Parity-time-symmetric whispering-gallery microcavities

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Optical systems combining balanced loss and gain provide a unique platform to implement classical analogues of quantum systems described by non-Hermitian parity-time (PT)-symmetric Hamiltonians. Such systems can be used to create synthetic materials with properties that cannot be attained in materials having only loss or only gain. Here we report PT-symmetry breaking in coupled optical resonators. We observed non-reciprocity in the PT-symmetry-breaking phase due to strong field localization, which significantly enhances nonlinearity. In the linear regime, light transmission is reciprocal regardless of whether the symmetry is broken or unbroken. We show that in one direction there is a complete absence of resonance peaks whereas in the other direction the transmission is resonantly enhanced, a feature directly associated with the use of resonant structures. Our results could lead to a new generation of synthetic optical systems enabling on-chip manipulation and control of light propagation.

Our system consists of two directly coupled microtoroidal whispering-gallery-mode resonators (WGMRs), each coupled to a different fibre-taper coupler (Fig. 1a–c). This system is PT symmetric because under parity reflection P the WGMRs become interchanged and under time reversal T loss becomes gain and gain becomes loss. The first microtoroid ($\mu R_1$) is an active resonator made from Er$^{3+}$-doped silica; the second microtoroid ($\mu R_2$) is a passive (no-gain-medium) resonator made from silica without dopants. Gain in $\mu R_1$ was provided in the 1,550 nm wavelength band by optically pumping Er$^{3+}$ ions with a pump laser in the 1,460 nm band. The Q-factors of $\mu R_1$ and $\mu R_2$ in the 1,550 nm band were $3 \times 10^5$ and $3 \times 10^7$, respectively, and $\mu R_2$ had a Q-factor of $2.4 \times 10^6$ in the 1,460 nm band (Fig. 1d). The microtoroids were fabricated at the edges of two separate chips placed on nanopositioning systems to control precisely the distance and hence the coupling between the microtoroids (Supplementary Sections A and B1). We mediated the coupling between $\mu R_1$ and $\mu R_2$ in the 1,550 nm band by controlling the detuning between their resonant wavelengths through the tuning of the resonance wavelength of $\mu R_2$ via the thermo-optic effect of silica. There was no coupling between the resonators in the 1,460 nm band; thus, the pump existed only in $\mu R_1$. Compensation of the $\mu R_2$ losses in the 1,550 nm band with the optical gain provided by Er$^{3+}$ was confirmed by the narrowing of resonance linewidth with increasing pump power (Fig. 1e) and by the emergence of a strong resonance peak (Fig. 1f) due to the amplification of a very weak probe by the gain.

We conducted two sets of experiments using the apparatus in Fig. 1. The first set determined the broken and unbroken PT phases in coupled microcavities with balanced loss and gain as a function of the coupling strength $\kappa$, unlike previous experiments where $\kappa$ was fixed and the gain and loss ratio was varied. We studied the...
system using only the waveguide (WG1) coupled to $\mu R_1$. The pump and the weak probe lasers were input at port 1 and the output transmission spectra were monitored at port 2 in the 1,550 nm band. Without the pump, the coupled-resonator system acted as a passive photonic molecule\(^{11}\) characterized by two supermodes whose spectral distance increases with increasing $\kappa$ as seen in Fig. 2a ($\kappa$ decreases exponentially with increasing distance between $\mu R_1$ and $\mu R_2$; see Supplementary Fig. 5). This system became PT symmetric when $\mu R_1$ was optically pumped to provide gain and $\mu R_2$ had a balanced loss. At fixed gain–loss ratio, we monitored the output port as a function of $\kappa$ and observed the PT phase transition at the threshold coupling strength $\kappa_{\text{TR}}$ (Fig. 2a,b). For $\kappa/\kappa_{\text{TR}} < 1$, the system is in a broken-symmetry phase, as seen in both the coalescence of the real parts of the eigenfrequencies (Fig. 2a) and the non-zero difference in their imaginary parts (Fig. 2b). As $\kappa/\kappa_{\text{TR}}$ approaches 1 from below, the difference in the imaginary parts of the eigenfrequencies decreases and their real parts bifurcate (mode-splitting).

Next, we chose two different WGMs with Q-factors $2.0 \times 10^7$ and $3.0 \times 10^7$ in $\mu R_2$ and adjusted the pump power so that the loss–gain ratio was nearly balanced. We observed the transition from the broken to unbroken phase occurring at different coupling strengths for modes with different $Q$, that is, different initial loss (Fig. 2c,d). The PT phase transition occurs at higher $\kappa_{\text{TR}}$ for lower Q-factors.

The PT phase transition can be understood intuitively as follows. If the coupling between the resonators is weak, the energy in the active resonator cannot flow fast enough into the passive resonator to compensate the absorption. Thus, the system cannot be in equilibrium and the eigenfrequencies are complex, implying exponential growth or decay. However, if the coupling strength exceeds a critical value, then the system can attain equilibrium because the energy in the active resonator can flow rapidly enough into the passive one to compensate the dissipation.

In our experiments the frequency bifurcation (splitting) is not in orthogonal directions (Fig. 2) as would be expected for ideal systems with exactly balanced gain and loss. Instead, the bifurcation is smooth and the degree of smoothness (how much the system deviates from the exactly balanced case) depends on the pump power. To understand the origin of this behaviour, we revisited the equations of motion for coupled oscillators, which showed that for unbalanced gain and loss, the eigenfrequencies are never exactly real\(^{27,33,38}\). Instead, there is a region of $\kappa$ where the difference in imaginary parts is large but the difference in real parts is small (but non-zero), and a second region where the difference in imaginary parts is small but non-zero, and the splitting is large. In practical implementations it is impossible to balance the loss and gain exactly, so the mathematical prediction of a smooth bifurcation is physically realistic and consistent with our experiments (Fig. 2).

