Implications for Primordial Non-Gaussianity ($f_{NL}$) from weak lensing masses of high-z galaxy clusters.

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The recent weak lensing measurement of the dark matter mass of the high-redshift galaxy cluster XMMU J2235.3-2557 of $(8.5 \pm 1.7) \times 10^{14} M_\odot$ at $z = 1.4$, indicates that, if the cluster is assumed to be the result of the collapse of dark matter in a primordial gaussian field in the standard LCDM model, then its abundance should be $< 2 \times 10^{-3}$ clusters in the observed area. Here we investigate how to boost the probability of XMMU J2235.3-2557 in particular resorting to deviations from Gaussian initial conditions. We show that this abundance can be boosted by factors $> 3 - 10$ if the non-Gaussianity parameter $f_{NL}$ is in the range $150 - 200$. This value is comparable to the limit for $f_{NL}$ obtained by current constraints from the CMB. We conclude that mass determination of high-redshift, massive clusters can offer a complementary probe of primordial non-Gaussianity.

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Introduction.— It has been recognized for almost a decade that the abundance of the most massive and/or high-redshift collapsed objects could be used to constrain the nature of the primordial fluctuation field $\Phi$ [1,2,3,4]. The subject has recently received renewed attention [3,6,7,9] possibly sparked by a claimed detection of deviations from Gaussianity on CMB maps [10]. Depending on the sign of the non-Gaussian perturbation, the abundance of rare objects will be enhanced or depleted. In [1] we developed the necessary theoretical tools to interpret any enhancement (depletion) in the abundance of rare-peak objects over the gaussian initial conditions case. Working with ratios of non-Gaussian over Gaussian case makes the theoretical predictions more robust. Later on, Ref. [2] generalised the procedure to more modern mass-functions and type of non-gaussianity including scale dependence. The validity of the analytical formulas developed in [1] has been recently confirmed by detailed N-body numerical simulations with non-gaussian initial conditions [7]. These authors have shown that the analytical findings in [1] provide an excellent fit to the non-Gaussian mass function found in N-body simulations as analytical findings in [1] provide an excellent fit to the initial conditions [7]. These authors have shown that the detailed N-body numerical simulations with non-gaussian including scale dependence. The validity of the analytical findings in [1] provides an excellent fit to the initial conditions case. Working with ratios of non-Gaussian over Gaussian abundance of rare-peak objects over the gaussian initial conditions case makes the theoretical predictions more robust. Later on, Ref. [2] generalised the procedure to more modern mass-functions and type of non-gaussianity including scale dependence. The validity of the analytical formulas developed in [1] has been recently confirmed by detailed N-body numerical simulations with non-gaussian initial conditions [7]. These authors have shown that the analytical findings in [1] provide an excellent fit to the non-Gaussian mass function found in N-body simulations with a simple “calibration” procedure.

Ref. [12] have recently reported a weak-lensing analysis of the $z = 1.4$ galaxy cluster XMMU J2235.3-2557 based in HST (ACS) images. Assuming a NFW [11] dark matter profile for the cluster, they estimate a projected mass within 1 Mpc of $(8.5 \pm 1.7) \times 10^{14} M_\odot$. Adopting a LCDM cosmology with cosmological parameters given by WMAP 5 yr data [13] and assuming Gaussian initial conditions they estimate that in the surveyed 11 sq. deg. there should be 0.005 clusters above that mass. Therefore the observed cluster is unlikely at the 3$\sigma$ level. In this Letter we explore what level of non-Gaussianity is required to boost this abundance by a factor $\sim 10$ and how this relates to the available constraints obtained from the CMB. We show that with $f_{NL}$ in the range $150 - 200$ it is possible significantly enhance the abundance expected for such a massive cluster. This value of $f_{NL}$ is comparable with current limits from the CMB [13,10].

High Redshift and/or Massive Objects.— While there are in principle infinite types of possible deviations from Gaussianity, it is common to parameterize these deviations in terms of the dimensionless parameter $f_{NL}$ (e.g., $f_{NL}^{local}$, $f_{NL}^{global}$).

$$\Phi = \phi + f_{NL}^{local}(\phi^2 - \langle \phi^2 \rangle).$$

(1)

where $\Phi$ denotes the primordial Bardeen potential [29] and $\phi$ denotes a Gaussian random field. With this convention a positive value of $f_{NL}^{local}$ will yield to a positive skewness in the density field and an enhancement in the number of rare, collapsed objects.

Although not fully general, this model (called local-type) may be considered as the lowest-order terms in Taylor expansions of more general fields. Local non-gaussianity arises in standard slow roll inflation (although in this case $f_{NL}^{local}$ is unmeasurably small), and in multi-field models (e.g., $f_{NL}^{global}$ is unmeasurably large). For other types of non-gaussianity (as we will see below) an “effective” $f_{NL}$ can be defined and related to this model.

