Revisiting lepton-specific 2HDM in light of muon g-2 anomaly

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Abstract

We examine the lepton-specific 2HDM as a solution of muon $g-2$ anomaly under various theoretical and experimental constraints, especially the direct search limits from the LHC and the requirement of a strong first-order phase transition in the early universe. We find that the muon $g-2$ anomaly can be explained in the region of $32 < \tan \beta < 80$, $10 \text{ GeV} < m_A < 65 \text{ GeV}$, $260 \text{ GeV} < m_H < 620 \text{ GeV}$ and $180 \text{ GeV} < m_{H^\pm} < 620 \text{ GeV}$ after imposing the joint constraints from the theory, the precision electroweak data, the 125 GeV Higgs data, the leptonic/semi-hadronic $\tau$ decays, the leptonic $Z$ decays and $\text{Br}(B_s \to \mu^+\mu^-)$. The direct searches from the $h \to AA$ channels can impose stringent upper limits on $\text{Br}(h \to AA)$ and the multi-lepton event searches can sizably reduce the allowed region of $m_A$ and $\tan \beta$ ($10 \text{ GeV} < m_A < 44 \text{ GeV}$ and $32 < \tan \beta < 60$). Finally, we find that the model can produce a strong first-order phase transition in the region of $14 \text{ GeV} < m_A < 25 \text{ GeV}$, $310 \text{ GeV} < m_H < 355 \text{ GeV}$ and $250 \text{ GeV} < m_{H^\pm} < 295 \text{ GeV}$, allowed by the explanation of the muon $g-2$ anomaly.

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I. INTRODUCTION

The muon anomalous magnetic moment \((g - 2)\) is a very precisely measured observable. The muon \(g - 2\) anomaly has been a long-standing puzzle since the announcement by the E821 experiment in 2001 [1, 2]. The experimental value has an approximate 3\(\sigma\) discrepancy from the SM prediction [3–5]. As a popular extension of the SM, the two Higgs doublet models (2HDM) have been applied to explain the muon \(g - 2\) anomaly in the literature [6–31]. Among these extensions, the lepton-specific 2HDM (L2HDM) provides a simple explanation for the muon \(g - 2\) anomaly [11, 14–17, 22]. This model includes two neutral CP-even Higgs bosons \(h\) and \(H\), one neutral pseudoscalar \(A\), and a pair of charged Higgs bosons \(H^\pm\). The lepton Yukawa couplings can be sizably enhanced by a large tan\(\beta\). The pseudoscalar can give positive contributions to muon \(g - 2\) via the two-loop Barr-Zee diagrams, and the muon \(g - 2\) excess favors a light pseudoscalar with a large coupling to lepton.

After the discovery of the SM-like Higgs boson at the LHC, it was found [15] that the muon \(g - 2\) explanation favors the lepton Yukawa couplings of the SM-like Higgs to have an opposite sign with respect to the SM couplings. The observation of \(\text{Br}(B_s \rightarrow \mu^+\mu^-)\) gives a new constraint on the parameter space of L2HDM [15]. Further, it was found [16] that the leptonic \(Z\) decays and leptonic/semi-hadronic \(\tau\) decays can also give strong constraints on the parameter space of L2HDM, and a more precise calculation is performed in [22]. The L2HDM can lead to \(\tau\)-rich signatures at the LHC in the parameter region favored by muon \(g - 2\). The study in [17] derived the constraints on the model by using the chargino/neutralino search at the 8 TeV LHC and analyzed the prospects at the 14 TeV LHC.

In this work we examine the parameter space of L2HDM by considering the joint constraints from the theory, the precision electroweak data, the 125 GeV Higgs signal data, the muon \(g - 2\) anomaly, the lepton universality in the \(\tau\) and \(Z\) decays, the measurement of \(\text{Br}(B_s \rightarrow \mu^+\mu^-)\), as well as the direct search limits from the LHC (the data of some channels analyzed by the ATLAS and CMS are corresponding to an integrated luminosity up to about 36 \(fb^{-1}\) recorded in proton-proton collisions at \(\sqrt{s} = 13\) TeV). On the other hand, it is known that the 2HDM can trigger a strong first-order phase transition (SFOPT) in the early universe [32, 33], which is required by a successful explanation of the observed baryon asymmetry of the universe (BAU) [34] and can produce primordial gravitational-wave (GW) signals [35] potentially detectable by future space-based laser interferometer detectors like...
eLISA [36]. Due to the importance of SFOPT in cosmology, we will also analyze whether a SFOPT is achievable in the parameter space in favor of the muon g \( -2 \) explanation.

Our work is organized as follows. In Sec. II we recapitulate the L2HDM. In Sec. III we discuss the muon g \( -2 \) anomaly and other relevant constraints. In Sec. IV, we constrain the model using the direct search limits from the LHC. In Sec. V, we discuss some benchmark scenarios leading to a SFOPT. Finally, we give our conclusion in Sec. VI.

II. THE LEPTON-SPECIFIC 2HDM

The general Higgs potential is given as [37]

\[
V = m_{11}^2 (\Phi_1 \dagger \Phi_1) + m_{22}^2 (\Phi_2 \dagger \Phi_2) - \left[ m_{12}^2 (\Phi_1 \dagger \Phi_2 + \text{h.c.}) \right] \\
+ \frac{\lambda_1}{2} (\Phi_1 \dagger \Phi_1)^2 + \frac{\lambda_2}{2} (\Phi_2 \dagger \Phi_2)^2 + \lambda_3 (\Phi_1 \dagger \Phi_1)(\Phi_2 \dagger \Phi_2) + \lambda_4 (\Phi_1 \dagger \Phi_2)(\Phi_2 \dagger \Phi_1) \\
+ \left[ \frac{\lambda_5}{2} (\Phi_1 \dagger \Phi_2)^2 + \text{h.c.} \right] + \left[ \lambda_6 (\Phi_1 \dagger \Phi_1)(\Phi_1 \dagger \Phi_2) + \text{h.c.} \right] \\
+ \left[ \lambda_7 (\Phi_2 \dagger \Phi_2)(\Phi_2 \dagger \Phi_2) + \text{h.c.} \right]. \tag{1}
\]

In this paper we focus on the CP-conserving case where all \( \lambda_i \) and \( m_{12}^2 \) are real. In the L2HDM, a discrete \( Z_2 \) symmetry is introduced to make \( \lambda_6 = \lambda_7 = 0 \), and allow for a soft-breaking term with \( m_{12}^2 \neq 0 \). The two complex scalar doublets have the hypercharge \( Y = 1 \),

\[
\Phi_1 = \begin{pmatrix} \phi_1^+ \\ \frac{1}{\sqrt{2}} (v_1 + \phi_1^0 + ia_1) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^+ \\ \frac{1}{\sqrt{2}} (v_2 + \phi_2^0 + ia_2) \end{pmatrix}, \tag{2}
\]

where the vacuum expectation values (VEVs) \( v^2 = v_1^2 + v_2^2 = (246 \text{ GeV})^2 \), and the ratio of the two VEVs is defined as usual to be \( \tan \beta = v_2/v_1 \). There are five mass eigenstates: two neutral CP-even states \( h \) and \( H \), one neutral pseudoscalar \( A \), and two charged scalars \( H^\pm \).

