Thermo-magnetic convection in a circular annulus filled with magnetocaloric nanofluid

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ABSTRACT

The present work is an effort to find a feasible design proposal which can ensure the continuous flow and self-sustained heat transfer within a circular annulus, solely because of magnetic forces. The objective is to study the change in flow characteristics because of magnetocaloric response of the nanoparticles used. In the present work, Mean Field Theory is used to calculate the specific magnetization and an improved viscosity model is used for considering its dependency on applied magnetic field. For better accuracy, individual mechanisms are considered while calculating the specific heat of magnetocaloric material. To have a better understanding, three different values of the remanent flux density (Br) and three distinct geometrical configurations are compared for their flow and thermal characteristics. For all cases, a periodic clockwise rotation of the fluid is observed and it is noticed that, the frequency of this rotation varies with both geometrical dimensions and Br. The main purpose of the current analysis is to contribute the preliminary data, which will further enhance our understanding about applications of Thermo-Magnetic Convection and Magnetocaloric Effect in heat transport devices.

1. Introduction

Magnetocaloric cooling is a rapidly growing technology in the past few years, which is being used for room temperature cooling [1–3]. Most of the modern household magnetic cooling systems work on the principle of active magnetocaloric regenerator (AMR) [4,5]. In such an AMR system, a continuous flow of heat carrier fluid needs to be maintained inside the system which requires either reciprocating or rotary arrangements. Due to this mechanical motion, components are subjected to periodic vibrations which can result in frequent maintenance cost and a less reliable system [3]. Also, because of the high initial cost involved for such an assembly, constant efforts are being made by researchers to look for better substitutes. One such alternative for the aforementioned problem is to use ferro-fluids as a heat transport liquid. Ferro-fluids are stable colloidal suspensions of mono-dispersed ferromagnetic nanoparticles of size about 10 nm in a carrier liquid such as kerosene or water [6]. These fluids are widely acknowledged for their various potential and existing applications [7,8]. Being responsive to magnetic fields, they exhibit certain characteristics different from conventional nanofluid, when subjected to magnets [9]. In the presence of an externally applied magnetic field, the behaviour of these fluids is influenced by the complex interactions of their magnetic and hydrodynamic properties. Due to their magnetic properties, when exposed to a magnetic field with a non-uniform temperature distribution, ferro-fluids illustrate a unique type of convection called as Thermo-Magnetic Convection (TMC). This occurs because thermal gradients inside the fluid alter the magnetic susceptibility. Consequently, the distribution of the Kelvin body force changes inside the fluid domain which results in TMC. Additionally, implementation of an externally applied field decreases (increases) the magnetic order and thus increases (decreases) the magnetic entropy which eventually leads to an adiabatic decrease (increase) of temperature (Tm). This effect is widely known as the Magnetocaloric Effect (MCE) and has been extensively used for the past few years to achieve refrigeration around and below room temperature [10].

Several studies have been performed to explore the influence of magnetic interaction on flow and thermal characteristics using both numerical and experimental methods. Jue [11] performed a numerical study for both magnetic and thermal convection in a 2D square cavity for various values of Rayleigh Number (Ra) and Prandtl Number (Pr). It is concluded that, various locations of the magnet along with its intensity can be used for flow regulation in ferrofluids. Ashouri et al. [12] investigated a differentially heated 2-D square cavity for magnetically induced heat transfer in the absence of gravity. Authors proposed a correlation of the averaged Nusselt Number (Nuavg) for a broad range of various geometrical and thermophysical parameters. A similar study for...
high Pr ferrofluid in a 2D cavity with a centrally inserted permanent magnet is conducted by Ashouri and Shafii [13]. Two dominant circulations are observed by the authors which then depend on the size of the magnet as well. Furthermore, an improved heat transfer is observed for higher wall temperatures. Jafari et al. [14] simulated the effect of uniform magnetic field and different temperature gradients for a ker- osene based ferrofluid in 3D cylindrical domain using CFD techniques. Authors proposed that increased heat transfer rates are observed, when the applied magnetic field is perpendicular to the direction of temperature gradient. Zablotsky et al. [15] made a comparison between experimental and numerical results obtained for a ferrofluid cell having permanent magnets attached to its walls. The K-o SST model is used to obtain the flow field for a 3D rectangular domain. Authors put forward that maximum thermo-magnetic effect is obtained when the magnets are placed close to the hot end. Wrobel et al. [16] numerically and experimentally studied TMC for a paramagnetic fluid in an annulus between two co-axial vertical cylinders. Authors compared the magnitude of $Nu$ for different orientations and reported that, for improved heat transfer a strong magnetic field can be used as a promising substitute.

Numerical and experimental observations are done by Yamaguchi et al. [17] to study the convection for a thermo-sensitive magnetic fluid (TSMF) within a cubic container having a heat generating object placed inside. Owing to their experimental results, it is put forward that in addition to the heat generating object, thermal characteristics of the fluid are increased because of the applied magnetic field. Gerdroodbary et al. [18] numerically conducted an extensive parametric study for the influence of non-uniform magnetic field on a ferrofluid within a T junction. Authors concluded that flow vortices are generated because of an applied magnetic field which eventually increase overall heat transfer. Dogonchi and Hashim [19] implemented a novel method named as Control Volume Finite Element Method (CVFEM) to investigate TMC along with radiation in a wavy circular cylinder and rhombus. A similar method is used by Sheikhholesami et al. [20] to study combined effects of Magnetohydrodynamics and Ferrohydrodynamics in a semi annulus area filled with ferrofluid including thermal radiation. Authors concluded that $Nu$ shows an increment for increased values of $Ra$ and volume fraction ($\phi_v$). Zanella et al. [21] studied both experimentally and numerically TMC in a cylindrical container with a heated solenoid in addition to the influence of a ferromagnetic core. Different volume fractions of nanoparticles are studied for their effect on cooling of a heated coil. It is concluded that, the use of ferromagnetic core increases heat transfer and maximum cooling is observed for the highest $\phi_v$. Lately, Vatani et al. [22] did experimental as well as numerical analysis for ferro-fluid around an electrically heated vertical wire. It is found that, because of magnetic forces, local vortices are generated within the domain which increase heat dissipation from the wire.

