Dirac neutrinos in a $SU(2)$ left-right symmetric model

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(Dated: 08/07/2019)

Abstract

In a left-right symmetric model, with the scalar sector consisting of several bi-doublets and two doublets, neutrinos remain as Dirac fermions in all order in perturbation theory. Although with only two bi-doublets the neutrino masses need still a fine tuning, this is not the case when a third bi-doublet is added. One of the scalar doublet may be of the inert type since it is protected by the left-right symmetry.

PACS numbers: 12.60.Fr 12.15.-y 14.60.Pq

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I. INTRODUCTION

At present it is very well established that neutrinos are massive particles and that there is mixing in the leptonic charged currents [1]. However, the nature of neutrinos is still unknown. They may be purely Majorana (equal to their charge conjugated fields) at all order in perturbation theory, purely Dirac (different from their charge conjugated fields) at any order of perturbation theory, quasi-Dirac when two active (left-handed) Majorana neutrinos are mass degenerate, or pseudo-Dirac when the mass degeneration occurs with an active (left-handed) and a sterile (right-handed) neutrino, see [2, 3] and references therein. In the last two cases, the mass degeneracy occurs at tree level but quantum corrections usually imply an additional small Majorana mass and, eventually neutrinos become Majorana particles. In fact, it is difficult to keep the lepton number $L$ automatically conserved in most extensions of the standard model (SM) and neutrinos are in these models Majorana particles. With Majorana neutrinos it is possible to explain the smallness of their masses, even at the tree level, using the so-called type I and II seesaw mechanism in $SU(2)_L \otimes U(1)_Y$ models, if complex ($Y=2$) scalar triplets and right-handed neutrinos $\nu_R$ are added [4, 5]. These mechanism can be implemented, for instance if new physics does exist at the TeV scale, in the context of models with $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ [6], in unified theories [7], and in models with $SU(n)_F \otimes SU(2) \otimes U(1)_Y$ symmetries [8]. The type III seesaw mechanism requires the introduction of a self conjugate ($Y=0$) triplet of fermions of $SU(2)$ and can be implemented in $SU(2) \otimes U(1)_y$ or $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ models [9].

One of the most motivated extensions of the electroweak standard model (ESM) are those with $SU(2)_L \otimes SU(2)_R \otimes U'(1)$ gauge symmetry with left-right parity symmetry [10–12]. We will call them LR symmetric models for short. Although it is possible to introduce instead of a parity a generalized charge conjugation symmetry [13] here we will consider only the case of parity. In particular, in these models the parity may be spontaneously broken [14] and, moreover the $U'(1)$ factor can be identify with $B-L$ allowing to implement quarks and leptons correspondence since they are only distinguished by the $B-L$ quantum number [6, 15]. We must bear in mind that this correspondence would be stronger if neutrinos were Dirac particles. However, in the minimal LR model with one bi-doublet and two doublets the smallness of the neutrino masses is not easily explained [16]. In the context of this model quarks and leptons are those of the SM plus three right-handed neutrinos which are
incorporated naturally in a doublet together the right-handed charged leptons.

In LR symmetric models Majorana neutrinos and the seesaw mechanism are obtained if, instead of scalar doublets $\chi_{L,R}$, scalar triplets $\Delta_{L,R}$ are introduced. However, as we said above, we already do not known the nature of neutrinos purely Dirac or Majorana (with or without seesaw mechanism). Hence, we may wonder ourselves, what would happen if the neutrinos are, in reality, pure Dirac fermions? In addition, can we have Dirac neutrinos if the only additional neutral fermions were right-handed neutrinos? After all it would be interesting if the lepton-quark correspondence is maintained when all particle gain masses but this implies that neutrinos have to be Dirac fermions. In LR models with the scalar sector consisting of only one bi-doublet and two doublets it is possible at least to accommodated Dirac neutrino masses.

Models have been proposed with Dirac neutrinos in which the smallness of theirs masses can be explained. For instance, calculable Dirac neutrino masses in the context of LR symmetric models were obtained but only by introducing extra heavy singlet leptons and/or charged and neutral scalars, or even doubly charged scalars. Recently, anomaly free models that allow Dirac or inverse seesaw neutrino masses, which include sterile neutrinos with exotic lepton number assignment was proposed in Ref.\cite{22}. In scotogenic models in order to obtain calculable Dirac masses for neutrino we have for instance, i) to add, besides the right-handed neutrinos two neutral leptons $N, N^c$ per family, or ii) two new fermion singlets and one fermion doublet. It is also possible to implement the inverse seesaw mechanism, without the introduction of triplets but we must add more neutral singlet leptons. An alternative formulation of the LR symmetric models in which $B - L$ is a global unbroken symmetry and in which neutrinos are Dirac particles has been formulated recently. However the price to be paid is the introduction of extra quarks and charged leptons. It means that, with only the known leptons plus right-handed neutrinos and renormalizable interactions, purely Dirac neutrinos do not arise easily in any model. Hence, it is interesting to search mechanisms that at least allow to accommodate light Dirac neutrinos in the context of a renormalizable electroweak model with a representation content in a completely analogy with the SM regarding the charged fermions, being the only extra neutral fermions three right-handed neutrinos.

On the other hand, although the resonance discovery at LHC is consistent with the neutral scalar of the SM, it does not discard the existence of more neutral scalars (and
their charged partners if they are not singlets of the gauge symmetry). Since the scalar content in any model is not fixed by the gauge symmetry, and also we do not know yet the complete spectra in the scalar sector we can add, in any model, more scalar multiplets. Hence, the issue of the number of scalars is added to the generation problem: how many scalars? The interesting possibility is that this number is equal to the number of fermion generations, i.e., three \[29\]. Although in the context of the SM, the introduction of three doublets is well motivated, say, for implementing CP violation \[30\] and/or dark matter candidates \[31\], in models with larger gauge symmetries a given number of scalar multiplets is not, necessarily, well motivated. It will depend on the phenomenological results. This is the case in the LR symmetric electroweak models in which there are several way to introduce scalar multiplets. Here, we will consider an extension of the minimal LR model by adding more bi-doublets and no triplets. In particular we show that the case of three bi-doublets it is possible to avoid a fine tuning in the neutrino Yukawa couplings. However, the details of the scalar potential are given only for the case of two bi-doublets. 

The outline of this work is as follows. In the next section we consider the model and the symmetries that make it invariant under a generalized parity and other discrete symmetries in such a way that one bi-doublet is coupled only with leptons and the other only with quarks. In Sec. \[\text{III}\] we consider the most general scalar potential invariant under the symmetries of the model. We show that for the two bi-doublets case an approximate \(Z_5\) symmetry allows to consider a more simplified potential. The gauge vector boson sector is analyzed in Sec. \[\text{IV}\] while Yukawa interactions and fermion masses are considered in Sec. \[\text{V}\]. Next, in Sec. \[\text{VI}\] the fermion-vector boson interactions are given, while in Sec. \[\text{VII}\] we analyze the case when we add a third bi-doublet. Some phenomenological consequences appear in Sec. \[\text{VIII}\] and in Secs. \[\text{IX}\] the case when parity is breakdown first is consider. Finally our conclusions appear in the last section.

\section{The Model}

The model to be consider has the following electroweak symmetry:

\[SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L} \otimes \mathcal{P}\, , \quad (1)\]
We omit the $SU(3)_C$ factor because is as in the standard model (SM). The electric charge operator defined as usual $Q = T_{3L} + T_{3R} + (B - L)/2$.

The left- and right-handed fermions transform non-trivially under different $SU(2)$ transformation. In the lepton sector $L_i^T = (\nu'_l)_{L} \sim (2_L, 1_R, -1)$ and $R_i^T = (\nu'_l)_{R} \sim (1_L, 2_R, -1)$, with $l = e, \mu, \tau$ and the primed states denote symmetry eigenstates. Similarly in the quark sector, $L_i \sim (2L, 1R, -1/3)$ and $R_i \sim (1L, 2R, -1/3)$. The scalar sector consists of at least two and three bi-doublets transforming as $(2L, 2^* R, 0)$ and two doublets $\chi_L^T = (\chi^+_L, \chi^0_L) \sim (2L, 1R, +1)$ and $\chi_R^T = (\chi^+_R, \chi^0_R) \sim (1L, 2R, +1)$ to break the parity and the gauge symmetry down to $U(1)_Q$.

We also impose a generalized parity under which

$$g_L \leftrightarrow g_R, \quad W_{L\mu} \leftrightarrow W^\mu_{R}, \quad f_L \leftrightarrow f_R, \quad \chi_L \leftrightarrow \chi_R, \quad \Phi_i \leftrightarrow \Phi_i^\dagger, \quad \tilde{\Phi}_i \leftrightarrow \tilde{\Phi}_i^\dagger,$$

where $\Phi_i = \tau_2 \Phi_i^* \tau_2; \quad W_{\mu L,R}$ are the gauge bosons of the factors $SU(2)_{L,R}$, respectively; $f$ denotes a quark or a lepton doublet, and $\Phi_i$ and $\chi_{L,R}$ are the scalar multiplets introduced above. The coupling constants $g_{L,R}, g'$ correspond the the groups $SU(2)_{L,R}$ and $U(1)_{B-L}$, respectively. However, the invariance under $\mathcal{P}$ implies equality of gauge couplings $g_L = g_R \equiv g$ at the energy at which these symmetries are realized. Under this condition the model has only two gauge couplings, $g$ and $g'$. Although as a result of running couplings we have $g_L \neq g_R$, we will consider in this paper the case when these two couplings are equal at any energy scale but this has to be seen just as an approximation.

