Bound on nonlocal scale from $g - 2$ of muon in a nonlocal WS model

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Abstract

We consider the nonlocal version of the Weinberg Salam model (following Kleppe et al. ) with a finite parameter $\Lambda$ signifying a fundamental length scale. We calculate the extra contributions to the anomalous magnetic moment of the muon coming from the nonlocal structure in this model. We find that the nonlocal contribution can be comparable to weak contributions and goes to increase theoretical estimates. We use this calculation to determine the limit on the scale of nonlocality. We obtain the result $1/\Lambda \lesssim 3 \times 10^{-16}$ cm, which could be improved when present experimental errors narrow down.

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I. INTRODUCTION

The presently successful theories of strong weak and electromagnetic interactions are all local quantum field theories: they assume a local interaction. One of the features of local QFTs is the presence of ultraviolet divergences that arise in the large momentum region in the loop integrations or equivalently, at short distances. Operator products of fields at the same $x$ are ill-defined due to such ultraviolet divergences. A natural procedure for avoiding such ultraviolet divergences has been to introduce a momentum cut-off $\Lambda$ or equivalently a small space-time separation $\epsilon_{\mu}(\sim \frac{1}{\Lambda})$ into the theory. Until recently, all attempts to introduce such cutoffs into the local QFT led to theories that were unitary only as $\Lambda \to \infty$. Hence such procedures were used only as regularizations: a physically meaningful theory required that renormalization be carried out and then $\Lambda$ let go to infinity. Hence such a introduction of a cut off was looked upon as a mathematical necessity only, without a direct physical significance.

Recently, a way of regularizing quantum field theories was introduced \cite{1} and developed further \cite{2,3} by the introduction of a cut off parameter $\Lambda$ in such a manner that (i) the theory is unitary for any finite $\Lambda$, (ii) all continuous symmetries of the theory are preserved in some (altered) form. Such theories necessarily contain derivatives of fields to all orders and hence are essentially nonlocal. Such a regularization has been labelled “nonlocal regularization”.

Since the theories are physically meaningful even for finite $\Lambda$, it has been suggested by Kleppe and Woodard \cite{2}, that one could look upon the regularized theory, containing the parameter $\Lambda$, as the actual physical theory with an inherent distance scale $\sim \frac{1}{\Lambda}$ in it. Thus, according to this viewpoint, the nonlocal version of the theory is not a regularization (i.e. a mere mathematical necessity), but a physical action that could embody a granular structure of space-time through a parameter $\Lambda$.

A natural question one can ask at this stage is what are the experimental limits on this parameter $\Lambda$. From this viewpoint, we consider using the anomalous magnetic moment $(g - 2)$ of the muon, \cite{10}; which is known to one part in $10^9$. The contributions to $(g - 2)$ in the local theory (standard model) have been well discussed and accurately calculated \cite{11} and are known to agree very well with experiment \cite{10}.

It is the uncertainties in the experimental and theoretical values of $(g - 2)$ that can set an upper limit on the nonlocal contribution to $(g - 2)$. From the theoretically calculated contribution to $(g - 2)$, one can then set a lower bound on the nonlocality parameter $\Lambda$. This is the purpose of the present work.

One can also expect nonlocality to make much difference in the operator product expansion at short distances, when applied to deep inelastic scattering and electroproduction. We expect to report it elsewhere.

While we can look upon this calculation as a limit on the scale of granularity of
space time, alternative interpretations of the result are also possible. For example, muon may not be a point particle as assumed in the local theory, but may have an internal structure, which may directly introduce nonlocality in its interaction with electromagnetic and weak bosons. In such a case, if we assume that our nonlocal theory is an effective theory, these results tell us about the scale of compositeness of the muon. Now such calculations in composite models have been performed in literature \[6,9\]; these calculations however, necessarily require a number of ad hoc assumptions in order to perform them. The calculations presented here on the other hand, are based on the self consistent unitary BRS invariant model involving the scale \( \Lambda \) and does not involve ad hoc procedures. Of course, our procedure while it gives a mathematically rigorous way of calculating nonlocal contributions to \( (g - 2) \), cannot shed light on how the compositeness scale \( \Lambda \) has arisen.

We should also mention that calculation of fundamental scale of space-time has been attempted in a similar manner in another formulation of nonlocal quantum field theory, viz. the stochastic quantization method, and similar results have been obtained \[8\]. However, these nonlocal methods are much more cumbersome and require extra regularizations in addition to the introduction of nonlocality. The formulation used in our work \[2\] is neat and lends easily to calculations. Further, the calculation done for the stochastic formulation is for the photon exchange diagram only. In our formulation, we on the other hand calculate all contributions including W, Z exchanges. While the latter contributions in local theory are small compared to the \( \gamma \)-exchange diagram the nonlocal contributions from all the \( \gamma \)-exchange, W-exchange and Z-exchange (and the related diagrams) are of the same order [i.e. \( O(m^2/\Lambda^2) \)].

