Electroweak penguins and SUSY $K^0$-$\bar{K}^0$ mixing with Neuberger quarks

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We present results for $\Delta I=3/2$ and $\Delta S=2$ matrix elements relevant for CP violation in $K \to \pi\pi$ decays and for the $K_S$-$K_L$ mass difference in the standard model and beyond. They were obtained with Neuberger fermions on quenched gauge configurations generated with the Wilson plaquette action at $\beta=6.0$ on an $18^3 \times 64$ lattice.

1. Introduction

CP violation in $K \to \pi\pi$ decays and the $K_S$-$K_L$ mass difference provide important constraints on the standard model (SM) and extensions, once the relevant hadronic matrix elements are accurately computed. In the following we present results for the chirally leading $\Delta I=3/2$ contributions to direct CP violation, and for the $\Delta S=2$ matrix elements which contribute to $K^0$-$\bar{K}^0$ mixing in SM extensions. These are obtained using the same Neuberger propagators as for the spectroscopy work presented in [1], where details of the simulation are given. We work with degenerate $u$, $d$, and $s$ quarks of bare masses $am_q = 0.03, 0.04, 0.06, 0.08, 0.10$. The chiral symmetry of Neuberger fermions guarantees that the mixing on the lattice is identical to that in the continuum and that our results are fully $O(a)$-improved.

2. Electroweak penguins

At leading chiral order, the $\Delta I=3/2$ contribution to direct CP violation in $K \to \pi\pi$ is determined by the matrix elements $\langle (\pi\pi)_{I=2}|Q_{7,8}|K^0\rangle$, of the electroweak penguin operators $Q_{7,8}$. Soft pion theorems can be used to reduce one of the pions, so that

$$\langle (\pi\pi)_{I=2}|Q_{7,8}|K^0\rangle \propto \frac{1}{F_{\pi}} \langle \pi^+|Q_{7,8}^{3/2}|K^+\rangle$$  \hspace{1cm} (1)

in the chiral limit, where $F_{\pi}$ is the chiral limit value of the pion decay constant ($F_{\pi} = 92\text{ MeV}$) and where $Q_{7,8}^{3/2}$ are the $\Delta I=3/2$ components of $Q_{7,8}$, given by

$$Q_{7,8}^{3/2} = \frac{1}{2} [ (\bar{s}d)_{V-A}(\bar{u}d)_{V+A} + (\bar{s}u)_{V-A}(\bar{d}u)_{V+A} ]$$ \hspace{1cm} (2)

with $Q_{7,8}^{3/2}$ color diagonal and $Q_{8}^{3/2}$ color mixed. Thus, we calculate $\langle \pi^+|Q_{7,8}^{3/2}|K^+\rangle$ on the lattice using Neuberger fermions at finite quark mass and extrapolate the results to the chiral limit.

Because we work in the isospin limit, eye contractions cancel and lower dimensional operators are absent. This greatly simplifies the calculation. We begin by constructing the following ratios of correlation functions ($a = 1$):

$$B_{Ia,\Gamma}^{7,8} \equiv \frac{N_{7,8} \sum_{x_i,x_j} \langle J_{Ia}^{7,8}(x_i)Q_{7,8}^{3/2}(0)J_{Ia}^{7,8}(x_i) \rangle}{\sum_{x_j} \langle J_{Ia}^{7,8}(x_j)J_{Ia}^{7,8}(0) \rangle \sum_{x_i} \langle J_{Ia}^{7,8}(0)J_{Ia}^{7,8}(x_i) \rangle}$$ \hspace{1cm} (3)

\[ T \gg t, \frac{3}{T} \gg t_j \gg 1 \quad \rightarrow \quad B_{7,8}^{3/2}, \]

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\[ \text{The proportionality constant is convention dependent as is the overall phase of } \langle \pi^+|Q_{7,8}^{3/2}|K^+\rangle. \]
with $N_f=3$ and $N_s=1$. $B_{7/8}^{3/2}$ measures deviation from the VSA of $(\pi^+|Q_{7/8}^3|K^+)$ in the chiral limit.

In Eq. (3), $J_{\bar{q}q}^{\Gamma}=\bar{q}\gamma\Gamma q$ with $\hat{\gamma}=1-(1-aD/2p)q$. Similarly, $Q_{7/8}^{3/2}$ are given by Eq. (2), with quark fields $q$ replaced by $\hat{q}$ and with the appropriate rewriting in terms of Euclidean Dirac matrices.

