Highlights

**DFC: Deep Feature Consistency for Robust Point Cloud Registration**

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- Investigate the pairwise registration problem between two scans with local sparsity and partial correspondences.
- Assign weights for correspondences to complete robust and efficient registration.
- Focus on the consistency between features of the correspondences.
- Achieve state-of-the-art performance on real-world scenarios, compared with other advanced registration approaches.
DFC: Deep Feature Consistency for Robust Point Cloud Registration

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ARTICLE INFO

Keywords:
3D point cloud registration
Multiscale graph feature merging
Deep feature matching
Deep feature consistency

ABSTRACT

How to extract significant point cloud features and estimate the pose between them remains a challenging question, due to the inherent lack of structure and ambiguous order permutation of point clouds. Despite significant improvements in applying deep learning-based methods for most 3D computer vision tasks, such as object classification, object segmentation and point cloud registration, the consistency between features is still not attractive in existing learning-based pipelines. In this paper, we present a novel learning-based alignment network for complex alignment scenes, titled deep feature consistency and consisting of three main modules: a multiscale graph feature merging network for converting the geometric correspondence set into high-dimensional features, a correspondence weighting module for constructing multiple candidate inlier subsets, and a Procrustes approach named deep feature matching for giving a closed-form solution to estimate the relative pose. As the most important step of the deep feature matching module, the feature consistency matrix for each inlier subset is constructed to obtain its principal vectors as the inlier likelihoods of the corresponding subset. We comprehensively validate the robustness and effectiveness of our approach on both the 3DMatch dataset and the KITTI odometry dataset. For large indoor scenes, registration results on the 3DMatch dataset demonstrate that our method outperforms both the state-of-the-art traditional and learning-based methods. For KITTI outdoor scenes, our approach remains quite capable of lowering the transformation errors. We also explore its strong generalization capability over cross-datasets.

1. Introduction

Point cloud registration is playing an increasingly important role in many applications, including simultaneous localization and mapping (SLAM), 3D reconstruction and autonomous driving. Point clouds have many specific characteristics that may increase the complexity of registration problems, including local sparsity, noise caused by acquisition equipment and a large number of points. On the one hand, point cloud sparsity and noise make it unrealistic to find correct correspondences between the source and target point clouds. On the other hand, the considerable number of points inevitably requires efficient algorithms and large computing resources.

Traditional point cloud registration pipelines (Besl and McKay, 1992; Yang et al., 2016) start with a coarse initial pose obtained by odometers and iterate until the optimal condition is satisfied. However, the registration result is highly dependent on a good initial estimation, which directly tends to cause these pipelines to become stuck in local minima. To increase registration accuracy and efficiency, researchers have proposed learning-based algorithms to replace the individual parts in the classical registration pipeline, including feature descriptors (Choy et al., 2019; Wang and Solomon, 2019) and pose optimization algorithms (Zhou et al., 2016; Yang et al., 2016). Specifically, end-to-end registration networks, such as DCP (Wang and Solomon, 2019), PointNetLK (Aoki et al., 2019), and VCR-Net (Qiao et al., 2020), have gradually emerged in recent years. Compared with other classical registration pipelines (Besl and McKay, 1992; Yang et al., 2016; Fischler and Bolles, 1981), the high efficiency of end-to-end neural networks has been fully verified. However, the robustness and application ability of end-to-end registration pipelines cannot achieve the expected effect, especially in some complex scenes. In summary, 3D point cloud registration is still an extremely challenging topic in 3D computer vision due to the point cloud characteristics mentioned above.

In this work, we present a novel method titled deep feature consistency network to jointly solve the inlier correspondences and estimate the rigid transformation in the absence of initial transformation by leveraging the feature consistency during the feature matching stage.

The main ideas of this paper are as follows. First, a multiscale graph feature merging network is proposed to extract the features of the correspondence set. Second, to effectively filter the outliers, we present a correspondence weighting module to estimate the confidence of each correspondence based on their features and then select a series of correspondences with a high confidence level as candidate inliers based on the confidence. These candidate inliers form a series of inlier subsets in the feature space as the input to the subsequent deep feature matching module. Finally, we present a fresh deep feature matching module, which constructs the feature consistency matrix of each inlier subset in parallel, calculates the principal vectors of the feature consistency matrix with principal component analysis (PCA) (Wold et al., 1987), and then obtains the corresponding rigid transformation of each inlier subset by the weighted singular value decomposition (SVD) optimization method. The determination of the optimal rigid transformation among the above obtained rigid transformations ultimately depends on maximizing the geometric consistency.

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Z. Xu et al.: Preprint submitted to Elsevier
To summarize, the key contributions of our work are as follows.

First, we investigate the point cloud pairwise registration problem between two fragments with local sparsity and only a part of the correspondences.

Second, we present a novel feature embedding method called multiscale graph feature merging network. This method is used to extract the final features of the correspondence set and can well make full use of the geometric connection information between the nearest correspondences.

Third, we propose a deep feature matching module that is designed to predict the rigid transformation used to align the point clouds to boost the registration performance.

