GINZBURG-LANDAU THEORY OF VORTEX PHASE DIAGRAM IN LAYERED TYPE II SUPERCONDUCTOR

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1. INTRODUCTION

Various macroscopic phenomena seen in high temperature superconductors (HTSC) under nonzero magnetic fields have inspired much interest in studying the phase diagram of vortex states of three dimensional (3D) layered type II superconductors. Experimentally, thermodynamic and transport phenomena have been intensively examined mainly in YBCO [1-3] and BSCCO [4]. The resistive broadening [5] in the so-called vortex liquid regime below the crossover line $H_{c2}(T)$ is common to these materials with strong fluctuation effect and is well described in terms of the non-Gaussian superconducting fluctuation theory[6]. Since, on approaching a phase transition, macroscopic behaviors tend to become insensitive to microscopic details of each material, one expects that the phase diagram itself should be qualitatively the same between these materials. However, recent data [7-10] in YBCO seem to have shown some details of transition lines in real systems with pinning disorders which have been unseen in BSCCO. It is important to find how such phenomena apparently dependent on the materials are explained within a single GL theory.

Theoretically, the phase diagram of vortex states has been tackled from two different points of view. In the literature [11-13] based on the elastic theory for the vortex systems with pinning disorder, one first starts from low fields and low temperatures and hence, works in the London (phase-only) limit of the GL model. Consequently, a glassy solid phase, named Bragg-glass (BrG) phase, was proposed as the ground state in cleaner real systems. Then, a melting line of BrG phase is estimated in terms of some kind of Lindemann criterion and is usually identified with a simultaneous destruction of positional and glass (superconducting) orders. Since the positional ordering of field-induced vortices in clean bulk systems is believed to occur through
a first order transition \[14\], however, this picture of glass ordering \[11-13\] does not permit a continuous disappearance of linear resistance and hence, is incompatible with transport phenomena in fields lower than a lower critical point of clean YBCO samples \[7-9\] and in systems with a continuous glass transition \[15\] due to strong correlated (line-like) disorder.

By contrast, the approach \[16-19\] from higher temperatures describes the vanishing of linear resistance according to a superconducting glass ordering proposed by Fisher et al. \[20\] for homogeneous (nongranular) type II superconductors in nonzero fields. In layered materials, this ordering occurs when the glass susceptibility, which is the spatial average of correlation function \[20\]

\[
G_G(md, R) = d \sum_j \int d^2r |< \psi^*_j(r)\psi_{j+m}(r + R) >|^2
\]  

expressed in terms of the pair-field (superconducting order parameter) \(\psi_j(r)\) at \(j\)-th layer, becomes divergent. In eq.(1.1), the angular bracket and the overbar denote, respectively, the thermal and the random averages, and \(d\) is the interlayer spacing. This approach can be formulated \[17-19\] as a natural extension of the nonGaussian superconducting fluctuation theory \[6\] to lower temperature at which the vortex pinning due to structural disorder is not negligible even in clean systems. Previously, this approach was criticized in a review paper \[21\] because it was not easy \[16\] for this approach to justify the phenomenological guess \[20,21\] that, in thermodynamic limit, the vortex solid in the pinning-free case with nonzero vortex flow resistance should be replaced in clean limit of real systems by a vortex glass with zero linear resistance. This obstacle was overcome \[17,22\] at least in high field case by combining properties of the pinning-free Abrikosov solid in 2D limit with the framework of vortex glass fluctuation based on eq.(1.1). Further, since eq.(1.1) is an expression independent of the detail of random average, this approach is easily extended \[19\] for describing continuous glass transitions \[15\] induced by correlated disorder. On the other hand, it is unclear at present to what extent this approach can be extended into the resulting glass phases.

In this article, a theoretical development on the vortex glass transitions based on eq.(1.1) is reviewed. Consistently with eq.(1.1), the GL model for the layered system, i.e., the Lawrence-Doniach (LD) model \[23\], will be used throughout this paper:

\[
\mathcal{H}_{LD} = d \sum_j \int d^2r \left[ \left( \frac{T}{T_c} - 1 \right)|\psi_j|^2 + \xi_0^2 \left( -i \nabla_\perp + \frac{2\pi}{\phi_0} A_{\perp,j} \right)|\psi_j|^2 \right. \\
+ \Gamma^{-1} \left( \frac{\xi_0}{d} \right)^2 |\psi_j - \psi_{j+1} \exp \left( \frac{2\pi d\delta A_{\parallel}}{\phi_0} \right)|^2 + b \frac{1}{2} |\psi_j|^4 \right] 
\]  

(1.2)
with disorder terms
\[
\mathcal{H}_{rp} = d \sum_l \int d^2 r \left[ u_l(r)|\psi_l(r)|^2 + f_l(r)\xi_0^2(\nabla \times j_l)\right],
\]  
(1.3)

where \( \xi_0 \) the in-plane coherence length, \( \phi_0 \) the flux quantum, \( b > 0 \), \( \Gamma \) the mass anisotropy, and the vector indices \( \perp \) and \( \| \) imply the directions, respectively, parallel and perpendicular to the layer plane. Throughout this paper, internal gauge fluctuations except the external disturbance \( \delta A \) are neglected by focusing on the type II limit so that the applied field is given by \( \text{curl} A_{\text{ext}} \) where \( A_{\text{ext}} = A - \delta A \). In eq.(1.3), \( j_l = \psi_l^\ast(-i\nabla + 2\pi A_{\text{ext}}/\phi_0)\psi_l + \text{c.c.} \), and the structural disorder is described by a random potential expressing \( T_{c0} \)-variations, \( u_j(r) \), and a randomness of flux \( f_j(r) \) [18,19]. We note that, in the phase-only limit (\( |\psi| \) const.), the second term of eq.(1.3) expresses the pinning of vortex cores [11,12], while its first term becomes negligible.

