Consistent Kaluza-Klein Reductions for General Supersymmetric $AdS$ Solutions

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Abstract

For the most general supersymmetric solutions of type IIB supergravity consisting of a warped product of $AdS_5$ with a five-dimensional manifold $M_5$, we construct an explicit consistent Kaluza-Klein reduction on $M_5$ to minimal $D = 5$ gauged supergravity. Thus, any solution of the gauged supergravity can be uplifted on $M_5$ to obtain an exact solution of type IIB supergravity. We also show that for general $AdS_4 \times SE_7$ solutions, where $SE_7$ is a seven-dimensional Sasaki-Einstein manifold, and for a general class of supersymmetric solutions that are a warped product of $AdS_4$ with a seven-dimensional manifold $N_7$, there is an analogous consistent reduction to minimal $D = 4$ gauged supergravity.
1 Introduction

A powerful method to construct solutions of $D = 10$ or $D = 11$ supergravity is to uplift solutions of simpler theories in lower-dimensions. For this to work it is necessary that there is an appropriate consistent Kaluza-Klein (KK) reduction on some internal manifold $M$ from $D = 10$ or $D = 11$ down to the lower-dimensional theory. In general, a KK expansion on $M$ leads to a lower dimensional theory involving an infinite tower of fields. Splitting these fields into a finite number of “light” fields and an infinite tower of “heavy” fields\(^1\), the KK reduction is called consistent if it is in fact consistent to set all of the heavy fields to zero in the equations of motion, leaving equations of motion for the light fields only. Clearly this is only possible if the on-shell light fields do not source the heavy fields.

KK reductions on a circle or more generally on an $n$-dimensional torus are always consistent. The heavy fields, which arise from modes with non-trivial dependence on the coordinates of the torus, are all charged under the $U(1)^n$ gauge symmetry, while the light fields, which in this case are actually massless fields, are independent of these coordinates and hence uncharged under the gauge symmetry. As a consequence, the heavy fields can never be sourced by the light fields alone and so the truncation to the light fields is consistent. Since this argument also extends to fermions, one concludes that a KK reduction of a higher-dimensional supergravity theory on a torus can always be consistently truncated to a lower-dimensional supergravity theory. Moreover, solutions of the lower dimensional supergravity theory that preserve supersymmetry will uplift to supersymmetric solutions of the higher dimensional supergravity theory.

More generally, however, consistent KK reductions are very much the exception rather than the rule. For example, it is only in very special circumstances that there is a consistent KK reduction on a sphere (for further discussion see [1]). An interesting class of examples are those associated with the maximally supersymmetric solutions of $D = 10$ and $D = 11$ supergravity that consist of products of $AdS$ spaces and spheres. Corresponding to the $AdS_4 \times S^7$ and $AdS_7 \times S^4$ solutions of $D = 11$ supergravity, there are consistent KK reductions on $S^7$ [2] and $S^4$ [3, 4] to $D = 4$ $SO(8)$ gauged supergravity and $D = 7$ $SO(5)$ gauged supergravity, respectively. Similarly, starting with the $AdS_5 \times S^5$ solution of type IIB supergravity there is expected to be a consistent KK reduction to $SO(6)$ gauged supergravity: various additional truncations were shown to be consistent in [5, 6, 7] and an ansatz for the full metric was constructed in [8].

\(^1\)In general there is not a sharp separation of energy scales, and hence the quotation marks.
We would like to view these examples as special cases of the following conjecture:

For any supersymmetric solution of $D = 10$ or $D = 11$ supergravity that consists of a warped product of $d + 1$ dimensional anti-de-Sitter space with a Riemannian manifold $M$, $AdS_{d+1} \times_a M$, there is a consistent Kaluza-Klein truncation on $M$ to a gauged supergravity theory in $d + 1$-dimensions for which the fields are dual to those in the superconformal current multiplet of the $d$-dimensional dual SCFT.

Equivalently, one can characterise the fields of the gauged supergravity as those that contain the $d + 1$-dimensional graviton and fill out an irreducible representation of the superisometry algebra of the $D = 10$ or $D = 11$ supergravity solution. This conjecture is essentially a restricted version of one that appeared long ago in [9], for which general arguments supporting it were put forward in [10].

For example the $AdS_5 \times S^5$ solution of type IIB, which has superisometry algebra $SU(2,2|4)$, is dual to $N = 4$ superYang-Mills theory in $d = 4$. The superconformal current multiplet of the latter theory includes the energy momentum tensor, $SO(6)$ R-symmetry currents, along with scalars and fermions. These are dual to the metric, $SO(6)$ gauge fields along with scalar and fermion fields, and are precisely the fields of the maximally supersymmetric $SO(6)$ gauged supergravity in five-dimensions.

As we have phrased the conjecture above, it is natural to try and prove the conjecture directly from the SCFT point of view. For the case of $AdS_3$ solutions, an argument has been made by [11, 12], but this needs to be modified for higher dimension $AdS$ solutions. While we think that this is an interesting avenue to pursue, in this paper we will verify the conjecture for a number of cases by constructing an explicit consistent KK reduction ansatz. By this we mean an explicit ansatz for the higher-dimensional fields that is built from the fields of the lower-dimensional theory with the property that it solves the equations of motion of the higher-dimensional theory provided that the equations of the lower-dimensional theory are satisfied. This approach has the advantage that it allows one to uplift an explicit solution of the lower-dimensional gauged supergravity to obtain an explicit solution\(^2\) of $D = 10$ or $D = 11$ supergravity.

Often, for simplicity, such explicit KK reduction ansätze are constructed for the bosonic fields only. This is thought to provide very strong evidence that the ansatz can be extended to the fermionic fields also. In fact an argument was constructed in

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\(^2\)Note that since the uplifting formulae are local, in general, even if the lower-dimensional solution is free from singularities one still needs to check that the higher dimensional solution is also.
[1], based on [10], which shows that if a consistent KK reduction has been constructed for the bosonic fields, then the supersymmetry of the higher dimensional theory will guarantee that the reduction can be consistently extended to the fermionic sector. In any event, a bosonic KK ansatz certainly allows one to uplift bosonic solutions which is the most interesting class of solutions. One can go further and construct an ansatz for the fermion fields and demand that the supersymmetry variation of a bosonic configuration in higher-dimensions leads to the correct supersymmetry variation of the bosonic configuration in lower-dimensions. This explicitly demonstrates that a supersymmetric bosonic solution of the lower-dimensional theory will uplift to a supersymmetric solution of $D = 10$ or $D = 11$ supergravity which will preserve at least the same amount of supersymmetry as in the lower-dimensional theory.

