Current induced distortion of a magnetic domain wall

Xavier Waintal and Michel Viret
CEA, Service de physique de l’État condensé,
Centre d’étude de Saclay F-91191 Gif-sur-Yvette cedex, France

(Dated: January 12, 2022)

With the advent of "spintronics", which aims at using the spin of charge carriers in devices, electronic transport in ferromagnets is being revisited from a different viewpoint. The focus has been for a long time on the effect of magnetism on transport properties (e.g. Magneto Resistance, MR) \[1\], but it is now realized that the electronic current can be a tool to change the magnetization direction. The relevant effect, known as spin torque has attracted considerable interest recently in the context of ferromagnetic–normal metal–ferromagnetic trilayers \[2\]. There, the first magnetic layer acts as a spin filter, and the incident polarized electrons exert a torque on the second layer. At the heart of this physical effect is the fact that spin currents are not preserved when electrons cross a magnetic layer, and as a result some angular momentum is transferred to the magnetization. It was shown experimentally that for strong enough current densities this mechanism can lead to magnetic reversal \[3\]. This demonstrates the feasibility of current controlled magnetic memory cells, but the current needed for complete reversal might be too high for industrial implementation. An alternative would be to use the current to move a domain wall (DW) in between two stable positions. The idea that a current can apply a force on a DW is due to Berger \[4\] in the seventies. It is the aim of this letter to study in detail the spin torque exerted on a DW in the presence of an electric current. Our main finding is that in addition to a global pressure, the torque has a spatially dependent component that will lead to a deformation of the DW in a periodic sub-structure.

At the root of understanding the spin torque in a DW is the question of what happens to the spin of a conducting electron when going through a DW. Two extreme cases can be considered called "interface" and "adiabatic". A very sharp domain wall (expected in constrictions for example), can be treated as an interface on which the electrons can be specularly reflected \[5\]. This leads to the giant magnetoreistance effect as a magnetic field will remove the DW and the associated extra resistance. Because the electronic spin is conserved during this process, no spin torque is exerted on the DW. On the other hand, in a very long domain wall, the electron’s spin will adiabatically adapt itself to remain aligned with the local magnetization under the effect of the "Larmor" precession \[6\]. There, no MR is observed \[7\] but each electron going through the DW will flip its spin and give a quantum of angular momentum to the wall inducing a global pressure.

Experimentally, the wall resistance (DWR) has been measured in macroscopic systems where it is of the order of a few percent per wall. This resistance results from a slight mistracking of the conduction electrons’ spins which mixes the majority and minority channels within the wall \[8\]. Numerous measurements have been reported in the last few years finding both positive \[9\] or negative \[10\] effects. The spread in the experimental results probably reflects how difficult it is to extract the DWR among other contributions in series coming from domains (like the Anisotropic MR). Other (ballistic) models were developed \[11\] to explain negative effects, but it was later recognized that proper band structure calculations \[12\] are needed to get reliable quantitative results. Nevertheless, recent clean experiments \[13\] in nanostructures of perpendicularly magnetized materials (FePd) have demonstrated that the spin scattering models developed in \[12\] can account for the MR in domain walls in 3d metals. Concerning torque effects, very recently several groups have been able to push DWs with a current \[15\].

In this letter, we proceed as follows. First, an heuristic argument is given to explain the origin of the spatial structure of the torque. Then, we introduce our (ballistic) model and point out that some key features of the band structure must be taken into account. Finally we calculate the spin torque and evaluate the corresponding distortion of the DW profile.

**Heuristic argument:** Let us follow an electron going through the (Neel) DW of size \(\lambda_w\) sketched in Fig.\[1\] (a Bloch wall is completely equivalent for MR and torque). The process is described in Fig.\[2\]. Before the wall, the electron’s spin is aligned with the local magnetization \(\vec{m}\) (a). Then, when entering the DW, \(\vec{m}\) begins to rotate, and a small angle \(\alpha\) starts to build up between the spin and the local magnetization (b). As soon as \(\alpha\) is not zero, the electron’s spin starts to precess around the direction of an effective magnetization with a period equal to the
"Larmor" precession length $\lambda_L$ [(c) and (d)]. At the end of every period, the electron's spin is back onto the local magnetization direction $\vec{m}$. At $x = \lambda_L / 2$, $\alpha$ reaches a maximum. Over this distance, $\vec{m}$ has rotated by an angle $\pi \lambda_L / 2 \omega$ which thus gives an upper bound for $\alpha$. Hence, $\eta = \lambda_L / \lambda_w$ is the parameter that controls the crossover from the adiabatic limit ($\eta \ll 1$) to the interface one ($\eta \gg 1$).