This paper is organized as follows. In §2, the phase diagram of real systems in the case with \( B \perp \) layers and, primarily with only point (uncorrelated) disorder, is discussed according to recent works [17-19, 24], which are based on the theoretical findings that, in the pinning-free case, the vortex liquid region of the normal metal phase discontinuously freezes to change into the Abrikosov vortex solid with long-ranged positional order and quasi long-ranged [25] (conventional) phase coherence and that, in real systems with pinning disorder, a static glass ordering defined using eq.(1.1) occurs while the conventional phase coherence remains short-ranged because of a disorder-induced partial destruction [20,21] of positional long-ranged order. The above second statement may be subtle if the BrG phase [11-13] will occur just below the first order freezing of the vortex liquid. In §3, the phase diagram in the case \( B \parallel \) layers of real systems with point disorder is discussed by applying [26,27] the treatment sketched in §2 to this case and is briefly compared with existing data [28,29]. In §4, a relevance of results in §2 to BSCCO in low fields perpendicular to the layers is discussed together with an issue of Hall conductivity near glass transitions.

2. PHASE DIAGRAM IN FIELDS PERPENDICULAR TO LAYERS

First, the model (1.2) will be rewritten within the subspace of the lowest Landau level (LLL) of the \( \psi \)-fluctuation by invoking a high field approximation valid far from a critical region of the normal-Meissner transition at \( T_{c0} \). Further, the 2D case will be considered [22] for a while for our convenience of description. Using a Laudau-gauge and expressing \( \psi \) as \( \psi(r) = \sum_p \phi_p u_p(r) \) in terms of the LLL eigenfunction \( u_p \), the model \( \mathcal{H} \equiv \mathcal{H}_{LD} + \mathcal{H}_{rp} \) takes the
form
\[ H = \sum_p \mu_0 |\phi_p|^2 + \sum_k \left( \frac{b}{2\xi_0^2} v_k |\tilde{\rho}_k|^2 + (u_{-k} + f_{-k} k^2 \xi_0^2) v_k^{1/2} \tilde{\rho}_k \right), \]

where \( h = 2\pi \xi_0^2 B/\phi_0, \mu_0 = -1 + h + T/T_c, \) \( B \) the magnitude of the applied field, \( v_k = \exp(-k^2/(2h)) \), and

\[ \tilde{\rho}_k = \frac{\xi_0}{L} \sum_p \exp(i p k_1/h) \phi^*_{p-k_2/2} \phi_{p+k_2/2} \]

with linear system size \( L \). Since the characteristic microscopic length of a vortex state is the magnetic length \( r_B \equiv (\phi_0/(2\pi B))^{1/2} \), the factor \( k^2 \) of the random-flux term implies that the random potential \( f \) is accompanied by the factor \( h \) and hence that the pinning disorder will be enhanced with increasing field. This trend is also valid in the phase-only model with no \( u \)-potential term and will be valid in general at least in type II limit \([20,21]\). The Gaussian ensemble for the \( u \)-potential will be assumed:

\[ u(r) u(r') = \Delta \delta^{(2)}(r - r'). \]

For just simplicity of our presentation, the \( f \)-potential term will be omitted for a while.

First, let us consider, as typical quantities appearing even in the pinning-free case, the pairing entropy density (pair-field propagator)

\[ < |\phi_p|^2 > = N_v^{-1} \int d^2 r < |\psi|^2 > \]

and the Abrikosov factor

\[ \beta_A = \frac{2\pi}{h} N_v \left( \int d^2 r < |\psi|^2 > \right)^{-2} \int d^2 r < |\psi|^4 >, \]

where \( N_v = L^2/(2\pi r_B^2) \) is the number of vortices. The former satisfies a Dyson equation

\[ \mu = \frac{k_B T}{< |\phi_p|^2 >} = \mu_0 + \mu x (\beta_A - \Delta_{\text{eff}}(T)(\beta_A - 1)), \]

where \( x = k_B T \mu h/(2\pi \xi_0^2 d\mu^2) \), and \( \Delta_{\text{eff}}(T) = \Delta d \xi_0^2/k_B T b \) is the pinning strength relative to the thermal fluctuation strength. The Abrikosov factor [30] is expressed in the form

\[ \beta_A = 1 + N_v^{-1} \sum_k v_k (1 - 2 x V_k), \]

where \( V_k \) is the fully-renormalized four-point vertex corresponding to the bare one \( v_k \) and also depends on \( \Delta_{\text{eff}} \). If the freezing to the solid is of first
order, a precursor of the positional ordering of vortices (zero points of $\psi$) will appear only in the vicinity of the transition. Then, sufficiently above the transition, $V_k$ will take a form of RPA type such as

$$V_{k}^{\text{liq}} \simeq \frac{\psi_k}{1 + 2x \psi_k}. \tag{2.7}$$

On the other hand, one will notice by comparing eq.(2.6) with its mean field expression [30] that $V_k$ in the limit of a perfect solid takes the form

$$V_{k}^{\text{sol}} = V_{k}^{\text{liq}}(x \gg 1) + \delta V_k = \frac{1}{2x} \left(1 - N_v \sum_{G \neq 0} \delta_k, G\right), \tag{2.8}$$

where $G$'s are the reciprocal lattice vectors of the vortex solid. Note that the first term corresponds to the $x \to \infty$ (i.e., low $T$) limit of $V_{k}^{\text{liq}}$, while the second term $\delta V_k$ corresponds to the structure factor of a vortex state. In a solid-like vortex state with a long but finite positional correlation length, $\delta V_k$ near $G$ will take, for instance, a Gaussian form like

$$\delta V_k \sim -\frac{1}{2x \epsilon} \exp \left(-\frac{(k - G)^2}{2\epsilon h}\right), \tag{2.9}$$

where $\epsilon^{-1} (\gg 1)$ corresponds to the positional correlation area $N_{\text{cor}}$. One can verify by substituting eq.(2.9) into eq.(2.6) that the $\beta_A$-value does not depend remarkably on $N_{\text{cor}}$ at least at low enough $T$. In addition, we note that, at low $T$, eq.(2.5) reduces to the mean field result

$$<|\psi|^2>_{sp} = -(b \beta_A)^{-1} \mu_0, \tag{2.10}$$

where $< >_{sp}$ denotes space average, and the small $O(\Delta_{\text{eff}})$ correction to $\beta_A$ was neglected.

