Experimental Realization of One-Way Quantum Computing with Two-Photon Four-Qubit Cluster States

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We report an experimental realization of one-way quantum computing on a two-photon four-qubit cluster state. This is accomplished by developing a two-photon cluster state source entangled both in polarization and spacial modes. With this special source, we implemented a highly efficient Grover’s search algorithm and high-fidelity two qubits quantum gates. Our experiment demonstrates that such cluster states could serve as an ideal source and a building block for rapid and precise optical quantum computation.

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Highly entangled multipartite states, so-called cluster states, have recently raised enormous interests in quantum information processing (QIP) and quantum computation. This sort of states are crucial to be a fundamental resource and a building block aiming at one-way universal quantum computing [1, 2, 3]. They are also the essential elements for various quantum error correction codes and quantum communication protocols [4, 5]. Moreover cluster states are shown to be robust against decoherence, be persistent against loss of qubits, and thus are exceptionally well suitable for quantum computing and many quantum information tasks [1, 2, 3, 4, 5]. Considerable efforts have been stepped toward generating and characterizing cluster states in linear optics [6, 7, 8, 9, 10, 11, 12]. Recently the principal feasibility of one-way quantum computing model has been experimentally demonstrated through 4-photon cluster state successfully [8, 11, 13].

So far, with current method, preparing photonic cluster state still suffers from several serious limitations. Due to the probabilistic nature and Poissonian distribution of the parametric down-conversion process, the generation rate of 4-photon cluster states is quite low [7, 8, 9, 11], and thus largely restricts speed of computing. Besides, the quality and fidelity of prepared cluster states are relatively low [8, 11, 11], which are difficult to be improved substantially. These disadvantages consequently impose great challenges of advancement even for few-qubit quantum computing.

Fortunately, motivated by the progress that an important type of states termed hyper-entangled states or double entangled states have been experimentally generated in various ways [14, 15, 16, 17], we have the possibility to generate a new type of cluster state (2-photon 4-qubit cluster state) with nearly perfect fidelity and high generation rate. The hyper-entangled states that are entangled in more than one degree of freedom, has been used to test “All-Versus-Nothing” (AVN) quantum nonlocality [14, 15, 16, 19], and is shown to lead to an enhancing violation of local realism [20, 21]. The states also enable to perform complete deterministic Bell state analysis [22] as demonstrated in [17, 23].

In this Letter we report an experimental realization of one-way quantum computing with such a 2-photon 4-qubit cluster state. The key idea is to develop and employ a bright cluster state source which produces a 2-photon state entangled both in polarization and spacial modes. We are thus able to implement the Grover’s search algorithm and quantum gates with excellent performances. The genuine four-partite entanglement and high fidelity of better than 88% for this cluster state are characterized and verified by measurement of an optimal entanglement witness with a minimal number of measurement settings. Inheriting the intrinsic two-photon character, compare with the one using multi-photon, our scheme promises a brighter source in quantum computing by more than 4 orders of magnitude, which offers a significantly high efficiency for optical quantum computing. It thus provides a simple and fascinating alternative to complement the usual multi-photon cluster state. With ease of manipulation and control, the nearly perfect quality of this source allow to perform a highly faithful and precise quantum computing with fidelities about 95% on average for quantum gates outputs.

