From Planck to GUT via Dimensional Transmutation

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Abstract

Consider a gauge singlet superfield $S$ coupled to a pair of adjoint fields in a SUSY-GUT. If the tree-level vacuum is flat in $S$, the vev $\langle S \rangle$ which defines the GUT scale will be determined via dimensional transmutation at a scale $M$ where the soft-breaking $(\text{mass})^2$ vanishes as a result of running from $M_P = (8\pi G_N)^{-1/2}$. Because of the large number of adjoint fields $N_A$ coupled to $S$, one finds that $M$ can be generically close to $M_{\text{GUT}} = 2 \times 10^{16}$ GeV:

$$M \simeq M_P \exp[-16\pi^2 \log(3/2)/(N_A + 4)\lambda^2],$$

where $\lambda$ is a Yukawa $\sim 0.7$. This work examines the symmetries and dynamical constraints required in a SUSY-GUT in order that the desired flatness in $S$ is achieved, and that this flatness may survive in a supergravity framework.
1 Introduction

The realization of a cogent supersymmetric grand unified theory (SUSY-GUT) has constituted an important goal in particle theory for well over a decade. Several impediments to achieving such a goal were already evident in the earliest papers on the subject [1, 2]. In SU(5), these consist of the lack of a mechanism within the theory to (a) lift the degeneracy of (supersymmetric) SU(5)-, SU(4)×U(1)-, and SU(3)×SU(2)×U(1)-invariant ground states and (b) implement a hierarchical splitting of the massless Higgs doublets from the massive triplets (D/T splitting) in the 5 + 5 representations. The lifting of the degeneracy is generally ascribed to the perturbation of the various ground states by the soft-breaking terms [1, 3], while the doublet-triplet splitting can be effected through a judicious choice of the Higgs sector [4].

When the SUSY-GUT is considered in the context of string theory, an additional problem emerges – the origin of the GUT scale as distinct from the Planck or string scale [5]. Generally speaking, the construction of the superpotential in string theory does not in any obvious way allow for the introduction of an additional scale $M$, as is manifest in the SU(5) superpotential

$$W = M \mathrm{Tr} A^2 + \lambda \mathrm{Tr} A^3,$$

where $A$ is the SU(5) adjoint chiral superfield. It is $M$ that sets the scale $M_{\text{GUT}} \simeq 10^{16} - 10^{17}$ GeV of SU(5)-breaking, and there is no obvious relation between $M_{\text{GUT}}$ and the Planck scale $M_P = (8\pi G_N)^{-1/2} = 2.44 \times 10^{18}$ GeV [5].

An obvious possibility is to consider, instead of (1)

$$W' = \lambda S \mathrm{Tr} A^2 + \lambda' \mathrm{Tr} A^3 + w(S),$$

where $S$ is an SU(5) singlet chiral superfield, and search for a mechanism which yields $\langle S \rangle \neq 0$, and $M = \lambda \langle S \rangle \sim M_{\text{GUT}}$. It is clear that, without additional input, (2) as it stands is not viable, since any $Z_N$ (or U(1)) invariance invoked in order to restrict $w(S)$ will allow $w(S) \sim S^3$, which, for arbitrary Yukawas, forces $\langle S \rangle = 0$ at tree level.

In this work, I will examine the possibility of generating $M_{\text{GUT}}$ from $M_P$ through radiative corrections in the soft-breaking sector, with a resulting dimensional trans-
mutation at the scale \( M \simeq M_{\text{GUT}} \). This will turn out to be possible, perhaps even inevitable, under certain simple, well-defined constraints placed on the superpotential. These constraints serve to effect the necessary flatness of the effective potential near the origin of \( S \), so that the minimum is free to wander off to the point of dimensional transmutation, \( \langle S \rangle \sim M_{\text{GUT}} \).

