Peak Effect of $J_c$ and Nonlinear $E − J$ Characteristics of High-$T_c$ Superconductors *

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The critical current density $J_c$ in high-$T_c$ superconductors (HTS) often shows a maximum at field far above the self-field. We study this peak effect (PE) with the nonlinear $I − V$ response of type-II superconductors and find analytical equation of $J_c$ in the dependence of field and temperature. This equation is compared with some experimental data of R-Ba-Cu-O single crystals with fair agreement.

PACS: 74.60.Jg, 74.60.Ge, 74.62.b, 74.72.-h

The vortex matter becomes mobile when the driving current density exceeds some critical value $J_c$. In high-$T_c$ superconductor RBa$_2$Cu$_3$O$_7$-$\delta$ (R=Y, Gd, Tm and Nd) $J_c$ often shows a maximum at applied magnetic fields far above the self-field. This peak effect is of great interest from both fundamental and technological aspects and studied extensively [1]-[6]. Scaling behavior in the form $J_c(B, T)/J_c(B_p, T) = f(B/B_p)$ with peak field $B_p$ has been observed in twinned and twin-free YBa$_2$Cu$_3$O$_7$-$\delta$ single crystals [3]. Prominent PE of $J_c$ with the scaling behavior of the pinning force $F_p$ in the form $F_p/F_{p\text{max}} = f(B/B_{irr})$ has been observed in NdBa$_2$Cu$_3$O$_7$-$\delta$ single crystals [4], where $B_{irr}$ is the irreversibility field. Gurevich and Vinokur showed that PE could be interpreted by the nonlinear transport properties of type-II superconductors with macroscopic inhomogeneities [5]. Otterlo et al. explained PE by phase transitions between a stiffer and softer vortex phase [6]. Nevertheless, comparison of models study with pertinent experimental data is still lacking. In present work we derive the analytical functional of $J_c(B)$ from the nonlinear materials equation of HTS and compare it with experiments.

One of the central issues of physics of the mixed state in type-II superconductors is thermally activated vortex creep characterized by highly nonlinear electric-current density ($E − J$) characteristics below the critical current density $J < J_c$. This kind of characteristics is usually expressed as

$$E(J) = J \rho f e^{-U(J,B,T)/kT}$$

(1)

with $\rho f \approx \rho_n B/\ objectively$ the flux-flow resistivity estimated by Bardeen and Stephen [3].

Different models used different types of $U(J)$ which are suggested to approximate the real barrier. For instance, the Anderson-Kim model with $U(J) = U_c(1 - 1/(\gamma J_{c0}))$ [3], the logarithmic barrier $U(J) = U_c \ln(1/(\gamma J_{c0}/J))$ [3], and the inverse power-law $U(J) = U_c \ln^\mu((\gamma J_{c0}/J) - 1)$ [11] [12]. It is recently shown [4] [5], if one makes a common modification to the different model barriers $U(J)$ as

$$U(J) \rightarrow U(J_p \equiv J - E(J)/\rho f)$$

(2)

The corresponding modified materials equation. (1) leads to a common nonlinear form as

$$y = x \exp[-\gamma(1 + y - x)^p]$$

(3)

with $x$ and $y$ the normalized current density and electric field respectively, $\gamma$ is a parameter characterizing the symmetry breaking of the pinned vortices system and $p$ is an exponent.

In connection with the Anderson-Kim model

$$E(J) = 2\nu_0 B \exp[-U_0 - W_V]/kT \sinh(W_L/kT)$$

or

$$E(J) = J \rho f \exp[-U_0 - W_V + W_L]/kT]$$

(4)

We have in Eq. (3) $p = 1$, $\gamma = U_0/kT$, $x = W_V/kT$ and $y = W_L/kT$. $W_V$ is the dissipation energy due to flux moving. From the result of Bardeen and Stephen [13]:

$$W_V = \eta \cdot v \cdot A = E(J) \cdot B \cdot A / \rho f$$

(5)

with the viscous drag coefficient $\eta = B \cdot B_{c2}/\rho_n = B^2/\rho f$. $A$ is the product of the volume of vortex bundles and the range of pinning force.

