frac{\partial^2}{\partial \zeta^2} G^{(0)-+}(\omega, \zeta) \right]_{\zeta = 0} \]
\[ = \frac{\partial^2}{\partial \zeta^2} \left[ g(z, k)\phi(z, k) \right]_{\zeta = 0} = -\frac{\partial^2}{\partial \zeta^2} \left[ g(z, k)\phi(z, k) \right]_{\zeta = 0} \]
\[ = -\frac{\partial^2}{\partial \zeta^2} \left[ g(z, k)\phi(z, k) \right]_{\zeta = 0} \]

Note, that $\phi(z, k) = k$ is true behind the barrier.

The problem of tunneling collision between particles $m_1$ and $m_2$ can be reduced to the one-dimensional motion of a particle with reduced mass $m_r = m_1m_2/(m_1 + m_2)$ in the external potential $U(r)$ [19]. We make this reduction in the original Hamiltonian and then second quantize it.

We need to perform calculations of $G^{(0)-+}$ and $K^{(0)}$ similar to those outlined above. However, now we have to do 3-D calculations, using spherical coordinates. So we replace \( \hat{L}(z) \) operator with \( \hat{L}_r \) operator, denote its radial part in spherical coordinates by \( \hat{L}_r \) and introduce the function $g(x)$ according to
\[ g(x) = \int \frac{k^2 dk}{(2\pi)^3} g_k(x) \]
\[ g_k(x) = \sum_{lm} g_{klm}(r)Y_{lm}(\theta \phi) \]
\[ (\hat{L}r - U(r)) g_k(r) = -\epsilon_k g_k(r), \]
\[ \psi(x, t_1) = \sum_{k,l,m} \hat{a}_{k1l1} g_k(r)Y_{lm}(\theta \phi)e^{-i\epsilon_k t_1}. \]

Now using Eqs. (8), (16), (17) and taking into account the contribution only from spherically symmetric modes, we obtain $G^{(0)-+}$
\[ G^{(0)-+}_{\omega k}(x_1, x_2) = 2\pi i \int_0^{\infty} \frac{k^2 dk}{(2\pi)^3} \]
\[ \delta(\omega - \epsilon_k + \mu)n_F(\epsilon_k)\phi_k(x_1)\phi_k^*(x_2). \]

Rewrite the rate $K^{(0)}$, Eq. (3), as
\[ K^{(0)}(x) = \frac{i}{2m_r} \int d\omega \left[ \Delta_{x_1} G^{(0)-+}_{\omega k}(x_1, x_2) \right]_{x_1 \rightarrow x_2} \]
In the WKB approximation we get for the function $g_k(r)$ the following formula

$$g_k(r) = \frac{1}{r} \chi(r)$$  \hspace{1cm} (20)

where the bar denotes dependence on the reduced mass, $\bar{\epsilon}_k = k^2/2m_r$, $\bar{C}_k^2 = \bar{A}_k^2 W(k)$, $\alpha \equiv Z_1 Z_2 e^2 e_2$ and $\bar{A}_k^2 = r_0^2 S(\bar{\epsilon}_k)/\bar{\epsilon}_k$. We evaluate the derivative of Eq. (19) in the quasiclassical sense

$$[g_k^r(x_2) \Delta_{x_1} g_k(x_1)]_{x_1 \rightarrow x_2 = r_0} = -k \bar{C}_k^2 r_0^{-2}$$  \hspace{1cm} (24)

and obtain the final result for $K^{(0)}$

$$K^{(0)} = n_0 \int_0^\infty \frac{4\pi k^2 dk}{(2\pi)^3} \tilde{n}_M(k) \frac{k}{m_r} S(\bar{\epsilon}_k) W(k),$$  \hspace{1cm} (25)

where $\tilde{n}_M(k)$ is the Maxwell distribution which depends on $m_r$.

It agrees with the answer obtained in [8] by averaging the quasiclassical tunneling factor $W(k)$ over the Maxwell distribution, $\langle n_M(v) \sigma(m_r v^2/2)v \rangle$. Hence, we see, that the dispersion relation $\delta(\omega - \epsilon_q)$ was implicitly used in [8], which corresponds to an approximation of instantaneous, two-body collisions. Now we will not make this assumption and proceed with finding the tunneling rate, $K$, of the particles, which always collide and hence are never “in between collisions”.

By using [13,14], we find

$$\left( \frac{q}{m} \frac{\partial}{\partial z} + U'(z) \frac{\partial}{\partial q} \right) G_{\omega q}^-(z) = -\gamma_{\omega q}(z) G_{\omega q}^-(z)$$

$$- \gamma_{\omega q}(z) n_F(\omega) \left( G_{\omega q}^R(z) - G_{\omega q}^A(z) \right),$$  \hspace{1cm} (26)

where we denote $\Sigma^R - \Sigma^A \equiv \gamma_{\omega q}(z) \equiv i\gamma_{\omega q}(q,z)$. Note, that $\gamma_{\omega q}(z)$ is a non-linear function of $G^{\alpha\beta}$ [13]. We now analyze Eq. (26) qualitatively, which will help us to find that part of the solution, which makes the largest contribution to $K$.