In this paper we will verify the conjecture for a general class of $AdS_5$ solutions which are dual to $N = 1$ SCFTs in $d = 4$ dimensions. For this case, the bosonic fields in the superconformal current multiplet are the energy momentum tensor and the abelian R-symmetry current. Thus we seek a consistent truncation to minimal $D = 5$ gauged supergravity whose bosonic fields are the metric (dual to the energy momentum tensor of the SCFT) and an abelian gauge field (dual to the R-symmetry current). For the special class of solutions of type IIB of the form $AdS_5 \times SE_5$, where $SE_5$ is a five-dimensional Sasaki-Einstein manifold, and only the self-dual five-form is non-vanishing, a consistent KK reduction was constructed in [13] (see also [14]). Here we will extend this result by showing that for the most general $AdS_5 \times_w M_5$ supersymmetric solution of type IIB supergravity with all of the fluxes active, that were analysed in [15], the KK reduction is also consistent. We will construct a KK ansatz for the bosonic fields and we will also verify the consistency of the supersymmetry variations. The analogous result for the most general supersymmetric solutions of $D = 11$ supergravity of the form $AdS_5 \times_w M_6$ with non-vanishing four-form flux [16], was shown in [17]. Given that any $AdS_5$ solution of type IIA supergravity can be considered to be a special case of one in $D = 11$, if we are assume that there are no $AdS_5$ solutions in type I supergravity, the results here combined with [13, 17] covers all $AdS_5$ solutions in $D = 10$ and $D = 11$ dimensions.

We will also prove similar results for two classes of $AdS_4$ solutions of $D = 11$ supergravity, both of which are dual to $N = 2$ SCFTs in $d = 3$. The first, and the simplest, is the Freund-Rubin class of solutions which take the form $AdS_4 \times SE_7$ where $SE_7$ is a seven-dimensional Sasaki-Einstein manifold and the four-form flux is proportional to the volume form of the $AdS_4$ factor. A discussion of this case appears in [18]. Furthermore, our analysis is a simple extension of the analysis in [19]
which considered the seven-sphere viewed as a $U(1)$ fibration over $CP^3$. The second is the class of $AdS_4 \times w N_7$ solutions, corresponding to M5-branes wrapping SLAG 3-cycles, that were classified in [20]. It is very plausible that this class of solutions are the most general class of solutions with this amount of supersymmetry and with purely magnetic four-form flux. In both cases we show that there is a consistent KK reduction on the $SE_7$ or the $N_7$ to minimal gauged supergravity in four spacetime dimensions. The bosonic fields of the latter theory again consist of a metric and a $U(1)$ gauge field which are dual to the bosonic fields in the superconformal current multiplet. For these examples, we will be content to present the KK ansatz for the bosonic fields only.

The general classes of supersymmetric solutions that we consider have been analysed using $G$-structure techniques [21, 22]. In particular, the $G$-structure can be characterised in terms of bi-linears constructed from the Killing spinors. Since the results we obtain only assume supersymmetry and $AdS$ factors one might expect that the explicit KK reduction ansatz involves these bi-linears, and this is indeed the case. In fact it might be illuminating to recast the known consistent KK truncations on spheres in terms of this language, but we shall not investigate that here.

The plan of the rest of the paper is as follows. We begin in sections 2 and 3 by considering the $AdS_4$ solutions of $D = 11$ supergravity. In section 4 we consider the general class of $AdS_5$ solutions of type IIB supergravity. Only for the latter class we will present details of our calculations and these can be found in the appendices. In section 5 we briefly conclude.

## 2 Reduction of $D = 11$ supergravity on $SE_7$

Our starting point in this section is the class of supersymmetric solutions of $D = 11$ supergravity of the form $AdS_4 \times SE_7$ where $SE_7$ is a Sasaki-Einstein 7-manifold:

$$
\begin{align*}
\text{ds}_{11}^2 &= \frac{1}{4}\text{ds}^2(AdS_4) + \text{ds}^2(SE_7) \\
G &= \frac{3}{8}\text{vol}(AdS_4).
\end{align*}
$$

Here vol$(AdS_4)$ is the volume 4-form of the unit radius $AdS_4$ metric $\text{ds}^2(AdS_4)$ and we have normalised the Sasaki-Einstein metric $\text{ds}^2(SE_7)$ so that Ric$(SE_7) = 6g(SE_7)$ (the same as for the unit radius metric on the round seven-sphere). The Sasaki-Einstein metric has a Killing vector which is dual to the R-symmetry of the dual $N = 2$ SCFT in $d = 3$. Introducing coordinates so that this Killing vector is $\partial_\psi$,
locally, the Sasaki-Einstein metric can be written

$$ds^2(SE_7) = (d\psi + \sigma)^2 + ds^2(M_6) \quad (2.2)$$

where $ds^2(M_6)$ is locally Kähler-Einstein with Kähler form $J$, normalised so that $Ric(M_6) = 8g(M_6)$ and $d\sigma = 2J$.

We now construct an ansatz which leads to a consistent truncation, at the level of bosonic fields, to gauged supergravity in $D = 4$. Specifically, we consider

$$ds^2_{11} = \frac{1}{4} ds^2_d + (d\psi + \sigma + \frac{1}{4} A)^2 + ds^2(M_6)$$

$$G = \frac{3}{8} \text{vol}_4 - \frac{1}{14} \ast_4 F_2 \wedge J \quad (2.3)$$

where $ds^2_d$ is an arbitrary metric on a four-dimensional spacetime, $\text{vol}_4$ is its associated volume form, and $A$ and $F_2 = dA$ are one- and two-forms on this spacetime with a normalisation chosen for convenience. Substituting this into the $D = 11$ equations of motion [23] (we use the conventions of [22]),

$$R_{AB} - \frac{1}{12} (G_{AC_1C_2C_3} G_B^{C_1C_2C_3} - \frac{1}{12} g_{AB} G^2) = 0$$
$$d \ast_{11} G + \frac{1}{2} G \wedge G = 0$$
$$dG = 0 \quad (2.4)$$

we find that the metric $g_{\mu\nu}$, corresponding to $ds^2_d$, and $F_2$ must satisfy

$$R_{\mu\nu} = -3g_{\mu\nu} + \frac{1}{2} F_{\mu\rho} F_{\nu}^\rho - \frac{1}{2} g_{\mu\nu} F_{\rho\sigma} F^{\rho\sigma}$$
$$d \ast_4 F_2 = 0 \quad (2.5)$$

These are precisely the equations of motion of minimal gauged supergravity in $D = 4$ [24, 25].

