An intrinsic parameter calibration method for R-LAT system based on CMM

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Abstract
Rotary-laser automatic theodolite (R-LAT) system is a distributed large-scale metrology system, which provides parallel measurement in scalable measurement room without obvious precision loss. Each of R-LAT emits two nonparallel laser planes to scan the measurement space via evenly rotations, while the photoelectric sensors receive these laser plane signals and perform the coordinate calculation based on triangulation. The accurate geometric parameters of the two laser planes play a crucial role in maintaining the measurement precision of R-LAT system. In practice, the geometry of the two laser planes, which is termed as intrinsic parameters, is usually unknown after assembled. Therefore, how to figure out the accurate intrinsic parameter of each R-LAT is a fundamental question for the application of R-LAT system. This paper proposed an easily operated intrinsic parameter calibration method for R-LAT system by adopting coordinate measurement machine. The mathematical model of laser planes and the observing equation group of R-LAT are established. Then, the intrinsic calibration is formulated as a nonlinear least-square problem that minimizes the sum of deviations of target points and laser planes, and the ascertainment of its initial guess is introduced. At last, experience is performed to verify the effectiveness of this method, and simulations are carried out to investigate the influence of the target point configuration in the accuracy of intrinsic parameters.

Keywords R-LAT system · Intrinsic parameter · Calibration · CMM · Initial guess · Precision influence

1 Introduction
Rotary-laser automatic theodolite (R-LAT) system is a distributed large-scale metrology system with adopting rule of triangulation, which is also called indoor GPS by iGPS. Nikon Metrology [EB/OL] [1] and wMPS [2]. Compared with traditional large-scale metrology instruments like laser tracker (LT), photogrammetry, theodolite system, total station, and CMM arm, the R-LAT system shows advantages in several points: (i) accomplish parallel measurement tasks, (ii) provide submillimeter measurement with the precision of 0.2 mm + 10 ppm, and (iii) obtain scalable measurement volume with increasing more R-LATs and without losing measurement precision. Moreover, it can collaborate with other instruments to deal with complex measurement tasks. Nowadays, R-LAT system has been widely used in industrial manufacture such as aircraft fuselage assembly [3, 4], AGV navigation [5, 6], and robot motion control [3]. Muelaner et al. [7–9] investigated the angular uncertainty of iGPS, Maisano et al. [10] studied the function of iGPS, Schimitt et al. [11] evaluated its dynamic working performance compared with LT. These studies show R-LAT system is capable to provide submillimeter precision measurement and its promising application prospect in industrial manufacturing.

Essentially, accurate intrinsic and extrinsic parameters are the fundamental to ensure the work performance of R-LAT system. Particularly, the intrinsic parameters, which are fixed after assembled, directly decide the measurement precision of R-LAT system. Zhao et al. [12] proposed a calibration method for R-LAT system by adopting spatial coordinate control network. Coplanar equations are established by measuring the coordinate known as target points, and the equation group is used to calculate both the intrinsic and extrinsic parameters.
The target points are fixed on a huge mechanic frame and are calibrated by LT; however, the spherical shape the same as the spherically mounted retroreflector (SMR) of LT is necessary for photoelectric sensor to maintain the precision of coordinate field. It is inconvenient for field application, and manufacturing high-precision spherical sensor is expensive. Particularly, in our self-made R-LATs system, this method cannot support the cube or conical shape sensors to finish the intrinsic parameter calibration. Therefore, practical intrinsic parameter calibration method is required to satisfy the easy achievement and application in both the laboratory and industrial field. Besides, in photogrammetry system which is a distributed metrology system, intrinsic parameters calibration is necessary for the cameras. Plenty of studies have been devoted; specifically, the flexible calibration method developed by Zhang [13] has been widely applied since its elegant operation that only a checkerboard is involved with combining several times measurement. This convenient calibration model could provide inspiration to simplify the calibration process for R-LAT system.