In the region away from the barrier we can neglect the LHS of Eq. (26), and obtain:

$$\tilde{G}_{\omega q}^{-}(z) = -n_F(\omega) \left( G_{\omega q}^R(z) - G_{\omega q}^A(z) \right)$$  \hspace{1cm} (27)

Therefore, Eq. (26) will allow us to propagate this solution into the region behind the barrier.

We can find the first integral and formally integrate Eq. (26) along trajectories, which will lead to a sum of solutions of homogeneous and “inhomogeneous” equations (due to the last term in (26)). As can be seen from Eq. (26), “inhomogeneous” solution will involve the integral of the product of $\gamma_{\omega q}(z)$ and $G_{\omega q}^R(z)$ or $G_{\omega q}^R(z)$.

Qualitatively, it means, that in the region of $z$ behind the barrier it will be proportional to a product of at least two exponential factors $W(k)$. This is so, because both $\gamma_{\omega q}(z)$ and $G_{\omega q}^R(z)$ will depend on the integral of the product $g(z_1,k)g^*(z_2,k)$ and such product is $\propto W(k)$ as can be seen from Eqs. (11)-(12).

Explicitly, the expression for the retarded Green function can be obtained through the usual resummation

$$G_{\omega q}^R(z) = \frac{1}{\omega - \epsilon_q - \Sigma_{\omega q}^R(z)}$$  \hspace{1cm} (28)

Since the advanced function, $G_{\omega q}^A$, is related to $G_{\omega q}^R$ by $G_{\omega q}^A = G_{\omega q}^R$, we find that their difference is

$$G_{\omega q}^R - G_{\omega q}^A = 2Im \left( \frac{1}{(G_{0}^{R})^{-1} - \Sigma^R} \right)$$

$$= \frac{\gamma}{(G_{0}^{R})^{-2} + (Im\Sigma^R)^2}$$  \hspace{1cm} (29)

Now we will show that $\gamma$ is $\propto W(k)$, which will allow us to neglect the “inhomogeneous” term in Eq. (26), as being second order in $W(k)$ since $G_{\omega q}^R - G_{\omega q}^A$ is also proportional to $W(k)$. First,
note that Eq. (15) is written in the Furry representation. The Furry representation is needed, since we are interested in the behaviour of all the functions in the external field $U(z)$. This means that the diagrammatic expansion of the operator $\gamma = 2Im \Sigma^R$ in Eq. (20) is built upon zero-th order functions, which incorporate dynamics in the external potential and hence are exponentially suppressed in the region behind the barrier. This is in contrast to the usual expansion which utilizes zero-th order functions of free particles. (Such representation is usually called Furry representation in the literature.) In other words, these functions resolve the operator

$$\left( \frac{q}{m} \frac{\partial}{\partial z} + U'(z) \frac{\partial}{\partial q} \right)$$

(30)

which is the left hand side of Eq. (20). Therefore, they are exponentially suppressed in the region of $z$ behind the barrier. This can be seen from the explicit expressions for the Green functions. The function $G^{(0)-+}$, is written in Eq. (8), and is proportional to the product $g(z_1,k)g^*(z_2,k)$. As discussed above, this product is exponentially suppressed behind the barrier. Analogously, the function $G^{(0)+-}$ can be obtained from $G^{(0)-+}$ by substitution $n(q \perp k) \rightarrow (1 - n(q \perp k))$, and is also suppressed exponentially, since its dependence on $z$ is the same. This is an important property, which we will make use of, while analyzing the behaviour of $\gamma$ in the region behind the barrier.

Now we can write explicitly expression of the first Born diagram for $\gamma_{\omega q \perp}(z_1, z_2)$ in the Furry representation from the Fig. 1 b) of Appendix A. There is a slight difference from the case of free zero-th order functions. Namely, there are additional integrals over intermediate $z_3, z_4$-points in the loop (see Appendix A for notations):

$$- i \Sigma_{\omega p \perp}^{-+}(z_1, z_2) = \int G_{a}^{+}(-\omega, p \perp - q \perp; z_1, z_2)G_{b}^{+}(-\omega, p_1 \perp; z_4, z_3)G_{b}^{+}(-\omega, p_1 \perp - q_1 \perp; z_3, z_4) V(q \perp; z_1 - z_4) V(q \perp; z_2 - z_3) \frac{d\omega_p dp_1^+ dq_1^+}{(2\pi)^3} \frac{d\omega_q dq_1^+}{(2\pi)^3} dz_3 dz_4.$$  

(31)

Since all the Green functions in Eq. (31) are exponentially suppressed behind the barrier, we will obtain the largest contribution, if we restrict integration over $z_3$ and $z_4$ to the region away from the barrier, $z > z_*$. The functions $G_{b}^{+}, G_{b}^{-}$ are not exponentially suppressed there. However, note that $\Sigma_{\omega p \perp}^{-+}(z_1, z_2)$ depends on $z_1, z_2$ through the function $G_{a}^{-}$ in Eq. (31).

$$\gamma_{\omega q \perp}(q, z) = \int exp[-iq(z_1 - z_2)] \left( \Sigma_{\omega q \perp}^{-+}(z_1, z_2) - \Sigma_{\omega q \perp}^{+}(z_1, z_2) \right) d(z_1 - z_2)$$

(32)

will depend on the behaviour of these Green functions behind the barrier.

