Fisher information as an indicator of quantum phenomena

Su-Yong Lee,1 Jeongho Bang,1 and Jaewan Kim1

1School of Computational Sciences, Korea Institute for Advanced Study, Hoegi-ro 85,Dongdaemun-gu, Seoul 02455, Korea
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Fisher information quantifies how well we can detect small changes in a parameter. According to the parameter that we focus on, the Fisher information presents different quantum phenomena. Here we investigate quantum interference of two particles in a two-wave mixing operation. Due to the symmetry of the two-wave mixing operation, we construct the Fisher information only by counting the number of particles in one of the output modes. When we focus on an input state parameter, we cannot discriminate particle species, i.e., boson and fermion, but can observe Hong-Ou-Mandel effect. When we focus on a parameter of the two-wave mixing operation, on the other hand, we can discriminate the particle species and extend it to detect two-particle entanglement scenario.

I. INTRODUCTION

Fisher information, introduced by R.A. Fisher, is called a measure of indeterminacy [1] and has been used to derive the lower bound for the mean-squared error of the unbiased estimator of a parameter [2]. In quantum metrology, the Fisher information is exploited to determine a small deviation from a true value of a parameter [3] while depending on a specific measurement basis. Over all possible measurement bases, the Fisher information is maximized to quantum Fisher information. The quantum Fisher information is an intrinsic quantity for a quantum state, which requires the optimal measurement basis to implement in a laboratory. However, given any measurement basis, we can readily obtain the Fisher information by performing measurements in the output state and processing the measurement outcomes. Except for the quantum metrology, the Fisher information was applied only to few other quantum tasks [4, 5] whereas the quantum Fisher information was widely applied to other quantum tasks [6–18].

Fisher information is related with the relative entropy, which is a measure of the statistical difference between two probability distributions [2]. When the two probability distributions are differed by a small variation of a parameter \( \Delta X \equiv X - X_0 \), the relative entropy is approximated as

\[
D(P(X)||P(X_0)) = \sum_y P(y|X) \ln \frac{P(y|X)}{P(y|X_0)} \approx \frac{(\Delta X)^2}{2} F(X_0),
\]

where \( X_0 \) is the initially known parameter value, \( P(y|X) \) is a probability distribution of measurement outcome \( y \) in a parameter \( X \), and \( F(X_0) \) is the Fisher information of \( X_0 \). In terms of the probability distributions of measurement outcomes \( y \) in a parameter \( X_0 \), the Fisher information of \( X_0 \) is defined as

\[
F(X_0) = \sum_y \frac{1}{P(y|X_0)} \left( \frac{\partial P(y|X_0)}{\partial X_0} \right)^2.
\]

From the two neighborhood probability distributions, thus, the relative entropy is approximated as the Fisher information of \( X_0 \) [19] multiplied by \((\Delta X)^2/2\). Since the Fisher information is the second-order derivative of the relative entropy, it implies a curvature of the relative entropy.

![FIG. 1. Framework for indicating quantum phenomena via Fisher information. The Fisher information, which is derived from the measurement outcomes in one of the output modes, provides the information on how well we can detect small changes in a parameter. \( \theta \) is encoded into one of the input modes \((|\varphi\rangle_a|\varphi\rangle_b)\), and \( \phi \) is a parameter of a two-wave mixing operation. We are interested in the input state parameter \( \theta \) and the parameter \( \phi \) of the two-wave mixing operation.

From the viewpoint of the statistical difference, we propose Fisher information as an indicator of quantum phenomena, such as the Hong-On-Mandel effect and a two-particle entanglement detection. In Fig. 1 we show our framework that identical two particles interact by a two-wave mixing operation, where a parameter \( \theta \) is encoded into one of the input particles and a parameter \( \phi \) is encoded into both particles. For simplicity, we consider the parameter \( \theta \) as an input state parameter. After interacting the two particles by the two-wave mixing operation, we evaluate the Fisher information by counting the number of particles in one of the output modes. Due to the symmetry of the two-wave mixing operation, the local Fisher information is the same as the global Fisher information which is constructed by counting the number of particles in both output modes. Thus we only consider the local Fisher information. According to the parameter that we are interested in, we can observe quantum phenomena while discriminating particle species, i.e., boson and fermion, or not.