In the L2HDM, the quarks obtain masses from \( \Phi_2 \) field, and the leptons from \( \Phi_1 \) field [38, 39]. The Yukawa interactions are given by

\[
-\mathcal{L} = Y_{u2} \overline{Q}_L \tilde{\Phi}_2 u_R + Y_{d1} \overline{Q}_L \Phi_1 d_R, + Y_{e1} \overline{L}_L \Phi_1 e_R + \text{h.c.}, \tag{3}
\]

where \( \overline{Q}_L = (u_L, d_L) \), \( \overline{L}_L = (\nu_L, l_L) \), \( \tilde{\Phi}_{1,2} = i\tau_2 \Phi_{1,2}^* \), and \( Y_{u2}, Y_{d1} \) and \( Y_{e1} \) are \( 3 \times 3 \) matrices in family space.
The Yukawa couplings of the neutral Higgs bosons normalized to the SM are given by

\[ y_h^V = \sin(\beta - \alpha), \quad y_h^f = [\sin(\beta - \alpha) + \cos(\beta - \alpha)\kappa_f], \]
\[ y_H^V = \cos(\beta - \alpha), \quad y_H^f = [\cos(\beta - \alpha) - \sin(\beta - \alpha)\kappa_f], \]
\[ y_A^V = 0, \quad y_A^f = -i\kappa_f \text{ (for } u), \quad y_A^f = i\kappa_f \text{ (for } d, \ell), \]

where \( V \) denotes \( Z \) or \( W \), \( \kappa_\ell \equiv -\tan\beta \), \( \kappa_d = \kappa_u \equiv 1/\tan\beta \) and \( \alpha \) is the mixing angle of the two CP-even Higgs bosons.

III. MUON \( g - 2 \) ANOMALY AND RELEVANT CONSTRAINTS

A. Numerical calculations

In this paper, we take the light CP-even Higgs \( h \) as the SM-like Higgs, \( m_h = 125 \) GeV. Since the muon \( g - 2 \) anomaly favors a light pseudoscalar with a large coupling to lepton, we scan over \( m_A \) and \( \tan\beta \) in the following ranges:

\[ 10 \text{ GeV} < m_A < 120 \text{ GeV}, \quad 20 < \tan\beta < 120. \]

In our calculation, we consider the following observables and constraints:

1. Theoretical constraints and precision electroweak data. The 2HDMC [40] is employed to implement the theoretical constraints from the vacuum stability, unitarity and coupling-constant perturbativity, as well as the constraints from the oblique parameters \( (S, T, U) \).

2. The signal data of the 125 GeV Higgs. Since the 125 GeV Higgs couplings with the SM particles in this model can deviate from the SM ones, the SM-like decay modes will be modified. Besides, for \( m_A \) is smaller than 62.5 GeV, the invisible decay \( h \rightarrow AA \) is kinematically allowed, which will be strongly constrained by the experimental data of the 125 GeV Higgs. We perform \( \chi^2 \) calculation for the signal strengths of the 125 GeV Higgs in the \( \mu_{ggF + th}(Y) \) and \( \mu_{VBF + Vh}(Y) \) with \( Y \) denoting the decay mode \( \gamma\gamma \),
$ZZ, WW, \tau^+\tau^-$ and $b\bar{b}$,
\[
\chi^2(Y) = \begin{pmatrix}
\mu_{ggH+ttH}(Y) - \hat{\mu}_{ggH+ttH}(Y) \\
\mu_{VBF+VH}(Y) - \hat{\mu}_{VBF+VH}(Y)
\end{pmatrix}^T \begin{pmatrix}
a_Y & b_Y \\
b_Y & c_Y
\end{pmatrix} 
\begin{pmatrix}
\mu_{ggH+ttH}(Y) - \hat{\mu}_{ggH+ttH}(Y) \\
\mu_{VBF+VH}(Y) - \hat{\mu}_{VBF+VH}(Y)
\end{pmatrix},
\]
(6)

where $\hat{\mu}_{ggH+ttH}(Y)$ and $\hat{\mu}_{VBF+VH}(Y)$ are the data best-fit values and $a_Y, b_Y$ and $c_Y$ are the parameters of the ellipse, which are given by the combined ATLAS and CMS experiments \[41\].

(3) Lepton universality in the $\tau$ decays. The HFAG collaboration reported three ratios from pure leptonic processes, and two ratios from semi-hadronic processes, $\tau \to \pi/K\nu$ and $\pi/K \to \mu\nu$ \[42\]:
\[
\left(\frac{g_\tau}{g_\mu}\right) = 1.0011 \pm 0.0015, \quad \left(\frac{g_\tau}{g_e}\right) = 1.0029 \pm 0.0015, \quad \left(\frac{g_\mu}{g_e}\right) = 1.0018 \pm 0.0014,
\]
\[
\left(\frac{g_\tau}{g_\mu}\right)_\pi = 0.9963 \pm 0.0027, \quad \left(\frac{g_\mu}{g_e}\right)_K = 0.9858 \pm 0.0071,
\]
(7)

with
\[
\left(\frac{g_\tau}{g_\mu}\right)^2 \equiv \frac{\bar{\Gamma}(\tau \to e\nu\bar{\nu})}{\bar{\Gamma}(\mu \to e\nu\bar{\nu})},
\left(\frac{g_\tau}{g_e}\right)^2 \equiv \frac{\bar{\Gamma}(\tau \to \mu\nu\bar{\nu})}{\bar{\Gamma}(\mu \to e\nu\bar{\nu})},
\left(\frac{g_\mu}{g_e}\right)^2 \equiv \frac{\bar{\Gamma}(\tau \to \mu\nu\bar{\nu})}{\bar{\Gamma}(\tau \to e\nu\bar{\nu})}.
\]
(8)
Here $\bar{\Gamma}$ denoting the partial width normalized to its SM value. The correlation matrix for the above five observables is
\[
\begin{pmatrix}
1 & +0.53 & -0.49 & +0.24 & +0.12 \\
+0.53 & 1 & +0.48 & +0.26 & +0.10 \\
-0.49 & +0.48 & 1 & +0.02 & -0.02 \\
+0.24 & +0.26 & +0.02 & 1 & +0.05 \\
+0.12 & +0.10 & -0.02 & +0.05 & 1
\end{pmatrix}.
\]
(9)

