An aerodynamic surrogate model of launch vehicle based on relevance vector machine

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ABSTRACT. In the process of launch vehicle multidisciplinary design optimization, aerodynamic calculation takes a long time, which affects the overall design cycle. In order to solve the above problems, based on the idea of machine learning, this paper constructs the surrogate model of relevance vector machine and calculates the aerodynamic coefficients of launch vehicles quickly. Firstly, the aerodynamic model of launch vehicle is established, and the orthogonal design method is used to generate test sample points. Then, the aerodynamic coefficients of the sample points are calculated by using Fluent software, and the training data of the surrogate model are obtained. On this basis, the relevance vector machine model is trained with training data, generating correlation vector machine agent model. Finally, the calculation accuracy of the surrogate model is evaluated by simulation, and the feasibility and validity of the method are verified.

1. Introduction
In the overall design process of the launch vehicle, it is necessary to consider the design schemes of multiple disciplines such as ballistics, aerodynamics, attitude control, dynamics, structure, load, cost analysis, and their mutual coupling. It is a very complex multi-disciplinary modeling and process of searching optimization analysis[1]. Aerodynamic design is an important part of the overall design of the launch vehicle. It not only affects the aerodynamic layout and overall program selection of the launch vehicle, but also is the original input conditions for the design of multiple disciplines and sub-systems, such as structural design, attitude control design, and engine design [2]. If the aerodynamic characteristics of the launch vehicle cannot meet the design requirements, the shape of launch vehicle will be redesigned or adjusted. In the subsequent design stage of the model, the pros and cons of aerodynamic characteristics directly reflect the overall level of launch vehicle performance. Therefore, aerodynamic design and evaluation run through the entire process of launch vehicle design and demonstration.

In the early aerodynamic design, wind tunnel tests were basically the only means to obtain the reliable aerodynamic data. The "trial and error method" was used by the designer to obtain a series of different shapes, and then optimize the best one after a large number of wind tunnel tests on these shapes. With the rapid development of Computation Fluid Dynamics (CFD) and the rapid improvement of computer performance, CFD numerical simulation is widely used. However, in the overall design process of the launch vehicle, the efficiency of the iterative optimization of the overall small loop was seriously affected by the longer counting period, if the CFD calculation was directly used by aerodynamics major.
In order to solve the above problems, an effective method is used which is called the surrogate model. Under the premise of ensuring accuracy, an approximate analysis model with a small amount of calculation is constructed. The calculation results of this model can approximately replace the results of CFD calculations or physical test results of aerodynamics, which is aimed at significantly reducing the calculation cycle under the premise of ensuring a certain accuracy.

2. Relevance vector machine

The approximation methods of surrogate model include response surface model, radial basis function model, Kriging model, neural network model, etc. The surrogate model has been widely valued in aerospace and other fields because of the advantage of improving the efficiency of engineering optimization problems based on high-confidence numerical simulation analysis[3]. The dynamic Kriging model was applied to the multi-objective optimization problem with constraints by Shinkyu Jeong[4] and others. Han[5][6] proposed a variety of improved surrogate models and applied them to aerodynamic shape optimization. Yuchao Li[7] studied the application of surrogate model in multidisciplinary design optimization of transport aircraft wings. Zeping Wu[8] et al. designed the solid launch vehicle motor charge based on the surrogate model.

The correlation vector machine was proposed by Tipping[9] in 2000. It is a nonlinear probability model based on Bayesian theory, and trained in a Bayesian framework, under the structure of prior parameters, the sparse model was obtained by removing the irrelevant points based on ARD (Automatic Relevance Determination). The relevance vector machine and the support vector machine have the same functional form, the kernel function used in the model does not need to meet the Mercer conditions, thus reducing the number of kernel involved in prediction calculation. Because of the sparsity of the algorithm and the learning characteristics based on probability, it reduces time of forecasting calculation, and can provide probabilistic forecasts.

