Optimal Design of Sparse Array for Ultrasonic Total Focusing Method by Binary Particle Swarm Optimization

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ABSTRACT Ultrasonic phased array technology is used in various fields. Traditional full phased arrays place elements in every position of a uniform lattice with half-wavelength spacing between the lattice points, so the hardware cost is very high. This paper introduces an automatically method to sparsify the full array method with well-controlled sidelobes and the main lobe. By calculating one-dimensional phased array patterns that can reflect phased array performance, the binary particle swarm optimization (BPSO) algorithm is used to optimize the array layout. The method initialized form full array and decreased several elements step by step, then, a sparse array with comprehensive acoustic performance close to the reference full array is obtained. By applying the proposed method to the sparse array design of total focusing method (TFM), the simulation results indicate that the proposed sparse total focusing method can greatly increase computational efficiency while providing significantly higher image quality. The BPSO can provide effective optimization design for sparse arrays.

INDEX TERMS Binary particle swarm optimization, sparse array, total focusing method, ultrasonic phased array.

I. INTRODUCTION

The phased array technology originated from the advanced phased array radar technology, which is widely used in marine landform detection and advanced anti-submarine sonar. In phased array radar technology, a large number of sub-antenna arrays are arranged according to a certain rule or shape combination, and then by controlling the delay and amplitude of the electromagnetic beam emitted by each sub-array, a flexible and variable radar focus can be formed within a certain time and space beam. Ultrasonic phased array is a transducer array composed of several piezoelectric transducer elements. Its basic function is to realize phase-control transmission or reception of ultrasonic waves.

In the early stage, the ultrasonic phased array was mainly used in the medical field. In medical ultrasound imaging, the phased array transducer was used to quickly move the sound beam to image the organ to be examined [1]. In ultrasound therapy, ultrasonic phased array allows one to vary the position of the focusing area without mechanical movement of the array itself, and to create several focal at the same time [2]. High-intensity focused ultrasound technique can make the ultrasound energy highly concentrated in the deep tissues of the human body, thereby causing acute thermal damage to the diseased tissue in a short time without affecting the surrounding normal tissues [3]. Nowadays, various High-intensity focused ultrasound array transducers have been developed and applied in a wide range of medical fields to ablate various types of lesions [2], [4]. Further, the application of ultrasonic phased array in the field of micromanipulation has shown initial effects. In 2015, Marzo et al proposed the acoustic holography technology to manufacture dynamic Wells to flexibly manipulate suspended particles [5]. However, the presence of beam sidelobes in the ultrasonic phased array reduces the performance of the phased array in its corresponding application field. For example, the damage to normal tissues during high-intensity focused ultrasound treatment or the decline of imaging quality in ultrasound diagnosis.

With the development of piezoelectric materials and computers, phased array technology has also gradually been applied to industrial inspection, and has made considerable
This paper proposes an optimized design method for the sparse array. By calculating the array beam directivity diagram, the element layout of the sparse array is optimized by binary particle swarm optimization (BPSO). I.e. removed some elements from the reference full array by minimizing the fitness value. Unlike previous sparse array research, this paper does not need to set the sparse rate of the target sparse array and can finally achieve a balance between array performance and sparse rate. Simulation results show that the optimized sparse array has better sound field characteristics. We use the optimized sparse array for TFM, the results show that its imaging resolution is close to that of the reference full array, with few sidelobe artifacts, high image signal noise ratio (SNR), and less imaging time consumption. The proposed method is so universal that the full array of any number of array elements can use the proposed algorithm to improve the performance of a phased array. The proposed method can be applied to sparse phased array design in many fields. For example, the proposed method will apply to TFM in the industrial ultrasonic testing, further improve imaging quality and real-time testing efficiency based on [15], [16].

The rest of the paper is organized as follows: Section II presents the details of the core idea of FMC and TFM technology, further, we elaborate the method of optimizing the element layout of the linear sparse phased array by BPSO. Section III presents the optimization results of arrays, then the optimized sparse array performance was tested by comparison of TFM simulation results. Section IV concludes the paper.