II. THE METHOD OF NONLOCALIZATION

In this section, we briefly review the method of nonlocalization \[1,2\] for obtaining a nonlocal theory from a local one. The method has been discussed extensively in many of the works on nonlocal regularization \[2–5\] but we recapitulate the salient features here and also introduce our notations and conventions. The first step is the nonlocalization of the local action using ‘smearing operators’ and is applicable to any local theory which can be formulated perturbatively. With this construct, for every local symmetry of the local action there is a corresponding nonlocal symmetry for the nonlocal action. This symmetry reduces to the local symmetry in the local limit of the nonlocal theory \[3\]. The classical nonlocal theory is quantized using the functional formalism, and in order that the quantized theory respects the nonlocal symmetry it is necessary that there should (except for anomalous theories) exist a measure factor which makes the path integral measure invariant under the nonlocal symmetry. For nonabelian gauge theories for example, the required measure factor is nontrivial. The second step in nonlocalization
is the construction of such a measure factor such that the resultant theory is finite, Poincare invariant and perturbatively unitary. In section II A we outline the method of nonlocalization using $\phi^4$ theory as an example. In section II B we non-localize the Weinberg Salam model, gauge fixed in the Feynman gauge and write down the Feynman rules needed to evaluate the anomalous magnetic moment of the muon in the nonlocal theory.

A. Nonlocal $\phi^4$ theory

Consider a local theory with the action written as a sum of the free and interacting parts

$$S[\phi] = \frac{1}{2} \sum \phi \int d^4x \phi F \phi + I[\phi],$$

where $\phi$ represents the fields (fermionic, bosonic) of the theory with the appropriate spacetime and internal symmetry group indices. $F$ is the 'kinetic' operator for the field $\phi$ and $I[\phi]$ is the interaction part of $S[\phi]$. For the $\phi^4$ theory,

$$F = (-\partial^2 - \mu^2)$$

and

$$I[\phi] = -\frac{\lambda}{4!} \phi^4$$

Nonlocalization of $S[\phi]$ is carried out using a 'smearing' operator defined in terms of the kinetic operator $F$ of the theory as

$$\mathcal{E} \equiv \exp[-\frac{F}{2\Lambda^2}].$$

$\Lambda$ is the scale of nonlocality. With the help of the smearing operator $\mathcal{E}$ we define a smeared field $\hat{\phi}$ by

$$\hat{\phi} = \mathcal{E}^{-1} \phi$$

Next, for every field $\phi$ we introduce an auxiliary 'shadow' field $\phi^{sh}$ of the same type as $\phi$ which couples to $\phi$ through an auxiliary action $S$ given by

$$S[\phi, \phi^{sh}] \equiv \frac{1}{2} \sum \phi \int d^4x \hat{\phi} F \hat{\phi} - \frac{1}{2} \sum \phi^{sh} \int d^4x \phi^{sh} O^{-1} \phi^{sh} + I[\phi + \phi^{sh}]$$

where $O$ is the 'shadow' kinetic operator defined as

$$O \equiv (\mathcal{E}^2 - 1)F^{-1}$$

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The action for the nonlocalized theory \( \hat{S}[\phi] \) is then defined by

\[
\hat{S}[\phi] \equiv S[\phi, \phi^{sh}(\phi)]
\]

where \( \phi^{sh}[\phi] \) is the solution of the classical shadow field equation

\[
\frac{\delta S}{\delta \phi^{sh}[\phi, \phi^{sh}]} = 0
\]

Quantization is carried out in the path integral formulation. The quantization rule is given by

\[
\langle T^* (O[\phi]) \rangle_{\mathcal{E}} \equiv \int D[\phi] \mu[\phi] O[\hat{\phi}] e^{i\hat{S}[\phi]}
\]

Here \( O \) is any operator taken as a functional of fields. \( \mu[\phi] \) is the measure factor defined such that \( D[\phi] \mu[\phi] \) is invariant under the nonlocal generalization of the local symmetry. For the \( \phi^4 \) theory \( \mu[\phi] = 1 \). For nonlocalized nonabelian gauge theories for example this measure factor is nontrivial.

The Feynman rules for the nonlocal theory are simple extensions of the rules of the local theory. In the local theory we leave the local propagator given by

\[
\frac{i}{\mathcal{F} + i\epsilon} = -i \int_{0}^{\infty} \frac{d\tau}{\Lambda^2} e^{\frac{\tau}{\Lambda^2}}
\]

In the nonlocal formulation there are two kinds of propagators, 'smeared' or an 'unbarred' propagator

\[
\frac{i\mathcal{E}^2}{\mathcal{F} + i\epsilon} = -i \int_{1}^{\infty} \frac{d\tau}{\Lambda^2} e^{\frac{\tau}{\Lambda^2}}
\]

and a 'barred' or shadow propagator

\[
\frac{i(1 - \mathcal{E}^2)}{\mathcal{F}} = -i\mathcal{O} = -i \int_{0}^{1} \frac{d\tau}{\Lambda^2} e^{\frac{\tau}{\Lambda^2}}
\]

For \( \lambda \phi^4 \) theory, these are represented graphically in Fig. 1. The vertices are the same as those of the local theory except that in the nonlocal theory we have additional vertices coming from the measure factor, whenever this factor is different from unity. For computing Feynman diagrams in the nonlocal theory the following points are to be noted.

1In this and the following sections the Feynman rules are those of a Minkowski space formulation. The loop integrations are well defined only in Euclidean space, however, and we evaluate them by a formal Wick rotation.
(a) the external lines of a given diagram can only be 'smeared' lines.