But we know that, since both $G_{a}^{(0)-+}$ and $G_{a}^{(0)+-}$ resolve the operator (30), they are exponentially suppressed behind the barrier. (This is also proven by their explicit expressions, as explained above). Therefore, $\gamma_{\omega q \perp}(q, z)$ is also exponentially suppressed there.

More formally, we can obtain explicit expression for the $\gamma$ as far as its dependence on $z$ is concerned. First, we have to simplify Eq. (32); then apply similar arguments to $\Sigma^{+-}$, which will allow to find $\gamma$.

We are interested in the region of $z \equiv (z_1 + z_2)/2$ behind the barrier. We can make this region rather small, compared to the region away from the barrier, by choosing potential $U(z)$ to be infinite for $z < z_0$. Here $z_0$ is an arbitrary, fi-
nite point behind the barrier. This procedure does not restrict generality of this argument, since the actual region behind the Coulomb barrier of a fusing particle is much smaller than the region away from the barrier. Then, we can approximate $V(q_1, z_1 - z_4)$ by $V(q_1, z - z_4)$, as well as $V(q_1, z_2 - z_3)$ by $V(q_1, z - z_3)$ in Eq. (21). As we explained above, the main contribution to the integral over $z_3, z_4$ comes from the region of $z > z_*$ (the region away from the barrier).

$$-i\Sigma^+_{\omega p} (z_1, z_2) = -4\pi i \int \int \frac{dp_1}{(2\pi)^3} \frac{dq}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{dk}{(2\pi)} n^a_{p_1-q_1} g_a(z_1, k^*) g_a^*(z_2, k) V_q^2 \delta(\omega_p - \epsilon^b_{p_1} + \epsilon^b_{p_1-q} - \epsilon^a_{p_1-q^+})$$

In a similar manner, we find the answer for the $\Sigma^{-+}$:

$$-i\Sigma^{-+}_{\omega p} (z_1, z_2) = -4\pi i \int \int \frac{dp_1}{(2\pi)^3} \frac{dq}{(2\pi)^3} \int_{-\infty}^{\infty} \frac{dk}{(2\pi)} (1 - n^a_{p_1-q_1} k^*) n^b_{p_1-q} g_a(z_1, k) g_a^*(z_2, k) V_q^2 \delta(\omega_p - \epsilon^b_{p_1} + \epsilon^b_{p_1-q} - \epsilon^a_{p_1-q^+})$$

The expression for $\gamma$ follows from Eqs. (32, 33, 34). We see that its dependence on $z$ is determined by functions $g(z, k)$, which are exponentially suppressed behind the barrier. This completes the proof and allows us to neglect the “inhomogeneous” term in the Eq. (20).

The solution of the ”homogeneous” equation, $G_f^{++}$, involves only one factor of $W(k)$, since $G_f^{++} \propto g g^*$, see Eq. (23). Therefore, to obtain $K$ we will find only the solution to the ”homogeneous” equation. We supplement this homogeneous equation with the boundary condition, $\tilde{G}^--$ from Eq. (27) imposed at $z = z_*$ away from the barrier.

Since the largest contribution to $K$ is made by $G_f^{++}$ one can see, that we can use Eq. (4) to find $K$ if we substitute $G_f^{++}$ instead of $G^{(0)+}$ in (4). Therefore, we find $G_f^{++}$ with dependence on $z_1, z_2$ from the very begining by acting on Eq. (2) with the operator

$$\int \int g(z_1, k_1) g^*(z_2, k_2) \frac{dk_1}{2\pi} \frac{dk_2}{2\pi}$$

We use the boundary condition, Eq. (27) and obtain the following answer for $G_f^{++}$:

$$\text{in}_F(\omega) \int g_k(z_1) \delta(\omega - \epsilon_{q_1}^*) g_k^*(z_2) dk$$

where $g_k(z) \equiv g(z, k)$ and we used for $\delta_{\gamma}$ [7]

$$2\text{Im} G^{R}_{\omega q_1} (k) = \frac{\gamma_{\omega q_1} (k)}{(\omega - \epsilon_{q_1})^2 + (\gamma_{\omega q_1} (k))^2}$$

