Nonreciprocal devices, featuring asymmetric flow of counterpropagating lights, are indispensable in optical information processing and communications [1]. Giant nonreciprocity and optical isolations have been achieved in various systems, such as atomic gases [2], solid-state nonlinear devices [3–7], and moving media [8–10] or synthetic structures [11–13]. Besides steering classical transmission rates, nonreciprocal quantum control of single photons has also been demonstrated, ranging from single-photon diodes [14–16] or circulators [17] to unidirectional photon blockade [18–20]. However, as far as we know, there are still lacking studies of properties of nonreciprocal entanglement and its unique applications in e.g., chiral quantum control [21–24] or entanglement-assisted quantum information processing [25–27].

In this work, we study the creation and exotic properties of nonreciprocal photon-phonon entanglement in a spinning resonator. We note that, based on radiation-pressure-induced coupling of light and mechanical motion [28–30], cavity optomechanical (COM) systems have provided versatile tools to realize ground-state motion cooling [31], quantum state transfer [32–34], mechanical squeezing [35], COM entanglement [36–41], and gravitational-wave sensing [42]. In a very recent advance, quantum correlations are demonstrated even between light and 40 kg mirrors at room temperature, enabling ultra-weak force sensing beyond the standard quantum limit [43]. Here, by combining quantum control and nonreciprocal engineering, we show that one-way COM entanglement can be created in a spinning resonator, which, surprisingly, provides a feasible way to protect the quality of entanglement against backscattering losses. Specifically, we find that in such a nonreciprocal device, for the same material imperfections, COM entanglement in a chosen direction can be significantly enhanced, in comparison with that in conventional COM systems. Our findings, indicating a new way to protect fragile quantum resources by utilizing the power of nonreciprocal devices, are useful for applications in e.g., chiral quantum information processing [21–24] and backaction-immune quantum sensing [25–27].

As shown in Figs. 1(a) and 1(b), we consider a spinning resonator, which is evanescently coupled with a tapered fiber. In a recent experiment [10], nonreciprocal propagation of light with 99.6 % isolation was demonstrated by using such a spinning device. Due to the rotation, the optical paths of counterpropagating lights in the resonator become different, which results in an irreversible effective refractive index for the clockwise (CW) and counter-clockwise (CCW) optical modes [10], i.e., $n_{\text{CCW}} = n_{0}[1 \pm n_{0} r^{2}(n_{0}^{2} - 1)/c]$, where $n_{0}$ is the refractive index of the material, $\Omega$ is the angular velocity of the resonator with radius $r$, and $c$ is the speed of light in the vacuum. Correspondingly, the resonance frequencies of the counter-propagating modes experience an opposite Sagnac-Fizeau shift [44], i.e., $\omega_{z} \rightarrow \omega_{z} + \Delta_{F}$ with

$$
\Delta_{F} = \pm \Omega \frac{n_{0} a_{0} c}{c} \left(1 - \frac{1}{n_{0}^2} - \frac{\lambda}{n_{0} d l} \right),
$$

where $\omega_{z}$ is the optical frequency of the stationary resonator, and the dispersion term $d n/d \lambda$, characterizing the relativistic origin of the Sagnac effect, is relatively small in typical materials (up to $\sim 1\%$) [10]. Figure 1(c) shows the frequency spectrum of such a spinning COM system. For the resonator spinning along the CW direction, we have $\Delta_{F} > 0$ or $\Delta_{F} < 0$ for the case with the driven laser on the left or right side, and the effective optical frequency is $\omega_{j} = \omega_{z} \pm | \Delta_{F} | (j = C, C)$, respectively. In addition, the resonator can support a mechanical breathing mode with frequency $\omega_{m}$. The Hamiltonian of this COM system, in a frame rotating with a driven frequency $\omega_{1}$, with the driven laser on the right side, is ($\hbar = 1$):

$$
\hat{H} = \hat{H}_{c} + \frac{\omega_{m}}{2}(\hat{p}^{2} + \hat{q}^{2}) - G_{0}(\hat{a}_{1}^{\dagger}\hat{a}_{C} + \hat{a}_{1}\hat{a}_{C}^{\dagger})\hat{q},
$$

$$
\hat{H}_{c} = \sum_{j = C, C'} \Delta_{j} \hat{a}_{j}^{\dagger}\hat{a}_{j} + J(\hat{a}_{1}^{\dagger}\hat{a}_{C} + \hat{a}_{1}\hat{a}_{C}^{\dagger}) + i \varepsilon(\hat{a}_{1}^{\dagger} - \hat{a}_{C}),
$$

