Plasma columns generated by the propagation of an electromagnetic surface wave have no effect on the properties of the wave, which depend only on operating conditions: a shift in paradigm

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Abstract
Surface-wave (SW) sustained plasmas belong to the category of gaseous discharges provided by the electric field component of an electromagnetic wave. Tubular plasmas of this kind can be achieved over an unrivalled wide range of operating conditions. Analytical and numerical simulation modelling of SW produced plasmas has rest till now on considering the wave and the generated plasma column as a whole. Doing so implies that the plasma column along which the SW travels influences the SW properties. In contrast, the current paper advocates that there is no coaction of the plasma column on the characteristics of the surface wave that sustains it, hence a proposed shift in paradigm. It arose initially from observing that the axial distribution of electron density along SWDs is linear (constant slope) till the very end of the plasma column, despite plasma diameter axial variations. It opposes directly the curving down of the axial electron density profile toward the end of the plasma column inferred by modellers. Truthful linearity of the experimental data (from various laboratories) is supported statistically by least squares regressions yielding high correlation coefficients. Additionally, the axial distribution of electron density is shown experimentally conforming to a similarity law expressed by the product of SW frequency times electron-neutral collision frequency over discharge tube inner radius (operating conditions). The physical justification behind the axial linearity of electron density stems from a stability criterion governing travelling-wave sustained discharges, which requires that the electron density decreases monotonously as the wave loses power as it propagates. This rule sets by itself the SW power flow axially delivered to the discharge gas expressed in terms of the density of electrons generated in any given axial segment of the plasma column. The plasma column properties are thus entirely established by the SW features, having no influence on the wave.

Keywords: Plasma physics, RF and microwave discharges, surface-wave discharges, axial distribution of electron density, discharge stability criterion

+ This paper is dedicated to the memory of Professor Ivan Zhelyazkov (1938–2021), a plasma physics theorist (St. Clement of Ohrid University, Sofia, Bulgaria).
1. Introduction

Gaseous discharges sustained by the electromagnetic (EM) field of a propagating surface wave (SW) can be achieved over an unequaled wide range of operating conditions. Specifically, in the case of tubular discharges, these are: gas nature and pressure (from few mTorr to at least ten times atmospheric pressure) accounted for by \( \nu \), the electron-neutral collision frequency for momentum transfer, then discharge (dielectric) tube inner radius \( R \) (from a mm to at least 150 mm) as well as EM field frequencies \( f \) that run from radio-frequencies (RF) (as low as a few MHz) to microwaves (MW) (as high as 40 GHz). Surface-wave discharges (SWDs) have been widely investigated both experimentally and theoretically since the 1980’s, incidentally resulting in applications. The work carried out expressly on the axial distribution of electron density can be found in a comprehensive paper by Zhelyazkov and Atanassov [1]. In their review, analytical expressions and results from numerically computed models are proposed to try fitting, through modifications of wave and plasma parameters, the observed linear axial distribution of electron density: whatever the models considered (excluding approximations), linearity is not recovered toward the end of the plasma column.

The way that modelling the SW plasma column has been envisaged hitherto can be depicted by evoking excerpts from literature, such as: "Clearly, SW propagation characteristics and the axial distribution of plasma density are interrelated" [2] or "mutually determined" [3]. A further similar assertion: "...the wave heats the electrons which ionize the medium ensuring in this a way for further wave propagation" [4]. Finally, from our group at the Université de Montréal: "The produced plasma column thereby constitutes an essential part of the waveguiding structure: the wave and plasma properties are interdependent" [5]. These statements imply that the plasma column is coacting on the SW properties, which is incorrect as will be further contended.

The starting point of the current paper stands in the experimental fact that, for given operating conditions, the axial gradient of electron density turns out to be constant all along the plasma column. It means that, at any axial position referenced to the column end, the electron density can be linearly extrapolated from even the shortest plasma column segment obtained experimentally: the occurrence of such a constant axial gradient of electron density is, already at first sight, incompatible with axially varying characteristics of the plasma column. Doubts being raised (essentially by modellers) about the true linearity of the axial profile of electron density, experimental results from five different laboratories (and more operators) have been gathered to confirm linearity through least squares regressions yielding (very) high correlation coefficients. Then, an analytical derivation of the SW power flow expression along the plasma column introduces a (novel) condition for the axial gradient of electron density to be constant till the very end of the plasma column even though, for instance, abrupt axial structural changes occur at the end of the SW plasma column.

The dependence of the SWD axial gradient of electron density on operating conditions was figured out analytically by Aliev et al. [2], bringing out more insight into its linearity. For instance, their model confirms (what had been demonstrated numerically earlier [6]) that the axial gradient of electron density is linearly dependent on \( f, \nu, \) and \( R \). Going a step further, Aliev et al. established that a similarity law of the form \( f \nu/R \) governs the axial gradient of electron density. Although their relationship fully reveals the dependence of SWDs on operating conditions, it does not lead to the basic physical mechanism(s) at the origin of such a linear electron density profile, which the current paper further discloses.

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The paper is organized as follows. Experimental data firmly supporting linearity of the axial distribution of electron density along SWDs (and its dependence on $f/\nu R$ for tubular discharges) are collected in Section 2. Section 3 relates these results with Mateev et al. [23] and Aliev et al. [2] analytical relationships predicting the axial gradient of electron density, this document extrapolating these experimentally to atmospheric pressure. Section 4 develops further SWD features, which include the axial dependence of the SW power attenuation coefficient on electron density as determined by experiments. It also introduces the discharge stability criterion obeyed by travelling wave (TW) sustained discharges: it that provides, on sound physical grounds, a justification to the observed constant axial gradient of electron density. Section 5 constitutes the summary and conclusion.

2. Observed axial distribution of electron density along SW generated plasmas (past the EM radiation region of the field applicator) for both tubular plasmas and plasma torch systems

Two distinct SW discharge configurations are considered therein: plasma is sustained inside a dielectric tube (surfatron, surfaguide: used as field applicators [7]), namely tubular plasmas, and plasma is free-standing in its flowing carrier-gas envelope (TIA/TIAGO plasma torch systems). referred to as dense plasma filaments.

2.1 SWD tubular plasmas

2.1.1. Low gas pressure (low collisional-regime). This case corresponds to $\nu/\omega \ll 1$ where $\omega = 2\pi f$. Such a condition causes that the SW stops propagating when the electron density falls below the minimum value $\bar{n}_{e(re)}$, setting the end of the plasma column. In figure 1, electron density, averaged over the radial cross-section of the plasma column, was determined with a TM$_{010}$ mode resonant-cavity method [8]. The figure exhibits the electron density as displayed axially from the end of the column, $\bar{n}(z)$, at four SW field frequencies running from RF (27 and 50 MHz) to MWs (100-200 MHz). The set of data points corresponding to a given wave frequency fit the straight line resulting from the least squares regression, with some coefficient of determination $r^2$: the closer $r^2$ to unity, the higher the confidence in the statistical regression (see figure captions for $r^2$ values). The slope of these straight lines increases with increasing frequency.

A correlative outcome of the linearity of the SWD axial distribution of electron density, noted from the very beginning of SWD studies, is the fact that increasing EM power communicated to the field-applicator leads to an increase in the length of the plasma column without modifying the slope of the axial distribution of its electron density. This is best seen when plotting electron density with reference to the end of the plasma column. It is exemplified with the 100 MHz curve in the figure: the arrow pointing at 36 W represents the axial position of the applicator relatively to the end of the SW

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1The relation $\bar{n}_{e(re)}$ (cm$^{-3}$) = $1.2 \times 10^4 (1 + \varepsilon_g) f^2$ (MHz) ($\varepsilon_g$ is the relative dielectric permittivity of the discharge tube material, 3.78 for fused silica and 4.52 for most Pyrex brands) determines the minimum electron density allowing SW propagation along the plasma column [3]. When $\nu/\omega$ is no longer much smaller than unity, the electron density is above $\bar{n}_{e(re)}$, allowing SW propagation, but an axial point is then reached where not enough wave-power flow is left to provide a further plasma segment, defining the end of the plasma column.
plasma column at that power value and, as just mentioned, the slope of the already existing segment of the plasma column is not affected when MW power is raised (past a μs transition period: Sec. 4.2), here, up to 58 W.²

Figure 1. Measured axial variation, referenced from the end of the plasma column, of the electron density averaged over its radial cross-section. The discharge is sustained by the propagation of an EM surface wave at four different field frequencies, in argon gas at a pressure of 30 mTorr (≈ 4 Pa) in a tube of 32 mm inner radius [8]. The straight lines result from least squares regressions on the electron density experimental values yielding as coefficients of determination $r^2 = 0.990, 0.999, 0.998, 0.998$, successively with increasing frequency.

