Development of the formulas for estimating the travel time of rainwater slope runoff and channel flow for the anthropogenic territories of small catchment basins

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Abstract. The article presents developed formulas for estimating the travel time of rainwater slope runoff and channel flow based on a unified schematization of mixed catchment basins with wide anthropogenic territories together with diversion channels. The estimation of common travel time for the rainwater overland flow on the adjacent watershed and further along the diversion channel is derived by modeling the catchment basins preserving their natural configurations. The necessary hydrological and hydraulic equations for describing the unsteady rainwater slope runoff and channel flow formation are chosen and proposed taking into consideration the proper factors affecting the flow. When developing a general formula for calculating the preliminary total travel time of the slope and channel flow have been taken into consideration the proper patterns of rainwater runoff on the inclined surfaces and stepped sectors applicable for estimation of the unsteady slope runoff and channel discharge (flow rate) in case of using the differential equations of one-dimensional hydraulics. The developed formula can be used for determining the travel time in predicting rainwater runoff for designing the system of road surface drainage and culverts facilities in mixed catchment basins with terraced paddy fields and other slope areas. Authors have proposed some recommendations and drawn the relevant conclusions.

Key words: terraced catchment basins, the principle of “extreme intensities”, rainwater slope runoff and channel flow, the travel time of the slope runoff and channel flow, the total estimated rainfall duration.

1. Introduction

Much of the agricultural land of tropics and subtropics of Asia, Africa, Europe and America is used for growing rice. For this purpose, the natural inclined surfaces of the slopes are transformed into a system of terraces (steps) with zero gradient of their bottom and small stable earth dykes (levees or bunds) at their edges to preserve the water for a long period of time. A huge part of the paddy fields is located in the most rain-prone regions, forming a kind of extensive catchment basins with significant areas of anthropogenic territories. Very often, terraced basins are located in priority areas of road transport.

One of the causes of damaging and destroying the small culverts and other drainage facilities on the roads in these regions, often causing significant economic losses, is insufficient reliability of the prediction of rainwater runoff for selecting rational structures and organizing the optimal surface drainage system. Such a situation requires an adequate modeling of the catchment areas using more
accurate hydrological and hydraulic equations for estimating rainwater slope runoff and channel flow rate.

The present article discusses the schematization of the surface of mixed catchment basins with differentiated consideration of inclined and stepped slope surface segments in their uninterrupted connection with the diversion channel. The main factors and characteristics of such mixed catchment basins causing the formation of rainwater surface runoff for determining the more accurate travel time of the slope and diversion channel flow are taken into account in traditional assumptions. The necessary transitional approaches have been taken for estimating rainwater runoff when designing drainage facilities by using the differential equations of unsteady slope runoff and channel flow. For describing the unsteady process of forming the rainwater slope runoff and channel discharge, changing in course of time and length the of the catchment area and diversion channel through the mixed surfaces of the catchment basin, it is necessary to determine the possibly utmost accurate time duration for the concentration of rainwater runoff. When using the differential equations of one-dimensional hydraulics, the maximum rainwater runoff at the given point can be determined depending on the estimated time duration that is sufficiently accurate for establishing the extreme discharge, variable along the channel. The proposed formulas for estimating travel time of the mixed slope and diversion channel are substantiated by the logically correct schematization of the catchment basin and the diversion channel, are proposed the hydraulic relations verified in practice.

Applied in estimating rainwater runoff in small catchment basins the principle of “extreme intensities” represents itself the travel time of a portion of water through the slope and channel from a hydraulically most distant point of watershed boundary to the given point (closing cross section). When determining the total travel time for catchment basin with anthropogenic terraced areas, it is important to compute differentially the travel time of separate sectors of the inclined and terraced slopes and diversion channel along the catchment area with sequent conjugation by using the relevant hydraulic laws and principles of rainwater runoff and schematization of the catchment basin.

2. Modeling a mixed catchment basin and equations for describing the rainwater slope runoff

According to the method developed by Professor O.V. Andreev and A.F. Shahidov [1] we get too short estimated time durations of rainfall for determining the flow rate in small drainage basins. Such short durations cannot be used for determining the extreme discharges when using the differential equations of the unsteady flow of rainwater proposed below in a continuous process of slope runoff and channel flow in small catchment basins.

