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A Joint Matrix Completion and Filtering Model for Influenza Serological Data Integration

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Abstract

Antigenic characterization based on serological data, such as Hemagglutination Inhibition (HI) assay, is one of the routine procedures for influenza vaccine strain selection. In many cases, it would be impossible to measure all pairwise antigenic correlations between testing antigens and reference antisera in each individual experiment. Thus, we have to combine and integrate the HI tables from a number of individual experiments. Measurements from different experiments may be inconsistent due to different experimental conditions. Consequently we will observe a matrix with missing data and possibly inconsistent measurements. In this paper, we develop a new mathematical model, which we refer to as Joint Matrix Completion and Filtering, for HI data integration. In this approach, we simultaneously handle the incompleteness and uncertainty of observations by assuming that the underlying merged HI data matrix has low rank, as well as carefully modeling different levels of noise in each individual table. An efficient blockwise coordinate descent procedure is developed for optimization. The performance of our approach is validated on synthetic and real influenza datasets. The proposed joint matrix completion and filtering model can be adapted as a general model for biological data integration, targeting data noises and missing values within and across experiments.

Introduction

Influenza virus causes both seasonal epidemics and pandemics, and continues to present a threat to public health.抗原特性变化由漂移或变异所致，影响抗原特性的突变，从而导致流感病毒逃逸免疫积累的种群。抗原变异可以是由于抗原基因突变或基因重排引起的。抗原变异导致抗原表位的改变，从而影响流感病毒的免疫学特性。

A serological data can be viewed as an $n \times m$ matrix, where $n$ and $m$ are the numbers of antigens and antisera in the assays, respectively. This matrix is used to quantify (direct or indirect) binding reactions between the two comparing antigen and antiserum. Each matrix entry (titre) can be a numeric value (high reactor), or a low reactor, or a missing value [4]. The low reactors are results of the detection limit of a serological assay, marked as $\leq \theta$, where $\theta$ is a threshold indicating the detection limit. The missing values are generally caused by the limitation of resources.

In many cases, it would not be possible to perform all pairwise comparisons between antigen and antiserum in each individual experiment. Thus, we have to combine and integrate the datasets from a number of individual experiments, each of which could be a separate dataset (matrix with missing values) by itself. Measurements from different experiments may be inconsistent due to different experimental conditions. Consequently we will observe a matrix with missing data and possibly inconsistent measurements. Taking the HI assay as an example, typically less than 15 reference antisera (antibodies) are used in each assay but the number of test antigens (viruses) can be more than 100. It is not possible to perform HI for all pairs of antigen and antiserum reactions. Thus, the resulting HI table is generally incomplete. The data absence in these low throughput biological experiments could be up to 95%.

Integration of influenza serological data is critical for vaccine strain selection and pandemic preparedness by providing an antigenic “bluemap” for influenza viruses, including contemporary and historical human influenza viruses and zoonotic influenza viruses. However, integration of influenza HI datasets is not trivial because HI data are notoriously noisy within and across the experiments. A number of factors can affect the robustness of HI assays [3]:

- Reference antisera. The antisera batch (from different challenge experiments), storage conditions (temperature time), and frozen-thawing can affect HI titre values.
- Types and batches of red blood cells. The red blood cells from different animal species affect the HI titre values dramatically, and the red blood cells from the same species can affect the HI titre values.
- Variations in supplies. The types of plates (e.g., “U” or “V” shape) can affect the HI data interpretation.
- Error from personnel.
Among these parameters, the antisera and red blood cells will probably most significantly affect the results from HI assays [3]. To date, there is still lack of robust computational models for integrating influenza HI data.

This paper proposes a new mathematical model called Joint Matrix Completion and Filtering Model, for influenza serological data integration. We address the major challenges caused by the incompleteness and uncertainty in the observation. The proposed model simultaneously handles these two challenges by assuming that the underlying joint table is low rank and carefully modeling different levels of random effects in each individual data table. We develop an efficient blockwise coordinate descent procedure for optimization. The performance of our model is validated on synthetic and real influenza datasets.

