Tool Orientation Planning Based on Kinematic Relation of Five-Axis CNC Machining Tools in Sculptured Surface Machining

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Abstract. Five-axis machining has been applied in sculptured surface manufacturing more and more widely. Tool orientation planning becomes important for higher machining precision. For this purpose, planning method based on kinematic relation of five-axis CNC machining tools is proposed to control the variations of the cutter-axis vectors and the motion axes. Through establishing smoothness index for reflecting the variations of the cutter-axis vectors and the motion axes and the kinematics model of machine tool with general structure, the overall smoothness model of the cutter axis vector and motion axes considering the kinematics spatial mapping is established, and the tool orientation planning is completed. Determination and simulation of the tool orientation are conducted through machining a complex blade surface. The results of the simulation result showed that the tool orientation planning method improved the smoothness of the processing and the efficiency.

1. Introduction

With the increasing application of complex free-form surface parts in the industrial field, more and more demands are placed on the tool orientation planning for complex free-form surface parts by using multi-axis NC machining. Many scholars have done a lot of research on the research of tool orientation planning considering surface geometry and kinematic smoothness.

Canadian scholar Bedi [1] proposed the main curvature method to adjust the tool orientation to establish the tool orientation planning model. Zhu [2-4] proposed a three-order shearing method based on the local reconstruction principle of the tool envelope surface to optimize the tool orientation of five-axis NC machining. Fan [5] et al. optimized the maximum cutting distance of the five-axis machining quadric surface of the flat-bottomed knives by avoiding the machining of local grooves, and determined the tool orientation and tool path generation based on this optimization. These methods only plan the tool pose from a single geometric aspect. British scholar Nassehi A [6] et al. proposed a new algorithm for creating and adapting tool path optimization evolution to generate continuous tool motion trajectories.
However, tool orientations planning for five axis CNC machining considering the variations of the cutter-axis vectors and the motion axes is still facing many challenges. This paper proposes to establish an overall smoothing model of the cutter axis vector, based on the kinematics spatial mapping, and complete the tool orientation planning.

2. Overall smoothing index establishing considering the kinematics mapping

2.1. Overall smoothing index

The tool axis vector is defined as mathematical description of the tool axis direction under the workpiece coordinate system. The indicators proposed in this paper are the variations of the cutter axis vectors $v_{i-1}$ and $v_{i+1}$ at the cutter position $P_{L,i}$ at the same tool path, and the adjacent tool positions $P_{L,i-1}$ and $P_{L,i+1}$, which could be build as

$$ W_1^v = \sqrt{\sum_{i=1}^{N_L} (\arccos(v_i, v_{i+1}))^2}. \quad (1) $$

Where, $i$ represents the sequence of tool location point number on the same tool path given by the tool location file; and $N_L$ is the total number of tool location points on the same tool path. Considering the change of the tool axis vectors between adjacent tool paths, the indicator in Eq(1) could be updated to

$$ W_2^v = \sqrt{\sum_{j=1}^{N_P} \sum_{i=1}^{N_L} \left( (\arccos(v_{i,j}, v_{i+1,j}))^2 + (\arccos(v_{i,j}, v_{i,j+1}))^2 \right)}. \quad (2) $$

Where $i$ is the sequence of the tool location points on the $j$-th tool path in the tool location file, $N_L$ is the total number of tool points on the same tool path, and $N_P$ is the total number of tool paths. Further, $W_1^v$ and $W_2^v$ only reflect the change in the tool space under the workpiece coordinate system, and fail to reflect the change of the motion of each axis caused by the change of the tool axis vector in the machine axis space. To this problem, kinematic modeling of spatial mapping is required to incorporate changes of machine axis space into the index.

As shown in Figure 1, the kinematics of the five-axis machine tool in the motion axis space includes three translational axes $(x, y, z)$ and two rotation axes $(a, c)$, which can be expressed as $P_M$.

$$ P_M = (x, y, z, a, c)^T. \quad (3) $$

When mapped to the tool space, the descriptions are the tool location point $P_L$ and the tool axis vector $v$. 

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The following section is to establish the kinematic mapping relationship of two spaces. Firstly, according to the basic theory of multi-body kinematics, the matrix formulas of various kinematic transformations are given. Let $a$ and $c$ be the rotation angles rotating about the $x$ and $z$ axes respectively. The composite matrix descriptions of the five kinematic transformation elements are:

$$
T_i(a) = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & \cos a & -\sin a & 0 \\
0 & \sin a & \cos a & 0 \\
0 & 0 & 0 & 1
\end{pmatrix},
$$

$$
T_2(c) = \begin{pmatrix}
\cos c & -\sin c & 0 & 0 \\
\sin c & \cos c & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix},
$$

$$
T_3(x) = \begin{pmatrix}
1 & 0 & 0 & x \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix},
$$

$$
T_4(y) = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & y \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix},
$$

$$
T_5(z) = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & z \\
0 & 0 & 0 & 1
\end{pmatrix}.
$$

The above five kinematic transformation elements are synthesized, that is, two rotations are performed around two coordinate axes and three translations are performed along each coordinate axis, and the composite transformation matrix is described as:

$$
T_6(a,b,c) = T_i(a)T_2(c)T_3(x)T_4(y)T_5(z)
$$

$$
= \begin{pmatrix}
\cos c & -\cos a \sin c & \sin a \sin c & x \\
\sin c & \cos a \cos c & -\sin a \cos c & y \\
0 & \sin a & \cos a & z \\
0 & 0 & 0 & 1
\end{pmatrix}
$$

