We report on the realization and application of nondestructive three-qubit parity measurements on nuclear spin qubits in diamond. We use high-fidelity quantum logic to map the parity of the joint state of three nuclear spin qubits onto an electronic spin qubit that acts as an ancilla, followed by a single-shot nondestructive readout of the ancilla combined with an electron spin echo to ensure outcome-independent evolution of the nuclear spins. Through the sequential application of three such parity measurements, we demonstrate the generation of genuine multipartite entangled states out of the maximally mixed state. Furthermore, we implement a single-shot version of the Greenberger-Horne-Zeilinger experiment that can generate a quantum versus classical contradiction in each run. Finally, we test a state-independent noncontextuality inequality in eight dimensions. The techniques and insights developed are relevant for fundamental tests as well as for quantum information protocols such as quantum error correction.

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Parity measurements—measurements that reveal whether the sum of a (quantum) bit string is even or odd—are a prime example of the radically different roles of measurement in quantum physics and classical physics. In contrast to classical parity measurements, a quantum parity measurement is able to extract only the parity information from the system without revealing any additional information about the individual qubit states. Therefore, the coherences within the parity subspace into which the system is projected remain unaffected. Thanks to these unique quantum properties, parity measurements are at the heart of many quantum information protocols—for example, as stabilizer measurements in quantum error correction [1,2] or to generate entangled states [3–7]. In addition, their strikingly nonclassical behavior features in tests of the foundations of quantum mechanics [8,9].

Experimentally, realizing parity measurements that project a system on a parity subspace but are otherwise nondestructive is challenging: uncontrolled interactions with the environment as well as cross talk between system and measurement device lead to a leakage of information out of the measured system. Several types of parity measurements have been implemented in circuit quantum electrodynamics, trapped ions, and nuclear spins. Two-qubit parity measurements were realized nondestructively and repeatedly [7,10–12], and they were used for demonstrations of multiple-round quantum error correction [7,11], to test quantum contextuality [13] and for the preparation and stabilization of entangled states [4–7,10–12,14]. Multiqubit parity measurements have thus far been limited to either destructive measurements as a benchmark for quantum processors [15] or to a single nondestructive measurement [3,7,16] used to generate three-qubit [7] and four-qubit entangled states [3]. The ability to sequentially and nondestructively apply multiqubit parity measurements would open up new opportunities for quantum error detection and correction codes, state preparation, and fundamental tests.

In this Letter, we experimentally realize repeated three-qubit parity measurements on nuclear spin qubits in diamond while minimizing the disturbance of the state of the qubits. We exploit these nondestructive measurements to deterministically generate a three-qubit Greenberger-Horne-Zeilinger (GHZ) state from any input state with three consecutive parity measurements. Finally, we demonstrate the usefulness of these measurements for fundamental tests by performing two contextuality experiments.

We implement the parity measurements on $^{13}$C nuclear spins in diamond that are weakly coupled via hyperfine interaction to the electron spin of a nitrogen-vacancy (NV) center. These nuclear spins are an excellent workhorse for multiqubit protocols [7,17,18] thanks to their long coherence times and their insensitivity to the optical and microwave fields that are used to control the NV center electron. We use conditional quantum logic to map the parity of multiple nuclear spin states onto the electron spin that acts as an ancilla qubit. The electron is then read out in a single shot [19,20] (Fig. 1). In this way, only the parity of the nuclear spin is projected and no information about the individual state of the nuclei is extracted, ensuring the nondestructive nature of the measurement.

The measurement of the electron spin state is performed by optical excitation of a spin-dependent transition and detection of emitted photons. The cycling nature of the
transitions of the electron-controlled gate deviates from a CNOT gate, the nuclear spins undergo extra π/2 rotations (Fig. S2 of the Supplemental Material [21]). To remove these, we reverse the unitary electron-controlled gates, in a way that is independent of the measurement outcome [Fig. 1(c)]. This also resets the electron spin state to the initial state \( m_s = 0 \) that dephases the nuclear spins. We note that this parity preservation by itself does not guarantee preservation of the coherences within the parity subspace.

