Strategic programming on graph rewriting systems

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We describe a strategy language to control the application of graph rewriting rules, and show how this language can be used to write high-level declarative programs in several application areas. This language is part of a graph-based programming tool built within the port-graph transformation and visualisation environment PORGY.

1 Introduction

Rewriting [4] is a computation model used in computer science, algebra, logic and linguistics, amongst others. Its purpose is to transform syntactic objects (words, terms, programs, proofs, graphs, etc., which we will call generally expressions), by applying rewrite rules until a suitable simplified form is obtained.

Given an expression and a set of rewrite rules, it is often the case that several different rules can be applied, and the same rule can be applied to different sub-expressions. To control the rewriting process, a strategy of application of rules is used.

In this paper we focus on graph rewriting [11, 27]. Graphs are widely used for describing complex structures in a visual and intuitive way, e.g., UML diagrams, representation of proofs, microprocessor design, XML documents, communication networks, data and control flow, neural networks, biological systems, etc. Graph rewriting has many applications in specification, programming, and simulation tools, amongst others [12, 13]. Several graph-transformation languages and tools have been developed, such as PROGRES [28], AGG [14], Fujaba [23], GROOVE [26], GrGen [17] and GP [25], only to mention a few.

When the graphs are large or growing along transformations, or when the number of graph rewriting rules is large, visualisation becomes crucial to understand the graph evolution. PORGY [1] is a visual environment that allows users to define graphs and graph rewriting rules, and to experiment with a graph rewriting system in a visual and interactive way. For instance, one may want to apply a rule \( r \) on a graph \( G \) at the positions described by the subgraph \( P \) to study different rewriting derivations for \( G \). To control the application of graph rewriting rules, PORGY uses a strategy language. In this paper we describe this strategy language and show how it can be used to write high-level declarative programs in several application areas.

Strategies for rewriting have been well studied in the case of terms: there are languages that allow the user to specify and apply strategies controlling the use of term rewrite rules (see for instance [9, 22, 6]). For graph rewriting, some of the languages and tools mentioned above also allow users to guide the rewriting engine, for instance by specifying the order in which rules will
be applied; PROGRESS, Fujaba and GP offer control structures to guide the application of rules. PORGY’s strategy language draws inspiration from these languages, and also from the strategy languages available for term rewriting. A distinctive feature of our strategy language is that it allows users to describe not only the rules that need to be applied, but also the location in the graph where they apply, and the propagation mechanism controlling successive applications of rules (the latter is not trivial in the case of graphs, since there is no notion of a root, so standard term rewriting strategies based on top-down or bottom-up traversals do not make sense in this setting).

The syntax of PORGY’s strategy language includes constructs to update the positions where rewriting rules can apply, and to specify the way rules should be applied. After giving the syntax of the language, and a short, informal description of its semantics, in this paper we show how the language can be used to write numerical programs, geometric applications where visualisation plays an important role (we give a program to draw a Von Koch fractal), and also applications to game design.

Summarising, in this paper we describe a language to define graph rewriting systems and strategies to control the application of rules. Programs in this language operate on graphs, which are transformed by application of rules at positions described explicitly in the program. We give examples to illustrate the features of the language, and in particular we show how the ability to control not only the rules and sequences of rules to be applied but also the location in the graph where rules are applied, allows programmers to write concise, visual programs, in several application areas.

In a companion paper [15] we give the formal semantics of the language. For more details about the PORGY tool for graph visualisation and transformation, for which the strategy language was developed, we refer the reader to [1].

The paper is organised as follows: In Section 2 we give a short overview of the graph rewriting formalisms used in this paper. We describe the strategy language in Section 3 and show applications in Section 4. In Section 5 we discuss related work. Section 6 concludes and gives some directions for future work.

2 Background

A graph rewriting system is a set of rewrite rules of the form $L \Rightarrow R$ where $L$ and $R$ are graphs. Such a rule applies to a graph $G$ if $G$ contains at least one instance of the left-hand side $L$, i.e. a subgraph $A$ isomorphic to $L$. Each rule specifies an interface that is used in order to rewrite $A$ in $G$: $G$ rewrites to a new graph $G'$ obtained by replacing the instance $A$ of $L$ by an instance $B$ of $R$, where edges that were connected to nodes in $A$ are connected to $B$ as specified by the rule’s interface. This rewriting process induces a transitive relation on graphs. Each rule application is a rewriting step and a derivation is a sequence of rewriting steps, that we will sometimes call a computation, referring to application of rewriting to programming languages.

