Since January 2020 Elsevier has created a COVID-19 resource centre with free information in English and Mandarin on the novel coronavirus COVID-19. The COVID-19 resource centre is hosted on Elsevier Connect, the company's public news and information website.

Elsevier hereby grants permission to make all its COVID-19-related research that is available on the COVID-19 resource centre - including this research content - immediately available in PubMed Central and other publicly funded repositories, such as the WHO COVID database with rights for unrestricted research re-use and analyses in any form or by any means with acknowledgement of the original source. These permissions are granted for free by Elsevier for as long as the COVID-19 resource centre remains active.
A novel mathematical approach of COVID-19 with non-singular fractional derivative

Sachin Kumar, Jinde Cao, Mahmoud Abdel-Aty

Department of Mathematics, Govt. M.G.M PG College, Itarsi 461111, India
School of Mathematics, Southeast University, Nanjing, China
Department of Mathematics, Faculty of Sciences, Sohag University, Sohag 82524, Egypt

A R T I C L E   I N F O

Article history:
Received 11 June 2020
Revised 13 June 2020
Accepted 22 June 2020
Available online 1 July 2020

Keywords:
Fractional mathematical model
Fractional derivative with Mittag-Leffler kernel
COVID-19 virus
Spectral method

A B S T R A C T

We analyze a proposition which considers new mathematical model of COVID-19 based on fractional ordinary differential equation. A non-singular fractional derivative with Mittag-Leffler kernel has been used and the numerical approximation formula of fractional derivative of function $(t-a)^q$ is obtained. A new operational matrix of fractional differentiation on domain $[0, a]$, $a > 1$, $a \in \mathbb{N}$ by using the extended Legendre polynomial on larger domain has been developed. It is shown that the new mathematical model of COVID-19 can be solved using Legendre collocation method. Also, the accuracy and validity of our developed operational matrix have been tested. Finally, we provide numerical evidence and theoretical arguments that our new model can estimate the output of the exposed, infected and asymptotic carrier with higher fidelity than the previous models, thereby motivating the use of the presented model as a standard tool for examining the effect of contact rate and transmissibility multiple on number of infected cases are depicted with graphs.

1. Introduction

The fractional calculus is a classical branch which has been developed recently to deal with a new discovered problems (see, e.g. J. Liouville and N. H. Abel) [1]. Also, more information and detail description have been discussed in Kilbas et al. [2], Podlubny [3], Machado et al. [4]. Fractional calculus generalize the differentiation and integration of integer order to real or fractional order. Nowadays, this generalization is extended to the variable order and the differential equations and integral equation have real and variable orders have been discovered with a beautiful physical interpretation. Control theory and stochastic process have many applications of fractional order differential equation [5]. The researchers have found many type of fractional derivative such as Riemann-Liouville, Caputo, Riesz, Hadamard and Grunwald-Letnikov derivatives. The fractional differential equations (FDEs) are obtained from ordinary ones by replacing the integer order to real order. FDEs have many applications in science, engineering, biology, medical, finance, economics and groundwater flow [6,7].

The applications of FDEs are increasing day by day which leads to an urgent need to find the general solution of these FDEs. To find out the analytical solution of these complicated FDEs, one needs to invoke a numerical solution treatment. Large number of methods have been discovered to deal with FDEs and FPDEs. Some of them are eigen-vector expansion, homotopy perturbation method [8], Adomain decomposition method [9], predictor-corrector method [10], fractional differential transform method [11] and generalized block pulse operational matrix method [12] etc. The spectral method known as operational matrix method is very efficient method and easy to apply. It has a very desirable accuracy. Some operational matrix based upon Legendre wavelets [13], Chebyshev wavelets [14], sine wavelets, Haar wavelets [15] are available in literature. Operational matrix based upon orthogonal and nonorthogonal polynomial are given in Legendre polynomial [16], Laguerre polynomial [17], Chebyshev polynomial and semi-orthogonal polynomial as Genocchi polynomial [18].

The novel corona virus was emerged first time in December, 2019, in Wuhan city of China. The virus is a new type in its family. Later world health organization (WHO) named it COVID-19.

