Multi-objective Optimisation of McPherson Strut Suspension Mechanism Kinematics using Random Search Method

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Abstract

McPherson suspension mechanism is one of the widespread mounted mechanisms in front axle of Front Wheel Drive (FWD) vehicles with transverse engine. In this study the kinematics of McPherson suspension mechanism is optimised in order to achieve the desired kinematic behavior and improve vehicle stability. First, the mechanism was modeled in Mechanical Desktop software package and the model transferred to Working Model 3D software for kinematic analysis. Then results of kinematic simulation compared to design criteria and as target function is established, by choosing the optimal amount of optimisation variables the amount of cost function has been minimized. Because of simple implementability and also because of having the least influence on other specifications of the vehicle, the spatial location of joints selected as optimisation variables and changing range of variables is set in a rational interval. The aforementioned procedure done for determined numbers of iteration by use of Random Search Method to obtain the optimum results. Results show that the employed method has the ability to optimise the mechanism kinematics in suitable spent time. In addition, it is seen that after optimisation the alteration of toe angle, camber angle, and track width were improved noticeably. Finally, in order to validation the optimised mechanism modeled in ADAMS.

Keywords: McPherson Strut, Multi-objective Optimisation, Random Search Method, Suspension Kinematics, Suspension Mechanism

1. Introduction

Suspensions link the wheel to the body of a vehicle and the desired movements of the wheel in order to improve dynamic characteristics such as stability, steerability, and road holding are accomplished using suspension mechanism. Hence, suspension is one of the most important systems in a vehicle and plays a determining role in ride and handling quality. General requirements of the suspension system can be listed as¹–³:

- Independent movement of each wheel of an axle (not guaranteed in rigid axles)
- Provide vertical compliance so that the body isolated from road disturbances and consequently improving ride comfort
- Improve steerability of a vehicle
- Keep the contact between tire and road with minimal vertical load variations so that the road holding is increased
- Resist roll movement of the body
- Small unsprung mass in order to minimize wheel load fluctuations
- React to applied forces and/or torques to the tire

In the vehicles suspension system can be categorized as: rigid axle, independent suspension, and semi-rigid axle. For the first time leaf springs utilized in vehicle suspension because of simplicity in manufacturing of Iron⁴. Also, according to Figure 1 a rigid axle links two wheels. However, the rigid axle is still used in commercial vehicles due to high strength, economic purposes, and
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simple design, but its utilization in passenger vehicles (except SUVs) is not acceptable in recent years because of potential disadvantages. Some characteristics of rigid axles are\textsuperscript{1,2–4}:

- Mutual wheel influence
- Limited potential for desired kinematics and elas to kinematics
- Undesired bump and roll steer when leaf springs are utilized
- Large unsprung mass which decreases road holding and ride comfort consequently.
- Relatively less resistance to roll movement

In the case of the passenger vehicles independent suspension is widely used in order to improvement of ride, handling, and stability in comparison with rigid axle. Initial designs of independent suspension for mounting on front axle were presented in 1920s\textsuperscript{4}. In addition to independent movement of each wheel, independent suspension has some other advantages\textsuperscript{1,2}:

- Good potential for desired kinematics and elas to kinematics (e.g. camber and toe alteration)
- Improved road holding and ride comfort as a result of low weight of unsprung mass
- Requires little space

Various mechanisms of independent suspension system have been introduced. Recently, McPherson mechanism is mounting widely on front axle of passenger (also light commercial) vehicles because of its characteristics. The primary advantages of McPherson mechanism are: ability to achieve desired kinematics, low unsprung mass, small forces in links and bearings, and low space requirement in the transversal direction which simplifies mounting of transverse engines. Especially the last mentioned characteristic is of vital importance in most of passenger vehicles that are front drive with transverse mounted engine and transmission. McPherson suspension mechanism of a steerable front drive axle is illustrated in Figure 2.