II. THEORY
A. FULL FOCUS ALGORITHM MODEL
The Full Matrix Capture (FMC) is an advanced data acquisition method based on array transducers. As shown in Fig. 1, the FMC excites on a single array element, and all array elements receive at the same time, which is performed in sequence until all the excitation and reception combinations are obtained, and then complete detection information is obtained.

A one-dimensional linear array probe considering \( N \) array elements is placed on the surface of a two-dimensional isotropic homogeneous medium, and point defects are located inside the medium. Establish a two-dimensional coordinate system \( Oxz \), as shown in Fig. 2.

The x-axis is along the array direction and parallel to the surface of the medium. The z-axis is perpendicular to the surface of the medium and points to the inside of the medium.
The array is arranged on the x-axis, and the imaging area is below the array. Assuming that the number of array elements is N and the echo received at the center point of each array element is recorded as $u(x_i, x_j, t)$, the data collected by the full matrix is a three-dimensional matrix, as shown in (1).

$$
\begin{align*}
\begin{bmatrix}
  u(x_1, x_1, t) & u(x_1, x_2, t) & \cdots & u(x_1, x_N, t) \\
  u(x_2, x_1, t) & u(x_2, x_2, t) & \cdots & u(x_2, x_N, t) \\
  \vdots & \vdots & \ddots & \vdots \\
  u(x_N, x_1, t) & u(x_N, x_2, t) & \cdots & u(x_N, x_N, t)
\end{bmatrix}
\end{align*}
\tag{1}
$$

where $i, j = 1, 2, \ldots, N$, $x_i$ and $x_j$ denote the coordinates of the excitation and receiving elements, respectively, and $t$ is time.

There is a scattering point $P(x_n, z_m)$ below the array, where $n = 1, 2, \ldots, M_1, m = 1, 2, \ldots, M_2$. $M_1$ is the number of pixels in the x-direction in the imaging area, $M_2$ is the number of pixels in the z-direction in the imaging area. According to the geometric relationship of Fig. 2, the distances from the excitation element $(x_i, 0)$ and the receiving element $(x_j, 0)$ to the scattering point $P(x_n, z_m)$ are given by

$$
\begin{align*}
  r_{ip} &= \sqrt{(x_i - x_n)^2 + z_m^2} \\
  r_{jp} &= \sqrt{(x_j - x_n)^2 + z_m^2}
\end{align*}
\tag{2}
$$

Then, the $i^{th}$ element is excited, and after reflecting at the focal point $P(x_n, z_m)$, the acoustic propagation delay $t_{ij}$ when it is received by the $j^{th}$ element can be denoted as

$$
  t_{ij} = \frac{r_{ip} + r_{jp}}{c} = \frac{\sqrt{(x_i - x_n)^2 + z_m^2} + \sqrt{(x_j - x_n)^2 + z_m^2}}{c}
\tag{4}
$$

where $c$ is the longitudinal velocity of sound. In Fig. 2, the pixel intensity value $I(x_n, z_m)$ of any imaging point $P(x_n, z_m)$ can be expressed as

$$
I(x_n, z_m) = \sum_{i=1}^{N} \sum_{j=1}^{N} h(x_i, x_j, t_{ij}) = \sum_{i=1}^{N} \sum_{j=1}^{N} h_{ij}(\frac{r_{ip} + r_{jp}}{c})
\tag{5}
$$

where $h_{ij}$ is the analytical version of the echo received signal.

**B. BEAM DIRECTIVITY OF THE SPARSE ARRAY**

It can be seen from (1) and (5) that TFM pixel synthesis needs to process a large amount of data information, which increases the processing time of the processor and reduces the frame rate of real-time imaging. The sparse array can make the performance of the sparse-TFM as close to the full array as possible while reducing the array elements. Although the number of effective array elements of the sparse array is reduced, due to the sparse arrangement of the array element position, the grating lobes are eliminated. And because its effective aperture can be the same as a full array, so they have a similar mainlobe characteristic, i.e. nearly the same lateral resolution [9]. To get the performance index that can evaluate the sparse array, we established an optimization model firstly. For a linear array as shown in Fig.1, the sound pressure of a single element can be defined as [17]