(b) The symmetry factor for any diagram is computed without distinguishing between barred and unbarred lines.

c) The loop integrations are well defined in the Euclidean space because of the exponential damping factors coming from propagators within loops.

d) The internal lines within a loop can either be smeared or barred. We sum over all possibilities excluding the diagrams with loop(s) made up entirely of 'barred' lines since shadow fields by construct obey Eq. 2.13 and we do not functionally integrate over them. Note also that including 'shadow loops' would give a theory which in effect is the same as the unregulated local theory. This is clarified by the following example: Consider the tadpole self energy diagram in nonlocal $\phi^4$ theory (Fig. 2) which using Eqs. 2.12 and 2.13:

$$= (-i)\left(-\frac{i\lambda}{2}\right) \int \frac{d^4l}{(2\pi)^4} \int_1^\infty \frac{d\tau}{\Lambda^2} \frac{1}{\lambda^2} e^{\tau \lambda (l^2 - \mu^2)} \tag{2.14}$$

the vertex factor is $(-i\lambda)$ and $l$ is the loop momentum. The shadow loop self energy diagram shown in Fig. 3 is given by (using Eqs. 2.12 and 2.13):

$$= (-i)\left(-\frac{i\lambda}{2}\right) \int \frac{d^4l}{(2\pi)^4} \int_0^1 \frac{d\tau}{\Lambda^2} \frac{1}{\lambda^2} e^{\tau \lambda (l^2 - \mu^2)} \tag{2.15}$$

If we take into consideration the 'shadow loop' also, we would obtain, upon adding Eqs. 2.14 and 2.15

$$\frac{i\lambda}{2} \int \frac{d^4l}{(2\pi)^4} \frac{1}{l^2 - \mu^2 + i\epsilon} \tag{2.16}$$

which is the tadpole diagram of the local theory. Note too that the shadow loop diagram here is ill-defined. In fact this example also shows how the finiteness of the nonlocalized theory arises. The divergences in the diagrams of the local theory can be seen as arising from the inclusion of the region around the origin of integration in the Schwinger parameter ($\tau$) space (see Eq. 2.11). The nonlocalized theory is finite because of the exclusion of the unit hypercube at origin from the region of integration in parameter space. This is effected by the exclusion of shadow loops.

\section*{B. Nonlocal WS model in Feynman gauge}

In this section we nonlocalize the WS model which has been gauge fixed in the Feynman gauge. In this paper since we are interested in evaluating the one

\footnote{The notations for the Feynman rules for the local theory are those given in Appendix(B) of Ref. [7].}
loop nonlocal electroweak contribution to the magnetic moment anomaly of the muon, \( a_\mu \), we will write down explicitly only the Feynman rules necessary for the calculation of \( a_\mu \). However, it is understood that we are nonlocalizing the full local theory and the complete set of Feynman rules are given accordingly, as outlined in section II A. In order to introduce the notations and definitions we first write down the action for the local WS model \([7]\) in the \( R_\xi \) gauge, as a sum of the free and interacting parts:

\[
S = \int d^4 x \mathcal{L}
\]

where

\[
\mathcal{L} = \frac{1}{2} \sum_{+,-} W_\mu \mathcal{F}^{\mu\nu}_W W_\nu + \frac{1}{2} Z_\mu \mathcal{F}^{\mu\nu}_Z Z_\nu + \frac{1}{2} A_\mu \mathcal{F}^{\mu\nu}_A A_\nu \\
+ \frac{1}{2} \sum_{+,-,1,2} \phi_i \mathcal{F}^i \phi_i + \mu^- (i \not\partial - m) \mu^- + \cdots \\
+ I[W^\pm, A, Z, \phi^\pm, \phi_1, \phi_2, \text{ghosts, leptons, quarks}]
\]

The dots in the above equation stand for the kinetic terms for the other leptons, quarks, ghosts. Here \( W^\pm, Z_\mu, A_\mu \) are the vector bosons, \( \phi^\pm \) and \( \phi_2 \) are the three would be goldstone bosons and \( \phi_1 \) is the Higg’s scalar. \( I \) represents the interaction part of the lagrangian.

The kinetic operators for the fields shown in the above equation are:

\[
\mathcal{F}^{\mu\nu}_W = (\partial^2 + m_W^2) g^{\mu\nu} + (1/\xi - 1) \partial^\mu \partial^\nu \\
\mathcal{F}^{\mu\nu}_Z = (\partial^2 + m_Z^2) g^{\mu\nu} + (1/\xi - 1) \partial^\mu \partial^\nu \\
\mathcal{F}^{\mu\nu}_A = \partial^2 g^{\mu\nu} + (1/\xi - 1) \partial^\mu \partial^\nu \\
\mathcal{F}_{\phi^\pm} = -\partial^2 - \xi m_W^2 \\
\mathcal{F}_{\phi_2} = -\partial^2 - \xi m_Z^2 \\
\mathcal{F}_{\phi_1} = -\partial^2 - 2 \mu^2 (\mu^2 > 0)
\]

We now nonlocalize the theory defined above, according to the procedure outlined in section II A. For simplicity we consider the theory in the Feynman gauge \((\xi = 1)\). The smearing operators for all bosonic fields are given by Eq. 2.4 where the kinetic operators \( \mathcal{F} \) are defined in the equations above with \( \xi = 1 \). For the fermions, it is simplest to define the smearing operator as a scalar operator

\[
\mathcal{E}_\psi = e^{-\partial^2 - \mu^2}.
\]

Having defined the smearing operators for all fields we define smeared fields according to Eq. 2.5 and also introduce shadow fields which couple to the smeared
fields via the auxillary action given by Eq. 2.4. The nonlocal action is finally defined using Eq. 2.8. We assume that the measure factor which make the quantized theory perturbatively unitary and Poincare invariant, exists. However, we do not discuss it in this paper at all since its vertices are not needed in the evaluation of \( a_\mu \) to one loop order in the nonlocal theory. (The measure factor vertices are necessarily of order \( \hbar \) and have external gauge boson lines only, and hence they can contribute to \( a_\mu \) only at two and higher loop order.) The Feynman rules needed for computing \( a_\mu \) are as follows. The propagators are shown in Fig. 4. The vertex factors are the same as those of the local theory and can be found in Appendix B of Ref. [7]. Of course, it must be borne in mind that the vertices connect smeared and/or shadow lines and not local ones.