Comparing Eq. (8) with (36), (37) we see, that the effect of collisions, which makes the largest contribution to $K$ can be described as $\delta(\omega - \epsilon_{q_1}) \Rightarrow \delta_{\gamma} (\omega - \epsilon_{q_1})$, as can be expected on the intuitive grounds. Now we substitute Eqs. (36), (37) into (4) and integrate $d\omega/2\pi$ by using the result of [4] for large momenta, $q \gg q_T$:

$$\int \frac{d\omega}{2\pi} n_M(\omega) \delta_{\gamma} (\omega - \epsilon_{q_1}^*) = n_M (\epsilon_{q_1}^*) + \delta n_{\gamma} (q_1^*)$$

where $\delta n_{\gamma} (q_1^*)$ is the “quantum tail”. To describe the tunneling of particles $m_1$ and $m_2$ we perform the same steps as those leading to Eq. (23). Note that this procedure implies quantization of the hypothetical particle with the mass equal to the reduced mass of fusing...
particles. Strictly speaking, this approximation is correct in the limit when one particle is light and the other is heavy. Keeping this in mind, we arrive at the answer for $K$:

$$K = n_0 \int_{0}^{\infty} \frac{d\varepsilon_p}{2\pi} \int_{0}^{\infty} \frac{4\pi p^2 dp S(\varepsilon_p)}{(2\pi)^3} \frac{\delta_{\gamma}(\omega - \varepsilon_p)}{\varepsilon_p}$$

We see that it involves integrals over both the energy and momentum. The width of the resonance is described by the zero-th order Green functions, lead to the analytical answer found in [4] (see Appendix A). We use this answer for $\gamma$ and obtain

$$[\bar{n}_M(p) + \delta\bar{n}_\gamma(p)] \equiv K_M + K_\gamma. \quad (39)$$

Finally, we evaluate $\Sigma^R$ through the collision frequency, $\nu_T$, carry out the integral in [11] and form the ratio $r_{12} = K_\gamma/K_M$ (see Appendices A and B):

$$r_{12} = \frac{3^{19/2}}{8\pi^{3/2}} \sum_j \frac{h\nu_T(m_{coll})}{T} \left( \frac{m_{coll}}{m_r} \right)^2 \frac{e^{\tau_{12}}}{\tau_{12}^8} \quad (40)$$

Here $m_{coll} = m_r m_j/(m_r + m_j)$, $m_r = m_1 m_2/(m_1 + m_2)$, $\tau_{12} = 3(\pi/2)^{2/3} \left( \frac{100Z^2 Z'^2 A_{12}}{T_k} \right)^{1/3}$, $A_{12} = A_1 A_2/(A_1 + A_2)$, $T_k$ is the temperature in keV; $A_1, A_2$ are the atomic numbers of tunneling particles, $m_j$ is the mass of the background particles colliding with tunneling particles. Summation is over all species of colliding background particles, including tunneling species. This result, [11], supports the idea of averaging of $\sigma v$ over momentum distribution postulated in [3].

Note an important feature of the result [11]: this ratio goes to zero as $h\nu_p/T \to 0$. As we show in Appendix A, the width, $\gamma$, can be approximated as $h\nu_p$. Therefore, this means that in the limit of negligible width, the correction to the rate becomes very small, and the reaction rate coincides with the usual Gamow rate. This is expected, since the smaller the width, the closer the particle to the mass shell. And the Gamow rate is derived for on-shell particles. On the other hand, the ratio becomes large, if $h\nu_p/T$ is not very small, yet still $h\nu_p/T < 1$. It is this condition, which allows us to use the result of [4] for $\gamma$ in Eqs. [39], [40]. In the opposite case, $h\nu_p/T \geq 1$, corresponding to more strongly coupled plasma, the theory presented here does not apply.

Consider a $DT$ plasma of density $\rho = 10\, g/cm^3$ and temperature $T = 0.1\, keV$. The ratio $h\nu_p/T = 0.0075$ for these conditions. Then Eq. [11] will lead to the rate accelerated by five orders of magnitude as compared to the conventional answer. We find the ratio $r_{DT} = K_{DT}/K_{DT}^M = 1.8 \cdot 10^5$, with $K_{DT}^M = 2.6 \cdot 10^{-30} cm^3/sec$, $K_{DT} = 4.8 \cdot 10^{-25} cm^3/sec$.

It is also interesting to consider how the reaction rates are changed under conditions relevant to astrophysics, e.g. in the Sun interior. Modification of several reaction rates and their influence on neutrino fluxes from the Sun was considered in [8]. Here we give examples of $p + Be^7$ and $p + p$ reactions, occuring at the center of the Sun, $r = 0$, where the temperature is $T = 1.336\, keV$ and the density $\rho = 156\, g/cm^2$, according to the solar model of [21]. The estimate for $\gamma_{17}/T$ is $h\nu_T/T \sim 0.0056$, while the rates and the ratio, $r_{17}$, are $K_{17}^M = 9.6 \cdot 10^{-36} cm^3/sec$, $K_{17}^\gamma = 4.0 \cdot 10^{-30} cm^3/sec$, $r_{17} = 4.17 \cdot 10^5$.

