Tests of the Parton Model for Inclusive Semileptonic B Decays with the Heavy Quark Effective Theory

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Abstract

In order to study the end-point spectrum of inclusive semileptonic $b \to u$ decays, we investigate the parton model motivated by deep inelastic scattering process. The reliability of the model is carefully examined by comparing the decay rates and the lepton spectra for $b \to c$ decay with those of the heavy quark effective theory. We also compute the decay rates of the inclusive semileptonic decay into tau lepton in terms of the parton model, which give another test of the model independently. We conclude that one can make the parton model consistent with the heavy quark effective theory, and it would be phenomenologically useful in extracting $|V_{ub}|$. 

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Semileptonic decays of B mesons play an important role for understanding the Standard Model (SM). They are essential for testing and determining the parameters in the SM, and also provide valuable informations on the bound state structure of the mesons.

Recently, there has been much progress in the study of inclusive semileptonic decays of b hadrons with the help of heavy quark effective theory (HQET) [1–3]. One can get a QCD-based expansion in powers of $1/m_Q$ by performing an operator product expansion on the hadronic tensor. It has been shown that at leading order the contributions to the decay rates in the expansion coincide with those of the free quark decay. The terms of order of $1/m_Q^2$ are the leading corrections of the nonperturbative effects because terms of order of $1/m_Q$ vanish. These enable one to obtain model-independent calculations of the decay rates and lepton spectra for most region of phase space.

However, a model study is still required. The study of the end-point region of the electron spectrum in $b \rightarrow u$ decay where $\bar{B} \rightarrow X_c l \bar{\nu}$ decays are forbidden is of great interest in the decay of $\bar{B} \rightarrow X_u l \bar{\nu}$, because it is potentially useful to determine the Cabibbo-Kobayashi-Maskawa (CKM) angle $|V_{ub}|$. Unfortunately, the HQET calculation of the lepton spectrum for the inclusive semileptonic decay gives singularities in the end-point region [2,3] and modelling the end-point is inevitable for the extraction of $|V_{ub}|$ at present. Thus the following strategy may be presented: one picks a model, fixes its shape parameters from fitting the spectrum in $\bar{B} \rightarrow X_c l \bar{\nu}$ and applies it to the end-point region from where one extracts $|V_{ub}/V_{cb}|$. Up to now the most popular model for the inclusive semileptonic decays of heavy mesons is the model presented by Altarelli et al. [4], to be referred as ACCMM. They improve the free quark decay by considering the Fermi motion of the spectator quark in the meson. The ACCMM model has been reanalysed in the framework of the HQET recently by several authors [5]. They showed that the ACCMM model can be made to be consistent with the HQET by properly redefining the model.

In the recent publication [6], a parton model motivated by the deep inelastic scattering process (DIS) was developed and the results of the model was compared with experimental results. It is well known that inclusive semileptonic decays of heavy flavours are intimately
related to DIS via channel crossing. The model shows a good agreement with the experimental data and yields well-behaved lepton energy spectrum at the end-point region where it may serve as a useful model. The parton model seems to be one of the useful models that can describe this region. In this letter, we investigate the reliability of the parton model approach by explicitly comparing the results of the model with those of HQET. For the model to be phenomenologically reliable, we require that the decay rates and the lepton spectra (at least far from the end-point region) coincide with those of the HQET. This requirement would fix the parameter of the model and we can compare the lepton spectra of the parton model with those of the HQET. For the test of consistency of the model, we also study the lepton mass effect. Theoretically the lepton mass does not affect the picture of the parton model and we expect that the parameters which we have fixed in the case of the decay into an electron as a final state should be also valid in the decay into a massive lepton.

The lepton mass effects are significant in the semileptonic decay involving a tau lepton. The tau channel in semileptonic decays of B mesons is one of the latest measured process. Recently a branching ratio $\text{Br}(\bar{B} \to X\tau\bar{\nu})$ has been measured in LEP [8].

$$\text{Br}(\bar{B} \to X\tau\bar{\nu}) = 2.4 \pm 0.7 \text{ (stat.)} \pm 0.8 \text{ (syst.)} \%$$

This mode is of some theoretical interest by several reasons, since the decay kinematics and dynamics become complicated in the decay involving a tau lepton. There are more structure functions which does not contribute to the decays involving an electron. Since these changes are independent of the CKM angle $|V_{cb}|$ and $|V_{ub}|$, computation of the ratio of the decay rates for the tau lepton channel to those for the electron channel is useful in testing the model. Phenomenologically the study of $B \to X\tau\bar{\nu}$ is useful to determine the CKM angle $|V_{cb}|$ independently of the decay into an electron. This channel is also interesting as being the window to show the effects of physics beyond the SM. It has been used to probe the charged Higgs boson signal and gives some constraints on parameters of the minimal supersymmetric standard model [9].

