Study on Mean Time Between Failures Prediction Algorithms Based on Weibull Distribution

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Abstract. The system reliability is significant importance because it is involved into most stages of the whole life-cycle of product. According to the existing literature, MTBF is the most frequently used index to indicate the system reliability. However, in engineering calculation, early failure data are always involved directly into MTBF's estimation, which results in that the estimated MTBF is always smaller than the actual MTBF, especially for the small samples. In order to alleviate this issue, the paper presents three integration algorithms to reconstruct PDF based on Weibull distribution, by reallocating the early failure data. Then relevant prediction expressions are derived by the reconstructed PDFs. Eventually, three new prediction methods are illustrated by a real example. As a result, the prediction value of Method 3 is very close to the actual observed value, moreover the prediction deviation rate is only 5.1%, which proves the proposed algorithm is reasonable. Finally the proposed methods are suggested applying to other distribution patterns.

1. Introduction

The system reliability is of significant importance because it is involved into most stages of the whole life-cycle of product. Thus, study on the evaluation of the system reliability has attracted more and more attentions in recent decades. Based on the existing literature, MTBF (mean time between failures) is currently the most frequently used index to indicate the system reliability [1]. For instance, Wang and Chu discussed an algorithm of determining the best-fitted distribution for MTBF assessment, and then demonstrated the proposed algorithm by a sample of 1.8 million LCD panels [2]. Braglia et al., presented a multivariate statistical approach to define MTBF of analyzed components and to identify the working parameters, depending on 126 centrifugal pumps [3].

In terms of the analysis above, the MTBF evaluation always requires plenty of failures. In contrast, small sample is the more common status [4]. Therefore, study on how to obtain MTBF for small sample is more important and urgent. Some scholars have already made some significant research on MTBF calculation for small samples. For example, Fulton and Abernethy, developed new methods to improve reliability prediction accuracy when using maximum likelihood estimates (MLE) for small samples [4]. Yang et al., developed a reliability modeling and assessment method for two-parameter Weibull distribution based on small sample data [5].
The preceding methods indicate that, existing prediction methods aims to obtain the optimizing parameter estimators of PDF (probability density function). Then the mathematical expectation of the lifetime are computed through numerical integration. However, one basic fact is that the small sample failure data mostly come from the failure high incidence area, whose failures could be ameliorated by product process improvement [6]. So the existing prediction methods, without redistributing the early fault date, maybe cause a not negligible deviation against the real value.

In general, when estimating MTBF of products, early failure data should be excluded and only fault data during normal operation period should be used. However, if the early fault data are removed for small samples, the density function fitting will become more severe. Therefore, how to fully mine and utilize existing fault data (including early fault data) and reasonably evaluate MTBF of products becomes very important. Therefore, this paper proposes some reconstructing PDF methods by reallocating the early failure data, further, the MTBF prediction expressions shall be derived by the reconstructed PDFs, by substituting the original PDF.

2. Background Review

The failure rate, $\lambda (t)$, defined as the number of failures per unit time per number of non-failed products remaining at time $t$ [7], is the foundation of research for the reliability assessment and lifetime analysis. According to the existing studies, $\lambda (t)$ branches out into three categories: (a) monotonic failure rates; (b) bathtub failure rates, where $\lambda (t)$ has a bathtub or a U shape; and (c) generalized bathtub failure rates [8]. As most complex systems have non-monotonic failure rate functions, bathtub-shaped life distribution is utilized widely, which can be divided into three periods: early failure period, normal operating period and wear-out period [1, 7, 8].

3. Algorithms Study

Figure 1 illustrates the bathtub-shaped Rate-of-Failure curve of a typical two-parameter Weibull distribution, in which the cut-off points, T1 and T2, are denoted. The bathtub-shaped curve is interrupted by cut-off points T1 and T2.

![Bathtub Curve of Typical Products](image)

**Figure 1. Rate of Failure**

For calculating the MTBF, accumulated failure rate in the early failure period is required to transfer from failure-rate-function to possibility-density-function. Firstly, the accumulated failure rate during
early failure period, \( F (t_1)\), remains unchanged. Then the corresponding time point \( t_1 \) in CDF shall be calculated. That is, the early accumulated failure rate from zero to \( T_1 \) in \( \lambda (t) \), equals to the rate from zero to \( t_1 \) in \( f(t) \).

**Method 1:** The early accumulated failure \( F (t_1)\), is deleted from the initial PDF \( f(t)_0\), then be reallocated to the whole serves lifetime in accordance with \( f(t)_0\). It is sure that the whole accumulated failure probability equals to unity. In result, the compensation expression is defined as:

\[
\delta (t)_1 = \int_0^{t_1} f(t)_0 dt \times f(t)_0 = F(t)_0 f(t)_0.
\]

Thus the new PDF is expressed:

\[
f(t)_1 = \begin{cases} 
\delta(t)_1, & t \leq t_1 \\
\delta(t)_1 + f(t)_0, & t > t_1 
\end{cases}
\]