Therefore, this paper proposed a flexible intrinsic calibration method for R-LAT system by adopting a CMM which make it easier to provide the target points and better at reducing cost. The rest of the part is arranged as follows: Sect. 2 briefly introduces the mathematics model of R-LAT system, Sect. 3 presents the intrinsic parameters calibration model and describes the ascertain method of its initial guess, and Sect. 4 carries out experiments and simulations to verify the accuracy of this method and to investigate the influence of target distribution on the result accuracy.

2 Mathemetic model of R-LAT system

The intrinsic parameter calibration method is based on a R-LAT system which is a distributed measurement system designed by the Xi’an Jiaotong University, China. R-LAT system consists of rotary-laser transmitters, photoelectric receivers, data process unit, and relevant equipment like scale bar and handheld probe. As shown in Fig. 1a, the transmitters which are the R-LAT are distributed around the measurement object, and the coordinate measurement of photoelectric sensors can be obtained via the rule of triangulation. During working process, as shown in Fig. 1b, the rotating head of each transmitter generates signals from the line laser modules and emits a pulse signal as the timing reference. The unique rotation speed of each R-AT is their identification information. At the same time, photoelectric sensors capture such triggered signals of laser planes, record their triggered time, and transfer them to the data process by Bluetooth network. Subsequently, the data process converts these time sequence data into angle information and calculates the spatial coordinates of these receivers. As one key operation, the time moments for each $k$th R-LAT are identified as $t_{k,0}$ for reference time and $t_{k,1}$ and $t_{k,2}$ for the two laser planes in sequence. As a result, according to the specified rotation speed $n_k$ of each $k$th R-LAT, the rotation angles $\theta_{k,1}$ and $\theta_{k,2}$ of the two laser planes that go through the photoelectric sensor relative to the reference phase can be given as follows:

![Fig. 1 The basic working model of R-LAT system. a The triangulation of R-LAT system and b the laser plane model and the measurement time of one R-LAT](image)
Obviously, the two rotation angles decide the geometry of the two laser planes for the R-LAT, while the two planes generate an intersection line that goes through the corresponding photoelectric sensor. This is the basic measurement model of one R-LAT for a photoelectric sensor. On this basis, all the R-LATs of one R-LAT system are performed lines that go through the same photoelectric sensor as shown in Fig. 1a, establishing a spatial triangulation model. Consequently, the coordinate of the target photoelectric sensor can be figured out once the precise laser planes of all the R-LATs are known. Essentially, each R-LAT encodes a measurement space by the intersection spatial line of its two laser planes with corresponding rotation angles. Therefore, the two laser planes of one R-LAT need delicate design to ensure all the spatial lines are unique. As provided in Fig. 1b, the Cartesian coordinate system of the transmitter is established and defined as $O_TX_TY_TZ_T$. In this coordinate system, the emit center of laser planes is defined as the origin $O_T$, the rotating shaft of the R-LAT is defined as the $Z_T$ axis, and the references phase of R-LAT is defined as the $X_T$ axis. Based on the right-hand rule, the $Y_T$ axis is defined.

It is known that the normal vector of $X_T-OT-Z_T$ plane is $\begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi_1) - \sin(\phi_1) & 1 \\ 0 & \sin(\phi_1) & \cos(\phi_1) \end{pmatrix}$, where $\text{\textbf{n}}_{10}$ is the normal vector of the $X_T-OT-Z_T$ plane in the R-LAT coordinate frame $\text{\textbf{n}}_{20} = \begin{pmatrix} \cos(\phi_0) - \sin(\phi_0) & 0 & 1 \\ \sin(\phi_0) & \cos(\phi_0) & 0 \\ 0 & 0 & 1 \end{pmatrix}$ is the normal vector of the $X_T-OT-Z_T$ plane in the R-LAT coordinate frame. Therefore, the corresponding normal vectors ($\text{\textbf{n}}_{10}$ and $\text{\textbf{n}}_{20}$) of the two laser planes are respectively given by the following.

\[
\begin{align*}
\text{\textbf{n}}_{10} &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi_1) - \sin(\phi_1) & 1 \\ 0 & \sin(\phi_1) & \cos(\phi_1) \end{pmatrix} \\
\text{\textbf{n}}_{20} &= \begin{pmatrix} \cos(\phi_0) - \sin(\phi_0) & 0 & 1 \\ \sin(\phi_0) & \cos(\phi_0) & 0 \\ 0 & 0 & 1 \end{pmatrix}
\end{align*}
\]