$W_L$ is the work done by Lorentz force.

$$W_L = J \cdot B \cdot A$$

(6)

The critical current density $J_c$ is defined by a certain criterion $E(J_c) \equiv E_c$. From upper equation. (3), (4) and (5) we get:

$$J_c = J_{c0}(1 - kT/\nu_0 B \ln\left(\frac{U_c E_c}{\rho f J_{c0}}\right))$$

(7)

*This work is supported by the Ministry of Science and Technology of China (NKBSF-G 19990640) and the Chinese NSF.
where $J_{c0}$ is the critical current density without the help of thermal activation.

$$J_{c0} = U_0/BA,$$

(8) and $\ln(\nu_0 B/E_c)$ is a slow varying function of the magnetic field $B$; therefore, we set $\ln(\nu_0 B/E_c) = \ln(E_0/E_c)$ as a constant. From Eq. (7) and according to the criterion about $E_c$, the irreversible field $B_{irr}$ can be defined as:

$$U_0(T, B_{irr}) = kT \ln(E_0/E_c).$$

When $B \approx B_{irr}$, $J_c \approx E_c/\rho_f$, the system will turn into flux flow region. The critical current density can be expressed as

$$J_c = J_{c0} \left[1 - \frac{U_0(T, B_{irr})}{U_0(T, B)} + \frac{E_c B_{irr}}{\rho_n BJ_{c0}}\right].$$

(9)

$$J_c = \alpha'(T) \left[1 - \zeta(C - 1)^m \left(\frac{B}{B_{irr}}\right)^{-l} \left(C - \frac{B}{B_{irr}}\right)^{-m}\right]^{-3\nu} \left(\frac{B}{B_{irr}}\right)^{l} \left(C - \frac{B}{B_{irr}}\right)^{-m} + \frac{E_c}{J_{c0}\rho_n b}$$

(12)

where $C \equiv B_{c2}/B_{irr}$ and $\alpha'(T) \equiv C^{-m} B_{irr}^{l} \alpha(T)$. The scaling form of the numerical solutions of Equation (12) are shown in Fig. 2 with prominent peaks form like that observed in Fig. 1. The comparison of Eq. (12) with the experimental data of Ref. [3] in the scaling form is shown in Fig. 2 where we see a fair agreement.

In summary, we show a common nonlinear electric field-current density $(E-J)$ characteristics equation. And from this equation we can get a clear expression of critical current density $J_c$. The widely observed peak effect can be well understood by this equation. Furthermore, we found that the peak effect under different temperature can be scaled as well.

This work is supported by the Ministry of Science and Technology of China (NKBRSF-G19990640) and the Chinese NSF.

We assume that the dependence of the pinning potential $U_0$ on temperature and field can be separated as

$$U_0(T, B) = \alpha(T) B^l (1 - b)^m, \ b \equiv B/B_{c2}.$$  

(10)

and note that the factor $BA$ in Eq. (8) has the form

$$BA \propto \left[1 - \frac{T}{T^*(B)}\right]^{-3\nu} \approx \left[1 - \zeta \left(\frac{B}{B_{irr}}\right)^{-l} \left(1 - \frac{b}{b^*}\right)^{-m}\right]^{-3\nu}$$

(11)

where $\zeta \equiv \alpha(T)/[\alpha(T^*(B))]$ is a slow variable and $b^* \equiv B_{irr}/B_{c2}$. Substitute Eqs. (10), (11) into (8), we have

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FIG. 1. The experimental data of the $J_c - B$ curves which is in Semi-logarithmic form [3].

FIG. 2. The numerical solutions of Eq. (12) with $3\nu = 2$, $l = 0.68$, $m = 9.8$.

FIG. 3. The scaling behavior of peak effect with $B/B_{irr}$ and $J_c/J_{cpeak}$. The lines are data from Fig.1 and the open symbols are datas from Fig.2.