Now, let us turn to the glass susceptibility in 2D, which is expressed within LLL in the form

$$\chi_G = N_v^{-1} \left< |\phi_p|^2 \right> - 2 \sum_{p,p'} \left< \phi_p \phi_{p'}^\ast \right>^2. \tag{2.11}$$

In clean limit, $\chi_G$ is given as a ladder-series of the irreducible vertex represented in Fig.1, i.e.,

$$\chi_G = 1 + I_{\text{irr}} + I_{\text{irr}}^2 + I_{\text{irr}}^3 + \cdots, \tag{2.12}$$

where

$$I_{\text{irr}} = \frac{\Delta h}{2N_v \mu^2} \sum_k \psi_k (1 - 2x V_k)^2. \tag{2.13}$$
Far above the freezing transition where $V_k$ is dominated by the RPA term (2.7), we obtain $I_{\text{irr}} \simeq \Delta_{\text{eff}} / 2(1 - O(x^{-1}))$. Since this expression is insensitive to $B$, one may conclude the absence [31] of 2D glass transition even at the mean field level. However, once the quantum superconducting fluctuation is taken into account, a glass transition line at the mean field level can exist above a (if any) first order transition line [32].

By contrast, when taking account of $\delta V_k$ illustrated by eq.(2.9) which becomes rather remarkable in the vicinity of the (if any) first order line, one obtains a result suggestive of a glass transition [17,22] induced by the vortex solidification (i.e., by the first order transition).

For instance, if substituting the expression of the perfect solid eq.(2.8) into eq.(2.13), one finds

$$I_{\text{irr}} = \frac{\Delta h}{2\pi \mu^2} \left( \beta_A - 1 \right) N_v$$

(2.14)

proportional to the total number $N_v$ of vortices. The factor $\beta_A - 1$ $(> 0)$ implies that a spatial variation of $|\psi|$ due to the vortices is crucial in obtaining a glass ordering at or below the (if any) first order line. The factor $N_v$ is a consequence of the assumption of a perfect solid in the case with a small but finite $\Delta$ and must be replaced by the positional correlation area $N_{\text{cor}}(\Delta_{\text{eff}})$ in terms of, say, eq(2.9). Note that the origin of the large factor $N_v$ or $N_{\text{cor}}$ is the vertex correction to the impurity line (the semicircles in Fig.1). If, as an estimation of $N_{\text{cor}}$, identifying it with the correlation area resulting from the collective pinning theory [33], we find $N_{\text{cor}} \simeq (\Delta_{\text{eff}})^{-1}$. Further, since the mean field glass transition line $T_{\text{mf}}^{(2d)}(B)$ in clean limit will lie just below the freezing transition, or crossover, line $T_{\text{m}}^{(2d)}(B)$ in 2D LLL, $\mu^2$ in eq.(2.13) may be replaced by its value at $T_{\text{m}}^{(2d)}(B) 10^{-2}h b k_B (2\pi \xi_0^2 d)^{-1}$. Then, since the $\beta_A$-value is insensitive to material parameters, the resulting $I_{\text{irr}}$ is almost independent of material and physical parameters. At least, it does not vanish in clean limit ($\Delta_{\text{eff}} \to 0$), implying that the disorder-free theory cannot be used even in clean limit below the expected $T_{\text{m}}^{(2d)}(B)$.

Next, let us extend the above analysis to 3D case in which the glass transition will occur more easily. For a while, the case of point disorder will be considered in which the random potentials satisfy $u_j(r)u_l(r') = d^{-1}\Delta(\rho)\delta(\rho)(r - r')\delta_{j,l}$ and $f_j(r)f_l(r') = d^{-1}\Delta(\rho)\delta(\rho)(r - r')\delta_{j,l}$. Roughly speaking, $\chi_G$ in 3D case is given by replacing $\Delta/\mu^2$ in eq.(2.13) by $\Delta(\rho)/\mu^{3/2}$ when, for simplicity, neglecting a $\Delta\Phi$ term. If, as in 2D case, $N_{\text{cor}}$ is identified with the dimensionless positional correlation area perpendicular to $B$ found in the collective pinning theory (in type II limit) [33], we obtain

$$I_{\text{irr}} \simeq \Delta_{\text{eff}}(\beta_A - 1) \exp(c_1 \Delta_{\text{eff}}^{-1})$$

(2.15)
along the first order line expected in LLL (see eq.(2.17) below), where \( c_1 \) is a positive constant. Since eq.(2.15) is divergent in \( \Delta_{\text{eff}} \to 0 \) limit, it implies that the glass transition due to point disorder, the so-called vortex-glass (VG) transition, will occur more suddenly in cleaner systems with smaller \( \Delta_{\text{eff}} \) and that the first order transition in clean limit should be a glass transition simultaneously. Further, the exponential \( \Delta_{\text{eff}} \)-dependence of \( N_{\text{cor}} \) suggests that this pinning dependence is stronger than the prefactor \( \Delta_{\text{eff}} \) in eq.(2.15). Since \( N_{\text{cor}} \) will decrease down to a constant of order unity with increasing \( \Delta_{\text{eff}} \), and \( \beta_A \) depends only weakly on \( \Delta_{\text{eff}} \), it is expected that \( I_{\text{irr}} \) monotonically decreases with increasing \( \Delta_{\text{eff}} \) down to a value \( \approx \Delta^{(p)} h(\beta_A - 1)/(2\pi \mu^{3/2}) \). Namely, the resulting VG transition line is expected to deviate from the first order line to lower temperature with increasing \( \Delta_{\text{eff}} \) and, in high \( \Delta_{\text{eff}} \) limit, approach \( B_{\infty_{\text{VG}}}(T) \approx H_{c2}(0)(-\mu_0/\theta_f(T))^{3/2}(\Delta^{(p)})^{1/2} \). (2.16)

Although this line [16] satisfies the LLL scaling \( B \sim (T_c(B) - T)^{3/2} \) as well as the first order line [34] in 3D and LLL

\[ B_{m}(T) \approx H_{c2}(0)(-\mu_0)^{3/2}/\theta_f(T), \]  

the dependences on the (anisotropic) 3D fluctuation strength \( \theta_f = b k_B T \sqrt{T}/\xi_0^3 \) of \( B_{\infty_{\text{VG}}} \) and \( B_{m} \) are different from each other.

