The importance of conceptualization in the use of Wolfram Mathematica 10 for a course in differential calculus

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Abstract. With the continuous advance in the technological tools, both Hardware and Software, a massive utilization has started in different fields of society. One of the fields in which this type of tool is used is the academic one, given the importance in the generation of spaces for discussion in teacher-student mediation, using the computer as a tool that supports the process in the classroom. In particular, the use of specialized software has made the teaching work change depending on the support in the resource, however, you cannot depend exclusively on the resource, but the teacher must be able to use the tool recognizing the benefits and limitations that the software You can submit. The objective of this work is to socialize a classroom experience developed with the students of Differential Calculus in the thematic axis of inverse functions using Wolfram Mathematica 10 software, in which it is possible to show a program limitation with respect to the Cartesian representation of certain functions inverse, which can lead to misinterpretations about their behavior.

1. Introduction

In some moments of any traditional course of differential calculus it is important to realize a Cartesian model (graphic) that represents the behavior of a given function and that of its inverse function, for this it is possible to resort to a specialized Software that allows to glimpse such functions when they are not trivial or typical elementary functions. Although it is possible to know the behavior of the inverse of a function from the same function by applying a geometric criterion that consists of making a rotation of its graph \(90^\circ\) anti-clockwise followed by a reflection with respect to the new vertical axis, in some cases It is convenient to obtain the inverse directly. Thus, through the classroom experience that we present below, we justify the importance of conceptualizing and internalizing the most elementary notions of calculus and of course of many other areas of knowledge, also of a theoretical nature, which are currently studied in the curricula of science or engineering.

2. Basic definitions

Currently there are several academic software tools that support teaching inside and outside the classroom. In the area of natural, exact and technical sciences in mathematics, it is possible to find a wide range of programs with which exercises, simulations, images are created, etc., and in each one of the different branches of mathematics such as algebra, calculus, geometry, among others. For the subjects of basic university cycle the use of ICT tools is of great help for teacher mediation, both in...
hardware and software. It is the duty of the teacher to review these ICT tools in order to be sure of their optimal and appropriate use. To contextualize the experience in the use of Mathematica 10 software, it is necessary to present some elementary definitions about the functions.

Definition 1.1. A function between two non-empty sets $A$ and $B$ is a relation between such sets, such that every element of the output set $A$ is related to one and only one element of the arrival set $B$.

The above definition is usually presented at the beginning of any course of Differential Calculus (Calculus I), however this definition has had several changes in its writing over time by example the concept of function according to Lagrange in 1797 is any expression of calculation in which such quantities enter in any way, mixed with other quantities or not with other quantities that have given and invariable values, while the quantities of the function can receive all possible values [1]. With the definition 1.1. it is intended that the student has a more formal notion of what a function is and thus avoid generating confusing ideas that usually associate such a concept with the "rule of correspondence", "equation", "formula" etc., the above can be evidenced during the didactic transposition that is generated in teacher mediation wrongly, understanding didactic transposition as a process through which the action of transposing a knowledge to a didactic site takes place [2] and not as an analogy of what is wanted to understand. Specifically, Chevaliard refers to didactic transposition to the adaptation of mathematical knowledge to transform it into knowledge to be taught [3], however this transposition produces certain effects such as simplification and delegating, creation of artifacts or production of totally new objects [4]. The contributions of the didactic transposition are to allow a didactic intervention that prevents, in the limit of the possible, the formation of inadequate or even erroneous concepts, and on the other hand allows the teacher to recognize his / her own implicit conceptions as far as the mathematics is concerned [5] and from this arises the idea of didactic engineering [6].

A mathematical object, as a function, does not exist independently of the totality of its representations, but it should not be confused with any particular representation [7]. Such ideas are expressed by students in courses after differential calculus, evidencing the lack of conceptualization in this area of mathematics. An analogous definition of function widely used based on the didactic transposition is "the variable (dependent) $y$ it is a function of the variable (independent) $x$ if there is a rule by which to each value of $x$, belonging to a certain set of numbers, corresponds a defined value of $y$" [8]. Another reason why some people still call a function a formula, is because in the mid-nineteenth century, mathematicians understood only the term function as an analytical expression, which had its support given that they considered a graph discontinuous corresponded no algebraic expression [8].