The (almost) adiabatic case is the most interesting regarding spin torque because the incident spin up electron ends up with the (almost) opposite spin after crossing the wall, losing an angular momentum $\hbar$ in the process. In return, since the total angular momentum is conserved, a torque of equal magnitude and opposite sign is exerted on the magnetization within the wall. This is the global pressure mentioned earlier. The spin precession around the slowly rotating effective magnetization does not conserve the angular momentum either, and, as detailed below, it gives rise to the spatially dependent part of the torque inducing a deformation with a period $\lambda_L$.

**Model.** Let us introduce a simple model for the conducting electrons through the DW of Fig. 1. The system is a ferromagnetic wire running along $x$ and of typical transverse dimension $d$. We approximate the wall by a linear rotation of the local magnetization on a scale $\lambda_w$. The unit vector $\vec{m}(x)$ lies inside the $xz$-plane and makes an angle $\theta(x) = \pi x / \lambda_w$ with the $z$ axis inside the DW.

The Hamiltonian reads,

$$ H = -\frac{\hbar^2}{2 m^*} \Delta - J_{\text{exc}} \frac{\vec{m}(x) \cdot \vec{\sigma}}{2}. $$  \hspace{1cm} (1)

Here $J_{\text{exc}}$ is the exchange energy, $m^*$ the effective mass, $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ the vector of pauli matrices and the Fermi energy is noted $E_F$. The spin dependant part of $H$ reads,

$$ \vec{m}(x) \cdot \vec{\sigma} = \begin{cases} \sigma_z, & x < 0 \\ -\sigma_z, & \lambda_w < x \end{cases} $$

with the rotation matrix defined as $R_{\theta(x)} = e^{-i \sigma \theta(x)/2}$. In the region outside the wall, the eigenstates of $H$ are plane waves in both the transverse $yz$-plane, with total momentum $k^\perp$, and in the $x$-direction, with momentum $k^x \parallel (k^x \perp)$ respectively for majority and minority electrons. In the transverse direction, $k^\perp$ is quantized in units of $2 \pi / d$ and for the corresponding eigenstate to be propagating in the $x$-direction, one needs

$$ k_{\parallel \perp}^\perp = \sqrt{2m^*/\hbar^2} (E_F \pm J_{\text{exc}}/2) - (k^x)^2 > 0 $$

which leads to $N_\uparrow (N_\downarrow)$ propagating channels for the majority (minority) spin. Once these propagating channels are defined, the natural way to calculate physical quantities such as the current or the spin current is to use the Landauer-Buttiker theory and fill up the different eigenstates of the system. This has been widely used in the literature and eventually leads to the Landauer formula for the conductance $g = e^2 / h \text{Tr} tt^\dagger$ where $t$ is the transmission matrix of the system.

The arrows stand for the magnetization direction $\hat{m}$, that makes an angle $\theta$ with the $z$ axis. The same $\lambda$ from the adiabatic limit ($\eta \ll 1$) to the interface one $(\eta \gg 1)$.
equal perpendicular energy, and the polarization of the current is entirely due to the remaining \( N_a \equiv N_\uparrow - N_\downarrow = N_\uparrow - N_\downarrow \) up channels that have large perpendicular energy (see Fig. 3 (a)). Hence their longitudinal energy \( \sim \sqrt{J_{\text{exc}}/m^*} \), is low and the corresponding minority channels may not propagate. This stems from the assumption that the up and down bands have exactly the same shape, which is obviously erroneous. It was recently clearly underlined by Mazin [16] that by oversimplifying the band structure of magnetic metals, one introduces symmetries in the system that ought not to be there. Indeed, when the polarization of the current \( P_I \) is taken equal to the polarization \( P_N \) i.e.,

\[
P_I \equiv \frac{I_\uparrow - I_\downarrow}{I_\uparrow + I_\downarrow} = P_N \equiv \frac{N_a}{N_\uparrow + N_\downarrow} = \frac{J_{\text{exc}}}{E_f} \quad (3)
\]

