Goldstino superfields for spontaneously broken $\mathcal{N} = 2$ supersymmetry

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Abstract

We examine spontaneously broken $\mathcal{N} = 2$ supersymmetry in four dimensions and associate a spinor superfield with each Goldstino via a finite supersymmetry transformation with parameters that are the Grassmann coordinates of $\mathcal{N} = 2$ superspace. Making use of a special choice of coset parametrization allows us to develop a version of nonlinearly realized $\mathcal{N} = 2$ supersymmetry for which the associated Goldstino superfields are defined on harmonic superspace, thereby providing a natural mechanism for construction of a Goldstino action. The corresponding superfield Lagrangian is an $O(4)$ multiplet. This property is used to reformulate the Goldstino action in projective superspace and in conventional $\mathcal{N} = 2$ superspace. We show how to generate matter couplings of the Goldstinos to supersymmetric matter using the $\mathcal{N} = 2$ harmonic, projective and full superspaces. As a bi-product of our consideration, we also derive an $\mathcal{N} = 2$ chiral Goldstino action.

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1 Introduction

The absence of observed superpartners for the particles in the Standard Model of particle physics suggests that if supersymmetry is a symmetry of Nature, then it is realized in a spontaneously broken form. It is somewhat ironic that the second oldest realization of a supersymmetry algebra in a field-theoretic context by Volkov and Akulov [1, 2] was indeed nonlinear.

Most of the focus of work on nonlinear realizations of extended supersymmetries has been on partially broken global supersymmetry [3–7, 8, 9], motivated by the relevance of these constructions to low-energy effective actions for branes in superstring theory.

In this paper we examine fully broken $\mathcal{N} = 2$ supersymmetry, so that there are two Goldstinos. We use the technique introduced by Ivanov and Kapustnikov [10, 11, 12] (see also [13]), and further elaborated by Samuel and Wess [14], to associate a superfield with each Goldstino via a finite supersymmetry transformation with parameters that are the fermionic coordinates of $\mathcal{N} = 2$ superspace. We find, using standard techniques for construction of nonlinear realizations, that a special choice of coset parametrization
yields a version of non-linearly realized $\mathcal{N} = 2$ supersymmetry for which the associated Goldstino superfields are defined on $\mathcal{N} = 2$ harmonic superspace, thereby providing a natural mechanism for construction of an $\mathcal{N} = 2$ Goldstino action.

The plan of the paper is as follows. In section 2, we review the non-linear realization of $\mathcal{N} = 1$ supersymmetry pioneered by Volkov and Akulov, and the mechanism by which an $\mathcal{N} = 1$ superfield can be associated with the corresponding Goldstino. It is shown that a different choice of coset parametrization from that used by Volkov and Akulov gives rise to an anti-chiral Goldstino superfield, which allows the construction of a Goldstino action via integration over the anti-chiral subspace of $\mathcal{N} = 1$ superspace. The $\mathcal{N} = 2$ version of this chiral superspace construction is discussed in the appendix. In section 3, we analyze the case of spontaneously broken $\mathcal{N} = 2$ supersymmetry, and show that via a particular choice of coset parametrization, it is possible to find Goldstone fields that are naturally adapted to the structure of $\mathcal{N} = 2$ harmonic superspace. The operators $D^{++}$ and $D^{--}$ associated with the $SU(2)$ degrees of freedom in harmonic superspace are analyzed in section 4, and it is shown that they have a non-linear action on the Goldstinos. In section 5, we demonstrate that analytic superfields can be associated with one of the Goldstinos, thus allowing the construction of a Goldstino action by integration over the analytic subspace of $\mathcal{N} = 2$ superspace. Several reformulations of the Goldstino action are given, and Goldstino-matter couplings are introduced. In section 6, we briefly discuss composite Goldstino superfields that can be used to construct higher-derivative Goldstino actions. Finally, the $\mathcal{N} = 2$ chiral construction is presented in the appendix.

2 The nonlinear realizations of $\mathcal{N} = 1$ supersymmetry revisited

Non-linearly realized internal symmetries can be treated systematically using coset constructions [15, 16, 17]. Volkov [18] extended the coset construction to include both broken and unbroken spacetime symmetries (see also [19]), and Volkov and Akulov [1, 2] treated the case of broken $\mathcal{N}$-extended supersymmetries.

The four-dimensional $\mathcal{N} = 1$ construction of Volkov and Akulov [1, 2] is associated with the supersymmetry algebra

\[ \{ Q_\alpha, \bar{Q}_{\dot{\alpha}} \} = 2 P_{\alpha \dot{\alpha}}. \]  

The coset construction of non-linearly realized supersymmetry on the Goldstone field $\lambda_\alpha(x)$
Based on the group element
\[ g(x, \lambda(x), \bar{\lambda}(x)) = e^{i(-x^a P_a + \lambda^a(x) Q_a + \bar{\lambda}_\dot{a}(x) \bar{Q}^\dot{a})}. \] (2.2)

Supersymmetry transformations are generated by left action by the group element
\[ g(\epsilon, \bar{\epsilon}) = e^{i(\epsilon^a Q_a + \epsilon_a Q^a)}. \] (2.3)

In infinitesimal form, the supersymmetry transformation law is
\[ \delta \lambda_\alpha = \epsilon_\alpha - i \nu^{\beta\dot{\beta}} \partial_{\beta\dot{\beta}} \lambda_\alpha, \quad \delta \bar{\lambda}_{\dot{\alpha}} = \bar{\epsilon}_{\dot{\alpha}} - i \nu^{\dot{\beta}\beta} \partial_{\dot{\beta}\beta} \bar{\lambda}_{\dot{\alpha}} \] (2.4)
with \( \nu^{\beta\dot{\beta}} = \lambda^\beta \bar{\epsilon}^{\dot{\beta}} - \epsilon^\beta \bar{\lambda}^{\dot{\beta}}. \)

An alternative nonlinear realization, first introduced by Zumino \[20\] and further developed by Samuel and Wess \[14\], involves a Goldstino \( \xi_\alpha \) which mixes only with itself under supersymmetry transformations:
\[ \delta \xi_\alpha = \epsilon_\alpha - 2i \xi_\beta \bar{\epsilon}^{\dot{\beta}} \partial_{\beta\dot{\beta}} \xi_\alpha. \] (2.5)

This nonlinear realization is related to the coset parametrization
\[ g(x, \xi(x), \bar{\chi}(x)) = e^{i(-x^a P_a + \xi^a(x) Q_a) + i\bar{\psi}_\dot{a}(x) \bar{Q}^\dot{a}}. \] (2.6)

Left action by the group element \( \text{[2.3]} \) generates the supersymmetry transformation \( \text{[2.5]} \), as well as the transformation
\[ \delta \bar{\psi}_\dot{a} = \bar{\epsilon}_{\dot{a}} - 2i \xi_\beta \bar{\epsilon}^{\dot{\beta}} \partial_{\beta\dot{\beta}} \bar{\psi}_{\dot{a}}. \] (2.7)

The Goldstinos \( \xi_\alpha, \bar{\psi}_{\dot{\alpha}} \) are related to the Volkov-Akulov Goldstinos \( \lambda_\alpha, \bar{\lambda}_{\dot{\alpha}} \) by
\[ \xi_\alpha(x) = \lambda_\alpha(y), \quad \bar{\psi}_{\dot{\alpha}}(x) = \bar{\lambda}_{\dot{\alpha}}(y), \] (2.8)
where \( y^a = x^a - i\lambda(y)\sigma^a\bar{\lambda}(y) \). Conversely,
\[ \lambda_\alpha(x) = \xi(z), \quad \bar{\lambda}_{\dot{\alpha}}(x) = \bar{\psi}_{\dot{\alpha}}(z), \] (2.9)
where \( z^a = x^a + i\xi(z)\sigma^a\bar{\psi}(z) \).

