Temperature dependence of trapped magnetic field in MgB$_2$ bulk superconductor

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Based on DC magnetization measurements, the temperature dependencies of the trapped magnetic field have been calculated for two MgB$_2$ samples prepared by two different techniques: the high-pressure sintering and the hot pressing. Experimentally measured trapped field values for the first sample coincide remarkably well with calculated ones in the whole temperature range. This proves, from one side, the validity of the introduced calculation approach, and demonstrates, from another side, the great prospects of the hot pressing technology for large scale superconducting applications of the MgB$_2$.

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Magnesium diboride is a new promising superconducting material with a critical temperature of about 40 K. High $T_c$ values and simple chemical composition of MgB$_2$ have made it an interesting object for applied investigations with great perspectives of use in superconducting motors, flywheels and bearings. The key parameter for such applications is the maximum trapped field in the sample and its temperature dependence. This, in turn, is closely connected both with the critical current density and with the size of superconductor.

Since the very discovery of superconductivity in MgB$_2$ the researchers dealt with tiny samples which were not suitable for large scale applications. Now, the situation is changed and several techniques have been elaborated to produce high quality MgB$_2$ bulk polycrystalline samples of a few centimeters in diameter and with the critical current densities up to $10^5$ - $10^6$ A/cm$^2$ [1, 2].

It is remarkable that, in contrast to the high temperature superconductors, the grain boundaries in bulk magnesium diboride superconductors do not act as weak links [3, 4]. This significantly simplifies the growth of bulk samples suitable for large scale applications.

In this letter we focus on an important property of the bulk superconductor – the trapped magnetic field. We calculate its temperature dependencies from DC magnetization data using an iteration approach. We show that the trapped field values measured experimentally are in a perfect agreement with the calculated ones. This makes us able to predict the expected values of the trapped magnetic field in a sample which is not yet grown to an appropriate size but which preparation technique looks the most promising today.

We have studied two MgB$_2$ bulk samples prepared by different techniques. One of them was sintered under high pressure as described in [5]. The sample has an uniform polycrystalline structure without cracks and has 28 mm in diameter and 11 mm in height. The trapped magnetic field in the centre of the sintered sample was measured by a Hall probe for different temperatures from 6 to 33 K. The resulting temperature dependence of the trapped field has a negative curvature and trends to saturation at low temperatures (see Fig. 1, cross symbols).

**FIG. 1:** Temperature dependencies of maximum trapped field in bulk MgB$_2$ samples prepared under high-pressure sintering and ball-milled techniques.
The other sample was prepared by ball milling of Mg and B powders at ambient temperatures followed by hot pressing, which we call "ball-milled". It consists of spherical nanocrystalline grains about 40 - 100 nm in size, that distinctly improve pinning due to the large number of grain boundaries. For this sample the trapped field can not be measured because of its small size.

To calculate the trapped field one should know temperature and field dependencies of the critical current density $J_c$ of the both superconductors. For DC magnetization measurements small bar shaped pieces of the both samples have been used. Then, the $J_c(H)$ curves were obtained from the magnetization loops $M(H)$ using the conventional expression $j_c(H) = 20 \cdot \Delta M (b - b^2 / 3l)$, where $\Delta M$ is the difference of the magnetization (in emu/cm²) measured for ascending and descending applied field, $b$ and $l$ are the sample width and length in cm, respectively, and $j_c$ is obtained in A/cm².

For the high-pressure sintered sample a set of $J_c(H)$ curves presented in Fig. 2 in logarithmical scale are straight lines for low fields up to the level of $J_c = 10^4$ A/cm². This means an exponential decay of critical current density with increasing field

$$J_c(H, T) = J(T) \exp(-H/b(T)),$$

where $J(T)$ and $b(T)$ are temperature dependent coefficients.

The dependencies of $J(T)$ and $b(T)$ for the sintered sample, shown in Fig. 3 by circles, can be well fitted to the expressions

$$b(T) = b_0[1 - (T/T_c)^2], J(T) = J_0[1 - (T/T_c)]. \quad (2)$$

Here $T_c = 37.5$ K is the critical temperature, whereas $b_0 = 0.9$ T and $J_0 = 3.1 \times 10^5$ A/cm² are fitting parameters, which characterize an effective field of supercurrent decay and a maximum critical current density at zero field and temperature, respectively.

For the ball-milled MgB₂ sample $J_c(H, T)$ can be described by Eq. (1) too (see Fig. 4 in [6]), but the functions $b(T)$ and $J(T)$ are different:

$$b(T) = b_0[1 - (T/T_c)^2]^{3/2}, J(T) = J_0[1 - (T/T_c)]^{3/2}, \quad (3)$$

where the corresponding parameters are $T_c = 34$ K, $b_0 = 2.78$ T and $J_0 = 8.5 \times 10^5$ A/cm² (see Fig. 3, squares).

