ElectroWeak precision data: the minimal set of parameters

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We show how precision electroweak data, assuming CP conservation and flavor universality, despite being sensitive to 20 dimension 6 operators, constrain severely only 9 combinations of them. We define a set of 7 oblique parameters $\hat{S}, \hat{T}, \hat{U}, \hat{V}, \hat{X}, \hat{W}, \hat{Y}$ which fully describe corrections to the leptonic observables. We add two additional parameters $\delta \epsilon_q, \delta C_q$ to take into account the most precise hadronic observables. Another parameter $\delta \epsilon_b$ is considered for the bottom quark if flavor universality is not assumed for the third generation of quarks.

We show that the approximation is extremely satisfactory testing it on a set of random new physics models and on some famous $Z'$ gauge bosons.

1 Introduction

To make the Standard Model (SM) a natural low energy effective field theory some new physics is expected at the TeV scale. Such new physics has to cut-off the quadratic divergences which plague the SM, destabilizing the Higgs boson potential. A lot of possibilities have been explored in the last two decades. Some of them try to enhance the symmetries of the SM (like supersymmetry), others try to make the Higgs a composite particle of the size of an inverse TeV (like technicolor and its 5-dimensional duals), others want to interpret the Higgs as a pseudo-Goldstone boson of some approximate global symmetry. The astonishing success of the SM in the precision experiments performed in nineties mainly at LEP and SLD has pushed most of these models in rather innatural corners of their parameter space.

Thus it is important to analyze in a complete, but possibly minimal way, what such precision experiments say. Global fits to all measured observables are of course a correct approach, but they don’t give much intuition into the true origin of the strongest constraints. In fact, the number of severely constrained parameters is typically smaller than the total number of measured quantities.

Our goal is to define a minimal number of parameters which catch most of the effects on the observables measured so far. In this way a necessary and sufficient criterion of compatibility with
the SM of a given new physics model can be defined. Possible large corrections to combinations of parameters which are accidentally poorly constrained are automatically taken care of if the parameters are properly defined.

In Sec. 2 we define the full set of the 20 dimension 6 operators $O_i$‘s which affect precision observables. In Sec. 3 we show how only about half of them is severely constrained, and give an operative definition of $\tilde{S}, \tilde{T}, \tilde{U}, V, X, W, Y, \delta C_q, \delta \epsilon q$ (and possibly $\delta \epsilon b$ for the third quark generation). These are the only severely constrained parameters. In Sec. 4 we verify quantitatively the goodness of our approximation. To be concrete, in Sec. 5 we apply our analysis to the case of $Z'$ gauge bosons. In Sec. 6 we draw our conclusions.

2 The operators affecting EW precision observables

The set of precision experiments we refer to are $e^+ e^-$ collisions performed in the nineties at LEP and SLD\cite{1,2}. The experimental activity can be logically split into two parts. Collisions performed at the $Z$ resonance (LEP1 and SLD) and measurements above the $Z$ pole, up to energies $\sqrt{s} = 209$ GeV (LEP2).

Effects of heavy new physics can be parametrized through a set of dimensions 6 operators $O_i$ added to the SM lagrangian:

$$\mathcal{L} = \mathcal{L}_SM + \sum_i c_i O_i$$

To set our notation, we define the Higgs and fermion currents as

$$J_{\mu H} = H^\dagger i D_\mu H, \quad J^{a}_{\mu H} = H^\dagger \tau^a i D_\mu H, \quad J_{\mu F} = \sum \bar{F} \gamma_\mu F, \quad J^{a}_{\mu D} = \sum \bar{D} \gamma_\mu \tau^a D$$

where $F = \{E, L, Q, U, D\}$ and $D = \{L, Q\}$. Assuming CP conservation and an $U(3)^6$ flavor symmetry, the relevant set of operators consists into\cite{3}

- 7 operators involving one fermion and one Higgs current (vertex operators): $O_{HF} = J_{\mu H} J_{\mu F} + \text{h.c.}$ with $F = \{L, E, Q, U, D\}$, and $O_{HD} = J_{\mu H} J_{\mu D} + \text{h.c.}$, where $D = \{L, Q\}$.
- 11 4 fermion operators $O_{FF'} = J_{\mu F} J_{\mu F'}/(1 + \delta_{FF'})$ and $O_{DD'} = J_{\mu D} J_{\mu D'}/(1 + \delta_{DD'})$ involving at least one charged lepton current.
- 2 oblique operators, i.e. not involving fermions. They are $O_{WB} = (H^\dagger \tau^a H) W^{a}_{\mu \nu} B^{\mu \nu}$, $O_{HH} = |J_{\mu H}|^2$.

