Towards the modelling of uplift resistance of skirted offshore mudmat foundations

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Abstract. A well-planned retrieval or removal operation of deep-water subsea foundations, such as mudmats, resting on soft soil requires estimation of the uplift forces required for the execution of the retrieval. In this paper, mathematical functions that are tuned to describe the uplift resistance due to skirt-soil friction and suction are proposed. Furthermore, possible benefit from inclined pull is deduced.

1. Introduction

Skirted mudmats are one of the frequently used foundations for subsea structures and platforms. They may be retrieved for a relocation or decommissioning at the end of their service life. For planning a safe and cost-effective execution of retrieval/removal, realistic assessment of the various anticipated forces that need to be overcome is required. In the past, researchers have investigated the problem using numerical models and model tests in centrifuge and prototype tests, e.g., [1-4]. Various factors that contribute to the uplift resistance of such structures have been investigated and identified. The aim of this paper is synthesizing the various observed trends and establishing mathematical functions that are tuned towards the description of the uplift resistance of short skirted shallow foundations. Towards this goal, empirical evidences are considered to a large extent, and when they are lacking speculative approaches have been employed.

2. Uplift from a sand bedding

Consider a pull force, \( P \), applied at the centre of a skirted mudmat foundation on a sand bedding, see Figure 1. During retrieval, the applied pull force must overcome the self-weight of the structure and the shear resistance at the skirts and possibly also some suction. But, the suction will be short lived for a sand bedding and waiting for some minutes might release it and hence suction may be disregarded for all practical purposes. As the applied pull force increases beyond the self-weight of the structure, shear stresses begin to mobilize at the skirt soil interface. The structure then begins to slip through the soil mass and move up developing more shear strains and hence mobilizing correspondingly higher shear stresses. Finally, when the pull increases such that the ultimate shear resistance is mobilized, the structure moves up and retrieval is successful.
It is assumed that the yielding of the soil around the skirts due to the mobilization of shear during the pull is governed by the yield function (modified from [5]):

$$f_s = \frac{\tau_{\text{max}}}{G_{50}} \frac{\tau_m}{\tau_{\text{max}} - \tau_m} - \frac{2\tau_m}{G_{ur}} - \gamma = 0,$$  

(1)

where $\tau_m$ is the mobilized shear, $\tau_{\text{max}}$ is the high estimate shear strength, $G_{50}$ is the secant shear modulus at which the shear mobilization is 50% of the ultimate shear strength, $G_{ur}$ is the unload-reload shear modulus and $\gamma$ is the shear strain.

It is further assumed that the mobilized shear strain can be approximated by:

$$\gamma = \frac{u_v}{L^*}$$  

(2)

where $u_v$ is the vertical uplift displacement and $L^*$ is the thickness of the sheared zone around the skirts, Figure 1. The relationship between vertical displacement and shear stresses expressed through Eqs.(1) and (2) is then illustrated in Figure 2.

The high estimate shear strength may also be obtained from Cone Penetration tests according to the methodology of DNV-GL or for drained fictional materials, which sand reasonably falls in the category of, the maximum shear resistance may be estimated using the Coulomb theory according to:

$$\tau_{\text{max}} = K \sigma'_{vo} \tan \delta$$  

(3)

where $K$ is the effective earth pressure coefficient (the high estimate is to be used), $\sigma'_{vo}$ is the in-situ vertical stress and $\delta$ soil-structure friction angle (the high estimate is to be used).

Rearranging Eq.1 the mobilized shear stresses are solved analytically as:

$$\tau_m = \frac{1}{4} \left( \sqrt{a^2 + 8b} - a \right),$$  

(4)

where

$$a = \frac{(L^* G_{ur} \tau_{\text{max}} - 2L^* G_{50} \tau_{\text{max}} + G_{ur} G_{50} u_v)}{L^* G_{50}}$$

and

$$b = \frac{G_{ur} G_{50} \tau_{\text{max}} u_v}{L^* G_{50}}.$$

Now, the mobilized resistance, $R$, can be obtained by integrating the average mobilized shear stress over the total area of the skirts as

$$R = P - W = \tau_m \sum A_{imb-s},$$  

(5)
Figure 2: The backbone shear stresses versus vertical displacement curve

where \( P \) is the total vertical pull force, \( W \) is the submerged weight of the mudmat and \( A_{imb-z} \) is the embedded part of the skirts. If all skirts have the same depth, then

\[
R = 2\tau_m (d_s - u_v) \sum L_s
\]  

holds, where \( u_v \) is the vertical displacement, \( d_s \) is the skirt depth and \( \sum L_s \) is the total length of the skirts.

Uplift resistance versus vertical displacement is shown in Figure 3a. The uplift resistance first increases with vertical displacement and reaches a peak. Then post peak reduction of the uplift resistance occurs.