This paper is organized as follows. In Section II, we focus on an input state parameter whose Fisher information can exhibit the Hong-Ou-Mandel effect. In Section III, we look into a parameter of a two-wave mixing operation whose Fisher information is used to detect two-particle entanglement. It is summarized in Sec. IV.
FIG. 2. Fisher information as a function of an input state parameter (Δ). (a) Fisher information as a function of the similarity of the input bosonic (fermionic) states |⟨φ1|φ2⟩|^2. (b) Fisher information as a function of the input-output fidelity |⟨φ1⟩a⟨φ2⟩a| of single input particles. The blue (red) circle represents the completely different (same) input particles. The input particles are in the states of (c) |φ1⟩a|φ2⟩b = (√−2gΔ)|H⟩a + \sqrt{1 - e^{-2gΔ}}|V⟩a)|H⟩b for photons, and of (d) |φ1⟩a|φ2⟩b = (√−2gΔ)|↓⟩a + \sqrt{1 - e^{-2gΔ}}|↑⟩a)|↓⟩b for electrons. The parameter g is an arbitrary constant and determines the width of the dip.

II. FISHER INFORMATION OF AN INPUT STATE PARAMETER

We focus on a parameter encoded into one of the two identical particles. A small difference between the two particles is quantifiable by the Hong-Ou-Mandel effect which exhibits bunching effect of two bosonic particles. Initially two identical single photons enter coherently into a 50:50 beam splitter, one on each side. After the beam splitting transformation which is represented by \(\hat{a}^\dagger \rightarrow (\hat{a}^\dagger + \hat{b}^\dagger)/\sqrt{2}\) and \(\hat{b}^\dagger \rightarrow (\hat{b}^\dagger - \hat{a}^\dagger)/\sqrt{2}\), the output two-mode state is transformed into \(B_{ab}(|1⟩_a|1⟩_b = (1/\sqrt{2})(|0⟩_a|2⟩_b - |2⟩_a|0⟩_b)\). Given a time delay between the two input single photons, we have a half probability of counting each photon in both output modes. With no time delay, each photon counting event drops into zero by bunching effect. Using the probability of each photon counting event, thus, we construct the HOM dip as a function of the time delay between the two input single photons.

Here we reproduce the HOM dip via Fisher information of an input state parameter. Instead of the time delay, we employ the photon polarization degree of freedom as a control parameter. By counting the number of photons only in one output mode, we derive Fisher information of the input state parameter in terms of the measurement outcome probabilities. For a 50:50 beam splitting operation, we show that the Fisher information is proportional to a similarity of the input particles with no discrimination of boson and fermion. Here we define the similarity of the input particle states as |⟨φ1|φ2⟩|^2. Initially we inject two polarized single photons into a 50:50 beam splitter as \(B_{ab}(\sqrt{e^{-gΔ^2}}|⟩_a + \sqrt{1 - e^{-gΔ^2}}|⟩_a)|⟩_b\), where g is an arbitrary constant and Δ is a control parameter. By counting the number of photons in the output mode b with no discrimination of polarization, then, we construct the probability distributions, \(P_b(0|Δ) = P_b(2|Δ) = 1/2(1 + e^{-gΔ^2})\), \(P_b(1|Δ) = 1/2(1 - e^{-gΔ^2})\). Using the formula of \(F(Δ) = \sum_{n=0}^2 \frac{1}{P_b(n|Δ)} |\partial P_b(n|Δ)/\partial Δ|^2\), we derive the Fisher information of the input state parameter (Δ) as

\[F(Δ) = \frac{4g^2Δ^2}{e^{2gΔ^2} - 1}.\] (2)

When the two input single photons are in the completely same polarized state, the Fisher information exhibits a maximum value of \(F(0) = 2g\) whereas the Fisher information goes to zero for the two input single photons having completely different polarizations. It is explained with the similarity of the input single-photon states |⟨φ1|φ2⟩|^2. In Fig. 2(a), we observe that the Fisher information increases with the similarity of the two single-photon states. However, the input-output fidelity exhibits the opposite tendency, as shown in Fig. 2(b). The more similar the two single-photon states are, the more different the output state is from the input state due to the bunching effect. From the result, we reproduce the HOM dip by taking the minus sign on the Fisher information that is a function of Δ, as shown in Fig. 2(c). The width of the dip is determined by the parameter g which imitates the length of the photon wave packet in the original HOM dip.

