Spontaneous excitation of an accelerated multilevel atom in dipole coupling to the derivative of a scalar field

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Abstract

We study the spontaneous excitation of an accelerated multilevel atom in dipole coupling to the derivative of a massless quantum scalar field and separately calculate the contributions of the vacuum fluctuation and radiation reaction to the rate of change of the mean atomic energy of the atom. It is found that, in contrast to the case where a monopole-like interaction between the atom and the field is assumed, there appear extra corrections proportional to the acceleration squared, in addition to corrections which can be viewed as a result of an ambient thermal bath at the Unruh temperature, as compared with the inertial case, and the acceleration induced correction terms show anisotropy with the contribution from longitudinal polarization being four times that from the transverse polarization for isotropically polarized accelerated atoms. Our results suggest that the effect of acceleration on the rate of change of the mean atomic energy is dependent not only on the quantum field to which the atom is coupled, but also on the type of the interaction even if the same scalar quantum field is considered.

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1. Introduction

There has been considerable interest in the radiative properties of uniformly accelerated atoms recently [1–8] and a new physical picture for the spontaneous excitation of a uniformly accelerated atom has emerged from works based upon the formalism to separate the contributions of vacuum fluctuations and radiation reaction proposed by Dalibard, Dupont-Roc and Cohen-Tannoudji (DDC) [1, 2] and subsequently generalized by others. In particular, using DDC’s formalism which demands a symmetric operator ordering of atom and field variables, one can show that, for ground-state atoms, the contributions of vacuum fluctuations...
and radiation reaction to the rate of change of the mean excitation energy cancel exactly and this cancellation forbids any transitions from the ground state and thus ensures atom’s stability in vacuum; while for any initial excited state, the rate of change of atomic energy acquires equal contributions from vacuum fluctuations and from radiation reaction and two contributions add up to the well-known spontaneous emission rate [1]. On the other hand, by generalizing the formalism of DDC [1, 2] to evaluate vacuum fluctuations and radiation reaction contributions to the spontaneous excitation rate of an accelerated two-level atom interacting with a scalar field in a linear monopole-like coupling, Audretsch, Müller and Holzmann [3, 5] have shown that if the atom is accelerated, then the delicate balance between vacuum fluctuations and radiation reaction is altered since the contribution of vacuum fluctuations to the rate of change of the mean excitation energy is modified while that of the radiation reaction remains the same. Thus transitions to excited states for ground-state atoms become possible even in vacuum. This result not only is consistent with the Unruh effect [9], which is closely related to the Hawking radiation of black holes, but also provides a physically appealing interpretation of it. Let us note here that recently a non-perturbative approach has been adopted to study the interaction of a uniformly accelerated detector, modelled by a harmonic oscillator which may be regarded as a simple version of an atom, with a quantum field in (3 + 1)-dimensional spacetime and some interesting new insights have been gained [10].

However, for a polarized neutral atom with an electric dipole moment, besides the monopole-like interaction considered in [3], the following manifestly invariant interaction Hamiltonian, in which the atomic dipole moment is coupled to the derivative of a scalar field, can also be introduced

\[ H_I(\tau) = m^\mu(\tau) \partial_\mu \phi(x(\tau)) = -e r^\mu(\tau) \partial_\mu \phi(x(\tau)), \]  

(1)

where \( e \) is the electron electric charge, \( m^\mu(\tau) = -e r^\mu(\tau) \), the atomic electric dipole moment, \( x(\tau) \leftrightarrow (t(\tau), x(\tau)) \), the spacetime coordinates of the atom and \( r^\mu(\tau) \), a 4-vector such that its temporal component in the frame of the atom (proper reference frame) vanishes and its spatial components are given by \( r(\tau) \). This choice of \( r^\mu(\tau) \) is based upon the atom’s stationarity in the proper reference frame and it ensures that if each component of the dipole moment is met to evolve in time in the same way as the Unruh-DeWitt’s monopole, then the direction of the dipole moment is kept fixed with respect to this frame and no extra time dependence will be brought in by the rotation of the dipole moment besides the intrinsic time evolution [11].

One may wonder happens to the radiation properties of accelerated atoms found in [3] when the monopole-like coupling is replaced by the above dipole derivative coupling. This is what we are going to examine in the present paper. We will separately calculate the contributions of vacuum fluctuations and radiation reaction to the rate of variation of the atomic energy of a multilevel atom in dipole coupling to the derivative of a real massless scalar field. Let us note here that the derivative coupling of the form (1) was first considered for the dipole detector as an alternative to the Unruh-DeWitt monopole one [12].

