Radiative Generation of dS from AdS

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Abstract

The large volume scenario (LVS) of type IIB string compactifications has a robust supersymmetry breaking minimum with a negative cosmological constant (CC). We argue that radiative corrections below the Kaluza-Klein (KK) scale can result in a positive CC, though the string theory generated CC is negative, if some mild conditions on the spectrum of low energy fluctuations are satisfied. This would make the so-called deSitter swampland conjecture (even if true at a high scale) physically irrelevant.

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1 Introduction

The difficulties of getting a classical solution of string theory that corresponds to a 4D dS space are well-known. Even with the inclusion of non-perturbative effects in type IIB compactifications on a CYO as in [1] (KKLT), and [2] (LVS), unless one adds additional structure (such as Dbar branes, T branes etc.), which makes these constructions more complicated and less well-controlled, the background (vacuum) solutions are AdS. That said, there is an important difference between the (pre-uplift) KKL T and the L VS vacua. While the former preserves (N=1) supersymmetry, the latter breaks it. We will therefore focus on the AdS vacuum of LVS. The point of this note is that subject to some mild conditions discussed below, the SUSY breaking LVS minimum with a negative CC can be lifted to a positive CC, by radiative corrections coming from the fluctuations of states below the compactification scale.

2 Review of LVS

The Kaehler and superpotential for the moduli sector (setting $M_P = 1$) are

$$\tilde{K} = -2 \ln \left( V + \frac{\xi}{2} \left( \frac{S + S}{2} \right)^{3/2} \right) - \ln \left( i \int \Omega \wedge \bar{\Omega}(U, \bar{U}) \right) - \ln (S + \bar{S}), \quad (1)$$

$$\tilde{W} = \int G_3 \wedge \Omega + \sum_i A_i e^{-a_i T_i} \equiv W_0 + W_{np}. \quad (2)$$

$\xi = -\chi \zeta(3)/(2\pi)^3$, $\chi$ is the Euler character of the CYO $S$ the axidilaton, $U$ the complex structure moduli, $V$ the volume of the CYO (taken to be of the “Swiss cheese” type in Planck units, $T^i$ are Kaehler moduli whose real parts determine 4-cycle volumes. $W_0$ is the flux superpotential and $W_{np}$ is that generated by non-perturbative effects. The $a_i$ a constants determined by string instantons, or from confinement/gaugino condensation effects - for a pure $SU(n)$ gauge theory on D7 branes $a = 8\pi^2/n$. In the simplest Swiss cheese constructions $V = k_b \tau_b^{3/2} - k_s \tau_s^{3/2}$ where $\tau_b = \Re T_b$ is a big cycle determining the volume and $\tau_s = \Re T_s$ is a small cycle $k_{b,s}$ are order one numbers. In a realistic model we would have more than this minimum number of 4 cycles and furthermore there would be additional terms in $K$, and $W$ coming from the visible matter (MSSM/SGUT) sector. The latter play no role in determining the potential for the moduli and the background values of these fields are effectively zero at this stage.

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1 For a related discussion see [3].
2 These come from Euclidean D3 branes wrapping a 4 cycle [4] in which case $a = 2\pi$. 

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The axi-dilaton $S$ and the complex structure moduli are fixed super-symmetrically in terms of the internal fluxes i.e. as solutions of $D_S W = 0$, $D_T W = 0$. Then one gets the following potential for the $T_i$,

$$V = V_F + V_D$$

$$V_F = \frac{4}{3} g_s (a|A|)^2 \frac{\sqrt{\tau_s} e^{-2a\tau_s}}{V} - 2 g_s a |A W_0| \frac{\tau_s e^{-a\tau_s}}{V^2} + \frac{3 \xi |W_0|^2}{8 g_s^{1/2} V^3} + \ldots$$

$$V_D = \frac{f}{2} D^2, \quad D = f^{-1} k^i K_i$$

The ellipses in the second equation represent higher powers of $1/V$ and the expansion is justified for large volume compactifications, which will be obtained by appropriate choice of fluxes. The minimization conditions give,

$$e^{-a\tau_s} \simeq \frac{3 W_0}{4 a A V \sqrt{\tau_s}} \left(1 - \frac{3}{4a\tau_s}\right),$$

$$\tau_s^{3/2} \simeq \frac{\hat{\xi}}{2} \left(1 + \frac{1}{2a\tau_s}\right) \simeq \frac{\hat{\xi}}{2},$$

where we’ve written $\hat{\xi} = (\frac{S + \bar{S}}{2})^{3/2} \xi = \xi / g_s^{3/2}$. Note that extremizing with respect to $\tau_s$ gives us an exponentially large volume and the three displayed terms in $V_F$ are all of order $V^{-3}$. This would mean that that at the classical (negative) minimum found in [2], $V_D = 0$ since it is positive definite and of order $1/V^2$. This would also be the case with the contribution to the F-term potential from the dilaton and complex-structure moduli.