Joint Matrix Completion and Filtering Model for Biological Data Integration

In this study we focus on influenza HI data. There are two major challenges in HI table integration: (i) to complete missing entries; (ii) to handle noisy observations. We propose a joint matrix completion and filtering method to simultaneously address these two challenges. The basic idea of this model is to approximate the merged data matrix using low-rank matrix decomposition, and to incorporate the experimental bias of individual tables. The challenge of missing data is handled by the low-rank matrix model, where we have assumed that the antigens and antisera can be represented by latent low-dimensional vectors, and the pairwise inner product of these vectors approximate HI titre values (including missing entries). The low-rank assumption of the merged HI table is motivated from the intuition that its rows corresponding to those antiviruses with similar antigenic characterization are expected to be linearly dependent. Such a low-rank assumption has also been made in our previous work [4] which was shown to work quite well in completing a manually merged HI table from [5]. The challenge of noise is handled by a two-level hierarchical model which incorporates the biases of antisera in different experiments and the system noise. The antisera-level bias should be considered because the antisera batch (from different challenge experiments), storage conditions (temperature time), and frozen-thawing can affect HI titre values. The system noise comes from a number of sources such as types and batches of red blood cells, variations in supplies, and error from personnel.

By combining the above two ideas, the joint model can be mathematically posed as follows:

\[ h_{ij,k} = u_i^T v_j + r_{j,k} + s_{i,j,k}, \]  

where \( h_{ij,k} \) are the observed HI titre values between the antigen-antisera pairs \((i,j)\) in table \(k\), \(u_i, v_j \in \mathbb{R}^d\) are latent representing vectors for antigen \(i\) and antisera \(j\), respectively, \(r_{j,k}\) are the (deterministic) bias of antisera \(j\) in table \(k\), and \(s_{i,j,k} \sim N(0, \sigma^2)\) are the i.i.d. Gaussian random noises with unknown variance \(\sigma\). Our goal is to estimate the parameters \(u_i, v_j, r_{j,k}, \sigma\) from the observations \(h_{ij,k}\).

We propose to estimate the parameters via solving the maximum log-likelihood objective which is equivalent to the following minimization problem:

\[ \min_{U, V, R} \sum_{i,j \in \Omega, k \in K} \frac{(h_{ij,k} - u_i^T v_j - r_{j,k})^2}{\sigma^2} + \log(\sigma^2), \]  

where \( U = (u_i)_{i \in \mathbb{R}^d \times |\Omega|}, V = (v_j)_{j \in \mathbb{R}^d \times |\Omega|}, \) and \( R = (r_{j,k})_{j \in \mathbb{R}^{|\Omega| \times |K|}} \). The summation of \((i,j)\) is taken over \(\Omega \subseteq |I \times J|\) which is the set of antigen-antisera pairs observed in at least one table. The summation of \(k\) is taken over \(K \subseteq |I \times J|\) which is the set of tables in which the HI titre between antigen \(i\) and antisera \(j\) are observed. Note that the estimation of \(U, V, R\) is separated from that of \(\sigma\) in model (2). That is, we may first estimate \(\{U, V, R\}\) via the following least square formulation:

\[ \{U, V, R\} = \arg \min_{U, V, R} \sum_{i,j \in \Omega, k \in K} \frac{(h_{ij,k} - u_i^T v_j - r_{j,k})^2}{\sigma^2}, \]  

Table 1. Simulated HI dataset: RMSE of HI titre and antiserum bias under different noise level configurations.

| Measurements | Methods | \(|\sigma; \sigma| = (1,1)\) | (2,1) | (2,2) | (3,3) |
|--------------|---------|----------------------------|---------|---------|---------|
| RMSE         | Our Model | 1.213 (0.049) | 1.336 (0.043) | 1.414 (0.066) | 1.624 (0.061) |
| (in HI titre) | Average Model | 1.228 (0.065) | 1.602 (0.052) | 1.663 (0.058) | 2.004 (0.068) |
| RMSE         | Our Model | 1.054 (0.022) | 0.984 (0.016) | 1.005 (0.013) | 1.091 (0.026) |
| (in antiserum bias) | Average Model | 1.078 (0.026) | 1.225 (0.031) | 1.225 (0.028) | 1.403 (0.035) |

These figures are generated by setting rank values as the optimal ones according to Figure 1.

