No Cosmic Rays from Curvature Oscillations during Structure Formation with $F(R)$-gravity

Dmitry Gorbunov$^{1,2}$, Anna Tokareva$^{1,3}$

$^1$Institute for Nuclear Research of Russian Academy of Sciences, 117312 Moscow, Russia
$^2$Moscow Institute of Physics and Technology, 141700 Dolgoprudny, Russia
$^3$Faculty of Physics of Moscow State University, 119991 Moscow, Russia

Abstract

The Starobinsky model of modified gravity adopted to explain dark energy may be also considered in the astrophysical context. Recently it has been pointed out that in contracting regions curvature oscillations around the GR value may lead to the production of high energy particles that contribute to the cosmic ray flux. We revisited these calculations in the Einstein frame and obtained that such curvature oscillations may decay only to soft photons.

1 Introduction

Theories of modified gravity has been suggested as models capable of explaining the accelerated expansion of the present Universe. The model of $F(R)$-gravity proposed by Starobinsky [1] provides a close to what cosmological constant gives evolution of the late-time Universe, which however approaches the Minkowsky flat space in the limit of infinite time. The corresponding action for gravity reads

$$S = -\frac{M_P^2}{2} \int d^4x \sqrt{-g} (R + F(R)), \quad (1)$$

where the reduced Planck mass $M_P$ is expressed via the Newtonian gravitational constant $G_N$ as follows $M_P = \sqrt{8\pi/G_N} = 2.4 \times 10^{18}$ GeV, and

$$F(R) = \lambda R_0 \left( \left( 1 + \frac{R^2}{R_0^2} \right)^{-n} - 1 \right) - \frac{R^2}{6m^2}. \quad (2)$$
Here $m$ is the maximal mass of additional scalar degree of freedom, scalaron, that appears in $F(R)$ gravity, and parameter $R_0$ fixes the scale of energy density associated with cosmological constant $\rho_\Lambda$ at present, when the Hubble parameter equals $H_0 = 1.45 \times 10^{-42} \text{GeV}$,\(^1\)

$$R_0 = -\frac{2 \rho_\Lambda}{\lambda M_p^2} = -\frac{6 \Omega_\Lambda H_0^2}{\lambda}, \quad (3)$$

with $\Omega_\Lambda \equiv \rho_\Lambda/\rho_c$ and critical energy density $\rho_c$ determined by the Friedman equation,

$$3 M_p^2 H_0^2 = \rho_c. \quad (4)$$

The main question is how to distinguish $F(R)$-gravity from the cosmological constant and other Dark Energy models? One way is to probe the Dark Energy equation of state: $F(R)$-gravity may predict Dark Energy with time-dependent and even phantom equation of state (see e.g. [2, 3, 4] for reviews). Another way is to look for possible observable effects in astrophysics.

$F(R)$-gravity is equivalent to usual gravity with the additional scalar field coupled to matter fields and having a non-trivial potential. In papers [5, 6] it was pointed out that in space regions with rising matter density growing curvature oscillations decay into high energy particles, thus contributing to the Cosmic Ray spectrum. These oscillations are oscillations of classical scalaron field. In this paper we address a question: where do this scalaron oscillations come from? Usually scalar field oscillates around minimum if for some reason it is shifted or pushed from the vacuum. In papers [5, 6] the initial amplitude of the oscillations is arbitrary while actually there is no arbitrariness because the history of the expanding Universe exactly defines the initial conditions for scalaron field. The right way is to state the absence of scalaron oscillations at present time, while scalarons may be produced through the quantum processes [8]. This implies zero initial amplitude. We show below that numerical estimates in [5, 6], performed for the adopted there initial conditions, correspond to very unrealistic situation of huge number density (and noticeable energy density) of the initial scalaron configuration.

Actually, as it was noticed in [5], the oscillations arise even if the field is settled in the vacuum because the scalaron minimum itself moves with rising density. This oscillation source leads to much smaller amplitudes than those considered in [5, 6] and for natural choice of the Starobinsky model parameters does not yield high energy particles. For matter densities close to the present dark energy (and for parameters in a particular region) the

\(^1\)This simple relation is valid for $\lambda \gg 1$. 

