Constraints on neutron skin thickness in $^{208}$Pb and density-dependent symmetry energy

Jianmin Dong,$^{1,*}$ Wei Zuo,$^{1,†}$ and Jianzhong Gu$^{2,‡}$

$^1$Institute of Modern Physics, Chinese Academy of Sciences, Lanzhou 730000, China
$^2$China Institute of Atomic Energy, P. O. Box 275(10), Beijing 102413, China

Abstract

Accurate knowledge about the neutron skin thickness $\Delta R_{np}$ in $^{208}$Pb has far-reaching implications for different communities of nuclear physics and astrophysics. Yet, the novel Lead Radius Experiment (PREX) did not yield stringent constraint on the $\Delta R_{np}$ recently. We employ a more practicable strategy currently to probe the neutron skin thickness of $^{208}$Pb based on a high linear correlation between the $\Delta R_{np}$ and $J - a_{sym}$, where $J$ and $a_{sym}$ are the symmetry energy (coefficient) of nuclear matter at saturation density and of $^{208}$Pb. An accurate $J - a_{sym}$ thus places a strong constraint on the $\Delta R_{np}$. Compared with the parity-violating asymmetry $A_{PV}$ in the PREX, the reliably experimental information on the $J - a_{sym}$ is much more easily available attributed to a wealth of measured data on nuclear masses and on decay energies. The density dependence of the symmetry energy is also well constrained with the $J - a_{sym}$. Finally, with a ‘tomoscan’ method, we find that one just needs to measure the nucleon densities in $^{208}$Pb starting from $R_m = 7.61 \pm 0.04$ fm to obtain the $\Delta R_{np}$ in hadron scattering experiments, regardless of its interior profile that is hampered by the strong absorption.

PACS numbers: 21.65.Ef, 21.10.Gv, 21.65.Cd, 21.60.Jz
I. INTRODUCTION

The nuclear physics overlaps and interacts with astrophysics not only expands its research space but also promotes the development of fundamental physics. A great of attention has been paid to the equation of state (EOS) of isospin asymmetric nuclear matter in both the two fields as the development of radioactive beam facilities and astronomical observation facilities over the past decade. The symmetry energy that characterizes the isospin dependence of the EOS, is a quantity of critical importance due to its many-sided roles in nuclear physics [1–7] and astrophysics [8–14]. Although great efforts have been made and considerable progresses have been achieved both theoretically and experimentally, its density dependence ultimately remains unsolved because of the incomplete knowledge of the nuclear force as well as the complexity of many-body systems. Nevertheless, many important and leading issues in nuclear astrophysics require the accurate knowledge about it urgently at present.

The symmetry energy $S(\rho)$ of nuclear matter is usually expanded around saturation density $\rho_0$ as

$$S(\rho) = J + \frac{L}{3} \left( \frac{\rho - \rho_0}{\rho_0} \right) + \frac{K_{\text{sym}}}{18} \left( \frac{\rho - \rho_0}{\rho_0} \right)^2 + ..., \quad (1)$$

where $J = S(\rho_0)$ is the symmetry energy at $\rho_0$. The slope parameter $L = 3\rho_0 \partial S(\rho)/\partial \rho|_{\rho_0}$, and curvature parameter $K_{\text{sym}} = 9\rho_0^2 \partial^2 S/\partial \rho^2|_{\rho_0}$ characterize the density-dependent behavior of the symmetry energy around $\rho_0$. Extensive independent studies have been performed to constrain the slope $L$, but the uncertainty is still large [15–18].