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and then estimate \( \sigma \) according to

\[
\sigma = \sqrt{\frac{1}{n} \sum_{i,j,k} (h_{i,j,k} - u_i^T v_j - r_{i,j,k})^2},
\]

where \( n = \sum_{i,j,k} \Omega_{i,j,k} \) is the total number of observations in all HI tables.

We present a blockwise coordinate descent procedure to solve the problem (3). Starting from an initialization \( R^{(0)} \), the algorithm alternates between solving the following two subproblems on \( U, V \) and on \( R \) respectively:

\[
\begin{align*}
\{U^{(t+1)}, V^{(t+1)}\} &= \arg \min_{U, V} \sum_{i,j,k} (h_{i,j,k} - u_i^T v_j - r_{i,j,k})^2, \quad \text{(4)} \\
R^{(t+1)} &= \arg \min_R \sum_{i,j,k} (h_{i,j,k} - (u_i^{(t)})^T v_j^{(t)} - r_{i,j,k})^2. \quad \text{(5)}
\end{align*}
\]

In our implementation, we initialize the algorithm with \( R^{(0)} = 0 \) which works satisfactory in the empirical study. We next describe how to solve the subproblems (4) and (5).

Let us consider the minimization of subproblem (4), which is equivalent to the following problem of rank-\( d \) matrix factorization with missing values:

\[
\min_{Z, U, V} \|(Z - U^T V) - W\|_F^2, \quad \text{s.t.} \quad z_{ij} = z_{ij}, \quad i,j \in \Omega,
\]

where \( \| \cdot \|_F \) denotes the Frobenius norm of matrix, \( z_{ij} = 1/|\Omega| \sum_{k} (h_{i,j,k} - r_{i,j,k}) \), \( W_{ij} = |\Omega| \) if \( i,j \in \Omega \) and \( W_{ij} = 0 \) otherwise. This problem is well studied in the matrix completion literature and a number of efficient algorithms are available online [6–10]. Here we utilize the LMaFit solver developed by [6] which works well for our problem.

Concerning the minimization problem (5), it is not difficult to check that \( R^{(t+1)} \) has the following closed-form solution: for each \( j \),

\[
R^{(t+1)} = \arg \min_R \sum_{i,j,k} (h_{i,j,k} - (u_i^{(t)})^T v_j^{(t)} - r_{i,j,k})^2.
\]

Figure 2. Antiserum bias distribution. Top row: the ground truth antiserum bias added to the simulated tables for antiseras #1, #20, #40, #80. Bottom row: the recovered antiserum bias by our algorithm for the same four antiseras. The noise level configuration for this group of experiment is \((r,\sigma) = (3,1)\). The rank value \( d = 5 \) is selected in our model.

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Figure 3. Antigenic cartographies constructed by the considered models on the simulated data. Here we use MC-MDS for cartography construction.
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and report the mean and standard deviation of RMSE. Figure 1 shows the RMSE versus rank curves obtained under different noise level configurations. From this group of curves we can make the following observations: (i) RMSE increases as the noise level increases, especially when rank is relatively large; (ii) the lowest RMSE is achieved at the rank close to ground truth rank 10; and (iii) the lowest RMSE is insensitive to noise level. The second and third observations confirm that low-rank approximation and hierarchical noise model are effective for accurately and robustly imputing the missing entries.

We now compare the proposed model to a baseline method which simply averages the measurements observed at each entry and then perform matrix completion over the resultant mean table. We call this method average model. This can be viewed as a simplified model of ours by setting $r_{jk} \equiv 0$ in Equation (2) (see Materials and Methods section). Table 1 lists the results for two methods respectively. Note that the only difference between our model and the average model is whether or not the antiserum bias is explicitly modeled. These results clearly showed that the explicit estimation of the antiserum bias improves the matrix completion accuracy. Figure 2 shows the ground truth antiserum biases we add to the data and the recovered biases $\hat{r}_{jk}$ by our model with the noise level configuration $(r,\sigma)=(3,1)$. The rank value parameter is set to be $d=5$ which is optimal according to Figure 1. We can see from this qualitative result that the recovered antiserum bias is close to the ground truth in terms of bias distribution and direction. The RMSE values between the estimated antiserum bias and the ground truth antiserum bias are listed in the row 4 and row 5 of Table 1, respectively for our model and the average model. Here the RMSE of antiserum bias by the average model is estimated in a similar way as in our model (recall that the average model can be taken as a special case of our model). This group of quantitative comparisons confirm the advantage of our model over the average model in recovering antiserum bias.