2
oscillations might contribute a noticeable amount to the energy density of Universe (this situation, if realistic, needs a special study). However, this energy density does not release as high energy particles. We revisit calculations of [5, 6] and in opposite to them obtain the flux of produced cosmic rays to be negligible.

2 Scalaron density in contracting objects

Classical oscillations of scalaron field \( \phi \) may be described in terms of scalaron condensate and the number density of scalarons \( n_\phi \) is defined through the amplitude of oscillations as \([9]\)

\[
n_\phi = \omega \langle \phi^2 \rangle .
\] (5)

Here \( \phi \) is a canonically normalized scalaron field and \( \omega \) is the energy of each particle in the condensate, i.e. the scalaron effective mass. It depends on the surrounding matter density \( \rho_m \) as (valid if \( \omega \ll m \))

\[
\omega = \frac{H_0}{\sqrt{2n\lambda(2n + 1)}} \left( \frac{\rho_m}{\rho_c} \right)^{n+1} \left( \frac{\lambda}{2\Omega^2} \right)^{n+1/2}.
\] (6)

While an astrophysical object (i.e. halo) contracts, \( \rho_m \) grows. In [5, 6] matter density \( \rho_m \) changes linearly with time as \( \rho_m = \rho_{m0}(1 + t/t_J) \) for \( t < t_J \) with \( t_J \) being the Jeans time of contraction.

The scalaron field is related to the scalar curvature and may be defined through the derivative \( F'(R) \equiv dF/dR \) as

\[
\phi = \frac{\sqrt{6}}{2} M_P \log(1 + F'(R)) \approx \frac{\sqrt{6}}{2} M_P F'(R).
\] (7)

The equations of motion for action (1) provide with the equation of motion for scalaron. It is convenient to write this equation in terms of dimensionless variables which first have been done in [7]. Later calculations in [5, 6] are performed in terms of variable \( \xi \) connected to scalaron field \( \phi \) as follows,

\[
\xi = -\frac{1}{2\lambda n} \left( \frac{\rho_{m0}}{M_P^2 R_0} \right)^{2n+1} F'(R) = -\frac{1}{2\lambda n} \left( \frac{\lambda \rho_{m0}}{2\Omega \rho_c} \right)^{2n+1} F'(R).
\] (8)

The equation of motion for \( \xi \), that defines the time dependence of scalaron field, reads [5]

\[
\xi'' + \frac{\rho_m}{\rho_{m0}} - y = 0,
\] (9)
where \( y = y(\xi) \) is obtained from equation \(^2\)

\[
\frac{1}{y^{2n+1}} - gy = \xi, \quad g \equiv \frac{H_0^2}{2n\lambda m^2} \left( \frac{\rho_{m0}}{\rho_c} \right)^{2n+2} \left( \frac{\lambda}{2\Omega_\Lambda} \right)^{2n+1} \tag{10}
\]

Primes in (9) correspond to the derivatives with respect to dimensionless time \( \tau \equiv m\sqrt{gt} \), and numerically \( g \ll 1 \).

Scalaron field \( \phi \) may be expressed in terms of function \( \xi \) by using (7) and (8)

\[
\phi = \sqrt{6}\lambda nM_P \left( \frac{2\Omega_\Lambda \rho_c}{\lambda \rho_{m0}} \right)^{2n+1} \xi \tag{11}
\]

As it is shown in paper [5] when initial conditions expected in General Relativity are imposed, \( dF/dR = 0 \), the amplitude of \( \xi \)-oscillations becomes

\[
\delta \xi = (\kappa - y'_0)(2n + 1)^{3/2}, \quad \kappa = \sqrt{6}\lambda n \left( \frac{\rho_{m0}}{\rho_c} \right)^{-n-1/2} \left( \frac{\lambda}{2\Omega_\Lambda} \right)^{-n-1/2} \tag{12}
\]

Here (in the limit of small \( g \)) \( y'_0 \) is proportional to the derivative of curvature \( R \) and is considered as a free parameter in [5]. Since the case of \( y'_0 = \kappa \) is recognized in [5] as fine tuning, the initial amplitude is found to be of order \( \delta \xi = \kappa (2n + 1)^{3/2} \). Putting all things together we obtain the initial energy density of the scalaron condensate:

\[
\rho_\phi(t = 0) = \omega^2(t = 0)\langle \phi^2 \rangle = 9\lambda^2 n^2(2n + 1)^2 M_P^4H_0^2 \left( \frac{2\Omega_\Lambda}{\lambda} \right)^{4n+2} \left( \frac{\rho_c}{\rho_{m0}} \right)^{4n+1}. \tag{13}
\]