III. CALCULATION OF NONLOCAL CONTRIBUTION TO \( A_\mu \)

In this section we evaluate the one-loop nonlocal corrections to \( a_\mu \) arising in the nonlocal electroweak theory discussed in Section II B. The leading order corrections are of order \( \alpha m^2/\Lambda^2 \) where \( \alpha \) is the fine structure constant and \( m \) the muon mass. We assume that the scale of nonlocality \( \Lambda \gg M_W \), and therefore neglect corrections of order \( (\Lambda^2)^{-n}, \ n > 1 \).

A. Feynman diagrams contributing

The one loop electromagnetic contributions to \( a_\mu \) in the nonlocal theory will come from the diagrams in Fig. 5 where the unbarred (barred) lines are the smeared (shadow) muon and photon lines. External lines are always smeared lines as has been pointed out in Section II A. For compactness of notation we represent the diagrams of Fig. 5 as shown in Fig. 4. The term 'barred variations' stands for all diagrams obtained from the unbarred diagram by replacing one or more of the internal lines with barred, i.e., shadow lines, excluding the case where all the lines in a loop are barred.

To calculate the one loop weak contributions we have to consider the diagrams shown in Fig. 7 along with the barred variations of each diagram as explained above. From these diagrams we are interested only in (on shell) contributions proportional to \( \bar{u}(q)\sigma_{\mu\nu}(p - q)^\nu u(p) \) (where \( q \) and \( p \) are the final and initial momenta, respectively) and will ignore the rest of the terms. Firstly, notice that the sum of all diagrams given in Fig. 5 for example, and the shadow loop diagram in Fig. 8 gives the diagram of the local theory (Fig. 9). (Refer also to the discussion at the end of section II A and Eqs. 2.14, 2.16) Now, the local contributions for \( a_\mu \) coming from the diagram in Fig. 9 of the local theory is finite. The nonlocal
contributions come from diagrams of Fig. 5 are also finite\(^3\). Hence the contributions for \(a_\mu\) from the shadow loop have to be finite. \(^4\) Therefore instead of considering all the diagrams of Fig. 5 we need consider only the single shadow loop diagram of Fig. 5. Then the contribution to \(a_\mu\) in the nonlocal theory will simply be the difference between the local and shadow loop contributions. Hence, the correction to \(a_\mu\) due to nonlocality will be given by the negative of the shadow loop contribution.

The same is true for weak contributions. Therefore instead of considering the diagrams of Fig. 7 along with their barred variations we need consider only the weak boson exchange shadow loop diagrams of Fig. 10 along with the shadow diagram of Fig. 8 of \(\gamma\) exchange. We will evaluate these in the next section.

### B. Calculation for \(a_\mu\)

In order to calculate the nonlocal contributions to \(a_\mu\), we have seen that we only have to consider the shadow diagrams of Figs. 5 and 10, and from these we need to extract the on-shell contribution proportional to \(\bar{u}(q)\sigma_{\mu\nu}u(p)\). We will work in the Feynman gauge (\(\xi = 1\)). We shall be brief in our discussions and only elaborate those points which are new to the reader because of the use of a nonlocal theory.

We shall ascribe a common momentum flow to all diagrams as shown in Fig. 11. The \(\gamma\)-exchange shadow diagram of Fig. 8 can be written as

\[
i \Delta^{(1)} \Gamma_{\rho}(p, q) = -e^3 \int \frac{d^4 l}{(2 \pi)^4} \int_0^1 \frac{d \tau_1}{\Lambda^2} \int_0^1 \frac{d \tau_2}{\Lambda^2} \int_0^1 \frac{d \tau_3}{\Lambda^2} \gamma_{\lambda}(I - \bar{p} + \bar{q} + m) \gamma_{\mu}(I + m) \gamma^\lambda \times \exp \left[ \frac{1}{\Lambda^2} \left( \tau_1 (l - p + q)^2 + \tau_2 l^2 + \tau_3 (p - l)^2 - (\tau_1 + \tau_2) m^2 \right) \right] (3.1)
\]

where an extra minus sign has been added because it is the -ve of the shadow diagram that gives the nonlocal contribution to \(a_\mu\). The above expression is un-

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\(^3\)Since the nonlocal theory by construct is a finite theory (\(\Lambda\) finite), all diagrams of the theory are finite. Further, for contributions to \(a_\mu\) there is no contribution proportional to \((\Lambda^2)^n, n > 0\), integer. This is because in the limit \(\Lambda \to \infty\), the nonlocal theory reduces to the local theory and nonlocal contributions to \(a_\mu\) reduce to the (finite) local contributions. Therefore all nonlocal contributions will be proportional to \((\Lambda^2)^n\) where \(n\) is an integer \(\leq 0\).

