Electro-Optically Tunable Low Phase-Noise Microwave Synthesizer in an Active Lithium Niobate Microdisk

Renhong Gao, Botao Fu, Ni Yao, Jianglin Guan, Haisu Zhang, Jintian Lin,* Chuntao Li, Min Wang, Lingling Qiao, and Ya Cheng*

Photonic-based low-phase-noise microwave generation with real-time frequency tuning is crucial for a broad spectrum of subjects, including next-generation wireless communications, radar, metrology, and modern instrumentation. Here, for the first time to the best of the authors’ knowledge, narrow-bandwidth dual-wavelength microlasers are generated from nearly-degenerate polygon modes in a high-Q active lithium niobate microdisk. The record-high-Q ($\approx 10^7$) nearly-degenerate polygon modes formation with independently controllable resonant wavelengths and free spectral ranges is enabled by the weak perturbation of the microdisks using a tapered fiber. Moreover, because a high spatial overlap factor between the pump and the dual-wavelength laser modes is achieved, the gain competition between the two lasing modes spatially separated with a $\pi$-phase difference is suppressed, leading to stable dual-wavelength laser generation with low threshold, and in turn, the low noise microwave source. The stable beating signal confirms the low phase-noise achieved in the tunable laser. Without the need of external phase stabilizers, the measured microwave signal shows a phase noise of $-123$ dBc Hz$^{-1}$ and an electro-optic tuning efficiency of $-1.66$ MHz V$^{-1}$. The linewidth of the microwave signal is measured as $6.87$ kHz, which is more than three orders of magnitude narrower than current records based on integrated dual-lasers.

1. Introduction

A frequency-tunable low-phase-noise microwave source is highly desirable for many applications such as radar, wireless communications, software-defined radio, and modern instrumentation.[1–3] Conventional microwave signals are generated using complicated and costly electronic circuits, severely suffering from low bandwidth and high-loss propagation in the electrical lines. In contrast, the generation of optically carried microwaves based on optical heterodyning offers superior solutions with broad bandwidth and low-loss microwave signal distribution. To this end, compact on-chip integrated microlasers have been developed for the generation of high-performance microwave signals featuring low phase-noise performance (e.g., linewidth on the order of magnitude of 10 MHz), compact device footprint, low energy consumption, ease of tunability, and broad wavelength coverage range.[4–9] Among the
Figure 1. The experimental setup for generation of microwave signal from dual-wavelength lasing in a single microdisk resonator. Inset: SEM image and optical micrograph of the microdisk integrated with microelectrodes.

demonstrated dual-wavelength coherent light sources, on-chip dual-mode microlasers generated in common microcavities have attracted significant attention as the two laser modes in a single cavity are naturally phase locked without the need of external phase stabilizers. The miniaturized microwave synthesizers also provide indispensable high performance microwave sources for monolithically integrated fully functional microwave photonics.

In the past, stable and frequency-tunable microwave signals with center frequencies ranging from several to hundreds of gigahertz have been successfully synthesized from dual-mode microlasers in single strongly deformed microcavities. To avoid longitudinal mode competition in the cavity, transverse polygon modes are generated by introducing large cavities deformation which breaks the cavities azimuthal symmetry. Yet, it remains a challenge to achieve high-Q modes in the strongly deformed microcavities, resulting in the relatively broad linewidth and low coherence of the synthesized microwave. In addition, rapid tuning of the microwave signals is highly desired but still missing due to the lack of the suitable electro-optic (EO) substrates such as lithium niobate for microcavity fabrication.

Recently, thin-film lithium niobate (TFLN) has shown great potential for integrated photonic devices due to the excellent optical properties including large EO coefficient and nonlinear optical coefficient, broad transparency window, and piezo-electric effect. Using the TFLN platform, integrated EO modulators, nonlinear optical frequency converters, compact frequency comb, as well as on-chip microlasers and optical amplifiers have been demonstrated, showing unparalleled device performances in terms of optical loss, speed, nonlinear wavelength conversion efficiency, optical gain coefficient, and tunability. Recently, we have reported a narrow linewidth microlaser in a weakly perturbed erbium ion-doped TFLN microdisk employing an ultra-high Q polygon mode. As the polygon modes are coherently synthesized with combination of the whispering gallery modes (WGMs) in a circular microdisk evanescently coupled with a tapered fiber, loaded Q factors as high as 10^7 are achieved due to the weak perturbation from the tapered fiber which leads to slight degradation of the high Q factors of the WGMs. The unique advantage gives rise to the ultra-narrow linewidth single-mode lasing. One may expect to generate stable dual-mode microlasers from two longitudinal polygon modes by merely increasing the cavity size (i.e., reducing the mode wavelength spacing), which is not possible because of the longitudinal mode competition.

