PROPERTIES OF STRONGLY PRIME IDEALS AND $C$-IDEALS IN POSETS

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Abstract. In this paper, the concepts of $C$-ideal are defined and explored the various properties $C$-ideals in posets. The equivalent conditions for an ideal to be a $C$-ideal is obtained. Further the relations between strongly prime ideals and $C$-ideals are discussed.

Keywords: poset; ideals; strongly prime ideal; strongly $m$-system; $C$-ideals.

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1. INTRODUCTION

The concept of $z$-ideals, which are both algebraic and topological objects played a fundamental role in studying the ideal theory of $C(X)$, the ring of continuous real-valued functions on a completely regular Hausdorff space $X$.

In 1973, Mason[6] studied $z$-ideals of commutative rings and he proved that maximal ideals, minimal prime ideals and some other important ideals in commutative rings are $z$-ideals.

An ideal $I$ of a commutative ring $R$ is called a $z$-ideal if for each $a \in I$, the intersection of all maximal ideals containing $a$ is contained in $I$.

The concept of $z^0$-ideals is nothing but the generalization of $z$-ideals. In 2006, K.Samei[7] studied $z^0$-ideals and some special commutative ring.

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Let $I$ and $J$ be two ideals of a commutative ring $R$. $I$ is said to be a $z^l$-ideal if $M_a \cap J \subseteq I$, for every $a \in I$, where $M_a$ is the intersection of all maximal ideals containing $a$.

Whenever $J \not\subseteq I$ and $I$ is a $z^l$-ideal, we say that $I$ is a relative z-ideal. This special kind of $z$-ideals introduced and investigated by F. Azarpanah and A. Taherifar in [2].

In 2013, A.R. Aliabad, et.al., have shown that $I$ is a relative z-ideal and the converse is also true for each finitely generated ideal in $C(X)$.

Hence it is natural to study the analogues concept of $z$-ideals and $Z^0$-ideal in lattices and posets. In this paper, we introduced and studied $\mathcal{C}$-ideals in posets. We discussed the relation between $z^l$ ideal and $\mathcal{C}$-ideals in posets and obtained some characterizations.

2. Preliminaries

Throughout this paper $(X, \leq)$ denotes a poset with least element 0. For basic terminology and notation for posets, we refer [5] and [4]. For $E \subseteq X$, let $E^l = \{x \in X : x \leq e \text{ for all } e \in E\}$ denotes the lower cone of $E$ in $X$ and dually, let $E^u = \{x \in X : e \leq x \text{ for all } e \in E\}$ be the upper cone of $E$ in $X$.

Let $E,F \subseteq X$, we shall write $(E,F)^l$ instead of $(E \cup F)^l$ and dually for the upper cones. If $E = \{e_1,e_2,\ldots,e_n\}$ is finite, then we use the notation $(e_1,e_2,\ldots,e_n)^l$ instead of $(\{e_1,e_2,\ldots,e_n\})^l$ (and dually).

It is clear that for any subset $E$ of $X$, we have $E \subseteq E^{ul}$ and $E \subseteq E^{lu}$. If $E \subseteq F$, then $F^l \subseteq E^l$ and $F^u \subseteq E^u$. Moreover, $E^{lul} = E^l$ and $E^{ulu} = E^u$.

Following [8], a non-empty subset $K$ of $X$ is called semi-ideal if $b \in K$ and $a \leq b$, then $a \in K$. A subset $K$ of $X$ is called ideal if $a,b \in K$ implies $(a,b)^{ul} \subseteq K$[5].

A proper semi-ideal (ideal) $K$ of $X$ is called prime if $(a,b)^l \subseteq K$ implies that either $a \in K$ or $b \in K$ [4].

An ideal $K$ of $X$ is called semi-prime if $(a,b)^l \subseteq K$ and $(a,c)^l \subseteq K$ together imply $(a,(b,c)^u)^l \subseteq K$[5]. Given $e \in X$, $(e) = L(e) = \{x \in X : x \leq e\}$ is the principal ideal of $X$ generated by $e$.