Few studies are also available in the literature that investigates the effect of applied magnetic field on viscosity. A non - linear stability analysis is carried out by Sunil et al. [23] to observe the behaviour of thermoconvective ferrofluid subjected to magnetic field dependent (MFD) viscosity and found possibilities for the existence of sub-critical instabilities. Nanjundappa et al. [24] used the Galerkin technique and regular perturbation method to analyse the performance of ferrofluid in a porous horizontal layer. It is put forward that, increase in MFD viscosity can retard the onset of ferroconvection. A linear stability analysis of mixed TMC is done by Mehmood [25] for two lid driven trapezoidal cavities using Galerkin weighted residual scheme. It is concluded that, concentration of ferromagnetic particles have significant influence on fluid flow along with Darcy, Grashof, and Hartman numbers. Recently, Dogonchi et al. [26] implemented a modified Fourier approach for investigating heat generation inside a semi-circular enclosure with MFD viscosity. Authors used the CVFEM technique for computations and concluded that magnetic field can be used as regulatory factor.

For the past few years, a lot of work is done to use these magnetically induced body forces for operating automated energy transport devices. Several researchers performed numerous experimental and numerical studies to investigate different possible configurations. Fumoto et al. [27] performed various experiments on an oval loop of 2 mm inner diameter for small scale heat transport, using TMC principles. It is stated that, by changing position and strength of the magnet, flow velocity of TSMF can be controlled. Lian et al. [28] developed a method, which uses TMC and illustrated that it can be used for automatic energy transfer. Authors compared experimental data with numerical results and studied the effect of different heat loads, temperature and magnetic field distributions. Xuan and Lian [29] extended this work and came up with an improved experimental design which can be used for cooling of electronic equipments using thermo-magnetic effect of ferrofluid. Authors concluded that a higher heat dissipation rate can be achieved with increasing heat load as it leads to greater TMC. Lian et al. [30] experimentally examined a small automatic energy transport device with micro-scale PIV technique. It is shown that an internal thermal gradient and an external magnetic field can be used to regulate energy transport in such devices. Among all three configurations studied by them, a circular configuration with two magnets is found to be optimal. Iwamoto et al. [31] used binary TSMF to operate a heat transport device which is solely driven by magnetic body force. With the help of experimental and theoretical formulations, it is put forward that buoyancy of gas phase helps in increasing magnetic body force at low magnetic fields. Yamaguchi and Iwamoto [32] further redesigned the device for its performance under various magnetic field orientations. It is experimentally demonstrated that, the device is able to transfer thermal energy over a distance of 5 m. A toroidal loop is numerically investigated for its heat transfer and flow characteristics in the presence of magnetic field by Ballare and Hangi [33]. Different parameters including the amount of heat flux in the heat source, temperature of the heat sink, magnet position and its magnetic strength are studied.

As it can be assessed from literature, very few studies have been performed that investigate the combined influence of TMC and MCE on the characteristics of magnetocaloric nanofluids. Also, less attention is given to observe the behaviour of such fluids near Curie Temperature ($T_C$). Thus, in continuation with the previous work of automated heat transport devices, and to improve our understanding of magnetocaloric nanofluids, a numerical study is carried out for a circular annulus, where simulations are performed for three different values of remanent flux density ($Br = 0.5\ T, 1\ T, and\ 1.5\ T$) and three different $L/D$ ratios ($L/D = 0.125, 0.25, 0.5$, refer Fig. 1). The flow field is closely analysed with the help of local as well as averaged velocity distributions, and the magnitude of $Nu$ is also compared for all distinct variations. Weiss Mean
2. Problem description

2.1. Geometrical details

For the present analysis, a magnetocaloric fluid is studied in a 2-D circular annulus with three different L/D ratios, where L represents the width between outer and inner radius, and D is the inner diameter ($D = 2R_i$) of the enclosure having a value of 0.1 m. The fluid consist of a colloidal suspension of 5% Gadolinium (Gd) in kerosene. The circular enclosure is placed at the centre of a square domain with a permanent magnet at the top of it as shown in Fig. 1. To cover the whole extent of field generated by the permanent magnet, a square domain of length $4D$ is considered as bounding region. The magnet is located at an immediate distance of 0.075D with its north pole facing towards the annulus. The permanent magnet used to produce the magnetic field is 1.6D long and has a square pole face with width of 0.4D. As the main interest of the present analysis is to investigate the TMC in the context of MCE. To achieve the maximum magnitude of MCE, which has higher value around ($T_C$) [10], the initial temperature of magnetocaloric nanofluid is taken as $T_C$ of Gd which is 292.15 K [34].