The methods proposed in [2] and [3] reflect more reasonably the physical processes of the formation and flow of rainwater. However, these standards include a variety of approximate nomograms, tabular coefficients and various simplified multipliers. For example, in [2] the fundamental parameters are based mainly on the results of calculations with a single rainfall intensity of 0.5 mm/min only for the inclined slope surfaces and later correlated to other cases. In [3] the travel time of the slope runoff is determined taking into account the natural zones depending on the grouped hydro-morphological characteristics of the slopes. This norm is also not acceptable when calculating the travel time of slope runoff for terraced areas.

Along the slopes of the catchment basins in the tropics and subtropics, we meet mixed surfaces, alternating the areas with ordinary inclined surfaces (with different vegetation cover and gradients) and terraced areas of paddy fields with zero gradients at the bottom of the steps (Figure 1). The terraces along their edges have stable small earth dykes (levees or bunds) that retain water for a long time. The runoff passes over them like a weir and the flow is broken after passing them, which requires the use of weir formulas.

The total travel time of slope in such mixed catchment areas must be determined by successive consideration of each alternating inclined (Figure 2) and stepped (Figure 3) sectors of the slope strip (since the inclined sector is always followed by a stepped one and vice versa) separately, taking into account the fundamentally different formulas (1) and (2) for the formation and flow of rainwater runoff on them. For the convenience of schematizing such different sectors, it is better to introduce a
unified numbering system for these successive sectors along the modeled elementary strip stretching from the beginning of watershed to the diversion channel, taking the beginning sector as inclined (let it be even with just 1m length), followed by a stepped one, and so forth.

The process of the formation and flow of unsteady rainwater runoff with shallow depth along an elementary strip (1m wide) of the inclined slope of catchment surface can be described with sufficient accuracy by the following differential equation of one-dimensional hydraulics [1, 5–10]:

\[
\frac{\partial q_{sl}}{\partial x} + \frac{\partial h_{sl}}{\partial t} = a - b; \quad q_{sl} = C h_{sl}^{\frac{i}{2}}; \quad C = \frac{n_{slav}}{s_{slav}}; \quad z = 1.5 + y_{sl}^{1.5}.
\]

where \( q_{sl} \) – elementary flow rate on the elementary modeling slope strip of 1m width; \( x \) – current coordinate, directed from the beginning point of the watershed down the slope; \( h_{sl} \) – depth of the slope runoff on the strip; \( t \) – time; \( a, b \) – intensities of rainfall and runoff losses; \( C \) – Chezy coefficient; \( i_{slav}, n_{slav} \) – weighted average values of the gradient and roughness coefficient of the considered slope strip; \( z \) – an exponent; \( y_{sl} \) – an exponent depending on the flow depth and roughness coefficient. For shallow depths, it is recommended \( y_{sl} = 1.5(n_{slav})^{1/2} \ldots 1.7(n_{slav})^{1/2} \) [4]. However, this issue requires serious study and correction.

![Figure 1. Schematization of the formation of rainwater runoff on mixed slopes: \( L_{sl(i)} \) – length of the \( i \)-th inclined section of the slope strip; \( X_{sl(i)} \) – horizontal projection of the inclined length \( L_{sl(i)} \); \( X_{st(i+1)} \) – length of the \((i+1)\)-th stepped sector of the slope strip; \( a, b \) – intensities of rainfall and runoff losses](image)

The process of the formation and flow of rainwater runoff on terraced slopes with a zero gradient at the bottom of the steps (terrace) with a shallow pond differs from the above-said case and can be described (for a selected model strip of 1m width) by the differential equation of rainwater flow and a scheme of "continuous layer" taking into account the waterfall over the small earth dykes (levees or bunds) by the following equations:
\[
\frac{\partial q}{\partial x} + \frac{\partial h}{\partial t} = a;
\]
\[
q = m \sqrt{2g h^{3/2}},
\]
where \(q\) – the linear flow rate along the model strip; \(x\) – the current coordinate, directed from the beginning point of the watershed down the terraces, equals to the width of the step; \(t\) – time; \(a\) – rainfall intensity without losses (steps are saturated with water); \(m\) – weir coefficient; \(h\) – the flow depth over the mud dyke of the considered terrace; \(g\) – gravitational acceleration.

When solving the equation (1) numerically to obtain a flow rate with permissible accuracy, for example in finite differences, it is necessary to consider the process in course of time with the optimal time intervals \(\Delta t\) and length steps \(\Delta x\) until a full flow rate is established at the end of a given inclined sector. For this, it is necessary to use a previously calculated in some way the time duration, approximately equaling to the travel time. An approximate travel time of the runoff can be determined, based on the proper schematization of catchment basins and the simulation of slope runoff formation, followed by the use of acceptable hydrological and hydraulic formulas.