A linear static dielectric system, even with gain and loss, cannot have non-reciprocal response\(^{27,33,38}\). However, a nonlinear system can exhibit strong non-reciprocity. In our second set of experiments we tested this in our PT-symmetric system (Fig. 1a with transmission from input port 1 (4) to output port 4 (1) defined as the forward $T_{\text{forw}}$ (backward $T_{\text{back}}$)) and demonstrated strong non-reciprocal light transmission associated with nonlinearity enhancement induced by PT-symmetry breaking.

We first monitored the output spectra at port 1 as the power of input probe at port 4 was varied while the system was in broken- or unbroken-symmetry phases. A clear nonlinear response was observed in the symmetry-broken phase in contrast to the linear response in the unbroken phase (Fig. 3a). At low power levels where the input–output relation was linear, the system was reciprocal in both the broken- and unbroken-symmetry phases (Fig. 3b,c). Thus, we have direct experimental clarification of reciprocity in PT-symmetric systems; PT symmetry alone is not sufficient for non-reciprocal light transmission. As we increased the input power, the system remained in the linear regime for the unbroken-symmetry phase, whereas the input–output relation became nonlinear in the broken phase (Fig. 3a). These results indicate nonlinearity enhancement (that is, lower threshold for nonlinearity) in the...
broken-symmetry phase, due to the stronger field localization in the resonator with gain, as compared with the unbroken-symmetry phase (Supplementary Section 7 and Fig. 9).

As a result of stronger nonlinearity in the broken-symmetry case, the PT transition is associated with a transition from reciprocal to non-reciprocal behaviour. When the pump at port 1 was OFF (μR1 and μR2 are passive) and a weak probe light was input at port 1 or 4, we observed resonance peaks in the forward or backward transmissions (Fig. 4a(i),b(i)) with no resolvable mode splitting. When the pump was set ON and the gain and loss were balanced so as to operate in the unbroken-PT-symmetric region, transmission spectra showed amplified signals with clearly resolved split peaks (Fig. 4a(ii),b(ii)). However, when the coupling strength was decreased so that the system transited into the broken-symmetry region, forward transmission reduced to zero $T_{1\rightarrow4}\approx0$ (Fig. 4a(iii)) but the backward transmission remained high (Fig. 4b(iii)). The transmission spectra showed a single resonance peak, as expected from the theory. Thus, in the broken-symmetry region the input at port 4 was transmitted to port 1 at resonance; however, the input at port 1 could not be transmitted to port 4, in stark contrast with what was observed for the unbroken-symmetry region. This indicates non-reciprocal light transport between ports 1 and 4.

Unlike previous experiments demonstrating non-reciprocal transport in non-PT structures and asymmetric behaviour in PT electronics\(^9\), we observed a complete absence of resonance peak in the forward transmission. The advantages of the present design, which brings together PT-symmetric concepts with nonlinearity-induced non-reciprocal light transmission, over the non-PT schemes utilizing nonlinearity are: a significant reduction in the input power to observe non-reciprocity (~1 μW in this work versus 3 W in ref. 34, 0.310 W in ref. 35 and ~85 μW in ref. 37); higher contrast; small footprint; and a complete absence of the signal in one direction but resonantly enhanced transmission in the other direction. This is in stark contrast with other non-reciprocal devices where transmission suffers from high losses (see Supplementary Section B11 for a detailed comparison of this work with other works in the literature). Also, the transmitted signal here was not from spontaneous emission of the gain medium. Without the weak injected signal at input port 4, the output at port 1 was at the noise level, and no resonance peak was observed (inset of Fig. 4b(ii),b(iii)).

Resonance enhancement (the peak) was observed only when the injected signal at input port 4, the output at port 1 was at the noise level, and no resonance peak was observed (inset of Fig. 4b(ii),b(iii)).
coherent-perfect-absorption lasers, topologically protected optical diodes, enhanced nonlinearities and light–matter interactions) that can be achieved only with resonant structures and resonant enhancement. Although we used fibre-taper-coupled microtoroids, our techniques can be extended to other WGMRs and to photonic crystal cavities. Similarly, gain could be provided by quantum dots or other rare-earth ions, and also through nonlinear processes, such as Raman or parametric amplification. Like any non-reciprocal device utilizing resonant effects, our PT-symmetric all-optical diode is bandwidth-limited. However, by thermally tuning resonance wavelengths and by using active resonators doped with multiple rare-earth ions, operation over large wavelength bands should be possible. Coupled WGMRs provide a comprehensive framework for understanding resonance effects in PT-symmetric optical systems and could thereby aid in developing on-chip synthetic structures to harness the flow of light. For example, the electromagnetically induced transparency in coupled passive resonators may benefit from PT-symmetric resonators through lossless modulation of the transparency for slowing and stopping of light. Similarly, PT-symmetric microresonators can be used for studying nonlinear Fano resonances that may give rise to ultralow-power and high-contrast switching and non-reciprocity due to their sharp asymmetric line shapes. Moreover, there has been an emerging interest in exploring PT symmetry in various fields.
such as microlasers, sensing, plasmonics, optomechanics and cavity-quantum electrodynamics, where passive WGMRs have been traditionally used. This may bring about new results and physical insights into these fields.

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Author contributions
S.K.O. and L.Y. conceived the idea and designed the experiments; B.P. performed the experiments with help from F.L., F.M. and S.K.O. Theoretical background and simulations were provided by E.L., F.M., M.G., C.M.B., S.F. and F.N. All authors discussed the results, and S.K.O. and L.Y. wrote the manuscript with inputs from all authors. L.Y. supervised the experiment.

Additional information
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Competing financial interests
The authors declare no competing financial interests.