1. INTRODUCTION

A large class of 2D conformal field theories is described by gauged Wess-Zumino-Novikov-Witten models with the following action:

\[ S(g, A) = S_{WZNW}(g) + \frac{k}{2\pi} \int d^2z \text{Tr}[Ag^{-1}A] \]

where

\[ S_{WZNW}(g) = \frac{k}{8\pi} \int d^2z g^{-1} \partial \mu g^{-1} \partial_\mu g \]

and \( g \in G, \ A, \bar{A} \) are the gauge fields taking values in the algebra \( \mathcal{H} \) of the diagonal group of the direct product \( H \times H, \ H \subset G \). From now on we shall be dealing with the orthogonal decomposition of the Lie algebra \( G \) as

\[ \mathcal{G} = \mathcal{H} \oplus \mathcal{M}, \ [\mathcal{H}, \mathcal{H}] \subset \mathcal{H}, \ [\mathcal{H}, \mathcal{M}] \subset \mathcal{M}. \]

There are different methods of studying correlation functions of gauged WZNW models such as the free field realization approach or the fermionization technik. We shall pursue a different idea which is parallel to the analysis of the gravitational dressing of 2D field theories.

Our starting point are the equations of motion of the gauged WZNW model:

\[ \nabla(\nabla g^{-1}) = 0, \]

\[ \nabla g^{-1}|_\mathcal{H} = 0, \]

\[ \bar{\partial}A - \partial \bar{A} + [A, \bar{A}] = 0. \]

where

\[ \nabla = \partial + \bar{A}, \ \nabla = \partial + A. \]

Under the gauge symmetry, the WZNW primary fields \( \Phi_i \) and the gauge fields \( A, \bar{A} \) transform respectively as follows:

\[ \delta \Phi_i = \epsilon^a (t_a^i + \bar{t}_a^i) \Phi_i, \]

\[ \delta A = -\partial \epsilon - [\epsilon, A], \]

\[ \delta \bar{A} = -\bar{\partial} \epsilon - [\epsilon, \bar{A}], \]

where \( t_a^i \in \mathcal{H} \).

In order to fix the gauge invariance, we impose the following condition:

\[ \bar{A} = 0. \]

Alternatively, one can choose any other gauge condition, say

\[ A = 0. \]

The physical sector of the quantum theory must not depend on the gauge choice. The gauge fixing (7) gives rise to the corresponding Faddeev-Popov ghosts with the action:

\[ S_{\text{ghost}} = \int d^2x \text{Tr}(b \partial c). \]

In the gauge (7), the equations of motion take the following form:

\[ \bar{\partial}J = 0, \]

\[ \bar{\partial}A = 0, \]

\[ J|_\mathcal{H} = 0. \]
where

\[ J = -\frac{k}{2} \partial gg^{-1} - \frac{k}{2} \partial Ag^{-1}. \] (11)

Thus, \( J \) is a holomorphic current in the gauge (7). Moreover, it has canonical commutation relations with the field \( g \) and itself:

\[ \{ J^a(w), g(z) \} = \ell^a g(z) \delta(w, z), \] (12)

\[ \{ J^a(w), J^b(z) \} = f^{abc} J^c(z) \delta(w, z) + \frac{k}{2} \delta^{ab} \delta'(w, z). \]

The given commutators follow from the symplectic structure of the gauged WZNW model in the gauge (7). In this gauge, the field \( A \) plays a role of the parameter \( \nu_0 \) of the orbit of the affine group \( \tilde{G} \). Therefore, the symplectic structure of the original model in the gauge (7) follows from the symplectic structure of the original WZNW model [4].

The crucial point is that there are residual symmetries which survive the gauge fixing (7). Under these symmetries the fields \( \Phi_i \) and the remaining field \( A \) transform according to

\[ \delta \Phi_i = (\epsilon_L^A \Phi_i + \epsilon_R^A \bar{\Phi}_i), \]

\[ \delta A = -\partial \epsilon_R - [\epsilon_R, A], \] (13)

where the parameters \( \epsilon_L \) and \( \epsilon_R \) are arbitrary holomorphic functions,

\[ \partial \epsilon_{L,R} = 0. \] (14)

In eqs. (13) the generators \( t^A \) act on the left index of \( \Phi_i \), whereas \( \bar{t}^a \) act on the right index of \( \Phi_i \). One can notice that the left residual group is extended to the whole group \( G \), whereas the right residual group is still the subgroup \( H \).