this artificial symmetry can lead to erroneous physics. For instance, after an “interface” wall, all the channels with high perpendicular energy are blocked (the \( N_a = (N_\uparrow - N_\downarrow) \) in Fig. 3 (a)) and the polarization of the current drops to almost zero. In a real system however (sketched in Fig. 4 (b)), the number \( N_a \) of blocked channels would not be equal to \( (N_\uparrow - N_\downarrow) \) and the system would retain some polarization \( P_I \approx (N_\uparrow - N_\downarrow)/(N_\uparrow) \). A complete treatment of the problem would require additional calculations as described in [16]. Here, we break this artificial symmetry by giving a different weight to the different channels in the Landauer formula, i.e., allowing \( P_I \) and \( P_N \) to be independent numbers. This can be viewed as replacing the ferromagnet by a non-magnetic reservoir in series with a normal-ferromagnetic interface that acts as a spin filter. It leads to the following modified Landauer formula,

\[
g = \frac{e^2}{h} \text{Tr} \left[ 1 + \frac{P_I - P_N}{1 - P_N} \sigma_z \right] t t^\dagger \quad (4)
\]

where the \( \sigma_z \) matrix applies in spin space and only to those channels that can propagate for both majority and minority spins. We emphasize that we introduce this modification to take into account the band structure of the magnet and do not question the Landauer-Buttiker formalism itself. This correction is crucial here since, as will be seen later, by symmetry the spin torque vanishes when \( P_N = P_I \).

For a given spinor wave function \( \Psi(x) \), the spin current flowing along the \( x \)-direction is defined as [17],

\[
\vec{J}_s(x) = \frac{\hbar^2}{2m^*} \text{Im} \int dydz \Psi^\dagger(x) \sigma \frac{\partial}{\partial x} \Psi(x) \quad (5)
\]

This is not a conserved quantity, and the corresponding loss of spin current is identified with the torque \([17]\).

The global torque is then \( \vec{T}_{\text{tot}} = \vec{J}_s(x = 0) - \vec{J}_s(x = \lambda_w) \) while the local torque (per unit distance) is,

\[
\vec{\tau} = \frac{\partial}{\partial x} \vec{J}_s(x) \quad (6)
\]

From the last equation, we derive for the torque per unit voltage,

\[
\frac{\partial \vec{\tau}}{\partial V} = -\frac{e}{4\pi \hbar} \text{Re} \text{ Tr} \left[ 1 + \frac{P_I - P_N}{1 - P_N} \sigma_z \right] t(x) \vec{\sigma} t^\dagger(x) \quad (7)
\]

where the generalized transmission matrix \( t(x) \) gives the amplitudes of the different modes inside the wall.

Domain wall close to the adiabatic limit. We now proceed with the treatment of Eq. 11 and Eq. 12 in the limit of a long wall. The first step consists in writing the Schrödinger equation in the basis aligned with the local magnetization. An eigenstate \( \Psi(x) \) is then written as,

\[
\Psi(x) = R_{\theta(x)} \Phi(x) \quad (8)
\]

An effective equation is obtained for \( \Phi(x) \) (see [3] for a similar treatment, the exact solution of the model can be found using the “spin spiral” state, see [12]). The solution is then formally expanded in series of \( (1/\lambda_w) \) and we match \( \Phi(x) \) and its derivative at \( x = 0 \) and \( x = \lambda_w \).

In first order in \( (1/Q_k \lambda_w) \) where \( Q_k = \sqrt{k_\parallel^2 + k_\perp^2} \), we get for incoming majority electrons in the wall:

\[
\Phi_\uparrow(x) = \frac{e^{ik_\parallel x}}{\sqrt{k_\parallel}} \quad \text{and} \quad \Phi_\downarrow(x) = \frac{i\pi}{4Q_k \lambda_w} \sqrt{\frac{1}{k_\parallel}} \times 
\]

\[
\left( \frac{1}{P_k} [\frac{e^{ik_\parallel x}}{e^{-ik_\parallel x}}] + P_k [\frac{e^{-ik_\parallel x}}{e^{ik_\parallel x}} \xi e^{-ik_\parallel x}] \right)
\]

and similar solutions for incoming minority spins. The wave functions are normalized to carry unit fluxes and only the longitudinal part has been written. We have defined \( P_k = (k_\parallel^2 - k_\perp^2)/(k_\parallel^2 + k_\perp^2) \), and \( \xi = e^{i(k_\parallel + k_\perp)\lambda_w} \). We point out that the oscillatory first term in the brackets is the expression of the Larmor precession while the second one is a reflected wave. In the region \( x < 0 \) the reflected wave takes the form,

\[
\Phi_\downarrow(x) = \frac{i\pi P_k}{4Q_k \lambda_w} (\xi - 1) \frac{e^{-ik_\parallel x}}{\sqrt{k_\parallel}} \quad (10)
\]