Due to this virus, any infected person faces many symptoms like as respiratory illness, cough, fever and difficulty in breathing [19,20]. This spreads when a healthy person comes in a contact with the virus carried out by a infected person specially contact with the drops of cough and sneeze of infected person. Some approximate solution of the time-fractional equations involving fractional integrals without singular kernel can be used to heed some
light on the expected time development [21–23]. WHO has declared it as a pandemic due to widely spread of this virus. Yet there is no medicine or vaccine to cure this virus infected people. Only precautions can be adopted to keep ourselves safe. Till the date 4, April, 2020, the number of confirmed COVID-19 infected cases is 1,118,045 and 59,201 are dead due to this. The effect of this virus is more on the people of age greater than 40. The only cure is our precautions, we should have to quarantine ourselves in our homes to decreases contact rate, transmissible multiple. The human kind has the power to change the environment around us. There are some boundaries that should not be violated. In this present era, the intention of competition between humans, countries has developed so many powerful instruments to control on sea, air and ground. The human has created so many weapons like guns, atom bomb, dangerous chemicals and nuclear bomb. So they violated the fundamental law of nature and led to so many natural disasters. We have forgotten that without nature we can not exist and we are just passenger. In this paper, we introduce a new and novel mathematical approach to study the behavior and dynamics of COVID-19 with a new non-singular fractional derivative called Mittag-Leffler kernel’s derivative. To solve the presented model, we use of a newly derived matrix with Legendre collocation method. We will present some numerical treatments based on the number of infected people increases with increment in contact rate.

The organization of this article is as follows. In Section 2, some preliminary definition of fractional derivative and ABC derivative are briefly discussed. The derivation of operational matrix of fractional differentiation based on orthogonal Legendre polynomial on interval [0, a] is derived in Section 3. The description of COVID-19 model, its related data and procedure of numerical solution are given in Section 4. The results and discussion are presented in Section 5 and the conclusion of all over article is given in Section 6.

2. Fractional calculus

The definition of fractional integration and differentiation are available in literature [23]. There are mainly two types i.e., Riemann-Liouville and Caputo [24,25]. In starting, fractional derivatives with power law kernel are introduced. In recent years, many fractional derivative definitions with non-singular kernel are introduced as exponential kernel and Mittag-Leffler kernel.

2.1. Definitions of fractional differentiation and integration with power law kernel

Definition 1. We define the fractional integration of $\Xi(x)$ of order $\beta$

$$I_0^{\beta} \Xi(x) = \frac{1}{\Gamma(\beta)} \int_0^x (x - \omega)^{\beta - 1} \Xi(\omega) d\omega, \quad x > 0, \quad \beta \in \mathbb{R}^+.$$  

The definition of Riemann-Liouville integration is given as follows

$$D_0^{\beta}_{\gamma} \Xi(x) = \left( \frac{d}{dx} \right)^n (I_0^{\gamma - n} \Xi)(x), \quad (\gamma > 0, \quad n - 1 < \gamma < n).$$  

Definition 2. The definition of fractional differentiation with power law kernel is given in literature as follows

$$D_0^{\beta}_{\gamma} \Xi(x) \begin{cases} \frac{d^n \Xi(x)}{dx^n} \prod_{z=0}^{n-1} (x - \zeta), \quad \gamma = n \in \mathbb{N} \quad \zeta < N \cup 0 & \quad \zeta < \lfloor \gamma \rfloor \land \zeta \geq \lceil \gamma \rceil, \\ \frac{d^n \Xi(x)}{dx^n} \prod_{z=0}^{n-1} (x - \zeta), \quad \gamma = n - 1 < \gamma < n. \end{cases}$$

With $z \in [0, \infty]$ and $n$ is an integer. The Caputo definition has a similarity with integer derivative that is

$$D_0^{\beta}_{\gamma} M = 0,$$  

with $M$ is a constant.

$$D_0^{\beta}_{\gamma} \Xi(x) = \begin{cases} 0, \quad \gamma \in N \cup 0 \land \gamma < \lfloor \gamma \rfloor \land \gamma \geq \lceil \gamma \rceil, \\ \frac{d^n \Xi(x)}{dx^n} \prod_{z=0}^{n-1} (x - \zeta), \quad \gamma \in N \cup 0 \land \gamma \geq \lceil \gamma \rceil, \\ \gamma \notin N \land \gamma > \lfloor \gamma \rfloor. \end{cases}$$  

where $\lfloor \gamma \rfloor$ is floor function. All fractional operator are linear in nature as they follow the linearity property

$$D_0^{\beta}_{\gamma}(M_1 \Xi_1(t) + M_2 \Xi_2(t)) = M_1 D_0^{\beta}_{\gamma} \Xi_1(t) + M_2 D_0^{\beta}_{\gamma} \Xi_2(t),$$

with $M_1$ and $M_2$ are constants.

