Effects of Anisotropic Stress in Interacting Dark Matter - Dark Energy Scenarios

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We study a novel interacting dark energy – dark matter scenario where the anisotropic stress of the large scale inhomogeneities is considered. The dark energy has a constant barotropic state parameter and the interaction model produces stable perturbations in the large scale of the universe. The resulting picture has been constrained using different astronomical data in a spatially flat Friedmann-Lemaître-Robertson-Walker (FLRW) universe. We perform different combined analyses of the astronomical data to measure the effects of the anisotropic stress on the strength of the interaction and on other cosmological parameters as well. The analyses from several combined data show that a non-zero interaction in the dark sector is favored while a non-interaction scenario is still allowed within 68% confidence-level (CL). The anisotropic stress measured from the observational data is also found to be small, and its zero value is permitted within the 68% CL. The constraints on the dark energy equation of state, \( w \), also point toward its \(-1\) value and hence the resulting picture looks like a non-interacting \( w \),CDM as well as ACDM cosmology. However, from the ratio of the CMB TT spectra, we see that the model has a deviation from the standard ACDM cosmology which is very hard to detect from the CMB TT spectra only. Although the deviation is not much significant, but from the present astronomical data, we cannot exclude such deviation. Overall, we find that the model is very close to the ACDM cosmology. Perhaps, a more accurate conclusion can be made with the next generation of surveys that are not so far. We also argue that the current tension on \( H_0 \) might be released for some combinations of the observational data. In fact, the allowance of \( w_\Lambda \) in the phantom region is found to be more effective to release the tension on \( H_0 \).

PACS numbers: 98.80.-k, 95.36.+x, 95.35.+d, 98.80.Es

I. INTRODUCTION

A remarkable revolution in the dynamical history of the universe has been witnessed in the last several years. Around 20 years back, the distant Supernovae of Type Ia (SN Ia) first indicated an accelerating expansion of the universe and thereafter a lot of distinct astronomical observations have strengthened such observational prediction. To interpret this acceleration a hypothetical fluid with negative pressure became necessary and subsequently, cosmological constant was revived into the picture. The cosmological constant, \( \Lambda \), has a negative equation of state, \( P_\Lambda = -\rho_\Lambda \) and together with cold-dark-matter, the joint scenario ACDM has been found to be the best cosmological model, at least according to a series of astronomical measurements. In ACDM scenario, both the cosmological constant and the cold-dark-matter remain conserved separately, as if they are two disjoint sectors. Such model of the universe is widely referred to as the non-interacting cosmological scenario. Unfortunately, the cosmological constant suffers from the fine-tuning problem (also known as the cosmological constant problem), where being time-independent, it reports an unimaginable difference (of the order of \( 10^{121} \)) in its value determined in the Planck and low energy scales. The problem associated with the cosmological constant is not a new detection, it is persisting since long back ago \( [1] \), even before the late-accelerating phase. Thus, attempts have been made aiming to provide with a reasonable justification on the fine-tuning problem \( [2] \). While on the other hand, people have tried to bypass this problem through the introduction of dark energy models \( [3–5] \) (we also refer to some specific scalar field dark energy models that may also account for the early scenarios of the universe \( [6, 7] \)). However, although the introduction of dark energy models relieves the cosmological constant problem but they raised another serious issue which is widely known as the cosmic coincidence problem \( [5] \). Such coincidence problem led to another class of cosmological theories which is the theory of non-gravitational interaction between dark matter and dark energy.

The non-gravitational interaction in the dark sector, precisely between dark matter and dark energy is a phenomenological concept that was originally thought to explain the different values of the time-independent cosmological constant \( [2] \), but later on, such concept was found to be very useful to explain the cosmic coincidence problem \( [8, 9] \). Certainly, this led to a large amount of investigations towards this direction where the dark
sectors have direct interaction [15][35] (also see [37][12]). Such interacting scenarios have good motivation if the particle physics theory is considered, because from the particle physics view, mutual interaction between any two fields, is a natural phenomenon, irrespective of the nature of the fields. Although the interacting dynamics is complicated and a generalized cosmic scenario, but it recovers the non-interaction cosmology as a special case. Thus, the theory of non-gravitational interaction between dark matter and dark energy is a generalized version of the non-interacting dark matter and dark energy cosmologies. Interestingly enough, the observational data at recent time found that the direct interaction between dark matter and dark energy cannot be excluded [43–51]. Moreover, very recently, it has been reported that the current tension on the local Hubble constant can be alleviated with the introduction of dark matter and dark energy interactions [46,52]. Additionally, the crossing of phantom barrier has also been found to be an easy consequence of the non-gravitational interaction. Thus, the theory of interacting dark energy might be considered to be an appealing field of research and indeed a hot topic for the next generation of the astronomical surveys.

The current work presents a general interacting scenario where aside from the non-gravitational interaction between dark matter and dark energy, we include the possibility of an anisotropic stress. The anisotropic stress appears when the first order perturbation is considered, and in most of the cases, it is generally neglected. From both theoretical and the observational grounds, the possibility of an anisotropic stress cannot be excluded at all. Although the dimension of the resulting parameter space is increased, but, due to advancements of the astronomical data, the measurement of the anisotropic stress becomes important. The most important thing is to measure the effect of this quantity on the large scale structure evolution of the concerned cosmological scenario. This is the primary motivation of this work. In particular, looking at the perturbation equations (see section II) one can realize that a nonzero value of the anisotropic stress, can affect the temperature anisotropy in the cosmic microwave background spectra and also on the matter power spectra as well. Thus, for a detailed understanding of the interacting scenario in its large scale structure, the anisotropic stress plays a significant role.

Thus, following the above motivation, in a spatially flat Friedmann-Lemaître-Robertson-Walker universe, we study an interacting dark matter-dark energy scenario in presence of an anisotropic stress and then constrain this model using a series of latest astronomical data from cosmic microwave background radiation, Joint Light Curve analysis (JLA) sample of Supernovae Type Ia, baryon acoustic oscillations (BAO) distance measurements, Hubble parameter measurements from cosmic chronometers (CC), weak gravitational lensing and the local Hubble constant value from the Hubble Space Telescope (HST). The analyses are based on the use of publicly available markov chain monte carlo package cosmomc where the convergence of the cosmological parameters follows the well known Gelman-Rubin statistics.

The work has been organized in the following way. In section II we describe the background and the perturbation equations for the coupled dark energy in presence of the matter-sourced anisotropic stress. Section III describes the observational data that we use to analyze the present models. In section IV we discuss the constraints on the current model. Finally, section V closes the work with the main findings of this investigation.

