Competition of static magnetic and dynamic photon forces in electronic transport through a quantum dot

Nzar Rauf Abdullah¹,², Chi-Shung Tang¹, Andrei Manolescu¹ and Vidar Gudmundsson²

¹ Physics Department, Faculty of Science and Science Education, School of Science, University of Sulaimani, Kurdistan Region, Iraq
² Science Institute, University of Iceland, Dunhaga 3, IS-107 Reykjavik, Iceland
³ Department of Mechanical Engineering, National United University, 1, Lienda, Miaoli 36003, Taiwan, People’s Republic of China
⁴ Reykjavik University, School of Science and Engineering, Menntavegur 1, IS-101 Reykjavik, Iceland

E-mail: nzar.rafulah@gmail.com

Received 4 April 2016, revised 21 June 2016
Accepted for publication 27 June 2016
Published 15 July 2016

Abstract
We investigate theoretically the balance of the static magnetic and the dynamical photon forces in the electron transport through a quantum dot in a photon cavity with a single photon mode. The quantum dot system is connected to external leads and the total system is exposed to a static perpendicular magnetic field. We explore the transport characteristics through the system by tuning the ratio, $\frac{c}{\omega_{e\gamma}}$, between the photon energy, $\hbar\omega_{\gamma}$, and the cyclotron energy, $\hbar\omega_{e}$. Enhancement in the electron transport with increasing electron–photon coupling is observed when $\frac{c}{\omega_{e\gamma}} > 1$. In this case the photon field dominates and stretches the electron charge distribution in the quantum dot, extending it towards the contact area for the leads. Suppression in the electron transport is found when $\frac{c}{\omega_{e\gamma}} < 1$, as the external magnetic field causes circular confinement of the charge density around the dot.

Keywords: cavity quantum electrodynamics, electro-optical effects, electronic transport in mesoscopic systems, quantum wires

(Some figures may appear in colour only in the online journal)

1. Introduction

Electron systems coupled to a quantized electromagnetic field are a common component of semiconductor and superconducting nanoscale devices. An optoelectronic system is formed by adding electronic and photonic sources. In optoelectronic circuits, electrons inelastically tunnel between two connected systems [1]. The characteristics of electron tunneling is modified by the electron–photon interaction influencing the electron motion [2, 3]. The electron tunneling is the so called photon-assisted transport (PAT) [4]. The PAT is inelastic electron tunneling in which the energy of electrons is changed by photon emission and absorption processes. These two processes can enhance or suppress the electron transport [5–7]. A suitable electronic structure for investigation of PAT is a quantum dot because of its potential application in quantum information processing [8–11]. The PAT of both charges [2] and spins [12] in quantum dots showing enhanced transport has been investigated.

If the electronic system is exposed to an external perpendicular magnetic field, the electron motion is also influenced by the magnetic field. It may form edge states [13], or localized states [14] leading to decreased conductivity. Magnetic field has been considered to control electron-switching processes in qubits [15], and to enable magnetic resonance imaging in biology [16] to mention two totally different applications.

In the presence of both external magnetic and photon fields, magneto-phototransport emerges in which the electrons are...
influenced by both fields. Magneto-phototransport has been studied in superconducting complementary split-ring resonators coupled to the cyclotron transition of two-dimensional electron gases [17] where blue-shifting of polaritons due to the diamagnetic term of interaction Hamiltonian was observed. In addition, the magneto-phototransport has been investigated in graphene coupled to cavity photons when the vacuum Rabi frequency is comparable to, or even larger than the cyclotron transition of Dirac fermions [18]. The magneto-phototransport has not been investigated in quantum dots in a photon cavity in the presence of several photons.

In a previous publication we investigated PAT in a quantum dot (QD) system coupled to cavity photons [19]. In this work, we study magneto-phototransport in a QD system coupled to a photon cavity using a generalized master equation (GME) [20]. We assume a QD embedded in a two-dimensional quantum wire in an external perpendicular magnetic field. The QD system is weakly connected to external leads and strongly coupled to the photon cavity with a single photon mode. We show how the external magnetic and photon fields influence the electron transport in the QD system. We consider the cavity initially containing two photons polarized either parallel (x-direction) or perpendicular (y-direction) to the direction of the electron transport. For the x-polarization, the electron transport is enhanced when the cyclotron energy is smaller than the photon energy while a suppression in the transport is noticed in the case of the cyclotron energy larger than the photon energy. No such transport characteristic is found in the case of y-polarization due to the anisotropy of the central system and the chosen energy for the photon.

This paper is organized as following: in section 2 description of the model and the theoretical formalism are shown. In section 3 we present the results and conclusions.