2 Toy Model

To illustrate some of the salient points, I will begin with the case of only two visible sector superfields, the singlet \( S \) and an adjoint \( A \). For the moment, I will impose a continuous \( R \)-symmetry where all superfields have \( R \)-character \( \frac{1}{3} \), so that all terms in the superpotential \( W \) are trilinear. In the case of where the GUT is SU(\( N \)) the most general superpotential consistent with these requirements is (omitting couplings)

\[
W = S \text{Tr} A^2 + \text{Tr} A^3 + S^3 .
\]

In the absence of soft-breaking, the vacuum is given by \( \langle S \rangle = \langle A \rangle = 0 \). With soft-breaking, even at a point where the soft-breaking \( m^2_S = 0 \), the vevs will be shifted to \( \langle S \rangle \sim \langle A \rangle \sim m_{3/2} \). This could be avoided if the \( S^3 \) term were absent. However, as remarked in the introduction, any symmetry prohibiting the \( S^3 \) term will also forbid the \( A^3 \) term. Let us accept this for now, so that one can impose a \( Z_4 \) symmetry with charges \( q_4(S) = 2, q_4(A) = 1 \). As a consequence of this and the \( R \)-symmetry, the superpotential is determined uniquely:

\[
W_0 = \lambda S \text{Tr} A^2 = \frac{1}{2} \lambda S \sum_{a=1}^{N_A} A_a A_a ,
\]

where \( N_A = N^2 - 1 \) is the dimension of the adjoint of SU(\( N \)). In this toy model, it is easy also to include SO(10) in the discussion, in which case \( N_A = 45 \).

At tree level, in the absence of soft-breaking, the vacuum corresponding to (4) is \( \langle A \rangle = 0 \) with \( \langle S \rangle \) undetermined. I now introduce the soft-breaking potential

\[
V_{\text{soft}} = m^2_S S^* S + m^2_A \sum_{a=1}^{N_A} A^*_a A_a + \frac{1}{2} \lambda A S \sum_{a=1}^{N_A} A_a A_a + h.c. + \frac{1}{2} M \sum_{a=1}^{N_A} \lambda^T_a \lambda_a ,
\]
where $\lambda_a$ is the adjoint gaugino, and the standard trilinear coupling parameter $A$ is (hopefully) not to be confused with the adjoint field $A_a$. This work will focus on the case where the RG evolution of $m_S^2$ down from $M_P$ leads to its vanishing at some scale $Q = M$. In that case the 1-loop improvement to the effective potential at scales near $M$ leads to the replacement in $V_{soft}$

$$m_S^2 S^* S \rightarrow m'^2 S^* S \ln(S^* S/M^2) \ ,$$

where $m'^2 = -\frac{1}{2} \left. \frac{d m_S^2}{dt} \right|_M$, $t \equiv \ln(M_P/Q)$. The potential to be minimized is

$$V = \left| \frac{\partial W}{\partial S} \right|^2 + \left| \frac{\partial W}{\partial A_a} \right|^2 + \frac{1}{2} m'^2 S^* S \ln(S^* S/M^2) + m_A^2 A_a^* A_a + \frac{1}{2} \lambda S A_a A_a + h.c. \ ,$$

where in accordance with $D$-flatness the adjoint field is chosen along one of the directions of the Cartan subalgebra. There is no sum on $a$ in Eq. (7).

It is a matter of algebra to see that even in the presence of the soft breaking, $V$ is minimized for $\langle A_a \rangle = 0$. However, $\langle S \rangle$ is now determined: one obtains $\langle S \rangle = M/\sqrt{e}$, so that although the gauge symmetry remains unbroken, the adjoint field grows a mass $\lambda M/\sqrt{e}$. This is dimensional transmutation, the breaking of scale invariance due to renormalization effects. It will now be seen that for generic choices of parameters, the RG equations will drive $m_S^2$ negative at a scale $M \sim M_{GUT}$.