Let \( \{ x_i \}_{i=1}^{N} \) is a set of training sample points, \( t = \begin{bmatrix} t_1, t_2, \ldots, t_N \end{bmatrix} \) is the response value of the sample points, the correlation vector machine model can be expressed as:

\[
\begin{align*}
t_i &= \sum_{i=1}^{N} \omega_i \phi(x, x_i) + \omega_0 + \epsilon_i \\
&= y(x_i; w) + \epsilon_i \\
&= w^T \Phi + \epsilon_i
\end{align*}
\]

Set \( t_i \) as the predicted response value, \( x \) is the input sample point vector, \( \omega_i \) is the weight of the sample, \( w = [\omega_0, \omega_1, \omega_2, \ldots, \omega_N] \) is the weight vector corresponding to the sample, and \( \Phi = \begin{bmatrix} \Phi(x_1), \Phi(x_2), \ldots, \Phi(x_N) \end{bmatrix} \) is the kernel function vector. Assuming that there is noise \( \epsilon_i \) and with a mean value of 0, the variance \( \sigma^2 \) obeys a Gaussian distribution, the probability density of \( t_i \) is:

\[
p(t_i) = N(t_i | y(x_i; w), \sigma^2)
\]

From the Bayesian formula, the posterior probability of \( w \) and \( t \) is:

\[
p(w | t, \alpha, \sigma^2) = (2\pi)^{\frac{N}{2}} | \Omega |^{-\frac{1}{2}} \exp \left\{ -\frac{1}{2} (w - \mu)^T \Omega^{-1} (w - \mu) \right\}
\]

\[
p(t | \alpha, \sigma^2) = (2\pi)^{\frac{N}{2}} | \Omega |^{-\frac{1}{2}} \exp \left\{ -\frac{1}{2} t^T \Omega^{-1} t \right\}
\]

Where:
\begin{equation}
\begin{aligned}
\Gamma &= \left( \sigma^2 \Phi^T \Phi + A \right)^{-1} \\
\mu &= \sigma^2 \Gamma \Phi^T t \\
A &= \text{diag} \left( \alpha_0, \alpha_1, \alpha_2, \ldots, \alpha_N \right) \\
\Omega &= \sigma^2 I + \Phi A^{-1} \Phi^T 
\end{aligned}
\end{equation}

using the maximum likelihood method and taking logarithms on both sides, we can get:

\begin{equation}
L(\alpha) = \log p(t | \alpha, \sigma^2) = -\frac{1}{2} \left[ N \left( 2\pi \right) + \log |\Omega| + t^T \Omega^{-1} t \right]
\end{equation}

Use the iterative method to find the point estimates of \( \alpha \) and \( \sigma^2 \), give the initial values of \( \alpha \) and \( \sigma^2 \), and then update it with the following formula:

\begin{equation}
\alpha_{i}^\text{new} = \frac{1}{\Gamma_{ii}} + \mu_{i}^2 
\end{equation}

\begin{equation}
\left( \sigma^2 \right)^\text{new} = \frac{1}{N - \sum_{i=0}^{N} \gamma_{i}} \left| t - \Phi \mu \right| 
\end{equation}

\begin{equation}
\gamma_{i} = 1 - \alpha_{i} \Gamma_{ii} 
\end{equation}

After the number of updates reaches a certain level, most of \( \alpha_{i} \) will approach to infinity, which corresponds \( \omega_{i} \) to 0, and the remaining few \( \alpha_{i} \) will close to finite values, and the corresponding \( x_{i} \) is the correlation vector. Calculate the correlation vector \( x_{i} \) corresponding to \( \omega_{i} \), then the expression of the correlation vector machine model can be obtained.

3. The example of correlation vector machine surrogate model

3.1 Process of building

The steps of constructing the launch vehicle aerodynamic correlation vector machine surrogate model can be carried out as follows:

1) Experimental design: Determine the variables and space of design, establish the launch vehicle aerodynamic shape and geometric model, and complete the CFD modeling in the CFD calculation platform; use the Design of Experiment (DOE) method to generate a set of design test points; The CFD calculation was carried out with Fluent and other software to obtain the response value of the test point, such as the resistance coefficient, etc.;

2) Model training: Use the test sample points and their response values to select the appropriate correlation vector machine model parameters, and the function expression is the surrogate model we need;

3) Accuracy evaluation: Randomly generate a number of test points in the design space which are different from the elements in the test point set, use the CFD model and the surrogate model to calculate the relevant aerodynamic coefficients, and compare the root mean square error (RMSE) of the two calculation results, which is used for testing whether the calculation accuracy of the surrogate model can meet the requirements.