Figure 3 shows the constructed antigenic cartographies from the ground truth table, the integrated tables by our model as well as those by the average model. For each model, we show the cartographies constructed by MC-MDS [4] which is a representative MDS method for antigenic cartography construction. We show the results under noise level configuration $(r,\sigma)=(2,1)$ (see Figure 3(b)&3(c)) and $(r,\sigma)=(3,1)$ (see Figure 3(d)&3(e)). For each noise level configuration, we select the optimal rank $d$ in our model based on the RMSE curves in Figure 1. Visual inspection on Figure 3 shows that the cartographies constructed from our model are closer to the ground truth than those constructed from the average model.

Data Integration on HI Datasets of 2009 H1N1 Influenza A Viruses

The H1N1 2009 HI dataset contains 6 HI tables collected from May of 2009 to August of 2009. The merged table is of size $253 \times 28$, i.e., 253 viruses versus 28 antisera. The RMSE versus rank curves for H1N1 2009 are shown in Figure 4. The mean and deviation of RMSE are calculated by 10-fold cross validation. The RMSE values between the estimated antiserum bias and the ground truth in terms of bias distribution and direction. The RMSE values between the estimated antiserum bias and the ground truth antiserum bias are listed in the row 4 and row 5 of Table 1, respectively for our model and the average model. Here the RMSE of antiserum bias by the average model is estimated in a similar way as in our model (recall that the average model can be taken as a special case of our model). This group of quantitative comparisons confirm the advantage of our model over the average model in recovering antiserum bias.

Figure 5 shows the constructed antigenic cartographies from the ground truth table, the integrated tables by our model as well as those by the average model. For each model, we show the cartographies constructed by MC-MDS [4] which is a representative MDS method for antigenic cartography construction. We show the results under noise level configuration $(r,\sigma)=(2,1)$ (see Figure 5(b)&5(c)) and $(r,\sigma)=(3,1)$ (see Figure 5(d)&5(e)). For each noise level configuration, we select the optimal rank $d$ in our model based on the RMSE curves in Figure 1. Visual inspection on Figure 3 shows that the cartographies constructed from our model are closer to the ground truth than those constructed from the average model.

After recovering the merged HI table, we will project the influenza antigens onto a two-dimensional antigenic cartography. Antigenic cartography presents an intuitive way to validate our computational models with biological domain knowledge. The details can be found in the Methods and Materials section.

**Results**

**Parameter Selection**

There is one free parameter, the rank $d$ for matrix completion, in our model. For the simulated data and the influenza HI data of 2009 H1N1 an H3N2 used in this study, a 10-fold cross validation suggested a value of $d$ between 3 and 20 for achieving low Root Mean Squared Error (RMSE). For model initialization, we simply use the mean table as the starting point. We have also tried random initialization in the vicinity of the mean table. The unreported experimental results suggest that the solution of our model is insensitive to these initialization schemes.

**Data Integration on Simulated HI Dataset**

To demonstrate the effectiveness of our model in influenza data integration, we performed Monte Carlo simulation using 100 randomly generated HI tables according to the joint model (1) (see Materials and Methods section). The ground truth full HI table contains 600 antigens versus 100 antiseras and its rank is 10. We further respectively group antigens and antiseras into 10 groups. The intra group HI titre is set to be high while the inter group HI titre is set to be low. Six noise level configurations $(r,\sigma)=((0,0),(1,1),(2,1),(2,2),(3,1),(3,3))$ are considered in the simulation.

