Temporal behaviour of field in high quality factor photonic crystal microcavity structure

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Abstract: Temporal behaviour of incident pulse in high-quality (Q) factor photonic crystal microcavities are studied by two dimensional finite difference time domain calculations. For high-Q mode excitation, two periods of oscillation are observed in addition to the exponential decay corresponding to the cavity mode photon life time. Long and short period oscillations correspond to beats with low-Q mode (for short pulse widths) and to lattice periodicity for all pulse widths, respectively. For low-Q mode and off-resonant excitations, long period oscillations correspond to coupling to bandedge states.

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1. Introduction

Photonic crystal (PC) microcavities that can exhibit high quality factor (Q) and small modal volumes (V) are considered ideal for both fundamental cavity Quantum Electrodynamics (cQED) and various device applications [1, 2, 3, 4]. With recent demonstration of high-Q cavities [4, 5, 6], it may be interesting to study the possible effects of these structures on the incident field. In order to characterize the microcavities, transmission measurements to estimate the coupling efficiency and the spectrum are studied. The cavity Q is determined from the spectral width (\(\Delta \lambda\)) of the transmission spectrum (Q \(\approx\) \(\lambda/\Delta \lambda\) for the cavity mode resonance at \(\lambda\)). While this is valid for a low-Q cavity, in general, the cavity Q may be obtained, by estimating the cavity photon life time (\(\tau\)) from Q = \(\omega_0\tau\) where \(\omega_0\) is the resonant cavity mode frequency. However, estimation of \(\tau\) by the use of a short pulse may not be trivial due to the possible excitation of multiple modes (due to the finite spectral width of the short pulse). Thus, the validity of estimation of Q-factor from the cavity mode photon life time using a short pulse is suspected. In addition, to study the feasibility of applying these cavities for quantum information processing and photonic switching applications, it would be interesting to study the effect of cavity on the time dependence of the incident field. Thus, we numerically studied the pulse width dependence of the pulse evolution. Though this may be of interest, to our knowledge, the time dependence of the incident pulse profile is not studied till date. Considerable work has been reported on the nonlinear propagation in coupled-resonator optical waveguide (CROW) structures [7, 8]. However, the pulse profile is not reported in these structures also.

Several high-Q PC microcavity designs based on fractional edge dislocations or phase slips, graded PC lattices, super defects, and hexapole modes have been reported till date [1, 9, 4, 10, 11, 12]. Though super defects, with modified set of holes as the defect, may lead to high-Q but at the expense of large modal volume (V) [10, 11]. Recent demonstration shows a defect with 3 missing holes and suppressed edge effects leading to a large Q of about 45,000 and a reasonable mode volume [5]. However, these may not be suitable to introduce a nonlinear medium with refractive index different from the substrate material or Quantum Dots (QDs) or atoms for cQED studies. Similarly, the high-Q and small V hexapole modes suffer from field cancellation (a node for the electric field) at the center of the defect [12]. To overcome these aspects, it would be essential to design PC structures that demonstrate high-Q and small V and which can employ an acceptor defect with variable refractive index.

Defects in photonic crystals are generally referred to as acceptor (reduced air hole regions) and donor type (increased air hole region) in a regular PC lattice. While acceptor defects are found to confine the field in the dielectric regions, the mode volume is increased [1]. Figure 1 shows a schematic of the 2D PC structure, being studied here, that is formed by a hexagonal lattice of air holes defined in a dielectric medium. In this structure, we have increased the radius of the central hole but filled it with a dielectric material to form the central defect where the incident field is concentrated. Thus, we effectively have an acceptor type defect (increased dielectric region or reduced air region) so that the field is confined in the dielectric region. In addition, the size and shape of the neighbouring holes are modified to reduce the in-plane radiation losses to increase the Q-factor. We studied the dependence of Q on the radius and the refractive index of the dielectric defect material (central hole) to further optimize the structure for high-Q. The optimum defect radius is about 0.2 \(\mu\)m (1.2 \(\times\) r, r is the radius of regular air
Fig. 1. A schematic of the 2-D photonic crystal microcavity structure. The typical size of this structure is about 15a^2, where a is the lattice periodicity.