Through the above findings on the glass transition at or below \( B_{m}(T) \), a picture on the phase diagram above a lower critical point \( B_{\text{lc}} \) (defined below) of the thermal first order transition is easily obtained [17]. First, since most of the above expressions depend not on \( \Delta^{(p)} \) but on the relative pinning strength \( \Delta_{\text{eff}}(T) \propto 1/T \), a cooling along \( B_{m}(T) \) (i.e., an increase of \( B \)) will imply an effective enhancement of random pinning effect. Further, as mentioned below eq.(2.2), an inclusion of nonzero \( \Delta_{\Phi} \) additionally induces a pinning effect enhanced by an increase of \( B \) [18,19]. For these reasons, the first order transition along \( B_{m}(T) \) will be weakened with increasing \( B \) and will disappear at some upper critical point \( B_{ucp} \). Then, it is clear that the vortex state just below the thermal first order transition and below \( B_{ucp} \) need not have a positional order. Actually, as explained above, the glass transition line begins to deviate from \( B_{m}(T) \) to lower temperature with increasing \( \Delta_{\text{eff}}(T) \) or \( B \) and approaches \( B_{\infty_{\text{VG}}}(T) \) at high enough fields. Note that \( B_{\infty_{\text{VG}}}(T) \) decreases with reducing the pinning strength and, as suggested below eq.(2.17), decreases more rapidly than \( B_{m}(T) \) with increasing \( \theta_f \). Hence, there is a possibility [17] of a wider window of the so-called vortex slush regime [35] in cleaner
systems as far as it is not masked by the BrG phase which may exist at lower fields (see Fig.3 below).

Of course, the vortex slush regime is a part of the vortex liquid region and hence, of the normal metal phase because the linear resistance is finite there. In the present theory, the in-plane resistivity \( \rho_{xx} \) (\( = \rho_{yy} \)) in this regime vanishes algebraically on approaching the glass transition, as in the strong disordered case, but with a field-dependent smaller exponent [18]. To show this, let us briefly explain how to evaluate the conductivity near a VG transition. As accepted even through the studies [6] at higher temperatures, the conductivity \( \sigma_{ij} \) may be separated into the quasiparticle part \( \sigma_{n,ij} \) and the superconducting fluctuation part \( \sigma_{s,ij} \), and, deep in the liquid regime of clean systems, \( \sigma_{s,ij} \) can be expressed as a sum of the pinning-free contribution \( \sigma_{F,ij} \) and the glass fluctuation part \( \sigma_{G,ij} \) so that \( \sigma_{ij} \simeq \sigma_{n,ij} + \sigma_{F,ij} + \sigma_{G,ij} \) [17]. The dynamics of \( \psi \)-field is incorporated according to the TDGL equation, or equivalently the quantum TDGL action [17,36]

\[
\frac{S_{\text{QLD}}}{\hbar} = d \sum_j \int d^2r \beta \sum_\omega (\gamma_1 |\omega| + i \gamma_2 |\omega|) |\psi_j(r,\omega)|^2 + \int_0^{\hbar/\beta} \frac{d\tau}{\hbar} [\mathcal{H}_{\text{LD}}(\psi \rightarrow \psi(\tau))
\]

\[
+ \mathcal{H}_{\text{rp}}(\psi \rightarrow \psi(\tau))], \tag{2.18}
\]

for the LD model, where \( \beta = 1/(k_B T) \), \( \gamma_1 > 0 \), and \( \omega \) denotes Matsubara frequency. As first found in Ref.37, the vortex flow conductivities

\[
\sigma_{F,xx} = R_q^{-1} \frac{\gamma_1 \phi_0}{b B} (-\mu_0) \tag{2.19}
\]

and \( \sigma_{F,xy} = \gamma_2 \sigma_{F,xx} / \gamma_1 \) are obtained as the low \( T \) limit of the renormalized Aslamasov-Larkin (AL) fluctuation conductivities [38], where \( R_q \) is the resistance quantum \( \pi \hbar/2e^2 \). According to eq.(4.1) of ref.6, the corresponding diagonal AL conductivity parallel to \( B \) has the following low \( T \) form obeying the LLL scaling

\[
\sigma_{F,zz} \simeq \sigma_{F,xx} \left( -\frac{2\pi \xi_0^2 r_B \mu_0}{bk_B T^3} \right)^2 \propto \frac{(T_c(B) - T)^3}{(Bk_B T)^2}. \tag{2.20}
\]

The same relation was derived later in ref.39 in terms of the phase-only model. It suggests that the behavior (2.20) of \( \rho_{zz} \) is also valid in the liquid regime in lower fields.

Below, we focus on the glass fluctuation term \( \sigma_{G,xx} \). The Feynman diagrams expressing \( \sigma_{G,xx} \) are illustrated in Fig.2. In weak enough pinning case, the resistive vanishing can be described just by Fig.2 (a), while Fig.2 (b) becomes necessary in order to derive the universal VG scaling.
Actually, assuming the glass transition to be continuous, Fig.2 (a) gives [17]

$$\sigma_{G,xx}^{(a)} \sim (T - T_{VG})^{(3-z)\nu}, \tag{2.21}$$

while the diagrams such as Fig.2 (b) result in the scaling behavior [20] argued by Fisher et al.

$$\sigma_{G,xx}^{(b)} \sim (T - T_{VG})^{(1-z)\nu}, \tag{2.22}$$

where $T_{VG}(B)$ is the VG transition line (in type II limit), and $z (> 4)$ and $\nu$ are, respectively, dynamical exponent and the exponent of correlation length $\xi_{VG}(T)$ in VG critical region (in type II limit). $\sigma_{G,xx}$ is given by a sum of eqs. (2.21) and (2.22). A crossover between the behaviors (2.21) and (2.22) occurs when [18]

$$\xi_{VG}(T) \sim L_{cr}(B) = \sqrt{\frac{\phi_0}{B \Delta_{eff}(T)}} \left(1 + \frac{\Delta^{(p)}}{4h^2\Delta_\Phi}\right). \tag{2.23}$$

Namely, if $L_{cr}(B)$ is beyond the system size, the behavior (2.21) may be seen like a true critical behavior in type II limit in cleaner systems, and, in a clean sample, the exponent of vanishing resistivity is $B$-dependent and will increase from $\nu(z - 3)$ to $\nu(z - 1)$ with increasing $B$ in an apparently continuous manner [18]. In fact, a $B$-dependent exponent was observed in the vortex slush regime of a YBCO sample, and the expected universal behavior (2.22) seems to have been found in higher fields than $B_{ucp}$ [40]. A similar behavior was observed previously in other experiments: the resistivity exponent in a moderately disordered sample was $B$-dependent and smaller than the expected one (2.22) [41], while a $B$-independent scaling behavior has been observed in dirtier samples [42]. This expectation [18] of an algebraic and $B$-dependent scaling of resistance in the vortex slush regime is different from the thermally-activated vanishing in the slush regime argued by Worthington et al. [35].