2. Interaction between a multilevel atom and the derivative of a scalar field

So, we will consider, in this paper, a multilevel atom interacting with the derivative of a scalar field. The Hamiltonian that controls the time evolution of the atom with respect to the proper time \( \tau \) is written as

\[ H_A(\tau) = \sum_n \omega_n \sigma_{nn}(\tau), \]  

(2)
where $|n\rangle$ denotes a series of stationary atomic states with energies $\omega_n$ and $\sigma_{nn}(\tau) = |n\rangle\langle n|$, ($\hbar = c = 1$). The free Hamiltonian of the quantum field is given by

$$H_F(\tau) = \sum_k \omega_k a_k^\dagger a_k \frac{d\tau}{d\tau},$$

where $\vec{k}$ denotes the wave vector and polarization of the field modes. The atom–field interaction Hamiltonian $H_I$ is given by (1), which, in the frame of the atom, reduces to

$$H_I(\tau) = -er_i(\tau)\partial_i\phi(x(\tau)) = -e\sum_{mn}\{r_i(\tau)\sigma_{mn}(\tau)\partial_i\phi(x(\tau)).$$

In what follows, we choose to work in this reference frame. The Heisenberg equations of motion for the dynamical variables of the atom and the scalar field can be derived from the Hamiltonian $H = H_A + H_F + H_I$:

$$d\frac{d}{d\tau}\sigma_{mn}(\tau) = i(\omega_m - \omega_n)\sigma_{mn}(\tau) - ie\partial_i\phi(x(\tau))[r_i(\tau), \sigma_{mn}(\tau)],$$

$$d\frac{d}{d\tau}a_k(\tau) = -i\omega_k a_k(\tau) - ier_i[\partial_i\phi(x(\tau), a_k(\tau))]\frac{d\tau}{d\tau}.\ (6)$$

In the solutions of the equations of motion, we can separate the ‘free’ and ‘source’ parts,

$$\sigma_{mn}(\tau) = \sigma^f_{mn}(\tau) + \sigma^s_{mn}(\tau), a_k(\tau) = a^f_k(\tau) + a^s_k(\tau),$$

where

$$\sigma^f_{mn}(\tau) = \sigma^f_{mn}(\tau_0) e^{i(\omega_m - \omega_n)(\tau - \tau_0)}, \sigma^s_{mn}(\tau) = -ie\int_{\tau_0}^{\tau} d\tau'[\partial_j\phi(x(\tau'))[r_j(\tau'), \sigma^f_{mn}(\tau)],$$

$$a^f_k(\tau) = a^f_k(\tau_0) e^{-i\omega_k(\tau - \tau_0)}, a^s_k(\tau) = -ie\int_{\tau_0}^{\tau} d\tau'[r_j(\tau')][\partial_j\phi(x(\tau')), a^f_k(\tau)].$$

3. The contributions of vacuum fluctuation and radiation reaction to the rate of variation of the atomic energy

We assume that the initial state of the field is the vacuum $|0\rangle$, while the atom is in the state $|b\rangle$. The equation of motion in the interaction representation for an arbitrary atomic observable $G(\tau)$, using symmetric ordering [1], can be split into the vacuum fluctuations and the radiation reaction contributions,

$$\frac{d}{d\tau}G(\tau) = \left(\frac{d}{d\tau}G(\tau)\right)_{\text{VF}} + \left(\frac{d}{d\tau}G(\tau)\right)_{\text{RR}},\ (10)$$

where

$$\left(\frac{d}{d\tau}G(\tau)\right)_{\text{VF}} = -ie\{[\partial_i\phi(x(\tau))[r_i(\tau), G(\tau)] + [r_i(\tau), G(\tau)]\partial_i\phi(x(\tau))],$$

$$\left(\frac{d}{d\tau}G(\tau)\right)_{\text{RR}} = -ie\{[\partial_i\phi(x(\tau))[r_i(\tau), G(\tau)] + [r_i(\tau), G(\tau)]\partial_i\phi(x(\tau))],$$

