Queue-Aware Beam Scheduling for Half-Duplex mmWave Relay Networks

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Abstract—We focus on two basic millimeter wave (mmWave) relay networks and for each network, we propose three beam scheduling methods to approach the network information theoretic capacity. The proposed beam scheduling methods include the deterministic horizontal continuous edge coloring (HC-EC) scheduler, the adaptive back pressure (BP) scheduler and the adaptive low-delay new back pressure (newBP) scheduler. With the aid of computer simulations, we show that within the network capacity range, the proposed schedulers provide good guarantees for the network stability, meanwhile achieve very low packet end-to-end delay.

Index Terms—mmWave, relay network, network stability, end-to-end delay, network capacity

I. INTRODUCTION

Millimeter wave (mmWave) bands are considered a powerful key enabler for next generation (5G) mobile networks by providing multi-Gbps data rates. However, mmWave transmissions suffer from high propagation and penetration losses, resulting in very limited range/coverage and very high susceptibility to blockage. One effective way to mitigate these effects and to increase the communication range is beamforming in combination with relaying [1]. The former is achieved by utilizing large antenna arrays on the transceivers and pointing their beams towards each other as illustrated in our previous work [2–4]. The latter refers to using intermediate nodes to relay the source signal to the destination [5].

Given a set of static relays between a source-destination pair, several relay selection schemes for two-hop or multi-hop mmWave settings have been proposed in recent literature [1, 5–7]. However, these works are limited to the selection of a single relay path that achieves the largest end-to-end signal to noise ratio (SNR). The utilization of multiple relay paths in combination with an efficient beam scheduling scheme to further increase the network throughput is yet unexplored.

A recent study of 1-2-1 model in [8, 9] provides an idealized and simplified information theoretic relay network model, where a potential link is active only if the transmitter beam and the receiver beams are pointing at each other which is termed as 1-2-1 link. Following [8, 9], in this paper, we focus on two basic mmWave relay networks, i.e., the line network and the diamond network as shown in Fig. 1 (a) and (b), respectively. In both cases, the relay nodes are assumed to work in half duplex (HD) mode and a link is active only if both nodes focus their beams to face each other.

We go beyond the single-relay-path limitations [1, 5–7] and take the general optimality of the beam scheduling approach, aiming at finding actual computational algorithms (both offline and online) to approach the beam scheduling theory in [8, 9]. We propose three beam scheduling methods: the deterministic horizontal continuous edge coloring (HC-EC) scheduler, the adaptive back pressure (BP) scheduler and the adaptive low-delay new back pressure (newBP) scheduler. We provide a joint performance evaluation of the network stability and the end-to-end delay in terms of different source input rates. It is shown through simulations that the proposed schedulers are throughput optimal and achieve very low end-to-end delay. Particularly, each of the proposed beam scheduling methods has its own superiority in terms of different source input rates and computation complexity.

II. SYSTEM MODEL

In this section, we present the formulation of two 1-2-1 network topologies considered in this paper, which can be seen as basic building blocks for more general topologies.

A. The line network \( \mathcal{L} \)

For the line network, we consider the \( N \)-relay Gaussian HD channel model as shown in Fig. 1 (a), where a source node (node 0) wishes to communicate to a destination node (node \( N + 1 \)) through a route of \( N \) relays. Each relay is operating in HD. Denoted by \( h_{i,j} \) as the complex channel coefficients from node \( j \) to node \( i \) and \( h_{i,j} = 0 \) whenever \( j \neq i - 1 \). We assume that \( h_{i,j} \) are constant for the whole transmission duration and the channel inputs satisfy an unit average power constraint. Hence the point-to-point link capacity from node \( i - 1 \) to node \( i \) can be written as

\[
l_i = \log(1 + |h_{i,i-1}|^2), \quad \forall i \in [N + 1],
\]
where we assume the additive white Gaussian noise at each nodes are independent and identically distributed (i.i.d.) as $CN(0, 1)$, and where $[N]$ indicates the set of non-negative integers $\{1, ..., N\}$. Following [8], the capacity of the Gaussian HD line network can be described within a constant gap $\text{GAP} = O(N)$ as

$$C_L = \max_{\lambda \in \Lambda} \min_{A \subseteq [N]} \sum_{s \in \{0, 1\}^N} \lambda_s \sum_{i=1}^{\infty} I_i. \quad (2)$$