In the following we briefly review the parton model for inclusive semileptonic decays of
B mesons and describe its extension to the decay of \( B \to X\tau\bar{\nu} \). The main ideas of this model are followings: i) it pictures the mesonic decay as the decay of the partons. We consider that the parton in the B meson carries a fraction of the B meson momentum. ii) The distribution of a parton in the B meson is determined by a single function, which will be discussed later.

The hadronic tensor is defined as

\[
4\pi W^{\mu\nu} = \int d^4x \ e^{iqx} < B|J^\mu(x)J^\nu(x)|B >
\]

\[
= \int \frac{d^3P_X}{(2\pi)^32E_X}(2\pi)^4\delta^4(P_B - P_X - q)\tilde{W}^{\mu\nu},
\]

where \( P_X \) are the momentum of the final state hadrons. Using standard definitions of the structure functions

\[
W^{\mu\nu} = -g^{\mu\nu}W_1 + P_B^\mu P_B^\nu \frac{W_2}{m_B^4} - i\epsilon^{\mu\nu\alpha\beta}P_B^{\alpha}q_\beta \frac{W_3}{2m_B^2}
+ q^\mu q^\nu \frac{W_4}{m_B^2} + (P_B^\mu q^\nu + P_B^\nu q^\mu) \frac{W_5}{2m_B^2} + i(P_B^\mu q^\nu - P_B^\nu q^\mu) \frac{W_6}{2m_B^2},
\]

we obtain

\[
W_1 = \frac{1}{2} (f(x_+) - f(x_-)),
\]

\[
W_2 = \frac{2}{m_B^2(x_+ - x_-)} (x_+ f(x_+) + x_- f(x_-)),
\]

\[
W_3 = -\frac{1}{2m_B^2(x_+ - x_-)} (f(x_+) + f(x_-)),
\]

\[
W_5 = -\frac{1}{2m_B^2(x_+ - x_-)} (f(x_+) + f(x_-)),
\]

\[
W_6 = -\frac{1}{2m_B^2(x_+ - x_-)} (f(x_+) + f(x_-)),
\]

where

\[
x_\pm = \frac{q_0 \pm \sqrt{q^2 + m_1^2}}{m_B}.
\]

The kinematics give the momentum transfer \( q = P_B - P_X \) which satisfies the following relations in the B-rest frame.
$2m_Bq_0 = m_B^2 - m_X^2 + q^2,$

$2m_Bp = (m_B^4 + m_X^4 + q^4 - 2m_B^2m_X^2 - 2m_B^2q^2 - 2m_X^2q^2)^{1/2},$  \hspace{1cm} (5)

where $q = (q_0, p)$ and $p = |p|$. $f(x)$ is the distribution function of $b$ quark in the $B$ mesons and $m_1$ a final state parton mass and $m_X$ an invariant mass of the final state hadrons. In the integrations the Dirac delta function has two positive roots $x_+, x_-$ because the momentum transfer $q^2 > 0$ in the decay process. On the other hand, in the deep inelastic scattering process where $q^2 < 0$ the Dirac delta function has only one positive root. In the eq. (3) the structure function $W_5$ contributes when the final lepton mass is nonvanishing.

The $b$ quark distribution inside $B$ mesons is related to the fragmentation function by crossing symmetry of the light quark followed by a time reversal transformation. Thus we use the measured fragmentation function of $B$ mesons as the distribution function of $b$ quark. Specifically we take the Peterson form [10] following the ref. [6,7], which is the fragmentation function usually used in Lund Monte Carlo programs:

$$f(x) = \frac{N}{x(1 - \frac{1}{x} - \frac{\epsilon_p}{1-x})^2}. \hspace{1cm} (6)$$

This is a single parameter function with a parameter $\epsilon_p$ and $N$ is the normalization factor.