Thus we have shown the consistency of the KK reduction at the level of the bosonic fields. In particular, any solution of the minimal gauged supergravity, which were systematically studied in [26], can be uplifted on an arbitrary seven-dimensional Sasaki-Einstein manifold to a solution of $D = 11$ supergravity.

### 3 Reduction of $D = 11$ supergravity on a SLAG-3 Flux Geometry

Let us now consider the general class of supersymmetric warped product solutions of the form $AdS_4 \times_w N_7$ with purely magnetic four-form flux which are dual to $N = 2$.
SCFTs in $d = 3$ [20]. We call these geometries SLAG-3 flux geometries, since they can be derived from a class of geometries that correspond to M5-branes wrapping special lagrangian (SLAG) three-cycles in a $SU(3)$ holonomy manifold - for further details see [20]. It is quite possible that this class of geometries is the most general class of $AdS_4$ geometries with this amount of supersymmetry and with purely magnetic four-form flux, but this has not been proven.

The $D = 11$ metric of the SLAG-3 flux geometry is given by

$$ ds_{11}^2 = \lambda^{-1} ds^2(AdS_4) + ds^2(\mathcal{N}_7) $$

(3.1)

where $ds^2(AdS_4)$ has unit radius and the warp factor $\lambda$ is independent of the coordinates of $AdS_4$. $\mathcal{N}_7$ has a local $SU(2)$ structure which is specified by three one-forms and three self-dual two-forms $J^1, J^2, J^3$. One of the one-forms is dual to a Killing vector that also preserves the flux: this is dual to the R-symmetry of the corresponding $N = 2$ SCFT. Introducing local coordinates so that this Killing vector is given by $\partial_\phi$ we have

$$ ds^2(\mathcal{N}_7) = ds^2(\mathcal{M}_{SU(2)}) + w \otimes w + \frac{\lambda^2 d\rho^2}{4(1 - \lambda^3 \rho^2)} + \frac{\lambda^2 \rho^2}{4} d\phi^2, $$

(3.2)

$\mathcal{M}_{SU(2)}$ is a four-dimensional space where the $J^a$ live. The three one-forms mentioned above are $w, (\lambda/2 \sqrt{1 - \lambda^3 \rho^2}) d\rho$ and $(\lambda \rho/2) d\phi$. In addition we must have

$$ d[\lambda^{-1} \sqrt{1 - \lambda^3 \rho^2} w] = \lambda^{-1/2} J^1 + \frac{\lambda^2 \rho}{2 \sqrt{1 - \lambda^3 \rho^2}} w \wedge d\rho, $$

$$ d \left( \lambda^{-3/2} J^3 \wedge w - \frac{\lambda \rho}{2 \sqrt{1 - \lambda^3 \rho^2}} J^2 \wedge d\rho \right) = 0, $$

$$ d \left( J^2 \wedge w + \frac{1}{2 \lambda^{1/2} \rho \sqrt{1 - \lambda^3 \rho^2}} J^3 \wedge d\rho \right) = 0. $$

(3.3)

Finally the 4-form flux is given by

$$ G = d\phi \wedge d \left( \frac{1}{2} \lambda^{-1/2} \sqrt{1 - \lambda^3 \rho^2} J^3 \right). $$

(3.4)

An explicit example of a solution to these equations was given in [27] as discussed in [20].

We now consider the KK reduction ansatz:

$$ ds_{11}^2 = \lambda^{-1} ds^2_4 + ds^2(\hat{\mathcal{N}}_7) $$

$$ G = \hat{G} + F_2 \wedge Y + *_4 F_2 \wedge X $$

(3.5)
where $ds^2_4$ is a line element and $F_2 = dA$ is a two-form on a four-dimensional spacetime. In addition $ds^2(\hat{N}_7)$ is the expected deformation of $ds^2(N_7)$, given by

$$ds^2(\hat{N}_7) = ds^2(M_{SU(2)}) + w \otimes w + \frac{\lambda^2 \rho^2}{4(1 - \lambda^2 \rho^2)} + \frac{\lambda^2 \rho^2}{4}(d\phi + A)^2,$$

(3.6)

$\hat{G}$ is the expected deformation of the four-form flux appearing in (3.4)

$$\hat{G} = (d\phi + A) \wedge d\left(\frac{1}{2} \lambda^{-1/2} \sqrt{1 - \lambda^3 \rho^2} J^3\right),$$

(3.7)

and the two-forms $X$ and $Y$ are given by

$$X = -\frac{1}{2}(\lambda^{-1/2} J^1 + \frac{\lambda^2 \rho}{2 \sqrt{1 - \lambda^3 \rho^2}} \omega \wedge d\rho),$$

$$Y = -\frac{1}{2}\lambda^{-1/2} \sqrt{1 - \lambda^3 \rho^2} J^3.$$

(3.8)

Substituting this ansatz into the equations of motion of $D = 11$ supergravity (2.4) and using (3.3) we find that all equations are satisfied provided that the equations of motion (2.5) of minimal gauged supergravity in $D = 4$ are satisfied. This again shows the consistency of the truncation, at the level of the bosonic fields.

4 Reduction of IIB on general $M_5$

We now turn to the general class of supersymmetric $AdS_5 \times M_5$ solutions of IIB supergravity with all fluxes active that were analysed in [15]. Such solutions are dual to $N = 1$ SCFTs in $d = 4$ which all have a $U(1)$ $R$-symmetry. We will show that there is a consistent KK reduction on $M_5$ to minimal gauged supergravity in $D = 5$. This case is more involved than the previous two and so we have included some details of the calculation in the appendices.

4.1 Internal geometry and fluxes

We begin by summarising the results of [15]. The ten-dimensional metric is a warped product of $AdS_5$ with a five-dimensional Riemannian manifold $M_5$,

$$ds^2_{10} = e^{2\Delta} \left[ds^2(AdS_5) + ds^2(M_5)\right],$$

(4.1)

where the warp factor $\Delta$ is a real function on $M_5$. All fluxes are active: in order to preserve the spatial $SO(4,2)$ isometry, the one-forms $P$, $Q$ and the complex three-form $G$ lie entirely on the internal $M_5$, and the five-form is taken to be

$$F = f \left(\text{vol}_{AdS_5} + \text{vol}_{M_5}\right),$$

(4.2)
where \( f \) is a constant and \( \text{vol} \) is the volume form corresponding to each of the metrics in the r.h.s. of (4.1). We use the same conventions as in [15] and some of this is recorded in appendix A.