In case of heavier elements and not very small $h\nu_T/T$, the ratio $r_{ij}$ becomes much higher. On the other hand, the corresponding numbers for the $p + p$ reaction under the same conditions are: $\gamma/T \sim h\nu_T/T \sim 0.00054$, $K_{11}^M = 1.4 \cdot 10^{-43} cm^3/sec$, $r_{11} = 3 \cdot 10^{-3}$. The reaction rate is essentially the same. Note that $h\nu_T/T \sim 0.00054$ is rather small. This confirms the conclusion of the discussion above, that small $\gamma$ lead to insignificant changes in the reaction rates. Note, however, that we need to have some gauge...
to determine which $\gamma$ is small. Naturally, such a gauge should be a ratio of the height of the Coulomb barrier to the average particle energy. As we see from Eq. (40) this ratio enters the answer for $r_{12}$ through $\tau_{12}$.

In conclusion, we derived the momentum distribution in equilibrium through real-time technique of Keldysh and Korenman which coincides with the distribution, obtained in imaginary time \[4\]. This distribution takes into account off-shell effects which lead to the quantum tail with power-like dependence on momentum in contrast to the distribution for on-shell particles which is Fermi-Dirac or Maxwellian. We calculated the rates of fusion reactions for particles in a weakly coupled plasma and showed that off-shell effects may significantly enhance the reaction rates.

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APPENDIX A

CALCULATION OF THE MASS OPERATOR, $\Sigma^R$, AND THE DISTRIBUTION FUNCTION OVER MOMENTUM

Galitski and Yakimets found how off-shell processes change the distribution function over momentum in equilibrium, using imaginary time technique \[4\]. In this Appendix we show how to find the same result through real time diagram method. We also generalize it slightly by allowing for collisions between different types of particles.

Dyson equations for the function $G^{-+}$ can be written as \[13\]

$$
\left( \frac{i}{\partial t} + \frac{p}{m} \cdot \frac{\partial}{\partial R} \right) G^{-+}(R, t; \omega, p) = -\Sigma^{-+}G^{++} + \Sigma^{++}G^{-+}
$$

To simplify notation we do not write explicitly the variables the functions $\Sigma$ and $G$ depend upon. We set to zero the LHS of this equation in equilibrium and obtain

$$
G^{-+} = \frac{\Sigma^{-+}}{\Sigma^{++}} \left( G^R - G^A + G^{-+} \right),
$$

(42)

which follows from the relation between Green functions \[13\]. The result for $G^{-+}$ follows by substituting Eq. (42) into (28):

$$
G^{-+} = -2\pi i n_F(\omega) \frac{Im\Sigma^R}{(\omega - \epsilon_p - Re\Sigma^R)^2 + (Im\Sigma^R)^2}
$$

(43)

We use this equation to find the distribution function over momentum which is

\[
f(p) = \int_{-\infty}^{\infty} \frac{d\omega p}{2\pi} G^{-+}(\omega_p, p).
\] (44)

First we calculate the mass-operator $\Sigma^R$. It can be expressed using the relation

$$
\Sigma^R = \frac{1}{2}(\Sigma^{-+} - \Sigma^{++}) - \frac{1}{2}(\Sigma^{+-} - \Sigma^{-+})
$$

(45)

The first term in this equation corresponds to the real part $Re\Sigma^R$, while the second term is imaginary part $Im\Sigma^R$. Since it is the imaginary part of $\Sigma^R$ which is responsible for the quantum tail \[4\] we calculate only $Im\Sigma^R$ and ignore $Re\Sigma^R$ altogether.

The first two terms of the series for the operators $\Sigma_{\alpha\beta}$ are shown in the Fig. 1. One has to assign all possible combination of signs $-, +$ to the vertices, keeping in mind that the sign does not change along the dashed line \[4\].

![Fig. 1](image)

**FIG. 1.** First and second-order terms in the diagram expansion of $\Sigma^R$. 8
In a more rigorous approach, based on the resummation of the series of graphs, one can show that, solid lines in the diagrams of Fig. 1 correspond to exact Green-functions, while the dashed lines represent scattering amplitudes of the particles in the media. However, such resummations lead to intractable non-linear integro-differential equations.

As a first approximation we assume that, the solid lines correspond to zero-order Green functions, while the dashed lines correspond to the interaction potential between the particles. These diagrams represent quite different physical phenomena. Diagram “a)” describes the screening effects and formation of the quasiparticles in the plasma. Therefore, an imaginary part of this graph represents decay of the quasiparticles. This term is conventionally put into the LHS of the kinetic equation, signifying that it has nothing to do with the collisional integral. Diagram “b)”, on the other hand, if evaluated has nothing to do with the collisional integral.

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This obviously reflects the fact that only identical particles are subject to exchange. Taking into account these remarks, we write the analytical expression for the diagramm b) in the form

$$-i\Sigma^{-+} = \int G_{a}^{-+}(P-Q)G_{b}^{+-}(P_{1})G_{b}^{+-}(P_{1}-Q) [V(q)]^{2} \frac{da_{p_{1}}dp_{1}d\omega_{q}dq}{(2\pi)^{4}} (2\pi)^{4}, \quad (46)$$

where capital letters denote four-momentum, e.g. $P = (\omega_{p}, p)$ and $V(q)$ is the interaction potential between the particles. We substitute here equilibrium zero-th order Green functions [13]

$$G^{-+}(\omega_{p}p) = 2\pi i n_{p} \delta(\omega_{p} - \epsilon_{p} + \mu), \quad (47)$$

$$G^{+-}(\omega_{p}p) = -2\pi i (1 - n_{p})\delta(\omega - \epsilon_{p} + \mu). \quad (48)$$

Analogously, we obtain an expression for the diagram c).

