THE GIESSEN MODEL - VECTOR MESON PRODUCTION ON THE NUCLEON IN A COUPLED CHANNEL APPROACH

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In ref. 1 we developed a unitary gauge invariant effective Lagrangian model including the final states $\gamma N$, $\pi N$, $2\pi N$, $\eta N$, $K\Lambda$, and $K\Sigma$ (ref. 2) for a simultaneous analysis of all available experimental data for photon- and pion-induced reactions on the nucleon. In ref. 3 this analysis was extended to $K^-$ induced reactions. In this paper we discuss an extension of this method to vector meson nucleon final states, outline a generalization of the standard partial wave formalism, that is applicable to any meson-/photon-baryon reaction, and show first results for $\omega N$ production.

1 Introduction

The determination of nucleon resonance properties from experiments where the nucleon is excited either via hadronic or electromagnetic probes is one of the major issues of hadron physics. The goal is to be finally able to compare the extracted masses and partial decay widths with predictions from lattice QCD (e.g. ref. 4) and/or quark models (e.g. ref. 5).

As has been shown in ref. 1 for a reliable extraction of these properties it is inevitable to analyze photon and pion induced experimental data simultaneously for as many channels as possible. Therefore our coupled channel model developed in ref. 1 incorporates the final states $\gamma N$, $\pi N$, $2\pi N$, $\eta N$, and $K\Lambda$, where the $2\pi N$ channel was modelled for simplicity by an isovector $0^+$ meson.

But as soon as we try to extend the model to CMS energies up to $\sqrt{s} = 2$ GeV for an investigation of higher and so-called missing nucleon resonances, the inclusion of the $\omega N$ final state becomes unavoidable due to unitarity. This can be seen from the left panel of Fig. 1. Furthermore, $\omega$ production on the nucleon represents a possibility to project out $I = \frac{1}{2}$ resonances in the reaction mechanism. Due to its intrinsic spin the inclusion of the $\omega N$ final state in our coupled channel model requires an extension of the standard partial wave decomposition (PWD) method developed for $\pi N/\gamma N \rightarrow \pi N$ and $\gamma N \rightarrow \gamma N$ (see e.g. ref. 6). Such an extension is provided in section 4. In addition, this formalism enables us to achieve a realistic treatment of the most dominant inelastic channel in $\pi N$ scattering, i.e. the $2\pi N$ state via $\rho N$, $\pi \Delta$, and $\sigma N$. 
Figure 1. Left: Total cross sections for the reactions $\pi^- p \to X$ with $X$ as given in the figure. Data are from ref. 6. Right: Total partial wave cross sections for $\pi N \to 2\pi N$.

2 The Model

Our method to solve the Bethe-Salpeter (BS) equation is the so called $K$ matrix Born-approximation, which is equivalent to setting the intermediate particles in the BS propagator on-shell; for more details cf. ref. 1. Then the reaction matrix $T$, defined by $S = 1 + 2iT$, can be calculated from the potential $V$ after a PWD in total spin $J$, parity $P$, and isospin $I$ via matrix inversion:

$$T(p', p; \sqrt{s}) = \frac{V(p', p; \sqrt{s})}{1 - iV(p', p; \sqrt{s})}. \quad (1)$$

and unitarity is fulfilled as long as $V$ is hermitian.

The potential $V_{fi}$ is built up by $s$-, $u$-, and $t$-channel Feynman diagrams by means of effective Lagrangians which can be found in refs. 1, 3. The advantage of this method is that the background contributions are created dynamically and the number of parameters is greatly reduced, i.e. as compared to Breit-Wigner driven models or those including only pointlike interactions.

3 Results on (pseudo)scalar meson production

As an example for the quality of the calculations we show in the right panel of Fig. 1 the total partial wave cross sections, as extracted by the standard PWD, for $\pi N \to 2\pi N$ in comparison with the inelasticities from $\pi N \to \pi N$ ($\times$) and a $\pi N \to 2\pi N$ analysis ($\circ$). The necessity of the inclusion of a large set of final states in a coupled channel calculation can be seen in various partial waves. In the $S_{11}$ partial wave the difference between the inelasticity and the $2\pi$ analysis around $\sqrt{s} = 1.5$ GeV is easily explained by the opening of the $\eta N$ final state, the same holds true for $P_{11}$ above $\sqrt{s} = 1.6$ GeV and $K\Lambda$ and for
above $\sqrt{s} = 1.7$ GeV and $K\Sigma$. However, there still is a discrepancy left in the $P_{13}$ partial wave arising around 1.7 GeV. Since this particular calculation did not include the $\omega N$ final state yet, this problem might be solved upon a reanalysis including the $\omega N$ final state.