We use RMSE between the recovered HI titre and the true HI titre as a measure of accuracy. We repeat the experiment 10 times, and report the mean and standard deviation of RMSE. Figure 1 shows the RMSE versus rank curves obtained under different noise level configurations.
colored by month in which they are collected, while the antigens are colored by tables in Figure 5(c)&5(d). Here we select rank $d = 3$ in our model through 10-fold cross-validation (see Figure 4).

The antigenic cartography by the average model showed that the viruses are separated by strips, indicating the viruses from different months are antigenically different. This is not true because there is no significant evidence demonstrating influenza antigenic drift in the first wave of 2009 H1N1 pandemic. In contrast, our joint matrix completion and filtering model was able to integrate these data, and the viruses from different months are mixed altogether. This is biologically sound.

**Data Integration on HI Datasets of 2000–2007 H3N2 Human Seasonal Influenza Viruses**

The H3N2 2000–2007 data used in our study contains a total of 369 individual HI tables performed from 2000 to 2007, including 11383 viruses and 393 antisera. All HI experiments were performed using turkey red blood cells. The RMSE versus rank curves are plotted in Figure 6. The mean and deviation of RMSE are calculated by 10-fold cross validation. Once again, the curves suggest that our model outperforms average model in table integration accuracy.

Figure 7 shows the constructed antigenic cartographies for both our model and average model. These antigens are labeled according to their collected years. Visualization shows that the constructed cartography from our automatic model is more compact than that from average model. It is interesting to note that our cartography is consistent to that created from a manually curated HI table [11]. The Matlab code for reproducing the results in this paper will be available upon request.

**Figure 5. Antigenic cartography of H1N12009 dataset.** Here the rank value $d = 3$ is selected in our model for matrix completion and MC-MDS is used for cartography construction. Top row: the antiviruses are colored by collected date. Bottom row: the antiviruses are colored by table.

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Discussion

In this work, we investigated the problem of combining a set of HI tables from different experiments in the same laboratories or even from different laboratories into an integrated table. With such a fusion of data, the rich but potentially redundant information contained in individual tables can be compactly and robustly represented in a single table which will in turn facilitate data visualization and higher level biological data mining tasks such as influenza surveillance. We note that each HI table (experiment) only includes a dozen of reference antisera, which are updated at each influenza season or even each month within the same influenza season. In addition, it is common for individual laboratories to use different antisera. Therefore, the integrated HI table is typically an incomplete matrix. For each pair of antigen-antiserum, its observed HI titre values in individual tables could be varying due to the random effects of antisera and the system noise. Therefore, the incompleteness and uncertainty of observations present two major challenges in combining tables into an integrated table.

We proposed a computational method for integrating multiple HI tables, and demonstrated its usefulness in antigenic characterization. We paid special attention to the major challenges caused by the incompleteness and the uncertainty of data. It is shown by extensive experiments that the standard approach by averaging the observations does not perform well. We proposed a simultaneous matrix completion and filtering model which carefully models different levels of random effects in each individual table. Our experimental results showed that more accurate results can be obtained through this new method. After table merging, we can use MDS to construct its antigenic cartography. In our experiments, we constructed antigenic cartography by using MC-MDS [4]. We have also tried another representative MDS method, the metric MDS [5], for cartography construction. The main difference between metric MDS and MC-MDS lies in that the former directly fits the HI titre (with proper transformation) of antigen-antiserum pairs while the latter regards each antigen as a row vector of data matrix and fits the pairwise Euclidean distances between antigens. Our unreported results suggest that both MDS methods work well although the MC-MDS tends to output more compact cartographies than metric MDS, as described elsewhere [4].

As we have shown in Figure 4 and Figure 6 that the prediction performance deteriorates as the rank parameter of matrix completion increases. In fact, the same trend has also been observed in the constructed antigenic cartography. Figure 8 shows the antigenic cartographies of H1N1 2009 constructed by MC-MDS under rank 3, 10, 20. It can be seen that as rank increases, the antigenic cartography becomes more and more separative. This verifies that low rank assumption is vital for HI table integration.