Following [3], an ideal $K$ of $X$ is called strongly prime if $(A^*,B^*)^l \subseteq K$ implies that either $A \subseteq K$ or $B \subseteq K$ for any different proper ideals $A, B$ of $K$, where $A^* = A \setminus \{0\}$.

Following [3], a non-empty subset $E$ of $X$ is called $m$-system if for any $e_1,e_2 \in E$, there exists $r \in (e_1,e_2)^l$ such that $r \in E$. 
As a generalization of $m$-system, we define the notion of strongly $m$-system as follows, a non-empty subset $E$ of $X$ is called strongly $m$-system if $A \cap E \neq \phi$ and $B \cap E \neq \phi$ implies $(A^*, B^*) \cap E \neq \phi$ for any proper ideals $A, B$ of $X$.

It is clear that an ideal $K$ of $X$ is strongly prime if and only if $X \setminus K$ is a strongly $m$- system of $X$. Also every strongly $m$-system is $m$-system. But the converse need not be true in general.

For an ideal $K$ of $X$, a strongly prime ideal $Q$ of $X$ is said to be a minimal strongly prime ideal of $K$ if $K \subseteq Q$ and there exists no strongly prime ideal $R$ of $X$ such that $K \subset R \subset Q$.

The set of all strongly prime ideal of $X$ is denoted by $S\text{spec}(X)$ and the set of minimal strongly prime ideals of $X$ is denoted by $S\text{min}(X)$. For any ideal $K$ of $X$, $SP(K)$ denotes the intersection of all strongly prime ideals of $X$ containing $K$ and $SP(X)$ denotes the intersection all strongly prime ideal of $X$.

If $K = \{0\}$, then we denote $SP(K) = SP(X)$. From [4], the intersection of all prime semi-ideal of $X$ containing $K$ is $K$ for any semi-ideal $K$ of $X$. But the intersection of all strongly prime ideal of $X$ containing $K$ need not to be $K$ for any ideal $K$ of $X[3]$.

For any subset $K$ of $X$, we define $\psi(K) = \{Q \in S\text{spec}(X) : K \subseteq Q\}$, $\phi(K) = S\text{spec}(X) \setminus \psi(K)$, $\psi'(K) = \psi(K) \cap S\text{min}(X)$, $\phi'(K) = \phi(K) \cap S\text{min}(X)$ and $[K]$ is the smallest ideal of $X$ containing $K$. Also $SP(a) = \bigcap_{a \in \psi} \psi.$

For each $a \in X$ and an ideal $K$ of $X$, we define $X_a(K) = \cap\{Q \in S\text{spec}(X) : Q \in \psi'(K) \cap \psi'(a)\}$.

Following [3], let $J$ be an ideal of $X$. An ideal $I$ of $X$ containing $J$ is called $z'$-ideal if for each $a \in I$, we have $X_a(J) \subseteq I$. Also if $I$ is a $z'$ - ideal of $X$, then $X_a(J) \neq X$ for any $a \in I$. Clearly every strongly prime ideal of $X$ is $z'$-ideal. But the converse need not be true always.

3. **Main Results**

**Definition 3.1.** Let $X$ be a poset and $I$ be an ideal of $X$. Then $I$ is called $C$-ideal of $X$ if $\psi(a) \subseteq \psi(b)$ and $a \in I$ implies $b \in I$.

**Theorem 3.1.** Every strongly prime ideal is a $C$-ideal of $X$. 
Proof: Let $S$ be a strongly prime ideal of $X$ and $\psi(a) \subseteq \psi(b)$, $a \in S$. Since $a \in S$, we have $S \in \psi(a)$ which implies $S \in \psi(b)$. Then $b \in S$. Hence $S$ is $\mathcal{C}$-ideal. \hfill \Box

Corollary 3.1. Let $I$ be a maximal strongly semi-prime ideal of $X$. Then $I$ is $\mathcal{C}$-ideal.

The following example gives the converse of the theorem 3.1 is need not be true in general.