Similarly to the bosonic particle case, the HOM dip is also reproduced with fermions in rotated spin states. We inject each spin-rotated electron into a 50:50 electronic beam splitter as \(B_{ab}(\sqrt{e^{-gΔ^2}}|⟩_a + \sqrt{1 - e^{-gΔ^2}}|⟩_a)|⟩_b\), where the electronic beam splitting transformation is given by \(f^+_a \rightarrow (f^+_a + f^-_b)/\sqrt{2}\) and \(f^-_b \rightarrow (f^+_a - f^-_b)/\sqrt{2}\). By counting the number of electrons with no discrimination of spin, we construct the probability distributions, \(P_b(0|Δ) = P_b(2|Δ) = 1/2(1 - e^{-gΔ^2})\), \(P_b(1|Δ) = 1/2(1 + e^{-gΔ^2})\), and derive the same Fisher information of Eq. (2) as the bosonic particle case. In the viewpoint of the similarity of the input single-electron states, however, the interpretation is contrary to the bosonic particle case. While the Fisher information increases with the similarity of the single-electron states, the input-output fidelity is also proportional to the similarity, as shown in Fig. 2(d). The similarity and the fidelity exhibit the same tendency for fermionic particles whereas they show the opposing tendency for bosonic particles. The more similar the two fermionic states are, the less different the output state is from the input state due to the anti-bunching effect. By taking the minus sign on the Fisher information, in Fig. 2(d), we reproduce the HOM dip near the completely same spin state of the two input single electrons. When the two input single elec-
the input state parameter is given by the same function for both bosonic and fermionic cases, the Fisher information of the bosonic (fermionic) particles increases with the decrement (increment) of the input-output fidelity due to the (anti) bunching effect. The highest sensitivity of \( \theta \) is exhibited at the value of \( \theta = \pi/2 \) which represents two indistinguishable particles.

III. FISHER INFORMATION OF A PARAMETER OF A TWO-WAVE MIXING OPERATION

A. Mach-Zehnder interferometer

We turn into a parameter of a two-wave mixing operation, specifically for a phase shifting parameter in the Mach-Zehnder interferometer (MZI) \((U_{MZI} = B_{ab} e^{i\phi} a^\dagger b a)\). Starting with different input states, we derive the Fisher information of the phase shifting parameter. From this approach, we can distinguish boson from fermion because the particle interference is included into the Fisher information of the phase shifting parameter in the MZI.

We inject input photons in the state of \(|\varphi_1\rangle_a |\varphi_2\rangle_b = (\sqrt{1 - |\beta|^2} |V\rangle_a + \beta |H\rangle_a)|H\rangle_b\) into the MZI. Then we construct the probability distributions \(P(n_{H_\alpha}, m_{V_\alpha}; k_{H_\alpha}, l_{V_\alpha})\) by counting the number of photons only in one output mode, while discriminating the horizontal and vertical polarizations \(|H\rangle\) and \(|V\rangle\). Using the formula of \(F(\phi) = \sum_{n=0}^{\infty} \frac{1}{\beta_n(\beta_0^{2n})} |\beta_0|^{2n} |\beta|^{2n} (\frac{\partial^2 P(n,|\phi,|\beta|^2)}{\partial \phi^2})^2\), we derive the Fisher information of a phase parameter \(\phi\) as

\[
F(\phi) = \frac{(1 - |\beta|^2) + (1 + 3|\beta|^2) \cos^2 \phi + 2(1 - |\beta|^2) \sin \phi (1 + 3|\beta|^2) \sin 2\phi^2}{4(1 - |\beta|^2)(1 - \cos^2 \phi) + 8|\beta|^2(1 + \cos 2\phi)}. \tag{3}
\]

which provides different values as a function of \(|\beta|^2\) for each \(\phi\). The sensitivity of \(\phi\) is more pronounced around the value of \(\phi = 0/\pi\). Particularly, for each parameter \(|\beta|^2\), maximizing the Fisher information over \(\phi\), we find the maximum value of the Fisher information monotonically increasing with the similarity \(|\beta|^2\), as shown in Fig. 4 (a).