The equivalent normalized tangent spring stiffness per unit skirt length can then be obtained by differentiating \( R \) wrt \( u_v \) which yields:

\[
k_{vt} = \frac{1}{\sum L_s} \frac{\partial R}{\partial u_v} = (d_s - u_v) \frac{g_{ur}}{4L' \sqrt{a^2 + 8b}} \{ a - 1 + 4\tau_{max} \} - \frac{1}{4} \left( \frac{1}{\sqrt{a^2 + 8b}} - a \right)
\]  

Note also that the maximum resistance to uplift mobilizes when the tangent stiffness is zero, i.e.,

\[
(d_s - u_v) \frac{g_{max} \{ a - 1 + 4\tau_{max} \}}{L' \sqrt{a^2 + 8b}} - \left( \frac{1}{\sqrt{a^2 + 8b}} - a \right) = 0.
\]  

Eq. (8) may be solved analytically. However, a simple excel plot of \( R \) versus \( u_v \) of Eq. (7) can give the desired value. Note that the higher the value of the \( L' \) (the wider the shear zone) the gentler is the mobilization of the uplift resistance with vertical displacement.

Similarly, the secant spring stiffness per unit skirt length to any desired resistance (uplift) can be obtained using:

\[
k_{vs} = \frac{R}{u_v \sum L_s}
\]  

See Figure 3b for illustration. Interest can be the secant spring stiffness through the peak uplift resistance, for instance. If the soil springs are to be distributed over the total projected area of the foundation, the quantity in Eq. (9) should be divided by the ratio of the foundation area to the total length of the skirts.
3. Uplift from a clay bedding

Consider a pull force applied vertically at the centre of the foundation structure on a clay bedding. In clays, the resistance due to suction comes into play and will stay for a significant amount of time. Under slower rates of uplift, the resistances due to shear at the skirt soil interface may need to be overcome in addition.

Centrifuge studies done on circular and rectangular mudmats have shown that:

- the undrained strength and hence the uplift resistance increases with the rate of uplift [6, 7].
- the uplift resistance first increases with displacement up to the maximum uplift resistance and beyond the maximum resistance the uplift resistance decreases with vertical displacement [1, 6], Figure 4.
- the uplift resistance decreases with sustained uplift [2-4].
- the displacement to the peak suction linearly increases with skirt length [6].
- the secant stiffness through the peak suction decreases lineally with skirt length [6].
- perforations [6] and eccentric uplift [3] may decrease the maximum uplift resistance.

Figure 3: Schematics of a) uplift resistance versus displacement, b) secant stiffness

Figure 4. Typical measurement of uplift resistances versus displacement in uplift tests performed in a centrifuge [4]. The model mudmat was of length 100 mm and width 50 mm which represents a prototype mudmat of 15 m long and 7.5 m wide, i.e., a scale of 150.
3.1 Suction resistance

According to numerical and centrifuge investigations, e.g., [2, 4], the average suction resistance may be written as a function of the rate of uplift, duration of the uplift force and the magnitude of the vertical displacement as

\[ \bar{s} = \bar{s}_{\text{max,ref}} f(\dot{u}_v) f(t) f(u_v), \]  

(10)

where \( \bar{s}_{\text{max,ref}} \) is the average maximum suction resistance at a reference rate of pulling, \( f(\dot{u}_v) \) is a function that takes into account the rate of pulling and hence here called velocity factor, \( f(t) \) is here called duration factor and describes the decay of the uplift resistance with sustained pull/creep and \( f(u_v) \) here called displacement factor is a function that describes the increase/decrease of the suction pressure due to increase in the vertical displacement during the uplift.

Next, we will establish the various functions in accordance with the key observations presented here before.

3.2 Velocity factor

The rate of uplift is considered through \( f(\dot{u}_v) \)-here called velocity factor. Data from literature [4] can be fit by a back bone curve of the form

\[ f(\dot{u}_v) = \frac{\dot{u}_v}{B + \dot{u}_v} \]  

(11)

where \( a \) is a fitting parameter, \( B \) is the width of the foundation and \( c_v \) is the coefficient of consolidation. The parameter \( a \) is set to 80 for fitting the data presented in Figure 5.

![Normalized average suction vs Normalized uplift velocity](image)

Figure 5. Dependence of uplift resistance on the rate of uplift, data from [4], B is breadth, D is diameter and \( c_v \) is coefficient of consolidation.

3.3 Duration factor

The decay in suction due to a sustained pull force may be realistically modelled when the change in time of the vertical displacement is described in terms of the vertical pull force and possibly of temperature such that it increases with displacement/magnitude of the pulling force and temperature.

