An Improved Algorithm Based on Landweber-Tikhonov Alternating Iteration for ECT Image Reconstruction

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Abstract. Aiming at the problem of "soft field" and ill state existing in image reconstruction proceeding of electrical capacitance tomography (ECT) system. An improved image reconstruction algorithm is proposed in this paper. For which, the pseudo-inverse matrix instead of transport matrix of sensitivity is directly used as the initial image value, and an alternate iteration procedure based on Landweber-Tikhonov algorithm is adopted to perform direct image reconstruction. Firstly, the Landweber algorithm is used to improve the initial value, and then the results are used as the inputs of Tikhonov regularization algorithm. The procedure is repeated alternately to search an ideal reconstructed image. This strategy can effectively solve the problem caused by the "soft field" and ill-state problems which always grows out poor quality of the reconstructed image. To test the availability and the effectiveness of the proposed algorithm, it is used to the image reconstruction of ECT system. The correlation coefficient and relative error are used as evaluation criteria for image quality, and the representative algorithms (e.g., LBP, Tikhonov, and Landweber ) are selected for comparison. Simulation experiments show that our algorithm is with well performance both in terms of subjective imaging effect and objective criteria under the same iteration steps with the compared algorithms. Moreover, the convergence speed of the proposed algorithm outperforms the compared algorithms.

1. Introduction

Electrical capacitance tomography (ECT) is a nondestructive testing technique for multiphase flow. This technology has the advantages of no radiation and fast response, so it has been widely used in the field of multiphase flow detection. The ECT system mainly includes capacitance sensing, data acquisition and image reconstruction. For the system, the independent capacitance value between electrodes is measured by the uniform distribution of electrodes outside the pipeline; when the corresponding sensitivity matrix is calculated, a certain image reconstruction algorithm can be used to reconstruct the section image of certain flow in the pipe. In practical application, image reconstruction algorithm is a very important [1], and it is one of key factors for the industry testing occasion. Generally, image reconstruction algorithms can be divided into three categories: direct algorithm, iterative algorithm, and intelligent algorithm. The linear back projection algorithm (LBP) [2] is one of the classic direct algorithms. It uses the transport matrix of sensitivity to replace the inverse matrix for imaging. The algorithm has the advantages of simple, lower computation, fast speed and so on. For the iterative algorithm, it mainly includes Landweber method [3-4], conjugate gradient method (CG) [5]. Because neural network can link input and output data with nonlinear mapping, it can be used as one of the representative intelligent algorithms to solve the image reconstruction problem of ECT system.
But the neural network needs a large number of image samples for training, and because of the complexity of the multiphase flow, the samples are not easy to train. Among them, the Landweber algorithm has a better compromise in the iterative algorithm. The classical Landweber algorithm is with higher imaging accuracy than the direct algorithm. However, the iteration numbers needed to get the best reconstructed image is always not consistent for different flow patterns, and the step needed to get the desired image is longer. Continuing to iterate, the algorithm presents divergent, which results in slower reconstruction processing [6]. For the existence of semi-convergent problems, Zhao et al. adopt the compression operator to make the Landweber algorithm stable and convergent. This method constructs a new operator $P$ by adding parameter $\beta$ to the residual matrix. However, the parameters need to be adjusted by themselves, and an inappropriate parameter will be maladjusted to the real image reconstruction [7]. Also, for different flow patterns, different parameters need to be set. Based on Runge-Kutta algorithm, Chen et al. proposed an improved Landweber algorithm. This method derives a mathematical model to solve the ECT imaging problem. It improves the imaging quality efficiently, but the imaging time is also longer [8]. Some new methods also are proposed, such as improving initial value, introducing damping operators and adding a unit matrix to the parameters. These methods can improve the accuracy of the reconstructed image, but the iteration numbers are not reduced [9].