Experimentally, comparison results on the 3DMatch dataset reveal that our approach achieves state-of-the-art performance, compared with both classical (Fischler and Bolles, 1981; Besl and McKay, 1992) and learning-based approaches (Bai et al., 2021; Choy et al., 2020). In addition, our method shows strong generalization ability over different datasets.

2. Related Work

2.1. Feature Extraction

Generally, we have to take measures to extract point-wise features before performing registration. There are four mainstream methods for extracting point features. The first is to convert a point cloud into a volumetric representation and then to apply a 3D convolution neural network (CNN) to extract features (Wu et al., 2015; Maturana and Scherer, 2015). The volume representation retains relatively complete structural information of a point cloud, but this method is time-consuming and requires high computing memory costs. To this end, octree-based methods have been proposed to reduce computational costs (Wang et al., 2017; Riegler et al., 2017). The second method is called multiview-based methods (Su et al., 2015; Yu et al., 2018; Feng et al., 2018; Wei et al., 2020). These methods project an unstructured 3D point cloud into pixel-based 2D maps (e.g., LiDAR front view, bird’s eye view (BEV), and the spherical map) and then use a well-established 2D-CNN to extract map features and fuse mapwise features from different view maps. The third method is to learn features directly from raw point clouds without any voxelization or projection. PointNet (Charles et al., 2017) was the first work to take raw point clouds as input. Specifically, PointNet extracts pointwise features with a multilayer perceptron (MLP) layer and then uses a max-pooling function to generate global features. As a pioneering work, PointNet achieves state-of-the-art performance on the classification and segmentation task. However, this approach ignores local structural relationships between keypoints. Therefore, [36] proposed another hierarchical network, PointNet++, to obtain geometric structure information from the nearest neighbor of each point. The last is graph-based methods. Graph-based networks treat each point as graph vertices of a graph, and generate edges based on the nearest neighbors of each point. [46] proposed an unsupervised multitask algorithm DGCNN that constructs a local graph neural network and applies channelwise symmetric aggregation onto edges to connect the neighbors of each point.

2.2. Outlier Removal

Due to the acquisition equipment and the environmental noise where the target object is located, the correspondences inevitably contain noise, which may cause some correspondences to become outliers. The existence of outliers may reduce the point cloud alignment accuracy, so it is necessary to take measures to filter out these outliers. The task of filtering outliers is also called inlier/outlier classification (Pais et al., 2020), where a correspondence is identified as whether an inlier or an outlier according to a specific criterion.

The traditional outlier removal methods include Random SAmple Consensus (RANSAC) (Fischler and Bolles, 1981) and its variants (Barath and Matas, 2018). The RANSAC method iteratively samples a small set of correspondences to ensure that the outliers are filtered out as much as possible. Other methods accomplish the task of outlier removal based on branch-and-bound (BnB) (Yang et al., 2016), semidefinite programming (SDP) (Le et al., 2019; Ahmed et al., 2019) or maximum clique schemes (Bustos et al., 2019; Perera and Barnes, 2012). These methods generally require more sampling iterations and higher computational costs. However, the robustness of FGR (Zhou et al., 2016) and TEASER (Yang et al., 2021) remains strong in the presence of high outlier rates. The main learning-based outlier filtering schemes are DGR (Choy et al., 2020) and 3DRegNet (Pais et al., 2020). The DGR algorithm uses a 6D convolutional network to classify the inliers and outliers, while the 3DRegNet algorithm uses multilayer ResNets as the classifier.

2.3. Point Cloud Registration

The iterative closest point (ICP) (Besl and McKay, 1992), which alternately performs correspondence searching and least squares optimization to update the alignment state, is the best-known algorithm used for solving rigid registration problems. The literature (Pomerleau et al., 2015; Rusinkiewicz and Levoy, 2001) summarized ICP and its variants in the last 20 years. The performance of the ICP algorithm is highly dependent on the accuracy of the initial estimated pose. However, the initial estimated information obtained from the odometers is not always reliable and may easily fall into local optima. To find an optimal transformation with ICP, [52] proposed the Go-ICP algorithm to determine the global optimal poses. It outperforms the ICP algorithm when point cloud registration requires the provision of a globally optimal solution. In addition, another algorithms attempt to apply convex relaxation (Maron et al., 2016), Riemannian optimization (Rosen et al., 2019), graph-matching-based correspondence search method (Chang et al., 2020), and mixed-integer programming (Izatt et al., 2020) to determine the global optimal pose, but these methods are computationally expensive and cannot meet the practical application requirements well.
In recent years, the application of deep learning has made great progress in point cloud registration. PointNetLK (Aoki et al., 2019) combines global feature descriptors based on PointNet (Charles et al., 2017) and the Lucas/Kanade optimization algorithm (Lucas and Kanade, 1981) and then iteratively solves the relative rigid transformation. DGR (Choy et al., 2020) uses a ConvNet to estimate the inlier likelihood of each correspondence and then applies a weighted Procrustes method to align point clouds. However, 3D spatial relations are omitted in these registration pipelines (Choy et al., 2020; Fischler and Bolles, 1981). To focus on leveraging the spatial consistency in outlier rejection, [4] proposes a spatial-consistency guided nonlocal module for geometric feature embedding of the correspondences and then uses a neural spectral matching (NSM) module to compute the rigid transform for each seed. Different from the above networks, Predator (Huang et al., 2021) uses a parallel encode-decode structure and proposes a deep attention mechanism for the overlapping regions to exchange information about two unaligned point clouds.