For the case of two bi-doublets we will impose also the discrete symmetries $\mathbb{Z}_2 \times \mathbb{Z}_2'$ in such a way that under the first factor $L_R, \Phi_1 \rightarrow -L_R, -\Phi_1$, and under the second one $Q_R, \Phi_2 \rightarrow -Q_R, -\Phi_2$, while all the other fields transform trivially under both factors. This symmetry implies that the bi-doublet $\Phi_1$ couples only to leptons and the other, $\Phi_2$, only to quarks. Notice that as usual in this sort of models, as a consequence of the transformation under the $SU(2)_{L,R}$ factors, none of the doublets is coupled with the fermions.

III. THE SCALAR POTENTIAL

Firstly, we consider the most general scalar potential invariant under the gauge symmetries and parity and then we see the effect of imposing discrete symmetries. Since some of our results are valid for an arbitrary number of bi-doublets we consider the scalar po-
potential involving \( n \) bi-doublets and two doublets. In general a bi-doublet transforms under the \( SU(2)_L \otimes SU(2)_R \) symmetry as \( \Phi \to U_L\Phi U_R^\dagger \), \( \Phi_i \to U_R\Phi_i U_L^\dagger \), and \( \bar{\Phi} \to U_L\bar{\Phi} U_R^\dagger \), \( \bar{\Phi}_i \to U_R\bar{\Phi}_i U_L^\dagger \). Under these conditions, the scalar potential is given by:

\[
V = V^{(2)} + V^{(4a)} + V^{(4b)} + V^{(4c)} + V^{(d)},
\]

where

\[
V^{(2)} = \frac{1}{2} \sum_{i,j=1}^{n} \left[ \mu_{ij}^2 \text{Tr}(\Phi_i^\dagger \Phi_j) + \mu_{ij}^2 \text{Tr}(\bar{\Phi}_i^\dagger \bar{\Phi}_j) + H.c. \right] + \mu_{LR}^2 (\chi_L^\dagger \chi_L + \chi_R^\dagger \chi_R),
\]

\[
V^{(4a)} = \frac{1}{2} \sum_{i,j=1}^{n} \left[ \lambda_{ij} \text{Tr}(\Phi_i^\dagger \Phi_j)^2 + \tilde{\lambda}_{ij} \text{Tr}(\bar{\Phi}_i^\dagger \bar{\Phi}_j)^2 + H.c. \right],
\]

\[
V^{(4b)} = \frac{1}{2} \left[ \sum_{i,j=1}^{n} \lambda'_{ij} (\text{Tr}\Phi_i^\dagger \Phi_j)^2 + \tilde{\lambda}'_{ij} (\text{Tr}\bar{\Phi}_i^\dagger \bar{\Phi}_j)^2 + H.c. \right],
\]

\[
V^{(4c)} = \sum_{ij} \rho_{ij} \text{Tr}(\Phi_i^\dagger \Phi_j^\dagger \bar{\Phi}_i \bar{\Phi}_j) + \bar{\rho}_{ij} \text{Tr}(\bar{\Phi}_i^\dagger \bar{\Phi}_j^\dagger \Phi_i \Phi_j),
\]

\[
V^{(4d)} = \frac{1}{2} \left[ \sum_{i,j=1}^{n} (\Lambda_{ij} \text{Tr}(\Phi_i^\dagger \Phi_j^\dagger \Phi_j) + \tilde{\Lambda}_{ij} \text{Tr}(\bar{\Phi}_i^\dagger \bar{\Phi}_j^\dagger \bar{\Phi}_j) (\chi_L^\dagger \chi_L + \chi_R^\dagger \chi_R)
\right.

\left. + \tilde{\Lambda}_{ij} (\chi_L^\dagger \Phi_i \Phi_j^\dagger \chi_L + \chi_R^\dagger \Phi_i \Phi_j^\dagger \chi_R) + \Omega_{ij} (\chi_L^\dagger \bar{\Phi}_i ^\dagger \bar{\Phi}_j \chi_L + \chi_R^\dagger \bar{\Phi}_i ^\dagger \bar{\Phi}_j \chi_R), \right]
\]

\[
V^{(4c)} = \lambda_{LR} [(\chi_L^\dagger \chi_L)^2 + (\chi_R^\dagger \chi_R)^2].
\]

We have omitted the redundant terms, for instance \( \text{Tr}(\Phi_i^\dagger \Phi_j) = \text{Tr}(\Phi_j^\dagger \Phi_i) \), and so on.

Let us consider explicitly the case of two bi-doublets, \( n = 1, 2 \) in (4) with

\[
\Phi_1 = \begin{pmatrix} \phi_1^0 \eta_1^+ \\ \phi_1^- \eta_1^0 \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \phi_2^0 \eta_2^+ \\ \phi_2^- \eta_2^0 \end{pmatrix}.
\]

The vacuum expectation values (VEVs) are \( \sqrt{2} \langle \Phi_1 \rangle = \text{Diag}(k_1 k'_1) \), \( \sqrt{2} \langle \Phi_2 \rangle = \text{Diag}(k_2 k'_2) \), \( \sqrt{2} \langle \chi_L \rangle = \text{Diag}(0 \ 0) \), and \( \sqrt{2} \langle \chi_R \rangle = \text{Diag}(0 \ 0) \). In general we will write the neutral components of the scalars as \( \eta_i^0 = \frac{1}{\sqrt{2}} (v_i + R_i + i I_i) e^{i\theta_i} \), where \( v_i, \theta_i \) may be complex numbers and \( R_i, I_i \) Hermitian fields. However, here we will consider all VEVs real, i.e., \( \theta_i = 0 \) for all \( i \) running over the bi-doublets and doublets.

In this case, the invariance under the parity transformations defined in (2) imply \( \mu_{12} = \mu_{21} \equiv \mu^2, \tilde{\mu}_{12} = \tilde{\mu}_{21} \equiv \nu^2; \lambda_{12} = \lambda_{21}, \tilde{\lambda}_{12} = \tilde{\lambda}_{21}, \lambda'_{12} = \lambda'_{21}, \Lambda_{12} = \Lambda_{21}, \tilde{\Lambda}_{12} = \tilde{\Lambda}_{21}, \Lambda'_ {12} = \Lambda'_ {21} \), and that \( \tilde{\lambda}_{ij}, \Omega_{ij} \) are real. Notice that the \( \mathbb{Z}_2 \otimes \mathbb{Z}_2' \) implies
\[ \mu^2 = \nu = 0 \] and \( \Lambda_{12} = \Lambda_2 = \bar{\Lambda}_{12} = \bar{\Lambda}_{21} = \bar{\Lambda}'_{12} = \bar{\Lambda}'_{21} = 0. \] However, we will allow for the moment a soft broken of these symmetries and use \( \mu^2 \neq 0. \)

The constraint equations \( t_X = \partial V / \partial X, \) \( X = k_1, k', k_2, k'_2, v_L, v_R \) (considered real), are

\[
t_1 = k_1 \left[ \mu_{11}^2 + (\lambda_{11} + \lambda'_1) k^2_1 + k^2_1 \left( \lambda'_{11} + \bar{\lambda}_{11} + 2 \bar{\lambda}'_{11} \right) + \frac{1}{2} \left( v^2_R H + \bar{\lambda}_{21} k^2_2 \right) \right. \\
+ \left( (\bar{\lambda}_{12} + \bar{\lambda}'_{12}) k^2_2 + \frac{1}{2} v^2_L H + k^2_2 \left( \lambda''_{12} + \lambda'_{12} + \lambda_{12} + \lambda_{21} + \rho_{12} \right) \right) \\
+ \frac{1}{4} (v^2_R + v^2_L) (k'_1 D + k'_2 F + k_2 G) + \frac{k_2 k_1 k'_1}{2} \left( \bar{\lambda}'_{21} + \bar{\lambda}'_{12} + \lambda_{12} + \lambda'_{21} + \bar{\rho}_{12} \right) \\
+ k_2 \mu^2 + \bar{\mu}_{11}^2 k_1, \\
\]

\[ \quad \text{(6)} \]

\[
t'_1 = k'_1 \left[ \mu_{11}^2 + (\lambda_{11} + \lambda'_1) k^2_1 + k^2_1 \left( \lambda'_{11} + \bar{\lambda}_{11} + 2 \bar{\lambda}'_{11} \right) + \frac{1}{2} \left( v^2_R H + \bar{\lambda}_{12} k^2_2 \right) \right. \\
+ \left( (\bar{\lambda}'_{21} + \bar{\lambda}_{12} + \bar{\lambda}_{21}) k^2_2 + \frac{1}{2} v^2_L H + k^2_2 \left( \lambda''_{21} + \lambda'_{21} + \lambda_{12} + \lambda_{21} + \rho_{12} \right) \right) \\
+ \frac{1}{4} (v^2_L + v^2_R) (k_1 D + k_2 F + k'_2 G) + \frac{k_2 k_1 k'_1}{2} \left( \bar{\lambda}'_{12} + \bar{\lambda}'_{21} + \lambda_{12} + \lambda'_{21} + \bar{\rho}_{12} \right) \\
+ k'_2 \mu^2 + \bar{\mu}_{11}^2 k_1, \\
\]