2.2. Magnetocaloric properties

The magnitude of MCE is mainly dependent on the type of magnetocaloric material (MCM), temperature ($T$), and strength of magnetic field ($H$). For the present study, to obtain maximum MCE, we are only interested in a narrow range around $T_C$. At $T_C$, materials lose their preferred spin axis which results in decreased magnetization and undergo a phase transition from the ferromagnetic to paramagnetic behaviour. Due to this phase transition, qualitative or many times quantitative changes are observed in the characteristics properties of MCM. For certain MCM's, (such as Gd) a database is available that explains the variation of these properties at phase transition, but such data does not ensure overall stability and energy conservation [37]. Thus, to obtain a continuous and an approximate solution near this transition, Mean Field Theories (MFT) are widely used [35,36].

In order to determine the characteristic properties of MCM in the present analysis, Weiss Mean Field Theory is used [35,38]. These properties are magnetization ($M = \rho m$), partial derivative of magnetization over temperature ($\left(\frac{\partial M}{\partial T}\right)_H$) and specific heat ($c_H$). Theoretical specific magnetization ($m$) and also its partial derivative with temperature are calculated as mentioned in literature [35–39]. It is clear from Eqs. (1)–(3), that the magnitude of $m$ varies significantly with both $T$ and $B$, so its variation is plotted as shown in Fig. 3. These characteristic properties are then used as an input function during calculation of source terms in continuity and momentum equations (see Eqs. (21)–(23)).

$$m = N_i g_i J \mu_B B_i(\chi)$$

(1)

In Eq. (1), $B_i(\chi)$ refers to the Brillouin function, which is a set of implicit equations (Eq. (2) and Eq. (3)) as shown below. Here, $\chi$ represents the ratio of the Zeeman energy to the thermal energy, $g_i$ is the Lande factor, $J$ is angular momentum. The values of all constant parameters used in the present study are mentioned in Table 1.

$$B_i(\chi) = \frac{2J + 1}{2J} \coth \frac{2J + 1}{2J} \chi(B_i) - \frac{1}{2J} \coth \frac{1}{2J} \chi(B_i)$$

(2)

$$\chi(B_i) = \frac{g_i \mu_B H}{k_B \beta} + \frac{3T_C \chi_i J}{T(J + 1)} B_i(\chi)$$

(3)

2.3. Thermo-physical properties

All thermo-physical properties except for the viscosity are calculated according to the literature [40,41]. Density ($\rho_{nf}$), specific heat ($c_{nf}$) and coefficient of thermal expansion ($\beta_{nf}$) are evaluated as the sum of corresponding contributions (see Table 2) from fluid and solid

![Fig. 3. Variation of magnetocaloric properties for different $T$ and $B$ (a) $m$ (b) $\left(\frac{\partial M}{\partial T}\right)_H$.](image)

Enlarged view of the discretized circular annulus.
particles based on the value of volume fraction ($\varphi_v$). Here, subscript $f$ and $s$ represent base fluid and solid nanoparticles, respectively. 

\[ \rho_{nf} = (1 - \varphi_v)\rho_f + \varphi_v\rho_s \]  

(4)

\[ \langle \rho \text{Cl} \rangle_{nf} = (1 - \varphi_v)\langle \rho \text{Cl} \rangle_f + \varphi_v \langle \rho \text{Cl} \rangle_s \]  

(5)

\[ \langle \rho \text{Be} \rangle_{nf} = (1 - \varphi_v)\langle \rho \text{Be} \rangle_f + \varphi_v \langle \rho \text{Be} \rangle_s \]  

(6)

The thermal conductivity ($k_{nf}$) is approximated by the Hamilton-Crosser model \([40,41]\) with $n = 3$ for spherical particles as shown below:

\[ k_{nf} = k_f \left( \frac{k_s + (n - 1)k_f - (n - 1)(k_f - k_v)\varphi_v}{k_s + (n - 1)k_f + (k_f - k_v)\varphi_v} \right) \]  

(7)

In the current work, as suspended colloidal particles are magnetically responsive, they will exhibit movement in presence of an externally applied magnetic field. This will eventually result in producing hindrance for fluid motion. Thus, influence of the magnetic field is taken into consideration while modeling the viscosity \([42]\). The new viscosity value will be addition of the magnetically induced viscosity ($\mu_{(H=0)}$) and the nanofluid viscosity in the absence of magnetic field ($\mu_{(H=0)}$)

\[ \mu_{nf} = \mu_{(H=0)} + \mu_{(H\neq 0)} \]  

(8)

\[ \mu_{(H=0)} = \frac{\mu_f}{(1 - \varphi_v)^{\frac{1}{S}}} \]  

(9)

\[ \mu_{(H\neq 0)} = \frac{3}{2} \varphi_f \langle \mu_{(H=0)} \rangle \left( \frac{\alpha - \tanh(\alpha)}{\alpha + \tanh(\alpha)} \right) \]  

(10)

where $\mu_{(H=0)} = \frac{\mu_f M_H}{k_B T}$ and $\varphi_f = \varphi_f \left( \frac{d + 2s}{d} \right)^3$  

(11)

The total specific heat ($C_{nf}$) for any material is the summation of magnetic ($C_{\text{magnetic}}$), lattice ($C_{\text{lattice}}$), and free electron ($C_{\text{electrons}}$) mechanisms \((\text{Eq. (12)})\). And as for the present analysis, we are dealing within a very precise range of $T$, it is desired to achieve maximum accuracy and reduce possible modeling errors. Thus, the individual contribution from all three mechanisms is taken into consideration by implementing different models as shown in Table 3.

\[ C_{nf} = C_{\text{magnetic}} + C_{\text{electrons}} + C_{\text{lattice}} \]  

(12)

To have a better understanding, variation of all three mechanisms with temperature is plotted for Gd at $B = 0.5 T$. It can be clearly seen from Fig. 4 that near $T_{C_{nf}}$ a considerable difference in the magnitude of $C_{\text{magnetic}}$ and $C_{nf}$ is present. Hence, the total specific heat for Gd ($C_{nf \ Gd}$) at different $T$ and $B$ is plotted as shown in Fig. 5. These values are then used as an input function for the second term of Eq. (5) which eventually calculates the specific heat of magnetic fluid ($C_{nf \ sy}$) at a particular temperature and magnetic field.