There are several formal definitions of graph rewriting systems, depending on the kind of graphs and rewriting rules that can be defined in the system (see, for instance, [7, 24, 8, 19]). As a particular example of graph rewriting formalism of interest, in this paper we consider port graph rewriting, of which interaction nets [19] are a particular case. We recall below the main notions of port graph rewriting, and refer to [2] for more details.
**Port graphs.** A port graph [3] is a graph where nodes have explicit connection points called ports and the edges are attached to specific ports of nodes. Each edge connects at most two ports. Nodes and ports are labelled using \( \mathcal{N} \) and \( \mathcal{P} \), which are two disjoint sets of node names and port names respectively. A p-signature over \( \mathcal{N} \) and \( \mathcal{P} \) is a mapping \( \mathcal{V} : \mathcal{N} \to 2^{\mathcal{P}} \) which associates a set of port names to a node name. Without loss of generality, we assume that the port names associated to different node names are different. In addition, ports may have state information, which is formalised using a mapping from port names to port states.

Let \( G \) and \( H \) be two port graphs over the same p-signature. A port graph morphism \( f : G \to H \) maps elements of \( G \) to elements of \( H \) preserving sources and targets of edges, as well as node names and associated port name sets.

A port graph rewrite rule \( L \Rightarrow R \) is itself represented as a port graph consisting of two port graphs \( L \) and \( R \) over the same p-signature and one special node \( \Rightarrow \), called arrow node connecting them. \( L \) and \( R \) are called the left- and right-hand side respectively. The arrow node is used to specify the correspondence between free ports in the left- and righ-hand sides of the rules, i.e., the interface of the rule. For each port \( p \) in \( L \) to which corresponds a non-empty set of ports \( \{p_1, \ldots, p_n\} \) in \( R \), the arrow node has a unique port \( r \) and the incident directed edges \((p, r)\) and \((r, p_i)\), for all \( i = 1, \ldots, n \); all ports from \( L \) that are deleted in \( R \) are connected to the black hole port of the arrow node. When the correspondence between the ports in the left- and right-hand sides is obvious we omit the ports and edges involving the arrow node.

We illustrate the notions of port graph and port graph rewriting rule with examples in Section 4.

Let \( L \Rightarrow R \) be a port graph rewrite rule and \( G \) a port graph such that there is an injective port graph morphism \( g \) from \( L \) to \( G \); hence \( g(L) \) is a subgraph of \( G \). Replacing \( g(L) \) by \( g(R) \) and connecting it with the rest of the graph as specified by the interface of the rule, we obtain a port graph \( G' \) which represents a result of one-step rewriting of \( G \) using the rule \( L \Rightarrow R \), written \( G \Rightarrow_{L \Rightarrow R} G' \). There may be different injective morphisms \( g \) from \( L \) to \( G \); they are built as solutions of a matching problem from \( L \) to a subgraph of \( G \). If there is none, we say that \( G \) is irreducible by \( L \Rightarrow R \). Given a finite set \( \mathcal{R} \) of rules, a port graph \( G \) rewrites to \( G' \), denoted by \( G \Rightarrow_{\mathcal{R}} G' \), if there is a rule \( r \) in \( \mathcal{R} \) such that \( G \Rightarrow r G' \). This induces a transitive relation on port graphs. Each rule application is a rewriting step and a derivation, or computation, is a sequence of rewriting steps. A port graph on which no rule is applicable is in normal form. Rewriting is intrinsically non-deterministic since it may be possible to rewrite several subgraphs of a port graph at the same time with different rules or the same one at different places, possibly getting different results.

**Interaction nets.** Interaction nets were introduced by Lafont in 1990 [19] and later used as a target language for implementation of efficient \( \lambda \)-calculus evaluators (see for instance [18, 21]).

A system of interaction nets is specified by a set \( \Sigma \) of symbols with fixed arities, and a set \( \mathcal{R} \) of interaction rules. An occurrence of a symbol \( \alpha \in \Sigma \) is called an agent. If the arity of \( \alpha \) is \( n \), then the agent has \( n + 1 \) ports: a principal port depicted by an arrow, and \( n \) auxiliary ports. Figure 1(a) shows a typical drawing of such an agent. Intuitively, a net \( N \) is a port graph (not necessarily connected) where the nodes are agents. The ports that are not connected to another agent are said to be free. The interface of a net is its set of free ports.

Interaction rules are port graph rewrite rules where the left-hand side consists of two agents \((\alpha, \beta) \in \Sigma \times \Sigma\) connected together on their principal ports (an active pair or redex) and the right-
hand side is a net $N$ with the same number of free ports as the left-hand side. In addition, at most one rule can be given for each pair of agents. The diagram depicted in Fig. 1(b) shows the format of interaction rules ($N$ can be any net built from $\Sigma$).

