A New Methodology for Estimating Internal Credit Risk and Bankruptcy Prediction under Basel II Regime

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Abstract Credit estimation and bankruptcy prediction methods have utilized Altman’s Z-score method for the last several years. It is reported in many studies that Z-score is sensitive to changes in accounting figures. Researchers have proposed different variations to conventional Z-score that can improve the prediction accuracy. In this paper, we develop a new multivariate nonlinear model for computing the Z-score. In addition, we develop a new credit risk index by fitting a Pearson type 3 distribution to the transformed financial ratios. The results of our study have shown that the new Z-score can predict the bankruptcy with an accuracy of 98.6% as compared to 93.5% by Altman’s Z-score. Also, the discriminate analysis revealed that the new transformed financial ratios could predict the bankruptcy probability with an accuracy of 93.0% as compared to 87.4% using the weights of Altman’s Z-score.

Keywords credit risk · bankruptcy · prediction · Pearson type 3 distribution · Z score · non-linear models · Type II errors · Type I errors

1 Introduction

Credit ratings have become an integral part of today’s capital markets as they help in the evaluation and assessment of credit risk, benchmark issues and

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create secondary markets for those aspects. Credit risk exists virtually in all income-producing activities and their inappropriate evaluation or inadequate mitigation would result in failure of institutions. In general, the credit ratings given by agencies such as Standard and Poor, Moody and Fitch are based on the probability of default and recovery rate taking into account not only the variables in the financial statement of the firms but also the market cues. Predicting bankruptcy for firms in financial distress from the financial statement history is an important problem studied widely by the researchers (Bartual et al. 2012; Hernandez & Wilson 2013; Mendes 2014; Zaghdoudih 2013). Among them the Altman’s Z-score (Tony et al. 2005; Radu et al. 2009; Altman 1968; Altman et al. 1977) is the most popular and widely accepted metric for predicting the bankruptcy. The popularity of Z-score may be attributed to its simplicity in computation and ease in its application (Ali & Kim-Soon 2012; Khalid et al. 2008; Allen et al. 2006; Landsman et al. 2009).

Altman’s Z-score uses mainly accounting figures in the financial statement as variables in the computation. The Z-score is highly sensitive to small variations in these figures due to its dependency on them. This leads to an exaggerated Z-score in case they are manipulated as it does not include the past accounting profile of the business into consideration during its computation. Therefore, the bankruptcy probability predictions using Altman’s Z-score would cause significant levels of type-I errors (classifying bankrupt firms as non-bankrupt). Beaver et al. (2009) demonstrated the bias of Z-score in predicting the bankruptcy. In addition, the models have the weakness of not being immune to false accounting practices (Aasen 2013). It is stated by Altman that the retained earnings account is subject to manipulation via corporate quasi-reorganizations and stock dividend declarations, which may cause a bias (Altman 2000). Moreover the weights used by Altman are still prevalent even though the financial reporting environment has changed drastically from a rule-based approach to a principle-based set of standards aiming for harmonization with the International Financial Reporting Standards (IFRSs) (Benston et al. 2006; Karim & Tan 2010; Jamal et. al. 2010).

The Z-score, which is a derivative of financial ratios may not represent different risks using same quantitative figures or same business risks for different financial statement figures. To account for this asymmetry, the Z-score may be adjusted by including the earnings management in the computation procedure (Seong et al. 2012) but it may also be exposed to manipulations in the accounting data. These models should not be applied to financial firms due to their frequent use of off balance-sheet items (Altman & Edith 2006). Also, the results of the model may vary over time, which may be explained by the uncertainty of the stock prices as they are subject to the stock market opinion. During periods when the stock market is relatively high, the Z-score outcomes will be higher than in times when stock prices are low.

The financial scores are linearly combined to obtain the Z-score using the weight functions derived from the multivariate discriminate analysis. In realistic scenarios the financial ratios used as independent variables may not be linearly related. Also, the score is biased to small variations in the financial
scores. Moreover, it is not possible to compare the performance of different firms such as non-manufacturing, manufacturing as the weights of the financial ratios would differ among different firms. Further, it is difficult/not feasible to develop specific models tailored to address the scenarios of each type of industry (retailers, airlines etc.) even though it may look ideal (Altman & Edith 2006).

Keeping in view the shortcomings of Altman’s Z-score and the adjusted Z-score (Seong et al. 2012), we frame the following objectives for the present study:

1. developing a score based method using a nonlinear form of financial ratios;
2. designing an index using an equi-probability transformation by fitting a Pearson type 3 (P3) distribution to the newly developed Z-score say \( Z_M \);
3. formulating a rating scheme based on the index;
4. comparing the \( Z_M \) with Altman’s Z-score and the proposed rating scheme with those being employed widely by financial institutions for bankruptcy predictions.

The present paper is organized as follows. In Section 2 we propose a new nonlinear transformation model for computing the \( Z \)-score while in Section 3 a new index (based on the \( Z_M \)) using a P3 distribution is developed. The methodology followed for predicting the bankruptcy probability of firms is presented in Section 4. The datasets and the results are described in Section 5. Conclusions and discussion are deferred to Section 6.

2 A generalized nonlinear score for modeling the financial ratios

The standard Z-score is a statistical measurement of a score’s relationship to the mean in a group of scores generally measured by the formula

\[
Z = \frac{(X - \mu)}{\sigma},
\]

in which \( X \) denotes the set of measurements, \( \mu \) and \( \sigma \) respectively denote the mean and standard deviation of the data in the set \( X \). The Z-score is a very useful statistic for obtaining the probability of a score occurring within a normal distribution and comparison of two scores that are from different normal distributions. Altman (1968) first proposed a Z-score measuring a company’s financial strength using a weighted sum of several factors among the variables (financial ratios) that gives an approximate description of the bankruptcy probability.