King pin axis which wheel rotates around is projected in all three orthogonal planes. Because of this and also due to suspension linkage, the position of wheel alters in different planes when the wheel subjects to vertical move and/or steer angle. According to SAE definition upward and downwards movement of the wheel referred to as compression and rebound respectively\textsuperscript{5}. Desired suspension mechanism kinematics imply detailed and accurate wheel motion during compression and rebound in a manner that vehicle handling and stability improve.

Suspension optimisation is of interest for improving different features such as ride comfort and handling as well as kinematics\textsuperscript{6}. Suspension kinematics, due to playing a vital role in vehicle stability, has been widely investigated specially beyond the scope of optimisation. Mantaras et al.\textsuperscript{7} performed a 3D model of suspension and steering mechanism and studied alteration of suspension geometry as a function of damper displacement and steering wheel angle. The model simulated in MATLAB/Simulink and the research verified experimentally. Habibi et al.\textsuperscript{8} studied roll steer -which is referred to as toe angle alteration due to roll motion- of a McPherson mechanism and performed an optimisation process to minimize the roll steer. Genetic algorithm is used to determine the optimum length and orientation of the members so that the alteration of camber, caster, and toe is minimized. The

Figure 1. Rigid axle suspension with leaf spring.\textsuperscript{1}

Figure 2. McPherson suspension mechanism.\textsuperscript{1}
variable parameters consist of length and initial angle of the control arm, King-pin inclination, and initial length of strut. Lee et al.\textsuperscript{9} investigated sensitivity analysis of a McPherson mechanism and determined dominant design variables to wheel angles and forces or moments. Then, multi-objective index i.e. camber, toe, King-pin inclination, and caster is proposed and the results show significant improvement in alteration of wheel angles. Thoresson et al.\textsuperscript{10} proposed a simplified mathematics method to optimise handling and ride comfort. Combined optimisation of ride comfort and handling is performed as well as individual. Results show that although simplified method took more iteration, the computing time has considerably reduced. Esfahani et al.\textsuperscript{11} optimised the camber angle of a double wishbone mechanism. The modelled mechanism had the ability to change the camber angle by the amount of $\pm 5$ degrees using a hydraulic actuator. Simulation is carried out in MSC Nastran and it is shown that the response time of variable camber mechanism is suitable.

In the present paper the kinematics of a McPherson mechanism is optimised via varying the spatial location of joints using Random Search method, while design parameters such as caster angle and length of links are almost maintained. This procedure ensures that optimisation has the least influence on other specifications e.g. cross wind disturbances, steering effort, and stress in the components. Then the kinematics of optimised and initial mechanisms is compared and validation of the optimisation performed by modeling the mechanisms in ADAMS software package.

2. Suspension Kinematics

When the wheel subjects to vertical move, position and angle of wheel would change in three planes of cartesian coordinate system. Suspension kinematics describes the movement caused in the wheels during vertical suspension travel and steering\textsuperscript{1}. The changes in wheel position include camber angle alteration, toe angle alteration, track width alteration, and caster angle alteration while wheel travelling. A suspension mechanism should be designed and fabricated for achieving desired kinematic features. In other words, the alteration of camber, caster, toe and track width while wheel travelling should lead to improve the stability, handling and other dynamic specification of the vehicle. However, the alteration of above mentioned parameters cause disturbing lateral force which consequently will decrease directional stability and raises tire wear.

Figure 3 shows the front view of a McPherson strut suspension mechanism and its camber alteration versus wheel travel. As it is illustrated, the camber angle would be more negative during compression and the camber angle would be less negative and gradually tend to be positive while rebounding. This is a desired feature for a suspension mechanism because while a car is turning the outside and inside wheels will compressed and rebound respectively. Because of body roll and weight transfer, the most lateral force generates by outside wheels. Moreover, in critical conditions when road holding of inside wheels is extremely decreasing, making more negative camber angle for outside wheels lead to generate more lateral force (see Figure 4) and consequently more stability and more handling for the car.