It has been established that the slope parameter $L$ is strongly correlated linearly with the neutron skin thickness $\Delta R_{np}$ of heavy nuclei [19–21]. Although the theoretical predictions on $L$ and $\Delta R_{np}$ are extremely diverse, this linear correlation is universal in the realm of widely different mean-field models [22]. Accordingly, a measurement of $\Delta R_{np}$ with a high accuracy is of enormous significance to constrain the density-dependent behavior of $S(\rho)$ around $\rho_0$. Actually, many experimentalists have been concentrating on it with different methods including the x-ray cascade of antiprotonic atoms [23], pygmy dipole resonance [24, 25], proton elastic scattering [26], proton inelastic scattering [27] and electric dipole polarizability [28]. However, systematic uncertainties associated with various model assumptions are unavoidable. The parity-violating electron elastic scattering measurement in the parity radius experiment (PREX) at the Jefferson Laboratory combined with the fact that the parity-
violating asymmetry $A_{PV}$ is strongly correlated with the neutron rms radius, determined the $\Delta R_{np}$ to be $0.33^{+0.16}_{-0.18}$ fm with a large central value compared to other measurements and analyses [29]. Although it was suggested that ruling out a thick neutron skin in $^{208}$Pb seems premature [30], in any case, the large uncertainty seems to be not of much help to explore the symmetry energy and other interesting issues. In this work, a more practicable strategy compared with the PREX at current is introduced to probe the $\Delta R_{np}$ of $^{208}$Pb together with nuclear matter symmetry energy. A new insight into the neutron skin is also provided.

II. NEUTRON SKIN THICKNESS $\Delta R_{np}$ PROBED BY THE $J - a_{\text{sym}}$

The neutron skin thickness of nuclei is given as $\Delta R_{np} = \sqrt{\frac{2}{3} \left( \frac{2n_{p}}{A^{1/3}} \cdot (I - I_{c}) - e^{2}Z/(70J) \right)} + S_{sw}$ in the nuclear droplet model [31, 32] with isospin asymmetry $I$, nuclear radius $R = r_{0}A^{1/3}$ and a correction $I_{c} = e^{2}Z/(20JR)$ due to the Coulomb interaction. $Z$, $A$ are the proton and mass numbers, respectively. $S_{sw}$ is a correction caused by an eventual difference in the surface widths of nucleon density profiles. $a_{\text{sym}}(A)$ is symmetry energy (coefficient) that has been received great interest because with the help of it one may obtain some information on the density dependence of $S(\rho)$ [33-35]. Centelles et al. showed that the neutron skin thickness $\Delta R_{np}$ correlates linearly with $J - a_{\text{sym}}(A)$ based on different mean-field models, where the symmetry energy (coefficient) $a_{\text{sym}}(A)$ is obtained within the asymmetric semi-infinite nuclear matter (ASINM) calculations [32]. In our previous work, instead of using the ASINM calculations, the $a_{\text{sym}}(A)$ was obtained in the framework of the Skyrme energy-density functional approach by directly integrating the density functional of the symmetry energy after subtracting Coulomb polarization effect without introducing additional assumptions [33]. In the present work, the $a_{\text{sym}}(A)$ of $^{208}$Pb, marked as $a_{\text{sym}}$, is extracted with both the Skyrme effective interactions and relativistic effective interaction Lagrangians, and the local density approximation is adopted by dropping the negligible non-local terms compared to [33]. As done in Ref. [22], to prevent eventual biases, we avoid including more than two models of the same kind fitted by the same authors and protocol and avoid models providing a charge radius of $^{208}$Pb away from experiment data by more than 1%.

The calculated neutron skin thickness $\Delta R_{np}$ of $^{208}$Pb and $J - a_{\text{sym}}$ with different mean-field models are presented in Fig. 1, in which a close dependence of $\Delta R_{np}$ on $J - a_{\text{sym}}$
predicted by the droplet model is displayed. By performing a two-parameter fitting, the correlation is given by

$$\Delta R_{np} = (0.0138 \pm 0.0003)(J - a_{sym}) + (0.0376 \pm 0.0041),$$

with the correlation coefficient $r = 0.989$, where $\Delta R_{np}$ and $J - a_{sym}$ are in units of fm and MeV, respectively. Here the empirical saturation density $\rho_0 = 0.16$ fm$^{-3}$ is used uniformly. If the symmetry energy is calculated at their own saturation densities from the mean-field models, the linear correlation vanishes due to the fact that the relativistic interactions provide smaller saturation densities compared with the non-relativistic ones.

