Approaches to solving the problem of fuzzy parametric programming in weakly structured objects

D T Muhamediyeva¹, N A Niyozmatova²
Tashkent University of Information Technologies named after Muhammad al-Khwarizmi, Tashkent, Uzbekistan

Abstract. The article discusses approaches to solving parametric programming problems with fuzzy initial information, in the case when the constraints and the objective function depend on many parameters. The paper considers an approach in which parameters can be expressed in a more general form of fuzzy sets of their possible values. In this way, we get a new type of mathematical optimization problems containing fuzzy parameters. In this chapter, the consideration of linear optimization problems based on this approach is the essence of fuzzy linear programming.

1. Introduction

In weakly formalized processes, there is often uncertain information about the coefficients of a mathematical model. As a mathematical tool that allows you to formalize uncertain a priori information, the elements of fuzzy mathematics are used in the work [1]. When building applied fuzzy systems, it is assumed that the initial knowledge base is generated by an expert who knows the object of modelling well. The essence of the proposed method lies in the selection of such membership functions of fuzzy terms and such weights of fuzzy rules that minimize the difference between the results of fuzzy inference and experimental data [2].

Parametric programming allows you to choose the best option, taking into account all the possible parameters of the problem.

In mathematical optimization, a preference between possible options is described using the objective functions for the given set of options. Values of the objective function describing the impact (importance) of each option, so the options are more preferable to have larger values than the less preferred. The set of valid options in the optimization problems described with the aid of constraints - equations or inequalities that represent necessary connections between the options. The results of the analysis greatly depend on how adequately various factors of the real system are reflected in the description of the target function (or functions) and limitations.

Mathematical formulation of objective functions and constraints in optimization problems usually involves some parameters. The values of such parameters depend on many factors, not usually included in the formulation of the problem. Trying to make the model more representative, we often input to the complex relationships, making the model more cumbersome and analytically unsolvable. Often such attempts to increase "accuracy" of the model is practically useless due to the inability of accurate measurement of its parameters. On the other hand, the model with fixed values of parameters may be too crude, since these values are often chosen quite arbitrarily.

Objectives fuzzy linear programming (and related tasks) has been intensively studied in many works, demonstrating the variety of formulations and approaches. Most of the approaches to the problems of fuzzy linear programming is based on the direct use of the intersection of fuzzy sets

¹ Tashkent University of Information Technologies named after Muhammad al-Khwarizmi, Tashkent, Uzbekistan
² Tashkent University of Information Technologies named after Muhammad al-Khwarizmi, Tashkent, Uzbekistan
representing goals and constraints, and then maximising the resulting function. This approach was considered by Bellman and Zadeh [1]. Later the team studied the problem of linear programming with fuzzy parameters, known under different names, mostly as fuzzy linear programming, sometimes known as probabilistic linear programming, agile, linear programming, linear programming under uncertainty, inexact linear programming.

2. Formulation of the problem

We need to find the optimal production structure of the enterprise, ensuring maximum net income, taking into account weather conditions.

Optimality criterion: maximizing net income.

\[ z(t) = \sum_{r \in R} \alpha_r t_r (x_r - \bar{x}_r) \rightarrow \max, 0 \leq t_r \leq 1. \]  \hspace{1cm} (1)

It is required to determine effective strategies for the development of enterprises consisting of various combinations of sectors of activity. For this purpose, a simulation model is proposed, which consists of the following blocks.

1. Silk complex consumption unit.

   The consumption function consists of a structural equation

   \[ D = \lambda_1 + \lambda_2 P + \lambda_3 P_{-1} + \lambda_4 (W' + W''), \]

   where \( D \) - consumption; \( P \) - profit; \( P_{-1} \) - prior period earnings; \( W' \) – wages in the agricultural sector of the silk complex; \( W'' \) – wages in the industrial sector of the silk complex.

   The payroll function consists of a structural equation

   \[ W = \gamma_1 + \gamma_2 (x_r + T) + \gamma_3 (x_r + T)_{-1}, \]

   here \( x_r \) – silk production; \( T \) – indirect taxes.