The manifold \( M_5 \) is equipped with two spinors \( \xi_1, \xi_2 \) of \( \text{Spin}(5) \) subject to a set of differential and algebraic constraints arising from the IIB Killing spinor equations. The spinors \( \xi_1, \xi_2 \) define a local identity structure on \( M_5 \), which can be conveniently characterised in terms of a set of forms, bi-linear in \( \xi_1, \xi_2 \), consisting of a real scalar \( \sin \zeta \), a complex scalar \( S \), a real one-form \( K_5 \), and two complex one-forms \( K, K^* \). These satisfy the following differential conditions

\[
e^{-4\Delta} d(e^{4\Delta} S) = 3iK \]
\[
e^{-6\Delta} D(e^{6\Delta} K_3) = P \wedge K_3^* - 4iW - e^{-2\Delta} * G \]
\[
e^{-8\Delta} d(e^{8\Delta} K_5) = 4 \sin \zeta V - 6U \tag{4.3}\]

where \( D(e^{6\Delta} K_3) \equiv d(e^{6\Delta} K_3) - iQ \wedge e^{6\Delta} K_3 \). In (4.3), \( U, V \) are real two-forms and \( W \) is a complex two-form that can be constructed as bi-linears in \( \xi \) and moreover can be expressed in terms of the identity structure:

\[
U = \frac{1}{2(\cos^2 \zeta - |S|^2)} \left( i \sin \zeta K_3^* \wedge K_3^* + iK \wedge K^* - 2 \text{Im} S^* K \wedge K_5 \right),
\]
\[
V = \frac{1}{2 \sin \zeta (\cos^2 \zeta - |S|^2)} \left( i \sin \zeta K_3 \wedge K_3^*ight.
\]
\[
+ i[\sin^2 \zeta + |S|^2] K \wedge K^* - 2 \text{Im} S^* K \wedge K_3 \big),
\]
\[
W = \frac{1}{\sin \zeta (\cos^2 \zeta - |S|^2)} \left( \cos^2 \zeta K_5 + \text{Re} S^* K + i \sin \zeta \text{Im} S^* K \right) \wedge K_3. \tag{4.4}\]

In addition, one also has the algebraic constraint

\[iK_3 \cdot P = 2iK_3 d\Delta, \tag{4.5}\]

the five-form flux is given by (4.2) with

\[f = 4e^{4\Delta} \sin \zeta, \tag{4.6}\]

the three-form flux is given by

\[
\left( \cos^2 \zeta - |S|^2 \right) e^{-2\Delta} * G
\]
\[
= 2P \wedge K_3^* - (4d\Delta + 4iK_4 - 4i \sin \zeta K_5) \wedge K_3 \tag{4.7}
\]
\[
+ 2 * (P \wedge K_3^* \wedge K_5 - 2d\Delta \wedge K_3 \wedge K_5) ,
\]
where \( \sin \zeta K_4 = K_5 + \text{Re}(S^*K) \), and the metric can be written
\[
ds^2(M_5) = \frac{(K_5)^2}{\sin^2 \zeta + |S|^2} + \frac{K_3 \otimes K_3^*}{\cos^2 \zeta - |S|^2} + \frac{|S|^2}{\cos^2 \zeta - |S|^2} \left( \text{Im } S^{-1}K \right)^2 \\
+ \frac{|S|^2}{\sin^2 \zeta} \frac{\sin^2 \zeta + |S|^2}{\cos^2 \zeta - |S|^2} \left( \text{Re } S^{-1}K + \frac{1}{\sin^2 \zeta + |S|^2} K_5 \right)^2.
\] (4.8)

Finally, the vector dual to \( K_5 \) is a Killing vector of the metric (4.8) that also generates a symmetry of the full solution: \( \mathcal{L}_{K_5} \Delta = i_{K_5} P = \mathcal{L}_{K_5} G = 0 \). The above constraints arising from supersymmetry ensure that all equations of motion and Bianchi identities are satisfied.

### 4.2 KK reduction

We now construct the ansatz for a KK reduction from type IIB on the general \( M_5 \) that we discussed in the last subsection. We shall show that there is a consistent reduction to minimal \( D = 5 \) gauged supergravity.

On \( M_5 \) the vector field dual to the one-form \( K_5 \) is Killing and corresponds to the R-symmetry in the \( d = 4 \) dual SCFT. If one introduces coordinates such that this dual vector field is \( 3 \partial \psi \), we would like to shift \( d\psi \) by the gauge field \( A \): noting that \( ||K_5||^2 = (\sin^2 \zeta + |S|^2) \) this means that we should make the shift
\[
K_5 \quad \rightarrow \quad \hat{K}_5 = K_5 + (\sin^2 \zeta + |S|^2) \frac{A}{3}.
\] (4.9)

In particular, given (4.1), our ansatz for the \( D = 10 \) type IIB metric is then
\[
ds_{10}^2 = e^{2\Delta} \left[ ds_5^2 + ds^2(\hat{M}_5) \right]
\] (4.10)
where \( ds_5^2 \) is an arbitrary metric on five-dimensional spacetime, and \( ds^2(\hat{M}_5) \) is the metric \( ds^2(M_5) \) in (4.8) after the shift (4.9).

The KK ansatz for the five-form and the complex three-form of type IIB reads:
\[
F_5 = \hat{F}_5 + F_2 \wedge \frac{1}{3} e^{4\Delta} \hat{\ast}_5 V + \ast_5 F_2 \wedge \frac{1}{3} e^{4\Delta} V \\
G = \hat{G} + F_2 \wedge \frac{1}{3} e^{2\Delta} K_3
\] (4.11)

where \( F_2 = dA, \hat{F}_5 \) and \( \hat{G} \) are the five-form and three-form flux of the undeformed solution on \( M_5 \) after we make the shift (4.9), \( V, K_3 \) are the bi-linears on \( M_5 \) introduced in the previous subsection\(^3\), and \( \hat{\ast}_5 \) and \( \ast_5 \) are, respectively, the Hodge duals with

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\(^3\)The bi-linear \( V \) is not affected by the shift (4.9): choosing the convenient frame of Appendix B of [15] one can check that all \( K_5 \) dependence of \( V \) in equation (4.4) drops out.
respect to the metrics $ds^2(M_5)$ and $ds_5^2$ in (4.10). Notice that Since the one-forms $P$ and $Q$ of the undeformed solution on $M_5$ are independent of $K_5$, they remain the same as they were.