To quantify possible unwanted contributions from finite-volume zero modes, we vary the source and sink by letting $\Gamma_{i,f}$ be one of $\gamma_5$, $\gamma_0\gamma_5$, $(1\pm\gamma_5)$ or $\gamma_0(1\pm\gamma_5)$. As shown in Fig. 1 for our next to lightest quark mass, $am_q=0.04$, which corresponds roughly to $m_q/2$, we see no dependence of $B_{7/8}^{3/2}$ on source and sink in the fit region. $B_{7/8}^{8}$ displays very similar behavior. We interpret this as meaning that zero-mode effects are negligible around the kaon mass. The fit region, $43 \leq t_i \leq 51$ and $13 \leq t_f \leq 21$, is chosen such that the ratios of Eq. (3) are asymptotic and free from wrap-around contributions. $B_{7/8}^{3/2}$ yield the results for $B_{7/8}^{3/2, bare}$ with the smallest error bars and we take these as the starting point for subsequent analysis.

Before proceeding with the chiral extrapolation of the matrix elements, we have chosen to renormalize them. We do so non-perturbatively in the RI/MOM scheme following [2]. $Q_7$ and $Q_8$ mix under renormalization. We fix gluon configurations to Landau gauge and numerically compute the relevant amputated forward quark q Green functions with legs of momentum $p = \sqrt{p^2}$, $\Lambda_{Q_{7,8}}(m_q, p^2)$. Then we determine the renormalization constants by requiring that the renormalized vertex functions have their tree-level values. Thus, we define the ratio:

$$R_{ij}^{RI}(m_q, p^2) \equiv Z_A^2 \frac{\text{Tr} \{ \Lambda_V(m_q, p^2)P_V \}^2}{\text{Tr} \{ \Lambda_{Q_i}(m_q, p^2)P_{Q_i} \}},$$

(4)

where $i,j \in \{7,8\}$ and the $P_O$ are normalized projectors onto the spin-color structure of tree-level $O = Q_{7,8}, V$. We extrapolate these ratios to $m_q = 0$ and fit the results to the OPE form (including discretization error terms) [3]:

$$R_{ij}^{RI}(0, p^2) = \cdots + \frac{A_{ij}}{p^2} + \frac{B_{ik}}{(p^2)} U_{ik}^{RI}(p^2) Z_{k,j}^{RGI} + B_{ij}(ap)^2 + \cdots,$$

(5)

where $U_{ik}^{RI}(p^2)$ describes the running of the renormalization constants in the RI/MOM scheme, $Z_{ij}^{RI}(p^2) = Z_{k,j}^{RGI} U_{ik}^{RI}(p^2)$, implemented at 2-loops [4,5] with $N_f = 0$ and $\alpha_s$ from [6]. For $B$-parameters, of course, the VSA’s must be appropriately renormalized.

In Fig. 2 we show the $p^2$-dependence of the ratios $R_{ij}^{RI}(0, p^2)$ and the fits, for $1.5 \text{GeV} \leq p \leq 3.2 \text{GeV}$, to the OPE expressions of Eq. (5) to the orders displayed in the caption.

We now turn to the chiral extrapolation. Here, complications arise due to the fact that we are working in the quenched approximation. Indeed, the VSA’s of $(\pi^+|Q_{7,8}^3|K^+)$ are ill-defined in the
chiral limit due to the presence of quenched chiral logs [7]. We could consider \( \langle \pi^+ | Q_{7,8}^{3/2} | K^+ \rangle / F^2 \), with \( F \) the decay constant of our degenerate \( K^+ \) and \( \pi^+ \). Instead, we choose to extrapolate \( \langle \pi^+ | Q_{7,8}^{3/2} | K^+ \rangle / F^2 \), which have similar mass dependences in the quenched and unquenched theories. Indeed, at 1-loop in \( \chi \)PT,

\[
\frac{\langle \pi^+ | Q_{7,8}^{3/2} | K^+ \rangle}{F^2} = \alpha_{7,8} \left( 1 + \frac{\beta_{7,8}}{4\pi F_\chi} \right),
\]

for \( N_f = 3 \) [8,9] and \( N_f = 0 \) [7,9]. This similarity in mass dependence may be an indication that some quenching errors cancel in these ratios. These ratios have the added advantage that they are free of chiral logarithms at 1-loop, and should hence have smoother chiral behaviors. We show the corresponding chiral extrapolations in Fig. 3, where we have added to Eq. (6) terms proportional to \( (M/4\pi F_\chi)^2 \) to account for higher orders. We obtain, in the NDR scheme at 2 GeV

\[
\lim_{m_s \to 0} \frac{\langle \pi^+ | Q_{7,8}^{3/2} | K^+ \rangle}{F^2} = 1.9 \pm 0.3 \pm ?? \text{ GeV}^2 \quad (7)
\]

\[
\lim_{m_s \to 0} \frac{\langle \pi^+ | Q_{8}^{3/2} | K^+ \rangle}{F^2} = 12. \pm 2. \pm ?? \text{ GeV}^2 \quad (8)
\]

where ?? stands for systematic errors which have yet to be determined. We postpone to a later publication the comparison of our results with non-lattice [10–13] and quenched Wilson [14], domain-wall [15,16] and overlap [17] lattice results.

