Insertion of a Contra-Continuous Function between two Comparable Contra-Precontinuous (Contra-Semi–Continuous) Functions

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Abstract: A necessary and sufficient condition in terms of lower cut sets are given for the insertion of a contra-continuous function between two comparable real-valued functions on such topological spaces that kernel of sets are open.

Keywords: Insertion, Strong binary relation, Semi-open set, Preopen set, Contracontinuous function, Lower cut set.

1. INTRODUCTION

The concept of a preopen set in a topological space was introduced by H.H. Corson and E. Michael in 1964 [4]. A subset $A$ of a topological space $(X,\tau)$ is called preopen or locally dense or nearly open if $A \subseteq \text{Int} (\text{Cl}(A))$. A set $A$ is called preclosed if its complement is preopen or equivalently if $\text{Cl} (\text{Int}(A)) \subseteq A$. The term preopen, was used for the first time by A.S. Mashhour, M.E. Abd El Monsef and S.N. El-Deeb [20], while the concept of a , locally dense, set was introduced by H.H. Corson and E. Michael [4].

The concept of a semi-open set in a topological space was introduced by N. Levine in 1963 [17]. A subset $A$ of a topological space $(X,\tau)$ is called semiopen [10] if $A \subseteq \text{Cl} (\text{Int}(A))$. A set $A$ is called semi-closed if its complement is semi-open or equivalently if $\text{Int}(\text{Cl}(A)) \subseteq A$.

A generalized class of closed sets was considered by Maki in [19]. He investigated the sets that can be represented as union of closed sets and called them V−sets. Complements of V−sets, i.e., sets that are intersection of open sets are called A−sets [19].

Recall that a real-valued function $f$ defined on a topological space $X$ is called $A$−continuous [24] if the preimage of every open subset of $R$ belongs to $A$, where $A$ is a collection of subsets of $X$. Most of the definitions of function used throughout this paper are consequences of the definition of $A$−continuity. However, for unknown concepts the reader may refer to [5, 11]. In the recent literature many topologists had focused their research in the direction of investigating different types of generalized continuity.

J. Dontchev in [6] introduced a new class of mappings called contracontinuity. A good number of researchers have also initiated different types of contra-continuous like mappings in the papers [1, 3, 8, 9, 10, 12, 13, 23]. Hence, a real-valued function $f$ defined on a topological space $X$ is called contra-continuous (resp. contra-semi–continuous , contra-precontinuous) if the preimage of every open subset of $R$ is closed (resp. semi–closed , preclosed) in $X$[6].

Results of Kat’etov [14, 15] concerning binary relations and the concept of an indefinite lower cut set for a real-valued function, which is due to Brooks [2], are used in order to give a necessary and sufficient conditions for the insertion of a contra-continuous function between two comparable realvalued functions on such topological spaces that $A$−sets or kernel of sets are open [19].

If $g$ and $f$ are real-valued functions defined on a space $X$, we write $g \leq f$ (resp. $g < f$) in case $g(x) \leq f(x)$ (resp. $g(x) < f(x)$) for all $x$ in $X$. 




The following definitions are modifications of conditions considered in [16].

A property $P$ defined relative to a real-valued function on a topological space is a \textit{cc–property} provided that any constant function has property $P$ and provided that the sum of a function with property $P$ and any contracontinuous function also has property $P$. If $P_1$ and $P_2$ are cc–properties, the following terminology is used:(i) A space $X$ has the weak cc–insertion property for $(P_1,P_2)$ if and only if for any functions $g$ and $f$ on $X$ such that $g \leq f$,$g$ has property $P_1$ and $f$ has property $P_2$, then there exists a contra-continuous function $h$ such that $g \leq h \leq f$.(ii) A space $X$ has the cc–insertion property for $(P_1,P_2)$ if and only if for any functions $g$ and $f$ on $X$ such that $g < f$,$g$ has property $P_1$ and $f$ has property $P_2$, then there exists a contra-continuous function $h$ such that $g < h < f$.(iii) A space $X$ has the weakly cc–insertion property for $(P_1,P_2)$ if and only if for any functions $g$ and $f$ on $X$ such that $g < f$,$g$ has property $P_1$ and $f$ has property $P_2$, then there exists a contra-continuous function $h$ such that $g < h < f$.