We are going to simplify the question by asking what is the maximum suction that needs to be overcome when that much load must be sustained over a given period. In other words, we ask the question that if we sustain a given uplift load, when does the uplift resistance come to the level of the sustained load
such that retrieval or removal is successful. Looking at the trends from tests performed in centrifuge, e.g., Gourvenec et al. [3], the following two major observations may be considered.

**Observation 1:** the higher the skirt length, and the higher the preloading and the longer a given load must be sustained for retrieval to be successful under sustained uplift force.

**Observation 2:** The lower the consolidation after installation, the less the time it takes for successful retrieval under a given sustained uplift force.

Observation 1 may be treated in the same manner as a consolidation process. The second observation implies that the level of pore pressure available prior to the operation of retrieval plays a role in the magnitude of the suction that develops in the clay soil during the uplift operation. This also implies that retrieval after a short time of installation requires relatively lower retrieval forces than a removal operation after a long period. The decay in time of the ultimate uplift force after consolidation may be captured using the following function

\[
f(t) = \beta \left( \frac{D}{T u_{v0}} \right)^{\alpha} \leq 1, \quad T = \frac{c_v t}{d_z}
\]

(12)

where \(T\) is the time factor, \(u_{v0}\) is the initial displacement, \(c_v\) is the coefficient of consolidation, and \(\alpha\) and \(\beta\) are fitting parameters. For the data by Gourvenec et al. [3], presented in Figure 6, \(\beta = 0.61\), and \(\alpha = 0.34\) gave a good fit. There are also indications that \(\beta\) depends on the skirt length.

**Figure 6:** Normalized uplift resistance versus time factor normalized by vertical displacement ratio (for \(ds/D=0.15\)), based on data from Gourvenec et al. [3]

3.4 **Displacement factor**

The function \(f(u_v)\) defines the release of suction and some weak bonds by the action of tensile displacements. The mathematical function

\[
f \left( \frac{u_v}{d_z} \right) = \frac{m+n}{n} \left( \frac{m+n}{m} \right)^{\frac{m}{n}} \left\{ 1 - \exp \left( - \frac{u_v}{md_z} \right) \right\} \exp \left( - \frac{u_v}{nd_{v0}} \right)
\]

has the desired property such that \(\bar{s} = \bar{s}_{max,ref}(u_v)\) is obtained as the peak value. A non-zero skirt length is considered.

Let the displacement at the maximum suction beyond which softening occurs be \(u_{v0}\). See Figure 7. This gives a unique relation between \(m\) and \(n\) according to

\[
n = m \left\{ \exp \left( \frac{u_{v0}}{d_{zm}} \right) - 1 \right\}.
\]

(14)

For central uplifts:
The study by Li et al. [6] shows that the vertical displacement to the peak uplift resistance linearly increases with skirt length, i.e., \( u^* = u_{0}^* + a_u d_s \) in which \( u_{0}^* \) is the displacement at the peak uplift resistance when the mudmat foundation is not provided with skirts and \( a_u \) is a fitting parameter.

- The uplift tests performed in a centrifuge by Li [4] indicate that \( u^* \) decreases with increasing rate of loading.

The secant modulus, \( E_s \) through the peak uplift resistance due to suction may then be derived from Eq. (13) as:

$$
\frac{E_s}{s_{max}} = \frac{1}{d_s m \ln\left( \frac{a}{b} \right)}
$$

According to the investigation [6] using model tests in a centrifuge, the secant stiffness decreases with increasing skirt length and increases with rate of uplift. In Figure 7, the trend of the displacement factor is compared with normalize curves from uplift tests performed in a centrifuge [4].

![Figure 7](image_url)

**Figure 7**: Displacement factor due to suction according to Eq. 13 (data from model tests in centrifuge [4]).

### 3.5 Total uplift resistance

In the previous sections, the various factors that determine the magnitude the suction resistance are synthesized. The following two conditions may be required for successful retrieval and the total uplift resistance need to be estimated accordingly.

**Condition 1**: If retrieval is carried out so fast that the suction resistance is very high, it can result in a reversed bearing capacity failure and the maximum retrieval resistance may be estimated using bearing capacity equations for undrained condition and a rate dependent undrained shear strength. In this case, for all partial purposes, the average suction resistance can be set to be equal to the reversed bearing capacity of the clay soil.

**Condition 2**: If retrieval is carried out at a relatively slower rates of pulling with pull loads sustained over time, the suction resistance subsides with time, the soil beneath undergoes creep under tensile loads gradually releasing the suction grip and as the foundations pulls away from the soil mass, the shear resistance at the skirt soil interface mobilizes.