The minimum found in [2] gives a negative value for the (classical) cosmological constant

$$V_0 \simeq - \frac{3 \hat{\xi}}{16 a \tau_s} \frac{m_{3/2}^2}{V} \simeq - \frac{m_{3/2}^2}{\ln m_{3/2}} \frac{1}{V},$$

where $m_{3/2}$ is the mass of the gravitino. Since the F-terms of the Kaehler moduli are non-zero at this minimum this solution breaks supersymmetry spontaneously.

### 3 Radiative corrections to the CC

Now we wish to estimate the radiative corrections due to quantum fluctuations around this minimum. These would be due to the moduli fluctuations around the background values fixed by the LVS minimum as well as the supersymmetric standard model and (super) GUT states (if present).
The RG improved one-loop contribution (in a spontaneously broken SUSY theory) to the cosmological constant evaluated in the deep IR (i.e. at cosmological scales) due to these states is

$$\Lambda_{cc}(\mu \ll m) \simeq \Lambda_{cc}(M_{KK}) - \frac{\bar{M}^2 - M_{KK}^2}{32\pi^2} \text{Str}_\mathcal{M} \mathcal{M}^2 - \frac{\bar{m}^2 - \bar{M}^2}{32\pi^2} \text{Str}_\mathcal{m} \mathcal{m}^2$$

$$+ \frac{1}{64\pi^2} \text{Str}_\mathcal{M} \mathcal{M}^4 \ln \left( \frac{\bar{M}^2}{M_{KK}^2} \right) + \frac{1}{64\pi^2} \text{Str}_\mathcal{m} \mathcal{m}^4 \ln \left( \frac{\bar{m}^2}{M^2} \right) + \ldots . \quad (9)$$

The cutoff of the effective field theory for these fluctuations is the Kaluza-Klein scale $M_{KK} = \mathcal{O}(1)$.

We’ve split up the states below this scale into two sectors, one characterized by the scale $\bar{M}$ refers to all the heavy moduli as well as matter supermultiplets at some high scale such as the GUT scale. The other set would contain the MSSM states as well as the light moduli. We’ve assumed for convenience that the splitting within a supermultiplet is smaller than the separation between these two scales.

The RHS of this equation should be identified with the observed CC, i.e. $\Lambda_{cc}(\mu \ll m) \simeq 10^{-122}$. The first term on the LHS however is the cosmological constant derived in LVS, i.e. we identify it with the LVS minimum (8), so (using also (7)),

$$\Lambda_{cc}(M_{KK}) = V_0 \sim -\frac{3\xi}{16\alpha_s} \frac{m_{3/2}^3}{V} = -\frac{3}{16\alpha} \left( 4\xi \right)^{1/3} \frac{m_{3/2}^3}{\sqrt{g_s} V}. \quad (10)$$

The CC at this scale is of course negative. The question is whether the radiative corrections can do the job of lifting this (SUSY broken) AdS vacuum to the observed value.

The validity of the EFT implies that the largest mass scale allowed is much less than the cutoff scale i.e. $\bar{M} \ll M_{KK}$. Also supertraces are essentially fixed by the gravitino mass and the number of states at or below this scale. In fact we have (see for example [8] and [9] for the correction when the background is not Minkowski),

$$\frac{1}{2} \text{Str}_\mathcal{M} \mathcal{M}^2 = (N - 1)(m_{3/2}^2 + V_0) - F^I(\mathcal{R}_{IJ} + S_{IJ})F^J, \quad (11)$$

where $N$ is the number of supermultiplets contributing to the supertrace, $\mathcal{R}_{IJ} = \partial_I \partial_J \ln \det K_{MN}$ and $S_{IJ} = -\partial_I \partial_J \ln \det \Re f_{ab}$ with $K_{MN}$ being the matter sector Kaehler metric and $f_{ab}$ the gauge coupling superfield. The sums over $I, J$ effectively go only over the SUSY breaking directions whose dimension is typically a number of order one (in our case it is essentially just the $T_b$ direction with a small contribution from $T_s$). In other words it does not scale with $N$. Hence for $N \gg 1$ we have,

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3 The original Coleman-Weinberg formula [2] was adapted for SUSY/SUGRA theories in [6]. The version below which incorporates one-loop RG running from the KK scale is taken from [7].