In practical, the second laser plane likely departures the origin $O_T$ along Z axis with a tiny $e$ due to manufacture/assembly errors. As a result, the equations of the two laser planes are given as follows:

\[
\begin{align*}
f_1 : \cos(\phi_1)y + \sin(\phi_1)z &= 0 \\
f_2 : -\sin(\phi_0)x + \cos(\phi_0)\cos(\phi_2)y + \sin(\phi_2)z + e &= 0
\end{align*}
\]

Equation (4) represents the geometrical constraint for one R-LAT that is the target sensor placed on the intersection of two laser planes when they pass though the sensor successively. Clearly, one can figure out that the intrinsic parameters of the R-LAT are the only unknown variables of this coplanar equation group once the coordinate value of these target photoelectric sensors is known. Therefore, using enough receiver points, each of them satisfies (4), and then

\[
\begin{align*}
\begin{pmatrix} n_{10} \\ 0 \end{pmatrix} \cdot \begin{pmatrix} R_z(\phi_1) \\ 0 \end{pmatrix} \cdot \begin{pmatrix} x_p \\ y_p \\ z_p \end{pmatrix} &= 0 \\
\begin{pmatrix} n_{20} \\ e \end{pmatrix} \cdot \begin{pmatrix} R_z(\phi_2) \\ 0 \end{pmatrix} \cdot \begin{pmatrix} x_p \\ y_p \\ z_p \end{pmatrix} &= 0
\end{align*}
\]

where $[x_p, y_p, z_p]^T$ is the coordinate of target photoelectric sensor $P$ in the R-LAT coordinate frame $(O_TX_TY_TZ_T)$ and $R_z(\theta)$ is the rotation matrix around Z axis given as follows:

\[
R_z(\theta) = \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix}
\]
the multiplane constraint can be established by processing the equation group as a nonlinear least-square problem.

In practical calibration process, the precise coordinate values of the target photoelectric sensor can be provided by precise coordinate measurement instrument like CMM and laser tracker. As shown in Fig. 2, after installing the R-LAT on the ground, the target photoelectric sensor that attaches to the probe of a CMM is moved to \( M \) positions for measuring. Finally, an observing equation group compose of 2 \( M \) coplanar planes equations is obtained. However, it is noteworthy that all the coordinate values of target photoelectric sensor are in the measurement coordinate frame of CMM (O_c-X_cY_cZ_c). They should be transformed into the R-LAT coordinate frame by a 3×3 rotation matrix \( R_c \) and a 3×1 translation vector \( T_c \) to satisfy the formulation as in Eq. (3).

As a result, the \( i \)th target photoelectric sensor position, of which the measurement coordinate value is \( P_i = [x_i, y_i, z_i, 1]^T \) and the corresponding rotation angles are \( \theta_{i1} \) and \( \theta_{i2} \), defines the observing equations as follows:

\[
\begin{align*}
\begin{bmatrix} n_{10} & 0 \end{bmatrix} \cdot \begin{bmatrix} R_c \theta_{i1} & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} R_c & T_c \end{bmatrix} \cdot P_i &= 0 \\
\begin{bmatrix} n_{20} & e \end{bmatrix} \cdot \begin{bmatrix} R_c \theta_{i2} & 0 \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} R_c & T_c \end{bmatrix} \cdot P_i &= 0
\end{align*}
\]

(6)