II. THE INTERACTING UNIVERSE

In this section we shall describe an interacting cosmological scenario both at background and perturbative levels. To do this, we assume the most general metric for the underlying geometry of the universe which is characterized by the Friedmann-Lemaître-Robertson-Walker (FLRW) line element. We also assume that the metric is spatially flat. The evolution equations for a pressureless dark matter and a dark energy fluid in this universe obey the following conservation equations

\[
\rho_c' + 3\mathcal{H}\rho_c = aQ_c = -aQ, \quad (1)
\]

\[
\rho_x' + 3\mathcal{H}(1 + w_x)\rho_x = aQ_x = aQ, \quad (2)
\]

where the prime denotes the differentiation with respect to the conformal time \( \tau \) (i.e. “\( \tau' = \frac{d\tau}{a^2} \)”; \( \mathcal{H} = a'/a \), is the conformal Hubble parameter; \( \rho_c, \rho_x \) are respectively the energy densities of cold dark matter and dark energy; \( w_x \) is the barotropic equation of state of the dark energy, that means \( w_x = p_x/\rho_x \), here \( p_x \) is the pressure of the dark energy fluid and as usual zero pressure is attributed to cold-dark-matter sector. The quantity \( Q = Q_x = -Q_c \) in the right hand sides of (1) and (2) is the energy transfer rate between the dark sectors and depending on its sign the direction of energy flow is determined. To be precise, a positive interaction rate \( (Q > 0) \) assigns the energy flow from dark matter to dark energy while the negative interaction rate reverses its direction of energy flow. In addition, we consider the presence of non-relativistic baryons (\( \rho_b \)) and relativistic radiation (\( \rho_r \)) which follow the standard evolution equations, that means they do not take part in the interaction. The dynamics of the spatially flat universe is thus constrained by the Hubble equation

\[
\mathcal{H}^2 = \frac{8\pi G}{3}a^2\rho_{tot}, \quad (3)
\]

where \( \rho_{tot} = \rho_c + \rho_x + \rho_b + \rho_r \), is the total energy density of the universe. Hence, if the energy transfer rate, \( Q \), is specified, then the evolution equations for \( \rho_c \) and \( \rho_x \) can fully be determined using the conservation equations (1) and (2) together with the Hubble constraint (3). However, the presence of interaction in the dark sector may significantly affect the large scale structure of the
universe and hence it becomes necessary to consider the evolution equations for the interacting model at the perturbative levels. Thus, in order to do so we consider the perturbed FLRW metric \[ ds^2 = a^2(\tau) \left[-(1 + 2\phi)d\tau^2 + 2\partial_i B d\tau dx^i + (1 - 2\psi)\delta_{ij} + 2\partial_i \partial_j \theta \right] dx^j. \] (4)

Here, the quantities appearing in the above metric [4], namely, \( \phi, B, \psi \) and \( \theta \) represent the gauge-dependent scalar perturbations. For this metric one can find the perturbation equations and following [56, 59, 60], the perturbed energy and momentum balance equations for the interacting dark matter and dark energy scenario can be found to be

\[
\begin{align*}
\delta \rho_A' &+ 3H(\delta \rho_A + \delta \rho_A) - 3(p_A + p_A)\psi' \\
- k^2(\rho_A + p_A)(v_A + E') &\equiv aQ_\rho \phi + a'Q_\rho A, \\
\delta \rho_A &+ [(\rho_A + p_A)(v_A + B)]' + 4H(\rho_A + p_A)(v_A + B) \\
+ (\rho_A + p_A)\phi &- \frac{2}{3} k^2 \rho_A \pi_A = aQ_A(v + B) + af_A,
\end{align*}
\] (5)

where \( A \) is the characterization symbol for any fluid (dark matter or dark energy) and prime is the representation of the differentiation with respect to the conformal time, mentioned earlier. The relation between the peculiar velocity potential \( v_A \) and the local volume expansion rate \( \theta_A \) is, \( \theta_A = - k^2(v_A + B) \) in the Fourier space with mode \( k \) [53, 56, 58]. Here by \( \delta_A = \delta \rho_A/\rho_A \), we mean the density perturbation for the fluid \( A \), and momentum transfer potential has been assumed to be the simplest physical choice, that gives its value to zero in the rest frame of dark matter [56, 59, 60]. Hence, the momentum transfer potential takes the expression \( k^2 f_A = Q_A(\theta - \theta_A) \) [27].

Now, the pressure perturbation \( \delta p_A \), for any fluid \( A \), is related as \( \delta p_A = c_{sA}^2 \delta \rho_A + (c_{sA}^2 - c_{sA}^2) \rho_A (v_A + B) \) [56] which for the dark energy fluid turns out to be \( \delta p_x = c_{sx}^2 \delta \rho_x + (c_{sx}^2 - c_{sx}^2) [3H(1 + w_x)\rho_x - aQ] \theta_x/k^2 \).

The evolution of fluid perturbations could be described by the adiabatic speed of sound \( c_{sA}^2 \equiv \rho_A^' / \rho_A = w_A - w_A' / [3H(1 + w_A)] \). In this adiabatic case, the relation between the perturbations of \( \delta \rho_A \) and \( \delta \rho_A \) is related by \( \delta p_A = c_{sA}^2 \delta \rho_A \). However, for an entropic fluid, the pressure might not be a unique function of the energy density \( \rho_A \). Therefore, there would be another degree of freedom to describe the microproperties of a general fluid. That is the physical speed of sound in the rest frame \( c_{sx}^2 \equiv [\partial \delta / \partial \rho_A]/[\partial \rho_A] \) \( \rho_A \) (\( r f \) represents the rest-frame) which is defined in the comoving frame of the fluid. Now, we note that when the entropic perturbation vanishes, the physical sound speed and the adiabatic sound speed vanishes, that means, \( c_{sx}^2 = c_{sA}^2 \). Hence, in the case of entropic fluid such as scalar fields, one needs both its equation of state and its sound speed, to have a complete description of dark energy and its perturbations. However, in order to fully describe a dark energy fluid and its perturbations, one should also consider the possibility of an anisotropic stress, even in an isotropic and homogenous FLRW universe, where the anisotropic stress \( \sigma_A = \frac{2w_A}{3(1 + w_A)} \pi_A \), can be taken as a spatial perturbation.

In the synchronous gauge (\( \phi = 0, \psi = \eta \), and \( k^2 E = -h/2 - 3\eta \)), the evolution equations for density perturbations and velocity perturbations equations for dark energy and dark matter respectively read

\[
\begin{align*}
\delta x_x &= -(1 + w_x) \left( \theta_x + \frac{h'}{2} \right) - 3H w_x \frac{\theta_x}{k^2} \\
-3H(c_{sx}^2 - w_x) \left( \delta x_x + 3H(1 + w_x)\frac{\theta_x}{k^2} \right) + \frac{\alpha Q}{\rho_x} \left[ -\delta x_x + \frac{\delta Q}{Q} + 3H(c_{sx}^2 - w_x)\frac{\theta_x}{k^2} \right], \\
\theta_x &= -H(1 - 3c_{sx}^2)\theta_x + \frac{c_{sx}^2}{(1 + w_x)} k^2 \delta x_x - k^2 \sigma_x \\
+ \frac{\alpha Q}{\rho_x} \left[ \theta_x - (1 + c_{sx}^2)\frac{\theta_x}{1 + w_x} \right], \\
\delta \rho_x' &= -\left( \theta_x + \frac{h'}{2} \right) + \frac{\alpha Q}{\rho_x} \left( \delta c_x - \frac{\delta Q}{Q} \right), \\
\theta_x' &= -H\theta_x,
\end{align*}
\] (7)

where the factor \( \delta Q/Q \) includes the perturbation term of Hubble expansion rate \( \delta H \) (61). In the next section, the anisotropic stress of dark energy \( \sigma_x \) would be assumed as an appropriate expression.

