Cosmological magnetogenesis

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Abstract. Observations indicate the presence of a magnetic field at galactic and cosmological scales. However, the origin of these magnetic fields is not well understood. There is enough motivation to look into the primordial origin of magnetic field, which essentially requires the breaking of conformal invariance of Maxwell’s theory. Several mechanisms to generate primordial magnetic field have been proposed. A brief overview of those models has been presented. Central problem of the models within inflationary paradigm has been addressed. Possibilities to generate primordial magnetic field beyond inflationary framework are mentioned. A toy model for bouncing cosmology has been presented to understand the idea of magnetogenesis in such models.

1. Introduction

Universe is magnetized at all scales. Magnetic field of the order of nano G has been observed in intergalactic medium (IGM). This has been confirmed by various measurements, like Faraday rotation measurements and CMBR polarization. The origin of magnetic fields at cosmological scales is an unsolved problem. Inflation becomes the natural choice to produce effects at scales larger than the Hubble horizon. The required strength of the cosmic magnetic field at the epoch of structure formation, adiabatically re-scaled to the present time, should be $10^{-10}$ to $10^{-7}$ G. On the other hand, if the galactic dynamo mechanism is at play, then the seed fields of strength $10^{-22}$ to $10^{-16}$ G is required at the end of inflation [1, 2]. (We have used the metric signature $(-,+,+,+)$. Natural units have been used for all calculations, i.e., $\hbar = c = G = 1$.)

2. Magnetogenesis in FRW-universe

Consider a homogeneous, isotropic and flat universe described by Friedman-Robertson-Walker (FRW) metric:

$$ds^2 = a^2(\eta) (d\eta^2 - \delta_{ik}dx^i dx^j), \quad \text{where} \quad \eta = \int \frac{dt}{a(t)}. \quad (1)$$

Here, $x, y, z$ are comoving coordinates and $\eta$ is the conformal time. The general form of the standard electromagnetic field action is given by:

$$S = -\frac{1}{16\pi} \int F_{\mu\nu} F^{\mu\nu} \sqrt{-g} d^4x, \quad (2)$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$. This action is conformally invariant. It turns out that electromagnetic wave fluctuations can not be amplified in FRW universe, as $B \propto 1/a^2(t)$. Therefore, mechanisms for magnetic field generation require the breaking of conformal invariance.
of the electromagnetic action. If one can construct a model in which $B \propto 1/\epsilon^\alpha$, with typically $\epsilon << 1$, one would have achieved a way to obtain strong magnetic fields.

3. Models within inflationary paradigm

Inflation becomes a natural choice to bring the effects at cosmological scales. There have been several mechanisms proposed to break conformal invariance of electromagnetic action \[3, 4, 8\]. A wide variety of models can be incorporated together into a general action of the form \[5\]:

$$S = -\int \left[ \frac{1}{16\pi} f^2(\phi) F_{\mu\nu} F^{\mu\nu} + \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + V(\phi) + M^2 A_\mu A^\mu \right] \sqrt{-g} dx,$$

(3)

where $f(\phi) \to f(\phi, R, R^2, R_{\mu\nu} R^{\mu\nu} \ldots)$.

$M$ is the mass associated with the massive vector field, and $\phi$ is the scalar field. Conformal invariance is broken by the presence of a time dependent coupling of electromagnetic field with scalar fields, curvature, and other such terms. This can be achieved also by the presence of a massive vector field. Here, it is important to mention that the mass term is rather put in by hand and not been generated through Higgs mechanism.

3.1. Evolution of magnetic field

For the gauge $A_0 = 0$, $\nabla \cdot A = 0$, we have found that the Fourier coefficients $A \equiv a(\eta) f(\eta) A(\eta, k)$ satisfy:

$$A''(\eta, k) + (k^2 - f'') A(\eta, k) = 0.$$

(4)

From the above equation, one can see that if $f$ is a constant in time, one gets the usual harmonic oscillator solution, and particularly $f = 1$ gives the results of usual Maxwell’s equations. For a growing or decaying mode solution, we require, $f''/f > k^2$. The electric and magnetic fields can be obtained from:

$$E_i = \dot{A}_i, \quad \text{and} \quad B_i = \frac{1}{\alpha} \epsilon_{ijk} \partial^j A^k.$$

(5)

3.2. Solution for power law inflation

We have worked with the form of scale factor as \[6, 7\]:

$$a(\eta) = a_0 \left[ \frac{\eta}{\eta_0} \right]^{1+\beta},$$

(6)

and the coupling function is taken as $f \propto a^\alpha$, where $\alpha$ and $\beta$ are parameters and $\beta = -2$ corresponds to the de Sitter solution. So, the equation for vector field takes the form:

$$A''(k, \eta) + \left[ k^2 - \frac{\gamma(\gamma - 1)}{\eta^2} \right] A(k, \eta) = 0,$$