The obtained results show that the critical current density strongly depends on temperature. For the high-pressure sintered sample the values of $J(T)$ and $b(T)$ are about 2.5 times lower than those of for the ball-milled sample. From the other hand, the latter sample has a much stronger power dependence of $J(T)$ and $b(T)$ on temperature.

The further calculation of the trapped field from a known $J_c(H, T)$ function is a nontrivial task. According to the Biot-Savart law, the magnetic field $H$ generated by the supercurrent $J_c$ flowing in the volume of sample $V$ is

$$H(r) = \frac{1}{c} \int_V J_c(H, \rho) \times \frac{r - \rho}{|r - \rho|^3} d^3 \rho. \quad (4)$$

As far as $J_c$ is a function of $H$, one needs to solve this integral equation in order to calculate the trapped field values.

To simplify the problem we assume the homogeneity of the sample, i.e. (1) that the critical current density does not depend on coordinate $\rho$ explicitly and (2) that the currents flow along concentric circles. Then, the field component normal to the sample surface, $H_z$, can be written in cylindrical coordinates $r = (r, \phi, z)$ and
\( \rho = (\rho, \psi, \zeta) \) as follows

\[
H_z(r) = \frac{1}{c} \int_V J_c(H_z)f(r, \rho, \psi, \zeta)d^3\rho,
\]

where \( f(r, \rho, \psi, \zeta) = \frac{\rho - r \cos \psi}{((r - \rho \cos \psi)^2 + (\rho \sin \psi)^2 + (z - \zeta)^2)^{3/2}}, \) \( d^3\rho = d\rho d\psi d\zeta. \) The resulting field profile has a cylindrically symmetric form with the maximum at \( r = 0. \)

The integral equation (5) can be solved for \( H(r) \) numerically using an iterative procedure (7)

\[
H_{i+1}(r) = \frac{1}{c} \int_V J_c(H_i)f(r, \rho, \psi, \zeta)d^3\rho,
\]

until \( |H_{i+1} - H_i|/H_i < \varepsilon. \) Here the obtained profile is used to calculate current in the next step of iterations. We chose an initial profile \( H_0(r) = \text{const} \) to start the iterations.

One should note that we calculate the trapped field profile numerically in each step and cannot define \( H(r) \) function in every point. Therefore, we should discretize Eq. (6) and divide the cylindrical sample into \( N \) concentric tubes of width \( \Delta = \frac{R_s}{N} \) and then calculate the field in each point \( r_k, k = 0 \ldots N - 1 \) as a sum of fields from all tubes

\[
H(r_k) = \frac{\Delta}{c} \sum_{j=0}^{N-1} \int J_c(H(\rho_j))f(r_k, \rho_j, \psi, \zeta)\rho_j d\psi d\zeta,
\]

where \( r_k = k\Delta, R_s \) is the radius, \( L \) is the height of the cylindrical sample and the integration here is made over \( \psi \) from 0 to \( 2\pi \) and over \( \zeta \) from \(-L\) to \( 0. \)

From equations (7) one obtains the field profile for a given \( J_c(H) \) function using iteration (6). In the case of \( J_c(H) = \text{const} \) we will obtain a conical Bean profile with the maximum in the center of the sample. To reach the accuracy of \( \varepsilon = 0.1 \% \) it is sufficient to make just 16 iterations with \( N = 14 \) tubes. Moreover, the method of calculation has no fitting parameter and uses only experimental \( J_c(H, T) \) data. The calculated temperature dependence of maximum trapped field for high-pressure sintered and ball-milled MgB\(_2\) samples is represented on Fig. 1 by the solid and dashed lines respectively.

We obtained a very good coincidence with experimental points for the high-pressure sintered sample. Remarkably, the trapped field, which is determined by the current distribution in the whole sample volume can be correctly calculated on the base of \( J_c \) measured locally in a small piece of the sample. The ball-milled (nanocrystalline) sample was found to have much higher trapped fields than the sintered sample. Thus, the development of large size nanocrystalline MgB\(_2\) samples is very attractive for future applications.

To sum up, we calculated the temperature dependence of the trapped magnetic field for two MgB\(_2\) bulk samples prepared under different techniques. We used DC magnetization data and an iteration approach to solve the Biot-Savart equation. Experimentally measured trapped field values for the high-pressure sintered sample coincide excellent with calculated ones in the whole temperature range. The correctness of the calculation allowed us to predict the expected values of the trapped field in the ball-milled sample and revealed the great prospects of this technology for large scale applications of superconducting MgB\(_2\).

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