So far, our discussion has been limited to the field range in which the VG transition occurs (at or) below the first order line or its extrapolation to higher fields than $B_{ucp}$. Since, as mentioned above, the pinning disorder becomes effectively weaker in lower fields, it may be natural to expect the first order transition not to terminate at lower fields. However, the thermal first order transition should not occur any longer in fields where a glass transition line exists above $T_m(B)$, and a lower critical point $B_{lcp}$ of the first order line should appear if the glass transition lies above $B_m(T)$ in lower fields [19].

Actually, this situation is realized even at weak disorder in the present LLL approach which may be qualitatively valid far above the $H_{c1}(T)$. To
see this, let us first consider the case with only line-like disorder parallel to $B$ [19,43], defined by $u_j(r)u_l(r') = \Delta^{(l)}(r-r')$ and a similar one for the $f$-potential. The resulting glass transition due to such correlated defects $\parallel B$ is called in the literature the Bose-glass (BG) transition [44]. Assuming the BG transition to occur above $B_m(T)$, the $\delta V_k$-contribution to the vertex correction to the impurity line, carrying $\Delta^{(l)}$ in this case, in Fig.1 can be neglected, and the vertex correction consists only of the RPA term (a 3D version of eq.(2.7)). Then, $I_{irr}$ is easily obtained in the lowest order in $\Delta^{(l)}$, and a BG-line results in. When $l_{ph} = \xi_0/\sqrt{\Gamma \mu}$ (the usual phase coherence length $\parallel B$) $\gg d$, it is expressed as

$$B_{BG}(T) \simeq H_{c2}(0)\frac{\Delta^{(l)}}{(\theta_f(T))^2}(-\mu_0), \quad (2.24)$$

which is linear in $T_{c0} - T$ near $T_{c0}$, as observed in ref.15. Since $B_m(T)$ (2.17) has a vanishing curvature near $T_{c0}$, one finds that the resistivities vanish on cooling continuously on $B_{BG}(T)$ above $B_m(T)$ in $B < B_{icp}^{(l)}$, where

$$B_{icp}^{(l)} \simeq \frac{(\Delta^{(l)})^3}{(\theta_f(T))^4} H_{c2}(0), \quad (2.25)$$

increasing with reducing the fluctuation or enhancing the pinning. Accompanying this BG transition, not only the diagonal conductivities $\sigma_{xx}$ and $\sigma_{zz}$ but also the tilt modulus (the diamagnetic susceptibility to a transverse field $\perp B$) show a critical divergence, implying the presence of a transverse Meissner effect in the BG phase [44]. The details of calculations of these response quantities near a BG transition will not be given here and can be found in ref.19 together with the corresponding results in the Gaussian splayed-glass transitions [45].

Interestingly, the corresponding situation with $B_{VG}(T) > B_m(T)$ also occurs in the case [24] with only point disorder. Under the same assumption as that used above for $B_{BG}(T)$, we find a $B_{VG}(T)$-line [17]

$$B_{VG}(T) \simeq \left(\frac{\Delta^{(p)}}{\theta_f(T)}\right)^2 B_{dc}(T) = H_{c2}(0)\frac{\xi_0(\Delta^{(p)})^2}{\sqrt{\Gamma(\theta_f(T))^3}}(-\mu_0) \quad (2.26)$$

linear in $-\mu_0$, where $B_{dc}(T)$ is the so-called decoupling crossover line [34]. In this case, the resulting lower critical point $B_{lcp}^{(p)}$

$$B_{lcp}^{(p)} \sim 10^{-2}H_{c2}(0) \frac{\xi_0}{\sqrt{\Gamma d}} \frac{(\Delta^{(p)})^6}{(\theta_f(T))^7}, \quad (2.27)$$
is usually much smaller than but has similar dependences on the pinning and fluctuation strengths to the corresponding $B_{\text{lcp}}^{(l)}$. We note that the above expression was derived in the lowest order in $\Delta^{(p)}$. An inclusion of the next order contribution to $I_{\text{irr}}$ tends to increase $B_{\text{lcp}}^{(p)}$-value, although the resulting dependences on the pinning and fluctuation strengths become complicated [46]. Hence, eq. (2.27) should be seen as a lower limit of the expected lower critical point. In any case, the resistivities in $B < B_{\text{lcp}}^{(p)}$ are expected to vanish continuously at the second order VG transition, and the thermal first order transition should not occur in these low fields where $B_m(T)$ lies in the glass phase. It is important to note that, as clear from the above discussion, a lower critical point is not due to an enhancement of pinning effect accompanying a lowering of the field: A lower critical point was observed in a couple of experiments [7-9] in tesla range of YBCO clean samples where the type II limit neglecting fluctuations of flux density is safely valid. It is theoretically difficult to expect a pinning-enhancement accompanying a field-lowering in type II limit. Actually, for this reason, the appearance of a lower critical point was not predicted from treatments based on the vortex elasticity.

