Development of a method for calculating time and solidification coefficient of castings in sand-clay molds

V B Deev1,2,3, O G Prikhodko3, E S Prusov4, E V Protopopov3, M V Temlyantsev3, A I Kutsenko3, Mei Shunqi1, E Kh Ri5, S V Smetanyuk6, K V Ponomareva3 and G N Gavrilov7
1Wuhan Textile University, Wuhan, China
2National Research Technological University “MISiS”, Moscow, Russia
3Siberian State Industrial University, Novokuznetsk, Russia
4Vladimir State University n.a. Alexander and Nikolay Stoletovs, Vladimir, Russia
5Pacific State University, Khabarovsk, Russia
6LLC Litmash, Shuya, Russia
7Nizhny Novgorod State Technical University n.a. R.E. Alekseev, Nizhny Novgorod, Russia
E-mail: deev.vb@mail.ru

Abstract. The mathematical model for calculation of the coefficient and solidification time of castings in sandy-clay form was developed. It takes into account the basic thermophysical parameters of the casting metal and mold material, heat transfer conditions at the crystallization front, at the ‘casting-mold’ interface and on the mold surface.

1. Introduction
In traditional foundry practice, it is customary to describe the solidification process of castings according to the square root law, which determines the thickness of the hardened rim:

$$\varepsilon = k \sqrt{\tau},$$

(1)

where $k$ is a constant value that received the name of the solidification constant in relation to foundry processes.

However, the square root law cannot be considered as a universal formula for calculating the solidification of castings and ingots. The assumptions of the authors who investigated this problem are reduced to the possibility to neglect the inhomogeneity of the temperature field in the casting and to take the form by the semi-bounded body having tight contact with the casting.

Thus, the use of the square root law in the analysis of real hardening processes is possible by adjusting the hardening coefficient, the value of which for different authors, even for the same mold materials and castings, varies widely (for example, [1-3], etc.). Therefore, the calculations performed according to the square root law often do not coincide with the actual experimental data.

During crystallization, the thermophysical parameters of the casting materials and molds are constantly changing and they must be taken into account when studying the process of solidification and crystallization of the casting [4-6]. A number of thermophysical parameters for calculation according to existing models are either taken as constant values or given by a piecewise constant
function, which is not true. The regression models proposed in the literature give true results about the process under study only for a certain type of castings and under certain technological limitations. In this regard, the search for directions for improving methods of calculating various indicators of castings solidification process is an urgent task in the field of foundry.

The purpose of the work is to develop a methodology for calculating the time and solidification coefficient of castings in sandy-clay form, taking into account the main thermophysical parameters of the casting metal and mold material, heat transfer conditions at the crystallization front, at the ‘casting-mold’ interface and on the mold surface. This work is the continuation of a many years of research in the field of alloys crystallization and castings solidification modeling, performed under the guidance of Prof. I.F. Selyanin [7, 8].

2. Methods

Figure 1 presents a diagram of the temperature distribution in the mold and hardened casting. We describe the temperature distribution curve in the casting section in the form of an $n$th-order parabola [9]:

$$
\frac{TCR - TM}{TCR - TC} = \left(\frac{y}{\varepsilon}\right)^n = \left(1 - \frac{x}{\varepsilon}\right)^n,
$$

(2)

where $TCR$ is the crystallization temperature of the alloy (for alloys crystallizing in the temperature range – liquidus temperature), K; $T_C$ — contact temperature at the “cast-mold” interface, K; $TM$ is the temperature of the hardened metal at time $\tau$ at the point with coordinate $y$, K; $y$ is the coordinate measured from the solidification front in the direction of the surface of the casting, m; $\varepsilon$ is the thickness of the hardened rim, m; $x$ is the coordinate, measured from the surface of the casting in the direction of the solidification front, m

In the formulation of the Stefan-Schwartz problem $TC$ at the ‘casting-mold’ boundary remains constant during the entire crystallization period, therefore, a quasistationary approach can be applied to the problem.
The contact temperature $T_k$ at the casting-mold interface is determined by the formula proposed by N.I. Khvorinov [10]:

$$T_C = T_{mo}^0 + \frac{T_{CR} - T_f^0}{1 + b_m},$$

where $T_f^0$ – the initial temperature of the form, K; $b_{mo}$, $b_m$ – heat storage coefficient of the mold and metal, W.s.0.5/(m².K).

$$b_{mo} = \frac{c_{mo} \cdot \rho_{mo} \cdot \lambda_f}{b_m} = \frac{c_m \cdot \rho_m \cdot \lambda_m}{b_m}.$$  