Accounting for the Friedman equation (4) we estimate for the set of parameters considered in [5] \( (n = 2, \lambda = 1, \kappa = 0.04) \) the initial scalaron energy density

\[
\rho_\phi(t = 0) = 1.5 \times 10^{-4} \rho_c, \tag{14}
\]

that actually exceeds the radiation (CMB) energy density at present. Since the scalaron effective mass (6) is tiny, the number density of scalarons is enormous, that is forbidden by the early time cosmology (e.g. from Big Bang Nucleosynthesis). Likewise one cannot expect such a large contribution of scalarons to the present energy density because less than one particle inside the horizon may be created in the present (or recent) Universe [8] while scalarons created in the very early Universe were very heavy (with mass \( m \)) and so decayed to the SM particles. So the initial conditions in [5] seem irrelevant and the result of significant particle production is obtained because these particles have been already present before the start of the process.

\(^2\)Here we use the value of \( R_0 \) defined by (3); in [5] it is taken approximately as \( R_0 = -1/t_U^2 \) with \( t_U \) being the Universe age. This results in different dependence of \( g \) on \( \lambda \) and \( \Omega_\Lambda \), as compared to [5], which is not important, however.
3 Relevant initial conditions for scalaron field

When density changes in a contracting object the form of scalaron potential changes: its minimum goes closer to $\phi = 0$ and its mass rises. The initial condition $y_0 = 0$ in [6] implies that scalaron at $t = 0$ is put into the moving minimum with zero ‘velocity’. But actually we should expect that at $t < 0$ (i.e. before the contraction starts) there were no excitations and scalaron was in the vacuum state. Also we should propose that in the real situation contraction starts in a smooth way providing the adiabatic evolution near $t = 0$ [8] and oscillations should be excited with the minimal possible amplitude. In what follows it is convenient to introduce dimensionless time $\tau = t/(\kappa t_J)$, where $t_J$ is the Jeans time, and new variables

$$\bar{\xi} = \xi - \xi_{\text{min}},$$

(15)

where

$$\xi_{\text{min}} = (1 + \kappa \tau)^{-(2n+1)}.$$  

(16)

So the adiabatic solution of the scalaron equation of motion

$$\bar{\xi''} + \Omega^2 \bar{\xi} = -\xi''_{\text{min}}$$

(17)

with such (zero) initial conditions that at $t = 0$ one has $\bar{\xi} = 0$, $\bar{\xi'} = 0$ looks to be the closest to the realistic physical situation. Here we use notifications of paper [5],

$$\Omega \equiv \frac{(1 + \kappa \tau)^{n+1}}{\sqrt{2n + 1}},$$

(18)

and primes correspond to derivatives with respect to $\tau$. 

Note that the source in the r.h.s. of (17) even with zero initial conditions leads to oscillations of $\bar{\xi}$. The adiabatic solution of (17) may be obtained by the standard technique in the form

$$\bar{\xi} = -\xi_1 \int \xi_2 \xi''_{\text{min}} d\tau + \xi_2 \int \xi_1 \xi''_{\text{min}} d\tau,$$

(19)

where

$$\xi_1 = \frac{1}{\sqrt{\Omega}} \sin \int \Omega d\tau, \quad \xi_2 = \frac{1}{\sqrt{\Omega}} \cos \int \Omega d\tau.$$  

(20)

The solution (19) may be rewritten in the form (s.f. eq. (12) of Ref. [5])

$$\bar{\xi} = \alpha(\tau) \sin \left( \int \Omega d\tau + \delta(\tau) \right).$$

(21)

3We treat maximal scalaron mass $m$ as being very large compared to the scale of cosmological constant so the parameter $g$ used in [5] may be set to zero in this notifications.
After some calculations one finds the amplitude $\alpha(\tau)$ of generated oscillations in the limit of small $\kappa \ll 1$,

$$\alpha(\tau) \simeq C_n \kappa^2 (1 + \kappa \tau)^{-\frac{n+1}{2}}, \quad C_n = 2(n + 1) (2n + 1)^2. \quad (22)$$

