Adapting solution: mathematical thinking abilities that facilitate problem solving ability in students

J V Septriwanto¹, E Retnowati¹ and B A O Rampean²

¹ Graduate School of Mathematics Educations, Universitas Negeri Yogyakarta, Indonesia
² Graduate School of Chemistry Educations, Universitas Negeri Yogyakarta, Indonesia

E-mail: Jewishvan.2019@student.uny.ac.id

Abstract. This article aims to present the results of the synthesis of adapting solutions in helping students solve mathematical problems. Adapting solution is a transfer strategy that adapts solutions provided in problem-solving. The selection of the right solution helps solve students’ mathematical problems by adapting to see the relationship between examples of identical problems or not identical to the given problem solutions. It is easy for students to adapt solutions to similar types of problems and the same solutions. However, what if students find two or more examples, one of which is more complicated than their assignment problem, and one of them is less complicated. Which should they choose as a more helpful guide? Alternatively, the difficulty is due to the inability to modify examples of simple problems to complex problems or vice versa. To answer the above problem formulation, in this article we will propose the use of learning strategies with adapting solutions and the proposed learning procedures of adapting solutions that are focused on facilitating students’ problem-solving abilities.

1. Introduction
Problem-solving is a critical component of many sciences, mathematics, and engineering classes. Problem-solving considered being an essential 21st-century skill that combines cognitive and metacognitive processes. As stated by Gagne, that problem-solving is the highest learning stage. Students use problem-solving skills to analyze problems [1] where students must determine relevant information, how the relationship is related to the problem, and what steps must take to solve the problem. Because of the importance of problem-solving and mathematical problems, students need strategies to develop problem-solving abilities to approach and answer various types of mathematical problems, especially Word Problems in mathematics [2]. Word problems in mathematics are one of the most challenging parts of mathematics for many students [3]. The best way to teach children how to solve word problems in mathematics is to give them many problems to solve [4] or practice the primary independent variable in acquiring problem-solving abilities [5]. Strategies for solving problems that play a role in all stages of problem-solving include understanding problems, solving problems, reflecting answers, and solutions [6][7]. One effective way or strategy for all stages of problem-solving is to learn adapting solutions. Adapting solution is a strategy used to solve a problem by looking at resolving other problems that have a similar solution. The selection of the right solution is very helpful in solving students’ mathematical problems by adapting to see relationships between...
identical or non-identical examples of solutions to the problem given and modifying simple problems to complex problems or vice versa [8].

It is easy for students to adapt solutions of similar types of problems and the same solutions. What if they find two examples, one of which is more complicated than their assignment's problem and one of which is less complicated. Which should they choose as a more helpful guide? The difficulty here is that some of the relationships between the two problems are not identical, and students often have difficulty modifying their relationship. Because of this difficulty, learning is created by increasing students' ability to adapt solutions of match given problems. Students tend to use problem-solving strategies such as trial and error, while students are often presented with examples of problem solutions before solving tests. It is considered necessary for students to use problem-solving strategies by learning adapting solutions and learning procedures that are more efficient and focus on structure problem [9].

Learning Adapting Solution in learning mathematics is one that has not been much focus on various mathematical studies. So the use of learning strategies for adapting solutions needs to be studied to be understood in depth how they relate to mathematical problem-solving abilities. The purpose of this paper is to examine the use of learning strategies adapting solutions in facilitating mathematical problem-solving abilities by using the literature review method, where researchers refer to relevant international journals to study the results of research adapting solutions.

2. Method
The sources used are relevant and valid journals and books that discuss adaptation solutions in problem-solving. In this study, the five steps of identification with the synthesis technique [8] are described as follows: (1) identifying and defining the topic of adapting solutions in mathematics, (2) identifying sources that cover essential aspects of characteristics and the conceptual model of adaptive problem-solving or factors related to adapting solutions in mathematics, (3) reviewing the findings to be analyzed and interpreted, (4) writing down the findings, and (5) applying the findings to recommend and offering learning procedures for adapting solutions to facilitate students' problem-solving abilities.