\[ \quad \text{(7)} \]

where

\[
A = \Lambda_{11} + \bar{\Lambda}_{11}, \quad B = \Lambda_{12} + \Lambda_{21} + \bar{\Lambda}_{12} + \bar{\Lambda}_{21}, \quad C = \bar{\Lambda}_{12} + \bar{\Lambda}_{21} + \Omega'_{12} + \Omega_{21} \]
\[
D = \Omega'_{11} + 2 \bar{\Lambda}_{11} + \Omega_{11}, \quad E = \Omega'_{22} + 2 \bar{\Lambda}_{22} + \Omega_{22}, \quad F = \Omega'_{21} + \bar{\Lambda}_{21} + \bar{\Lambda}_{12} + \Omega_{12}, \]
\[
G = \bar{\Lambda}_{21} + \bar{\Lambda}_{12} + \Lambda_{21} + \Lambda_{12}, \quad H = \Lambda_{11} + \bar{\Lambda}'_{11}, \]
\[
I = \Lambda_{22} + \bar{\Lambda}'_{22}, \quad J = \Lambda_{22} + \bar{\Lambda}_{22}, \quad \text{(8)}\]

and similarly we obtain \( t_2 \) and \( t'_2 \) for \( k_2 \) and \( k'_2 \), respectively, but we will not write them explicitly. Finally, we have

\[
t_L = \frac{v_L}{2} (2 \mu^2_{LR} + 2 \lambda_{LR} v^2_L + \Delta), \quad t_R = \frac{v_R}{2} (2 \mu^2_{LR} + 2 \lambda_{LR} v^2_R + \Delta). \]

\[ \quad \text{(9)} \]

where

\[
\Delta = k^2_1 A + k'_1 k^2_2 B + k'_1 k_2 C + k_1 k'_1 D + k'_2 k_2 E + k_1 k'_2 F + k_1 k_2 G + k^2_1 H + k^2_2 I + k^2_2 J. \]

\[ \quad \text{(10)} \]

Notice that only \( v_L \) and \( v_R \) can be zero, however this solution is not accepted for \( v_R \). We assume also that \( v_R \gg k_2 \gg k_1, k', k'_2 \gg v_L \), and if

\[
D, F, G \ll 1, \quad \bar{\lambda}'_{12} + \bar{\lambda}'_{21} + \lambda_{21} + \lambda_{12} + \bar{\rho}_{12} \ll 1, \quad \text{(11)}
\]
then we obtain from Eqs. (6) and (7), respectively,

$$k_1 \approx \frac{\mu_2^2}{\mu_{11}^2 + v_R^2 H} k_2 \ll k_2, \quad k'_1 \approx \frac{\mu_2^2}{\mu_{11}^2 + v_R^2 H} k'_2 \ll k'_2,$$

with $v_R^2 H, v_R^2 H' > |\mu_{11}^2|$. This shown that there is a range of the parameter space in which we can have $k'_1 \ll k_1 \ll k'_2 < k_2$. Moreover, if

$$D = F = G = 0, \quad \lambda_{ij} = \lambda'_{ij} = \bar{\lambda}_{ij} = \bar{\rho}_{ij} = 0, \quad i \neq j; \quad \mu^2 = \bar{\mu}_{11}^2 = 0,$$

the constraint equations become

$$
\begin{align*}
t_1 &= k_1 \left( \mu_{11}^2 + (\lambda_{11} + \lambda'_{11})k_1^2 + \lambda'_{11}k_1^2 + \frac{1}{2}(v_L^2 + v_R^2)H + (\lambda_{12} + \lambda'_{12} + \rho_{12})k_1^2 \right), \\
t_1' &= k_1' \left( \mu_{11}^2 + \lambda'_{11}k_1^2 + (\lambda_{11} + \lambda'_{11})k_1^2 + \frac{1}{2}[(v_L^2 + v_R^2)I + (\lambda_{12} + \lambda'_{12} + \rho_{12})k_1^2] \right), \\
t_2 &= k_2 \left( \mu_{22}^2 + (\lambda_{22} + \lambda'_{22})k_2^2 + \lambda'_{22}k_2^2 + \frac{1}{2}[(v_L^2 + v_R^2)H + (\lambda_{12} + \lambda'_{12} + \rho_{12})k_1^2] \right), \\
t_2' &= k_2' \left( \mu_{22}^2 + (\lambda_{22} + \lambda'_{22})k_2^2 + \lambda'_{22}k_2^2 + \frac{1}{2}[(v_L^2 + v_R^2)J + (\lambda_{12} + \lambda'_{12} + \rho_{12})k_1^2] \right), \\
t_L &= \frac{v_L}{2} \left[ 2\mu_{LR}^2 + 2\lambda_{LR}v_L^2 + k_1^2 A + k_1^2 H + k_2^2 I + k_2^2 J \right], \\
t_R &= \frac{v_R}{2} \left[ 2\mu_{LR}^2 + 2\lambda_{LR}v_R^2 + k_1^2 A + k_1^2 H + k_2^2 I + k_2^2 J \right].
\end{align*}
$$

(14)

In fact, we further restrict the Higgs potential so that it is invariant under the $\mathbb{Z}_5$ symmetry, (defined as $\omega_i = e^{-2\pi n/5} \quad n = 1, \cdots, 5$) under which $\Phi_1 \to \omega_1 \Phi_1$, $\Phi_2 \to \omega_2 \Phi_2$ while also other fields are invariant, the scalar potential in Eq. (11) becomes:

$$
\begin{align*}
V^{(2)} &= \frac{1}{2} \sum_{i=1,2} \left[ \mu_{ii}^2 \Tr(\Phi_i^\dagger \Phi_i) + H.c. \right] + \mu_{LR}^2(\chi_L^\dagger \chi_L + \chi_R^\dagger \chi_R), \\
V^{(4a)} &= \frac{1}{2} \sum_{i=1,2} \left[ \lambda_{ii} \Tr(\Phi_i^\dagger \Phi_i)^2 + H.c. \right], \\
V^{(4b)} &= \frac{1}{2} \sum_{i=1,2} \lambda'_{ii} \Tr(\Phi_i^\dagger \Phi_i), \\
V^{(4c)} &= \rho_{12} \Tr(\Phi_1^\dagger \Phi_1 \Phi_2^\dagger \Phi_2), \\
V^{(4d)} &= \frac{1}{2} \left[ \sum_{i=1,2} (\Lambda_{ii} \Tr \Phi_i^\dagger \Phi_i + \bar{\Lambda}_{ii} \chi_L^\dagger \Phi_i^\dagger \Phi_i \chi_L + \chi_R^\dagger \Phi_i^\dagger \Phi_i \chi_R) + \bar{\Lambda}_{ii} \chi_L^\dagger \Phi_i^\dagger \Phi_i \chi_L + \chi_R^\dagger \Phi_i^\dagger \Phi_i \chi_R \right], \\
V^{(4e)} &= \lambda_{LR} \left[ (\chi_L^\dagger \chi_L)^2 + (\chi_R^\dagger \chi_R)^2 \right].
\end{align*}
$$

(15)

and the constraints in Eq. (13) arise from this potential. It means that these conditions are protected by the $\mathbb{Z}_5$ symmetry and may be naturally small. We can consider the potential in
Eq. (15), and the respective mass spectra, as a good approximation. Notice that all VEVs may be zero, in particular the solutions \( k_1', k_2' = 0 \) and \( v_L = 0 \) are allowed. The SM-like Higgs scalar is in the bi-doublet \( \Phi_2 \).

It is important to note that since the doublet \( \chi_L \) was introduced just to implement the invariance of the Lagrangian under parity and it does not couple to fermions, if the respective VEV is zero it is an inert doublet an the left-right symmetry protects its inert character, hence it is a candidate for dark matter. However, notice that \( v_L \neq 0 \) is also a solution hence the possibility to have a model without any bi-doublet, with fermion masses arisen from non-renormalizable interactions [33], in which case \( A = H = I = J = 0 \) in Eq. (14), it is possible. However, in this case the model needs an ultraviolet completion. We stress that although the constraint equations in Eq. (14) were obtained using the potential in Eq. (15) by considering the most general potential (without the \( \mathbb{Z}_2 \otimes \mathbb{Z}_2' \) symmetry) we still obtain

\[
t_L = v_L (\mu^2_{LR} + \lambda_{LR} v_L^2 + \text{bi-doublet contributions}),
\]

and the solution \( v_L = 0 \) is still allowed even without a soft breaking of parity symmetry [34].