In the work of Samuel and Wess, the field \( \bar{\psi}_{\dot{\alpha}} \) was not exploited. Instead, they used \( \bar{\xi}_{\dot{\alpha}} \), defined as the Hermitian conjugate of \( \xi_\alpha \), with the supersymmetry transformation
\[ \delta \bar{\xi}_{\dot{\alpha}} = \bar{\epsilon}_{\dot{\alpha}} + 2i \epsilon^\beta \bar{\xi}^{\dot{\beta}} \partial_{\beta\dot{\beta}} \bar{\xi}_{\dot{\alpha}}. \] (2.10)
However, there is a benefit to be derived from working with the fields \((\xi_\alpha, \bar{\psi}_\dot{\alpha})\) rather than \((\xi_\alpha, \bar{\xi}_\dot{\alpha})\). As first demonstrated by Ivanov and Kapustnikov \[10, 11, 12\] (see also \[14\]), it is possible to associate a superfield with a Goldstino via a finite supersymmetry transformation for which the parameter is the corresponding fermionic superspace coordinate. When the Goldstinos \((\xi_\alpha, \bar{\psi}_\dot{\alpha})\) are promoted to superfields, the superfield corresponding to \(\bar{\psi}_\dot{\alpha}\) is antichiral. This allows the formulation of a Goldstino action by integration over the antichiral subspace of \(\mathcal{N} = 1\) superspace.

Explicitly, the supersymmetry transformations (2.5) and (2.7) imply

\[
i Q_\alpha \xi_\beta = -\epsilon_{\alpha\beta} \tag{2.11a}
\]
\[
i \bar{Q}_\dot{\alpha} \xi_\beta = -2i \xi^\alpha \partial_{\alpha\dot{\alpha}} \xi_\beta \tag{2.11b}
\]

and

\[
i Q_\alpha \bar{\psi}_\dot{\beta} = 0 \tag{2.12a}
\]
\[
i \bar{Q}_\dot{\alpha} \bar{\psi}_\dot{\beta} = \bar{\epsilon}_{\dot{\alpha}\dot{\beta}} - 2i \xi^\alpha \partial_{\alpha\dot{\alpha}} \bar{\psi}_\dot{\beta} \tag{2.12b}
\]

The superfields associated with the Goldstinos \(\xi_\alpha, \bar{\psi}_\dot{\alpha}\) are

\[
\Xi_\alpha(x, \theta, \bar{\theta}) = e^{iX} \xi_\alpha(x), \quad \bar{\Psi}_\dot{\alpha}(x, \theta, \bar{\theta}) = e^{iX} \bar{\psi}_\dot{\alpha}(x), \tag{2.13}
\]

where \(X = \theta^\alpha Q_\alpha + \bar{\theta}_{\dot{\alpha}} \bar{Q}_{\dot{\alpha}}\). Using

\[
D_\alpha e^{iX} = e^{iX} i Q_\alpha, \quad \bar{D}_{\dot{\alpha}} e^{iX} = e^{iX} i \bar{Q}_{\dot{\alpha}}, \tag{2.14}
\]

it follows that

\[
D_\alpha \Xi_\beta = -\epsilon_{\alpha\beta} \tag{2.15a}
\]
\[
\bar{D}_{\dot{\alpha}} \Xi_\beta = -2i \Xi^\alpha \partial_{\alpha\dot{\alpha}} \Xi_\beta \tag{2.15b}
\]

and

\[
D_\alpha \bar{\Psi}_\dot{\beta} = 0 \tag{2.16a}
\]
\[
\bar{D}_{\dot{\alpha}} \bar{\Psi}_\dot{\beta} = \epsilon_{\dot{\alpha}\dot{\beta}} - 2i \Xi^\alpha \partial_{\alpha\dot{\alpha}} \bar{\Psi}_\dot{\beta} \tag{2.16b}
\]

For completeness, we also introduce, following Wess and Bagger \[21, 22\], the spinor superfield associated with the Volkov-Akulov Goldstino \(\lambda_\alpha\)

\[
\Lambda_\alpha(x, \theta, \bar{\theta}) = e^{iX} \lambda_\alpha(x). \tag{2.17}
\]
It obeys the constraints \[ D_\alpha \Lambda_\beta = -\varepsilon_{\alpha\beta} + i\tilde{\Lambda}_\alpha \partial_\alpha \Lambda_\beta , \quad \bar{D}_\dot{\alpha} \Lambda_\beta = -i\Lambda^\alpha \partial_{\dot{\alpha}\dot{\alpha}} \Lambda_\beta . \tag{2.18} \]

It can be shown using (2.9) that the Goldstino superfields \( \Xi_\alpha \) and \( \Psi_\alpha \) are related to each other as follows:

\[ \Xi_\alpha(z) = \Psi_\beta(z) B^\beta_\alpha(z) , \quad z^A := (x^a, \theta^\alpha, \bar{\theta}^{\dot{\alpha}}) , \tag{2.19} \]

for some nonsingular \( 2 \times 2 \) matrix \( B \) such that

\[ B = 1 + \text{nonlinear Goldstino-dependent terms} . \]

Eq. (2.16a) means that the superfield \( \bar{\Psi}_{\dot{\alpha}} \) is indeed antichiral. This allows the construction of an action

\[ S_\Psi = -\frac{1}{2} \int d^4 x d^2 \theta d^2 \bar{\theta} \Psi_\alpha \bar{\Psi}_\bar{\alpha} \tag{2.20} \]

which can be used to describe the Goldstino’s dynamics, instead of the action proposed by Samuel and Wess \[14]\]

\[ S_{SW} = -\frac{1}{2} \int d^4 x d^2 \theta d^2 \bar{\theta} \Xi_\alpha \Xi_{\bar{\alpha}} \Xi_{\bar{\beta}} \Xi_{\bar{\gamma}} \tag{2.21} \]

or the superfield version \[22\] of the Volkov-Akulov action

\[ S_{VA} = -\frac{1}{2} \int d^4 x d^2 \theta d^2 \bar{\theta} \Lambda_\alpha \bar{\Lambda}_{\dot{\alpha}} \Lambda_{\dot{\beta}} \bar{\Lambda}_{\dot{\gamma}} . \tag{2.22} \]

The component form of the action (2.20) is

\[ S_\Psi = -\int d^4 x \left( \frac{1}{2} + i \xi^\alpha \partial_{\dot{\alpha}\dot{\alpha}} \bar{\psi}_{\dot{\alpha}} - \xi^\alpha (\partial_{\alpha\dot{\alpha}} \xi_{\beta}) \bar{\psi}_{\dot{\alpha}}^2 - \frac{1}{4} \xi^\alpha \partial_{\dot{\alpha}\dot{\alpha}} \partial_{\alpha\dot{\alpha}} \bar{\psi}_{\dot{\alpha}}^2 + \text{c.c.} \right) . \tag{2.23} \]

The striking feature of this action is that it is at most quartic in the Goldstino fields, unlike the other Goldstino models which in general contain terms to eighth order in the fermionic field \[3\] (see [24] for a detailed analysis of the known Goldstino models and their relationships). This unusual feature is deceptive, however, since the action is given in terms of two different sets of fields, \( (\xi_\alpha, \bar{\xi}_{\dot{\alpha}}) \) and \( (\psi_\alpha, \bar{\psi}_{\dot{\alpha}}) \) related to each other as in via (2.8, 2.9). Sixth- and eighth-order terms will inevitably appear once the action is expressed via one set of fields or the other. If we express the action entirely in terms

\[ ^3\text{The Volkov-Akulov action does not contain any terms of eighth order in } \lambda \text{ and } \bar{\lambda} \[23]. \]
of the fields \((\xi_\alpha, \bar{\xi}^{\dot{\alpha}})\), we must end up with (the component version of) the Samuel-Wess Goldstino action. On the other hand, expressing (2.23) entirely in terms of the fields \((\psi_\alpha, \bar{\psi}^{\dot{\alpha}})\), it turns out that we end up with Roček’s Goldstino superfield \[26\]. This claim can be justified as follows.