Altman (1968) utilized a data set composed of sixty-six corporations with thirty-three firms in each of the two risk groups and financial ratios given in Table 1 to obtain a set of ratios that influence the bankruptcy prediction. The mean asset size of these firms was $6.4 million, with a range between $0.7 million and $25.9 million. Also, the financial ratios \( R_{31} \) \( x_1 \), \( R_{26} \) \( x_2 \), \( R_{21} \) \( x_3 \), \( R_{15} \) \( x_4 \) and \( R_{19} \) \( x_5 \) were identified as key variables for bankruptcy
Table 1: Financial Ratios previously used in bankruptcy prediction studies

| Ratio# | Name                                      | Type        |
|--------|-------------------------------------------|-------------|
| R1     | Cash/Current Liabilities                  | liquidity   |
| R2     | Cash Flow/Current Liabilities             | liquidity   |
| R3     | Cash Flow/Total Assets                    | liquidity   |
| R4     | Cash Flow/Total Debt                      | liquidity   |
| R5     | Cash/Net Sales                            | liquidity   |
| R6     | Cash/Total Assets                         | liquidity   |
| R7     | Current Assets/Current Liabilities        | liquidity   |
| R8     | Current Assets/Net Sales                  | liquidity   |
| R9     | Current Assets/Total Assets               | liquidity   |
| R10    | Current Liabilities/Equity                | liquidity   |
| R11    | Equity/Fixed Assets                       | solidity    |
| R12    | Equity/Net Sales                          | solidity    |
| R13    | Inventory/Net Sales                       | liquidity   |
| R14    | Long Term Debt/Equity                     | solidity    |
| R15    | Market Value of Equity/Book Value of Debt | solidity    |
| R16    | Total Debt/Equity                         | solidity    |
| R17    | Net Income/Total Assets                   | profitability|
| R18    | Net Quick Assets/Inventory                | liquidity   |
| R19    | Net Sales/Total Assets                    | profitability|
| R20    | Operating Income/Total Assets             | profitability|
| R21    | Earnings Before Interest & Tax/Total Interest Payments | liquidity |
| R22    | Quick Assets/Current Liabilities          | liquidity   |
| R23    | Quick Assets/Net Sales                    | liquidity   |
| R24    | Quick Assets/Total Assets                 | liquidity   |
| R25    | Rate of Return to Common Stock            | profitability|
| R26    | Retained Earnings/Total Assets            | profitability|
| R27    | Return on Stock                           | profitability|
| R28    | Total Debt/Total Assets                   | solidity    |
| R29    | Working Capital/Net sales                 | liquidity   |
| R30    | Working Capital/Equity                    | liquidity   |
| R31    | Working Capital/Total Assets              | liquidity   |

Prediction and their weights are obtained by applying multivariate discriminate analysis. The final discriminate function obtained by Altman (say $Z_A$) is given by

$$Z_A = 1.2x_1 + 1.4x_2 + 3.3x_3 + 0.6x_4 + 0.999x_5.$$ (2)

The present condition of the firms may be assessed based on the $Z_A$ values as follows:

$$Z_A = \begin{cases} 
\text{Safe Zone}, & Z_A \geq 2.99; \\
\text{Grey Zone}, & 1.81 \leq Z_A < 2.99; \\
\text{Distress Zone}, & Z_A < 1.81. 
\end{cases}$$

The companies in the safe zone may be considered financially healthy where as those in grey zone could go either way and if in the distress zone there is a greater risk that the company will go bankrupt within two years. Altman (2000) refined the weights of the Z-scores by revisiting the Zeta analysis and obtained the discriminate function as

$$Z_U = 0.72x_1 + 0.85x_2 + 3.1x_3 + 0.42x_4 + x_5.$$ (3)
In a linear model, the financial ratios \( x_i \) would influence Z-score in a linear way. In the context of risk a change of a financial ratio by 1% may not have the same influence on the score (Tony et al. 2005; Atiya 2001; Baesens et al. 2003). Also, the studies in (Altman & Edith 2006; Aasen 2013) have found that financial ratios may be overstated due to accounting practices or manipulations. Therefore estimating nonlinear transformations for some of the independent variables would improve the bankruptcy predictions. The power transformations due to Box and Cox (Box & Cox 1964) are a popular method for nonlinear transformations to improve the symmetry and normality of the model fit. However, these transformations are proposed only on positive values whereas the financial ratios could be negative. An alternative family of transformations is proposed in (Yeo & Johnson 2000) which may be applied even on negative values.

In the present work we propose a nonlinear mapping \( x_i \mapsto f(x_i) \) of the form

\[
f(x_t) = \begin{cases} 
-\ln(-x_t + 1), & x_t \leq 0; \\
\ln(x_t + 1), & x_t > 0,
\end{cases}
\]

(4)

for transforming financial ratios \( (x_t) \) before deriving the Z-score. The loglinear models make the differences between large values less important and those between small values more important. They are employed in (McLeay et al. 2002; Ashton et al. 2004) to assume proportionate growth in accounting variables that may be restricted to firm growth and are employed in (Senteney et al. 2006) for predicting the impending bankruptcy.

We first transform the financial ratios utilizing the nonlinear function given in Equation 4 and then compute the new Z-score, \( Z_M \) as

\[
Z_M = \lambda_1 f(x_1) + \lambda_2 f(x_2) + \lambda_3 f(x_3) + \lambda_4 f(x_4) + \lambda_5 f(x_5) + \ldots + \lambda_t f(x_t),
\]

(5)

in which \( \lambda_1 \ldots \lambda_t \) denote the parameters of the financial ratios \( x_1 \ldots x_t \) respectively.

A more generalized form of \( Z_M \) is given by

\[
Z_M = \sum_{k=1}^{t} \lambda_k f(x_k),
\]

(6)

in which \( x_k \) denotes the financial ratio for each \( k = 1, 2, \ldots, t \) and \( \lambda_k \) denotes the weight of the \( x_k \). The weights \( \lambda_k \) are estimated using multivariate discriminate analysis (MDA).