For electron, we inject input electrons in the state of \(|\varphi_1\rangle_a |\varphi_2\rangle_b = (\sqrt{1 - |\beta|^2} |\uparrow\rangle_a + \beta |\downarrow\rangle_a)|\downarrow\rangle_b\) into the electronic MZI [21]. By counting the number of electrons only in one output mode while discriminating the spin up-and-down, we derive the Fisher information of a phase parameter \(\phi\) as

\[
F(\phi) = |\alpha|^2 (1 + \cos^2 \phi) + \frac{|\alpha|^2(1 - \cos \phi) \sin 2\phi}{|\alpha|^2(1 - \cos^2 \phi) + 4|\beta|^2}. \tag{4}
\]

where \(|\alpha|^2 = 1 - |\beta|^2\). The sensitivity of \(\alpha\) is also more pronounced around the value of \(\phi = 0/\pi\). Maximizing the Fisher information over \(\phi\), for each parameter \(|\beta|\), we find the maximum value of the Fisher information monotonically decreasing with the parameter \(|\beta|^2\), as shown in Fig. 4 (b).
For the single-photon in a mixed state $|\alpha|^2|V\rangle_a + |\beta|^2|H\rangle_a$, the corresponding Fisher information is given by

$$F_{\text{mix}}(\phi) = 2c_0^2 \sin^2(2\phi) \left[ \frac{1}{c_0(1-\cos(2\phi))} + \frac{1}{2 - 2c_0 \sin^2 \phi} \right].$$

Comparing the Fisher informations of the input pure and mixed states, we figure out the difference in the existence of the interference term $c_3$ which includes the phase information of the bosonic pure state.

For fermionic particles, we start with the single-electron of mode $b$ in the superposed state $(\cos \theta | \uparrow \rangle + \sin \theta | \downarrow \rangle)_b$. When the single-electron of mode $a$ is in a pure state $(\alpha | \uparrow \rangle_a + \beta | \downarrow \rangle_a)$, we obtain the probability distributions by counting the number of electrons only in the output mode $b$, such as $P_b(0|\phi) = P_b(2|\phi) = \frac{1-\cos(2\phi)}{4}(c_0 + c_3)$ and $P_b(1|\phi) = \frac{1}{2}[1 + \cos(2\phi)c_0 + \cos^2(\phi)c_2 - c_4]$. Then the Fisher information of the phase parameter is given by

$$F_{\text{pure}}(\phi) = (c_2 + c_3)^2 \sin^2(2\phi) \left[ \frac{1}{(c_2 + c_3)(1-\cos(2\phi))} \right] + \frac{1}{4 - 2(c_2 + c_3) \sin^2 \phi}.$$  

For a mixed state $|\alpha|^2|\uparrow \rangle_a|\uparrow \rangle + |\beta|^2|\downarrow \rangle_a|\downarrow \rangle$ of the single-electron, the corresponding Fisher information is given by

$$F_{\text{mix}}(\phi) = c_2^2 \sin^2(2\phi) \left[ \frac{1}{c_2(1-\cos(2\phi))} + \frac{1}{4 - 2c_2 \sin^2 \phi} \right].$$

Comparing the Fisher informations of the input pure and mixed states, we figure out the difference in the existence of the interference term $c_3$ which includes the phase information of the fermionic pure state.

### B. Detecting two-particle entanglement

Previously, there were some works [3,15] about detecting entanglement by quantum Fisher information (QFI). However the QFIs were evaluated without considering physical measurement setups because the QFI assumes optimal measurement bases. Here we consider the entanglement detection by Fisher information which is evaluated simply with considering particle-counting events. In the two-dimensional systems, it is only required to count the number of particles up to two.

Using the monotonic relation between the maximum Fisher information and the similarity of the input particle states in Fig. 4, we can devise the scenario of entanglement detection for pure bipartite states in a two-dimensional system. We prepare an unknown entangled two-particle and a known two-particle. A pair of the unknown and known particles is injected into each MZI and
are processed to derive Fisher information of a phase $\frac{F_{a}}{\beta}$. MZI only one output mode in each side is measured by the states of (a) entanglement for the two-qubit Bell states. We denote by manipulating the amount of Fisher information $\frac{F_{a}}{\beta}$ with respect to the maximum entanglement point $\frac{F_{a}}{\beta} = 1$.