3. Result and Discussion
3.1. Adapting Solution
Adapting solution is a strategy used to solve a problem by looking at the resolution of other problems that have a similar solution [10]. Adapting solution is a transfer strategy that adapts the solutions provided in problem-solving [8]. Adapting solution is a form of problem-solving that requires a series of reformulations of the problem or continuously re-evaluating changes in the problem situation [11]. In addition to adapting solutions in Word Problems, several other strategies can be developed in learning by abstracting solutions and representing solutions to transfer students' mathematical knowledge [8] [12].

3.2. Adapting Solutions in Problem Solving
Problem-solving is thinking directed at goals and actions in situations where there is no solution procedure available [13]. Students develop problem-solving from concrete problems to abstracts during mathematical activity [14]. Problem-solving leads to solutions to problems that can increase the effectiveness of mathematics learning [15]. Students usually find it challenging to adapt solutions because they often rely too much on sample solutions without making necessary adjustments. Solutions sometimes have to be adjusted. How well students choose the right example to be adapted? Will the solution to a simple problem or the solution to a more complex problem be more helpful in solving the same problem? Consider the following three problems showing the increasing complexity (inclusive) of the problem.
Example problem:

| Problem 1                                                                 | Problem 2                                                                 | Problem 3                                                                 |
|---------------------------------------------------------------------------|---------------------------------------------------------------------------|---------------------------------------------------------------------------|
| A group of people pay 238 dollars to buy a play ticket. How many people   | A group of people pay 306 dollars to buy theatre tickets. When seven      | A group of people pay 70 dollars to watch a basketball game. When eight   |
| are in the group if the ticket costs 14 dollars each?                      | people join the group, the total cost is 425 dollars. How many people     | people join the group, the total cost is 120 dollars. How many people    |
|                                                                           | are in the first group if all tickets have the same price?                | are in the first group if the largest group                                |
|                                                                           |                                                                           | receives a 20% discount?                                                  |

The first problem is less inclusive than the second problem because the information is reduced or simple problems. The third problem looks more inclusive than the second problem because it has excess information or more complex problems. Students are twice as successful at building equations for the example problem above when given a more inclusive solution as when given a less inclusive solution. Students are shown solutions to simple problems and complex problems. Then, they are asked to solve test problems. A study shows that students often use complex solutions more often than simple solutions to solve test problems [16]. Reed [8] found that students were very good at adapting complex solutions obtained with adaptations that were quite difficult, even when adapting complex solutions to fit the given simple problem. A good strategy will always choose a simple example when a problem assignment is equivalent to a simple example, and always choose a complex example when a problem is equivalent to a complex example. Nevertheless, it will also be easier to modify complex solutions to fit more specific problems than modifying simple solutions to fit more complex problems. It enhances students’ skills in using analogy solutions.

Effective learning uses several examples for each type of conceptual problem; the format of examples varies in the type of problem and uses fundamental concepts to guide the structure of deeper problems. Also, the examples must be present almost identical to the problem of the corresponding practice. Besides, students can be encouraged through hands-on training or sample structures that actively explain their sample solutions.

Learning Adapting Solution shows the process involved in concept formation [17] [18]. Examples usually present solutions in step-by-step mode and match between identical or non-identical problems with the concept to find the relationship between the two [8].

Consider an example of an identical problem. How to match the concept?

| Concepts | Matching Concept |
|----------|-----------------|
| 1. 40 pounds | 1. 65 kg |
| 2. d feet | 2. d meters |
| 3. 50 pounds × d feet | 3. 80 kg × d meters |
| 4. 6 feet | 4. (4-d) meters |
| 5. 50 pounds | 5. 80 kg |
How are the two problems the same?
Both problems are examples of fulcrum problems. Weight and distance in the first problem correspond to weight and distance in the second problem.

How are the two problems different?
In the first problem, it was told that Laurie sat 6 feet from the fulcrum. In the second problem, we must represent Mary’s distance from the fulcrum to Tom’s distance. Mary and Tom sat 4 meters apart, and Tom sat d meters from the fulcrum so that Mary sat for sure (4 - d) meters from the fulcrum.

Difficulty students do not know what to do on the test problem section when the test problem does not match the example. A study shows that combining a symbolic representation of one’s knowledge is successful in mapping isomorphic relationships by adjusting the solution to the test problem but not succeeding in adapting the solution [19]. So, what do instructors do to help students adapt solutions when problems are not identical? Different instructions will determine whether students’ ability to adapt solutions can be improved. The ability to construct equations for problem-solving can be improved by combining examples and giving them a list of rules from example problems [20]. The aim of the rules is to show how different mathematical values could replace each quantity in the equation.