Interaction rules can be seen as a particular kind of port graph rewriting rules with constraints that ensure good computational properties: pattern-matching is particularly easy in interaction nets, since patterns are graphs containing just two nodes, and the transformations are local and strongly confluent.

In some contexts, for instance when using interaction rules to program functions, we need a weaker notion of normal form corresponding to the notion of weak head normal form in $\lambda$-calculus. Interface normal form, defined in [16], can be computed by a strategy which applies rules at the interface (if possible) in order to minimise the length of the reduction sequence. We give examples below.

3 Strategy language

Since the application order of a set of port graph rewrite rules on a port graph may alter the result and the resources (e.g., time and space) needed to reach the result, a rewriting strategy is needed to control the rewriting process. As mentioned in the introduction, strategic rewriting has been well studied for term rewriting systems (see, e.g., [10, 29, 6]). Here we present a strategy language for graph rewriting systems, where strategies take into account rule selection and position selection in a graph.

The notion of position in a graph is rather delicate since there is no notion of root. Hence standard term rewriting strategies based on top-down or bottom-up traversals do not make sense in the case of graphs. A specific language is needed to define strategies for graph rewriting systems. We propose a strategy language where expressions include operators to select rules and the positions where the rules are applied, and also to change the positions along the derivation.

Let $G$ be the graph to be rewritten. Then a position $P$ is a subgraph of $G$ specifying where rewrite rules are allowed to apply. More precisely, we require that the homomorphic image of the left-hand side of the rule ($L \Rightarrow R$) has a non-empty intersection with this subgraph, that is at least one node in $g(L)$ is in $P$. A simple and intuitive example is to define $P$ to be a specific node and in this case, only the rules having this node in the homomorphic image of their left-hand

Figure 1: General format of an agent and an interaction rule
The structure $G[P]$ consisting of a graph and a position a located graph. A program is composed of a located graph and a strategy expression. Programs act on graphs; the result of a terminating program is a new graph.

The syntax of the strategy language is given in Figure 2, where we show the grammars $S$, $A$ and $T$ for generating strategy expressions, rule applications and position transformations, respectively. The main idea is that given a set of graph rewrite rules, an initial located graph and a strategy expression in this language, rewriting sequences (i.e., derivations) are generated, starting from the initial graph, by applying the rules as specified by the strategy expression. The most basic expressions generated by the grammar are $Id$ and $Fail$, namely identity and failure. A position transformation (see the grammar for $T$ in Figure 2) applied on a located graph $G[P]$ only affects the subgraph specifying the positions (i.e., $P$). An application (see the grammar for $A$ in Figure 2) on a located graph may change both the graph and the positions. A strategy expression embeds the previous constructs and combines them using sequential composition, iteration and conditional choice (see the grammar for $S$ in Figure 2). Below we describe the constructs of the strategy language and their effect on a given located graph; we finish this section with some examples.

**Position transformations.** These allow programmers to focus on different parts of the graph during the rewriting process. A position transformation $T$ applies to a located graph $G[P]$ to produce a new graph $G[P']$. The new position $P'$ computed by $T$ is defined as follows. $CrtPos$ returns the current position in the located graph (it is akin to the identity transformation). $AllSuc$ returns immediate successors of all nodes of $P$, where an immediate successor of a node $A$ is a node that has a port connected to a port of $A$. $OneSuc$ looks for all the immediate successors of all nodes in $P$ and picks one of those randomly. $NextSuc$ computes successors of nodes in $P$ using a designated port for each node (for Interaction Nets, this is the principal port). $SetPos(P')$ changes the position $P$ to $P'$, where $P'$ is a subgraph of $G$ explicitly defined (for instance by selecting nodes through PORGY’s visual interface). $Property(\rho, G')$ updates $P$ to contain only the nodes from the graph $G'$ that satisfy the property $\rho$ ($G'$ will generally be the given located graph or the subgraph $P$). The set theory operators union, intersection, complement and subtraction apply to positions since they are graphs considered as sets of nodes and edges.