For a moment, we omit the indices $a$ and $b$ to simplify the formulas, but write them explicitly in the final formulas. Then the result for $\Sigma^{-+}$ is

$$\Sigma^{-+}(\omega_{p}p) = (-2\pi i) \int \frac{dP_{1}}{(2\pi)^{3}} \int \frac{dq}{(2\pi)^{3}} \left(2V_{q}^{2} - V_{q}V_{q-p-p_{1}}\right) n_{p-q} n_{p_{1}-q} \delta(\omega_{p} - \epsilon_{p-q} + \epsilon_{p_{1}} - \epsilon_{p_{1}-q}) \quad (49)$$

Proceeding in a similar fashion with $\Sigma^{+\gamma}$ and using Eq. (50) we obtain the final result for $Im\Sigma^{R} \equiv \gamma(\omega_{p}, p)$:

$$\gamma(\omega_{p}, p) = \pi i \int \frac{dq}{(2\pi)^{3}} \frac{dP_{1}}{(2\pi)^{3}} V_{q}(2V_{q} - V_{q-p-p_{1}}) \left[n_{p-q} (n_{p_{1}-q} - n_{p_{1}}) + n_{p_{1}} (1 - n_{p_{1}-q})\right] \delta(\omega_{p} - \epsilon_{p-q} + \epsilon_{p_{1}} - \epsilon_{p_{1}-q}) \quad (50)$$

This answer coincides with the result of [24] obtained in the imaginary-time technique.

Note that we could improve this relation by substituting $\delta_{\gamma}$ of Eq. (57) instead of the delta-function. This, however, would lead to an integral equation for $\gamma$, and its first iteration will be just Eq. (50).

Now we can find the distribution function over momentum from Eqs. (14), (50), (13)

$$f(p) = \frac{1}{(2\pi)^{4}} \frac{n_{F}(\omega)}{\gamma(\omega_{p}, p)} \frac{d\omega_{p}}{(2\pi)^{3}} \frac{dP_{1}}{(2\pi)^{3}} \frac{d\omega_{q}dq}{(2\pi)^{4}} \quad (51)$$

by substituting Eq. (50) into Eq. (13) * . The integral $d\omega$ can be rewritten as a sum of two terms

*From now on we follow closely the work of [8]
\[ f(p) = \int_{-\infty}^{\epsilon_0} \frac{d\omega_p}{2\pi} n_F(\omega) \delta_\gamma(\omega_p - \epsilon_p) + \int_{\epsilon_0}^{\infty} \frac{d\omega_p}{2\pi} n_F(\omega) \delta_\gamma(\omega_p - \epsilon_p), \] (52)

where \( \epsilon_0 \) is the characteristic energy. In the degenerate case, \( T \ll \mu \), we have \( \epsilon_0 = \mu \), in the non-degenerate case, \( T \gg \mu \) and \( \epsilon_0 = T \). We define \( p_0 \) as \( \epsilon_0 = p_0^2/2m \).

Consider the case of large momenta, \( p \gg p_0 \). The function \( \delta_\gamma(\omega_p - \epsilon_p) \) has a maximum at \( \omega_p = \epsilon_p \gg \epsilon_0 \), so that \( \omega_p \in [\epsilon_0, \infty] \), and the contribution of the second integral can be approximated as \( n_F(\omega) \approx 1 \). In the region of integration of the first integral in Eq. (52) we can set \( n_F(\omega) \approx 1 \). Then Eq. (52) becomes for \( p \gg p_0 \)

\[ f(p) = n_p + \int_{-\infty}^{\epsilon_0} \frac{d\omega_p}{2\pi} \gamma(\omega_p, p) (\omega_p - \epsilon_p)^2 \equiv n_p + \delta n_\gamma(p), \] (53)

We carry out the integral in the second term of (53), use Eq. (50) and obtain

\[ \delta n_\gamma(p) = \int \frac{d\mathbf{q}}{(2\pi)^3} \frac{d\mathbf{p}_1}{(2\pi)^3} V_q(2V_q - V_{-\mathbf{p} - \mathbf{p}_1}) \frac{n_{p-q}(n_{p_1-q} - n_{p_1}) + n_{p_1}(1 - n_{p_1-q})}{(\epsilon_p - \epsilon_{p-q} + \epsilon_{p_1} - \epsilon_{p_1-q})^2}, \] (54)

with the condition on momenta

\[ -\infty < \epsilon_{p-q} - \epsilon_p + \epsilon_{p_1} - \epsilon_{p_1-q} < \epsilon_0 \] (55)

due to the integration of the delta-function from \( \gamma(\omega_p, p) \).

