Form factors and branching ratio for the \( B \to l\nu\gamma \) decay

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Abstract

Form factors parameterizing radiative leptonic decays of heavy mesons \((B^+ \to \gamma l^+ \nu_l)\) for photon energy are computed in the language of dispersion relation. The contributing states to the absorptive part in the dispersion relation are the multiparticle continuum, estimated by quark triangle graph and resonances with quantum numbers \(1^-\) and \(1^+\) which includes \(B^*\) and \(B^*_\lambda\) and their radial excitations, which model the higher state contributions. Constraints provided by the asymptotic behavior of the structure dependent amplitude, Ward Identities and gauge invariance are used to provide useful information for parameters needed. The couplings \(g_{BB^*\gamma}\) and \(f_{BB^*\gamma}\) are predicted if we restrict to first radial excitation; otherwise using these as an input the radiative decay coupling constants for radial excitations are predicted. The value of the branching ratio for the process \(B^+ \to \gamma \mu^+ \nu_\mu\) is found to be in the range \(0.5 \times 10^{-6}\). A detailed comparison is given with other approaches.

1 Introduction

In spite of small branching ratio, the radiative \(B\)-meson decay \((B \to l\nu\gamma)\) is of viable interest because it contains important information about weak
and hadronic interactions of $B$-meson. Furthermore, with the introduction of $B$-factories LHCb, BaBar, Belle and CLEO, the radiative $B$-meson decay can be studied with enough statistics. Preliminary data from the CLEO collaboration indicates the limit on the branching ratio $\mathcal{B}(B \to l\nu\gamma)$ which is:

$$\mathcal{B}(B \to e\nu\gamma) < 2.0 \times 10^{-4}$$
$$\mathcal{B}(B \to \mu\nu\gamma) < 5.2 \times 10^{-5}$$

at 90% confidence level [1]. With the better statistics expected from the upcoming $B$ factories, the observation and experimental study of this decay could become soon feasible. It is therefore of some interest to have a good theoretical control over the theoretical uncertainties affecting the relevant matrix elements.

The radiative leptonic decay $B^+ \to l^+\nu\gamma$ has received a great deal of attention in the literature [2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13] as a means of probing aspects of the strong and weak interactions of a heavy quark system. The presence of the additional photon in the final state can compensate for the helicity suppression of the decay rate present in purely leptonic mode. As a result, the branching ratio for the radiative leptonic mode can be as large as $10^{-6}$ for the $\mu^+$ case [10], which would open up a possibility for directly measuring the decay constant $f_B$ [7]. A study of this decay can offer also useful information about the Cabibbo-Kobayashi-Maskawa (CKM) matrix element $|V_{ub}|$ [14, 15].

In the radiative $B$-decay process, there are two contributions to the amplitude:

1. inner bremsstrahlung (IB) and

2. the structure dependent (SD) contribution which depends on the vector and axial vector form factor $F_V$ and $F_A$ respectively.

The IB contribution to the decay amplitude is associated with the tree diagrams shown in Figs. 1a and 1b, and SD contribution is associated with Fig. 1c.

In this paper, we will study the radiative leptonic $B$ decays of $B^+ \to l^+\nu\gamma$. The IB part is still helicity suppressed [2], while the SD one is free of the suppression [16]. Therefore, the radiative decay rates of $B^+ \to l^+\nu\gamma$
\((l = e, \mu)\) could have an enhancement with respect to the purely leptonic modes of \(B^+ \rightarrow l^+\nu_l\) due to the SD contributions in spite of the electromagnetic coupling constant \(\alpha\). With the possible large branching ratios, the radiative leptonic \(B\) decays could be measured in the future experiments at hadronic colliders, such as BTeV and CERN Large Hadron Collider (LHC-B) experiments [17].

The paper is organized as follows. In Sec. 2, we present the decay kinematics and current matrix elements for \(B^+ \rightarrow l^+\nu_l\gamma\). In section 3, we discuss the various contributions to the absorptive part of the SD amplitude \(iH_{\mu\nu}\), needed in the dispersion relation. This include multiparticle continuum and resonances with quantum numbers \(1^-\) and \(1^+\). The resonances include \(B^*\) and \(B^*_A\) mesons and their radial excitations, which model the higher states. The continuum is estimated by quark triangle graphs. In Sec. 4, the asymptotic behavior of the SD amplitude is studied. This provides a usual constraint on the residues of the resonance contribution, in terms of the continuum contribution. In Sec. 5, we discuss Ward Identities which together with gauge invariance relates various form factors. These identities which are expected to hold below the resonance regime, fix the normalization of the forms at \(q^2 = 0\) in terms of universal function \(g_+(0)\) as well as another constraint on the residues. Thus in our approach, a parametrization of \(q^2\) dependence of form factors is not approximated by single pole contribution. But this parametrization is dictated by considerations mentioned above and also predict the coupling constants of \(1^-\) and \(1^+\) resonances with photon if we restrict to one radial excitation; otherwise using these as input, the radiative coupling constants of radial excitations are predicted. In this and other aspects our approach is different from the others mentioned previously. Our approach is closest to the one used in [18] for \(B \rightarrow \pi l\nu_l\). We calculate the decay branching ratios in Sec. 5. We give our conclusions in Sec. 6.

## 2 Decay kinematics and current matrix elements

We consider the decay

\[ B^+(p) \rightarrow l^+(p_l)\nu_l(p_\nu)\gamma(k), \quad (1) \]
where $l$ stands for $e$ or $\mu$, and $\gamma$ is a real photon with $k^2 = 0$. The decay amplitude for radiative leptonic decay of $B^+ \rightarrow l^+\nu_l\gamma$ can be written in two parts, $M_{IB}$ and $M_{SD}$, as follows:

$$M(B^+ \rightarrow l^+\nu_l\gamma) = M_{IB} + M_{SD}$$

in terms of two emission types of real photon from $B^+ \rightarrow l^+\nu_l$. They are given by [19, 20, 21, 22]

$$M_{IB} = i e G_F \sqrt{2} V_{ub} f_B m_l \epsilon_\mu^* L^\mu$$

$$M_{SD} = -i G_F \sqrt{2} V_{ub} f_B m_l \epsilon_\mu^* \tilde{H}^{\mu\nu} l_\nu$$

with

$$L^\mu = m_l \bar{u}(p_\nu) (1 + \gamma_5) \left( \frac{2p_\mu}{2p \cdot k} - \frac{2p_l^\mu + k^\mu}{2p_l \cdot k} \right) v(p_l, s_l),$$

$$l^\mu = \bar{u}(p_\nu) \gamma^\mu (1 + \gamma_5) v(p_l, s_l),$$

$$\tilde{H}^{\mu\nu} = i F_V(q^2) \epsilon^{\mu\alpha\beta} k_\alpha p_\beta - F_A(q^2) (p \cdot k) (p^\mu - p^\nu k^\nu),$$

$$q^\mu = (p - k)^\mu = (p_l + p_\nu)^\mu.$$  

Here $\epsilon^*_\mu$ denotes the polarization vector of the photon with $k^\mu \epsilon^*_\mu(k) = 0$. $p$, $p_l$, $p_\nu$, and $k$ are the four momenta of $B^+$, $l^+$, $\nu$, and $\gamma$, respectively, $s_l$ is the polarization vector of the $l^+$, $f_B$ is the $B$ meson decay constant, and $F_A$, $F_V$ stand for two Lorentz invariant amplitudes (form factors).