3.2 Experimental design

The research object of this article considers a launch vehicle with two boosters. The aerodynamic shape diagram of the launch vehicle is shown in Figure 1.
The total arrow aerodynamic drag coefficient of launch vehicle is used to multi-disciplinary optimization design. According to the characteristics of the launch vehicle, 4 parameters are used as the variate in aerodynamic shape design, such as: the diameter of the fairing column DT, the length of the fairing column LT, the length of the launch vehicle body LX and the Booster column diameter DZ. On the basis of the coupling relationship between aerodynamics and ballistics and other disciplines, the Mach number Ma and the angle of attack $\alpha$ are selected as input design variables. The unit of DT, DZ, LT and LX is mm, the unit of angle of attack $\alpha$ is angle, and the Mach number is a dimensionless number. Considering that the number of calculations required for the orthogonal experiment design is less, and the generated test point set has characteristics of orthogonality, dispersible uniformity and comprehensive comparability, 5 levels of design variables are selected, and the orthogonal method is used for the experimental design, L25 (5^6) orthogonal table with 6 factor 5 level were applied to generate 25 test sample points. The design space table and the experimental design table are shown in Table 1 and Table 2.

**Table 1 Design variables, spaces and levels**

| Design variable | Lower bound | Upper bound | Number of levels | Horizontal value |
|-----------------|-------------|-------------|------------------|------------------|
| DT              | 3598        | 4010        | 5                | 3598/3701/3804/3907/4010 |
| DZ              | 1998        | 2314        | 5                | 1998/2077/2156/2235/2314 |
| LT              | 3510        | 4314        | 5                | 3510/3711/3912/4113/4314 |
| LX              | 41204       | 42016       | 5                | 41204/41407/41610/41813/42016 |
| Ma              | 0.4         | 0.8         | 5                | 0/0.4/1.45/2.5/3.55/4.6 |
| $\alpha$        | 0           | 8           | 5                | 0/2/4/6/8  |

**Table 2 Orthogonal table of experimental design**

| Number | DT(mm) | DZ(mm) | LT(mm) | LX(mm) | Ma | $\alpha$ (°) |
|--------|--------|--------|--------|--------|----|--------------|
| 1      | 3598   | 1998   | 3510   | 41204  | 0.4| 0            |
| 2      | 3598   | 2077   | 3711   | 41407  | 1.45| 2            |
| 3      | 3598   | 2156   | 3912   | 41610  | 2.5| 4            |
| 4      | 3598   | 2235   | 4113   | 41813  | 3.55| 6            |
| 5      | 3598   | 2314   | 4314   | 42016  | 4.6| 8            |
| 6      | 3701   | 2156   | 3510   | 41407  | 3.55| 8            |
| 7      | 3701   | 2235   | 3711   | 41610  | 4.6| 0            |
| 8      | 3701   | 2314   | 3912   | 41813  | 0.4| 2            |
| 9      | 3701   | 1998   | 4113   | 42016  | 1.45| 4            |
| 10     | 3701   | 2077   | 4314   | 41204  | 2.5| 6            |
| 11     | 3804   | 2314   | 3510   | 41610  | 1.45| 6            |
| 12     | 3804   | 1998   | 3711   | 41813  | 2.5| 8            |
| 13     | 3804   | 2077   | 3912   | 42016  | 3.55| 0            |
| 14     | 3804   | 2156   | 4113   | 41204  | 4.6| 2            |
| 15     | 3804   | 2235   | 4314   | 41407  | 0.4| 4            |
| 16     | 3907   | 2077   | 3510   | 41813  | 4.6| 4            |
| 17     | 3907   | 2156   | 3711   | 42016  | 0.4| 6            |
| 18     | 3907   | 2235   | 3912   | 41204  | 1.45| 8            |
| 19     | 3907   | 2314   | 4113   | 41407  | 2.5| 0            |
| 20     | 3907   | 1998   | 4314   | 41610  | 3.55| 2            |
| 21     | 4010   | 2235   | 3510   | 42016  | 2.5| 2            |
| 22     | 4010   | 2314   | 3711   | 41204  | 3.55| 4            |
| 23     | 4010   | 1998   | 3912   | 41407  | 4.6| 6            |
| 24     | 4010   | 2077   | 4113   | 41610  | 0.4| 8            |
| 25     | 4010   | 2156   | 4314   | 41813  | 1.45| 0            |
Taking the set of test sample point as input, using Fluent software to calculate the drag coefficient of the full launch vehicle corresponding to each set of sample points. The calculation results are shown in Table 3.