3 A New Indexing Measure for Credit Rating

To predict bankruptcy of a firm it is required to identify the bounds on Z-score that can be estimated through empirical studies. These bounds are not comparable and they vary from business to business and also on countries’ economic
situations. Therefore, there is a need to standardize the Z-score to a distribution before deriving useful indices for predicting the credit risk and bankruptcy of the firms. The present procedure generalizes the bankruptcy prediction that can be utilized by all the companies around the world. In (Hans et al. 2007) full credibility theory approach is used to estimate the parameters of the frequency (Poisson) and severity (Pareto) distributions for low frequency, high impact operational risk losses exceeding some threshold for each risk cell. An extreme value theory that provided fundamentals needed for the statistical modelling has been applied in stock market indices (Gilli & Kllezi 2006) to compute tail risk measures and extreme market events (Carvalhal & Mendes 2003). A new set of assessment models for long-term credit risk is studied in (Kubo & Sakai 2011) which does not include stock prices and incorporates business cycles.

The final objective of any risk model is to build the probability density function (PDF) of future losses in a portfolio. Renzo et al. (2006) developed simplest model using Bernoulli-distributed events and Poisson distribution. Probability distributions such as Poisson and Gamma have been employed to analyze aggregate loss distributions associated with operational risk (Degen 2006; Dutta & Perry 2006; Embrechts et al. 2006).

Credit risk models assume that the risk factors are independent gamma distributed random variables with mean 1 and variances $\sigma^2$ (Matthias & Alexander 2012). Whereas, the P3 distribution with three parameters, location, shape and scale improves the goodness of fit to the data and can provide better estimates on the ratings. Moreover, Pearson family of distributions is employed in a wide range of applications such as financial time series modeling (Stavros 2014), distribution of stock returns (Pizzutilo 2002), flood risk modeling due to climate change (Miley et al. 2001). This distribution can fit a wide range of shapes with positive or negative skewness including a good approximation to the normal distribution. This motivated us to develop a new methodology for credit risk rating using three parameter P3 distribution. In addition, our methodology is universal and can apply to a wide range of distributions that are popularly being employed in credit risk applications.

Presently, methods employed in credit risk applications use conventional moments such as mean, standard deviation, skewness and kurtosis to fit a distribution to the observations. These moments involve nonlinearities that are influenced by the presence of outliers and would result in over or under-estimation of the credit ratings. Therefore, we propose an approach based on the method of linearized moments popularly known as L-moments where in the parameters of the distribution can be expressed in a linear form. These L-moments can be computed using probability weighted moments (PWM) presented in Section 3.1. The parameters of the P3 distribution (Hosking 1989) from L-moments can be computed using the procedure discussed in Section 3.2.

In our methodology we first propose to fit a P3 distribution to the $Z^M$ score by computing L-moments and the parameters of the distribution. An index is then computed by measuring the deviations of the data using parameters of the P3 distribution. The ratings are then obtained by classifying the index into intervals ranging from highest safety (AAA) to high risk (CCC) based
on whether the value of the index is on positive or negative extreme of the distribution respectively. The details of the computations are presented in the Section 4.

3.1 Probability Weighted Moments and L-moments

The L-moments are analogous to conventional central moments, but can be estimated by linear combinations of order statistics. The L-moment estimates are found to be more robust compared to the conventional moments in the presence of outliers (Sankarasubramanian & Srinivasan 1999; Royston 1992; Ulyrch et al. 2000). The L-moments are less sensitive to the effects of sampling variability, and are used to characterize a wide range of distributions than the conventional moments. Practically, they are less subject to bias in estimation and they approximate their asymptotic normal distribution more closely. The parameters estimated through L-moments are more accurate than the maximum likelihood and least square estimates. The L-moment estimates using PWM has been used in applications such as floods (Tai et al. 2012), drought (Eslamian et al. 2003) and financial risk (Maillet & Michel 2003).

The probability weighted moments are defined in terms of the cumulative distribution function $F(y)$ (Greenwood et al. 1979)

$$M_{p,r,s} = \int_0^1 F_{-1}(y)^p F(y)^r (1 - F(y))^s dF,$$  

in which $p, r, s$ are positive integers, $F_{-1}(y)$ denotes the inverse cumulative distribution function of the random variable $Y$. The term $M_{p,r,s}$ can now be used for describing the probability distribution. In a particular case where $p = 1$, and $s = 0$ the variable $y$ becomes linear and the moment $\beta_r$ is defined as

$$\beta_r = M_{1,r,0} = \int_0^1 F_{-1}(y) F(y)^r dF.$$  

The first three L-moments expressed as the linear combinations of the PWM as

$$\theta_1 = \beta_0,$$

$$\theta_2 = 2\beta_1 - \beta_0,$$

$$\theta_3 = 6\beta_2 - 6\beta_1 + \beta_0,$$

in which $\theta_1$ known as L-mean, is a measure of central tendency and $\theta_2$ known as L-standard deviation is a measure of dispersion. The ratios of L-moments are defined as

$$\tau_2 = \theta_2/\theta_1,$$

$$\tau_3 = \theta_3/\theta_2,$$

in which $\tau_2$ is termed as L-coefficient of variation and $\tau_3$ is known as L-skewness and they are employed in estimating the parameters of the P3 distribution.
3.2 Pearson Type 3 (P3) Distribution

In particular, the P3 probability density function \( g \) of the random variable \( \Xi \) is defined as

\[
g(\xi) = \left| \alpha \right| \frac{\alpha}{\Gamma(\eta)} \left[ \alpha(\xi - c) \right]^{\eta-1} e^{-\alpha(\xi - c)},
\]

(11)

in which \( c, \alpha \) and \( \eta \) are location, scale and shape parameters of the distribution respectively. When the parameter \( \alpha > 0 \), \( \xi \) has positive skewness leading to \( c \leq \xi \leq +\infty \) and when \( \alpha < 0 \), \( \xi \) has negative skewness leading to \( -\infty \leq \xi \leq c \). Hence, \( c \) is a lower bound for positively skewed and an upper bound for negatively skewed P3 random variable \( \Xi \).