The RG equations for this model are straightforward to obtain:

$$16\pi^2 \frac{dg}{dt} = - \left( \sum S_2(R) - 3C_2(adj) \right) g^3$$

$$16\pi^2 \frac{dM}{dt} = -2 \left( \sum S_2(R) - 3C_2(adj) \right) g^2 \tilde{M}$$

$$16\pi^2 \frac{d\lambda}{dt} = -\frac{1}{2} \lambda \left[ (\mathcal{N}_A + 4)\lambda^2 - 8g^2C_2(adj) \right]$$

$$16\pi^2 \frac{dA}{dt} = -\lambda \left[ (\mathcal{N}_A + 4)\lambda^2 A + 8g^2C_2(adj) \tilde{M} \right]$$

$$16\pi^2 \frac{dm_S^2}{dt} = -\mathcal{N}_A \lambda^2 (m_S^2 + 2m_A^2 + A^2)$$

$$16\pi^2 \frac{dm_A^2}{dt} = -2\lambda^2 (m_S^2 + 2m_A^2 + A^2) + 8g^2C_2(adj) \tilde{M}^2 \ ,$$

(8)
where $C_2(adj) = N$ for SU($N$), 8 for SO(10), and $S_2(R)$ is the Dynkin index of any gauge-coupled field. As defined previously, $N_A$ is the dimension of the adjoint. Standard initial conditions are imposed on the soft scalar masses: $m^2_S(M_P) = m^2_A(M_P) = m^2_0$. For simplicity of discussion, I will assume in all that follows that the quantity $\sum S_2(R)$ is such that the gauge coupling is essentially constant between $M_P$ and the scale $M$. As a result, the gaugino mass $\tilde{M}$ will also be constant.

Examination of the evolution equation for $m^2_S$ in (8) reveals immediately why dimensional transmutation is likely to occur at scale $M$ not far below $M_P$:

- there is a large factor of $N_A$ multiplying the right hand side:
- there is no gaugino contribution serving to retard the decrease of $m^2_S$ with momentum scale.

Neither of these properties characterize the $m^2_A$ equation, and both are directly tied to the gauge singlet nature of $S$. A simple analytic treatment is heuristic: take $A = \tilde{M} = 0$, $\lambda = constant$. (The latter will be strictly true only at the fixed point.) Then a simple integration of the last two of Eqs. (8) gives the solution

$$\frac{m^2_S}{m^2_0} = 1 - 3 \left( \frac{N_A}{N_A + 4} \right) \left( 1 - e^{-\kappa t} \right),$$

where $\kappa = ((N_A + 4)\lambda^2/16\pi^2)$. Thus, to a good approximation, $m^2_S = 0$ at $t_1 = \ln(3/2)/\kappa$, or

$$Q_1 = M = M_P \ e^{-16\pi^2 \log(3/2)/(N_A+4)\lambda^2}.$$  

(10)

Because of the large size of $N_A + 4$, the evolution to the point of dimensional transmutation is rapid: from Eq. (10), $M = M_{GUT}$ in SU(5) for $\lambda \simeq 0.7$.

For $A$, $\tilde{M} \neq 0$, some representative numbers can be given. With $g^2/4\pi = 1/24$, $A(M_P) = \tilde{M}(M_P) = m_0$, I find $m^2_S = 0$ at $M = M_{GUT}$ for $\lambda(M_P) = 0.57$ in the case of SU(5), and $\lambda(M_P) = 0.34$ in the case of SO(10). There are sizeable arrays of parameter space for which $M \simeq M_{GUT}$, and I will present more detail in the discussion of a more realistic model. Two points may be noted before proceeding:
• The GUT scale is triggered by dimensional transmutation in the soft-breaking sector, but it has no explicit or implicit dependence on $m_{3/2}$; it is essentially given by Eq. (10).

• Renormalizability is crucial to the dynamical mechanism proposed here. Thus it is unclear how to relax the requirement of continuous $R$-invariance so as to allow higher dimension operators (such as $(S \text{Tr} A^2)^n/M_P^{3(n-1)}$) which respect the $Z_4$ or $U(1)$ symmetry requirement. Such terms may also vitiate $F$-flatness in $S$. For this paper, I maintain the strict $R$-invariance of the superpotential.