2. WARD IDENTITIES

Let us define dressed correlation functions

\[ \langle \cdots \rangle = \int D\tilde{A} D\tilde{A}^a \langle \cdots \rangle \exp[-\frac{k}{2\pi} \int \partial^2 z \]

\[ \times \text{Tr}\{ \tilde{A} g^{-1} \partial g + A \tilde{g} g^{-1} + A g \tilde{A} g^{-1} + A A \}, \]

where \( \langle \cdots \rangle \) is the correlation function before gauging, which is found as a solution to the Knizhnik-Zamolodchikov equation.

In the gauge (7), the dressed correlation functions can be presented as follows

\[ \langle \Phi_1(z_1, \bar{z}_1) \Phi_2(z_2, \bar{z}_2) \cdots \Phi_N(z_N, \bar{z}_N) \rangle = \int Dh Dc \exp(-S_{\text{ghost}}) \]

\[ \times \int Da \exp[-S_{\text{eff}}(A)] \]

\[ \times \int Dg \Phi_1(z_1, \bar{z}_1) \Phi_2(z_2, \bar{z}_2) \cdots \Phi_N(z_N, \bar{z}_N) \]

\[ \times \exp[-\Gamma(g, A)], \]

where \( S_{\text{eff}}(A) \) is the effective action of the field \( A \) and \( \Gamma(g, A) \) is formally identical to the original gauged WZNW action in the gauge (7). The action \( S_{\text{eff}} \) is non-local and can be obtained by integration of the following variation (which follows from the Wess-Zumino anomaly condition)

\[ \frac{\delta S_{\text{eff}}}{\delta A^a} + f^{abc} A^c \frac{\delta S_{\text{eff}}}{\delta A^b} = \tau \partial A^a. \] (17)

Here the constant \( \tau \) is to be defined from the consistency condition of the gauge (7), which is

\[ J_{\text{tot}} \equiv \frac{\delta Z}{\delta A^a} = 0, \quad a = 1, 2, \ldots, \text{dim} \, H, \] (18)

at \( \tilde{A} = 0 \). Here \( Z \) is the partition function of the gauged WZNW model. Condition (18) amounts to the vanishing of the central charge of the affine current \( J_{\text{tot}} \) which is a quantum analog of \( J_{\text{H}} \). This in turn means that \( J_{\text{H}} \) is a first class constraint 4. In order to use this constraint, we need to know the OPE of \( A \) with itself. This can be derived as follows. Let us consider the identity

\[ \frac{\delta \langle \partial A(z) A(z_1) \cdots A(z_N) \rangle}{\delta A^a(z)} = \int \text{DAA}(z_1) \cdots A(z_N) \]

\[ \times \left[ \partial \frac{\delta S_{\text{eff}}}{\delta A^a(z)} + f^{abc} A^c \frac{\delta S_{\text{eff}}}{\delta A^b(z)} \right] e^{-S_{\text{eff}}}. \] (19)

Here we used relation (17). Integrating by parts in the path integral, we arrive at the following formula

\[ \tau \langle A^a(z) A^{a_1}(z_1) \cdots A^{a_N}(z_N) \rangle \]

\[ = \frac{1}{2\pi i} \sum_{k=1}^{N} \frac{-\delta^{a_k}}{z-z_k} \langle A^{a_1}(z_1) \cdots \hat{A}^{a_k}(z_k) \cdots A^{a_N}(z_N) \rangle \]
A to the following rule

must be zero, we obtain the following relation eq. (16) under transformations (13). Because it

the expression for

Along with condition (18), the equation (22) gives the expression for \( \tau \)

where the nominator is understood as OPE (26).

\[
\frac{1}{z-z_k} = 2\pi i \delta(z-z_k). \tag{21}
\]

From eq. (20), it follows that

\[
\tau A^a(z)A^b(0) = \frac{1}{2\pi i} \left[ -\frac{\delta_{ab}}{z^2} + f^{abc} A^c(0) \right] + \text{reg}. \tag{22}
\]

Along with condition (18), the equation (22) gives the expression for \( \tau \)

\[
\tau = \frac{i(k + 2 c_V(H))}{4\pi}. \tag{23}
\]

We proceed to derive the Ward identity associated with the residual symmetry (13). The

Ward identity comes along from the variation of eq. (16) under transformations (13). Because it

must be zero, we obtain the following relation

\[
\sum_{k=1}^{N} \frac{t_k^a}{z-z_k} \delta(z, z_k) \langle \Phi_1(z_1, \bar{z}_1) \Phi_2(z_2, \bar{z}_2) \cdots \Phi_N(z_N, \bar{z}_N) \rangle \tag{24}
\]

\[
+ \tau \langle \partial \Phi_1(z_1, \bar{z}_1) \Phi_2(z_2, \bar{z}_2) \cdots \Phi_N(z_N, \bar{z}_N) \rangle = 0.
\]