The Caputo and Riemann-Liouville operator can be relate by the following expression

$$D_0^{\beta}_{\gamma} \Xi(z) = \Xi(z) - \sum_{j=0}^{l - 1} \Xi(0^+) \frac{z^j}{j!}, \quad l - 1 < \gamma \leq l.$$  

2.2. Definition of fractional derivative with Mittag-Leffler kernel [26–28]

Let a function $\Xi(x, t)$ belongs to the Sobolev space $H^0(0, 1)$. Then this fractional derivative with Mittag-Leffler kernel which is also known as ABC fractional derivative can be defined as

$$A^{\beta}_{\gamma} D_0^{\beta}_{\gamma} \Xi(x, t) = \frac{B(\gamma)}{n - \gamma} \int_0^t \frac{\partial^n \Xi(w, t)}{\partial w^n} E_\gamma \left( \left\lfloor -\gamma \right\rfloor (t - w)^\gamma \right) dw, \quad n - 1 < \gamma \leq n.$$  

The function $B(\gamma)$ is normalization function which follow the property $B(0) = B(1) = 1$ and $E_\gamma(x)$ is Mittag-Leffler function defined as follows

$$E_\gamma(x) = \sum_{j=0}^{\infty} \frac{x^j}{\Gamma(j\gamma + 1)}.$$  

3. Derivation of operational matrix of fractional differentiation having Mittag-Leffler kernel

3.1. Fractional derivative of $t^l$

In this section we derive the Legendre operational matrix of fractional differentiation on domain $[0, a], a > 1, a \in N$ of Mittag-Leffler kernel derivative which is known as ABC derivative.

**Theorem 1.** The value of ABC fractional differentiation of order $n - 1 < \rho < n$ of function $f(y) = (y - b)^l$ with $l \geq \lfloor \rho \rfloor$ can be found by the following approximation formula

$$A^{\beta}_{\gamma} D_0^{\beta}_{\gamma} (y - b)^l = \frac{B(\rho)}{n - \rho} \times y^l \frac{\partial^{l - n} E_\rho_{1, l - n} (\frac{-\rho}{n - \rho} y^n)}{\partial y^n}.$$  

**Proof.** From the definition (9) $D_0^{\beta}_{\gamma} y = 0, i = 0, 1, \ldots, n - 1$ and for the condition $l \geq \lfloor \rho \rfloor$ we have

$$A^{\beta}_{\gamma} D_0^{\beta}_{\gamma} (y - b)^l \begin{cases} \frac{B(\rho)}{n - \rho} \int_0^y D_\rho (p - b) \frac{\partial^{l - n} E_\rho_{1, l - n} (\frac{-\rho}{n - \rho} (y - p)^n)}{\partial y^n} dp, \\ \frac{B(\rho)}{n - \rho} \int_0^1 \frac{\partial^{l - n} E_\rho_{1, l - n} (\frac{-\rho}{n - \rho} (y - p)^n)}{\partial y^n} dp. \end{cases}$$

Now, we use the series expansion formula of Mittag-Leffler function and evaluate the above integral as follows
\[ \frac{D^\mu}{\partial y^\mu}(y-b)^l = \frac{B(\rho)}{n-\rho} \int_0^y p^{l-n} E_{\rho} \left( \frac{-\rho}{n-\rho} (y-p)^n \right) dp, \]
\[ = \frac{B(\rho)}{n-\rho} \times \frac{\Gamma(1+l)}{\Gamma(1+l-n)} \int_0^y p^{l-n} \sum_{j=0}^{\infty} \frac{1}{\Gamma(j+\rho+1)} \left( \frac{-\rho}{n-\rho} \right)^j (y-p)^j dp, \]
\[ = \frac{B(\rho)}{n-\rho} \times \frac{\Gamma(1+l)}{\Gamma(1+l-n)} \sum_{j=0}^{\infty} \frac{1}{\Gamma(j+\rho+1)} \left( \frac{-\rho}{n-\rho} \right)^j \frac{\Gamma(1+j+\rho)}{\Gamma(2+j+\rho+l-n)} y^{l+j-n+1}, \]

\[ \frac{D^\mu}{\partial y^\mu}(y-b)^l = \frac{B(\rho)}{n-\rho} \times \frac{\Gamma(1+l)}{\Gamma(1+l-n)} \sum_{j=0}^{\infty} \frac{1}{\Gamma(j+\rho+1)} \left( \frac{-\rho}{n-\rho} \right)^j \frac{\Gamma(1+j+\rho)}{\Gamma(2+j+\rho+l-n)} y^{l+j-n+1}, \]

This is the desired approximation expression for ABC derivative of function \((y-b)^l\). \qed

3.2. Extended Legendre polynomials on interval \([0, a]\)

Here, we give the brief definition and property of extended Legendre polynomial. We know that the Legendre polynomial are orthogonal polynomial defined on interval \([-1, 1]\). By using the transformation \(z = \frac{2y-a}{a}\), we transform the Legendre polynomial from the interval \([-1, 1]\) to the interval \([0, a]\). The series form of this polynomial is given in the following expression
\[ \Psi_i(y) = \frac{a^-}{a} \int_0^a \left( z \right)^i \left( \frac{a}{y} \right)^{\frac{a}{i} - 1} dp. \]