\(^4\)In general, the shadow loops are ill-defined whenever the corresponding loops in the local theory are, and we cannot consider them for manipulations (see end of Section 11A for an example).
understood to be sandwiched between $\bar{u}(q)$ and $u(p)$. Then $\Delta^{(1)}\Gamma$ is a sum of $\gamma_\rho$ and $\sigma_{\rho\nu}(p - q)^\nu$ type terms. The following common features are noted.

1. A shift of the momentum variable

$$l \rightarrow l + \sum \tau_i p - \sum \tau_i q$$  \hspace{1cm} (3.2)

removes from the exponent cross terms in $l$, and further, the change of variables above is common to all diagrams in view of the common momentum routing in Fig. [11].

2. The exponent then assumes the form

$$\exp \left[ \frac{\tau l^2}{\Lambda^2} + \frac{1}{\Lambda^2} f(m^2, p, q; \tau_i) \right]$$

$$= \exp \left[ \frac{\tau l^2}{\Lambda^2} \right] \left[ 1 + \frac{1}{\Lambda^2} f + O\left( \frac{1}{\Lambda^4} \right) + \cdots \right]$$  \hspace{1cm} (3.3)

As we shall see, the first term in the above expansion itself contributes the leading nonlocal correction of $O(m^2/\Lambda^2)$ and when this happens the contributions from further terms in the series (which are all well defined: recall that $f_i$ are polynomials in $\tau_i$’s and the range of $\tau_i$’s is 0 to 1, i.e., finite) only give nonleading contributions and hence can be dropped.

3. Note next that in an expression such as

$$\int_0^1 d\tau_1 \int_0^1 d\tau_2 \int_0^1 d\tau_3 \exp \left[ \frac{\tau}{\Lambda^2} l^2 \right] h(p, q, \tau_i)$$  \hspace{1cm} (3.4)

as the integral ranges as well as the exponential are symmetric under simultaneous interchange of any two $\tau_i$’s, the $h(\tau_i)$’s can be symmetrized.

4. We need only the following $l$ integral which can be done following a Wick rotation [1]:

$$\int \frac{d^4 l}{(2\pi)^4} \exp \left[ \frac{\tau l^2}{\Lambda^2} \right] = \frac{i \Lambda^4}{16\pi^2 \tau^2}$$  \hspace{1cm} (3.5)

5. When sandwiched between $\bar{u}(q)$ and $u(p)$ on mass shell for the muon, the following expressions contribute, as indicated below. Here, we have used Gordon decomposition and dropped $\gamma_\rho$ type terms (that contribute to the electric form factor)

$$\gamma_\rho \not{p} \approx 0 ; \hspace{0.5cm} \not{q} \gamma_\rho \approx 0$$

$$\gamma_\rho \not{q} \approx -2i\sigma_\rho\nu q_\nu ; \hspace{0.5cm} \not{p} \gamma_\rho \approx 2i\sigma_\rho\nu p_\nu$$

$$\not{p} \gamma_\rho \not{q} \approx -2im\sigma_{\rho\nu}(q - p)^\nu$$  \hspace{1cm} (3.6)

Using the statements made above, we can evaluate $\Delta^{(1)}\Gamma_\rho$ of Eq. (3.1) and we find
\[ i \Delta^{(1)} \Gamma_{\rho}(p, q) = \left( \frac{e}{2m} \sigma_{\rho \mu}(p - q)^\mu \right) \frac{e^2 m^2}{16 \pi^2 \Lambda^2} \times (-8) \]
\[ \times \int_0^1 d \tau_1 \int_0^1 d \tau_2 \int_0^1 d \tau_3 \frac{1}{\tau_2^3} \left[(1 - \tau_2/\tau)(1 - \tau_1/\tau) - (1 - \tau_2/\tau)(\tau_2/\tau) + 2(1 - 2\tau_2/\tau)\right] \]
\[ + O.T. \tag{3.7} \]

A straightforward evaluation yields

\[ i \Delta^{(1)} \Gamma_{\rho}(p, q) = \left( \frac{e}{2m} \sigma_{\rho \mu}(p - q)^\mu \right) \frac{\alpha m^2}{4 \pi \Lambda^2} \times \left[-6.2986\right] + O.T. \tag{3.8} \]

In a similar manner, we evaluate the diagrams 10(a), 10(b), and 10(c). We give the results. For the diagram of Fig. 10(a), we obtain

\[ i \Delta^{(2)} \Gamma_{\rho}(p, q) = \left( \frac{e}{2m} \sigma_{\rho \mu}(p - q)^\mu \right) \frac{e^2 m^2}{16 \pi^2 \Lambda^2} \]
\[ \times \left[ \frac{-1}{\sin^2 \theta_w} \right] \int_0^1 d \tau_1 \int_0^1 d \tau_2 \int_0^1 d \tau_3 \frac{1}{\tau_2^2} \left[2 + 4 \tau_1 \tau_2 - \frac{12 \tau_1}{\tau} \right] + O.T. \tag{3.9} \]

On simplification

\[ i \Delta^{(2)} \Gamma_{\rho}(p, q) = \left[ \frac{-1}{\sin^2 \theta_w} \right] \int_0^1 d \tau_1 \int_0^1 d \tau_2 \int_0^1 d \tau_3 \frac{1}{\tau_2^2} \left[2 + 4 \tau_1 \tau_2 - \frac{12 \tau_1}{\tau} \right] + O.T. \]
\[ = \left[ \frac{-1}{\sin^2 \theta_w} \right] \int_0^1 d \tau_1 \int_0^1 d \tau_2 \int_0^1 d \tau_3 \frac{1}{\tau_2^2} \left[2 + 4 \tau_1 \tau_2 - \frac{12 \tau_1}{\tau} \right] + O.T. \tag{3.10} \]