**Applications.** The application of $Id$ never fails and leaves the graph unchanged whereas $Fail$
always fails (it leaves the graph unchanged and returns failure). \((L \Rightarrow R)_M\) represents the application of the rule \(L \Rightarrow R\) in \(G[P]\) if \(L \cap P\) is not empty (i.e., the graph reduced must overlap with the position \(P\) specified in the located graph); \(M\) is the subgraph of \(R\) that is added to position \(P\) (in the copy of \(R\) added to \(G\) in the rewrite step, the nodes in \(M\) become part of \(P\); the rest of \(R\) is only added to \(G\) and not to \(P\)). \(A \parallel A'\) represents simultaneous application of \(A\) and \(A'\) on disjoint subgraphs of \(G\) and returns \(Id\) only if both applications are possible and \(Fail\) otherwise. \(A \parallel A'\) is a weaker version of \(A \parallel A'\) as it returns \(Id\) if at least one application of \(A\) or \(A'\) is possible. \(A^{\parallel(m,n)}\) applies \(A\) simultaneously a minimum of \(m\) and a maximum of \(n\) times. If the minimum is not satisfied then \(Fail\) is returned and \(Id\) otherwise. If \(n\) is a negative integer then no maximum is considered.

**Strategies.** The expression \(S;S'\) represents sequential application of \(S\) followed by \(S'\), and \(S + S'\) implements McCarthy’s AMB operator [22]. The strategy \(ppick(S,S')\) is a weaker version of \(S + S'\) as it randomly picks one of the strategies for application. For iterations, we have expressions of the form \(while(S) \ do(S') \ min(m) \ max(n)\) which keep on sequentially applying \(S'\) while the expression \(S\) evaluates to \(Id\); if the minimum of \(m\) successful applications of \(S'\) is not satisfied then it returns \(Fail\) or else \(Id\) is returned. Similar to \(A^{\parallel(m,n)}\), setting \(n\) to a negative integer eliminates the maximum. The strategy \(if(S) \ then(S') \ else(S'')\) checks if the application of \(S\) to \(G[P]\) returns \(Id\) in which case \(S'\) is applied to \(G[P]\) otherwise \(S''\) is applied (\(S\) is checked on a copy of \(G\) and not on \(G\) itself so the graph is not affected during the checking process). \(PnotEmpty\) returns \(Fail\) if \(P\) is empty and \(Id\) otherwise. This can be used for instance inside the condition of an \(if\) or \(while\), to check if the strategy makes \(P\) empty or not, instead of checking if the strategy itself can be applied. The strategy \(⟨S⟩\) applies \(S\) and considers \(S\) as one atomic rewriting step in the derivation tree. This is useful to abstract several reduction steps as one for visualisation purposes.

The semantics of the strategy constructs defined by the grammars in Fig. 2 has been formally defined in [15] using rewrite rules that apply to programs \([S,G[P]]\) where \(S\) is a strategy and \(G[P]\) a located graph. These rewrite rules are governed by a top-down, left-right meta-strategy, which ensures the following property (we refer the reader to [15] for details and proofs).

When the strategy \(S\) is grammatically correct, an expression \([S,G[P]]\) either rewrites indefinitely (when the strategy \(S\) does not terminate) or rewrites to an expression of the form \([E,G'[P']]\), where \(E\) is \(Id\) or \(Fail\) and \(G'[P']\) is a new located graph.

We will now illustrate the strategy language with a few examples.

**Example 1** In many applications, we need to repeatedly apply a rule (or set of rules, or more generally, we need to iterate a strategy) as long as possible. This can be easily done in the language by defining the expression \(repeat_1(S)\) as shown below. The expression \(repeat_1(S)\) applies \(S\) at least once.

- \(repeat_1(S) \triangleq while(S) \ do(S) \ min(-1) \ max(-1)\)
- \(repeat_1(S) \triangleq while(S) \ do(S) \ min(1) \ max(-1)\)

More concretely, the following expression can be used if \((A \Rightarrow B)\) and \((C \Rightarrow D)\) are two chemical reactions that must take place together (represented as graph rewriting rules).

\(repeat_1((A \Rightarrow B)||(C \Rightarrow D))\)

Other well-known strategy operators, such as Not, orelse and Try are defined below, together with a strategy to compute interface normal forms of interaction nets.
• Not$(S) \triangleq \text{if } (S) \text{ then } (\text{Fail}) \text{ else } (\text{Id}) \text{ is a strategy that fails if } S \text{ succeeds, and conversely, it succeeds if } S \text{ fails.}$

• S orelse $S'$ $\triangleq \text{if } (S) \text{ then } (S) \text{ else } (S') \text{, applies } S \text{ if possible, otherwise it applies } S' \text{ and fails if neither strategy is applicable.}$

• Try$(S) \triangleq \text{if } (S) \text{ then } (S) \text{ else } (\text{Id}) \text{ is a strategy that behaves like } S \text{ if } S \text{ succeeds, but if } S \text{ fails then it behaves like Id.}$

• The interface of a graph $G$ is the set of nodes of $G$ that have a free port. They can be selected by defining the position Property$(\text{interface},G)$, denoted Int. Then, if $G$ is an interaction net and $P$ its interface, the program $\text{[repeat}_t\text{((R}_1\text{;Int orelse Next),G[P]])}$ computes the Interface Normal Form of $G$ with respect to $R_1$.