Because the implementation of the electron-controlled gate deviates from a CNOT gate, the nuclear spins undergo extra π/2 rotations (Fig. S2 of the Supplemental Material [21]). To remove these, we reverse the unitary electron-controlled gates, in a way that is independent of the measurement outcome [Fig. 1(c)]. This also resets the electron spin state to the initial state \( m_s = 0 \). The sequence is compiled where possible: we remove unnecessary gates and adapt the gates based on the phase accumulated by the nuclear spins [21].

The phase evolution of the nuclear spins depends on the electron spin readout outcome. A solution is to track the phase acquired for readout outcomes \( m_s = 0 \) and \( m_s = -1 \) separately and use feedback to switch to a different branch of the control sequence after each readout [7] [Fig. 1(d)].
However, this leads to an outcome dependence of the control sequence that complicates the interpretation of contextuality experiments. In addition, if each branch is preprogrammed, this leads to memory requirements that are exponential in the number of measurements. To avoid branching of the control sequences, we implement a spin echo [35] after the readout [Fig. 1(e)]. This removes any measurement outcome dependence of the remainder of the sequence. In addition, it reduces the memory required to store control sequences from exponential to linear in the number of readouts, which is important for more complex protocols with more subsequent readouts.

We now use these three-qubit parity measurements for the creation of a maximally entangled three-qubit GHZ state [36]. Preservation of the coherences within the parity subspaces after the parity measurement is crucial here: generating a GHZ state with three consecutive parity measurements is possible only if they are highly nondestructive. We consecutively apply the following parity measurements:

\[
p_1 = \sigma_{x,1} \otimes \sigma_{y,2} \otimes \sigma_{y,3},
\]

\[
p_2 = \sigma_{y,1} \otimes \sigma_{y,2} \otimes \sigma_{y,3},
\]

\[
p_3 = \sigma_{y,1} \otimes \sigma_{y,2} \otimes \sigma_{x,3},
\]

where \(\sigma_{x,k}\) and \(\sigma_{y,k}\) are the \(x\)- and \(y\)-Pauli matrices on the \(k\)th qubit. These measurements ideally project any input state of the three nuclear spins onto one of the eight GHZ states [e.g., \(\frac{1}{\sqrt{2}}(000 + 111)\)], depending on the measurement outcomes. To demonstrate this, we prepare the nuclear spins in the maximally mixed state before each measurement round by using resonant lasers that induce electron spin flips, and thereby dephase the nuclear spin states.

Each parity measurement contains four or five electron-controlled nuclear spin rotations that each consist of around 40 electron \(\pi\) pulses [21]. As a result, the measurement sequence for GHZ state generation and verification spans a total time of approximately 10 ms. The dephasing times of the nuclear spin states are of the same order \(T_2^* = 9.9(2), 11.2(3),\) and \(17.3(6)\) ms for nuclear spins 1, 2, and 3, respectively). However, dephasing is suppressed by the quantum Zeno effect [37,38]: repeated measurements project the state, restricting its evolution.

Measurements of the nonzero components of the resulting GHZ states on three nuclear spins are shown in Fig. 2. As expected from the readout characterization, we find that the best fidelity with a GHZ state \(F_{\text{GHZ}} = 0.68(1)\) is obtained when positive parity (corresponding to the electron spin state \(m_s = 0\)) is found three times in a row. But even when obtaining negative parity three times, the nuclear state has a fidelity \(F_{\text{GHZ}} = 0.57(1)\), still demonstrating genuine multipartite entanglement [39,40]. The observed fidelities are limited mainly by electron spin readout errors and by imperfections in the electron-controlled nuclear spin gates. The average fidelity for all eight states is \(0.634(3)\). With these nondestructive parity measurements, a multipartite entangled state can thus be deterministically prepared, as the long coherence times enable the application of fast feedback based on the measurement outcomes [7]. Importantly, these results show that the three-qubit parity measurements do not destroy coherences within the parity subspace.