Table 1

| Symbol | Description | Value |
|--------|-------------|-------|
| $\mu_0$ | Permeability of free space | $1.256 \times 10^{-6}$ N/A² |
| $\mu_B$ | Bohr magneton | $9.274 \times 10^{-24}$ J/T |
| $\gamma_e$ | Sommerfeld constant | $5.93 \times 10^{-23}$ J/K² |
| $N_s$ | Number of magnetic spins per kg | $8.8 \times 10^{-6}$ m³/kg |
| $k_B$ | Boltzmann constant | $1.381 \times 10^{-23}$ (m kg)/(s² K) |
| $\chi_{sv}$ | Volume magnetic susceptibility of fluid | $8.8 \times 10^{-6}$ m³/kg |

Table 2

| Base fluid        | Properties | Value |
|-------------------|------------|-------|
| Kerosene          | $c_p$ (J/kg.K) | 1010 |
|                   | $\beta$ (K²) | 11.94 |
|                   | $\rho$ (kg/m³) | 795 |

Table 3

| Type               | Model            | Equation |
|--------------------|------------------|----------|
| $C_{\text{magnetic}}$ | Weiss Mean Field Theory | $C_{\text{magnetic}} = \frac{1}{2} \left( \frac{\mu_f H}{k_B T} \right)^2 - \frac{1}{2} \chi_{sv} (\frac{d + 2s}{d})^3 \frac{d}{d T}$ |
|                    |                   |          |
| $C_{\text{lattice}}$ | Debye Model       | $C_{\text{lattice}} = 9N_s \left( \frac{T_{Debye}}{k_BT} \right)^2 - \frac{1}{2} \chi_{sv} (\frac{d + 2s}{d})^3 \frac{d}{d T}$ |
| $C_{\text{electrons}}$ | Sommerfeld Model | $C_{\text{electrons}} = \gamma_e T$ |

Fig. 4. Variation of different specific heats for Gd with temperature at $B = 0.5 T$. Fig. 5. $C_{nf \ Gd}$ of Gd for various temperature and magnetic fields.
with this, Maxwell’s Equations are solved in its magnetostatics form [46] for the present analysis as shown below.

**Gauss’s Law**
\[ \nabla \cdot B = 0 \quad (13) \]

**Ampere’s Law**
\[ \nabla \times H = 0 \quad (14) \]

Here, \( B \) is magnetic induction, \( H \) is magnetic field, and both are coupled via the magnetic permeability (\( \mu_0 = 4\pi \times 10^{-7} \)) and remanent flux density (\( Br \)) as
\[ B = \mu_0 H + Br \quad (15) \]

Additionally, the flow is considered laminar, incompressible, and of Newtonian rheological behaviour. In the current study, as the volume fraction of Gd is small thus, it is assumed that nanoparticles and base fluid are in thermal equilibrium, therefore the single phase approach is used. The governing equations used to calculate the magnitude of all field variables are written in conservation form as shown below:

**Continuity Equation**
\[ \frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} = 0 \quad (16) \]

**x - Momentum Equation**
\[ \frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} = -\frac{\partial p}{\partial x} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + F_{Fx} \quad (17) \]

**y - Momentum Equation**
\[ \frac{\partial \rho}{\partial t} + \frac{\partial (\rho u)}{\partial x} + \frac{\partial (\rho v)}{\partial y} = -\frac{\partial p}{\partial y} + \mu \left( \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) + F_{Fy} \quad (18) \]

**Energy Equation**
\[ (\rho c_H)_{ef} \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k_{ef} \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right) + Q_{MCE} \quad (19) \]

Here, \( u, v \) denote \( x \) and \( y \) components of velocity respectively, \( T \) denotes temperature, and \( p \) is pressure. Density variation of the present magnetic fluid is taken into account using the Boussinesq Approximation which will generate a buoyancy force (\( F_B \)) as shown below:
\[ F_B = \rho_{ef} g \left( T - T_{ref} \right) \quad (20) \]

Moreover, due to the response of MCM to the applied magnetic field, additional source terms such as the Kelvin force density (\( F_K \)) and the magnetocaloric heating (\( Q_{MCE} \)) are considered in both momentum (Equations (17) and (18)) and energy equation (Equation (19)) respectively. \( F_K \) represents the magnetic body force experienced by fluid elements [6] and when the domain is considered to be free from electric current it is calculated as shown in Eq. (21). Being a vector quantity, \( F_K \) acts individually in both \( x \)-momentum and \( y \)-momentum equations as \( F_{Fx} = \left( \mu_0 M_{x} \frac{\partial H}{\partial x} \right) \) and \( F_{Fy} = \left( \mu_0 M_{y} \frac{\partial H}{\partial y} \right) \), respectively.

\[ F_K = \mu_0 M_V H \quad (21) \]

Again, to maintain the highest accuracy and replicate exact behaviour, individual contributions from fluid and solid parts are considered while calculating \( M \), as presented in Eq. (22).
\[ M = M_f + M_s = \left( 1 - \frac{\chi_f}{\chi} \right) \rho F \times \nabla T + \rho \rho H \quad (22) \]