Such constant gap result is referred to as the network approximate capacity in [9]. Here the schedule $\lambda \in C^{2N}$ determines the fraction of time $x_s$ for each network state $s \in \{0, 1\}^N$, $\Lambda = \{ \lambda : \lambda \in C^{2N}, \lambda \geq 0, ||\lambda||_1 = 1 \}$ is the set of all possible schedules, $R_s$ (w.r.t. $T_s$) represents the set of indices of receiving (w.r.t. transmitting) relays in the state $s$, and $A^c = [N] \setminus A$. In [8], the authors propose a simple edge coloring (EC) scheduler that achieves $C_L$ and shows that the approximate capacity in (2) is given explicitly by

$$C_L = \min_{i \in [N]} \left\{ \frac{l_i \cdot l_i + 1}{l_i + l_i + 1} \right\}. \quad (3)$$

The EC scheduler presented in [8] leverages the similarities between network states in HD and edge coloring in a graph. Although this EC scheduler achieves $C_L$, it ignores the average queues over each network node, which may result in a very large transmission delay. In Section III, we will provide a modified version of the EC scheduler, which can significantly reduce the transmission delay.

**B. The diamond network $D$**

Similarly for the diamond network $D$, we consider the $N$-relay Gaussian HD channel model as shown in Fig. 1(b), where $N$ relays assist the communication between a source node (node 0) and a destination node (node $N + 1$). Denoted by $h_{p,1}$ and $h_{p,2}$, $p \in [N]$, as the complex channel coefficients in the first and the second hops, respectively. Consider the unite input power constraints and the i.i.d. Gaussian noise of $CN(0, 1)$ at each nodes, the point-to-point link capacity in the diamond network can be written as

$$l_{p,j} = \log(1 + |h_{p,j}|^2), \quad \forall p \in [N], \quad \forall j \in [2]. \quad (4)$$

Following [9], the approximate capacity of the $N$-relay HD Gaussian diamond network $D$ can be written as

$$C_D = \max_{p \in [N]} \sum_{j=1}^{2} x_p C_p$$

$$s.t. \quad 0 \leq x_p \leq 1, \quad \forall p \in [N]$$

$$\sum_{p \in [N]} x_p \frac{C_p}{l_{p,j}} \leq 1, \quad \forall j \in [2], \quad (5)$$

where $x_p$ represents the fraction of time that the $p$-th path is utilized in the network, and $C_p$ is the capacity of $p$-th path, given by $C_p = \frac{l_{p,1} \cdot l_{p,2}}{l_{p,1} + l_{p,2}}$. As has been proved in [9], the approximate capacity $C_D$ of such network $D$ can always be achieved by activating at most 3 relays, independently of $N$.

**C. Network stability and end-to-end delay**

We say that a network is stable for a source arrival rate $A_0$ if there exists a transmission strategy such that the average backlog of all queues is finite [10]. In this paper, we assume that the network operates in slotted time, denoted by $t \geq 0$. Considering a first-in-first-out (FIFO) system, we assume that only the packets currently in node $i$ at the beginning of slot $t$ can be transmitted during that slot. Denoted by $A_i(t)$ and $D_i(t)$ as the number of arrival packets and the number of departure packets at node $i$, respectively. The slot-to-slot dynamics of the queuing backlog $U_i(t)$ satisfies the following:

$$U_i(t + 1) = \max \left\{ U_i(t) - D_i(t), 0 \right\} + A_i(t) \quad (6)$$

To evaluate the network stability under a specific scheduler, we define the network average sum backlog as

$$U = \sum_{i=0}^{N} \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} U_i(t) = \sum_{i=0}^{N} \bar{U}_i \quad (7)$$

where $\bar{U}_i = \lim_{T \to \infty} \frac{1}{T} \sum_{t=0}^{T-1} U_i(t)$ denotes the time average backlog in the queue of node $i$ and $U_{N+1}(t) = 0$.