In this paper, for a topological space whose $\Lambda$–sets or kernel of sets are open, is given a sufficient condition for the weak cc–insertion property. Also for a space with the weak cc–insertion property, we give a necessary and sufficient condition for the space to have the cc–insertion property. Several insertion theorems are obtained as corollaries of these results.

2. THE MAIN RESULT

Before giving a sufficient condition for insertability of a contra-continuous function, the necessary definitions and terminology are stated. \textbf{Definition 2.1.} Let $A$ be a subset of a topological space $(X,\tau)$. We define the subsets $A^A$ and $A^V$ as follows:

\[ A^A = \cap \{O: O \supseteq A, O \in (X,\tau)\} \text{ and } A^V = \cup \{F: F \subseteq A, F^c \in (X,\tau)\}. \]

In [7, 18, 22], $A^A$ is called the kernel of $A$.

The family of all preopen, preclosed, \textit{semi}–open and \textit{semi}–closed will be denoted by $pO(X,\tau)$, $pC(X,\tau)$, $sO(X,\tau)$, and $sC(X,\tau)$, respectively.

We define the subsets $p(A^A), p(A^V), s(A^A)$ and $s(A^V)$ as follows: $p(A^A) = \cap \{O: O \supseteq A, O \in pO(X,\tau)\}$, $p(A^V) = \cup \{F: F \subseteq A, F^c \in pC(X,\tau)\}$, $s(A^A) = \cap \{O: O \supseteq A, O \in sO(X,\tau)\}$ and $s(A^V) = \cup \{F: F \subseteq A, F^c \in sC(X,\tau)\}$. $p(A^A)$ (resp. $s(A^A)$) is called the prekernel (resp. \textit{semi} – kernel) of $A$.

The following first two definitions are modifications of conditions considered in [14, 15].

\textbf{Definition 2.2.} If $\rho$ is a binary relation in a set $S$ then $\rho^-$ is defined as follows: $x \rho^- y$ if and only if $y \rho v$ implies $x \rho v$ and $u \rho x$ implies $u \rho y$ for any $u$ and $v$ in $S$.

\textbf{Definition 2.3.} A binary relation $\rho$ in the power set $P(X)$ of a topological space $X$ is called a \textit{strong binary relation} in $P(X)$ in case $\rho$ satisfies each of the following conditions:

- If $A, \rho B$, for any $i \in \{1,\ldots,m\}$ and any $j \in \{1,\ldots,n\}$, then there exists a set $C$ in $P(X)$ such that $A, \rho C$ and $C, \rho B$ for any $i \in \{1,\ldots,m\}$ and any $j \in \{1,\ldots,n\}$.

- If $A \subseteq B$, then $A \rho^- B$.

- If $A, \rho B$, then $A^A \subseteq B$ and $A \subseteq B^V$.

The concept of a lower indefinite cut set for a real-valued function was defined by Brooks [2] as follows:

\textbf{Definition 2.4.} If $f$ is a real-valued function defined on a space $X$ and if $\{x \in X : f(x) < v\} \subseteq A(f,^-) \subseteq \{x \in X : f(x) \leq v\}$ for a real number $v$, then $A(f,^-)$ is called a \textit{lower indefinite cut set} in the domain of $f$ at the level.

We now give the following main result:

\textbf{Theorem 2.1.} Let $g$ and $f$ be real-valued functions on the topological space $X$, in which kernel sets are open, with $g \leq f$. If there exists a strong binary relation $\rho$ on the power set of $X$ and if there exist lower indefinite cut sets $A(f,t)$ and $A(g,t)$ in the domain of $f$ and $g$ at the level $t$ for each rational number $t$ such that if $t_1 < t_2$ then $A(f,t_1) \rho A(g,t_2)$, then there exists a contra-continuous function $h$ defined on $X$ such that $g \leq h \leq f$.
Proof. Let \( g \) and \( f \) be real-valued functions defined on the \( X \) such that \( g \leq f \). By hypothesis there exists a strong binary relation \( \rho \) on the power set of \( X \) and there exist lower indefinite cut sets \( A(f,t) \) and \( A(g,t) \) in the domain of \( f \) and \( g \) at the level \( t \) for each rational number \( t \) such that if \( t_1 < t_2 \) then \( A(f,t_1) \rho A(g,t_2) \).