The instantaneous average velocity \(v_{slav(i)}\) on the considered \(i\)-th segment of the inclined slope surface (Figure 2) can be determined by the Chezy-Manning formula [7, 5, 8]:

\[
v_{slav(i)} = \frac{L}{n_{slav(i)} i_{slav(i)}^{1/2} h_{slav(i)}^{2/3}},
\]
where \(n_{slav(i)}\), \(i_{slav(i)}\) – weighted average values of roughness coefficient and surface gradient of the \(i\)-th sector of inclined strip; \(h_{slav(i)}\) – average depth of the runoff layer on the \(i\)-th section of the slope strip lengthening \(L_{sl(i)}\).

The elementary discharge at the entrance of the \(i\)-th inclined sector is equal to the elementary discharge at the end of the preceding stepped sector, because the inclined and stepped sectors alternate each other, starting from the inclined one, i.e.:

\[
q_{slen(i)} = q_{st(i-1)}.
\]

When schematizing the trajectory of the rainwater runoff along an isolated inclined strip from the extent watershed point down to the diversion channel, we can adopt the mean values of the velocity \(v_{slav(i)}\), depth of the runoff layer \(h_{slav(i)}\) and the unit discharge rate \(q_{slav(i)}\) within the length of the declined sector \(L_{sl(i)}\) for determining the slope runoff travel time \(t_{sl(i)}\) [4, 6, 10–12]. Since, at the beginning point of the model strip with 1m width (at the boundary line of the watershed) the discharge is zero and reaches the maximum at the end of the strip with a length \(X_{sl}\) equaling to \(aX_{sl}\), formed due to the rainfall intensity \(a\). The designed intensity of the rainfall duration should be defined as the exceeding intensity, remained after subtracting the losses and transformed from the hourly intensity to the

Figure 2. Schematization of runoff formation on the inclined surface: \(q_{slen(i)}, q_{slex(i)}\) – runoff rates at the entrance and exit
estimated one multiplying by the transform factor, depended on the travel time. Based on this, it can be taken that along the \(i\)-th sector of the inclined slope an average runoff rate equaling to \(q_{\text{slav}(i)} \approx 0.5a X_{\text{sl}(i)}\) is formed due to the estimated intensity \(a\). Taking into account the discharge at the beginning point (at the entrance) of the \(i\)-th slope sector \(q_{\text{slen}(i)}\) from the preceding one and an additionally formed average discharge due to the intensity \(a\), equaling to \(0.5a X_{\text{sl}(i)}\) on the very sector and using the expression (3) and (4) for the modeled strip we get the following formulas:

\[
h_{\text{slav}(i)} = \left( \frac{q_{\text{slav}(i)} n_{\text{slav}(i)}}{i^{1/2} / \text{slav}(i)} \right)^{3/5};
\]

\[
q_{\text{slav}(i)} = q_{\text{slen}(i)} + 0.5a X_{\text{sl}(i)};
\]

\[
v_{\text{slav}(i)} = \frac{q_{\text{slav}(i)}}{h_{\text{slav}(i)}} = \left( \frac{q_{\text{slav}(i)} + 0.5a X_{\text{sl}(i)}}{i^{3/10} / \text{slav}(i)} \right)^{3/5} n_{\text{slav}(i)};
\]

where \(q_{\text{slav}(i)}\) – the average linear discharge on the sector \(L_{\text{sl}(i)}\); \(q_{\text{slen}(i)}\) – discharge at the entrance of the sector, equaling to the discharge \(q_{\text{stex}(i-1)}\) at the end of the \((i-1)\)-th sector.

Using the formula (5), we get the travel time \(t_{\text{sl}(i)}\) for the inclined section of slope \(L_{\text{sl}(i)}\):

\[
t_{\text{sl}(i)} = \frac{L_{\text{sl}(i)}}{v_{\text{slav}(i)}} = \frac{L_{\text{sl}(i)} n^{3/5} / \text{slav}(i)}{i^{3/10} / \text{slav}(i)} \left( q_{\text{slen}(i)} + 0.5a X_{\text{sl}(i)} \right)^{3/5};
\]