2. Method and theory

In this section, we introduce the Hamiltonian of the system and the potential that forms the quantum dot. The QD system is exposed to a uniform perpendicular magnetic field and in a quantized electromagnetic cavity with a single photon mode. The electron-electron and the electron–photon interactions are explicitly taken into account. The photons in the cavity are linearly polarized. We use a non-Markovian generalized master equation to investigate the non-equilibrium electron transport in the system.

The central system is hard-wall confined in the x-direction and parabolically confined in the y-direction. The QD potential shown in figure 1(a) can be described by

$$V_{QD} = V_0 \exp(-\alpha_x^2 x^2 - \alpha_y^2 y^2),$$  \hspace{1cm} (1)

where $V_0$ is the strength of the potential, and $\alpha_x$ and $\alpha_y$ are constant values that define the diameter of the QD.

Figure 1(b) is a schematic representation of the QD system under the static external magnetic (red dashed line) and the dynamic photon field (blue dashed line). The magnetic field induces the Lorentz force ($F_L$), and the photon field induces a force ($F_P$).

The Hamiltonian of the QD system coupled to a single photon mode in an external perpendicular magnetic field in the z-direction is [21, 22]

$$\hat{H}_S = \int d^2r \psi^\dagger(r) \left[ \frac{1}{2m^*} \left( \vec{\nabla} + \frac{e}{\hbar c} [A(r) + \hat{A}_r(r)] \right)^2 + eV_{pg} + V_{QD}(r) \right] \psi(r) + H_Z + \hat{H}_{ee} + \hbar \omega_c \hat{a} \hat{a}^\dagger.$$  \hspace{1cm} (2)

Herein, $\psi$ is the field operator, $A(r) = -B y \vec{x}$ is the vector potential of the external magnetic field defined in the Landau gauge, and $\hat{A}_r$ is the vector potential of the photon field given by $\hat{A}_r(r) = A(\hat{a} + \hat{a}^\dagger) e$ with $A$ the amplitude of the photon field with the electron–photon coupling constant $g_e = e A a \omega_c$. $e$ is the electric field of the parallel polarized photon field $(TE_{011})$, $e = e_x$ for parallel polarized photon field $(TE_{011})$, and $\hat{a}(\hat{a}^\dagger)$ annihilation (creation) operators of the photon in the cavity, respectively. The effective confinement frequency is $\Omega_w = \sqrt{\Omega_0^2 + \omega_c^2}$ with $\Omega_0$ being electron confinement frequency due to the lateral parabolic potential and $\omega_c$ the cyclotron frequency due to external magnetic field. The effective confinement supplies a natural length scale $a_w^2 = \hbar(\Omega_w m^*)$. In addition, $V_{pg}$ is the plunger gate voltage that controls the energy levels of the QD system with respect to the chemical potential of the leads. The second term is the Zeeman Hamiltonian describing the interaction between the external magnetic field and the magnetic moment of an
The electron Hamiltonian is given by $H_e = \pm g^* \mu_B B/2$ with $\mu_B$ the Bohr magneton and $g^* = -0.44$ the effective g-factor for GaAs. The third term of equation (2) $(\hat{H}_C)$ is the Coulomb repulsion between the electrons in the QD system [19]. Finally, the last term is the quantized photon field, with $\hbar \omega_0$, the photon energy. We do not include terms describing leakage or pumping of the cavity.

The QD system is coupled to the leads at $t = 0$ via the time-dependent Hamiltonian

$$H(t) = \chi(t) \sum_{q,a} [T_{qa}^a d^a_q + T_{qa} d^a_q],$$

where $l \equiv \{L,R\}$, and $\chi(t) = 1 - 2(e^{\alpha t} + 1)$ is the switching function between the leads and the QD system with a switching parameter $\alpha$. $d^a$ and $c^a$ are the creation (annihilation) operators of the QD system and the leads, respectively. The momentum of a state, $q$, and its subband index, $n_{q\alpha}$, in the leads are combined into a single dummy variable $q = (n_{q\alpha}, q)$ for simplicity. The coupling strength tensor $T_{qa}$ is modeled as $[T_{qa} = 1/2 \int dr dr' \psi_a^*(r') \phi_{qa}^*(r) \psi_a(r') \phi_{qa}(r)]$ indicating state-dependent coupling coefficients that define the electron transfer between the single-electron state $|a\rangle$ in the QD system and the extended single-electron state $|q\rangle$ in the leads, where $\psi_{qa}(r)$ and $\phi_{qa}(r)$ are the single electron wavefunctions in the QD system and the leads, respectively.

