Asymptotic effects in jet production at high energies*

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Monte Carlo event simulation with BFKL evolution is discussed. We report current status of a Monte Carlo event generator ULYSSES with BFKL evolution implemented. The ULYSSES, based on Pythia Monte Carlo generator, would help to reveal BFKL effects at LHC energies. In particular, such an observable as dijet k-factor can serve as a source of BFKL dynamics at the LHC, and it would also help to search for new physics.

With advent of the Large Hadron Collider (LHC) at CERN we have an opportunity to probe the Standard Model far beyond the explored domains. QCD is an essential ingredient of the Standard Model, and it is well tested in hard processes when transferred momentum is of the order of the total collision energy (Bjorken limit: \(Q^2 \sim s \rightarrow \infty\)). The cornerstones of perturbative QCD at this kinematic regime (QCD-improved parton model) are factorization of inclusive hard processes \cite{1} and Gribov–Lipatov–Altarelli–Parisi–Dokshitzer (GLAPD) evolution equation \cite{2}. They provide a basis for the successful QCD-improved parton model. The factorization theorem \cite{1} for inclusive hard processes ensures that the inclusive cross section factorizes into partonic subprocesses and parton distribution function(s). The GLAPD-evolution equation governs the log \(Q^2\)-dependence (at \(Q^2 \rightarrow \infty\)) of the inclusive hard process cross-sections at fixed scaling variable \(x \sim Q^2/s\).

Another kinematic domain that is very important at high-energy is given by the Balitsky–Fadin–Kuraev–Lipatov (BFKL) limit \cite{3,4,5,6}, or QCD-Regge limit, whereby at fixed \(Q^2 \gg \Lambda^2_QCD, s \rightarrow \infty\). In the BFKL limit, the BFKL evolution in the leading-log approximation (LLA) governs log \((1/x)\) evolution (at \(x \rightarrow 0\)) of inclusive processes.

One of the key BFKL features is a relation of the highest eigenvalue, \(\omega^{\text{max}}\), of the BFKL equation \cite{3,4,5,6} (with \(s/s_0\)) to the intercept of the Pomeron, which in turn governs the high-energy asymptotics of the total cross-sections: \(\sigma \sim (s/s_0)^{\alpha_P} = (s/s_0)^{\omega^{\text{max}}}\), where the Regge parameter \(s_0\) defines the approach to the asymptotic regime. The BFKL Pomeron intercept in the LLA turns out to be rather large: \(\alpha_P - 1 = \omega^{\text{max}}_\text{LLA} = 12 \log 2 (\alpha_S/\pi) \approx 0.54\) for \(\alpha_S = 0.2\); hence, it is very important to take into account NLLA corrections \cite{7,8,9,10,11} to the BFKL. Note that the BFKL evolution in the next-to-leading-log approximation (NLLA) \cite{8,12}, unlike the LLA BFKL \cite{3,4,5}, includes GLAPD evolution with the running coupling constant of the leading-order (LO) GLAPD, \(\alpha_S(Q^2) = 4\pi/\beta_0 \log(Q^2/\Lambda^2_QCD)\).

One of the striking features of the NLLA BFKL analysis \cite{8} is that the NLLA value for the intercept of the BFKL Pomeron, improved by the BLM procedure \cite{13}, has a very weak dependence on the gluon virtuality \(Q^2\): \(\alpha_P - 1 = \omega^{\text{max}}_\text{NLLA} \approx 0.13 - 0.18\) at \(Q^2 = 1 - 100\ \text{GeV}^2\). This agrees with the conventional Regge theory where one expects a universal intercept of the Pomeron without any \(Q^2\)-dependence. The value of NNLA BFKL Pomeron intercept \cite{8} becomes compatible with the available data on hard QCD Pomeron. So, NLLA BFKL approach \cite{8,12} possesses all basic features of BFKL, but includes also running coupling effects and realistic value of the hard Pomeron intercept.

Therefore, the BFKL and especially the NLLA BFKL \cite{7,8,12} are anticipated to be important tools for exploring the high-energy limit of QCD.

It should be stressed that in contrast to the GLAPD, BFKL dynamics involves parton distributions unintegrated over \(k_t\). So, to reveal BFKL effects one needs to deal, e.g., with parton (jet) production. For jet production in the LLA BFKL within \(k_t\)-factorization \cite{14} one can use effective Feynman-like rules for inclusive jet cross sections \cite{15}.