The parameters \( c, \alpha \) and \( \eta \) of P3 distribution are related to the L-moments as

\[
\eta = \begin{cases} 
1 + 0.2906 \delta & \text{for } 0 < \tau_3 < 0.3333; \\
0.96067 \zeta - 0.5967 \zeta^2 + 0.2536 \zeta^3 & \text{for } 0.3333 \leq \tau_3 < 1
\end{cases}
\]

\[
\alpha = \sqrt{\pi} \theta_2 e^{(\Gamma(\eta) - \Gamma(\eta+0.5))},
\]

\[
c = \theta_1 - (\alpha \eta),
\]

(12)

in which \( \delta = 3 \pi \tau_3^2 \) and \( \zeta = 1 - \tau_3 \).

4 Methodology

The steps involved in the computation of the index are shown in Fig. 1. The dataset consisting of the financial ratios is first transformed into new variables using the nonlinear function proposed in Equation 6. In the next step we convert the credit ratings given in the dataset into a binary variable \( b_\phi \) henceforth known as bankruptcy index as follows. Define

\[
b_\phi = \begin{cases} 
1, & \forall R_\phi \in \{B, BB, BBB, CCC\}; \\
0, & \forall R_\phi \in \{A, AA, AAA\},
\end{cases}
\]

(13)

in which \( R_\phi \) is the credit rating of the record \( \phi \) in the data set of \( m \) records, i.e., \( \phi \) takes values from 1, \ldots, \( m \). As per CRISIL the credit ratings AAA denotes highest safety, AA denotes high safety, A denotes adequate safety, BBB denotes moderate safety, BB denotes moderate risk, B denotes high risk and CCC denotes very high risk. Clearly, from the Equation 13 one can infer that \( b_\phi = 1 \Rightarrow \) bankruptcy or high risk category and \( b_\phi = 0 \Rightarrow \) non-bankruptcy or high safety category.

Subsequently, the weights \( \lambda_1, \ldots, \lambda_t \) of the transformed financial ratios are estimated using MDA with \( b_\phi \) as dependent variable and \( f(x_1), f(x_2), \ldots, f(x_t) \) as independent variables.
\[ b_1 = \lambda_1 f(x_1) + \lambda_2 f(x_2) + \ldots + \lambda_t f(x_t), \]
\[ b_\varphi = \lambda_1 f(x_1) + \lambda_2 f(x_2) + \ldots + \lambda_t f(x_t), \]
\[ b_m = \lambda_1 f(x_1) + \lambda_2 f(x_2) + \ldots + \lambda_t f(x_t), \]

in which the variables \( f(x_1), f(x_2), \ldots, f(x_t) \) denote the financial ratios after the application of the function \( f \) on the financial ratios \( x_1, x_2, \ldots, x_t \) respectively as defined in Equation 4. The weights obtained from the Equation 14 are substituted in Equation 5 to obtain the score \( Z_M \).

The new Z-score \( Z_M \) is then split into subsets \( Z_M = \{Z_1, \{1\ldots j\}, Z_2, \{1\ldots j\}, \ldots, Z_i, \{1\ldots j\}\} \) where in \( j \) denotes the year of observation for the \( i^{th} \) industry type. For each of these subsets the PWM are computed using the Equation 8. The L-moments \( \theta_1, \theta_2, \theta_3 \), L-moment ratios \( \tau_2 \) and \( \tau_3 \) are computed using Equations 9, 10 respectively. The parameters \( c, \eta, \alpha \) of P3 distribution are obtained from L-moments and L-moment ratio (\( \tau_3 \)) by employing Equation 12.

**Fig. 1** Methodology for computing the new index measure
In the next step we standardize the dataset $Z_{M_{i,j}}$ with respect to the origin (parameter $c$) of the P3 distribution as $v_{i,j} = (Z_{M_{i,j}} - c)/\alpha$. The new index $H_{i,j}$ is then obtained as $H_{i,j} = \left(\left(\frac{v_{i,j}}{\eta}\right)^{0.33} + \frac{1}{(9\eta)} - 1\right)\left(9\eta\right)^{0.5}$. Based on the index the credit ratings are assigned to the dataset. The details of the procedure are presented in the Algorithm 1.
Algorithm 1 Algorithm for Computing Ratings based on the novel Credit Risk Index

Require: 1. Data set  \(S(m, n)\) where \(m\) and \(n\) denotes the number of rows and number of columns respectively. The attributes consist of financial ratios \(x_{1}(m, 1), \ldots, x_{t}(m, 1)\) and credit ratings \(R(m, 1)\) as per CRISIL. Clearly the number of columns in the dataset \(S\) is \(n = t + 1\).

2. A set \(E(1, m)\) consisting of the type of industry type as in Ameya (2013) numbered from 1, \ldots, 12 to which each row in the data set \(S\) belongs.

3. A set \(Y(1, m)\) consisting of the year in which the ratings are observed for each row in the dataset \(S\).

Ensure: 1. Ratings \(W(1, m)\) for each row in the dataset \(S\).

Algorithm

1. Transform the financial ratios given in the dataset \(S\) using the nonlinear transformation function \(f\) given in Equation 4 to obtain a set with columns \(f(x_{1}), \ldots, f(x_{t})\). Designate the set as \(D(m, t)\). The set \(D\) has \(m\) records and \(t\) attributes.

2. Convert the credit ratings in column \(R\) in the dataset \(S\) to a bankruptcy index variable \(b\) using the transformation in Equation 13. We designate this set as \(\Theta\). Obtain the set \(\hat{\Theta} = \Theta \cup D\). The set \(\hat{\Theta}(m, n)\) has \(m\) rows and \(n = t + 1\) columns.