As a test of the present explanation [17,24] on the existence of the vortex slush regime and of $B_{\text{lcp}}$, it is interesting to compare the above results with the oxygen-deficiency dependence [9] of YBCO phase diagram. As systematically examined by Nishizaki et al., both the upper and lower critical points of first order line tend to decrease with underdoping [9,10]. It is well known at present through the doping dependences of penetration depth [47] and heat capacity jump [48] that the thermal superconducting fluctuation is enhanced with underdoping. On the other hand, effects of point disorder due to oxygen deficiency also become more remarkable with underdoping. Namely, both $\theta_f(T_c0)$ and $\Delta^{(p)}$ increase with underdoping. It is natural to interpret the doping dependence of $B_{\text{ucp}}$ as being due to a $\Delta_{\text{eff}}(T) = \Delta^{(p)}/\theta_f(T)$-increase (i.e., a relative enhancement of pinning) with underdoping. However, additional dependences on the anisotropy $\Gamma$ and on $\theta_f$ of $B_{\text{lcp}}^{(p)}$ (2.27) imply that $B_{\text{lcp}}^{(p)}$ can decrease with increasing $\Delta_{\text{eff}}$, because $\Gamma$ remarkably increases with underdoping [48]. An explanation of other experimental findings on the lower critical points was given in ref.24. Now, we are in a position of discussing possible phase diagrams under $B_{\perp}$ layers, which are described in Fig.3 [24]. Before explaining Fig.3, we need to mention a bit about results of the elastic approach. As emphasized in, for instance, ref.12, the melting line of the BrG phase should be of first order in general. However, the superconducting transition observed in $B < B_{\text{lcp}}$ is of second order, and it is difficult to explain the presence of $B_{\text{lcp}}$ consistently with the argument favoring the BrG phase. As far as the lower critical point is unrelated to the BrG melting, there is no
reason why all of the thermal first order line in $B_{lc}<B<B_{uc}$ is included in the BrG melting line. Through some consideration including the above statements, we have concluded that a generic 3D phase diagram in $B$ perpendicular layers with intermediate strengths of fluctuation and pinning will be of the form Fig.3 (a), in which the BrG melting is separated from the thermal first order transition occurring along $B_m(T)$, and the vortex slush regime exists entirely in $B_{lc}<B<B_{uc}$. An evidence of a BrG melting line lying much below $B_{VG}(T)$ was recently found in data of NbSe$_2$ [49] and (K, Ba)BiO$_3$ [50]. However, in HTSC with strong fluctuation, the thermal first order line may be pushed down to lower temperature and, in part, merge the BrG melting line. Then, the only possible phase diagram will be of the form Fig.3 (b), in which the glass phase just below the first order transition in $B_{lc}<B<B^*$ is BrG, and the vortex slush regime exists only in $B^*<B<B_{uc}$. A recent magnetization measurement in heavily overdoped YBCO [51] seems to have shown the presence of the BrG melting line in $B<B_{lc}$ approaching $T_c$ with decreasing $B$ just like what we expect through Fig.3 (b). The dashed curves in both Fig.3 (a) and (b) indicate $B_{xG}(T)$. In passing, we note that the phase diagrams given in Fig.3 are valid even for real systems including weak line-like disorder, although in this case the glass phases have the transverse Meissner effect, and the vortex slush regime is much narrower [24] than in the case with no line disorder.

3. PHASE DIAGRAM IN FIELDS PARALLEL TO LAYERS

In this section, we briefly explain results found in LLL approach to the phase diagram of model (1.1) in $B \parallel$ layers. In this field configuration, the field strength is usually measured by the combination [26,52]

$$p \equiv \frac{2\pi d^2}{\phi_0} \sqrt{\Gamma} B,$$

as far as the relation $\xi_0 < \sqrt{\Gamma d}/\sqrt{2}$ is satisfied. Below, we primarily focus on the strong field region satisfying $\exp(-p) \ll 1$ in such a layered system. In such high fields, the mean field transition line $T_c(B)$ approaches a limiting behavior [53]

$$T_c(B) \to T_{c0} \left(1 - \frac{2\xi_0^2}{\Gamma d^2} \right)$$

independent of $p$, and the action (2.18) written in terms of LLL modes takes the simple form [26,27]

$$\frac{S_{QLD}}{\hbar} = \beta \sum_Q \sum_\omega \gamma_1 |\omega||\phi_\omega(Q)|^2 + \int_0^{\beta} d\tau \sum_Q \left[ \left( \mu_0 + \xi_0^2 \sum_{\mu=x,y} \left( q_\mu + \frac{2\pi}{\phi_0} \delta A_\mu(\tau) \right)^2 \right) \frac{r_\beta}{\hbar} \sum_\omega \sum_\xi_0 \right]$$
\[ \times |\phi(Q, \tau)|^2 + \frac{b}{2dL^2} \sum_{Q_1, Q_2, Q_3} V_0(n_1 - n_3, n_2 - n_3; q_{y,i}) \]
\[ \times \phi^*(Q_1, \tau)\phi^*(Q_2, \tau)\phi(Q_3, \tau)\phi(Q_1 + Q_2 - Q_3, \tau) \] (3.3)

where \( \phi_\omega \) is the Fourier transform of the LLL fluctuation field \( \phi(\tau) \), \( Q_j = q_{x,j}\hat{x} + Q_{y,j} \hat{y}, Q_j = q_{y,j} + \tau B d n_j \) with integer \( n_j \), \( \mu_0 = (T - T_c(B))/T_{c0} \) with eq.(3.2), and the disorder energy term \( \mathcal{H}_{rp} \) was dropped for convenience of our presentation. The bare vertex \( V_0 \) is an even function [26] of \( n_1 - n_3 \) and \( n_2 - n_3 \), implying that the partial LLL degeneracy of degree \( N_d = L/d \) is measured by \( n_j \). Further, the gauge disturbance \( \delta A \) necessary in deriving a conductivity at a uniform current was assumed to be spatially uniform. The corresponding action useful in deriving tilt modulus can be seen in ref.27.