representing the contribution of the vacuum fluctuations and denoting that of the radiation reaction.
Our main aim now is to identify the contributions of vacuum fluctuations and radiation reaction in the evolution of the atom’s excitation energy, which is given by the expectation value of $H_A$. Separating $r_i(\tau)$ and $\sigma_{nn}(\tau)$ into their free part and source part and taking the vacuum expectation value, we can obtain, in a perturbation treatment up to order $e^2$,

\[
\langle 0| \frac{dH_A(\tau)}{d\tau} |0\rangle_{VF} = -e^2 \int_{\tau_0}^{\tau} d\tau' C^F_{ij}(x(\tau), x(\tau')) \left[ r^f_j(\tau'), \left[ r^f_i(\tau), \sum_n \omega_n \sigma^f_{in}(\tau) \right] \right],
\]

\[
\langle 0| \frac{dH_A(\tau)}{d\tau} |0\rangle_{RR} = e^2 \int_{\tau_0}^{\tau} d\tau' C^F_{ij}(x(\tau), x(\tau')) \left[ r^f_i(\tau'), \left[ r^f_i(\tau), \sum_n \omega_n \sigma^f_{ni}(\tau) \right] \right].
\]

The statistical functions, $C^F_{ij}(x(\tau), x(\tau'))$ and $\chi^F_{ij}(x(\tau), x(\tau'))$, of the field derivative are written as

\[
C^F_{ij}(x(\tau), x(\tau')) = \frac{1}{2} \langle 0 | \{ \partial_i \phi^f(x(\tau)), \partial_j \phi^f(x(\tau')) \} | 0 \rangle,
\]

\[
\chi^F_{ij}(x(\tau), x(\tau')) = \frac{1}{2} \langle 0 | \{ \partial_i \phi^f(x(\tau)), \partial_j \phi^f(x(\tau')) \} | 0 \rangle.
\]

Since we are interested in the evolution of expectation values of atomic observables, we take the expectation value of equations (13) and (14) in the atom’s state $|b\rangle$. Using the Heisenberg equation of motion, we can replace the commutator $[r^f_i(\tau), \sum_n \omega_n \sigma^f_{in}(\tau)]$ by $i \frac{d}{d\tau} r^f_i$, and obtain

\[
\langle \frac{dH_A(\tau)}{d\tau} \rangle_{VF} = 2ie^2 \int_{\tau_0}^{\tau} d\tau' C^F_{ij}(x(\tau), x(\tau')) \frac{d}{d\tau} \langle \chi^A_{ij} \rangle _b(\tau, \tau'),
\]

\[
\langle \frac{dH_A(\tau)}{d\tau} \rangle_{RR} = 2ie^2 \int_{\tau_0}^{\tau} d\tau' C^F_{ij}(x(\tau), x(\tau')) \frac{d}{d\tau} \langle C^A_{ij} \rangle _b(\tau, \tau'),
\]

where $|\rangle = |b, 0\rangle$. Here the symmetric correlation function and linear susceptibility of the atom are given analogously to equations (15) and (16) by

\[
(C^A_{ij})_b(\tau, \tau') = \frac{1}{2} \langle b | [r^f_i(\tau), r^f_j(\tau')] | b \rangle,
\]

\[
(\chi^A_{ij})_b(\tau, \tau') = \frac{1}{2} \langle b | [r^f_i(\tau), r^f_j(\tau')] | b \rangle.
\]

These functions do not depend on the trajectory of the atom and they only characterize the atom itself. Now we give the explicit forms of the statistical function of the atom

\[
(C^A_{ij})_b(\tau, \tau') = \frac{1}{2} \sum_d \left[ \langle b | r^f_i(\tau_0) | d \rangle \langle d | r^f_j(\tau_0) | b \rangle e^{i\omega_{bd}(\tau-\tau')} + \langle b | r^f_j(\tau_0) | d \rangle \langle d | r^f_i(\tau_0) | b \rangle e^{-i\omega_{bd}(\tau-\tau')} \right],
\]

\[
(\chi^A_{ij})_b(\tau, \tau') = \frac{1}{2} \sum_d \left[ \langle b | r^f_i(\tau_0) | d \rangle \langle d | r^f_j(\tau_0) | b \rangle e^{i\omega_{bd}(\tau-\tau')} + \langle b | r^f_j(\tau_0) | d \rangle \langle d | r^f_i(\tau_0) | b \rangle e^{-i\omega_{bd}(\tau-\tau')} \right],
\]

where $\omega_{bd} = \omega_b - \omega_d$ and the sum extends over a complete set of atomic states.
4. Spontaneous emission from a uniformly moving atom