Figure 3 also shows that in addition to camber alteration, track width alteration will occur as well. Figure 5 illustrates track width alteration for three suspension mechanisms. As it can be seen, track width increases during compression and decreases while rebounding. This feature is desired as well, because increase of vehicle mass widens track width and according to Figure 6 and Equation 1 and 2, could decrease the roll motion of body. However graph no. 3, in which the track width faces with decrease if the wheel compresses more than 40mm, is an exception. This shows potential weakness of the designed mechanism that usually occurs in McPherson strut mechanisms. Furthermore, according to Figure 4, camber angle alteration while wheel travelling inevitably leads to

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{camber.png}
\caption{Camber angle alteration due to wheel travel.}
\end{figure}
track width alteration. And, track width alteration would be more important because of the roll center height. Roll center is the point that body roll occurs around it and it is defined for both rear and front suspension mechanisms. Connecting these two points together will result in a line is called roll axis. Roll center can also define as the point that is the instantaneous center for body roll relative to ground so that its position depends on the suspension mechanism geometry.

\[
\varphi = \frac{M_g}{K_p} 
\]

\[
K_p = \frac{1}{2} K_T T_p^2
\]

According to Figure 6, the roll moment depends on lateral force and lateral force lever; the distance between roll center and the center of gravity. Hence, lateral force lever is an important factor for body roll, the less lateral force lever is, the more stability and ride comfort the vehicle has. According to kinematics of a suspension mechanism, if there is a track width alteration as shown in Figure 5, the roll center will be positioned between ground level and the center of gravity. But, if there is no track width alteration while wheel traveling, the roll center will be positioned at the ground level that leads to increase of roll motion and decrease of stability and ride comfort as a consequence of lateral force lever enlargement. Therefore, using a suitable design for a suspension mechanism results in track width alteration in a manner that it will alter positive while compression. Consequently, by limiting the roll motion of body, vehicle stability increases.

Another parameter that alters while wheel travelling, is the toe angle. Figure 7 shows toe angle alteration versus wheel travel. According to the Figure, in compression the toe angle tends to outside and vice versa. As it shown in Figure 8, because of outer wheel compression and rebound of inner wheel, the steer angle reduces in corners, which is known as roll understeering. The case can be a good property of a suspension mechanism because it increases the vehicle stability in maneuvers.

### 3. Modelling and Optimisation Procedure

One of the design methods of the McPherson strut suspension mechanism is path generation. So that it is expected that some specific points of the mechanism
should follow a desired path. In this research, achieving suitable alteration of camber angle, toe angle, and track width versus wheel travel is the target of optimisation. To model the mechanism, parametric design of the mechanism is done in Mechanical Desktop (MDT) software package. In modeling, all parameters of the steering mechanism are considered as input variables and variables for optimising the suspension mechanism are as follows:

- Spatial position of damper mounting
- Spatial position of revolute joint of lower control arm
- Spatial position of spherical joint of lower control arm to knuckle arm

After that, by setting the input variables and initial values of optimising variables, the model is created in MDT. Then, the model is sent to Working Model 3D software package for kinematic analysis. Based on the results of kinematic analysis as graphs of camber angle, toe angle, and track width alteration versus wheel travel and in comparison with target graphs using least square theorem, cost function is made. To minimize the cost function, the optimising variables are changed in defined and rational intervals by utilization of random search method. Figure 9 shows parametric design and optimisation procedure of the mechanism. Figures 10 and 11 illustrate the model in MDT and Working Model 3D software package respectively.
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Hence, three scale factors regarding to camber angle, toe angle, and track width alteration are achieved. In the process of designing a suspension mechanism, the importance and the value of each designing criteria may be different concerning to designer opinion. Therefore, using weight factor proposes a suitable way so that more important criterion can get larger weight factor. Thus, the total cost function can be defined as Equation 5.

$$F_{f} = \frac{F_{c}}{\max(F_{c})}$$  \hspace{1cm} (4)

To optimise the suspension mechanism, it should be minimized the total cost function. Random Search can be a good choice because of its precision and speed. Figure 12 shows the procedure of achieving the minimized cost function by using Random Search. To optimise the suspension mechanism, it should be minimized the total cost function. Random Search can be a good choice because of its precision and speed. Figure 12 shows the procedure of achieving the minimized cost function by using Random Search. Table 1 illustrates the design parameters used in kinematic analysis of the mechanism.