2. Algorithm Principle reconstruction Algorithm

2.1. The Basic Principle of Landweber Algorithm and Tikhonov Algorithm

As one of the typical iterative algorithm for ECT image reconstruction, the Landweber algorithm launches the iteration steps under the least mean square criterion. The cost function of image reconstruction is as follows:

$$\min \frac{1}{2} \| S \cdot G - C \|^2$$  \hspace{1cm} (1)

Where $S$ is known as the sensitivity matrix, $G$ is the unknown permittivity vector, and $C$ denotes the capacitance vector. According to the definition of vector norm, equation (1) can be rewritten as:

$$f(G) = \frac{1}{2} \| S \cdot G - C \|^2 = \frac{1}{2} \left( G^T \cdot S^T \cdot S \cdot G - 2G^T \cdot S^T \cdot C + C^T \cdot C \right)$$  \hspace{1cm} (2)

According to the principle of steepest descent, the negative gradient direction is used as the search direction for optimization. The iterative formula of image reconstruction is followed as:

$$G_{k+1} = G_k - \partial f(G_k) = G_k - \partial \| S \cdot G - C \|^2$$  \hspace{1cm} (3)

Tikhonov regularization algorithm is an effective method to solve ill state problems. It has been used in image reconstruction of ECT system. The basic form is as follows:

$$J(G) = \min \{ \| S \cdot G - C \|^2 + u \| L \cdot G \|^2 \}$$  \hspace{1cm} (4)

In equation (4), $u$ is a Regularization parameter; $\| L \cdot G \|^2$ is a regularization term, and $L$ is a regularization operator.

$$(S^T \cdot S + u \cdot I) \cdot G = S^T \cdot C$$  \hspace{1cm} (5)

Regularization parameter $u$ has a crucial impact on image reconstruction results. A larger $u$ always gives a higher regularization degree, and then brings out more ambiguous reconstruction result; otherwise, a smaller $u$ may procedure more serious noise interference.
2.2. Landweber-Tikhonov Algorithm

From above analysis, the Landweber algorithm uses least mean squares criterion for image reconstruction, and achieves well trade-off between the accuracy and speed of the reconstruction processing. However, Landweber algorithm has some problems such as semi-convergence and long iteration steps. Tikhonov regularization algorithm is based on the least mean square criterion and the smoothing criterion. By adding a regularized matrix, a set of close fitting problems is used to approximate the solution, and can eliminate the discomfort of the original problem. Moreover, by adjusting the regularization parameters, the iteration steps can be reduced effectively. Therefore, adding the Tikhonov regularization algorithm to the Landweber algorithm can reduce the iteration step, and the regularization method can effectively maintain the stability of the solution. The Landweber algorithm is iterated under the improved initial value, and the result is deeply used as the input of Tikhonov algorithm for next iteration. The procedure is launched under an alternate iteration method, so that the scope of the true solution can be reduced. The steps of the proposed algorithm are described as figure 1.

Setting the parameters and the iteration number

Preprocessing the data

Using the Landweber algorithm to iterate

Tikhonov regularization algorithm

Satisfy iterate condition?

Yes

End

No

Figure 1. Flow chart of the proposed algorithm.

3. Numerical Experiment and Analysis

In this paper, we take two phase flow as the research object, and complete ECT image reconstruction to test the convergence and effectiveness of the improved algorithm. In the experiment, four common flow patterns are selected, they are foam, core, laminar, and annular flow. For a concise representation, the flow pattern is notated as 1, 2, 3, and 4, respectively, as shown in figure 2.

Figure 2. Four flow patterns shown in 2D mode. Shown from left to right are foam flow, core flow, laminar flow, and annular flow.
In this section, LBP, Landweber, and Tikhonov algorithm are used to reconstruct the image and the results are compared with that of the proposed Landweber-Tikhonov algorithm. Table 1 list the subjective results of the reconstructed image of each algorithm, and the objective evaluation indexes are calculated and listed in table 2.

**Table 1. Comparison of the reconstructed image of each algorithm.**

| Flow | LBP | Landweber | Tikhonov | The proposed |
|------|-----|-----------|----------|-------------|
| 1    | ![Image 1](image1.png) | ![Image 2](image2.png) | ![Image 3](image3.png) | ![Image 4](image4.png) |
| 2    | ![Image 5](image5.png) | ![Image 6](image6.png) | ![Image 7](image7.png) | ![Image 8](image8.png) |
| 3    | ![Image 9](image9.png) | ![Image 10](image10.png) | ![Image 11](image11.png) | ![Image 12](image12.png) |
| 4    | ![Image 13](image13.png) | ![Image 14](image14.png) | ![Image 15](image15.png) | ![Image 16](image16.png) |