The $\Delta R_{np}$ of $^{208}$Pb is found to have a high linear correlation with $J - a_{sym}$ as that with the slope $L$ (not shown here). It is thus indisputable that the $J - a_{sym}$ with a high accuracy places a stringent constrain on the $\Delta R_{np}$. As the primary advantage, the reliably experimental information about the $J - a_{sym}$ is much more easily available compared with that about the parity-violating asymmetry $A_{PV}$ in the PREX. Recently, the symmetry energy $J$ at saturation density $\rho_0$ has been well determined to rather narrow regions, in particular, $32.5 \pm 0.5$ MeV from the mass systematics and $32.10 \pm 0.31$ MeV from the double differences of experimental symmetry energies agreeing with that of the mass systematics. These results are very useful in exploring the density-dependent symmetry energy as inputs.
Here we adopt the union of the two values, i.e. \( J = 32.4 \pm 0.6 \text{ MeV} \), and hence the central issue is to determine the symmetry energy \( a_{\text{sym}} \) of \(^{208}\text{Pb} \) accurately. We extract the mass dependent symmetry energy \( a_{\text{sym}}(A) = J/(1+\kappa A^{1/3}) \) [40, 41] with \( \beta^- \) -decay energies \( Q_{\beta^-} \) of heavy odd-\( A \) nuclei and with mass differences \( \Delta B \) between \( ^A(Z-1) \) and \( ^A(Z+1) \) as our previous calculations [42, 43] but with a new input quantity \( J \), and then derive the \( a_{\text{sym}} \) of \(^{208}\text{Pb} \). The merit of these two approaches is that only the well known Coulomb energy survives in \( Q_{\beta^-} \) and in \( \Delta B \) when determining the unknown \( a_{\text{sym}} \), where the \( Q_{\beta^-} \) and \( \Delta B \) are all taken from experimental data. Consequently, the \( a_{\text{sym}} \) is extracted to be \( 22.4 \pm 0.4 \text{ MeV} \) accurately, which is quite insensitive to the input quantity \( J \). As a result, the derived \( J - a_{\text{sym}} \) is \( 10.0 \pm 1.0 \text{ MeV} \) (solid circle in Fig. 1), which allows us to constrain the neutron skin thickness as well as the slope \( L \) in our subsequent calculations.

The neutron skin thickness in \(^{208}\text{Pb} \) is predicted to be \( \Delta R_{np} = 0.176 \pm 0.021 \text{ fm} \) (solid square in Fig. 1), where the estimated error stems from the uncertainties of the \( J - a_{\text{sym}} \) as well as Eq. (2). To reach such an error level, the \( A_{PV} \) in the PREX should be measured at least up to 2% accuracy, which is hardly implemented at present. This fact indicates the \( J - a_{\text{sym}} \) is much more effective to probe the \( \Delta R_{np} \) currently. The precise information about the \( \Delta R_{np} \) is of fundamental importance and has far-reaching implications in neutron star physics, such as the structure, composition and cooling. As an example, a relation of \( \rho_c \approx 0.16 - 0.39\Delta R_{np} \) was put forward to describe the relation between the \( \Delta R_{np} \) of \(^{208}\text{Pb} \) and the transition density \( \rho_c \) from a solid neutron star crust to the liquid interior [44], where the \( \rho_c \) is estimated to be \( 0.091 \pm 0.008 \text{ fm}^{-3} \). The properties of the crust-core transition is of crucial importance in understanding of the pulsar glitch [45].