4 Vector Meson Production

Since the orbital angular momentum $\ell$ is not conserved in, e.g., $\pi N \rightarrow \omega N$ the standard PWD becomes really clumsy for many of the channels that have to be included. A more elegant and in particular uniform PWD for all channels would be desirable. Hence we use here a generalisation of the standard PWD method which represents a tool to analyze any meson- and photon-baryon reaction on an equal footing.

We start with the decomposition of a two-particle momentum states into states characterized by the total spin $J$ and its $z$-component $M$ (see ref. 8): 

$$ |p; JM, \lambda_1 \lambda_2 \rangle = \sqrt{\frac{2J + 1}{4\pi}} \int e^{i(M-\lambda)x} d^J_{M\lambda}(\theta) |p; \lambda_1 \lambda_2 \rangle d\Omega, \quad \lambda = \lambda_1 - \lambda_2, $$

where $\lambda_1$ and $\lambda_2$ are the helicities of the two particles and the $d^J_{M\lambda}(\theta)$ are Wigner functions. For the incoming CMS state ($\theta_0 = \varphi_0 = 0 \Rightarrow \ell_z = 0$) one gets

$$ \langle JM, \lambda_1 \lambda_2 | \theta_0 \varphi_0, \lambda_1 \lambda_2 \rangle \sim \delta_{M\lambda}, $$

hence $M = \lambda$ and one can drop the index $M$.

By using the parity property (cf. ref. 8) $\hat{P} |J, \lambda \rangle = \eta_1 \eta_2 (\lambda) \eta_1 \eta_2 (\lambda) ^{-1} |J, -\lambda \rangle$, where $\eta_1$, $\eta_2$, and $s_1$, $s_2$ are the intrinsic parities and spins of the two particles, the construction of states with a well defined parity is straightforward:

$$ \hat{P} |J, \lambda; P \pm \rangle := \hat{P} \frac{1}{2} (|J, +\lambda \rangle \pm |J, -\lambda \rangle) = \pm \eta_1 \eta_2 (1) ^{J-s_1-s_2} |J, \lambda; P \pm \rangle, $$

and we can use them to project out helicity amplitudes with definite parity:

$$ T^{\pm}_{\lambda \lambda} := \langle \lambda | \hat{T} | J; P \pm \rangle \quad (2) $$

$$ = \frac{1}{2} (T^J_{\lambda \lambda} \pm T^J_{\lambda \lambda} ^{-1}) \quad \text{with} \quad T^J_{\lambda \lambda} = \frac{1}{2} \int T^J_{\lambda \lambda}(x) d^J_{\lambda \lambda}(x) dx, \quad x = \cos \theta. $$

These helicity amplitudes $T^J_{\lambda \lambda}$ have definite, identical $J$ and definite, but opposite $P$. As is quite obvious this method is valid for any meson-baryon final state combination, even cases as e.g. $\omega N \rightarrow \pi \Delta$. In the case of $\pi N \rightarrow \pi N$ the $T^J_{\lambda \lambda}$ coincide with the conventional partial wave amplitudes: $T^J_{\lambda \lambda} \equiv T^J_{\ell \ell}$.

This PWD has been used for calculating pion- and photon-induced $\omega$ production on the nucleon. For our first results, we have applied the couplings set SM95-pt-3 from ref. 1, i.e. the $\omega$ only couples to the nucleon. However, as can be seen in the left panel of Fig. 5 for a reliable calculation of $\omega$ production on the nucleon the inclusion of rescattering is a basic requirement.


5 Outlook

The next step of the extension of our coupled channel $K$ matrix model will be the inclusion of $\omega N$ data in the determination of resonance properties. Furthermore, since the partial wave formalism is now settled, the inclusion of additional final states, in particular for a more sophisticated description of the $2\pi N$ final state, such as $\rho N$, $\sigma N$, and $\pi\Delta$ and accounting for their spectral function is straightforward.

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