If \( x_0 = \phi^0_{1,2}, \eta^0_{1,2} \) (we omit the respective VEV), we denote the symmetry eigenstate as \( x^0_i = R_i + i I_i \) they are related with the mass eigenstates, \( H^0, A^0 \), through out the orthogonal \( 4 \times 4 \) matrices, say \( R_i = O_{ij} H_j \) and \( I_i = O_{ij} A_j \). Recall that we are assuming \( CP \) violation, hence \( O \) and \( \mathcal{O} \) are orthogonal matrices.

**IV. GAUGE BOSONS MASS EIGENSTATES**

The covariant derivative for the bi-doublets \( \Phi_i, \ i = 1, 2 \) and for the doublets \( \chi_L \) and \( \chi_R \) are given by

\[
\mathcal{D}_\mu \Phi_i = \partial_\mu \Phi_i + ig \left[ \frac{\bar{\tau}}{2} \cdot \bar{W}_L \Phi_i - \Phi_i \bar{\tau} \cdot \bar{W}_R \right], \quad (a)
\]
\[
\mathcal{D}_\mu \chi_L = \left( \partial_\mu + ig \frac{\bar{\tau}}{2} \cdot \bar{W}_L + g' B_\mu \right) \chi_L, \quad (b)
\]
\[
\mathcal{D}_\mu \chi_R = \left( \partial_\mu + ig \frac{\bar{\tau}}{2} \cdot \bar{W}_R + g' B_\mu \right) \chi_R, \quad (c)
\]

where we have already did \( g_L = g_R = g \). (However, see Sec. III) With the VEVs given in Sec. III we obtain for the charged vector bosons:

\[
M_{CB}^2 = \frac{g^2 v^2}{4} \begin{pmatrix}
x + y & -2z \\
-2z & 1 + x
\end{pmatrix}.
\]

(18)
where \( x = K^2/v_R^2, z = K^2/v_R^2, y = v_L^2/v_R^2, \) and \( K^2 = k_1^2 + k_1'^2 + k_2^2 + k_2'^2, \) \( K^2 = k_1k_1' + k_2k_2', \) and the respective eigenvalues are given by

\[
M_{W_1}^2 = \frac{g^2v_R^2}{4} \left( x + \frac{1 + y}{2} - \sqrt{\Delta} \right), \quad M_{W_2}^2 = \frac{g^2}{4} \left( x + \frac{1 + y}{2} + \sqrt{\Delta} \right),
\]

(19)

where \( \Delta = 4z + \frac{1}{4}(y - 1)^2. \) These expressions can be generalized for an arbitrary number of bi-doublets \( K^2 = \sum_i^n k_i^2 \) and \( \bar{K}^2 = \sum_i^n k_i'k_i' \) and the results of this section are valid for \( n \) bi-doublets.

Symmetry and mass eigenstates are related by an orthogonal matrix:

\[
\begin{pmatrix}
W_{1\mu}^+ \\
W_{2\mu}^+
\end{pmatrix} = \begin{pmatrix}
c_\xi & s_\xi \\
-s_\xi & c_\xi
\end{pmatrix}
\begin{pmatrix}
W_{L\mu}^+ \\
W_{R\mu}^+
\end{pmatrix}
\]

(20)

\[
c_\xi = \frac{Y}{\sqrt{16z + Y^2}}, \quad s_\xi = \frac{\sqrt{16z}}{\sqrt{16z + Y^2}}
\]

(21)

where \( Y = 1 - y + 2\sqrt{\Delta}, \) and \( c_\xi = \cos \xi, \) etc. Notice that since we will always consider that \( k_{1,2}, k_{1,2}' \ll v_R \) i.e., \( z \ll Y, \) the mixing angle between \( W_L - W_R. \)

Notice that \( M_{W_2}^2 \gg M_{W_1}^2, \) and we identify the \( W^\pm \) of the standard model with \( W_1^\pm. \) In the limit \( v_R \to \infty (x, z, y \ll 1), \) we obtain

\[
M_{W_1}^2 \approx \frac{g^2}{4} \left( K^2 + \frac{v_R^2}{2} \right), \quad M_{W_2}^2 \approx \frac{g^2}{4}v_R^2.
\]

(22)

In the neutral vector bosons we have the massa matrix:

\[
M_{NB}^2 = \frac{g^2v_R^2}{4} \begin{pmatrix}
x + y & -x & -ry \\
-x & 1 + x & -r \\
r^2(1 + y) & -r & r^2(1 + y)
\end{pmatrix},
\]

(23)

where we have defined \( x \) and \( y \) as before and \( r \equiv g'/g. \)

The determinant of the matrix in (23) is zero and its eigenvalues are without any approximation

\[
M_A = 0, \quad M_{Z_1}^2 = \frac{g^2v_R^2}{4} \left[ x + \frac{1}{2}(1 + r^2)(1 + y) - \frac{1}{2}\sqrt{\Omega} \right], \quad M_{Z_2}^2 = \frac{g^2v_R^2}{4} \left[ x + \frac{1}{2}(1 + r^2)(1 + y) + \frac{1}{2}\sqrt{\Omega} \right],
\]

(24)
we have defined
\[ \Omega = (1 + r^2)^2 (1 + y^2) - 4(1 + 2r^2)y - 4xr^2 (1 + y^2) + 4x^2. \] (25)

The symmetry eigenstates \((W_{3L}, W_{3R}, B)\) are linear combinations of the mass eigentates \((A, Z_1, Z_2)\) as follows:
\[
\begin{pmatrix}
W_{3L} \\
W_{3R} \\
B
\end{pmatrix} =
\begin{pmatrix}
n n_{12} n_{13} \\
n n_{22} n_{23} \\
n' n_{32} n_{33}
\end{pmatrix}
\begin{pmatrix}
A \\
Z_1 \\
Z_2
\end{pmatrix}
\] (26)

Although we have all the elements of \(n_{ij}\) exactly calculated, here we write for the sake of space only \(n\) and \(n'\) exactly, while the other entries are in the approximation \(v_R \gg K, v_L\) \((x, y, z \ll 1)\) up to \(O(1/v_R^2)\) terms,
\[
n = \sin \theta, \quad n' = \sqrt{\cos 2\theta}, \quad n_{12} \approx \cos \theta, \quad n_{13} \approx \phi,
\]
\[
n_{22} \approx -t_\theta s_\theta \left(1 - \phi \frac{\sqrt{c_{2\theta}}}{s_\theta^2 c_\theta}\right), \quad n_{23} \approx \frac{\sqrt{c_{2\theta}}}{c_\theta} \left(1 + \phi \frac{c_{\theta t_\theta^2}}{\sqrt{c_{2\theta}}}\right),
\]
\[
n_{32} \approx -t_\theta \sqrt{c_{2\theta}} \left(1 + \phi \frac{1}{c_\theta \sqrt{c_{2\theta}}}\right), \quad n_{33} \approx -t_\theta \left(1 - \phi \frac{\sqrt{c_{2\theta}}}{c_\theta}\right).
\] (27)

where we have defined \(\phi = (x - r^2 y)/(1 + r^2)^{3/2}\). The angle \(\theta\) is defined below, see Eq. (46).

In the limit \(v_R \to \infty\) i.e., \(x, y \to 0\) \((\phi \to 0\) also), the matrix in Eq. (27) becomes to the usual form in literature:
\[
\begin{pmatrix}
W_{3L} \\
W_{3R} \\
B
\end{pmatrix} =
\begin{pmatrix}
s_\theta & c_\theta & 0 \\
s_\theta & -s_\theta t_\theta & -\sqrt{c_{2\theta}} \\
\sqrt{c_{2\theta}} & -t_\theta \sqrt{c_{2\theta}} & -t_\theta
\end{pmatrix}
\begin{pmatrix}
A \\
Z_1 \\
Z_2
\end{pmatrix}.
\] (28)

Going back to the masses of vector bosons we note that in the limit \(v_R \gg v\) where \(v\) is any VEVs, we obtain from Eq. (24)
\[
M^2_{Z_1} \approx \frac{g^2}{4 \cos^2 \theta} \left(K + \frac{v_L^2}{2}\right), \quad M^2_{Z_2} \approx \frac{g^2 + g'^2}{4} v_R^2,
\] (29)

and we see from (22) and (29) that
\[
M_{Z_1} \approx \frac{M_{W_L}}{\cos \theta} + O(x).
\] (30)

Notice that only in the limit \(v_R \to \infty\) the angle \(\theta\) in this model has a relation with the \(\theta_W\) of the SM. However, it is important that \(v_R\) is keep to be large but finite in order to obtain a
lower bound on the right-handed vector bosons, $W_2$ and $Z_2$ and the respective coupling with fermions. If $\chi_L$ is an inert doublet, we simply put $v_L = 0$, or equivalently $y = 0$, in the expressions above.