Define functions \( f \) and \( G \) mapping the rational numbers \( Q \) into the power set of \( X \) by \( F(t) = A(f,t) \) and \( G(t) = A(g,t) \). If \( t_1 \) and \( t_2 \) are any elements of \( Q \) with \( t_1 < t_2 \), then \( F(t_1) \rho F(t_2), G(t_1) \rho G(t_2), \) and \( F(t_1) \rho G(t_2) \). By Lemmas 1 and 2 of [15] it follows that there exists a function \( H \) mapping \( Q \) into the power set of \( X \) such that if \( t_1 \) and \( t_2 \) are any rational numbers with \( t_1 < t_2 \), then \( F(t_1) \rho G(t_2), H(t_1) \rho H(t_2) \) and \( H(t_1) \rho G(t_2) \). For any \( x \) in \( X \), let \( h(x) = \inf\{ t \in \mathbb{Q} : x \in H(t) \} \).

We first verify that \( g \leq h \leq f \). If \( x \) is in \( H(t) \) then \( x \) is in \( G(t^0) \) for any \( t^0 > t \); since \( x \) is in \( G(t^0) = A(g,t^0) \) implies \( g(x) \leq t^0 \), it follows that \( g(x) \leq t \). Hence \( g \leq h \). If \( x \) is not in \( H(t) \), then \( x \) is not in \( F(t^0) \) for any \( t^0 < t \); since \( x \) is not in \( F(t^0) = A(f,t^0) \) implies \( f(x) > t^0 \), it follows that \( f(x) \geq t \). Hence \( h \leq f \).

Also, for any rational numbers \( t_1 \) and \( t_2 \) with \( t_1 < t_2 \), we have \( h^{-1}(t_1,t_2) = H(t_2) \setminus H(t_1) \). Hence \( h^{-1}(t_1,t_2) \) is closed in \( X \), i.e., \( h \) is a contra-continuous function on \( X \).

The above proof used the technique of theorem 1 in [14].

**Theorem 2.2.** Let \( P_1 \) and \( P_2 \) be \( cc \)–property and \( X \) be a space that satisfies the weak \( cc \)–insertion property for \((P_1,P_2)\). Also assume that \( g \) and \( f \) are functions on \( X \) such that \( g \leq f \). If \( g \) has \( P_1 \) and \( f \) has \( P_2 \), the space \( X \) has the \( cc \)–insertion property for \((P_1,P_2)\) and only if there exist lower cut sets \( A(f - g,3^{−\alpha}) \) and there exists a decreasing sequence \( \{D_n\} \) of subsets of \( X \) with empty intersection and such that for each \( n \), \( X \setminus D_n \) and \( A(f - g,3^{−\alpha}) \) are completely separated by contra-continuous functions.

**Proof.** Theorem 2.1 of [21].

**3. APPLICATIONS**

The abbreviations \( cpc \) and \( csc \) are used for contra-precontinuous and contrasemi–continuous, respectively.

Before stating the consequences of theorems 2.1, 2.2, we suppose that \( X \) is a topological space whose kernel sets are open.

**Corollary 3.1.** If for each pair of disjoint preopen (resp. semi–open) sets \( G_1,G_2 \) of \( X \), there exist closed sets \( F_1 \) and \( F_2 \) of \( X \) such that \( G_1 \subseteq F_1 \), \( G_2 \subseteq F_2 \) and \( F_1 \cap F_2 = \emptyset \) then \( X \) has the weak \( cc \)–insertion property for \((cpc,cpc)\) (resp. \((csc,csc)\)).