In appendix B we provide some details of how we constructed this particular ansatz. In particular, a long calculation shows that the ansatz (4.10), (4.11) with $P, Q$ unchanged satisfies all of the IIB equations of motion and Bianchi identities, provided that $ds_5^2$ and $F_2$ satisfy

\begin{align*}
R_{\mu\nu} &= -4g_{\mu\nu} + \frac{1}{6}F_{\mu\lambda}F^\lambda_{\nu} - \frac{1}{36}g_{\mu\nu}F_{\lambda\rho}F^{\lambda\rho} \quad (4.12) \\
d^*F_2 - \frac{1}{3}F_2 \wedge F_2 &= 0. \quad (4.13)
\end{align*}

These are precisely the equations of motion of minimal $D = 5$ gauged supergravity [28]. This shows the consistency of the truncation of the bosonic sector.

The truncation is, moreover, consistent at the level of the variations of the IIB fermion fields (see Appendix C for the details). On the one hand we find that the supersymmetry variations of the dilatino $\lambda$ and of the internal components of the gravitino $\Psi_M$ identically vanish. On the other hand, the external components of the IIB gravitino variation reduce to

\begin{equation}
\delta\psi_\alpha = D_\alpha \varepsilon - \frac{1}{2} \rho_\alpha \varepsilon + \frac{i}{2} A_\alpha \varepsilon + \frac{i}{24} F_{\beta\gamma} (\rho_\alpha^{\beta\gamma} - 4\delta^{\beta\gamma}_\alpha) \varepsilon, \quad (4.14)
\end{equation}

where $\psi_\alpha$ is the $D = 5$ gravitino and $\varepsilon$ a $D = 5$ spinor. This is the gravitino variation corresponding to minimal $D = 5$ gauged supergravity.

To summarise, we have shown that any bosonic solution of $D = 5$ supergravity can be uplifted to $D = 10$ using a general supersymmetric solution by means of the KK ansatz (4.10), (4.11). Moreover, if the five-dimensional bosonic solution is supersymmetric\(^4\) then so will be the uplifted ten-dimensional solution.

## 5 Conclusion

In this paper we have constructed explicit consistent KK reduction ansätze for general classes of $AdS_5$ solutions in type IIB supergravity and $AdS_4$ solutions in $D = 11$ supergravity. Our results can be extended to other classes of supersymmetric solutions that have been classified. It would be nice to show for the $AdS_5 \times M_6$ solutions of $D = 11$ supergravity, classified in [30], which are dual to $N = 2$ SCFTs in $d = 4$, that there is a consistent KK reduction to the $SU(2) \times U(1)$ gauged supergravity of

\(^4\)Such solutions were classified in [29].
A similar result in type IIB requires an analogous classification of $AdS_5 \times w M_5$ solutions that are dual to $N = 2$ SCFTs in $d = 4$, which has not yet been carried out.

There are several classes of $AdS_4$ solutions of $D = 11$ supergravity that can be considered. For example, one can consider $AdS_4 \times N_7$ solutions of $D = 11$ where $N_7$ has weak $G_2$ holonomy [32, 33] or the $AdS_4 \times w N_7$ solutions that arise from $M5$-branes wrapping associative 3-cycles that were analysed in [20]. These solutions are dual to $N = 1$ SCFTs in $d = 3$, which have no $R$-symmetry, and so one expects a consistent KK reduction on $N_7$ to a $N = 1$ supergravity whose field content is just the metric and fermions. In fact it is easy to show that there is a consistent reduction to the $N = 1$ supergravity of [34]. Similarly, the $AdS_4 \times N_7$ solutions of $D = 11$ where $N_7$ is tri-Sasaki [32, 33], are dual to $N = 3$ SCFTs in $d = 3$ and there should be a consistent KK reduction to an $SO(3)$ gauged supergravity in $D = 4$. Additional $AdS_3$ and $AdS_2$ solutions of $D = 11$ supergravity studied in [20, 35, 36] can also be considered.

The consistency of the KK truncation makes it manifest from the gravity side that SCFTs with type a IIB or $D = 11$ dual share common sectors. For example, if we consider such SCFTs in $d = 4$, the black hole solutions of minimal gauged supergravity constructed in [37] should be relevant for any of the SCFTs. It would be interesting to pursue this further.

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A IIB supergravity conventions

We quote here our conventions for IIB supergravity [38, 39], that follow those of [15]. The bosonic ten-dimensional fields consist of a metric and the following set of form
field strengths: a complex one-form $P$, a complex three-form $G$ and a real five-form $F_5$, subject to the following equations of motion:

$$R_{MN} = P_M P_N^* + P_N P_M^* + \frac{1}{96} F_{MP} P_1 P_2 P_3 F_N P_4 P_5 P_6 + \frac{1}{8} \left( G_M P_1 P_2 G_{NP_1 P_2} + G_N P_1 P_2 G_{MP_1 P_2} - \frac{1}{6} g_{MN} G_{P_1 P_2 P_3 G} P_1 P_2 P_3 G^* \right); \quad \text{(A.1)}$$

$$*F_5 = F_5,$$ \quad \text{(A.2)}

$$D*G - P \wedge *G^* + iG \wedge F_5 = 0,$$ \quad \text{(A.3)}

$$D*P + \frac{1}{4} G \wedge *G = 0.$$ \quad \text{(A.4)}

We are working in the formalism where $SU(1,1)$ is realised linearly. In particular there is a local $U(1)$ invariance and $Q_M$ acts as the corresponding gauge field. Note that $Q_M$ is a composite gauge field with field strength given by $dQ = -iP \wedge P^*$. Since $G$ has charge 1 and $P$ has charge 2 under this $U(1)$ we have the covariant derivatives:

$$D*G \equiv d*G - iQ \wedge *G^*,$$

$$D*P \equiv d*P - 2iQ \wedge *P.$$ We also need to impose the Bianchi identities

$$dF_5 - \frac{i}{2} G \wedge G^* = 0,$$

$$DG + P \wedge G^* = 0,$$

$$DP = 0.$$ \quad \text{(A.5)}

The IIB fermionic fields consist of a gravitino $\Psi_M$ and a dilatino $\lambda$. For supersymmetric bosonic solutions, the variations under supersymmetry of the fermion fields,

$$\delta \lambda = i \Gamma^M P_M \epsilon^c + \frac{i}{24} \Gamma^{P_1 P_2 P_3} G_{P_1 P_2 P_3} \epsilon,$$ \quad \text{(A.6)}

$$\delta \Psi_M = D_M \epsilon - \frac{1}{96} \left( \Gamma^M P_1 P_2 P_3 G_{P_1 P_2 P_3} - 9 \Gamma^{P_1 P_2} G_{MP_1 P_2} \right) \epsilon^c + \frac{i}{192} \Gamma^{P_1 P_2 P_3 P_4} F_{MP_1 P_2 P_3 P_4} \epsilon,$$ \quad \text{(A.7)}

must vanish. The spinor $\epsilon$ has composite $U(1)$ charge $+1/2$ so that $D_M \epsilon = (\nabla_M - \frac{i}{2} Q_M) \epsilon$.