$$
p(r, \theta, t) = (\frac{p_0}{r})^{1/2} \sin(ka \sin \theta / 2) \cdot \exp(j\omega t - kr)
\tag{6}
$$

where $r$ is the distance between the imaging point and the array element, $k$ is the wavenumber, $\theta$ is the direction angle, $\omega$ is the circular frequency, $a$ is the element width. For the sparse array, the synthesized sound pressure can be defined as

$$
p(r, \theta, t) = \sum_{i=1}^{N} g_i p_i(r, \theta, t)
\tag{7}
$$

where $g_i$ is the binary coefficient, $g_i = 1$ denotes the $i^{th}$ element is active and $g_i = 0$ denotes it is inactive. Then we can get the beam directivity function under the sparse array distribution by

$$
H(\theta) = \frac{\left| p(r, \theta, t) \right|}{\left| p(r, \theta_m, t) \right|}
\tag{8}
$$

where, $\theta_m$ is the steering angle of the phased array, which is set to 0 here. The directivity diagram of a sparse array can reflect the imaging performance of the array. Hence, we can optimize the array according to the characteristics of the directivity diagram.

**C. OPTIMIZATION PROBLEM FORMULATION**

Because the particle swarm algorithm has the characteristics of simple operation steps, easy programming, high search efficiency, and wide applicability. And the algorithm can face the problem to be solved, it reduces the coding and decoding process of the problem solution in the genetic algorithm, which is very suitable for the optimization of constraint problems [18], [19].
We aim to reduce the number of effective array elements to reduce the calculation time of TFM, at the same time, we want to guarantee the SNR and resolution of the imaging. Therefore, our optimization objectives are the contrast resolution and spatial resolution of the sparse array. The contrast resolution and spatial resolution can be characterized by the maximum sidelobe level (MSL) and the -6 dB width of the mainlobe of the directivity diagram of a sparse array, respectively [11]. The sparse array directivity diagram can be obtained by (8). Since the main-lobe width (MLW) and MSL cannot be perfect optimized simultaneously perfectly in the directivity diagram, we define the fitness function as follows

\[
Fit = \psi_1 \cdot MSL + \psi_2 \cdot MLW
\]  
(9)

where, \(\psi_1\) and \(\psi_2\) are coefficient values selected according to different optimization objectives. Since we want to take into account the characteristics of both the mainlobe and sidelobes, both \(\psi_1\) and \(\psi_2\) are set to 1.

This paper is to optimize the element layout of the linear sparse phased array by minimizing the fitness value. That is, each array element is first placed at equal intervals according to grating lobe suppression rule, then based on reference full array, some array elements are closed by minimizing the objective function to achieve the purpose of the sparse array. Finally, a sparse array of the combined good performance of the MSL and MLW of the beam can be obtained. The optimization of the full array means the optimization of the parameter \(g_i\), for \(i = 1, 2, \ldots, N\). \(g_i\) is a binary parameter. Therefore, this paper uses a BPSO as the optimization method of the one-dimensional sparse array. Each particle carries a binary code of dimension N, which indicates the distribution of the N array element. The value of the \(k\)th dimension of the \(j\)th particle after the \(t\)th iteration is updated by

\[
Y_{jk}(t + 1) = g_i
\]  
(10)

Particle velocity update formula is given by

\[
v_{jk}(t + 1) = \omega(t) \cdot v_{jk}(t) + c_1 R_{1k}(t)[Y_P - Y_{jk}(t)] + c_2 R_{2k}(t)[Y_G - Y_{jk}(t)]
\]  
(11)

where \(c_1\) and \(c_2\) are acceleration factors both equalling to 2, \(Y_{jk}(t)\) and \(v_{jk}(t)\) represent the position and velocity of the \(k\)th dimension of the \(j\)th particle at the \(t\)th iteration, respectively. \(R_{1k}(t)\) and \(R_{2k}(t)\) are uniformly-distributed random numbers between 0 and 1. \(Y_P\) and \(Y_G\) are personal best and global best, respectively. \(w\) is the inertia coefficient, and its value changes with the number of iterations, which is given by

\[
w(t) = w_{\text{max}} - (w_{\text{max}} - w_{\text{min}}) \frac{t}{T_{\text{max}}}
\]  
(12)

where, \(w_{\text{max}}\) and \(w_{\text{min}}\) are the maximum and minimum inertia coefficients, respectively. \(T_{\text{max}}\) represents the maximum number of iterations. In this way, the particle velocity can be controlled and premature convergence can be avoided.