(2)
where $\hat{a}_j (\hat{a}_j^\dagger)$ is the optical annihilation (creation) operator, $\Delta_j = \omega_j - \omega_i$, and $\hat{q} (\hat{p})$ is the dimensionless mechanical displacement (momentum) operator. The field amplitude of the driving laser is $|\epsilon| = \sqrt{2\kappa P/\hbar \omega_i}$, where $P$ and $\kappa$ are the input laser power and the optical decay rate, respectively. The optical coupling strength $J$ denotes the backscattering process which is inevitable for practical materials with inherent imperfections, and $G_0 = (\omega_c/r)(m \omega_m)^{-1/2}$ is the single-photon COM coupling rate, with $m$ denoting the mass of the resonator.

The quantum Langevin equations of this COM system then read:

$$\dot{\hat{a}}_\circ = - (i\Delta_\circ + \kappa) \hat{a}_\circ - iJ\hat{a}_\circ + iG_0 \hat{a}_\circ \hat{q} + \sqrt{2\kappa}\hat{a}_\circ^{\dagger},$$
$$\dot{\hat{q}} = \omega_m \hat{p},$$
$$\dot{\hat{p}} = -\omega_m \hat{q} - \gamma_m \hat{p} + G_0 (\hat{a}_\circ^{\dagger} \hat{a}_\circ + \hat{a}_\circ \hat{a}_\circ^{\dagger}) + \xi,$$

where $\gamma_m$ is the mechanical damping rate, and $\hat{a}_\circ^{\dagger}$ ($\xi$) is the zero-mean input operator for the optical (mechanical) mode, characterized by the following correlation functions [45]:

$$\langle \hat{a}_\circ^{\dagger} (t) \hat{a}_\circ^{\dagger} (t') \rangle = \delta(t - t'),$$
$$\langle \xi(t) \xi(t') \rangle \approx \gamma_m (2n_m + 1) \delta(t - t'), \text{ for } \omega_m/\gamma_m \gg 1,$$

where $n_m = [\exp(\hbar \omega_m/k_B T) - 1]^{-1}$ is the thermal phonon number, $k_B$ is the Boltzmann constant, and $T$ is the bath temperature. The zero-mean features of quantum noise preserve the Gaussian nature of any initial state of the system [36]. By writing each operator as a sum of its steady-state mean value and a small fluctuation around it, i.e.,

$$\hat{a}_j = \alpha_j + \delta \hat{a}_j, \quad \hat{q} = q_s + \delta \hat{q}, \quad \hat{p} = p_s + \delta \hat{p},$$

and defining the vectors of quadrature fluctuations and input noises as $u^j(t) = (\delta \hat{X}_j, \delta \hat{Y}_j, \delta \hat{X}_\circ, \delta \hat{Y}_\circ, \delta \hat{q}, \delta \hat{p})$, $n^j(t) = (\sqrt{2\kappa} X_j^{\circ}, \sqrt{2\kappa} Y_j^{\circ}, \sqrt{2\kappa} X_\circ, \sqrt{2\kappa} Y_\circ, 0, \xi)$, with the components:

$$\delta \hat{X}_j = \frac{1}{\sqrt{2}} \left( \delta \hat{a}_j + \delta \hat{a}_j^\dagger \right), \quad \delta \hat{Y}_j = \frac{i}{\sqrt{2}} \left( \delta \hat{a}_j^\dagger - \delta \hat{a}_j \right),$$
$$\hat{X}_j^{\circ} = \frac{1}{\sqrt{2}} \left( \hat{a}_j^{\dagger} + \hat{a}_j \right), \quad \hat{Y}_j^{\circ} = \frac{i}{\sqrt{2}} \left( \hat{a}_j^{\dagger} - \hat{a}_j \right).$$