To begin with, we first apply the above-developed formalism to consider the spontaneous emission from an inertial atom moving in the \( x \)-direction with a constant velocity \( v \). The atom’s trajectory is given by

\[
\begin{align*}
\tau(t) &= \gamma t, \\
x(t) &= x_0 + v\gamma t, \\
y(t) &= z(t) = 0,
\end{align*}
\]

where \( \gamma = (1 - v^2)^{-1/2} \). The correlation function of the field derivative in the frame of the atom can be readily calculated as follows:

\[
\langle 0 | \partial_i \phi^f (x(\tau)) \partial_j \phi^f (x(\tau')) | 0 \rangle = \frac{\delta_{ij}}{2\pi^2 (\tau - \tau' - i\epsilon)^4}. \tag{24}
\]

From equation (24), we get the symmetric function

\[
C_{ij}^F (x(\tau), x(\tau')) = \frac{\delta_{ij}}{4\pi^2} \left[ \frac{1}{(\tau - \tau' - i\epsilon)^4} + \frac{1}{(\tau - \tau' + i\epsilon)^4} \right], \tag{25}
\]

and the linear susceptibility

\[
\chi_{ij}^F (x(\tau), x(\tau')) = -\frac{i}{12\pi} \delta_{ij} \delta''''(\tau - \tau'), \tag{26}
\]

where \( \delta''''(\tau - \tau') \) is the third derivative of the Dirac delta function. With a substitution \( u = \tau - \tau' \), we obtain the contribution of the vacuum fluctuations to the rate of change of the atomic energy with equation (17)

\[
\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{VF} = -\frac{e^2}{4\pi^2} \sum_d |\langle b | r^f (\tau_0) | d \rangle|^2 \omega_{bd} \int_{-\infty}^{+\infty} du e^{iu\omega_{bd}} \left[ \frac{1}{(u - i\epsilon)^4} + \frac{1}{(u + i\epsilon)^4} \right]
\]

\[
= -\frac{e^2}{12\pi} \sum_{\omega_{bd} > 0} |\langle b | r^f (\tau_0) | d \rangle|^2 \omega_{bd}^4 + \frac{e^2}{12\pi} \sum_{\omega_{bd} < 0} |\langle b | r^f (\tau_0) | d \rangle|^2 \omega_{bd}^4, \tag{27}
\]

and that of the radiation reaction

\[
\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{RR} = \frac{ie^2}{12\pi^2} \sum_d |\langle b | r^f (\tau_0) | d \rangle|^2 \omega_{bd} \int_{-\infty}^{+\infty} du e^{iu\omega_{bd}} \delta''''(u)
\]

\[
= -\frac{e^2}{12\pi} \left( \sum_{\omega_{bd} > 0} |\langle b | r^f (\tau_0) | d \rangle|^2 \omega_{bd}^4 + \sum_{\omega_{bd} < 0} |\langle b | r^f (\tau_0) | d \rangle|^2 \omega_{bd}^4 \right), \tag{28}
\]

where we have extended the range of integration to infinity for sufficiently long times. After the calculation of the integrals, we obtain

\[
\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{tot} = -\frac{e^2}{6\pi} \sum_{\omega_{bd} > 0} |\langle b | r^f (\tau_0) | d \rangle|^2 \omega_{bd}^4. \tag{29}
\]

This result, except for a numerical factor, is the same as that obtained in [3] where a monopole-like interaction with the scalar field is assumed. This shows that, in terms of the spontaneous emission rate, an inertial atom does not tell the difference of the type of interaction between the atom and the quantum field, be it of the linear monopole-field or dipole-derivative type. However, things will be different when we come to the case of an accelerated atom, as we will demonstrate next.
5. Spontaneous excitation from a uniformly accelerated atom

Let us now turn our attention to the case in which the atom is uniformly accelerated in the $x$-direction. The atom’s trajectory is now given by

$$t(\tau) = \frac{1}{a} \sinh a \tau, \quad x(\tau) = \frac{1}{a} \cosh a \tau, \quad y(\tau) = z(\tau) = 0. \quad (30)$$