For the diagrams of Figs. 10(b) and 10(c) together, we find

\[ i \Delta^{(3)} \Gamma_{\rho}(p, q) = \left[ \frac{-1}{\sin^2 \theta_w} \right] \int_0^1 d \tau_1 \int_0^1 d \tau_2 \int_0^1 d \tau_3 \frac{\tau_2}{\tau^2} \left[2 + 4 \tau_1 \tau_2 - \frac{12 \tau_1}{\tau} \right] + O.T. \]
\[ = \left[ \frac{-1}{\sin^2 \theta_w} \right] \int_0^1 d \tau_1 \int_0^1 d \tau_2 \int_0^1 d \tau_3 \frac{\tau_2}{\tau^2} \left[2 + 4 \tau_1 \tau_2 - \frac{12 \tau_1}{\tau} \right] + O.T. \tag{3.11} \]

On simplification.

The diagram of Fig. 10(d) clearly involves exactly one \(\gamma\)-matrix and hence does not contribute to the anomalous magnetic moment.

The Z-exchange diagram of Fig. 10(e) yields a contribution

\[ i \Delta^{(4)} \Gamma_{\rho}(p, q) = \left( \frac{e}{2m} \sigma_{\rho \mu}(p - q)^\mu \right) \frac{e^2 m^2}{16 \pi^2 \Lambda^2} \frac{1}{\sin^2 \theta_w \cos^4 \theta_w} \times \frac{1}{2} \]
\[ \times \left[-1 + 4 \sin^2 \theta_w\right] \int d \tau_1 d \tau_2 d \tau_3 \frac{1}{\tau_2^2} \left(1 - \tau_2/\tau\right)(1 - \tau_1/\tau - \tau_2/\tau) \]
\[ + \left((-1 + 4 \sin^2 \theta_w)^2 - 1\right) \int d \tau_1 d \tau_2 d \tau_3 \frac{1}{\tau_2^2} \left(1 - \tau_2/\tau\right) \]
\[ + O.T. \tag{3.12} \]
\[
\begin{align*}
&= \left[ \frac{e^2}{2m} \sigma_{\mu}(p - q)^\mu \right] \times \frac{e^2 m^2}{16\pi^2 \Lambda^2} \times \frac{1}{\sin^2 \theta_w \cos^4 \theta_w} \times \\
&\quad \left[ 0.7876(8 \sin^4 \theta_w - 4 \sin^2 \theta_w) + 0.1060 \right] + \text{O.T.} \quad (3.13) \\
&= \left( \frac{e}{2m} \sigma_{\mu}(p - q)^\mu \right) \frac{\alpha m^2}{4\pi \Lambda^2} \left[ -2.0873 \right] + \text{O.T.} \quad (3.14)
\end{align*}
\]

Finally we note that the diagrams of Figs. 10(f) and 10(g) are suppressed by factors of \(m^2/m^2_W\). These diagrams contribute finitely to the anomalous magnetic moment and both the local and nonlocal contributions are suppressed by this additional factor; as an inspection of the contribution will show. We neglect them for present considerations.

The total shadow loop contribution for \(a_\mu\) is given by the sum of contributions from Eqs. 3.8, 3.10, 3.11, 3.14 as

\[
\begin{align*}
&= \left( \frac{e}{2m} \sigma_{\mu}(p - q)^\mu \right) \frac{\alpha m^2}{4\pi \Lambda^2} \left[ -10.0961 \right] \\
&= \left( \frac{e}{2m} \sigma_{\mu}(q - p)^\mu \right) \frac{65.45(Mev)^2}{\Lambda^2} \quad (3.15)
\end{align*}
\]

From this we can read off the nonlocal electroweak corrections to \(a_\mu\) of order \(m^2/\Lambda^2\)

\[
(\Delta a_\mu)_{th}^{nl} = \frac{(65.45)(Mev)^2}{\Lambda^2} \quad (3.16)
\]

The total theoretical contributions to \(a_\mu\) from local theory is \[10\]

\[
a_\mu^{th} = 1165918(2) \times 10^{-9} \quad (3.17)
\]

The experimental value of \(a_\mu\) is presently \[10\]

\[
a_\mu^{exp} = 1165923(8.5) \times 10^{-9} \quad (3.18)
\]

Therefore the contribution which may be attributed to nonlocal corrections, \(\Delta\), is

\[
\Delta = 5 \pm 8.5 \times 10^{-9} \quad (3.19)
\]

We note that the nonlocal contribution to the anomalous magnetic moment is positive and such as to close the gap between the local theoretical value and the experimental one (if the numbers are taken literally, ignoring error bars). But in view of the fact that the scale of nonlocality \((1/\Lambda)\) if it exists, may be quite small, this contribution is not quite large enough to explain the (literal) difference. We further note that even for a quite small \(1/\Lambda\) (of the order of \((500 \text{ Gev})^{-1}\)), the nonlocal contribution is comparable to the weak contribution. However, in view of the error bars in \[3.19\] we are unable at present to obtain a stringent enough bound on \(\Lambda\): we obtain
A similar bound has been obtained from nonlocal formulations using stochastic quantization.