This yields

\[
2\pi \tau \langle A^a(z) \Phi_1(z_1, \bar{z}_1) \Phi_2(z_2, \bar{z}_2) \cdots \Phi_N(z_N, \bar{z}_N) \rangle \tag{25}
\]

\[
= i \sum_{k=1}^{N} \frac{t_k^a}{z-z_k} \langle \Phi_1(z_1, \bar{z}_1) \Phi_2(z_2, \bar{z}_2) \cdots \Phi_N(z_N, \bar{z}_N) \rangle,
\]

which in turn gives rise to the OPE between the gauge field \( A^a \) and \( \Phi_i \)

\[
A^a(z) \Phi_i(0) = \frac{2}{k + 2 c_V(H)} \frac{t_i^a}{z} \Phi_i(0). \tag{26}
\]

Now we are in a position to define the product \( A(z) \Phi_i(z, \bar{z}) \). Indeed, we can define this according to the following rule

\[
A^a(z) \Phi_i(z, \bar{z}) = \oint \frac{d\zeta}{2\pi i} A^a(\zeta) \Phi_i(z, \bar{z}), \tag{27}
\]

where the nominator is understood as OPE (26).

Formula (27) is a definition of normal ordering for the product of two operators.

3. GAUGE DRESSING

Let us come back to equation (11). At the quantum level it can be presented in the following form

\[
\partial g + \eta g A + (2/\kappa) J g = 0. \tag{28}
\]

Here \( \eta \) and \( \kappa \) are some renormalization constants due to regularization of singular products \( gA \) and \( Jg \).

Variation of (28) under the residual symmetry with the parameter \( \epsilon_R \) gives rise to the following relation

\[
\left[ 1 - \eta \left( 1 - \frac{c_V(H)}{k + 2 c_V(H)} \right) \right] \partial \epsilon_R(z) g(z) = 0. \tag{29}
\]

From this relation we find the renormalization constant \( \eta \)

\[
\eta = \frac{k + 2 c_V(H)}{k + c_V(H)}. \tag{30}
\]

In the classical limit \( k \to \infty, \eta \to 1 \).

In the same fashion, we can compute variation of (28) with respect to the residual symmetry with the parameter \( \epsilon_L \). This fixes the constant \( \kappa \) as follows

\[
\kappa = \frac{1}{k + c_V(G)}. \tag{31}
\]

Note that the given expression for \( \kappa \) is consistent with the condition that the combination \( \partial + \eta A \) acted on \( g \) as a Virasoro generator \( L_{-1} \):

\[
L_{-1} g = \frac{2J^A_i t_i^A}{k + c_V(G)} g. \tag{32}
\]

All in all, with the regularization given by eq. (27) and the Ward identity (25) the equation (28) gives rise to the following differential equation

\[
\left\{ \frac{1}{2} \frac{\partial}{\partial \bar{z}_i} + \sum_{j \neq i}^{N} \left[ \frac{t_i^A t_j^A}{k + c_V(G)} - \frac{\bar{t}_i^A \bar{t}_j^A}{k + c_V(G)} \right] \frac{1}{z_i - z_j} \right\}
\]

\[
\times \langle \Phi_1(z_1, \bar{z}_1) \Phi_2(z_2, \bar{z}_2) \cdots \Phi_N(z_N, \bar{z}_N) \rangle = 0, \tag{33}
\]

where \( t_i^A \in \mathcal{G} \) and \( \bar{t}_i^A \in \mathcal{H} \).

Note that one could have started with the gauge (8). In this case one would have derived
a similar equation for the antiholomorphic coordinate $\bar{z}$:
\begin{equation}
\frac{1}{2} \frac{\partial}{\partial \bar{z}_i} + \sum_{j \neq i}^N \left[ \frac{t_i^A t_j^A}{k + c_V(G)} - \frac{\bar{t}_i^A \bar{t}_j^A}{k + c_V(H)} \right] \frac{1}{\bar{z}_i - \bar{z}_j}
\end{equation}

\times \langle \langle \Phi_1(z_1, \bar{z}_1) \Phi_2(z_2, \bar{z}_2) \ldots \Phi_N(z_N, \bar{z}_N) \rangle \rangle = 0. \tag{34}

Because correlation functions do not depend on the gauge, we arrive at the conclusion that $\langle \langle \Phi_1(z_1, \bar{z}_1) \Phi_2(z_2, \bar{z}_2) \ldots \Phi_N(z_N, \bar{z}_N) \rangle \rangle$ obey both equation (33) and (34).