To optimize the BPSO algorithm, we set \(w_{\text{max}} = 0.9\), \(w_{\text{min}} = 0.4\). In order to control the global search behavior of particles, the particle speed is clamped in a bounded range. A positive integer \(v_{\text{max}}\) is introduced so that \(v_{jk}(t)\) satisfies

\[
v_{jk}(t + 1) = \begin{cases} 
\max(0, v_{jk}(t) + \min(0, v_{\text{max}})), & \text{if } v_{jk}(t + 1) > v_{\text{max}} \\
\min(0, v_{\text{max}}), & \text{if } v_{jk}(t + 1) < -v_{\text{max}} \\
v_{jk}(t + 1), & \text{otherwise}
\end{cases}
\]  
(13)

If the speed converges to close near \(v_{\text{max}}\) or \(-v_{\text{max}}\), it will be hard to change the corresponding position at a small change of velocity, which makes it difficult to escape from the good local optimal value of BPSO. To solve this problem, we introduce the following operation after the velocity update (13).

\[
v_{jk}(t + 1) = \begin{cases} 
1 - v_{jk}(t + 1), & \text{if } R_{3k}(t) < r_{\text{mu}} \\
v_{jk}(t + 1), & \text{otherwise}
\end{cases}
\]  
(14)

where \(R_{3k}(t)\) indicates the random number between 0 and 1. \(r_{\text{mu}}\) is the probability that the operation is conducted in the \(k\)th dimension of the \(j\)th particle.

Use the Sigmoid function to normalize the speed obtained by (14), given by (15).

\[S(v_{jk}(t + 1)) = \frac{1}{1 + \exp(-v_{jk}(t + 1))}
\]  
(15)

where \(S(v_{jk}(t))\) represents the probability that the \(k\)th dimension of the \(j\)th particle changes from one state to another at the \(t\)th iteration. Each particle updates the position vector according to (16).

\[Y_{jk}(t + 1) = \begin{cases} 
1, & \text{if } R_{4k}(t) < S(v_{jk}(t + 1)) \\
0, & \text{otherwise}
\end{cases}
\]  
(16)

where \(R_{4k}(t)\) indicates the random number between 0 and 1. The BPSO algorithm steps are as follows:

Step 1: Initialize particle population, including initialization of particle position and velocity.

Step 2: Calculate the fitness value of the first-generation particle by (9).

Step 3: Update the personal best \(Y_P\) and global best \(Y_G\) according to the return value of the fitness function.

Step 4: Update particle speed by (11), (13), and (14), then update the particle position by (15).

Step 5: Calculate the fitness value of the current-generation particle by (9).

Step 6: Update the personal best \(Y_P\) and global best \(Y_G\) according to the return value of the fitness function.

Step 7: If \(t = T_{\text{max}}\), it ends and output global best \(Y_G\). Otherwise, go to step 4.

III. SIMULATION AND COMPARISON

A. DIRECTIVITY DIAGRAM

Taking 64-element full array transducer as an example, the element layout of the linear sparse phased array was optimized by the proposed method. For comparison purposes, another full array with the same number of elements as the optimized sparse array is also calculated. The full array transducer parameters are shown in Table 1, where the parameters
TABLE 1. The full array parameters.

| Parameter               | Value          |
|-------------------------|----------------|
| Element width           | 0.53 mm        |
| Element pitch           | 0.63 mm        |
| Central frequency       | 5 MHz          |
| Sound velocity          | 6300 m/s       |
| Bandwidth (-6 dB)       | 50%            |

satisfy \( d = \frac{\lambda}{2} \) to prevent the generation of periodic grating lobes, \( \lambda \) is the wavelength.