We can associate with the Goldstino superfields two composite nilpotent objects which contain all the information about the original superfields, specifically:

\[
\Phi := \Psi^\alpha \Psi_\alpha ; \quad (2.24)
\]

\[
\Sigma := \bar{\Xi}^{\dot{\alpha}} \bar{\Xi}^{\dot{\alpha}} . \quad (2.25)
\]

The superfield \(\Sigma\) is equivalent to the complex linear Goldstino superfield introduced in \[25\]. Its properties are

\[
- \frac{1}{4} \bar{D}_2 \Sigma = 1 , \quad \Sigma^2 = 0 , \quad - \frac{1}{4} \Sigma \bar{D}^2 D_\alpha \Sigma = D_\alpha \Sigma . \quad (2.26)
\]

The superfield \(\Phi\) proves to be equivalent to Roček’s chiral Goldstino superfield \[26\] as its properties are

\[
\bar{D}_\alpha \Phi = 0 , \quad \Phi = 0 , \quad - \frac{1}{4} \Phi \bar{D}^2 \Phi = \Phi . \quad (2.27)
\]

Eq. (2.19) has to be used to derive the last constraint in (2.27). This confirms that (2.20) is equivalent to Roček’s Goldstino action

\[
S_R = - \int d^4x \, d^2\theta \, \Phi = - \int d^4x \, d^2\theta \, d^2\bar{\theta} \, \bar{\Phi} \Phi . \quad (2.28)
\]

Our realization (2.20) shows that \(S_\Psi\) may be interpreted as a square root of \(S_R\).

Using eqs. (2.26) and (2.27), it is not difficult to show that the Goldstino action (2.20) can be expressed only in terms of the Goldstone superfields (2.13) as

\[
S_{\Xi \Psi} = - \int d^4x \, d^2\theta \, d^2\bar{\theta} \, \Xi_\alpha \Xi_\alpha \, \bar{\Psi}_\dot{\alpha} \bar{\Psi}^{\dot{\alpha}} . \quad (2.29)
\]

This action proves to be real, as a consequence of the third constraint in (2.27).

3 Analytic realization of spontaneously broken \(\mathcal{N} = 2\) supersymmetry

In the case of \(\mathcal{N} = 2\) supersymmetry, one can also introduce a nonlinear realization that gives rise to Goldstino superfields that are antichiral, as explicitly described in the
Appendix. However, for reasons discussed above in the $\mathcal{N} = 1$ case, such a realization appears to be less useful than the $\mathcal{N} = 2$ analogue of the construction by Samuel and Wess [14]. This motivates seeking a new approach to the description of spontaneously broken $\mathcal{N} = 2$ supersymmetry.

Harmonic superspace [27] (see [28] for a review) extends conventional $\mathcal{N} = 2$ superspace by the two-sphere $S^2 = SU(2)/U(1)$ parametrized by the group elements

$$(u_i^-, u_i^+) \in SU(2), \quad u_i^+ := \epsilon_{ij} u_j^+, \quad \overline{u_i^+} = u_i^- \ , \quad u_i^+ u_i^- = 1. \quad \text{The } \mathcal{N} = 2 \text{ supersymmetry algebra,}$$

$${\{Q^{\dot{a}}_\alpha, Q_{\dot{a}j}\}} = 2 \delta_j^i P_{a\dot{a}} \ , \quad (3.1)$$
can be re-cast in the form

$${\{Q^\pm, \hat{Q}^\pm\}} = \pm 2 \ P_{a\dot{a}} \ , \quad (3.2)$$

where

$$Q^\pm_a = Q^i_a u_i^\pm, \quad \hat{Q}^\pm_a = \hat{Q}^i_a u_i^\pm. \quad (3.3)$$

To examine the complete breaking of $\mathcal{N} = 2$ supersymmetry in a harmonic superspace context, we choose a very particular coset parametrization

$$g(x, \lambda(x), \bar{\lambda}(x)) = e^{i(-x^a P_a + \lambda^\alpha(x) Q^\alpha_a - \bar{\lambda}^a(x) Q^a_{\dot{a}})} e^{i(-\lambda^\alpha(x) Q^\alpha_a + \bar{\lambda}^a(x) Q^a_{\dot{a}})} \ . \quad (3.4)$$

The fields in (3.4) are related to the Volkov-Akulov Goldstinos, eq. (A.1), as follows:

$$\lambda^\alpha_i(x') = \lambda^\alpha(x) u^+_i - \lambda^\alpha(x) u^-_i , \quad \bar{\lambda}^{\dot{a}}_i(x') = \bar{\lambda}^{\dot{a}}(x) u^+_i - \bar{\lambda}^{\dot{a}}(x) u^-_i, \quad (3.5)$$

where

$$x'^a = x^a - i\lambda^\alpha(x) \sigma^a \bar{\lambda}^\alpha(x) - i\lambda^\alpha(x) \sigma^a \bar{\lambda}^- (x) \ . \quad (3.6)$$

Left action by the group element

$$g(\epsilon, \bar{\epsilon}) = e^{i(\epsilon^\alpha Q^\alpha_a - \bar{\epsilon}^a Q^{a\dot{a}} - \epsilon^a + \bar{\epsilon}_a + i Q_{a\dot{a}})}$$

gives rise to the infinitesimal supersymmetry transformations

$$\delta \lambda^+_\alpha = \epsilon^+_{\alpha} - 2i \epsilon^\beta \bar{\lambda}^\dot{a} - \partial_\beta \lambda^+_\alpha + 2i \bar{\epsilon}^\dot{b} \lambda^\beta_{\dot{a}} - \partial_\dot{b} \lambda^+_\alpha \quad (3.7a)$$

$$\delta \bar{\lambda}^+_\dot{a} = \bar{\epsilon}^+_{\dot{a}} - 2i \epsilon^\beta \bar{\lambda}^\dot{a} - \partial_\beta \bar{\lambda}^+_\dot{a} + 2i \bar{\epsilon}^\dot{b} \lambda^\beta_{\dot{a}} - \partial_\dot{b} \bar{\lambda}^+_\dot{a} \quad (3.7b)$$

$$\delta \lambda^-_\alpha = \epsilon^-_{\alpha} - 2i \epsilon^\beta \bar{\lambda}^\dot{a} - \partial_\beta \lambda^-_\alpha + 2i \bar{\epsilon}^\dot{b} \lambda^\beta_{\dot{a}} - \partial_\dot{b} \lambda^-_\alpha \quad (3.7c)$$

$$\delta \bar{\lambda}^-_{\dot{a}} = \bar{\epsilon}^-_{\dot{a}} - 2i \epsilon^\beta \bar{\lambda}^\dot{a} - \partial_\beta \bar{\lambda}^-_{\dot{a}} + 2i \bar{\epsilon}^\dot{b} \lambda^\beta_{\dot{a}} - \partial_\dot{b} \bar{\lambda}^-_{\dot{a}} \ . \quad (3.7d)$$
The significance of the coset parametrization chosen is that \( \delta \lambda_\alpha^+ \) and \( \delta \bar{\lambda}_\dot{\alpha}^+ \) have no dependence on \( \epsilon^{\alpha-} \) and \( \bar{\epsilon}_{\dot{\alpha}}^- \), meaning that \( \lambda_\alpha^+ \) and \( \bar{\lambda}_{\dot{\alpha}}^+ \) are annihilated by \( Q_\alpha^+ \) and \( \bar{Q}^{\dot{\alpha}+} \):

\[
Q_\beta^+ \lambda_\alpha^+ = 0 \quad (3.8a) \\
\bar{Q}_{\dot{\beta}}^+ \lambda_\alpha^+ = 0 \quad (3.8b) \\
Q_\beta^- \lambda_\alpha^+ = i \epsilon_{\alpha\dot{\beta}} + 2 \bar{\lambda}_\dot{\beta}^- \partial_{\dot{\beta}} \lambda_\alpha^+ \quad (3.8c) \\
\bar{Q}_{\dot{\beta}}^- \lambda_\alpha^+ = -2 \lambda_{\beta}^- \partial_{\beta} \lambda_\alpha^+ \quad (3.8d)
\]

and

\[
Q_{\bar{\beta}}^+ \bar{\lambda}_{\dot{\alpha}}^+ = 0 \quad (3.9a) \\
\bar{Q}_{\dot{\beta}}^+ \bar{\lambda}_{\dot{\alpha}}^+ = 0 \quad (3.9b) \\
Q_{\bar{\beta}}^- \bar{\lambda}_{\dot{\alpha}}^+ = 2 \bar{\lambda}_{\dot{\beta}}^+ \partial_{\dot{\beta}} \bar{\lambda}_{\dot{\alpha}}^+ \quad (3.9c) \\
\bar{Q}_{\dot{\beta}}^- \bar{\lambda}_{\dot{\alpha}}^+ = i \epsilon_{\alpha\dot{\beta}} - 2 \lambda_{\beta}^+ \partial_{\beta} \bar{\lambda}_{\dot{\alpha}}^+ \quad (3.9d)
\]

When we come to construct superfields from \( \lambda_\alpha^+ \) and \( \bar{\lambda}_{\dot{\alpha}}^+ \), this will ensure that the corresponding superfields are analytic.