3. Problem Formulation

In general, point cloud fragments are obtained with the light detection and ranging (LiDAR) scanners by receiving laser beams reflected by objects in the surrounding environment (An et al., 2022). The task to align two or more point clouds by estimating the relative transformation between them is named the point cloud registration or pose estimation. Given two point clouds:

\[ P = \{x_i, i = 1, 2, \cdots, M \} \]
\[ Q = \{y_i, i = 1, 2, \cdots, N \} \]

where \( P \) and \( Q \) denote the source and target point clouds, respectively, and \( x_i \in \mathbb{R}^3 \) and \( y_i \in \mathbb{R}^3 \) are the 3D point coordinates of the source and target point cloud fragments. For ease of notation, we only describe the simplest case of point cloud registration, in which \( M = N \) and \( \{(x_i, y_i)\}_{i=1}^N \).

The object of point cloud registration is to estimate the relatively rigid transformation that can correctly align two point clouds. We denote the rigid transformation as \([R, t]\), which can be represented as follows:

\[
[R, t] = \arg\min_{R, t} \frac{1}{N} \sum_{i=1}^{N} w_i \| Rx_i + t - y_i \|^2
\]

in which \( R \in SO(3) \) and \( t \in \mathbb{R}^3 \) denotes the rotation matrix and the translation vector, \((x_i, y_i)\) is a pair of matched correspondence points, and \( w_i \) indicates an inlier likelihood for a certain correspondence \((x_i, y_i)\).

First, the weighted centroids of \( P \) and \( Q \) are defined as:

\[
\bar{x} = \frac{\sum_{i=1}^{N} w_i x_i}{\sum_{i=1}^{N} w_i}, \quad \bar{y} = \frac{\sum_{i=1}^{N} w_i y_i}{\sum_{i=1}^{N} w_i}
\]

Then, the next step is to compute cross-covariance matrix \( H \):

\[
H = \sum_{i=1}^{N} w_i (x_i - \bar{x})(y_i - \bar{y})^T
\]

Last, we need to use singular value decomposition(SVD) method to decompose \( H \):

\[
[U, S, V] = \text{SVD}(H)
\]

Eq. 3 gives a closed-form solution to the rigid transformation by minimizing the mean-square error (MSE) function:

\[
E_1 = \frac{1}{N} \sum_{i=1}^{N} w_i \| Rx_i + t - y_i \|^2
\]
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Fig. 2. Overall framework of the proposed DFC pipeline. It takes the correspondence set $\mathcal{M}$ as input, outputs the best rigid transformation and classifies the correspondences as inliers and outliers.

The rigid transformation $[R, t]$ can be obtained as follows:

$$R = \text{V} \text{diag}(1, 1, \ldots, \text{det}(\text{VU}^T)) \text{U}^T \quad (8)$$

$$t = -R \bar{x} + \bar{y} \quad (9)$$

where $\text{det}(\cdot)$ denotes the determinant.

4. Deep Feature Consistency

Having demonstrated preliminaries about point cloud registration, we are now equipped to present the architecture of the proposed deep feature consistency, abbreviated as DFC. The overall architecture of our DFC pipeline is shown in Fig. 2 with three modules for feature embedding, correspondence weighting and deep feature matching. The goal of the proposed DFC method for registration is to provide an excellent end-to-end solution for correctly classifying correspondences into outliers/inliers and estimating the relative transformation between two unaligned point clouds in the absence of initial transformation prediction.

In summary, our pairwise registration pipeline begins with embedding deep features of the putative correspondences into high dimensional space. Then, we apply the correspondence weighting module to predict the veracity of each correspondence and deep feature matching. The goal of the proposed DFC method for registration is to provide an excellent end-to-end solution for correctly classifying correspondences into outliers/inliers and estimating the relative transformation between two unaligned point clouds in the absence of initial transformation prediction.

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4.1. Feature Embedding

The input of other state-of-the-art learning-based registration methods such as PointNetLK (Aoki et al., 2019), DCP (Wang and Solomon, 2019) and DeepVCP (Lu et al., 2019) only includes the coordinates of the key points located in the source and target point clouds, i.e., these methods independently extract the key-point features, thus ignoring the internal relation of the correspondences. In contrast to these methods, the input of our DFC method is not only the
key points, but also the correspondence set $\mathcal{M}$:

$$\mathcal{M} = \{(x_i, \arg \min_{y_j} \|f_{x_i} - f_{y_j}\|^2) | i, j \in [1, 2, \cdots, N]\}$$

(12)

where $N$ denotes the number of correspondences. A correspondence $(x_i, y_j)$ can be represented as a specific point in 6-dimensional space $[x^T_i, y^T_j] \in \mathbb{R}^6$ (Choy et al., 2020). Therefore, it is out of question that we can consider embedding features of all correspondences along with the idea of pointwise feature extraction.