According to the reasonable selection range of the parameters, the size of the initial population group was set to 50 after several attempts, the number of iterations was set to 200, and \( r_{mu} \) set to 0.2. And carry out simulation experiments with Matlab software, then the directivity diagrams of the 64-element full array and the optimized sparse array are shown in Fig. 3 (a) and Fig. 3 (b), respectively.

FIGURE 3. The directivity diagram of (a) 64-element full array and (b) optimized sparse array.

In the directivity diagram of 64-element full array, there are large sidelobes around the main lobe. The MSL and MLW are \(-13.2455 \text{dB} \) and \(2.1557^\circ\), respectively. In the optimized sparse array, the sidelobes suppression effect is obvious, the MSL is \(-19.0035 \text{dB} \), and the MLW of \(-6\text{dB} \) is \(2.4620^\circ\). The number of array elements has been reduced by 13, which will greatly improve the FMC/TFM efficiency. For comparison purposes, we plotted the directivity diagram of the 51-element full array with the same number of elements as the optimized sparse array, the array parameters are also the same as those in Table 1. The directivity diagram of the 51-element full array is shown in Fig. 5.

FIGURE 5. The directivity diagram of the 51-element full array.

Compared with the 64-element full array, the MSL is almost invariable in Fig. 5, but the MLW is significantly increased. By measurement, the MSL and MLW of \(-6\text{dB} \) of the 51-element full array are \(-13.2476 \text{dB} \) and \(2.7064^\circ\), respectively. As the number of elements of the full array decreases by 13, the MLW increases by \(0.5507^\circ\). This is very natural because the reduction of the number of array elements will inevitably lead to the reduction of the size of the effective aperture of the full array transducer. However, the optimized sparse array is obtained based on the full array
by minimizing fitness value, which guarantees the size of the effective aperture so that imaging resolution hardly affected.

B. TFM
To verify the TFM effect of optimized sparse arrays, a simulation by MATLAB was performed to compare these three arrays mentioned above. The point spread function (PSF) is the response of the imaging algorithm to a single ideal scattering point. In a linear sound field, the imaging result of any defect can be regarded as the convolution process of the actual scattering function of the defect and PSF [20]. Therefore, in this paper, we use PSF to characterize the spatial imaging characteristics of the TFM algorithm.
The full array parameters are in Table 1. The array element distribution of the sparse array is determined based on the optimization results of the directivity diagram above. The output of each element was a five cycle, Gaussian windowed tone burst with a 5 MHz center frequency and a −6 dB bandwidth of 50%. Fig. 6 shows the results scaled in dB to display a point-like scatter at (0,20λ) after the generation of pixels based on the FMC signals. The imaging area is 30 mm × 40 mm. It is divided into a 0.2 mm grid on the horizontal and vertical axes, and the grid size is 150 × 200 pixels.

As can be seen from the Fig.6, in the imaging area of the 64-element full array and the 51-element full array, there are “ear” shaped areas on both sides of the ideal point oval area, which is generated by the side beam lobe scanning. And the size of the point defect of the 51-element full array is larger than that of the 64-element full array. Interestingly, the artifacts of optimized sparse array imaging almost disappeared, and single-point defects were clearer, the size of the point defect is close to that of the 64-element full array. The imaging effects of the three arrays are consistent with the theoretical analysis of their corresponding directivity diagram synthetic beams.

For the purpose to evaluate quantitatively the imaging performance of the array, we need to introduce the array performance indicator API [13], which is given by

\[
API = \frac{A_{-6dB}}{\lambda^2} \tag{18}
\]

where \(A_{-6dB}\) is the area of the PSF which is greater than −6dB down from its maximum value. The smaller the API, the better the imaging resolution of the array.