**III. DENSITY DEPENDENCE OF THE SYMMETRY ENERGY PROBED BY THE \( J - a_{\text{sym}} \)**

Since the neutron skin thickness \( \Delta R_{np} \) correlates linearly with both the slope \( L \) and \( J - a_{\text{sym}} \), the slope \( L \) naturally correlates linearly with the \( J - a_{\text{sym}} \), which is displayed in Fig. 2(a). The linear relation is \( L = (9.682 \pm 0.285)(J - a_{\text{sym}}) + (-42.694 \pm 3.441) \), where \( L \) and \( J - a_{\text{sym}} \) are in units of MeV. Imposing the above obtained \( J - a_{\text{sym}} \), the slope parameter is estimated to be \( L = 54 \pm 16 \text{ MeV} \). Recently, the properties of nuclear matter at subsaturation density \( \rho \approx 0.11 \text{fm}^{-3} \) have attracted considerable attention because it has been shown that the \( \Delta R_{np} \)
is uniquely fixed by the slope $L(\rho \approx 0.11 \text{ fm}^{-3})$ \cite{46} and the giant monopole resonance of heavy nuclei is constrained by the nuclear matter EOS at this density \cite{47}. Fig. 2(b) shows that the slope $L(\rho = 0.11 \text{ fm}^{-3})$ (labeled $L_{0.11}$ for short) and $J - a_{\text{sym}}$ have a higher linear dependence $L_{0.11} = (4.542 \pm 0.073)(J - a_{\text{sym}}) + (2.140 \mp 0.885)$ with the correlation coefficient $r = 0.995$. Accordingly, the $L_{0.11}$ is evaluated to be $48 \pm 6$ MeV.

![Graph showing the correlation between $L$ and $J - a_{\text{sym}}$ at different densities.](image)

**FIG. 2:** (Color Online) Correlation of the slope parameter $L$ at densities $\rho = 0.16 \text{ fm}^{-3}$ and $\rho = 0.11 \text{ fm}^{-3}$ with the $J - a_{\text{sym}}$.

The slope $L$ is constrained with the $J - a_{\text{sym}}$ in another way for comparison. Centelles et al. found that the symmetry energy $a_{\text{sym}}$ of $^{208}\text{Pb}$ is approximately equal to the nuclear matter symmetry energy $S(\rho_A)$ at a reference density $\rho_A \simeq 0.1 \text{ fm}^{-3}$ \cite{32}. This important relation bridges the symmetry energies of nuclear matter and the finite nucleus. We calculate the reference density $\rho_A$ for $^{208}\text{Pb}$ and find that the interactions which provide the values of $J$ and $a_{\text{sym}}$ agreeing with the ones extracted from experimental information, give $\rho_A \simeq 0.088 \text{ fm}^{-3} = 0.55\rho_0$. It should be noted that the $a_{\text{sym}}$ does not equal the symmetry energy at the mean density of $^{208}\text{Pb}$ as a result of the extremely inhomogeneous isospin asymmetry distribution in the nucleus as shown in \cite{33}. Since the accurate value of the reference density $\rho_A$ is of crucial importance for determining the slope parameter $L$ \cite{42, 43}, we further examine it. Instead of the DDM3Y-shape expression used before \cite{42, 43}, Eq. (1) is employed directly to describe the density dependent symmetry energy to reduce the uncertain factors as far
as possible. The $K_{\text{sym}}$ term that contributes weakly to the symmetry energy nearby $\rho_0$ is estimated with the relation $K_{\text{sym}} = 39 + 5L - 15J$ \cite{48} obtained from the DDM3Y-shape expression without loss of accuracy. In terms of $J - S(\rho_A) = 10.0 \pm 1.0$ MeV and $\rho_A = 0.55\rho_0$, the slope $L$ at the saturation density $\rho_0$ is predicted to be $53 \pm 10$ MeV according to Eq. (1), which is in excellent agreement with that from Fig. 2(a). At the density of $\rho = 0.11$ fm$^{-3}$, the slope $L_{0.11} = 49 \pm 4$ MeV, being also particularly consistent with the value of $48 \pm 6$ MeV from Fig. 2(b). The consistency of the two approaches not only indicates the reliability of the present methods but also further verifies the accuracy of the reference density $\rho_A = 0.55\rho_0$. As an important conclusion, the $a_{\text{sym}} = S(\rho = 0.55\rho_0) \simeq 22.4$ MeV will be a very useful reference to calibrate the effective interactions in nuclear energy density functionals.