In the L2HDM we have the ratios
\[
\left(\frac{g_\tau}{g_\mu}\right) \approx 1 + \delta_{\text{loop}}, \quad \left(\frac{g_\tau}{g_e}\right) \approx 1 + \delta_{\text{tree}} + \delta_{\text{loop}}, \quad \left(\frac{g_\mu}{g_e}\right) \approx 1 + \delta_{\text{tree}},
\]
\[
\left(\frac{g_\tau}{g_\mu}\right)_\pi \approx 1 + \delta_{\text{loop}}, \quad \left(\frac{g_\tau}{g_e}\right)_K \approx 1 + \delta_{\text{loop}},
\]
(10)
where $\delta_{\text{tree}}$ and $\delta_{\text{loop}}$ are respectively corrections from the tree-level diagrams and the one-loop diagrams mediated by the charged Higgs. They are given as \([16, 22]\)

$$
\delta_{\text{tree}} = \frac{m_\tau^2 m_\mu^2 t_\beta^4}{8 m_{H^\pm}^4 t_\beta} - \frac{m_\mu^2}{m_{H^\pm}^2} t_\beta^2 \frac{g(m_\mu^2/m_\tau^2)}{f(m_\mu^2/m_\tau^2)},
$$

$$
\delta_{\text{loop}} = \frac{1}{16\pi^2} \frac{m_\tau^2}{v^2} \frac{t_\beta^2}{t_\beta} \left[1 + \frac{1}{4} (H(x_A) + s_{3-\alpha}^2 H(x_H) + c_{3-\alpha}^2 H(x_h))\right],
$$

where $f(x) \equiv 1 - 8x + 8x^3 - x^4 - 12x^2 \ln(x)$, $g(x) \equiv 1 + 9x - 9x^2 - x^3 + 6x(1 + x) \ln(x)$ and $H(x) \equiv \ln(x)(1 + x)/(-1 - x)$ with $x = m_\phi^2/m_{H^\pm}^2$.

We perform $\chi^2$ calculation for the five observables. The covariance matrix constructed from the data of Eq. (7) and Eq. (9) has a vanishing eigenvalue, and the corresponding degree is removed in our calculation.

(4) Lepton universality in the $Z$ decays. The measured values of the ratios of the leptonic $Z$ decay branching fractions are given as \([43]\):

$$
\frac{\Gamma_{Z \rightarrow \mu^+\mu^-}}{\Gamma_{Z \rightarrow e^+e^-}} = 1.0009 \pm 0.0028, \\
\frac{\Gamma_{Z \rightarrow \tau^+\tau^-}}{\Gamma_{Z \rightarrow e^+e^-}} = 1.0019 \pm 0.0032,
$$

with a correlation of $+0.63$. In the L2HDM, the width of $Z \rightarrow \tau^+\tau^-$ can have sizable deviation from the SM value by the loop contributions of the extra Higgs bosons, because they strongly interact with charged leptons for large $\tan\beta$. The quantities of Eq. (13) are calculated in the L2HDM as \([16, 22]\)

$$
\frac{\Gamma_{Z \rightarrow \mu^+\mu^-}}{\Gamma_{Z \rightarrow e^+e^-}} \approx 1.0 + \frac{2g_L^2 \text{Re}(\delta g_{L}^{2\text{HDM}}) + 2g_R^2 \text{Re}(\delta g_{R}^{2\text{HDM}})}{g_L^2 + g_R^2} \frac{m_\mu^2}{m_\tau^2},
$$

$$
\frac{\Gamma_{Z \rightarrow \tau^+\tau^-}}{\Gamma_{Z \rightarrow e^+e^-}} \approx 1.0 + \frac{2g_L^2 \text{Re}(\delta g_{L}^{2\text{HDM}}) + 2g_R^2 \text{Re}(\delta g_{R}^{2\text{HDM}})}{g_L^2 + g_R^2} \frac{m_\tau^2}{m_\mu^2}
$$

where the SM value $g_L^e = -0.27$ and $g_R^e = 0.23$. $\delta g_{L}^{2\text{HDM}}$ and $\delta g_{R}^{2\text{HDM}}$ are from the
one-loop corrections of L2HDM, which are given as

\[
\delta g_{L}^{2\text{HDM}} = \frac{1}{16\pi^{2} v_{L}^{2} f_{\beta}} \left\{ -\frac{1}{2} B_{Z}(r_{A}) - \frac{1}{2} B_{Z}(r_{H}) - 2C_{Z}(r_{A}, r_{H}) + s_{W}^{2} \left[ B_{Z}(r_{A}) + B_{Z}(r_{H}) + \tilde{C}_{Z}(r_{A}) + \tilde{C}_{Z}(r_{H}) \right] \right\},
\]

\[
\delta g_{R}^{2\text{HDM}} = \frac{1}{16\pi^{2} v_{L}^{2} f_{\beta}} \left\{ 2C_{Z}(r_{A}, r_{H}) - 2C_{Z}(r_{H^{\pm}}, r_{H^{\pm}}) + \tilde{C}_{Z}(r_{H^{\pm}}) - \frac{1}{2} \tilde{C}_{Z}(r_{A}) - \frac{1}{2} \tilde{C}_{Z}(r_{H}) + s_{W}^{2} \left[ B_{Z}(r_{A}) + B_{Z}(r_{H}) + 2B_{Z}(r_{H^{\pm}}) + \tilde{C}_{Z}(r_{A}) + \tilde{C}_{Z}(r_{H}) + 4C_{Z}(r_{H^{\pm}}, r_{H^{\pm}}) \right] \right\},
\]

where \( r_{\phi} = \frac{m_{\phi}^{2}}{m_{Z}^{2}} \) with \( \phi = A, H, H^{\pm} \), and

\[
B_{Z}(r) = -\frac{\Delta_{r}}{2} - \frac{1}{4} + \frac{1}{2} \log(r),
\]

\[
C_{Z}(r_{1}, r_{2}) = \frac{\Delta_{r}}{4} - \frac{1}{2} \int_{0}^{1} dx \int_{0}^{x} dy \log[r_{2}(1 - x) + (r_{1} - 1)y + xy],
\]

\[
\tilde{C}_{Z}(r) = \frac{\Delta_{r}}{2} + \frac{1}{2} - r[1 + \log(r)] + r^{2} \left[ \log(r) \log(1 + r^{-1}) - \text{Li}_{2}(-r^{-1}) \right] - \frac{i\pi}{2} \left[ 1 - 2r + 2r^{2} \log(1 + r^{-1}) \right].
\]

(5) The muon \( g - 2 \). The recent measurement is \( a_{\mu}^{\text{exp}} = (116592091 \pm 63) \times 10^{-11} \) [44], which has approximately 3.1σ deviation from the SM prediction [45], \( \Delta a_{\mu} = a_{\mu}^{\text{exp}} - a_{\mu}^{\text{SM}} = (262 \pm 85) \times 10^{-11} \). In the L2HDM, the muon \( g - 2 \) obtains contributions from the one-loop diagrams induced by the Higgs bosons and also from the two-loop Barr-Zee diagrams mediated by \( A, h \) and \( H \). For the one-loop contributions [6], we have

\[
\Delta a_{\mu}^{2\text{HDM}}(1\text{loop}) = \frac{G_{F} m_{\mu}^{2}}{4\pi^{2} \sqrt{2}} \sum_{j} (y_{\mu}^{j})^{2} \frac{f_{j}(r_{\mu}^{j})}{r_{\mu}^{j}},
\]

where \( j = h, H, A, H^{\pm} \), \( r_{\mu}^{j} = m_{\mu}^{2}/M_{j}^{2} \). For \( r_{\mu}^{j} \ll 1 \) we have

\[
f_{h,H}(r) \simeq -\ln r - 7/6, \quad f_{A}(r) \simeq \ln r + 11/6, \quad f_{H^{\pm}}(r) \simeq -1/6.
\]