The strategy language described in this section has been implemented into PORGY in the form of a plug-in. The user types in a strategy expression which is first parsed by an algorithm to create a strategy tree. A strategy engine then takes the tree and applies a series of rewrite rules on the tree, following the meta-strategy defined by the formal semantics of the language in [15]. Some of these rules will call for an application or a transformation to be performed on the graph. The strategy engine terminates when the tree is reduced to only its root which will either be Id or Fail (a successful strategy or a failed one, respectively). The user will then have a visual representation of the final state of the graph as well as a step by step trace of the strategy applied.

4 Applications

4.1 Arithmetic programs with Interaction Nets

In term rewriting systems with a finite signature, natural numbers are often represented using two function symbols: $S$ and $0$. Then the number $n$ is represented by a term $S(S(\ldots S(0)\ldots))$ with $n$ occurrences of $S$. This representation is inefficient, but in [20] it is shown that using Interaction Nets we can implement efficiently arithmetic operations on integers, with a finite signature, by representing a number $z$ in the form of a difference list $p - q$. The $I$ agent is used as head of a number, and holds two lists of $S$ agents: a left list containing $p$ and a right list containing $q$; see Figure 3 for an example of the number 1 represented as 4 $- 3$. We also note that there are infinite representations for each number in this way: $1 = 4 - 3 = 6 - 5 = 7 - 6 = \ldots$

The open rule given in Figure 3 extracts both lists from a number so that they can be used for arithmetic operations. If two lists are put head to head, the reduce rule will eventually return a single list containing the absolute value of the difference of the lists. The right-hand sides of these two rules are depicted as wires, that is, when the rule reduce is applied, the ports connected to the $S$ agents being reduced will be connected together, and similarly for the open rule. The negate rule switches the left and right list of a number, giving us its negative. The negate rule has its entire right hand side in its subgraph $M$, so if $P$ is initially the whole graph then no matter which of these three rules we apply, $P$ will always be the entire graph.

Using these three rules we can model Addition, Negation and Subtraction, as seen in Figure 4. If we liken the size of a graph to memory space, we could then prioritise the reduce rule so that the graph is always kept at its smallest. A useful strategy to design would then be:

$$\text{ArithStrategy: repeat} \times (\text{repeat} \times (\text{reduce} \times \text{Try} \times (\text{negate} \times \text{Try} \times (\text{open}))}$$
Here, we apply first the rule \textit{reduce} as much as possible, to simplify the representation of the numbers, followed by applications of negate and open to perform the arithmetic operations.

4.2 Von Koch Fractal

To draw a Von Koch Fractal (see Figure 5), we only need one agent type and one rule. Our initial graph is a triangle and has one (and only one) of its nodes in \( P \). We define a rule \textit{vonKoch} of type \(( L \Rightarrow R )_M\) (see \( VKF \) in Figure 5) such that \( M \) contains the right-most agent from the right-hand side of the rule. This means that after each application of the rule \textit{vonKoch} the subgraph \( P \) is updated to restrict the next application of rules to the neighbouring segment. Visually, our rule will \textit{travel} round the triangle, segment by segment, gradually creating a more complex fractal after each round trip.

In Figure 6 we can see three successive applications of \textit{vonKoch}. Agents drawn with dashed lines are agents that are in \( P \). The VKF strategy used is simply:

\[
\text{while(}\text{vonKoch})\text{do(}\text{vonKoch})\text{min(0)max(m)}
\]

where \( m \) is the number of iterations required.

Without the notion of position in the strategy language, the application of the VKF rule would have been random and the fractal generated would not necessarily be balanced. The strategy language allows us to define for each rewrite rule the nodes in the right-hand side that will become part of the position subgraph. The ability to update positions directly is exploited in this example to obtain a simple and concise program that goes round the triangle creating the fractal in a balanced way.

For this example, it is important that the shape and layout of the right hand side of the rules is preserved during the rewriting step to ensure the proper shape of the fractal. In PORGY the user can specify for each rule whether the shape of the right hand side is preserved or not.
Figure 4: Modelling Addition, Negation and Subtraction.

Figure 5: Modelling the Von Koch Fractal.
4.3 Game example: Pacman

To simulate a game of pac-man, we use the initial graph in Figure 7 with the five types of nodes depicted. We assume all nodes in this system to have four ports each, one for each direction. For visual simplicity we will not draw any free ports or ports whose state does not affect a rewrite rule.