| Number | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 | 21 | 22 | 23 | 24 | 25 |
|--------|---|---|---|---|---|---|---|---|---|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| Coefficient | 1.19 | 1.63 | 1.19 | 0.97 | 0.85 | 0.99 | 0.78 | 1.24 | 1.61 | 1.61 | 1.97 | 1.23 | 0.92 | 0.79 | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 | 1.25 |

3.3 Model training

The set of experimental design sample points is \( \{(x_i, y_i)\}_{i=1}^l \), \( x_i \in \mathbb{R}^k \), where \( x_i \) is a 6-dimensional real number, which corresponds to the 6 design variables, such as \( DT, DZ, LT, LX, Ma, \) and \( \alpha \), where \( y_i \) is the resistance coefficient, and \( l = 25 \) is the number of experimental sample points. The regression function of the correlation vector machine is in the form:

\[
 f(x) = \sum_{i=1}^l (\alpha_i - \alpha_i^*) k(x_i, x_j) + b \tag{10}
\]

The drag coefficient of the whole arrow is used as the output vector, and the Gauss kernel is served as the kernel function to iteratively solve the point estimation of \( \alpha \) and \( \sigma^2 \) in the correlation vector machine model. The trained model is the desired surrogate model.

3.4 Accuracy evaluation

Randomly select 10 points different from the sample points in the design space as the test sample points, as shown in Table 4.

| Number | D1 | D2 | L1 | L2 | Ma | \( \alpha \) |
|--------|----|----|----|----|----|--------|
| 1      | 3618 | 2132 | 3854 | 41815 | 0.8 | 4.5    |
| 2      | 3902 | 2250 | 3516 | 41517 | 1.6 | 3.9    |
| 3      | 3697 | 2231 | 3765 | 41847 | 3   | 2.1    |
| 4      | 3777 | 2050 | 3840 | 41804 | 3.3 | 4.3    |
| 5      | 3875 | 2260 | 3716 | 41502 | 3.2 | 1.6    |
| 6      | 3744 | 2297 | 3657 | 41373 | 1.3 | 0.5    |
| 7      | 3895 | 2154 | 4157 | 41832 | 2.1 | 5.7    |
| 8      | 3758 | 2265 | 3844 | 41960 | 0.4 | 2.9    |
| 9      | 3873 | 2176 | 4210 | 41462 | 0.4 | 6.6    |
| 10     | 3882 | 2046 | 3813 | 41740 | 1.5 | 5.1    |

Taking the test samples as input, using commercial software, calculate the resistance coefficient of each group of test sample points, and the results are shown in Table 5.

| Number | Resistance coefficient |
|--------|------------------------|
| 1      | 1.0856                 |
| 2      | 1.3431                 |
| 3      | 0.7852                 |
| 4      | 0.7371                 |
| 5      | 0.7675                 |
| 6      | 1.4185                 |
| 7      | 1.0587                 |
| 8      | 0.5842                 |
| 9      | 0.8742                 |
| 10     | 1.2875                 |
The trained relevance vector machine surrogate model is used to predict the response value of the test sample points, and calculate the prediction error between the surrogate model calculation result and the CFD model calculation result, RSME=0.0204. The error range of the correlation vector machine for predicting the aerodynamic drag coefficient meets the accuracy requirements in the launch vehicle aerodynamic design. The calculation efficiency of the aerodynamic surrogate model of the correlation vector machine is very high, and the single calculation time is only 0.0035s, which fully meets the requirements of the iterative calculation of the multidisciplinary optimization design of the launch vehicle.

4. Conclusion
The speed of calculation of aerodynamic coefficients by the launch vehicle aerodynamics discipline using the relevant vector machine surrogate model is much higher than that of the CFD calculation, which provides an effective way for the aerodynamics discipline to integrate into the overall MDO of the launch vehicle. Taking the calculation of aerodynamic drag coefficient of a certain launch vehicle as an example, this paper discusses the construction process of aerodynamic surrogate model based on correlation vector machine. The generated surrogate model meets the requirements of sample prediction accuracy and can be used in the overall design of the launch vehicle.

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