The Nijmegen Potentials

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Abstract. A review is given of the various Nijmegen potentials. Special attention is given to some of the newest developments, such as the extended soft-core model, the high-quality potentials, and the Nijmegen optical potentials for NN.

1 Introduction

A large part of the efforts of the Nijmegen group in the last decennia has been concentrated on the study of the baryon-baryon [1, 2, 3], as well as the antibaryon-baryon interaction [4, 5]. In first instance this has been the construction of potentials [6, 7, 8, 9, 10], but later also partial-wave analyses (PWA) of the experimental scattering data [11, 12] were performed. The knowledge obtained in these PWA’s was then applied again in the construction of new, improved potentials [13, 14, 15]. This interplay between potential construction and PWA has has turned out to be very fruitful.

We considered extensively the baryon-baryon channels with strangeness S = 0, 1, and 2. The potentials we constructed for the (non-strange) NN-channels were in the beginning all “NN-potentials”. We say here explicitly “NN-potentials”. We mean with this something else then when we say “pp- or np-potentials”. In NN-potentials charge-independence is assumed for the nuclear part of the potential. For the exchanged mesons and for the nucleons averaged iso-multiplet masses are used, such as the average pion mass $m = (2m_\pi + m_0)/3 = 138.4$ MeV, the nucleon mass $M = (M_p + M_n)/2 = 938.93$ MeV, etc. The $I = 1$ part of the NN-potentials was always obtained by fitting the pp-data. Lately we have been constructing pp-, as well as np-potentials. In such potentials the mass differences are properly taken into account. In pp-scattering there is only $\pi^0$-exchange, while in np-scattering one must introduce $\pi^+$- as well as $\pi^0$-exchange.

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In our efforts to describe and understand the inelasticity in $pp$-scattering above the various pion production thresholds, we have studied also the coupled $I = 1$ NN- and NΔ-channels.

In order to describe the elastic $Ap$ scattering below, as well as above the $\Sigma$-production threshold, and the elastic and inelastic $\Sigma p$ scattering, we have been constructing hyperon-nucleon (YN-) potentials [2, 3, 10]. Charge independence breaking was partially taken into account by using the correct $\Sigma$-thresholds, and by introducing explicit $\pi\Lambda\Lambda$-couplings via the mechanism of $\Lambda\Sigma^0$-mixing. At various times also the $Y = 0$ potentials [16, 17, 18] such as $\Lambda\Lambda$, $\Xi N$, $\Sigma\Sigma$, etc. were considered.

The baryon-antibaryon (BB) potentials [4, 15, 5] were constructed to describe the large amount of elastic $\bar{p}p$-scattering data, and the quasi-elastic $\bar{p}p \to \bar{n}n$ charge exchange data. Potentials [13, 20] were also constructed to describe the various strangeness exchange reactions, such as $\bar{p}p \to \Lambda\Lambda$, etc.

All Nijmegen NN-potentials (except the HQ-potentials) were developed with in the back of our minds the extension to the YN-channels. This required treatments which could be generalized to the other channels with the help of SU(3). One needs therefore to include exchanges of all mesons of the same meson nonet. Next to the $\pi$-meson one needs to include the exchanges of $\eta$ and $\eta'$-mesons. Next to the $\rho$- and $\omega$-meson one needs to take account of the $\phi$-meson.

The various Nijmegen potentials can be grouped into several classes. These are

- **Hard core potentials.**
  Important examples are the potentials Nijm D [6, 7] and Nijm F [8].

- **Soft core potentials.**
  We think here of Nijm78 [9] and its update Nijm93 [13].

- **Extended soft core potentials.**

- **High-Quality $pp$- and $np$-potentials.**
  We would like to call a potential a HQ-potential, when compared with the experimental data it has a $\chi^2/N_d < 1.05$. The only examples [13, 14] of this class are Nijm I, Nijm II, and Reid93.