The coupling function that defines the geometrical coupling between the waveguide and the leads is

$$g_{qa}(r,r') = g_0 \exp[-4 \delta_{l}^x (x-x')^2 - 4 \delta_{y}^y (y-y')^2] \times \exp(-\Delta_{q}^q / \hbar).$$

Here, $g_0$ is the overall coupling strength, $\delta_{l}^x$ and $\delta_{y}^y$ define the contact region parameters for lead $l$ in $x$- and $y$-direction, respectively. In addition, $\Delta_{q}^q$ indicates the electron affinity between the QD system single-electron states ($E_a$) and the lead states ($E_q$).

The QD system is coupled to two leads with the chemical potential of the left lead ($\mu_L$) higher than that of the right lead ($\mu_R$). The bias difference of the leads causes electrons to be transferred from the left to the right lead when the system has reached a steady state. The leads are semi-infinite with the same parabolic confinement as the central system and hard-wall confinement at the contact end. They are in the same external magnetic field as the central system and their non-interacting electrons have a continuous energy spectrum displaying characteristic quasi-one-dimensional subbands. The density operator of the $l$ lead before connection to the central system is

$$\hat{\rho}_l = \frac{\exp(-\beta (\hat{H}_l - \mu^N_l))}{\text{Tr}[\exp(-\beta (\hat{H}_l - \mu^N_l))]},$$

where $l \equiv \{L,R\}$, $\beta = 1/(k_B T)$ with $k_B$ is the Boltzmann constant, $\mu^N_l$ is the electron number operator and the Hamiltonian of lead $l$, respectively [23].

Once the QD system and the leads are connected, the reduced density operator (RDO) $\hat{\rho}_S$ describing the state of the electrons in the QD system under the influence of the leads can be obtained by taking the trace over the Fock space of the leads. Using a projection formalism for the weak coupling regime, the RDO of the system,

$$\dot{\hat{\rho}}_S(t) = \text{Tr}_{\text{R}}[\hat{H}_l] \hat{\rho}_l,$$

evolves in time as [24]

$$\frac{\partial \hat{\rho}_S(t)}{\partial t} = -i [\hat{H}_S, \hat{\rho}(t)] + \int_{t_0}^{t} \hat{K}(t,t') \hat{\rho}_S(t'),$$

where $\hat{K}(t,t') = [\hat{H}_S, \hat{\rho}(t')].$

Now, we can calculate current carried by the electrons in the system from the reduced density operator. We introduce net charge current ($I_{net}$)

$$I_{net} = I_L + I_R,$$

where $I_L$ is the current from the left lead to the QD system defined as

$$I_L(t) = \text{Tr} [\hat{\rho}_S(t) \hat{Q}],$$

where the charge operator is $\hat{Q} = e \hat{N}$ with the electron number operator $\hat{N}$. The current from the QD system to the right lead ($I_R$) is

$$I_R(t) = -\text{Tr} [\hat{\rho}_S(t) \hat{Q}].$$

with $\hat{\rho}_S(t)$ given by the equation of motion, (7). (The trace of the first term on the right hand side of equation (7) vanishes and the integral kernel can be split into two parts, each one pertaining to one lead [19].)

In a steady state the right and left currents are of same magnitude. The time needed to reach the steady state depends on the chemical potentials in each lead, the bias window, and their relation to the energy spectrum of the system. In anticipation that the operation of an optoelectronic circuit can be sped up by not waiting for the exact steady state we integrate the GME to $t = 220$ ps, a point in time late in the transient regime when the system is approaching the steady state. We will address the issue of the time-scales below.

Essential information about the partial current through a state can be obtained by taking an expectation value with respect to that state instead of the trace over all states as is done in equations (9) and (10).

To show the dynamic motion of the central electrons in the central system the electron charge density is presented in the result section [19]. Conclusions about the electron motion always
needs the simultaneous checking of the corresponding local current that will not be displayed here.

We like to stress that for the time interval (∼ 300 ps) in which we calculate the time evolution of the system here mostly nonradiative transitions are active. Radiative transitions can play a large role in the nano- to the microsecond regime. The states in the central system are cavity photon dressed electron states. If the photon field happens to be in resonance with a Bohr frequency in the electron system then the dressed electron states can contain a photon component that is composed of several eigenstates of the photon number operator. In the nonresonant case the photon component tends to be a single photon state with a definite photon number, and the perturbational concept of a one-, two- or higher order photon replica is a proper representation of the state. If initially there are two photons in the cavity then an entering electron can be transferred through a dressed state with a high two-photon component. A state with a vanishing two-photon component in the bias window would remain inactive.