However, extremely sophisticated design of contemporary high-energy detectors requires from theory and phenomenology a Monte Carlo event generator implementation. Widely used Monte Carlo event generator Pythia has been modified in parton shower kernel and parton distribution functions.

For Monte Carlo event simulations of GLAPD evolution it is convenient to use the Dokshitzer–Diakonov–Troyan (DDT) \cite{17} representation:

\[
\frac{\partial a(x, k_t^2)}{\partial \log k_t^2} = \frac{\alpha_S(k_t^2)}{2\pi} \sum b \int_x^1 dz P_{ab}(z) b \left( \frac{z}{x}, k_t^2 \right) - \frac{a(x, k_t^2)}{T_a(k_t^2, \mu^2)} \frac{\partial T_a(k_t^2, \mu^2)}{\partial \log k_t^2},
\]

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where \(P_{ab}(z)\) are kernels (splitting functions) of GLAPD equation for partons \(a\) and \(b\),

\[
a(x, \mu^2) = \int d^2 k_t^2 f_a(x, k_t^2, \mu^2), \quad a = g, q, \bar{q}
\]
is GLAPD parton distribution for parton \(a\), \(f_a(x, k_t^2, \mu^2)\) is so-called unintegrated parton distribution used in BFKL approach and

\[
T_a(k_t^2, \mu^2) = \exp \left( - \int_{k_t^2}^{\mu^2} d^2 k_t^2 \frac{\alpha_s(k_t^2)}{2\pi} \sum_b \int_0^1 d\zeta \, \zeta P_{ba}(\zeta) \right)
\]
is QCD analog for Sudakov form factor (DDT form factor \[17\]), which defines the probability not to emit a gluon.

In terms of unintegrated parton distribution the GLAPD equation can be represented as:

\[
f_a(x, k_t^2, \mu^2) = \frac{\partial}{\partial \log k_t^2} \left[ a(x, k_t^2) T_a(k_t^2, \mu^2) \right]
\]

\[
= T_a(k_t^2, \mu^2) \frac{\alpha_s(k_t^2)}{2\pi} \sum_b \int_x^1 dz P_{ab}(z) b \left( \frac{x}{z}, k_t^2 \right)
\]

Then, a unified BFKL-GLAPD equation can be presented in the following way \[18\]:

\[
f_a(x, k_t^2, \mu^2) = T_\gamma(k_t, \mu) \frac{\alpha_s(k_t^2)}{2\pi} \left\{ \int_x^{\mu/(\mu+k_1)} dz \int_{k_t^2}^{\mu^2} \frac{d^2 k_t^2}{k_t^2} \bar{P}(z) h_g \left( \frac{x}{z}, k_t^2 \right) \right. \\
+ \left. P_{gq}(z) \sum h_q \left( \frac{x}{z}, k_t^2 \right) \right\} + 2 N_C \int_x^{\mu/(\mu+k_1)} dz \int \frac{2 g^2}{\pi q^2} \\
\times \left[ \frac{k_t^2}{k_t^2} h_g \left( \frac{x}{z}, k_t^2 \right) - \Theta(k_t^2 - q^2) h_g \left( \frac{x}{z}, k_t^2 \right) \right], \quad \bar{P}(z) = P_{gg}(z) - \frac{2 N_C}{z}
\]

(1)

The Eq. (1) is similar to Ciafaloni-Catani-Fiorani-Marchesini (CCFM) \[19\] equation, which is a version of BFKL equation with imposed angle ordering.

![Inclusive to exclusive ratio](image)

**FIG. 1:** Dijet K-factor at LHC: inclusive dijet cross section over only-two-jet cross section ratio. The results are shown with full CMS detector simulation (arbitrary normalization) for luminosity 20 pb\(^{-1}\).
Monte Carlo event generator Ulysses contains an implementation of the Eq. into Pythia partron shower simulation and embedded BFKL unigreted partron distributions. Preliminary results for dijet K-factor at CMS detector of the LHC are shown in Fig. 1, where dijet K-factor for jets with $E_\text{t} > E_\text{t}^\text{min} = 60$ GeV is defined as ratio of inclusive dijet cross section to "exclusive" one. "Exclusive" dijet event means the event when only two jets with $E_\text{t}$ above $E_\text{t}^\text{min}$ are available ("Born" cross section).

To summarize, BFKL Monte Carlo should help to look at LHC for BFKL dynamics as a new feature of asymptotic QCD. It also has a potential for search of new physics, e.g., graviton production in trans-Planckian regime [21].

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