4. Results of Optimisation and Simulation

Spatial positions of joints for initial and optimised model are presented in Table 1. Also, Figure 13 shows components and joints of the mechanism. Simulation results of the optimised and initial mechanisms are illustrated in Figures 14 to 16. As it can

$$F_{c,\text{Tot}} = [f_{w} \cdot f_{c} \cdot F_{c}^2] + [f_{w} \cdot f_{s} \cdot F_{s}^2] + [f_{w} \cdot f_{c} \cdot F_{c}^2]$$  \hspace{1cm} (5)

In order to kinematic optimisation of the mechanism based on least square theorem, cost function is defined as Equation 3.

$$F_{c} = \sum_{j=1}^{n} \left(y_{j} - g(x)\right)^2$$  \hspace{1cm} (3)

To optimise the mechanism, it should be tried to minimize the cost function. Because this function shows the error value between graphs of real model and those of target model. And, minimizing the error value shows that the error value between graphs of real model and those of target model is diminishing. Considering three criteria for kinematic optimisation of the mechanism, hence three cost functions should be defined. To sum these cost functions and achieve the total cost function, scale factor, given by Equation 4, should be defined in order to make it dimensionless and the same scale all cost functions.

Figure 10. Model in MDT.

Figure 11. Model in Working Model 3D.

Figure 12. Random Search theorem flow chart.
be seen, the kinematics of the mechanism has been gotten closer to the target graphs. This will lead to improve vehicle dynamics.

Table 1. Design parameters

| Unit | Value | Parameter          | Value |
|------|-------|--------------------|-------|
| mm   | 1400  | Track width        | 1     |
| ---  | 0.65  | Aspect ratio of tire | 2     |
| mm   | 272   | Tire radius        | 3     |
| mm   | 165   | Tire width         | 4     |
| mm   | -7    | Scrub radius       | 5     |
| deg  | 2.02  | Caster angle      | 6     |
| deg  | 0     | Camber angle      | 7     |

Table 2. Spatial positions of joints

| Optimised position (x,y,z) mm | Initial position (x,y,z) mm | Joint |
|------------------------------|-----------------------------|-------|
| (0,700,0)                    | (0,700,0)                   | J₁    |
| (126.5, 268.9, 60.7)         | (126.5, 268.9, 60.7)        | J₂    |
| (23, 510, 476.8)             | (11.2, 519.6, 482.4)        | J₃    |
| (73.4, 382.5, -73.8)         | (52.8, 398.7, -76.4)        | J₄    |
| (109, 614, 70)               | (109, 614, 70)              | J₅    |
| (3, 660.7, 10.2)             | (-9.2, 663.2, 10.2)         | J₆    |

Figure 13. Mcpherson strut mechanism with rack and pinion steering; 1: Tire, 2: Damper strut, 3: Lower control arm, 4: Tie rod, 5: Rack, 6: Body.

Figure 14. Track width alteration.

Figure 15. Camber angle alteration.

Figure 16. Toe angle alteration.
The difference between graphs of the optimised mechanism and the target is because of three following reasons:

The first reason is that some parameters which have considerable effect on the kinematic properties of the mechanism such as caster angle, the length and position of tie rod, and the initial value of camber angle are input and as constant values to the optimisation program and just the spatial position of three joints are selected as variable parameters for optimisation program. In the case optimisation procedure is restricted, although the implementation of the mechanism would be simplified.