The process of generating the correspondence set can filter out the noise at a certain rate (Zhou et al., 2016), thus effectively improving registration performance. It has been proven that fusing feature maps from different scales can significantly improve the performance on a set of computer vision tasks (Hou et al., 2017; Lin et al., 2017; Liang et al., 2021; Choy et al., 2019; Li et al., 2019), so we follow the idea of these excellent methods to present multiscale graph feature merging (GFM) network, which is designed for embedding features of the putative correspondences and compensating for the disadvantage of FCGF feature descriptors without geometric information of correspondences. The overall structure of the multiscale GFM network is shown in Fig. 3, which consists of the graph neural network (GNN) (Wang et al., 2019) and multiscale feature merging (FM) block.

**Graph Neural Network.** Let $m_i$ and $m_j$ denote a pair of adjacent correspondences, and $(i, j) \in \varepsilon$ the graph edge between $m_i$ and $m_j$. The correspondence $m_i$ is used as the input of the GNN layer to obtain its graph feature representation using the $k$-nearest neighbor ($k$-NN) in Euclidean space:

$$F^e_{g_{m_i}} = h_\theta (m_i, m_j) = \tilde{h}_\theta (m_i, m_j - m_i)$$

(13)

where the size of $F^e_{g_{m_i}}$ is $12 \times 1 \times 100$ and $\tilde{h}_\theta (\cdot)$ denotes a nonlinear function with a series of learnable parameters $\theta$.

By performing such steps for each correspondence among $\mathcal{M}$ in parallel, the final GNN features of all correspondences can be gained as $F^e_{g}$ with the size of $12 \times N \times 100$.

**Multiscale Feature Merging.** The multiscale FM block consists of three scale layers, each of which consists of two blocks of a $1 \times 1$ convolution function followed by batchnormalization (BN) (Ioffe and Szegedy, 2015) and ReLU activation (Nair and Hinton, 2010). The outputs $F^m_{g}$ of GNN are then fed to the multiscale FM block. For the sake of description, let $F^m_{g_1}, F^m_{g_2}$ and $F^m_{g_3}$ define the different output graph features of the three scale layers, respectively. The spatial size between the last adjacent scale layers is reduced by half with stride 2, and the channel number between the first and second scale layers is always maintained at 64 while it becomes twice as large as 128 under the third scale layer. To smoothly merge the graph features of the three scale layers, the feature maps $F^m_{g_1}$ and $F^m_{g_2}$ are upsampled to the same spatial size as $F^m_{g_3}$ is. Finally, the graph features from the three scale layers mentioned above are merged and fed into another convolution with a $1 \times 1$ kernel and 256 channels, followed by BN and ReLU layer to obtain the final features $F^m_{\mathcal{M}}$ with the size of $N \times D$, where $D$ means the feature embedding dims and is set to 256.

Let $F^m_{\mathcal{M}} = \{f_1, f_2, \cdots, f_N\}$ denote the final features of $\mathcal{M}$ and $f_i$ represent a feature representation for the $i$-th correspondence $(x_i, y_i)$. The feature representations $F^m_{\mathcal{M}}$ contain both high-level semantic information and low-level detail, which allows us to set a better balance between invariance and discriminability (Liang et al., 2021).
the candidate inliers have much more probabilities to be good candidates, which can better guarantee the registration success ratio.

As shown in Fig. 2, First, we decide to apply an MLP to estimate a likelihood for each correspondence among $\mathcal{M}$ based on the features $\mathcal{F}_M$ extracted in the previous multi-scale GFM network. This estimation can be viewed as likely if a correspondence paired is an inlier. The higher the confidence predicted by the MLP, the more likely the counterpart correspondence may be a good candidate to be an inlier. As the core of the correspondence weighting module, the MLP layer is composed of the three fully connected layers, where the first two layers consist of a convolution function followed by ReLU, and the last layer only consists of the convolution function, excluding ReLU activation. Then in the sampling step, we select a series of correspondences $S$ ($S \subseteq \mathcal{M}$) based on the confidence ranking in descending order by direct one-shot sampling, unlike the iterative sampling optimization of the RANSAC method. Here, we finally determine that the total sampling number $N_S$ of $S$ is set to 200. These selected correspondences have many unique characteristics that may enhance the registration performance, including high confidence and wide distribution. These selected correspondences are referred to as candidate inliers for the sake of distinction.

In the process of sampling candidate inliers, the correspondences among $\mathcal{M}$ with low confidence are identified as latent outliers, which completes the task of coarse filtering outliers. In the subsequent Sec. 5.1, we conduct experiments to explore the effectiveness of outlier removal and how it affects the alignment accuracy.