- **Optical potentials.**

We would like to point out here the existence of the NN-OnLine facility of the Nijmegen group [21], which is accessible via the World-Wide Web. There one can obtain the various Nijmegen e-prints, the fortran codes for some of the Nijmegen potentials, the deuteron parameters and the deuteron wave functions, the phases obtained from the Nijmegen PWA and from the Nijmegen potentials, and predictions for many of the experimental quantities. A direct comparison of these predictions with the Nijmegen NN-data base is also possible.
2 Hard Core Potentials

The Nijmegen D potential [6] was one of our first hard core potentials that had an acceptable $\chi^2$ with respect to the experimental NN-data. The extension [7] to the YN-channels did describe the YN-data well. Because of the success of this potential in hypernuclear physics [22] we used this potential also for the construction of BB-potentials [4, 19]. The elastic and charge exchange $\vec{p}p$-scattering data are well described in the Nijmegen coupled-channel model [4]. Our wish to study also the strangeness exchange reaction $\vec{p}p \rightarrow \Lambda\Lambda$ was the main reason for using the Nijmegen D model in these anti-particle reactions, because this Model D had been tested in the YN-channels. The Nijmegen soft-core potential [9] was then already available, but was not tested yet in YN.

It is very interesting to look at some of the parameters of this, now 20 years old, Nijm D potential. The $\pi$NN-coupling constant was determined by fitting to the experimental NN-scattering data from before 1969, using the PWA of the Livermore group [23]. We obtained $f^2/4\pi = 0.074$. This must be compared with the present best value $f^2/4\pi = 0.0748$. This 20 year old result shows, that the recently obtained low value for the $\pi$NN-coupling constant is not due to recent experimental data, but results from our more sophisticated handling of the data. It was unfortunate that at that time in 1975, when we found this low value, we were so brainwashed by the $\pi$N-community in thinking that $f^2/4\pi \simeq 0.080$, that we did not take this result very seriously. It lasted till 1984 before we were convinced that the $\pi$NN-coupling constant was indeed so low [24].

This low value for the $\pi$NN-coupling constant resulted in a very good deuteron in Nijm D. For the $d/s$ ratio we then found $\eta = 0.0251$. This must be compared with the present best value $\eta = 0.0252(1)$. The value $\rho(\varepsilon, -\varepsilon) = 1.776$ fm for the effective range at the deuteron pole must be compared with the present value $\rho(-\varepsilon, -\varepsilon) = 1.764(3)$ fm.

Other consequences of this all are: the pretty good values for the $d$-state probability $p_d = 5.9\%$, and the uncorrected value for the electric quadrupole moment $Q_0 = 0.272$ fm$^2$. These are in good agreement with the most recent guesses $p_d = 5.7(1)\%$ and $Q_0 = 0.271(1)$ fm$^2$.

Another Nijmegen hard core potential that is worth looking at, is the Nijm F model. This model has an NN as well as an YN-version. In fact even an YY-version [17] was constructed, but unfortunately never published. Recent calculations [24] show, that the YN-version of Model F reproduces many of the features in hypernuclear physics.

3 Soft Core Potentials

The Nijmegen soft-core OBE-potential Nijm78 [9] and its updated version Nijm93 [13] are based upon Regge pole theory. The corresponding YN-version [10] was published in 1989. This potential has also been applied to antibaryon-baryon scattering [12, 13, 20], where very good descriptions of the
various reactions, such as elastic scattering, charge exchange scattering, and strangeness exchange scattering, have been obtained.

One of the attractive features of this potential is that the coordinate space version and the momentum space version are exactly equivalent [26]. This at the cost of having only a minimal form of non-locality. In the triton this minimal non-locality has a 100 keV effect on the binding energy [14].

4 The Extended Soft Core Model

We next mention an important improvement on the soft-core OBE-model. Inspired by the chiral quark model, see for example [27], and duality [1, 28, 29], recently there has been constructed the extended soft-core model (ESC model) for the NN interaction. The first results with this ESC model were reported in Refs. [30, 31]. The ESC-model contains, besides soft OBE potentials of [9], also contributions from two-meson exchange diagrams ($\pi\pi$, $\pi\rho$, $\pi\varepsilon$, etc.) [32, 33], and from one-pair and two-pair diagrams. The latter are generated through pair-vertices ($\pi\pi$, $\pi\rho$, $\pi\varepsilon$ etc.) [34]. These meson-pair vertices are, except for a few, all fixed by heavy meson saturation. This way an excellent fit to the NN single energy PWA is achieved with a restricted set of free parameters. This model is still under construction. A preliminary fit is reached with $\chi^2_{p.d.p.} = 1.08$.