With the help of these extended Legendre polynomial a function \(\chi(y)\) belonging to \(L^2[0, a]\) can be written as a finite linear combination as
\[ \chi(y) = \chi_N(y) = \sum_{i=0}^{n} c_i \Psi_i(y). \]

The coefficient \(c_i\) are determined as follows with the help of orthogonality condition
\[ c_i = \int_0^a \chi(y) \Psi_i(y) dy = A^T \Pi_N(y). \]

where
\[ A^T = (a_0, a_1, \ldots, a_{n-1}), \]
\[ \Pi_N(x) = (\psi_0(x), \psi_1(x), \ldots, \psi_{n-1}(x)). \]

Now in next theorem we will develop the Legendre operational matrix of fractional differentiation on the interval \([0, a]\) with the help of Eq. (13).

**Theorem 2.** If we denote the column vector of extended Legendre polynomial by \(\Pi_N(y)\) then fractional differentiation of order \(n-1 < \gamma < n\) is given by the formula,
\[ \frac{D^\mu}{\partial y^\mu} \Pi_N(y) = R^\gamma \Pi_N(y). \]

Here \(R^\gamma\) represents the operational matrix of fractional differentiation of order \(N \times N\). We can obtain this as the following
\[ R^\gamma = \begin{bmatrix} 0 & 0 & \cdots & 0 \\
0 & 0 & \cdots & 0 \\
\vdots & \vdots & \ddots & \vdots \\
0 & 0 & \cdots & 0 \\
\end{bmatrix} + \sum_{i=0}^{l} \sum_{j=0}^{n} \frac{\Gamma(1+j+\rho)}{\Gamma(2+j+\rho+1-n)} y^{l+j-n+1}, \]

where \( \xi_{ij} \) is defined by the following expression
\[ \xi_{ij} = \left( \frac{l}{i} \right) \left( \frac{l+j}{i+j} \right) \frac{1}{\Gamma(l+\rho)} \frac{1}{\Gamma(l+j+\rho)} \frac{1}{\Gamma(l+j+\rho+1-n)} y^{l+j-n+1}. \]

Here range of \(i\) and \(j\) varies from \(i = [0] \ldots, N-1\) and \(j = 0, 1, \ldots, N-1\) respectively.

**Proof.** Using the value of \(\frac{D^\mu}{\partial y^\mu}(y-b)^l\) from the Theorem 1
\[ \frac{D^\mu}{\partial y^\mu}(y-b)^l = \frac{B(\rho)}{n-\rho} \times y^{l+1-n} E_{\rho,1+1-n} \left( \frac{-\rho}{n-\rho} y^\rho \right). \]

Taking the help from the series expansion of legendre polynomial and the definition of ABC derivative
\[ \frac{D^\mu}{\partial y^\mu} \Psi(y) = \sum_{i=0}^{l} \left( \frac{l}{i} \right) \left( \frac{l+j}{i+j} \right) \frac{1}{\Gamma(l+\rho)} \frac{1}{\Gamma(l+j+\rho)} \frac{1}{\Gamma(l+j+\rho+1-n)} y^{l+j-n+1}. \]

To find out the \((i, j)\)th element \(\alpha_{ij}\) of operational matrix \(R^\gamma\), we perform the inner product as follows
\[ \frac{D^\mu}{\partial y^\mu} \Psi_i(y) = \sum_{j=0}^{m-1} \alpha_{ij} \Psi_j(y). \]

where
\[ \alpha_{ij} = \left( \frac{D^\mu}{\partial y^\mu} \Psi_i(y), \Psi_j(y) \right). \]

\[ = \sum_{i=0}^{l} \left( \frac{l}{i} \right) \left( \frac{l+j}{i+j} \right) \frac{1}{\Gamma(l+\rho)} \frac{1}{\Gamma(l+j+\rho)} \frac{1}{\Gamma(l+j+\rho+1-n)} y^{l+j-n+1} \]
By using the orthogonal property of Legendre polynomial we determined the above inner products value as follows

\[
(y^{1+i-n}E_{ρ,1+i-n}(−ρ/n − ρ)y^j, ψ_j(y)) = \frac{(2j + 1)}{b} \int_0^b y^{1+i-n}E_{ρ,1+i-n}(−ρ/n − ρ)y^j(y)dy, \quad k = 0, 1, \ldots, M
\]

\[
= \frac{(2j + 1)}{b} \int_0^b y^{1+i-n}E_{ρ,1+i-n}(−ρ/n − ρ)y^j(y)dy = \frac{(2j + 1)}{b} \sum_{q=0}^j \binom{j + q + j}{q} \left(\frac{q}{b}\right) (y - b)^qdy
\]