To generate the 2-photon 4-qubit cluster state we use the technique developed in previous experiments [15] with type-I spontaneous parametric down-conversion (SPDC) source [24]. The experimental setup for generating 2-photon 4-qubit cluster state is shown in Fig. 1. A pulse of ultraviolet (UV) light passes twice through two contiguous beta-barium borate (BBO) with optic axes aligned in perpendicular planes to produce one polarization entangled photon pair, with one possibility in the forward direction of generating a state $(|H\rangle_A |H\rangle_B +$
FIG. 1: Schematic of experimental setup. By pumping a two-crystal structured BBO by a UV pulse in a double pass configuration, one polarization entangled photon pair is generated by Type-I-SPDC with two possibilities in the forward direction and in the backward direction, respectively, to perform the preparation of 2-photon 4-qubit cluster state. The UV laser with a central wavelength of 355 nm has pulse duration of 5 ps, a repetition rate of 80 MHz, and an average pumping power of 200mW. Two quarter-wave plates (QWPs) are tilted along their optic axis to vary relative phases between polarization components to attain two desired possibilities for entangled pair creation. Concave mirror and prism are used to optimize interference and overlapping on two beam splitters (BS1, 2) or two polarization beam splitters (PBS1, 2) for achieving the target cluster state. Half-wave plates (HWPs) together with polarizing beam splitters (PBS) and 8 single-photon detectors (D1-D8) are used for polarization analysis of the output state. Finally, we observe a cluster state generation rate of about $1.2 \times 10^7$ per second behind 3 nm filters (IF) of central wavelength 710 nm.

$|V\rangle_A |V\rangle_B)/\sqrt{2}$ on spacial (path) modes $L_{A,B}$, and another possibility in the backward direction of producing a state $(|H\rangle_A |H\rangle_B - |V\rangle_A |V\rangle_B)/\sqrt{2}$ on spacial modes $R_{A,B}$. Here $|H\rangle (|V\rangle)$ stands for photons with horizontal (vertical) polarization.

Through perfect temporal overlaps of modes $R_A$ and $L_A$ and of modes $R_B$ and $L_B$, one can obtain a state with coherent superposition

$$\left(\left(|H\rangle_A |H\rangle_B + |V\rangle_A |V\rangle_B\right) |L\rangle_A |L\rangle_B + e^{i\theta}\left(|H\rangle_A |H\rangle_B - |V\rangle_A |V\rangle_B\right) |R\rangle_A |R\rangle_B\right)/2. \quad (1)$$

By properly adjusting the distance between the concave mirror and the crystal, so that $\theta = 0$, the generated state is exactly the desired 2-photon 4-qubit cluster state

$$|C_4\rangle = \left(|0000\rangle_{1234} + |0011\rangle_{1234} + |1100\rangle_{1234} - |1111\rangle_{1234}\right)/2, \quad (2)$$

if we identify photon A to be qubits 2,3 and photon B to be qubits 1,4 and encode logical qubits as $|H(V)\rangle_B \leftrightarrow |0(1)\rangle_1, |H(V)\rangle_A \leftrightarrow |0(1)\rangle_2, |L(R)\rangle_A \leftrightarrow |0(1)\rangle_3, |L(R)\rangle_B \leftrightarrow |0(1)\rangle_4$. We observe a cluster state generation rate about $1.2 \times 10^4$ per second for 200mw UV pump, which is 4 order of magnitude more than the usual 4-photon cluster state production $[3, 11]$. To evaluate the quality of the state, we apply an optimal entanglement witness given in $[23]$ to characterize genuine four-partite entanglement for $|C_4\rangle$. The witness is of form

$$W = \left(4 \cdot I^\otimes 4 - (XXIZ + XXZI + IIZZ + IZXX + ZIXX + ZZII)\right)/2, \quad (3)$$