   Identity

   \[ x_r + T = D + I, \]

   where \( I \) – investment.

2. Block production activities of enterprises for the production of silk products.

   Production activities of enterprises are described by the neoclassical production function

   \[ Y_m = A_m^w L_m^w B_m^v I_m^\nu \gamma_m^p, (a + l + b) \leq 1, \]

   where \( A_m^w, L_m^w, B_m^v \) – respectively, production assets, labor, raw materials and supplies.

3. Block of commercial activities. This block describes the results of the company's activities from the sale of a part of the j-th product in cooperation and commercial activities. Given this circumstance, the volume of production of the j-th type \( Y^{jk} \), sold by an enterprise in the order of its interaction with associations, is calculated by the formula

   \[ Y^{jk} = \min \{ d^{jk}, Y^{jop}, \Phi^{jk} / S^{j'i} \} \delta^{ek}. \]

   Where \( d^{jk} \) – the proportion of j-th product sold through the enterprise; \( Y^{jop} \) – production volume of the j-th product in the enterprise in terms of value, \( \Phi^{jk} \) – enterprise, designed for business fund; \( S^{j'i} \) – unit costs for the sale of one item of the j-th product; \( \delta^{ek} \) – boolean variable.

   The costs of commercial activities (advertising, transportation costs, other costs) that are assumed to be proportional to the turnover of commercial activities, for product j has the form

   \[ C^{j'i} = Y^{jk} S^{j'i} \delta^{ek}, \]

   where \( S^{j'i} \) – unit costs for commercial activities.
Commercial profit $M^j$ is formed as the difference between income and costs for the implementation of $C^j$

$$M^j = (P^j_p - P^{iopp}_p) \min\{D^j, Y^j_k\} - C^j,$$

where $P^j_p$ and $P^{iopp}_p$ — respectively, the market (sales) and the selling price of the enterprise; $\min\{D^j, Y^j_k\}$ — actually sold products of type $j$, determined by the minimum between the demand for it and supply $Y^j_k$.

4. Block of innovation activities. In this block incomes from innovation are calculated. [1]. The amount of capital investment $F^q$, aimed at developing new technologies for the production of products $q$,

$$F^q = \sum_{t=1}^{Q} F^q_{in} \left(1/(1+E)^t\right),$$

where $\Theta_1$ — project implementation time until its liquidation; $q = 1, \ldots, Q$.

The profitability index $ID^q$ for the $q$-th project, according to [1], is calculated by the formula

$$ID^q = (1/F^q) \sum_{t=1}^{Q} (u^q_t - v^q_t)(1/(1+E)^t),$$

where $u^q_t$ and $v^q_t$ — respectively, the results and costs of the project $q$ at time $t$.

$M^q_t$ arrived at the project $q$ at the moment of time $t$, taking into account the profitability index $ID^q$ and the time of the project $\Theta_2$

$$M^q_t = \begin{cases} 0, t < \Theta_2, \\
ID^q F^q_t, t \geq \Theta_2. 
\end{cases}$$

5. Block of formation and distribution of profits.

The total profit of SM for all activities is determined by the formula

$$SM = OM^p + OM^e + OM^{ia},$$

where $OM^p$ — total profit from the production activities of the enterprise; $OM^e$ — total profit from the business of the enterprise; $OM^{ia}$ — total profit from innovation.

The amount of tax deductions $Nal^p$ according to the results of the production activity of the enterprise

$$Nal^p = HDC + PR,$$

where

$$HDC = n' \sum_{i=1}^{n} (l^i + b^i) x^i,$$

$$PR = n'' OM^e.$$

Where $n'$ and $n''$ accordingly tax rates, HDC- Value Added Tax, PR — income tax.