(17)

where

\[
M_{ρ,1}(y) = y^{1+i-n}E_{ρ,1+i-n}(−ρ/n − ρ)y(y - b)^q.
\]

We use a numerical integration scheme known as Simpson 1/3 rule and the interval of integration [0, b] is divided into m equal sub parts with length of segment width h. Putting the value of both of inner product in Eq. (16) we obtained the following expression of \(a_{ij}\)

\[
a_{ij} = \sum_{l=0}^{m} \binom{l}{i} \left(\frac{1}{l}\right) \frac{1}{b} \times \left(\frac{2j + 1)}{b} \sum_{q=0}^j \binom{j + q + j}{q} \left(\frac{q}{b}\right) \right) \left[\frac{h}{3} - \frac{M_{ρ,1}(p_0) + M_{ρ,1}(p_m)}{2} + \frac{4[M_{ρ,1}(p_1) + M_{ρ,1}(p_3) + \ldots + M_{ρ,1}(p_{m-1})]}{3} + 2[M_{ρ,1}(p_2) + M_{ρ,1}(p_4) + \ldots + M_{ρ,1}(p_{m-2})]\right]
\]

(18)

Assuming \(a_{ij} = \sum_{l=0}^{m} \xi_{i,j,l}\) we get the final desired result

\[
\xi_{i,j,l} = \binom{j + q + j}{q} \left(\frac{q}{b}\right) \times \left(\frac{2j + 1)}{b} \sum_{q=0}^j \binom{j + q + j}{q} \left(\frac{q}{b}\right)\right) \left[\frac{h}{3} - \frac{M_{ρ,1}(p_0) + M_{ρ,1}(p_m)}{2} + \frac{4[M_{ρ,1}(p_1) + M_{ρ,1}(p_3) + \ldots + M_{ρ,1}(p_{m-1})]}{3} + 2[M_{ρ,1}(p_2) + M_{ρ,1}(p_4) + \ldots + M_{ρ,1}(p_{m-2})]\right]
\]

We have derived the operational matrix of differentiation for fractional order on domain [0, b]. But for the integer order elements of operational matrix is obtained as follows

\[
ρ_{i,j} = \begin{cases} \frac{ζ_j}{j} & \text{if } j = i - l, \\ 0 & \text{otherwise}, \end{cases}
\]

(19)

where \(l = 1, 3, \ldots, m\) if \(m\) is odd and \(l = 1, 3, \ldots, m - 1\) if \(m\) is even. The function \(ζ_j\) is defined as

\[
ζ_j = 2 \times \frac{(2j + 1)}{b}.
\]

\[
\square
\]

4. COVID-19 mathematical model, used parameters and taken data

To study the measurement-induced by COVID-19 transition due to the time development with different parameters and data, we performed the time-evolution using exact diagonalization. In this section we focus on cases dynamics interspersed with fractional order measurements in the two-dimension basis, and demonstrate the fact that there is a qualitative difference once initial values are introduced differently. We suppose that \(N_p\) denotes the total population of people. This is divided into 5 categories as

(i) \(S_p\) - Susceptible people.

(ii) \(E_p\) - Exposed people

(iii) \(I_p\) - Infected people

(iv) \(A_p\) - Asymptotically infected people

(v) \(R_p\) - Recovered people

and \(N_p = S_p + E_p + I_p + A_p + R_p\). The parameter \(P_p\) denotes the birth rate of people and the parameter \(μ_p\) represent the death rate of people in each case. The term \(η_p S_p P_p\) represents the susceptible people will infected with a sufficient contact with infected people \(I_p\). Here, \(η_p\) is disease transmissibility coefficient. And the term \(ψ_0 S_p P_p\) denotes that susceptible people will be infected from asymptotically infected people with \(ψ_0\) transmissibility multiple of \(A_p\), and \(ψ_1\) to \(I_p\), and values of \(ψ\) belong to the closed interval [0,1]. The \(ψ = 0\) states no transmissibility and \(ψ = 1\) then this contact with asymptotically infected people will be treated as contact with infected people. The rate of susceptible people, from which they join the class of infected and asymptomatic class are denoted by \(ω_p\) and \(q_p\) respectively. The removal and recovery rate from the class \(I_p\) and \(A_p\) to the class \(R_p\) are depicted by the parameters \(τ_p\) and \(τ_{ap}\) respectively. The unknown function \(M\) is related to the seafood market or reservoir. The parameter \(η_w\) is disease transmission coefficient from \(M\) to \(S_p\) with term \(η_w S_p M\). The contribution of virus from asymptotically infected and symptomatic infected to the reservoir is denoted by \(q_p\) and \(ω_p\). The parameter \(λ\) denotes the removing rate of virus from reservoir. We present the model as follows [29].

\[
ABCD_0^γ^i S_p(t) = -\frac{η_p(ψ_0 A_p + I_p) S_p}{N_p} - μ_p S_p - η_w S_p M + P_p,
\]

(20)

The prescribed initial conditions for the above model are

\[
S_p(0) = s_0 ≥ 0, \quad I_p(0) = i_0 ≥ 0, \quad E_p(0) = e_0 ≥ 0,
\]

(21)

The value of used parameters are taken from the literature [29] (Table 1).