Spin torque in a macroscopic DW. To proceed with the calculation of the conductance and torque, one incorporates the expression for the wavefunctions into Eq. 11 and Eq. 12 and performs the average over transverse momentum. This average is done in two steps, first over \( \xi \), which is taken to be a random phase and then over \( k_\perp \). Also, the Fermi wave length \( \lambda_F \) and Larmor precession length \( \lambda_L \) are defined as,

\[
\lambda_F = 2\pi \sqrt{\frac{\hbar^2}{2m^* E_F}}, \quad \lambda_L = \pi \sqrt{\frac{E_f}{J_{\text{exc}}}} \quad \sqrt{\frac{\hbar^2}{2m^* J_{\text{exc}}}} \quad (11)
\]

We get for the correction to the conductivity due to the presence of the DW, \( \Delta g/g = -P_N \lambda_F/(64\lambda_w^2) \). This
correction is (too) small because in a ballistic model, the calculated quantity is the reflected part of the wavefunction due to the potential step. Diffusive models, on the other hand, neglect this contribution and estimate the resistance due to spin mixing between the up and down electrons. It turns out that in the macroscopic case, the latter dominates. In the interface limit however, the ballistic contribution becomes important. We calculate the torque per current along the local frame \((a, v, w)\) from the wavefunctions expressions as,
\[
\frac{\partial \tau_u(x)}{\partial I} = \frac{\hbar}{e 2 \lambda_w} \left[ P_1 + (P_1 - P_N) \cos \left( \frac{2 \pi x}{\lambda} \right) \right].
\]
\[
\frac{\partial \tau_v(x)}{\partial I} = -\frac{\hbar}{e 2 \lambda_w} (P_1 - P_N) \sin \left( \frac{2 \pi x}{\lambda} \right).
\]
Equation (12) is the central result of this letter. The torque consists of two terms. The first one, proportional to \(P_1\) pushes the wall in the direction of the electrons and does not depend on \(x\). The second part of Eq.(12) is much more interesting since it leads to a deformation of the wall on the scale of \(\lambda_L\). Note that although its net contribution to the global torque is small, its intensity is of the same magnitude as the first term. However, the \(N_a\) channels where the perpendicular energy is high (hence the down electrons cannot propagate) do not contribute to the torque since no precession is possible inside those channels. As a result, this component of the torque vanishes when \(P_1 = P_N\), hence the necessity of taking into account the fact that \(P_1 \neq P_N\) in real materials. We point out that although our calculation is done for a ballistic model, the result should hold for realistic domain walls. Indeed, both the spin diffusion length (about 50 nm for Ni) and the mean free path (a few nm) are larger than \(\lambda_L\) (of the order of 3 nm for Ni) and at the Larmor length scale, the electrons can thus be considered ballistic. In a system where the mean free path would be smaller than \(\lambda_L\) but still with a (rather) large spin diffusion length, our conclusions would remain qualitatively correct, though a quantitative treatment might be needed.

In order to quantify the expected deformation of the wall, one would have to solve self-consistently the torque equation together with the reaction from \(\dot{\vec{m}}(x)\) linked to the "stiffness" of the wall (using for instance the Landau-Lifshitz-Gilbert equation). Here we simply evaluate the magnitude of the deformation by comparing the energy given to the wall via the torque to the total wall energy. Typical quantities for Ni are \(\lambda_w = 100 nm\), \(\lambda_f = 0.2 nm\), \(\lambda_L = 3nm\), and \(P_1 - P_N \approx 0.5\). A current density \(j = 10^{10} A/m^2\) would then leave in a Ni wall \(10^{-3} J/m^2\), which is one tenth of the wall surface energy. Hence the distortion in angular gradient should be of the order of several \%. This is a significant effect considering that the chosen current density is one order of magnitude below that used in tri-layer spin torque experiments.