Example 3.1. Consider $X = \{0, 1, 2, 3, 4\}$ and define a relation $\leq$ on $X$ as follows.

\[
\begin{array}{c}
4 \\
3 \\
1 \\
0 \\
2 \\
\end{array}
\]

Then $(X, \leq)$ is a poset and $I_1 = \{0, 1\}$ is a $\mathcal{C}$-ideal of $X$. But not a Strongly prime ideal as we take $I_2 = \{0, 1, 2\}$ and $I_3 = \{0, 1, 3\}$, we have $L(I_2, I_3) \subseteq I_1$ with $I_2 \not\subseteq I_1$ and $I_3 \not\subseteq I_1$. \hfill \Box

Theorem 3.2. Let $S$ be a unique strongly prime ideal of $X$ and an ideal $I$ of $X$ such that $I \subset S$. Then $I$ is not a $\mathcal{C}$-ideal of $X$.

Proof: Let $I \subset S$. Then there exists a $x \in S \setminus I$. Since $I$ is a unique strongly prime ideal of $X$, we have $\psi(x) = \psi(i)$ for all $i \in I$ which gives $I$ is not a $\mathcal{C}$-ideal of $X$. \hfill \Box

Example 3.2. Consider $X = \{0, a, b, c, d, e\}$ and define a relation $\leq$ on $X$ as follows.

\[
\begin{array}{c}
e \\
d \\
c \\
b \\
a \\
0 \\
d \\
c \\
a \\
b \\
\end{array}
\]

Then $(X, \leq)$ is a poset and $I_1 = \{0, a, b, c\}$ is the only strongly prime ideal of $X$ and if we take any proper ideal $I_1$ like $K = \{0, b\} \subset I$ which is not $\mathcal{C}$-ideal. \hfill \Box
Theorem 3.3. Let $X$ be a poset and $a, b \in X$. Then the following statements hold.

(i) $SP((a, b)^l) = SP(a) \cap SP(b)$.

(ii) If $\psi(b) \subseteq \psi(a)$, then $\psi((b, c)^l) \subseteq \psi((a, c)^l)$ for any $c \in X$.

Proof: (i) Let $t \in SP((a, b)^l)$ and $t \notin SP(a) \cap SP(b)$. Without loss of generality, assume that $t \notin Q_1$ for a strongly prime ideal $Q_1$ containing $a$. Since $t \in SP((a, b)^l) \subseteq Q_1$, a contradiction. Hence $SP((a, b)^l) \subseteq SP(a) \cap SP(b)$.

Now, let $r \in SP(a) \cap SP(b)$ and $r \notin SP((a, b)^l)$. Then there exists a strongly prime ideal $Q_2$ containing $(a, b)^l$ and $r \notin Q_2$. Since $Q_2$ is strongly prime ideal and $((a)^*, (b)^*)^l \subseteq (a, b)^l \subseteq Q_2$, we have $(a) \subseteq Q_2$ or $(b) \subseteq Q_2$. Without loss of generality, assume that $a \in Q_2$. As $r \in SP(a) \subseteq Q_2$, a contradiction. Hence $SP((a, b)^l) = SP(a) \cap SP(b)$.

(ii) Let $\psi(b) \subseteq \psi(a)$ for $a, b \in X$ and $S$ be a strongly prime ideal of $X$ containing $(b, c)^l$. Then $S \in \psi((b, c)^l)$ which implies $((b)^*, (c)^*)^l \subseteq S$. Since $S$ is a strongly prime ideal of $X$, we have $(b) \subseteq S$ or $(c) \subseteq S$.

Case 1: If $(c) \subseteq S$, then $(a, c)^l \subseteq S$ which implies $S \in \psi((a, c)^l)$.

Case 2: If $(b) \subseteq S$, then $S \in \psi(b) \subseteq \psi(a)$ which gives $a \in S$ and $(a, c)^l \subseteq S$. Hence $S \in \psi((a, c)^l)$.

\[ \square \]

Theorem 3.4. Let $X$ be a poset and $a, b \in X$. Then $a \in SP(b)$ if and only if $SP(a) \subseteq SP(b)$ if and only if $\Psi(b) \subseteq \psi(a)$.