Consider the following example problem.
Ann can type the script in 10 hours, and Florence can type it in 5 hours. How long will it take if the two of them work together?

Note the rules in the example problem represent time.
1. Time refers to the amount of time each worker contributes to the task. Suppose this value is stated in the problem and entered into the equation. For example, if one person works for 5 hours, enter 5 hours into the equation for that worker.
2. Time is an unknown variable in the problem. Be sure to represent the time of the object measured from another object. For example, if one worker works 3 hours more than the value (h) the time indicated is h + 3 for that worker.

3.3. Learning procedures of Adapting Solution
Strategies to explain how people adapt to their problem solutions in problem-solving abilities is part of the adaptive process [21]. They stated that a framework was needed and developed into a more formal process so that it was real problem solving is better. A small illustration of adaptive problem solving to help understand the concepts is included in the following chart.

The purpose of the adapting solution strategy is to show how examples should be selected, presented, and ordered [22] to facilitate concept learning and improve students’ problem-solving abilities to more complex forms of knowledge and learning [23]. Learning with adapting solutions provides opportunities to develop cognitive abilities from problem solutions [24]. The solution being studied challenges students to build their understanding, including their representation when test problems are presented, by maintaining the level of test difficulty and by giving students the right time and support to solve their assignments [25].
Some things to consider in the learning procedure of *adapting solution* include:

a. understanding the core characteristics of the problem, including the context, scope, and complexity of the problem;
b. after choosing an analogy solution, it is necessary to map the object and the relationship of the solution to test problems; and

c. one approach is to provide specific instructions or rules about the relationship between problems.

**Figure 1.** A model for adaptive problem solving [21].

**Figure 2.** Conceptual model of adaptive problem solving [26].

**Stages of adaptive problem solving:**

*Adapting solutions* involve three main stages, namely, defining problems, finding solutions, and implementing solutions. These three stages involve cognitive processes and metacognition processes [26]

1) **Defining problems**

- Searching for information
- Selecting, organizing, and integrating problem information into a mental model
- Generating relevant background information
- Forming an external representation of the problem

2) **Searching**

- Setting goals
- Monitoring comprehension
- Searching information in the mind and environment
- Evaluating the quality of information
- Attending to information relevant to goals
- Changing goals and strategies based on feedback

3) **Application**

- Applying plans to solve problems
- Executing operators
- Evaluating the soundness of feedback from people and environment
- Distinguishing between surface and structural features
- Monitoring progress
- Regulating the application process
- Comparing alternative solutions
- Reflection

**INTERNAL WORLD** (involves manipulation and storage of information in memory)

*Definition* | *Searching* | *Application*
---|---|---
- Setting goals | - Searching for information | - Applying plans to solve problems
- Monitoring comprehension | - Selecting, organizing, and integrating problem information into a mental model | - Executing operators
- Generating relevant background information | - Evaluating the quality of information | - Evaluating the soundness of feedback from people and environment
- Forming an external representation of the problem | - Attending to information relevant to goals | - Distinguishing between surface and structural features

**INTERFACE** (contains the link to external knowledge)

**INFORMATION ENVIRONMENT** (PHYSICAL, SOCIAL, DIGITAL)
The processes of defining problems require one to build a situation model described in the problem [27]. Creating a situation model requires sufficient factual and conceptual knowledge to determine and understand all further problem-solving activities by finding information and gathering information that may be relevant to the Solution of the problem.

2) Finding a solution

The solution to a problem can be explained in terms of the sequence of steps needed in the solution. Planning a solution will likely make the problem solver aware of the fact that there is important information missing from the problem to devise a solution method. Adapting Solution determines when to use the same solution again. This Adapting Solution is enough to show how students can generalize complex solutions to solve simple problems or vice versa.

3) Implementing the solution

This stage depends on the available procedural knowledge. In applying the solution, alternative solutions will be found from the example problems. Most students get a reflection when solving mathematical problems forming their understanding [28]. Through the application of the solution students are expected to be able to establish a connection between the solution of the given problem sample and the problem test.

