Fluid-structure coupling topology optimization based on added mass method

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Abstract A wet model topology optimization method based on added mass method and bidirectional evolutionary structure optimization algorithm is proposed to maximize the natural frequency of fluid-structure coupling system. The fluid-structure coupling equation of the finite element method and the sensitivity formula of the element are analyzed simultaneously, and then the coupling optimization model is established. Under the constraints of structure volume, the weighted natural frequencies of single-objective and multi-objective of the coupled system are maximized to optimize the underwater coupled structure. The numerical results show that the evolutionary methods can be applied to this kind of problem effectively and efficiently. Besides, the optimization results of cases have smooth boundaries and the numerical stability of the optimization algorithm is good. Under constant volume constraints, the single-order natural frequency is increased by more than 380%. The results of multi-objective weighted optimization show that the weighted coefficients are final. The larger the weighting coefficient is, the closer the topological form is to the optimal result of the model.

1. Introduction

The interaction between the structure of fluid-structure coupled system and the flow field caused by vibration is one of the main sources of noise. Therefore, it is very important to reduce structural noise through dynamic design in the design stage[1]. Topology optimization is a practical method for structural vibration and noise reduction. By optimizing the structure topology, this method can change the natural frequency of the structure and make the natural frequency of the structure far away from the vibration response frequency, thus reducing the noise[2-4]. Therefore, on the basis of meeting the functional requirements of the structure, it is of great practical significance and application value to use topology optimization algorithm to optimize the structure and change the natural frequency of the structure. Scholars at home and abroad have made some achievements in this regard.

Ye[5] used ICM topology optimization method to study the fluid-structure coupling frequency optimization of laminated structures. Sigmund[6] proposed an acoustic model controlled by Helmholtz equation and related boundary conditions, but did not consider the fluid-structure coupling effect. Then a hybrid finite element model combining displacement/pressure formula[7] is proposed, which could solve the problem of acoustic structure interaction without explicit coupling boundary. Akl[8] combined the MMA method with the vibration optimization of closed flexible plates. The numerical results agree well with the experimental data. Vicente et al. developed an extended optimization method for the structural problem of artificial fluid loading and applied it to the frequency optimization problem[9].

In order to optimize the natural frequencies of fluid-structure coupling systems, a wet mode
A topology optimization algorithm is proposed by combining the added mass method with the bidirectional evolutionary structure optimization algorithm. Based on the mathematical model of finite element added mass theory, the element sensitivity in wet mode is analyzed. It is proved that the optimization methods of single-order and multi-order weighted natural frequencies have good applicability.

2. Fluid-structure coupling vibration equation

2.1 Fluid-structure coupling model

The wave propagation of mechanical vibration is related to the particle which generates mechanical vibration and the motion state of the propagating medium. Here, it is assumed that the sound wave is a small amplitude wave. For non-viscous, non-rotating fluid region without considering heat transfer and volume force, the acoustic equation\(^\text{[10]}\) is

\[
\nabla^2 p_f + \frac{1}{c_0^2} \frac{\partial^2 p_f}{\partial t^2} = 0
\]

where \(c_0\) is the speed of sound in the fluid, \(p_f\) is the sound pressure.

The structural domain is expressed by the continuum motion balance equation considering small deformation. For the coupling boundary (the external force is the force of the fluid on the coupling surface), the equilibrium equation of the structure is

\[
\nabla \cdot \sigma_s n_s + \rho_s \ddot{u}_s = f_s
\]

where \(\sigma_s\) is the structural stress matrix, \(\rho_s\) is the density of structure, \(n_s\) is the boundary normal vector.

Write the equivalent weak integral form of its equilibrium equation and force boundary condition, and then apply the Gauss-Green formula to reduce the dimension of the integral domain to obtain the following structural static equilibrium equation.

\[
M_s \ddot{u}_s + K_s u_s = f_s
\]

On the basis of the displacement and pressure continuity on the coupling surface between the fluid domain and structural domain, the following conditions are satisfied on the coupling surface.

\[
\begin{align*}
\rho_f \ddot{u}_f &= \rho_s \ddot{u}_s \\
\sigma_s n_s &= \rho_s \ddot{u}_s \\
p_n &= p_f
\end{align*}
\]

where \(\rho_f\) is the fluid node displacement vector, \(\Omega_f\) and \(\Omega_s\) represent coupling surface.