Proof: Let $SP(a) \subseteq SP(b)$. Since $a \in SP(a)$, we have $a \in SP(b)$.

Now, suppose that $a \in SP(b) = \bigcap_{b \in Q \in \Psi} Q$ and $t \in SP(a)$.

Then $t \in \bigcap_{a \in Q \in \Psi} Q$.

Let $Q_1$ be any strongly prime ideal of $X$ and $b \in Q_1$.

As $a \in SP(b)$, we have $a \in Q_1$ which implies $t \in Q_1$ for all strongly prime ideals containing $b$. Hence $t \in SP(b)$ and $SP(a) \subseteq SP(b)$.

Let $SP(a) \subseteq SP(b) \iff \bigcap_{a \in Q_1} Q_1 \subseteq \bigcap_{b \in Q_2} Q_2$

$\iff \{Q_2 : b \in Q_2\} \subseteq \{Q_1 : a \in Q_1\}$

$\iff \psi(b) \subseteq \psi(a)$

\[ \square \]
**Theorem 3.5.** Let $J$ be an ideal of $X$. Then the following statements are equivalent

(i) $J$ is a $C$-ideal of $X$.

(ii) If $\psi(a) = \psi(b)$ and $b \in J$ implies $a \in J$.

(iii) $SP(a) \subseteq J$ for all $a \in J$.

(iv) If $SP(b) \subseteq SP(a)$ and $a \in J$ implies $b \in J$.

**Proof:** (i) $\Rightarrow$ (ii) It is obvious.

(ii) $\Rightarrow$ (iii) Let $t \in SP(a)$. Then by Theorem 3.4, $SP(t) \subseteq SP(a)$. Hence $SP(t) = SP(t) \cap SP(a)$ and by Theorem 3.3, $SP(t) = SP(L(a,t))$ which implies $\psi(t) = \psi(L(a,t))$. If $a \in J$, then $L(a,t) \subseteq J$. By (ii), $t \in J$.

(iii) $\Rightarrow$ (iv) Let $a \in J$. Then by (iii), $SP(a) \subseteq J$. Suppose $SP(b) \subseteq SP(a)$, then $b \in SP(b) \subseteq J$.

(i) $\Rightarrow$ (ii) It follows from Theorem 3.4. $\square$

**Theorem 3.6.** Let $X$ be a poset. If $I \cap M = \emptyset$ for a $C$-ideal $I$ and a strongly $m$-system $M$ of $X$. Then there exists a $C$-ideal $K$ of $X$ containing $I$ and disjoint from $M$ and $K$ is a strongly prime ideal of $X$.

**Proof:** Let $\mathcal{F} = \{J : J$ is an $C$-ideal containing $I$ and $J \cap M = \emptyset\}$. Since $I \in \mathcal{F}$, $\mathcal{F} \neq \emptyset$.

Let $\mathcal{X}$ be a chain $\mathcal{F}$ and $R = \bigcup_{J \in \mathcal{X}} J$.

To show that $R$ is a $C$-ideal of $X$, let $\psi(a) \subseteq \psi(b)$ and $a \in R$. Then $a \in J_i$ for some $i$. Since $J_i$ is a $C$-ideal of $X$, we have $b \in J_i$ and $b \in R$. Thus $R$ is a $C$-ideal of $X$.

By Zorn’s Lemma, there exists a maximal $C$-ideal $K$ such that $K \cap M = \emptyset$. Let $(A^*, B^*) \subseteq K$ and $A, B \notin K$. Then $[K \cup A] \cap M \neq \emptyset$ and $[K \cup B] \cap M \neq \emptyset$. Since $M$ is strongly $m$-system we have $([K \cup A], [K \cup B]) \subseteq M \neq \emptyset$ which implies $K \cap M \neq \emptyset$, a contradiction. So $A \subseteq K$ or $B \subseteq K$. Hence $K$ is a strongly prime ideal of $X$. $\square$

**Theorem 3.7.** Every $C$-ideal is a $z^I$-ideal of $X$.

**Proof:** Let $I$ be a $C$-ideal of $X$. To prove $I$ is $z^I$-ideal, for all $a \in I$ and $J \subseteq I$, let $x \in X_a(J)$.