The integral in Eq. (54) should be carried out numerically. We can, however, obtain a reasonable estimate in the case of large momentum, \( p \gg p_0 \). Observe that the first two terms in the brackets [...] should have the same dependence on \( p \). To obtain an estimate consider the product of the first term in (54), \( V_q^2 \), and the first term in [...] which involves the product \( n_{p-q}n_{p_1-q} \). We denote it \( I_1(p) \). Note that there is a region of non-exponential contribution to the integral, which is limited to the region of momenta

\[ |\mathbf{p} - \mathbf{q}| \sim |\mathbf{p}_1 - \mathbf{q}| \sim p_0, p \sim p_1 \sim q \gg p_0 \] (56)

The inequality (53) is satisfied which, in turn, means that the dominant contribution to this product can be approximated as

\[ I_1(p) = \int \frac{d\mathbf{q}}{(2\pi)^3} \int \frac{d\mathbf{p}_1}{(2\pi)^3} 2V_q^2 n_{p-q}(n_{p_1-q}n_{p_1} - q^2 - \epsilon_p^2), \] (57)

where \( \epsilon_p \equiv p^2/2m_{ab} \) and we reinstated indices \( a \) and \( b \). Consider non-degenerate case.

We change variables, \( \mathbf{p} - \mathbf{q} = \mathbf{q}', \mathbf{p}_1 - \mathbf{q} = \mathbf{p}_1' \) and obtain an estimate

\[ I_1(p) = e^{a^2} e_b^2 \left( \frac{2m_{ab}}{p^2} \right)^2 \left( \frac{m_a}{2\beta \pi} \right)^{3/2} \left( \frac{m_b}{2\beta \pi} \right)^{3/2} e^{\beta(\mu_a + \mu_b)} \] (58)

We substitute the relation

\[ e^{\beta(\mu_a + \mu_b)} = n^{(a)}(\bar{\lambda}_a^3 n^{(b)} \bar{\lambda}_b^3 \] (59)

\[ \bar{\lambda}_a^3 = \left( \frac{m_a}{2\beta \pi} \right)^{3/2}, \bar{\lambda}_b^3 = \left( \frac{m_b}{2\beta \pi} \right)^{3/2} \] (60)

into Eq. (58) and find

\[ I_1(p) = \frac{(2m_{ab})^2}{p^8} (e_a e_b)^2 n^{(a)} n^{(b)} \] (61)

Since the physical reason for non-zero \( \gamma(\omega_p, \mathbf{p}) \) is collisions, we rewrite (61) in terms of the collision frequency

\[ \nu_p = \left( \frac{\pi}{2} \right)^{3/2} \left( \frac{\pi}{2} \right)^{1/2} n^{(a)}(e_a e_b)^2 m_{ab}^{-1/2} T^{3/2} \frac{(2m_{ab}T)^{3/2}}{p^3} \] (62)

as

\[ I_1(p) = \frac{2}{\pi^{3/2} e_p^2} \frac{\hbar \nu_p T \tilde{p}_T n^{(b)}}{p^3}, \] (63)

where \( \tilde{p}_T \equiv (2m_{ab}T)^{1/2} \). This is essentially the result of \( [1] \) for the degenerate case, sligtly generalized to the case of colliding particles with different masses.

In the non-degenerate case we can keep only the term linear in \( n_p \) in Eq. (54), because the quadratic terms are smaller. We denote this
term by $I_3$. Since there is no exponential suppression in the $q$-integral for this term, we have to evaluate the limits of the $q$-integration carefully. We can obtain these limits from Eq. (55). After some algebra it leads to
\[
\frac{q^2}{2m_{ab}} - qx \left[ \frac{p}{m_a} + \frac{p_1}{m_b} \right] > \epsilon_p - T,
\]
where $x$ is the cosine of the angle between $q$ and $\frac{p}{m_a} + \frac{p_1}{m_b}$. We resolve this inequality in the limit of large $p \gg p_T$, neglecting all terms on the order of $p_T^2$ in comparison to $p^2$. Note that since the term in question, $I_3$, is proportional to $n_{p_1}$, the contribution from the region $p_1 \geq p_T$ is exponentially suppressed. Since we are interested in the non-exponential contribution only, we can assume that $p_1 \sim p_T$ in Eq. (64). Then we finally arrive at
\[
q > q_* = \frac{m_{ab}}{m_a} p (1 + \sqrt{1 + \frac{m_a}{m_b}})
\]
To obtain an estimate for $I_3$, we carry out the integral of the linear term in $n_{p_1}$ from Eq. (64), multiplied by the first term in $(..)$, $V_q^2$:
\[
I_3(p) = 2\pi \int dp_1 \int dx \int_{-1}^{\infty} q^2 dq
\]
\[
\frac{(e_a e_b)^2}{q^4} \frac{e^{-p_1^2/2m_0 T} e^{\beta(\mu_a + \mu_b)}}{\left( \frac{q^2}{2m_{ab}} - qx \left[ \frac{p}{m_a} + \frac{p_1}{m_b} \right] \right)^2}
\]
After performing the integral over $x$, we are left with the integral over $y$, which is the cosine of the angle between $p$ and $p_1$. The $y$-dependent term becomes
\[
\int dy \left( \frac{q^2}{2m_{ab}} - q^2 \left( \frac{p^2}{m_a} + \frac{p_1^2}{m_b} + \frac{2p p_1}{m_a m_b} \right) \right)
\]
We carry out this integral, neglect all the terms of the order of $p_T^2$ in comparison to $p^2$ and obtain
\[2m_{ab}^2/(q^2 p^2).\]
The remaining integrals over $\int_0^\infty dp_1$ and $\int dq$ are trivial. Using Eq. (65) we get the result for $I_3$
\[
I_3(p) = \frac{2}{3} \frac{1}{(2\pi)^3} \frac{m_{ab}^2}{p^3} \left( \frac{m_a}{m_{ab}} \right)^3 \frac{(e_a e_b)^2 n_{\mu_a}^2}{1 + \sqrt{1 + \frac{m_a}{m_{ab}}}}
\]
Now we can see that different terms in Eq. (64) have a different dependence on $p$. Note however that the answers for these terms are really estimates. To find the correct answer one should solve non-linear equation for $\gamma$ [5], which is possible only through numerical methods.