The two-point function of the field derivatives on the atom’s trajectory can be evaluated in the frame of the atom to get

$$\langle 0 | \partial_x \phi^f (x(\tau)) \partial_x \phi^f (x(\tau')) | 0 \rangle = \frac{a^4}{32\pi^2 \sinh^4 \left( \frac{a(\tau - \tau')}{2} \right)} \left\{ 1 - 2 \sinh^2 \left( \frac{a(\tau - \tau')}{2} \right) \right\}, \quad (31)$$

$$\langle 0 | \partial_y \phi^f (x(\tau)) \partial_y \phi^f (x(\tau')) | 0 \rangle = \langle 0 | \partial_z \phi^f (x(\tau)) \partial_z \phi^f (x(\tau')) | 0 \rangle = \frac{a^4}{32\pi^2 \sinh^4 \left( \frac{a(\tau - \tau')}{2} \right)} \sinh^4 \left( \frac{a(\tau - \tau')}{} + i \epsilon \right) \sinh^4 \left( \frac{a(\tau - \tau')}{} - i \epsilon \right), \quad (32)$$

and

$$\langle 0 | \partial_i \phi^f (x(\tau)) \partial_j \phi^f (x(\tau')) | 0 \rangle_{i \neq j} = 0. \quad (33)$$

Consequently, we obtain the symmetric correlation function

$$C_{xx}^F(x(\tau), x(\tau')) = \frac{a^4}{64\pi^2} \left[ 1 - 2 \sinh^2 \left( \frac{a(\tau - \tau')}{} \right) \frac{1}{\sinh^4 \left( \frac{a(\tau - \tau')}{} + i \epsilon \right)} + \frac{1}{\sinh^4 \left( \frac{a(\tau - \tau')}{} - i \epsilon \right)} \right], \quad (34)$$

$$C_{yy}^F(x(\tau), x(\tau')) = C_{zz}^F(x(\tau), x(\tau')) = \frac{a^4}{64\pi^2} \left[ 1 + \frac{1}{\sinh^4 \left( \frac{a(\tau - \tau')}{} + i \epsilon \right)} + \frac{1}{\sinh^4 \left( \frac{a(\tau - \tau')}{} - i \epsilon \right)} \right], \quad (35)$$

and the linear susceptibility

$$\chi_{xx}^F(x(\tau), x(\tau')) = -\frac{i}{2\pi \cosh \left( \frac{a(\tau - \tau')}{} \right)} \left( 5 + \cosh^2 \left( \frac{a(\tau - \tau')}{} \right) \right) \delta'''(\tau - \tau'), \quad (36)$$

and

$$\chi_{yy}^F(x(\tau), x(\tau')) = \chi_{zz}^F(x(\tau), x(\tau')) = -\frac{i}{2\pi \cosh \left( \frac{a(\tau - \tau')}{} \right)} \left( 5 + \cosh^2 \left( \frac{a(\tau - \tau')}{} \right) \right) \delta'''(\tau - \tau'). \quad (37)$$

Since the polarization direction of the atom can be arbitrary, in general, the polarization can have non-zero components in both the direction of acceleration and that which is perpendicular to it. So calculations have to be carried out for all non-zero field derivative statistical functions.

Then, it is easy to show the contribution of the vacuum fluctuations associated with the $xx$ component of the statistical functions is given by

