Kinetic Energy, Condensation Energy, Optical Sum Rule and Pairing Mechanism in High-$T_c$ Cuprates

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The mechanism of high-$T_c$ superconductivity is investigated with interests on the microscopic aspects of the condensation energy. The theoretical analysis is performed on the basis of the FLEX approximation which is a microscopic description of the spin-fluctuation-induced-superconductivity. Most of phase transitions in strongly correlated electron system arise from the correlation energy which is competitive to the kinetic energy. However, we show that the kinetic energy cooperatively induces the superconductivity in the underdoped region. This unusual decrease of kinetic energy below $T_c$ is induced by the feedback effect. The feedback effect induces the magnetic resonance mode as well as the kink in the electronic dispersion, and alters the properties of quasi-particles, such as mass renormalization and lifetime. The crossover from BCS behavior to this unusual behavior occurs for hole dopings. On the other hand, the decrease of kinetic energy below $T_c$ does not occur in the electron-doped region. We discuss the relation to the recent observation of the violation of optical sum rule.

KEYWORDS: High-$T_c$ superconductivity; spin fluctuation; feedback effect; kinetic energy; condensation energy; optical sum rule

1. Introduction

The mechanism of high-$T_c$ superconductivity in cuprate materials has been one of the most appealing subject in the condensed matter physics over the last couple of decades. Through intensive studies from theoretical and experimental researchers, the “magnetic mechanism” is believed most predominantly.

The “magnetic mechanism” is represented by the spin fluctuation theory which takes into account the interaction between quasi-particles exchanging the spin fluctuations. Among the microscopic descriptions beyond the original phenomenology the fluctuation-exchange (FLEX) approximation is adopted most widely. The qualitative validity of this theory is highly expected from weak to intermediate coupling region. For example, the spin fluctuation theory is robust for the vertex corrections arising from the non-RPA terms as well as those from the multiple spin fluctuation exchange terms. On the other hand, in the expansion from the strong coupling limit, the resonating valence bond (RVB) theory and some numerical methods have concluded the $d_{x^2-y^2}$-wave superconductivity where the super-exchange interaction plays an essential role.

In the spin fluctuation theory, Eliashberg equation is used for an analysis of the superconductivity. The Eliashberg equation provides a clear understanding for the mechanism of superconductivity on the basis of the BCS picture. Then, the attractive interaction leading to the Cooper pairing is represented by the irreducible four point vertex. The momentum dependence of this vertex induces the non-s-wave superconductivity. In the spin fluctuation theory for high-$T_c$ cuprates, the irreducible four point vertex is described by the anti-ferromagnetic spin fluctuation and the attractive interaction is most effective in the $d$-wave channel.

The goal of this paper is to investigate the mechanism of high-$T_c$ superconductivity from another point of view. We study how the energy is gained below $T_c$ owing to the superconductivity. Of course, the ground state is determined as a result of the energetic optimization. The understanding from the energetics will be complementary to the analysis of the interaction leading to the pairing. Some interesting aspects are clarified from this point of view.

This study is partly motivated by the theoretical proposal for “kinetic energy driven pairing” and by the recent experimental supports for this proposal. In the conventional BCS theory, the kinetic energy increases owing to the superconductivity. This increase is slightly over-compensated by the decrease of correlation energy. Contrary to the BCS theory, the “kinetic energy driven pairing” attributes the mechanism of superconductivity to the gain of kinetic energy. This mechanism has been considered to be highly unconventional and the discrepancy to the spin-fluctuation-induced-superconductivity has been noted. On the other hand, the consistency to the RVB theory has been discussed, where the superconducting transition is triggered by the coherence of charge carriers. As for numerical studies, the dynamical cluster approximation and variational Monte Carlo simulation for Hubbard model have shown a decrease of kinetic energy owing to the superconductivity, and then implications for the RVB state have been noted.

In this paper, we study these problems on the basis of the microscopic and strong coupling theory on the spin-fluctuation-induced-superconductivity, namely the FLEX approximation. This subject has been investigated by several phenomenological theories assuming the non-Fermi liquid normal state, spin-Fermion coupling, superconducting phase fluctuation, and electron-phonon coupling. In contrast to these theo-
ries, the FLEX approximation is a “conserving approximation” which is highly suitable for a discussion of thermodynamic properties. In the conserving approximation formulated by Luttinger and Ward, all quantities are self-consistently derived from the differential of thermodynamic potential without any phenomenological assumption. Unphysical results inherent in the phenomenological theory are considerably excluded in the microscopic theory adopted here. It should be stressed that highly careful treatment is needed for thermodynamic properties rather than for magnetic or single-particle properties. This is partly because the condensation energy of superconductivity is much smaller than the energy scale of electrons. For example, the condensation energy is in the order of 0.1meV, while the band width is in the order of 1eV.

Note that kinetic energy along c-axis has attracted much interests in the early stage because it is related to the 'interlayer tunneling mechanism' (ILT) proposed by Anderson. Optical measurements have supported the decrease of c-axis kinetic energy. However, it has been shown that the gain of c-axis kinetic energy is much smaller than the condensation energy, and therefore this subject is not essential for the mechanism of superconductivity. In this paper we focus our attention on the kinetic energy along the plane.

In §2, we formulate a conserving approximation in the superconducting state and provide the expressions of FLEX approximation. Results on the kinetic energy are shown in §3.1. We show that the kinetic energy decreases below \( T_c \) in the under-doped region while it increases like BCS theory in the over-doped region and in the electron-doped region. It will be stressed that the concepts of “spin-fluctuation-induced-superconductivity” and “kinetic energy driven pairing” are not incompatible. The relation between the kinetic energy and the optical sum rule is discussed in §3.2. In §3.3, we discuss thermodynamic properties in more details. Then, we propose another interpretation of the condensation energy by considering the free energy arising from the spin fluctuation. Some discussions are given in §4.

2. Thermodynamic Property and FLEX Approximation below \( T_c \)

In this paper, we analyze the two-dimensional Hubbard model which is expressed as,

\[
H = \sum_{k,\sigma} \varepsilon(k)c_k^{\dagger}c_{k\sigma} + U \sum_i n_{i\uparrow}n_{i\downarrow}.
\]

We consider the square lattice and choose the following tight-binding dispersion,

\[
\varepsilon(k) = -2t(\cos k_x + \cos k_y) + t' \cos k_x \cos k_y.
\]

In the following, the unit of energy is chosen as \( 2t = 1 \). The next nearest neighbor hopping \( t' \) is necessary and sufficient to reproduce the Fermi surface of high-\( T_c \) cuprates. The typical value is estimated to be \( t'/t = 0.1 \sim 0.4 \). Qualitative results in this paper are not altered by this value. We fix \( t'/t = 0.25 \) in the hole-doped region and \( t'/t = 0.35 \) in the electron-doped region, respectively. The concentration of hole doping is expressed as \( \delta = 1 - n \) where \( n \) is the density of electrons per sites.