### 4.3. Deep Feature Matching

![Fig. 4. The detailed steps of the deep feature matching module used for estimating a rigid transformation for each candidate inlier subset.](image)

Obtaining a series of candidate inliers $S$ from the previous correspondence weighting module, we construct a series of candidate inlier subsets $C$ for each latent inlier in the feature space using the $k$-NN method, where $|C| = k$ and $k$ takes the value of 40. As shown in Fig. 4, the overall procedure of the deep feature matching module is divided into two steps: constructing a deep feature consistency matrix and estimating the rigid transformation by the weighted SVD method. First, we need to establish a deep feature consistency matrix to estimate the inlier probability for each candidate inlier subset among $C$ and then to apply the weighted SVD method to solve the corresponding rigid transformation.

The core of the deep feature matching module is constructing the feature consistency matrix $\mathbf{M}$ that derived from the literature (Leordeanu and Hebert, 2005) and its elements can be calculated according to Eq. 14:

$$e_{ij} = \left[1 - \frac{1}{\sigma^2} \left\| \mathbf{f}_i - \bar{\mathbf{f}}_j \right\|^2 \right]$$  \hspace{1cm} (14)

where $\mathbf{f}_i$ and $\bar{\mathbf{f}}_j$ are the L2-normalized feature vectors of $\mathbf{f}_i$ and $\mathbf{f}_j$, respectively, and $\sigma$ is a balanced parameter to control the sensitivity to the feature difference (Bai et al., 2021).

The calculation to solve $e_{ij}$ makes $\mathbf{M}$ nonnegative, serving as a role to ensure the consistency between the correspondence features. After computing $\mathbf{M}$ by Eq. 14, the PCA method is then used to estimate its principal vectors. Here we denote the principal vectors as $\mathbf{w} = \{w_1, w_2, \ldots, w_k\}$. In Sec. 5.4, we will give an ablation study on another method named eigenvalues to calculate principal vectors. The principal component $\mathbf{w}$ can be considered as the inlier probability corresponding to each candidate inlier subset. Once we compute the inlier probability, the rigid transformation can be completed with Eq. 3. Similarly, by performing the same steps for each candidate inlier subset in parallel, the rigid transformations $[\mathbf{R}, \mathbf{t}]$ corresponding to each subset can be obtained simultaneously.

### 4.4. Hypothesis Verification

The final step of the proposed DFC method is the same as the literature (Bai et al., 2021), i.e., to determine the optimal transformation $[\mathbf{R}^*, \mathbf{t}^*]$ among a series of rigid transformations $[\mathbf{R}, \mathbf{t}]$ generated by the deep feature matching module according to a certain rule that the number of inliers computed by each transformation maximizes. The process of choosing the optimal transformation can be accomplished by maximizing the following objective function:

$$E_2 = \max_{[\mathbf{R}, \mathbf{t}]} \sum_{i=1}^{k} \left[ \left\| \mathbf{R}x_i + \mathbf{t} - y_i \right\| < \tau \right]$$  \hspace{1cm} (15)

where $\tau$ denotes the given inlier threshold and $[\cdot]$ denotes the Iverson bracket. When $\left\| \mathbf{R}x_i + \mathbf{t} - y_i \right\|$ is less than the given threshold $\tau$, the correspondence $(x_i, y_i)$ is considered an inlier (labeled as one), and the total number of inliers will be increased by one; otherwise, $(x_i, y_i)$ will be identified as an incorrect correspondence (labeled as zero).

### 4.5. Loss Function

The loss function of the proposed DFC method consists of two independent loss terms called classification loss and transformation loss.

#### Classification Loss

The classification loss is a common metric to evaluate incorrect correspondences using binary cross entropy (BCE) (Pais et al., 2020; Choy et al., 2020):

$$\mathcal{L}_c = \text{BCE}(c, I)$$  \hspace{1cm} (16)
Transformation Loss. The transformation loss is commonly used to assess the agreement between the ground-truth rigid transformation $[R^g, t^g]$ and estimated rigid transformation $[R^e, t^e]$:

$$L_i = \| (R^e)^T R^g - I \|^2 + \| t^e - t^g \|^2$$ (17)

The total loss is a weighted sum of the above loss functions:

$$L = L_c + \lambda L_i$$ (18)

where $\lambda$ is a hyperparameter that can be manually set to balance these two losses.

5. Experiments

In this section, we analyze the robustness and generalization of the proposed DFC method in indoor, outdoor and multiway registration scenarios. As the name suggests, pairwise registration means estimating the rigid transformation between two point cloud scans as shown in Fig. 1, and multiway registration produces a final global reconstruction map and pose estimation for all point cloud fragments. Pairwise registration plays an extremely important role in the multiway registration task. Before performing multiway registration, we need to use a pairwise registration method to estimate the initial poses and then obtain the optimal poses with robust pose graph optimization.