For convenience, the translation vector \( T_c \) is specified as \( [t_x, t_y, t_z]^T \); the rotation matrix \( R_c \) is formulated by Euler angles (rotated \( \beta \) around z axis, rotate \( \beta \) around y axis, and rotated \( \alpha \) around x axis in sequence) as follows:

\[
R_c = \begin{bmatrix}
\cos(\beta)\cos(\gamma) & \sin(\alpha)\sin(\beta)\cos(\gamma) - \cos(\alpha)\sin(\gamma) & \cos(\alpha)\sin(\beta)\sin(\gamma) + \cos(\alpha)\sin(\gamma)
\sin(\beta)\sin(\gamma) & \sin(\alpha)\sin(\beta)\cos(\gamma) + \cos(\alpha)\sin(\gamma) & \cos(\alpha)\sin(\beta)\sin(\gamma) - \sin(\alpha)\cos(\gamma)
-\sin(\beta) & \sin(\alpha)\cos(\beta) & \cos(\alpha)\cos(\beta)
\end{bmatrix}
\]

(7)

Herein, the observing equation group includes 10 unknown variables, i.e., 4 intrinsic parameters of R-LAT (\( \phi_0, \phi_1, \phi_2, \) and \( e \)) and 6 elements of \( T_c \) and \( R_c \) (\( t_x, t_y, t_z, \alpha, \beta, \) and \( \gamma \)). To ensure the unique solution exists in this observing equation group, the equation number \( 2M \) must no less than the number of 10 unknown variables, which denotes \( M \geq 5 \). Actually, due to the rotation error of R-LAT and the timing error of photoelectric sensor as well as the disturbances of environment, the \( i \)th target photoelectric sensor will deviate to the \( j \)th laser plane, and this error \( d_{ij} \) can be defined as follows:

\[
\begin{align*}
\begin{cases}
d_{i1} = [0, \cos(\phi_1), \sin(\phi_1), 0] \cdot \begin{bmatrix} R_c & T_c \end{bmatrix} \cdot \begin{bmatrix} R_c \theta_{i1} & 0 \\ 0 & 1 \end{bmatrix} \cdot P_i - 0 \\
d_{i2} = [-\sin(\phi_0), \cos(\phi_0)\cos(\phi_2), \sin(\phi_2), e] \cdot \begin{bmatrix} R_c & T_c \end{bmatrix} \cdot \begin{bmatrix} R_c \theta_{i2} & 0 \\ 0 & 1 \end{bmatrix} \cdot P_i - 0
\end{cases}
\end{align*}
\]

(8)

For all the laser coplanar equations, the object function can be defined as follows:

\[
F = \min \left( \sum_{i=1}^{M} \sum_{j=1}^{2} d_{ij}^2 \right)
\]

(9)

The 10 unknown variables can be figured out by minimizing the object function and by adopting Levenberg–Marquardt (L-M) method [15] in consideration of its excellent performance in solving nonlinear least-square problem. In practical, looking forward to improve the accuracy of the solution, the observing equation group is overdefined by measuring plenty of target photoelectric sensor.

### 3.2 Initial guess ascertaining

Appropriate initial guess is significant to solving nonlinear least-square problem, which ensures that the iterative searching is converged to the true solution instead of trapping in local optimum. In fact, it is not easy to get a reasonable initial guess for the unknown variables of equation group (7). Particularly, the rotation matrix \( R_c \) and translation vector \( T_c \) are hard to assess because \( X_T \) axis of each R-LAT is invisible; even the approximate rotation angles of laser planes can be measured manually, while the specification deviation \( e \) is given as the designed/measured value directly.

In the basis of the working principle of R-LAT system, one can find similarity between the R-LATs and cameras in photogrammetry, in which the 3D points are mapped to a 2D formu-
As shown in Fig. 3, the frame of the R-LAT set coincides with the camera coordinate frame, while the image plane goes through point \([0, 1, 0]^T\) and perpendicular to the \(Y_T\) axis. The direction of the \(X\) axis and \(Z\) axis of the image plane coordinate frame \((O_I-X_IY_I)\) is in accordance with the R-LAT coordinate frame.