hole) and the defect refractive index is about 3.9 (Sb-based semiconductor materials for GaAs PCs and PbSe or PbS for PCs fabricated on Si-slab). Details of these studies will be presented elsewhere. Briefly, from the calculated mode profiles, the Q value peaks when the defect size is comparable to that of the mode (about 1.2×r). That is, the mode is nicely confined in the defect region. For a reasonable 2D PC of size (15a)^2, a being the lattice periodicity, an in-plane Q-factor of about 1.7×10^5 and a small mode volume, in terms of the cubic wavelength inside the medium, of about 0.42×(λ/n)^3 were observed. These designs were extended to 3D structures and a Q-factor of about 5,000 was achieved by modifying the neighbouring holes to reduce the vertical radiation losses. In this paper, we present two-dimensional finite difference time domain (FDTD) calculations on this structure to study the time dependence of the incident power to study the temporal behaviour of the pulse profile.

Fig. 2. The calculated band structure for TM mode excitation. The hashed region shows the photonic bandgap region with the two defect modes at 1.53 μm and 1.414μm wavelengths. Figure on the right shows the transmission spectrum.

2. Results

Temporal studies in photonic crystals performed till date are to study the group velocity and more recently the cavity Q [13]. In the two-dimensional PC structure studied here, the high-Q features are achieved by modifying the central hole (defect) and the six nearest neighbour holes (Fig. 1). Figure 2 shows the band structure of the 2D structure of air holes for transverse
magnetic mode excitation. The band structure calculations show two defect modes at 1.53 and 1.414 \( \mu \text{m} \) wavelengths in the PBG region. This is verified to be equivalent to a 3D structure of air holes in a Si-slab of thickness 0.2 \( \mu \text{m} \). Figure 2 also shows the transmission spectra with the photonic bandgap (PBG) in the region of 1.3 to 1.8 \( \mu \text{m} \) and a defect mode peak at about 1.53\( \mu \text{m} \). The second peak at 1.414 \( \mu \text{m} \), however, is not seen in the spectrum due either to a weak peak or due to the position of the detector. It would further be interesting to note that while the cavity mode photon life time may be estimated unambiguously from the temporal data (inset of Figure 3(a)), to calculate the spectral width accurately, one requires a longer calculation. For temporal studies, we varied the gaussian pulse width in the range of 16fsec to 3psec to study the effect of pulse width on the cavity mode photon lifetime and the field (due to the possible excitation of multiple modes). By using 2D FDTD calculations, we calculated the transmitted power and the power at the center of the PC structure (defect region). The grid size employed in these calculations is 0.01x0.01 \( \mu \text{m} \). Broad pulse calculations indicate a high-Q at 1.53\( \mu \text{m} \) wavelength. Figure 3(a) compares the time evolution of the transmitted pulse intensity for two pulse widths resonant at 1.53 \( \mu \text{m} \). While the 100fsec pulse excitation shows a non-oscillatory, single exponential decay in the long time limit (inset of Fig. 3(a)) with a decay constant of about 133psec, the 17fsec pulse excitation shows periodic oscillations even in the long time limit.

![Figure 3](image)

Fig. 3. (a) Temporal evolution of 100 fsec (dashed line) and 16.7 fsec (solid line) incident pulses resonant at 1.53 \( \mu \text{m} \). Inset shows the close up of the exponential decay for the 100fsec pulse (dashed line) along with an exponential fit (solid line). (b) A close up of the long period oscillations observed for short pulse excitation (16.7 fsec pulse width) at 1.53 \( \mu \text{m} \) is shown by the dotted line. The oscillation period corresponds to a period of 33.33fsec or a beat energy separation of 63 meV. The solid line is a fit with the energy separation between the two modes to be 65 meV.