3. Results

We study the effects of the external magnetic and photon fields on electron transport in a non-equilibrium system. The QD system and the leads are made of GaAs with electron effective mass \( m^* = 0.067 m_e \) and relative dielectric constant \( \kappa = 12.4 \). The parameters that specify the radius of the dot are \( \alpha_0 = 0.03 \text{ nm}^{-1} \), \( \alpha_\gamma = 0.03 \text{ nm}^{-1} \), and \( V_0 = -3.3 \text{ meV} \). The radius of the dot is thus \( R_{\text{QD}} \approx 33.33 \text{ nm} \). The cavity has a single photon mode with the photon energy \( h \omega_c = 0.3 \text{ meV} \), and it initially contains two photons \( N_c = 2 \). The confinement energy of the QD system is equal to that of the leads \( h \Omega_0 = h \Omega_L = 2.0 \text{ meV} \). The switching parameter is \( \alpha' = 0.3 \text{ ps}^{-1} \). In addition, the temperature of the leads before coupling to the QD system is assumed to be \( T = 0.001 \text{ K} \) (in order to avoid numerical instabilities at \( T = 0 \)).

3.1. Photon cavity with x-polarization

In this section we assume the photons in the cavity are polarized in the x-direction. We vary the cyclotron energy with the strength of the external magnetic field, \( h \omega_c = e \beta B / m^* c \), and fix the photon energy at \( h \omega_c = 0.3 \text{ meV} \).

In order to calculate the energy spectrum of the QD-cavity system, exact-diagonalization technique is utilized to diagonalize the matrix elements of the Hamiltonian equation (2) stepwise in truncated Fock-spaces delivering the many-body states of the central system, the dressed electron states and the pure photon states, \( |\mu\rangle \) [20]. Figure 2 shows the energy spectra as a function of the cyclotron energy for the QD system without (a) and with (b) the photon cavity, including zero-electron states (0ES, blue dots), one-electron states (1ES, golden squares). Two-electron states have higher energies because of the Coulomb repulsion.

The chemical potential of the left and the right leads (black lines) are \( \mu_L = 1.2 \text{ meV} \) and \( \mu_R = 1.1 \text{ meV} \), respectively. The bias window is thus \( \Delta \mu = \mu_L - \mu_R = 0.1 \text{ meV} \). The plunger-gate voltage is assumed to be \( V_{\text{pg}} = 0.4 \text{ meV} \), \( h \omega_c = 0.3 \text{ meV} \), \( \gamma_c = 0.10 \text{ meV} \). The 1ES state in the bias window is almost doubly degenerate due to the small Zeeman energy.

![Figure 2](image.png)

Figure 2. Energy spectra versus cyclotron energy plotted for the QD system without (a) and with (b) the photon field, including zero-electron states (0ES, blue dots), one-electron states (1ES, golden squares). The chemical potentials are \( \mu_L = 1.2 \text{ meV} \) and \( \mu_R = 1.1 \text{ meV} \). The plunger gate voltage \( V_{\text{pg}} = 0.4 \text{ meV} \), \( h \omega_c = 0.3 \text{ meV} \), \( \gamma_c = 0.10 \text{ meV} \). The 1ES state in the bias window is almost doubly degenerate due to the small Zeeman energy.

In figure 2(a) single-electron energy spectrum versus the cyclotron energy \( (h \omega_c) \) is plotted. The first-excited state is found in the bias window for the selected range of the cyclotron energy while the ground state is located below 1.0 meV (not shown). At \( V_{\text{pg}} = 0.4 \text{ meV} \), the first-excited state is getting into resonance with the first subband energy of the leads (not shown). Varying the cyclotron energy, the first-excited state stays in the bias window and actively contributes to the electron transport in the QD system.

In our approach we first diagonalize the Hamiltonian for the Coulomb interacting electrons in a large truncated many-electron basis of 1227 states, secondly we form a new basis as a tensor product of the 120 lowest Coulomb interacting ME states and 16 eigenstates of the photon number operator. This basis is used to diagonalize the interacting electron–photon system using both the para- and the diamagnetic part of the electron–photon interaction [20]. The transport calculation is performed for the lowest 120 many-body states obtained from this last diagonalization. This approach has its correspondence in a method using Green functions. First, the Green function for the Coulomb interacting electrons is found, and subsequently dressed by the photon interaction. We thus obtain cavity-photon dressed many-electron states, in a perturbational description extra photon replica states are formed for each many-electron (ME) state with different photon content as is displayed in figure 2(b). Off resonance, or for weak coupling, the energy difference between the photon replica states is approximately equal to the photon energy. Where the photon number of a state does not deviate strongly from an integer we use the replica picture, but having in mind its limitations. For instance, the states around 1.40–1.45 meV and 1.70–1.76 meV can be seen as photon replicas of the first-excited state containing approximately one and two photon(s), respectively. Other photon replicas of the ground-state around
The current from the left lead $I_L$ and the current into the right lead $I_R$ as a function of time for the system without a photon cavity (solid and dashed blue curves, respectively), and in case of the system in a photon cavity for $g_c = 0.1$ meV (red), and $g_c = 0.15$ meV (green). The cyclotron energy is 0.31 meV corresponding to the crossing point in figure 4.