## B IIB reduction: bosonic sector

We now derive the KK reduction ansatz (4.11) for the type IIB bosonic fields Recall that the vector field dual to the bi-linear $K_5$ is Killing and that $||K_5||^2 = (\sin^2 \zeta + |S|^2)$. We therefore need to make the shift

$$K_5 \rightarrow \hat{K}_5 = K_5 + (\sin^2 \zeta + |S|^2) \frac{A}{3} \quad \text{(B.1)}$$

in the metric of the undeformed solution to obtain:

$$ds_{10}^2 = e^{2\Delta} \left[ ds_5^2 + ds^2(\hat{M}_5) \right]. \quad \text{(B.2)}$$
In fact for any $p$-form $\beta_p$ on $M_5$ we can define a $\hat{\beta}_p$ in $\hat{M}_5$ via

$$\hat{\beta}_p = \beta_p + \frac{1}{3} A \wedge i_{K_5} \beta_p,$$

(B.3)

where $i_{K_5}$ is the interior product with respect to the vector dual to the one-form $K_5$. If we restrict to forms $\beta_p$ whose Lie-derivative with respect to the Killing vector dual to $K_5$ vanish, it is useful in the calculations below to note that

$$d\hat{\beta}_p = d\beta_p + \frac{1}{3} A \wedge i_{K_5} \beta_p + \frac{1}{3} F_2 \wedge i_{K_5} \beta_p + \frac{1}{3} F_2 \wedge i_{K_5} \beta_p$$

(B.4)

We now propose the following KK ansatz for the five-form and complex three-form field strengths. We first take the fluxes of the undeformed $AdS_5 \times_{w} M_5$ solution, and make the shift (B.3) to obtain $\hat{F}_5$ and $\hat{G}$. We then introduce a set of forms $\beta_3$, $\beta_2$, $\alpha_1$, $\alpha_0$ on $M_5$, which we take to be invariant under the action of the Killing vector

$$F_5 = \hat{F}_5 + F_2 \wedge \hat{\beta}_3 + \ast_5 F_2 \wedge \hat{\beta}_2,$$

$$G = \hat{G} + F_2 \wedge \hat{\alpha}_1 + \ast_5 F_2 \hat{\alpha}_0.$$

(B.5)

The IIB forms $P$ and $Q$ in the KK ansatz are taken to be the same as those in the undeformed solution.

For the KK reduction ansatz (B.2), (B.5) to be consistent, it must satisfy the IIB field equations (A.1)–(A.5) when the $D = 5$ equations (4.12) and (4.13) for $ds_5^2$, $F_2$ are satisfied. To carry out these calculations it is useful to use the orthonormal frame $e^a, a = 1, \ldots, 5$, on $M_5$ that was introduced in appendix B of [15] which, in particular, contains

$$e^1 = \frac{3}{h} K_5, \quad h = \frac{1}{3} \sqrt{\sin^2 \zeta + |S|^2}.$$

(B.6)

For $\hat{M}_5$ we use the corresponding frame obtained by the prescription (B.1).

The requirement that the fields obey the field equations (A.2)–(A.5) translates into a set of differential and algebraic equations relating the undeformed forms $\beta_3$,  

\footnote{This is a natural condition to impose. If we introduce coordinates so that the Killing vector field dual to $K_5$ is $3\partial_\psi$, then the condition says that the components of the forms must be independent of $\psi$.}
\[ \beta_2, \alpha_1, \alpha_0 \to \text{the undeformed fluxes } G, F_5, P, Q \text{ and metric on } M_5: \]
\[
\begin{align*}
    d\beta_2 &= \frac{i}{2}(\alpha_0^* G - \alpha_0 G^*), \\
    \frac{1}{3}i\kappa_5 \beta_3 &= -\frac{1}{3}\beta_2 + \frac{i}{3} \alpha_1 \wedge \alpha_1^*, \\
    d\beta_3 &= \frac{i}{2}(G \wedge \alpha_1^* - G^* \wedge \alpha_1) - \frac{1}{3}i\kappa_5 F_5, \\
    \frac{1}{3}i\kappa_5 \beta_2 &= \frac{i}{2}(\alpha_0^* \alpha_1 - \alpha_0 \alpha_1^*), \\
    D\alpha_1 + P \wedge \alpha_1^* + \frac{1}{3}i\kappa_5 G &= 0, \\
    D\alpha_0 + P \alpha_0^* &= 0, \\
    i\kappa_5 \alpha_1 &= -\alpha_0, \\
    \beta_3 &= *_5 \beta_2, \\
    \frac{1}{3}e^{4\Delta} i\kappa_5 *_5 \alpha_1 &= -i\alpha_1 \wedge \beta_2 + i\alpha_0 \beta_3, \\
    - \frac{1}{3}e^{4\Delta} *_5 \alpha_1 &= \frac{1}{3}e^{4\Delta} \alpha_0 i\kappa_5 \text{vol} M_5 + i\alpha_1 \wedge \beta_3, \\
    D(e^{4\Delta} *_5 \alpha_1) - P \wedge e^{4\Delta} *_5 \alpha_1^* + iG \wedge \beta_2 - i\alpha_0 f \text{vol} M_5 &= 0, \\
    \alpha_1 \wedge *_5 \alpha_1 &= \alpha_0^2 \text{vol} M_5,
\end{align*}
\]
\[(B.7)\]

where \(\alpha_0, \alpha_1\) both carry charge 1 under the composite \(U(1)\) gauge-field so that e.g. \(D\alpha_1 \equiv d\alpha_1 - iQ \wedge \alpha_1\).