The supersymmetry transformations of \( \lambda_\alpha^- \) and \( \bar{\lambda}_{\dot{\alpha}}^- \) are given by

\[
Q_{\beta}^+ \lambda_\alpha^- = -i \epsilon_{\alpha\beta} \quad (3.10a) \\
\bar{Q}_{\dot{\beta}}^+ \lambda_\alpha^- = 0 \quad (3.10b) \\
Q_{\beta}^- \lambda_\alpha^- = 2 \bar{\lambda}_{\dot{\beta}}^- \partial_{\dot{\beta}} \lambda_\alpha^- \quad (3.10c) \\
\bar{Q}_{\dot{\beta}}^- \lambda_\alpha^- = -2 \lambda_{\beta}^- \partial_{\beta} \lambda_\alpha^- \quad (3.10d)
\]

and

\[
Q_{\bar{\beta}}^+ \bar{\lambda}_{\dot{\alpha}}^- = 0 \quad (3.11a) \\
\bar{Q}_{\dot{\beta}}^+ \bar{\lambda}_{\dot{\alpha}}^- = -i \epsilon_{\alpha\dot{\beta}} \quad (3.11b) \\
Q_{\bar{\beta}}^- \bar{\lambda}_{\dot{\alpha}}^- = 2 i \bar{\lambda}_{\dot{\beta}}^- \partial_{\dot{\beta}} \bar{\lambda}_{\dot{\alpha}}^- \quad (3.11c) \\
\bar{Q}_{\dot{\beta}}^- \bar{\lambda}_{\dot{\alpha}}^- = -2 i \lambda_{\beta}^+ \partial_{\beta} \bar{\lambda}_{\dot{\alpha}}^- \quad (3.11d)
\]

It should be pointed out that the fields \( \lambda_{\alpha}^+ \) and \( \bar{\lambda}_{\dot{\alpha}}^+ \) are conjugate with respect to the analyticity-preserving conjugation [27]:

\[
\tilde{\lambda}_{\alpha}^+ = \bar{\lambda}_{\dot{\alpha}}^+ \quad , \quad \tilde{\lambda}_{\alpha}^+ = -\lambda_{\alpha}^+ . \quad (3.12a)
\]
Similar relations hold for the fields $\tilde{\lambda}_\alpha$ and $\bar{\lambda}_\alpha$, that is
\[
\tilde{\lambda}_\alpha = \tilde{\lambda}_\alpha, \quad \bar{\lambda}_\alpha = -\lambda_\alpha.
\] (3.12b)

This is the key feature distinguishing the nonlinear realization in this section from the antichiral one detailed in the Appendix. In the latter, there is no simple conjugation relation between the Goldstinos associated with dotted and undotted supersymmetry generators (unlike the Volkov-Akulov realization (A.1)). In the parametrization (3.4) adapted to harmonic superspace, Goldstinos are related by analyticity-preserving conjugation.

### 4 Including the harmonic operators $D^{++}$ and $D^{--}$

In harmonic superspace, a key role is played by the SU(2) left-invariant vector fields
\[
D^{++} = u^{+i} \frac{\partial}{\partial u^{-i}}, \quad D^{--} = u^{-i} \frac{\partial}{\partial u^{+i}}, \quad D^0 = u^{+i} \frac{\partial}{\partial u^{+i}} - u^{-i} \frac{\partial}{\partial u^{-i}}.
\] (4.1)

Here $D^0$ is the operator associated with harmonic U(1) charge. Given a scalar function $\varphi^{(n)}(u^+, u^-)$ on SU(2) of U(1) charge $n$,
\[
\varphi^{(n)}(e^{i\alpha} u^+, e^{-i\alpha} u^-) = e^{i\alpha} \varphi^{(n)}(u^+, u^-), \quad \alpha \in \mathbb{R},
\] (4.2)

we have $D^0 \varphi^{(n)} = n \varphi^{(n)}$. Of particular importance for our considerations is the operator $D^{++}$ which acts on the supersymmetry generators as
\[
[D^{++}, Q_\alpha^+] = 0, \quad [D^{++}, Q^-_\alpha] = Q_\alpha^+, \quad [D^{++}, \bar{Q}_\alpha^+] = 0, \quad [D^{++}, \bar{Q}_\alpha^-] = \bar{Q}_\alpha^+.
\] (4.3)

At first glance, one might expect
\[
D^{++} \lambda_\alpha^+ = 0, \quad D^{++} \lambda_\alpha^- = \lambda_\alpha^+, \quad D^{++} \bar{\lambda}_\alpha^+ = 0, \quad D^{++} \bar{\lambda}_\alpha^- = \bar{\lambda}_\alpha^+.
\] (4.4)

However, these are inconsistent with the nonlinear action of the supersymmetry generators on the Goldstinos. We can see this with a simple example. Assuming $[D^{++}, Q^-_\beta] = Q^+_\beta$ and the nonlinear supersymmetry transformations of the $\lambda$'s,
\[
0 = Q^+_\beta \lambda_\alpha^+ \\
= [D^{++}, Q^-_\beta] \lambda_\alpha^+ \\
= D^{++}(-i\epsilon_{\alpha\beta} + 2\tilde{\lambda}^-_\beta \partial_{\beta\tilde{\lambda}}^+ \lambda_\alpha^+) - Q^-_\beta (D^{++} \lambda_\alpha^+) \\
= 2(D^{++} \bar{\lambda}_\beta^-) \partial_{\beta\tilde{\lambda}}^+ \lambda_\alpha^+ + 2\tilde{\lambda}^- \partial_{\beta\tilde{\lambda}}^+ (D^{++} \lambda_\alpha^+) - Q^-_\beta (D^{++} \lambda_\alpha^+),
\]
which yields a contradiction if we set $D^{++} \lambda^+_\alpha = 0$.

In order to determine how the operator $D^{++}$ acts on the Goldstinos, note that the action of the supersymmetry generators on the Goldstinos can be represented schematically in the form:

$$Q^+_\beta = -i \frac{\partial}{\partial \lambda^-_\beta}$$  \hspace{1cm} (4.5a)
$$\hat{Q}^+_\beta = -i \frac{\partial}{\partial \lambda^-_\beta}$$  \hspace{1cm} (4.5b)
$$Q^-_\beta = i \frac{\partial}{\partial \lambda^+_\beta} + 2 \lambda^\beta \partial_{\beta\beta}$$  \hspace{1cm} (4.5c)
$$\hat{Q}^-_\beta = i \frac{\partial}{\partial \lambda^+_\beta} - 2 \lambda^\beta \partial_{\beta\beta}.$$  \hspace{1cm} (4.5d)

This is similar to the superspace representation of the supersymmetry generators in the analytic basis, and the corresponding central basis version is

$$\hat{Q} = e^Z Q e^{-Z}, \quad Z := -i \lambda^\beta \bar{\lambda}^{\bar{\beta}} \partial_{\beta\beta} - i \lambda^\beta \partial_{\beta\beta}.$$  \hspace{1cm} (4.6)

One finds

$$\hat{Q}^+ = -i \frac{\partial}{\partial \lambda^-_\beta} + \bar{\lambda}^{\bar{\beta}} \partial_{\bar{\beta}\bar{\beta}}$$  \hspace{1cm} (4.7a)
$$\hat{Q}^-_\beta = -i \frac{\partial}{\partial \lambda^-_\beta} - \lambda^\beta \partial_{\beta\beta}$$  \hspace{1cm} (4.7b)
$$\hat{Q}^-_\beta = i \frac{\partial}{\partial \lambda^+_\beta} + \bar{\lambda}^{\bar{\beta}} \partial_{\bar{\beta}\bar{\beta}}$$  \hspace{1cm} (4.7c)
$$\hat{Q}^-_\beta = i \frac{\partial}{\partial \lambda^+_\beta} - \lambda^\beta \partial_{\beta\beta}.$$  \hspace{1cm} (4.7d)