Equations (33) and (34) are our main result. By solving them, one can find dressed correlation functions in the gauged WZNW model. The solution can be expressed as products of correlation functions in the gauged WZNW model. The solution has the following form

\begin{equation}
\langle \langle \Phi(z, \bar{z}) \Phi(0) \rangle \rangle
\end{equation}

\begin{equation}
= -2 \left[ \frac{t_i^A t_j^A}{k + c_V(G)} - \frac{\bar{t}_i^A \bar{t}_j^A}{k + c_V(H)} \right] \frac{1}{z} \langle \langle \Phi(z, \bar{z}) \Phi(0) \rangle \rangle.
\end{equation}

By the projective symmetry, the two-point function has the following form

\begin{equation}
\langle \langle \Phi(z, \bar{z}) \Phi(0) \rangle \rangle = \frac{G_{ij}}{|z|^{4\Delta_i}},
\end{equation}

where $\Delta_i$ is the anomalous conformal dimension of $\Phi_i$ after the gauge dressing and $G_{ij}$ is the Zamolodchikov metric which can be diagonalized. After substitution of expression (36) into eq. (35), and using the fact that, as a consequence of the residual symmetry (13), the dressed correlation functions must be singlets of both the left residual group $G$ and the right residual group $H$, we find

\begin{equation}
\Delta_i = \frac{c_i(G)}{k + c_V(G)} - \frac{c_i(H)}{k + c_V(H)}.
\end{equation}

where $c_i(G) = t_i^A t_i^A$, $c_i(H) = \bar{t}_i^A \bar{t}_i^A$.

Eqs. (33) and (34) describe the gauge dressing of correlation functions of primary operators. In an ordinary WZNW model, correlators of primary operators contain complete information about the system as correlators of any descendant operators can be expressed in terms of correlation functions of primary operators alone \cite{8}. The point to be made is that descendents of the WZNW model may become primaries of the gauged theory. In the case of a gauged WZNW model, the Ward identities of the un-gauged theory are no longer applicable. Nevertheless, dressed correlation functions of descendant operators can still be represented in terms of dressed correlators of primary operators. This is due to the equation (28), which allows us to express $\partial_{gg}^{-1}$ as a combination of the affine current $J$ and the product $A^a \phi^{a\bar{a}}$, where $\phi^{a\bar{a}} = Tr t^a g^{-1} t^\alpha g$. Since we know that there are the Ward identities for $J$ and $A$, we can use them to derive a Ward identity for $\partial_{gg}^{-1}$, which will relate dressed correlation functions of descendents with correlators of primaries.

4. CONCLUSION

For the gauged WZNW models, which form a particular class of 2D gauged theories, we have derived differential equations which, in principle, allow one to find correlation functions of these models. In \cite{8} we have used our equations to show that the $SL(2)/U(1)$ conformal blocks can be expressed as products of $SL(2)$ and $U(1)$ conformal blocks. The detailed analysis of the dressed four-point function in the $SL(2)/U(1)$ coset construction can also be found in \cite{8}.

REFERENCES

1. K. Gawedzki and A. Kupiainen, Phys. Lett. B215 (1988) 119; Nucl. Phys. B320 (1989) 649; D. Karabali, Q.-H. Park, H. J. Schnitzer and Z. Yang, Phys. Lett. B216 (1989) 307.
2. S. G. Naculich and H. J. Schnitzer, Nucl. Phys. B332 (1990) 583.
3. A. Polyakov, Mod. Phys. Lett. A2 (1987) 893; I. R. Klebanov, I. I. Kogan and A. M. Polyakov, Phys. Rev. Lett. 71 (1993) 3243.
4. A. Alekseev and S. Shatashvili, Nucl. Phys. B323 (1989) 719.
5. L. D. Faddeev, Quantum symmetry in CFT.
by Hamiltonian methods, Helsinki preprint, HU-TFT-92-5, 1992.

6. D. Karabali and H. J. Schnitzer, Nucl. Phys. B329 (1990) 649.

7. V. G. Knizhnik and A. B. Zamolodchikov, Nucl. Phys. B247 (1984) 83.

8. I. I. Kogan, A. Lewis and Oleg A. Soloviev, *Gauge dressing of 2D field theories*, to appear in Mod. Phys. Lett. A.