The term proportional to $L^\mu$ in (5) does not contain unknown quantities—it is determined by the amplitude of the non-radiative decay $B^+ \rightarrow l^+\nu_l$. This part of the amplitude is usually referred as “inner bremsstrahlung (IB) contribution”, whereas the term proportional to $H^{\mu\nu}$ is called “structure dependent (SD) contribution”.

The form factor $F_A(F_V)$ is related to the matrix element of the axial (vector) current. The factors $f_B$ and $F_{V,A}$ are defined by

$$\langle 0 | \bar{u} \gamma^\mu \gamma_5 b | B(p) \rangle = -i f_B p^\mu$$

$$\langle \gamma(k) | \bar{u} \gamma^\mu \gamma_5 b | B(p) \rangle = -[ (\epsilon^*_\mu \cdot p) k^\mu - \epsilon^*_\mu (p \cdot k) ] F_A(q^2)$$

$$\langle \gamma(k) | \bar{u} \gamma^\mu b | B(p) \rangle = -i \epsilon^{\mu\alpha\beta} \epsilon^*_\nu p_\alpha k_\beta F_V(q^2).$$
In our phase convention, the form factors $F_A$ and $F_V$ are real in the physical region

$$m_l^2 \ll q^2 \ll M_B^2$$

where $q$ is the momentum transfer. The kinematics of the decay needs two variables, for which we choose the conventional quantities and in the rest frame of $B$

$$x = \frac{2p \cdot k}{M_B^2} = \frac{2E_\gamma}{M_B}$$

$$y = \frac{2p \cdot p_l}{M_B^2} = \frac{2E_l}{M_B}$$

(12)

and the angle $\theta_{l\gamma}$ between the photon and the charged lepton is related to $x$ and $y$ by

$$x = \frac{1}{2} \frac{(2 - y + \sqrt{y^2 - 4r_l})(2 - y - \sqrt{y^2 - 4r_l})}{2 - y + \sqrt{y^2 - 4r_l} \cos \theta_{l\gamma}}.$$  

(14)

In terms of these quantities, one can write the momentum transfer as

$$q^2 = M_B^2 (1 - x), \quad (k^2 = 0).$$

(15)

We write the physical region of $x$ and $y$ as

$$0 \leq x \leq 1 - r_l, \quad (16)$$

$$1 - x + \frac{r_l}{1 - x} \leq y \leq 1 + r_l, \quad (17)$$

where

$$r_l = \frac{m_l^2}{M_B^2} = \left\{ \begin{array}{cl} 9.329 \times 10^{-9} & (l = e), \\ 4.005 \times 10^{-4} & (l = \mu). \end{array} \right.$$  

(18)

3 Dispersion Relations

The structure dependent part, $H^{\mu\nu}$ is given by

$$iH^{\mu\nu} = i \int d^4x e^{ik \cdot x} \langle 0 | T (j_{\text{em}}^\mu(x) J_{2\nu}^\nu(0)) | B(p) \rangle.$$  

(19)

We note that [23]

$$ik_\mu H^{\mu\nu} = if_B p_\nu.$$  

(20)
so that for the real photon we can write

\[ H^{\mu\nu} = \tilde{H}^{\mu\nu} + f_B \frac{p^{\mu}p^{\nu}}{p \cdot k} \]  

(21)

where \( k^{\mu} \tilde{H}^{\mu\nu} = 0 \) and \( \tilde{H}^{\mu\nu} \) is parametrized as in Eq.(7). The second term in (21) is absorbed in \( M_{IB} \). The absorbive part is

\[
\text{Abs} [iH^{\mu\nu}] = \frac{1}{2} \int d^4x e^{ik \cdot x} \langle 0 | j_{em}^{\mu}(x), J_{2}^{\nu}(0) | B(p) \rangle \\
= \frac{1}{2} (2\pi)^4 \sum_n \langle 0 | j_{em}^{\mu}(0) | n \rangle \langle n | J_{2}^{\nu}(0) | B(p) \rangle \delta^4(k - p_n) \\
- \sum_n \langle 0 | J_{2}^{\nu}(0) | n \rangle \langle n | j_{em}^{\mu}(0) | B(p) \rangle \delta^4(k + p_n - p) \]  

(22)

The \( \delta \)-function in the first term implies \( p_n^2 = k^2 = 0 \) and since there is no real particle with zero mass, the first term does not contribute. Thus contributing to the absorbive part are all possible intermediate states that couple to \( B\gamma \) and annihilated by the weak vertex \( \langle 0 | J_{2}^{\nu}(0) | n \rangle \). These include the multiparticle continuum as well as resonances with quantum numbers \( 1^- \) and \( 1^+ \). Thus \( (t = q^2) \)

\[
F_V(t) = \frac{g_{BB^*\gamma}}{M_{B^*}^2 - t} f_{B^*} + \cdots \\
F_A(t) = \frac{f_{B_A^*\gamma}}{M_{B_A}^2 - t} f_{B_A^*} + \cdots 
\]

(23)

The ellipses stand for contributions from higher states with the same quantum numbers. The couplings \( g_{BB^*\gamma} \) and \( f_{B_A^*\gamma} \) are defined as

\[
\langle B^\gamma (q, \eta) \gamma (k, \epsilon) | B^- (P) \rangle = ig_{B^*B\gamma} \tilde{\varepsilon}_{\alpha\mu\sigma} \epsilon^{*\alpha} q^\sigma \eta^\mu p^\sigma \\
\langle B_A^\gamma (q, \eta) \gamma (k, \epsilon) | B^- (P) \rangle = ig_{B_A^*B\gamma}(\epsilon^* \eta^*) - if_{B_A^*B\gamma}(q.\epsilon^*)(k.\eta^*) \\
\langle 0 | i\bar{u} \gamma^\mu b | B^\gamma (q, \eta) \rangle = f_{B^\gamma} \eta^\mu \\
\langle 0 | i\bar{u} \gamma^\mu \gamma_5 \gamma_5 | B_A^\gamma (q, \eta) \rangle = f_{B_A^\gamma} \eta^\mu 
\]