In this central basis, $\hat{D}^{++} = u^{++} \frac{\partial}{\partial u^-}$ satisfies

$$[\hat{D}^{++}, \hat{Q}^+] = 0, \quad [\hat{D}^{++}, \hat{Q}^-] = \hat{Q}^+.$$  \hspace{1cm} (4.8)

It follows that in the analytic basis,

$$D^{++} = e^{-Z} \hat{D}^{++} e^Z = \hat{D}^{++} - 2i \lambda^\beta \bar{\lambda}^{\bar{\beta}} \partial_{\beta\beta}.$$  \hspace{1cm} (4.9a)

Thus $[D^{++}, Q^+_\alpha] = 0, [D^{++}, Q^-_\alpha] = Q^+_\alpha$ etc, and

$$D^{++} \lambda^+_\alpha = -2i \lambda^\beta \bar{\lambda}^{\bar{\beta}} \partial_{\beta\beta} \lambda^+_\alpha$$  \hspace{1cm} (4.9b)
$$D^{++} \lambda^-_\alpha = \lambda^+_\alpha - 2i \lambda^\beta \bar{\lambda}^{\bar{\beta}} \partial_{\beta\beta} \lambda^-_\alpha$$  \hspace{1cm} (4.9c)
$$D^{++} \bar{\lambda}^+_\alpha = -2i \bar{\lambda}^{\bar{\beta}} \bar{\lambda}^{\bar{\beta}} \partial_{\bar{\beta}\bar{\beta}} \bar{\lambda}^+_\alpha$$  \hspace{1cm} (4.9d)

$$D^{++} \bar{\lambda}^-_\alpha = \bar{\lambda}^+_\alpha - 2i \lambda^\beta \bar{\lambda}^{\bar{\beta}} \partial_{\beta\beta} \bar{\lambda}^-_\alpha.$$  \hspace{1cm} (4.9d)
In particular, this shows that the Goldstinos $\lambda^\pm_\alpha$ and $\bar{\lambda}^\pm_\alpha$ in the coset parametrization \((3.4)\) are not simply of the form $\lambda^\pm_i u^\pm_i$ and $\bar{\lambda}^\pm_i u^\pm_i$, with $\lambda^i_\alpha$ and $\bar{\lambda}^i_\alpha$ conventional $\mathcal{N} = 2$ Goldstinos independent of the harmonic coordinates.

The equations \((4.9b)\) and \((4.9d)\) are very important, since they explicitly relate the Goldstino fields $(\lambda^-_\alpha, \bar{\lambda}^-_\alpha)$ to $(\lambda^+_\alpha, \bar{\lambda}^+_\alpha)$.

In a similar manner, one can prove

\[
D^{--}\lambda^+_\alpha = \lambda^-_\alpha - 2i \lambda^\beta - \bar{\lambda}^\beta - \partial_{\beta\bar{\beta}} \lambda^+_\alpha \quad (4.10a)
\]
\[
D^{--}\lambda^-_\alpha = -2i \lambda^\beta - \bar{\lambda}^\beta - \partial_{\beta\bar{\beta}} \lambda^-_\alpha \quad (4.10b)
\]
\[
D^{--}\bar{\lambda}^+_\alpha = \bar{\lambda}^-_\alpha - 2i \bar{\lambda}^\beta - \lambda^\beta - \partial_{\beta\bar{\beta}} \bar{\lambda}^+_\alpha \quad (4.10c)
\]
\[
D^{--}\bar{\lambda}^-_\alpha = -2i \bar{\lambda}^\beta - \lambda^\beta - \partial_{\beta\bar{\beta}} \bar{\lambda}^-_\alpha . \quad (4.10d)
\]

It follows from \((4.9)\) that

\[
(D^{++})^4 \lambda^+_\alpha = 0 , \quad (D^{++})^4 \bar{\lambda}^+_\alpha = 0 , \quad (4.11)
\]

and therefore $\lambda^+_\alpha$ and $\bar{\lambda}^+_\alpha$ have simple harmonic expansions in powers of $u^+_i$ and $u^-_i$, in particular

\[
\lambda^+_\alpha(u^+, u^-) = \lambda^i_\alpha u^+_i + \sum_{n=1}^3 \lambda^{(i_1\ldots i_n+1j_1\ldots j_n)}_\alpha u^+_i \ldots u^+_i u^-_{j_1} \ldots u^-_{j_n} . \quad (4.12)
\]

Here $\lambda^i_\alpha$ is the $\mathcal{N} = 2$ Volkov-Akulov Goldstino, eq. \((A.1)\). The other fields in \((4.12)\) are nonlinear functions of $\lambda^i_\alpha$ and its conjugate.

## 5 Construction of Goldstino superfields

Similarly to the $\mathcal{N} = 1$ case discussed in section 2, to each of the Goldstinos $\lambda^\pm_\alpha$ and $\bar{\lambda}^\pm_\alpha$ we can associate an $\mathcal{N} = 2$ superfield depending on fermionic superspace coordinates $\theta^\pm_\alpha$ and $\bar{\theta}^\pm_\alpha$. In general form,

\[
\Lambda(x, \theta, \bar{\theta}) = e^{iX} \lambda(x) , \quad X := \theta^{\alpha-}Q^+_{\alpha} - \bar{\theta}^{-\bar{\alpha}}\bar{Q}^{\dot{\alpha}+} - \theta^{\alpha+}Q^-_{\alpha} + \bar{\theta}^{\bar{\alpha}+}\bar{Q}^{\dot{\alpha}-} . \quad (5.1)
\]

In the central basis for harmonic superspace\(^4\), the supercovariant derivatives take the form

\[
D^+_\beta = \frac{\partial}{\partial \theta^{\beta-}} + i \bar{\theta}^{\beta+} \partial_{\beta\dot{\beta}} , \quad \bar{D}^+_\beta = \frac{\partial}{\partial \bar{\theta}^{\beta-}} - i \theta^{\beta+} \partial_{\beta\dot{\beta}}
\]
\[
D^-_{\beta} = -\frac{\partial}{\partial \theta^{\beta+}} + i \bar{\theta}^{\beta-} \partial_{\beta\dot{\beta}} , \quad \bar{D}^-_{\beta} = -\frac{\partial}{\partial \bar{\theta}^{\beta+}} - i \theta^{\beta-} \partial_{\beta\dot{\beta}}
\]

\(^4\)Our use of harmonic-superspace terminology is somewhat unorthodox.
and satisfy the algebra
\[ \{ D^\pm_\beta, \bar{D}^\mp_\bar{\beta} \} = \mp 2i \partial_{\beta\bar{\beta}} . \]  
(5.2)

The action of the spinor covariant derivatives on the Goldstino superfields\(^5\) replicates the action of the supersymmetry generators on the Goldstinos via
\[ D^\pm_\beta \Lambda = e^{iX} i Q^\pm_\beta \lambda , \quad \bar{D}^\pm_\bar{\beta} \Lambda = e^{iX} i \bar{Q}^\pm_{\bar{\beta}} \lambda . \]  
(5.3)

In particular we obtain
\[ D^+_\beta \Lambda^+_\alpha = 0 \]  
(5.4a)
\[ \bar{D}^+_\bar{\beta} \Lambda^+_\alpha = 0 \]  
(5.4b)
\[ D^-_\beta \Lambda^+_\alpha = -\epsilon_{\alpha\beta} + 2i \bar{\Lambda}^\beta - \partial_{\beta\bar{\beta}} \Lambda^+_\alpha \]  
(5.4c)
\[ \bar{D}^-_\bar{\beta} \Lambda^+_\alpha = -2i \Lambda^\beta - \partial_{\beta\bar{\beta}} \Lambda^+_\alpha , \]  
(5.4d)

meaning that \( \Lambda^+_\alpha \) is an analytic superfield. Also
\[ D^+_\beta \bar{\Lambda}^+_\alpha = 0 \]  
(5.5a)
\[ \bar{D}^+_\bar{\beta} \bar{\Lambda}^+_\alpha = 0 \]  
(5.5b)
\[ D^-_\beta \bar{\Lambda}^+_\alpha = 2i \bar{\Lambda}^\beta - \partial_{\beta\bar{\beta}} \bar{\Lambda}^+_\alpha \]  
(5.5c)
\[ \bar{D}^-_\bar{\beta} \bar{\Lambda}^+_\alpha = -\epsilon_{\alpha\bar{\beta}} - 2i \Lambda^\beta - \partial_{\beta\bar{\beta}} \bar{\Lambda}^+_\alpha , \]  
(5.5d)

so \( \bar{\Lambda}^+_\alpha \) is also an analytic superfield.