In the superconducting state, statistical quantum field theory is described by normal and anomalous Green functions, \( G(k) \) and \( F(k) \). The Dyson-Gorkov equation describes the Green functions through the normal and anomalous self-energies, which are denoted as \( \Sigma^0(k) \) and \( \Delta(k) \), respectively.

\[
\begin{pmatrix}
G(k) & F(k) \\
F^\dagger(k) & -G(-k)
\end{pmatrix} = \begin{pmatrix}
G(0) - \Sigma^0(k) & \Delta(k) \\
-\Delta^*(k) & -G(0) + \Sigma^0(-k)
\end{pmatrix}^{-1}
\]

Here, \( G(0)(k) \) is the Green function in the non-interacting case,

\[
G(0)(k) = \frac{1}{\omega_n - \varepsilon(k) + \mu},
\]

where \( \mu \) is the chemical potential. The superconducting gap \( \Delta(k) \) is obtained by the anomalous self-energy as

\[
\Delta(k) = z(k)|\Delta(k)| \quad \text{where} \quad z(k)^{-1} = 1 - \partial \Re G(\omega) / \partial \omega |_{\omega = 0}.
\]

In order to discuss the thermodynamic quantities, we first formulate a general expression for the thermodynamic potential in the superconducting state and derive the self-energy, momentum distribution function and kinetic energy on the basis of the functional derivatives. In the following, we describe the formulation in case of the spin singlet pairing.

The conserving form of the thermodynamic potential in the normal state was formulated by Luttinger and Ward and developed by Baym and Kadanoff. Then, the self-energy is obtained by the functional derivative of generating function \( \Phi \) as

\[
\sigma G(\sigma) = \delta \Phi [\sigma G(\sigma)] / \delta G(\sigma),
\]

The thermodynamic potential is obtained by the generating function and self-energy as,

\[
\Omega(T, \mu) = \Omega_0(T, \mu) - \sum_k \sum_{\sigma} \log \left( \frac{\delta \Phi (\sigma) G(\sigma)}{\delta G(\sigma)} \right) + G(\sigma) \Sigma^0(\sigma) + \Phi[\sigma G(\sigma)].
\]

Here, \( \Omega_0(T, \mu) = -2T \sum_k \log[1 + \exp(-\beta(\varepsilon(k) - \mu))] \) is the thermodynamic potential in the non-interacting case. Although we have formally introduced the index of spin \( \sigma \), indeed, \( G(\sigma)(k) = G(k) \) and \( \Sigma(\sigma)(k) = \Sigma(k) \) since we consider the paramagnetic state or spin singlet superconducting state.

It is straightforward to generalize this formulation to the superconducting state. We obtain the normal and anomalous self-energies from the generating function \( \Phi[\sigma G, F, F^\dagger] \) as,

\[
\sigma \Sigma^0(\sigma) = \frac{\delta \Phi}{\delta G(\sigma)},
\]

\[
\Delta = - \frac{\delta \Phi}{\delta F(\sigma)},
\]

\[
\Delta^*(\sigma) = - \frac{\delta \Phi}{\delta F^\dagger(\sigma)}.
\]

Note that eq. (7) (equivalently eq. (8)) is a self-consistent equation determining the second order phase transition.
The linearized version of eq. (7) has been used in order to determine the superconducting instability. We obtain the general expression of thermodynamic potential as,
\[ \Omega(T, \mu) = \Omega_0(T, \mu) + \Omega_F + \Omega_B, \]  
\[ \Omega_F = -\sum_k \left[ \log \left\{ \frac{\Sigma^2_c(k) - G^{(0)}(k)^{-1} + |\Delta(k)|^2}{| - G^{(0)}(k)|^{-1}} \right\} \right. 
\left. + \sum \sigma G_\sigma(k) \Sigma^n_\sigma(k) - F(k) \Delta^*(k) - F^\dagger(k) \Delta(k) \right], \]  
\[ \Omega_B = \Phi[G_\sigma, F, F^\dagger]_{\text{st}}. \]  
The derivation of eqs. (9-11) is summarized in Appendix. According to eqs. (6-11), the variational conditions with respect to the self-energy are satisfied as,
\[ \frac{\delta \Omega}{\delta \Sigma_n(k)} = \frac{\delta \Omega}{\delta \Delta(k)} = 0. \]  
Therefore, the self-energies obtained by eqs. (6-8) provide a stationary value of thermodynamic potential. According to the thermodynamics, we obtain the number density as
\[ n = -\frac{\delta \Omega}{\delta \mu} = 2 \sum_k n(k), \]  
where the momentum distribution function is also obtained by the functional derivatives as,
\[ n(k) = \frac{1}{2} \frac{\delta \Omega}{\delta \varepsilon(k)}. \]  
By performing the functional derivatives, eq. (14) is reduced to the usual definition of \( n(k) \),
\[ n(k) = \sum_\omega G(k) e^{i \omega_n \delta} = \sum_\omega [G(k) - G^{(0)}(k)] + f(\varepsilon(k) - \mu), \]  
where we have eliminated the ultra-violet divergence by subtracting the Fermi distribution function \( f(\varepsilon(k) - \mu) \). Finally, the kinetic energy is obtained as,
\[ E_k = \sum_k \varepsilon(k) \frac{\delta \Omega}{\delta \varepsilon(k)} = 2 \sum_k \varepsilon(k) n(k). \]  

When these relations are self-consistently satisfied in an approximation, the approximation is classified into the "conserving approximation". The FLEX approximation is one of them. In the following, we fix the number density \( n \) instead of the chemical potential \( \mu \). Therefore, the free energy \( F(T, n) = \Omega + \mu n \) is a more convenient quantity.

Hereafter, the superscripts \( S \) and \( N \) denote the superconducting and normal states, respectively. The former is defined by the stationary solution with finite value of \( \Delta(k) \) and the latter is defined by the solution with \( \Delta(k) = 0 \). Both solutions satisfy the variational conditions eq. (12), but only the former satisfies the stable condition below \( T_c \). The difference between normal and superconducting states are denoted as \( \delta A = A^N - A^S \). For example, the condensation energy is expressed as \( \delta F = F^N - F^S \).