where $I$ is a 2-dimensional identity matrix while $Z = (|0\rangle\langle 0| - |1\rangle\langle 1|), X = (|0\rangle\langle 1| + |1\rangle\langle 0|)$ are Pauli matrices. A negative value for the witness implies 4-partite entanglement for a state close to $|C_4\rangle$ and will be optimally as -1 for a perfect cluster state. Only two experimental settings of $XXXZ$ and $ZZXX$ are needed. As for our cluster state of Eq. (2), $XXXX$ can be retrieved out by measuring in the +/− basis for the polarization in each output arm where BS1,2 and PBS1,2 are removed, while $ZZXX$ can be realized by measuring in the $H/V$ basis for the polarization in each output arm after BS1,2. This is because BS acts exactly as a Hadamard transformation for the path modes to change $Z$ basis to $X$ basis for measurement, namely, $|L\rangle_A |B \rightarrow (|R\rangle_A |B + |L\rangle_A |B))/\sqrt{2}$, $|R\rangle_A |B \rightarrow (|R\rangle_A |B - |L\rangle_A |B))/\sqrt{2}$. All of the observables needed to evaluate the witness are listed in Table I. Substituting their experimental values into Eq. (3) yields $|W|_{exp} = -0.766 \pm 0.004$, which clearly proves the genuine four-partite entanglement by about 200 standard deviations. As shown in $[23]$, one can obtain a lower bound for fidelity of experimental prepared state to $|C_4\rangle$

$$F \geq 1 - \frac{1}{2} \left|W\right|_{exp} = 0.883 \pm 0.002. \quad (4)$$

We attribute impurity of our state to imperfect overlapping on BS, deviations of the beam splitters from 50%, as well as imperfections in the polarization and path modes analysis devices.

A cluster state can be represented by an array of nodes, where each node is initially in the state of $|+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$. Every connected line between nodes experiences a controlled-phase (‘CPhase’) gates acting as

| Observable | Value          | Observable | Value          |
|------------|----------------|------------|----------------|
| XXIZ       | 0.9070 ± 0.0036 | IZXX       | 0.9071 ± 0.0037 |
| XXZI       | 0.9076 ± 0.0035 | ZIXX       | 0.8911 ± 0.0040 |
| IIZZ       | 0.9812 ± 0.0016 | ZZII       | 0.9372 ± 0.0030 |

**TABLE I:** Experimental values of all the observable on the cluster state $|C_4\rangle$ for the entanglement witness $W$ measurement. Each experimental value corresponds to measure in an average time of 1 sec and considers the Poissonian counting statistics of the raw detection events for the experimental errors.
measurement output of a Hadamard operation determines a rotation. The state $|00\rangle$ can be represented by a box type graph shown in Fig. 2, up to a local unitary transformation.

**Grover’s algorithm.** For an unsorted database with $N$ entries generally, Grover’s algorithm would give a quadratic speed-up on average for search compared with the one classically [20]. Linear optics implementations have been achieved in [27] through single photon and in [4] [11] through one-way realization. A single quantum search will find the marked element for the case of four entries $|00\rangle, |01\rangle, |10\rangle, |11\rangle$. An execution of quantum search goes as follows: an oracle encodes a desired entry by changing its sign through a black box with initial states $|++\rangle$. After an inversion-about-the-mean operation, the labelled element will be found with certainty by readout measurements. It is shown in [4] that this can be exactly finished with the box cluster state shown in Fig. 2a. For demonstration, we experimentally tag the element $|00\rangle$ in qubits 2, 3 and make the final readout measurements on qubits 1, 4 all along basis $B(\pi)$. By noting the fact that the state Eq. (2) differs from the box cluster state up to a $H$ transformation on every qubit and a swap between qubits 2 and 3, this amounts to measure along the $\{|V\rangle, |H\rangle\}$ basis for the polarization in each output arm after PBS$_1$ and PBS$_2$. The output of the algorithm is two bits $\{s_3\oplus s_4, s_1\oplus s_2\}$ in lab basis by feed-forwarding outcomes of qubits 2, 3, where $s_i$ are measurement outcomes for qubits $i$. The experimental results are sketched in Fig. 2.