The amount of tax deductions for the results of commercial activities

$$Nal^k = n^3 \sum_{j=1}^{J} \min\{D^j, Y^j_k\} + n^2 OM^k,$$

where $n^3$ and $n^2$ — respectively, the tax rate on turnover and profit of commercial activities. Since it is considered that innovation activity is not taxed, we assume tax deductions from this type of work to be zero.

The total value of Snal taxes for enterprises in general is
\[
S_{Nal} = Nal^n + Nal^k.
\]

Distribution of profits of enterprises for taxes \(S_{Nal}\), rent

\[
ar_{n} (k_n \sum_{i=1}^{m} A^i + \sum_{q=1}^{Q} A^q),
\]

Dividend \(Div\) and POM residual income aimed at the development of enterprises, is carried out according to the formula

\[
POM = SM - S_{Nal} - ar(k_n \sum_{i=1}^{m} A^i + \sum_{q=1}^{Q} A^q) - Div,
\]

where \(ar\) – cost of renting a unit of enterprise space, \(k_n^i\) – the size of the area per unit cost of equipment.

6. The block of calculation of the main indicators of the enterprise.

In this section, the main indicators of enterprises are considered – profitability and profitability. The profitability of the enterprise is determined by the formula

\[
R^{IP} = (OM) / (\delta^{M} \sum_{i=1}^{M} (A^i + \gamma B^i) + \delta^{F} \sum_{j=1}^{F} F^j + \delta^{inf} \sum_{q=1}^{q} F^{inf}),
\]

where \(\gamma\) – coefficient of recalculation of the cost of raw materials and materials into the cost of working capital, taking into account their turnover and the proportion of work in progress and expenses for the future period.

The profitability of the product produced in the sector of industrial activity of the enterprise, can be calculated by the formula

\[
R^{s} = OM^{s} / \sum_{i=1}^{M} x^i.
\]

The developed simulation model allows simulation experiments, i.e. by varying the values of the control parameters, it is possible to obtain various lines of activity of enterprises, contributing to an increase in production efficiency.

Solving a fuzzy parametric programming problem (1) is reduced to solving the following problem

\[
\begin{align*}
\min & \ z = (\bar{a}_0 + \bar{b} t) x + \bar{c} t \\
\text{s.t} & \ (a + ct)x \subset K, \\
& \ t \in E_t
\end{align*}
\]

Here \(K = \{ y : y \in R^n, y \leq a_0 + dt \} \) – given convex subset of space \(R^n\). The coefficients are given in fuzzy form:

\[
\begin{align*}
a_q = & \sum_{i=1}^{q} a_{i} \mu^{i} / \sum_{i=1}^{q} \mu_i, \\
b_{ij} = & \sum_{i=1}^{q} b_{i} \eta^{i} / \sum_{i=1}^{q} \eta_i, \\
c_{ij} = & \sum_{i=1}^{q} c_{i} \zeta^{i} / \sum_{i=1}^{q} \zeta_i.
\end{align*}
\]

3. Implementation of parametric programming tasks

The solution to problem (2) is:

\[
z(t) = \min \{ z = (\bar{a}_0 + \bar{b} t) x + \bar{c} t \ | \ (a + ct)x \leq (a_0 + dt); x \geq 0 \}.
\]
Denote by \( O_i \), the set of solutions \( t \) satisfying (2). Due to the fact that the values of the coefficients are not clearly defined, \( O_i \) is a fuzzy set. Let its membership function be \( \varphi_{0i}(t) \). Thus
\[
O_i = \{t, \varphi_{0i}(t), t \in \mathbb{R}^s\}.
\]
Fuzzy solution corresponds to a fuzzy maximum value
\[
\varphi_{zi} = \sup_{t \in O_i} \varphi_{0i}(t).
\]
For any admissible vector \( x \), the inequalities are true
\[
z_i(t) \leq g(t),
\]
where \( g(t) = (\bar{a}_i + \bar{b}_i t)x + \bar{e}_i t \).