Then $x \in \bigcap\{Q \in Sspec(X) : Q \in \psi(J) \cap \psi(a)\}$

$\Rightarrow x \in Q$ for all $Q \in \psi(J) \cap \psi(a) \subseteq \psi(a)$. 

⇒ \( x \in \psi(a) \) for all \( a \in I \)
⇒ \( x \in SP(a) \) for all \( a \in I \).

By Theorem 3.4, \( SP(x) \subseteq SP(a) \) which gives \( \psi(a) \subseteq \psi(x) \). Since \( I \) is a \( C \)-ideal of \( X \) and \( a \in I \), we have \( x \in I \). Hence \( X_a(J) \subseteq I \) for all \( a \in I \). So \( I \) is \( z^J \)-ideal.

Remark 3.1.

(1) In the above Example 3.1, \( I_1 = \{0, 1\} \) is both \( C \)-ideal and \( z^J \)-ideal of \( X \) if we take \( J = \{0\} \).

(2) In Example 3.2, \( I_1 = \{0, a, d\} \) is neither \( C \)-ideal nor \( z^J \)-ideal of \( X \).

The converse of the Theorem 3.7 need not be true in general. The below example gives a \( z^J \)-ideal of \( X \) which is not \( C \)-ideal.

**Example 3.3.** Consider \( X = \{0, a, b, c, d, e\} \) and define a relation \( \leq \) on \( X \) as follows.

```
\begin{array}{cccccc}
0 & a & b & c & d & e \\
\hline
a & 0 & a & b & c & d \\
b & 0 & b & e & d & c \\
c & 0 & c & b & e & d \\
d & 0 & d & c & b & e \\
e & 0 & e & d & c & b \\
\end{array}
```

Then \( (X, \leq) \) is a poset and \( I_1 = \{0, a, b, c\} \) and \( I_2 = \{0, a, b, d\} \) are the strongly prime ideals of \( X \). \( I = \{0, b\} \) is a \( z^J \)-ideal of \( X \) for \( J = \{0\} \). But \( I \) is not a \( C \)-ideal of \( X \) as \( \psi(b) \subseteq \psi(a) \) with \( b \in I \) and \( a \notin I \).

**Remark 3.2.** For any ideal \( J \) of \( X \), \( J^C = \bigcap \{K : K \text{ is a } C \text{-ideal of } X \text{ and } K \supseteq J\} \).

**Theorem 3.8.** For an ideal \( J \) of \( X \), \( J^C \) is the least \( C \)-ideal Containing \( J \).

**Proof:** Let \( \psi(b) \subseteq \psi(a) \) and \( b \in J^C \). Then any arbitrary \( C \)-ideal \( Q_1 \) containing \( J \) and \( b \in Q_1 \) which implies \( a \in Q_1 \). So \( a \in J^C \). Hence \( J^C \) is a \( C \)-ideal of \( X \).

Let \( R \) be any \( C \)-ideal of \( X \) such that \( R \subseteq J^C \) and \( x \in J^C \). Then \( x \in R \). So \( J^C \subseteq R \) for all \( R \). Hence \( J^C \) is the least \( C \)-ideal of \( X \).
**Theorem 3.9.** Let $A$ and $B$ be any two ideals of $X$, then the following statements hold

(i) if $A \subseteq B$, then $A_{\mathcal{C}} \subseteq B_{\mathcal{C}}$.

(ii) $(A_{\mathcal{C}})_{\mathcal{C}} = A_{\mathcal{C}}$.

(iii) $(A \cup B)_{\mathcal{C}} \subseteq A_{\mathcal{C}} \cap B_{\mathcal{C}} \subseteq (A \cap B)_{\mathcal{C}}$

**Proof:** (i) Let $A \subseteq B$ and $t \in A_{\mathcal{C}} = \bigcap_{K \supseteq I} K$, where $K$ is a $\mathcal{C}$-ideal of $X$. If $t \notin B_{\mathcal{C}}$, then there exists a $\mathcal{C}$-ideal $J_1$ such that $t \notin J_1$ and $B \subseteq J_1$ which gives $A \subseteq J_1$. Since $t \in A_{\mathcal{C}}$, we have $t \in J_1$, a contradiction.