Figure 2a exhibits, with respect to the plasma column end, the measured axial distribution of the radially averaged electron density $\bar{n}_e(z)$, at five argon gas pressures (almost doubling to the next one). The data points all fit, again, a straight line resulting from least squares regressions with high correlation coefficients $r^2$ (see captions) for given gas pressures, this line taking a more inclined slope as pressure is increased. As for the "curved line" segments of electron density occurring at higher axial positions than the straight line, they belong to the antenna-like radiation region at the exit of the field applicator [9]. Measurement of the electron density, as in figure 1, is based on a TM$_{010}$ mode resonant-cavity technique (see [6] for details).

² Noteworthy is that at a frequency of 27 MHz and a pressure of 30 mTorr (4 Pa) in a 32 mm inner radius discharge tube, the SW argon plasma column expands up to 4.5 m with less than 40 W supplied to the surfatron!
Figure 2a. Measured axial distribution, plotted from the end of the plasma column, of the radially averaged electron density along the discharge sustained by the propagation of a SW at 360 MHz, at five argon gas pressures in a fused silica discharge tube of inner radius 12.5 mm [6]. As an example, at 80 mTorr the coefficient of determination on the straight segment of the data points reaches to $r^2 = 0.9996$!

Figure 2b is an enlargement of the 0-50 cm axial portion in figure 2a, additionally exhibiting $\bar{n}_{e(\text{re})}$, the minimum electron density required for the SW to propagate at 360 MHz under low-collisional regime (footnote 1). The axial distributions of electron density in the figure are undoubtedly straight lines till the very end of the plasma column while most modeling calculations till now reveal a bending down of the straight line toward the column end ([1] and, for example, figure A4 in the Appendix). Noteworthy is that these straight lines lead (or would graphically extend) to $\bar{n}_{e(\text{re})}$ at the column end.

Figure 2b. Enlargement of the 0-50 cm axial segment in figure 2a, with indication (arrow) of $\bar{n}_{e(\text{re})}$, the minimum electron density for the SW to propagate at 360 MHz under a low-collisional regime. The experimental data points for a given gas pressure all fit a straight line (obtained from a least squares regression) till the very end of the plasma column.
Figure 3 reports the measured axial variation of the radially averaged electron density, laid out from the end of the plasma column sustained by a SW at 100 MHz, at two values of the inner radius of the discharge tube, in argon gas at a pressure of 1.8 Pa (10 mTorr). Anew, the electron density experimental points all distinctly fit, following a least squares regression, a straight line for a given tube radius \( R \). Scaling down the value of \( R \) renders the corresponding slope steeper [8].

![Figure 3. Measured axial variation of the radially averaged electron density referenced to the end of the plasma column sustained by the propagation of a SW at 100 MHz for two values of the inner radius of the discharge tube, in argon gas at a pressure of 1.8 Pa (10 mTorr) [8]. Coefficients of determination of the least squares regression \( r^2 = 0.993 \) and \( 0.986 \) at \( R = 3.2 \) cm and 6.2 cm, respectively. Electron density in the 32 mm inner radius tube was ascertained with a TM\(_{010}\) resonant-cavity method while for \( R = 62 \) mm the SW axial phase variation technique was used [10].](image)

2.1.2. Atmospheric gas pressure (collisional-regime). Figure 4 represents the measured axial distribution of the radially averaged electron density along a SW sustained plasma column at atmospheric pressure in argon gas [11]. While the three preceding figures concerned a low-collisional plasma \( (\nu/\omega<1) \), this time, \( \nu > 10 \omega \). Once more, nonetheless, the data points fit a straight line (except the one point for \( R = 0.97 \) mm situated in the antenna radiation region of the field applicator (surfatron)). As in figure 3, the smaller the value of \( R \), the steeper the corresponding slope of the axial distribution of electron density.

The main difference with the low-pressure case is that at atmospheric pressure, the electron density at \( z=0 \) largely exceeds \( \bar{n}_{	ext{ion}} \), which means that the SW stops propagating (marking the end of the plasma column) at a higher electron density than with low-pressure plasmas (figure 2). This is because the higher the discharge gas collision frequency, the more rapidly the SW power is expanded as the SW travels along the plasma column; it then happens that at some axial position, the level of the remaining SW power flow is too low to ionize further the discharge gas.
Figure 4. Measured axial distribution of the radially averaged electron density, displayed from the end of the plasma column sustained by a SW at 915 MHz in (fused silica) discharge tubes of three different inner radii, in argon gas at atmospheric pressure (after [11]). Regression factor $r^2 = 0.935, 0.999, 0.977$ with increasing tube radius. Electron density was determined from the broadening of the $H_\beta$ line (486.1 nm) with an argon-hydrogen gas mixture containing 0.5 % hydrogen [11].

Figure 5 illustrates the measured radially averaged electron density along the plasma column sustained by a SW at 2450 MHz in (fused silica) discharge tubes of four different inner radii in neon gas and, additionally, in a 3 mm inner radius tube in helium, all at atmospheric pressure. It shows that the smaller the tube radius, the steeper the slope of the axial distribution of electron density, as in the low-pressure case (figure 3). However, this time for large enough tube radius, the slope can become negative ($R = 3$ mm): this is attributed to the contraction phenomenon affecting discharges at high enough gas pressures$^3$ (see further below). Nonetheless, also in a $R = 3$ mm tube but this time in helium, the slope is fully positive since atmospheric pressure SWDs sustained in helium gas incur contraction only in a much larger inner radius tube [12].

$^3$ The discharge stability criterion (Sec. 4.3) requires electron density to decrease as the wave power expands through its propagation. However, in the case of a plasma column experiencing an axially decreasing diameter (contraction), one has to consider, more generally, that it is the total number of electrons generated that needs to drop axially as the wave loses power.
Figure 5. Measured axial distribution of the radially averaged electron density along plasma columns sustained by a SW at 2450 MHz in (fused silica) discharge tubes of four different inner radii \( R \) in neon gas and in a 3 mm inner radius tube in helium, all at atmospheric pressure. The data points for \( R = 2 \) and 3 mm in neon originate from both the Cordoba (Spain) [13] and the Université de Montréal laboratories [14], underlining the perfect reproducibility of SWDs for given operating conditions. Coefficients of determination of the least squares regression \( r^2 = 0.961, 0.967, 0.949 \) and 0.996 for \( Ne \) with increasing tube inner radius and 0.821 for \( He \). Electron density was determined from the broadening of the \( H_\beta \) line (486.1 nm), hydrogen atoms [14] being provided by a minimal amount of water vapor in the discharge gas.

Figure 6 presents the dependence on SW frequency of the axial distribution of radially averaged electron density in neon gas at atmospheric pressure: the higher the frequency, the steeper the electron density slope [15], as is the case also at reduced pressure (figure 1).

Figure 6. Measured axial distribution of the radially averaged electron density displayed from the end of the plasma columns sustained by a SW at 915 MHz and 2450 MHz for a (fused silica) discharge tube of 3 mm inner radius, in neon gas at atmospheric pressure [15]. Coefficients of determination of the least squares regression \( r^2 = 0.800 \) and 0.946 for 915 and 2450 MHz, respectively. Electron density was determined from the broadening of the \( H_\beta \) line (486.1 nm), hydrogen atoms being provided by a minimal amount of water vapor in the discharge gas [15].