The remainder of the \( D \)-algebra is given by
\[ D^+_\beta \Lambda^-_\alpha = \epsilon_{\alpha\beta} \]  
(5.6a)
\[ \bar{D}^+_\bar{\beta} \Lambda^-_\alpha = 0 \]  
(5.6b)
\[ D^-_\beta \Lambda^-_\alpha = 2i \bar{\Lambda}^\beta - \partial_{\beta\bar{\beta}} \Lambda^-_\alpha \]  
(5.6c)
\[ \bar{D}^-_\bar{\beta} \Lambda^-_\alpha = -2i \Lambda^\beta - \partial_{\beta\bar{\beta}} \Lambda^-_\alpha \]  
(5.6d)

and
\[ D^+_\beta \bar{\Lambda}^-_\alpha = 0 \]  
(5.7a)
\[ \bar{D}^+_\bar{\beta} \bar{\Lambda}^-_\alpha = \epsilon_{\alpha\bar{\beta}} \]  
(5.7b)
\[ D^-_\beta \bar{\Lambda}^-_\alpha = 2i \bar{\Lambda}^\beta - \partial_{\beta\bar{\beta}} \bar{\Lambda}^-_\alpha \]  
(5.7c)
\[ \bar{D}^-_\bar{\beta} \bar{\Lambda}^-_\alpha = -2i \Lambda^\beta - \partial_{\beta\bar{\beta}} \bar{\Lambda}^-_\alpha . \]  
(5.7d)

\(^5\)The construction \( \Lambda(x, \theta, \bar{\theta}) = e^{iX} \lambda(x) \) with \( X = \theta^a Q^+_a - \bar{\theta}^\dot{a} \bar{Q}^{\dot{a}+} - \theta^a Q^-_a + \bar{\theta}^\dot{a} \bar{Q}^{\dot{a}-} \) is adapted to the central basis for the supercovariant derivatives. The analytic basis is adapted to \( \Lambda(x, \theta, \bar{\theta}) = e^{iX_1} e^{iX_2} \lambda(x) \) with \( X_1 = -\theta^a Q^-_a + \bar{\theta}^\dot{a} \bar{Q}^{\dot{a}-} \) and \( X_2 = \theta^a Q^+_a - \bar{\theta}^\dot{a} \bar{Q}^{\dot{a}+} \).
To find the action of the $SU(2)$ generator $D^{++}$ on the Goldstino superfields, represented schematically in the form $\Lambda(x, \theta, \bar{\theta}) = e^{iX} \lambda(x)$ with $X = \theta^\alpha Q^+ - \bar{\theta}^\dot{\alpha} \bar{Q}^+ - \theta^{\dot{\alpha}} Q^+ - \bar{\theta}^\alpha \bar{Q}$, we note $D^{++} X = 0$, so

$$D^{++} \Lambda(x, \theta, \bar{\theta}) = e^{iX} D^{++} \lambda(x).$$

Thus the transformation properties of the superfield $\Lambda(x, \theta, \bar{\theta})$ are determined by those of the corresponding field $\lambda(x)$,

$$D^{++} \Lambda^+ = -2i \Lambda^{\alpha+} \bar{\Lambda}^{\dot{\alpha}+} \partial_{\beta \dot{\beta}} \Lambda^+_\alpha, \quad (5.8a)$$
$$D^{++} \Lambda^- = \Lambda^+ - 2i \Lambda^{\alpha+} \bar{\Lambda}^{\dot{\alpha}+} \partial_{\beta \dot{\beta}} \Lambda^-_\alpha, \quad (5.8b)$$
$$D^{++} \bar{\Lambda}^+ = -2i \Lambda^{\dot{\alpha}+} \bar{\Lambda}^{\alpha+} \partial_{\beta \dot{\beta}} \bar{\Lambda}^+_\dot{\alpha}, \quad (5.8c)$$
$$D^{++} \bar{\Lambda}^- = \bar{\Lambda}^+ - 2i \Lambda^{\dot{\alpha}+} \bar{\Lambda}^{\alpha+} \partial_{\beta \dot{\beta}} \bar{\Lambda}^-_{\dot{\alpha}}. \quad (5.8d)$$

Similarly, one can prove

$$D^{-} \Lambda^+ = \Lambda^- - 2i \Lambda^{\beta-} \bar{\Lambda}^{\dot{\beta}-} \partial_{\beta \dot{\beta}} \Lambda^+_\alpha, \quad (5.9a)$$
$$D^{-} \Lambda^- = -2i \Lambda^{\dot{\beta}-} \bar{\Lambda}^{\beta-} \partial_{\beta \dot{\beta}} \Lambda^-_\alpha, \quad (5.9b)$$
$$D^{-} \bar{\Lambda}^+ = \bar{\Lambda}^- - 2i \Lambda^{\dot{\beta}-} \bar{\Lambda}^{\beta-} \partial_{\beta \dot{\beta}} \bar{\Lambda}^+_{\dot{\alpha}}, \quad (5.9c)$$
$$D^{-} \bar{\Lambda}^- = -2i \Lambda^{\beta-} \bar{\Lambda}^{\dot{\beta}-} \partial_{\beta \dot{\beta}} \bar{\Lambda}^-_{\dot{\alpha}}. \quad (5.9d)$$

Since $\Lambda^+_\alpha$ and $\bar{\Lambda}^+_{\dot{\alpha}}$ are analytic superfields, it is possible to consistently define an action

$$S = \int du \int d\zeta^{(4)} L^{(+4)}, \quad L^{(+4)} = -\frac{1}{2} \Lambda^{\alpha+} \bar{\Lambda}^{\alpha+} \bar{\Lambda}^{\dot{\alpha}+}, \quad (5.10)$$

where the integration is over the analytic subspace of harmonic superspace,

$$d\zeta^{(4)} := d^4 x (D-)^4, \quad (D-)^4 := \frac{1}{16} (\bar{D})^2 (D-)^2. \quad (5.11)$$

In (5.10), $\int du$ denotes the integration over the group manifold SU(2) defined as in [28]

$$\int du 1 = 1 \quad \int du u^+_{i_1} \ldots u^+_{i_n} u^-_{j_1} \ldots u^-_{j_m} = 0 \quad n + m > 0. \quad (5.12)$$

Direct calculations give

$$\frac{1}{16} \int du (\bar{D})^2 (D-)^2 (\bar{\Lambda}^+)(\Lambda^+) = 1 + i \lambda^\alpha a \bar{\lambda}^{\dot{\alpha}i} + i \bar{\lambda}^{\dot{\alpha}i} \partial_{\alpha a} \lambda^\alpha_i + O(\lambda^4), \quad (5.13)$$

and thus the above action generates the correct kinetic term for the Goldstinos.
It is important to point out that the equations (5.8a) and (5.8c) imply the following nontrivial property

\[ D^{++} L^{(+4)} = 0 \iff L^{(+4)}(u) = L^{ijkl} u_i^+ u_j^+ u_k^+ u_l^+, \quad (5.14) \]

so that the Lagrangian is in fact independent of \( u_i^- \). Since \( L^{(+4)} \) is analytic, the superfield \( L^{ijkl} \) obeys the analyticity constraints

\[ D^{(m)}(L^{ijkl}) = \bar{D}^{(m)}(L^{ijkl}) = 0. \quad (5.15) \]