**Proof.** Let \( g \) and \( f \) be real-valued functions defined on \( X \), such that \( f \) and \( g \) are \( cpc \) (resp. \( csc \)), and \( g \leq f \). If a binary relation \( \rho \) is defined by \( A \rho B \) in case \( p(A^\lambda) \subseteq p(B^\lambda) \) (resp. \( s(A^\lambda) \subseteq s(B^\lambda) \)), then by hypothesis \( \rho \) is a strong binary relation in the power set of \( X \). If \( t_1 \) and \( t_2 \) are any elements of \( Q \) with \( t_1 < t_2 \), then \( A(f,t_1) \subseteq \{ x \in X : f(x) \leq t_1 \} \) and \( A(g,t_2) \subseteq \{ x \in X : g(x) < t_2 \} \) are preopen (resp. semi–open) set and since \( \{ x \in X : g(x) < t_2 \} \) is a preclosed (resp. semi–closed) set, it follows that \( p(A(f,t_1)^\lambda) \subseteq p(A(g,t_2)^\lambda) \) (resp. \( s(A(f,t_1)^\lambda) \subseteq s(A(g,t_2)^\lambda) \)). Hence \( t_1 < t_2 \) implies that \( A(f,t_1) \rho A(g,t_2) \). The proof follows from Theorem 2.1.

**Corollary 3.2.** If for each pair of disjoint preopen (resp. semi–open) sets \( G_1,G_2 \) of \( X \), there exist closed sets \( F_1 \) and \( F_2 \) such that \( G_1 \subseteq F_1 \), \( G_2 \subseteq F_2 \) and \( F_1 \cap F_2 = \emptyset \) then every contra-precontinuous (resp. contra-semi–continuous) function is contra-continuous.

**Proof.** Let \( f \) be a real-valued contra-precontinuous (resp. contra-semi–continuous) function defined on \( X \). Set \( g = f \), then by Corollary 3.1, there exists a contracontinuous function \( h \) such that \( g = h = f \).

**Corollary 3.3.** If for each pair of disjoint preopen (resp. semi–open) sets \( G_1,G_2 \) of \( X \), there exist closed sets \( F_1 \) and \( F_2 \) of \( X \) such that \( G_1 \subseteq F_1 \), \( G_2 \subseteq F_2 \) and \( F_1 \cap F_2 = \emptyset \) then \( X \) has the \( cc \)–insertion property for \((cpc,cpc)\).
**Lemma 3.1.** The following conditions on the space $X$ are equivalent:

- For each pair of disjoint subsets $G_1,G_2$ of $X$, such that $G_1$ is preopen and $G_2$ is $semi$–open, there exist closed subsets $F_1,F_2$ of $X$ such that $G_1 \subseteq F_1, G_2 \subseteq F_2$ and $F_1 \cap F_2 = \emptyset$.
- If $G$ is a $semi$–open (resp. preopen) subset of $X$ which is contained in a preclosed (resp. $semi$–closed) subset $F$ of $X$, then there exists a closed subset $H$ of $X$ such that $G \subseteq H \subseteq F^\Lambda \subseteq F$.

**Proof.** (i) $\Rightarrow$ (ii) Suppose that $G \subseteq F$, where $G$ and $F$ are $semi$–open (resp. preopen) and preclosed (resp. $semi$–closed) subsets of $X$, respectively. Hence, $F$ is a preopen (resp. $semi$–open) and $G \cap F = \emptyset$.

By (i) there exists two disjoint closed subsets $F_1,F_2$ such that $G \subseteq F_1$ and $F \subseteq F_2$. But $F^c \subseteq F_2 \Rightarrow F_2^c \subseteq F$, and

$$F_1 \cap F_2 = \emptyset \Rightarrow F_1 \subseteq F_2^c$$

hence

$$G \subseteq F_1 \subseteq F_2^c \subseteq F$$

and since $F_2^c$ is an open subset containing $F_1$, we conclude that

$$F_1^\Lambda \subseteq F_2^c \text{ i.e., } G \subseteq F_1 \subseteq F_1^\Lambda \subseteq F.$$ 

By setting $H = F_1$, condition (ii) holds.