Here \( \rho \) represents density of the corresponding part, \( \chi_f \) denotes volume susceptibility of the base fluid, and \( m \) is calculated using Eqs. (1)-(3). \( Q_{MCE} \) denotes cooling or heating of magnetic material in the presence of varying magnetic field [37] and it’s expression is given by Eq. (23). As the field generated by a permanent magnet does not vary significantly within the present simulation time \( \left( \frac{\partial H}{\partial t} \approx 0 \right) \) first term of \( Q_{MCE} \) is not considered while solving equations.
\[ Q_{MCE} = \left( \frac{\partial M}{\partial t} \right)_H \left( \frac{\partial H}{\partial t} \right) - \left( \frac{\partial M}{\partial H} \right)_H \left( \frac{\partial H}{\partial t} \right) \left( \frac{\partial H}{\partial x} + \frac{\partial H}{\partial y} \right) \quad (23) \]

### 3.2. Validation study

As the present work is in context with flow and heat transfer characteristics within a circular annulus section, a validation study is carried out to compare results with similar experiments performed by Sawada et al. [47]. Development of temperature distribution at various probe locations (A-D) with time in the non-dimensional form is plotted for \( Ra = 2.6 \times 10^4 \) as shown in Fig. 6. The observed differences between experimental and current numerical work can be justified because of 10% estimated error associated with positioning of thermal probes as mentioned by authors [47]. Value of \( Nu_{avg} \) is also compared for the same \( Ra \) between the present numerical model and experimental results in Table 4. All parameters, probe locations and \( Nu_{avg} \) calculations are modelled in accordance with the information provided by Sawada et al. [47].

Additionally, to make sure the magnetic field generated by a permanent magnet is modelled correctly in COMSOL environment, a comparison is made with Szabo and Früh [48] as shown in Fig. 7. The non dimensionalised magnetic field \( (H^* = \mu_0 H/Br) \) is calculated for a square enclosure with a permanent magnet placed on top of it. From both Figs. 6 and 7 and Table 4, it is clear that present numerical model is in good agreement with published literature.

### 3.3. Grid convergence and time independence test

To minimize possible discretization errors, three different grids are compared to ensure that the results are independent of mesh

| Table 4 | Comparison of \( Nu_{avg} \). |
|---------|-------------------------------|
| \( Ra \) | \( Nu_{avg} \) (Sawada et al.) | \( Nu_{avg} \) (present study) | % Error |
| \( 2.6 \times 10^4 \) | 2.5863 | 2.4675 | 4.59 |

![Fig. 6. Comparison of temperature distribution between the present numerical work and Sawada et al. [47] for \( Ra = 2.6 \times 10^4 \)](image-url)
distribution. These grids are compared for the case of Br 1.5 T and L/D of 0.125. Details of averaged values of velocity and Nu for considered grids are shown in Table 5. As it can be clearly seen, even after having a substantial difference in the mesh size, variation for both \( U_{\text{avg}} \) and \( Nu_{\text{avg}} \) is small. Thus, to maintain a balance between computational ease and numerical accuracy, Grid 2 (Fig. 2) is selected for all of the remaining simulations. Also, to ensure that the final solution is free from temporal errors, a time independent study is carried out for the finalized mesh size, and a time step of 0.2 s is used for all simulations.

### 3.4. Numerical procedure

For the present study, computations of all field variables are performed with the commercial software COMSOL - Multiphysics 5.4 using FEM technique [49]. Initially, the magnetic field is calculated for the whole square enclosure using Eqs. (13)-(15). This magnetic field \( H \) is then used along with \( M \) to solve all governing equations and calculate temperature and velocity field distributions throughout the computational domain. To handle the system of non-linear equations, a fully coupled approach is implemented and PARDISO is used as direct linear system solver. Being an unsteady analysis, four different probes are observed for their behaviour over time and all the data shown here are taken only when probes show converged values. Additionally, to ensure the continuous fluid flow, a linear gradient of temperature is implemented at both heat exchangers in horizontal direction as shown in Fig. 8. The gradients are applied to ensure that the maximum temperature inside the annulus is at rightmost region of HHEX and the minimum temperature is at leftmost region of CHEX. The model which is used to obtain such a temperature gradient is shown in Eq. (24).

\[
H_{\text{HEX}} = T_{\text{C}} + \left( \frac{x + |x_s|}{2|x_s|} \right), \quad H_{\text{CHEX}} = T_{\text{C}} - \left( \frac{x + |x_s|}{2|x_s|} \right) \quad \text{and} \quad X^* = \frac{x}{\pi D}
\]

Here, \( x \) is the local coordinate in the horizontal direction and \( x_s \) denotes the x coordinate at the starting point of the HEX length. The non-dimensionalised magnetic field \( H' = \mu_0 H / Br \) produced by the permanent magnet is also shown for the smallest \( L/D \) ratio considered in Fig. 9.