The temperature distribution curve in the cross section of the form will be described in the form of a parabola of $n'$-order

$$\frac{T_{mo}^0 - T_C}{T_{mo}^0 - T_{mo}} = \left(\frac{z}{\delta_{mo}}\right)^{n'},$$

where $T_{mo}$ is the temperature of the mold at time $\tau$, at the point with coordinate $z$, K; $z$ is the coordinate measured from the surface of the casting in the mold, m; $\delta_f$ – thickness of the heated layer of the mold, m. According to [9], the average temperature of the hardened rim $\bar{T}_M$ for an infinite flat casting

$$\bar{T}_M = \frac{T_C - T_{mo}}{n' + 1} + T_{CR},$$

for infinite cylindrical casting

$$\bar{T}_M = \frac{T_C - T_{CR}}{n' + 1} \cdot \frac{1 - \frac{R}{2R}}{1 - \frac{1}{2R}} + T_{CR},$$

for ball casting

$$\bar{T}_M = \frac{T_C - T_{CR}}{n' + 1} \cdot \frac{1 - 2\frac{R}{2R}}{1 - \frac{1}{2R}} + T_{CR},$$

where $R$ is the reduced size of the casting, $R = V_0/F_0$ (where $V_0$ and $F_0$ are the volume and surface area of the casting), the introduction of the reduced size of the casting leads the solidification of the casting to a one-dimensional version.

From (5) follows that the average values of the temperatures of the heated layer of the mold for an infinite flat casting can be expressed as

$$\bar{T}_F = \frac{T_C - T_{mo}^0}{n' + 1} + T_{mo}^0.$$  

For casting of infinite cylinder

$$\bar{T}_F = \frac{T_C - T_{mo}^0}{n' + 1} \cdot \frac{1 + \frac{\delta_{mo} R}{2 R}}{1 + \frac{2 \delta_{mo} R}{2 R}} + T_{mo}^0,$$

for ball casting

$$\bar{T}_{Mo} = \frac{T_C - T_{mo}^0}{n' + 1} \cdot \frac{1 + 2 \delta_{mo} R}{1 + \frac{2 \delta_{mo} R}{2 R}} + T_{mo}^0.$$  

The thickness of the growing rim and the thickness of the heated layer of the form change in the scale of the reduced size; in real scale, their values will be determined as follows:

$$\varepsilon_f = \frac{\varepsilon_l}{R}; \delta_{fl} = \frac{\delta_{mo l}}{R}.$$
where \( l \) is the actual size of the casting in the direction of the heat sink, m.

The heat balance equation during casting crystallization in the sand-clay mold has the following form:

\[
L_C \rho_m F_{CR} \, d \epsilon + c_m \rho_m F_{CR} \, d \epsilon (T_P - T_{CR}) + c_m \rho_m F_{CR} \, d \epsilon (T_{CR} - T_M) = c_{mo} \rho_{mo} F \, d \delta_{mo} (T_{Mo} - T_{mo}^0),
\]

where \( L_C \) – the heat of metal crystallization, J/kg; \( \rho_m \), \( \rho_{mo} \) – density of the alloy and the molding mixture, kg/m\(^3\); \( c_m, c_{mo} \) is the heat capacity of the alloy and the molding mixture, J/(kg .K); \( T_P \) – pouring temperature of the alloy, K; \( F_{CR}, F_{Mo} \) – surface area of the crystallization front and the heated layer of the mold, m\(^2\).

On the left side of equation (13), the heat input is recorded, which is the sum of the heat of crystallization and the heat of superheat, the heat accumulated by the hardened metal rim. On the right side of the equation, heat consumption is the heat spent on heating the mold layer.

To take into account the multidimensionality of the problem, the parameter \( F_{CR} ÷ F_O ÷ F_{Mo} \) was introduced, which takes into account the spreading of the heat flux from the crystallization front (\( F_O \) is the surface area of the casting). For infinite flat casting

\[
F_{CR} = F_O = F_{Mo}. \tag{14}
\]

For the infinite cylinder casting

\[
F_{CR} ÷ F_O ÷ F_{Mo} = \left(1 - \frac{\epsilon}{R}\right) + \frac{1}{1 ÷ \frac{\delta_{mo}}{R}}. \tag{15}
\]

For ball casting and casting of finite sizes of arbitrary shape (plate, cylinder, ball)

\[
F_{CR} ÷ F_O ÷ F_{Mo} = \left(1 - \frac{\epsilon}{R}\right)^2 + \frac{1}{1 ÷ \frac{\delta_{mo}}{R}^2}. \tag{16}
\]