(ii) $\Rightarrow$ (i) Suppose that $G_1,G_2$ are two disjoint subsets of $X$, such that $G_1$ is preopen and $G_2$ is $semi$–open. This implies that $G_2^c \subseteq G_1^c$ and $G_1^c$ is a preclosed subset of $X$. Hence by (ii) there exists a closed set $H$ such that

$$G_2 \subseteq H \subseteq F^\Lambda \subseteq G_1^c.$$ 

But $H \subseteq F^\Lambda \Rightarrow H \cap (H^\Lambda)^c = \emptyset$

and

$$H^\Lambda \subseteq G_1^c \Rightarrow G_1 \subseteq (H^\Lambda)^c.$$ 

Furthermore, $(H^\Lambda)^c$ is a closed subset of $X$. Hence $G_2 \subseteq H,G_1 \subseteq (H^\Lambda)^c$ and $H \cap (H^\Lambda)^c = \emptyset$. This means that condition (i) holds.
**Lemma 3.2.** Suppose that $X$ is a topological space. If each pair of disjoint subsets $G_1, G_2$ of $X$, where $G_1$ is preopen and $G_2$ is semi–open, can be separated by closed subsets of $X$ then there exists a contra-continuous function $h : X \rightarrow [0,1]$ such that $h(G_2) = \{0\}$ and $h(G_1) = \{1\}$.

**Proof.** Suppose $G_1$ and $G_2$ are two disjoint subsets of $X$, where $G_1$ is preopen and $G_2$ is semi–open. Since $G_1 \cap G_2 = \emptyset$, hence $G_2 \subseteq G'^{c}_1$. In particular, since $G'^{c}_1$ is a preclosed subset of $X$ containing the semi–open subset $G_2$ of $X$, by Lemma 3.1, there exists a closed subset $H_{1/2}$ such that $G_2 \subseteq H_{1/2} \subseteq H^{\Lambda}_{1/2} \subseteq G'^{c}_1$.

Note that $H_{1/2}$ is also a preclosed subset of $X$ and contains $G_2$, all $G'^{c}_1$ is a preclosed subset of $X$ and contains the semi–open subset $H_{1/2}^{\Lambda}$ of $X$. Hence, by Lemma 3.1, there exists closed subsets $H_{3/4}$ and $H_{3/4}$ such that

$$G_2 \subseteq H_{3/4} \subseteq H^{\Lambda}_{3/4} \subseteq H_{3/4}^{\Lambda} \subseteq G'^{c}_1$$

By continuing this method for every $t \in D$, where $D \subseteq [0,1]$ is the set of rational numbers that their denominators are exponents of 2, we obtain closed subsets $H_t$ with the property that if $t_1, t_2 \in D$ and $t_1 < t_2$, then $H_{t_1} \subseteq H_{t_2}$. We define the function $h$ on $X$ by $h(x) = \inf \{t : x \in H_t\}$ for $x \in G_1$ and $h(x) = 1$ for $x \in G_1$.

Note that for every $x \in X, 0 \leq h(x) \leq 1$, i.e., $h$ maps $X$ into $[0,1]$. Also, we note that for any $t \in D, G_2 \subseteq H^t$; hence $h(G_2) = \{0\}$. Furthermore, by definition, $h(G_1) = \{1\}$. It remains only to prove that $h$ is a contra-continuous function on $X$. For every $a \in R$, we have if $\alpha \leq 0$ then $\{x \in X : h(x) < \alpha\} = \emptyset$ and if $0 < \alpha$ then $\{x \in X : h(x) < \alpha\} = \bigcup \{H_t : t < \alpha\}$, hence, they are closed subsets of $X$. Similarly, if $\alpha < 0$ then $\{x \in X : h(x) > \alpha\} = X$ and if $0 \leq \alpha$ then $\{x \in X : h(x) > \alpha\} = \bigcup \{(H^t)^c : t \geq \alpha\}$, hence, every of them is a closed subset. Consequently $h$ is a contra-continuous function.