3 Model with Gauge Symmetry Breaking

A more realistic model requires some mechanism for the breaking of SU($N$). (The SO(10) case will receive comment later.) As already noted, simply extending the original SU(5) model by letting $M \rightarrow S$ is not possible, since undesirable $S^3$ terms are then permitted in the superpotential. Instead, it is necessary to introduce a second adjoint $A'$, and take as the superpotential

$$W = 2\lambda S \text{Tr} AA' + 2\lambda' \text{Tr} A^2 A' ,$$

with the $Z_4$ assignments $(S, A, A') = (1, 1, 2)$. Once more, a continuous $R$-symmetry is imposed, with $R$-character = $\frac{1}{3}$ for all fields, which will forbid terms such as $M \text{Tr} A'^2$, as well as all higher dimensional operators consistent with the $Z_4$ symmetry.

At tree level, the vacuum configuration for SU(5) in the direction of the standard model (the “24” direction) is given by

$$\langle A \rangle = (\lambda/\lambda') \langle S \rangle \text{diag}(2, 2, 2, -3, -3), \quad \langle A' \rangle = 0 ,$$

with $\langle S \rangle$ undetermined.

The soft-breaking potential is now generalized to

$$V_{\text{soft}} = m_S^2 S^* S + \sum_{a=1}^{N_A} \left( m_A^2 A_a^* A_a + m_{A'}^2 A'_a A'^*_a + \lambda A A'_a A'^*_a + \text{h.c.} \right) + \frac{1}{2} A' \lambda' \sum_{a,b,c} d_{abc} A_a A_b A'_c + \text{h.c.} + \frac{1}{2} \sum_{a=1}^{N_A} \tilde{M}_a \lambda_a^T \lambda_a ,$$

(13)
where the $d^{abc}$ is the symmetric SU($N$) tensor. The RG equations for this model are:

\[
\begin{align*}
16\pi^2 \frac{d\lambda}{dt} &= \frac{1}{2} \lambda \left[ (N^2 + 3)\lambda^2 + 3N'\lambda'^2 - 8g^2 N \right] \\
16\pi^2 \frac{d\lambda'}{dt} &= \frac{1}{2} \lambda' \left[ 6\lambda^2 + 5N'\lambda'^2 - 12g^2 N \right] \\
16\pi^2 \frac{dA}{dt} &= 2 \left[ (N^2 + 1)\lambda^2 A + N'\lambda'^2 A' + 4g^2 N \tilde{M} \right] \\
16\pi^2 \frac{dA'}{dt} &= 2 \left[ \lambda^2 A + \frac{3}{2}N'\lambda'^2 A' + 6g^2 N \tilde{M} \right] \\
16\pi^2 \frac{dm_S^2}{dt} &= (N^2 - 1)\lambda^2 \left[ m_S^2 + 2(m_A^2 + m_A'^2 + A'^2) \right] \\
16\pi^2 \frac{dm_A^2}{dt} &= 2 \left[ (\lambda^2 m_S^2 + (\lambda^2 + 2N'\lambda'^2)m_A^2 + (\lambda^2 + N'\lambda'^2)m_A'^2, \\
&\quad + \lambda^2 A^2 + N'\lambda'^2 A'^2 - 4g^2 N \tilde{M}^2 \right] \\
16\pi^2 \frac{dm_A'^2}{dt} &= 2 \left[ (\lambda^2 m_S^2 + (\lambda^2 + N'\lambda'^2)m_A^2 + (\lambda^2 + \frac{1}{2}N'\lambda'^2)m_A'^2, \\
&\quad + \lambda^2 A^2 + \frac{1}{2}N'\lambda'^2 A'^2 - 4g^2 N \tilde{M}^2 \right], 
\end{align*}
\]

where $N' = (N^2 - 4)/N$. Once again, one notes the large $N_A = N^2 - 1$ factor, as well as the absence of the gaugino mass term on the R.H.S. of the $m_S^2$ equation in (14), allowing, as in the toy model, a rapid evolution of $m_S^2$ toward zero. In the present case, there are factors of $O(N)$ enhancing the decrease of $m_A^2, m_A'^2$ in descending from $M_P$. Nevertheless, unless $\tilde{M} = 0$ and $\lambda'(M_P) \geq 1.5$, these quantities will not be driven negative in the region above $10^{16}$ GeV.