Here, we demonstrate the generation of frequency-tunable narrow linewidth microwave signals using dual-wavelength microlasers fabricated on erbium ion-doped TFLN. The two laser modes are characterized to be nearly-degenerate polygon modes with high-order excited states with a small wavelength spacing in the same microcavity, which has not yet been employed for lasing due to the severe degradation of the Q factors in traditionally largely deformed microcavities. Benefiting from the high-Q factors, the suppression of gain competition, and high spatial overlap factor of the polygon modes in the weakly perturbed microcavities, narrow linewidth dual-wavelength microlasers are demonstrated with low lasing threshold and the beating of the phase-stabilized dual-wavelength laser generates the low-phase-noise microwave signal with low power consumption. We also reveal the underlying physics using numerical simulation, which agrees well with the experiment results.

2. Fabrication of the Active TFLN Microdisks

In our experiment, the active microcavities were fabricated on a TFLN wafer using the photolithography assisted chemo-mechanical etching technique. The TFLN wafer was prepared by bonding a Z-cut erbium ion-doped TFLN with concentration of 0.1 mol% and a thickness of 710 nm onto a holder wafer with 2 μm thick SiO2 layer and 500 μm thick lithium niobate. The fabrication process of microdisks integrated with microwhisker electrodes begins from the preparation of erbium ion doped TFLN by ion slicing. The sample endures chromium (Cr) layer coating, hard mask patterning via femtosecond laser ablation, pattern transferring from the Cr hard mask to TFLN via chemo-mechanical etching, and chemical wet etching to completely remove the Cr layer and partially remove the silica layer underneath the TFLN disk into the supporting pedestal. The details of the fabrication process can be found in Refs. [31, 33]. The diameters of the fabricated TFLN microdisk supported by the silica pedestal and microelectrode on the top surface of the microdisk
Figure 2. Dual-wavelength microdisk lasers. a) The spectrum of the microlaser with fine structure. Inset: The spectrum of the microlaser shown in a wide wavelength range (left), indicating only one set of dual-wavelength signal, and the spectrum of the pump light (right). b) The optical microscope images of spatial intensity distributions of the up-conversion fluorescence (top middle), pump mode (bottom left), and lasing modes (bottom right). c) Output power dependence of the pump power. d) The other spectrum of dual-wavelength microlasers with the varied coupled position and pump wavelength. Inset: The optical microscope images of spatial intensity distributions of the up-conversion fluorescence (left) and lasing modes (right).

are 106.33 and 15 μm, respectively. The scanning electron microscope (SEM) image of the microdisk is shown in the inset of Figure 1, indicating an ultrasmooth surface.

3. Microwave Signal Synthesizer on Dual-Wavelength Microlasers

3.1. Experimental Setup

Figure 1 schematically shows the experimental setup for generating the frequency-tunable low-phase-noise microwave signals using dual-wavelength microdisk lasers fabricated on erbium ion-doped TFLN. The pump laser was provided by a narrow-linewidth tunable laser (Model TLB-6719, New Focus Inc.) together with a variable optical attenuator (VOA), and coupled into the microdisk through a tapered fiber with a diameter of 2 μm. An in-line fiber polarization controller could adjust the polarization state of the pump laser. The input power was measured using a power meter before the pump light was coupled into the microdisk at the input port of the tapered fiber. The relative position of the tapered fiber could be accurately adjusted and monitored by a 3D piezo-electric stage with a resolution of 20 nm and a real-time optical microscope imaging system, respectively. The optical microscope imaging system consisted of an objective lens with a numerical aperture of 0.42, long-pass or short-pass filters, and a visible or infrared charge-coupled device (CCD). To capture the up-conversion fluorescence and pump signals, short-pass filter (Model FES 800, Thorlabs Inc.) and long-pass filter (Model: FELH 800, Thorlabs Inc.) were inserted before the visible CCD, respectively, and the lasing signals were captured by long-pass filter (Model FELH1100, Thorlabs Inc.) and the infrared CCD. The dual-wavelength lasing signals were coupled out of the microdisk by the same tapered fiber once the pump power reached the lasing threshold. Experimentally, to efficiently excite the transverse-electrically polarized polygon modes, the relative position between the tapered fiber and the microdisk center was set to 47 μm. The lasing experiment was carried out at room temperature, and a thermoelectric cooler (TEC) was placed underneath the microdisk chip to overcome environmental temperature fluctuations with a TEC driver of 2 mK temperature.
stability. The generated laser signals were transverse-electrically polarized, and the output powers were measured from the output port of the tapered fiber after removing the pump light with long-pass filters. Then, the laser signals were separated by a beam splitter with a power ratio of 20/80. The minority part was sent into an optical spectral analyzer (OSA) for spectral analysis. Meanwhile, the majority part was first filtered with a long-pass filter (Model FELH 1100, Thorlabs Inc.), and then amplified by an erbium–ytterbium-doped fiber amplifier (EDFA), and finally detected by a fast photodetector (PD for short, bandwidth 30 GHz) to generate the beating signal. The beating signal and its phase noise were analyzed and recorded by a real-time electric spectral analyzer (ESA) with a bandwidth of 25 GHz.