3. Obtain the weights \(\lambda_{1}, \ldots, \lambda_{t}\) in Equation 14 by employing MDA with column \(b\) of the set \(\hat{\Theta}\) as dependent variable and the columns \(f(x_{1}), \ldots, f(x_{t})\) of the set \(\hat{\Theta}\) as independent variables.

4. Obtain the new Z-score using the computations given in Equation 5 for all the records in the dataset \(\hat{\Theta}\) with the weights obtained in Step 3. Designate the set as \(Z_{M}(1, m)\). Obtain the set \(\Omega = E \cup Y \cup \hat{\Theta} \cup Z_{M}\). Clearly the set \(\Omega(m, \hat{n})\) has \(m\) rows and \(\hat{n} = \hat{n} + 3\) columns.

5. For each type of industry \(i\) in column \(E\) of set \(\Omega\)
   (a) identify and collect all records in \(\Omega\) belonging to a particular type of industry \(E(i)\). The subset is designated as \(Q\) i.e. \(Q(m, \hat{n}, \hat{n}) = \{\Omega_{m, \hat{n}, \hat{n}} : m \in E(i)\}\). Clearly \(Q \subseteq \Omega\) and \(\hat{n} \leq m\).
   (b) obtain the PWM using the \(Z_{M}\) column in set \(Q\) and then compute L-moments and their ratios using Equations 9, 10.
   (c) compute the parameters \(c, \alpha, \eta\) of P3 distribution using Equation 12.
   (d) for each year \(j\) in column \(Y\) of set \(Q\)
      i. compute the quantity \(v_{i, j} = (Z_{M_{i, j}} - c)/\alpha\).
      ii. obtain the index \(H_{i, j} = ((v_{i, j}/\eta)^{0.33} + 1/(9\eta) - 1)(9\eta)^{0.5}\).
      iii. obtain the row identification \((d)\) corresponding to the \(j^{th}\) industry and \(i^{th}\) year i.e. \(d = Index(i, j)\) would return the row number in dataset \(Q\) for which the rating is being computed.
      iv. compute the rating as defined by
          \[
          W_{d} = \begin{cases} 
            AAA, & H_{i, j} > 2.0; \\
            AA, & 1.5 < H_{i, j} \leq 2.0; \\
            A, & 0 < H_{i, j} \leq 1.5; \\
            BBB, & -1.0 < H_{i, j} \leq 0.0; \\
            BB, & -1.5 < H_{i, j} \leq -1.0; \\
            B, & -2.0 < H_{i, j} \leq -1.5; \\
            CCC, & H_{i, j} \leq -2.0, 
          \end{cases} 
          \tag{15}
          \]
   (e) initialize the set \(Q\) i.e Let \(Q = \{\emptyset\}\).

6. RETURN \(W\).

7. END.
4.1 Toy Example

To illustrate our methodology, we utilize a toy dataset given in Table 2 with $m = 10$ records, financial ratios in columns $x_1, \ldots, x_5$ (say $t = 5$), and the credit ratings in column $R$ as per CRISIL. Clearly the total number of attributes $n = t + 1 = 5 + 1 = 6$. The dataset is designated as $S(m,n)$.

Table 2 Toy dataset

| $x_1$ | $x_2$ | $x_3$ | $x_4$ | $x_5$ | $R$  |
|-------|-------|-------|-------|-------|------|
| 0.121 | 0.203 | 0.046 | 1.219 | 0.286 | BBB  |
| -0.046| -0.164| 0.027 | 0.218 | 0.103 | B    |
| 0.481 | 0.696 | 0.099 | 3.969 | 0.532 | AAA  |
| 0.351 | 0.238 | 0.07  | 1.023 | 0.237 | BBB  |
| 0.217 | 0.326 | 0.045 | 2.522 | 0.295 | AA   |
| 0.105 | 0.236 | 0.053 | 1.566 | 0.216 | BBB  |
| 0.078 | 0.157 | 0.041 | 1.402 | 0.335 | BBB  |
| 0.189 | 0.437 | 0.059 | 5.043 | 0.452 | AAA  |
| 0.043 | -0.047| 0.041 | 0.287 | 0.114 | B    |
| 0.17  | 0.702 | 0.089 | 23.002| 1.183 | AAA  |

The dataset can as well be written as

$$S = \{(0.121, 0.263, 0.046, 1.219, 0.286, BBB), (-0.046, -0.164, 0.027, 0.218, 0.103, B), (0.481, 0.696, 0.099, 3.969, 0.532, AAA), \ldots\}$$

(16)

Clearly the cardinality of above set is $|S| = 10$. We now present the steps involved in Algorithm 1 for computing the credit ratings. We consider sets $E(1, m) = \{1, 1, 1, 1, 1, 1, 1, 1, 1, 1\}$ and $Y(1, m) = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ denoting the type of industry and the year of observation respectively, for each of the records in $S$. Clearly the cardinality of the sets $E$ and $Y$ is equal to the number of rows in $S$, i.e., $|E| = |Y| = 10$. A log-transformation function $f$ given in Equation 4 is applied to each of the financial ratios (i.e., for column $x_1$ of record 1 we have $f(x_{11}) = f(0.121) = \ln(0.121 + 1) = 0.114$). We designate this set as $D$. Clearly the elements of this set are

$$D = \{(0.114, 0.233, 0.045, 0.797, 0.252), (-0.045, -0.152, 0.027, 0.197, 0.098), (0.393, 0.528, 0.094, 1.603, 0.427), (0.301, 0.289, 0.042, 0.045, 0.127)\}$$

(17)

We obtain the bankruptcy index $b$ by applying Equation 13 to the column $R$ in the Table 2 (i.e., record 1, $R_1 \in BBB \Rightarrow b_1 = 1$). We designate this set as $\Theta$ and the elements of this set are $\{1, 1, 0, 1, 0, 1, 1, 0, 1, 0\}$. We then obtain a set $\tilde{\Theta} = D \cup \Theta$. The elements of this set are

$$\tilde{\Theta} = \{(0.114, 0.233, 0.045, 0.797, 0.252, 1), (-0.045, -0.152, 0.027, 0.197, 0.098, 1), (0.393, 0.528, 0.094, 1.603, 0.427, 0), (0.301, 0.289, 0.042, 0.045, 0.127)\}$$

(18)

We obtain the set of equations between the bankruptcy index $b$ and the financial ratios $x_1, \ldots, x_5$ as given in Equation 14.
By employing MDA on Equation 19 we obtain the weights as $\lambda_1 = 1.841, \lambda_2 = -0.856, \lambda_3 = -1.087, \lambda_4 = 3.390, \lambda_5 = -1.649$.