Figure 3(b) (dotted line) shows the oscillations observed for the short pulse excitation case in the time range 330 and 670 fsec. Considering that these oscillations correspond to beats between two states, the period of oscillation corresponds to 33.3fsec (\( \tau \approx 10\mu \text{m} \)) or to an energy separation of about 63meV. The solid line in Fig. 3(b) is a fit based on an expression for beats for a two-level system given by, \( \exp(-t\gamma_1) + \alpha^2 \exp(-t\gamma_2) + 2\alpha \cos(t\Delta E) \exp(-0.5t(\gamma_1 + \gamma_2)) \) where \( \gamma_1 \) and \( \gamma_2 \) are the decay rates of 1.53 \( \mu \text{m} \) and 1.414 \( \mu \text{m} \) modes, \( \Delta E \) is the energy separation between the two modes, and \( \alpha \) is the coupling strength. The fit parameters are 133 psec, 100 fsec, 65 meV, and 0.04 for \( \gamma_1^{-1}, \gamma_2^{-1}, \Delta E \), and \( \alpha \), respectively.

As shown in Fig. 2, the band structure calculated by the plane wave expansion technique reveals the corresponding defect mode. The calculated energy separation of about 64.5meV
between the two modes at 1.53 and 1.414 µm matches quite well with that estimated from the fit and the observed period of oscillation in Fig. 3(b) indicating that the oscillations observed for short pulse excitation could be due to beats between these two modes. We further verified this by studying the pulse width dependence. These studies show that the oscillations are observed only for pulse widths shorter than 90fsec. This is consistent with the fact that for pulse widths shorter than 90fsec, the spectral width is broad enough to simultaneously excite both the modes and we observe the oscillations in the time evolution similar to the oscillations observed in multi-level electronic, excitonic or atomic systems [14].

Fig. 4. Temporal evolution for 500fsec incident pulse resonant at 1.414 µm. Dashed lines are exponential fits. Inset shows the time evolution of a short (17 fsec) incident pulse resonant at 1.414 µm.

We also performed similar studies on the short wavelength mode. Figure 4 shows the calculated time evolution of transmitted field intensity for 0.5psec broad pulse that is resonant at the 1.414 µm mode in a PC structure of size 12a^2. It may be interesting to note that, though the pulse width is broad (thus the spectral width is narrow enough to not excite the 1.53 µm mode), we see oscillations in the time evolution. The time decay shows two exponential decays in addition to these oscillations. The dashed lines shown in Fig. 4 are exponential fits. The short time decay constant is about 0.68 psec. Even though most of the field decayed before the long time limit exponential decay begins, the long time decay constant shows about 22.3 psec decay constant similar to that observed for 1.53 µm mode. When we increase the size of the PC structure from (12a)^2 to (15a)^2, the short time exponential decay constant at 1.414 µm falls from 0.68 psec to 0.45 psec (within the pulse width). The long period time constant of 130 psec is still similar to that for the 1.53 µm mode. Though we do not understand the origin of this long time decay of 1.414 µm, it may be very interesting to study the possibility of coupling to the long wavelength mode.

Inset of Fig. 4 shows the time evolution for a short pulse (width is 17fsec) excitation resonant at 1.414 µm. As may be expected, we observe oscillations but a non-exponential long time limit decay. A closer inspection shows two time periods. The longer oscillation period matches with the beat frequency corresponding to the simultaneous excitation of both modes as observed for the short pulse excitation at 1.53 µm. However, the decay is very fast unlike for the 1.53 µm excitation case. It may be interesting to note that the 1.414 µm mode is separated by similar energy from both the high energy PBG edge at 1.3 µm (0.954 eV) and the 1.53 µm.
Consider that there are two decay paths for the 1.414 µm mode with time scales \( T_1 \) and \( T_2 \) corresponding to coupling to 1.53 µm and PBG edge, respectively. The decay time (T) of the 1.414 µm mode would be given by \( (C_1/T_1)+(C_2/T_2) \) where \( C_1 \) and \( C_2 \) are the coupling strengths. From the above results we see that the \( T_1 \) is about 130 psec and the \( T \) is about 100 fsec (comparable to the pulse width). So, for coupling to the 1.53 µm mode to be non-negligible and the decay to be a slower one, \( C_1 \) has to be, at least, a 100 times larger than \( C_2 \) (coupling to the band edge states). Thus, from the fast decay time constant one may infer that the radiative coupling to the PBG edge may be more effective than coupling to the high-Q mode. In other words, the observed long period oscillations for the low-Q mode excitation could correspond to the coupling to the PBG edge.