we can obtain a compact form of the linearized equations of fluctuations

$$\dot{u}(t) = Au(t) + n(t),$$

where

$$A = \begin{pmatrix}
-k \Delta_\circ & 0 & J & -G_0^e & 0 \\
-\Delta_\circ & -\kappa & -J & 0 & G_0^e \\
0 & J & -\kappa & \Delta_\circ & -G_0^e \\
-J & 0 & -\Delta_\circ & -\kappa & G_0^e \\
0 & 0 & 0 & 0 & \omega_m \\
G_0^o & G_0^o & G_0^o & G_0^o & -\omega_m - \gamma_m
\end{pmatrix},$$

and $\Delta_j = \Delta_j - G_0^o q_s$ is the effective optical detuning. $G_j^e$ ($G_j^o$) is the real (imaginary) part of the effective COM coupling

$$G_j = \sqrt{2} G_0^o \alpha_j = G_j^e + i G_j^o,$$

and the steady-state solutions of the dynamical variables are given by:

$$\alpha_\circ = \frac{-i J \varepsilon}{(i \Delta_\circ + \kappa)(i \Delta_\circ + \kappa) + J^2},$$
$$\alpha_\circ = \frac{(i \Delta_\circ + \kappa) \varepsilon}{(i \Delta_\circ + \kappa)(i \Delta_\circ + \kappa) + J^2},$$
$$q_s = \frac{G_0^o}{\omega_m} \left( |\alpha_\circ|^2 + |\alpha_\circ|^2 \right), \quad p_s = 0.$$
FIG. 2: (Color online). Nonreciprocal photon-phonon entanglement without backscattering. (a-b) The logarithmic negativity $E_N$ versus the scaled optical detuning $\Delta_c/\omega_m$ for different input directions. For the nonspinning case $\Delta_r = 0$, COM entanglement does not rely on the input directions and always appears around the resonance $\Delta_c/\omega_m \approx 1$ (gray dotted curve). In the presence of rotation, nonreciprocal entanglement emerges around $\Delta_c/\omega_m = 0.3$ or $1.2$, i.e., COM entanglement is created or prohibited for opposite input directions. (c-d) Classical nonreciprocity versus quantum nonreciprocity. For $\Delta_c/\omega_m \approx 0.27$, quantum nonreciprocity exists even when $N_c = N_r$, i.e., no classical nonreciprocity for the mean number of photons; in contrast, for $\Delta_c/\omega_m \approx 1$, classical nonreciprocity can appear for $E_{N,c} = E_{N,r}$. Here we choose $\Omega = 8 \text{kHz}$, $J = 0$, $P_m = 20 \text{mW}$, and other parameters are given in the text.

The solution of the fluctuations in Eq. (7) is given by $u(t) = \mathcal{M}(t)u(0) + \int_0^t d\tau \mathcal{M}(\tau)u(t - \tau)$, where $\mathcal{M}(t) = \exp\{A t\}$. The system is stable when all the real parts of the eigenvalues of $A$ are negative, as characterized by the Routh-Hurwitz criterion [46, 47]. Under this stability condition, we have $\mathcal{M}(\infty)$ at the steady state and

$$u_i(\infty) = \int_0^\infty d\tau \sum_k \mathcal{M}_{ik}(\tau) n_k(t - \tau).$$

(11)

This COM system finally evolves into a Gaussian state, which is fully characterized by a $6 \times 6$ correlation matrix $V$, with the component

$$V_{kl} = \langle u_k(\infty) u_l(\infty) + u_l(\infty) u_k(\infty) \rangle / 2.$$

(12)