2.2 Dense filament-like plasmas sustained at atmospheric pressure within the flowing carrier-gas as its confining structure as achieved with the TIA/TIAGO field applicator

Figure 7a is a schematic description of the TIA/TIAGO\(^4\) microwave field-applicator. A hollow conducting rod ending with a conical nozzle is emerging from a surfaguide [7], outlined by the front

\(^4\) Both TIA and TIAGO systems are waveguide-supplied MW field applicators. The TIA system has two impedance tuning means [16] while the TIAGO arrangement, much simpler, has only one [17]. However, the TIAGO field applicator requires the waveguide narrow wall to be tapered off [18], such that its impedance is close to that of the SW plasma column that goes across its aperture: the SW plasma column is viewed as a transmission line typified as such by a characteristic impedance [19].
(thinned) and back (regular thickness) wide-walls of the rectangular waveguide constituting it, defining the interstice with its open launching aperture at the front wall. The waveguide narrow-wall has been tapered off to achieve optimum impedance matching over large operating settings (MW power level, gas mixture and flow) [18]. The carrier gas flows through the inner part of the rod exiting at the tip of the nozzle. As pictured in figure 7a, the plasma flame, commencing at the nozzle tip, is comprised of the dart accompanied by the plume, the latter expanding both in length and volume with increasing microwave power (more details in [19-21]).

Figure 7b exhibits the axial electron density (obtained from the broadening of the $Hß$ line, without accounting for the van der Waals contribution) from the tip of the nozzle (1 mm diameter hole) [20]. Plasma is sustained at 2450 MHz in a He/H₂ 9/1 pre-mixed gas flow at a rate of 10 standard liters/min (slm), a high enough flow rate for ensuring a suitable confinement of the plasma flame developing in ambient atmosphere (mainly $N₂$). The $0 ≤ z ≤ 3$ mm region corresponds to the space-wave (non-guided wave) EM radiation region, customary to the initial segment of SWDs [9]. For $z > 3$ mm, the electron density decreases linearly with $z$ (as with the tubular SWDs described in the previous sections). Both (intense light-emission) regions constitute the dart. The weaker light-emission area, without clear contour, surrounding the dart and extending axially away from it has been termed the plume [20].

The odd case of a SWD generated without being enclosed in a dielectric tube is explained by the fact that the equivalent-permittivity of the dense plasma filament is negative while it is surrounded by a positive-value-permittivity medium, which in the circumstances results from a much lower-ionized gas, the plume. Such an atypical SWD is of the greatest interest as far as gaining insight into the features of SWs propagating along plasma columns. The plasma observed in the present case can also be comprehended as a long wire of finite-conductivity, inserted in a dielectric material, propagating a Sommerfeld SW [22].

Figure 7. a) Schematic representation of the TIA/TIAGO microwave field-applicator: a hollow conducting rod, terminated by a conical nozzle, emerges from a surfaguide field-applicator, represented by the front (thinned) and back wide-walls of a reduced-height section of the narrow walls from a regular rectangular waveguide. He/H₂ 9/1 pre-mixed gas flows at a rate of 10 slm through the inner part of the rod exiting at the tip of the nozzle. The plasma flame is constituted of the dart embedded in the plume, which extends farther; b) measured
electron density (through $H_{\beta}$ line broadening) as a function of axial position from the nozzle tip. The discharge is sustained at 2450 MHz with 700 W [20]. Excluding the first two axial data points belonging to the antenna-like radiation region and the two last ones assumed affected by ambient $N_2$ penetrating the plasma filament, the least squares regression ($r^2 = 0.9995$) reveals a truly linear axial decrease of the electron density starting at 3.5 mm away from the tip of the nozzle.

The experimental results just gathered on the axial distribution of electron density along both tubular SW plasmas and TIA/TIAGO plasma filaments allow to sum up the following: i) the electron density axial distributions of HF (RF and MW) discharges sustained in cylindrical dielectric tubes, either at low-gas pressure or at atmospheric pressure, as well as those observed in microwave discharges generated in an axially flowing gas as its "dielectric" confinement medium, all fit, through a highly confident least squares regression, a straight line for any given SW frequency and gas pressure (operating conditions). It is to be underlined that the electron density axial gradient remains constant till the very end of the plasma column; ii) since the axial gradient of electron density is linear all along the plasma column, it means, as already claimed, that the value of electron density is independent of the plasma column axial properties: the axial distribution of electron density and the SW attenuation coefficient are interrelated (next section), making that the axial distribution of electron density is entirely set by the SW; iii) the characteristics of SWDs depending only and strictly, in the end, on operating conditions, perfect reproducibility of these discharges is therefore ensured for given operating conditions.

Experimental results from other research groups supporting the features evinced in the present section are to be found in the Appendix.

Statistical backing of the true linearity of the axial distributions of electron density till the very end of the plasma column provides a fair reason to declare the published modelling results as incorrect. Section 3 next explores the dependence of the axial distribution of electron density on the product $fp/R$ as a similarity law to further validate its linearity.

3. The axial gradient of electron density along SW sustained tubular plasma columns depends linearly on $f\nu/R$ as a functional similarity law (operating conditions parameters)

3.1 Proposed analytical expressions for the axial gradient of electron density in terms of the product $f\nu/R$

Mateev et al. were the first to suggest an analytical expression for the axial distribution of electron density along a SW sustained cold-plasma column, considering a diffusion-controlled regime at low gas pressures [23]. Their derivation neglected the radial dependence of the electron density and assumed the plasma column to be weakly axially inhomogeneous and simply immersed in vacuum. For simplicity, they presume that the SW EM field components (supposed azimuthally symmetric) could be expressed within the electrostatic approximation. Their calculated axial distribution of electron density is slightly non-linear as it bends down toward the very end of the column. Notwithstanding this curving downward, they proposed, as a first approximation, a linearly decreasing axial distribution of electron density till the very column end in the form [23]:

$$\frac{d\eta}{dz} = 0.73 \times 10^{-2} \frac{f\nu}{R} \text{ (cm}^{-4})$$

(1)
Aliev et al. [2] went deeper into modeling of the axial distribution of electron density, but still assuming a cold plasma column enclosed in vacuum: i) looking for the influence of the SW properties on $\tilde{n}_e(z)$, they took into account the full set of Maxwell equations; ii) their analysis additionally provided an expression for the wave attenuation coefficient $\alpha_L$ at the very end of the plasma column which, they pointed out, determines from then on the behavior of $\tilde{n}_e(z)$ along the SW plasma column; iii) the actual regime of charged particle recombination materializes as a constant factor $1/A$ ahead of the similarity law expression and, as a result, whatever the specific recombination regime, it does not affect the linearity promoted by the similarity law. In the case of ambipolar diffusion, for example, $A=2$. As a whole, they ended up with:

$$\frac{d\tilde{n}_e}{dz} = \frac{m_e e_0 / A e^2 \langle \phi_{sw} \rangle}{\omega v / R}.$$  

(2)

Their demonstration entails that: i) the influence of the SW dispersion along the plasma column, expressed through the term $\langle \phi_{sw} \rangle$ (as an average), is negligible; ii) their expression for $\alpha_L$, the attenuation coefficient at the end of column, depends on operating conditions only and once more is in the form of an electrodynamic similarity law:

$$\alpha_L = \sqrt{\frac{v}{\omega R}}.$$  

(3)

3.2 Authentication from experiments of the linearity of $dn_e/dz$ as a function of the product $fv/R$ (expressions (1) and (2))

The experimental results presented in figures 1 to 6, which comprise low collisional and atmospheric pressure regimes, definitely substantiated that the axial gradient of the (radially averaged) electron density stays constant till the very end of the plasma column for given operating conditions.\(^5\) Although expressions (1) and (2) were derived for a low-pressure gas, experiments will confirm that their linear dependence on $fv/R$ also holds at atmospheric pressure.

Considering first low-pressure conditions, Table 1 displays, based on figure 1 for two successive SW frequencies, a comparison of their frequency ratio to the recorded ratio of the gradients of electron density (as the slope of the axial distributions of electron density). For example, the measured slope ratio for the 200/100 MHz frequency pair, 1.97, is within experimental error equal to their frequency ratio, namely 2. The discrepancy encountered below 100 MHz, specifically for the 100/50 and 50/27 ratios, is possibly due to modifications in the charged particle recombination regime (expressed by $A$ in (2)). Recall that the range of electron density steadily increases with $f^2$ under low-collision frequency regime (footnote 1). It can make diffusion varying from free diffusion (due to low electron density at frequencies as low as 27 and 50 MHz) to ambipolar diffusion (at high enough electron density, at and above 100 MHz) [19,24].