The two-loop contributions are given by

\[
\Delta a_{\mu}^{2\text{HDM}}(2\text{loop} - \text{BZ}) = \frac{G_{F} m_{\mu}^{2}}{4\pi^{2} \sqrt{2}} \frac{\alpha_{\text{em}}}{\pi} \sum_{i,j} N_{j}^{c} Q_{j}^{2} y_{j}^{i} y_{j}^{f} r_{j}^{i} g_{i}(r_{j}^{i}),
\]

where \( i = h, H, A \), and \( m_{f}, Q_{f} \) and \( N_{j}^{c} \) are the mass, electric charge and the number of color degrees of freedom of the fermion \( f \) in the loop. The functions \( g_{i}(r) \) are
\[ g_{h,H}(r) = \int_0^1 dx \frac{2x(1-x) - 1}{x(1-x) - r} \ln \frac{x(1-x)}{r}, \]
\[ g_A(r) = \int_0^1 dx \frac{1}{x(1-x) - r} \ln \frac{x(1-x)}{r}. \]

(25)

(26)

The contributions of the CP-even (CP-odd) Higgses to \( a_\mu \) are negative (positive) at the two-loop level and positive (negative) at one-loop level. As \( m_f^2/m_\mu^2 \) could easily overcome the loop suppression factor \( 1/\pi \), the two-loop contributions can be larger than one-loop ones. In the L2HDM, since the CP-odd Higgs coupling to the lepton is proportional to \( \tan \beta \), the L2HDM can sizably enhance the muon g-2 for a light CP-odd Higgs with a large \( \tan \beta \).

(6) \( B_s \to \mu^+\mu^- \). We take the formulas in [46] to calculate \( B_s \to \mu^+\mu^- \)

\[ \frac{\mathcal{B}(B_s \to \mu^+\mu^-)}{\mathcal{B}(B_s \to \mu^+\mu^-)_{\text{SM}}} = \left[ |P|^2 + \left( 1 - \frac{\Delta \Gamma_s}{\Gamma_s} \right) |S|^2 \right], \]

where the CKM matrix elements and hadronic factors cancel out, and

\[ P \equiv \frac{C_{10}}{C_{10}^{\text{SM}}} + \frac{M_{B_s}^2}{2M_W^2} \left( \frac{m_b}{m_b + m_s} \right) \frac{C_P - C_P^{\text{SM}}}{C_{10}^{\text{SM}}}, \]

\[ S \equiv \sqrt{1 - \frac{4m_\mu^2}{M_{B_s}^2} \frac{M_{B_s}^2}{2M_W^2} \left( \frac{m_b}{m_b + m_s} \right) \frac{C_S - C_S^{\text{SM}}}{C_{10}^{\text{SM}}}}. \]

(27)

(28)

(29)

The L2HDM can give the additional contributions to coefficient \( C_{10} \) by the Z-penguin diagrams with the charged Higgs loop. Unless there are large enhancements for \( C_P \) and \( C_S \), their contributions can be neglected due to the suppression of the factor \( M_{B_s}^2/M_W^2 \). For example, the \( C_P \) and \( C_S \) of type-II 2HDM can be dominant due to the enhancement of the large \( \tan^2 \beta \) terms [47]. Although such large \( \tan^2 \beta \) terms are absent in the L2HDM, \( C_P \) can obtain the important contributions from the CP-odd Higgs exchange diagrams for a very small \( m_A \). The experimental data of \( \text{Br}(B_s \to \mu\mu) \) is given as [48]

\[ \text{Br}(B_s \to \mu\mu) = (3.0 \pm 0.6^{+0.3}_{-0.2}) \times 10^{-9}. \]

(30)

(7) The exclusion limits from the searches for Higgs bosons at the LEP and \( h \to AA \) at the LHC. We employ HiggsBounds [51] to implement the exclusion constraints from the searches for the neutral and charged Higgs at the LEP at 95% confidence.
TABLE I: The upper limits at 95% C.L. on the production cross-section times branching ratio for $h \to AA$ channels at the LHC.

| Channel | Experiment | Mass range (GeV) | Luminosity |
|---------|------------|------------------|------------|
| $gg \to h \to AA \to \tau^+\tau^-\tau^+\tau^-$ | ATLAS 8 TeV | 4-50 | 20.3 fb$^{-1}$ |
| $pp \to h \to AA \to \tau^+\tau^-\tau^+\tau^-$ | CMS 8 TeV | 5-15 | 19.7 fb$^{-1}$ |
| $pp \to h \to AA \to (\mu^+\mu^-)(b\bar{b})$ | CMS 8 TeV | 25-62.5 | 19.7 fb$^{-1}$ |
| $pp \to h \to AA \to (\mu^+\mu^-)(\tau^+\tau^-)$ | CMS 8 TeV | 15-62.5 | 19.7 fb$^{-1}$ |

level. The searches for a light Higgs at the LEP can impose stringent constraints on the parameter space.

The ATLAS and CMS have searched for some exotic decay channels of the 125 GeV Higgs, such as $h \to AA$. In addition to the global fit to the 125 GeV Higgs signal data, the $hAA$ coupling will be constrained by the ATLAS and CMS direct searches for $h \to AA$ channels at the LHC. Table [I] shows several $h \to AA$ channels considered by us.

The 125 GeV Higgs signal data and the lepton universality data from $\tau$ decays include a large number of observables. We perform a global fit to the 125 GeV Higgs signal data and the lepton universality data from $\tau$ decays, and define $\chi^2$ as $\chi^2 = \chi^2_h + \chi^2_{\tau}$. We pay particular attention to the surviving samples with $\chi^2 - \chi^2_{\text{min}} \leq 6.18$, where $\chi^2_{\text{min}}$ denotes the minimum of $\chi^2$. These samples correspond to be within the 2$\sigma$ range in any two-dimension plane of the model parameters when explaining the signal data of the 125 GeV Higgs and the data of the lepton universality from $\tau$ decays.