(24)

We assume that the contributions from the radial excitations of \( B^\gamma \) and \( B_A^\gamma \) dominate the higher state contribution. Thus we write

\[
F_V(t) = \frac{R_V}{1 - t/M_{B^*}^2} + \sum_i \frac{R_{V_i}}{1 - t/M_{B_i^*}^2} + \frac{1}{\pi} \int_{s_0}^{s} \frac{F_V^{\text{Cont}}(s)}{s - t - i\varepsilon} ds + \cdots 
\]

6
\[ F_A(t) = \frac{R_A}{1 - t/M_{B_A}^2} + \sum_i \frac{R_{A_i}}{1 - t/M_{B_{A_i}}^2} + \frac{1}{\pi} \int_{S_0}^{M^2} \frac{\Im F_A^{\text{Cont}}(s)}{s - t - i\varepsilon} ds + \ldots \] (25)

where ellipses stands for the contributions from the region for much larger than the physical mass of heavy resonances up to \( \infty \). Here, \( M \) is a cut off near the first radial excitation of \( M_{B^*} \) or \( M_{B_{A^*}} \) and \( S_0 = M_B + m_\pi \), and

\[ R_{V_i} = \frac{g_{BB^*\gamma}}{M_{B^*}^2} f_{B^*} \]

\[ R_A = \frac{f_{B_{A^*}B^*}}{M_{B_{A^*}}^2} f_{B^*} \]

\( R_{V_i} \) and \( R_{A_i} \) are the corresponding quantities for the radial excitations with masses \( M_{B_{i^*}} \) and \( M_{B_{A_{i^*}}} \). In the next section we develop the constraints on some of the parameters appearing in the above equations.

If we model the continuum contribution by quark triangular graph (similar calculations exist in the literature [24]), we obtain

\[ F_V^{\text{Cont}} = F_A^{\text{Cont}} = \frac{f_B}{M_B} \left\{ \frac{Q_u - Q_b}{M_B} \cdot \frac{\lambda - \frac{Q_b}{M_B}(1 + \frac{\lambda}{M_B})}{1 - q^2/M_B^2} \right\} \] (27)

where

\[ \lambda = M_B - m_b, \] (28)

together with the term

\[ (Q_u - Q_b) f_B \frac{p^\mu p^\nu}{k \cdot p} = f_B \frac{p^\mu p^\nu}{k \cdot p} \]

which appears in Eq. (21). As is well known (see for example Ref. [25]), the pole at \( q^2 = M_B^2 \) in Eq. (27) arises due to \( u \) \( (\bar{b}) \) quark propagator which form one leg of quark \( \Delta \), the other legs are the part of \( B \) meson wave function.

### 4 Asymptotic Behavior

To get constraints on the residues \( R_i \), it is useful to study the asymptotic behavior of form factors \( F_V \) and \( F_A \). It has been argued that the behavior of form factor for very large values of \(|t|\) can be estimated reliably in
perturbative QCD processes \( \text{pQCD} \)[26, 27, 18]. For \( t \ll 0 \) and for \(|t| \) much larger than the physical mass of heavy resonances, pQCD should yield a very good approximation to the form factors. First we note that by vector meson dominance

\[
\langle \gamma \left( k, \varepsilon^* \left( k \right) \right) | \bar{u} \gamma^\mu \left( 1 - \gamma_5 \right) b | B \left( p \right) \rangle \simeq Q_u \frac{f_\rho}{m_\rho} \langle \rho \left( k, \varepsilon^* \left( k \right) \right) | \bar{u} \gamma^\mu \left( 1 - \gamma_5 \right) b | B \left( p \right) \rangle ,
\]

(29)

where \( f_\rho \), having dimensions of mass, is defined as

\[
\langle 0 | \bar{u} \gamma^\mu u | \rho \left( k, \varepsilon \left( k \right) \right) \rangle = \frac{f_\rho}{m_\rho} \varepsilon^\mu
\]

(30)

Then using the methods employed in [27], it is easy to calculate [only the diagram where gluon is emitted by the light quark in \((b\bar{u})\) bound state and absorbed by the heavy quark contributes and is by itself gauge invariant] \( F_{\text{pQCD}} \):

\[
F_{\text{pQCD}}^V = \frac{F_{\text{pQCD}}^A}{3} = \left( Q_u \frac{f_\rho}{m_\rho} \right) \frac{32\pi \alpha_s \left( t \right)}{3} \left( f_B f_\rho \right) m_B \left( \frac{1}{\varepsilon} \ln \varepsilon \right) \frac{1}{t^2}
\]

(31)

Here

\[
\varepsilon \sim \mathcal{O} \left( \frac{\Lambda_{\text{QCD}}}{m_B} \right)
\]

(32)

and is governed by the tail end of the \( B \) meson wave function characterized by \( \varepsilon \).

Now the asymptotic behavior of Eq. (25), is given by

\[
F \left( q^2 \right) \rightarrow -\frac{1}{q^2} \left[ R M^2 + \sum_i R_i M_i^2 + \frac{1}{\pi} \int_{S_0}^{M^2} 3 F_{\text{Cont}}(s) ds \right].
\]

(33)

Since \( F_{\text{pQCD}}(t) \) is a reliable approximation to the form factor for \( t \rightarrow -\infty \), and \( t F_{\text{pQCD}} \rightarrow 0 \) in this limit, it follows that

\[
RM^2 + \sum_i R_i M_i^2 + c \simeq 0,
\]

(34)

where we have defined

\[
c = \frac{1}{\pi} \int_{S_0}^{M^2} 3 F_{\text{Cont}}(s) ds.
\]

(35)
The convergence relation (34) is a model-independent result and constitutes a very binding constraint for model building. In other words, the various contributions in Eq. (33) may be individually much larger than the \( t^{F_{PQCD}} (t) \) due to \( \alpha_s (t)/t \) suppression, but there must be large cancellations among the non-perturbative contributions in (33). This is in the spirit of ref. [18]. We will explore the resonant contribution (in our model) in order to understand the effect of Eq. (34) on the behavior of form factors in the physical region.

