Evolution of anisotropic distributions of weakly charged heavy ions downstream collisionless quasiperpendicular shocks

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Abstract. Hybrid simulations of quasiperpendicular collisionless shocks (QRCS) in multicomponent plasmas have revealed that weakly charged ions can keep anisotropic nonequilibrium distributions at relatively long distances downstream the shock front. However, analytic considerations suggest that such distributions lead to growth of the Alfvén ion cyclotron (AIC) instability, which in turn governs isotropisation. Hence, we have conducted targeted hybrid simulations of QRCS with increased accuracy, which appeared to agree with the analytical predictions and suggest that the nonequilibrium distributions eventually relax due to the AIC instability. On another hand, downstream electromagnetic instabilities generated by anisotropic particle distributions may affect the energy balance, therefore influencing macroscopic shock parameters, such as the compression ratio. Simulations of QRCS with parameters close to those of the heliospheric termination shock measured by the Voyager mission, showed that an admixture of just 0.7\% (by mass) of oxygen decreases the compression ratio by 3-4\%. Thus, one might conclude that along with the effect of the pickup ions, the multi-species composition of the interplanetary plasma can add to the observed decrease of the shock compression.

1. Introduction
Shock waves provide a universal mechanism of conversion of kinetic energy of supersonic flows into the thermal energy of plasma, as well as into the energy of amplified magnetic fields and of suprathermal particles. In the strong enough collisionless shocks (typically, of a Mach number above a few depending on the shock obliquity) the electron resistivity cannot provide energy dissipation fast enough to create a quasi-steady shock transition on a microscopic scale \cite{1–3}. The structure of the shock transition is governed by the instabilities of the ion flows and such shocks are called supercritical\cite{see, e.g., [4]; [5]; [6]). At the microscopic scale the front of a supercritical shock is a transition region occupied by magnetic field fluctuations of an amplitude $\delta B/B \gtrsim 1$ and characteristic frequencies of about the ion gyro-frequency. Generation of the fluctuations is due to instabilities in the interpenetrating multi-flow ion movements. The width of the viscous transition region of a collisionless shock typically reaches a few hundred ion inertial lengths defined as $l_i = c/\omega_{pi} \approx 2.3 \times 10^7 n^{-0.5} \text{ cm}$. Here $\omega_{pi}$ is the ion plasma frequency and $n$ is the ionized ambient gas number density measured in $\text{ cm}^{-3}$. The width of the collisionless...
shock transition region is smaller by many orders of magnitude than the Coulomb mean free path in the interstellar or intergalactic medium [7], so particle relaxation is due to collective electromagnetic field fluctuations. In such an environment nonequilibrium states can arise and be kept for distances much larger than the characteristic shock transition lengths. Particularly, quasi-stationary anisotropic velocity distributions of a small admixture of weakly charged heavy ions downstream QRCS (i.e., the shocks with initial magnetic field directed nearly transversely to the shock normal) were found by [8,9] via hybrid plasma simulations.

Here we present a detailed modeling of AIC instability caused by such ion distributions. Due to the high nonlinearity of a collisionless shock and the strong coupling between electromagnetic fluctuations and the motions of charged particles, pure analytic studies of such processes are possible only in some special cases and still provide only approximate results. Hence we combine an analytic approach with numerical hybrid simulations of QRCS (see [8,9] for more detailed discussions of this approach). In section 2 the hybrid code and initial parameters of a simulation are briefly described. In section 3 the resulting nonequilibrium distributions are shown and both analytical and numerical treatments of AIC instability are discussed, as well as their possible application of to the physics of the heliospheric termination shock.

2. Hybrid Simulations of Shocked Plasma Flows

A 3d second-order accurate particle-mesh hybrid code with exact magnetic field divergence conservation was used for simulations of QRCS evolution. The main feature of a hybrid code is that it combines hydrodynamic and particle-in-cell approaches, treating ions as particles and electrons as a neutralizing massless fluid ([10]). This allows one to self-consistently model processes governed by ions at time scales up to several thousand inverted ion gyrofrequencies (Ω⁻¹. A standard system of equations for a hybrid code is given in [11]. These equations are solved within a cyclic leapfrog approach with particle coordinates and velocities given at the odd timesteps and fields – at the even ones. A “Reconstruct-Solve-Average” algorithm including finite-volume discretisation, piezewise-parabolic reconstruction and linearized Roe-type Riemann solver (see, e.g., [12]) was implemented in the code to achieve the exact magnetic field divergence conservation. Triangular-shaped cloud representation of the ions and Boris particle pusher (see, e.g., [13]) were employed to move particles and calculate induced currents with second-order accuracy in time and space. The shock was initialized via the standard piston method, where a supralfvenic flow reflects from a conducting wall and forms the front moving oppositely to the initial flow.