Similarly, for the stepped section of the slope \(X_{\text{st}(i+1)}\) (Figure 3) the travel time can be determined by taking into account the average speed of the waterfall over the dykes (levees or bunds) at the edges of the steps, neglecting the free waterfall time along almost vertical walls. By such a schematization of the steps (see Figure 3) with a waterfall threshold width of 1m, the traveling time \(t_{\text{st}(i+1)}\) of the stepped section, average speed \(v_{\text{stav}(i+1)}\), average depth \(h_{\text{stav}(i+1)}\) of the stream over the dykes (levees or bunds) and average linear discharge \(q_{\text{stav}(i+1)}\) on the stepped section \(X_{\text{st}(i+1)}\) can be described [7, 13] by formulas:

\[
t_{\text{st}(i+1)} = \frac{X_{\text{st}(i+1)}}{v_{\text{stav}(i+1)}}; \quad v_{\text{stav}(i+1)} = m \sqrt{2g h_{\text{stav}(i+1)}}^{1/2};
\]

\[
h_{\text{stav}(i+1)} = \left( \frac{q_{\text{stav}(i+1)}}{m \sqrt{2g}} \right)^{2/3};
\]

\[
q_{\text{stav}(i+1)} = q_{\text{stav}(i+1)} + 0.5a X_{\text{st}(i+1)};
\]

where \(m\) – discharge coefficient of the weir (for weirs of a practical profile with a small depth of head flow, it is usually adopted \(m = 0.32 \ldots 0.45\)); \(g\) – gravitational acceleration; \(q_{\text{stav}(i+1)} = q_{\text{stav}(i)}\) – unit discharge at the beginning of the \((i+1)\)-th stepped section \(X_{\text{st}(i+1)}\) (i.e., at the end of the \(i\)-th inclined section \(L_{\text{st}(i)}\) of the strip under consideration).

Taking the mean value of \(m = 0.38\) and \(g = 9.81\ \text{m/c}^2\) in the above formulas, we obtain the travel time \(t_{\text{st}(i+1)}\) for the stepped section \(X_{\text{st}(i+1)}\):
\[
t_{st(i+1)} = \frac{X_{st(i+1)}}{\nu_{stav(i+1)}} = \frac{0.71X_{st(i+1)}}{\nu_{stav(i+1)} + 0.5aX_{st(i+1)}}^{1/3}. \tag{7}
\]

Taking in view the above said, the total travel time \(T_{st}\) of the considered slope strip will be:
\[
T_{slt} = \sum_{i}^{n} t_{sl(i)} + \sum_{i+1}^{n+1} t_{st(i+1)}, \tag{8}
\]

where \(i = 1, 3, 5, \ldots, n\).

In the formula (8), the travel time \(t_{sl(i)}\) of every inclined section and \(t_{st(i+1)}\) of the stepped one in each strip under consideration should be determined in strict sequence by formulas (6) and (7).

Figure 3. Schematization of the formation of runoff on the stepped section: \((q_{sten(i+1)} = q_{stex(i)})\) – linear discharge at the entrance of the \((i+1)\)-th stepped section of the strip; \(q_{stex(i+1)}\) – the same at the exit of the \((i+1)\)-th stepped section; \(q_{sten(i)}\) – the same at the end of the \(i\)-th inclined sector of the strip.

The total travel time \(T_{n}\) for passing the trajectory from the hydraulically most distant point of watershed boundary line of mixed catchment basins to the closing cross section of the diversion channel consists of two components:
\[
T_{n} = T_{slt} + T_{ch}, \tag{9}
\]

where \(T_{ch}\) – duration of the flow time from the end of the considered slope strip to the closing cross section of the channel resulting the expected longest travel time.

After determining the travel time for each probable slope strip, the longest of them could have been taken as the longest duration of the downpour, since by that time the full runoff flow rate had already been established at the ends of all other strips. But the total estimated time to reach the closing cross section of the channel consists of two components: the travel time of the catchment basin slope and the time to reach the closing cross section of the channel from the last point of the assumed hydraulically longest slope strip along the single trajectory to the end of the channel. The maximal duration of travel time formed by the slope and channel components of the trajectory together may not necessarily include the strip of the slope with the longest running time.

According to the above schematization of mixed slopes of small catchment basins, the runoff with a continuous layer varying its thickness in time and length from the watershed boundary line downward (starting from zero at the beginning point) reaches its local maximum at its end point along the axis of the diversion channel. Consequently, a continuous lateral inflow with different rates is formed along the sides of the diverting channel at each point, varying in time and length.
If the diversion channel passes across a slope, like a mountain ditch, the inflow is formed only from its upper side. But in case of lowland with a ditch or similar diversion channel between two opposite slopes like the road upper side ditches it occurs from both sides. This continuous lateral inflow merges into the channel stream and flows along the single open channel according to other hydraulic laws and characteristics forming the channel discharge.