Which mechanism should be considered in the evaluation of the uplift depends on how fast the retrieval needs to be carried out according to the cost-benefit analysis. The problem may be analysed using advanced numerical methods with suitable models that include the key observations above. If \( s > N_e c_{u,ref} (u_0) \), where \( N_e \) is the undrained bearing capacity factor, **Condition 1** is prevailing, and the bearing capacity is the governing mechanism during retrieval, otherwise **Condition 2** can be
expected. In the case where the failure mechanism is governed by Condition 2, we assume that the hyperbolic relationship in Eq.1 governs the shear mobilization at the skirt-soil interface. The maximum shear resistance at the skirt-soil interface is then defined by $\tau_{\text{max}} = \alpha c_u$, where $\alpha$ is the interface coefficient and $c_u$ is the undrained shear strength of an intact sample. In some moderately and heavily overconsolidated clays, suction might also develop at the soil-skirt interface and could sustain longer than the suction at the foundation invert. This aspect needs to be investigated and is not considered here.

The total uplift resistance may then be set as the lesser of the reversed bearing capacity and the sum of the resistance due to suction and the shear resistance at the skirt-soil interface, say

$$R = \tau_m \sum A_{imb-s} + \bar{s}_{\text{max,ref}} f(\bar{u}_v) f(t) f(\bar{u}_v) A \leq N_c c_u,$$

where $A$ is the foundation area and other quantities are as defined elsewhere.

Figure 8 illustrates the typical suction dominated and shear dominated uplift resistance versus displacement curves according to Eq.16.

3.6 Possible advantages with inclined pull

Assume that Condition 1 is governing and the reversed bearing capacity (uplift resistance) can be sufficiently described by the Brinch-Hansen’s bearing capacity formula [8]. Let the load inclination factor,

$$i_c = 0.5 - 0.5 \sqrt{1 - \frac{H}{A c_u}},$$

hold as good for the uplift bearing capacity as the down push bearing capacity. The uplift bearing capacity due to inclined pull may then be considered through the bearing capacity formula (for simplicity neglecting other factors) as:

$$\frac{Q_v}{A} = N_c c_u (1 - i_c),$$

where $N_c$ is the bearing capacity factor, $H$ is the horizontal loading $A$ is the footprint area of the foundation, $c_u$ is the undrained shear strength of the soil, $Q_v$ is the vertical uplift resistance assuming Condition 1 is prevailing.

Let a pull force $Q$ be applied at an angle $\theta$ with the vertical such that it has a horizontal component of $H = Q \sin \theta$, and a vertical component of $Q_v = Q \cos \theta$. Substituting Eq.17 into Eq.18 and after some simple rearrangement we have:
\[
\frac{Q}{AN_{c\text{cu}}} = \min \left\{ \left( 1 - \frac{\pi+2}{4} \tan \theta \right) \frac{1}{\cos \theta} N_{c\text{u}} \sin \theta \right\}
\]  

(19)

The meaning of Eq. (19) is illustrated in Figure 9. Benefiting from inclined pull may be realistic for foundations with short skirt length. Otherwise, more passive resistances may be mobilized with the inclination of the pull. The mobilization of the passive resistance may be assumed to be a function of the load inclination factor as

\[
P_{pm} = (2i_c)^\omega P_p
\]  

(20)

where \( P_{pm} \) is the intermediate mobilized passive resistance, \( P_p \) is the fully mobilized passive resistance and \( \omega \) is a model parameter that controls the rate of the mobilization of the passive resistance. It is important to find the optimum inclination angle which in the overall reduces the undrained bearing capacity should the uplift be performed fast. However, some practical issues may arise when considering inclined uplift at large water depths.

![Figure 9: Theoretical reduction of uplift resistance with pull inclination](image-url)

4. Summary

In this paper, the uplift resistance of skirted mudmats in sands and clays has been described. Several mathematical functions that capture various observed trends are proposed. When the material is sand-like and no significant suction is developing, or the developed suction is short lived and can be ignored for all practical purposes, the uplift resistance can be approximated by integrating shear resistances mobilized at the skirt-soil interface. A yield function that describes the mobilization of the shear resistance at the skirt-soil interface is proposed in this paper. In clays, significant suction is likely to develop during uplift. The magnitude of the suction is known to depend on the velocity of the pull, the duration of the pull force is sustained and the magnitude of the vertical displacement. A mathematical model that considers the influence of the various factors on the suction resistance is proposed. The relationships are developed directly in terms of forces and displacements. However, they can be easily translated into stress-strain relationships for use in constitutive models that aim the modelling of the problem in a Finite Element or in a Finite Difference environment. Assuming reversed bearing capacity failure for describing uplift resistance of short skirted foundations on a clay bedding, the possible benefit of inclined pull is deduced. The practicality of application of inclined pull needs to be investigated further.
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