Figure 3. The current from the left lead $I_L$ and the current into the right lead $I_R$ as a function of time for the system without a photon cavity (solid and dashed blue curves, respectively), and in case of the system in a photon cavity for $g_c = 0.1$ meV (red), and $g_c = 0.15$ meV (green). The cyclotron energy is 0.31 meV corresponding to the crossing point in figure 4.

Figure 4. The net charge current is plotted as a function of the cyclotron energy at time $t = 220$ ps for the QD system without photon (w/o ph) (blue circles), and with photon (w ph) cavity in the case of the electron–photon coupling strength $g_c = 0.10$ meV (red squares) and 0.15 meV (green triangles). The plunger gate voltage is $V_{pg} = 0.4$ meV, $h\omega_p = 0.3$ meV, and $\Delta\mu = 0.1$ meV.

The current from the left lead $I_L$ and the current into the right lead $I_R$ as a function of time for the system without a photon cavity (solid and dashed blue curves, respectively), and in case of the system in a photon cavity for $g_c = 0.1$ meV (red), and $g_c = 0.15$ meV (green). The cyclotron energy is 0.31 meV corresponding to the crossing point in figure 4.

Figure 3. The current from the left lead $I_L$ and the current into the right lead $I_R$ as a function of time for the system without a photon cavity (solid and dashed blue curves, respectively), and in case of the system in a photon cavity for $g_c = 0.1$ meV (red), and $g_c = 0.15$ meV (green). The cyclotron energy is 0.31 meV corresponding to the crossing point in figure 4.

Figure 4. The net charge current is plotted as a function of the cyclotron energy at time $t = 220$ ps for the QD system without photon (w/o ph) (blue circles), and with photon (w ph) cavity in the case of the electron–photon coupling strength $g_c = 0.10$ meV (red squares) and 0.15 meV (green triangles). The plunger gate voltage is $V_{pg} = 0.4$ meV, $h\omega_p = 0.3$ meV, and $\Delta\mu = 0.1$ meV.

The current from the left lead $I_L$ and the current into the right lead $I_R$ as a function of time for the system without a photon cavity (solid and dashed blue curves, respectively), and in case of the system in a photon cavity for $g_c = 0.1$ meV (red), and $g_c = 0.15$ meV (green). The cyclotron energy is 0.31 meV corresponding to the crossing point in figure 4.

Figure 3. The current from the left lead $I_L$ and the current into the right lead $I_R$ as a function of time for the system without a photon cavity (solid and dashed blue curves, respectively), and in case of the system in a photon cavity for $g_c = 0.1$ meV (red), and $g_c = 0.15$ meV (green). The cyclotron energy is 0.31 meV corresponding to the crossing point in figure 4.

Figure 4. The net charge current is plotted as a function of the cyclotron energy at time $t = 220$ ps for the QD system without photon (w/o ph) (blue circles), and with photon (w ph) cavity in the case of the electron–photon coupling strength $g_c = 0.10$ meV (red squares) and 0.15 meV (green triangles). The plunger gate voltage is $V_{pg} = 0.4$ meV, $h\omega_p = 0.3$ meV, and $\Delta\mu = 0.1$ meV.

The current from the left lead $I_L$ and the current into the right lead $I_R$ as a function of time for the system without a photon cavity (solid and dashed blue curves, respectively), and in case of the system in a photon cavity for $g_c = 0.1$ meV (red), and $g_c = 0.15$ meV (green). The cyclotron energy is 0.31 meV corresponding to the crossing point in figure 4.

Figure 3. The current from the left lead $I_L$ and the current into the right lead $I_R$ as a function of time for the system without a photon cavity (solid and dashed blue curves, respectively), and in case of the system in a photon cavity for $g_c = 0.1$ meV (red), and $g_c = 0.15$ meV (green). The cyclotron energy is 0.31 meV corresponding to the crossing point in figure 4.

Figure 4. The net charge current is plotted as a function of the cyclotron energy at time $t = 220$ ps for the QD system without photon (w/o ph) (blue circles), and with photon (w ph) cavity in the case of the electron–photon coupling strength $g_c = 0.10$ meV (red squares) and 0.15 meV (green triangles). The plunger gate voltage is $V_{pg} = 0.4$ meV, $h\omega_p = 0.3$ meV, and $\Delta\mu = 0.1$ meV.