### 4. Results and discussion

For the current analysis, the behaviour of TMC is studied for different \( Br \) and \( L/D \) ratios based on their flow and thermal response. The comparison is made with the help of local as well as averaged velocity distribution for all cases. Being representative of flow characteristics, velocity contours are closely observed for their unsteady behaviour. Their variation over time is plotted in non-dimensionalised form.
\( t' = t/t_p \) for each case according to Eq. (25), where \( S \) represents surface area of the annulus and \( t_p \) is the total time period taken by a fluid particle to complete one cycle. As it can be observed from Fig. 10, the normalised surface averaged magnitude of velocity (\( U_{\text{avg}}^* \)) shows a periodic behaviour for all cases.

\[
U_{\text{avg}}^* = \frac{1}{S} \int_S \int U_{\text{avg}} dS, \quad U_{\text{avg}}^* = \frac{U_{\text{avg}}}{(U_{\text{avg}})_{\text{max}}}, \quad \text{and} \quad U_{\text{avg}} = \frac{1}{t_p} \int_0^{t_p} U_{\text{avg}} dt
\]

(25)

It is noticed from the power spectrum that, for a specific \( L/D \), frequency and strength increases with increase in magnitude of the applied field. It means that for the same geometrical configuration, less time is required for fluid particles to complete its cycles within the annulus. This eventually will improve heat transfer rate between both heat exchangers. It can be explained as, the value of \( (U_{\text{avg}}^*) \) depends considerably on the magnitude of the source terms in both momentum and energy equation. These source terms \( (F_k \text{ and } Q_{\text{MCE}}) \) again rely upon the strength of the externally applied magnetic field. Thus, for increased values of \( Br \) an improved TMC mechanism is observed for a particular \( L/D \) ratio. On the other hand, if we compare three different geometrical configurations of the same \( Br \), a totally contrary trend is observed. For all three magnitudes of \( Br \), frequency as well as strength of \( (U_{\text{avg}}^*) \) decreases with increase in \( L/D \) value. This is mainly because for a specific \( Br \), the influence of source terms reduces as the driving force gets distributed over a wider region with increase in \( L/D \). Among all cases, the maximum frequency is observed for the highest magnetic field and smallest \( L/D \) i.e. Fig. 10g.

For a complete insight of flow structures and their development, velocity contours for three different geometrical configurations of highest \( Br \) are compared over time in Fig. 11. Initially, for all configurations, fluid elements are randomly arranged throughout the annulus with no specific orientation preferences. But as time progresses, due to the combined effect of \( Q_{\text{MCE}} \) and the applied temperature gradient, fluid particles adjacent to the rightmost part of HHEX will gain some energy and reach to higher temperature relative to its surrounding fluid. Now, due to their thermo-magnetic characteristics, fluid particles with increased \( T \) experience a reduced value of the magnetic susceptibility. This susceptibility difference eventually results in movement of these fluid elements to the region of lower magnetic field which is in the bottom half of annulus as can be seen from Figure. When these fluid particles enter CHEX, they exchange heat and reach to the leftmost part.
with having temperature values smaller than their ambient fluid. It results in an increase in magnetisation and particles will try to reach to the high magnetic field zone i.e. the upper half. This is how the fluid elements within the annulus are exhibiting a periodic fluid motion in clockwise direction. In general, it is observed that the magnitude of the local velocity decreases with the increase in the width between outer and inner radius. Thus, its highest value is observed for $L/D$ of 0.125. For all three configurations, few fluid elements near to CHEX also try to reach to higher magnetic region along the inner circumference. This flow offers some resistance on the right half, while work in favour of mainstream velocity for the left half of annulus. As a result of this, some interesting flow patterns are observed for $L/D$ of 0.5. Due to its wider span, this combination of opposite flow streams along inner and outer circumference results in the formation of alternate clockwise and anticlockwise local vortices on the right half of the domain as displayed in Fig. 11f and Fig. 11i. This formation of alternative vortices is also responsible for the reduced magnitude of average velocities for all cases of 0.5 $L/D$. As it can be clearly noticed from Fig. 12, for a distinct magnetic field, the magnitude of $U_{avg}$ shows lower value for a wider geometrical configuration. On the other hand, a substantial increment is observed if we raise the magnetic strength for a particular $L/D$. This behaviour can be primarily attributed to the influence of $F_k$ term in the momentum equations.

Fig. 11. Development of velocity contours over time for different $L/D$ ratios at Br of 1.5 T

Fig. 12. Comparison of $U_{avg}$ with $L/D$ for different $Br$
Also, to have a better understanding about heat transfer mechanism, \( \text{Nu}_{\text{avg}} \) is plotted for all three variations of \( \text{Br} \) and \( \text{L/D} \). The time averaged value of \( \text{Nu} \) is integrated over the whole domain and compared for all cases. It is observed that, for a specific \( \text{L/D} \), the magnitude of \( \text{Nu}_{\text{avg}} \) increases with increase in \( \text{Br} \). It is mainly because, as mentioned in Eq. (26), \( \text{Nu}_{\text{avg}} \) is directly influenced by the magnitude of net convective heat flux (\( Q_c \)). This \( Q_c \) is further affected by the strength of \( Q_{\text{MCE}} \) which again increases with \( \text{Br} \). A similar trend of increment in \( \text{Nu}_{\text{avg}} \) is observed if we go on increasing \( \text{L/D} \) for a particular \( \text{Br} \). Thus, the maximum value of \( \text{Nu}_{\text{avg}} \) is obtained for the case of 0.5 \( \text{L/D} \) with highest \( \text{Br} \) as shown in Fig. 13.

\[
\text{Nu}_{\text{local}} = \frac{Q_c L}{k_d T}, \quad \text{Nu} = \frac{1}{S} \int_0^S \text{Nu}_{\text{local}} dS, \quad \text{and} \quad \text{Nu}_{\text{avg}} = \frac{1}{T} \int_0^T \text{Nu} dt
\]  

At the end, in order to closely analyse the exclusive contribution of the buoyancy force to the fluid rotation, a comparison is made with and without including the \( F_B \) term in the \( y \)-momentum equation (Eq. 18). Values of \( U_{\text{avg}} \) and \( \text{Nu}_{\text{avg}} \) are compared for the case of highest magnetic strength and smallest \( \text{L/D} \) along with their power spectrums. As it is clearly observed from Table 6 and Fig. 14, \( F_B \) has an insignificant contribution in flow development. Thus, it can be evidently said that for all the cases of current framework, the fluid flow is occurring entirely because of magnetic forces (\( F_K \) and \( Q_{\text{MCE}} \)) and is not an outcome of the density difference inside the annulus.