5 High Quality Potentials

In the various Nijmegen partial wave analyses [11] of the NN-scattering data we can describe these data with $\chi^2/N_d \simeq 1.0$. This means that with a potential model description this will also be about the limit. A measure for the quality of potentials is therefore the difference with 1.0 for the value of $\chi^2/N_d$. Let us define high quality (HQ) potentials as having $\chi^2/N_d < 1.05$ when compared directly with the experimental data. We constructed in Nijmegen three potentials of this HQ-type [13, 14]. These are the potentials Nijm I, Nijm II, and Reid93. They have all the excellent value $\chi^2/N_d = 1.03$.

Nijm I

The Nijmegen soft-core Nijmegen potential Nijm78 has been the starting point in the construction of the high quality NN-potentials. This soft-core potential gives already a reasonable good description of all partial waves. In each partial wave separately the description can be improved and made excellently, when we allow the parameters of the potential to be adjusted in each partial wave separately. This leads to the Nijm I potential. This potential has a minimal form of non-locality in the central potential only. There

$$V_c(r) = V(r) - \frac{1}{2m}(\Delta V' + V'\Delta)$$

Nijm II and Reid93

The Nijm II potential is similar to the Nijm I potential, but with all non-locality in each partial wave removed, i.e. $V' \equiv 0$. The Reid93 potential is an update
of the old Reid potential (see ref \[13\]). The singularities in this potential at the origin are removed by introducing formfactors. The main difference between the Nijm I and the Reid93 potential is just these formfactors. In the Nijmegen potentials an exponential form factor has been used

$$F(k^2) = \exp(-k^2/\Lambda^2),$$

while in the Reid93 potential we used

$$F(k^2) = (\Lambda^2/(\Lambda^2 + k^2))^2.$$

The presence of form factors is due to the spatial extension of the nucleons and the mesons. Quarkmodels using harmonic oscillator interactions between the quarks will lead quite naturally to exponential form factors \[35\].

6 The Deuteron

It is instructive to compare the tensor potentials of the 3 HQ-potentials. In doing this we must keep in mind that the quality of the description of the NN-data is in all three cases essentially the same. In Fig. 1 we plot the tensor potential connecting the $^3S_1$ and $^3D_1$ partial waves. We see that in the inner region, $r < 1$ fm, these potentials are quite different. In Fig. 2 we plot the deuteron $d$-state wave function $w(r)$. It is remarkable that these wave functions are essentially the same, despite the fact that the tensor potentials are different. In Fig. 3 we plot the deuteron $s$-state wave function $u(r)$. We see again the agreement between these wave functions for large values of $r$ but for values of $r < 0.6$ fm we spot some interesting differences.

There is a fantastic agreement between the values of the deuteron parameters as determined in our PWA’s and the values for the same parameters as given by the HQ-potentials. These deuteron parameters are \[36\]: the $d/s$ ratio $\eta = 0.0252(1)$ and the effective range at the deuteron pole $\rho(-\varepsilon, -\varepsilon) = 1.764(3)$ fm.

![Figure 1](image-url) **Figure 1.** The tensor potential connecting the $^3S_1$ and $^3D_1$ partial waves.
Figure 2. The deuteron $d$-state wave function $w(r)$.

Figure 3. The deuteron $s$-state wave function $u(r)$.
These values follow directly from the scattering data, and can be considered as the best experimental determinations. More surprising is, that the value of the $d$-state probability $p_d = 5.7(1)$% and the value of the uncorrected electric quadrupole moment $Q_0 = 0.271(1)$ fm$^2$ are more or less unique. This value of $Q_0$ does not agree at all with the experimental value $Q = 0.2859(3)$ fm$^2$. The difference $Q - Q_0 = 0.015$ fm$^2$ needs to be explained in terms of meson exchange currents, relativistic effects, contributions of $Q^0$-states, etc.