3. Evolution of Heavy Ion Distributions

3.1. Nonequilibrium phase spaces and distributions

According to [8,9], nonequilibrium heavy ion distributions appear for various QRCS parameters, provided that the ion charge-to-mass ratio and relative density is low enough. For an illustrative example shown in Fig. 3.1 plasma parameters typical for a supernova reverse shock were chosen: the upstream Alfven Mach number (in the downstream frame) MA = 9.0, the inclination angle (between the direction of the initial magnetic field and the shock normal) θ = 90° (so the shock was exactly perpendicular), and the ratio of thermal to magnetic pressure β = 1.0. The reflecting wall was at x = 0, the initial flow moved to the left, and the initial number of model particles per cell (ppc) was equal to 200. Plasma consisted of 99% O IV and 1% Si II (by mass). Figure 3.1 shows the resulting phase spaces of the main element (left panels) and heavy ion admixture (middle panels) at t = 100Ω⁻¹, where Ω = qmainB/mmainc is the main ions sort (O IV) gyrofrequency. The corresponding spatially averaged velocity distributions are given at the right panels. It is clear that while the main element phase spaces and distributions are isotropic and homogenous, the Si II ones show peculiar quasi-periodic structure, associated with heavy ions gyromotion downstream the shock. The averaged Si II velocity distributions
are substantially anisotropic with preferred magnetic field direction (which is along $z$ in this simulation).

Figure 1. The O IV and Si II $x-V$ phase spaces and velocities distributions in shock with $M_a = 9.0$, $\theta = 90^\circ$ and $\beta = 1.0$ at $t = 100\Omega_i^{-1}$. All lengths are given in the main element $l_i$, while velocities are in the far upstream $V_a = B_0/\sqrt{(4\pi\rho)}$.

Figure 2. Solution of the dispersion equation for medium with macroscopic parameters and SiII distribution anisotropy taken from the simulated downstream region of the same shock as in figure 3.1. Positive $\gamma$ indicate instability growth.

3.2. Alfven ion cyclotron instability
In paper [14] it was shown (with application to thermonuclear fusion) that temperature anisotropy of hot minority ions population with isotropic equilibrium background can drive the AIC instability, which leads to the hot minority distribution relaxation. In the kinetic
approach the distribution details appeared to be unimportant, so just the distribution moments are included in the following dispersion equation:

\[ \Omega^2 - c^2 k^2 + \omega^2 \frac{\Omega}{k V_T |e|} Z(\xi^+_{e}) + \omega^2 \frac{\Omega}{k V_T |i|} Z(\xi^+_{i}) + \omega^2 \frac{\Omega}{k V_T |a|} Z(\xi^+_{a}) + \omega^2 \frac{\Omega}{k V_T |\alpha|} (1 + \xi^+_{\alpha} Z(\xi^-_{\alpha})) = 0 \]  