The abundance of rare events (high-redshift and/or massive objects) is determined by the form of the high-density tail of the primordial density distribution function. A probability distribution function (PDF) that produces a larger number of $> 3\sigma$ peaks than a Gaussian distribution will lead to a larger abundance of rare events. Since small deviations from Gaussianity have deep impact on those statistics that probe the tail of the distribution (e.g., $f_{NL}$, rare events should be powerful probes of primordial non-Gaussianity. 

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The non-Gaussianity parameter $f_{NL}$ is effectively a “tail enhancement” parameter (c.f., [1]).

As shown in [1,2,7] when using an analytical approach to compute the mass function a robust quantity to use is the fractional non-gaussian correction to the Gaussian mass function $R_{NG}(M, z)$. This quantity was calibrated on non-gaussian N-body simulations in [7]. For our purpose here we want to compute a closely related quantity: the non-gaussianity enhancement i.e. ratio of the non-gaussian to gaussian abundance of halos above a mass threshold [3]. As the mass function is exponentially steep for rare events here we can safely make the identification of the non-gaussianity enhancement with $R_{NG}$.

To understand the effect of non-gaussianity on halo abundance let us recall that to first order the non-gaussianity enhancement is given by [3,7]:

$$R_{NG}(M, z) \sim 1 + S_{3, M} \frac{\delta_c^4(z)^3}{6 \sigma_M^2}$$  

(2)

where $S_{3, M}$ denotes the skewness of the density field linearly extrapolated at $z = 0$ and smoothed on a scale $R$ corresponding to the comoving Lagrangian radius of the halo of mass $M$, $\sigma_M$ denotes the rms if the linearly extrapolated at $z = 0$ density field also smoothed on the same scale $R$; $\delta_c^4(z) = \sqrt{\delta_c^2(z)}$ and $\delta_c(z)$ denotes critical collapse density at the formation redshift of the cluster $z_f$. Note that $\delta_c(z) = \Delta_c D(z = 0)/D(z)$ with $D(z)$ denoting the linear growth factor and $\Delta_c$ is a quantity slightly dependent on redshift and on cosmology, which only for an Einstein-de-Sitter Universe is constant $\Delta_c = 1.68$. The constant $q \simeq 0.75$ (which we will call “barrier factor”) can be physically understood as the effect of non-spherical collapse [21,22] lowering the critical collapse threshold of a diffusing barrier [26] see also [27], and has been calibrated on N-body simulations in Ref. [2]. The full expression for $R_{NG}$ is (cf Eqs. 6 and 7 in Ref. [2]):

$$R_{NG}(M, z) = \exp \left[ \frac{\delta_c^4(z)^3}{6 \sigma_M^2} S_{3, M} \right] \times$$

$$\left[ 1 - \frac{\delta_c^2}{6} \frac{dS_{3, M}}{d \ln \sigma_M} \frac{\delta_c}{\sqrt{1 - \delta_c S_{3, M}^2/3}} + \frac{\delta_c^2}{\delta_c} \frac{1 - \delta_c S_{3, M}^2/3}{\delta_c^2} \right].$$  

(3)

Let us re-iterate that in principle the enhancement factor should be computed by integrating the mass function $n(M, z, f_{NL})$ between the minimum and the maximum mass and for redshifts above the observed one [4]:

$$\bar{R}_{NG} = \frac{\int n(M, z, f_{NL}) dM dz}{\int n(M, z, f_{NL} = 0) dM dz}$$

(4)

but since the mass function, in the regime we are interested in, is exponentially steep, we can identify $\bar{R}_{NG} = R_{NG}$.