Using Eqs. (4) and (11) the steady-state correlation matrix $V$ is obtained as

$$V = \int_0^\infty d\tau \mathcal{M}(\tau) D(\tau) \mathcal{M}^T(\tau),$$

(13)

where $D = \text{Diag} \{c, \kappa, \kappa, \kappa, 0, \gamma_m(2n_m + 1)\}$ is a diagonal matrix characterizing the stationary noise correlations. When the stability condition is satisfied, Eq. (13) is determined by the Lyapunov equation [36]

$$AV + VA^T = -D.$$

(14)

By solving this linear Eq. (14), we can calculate quantum correlations of the fluctuations and also the logarithmic negativity, $E_N$, as a bipartite entanglement measure of continuous variables, i.e., [48]

$$E_N = \max[0, -\ln(2\nu^-)],$$

(15)

with

$$\nu^- = \frac{1}{\sqrt{2}} \left\{ \left( \sum (\mathcal{V}_{bp}) - \left[ \left( \sum (\mathcal{V}_{bp})^2 - 4 \det \mathcal{V}_{bp} \right)^{1/2} \right] \right)^{1/2},$$

(16)

and $\sum (\mathcal{V}_{bp}) = \det A + \det B - 2 \det C$. Here $\nu^-$ is the lowest symplectic eigenvalue of the partial transpose of a bipartite $4 \times 4$ correlation matrix, $\mathcal{V}_{bp}$, which is obtained by selecting the rows and columns of the interesting modes in $V$ and has a $2 \times 2$ block form

$$\mathcal{V}_{bp} = \begin{pmatrix} A & C \\ C^T & B \end{pmatrix}.$$  

(17)

We see from Eq. (15) that COM entanglement can be achieved only for $\nu^- < 1/2$.

In our numerical calculations, for ensuring the system staying in the stable regime, we use the following experimentally feasible parameters [10, 49]: $n_0 = 1.48$, $m = 10 \text{ng}$, $r = 1.1 \text{mm}$, $\lambda = 1.55 \mu \text{m}$, $Q = \omega_c/\kappa = 3.2 \times 10^7$, $\omega_m = 63 \text{MHz}$, $\gamma_m = 5.2 \text{kHz}$, $T = 130 \text{mK}$, and $\Omega = 8 \text{kHz}$ or $23 \text{kHz}$ [10]. We first consider the ideal case without optical backscattering, i.e., $J = 0$, and we plot the logarithmic negativity $E_N$ and the intracavity photon number $N_j$ in Fig. 2 as a function of the optical detuning $\Delta_c = \omega_c - \omega_q$. For a stationary resonator, $E_N$ remains the same independently of driving it from the left or right side; in contrast, for a spinning resonator, $E_N$ can be significantly different by reversing the driving direction. For example, as shown in Figs. 2(a) and 2(b), when the maximal COM entanglement is generated by driving the setup from one side, i.e., $E_N \sim 0.15$, no entanglement occurs by driving it from the other side, i.e., $E_N \sim 0$. This is a clear signature of quantum nonreciprocity, which is fundamentally different from that in classical devices showing only nonreciprocal transmission rates. In fact, as shown in Figs. 2(c) and 2(d), for $\Delta_c/\omega_m \approx 0.27$, nonreciprocal COM entanglement is significant even when there is no classical nonreciprocity (i.e., $N_c = N_r$); in contrast, for $\Delta_c/\omega_m \sim 1$, significant classical nonreciprocity can appear for $E_{N,c} \simeq E_{N,r}$. This difference between classical nonreciprocity and quantum nonreciprocity can be easily understood by the fact that the COM entanglement is created by the anti-Stokes scattering process between the mechanical mode and the driving field. Therefore, in a broader view, our work provides a new opportunity...
to explore the fundamental differences and practical applications of quantum nonreciprocity and classical nonreciprocity with a single device.

Surprisingly, we also find that nonreciprocal quantum control provides a feasible way to overcome various harmful effects of nonideal factors such as surface roughness or material inhomogeneity in a practical device. Indeed, as Fig. 3 shows, in a conventional COM system, for \( J \neq 0 \), \( E_N \) always decreases. In comparison, nonreciprocal entanglement can be more robust against the backscattering process. For example, for \( J/\kappa = 2 \), the maximal value of \( E_N \) in a spinning resonator can be enhanced by a factor of 2.5, approaching to that in an ideal device without backscattering. The other parameters are the same as in Fig. 2.

