Operational Effectiveness Evaluation of Antimissile Early Warning Based on Combination Weighted TOPSIS

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Abstract. Operational effectiveness evaluation is of great significance for early warning operation of antimissile, which provides quantitative basis for equipment development, operational planning, combat method research and operational management. In view of the shortcomings of TOPSIS, this paper improves the TOPSIS method. First, before calculating positive and negative ideal solutions, it establishes corresponding standardized methods for different evaluation indexes to solve the problems of inconsistent impact of each evaluation index on operational effectiveness and the "drowning" of small value indexes by large value indexes. Second, it calculates the weight of each index based on combination weighting method to overcome the shortcomings of single subjective or objective weighted method. On this basis, the operational effectiveness evaluation method of antimissile early warning based on combination weighted TOPSIS is proposed, and the quantitative evaluation and comparative analysis of different operational schemes are carried out. The simulation results show that the effectiveness evaluation result of antimissile early warning operation based on combination weighted TOPSIS method is scientific.

Keywords: Operational effectiveness evaluation, Combination weighted, TOPSIS, AHP.

1. Introduction
Due to the complexity of ballistic missile target characteristics, antimissile early warning operation is very complex and difficult, which requires space-based, air-based, land-based, sea based and other early warning platforms to form a system of cooperative detection. The research on the operational effectiveness evaluation can quantitatively evaluate the operational effectiveness of antimissile early warning system, analyze the deficiencies in the operational theory, operational plan and operational equipment of antimissile early warning system, seek optimization methods, provide scientific support for the construction and operational application of antimissile early warning system, and provide effective methods for improving the operational effectiveness of antimissile early warning system. This paper starts from the characteristics of early warning operation of antimissile, takes the evaluation requirements of early warning operation of antimissile as the traction, improves the TOPSIS [1] [2] method according to its shortcomings, and puts forward the evaluation method of early warning operation efficiency of antimissile based on combination weighted TOPSIS, which is applied to the optimization of early warning operation scheme of antimissile.
2. Operational Effectiveness Evaluation Method of Antimissile Early Warning Based on Combination Weighted TOPSIS

2.1. Standardization of operational Effectiveness Evaluation Data for Antimissile Early Warning

According to the characteristics of operational effectiveness evaluation index of antimissile early warning, different standardized methods are established for benefit index, cost index and fixed index[3]. There are $n$ evaluation indexes $f_j(1 \leq j \leq n)$, $m$ early warning operation schemes $p_i(1 \leq i \leq m)$, and $N$ indexes of $M$ early warning operation schemes. The matrix $X=(x_{ij})_{mn}$ is called evaluation matrix. In the evaluation matrix $X=(x_{ij})_{mn}$, for the benefit index $f_j$, let

$$y_j = \frac{x_{ij} - \min_{i} x_{ij}}{\max_{i} x_{ij} - \min_{i} x_{ij}}, \quad (1 \leq i \leq m, 1 \leq j \leq n)$$

(1)

For the cost index $f_j$, let

$$y_j = \frac{\max_{i} x_{ij} - x_{ij}}{\max_{i} x_{ij} - \min_{i} x_{ij}}, \quad (1 \leq i \leq m, 1 \leq j \leq n)$$

(2)

For the fixed index, if $\alpha_j$ is the optimal value, then

$$y_j = \begin{cases} \frac{x_{ij}}{\alpha_j}, & x_{ij} \in [\min_{i} x_{ij}, \alpha_j] \\ 1 + \frac{\alpha_j}{\max_{i} x_{ij} - \min_{i} x_{ij}} - \frac{x_{ij}}{\max_{i} x_{ij}}, & x_{ij} \in [\alpha_j, \max_{i} x_{ij}] \end{cases}$$

(3)

The resulting matrix $Y=(y_{ij})_{mn}$ is a standardized evaluation matrix, and each index in the standardized evaluation matrix is unified as a benefit index, that is, the larger the index is, the greater the combat effectiveness is, and the "drowning" problem of small index is eliminated.

2.2. Weight Combination Weighting Method of Antimissile Early Warning Operational Effectiveness Evaluation Index

This paper proposes a combination weighting method, which determine subjective weight by AHP and determine objective weight by entropy weight, and then obtain the final weight through fusing subjective weight and objective weight.