With the obtained $L_{0.11}$ and $L$ values, the curvature parameter is evaluated to be $K_{\text{sym}} = -152 \pm 70$ MeV. Currently, the symmetry energy at suprasaturation densities is extremely controversial. It was indicated that the three bulk parameters $J$, $L$ and $K_{\text{sym}}$ well characterize the symmetry energy at densities up to $\sim 2\rho_0$ while higher order terms contribute negligibly small \cite{49}. If true, the symmetry energy $S(\rho)$ at high densities up to $\sim 2\rho_0$ turns out to be not stiff, as shown in Fig. 3. The symmetry energy at $2\rho_0$ is estimated to be $S(2\rho_0) = 42 \pm 10$ MeV. In short, to characterize the symmetry energy at high densities, the accurate knowledge about its density dependence at the saturation density is crucial.

FIG. 3: (Color Online) Density dependent symmetry energy at high densities.
IV. FURTHER EXPLORATION ON THE MEASUREMENT OF THE $\Delta R_{np}$

Based on the above discussions on the neutron skin thickness $\Delta R_{np}$ and symmetry energy, we make an exploration on the measurement of the $\Delta R_{np}$ in $^{208}$Pb. To grasp richer information on the $\Delta R_{np}$, we formulate it as an integral of a distribution function

$$\Delta R_{np} = \sqrt{\langle r_n^2 \rangle} - \sqrt{\langle r_p^2 \rangle} = \int_0^\infty f(r) dr,$$

where $f(r) = 4\pi r^4 \left( \frac{\rho_n}{N} - \frac{\rho_p}{Z} \right) / \left( \sqrt{\langle r_n^2 \rangle} + \sqrt{\langle r_p^2 \rangle} \right)$ is defined as the radial distribution function which is actually determined by the nucleon densities and reflects the detailed information about the neutron skin. $\sqrt{\langle r_n^2 \rangle} + \sqrt{\langle r_p^2 \rangle} \simeq 11.1$ fm changes by less than 3% in the mean field model calculations, and can be taken as a known value. Fig. 4(a) illustrates the distribution function $f(r)$ in $^{208}$Pb as a function of distance $r$ generated by the SLy5 interaction as an example. It is a misleading idea to consider the neutron skin merely originating from the nuclear surface. The area enclosed by the x-axis and the curve $f(r)$ (colored regions) is exactly the neutron skin thickness $\Delta R_{np}$. We name this new method that dissects the $\Delta R_{np}$ with a distribution function as ‘tomoscan’ picturesquely here. As a new concept in nuclear physics, it could also be used to analyze other intriguing issues, such as the halo structure in exotic nuclei. The region of $r < R_0$ contributes negatively while that of $r > R_0$ contributes positively to the $\Delta R_{np}$. Thus, there exists a distance $R_m$ below which ($0 \leq r < R_m$) the contributions (red shaded regions) cancel each other out, and hence the $\Delta R_{np}$ can be calculated by the neutron and proton density distributions just starting from $R_m$ (blue filled region).