Small deviations from Gaussian initial conditions will lead to a non-zero skewness and in particular for local non Gaussianity $S_{3, M} = f^f_{NL} S_{3, M}$ where $S_{3, M}$ denotes the skewness produced by $f^l_{NL}$ = 1. Since non-Gaussianity comes in the expression for $R_{NG}$ only through the skewness, the same expression can be used for other types of non-Gaussianity such as the equilateral type (see e.g. Ref. [5,6] for example of applications). For example, at the scales of interest $R = 13 \text{Mpc}/h$, $S_{3,local} = 3.4 S_{3,R}$ thus when working on these scales to obtain the same non-Gaussian enhancement as a local model, an equilateral model needs a higher effective $f_{NL}$. We can make the identification $f_{NL}^{local} = 3.4 f_{NL}^{equil}$. Here we will use the full [1] expression, corrected for the “barrier factor”, for the non-gaussian mass function to compute the non-gaussianity enhancement. Note that the estimated mass and redshift of XMMUJ2235.3-2557, places it just outside the range where the mass function expressions of [1,3] have been directly reliably tested with non-Gaussian N-body simulations. Simulations seems to indicate that the [1] expression is a better fit than [3] at high masses/redshift and large $f_{NL}$, this is also supported by theoretical considerations [7].
Results. — Fig. 1 shows the enhancement factor $R_{NG}$ as a function of the mass of the galaxy cluster for different values of $f_{NL}^{local}$ and the redshift of collapse. The shaded area shows the error band for the mass determination of XMMUJ2235.3-2557 from Ref. [12] and the different lines have been computed using the [1] mass function, with the “barrier factor” correction. Ref. [7] show that it fits very well the N-body numerical simulations for the case of rare peaks, which is the one we are concerned with. The solid lines correspond to $f_{NL} = 260$, the lower one is for a cluster collapse redshift of $z_f = 1.4$ (i.e. assuming that the cluster forms at the observed redshift) and the upper one for $z_f = 2$. The two dashed lines also depict the mentioned collapse redshifts but for $f_{NL} = 150$. We see that the galaxy cluster abundance can be enhanced by a factor up to 10. In the mass range of interest, the same enhancement factor can be obtained for an equilateral-type non-gaussianity for $f_{NL}^{equl} = 884$ and 510 respectively.

We should bear in mind that XMMUJ2235.3-2557 is an extremely rare object, sampling the tail of the mass function which may not be well known and may be strongly affected by cosmology. Using the [28] mass function we estimate that in the WMAP5 LCDM model one should find 7 galaxy clusters in the whole sky with mass greater or equal than the lower mass estimate of XMMUJ2235.3-2557 $M = 5 \times 10^{14} M_\odot$ and $z > 1.4$ corresponding to a probability of 0.002 for the 11 deg$^2$ of the survey. This should be compared with the reported number of 0.005 obtained by [12] for a different cosmology and different mass function. Thus the effects of cosmology and uncertainty in the mass function can account for a factor $\sim 2$ uncertainty in the predicted halo abundance.

Note that in all our calculations we have used a conservative lower limit for the mass of the cluster. If instead we use the central or upper value for the mass, using the WMAP5 cosmology and the [28] mass function we expect to find zero such clusters in the whole sky, which will make our conclusions even stronger.

The survey area used in Ref. [12] is 11 deg$^2$, but the XMM serendipitous survey in 2006 covered 168 deg$^2$ and today covers $\sim$ 400 deg$^2$. Below we report the Ref. [12] numbers and in parenthesis the numbers we obtain. The probability of finding XMMUJ2235.3-2557 is thus 0.005 (0.002) if using 11 deg$^2$, to avoid as much as possible biases due to a posteriori statistics one could use 168 deg$^2$ obtaining a probability of 0.07 (0.03), or, as a limiting case, even 0.17 (0.07) if using 400 deg$^2$. Note that it is likely that there are more clusters as massive in the survey area [28] and therefore these numbers are conservative. If we use from Fig. 1 the factor 3 to 10 enhancement, we find that it would help boost the probability to $\sim 1$ in the surveyed areas.

The latest WMAP compilation [13] reports $-9 < f_{NL}^{local} < 111$ and $-151 < f_{NL}^{equl} < 253$ at 95% confidence, [10] reports $27 < f_{NL}^{local} < 147$. The CMB however probes much larger scales ($R > 120 Mpc/h$) than those probed by clusters such as XMMUJ2235.3-2557 $R \sim 13 Mpc/h$: a scale-dependent $f_{NL}$ with $k \sim 0.3$ can yield an effective $f_{NL}$ on dependence XMMUJ2235.3-2557 scales that is larger than the CMB one by a factor of 3.

The $f_{NL}^{local}$ values needed to accomodate the observed cluster at $z = 1.4$ is in the range 150 to 260. This is comparable to the limits quoted by Ref. [13] and [10].

Conclusions. — Accurate masses of high-redshift clusters are now becoming available through weak lensing analysis of deep images. As already discussed in previous papers [11, 2], their abundance can be used to put constraints on primordial non-gaussianity. $f_{NL}^{local}$ in the range 150 – 260 can boost the expected number of massive ($> 5 \times 10^{14} M_\odot$) high redshift ($z > 1.4$) clusters by factors of 3 to 10. Such large numbers would help make clusters like XMMUJ2235.3-2557 much more probable. The scales probed by clusters are smaller than the CMB scales, and in principle non-Gaussianity may be scale-dependent, making this a complementary approach.

The adopted error range in the mass determination of XMMUJ2235.3-2557 is 100%; even with such a large mass uncertainty and considering the pessimistic estimate of 7 such objects expected in the entire sky with a Poisson error of $\pm 2.6$, if the entire sky could be covered, $f_{NL}^{local} \sim 150$ could be detected at $\sim 4\sigma$ level.

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