3.4. Learning Adapting Solution recommendations

Different methods can solve most mathematical problem-solving. The learning method is Adapting Solution supported by many instructional approaches [29][30][31]. Some supportive include learning with multiple solutions; one of the solution methods helps learners to apply adapting solutions more effectively to use a combination of strategies rather than relying on one strategy to solve a problem [32][33]. We have the opportunity to use different representations in developing problem-solving skills and encourage students' understanding to focus on comparing different solutions or explaining appropriate learning instructions. The main advantages of this method are: (a) provides a representation of many problems by combining representations that contain complexity and simple information; (b) some representations can help to avoid misinterpretations; and (c) connecting some representations is useful to gain a deeper understanding [34] [35] [36]. Learning that supports others is learning with worked examples. This learning helps in problem-solving as students adapt and find solutions, thus enabling them to understand solutions and basic principles [9] [27]. This learning facilitates learning adapting solutions if designed appropriately and helps reduce students' cognitive load [37].

Problem-solving in learning schools mathematics is one of the mathematical competencies, which is an important 21st-century skill combining cognitive processes and students' metacognitive processes. Adapting solutions facilitate all these processes by adapting solutions to various types of mathematical problems. This learning strategy might have sufficient challenges for teachers to develop their learning procedures so that students' problem-solving abilities can be increased and facilitated because teacher's instruction in learning is the key to successful learning through adapting solutions. The teacher must facilitate student learning to build students' mathematical concepts and procedural knowledge in problem-solving by adopting solutions from examples of mathematical problems. It is not easy if the problem's complexity increases, so students must be familiar with the change from simple problems to complex mathematical problems or vice versa by modifying the solution and understanding the relationship between problem solutions with their problem tests.

4. Conclusion

Students problem-solving develop from simple problems to complex problems and demand solutions. Adapting solution strategies becomes an alternative to facilitate students' ability to solve these problems where the solution must sometimes be adjusted to how well students choose the right problem solution to be adapted. It is expected that the learning procedure of adapting solutions could be one of the learning strategies for solving mathematical problems used.
Reference

[1] Miller S and Hudson P 2007 Using Evidence-Based Practices to Build Mathematics Competence Related to Conceptual, Procedural, and Declarative Knowledge Learn. Disabil. Res. Pract. 22 47-57

[2] Schleppegrell M J 2007 The Linguistic Challenges of Mathematics Teaching and Learning: A Research Review Read. Writ. Q. 23 139-59

[3] Van Garderen D 2007 Teaching Students With LD to Use Diagrams to Solve Mathematical Word Problems J. Learn. Disabil. 40 540–53

[4] Davidson J E and Sternberg R J 2003 The Psychology of Problem Solving ed J E Davidson and R J Sternberg (Cambridge: Cambridge University Press)

[5] Hall G 2016 Mathematics Problem Solving Strategies

[6] Usman M, Abdul R and Ansari S A 2017 Analysis of the abilities in mathematical problem solving based on SOLO taxonomy and cognitive style World Trans. Eng. Technol. Educ. 15 68–73

[7] Bell A W 1979 The learning of process aspects of mathematics Educ. Stud. Math. 10 361–87

[8] Reed S K 1999 The studies in mathematical thinking and learning series Word problems: Research and curriculum reform (Lawrence Erlbaum Associates Publishers)

[9] Sweller J and Cooper G A 1985 The Use of Worked Examples as a Substitute for Problem Solving in Learning Algebra Cogn. Instr. 2 59–89

[10] Mwangi W and Sweller J 1998 Learning to Solve Compare Word Problems: The Effect of Example Format and Generating Self-Explanations Cogn. Instr. 16 173–99

[11] Mayer R 2014 What Problem Solvers Know: Cognitive Readiness for Adaptive Problem Solving doi: 10.1007/978-1-4614-7579-8_8

[12] Tabachneck H J M, Koedinger K R and Nathan M J 1994 Toward a theoretical account of strategy use and sense-making in mathematics problem solving In Proc. of the 16th annual conference of the cognitive science society (Erlbaum: Hillsdale, NJ) p836e841

[13] Retnowati E, Ayres P, Sweller J, Retnowati E, Ayres P and Sweller J 2016 Can Collaborative Learning Improve the Effectiveness of Worked Examples in Learning Mathematics? J. Educ. Psychol. 109. 10.1037/edu0000167