This makes a total of 20 parameters. Observables at and below the $Z$– resonance are sensitive to only 10 of them\cite{4}. They are all the 7 fermion-Higgs operators (vertex corrections), $O'_{LL}$.
corrections to the muon decay which sets the Fermi constant), and the two oblique operators \( O_{WB}, O_{HH} \). Going above the \( Z \) pole, i.e. considering the LEP2 data, one becomes sensitive to 10 more operators: the remaining 10 4-fermi operators.

At this point, given a certain model of New Physics, one can calculate the corrections to the 20 parameters and perform a global fit. However, one might wonder if all these 20 parameters are really constrained. For example, it has been recently pointed out that two combinations of these 20 parameters are almost unconstrained. For example, after calculating a global \( \chi^2 \) in terms of the coefficients of the 20 parameters, one can look to the eigenvalues of the error matrix. This automatically identifies all correlations of theoretical, experimental and accidental nature. The eigenvalues are shown in Fig. It is clear that only about 10 combinations of these operators are important, and that a few constraints often dominate the fit.

It is interesting, at this point, to try to define the mostly constrained parameters which appear in Fig. This will be done in the next section.

3 The minimal set of parameters

In order to identify the mostly constrained parameters it is convenient to use the equations of motion for the gauge bosons. In this way one obtains an equivalent set of operators, but in the new basis it will be more transparent what are the important parameters. At leading order, the equations of motions are

\[
\partial^\nu B_{\nu \mu} + \frac{M^2_W}{g^2} g'(g' B_{\mu} - g W^3_{\mu}) + g' \sum_F Y F J^f = 0 + \ldots 
\]

\[
\partial^\nu W^3_{\nu \mu} + \frac{M^2_W}{g^2} g(g W^3_{\mu} - g' B_{\mu}) + g \sum_f T_3 J^f_{\mu} = 0 + \ldots 
\]

\[
\partial^\nu W^\pm_{\nu \mu} + M^2_W W^\pm_{\mu} + \frac{g}{\sqrt{2}} \sum_F J^\pm_{\mu F} = 0 + \ldots 
\]

where we neglected on the r.h.s. operators that are poorly measured. We now solve the equations of motion in terms of

\[
J^{\mu R} \equiv J_{\mu E} = \bar{e}_R \gamma_\mu e_R, \quad J^{\mu L} \equiv \bar{e}_L \gamma_\mu e_L, \quad J^+_{L \mu} \equiv \bar{e}_L \gamma_\mu \nu_L 
\]

and plug the result into the Lagrangian generated by the new physics. In this way the charged lepton currents disappear from the Lagrangian. Vertex corrections and 4-fermi operators involving only leptons are now recast into oblique corrections. The basis of 20 parameters defined in the previous section is now mapped in the following basis.

- 7 oblique parameters \( \hat{S}, \hat{T}, \hat{U}, V, X, W, Y \). They can be expressed in terms of the gauge bosons self-energies \( \Pi_{ij}(p^2) \) as

\[
\hat{S} = \frac{g}{g'} \Pi'_{30}, \quad \hat{T} = \Pi'_{33} - \Pi_{WW}, \quad W = \frac{M^2_W}{2} \Pi''_{33}, \quad Y = \frac{M^2_W}{2} \Pi''_{00}, 
\]

\[
\hat{U} = \Pi_{WW} - \Pi'_{33}, \quad V = \frac{M^2_W}{2} (\Pi''_{33} - \Pi''_{WW}), \quad X = \frac{M^2_W}{2} \Pi''_{30}, 
\]