Using the exact results in Eqs. (19) and (24), using $r = g'/g \approx 0.6355$, $k_2 \sim k'_2 \approx v_{SM}/\sqrt{2}$ ($z = x/2, y = 0$) where $v_{SM} = 246$ GeV, and $M_W/M_Z = 0.88147 \pm 0.00013$ we obtain (using $2\sigma$ value of that ratio) that $v_R > 24$ TeV. With this lower limit on $v_R$ we can calculate the lower limit for the masses of $W_2^+$ and $Z_2$, using Eqs. (19) and (24), respectively, obtaining (in TeV):

$$M_{W_2} > 7.2, \quad M_{Z_2} > 9.28. \quad (31)$$

and the mixing angle $W_L - W_R$ defined in Eq. (21) has an upper limit $\sin \xi < 10^{-4}$. Recent analysis comparing the experimental limits to the theoretical calculations for the total $W_2$ resonant production and the decay $W_2 \rightarrow WZ$ implies that $\xi$ is between $10^{-4} - 10^{-3}$.

V. YUKAWA INTERACTIONS AND FERMION MASSES

The Yukawa interactions in the lepton sector are given by

$$- \mathcal{L}_Y = \bar{L}'_L (G \Phi_1 + F \tilde{\Phi}_1) L'_R + \bar{L}'_R (G^{\dagger} \Phi_1 + F^{\dagger} \tilde{\Phi}_1) L'_L, \quad (32)$$

where $L'$ and $R'$ are defined in Sec. II and we have omitted generations indices. Since $\Phi_i \leftrightarrow \Phi_i^{\dagger}$ under the left-right symmetry implies that $G^{\dagger} = G$ and $F^{\dagger} = F$.

With these interactions and the vacuum alignment the mass matrices in the lepton sector are

$$M' = G \frac{k_1}{\sqrt{2}} + F \frac{k_1^{*}}{\sqrt{2}}, \quad M' = G \frac{k'_1}{\sqrt{2}} + F \frac{k'_1^{*}}{\sqrt{2}}. \quad (33)$$

A similar expression arises in the quarks sector but now $\Phi_1 \rightarrow \Phi_2$ and $(\nu'_L, \nu'_R) \rightarrow (u'_L, d'_L)$.

Primed fields denote symmetry eigenstates and unprimed ones mass eigenstates. In general $G, F$ and VEVs are complex, and the mass matrices are diagonalized by bi-unitary transformations as follows:

$$V^*_L M' V^*_R = \hat{M}' \quad U^*_L M' U^*_R = \hat{M}' \quad (34)$$
where $\hat{M}^l = \text{diag}(m_e, m_\mu, m_\tau)$ and $\hat{M}^\nu = \text{diag}(m_1, m_2, m_3)$ for charged lepton and neutrinos respectively.

For given an appropriate mass to the quarks, we have introduce the bi-doublet $\Phi_2$, and it is possible to implement the analysis as in Ref. [37]. Notice that this means that the neutral scalar with VEV and mass about 174 and 125 GeV is part of this bi-doublet.

We will assume that $k_1' = 0$ (see Sec. III) and in this case the lepton mass matrices are given by

$$M^l_{ab} = F_{ab}\frac{k_1^*}{\sqrt{2}}, \quad M^\nu_{ab} = G_{ab}\frac{k_1}{\sqrt{2}}, \quad (35)$$

where $G$ and $F$ are symmetric complex matrices that are diagonalized by the bi-unitary transformation in Eq. (34). Hereafter we will consider, just for the sake of simplicity, all VEVs being real.

From these matrices and the lepton measured masses we found the Yukawa coupling matrices

$$G = \frac{\sqrt{2}}{k_1} U_L^I \hat{M}^\nu U_R^I, \quad F = \frac{\sqrt{2}}{k_1} V_L^I \hat{M}^l U_R^I, \quad (36)$$

and we use for numerical calculations $|k_1| = 2$ GeV since this VEV is the only one for generating the lepton masses. We will work for the sake of simplicity in the basis in which the charged lepton mass matrix is diagonal and consider the matrices $G, F$ and all VEVs real. In this case $U_L^\nu = U_R^\nu \equiv U^\nu$, and $U^\nu = V_{PMNS}^L = V_{PMNS}^R \equiv V_l$, and we have

$$G = \frac{\sqrt{2}}{k_1} V_l^I \hat{M}^\nu V_l^I, \quad F = \frac{\sqrt{2}}{k_1} \hat{M}^l, \quad (37)$$

being the unitary matrix $V_l$ parametrized in the same way for Dirac particles. We use the PDG parametrization for Dirac neutrinos, for the interactions with $W^L_{L,R}$:

$$V_l = \begin{pmatrix}
  c^l_{12} c^l_{13} & s^l_{12} c^l_{13} & s^l_{13} \\
  -s^l_{12} c^l_{23} - c^l_{12} s^l_{13} s^l_{23} & c^l_{12} c^l_{23} - s^l_{12} s^l_{13} s^l_{23} & c^l_{13} s^l_{23} \\
  s^l_{12} s^l_{23} - c^l_{12} s^l_{13} c^l_{23} & -c^l_{12} s^l_{23} - s^l_{12} s^l_{13} c^l_{23} & c^l_{13} c^l_{23}
\end{pmatrix}, \quad (38)$$

with $s_{ij}^l = \sin\theta_{ij}^l, \ldots$ and we have considered $\delta_l = 0$.

In the case the Yukawa interactions are

$$-\mathcal{L}^\gamma_Y = \frac{\sqrt{2}}{k_1} \{\bar{\nu}_L[(\hat{M}^\nu \phi^0_1 + V^l_I \hat{M}^I V^0_{l\eta}^I)\nu_R + (\hat{M}^\nu \nu^l_I \eta^+_1 + V^l_I \hat{M}^l \phi^+_1)l_R] \right. $$

$$+ \left. \bar{l}_L[(V^l_I \hat{M}^\nu \phi^-_1 + \hat{M}^l \nu^l_I \eta^-_1)\nu_R + (V^l_I \hat{M}^\nu \nu^-_I \eta^+ \phi^+_1)l_R] \right\} + H.c. \quad (39)$$
and $V_l$ given in (38). Notice that in this case (charged leptons in the diagonal basis), the Higgs $\phi^0_1$ is the one whose couplings with charged leptons are proportional to respective masses and the couplings with $\eta^0_1$ are suppressed by the neutrino masses in the charged lepton sector. In the neutrino sector the situation reverses the enhanced interactions among those with $\eta^0_1$ since they are proportional to the charged lepton masses. For instance, the vertex $\bar{\nu}_3 R \nu_1 L \eta^0_2$ has the strength proportional to $s_{13} c_{13} c_{23} m_\tau$ and $\eta^0_1$ can decay through its mixing with the other neutral scalar, into two of the other particles, bosons or fermions.

For completeness, we write the Yukawa interactions in the quark sector (mass matrices diagonalized by the unitary matrices $V^L_{C\nu KM} = V^R_{CKM} = V^{u\dagger} V^d$):

$$-\mathcal{L}^Y_q = \frac{\sqrt{2}}{k_1} \left\{ \bar{u}_L V^u \left[ (G_q \phi^0_2 + F_q \eta^0_2) V^u_R + \left( G_q \eta^0_2 - F_q \phi^0_2 \right) V^d_R \right] + \bar{d}_L V^d \left[ (G_q \phi^0_1 - F_q \eta^0_1) V^u_R + \left( G_q \eta^0_1 + \phi^0_2 \right) V^d_R \right] \right\} + H.c.$$  (40)

In the quark sector we shall not consider the solution $k'_2 = 0$ since for the case of generalized parity, $\mathcal{P}$, it has been shown that $k'_2 \ll k_2$ is ruled out by the $CP$ violating parameters $\epsilon$ and $\epsilon'$, however this hierarchy is allowed in the case of generalized $C$.  (38)

Notice that there are flavor changing neutral currents mediated by scalars in both, lepton and quark sectors.

VI. FERMION-VECTOR BOSON INTERACTIONS

The covariant derivatives are given by

$$\left( D_{\mu L}(R_i) \right) L'_i(R'_i) = \left( \partial_\mu + \frac{g}{2} \bar{\tau} \cdot \vec{W}_{\mu L}(R) - \frac{g'}{2} B_\mu \right) L'(R'_i),$$  (41)

and similarly for quarks. The lepton-gauge boson interactions are obtained from

$$\mathcal{L} = \bar{L}_i \gamma^\mu D_{\mu L} L_i$$

and similarly for the right-handed doublets.

A. Charged currents

The charged current interactions in the mass eigenstates basis are given by the Lagrangian

$$\mathcal{L}'_W = -\frac{g}{2} \left[ e^{i\phi_1} \bar{\nu}_L \gamma^\mu V_{iL} W^\pm_{1\mu} + \bar{\nu}_R \gamma^\mu V_{iR} W^\pm_{R_\mu} \right] + H.c.$$  (42)
where \( J^\mu_L = \bar{\nu}_L \gamma^\mu V_l L \) and \( J^\mu_R = \bar{\nu}_R \gamma^\mu V_l R \). In the general case where the Yukawa couplings in Eq. (33) are complex the right-handed CKM matrix is different from the left-handed one. This case was considered in Ref. [39].