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\(^5\) The level of RF and MW powers supplying the plasma column is not part of the operating conditions.
| Frequency (MHz) ratio | Corresponding slope ratio (Observed) |
|-----------------------|-------------------------------------|
| 50/27 = 1.86          | 1.17                                |
| 100/50 = 2            | 1.52                                |
| 200/100 = 2           | 1.97                                |

**Table 1.** Compared SW frequency ratio and observed corresponding slope ratio of the axial distribution of electron density (figure 1) under low-pressure collisional regime.

Table 2 reviews various pairs of SWD gas pressures comparing their ratio to their corresponding slope of axial electron density distribution, as in figure 2a. Rigorous comparisons with relations (1) and (2) would require correlating the values of $\nu$ rather than those of gas pressures $p$. The observed slope ratios nonetheless comply qualitatively with predictions.

| Pair of pressure (mTorr) | Pressure ratio | Observed corresponding slope ratio |
|--------------------------|----------------|-----------------------------------|
| 40/20                    | 2              | 1.78                              |
| 80/40                    | 2              | 2.14                              |
| 150/80                   | 1.9            | 1.42                              |
| 300/150                  | 2              | 1.51                              |

**Table 2.** Compared ratio of pairs of gas pressure (instead of collision frequency $\nu$) with the ratio of the observed corresponding slopes of electron density axial distributions ((data from figure 2a: low-pressure collisional regime).

Table 3 considers the observed slope ratio of the electron density axial distribution at two inner radius values of the discharge tube relatively to their $1/R$ dependence. In the present case, the measured value is higher than that of the model by approximately 15%.

| Observed slope ratio | Model slope ratio |
|----------------------|-------------------|
| $S(R=3.2)/S(R=6.2) = 2.2$ | $(1/3.2)/(1/6.2) = 1.92$ |

**Table 3.** Observed slope ratio of the axial distribution of electron density at two values of the tube inner radius $R$ compared to the $1/R$ ratio dependence as inferred from (1) and (2) (data from figure 3: low-pressure collisional regime).

Table 4 compares the slope ratio of the axial distribution of electron density of atmospheric pressure SWDs sustained at 2450 and 915 MH with their frequency ratio. The deviation of the observed slope from the model is approximately -15%, suggesting that expressions (1) and (2) apply too, within experimental error, to the atmospheric pressure collisional regime.
Table 4. Observed slope ratio of the axial distribution of electron density at 2450 MHz to that at 915 MHz (figure 6) compared to this frequency ratio, suggesting the validity of expressions (1) and (2) for atmospheric-pressure SW plasma columns.

Table 5 compares the observed slope ratio of the axial distribution of electron density at two successive inner radii of the discharge tube, assuming the $I/R$ model ratio to be valid also at atmospheric pressure. The observed ratio is close to that expected from the model for the 0.97 mm and 0.59 pair, but differs when considering the smallest tube radius: atmospheric-pressure discharges are susceptible to contraction effects. Specifically, for large enough discharge-tube radius, plasma does not radially fill the tube and its radius decreases toward the end of the plasma column (figure 8 farther). In contrast, in small enough tube radius, the plasma radius is that of the tube inner radius: comparing plasma columns in a "small" and in a "large" tube is therefore not conform to the derivation of (1) and (2) (more on this in Sec. 3.3, next).

| Observed slope ratio | Model slope ratio |
|----------------------|------------------|
| $S(f=2450 \text{ MHz})/S(f=915 \text{ MHz}) = 2.3$ | $(2450)/(915) = 2.7$ |
| $S(R=0.46)/S(R=0.59) = 2.1$ | $(1/0.46)/(1/0.59) = 1.2$ |
| $S(R=0.59)/S(R=0.97) = 1.8$ | $(1/0.59)/(1/0.97) = 1.7$ |

Table 5. Observed slope ratio of the electron density axial distribution corresponding to two successive $R$ values (figure 4) as per the $I/R$ slope ratio dependence expected from expressions (1) and (2), which implies assuming these relations to be also valid at atmospheric pressure.

### 3.3 Axial structure of SW plasma columns affected by the contraction phenomenon

At reduced gas pressure (low-collisional regime), the generated plasma radially fills the discharge tube, except at the column very end (figure 9b, farther). As the gas pressure reaches a high enough value, the diameter of the plasma column, in contrast, reduces continuously toward its end because of the contraction effect, as shown in figure 8: the plasma column is therefore structurally inhomogeneous as a result of its continuously varying diameter. It is a common case for rare gases at atmospheric pressure (much less for He).

**Figure 8.** Photograph of a neon atmospheric-pressure SWD at 915 MHz (12 mm id tube), indicating that the plasma column diameter continuously decreases axially toward its end, making it structurally inhomogeneous axially (after [25]).

A further related aspect is that the slope of the axial gradient of the (radially averaged) electron density of a SW plasma column not radially filling the tube is lower than when it does fully occupy it, as can be seen from figure 5: the slope progressively increases as the tube radius is reduced, i.e. as the plasma occupies a larger radial part of the tube cross-section. More generally for plasmas not radially filling the tube, then as the SW power flow increases toward the field applicator, only a limited part
(even none for the $Ne R = 3 \text{ mm}$ case) of the absorbed power serves to raise the electron density of the plasma column, the other (major) power contribution going to the widening of the plasma radius (figure 8). In contrast, when the plasma radially occupies the tube whole cross-section (the smaller 1.25 and 0.5 mm tube radii in figure 5), all the SW power absorbed goes into increasing electron density along the tube axis toward the EM field-applicator, yielding a very steep slope: the plasma column is then axially inhomogeneous due, this time, to a continuously varying electron density.

The two aforementioned types of axial inhomogeneity clearly have no influence on the observed axial distribution of electron density, as can be inferred from the fact that, for given operating conditions, its slope remains constant all along the plasma column whatever the form and type of inhomogeneity encountered axially. We surmise that this is because there is no influence, no co-action, of the plasma column on the SW properties as it propagates. In fact, the plasma column is generated as such directly from the power flowing out from the wave. The next section discloses more modelling outcomes along this line.

4. Further modelling results related to the axial distribution of electron density along SW plasma columns

Aliev et al. [2] were the first to establish analytically that the attenuation coefficient $\alpha_{L}$ (3) at the end of the SWD could be the starting point for determining the axial distribution of electron density, climbing up the plasma column. This has been undoubtedly confirmed experimentally in Sec. 2: the slope of the axial distribution of electron density, determined from an, even, extremely small axial segment of electron density at the column end, remains the same whatever the length of the plasma column achieved (and whatever the type and form of plasma column inhomogeneity encountered axially), for given operating conditions. This fact is central, as already indicated, in proving that there is no co-action of the plasma column on the SW properties. As a matter of fact, the axial distribution of electron density derives exclusively from its specific axial dependence on the SW power attenuation coefficient, as will be finally asserted.

4.1 Analytical derivation of the axial dependence of the SW power-flow attenuation coefficient $\alpha(z)$

Considering that the wave propagates in the $-z$ direction ($z = 0$ is the plasma column end in this model), the basic power flow equations are:

$$\alpha(z) = \frac{1}{2P(z)} \frac{dP(z)}{dz}, \quad (4)$$

$$L(z) = \frac{dP(z)}{dz} \quad (5)$$

where $\alpha(z)$ is the attenuation coefficient of the SW power flow along $z$ and $P(z)$ the corresponding power flow. $L(z)$ is the power lost by electrons per unit length through collisions of all kinds at $z$ [7], compensated by $dP/dz$, the absorbed wave power. $L(z)$ can be expressed as:

$$L(z) = \bar{n}_e(z) \theta(z) S(z), \quad (6)$$

where $\theta$ is the power per electron and $S$ the tube inner cross-sectional area, both depending a priori on $z$. 