Before closing the general formulation, we note some analytical expressions for the parameter dependence of the free energy. First, the number dependence of the free energy is given by the chemical potential \( \partial F/\partial n = \mu \). Therefore, the number dependence of the condensation energy is obtained as,
\[ \frac{\partial \delta F}{\partial n} = \mu^N - \mu^S. \]  
Second, the \( U \)-dependence of the free energy is given by the running coupling constant formula which is expressed as follows,
\[ \frac{\partial F}{\partial U} = \frac{1}{2U} \sum_k \sum_\sigma G_\sigma(k) \Sigma^n_\sigma(k) - F(k) \Delta^*(k) - F^\dagger(k) \Delta(k). \]  
The \( U \)-dependence of the condensation energy is simply obtained by the subtraction. These expressions are convenient to understand the qualitative behaviors of the condensation energy (see §3.3).

The formulation of FLEX approximation has been given in literatures,\(^5\,14\) The extension to the superconducting state is straightforward. Indeed, some authors have investigated the properties in the superconducting state by using the FLEX approximation. For instance, the temperature dependence of superconducting gap,\(^7\,8\) magnetic and single-particle properties\(^9\,13\) have been discussed. In this paper, we analyze the thermodynamic quantities and their relation to the optical sum rule.

The generating function \( \Phi[G_\sigma, F, F^\dagger] \) is obtained in the FLEX approximation as,
\[ \Phi[G_\sigma, F, F^\dagger] = \sum_q \left[ \frac{3}{2} \log \{1 - U \chi^0_S(q)\} + \frac{1}{2} \log \{1 + U \chi^0_c(q)\} \right. \]  
\[ \left. + \frac{1}{4} U^2 (\chi^0_S(q)^2 + \chi^0_c(q)^2) + U \left( \frac{3}{2} \chi^0_S(q) - \frac{1}{2} \chi^0_c(q) \right) \right]. \]  
Here, we have denoted the irreducible spin and charge susceptibilities as,
\[ \chi^0_{sc}(q) = -\sum_k [G(k + q)G(k) \pm F(k + q)F(k)]. \]  
Note that we have ignored the first order terms in the generating function since their roles are trivial and do not affect the following discussions.

We obtain the self-energy from eqs. (6-7) as,
\[ \Sigma^N(k) = \sum_q V_n(q) G(k - q), \]  
\[ \Delta(k) = -\sum_q V_\sigma(q) F(k - q), \]  
where \( V_n(q) \) and \( V_\sigma(q) \) are expressed as,
\[ V_n(q) = U^2 \left[ \frac{3}{2} \chi_S(q) + \frac{1}{2} \chi_c(q) - \frac{1}{2} (\chi^0_S(q) + \chi^0_c(q)) \right], \]  
\[ V_\sigma(q) = U^2 \left[ \frac{3}{2} \chi_S(q) - \frac{1}{2} \chi_c(q) - \frac{1}{2} (\chi^0_S(q) - \chi^0_c(q)) \right]. \]  
We have introduced the spin and charge susceptibilities.
obtained by the generalized RPA as,
\[
\chi_{s,c}(q) = \frac{\chi_{0,s,c}(q)}{1 \pm U\chi_{0,s,c}(q)}.
\]
(25)
The normal vertex \(V^a(q)\) and anomalous vertex \(V^c(q)\) are represented by the irreducible four point vertex in the particle-hole channel and in the particle-particle channel, respectively.

The superconducting transition from the normal state is determined by the appearance of non-trivial solution of eq. (22). In analogy with the gap equation in the BCS theory, we often denote \(V^c(q)\) as effective interaction leading to the pairing. Generally speaking, the momentum dependence of effective interaction results in the attractive interaction in a non-s-wave channel.\(^{14}\) In the FLEX approximation, the effective interaction \(V^c(q)\) is dominated by the spin fluctuation whose momentum dependence is favorable for the \(d_{x^2−y^2}\)-wave superconductivity. This is the ordinary understanding on the spin-fluctuation-induced superconductivity.

In this paper, we obtain another insights on the spin-fluctuation-induced superconductivity which are given by the analysis of energetics. The free energy is described as
\[
F = E_k + E_{\text{cr}} − TS
\]
where \(E_{\text{cr}} = < U n_{\uparrow} n_{\downarrow} >\) is the correlation energy and \(S\) is the entropy. At \(T = 0\), the condensation energy is obtained by the kinetic energy and correlation energy as
\[
\delta F = \delta E_k + \delta E_{\text{cr}}.
\]
In §3.1, we show that the kinetic energy increases the condensation energy cooperatively with the correlation energy. This is in sharp contrast with the weak coupling BCS theory where the kinetic energy remarkably decreases the condensation energy.

Note that the FLEX approximation provides a reasonable value of \(T_c\) not only in the hole-doped region but also in the electron-doped region. \(T_c\) and doping region with superconducting order is very small in the electron-doped region\(^{52–55}\) owing to the small DOS and the localized character of spin fluctuation in the momentum space.

It should be noticed that the FLEX approximation does not explain the pseudogap phenomena in the normal state of under-doped region. The superconducting fluctuation should be included to explain the pseudogap phenomena in this framework.\(^{53,56,57}\) However, it has been shown that the effects of superconducting fluctuation on the electronic state is rapidly suppressed below \(T_c\).\(^{58}\) Therefore, we believe that the FLEX approximation is appropriate for a description of superconducting state. We note that some interesting phenomena, such as magnetic resonance peak\(^{13}\) and kink in the electronic dispersion,\(^{59}\) are well explained within the FLEX approximation.

We have used the notations \(\sum_k = T/N \sum_{\omega_n,k}\) and \(\sum_q = T/N \sum_{\omega_n,q}\) where \(\omega_n = (2n + 1)\pi T\), \(\Omega_n = 2n\pi T\), \(T\) is the temperature and \(N\) is the number of sites. The unit \(\hbar = c = k_B = 1\) is used through this paper. Eqs. (20), (21) and (22) are estimated by using the fast Fourier transformation (FFT).

### 3. Results

Before showing the results, we note some cares involved in the numerical calculation because a careful treatment is highly needed for an estimation of thermodynamic quantities. This is partly because the electronic states far below Fermi level essentially contribute to the free energy, and partly because the condensation energy is a very small value in the order of \(10^{-4} \sim 0.1\text{meV}\).