B. Results and discussions

In Fig. [I] we project the surviving samples within 1$\sigma$, 2$\sigma$, and 3$\sigma$ ranges of $\Delta \chi^2$ on the planes of $\tan \beta$ versus $m_A$, $\tan \beta$ versus $m_{H^\pm}$, and $m_A$ versus $m_{H^\pm}$ after imposing the constraints from theory, the oblique parameters, the exclusion limits from searches for Higgs at LEP, the signal data of the 125 GeV Higgs, and the lepton universality in $\tau$ decays. We obtain a surviving sample with a minimal value of $\chi^2$ fit to the 125 GeV Higgs signal data and the lepton universality data in $\tau$ decays, $\chi^2_{\text{min}} = 16.99$. The upper-left panel of Fig. [I]
FIG. 1: The samples within the 1σ, 2σ and 3σ ranges of $\Delta\chi^2$ projected on the planes of $\tan\beta$ versus $m_{H^\pm}$, $\tan\beta$ versus $m_A$, $m_A$ versus $m_{H^\pm}$, and $m_A$ versus $m_H$ after imposing the constraints from theory, the oblique parameters, the exclusion limits from searches for Higgs at LEP, the signal data of the 125 GeV Higgs, and the lepton universality from the $\tau$ decays. The bullets (green), crosses (blue), and triangles (red) are respectively within the 3σ, 2σ and 1σ regions of $\Delta\chi^2$.

shows that the value of $\chi^2$ is favored to increase with $\tan\beta$ and with a decrease of $m_{H^\pm}$. This is because the lepton universality in $\tau$ decays is significantly corrected by the tree-level diagrams mediated by the charged Higgs. The lower-left panel of Fig. 1 shows that $\chi^2$ is favored to have a large value for a small $m_A$. For $m_A < m_{H^\pm}$, the large mass splitting between $m_A$ and $m_{H^\pm}$ can make the one-loop diagram to give sizable correction to the lepton universality in $\tau$ decays. The upper-right panel shows that, for a light pseudoscalar, such
FIG. 2: The surviving samples on the planes of $\text{Br}(h \to AA)$ versus $m_A$ after imposing the constraints from theory, the oblique parameters, the exclusion limits from searches for Higgses at LEP, the signal data of the 125 GeV Higgs, the lepton universality in $\tau$ decays, and the exclusion limits from $h \to AA$ channels at LHC. The bullets (green) and crosses (blue) are respectively within the $3\sigma$ and $2\sigma$ regions of $\Delta\chi^2$. The circles (pink) are within the $2\sigma$ regions of $\Delta\chi^2$ and allowed by the exclusion limits from $h \to AA$ channels at the LHC.

as $m_A < 25$ GeV, $\tan\beta$ is strongly imposed an upper limit. The main constraints are from the exclusion limits from the searches for Higgs at LEP. Most of regions of $m_A < 60$ GeV and $m_H < 300$ GeV are beyond the $2\sigma$ range of $\Delta\chi^2$, as shown in lower-right panel of Fig. 1. The main constraints are from the signal data of the 125 GeV Higgs and the theory.

In Fig. 2 we project the surviving samples on the planes of $\text{Br}(h \to AA)$ versus $m_A$ after imposing the constraints from ”pre-muon $g - 2$” (denoting the theory, the oblique parameters, the exclusion limits from the searches for Higgs at LEP, the signal data of the 125 GeV Higgs, the lepton universality in $\tau$ decays, and the exclusion limits from $h \to AA$ channels at LHC). The direct searches for $h \to AA$ channels at the LHC impose stringent upper limits on $\text{Br}(h \to AA)$ in the L2HDM, such as $\text{Br}(h \to AA) < 4\%$ for $m_A = 60$ GeV. Many samples within the $2\sigma$ range of $\Delta\chi^2$ are excluded.

In Fig. 3 we project the surviving samples on the planes of $\tan\beta$ versus $m_A$, $\tan\beta$ versus
FIG. 3: The surviving samples projected on the planes of $\tan \beta$ versus $m_A$, $\tan \beta$ versus $m_{H^\pm}$, $\tan \beta$ versus $m_H$, $m_A$ versus $m_{H^\pm}$, $m_A$ versus $m_H$, and $m_H$ versus $m_{H^\pm}$. All the samples are allowed by the constraints of ”pre-muon g − 2”. The triangles (pink) are excluded by the $\text{Br}(B_s \to \mu^+\mu^-)$ limits. The light bullets (sky blue) and dark bullets (royal blue) are excluded by the limits of the lepton universality in $Z$ decay. In addition, the light bullets accommodate the muon $g − 2$ anomaly and the dark bullets do not. The circles (black) are allowed by the constraints from ”pre-muon $g − 2$”, the lepton universality in $Z$ decay, and $\text{Br}(B_s \to \mu^+\mu^-)$.

$m_{H^\pm}$, $\tan \beta$ versus $m_H$, $m_A$ versus $m_{H^\pm}$, $m_A$ versus $m_H$, and $m_H$ versus $m_{H^\pm}$ after imposing the constraints from ”pre-muon g − 2”, muon g-2 anomaly, the lepton universality in $Z$ decays, and $\text{Br}(B_s \to \mu^+\mu^-)$. The lower-left and lower-middle panels show that the lepton universality in $Z$ decays excludes most of samples in the large $m_{H^\pm}$ and $m_H$ regions. This is because that the one-loop diagram can give sizable corrections to the lepton universality in $Z$ decays for $m_A < m_{H^\pm}$ ($m_H$). Certainly, because of the constraints from the oblique parameters, $H$ and $H^\pm$ are favored to have a small splitting mass for large $m_{H^\pm}$, as shown in the lower-right panel of Fig. 3. For $m_{H^\pm} < 250$ GeV, all the samples within 2σ region of
\( \Delta \chi^2 \) are consistent with the limits of the lepton universality in \( Z \) decay. This is because the lepton universality in \( \tau \) decays can give more stringent constraints on the L2HDM than the lepton universality in \( Z \) decays for a light charged Higgs.

The upper-left and lower-left panels of Fig. 3 show that the limits of \( \text{Br}(B_s \rightarrow \mu^+\mu^-) \) exclude most of regions of \( m_A \) around 10 GeV and \( m_{H^\pm} < 300 \) GeV. The \( A \) exchange diagrams can give sizable contributions to \( B_s \rightarrow \mu^+\mu^- \) for a very small \( m_A \). In the L2HDM, the lepton couplings are enhanced by \( \tan \beta \), while the quark couplings are suppressed by \( \cot \beta \). Therefore, the leading contributions are almost independent on \( \tan \beta \) for large \( \tan \beta \).

Fig. 3 shows that with the limits from "pre-muon \( g - 2 \)”, the lepton universality in \( Z \) decays and \( \text{Br}(B_s \rightarrow \mu^+\mu^-) \) being satisfied, the muon \( g - 2 \) anomaly can be explained in the regions of \( 32 < \tan \beta < 80, \) 10 GeV \( < m_A < 65 \) GeV, 260 GeV \( < m_H < 620 \) GeV, and 180 GeV \( < m_{H^\pm} < 620 \) GeV. The upper-left panel of Fig. 3 shows that in the range of \( 65 \) GeV \( < m_A < 100 \) GeV, the muon \( g - 2 \) anomaly can be explained for a large enough \( \tan \beta \). However, such a large \( \tan \beta \) is excluded by the lepton universality in \( Z \) decays. The contributions of \( A \) to the muon \( g - 2 \) anomaly have destructive interference with those of \( H \). Therefore, a large mass splitting between \( A \) and \( H \) is required to explain the muon \( g - 2 \) anomaly, as shown in the lower-middle panel of Fig. 3.