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The experimental axial distributions of electron density displayed in section 2 definitely reveal that:

\[ \bar{n}_e(z) = n_0 + bz, \]  

(7)

where \( n_0 \) is the electron density at the plasma column end\(^6\) and \( b \equiv \frac{d\bar{n}_e}{dz} \), the (constant) slope of the axial distribution of electron density. Expressing (6) fully as:

\[ L(z) = (n_0 + bz)\theta(z)S(z). \]  

(8)

then from (5), \( P(z) \) can be formulated as an integral over \( L(z) \) in the form:

\[ P(z) = \theta S \int_0^z (n_0 + bz)dz + P_0. \]  

(9)

As mentioned, the values of \( \theta \) and \( S \) possibly vary along the plasma column, but their product \( \theta S \) cannot otherwise, \( P(z) \) (and thus \( \bar{n}_e(z) \)) would vary with \( S(z) \), which opposes the demonstrated independence of electron density (7) from plasma diameter variations along the column (last paragraph of Sec. 3). After integration and calling on (4), one gets:

\[ P(z) = (n_0z + \frac{1}{2}bz^2)\theta S + \frac{n_0\theta S}{2\alpha_0} \]  

(10)

identifying the power flow remaining at the column end:

\[ P_0 = \frac{n_0\theta S}{2\alpha_0}. \]  

(11)

and finally:

\[ P(z) = (\frac{n_0}{2\alpha_0} + n_0z + \frac{1}{2}bz^2)\theta S. \]  

(12)

From (4) and (12) and recalling that the product \( \theta S \) is independent of \( z \), there comes:

\[ \alpha(z) = \frac{n_0 + bz}{\frac{n_0}{\alpha_0} + 2n_0z + bz^2}. \]  

(13)

Relation (13) can be also written as:

\[ \alpha(z) = \frac{b \bar{n}_e(z)}{\bar{n}_e(z)^2 + c} \]  

(14)

where:

\[ c = n_0 \left( \frac{b}{\alpha_0} - n_0 \right). \]  

(15)

Posing \( c = 0 \):

\[ b = n_0\alpha_0 \]  

(16)

then from (14):

\[ \alpha(z) = \frac{b}{\bar{n}_e(z)}. \]  

(17)

also:

\[ \alpha(z) = \frac{d\bar{n}_e}{dz} \frac{1}{\bar{n}_e(z)}. \]  

(18)

\(^6\) The end of the SWD, under low collision regime, is characterized by an electron density \( \bar{n}_e \) below which SW propagation along the dielectric tube, filled with plasma, stops (footnote 1). Denoting more generally \( n_0 \) as the electron density at the end of the plasma column whatever the collisional regime, the remaining power flow at that position \( P_0 \) (11) allows a different SW to go on propagating: it is a SW that runs this time along an empty (no plasma) dielectric tube. Such a possibility is discussed further on with figure 11.
or equivalently:

\[ \alpha(z)\bar{n}_e(z) = \frac{d\bar{n}_e}{dz}, \quad (19) \]

where \( \frac{d\bar{n}_e}{dz} \), recall, is experimentally constant till the very end of the plasma column (Sec. 2) and, therefore, such must be the product \( \alpha(z)\bar{n}_e(z) \), in contrast to model calculations [1]. Setting \( c = 0 \) in (14) is required to reach relation (19), which is the expression provided independently by the discharge stability criterion, as shown in Sec. 4.3.

The fact that the product \( \theta(z)S(z) \) is independent of \( z \) ensures linearity of the axial distribution of electron density till the very end of the plasma column. This observation is an essential (and original) feature of the current model. To illustrate how it materializes, consider the case of a low-pressure SWD. The value of \( \theta \) is, in this case, experimentally constant all along the plasma column, except at its very end where it rises abruptly, as depicted in figure 9a [26] ([19] for detailed explanations). For the product \( \theta(z)S(z) \) to remain constant axially, the observed sudden increase of \( \theta \) has to be compensated by a reciprocal reduction of the plasma cross-sectional area \( S \). This phenomenon actually occurs at the column end as can be seen from the photograph in figure 9b and confirmed in matching figure 9c from luminosity measurements of the plasma column diameter.

It must be stressed that modelers rather came out with the product \( \alpha(z)n_e(z) \) growing toward the column end (often even diverging) because, seemingly, they did not account for the related decrease of \( S(z) \) when approaching the column end [6]. The condition \( \theta(z)S(z) = \text{constant} \) makes that even though there is a marked decrease of the plasma column diameter, the axial distribution of electron density remains nonetheless linear till the column very end (figure 2, for instance)!

\[ \text{[a]} \quad \text{[b]} \]

\[ \text{[a]} \quad \text{[b]} \]

\[ 7 \text{ A rigorous check on } \theta(z)S(z) = \text{constant would necessitate determining } \theta(z) \text{ and } S(z) \text{ under the same operating conditions.} \]
Figure 9. a) Measured values of $\theta_A/p$ as functions of the axial position from the end of the SW plasma column sustained at 200 MHz in a tube of inner and outer radii 13 and 15 mm, for three different gas pressures $p$ (low collisional regime). For a given gas pressure, the power absorbed per electron $\theta_A$ is observed not to vary with axial position except at the column end [26]; b) photograph of a 0.05 Torr (6.7 Pa) argon SWD sustained at 915 MHz in a tube of inner radius 10.7 mm. The SW runs from left (surfatron interstice) to the plasma column end on the right-hand side. The diameter of the plasma column rounds off close to its end (alternate luminosity variations are due to SW reflections at the column end); c) measured luminous diameter of the plasma column indicating that it fills the discharge tube except at the very end of the plasma column where it decreases abruptly over approximately 10 mm.

Retrospective comments on the initial studies of the axial distribution of electron density along SW plasma columns. Analysis of the first published results (Glaude et al. [6]) and how they developed contribute to more insight into the matter. It all started with considering two adjacent differential slabs of the plasma column and observing that given $\alpha(z)$ and $\bar{n}_e(z)$ at $z$, their corresponding values at $z + dz$ could be readily calculated, in accord with experiments, a feature generally retained by others since then. They also demonstrated, again through numerical calculations, that the axial gradient of electron density varied linearly with the SW frequency, the electron-neutral collision frequency and as the inverse of the discharge-tube inner radius, specifically that the axial distribution of electron density truly depends on operating conditions. Glaude et al. further concluded that "the product $\alpha(z)\bar{n}_e(z)$ is a very slowly increasing function (from the launcher) with $z$, except at the very end of the column where it grows fast" [6]. This conclusion, reached in different forms by others afterwards until the current paper, is incorrect. It comes from not considering that $\theta(z)S(z)$ is independent of $z$.

4.2. Initial-time evolution of a SWD

It is worth examining as MW power is applied to the field applicator, how the wave power initially flows and how plasma evolves toward steady-state. Figure 10 depicts the initial formation of a SWD, as observed under gated-pulsed regime operation, at different time intervals once pulse power has been
switched on, specifically from 165 μs to 190 μs (approximately a 4 μs time interval between pulses), reaching a fully deployed SWD at 190 μs [27].

At 169 μs (2nd pulse down from top in figure 10), a (short) plasma column is generated in the discharge tube having its maximum of luminosity (proportional to electron density) along a given section of the discharge tube and leaning against the tube inner wall facing the surfaguide plunger (figure 6a in [19]). It results from the \( E \)-field of a standing wave, which is not of azimuthal symmetry (as well as for frame 3). However, starting from frame 5 and later in time, plasma displays an azimuthally symmetric luminosity pattern. The 6th and 7th pulses then exhibit an undivided plasma column with, according to plasma luminosity, a progressively decreasing electron density and plasma radius toward the plasma column end, as anticipated from a steady-state SWD. The observed transitory phases of the discharge illustrate the importance of the dielectric tube material, even limited parts of it, to assist at first EM field concentration and transmission and, possibly, even past the plasma column end, as described in the next section.

![Image](image_url)

**Figure 10.** Images taken with an intensified charged coupled device (iCCD) under gated-pulsed regime at a repetitive rate of 50 Hz in a helium SW discharge at 5 Torrs (\( \approx 670 \text{ Pa} \)) and 16 W average absorbed power, with the SW finally running from the right (surfaguide interstice) to the left on the picture [27]. The more intense the red colour, the higher the electron density. The bright red-coloured box formed on the far-right side corresponds to the space-wave radiation region related to the field applicator [9].