![Fig. 1. (a) Temperature dependence of free energy in the normal state calculated by fixing the cut-off frequency. The solid line is a fitting curve with use of the function \(F^N = a + bT^2 + cT^3\log T\). The inset shows the same quantity with use of fixed \(N_f = 1024, 2048\) and 4096. (b) Temperature dependence of \(\delta F\) for various \(N_f\). We choose the parameters as \(\delta = 0.1\) and \(U/t = 4.2\).](image-url)

We divide the first Brillouin zone into \(N \times N\) and take \(N_f\) Matsubara frequency. The numerical inaccuracy mainly arises from the cut-off of Matsubara frequency. It should be noticed that we have introduced expressions without ultra-violet divergence in eqs. (9) and (15). This procedure remarkably improves the numerical accuracy. However, further care is needed for a temperature dependence of thermodynamic quantities. If we fix the number of Matsubara frequency \(N_f\), the cut-off of frequency depends on the temperature as \(\omega_c = (N_f - 1)\pi T\). This induces an artificial temperature dependence which may smear the intrinsic temperature dependence. This difficulty is very serious for the free energy, as is shown in Fig. 1(a). The main fig-
ure shows the temperature dependence of free energy $F^N$ calculated with the cut-off frequency fixed to be $\omega_c = 38.6 \sim 10 W$. Then, the free energy is well fitted by the function $F^N = a + bT^2 + cT^2 \log T$, which is consistent with the nearly anti-ferromagnetic Fermi liquid state.\textsuperscript{4,5} On the other hand, the free energy obtained by fixing $N_\text{f}$ shows much larger temperature dependence (see inset in Fig. 1(a)) indicating the violation of thermodynamic third law. Thus, we have to fix the cut-off frequency instead of $N_\text{f}$ in order to obtain appropriate results. Unfortunately, owing to the computational constraint arising from the FFT, it is troublesome to fix the cut-off frequency $\omega_c = (N_\text{f} - 1)\pi T$ for various temperatures.

However, this difficulty does not matter for the differences between normal and superconducting states. This is because the superconductivity affects the low energy states while high energy states are not sensitive to the superconductivity. For instance, we show the temperature dependence of $\delta F$ in Fig. 1(b). It is clearly shown that $\delta F$ depends on $N_\text{f}$ only slightly, and the fixed-$N_\text{f}$ calculation is valid. Note that $N_\text{f}$-dependence of the calculated free energy is still in the order of $10^{-3}$ at $T = 0.005$. However, the difference $\delta F$ is estimated to an accuracy of $10^{-6}$. This circumstance is in common with the momentum distribution function, kinetic energy and internal energy. We have confirmed that 2048 Matsubara frequency is sufficient for the following results. We show the results obtained by using 64 $\times$ 64 meshes or 128 $\times$ 128 meshes in the first Brillouin zone in the hole-doped case. We have confirmed that the finite size effects are negligible in these calculations. In the electron-doped region, we use 256 $\times$ 256 meshes in order to ensure the numerical accuracy.

### 3.1 Kinetic energy

First, Fig. 2(a) show the typical $U$-dependence of $\delta E_k$ in the hole-doped region. We have also performed the weak coupling theory using the second order perturbation theory (SOP) and random phase approximation (RPA). While the SOP and RPA are performed at $T = 0$, the FLEX is performed at $T = 0.005$. In the FLEX approximation at finite temperature, there is a critical value of $U$ above which superconductivity occurs. In the present case, $U_{\text{cr}}/t = 2.54$ at $T = 0.005$. This temperature is far below $T_c$ if $U/t > 2.8$. For example, $T_c = 0.0102$ at $U/t = 4.2$.

It is clearly shown that the sign of $\delta E_k$ changes from negative to positive with increasing $U/t$. The negative sign is expected in the conventional BCS theory. In the BCS theory, the kinetic energy increases owing to the particle-hole mixing which is essential for the Cooper pairing. This increase of kinetic energy is logarithmically divergent for the cut-off of energy as $\delta E_k = -\rho \Delta^2 \log \frac{\omega_c}{\Delta}$, whose absolute value is much larger than the condensation energy $\delta F = \frac{1}{2} \rho \Delta^2$. Here, $\rho$ is the electronic DOS at the Fermi level. In the SOP and RPA, these weak coupling behaviors are reproduced. We see that the FLEX approximation also reproduces the weak coupling behaviors in the weak coupling region.

On the other hand, the positive sign in the strong coupling region is a remarkably unconventional. In order to clarify the microscopic origin of this behavior, we show the momentum distribution function in Fig. 3. We see that $n^S(k) - n^N(k)$ takes large absolute value around the Fermi surface. This is because the quasi-particles near the Fermi surface mainly contribute to the superconducting gap. The qualitatively different behavior of $\delta n(k)$ in the direction perpendicular to the Fermi surface is a key to understand the unusual behavior. It is shown that $n^S(k) - n^N(k)$ is positive (negative) below (above) Fermi surface at the cold spot around $k = (\pi/2, \pi/2)$. The situation is opposite at the hot spot around $k = (\pi, 0)$. This result means that the kinetic energy arising from the hot spot increases owing to the particle-hole mixing induced by the superconducting gap. On the other hand, the kinetic energy arising from the cold spot decreases owing to the feedback effect on the spin fluctuation.\textsuperscript{7,8} The gap in the magnetic excitation induced by the superconducting gap remarkably decreases the correlation effects on the low energy electron states. Therefore, quasi-particles recover their coherent character below $T_c$. Since the superconducting gap is small at the cold spot, this feedback effect dominates the role of particle-hole mixing.

![Fig. 2. The difference of kinetic energy between the normal state and the superconducting state. (a) The results of SOP, RPA and FLEX at $\delta = 0.1$. (b) The results for 10%, 15% and 20% hole-dopings as well as 10% electron-doping.](image-url)
The sign of $\delta E_k$ is determined by these two competing effects. In the weak coupling region, the particle-hole mixing is dominant and $\delta E_k$ is negative. On the other hand, the feedback effect is dominant in the strong coupling region where $\delta E_k$ is positive. The qualitatively similar effect has been discussed phenomenologically as a “quasiparticle undressing”. Here, the “undressing” is caused by the feedback effect on the spin fluctuation. Note that the positive sign of $\delta E_k$ has been reported in the spin-fermion model where the momentum dependence is simply neglected. However, our microscopic calculation shows that the momentum dependence plays an essential role. Note that the positive value of $\delta E_k$ is caused by the feedback effect on the spin fluctuation.