Following Hu [62], the evolution of the anisotropic stress, \( \sigma_x \), is considered to be

\[
\begin{align*}
\delta \rho_x' + 3H c_{sx}^2 \sigma_x &= \frac{8}{3} c_{visc} \left( \frac{\theta_x + \frac{h'}{2} + 3\eta'}{1 + w_x} \right), \\
\end{align*}
\] (11)

where \( c_{visc}^2 \) is the viscous speed of sound which controls the correspondence between the velocity (or metric) shear and the anisotropic stress. In particular, for a relativistic fluid, \( c_{visc}^2 = 1/3 \), while for a general dark energy fluid, \( c_{visc}^2 \) is a free parameter and it can be constrained through the observational data [63, 71].

The anisotropic stress can be linked to some either the overdensity of matter (dark matter (c) + baryons (b)) or dark energy as shown in [67]. The anisotropic stress that is linked to the overdensity of dark matter is also known as the matter-sourced anisotropic stress model having the form

\[
\begin{align*}
\sigma_x &= \frac{2}{3} \frac{1}{1 + w_x} c_{e} a' n \Delta m, \\
\end{align*}
\] (12)
while when the anisotropic stress is linked to the over-density of dark energy (similarly, it might be dubbed as the dark-energy-sourced anisotropic stress model)

\[
\sigma_x = \frac{2}{3} \frac{1}{1 + w_x} \frac{f_x}{1 + (g_x H / k)^2} \Delta_x, \tag{13}
\]

where \( \Delta_x = \delta_i - \frac{\rho_x}{\rho_c} \delta_i \), is the gauge invariant density perturbations for matter \((i = m)\) and dark energy \((i = x)\), respectively. The above relations are established on the fact that the anisotropic stress and the overdensity of dark matter (or, dark energy) may modify the gravitational slip in an effective way \( [71] \). The latest analysis on the observational constraints of dark energy with anisotropic stress can be found in \( [72, 73] \).

Let us come to the interaction model that we wish to study in this work. Before taking any typical interaction model we recall that the interaction function \( Q \) directly enters into the pressure perturbation for dark energy as \( [54] \)

\[
\delta p_x = c_{sx}^2 \delta p_x + (c_{sx}^2 - c_{ax}^2) [3H(1 + w_x)\rho_x - aQ] \frac{\theta_x}{k^2}. \tag{14}
\]

According to the qualitative analysis on the large-scale instability in the dark sector perturbations during the early radiation era \( [56] \), in the pressure perturbation of dark energy \( [11] \), the coupling term \( Q \) in the pressure perturbation \( \delta p_x \) can lead to a driving term \( w_x \frac{\theta_x}{k^2} \) which includes the factor \( \mathcal{H} \theta_x \), and it becomes very large if \( w_x \) is close to \( -1 \). This causes rapid growth of \( \theta_x \). Qualitatively, this is the source of the instability: in the presence of energy-momentum transfer in the perturbed dark fluids, momentum balance requires a runaway growth of the dark energy velocity. In order to avoid the perturbation instability, and based on the phenomenological consideration, we assume the constant equation of state \( w_x \) in the interacting dark energy with the energy transfer rate \( Q = 3H \xi (1 + w_x) \rho_x \). The presence of the factor \((1 + w_x)\) in the interaction function does not bother with the dark energy equation of state, and hence the stability of the interaction model in the large scale structure of the universe rests on the coupling parameter of the interaction. The perturbation equations \( [7] \) to \( [10] \) for the specific interaction model turn out to be

\[
\begin{align*}
\delta'_{c} &= -(1 + w_x) \left( \frac{\theta_x + h'/2}{2} \right) + 3H (c_{sx}^2 - w_x) \left[ \delta_{c} + 3H (1 + w_x) \frac{\theta_x}{k^2} \right] \\
&+ 3H \xi (1 + w_x) \left[ \frac{\theta + h'/2}{2} + 3H (c_{sx}^2 - w_x) \frac{\theta_x}{k^2} \right] ,
\end{align*}
\]

\[
\begin{align*}
\theta'_{c} &= -\mathcal{H} \delta_{c} - 3H \xi \left[ \theta_{c} - (1 + c_{sx}^2) \theta_x \right] - k^2 \sigma_x + 3H \xi \left[ \sigma_{c} - (1 + c_{sx}^2) \sigma_x \right] ,
\end{align*}
\]

where the matter-sourced anisotropic stress is \( \sigma_x = 2/3(1 + w_x)\xi a^3 \Delta_x \). In this work, we consider the matter-sourced model with \( n = 0 \), that means, \( \sigma_x = 2/3(1 + w_x)\xi a^3 \Delta_m \), as the simplest case in such complicated interacting dynamics. Although there is no such strict restriction to exclude the possibility of anisotropic stress sourced by dark energy, but, however, since the cluster effects of dark energy is smaller in compared to the dark matter, the effects of anisotropic stress sourced by dark energy must be weaker in respect to the anisotropic stress sourced by dark matter. As a result, the anisotropic stress sourced by dark matter might be more relevant in this context.

### III. OBSERVATIONAL DATA SETS AND THE STATISTICAL TECHNIQUE

In this section we describe the main observational data that we have used to constrain the cosmological scenarios and also we outline the statistical methodology. We use various astronomical data ranging from low redshifts to high redshifts, for our analysis. Below we summarize the data sets with their corresponding references.

1. Cosmic Microwave Background Radiation: The full Planck 2015 low–\( l \) temperature-plus-polarization and the high–\( l \) \( C_l^{EE} + C_l^{TT} \) likelihood (“Planck TT, TE, EE + lowTEB”) \( [74, 75] \) have been used. For the interacting dark energy with matter-sourced anisotropic stress, the amplitude of CMB at low multipole \( l < 30 \) is very sensitive to the values due to the fact that the anisotropic stress of dark energy is proportional to the velocity of dark energy directly. The summation of the Newtonian potentials becomes

\[
k^2 (\Phi + \Psi) = -8\pi G a^2 \left( \sum_A \rho_A \Delta_A + \sum_A \rho_A \Pi_A \right) \tag{15}
\]

where \( \Delta_A \) is the gauge invariant density contrast and \( \Pi_A \) is related to the anisotropic stress \( \sigma_A \) via \( \sigma_A = \frac{3}{4} \frac{w_A}{1 + w_A} \Pi_A \). Thus, an extra contribution to the integrated Sachs–Wolfe (ISW) effect due to the existence of anisotropic stress of dark energy is

\[
-k^2 ISW_{stress} = 8\pi G a^2 \sum_A \rho_A \Pi_A - 8\pi G a^2 H \left[ 4 \sum_A \rho_A \Pi_A \right. \\
+ \sum_A (3\rho_A - \rho_A) \Pi_A - \sum_A \frac{d\ln w_A}{d\ln a} \rho_A \Pi_A \right]. \tag{16}
\]
2. Joint Light-Curve Analysis: The Joint Light-curve Analysis (JLA) sample [80] containing 740 Supernovae Type Ia in the low-redshift range $z \in [0.01, 1.30]$ have been considered.