The rewrite rules for pac-man can be found in Figure 8. These rules, with the help of a strategy, will help simulate a basic behavior for pac-man. Pac-man’s first instinct will be to flee any nearby ghosts (rules flee1a, flee1b, flee2a and flee2b). If pac-man is not near any ghosts he will then seek out pac-dots (rule getPacDot) and then if not near any pac-dots, he will proceed to explore the level (rule explore).

The strategy for controlling pac-man is as follows:

- pacAI: if(nearGhost1 or else nearGhost2) then (Flee) else (Move)
- Flee: if(flee1a or else flee1b) then (flee1a or else flee1b) else (Try(flee2a or else flee2b))
- Move: if(getPacDot) then (getPacDot) else (Try(explore))

The rewrite rules for the ghosts can be found in Figure 8. Like for pac-mac, these rules and a set of strategies will help simulate the behaviour of the ghosts. A ghost’s first priority is to eat pac-man (rules kill1 and kill2). If pac-man is not nearby, then a ghost will try move to a space with no pac-dots (rules moveE1 and moveE2) since following empty spaces should lead the ghost to pac-man. If a ghost can only move to a space with a pac-dot then do so (rules moveP1 and moveP2).

The strategy for controlling ghosts is as follows:

- ghostAI: if(kill1 or else kill2) then (kill1 or else kill2) else (gMove)
Figure 7: The pac-man playing field.

- gMove: if (moveE1 orelse moveE2) then (moveE1 orelse moveE2) else (Try(moveP1 orelse moveP2))

The overall strategy called gameLoop that controls the game is as follows: We must first check that pac-man has not been eaten (by checking for the existence of a node of type End). We then call pacAI followed by ghostAI for each ghost (we do this by adding pacman and all ghosts to P at the start of each game loop using Property(Y,G) and making sure all the rules that involve ghosts have an empty M. This means every time a ghost performs an action, which removes the ghost from P, it cannot perform another one till the next game loop).

- gameLoop: repeat.(Property(Y,G);if(isGameOver)then(Fail)else(pacAI;repeat.(ghostAI))

- Y is: type=="ghost" or "pac-man" or "End"

As we can see, a basic pac-man game does not require many agents and just six relatively simple strategies are sufficient to model it using our language.

Adding a scoring system would be trivial: each time pac-man eats a pellet, a point agent would be created and added to a list of points which can then be counted at the end of a game.

If there is more than one possible application of a rule, the implementation will pick one of the possibilities at random. This will create a different game each time.

4.4 Labyrinth

We will now give a program to find a path in a labyrinth. The labyrinth is represented as a graph built out of Labyrinth agents, as shown in Figure 9 where Labyrinth agents are depicted
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Figure 8: The set of rules to control pac-man (left) and to control the ghosts (right).

as empty circles and exits are represented with an End agent. The initial located graph in this example has a Pather agent connected to the start of the maze.

A Labyrinth agent has five ports, one for each cardinal direction North, East, South and West and a Pather port, where a Pather agent can attach to (see Figure 10). The End agent has the same ports as a Labyrinth agent but will react differently when a Pather agent is connected to it. We will also have a Visited agent, which has the same ports as a Labyrinth agent but like the End agent, will react differently when a Pather agent connects to it. Lastly we have a PATH agent which has the same ports as the Labyrinth agent and will be used to replace Labyrinth agents so that a visible path will be drawn from the start to the exit of the labyrinth.

For the sake of clarity, in the following diagrams the four directional ports will not be labelled but will be drawn in the standard orientation on the agents. In the rules, a white port means that the port must be connected, a crossed port must be free and a black port means either connected or free.

A Pather agent has a Position port and a List port. The Position port connects to a Labyrinth agent and the List port will connect to a list of Direction agents (representing the path followed so far). We have four Direction agents N, E, S and W that each have two ports: a Next and a Prev port. We will also need a Drawer agent (with the same ports as a Pather agent) that will travel back to the start of the labyrinth, following a list of Directions and replacing Labyrinth agents with PATH agents.

Lastly, we have some management agents: two copy agents (cp2 and cp3) and a delete agent named $\epsilon$. The copy agents take a list of directions and duplicate (cp2) or triplicate (cp3) it. The
Figure 9: An example of a labyrinth.

\(\epsilon\) agent takes a list and deletes it. The rewrite rules for \(cp2\) can be found in Figure 12 (the \(cp3\) rules are similar but produce three copies) and the rewrite rules for \(\epsilon\) in Figure 13.