3.2. Dual-Wavelength Lasing From Polygon Modes

The dual-wavelength lasing signals with a wavelength interval of 8 pm were observed when the pump wavelength and power were tuned to 970.02 nm and higher than 80 μW, respectively, as demonstrated in Figure 2a. There is an error of the measured wavelength interval, resulting from the resolution of the OSA with 20 pm. Correspondingly, the up-conversion fluorescence, pump, and lasing modes emitted from the microdisk were captured by the optical microscope imaging system, as shown in Figure 2b. All the images featured dual-localization octagon modes, displaying polygon modes with similar spatial distributions. When the pump wavelength was tuned to be 970.02 nm, a wavelength of the localized polygon mode, only the nearly-degenerate polygon modes within the optical gain of erbium ion would be excited because of the considerable overlap between the pump and lasing modes. The green fluorescence possessed a broadband spectral envelope[31] with three fluorescence emission peaks at ≈490, ≈530, and ≈550 nm, which could be attributed to the transitions[21,22] of $^4F_{7/2} \rightarrow ^4I_{15/2}$, $^2H_{15/2} \rightarrow ^4I_{15/2}$, and $^4S_{3/2} \rightarrow ^4I_{15/2}$, respectively. Figure 2c gives laser output power recorded at different pump powers, indicating a lasing threshold of ≈80 μW. The dual-wavelength lasing signals with a wavelength interval of 70 pm were observed when the position of the tapered fiber and pump wavelength were tuned to a relative position of 47.5 μm.

Figure 3. Transmission spectra of the tapered fiber coupled with the microdisk. a) Transmission spectrum around the pump wavelength, where the dips labeled with black and blue dots are the families of the polygon mode and fundamental WGMs, respectively. b) The Q factor of the pump mode. c) Transmission spectrum around the lasing wavelengths. d) The Q factors of the nearly-degenerate lasing modes.
Figure 4. Microwave signal synthesizer and tuning. a) The intensity noise of the microwave signal. Inset: optical micrograph of the microdisk coupled with the tapered fiber, when the Cr/Au probe was contacted with the microelectrode. b) The phase noise of the microwave carrier. c) The tuning of the microwave signal. Inset: the tuning efficiency. d) The microwave signal synthesized from the other dual-wavelength microlaser with a large wavelength interval of 70 pm.

between the tapered fiber and the cavity center and 969.82 nm, respectively, to regulate the mode recombination by changing the perturbation strength for mode recombination, as shown in Figure 2d. We find the polygon patterns in Figure 2d give a more approximate octagon shape. Thus, the wavelength interval of dual-wavelength lasing signals can be adjusted utilizing the perturbation induced by the tapered fiber coupling.

3.3. Characterization of High-Q Nearly-Degenerate Polygon Modes

The mode structures of the microdisk around 970 and 1550 nm wavebands were characterized by tunable diode lasers (Models: TLB-6719 & TLB-6728, New Focus, Inc.) with an output power of 5 μW, at the same coupled position as the dual-wavelength lasing experiment of the wavelength interval of 8 pm. Figure 3a,c shows the transmission spectra measured by a photodetector with 125 MHz bandwidth around the pump and lasing wavelengths, respectively. The loaded Q factor of the mode at the pump wavelength is estimated to be $3.5 \times 10^6$ by Lorentz fitting of the resonant dip, as plotted in Figure 3b. As the coupling condition was tuned to the over-coupling state to introduce the weak perturbation, the intrinsic Q factor of the pump mode was calculated to be $4.7 \times 10^6$. Figure 3d demonstrates the magnified spectrum indicated by a dotted box in Figure 3c. Coincident dual-dip structures with a wavelength interval of 9.9 pm could be observed, and the Lorentz fitting curves indicated a pair of lasing modes with loaded Q factors of $8.7 \times 10^6$ and $9.0 \times 10^6$, respectively. The intrinsic Q factors of the two lasing modes were calculated to be $1.1 \times 10^7$ and $1.1 \times 10^7$, respectively. These record-high Q factors of the nearly-degenerate polygon modes would lead to ultra-narrow linewidth microwave synthesizer.