**IV. THE DIRECT SEARCH LIMITS FROM THE LHC**

Here we discuss the direct search limits from the LHC. In the parameter space in favor of muon \( g - 2 \) anomaly explanation, the production processes of extra Higgs bosons via the Yukawa interaction with quarks can be neglected due to the suppression of large \( \tan \beta \) in the L2HDM. For \( m_A \) smaller than 62.5 GeV, a pair of pseudoscalars can be produced via \( pp \rightarrow h \rightarrow AA \) at the LHC. In the above Section, we find that \( h \rightarrow AA \) channel at the LHC can exclude many samples within the 2\( \sigma \) region of \( \Delta \chi^2 \).

The extra Higgs bosons are dominantly produced at the LHC via the following electroweak
FIG. 4: The samples satisfying the constraints described in Sec. III projected on the planes of $m_H$ versus $m_A$ and $m_{H^\pm}$ versus $m_A$. The varying colors in each panel indicate the values of $m_{H^\pm}$, $\text{BR}(H \to \tau^+\tau^-)$, $\text{BR}(H \to ZA)$, $m_H$, $\text{BR}(H^\pm \to \tau^\pm\nu)$ and $\text{BR}(H^\pm \to W^\pm A)$, respectively.

processes:

$$pp \to W^{\pm*} \to H^\pm A, \quad (31)$$

$$pp \to Z^* / \gamma^* \to HA, \quad (32)$$

$$pp \to W^{\pm*} \to H^\pm H, \quad (33)$$

$$pp \to Z^* / \gamma^* \to H^+H^-. \quad (34)$$

In our scenario, the important decay modes of the Higgs bosons are

$$A \to \tau^+\tau^-, \mu^+\mu^-, \ldots \ldots, \quad (35)$$

$$H \to \tau^+\tau^-, ZA, \ldots \ldots, \quad (36)$$

$$H^\pm \to \tau^\pm\nu, W^\pm A, \ldots \ldots. \quad (37)$$
Here the light pseudo-scalar $A$ indeed decays into $\tau\tau$ essentially at 100% due to the enhanced lepton Yukawa couplings by large $\tan\beta$. The other decay branch ratios and mass spectrum for the samples satisfying constraints described in Sec. III are presented in Fig. 4 on the planes of $m_H$ versus $m_A$ and $m_{H\pm}$ versus $m_A$. We can see from the upper panels that $m_H$ increases from 260 GeV to 620 GeV with $m_{H\pm}$ increasing from 180 GeV to 620 GeV and the upper bounds of $m_A$ decreasing from 65 GeV to 30 GeV. The reason are discussed in Sec. III. As a result, the cross sections of processes in Eq. (31) and Eq. (32) are much larger than the two in Eq. (33) and Eq. (34). For example, the LO cross sections at 13 TeV LHC are $\sigma_{H^\pm A} + \sigma_{HA} = 15.33$ pb and $\sigma_{H^+H^-} + \sigma_{H^\pm H} = 2.15$ pb for a benchmark point of $(m_A = 24$ GeV, $m_H = 407$ GeV, $m_{H\pm} = 432$ GeV). The middle and right panels exhibit the decay branch ratios of $H/H^\pm$ to $\tau^+\tau^-/\tau^\pm v_\tau$ and $H/H^\pm$ to gauge boson and $A$. With an increase of $m_A$, the partial widths of $H^\pm/H$ to $AW^\pm/Z$ decrease due to the suppression of phase space. The muon $g-2$ anomaly favors a large $\tan\beta$ with $m_A$ increasing, which leads the partial widths of $H \rightarrow \tau^+\tau^-$ and $H^\pm \rightarrow \tau^\pm v_\tau$ to be enhanced since the Yukawa couplings are proportional to $\tan\beta$. Therefore, with an increase of $m_A$, $\text{Br}(H \rightarrow AZ)$ and $\text{Br}(H^\pm \rightarrow W^\pm A)$ decrease, and $\text{Br}(H \rightarrow \tau^+\tau^-)$ and $\text{Br}(H^\pm \rightarrow \tau^\pm v_\tau)$ increase. In conclusion, the dominated final states generated at LHC of our samples are 3 or 4 $\tau$s with or without gauge boson from

$$pp \rightarrow W^\pm \rightarrow H^\pm A \rightarrow 3\tau + v_\tau \text{ or } 4\tau + W^\pm \quad (38)$$

$$pp \rightarrow Z^* / \gamma^* \rightarrow HA \rightarrow 4\tau \text{ or } 4\tau + Z \quad (39)$$

In order to restrict the productions of the above processes at the LHC for our model, we perform simulations for the samples in Fig. 4 using MG5_aMC-2.4.3 [53] with PYTHIA6 [54] and Delphes-3.2.0 [55], and adopt the constraints from all the analysis for the 13 TeV LHC in version CheckMATE 2.0.7 [56]. Besides, the latest multi-lepton searches for electroweakino [57–61] implemented in Ref. [62] are also taken into consideration because of the dominated multi-$\tau$ final states in our model.

The results from CheckMATE are presented in Fig. 5 on the planes of $m_H$ versus $m_A$, $m_{H\pm}$ versus $m_A$, and $\tan\beta$ versus $m_A$. The colors stand for the $R$ values defined by [56]

$$R = \max_i \left\{ \frac{S_i - 1.96\Delta S_i}{S_{95i,\text{Exp}}} \right\}, \quad (40)$$

where $S_i$ and $\Delta S_i$ denote the number of signal events in signal region $i$, and $S_{95i,\text{Exp}}$ is the
FIG. 5: The samples satisfying the constraints described in Sec. III, projected on the planes of $m_H$ versus $m_A$, $m_{H^\pm}$ versus $m_A$, and $\tan \beta$ versus $m_A$ with colors indicating the $R$ values from CheckMATE. The orange stars and green dots stand for the samples excluded and allowed by the LHC Run-2 data at 95% confidence level, respectively.

experimentally measured 95% confidence limit on signal event in signal region $i$. Obviously, $R > 1$ means that the corresponding point is excluded at 95% confidence level by at least one search channel. We can see that the constraints from current LHC 13 TeV data shrink $m_A$ from [10, 65] GeV to [10, 44] GeV and $\tan \beta$ from [32, 80] to [32, 60]. For the samples excluded by current 13 TeV LHC data, the strongest constraint comes from the search for electroweak production of charginos and neutralinos in multilepton final states [58]. In this analysis, 7 categories of signal region are designed for event with $\tau$ in final state, SR–C to SR–F and SR–I to SR–K. The most sensitive signal region is SR–K for most of the parameter space. It requires at least two light-flavor leptons and two $\tau$ jets without b-tagged jet. The signal region is subdivided by missing energy $E_T$ to three bins, SR–K01, SR–K02, and SR–K03. The main contributions of our samples to the bins are from processes in Eq. (38) and Eq. (39) with at least two of the $\tau$s decaying hadronically. In Fig. 5, the points with relatively larger $m_H/m_{H^\pm}$ or lower $m_A$ can escape the direct searches. The $R$ value decreases gently with heavier $H/H^\pm$ because of the smaller cross sections. With higher luminosity and collision energy this region can be further detected. For
the light $A$, the $\tau$s from $A$ in Eq. (31) to Eq. (34) decays become too soft to be distinguished at detector, while the $\tau$s from $A$ in $H/H^\pm$ decays are collinear because of the large mass splitting between $A$ and $H/H^\pm$. Meanwhile, the $H/H^\pm \rightarrow AZ/W^\pm$ decay modes dominate the $H/H^\pm$ decays in the low $m_A$ region. Thus, in the region of $m_A < 20$ GeV, the acceptance of above signal region for final state of two collinear $\tau + Z/W$ boson quickly decreases. Specially designed signal regions are needed to detect this region.