The fluid-structure coupling equilibrium equation can be obtained as

\[
\begin{bmatrix}
M_s & 0 \\
-H^T M_f & p_f
\end{bmatrix}
\begin{bmatrix}
\ddot{u}_s \\
\ddot{p}_f
\end{bmatrix}
+ \begin{bmatrix}
K_s & H\
-H/\rho_f & K_f
\end{bmatrix}
\begin{bmatrix}
\ddot{u}_s \\
\ddot{p}_f
\end{bmatrix}
= \begin{bmatrix}
0 \\
0
\end{bmatrix}
\]

where \(H\) is the coupling matrix. If the fluid is assumed to be incompressible without regard to the effects of fluctuations in the free surface of the fluid, the above equation can be degraded into the form of additional water quality.

\[
\left( \begin{bmatrix}
M_s & HK_f^{-1} H^T/\rho_f
\end{bmatrix} \right) \ddot{u} + K_s u = 0
\]

2.2 Element coupling mode

Four-node quadrilateral linear element is used to discrete the domain, and four-node quadrilateral isoparametric fluid element is used to discrete the fluid domain. As shown in Figure 1, the figure is a four-node plane element with two typical coupling modes. The element node numbers of the numerical grid are in counterclockwise order, as shown by the arrows.

The fluid element variables at the two coupling nodes are \(p_1\) and \(p_2\). The structural element variables at the two nodes are displacements \(u_1, v_1, u_2, v_2\) in the local coordinate system \(x\) and \(y\), and the element shape functions \(n_s\) and \(n_f\), respectively. It can be determined by the relationship between these variables. It should be noted that the fluid cannot bear shear stress, so the structural
elements on the coupling surface can only bear the normal load of the fluid. Therefore, when generating the structural unit shape function on the coupling surface, in the shape function, the variables \( u_1 \) and \( u_2 \) of the x axis in the local coordinate system should be 0.

![Figure 1. Element coupling model](image)

### 3. Element sensitivity analysis and optimization algorithm

#### 3.1 Element sensitivity

The constraint for setting the natural frequency topology optimization is the volume constraint, and the objective function is to specify the maximum natural frequency of a certain order. The objective function and constraints are as follows

\[
C = \omega_j \\
V^* = \sum_{k=1}^{N} V_k x_k = 0, \ x_k \in [x_{\min}, 1]
\]

where \( V^* \) is the optimized volume, \( V_k \) is the volume of the \( k \)-th element, \( N \) is the total element number of design domain, \( x_k \) is the element state parameter, and only \( x_{\min} (1.0 \times 10^{-3}) \) and 1 value can be taken.

Equation (7) is written in the frequency domain form as

\[
-\omega_j^2 \left( M_x + H K_j H^T / \rho_f \right) + K_j u_j = 0
\]

The above formula is multiplied by \( u_j^T \) and obtaining the sensitivity of the objective function.

\[
\frac{d\omega_j}{dx_k} = u_j^T \left( \omega_j^2 \frac{\partial M}{\partial x_k} + \frac{\partial K}{\partial x_k} u_j \right) / 2\omega_j
\]

In order to prevent the local modality from being generated by the void element in the optimization process, the material property of the void element is changed based on the improved power law penalty function model.

\[
\begin{align*}
\rho_{(x_k)} &= x_k \rho^0 \\
E_{(x_k)} &= \frac{1 - x_{\min}}{1 - x_{\min}^p} (1 - x_k^p) + x_k^p E^0
\end{align*}
\]

The sensitivity of the objective function due to a structural element removal can be obtained by deriving \( \omega_j \) with respect to the design variables

\[
\alpha_k = \frac{u_k^T}{2\omega_j} \left( \frac{1-x_{\min}}{1-x_{\min}^p} p x_k^{p-1} K_k^0 - \omega_j^2 M_k^0 \right) u_k
\]

Multi-objective optimization is based on the weighted multi-order characteristic frequency as the objective function optimization, and its objective function is

\[
C = \mu_1 \omega_1 + \mu_2 \omega_2 + \ldots + \mu_i \omega_i
\]

Then the sensitivity of the \( k \)-th element under the objective function can be found as

\[
\alpha_k = \mu_1 \alpha_{1,k} + \mu_2 \alpha_{2,k} + \ldots + \mu_i \alpha_{i,k}
\]

where \( \mu_i \) is the weighting coefficient of the \( i \)-th natural frequency, and \( \alpha_{i,k} \) is the sensitivity of the \( k \)-th element in the \( i \)-th mode.
3.2 Grid sensitivity filtering

In order to avoid the problem of checkerboard mode and numerical divergence, grid sensitivity independent filtering method can be used to average the sensitivity of common node elements. After obtaining the sensitivity of nodes, the sensitivity of nodes must be redistributed to the element. The grid independence method is adopted. The element sensitivity is redistributed by calculating the weighted average of the sensitivity of \( N \) nodes in the \( x_{\text{min}} \) range of elements

\[
\alpha_{j,k} = \left( \sum_{m=1}^{N} \alpha_j^m \right) / N
\]  