3.4. Microwave Signal Synthesizer and Tuning

The majority part of the output power of the microlaser around 1551.66 nm wavelength was amplified to 0.047 mW to synthesize microwave signal with the fast photodetector by beating. The
quality of the generated beat note signal was analyzed by the ESA with a resolution of 80 Hz, indicating a near Lorentzian-shaped power signal with a center frequency of 1.23 GHz being generated, as exhibited in Figure 4a. As the frequency/phase noise consists of white noise and 1/f noise,[4] where f is the frequency, the spectral width of the microwave signal was estimated to be 20 dB down from the peak and the Lorentzian 3 dB linewidth was extracted to be 6870 Hz. The narrow linewidth is a result of using the single ultra-high Q microcavity for dual-wavelength lasing, the suppression of gain competition, and the utilizations of the TEC and the stable pump light with a small power fluctuation <0.01% to suppress the white noise and environmental noises. Figure 4b records the phase noise of the microwave carrier. When the frequency offset is more than 110 MHz, corresponding to a white-phase-noise floor of the carrier signal of −123 dBc/Hz, the phase noise is almost flat, indicating a highly stable microwave signal and dual-wavelength microlasers. Figure 4c shows the EO tuning of the microwave signal; here, linear EO coefficient \( r_{32} \) is employed[31] for tuning the wavelength interval of the nearly-degenerate modes. When the externally applied direct-current bias is tuned from −300 to 300 V, the microwave signal is red-shifted from 1.73 to 0.73 GHz, showing the tuning efficiency of −1.66 MHz V\(^{-1}\). To further reduce the bias voltage, the largest EO coefficient \( r_{33} \) (30.9 pm V\(^{-1}\)) should be utilized and the distance between the microelectrode pairs should be reduced to several micron. The dual-wavelength lasing signal with a wavelength interval of 70 pm was also utilized to synthesize the microwave signal, and correspondingly, the microwave signal was located at 8.72 GHz, as shown in Figure 4d. Thus, a broad frequency range tuning of the generated microwave frequencies can be directly realized by adjusting the perturbation location of the microdisk.

4. Numerical Simulations

The underlying physics behind the formation of nearly-degenerate polygon modes with high-order excited states[32] is further theoretically analyzed via a 2D simulation. The effective indices of the transverse-electrically polarized pump light and lasing signal used in the numerical simulation are 2.12 and 1.97, respectively. In our simulation, perturbation is introduced into the circular microcavity through coupling with the tapered fiber by adjusting the disk-taper distance,[30,31] other than cavity deformation.[34–36] With the help of the tapered fiber coupling, nearly-degenerate polygon modes are formed with the recombination of inherent nearly-degenerate eigenmodes of the thin circular microdisk. The intensity profiles of the first and the second excited states of the polygon modes at the lasing wavelengths are demonstrated in Figure 5a,b, exhibiting octagon patterns with \( \pi \)-phase difference. Here, the polygon mode exhibits dual-localization octagon pattern. It is noteworthy that the octagon pattern appears slightly incomplete because of the weak perturbation of the tapered fiber. The calculated 10 pm wavelength interval of the two nearly-degenerate modes agrees well with the measured result. The intensity profile of the pump mode is plotted in Figure 5c, showing a slightly incomplete octagon-shaped pattern as well. The spatial overlap factors \( \Gamma = \frac{\langle \langle I_1 \rangle \rangle}{\sqrt{\langle \langle I_1 \rangle \rangle^2 + \langle \langle I_2 \rangle \rangle^2}} \) between the pump mode \( I_1 \) and the nearly-degenerate lasing modes \( I_2 \)
are calculated. The corresponding overlap factors are 0.63 and 0.52 when the distributions of the integrand between the pump and lasing modes are calculated, as shown in Figure 5d,e respectively. Remarkably, the dual-wavelength signals have almost the same gain due to the overlap factors being close. Moreover, both the distributions of the integrand calculated in Figure 5d,e show a periodic wave-like structure but with a phase shift of π along the periphery of the microdisk. The maximum overlaps of Figure 5d always correspond to the zero points of Figure 5e and vice versa. This characteristic provides an extremely useful advantage for generating the stable dual-wavelength laser output, and in turn, for synthesizing the low phase noise microwave because of the strong suppression of the longitudinal mode competition.