For describing the unsteady flow of rainwater in open channels with continuous lateral inflow can be used a system of differential equations of mathematical physics [4, 5, 10, 12]:

the equation of continuity

$$\frac{\partial Q}{\partial t} + \frac{\partial \omega}{\partial t} = q$$  \hfill (10)

and the equation of dynamic equilibrium

$$i_{ch} = \frac{\partial h}{\partial t} + \frac{\alpha_0 q}{g \omega} (\theta - \nu) = \frac{\alpha}{2g} \frac{\partial \nu^2}{\partial t} + \frac{\alpha_0 \partial \nu}{\partial t} + \frac{Q^2}{K^2},$$  \hfill (11)

where $Q$ – discharge; $l$ – length along the channel axis; $\omega$ – area of the live cross section; $t$ – time; $q$ – lateral continuous inflow along the channel per unit of time; $i_{ch}$ – gradient of the channel bed; $h$ – depth of the flow in the channel; $g$ – gravitational acceleration; $\theta$ – projection of the lateral inflow velocity on the direction of the flow velocity in the channel; $\nu$ – the channel flow velocity; $\alpha_0$, $\alpha$ – the Coriolis and Boussinesq coefficients; $K$ – discharge characteristic.

The equations (10) and (11) can be solved more easily in finite differences. To ensure the accuracy of the channel flow discharge computation varying in time and length, it is necessary to determine the optimal intervals (finite elements) for the time $\Delta t$ and length $\Delta l$ taking in consideration the continuous process until the full discharge is established at the closing cross section of the channel (Fig. 4). This situation occurs only by the expiration of the total travel time.

3. Modelling a diversion channel in conjunction with the adjacent mixed catchment basin.

Equations for the description of rainwater channel flow

Based on the foregoing, it can be stated that the estimated duration of the rainfall of a given probability of exceedance for mixed catchment basins with terraced paddy fields represents the total travel time of a slope and channel flow of a portion of rainwater along a concrete trajectory, beginning from the hydraulically most distant point of the watershed to the closing cross section of the diversion channel.

When determining the travel time of channel flow $T_{ch}$, should be taken into account only the part $L_{ch(k)}$ of the channel, extending from its closing cross section to the mouth of the considered $k$-th model slope strip that stretches from the extreme boundary point of the watershed to the axis of the channel, assuming that the flow of a portion of water along this strip of catchment to the channel axis and further to its closing cross section creates the trajectory for forming the longest travel time. Within the length $L_{ch(k)}$, there can be many characteristic model slope strips located either on one side (on the left or right of it as in Figure 3) of the diversion channel (for example, in case of upland ditch) or on the both sides – as along the side ditch (Figure 4). Thus, a continuous lateral inflow exists along the diversion channel forming a variable flow rate at every point of it in course of time and length, establishing a maximum flow rate at the closing cross section by the end of the longest travel time.

In the equations (10) and (11) an average lateral inflow $q_{(mj)av}$ along the $m$-th interval of the diverting channel at the $j$-th interval of time from the left ($q_{(mj)av}^l$) and right ($q_{(mj)av}^r$) of it (for example, in case of a road side ditch) can be determined by the formula (12):

$$q_{(mj)av} = q_{(mj)av}^l + q_{(mj)av}^r.$$  \hfill (12)
The maximum flow rate at the closing cross section of the channel $Q_{\text{max}}$ can be determined, based on either of the following main hypothetical assumptions: 1) through the maximum inflows at the mouths of the model slope strips $q_{\text{max}}$, (see Figure 4); 2) through the mean values of the continuous lateral inflow rates $q_{\text{av}}$. This question should be analyzed by detailed simulations depended on the solution of the equations (2), (10) and (11). The analysis of various schemes of modeling and formation of the discharge along the diversion channel is also necessary. Here we can take either 1) an average area of the live cross section of the part $L_{\text{ch}(i)}$ taking the parabolicity of the curvature of free surface flow [4, 6], or 2) the average flow rate along the entire length of the channel. The later can be formed by the continuous lateral inflow (equaling to half of the maximum flow rate at the closing cross section), by the constant (boundary) flow rate at the beginning of the channel $Q_{\text{en}}$ (sometimes it may take place) and by other concentrated discharges along the channel (if there exist some springs or other concentrated tributaries). The second assumption may slightly increase the travel time. But at the same time this increases the reliability of the runoff computation when using the partial differential equations (2), (10) and (11), because the computed flow rates may also be underestimated due to an underestimated travel time. When the travel time exceeds its real value it causes no more influence on the maximal hydraulic outputs of the flow after the full runoff has been established, whatever may be the time duration and the rainfall intensity [4, 5, 10].