**Table 2. Relative error (%) /Correlation coefficient /Iteration steps.**

| Flow | Landweber | Tikhonov | The proposed |
|------|-----------|----------|-------------|
| 1    | 12.3909/0.8687/400 | 11.8565/0.8818/400 | 11.4401/0.8927/400 |
| 2    | 10.2602/0.8541/400 | 9.7479/0.8663/400 | 8.3664/0.9120/200 |
| 3    | 20.1898/0.8368/400 | 24.4945/0.7093/400 | 14.4201/0.9064/5 |
| 4    | 30.0898/0.6204/400 | 25.8341/0.6645/400 | 23.5412/0.6971/400 |

As shown in table 1, the reconstructed image of LBP algorithm is deformed and poorly distinguished. The original flow pattern can be roughly reconstructed, but it is not very clear. The results of other three algorithms are all approximate to the original flow pattern on the shape with little subjective differences. For flow pattern 3, Landweber algorithm and Tikhonov algorithm have a strong shadow. Compared with other algorithms, the proposed algorithm can reconstruct a variety of flow patterns in the field, especially the convective flow 1, 2. For further comparison, we count the iteration steps which bring out the results of Table 1 and 2. The step numbers of three iterative algorithms are also listed in Table 2. These results shown that, at the same iteration steps, Landweber and Tikhonov algorithm can both successfully reconstruct the original flow, and the Tikhonov precede the Landweber to a certain extent. The proposed algorithm obtains the superiority in the case of the iteration steps, especially for flow patterns 2 and 3. This results show that with an improved reconstruction quality the iteration step is half of the other two iterative algorithms, especially for flow pattern 3, only five steps can get the well reconstruction, which means a little computational cost, and attest the availability and the efficiency of the proposed algorithm.

4. Conclusion
Because of the problem of "soft field" and ill condition in the ECT, the type and the resolution of reconstructed image in ECT are not perfect. In this paper, an improved image reconstruction algorithm based on Landweber-Tikhonov alternative iteration is proposed. Firstly, the generalized inverse matrix is used as the initial value for Landweber algorithm, and the succedent Tikhonov iteration is used to process the result of Landweber algorithm, then the procedure keeps on an alternate way to search an ideal reconstructed image. It can effectively combine the advantages of these two algorithms, reduce the iteration steps and improve the imaging quality. Experimental results show that the proposed algorithm is effective for two phase flow pattern, and the reconstruction results are closer to the prototype.
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6. References
[1] Ye J M 2012 Image filtering with associative Markov networks for ECT with distinctive phase origins IEEE Sens. J. 12 (7) 2435 - 43.
[2] Xie C G, Huang S M and Hoyle B S 1992 Electrical capacitance tomography for flow imaging: system model for development of image reconstruction algorithm and design of primary sensors IEE Proceedings G - Circuits, Devices and Systems 139 (1) 89 - 98.
[3] Soleimani M, Lioheart W R B 2005 Nonlinear image reconstruction for electrical capacitance tomography using experimental data Meas. Sci. Technol. 16 (10) 1987-96.
[4] Yang W Q, Spink D M and York T A 1999 an image reconstruction algorithm based on Landweber's iteration method for electrical capacitance tomography Meas. Sci. Technol. 10(11)1065 - 69.
[5] Wang H X, Zhu X M and Zhang L F 2005 Conjugate gradient algorithm for electrical capacitance tomography J. Tianjin Univ. 38 (1) 1 - 4.
[6] Xiong X Y, Tang L and Wang C 2007 One-Step stable image reconstruction algorithm for electrical capacitance tomography Chinese Journal of Scientific Instrument 28 (11) 1982-86.
[7] Zhao Y L, Guo B L 2016 Multiphase flow detection based on ECT image reconstruction algorithm Transactions of the Chinese Society for Agricultural Machinery 47 (07) 368 -74.
[8] Feng Y H, Cao R X and He M Y 2011 Study of the measurement of two-phase flow based on the modified Landweber Algorithm Industrial Instrumentation & Automation 02 68 -71.
[9] Chen Y, Chen D Y 2014 Improved Runge-Kutta type Landweber image reconstruction algorithm for electrical capacitance tomography system Electric Machines and Control 18 (07) 107 -12.