The value of initial conditions are taken as follows

\[
s_0 = 8065518, \quad i_0 = 282, \quad e_0 = 2000000, \quad a_0 = 200, \quad r_0 = 0, \quad m_0 = 500000.
\]

We have derived the Legendre operational matrix of fractional differentiation on domain [0, a]. Now we will use this newly operational matrix to find the solution of COVID-19 model. So with the help of Eq. (12) we will approximate the unknown functions present in our model as follows
functions

\[ S_p(t) = \sum_{j=0}^{N-1} S_j(t) = S^T N_i P_i N_i = E_p(t) = \sum_{j=0}^{N-1} E_j(t) = E^T P_i N_i. \]

\[ I_p(t) = \sum_{j=0}^{N-1} I_j(t) = I^T P_i N_i, \]

\[ R_p(t) = \sum_{j=0}^{N-1} R_j(t) = R^T P_i N_i. \]

Table 1

| Used parameters | Description of parameters | Numerical value |
|-----------------|---------------------------|----------------|
| \( \Lambda \)   | Rate of removing virus from reservoir | 0.01 |
| \( \omega_p \)   | Contribution of virus from \( A_p \) to \( M \) | 0.001 |
| \( \nu_p \)   | Contribution of virus from \( I_p \) to \( M \) | 0.000398 |
| \( \tau_p \)   | Recovery rate of \( I_p \) | 0.09871 |
| \( \tau_{sp} \) | Recovery rate of \( A_p \) | 0.0854302 |
| \( \rho_p \)   | Incubation period | 0.005 |
| \( \phi_p \)   | Disease transmission coefficient | 0.0047876 |
| \( \psi \)   | Transmissibility multiple | 0.000001231 |
| \( \eta_p \)   | Contact rate | 0.02 |
| \( \mu_p \)   | Birth rate | 0.05 |
| \( \mu \)   | Death rate | 8,266,000 |
| \( N_p(0) \) | Initial population of city | 0.02 |

where \( S = [S_j(t)]_{j=0}^{N-1} \), \( E = [E_j(t)]_{j=0}^{N-1} \), \( A = [A_j(t)]_{j=0}^{N-1} \), \( M = [M_j(t)]_{j=0}^{N-1} \), \( I = [I_j(t)]_{j=0}^{N-1} \) is an row vector of unknowns and \( \Pi(x) = \left( \psi_0(x), \psi_1(x), \ldots, \psi_{N-1}(x) \right)^T \) is a column vector. Now for approximating the left hand parts of all equations presented in model (19) we are operating the fractional operator of derivative on these sides and using Eq. (23)

\[ s = \frac{\partial}{\partial t} S_p(t) = S^T R^\sigma \Pi N_i, \]

\[ \frac{\partial^\gamma}{\partial t^\gamma} E_p(t) = E^T R^\sigma \Pi N_i, \]

\[ \frac{\partial^\gamma}{\partial t^\gamma} A_p(t) = A^T R^\sigma \Pi N_i, \]

\[ \frac{\partial^\gamma}{\partial t^\gamma} M(t) = M^T R^\sigma \Pi N_i. \]

To find the solution we approximate the initial conditions by taking help of Eq. (12)

\[ S^T \Pi N_i = s_0 \geq 0, I^T \Pi N_i = i_0 \geq 0, E^T \Pi N_i = e_0 \geq 0, A^T \Pi N_i = a_0 \geq 0, R^T \Pi N_i = r_0 \geq 0, M^T \Pi N_i = m_0 \geq 0. \]

Now using the Eqs. (24) and (25) in our model we get the residual functions as

\[ S^T \Pi N_i = S_p(t) = \sum_{j=0}^{N-1} S_j(t) = S^T N_i P_i N_i, \]

\[ I^T \Pi N_i = I_p(t) = \sum_{j=0}^{N-1} I_j(t) = I^T P_i N_i, \]

\[ E^T \Pi N_i = E_p(t) = \sum_{j=0}^{N-1} E_j(t) = E^T P_i N_i, \]

\[ A^T \Pi N_i = A_p(t) = \sum_{j=0}^{N-1} A_j(t) = A^T P_i N_i, \]

\[ R^T \Pi N_i = R_p(t) = \sum_{j=0}^{N-1} R_j(t) = R^T P_i N_i, \]

\[ M^T \Pi N_i = M(t) = \sum_{j=0}^{N-1} M_j(t) = M^T P_i N_i. \]