From (2) it follows

\[
d\delta_{mo} = b \frac{F_{CR}}{F_{Mo}} \, d \epsilon, \tag{17}
\]

where \( b \) is a dimensionless coefficient showing how many times for an infinite plate the thickness of the heated layer of the mold \( \delta_{mo} \) is greater than the thickness of the hardened metal rim \( \epsilon \), and

\[
b = \frac{L_C \rho_m + c_m \rho_m (T_P - T_M)}{c_{Mo} \rho_{mo} (T_P - T_{mo}^0)}. \tag{18}
\]

The heat flow from the crystallization front is transferred to form heating through the hardened metal rim and a heated layer of the molding sand. In accordance with this, we can write the following equality

\[
K_H (T_P - T_{mo}^0) F \, d \tau = c_{Mo} \rho_{mo} F_{Mo} d \delta_{mo} (T_{Mo} - T_{mo}^0), \tag{19}
\]

where

\[
\delta_{mo} = b \cdot \epsilon,
\]

\( K_H \) is heat transfer coefficient for a flat multilayer wall, W/(m\(^2\)-K):

\[
K_H = \left(\frac{1}{\alpha_1} + \frac{\epsilon}{\alpha_m} + \frac{1}{\alpha_2} + \frac{\delta_{mo}}{\lambda_{mo}}\right)^{-1},
\]

\( \alpha_1, \alpha_2 \) – heat transfer coefficients at the crystallization front, at the boundary ‘casting-mold’, W/(m\(^2\)-K); \( \lambda_P, \lambda_m \) – the thermal conductivity of the mold and metal, W/(m-K).

The heat transfer coefficient at the crystallization front is determined as follows [1]
\[ \alpha_1 = N_u \cdot \lambda_{lm}/d, \]  

where \( N_u \) – the Nusselt criterion; \( \lambda_{lm} \) – thermal conductivity coefficient of liquid metal, W/(m·K); \( d \) – the reduced characteristic size of the liquid part of the casting, m.

Thus, the differential equation for determining the solidification time of castings in sand-clay molds is

\[ d\tau = B \cdot b \cdot \left( \frac{1}{\alpha_1} + \frac{\varepsilon}{\lambda_{lm}} + \frac{1}{\alpha_2} + \frac{b \varepsilon}{\lambda_f} \right) F_{ER}/F_o \, d\varepsilon, \]  

where \( B \) is the coefficient, J/(m³K);

\[ B = c_{Mo}p_{Mo} \frac{(T_{Mo} - T_{mo})}{(T_f - T_{mo})}. \]  

### 3. Results and discussion

Based on the mathematical transformations carried out, the solidification time of castings in the sand-clay molds can be determined by the formula:

\[ \tau = A_1 \varepsilon + A_2 \varepsilon^2 + A_3 \varepsilon^3 + A_4 \varepsilon^4, \]  

where \( A_1, A_2; A_3; A_4 \) – coefficients determined by the expressions below.

For casting of “infinite plate” type

\[ A_1 = B \cdot b \cdot \left( \frac{1}{\alpha_1} + \frac{1}{\alpha_2} \right), \text{s/m}; \]
\[ A_2 = \frac{b \cdot b}{2} \cdot \left( \frac{1}{\lambda_{lm}} + \frac{1}{\lambda_{mo}} \right), \text{s/m}^2; \]
\[ A_3 = 0; \]
\[ A_4 = 0. \]  

For casting of “infinite cylinder” type

\[ A_1 = B \cdot b \cdot \left( \frac{1}{\alpha_1} + \frac{1}{\alpha_2} \right), \text{s/m}; \]
\[ A_2 = \frac{b \cdot b}{2} \cdot \left( \frac{1}{\lambda_{lm}} + \frac{1}{\lambda_{f}} \right) - \frac{b \cdot b}{2R} \cdot \left( \frac{1}{\alpha_1} + \frac{1}{\alpha_2} \right), \text{s/m}^2; \]
\[ A_3 = \frac{b \cdot b}{3R} \cdot \left( \frac{1}{\lambda_{lm}} + \frac{1}{\lambda_{f}} \right), \text{s/m}^3; \]
\[ A_4 = 0. \]  

For ball casting

\[ A_1 = B \cdot b \cdot \left( \frac{1}{\alpha_1} + \frac{1}{\alpha_2} \right), \text{s/m}; \]
\[ A_2 = \frac{b \cdot b}{2} \cdot \left( \frac{1}{\lambda_{lm}} + \frac{1}{\lambda_{mo}} \right) - \frac{b \cdot b}{R^2} \cdot \left( \frac{1}{\alpha_1} + \frac{1}{\alpha_2} \right), \text{s/m}^2; \]
\[ A_3 = \frac{b \cdot b}{3R} \cdot \left( \frac{1}{\lambda_{lm}} + \frac{1}{\lambda_{mo}} \right) - 2 \left( \frac{1}{\lambda_{lm}} + \frac{b}{\lambda_{f}} \right), \text{s/m}^3; \]
\[ A_4 = \frac{b \cdot b}{4R^2} \cdot \left( \frac{1}{\lambda_{lm}} + \frac{b}{\lambda_{mo}} \right), \text{s/m}^4. \]