Constriction "interface" limit. In a constriction (small transverse direction \(d\)), the DW width is expected to scale with \(d^{18, 20}\) and the system could be driven to the interface limit. As explained earlier, the spin of the transferred electrons would remain mostly un-flipped and the torque would decrease as a result. Hence, while in this regime the MR gets larger, the torque described in this paper would drop. In addition, since in this limit the reflection is higher, another contribution to the total pressure could come from the transfer of momentum from the reflected electrons to the DW. Although it is not clear to us whether this change of momentum would actually push on the wall or on the atoms, its effect would be much smaller than that in a long wall.

Conclusion. The torque generated by a current on a domain wall in a ferromagnetic metal has been studied in unconstrained DWs. It is composed of two contributions of comparable magnitude. The first one is a global pressure resulting from the loss of angular momentum of electrons crossing the wall. The second is due to the precession of electrons' spins inside the wall which generate a periodic torque. The first effect can be used to move the DW with a current. Depending on pinning, the current density necessary to dislodge the DW could be smaller than that necessary to reverse a full magnetized layer in a spin valve. The effect can then potentially be useful in spin electronic devices where a DW switched between two stable positions could be used in the gate of a three terminal device. Moreover, the periodic torque will distort the wall’s internal structure in a significant manner when current densities of the order of \(10^{10} A/m^2\) are driven through it. This may help the depinning process and might also be able to switch the wall between different types (Bloch and Neel for example) in a similar manner to the predicted temperature effect in a constriction \(20\). The distortion may be measured by several techniques including polarized neutrons reflectivity (PNR) and domain wall resonance. We also infer that a current flowing parallel to the wall would produce a surstructure of the same period, which may be easier to measure with PNR.

Acknowledgment. It is a pleasure to thank R. Jalabert and D. Weinnmann for interesting discussions.

[1] A. Barthelemy, A. Fert, J.P. Contour, et al. J. Mag. Magn. Mat. 242: 68 (2002).
[2] J. Slonczewski, J. Magn. Magn. Mater. 159, L1 (1996).
[3] E. B. Myers, D. C. Ralph, J. A. Katine, and al, Science 285, 867 (1999). J. A. Katine, F. J. Albert, R. A. Buhrman, and al, Phys. Rev. Lett. 84, 3129 (2000).
[4] L. Berger, Phys. Lett. A 46A, 3 (1973).
[5] G. G. Cabrera and L. M. Falicov, Phys. Stat. Sol. 61,
[6] M. Viret, D. Vignoles, D. Cole and al, Phys. Rev. B 53, 8464 (1996).
[7] L. Berger, J. Appl. Phys. 49, 2156 (1978).
[8] P.M. Levy and S. Zhang Phys. Rev. Lett. 79, 5110 (1997).
[9] A. Brataas, G. Tatara and G.E.W. Bauer Phys. Rev. B 60, 3406 (1999).
[10] U. Ruediger, J. Yu, S. Zhang, A. D. Kent, and S. S. P. Parkin, Phys. Rev. Lett. 80, 5639 (1998).
[11] G. Tatara and H. Fukuyama, Phys. Rev. Lett. 78, 3773 (1997).
[12] R. P. van Gorkom, A. Brataas and G. E. W. Bauer Phys. Rev. Lett. 83, 4401 (1999).
[13] J. B. A. N. van Hoof, K. M. Schep, A. Brataas and al, Phys. Rev. B 59, 138 (1999).
[14] M. Viret, Y. Samson, P. Warin, et al. Phys. Rev. Lett. 85, 3962 (2000); R. Danneau, P. Warin, J.P. Attane, et al. Phys. Rev. Lett. 88, 157201 (2002).
[15] N. Vernier et al., D. Ravelosona et al., R. Cowburn et al., A. Fert et al. private communications.
[16] I. I. Mazin, Phys. Rev. Lett. 83, 1427 (1999).
[17] X. Waintal, E. B. Myers, P. W. Brouwer, and D. C. Ralph Phys. Rev. B 62, 12317 (2000). X. Waintal and P. W. Brouwer Phys. Rev. B 65, 054407 (2002).
[18] P. Bruno, Phys. Rev. Lett. 83, 2425 (1999).
[19] D. Weinmann, R. L. Stamps and R. A. Jalabert, in proceedings of the ‘36th Rencontres de Moriond, edited by T. Martin, G. bert and J. Trân Thanh Vân.
[20] Y. Labaye, L. Berger and J.M.D. Coey J. Appl. Phys. 91, 5341 (2002).