The program consists of the strategy expression \(LabStrat\) described below, and a located graph representing the labyrinth, where the only node in the initial subgraph \(P\) is the \(Pather\) agent marking the starting point in the labyrinth. The strategy has two main parts, which we call Step 1 and Step 2. Step 1 attempts to find a path to the exit of the labyrinth, by moving the starting \(Pather\) until a \(Pather\) agent positions itself onto the \(End\) agent. If a \(Pather\) agent is positioned onto an \(End\) agent, a path was found and the program will move onto the Step 2. When a \(Pather\) agent moves to a new position, it changes the \(Labyrinth\) agent it moved from into a \(Visited\) agent. This will ensure the \(Pather\) agent never backtracks.

The strategy will start by checking if a \(Pather\) agent is connected to four \(Labyrinth\) or \(End\) agents that have their \(Pather\) port free (rule \(split4\) in Figure 14, a special case of when the starting \(Pather\) is put on such a \(Labyrinth\) agent). This rule will remove the \(Pather\) agent and create four new \(Pather\) agents for each of the four positions and give each one of the new \(Pathers\) a corresponding \(Direction\) agent (to remember the step done).

If \(split4\) cannot be applied, the strategy will try to apply one of the four \(split3\) rules \(split3a\),
split3b, split3c or split3d. This rule deletes the original Pather agent and creates three new Pather agents, adding a corresponding Direction agent to each of their lists, and copying the original Pather agent’s list onto the end of the new Pather agents’ lists. See Figure 15 for the split3a rule (the other three split3 rules are similar, taking into account the remaining combinations).

If none of the split3 rules can be applied, the strategy then tries all six of the split2 rules and if none of the split2 rules can be applied, it tries one of the four split1 rules. These rules do the same thing as the split3 rules but only split to two and one Labyrinth agents respectively. See Figure 16 for split1a and split2a.

We need to apply the split rules in this specific order or a possible split might be missed. For example, split1a might be applicable somewhere where split3a is also applicable but by applying split1a first we would not then explore the labyrinth to the West or South. This could lead to ending up with a longer path to the exit or in the worst case not finding the exit at all.

All split rules have an empty $M$ subgraph. This will allow the strategy to move each Pather at most once per iteration. The strategy will do this and then use Property($\rho, G$) to add all the Pathers back to $P$ and then start over again. This ensures that no Pather is given priority and is needed to find the shortest path (as explained further down).

While trying to apply the split rules in that specific order, the strategy will constantly check if the found rule (in Figure 17) is applicable. If it is, it moves onto Step 2: drawing the path. If the End node is not reachable from the starting point, the program will not terminate.

Step 2 checks if the done rule (Figure 17) is applicable and if it is not it will attempt to apply the four draw rules drawN, drawE, drawS and drawW. See Figure 17 for the drawN rule; the other three draw rules are similar and cater to a different direction each.
If the \textit{done} rule is applicable, the program will terminate and our labyrinth will have the shortest path to the exit drawn on it.

- \textbf{LabStrat:} \texttt{Step1} ; \texttt{Step2}
- \texttt{Step1}: \texttt{while(Not(\texttt{found}))do(\texttt{repeat.(Step1Split)}; \texttt{Property(\texttt{Y,G})min(0)max(-1)}; \texttt{\texttt{found}})}
- \texttt{Y} is: \texttt{type=="Pather"}
- \texttt{Step1Split}: \texttt{split4 orelse split3a orelse split3b orelse split3c orelse split3d orelse split2a orelse split2b orelse split2c orelse split2d orelse split2e orelse split2f orelse split1a orelse split1b orelse split1c orelse split1d}
- \texttt{Step2}: \texttt{while(Not(\texttt{done}))do(\texttt{drawN orelse drawE orelse drawS orelse drawW})min(0)max(-1); done}

Since \textit{Labyrinth} agents are changed to \textit{Visited} agents when a \textit{Pather} moves from them, if a branching occurs in the labyrinth that later reconnects (as seen at the lower middle Figure 9), the branch with the shortest path will be picked (remembering that each \textit{Pather} can only take one step at most during each iteration so the \textit{Pather} in the shortest branch will get to the reconnecting \textit{Labyrinth} node first).

Branching that reconnects will cause stuck \textit{Pathers}. When the \textit{Pather} from the quickest branch gets to the reconnecting \textit{Labyrinth} node, it will split and go to the slowest branch. That
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\[ \alpha \text{ is } N, E, S \text{ or } W \]

\[ \varepsilon \quad \alpha \quad \Rightarrow \]

\[ \varepsilon \quad \alpha \quad 1 \quad \Rightarrow \quad \varepsilon \quad 1 \]

Figure 13: The set of rules for \( \varepsilon \).