We in fact expect the bound to improve substantially once the new experiment planned at Brookhaven National Laboratory [12] to determine $a_\mu$ is performed. The experimental error is expected to come down to $\pm 40 \times 10^{-11}$, i.e., by a factor of 20 compared to the presently available data. Once the results of this experiment are available, the hadronic contribution is calculated with greater accuracy [13], the bound of Eq. 3.20 could be improved by a factor of up to 5-6.

We should however point out that the result 3.16 has been obtained on certain assumptions made in the calculations. Thus, the numerical result of 3.16 is valid only if the actual scale of nonlocality $\Lambda^2 \gg M_W^2$; say $\Lambda > 300$ Gev. If $\Lambda$ were smaller than this, corrections of the orders $M_W^2/\Lambda^2$, $M_Z^2/\Lambda^2$ would become significant. We have not calculated these. However, we do expect these to alter the result 3.16 drastically so that the bound of 3.20 may still survive (modulo a factor not far from unity). Of course once the results of the upcoming experiment mentioned above are available, the bound on $\Lambda$ is expected to be raised so that our approximations can be sustained.

### IV. COMPARISON WITH NONSTANDARD CONTRIBUTIONS

As experimental accuracy with which $(g-2)$ of the muon is measured is improved in the latest experiments, comparison of these results with the results expected from (local) Standard Model (calculated with improved accuracy), is expected to reveal much new physics. Thus a discrepancy between these can be a signal of, say, nonlocality of the underlying Standard Model as considered in this work or a signal of new physics in addition to the Standard Model, viz. of the nonstandard effects of various kinds. In this section we shall compare our results with several other works on additional contributions to $(g-2)$ of the muon due to these nonstandard effects [14–16].

We shall compare our work successively with these works for their nonstandard physics considered in them and/or their methodology. While comparing these results we first make some general remarks.

(i) At the outset we point out a major difference: Our work uses Standard Model fields with Standard Model interactions and adds no other particles, and no arbitrary anomalous interactions to the Lagrangian except to nonlocalize the SM Lagrangian. And this nonlocalization is carried out in a way restricted by the preservation of a (nonlocal) BRS invariance, renormalizability and unitarity of the theory (and is thus mostly free from adhoc features). In most of the works on nonstandard effects, on the other hand, particles and/or couplings, foreign to Standard Model are introduced. This limits the scope of their comparison.
(ii) The second point one can make before going into specific works is as follows: In many of the nonstandard effects, a new (higher) mass scale \( \Lambda_{NS} \) is involved. (This could be the scale of compositeness, mass of additional new particles, scale introduced in form factors etc.) In most cases, the additional contribution to \( g-2 \) is of \( \mathcal{O}(m^2/\Lambda_{NS}^2) \). If, then, the coupling involved is also of \( \mathcal{O}(e) \), then, we would expect a bound on \( \Lambda_{NS} \) of the same order as on the scale of nonlocality \( \Lambda \) in our work.

(iii) Finally we make a remark on our methodology. Calculations done in our work involve a renormalizable gauge always, and are done without adhoc cutoff and renormalization procedures. In the works on nonstandard effects, a necessity for introducing adhoc cutoff on momentum integrals (however physical) and an adhoc subtraction procedure (however natural) arises.

With these remarks we proceed with specific comparisons.

The work of Mery et al. [14] takes into account nonstandard effects of various kinds that can arise at lower energies from a (local) composite model with a higher scale of compositeness. Compositeness of leptons and gauge bosons could lead to form factors for these. In addition, there could be excited states of the Standard Model particles (\( \mu^*, W^*, Z^* \) etc). There could be residual effective 4-fermion interactions below the scale of compositeness and nonstandard vector boson couplings. The effect of form factors is taken into account by introducing a phenomenological form factor \( (1 - k^2/\Lambda_F^2)^{-1} \) and performing the calculations in Unitary Gauge. The divergence of the integrals necessitates a physical cutoff \( \Lambda \sim \Lambda_F \). The remarks (iii) and (ii) made in the preceding paragraph directly apply here. Comparison with results for diagrams involving excited particles is difficult on account of remark (i) above except that remark (ii) directly applies here. As for the residual 4-fermion interactions, remarks (ii) and (iii) apply directly. It is, however, hard to compare the results for anomalous couplings of vector bosons as they have no analogue in our work.

The work of Carena et al. [15] discusses the corrections to \( g-2 \) in supersymmetric models. Here the corrections to \( g-2 \) arise from additional diagrams involving supersymmetric partners of various standard model particles, and depend on their masses. This work then explores bounds on these masses and couplings. In this sense this work is completely different from ours in that we accept standard model as essentially correct but for an allowed length scale \( 1/\Lambda \) and expect corrections from this scale. On account of the entirely different origins of the possible corrections to \( g-2 \) a direct comparison of the results seem difficult; except that one expects the scale \( \Lambda \) in nonlocal theories and \( \tilde{m} \), the scale of the masses of supersymmetric partners to be comparable. (Note remark (i) and (ii) made earlier.)

The work of Arzt et al. [16] explores the corrections to \( g-2 \) in a model independent way by formulating the ‘non-Standard Model’ terms as a series of dimension six operators in an Effective Lagrangian approach. It also naturally involves a scale \( \Lambda \) at which these nonstandard corrections become significant. The calculation does
require a cutoff procedure and additional renormalizations (involving extra renormalization conditions). As per remark (ii) we would expect the scale $\Lambda$ involved in the effective action to be comparable to our scale of nonlocality $\Lambda$ as the couplings have been assumed to be of the same order as $\epsilon$. We however note several differences in methodology.