The current from the left lead $I_L$ and the current into the right lead $I_R$ as a function of time for the system without a photon cavity (solid and dashed blue curves, respectively), and in case of the system in a photon cavity for $g_c = 0.1$ meV (red), and $g_c = 0.15$ meV (green). The cyclotron energy is 0.31 meV corresponding to the crossing point in figure 4.

Figure 3. The current from the left lead $I_L$ and the current into the right lead $I_R$ as a function of time for the system without a photon cavity (solid and dashed blue curves, respectively), and in case of the system in a photon cavity for $g_c = 0.1$ meV (red), and $g_c = 0.15$ meV (green). The cyclotron energy is 0.31 meV corresponding to the crossing point in figure 4.

Figure 4. The net charge current is plotted as a function of the cyclotron energy at time $t = 220$ ps for the QD system without photon (w/o ph) (blue circles), and with photon (w ph) cavity in the case of the electron–photon coupling strength $g_c = 0.10$ meV (red squares) and 0.15 meV (green triangles). The plunger gate voltage is $V_{pg} = 0.4$ meV, $h\omega_p = 0.3$ meV, and $\Delta\mu = 0.1$ meV.
density is affected by the photons, and therefore, the net charge current increases.

When the cyclotron energy is approximately equal to the photon energy $\hbar \omega \approx \hbar \gamma$, the effect of the Lorentz force matches that of the 'para- and diamagnetic forces' in the case of non-vanishing electron–photon coupling. The charge density is thus not effectively changed by the photon field as is displayed in figure 5(e). Therefore, the net charge current in the QD system without a photon field around $0.3 \hbar \omega \approx \hbar \gamma$ meV is almost equal to the net charge current in the presence of the photon cavity.

In the third case when the cyclotron energy is larger than the photon energy $\hbar \omega_c > \hbar \gamma$, the Lorentz force is dominant. The contribution of the photon replica state with two photons is increased to 93% while the photon replica state containing one photon is decreased to 3%. In absence of the photon cavity the energy difference between the first and second excitation of the one-electron state is 0.520 meV at $\hbar \omega_c = 0$, but 0.572 meV at $\hbar \omega_c = 0.86$ meV. So, with increased cyclotron energy the energy of this excitation is getting closer to be in resonance with a two-photon transition of the photon field. Concurrently, the one-photon transition loses importance. On the other hand, in the presence of the photon field a suppression in the current is seen at a high cyclotron energy compared to the net charge current in the absence of the photon cavity. The reason is that the increased Lorentz force induces a circular motion and collects the charge density around the QD as is shown in figure 5(f). As a result the net charge current is suppressed.

To show further the importance of the photon replica states on the electron transport, we tune the plunger-gate voltage to $V_{0.1pg} = 0.1$ meV in order to shift down the energy spectrum. Figure 6(a) shows the many-body (MB) energy spectrum versus the cyclotron energy at $V_{0.1pg} = 0.1$ meV for the QD system in a photon cavity. The chemical potential of the left and the right leads (black lines) are $\mu_L = 1.2$ meV and $\mu_R = 1.1$ meV, respectively. It is clearly seen that the one-photon replica of the first-excited state enters into the bias window and the rest of the energy spectrum is shifted down by 0.3 meV. In this case, the current is totally due to states with more than one photon, because all the states within the bias window are photon replica states. The net charge current would be zero in the absence of a cavity.

We now expect the photon replica of the first-excited state with approximately three photons to be participating in the transport. But three active one-electron states located between the green lines are found: The third-excited state in the energy

\[ \text{Figure 5. Distribution of charge density at } t = 220 \text{ ps of the QD system without (left panels) and with (right panels) the photon field for three different cyclotron energies } \hbar \omega_c \approx 10^{-4} \text{ meV (a)–(d), } 0.3 \text{ meV (b)–(c), and } 0.85 \text{ meV (c)–(f). Other parameters are } \hbar \omega_c = 0.3 \text{ meV, } g_s = 0.10 \text{ meV, } L_x = 300 \text{ nm, and } \Delta \mu = 0.1 \text{ meV.} \]
range $\sim 1.60$–1.63 meV which is out of resonance. The photon replica of the first-excited state with photon content 2.53 around 1.69–1.71 meV, and the photon replica of second-excited state containing approximately one photon at $\sim 1.74$–1.77 meV. In the low cyclotron energy range $h\omega_c < h\omega_\gamma$, all three channels mentioned above are active in the electron transport with a contribution of 30% for each state. This is because the photon replica of first-excited state with approximately three photons is entangled with the third-excited state. Activation of a photon replica of the first-excited state causes participation of the third-excited state in the electron propagation. The mean value of photons as a function of cyclotron energy and MB energy for the two entangled states is shown in figure 6(b). We see that the photon content of the third excited state decreases by tuning the cyclotron energy to $h\omega_c > h\omega_\gamma$, the photon content of the third excited state is approaching zero. So, the third excited state is blocked. The photon replica of the first-excited state with approximately three photons is also getting weak in transport because these two states are entangled. So, the main active state in the transport is the photon replica of second excited state containing one photon. Therefore, the net charge current drops. The characteristics of the net charge current at $V_{pg} = 0.1$ meV are totally opposite to those of the net charge current at $V_{pg} = 0.4$ meV where the current is enhanced by increased cyclotron energy.