Note that for a feasible source input $A_0 = \frac{1}{T} \sum_{t=0}^{T-1} A_0(t) < C_L$ w.r.t. $C_D$, a superior beam scheduling method should achieve a smaller average backlog (7) and therefore a smaller end-to-end delay. By Little’s theorem [11], the average delay time $\bar{W}$ that a packet spends in the network reads $\bar{W} = U/A$, where $A$ is the long-term average effective arrival rate with $A = \frac{1}{T} \sum_{t=0}^{T-1} A_0(t) = A_0$. Later by simulations we will evaluate the network stability and the end-to-end delay performance w.r.t. different schedulers.

**III. PROPOSED BEAM SCHEDULING METHODS**

In this section, we will present three beam schedulers which are applicable for both of the line network and the diamond network, only with some minor modifications depending on the specific network characteristics.

**A. The deterministic horizontal continuous edge coloring (HC-EC) scheduler**

1) The HC-EC scheduler for the line network $L$

We consider a running example with $N = 3$ and

$$l_1 = 8, \quad l_2 = 8, \quad l_3 = 12, \quad l_4 = 4, \quad (8)$$

where with a slight abuse of notation, we assume that the link capacity $l_i$ here are in the unit of packet per slot (packet/slot). Let $M$ be a common multiple of $l_i$. We will construct an associate graph $G_L$ w.r.t. the line network $L$, where the set of nodes is the same as in $L$ and each link with capacity $l_i$ is replaced by $n_i$ parallel edges, given by

$$n_i = \frac{M}{l_i}, \quad \forall i \in [N+1]. \quad (9)$$

For the running example, we have

$$M = 24, \quad n_1 = 3, \quad n_2 = 3, \quad n_3 = 2, \quad n_4 = 6. \quad (10)$$
Here $G_{\mathcal{L}}$ is a bipartite graph, since the vertices of $G_{\mathcal{L}}$ can be divided into two disjoint and independent sets with set $1 = \{0, 2, 4, \ldots \}$ and set $2 = \{1, 3, 5, \ldots \}$ such that every edge connects a vertex in set $1$ to one in set $2$. Hence an optimal coloring can be performed to $G_{\mathcal{L}}$ with $\Delta_{\mathcal{L}}$ colors, where $\Delta_{\mathcal{L}}$ is the graph maximum degree, given by $\Delta_{\mathcal{L}} = \max_{i \in [N]} \{ n_i + 1 \}$. In the example we have $\Delta_{\mathcal{L}} = 8$.

Denoted by $C_{\mathcal{L}}^{i}$ as the set of colors assigned to link $l_i$, $\Lambda_{l_i} \in \mathbb{C}^{\Delta_{\mathcal{L}} \times (N+1)}$ as the schedule matrix with each row corresponding to one network state $s$ w.r.t. link $l_1, l_2, \ldots, l_{N+1}$. Each state $s$ has an active time fraction $\omega_s = \frac{1}{\Delta_{\mathcal{L}}}$. The HC-EC scheduler contains $\max_{i \in [N+1]} \{ n_i \}$ loops. Each loop begins from link $1$ until link $N+1$. Over each link $i$, the HC-EC scheduler extract one available color that has the minimum label and then assign this color to one pending edge at link $i$. The underlying intuition is that, any data packets entering into the network $G_{\mathcal{L}}$ will be transmitted to the destination node as soon as possible. For the running example, the coloring result is shown in TABLE I which contains 6 loops. The assignment of the schedule matrix $\Lambda_{l_i}$ for color $1$ is $\Lambda_{l_i}[1,\cdot] = [1, 0, 1, 0]$, for color $2$ is $\Lambda_{l_i}[2,\cdot] = [0, 1, 0, 1]$, and the rest can be done in a similar way, where $\Lambda_{l_i}[n,\cdot] = 1$ indicates that link $i$ at $n$-th network state $s = \Lambda_{l_i}[n,\cdot]$ is activated. Having obtained $\Lambda_{l_i}$, the network scheduler would be a simple deterministic repetition among the $\Delta_{\mathcal{L}}$ states.