Following the terminology [29] of \( \mathcal{N} = 2 \) projective superspace [30] (see [31] for a modern review), the Lagrangian \( L^{(+4)} \) is an \( O(4) \) multiplet \( ^6 \). We can now reformulate the Goldstino action in projective superspace \( ^7 \). To pass from the harmonic to the projective formulation requires several steps. First of all, one should replace \( u^{+i} \rightarrow v^i \in \mathbb{C}^2 \setminus \{0\} \) where the isotwistor \( v^i \) provides homogeneous coordinates for \( \mathbb{CP}^1 \). Secondly, one should also replace \( u_i^- \rightarrow w_i \), where the isotwistor \( w_i \) is subject only to the condition \( (v, w) := v^i w_i \neq 0 \), and is otherwise completely arbitrary. Thirdly, the integration over \( SU(2) \) in (5.10) should be replaced by a closed contour integral in \( \mathbb{CP}^1 \). The projective-superspace realization of the Goldstino action is as follows \( ^8 \):

\[ S = -\frac{c}{2\pi} \oint \frac{v_i dv^i}{(v, w)^4} \int d^4 x (D^-(v)) 4 \frac{L^{(+4)}(v)}{c^{(+2)}(v)} , \quad c^{(+2)}(v) := c^{ij} v_i v_j , \quad (5.16) \]

with \( c^{ij} = c^{ji} \) a real non-zero constant isovector, and \( 2c^2 = c^{ij} c_{ij} \). One can show that this action is independent of \( w_i \) and \( c^{ij} \).

The properties of \( L^{(+4)} \) given above are such that the action (5.10) can be brought to the form:

\[ S = \frac{1}{80} \int d^4 x D^i D^j \bar{D}^k \bar{D}^l L_{ijkl} \]. \quad (5.17) \]

Thirty years ago, Sohnius, Stelle and West [33] realized that one can associate an \( \mathcal{N} = 2 \) super-action of the form (5.17) with any real symmetric iso-tensor superfield \( L^{ijkl} \) under the constraints (5.15). It is interesting that their supersymmetric action principle can be used to describe spontaneous supersymmetry breaking.

\(^6\)The fact that \( L^{(+4)} \) is a holomorphic tensor field over \( \mathbb{CP}^1 \), i.e. independent of \( u_i^- \), may be of significance in formulating a curved superspace version of the model under consideration.

\(^7\)The relationship between the \( \mathcal{N} = 2 \) harmonic and projective superspace approaches is spelled out in [31], [32].

\(^8\)The action (5.16) can be obtained from (5.10) by applying the reduction technique developed in [32].
The Goldstino action, eq. (5.10), can also be represented as an integral over full $\mathcal{N} = 2$ superspace

$$S = \int d^4 x \, d^4 \theta \, d^4 \bar{\theta} \, L , \quad L := -\frac{1}{2} (\Lambda^+)^2 (\Lambda^-)^2 (\bar{\Lambda}^+)^2 (\bar{\Lambda}^-)^2 .$$

(5.18)

There is no need to include the integration over SU(2), since the Lagrangian $L$ is independent of the harmonics,

$$D^{++} L = D^{--} L = 0 ,$$

(5.19)
as follows from (5.8) and (5.9).

We have constructed three different versions of the Goldstino action: (i) the harmonic superspace realization (5.10); (ii) the projective superspace realization (5.16); (iii) the full superspace realization (5.18). They allow us to generate three different types of Goldstino-matter couplings. Within the harmonic-superspace approach, couplings of the Goldstinos to $\mathcal{N} = 2$ supersymmetric matter can be described by actions of the form:

$$S_{\text{inter, harmonic}} = \int d\zeta^{(-4)} L^{(+4)}(u^+) L^{(0)}_{\text{matter}}(u^+, u^-) ,$$

(5.20)

where $L^{(0)}_{\text{matter}}$ is a real analytic superfield of U(1) charge zero,

$$D^{+} L^{(0)}_{\text{matter}} = \bar{D}^{+} L^{(0)}_{\text{matter}} = 0 , \quad L^{(0)}_{\text{matter}}(e^{i\alpha} u^+, e^{-i\alpha} u^-) = L^{(0)}_{\text{matter}}(u^+, u^-) .$$

(5.21)

In the projective-superspace approach, couplings of the Goldstinos to $\mathcal{N} = 2$ supersymmetric matter can be described by actions of the form:

$$S_{\text{inter, projective}} = \frac{1}{2\pi} \oint \frac{v_i dv^i}{(v, w)^4} \int d^4 x \,(D^-)^4 \left\{ L^{(+4)}(v) L^{(-2)}_{\text{matter}}(v) \right\} ,$$

(5.22)

where $L^{(-2)}_{\text{matter}}$ is a real projective superfield of weight $-2$,

$$D^{+} L^{(-2)}_{\text{matter}} = \bar{D}^{+} L^{(-2)}_{\text{matter}} = 0 , \quad L^{(-2)}_{\text{matter}}(a v) = a^{-2} L^{(-2)}_{\text{matter}}(v) .$$

(5.23)

We should point out that the interaction $L^{(-2)}_{\text{matter}}(v)$ is only required to be a holomorphic function of $v^i$ in the vicinity of the integration contour. Unlike the harmonic-superspace Lagrangian in (5.20), the $L^{(-2)}_{\text{matter}}(v)$ is not expected to be globally defined over $\mathbb{C}P^1$. The isotwistor $w_i$ in (5.22) has to be kept constant along the integration contour. One can show that the action (5.22) is independent of $w_i$, see [31] for more details. Finally, Goldstino-matter couplings can be described by actions of the form:

$$S_{\text{inter, full}} = \int d^4 x \, d^4 \theta \, d^4 \bar{\theta} \, L \, L_{\text{matter}} ,$$

(5.24)

\[9\] See [31] for the general definition of projective superfields.
where $L_{\text{matter}}$ is an ordinary $\mathcal{N} = 2$ superfield. We can also generate chiral Goldstino-matter couplings using the chiral Lagrangian in (A.6).

We would like to make a final comment. It appears that all information about the Goldstino superfields $\Lambda^\pm_\alpha$ and $\bar{\Lambda}^{\pm}_\dot{\alpha}$ is encoded in the Lagrangian $L^{(+)4}$ in the sense that these superfields can be obtained from $L^{(+)4}$ by applying various differential operators like $D^-_\alpha$, $\bar{D}^-_{\dot{\alpha}}$, $D^{--}$ etc. We believe that $L^{(+)4}$ should obey a system of constraints which involve only $L^{(+)4}$ that completely fix its structure, compare with (2.27). However, we have not been able to determine such constraints.

6 Composite Goldstino superfields

Here we briefly discuss composite Goldstino superfields that can be used to construct higher-derivative Goldstino actions.

All of the Goldstino superfields can be written as spinor covariant derivatives of a set of “prepotentials.” Defining

$$
\Sigma^{++} := \Lambda^{\alpha+} \Lambda^+_\alpha, \quad \Sigma := \Lambda^{\alpha-} \Lambda^+_\alpha, \quad \Sigma^{--} := \Lambda^{\alpha-} \Lambda^-_{\dot{\alpha}},
$$

and their conjugates, it follows that

$$
D^+_\beta \Sigma = \Lambda^{\beta+}_\beta \quad (6.2a)
$$
$$
D^+_\beta \Sigma^{--} = 2\Lambda^-_{\dot{\beta}} \quad (6.2b)
$$
$$
\bar{D}^+_{\dot{\beta}} \Sigma = -\bar{\Lambda}^+_{\dot{\beta}} \quad (6.2c)
$$
$$
\bar{D}^+_{\dot{\beta}} \Sigma^{--} = -2\bar{\Lambda}^-_{\dot{\beta}}. \quad (6.2d)
$$