- Vertex corrections for fermions other than charged leptons. They can be parametrized as

\[
\mathcal{L}_{\text{couplings}} = \sum_f (\bar{f} \gamma^\mu f) \left[ e A_\mu \frac{C^y_f}{M_W} p^2 + \sqrt{g^2 + g'^2} Z_\mu \left( \frac{C^Z_f}{M_W^2} (p^2 - M_Z^2) + \delta g_f \right) \right],
\]

\footnote{For detailed formulas about the connection between the two basis see Appendix A of [8].}
where \( f = u_L, d_L, u_R, d_R, \nu_L \). The \( \delta g \)'s are corrections to on-shell \( Z \) couplings, tested by measurements at the \( Z \) pole. The \( C^\gamma \) and \( C^Z \) are equivalent to 4-fermion contributions to \( e^+ e^- \rightarrow q \bar{q} \). \( \delta g_{L\nu} \) can be written as linear combination of oblique parameters \( 6 \). Thus it is properly included in the fit considering the full set of 7 oblique parameters. Furthermore, only 11 of the 12 are independent, since the relation \( (C^\gamma_{dL} - C^\gamma_{uL}) = \cos^2 \theta_W (C^Z_{dL} - C^Z_{uL}) + X/\tan \theta_W \) holds.

At this point we are left with 18 independent parameters. The 7 oblique ones, 4 \( \delta g \)'s for the quarks, 4 quark \( C^Z_q \)'s and 3 independent \( C^\gamma_q \)'s. The total is 18 parameters: 2 less then the 20 we started from. This is because we neglected the poorly constrained operators which affect only trilinear couplings among vectors \( 5 \).

In the new basis it is very transparent to understand which are the most important parameters, All the corrections involving only leptons are included in the 7 oblique parameters \( \hat{S}, \hat{T}, \hat{U}, V, X, W, Y \). These are

\[
\alpha_{em}, \Gamma(\mu), M_Z, M_W, \Gamma(Z \rightarrow \ell\bar{\ell}), A^i_{FB}, A^i_{LR}, A^c_{pol}, \sigma_{LEP2}(e\bar{e} \rightarrow \ell\bar{\ell}), ee \rightarrow ee
\] (9)

They are the most precisely measured observables, tested at the \textit{per mille} level. In most of the cases they are a sufficient set of parameters. However this approximation would fail if, for some reason, new physics is leptophobic, i.e. if quarks are affected much more strongly then leptons. In this case one has to add some additional parameter to include the hadronic observables, i.e.

\[
\Gamma(Z \rightarrow q\bar{q}), A^b_{FB}, A^b_{LR}, A^c_{LR}, A^c_{FB}, \sigma_{LEP2}(e\bar{e} \rightarrow q\bar{q}), Q_W \quad (10)
\]

It turns out that the most precise measurements are, at the \( Z \)-pole, the hadronic branching fraction of the \( Z \) and, above the pole, the total cross section \( e\bar{e} \rightarrow \) hadrons. One can show \( 6 \) that corrections to these two observables depend mainly on the two following combinations, which are the 2 additional parameters we include in our fit.

\[
\delta \varepsilon_q = \delta g_{uL} - \delta g_{dL} , \quad \delta C_q = C^Z_{uL} - C^Z_{dL} . \quad (11)
\]

Furthermore in many models of electroweak symmetry breaking the third generation of quarks is special due to the heavyness of the top quark, and it is differently affected by new physics. For this reason, one can relax the flavor universality for the bottom quark, and deal with it separately. This is also necessary since the bottom final state is well measured. The most relevant parameter is the left-handed bottom quark coupling \( \delta g_{bL} \) measured at LEP1. Thus we define \( \delta \varepsilon_b \) as

\[
\delta g_{bL} = -\frac{1}{2} \delta \varepsilon_b . \quad (12)
\]

Summarizing, we defined a set of \( 7 + 2 + 1 = 10 \) parameters. The 7 oblique parameters \( \hat{S}, \hat{T}, \hat{U}, V, X, W, Y \) fully describe the effects on all the leptonic observables, which are the best measured ones. 2 additional parameters \( \delta \varepsilon_q, \delta C_q \) describe the most constrained quantities for the hadronic observables, and 1 additional parameter \( \delta \varepsilon_b \) describes deviations in the left-handed bottom quark coupling to the \( Z \) if flavour universality is not assumed for the third generation of quarks. If Ref. best fit, errors and correlations for all the 10 parameters is given. In the case on universal models, only 4 parameters are non-vanishing: \( \hat{S}, \hat{T}, W, Y \).