2) The HC-EC scheduler for the diamond network $\mathcal{D}$

We consider a running example for the diamond network $\mathcal{D}$ with $N = 4$ and the link capacities (packet/slot) are given by

\[ l_{1,1} = 3, \quad l_{1,2} = 2, \quad l_{3,1} = 3, \quad l_{4,1} = 2, \]
\[ l_{1,2} = 3, \quad l_{2,2} = 3, \quad l_{3,2} = 2, \quad l_{4,2} = 2. \]  \tag{11}

The active time fraction for each path is calculated by (5) with

\[ x_1 = 1, \quad x_2 = 0.5, \quad x_3 = 0.5, \quad x_4 = 0. \]  \tag{12}

We assume that the total number of paths with non-zero active time fraction is denoted by $P$. Following [9] we have $P \leq \min\{3, N\}$. Without loss of generality, we assume that the first $P$ paths are activated with time fraction $x_1 \geq x_2 \geq \cdots \geq x_P$. In the running example, we have $P = 3$. Let $M$ be a common multiple of the $l_{p,j}$ and $x_P$. We will construct an associate graph $G_{\mathcal{D}}$ such that the set of nodes coincide with the $P$ activated nodes in $\mathcal{D}$ and each link with capacity $l_{p,j}$ in $\mathcal{D}$ is replaced by $n_{p,j}$ parallel edges, given by

\[ n_{p,j} = M \cdot x_P \cdot \frac{l_{p,3-j}}{l_{p,1} + l_{p,2}} \]  \tag{13}

For the running example, we have $M = 10$ and

\[ n_{1,1} = 5, \quad n_{2,1} = 3, \quad n_{3,1} = 2, \]
\[ n_{1,2} = 5, \quad n_{2,2} = 2, \quad n_{3,2} = 3. \]  \tag{14}

Again $G_{\mathcal{D}}$ is a bipartite graph with two disjoint vertex sets, i.e., set $1 = \{0, N + 1\}$ and set $2 = \{1, 2, 3, \ldots, N\}$, such that every edge connects a vertex in set $1$ to one in set $2$. Now we can perform an optimal coloring with $\Delta_{\mathcal{D}}$ colors, where $\Delta_{\mathcal{D}} = \max_{p \in [P]} \{ \sum_{j \in [P]} n_{p,j} \}$, $\sum_{p \in [P]} n_{p,2} = n_{p,1} + n_{p,2}$ is the graph maximum degree and $\Delta_{\mathcal{D}} = 10$ in the running example.

Let $C_{\mathcal{D}}^{j}$ denote the set of colors assigned to link $l_{p,j}$, $\Lambda_{l_i} \in \mathbb{C}^{\Delta_{\mathcal{D}} \times (2P)}$ denote the schedule with each row corresponding to one network state $s$ w.r.t. link $l_1, l_2, l_3, \ldots, l_{P,2}$. From now on, we will ignore the non-active nodes and paths. Each state $s$ has an activated fraction of time $\omega_s = \frac{1}{\Delta_{\mathcal{D}}}$. The HC-EC scheduler starts from path $1$ until path $P$. Over each path, the coloring procedure is exactly the same as for the line network in Section III-A1. TABLE II illustrates the coloring result. The assignment of $\Lambda_{l_i}$ for color $1$ is $\Lambda_{l_i}[1,\cdot] = [1, 0, 0, 1, 0, 0]$, for color $2$ is $\Lambda_{l_i}[2,\cdot] = [0, 1, 1, 0, 0, 0]$, and the rest can be done in a similar fashion.

**Remark 1.** Note that for the HD 1-2-1 diamond network with $P = 3$ activated paths, the HC-EC scheduler is applicable only if the network $\mathcal{D}$ satisfies either $n_{3,1} \leq \max\{n_{1,2} - n_{2,1}, 0\} + \max\{n_{2,2} - n_{1,1}, 0\}$ or $n_{3,2} \leq \max\{n_{1,1} - n_{2,2}, 0\} + \max\{n_{2,1} - n_{1,2}, 0\}$. Otherwise, the coloring procedure stays the same for the first 2 paths, but for the 3-rd path, the scheduler should exchange $\min\{n_{3,1}, n_{3,2}\}$ colors between the remaining available colors for path 3 and the union of already assigned colors for path 1 and path 2 at the same hop.