The latter indicates that the spin fluctuation is very weak. Actually, an anti-ferromagnetic instability occurs in the electron-doped region. This is mainly because the tendency to the superconductivity is remarkably weak in the under-doped region. Combined with small value of $T_c$, the superconducting gap $\Delta$ in the electron-doped region is much smaller than that in the hole-doped region. Therefore, the effect of superconducting gap on the spin fluctuation is not so significant. We find that neither the magnetic resonance peak nor the kink in the electronic dispersion, which are interesting subsequences of feedback effect, appear in the electron-doped region.

The positive sign of $\delta E_k$ means that the kinetic energy plays a role for lowering the internal energy below $T_c$. This is in sharp contrast to the BCS theory, but as shown here, the spin-fluctuation-induced superconductivity also gives kinetic energy gain. It should be noticed that the FLEX approximation is a microscopic description of the nearly anti-ferromagnetic Fermi liquid theory, where the Cooper pairing between quasi-particles occurs. Therefore, the kinetic energy gain is not a consequence of the non-Fermi liquid normal state as assumed in Ref. 39, and it is not a negative evidence for the concept of Cooper pairing between quasi-particles as argued in Refs. 28-30.

Generally speaking, the kinetic energy gain can occur in the strong coupling superconductors where the feedback effect is important. However, such a strong feedback effect as to change the sign of $\delta E_k$ is not expected in the low-$T_c$ superconductors. We point out that the gain of kinetic energy in Fig. 2 is due to (i) strong AF spin fluctuation, (ii) high-$T_c$, namely large superconducting gap and (iii) $d$-wave symmetry. The existence of line node due to (iii) is especially important as is shown in Fig. 3.

Next, we discuss the doping dependence. Fig. 2(b) shows the results of $\delta E_k$ for various dopings. We have shown the results in the electron-doped region as well as in the hole-doped region. Here, the temperature is fixed to be $T = 0.005$ in the hole-doped case and $T = 0.003$ in the electron-doped case, respectively. We see that the kinetic energy gain occurs in the hole-doped region and the crossover value of $U$ increases with increasing the doping. Thus, the kinetic energy gain is likely in the under-doped region.

It is interesting that the kinetic energy gain does not occur in the electron-doped region. The region of superconducting state is very narrow as $U/t = 3.8 \sim 4$, and the tendency to the superconductivity is remarkably weak in the electron-doped region. This is mainly because the electronic DOS is small and also because the spin fluctuation is sharply localized in the momentum space. The latter indicates that the spin fluctuation is very weak. Actually, an anti-ferromagnetic instability occurs at $U/t > 4$ if we introduce a weak three-dimensionality. The absence of kinetic energy gain is due to the significantly small magnitude of superconducting gap. If we define $\Delta$ as the maximum of superconducting gap $\Delta(k)$, we obtain nearly the BCS value $2\Delta/T_c = 4 \sim 5$ in the electron-doped region, while $2\Delta/T_c = 8 \sim 10$ in the under-doped region. Combined with small value of $T_c$, the superconducting gap $\Delta$ in the electron-doped region is much smaller than that in the hole-doped region. Therefore, the effect of superconducting gap on the spin fluctuation is not so significant. We find that neither the magnetic resonance peak nor the kink in the electronic dispersion, which are interesting subsequences of feedback effect, appear in the electron-doped region.

### 3.2 Optical sum rule

The role of kinetic energy has been discussed extensively with the relation to the optical sum rule which is measured experimentally. Here, we show that the decrease of kinetic energy can be observed by the measurement of optical integral, although they are different quantities.

In the isotropic system like $^3$He, the optical integral, namely the frequency integral of optical spectrum, is conserved through the superconducting transition as,

$$
\int_{-\infty}^{\infty} \sigma_{xx}(\omega) d\omega = \pi e^2 n/m.
$$

(26)

This is called Ferrell-Grover-Timkam sum rule. Here, $\sigma_{xx}(\omega)$ is the optical conductivity including the $\delta$-function at $\omega = 0$, which corresponds to the superfluid density. This sum rule is generally violated under the periodic potential, namely in the metals. According to the Kubo formula, the optical integral is related to the momentum distribution function $n(k)$ as,

$$
\int_{-\infty}^{\infty} \sigma_{xx}(\omega) d\omega = 2\pi e^2 \Sigma_k \frac{\partial^2 \varepsilon_k}{\partial k^2} n(k).
$$

(27)

If we assume $t' = 0$, eq. (27) is expressed by the kinetic
energy as,

\[ \int_{-\infty}^{\infty} \sigma_{xx}(\omega)d\omega = -\frac{\pi e^2}{2} E_K, \tag{28} \]

This relation enables the kinetic energy to be measured experimentally.

However, the optical integral is not expressed by the kinetic energy in more general case \( t' \neq 0 \). Therefore, we define the optical energy as \( E_{\text{op}} = -2\Sigma_k \left( \frac{\partial^2 \varepsilon(k)}{\partial k^2} \right) n_k \) so that the optical integral is expressed as,

\[ \int_{-\infty}^{\infty} \sigma_{xx}(\omega)d\omega = -\frac{\pi e^2}{2} E_{\text{op}}. \tag{29} \]

Note that \( E_{\text{op}} \) is equivalent to \( E_k \) when \( t' = 0 \).

We show the \( U \)-dependence of \( \delta E_{\text{op}} \) in Fig. 4. It is clearly shown that the qualitative behavior of \( \delta E_{\text{op}} \) is the same as that of \( \delta E_k \) for a realistic value of \( t'/t \). The crossover of the sign occurs in common. We see that the absolute value of \( \delta E_{\text{op}} \) is larger than \( \delta E_k \). However, this tendency depends on the value of long range hoppings.\(^{65} \)

Recently, a violation of FGT sum rule has been actually observed in measurements of optical integral.\(^{31,32,60,61} \) The results seem to be controversial, but some of them indicate a decrease of kinetic energy.\(^{31,32} \) It seems that a very accurate measurement is needed since the change of optical integral may be much smaller than its absolute value.\(^{62} \)

Note that the maximum value of \( \delta E_{\text{op}} \) shown in Fig. 4 is \( \delta E_{\text{op}} \sim 0.2 \text{ meV} \) if we adopt the band width \( W = 8 \text{t} = 2 \text{ eV} \). This value is smaller than the reported value, \( \delta E_{\text{op}} \sim 1 \text{ meV}. \)\(^{31} \) However, this can be a sufficient agreement because the experimental value significantly depends on the frequency cut-off\(^{32} \) which is needed for a validity of the single-band description.