where \( Z(\xi) = \pi^{-0.5} \int_{-\infty}^{\infty} dt \cdot \exp(-t^2) / (t - \xi) \), \( \xi_{j}^{\pm} = \Omega \pm \Omega_j/V_T |j|, \quad \Omega_j = q_j B/m_j \), \( \omega^2_{\alpha} = 4\pi n_j q_j^2/m_j \) are \( j \)th species gyro- and plasma frequencies and \( V_T |j|, T_{\parallel |j|}, T_{\perp |j|} \) - their thermal velocities and temperatures with respect to the averaged magnetic field direction, \( i \) index corresponds to the main sort, while \( \alpha \) - to the admixture. The solution of this equation (real and imaginary parts of \( \Omega \)) is plotted versus \( k \) in Figure 3.1 for parameters and SiII anisotropy from model downstream region. The wave vector is given in inversed \( k \) and frequencies in \( \Omega \). It can be seen that there persists positive increment area, which means the instability growth, though the maximum increment for such low SiII density is only \( 10^{-2} \Omega \), which gives the characteristic growth time about \( \tau \sim 500\Omega^{-1} \). In order to numerically confirm these analytical results we made the following test run of the hybrid code: one period of heavy ions phase space was placed in a periodic box with uniform averaged downstream magnetic field and equilibrium downstream main sort population. In order to avoid numerical effects and make visible the expected tiny AIC instability effect we increased number of \( ppc \) up to 2000. The resulting phase spaces and velocity distributions evolution for \( ppc=200 \) are shown in Figure 3.2, while the space-averaged magnetic field variance evolution for cases of \( ppc=200 \) and \( ppc=2000 \) is given in Figure 3.2. It is clear that for the case \( ppc=200 \) numerical noise is greater than the effect we consider, while for \( ppc=2000 \) there can be seen even two instabilities. The first one grows up very rapidly and is substantially damped up to \( t = 200\Omega^{-1} \). It gives rise to magnetic field fluctuations propagating along \( x \) axis, so it cannot be associated with AIC instability, which propagates along the averaged magnetic field direction (\( z \) in our case). Probably it is governed by heavy ions spatial inhomogeneity, which was not included in analytic studies and is valuably reduced with time, as can be seen from Figure 3.2. The second, slowly growing, type of instability gives rise to magnetic field fluctuations, transverse to the magnetic field direction and propagating along it with nearly alfven speed. Its visible growth is roughly consistent with characteristic time \( \tau \sim 500\Omega^{-1} \), obtained in kinetic theory approach. Figure 3.2 also indicates slow velocity distribution evolution towards isotropy. So it can be concluded that anisotropic heavy ion distributions drive AIC instability, which eventually leads to their relaxation on scales much less that Coulomb ones, but much greater than characteristic shock transition lengths.

3.3. Termination shock simulations

The effect of slow heavy ions temperature relaxation downstream QRCS leads to increased magnetic field fluctuations level in the vicinity of shock transition. This effect can therefore change the total energy balance and affect the Rankine-Hugonio relations. As Supernova remnant observations are indirect and cannot reveal such a local effect, we tested this hypothesis on the shock with parameters close to the Termination shock ones, measured by Voyager2 ( [15]): \( M_a = 4.0 \) in downstream frame, \( \theta = 82^\circ, \beta = 0.01 \). Actual Solar wind composition includes about 85% of H II, 14.2% of He III and about 0.7% of heavier ions (by mass). So three test runs were done to estimate the heavy ions impact on the shock compression ratio: the first – for the pure Hydrogen shock, the second – for the shock consisting of H II and He III with Solar abundancies and the third one with admixture of 0.7% of O VII. For all cases the averaged downstream density was measured. The resulting values for the proton and proton-helium shocks coincided within the uncertainty, while addition of only 0.7% of O VII appeared to decrease the downstream density by 3-4%. The O VII distributions appeared to be anisotropic
Figure 3. The O IV and Si II x – V phase spaces and velocity distributions evolution in the test run (see text)

in the latter case, and the corresponding level of magnetic field fluctuations was higher than without Oxygen. This may lead to the conclusion that pickup ions are not the only reason of the observed low Termination shock compression ratio ([15]), but plasma composition can also affect this parameter. However, to check this statement, the full model including both pickup ions and heavy ions is needed, which is out of the scope of this paper.

4. Conclusions
The two approaches such as solution of plasma kinetic dispersion equation and hybrid modeling with increased number of particles per cell (ppc = 2000) follow to the same conclusion that anisotropic heavy ions distribution, arising downstream QRCS, lead to emerging of alfvén ion cyclotron instability, which might grow rather slowly in case of small heavy ions relative density. Anyway, this instability has two consequences:

(i) Relaxation of anisotropic ion distributions to isotropic maxwellian ones on scales much less than the Coulomb length for typical astrophysical objects
(ii) Increasing level of magnetic field fluctuations in the vicinity of transition, which can lead to changing of macroscopic shock parameters. The latter effect was illustrated on the example of the Termination shock.

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