(16)

In addition, the historical average value of element sensitivity can effectively improve the numerical stability, that is, the current element sensitivity can be corrected by the average value of the current iteration step and the last iteration step.

\[
\alpha_{j,k} = \frac{1}{2} \left( \alpha_{j,k}^{\text{iter}} + \alpha_{j,k}^{\text{iter}-1} \right)
\]  

(17)

4. Numerical examples and analysis

4.1 Coupling optimization of monophonic cavity structure

The first example considers a rectangular acoustic cavity (as shown in Figure 2). The initial design domain is a rectangular part of 1.6m \( \times \) 0.6m, the non-design domain of the design domain width is 0.04m, and the bottom of the non-design domain is fixed. The structure has density of 2700 kg/m\(^3\), Young's modulus of 70 GPa and Poisson's ratio of 0.3. The fluid has density of 1000 kg/m\(^3\) and sound velocity of 1450 m/s. The thickness of fluid, structure and coupling elements is 0.1 m. The target volume is maximized to 40% of the total design area, the filter radius is 3 times of the element size, and the penalty factor is 3.

Figure 3 shows the iteration process of the objective function and volume of rectangular acoustic cavity structure. Under the constraints of constant volume, the convergence speed of the algorithm is faster, the optimized iteration curve is more stable, and the numerical stability is better. The optimized first-order frequency is 437.9 Hz, which is 78.5% higher than the original structure of 49.5 Hz. The optimized third-order natural frequency is 527.7Hz, which is 385% higher than the initial frequency 108.8Hz. The optimization effect is remarkable.

Figure 4 shows the final optimization results of the rectangular cavity structure. The light-colored part of the structure is the structural element. It can be seen from the figure that the structural materials are redistributed, the original topological structure has changed greatly, and the boundary of the structure is relatively smooth.

Figure 2. Schematic diagram of rectangular acoustic cavity structure

Figure 3. Iterative process of objective function and volume
4.2 Coupling optimization of fixed beam system

Figure 5 shows the fixed beam system. The flow field is a rectangular part of 1m×0.7 m, and the upper and lower ends of the structure are fixed brackets. The specific size is shown in the figure. The two side walls of all fluid regions are zero sound pressure boundary conditions, and the contact part with the structure is a coupling boundary, while the rest is a rigid wall. The middle rectangular part is the initial design area, and the rectangular part on both sides is the design area. The physical parameters of structure and fluid are the same as those in Section 3.1.

In this section, four scenarios are set up. Under the constraints of constant volume, case-1 takes the first-order maximum natural frequency as the objective function. Case 2 takes the maximization of third-order natural frequencies as the objective function. Case 3 focuses on weighted optimization of first and third natural frequencies. The first-order weighting coefficient is 0.3 and the third-order natural frequency weighting coefficient is 0.7. Case 4 also aims at weighted optimization of first and third natural frequencies. The first-order weighting factor is 0.7, and the third-order natural frequency unit weighting factor is 0.3.

Figure 6 shows the optimization iteration process. The results show that the weighted coefficients have a great influence on the results of the third-order optimization. Under the condition of third-order weighting coefficients of two conditions, the optimized third-order frequency shows a downward trend. It's just that the droplets are different in size. Two second-order weighting coefficients can effectively improve the final optimization of second-order natural frequencies. The bigger the weighting coefficient is, the more obvious the improvement is.

Figure 7 shows the final optimization results of the fixed beam system. After optimization, there are no checkerboard grids and isolated grids, and the coupling boundary is relatively smooth. By comparing the optimal topology forms under different weighting coefficients, it is found that CASE-3 and case-2 are quite different due to the higher third-order weighting coefficients, and CASE-4 is closer to case-1. The reason is that in the optimization process, the increase of weighting coefficient is closer to the optimization of natural frequency.
5. Conclusion
The bidirectional progressive structure optimization algorithm is applied to the topology optimization design of fluid-structure coupling system, and a wet mode natural frequency topology optimization algorithm is proposed. The weighted optimization problems of single objective and multiple objectives are studied. The results show that the optimization results have smooth boundary, no chessboard phenomenon and fast convergence speed, which indicates that the numerical stability of the optimization algorithm is good. Under the constraints of constant volume, the single-order natural frequencies have been greatly improved. The results of multi-order weighted optimization show that the weighted coefficients have a great influence on the final optimization of the topology structure. The larger the weighting coefficient is, the closer the topological form is to the topological structure of the optimization model.

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