**Quantum gates.** Non-trivial two-qubit quantum gates such as the CPhase gate are at the heart of universal quantum computation in addition to single qubits rotations, that can be realized by cluster states conveniently. Depending on the initial cluster state and measurement basis, states with different degrees of entanglement can be generated. The horseshoe or box cluster shown in Fig. 3a and 3c can realize such important gates. For the case of horseshoe cluster in Fig. 3a, depending on the outcomes for measurement along basis $B_{2,3}(0)$, the output state on qubits 1, 4 would be $|\Omega_{out}\rangle = (X^s \otimes X^{s_3}) (H \otimes H) (R_z(-\alpha) \otimes R_z(-\beta)) C\text{Phase} |\Omega_{in}\rangle$, where $|\Omega_{in}\rangle = |++\rangle$. The state $|\Omega_{out}\rangle$ is always a maximal entangled state. Consider outcomes ‘00’ in qubits 2, 3. This implies a final Bell state of $|\Omega_{out}\rangle = (|+\rangle \langle 0| + |-\rangle \langle 1|) / \sqrt{2}$. Note that the horseshoe cluster state is equivalent to our cluster state of Eq. (2) up to a $H H H$ transformation, in lab basis this amounts exactly the same state as $|\Omega_{out}\rangle$ that is invariant under $H H$ transformation. To characterize quality of quantum gates outputs and distinguish this state from other Bell states completely, we put a
birefringent crystal in path $R_B$ to make a transformation $|+\rangle \leftrightarrow |-\rangle$ for polarization of photon $B$ in the path. After BS$_2$, all the Bell states on qubits 1,4 will change as

$$
(\langle +|_1 |0\rangle_4 \pm |\rangle_1 |1\rangle_4)/\sqrt{2} \rightarrow |+\rangle_1 |\pm\rangle_4,
$$
$$
(\langle -|_1 |0\rangle_4 \pm |+\rangle_1 |1\rangle_4)/\sqrt{2} \rightarrow |-\rangle_1 |\pm\rangle_4,
$$

(5)

which can be completely and deterministically discriminated by measuring along $|\pm\rangle$ basis in each output arm after BS$_2$. The fidelities of the output states in the lab basis to the ideal Bell state are shown in Fig. 3. Similarly, for the box cluster state shown in Fig. 3c, measurements on qubits 2,3 along basis $\{B_2(\pi), B_3(0)\}$ will give an output state on qubits 1,4 with

$$
\Omega_{out} = (Z \otimes X)^\alpha (X \otimes Z)^\beta \text{CPhase}(H \otimes H)(R_x(-\alpha) \otimes R_z(-\beta)) \text{CPhase}(\Omega_{in})
$$

which is a product state when $\alpha = \pi$ and $\beta = 0$. Since we can completely distinguish 4 different products states, the fidelities of the output states to the desired states can be obtained directly as shown in Fig. 3b. By employing the techniques developed in [11] with active feed-forward, one can expect to achieve deterministically quantum computing with excellent quality outputs employing our cluster state source.

We remark that other type of 2-qubit entangled states can be accordingly generated, by achieving suitable basis measurements on qubits 2,3. In addition, single-qubit operation can be conveniently obtained through one-way realization. However, an arbitrary single-qubit rotation needs measurements on at least two nodes on a cluster, which is a large consuming of resource considering the exponential inefficiency for generation of both multi-photon and our source. Fortunately, arbitrary single-qubit rotation can be easily attained both for polarization and path modes by linear optical components. Therefore a hybrid framework would be more practical and resource-efficient, by means of one-way implementation of two-qubit gates and usual realization of single-qubit gates for a universal optical quantum computing.

In summary, we have developed a scheme for preparation of a two-photon four-qubit cluster state, designed and demonstrated the first proof-of-principle experimental realization of one-way quantum computing employing such a source. The excellent quality of the state with fidelity better than 88% is characterized by an optimal witness without using of a full state tomography. Moreover, high count rates of the state creation enable more efficient quantum computing by 4 orders of magnitude than previous methods. We have thus achieved implementation of Grover’s algorithm with a successful probability of about 96% and quantum gates with high fidelities of about 95% on average to ideal outputs. Our results help to make a significant advancement of QIP and allow our source to act as a promising candidate for an efficient and high quality one-way optical quantum computing. By using more photons and more degrees of freedom, one can expand the ability to generate many-qubit cluster states for performing quantum computing and other complex tasks. Our results can also find rapid applications in quantum error correction codes, multi-partite quantum communication protocols [4, 5], as well as novel types of “All-Versus-Nothing” tests for nonlocality [18, 19, 28].