In the disorder-free case, the only true transition is argued again to be a first order transition at \( T_m(B) \) between a vortex solid and a (narrow) vortex liquid regime below \( T_c(B) \) [26]. In low fields, \( T_m(B) \) is close to the corresponding one of the anisotropic 3D GL model, which is given by eq.(2.17) with \( \Gamma \) replaced by 1, while it approaches, as well as \( T_c(B) \), a \( p \)-independent value in large \( p \) limit [26,27]:

\[ T_m(p \gg 1) \simeq T_c(B) \left( 1 + c_m \theta_f^{(2d)}(T_{c0}) \right)^{-1} \] (3.4)

with eq.(3.2), where \( \theta_f^{(2d)}(T) = b k_B T/(2\pi\xi_0^2 d) \) is the fluctuation strength in 2D, and a constant of order unity \( c_m > 0 \) has not been determined analytically. The fact that \( T_m(B) \) becomes independent of \( p \) for high \( p \) values is a reflection of confinement of vortices between all interlayer spacings. Because an increase of \( p \) in \( p > 1 \) does not delocalize the vortices out of interlayer spacing any longer but just compresses each vortex row along the layers, the spatial variation of \( |\psi| \) on the superconducting layers diminishes with increasing \( p \). Namely, since the Abrikosov factor \( \beta_A \), eq.(2.4), approaches 1.0 with increasing \( p > 1 \) irrespective of the vortex lattice structure at lower temperatures, the first order transition becomes significantly weaker with increasing \( p \). In the simulation of 3D XY model [54] where \( T_c(B) \) is always identical with \( T_{c0} \), a similar melting line insensitive to \( p \) in \( p > 1 \) was detected from heat capacity data.

However, the high \( p > 1 \) portion of the melting transition was argued there not to be weakened first order mentioned above but to be continuous. This controversy may not be resolved by real experiments because, as discussed below, there is a reason why the disorder-free melting transition in higher \( p \) should be easily destroyed by disorder existing in real systems [27].
Here we merely note that $T_m$ of eq.(3.4) decreases with increasing the fluctuation strength $\theta_f^{(2d)}(T_{c0})$ or with decreasing the anisotropy $\Gamma$. The latter dependence seems to become dominant in the doping dependence in YBCO according to resistivity data in ref.55.

Below $T_m(B)$ of the *disorder-free* system, a Josephson-vortex-solid, i.e., a solid phase pinned by the layer structure, is created. This low $T$ phase has finite helicity moduli and thus, zero resistance for any direction on the layer, while it has a nonzero vortex flow resistance guaranteed by zero helicity modulus in the perpendicular direction to the layers. Consequently, there is a transverse Meissner effect for a tilt perpendicular to the layers but not for any tilt parallel to the layers. With a slight change of the $p$-value, a structural transition possibly mediated by a *unpinned* solid occurs between different pinned solids and is reflected on the $T_m(B)$ line in intermediated fields as its oscillating $B$-dependence [26]. This oscillating behavior has been suggested in YBCO data [55,56]. However, a description of $p$-dependences of such consecutive structure transitions is highly complicated and will not be given here.

The isotropic form of the gradient terms in eq.(3.3) within the layers (in $x$-$y$ plane) leads to a key insight on the physical picture in the liquid regime. It implies that, for high enough $p$ values, the linear responses, such as the resistance, measured along the layers are independent of the relative direction between $\delta A$ (i.e., the current) and $B$. This is the essence of the so-called in-plane Lorentz force-free behavior observed [57,58] in tesla range of BSCCO where $p > 1$ is safely valid. Simultaneously, the in-plane isotropic form of gradients implies that, on cooling, the phase coherence lengths grow isotropically on the layers, while, as in the case $B \perp$ layers [34], the phase coherence perpendicular to the layers above $T_m$ is not sensitive to cooling and remains microscopic as a result of the (partial) LLL degeneracy. Since the phase correlation must be compatible with the positional correlation of vortices [25,26], the above-mentioned anisotropy appearing in the phase correlation must be also satisfied by the positional correlation. Hence, the observed in-plane Lorentz-force free behavior proves that, above $T_m$, the positional correlation first grows along the layers rather than across the layers, which is an opposite trend to an argument favoring a vortex smectic liquid [59]. Namely, the observation [57,58] is incompatible with assuming the intermediate phase [59].

It is not easy to describe in details the glass transition due to point disorder in this case $B \parallel$ layers. In ref.27, a high field behavior of glass transition line, corresponding to eq.(2.16) in $B \perp$ layers, was found and its interpolated behavior to lower fields was conjectured.
Applying a similar analysis to that leading to eq.(2.14) to the present case, the irreducible vertex \( I_{\text{irr}} \) of \( \chi_G = (1 - I_{\text{irr}})^{-1} \), when \( N_{\text{cor}} \sim O(1) \) and \( e^{-p} \ll 1 \), is found to have the form

\[
I_{\text{irr}} \simeq \frac{\Delta}{2\pi \mu} \exp(-2p),
\]  

(3.5)

where \( \mu \) in the present case satisfies

\[
\mu \simeq \exp\left(\frac{2\mu_0}{\theta_f^{(2d)}(T)}\right)
\]

(3.6)

which is identical with eq.(2.10), and the random flux \( \Delta \Phi \) was again neglected. The \( e^{-p} \)-dependence of eq.(3.5) is a reflection of \( \beta_A - 1 \sim e^{-p} \), i.e., of a weak spatial variation of \( |\psi| \) and is also related to an exponentially small shear modulus of the pinning-free solid [60,26], while the exponential \( T \)-dependence in eq.(3.6) is a reflection of a 2D-like superconducting fluctuation weakened by a partial breaking, due to the layering, of the LLL degeneracy. Further, the \( p \)-insensitive melting line, eq.(3.4), is a consequence of eq.(3.6) which is also independent of \( p \).