For the $b \to u$ decays, the decay rates depends only on a single parameter $\epsilon_p$. Though we need two parameters $\epsilon_p$ and $m_c$ for the $b \to c$ decays, we may use the value of $m_c$ used in the calculation of HQET for the results to be consistent with those of HQET. The heavy quark mass was calculated up to $1/m_Q$ in terms of hadronic parameters with HQET and QCD sum rules [11,13]:

$$m_{HQ} = m_Q + \bar{\Lambda} - \frac{\lambda_1 + 3\lambda_2}{2m_Q}. \hspace{1cm} (7)$$

The parameter $\bar{\Lambda}$ is associated with the leading contribution to the mass of the light degrees of freedom of the heavy meson. The parameter $\lambda_1$ is the spin-averaged mass shift. The parameter $\bar{\Lambda}$ and $\lambda_1$ are related to some nonperturbative calculations. We choose the value of them from ref. [14], which are calculated by QCD sum rules,
0.45 GeV < \bar{\Lambda} < 0.60 GeV,

0 GeV^2 < -\lambda < 0.3 GeV^2. \quad (8)

The value of \(\lambda_2\) is obtained from the \(B - B^*\) mass splitting and we use the value \(\lambda_2 = 0.12\) GeV^2. The masses of B- and D- mesons are referred from the ref. [14].

\[m_B = 5.279\text{ GeV}, \quad m_D = 1.865\text{ GeV}\]

With these values we obtain the quark masses in eq. (7) as followings:

\[m_b = 4.776\text{ GeV}, \quad m_c = 1.414\text{ GeV}.\] \quad (9)

Now we fix the parameters of the model using the results of HQET. We have the total width in terms of HQET \[\Gamma_{HQET} = |V_{qb}|^2 \frac{G_F m_b^5}{192\pi^3} [(1 - K_b)z_0 + G_bz_1] \quad (10)\]

where \(z_0 = 1 - 8\rho + 8\rho^3 - \rho^4 - 12\rho^2 \ln \rho, \quad z_1 = 3 - 8\rho + 24\rho^2 - 24\rho^3 + 5\rho^4 + 12\rho^2 \ln \rho\) and \(\rho = m_q^2/m_b^2\). \(G_b\) and \(K_b\) are related to the terms of the operator product expansion and obtained from the parameters \(\lambda_1\) and \(\lambda_2\).

The parton model has the shape parameter \(\epsilon_p\). We show the decay rates of the decay \(\bar{B} \rightarrow Xe\bar{\nu}\) in the parton model, \(\Gamma_{\text{parton}}(\epsilon_p)\), with varying \(\epsilon_p\) in the table 1. In the numerical calculation we use above values of parameters and the mass of tau lepton

\[m_\tau = 1.777\text{ GeV}\]

for the value to be consistent with those of ref. [12]. We require the condition \(\Gamma_{HQET} = \Gamma_{\text{parton}}\) to fix the parameter \(\epsilon_p\). From fig. 1 we favor the values of \(\epsilon_p = 0.0035 - 0.004\). There has been some controversy for the value of \((m_X)_{\text{min}}\) in the decay of b to c, whether it corresponds to \(m_c\) or \(m_D\). If we measure the \(m_X\) spectrum in the experiment, it has the minimum at \(m_D\) as a resonance. But if we calculate the \(m_X\) spectrum we would not obtain such features as the resonant spectra of \(D, D^*\) etc. in the spectrum. It means that the \(m_X\) spectrum in our calculation is, in fact, corresponding to the momentum spectrum of
the final state parton instead of the invariant mass of the final state hadrons. Therefore we should take the \((m_X)_{\text{min}}\) as \(m_c\). We may compute the electron spectrum and compare it with those of the HQET. Figs. 2 shows normalized spectrum of the electron energy for the parameter \(\epsilon_p = 0.004\) in both of the cases \(b \to c\) and \(b \to u\) along with predictions by HQET in dotted curve. They agrees well with those of HQET far from the end-point region.