Figure 14: The split4 rule. \( \alpha \) is a Labyrinth or End agent.

newly split Pather will eventually meet the original Pather of that branch (going the other way). These two Pather agents will be positioned in two adjacent Labyrinth agents but won’t be able to move and remain stuck there. We could extend our graph program by creating a set of rules to eliminate these stuck Pather agents, using the \( \varepsilon \) agent. This is mainly an aesthetic improvement since the stuck Pather agents will not affect the functionality of the program.

5 Related Work: Graph Rewriting Tools

Several tools are available to edit graphs, and some of them allow users to model graph transformations. Below we review some of the systems that share some goals with PORGY.

GROOVE \[26\] is centered around the use of simple graphs for modeling the design-time, compile-time, and run-time structure of object-oriented systems. Visualisation is not the main objective, and after each rewrite step the user must update the layout of the graph by hand. GROOVE permits to control the application of rules, via a control language with sequence, loop, random choice, try(else) and simple (non recursive) function calls. These are similar to PORGY’s constructs, but GROOVE’s language does not include the notion of position; thus, it is not possible to specify directly a position for the application of rules.

The Fujaba \[23\] Tool Suite is an Open Source CASE tool providing developers with support
Figure 15: The split3\(\alpha\) rule. \(\alpha\) is a Labyrinth or End agent.

Figure 16: The split2\(\alpha\) and split1\(\alpha\) rule. \(\alpha\) is a Labyrinth or End agent.

for model-based software engineering and re-engineering. Fujaba has a basic strategy language, including conditionals, sequence and method calls. There is no parallelism, and again one of the main differences with PORGY is that Fujaba does not include a notion of location to guide the rule application.

AGG [14] is a rule based visual language supporting an algebraic approach to graph transformation. It aims at the specification and prototypical implementation of applications with complex graph-structured data. The application of rules can be controlled by defining layers and then iterating through and across layers. Again, there is no notion of position and there is no control on the search for redexes.

PROGRES [28] offers an executable specification language based on graph rewriting systems (graph grammars). The aim is to combine EER-like and UML-like class diagrams for the definition of complex object structures with graph rewrite rules for the definition of operations on these structures. PROGRES allows users to define the way rules are applied (it includes
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Figure 17: The *found*, *done* and *drawN* rules.

non-deterministic constructs, sequence, conditional and looping) but it does not allow users to specify the position where the rule is applied. It is a very expressive language and includes a tracing functionality through backtracking.

GrGen.NET [17] is a programming tool for graph transformation designed to ease the transformation of complex graph structured data as required in model transformation, computer linguistics, or modern compiler construction, for example. It is comparable to other programming tools like parser generators which ease the task of formal language recognition.

GP [25] is a rule-based, non-deterministic programming language. Programs are defined by sets of graph rewriting rules and a textual expression that describes the way in which rules should be applied to a given graph. The simplest expression is a set of rules, and this means that any of the rules can be applied to rewrite the graph. The language has three main control constructs: sequence, repetition and conditional (if-then-else), and it has been shown to be complete. It uses a built-in Prolog-like backtracking technique (users cannot easily handle the derivation tree or change the backtracking algorithm).

GReAT (Graph Rewriting and Transformation) [5] is a tool for building model transformation tools. Rule execution is sequential and there are conditional and looping structures.

PORGY and its strategy language allow a higher expressive power with its focus on position. Strategies are not limited to picking random applications but can travel through the graph in a dynamic and strategic manner to apply rules and sub-strategies. PORGY has also a strong focus on visualisation and scale, thanks to the TULIP back-end which can handle large graphs with millions of elements and comes with powerful visualization and interaction features. Some of
Tulip’s built-in functionalities, such as selecting a node in the trace for highlighting its “lifetime” within the trace, give the user an immediate visual feedback.

6 Conclusion and Future Work

The strategy language defined in this paper is part of the PORGY system, which is an environment that allows users to define graphs and graph transformation rules. PORGY is implemented using the TULIP platform, for the visualisation of graphs and graph transformation rules. PORGY provides also tools to visualise traces of rewriting, and the strategy language is used in particular to guide the construction of the traces.

Although PORGY and its strategy language were implemented specifically to work with port graphs (and interaction nets in particular), the strategy language could be applied to other graph formalisms (e.g., term graphs). This is a direction for future work. Verification and debugging tools for avoiding conflicting rules or non-termination are also planned for future work. The PORGY strategy language is not minimal and finding a set of minimal constructs will also be a subject for future work.

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