We thus can think of the scalar composites $\Sigma$, $\Sigma^{--}$ and their conjugates as fundamental building blocks. The rest of the $D$ algebra for $\Sigma$ is

$$
\bar{D}^+_a \Sigma = 0 \quad (6.3a)
$$
$$
D^-_a \Sigma = -\Lambda^-_a + 2i \bar{\Lambda}^{\dot{\beta}-} \Lambda^+_{\dot{\beta}} \partial_{\alpha\dot{\beta}} \Lambda^-_{\dot{\alpha}} + 2i \bar{\Lambda}^{\dot{\beta}-} \Lambda^+_{\dot{\beta}} \partial_{\alpha\dot{\beta}} \Lambda^+_\alpha \quad (6.3b)
$$
$$
\bar{D}^-_{\dot{a}} \Sigma = -2i \Lambda^{\alpha+} \partial_{\dot{a}\alpha} \Sigma. \quad (6.3c)
$$

The superfield $\bar{\Sigma}^{--}$ can be viewed as an $\mathcal{N} = 2$ extension of the $\mathcal{N} = 1$ complex linear Goldstino superfield [25], eq. (2.26), for its properties are are

$$
-\frac{1}{4}(\bar{D}^+)^2 \bar{\Sigma}^{--} = 1, \quad (\bar{\Sigma}^{--})^2 = 0, \quad -\frac{1}{4} \bar{\Sigma}^{--}(\bar{D}^+)^2 D^-_a \bar{\Sigma}^{--} = D^-_a \bar{\Sigma}^{--}. \quad (6.4)
$$
It is not difficult to see that $\Sigma$ and $\bar{\Sigma}$ can be expressed in terms of $\Sigma^{-}$ and $\bar{\Sigma}^{-}$. Therefore, $\Sigma^{-}$ and $\bar{\Sigma}^{-}$ contain all the information about the Goldstino superfields.

The superfield $\Sigma^{++}$ is analytic, $D^{+}_{\alpha} \Sigma^{++} = \bar{D}^{+}_{\dot{\alpha}} \Sigma^{++} = 0$, and obeys the covariant constancy condition

$$ (D^{++} + i \mathcal{V}^{++}) \Sigma^{++} = 0 \; , \quad \mathcal{V}^{++} := 2(\Lambda^{a+} \partial_{\alpha a} \bar{\Lambda}^{\dot{a}+} + \bar{\Lambda}^{\dot{a}+} \partial_{\dot{\alpha} a} \Lambda^{a+}) = \bar{\mathcal{V}}^{++} \; . \quad (6.5) $$

Using $\Sigma^{++}$, $\bar{\Sigma}^{++}$ and $\mathcal{V}^{++}$, we can generate reduced chiral superfields of the form

$$ W = \frac{1}{4} \int du (\bar{D}^{-})^{2} \left( c \Sigma^{++} + \bar{c} \bar{\Sigma}^{++} + r \mathcal{V}^{++} \right) \; , \quad (6.6) $$

for arbitrary complex $c$ and real $r$ parameters. The properties of $W$ are

$$ \bar{D}^{i}_{\dot{\alpha}} W = 0 \; , \quad D^{i\alpha} D^{j\dot{\alpha}} W = \bar{D}^{i}_{\dot{\alpha}} \bar{D}^{j\dot{\alpha}} W \; . \quad (6.7) $$

Using such chiral superfields, we can generate higher derivative Goldstino couplings.

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We are grateful to Simon Tyler for a question leading to (2.29). This work is supported in part by the Australian Research Council.

### Appendix A  $\mathcal{N} = 2$ chiral construction

In this appendix, we apply the technique for construction of an antichiral Golstino superfield given in section 2 to the case of spontaneously broken $\mathcal{N} = 2$ supersymmetry. The $\mathcal{N} = 2$ supersymmetry algebra is given in equation (3.1). In the Volkov-Akulov construction, there are two Goldstinos $\lambda_{ai}$ (and their Hermitian conjugates $\bar{\lambda}_{\dot{a}i}$) associated with the broken $\mathcal{N} = 2$ supersymmetry generators via the coset parametrization

$$ g(x, \lambda_{i}(x), \bar{\lambda}^{i}(x)) = e^{i(-x^{a} P_{a} + \lambda_{i}(x) Q^{a}_{i} + \bar{\lambda}^{i}(x) \bar{Q}^{a}_{i})} \; . \quad (A.1) $$

This yields the infinitesimal supersymmetry transformations

$$ \delta \lambda_{\alpha i} = \epsilon_{\alpha i} - i \nu^{\alpha \beta} \partial_{\beta \dot{\alpha}} \lambda_{\alpha i} \; , \quad \delta \bar{\lambda}^{i}_{\dot{\alpha}} = \bar{\epsilon}^{i}_{\dot{\alpha}} - i \nu^{\beta \dot{\alpha}} \partial_{\beta \alpha} \bar{\lambda}^{i}_{\dot{\alpha}} \; . \quad (A.2) $$

with $\nu^{\alpha \beta} = \lambda_{j}^{\alpha} \epsilon^{\beta \dot{\alpha}} - \epsilon_{\dot{\alpha}}^{\beta} \lambda^{\dot{\alpha} \beta}$. The analogue of the Samuel-Wess nonlinear realization, in which there is a pair of Golsdtone fields $\xi_{ai}(x)$ which mix only with themselves under supersymmetry transformation, is based on the alternative coset parametrization

$$ g(x, \xi_{i}(x), \bar{\psi}^{i}(x)) = e^{i(-x^{a} P_{a} + \xi_{i}(x) Q^{a}_{i})} e^{i\bar{\psi}^{i}_{\dot{\alpha}}(x) \bar{Q}^{a}_{i}} \; . \quad (A.3) $$
This yields the supersymmetry transformations

\begin{align}
\delta \xi_{\alpha i} &= \epsilon_{\alpha} - 2i \xi_{\beta j} \bar{\epsilon}^{\dot{\beta}j} \partial_{\beta \dot{\beta}} \xi_{\alpha i} \\
\delta \bar{\psi}_{\dot{\alpha} i} &= \bar{\epsilon}_{\dot{\alpha} i} - 2i \xi_{\beta j} \bar{\epsilon}^{\beta j} \partial_{\dot{\beta} j} \bar{\psi}_{\dot{\alpha} i}.
\end{align}

(A.4a)

(A.4b)

The construction of $N = 2$ superfields associated with the Goldstinos proceeds as in the $N = 1$ case, and the resulting superfields $\Xi_{\alpha i}$ and $\bar{\Psi}^{i}_{\dot{\alpha}}$ satisfy the following set of constraints involving the $N = 2$ supercovariant derivatives:

\begin{align}
D_{\alpha}^{i} \Xi_{\beta j} &= \epsilon_{\beta \alpha} \delta_{j}^{i} \\
\bar{D}_{\dot{\alpha} i} \Xi_{\beta j} &= -2i \Xi_{\alpha}^{\alpha} \partial_{\alpha \dot{\alpha}} \Xi_{\beta j} \\
D^{i} \bar{\Psi}^{j}_{\dot{\beta}} &= 0 \\
\bar{D}_{\dot{\alpha} i} \bar{\Psi}^{j}_{\dot{\beta}} &= -\epsilon_{\beta \dot{\alpha}} \delta_{j}^{i} - 2i \Xi_{\alpha}^{\alpha} \partial_{\alpha \dot{\alpha}} \bar{\Psi}^{j}_{\dot{\beta}}.
\end{align}

(A.5a)

(A.5b)

(A.5c)

(A.5d)

In particular, (A.5c) means that the superfields $\bar{\Psi}^{i}_{\dot{\alpha}}$ are antichiral, and so provide ingredients for an action obtained by integration over the antichiral subspace of $N = 2$ superspace:

\[ S \propto \int d^{4}x \, d^{4}\theta \, \Psi^{4} + \int d^{4}x \, d^{4}\bar{\theta} \, \bar{\Psi}^{4}, \]

(A.6)

where $\Psi^{4} := \frac{1}{3} \Psi^{i j} \Psi_{i j}$ and $\Psi^{i j} := \Psi^{\alpha i} \Psi_{\alpha j}$. The nilpotent chiral superfield $\Psi^{4}$ can be shown to satisfy a constraint

\[ \Psi^{4} \propto \Psi^{4} D^{4} \Psi^{4}, \]

(A.7)

which is similar to (2.27)

References

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