We must also demand that the KK ansatz satisfies the Einstein equations. After substitution of (B.5), and imposing for simplicity \(\beta_3 = *_5 \beta_2\) (one of the conditions in (B.7)) we find that the external, \(\mu \nu\), components of the Einstein equation (A.1) read
\[
R_{\mu \nu} = -4g_{\mu \nu} - k_1 F_{\mu \lambda} F^{\lambda \nu} - k_2 g_{\mu \nu} F_{\lambda \rho} F^{\lambda \rho}
\]
\[(B.8)\]

where \(k_1, k_2\) are functions on \(M_5\) given by
\[
\begin{align*}
    k_1 &= \frac{1}{4} \left[ e^{-8\Delta} \beta_{2ab} \beta_{2}^a + 2e^{-4\Delta} \alpha_0 \alpha_0^* + 2e^{-4\Delta} \alpha_1^a \alpha_1^a + 2h^2 \right], \\
    k_2 &= \frac{1}{16} \left[ e^{-8\Delta} \beta_{2ab} \beta_{2}^a + 3e^{-4\Delta} \alpha_0 \alpha_0^* + e^{-4\Delta} \alpha_1^a \alpha_1^a \right].
\end{align*}
\]
\[(B.9),(B.10)\]

Comparing with (4.12) we see that we require\(^6\) that \(k_1 = 1/6\) and \(k_2 = 1/36\).

The mixed, \(\mu a\), components of the Einstein equations (A.1) give:
\[
\nabla^\rho F_{\rho \mu} + \frac{k_3}{4} \epsilon_{\mu \nu \lambda \rho \sigma} F^{\nu \lambda} F^{\rho \sigma} = 0
\]
\[(B.11)\]

with
\[
\begin{align*}
    k_3 &= \frac{1}{84} \delta^{1a} \left[ e^{-8\Delta} \epsilon_{abcd} \beta_{2}^b \beta_{2}^d + 4e^{-4\Delta} (\alpha_0 \alpha_1^a + \alpha_0^* \alpha_1^a) \right].
\end{align*}
\]
\[(B.12)\]

\(^6\)The possibility that \(k_1, k_2\) and \(k_3\), below, cannot be chosen to be constant is a potential source of inconsistency of the KK reduction; a similar issue has been discussed for other reductions in [18, 40].
Comparing with (4.13) we see that we demand $k_3 = -1/3$.

Finally, the internal, $ab$, components of the Einstein equations (A.1) give one more relation among the unknown coefficients in the KK ansatz:

$$4e^{-8\Delta} \beta_{2ac} \beta_{2b}^c + 2e^{-4\Delta}(\alpha_{1a}\alpha_{1b}^* + \alpha_{1a}^* \alpha_{1b})$$
$$+ \delta_{ab}(e^{-8\Delta} \beta_{2cd} \beta_{2d} - e^{-4\Delta}(\alpha_0 \alpha_0^* - \alpha_1^* \alpha_1^*)) = 4h^2 \delta_{a1} \delta_{b1}.$$  \(\text{(B.13)}\)

After considering the spinor bi-linears that characterise the identity structure on $M_5$ [15], we find that all of the above conditions are satisfied if we choose

$$\alpha_0 = 0,$$
$$\alpha_1 = \frac{1}{3} e^{2\Delta} K_3,$$
$$\beta_2 = \frac{1}{4} e^{4\Delta} V,$$
$$\beta_3 = \frac{1}{3} e^{4\Delta} *_5 V.$$  \(\text{(B.14)}\)

The most convenient way to prove this is to again use the specific frame on $M_5$ introduced in Appendix B of [15].

C IIB reduction: fermions

Now we show that the KK ansatz (4.10), (4.11) is also consistent at the level of the supersymmetry variations of the fermions. For this we follow the spinor conventions of Appendix A of [15] which we refer the reader to for more details (we will correct a typo in [15] below).

The undeformed $AdS_5 \times_w M_5$ solution admits Killing spinors of the form

$$\epsilon = \psi \otimes e^{\Delta/2} \xi_1 \otimes \theta + \psi^c \otimes e^{\Delta/2} \xi_2^c \otimes \theta,$$  \(\text{(C.1)}\)

where $\psi$ is a Killing spinor on $AdS_5$, $\theta$ is a constant two-component spinor and, most importantly, $\xi_1$, $\xi_2$ are Spin(5) spinors on $M_5$ that satisfy two differential conditions

$$D_m \xi_1 + \frac{i}{4}(e^{-4\Delta} f - 2) \gamma_m \xi_1 + \frac{1}{8} e^{-2\Delta} G_{mnp} \gamma^{np} \xi_2 = 0$$
$$\bar{D}_m \xi_2 - \frac{i}{4}(e^{-4\Delta} f + 2) \gamma_m \xi_2 + \frac{1}{8} e^{-2\Delta} G^*_{mnp} \gamma^{np} \xi_1 = 0.$$  \(\text{(C.2)}\)
and four algebraic conditions

\[
\gamma^m \partial_m \Delta \xi_1 - \frac{1}{48} e^{-2\Delta} \gamma^{mnp} G_{mnp} \xi_2 - \frac{i}{4} \left( e^{-4\Delta} f - 4 \right) \xi_1 = 0
\]

\[
\gamma^m \partial_m \Delta \xi_2 - \frac{1}{48} e^{-2\Delta} \gamma^{mnp} G^*_{mnp} \xi_1 + \frac{i}{4} \left( e^{-4\Delta} f + 4 \right) \xi_2 = 0
\]

\[
\gamma^m P_m \xi_2 + \frac{1}{24} e^{-2\Delta} \gamma^{mnp} G^*_{mnp} \xi_1 = 0
\]

\[
\gamma^m P^* \xi_1 + \frac{1}{24} e^{-2\Delta} \gamma^{mnp} G^*_{mnp} \xi_2 = 0
\]

(C.3)

where \( \gamma^m \) generate Cliff(5) with \( \gamma_{12345} = +1 \). Note that \( \psi^c = C_{1,4} \psi^*, \xi^c_i = C_5 \xi_i^* \), \( i = 1, 2 \), where \( C_{1,4}, C_5 \) are charge conjugation matrices.