An empty (no plasma) dielectric discharge tube can serve to support SW propagation. Although the SW plasma column sustained on the dipolar \( (m = 1) \) mode\(^8\) ends at the vertical dotted line in figure 11, a SW is observed to go on propagating past it on approximately 20 more cm. The SW then travels along the dielectric wall of an empty (no plasma) discharge tube, the wave being driven by the power flow that remained (11) past the cut-off electron density \( \tilde{n}_{\text{e}(m)} \) (footnote 1) at which the plasma column ended. The SW, initially travelling on the dipolar \( (m = 1) \) mode, most probably transforms in the azimuthally symmetric \( (m = 0) \) mode along the empty dielectric tube. The wavelength values recorded in-between the two types of wave mode apparently exceeds \( \lambda_0 \), an artefact attributed to propagation mode conversion (from \( m = 1 \) to \( m=0 \)).

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\(^8\) Since the product \( f R \) = 3.18 GHz-cm is larger than \( \approx 2 \) GHz-cm, then only the dipolar \( (m = 1) \) mode can yield a stable plasma column [28].
Figure 11. (a) Recorded axial phase variation of the $E$-field radial component along a dipolar ($m = 1$) mode SW sustained plasma column operated at 2.45 GHz and with 275 W in argon gas at 200 mTorr ($\approx$27 Pa) in a 26 mm i.d. Pyrex™ tube. Past the end of the plasma column, the phase variation detected is assumed to be that of an azimuthally symmetric SW; (b) the (half) wavelength along the SWD is shorter, due to the presence of plasma in the tube, relatively to that in the empty (no plasma) discharge tube ($\lambda_0$ is the wavelength in vacuum) [28].

Two types of SWs can therefore propagate along the air (vacuum)-dielectric tube interface: i) when it is filled with plasma; ii) when the discharge tube is empty (no plasma); In the first case, the attenuation coefficient of the wave power flow is high and its phase velocity low due to the plasma column "dissipative load" whereas, in the second case, the SW attenuation coefficient is lower and the wave phase velocity higher.

4.3. The axial distribution of electron density along traveling-wave sustained discharges is set (under steady-state) by a stability criterion

Zakrzewski [29] foresaw, using physical arguments, that the axial distribution of electron density along traveling-wave (TW) supported discharges could be inferred from a stability (existence) criterion, which requires $d\bar{n}_e(z)/dz < 0$ where $z$ is assessed from the SWD start. The criterion requires the electron density to decrease monotonously with $z$ as the wave power-flow falls off along the plasma column. Analytically assumed expressions between the wave attenuation coefficient and the electron density at given axial positions were then sought for. Equation (20) is such a possible, relationship:

$$\alpha(n) = An^k,$$

where $k$ is an integer and $A$, a constant, the electron density $n$ being normalized to that at the SW plasma column start $n_{cs}$. Then starting from [6] with the expression:

$$\frac{dn}{dz} = -2\alpha(n)z(n)\left(1 - \frac{n(z)}{n(n)}\frac{d\alpha(n)}{dn}\right)^{-1},$$

from (20) together with (21):

$$\frac{n(z)}{n_{cs}} = \left(1 + 2A n_{cs}^k \frac{k}{1-k} z\right)^{-1/k} = \left(1 + \frac{2k}{1-k} \alpha_{cs} z\right)^{-1/k}$$

(22)
and
\[
P(z) = \left(\frac{n(z)}{n_{cs}}\right)^{1-k},
\]
where, as mentioned, normalized quantities \(n_{cs}, \alpha_{cs}\) and \(P_{cs}\) are those at the start of the plasma column. Figure 12 is a graphical plot of equation (22).

Figure 12. Electron density as a function of axial position as predicted from Zakrzewski’s TW discharge stability criterion (both quantities are normalized to their values at the start of the plasma column, \(z = 0\)), assuming \(\alpha(n) = An^k\) [29]. The \(k = -1\) curve corresponds to the experimentally observed linear profile (7).

Figure 12 shows that the axial profile of the electron density distribution can either be concave \((k = 0.5\) and 0\), linear \((k = -1)\) or convex \((k = -2)\), all of them strictly complying with the stability criterion \(d\bar{n}_e(z)/dz < 0\). Noteworthy is that these profiles are obtained for TW sustained discharges at large, i.e., without involving their specific dispersion properties. The \(k = -1\) case corresponds to the observed axial distribution of electron density for SWDs, thereby endorsing relation (19) since \(\frac{d\bar{n}_e}{dz}\) then comes out as a constant. On the other hand, recall that relation (19) is the end result of the analytical derivation of \(\alpha(z)\) (Sec. 4.1), therefore confirming the validity of setting \(c = 0\) in (15).

The axial distribution of electron density remaining linear till the very end of the plasma column comes from Zakrzewski’s stability criterion applying to the, even, shortest SW plasma column generated. Such a result should be emphasized as it is a further reason (besides \(\theta(z)S(z) = \text{constant}\)) to reject any nonlinear profiles of the very end-segment of axial electron density distributions: see figure A4 in the Appendix for example. This error is clearly the stumbling rock of the SW plasma column modeling. The fact that the SW wave properties do not depend on the plasma column properties is a different matter.

5. Summary and conclusion

Summary
The first part of the paper aimed at confirming, on statistical grounds (through least squares regressions), the linearity of the experimental axial distributions of electron density acquired along
SWDs, including its very end (Sec. 2). This fact was questioned (even denied) by modelers since it opposes results of their calculations, which yield a curving down, at the end of the column, of the axial distribution of electron density. It was further determined that the axial gradient of electron density depends only on operating conditions \((f, v \text{ (or } p) \text{ and } R)\) and, moreover, as a \(fv/R\) similarity law, whatever the gas pressure collisional regime (Sec. 3).

An expression for the axial dependence of the power flow attenuation coefficient \(\alpha(z)\) was derived analytically (Sec. 4.1). When confronted with the experimental fact that \(\alpha(z) n_e(z) = dn_e/dz\) is axially constant till the very end of the plasma column, it led to concede that the product \(\theta(z)S(z)\) (appearing in the \(P(z)\) power flow relationship (9)) needs to be constant all along the plasma column (for given operating conditions). Not having accounted for this feature (brought out for the first time in the current paper) explains the breakdown of linearity experienced by modelers at the end of the plasma column due to an abrupt drop in the plasma column cross-sectional area \(S\).

Finally, Zakrzewski’s stability criterion for TW sustained plasmas provides due justification, on solid physical grounds, for the observed intrinsic dependence between \(\alpha(z)\) and \(n_e(z)\) (in the form \(\alpha(z)n_e(z) = dn_e/dz\)), which consequently ascertains that the plasma column properties do not influence SW propagation (Sec. 4.3). A similar conclusion applies as a result of the requirement \(\theta(z)S(z) = \text{constant}\).

Conclusion
The curving down of the axial gradient of electron density at the column end erroneously proposed by modelers is due to omitting considering that \(\theta(z)S(z)\) needs to be axially constant to account for any steep plasma-diameter reduction at the column end. This feature results from the fact that plasma column properties are set by the propagating SW, not the reverse.

Describing the SW and the plasma column together (Maxwell and plasma equations) adequately depicts their features, as they exist. However, such a description does not provide indications as to how the plasma column has been formed. Since the SW power flow expenditure depends only on electron density in the form \(\alpha(z) n_e(z) = dn_e/dz\) where the axial gradient is constant, it implies that the plasma column is produced independently from any coaction with the SW; in particular, the plasma column characteristics are set such that \(\theta(z)S(z)\) is axially constant. The generally accepted paradigm stating that the generation of the SW plasma column results from coaction of the plasma column with the surface wave is therefore erroneous and must be abandoned.

There remains to determine the dispersion properties of a surface wave that is independent of the plasma column along which it propagates and to document experimentally under the same operating conditions the axial variation of \(\theta(z)\) and \(S(z)\) to confirm that their product \(\theta(z)S(z)\) is truly axially constant, even at the column end.