Figure 8 displays the charge density at $h\omega_c \approx 10^{-4}$ meV (a), 0.3 meV (b), and 0.85 meV (c) for the plunger gate $V_{pg} = 0.1$ meV and time $t = 220$ ps. In figure 8(a) the charge density is mostly distributed in the contact area to the leads indicating the dominance of the ‘para- and diamagnetic forces’ which cause its stretching.

By increasing the cyclotron energy to $h\omega_c \approx 0.85$ meV the Lorentz force is stronger than the ‘para- and diamagnetic forces’. The charge density indicates a circular motion around the QD as is demonstrated in figure 8(c). Consequently, the net charge current is reduced.

In the following, we shall show the influence of the photon cavity in the y-polarized photon field.

### 3.2. Photon cavity with y-polarization

In this section, we assume the photons in the cavity are polarized in the y-direction and the photon energy is fixed at $h\omega_c = 0.3$ meV with $\langle N_y \rangle \approx 2$. The energy spectrum of the central system in the y-polarized photon field is very similar to that of the x-polarization displayed in figure 2(b). Figure 9 shows the net charge current versus the cyclotron energy at $t = 220$ ps and $V_{pg} = 0.4$ meV of the QD system without (w/o) (blue circles) and with photon (w ph) cavity in the case of

![Figure 7](image-url)
N R Abdullah et al.

8

...net charge current in the absence of the cavity was explained in section 3.1. Now we can clearly see that the net charge current is not influenced by the $y$-polarized photon field for this selected photon energy due to the anisotropy of the system. The photon energy is far from resonance with any electron state representing excitations in the $y$-direction.

4. Conclusions

We have investigated the influences of a static magnetic and a dynamic photon fields on transport of electron through a quantum dot system in a quantized photon cavity in an external perpendicular magnetic field. The photons are assumed to be polarized either parallel or perpendicular to the electron propagation in the QD system. The quantum dot system is connected to two leads and a non-Markovian master equation is used to describe time evolution of the electrons in the system [23]. The motion of electrons in the system is influenced by the Lorentz force caused by the external magnetic field and the ‘para- and diamagnetic forces produced by the photon field. We have studied the characteristics of the net charge current in the system for three different cases: First, the ‘para- and diamagnetic forces’ are dominant where the cyclotron energy is less than the photon energy, Second, the static and the dynamic forces are approximately ‘equal’ when the cyclotron energy is ‘equal’ to the photon energy, Third, the Lorentz force is dominant where the cyclotron energy is larger than the photon energy. In the first situation, the net charge current is enhanced because the dia- and paramagnetic forces extend the electron charge outside the quantum dot into the contact area of the external leads. In the third case an opposite situation happens because the Lorentz force tends to shrink the electron charge towards the quantum dot inducing a circular motion. Consequently, the net charge current is suppressed.

Here, we present results for the cases when we have initially two photons in the cavity. We have confirmed that qualitatively the same phenomena are seen for the case of one photon initially in the cavity. Quantitatively, the crossing point for the strength of the cyclotron motion and the photon polarization happens for the same magnetic field in both cases. As has been noted before [20] two photons initially slightly reduce the net current through the system compared to one photon in the case the photons are not in resonance with the electron system.

This investigation shows that the transport through a cavity photon system depends on the balancing of the external forces and not only on the question which energy levels are close to resonance with the photon energy. Essential in these phenomena is the response of the charge density, or the information in the wavefunctions, to the external forces and thus the geometry of the system.

Acknowledgments

Financial support is acknowledged from the Icelandic Research and Instruments Funds, and the Research Fund of the University of Iceland. The calculations were carried out on the Nordic high Performance computer (Gardar). We acknowledge the University of Sulaimani, Iraq, the Nordic network NANOCONTROL, project No.: P-13053, and the
Ministry of Science and Technology, Taiwan through Contract No. MOST 103-2112-M-239-001-MY3.