The score $Z_M$ is computed using Equation 5 as

$$Z_M = 1.841 \times 0.114 - 0.233 \times 0.856 - 0.045 \times 1.087 + 0.797 \times 3.390 - 0.252 \times 1.649 = 2.249,$$

The computed $Z_M$ scores obtained for the years $j = 1, \ldots, 10$ using Equation 13 is given as $Z_M = \{2.249, 0.525, 4.900, 2.335, 3.914, 2.818, 2.464, 5.429, 0.750, 9.228\}$. We then obtain the dataset $\Omega = E \cup Y \cup \tilde{\Theta} \cup Z_M$ as given in Table 3.

The PWM are computed using Equation 8 to obtain the values $\beta_0 = 3.461, \beta_1 = 2.449,$ and $\beta_2 = 1.939$.

The L-moments and their ratios are computed from $\beta_0, \beta_1, \beta_2$ using Equation 8, 10 as

$$\theta_1 = \beta_0 = 3.461,$$
$$\theta_2 = 2 \times 2.449 - 3.461 = 1.437,$$
$$\theta_3 = 6 \times 1.939 - 6 \times 2.449 + 3.461 = 0.401,$$
$$\tau_2 = \theta_2/\theta_1 = 1.437/3.461 = 0.415,$$
$$\tau_3 = \theta_3/\theta_2 = 0.401/1.437 = 0.279.$$
The parameters of P3 distribution is obtained by substituting from L-moments obtained in Equation 21 in Equation 12. Since, \(\tau_3 < 0.333\) we apply

\[
\delta = 3 \times 3.146 \times (0.279)^2 = 0.7202,
\]

\[
\eta = \frac{3\times 0.1882 \times (0.7202)^2 + 0.0442 \times (0.7202)^3}{(1 + 0.2093)(0.7202 + 0.0976 + 0.0165)} = 1.449,
\]

\[
\alpha = \sqrt{3.1416 \times 1.437 \times e^{(1.449)} - (1.449 + 0.5)}
\]

\[
= 1.7725 \times 1.437 \times e^{(-0.1214)}
\]

\[
= 2.5488 \times e^{-0.109} = 2.3042,
\]

\[
c = 3.461 - 1.449 \times 2.3042 = 3.461 - 3.3398 = 0.121.
\] (22)

To compute the Index \(H\) for \(i = 1\) and \(j = 1\) we first compute

\[
v_{1,1} = (Z_{M_{1,1}} - c)/\alpha = (2.249 - 0.121)/2.3042
\]

\[
= 2.1273/2.3042 = 0.9232,
\]

\[
H_{1,1} = ((v_{1,1}/1.449)^{0.33} + 1/(9 \times 1.449) - 1) (9 \times 1.449)^{0.5}
\]

\[
= ((0.9232/1.449)^{0.33} + 1/(9 \times 1.449) - 1) \times (9 \times 1.449)^{0.5}
\]

\[
= (0.8604 + 0.0767 - 1) \times 3.6118
\]

\[
= -0.0629 \times 3.6118 = -0.2272.
\] (23)

The remaining indices of \(H\) for \(i = 1\) and \(j = 2, \ldots, 10\) can be obtained using the steps in Equation 23 as \(H_{i,j}\)

\[
= \{-1.549, 0.735, -0.186, 0.433, 0.028, -0.126, 0.880, -1.265, 1.711\}. \] (24)

To compute \(W_1\) we apply Equation 15 on \(H_{1,1} = -0.2272\) and find that it falls in the range \(-1.0 < -0.2272 \leq 0.0\), hence the rating BBB is assigned to \(W_1\).

Similarly, the credit ratings for \(i = 1\) and \(j = 2, \ldots, 10\) can be obtained using Equation 15 on values in \(H_{i,j}\) as \(W_{2,\ldots,10}\)

\[
= \{B, A, BBB, A, A, BBB, A, BB, AA\}. \] (25)

5 Experiments and Results

In this section we present the experiments conducted on the dataset and a comparison of our results with those of the earlier studies. A time series dataset Kubo & Sakai (2011) consisting of 3932 records with seven attributes as given in Table 4 is considered in our analysis. We have considered five financial ratios namely (i) working capital/total assets \((WC_{TA})\), (ii) retained earnings/total assets \((RE_{TA})\), (iii) earnings before interest and taxes/total
assets (\text{EBIT\_TA}), (iv) market value of equity/book value of the total debit (\text{MVE\_BVTD}) and (v) sales/total assets (\text{S\_TA}) for the analysis. There are 2392 bankrupt cases with credit ratings from \textit{BBB} to \textit{CCC} and 1540 non-bankrupt cases with credit ratings \textit{A} to \textit{AAA}.

Table 4 Description of the dataset

| Sno | Attribute | Description                                      | Type    |
|-----|-----------|--------------------------------------------------|---------|
| 1   | WC\_TA   | working capital/total assets                     | Real    |
| 2   | RE\_TA   | retained earnings/total assets                   | Real    |
| 3   | EBIT\_TA | earnings before interest and taxes/total assets  | Real    |
| 4   | MVE\_BVTD| market value of equity/book value of total debit | Real    |
| 5   | S\_TA    | sales/total assets                               | Real    |
| 6   | Industry | 1 to 12                                          | Categorical |
| 7   | Rating   | A, AA, AAA, B, BB, BBB, CCC                      | Categorical |

The dataset consists of the credit ratings belonging to 12 different industries with seven ratings ranging from highest safety (AAA) to very high risk (CCC).