The calculated values of $R_m$ with different interactions are marked in Fig. 4(b). The $R_m$ is found to be model dependent, which should be further constrained. The interactions generating smaller (larger) $\Delta R_{np}$ tend to yield slightly larger (smaller) $R_m$. As we mentioned above, one important conclusion of this work is that the $a_{sym} = S(\rho = 0.55\rho_0) \simeq 22.4$ MeV (along with $J \simeq 32.4$ MeV) serve important calibrations for effective interactions in nuclear energy density functionals. Thus we use those constraint conditions to filter those interactions. The eligible interactions give $R_m = 7.61 \pm 0.04$ fm (colored solid symbols), where the error bar of $0.04$ fm just leads to an uncertainty of the $\Delta R_{np}$ by about $0.005$ fm. The error bar of $0.005$ fm for the $\Delta R_{np}$ is so small that the obtained $R_m$ value should not be regarded as model dependent any more. This result leads to an intriguing
FIG. 4: (Color Online) (a) Radial distribution function \( f(r) \) of the neutron skin thickness in \( ^{208}\)Pb. The contributions from the two parts in the red shaded regions cancel each other out. The area under the curve of \( f(r) \) starting from \( R_m \) (blue filled region) is equal to the neutron skin thickness \( \Delta R_{np} \). (b) Calculated \( R_m \) values with different energy density functionals. The colored solid symbols are from the interactions generating the reference density \( \rho_A \simeq 0.55\rho_0, \ a_{sym} \simeq 22.4 \) MeV and \( J \simeq 32.4 \) MeV. (c) Error accumulation of the \( \Delta R_{np} \) measurement in hadron scattering experiments as a function of distance \( r \), where the nucleonic density distributions are from Tables III and IV in Ref. [26].

conclusion: one just needs to measure the rather dilute matter located in the nuclear surface to determine the neutron skin thickness of \( ^{208}\)Pb, namely, only measures the nucleon densities from \( r = R_m = 7.61 \pm 0.04 \) fm to about \( r = 12 \) fm. Thus, the measurement of the \( \Delta R_{np} \) would be substantially simplified in hadron scattering experiments which have been hampered by the strong absorption in the nuclear interior. We stress that, contrary to the usual understanding, the nuclear surface properties are in fact not well constrained by the nuclear mean-field models obtained by fitting nuclear masses and charge radii. For instance, both the SLy5 and NL3 interactions give \( R_m = 7.62 \) fm, but they provide a substantial difference in the \( \Delta R_{np} \) amounting to 0.12 fm. In other words, it is exactly the ambiguity of the nuclear surface profile that leads to the large uncertainty of the \( \Delta R_{np} \), because the radial distribution function \( f(r) \) relies on the fourth power of distance \( r \) according to Eq. (3), causing a drastic amplification of the error as \( r \) increases. Fig. 4(c) illustrates the error accumulation of the \( \Delta R_{np} \) in hadron scattering experiments for different regions, which is obtained by analyzing the data in Ref. [26] combined with the ‘tomoscan’ method. The
error accumulation at distance \( r \) is defined as the error generated by the region from the nuclear center to \( r \). It indicates that the error also primarily originates from the surface structure. Therefore, the surface profiles must receive particular attention and be measured with a much higher accuracy.