5. Conclusion

To conclude, we have demonstrated frequency-tunable narrow-linewidth microwave signals using dual-wavelength lasers from a single microdisk fabricated on erbium ion-doped TFLN. Nearly-degenerate polygon modes with high-order excited states, which possess ultra-high Q factors and π-phase difference were formed in the weakly perturbed circular microwavcy, triggering stable dual-wavelength lasing and the resulting stable microwave signal directly from the integrated platform for the first time. The linewidth of the highly coherent microwave signal was measured as narrow as 6.87 kHz, being at least three orders of magnitude narrower than the previously reported results from dual-wavelength integrated lasers. Such integrated microwave source was doomed to promote future advancements of microwave photonics and optical information processing.

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Conflict of Interest

The authors declare no conflict of interest.

Data Availability Statement

The data that support the findings of this study are available from the corresponding author upon reasonable request.

Keywords

integrated dual-wavelength microlasers, lithium niobate, miniaturized microwave synthesizers, perturbed microcavities

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[1] J. Yao, J. Lightwave Technol. 2009, 27, 314.
[2] D. Marpaung, J. Yao, J. Capmany, Nat. Photonics 2019, 13, 80.
[3] J. Li, H. Lee, K. J. Vahala, Nat. Commun. 2013, 4, 2097.
[4] J. Huang, C. Sun, B. Xiong, Y. Luo, Opt. Express 2009, 17, 20727.
[5] Y. -H. Lo, Y. -C. Wu, S. -C. Hsu, Y. -C. Hwang, B. -C. Chen, C. -C. Lin, Opt. Express 2014, 22, 13125.
[6] H. Wang, S. Liu, L. Chen, D. Shen, X. Wu, Sci. Rep. 2016, 6, 38053.
[7] H. Long, Y. -Z. Huang, X. -W. Ma, Y. -D. Yang, J. -L. Xiao, L. -X. Zou, B. -W. Liu, Opt. Lett. 2015, 40, 3548.
[8] H. -Z. Weng, Y. -Z. Huang, X. -W. Ma, F. -L. Wang, M. -L. Liao, Y. -D. Yang, J. -L. Xiao, IEEE Photonics Technol. Lett. 2017, 29, 1931.
[9] H. -Z. Weng, Y. -Z. Huang, Y. -D. Yang, X. -W. Ma, J. -L. Xiao, Y. Du, Phys. Rev. A 2017, 95, 013833.
[10] C. Wang, M. Zhang, X. Chen, M. Bertrand, A. Shams-Ansari, S. Chandrasekhar, P. Winzer, M. Lončar, Nature 2018, 562, 101.
[11] M. Xu, M. He, H. Zhang, J. Jian, Y. Pan, X. Liu, L. Chen, X. Meng, H. Chen, Z. Li, X. Xiao, S. Yu, S. Yu, X. Cai, Nat. Commun. 2020, 11, 3911.
[12] Y. He, Q.-F. Yang, J. Ling, R. Luo, H. Liang, M. Li, B. Shen, H. Wang, K. Vahala, Q. Lin, Optica 2019, 6, 1138.
[13] J. Lin, N. Yao, Z. Hao, J. Zhang, W. Mao, M. Wang, W. Chu, R. Wu, Z. Fang, L. Qiao, W. Fang, F. Bo, Y. Cheng, Phys. Rev. Lett. 2019, 122, 173903.
[14] M. Vazimani, S. Fathpour, Adv. Photonics 2022, 4, 034001.