5. Conclusion

A numerical analysis is performed to define the design criteria for a heat transport device filled with magnetocaloric nanofluid. For this reason, thermal and flow behaviour is studied in a circular annulus for different magnetic fields. Moreover, to evaluate the effect of geometrical configurations, three different aspect ratios (\( \text{L/D} \)) are also observed. Results show a periodic fluid rotation in clockwise direction for all cases and highest frequency of this rotation corresponds to the smallest \( \text{L/D} \) and maximum \( \text{Br} \). \( \text{Nu}_{\text{avg}} \) shows improved values for higher strength of the applied field and larger span between the annulus. It is also found that, for a specific \( \text{L/D} \) ratio, the flow velocity increases with intensity of the applied magnetic field whereas at a particular \( \text{Br} \), it decreases with increase in span-wise width.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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| Case       | \( U_{\text{avg}} \) [m/s] | \( \text{Nu}_{\text{avg}} \) [-] | % Error |
|------------|-----------------|-----------------|--------|
| With \( F_B \) | 0.026607        | 1.666349        | 1.153  |
| Without \( F_B \) | 0.026914       | 1.678811        | 0.747  |

Fig. 14. Comparison of Power Spectrum of \( U_{\text{avg}} \) for \( \text{L/D} \) of 0.125 at \( \text{Br} = 1.5 \) (a) With \( F_B \) (b) Without \( F_B \).
References