The KK ansatz for the \( D = 10 \) Killing spinor is then simply

\[
\epsilon = \varepsilon \otimes e^{\Delta/2} \xi_1 \otimes \theta + \varepsilon^c \otimes e^{\Delta/2} \xi^c_2 \otimes \theta.
\]

(C.4)

Here \( \varepsilon \) is an arbitrary \( D = 5 \) spacetime spinor and the rest is as in the undeformed case. For the gravitino, we shall only need a KK reduction ansatz for the external components, namely (in tangent space):

\[
\Psi_\alpha = \psi_\alpha \otimes e^{-\Delta/2} \xi_1 \otimes \theta + \psi^c_\alpha \otimes e^{-\Delta/2} \xi^c_2 \otimes \theta
\]

(C.5)

where \( \psi_\alpha \) is the \( D = 5 \) gravitino.

We now demand that the conditions for the KK ansatz to preserve supersymmetry, namely, that the supersymmetry variations of \( \lambda \) and \( \Psi_M \) vanish, is the same as the conditions for preservation of supersymmetry in the \( D = 5 \) gauged supergravity. We will use (B.5) but with \( \alpha_0 = 0 \) and \( \beta_3 = *_5 \beta_2 \).

First consider the variations of the dilatino and of the internal components \( \Psi_\alpha \) of the gravitino. After substituting (B.5) into (A.6), (A.7) and using (C.2), (C.3), we find that these variations vanish providing that

\[
\alpha_{1a} \gamma^a \xi_1 = 0
\]

\[
\alpha_{1a}^* \gamma^a \xi_2 = 0
\]

\[
-4h \delta_a \xi_1 + 2ie^{-4\Delta} \beta_{2ab}^* \gamma^b \xi_1 - ie^{-4\Delta} \gamma_{abc} \beta^c_{2ab} \xi_1 - e^{-2\Delta} \alpha_{1a}^b \gamma^b_1 \xi_2 + 3e^{-2\Delta} \alpha_{1a} \xi_2 = 0
\]

\[
-4h \delta_a \xi_2 - 2ie^{-4\Delta} \beta_{2ab} \gamma^b \xi_2 + ie^{-4\Delta} \gamma_{abc} \beta^c_{2ab} \xi_2 - e^{-2\Delta} \alpha_{1a}^b \gamma^b_1 \xi_1 + 3e^{-2\Delta} \alpha_{1a}^* \xi_1 = 0
\]

(C.6)

One can check that these relations are indeed satisfied\(^7\) given our expressions (B.14) for \( \alpha_1 \) and \( \beta_2 \).

\(^7\)This can be seen by using the basis of Cliff(5) and the frame for \( M_5 \) given in appendix B of [15].
Next consider the variation of the external components of the gravitino. After substituting (B.5) into (A.7), one finds
\[
\delta \Psi_\alpha = \frac{1}{2} e^{-\Delta/2} \rho_\alpha \epsilon \otimes \left( -\frac{1}{4} (e^{-4\Delta} f - 4) \xi_1 - i \gamma_\alpha \partial^\alpha \Delta \xi_1 + \frac{i}{48} e^{-2\Delta} \delta^{abc} \Gamma_{abc} \xi_2 \right) \otimes \theta
+ \frac{1}{2} e^{-\Delta/2} \rho_\alpha \epsilon^c \otimes \left( -\frac{1}{4} (e^{-4\Delta} f + 4) \xi_2^c - i \gamma_\alpha \partial^\alpha \Delta \xi_2^c + \frac{i}{48} e^{-2\Delta} \delta^{abc} \Gamma_{abc} \xi_1^c \right) \otimes \theta
+ e^{-\Delta/2} \left[ D_a \epsilon \otimes \xi_1 - \frac{1}{2} \rho_a \epsilon \otimes \xi_1 - A_a \epsilon \otimes \partial_\psi \xi_1
+ \frac{1}{16} F_{ab} \rho^b \epsilon \otimes \left( -4 i h \gamma_1 \xi_1 - e^{-4\Delta} \beta_{2ab} \gamma^{abc} \xi_1 - 3 i e^{-2\Delta} \alpha_{1a} \gamma^a \xi_2 \right)
+ \frac{1}{32} \rho_{ab} \gamma \epsilon \otimes \left( i e^{-2\Delta} \alpha_{1a} \gamma^a \xi_2 + e^{-4\Delta} \beta_{2bc} \gamma^{bc} \xi_1 \right) \right] \otimes \theta
\]

where we are using the coordinate $\psi$ so that the Killing vector dual to $K_5$ is $3 \partial_\psi$. In this expression the $\rho^a$ generate Cliff(4,1) and satisfy $\rho_{01234} = -i$ (this corrects a sign in [15]). We also have $\epsilon_{01234} = +1$.

We now observe that for the choice of forms given in (B.14) one has
\[
-4 i h \gamma_1 \xi_1 - e^{-4\Delta} \beta_{2ab} \gamma^{abc} \xi_1 - 3 i e^{-2\Delta} \alpha_{1a} \gamma^a \xi_2 = -\frac{8i}{3} \xi_1
\]

\[
i e^{-2\Delta} \alpha_{1a} \gamma^a \xi_2 + e^{-4\Delta} \beta_{2bc} \gamma^{bc} \xi_1 = \frac{4i}{3} \xi_1
\]

(C.8)

and similar expressions for the last two terms of (C.7). Using these results, the fact that $\partial_\psi \xi_1 = -\frac{i}{2} \xi_1$ and equations (C.3), after introducing the KK ansatz (C.5) for the gravitino we deduce that
\[
\delta \Psi_\alpha \otimes e^{-\Delta/2} \xi_1 \otimes \theta + \delta \Psi^c_\alpha \otimes e^{-\Delta/2} \xi_2^c \otimes \theta =

(D_a \epsilon - \frac{1}{2} \rho_a \epsilon + \frac{i}{2} A_a \epsilon + \frac{i}{24} F_{b\gamma} (\rho_{ab} \beta^\gamma - 4 \delta_{ab} \rho^\gamma) \epsilon) \otimes e^{-\Delta/2} \xi_1 \otimes \theta

(D_a \epsilon^c + \frac{1}{2} \rho_a \epsilon^c - \frac{i}{2} A_a \epsilon^c + \frac{i}{24} F_{b\gamma} (\rho_{ab} \beta^\gamma - 4 \delta_{ab} \rho^\gamma) \epsilon^c) \otimes e^{-\Delta/2} \xi_2^c \otimes \theta
\]

(C.9)

which implies
\[
\delta \Psi_\alpha = D_a \epsilon - \frac{1}{2} \rho_a \epsilon + \frac{i}{2} A_a \epsilon + \frac{i}{24} F_{b\gamma} (\rho_{ab} \beta^\gamma - 4 \delta_{ab} \rho^\gamma) \epsilon,
\]

(C.10)

as claimed in the text.
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