[15] G. -T. Xue, Y. -F. Niu, X. Liu, J. -C. Duan, W. Chen, Y. Pan, K. Jia, X. Wang, H. -Y. Liu, Y. Zhang, P. Xu, G. Zhao, X. Cai, Y. -X. Gong, X. Hu, Z. Xie, Z. Zhu, Phys. Rev. Appl. 2021, 15, 064059.
[16] J. F. Herrmann, V. Ansari, J. Wang, J. D. Wittmer, S. Fan, A. H. Safavi-Naeini, Nat. Photonics 2022, 16, 603.
[17] X. He, L. Cortes-Herrera, K. Opong-Mensah, Z. Zhang, Y. Song, G. P. Agrawal, J. Cardenas, Opt. Lett. 2022, 47, 5849.
[18] H. Wang, H. Heo, K. Ko, M. R. Nurrakhman, K. Moon, J. J. Ju, S.-W. Han, H. Jung, H. Lee, M. -K. Seo, Opt. Lett. 2022, 47, 6149.
[19] A. Shams-Ansari, M. Yu, Z. Chen, C. Reimer, M. Zhang, N. Picqué, M. Lončar, Commun. Phys. 2022, 5, 88.
[20] X. Sun, Y. Sheng, X. Gao, Y. Liu, F. Ren, Y. Tan, Z. Yang, Y. Jia, F. Chen, Small 2022, 18, 2203532.
[21] Z. Wang, Z. Fang, Z. Liu, W. Chu, Y. Zhou, J. Zhang, R. Wu, M. Wang, T. Lu, Y. Cheng, Opt. Lett. 2021, 46, 380.
[22] Y. Liu, X. Yan, J. Wu, B. Zhu, Y. Chen, X. Chen, Sci. China Phys., Mech. Astron. 2021, 64, 234262.
[23] Q. Luo, Z. Hao, C. Yang, R. Zhang, D. Zheng, S. Liu, H. Liu, F. Bo, Y. Kong, G. Zhang, J. Xu, Sci. China Phys., Mech. Astron. 2021, 64, 234263.
[24] T. Li, K. Wu, M. Cai, Z. Xiao, H. Zhang, C. Li, J. Xiang, Y. Huang, J. Chen, APL Photonics 2021, 6, 101301.
[25] R. Gao, J. Guan, N. Yao, L. Deng, J. Lin, M. Wang, L. Qiao, Z. Wang, Y. Liang, Y. Zhou, Y. Cheng, Opt. Lett. 2021, 46, 3131.
[26] Z. Chen, Q. Xu, K. Zhang, W.-H. Wong, D.-L. Zhang, E. Y.-B. Pun, C. Wang, Opt. Lett. 2021, 46, 1161.
[27] J. Zhou, Y. Liang, Z. Liu, W. Chu, H. Zhang, D. Yin, Z. Fang, R. Wu, J. Zhang, W. Chen, Z. Wang, Y. Zhou, M. Wang, Y. Cheng, Laser Photonics Rev. 2021, 15, 2100030.
[28] Y. Jia, J. Wu, X. Sun, X. Yan, R. Xie, L. Wang, Y. Chen, F. Chen, Laser Photonics Rev. 2022, 16, 2200039.
[29] J. Lin, F. Bo, Y. Cheng, J. Xu, Photonics Res. 2020, 8, 1910.
[30] Z. Fang, S. Haque, S. Farajollahi, H. Luo, J. Lin, R. Wu, J. Zhang, Z. Wang, M. Wang, Y. Cheng, T. Lu, Phys. Rev. Lett. 2020, 125, 173901.
[31] J. Lin, S. Farajollahi, Z. Fang, N. Yao, R. Gao, J. Guan, L. Deng, T. Lu, M. Wang, H. Zhang, W. Fang, L. Qiao, Y. Cheng, Adv. Photonics 2022, 4, 036001.
[32] C. C. Liu, T. H. Lu, Y. F. Chen, K. F. Huang, *Phys. Rev. E: Stat., Nonlinear, Biol., Soft Matter Phys*. **2006**, *74*, 046214.
[33] R. Wu, J. Zhang, N. Yao, W. Fang, L. Qiao, Z. Chai, J. Lin, Y. Cheng, *Opt. Lett*. **2018**, *43*, 4116.
[34] S. Shinohara, T. Harayama, T. Fukushima, M. Hentschel, T. Sasaki, E. E. Narimanov, *Phys. Rev. Lett*. **2010**, *104*, 163902.
[35] Q. H. Song, L. Ge, A. D. Stone, H. Cao, J. Wiersig, J.-B. Shim, J. Unterhinninghofen, W. Fang, G. S. Solomon, *Phys. Rev. Lett*. **2010**, *105*, 103992.
[36] X. Jiang, L. Shao, S.-X. Zhang, X. Yi, J. Wiersig, L. Wang, Q. Gong, M. Lončar, L. Yang, Y.-F. Xiao, *Science* **2017**, *358*, 344.