Finally, we remark that recent experiments show that COM system is an ideal platform in achieving entanglement at macroscopic level [37–41]. In order to verify the generated entanglement in these experiments, one needs to measure the corresponding correlation matrix \( V_{bp} \), which requires to access all the quadrature fluctuations in the optical and mechanical canonical variables. By applying homodyne or heterodyne detection of the cavity output field, the quadrature fluctuations of the optical mode can be measured straightforwardly [37]. The measurement of the mechanical quadratures involves a relatively complicated process—specifically, a weak probe laser is applied to the COM resonator at the red sideband frequency \( \omega_p = \omega_c - \omega_m \), allowing for mapping the mechanical motion to the anti-Stokes sideband of the probe field at cavity resonance [37, 39]. Then, the mechanical quadratures can be obtained through a similar homodyne process on the probe output field. In our system, nonreciprocal features of COM entanglement, due to the rotation-dependent frequency shifts of the output lights, can be experimentally verified in the same procedure.

In summary, we have investigated the controllable generation of nonreciprocal photon-phonon entanglement in a spinning COM resonator. In particular, we find that in such a nonreciprocal device, COM entanglement can be more robust against backscattering losses in comparison with that in conventional non-spinning systems. This indicates a new feasible way to protect fragile quantum resources by utilizing the power of nonreciprocal control, which, to our knowledge, has never been revealed in previous literatures. In a broader view, our work establishes a new bridge between such two active fields of nonreciprocal physics and quantum-state engineering. Our method of achieving and manipulating nonreciprocal nonclassical effects can also be implemented in a wide range of physical systems, such as microwave or electronic circuits [50–52], cavity atomic gases [53], Floquet time crystals [54], and optically-levitated nanomechanical [55, 56] or magnomechanical systems [57].

We thank Chang-Ling Zou and Chun-Hua Dong for helpful discussions. Y.-F. J. and L.-M. K. are supported by the National Natural Science Foundation of China (NSFC, 11775075 and 11434011). H. J. is supported by the NSFC (11474087, 11774086, 11935006). A. M. is supported by the Polish National Science Centre (NCN) under the Maestro Grant No. DEC-2019/34/A/ST2/00081. Y.-L. Z. is supported by the NSFC (11704370) and the China Postdoctoral Science Foundation (2019M652181).

---

* Electronic address: lmkuang@hunnu.edu.cn
† Electronic address: jinghui73@gmail.com

[1] Y. Shoji and T. Mizumoto, Magneto-optical nonreciprocal devices in silicon photonics, Sci. Technol. Adv. Mater. 15, 014602 (2014).
[2] H. Ramezani, P. K. Jha, Y. Wang, and X. Zhang, Nonreciprocal Localization of Photons, Phys. Rev. Lett. 120, 043901 (2018).
[3] Q.-T. Cao, H. Wang, C.-H. Dong, H. Jing, R.-S. Liu, X. Chen, L. Ge, Q. Gong, and Y.-F. Xiao, Experimental Demonstration of Spontaneous Chirality in a Nonlinear Microresonator, Phys. Rev. Lett. 118, 033901 (2017).
[4] K. Y. Xia, F. Nori, and M. Xiao, Cavity-Free Optical Isolators and Circulators Using a Chiral Cross-Kerr Nonlinearity, Phys.
[5] S. Manipatruni, J. T. Robinson, and M. Lipson, Optical Non-reciprocity in Optomechanical Structures, Phys. Rev. Lett. 102, 213903 (2009).

[6] Z. Shen, Y.-L. Zhang, Y. Chen, C.-L. Zou, Y.-F. Xiao, X.-B. Zou, F.-W. Sun, G.-C. Guo, and C.-H. Dong, Experimental realization of optomechanically induced non-reciprocity, Nat. Photonics 10, 657 (2016).