Another of the elementary concepts during the course is the injective function that we remember below:

Definition 1.2. Let $f$ a function from $A$ to $B$. We say that $f$ is an injective function if and only if every element of its path is the image of a single element of its domain.

It is convenient to mention that the previous definition is commonly confused by students with the concept of function due to the uniqueness of the elements that are related. On the other hand, it is important to know the previous definition well since it allows to study the inverse concept of a function since for every injective function $f$ it is possible to define another function from it and that is called the inverse of $f$ and that is usually noticed by the symbol $f^{-1}$ and that satisfies the following equalities:

$$f \circ f^{-1} = I \quad y \quad f^{-1} \circ f = I,$$

Being $I$ the identity function. That is, the composition of the two functions results in the identity function, which means, intuitively, that by applying $f$ followed by $f^{-1}$ to every element the same element is obtained, in other words, it is invariant.
Definition 1.3. The graph of a function $f$ is the locus of all related points by $f$.

3. Results and discussion

With the continuous advance in the technological tools and their easy acquisition, there is evidence of greater use of said resources in the classroom, so much that nowadays the teaching mediation is articulated with strategies supported in the TIC, such as the use of video beam, videos, social networks, etc., however the new technologies will end up eroding the curriculum and will demand a new one, articulated intimately with said technologies [9], not only with respect to the physical resource but also to the different applications that exist. The problem does not lie in the articulation of these tools, but on the contrary of the shortcomings that these may have at the programming level, which generate errors that are accepted by the learners. When a student has understood the arithmetic, algebraic and / or logical process, technological tools become a support in their cognitive process, however, if the artifact is considered as the means of solving a given problem, for example in mathematics, the unthinking use of the device can introduce distortions in the processes of teaching and learning, then the computer or the calculator does not come to demobilize the cognitive activity, but to enhance its action in new areas to reorganize the cognitive functioning of the student [9].

An integrated class gives meaning and practical significance to computer science, as it seeks the integration of knowledge in the education of the learner [10]. The experience basically consisted in using the software Mathematica 10 [11, 12] during 5 classes of differential calculus to represent any function whose inverse is not possible to express analytically explicitly [13], as is the case of the transcendental function. injective: $f(x) = e^x + \sin x$. The result is the one shown below (Figure 1).

![Figure 1. Cartesian model of function $f(x) = e^x + \sin x$, through Wolfram Mathematica 10.](image_url)

The response of the students as expected was accepted as they assume that the software is "advanced" and its computing power for this type of task is enough. Subsequently, the result of implementing the code to generate the inverse was shown (see Figure 2). Of course, the answer was the same, although they were surprised by the shape of the resulting curve, which was awakening a certain type of skepticism until the geometric criteria were recalled obtaining the inverse of the function from the same function and of how non-injectivity affects the definition of an inverse function.
4. Conclusions

In conclusion, as in any application based on numerical methods, there is some uncertainty in the information generated and it is also subject to adequate interpretation by the user, which depends to a large extent on the background that has the concepts that support the design of the same. It is important to recognize that the use of technological tools to support the cognitive process in each person must be subject to cognitive flexibility in terms of seeing the student with his artifact, integrating the conceptual formality with the competence in the use of technological tools that the medium provides him continuously. The foregoing will demand in the mediator, a preparation not only in the theoretical part but also be digitally literate, to guarantee reflection spaces mediated by technological tools. Finally, it is convenient to propose the following questions:

Question 1. Is it possible to design an algorithm to make a representation of the inverse of a function since it is not injective in its entire domain?

Question 2. Is it possible that the use of different technological tools can generate inappropriate learning due to its limitation? How to identify this limitation? What to do in those cases?

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