The imposition of this constraint will lead to a very distinct behavior of the photon momentum distribution, independently of how many resonances we choose to keep. As the radial excitations of \( B^* \) become heavier, they are less relevant to the form factors since the spacing between the consecutive radial excitations are expected to become narrower and narrower [28]. Thus, heavier resonances contribute with a smaller value even in the narrow width approximation. Furthermore, as finite widths are considered, the contribution of heavier and thus broader excitations are additionally suppressed. This shows that the truncation of the sum over resonances is a reasonable approximation.

For the resonances stated above we will study a constrained dispersive model where only the first two radial excitations are kept. This is mainly for the reason mentioned above. On the other hand, the “minimal” choice of keeping only one radial excitation will determine \( R_1 \) in terms of \( R \). The other necessary ingredient to specify the model is the knowledge of the spectrum of radial excitations. These resonances [(2S) and (3S) excitations of \( B^* \)] have not yet been observed in the \( B \) systems. We will then rely on potential model calculations for their masses [28]. These models have been very successful in predicting the masses of orbitally excited states and as such we are confident that the position of the radial excitations does not introduce a sizeable uncertainty. The resultant spectrum explicitly shows that the spacing among 1S, 2S, 3S states are, to leading order, independent of heavy quark mass and, therefore, constitutes the property of the light degrees of freedom. The spectrum of radial excitations is given in Table 1, where the subindices 1 and 2 correspond to the 2S and 3S excitation of the \( B^* \), etc. Thus the convergence condition (34) now reads

\[ RM^2 + R_1 M_1^2 + R_2 M_2^2 + c = 0, \]  

(36)

This condition leaves two free parameters \( R_1 \) and \( R_2 \) in the model. This results in the correct scaling of form factors with the heavy meson mass.
Solving Eq. (36) for $R_2$ and using in Eq. (33), we obtain

$$F(q^2) = \frac{RM^2 (M^2_2 - M^2)}{(M^2 - q^2) (M^2_2 - q^2)} + \frac{R_1 M^2_1 (M^2_2 - M^2)}{(M^2_1 - q^2) (M^2_2 - q^2)} + \frac{1}{M^2_2 - q^2} \int s_0 \frac{M^2_2 - s}{s - q^2} \Re F_{\text{Cont}}^\text{V}$$

(37)

If we model the continuum contribution by quark triangle graph as given in Eq. (27), we obtain

$$F(q^2) = \frac{RM^2 (M^2_2 - M^2)}{(M^2 - q^2) (M^2_2 - q^2)} + \frac{R_1 M^2_1 (M^2_2 - M^2)}{(M^2_1 - q^2) (M^2_2 - q^2)} + \frac{M^2_2 - M^2}{(M^2_2 - q^2) (M^2_2 - q^2)} c$$

(38)

where in the heavy quark limit $M_B = M_B^* = M$ and

$$c = f_B M_B \left[ \frac{Q_u}{\Lambda} + \mathcal{O} \left( \frac{1}{M_B} \right) \right]$$

(39)

5 Ward Identities Constraints

It is useful to define

$$\langle \gamma (k, \epsilon) | \bar{u} i\epsilon^{\mu\nu} q_\nu b | B(p) \rangle = -i \epsilon^{\mu\nu\alpha\beta} k_\alpha p_\beta F_1(q^2)$$

(40)

$$\langle \gamma (k, \epsilon) | \bar{u} i\epsilon^{\mu\nu} \gamma_5 q_\nu b | B(p) \rangle = [(q \cdot k) \epsilon^* \mu - (\epsilon^* \cdot q) k^\mu] F_3(q^2)$$

(41)

Now we will make use of Ward Identities and gauge invariance principle to relate different form factors.

Usually, the gauge invariance is implemented by means of the Ward Identities; another way, essentially the same, is to consider what happens if the polarization vector of an external (real) photon is replaced by its four-momentum. The result is zero, provided that one considers all diagrams where this particular photon is connected in all possible ways to a charge carrying line. In this way one understands the connection between gauge invariance and charge conservation. The Ward Identities\(^1\) used to relate different form factors appearing in our process are:

$$\langle \gamma (k, \epsilon) | \bar{u} i\epsilon^{\mu\nu} q_\nu b | B(p) \rangle = -(m_b + m_q) \langle \gamma (k, \epsilon) | \bar{u} \gamma^\mu b | B(p) \rangle + (p^\mu + k^\mu) \langle \gamma (k, \epsilon) | \bar{u} b | B(p) \rangle$$

\(^1\)See ref.[29] for a detailed derivation of these Ward Identities.
\[
\langle \gamma (k, \epsilon) | u \sigma_{\alpha \beta} b | B(p) \rangle = -(m_b + m_q) \langle \gamma (k, \epsilon) | \bar{u} \gamma^\mu b | B(p) \rangle
\]

\[
= (m_b - m_q) \langle \gamma (k, \epsilon) | \bar{u} \gamma^\mu \gamma_5 b | B(p) \rangle + (p^\mu + k^\mu) \langle \gamma (k, \epsilon) | \bar{u} \gamma_5 b | B(p) \rangle
\]

\[
= (m_b - m_q) \langle \gamma (k, \epsilon) | \bar{u} \gamma^\mu \gamma_5 b | B(p) \rangle
\]

(42)

where the matrix elements \( \langle \gamma (k, \epsilon) | \bar{u} b | B(p) \rangle \) and \( \langle \gamma (k, \epsilon) | \bar{u} \gamma_5 b | B(p) \rangle \) vanish for real photon due to gauge invariance.

Using the Ward Identities in Eqs.(40) and (41), and comparing the coefficients, we obtain

\[ p \cdot k = q \cdot k, \quad \epsilon^* \cdot p = \epsilon^* \cdot q \]

\[
F_V(q^2) = \frac{1}{m_b + m_q} F_1(q^2)
\]

(44)

\[
F_A(q^2) = \frac{1}{m_b - m_q} F_3(q^2)
\]

(45)

The results given in Eqs.(44) and (45) are model independent because these are derived by using Ward Identities.