[1] A. Kitanovski, J. Túnek, U. Tomc, U. Plaznik, M. Ozbolt, A. Poredos, Magnetocaloric Energy Conversion, Springer, 2016.
[2] B. Yu, M. Liu, P.W. Egolf, A. Kitanovski, A review of magnetic refrigerator and heat pump prototypes built before the year 2010, Int. J. Refrig. 33 (6) (2010) 1029–1060.
[3] K. Gschenlider Jr., V. Pecharsky, Thirty years of near room temperature magnetic cooling: where we are today and future prospects, Int. J. Refrig. 31 (6) (2008) 945–961.
[4] S. Choi, U. Han, H. Cho, H. Lee, Recent advances in household refrigerator cycle technologies, Appl. Therm. Eng. 132 (2018) 560–574.
[5] P. Bansal, E. Vineyard, D. Abdelaziz, Advances in household appliances—a review, Appl. Thermal Eng. 31 (17-18) (2011) 3748–3766.
[6] R.E. Rosensweig, Ferrohydrodynamics, Courier Corporation, 2013.
[7] M. Baihinei, M. Hangi, Flow and heat transfer characteristics of magnetic nanofluids: a review, J. Magn. Magn. Mater. 374 (2015) 125–138.
[8] M. Alshuwaikat, R. Boukhanouf, Y. Yan, Thermophysical properties and thermo-magnetic convection of ferrofluid, Appl. Therm. Eng. 88 (2015) 14–21.
[9] S.A. Zonouzi, H. Aminfar, M. Mohammadpourfard, A review on effects of magnetic fields and electric fields on boiling heat transfer and chf, Appl. Therm. Eng. 151 (2019) 11–25.
[10] J.R. Gomez, R.F. Garcia, A.D.M. Cotaora, M.R. Gomez, Magnetocaloric effect: A review of the thermodynamic cycles in magnetic refrigeration, Renew. Sustain. Energy Rev. 17 (2013) 74–82.
[11] T.-C. Jou, Analysis of combined thermal and magnetic convection ferrofluid flow in a cavity, Int. Commun. Heat Mass Transfer 33 (7) (2006) 846–852.
[12] M. Ashouri, B. Ebrahim, M. Shafii, M. Saidi, M. Saidi, Correlation for nusselt number in pure magnet convection ferrofluid flow in a square cavity by a numerical investigation, J. Magn. Magn. Mater. 322 (223) (2010) 3607–3613.
[13] M. Ashouri, M.B. Shafii, Numerical simulation of magnetic convection ferrofluid flow in a permanent magnet-inserted cavity, J. Magn. Magn. Mater. 442 (2017) 270–278.
[14] A. Jafar, T. Tynjala, S. Mousavi, P. Sarkomaa, Simulation of heat transfer in a ferrofluid using computational fluid dynamics technique, Int. J. Heat Fluid Flow 29 (4) (2008) 1197–1202.
[15] D. Zablotsky, A. Mezulis, E. Blums, Surface cooling based on the thermomagnetic convection: numerical simulation and experiment, Int. J. Heat Mass Transf. 52 (23–24) (2009) 5302–5308.
[16] W. Wrobel, E. Pomažil-Wejdi, J. Szymy, Experimental and numerical analysis of thermo-magnetic convection in a vertical annular enclosure, Int. J. Heat Fluid Flow 31 (6) (2010) 1019–1031.
[17] H. Yamaguchi, X.-R. Zhang, X.-D. Niu, K. Yoshikawa, Thermomagnetic natural convection of thermo-sensitive magnetic fluids in cubic cavity with heat generating object inside, J. Magn. Magn. Mater. 322 (6) (2010) 698–704.
[18] M.B. Gerrodobad, M. Sheikholeslami, S.V. Mousavi, A. Anazadehshayed, R. Moradi, The influence of non-uniform magnetic field on heat transfer intensification of ferrofluid inside a t-junction, Chem. Eng. Process. Process Intensification 123 (2018) 58–66.
[19] A. Dogonchi, et al., Heat transfer by natural convection of ferrofluid saturated porous layer heated from below and cooled from above with constant heat flux subject to mfd viscosity, Int. Commun. Heat Mass Transfer 37 (9) (2010) 1246–1250.
[20] Z. Mehmoond, Numerical simulations and linear stability analysis of mixed thermomagnetic convection through two lid-driven entrapped trapezoidal cavities enclosing ferrofluid saturated porous medium, Int. Commun. Heat Mass Transfer 109 (2019) 104345.
[21] A. Dogonchi, M. Waqas, S.M. Seyyedi, M. Hashemi-Tilehnooe, D. Ganji, A modified fouling approach for analysis of nanofluid heat generation within a semi-circular enclosure subjected to mfd viscosity, Int. Commun. Heat Mass Transfer 111 (2020) 104430.
[22] K. Pumoto, H. Yamaguchi, M. Ikegawa, A mini heat transport device based on thermo-sensitive magnetic fluid, Nanoscale Microscale Thermophys. Eng. 11 (1–2) (2007) 201–210.
[23] W. Lian, Y. Xuan, Q. Li, Design method of automatic energy transport devices based on the thermo-magnetic effect of magnetic fluids, Int. J. Heat Mass Transf. 52 (23–24) (2009) 5451–5458.
[24] Y. Xuan, W. Lian, Electronic cooling using an automatic energy transport device based on thermomagnetic effect, Appl. Therm. Eng. 31 (8–9) (2011) 1487–1494.
[25] W. Lian, Y. Xuan, Q. Li, Characterization of miniature automatic energy transport devices based on the thermomagnetic effect, Energy Convers. Manag. 50 (1) (2009) 35–42.
[26] Y. Iwamoto, H. Yamaguchi, X.-D. Niu, Magnetically-driven heat transport device using a binary temperature-sensitive magnetic fluid, J. Magn. Magn. Mater. 323 (10) (2011) 1378–1383.
[27] H. Yamaguchi, Y. Iwamoto, Energy transport in cooling device by magnetic fluid, J. Magn. Magn. Mater. 431 (2017) 229–236.
[28] M. Baihinei, M. Hangi, Automatic cooling by means of thermomagnetic phenomenon of magnetic nanofluid in a toroidal loop, Appl. Therm. Eng. 197 (2016) 700–708.
[29] C. Graham Jr., Magnetic behavior of gadolinium near the curie point, J. Appl. Phys. 36 (3) (1965) 1135–1136.
[30] M. Suzuki, Lecture Note on Solid State Physics: Mean Field Theory, Department of Physics State University of New York, Binghamton, 2006.
[31] C. Kittel, Introduction to Solid State Physics, Wiley, 2005.
[32] A.M. Tnishin, Y.I. Spichkin, The Magnetocaloric Effect and its Applications, CRC Press, 2016.
[33] M. Laousyenne, S. Mahjoub, M. Baaazouzi, E. Hill, M. Oumezzine, Magnetic entropy change by mean-field theory for the second-order phase transition magnetic nld 6sr0.3ca0.1mn0.975fe0.025o3, J. Supercond. Nov. Magn. 29 (5) (2016) 1151–1157.
[34] M. Darby, Tables of the brillouin function and of the related function for the spontaneous magnetization, Br. J. Appl. Phys. 18 (10) (1967) 1415.
[35] M. Sheikholeslami, M.D. Ganji, Ferrohydrodynamic and magnetohydrodynamic effects on ferrofluid flow and convective heat transfer, Energy 75 (2014) 400–410.
[36] K. Khanafar, K. Vafai, M. Lightstone, Buoyancy-driven heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids, Int. J. Heat Mass Transf. 46 (19) (2003) 3639–3653.
[37] S. Odenbach, Magnetoviscous Effects in Ferrofluids, Vol. 71 Springer Science & Business Media, 2003.
[38] R. Ganguly, S. Sen, I.K. Puri, Thermomagnetic convection in a square enclosure using a line dipole, Phys. Fluids 16 (7) (2004) 2228–2236.
[39] P.S. Szabo, M. Bekovic, W.G. Früh, Using infrared thermography to investigate thermomagnetic convection under spatial non-uniform magnetic field, Int. J. Therm. Sci. 116 (2017) 118–128.
[40] A. Mokhopadhyay, R. Ganguly, S. Sen, I.K. Puri, A scaling analysis to characterize thermomagnetic convection, Int. J. Heat Mass Transf. 48 (17) (2005) 3485–3492.
[41] J.D. Jackson, Classical Electrodynamics, (1999).
[42] T. Sawada, H. Kikura, A. Saito, T. Tanahashi, Natural convection of a magnetic fluid in concentric horizon- tal annuli under nonuniform magnetic fields, Exp. Thermal Fluid Sci. 7 (3) (1993) 212–220.
[43] P.S. Szabo, W.G. Früh, The transition from natural convection to thermomagnetic convection of a magnetic fluid in a non-uniform magnetic field, J. Magn. Magn. Mater. 447 (2018) 116–123.
[44] Comsol Multiphysics OR , User's Guide, Version 5.4.