**Numerical Study**

In Figure 1, I show some sample ranges of parameters which will give dimensional transmutation at $M = M_{\text{GUT}}$ in SU(5). The numerical data are presented as loci in the $\lambda(M_P)$-$\lambda'(M_P)$ space for the four sets of initial conditions $A(M_P) = A'(M_P) = (0, m_0)$, and $\tilde{M}(M_P) = (0, m_0)$. The gauge coupling $g^2/4\pi$ is again fixed at 1/24. The required values of the Yukawa $\lambda(M_P)$ are all in the range $0.4 - 0.6$, showing that the dynamics is effectively controlled by the physics already present in the toy model of the last section.
Figure 1: Loci in space of Yukawa couplings giving dimensional transmutation at $2 \times 10^{16}$ GeV, for various choices of gaugino mass $\tilde{M}(M_P)$ and trilinear parameters $A(M_P)$, $A'(M_P)$ at the Planck scale. Curve (a) : $(\tilde{M}, A, A') = (0.5, 0, 0) m_0$; (b) : $(\tilde{M}, A, A') = (0.5, 1, 1) m_0$; (c) : $(\tilde{M}, A, A') = (1, 0, 0) m_0$; (d) : $(\tilde{M}, A, A') = (1, 1, 1) m_0$.

It is straightforward to check the spectrum of this model at scales $Q < M$: There are 24 Dirac spinors and superpartners with GUT-scale masses, and one light standard model singlet chiral field. The scalar component of this field presents a potential Polonyi problem, which will receive some comment in the concluding section.
4 Effects of Supergravity

To what extent are the results presented here stable with respect to extension to local supersymmetry (supergravity)? In a $D$-flat direction, the tree-level potential in local supersymmetry (for the visible sector only) corresponding to a superpotential $W(Z_i)$ is given by

$$V_{\text{sugra}} = e^{K/M_P^2} \left[ \left( \frac{\partial W}{\partial Z_i} + \frac{\partial K}{\partial Z_i} \frac{W}{M_P^2} \right) (K^{-1})_{ij} \left( \frac{\partial W}{\partial Z_j} + \frac{\partial K}{\partial Z_j} \frac{W}{M_P^2} \right)^* - 3 \frac{W W^*}{M_P^2} \right],$$

where $K(Z_i, Z_i^*)$ is the Kähler potential and $K^{-1}$ is the inverse of the matrix $K_{ij} = \partial^2 K/\partial Z_i \partial Z_j$. The $R$-symmetry restriction to superpotentials of homogeneous degree 3 implies

$$\sum_i Z_i \frac{\partial W}{\partial Z_i} = 3W .$$

For a flat Kähler ($K = \sum_i Z_i Z_i^*$), one obtains on inserting (16) into (15)

$$V_{\text{sugra}} = \exp\left( \sum_i |Z_i|^2 / M_P^2 \right) \left[ \sum_i \left| \frac{\partial W}{\partial Z_i} \right|^2 + \left( 3 + \sum_i \frac{|Z_i|^2}{M_P^2} \right) \frac{|W|^2}{M_P^2} \right].$$

From (17), we find that $V_{\text{sugra}} \geq 0$. Eqs. (16) and (17) then ensure that the global symmetry condition $\partial W/\partial Z_i = 0$ provides a necessary and sufficient condition for the minimum ($V = 0$) to be obtained. From this, it follows that if $\langle S \rangle$ is not determined in the global theory (before soft-breaking), neither is it determined in the flat-Kähler local theory.

What about higher order terms in the Kähler potential? For an arbitrary Kähler, the $R$-symmetry (16) guarantees that $\partial V_{\text{sugra}}/\partial Z_i = V_{\text{sugra}} = 0$ at the ($S$-flat) field configuration corresponding to $\partial W/\partial Z_i = 0$; it does not, of course, guarantee that this field configuration provides a global minimum for the potential. There is an interesting case where it does: consider in the Toy Model of Section 2 a region of field space where $K = \rho + \frac{1}{2}a \rho^2 / M_P^2$, $\rho = S^* S + A^* A$. This is the $\text{U}(N)$ ($N = 2$) symmetric form suggested by graviton loop corrections [8]. If $a \geq 0$, then one can show that the minima of the global and local theories coincide, and $\langle S \rangle$ is still undetermined. Generally speaking, if $K$ is such as to destroy $S$-flatness, the vevs will be moved to $O(M_P)$, the
space will become anti-deSitter, and the entire $R$-symmetry must be dropped in order to cancel the resulting $O(M_P^4)$ cosmological constant.\footnote{The hidden sector does not, of course, respect an $R$-symmetry, because of the dual requirements of breaking SUSY and maintaining a zero cosmological constant.} For now, I will just assume that $K$ behaves in a manner such as to preserve the vacuum in the $S$-flat direction, and delay consideration of this point to future study. It must be noted, however, that even if $K$ behaves appropriately, the local theory is still not renormalizable, so that the dimensional transmutation requires ignoring the gravitational strength interactions in obtaining the running of the soft parameters.