In order to make use of Ward Identities to relate different form factors, we define

\[
\langle \gamma (k, \epsilon) | i \bar{u} \sigma_{\alpha \beta} b | B(p) \rangle = -i \varepsilon_{\alpha \beta \rho \sigma} \epsilon^\rho (k) [(p + k)^\sigma g_+ + q^\sigma g_-]
\]

\[ -i q \cdot \epsilon^* (k) \varepsilon_{\alpha \beta \rho \sigma} (p + k)^\rho q^\sigma h
\]

\[ -i [q_\alpha \varepsilon_{\beta \rho \sigma} \epsilon^\rho (k) (p + k)^\sigma q^\tau - \alpha \leftrightarrow \beta] h_1
\]

\[ -i [(p + k)_\alpha \varepsilon_{\beta \rho \sigma} \epsilon^\rho (k) (p + k)^\sigma q^\tau - \alpha \leftrightarrow \beta] h_2.
\]

(46)

Since we have a real photon, gauge invariance requires that if we replace \( \epsilon^\mu (k) \) by \( k^\mu \), the matrix element should vanish. This requires

\[ g_+ + g_- + 2 (q \cdot k) h = 0 \]  

(47)

From Dirac algebra

\[ \sigma^{\mu \nu} \gamma_5 = -\frac{i}{2} \varepsilon^{\mu \nu \alpha \beta} \sigma_{\alpha \beta}, \]

(48)

we can write

\[
\langle \gamma (k, \epsilon) | i \bar{u} \sigma^{\mu \nu} \gamma_5 b | B(p) \rangle
\]
The gauge invariance, namely, replacing $\epsilon^\mu$ by $k^\mu$, the matrix element should be zero, does not give any new relation other than (47). Using this relation and $2k \cdot q = M_B^2 - q^2$, we get

$$
\langle \gamma (k, \epsilon) | i \bar{u} \sigma_{\alpha \beta} g_{\beta \gamma 3} b | B(p) \rangle = (\epsilon^{\mu} k^{\nu} - \epsilon^{\nu} k^{\mu}) \left[ 2g_+ + \left( M_B^2 - q^2 \right) (h - h_1 - h_2) - 2q^2 h_1 - 2M_B^2 h_2 \right] - [2k \cdot q (\epsilon^{\nu} p^\nu - \epsilon^{\nu'} p^{\nu'}) + 2q \cdot \epsilon^* (p^\mu k^{\nu} - p^{\nu'} k^\mu)] (h - h_1 - h_2)
$$

Contrary to what is stated in some literature, the gauge invariance does allow a second tensor structure in addition to $(\epsilon^\mu k^{\nu} - \epsilon^{\nu} k^\mu)$.

This gives

$$
\langle \gamma (k, \epsilon) | i \bar{q} \sigma^{\mu \nu} q_\nu g_3 b | B(p) \rangle = 2 \left( g_+ - q^2 h - \left( M_B^2 - q^2 \right) h_2 \right) \times (q \cdot k \epsilon^* (k) - q \cdot \epsilon^* (k) k^\mu).
$$

This, in turn, gives [from Eq.(41)]

$$
F_3(q^2) = 2 \left[-g_+ + q^2 h - \left( M_B^2 - q^2 \right) h_2\right] \quad (52)
$$

Similarly, from Eq.(46), we get the relation

$$
\langle \gamma (k, \epsilon) | \bar{u} \sigma_{\alpha \beta} q_\beta g_{3 b} | B(p) \rangle = -i \epsilon_{\alpha \beta \rho \sigma} \epsilon^* \rho q^3 2 \left[ g_+ - q^2 h_1 - M_B^2 h_2 \right]
$$

Comparison of this equation with Eq.(40) gives

$$
F_1 \left( q^2 \right) = 2\left[ g_+ \left( q^2 \right) - q^2 h_1 \left( q^2 \right) - M_B^2 h_2 \left( q^2 \right) \right]
$$

Thus, finally we obtain

$$
F_V \left( q^2 \right) = \frac{2}{m_b + m_q} \left\{ g_+ \left( q^2 \right) - q^2 h_1 \left( q^2 \right) - M_B^2 h_2 \left( q^2 \right) \right\}, \quad (54)
$$

$$
F_A \left( q^2 \right) = \frac{2}{m_b - m_q} \left\{ g_+ \left( q^2 \right) - q^2 h \left( q^2 \right) - \left( M_B^2 - q^2 \right) h_2 \left( q^2 \right) \right\}. \quad (55)
$$
Therefore, the normalization of $F_V$ and $F_A$ at $q^2 = 0$ is determined by a universal form factor $(g_+ (0) - M_B^2 h_2)$. Now the form factor $h_2$ does not get any contribution from quark triangle graph nor from the pole and therefore we shall put it equal to zero. On the other hand, only $g_+ (q^2)$ gets contribution from quark $\Delta$-graph [24],

$$g_+ (q^2) = f_B \left\{ \frac{Q_u}{2\Lambda} - \frac{Q_b}{2M_B} \left( 1 - \frac{m_q}{M_B} \right) \right\} \frac{1}{1 - q^2/M_B^2}. \quad (56)$$

We expect the Ward Identities to hold at low $q^2$ below the resonance regime and as such we use the results obtained from them at $q^2 = 0$. Thus from Eqs. (54 and 55), we obtain

$$(m_b + m_q) F_V (0) = 2g_+ (0) = (m_b - m_q) F_A (0). \quad (57)$$

Further, using Eq. (28) in the above Eq. (57) and neglecting terms of the order of $(\Lambda \mp m_q) / M_B$, we obtain another constraint using Eqs. (38, 39) at $q^2 = 0$

$$R \left( 1 - \frac{M_B^2}{M_B^2} \right) + R_t \left( 1 - \frac{M_B^2}{M_B^2} \right) = \left( \frac{2g_+ (0)}{M} \right) \frac{M_B^2}{M_B^2}. \quad (58)$$

Now if we restrict to one radial excitation ($M_2 = M_1$) we obtain from Eq. (58)

$$R = \left( \frac{2g_+ (0)}{M_B^2/M_B^2 - 1} \right) \frac{M}{M_B^2} \quad (59)$$

$$F (q^2) = \frac{2}{M (1 - q^2/M^2)} \frac{g_+ (0)}{1 - q^2/M_1^2} \quad (60)$$

Restoring the subscripts and using the definitions (26)

$$g_{B^* B \gamma} = 2g_+ (0) \frac{M_{B^*}}{M_B \ f_{B^*} \left( M_{B^*}^2 / M_{B^*}^2 - 1 \right)}$$

$$\approx \frac{2g_+ (0)}{f_B \left( M_{B^*}^2 / M_{B^*}^2 - 1 \right)} \quad (61)$$

while

$$f_{B^\star_\Lambda B \gamma} = \frac{M_{B^*}^2}{M_B \ f_{B^*_\Lambda} \left( M_{B^* \Lambda}^2 / M_{B^*_\Lambda}^2 - 1 \right)} \quad (62)$$
Table 1: $B$-mesons masses in GeV [30]

|     | $J^P$ | $M$  | $M_1/M$ | $M_2/M$ |
|-----|-------|------|---------|---------|
| $M_B$ | $0^-$ | 5.28 | 1.14    | 1.24    |
| $M_{B^*}$ | $1^-$ | 5.33 | 1.14    | 1.24    |
| $M_{B^*_A}$ | $1^+$ | 5.71 | 1.12    | 1.22    |