FIG. 2. Creating a GHZ state by consecutive parity measurements. Conditional on the outcomes of three consecutive parity measurements, one out of eight maximally entangled GHZ states is created out of an initially maximally mixed state. The control sequences used to obtain the data (blue filled bars) include phase branching after each parity measurement [Fig. 1(d)]. To better estimate the nuclear spin state, the final readout is corrected for electron spin readout infidelity. Positive parity is mapped onto the \(\frac{1}{\sqrt{2}}(000 + 111)\). Black outlined bars indicate the ideal outcome for a GHZ state; dashed bars are outcomes of a simulation with independently characterized parameters [21]. Error bars (statistical standard error) on the data are indicated with black lines.
Phase-echoed [Fig. 1(e)] parity measurements (data in the Supplemental Material [21]) give an average GHZ state preparation fidelity of 0.600(3). We attribute the slight decrease in fidelity for the phase-echoed protocol to imperfect nuclear precession frequency calibration and imperfections in the electron echo pulse. We note that the good performance of the phase-echoed implementation is promising as it enables extension to protocols with more subsequent measurements, as required for, e.g., quantum error correction.

We next use sequential nondestructive parity measurements for a contextuality experiment. Quantum contextuality goes against the classical notion of noncontextuality: a measurement outcome does not depend on which other compatible measurements are performed jointly. Measurements are compatible if they can be measured jointly without disturbing each other’s measurement outcome; i.e., observables A and B are compatible if measuring consecutively A-B-A gives twice the same outcome for A.

These classical versus quantum contradictions can be probed experimentally in a GHZ test [36]. In the original version of this test, a system is prepared in a GHZ state and four sets of observables are measured: the three observables described in Eq. (1) and a fourth observable,

\[ p_4 = \sigma_{x,1} \otimes \sigma_{x,2} \otimes \sigma_{x,3}. \]  

If we measure the first three sets of observables on the GHZ state \( \sqrt{1/2(|000\rangle - |111\rangle)} \), we would, for ideal measurements, get the outcomes \( (P_1, P_2, P_3) = (+1, +1, +1) \), where \( P_j \) is the outcome of measurement \( p_j \). Given these three outcomes, a noncontextual theory predicts \( P_4 = +1 \). But quantum theory predicts \( P_4 = -1 \), thus showing a maximal contradiction with noncontextual models [36].

In previous experiments, the measurements \( p_j \) \( (j = 1, \ldots, 4) \) were implemented as classical parity measurements: each qubit is measured individually and the parity calculated using the classical outcomes. Because these measurements do not preserve coherences between the qubits, each measurement \( p_j \) needs to be performed separately on newly prepared GHZ states. In that case, the result can be formalized into an inequality as done by Mermin [41]. This GHZ experiment has been implemented in local [42–45] and distant setups [46,47].

An interesting variation of the GHZ experiment has been proposed in which the measurements \( p_j \) are performed as sequential quantum parity measurements on a single input state [48,49], enabling a test of quantum contextuality in a single-shot fashion, with a quantum or classical outcome after each sequence. Owing to the challenging requirement of multiple consecutive nondestructive single-shot parity measurements, experimental implementations have thus far remained elusive. In such an implementation, a maximal quantum versus classical contradiction is obtained in every measurement round since a noncontextual theory predicts \( (P_1 \times P_2 \times P_3 \times P_4) = 1 \), while quantum theory gives \( (P_1 \times P_2 \times P_3 \times P_4) = -1 \) [Fig. 3(a)]. Imperfections in measurement assignment fidelity reduce the expectation value of the product but, for compatible measurements, cannot cause a sign flip. This single-shot form of the GHZ test is state independent: the input of the measurement sequence does not have to be an entangled GHZ state but can be any state, even a maximally mixed state. Such state independence is a distinct feature of quantum contextuality tests [13,50–58].