3. Baryon Acoustic Oscillations Distance Measurements: For baryon acoustic oscillations (BAO) data, we mainly use four different data points. In particular, we use the CMASS and LOWZ samples from the latest Data Release 12 (DR12) of the Baryon Oscillation Spectroscopic Survey (BOSS) respectively at the effective redshifts $z_{\text{eff}} = 0.57$ and $z_{\text{eff}} = 0.32$ [76]. In addition, we include the 6dF Galaxy Survey (6dFGS) measurement at $z_{\text{eff}} = 0.106$ [77], and the Main Galaxy Sample of Data Release 7 of Sloan Digital Sky Survey (SDSS-MGS) at $z_{\text{eff}} = 0.15$ [78].

4. Redshift Space Distortion Data: We employ the redshift space distortion (RSD) measurements from two distinct galaxy samples, the one which includes the CMASS sample with an effective redshift of $z_{\text{eff}} = 0.57$ [79] while the other includes the LOWZ sample with an effective redshift of $z_{\text{eff}} = 0.32$ [79].

5. Hubble Parameter Measurements: We also employ the recently released cosmic chronometers (CC) data with 30 measurements of the Hubble parameter values in the redshift interval $0 < z < 2$ [83]. The cosmic chronometers are basically some galaxies which evolve passively and are the most massive. An accurate measurement of the differential ages of galaxies together with the spectroscopic estimation of $dz$ with high accuracy yield the Hubble parameter value through $H(z) = (1 + z)^{-1} dz/dt$. For more on the CC, we refer the readers to [83].

6. $H_0$ from the Hubble Space Telescope: The present Hubble constant value yielding $H_0 = 73.02 \pm 1.79 \text{km s}^{-1} \text{Mpc}^{-1}$ [84] from the Hubble Space Telescope (HST) has been used. We label this value as HST.

7. Weak Gravitational Lensing Data: Finally, we also use the weak gravitational lensing data (WL) along with the previous data sets. The sample is taken from the Canada-France-Hawaii Telescope Lensing Survey (CFHTLenS) which spans 154 square degrees in five optical bands. In this survey, 21 sets of cosmic shear correlation functions linked to six redshift bins have been presented, see Refs. [81] [82] for details. The tomographic correlation functions measured from the blue galaxy sample and consistent with zero intrinsic alignment nuisance parameter has been named as blue sample and we have used this blue sample for the present work. From the likelihood analysis of the CFHTLenS data one can extract the information of our universe. Here, the true inverse covariance matrix takes the form $\Sigma^{-1} = \alpha_A \hat{\Sigma}^{-1}$ in which $\alpha_A = (n_\mu - p - 2)/(n_\mu - 1)$, and $\hat{\Sigma}$ is the measured covariance matrix. The inclusion of the anisotropic stress of dark energy certainly modifies the summation of potentials given in Eq. (15). Moreover, for the presence of anisotropic stress, the lensing potential gains an extra contribution leading to the convergence power spectrum at angular wave number $\ell$ as

$$P_k^{ij}(\ell) = \int_0^{aH} \frac{d\eta}{f_K(\eta)} \left( 1 + \sum_A \frac{p_A \Pi_A}{\sum_A \rho_A \Delta_A} \right)^2 P_\delta \left( \frac{\ell}{f_K(\eta)} ; \eta \right),$$

(17)

where $\eta$ is the comoving distance; $f_K(\eta)$ is the angular diameter distance out to $\eta$ and it depends on the curvature scalar $K$. We note that in the present work we have assumed $K = 0$. The quantity $\eta_H$ is the horizon distance, and $q_i(\eta)$ represents lensing efficiency function for the redshift bin $i$, see [81] [82] for more discussions.

Now, for the interacting dark energy with matter-sourced anisotropic stress, the amplitude of CMB at low multipole ($\ell < 30$) is very sensitive to the values of $\epsilon_x$ due to the fact that the anisotropic stress of dark energy is proportional directly to the velocity of dark energy fluid.

The summation of the Newtonian potentials becomes

$$k^2 (\Phi + \Psi) = -8\pi G a^2 \sum_A \rho_A \Delta_A - 8\pi G a^2 \sum_A p_A \Pi_A$$

$$= -8\pi G a^2 (\rho_b \Delta_b + \rho_c \Delta_c + \rho_x \Delta_x + \rho_x \pi_x)$$

$$= -8\pi G a^2 \left[ \rho_b \delta_b + \rho_c \delta_c + \rho_x \delta_x + \left( 3H(1 + w_x) \rho_x - \frac{1}{2} \left( 5 + 3c_{sx}^2 \right) aQ \right) \theta_x \right] k^2 \right) \theta_x \right]$$

(18)

where $\pi_x$ is related to the anisotropic stress $\sigma_x$ through the relation $\pi_x = \frac{2w_x}{3(1 + w_x)\pi_x}$. For the influence of WL,
the convergence power spectrum will also be modified by the anisotropic stress in the same way, but in the spatial part of the Newtonian potentials.

The likelihood for our analysis is, $\mathcal{L} \propto e^{-\chi^2_{\text{tot}}/2}$, where $\chi^2_{\text{tot}} = \chi^2_{\text{HST}} + \chi^2_{\text{BAO}} + \chi^2_{\text{RSD}} + \chi^2_{\text{CMB}} + \chi^2_{\text{WL}}$. We modify the code CAMB [85] which is freely available and here we implement a numerical algorithm. This numerical algorithm is called to solve the background equations and after that corresponding to each data set we calculate the $\chi^2_{\text{tot}}$ values. Finally, we call another code known as cosmomc, a markov chain monte carlo package together with a convergence diagnostic [84] that is used to extract the cosmological parameters. The parameters space for our present model is, $P_1 = \{\Omega_c h^2, \Omega_b h^2, 100\theta_{MC}, \tau, e_\pi, w_x, \xi, n_s, \log[10^{10} A_s]\}$ (nine-dimensional space). Here, $\Omega_c h^2$ is the cold dark matter density, $\Omega_b h^2$ is the baryon density, $100\theta_{MC}$ is the ratio of sound horizon to the angular diameter distance, $\tau$ is the optical depth, $n_s$ is the scalar spectra index, $A_s$ is the amplitude of the initial power spectrum and the remaining $e_\pi, w_x, \xi$ are the model parameters described earlier. Certainly, the inclusion of both interaction (in terms of the strength $\xi$) and the anisotropic stress ($e_\pi$) extends the parameters space in compared to the minimum number of parameters space in $\Lambda$CDM, see [87] for a detailed discussion.