The proposed data integration model can be used as a generic data integration model for biological datasets, especially the binding affinity data. The study of biological systems typically involves quantification of the binding affinity amongst different types of biological agents, which could be DNA, RNA, ligands, proteins, cells, or even living organisms, such as viruses. The reactions could be DNA versus DNA, DNA versus RNA, DNA versus protein, protein versus ligand (e.g. carbohydrates), and protein versus protein. Similar to HI data, the data point in these binding affinity datasets also suffers from missing values. For example, the missing data problem is also very common in high throughput DNA microarrays, protein microarrays and glycan microarrays, and these missing values can result from different version of array, poor printed spots in the slides, insufficient resolution, or hybridization problems [12]. Integration of this binding affinity data from individual experiments can be formulated to a similar problem as HI data integration studied in this paper. Thus, our proposed joint matrix completion and filtering model can be adapted as a general model for biological data integration, targeting data noises, missing values, and low reactors within and across experiments.

Materials and Methods

Generation of Simulated HI Datasets

In Monte Carlo simulation study, we generated a set of HI tables according to the data model (1). More specifically, we utilized the following steps to generate tables:

- Generate $d = 10$ dimensional latent coordinates $U \in \mathbb{R}^{d \times |I|}$ and $V \in \mathbb{R}^{d \times |J|}$ for $|I| = 600$ antigens and $|J| = 100$ antisera, respectively. Both the antigens and antisera are equally partitioned into 10 blocks. In order to generate such $U, V$, we first generate a matrix $T \in \mathbb{R}^{[d \times |I|]}$ (the ground truth full table) with entry $T_{ij}$ is randomly drawn from $\{10-|t_i-t_j|, 10-|t_i-t_j|-1, 10-|t_i-t_j|-2, 10-|t_i-t_j|-3\}$ where $t_i, t_j$ are the corresponding block index. We then calculate $U, V$ by rank-$d$ decomposition of $T$ through singular value decomposition.

- Generate the antiserum bias $R \in \mathbb{R}^{[d \times |K|]}$ for $|K| = 100$ tables, with integer $r_{j,k}$ uniformly drawn from interval $[-r, +r]$. Choose variance $\sigma$ for the white Gaussian noise. We will consider different configurations of $r$ and $\sigma$ in our empirical study.

- Given $U, V, R$ and $\sigma$, we randomly generate $|K| = 100$ individual tables according to the linear model (1).

We generated 10 independent copies of the data and reported the mean and deviation of RMSE.

2009 H1N1 HI Dataset from Influenza Surveillance

The 2009 H1N1 HI dataset contains six individual HI tables spanning from April to August in 2009. The sizes of tables differ...
greatly, e.g., table #1 is of size 11 x 8, table #2 of 46 x 8 and table #6 of 92 x 9. Each table only includes up to 16 reference ferret antisera, only some of them appear in two or more tables. A total number of 253 antigens (viruses) and 28 antisera are included from these six tables. Figure 9 shows the occurrence of the six HI tables in the merged $253 \times 28$ antigen-antiserum array (left panel), as well as the distribution of observed entries in the merged array (right panel). Data statistics show that there are only 36.86% observed entries in the merged table.

Table 2 lists four selected antigen-antiserum pairs which have multiple observations of HI titre across the six HI tables in H1N1 2009. Note that one unit of HI titre represents 2-fold dilution of antiserum in the HI assay. It can be seen that the observed HI titre values are not consistent for 3 out of the 4 pairs. For example, there exists up to 4-fold divergence among the HI titers for the pair (A/New York/18/2009, A/California/7/2009). Therefore, uncertainty of observation presents another challenge for the table integration problem.