For indoor alignment scenarios, we choose the 3DMatch dataset (Zeng et al., 2017) to evaluate the performance of our method, where the scans are composed of 3D point clouds from different real-world scenes, and these point cloud scans also contain ground-truth transformations computed by the RGB-D reconstruction system. To verify the generalization ability of our pipeline across different datasets, we conduct another cross-dataset experiment on the augmented ICL-NUMIM (Choi et al., 2015; Handa et al., 2014) to quantify the average trajectory error (ATE). In addition, we use KITTI odometry (Geiger et al., 2012) as a benchmark dataset for large outdoor alignment scenarios. However, there is no clear official division labels for train/val/test splits, so we are determined to divide the KITTI odometry benchmark into train/val/test sets following FCGF (Choy et al., 2019).

During training, we apply Gaussian noise with a standard deviation of 0.03, random rotations $\in [0^\circ, 360^\circ]$ around a random axis. All experiments are performed on the platform with one single NVIDIA RTX 2080Ti graphics card and Intel Xeon E5-2630 v3 CPU. We construct our model in PyTorch, train it on the previous platform for 100 epochs and set the batch size to 8.

5.1. Pairwise Registration

The scheme of the train/test splits is the same as (Choy et al., 2019; Choy et al., 2020; Bai et al., 2021), i.e., 54 scenes among the 3DMatch dataset are used for training and validation, and the remaining 8 scenes are used for testing. The hyperparameter $\lambda$ and the inlier threshold $r$ are set to $10^{-2}$ and 1cm, respectively. A voxelized 5cm grid is first used to downsample the point cloud data, and then the FCGF descriptors are applied to extract the pointwise features to prepare to construct the input correspondences.

In this section, we compare and analyze the registration results on the test split of 3DMatch (Zeng et al., 2017), which contains 8 different indoor scenes, as shown in Fig. 5. We further evaluate the performance of our method on the 3DMatch dataset by computing RR, RE and TE based on Eq. 20, respectively. After obtaining the initial rigid transformation, we attempt to use the ICP algorithm (Besl and McKay, 1992) for subsequent optimization of the initial predicted transformation during the testing stage. By a slight abuse of notation, we define our pipeline without subsequent ICP refinement as DFC-v1 and the full model with ICP as DFC. Fig. 6 summarizes the detailed statistics on each test scene. Our method outperforms the other advanced classical and learning-based methods in terms of recall ratio and reaches the lowest RE and TE on most scenes, which implies that our method has much more robustness when dealing with different real scenes. Evaluation Metrics. To make a fair comparison with other state-of-the-art registration methods (Wang and Solomon, 2019; Choy et al., 2020; Bai et al., 2021; Zhou et al., 2016), we adopt the following three evaluation metrics to evaluate the performance of the proposed DFC method:

(1) Rotation error (RE) and translation error (TE). RE and TE penalize errors between estimated poses and ground-truth poses:

$$\text{RE} \left(R^e, R^g\right) = \arccos \left(\frac{\text{Tr} \left((R^e)^{-1} R^g\right) - 1}{2}\right)$$ (19)

$$\text{TE} \left(t^e, t^g\right) = \|t^e - t^g\|$$ (20)

where $R^e$ and $t^e$ represent the ground-truth rotation and translation, respectively, and $\text{Tr} \left(\cdot\right)$ represents the trace of one certain matrix. It is worth noting that RE and TE are calculated only when two point clouds are successfully aligned. This is because two point clouds that fail to align will return an incorrect pose estimate that differs significantly from the ground-truth transformation and makes the predictions for RE and TE unreliable.

(2) Registration Recall (RR) (Choi et al., 2015). The recall ratio metric represents the percentage of successful pairwise alignments. This means successful alignment when TE and RE are less than some thresholds at the same time. For the 3DMatch benchmark, the pairwise alignment result can be regarded as successful if $\text{RE} < 15^\circ$ and $\text{TE} < 30\text{cm}$. Candidate inliers sampling. The second step of our pipeline is weighting and sampling a set of correspondences referred to as candidate latent inliers in one shot, instead
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Fig. 5. The registration visualization results of our registration method in 8 different scenes of the 3DMatch benchmark (Zeng et al., 2017).

(a) Kitchen  (b) Home1  (c) Home2  (d) Hotel1  (e) Hotel2  (f) Hotel3  (g) Study  (h) Lab

Table 1: The performance of our method with different candidate inlier sampling strategies.

| Metrics      | Nₖ |
|--------------|----|
| RR(%)        |    |
| RE(Deg)      |    |
| TE(cm)       |    |

Table 1 shows the comprehensive assessment results of the proposed DFC method with different numbers of samples. We conclude that the sampling strategy can greatly improve the registration recall.

Traditional methods. To compare with traditional registration methods, we choose five typical traditional algorithms as benchmarks: FGR (Zhou et al., 2016), RANSAC (Fischler and Bolles, 1981), GC-RANSAC (Barath and Matas, 2018), Point2Point-ICP and Point2Plane-ICP. All registration algorithms are implemented with the Open3D library, except for GC-RANSAC. The assessment results are shown in Table 2.