The priors of specific model parameters are displayed in Table I.

| Parameter | Prior |
|-----------|-------|
| $\Omega_c h^2$ | [0.01, 0.99] |
| $\Omega_b h^2$ | [0.005, 0.1] |
| $100\theta_{MC}$ | [0.5, 1.0] |
| $\tau$ | [0.01, 0.8] |
| $n_s$ | [0.5, 1.5] |
| $\log[10^{10} A_s]$ | [2.4, 4] |
| $w_x$ | [-2.0] |
| $e_\pi$ | [-1.1] |
| $\xi$ | [0, 2] |

The interacting scenario in presence of the matter-sourced anisotropic stress is the main focus of the work. However, we have also constrained the interacting scenario where no matter sourced anisotropic stress is present. The motivation of the second analysis is to see how the presence of matter sourced anisotropic stress affects the cosmological dynamics. In order to constrain both the interacting scenarios we have used the following observational data:

1. CMB (Planck TTTEEE+lowTEB),
2. CMB+BAO+RSD,
3. CMB+BAO+RSD+HST,
4. CMB+BAO+WL+HST,
5. CMB+BAO+RSD+WL+HST+JLA+CC.

For the interacting scenario with matter sourced anisotropic stress, we have presented the observational summary in Table I where the constraints on the model parameters are shown at 68% and 95% confidence levels. In Fig. [we display the one-dimensional posterior distributions for some selected model parameters for the above observational data. Let us now analyze the observational constraints on the model parameters.

From Table I we see that the observational data favor a non-zero interaction between dark matter and dark energy. However, observing the 1σ error-bars of the coupling parameter, $\xi$, one can readily conclude that $\xi = 0$ is allowed by almost all observational data. That means within 1σ confidence-level, a non-interacting wCDM model is still allowed. From the dark energy equation of state we find that only CMB and the combined analysis CMB+BAO+RSD hint for its quintessence nature. One can see that the CMB data alone constrain the dark energy equation of state, $w_x = -0.9490^{+0.1670}_{-0.1070}$ at 68% CL ($-0.9490^{+0.3254}_{-0.3329}$ at 95% CL) while from the combination CMB+BAO+RSD, we find $w_x = -0.9494^{+0.0392}_{-0.0415}$ at 68% CL ($-0.9494^{+0.0832}_{-0.0838}$ at 95% CL). One may notice that the addition of BAO and RSD to CMB decreases the error bars in the dark energy equation of state, that means the parameter space for $w_x$ gets reduced. Interestingly enough, when the H0 prior from the HST is included to the other data sets, the dark energy equation of state moves toward the cosmological constant boundary. The combined analysis CMB+BAO+RSD+HST shows that $w_x = -1.0374^{+0.0556}_{-0.0420}$ at 68% CL ($-1.0374^{+0.0894}_{-0.0998}$ at 95% CL). The last two analyses, namely, CMB+BAO+WL+HST and CMB+BAO+RSD+WL+HST+CC+JLA infer the same about the dark energy equation of state, see the last two columns of the Table II. Thus, from the results, one can identify that the dark energy resembles with the cosmological constant. Hence, one may conclude that, although a non-zero deviation from the $\Lambda$CDM cosmology is favored by the observational data but effectively, such deviation is very minimal and hence the model is close to the $\Lambda$CDM model. In Fig. 2 we display the 68% and 95% confidence-level contour plots for the combinations ($w_x$, $H_0$) and ($w_x$, $\Omega_m$) and ($w_x$, $\xi$) using different combined analyses performed in this work. From the left panel of Fig. 2, we find that for lower values of $H_0$, the dark energy equation of state, $w_x$ has a shifting nature toward the quintessence regime while from the middle panel of
TABLE II: 68% and 95% confidence-level constraints on the model parameters of the interacting scenario with anisotropic stress using different combined analyses of the observational data. Here, $\Omega_{m0} = \Omega_{m} + \Omega_0$.

| Parameters | CMB | CMB+BAO+RSD | CMB+BAO+RSD+HST | CMB+BAO+WL+HST+JLA+CC |
|------------|-----|-------------|------------------|------------------------|
| $\Omega_{b} h^2$ | 0.1202 ± 0.0018 ± 0.0062 | 0.1223 ± 0.0015 ± 0.0031 | 0.1231 ± 0.0014 ± 0.0029 | 0.1201 ± 0.0018 ± 0.0053 |
| $\Omega_{c} h^2$ | 0.0567 ± 0.0017 ± 0.0032 | 0.0569 ± 0.0015 ± 0.0029 | 0.0570 ± 0.0014 ± 0.0027 | 0.0565 ± 0.0016 ± 0.0027 |
| $\Omega_{m0}$ | 0.3087 ± 0.0035 ± 0.0015 | 0.3060 ± 0.0033 ± 0.0014 | 0.3048 ± 0.0032 ± 0.0013 | 0.3021 ± 0.0030 ± 0.0013 |
| $\sigma_8$ | 0.8493 ± 0.0364 ± 0.0925 | 0.8493 ± 0.0364 ± 0.0925 | 0.8493 ± 0.0364 ± 0.0925 | 0.8493 ± 0.0364 ± 0.0925 |
| $H_0$ | 68.6361 ± 4.0644 ± 12.531 | 68.4256 ± 5.7559 ± 10.6949 | 68.5194 ± 4.0644 ± 12.531 | 68.6145 ± 4.0644 ± 12.531 |

FIG. 1: The plots show the one-dimensional posterior distributions for various cosmological parameters using different combined analysis of the observational data as displayed in Table II.

FIG. 2: 68% and 95% confidence-level contour plots in the two-dimensional ($H_0, w_x$) and ($\Omega_{m0}, w_x$) and ($\xi, w_x$) planes for different combined analyses have been shown. Left Panel: This shows that higher values of $H_0$ allow more phantom nature in the dark energy equation of state $w_x$, while the quintessence nature is favoured in $w_x$ for lower values of $H_0$. Middle Panel: Higher values of $\Omega_{m0}$ favor the quintessence character in the dark energy equation of state while the phantom character of $w_x$ is increased with the lower values of $\Omega_{m0}$. Right panel: The parameters $w_x$ and $\xi$ are almost uncorrelated with each other.
this figure we observe that, as \( w_x \) increases, that means when it shifts towards the quintessence regime, the density parameter for cold dark matter increases. From the right panel of Fig. 2 we show the dependence of coupling parameter \( \xi \) with the dark energy equation of state, \( w_x \), from which making any decisive conclusion between the dependence of \( \xi \) with \( w_x \) looks very hard, in fact, the parameters \( w_x \) and \( \xi \) look uncorrelated.

The inclusion of \( H_0 \) prior from HST also affects other cosmological parameters. For instance, from the constraints on the anisotropic stress displayed in Table II one can find the considerable changes in its constraints. The magnitude of the anisotropic significantly changes. The only CMB data constrains \( e_\pi = 0.0856^{+0.0438}_{-0.0495} \) at 68% CL (0.0856^{+0.0894}_{-0.0896} at 95% CL) while from CMB+BAO+WL+HST, \( e_\pi = -0.0014^{+0.0237}_{-0.0248} \) at 68% CL (-0.0014^{+0.0518}_{-0.0494} at 95% CL) and from CMB+BAO+RSD+WL+HST+JLA+CC, it is \( e_\pi = -0.0064^{+0.0194}_{-0.0277} \) at 68% CL (-0.0064^{+0.0215}_{-0.0243} at 95% CL). One may observe that there is no such significant changes in the error bars in the anisotropic stress. In Fig. 3 we present the 68% and 95% confidence-level contour plots where we show the effects of the anisotropic stress on some selected cosmological parameters.