Figure 2 illustrates the comparison of \( f(t)_I\) and \( f(t)_0\). Now the new PDF \( f(t)_I\) will be used to describe the MTBF expression, by substituting the initial PDF \( f(t)_0\). Hence, the cumulative distribution function \( F(t)_I\) and reliability function \( R(t)_I\) are got. It is noted the new reconstructed probability density function \( f(t)_I\) is discontinuous at \( t_1 \) time point.

\[
MTBF_I = \int_0^{\infty} R(t)_I dt = (1 + F(t)_1)MTBF_0
\]

**Method 2:** The start-time-point of the new probability density function is set for parallel moving backward \( t_1 \) hours. Viz., the new probability density function \( f(t)_II\) is supposed to start from \( t_1 \) to the end of life cycle. Then the early accumulated failure, \( F(t_1)_0\), is removed from the initial PDF \( f(t)_0\) and
considered as censored data of the new PDF \( f(t)_{II} \), referencing to literature [3]. As a result, the relative expressions are obtained as below.

The new PDF:

\[
f(t)_{II} = f(t + t_1)_0. \tag{4}
\]

The accumulated failure probability of the new PDF \( f(t)_{II} \) which starts from zero to infinite is verified as underlying.

\[
\int_0^\infty f(t)_{II} dt = \int_0^\infty f(t + t_1)_0 dt = \int_0^\infty f(t)_0 dt \bigg|_{t=t+t_1} = 1 - F(t)_0
\]

It could be found the accumulated failure probability of the new reconstructed PDF \( f(t)_{II} \) is not equal to one. Furthermore, the residual value between the new PDF and the original PDF, is \( F(t_1)_0 \), which should be included into MTBF calculation.

In result, the predicted MTBF\(_{III}\) is obtained, as the new reconstructed PDF \( f(t)_{III} \) is used on behalf of \( f(t)_0 \).

\[
MTBF_{III} = \frac{1}{1 - F(t)_0} (MTBF_0 - \int_0^{t_1} f(t)_0 dt - t_1 F(t)_0) \tag{5}
\]

### 4. Illustrative Example

The probability density function of the electronic control system of CNC grinders was fitted, which meets Weibull distribution with \( \eta = 658.5 \) and \( \beta = 0.3844 \) [9]. Hence, the real PDF, involving directly the early failures, shall be finalized as expression (6).

\[
f(t) = 0.03172 * t^{-0.6156} * \exp\left(-0.00152t\right)^{0.3844} \tag{6}
\]

According to the definition, MTBF\(_0 = \eta * \Gamma(1+1/\beta) = 2451.7 \) hours is obtained. This is the initial predicted MTBF, without redistributing the early fault data.

Referring to Figure 1, the cut point, \( T_1 \), is assigned as 3000 hours, considering that the total failure rate in this period is about 50%, according to the literature [10]. Furthermore, in terms of Equations (8), the cut-point of early failure period is calculated as 254 hour (t1) approximately. Additionally, a criterion should be fixed to appraise all of the above prediction methods. Here the real observed value of this EC system of CNC machines, MTBF\(_r\), is chosen as the comparison benchmark. Data is derived from the complete sample of 5 same CNC machines, with the same configuration observed for 5 years, the total failures of the EC systems are recorded as 31. Hence, the real observed value is obtained.

\[
MTBF_r = 5 \times 5 \times 250 \times 24 \div 31 = 4838.7h
\]

Finally, three MTBF prediction methods are implemented by proposed calculation expressions and the results are listed in Table 1 following.

| Method | MTBF\(_i\) calculation expressions | MTBF (h) | MTBF\(_r\) (h) |
|--------|----------------------------------|----------|---------------|
| 1      | Formula 3                        | 3677.8   |               |
| 2      | Formula 5                        | 2273.7   | 4838.7        |
| 3      | Formula 7                        | 4593.2   |               |
Based on Table 1, one can find, Method 3 is appropriate for MTBF prediction. Method 2 is unsuitable to be utilized to predicate MTBF. Moreover, the deviation rate of Method 3 can be got as: \((\frac{4838.7-4593.2}{4838.7}) \times 100\% = 5.1\%\).

![Prediction Performance Comparison](image)

**Figure 3.** Comparison of MTBF prediction for three methods

The comparison of three proposed methods can be plotted as Figure 3. The thick line indicates the real observed value of MTBF. The thin line represents the initial predicted MTBF, without reallocating the early fault data. The dash line depicts the predicted MTBF computed as per Method 3. The dot line describes the predicted MTBF obtained by Method 1. The dash dot line shows the predicted MTBF according to Method 2.

5. Conclusion

To predict MTBF for small sample size, taking account of the influence of the early failure data, the paper presents three integration methods to reconstruct PDF based on Weibull distribution. Then relevant prediction expressions are derived through reconstructed PDFs. Afterwards, the three methods are illustrated by an example. In result, the Method 3 is proposed for MTBF prediction, for those products subjecting to Weibull distribution.

Furthermore, the method of reconstructed PDF, by reallocating the early failure data for small samples, is suggested to apply to other distributions in future. Regarding some hypotheses in this exploratory study, such as the starting timeline parallel moving backward and T1 assumption, have to be investigated further.

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