Use $g_+(0)$ given in Eq. (56) with $Q_u = 2/3$, namely

$$g_+(0) = \frac{2f_B}{3\Lambda}$$  \hspace{1cm} (63)

we have the prediction

$$g_{B^*\gamma} = \frac{2}{3\Lambda} \frac{1}{(M_{B^*_1}/M_{B^*} - 1)}$$  \hspace{1cm} (64)

Further

$$F_V(q^2) = \frac{2}{M_B (1 - q^2/M_{B^*}^2) (1 - q^2/M_{B^*_1}^2)} g_+(0)$$  \hspace{1cm} (65)

$$F_A(q^2) = \frac{2}{M_B (1 - q^2/M_{B^*_A}^2) (1 - q^2/M_{B^*_A}^2)} g_+(0)$$  \hspace{1cm} (66)

This is the final expression for the form factors of our process $B \rightarrow \gamma l\nu_l$, if we restrict to the one radial excitation. We also observe the approximate equality $F_V(q^2) = F_A(q^2)$ of the form factors which also occur in some other models [12, 13]. For numerical work, we shall use $B$-meson masses given in Table 1 and $f_B = 0.180$ GeV.

This gives the prediction from Eq.(64)

$$g_{B^*\gamma} = \frac{2.2\bar{\Lambda}}{\bar{\Lambda}} = 5.6\text{GeV}^{-1},$$  \hspace{1cm} (67)

for $\bar{\Lambda} = 5.28 - 4.8 = 0.4$ GeV$^{-1}$ [see Eq. (28) and Table 1]. Also, we obtain from Eq. (63)

$$g_+(0) = \frac{3}{20} = 0.15.$$  \hspace{1cm} (68)
Further from Eq.(62)

\[
 f_{B\gamma} = \frac{f_{BM\gamma}}{\Lambda} 2.6
\]

\[
 = 6.5 \frac{f_{BM\gamma}}{f_{B\gamma}} GeV^{-1}
\]  

(69)

We now study the effect of the second radial excitation. We go back to Eq. (38) and use the constraint (58) to obtain

\[
 F(q^2) = R \frac{M^2 - 1}{M^2} \left( \frac{M^2 - 1}{M^2} \right) \frac{q^2}{M^2} + \frac{2g_+}{M} \left( 1 - \frac{q^2}{M^2} \frac{1}{M_B^2} - \frac{M^2}{M_B^2 M^2} \right)
\]

\[
 (1 - q^2/M_B^2) (1 - q^2/M_B^2) (1 - q^2/M^2)
\]

If we parametrize \( R \) as

\[
 R = \frac{2g_+ (0)}{M} \frac{1}{M^2} \left( 1 - \frac{q^2}{M^2} \right) \frac{1}{M^2} \frac{1}{M_B^2} \frac{1}{M^2} \frac{1}{M_B^2} \frac{1}{M^2}
\]

where \( A \) is a parameter which in principle can be obtained when \( g_{B\gamma} \) and \( f_{B\gamma} \) become known. Then

\[
 F(q^2) = \frac{2g_+ (0)}{M} \frac{1}{M} \left( 1 + \frac{1}{M^2} \frac{1}{M_B^2} \frac{1}{M^2} \frac{1}{M_B^2} \frac{1}{M^2} \right) \left( 1 - \frac{q^2}{M^2} \frac{1}{M_B^2} \frac{1}{M^2} \frac{1}{M_B^2} \frac{1}{M^2} \right)
\]

\[
 (1 - q^2/M_B^2) (1 - q^2/M_B^2) (1 - q^2/M^2)
\]

(70)

For \( M_1 = M_2 \) the above Eq. (70) reduces to Eq. (60). So the couplings of \( B \) with \( B^*\gamma \) and \( B^*\gamma \) become

\[
 g_{B\gamma} = \frac{2g_+ (0) M_B^{B\gamma}}{2} \left( 1 - \frac{1}{M_B^{B\gamma}} \right) \left( 1 - \frac{1}{M_B^{B\gamma}} \right) \left( 1 - \frac{1}{M_B^{B\gamma}} \right) \left( 1 - \frac{1}{M_B^{B\gamma}} \right)
\]

\[
 (1 - q^2/M_B^2) (1 - q^2/M_B^2) (1 - q^2/M^2)
\]

\[
 (1 - q^2/M_B^2) (1 - q^2/M_B^2) (1 - q^2/M^2)
\]

(71)

\[
 f_{B\gamma} \ = \ \frac{f_{BM\gamma}}{f_{B\gamma}} \frac{1}{M_B} \left( 1 - \frac{1}{M_B^{B\gamma}} \right) \left( 1 - \frac{1}{M_B^{B\gamma}} \right) \left( 1 - \frac{1}{M_B^{B\gamma}} \right) \left( 1 - \frac{1}{M_B^{B\gamma}} \right)
\]

\[
 (1 - q^2/M_B^2) (1 - q^2/M_B^2) (1 - q^2/M^2)
\]

(72)

and the corresponding form factors become

\[
 F_V(q^2) = \frac{2g_+ (0)}{M_B} \frac{1}{M_B} \left( 1 + \frac{1}{M_B^{B\gamma}} \right) \left( 1 - \frac{1}{M_B^{B\gamma}} \right) \left( 1 - \frac{1}{M_B^{B\gamma}} \right)
\]

\[
 (1 - q^2/M_B^2) (1 - q^2/M_B^2) (1 - q^2/M^2)
\]

(73)
$$F_A(q^2) = \frac{2g_+(0)}{M_B} \frac{1 - \frac{q^2}{M^2_{B^*_{A_1}}} \left(1 + \left(1 - \frac{M_{B^*_{A_2}}}{M^2_{B^*_{A_2}}} \left(1 - \frac{M_{B^*_{A_2}}}{M^2_{B^*_{A_2}}} \right) A \right) \right)}{\left(1 - \frac{q^2}{M^2_{B^*_{A_2}}} \right) \left(1 - \frac{q^2}{M^2_{B^*_{A_1}}} \right) \left(1 - \frac{q^2}{M^2_{B^*_{A_2}}} \right)} \quad (74)$$