7 Optical Potentials

Below the threshold(s) for pion-production the NN-potentials are real. When one wants to describe the inelasticities in the scattering above these thresholds, then one has to go to either a complicated coupled channel description or one has to introduce an optical NN-potential: $V = V_R - i V_I$.

The influence and the importance of the imaginary part of the optical NN-potential can clearly be seen in our PWA of the $np$-scattering data below $T_L = 500$ MeV. In Table 7.1 we give for various energy intervals the value of $\chi^2$ obtained in our PWA, and the increase in the value of $\chi^2$ when we omit the imaginary part of the potential.

Table 7.1. The values of $\chi^2$ obtained in our PWA of the $np$ data for various energy intervals. Also given are the number of data in that interval and the increase $\Delta \chi^2$ in the value of $\chi^2$, when we omit the imaginary part of the potential.

| Interval | # data | $\chi^2$ | $\Delta \chi^2$ |
|----------|--------|----------|-----------------|
| 0-350    | 2549   | 2519     | 11              |
| 350-400  | 254    | 306      | 13              |
| 400-450  | 319    | 337      | 105             |
| 450-500  | 866    | 905      | 651             |
| 0-500    | 3988   | 4067     | 880             |

Looking at this Table 7.1 we come to the conclusion that the use of a purely real potential works very well up to $T_L = 400$ MeV, it works well up to $T_L = 450$ MeV, and one can say that it works reasonably up to $T_L = 500$ MeV.

Let us next make from the real HQ-potentials optical potentials by adding to them the same imaginary part as was used in our PWA of the $np$-data below $T_L = 500$ MeV. When we compare now with the experimental $np$-data below 500 MeV we find for the Nijm I and Nijm II optical potentials $\chi^2/\# \text{data} \simeq 2.4$ to 2.45. This is quite a lot larger than the minimum $\chi^2$-values obtained in our PWA of these data. It turns out that especially the $^1F_3$ and $^1D_2$ partial waves are responsible for the big rise in $\chi^2$.

We are looking into the possibility to construct optical potentials that are good up to 1 GeV. In Fig. 4 we give the $^1S_0$ and the $^1D_2$-phase shifts as
determined in a preliminary PWA of all $np$-data below 1 GeV. In the same figures are also plotted the prediction of the optical Nijm I potential. For the $^1S_0$-phase the description is pretty good up to 1 GeV. For the $^1D_2$-wave one notices quite large differences. However, after refitting we get the modified Nijm I optical potential (Nijm I(mod)), which gives a very good fit to the $^1D_2$-phase shift up to 1 GeV. We think that it will be not too difficult to produce an optical potential which fit the $np$-data up to 1 GeV.