It may appear at first sight that the nonlocal W-S Lagrangian expanded to $\mathcal{O}(1/\Lambda^2)$ is actually a special case of the effective Lagrangian approach albeit with known operators and known coefficients. In this connection we point out two things. Firstly, if only $\mathcal{O}(1/\Lambda^2)$ terms were retained in our $\mathcal{L}$, the convergence of integrals that is present in our approach with full $\mathcal{L}$, would be lost and a need for adhoc cutoff and renormalization procedure would be necessiated as in [16]. Please note remark (iii) made earlier. Secondly, the operators of $\mathcal{O}(1/\Lambda^2)$ arising in such an expansion of the nonlocal W-S action, would not be gauge invariant as our action is invariant under a nonlocal BRS transformation (i.e transformations themselves contain terms of various orders in $1/\Lambda^2$.) Of course the total action is BRS invariant. In this sense, the assumptions of Arzt et al. about the gauge invariant nature of dimension six operators is not directly fulfilled in such an expansion.
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FIGURES

\[ = -i \int_{1}^{\infty} \frac{d\tau}{\Lambda^2} e^{\frac{\tau}{\Lambda^2}} (p^2 - \mu^2) \]

\[ = -i \int_{0}^{1} \frac{d\tau}{\Lambda^2} e^{\frac{\tau}{\Lambda^2}} (p^2 - \mu^2) \]

FIG. 1. unbarred and barred propagators in nonlocal $\phi^4$ theory

FIG. 2. self energy diagram in nonlocal $\phi^4$ theory

FIG. 3. shadow loop self energy diagram in nonlocal $\phi^4$ theory
\begin{align*}
\text{fermion(smeared)} & : -i \int_{1}^{\infty} \frac{d\tau}{\Lambda^2} e^{\frac{i\tau}{\Lambda^2}(k^2-m^2)} (k + m) \\
\text{fermion(barred)} & : -i \int_{0}^{1} \frac{d\tau}{\Lambda^2} e^{\frac{i\tau}{\Lambda^2}(k^2-m^2)} (k + m) \\
\phi_1(\text{smeared}) & : -i \int_{1}^{\infty} \frac{d\tau}{\Lambda^2} e^{\frac{i\tau}{\Lambda^2}(p^2-2\mu^2)} \\
\phi_1(\text{barred}) & : -i \int_{0}^{1} \frac{d\tau}{\Lambda^2} e^{\frac{i\tau}{\Lambda^2}(k^2-m^2)} \\
\phi_2(\text{smeared}) & : -i \int_{1}^{\infty} \frac{d\tau}{\Lambda^2} e^{\frac{i\tau}{\Lambda^2}(k^2-m^2)} \\
\phi_2(\text{barred}) & : -i \int_{0}^{1} \frac{d\tau}{\Lambda^2} e^{\frac{i\tau}{\Lambda^2}(k^2-m^2)} \\
\phi(\text{smeared}) & : -i \int_{1}^{\infty} \frac{d\tau}{\Lambda^2} e^{\frac{i\tau}{\Lambda^2}(k^2-m^2_W)} \\
\phi(\text{barred}) & : -i \int_{0}^{1} \frac{d\tau}{\Lambda^2} e^{\frac{i\tau}{\Lambda^2}(k^2-m^2_W)} \\
\gamma(\text{smeared}) & : \mu \overset{\nu}{\longrightarrow} +i \int_{1}^{\infty} \frac{d\tau}{\Lambda^2} e^{\frac{i\tau}{\Lambda^2}k^2} g_{\mu\nu} \\
\gamma(\text{barred}) & : \mu \overset{\nu}{\longrightarrow} +i \int_{0}^{1} \frac{d\tau}{\Lambda^2} e^{\frac{i\tau}{\Lambda^2}k^2} g_{\mu\nu} \\
Z(\text{smeared}) & : \mu \overset{\nu}{\longrightarrow} +i \int_{1}^{\infty} \frac{d\tau}{\Lambda^2} e^{\frac{i\tau}{\Lambda^2}(k^2-m^2_W)} g_{\mu\nu} \\
Z(\text{barred}) & : \mu \overset{\nu}{\longrightarrow} +i \int_{0}^{1} \frac{d\tau}{\Lambda^2} e^{\frac{i\tau}{\Lambda^2}(k^2-m^2_W)} g_{\mu\nu} \\
W(\text{smeared}) & : \mu \overset{\nu}{\longrightarrow} +i \int_{1}^{\infty} \frac{d\tau}{\Lambda^2} e^{\frac{i\tau}{\Lambda^2}(k^2-m^2_W)} g_{\mu\nu} \\
W(\text{barred}) & : \mu \overset{\nu}{\longrightarrow} +i \int_{0}^{1} \frac{d\tau}{\Lambda^2} e^{\frac{i\tau}{\Lambda^2}(k^2-m^2_W)} g_{\mu\nu}
\end{align*}

FIG. 4.
FIG. 5.
$+ \text{ barred variations}$

FIG. 6.
FIG. 7.
FIG. 8.

FIG. 9.
FIG. 10.
FIG. 11. momentum routing in the Feynman diagrams