4 I wish to thank Heliudson de Oliveira Bernardo for pointing out this reference to me.
since $V_0$ is suppressed by an extra factor of $\mathcal{V}$ compared to $m_{3/2}^2$ in LVS (unlike in KKLT where $m_{3/2}^2 + V_0 = -2m_{3/2}^2$),

$$\text{Str}M^2 \simeq Nm_{3/2}^2.$$  \hfill (12)

Thus for large $N$ it follows from (9) that the radiative contributions are positive and thus we have the possibility of lifting the negative CC coming from the string theory calculation to a positive value.

Let us consider whether such a scenario is actually plausible - starting from the LVS minimum of eqn. (9). Keeping just the dominant terms in (9) we get.

$$\Lambda_{cc}(\mu \ll m) \simeq \Lambda_{cc}(M_{KK}) + \frac{M_{KK}^2 - \bar{M}^2}{32\pi^2} \text{Str}_M M^2 + \frac{\tilde{M}^2 - m^2}{32\pi^2} \text{Str}_M m^2$$

$$\simeq \Lambda_{cc}(M_{KK}) + \frac{M_{KK}^2}{16\pi^2} N m_{3/2}^2 \equiv V_0 + \Delta_{\text{radiative}} V.$$ \hfill (13)

Here we’ve used $M_{KK} \gg \bar{M}$ and eqn. (12). To have a chance of cancelling the negative LVS CC with the radiative corrections we should have $\Delta_{\text{radiative}} V \gtrsim |V_0|$ which gives (using the estimate $M_{KK}^2 \simeq 1/\mathcal{V}^{4/3}$ and (10)) an upper bound

$$\mathcal{V}^{1/3} \leq \left( \frac{\sqrt{g_s}}{(4\xi)^{1/3}} \right) \left( \frac{N}{16\pi^2} \right) \left( \frac{16a}{3} \right).$$ \hfill (14)

For typical LVS compactifications $\chi \sim 10^2$ and the number of heavy moduli (the complex structure moduli) are also $O(10^2)$, and if we have a SGUT we can have an additional number of heavy states (i.e. with mass greater than the electro-weak scale but much less than the KK scale), so that one expects $N \sim 10^2$. So the first product of the first two factors can easily be a number of the order unity. The third factor can however be large. Taking for instance $a = 2\pi$ which is the value generated by D3 brane instantons (as in [4]) it is about 30 so we get an upper bound

$$\mathcal{V} \lesssim 10^3.$$

Note that one can get larger upper-bounds for smaller Euler characters (keeping $g_s < 1$) and larger $N$ as well as from larger values of $a$ coming from gaugino condensation. Now one might think that the species bound

$$M_{KK} = \frac{M_P}{\mathcal{V}^{2/3}} \lesssim \frac{M_P}{\sqrt{N}}$$

gives a constraint. However this translates to (using (14)),

$$\sqrt{N} \lesssim \mathcal{V}^{2/3} \lesssim \left( \frac{\sqrt{g_s}}{(4\xi)^{1/3}} \right)^2 \left( \frac{N}{16\pi^2} \right)^2 \left( \frac{16a}{3} \right)^2,$$
which just gives a lower bound on $N$. For the choices of $a = 2\pi$ and $\chi = 10^2$, this gives $N \gtrsim O(10)$.

Of course the mere fact that we get a radiatively generated dS vacuum in the IR, does not necessarily mean that we get the observed value of the CC. For that we need to use the Bousso-Polchinski [10] argument for getting the right CC by scanning over the landscape of LVS flux vacua, and in that respect we have added nothing new to the CC problem. The point of this note is to show that even if one can rigorously prove that it is impossible to get a positive CC at the KK scale, it does not mean that the the CC in the IR cannot be positive. In fact our argument shows that provided that the states below the KK scale satisfy some mild criteria, one can generate a positive CC in the IR.

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