### 3.3 Free energy and internal energy

At the last of this section, we analyze the thermodynamic properties of spin-fluctuation-induced superconductivity in more details. First, we show the temperature dependence of internal energy \( \delta E \), kinetic energy \( \delta E_k \), optical integral \( \delta E_{\text{op}} \) as well as the free energy \( \delta F \) in Fig. 5. We estimate the free energy by eq. (9) and \( F = \Omega + \mu n \). Since the FFT involves unphysical results around the cut-off frequency, we replace the summation of Matsubara frequency in eq. (9) as \( \sum_{N_{f}/2}^{N_{f}/4} \rightarrow \sum_{-N_{f}/4}^{N_{f}/4} \). This ingenuity significantly improves an accuracy of numerical calculation. Here, we estimate the entropy \( S = -\frac{\partial F}{\partial T} \) by polynomial fitting and obtain the internal energy as \( E = F + TS \). We show the result for \( \delta E \) only around \( T = T_c \) because the polynomial fitting is not so accurate at low temperatures.

As is shown in the figure, we obtain the condensation energy \( \delta F \sim 5 \times 10^{-4} \sim 0.25 \text{ meV} \) which is consistent with experimental value.\(^{56,67} \) As for temperature dependence, the gain of kinetic energy shows qualitatively similar behavior to that of internal energy. The decrease of \( \delta E_k \) as decreasing temperature is mainly owing to the decrease of kinetic energy in the normal state. The damping of quasi-particle in the normal state is reduced by decreasing temperature, especially at the cold spot. This temperature dependence of \( \delta E_k \) is qualitatively independent of the parameters. Although \( \delta E_k \) at \( T = 0 \) seems to be negative in Fig. 5, we find that \( \delta E_k \) at \( T = 0 \) is positive when \( U/t \) and \( t'/t \) are large.

It is shown that a considerable part of the gain of internal energy is attributed to the kinetic energy, especially around \( T = T_c \). However, we see \( \delta E_k < \delta E \) in the whole temperature region. This means that the correlation energy also plays a positive role for the gain of internal energy. These features are robust in the intermediate coupling region \( U/t = 3 \sim 5 \), but the contribution of kinetic energy increases with increasing \( U/t \). We find that most part of the condensation energy is attributed to the kinetic energy at \( t'/t = 0.35, U/t = 7 \) and \( T = 0 \), but the kinetic energy gain is still smaller than the condensation energy. In the variational Monte Carlo study for the Hubbard model,\(^{36} \) there is a parameter region \( 10 \leq U/t \leq 12 \) where the kinetic energy and correlation energy play cooperative role.
It should be noticed that the temperature dependence of optical integral $\delta E_{\text{op}}$ is different from $\delta E_{\text{kin}}$, qualitatively. Thus, it is necessary to distinguish the optical integral and kinetic energy when detailed properties of optical integral are discussed experimentally. As we have shown in Fig. 3, $\delta E_{\text{kin}}$ and $\delta E_{\text{op}}$ are determined by competitive contributions from the hot spot and cold spot. Therefore, the detailed properties are sensitive to the long range hoppings.

The specific heat over temperature is estimated from the free energy as $C/T = -\frac{\partial^2 \delta F}{\partial T^2}$ and we obtain a very large jump of specific heat as $(C^S - C^N)/C^N \sim 5.3$, while $(C^S - C^N)/C^N \sim 1$ in the weak coupling $d$-wave BCS theory. This enhancement of specific heat jump is basically caused by the rapid increase of superconducting gap below $T_c$, which is a characteristic property of strong coupling superconductors. It should be noted that the FLEX approximation is not appropriate in the under-doped region around $T = T_c$, because the superconducting fluctuation plays an important role.\(^{14}\) Therefore, the large jump of specific heat is not observed in the under-doped region.\(^6^6\) However, this large jump has been observed in optimally-doped region\(^6^7\) as well as in a heavy fermion resemblance CeMIn\(_5\),\(^6^8\) where the pseudogap phenomena hardly occur.

Here, we propose another interpretation of the condensation energy. According to eq. (9), the condensation energy is expressed as $\delta F = \delta \Omega_B + \delta \Omega_F + \delta \Omega_0 + \delta \mu$. Fig. 6 shows the contributions from the first and second terms. The first term is expressed by eqs. (11) and (19). If we ignore the contribution from the charge susceptibility, eq. (19) is equivalent to the free energy arising from the spin fluctuation discussed by Brinkman \textit{et al.}\(^6^9\) They have estimated this term within RPA and shown that the Anderson-Brinkman-Morel (ABM) state in $^3$He is stabilized by the feedback effect. The expression of eq. (19) is also equivalent to the free energy discussed in the SCR.\(^7^0\) Therefore, the first term $\delta \Omega_0$ can be regarded as a free energy arising from the spin fluctuation. Fig. 6 show that the first term $\delta \Omega_0$ positively contributes to the condensation energy, while the second term $\delta \Omega_F$ is negative at $T > 0.003$. We see that the magnitude of $\delta \Omega_F$ is very small at low temperatures and therefore the condensation energy is basically determined by the contribution from $\delta \Omega_0$.\(^7^1\) This result implies that the condensation energy mainly originates from the feedback effect on the spin fluctuation. Note that the large spin fluctuation generally lowers $\Omega_F$. Although the static spin susceptibility $\chi_S(q)$ at $\Omega_F = 0$ is reduced by the superconducting gap, the dynamical part at $\Omega_F \neq 0$ is enhanced by the feedback effect. In the present case, the contribution from the dynamical part over-compensates the static part and induces the condensation energy of superconductivity.

Note that this interpretation is simple, but not unique, since the classification of free energy in eq. (9) is somewhat arbitrary. The spin fluctuation also affects the second term $\delta \Omega_F$ through the self-energy, and this contribution is necessary so as to satisfy the conservation laws. Nevertheless, this interpretation of condensation energy is somewhat interesting. We see in eq. (22) that the effective interaction mediated by the spin fluctuation induces the $d$-wave superconductivity. Then, the static part of spin fluctuation mainly works as a de-pairing effect through the normal self-energy, while the dynamical part works as a pairing effect through the effective interaction. This frequency dependence is qualitatively consistent with the above interpretation on the condensation energy.