4 Goodness of the approximation

We now check how good our approximations are for guessing the bound on the scale \( \Lambda \) of new physics in generic models. To do that, we generated many random models by writing each coefficient \( c_i \) of eq. \( 14 \) as \( c_i = r_i/\Lambda^2 \), where \(-1 \leq r_i \leq 1\) are random numbers. We then
extract the bound on $\Lambda$ both from the exact fit and the approximate fits. In the following table we report the average value and the variance of $\Lambda_{\text{approx}}/\Lambda_{\text{true}}$ in the three following cases: the oblique approximation, then we add the two parameters $\delta C_q$ and $\delta \varepsilon_q$ for the quarks and finally we include all the parameters except $\delta C_q$ and $\delta \varepsilon_q$.

| Approximation             | $\Lambda_{\text{approx}}/\Lambda_{\text{true}}$ |
|---------------------------|-----------------------------------------------|
| Oblique                   | 0.95 ± 0.16                                   |
| Oblique plus $C_q$, $\delta \varepsilon_q$ | 0.98 ± 0.06                                   |
| All but $C_q$, $\delta \varepsilon_q$    | 0.98 ± 0.15                                   |

We see that the oblique approximation is already reasonable: in most of the cases the approximate bound is less than 25% away from the correct one. Adding the two parameters $\delta C_q$, $\delta \varepsilon_q$ improves the approximation significantly: in more than 90% of the cases the approximate bound reproduces the exact one within 10%. Furthermore, it is important to notice that considering a fit where all the parameters except $\delta C_q$, $\delta \varepsilon_q$ are added does not improve much the approximation with respect to the oblique case. This is telling us that in the quark sector it is indeed $\delta C_q$ and $\delta \varepsilon_q$ which are the most constrained parameters, while all the others are much less constrained (and mostly negligible). The arguments of Sec. 3 which led us to define the 9 parameters find here a quantitative confirmation: the 9 remaining parameters can be safely neglected.

5 An example: $Z'$s

To be concrete, we apply our analysis to a generic heavy non-universal $Z'$ vector boson. It is characterized by its mass $M_{Z'}$, its coupling $g_{Z'}$ and charges $Z_X$ under the various SM fields $X = \{H, E, L, Q, U, D\}$. In terms of these parameters one can easily calculate the corrections to the 9 parameters $\hat{S}, \hat{T}, \hat{U}, V, X, W, Y, \delta \varepsilon_q, \delta C_q$ defined above. In order to verify the goodness of our approximation it is interesting to compare the bounds on the ratio $M_{Z'}/g_{Z'}$ in the three following cases: i) exact case (i.e. including all the 20 parameters), ii) oblique approximation (i.e. including only $\hat{S}, \hat{T}, \hat{U}, V, X, W, Y$), and iii) adding $\delta \varepsilon_q, \delta C_q$ for the quarks. The results are shown in Table 1. It is interesting to notice that the approximate bounds reproduce the exact one accurately in almost all the cases. There are few exceptions where the effect of quarks is relevant, and the oblique bound is overestimated. On the other hand, the 9-parameter approximation is always successful.

6 Conclusions

The astonishing (and boring) success of the Standard Model has put many constraints on models aiming to solve the Higgs hierarchy problem. Most of them are today still compatible with the data only in rather unnatural regions of their parameter space. We have defined a minimal and exhaustive set of parameters which are constrained by the electroweak data. We have shown that experiments below, at and above the $Z$ pole are sensitive (assuming flavor universality and CP conservation) to 20 dimension 6 operators. However, only about half of them are severely constrained. Using the equations of motion for the gauge bosons we have defined a set of $7 + 2 + 1 = 10$ parameters. The 7 oblique parameters $\hat{S}, \hat{T}, \hat{U}, V, X, W, Y$ include all the effects on the purely leptonic observables. 2 additional parameters $\delta \varepsilon_q, \delta C_q$ include the effects on the two most precisely measured hadronic quantities: the $Z$ hadronic branching fraction and the $e\bar{e} \rightarrow$ hadrons total cross section. Furthermore, an additional parameter $\delta \varepsilon_b$ can be added, describing the deviations in the coupling of the left handed bottom quark to the $Z$, if flavor universality is not assumed for the third generation of quarks.