V. SUMMARY

We have developed alternative methods in the present study to explore the neutron skin thickness \( \Delta R_{np} \) of \(^{208}\text{Pb} \) and density dependence of symmetry energy. The main conclusions are summarized as follows. i) We have established a high linear correlation between the \( \Delta R_{np} \) and \( J - a_{\text{sym}} \) on the basis of widely different nuclear energy-density functionals. Accordingly, an accurate \( J - a_{\text{sym}} \) value sets a significant constrain on the \( \Delta R_{np} \), which turns out to be a much more effective probe than the parity-violating asymmetry \( A_{\text{PV}} \) in the current PREX. ii) The symmetry energy (coefficient) \( a_{\text{sym}} \) of \(^{208}\text{Pb} \) was extracted accurately with the experimental \( \beta^- \)-decay energies of heavy odd-\( A \) nuclei and with the experimental mass differences. Given that the symmetry energy \( J \) has been well determined recently, the \( \Delta R_{np} \) in \(^{208}\text{Pb} \) was thus predicted to be \( 0.176 \pm 0.021 \) fm robustly. This conclusion would be significantly meaningful to discriminate between the models and predictions relevant for the description of nuclear properties and neutron stars. iii) With the above derived \( J - a_{\text{sym}} \), the values of the slope \( L \) of the symmetry energy at the densities of \( \rho = 0.16 \) fm\(^{-3} \) and \( \rho = 0.11 \) fm\(^{-3} \) which are of great concern, are predicted to be \( 54 \pm 16 \) MeV and \( 48 \pm 6 \) MeV respectively. These results, together with the \( \Delta R_{np} \) of \(^{208}\text{Pb} \), can be applied to explore some intriguing problems in nuclear astrophysics. In particular, the derived \( a_{\text{sym}} \) and \( S(\rho_A) \) serve as important calibrations for a reliable construction of new effective interactions in nuclear many-body models. iv) The symmetry energy at suprasaturation densities up to \( \sim 2\rho_0 \) was predicted to be not stiff. v) With the firstly proposed ‘tomoscan’ method, we concluded that to obtain the \( \Delta R_{np} \) one needs to only measure the nucleon densities in \(^{208}\text{Pb} \) from \( R_m = 7.61 \pm 0.04 \) fm as the densities in the range of \( r < R_m \) have no contribution to the \( \Delta R_{np} \). Thus, the measurement on the \( \Delta R_{np} \) is significantly simplified in hadron scattering experiments which have been hampered by the strong absorption in the nuclear interior. Incidentally, the ‘tomoscan’ method could be employed to analyze the halo structure in exotic nuclei. vi) It has been widely believed that the nuclear surface structure is well constrained
in nuclear energy-density functionals and in experimental measurements. However, within the ‘tomoscan’ concept, we have showed that it is not true but a complete illusion. To grasp the $\Delta R_{np}$, one must especially concentrate on the dilute matter located in nuclear surface which results in the dominant uncertainty.

Acknowledgment

This work was supported by the National Natural Science Foundation of China under Grants No. 11405223, No. 11175219, No. 10975190 and No. 11275271, by the 973 Program of China under Grant No. 2013CB834405, by the Knowledge Innovation Project (KJCX2-EW-N01) of Chinese Academy of Sciences, by the Funds for Creative Research Groups of China under Grant No. 11321064, and by the Youth Innovation Promotion Association of Chinese Academy of Sciences.