It is also possible to obtain an estimate through dimensional analysis which leads to
\[
\gamma = 2Im \Sigma_R \sim 2h\nu_p
\]
This expression has to be substituted into Eq. (53). Note that we have to carry out the integral over energy from $-\infty$ to $T$. Since we do not know how $\gamma$ behaves at negative energies, we can only make an assumption that, this region does not contribute significantly to the integral, and the main contribution comes from $\omega_p \sim T$.

Then the integral is equal roughly to
\[
I(p) = \frac{h\nu_p T n^{(a)}}{2\pi^2 \beta^3 p^3}
\]
We can simplify it to
\[
I(p) = \frac{h\nu_p T e^{\beta \mu_a}}{2\pi^2 p^3}
\]
which is the result for the non-degenerate case found in [5]. We will use this estimate in our calculations of the reaction rate because of its simplicity.

**APPENDIX B**

**REACTION RATE: CALCULATION OF THE INTEGRAL OVER $\delta n_\gamma(p)$**
In this Appendix we find the rate $K_\gamma$ of Eq. (39). Observe, that in the main text we used approximation of a hypothetical particle with the mass equal to the reduced mass of fusing particles, $m_r$. It is this particle which undergoes collisions with the background, leading to its power-like distribution over momentum. We will need the result (69) for these particles. We write as

$$I(p) = \frac{\hbar \nu_p T}{2\pi \epsilon_p^2} \left( \frac{m_r}{m_{coll}} \right)^{3/2} e^{\beta \mu_a} \tag{71}$$

taking into account that

$$n^{(a)}(p_T^2) = \left( \frac{m_r}{m_{coll}} \right)^{3/2} e^{\beta \mu_a}. \tag{72}$$

Therefore, we substitute Eq. (71) into Eq. (39) and obtain $K_\gamma$

$$K_\gamma = \left( \frac{m_r}{m_{coll}} \right)^{3/2} \left( 2m_r \right)^3 \frac{\hbar \nu_T (m_{coll}) T}{2\pi} S(\epsilon^*) \int_0^\infty \epsilon^{-\pi p_G/p} \frac{1}{p_0^2} dp \tag{73}$$

where $S(\epsilon^*)$ is the astrophysical factor at some fixed value of energy. The integral is trivial and is equal to

$$\int_0^\infty \epsilon^{-\pi p_G/p} \frac{1}{p_0^2} dp = \frac{24}{\pi^3 p_G^2}. \tag{74}$$

For the sake of completeness, we also find here the usual Gamow rate:

$$K_M = 4\pi S(\epsilon^*) \int_0^\infty \epsilon^{-\pi p_G/p - p^2/2m_r T} p dp. \tag{75}$$

The result, obtained by using the steepest descend method, is

$$K_M = S(\epsilon^*) \frac{2}{3} \pi^{1/2} (m_r T)^{1/2} e^{-\tau}. \tag{76}$$

Strictly speaking, the astrophysical factors in Eqs. (73) and (76) need not be the same. In fact, one should consider a generalized astrophysical factor, $S(\omega_p, \epsilon_p)$, depending both on energy and momenta. Such factors would take into account the influence of the off-shell effects on nuclear transformations. However, as far as we know, they have not been studied. Therefore, we assume that the factors in Eqs. (73), (76) are the same.

With this in mind, we can form the ratio

$$r_{12} = \frac{K_\gamma}{K_M} = \frac{3^{19/2}}{2^2 5^{3/2}} \frac{\hbar \nu_T (m_{coll})}{T} \left( \frac{m_{coll}}{m_r} \right)^2 \frac{e^\tau}{\tau^8} \tag{77}$$

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