$\text{b}$For the analytical expressions see Sec. 4 of [6]
| U(1) universal? | $Z_H$ | $Z_L$ | $Z_D$ | $Z_U$ | $Z_Q$ | $Z_E$ | full approx oblique |
|-----------------|------|------|------|------|------|------|-------------------|
| $H$ yes          | 1    | 0    | 0    | 0    | 0    | 0    | 6.7 6.7 6.7       |
| $B'$ yes         | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | 1    | $\frac{1}{2}$ | 6.7 6.7 6.7       |
| $B'_F$ yes       | 0    | $-\frac{1}{2}$ | $\frac{1}{2}$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | 1    | 4.8 4.8 4.8       |
| $B-L$ no         | 0    | $-1$ | $-\frac{1}{2}$ | $\frac{1}{2}$ | $\frac{1}{2}$ | 1    | 6.7 7.1 7.1       |
| $L$ no           | 0    | 1    | 0    | 0    | 0    | $-1$ | 6.3 7.1 7.1       |
| 10 no            | 0    | 0    | 0    | 1    | 1    | 1    | 2.5 2.9 3.4       |
| 5 no             | 0    | 1    | 1    | 0    | 0    | 0    | 3.8 3.2 5.6       |
| $Y$ no           | $\frac{2}{3}$ | 1    | 1    | $-\frac{1}{3}$ | $-\frac{1}{3}$ | $-\frac{1}{3}$ | 4.8 5.0 6.0       |
| 16 no            | 0    | 1    | 1    | 1    | 1    | 1    | 4.4 4.7 6.5       |
| $R$ no           | 0    | 0    | $-\frac{1}{2}$ | $\frac{1}{2}$ | 0    | $-\frac{1}{2}$ | 1.6 1.5 1.7       |
| SLH no           | Simplest little Higgs | | | | | | 2.7 2.5 2.7       |
| SU6 no           | | | | | | | 3.1 3.3 3.3       |

Table 1: 99% CL bounds on the ratio $M_{Z'}/g_{Z'}$ in TeV for a set of frequently studied $Z'$. In the last two cases we report the bound on the scale $f$ of little-Higgs models.

We have then checked how good the various approximation is in two different ways. First, we have generated many random models by varying randomly the coefficients of the various parameters. Then, we applied our formalism to a set of famous $Z'$s. The result is that the purely oblique approximation (i.e. the 7 parameters $\hat{S}, \hat{T}, \hat{U}, V, W, Y$) is often satisfactory. Adding the two parameters $\delta \varepsilon_q, \delta C_q$ for the hadronic observables improves significantly the oblique approximation: in more than 90% of the randomly generated models the correct bound is reproduced within 10%.

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**References**

1. [LEP Collaborations], arXiv:hep-ex/0412015.
2. G. Abbiendi et al. [OPAL Collaboration], Eur. Phys. J. C 33 (2004) 173 [arXiv:hep-ex/0309053].
3. Z. Han and W. Skiba, Phys. Rev. D 71 (2005) 075009 [arXiv:hep-ph/0412166].
4. R. Barbieri and A. Strumia, Phys. Lett. B 462 (1999) 144 [arXiv:hep-ph/9905281].
5. C. Grojean, W. Skiba and J. Terning, Phys. Rev. D 73 (2006) 075008 [arXiv:hep-ph/0602154].
6. G. Cacciapaglia, C. Csaki, G. Marandella and A. Strumia, arXiv:hep-ph/0604111.
7. R. Barbieri, A. Pomarol, R. Rattazzi and A. Strumia, Nucl. Phys. B 703 (2004) 127 [arXiv:hep-ph/0405040].
8. M. Schmaltz, JHEP 0408 (2004) 056 [arXiv:hep-ph/0407143].
9. C. Csaki, G. Marandella, Y. Shirman and A. Strumia, Phys. Rev. D 73 (2006) 035006 [arXiv:hep-ph/0510294].