We now focus on the dynamics of the universe on the large scales for the current cosmological scenario. In Fig. 5 we have plotted the CMB TT power spectra (see the left panel of Fig. 5) and the ratio of the CMB TT power spectra (see the right panel of Fig. 5) for different values of the anisotropic stress \( e_\pi \) and compared the analyses with the base ΛCDM model. It is quite clear from this figure that at low angular scales, for large anisotropic stress, the model deviates vastly from the ΛCDM model while as the angular scale increases, the deviation reduces from the ΛCDM and at high angular scales, the anisotropic stress does not produce any effective changes in the power spectra. However, the right panel of Fig. 5 says something more which is not visible from the left panel of Fig. 5. From the ratio of CMB TT spectra displayed in the right panel of Fig. 5 one can see that the model still shows a slight deviation from ΛCDM in the large angular scale and even if a nonzero value of the anisotropic stress is allowed. Thus, the model has a slight difference from ΛCDM and such a difference is very small. However, we have a very interesting observation from Fig. 6 displaying the CMB TT spectra and the ratio of the CMB TT spectra for different strengths of the coupling parameters. From the left panel of Fig. 6 one can see that the a slight deviation of the stressed interacting scenario from the ΛCDM model is observed for a large value of the coupling parameter (\( \xi = 0.8 \), which is a very big value in compared to the observational estimation summarized in Table II) while from the right panel of Fig. 6 it is quite clear that the model definitely has a deviation from the base ΛCDM for any \( \xi \neq 0 \). However, the deviation is not much significant.

### A. Comparison with no-anisotropic stress

In the previous section we have studied the effects of the anisotropic stress on the cosmological parameters when the dark fluids are interacting with each other. A question that immediately appears in this context is, how the contribution of anisotropic stress affects the large scale structure of the universe and also in the estimation of the cosmological parameters? To answer these questions, we perform similar analyses making \( e_\pi = 0 \) in the evolution equations with the same priors presented in Table II. It is quite evident that the analysis with no-anisotropic stress will effectively present a quantitative and qualitative differences on the cosmological parameters. The observational constraints for this particular scenario have been shown in Table III. The 68% and 95% confidence level contour plots for some selected parameters have been presented in Fig. 7 where we also show their one-dimensional posterior distributions. From both the analyses, one can clearly notice that except for CMB data only, the exclusion of the anisotropic stress lowers the strength of the coupling parameter. This is an interesting observation in this work which clearly demonstrates that the addition of \( e_\pi \) is the measure of increment in the coupling strength, \( \xi \). However, from the analyses presented in Table II and Table III a clear conclusion that one might draw is, in both the scenarios (with and without anisotropic stress), the coupling strength recovers its zero value within the 68% CL, that means the model \( w_x \) + CDM + \( \xi + e_\pi \) may recover the non-interacting \( w_x \) + ΛCDM cosmology within this 68% CL and this model has a close resemblance with that of the ΛCDM cosmology. This might be considered to be a common behavior of the models. Furthermore, it should be mentioned that only the combination CMB+BAO+RSD with \( e_\pi \neq 0 \), does not recover the \( \xi = 0 \) limit in anyway and thus, this particular combination always indicates for a non-zero interaction in the dark sector.

Probably the most interesting observation comes from the relative deviation of the CMB TT and matter power spectra shown in Fig. 8. A quick look says that both the scenarios are close to ΛCDM but there is something more that we would like to describe here. Let us focus on the left panel of Fig. 8 where the relative deviation of the CMB TT spectra has been shown. One can notice that for \( l \lesssim 10 \), the scenario \( w_x \) + CDM + \( \xi + e_\pi \) significantly differs from \( w_x \) + CDM + \( \xi \), and with the increase of \( l \), the difference between the scenarios decreases. We find a particular \( 10 < l < 10^2 \) where the difference between the models becomes zero, but after that, for some certain \( l \), the difference again increases where the model \( w_x \) + CDM + \( \xi + e_\pi \) stays far from ΛCDM in compared to \( w_x \) + CDM + \( \xi \). Further, we again notice that, the model \( w_x \) + CDM + \( \xi + e_\pi \) approaches toward ΛCDM and become closer in compared to the \( w_x \) + CDM + \( \xi \). However, for large \( l \), both the models become indistinguishable from one another. We note that, for all \( l \), the quantity \( \Delta C_{TT} / C_{TT} \) that reports the difference of the model from the ΛCDM...
FIG. 3: 68% and 95% confidence-level contour plots in $(e_{\pi}, H_0)$, $(e_{\pi}, \xi)$, $(e_{\pi}, w_x)$ planes have been shown for several observational combinations. **Left Panel:** This shows the $(H_0, e_{\pi})$ plane. One can see that the combination CMB+ext (where ‘ext’ is the other data sets, for instance BAO, RSD,.. etc) decreases the error bars on the parameters. Although, one cannot find a clear relation between $e_{\pi}$ and the Hubble parameter values, but the plots for different combinations (except CMB) slightly show that $e_{\pi}$ has a very weak tendency to increase its values for lower values of $H_0$. We repeat that such tendency is very very weak according to the current data we employ. **Middle Panel:** This actually infers a low interaction scenario with a small anisotropic stress. However, one can clearly notice that the parameters $(e_{\pi}, \xi)$ are almost uncorrelated with each other. **Right Panel:** One can see that the phantom dark energy allows lower values of $e_{\pi}$ while for quintessence dark energy one may expect slightly higher values of $e_{\pi}$, although, it is clear that the observational data do not allow a large $e_{\pi}$.

FIG. 4: 68% and 95% confidence-level contour plots in $(\sigma_8, e_{\pi})$, $(\sigma_8, \xi)$ and $(H_0, \sigma_8)$ planes for several observational combinations. **Left Panel:** From the plot, we do not observe any significant effect on $\sigma_8$ for anisotropic stress. In fact, one may see that a small value of $\pi$ is allowed in agreement with the estimated value of $\sigma_8$ from Planck [SS]. **Middle Panel:** One may notice that $\sigma_8$ has a slight dependence on $\xi$, although such dependence is weak but this is not null. One can see that $\sigma_8$ has a tendency to take lower values for increasing strength of the interaction. **Right Panel:** A weakly dependence between $H_0$ and $\sigma_8$ is reflected from this plot.

is very small.

Now we concentrate on the relative deviation in the matter power spectra, $\Delta P(k)/P(k)$. We remark that for all $k$, the quantity $\Delta P(k)/P(k)$ is very small informing the closeness toward the $\Lambda$CDM model, but however, we notice some additional features. We find that for very small $k$, almost for $k \lesssim 10^{-3}$, the quantity $\Delta P(k)/P(k)$ for the model $w_x \Lambda$CDM+$\xi + e_{\pi}$ is slightly greater in compared to the model $w_x \Lambda$CDM+$\xi$, however, for $k \gtrsim 10^{-3}$, the reverse scenario is observed, and finally, both the models look indistinguishable.

