Impact of sterile neutrinos in lepton flavour violating processes

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Abstract. We discuss charged lepton flavour violating processes occurring in minimal extensions of the Standard Model via the addition of sterile fermions. We firstly investigate the possibility of their indirect detection at a future high-luminosity Z-factory (such as FCC-ee). Rare decays such as $Z \rightarrow \ell^- \ell^+$ can indeed be complementary to low-energy (high-intensity) observables of lepton flavour violation. We further consider a sterile neutrino-induced charged lepton flavour violating process occurring in the presence of muonic atoms: their (Coulomb enhanced) decay into a pair of electrons $\mu^- e^- \rightarrow e^- e^-$. Our study reveals that, depending on their mass range and on the active-sterile mixing angles, sterile neutrinos can give significant contributions to the above mentioned observables, some of them even lying within present and future sensitivity of dedicated cLFV experiments and of FCC-ee.

1. Introduction

Several extensions of the Standard Model (SM) add sterile neutrinos to the particle content in order to account for neutrino masses and mixings. These models are further motivated by anomalous (oscillation) experimental results, as well as by certain indications from cosmology (see [1,2] and references therein). The existence of these sterile states may be investigated in many fronts, among them at high-energy colliders. Motivated by the design study for a high luminosity circular $e^+ e^-$ collider (called FCC-ee) [3], we investigate the prospects for searches for sterile neutrinos by means of rare charged lepton flavour violating (cLFV) $Z$ decays [4]. Moreover, sterile neutrinos can be - indirectly - searched for also at high-intensity facilities, via numerous possible manifestations of cLFV: in addition to radiative and three-body decays, rare muon transitions can take place in the presence of nuclei - when a $\mu^-$ is stopped in matter, it can be trapped, thus forming a “muonic atom”. Here we consider the Coulomb-enhanced decay of a muonic atom into a pair of electrons, $\mu^- e^- \rightarrow e^- e^-$ [5].

2. SM extensions via sterile states

The effective “3+1 model” A simple approach to address the impact of sterile fermions on rare cLFV $Z$ decays consists in considering a minimal model where only one sterile Majorana state is added to the three light active neutrinos of the SM. This allows for a generic evaluation of the impact of the sterile fermions for these processes. In this simple toy model, no assumption is made on the underlying mechanism of neutrino mass generation. The addition of an extra neutral fermion to the particle content translates into extra degrees of freedom: the mass of the
new sterile state \( m_4 \), three active-sterile mixing angles \( \theta_{i4} \), three new CP phases (two Dirac and one Majorana).

**Inverse Seesaw**  The Inverse Seesaw (ISS) mechanism [6] is an example of low-scale seesaw realisation which in full generality calls upon the introduction of at least two generations of SM singlets. Here, we consider the addition of three generations of right-handed (RH) neutrinos \( \nu_R \) and of extra \( SU(2) \) singlets fermions \( X_t \), to the SM particle content. Both \( \nu_R \) and \( X_t \) carry lepton number \( L = +1 \). The ISS Lagrangian is given by 
\[
\mathcal{L}_{\text{ISS}} = \mathcal{L}_{\text{SM}} - Y_{\nu_R}^i \bar{\nu}_R \bar{H}^\dagger L_i - M_{Rij} \bar{\nu}_{Ri} X_j - \frac{1}{2} \mu_{Xij} X_i X_j + \text{h.c.},
\]
where \( i, j = 1, 2, 3 \) are generation indices and \( \bar{H} = i\sigma_2 H^* \). Lepton number \( U(1)_L \) is broken only by the non-zero Majorana mass term \( \mu_X \), while the Dirac-type RH neutrino mass term \( M_R \) does conserve lepton number. In the \( (\nu_L, \nu_R, X)^T \) basis, and after the electroweak symmetry breaking, the (symmetric) \( 9 \times 9 \) neutrino mass matrix \( \mathcal{M} \) is given by
\[
\mathcal{M} = \begin{pmatrix}
0 & m_D^T & 0 \\
m_D & 0 & M_R \\
0 & M_R^T & \mu_X
\end{pmatrix}, \tag{1}
\]