This result corresponds to the initial amplitude of oscillations equal to $\alpha_0 = C_n \kappa^2$, not of order $\kappa$ as it is proposed in [5]. So we obtained parametrically smaller amplitude for matter densities much larger than the critical one (astrophysical processes), $\rho_{m0} \gg \rho_c$, while for matter densities close to the critical one (cosmological processes), $\rho_{m0} \sim \rho_c$, the amplitude may be large, see eq. (13), providing (potentially) a significant contribution to the energy density of the Universe for some choice of parameters: e.g., $n = 2$ and $\lambda \approx 1$. In this region of model parameter space the detailed analysis of theoretical consistency and phenomenological and cosmological viability of $F(R)$-model is needed.

As noted in [5, 6] scalaron oscillations may produce massless particles. But when $\xi > 0$ (regular region) it cannot be efficient because of very small effective scalaron mass. Only if $\xi$ becomes negative (in a spike region as it is called in [6]) its effective mass may be increased because the scalaron potential at negative $\phi$ may be very steep. One can observe from (12), (15), (16), (22), that the spike region can be reached during the Jeans time only for matter densities $\rho_m$ close to $\rho_c$, and hence relevant only for the recent cosmic structure formation.

In the next Section we try to estimate the flux of particles produced by scalaron oscillations in the spike region.

4 Particle production in the spike region

Hereafter we consider the case of matter densities $\rho_m$ close to $\rho_c$ which may help $\xi$ to reach negative values. In this region of parameters the time of contraction is very large, of order of the Universe age and the contraction has been started not long ago. So we are interested in the evolution for short time only, $t < t_J$, and for that time $\xi$ reaches negative values only once or twice each time producing a spike in the solution for curvature [5, 6]. Below we try to estimate the particle production during one spike.

Homogeneous oscillations of classical scalaron field may produce non-conformal particles and production of scalars minimally coupled to gravity is the most efficient [10]. Scalaron is coupled to scalar field $h$ through kinetic mixing in lagrangian [10]

$$L_{int} = \frac{h}{\sqrt{6}M_P} \partial_\mu \phi \partial^\mu h \quad (23)$$
This coupling modifies the equation of motion for $h$:

$$\partial_\mu \partial^\mu h + \left( \ddot{\phi} \frac{\dot{\phi}}{\sqrt{6} M_P} + m_h^2 \right) h = 0,$$

(24)

where $m_h$ is the scalar mass. Scalaron field here plays a role of the external force producing particles $h$. The number of produced particles may be estimated by making use of the Bogoloubov transformations. From eq. (24) one obtains for the Fourier mode $h_k$ of 3-momentum $k$,

$$\ddot{h}_k + \left( k^2 + m_{\text{eff}}^2(t) + m_h^2 \right) h_k = 0,$$

(25)

where $m_{\text{eff}}^2(t) = \ddot{\phi}/(\sqrt{6} M_P)$.

Note that the spikes correspond to the negative values of $\phi$ and in that region the scalaron mass is maximal, so one has $\ddot{\phi} = -m^2 \phi$. The maximum value of $|\phi_m|$ may be extracted from the maximum $|\xi_m|$ estimated in [5]:

$$|\phi_m| = N \frac{H_0}{m} \left( \frac{\rho_{m0}}{\rho_c} \right)^{-n} z_1^{-n}, \quad N = \sqrt{\frac{\lambda}{2}} \left( \frac{2 \Omega \Lambda}{\lambda} \right)^{n+1/2}$$

(26)

Here $z_1 \equiv 1 + t_1/t_J$ where $t_1$ corresponds to the moment when $\xi$ crosses zero for the first time: $\alpha(\tau_1) = \xi_{\text{min}}(\tau_1)$. The initial amplitude of oscillations is encoded only in the parameter $z_1$. Note that for the case we consider variable $z$ is in the interval $1 < z_1 < 2$, so we put it to be $z_1 = 1$ hereafter having in mind an upper bound. Then the maximum possible value of $m_{\text{eff}}^2(t)$ is

$$M^2 = N m H_0 \left( \frac{\rho_{m0}}{\rho_c} \right)^{-n}$$

(27)

Obviously the particle production is possible only if $M > m_h$. Using as a reference number the appropriate for early-time $R^2$-inflation [12] value of $m = 3 \times 10^{13}$ GeV one can obtain $\sqrt{m H_0} = 6 \times 10^{-6}$ eV. If we treat $h$ as a Higgs boson we see that it is too massive to be produced. Only massless gauge bosons may be produced through the conformal anomaly but this process (being one-loop effect) is additionally suppressed by gauge couplings$^4$.