Thus, overall, one may conclude that indeed the scenarios $w_x \Lambda$CDM+$\xi + e_{\pi}$ and $w_x \Lambda$CDM+$\xi$ maintain differences amongst each other but for large $l$ (for CMB TT spectra) and large $k$ (matter power spectra), both the scenarios effectively approach toward the $\Lambda$-cosmology with almost no difference between them.
FIG. 5: The figure shows the CMB TT power spectra (Left Panel) and the ratio (also known as the relative deviation) of the CMB TT power spectra (Right Panel) for the interacting dark-energy scenario with and without the presence of anisotropic stress. Here, $\Delta C_{TT}^I = C_{TT}^I_{\text{model}} - C_{TT}^I_{\Lambda CDM}$ and $C_{TT}^I = C_{TT}^I_{\Lambda CDM}$. From the Left Panel, one may notice that at low angular scales, with the increase of $|e_{\pi}|$, the deviation from the non-interacting $\Lambda CDM$ becomes high, but however, at high angular scales, no such deviation in the CMB TT spectra for $|e_{\pi}|$ is observed. The similar behaviour is reflected from the Right Panel, although a non-zero deviation from the $\Lambda CDM$ even at high angular scales is observed here.

FIG. 6: The figure shows the CMB TT power spectra (Left Panel) and the ratio of the CMB TT power spectra (Right Panel) for the interacting dark-energy scenario in presence of the anisotropic stress. From the Left Panel we notice that even in presence of an anisotropic stress sourced by the matter field, the deviation in the CMB TT spectra mainly appears due to large values of the coupling parameter. The Right Panel confirms the observation in the Left Panel and additionally a deviation from the $\Lambda CDM$.

B. Easing the tension on $H_0$?

We now investigate whether the tension on $H_0$ is released in this context. The tension is one of the most talkative issues at current cosmological research. However, at the very beginning, we recall what exactly the tension on $H_0$ is. From the estimated values of $H_0$, one from Planck [88] (assuming $\Lambda CDM$ as the base model) and one from Riess et al. [84] (using the data from Hubble Space Telescope) one can see that the values conflict amongst each other with a sufficient difference between their measurements. From Planck, the estimation of the current Hubble constant is $H_0 = 67.27 \pm 0.66$ km s$^{-1}$ Mpc$^{-1}$, while the same is reported in [84] having $H_0 = 73.24 \pm 1.74$ km s$^{-1}$ Mpc$^{-1}$. That means the $H_0$ from [84] is about 3σ higher from Planck’s estimation. This is usually known as the tension on $H_0$. In the context of interacting dark energy, some latest articles [49, 52] argued that the allowance of such coupling in the dark sectors becomes efficient to release such tension. Indeed this is a very potential result because the allowance of extra degrees of freedom in terms of the coupling strength might be able to release such tension. The difference of the earlier works [49, 52] with the current one is very clear — here we consider the anisotropic stress into the picture, thus, perhaps one may expect slightly
FIG. 7: 68% and 95% confidence level contour plots for the interacting scenario with no-anisotropic stress have been shown using different combined analysis. The figure also contains the 2-dimensional posterior distributions for the parameters ($w_x$, $\xi$, $\Omega_{m0}$, $\sigma_8$, $H_0$). Here, the parameter $\Omega_{m0}$ is the current value of $\Omega_m = \Omega_c + \Omega_b$.

FIG. 8: The relative deviations in the CMB TT spectra (left panel) and matter power spectra (right panel) have been shown for the interacting scenarios with and without the anisotropic stress using the combined observational data CMB+BAO+RSD+WL+HST+JLA+CC.
TABLE III: 68% and 95% confidence-level constraints on the model parameters of the interacting scenario with no anisotropic stress for different combined analyses. Here, $\Omega_m = \Omega_0 + \Omega_b$.

| Parameters | CMB | CMB+BAO+RSD | CMB+BAO+RSD+HST | CMB+BAO+WL+HST | CMB+BAO+RSD+WL+HST+JLA+CC |
|------------|-----|-------------|-----------------|----------------|--------------------------------|
| $\Omega_0 h^2$ | $0.1214^{+0.0042+0.0052}_{-0.0032-0.0046}$ | $0.1808^{+0.0026+0.0042}_{-0.0014-0.0051}$ | $0.1206^{+0.0019+0.0085}_{-0.0013-0.0051}$ | $0.1190^{+0.0020+0.0044}_{-0.0015-0.0059}$ | $0.1183^{+0.0014+0.0030}_{-0.0013-0.0029}$ |
| $\Omega_b h^2$ | $0.0220^{+0.0016+0.0029}_{-0.0016-0.0032}$ | $0.0223^{+0.0015+0.0029}_{-0.0014-0.0029}$ | $0.0222^{+0.0015+0.0029}_{-0.0014-0.0029}$ | $0.0222^{+0.0016+0.0029}_{-0.0015-0.0029}$ | $0.0223^{+0.0014+0.0029}_{-0.0013-0.0029}$ |
| $10^5 y_0 h$ | $1.1040^{+0.0003+0.00064}_{-0.0003-0.00066}$ | $1.04055^{+0.0003+0.00065}_{-0.0002-0.00063}$ | $1.04058^{+0.0003+0.00065}_{-0.0002-0.00063}$ | $1.0404^{+0.0004+0.00069}_{-0.00036-0.00074}$ | $1.0406^{+0.0004+0.00060}_{-0.00039-0.00062}$ |
| $\sigma_8$ | $0.885^{+0.0032+0.0051}_{-0.0030-0.0049}$ | $0.805^{+0.0025+0.0046}_{-0.0024-0.0045}$ | $0.790^{+0.0022+0.0044}_{-0.0020-0.0043}$ | $0.789^{+0.0022+0.0044}_{-0.0020-0.0043}$ | $0.790^{+0.0022+0.0044}_{-0.0020-0.0043}$ |
| $H_0$ | $0.6980^{+0.0029+0.0068}_{-0.0030-0.0067}$ | $0.6980^{+0.0029+0.0068}_{-0.0030-0.0067}$ | $0.6980^{+0.0029+0.0068}_{-0.0030-0.0067}$ | $0.6980^{+0.0029+0.0068}_{-0.0030-0.0067}$ | $0.6980^{+0.0029+0.0068}_{-0.0030-0.0067}$ |

TABLE IV: 68% and 95% CL constraints on $H_0$ for the models with $e_0 \neq 0$ and $e_0 = 0$ for different combined analyses of the observational data.

| Parameters | CMB | CMB+BAO+RSD | CMB+BAO+RSD+HST | CMB+BAO+WL+HST | CMB+BAO+RSD+WL+HST+JLA+CC |
|------------|-----|-------------|-----------------|----------------|--------------------------------|
| $H_0 (e_0 \neq 0)$ | $68.64^{+4.07+1.12}_{-5.79-10.90} \times 10^{9}$ | $65.43^{+0.94+1.89}_{-0.53-2.38} \times 10^{9}$ | $65.52^{+0.94+1.89}_{-0.53-2.38} \times 10^{9}$ | $65.52^{+0.94+1.89}_{-0.53-2.38} \times 10^{9}$ | $65.52^{+0.94+1.89}_{-0.53-2.38} \times 10^{9}$ |
| $H_0 (e_0 = 0)$ | $69.08^{+2.79+9.10}_{-7.96-10.12} \times 10^{9}$ | $66.32^{+2.04+1.74}_{-1.86-2.83} \times 10^{9}$ | $66.32^{+2.04+1.74}_{-1.86-2.83} \times 10^{9}$ | $66.32^{+2.04+1.74}_{-1.86-2.83} \times 50^{9}$ | $66.32^{+2.04+1.74}_{-1.86-2.83} \times 50^{9}$ |