In the quark sector

\[
\mathcal{L}_W^q = -\frac{g}{2} \left[ e^{i\phi_q} \bar{u}_L \gamma^\mu V_{CKM}^d L W_{L\mu}^+ + \bar{u}_R \gamma^\mu V_{CKM}^d R W_{R\mu}^+ \right] + H.c.
\]

\[
= -\frac{g}{2} \left[ (e^{i\phi_q} c_\xi J^\mu_L + s_\xi J^\mu_R) W_{L\mu}^+ + (-e^{i\phi_q} s_\xi J^\mu_L + c_\xi J^\mu_R) W_{R\mu}^+ \right] + H.c.,
\]

(43)

where \( J^\mu_L = \bar{u}_L \gamma^\mu V_{CKM}^d L \) and \( J^\mu_R = \bar{u}_R \gamma^\mu V_{CKM}^d R \) with \( V_{CKM} \) being the same as in the left-handed sector with 3 angles and one physical phase.

The introduction of the phase \( \phi_l \) and \( \phi_q \) in Eqs. (42) and (43), respectively, need an explanation. In the mixing matrix for \( n \) Dirac fermions \( 2n - 1 \) phases are absorbed in the Dirac fields since one is a global phase. In the SM this is enough because there is only one charged currents and the global phase never appear in amplitudes. However, in this sort of models there are also right-handed charged currents and the there is a relative global phase between both charged currents. This phase is \( \phi_l \) for lepton and \( \phi_q \) for quarks.

**B. Electromagnetic interactions**

The interaction with the photon arises from the projection of \( W_{3L}, W_{3R} \) and \( B \) over \( A \) using the matrix in Eq. (26). Then, it is possible to verified that the electric charge is written in terms of \( g \) and \( g' \) as

\[
e = \frac{gg'}{\sqrt{g^2 + 2g'^2}},
\]

and we obtain

\[
\frac{1}{e^2} = \frac{2}{g^2} + \frac{1}{g'^2}, \quad \frac{1}{g_Y} = \frac{1}{g} + \frac{1}{g'},
\]

(45)

where \( g_Y \) is the coupling constant of the SM. These relations are valid only at the energy scale at which \( g_L = g_R \equiv g \). Hence, we have the relations

\[
g = \frac{e}{\sin \theta}, \quad g' = \frac{e}{\sqrt{\cos 2\theta}}
\]

(46)

From the relations in Eq. (46) we have \( r = g'/g = \sin \theta / \sqrt{\cos \theta} \). We have also that

\[
\frac{g'^2}{g^2} = \frac{s_\theta^2}{1 - 2s_\theta^2}
\]

(47)
and the model has a Landau-like pole in $g'$ when $s_{\theta}^2 = 1/2$ but it happens at energies larger than the Planck scale. However, this only implies that the energy scale at which $g_L(\mu) = g_R(\mu)$ must be below the scale at which $s_{\theta}(\Lambda) = 1/2$, $\mu < \Lambda$.

C. Neutral currents

Next, we parametrize the neutral interactions of a fermion $i$ with the $Z_{1\mu}$ and $Z_{2\mu}$ neutral bosons as follows:

$$\mathcal{L}_{NC} = -\frac{g}{2\cos \theta} \sum_i \bar{\psi}_i \gamma^\mu[(g_V^i - g_A^i \gamma^5)Z_{1\mu} + (f_V^i - f_A^i \gamma^5)Z_{2\mu}] \psi_i$$

(48)

Let us consider the case when the VEV of the doublet $\chi_L$ is not zero, $v_L \neq 0$. Using [26] and $r = s_{\theta}/\sqrt{2}c_{\theta}$ in Eq. (27), we obtain:

$$a_L^\nu \approx 1 + \frac{t_{\theta}^2 c_{2\theta}}{c_{\theta}} \left(x - \frac{s_{\theta}^2}{c_{2\theta}} y\right), \quad a_R^\nu \approx \frac{c_{2\theta}}{c_{\theta}} \left(x - \frac{s_{\theta}^2}{c_{2\theta}} y\right),$$

$$a_L^f \approx c_{2\theta} \left[1 - \frac{t_{\theta}^2}{c_{\theta}} \left(x - \frac{s_{\theta}^2}{c_{2\theta}} y\right)\right], \quad a_R^f \approx -2s_{\theta}^2 \frac{c_{2\theta}}{c_{\theta}} + \frac{x}{c_{\theta}}(s_{\theta}^4 + c_{2\theta}^2) - \frac{y x^2 c_{2\theta}}{c_{\theta}}.$$

(49)

Defining

$$g_V^f = \frac{1}{2}(a_L^f + a_R^f), \quad g_A^f = \frac{1}{2}(a_R^f - a_L^f),$$

(50)

where $a_L^f$ and $a_R^f$ are the couplings of the left- and right-handed components of a fermion $f$.

$$g_V^{\nu} = \frac{1}{2} + \frac{c_{2\theta}}{2 c_{\theta}^2} \left(x - \frac{s_{\theta}^2}{c_{2\theta}} y\right), \quad g_A^{\nu} = \frac{1}{2} - \frac{c_{2\theta}}{2 c_{\theta}^2} \left(x - \frac{s_{\theta}^2}{c_{2\theta}} y\right)$$

$$g_V^f \approx \frac{1}{2}(-1 + 4s_{\theta}^2) \left[1 + \frac{c_{2\theta}}{c_{\theta}^2} \left(x - \frac{y s_{\theta}^2}{c_{2\theta}}\right)\right], \quad g_A^f \approx -\frac{1}{2} + \frac{c_{2\theta}}{2 c_{\theta}^2} \left(x - \frac{y s_{\theta}^2}{c_{2\theta}}\right).$$

(51)

Notice that when $v_R \rightarrow \infty (x, y \rightarrow 0)$, we obtain

$$g_V^{\nu} = g_A^{\nu} = \frac{1}{2}, \quad g_V^f = -\frac{1}{2} + 2s_{\theta}^2, \quad g_A^f = \frac{1}{2}.$$  

(52)

and the same happens with the coefficients of the quark sector in that limit, we obtain

$$g_V^{\nu} = \frac{1}{2} - \frac{4}{3} s_{\theta}^2, \quad g_A^{\nu} = \frac{1}{2}, \quad g_V^d = -\frac{1}{2} + \frac{2}{3} s_{\theta}^2, \quad g_A^d = -\frac{1}{2},$$

$$f_V^\nu = \frac{1}{2} - \frac{4}{3} s_{\theta}^2, \quad f_A^\nu = -\frac{1}{2} c_{2\theta}, \quad f_V^d = -\frac{1}{2} + \frac{2}{3} s_{\theta}^2, \quad f_A^d = \frac{1}{2} c_{2\theta}.$$  

(53)

It is worth noting that the vector couplings are the same for $Z_1$ and $Z_2$. We see once again that only when $v_R$ is strictly infinite we can identify, at tree level, the angle $\theta$ with $\theta_W$ of
the SM. Assuming the measured values $g^L_v = 0.03783 \pm 0.00041$ does not imply a stronger lower bound on $v_R$ and the $W_2$ and $Z_2$ masses obtained from the $M_W/M_Z$ ratio in Eq. (31).

Recently, the CMS Collaboration using $W_2 \to B + t$ or $W_2 \to T + b$ ($T, B$ are vector-like quarks VLQ) a $W_2$ with a mass below 1.6 TeV is excluded at 95% CL assuming equal branching fractions for $W'$ boson to $tB$ and $bT$ and 50% for each VLQ to $qH$ where $H$ is a neutral scalar [40]. If $T, B$ are the known $t, b$ quarks and assuming $W_2$ with coupling to the SM particles equal to the SM weak coupling constant, masses below 3.15 TeV are excluded at the 95% confidence level [41].

Furthermore, if right-handed gauge bosons decay into a high-momentum heavy neutrino and a charged lepton at LHC has excluded values of the $W_R$ smaller than 3.85 TeV for $N_R$ in the mass range 0.11.8 TeV [42]. Of course, if there are no extra quarks like $T$ and $B$ and neither heavy right-handed neutrinos, these restrictions for the mass of $W_R$ are not valid anymore.

Only for illustration, we give the partial widths at tree level and neglecting the fermion masses,

$$\Gamma(W^+_2 \to l^+\nu) \approx \frac{G_F M^2_{W_2} M_{W_2}}{6\pi \sqrt{2}} \sim 31.57 \text{ GeV},$$

if $M_{W_2} = 7.8$ TeV. Summing over all fermions it means a full width $\Gamma \sim 24 \text{ GeV}$. Compare this with the case of the $W$ of the SM, $\Gamma_W = 2.085 \pm 0.042 \text{ GeV}$ [1] where $l$ denotes any of charged leptons i.e., $l = e, \mu, \tau$ without sum over them [1].

For the $Z_2$ and also neglecting the fermion masses we have

$$\Gamma(Z_2 \to f\bar{f}) \simeq N_c \left[ (a^f_L)^2 + (a^f_R)^2 \right] \frac{G_F M^2_{W_2} M_{Z_2}}{24\pi}$$

(55)

for leptons $N_c = 1$ and for quarks $N_c = 3$. In the case of leptonic decay, using the couplings in Eq. (49), we have $\Gamma(Z_2 \to l^-l^+) \sim 3.79 \text{ GeV}$ for the three charged leptons, if $M_{Z_2} = 9.28 \text{ TeV}$, while $\Gamma(Z \to l^-l^+) = 83.984 \pm 0.086 \text{ MeV}$ [1].