[1] V. Baran, M. Colonna, V. Greco, and M. Di Toro, Phys. Rep. 410, 335 (2005).
[2] B. A. Li, L. W. Chen, and C. M. Ko, Phys. Rep. 464, 113 (2008).
[3] P. Danielewicz, R. Lacey, and W. G. Lynch, Science 298, 1592 (2002).
[4] A. W. Steiner, M. Prakash, J. Lattimer, and P. J. Ellis, Phys. Rep. 411, 325 (2005).
[5] J. M. Pearson, N. Chamel, A. F. Fantina, and S. Goriely, Eur. Phys. J. A 50, 43 (2014).
[6] N. Wang, M. Liu, and X. Wu, Phys. Rev. C 81, 044322 (2010).
[7] J. Dong, W. Zuo, and W. Scheid, Phys. Rev. Lett. 107, 012501 (2011).
[8] H.-T. Janka, K. Langanke, A. Marek, G. Martínez-Pinedo, and B. Müller, Phys. Rep. 442, 38 (2007).
[9] J. M. Lattimer and M. Prakash, Phys. Rep. 333, 121 (2000); Phys. Rep. 442, 109 (2007).
[10] K. Hebeler, J. M. Lattimer, C. J. Pethick, and A. Schwenk, Phys. Rev. Lett. 105, 161102 (2010).
[11] A. W. Steiner and A. L. Watts, Phys. Rev. Lett. 103, 181101 (2009).
[12] D. H. Wen, B. A. Li, and P. G. Krastev, Phys. Rev. C 80, 025801 (2009).
[13] H. Sotani, K. Nakazato, K. Iida, and K. Oyamatsu, Phys. Rev. Lett. 108, 201101 (2012).
[14] L. F. Roberts et al., Phys. Rev. Lett. 108, 061103 (2012).
[15] M. B. Tsang et al., Phys. Rev. C 86, 015803 (2012).
[16] X. Viñas, M. Centelles, X. Roca-Maza, and M. Warda, Eur. Phys. J. A 50, 27 (2014).
[17] B. M. Santos, M. Dutra, O. Lourenço, and A. Delfino, Phys. Rev. C 90, 035203 (2014).
[18] Bao-An Li, Xiao Han, Phys. Lett. B727, 276 (2013).
[19] B. A. Brown, Phys. Rev. Lett. 85, 5296 (2000).
[20] S. Typel and B. A. Brown, Phys. Rev. C 64, 027302 (2001).
[21] R. J. Furnstahl, Nucl. Phys. A706, 85 (2002).
[22] X. Roca-Maza, M. Centelles, X. Viñas, and M. Warda, Phys. Rev. Lett. 106, 252501 (2011).
[23] A. Trzcińska et al., Phys. Rev. Lett. 87, 082501 (2001); B. Klos et al., Phys. Rev. C 76, 014311 (2007).
[24] Andrea Carbone et al., Phys. Rev. C 81, 041301(R) (2010).
[25] A. Klimkiewicz et al., Phys. Rev. C 76, 051603(R) (2007).
[26] J. Zenihiro et al., Phys. Rev. C 82, 044611 (2010).
[27] A. Tamii et al., Phys. Rev. Lett. 107, 062502 (2011).
[28] J. Piekarewicz, B. K. Agrawal, G. Colò, W. Nazarewicz, N. Paar, P.-G. Reinhard, X. Roca-Maza, and D. Vretenar, Phys. Rev. C 85, 041302(R) (2012).
[29] S. Abrahamyan et al., Phys. Rev. Lett. 108, 112502 (2012).
[30] F. J. Fattoyev, and J. Piekarewicz, Phys. Rev. Lett. 111, 162501 (2013).
[31] W. D. Myers and W. J. Swiatecki, Nucl. Phys. A336, 267 (1980).
[32] M. Centelles, X. Roca-Maza, X. Viñas, and M. Warda, Phys. Rev. Lett. 102, 122502 (2009).
[33] J. Dong, W. Zuo, and J. Gu, Phys. Rev. C 87, 014303 (2013).
[34] Junlong Tian, Haitao Cui, Kuankuan Zheng, and Ning Wang, Phys. Rev. C 90, 024313 (2014).
[35] Z. W. Zhang, S. S. Bao, J. N. Hu, and H. Shen, Phys. Rev. C 90, 054302 (2014).
[36] Z. X. Wang, nuclear matter, (Beijing, 2014).
[37] P. Möller, W. D. Myers, H. Sagawa, and S. Yoshida, Phys. Rev. Lett. 108, 052501 (2012).
[38] H. Jiang, G. J. Fu, Y. M. Zhao, and A. Arima, Phys. Rev. C 85, 024301 (2012).
[39] B. K. Agrawal, J. N. De, and S. K. Samaddar, Phys. Rev. Lett. 109, 262501 (2012).
[40] W. D. Myers and W. J. Światecki, Ann. Phys. (N.Y.) 55, 395 (1969); 84, 186 (1974).
[41] P. Danielewicz and J. Lee, Nucl. Phys. A818, 36 (2009).
[42] J. Dong, H. Zhang, L. Wang and W. Zuo, Phys. Rev. C 88, 014302 (2013).
[43] X. Fan, J. Dong, and W. Zuo, Phys. Rev. C 89, 017305 (2014).
[44] C. J. Horowitz and J. Piekarewicz, Phys. Rev. Lett. 86, 5647 (2001).

[45] N. Chamel, Phys. Rev. Lett. 110, 011101 (2013).

[46] Z. Zhang, Lie-Wen Chen, Phys. Lett. B726, 234 (2013).

[47] E. Khan, J. Margueron, and I. Vidaña, Phys. Rev. Lett. 109, 092501 (2012).

[48] J. Dong, W. Zuo, J. Gu, and U. Lombardo, Phys. Rev. C 85 034308 (2012).

[49] L. W. Chen, Sci. China Phys. Mech. Astron. 54, 124 (2011).