$$\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{VFF_{xx}} = -\frac{e^2 a^4}{64\pi^2} \sum_d \omega_d |b_d|^2 \int_{-\infty}^{\infty} du e^{iu\omega_d} \left( 1 - 2 \sinh^2 \left( \frac{a u}{2} \right) \right) \frac{1}{\sinh^4 \left( \frac{a u}{2} \right)} \sinh^4 \left( \frac{a u}{2} + i \epsilon \right), \quad (38)$$
and that with other non-zero components are
\[
\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{\text{VFI}i} = -\frac{e^2 a_i^4}{64\pi^2} \sum_d \omega_{bd} |\langle b|r_i^H(\tau_0)|d\rangle|^2 \int_{-\infty}^{+\infty} du \, e^{i\omega_{bd}u} \\
\times \left[ \frac{1}{\sinh^2\left(\frac{\eta u}{2} - i\epsilon\right)} + \frac{1}{\sinh^2\left(\frac{\eta u}{2} + i\epsilon\right)} \right],
\]
for \(i \neq x\). With the help of the following integrals which can be easily calculated using the residue theorem
\[
\int_{-\infty}^{+\infty} du \, e^{i\omega_{bd}u} \left(1 - 2\sinh^2\left(\frac{\eta u}{2} - i\epsilon\right) + \sinh^2\left(\frac{\eta u}{2} + i\epsilon\right) \right) = \frac{16\pi}{3a_i^2} |\omega_{bd}| \left(\omega_{bd}^2 + 4a_i^2\right) e\left(\frac{2\pi|\omega_{bd}|}{a_i}\right),
\]
we find
\[
\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{\text{VFI}x} = -\frac{e^2}{12\pi} \sum_{\omega_{bd} > 0} |\langle b|r_i^H(\tau_0)|d\rangle|^2 \omega_{bd}^4 \left(1 + \frac{4a_i^2}{\omega_{bd}^2}\right) \left[1 + \frac{2}{e^{\frac{2\pi|\omega_{bd}|}{a_i}} - 1}\right],
\]
and
\[
\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{\text{VFI}i} = -\frac{e^2}{12\pi} \sum_{\omega_{bd} < 0} |\langle b|r_i^H(\tau_0)|d\rangle|^2 \omega_{bd}^4 \left(1 + \frac{4a_i^2}{\omega_{bd}^2}\right) \left[1 + \frac{2}{e^{\frac{2\pi|\omega_{bd}|}{a_i}} - 1}\right],
\]
for \(i \neq x\).

Vacuum fluctuations tend to excite an accelerated ground-state atom and de-excite it in the excited state and the probability of these processes are enhanced by the acceleration-dependent correction terms as compared to the inertial case. Notice the appearance of nonthermal term proportional to \(a_i^2\) as compared with the purely thermal result obtained in [3] for the monopole–field interaction. An interesting feature worth being noted is that the nonthermal correction for a longitudinally polarized atom (polarized in the direction of acceleration) is four times that for a transversely polarized atom (polarized in the perpendicular direction).

Similarly, we can find the contributions of the radiation reaction as follows:
\[
\left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{\text{RRxx}} = \frac{ie^2}{2\pi} \sum_d |\langle b|r_i^H(\tau_0)|d\rangle|^2 \omega_{bd} \int_{-\infty}^{+\infty} du \, \delta''(u) \frac{e^{i\omega_{bd}u}}{\cosh\frac{\eta u}{2}(5 + \cosh^2\frac{\eta u}{2})} \\
= -\frac{e^2}{12\pi} \sum_{\omega_{bd} > 0} |\langle b|r_i^H(\tau_0)|d\rangle|^2 \omega_{bd}^4 \left(1 + \frac{4a_i^2}{\omega_{bd}^2}\right) \\
+ \sum_{\omega_{bd} < 0} |\langle b|r_i^H(\tau_0)|d\rangle|^2 \omega_{bd}^4 \left(1 + \frac{4a_i^2}{\omega_{bd}^2}\right),
\]
and
\[ \left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{RRii} = \frac{ie^2}{2\pi} \sum_d |(b| r_i^f (\tau_0) |d)|^2 \omega_{bd} \int_{-\infty}^{+\infty} du \delta''(u) \frac{e^{i\omega_{bd}u}}{\cosh \frac{au}{2} (5 + \cosh^2 \frac{au}{2})} \]
\[ = -\frac{e^2}{12\pi} \left\{ \sum_{\omega_{bd} > 0} |(b| r_i^f (\tau_0) |d)|^2 \omega_{bd}^4 f_i(a, \omega_{bd}) \left( 1 + \frac{a^2}{\omega_{bd}^2} \right) \right\}
\[ + \left\{ \sum_{\omega_{bd} < 0} |(b| r_i^f (\tau_0) |d)|^2 \omega_{bd}^4 f_i(a, \omega_{bd}) \left( 1 + \frac{a^2}{\omega_{bd}^2} \right) \right\}, \quad (45) \]
for \( i \neq x \). The above result shows that the contribution of radiation reaction to the rate of change of the atomic energy always leads to a loss of energy of atoms and it is affected by the acceleration. This is to be contrasted to the case with a monopole-like interaction between the atom and the field where it has been demonstrated that first for uniformly accelerated atoms [3] and then for accelerated atoms on arbitrary stationary trajectory [5], the contribution of radiation reaction is generally not altered from its inertial value. This along with what we have found elsewhere when interaction with electromagnetic fields is considered [8] suggests that the non-alteration of the contributions of radiation reaction to rate of change of the mean atomic energy from its inertial value is a property which is only unique to the case considered in [3]. Again, the acceleration-induced correction for the longitudinally polarized atoms is four times that for transversely polarized ones.