[7] N. R. Bernier, L. D. Töth, A. Kottandavidia, M. A. Ioannou, D. Malz, A. Nunnerenkamp, A. K. Feofanov, and T. J. Kippenberg, Nonreciprocal reconfigurable microwave optomechanical circuit, Nat. Commun. 8, 604 (2017).

[8] D.-W. Wang, H.-T. Zhou, M.-J. Guo, J.-X. Zhang, J. Evers, and S.-Y. Zhu, Optical Diode Made from a Moving Photonic Crystal, Phys. Rev. Lett. 110, 093901 (2013).

[9] Y. Yang, C. Peng, D. Zhu, H. Buljan, J. D. Joannopoulos, B. Zhen, and M. Soljačić, Synthesis and observation of non-Abelian gauge fields in real space, Science 365, 1021 (2019).

[10] S. Maayani, R. Dahan, Y. Kligerman, E. Moses, A. U. Hassan, H. Jing, F. Nori, D. N. Christodoulides, and T. Carmon, Flying couplers above spinning resonators generate irreversible refraction, Nature (London) 558, 569 (2018).

[11] D. L. Sounas and A. Alù, Non-reciprocal photons in time modulation, Nat. Photonics 11, 774 (2017).

[12] B. Peng, Ş. K. Özdemir, F. Lei, F. Monifi, M. Gianfreda, G. L. Long, S. Fan, F. Nori, C. M. Bender, and L. Yang, Parity-time-symmetric whispering-gallery microcavities, Nat. Phys. 10, 394 (2014).

[13] L. Chang, X. Jiang, S. Hua, C. Yang, J. Wen, L. Jiang, G. Li, G. Wang, and M. Xiao, Parity-time symmetry and variable optical isolation in active-passive-coupled microresonators, Nat. Photonics 8, 524 (2014).

[14] K. Y. Xia, G. W. Lu, G. W. Lin, Y. Q. Cheng, Y. P. Niu, S. Q. Gong, and J. Twamley, Reversible nonpassive single-photon isolation using unbalanced quantum coupling, Phys. Rev. A 90, 043802 (2014).

[15] L. Tang, J. Tang, W. Zhang, G. Lu, H. Zhang, Y. Zhang, K. Y. Xia, and M. Xiao, On-chip single-chip single-photon interface: Isolation and unidirectional emission, Phys. Rev. A 99, 043833 (2019).

[16] M.-X. Dong, Y.-C. Yu, Y.-H. Yo, W.-H. Zhang, E.-Z. Li, L. Zeng, G.-C. Guo, D.-S. Ding, and B.-S. Shi, Experimental realization of quantum non-reciprocity based on cold atomic ensembles, arXiv:1908.09242.

[17] M. Scheucher, A. Hilico, E. Will, J. Volz, and A. Rauschenbeutel, Quantum optical circulator controlled by a single chirally coupled atom, Science 354, 1577 (2016).

[18] R. Huang, A. Miranowicz, J.-Q. Liao, F. Nori, and H. Jing, Nonreciprocal Photon Blockade, Phys. Rev. Lett. 121, 153601 (2018).

[19] B.-J. Li, R. Huang, and X.-W. Xu, A. Miranowicz, and H. Jing, Nonreciprocal unconventional photon blocking in a spinning optomechanical system, Photon. Res. 7, 630 (2019).

[20] P. Yang, M. Li, X. Han, H. He, G. Li, C.-L. Zou, P. Zhang, and T. Zhang, Non-reciprocal cavity polariton, arXiv:1911.10300.

[21] P. Lodahl, S. Mahmodian, S. Stobbe, A. Rauschenbeutel, P. Schneeweiss, J. Volz, H. Pichler, and P. Zoller, Chiral quantum optics, Nature (London) 541, 473 (2017).

[22] C.-W. Qiu, H.-Y. Yao, L.-W. Li, S. Zouhdi, and T.-S. Yeo, Backward waves in magnetoelectrically chiral media: Propagation, impedance, and negative refraction, Phys. Rev. B 75, 155120 (2007).