V. THE STRONG FIRST-ORDER PHASE TRANSITION

Originally the 2HDM was proposed to spontaneously break CP-conservation \[63\]. Thus by combining with baryon number violation and departure from the thermal equilibrium, it is possible to explain BAU \[34\]. Baryon number conservation is broken by electroweak sphaleron process \[64\], and the departure from the thermal equilibrium can be realized by a SFOPT. The SM electroweak sector cannot trigger a SFOPT, because SFOPT in the SM requires a Higgs mass of around 70 GeV $\sim$ 80 GeV \[65\]. This problem can be solved by the extended Higgs sector in 2HDM which has more degrees of freedom \[32\]. In this section we study the possibility to obtain a parameter space in L2HDM that can trigger a SFOPT and explain muon $g−2$ anomaly at the same time.

A. Thermal effective potential

In order to know the strength of phase transition in our scenario, we need to study the effective potential with thermal correction included. The thermal effective potential $V(\phi_1, \phi_2, T)$ at temperature $T$ is composed of four parts:

$$V(\phi_1, \phi_2, T) = V^0(\phi_1, \phi_2) + V^{CW}(\phi_1, \phi_2) + V^{CT}(\phi_1, \phi_2) + V^T(\phi_1, \phi_2, T),$$

(41)

where $V^0$ is the tree-level potential, $V^{CW}$ is the Coleman-Weinberg potential, $V^{CT}$ is the counter term and $V^T$ is the thermal correction.

The tree-level potential $V^0(\phi_1, \phi_2)$ can be obtained by replacing scalar fields $\Phi_1(x)$ and $\Phi_2(x)$ in $V^0(\Phi_1, \Phi_2)$ with homogeneous values $\frac{1}{\sqrt{2}}(0, \phi_1)^T$ and $\frac{1}{\sqrt{2}}(0, \phi_2)^T$ \[73\]:

$$V^0(\phi_1, \phi_2) = \frac{1}{2} m^2_{12} \tan \beta \left( \phi_1 - \frac{1}{\tan \beta} \phi_2 \right)^2 - \frac{1}{4} \left( \lambda_1 \cos^2 \beta + \lambda_{345} \sin^2 \beta \right) v^2 \phi_1^2 - \frac{1}{4} \left( \lambda_2 \sin^2 \beta + \lambda_{345} \cos^2 \beta \right) v^2 \phi_2^2 + \frac{1}{8} \lambda_1 \phi_1^4 + \frac{1}{8} \lambda_2 \phi_2^4 + \frac{1}{4} \phi_1^2 \phi_2^2.$$
CP violation is not introduced here because its impact on phase transition is usually not significant \[66\].

The one-loop Coleman-Weinberg potential \(V^{CW}(\phi_1, \phi_2)\) \[67\] under \(\overline{\text{MS}}\) renormalization scheme is

\[
V^{CW}(\phi_1, \phi_2) = \frac{1}{64\pi^2} \sum_i n_i m_i^4(\phi_1, \phi_2) \left[ \ln \frac{m_i^2(\phi_1, \phi_2)}{\mu^2} - c_i \right],
\]

where the index \(i\) runs over all the massive particles, and \(n_i\) is the number of degrees of freedom, equal to -12, -4, 6, 3, 2, 1, and 1 for quark, lepton, \(W^\pm, Z, H^\pm, G^0, G^\pm\), and neutral scalars, respectively. Here we include the contribution from goldstone \(G^0\) and \(G^\pm\) because their mass can be non-zero for \((\phi_1, \phi_2) \neq (v_1, v_2)\). \(c_i\) is equal to \(\frac{5}{6}\) for gauge bosons, and is equal to \(\frac{3}{2}\) for other particles. \(m_i^2(\phi_1, \phi_2)\) is the field value dependent mass square of different particles. Renormalization scale \(\mu\) is set to zero temperature VEV \(v\).

The tree-level spectrum and mixing angles will be shifted by the Coleman-Weinberg correction. In order to offset the shift, a counter term \(V^{CT}(\Phi_1, \Phi_2)\) needs to be added to Lagrangian. For a CP-conserving 2HDM, \(V^{CT}(\Phi_1, \Phi_2)\) can be expressed as

\[
V^{CT}(\Phi_1, \Phi_2) = \delta m_1^2(\Phi_1^i \Phi_1 + \delta m_2^2(\Phi_2^i \Phi_2 - \delta m_{12}^2(\Phi_1^i \Phi_2 + h.c.) + \frac{\delta \lambda_1}{2}(\Phi_1^i \Phi_1)^2 + \frac{\delta \lambda_2}{2}(\Phi_2^i \Phi_2)^2

+ \delta \lambda_3(\Phi_1^i \Phi_1)(\Phi_2^i \Phi_2) + \delta \lambda_4(\Phi_1^i \Phi_2)(\Phi_2^i \Phi_1) + \frac{\delta \lambda_5}{2}(\Phi_1^i \Phi_2)^2 + h.c.

+ \delta t_1(v_1 + h_1) + \delta t_2(v_2 + h_2),
\]

where the \(\delta\)'s are determined by the “on-shell” renormalization conditions:

\[
\partial_{\psi_i}(V^{CT}(\Phi_1, \Phi_2) + V^{CW}(\Phi_1, \Phi_2)) = 0 \tag{45}
\]

\[
\partial_{\psi_i} \partial_{\psi_j}(V^{CT}(\Phi_1, \Phi_2) + V^{CW}(\Phi_1, \Phi_2)) = 0, \tag{46}
\]

with \(\psi_i\) denoting all the component scalar fields of \(\Phi_1\) and \(\Phi_2\). These conditions are evaluated at the minimum of scalar potential at zero temperature, where \(\Phi_1 = \frac{1}{\sqrt{2}}(0, v_1)^T\) and \(\Phi_2 = \frac{1}{\sqrt{2}}(0, v_2)^T\) \[74\]. Thus our tree-level input parameters can be preserved after including loop correction. And the corresponding potential \(V^{CT}(\phi_1, \phi_2)\) needs to be added to \(V(\phi_1, \phi_2, T)\).