A final word in favor of SWDs: the exceptional reproducibility of SWDs, due to their unique and only dependence on operating conditions, was clearly a key factor in reaching an accurate and high level of insight in their mechanisms.

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Appendix. Experimental axial distributions of electron density along SWDs from foreign research groups

Notwithstanding the fact that the linearity of the axial distribution of electron density of SWD has been amply substantiated experimentally in the main text, all the data points (with an exception only) originated from the Université de Montréal laboratory. The Appendix aims at clearing doubts from modelers by surveying similar results from other laboratories (Spain, France, Portugal) and considering, besides rare gas discharges, molecular ones.

Figures A1-a) and A1-b) display least squares linear regressions made on data points (Spain) laying out the axial distributions of electron density along a 2450 MHz SW sustained plasma column [24]. Discharge tubes of inner radius 1.5 mm (A1-a) and 2.5 mm (A1-b) have been exploited with a SWD operated at five different argon gas pressures (extended collisional regime). In this case, the electron density has been recovered by recording the SW axial phase variation (see e.g., figure 10) and using the SW dispersion relation to come out with the radially averaged electron density [10].

![Figure A1](image)

**Figure A1.** Axial distribution of electron density of a SWD sustained at 2450 MHz at different argon gas pressures in two sets of discharge tubes ($\varepsilon_g=4.8$): a) $R = 1.5 \text{ mm}$ where $r^2 = 0.9954$ and b) $R = 2.5 \text{ mm}$ where $r^2 = 0.9955$. Normalized variables $N$ and $Z$ are defined in [30] (adapted from [30]).

Figure A2 exhibits the linear regression achieved on the axial electron density data points (France) recorded as functions of axial position from the column end along SWDs sustained at 210 and 2450 MHz and 0.2 Torr (27 Pa) argon gas pressure [8,31]. Electron density measurements were performed through the SW axial phase variation method [10]. It happens fortuitously that the two axial distributions have the same slope!
Figure A2. Axial distribution of electron density of a SWD sustained in argon gas at 27 Pa in a $R = 9$ mm inner radius discharge tube at 200 and 2450 MHz, the least squares linear regression yielding $r^2 = 0.9968$ and 0.9893, respectively (adapted from [8]).

The axial distributions of electron density reported up to this point concerned rare gas SW discharges (He, Ne, Ar). Figures A3 and A4 (Portugal) prove that the axial distribution of electron density along a SWD in a molecular gas ($N_2$ and $H_2$, respectively) is also characterized by a constant axial gradient of electron density.

The straight line drawn in figure A3 results from a least squares regression carried out on the experimental points [32]. The authors concluded "that the electron density is approximately a linear function of the axial coordinate". Since $r^2 = 0.986$, they could have been more affirmative statistically speaking!

Figure A3. Axial distribution of electron density of a SWD sustained in $N_2$ gas at 67 Pa in a discharge tube ($\varepsilon_g=4.52$) of inner radius $R = 22.5$ mm at 500 MHz. The data linear fit from a least squares regression yields $r^2 = 0.986$ (adapted from [32]).

In figure A4, a least squares linear regression was performed on the experimental data of the axial distribution of electron density, ignoring however the highest $z$ value point related to the antenna-
like EM radiation of the field applicator [9]. The dotted line is the axial distribution of electron density calculated by the authors. The linearity of the calculated axial distribution fails, with respect to experiment, toward the plasma column end, where it markedly curves down. Such, more or less pronounced, curled behaviour is a general feature of the current models. In contrast, the minimum $r^2$ value encountered experimentally for linearity is 0.98!

**Figure A4.** Axial distribution of electron density of a SWD sustained at 500 MHz in $H_2$ gas at 27 Pa in a Pyrex™ discharge tube ($\varepsilon_g=4.8$) of 22.5 mm inner radius. The least squares linear regression, excluding the value at the largest $Z$ value (related to the antenna-like region of the field applicator), yields $r^2 = 0.948$. The dotted curve is theoretical, calculated by the authors (adapted from [33]).

More recently, modellers have (at last) integrated the experimental evidence that the axial gradient of electron density is linear till the very end of the SW plasma column. Therefore, they acknowledged that their calculations must recover such linearity, avoiding any curving down of the axial gradient of electron density at the end of the column, in contrast to publications promoting such a feature. In that perspective, Kovačević et al. [34] figured out a linear axial profile of electron density using what they called the *square root approximation* in the region of small (normalized) wavenumbers. Nonetheless, this solution remains basically unsatisfactory as it requires a specific approximation.

**Comments on observed deviations from a linear axial distribution of electron density**

1- There is the case where SW power is reflected at the end of the column (from a vacuum metallic cap, for example), generating a standing (sine form) wave pattern of electron density as a function of axial position. The most pronounced electron density variation is then located, as expected, close to the end of the plasma column [35].

2- The plasma column emerging from the EM field applicator is constituted, immediately past it from the space-wave radiation region and then, by the SWD. The axial extent of the space-wave region decreases as SW frequency is raised. Typically [9], for a surfatron SWD, this region is approximately 43 mm long at 915 MHz, coming down to 28 mm at 2450 MHz. It is only past this distance that a SW discharge is established, thus that a linear axial distribution of electron density is to be expected.
3- Experimental flaws can lead to deviation from a linear profile. Such is the case, for instance, with the 270 W recording made in atmospheric pressure argon gas at 915 MHz where curving down of the axial electron density toward the column end was reported (figure 7 in [11]). Later measurements made with a similar SW discharge arrangement did not reproduce the noted absence of linearity [24].

Determining the electron density along SWDs. The techniques chosen for that purpose are adapted to their specific operating conditions, namely discharge tube diameter, gas pressure and SW frequency range. They involve means such as resonant cavity, SW axial phase detection, broadening of spectroscopic lines, even Langmuir probes. Owing to approximations made in modeling the corresponding density measuring systems, the values obtained with different techniques are estimated to vary possibly by 10 to 20%.

- A first method is to have the plasma tube going across a TM$_{010}$ resonant cavity and display the frequency shift of the resonant peak as a function of RF/MW power incoming to the field applicator, which for SWDs translates into plasma column length. In such a case, the plasma is represented by its (equivalent) dielectric permittivity value, which can be calculated and/or calibrated with the (opposite sign) TM$_{010}$ shift on a scope obtained with respect to a known dielectric liquid (e.g., benzene). Then, the axial distribution of electron density can be ascertained, segment by segment, by having the plasma column sliding across the cavity past the field applicator varying HF power to the launcher [6]. An alternative is to calibrate the light-emission intensity observed in the cavity with the corresponding electron density and do the same with varying HF power [8]. The resonant cavity approach is limited by the fact that above a certain value of electron density and gas pressure, it becomes impossible to obtain an accurate resonance peak (too much damped) and, additionally, that for large diameter tubes the resonant cavity (which must be much wider in diameter than the hole through which goes the discharge tube) becomes cumbersome.

- Electron density was also determined from the broadening of the $H_\beta$ emission line (486.1 nm), hydrogen atoms being provided by a minimal amount of water vapor in argon gas [12].(figs. 5 and 6) or directly with an argon-hydrogen gas mixture containing 0.5 % hydrogen [11] (fig. 4)

- A third possible method is to record the SW axial phase variation and use the experimental or calculated SW phase diagram ($\omega/\omega_{pe}$ vs. $\beta$, where $\omega_{pe}$ is the angular plasma frequency and $\beta$ the axial SW wavenumber with $\omega$ being kept constant) to relate the recorded SW wavelength to electron density [10]. This method has the advantage of being possibly used with larger discharge tube diameters and at higher gas pressures. However, with atmospheric pressure gas discharges, accurate determination of the electron number density becomes limited whenever the SW dispersion relation ceases depending significantly on the SW wavenumber $\beta$ (e.g. [11]).

- A fourth possible method, used by some authors to characterize SWDs, is the Langmuir probe diagnostic technique. A radially directed tungsten wire is immersed in the discharge. In order to avoid perturbation caused by microwave field influence on the electronic component, the ion saturation part of the probe characteristic is used for processing [32]. However, even though the probe is directed perpendicularly to the discharge tube axis, despite the authors' assertion, the presence of a conducting wire is bound to modify the EM field distribution of the SW and thus the local value of electron density.