5 Summary and Remarks

(1) In this work, I have demonstrated how the GUT scale $M_{\text{GUT}}$ could arise through dimensional transmutation at a scale $M$ where the soft-breaking $m_S^2$ of a gauge-singlet field $S$ becomes negative and a vev $\langle S \rangle = M/\sqrt{\epsilon}$ develops. The scale $M$ does not depend numerically on the SUSY-breaking scale $m_{3/2}$, and is of $O(M_{\text{GUT}})$ because the rate of decrease of $m_S^2$ on descending from $M_P$ is proportional to a large number, the dimension of the adjoint Higgs representation $A$. At the scale $M$, the adjoint develops a mass $\sim M$, and if there is self-coupling, a non-zero vev.

(2) This scenario requires $S$-flatness of the effective potential before radiative corrections. In this work, this has been implemented by two symmetries: a continuous $R$-symmetry which enforces all terms in the superpotential to be trilinear, and a discrete or continuous symmetry which forbids more than a linear dependence on $S$ for any term in the superpotential. Except for possible gravitational effects discussed above, the $R$-symmetry allows the theory to be renormalizable between $M_P$ and $M$, while the additional symmetry keeps $\langle S \rangle$ indeterminate at tree level, allowing dimensional transmutation to take place at the high scale $\sim M_{\text{GUT}}$.

(3) An extension to SO(10) of the second model discussed in this paper would require a third adjoint (or a symmetric $\bf{54}$) in order to create a trilinear term besides $SAA'$. Such an extension, and other non-trivial modifications (such as those required to ac-
commodate the doublet triplet splitting) are the subjects of future study. It should be noted that every field coupled to the singlet $S$ will tend to drive the transmutation scale $M$ closer to $M_p$. This will limit the number and dimension of such fields.

(4) Many SO(10) models require a set of heavy $\mathbf{16}+\overline{\mathbf{16}}$ pairs of superfields to effect the SO(10)$\rightarrow$SU(5) breaking, and in order to obtain realistic low energy Yukawa matrices $\mathbf{1}$. By allowing the singlet $S$ to couple to such pairs, the dimensional transmutation will automatically force them to grow a mass $M$.

(5) The development of a vev for $S$ will break the $Z_4$ (or U(1)) symmetry used in order to forbid the $S^3$ term. In the $Z_4$ case, the resulting domain walls can be rendered harmless by a period of post-GUT inflation. In the U(1) case, the undesirable GUT-scale axion $\mathbf{11}$ is not present if the U(1) is gauged. The final cosmological problem is presented by the scalar component of the field $S' = S - \langle S \rangle$, which has a mass $\sim |dm_S^2/dt|^{1/2} \sim m_{3/2}$. If these particles survive to the post-inflation era as a long-wavelength classical field with amplitude of $S' \sim O(M_{GUT})$, then the familiar Polonyi problem results $\mathbf{11}$. After the onset of inflation, $S'$ has a mass $\sim H \mathbf{12}$, and is localized at $S' = 0 \mathbf{12,13}$. Whether or not it remains localized depends on its Kähler couplings to the inflaton and to the fields of the hidden sector $\mathbf{14}$. Discussion of this awaits a fuller understanding of Planck scale physics.

Acknowledgement

I would like to thank Stuart Raby for helpful comments. This research was supported in part by Grant No PHY9411546 from the National Science Foundation.

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