V. SUMMARY AND CONCLUSIONS

For the first time, we consider an interaction scenario between pressureless dark matter and dark energy when a matter-sourced anisotropic stress is present into the formalism. In general, the contribution from the anisotropic stress is often excluded from the cosmic picture, but however, a complete cosmological scenario must include all the associated parameters where theoretically, there is no such strong reason to exclude such anisotropic stress. And from the observational point of view, only the analyses might tell us whether the inclusion of anisotropic stress is necessary or not. Thus, keeping the anisotropic stress into our discussions, we try to explore this general interacting scenario. The dark energy equation of state, $w_0$, in this work has been considered to be time independent, and the interaction rate, $Q$, has been taken to be of the form $Q = 3H_2(1 + w_0)e_0$, in order to investigate the entire parameter space for $w_0$ unlike other interaction models where two separate regions for the dark energy equation of state are considered, see [30, 31] for a detailed motivation behind the choice of the above interaction rate. The cosmological scenario has been constrained for different combinations of the astronomical data with latest compilation (see Table V).

Our analyses show that the current observational data definitely indicate for a nonzero interaction in the dark sectors with a nonzero anisotropic stress in addition. That means, from the observational base, $e_0$ should not be identicaly taken to be zero to explore the dynamical features of the universe. Interestingly, most of the combined analyses include $e_0 = 0$ and $e_0 = 0$ within the

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TABLE V: For different regions of the dark energy state parameter, \( w_x \), we constrain the \( w_x \) CDM + \( \xi \) + \( e_x \) scenario using the combined observational data CMB + BAO + RSD + WL + HST + JLA + CC. The table shows the mean values of the cosmological parameters with their errors at 68% and 95% confidence-levels.

| Parameters | \( w_x \in [-2,-1.2] \) | \( w_x \in [-2,-1] \) | \( w_x \in [-2,-0.9] \) |
|-----------|-----------------|-----------------|-----------------|
| \( \Omega_{h}h^{2} \) | 0.02206\( \pm \)0.00027 | 0.02228\( \pm \)0.00029 | 0.02239\( \pm \)0.00029 |
| \( \Omega_{b}h^{2} \) | 0.0049\( \pm \)0.0011 | 0.0046\( \pm \)0.0039 | 0.0046\( \pm \)0.0039 |
| \( \tau \) | 0.0162\( \pm \)0.0317 | 0.0162\( \pm \)0.0317 | 0.0162\( \pm \)0.0317 |
| \( \nu \) | 0.0367\( \pm \)0.0075 | 0.0367\( \pm \)0.0075 | 0.0367\( \pm \)0.0075 |
| \( \ln(10^{10}A_{s}) \) | 3.0322\( \pm \)0.0322 | 3.0702\( \pm \)0.0323 | 3.0726\( \pm \)0.0325 |
| \( \epsilon_{e} \) | -0.0186\( \pm \)0.0196 | -0.0192\( \pm \)0.0194 | -0.0196\( \pm \)0.0194 |
| \( w_{x} \) | -1.2095\( \pm \)0.0975 | -1.0601\( \pm \)0.062 | -1.0010\( \pm \)0.048 |
| \( \xi \) | 0.0541\( \pm \)0.0089 | 0.0431\( \pm \)0.0060 | 0.0399\( \pm \)0.0058 |
| \( \Omega_{m0} \) | 0.0304\( \pm \)0.0147 | 0.0377\( \pm \)0.0152 | 0.0306\( \pm \)0.0138 |
| \( \sigma_{8} \) | 0.7811\( \pm \)0.0572 | 0.8074\( \pm \)0.0352 | 0.8079\( \pm \)0.0352 |
| \( H_{0} \) | 71.3016\( \pm \)0.6416 | 69.0966\( \pm \)0.6416 | 68.6800\( \pm \)0.6416 |

FIG. 9: 68% and 95% confidence level contour plots in the fixed plane \( (H_{0}, w_{x}) \) for different regions of the dark energy state parameter \( w_{x} \). The combined analysis for this analysis has been fixed to be CMB + BAO + RSD + WL + HST + JLA + CC.

68% confidence-region which means that the model could mimic the non-interacting \( w_x \) CDM model. And moreover, from the estimated values of \( w_x \) from different combined analyses, one can also see that \( w_x \) is very close to \(-1\) boundary meaning that the model is actually close to the \( \Lambda \)CDM cosmological model as well. However, the most striking result is observed from the perturbative analysis which shows that, the model is actually different from the \( \Lambda \)CDM model. We find that if we allow \( \xi \) to be very small (even if we assume \( \xi = 0 \)) but consider the anisotropic stress whatever small its strength be, a deviation from \( \Lambda \)-cosmology is pronounced from the ratio of CMB TT spectra (see the right panel of Fig. [5]). On the other hand, even if we make \( e_x = 0 \) and consider different strengths of the interactions (see Fig. [6]), then the interaction model shows a deviation from the \( \Lambda \)-cosmology that is only perfectly realized from the ratio of the CMB TT spectra displayed in the right panel of Fig. [6]. This is an interesting result because from the background analysis we could not distinguish the interaction model from the base \( \Lambda \)CDM while only the analysis at the perturbative level became able to find out such differences.

We also find that the tension on \( H_{0} \) can be alleviated. Actually, whenever interaction is present, then the re-
lease of tension on $H_0$ is possible as found in some latest investigations [46, 52] where the authors show that the coupling into the dark sector shifts the Hubble parameter value toward its local measurement. Since in the current work we consider the anisotropic stress into the picture, hence, we have investigated how the presence of an anisotropic stress controls the tension on $H_0$. The values of $H_0$ from different analyses have been shown in Table IV which clearly shows that the combined analysis CMB+BAO+WL+HST could alleviate the tension on $H_0$. The analysis with only CMB allow a very large region of $H_0$ even at 68% confidence-level and thus naturally, the tension is found to be released. While for the other combinations, we do not observe anything similar to that. But, interestingly enough, we find that if we allow $w_x$ to lie within the phantom region only, the tension is surely released (see the second column of Table IV). This result coincides with a latest investigation [52], although the major difference with this work is that, here we have an extra degrees of freedom in terms of the anisotropic stress.

As a closing remark, a number of investigations might be performed following the present work. In particular, it is interesting to see the behavior of the interacting scenario in presence of a dynamical $w_x$ instead of its constant value. The inclusion of massive neutrinos is another important addition in this picture. As the consideration of anisotropic stress is new in the context of coupled dark matter — dark energy models, one can explore some more interesting and important ideas. We hope to address some of them in near future, although such investigations are open to all.

ACKNOWLEDGMENTS

The authors acknowledge the use of publicly available markov chain monte carlo package cosmomc. WY acknowledges the support from the National Natural Science Foundation of China under Grants No. 11705079 and No. 11647153. LX acknowledges the support from the National Natural Science Foundation of China under Grant No. 11275035, No.11675032, and “the Fundamental Research Funds for the Central Universities” under Grant No. DUT16LK31. DFM acknowledges the support from the Research Council of Norway, and this paper is based upon work from COST action CA15117 (CANTATA), supported by COST (European Cooperation in Science and Technology).

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