ICP variants, including point-to-point and point-to-plane ICP methods, fail in most indoor scenes, because the ICP algorithm is highly dependent on the initial pose estimation so that it easily falls into the local optimum and partly because the overlap ratio between 3D scans is low. The lower the overlap between two point clouds is, the more likely the point clouds fail to align. The performances of FGR, RANSAC and GC-RANSAC are better than that of the ICP variants. The FGR network achieves a recall as high as 78.62% when applied with FCGF feature descriptors, and even RANSAC reaches 91.99% registration recall. When aligning point clouds with 200k sampling iterations, RANSAC can still maintain strong robustness. This conclusion is absolutely different from the literature (Choy et al., 2020), which is due to the fast and compact characteristics of FCGF feature descriptors compared to classical FPFH descriptors. However, it is worth noting that our method is approximately 10 times faster than RANSAC-200k, and our method achieves much higher registration recall by a significant margin. Another method named GC-RANSAC only achieves a 91.68% recall ratio, and our method exceeds 92.98%.

Learning-based methods. In addition, we choose another three state-of-the-art learning-based algorithms named 3DRegNet (Pais et al., 2020), DGR (Choy et al., 2020) and PointDSC (Bai et al., 2021) as our comparison benchmarks, and the registration results of DGR are also recorded in the absence of a protection mechanism (i.e., the RANSAC algorithm is used to optimize the initial poses during the evaluation phase). The registration recall of our method, especially without applying the ICP algorithm to optimize the estimated initial poses, has already exceeded 1.24% compared to DGR. DGR only achieves a recall ratio of 86.2% in the absence of RANSAC optimization (called the safeguard mechanism). Compared with PointDSC, our method has a slightly higher registration recall than PointDSC after optimizing the estimated poses using the ICP algorithm, and the model runtime is only 0.08s without ICP refinement.

of randomly sampling minimal subsets iteratively, such as RANSAC (Fischler and Bolles, 1981). These selected candidate inliers are characterized by high probability, thus making them have higher probabilities of becoming inliers compared to other correspondences.

Table 1 shows the comprehensive assessment results of the proposed DFC method with different numbers of samples. We conclude that the sampling strategy can greatly improve the registration recall. To some extent, registration recall is higher when the number of candidate inlier samples is smaller; specifically, RR is optimal when Nₖ is set to 200, which indicates that taking a portion of the correspondences with high confidence as candidate inliers can greatly improve the alignment effect and filter out the outliers effectively. It may be possible to achieve even higher
### Table 2
Quantitative comparisons of different registration algorithms on the 3DMatch benchmark. Time excludes the construction of matched correspondences.

| Methods | RR(%) ↑ | RE(deg ↓) | TE(cm ↓) | Time(s) |
|---------|---------|-----------|----------|---------|
| FGR(Choy et al., 2016) | 85.50 | 2.28 | 7.28 | 0.42 |
| RANSAC-1k(Fischler and Bolles, 1981) | 87.83 | 2.25 | 7.40 | 0.15 |
| RANSAC-20k | 91.99 | 2.47 | 7.53 | 10.89 |
| GC-RANSAC-1M(Barath and Matas, 2018) | 91.68 | 2.29 | 7.09 | 0.42 |
| ICP(Point2Point)(Zhou et al., 2018) | 10.10 | 4.06 | 10.21 | 0.10 |
| ICP(Point2Plane)(Zhou et al., 2018) | 11.34 | 2.40 | 6.79 | 0.71 |
| DGR w/o safeguard | 85.20 | 2.58 | 7.73 | 0.70 |
| DGR(Choy et al., 2020) | 91.30 | 2.43 | 7.34 | 1.21 |
| PointDSC(Bai et al., 2021) | 93.28 | 2.06 | 6.55 | 0.09 |
| DFC-v1(Ours) | 92.54 | 2.04 | 6.56 | 0.08 |
| DFC(Ours) | 93.47 | 1.67 | 6.04 | 0.14 |

### Table 3
ATE(cm) on the augmented ICL-NUIM dataset with simulated depth noise. The last column shows the average ATE of all four scenes. For BAD-SLAM, this method fails in the scene "Living room 1", so we do not compute its average ATE.

| Method | Living1 | Living2 | Office1 | Office2 | Average |
|--------|---------|---------|---------|---------|---------|
| ElasticFusion(Whelan et al., 2015) | 66.61 | 24.33 | 13.04 | 35.02 | 34.75 |
| InfiniTAM(Kähler et al., 2016) | 46.07 | 73.64 | 113.8 | 105.2 | 84.68 |
| BAD-SLAM(Schops et al., 2019) | - | 40.41 | 18.53 | 26.34 | - |
| Multiway+FGR(Zhou et al., 2016) | 78.97 | 24.91 | 14.96 | 21.05 | 34.98 |
| Multiway+RANSAC(Fischler and Bolles, 1981) | 110.9 | 19.33 | 14.42 | 17.31 | 40.49 |
| Multiway+DGR(Choy et al., 2020) | 21.06 | 21.88 | 15.76 | 11.56 | 17.57 |
| Multiway+PointDSC(Bai et al., 2021) | 20.25 | 15.58 | 13.56 | 11.30 | 15.18 |
| Multiway+DFC(Ours) | **18.15** | **15.28** | **12.76** | **32.44** | **19.66** |

In conclusion, the DFC method proposed in this paper provides an efficient and robust registration method in terms of registration recall, and achieves a better balance between efficient computation and robustness at the same time.