**Lemma 3.3.** Suppose that $X$ is a topological space such that every two disjoint semi–open and preopen subsets of $X$ can be separated by closed subsets of $X$. The following conditions are equivalent:

- Every countable covering of semi–closed (resp. preclosed) subsets of $X$ has a refinement consisting of preclosed (resp. semi–closed) subsets of $X$ such that for every $x \in X$, there exists a closed subset of $X$ containing $x$ such that it intersects only finitely many members of the refinement.
- Corresponding to every decreasing sequence $\{G_n\}$ of semi–open (resp. preopen) subsets of $X$ with empty intersection there exists a decreasing sequence $\{F_n\}$ of preclosed (resp. semi–closed) subsets of $X$ such that $\bigcap_{n=1}^{\infty} F_n = \emptyset$ and for every $n \in \mathbb{N}, G_n \subseteq F_n$.

**Proof.** (i) $\Rightarrow$ (ii) Suppose that $\{G_n\}$ is a decreasing sequence of semi–open (resp. preopen) subsets of $X$ with empty intersection. Then $\{G'^{c}_n : n \in \mathbb{N}\}$ is a countable covering of semi–closed (resp. preclosed) subsets of $X$. By hypothesis (i), we have a countable covering of semi–closed (resp. preclosed) subsets of $X$. By Lemma 3.1, there exists a refinement $\{V_n : n \in \mathbb{N}\}$ such that every $V_n$ is a closed subset of $X$ and $V^{\Lambda}_n \subseteq G'^{c}_n$. By setting $F_n = (V^{\Lambda}_n)^c$, we obtain a decreasing sequence of closed subsets of $X$ with the required properties.

(ii) $\Rightarrow$ (i) Now if $\{H_n : n \in \mathbb{N}\}$ is a countable covering of semi–closed (resp. preclosed) subsets of $X$, we set $G_n = (\bigcup_{i=1}^{n} H_i)$ for $n \in \mathbb{N}$. Then $\{G_n\}$ is a decreasing sequence of semi–open (resp. preopen) subsets of $X$ with empty intersection. By (ii) there exists a decreasing sequence $\{F_n\}$ consisting of preclosed (resp. semi–closed) subsets of $X$ such that $\bigcap_{n=1}^{\infty} F_n = \emptyset$ and for every $n \in \mathbb{N}, G_n \subseteq F_n$. Now we define the subsets $W_n$ of $X$ in the following manner:

- $W_1$ is a closed subset of $X$ such that $F'^{c}_1 \subseteq W_1$ and $W^{\Lambda}_1 \cap G_1 = \emptyset$.
- $W_2$ is a closed subset of $X$ such that $W^{\Lambda}_1 \cap F'^{c}_2 \subseteq W_2$ and $W^{\Lambda}_2 \cap G_2 = \emptyset$, and so on. (By Lemma 3.1, $W_n$ exists).

Then since $\{F'^{c}_n : n \in \mathbb{N}\}$ is a covering for $X$, hence $\{W_n : n \in \mathbb{N}\}$ is a covering for $X$ consisting of closed sets. Moreover, we have
Now by Lemma 3.3, there exists a decreasing \((csc,cpc)\) of \(D\) and \(3\), i.e.,
\[
A(3,3-1) = \{x \in X: (f-g)(x) \leq 3-1\}.
\]
Since \(f-g\) is \((csc,cpc)\), hence \(A(f-g,3-1)\) is a \((semi)\)–open (resp. preopen) subset of \(X\). Consequently, \(A(f-g,3-1)\) is a decreasing sequence of \((semi)\)–open (resp. preopen) subsets of \(X\) and furthermore since
\[
0 < f-g, \text{ it follows that } \bigcap_{n=1}^{\infty} A(f-g,3-n-1) = \emptyset.
\]
Now by Lemma 3.3, there exists a decreasing sequence \(\{D_n\}\) of \((semi)\)–open (resp. \(csc,cpc)\) subsets of \(X\) such that \(A(f-g,3-n-1) \subseteq D_n\) and \(\bigcap_{n=1}^{\infty} D_n = \emptyset\). But by Lemma 3.2, the pair \(A(f-g,3-n-1)\) and \(X\backslash D_n\) of \((semi)\)–open (resp. preopen) and preopen (resp. \(csc,cpc)\) subsets of \(X\) can be completely separated by contra-continuous functions. Hence by Theorem 2.2, there exists a contracontinuous function \(h\) defined on \(X\) such that \(0 < h < f\), i.e., \(X\) has the weakly \((csc,cpc)\).

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