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Note added.– During the preparation of our manuscript, we are aware of one related experiment for realization of a linear cluster state entangled in both polarization and linear momentum [20].

[1] H.J. Briegel and R. Raussendorf, Phys. Rev. Lett. 86, 910 (2001).
[2] R. Raussendorf and H.J. Briegel, Phys. Rev. Lett. 86, 5188 (2001).
[3] R. Raussendorf, D.E. Browne, and H.J. Briegel, Phys. Rev. A 68, 022312 (2003).
[4] D. Schlingemann and R.F. Werner, Phys. Rev. A 65, 012308 (2002).
[5] R. Cleve, D. Gottesman, and H.-K. Lo, Phys. Rev. Lett. 83, 648 (1999).
[6] D.E. Browne, and T. Rudolph, Phys. Rev. Lett. 95, 010501 (2005).
[7] P. Walther, M. Aspelmeyer, K.J. Resch, and A. Zeilinger, Phys. Rev. Lett. 95, 020403 (2005).
[8] N. Kiesel et al., Phys. Rev. Lett. 95, 210502 (2005).
[9] P. Walther et al., Nature (London) 434, 169 (2005).
[10] T.P. Bodiya, and L.-M. Duan, Phys. Rev. Lett. 97, 143601 (2006).
[11] R. Prevedel et al., Nature (London) 445, 65 (2007).
[12] C.-Y. Lu et al., Nature Physics 3, 91-95 (2007).
[13] M.S. Tume et al., Phys. Rev. Lett. 98, 140501 (2007).
[14] C. Cinelli, M. Barbieri, R. Ferris, P. Mataloni, and F. De Martini, Phys. Rev. Lett. 95, 240405 (2005);
[15] T. Yang et al., Phys. Rev. Lett. 95, 240406 (2005).
[16] J.T. Barreiro, N.K. Langford, N.A. Peters, and P.G. Kwiat, Phys. Rev. Lett. 95, 260501 (2005).
[17] C. Schuck, G. Huber, C. Kurtsiefer, and H. Weinfurter, Phys. Rev. Lett. 96, 190501 (2006).
[18] A. Cabello, Phys. Rev. Lett. 87, 010403 (2001).
[19] Z.-B. Chen, J.-W. Pan, Y.-D. Zhang, C. Brukner, and A. Zeilinger, Phys. Rev. Lett. 90, 160408 (2003).
[20] A. Cabello, Phys. Rev. Lett. 97, 140406 (2006).
[21] M. Barbieri, F. De Martini, P. Mataloni, G. Vallone, and A. Cabello, Phys. Rev. Lett. 97, 140407 (2006).
[22] P.G. Kwiat, and H. Weinfurter, Phys. Rev. A 58, R2623 (1998).
[23] M. Barbieri, G. Vallone, P. Mataloni, and F. De Martini, e-print, arXiv:quant-ph/0609080v2.
[24] P.G. Kwiat, E. Waks, A.G. White, I. Appelbaum, and P.H. Eberhard, Phys. Rev. A 60, R773 (1999).
[25] G. Tóth, O. Gühne, Phys. Rev. A 72, 022340 (2005).
[26] L.K. Grover, Phys. Rev. Lett. 79, 325 (1997).
[27] P.G. Kwiat, J.R. Mitchell, P.D.D. Schwindt, and A. G. White, J. Mod. Opt. 47, 257 (2000).
[28] A. Cabello, Phys. Rev. Lett. 95, 210401 (2005).
[29] G. Vallone et al., e-print, arXiv:quant-ph/0703191v1.