Let us estimate the upper bound on the massless particle production using for simplicity the equation (25) because we may be sure that the real result for the gauge bosons is always smaller. Using the dimensional analysis we may approximate the release of energy density as

$$\rho \sim M^4 = N^2 m^2 H_0^2 \left( \frac{\rho_{m0}}{\rho_c} \right)^{-2n}.$$

(28)

$^4$For recent discussion of such situation in the inflationary context see [13].
The corresponding flux is of the same order \((m^2H_0^2)\) as in [5, 6] but we should mention two important things. First, the energy of created particles is of order \(M = \sqrt{mH_0}\) but not \(m\) as it was assumed in [5, 6]. Such energy corresponds to 3 cm radio waves for reference value of \(m = 3 \times 10^{13}\) GeV, not to typical energies of gamma ray bursts, discussed in [5, 6]. Second, such energy flux is emitted only during the spike, which width is of order \(1/m \ll t_J\). So the average energy flux seems negligibly small.

Presumably the naive picture that the scalaron condensate of particles with mass \(m\) perturbatively decays to massless scalars doesn’t work in the spike region. It is not surprising because scalaron oscillations are strongly nonharmonic and asymmetric. Thus the method of calculation used in [5, 6] comprising the Fourier decomposition of spikes in curvature and using the formula obtained for \(\text{harmonic}\) oscillations of curvature [11] is not justified in the spike regions.

To summarize, we have shown that contracting objects in modified \(F(R)\)-gravity do not contribute to the cosmic ray spectrum via scalaron oscillations.

We thank A.Dolgov, E.Arbuzova and L.Reverberi for discussions. The work has been supported by Russian Science Foundation grant 14-12-01430.

References

[1] A. A. Starobinsky, JETP Lett. 86, 157 (2007) [arXiv:0706.2041 [astro-ph]].

[2] S. Nojiri and S. D. Odintsov, eConf C 0602061 (2006) 06 [Int. J. Geom. Meth. Mod. Phys. 4 (2007) 115] [hep-th/0601213].

[3] T. P. Sotiriou and V. Faraoni, Rev. Mod. Phys. 82, 451 (2010) [arXiv:0805.1726 [gr-qc]].

[4] S. Nojiri and S. D. Odintsov, Phys. Rept. 505 (2011) 59 [arXiv:1011.0544 [gr-qc]].

[5] E. V. Arbuzova, A. D. Dolgov and L. Reverberi, Eur. Phys. J. C 72, 2247 (2012) [arXiv:1211.5011 [gr-qc]].

[6] E. V. Arbuzova, A. D. Dolgov and L. Reverberi, Phys. Rev. D 88, no. 2, 024035 (2013) [arXiv:1305.5668 [gr-qc]].

[7] E. V. Arbuzova and A. D. Dolgov, Phys. Lett. B 700, 289 (2011) [arXiv:1012.1963 [astro-ph.CO]].
[8] D. Gorbunov and A. Tokareva, “Scalaron production in contracting astrophysical objects,” arXiv:1412.3413 [astro-ph.CO].

[9] D. S. Gorbunov and V. A. Rubakov, Hackensack, USA: World Scientific (2011) 489 p

[10] D. S. Gorbunov and A. G. Panin, Phys. Lett. B 700, 157 (2011) [arXiv:1009.2448 [hep-ph]].

[11] E. V. Arbuzova, A. D. Dolgov and L. Reverberi, JCAP 1202, 049 (2012) [arXiv:1112.4995 [gr-qc]].

[12] A. A. Starobinsky, Phys. Lett. B 91 (1980) 99.

[13] D. Gorbunov and A. Tokareva, JCAP 1312, 021 (2013) [arXiv:1212.4466 [astro-ph.CO]].