[23] J. Petersen, J. Volz, and A. Rauschenbeutel, Chiral nanophotonic waveguide interface based on spin-orbit interaction of light, Science 346, 67 (2014).

[24] C. Gonzalez-Ballestero, A. Gonzalez-Tudela, F. J. Garcia-Vidal, and E. Moreno, Chiral route to spontaneous entanglement generation, Phys. Rev. B 92, 155304 (2015).

[25] F. Wolfgaam, C. Vitelli, F. A. Beduini, N. Godbout, and M. W. Mitchell, Entanglement-enhanced probing of a delicate material system, Nat. Photonics 7, 28 (2013).

[26] Y. Ma, H. Miao, B. H. Pang, M. Evans, C. Zhao, J. Harms, R. Schnabel, and Y. Chen, Proposal for gravitational-wave detection beyond the standard quantum limit through EPR entanglement, Nat. Phys. 13, 776 (2017).

[27] S. Qvarfort, A. Serafini, P. F. Barker, and S. Bose, Gravimetry through non-linear optomechanics, Nat. Commun. 9, 3690 (2018).

[28] M. Aspelmeyer, P. Meystre, and K. Schwab, Optomechanics, Phys. Today 65, 29 (2012).

[29] M. Aspelmeyer, T. J. Kippenberg, and F. Marquardt, Cavity optomechanics, Rev. Mod. Phys. 86, 1391 (2014).

[30] E. Verhagen, S. Deléglise, S. Weis, A. Schliesser, and T. J. Kippenberg, Quantum-coherent coupling of a mechanical oscillator to an optical cavity mode, Nature (London) 482, 63 (2012).

[31] A. Schliesser, R. Rivière, G. Anetsberger, O. Arcizet, and T. J. Kippenberg, Resolved-sideband cooling of a micromechanical oscillator, Nat. Phys. 4, 415 (2008).

[32] K. Stannigel, P. Rabl, A. S. Sorensen, P. Zoller, and M. D. Lukin, Optomechanical Transducers for Long-Distance Quantum Communication, Phys. Rev. Lett. 105, 220501 (2010).

[33] J. Bochmann, A. Vainsencher, D. A.Awschalom, and A. N. Cleland, Nanomechanical coupling between microwave and optical photons, Nat. Phys. 9, 712 (2013).

[34] T. A. Palomaki, J. W. Harlow, J. D. Teufel, R. W. Simmonds, and K. W. Lehnert, Coherent state transfer between itinerant microwave fields and a mechanical oscillator, Nature (London) 495, 210 (2013).

[35] E. E. Wollman, C. U. Lei, A. J. Weinstein, J. Suh, A. Kronwald, F. Marquardt, A. A. Clerk, and K. C. Schwab, Quantum squeezing of motion in a mechanical resonator, Science 349, 952 (2015).

[36] D. Vitali, S. Gigan, A. Ferreira, H. R. Böhm, P. Tombesi, A. Guerreiro, V. Vedral, A. Zeilinger, and M. Aspelmeyer, Optomechanical Entanglement between a Moveable Mirror and a Cavity Field, Phys. Rev. Lett. 109, 030405 (2007).

[37] T. A. Palomaki, J. D. Teufel, R. W. Simmonds, and K. W. Lehnert, Entangling Mechanical Motion with Microwave Fields, Science 342, 710 (2013).

[38] R. Riedinger, S. Hong, R. A. Nortec, J. A. Slater, J. Shang, A. G. Krause, V. Anant, M. Aspelmeyer, and S. Gröblacher, Nonclassical correlations between single photons and phonons from a mechanical oscillator, Nature (London) 530, 313 (2016).

[39] C. F. Ockeloen-Korppi, E. Damskägg, J.-M. Pirkkalainen, M. Asjad, A. A. Clerk, F. Massel, M. J. Woolley, and M. A. Sillanpää, Stabilized entanglement of massive mechanical oscillators, Nature (London) 556, 478 (2018).