We think that this interpretation of condensation energy is qualitatively similar to the previous proposal\(^7^2\) on the anti-ferromagnetic exchange energy arising from the magnetic resonance peak. In the FLEX approximation, the magnetic resonance peak clearly appears.\(^1^3\) Then, the frequency dependence of spin susceptibility on the real axis is determined by the frequency dependence on the imaginary axis. The latter has been discussed above and then we found that the dynamical part with $\Omega_n \neq 0$ increases owing to the superconductivity. This increase is an origin of magnetic resonance peak appearing in the spin susceptibility on the real axis. Therefore, the interpretation of condensation energy discussed above is directly related to the appearance of the magnetic resonance peak.

At the last of this section, we discuss the doping dependence of condensation energy. As shown in Fig. 7, the condensation energy has a dome shape. This behavior should be contrasted to the fact that $T_c$ increases with under-doping. The decrease of condensation energy in the under-doped region is basically due to the existence of competing order, namely the anti-ferromagnetism in the present case. In the under-doped region, the static spin correlation remarkably decreases the free energy in the normal state $F^N$, and therefore the condensation energy $\delta F = F^N - F^S$ is reduced. As shown in the inset, $\delta F$ increases monotonically as a function of $U$. Although the phenomenological treatment has concluded that the dome shape appears by increasing the effective coupling constant,\(^4^0\) our microscopic theory provides a qualitatively different result. We have confirmed that these parameter dependences are consistent with the numerical estimation of eqs. (17) and (18). For example, $\mu^N - \mu^S$ of optical integral $C/T$ and contributions from $\delta \Omega_F$ and $\delta \Omega_B$. We choose $\delta = 0.1$ and $U/t = 4.2$. Fig. 6. Temperature dependence of condensation energy $\delta F$, and contributions from $\delta \Omega_F$ and $\delta \Omega_B$.
in eq. (17) changes its sign around $\delta = 0.125$.

The dome shape of the condensation energy is qualitatively consistent with experimental observations.$^{66,67}$ However, we wish to stress that our definition of condensation energy is somewhat different from that used in the experimental analysis. Since a pseudogap exists above $T_c$, the extrapolation of the normal state free energy to $T < T_c$ needs many cares. If the superconducting fluctuation is an origin of pseudogap phenomena,$^{53}$ the free energy is reduced by the fluctuation above $T_c$. In this case, the condensation energy may be under-estimated if the role of pseudogap is neglected.$^{75}$ Actually, the experiments have observed a remarkable suppression of condensation energy in the under-doped region.$^{66,67}$

4. Summary and Discussions

In this paper, we have analyzed a mechanism of high-$T_c$ superconductivity with a main interest on the energetics. The Hubbard Hamiltonian was analyzed on the basis of the FLEX approximation. It is shown that the kinetic energy is decreased below $T_c$ in the under-doped region, while it is increased in the over-doped region. Interestingly, the gain of kinetic energy can not occur in the electron-doped region.

These findings are related to the violation of the optical sum rule. The recent observation in the under-doped region has reported incompatible results to the BCS theory. However, as shown here, the microscopic theory on the spin-fluctuation-induced-superconductivity reproduces the decrease of kinetic energy in relatively strong coupling region, $U/t > 4$. The decrease of kinetic energy in the ordered state is quite unusual, because the phase transition from the normal state is usually induced by the correlation energy which competes with the kinetic energy. The microscopic origin of the decrease of kinetic energy is the feedback effect.$^{7,8}$ The low energy spin fluctuation significantly suppresses the coherent motion of quasi-particles above $T_c$, and therefore quasi-particle lifetime is short. Below $T_c$, the low energy spin fluctuation is suppressed by the opening of superconducting gap. Therefore, the coherence of quasi-particles is recovered below $T_c$ especially around $(\pi/2, \pi/2)$, and the kinetic energy decreases.

We have shown that another interpretation of condensation energy is possible. As discussed in §3.3, the condensation energy is dominated by $\delta \Omega_2$ which is expressed as eq. (19). This term has been discussed as a contribution from the spin fluctuation to the condensation energy.$^{69}$ We find that while the static part of spin fluctuation plays a negative role for the condensation energy, the dynamical part plays a positive role. This interpretation is complementary with the analysis of the kinetic energy discussed above.

Although we have discussed the mechanism of high-$T_c$ superconductivity from the viewpoint of the condensation energy, we wish to stress that the understanding obtained from the analysis of effective interaction is clear and useful. As shown in this paper, the microscopic origin of the energy gain depends on the doping or $U/t$. For example, $\delta E_k$ is positive in the under-doped region, while it is negative in the over-doped region. On the other hand, the dominant scattering process leading to the superconductivity is universal, namely the anti-ferromagnetic spin fluctuation induces the strong scattering from $(\pi, 0)$ to $(0, \pm \pi)$ which is attractive in the $d$-wave channel. In other words, even if the behaviors of kinetic energy are different between under-doped and over-doped region, it does not mean that pairing mechanism changes as a function of doping.

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Appendix: Derivation of eqs. (9-11)

Here, we provide a derivation of eqs. (9-11) which are the general relation between the thermodynamic potential and Green function in the superconducting state. The derivation in the normal state has been given by Luttinger and Ward.$^{43}$ This can be extended to the superconducting state in a straightforward way.

We adopt the “Bogolyubov’s trick” in order to formulate the perturbation theory in the symmetry broken state. We add an infinitesimal symmetry-breaking term in the Hamiltonian as $H \rightarrow H'$.

$$H' = H'_0 + H_1,$$

$$H'_0 = \sum_{k, \sigma} \varepsilon(k)c^\dagger_{k\sigma}c_{k\sigma} - \Delta_0 c^\dagger_{k\uparrow}c_{-k\downarrow} - \Delta_0 c^\dagger_{-k\downarrow}c_{k\uparrow} \quad (A.1)$$

$$H_1 = U \sum_i n_i^\uparrow n_i^\downarrow. \quad (A.2)$$

Here, $\Delta_0$ is an infinitesimal value which breaks the $U(1)$ gauge symmetry. We take the limit $\Delta_0 \rightarrow 0$ at the last
of the derivation.