For numerical values we shall use $A = 0$ [i.e., $M_1 = M_2$] and $A = 3$ and $A = 4.8$. The second value of $A (= 3)$ corresponds to estimate of $g_{B^*B\gamma}$ from vector meson dominance

$$g_{B^*B\gamma} = \frac{2}{3} g_{B^*B\rho} \frac{f_{\rho^-}}{m_\rho^2} = 2.76 \text{GeV}^{-1}$$

where $g_{B^*B\rho^-} = \sqrt{2}(11) \text{ GeV}^{-1}$ obtained in [31] and $f_{\rho^-}/m_\rho = 205 \text{ MeV}$. The third value of $A (= 4.8)$ gives more or less the width for $B^* \to B\gamma$ obtained from MI transition in non relativistic quark model (NRQM). These values give decay width for $B^* \to B\gamma$ transition 23 keV, 5.5 keV and 0.8 keV respectively while MI transition in NRQM predicts it to be 0.9 keV. These predictions are testable when above decay width is experimentally measured.

6 Decay distribution

The Dalitz plot density

$$\rho(x, y) = \frac{d^2\Gamma}{dxdy} = \frac{d^2\Gamma_{IB}}{dxdy} + \frac{d^2\Gamma_{SD}}{dxdy} + \frac{d^2\Gamma_{INT}}{dxdy} = \rho_{IB}(x, y) + \rho_{SD}(x, y) + \rho_{INT}(x, y) \quad (75)$$

is a Lorentz invariant which contains the form factors $F_V$ and $F_A$ in the following form [19, 20, 22]

$$\rho_{IB}(x, y) = A_{IB} f_{IB}(x, y)$$
$$\rho_{SD}(x, y) = A_{SD} M_B^2 \left[ (F_V + F_A)^2 f_{SD^+}(x, y) + (F_V - F_A)^2 f_{SD^-}(x, y) \right]$$
$$\rho_{INT}(x, y) = A_{INT} M_B \left[ (F_V + F_A) f_{INT^+}(x, y) + (F_V - F_A) f_{INT^-}(x, y) \right]$$

where

$$f_{IB}(x, y) = \left( \frac{1 - y + r_t}{x^2(x + y - 1 - r_t)} \right)$$
\[
\begin{align*}
\times \left( x^2 + 2 (1 - x) (1 - r_l) - \frac{2 x r_l (1 - r_l)}{(x + y - 1 - r_l)} \right)
\end{align*}
\]

\[
\begin{align*}
f_{SD^+}(x, y) &= (x + y - 1 - r_l) (x + y - 1) (1 - x) - r_l \\
f_{SD^-}(x, y) &= (1 - y + r_l) (1 - x) (1 - y) + r_l \\
f_{INT^+}(x, y) &= \left( \frac{1 - y + r_l}{x(x + y - 1 - r_l)} \right) ((1 - x) (1 - x) + r_l) \\
f_{INT^-}(x, y) &= \left( \frac{1 - y + r_l}{x(x + y - 1 - r_l)} \right) (x^2 - (1 - x) (1 - x) - r_l)
\end{align*}
\]

and

\[
\begin{align*}
A_{IB} &= 4r_l \left( \frac{f_B}{M_B} \right)^2 A_{SD} \\
A_{SD} &= \frac{G_F^2 |V_{ub}|^2 \alpha}{32 \pi^2} M_B^5 \\
A_{INT} &= 4r_l \left( \frac{f_B}{M_B} \right) A_{SD}
\end{align*}
\]

The \(SD^+\) term reaches its maximum at \(x = 2/3, y = 1\), which corresponds to \(\theta_{l\gamma} = \pi\). The \(SD^-\) term reaches its maximum at \(x = 2/3, y = 1/3\), corresponding to \(\theta_{l\gamma} = 0\). Indeed, for lepton of maximal energy (\(y = 1\)), only “right-handed” photons contribute. In this situation, the photon and the neutrino must be emitted in the direction opposite to that of the lepton. Angular momentum conservation forces the photon spin to be opposite to the total lepton spin and the photon helicity has the same sign as that of the lepton. Then the photon and the neutrino are emitted parallel. This configuration corresponds to a neutrino of maximal energy \((E_\nu = E_\nu^{\text{max}}\) when \(x + y = 1\)). In this case, only the “left-handed” photon contributes. When \(x + y = 1\), the \(IB\) contribution becomes very large: this corresponds to \(\theta_{l\gamma} = 0\). Consequently, it is very difficult to distinguish experimentally between the \(IB\) and the \(SD^-\) contribution. To summarize, an experiment performed in the region \(\theta_{l\gamma} \simeq \pi\) is essentially sensitive to \((F_V + F_A)^2\).

The form factors calculated in Eq. (60) can be expressed in terms of dimensionless variable \(x\),

\[
F(x) = \frac{F(0)}{x \left[ 1 - (1 - x) / (M_1/M)^2 \right]}, \quad (76)
\]
where $x$ is defined in Eq. (12) and $q^2$ in Eq. (15). After restoring subscripts, the form factors $F_V (q^2)$ [Eq. (65)] and $F_A (q^2)$[Eq. (66)] can be written as

$$F_V(x) = \frac{F_V(0)}{x \left[ 1 - (1 - x) / \left(M_{B^*} / M_B \right)^2 \right]}$$

(77)

$$F_A(x) = \frac{F_V(0)}{x \left[ 1 - (1 - x) / \left(M_{A^*} / M_B \right)^2 \right]},$$

(78)

where

$$F_{V,A}(0) = \frac{2g_+(0)}{M_B}$$

We use these in Eq. (75) and integrate over $x$ and $y$ in the limit as mentioned in Eqs. (16, 17). IB contribution diverges for the minimum value of $x$, we take an arbitrary lower limit for $x$ i.e. $x_{\text{min}} \approx r_l$ for which the divergence problem is cured and the IB part gives some definite value $O(10^{-20})$. But as the energy of the photon is increased, it approaches zero at $x_{\text{max}}$. Therefore in the total decay width, this does not contribute much. The $SD$ part is the most dominant part of the decay width which provides almost the whole contribution. This part increases initially with increasing $x$, reaches its peak value and then starts decreasing. The $INT$ part of the decay width is an increasingly vanishing contribution and can be neglected in comparison to the $SD$ part, because it is suppressed by $O(10^{-21})$ and becomes flat (approaches zero) as $x$ (the photon energy) approaches 1 (its maxima). Therefore, this does not contribute fairly to the total decay width of the process.