with \( m_D = Y^{\nu} v \) the Dirac mass term, \( v \) being the vacuum expectation value of the SM Higgs boson. Under the assumption that \( \mu_X \ll m_D \ll M_R \), the diagonalization of \( \mathcal{M} \) leads to an effective Majorana mass matrix for the active (light) neutrinos [7], \( m_\nu \approx m_D^T M_R^{-1} \mu_X M_R^{-1} m_D \). The remaining six (mostly) sterile states form nearly degenerate pseudo-Dirac pairs. In our analysis, and for both hierarchies of the light neutrino spectrum, we scan over the following range for the sterile neutrino mass: \( 10^{-9} \text{ GeV} \lesssim m_4 \lesssim 10^6 \text{ GeV} \), while the active-sterile mixing angles are randomly varied in the interval \([0, 2\pi]\). All CP phases are also taken into account.

3. **Constraints on sterile neutrino extensions of the SM**

The introduction of sterile fermion states, which have a non-vanishing mixing to the active neutrinos, leads to a modification of the leptonic charged current Lagrangian: 
\[
-\mathcal{L}_{cc} = \frac{\sqrt{2}}{v} U^{ji} \bar{\ell}_j \gamma^\mu P_L \nu_i W^-_\mu + \text{c.c.},
\]
where \( U \) is the leptonic mixing matrix, \( i = 1, \ldots, n_\nu \) denotes the physical neutrino states and \( j = 1, \ldots, 3 \) the flavour of the charged leptons. In the standard case of three neutrino generations, \( U \) corresponds to the unitary matrix \( U_{\text{PMNS}} \). For \( n_\nu > 3 \), the mixing between the left-handed leptons, which we denote by \( \bar{U}_{\text{PMNS}} \), corresponds to a \( 3 \times 3 \) sub-block of \( U \), which can show some deviations from unitarity. One can parametrise [8] the \( \bar{U}_{\text{PMNS}} \) mixing matrix as \( U_{\text{PMNS}} \to \bar{U}_{\text{PMNS}} = (1 - \eta) U_{\text{PMNS}} \), where the matrix \( \eta \) encodes the deviation of the \( U_{\text{PMNS}} \) from unitarity [9,10], due to the presence of extra neutral fermion states. One can also introduce the invariant quantity \( \tilde{\eta} \), defined as \( \tilde{\eta} = 1 - |\text{Det}(\bar{U}_{\text{PMNS}})| \), particularly useful to illustrate the effect of the new active-sterile mixings (corresponding to a deviation from unitarity of the \( \bar{U}_{\text{PMNS}} \)) on several observables. The non-unitarity of \( \bar{U}_{\text{PMNS}} \) will induce a departure from the SM expected values of several observables. In turn, this is translated into a vast array of constraints which we will apply to our analysis (see details and references in [4]). We require compatibility with: the \( \nu \)-oscillation data best-fit intervals [11]; unitarity bounds on the (non-unitary) matrix \( \eta \) [12]; electroweak precision observables; LHC data on invisible Higgs decays; laboratory searches for monochromatic lines in the spectrum of muons from \( \pi^\pm \to \mu^\pm \nu \) decays; searches for neutrinoless double beta decay; leptonic and semileptonic decays of pseudoscalar mesons \( K, D, D_s, B \). Other than the rare decays occurring in the presence of nuclei, the new states can contribute to several charged lepton flavour violating processes such as \( \ell \to \ell' \gamma \).

\(^2\) In all cases we ensure that the perturbative unitary bound on the sterile masses and their couplings to the active states is respected.
\[ \ell \rightarrow \ell_1 \ell_1 \ell_2. \] We compute the contribution of the sterile states to all these observables imposing compatibility with the current experimental bounds. Finally, cosmological observations \cite{2} put severe constraints on sterile neutrinos with a mass below the GeV.