![Figure 4](image)

**Figure 4.** Unified schematization of the slopes and diversion channel with lateral inflow (a road side ditch is presented for an example)

### 4. Development of a formula for estimating the travel time of rainwater channel flow in mixed catchment basins with terraced surface plots

When the travel time along the model slope strip under consideration beginning from the hydraulically most remote point of the watershed boundary to its mouth at the axis of the diversion channel is reached, the full inflow rate is already established at the mouths of all other strips. Therefore, simplifying the task, for a maximum discharge $Q_{\text{max}}$ at the closing cross section of the channel can be used the sum of the average products of lateral inflow rates of each adjacent pair of model slope strips and the distances between them $L_{\text{ch}(i)}$ at their mouths (Figure 4) taking into account the input discharge $Q_{\text{en}}$ at the beginning of the channel and the springs or other concentrated tributaries $Q_{\text{trib}}$ along it (if they exist) as shown in the formula (13):

$$Q_{\text{max}} = \sum q_i \cdot L_{\text{ch}(i)},$$

where $q_i$ is the lateral inflow rate and $L_{\text{ch}(i)}$ is the distance between the mouths of the model slope strips.
\[
Q_{\text{max}} = Q_{\text{en}} + Q_{\text{trib}} + \frac{1}{2} \sum_{i=1}^{n_1} \left( q^l_{(i)} + q^l_{(i+1)} \right) L_{x(i)} + \frac{1}{4} \sum_{m=1}^{n_2} \left( q^r_{(m)} + q^r_{(m+1)} \right) L_{x(m)},
\]

where \( i = 1, 2, 3 \ldots n_1 \) - numbers of the strips on the left slope; \( m = 1, 2, 3 \ldots n_2 \) - the same on the right side; \( Q_{\text{en}} \) - constant inflow rate at the inlet of the diversion channel (if it exits), \( m^3/c \); \( Q_{\text{trib}} \) - discharge from the springs or other concentrated tributaries; \( L_{x(i)} \) - length of the \( i \)-th section of the channel between adjacent strips of the left slope, \( m \); \( L_{x(m)} \) - the same of the \( m \)-th section of right slope, \( m \); \( q^l_{(i)} \), \( q^l_{(i+1)} \) - inflow rates at the mouths of adjacent elementary strips on the \( L_{x(i)} \) sector from the left slope, \( m^2/s \); \( q^r_{(m)} \), \( q^r_{(m+1)} \) - the same from the right-hand slope \( L_{x(m)} \), \( m^2/s \).

Taking for the average flow rate \( Q_{\text{chav}} \) over the entire length of the diversion channel as half of the maximal flow rate \( Q_{\text{max}} \) (14) at its closing cross section formed by the lateral inflows that can be obtained by the formula (13) plus the flow rate at the inlet of the channel and other concentrated tributaries, we get:

\[
Q_{\text{chav}} = Q_{\text{en}} + Q_{\text{trib}} + \frac{1}{4} \sum_{i=1}^{n_1} \left( q^l_{(i)} + q^l_{(i+1)} \right) L_{x(i)} + \frac{1}{4} \sum_{m=1}^{n_2} \left( q^r_{(m)} + q^r_{(m+1)} \right) L_{x(m)}. \tag{14}
\]

In a diversion channel the average instantaneous flow velocity can be determined by the formula:

\[
v_{\text{chav}} = \frac{1}{m_{\text{chav}}} \frac{1}{2} \frac{1}{n_{\text{chav}}} \frac{1}{2} R_{\text{chav}}^{2/3},
\]

where \( n_{\text{chav}} \), \( I_{\text{chav}} \) - weighted average values of the roughness coefficient and bed gradient; \( R_{\text{chav}} \) - hydraulic radius of live cross section of the channel.