### B. The adaptive back pressure (BP) scheduler

The above HC-EC scheduler is rather simple, since once the schedule matrix $\Lambda_{l_i}$ ($\Lambda_{\mathcal{D}}$) is obtained, the network activate states become deterministic. The scheduler just needs to periodically repeat the network states decided by $\Lambda_{l_i}$ ($\Lambda_{\mathcal{D}}$). However, since the HC-EC scheduler is one-time predetermined by the network link capacities, the scheduler cannot handle seamlessly variations in the link capacities. As an alternative approach, we can consider “online” dynamic scheduling policies that are guaranteed to achieve stability for all $\lambda_0 < C_{\mathcal{L}}$ (w.r.t. $\lambda_0 < C_{\mathcal{D}}$) without knowing explicitly the link capacities, and therefore can potentially adapt to (sufficiently slow) variations of the link capacities. In particular, we consider the well-known back pressure (BP) algorithm [10] which is well understood to stabilize the network whenever the input rate lies within the capacity region of the network.

1) The BP scheduler for the line network $\mathcal{L}$

Let $\Lambda_{l_i}(t) \in \mathbb{C}^{N+1}$ denote the scheduling decision at slot $t$. We define the differential backlog weight matrix $W(t) \in \mathbb{C}^{N+1}$ with elements given by

\[ W(t)_i = \max\{U_{i-1}(t) - U_i(t), 0\}, \quad i \in [N + 1]. \]  \tag{15}
Then choose the scheduling matrix \( \Lambda_L(t) \) as the solution of the following binary integer programming (BIP)

\[
\Lambda_L(t) = \arg \max_{i=1}^{N+1} \sum_i W(t)_{[i]} \cdot r_i(t)
\]

\[
s.t. \quad r_i(t) = \bar{r}_i(t) \cdot \Lambda_L(t)_{[i]}
\]

\[
\bar{r}_i(t) = \min \{ U_{i-1}(t), l_i \}
\]

\[
\Lambda_L(t)_{[i]} \in \{0, 1\}
\]

\[
\Lambda_L(t)_{[i,j]} + \Lambda_L(t)_{[j+1]} \leq 1, j \in [N],
\]

where (17) indicates the actual transmit rate for link \( i \), (18) indicates that each link rate at slot \( t \) should not exceed the current backlog of the last departure node, (19) is the binary scheduling decision and (20) indicates the HD operating mode.

Then the slot-to-slot queuing evolution is given by (6) with \( D_i(t) = r_{i+1}(t) \) and \( A_i(t) = r_{i-1}(t) \). At node 0 the number of arrival packets is the source input.

2) The BP scheduler for the diamond network \( \mathcal{D} \)

For the diamond network, let \( \Lambda_D(t) \in \mathbb{C}^{N \times 2} \) denote the scheduling decision at slot \( t \), the differential backlog weight matrix \( W(t) \in \mathbb{C}^{N \times 2} \) is given by

\[
W(t)_{[i,1]} = \max \{ U_0(t) - U_i(t), 0 \}, \quad W(t)_{[i,2]} = U_i(t).
\]

Then choose the scheduling matrix \( \Lambda_D(t) \) as the solution of the following BIP optimization problem

\[
\Lambda_D(t) = \arg \max_{i=1}^{N} \sum_i \sum_{j=1}^{2} W(t)_{[i,j]} \cdot r_{i,j}(t)
\]

\[
s.t. \quad r_{i,j}(t) = \bar{r}_{i,j}(t) \cdot \Lambda_D(t)_{[i,j]}
\]

\[
\bar{r}_{i,1}(t) = \min \{ U_0(t), l_{i,1} \}
\]

\[
\bar{r}_{i,2}(t) = \min \{ U_i(t), l_{i,2} \}
\]

\[
\Lambda_D(t)_{[i,j]} \in \{0, 1\}
\]

\[
\| \Lambda_D(t)_{[i,j]} \|_1 \leq 1, i \in [N],
\]

\[
\| \Lambda_D(t)_{[i,j]} \|_1 \leq 1, j \in [2],
\]

where (23), (24), (25), (26), (27) have the same meaning correspondingly as (17), (18), (19), (20). (28) indicates that the source (destination) node can only point its beam to one relay node in order to achieve the full beamforming gain [4].