### 5.2. Multiway Registration

To evaluate the generalization capability of our method to new datasets, we use the training model on the 3DMatch dataset again and then evaluate the performance of multiway registration on the augmented ICL-NUIM dataset, which is also called cross-dataset generalization capability analysis.

Following (Bai et al., 2021; Choy et al., 2020), we apply our method to roughly align all scan fragments and to obtain the initial poses and then trim the initial poses with multiway registration using the pose graph optimization algorithm which can be implemented with the open-source library Open3D (Zhou et al., 2018). To assess multiway registration, we measure the absolute trajectory error (ATE) on the augmented ICL-NUIM dataset with simulated depth noise, and the results are shown in Table 3. Compared with current state-of-the-art online SLAM methods and offline reconstruction systems, our method achieves the lowest level of ATE in the first three scenes.

### 5.3. Outdoor LIDAR Registration

Following (Choy et al., 2019), we use outdoor LiDAR scans from the KITTI odometry (Geiger et al., 2012) for registration. Similar to pairwise registration in indoor scenes, we set the inlier threshold \( \tau \) to 60 cm, downsample the point clouds using a voxelization filter with a 30 cm voxel and then extract the pointwise features by applying FCGF descriptors to form the correspondence set as the input of our pipeline. As illustrated in the literature (Choy et al., 2020; Bai et al., 2021; Choy et al., 2019), the thresholds of TE and RE are set to 60 cm and 5°, respectively. When both TE and RE are less than the abovementioned thresholds, the alignment of two point clouds in KITTI odometry can be considered successful. The quantified results of our network on the KITTI odometry dataset are shown in Table 4, and the visualization results are shown in Fig. 7. Although the recall ratio of our method is slightly below another learning-based method PointDSC, our network still appears to be strongly competitive in decreasing the transformation errors.

### 5.4. Ablation Studies

We designed some ablation experiments by dividing our pipeline into multiple modules and replacing each module with another different counterpart to study the role of each...
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Fig. 6. Registration results per scene of the 3DMatch benchmark. **Row 1-3:** Registration recall (higher is better), TE and RE measured on successfully aligned pairs (lower is better). Our method consistently performs better on most scenes. The last column is the average registration recall, TE and RE on all eight scenes. A certain missing bar means its value is zero and there are no successful alignments in the corresponding scene.

part. All experiments are performed on the same condition as the experiments in Sec 5.1.

**Ablation on feature embedding.** To discuss the effectiveness of the proposed multiscale GFM network, we design ablation experiments on the 3DMatch benchmark to make a comparison. We choose two methods: PointNet and DGCNN. The comparison results are shown in Table 5. PointNet (Charles et al., 2017) achieves state-of-the-art performance for classification and segmentation and provides a new research perspective on processing raw point clouds while DGCNN (Wang et al., 2019) constructs local graph geometric features by using a local neighborhood graph and convolution operations on edges. Our method performs consistently better with the proposed multiscale GFM network than PointNet and DGCNN, which provides strong evidence that our multiscale GFM module can help to improve the recall ratio.

**Ablation on principal vectors.** In this experiment, we subsequently design ablation experiments to explore whether the eigenvalue or PCA method is better for registration. We compare PCA and Eigenvector with both DFC-v1 and DFC networks on the 3DMatch benchmark. Table 6 shows both DFC-v1 and DFC perform better with PCA than Eigenvector. Therefore, we can conclude that the PCA algorithm is helpful to boost the registration performance.

6. Conclusion

We propose deep feature consistency, a learning-based framework for robust, accurate and efficient point cloud registration by jointly embedding the correspondence features with a multiscale GFM network, weighting and sampling
correspondences with a correspondence weighting module, and solving the rigid transformation for alignment with a deep feature matching module. The results on 3DMatch indoor scenes and KITTI odometry outdoor scenes show that our methodology outperforms the traditional and learning-based algorithms and can effectively and quickly align the point clouds. The low transformation errors and high robustness of our method make it attractive for many applications relying on the point cloud registration task. In a further extension of this work, we will explore new methods for improving the generalization capability in broader applications and attempt to extend our method to other 3D computer visual tasks, such as 3D reconstruction and mapping, object pose estimation.

Acknowledgements

This work was supported by the Yunnan provincial major science and technology special plan projects: digitization research and application demonstration of Yunnan characteristic industry, under Grant: 202002AD080001, and the Practice&Innovation Foundation for Professional Degree Graduates of Yunnan University, under Grant: 2021Y168.

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Fig. 7. Registration visualization examples on KITTI. Our method can effectively and accurately align the outdoor point cloud scans.

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