According to this camera model, the ray \(L_i\) from \(O_T\) to the \(i\)th spatial target \(P_i\) intersects the image plane at \(Q_i\). With the omission of the tiny deviation \(e\), the ray vector \(L = [l_x, l_y, l_z]^T\) can be identified as the cross product of the normal vectors \(N_1\) and \(N_2\) of the two laser planes that go through the corresponding target \(P_i\):

\[
L = \frac{N_1 \times N_2}{|N_1 \times N_2|} \tag{10}
\]

where normal vectors \(N_1\) and \(N_2\) are given as follows:

\[
\begin{align*}
N_1 &= [\cos(\phi_1), \sin(\phi_1)] \cdot R_c(\theta_1) \\
N_2 &= [-\sin(\phi_0), \cos(\phi_0) \cos(\phi_2), \sin(\phi_2)] \cdot R_c(\theta_2) \tag{11}
\end{align*}
\]

Meanwhile, the coordinate value \([u, v]\) for point \(Q_i\) in image plane coordinate system can be given as below:

\[
\begin{bmatrix}
  u \\
  v
\end{bmatrix} = \begin{bmatrix}
  -\frac{l_x}{l_z} \\
  \frac{l_y}{l_z}
\end{bmatrix} \tag{12}
\]

i. Herein, all the target points for \(P_i\) and their corresponding 2D coordinate values \([u, v]\) on image plane are known. Obviously, to identify the rotation matrix \(R_c\) and translation vector \(T_c\), a PnP question is given as below:

\[
\begin{bmatrix}
  u_i \\
  v_i
\end{bmatrix} = \begin{bmatrix}
  R_c & T_c
\end{bmatrix} \begin{bmatrix}
  x_i \\
  y_i \\
  z_i
\end{bmatrix} \tag{13}
\]

The results, i.e., \(R_c\) and \(T_c\), can be figured out with adopting DLT method directly.

iii. Finally, according to Eq. (6), the components of the Euler angles \((\alpha, \beta, \gamma)\) can be extracted from the rotation matrix \(R_c\) as below:

\[
\begin{align*}
\alpha &= \arctan2(m_{32}, m_{33}) \\
\beta &= \arctan2(-m_{31}, \sqrt{m_{32}^2 + m_{33}^2}) \\
\gamma &= \arctan2(m_{21}, m_{11}) \tag{14}
\end{align*}
\]

where \(m_g\) is the element of matrix \(R_c\) at \(i\)th row \(j\)th column.

### 3.3 Procedure of intrinsic parameters calibration with CMM for R-LAT

With combining the aforementioned intrinsic parameters calibration model and the initial guess ascertaining method, the complete calibration procedure is organized as shown in Fig. 4.

### 4 Examples and investigation on calibration precision

The proposed method is used to calibrate three R-LATs, and then a scale bar is measured by R-LAT system composed by these R-LATs to evaluate the accuracy of this method. In addition, numerical calibration simulations are carried out to investigate the influence of target position distribution on the accuracy of calibrated result, and discussion is given at last.
4.1 Calibration experiment

As shown in Fig. 5a, 100 target points were sampled in 400 mm x 400 mm x 400 mm by a CMM (type: Hexagon Global 575, working room 500 mm x 700 mm x 500 mm) and use the same photoelectric sensor for three R-LATs, respectively. The measured data of these target points established a least-square problem, and it was solved by L-M method. The calibrated intrinsic parameters of the R-LATs are demonstrated in Table 1. With substituting the result of intrinsic parameters into Eq. (7), the distances of all target points that deviate to the two laser planes were figured out as in Fig. 6a, respectively. The maximum deviations for both two laser planes are no more than 0.08 mm. It indicates the result intrinsic parameters accurately defined the laser planes of these R-LATs.

Furthermore, as provided in Fig. 5b, a scale bar with a calibrated length of 969.467 ± 0.005 mm was measured at 30 positions by a R-LAT system that is composed by aforementioned three R-LATs after delicately extrinsic calibration. The deviations of the measured lengths are shown in Fig. 6b. The maximum length deviation is no larger than 0.22 mm, which shows the same precision grade as iGPS provided by iGPS, Nikon Metrology [EB/OL] [1]. The result proof this calibration model is effect in industrial application.