2.2.1. Computing subjective weight by Analytic Hierarchy Process. The core of AHP is to build a paired comparison matrix for each evaluation index[4][5]. With the paired comparison matrix, we only need to give the importance of each two indexes, and then calculate the weight value of all indexes. Before constructing the paired comparison matrix, it is needed to quantify the paired comparison results, as shown in Table 1.

**Table 1.** Quantitative representation of relative importance of indicators.

| Value | Significance |
|-------|--------------|
| 1     | The last indicator is as important as the next one |
| 3     | The last indicator is slightly more important than the next one |
| 5     | The last indicator is more important than the next one |
| 7     | The last indicator is significantly more important than the next one |
| 9     | The last indicator is absolutely more important than the next one |
| 2, 4, 6 and 8 | are other intermediate values that can be used |
Suppose there are four evaluation indicators: first point discovery time, target recognition, tracking accuracy and tracking time[6]. A pairwise comparison matrix is constructed by pairwise comparison, as shown in Table 2. The values in the matrix indicate relative importance of indicators in the row relative to the indicators in the column. For example, element 3 in the first row and second column indicates that the first point discovery time is slightly more important than the target recognition.

|                 | First discovery time | Target recognition | Tracking accuracy | Tracking time |
|-----------------|----------------------|--------------------|-------------------|--------------|
| First discovery time | 1                    | 3                  | 3                 | 7            |
| Target recognition   | 1/3                  | 1                  | 3                 | 5            |
| Tracking accuracy          | 1/3                  | 1/3                | 1                 | 5            |
| Tracking time              | 1/7                  | 1/5                | 1/5               | 1            |

After establishing the pairwise comparison matrix, its consistency should be tested. Note the pairwise comparison matrix as \( B \), and calculate its maximum eigenvalue and eigenvector

\[
\lambda_{\text{max}} W = \lambda_{\text{max}} W
\]

\( \lambda_{\text{max}} \) is the largest eigenvalue of \( B \) and \( W \) is the corresponding eigenvector. The consistency index is calculated from the maximum eigenvalue

\[
CR = \frac{\lambda_{\text{max}} - n}{(n-1)RI}
\]

Here \( n \) is dimension of pairwise comparison matrix, \( RI \) is a constant, which is determined by \( N \). The specific corresponding relationship is shown in the Table 3.

| \( n \) | 3   | 4   | 5   | 6   | 7   | 8   | 9   | 10  | 11  | 12  | 13  | 14  | 15  |
|--------|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| \( RI \) | 0.58 | 0.96 | 1.12 | 1.24 | 1.32 | 1.41 | 1.45 | 1.49 | 1.52 | 1.54 | 1.56 | 1.58 | 1.59 |

When the number of evaluation indicators is further increased and the dimension of pairwise comparison matrix is greater than 15, the indicators can be grouped or layered. Generally, when \( CR < 0.1 \), the pairwise comparison matrix is considered to have better consistency, and the weight can be calculated. The normalized feature vector \( W = [W_1, W_2, \cdots, W_n] \) is \( W \) of each index.

\[
W = \left[ \frac{W_1}{|W|}, \frac{W_2}{|W|}, \cdots, \frac{W_n}{|W|} \right]
\]

### 2.2.2. Computing Objective Weight by Entropy Weight

**Step 1: Standardization of Data**

For the effectiveness evaluation of antimissile early warning operation, \( A = (a_{ij})_{n \times n} \) is constructed[7]. Here, \( n \) is the number of antimissile early warning operation schemes, \( M \) is the number of evaluation indexes, and \( a_{ij} \) is the attribute value of the i-th scheme under the j-th index. After the matrix \( A \) is normalized, the normalized matrix \( R = (Y_{ij})_{n \times n} \) is obtained.