Notice that scalar doublets $\chi_{L,R}$ do not couple to fermions and we will assume that vacuum alignment is such that $v_L = 0$, and this scalar field does not contribute to the gauge boson masses, hence $\chi_L$ is an inert doublet [43]. Hence, in this case the inert character is protected by the left-right symmetry.
VII. LEPTON MASSES AND MIXING

Here we will obtain, assuming the measure matrix elements of the PMNS matrix, the Yukawa coupling generating the correct charged lepton and neutrino masses. Firstly, considering the present case, i.e., two bi-doublets and then we briefly discuss the case with three bi-doublets.

A. The two bi-doublet case

Firstly, we neglect $CP$ violation which means that the matrices $F$ and $G$ are real and if we consider $F$ diagonal, $U^\nu = V_t$, see Eq. (38). Concerning the lepton masses, in the charged lepton sector we will use the central values given in PDG \cite{1} and in the neutrino sector we will use the several possibilities: i) normal mass hierarchy (NH), $m_1 \ll m_2 < m_3$

$$m_1 = 0, \quad m_2 \simeq (\Delta m^2_{21})^{1/2} \simeq 0.0086 \text{ eV}, \quad m_3 \simeq |\Delta m^2_{31}|^{1/2} \simeq 0.0506 \text{ eV}. \quad (56)$$

or ii) the inverted hierarchy (IH) $m_3 \ll m_1 < m_2,$

$$m_3 = 0, \quad m_1 \simeq 0.0497, \quad m_2 \simeq 0.0504 \text{ eV}, \quad (57)$$

or even iii) the quasi-degenerate case (QD) $m_1 \simeq m_2 \simeq m_3 \simeq m_0 \[1\],

$$m_j \gg |\Delta m^2_{(31)32}|^{1/2}, \quad m_0 \lesssim 0.10 \text{ eV}. \quad (58)$$

Recall that in the case we are considering here, the PMNS mixing matrix is given by $V_t$.

Using the numerical values for the neutrinos masses in Eq. (56) and the PDG’s angles,

$$s_{12}^2 = 0.307, \quad s_{23}^2 = 0.512 \ (\text{Normal order, octant I}), \quad s_{13}^2 = 0.00218, \quad k_1 = 2 \text{ GeV}, \ (59)$$

With this conditions, from Eq. (37) in the normal hierarchy $\hat{M}^\nu = \text{Diag}(0, (\Delta m^2_{12})^{1/2}), (\Delta m^2_{31})^{1/2}$, and using the matrix in Eq. (38) we obtain (up to a factor $10^{-11}$):

$$G_{11} \approx 0.2606, \quad G_{12} \approx 0.5481 \quad G_{13} \approx 0.1474, \quad G_{21} \approx 0.5481, \quad G_{22} \approx 1.9583, \quad G_{23} \approx 1.5418, \quad G_{31} \approx 0.1474, \quad G_{32} \approx 1.5418, \quad G_{33} \approx 1.8671. \quad (60)$$
Similarly in the charged lepton sector we have, for instance using the central value of the charged lepton masses \[1\]

\[F_e \approx 3.613 \times 10^{-4}, \quad F_\mu \approx 0.075, \quad F_\tau \approx 1.25. \quad \text{(61)}\]

Using the inverse mass hierarchy in Eq. (57) we obtain (up to a factor $10^{-11}$):

\[G_{11} \approx 3.4526, \quad G_{12} \approx -0.3530, \quad G_{13} \approx -0.3762, \quad G_{21} \approx -0.3530, \]
\[G_{22} \approx 1.7677, \quad G_{23} \approx -1.7352, \quad G_{31} \approx -0.3762, \quad G_{32} \approx -1.7352, \]
\[G_{33} \approx 1.8568. \quad \text{(62)}\]

In the quasi-degenerate case, in (58), we obtain also up to a factor $10^{-11}$,

\[G_{11} = G_{22} = G_{33} \approx 7.0711, \quad \text{(63)}\]

in this case all the other $G$’s vanish for all practical purposes.

Although this model with two bi-doublets contains a fine adjustment as in the SM, which is avoided if we introduce the scalar triplets, this would be the price to pay for having Dirac neutrinos and only the known charged leptons plus the right-handed neutrinos. However, we will show that when three bidoublet is considered it seems possible to avoid a fine tuning in the lepton masses.

**B. Three bi-doublets case**

It is interesting that one of the natural hierarchy in field theories are those in the values of the VEVs which are responsible by the spontaneously breaking of symmetries. This is because their values depend on the vacuum alignment and heavy scalars may have small VEVs. Probably this was first noted by Ma [44] and we have seen an example in Sec. III in the case of $k_1'$. Moreover, as we have stressed before, we already do not known the number and sort of scalars and we can think of an extension of the present model in which three bi-doublets (and the two doublets $\chi_{L,R}$) are introduced.

In this case, the Yukawa interactions is the sector of the model which is more affected by the existence of a third bi-doublet is the Yukawa. Let us denote $\Phi_\nu, \Phi_l$ and $\Phi_q$ the three bi-doublets. We introduce the discrete symmetries $D$ under which \[45\]

\[D: \quad \Phi_\nu, \Phi_l \rightarrow -i\Phi_\nu, -i\Phi_l, \quad L_R' \rightarrow iL_R', \quad \text{(64)}\]
and all the other fields being invariant under $D$. Under this conditions the Yukawa interactions are given by

$$L_Y = \bar{L}'_i (G^\nu \Phi_\nu + \bar{L}'_i G^l \Phi_l) R'_i + \bar{Q}'_L (G^q \Phi_q + F^q \Phi_q) Q'_R + H.c.$$  \hspace{1cm} (65)$$

Notice that the $D$ symmetry forbid the interactions as $\bar{L}'_\nu \tilde{\Phi}_\nu R'_i$ and $\bar{L}'_l \tilde{\Phi}_l R'_i$, where $\tilde{\Phi} = \tau_2 \Phi^* \tau_2$.

Denoting the respective VEVs $k_\nu, k'_\nu, k_l, k'_l$ and $k_q, k'_q$, we see that if the vacuum alignment allows that $k_\nu, k'_\nu, k_l \ll k'_l \ll k_q, k'_q$, then neutrino masses arise from $k_\nu$, the charged lepton masses from $k'_l$ (these leptons receive a small contributions from $k'_\nu$). If $k_\nu \gtrsim \sqrt{\Delta m^2_{31}}$ then all entries of the matrix $G^\nu$ are of order of the unity. The same happens in the Yukawa couplings in the charged lepton sector if $k'_l \gtrsim m_\tau$. Recall that hierarchy among VEVs are more easily to be justified than in the Yukawa couplings. The quark sector follows as usual.

VIII. SOME PHENOMENOLOGICAL CONSEQUENCES

Many of the features of the present model are as those in muti-Higgs models, say, flavor changing neutral currents (FCNC) mediated by scalars, several $CP$ violating phases, etc. The existence of FCNC in the scalar sector means that there are contributions to $\Delta a_\mu$ and $\Delta a_e$. For instance, taking the present data for the case of the muon $\Delta a_\mu = a^{exp} - a_\mu^{SM} = 288(63)(49) \times 10^{-11}$ \cite{1} and one contribution of a scalar or a pseudoscalar \cite{46}

$$\Delta a^\mu_{H,A}(\tau) = \frac{m_\mu^2}{8\pi^2 m_X^2} |O|^2 \int_0^1 dx \frac{Q_{S,A}(x)}{(1 - x)(1 - \lambda^2) + (\epsilon \lambda)^2 x}, \hspace{1cm} (66)$$

with $X = S, A$, $\epsilon = m_\tau/m_\mu$, $\lambda = m_\mu/M_X$ and $Q_S(x) = x^2(1 + \epsilon - x)$ for a scalar $S$, and $Q_A = x^2(1 - \epsilon - x)$ for a pseudo-scalar $A$; $O$ denotes a matrix element in the scalar or pseudo-scalar sector and use $O = 1$ for the sake of simplicity. In order to fit the muon anomaly we need $m_H \gtrsim 4.318$ TeV and $m_A \gtrsim 4.321$ TeV. However, lower masses are allowed if we consider the contributions of all scalar and pseudo-scalars in the model. We recall that vector boson contributions $W_1, W_2$ are suppressed by neutrino masses and the unitarity of the PMNS mixing matrices and neutral vector bosons have diagonal interactions with leptons.

Below, we will consider mainly the difference with the case of the model with Majorana neutrinos (with triplets in the scalar sector), in particular when heavy neutrino do exist, with the present model with Dirac neutrinos.
• Obviously, in the present case there is no heavy neutrinos that can decay into a Higgs boson plus an active neutrino, \( N_R \rightarrow H + \nu \).

• Although the processes \( \mu \rightarrow e + \gamma \) occurs in this model, they have not the (logarithmic) enhancement produced by the doubly charge scalar bosons \([47, 48]\).