In addition, one can find that the calibrated axial eccentricities e of these R-LATs are deviated seriously to the desired 0 mm. Several factors might contribute to the following: (i) the deflections of the laser planes, which are produce since the laser beams are not align to the axis of the cylindrical prisms, and (ii) machining and assembly errors of the two laser devices. Besides, the vibration of

### Table 1 The calibrated intrinsic parameters of three R-LATs

| R-LAT no | $\varphi_0$ (°) | $\varphi_1$ (°) | $\varphi_2$ (°) | e (mm) |
|----------|----------------|----------------|----------------|--------|
| #1       | −31.750        | 32.048         | 89.957         | −38.653|
| #2       | −30.070        | 33.990         | 89.599         | 25.907 |
| #3       | −28.737        | 32.565         | 89.514         | 31.502 |

Fig. 4 The procedure of intrinsic parameters calibration

Fig. 5 Experiment. a The intrinsic parameters calibration for one R-LAT by CMM and b scale-bar measurement by R-LAT system
Fig. 6 The deviations of two laser planes for all the target points. a Deviations of target point to the two laser planes and b the measurement length deviation of the scale bar.

Fig. 7 The configurations of simulations for intrinsic parameters calibration. a The layout of the R-LAT and CMM and b the spatial configuration of target points.
the head of the R-LAT also leads to measurement errors rather than the simplified rotation angle errors.

### 4.2 Influence of the target distribution on calibration precision

The spatial configuration of target points probably affects the precision of calibrated result since it decides the laser planes in the observe equation group. Hence, figuring out the influence of the target points configuration on the calibration result will provide optimal guidance for intrinsic parameter calibration of R-LAT and benefit the work performance of R-LAT system.

Since the truth values of the intrinsic parameters of every R-LAT are never known, the deviation is difficult to evaluate. Instead, numerical calibration simulations are adopted taking into account the truth values that are known. The measurement data for numerical calibration simulations is generated by the following steps: (i) sampling desired target points with known coordinate value in CMM coordinate frame; (ii) identifying the truth rotation angles of the two laser planes for each target point according to Eq. (3) with known truth intrinsic parameters and rotation matrix as well as translation vector; and (iii) taking account of the rotation speed fluctuation of the R-LAT, these nominal rotation angles are deliberately distorted by adding zero-mean normally distributed error with known variance ($\sigma = 3$ arcsec). Then the measurement rotation angles and corresponding target points are used in intrinsic parameter calibration.

The number and the distributed spatial size of the target points are the two main factors in deciding the observing equation group. Therefore, two groups of simulations were carried out to investigate their influence on the calibrated results. All the simulations shared the same specification truth intrinsic parameters, i.e., $\phi_0 = 90^\circ$, $\phi_1 = -30^\circ$, $\phi_2 = 30^\circ$, and $e = 0.8$ mm, and the layout of the R-LAT and CMM is illustrated as in Fig. 7a, while the sampling space was designed as a cube according to the working space of CMM as shown in Fig. 7b.

The first group of simulations was deployed with the variable number of target points in constant sampling room. As provided in the upper of Fig. 7b, within the $4\,m \times 4\,m \times 4\,m$ room, the target points for every simulation were evenly sampled in every direction respectively with following the rule that the target count in one direction was set up as 3, 5, 7, 9, and 11 in sequence, while in the rest two directions, the target count was constant to 5. Afterward, the intrinsic calibration is implemented 500 times for every configuration. The deviations between the estimated intrinsic parameters and the nominal values are figured out as in Figs. 8a.

In contract, the second group of simulations arranges constant target point count in variable sampling room. The sampling target count in the three directions is 5. With constant the center of sampling room, in each direction respectively, the distance between two adjacent targets is set as 0.4 m, 0.7 m, 1.0 m, 1.3 m, and 1.6 m in sequence. The intrinsic calibration is carried out 500 times for every configuration. The statistical analysis results of the estimated result compared to the nominal values are provided as in Figs. 8b.