The map \( q \) defined by (3) is described by the expression
\[
\varphi_q(z_i(t), g(t)) = \begin{cases} 
1, & \text{if } g(t) \geq z_i(t), \\
0, & \text{if } g(t) < z_i(t).
\end{cases}
\]
(4)

Since \( z_i(t) \) – belongs to the fuzzy set \( Z_i \), \( g(t) \) is an element of some fuzzy set \( G_i \), which is the image of \( Z_i \) under the map \( q \). According to [1], the membership function of the set \( Z_i \) and its image \( G_i \) are related by
\[
\varphi_{q_i}(g(t)) = \max\{\varphi_{z_i}(z_i(t)), \varphi_q(z_i(t), g(t))\}.
\]
(5)

For \( \varphi_q(z_i(t), g(t)) \), we get
\[
\varphi_{q_i}(g(t)) = \max\{\varphi_{z_i}(z_i(t)) : z_i(t) \leq g(t)\}.
\]
(6)

Since the function \( g(t) \), the set \( O_i \) is the prototype of \( G_i \), according to [1], we have
\[
\varphi_{0i}(t) = \varphi_{q_i}(g(t)) = \varphi_{q_i}((\bar{a}_0 + \bar{b}_0 t)x + \bar{e}_0 t).
\]
(7)

Substituting (6) into (7), we get
\[
\varphi_{0i}(t) = \max\{\varphi_{z_i}(z_i(t)) : z_i(t) \leq (\bar{a}_0 + \bar{b}_0 t)x + \bar{e}_0 t\}.
\]
(8)

Solution (8) for \( \varphi_{z_i}(z_i(t)) \) is
\[
\varphi_{z_i}(t) = \exp(-\frac{k}{2}[\max(0, z_i(t) - (\bar{a}_0 + \bar{b}_0 t)x - \bar{e}_0 t)]^2).
\]
(9)
The membership function of a fuzzy mapping depends on \( s \) variables
\[
\varphi_{z_i}(t) = \prod_{i=1}^{s} \varphi_{z_i}(t),
\]
(10)

where \( t = t_1 \times \cdots \times t_s \) – cartesian product. From (9) and (10) we have
\[
\varphi_{z_i}(t) = \exp(-\Phi(t)).
\]
(11)

Here \( \Phi(t) = \frac{1}{2} \sum_{i=1}^{s} k_i [\max(0, z_i(t) - (\bar{a}_0 + \bar{b}_0 t)x - \bar{e}_0 t)]^2 \).
We have proved the following statement: let $z_t(t)$ be a fuzzy solution of problem (2). Then the solution set has the membership function (11).

If there is fuzzy information, then it is necessary to choose such an estimate of the parameters $t$, so that it minimizes $z_t(t)$ and at the same time its degree of belonging to the permissible set $Z_t$ was maximum, that is:

$$
z(t) \to \min, \varphi_t(t) \to \max. \quad (12)
$$

According to Lemma 2 [2], by virtue of positiveness of $\varphi_t(t)$, the second criterion in (12) can be replaced by $\ln \varphi_t(t)$. Thus, (12) is equivalent to the problem

$$
z(t) \to \min, \Phi(t) \to \min. \quad (13)
$$

The preferred solutions to this problem are those that cannot be improved by one criterion without increasing the other criterion in (13). Such solutions are called Pareto optimal.

Since $z(t)$ and $\Phi(t)$ are convex functions, Pareto optimal solutions are the solution to the problem

$$L(t) = z(t) + r\Phi(t) \to \min, \quad r \in [0, \infty]. \quad (14)
$$

In the case of formalization of information in the form of fuzzy sets, not one solution is determined, but some of them are many. They are a function of $r$. Denote them by $t(r)$.

By $L(t)$ is convex to have $r_2 > r_1 \geq 0$

$$z(r_1) + r_1\Phi(r_1) < z(r_2) + r_2\Phi(r_2),$$

$$z(r_2) + r_2\Phi(r_2) < z(r_1) + r_1\Phi(r_1).$$

From here we get

$$r_2(\Phi(r_2) - \Phi(r_1)) + r_1(\Phi(r_1) - \Phi(r_2)) < 0,$$

$$\Phi(r_2) - \Phi(r_1))(r_2 - r_1) < 0.$$

This inequality implies the following statement: if $L(t)$ is a convex function, then $\varphi_t(t)$ is strictly monotonously decreasing as $r \geq 0$.