5.1 Results

In this section we present the results of our study on the dataset. A comparison of skewness and the kurtosis of the original and transformed financial ratios are shown in Table 5.

Table 5 Comparison of skewness and kurtosis of old and transformed financial ratios

| Financial ratio | Skewness old | Skewness transformed | Kurtosis old | Kurtosis transformed |
|-----------------|--------------|----------------------|--------------|---------------------|
| WC\_TA          | -1.152       | -0.458               | 17.944       | 4.637               |
| RE\_TA          | -2.476       | -1.591               | 17.181       | 6.462               |
| EBIT\_TA        | -4.665       | -3.760               | 74.310       | 51.487              |
| MVE\_BVTD       | 12.992       | 1.415                | 269.574      | 3.357               |
| S\_TA           | 9.160        | 2.129                | 206.135      | 12.598              |

From Table 5 we can infer that the log transformation has reduced the skewness and kurtosis of the original variables, thereby improving the normality of the financial ratios.

We then estimate the weights of the loglinear model by performing MDA analysis between the bankruptcy index obtained by applying Equation 13 and
log transformed financial ratios. We have obtained the parameters given in the Equation 6 as $\lambda_1 = 0.375$, $\lambda_2 = 0.028$, $\lambda_3 = -0.316$, $\lambda_4 = 1.126$, and $\lambda_5 = -0.236$.

Altman’s Z-score ($Z_A$) and the revised Z-score ($Z_U$) is computed as given in Equations 2 and 3. The $Z_M$ score of the transformed variables is computed and comparison of the descriptive statistics among the $Z_M$, original Z-score and revised Z-score score is shown in Table 6.

### Table 6 Descriptive Statistics among the new score ($Z_M$), Altman’s Z-score ($Z_A$) and revised Z-score ($Z_U$)

| Scoring Method | Mean | Standard Deviation | Quantile (25%) | Median | Quantile (75%) |
|----------------|------|--------------------|----------------|--------|----------------|
| $Z_M$          | 1.007| 0.686              | 0.529          | 0.883  | 1.374          |
| $Z_A$          | 2.172| 3.060              | 0.926          | 1.626  | 2.679          |
| $Z_U$          | 1.609| 2.178              | 0.714          | 1.207  | 1.973          |

We performed F-Test on the standard deviation of the $Z_M$ and other Z-score methods. The estimated $F$ value ($F_{cal}$) is 30.664 and the tabulated $F$ value ($F_{tab}$) is 39.863 at 0.01 significance. Since $F_{cal}$ is less than $F_{tab}$ we accept the null hypothesis that the two standard deviations are equal. The original $Z_A$ and $Z_M$ are found to have good correlation with Spearman $\rho$ coefficient 0.963 significant at 0.01 (2 tailed) level (see Table 7).

### Table 7 Correlation coefficients between $Z_M$ and $Z_A$

|       | $Z_M$ | $Z_A$ |
|-------|-------|-------|
| $Z_M$ | 1.0   | 0.963 |
| $Z_A$ | 0.963 | 1.0   |

The estimates of the parameters of the P3 distribution obtained using the methodology described in Section 4 are shown in Table 8 for each of the industry types.
Table 8  P3 distribution parameter estimates using probability weighted moments

| Industry Type | Location (ε) | Shape (β) | Scale (α) |
|---------------|--------------|-----------|-----------|
| 1             | -0.120815    | 0.434659  | 2.588683  |
| 2             | -0.141720    | 0.387980  | 3.053867  |
| 3             | -0.466324    | 0.279429  | 5.150730  |
| 4             | -0.168323    | 0.422125  | 2.898292  |
| 5             | -0.157205    | 0.466022  | 2.546405  |
| 6             | -0.469409    | 0.290401  | 5.071536  |
| 7             | -0.640034    | 0.261942  | 6.265737  |
| 8             | -0.279976    | 0.341867  | 3.639809  |
| 9             | -0.062825    | 0.456268  | 2.352211  |
| 10            | -0.117848    | 0.396729  | 2.796561  |
| 11            | -0.303738    | 0.344976  | 3.710743  |
| 12            | -0.186726    | 0.382682  | 3.192083  |

The credit ratings are computed after obtaining the standardized index of the data set using the P3 distribution parameters. The credit ratings are then converted to bankruptcy index using the Equation 13. A binary logistic regression, classification is carried out with the bankruptcy index as dependent variable and the $Z_A$, $Z_M$ or the $Z_U$ scores as an independent variable as given in Equation 26

\[
A_a = \nu_a + \nu_1 Z_A + \epsilon_1,
\]

\[
A_m = \nu_m + \nu_2 Z_M + \epsilon_2,
\]

\[
A_u = \nu_u + \nu_3 Z_U + \epsilon_3,
\]

in which $A_a$, $A_m$, $A_u$ denotes the bankruptcy index of the Z-scores $Z_A$, $Z_M$ and $Z_U$ respectively.

The parameters obtained from the three models are shown in Table 9.