As demonstrated in Fig. 8a, the results of the first simulation group indicate the estimated intrinsic parameters are closer to the nominal value with the increase of the target point count. These trends are consistent in x-, y-, and z-axis directions. In addition, by taking the maximum deviations, the approximate relative accuracy of the four intrinsic parameters is 0.006%, 0.008%, 0.008%, and 130% for $\phi_0$, $\phi_1$, $\phi_2$, and $e$. Obviously, the angular components can be calibrated accurately; however, a large uncertainty was shown in the axial deviation $e$. The results of the second group simulation shown in Fig. 8b point out the precisions of estimated intrinsic parameters are affected by the spatial layout of target points. The increases of the interval length of adjacent target points in x-axis and z-axis directions provide great improvement on the precision of angular component of intrinsic parameters; however, the variation in y axis shows less contribution. Meanwhile, the change in all the directions provides consistent contributions on the component $e$.

In all, it can be figured out the first group simulation performs higher precision than the second group simulation since more target points were used, which agrees with increasing the target point count benefits of precision improving intrinsic parameters. Meanwhile, the second group simulation shows even fewer target points were adopted; enlarging the distribution range of target points along non-depth directions also contributes to intrinsic parameter precision promotion. To improve the calibration accuracy of intrinsic parameters, we suggest two manners: one is enlarging the room of target points, especially around the non-depth direction of R-LAT, where a spatial coordinate control network is established [12]; the other one is increasing the target points number when sampling is feasible in a small room. Besides, considering the large relative errors of eccentricity $e$ in the simulation, we also suggest a variant calibration method. The eccentricity $e$ is not performed as an unknown variant but a constant input element which has been figured out by other inspection methods.
4.3 Discussion

For the intrinsic parameters calibration requirement for self-made R-LAT system, firstly, the maximum deviation of the measured scale-bar length is no more than 0.22 mm in experimental proofs that this method has the same precision performance as Nikon iGPS. Secondly, this method can be implemented by a CMM, and SMR-like photoelectric sensor is unnecessary. It shows not only better generality for cube-like sensors but also excellent economy that establishes a huge coordinate field. Besides, it is easy to implement both research and development of R-LAT system in industrial field.

However, this method is still subject to some drawbacks in practical application, such as the measurement room of CMM limits the sampling, which conflict to the suggestion that large sampling room provides better precision. Moreover, how to improve the calibration precision for eccentricity $e$ and look for better compensation model for intrinsic parameter of R-LAT are needed in further study.

5 Conclusions

This study developed an intrinsic parameters calibration method for R-LAT system by adopting a CMM. The main conclusions are summarized as follows:

i. The measurement model of R-LAT system is established as coplanar equations of every R-LAT. On this basis, the 4 intrinsic parameters are defined to formulate the two laser planes of R-LAT considering the axial deviation induced by assembly error.

ii. In intrinsic parameter calibration, the coordinate values of target points are provided by a CMM. All the coplanar equations of laser planes compose an observing equation group, which is formulated as a nonlinear least-square problem to minimize the sum of deviations of target points to corresponding laser plane. The unknown variables include 4 intrinsic parameters and 6 elements of the rotation matrix and translation vectors between the coordinate frames of R-LAT and CMM.

iii. An initial solute ascertaining method is given to improve the calculation of this nonlinear least-square problem, which enhances the robustness of this calibration method. Through transforming the measurement model of one R-LAT into the image plane coordinate of a camera, figuring out the rotation matrix and translation vector is formulated as a PnP question and performed by the DTL method.

iv. Both increasing the count of target points and positioning the target point widely along the non-depth direction of R-LAT improve the accuracy of intrinsic parameters. They are suitable for sampling target with a CMM and establishing a spatial coordinate control network by laser tracker. Furthermore, the investigations indicate that the angular components are easy to reach a higher accuracy level rather than the eccentricity $e$.

Author contribution WS proposed the detailed method and carried out the formula derivation and theoretical analysis. JG was responsible for the experimental work and part of data analysis. ZL was involved in the discussion and significantly contributed to making the final draft of the article. KJ proposed the research idea and technical scheme. All the authors read and approved the final manuscript.

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Data availability The authors confirm that the data supporting the findings of this study are available within the article.

Declarations

Ethics approval Not applicable.

Consent to participate Not applicable.

Consent for publication Not applicable.

Conflict of interest The authors declare no competing interests.

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