2000–2007 H3N2 Human Seasonal Influenza Viruses HI dataset

In Figure 10, we show the distribution of observed entries in another dataset used in this study, the H3N2 2000–2007 datasets, in which the tables are collected from year 2000 to year 2007. This dataset contains 369 tables and the merged table has 11,383

Figure 7. Antigenic cartography of H3N2 2000–2007 dataset. Here the rank value $d = 3$ is selected in our model for matrix completion and MC-MDS is used for cartography construction. doi:10.1371/journal.pone.0069842.g007

Figure 8. Antigenic cartography of H1N12009 data constructed by our model with different ranks. doi:10.1371/journal.pone.0069842.g008

Figure 10. Antigenic cartography of H3N2 2000–2007 dataset.
antigens as rows and 393 antisera as columns. Visual inspection of
Figure 10 shows that a large portion of the entries in the combined
H3N2 table are missing, and there are only 27.4% of observed
entries. Similar to the H1N1 2009 dataset, the observed HI titre
values of the same pair of antigen-antiserum in individual tables
could be varying. One important task is to fill out the missing
entries based on the noisy observations in individual tables.

Performance Evaluation
In this study, the performance of our data integration model is
evaluated using the following two criteria: root mean squared error
and biological interpretation, which is mainly through antigenic
cartography.

Quantitative Evaluation by RMSE
Given N true values \( \{x_n\}_{n=1}^{N} \) and prediction values \( \{y_n\}_{n=1}^{N} \),
the root mean squared error is defined by:

\[
RMSE = \sqrt{\frac{\sum_{n=1}^{N} (x_n - y_n)^2}{N}}
\]

If a prediction scheme has a small RMSE value, then the
predicted values are close to the true values. For real data, since we
have no ground truth for the merged HI table, we calculate the
RMSE values on individual tables through 10-fold cross-valida-
tion. The observed HI titre values \( h_{ij,k} \) from the tables are
partitioned into 10 equal parts. Each time, one part is used for
testing and the remaining nine parts for training. That is, each
time we estimate the parameters of model (2) using 9 parts as
observed values, and then calculate RMSE between the predicted
values and the observed values on the remaining part. The process
is repeated for every part and we report the mean and standard
deviation of RMSE. It is known that standard deviation
calculation based on cross validation is often smaller than the
true standard deviation; however, the numbers still provide
meaningful indications and hence are included.

Qualitative Evaluation by Antigenic Cartography
The biological interpretation is based on quantification of the
reported antigenic variant groups in the influenza antigenic
cartography. An antigenic cartography is a geometric representa-
tion of HI assay data. In such a cartography, the relative positions
of antigens and antisera are adjusted such that the distances

Table 2. H1N1 2009 dataset: examples of HI titre
observations across multiple tables.

| Antigen – Antiserum | T1 | T2 | T3 | T4 | T5 | T6 |
|---------------------|----|----|----|----|----|----|
| A/Wisconsin/10/1998 – A/Wisconsin/10/1998 | 9  | 8  | 8  | –  | –  | –  |
| A/Brisbane/59/2007 – A/California/7/2009 | –  | –  | 0  | 0  | 0  | 1  |
| A/New York/18/2009 – A/California/7/2009 | –  | –  | 8  | 7  | 9  | 7  |
| A/Mexico/4108/2009 – A/TEXAS/15/2009 | –  | –  | 9  | 9  | 9  | 8  |

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between antigens and antisera in the map represent the corresponding HI measurements with least error. Distance in the map thus represents antigenic distance and the closer antigens are to each other in the map the more similar they are antigenically.

Multidimensional scaling (MDS) is a statistical technique widely used in information visualization [13]. It attempts to embed a set of data into low dimensional vectors while preserving their pairwise distances. This technique has been applied to the construction of influenza antigenic cartography [4,5,14,15]. Particularly, the MC-MDS method was developed in [4] to visualize antigenic distances. The goal is to minimize the error $\sum_{ij} (d_{ij} - |a_i - a_j|)^2$ in which $a_i$ and $a_j$ are two or three dimensional representation of antigen $i$ and antigen $j$. Here we conventionally define the antigenic distance $d_{ij}$ as the Euclidean distance between the row vectors associated with antigens $i$ and $j$ in the merged HI titre table. One unit of such an antigenic distance corresponds to a 2-fold dilution of antiserum in the HI assay.

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Author Contributions

Conceived and designed the experiments: XFW TZ. Performed the experiments: XTY. Analyzed the data: TZ XFW XTY. Contributed reagents/materials/analysis tools: XFW. Wrote the paper: XTY XFW TZ.

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