References

[1] Fujisawa T, van der Wiel W G and Kouwenhoven L P 2000 Inelastic tunneling in a double quantum dot coupled to a bosonic environment Physica E 7 413–9
[2] Ishibashi K and Aoyagi Y 2002 Interaction of electromagnetic wave with quantum dots Physica B 314 437–43
[3] Shibata K, Umeno A, Cha K M and Hirakawa K 2012 Photon-assisted tunneling through self-assembled InAs quantum dots in the terahertz frequency range Phys. Rev. Lett. 109 077401
[4] Kouwenhoven L P, Jauhar S, McCormick K, Dixon D, McEuen P L, Nazarov Yu V, van der vaart N C and Foxon C T 1994 Photon-assisted tunneling through a quantum dot Phys. Rev. B 50 2019–22
[5] Stooft T H and Nazarov Yu V 1996 Time-dependent resonant tunneling via two discrete states Phys. Rev. B 53 1050–3
[6] Torres L E F F 2005 Mono-parametric quantum charge pumping: interplay between spatial interference and photon-assisted tunneling Phys. Rev. B 72 245339
[7] Reichl L E and Snyder M G 2005 Coulomb entangler and entanglement-testing network for waveguide qubits Phys. Rev. A 72 032330
[8] Imamoglu A and Yamamoto Y 1994 Turnstile device for heralded single photons: Coulomb blockade of electron and hole tunneling in quantum confined p-i-n heterojunctions Phys. Rev. Lett. 72 210–3
[9] Loss D and DiVincenzo D P 1998 Quantum computation with quantum dots Phys. Rev. A 57 120
[10] DiVincenzo D P 2005 Double quantum dot as a quantum bit Science 309 2173
[11] Chuang L I and Nielsen M A 2010 Quantum Computation and Quantum Information (Cambridge: Cambridge University Press)
[12] Souza F M, Carrara T L and Vernek E 2011 Enhanced photon-assisted spin transport in a quantum dot attached to ferromagnetic leads Phys. Rev. B 84 115322
[13] Ihn T 2010 Semiconductor Nanostructures (New York: Oxford University Press)
[14] Abdullah N R, Tang C-S and Gudmundsson V 2010 Time-dependent magnetotransport in an interacting double quantum wire with window coupling Phys. Rev. B 82 195325
[15] Harris J, Akis R and Ferry D K 2001 Magnetically switched quantum waveguide qubit Appl. Phys. Lett. 79 2214
[16] De Wilde J P, Rivers A W and Price D L 2005 A review of current use of magnetic resonance imaging in pregnancy and safety implication for fetus Prog. Biophys. Mol. Biol. 87 335
[17] Maisen C, Scalari G, Valmorra F, Beck M, Faist J, Cibella S, Leoni R, Reichl C, Charpentier C and Wegscheider W 2014 Ultrastrong coupling in the near field of complementary split-ring resonators Phys. Rev. B 90 205309
[18] Hagemüller D and Ciuti C 2012 Cavity QED of the graphene cyclotron transition Phys. Rev. Lett. 109 267403
[19] Abdullah N R, Tang C S, Manolescu A and Gudmundsson V 2013 Electron transport through a quantum dot assisted by cavity photons J. Phys.: Condens. Matter 25 465302
[20] Gudmundsson V, Jonasson O, Arnold Th, Tang C-S, Goan H-S and Manolescu A 2013 Stepwise introduction of model complexity in a general master equation approach to time-dependent transport Fortschr. Phys. 61 305
[21] Arnold T, Tang C-S, Manolescu A and Gudmundsson V 2015 Excitation spectra of a quantum ring embedded in a photon cavity J. Opt. 17 015201
[22] Jonasson O, Tang C-S, Goan H-S, Manolescu A and Gudmundsson V 2012 Nonperturbative approach to circuit quantum electrodynamics Phys. Rev. E 86 046407
[23] Gudmundsson V, Guinan C, Tang C-S, Moldoveanu V and Manolescu A 2009 Time-dependent transport via the generalized master equation through a finite quantum wire with an embedded subsystem New J. Phys. 11 113007
[24] Haake F 1973 Quantum statistics in optics and solid-state physics Quantum Statistics in Optics and Solid-state physics (Springer Tracts in Modern Physics vol 66) ed G Hohler and E A Niekisch (Berlin: Springer) p 98
[25] Bednorz A and Belzig W 2008 Formulation of time-resolved counting statistics based on a positive-operator-valued measure Phys. Rev. Lett. 101 206803
[26] Gudmundsson V, Jonsson T H, Bermudsson M L, Abdullah N R, Sitek A, Goan H S, Tang C-S and Manolescu A 2016 arXiv:1605.08248