The axial distributions of electron density in figures (paradigm paper) 1, 2 and 3 (for $R=32$ mm) of the current paper were determined using a TM$_{010}$ resonant cavity method, then with the
broadening of the $H_β$ emission line in figure 7b while others were obtained by recording the SW axial phase variation. Comparing electron density data from different techniques for a given SWD operating condition can procure a more or less extended interval of density values depending on the accuracy of the measuring system used and on the validity (exactness) of the calculations that connect them with actual electron densities. As an example, a similarity law (with a logarithmic linear slope) can be shown to be always obeyed (provided $pR$ remains smaller than 1 Torr-cm), but with a different slope according to whether the electron density is underestimated or overestimated (the set of values shifted upward or downward) depending on the way electron density has been inferred.

References
[1] Zhelyazkov I and Atanassov V 1995 Axial structure of low-pressure high-frequency discharges sustained by travelling electromagnetic surface waves Physics Reports 255 79.
[2] Aliiev Y M, Ivanova K M, Moisan M and Shivarova A 1993 Analytical expression for the axial structure of surface wave sustained plasmas under various regimes of charged particle loss Plasma Sources science and Technology 2 145.
[3] Aliiev Y M, Schlüter H and Shivarova A 2000 Guided-wave-produced plasmas (Berlin: Springer).
[4] Zhelyazkov I, Benova E and Atanassov V 1986 Axial structure of a plasma column produced by a large-amplitude electromagnetic surface wave Journal of Applied Physics 59 1466.
[5] Kabouzi Y, Graves D B, Castaños-Martínez E and Moisan M 2007 Modeling of atmospheric-pressure plasma columns sustained by surface waves Journal of Applied Physics 75 016402.
[6] Glaude V M M, Moisan M, Pantel R, Leprince P and Marec J 1980 Axial electron-density and wave power distributions along a plasma-column sustained by the propagation of a surface microwave Journal of Applied Physics 51 5693.
[7] Moisan M and Zakrzewski Z 1991 Plasma sources based on the propagation of electromagnetic surface waves Journal of Physics D: Applied Physics 25 1025.
[8] Chaker M, Moisan M and Zakrzewski Z 1986 Microwave and RF surface-wave sustained discharges as plasma sources for plasma chemistry and plasma processing Plasma Chemistry and Plasma Processing 6 79.
[9] Moisan M, Levif P and Nowakowska H 2019 Space-wave (antenna) radiation from the wave launcher (surfatron) before the development of the plasma column sustained by the EM surface wave: a source of microwave power loss AMPERE Newsletter 9.
[10] Margot-Chaker J, Moisan M, Zakrzewski Z, Glaude V M and Sauvé G 1988 Phase sensitive methods to determine the wavelength of electromagnetic waves in lossy nonuniform media: The case of surface waves along plasma columns Radio Science 23 1120.
[11] Moisan M, Pantel R and Hubert J 1990 Propagation of surface wave sustaining a plasma column at atmospheric pressure Contributions to Plasma Physics 30 293.
[12] Moisan M and Pelletier J 2012 Physics of collisional plasmas: Introduction to high-frequency discharges (Berlin: Springer).
[13] Sáinz A and García M C 2008 Spectroscopic characterization of a neon surface-wave sustained (2.45 GHz) discharge at atmospheric pressure Spectrochimica Acta Part B: Atomic Spectroscopy 63 948.
[14] Kabouzi Y, Calzada M D, Moisan M, Tran K C and Trassy C 2002 Radial contraction of microwave-sustained plasma columns at atmospheric pressure Journal of Applied Physics 91 1008.
[15] Castaños-Martínez E 2004 Influence de la fréquence d’excitation sur les phénomènes de contraction et de filamentation dans les décharges micro-ondes entretenues à la pression atmosphérique Mémoire de Maîtrise (M. Sc.) Université de Montréal).
[16] Moisan M, Sauvé G, Zakrzewski Z and Hubert J 1994 An atmospheric pressure high power microwave plasma torch: the TIA design Plasma Sources Science and Technology 3 584.
[17] Moisan M, Zakrzewski Z and Rostaing J C 2001 Waveguide-based single and multiple nozzle plasma torches: the TIAGO concept Plasma Sources science and Technology 10 387.
[18] Fleisch T, Kabouzi Y, Moisan M, Pollak J, Castaños-Martínez E, Nowakowska H and Zakrzewski Z 2007 Designing an efficient microwave-plasma source, independent of operating conditions, at atmospheric pressure Plasma Sources science and Technology 16 173.
[19] Moisan M and Nowakowska H 2018 Contribution of surface-wave (SW) sustained plasma columns to the modeling of RF and microwave discharges Plasma Sources science and Technology 27 073001.
[20] Ricard A, St-Onge L, Malvos H, Gicquel A, Hubert J and Moisan M 1995 Torche à plasma à excitation micro-onde : deux configurations complémentaires Journal de Physique III France 5 1269.
[21] Rincón R, Muñoz J, Sáez M and Calzada M D 2013 Spectroscopic characterization of atmospheric pressure argon plasmas sustained with the torche à injection axiale sur guide d’ondes Spectrochimica Acta Part B: Atomic Spectroscopy 81 26.
[22] Nowakowska H, Zakrzewski Z and Moisan M 2001 Propagation characteristics of electromagnetic waves along a dense plasma filament Journal of Physics D: Applied Physics 34 1474.
[23] Mateev E, Zhelyazkov I and Atanassov V 1983 Propagation of a large-amplitude surface wave in a plasma column sustained by the wave Journal of Applied Physics 54 3049.
[24] Moisan M Concept of power absorbed and lost per electron and its contribution to the advanced understanding and modeling of microwave discharges (submitted) Physical Review E.
[25] Castaños-Martínez E and Moisan M 2011 Expansion/homogenization of contracted/filamentary microwave discharges at atmospheric pressure IEEE Transactions on Plasma Science 39 2192.
[26] Moisan M, Barbeau C, Claude R, Ferreira, Margot J, Paraszczał J, Sá A B, Sauvé G and Wertheimer M R 1991 Radio-frequency or microwave plasma reactors - factors determining the optimum frequency of operation Journal of Vacuum Science and Technology B 9 8.
[27] Hamdan A, Valade F, Margot J, Vidal F and Matte J P 2017 Space and time structure of helium pulsed surface-wave discharges at intermediate pressures (5–50 Torr) Plasma Sources science and Technology 26 015001.
[28] Margot-Chaker J, Moisan M, Chaker M, Glaude V M M, Lauque P, Paraszczał J and Sauvé G 1989 Tube diameter and wave frequency limitations when using the electromagnetic surface wave in the m=1 (dipolar) mode to sustain a plasma column Journal of Applied Physics 66 4134.
[29] Zakrzewski Z 1983 Conditions of existence and axial structure of long microwave discharges sustained by travelling waves Journal of Physics D: Applied Physics 16 171.
[30] Sola A, Cotrino J and Colomer J 1988 Reexamination of recent experimental results in surface-wave-produced argon plasmas at 2.45 GHz: Comparison with the diffusion-recombination model results Journal of Applied Physics 64 3419.
[31] Chaker M, Nghiem P, Bloyet E, Leprince P and Marc J 1981 Étude d’une décharge créée par une onde de surface (Orsay, France: Université Paris-XI).
[32] Dias F M, Tatarova E and Ferreira C M 1998 Spatially resolved experimental investigation of a surface wave sustained discharge in nitrogen Journal of Applied Physics 83 4602.
[33] Gordiets B, Pinheiro M, Tatarova E, Dias F M, Ferreira C M and Ricard A 2002 A travelling wave sustained hydrogen discharge: modelling and experiment Plasma Sources science and Technology 9 295.
[34] Kovačević M S, Kuzmanović L, Milošević M M and Djordjević A 2021 An estimation of the axial structure of surface-wave produced plasma column Physics of Plasmas 28 023502.
[35] Henriques J, Tatarova E, Dias F M and Ferreira C M 2002 Wave driven N₂ – Ar discharge. II. Experiment and comparison with theory Journal of Applied Physics 91 5632.