The second is that the design of a suspension mechanism is a multi-task case. For instance, caster angle has a primary effect on steering effort, self-steering torque, and sensitivity to lateral disturbances as lateral wind. Therefore, the optimising variables of the mechanism are selected by this fact that optimisation has the least effect on other aspects of the suspension operation.

The third one is that the target graphs are related to an ideal multi-link suspension mechanism. This mechanism is a complex and an expensive mechanism that is suitable for luxury and high performance vehicles and obviously the kinematic properties for these kinds of mechanism are much better than a simple mechanism e.g. McPherson strut mechanism. In other words, McPherson mechanism does not seem to have the potential to meet the kinetics of a multi-link mechanism.

Increasing track width during compression is a suitable factor as promoting the kinematic property of the mechanism. According to Figure 14, one of the weaknesses of the initial model of the mechanism is decrease of track width while the wheel is compressing for more than 50 mm. This means in severe maneuvering or turning, when wheel compression would be more than 50 mm, the stability of the car would be in great danger. The reason is that in this situation the distance between the roll center and the center of gravity would considerably increase. Although in the optimised model of the mechanism the problem is not completely solved, it would happen when wheel compression exceeds 62 mm. This shows the issue is considerably developed.

Camber angle alteration is completely vital in corners, especially for outer wheel. As it can be seen in Figure 15, while the wheel compresses in maneuvering or turning the camber angle of the optimised model becomes more negative. Thus, camber thrust and lateral force in outer wheel will increase and this case leads to promote lateral stability of the vehicle.

Figure 16 shows toe angle alteration that is important to roll understeering of the vehicle. According to the Figure, it can be seen that the initial model is more understeer than the target model and after optimisation the understeer gradient of the model has been decreased. Since during maneuvering, the more a vehicle is understeer the more stable vehicle is, but it affects steerability as well. It is the reason that the target graph has less understeer gradient than the initial model. So, the optimised mechanism produces a rapider reaction to driver`s steering.

5. Validation

To validate the procedure of the optimisation, the optimised mechanism is modeled in ADAMS software package and its kinematics is simulated.

Figure 17. Model in ADAMS.

Figure 18. Track width alteration.
Figure 17 illustrates the mechanism model in ADAMS. The results of the simulation are also shown in Figures 18 to 20. As it can be seen, the results of kinematic simulation of the mechanism for both methods verify each other.

6. Conclusion

In this paper, kinematic optimisation of a McPherson strut suspension mechanism is fulfilled by using Random Search method. First, the mechanism is modeled in Mechanical Desktop software package and then, the simulation is performed in Working Model 3D. Spatial position of damper mounting, revolute joint of lower control arm, and spherical joint of lower control arm are selected as optimisation variables. Other parameters of the mechanism are considered as constant factors. The cost function is composed of error of camber angle, toe angle, and track width. Also, kinematic properties of an ideal mechanism are considered as the target function. Finally, validation of the optimisation results is done by modeling the optimised model in ADAMS software package so that the results of both methods verify each other. Although the kinematic properties of a McPherson strut are less favourable than the target mechanism and also some parameters of the mechanism are selected as constant input values, results show that the kinematic properties of the optimised mechanism are considerably developed. But, to achieve more capability for the mechanism, it is suggested to utilize more complex mechanisms such as double wishbone and multi-link.

7. References

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## Appendix

| Nomenclature                | Subscripts |
|-----------------------------|------------|
| $f$ : coefficient           | $c$ : cost |
| $F$ : function              | $s$ : scale|
| $g$ : present function      | $sp$ : spring|
| $H$ : center of gravity height, m | $T$ : track width |
| $K$ : stiffness, Nm$^{-1}$   | $Tot$ : total |
| $M$ : moment, Nm            | $w$ : weight |
| $T$ : track width, m        | $\delta$ : toe angle |
| $y$ : target function       | $\epsilon$ : camber angle |
| $\varphi$ : roll angle, rad | $\varphi$ : roll |