4. cLFV processes

Lepton-flavour changing \(Z\) decays These processes are forbidden in the SM due to the GIM mechanism \cite{13}, and their rates remain extremely small even when lepton mixing is introduced. The observation of such a rare decay would therefore serve as an indisputable evidence of new physics \cite{14,15}. The mixing in the neutral lepton sector induced by the sterile Majorana fermions also opens the possibility for flavour violation in \(Z\nu \mu_j\) interactions (flavour-changing neutral currents), coupling both the left- and right- handed components of the neutral fermions to the \(Z\) boson. Together with the charged-current LFV couplings, these interactions will induce an effective cLFV vertex \(Z \ell_i^\pm \ell_j^\mp\).

Decay of muonic atoms to \(e^-e^-\) pairs A new cLFV process was recently proposed in \cite{16}. It consists in the flavour violating decay of a bound \(\mu^-\) in a muonic atom into a pair of electrons, and has been identified as potentially complementary to other cLFV muon decays: \(\mu^- e^- \rightarrow e^- e^-\).

In the above transition, the initial states are a \(\mu^-\) and a 1s atomic \(e^-\), bound in the Coulomb field of a nucleus \cite{16}. Although the underlying sources of flavour violation giving rise to this observable are the same as those responsible for other non-radiative \(\mu - e\) transitions (such as \(\mu \rightarrow ee\)), the \(\mu^- e^- \rightarrow e^- e^-\) decay in a muonic atom offers significant advantages. For instance, the rate of this process can be enhanced due to the Coulomb attraction from the nucleus, which increases the overlap of the 1s electron and muon wavefunctions. The muonic atom decay rate is thus enhanced by a factor \(\sim (Z - 1)^3\), which can become important for nuclei with large atomic numbers. The \(\mu^- e^- \rightarrow e^- e^-\) process could be investigated by the COMET collaboration \cite{17} (possibly being part of its Phase II programme).

5. Results

cLFV \(Z\) decays

The prospects for the observation of cLFV \(Z\) decays in the ISS are summarised in the left plot of Fig. 1 by considering the values of \(\text{BR}(Z \rightarrow \ell_i^\pm \ell_j^\mp)\) in the \((\tilde{\eta}, (m_{4-9}))\) parameter space of this specific realisation, where \((m_{4-9})\) is the average of the absolute masses of the mostly sterile states, \((m_{4-9}) = \frac{1}{9} \sum_{i=4-9} |m_i|\). We identify as grey points the solutions which fail to comply with (at least) one of the constraints listed in Section 3. These results indicate that this ISS realisation can account for sizeable values of cLFV \(Z\)-decay branching ratios, within the reach of FCC-ee (whose expected sensitivity is \(\mathcal{O}(10^{-13})\)). As to the minimal extension of the SM by one sterile neutrino, the “3+1 model” can also account for values of \(\text{BR}(Z \rightarrow \ell_i^\pm \ell_j^\mp)\) within the sensitivity of a high luminosity \(Z\)-factory, such as the FCC-ee. Larger cLFV \(Z\) decay branching fractions (as large as \(\mathcal{O}(10^{-6})\)) cannot be reconciled with current bounds on low-energy cLFV processes. Indeed, sterile neutrinos also contribute via \(Z\) penguin diagrams to cLFV 3-body decays and \(\mu - e\) conversion in nuclei, which severely constrain the flavour violating \(Z \ell_i^\pm \ell_j^\mp\) vertex (see also \cite{15}). Moreover, the recent MEG result on \(\mu \rightarrow e\gamma\) also excludes important regions of the parameter space. These constraints are especially manifest in the case of \(Z \rightarrow e\mu\) decays, since the severe limits from \(\text{BR}(\mu \rightarrow 3e)\) and \(\text{CR}(\mu - e, \text{Au})\) typically preclude \(\text{BR}(Z \rightarrow e\mu) \gtrsim 10^{-13}\).