This means that in the set $O$ there is no such solution for which the inequalities

$$\varphi_t(t) > \varphi_t(r) > 0 \text{ and } z(t) > r.$$

If the decision maker prefers to choose a specific solution $t \in T$, then his choice should be based not only on the degree to which the alternative belongs to the fuzzy set $\varphi_t(t)$, but also on the corresponding values of the function $z(t)$.

**4. Algorithm for the implementation of tasks**

I. Implementation of the fuzzification operator. A maximized linear form $z$ is given, in which the values of the coefficients are fuzzy sets.

In addition, constraints in the form of fuzzy sets are given. It is required to make a rational choice of vector $x \in R^n$, which in a certain sense “maximizes” a given fuzzy linear form.

If the membership function $\mu, \nu, \eta, \xi$ is set, then we can consider

$$\mu^k / \sum_{k=1}^{K} \mu^k, \eta^k / \sum_{k=1}^{K} \eta^k, \nu^k / \sum_{k=1}^{K} \nu^k \text{ and } \xi^k / \sum_{k=1}^{K} \xi^k,$$

as reduced subjective probability distributions of the components of the $\mu, \eta, \nu$ and $\xi$ membership function.
Determined by \( a_{ij}, b_{ij}, c_{ij}, d_{ij} \).

The solution of parametric programming will give the optimal separation of variables into free and basic in the task of fuzzy parametric programming.

II. Finding the optimal solution to the problem. Based on a certain value of \( t = t^0 \), it is determined whether there is an optimal solution for problem (2) for this value, i.e. is the function \( z_0(t) \) defined at a point \( t^0 \).

III. Putting \( p = 0 \) and \( v = 0 \), we present the coefficients of the \( r \)-th table as rational functions \( a^{(r)}_{ij} \) and find all the roots of the equations \( D^{(r)}_{ij} = 0; i = 0,1,\ldots, m; j = 0,1,\ldots, m+n \) and \( N^{(r)}_{ij} = 0; i = 0,1,\ldots, m; j = 0,1,\ldots, m+n \) for \( t < t^0 \). By ordering these roots according to the rule

\[
st^{(r+1)} > st^u, u = 0,1,\ldots, H,
\]

forming a plurality of

\[
T^{(r)}_s = \{ t^u | u = 0,1,\ldots,H \}
\]

and the condition is checked \( v = H \).

IV. Let the value of \( t = t^0 \) be known for which there exists an optimal solution to problem (2). If such a value is unknown, use the method described above to obtain it or to find out that such a value does not exist, i.e. that \( Q \) is empty. Putting \( s = s^* = -1, p = s, t^* = t^0, R = 0 \), proceeds to point V.

V. Critical value determined \( t_0 = k^{(r)}_s \).

VI. Putting \( t^0 = t^*_p \), proceeds to point VII.

VII. Putting \( p=0, v=0 \), define the set \( T^{(r)}_s \):

\[
T^{(r)}_s = \{ t^u | u = 0,1,\ldots,H \}
\]

5. Conclusion

Thus, the dependence of the effectiveness of managerial decisions in a particular subject area, especially in agriculture, on the external environment is as noticeable that, with significant fluctuations, many of the decisions made earlier and the costs incurred are replaced by others. Consequently, the effectiveness of previous decisions is much underestimated. In this regard, to improve management efficiency, it is necessary to make management decisions taking into account the influence of random, unregulated human factors, such as rainfall and their distribution over periods of the year, air temperature, etc. For this purpose, parametric programming methods are used, which take into account random parameters. The influence of random parameters are taken into account through fluctuations of crop yields. Consequently, resource costs and output, which are reflected in the model through technical and economic coefficients, also have varying values.

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