5. Results and discussion

We study the dynamics of susceptible, exposed, infected, and asymptotically infected people using different fractional order. Fig. 1(a) shows the behavior between susceptible people versus time and Fig. 1(b) indicates the relation between exposed people versus time. We see in Fig. 1(b) that number of exposed people increases with time. We observe that this growth increases as
we increases the fractional order $y_2$ from 0.7 to 1 while Fig. 1(a) shows that number of susceptible people decreases with time because they are getting into exposed or infected class. Fig. 2(a) and (b) are plotted between infected people $I_p(t)$ versus time and asymptotically infected people versus time respectfully. Fig. 2(a) predicts that number of infected people will increase with time. Similar nature is also seen for the asymptotically infected people. The effect of fractional exponent on $I_p(t)$ and $A_p(t)$ is that it increases with increment in $y_3$ and $y_4$ from 0.7 to 1. As COVID-19 spread with social contacting, touch with infected people and infected surfaces. We see that number of infected people increase exponentially with time. This behavior can be seen by taking data of Italy and USA till today 4 April, 2020 as there infected people are increasing followed by this exponentially behavior.

To study the behavior of this virus with contacting to infected or asymptotically infected people, we plotted the graph between the infected people $I_p(t)$ versus time and asymptotically infected people $A_p(t)$ versus time. We see in Fig. 3(a) that number of
asymptotically infected people increases with time. And an important fact can be seen that it increases as contact rate \( \eta_p \) increases.

Fig. 3(b) shows that number of infected people increases rapidly like exponentially behavior and this number increases as contact rate increases.

Fig. 4(a) and (b) show that the behavior of \( I_p(t) \) and \( A_p(t) \) with transmissibility condition \( \psi \). We can observe that number of infected people and asymptotically infected people increases with increment in \( \psi \). To study the behavior of model with different initial condition, we plotted two graphs Fig. 5(a) and (b).

Fig. 5(a) is plotted between infected people versus time with different \( A_p(0) \). We see as initial number of \( A_p(t) \) increases the number of infected people also increases. Similarly, from Fig. 5(b), if at initially stage number of infected people is more than number of asymptotically infected people increases with this initially increment of infected people.

6. Conclusion

Measurement-induced COVID-19 transitions represent an interesting new class of phase transition which shine light on the resilience of the present kind of viruses against a known one. They were initially explored for systems at ordinary differential equations dynamics and integrable models. In this work we have demonstrated that the nature of the measurement induced COVID-19 transition can be well described by newly fractional calculus systems. The measurements have been made in a basis which is scrambled by different parameters and new controllers of the dynamics, then the transition from infected case to recovered case occurs at a nonzero measurement probability and can be controlled by changing the significant parameter, generalizing the previously studied chaotic systems. It is worth noting that one key difference between the model considered in this paper and previous models is that there are more variables (spatial) disorder in the unitary part of the dynamics. This is noteworthy because we could derive an approximation formula for the fractional derivative of ABC type of function \( f(a) \) on domain \([0; a]; a \geq 1; a \in N:\) for the first time (as far as we know) and have developed the operational matrix of fractional differentiation with Mittag-Leffler kernel. The use of this newly derived matrix with Legendre collocation method is applied to solve a system of fractional ordinary differential equation. We find out the dynamics of susceptible, exposed, infected and asymptotically infected people, that how is behave with different fractional fractional order. It is shown that the number of infected people increases with increment in contact rate. So if we want to stop this outbreak pandemic we should be quaran-
tine to reduce the contact rate. The effect of transmissibility multiple is shown by graphical representation. Effect on number on infected people and asymptotically infected people with different initial conditions.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

CRediT authorship contribution statement

Sachin Kumar: Conceptualization, Methodology, Software, Data curation, Writing - original draft. Jinde Cao: Visualization, Investigation, Supervision. Mahmoud Abdel-Aty: Writing - review & editing.

References

[1] Gorenflo R, Mainardi F. Fractional calculus in continuum mechanics. Springer; 1997. p. 223–76.

[2] Kilbas A, Srivastava H, Trujillo JJ. Theory and applications of the fractional differential equations, 204. Elsevier (North- Holland; Amsterdam Amsterdam; 2006.

[3] Podlubny I. Fractional differential equations, to methods of their solution and some of their applications. Fractional differential equations: an introduction to fractional derivatives. San Diego, CA: Academic Press; 1998.