(7)

where $\gamma = \alpha(1 + \beta)$. At much smaller length scale ($-k \eta >> 1$), one can neglect the effect of spacetime curvature and the solution will be similar to that found in Minkowski space time, i.e., a plane wave solution. Thus, we have obtained the initial conditions as:

$$A(k, \eta) \to \frac{1}{\sqrt{2k}} e^{-i k \eta}, \quad \text{and} \quad A'(k, \eta) \to -i \sqrt{\frac{k}{2}} e^{-i k \eta}.$$

(8)
3.3. Energy-momentum tensor for EM fields

The energy-momentum tensor can be calculated by varying the EM action with respect to background metric. This gives:

\[ T_{\mu\nu} = -\frac{2}{\sqrt{\gamma}} \frac{\delta [\sqrt{\gamma} g_{EM}]}{\delta g_{\mu\nu}} = \frac{f^2}{4\pi} \left[ g^{\gamma\beta} F_{\mu\gamma} F_{\nu\beta} - g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \right]. \] (9)

Vacuum expectation values of energy densities for electromagnetic field can be calculated by:

\[ \rho_B = \langle 0 | T^B_{\mu\nu} u^\mu u^\nu | 0 \rangle, \] and \[ \rho_E = \langle 0 | T^E_{\mu\nu} u^\mu u^\nu | 0 \rangle, \] (10)

where, \( T^B_{\mu\nu} u^\mu u^\nu = \frac{f^2 B^i B_i}{8\pi} \) and \( T^E_{\mu\nu} u^\mu u^\nu = \frac{f^2 E^i E_i}{8\pi} \).

3.4. Scale invariant power spectrum

The power spectrum is defined as: \( d\rho/d\ln k \). Power spectra for electric (PSE) and magnetic (PSM) fields are obtained from:

\[ \frac{d\rho_B}{d\ln k} = \frac{1}{2\pi^2} \left( \frac{k}{a} \right)^4 k |A(k, \eta)|^2, \] and \[ \frac{d\rho_E}{d\ln k} = \frac{f^2}{2\pi^2} \frac{k^3}{a^4} \left| \frac{A(k, \eta)}{f} \right|^2. \] (11)

It is possible to obtain scale invariant spectrum for magnetic field for \( \gamma = -2 \) and \( \gamma = 3 \). In fact, \( \gamma = -2 \) seems a plausible solution, as it can be shown that electric field decays for this solution. For the other case \( \gamma = 3 \), the electric field will grow rapidly, which is not an acceptable solution [6, 7].

4. Problems and possibilities

Getting a scale invariant power spectrum is not enough. One has to check whether magnetic field can survive through the expansion. It is shown that in such models, inflation cannot generate a seed magnetic field of sufficient strength. The first reason is the back-reaction. Once we start with small coupling regime, inflation will halt before the universe has expanded to the required number of e-foldings. On the other hand, if we start with large coupling regime, it is possible to generate sufficient amount of magnetic field. But, this is not a very acceptable scenario, as large couplings theories are not trusted at all. The case for massive vector field has also been ruled out [5].

4.1. Magnetogenesis beyond inflationary paradigm

There are possibilities to look within inflationary paradigm, as well as there are enough motivations to look beyond the standard inflationary models. As inflation is not the only
outcome of possible cosmological theories, possibilities of generating primordial magnetic fields are being probed into models based upon \( f(R) \) (modified gravity theories), higher dimensional cosmology and bouncing cosmological theories [8].

Here, we have presented a toy model for bouncing cosmology with the idea of magnetogenesis.

4.2. A toy model

\[
\begin{align*}
\text{Figure 3.} & \quad \text{The solid curve shows the evolution of scale factor and the dotted curve correspondingly to Hubble parameter. This toy model represents a symmetric bounce and has no singularity.}
\end{align*}
\]

A toy model for symmetric bouncing universe is taken as:

\[
a(t) = a_0 \left[ 1 + \left( \frac{t}{t_0} \right)^2 \right]^p.
\]

Problem of big bang singularity gets resolved automatically in bouncing cosmological models. Universe goes through a contracting phase up to finite size, and then bounces back expanding. This also solves the horizon problem as the modes which are seen outside horizon could be easily brought back inside during the contracting phase. For magnetogenesis, the idea is such that magnetic field can grow rapidly during contracting phase (through flux conservation) and evolve through the bounce. It again decays during the expansion phase as in FRW universe. A more plausible scenario should be asymmetric bounce.

5. Conclusion

Conformal invariance needs to be broken for electromagnetic fields for the generation of magnetic fields. Array of models based upon inflation can produce a seed magnetic field. Most of the models are unable to generate seed of desired magnitude. Back reaction problem is there in almost all models based upon inflation. Conformal invariance can be broken through non-inflationary models also. There are motivations to look beyond the standard cosmological models like bouncing cosmology, \( f(R) \), and brane-world cosmology.

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