![FIG. 5. Entanglement detection for pure bipartite states in a two-dimensional system, in terms of maximum Fisher information. MZI represents the Mach-Zehnder interferometer. The unknown entangled two-bosonic-particles are in the states of (a) $|\psi\rangle_{ab} = \sqrt{(1 - |\beta|^2)}|H\rangle_a|V\rangle_b + \beta|V\rangle_a|H\rangle_b$ and (b) $|\psi\rangle_{ab} = \sqrt{(1 - |\beta|^2)}|H\rangle_a|H\rangle_b + \beta|V\rangle_a|V\rangle_b$ with $(\text{anc})_{cd} = |1\rangle_c|1\rangle_d$. The unknown entangled two-fermionic-particles are in the states of (c) $|\psi\rangle_{ab} = \sqrt{1 - |\beta|^2}|\uparrow\rangle_a|\downarrow\rangle_b + |\downarrow\rangle_a|\uparrow\rangle_b$ and (d) $|\psi\rangle_{ab} = \sqrt{1 - |\beta|^2}|\uparrow\rangle_a|\uparrow\rangle_b + |\downarrow\rangle_a|\downarrow\rangle_b$ with $(\text{anc})_{cd} = |1\rangle_c|1\rangle_d$.]

Then we detect only in one output mode, as shown in Fig. (b).

By manipulating the amount of Fisher information on each side, we can detect whether the unknown two-particle is entangled or not, and classify the detection of entanglement for the two-qubit Bell states. We denote that the classification of the two-particle entanglement is transformed by choosing the different state $(\text{anc})_{cd}$ of the known particles. Here we consider only one state of the known particles for each particle species.

Initially an unknown two-photon is prepared in the state of $|\psi\rangle_{ab} = \sqrt{(1 - |\beta|^2)}|H\rangle_a|V\rangle_b + \beta|V\rangle_a|H\rangle_b$ and a known two-photon is in the horizontal polarization state of $(\text{anc})_{cd} = |1\rangle_c|1\rangle_d$. After the interaction with the MZIs, only one output mode in each side is measured by a photon counter and then the measurement outcomes are processed to derive Fisher information of a phase parameter. At $|\beta|^2 = 0$ (1), the maximum Fisher informations correspond to $F_{a}^{\text{max}} = 4$ (2) and $F_{b}^{\text{max}} = 2$ (4). At $|\beta|^2 = 1/2$, the maximum Fisher informations become $F_{a}^{\text{max}} = F_{b}^{\text{max}} = 3$. To detect entanglement in the type of the polarization state $HV \pm VH$, we multiply their maximum Fisher informations as $F_{a}^{\text{max}} F_{b}^{\text{max}}$. Then, we figure out that the maximum entanglement is represented by $F_{a}^{\text{max}} F_{b}^{\text{max}} = 9$ and no entanglement is by $F_{a}^{\text{max}} F_{b}^{\text{max}} = 8$. In Fig. (a), the multiplication of the maximum Fisher informations is symmetric with respect to the maximum entanglement point $|\beta|^2 = 1/2$ and we can detect entanglement in the condition of $F_{a}^{\text{max}} F_{b}^{\text{max}} > 8$. Otherwise, the subtraction of the maximum Fisher informations, $F_{a}^{\text{max}} - F_{b}^{\text{max}}$, increases monotonically with respect to $|\beta|^2$ and we can also detect entanglement in the condition of $-2 < F_{a}^{\text{max}} - F_{b}^{\text{max}} < 2$.

Replacing the unknown bipartite state by the state of $|\psi\rangle_{ab} = \sqrt{(1 - |\beta|^2)}|H\rangle_a|H\rangle_b + \beta|V\rangle_a|V\rangle_b$, we obtain the maximum Fisher informations, such as $F_{a}^{\text{max}} = F_{b}^{\text{max}} = 4$ (2) at $|\beta|^2 = 0$ (1) and $F_{a}^{\text{max}} = F_{b}^{\text{max}} = 3$ at $|\beta|^2 = 1/2$. To detect entanglement in the type of the polarization state $HH \pm VV$, we take addition of the maximum Fisher informations as $F_{a}^{\text{max}} + F_{b}^{\text{max}}$. Then, we observe that the maximum entanglement is represented by $F_{a}^{\text{max}} + F_{b}^{\text{max}} = 6$ and no entanglement is by $F_{a}^{\text{max}} + F_{b}^{\text{max}} = 4$ or 8. In Fig. (b), the addition of the maximum Fisher informations increases monotonically with respect to $|\beta|^2$, and we can detect entanglement in the condition of $4 < F_{a}^{\text{max}} + F_{b}^{\text{max}} < 8$.