[40] S. Barzanjeh, E. S. Redchenko, M. Peruzzo, M. Wulf, D. P. Lewis, G. Arnold, and I. M. Fink, Stationary entangled radiation from micromechanical motion, Nature (London) 570, 480 (2019).

[41] J. Chen, M. Rossi, D. Mason, and A. Schliesser, Entanglement of propagating optical modes via a mechanical interface, Nat. Commun. 11, 943 (2020).

[42] D. E. McClelland, N. Mavalvala, Y. Chen, and R. Schnabel, Advanced interferometry, quantum optics and optomechanics in gravitational wave detectors, Laser Photonics Rev. 5, 677 (2011).
[43] H. Yu et al. (LIGO Scientific Collaboration and Virgo Collaboration), Quantum correlations between the light and kilogram-mass mirrors of LIGO, arXiv:2002.01519.
[44] G. B. Malykin, The Sagnac effect: Correct and incorrect explanations, Phys. Usp. 43, 1229 (2000).
[45] C. W. Gardiner and P. Zoller, Quantum Noise (Springer, Berlin, Germany, 2000).
[46] E. X. DeJesus and C. Kaufman, Routh-Hurwitz criterion in the examination of eigenvalues of a system of nonlinear ordinary differential equations, Phys. Rev. A 35, 5288 (1987).
[47] See the Supplemental Material for detailed derivations of our main results.
[48] G. Adesso, A. Serafini, and F. Illuminati, Extremal entanglement and mixedness in continuous variable systems, Phys. Rev. A 70, 022318 (2004).
[49] G. C. Righini, Y. Dumeige, P. Feron, M. Ferrari, G. N. Conti, D. Ristic, and S. Soria, Whispering gallery mode microresonators: Fundamentals and applications, Riv. Nuovo Cimento Soc. Ital. Fis. 34, 435 (2011).
[50] C. C. Zhong, Z. X. Wang, C.-L. Zou, M. Z. Zhang, X. Han, W. Fu, M. R. Xu, S. Shankar, M. H. Devoret, H. X. Tang, and L. Jiang, Proposal for Heralded Generation and Detection of Entangled Microwave-Optical-Photon Pairs, Phys. Rev. Lett. 124, 010511 (2020).
[51] P. Kurpiers, P. Magnard, T. Walter, B. Royer, M. Pechal, J. Heinsoo, Y. Salathé, A. Akin, S. Storz, J.-C. Besse, S. Gasparinetti, A. Blais, and A. Wallraff, Deterministic quantum state transfer and remote entanglement using microwave photons, Nature (London) 558, 264 (2018).
[52] L. DiCarlo, M. D. Reed, L. Sun, B. R. Johnson, J. M. Chow, J. M. Gambetta, L. Frunzio, S. M. Girvin, M. H. Devoret, and R. J. Schoelkopf, Preparation and measurement of three-qubit entanglement in a superconducting circuit. Nature (London) 467, 574 (2010).
[53] B. Julsgaard, A. Kozhekin, and E. S. Polzik, Experimental long-lived entanglement of two macroscopic objects, Nature (London) 413, 400 (2001).
[54] Y. Mei, Y. Zhou, S. Zhang, J. Li, K. Liao, H. Yan, S.-L. Zhu, and S. Du, Einstein-Podolsky-Rosen Energy-Time Entanglement of Narrow-Band Biphotons, Phys. Rev. Lett. 124, 010509 (2020).
[55] D. E. Chang, C. Regal, S. Papp, D. Wilson, J. Ye, O. Painter, H. J. Kimble, and P. Zoller, Proc. Natl. Acad. Sci. U.S.A. 107, 1005 (2010).
[56] U. Delić, M. Reisenbauer, K. Dare, D. Grass, V. Vuletić, N. Kiesel, and M. Aspelmeyer, Cooling of a levitated nanoparticle to the motional quantum ground state, Science 367, 892 (2020).
[57] J. Li, S.-Y. Zhu, and G. S. Agarwal, Magnon-Photon-Phonon Entanglement in Cavity Magnomechanics, Phys. Rev. Lett. 121, 203601 (2018).