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References

1. J.J. de Swart, M.M. Nagels: Fortschritte der Physik 28, 215 (1978)
2. J.J. de Swart, M.M. Nagels, T.A. Rijken, P.A. Verhoeven: Springer Tracts in Modern Physics 60, 138 (1971)
3. J.J. de Swart, W.A. van der Sanden, W. Derks: Nucl. Phys. A 416, 2992 (1984)
4. P.H. Timmers, W.A. van der Sanden, J.J. de Swart: Phys. Rev. D 29, 1928 (1984); Erratum: D 30, 1995 (1984)
5. J.J. de Swart, R. Timmermans: The antibaryon-baryon interactions, Proceedings LEAP94, Bled, Slovenia
6. M.M. Nagels, T.A. Rijken, J.J. de Swart: Phys. Rev. D 12, 744 (1975)
7. M.M. Nagels, T.A. Rijken, J.J. de Swart: Phys. Rev. D 15, 2547 (1977)
8. M.M. Nagels, T.A. Rijken, J.J. de Swart: Phys. Rev. D 20, 1633 (1979)
9. M.M. Nagels, T.A. Rijken, J.J. de Swart: Phys. Rev. D 17, 768 (1978)
10. P.M.M. Maessen, Th.A. Rijken, J.J. de Swart: Phys. Rev. C 40, 2226 (1989)
11. V.G.J. Stoks, R.A.M. Klomp, M.C.M. Rentmeester, J.J. de Swart: Phys. Rev. C 48, 792 (1993)
12. R. Timmermans, Th.A. Rijken, J.J. de Swart: Phys. Rev. C 50, 48 (1994)
13. V.G.J. Stoks, R.A.M. Klomp, C.P.F. Terheggen, J.J. de Swart: Phys. Rev. C 49, 2950 (1994)
14. J.L. Friar, G.L. Payne, V.G.J. Stoks, J.J. de Swart: Phys. Lett. B 311, 4 (1993)
15. R. Timmermans, Antiproton-proton scattering at LEAR energies, Thesis University of Nijmegen, 1991
16. J.J. de Swart, Phys. Lett. 5, 58 (1963)
17. W.M. Macek, M.M. Nagels, J.J. de Swart (1978) unpublished
18. P.M.M. Maessen, Th.A. Rijken, J.J. de Swart (1990) unpublished
19. P.H.A. Timmers, Nucleon-antinucleon interaction, Thesis University of Nijmegen, 1985
20. R.G.E. Timmermans, Th.A. Rijken, J.J. de Swart: Phys. Rev. D 45, 2288 (1992)
21. The URL is: http://NN-OnLine.sci.kun.nl
22. J. Rozynek, J. Dabrowski: Phys. Rev. C 20, 1612 (1979); C.B. Dover, A. Gal: in Progress in Particle and Nuclear Physics, (ed. D. Wilkinson: Pergamon, Oxford, 1984), Vol. 12, pp. 17; H. Bandō, S. Nagata: Prog. Theor. Phys. 67, 522 (1982)
23. M.H. MacGregor, R.A. Arndt, R.M. Wright: Phys. Rev. 182, 1714 (1969)
24. J.J. de Swart, W.A. van der Sanden, W. Derks: Nucl. Phys. A 416, 2992 (1984)
25. Y. Yamamoto, T. Motoba, H. Himeno, K. Ikeda, S. Nagata: Progr. Theor. Phys. Suppl. 117, 361 (1994)
26. Th.A. Rijken, R.A.M. Klomp, J.J. de Swart: A Gift of Prophecy (R.E. Marshak Memorial Volume), World Scientific, 1995
27. J. Schwinger: Phys. Rev. 167, 1432 (1968); S. Weinberg: ibid. 166, 1568 (1968) and 177, 2604 (1969)
28. R. Dolen, D. Horn, C. Schmid: Phys. Rev. 166, 1768 (1968);  
   H. Harari: Phys. Rev. Lett. 20, 1395 (1968);  
   F.J. Gilman, H. Harari, Y. Zarmi: Phys. Rev. Lett. 21, 323 (1968)

29. J.J. de Swart, Th.A. Rijken, P.M. Maessen, R.G. Timmermans: Nuovo Cim. 102A, 203 (1989)

30. Th.A. Rijken, in Proceedings of the XIVth European Conference on Few-Body Problems in Physics, Amsterdam, 1993, edited by B. Bakker and R. von Dantzig, Few-Body Systems, Suppl. 7, 1 (1994)

31. V.G.J. Stoks, Th.A. Rijken: contributed paper to the 14th International Conference on Few-Body Problems, Williamsburg, 1994, edited by F. Gross, p. 193

32. Th.A. Rijken: Ann. Phys. (N.Y.) 208, 253 (1990).

33. Th.A. Rijken, V.G.J. Stoks: (submitted for publication)

34. Th.A. Rijken, V.G.J. Stoks: (submitted for publication)

35. F. Fernandez, E. Oset: Nucl. Phys. A 455, 720 (1986)

36. C.P.F. Terheggen, R.A.M.M. Klomp, M.C.M. Rentmeester, V.G.J. Stoks, J.J. de Swart (in preparation)

37. D.M. Bishop, L.M. Cheung: Phys. Rev. A 20, 381 (1979)

38. R.A.M.M. Klomp, J.J. de Swart (in preparation)