![Graph showing power vs time for 1.53 µm and 1.414 µm excitation](image)

**Fig. 5.** A close-up of the short period oscillations observed for 1.53 µm (solid line) and 1.414 µm (dashed line) excitation.

The solid line in Fig. 5 shows the close-up of short period oscillations in the time scale 333 and 367 fsec for excitation resonant at 1.53 µm. The oscillation period is about 0.45±0.03 µm and matches quite well with the lattice constant. Dashed line in Fig. 5 shows the corresponding close-up of short period oscillations for short pulse excitation resonant at 1.414 µm with the oscillation period (0.45±0.05 µm) similar to that observed at 1.53 µm. This indicates that the short period oscillations observed could be related to the lattice periodicity (\( a=0.49 \) µm).

We also studied the time evolution of field at two non-resonant wavelengths above and below the 1.53 µm wavelength. Fig. 6 shows the time dependence for 1.5 and 1.57 µm excitation. The pulse width of 167 fsec is chosen such that the two defect modes are not within the pulse spectral width. However, a small component (at 1.53 µm) in the tail of the Gaussian excitation pulse excites the 1.53 µm mode. Correspondingly, a very weak peak at 1.53 µm is visible in the Fourier transform of the temporal data shown in Fig. 6. For the purpose of studying the coupling behaviour, from the temporal data we evaluate the beat frequency and the corresponding energy separation. This information is not available from the spectrum (Fourier transform of the temporal data). As expected, the time evolution shows a fast decay at both the non-resonant wavelengths. In addition, the beat period of about 0.6 psec for 167 pulse excitation (solid line in Fig. 6) corresponds to an energy separation of about 93 meV for 1.5 µm (0.827 eV) excitation indicating beats with the low energy edge of the PBG at \( \approx 1.8 \) µm (0.689 eV). Though a short pulse width (67 fsec) at these off-resonant wavelengths is considered such that the high-Q mode
at 1.53 \( \mu \text{m} \) is within the pulse spectral width, the beats observed still correspond to coupling to the PBG edge (data not shown). Similarly, for both the broad (167fsec- dashed line in Fig. 6) and short (67fsec) pulse excitations at 1.57 \( \mu \text{m} \) (0.79eV), long period oscillations correspond to beats with the low energy PBG edge. Clearly, the off-resonant and the resonant low-Q mode excitations, for all pulse widths, show a fast decay (similar to the pulse width). Only the resonant excitation of the high-Q mode at 1.53 \( \mu \text{m} \) shows a long exponential decay in the long time limit. This addresses, thus, the question about the use of short pulse to estimate the \( \tau \) of a high-Q mode.

3. Conclusion

In conclusion, we have studied, by 2D FDTD calculations, the time dependence of the incident pulse in a high-Q photonic crystal cavity. These studies give an insight into the coupling behaviour of defect modes in high-Q PC cavities. Though the pulse spectral width covers both the modes, only the pulse resonant at the high-Q mode shows the long cavity mode decay in addition to the oscillations corresponding to beats with the low-Q defect mode. While long period oscillations in case of simultaneous excitation of both the defect modes correspond to beats, the short period oscillation for all pulse widths in the range 16fsec to 3 psec correspond to the lattice periodicity. Low-Q mode and off-resonant excitations couple to the band edge states and decay within the short pulse width. From the beat frequency (long period oscillations) and the fast decay, it may be inferred that the low-Q modes in this cavity couple to the band edge states. It may be interesting to study the effect of pulse widths in structures in which the two defect modes are close to each other and far away from the bandedge. These results also indicate that in spite of the apprehensions in using the more general definition based on the cavity mode life time to estimate Q, in addition to the oscillations corresponding to beats, only the resonant excitation of the high-Q mode shows the long time exponential decay corresponding to the cavity mode life time.
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