The thermal correction including daisy resummation \[68\, 69\] is

\[
V^T(\phi_1, \phi_2, T) = \frac{T^4}{2\pi^2} \sum_i n_i J_B \left( \frac{m_i^2(\phi_1, \phi_2)}{T^2} \right) + \frac{T^4}{2\pi^2} \sum_j n_j J_F \left( \frac{m_j^2(\phi_1, \phi_2)}{T^2} \right)

- \frac{T^4}{12\pi} \sum_k n_k \left[ \frac{\tilde{m}_k^2(\phi_1, \phi_2, T)}{T^2} \right]^{3/2} - \left( \frac{m_k^2(\phi_1, \phi_2)}{T^2} \right)^{3/2}, \tag{47}
\]
where the index $i$ denotes all gauge bosons and scalars, $j$ denotes leptons and quarks, and $k$ denotes scalars and longitudinal component of gauge bosons. The integral functions $J_{B,F}$ are given by

$$J_B(x) = \int_0^\infty dk \, k^2 \ln \left[ 1 - \exp \left( -\sqrt{k^2 + x} \right) \right], \quad (48)$$

$$J_F(x) = \int_0^\infty dk \, k^2 \ln \left[ 1 + \exp \left( -\sqrt{k^2 + x} \right) \right]. \quad (49)$$

And $\tilde{m}_k^2(\phi_1, \phi_2, T)$ is thermal Debye mass. Expression of thermal Debye mass can be found in literature [70].

B. Numerical results

The condition for a SFOPT is usually taken to be [71]:

$$\xi_c \equiv \frac{v_c}{T_c} \geq 1.0 \quad (50)$$

where $T_c$ is the critical temperature at which a second minimum of $V(\phi_1, \phi_2, T)$ with non-zero VEV appears, and $v_c = \sqrt{\phi_1^2 + \phi_2^2}$ is the corresponding VEV at $T_c$. Due to the complicated form of $V(\phi_1, \phi_2, T)$, a numerical calculation is always required to analyze the geometry evolution of $V(\phi_1, \phi_2, T)$. In this work we use package BSMPT [72] to do the analysis. In BSMPT, the critical temperature $T_c$ is determined when the minimization point $v = v_c$ at critical temperature $T_c$ jumps to the origin $v = 0$ at a slightly higher temperature $T > T_c$.

In Sec. IV, we obtained a large number of points which can explain the muon $g - 2$ anomaly and accommodate various relevant theoretical and experimental constraints. Those points are used as inputs to calculate $\xi_c$. The points with $\xi_c \geq 1.0$, which can trigger a SFOPT, are projected on the planes of tan $\beta$ versus $m_{12}^2$, $m_H$ versus $m_{H^\pm}$, $m_A$ versus $m_{H^\pm}$, and $m_A$ versus $m_H$ in Fig. 6. Out of 4056 input points, there are only 23 points that can lead to a SFOPT. And those points locate in a small subset of 2HDM parameter space: $14$ GeV $< m_A < 25$ GeV, $310$ GeV $< m_H < 355$ GeV, and $250$ GeV $< m_{H^\pm} < 295$ GeV. As pointed out in [70], $m_A < 100$ GeV is not favored by SFOPT in 2HDM. Thus a certain level of fine-tuning is required if one wants to explain the muon $g - 2$ anomaly, which needs a light $m_A$, and SFOPT in the meantime. For the narrow parameter space which can achieve SFOPT and accomodate the muon $g - 2$ anomaly, we list several benchmark points in Table II. A thorough study of its observability at the LHC or future colliders will be performed elsewhere.
FIG. 6: The samples achieving SFOPT, projected on the planes of $\tan \beta$ versus $m_{12}^2$, $m_H$ versus $m_{H^\pm}$, $m_A$ versus $m_{H^\pm}$, and $m_A$ versus $m_H$. All samples can accommodate the muon $g - 2$ anomaly and satisfy the relevant theoretical and experimental constraints.

VI. CONCLUSION

The L2HDM can provide a simple explanation for the muon $g - 2$ anomaly. We performed a scan over the parameter space of L2HDM to identify the ranges in favor of the muon $g - 2$ explanation after imposing various relevant theoretical and experimental constraints, especially the direct search limits from LHC and a SFOPT in the early universe. We found that the muon $g$-2 anomaly can be accommodated in the region of $32 < \tan \beta < 80$, $10$ GeV $< m_A < 65$ GeV, $260$ GeV $< m_H < 620$ GeV and $180$ GeV $< m_{H^\pm} < 620$ GeV
| Benchmark points | $A$      | $B$      | $C$      | $D$      | $E$      |
|------------------|---------|---------|---------|---------|---------|
| $\sin(\beta - \alpha)$ | 0.999   | 0.999   | 0.999   | 0.999   | 0.999   |
| $\tan \beta$     | 48.57   | 46.09   | 53.66   | 41.46   | 48.61   |
| $m_h$ (GeV)       | 125.0   | 125.0   | 125.0   | 125.0   | 125.0   |
| $m_H$ (GeV)       | 314.96  | 322.95  | 330.88  | 342.27  | 352.19  |
| $m_A$ (GeV)       | 18.22   | 20.3    | 18.24   | 20.45   | 24.13   |
| $m_{H^\pm}$ (GeV) | 253.27  | 259.89  | 264.59  | 284.7   | 290.61  |
| $m_{12}^2$ (GeV$^2$) | 2041.32 | 2261.78 | 2039.42 | 2823.33 | 2549.66 |
| $\text{Br}(H \to \tau^+\tau^-)$ | 0.16    | 0.14    | 0.17    | 0.10    | 0.13    |
| $\text{Br}(H \to AZ)$ | 0.83    | 0.85    | 0.82    | 0.89    | 0.86    |
| $\text{Br}(H^\pm \to \tau^\pm \nu)$ | 0.24    | 0.21    | 0.26    | 0.14    | 0.18    |
| $\text{Br}(H^\pm \to W^\pm A)$ | 0.75    | 0.78    | 0.73    | 0.85    | 0.81    |

TABLE II: Several benchmark points achieving the SFOPT.

after imposing the joint constraints from the theory, the precision electroweak data, the 125 GeV Higgs signal data, the lepton universality in $\tau$ and $Z$ decays, and the measurement of $\text{Br}(B_s \to \mu^+\mu^-)$. The direct search limits from the LHC can give stringent constraints on $m_A$ and $\tan \beta$ for small $m_H$ and $m_{H^\pm}$: $10 \text{ GeV} < m_A < 44 \text{ GeV}$ and $32 < \tan \beta < 60$. The direct search limits from the $h \to AA$ channels at the LHC can impose stringent upper limits on $\text{Br}(h \to AA)$. Finally, we found that a SFOPT can be achievable in the region of $14 \text{ GeV} < m_A < 25 \text{ GeV}$, $310 \text{ GeV} < m_H < 355 \text{ GeV}$, and $250 \text{ GeV} < m_{H^\pm} < 295 \text{ GeV}$ while the muon $g - 2$ anomaly is accommodated.

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In this paper we use \( F(\Phi_1, \Phi_2) \) to represent a function composed by field operator \( \Phi_i \), and use \( F(\phi_1, \phi_2) \) to represent a function composed by homogeneous field value \( \phi_i \).

2nd order derivative of \( V^{CW} \) suffer from infrared divergence originating from the massless goldstone when \( T = 0 \). This problem can be solved by introducing a IR cut-off mass \[33\].