Then the queuing evolution is given by (6) with \( D_i(t) = r_{i,2}(t) \) and \( A_i(t) = r_{i,1}(t), i \in [N] \). At node 0, \( A_0(t) \) is the source input and \( D_0(t) = \sum_{i=1}^{N} r_{i,1}(t) \).

Remark 2. According to Edmonds’ matching polytope theorem [12], one can easily prove that the BIP problem in (16) (w.r.t. (22)) are identical to the Matching polytope. Namely we can replace the binary constraint (19) and (26) with the linear constraint \( \Lambda_L(t)_{[i,j]} \in [0, 1] \) and \( \Lambda_D(t)_{[i,j]} \in [0, 1] \), respectively, such that the BIP problem simply boils down to a linear programming with all the extreme points are binary integral.

C. The adaptive low-delay back pressure (newBP) scheduler

Several variants of BP have appeared in the recent literature [13, 14] with the goal of improving the delay properties of the basic BP scheme. In this section, we present the newBP beam scheduler, which is based on the state-of-the-art new BP algorithm proposed in [13]. In this new BP algorithm, the “back pressure” quantities are with respect to virtual queue lengths aiming at reducing the implementation complexity for large networks. Moreover, the new BP algorithm involves a quadratic term for a further reduction of the end-to-end delay.

1) The newBP scheduler for the line network \( \mathcal{L} \)

Choose parameters \( \rho > 0, \tau > 0 \) and \( \beta_{ij} > 0 \), with \( j \in [N+1] \). Let \( U_i(t) \) and \( V_i(t) \) denote the physical and virtual queues (backlog), respectively, which are empty at the initial state \( U_0(0) = V_0(0) = V_1(-1) = 0 \). At the beginning of each slot \( t \), calculate the new weight

\[
z_i(t) = (1 + \frac{1}{\tau}) V_i(t-1) - \frac{1}{\tau} V_i(t-2),
\]

where for the destination node, we have \( z_{N+1}(t) \equiv 0 \). Denoted by \( \bar{W}(t) \in \mathbb{C}^{N+1} \) as the new differential backlog weight matrix with elements given by

\[
\bar{W}(t)_{[i,j]} = z_{i-1}(t) - z_i(t).
\]

Then choose the scheduling matrix \( \Lambda_L(t) \) as the solution of the following BIP problem

\[
\Lambda_L(t) = \arg \max_{i=1}^{N+1} \sum_i \bar{W}(t)_{[i]} r_i(t) - \frac{\beta_{i,j}}{2} [r_i(t) - r_i(t-1)]^2
\]

\[
s.t. \quad (17), (18), (19), (20),
\]

Then update the virtual queue length \( V_i(t) \) by

\[
V_i(t) = V_i(t-1) - \rho \tau D_i(t) + \rho \tau A_i(t),
\]

where \( D_i(t) = r_{i+1}(t) \), \( A_i(t) = r_{i-1}(t) \) and at node 0 the number of arrival packets is the source input. The evolution of the physical queue \( U_i(t) \) is given by (6).

2) The newBP scheduler for the diamond network \( \mathcal{D} \)

For the diamond network \( \mathcal{D} \), choose parameters \( \rho > 0, \tau > 0 \) and \( \beta_{ij} > 0 \), with \( i \in [N] \) and \( j \in [2] \). Denoted by \( U_i(t) \) and \( V_i(t) \) as the physical and virtual queues (backlog), respectively, and let \( U_i(t) \) and \( V_i(t) \) be empty at the initial state \( U_i(0) = V_i(0) = V_1(-1) = 0 \). At the beginning of each slot \( t \), calculate the new weights

\[
z_i(t) = (1 + \frac{1}{\tau}) V_i(t-1) - \frac{1}{\tau} V_i(t-2),
\]

where for the destination node, we have \( z_{N+1}(t) \equiv 0 \). Denoted by \( \bar{W}(t) \in \mathbb{C}^{N \times 2} \) as the new differential backlog weight matrix with elements given by