We also compute the decay rates of \(\bar{B} \to X\tau\bar{\nu}\) in terms of the parton model. Several authors have studied this channel in the framework of HQET \[12,15\]. It is convenient to use the ratio of the branching ratios of the decay with a tau lepton as a final state to those of the decay with an electron in analyzing this channel, because we are free from uncertainties of CKM angle \(|V_{cb}|\) and \(|V_{ub}|\). Furthermore the shape parameter \(\epsilon_p\) is associated with bound state structure of B meson and must be independent of the lepton flavours. Therefore the results for this channel which we compute with the value of \(\epsilon_p\) obtained from the results of table 1 should also agree with those of the HQET. Falk et al. \[12\] obtain the ratio

\[
\frac{\Gamma(\bar{B} \to X\tau\bar{\nu})}{\Gamma(\bar{B} \to Xe\bar{\nu})} = 0.215 \pm 0.035
\]

using the HQET and our result is

\[
\frac{\Gamma(\bar{B} \to X\tau\bar{\nu})}{\Gamma(\bar{B} \to Xe\bar{\nu})} = 0.213
\]

They also show a good agreement with each other. The QCD corrections are incorporated like ref. \[16\]. With the value \(\alpha_s(m_b) \approx 0.22\) the total rates are corrected by the factors 0.90 for \(\bar{B} \to X\tau\bar{\nu}\) and 0.88 for \(\bar{B} \to Xe\bar{\nu}\) respectively. In the tau channel the tau lepton spectrum is not measured directly and it is less valuable phenomenologically.

In conclusion the parton model provides a reliable method for analysing inclusive semileptonic decays of B mesons. The parton model has one single parameter. Even though it also depends upon the c-quark mass, \(m_c\), we may take its value from the HQET since the results of the HQET also contain \(m_c\). We fix the value of the parameter \(\epsilon_p\) by requiring the condition \(\Gamma_{\text{HQET}} = \Gamma_{\text{parton}}\). As independent test of our choice of the parameter, we also calculate the electron energy spectrum and compare the results of the model with those
of HQET explicitly and it shows a good agreement with HQET. The parton model gives well-behaved lepton spectra in the end-point region of the electron energy by a compact formular with a single parameter. We also study the semileptonic B decay into tau lepton in terms of the parton model and it provides another test of the model. It was shown that we can consistently fix the model by investigating three quantities, decay rates of $b$ to $c$ and $b$ to $u$ into the electron and the ratio of the decay rates into the electron to those into the tau lepton, which are independent of one another. From these we assert that the parton model is very useful in analyzing the B decay process, especially near the end-point region where the systematic expansion of HQET breaks down.

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TABLE I. The decay width of $\bar{B} \to Xe\bar{\nu}$ computed by the parton model with varying the parameter $\epsilon_p$.

| $\epsilon_p$ | $\Gamma(b \to c)/|V_{cb}|$ ($\times 10^{-11}$) | $\Gamma(b \to u)/|V_{ub}|$ ($\times 10^{-11}$) |
|-------------|--------------------------------------|--------------------------------------|
|             | parton model                         | HQET                                 | parton model                         | HQET                                 |
| 0.0030      | 2.901                                | 5.375                                |
| 0.0035      | 2.795                                | 5.200                                |
| 0.0040      | 2.700                                | 2.718                                | 5.042                                | 5.189                                |
| 0.0045      | 2.620                                | 4.906                                |
| 0.0050      | 2.544                                | 4.779                                |
| 0.0055      | 2.477                                | 4.664                                |
| 0.0060      | 2.416                                | 4.559                                |
| 0.0065      | 2.359                                | 4.462                                |
| 0.0070      | 2.305                                | 4.370                                |
| 0.0075      | 2.257                                | 4.285                                |
| 0.0080      | 2.211                                | 4.206                                |
| 0.0085      | 2.168                                | 4.130                                |
| 0.0090      | 2.128                                | 4.060                                |
FIGURES

FIG. 1. This is the ratios of decay rates calculated in the parton model with varying $\epsilon_p$ to those of the HQET, $\Gamma_{\text{parton}}/\Gamma_{\text{HQET}}$. The solid line is of the decay $b \to c$ and the dashed line $b \to u$.

FIG. 2. (a) The electron energy spectra for the decays of $b \to c$. (b) The electron energy spectra for the decays of $b \to u$. The solid line is of the parton model and the dashed line of the heavy quark effective theory. The parton model is computed with the parameter $\epsilon_p = 0.004$. 
Fig. 2 (a)

\[ \frac{1}{\Gamma} \frac{d\Gamma}{dE_e} \]

\( E_e \) (GeV)

Fig. 2 (b)

\[ \frac{1}{\Gamma} \frac{d\Gamma}{dE_e} \]

\( E_e \) (GeV)