Simplifying the shape of the diversion channel as triangular with weighted average side gradients \( m_{1\text{chav}} \) and \( m_{2\text{chav}} \) the average values of the area of channel cross section \( \omega_{\text{chav}} \), the flow depth \( h_{\text{chav}} \) and the wetted perimeter \( \chi_{\text{chav}} \) will be:

\[
\omega_{\text{chav}} = \frac{1}{2} \left( m_{1\text{chav}} + m_{2\text{chav}} \right) h_{\text{chav}}^2 = m_{\text{chav}} h_{\text{chav}}^2,
\]

\[
h_{\text{chav}} = \sqrt{\frac{\omega_{\text{chav}}}{m_{\text{chav}}}}; \quad \chi_{\text{chav}} = h_{\text{chav}} \left( \sqrt{1 + m_{1\text{chav}}^2} + \sqrt{1 + m_{2\text{chav}}^2} \right).
\]

Based on this, the average instantaneous flow velocity will be:

\[
v_{\text{chav}} = \frac{n_{\text{chav}}}{m_{\text{chav}}} \left( \frac{\omega_{\text{chav}}}{\chi_{\text{chav}}} \right)^{2/3} \left( \frac{m_{\text{chav}} h_{\text{chav}}}{m_{\text{chav}} h_{\text{chav}}} \right)^{2/3}, \tag{15}
\]

Taking into account the above said and \( v_{\text{chav}} \omega_{\text{chav}} = Q_{\text{chav}} \), the equation for the average depth of the flow becomes:

\[
h_{\text{chav}} = \frac{Q_{\text{chav}}}{m_{\text{chav}} v_{\text{chav}}} = \sqrt{n_{\text{chav}} Q_{\text{chav}} \left( \sqrt{1 + m_{1\text{chav}}^2} + \sqrt{1 + m_{2\text{chav}}^2} \right)^{2/3} \frac{m_{\text{chav}}^{5/3} h_{\text{chav}}^{1/2}}{m_{\text{chav}}^{5/3} h_{\text{chav}}^{1/2}}, \tag{16}
\]

or

\[
h_{\text{chav}} = \left( \frac{n_{\text{chav}} Q_{\text{chav}}}{m_{\text{chav}}^{5/8} h_{\text{chav}}^{3/16}} \right)^{1/4} \left( \sqrt{1 + m_{1\text{chav}}^2} + \sqrt{1 + m_{2\text{chav}}^2} \right)^{1/4} \left( m_{\text{chav}}^{5/8} h_{\text{chav}}^{3/16} \right)^{1/4}.
\]

where \( Q_{\text{chav}} \) - average discharge computed by using the formula (14).
The duration of the travel time of channel flow $T_{ch}$ is the time that takes for the water to reach from mouth of the the considered $k$-th slope strip to the closing cross section of the channel with a length $L_{ch(k)}$ and an average velocity $\nu_{chav}$, computed by the formula (15). The average depth of the flow is calculated by formula (16). Having the distance $L_{ch(k)}$ and the flow velocity $\nu_{chav}$, we can easily determine $T_{ch}$:

\[
T_{ch} = \frac{L_{ch(k)}}{\nu_{chav}} = \frac{n^{3/4} L_{chav} L_{ch(k)}^{3/8}}{m_{chav}^{3/8} Q_{chav}^{1/4}} \left( \sqrt{1 + m_{1chav}^2} + \sqrt{1 + m_{2chav}^2} \right)^{1/2}.
\]  

(17)

Taking in view the above said, the formula (9) is used to compute the total travel time $T_t$ for rainwater runoff from small catchments with a mixed surface and the $T_{slt}$ and $T_{ch}$ are computed by formulas (6), (7), (8) and (17).

5. Some recommendations

1. When determining the travel time by the proposed method for a considered part of the slope, should be used the weighted average gradients and surface roughness coefficients of the model strips taking into account their differences in different segments of the given distance.

2. As the small catchment basins in rain-prone regions with a long rainy season are taken in account, it is permissible to neglect the various kinds of losses due to the constant saturation of soils with moisture, wetted vegetations and filled up microrelief cavities because of the frequent precipitation.

3. When calculating total travel time $T_{slt}$ for the slope, it is necessary to convert the intensity of a one-hour-long shower to the intensity of the estimated duration by using a transformation factor $K_t = (60/T_t)^{2/3}$, where the $T_t$ is taken in minutes. This requires determining the travel time $T_{slt}$ on the finally adopted design slope strip based on the most probable shower intensities and the remained part $L_{ch(k)}$ of the diversion channel.

4. The main goal of the present method is to determine the approximate travel time of the slope and channel flow and use it when working out and testing a simulation model for estimating the rainwater slope runoff and channel flow based on the differential equations of one-dimensional hydraulics. Therefore, the estimation formulas of the slope runoff and the channel flow as well as the cross section of the diversion channel are taken permissibly simplified.

5. Having obtained the travel time by this method, it is necessary to conduct numerical simulation studies of the real catchments and forms of the diversion channels with their own characteristics based on the above proposed hydrological and hydraulic equations until establishing the full flow on the slopes and channels separately and clarify the derived formulas.