**Step 2: calculation of information entropy for each index**

The calculation formula of information entropy is as follows

\[
E_j = - \left( \sum_{i=1}^{n} p_{ij} \ln p_{ij} \right) \ln (n)^{-1}
\]

where

\[
p_{ij} = Y_{ij} / \sum_{i=1}^{n} Y_{ij}
\]

**Step 3: Compute weight for each index**
The information entropy of each index is $E_1, E_2, \cdots, E_k$. The weight vector $\omega = (W_1, W_2, \cdots, W_k)$ of each index can be calculated with information entropy.

$$W_i = \frac{1-E_i}{k-\sum E_i}, \quad (i = 1, 2, \cdots, k) \quad \text{(9)}$$

2.2.3. **Combination for subjective weight and objective weight.** Use $W_{i}^*$ for subjective weight by analytic hierarchy process, $W_{i}^{**}$ for objective weight by entropy weight, and $W_{i}$ for weight of the $i$-th indictor after the combination of $W_{i}^*$ and $W_{i}^{**}$. Then, $W_{i}$ can be computed by a linear combination of $W_{i}^*$ and $W_{i}^{**}$:

$$W_{i} = (1-\lambda)W_{i}^{**} + \lambda W_{i}^* \quad \text{(10)}$$

Here, $\lambda$ is subjective weight coefficient, $1-\lambda$ is the objective weight coefficient.

2.3. **Operational effectiveness evaluation process of antimissile early warning based on combination weighted TOPSIS**

The operational effectiveness evaluation process of antimissile early warning based on combination weighted TOPSIS is shown in Figure 1.

![Diagram](image.png)

**Figure 1.** Operational effectiveness evaluation process of antimissile early warning based on combination weighted TOPSIS.

3. **Simulation and Result Analysis**

To verify the scientificity and rationality of our algorithm, the paper takes the ground-based radar's early warning operation for a certain range ballistic missile as an example to evaluate the influence of each radar deployment scheme on the operational effectiveness. As shown in Figure 2, a submarine launched ballistic missile strikes Hubei Province and deploys ground-based radars at five different locations around Hubei Province. The operational effectiveness of these five radars is evaluated and the impact of deployment location on operational effectiveness is analyzed.
Solution 1: 105.111E, 33.479N
Solution 2: 111.14E, 33.393N
Solution 3: 116.716E, 34.211N
Solution 4: 107.329E, 29.432N
Solution 5: 114.24E, 28.463N

Figure 2. Different deployment schemes of antimissile early warning operational equipment. In order to obtain the operational data, the antimissile early warning operational simulation system is developed to simulate the operational process, as shown in Figure 3.

Figure 3. Simulation deduction system of antimissile early warning operation. In order to evaluate the operational effectiveness of these five deployment schemes, an evaluation index system is established. Ten operational effectiveness evaluation indexes are established from four aspects of target detection ability, target tracking ability, target recognition ability and trajectory calculation ability.

Based on our evaluation algorithm, the operational effectiveness of five radar deployment schemes is evaluated. The order of operational effectiveness of the five deployment solution is 3 > 5 > 2 > 4 > 1. The evaluation conclusion is in line with the objective facts, because it can be seen from Figure 2 that solution 3 is the closest to the ballistic missile launch point and has the best operational efficiency, which is consistent with the principle of radar pre deployment in early warning operations; solution 1 is the farthest from the ballistic missile launch point or the landing point, and it has the lowest ballistic coverage through geometric analysis, so it has the worst operational efficiency.

4. Conclusion
TOPSIS operational effectiveness evaluation method is widely used because of its simple principle, simultaneous evaluation of multiple objects, fast calculation, high resolution, objective evaluation, good rationality and applicability. However, it cannot be directly applied to the effectiveness
evaluation of antimissile early warning operation because it does not give a specific standardized method suitable for the characteristics of antimissile early warning operation effectiveness evaluation index and lacks a specific calculation method for the weight of each index. Starting from the characteristics and requirements of operational effectiveness evaluation of early warning of antimissile, this paper improves the operational effectiveness evaluation method of TOPSIS in view of the shortcomings of TOPSIS, and puts forward an operational effectiveness evaluation method of early warning of antimissile based on combination weighted TOPSIS, which solves the problems of inconsistent impact of each evaluation index on operational effectiveness, small value index being "submerged" by large value index, single subjective or objective The objective weighting method is insufficient. The results show that the combination weighted TOPSIS method is scientific and reasonable, which is suitable for antimissile early warning operational effectiveness evaluation.

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