By calculating, the API in Fig. 6 (a),(b),(c) are 0.6160, 0.6595, and 0.7180, respectively, and the SNR are 39.9085 dB, 39.9452 dB, and 39.9125 dB, respectively. The optimized sparse array has the highest SNR and has a smaller API value than the 51-element full array. What is more, in the whole simulation process, the imaging time of the optimized sparse array is reduced by more than 20% compared with the 64-element full array, which means the sparse array can improve the efficiency of ultrasonic detection. Although the API of the optimized sparse array is slightly increased than the reference full array. On the one hand, the error is small, which guarantees image resolution of the sparse array close to that of the reference full array. On the other hand, the performance of the optimized sparse array can be biased toward the resolution by changing the coefficient \(\psi_1, \psi_2\) of the fitness function according to the actual situation.

To study the imaging performance of the three arrays for the closely spaced scatterers, we set up three point-like scatterers with a spacing of 2mm. i.e., the scatter positions are (−1.59λ, 20λ), (0, 20λ), and (1.59λ, 20λ), respectively. The TFM results of three arrays as shown in Fig.7.

It can be seen from Fig. 7 that the 64-element full array has the strongest sidelobe energy near the main lobe, and the obvious artifacts. The ellipse-shaped image area at the point-like scatterers of the 51-element full array is the biggest, which indicates that it has the lowest lateral resolution within the three arrays. Interestingly, The imaging quality of the optimized sparse array with three point-like scatterers is also very great, the artifacts near the defect points are significantly reduced, and the scatter outline is visible clearly. Some 1D patterns extracted from Fig.7 are plotted in Fig. 8.
FIGURE 8. The simulated imaging intensity distribution in the lateral direction in comparison to that with the full array.

which are distributed along the lateral direction and crossing
the mainlobe peak point.

Peak to Centre Intensity Difference represents the level of
drop in the x-direction at the depth of the z-axis
where two adjacent scatterers are located in the image, which
is recorded as $\Delta A$. In Fig.8, we found that the horizontal
width of the mainlobe of three arrays is very close to each
other, but the $\Delta A$ of the optimized sparse array is minimum,
which indicates that two adjacent scatterers can be better
distinguished. And the sparse array has the lowest side beams
of three array, so there are fewer imaging artifacts and a higher
SNR.

C. THE PERFORMANCE OF DIFFERENT ARRAY

To study the generality of the proposed method, we calculated
the imaging performance of several one-dimensional full
arrays with different numbers of array elements, corresponding
optimized sparse arrays, and the full array with the same
number of array elements as the optimized sparse arrays. The
array parameters are the same as those of the above example
array. The time-consuming is measuring with the processor
identified by Intel(R) Core(TM) i7-8750 CPU @2.20GHz
and the runtime environment is MATLAB R2015b. The
results are shown in Table 2, which can be summarized as
follows.

1). Although the number of elements in the reference full
array is different, the optimized sparse array always has a
lower MSL and higher SNR than that of the corresponding
reference full array. When the number of array elements is
the same, compared with the full array, the sparse array has
lower MSL, lower API, and higher SNR.

2). The run time consumed by FMC/TFM simulation is
proportional to the number of array elements. The sparse
array has less time than the reference full array.

3). The total size of the active elements of the optimized
sparse array is about 0.7 of the reference full array aperture.

4). In general, as the number of elements of the full array
increases, both the API and the SNR are decreases. However,
through optimization by BPSO, the imaging SNR can be
improved to a certain extent for each reference full array.

In short, no matter how many the number of elements
in the reference full array, the optimized sparse array can
improve the imaging SNR. Although the imaging resolution
of the array is slightly reduced after optimization, the imaging
efficiency can be improved, the method is feasible.

IV. CONCLUSION

This paper introduces a design method of the sparse array.
Based on the reference full array, the BPSO is used to
automatically sparsify the array by calculating phased array
patterns. Finally, the array is sparse to a suitable level without reducing the sound field characteristics. On the whole, the optimized sparse array has better performance than the reference full array. The ultrasonic imaging results by TFM indicate that the optimized sparse array not only has better imaging quality than the reference full array but also can improve the FMC/TFM efficiency. The method has certain universality and has certain reference values for the design of the phased array system. Future experimental validation will be performed to verify the imaging performance of sparse arrays. Further, apply this method to the optimal design of various phased array systems.

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