\[
\bar{W}(t)_{[i,1]} = z_0(t) - z_i(t), \quad \bar{W}(t)_{[i,2]} = z_i(t)
\]

for the first and the second hops, respectively. Then choose the scheduling matrix \( \Lambda_D(t) \) as the solution of the following BIP optimization problem

\[
\Lambda_D(t) = \arg \max_{i \in [N], j \in [2]} \sum \bar{W}(t)_{[i,j]} r_{i,j}(t) - \frac{\beta_{ij}}{2} [r_{i,j}(t) - r_{i,j}(t-1)]^2
\]
Fig. 2: The averaged sum backlog \( U \) w.r.t. different source input rates \( \bar{A}_0 \) for (a) the line network \( L \), (b) the diamond network \( D \).

![Graph showing averaged sum backlog](image)

Fig. 3: The end-to-end delay comparison of the proposed three beam schedulers for the line network \( L \).

![Graph showing end-to-end delay](image)

IV. NUMERICAL RESULTS

In this section, we investigate the numerical performance of the proposed HC-EC, BP and newBP beam schedulers. We consider the same running examples as in Section III-A. The information theoretic capacities for the line network \( L \) and the diamond network \( D \) are given by (3) and (5) with \( C_L = 3 \) and \( C_D = 2.7 \), respectively. The number of packets arriving at the source node obey the Poisson distribution with mean \( \bar{A}_0 \).

A. The evaluation of network stability

For the line network \( L \) as shown in Fig. 2 (a), within the network capacity range \([0,\bar{C}_L] \) (as indicated by the vertical dashed line), all the physical queue lengths w.r.t. the proposed three schedulers (i.e., HC-EC, BP and newBP) are finite, which indicates the guarantee for the network stability. In particular, the average sum backlog of the proposed schedulers are significantly smaller than that achieved by the original EC scheduler in [8] for rates \( \bar{A}_0 \) not too close to \( \bar{C}_L \).

To the best of our knowledge, there are no reference baseline schedulers for the diamond or similar mmWave relay networks in the literature. Therefore, we can only present simulations comparing our proposed approaches. As shown in Fig. 2 (b), within the network capacity range \([0,\bar{C}_D] \), the average backlog with the proposed schedulers are very similar, concluding that for the diamond 1-2-1 network these methods are quite equivalent and they all achieve stability for all \( \bar{A}_0 < \bar{C}_D \), in accordance with the theory.

B. The evaluation of end-to-end delay

For the line network \( L \) as shown in Fig. 3 (a), with a low source input rate \( \bar{A}_0 = 2.5 \), the packets with the newBP scheduler experience the smallest delays, followed by the BP and the HC-EC. When \( \bar{A}_0 \) is close to the network capacity \( \bar{A}_0 = 2.97 \) as in Fig. 3 (b), the schedulers perform very similar with the HC-EC slightly superior than the others. Very similar performances are shown in Fig. 4 for the diamond network \( D \).

Note that, each of the proposed three schedulers has its own advantages in terms of different implementation regards. From the computation complexity point of view, the HC-EC scheduler is preferred since it is one-time computation and then repetition, which results in less adaptivity to channel changes. The computation complexity of the adaptive BP scheduler is slightly less than the adaptive newBP scheduler since the former is identical to the Matching polytope and can be solved by a simple linear programming. Regarding the packet end-to-end delay, in low source input rate range, the newBP scheduler performance better than the other two, while near to the network capacity, the HC-EC scheduler outperforms the others.

V. CONCLUSION

In this paper, we focused on two typical relay models, i.e., the line network and the diamond network, which provides the basic topology for more general mmWave relay networks. We proposed three beam schedulers to approach the network information theoretic capacity, i.e., the deterministic horizontal continuous edge coloring (HC-EC) scheduler, the adaptive back pressure (BP) scheduler and the adaptive low-delay new back pressure (newBP) scheduler. Within the network capacity range, all the schedulers can guarantee the network stability and achieve very low packet delay. Particularly, we have shown that each scheduler has its own advantages in terms of different implementation regards.
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