• Flavour lepton number processes as \( \mu \rightarrow ee\bar{e} \) and \( \mu - e \) conversion cannot occur in this model.

• There is no neutrinoless double beta decay (\( \beta \beta_{0\nu} \)) and other \(|\Delta L| = 2\) processes.

• There is no Keung-Senjanovic (KS) production of same-sign charged lepton pairs plus jets: \( pp \rightarrow W^+_R \rightarrow l^+_R N^c_L \rightarrow l^+_R W^-_R l^+_R \rightarrow l^+_R l^+_R j j \) \([49]\).

Notice that in the KS process there are no missing energy. However, in both Majorana and Dirac neutrino cases there are processes like \( W^+_R \rightarrow l^+_R l^+_R l^+_R (\nu^c)_L \) (Majorana) if \( M_{W^+} > m_{N_R} \), and \( W^+_R \rightarrow l^+_R l^+_R l^-_R (\nu^c)_R \) (Dirac). We recall that in the SM the processes mediated by the \( W^\pm \), for instance \( pp \rightarrow W^+ \rightarrow \nu^c_R l^+_L \rightarrow l^+_L l^+_R l^+_R \nu^c_R \) are possible. Hence, at least in principle, it is possible to use these processes to distinguish the Dirac from the Majorana case. It is worth to note that if neutrino are Majorana particles through the type II seesaw mechanism implemented in the SM plus a scalar triplet with \( Y = 2 \) the processes are \( pp \rightarrow H^+ \rightarrow l^+_R l^+_R l^+_R:\nu^c_L \), if a charged scalar \( H^+ \) is in the second lepton vertex; or \( pp \rightarrow H^+ \rightarrow \nu^c_L l^+_L \rightarrow l^+_L l^+_R l^+_R \nu^c_L \), if a \( W^+_R \) is in the second leptonic vertex. The case of trimuons, \( l = \mu \), could be the more interesting in all these processes.

Finally, but not least, we note that in LR or other models with s second chargecd current, there is a contribution to the electric dipole moment (EDM) of an elementary particle, say electron or quarks (neutron), at the 1-loop level. The phase in the CKM or PMNS matrix do not contribute at this level since in this the diagram \( CP \) violating phase cancel out because on vertex in the complex conjugate of the other, say \( V_{CKM} V_{CKM}^* \) in the quark secto, and the diagram is real \([50]\). However, if there is a second charged currents as in LR models, the relative phase \( \phi_l \) and \( \phi_q \) in Eqs. \([42]\) and \([43]\) cannot be absorbed. In particular it implies a contribution to the EDM a quark:

\[
\mu_E^q = e \frac{m_q}{M_{W^i}^2} c_\xi s_\xi \sin \phi_q \times \text{logarithmic corrections} \tag{67}
\]
where $M_{W_1}^2 = M_{W_1,W_2}^2$, and the larger contributions is that of $W_1^+$. As an illustration, the EDM of a light quarks with $m_q \sim 10^{-3}$ GeV and $M_{W_1} \sim 80$ GeV, $s_\xi \sim 10^{-3}$ we obtain $\mu_E^q \approx 3.08 \times 10^{-26} \sin \phi_q e \text{cm}$, which is almost the present limit for the EDM of the neutron $\mu_E^n < 0.30 \times 10^{-25} e \text{cm} \text{ CL } 90\%$. Hence, the phase $\phi_q$ is not restricted with the present experimental data. In the lepton sector the most restringent EDM is that of the electron $\mu_E^e < 0.87 \times 10^{-28} e \text{cm} \text{ CL } 10\%$ which implies $\sin \phi_l < 10^{-2}$. In the lepton sector the 1-loop contribution (induced by the $W_L - W_R$ mixing) to the electron EDM is suppressed by the neutrino masses and the phase $\phi_l$ is not suppressed by this observable. On the other hand, a neutrinos have magnetic and electric dipole moment (induce by the mixing) which are not suppressed since they are proportional to the heavy charged lepton.

IX. BREAKING PARITY FIRST

Any model beyond the standard model (BSM) must match with that model at a given energy, say the $Z$-pole. In the SM coupling constants $g$ and $g_Y$ have different running with the energy. In the case of LR symmetric models, the same happens with $g$ and $g' \equiv g_{B-L}$ as was noted in Ref. [32]. It means that we cannot keep $g_L = g_R$ for all energies since quantum corrections imply a finite $\Delta g = g_L - g_R \neq 0$. This is due to the fact that both constants feel different degrees of freedom. Hence, it is interesting to search for models with gauge symmetries as in Eq. (1) but in which parity is broken spontaneously by non-zero VEVs [32] or softly if quadratic terms in the scalar potential are different $\mu^2_L \neq \mu^2_R$ as in Ref. [12].

Let us consider as in [32] the possibility that the symmetries in Eq. (1) are broken spontaneously but in the following way: First the parity $P$ is breaking at an energy scale $\mu_P$ by introducing a neutral pseudoscalar singlet, $\eta \sim (1, 1, 0)$ with $\eta \rightarrow -\eta$ under parity. Then, the doublet $\chi_R$ breaks the $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$ symmetry to $SU(2)_L \otimes U(1)_Y$. The relevant terms in the scalar potential involving the doublets $\chi_L, \chi_R$ and the isosinglet $\eta$ are the following:

$$\mu_\eta^2 \eta^2 + \lambda_\eta \eta^4 + \mu_{LR}^2 (\chi^+_L \chi_L + \chi^+_R \chi_R) + f_\eta (\chi^+_L \chi_L - \chi^+_R \chi_R) + \lambda'_\eta \eta^2 (\chi^+_L \chi_L + \chi^+_R \chi_R) \subset V. \quad (68)$$

At the energy $\mu_P$, $\mu_\eta^2 < 0$ with $\langle \eta \rangle = v_\eta \simeq \mu_P$, and all the other VEVs are still zero. We
obtain
\[ \mu_L^2 = \mu_{LR}^2 + f v_\eta + \lambda_\eta v_\eta^2, \quad \mu_R^2 = \mu_{LR}^2 - f v_\eta + \lambda_\eta v_\eta^2, \]
(69)
with the singlet VEV \( v_\eta = \sqrt{-\mu_\eta^2/2\lambda_\eta} \). Next, if \( \mu_L^2 < 0 \) and \( |\mu_R^2| \ll v_\eta \) we have that \( \langle \chi_R \rangle = v_R \neq 0 \). This leads to the interesting case in which the \( SU(2)_R \) symmetry breaking scale is induced by the parity breaking scale as noted in Refs. [32]. It happens also that \( g_L \neq g_R \), for energies in the range \( v_R < \mu < v_\eta \), and also \( V_{PMNS}^L \neq V_{PMNS}^R \), with \( V_{PMNS}^L = V_L^\dagger U_L^\nu \), \( V_{PMNS}^R = V_R^\dagger U_R^\nu \). In this case we have to consider the most scalar potential involving two or three bi-doublets, \( \Phi_i \), two doublets \( \chi_{L,R} \), and the singlet \( \eta \).

X. CONCLUSIONS

In the context of the SM with three right-handed neutrinos the Yukawa couplings have the hierarchy (using the normal hierarchy): \( \Delta y_{31} = (\Delta m_{31}^2)^{1/2}/v_{SM} \approx 2 \times 10^{-13} \) and \( \Delta y_{21} = (\Delta m_{21}^2)^{1/2}/v_{SM} \approx 2 \times 10^{-14} \) which we compare with those in the present model given in Eq. (60). Although the later values values are smaller than the Yukawa sector in the charged lepton sector, see Eq. (61), we note that the dispersion in the neutrino Yukawa couplings are in the range \( 0.25 - 2.2 \) (up to a factor of \( 10^{-11} \)). In this model, as in the old left-right symmetric models without scalar triplets and also no extra charged leptons, neutrinos gain arbitrary small masses. Notice, however that the Yukawa coupling are all almost of the same order of magnitude and about two order of magnitude larger compared with those in the context of the SM with three right-handed neutrinos. However, we shown in Sec. VII how this fine tuning in the lepton sector can be avoided at the price of introducing a third bi-doublet and the discrete \( D \) symmetry. In the latter case, all Yukawa couplings in the lepton and quark sector may be of order \( \mathcal{O}(1) \) if there is a hierarchy in the VEVs of the three bi-doublets.

We stress that, since the early eighties most phenomenology of the left-right symmetric models includes triplets and Majorana neutrinos [17]. Since then, the model with the following scalar multiplets: one bi-doublet and two triplets was considered the minimal left-right symmetric model. There is no doubt that this proposal was, and still is, well motivated [51]. However, if the neutrinos ultimately turn out to be Dirac particles, all that efforts will have been in vain. For this reason we have to pay attention to Dirac neutrinos, even in the context of the left-right symmetric models.
ACKNOWLEDGMENTS

HD thanks to CONCYTEC for financial support and to the IFT-UNESP for the kind hospitality where part of this work was done. VP would like to thanks for partial financial support to CNPq and FAPESP under the funding Grant No. 2014/19164-6 and last, but not least, to the the Faculty of Sciences of the Universidad Nacional de Ingeniera (UNI) for the kind hospitality.

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