(ii) Clearly, $A_{\mathcal{C}} \subseteq (A_{\mathcal{C}})_{\mathcal{C}}$. Now, let $r \in (A_{\mathcal{C}})_{\mathcal{C}} = \bigcap_{K \supseteq A_{\mathcal{C}}} K$, where $K$ is a $\mathcal{C}$-ideal containing $A_{\mathcal{C}}$. But $A_{\mathcal{C}}$ is the least $\mathcal{C}$-ideal containing $A_{\mathcal{C}}$. Therefore $r \in A_{\mathcal{C}}$. Hence $(A_{\mathcal{C}})_{\mathcal{C}} = A_{\mathcal{C}}$.

(iii) It is trivial. □

**Remark 3.3.** For any ideal $J$ of $X$, $J_{\mathcal{C}} = \bigcup\{K : K$ is a $\mathcal{C}$-ideal of $X$ and $K \supseteq J\}$. If union of any two ideals of $X$ is again an ideal in $X$, then we can say that $X$ has $\xi$ property.

**Theorem 3.10.** Let $J$ be an ideal of $X$ and $X$ has $\xi$ property. Then $J_{\mathcal{C}}$ is the greatest $\mathcal{C}$-ideal Containing $J$.

**Proof:** Let $\psi(b) \subseteq \psi(a)$ and $b \in J_{\mathcal{C}}$. Then there exists a $\mathcal{C}$-ideal $Q_1$ of $X$ containing $J$ and $b \in Q_1$ which implies $a \in Q_1$. So $a \in \bigcup\{K : K$ is a $\mathcal{C}$-ideal of $X$ and $K \supseteq J\} = J_{\mathcal{C}}$. Hence $J_{\mathcal{C}}$ is a $\mathcal{C}$-ideal of $X$.

Let $A$ be any $\mathcal{C}$-ideal of $X$ such that $J_{\mathcal{C}} \subseteq A$ and $l \in A$. Then $l \in \bigcup\{K : K$ is a $\mathcal{C}$-ideal of $X$ and $K \supseteq J\}$. So $x \in J_{\mathcal{C}}$. Hence $J_{\mathcal{C}}$ is the greatest $\mathcal{C}$-ideal of $X$. □

**Theorem 3.11.** Let $E$ and $F$ be any two ideals of $X$, then the following statements hold

(i) if $E \subseteq F$, then $F_{\mathcal{C}} \subseteq E_{\mathcal{C}}$.

(ii) $(E_{\mathcal{C}})_{\mathcal{C}} = E_{\mathcal{C}}$.

(iii) $E_{\mathcal{C}} \subseteq E_{\mathcal{C}}$.

(iv) $(E \cup F)_{\mathcal{C}} \subseteq E_{\mathcal{C}} \cap F_{\mathcal{C}}$.
Proof: (i) Let $E \subseteq F$ and $t \in F^{E^c} = \bigcup_{K \supseteq F} K$, where $K$ is a $\mathcal{C}$-ideal of $X$. Then $t \in K_i$ for some $\mathcal{C}$-ideal $K_i$ of $X$ and $K_i \supseteq F \supseteq E$ which implies $t \in E^{E^c}$.

(ii) Clearly, $E^{E^c} \subseteq (E^{E^c})^{E^c}$. Now, let $r \in (E^{E^c})^{E^c} = \bigcup_{K \supseteq E^{E^c}} K$, where $K$ is a $\mathcal{C}$-ideal containing $E^{E^c}$. But $E^{E^c}$ is the greatest $\mathcal{C}$-ideal containing $E^{E^c}$. Therefore $r \in E^{E^c}$. Hence $(E^{E^c})^{E^c} = E^{E^c}$.

(iii) It is follows from Theorem 3.8 and Theorem 3.10.

(iv) It is trivial. □

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CONFLICT OF INTERESTS

The author(s) declare that there is no conflict of interests.

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