Based on the kinematic mapping relationship between the five-axis machine tool axis space and the tool space, the kinematic mapping model of the five-axis machine tool with general structure is established. The index reflecting the variations of tool axis vectors by mapping to the machine axis space can be proposed as follows.
where, $\Delta x_i$, $\Delta c_i$ are the motion variations of each axis. $\mu_1$, $\mu_5$ are the weights of $\Delta x_i$, $\Delta c_i$, set in $[0,1]$, and $\mu_1+\mu_2+\mu_3+\mu_4+\mu_5=1$. Value of the weights needs to be set according to the machine structure parameters and workpiece characteristics.

$$
M \Delta v_3 = \sqrt{\sum_{i=1}^{N_o} \left( \mu_1^2 \Delta x_i + \mu_2^2 \Delta y_i + \mu_3^2 \Delta z_i + \mu_4^2 \Delta a_i + \mu_5^2 \Delta c_i \right)^2}.
$$

Similarly, considering the mapping of the tool axis vector variation between adjacent toolpaths in the axis space, the Eq (6) could be updated to

$$
M \Delta v_4 = \sqrt{\sum_{i=1}^{N_o} \sum_{j=1}^{N_o} \left( \mu_1^2 \Delta x_{i,j} + \mu_2^2 \Delta y_{i,j} + \mu_3^2 \Delta z_{i,j} + \mu_4^2 \Delta a_{i,j} + \mu_5^2 \Delta c_{i,j} \right)^2}.
$$

Where, $\Delta x_{i,j}$, $\Delta c_{i,j}$ are the motion variations of each axis corresponding to the tool point of the $i$-th row and the $j$-th column.

$$
\Delta x_{i,j} = (x_{i+1,j} - x_i)^2 + (y_{i+1,j} - y_i)^2,
\Delta y_{i,j} = (y_{i+1,j} - y_i)^2 + (z_{i+1,j} - z_i)^2,
\Delta z_{i,j} = (z_{i+1,j} - z_i)^2 + (a_{i+1,j} - a_i)^2,
\Delta a_{i,j} = (a_{i+1,j} - a_i)^2 + (c_{i+1,j} - c_i)^2,
\Delta c_{i,j} = (c_{i+1,j} - c_i)^2 + (d_{i+1,j} - d_i)^2.
$$

From equations (1) and (6), the index considering the change of the tool axis vector in the tool space and the machine axis space is

$$
\Delta v_5 = \mu_w \left( M \Delta v_3 \right) + \mu_m \left( M \Delta v_3 \right).
$$

Where, $\mu_w$ and $\mu_m$ are weights of the indexes in the tool space and the machine axis space, and $\mu_w + \mu_m = 1$.

From formula (2) and formula (8), it is possible to consider both the tool space and the machine tool axis space, and establish the index of the tool axis vector variations between adjacent tool paths as

$$
\Delta v_6 = \mu_w \left( w \Delta v_5 \right) + \mu_m \left( M \Delta v_4 \right).
$$

In summary, equations (1), (2), (6), (8), (10), and (11) give the tool axis vector smoothing indexes under different operating conditions.
2.2. Tool axis vector subspace considering interference avoidance

The interference, usually referred to as global interference, between the tool envelope surface $S_{Ts}$ and the workpiece real-time surface $S_{W}$ is necessary to be avoided. To avoid the global interference, it is necessary to control the distance between the tool envelope surface and the real-time surface of the workpiece as a certain safety distance $\delta_{Ts}$. The established tool axis vector subspace expression is

$$V_{Ts}(P_{i,j}) = \{v \mid d_{\min}(S_{Ts}(P_{i,j}, v), S_{W}) > \delta_{Ts}\}.$$  \hspace{1cm} (12)

3. Tool orientation planning for overall smoothing

Based on kinematics model of the machine tool with general structure and the overall smoothing index, the overall smoothing model of the tool axis vector considering the kinematics space mapping can be established as follows

$$\min \Delta v_{e}(\alpha, \beta)$$

s.t.

$$v(P_{i,j}, \alpha, \beta) \in V_{Ts}(P_{i,j}).$$  \hspace{1cm} (13)

Where, the optimization goal is to consider the tool axis vector variations in the tool space and the machine tool axis space and the tool axis vector variations between adjacent tool paths simultaneously.

The tool axis vector smoothing method is applied in tool orientation planning for machining a complex surface, to evaluate the application effect.

The machining surface selected in the example is a partial area of a certain type of rotor blade. In the example, the maximum number of iteration steps is set to 100, and the discrete time difference $\Delta t$ is taken as 0.5. The reduction of the discrete time difference can improve the smoothing of the tool axis vector, but it will increase the number of iterations significantly.

Fig. 2. Cutter axis vectors before and after the overall smoothing

It can be seen from Fig. 2 that the result of the tool axis vector planning is the angle between the front and rear tool axis vector of the same tool path and the angle of the left and right tool axis vectors of the adjacent tool path.

4. Conclusion

In this paper, the tool orientation planning is completed by establishing an overall smoothing model of the cutter axis vector considering the kinematics spatial mapping. Calculation for machining example of the blade surface is conducted and simulated. It ensures smooth and excessive tools in the complex
free surface and increases machining accuracy and efficiency. It is a feasible and effective tool orientation planning method.

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