Finally, we add up the contributions of vacuum fluctuations and radiation reaction to obtain the total rate of change of the atomic excitation energy:
\[ \left\langle \frac{dH_A(\tau)}{d\tau} \right\rangle_{tot} = -\frac{e^2}{6\pi} \sum_{\omega_{bd} > 0} \sum_i |(b| r_i^f (\tau_0) |d)|^2 \omega_{bd}^4 f_i(a, \omega_{bd}) \left( 1 + \frac{1}{e^{(2\pi \omega_{bd}/a)} - 1} \right) \]
\[ + \frac{e^2}{6\pi} \sum_{\omega_{bd} < 0} \sum_i |(b| r_i^f (\tau_0) |d)|^2 \omega_{bd}^4 f_i(a, \omega_{bd}) \frac{1}{e^{(2\pi |\omega_{bd}|/a)} - 1}, \quad (46) \]
where functions \( f_i(a, \omega_{bd}) \) are defined as follows:
\[ f_x(a, \omega_{bd}) = 1 + \frac{4a^2}{\omega_{bd}^2}, \quad f_y(a, \omega_{bd}) = f_z(a, \omega_{bd}) = 1 + \frac{a^2}{\omega_{bd}^2}. \quad (47) \]
For a ground-state atom, although both contributions of the vacuum fluctuations and radiation are altered for accelerated atoms with the dipole–derivative coupling, as opposed to no change in the contribution of radiation reaction in the monopole–field coupling [3], they conspire to change in such a way that the delicate balance between the vacuum fluctuations and radiation reaction no longer exists. There is a positive contribution from the \( \omega_b < \omega_d \) term, therefore transitions of ground-state atoms to excited states are allowed to occur even in vacuum.

6. Conclusions

In conclusion, assuming a linear coupling between the dipole moment of a multi-level atom and the derivative of a massless quantum scalar field, we have calculated the contributions of vacuum fluctuations and radiation reaction to the rate of change of the atomic energy of a uniformly accelerated atom. In contrast to the case where a monopole-like interaction is assumed [3], there appear extra corrections proportional to \( a^2 \), in addition to corrections which can be viewed as a result of an ambient thermal bath at the Unruh temperature \( T = a/2\pi \).
as compared with the inertial case. Let us note that similar terms have also been found in
the response function for a free falling Unruh detector in de Sitter space interacting with
minimally coupled massless scalar fields [13]. The deviation from pure thermal behaviour
of the spontaneous excitation rate of the uniformly accelerated atom in the our case and the
response functions of a free falling Unruh detector in de Sitter space by no means imply the
exact final thermal equilibrium is not achieved [13, 14]. As a matter of fact, with the transition
probabilities for the uniformly accelerated atom which can be found from our calculation in
the preceding section, one can show, by the same argument as that in [13], that exact thermal
equilibrium will be established at the Unruh temperature. However, although a discrepancy of
the excitation rate in the present case with the pure thermal one neither leads to thermal non-
equilibrium nor a different thermal equilibrium temperature, but, with different behaviours
of the transition probabilities of the atoms, it does imply a clear difference in how atomic
transitions occur and how the equilibrium is reached.

It is interesting to note that even if the atom is isotropically polarized, the acceleration
induced correction terms in addition to the pure thermal one show anisotropy with the
contribution arising from longitudinal polarization being four times that from the transverse
polarization. This can probably understood as a result of the fact that there is one distinguished
spatial direction, namely, the direction of acceleration. This is to be contrasted with the case in
which dipole coupling with electromagnetic field is considered [8] and there one finds that the
acceleration induced terms are isotropic. However, it should be pointed out that the isotropy
associated with electromagnetic field is probably a property unique to the four dimensions. An
example of this is that the electromagnetic vacuum noise is isotropic in four dimensions but
not in higher dimensions [11]. Finally, our results show that the effect of acceleration on the
rate of change of the mean atomic energy is dependent not only on the quantum field to which
the atom is coupled (electromagnetic versus scalar), but also on the type of the interaction
(monopole-field versus dipole-derivative) even if the same scalar quantum field is considered.

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