Table 9  Comparison of $Z_A$, $Z_U$ and the new score $Z_M$ for bankruptcy prediction using Wald statistics

| Parameter | $Z_A$       | $Z_M$       | $Z_U$       |
|-----------|-------------|-------------|-------------|
| $\nu_a$  | 11.016(594.168) | -           | -           |
| $\nu_m$  | -           | 77.15(106.313) | -           |
| $\nu_u$  | -           | -           | 11.956(574.677) |
| $\nu_1$  | -5.423(572.703) | -           | -           |
| $\nu_2$  | -           | -78.534(106.569) | -           |
| $\nu_3$  | -           | -           | -7.993(555.379) |

The estimated coefficients of $Z_A$ and $Z_M$ are both negative and statically significant at 0.01% indicating that both measures are useful in predicting bankruptcy risk and lower the score, the higher the risk of bankruptcy. The coefficient of $Z_M$ is far lower than $Z_A$ indicating the fact that the predictive power of $Z_M$ is far better than Altman’s $Z_A$ score and revised Altman’s $Z_U$ score.
A hold-out classification between $Z_M$, $Z_A$, $Z_U$ and bankruptcy index $b$ is carried out using MDA and the prediction accuracy of the proposed methodology is found to be 98.6% which is higher by 5% than any of the models proposed by Altman. This confirms that the proposed methodology is universal and serves as a generalized tool that can improve the estimations of the existing methods/procedures in vogue and can predict the bankruptcy risk in an effective manner.

Discriminate analysis on the data set generated using the new transformation $Z_M$ has resulted in an accuracy of 93.7% in cross validated grouped cases correctly classified where as Altman’s $Z$-score $Z_A$ has resulted only in an accuracy of 87.4%.

MDA is carried out on $Z_M$ score and the credit ratings obtained from P3 and Pareto distributions. The proposed method with P3 distribution resulted in an accuracy 92.2% whereas the model with the Pareto distribution has resulted in only 80% accuracy.

To understand the sensitivity on the choice of thresholds for different ratings in predicting bankruptcy, we first construct a classification matrix or accuracy matrix (Table 10) based on the number of agreements and disagreements between the predicted group membership (estimated from the model) and the actual group membership (as present in the dataset) of bankruptcy for the thresholds given in Equation 15.

| Actual group membership | Predicted group membership |
|-------------------------|----------------------------|
| Bankrupt                | Bankrupt: $N_1$ (1966) | Non-Bankrupt: $M_1$ (426) |
| Non-Bankrupt            | Bankrupt: $M_2$ (14) | Non-Bankrupt: $N_2$ (1526) |

The actual group membership is equivalent to the a priori grouping and the predicted group refers to the cases wherein the proposed methodology attempts to classify them correctly. In the Table 10 $N_1$, $N_2$ denotes the correct classifications (Hits) and $M_1$, $M_2$ denotes the misclassifications (Misses). $N_1$ (1966) gives the number of cases of actual bankruptcy correctly classified as bankrupt by the proposed method. $M_1$ (426) is the Type-I error that gives the number of cases wherein the actual group membership is bankrupt whereas the proposed model misclassified them as non-bankrupt. $M_2$ (14) is the Type-II error, that denotes the number of actual cases belonging to non-bankrupt group misclassified as bankrupt by the proposed model. $N_2$ (1526) are the number of cases wherein the proposed model correctly labels the actual cases as non-bankrupt.

The accuracy of the proposed methodology is computed as $(N_1 + N_2)/(N_1 + N_2 + M_1 + M_2) = (1966 + 1526)/(1966 + 1526 + 14 + 426) = 3492/3932 = 0.88$ = 88%. The Type-I error is the ratio of misclassified cases of actual bankrupt cases declared as non-bankrupt by the model with total bankrupt cases i.e
Type-I = \frac{M_1}{(N_1 + M_1)} = \frac{426}{(1966 + 426)} = \frac{426}{2392} = 0.177 = 17.7%.

The Type-II error is the ratio of misclassified cases of actual non-bankruptcy cases declared as bankrupt by the model with total non-bankruptcy cases i.e

Type-II = \frac{M_2}{(N_2 + M_2)} = \frac{14}{(14 + 1526)} = \frac{14}{1540} = 0.009 = 0.9%.

The proposed method with thresholds as given in Equation 15 is accurate in classifying 88.8% of total samples with Type I error to be only 17% while the Type II error was even better at 0.9%. Therefore, there is a positive upward bias which can be addressed by adjusting the thresholds between the credit ratings A and BBB as the boundaries fall in the grey zone. Keeping the other thresholds unchanged, we updated the thresholds of BBB as \(-1.0 < H_{i,j} \leq 0.25\) and A as \(0.25 < H_{i,j} \leq 1.5\) from the classification table, we obtained the Type I error as 4% and Type II error as 5% with overall accuracy of 95%. To see if the sensitivity be further improved we updated the thresholds of BBB as \(-1.0 < H_{i,j} \leq 0.5\) and A as \(0.5 < H_{i,j} \leq 1.5\) keeping the others unchanged. We found from the classification table the Type I error as 0.16% whereas Type II error has increased to 21.7% with overall accuracy of 91.4%. Therefore, the choice of thresholds for transition from bankruptcy to non-bankruptcy should be chosen with caution so that both Type I and Type II errors are at minimum.

Even though the samples are disproportionate the Algorithm 1 has outperformed the accuracies obtained using the Altman’s Z-score methods.

6 Conclusions and Discussion

A new nonlinear transformation procedure for building a log linear model for computing the Z-score is proposed. Based on the new Z-score (\(Z_M\)) a new indexing measure is proposed by fitting the data to a P3 distribution and then obtaining the deviations of the given dataset from the standard normal using an equi-probability transformation. The multivariate discriminate analysis (MDA) for predicting the bankruptcy index has shown that the proposed methodology has given highest accuracy of 98.5% which is higher by 5% as compared with Altman’s Z-score. The classification accuracies of the transformed financial ratios in predicting the bankruptcy is around 93.7% as compared to 87.4% obtained by using the factors of Altman’s procedure. The accuracies of the proposed method with P3 distribution was 92.2% where as a model with Pareto distribution resulted in an accuracy of 80%. Though the methodology is universal and serves as a generalized tool, there is an immense need to validate with global datasets. Also, mutual interference among financial ratios is an important aspect that requires further investigation. We defer our ongoing work in this direction to a subsequent exposition.

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