6. It is necessary to simultaneously conduct researches aimed to working out formulas for calculating optimal intervals of time and length to solve the proposed differential equations in finite differences in order to obtain the hydrological and hydraulic characteristics of the rainwater slope runoff and channel flow with a given accuracy.

6. Main conclusions

1. A new method is proposed for unified schematization of a mixed catchment basin with natural and terraced rice field sectors of the slopes in conjunction with a diversion channel, taking into account the peculiarities of small drainage areas with anthropogenic plots.

2. The equations of one-dimensional hydraulics based on the laws of conservation of matter and energy are proposed for describing the rainfall slope runoff as a sequent and uninterrupted process.

3. The main characteristics of the slope surfaces of mixed catchment basins and chosen diversion channels influencing the formation of rainwater runoff and travel time of the slope and diversion channel are taken into account.
4. A unified formula has been developed for determining the total travel time of the slope runoff and channel flow on mixed drainage basins with terraced areas, which can be used in calculating slope runoff and channel discharges based on the differential equations of unsteady flow of storm water.

5. The developed formula based on the methods of mathematical modeling, gives a more accurate travel time of slope and channel flow than the methods recommended by existing standards, and is a novelty for the case of anthropogenic territories.

References
[1] Andreev O V and Shahidov A F 1977 Metodicheskie razrabotki po raschyotu livnevogo stoka s malych vodosborov (Moscow: MADI) 8 p
[2] VSN 63-76 1976 Instrukciya po raschetu livnevogo stoka vody s malych bassejnov (Moscow: Orgstroj Mintransstroya SSSR) 101 p
[3] SP 33-101-2003 2004 Opredelenie osnovnyx raschetnyx gidrologicheskih harakteristik (Moscow: Gosstroya Rossii, FGUP CPP) 36 p
[4] Bolshakov V A and Kurganovich A A 1983 Gidrologicheskie i gidravlicheskie raschyoty malych iskusstvennykh sooruzhenij: uchebnoe posobie dlya vuzov (Kiev: Golovnoe izd-vo ob"edineniya “Vyssha shkola”) 280 p
[5] Paudyal S P 1990 Proektirovanie optimal'nogo poverhnostnogo dorozhnogo vodootvoda Cand. dissertation thesis (Moscow: MADI (GTU)) 20 p
[6] Paudyal S P 2005 Opredelenie raschyotnoy prodolzhitel'nosti livnevogo stoka s malych vodosborov dlya proektirovaniya optimal'nyh konstrukcij sistem poverhnostnogo dorozhnogo vodootvoda Proektirovanie avtomobil'nyh dorog (Moscow: MADI (GTU)) pp 117–26
[7] Paudyal S P 2011 Osobennosti formirovaniya raskhodov livnevogo sklonovogo stoka v terrasirovanych risovyh dolinah Nepala s zemlyanymi reguljaciynymi ustroystvami Proektirovanie avtomobil'nyh dorog (Moscow: MADI (GTU)) pp 43–8
[8] Fedotov G A 1986 Avtomatizirovannoe proektirovanie avtomobil'nyh dorog (Moscow: Transport) 317 p
[9] ChEbotaryov A I, Serpik B I 1973 O raschyote parametrov gidrologicheskogo rezhima Meteorolog. i gidrolog., 1 55–67
[10] Chistjakov I V 1988 Metodika detal'nogo raschyota livnevogo stoka s malych vodosborov pri proektirovanii malych mostov i vodopropusknyh trub na avtomobil'nyh dorogah Cand. dissertation thesis (Moscow: MADI) 18 p
[11] Paudyal S P 2014 Issledovanie vliyania harakteristik poverhnosti smeshannyh vodosbornyh bassejnov s terrasirovannymi uchastkami na vremya sklonovogo dobeganiya livnevogo stoka v terrasirovanych risovyh dolinah Nepala s zemlyanymi reguljaciynymi ustroystvami Vestn. Moskovsk. avtomobil'no-dorozhn. Gosudarstv. Tehnichesk. Univer. (MADI) 1(36) 92–9
[12] Gavrilenko S M 1980 Issledovaniya neustanovivshegosya dvizheniya livnevych vod v otkrytyh ruslah s bokovoj pritochnost'yu Cand. dissertation thesis (Kiev) 21 p
[13] Chugaev R R 1982 Gidravlika Uchebnik (Leningrad: Energoizdat) 672 p