We turn into detecting fermionic entanglement for an unknown two-electron by manipulating the amount of Fisher information. The unknown two-electron is prepared in the state of $|\psi\rangle_{ab} = \sqrt{(1 - |\beta|^2)}|\uparrow\rangle_a|\downarrow\rangle_b + |\downarrow\rangle_a|\uparrow\rangle_b$ and a known two-electron is in $(\text{anc})_{cd} = |1\rangle_c|1\rangle_d$. After the interaction with the electronic MZIs, we derive Fisher information of a phase parameter by counting the number of electrons of only one output mode in each side. At $|\beta|^2 = 0$ (1), the maximum Fisher informations correspond to $F_{a}^{\text{max}} = 2$ (0) and $F_{b}^{\text{max}} = 0$ (2). At $|\beta|^2 = 1/2$, the maximum Fisher informations become $F_{a}^{\text{max}} = F_{b}^{\text{max}} = 1$. To detect entanglement in the type of the spin state $\uparrow\downarrow \pm \downarrow\uparrow$, we multiply their maximum Fisher informations as $F_{a}^{\text{max}} F_{b}^{\text{max}}$. In terms of the multiplication, we obtain that the maximum entanglement is represented by $F_{a}^{\text{max}} F_{b}^{\text{max}} = 1$ and no entanglement is by $F_{a}^{\text{max}} F_{b}^{\text{max}} = 0$. In Fig. (c), the multiplication of the maximum Fisher informations is symmetric with respect to the maximum entanglement point $|\beta|^2 = 1/2$ and we can detect entanglement in the condition of $F_{a}^{\text{max}} F_{b}^{\text{max}} > 0$. Taking the subtraction of the maximum Fisher informations, we can also detect entanglement in the condition of $-2 < F_{a}^{\text{max}} - F_{b}^{\text{max}} < 2$ while decreasing monotonically with respect to $|\beta|^2$.

For the different type of an unknown bipartite state $|\psi\rangle_{ab} = \sqrt{(1 - |\beta|^2)}|\uparrow\rangle_a|\uparrow\rangle_b + \beta|\downarrow\rangle_a|\downarrow\rangle_b$, we derive the maximum Fisher informations, such as $F_{a}^{\text{max}} = F_{b}^{\text{max}} = 2$ (0) at $|\beta|^2 = 0$ (1) and $F_{a}^{\text{max}} = F_{b}^{\text{max}} = 1$ at $|\beta|^2 = 1/2$. To detect entanglement in the type of the spin state $\uparrow\uparrow \pm \downarrow\downarrow$, we take the addition of their Fisher informations as $F_{a}^{\text{max}} + F_{b}^{\text{max}}$. Then, we obtain that the maximum entanglement is represented by $F_{a}^{\text{max}} + F_{b}^{\text{max}} = 2$ and no entanglement is by $F_{a}^{\text{max}} + F_{b}^{\text{max}} = 0$ or 4. In Fig. (d), the addition of the maximum Fisher informations decreases monotonically with respect to $|\beta|^2$, and we can detect entanglement in the condition of $0 < F_{a}^{\text{max}} + F_{b}^{\text{max}} < 4$.

To distinguish the pure bipartite entangled states from a mixture of bipartite pure states, we can test the entanglement by rotating the polarization (spin) of the known particle states. Starting with a superposed state of the known particles, we can detect the interference parts of the pure bipartite states in terms of different values of...
the Fisher information.

IV. SUMMARY

We proposed Fisher information as an indicator of quantum phenomena, such as the Hong-Ou-Mandel effect and a two-particle entanglement detection. Interacting two particles by a two-wave mixing operation, we constructed the Fisher information which is obtained simply by counting the number of particles in one of the output modes. Due to the symmetry of the two-wave mixing operation, we only consider the local Fisher information which is the same as the global Fisher information. According the parameter that we are interested in, we found whether the Fisher information includes the information on particle species, i.e., boson and fermion. When we focus on an input state parameter, we reproduced the Hong-Ou-Mandel effect with the ignorance of the particle species. The highest sensitivity of the input state parameter was shown in the case of two indistinguishable input particles. When we focus on a parameter of a two-wave mixing operation, we devised a protocol for detecting two-particle entanglement while discriminating the particle species.

As further works, it is worthwhile to consider extending the idea into many-particle scenarios and taking different unitary operations.

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