[14] Gravemeijer K 1999 How Emergent Models May Foster the Constitution of Formal Mathematics Math. Think. Learn. 1 155–77

[15] Collins A, Brown J S and Newman S E 1989 Cognitive Apprenticeship:Teaching the crafts of reading, writing and mathematics Knowing, learning, and instruction ed L B Resnick (Hillsdale, NJ: Erlbaum) p 453e493

[16] Reed S K, Willis D and Guarino J 1994 Selecting examples for solving word problems J. Educ. Psychol. 86 380–88

[17] Bourne L E, Goldstein S and Link W E 1964 The concept of learning as a function of availability of previously presented information J. Exp. Psychol. 67 439–48

[18] Tennyson R D, Woolley F R and Merrill M D 1972 Exemplar and nonexampler variables which produce correct concept classification behavior and specified classification errors J. Educ. Psychol. 63144–52

[19] Holyoak K J, Novick L R and Melz E R 1994 Component processes in analogical transfer: Mapping, pattern completion, and adaptation Advances in connectionist and neural computation theory (Vol. 2. Analogical connections) ed K J Holyoak and J A Barnden (Norwood, N.J.: Ablex) pp 113–80

[20] Reed S K and Bolstad C A 1991 Use of examples and procedures in problem solving J. Exp. Psychol. Learn. Mem. Cogn. 17 753-766
[21] Tennyson R D and Cocchiarella M J 1986 An Empirically Based Instructional Design Theory for Teaching Concepts Rev. Educ. Res. 56 40–71
[22] Kalyuga S, Chandler P, Tuovinen J and Sweller J 2001 When problem solving is superior to studying worked examples J. Educ. Psychol. 93 579–88
[23] Van Gog T, Kester L and Paas F 2011 Effects of worked examples, examples, problems and examples of pairs on novices’ learning Contemp. Educ. Psychol. 36 212–8
[24] Barnett SM and Ceci S J 2002 When and where do we apply what we learn? A taxonomy for far transfer Psychol. Bull 128 612–37
[25] Gick M L 1986 Problem-Solving Strategies Educ. Psychol. 21 99-120
[26] Nathan M J, Kintsch W and Young E 1992 A Theory of Algebra-Word-Problem Comprehension and Its Implications for the Design of Learning Environments Cogn. Instr. 9 329–389
[27] Brown A L and Kane M J 1988 Preschool children can learn to transfer: Learning to learn and learn from examples Cogn. Psychol. 20 493-523
[28] Wijaya A, Heuvel-panhuizen M Van Den, Doorman M and Robitzch A 2014 Difficulties in solving context-based PISA mathematics tasks: An analysis of students’ errors Math. Enthus. 11 555–84
[29] Van Gog T 2011 Effects of identical examples-problems and problems-examples pairs on learning Comput. Educ. 57 1775-9
[30] Polya G 1975 How to Solve It: A New Aspect of Mathematical Method (New Jersey: Princeton University Press)
[31] Van Gog T, Kester L and Paas F 2011 Effects of worked examples, examples, problems and examples of pairs on novices’ learning Contemp. Educ. Psychol. 36 212–8
[32] Greer B and Harel G 2002 The role of isomorphisms in mathematical cognition J. Math. Behav 17 5–24
[33] Retnowati E and Marissa 2018 Designing worked examples for learning tangent lines to circles J. Phys.: Conf. Ser. 983 012124
[34] Rohman H M H and Retnowati E 2018 How to teach geometry theorems using worked examples: A cognitive load theory perspective J. Phys.: Conf. Ser. 1097 012104
[35] Johanning and Debra I 2010 Helping Students Develop Capacity for Transfer Mathematics Teaching in the Middle School 16 260–64
[36] Greiff S, Scheiter K, Scherer R, Borgonovi F, Britt M, Graesser A, Kitajima M and Rouet J-F 2017 Adaptive problem solving: Moving towards a new assessment domain in the second cycle of PIAAC OECD Education Working Papers no. 156 OECD Paris
[37] Große C and Renkl A 2006 Effects of multiple solution methods in mathematics learning Learn. Instr. 16 122-38