In the Fig. 2, differential decay width of the process is plotted against $x$ and we see that for our calculations, the peak is shifted to lower value of $x$ as compared to those for Eilam et al., [10], Korchemsky et al. [12] and Chełkov et al. [13]. So, for the process $B \to \gamma l \nu_l$ the branching ratios obtained is

$$B(B \to \gamma l \nu_l) = 0.5 \times 10^{-6} \quad (l = \mu)$$

(79)

This value is for the form factors given in Eqs. (77, 78) which are obtained by restricting to the first radial excitation only. Now if we consider the effect of second radial excitation the expression for the form factors are given in Eqs. (73, 74). The branching ratio thus obtained are

$$B(B \to \gamma l \nu_l) = 0.38 \times 10^{-6} \quad (l = \mu, A = 3.0)$$

$$B(B \to \gamma l \nu_l) = 0.32 \times 10^{-6} \quad (l = \mu, A = 4.8)$$

18
for two representative cases of $A = 3$ and $A = 4.8$ respectively. These are not sensitive to the values of $A$ in contrast to the decay width of $B^* \rightarrow B\gamma$. The CLEO collaboration indicate an upper limit on the branching ratio $\mathcal{B}(B^+ \rightarrow \gamma \nu e^+)$ of $2.0 \times 10^{-4}$ at the 90% confidence level [1]. The predicted values are within the upper limit provided by CLEO collaboration but differ from those predicted in [12, 13], namely $(2 - 5) \times 10^{-6}$ and $0.9 \times 10^{-6}$, respectively. The Monte-Carlo simulation results are given in [32] where the upper limit on the branching ratio for this process is predicted to be $5.2 \times 10^{-5}$.

7 Conclusions

Preliminary data from the CLEO Collaboration indicate an upper limit on the branching ratio $\mathcal{B}(B^+ \rightarrow \gamma \nu e^+)$ of $2.0 \times 10^{-4}$ at the 90% confidence level [1]. With the better statistics expected from the upcoming $B$ factories, the observation and experimental study of this decay could become soon feasible. It is therefore of some interest to have a good theoretical control over the theoretical uncertainties affecting the relevant matrix elements.

We have studied $B \rightarrow \gamma \nu l$ decay using dispersion relations, asymptotic behavior of form factors and Ward Identities. The dispersion relation involves ground state $B^*$ and $B^{*}_{A}$ resonances and their radial excitations which model contributions from higher states and continuum contribution, which is calculated from quark triangle graph. The asymptotic behavior of form factors and Ward Identities fix the normalization of the form factors in terms of universal function $g_{+}(0)$ at $q^2 = 0$ and put constraints on the residues. Thus in our approach, a parameterization of $q^2$ dependence of form factors is not approximated by single pole contributions. This parameterization is dictated by considerations mentioned above and also the coupling constants of $1^-(B^*)$ and $1^+(B^*_{A})$ resonances with photon are predicted if we restrict to one radial excitation. By using $\Lambda = 0.4$ GeV$^{-1}$ we have calculated $g_{+}(0) = 0.15$ and predicted the value of $g_{B^*B\gamma} = 5.6$ GeV$^{-1}$ (cf. Eq. (67)) and $f_{B^{*}_{A}B\gamma} = 6.5 f_{B} M_{B^{*}_{A}} / f_{B^*}$ GeV$^{-1}$ (cf. Eq. (69)). Taking into account one radial excitation the form factors are summarized in Eq. (65, 66). Branching ratio for the process is then calculated to be $\mathcal{B}(B \rightarrow \gamma \nu l) = 0.5 \times 10^{-6}$ which lies within the upper limit predicted by CLEO Collaboration at 90% confidence level [1]. Then we study the effect of second radial excitation in terms of a single parameter $A$, which in principle is determined once $g_{B^*B\gamma}$.
and $f_{B^*_A B \gamma}$ are known (cf. Eq. (71, 72)). The resulting form factors are given in Eqs. (73, 74). By using these form factors the branching ratio is $\mathcal{B}(B \to \gamma \nu l) = 0.38 \times 10^{-6}$ and $\mathcal{B}(B \to \gamma \nu l) = 0.32 \times 10^{-6}$ for two representative cases $A = 3.0$ and $A = 4.8$ respectively. These branching ratios are not sensitive to the value of $A$ in contrast to radiative coupling constants which give respectively $B^* \to B \gamma$ width as 23 keV ($A = 0$), 5 keV ($A = 3.0$) and 0.8 keV ($A = 4.8$). One can also predict radiative widths of radial excitation in terms of $B^*$ and $B^*_A$ radiative widths by using relations (36), (39) and (58).

The differential decay width versus photon energy is plotted in Fig. 2 to compare our results with the existing calculations in the light-cone QCD approach [10, 12] and in the instantaneous Bethe-Salpeter approach [13]. The results for $B \to \gamma \nu l$ have been reproduced by using Suddakov resummation [12] and have also been shown graphically. In our calculations as well as in [10], the position of the peak of differential decay width is shifted towards the lower value of photon energy spectrum. This is due to the double pole in the form factors. The overall effect of radial excitations is to soften the $q^2$-behavior of differential decay distribution while in [12] it is due to Suddakov resummation.

Our main inputs have been dispersion relations, asymptotic behavior and Ward Identities, all of which have strong theoretical basis and in these aspects it differs from other approaches. Our approach is closer to the one followed in [18] for $B \to \pi l \nu$. Only external parameters involved are $f_B$, resonance masses (which are determined in potential models) and $g_{B^* B \gamma}$ and $f_{B^*_A B \gamma}$ which are either predicted or on which we have some theoretical information. The radiative widths of radial excitations are predicted in terms of the above coupling constants. Thus our approach has predictive power and can be tested by future experiments.

The experiments at the B-factories, BaBar at SLAC and Belle at KEK (Japan) and the planned hadronic accelerators are capable to measure the branching ratio as low as $10^{-8}$ [33].

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**Figure Captions:**

1. $B \to l\nu\gamma$ radiative leptonic decay diagrams.

23
2. Differential decay rate versus photon energy $x$ is plotted and comparison is given with various approaches. The solid line (for $A = 0$), dashed-trippledotted line (for $A = 3.0$) and dotted line (for $A = 4.8$) are our calculation, dash-dot-dot line [10], dashed line [12] and dash-dotted line [13]. The thin-solid line is the Sudekov resummation calculation result from Ref. [12].