[4] Machado JT, Kiryakova V, Mainardi F. Recent history of fractional calculus. Commun Nonlinear Sci Numer Simul 2011;16(3):1140–53.

[5] Matsuo R. Application of fractional order calculus to control theory. Int J Math Models Methods Appl Sci 2011;5(7):1162–9.

[6] Atangana A. Derivative with two fractional orders: a new avenue of investigation toward revolution in fractional calculus. Eur Phys J Plus 2016;131(10):373.

[7] Kumar S, Atangana A. A numerical study of the nonlinear fractional mathematical model of tumor cells in presence of chemotherapeutic treatment. Int J Biomath 2020;13(01):2050021.

[8] Hashim I, Abdulaziz O, Momani S. Homotopy analysis method for fractional IVPs. Commun Nonlinear Sci Numer Simul 2009;14(3):674–84.

[9] Suarez L, Shokooh A. An eigenvector expansion method for the solution of motion containing fractional derivatives. ASME, J. Appl. Mech. 1997;64(3):629–35, doi:10.1115/1.2788939.

[10] Dietheilm K, Ford NJ, Freed AD. A predictor-corrector approach for the numerical solution of fractional differential equations. Nonlinear Dyn 2002;29(1–4):3–22.

[11] Darania P, Ebadian A. A method for the numerical solution of the integro-differential equations. Appl Math Comput 2007;188:657–68.

[12] Li Y, Sun N. Numerical solution of fractional differential equations using the generalized block pulse operational matrix. Comput Math Appl 2011;62(3):1046–54.

[13] Jafari H, Yousefi S, Firoozjaee M, Momani S, Khalique CM. Application of Legendre wavelets for solving fractional differential equations. Comput Math Appl 2011;62(3):1038–45.

[14] Yuanlu L. Solving a nonlinear fractional differential equation using Chebyshev wavelets. Commun Nonlinear Sci Numer Simul 2010;15(9):2284–92.
[15] Li Y, Zhao W. Haar wavelet operational matrix of fractional order integration and its applications in solving the fractional order differential equations. Appl Math Comput 2010;216(8):2276–85.

[16] Odibat Z. On Legendre polynomial approximation with the vim or ham for numerical treatment of nonlinear fractional differential equations. J Comput Appl Math 2011;235(9):2956–68.

[17] Gürbüz B, Sezer M. Laguerre polynomial solutions of a class of initial and boundary value problems arising in science and engineering fields. Acta Phys Pol A 2016;130(1):194–7.

[18] Araci S. Novel identities for Genocchi numbers and polynomials. J Funct Spaces Appl 2012;2012:13 pages.

[19] Alnaser WE, Abdel-Aty M, Al-Ubaydli O. Mathematical prospective of coronavirus infections in Bahrain, Saudi Arabia and Egypt. Inf Sci Lett 2020;9(2):51–64.

[20] Atangana A. Modelling the spread of COVID-19 with new fractal-fractional operators: can the lockdown save mankind before vaccination? Chaos Solitons Fractals 2020;136:109860.

[21] Khan MF. Some new hypergeometric transformations via fractional calculus technique. Appl Math Inf Sci 2020;14(2):177–90.

[22] Srivastava HM, Saad K. New approximate solution of the time-fractional Nagumo equation involving fractional integrals without singular kernel. Appl Math Inf Sci 2020;14(1):1–8.

[23] Chatzarakis GE, Deepa M, Nagajothi N, Sadhasivam V. Oscillatory properties of a certain class of mixed fractional differential equations. Appl Math Inf Sci 2020;14(1):141–9.

[24] Cernea A. Continuous family of solutions for fractional integro-differential inclusions of Caputo-Katugampola type. Progr Fract Differ Appl 2020;5(1):37–42.

[25] Mouzakis DE, Lazopoulos AK. Fractional modelling and the Leibniz (L-fractional) derivative as viscoelastic respondents in polymer biomaterials. Progr Fract Differ Appl 2020;5(1):43–8.

[26] Atangana A, Koca I. Chaos in a simple nonlinear system with Atangana–Baleanu derivatives with fractional order. Chaos Solitons Fractals 2016;89:447–54.

[27] Atangana A, Gómez-Aguilar J. Numerical approximation of Riemann-Liouville definition of fractional derivative: from Riemann-Liouville to Atangana-Baleanu. Numer Methods Partial Differ Equ 2018;34(5):1502–23.

[28] Atangana A, Khan MA. Validity of fractal derivative to capturing chaotic attractors. Chaos Solitons Fractals 2019;126:50–9.

[29] Khan MA, Atangana A. Modeling the dynamics of novel coronavirus (2019-nCoV) with fractional derivative. Alexandria Eng J 2020 in press.