In the right plot of Fig. 1 we illustrate the complementary rôle of a high-luminosity \(Z\)-factory with respect to low-energy (high-intensity) cLFV dedicated experiments. We display the sterile neutrino contributions \(\text{BR}(Z \rightarrow \mu\tau)\) versus \(\text{BR}(\tau \rightarrow \mu\gamma)\). We depict in red the points that survive all other bounds but are typically disfavoured from standard cosmology arguments. Finally, blue points are in agreement with all imposed constraints. We further highlight in dark
Figure 1. On the left: maximal values (in log scale) of BR($Z \rightarrow \ell_1^{\pm} \ell_2^{\pm}$) in the ISS, on the ($\tilde{\eta}$, $\langle m_{4-9} \rangle$) parameter space (right) for a NH light neutrino spectrum, from larger (dark blue) to smaller (orange) values. Cyan denotes values of the branching fractions below $10^{-18}$. FCC-ee expected sensitivity is $O(10^{-13})$. On the right, BR($Z \rightarrow \mu \tau$) vs BR($\tau \rightarrow \mu \gamma$) in the “3+1 model”. The additional green vertical lines denote the current bounds (solid) and future sensitivity (dashed), and dark-yellow points denote an associated $|m_{ee}|$ within experimental reach.

yellow solutions which allow for a third complementary observable within future sensitivity, which is the effective neutrino mass in $0\nu2\beta$ decays.

**Decay of muonic atoms to $e^-e^-$ pairs**

As emphasised in the discussion of Section 4, the process’ rate can be significantly enhanced in large $Z$ atoms (in particular the contributions from contact interactions). We thus compare the prospects for two different nuclei; this is illustrated in Fig. 2 for Aluminium ($Z = 13$, dark blue) and Uranium ($Z = 92$, cyan). Grey points correspond to the violation of at least one experimental bound: the most stringent constraints arise, as expected, from $\mu \rightarrow eee$ (and also from CR($\mu^-e$, Au)). The Coulomb enhancement is clearly visible: should this process be included in COMET’s physics programme, the cLFV muonic atom decay should be within reach of COMET’s Phase II, even for light nuclei, such as Aluminium (in the regime $m_4 \gtrsim 200$ GeV); for heavier atoms, such as Uranium, branching ratios above $10^{-15}$ render this process experimentally accessible (a similar situation occurs for Lead nuclei - albeit suppressed by a factor $\sim 7/9$ when compared to Uranium).

6. Conclusions

We have considered two extensions of the SM which add to its particle content one or more sterile neutrinos. We have explored indirect searches for these sterile states at a future circular collider like FCC-ee, running close to the $Z$ mass threshold. We have considered the contribution of the sterile states to rare cLFV $Z$ decays in these two classes of models and discussed them taking into account a number of experimental and theoretical constraints. Among these, low-energy LFV observables like cLFV 3-body decays and $\mu-e$ conversion in nuclei impose strong constraints on the sterile neutrino induced BR($Z \rightarrow \ell_1^{\pm} \ell_2^{\pm}$). Our analysis emphasises the underlying synergy between a high-luminosity $Z$ factory and dedicated low-energy facilities: regions of the parameter space of both models can be probed via LFV $Z$ decays at FCC-ee, at low-energy cLFV dedicated facilities and also via searches for $0\nu2\beta$. Notably, FCC-ee could better probe LFV in the $\mu-\tau$ sector, in complementarity to the reach of low-energy experiments like COMET. We have further investigated the impact of sterile fermions on cLFV observables which occur in the presence of “muonic atoms” such as the (Coulomb enhanced) decay of muonic atoms into $e^-e^-$ pairs. The
Figure 2. Effective “3+1 model”: BR(µ−e− → e−e−, N) as a function of the mass of the mostly sterile state m4, for two distinct muonic atoms, Aluminium (dark blue) and Uranium (cyan). Grey points correspond to the violation of at least one experimental bound; dashed horizontal lines denote the future sensitivity of COMET (Phase I and II) [17].

experimental relevance of this observable is manifest even for the simple “3+1” toy model: sterile neutrinos with masses m4 ≳ 800 GeV, lead to BR(µ−e− → e−e−, Al) within the reach of COMET, and the contributions would be further enhanced for heavier atoms, such as Lead or Uranium, thus improving the experimental potential.

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