We realize this single-shot GHZ experiment using parity measurements on nuclear spins with both the conventional phase-branched [Fig. 1(d)] and new phase-echoed readout methods [Fig. 1(e)]. We find results contrasting the classically expected outcomes with both readout methods: \( (P_1 \times P_2 \times P_3 \times P_4) = -0.58(6) \) and \(-0.5(1)\), respectively, as shown in Figs. 3(b) and 3(c).

The single-shot version of the GHZ experiment assumes that the parity measurements probe the individual underlying observables, e.g., that \( \sigma_{x,1} \otimes \sigma_{y,2} \otimes \sigma_{z,3} \) probes \( \sigma_{x,1} \), \( \sigma_{y,2} \), and \( \sigma_{z,3} \). The experiment can be extended to explicitly measure this, and to formalize the result through violation

\[
\begin{align*}
P_1 &\equiv \sigma_{x,1} \\
P_2 &\equiv \sigma_{y,2} \\
P_3 &\equiv \sigma_{z,3} \\
P_4 &\equiv \sigma_{x,1} \otimes \sigma_{y,2} \otimes \sigma_{z,3}.
\end{align*}
\]

FIG. 3. A single-shot GHZ experiment. (a) For the consecutive application of four parity measurements, a quantum versus classical contradiction is predicted in each measurement run. (b), (c) Both the (b) phase-branched and (c) phase-echoed implementations of the protocol are in contrast with the classical expectation (black outlined bars). Dashed bars are simulated outcomes [21]; error bars are indicated with black lines.
of an inequality. This is done by measuring four additional contexts [Fig. 4(a)], testing a noncontextuality inequality (NCI) as proposed by Cabello [59]:

\[
C = \langle C_1 \rangle + \langle C_2 \rangle + \langle C_3 \rangle + \langle C_4 \rangle - \langle C_5 \rangle \leq 3,
\]

where \(X_k\) and \(Y_k\) are the measurement outcomes corresponding to \(\sigma_{x,k}\) and \(\sigma_{y,k}\). The bound of 3 is found for noncontextual hidden-variable models, while for an ideal quantum system with perfectly nondestructive quantum measurements, \(C = 5\) is predicted. Like the single-shot GHZ experiment, this NCI is state independent. It requires the lowest number of contexts known of any state-independent NCI. To test it requires the application of up to four consecutive three-qubit parity measurements in an eight-dimensional system. So far, the highest-dimensional state-independent NCI that has been tested featured three sequential two-qubit parity measurements in a four-dimensional system [13].

We implement the NCI using the phase-echoed nuclear spin parity measurements [Fig. 4(b)] and observe a violation of the noncontextual bound, \(C = 3.19(2)\), rejecting the hypothesis that our experiment is described by a noncontextual model with a \(p\) value of \(1.21 \times 10^{-14}\) [21,60]. Note that, as in any such contextuality test, the measurements must be assumed to be compatible in order to reach this conclusion [61]. Since we efficiently detect the observables, no fair sampling assumption is necessary. With improved experimental parameters, e.g., using refocusing pulses on the nuclear spin states, decoherence-protected subspaces [33], or isotopic purification of the nuclear environment [62], experiments may be designed in which theories that include specific models for measurement incompatibility can be further restricted [61].

In conclusion, we achieve in this Letter repeated nondestructive three-qubit parity measurements, which constitutes the first realization in any experimental platform. We use a readout echo pulse to prevent nuclear phase branching, enabling a memory-efficient and outcome-independent implementation of sequential measurements. We apply three-qubit parity measurements on a maximally mixed state to generate genuine multipartite entanglement, and we realize a first test of quantum contextuality in a single shot. Furthermore, we push the implementation of noncontextuality tests to higher-dimensional systems than previously reported. The techniques and insights developed here can be directly applied to parity-measurement-based quantum computing protocols such as quantum error correction [1,2,7].

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