We denote the Green functions and self-energy in a matrix form.

$$\hat{G}(k) = \begin{pmatrix} G(k) & F(k) \\ F^\dagger(k) & -G(-k) \end{pmatrix}, \quad (A-4)$$

$$\hat{\Sigma}(k) = \begin{pmatrix} \Sigma^r(k) & -\Delta(k) \\ -\Delta^*(k) & -\Sigma^i(k) \end{pmatrix}. \quad (A-5)$$

Here, the non-interacting Green function is obtained as

$$\hat{G}_0(k) = \begin{pmatrix} (G^{(0)}(k)^{-1}) & \Delta_0 \\ \Delta_0^* & -(G^{(0)}(-k)^{-1}) \end{pmatrix}. \quad (A-6)$$

Then, Dyson-Gorkov equation is described as,

$$\hat{G}(k) = \hat{G}_0(k) + \hat{G}_0(k)\hat{\Sigma}(k)\hat{G}(k). \quad (A-7)$$

Following eqs. (A-8) and (A-11),

$$\frac{d\Omega}{dU} - \frac{d\Phi}{dU} = -\sum_k \text{Tr}(\hat{\Sigma}(k)) \frac{d\hat{G}(k)}{dU}. \quad (A-12)$$

Integrating the right hand side of eq. (A-13) with respect to $U$, we obtain

$$\Omega - \Phi - \Omega_0 = -\sum_k [\log \left( \frac{\text{det}\hat{G}(k)^{-1}}{\text{det}\hat{G}_0(k)^{-1}} \right) + \text{Tr}\hat{\Sigma}(k)\hat{G}(k)]. \quad (A-14)$$

Here, we have used the relations $\Omega = \Omega_0$ and $\Phi = 0$ at $U = 0$. Taking the limit $\Delta_0 \to 0$, we obtain eqs. (9-11).
44) G. Baym and L. P. Kadanoff, Phys. Rev. 124 (1961) 287; G. Baym and L. P. Kadanoff, Phys. Rev. 127 (1962) 1391.
45) P. W. Anderson, Science 268 (1995) 1154.
46) D. N. Basov, S. I. Woods, A. S. Katz, E. J. Singley, R. C. Dynes, M. Xu, D. G. Hinks, C. C. Homes and M. Strongin, Science 283 (1999) 49.
47) A. S. Katz, S. I. Woods, E. J. Singley, T. W. Li, M. Xu, D. G. Hinks, R. C. Dynes and D. N. Basov, Phys. Rev. B 61 (2000) 5930.
48) D. N. Basov, C. C. Homes, E. J. Singley, M. Strongin, T. Timusk, G. Blumberg and D. van der Marel, Phys. Rev. B 63 (2001) 134514.
49) K. A. Moler, J. R. Kirtley, D. G. Hinks, T. W. Li, M. Xu, Science 279 (1998) 1193.
50) A. A. Tsvetkov, D. van der Marel, K. A. Moler, J. R. Kirtley, J. L. de Boer, A. Meetsma, Z. F. Ren, N. Koleshnikov, D. Dulic, A. Damascelli, M. Grüninger, J. Schützmann, J. W. van der Eb, H. S. Somal and J. H. Wang, Nature, 395 (1998) 360.
51) J. R. Kirtley, K. A. Moler, G. Villard and A. Maignan, Phys. Rev. Lett. 81 (1998) 2140.
52) D. Manske, I. Eremin and K. H. Bennemann, Phys. Rev. B. 62 (2000) 13922.
53) Y. Yanase and K. Yamada, J. Phys. Soc. Jpn. 70 (2001) 1659.
54) H. Kondo and T. Moriya, J. Phys. Chem. Solids, 63 (2002) 1399.
55) H. Yoshimura and D. S. Hirashima, J. Phys. Soc. Jpn. 73 (2004) 2057.
56) Y. Yanase, J. Phys. Soc. Jpn 71 (2002) 278.
57) Y. Yanase, J. Phys. Soc. Jpn 73 (2004) 1000.
58) Y. Yanase, T. Jujo and K. Yamada, J. Phys. Soc. Jpn. 69 (2000) 3664.
59) We have confirmed that the kink structure in the electronic dispersion clearly appears in the superconducting state of under-doped region. This is not the case in the electron-doped region.
60) A. V. Boris, N. N. Kovaleva, O. V. Dolgov, T. Holden, C. T. Lin, B. Keimer and C. Bernhard, Science 304 (2004) 708.
61) S. Tajima, Y. Fudamoto, T. Kakeshita, B. Gorshunov, V. Zelezny, K. M. Kojima, M. Dressel and S. Uchida, cond-mat/0401447.
62) C. C. Homes, S. V. Dordevic, D. A. Bonn, R. Liang and W. N. Hardy, Phys. Rev. B 69 (2004) 024514.
63) M. Tinkham, Introduction to Superconductivity (McGraw-Hill, 1975).
64) D. J. Scalapino, S. R. White and S. C. Zhang, Phys. Rev. B 47 (1993) 7995.
65) We have confirmed that crossover in $\delta E_{op}$ occurs at $t'/t = 0.4$, but its absolute value is smaller than the gain of kinetic energy.
66) J. W. Loram, J. Luo, J. R. Cooper, W. Y. Liang, J. L. Tallon, J. Phys. Chem. Solids 62 (2001) 59.
67) N. Momono, T. Matsuzaki, M. Oda and M. Oda, J. Phys. Soc. Jpn. 71 (2002) 2832; T. Matsuzaki, N. Momono, M. Oda and M. Oda, J. Phys. Soc. Jpn. 73 (2004) 2232.
68) H. Hegger, C. Petrovic, E. G. Moshopoulou, M. F. Hundley, J. L. Sarrao, Z. Fisk, and J. D. Thompson, Phys. Rev. Lett. 84 (2000) 4986. C Petrovic, P G Pagliuso, M F Hundley, R Movshovich, J L Sarrao, J D Thompson, Z Fisk and P Monthoux, J. Phys. Condens. Matter 13 (2001) L337.
69) W. F. Brinkman, J. W. Seren and P. W. Anderson, Phys. Rev. A 10 (1974) 2386.
70) K. Makoshi and T. Moriya, J. Phys. Soc. Jpn. 38 (1975) 10.
71) The fraction of $\delta E_B$ of condensation energy depends on the parameters, however it is larger than 40% in the under-doped region and for realistic values of $t'/t$.
72) D. J. Scalapino and S. R. White, Phys. Rev. B 58 (1998) 8222.
73) E. Demler and S. C. Zhang, Nature 396 (1998) 733.
74) A. Abanov and A. V. Chubukov, Phys. Rev. B 62 (2000) R787.
75) D. van der Marel, A. J. Leggett, J. W. Loram and J. R. Kirtley, Phys. Rev. B 66 (2002) 140501.