3. \( \Delta S=2 \) transitions beyond the SM

In extensions of the SM, analysis of \( K^0 \) mixing generically requires knowledge of \( \langle \bar{K}^0 | O_i | K^0 \rangle \) with

\[
\begin{align*}
O_1 & = \langle \bar{s}d | V_A | s \rangle \langle \bar{s}d | V_A | d \rangle \\
O_{2,3} & = \langle \bar{s}d | S_P | s \rangle \langle \bar{s}d | S_P | d \rangle \quad \text{(unmix, mix)} \quad (9)
\end{align*}
\]

where:

- \( \langle \bar{K}^0 | O_1 | K^0 \rangle \) is the SM contribution, best given in terms of

\[
B_K = \frac{3}{16} \frac{\langle \bar{K}^0 | O_1 | K^0 \rangle}{F_K^2 M_K^2}
\]

- \( \langle \bar{K}^0 | O_5 | K^0 \rangle = 2 \times \langle \pi^+ | Q_{7,8}^{3/2} | K^+ \rangle \) for \( m_s = m_u = m_d \)

- \( \langle \bar{K}^0 | O_i | K^0 \rangle \) is \( O(p^2) \) in the chiral expansion for \( i \neq 1 \).

The last point indicates that non-SM matrix elements are expected to be larger than \( \langle \bar{K}^0 | O_1 | K^0 \rangle \), which is \( O(p^2) \). To make this explicit, we define the ratios \((i=2, \ldots, 5)

\[
R_i^{\text{BSM}}(M^2) \equiv \left[ \frac{F_K^2}{M_K^2} \right]_{\text{expt}} \left[ \frac{M^2 \langle \bar{K}^0 | O_i | K^0 \rangle}{F_K^2 \langle \bar{K}^0 | O_1 | K^0 \rangle} \right]_{\text{lat}},
\]

where BSM stands for “beyond the SM” and where \( M \) and \( F \) are the mass and “decay constant” of the lattice \( K^0 \). These ratios have a number of advantages:

- \( [ \ ]_{\text{lat}} \) is dimensionless

- \( [ \ ]_{\text{lat}} \) is finite in the chiral limit

- \( R_i^{\text{BSM}}(M_k^2) \) measures directly the ratio of BSM to SM contributions.

We obtain the required matrix elements from ratios of 3-point to two 2-point functions, as we did for \( \langle \pi^+ | Q_{7,8}^{3/2} | K^+ \rangle \). Pseudoscalar sources and sinks were used, except for \( \langle \bar{K}^0 | O_1 | K^0 \rangle \) where left-handed current sources and sinks were taken to completely eliminate zero-mode contamination [3]. We also perform an RI/MOM non-perturbative renormalization as above: \( O_1 \) renormalizes multiplicatively; \( O_{2,3} \), mix; the \( O_{4,5} \) mixing matrix was already shown.

In Fig. 4, we present new results for \( B_K^{\text{NDR}}(2 \text{ GeV}) \) as a function of pseudoscalar mass.
squared $M^2$, together with our published results obtained at the same lattice spacing but on a smaller $16^3 \times 32$ lattice [3]. The results are entirely compatible, indicating that finite-volume effects are small around the physical kaon mass. We find $B^{\text{NDR}}_K(2\text{ GeV}) = 0.60(5)(1)$, which is to be compared with the value of $0.63(6)(1)$ that we found on the smaller lattice. Our results are compatible with the quenched benchmark result of JLQCD [18], as well as with the overlap results of [17].

In Fig. 5 we plot results for $R_i^{\text{BSM}}(M^2)$ in the RI/MOM scheme at $2\text{ GeV}$ as a function of $M^2$. We find that the BSM matrix elements are enhanced at $M=M_K$ by factors ranging from approximately 5 to 20 compared to the chirally suppressed SM one. This enhancement is significantly larger than the one observed in the quenched, Wilson fermion calculation of [14].

4. Outlook

We are investigating systematic errors on the results presented above. In particular, we are performing the same calculation at $\beta = 5.85$ on a $14^3 \times 48$ lattice to quantify discretization errors.

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