Transforming formula (23), we can obtain an expression for determining thickness of the hardened rim during crystallization of castings:

\[ \varepsilon = \sqrt{\frac{A_1}{A_2} + A_3 \varepsilon + A_4 \varepsilon^2} \cdot \sqrt{\tau}. \]  

Formula (27) is equivalent to the square root law (1). From it follows that the solidification constant

\[ K = \frac{1}{\sqrt{A_1 + A_2 \varepsilon + A_3 \varepsilon^2 + A_4 \varepsilon^2}}. \]  

The full solidification time (\( \tau_f \)) with perfect contact at the crystallization front (\( \alpha_1 \rightarrow \infty \)) and at the casting-mold boundary (\( \alpha_2 \rightarrow \infty \)) for casting of any configuration with a given size R can be calculated by the formula
\[ \tau_S = \frac{B_b}{2a} \left( \frac{1}{\lambda_m} + \frac{b}{\lambda_m b} \right) R^2, \]  
\hfill (29)

where \( a \) is a coefficient depending on the configuration of the casting: for an infinite plate \( a = 1 \); for an infinite cylinder \( a = 3 \); for the ball \( a = 6 \).

The coefficient of solidification (\( K \)) in the square root law, respectively, is determined from the expression

\[ K = \frac{2a\lambda_m\lambda_{mo}}{B_b(b\lambda_m+\lambda_{mo})}. \]  
\hfill (30)

When determining the solidification time of a casting with a complex configuration, it is divided into elements, each of which approaches a plate, cylinder and ball. Analysis of the design of castings of the industrial nomenclature shows that flat walls are the predominant of the forming elements, cylinders or prisms are less common, balls or cubes are even less common.

In the initial period of solidification, the thickness of the growing rim is relatively small; therefore, the process can be considered as plate solidification (the rim is considered as a thin plate). Based on the above formulas, the dependence of the thickness of the hardened rim at the surface of the ‘casting-mold’ section (the beginning of the process of solidification of the casting) is linear, since at \( \varepsilon \to 0 \) the most significant member of equation (23) will be \( A_1 \varepsilon \) will be. That is, at \( \varepsilon \to 0, \tau = A_1 \varepsilon. \) At the final stage of solidification, the castings of final dimensions harden as cylinders if their width (or height) is less than their length, or like balls if their width (or height) does not differ significantly from their length. This is due to the dissipation of the heat flow, while the solidification front in the middle part of the hardened casting is a surface of revolution.

The developed mathematical model takes into account the basic thermophysical parameters of the casting metal and the mold material, heat transfer conditions at the crystallization front, at the ‘casting-mold’ interface and on the mold surface, and can be used to determine theoretical values of the coefficient and solidification time of castings in sand-clay mold.

4. Conclusion

A method for calculating the time and solidification coefficient of castings in sand-clay mold was developed. A distinctive feature of the technique is the possibility to take into account both the thermophysical parameters of the casting material and the mold, as well as the heat transfer conditions at the crystallization front, at the ‘casting-mold’ interface and on the mold surface, and can be used to determine theoretical values of the coefficient and solidification time of castings in sand-clay mold.

Acknowledgments

The work was performed within state task of the Ministry of Science and Higher Education of Russia in the field of scientific activity for 2017-2019 “Organization of Scientific Research” (task No.11.5684.2017/6.7).

References

[1] Shrewe H 1989 Continuous Casting of Steel (Germany: Stahleisen)
[2] Mehrara H, Santillana B et al 2011 IOP Conf. Series: MSE 27 012046
[3] Xiao C, Zhang J M et al 2012 Journal of University of Science and Technology Beijing 34(9) 1011-16
[4] Campbell J 2003 Casting (Butterworth-Heinemann, Elsevier)
[5] Dantzig J A and Rappaz M 2009 Solidification (Taylor & Francis Group, CRS Press)
[6] Stefanescu D M 2015 Science and Engineering of Casting Solidification (Springer International Publishing AG Switzerland)
[7] Selyanin I F, Deev V B et al 2015 Russian J. of Non-Ferrous Metals 56(4) 434–436
[8] Deev V B, Selyanin I F et al 2015 Metallurgist 58(11-12) 1123–27
[9] Veinik A I 1960 Theory of Solidification of a Casting (Moscow) 1960
[10] Chvorinov N I 1954 Crystallization and Non-Homogeneity of Steel (Prague: Publishing Czechoslovak Academy of Sciences)