Here we will assume the first order transition line \( T_m(B) \) to terminate at some \( p > 1 \) and hence to have a upper critical point indicated as \( p_c \) in Fig.4. This is reasonable, because the above-mentioned exponentially small shear modulus in \( p > 1 \) likely results in a stronger random-pinning effect with increasing \( p \). Then, even the first order transition in \( p < p_c \) may not be accompanied by an ordinary superconducting ordering signaled by \( \mu \to 0 \), and eq.(3.6) becomes valid even below \( T_m \). Consequently, using the above expressions, one obtains a Josephson-vortex glass (JG) transition line

\[
B_{\text{JG}}(T) \simeq \frac{\phi_0}{2\pi d\xi_0\theta_f(T)} \left[ 1 - \frac{2}{\Gamma d^2} + \frac{T}{T_{c0}} \left( 1 + \frac{\theta_f^{(2d)}(T_{c0})}{2} \ln \frac{2\pi \Delta}{\Gamma} \right) \right],
\]

(3.7)

which has a similar \( T \)-dependence to eq.(2.26). Note that the prefactor decreases with increasing 3D fluctuation strength \( \theta_f \propto \sqrt{T} \) and that, due to the assumption \( N_{\text{cor}} \sim 1 \), the \( \Delta \to 0 \) limit cannot be taken in eq.(3.7). In the sketched phase diagram Fig.4, eq.(3.7) is useful just in \( p > p_{c1} \). In \( p_c < p < p_{c1} \), an exponential decay of \( N_{\text{cor}} \) resulting from a similar \( p \)-dependence of shear modulus will not be negligible with increasing \( p \). Then, the prefactor of eq.(3.7) effectively diminishes in this field range, and the resulting \( B_{\text{JG}}(T) \), as described in Fig.4, becomes more flat. This phase diagram Fig.4 will be compared with existing data in HTSC. An a.c. susceptibility measurement for examining the onset of lock-in phenomena (i.e., of the transverse Meissner effect) in BSCCO has been performed [28], and the
onset line of lock-in behavior found there has shown a relation similar to eq.(3.7) appearing in $p > p_{c1}$ in Fig.4. A detailed resistivity measurement has been performed in an optimally-doped BSCCO above 60 (K) [29] and has shown, at lower temperatures, a continuous vanishing of resistance along a flat curve consistent with $B_{JG}(T)$ of Fig.4 but, near $T_{c0}$, a discontinuous resistivity vanishing at $B_{IG}(T)$ just on or below the disorder-free melting line $T_m(B)$. Further, recent resistivity data in 60K YBCO have suggested the presence of a remarkable slush regime and of a $B_{lcp} \simeq 7$ (T) above which $T_m$ is roughly independent of $p$ [56]. The occurrence of a $B_{lcp}$ in $p \sim 1$ of the case $\mathbf{B} \parallel \text{layers}$ is not surprising if the system is moderately dirty so that the VG transition line in $p < 1$ lies above the $T_m(B)$-line. In fact, the ”vertical” $T_m(B)$ in $p > 1$ suggests that a situation with $B_{lcp}$ and with the first order transition at higher $p$ occurs more easily than in the case $\mathbf{B} \perp \text{layers}$. The continuous resistivity vanishing in 90K YBCO reported previously [61] might be a phenomenon below a $B_{lcp}$.

In the case with disorder, the resistances for a current in all directions vanish simultaneously at $T_{IG}(B)$. Further, the transverse Meissner effect signalled by a critical divergence of tilt modulus for a tilt across the layers is expected to occur for any $p$-value, i.e., even if the vortex lattice in the pinning – free case is a unpinned solid. A detailed analysis leading to these conclusions on response properties is seen in ref.27.

4. SUMMARY AND DISCUSSION

We have briefly explained the existing theory of phase diagram and physical properties deep in the vortex liquid regime based on the LLL approximation of the GL model. This theory can describe the vanishing behaviors of resistance in various situations in a way consistent with the phase diagram, while it is unclear to what extent the glass phases and possible transitions between them are described starting from the original GL model.

In §2, a comparison of the theory with experimental data was done for YBCO. In BSCCO with stronger fluctuation and much larger anisotropy, it is known through the angular dependence [62] that the internal gauge fluctuation (leading to a magnetic screening changing features of interaction between the vortices) is no longer negligible in the low fields where the first order transition is realized, and hence, the present theory will not be directly applicable.

Nevertheless, we note that, due to the strong dependence on the anisotropy of $B_{lcp}^{(p)}$ in eq.(2.27), the absence of a lower critical point in BSCCO is not surprising. The position of the upper critical point and the presence of slush
regime in BSCCO are not clear yet to us. We simply guess here that, like in Fig.4, the VG transition curve and its high field limit (corresponding to eq.(2.16)) in BSCCO should lie extremely below an extrapolated curve of $T_m(B)$ to higher fields.

Recently, the presence of pinning-induced growth of $|\sigma_{xy}|$ near a glass transition was shown theoretically [63] and experimentally [64,65] and cannot be explained correctly based on the mean field vortex dynamics neglecting thermal fluctuation [66]. This should be the case, because the present extension of the fluctuation theory [6] has explained the details of phase transition lines at which the linear dissipation vanishes. The glass fluctuation contribution $\sigma_{G,xy}$ to the Hall conductivity is expressed, as well as $\sigma_{G,xx}$, by Fig.2, and it is expected that the magnitude of $\sigma_{G,xy}$ grows on approaching a glass transition with its sign, relative to that of $\sigma_{F,xy}$, dependent on the dimensionality of a dominant pinning disorder [63]. The phenomena in the case with only line disorder are the best understood ones, and we find for this case that $\sigma_{G,xy} \cdot \sigma_{F,xy} < 0$ [63,65] and that the ratio $\sigma_{G,xy}/\sigma_{G,xx}$ (i.e., the Hall angle near the transition) does not vanish but seems to approach a constant at the transition [65,66]. Its details will be reported elsewhere.

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Figure 1: Diagram representing $I_{\text{irr}}$. The straight line denotes the LLL propagator, the dashed line is the "impurity" line carrying $\Delta$, and the semicircles imply vertex correction.

Figure 2: Two typical Feynman diagrams contributing to $\sigma_{G,xx}$, in which the chain line denotes the next lowest Landau mode of $\psi$, and the hatched rectangle implies the correlation function (1.1).
Figure 3: Schematic phase diagrams conjectured for $\mathbf{B} \perp$ layers. The specific case (b) follows from the generic one (a) as a consequence of strong fluctuation. The solid curve denotes the second order glass transition, while the chain curves include both the thermal first order line and the BrG melting line.
Figure 4: Schematic phase diagram for the case $\mathbf{B} \parallel$ layers. $T_m(B)$ and $T_c(B)$ denote, respectively, the melting curve in the pinning-free case and the $H_{c2}(T)$ crossover line. The $B_{JG}(T)$ curve and the first order transition line are expressed by, respectively, the solid and chain curves. Any reflection of possible structural transitions below $T_m(B)$ is not described here.