Topological gravity on plumbed V-cobordisms

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Abstract
An ensemble of cosmological models based on generalized BF-theory is constructed where the role of vacuum (zero-level) coupling constants is played by topologically invariant rational intersection forms (cosmological-constant matrices) of four-dimensional plumbed V-cobordisms which are interpreted as Euclidean spacetime regions. For these regions describing topology changes, the rational and integer intersection matrices are calculated. A relation is found between the hierarchy of certain elements of these matrices and the hierarchy of coupling constants of the universal (low-energy) interactions.

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1. Introduction

The problem of topology and signature changes has been discussed from different viewpoints in both classical and quantum gravity (see, e.g., [1, 2]). From the conceptual point of view, the future quantum theory of gravitation and unification theories are dependent on the resolution of this problem. In late 1980s the hopes of solution of the cosmological-constant problem were related to the theory of topology changes [3–6]; similarly the problem of fixing other fundamental constants of nature was posed [7, 8]. Here we undertake an attempt to solve the problem of coupling constants’ hierarchy on the basis of topological invariants of four-dimensional cobordisms describing topological changes between vacuum states in the instanton-like models. The gravitational (cosmological) instanton is regarded as a region of spacetime with the Euclidean signature (+++). In other words, we describe topology changes as a quantum tunnelling effect from initial \((M_{\text{in}})\) to final \((M_{\text{out}})\) three-dimensional sections of the Euclidean spacetime region. The topology change really takes place when the three-dimensional topological spaces \(M_{\text{in}}\) and \(M_{\text{out}}\) are not homeomorphic. These spaces may consist of a finite number of disconnected components. In mathematical terms, the topological change...
is naturally described by means of a cobordism [9–11]. If the boundary of a compact four-
dimensional topological space $X$, $\partial X$, is a disjoint sum of two three-dimensional topological
spaces $M_{\text{in}}$ and $M_{\text{out}}$, the 3-tuple $(X, M_{\text{in}}, M_{\text{out}})$ is called a cobordism [12]. Both $M_{\text{in}}$ and $M_{\text{out}}$ are often merely implied, so the very topological space $X$ is called a cobordism. In our
model the plumbed V-cobordism (pV-cobordism), a well-known algebraic topology type of
cobordism, is used; see [13–18].

In this paper, we intend to develop a purely topological approach to unification of
(pre-)interactions (section 4.2) shifting the complexity of description of the fundamental
interactions to the non-trivial topology of the Euclidean spacetime regions (pV-cobordisms),
leaving the gauge group as Abelian, $U(1) \times \cdots \times U(1)$, thus only its rank is being changed
while different versions of modified BF-theory are being proposed (see section 3). Thereby in
our model the topology changes are primarily related to changes of the number of fundamental
(pre-)interactions and to changes of the respective coupling constants. The information about
the latter ones is contained (as we guess) in the basic topological invariants of pV-cobordisms,
namely in the intersection forms of two-dimensional homology classes (intersection forms are
four-dimensional analogues of linking matrices for 3-manifolds [19]).

The pV-cobordisms we are dealing with are bounded by $\mathbb{Z}$-homology spheres and a
disjoint set of lens spaces. These 3-manifolds possess topological invariants with non-trivial
numerical values, and at the same time they are next of the ‘topological kin’ of the ordinary
3-sphere $S^3$. The $\mathbb{Z}$-homology spheres are spliced of more elementary objects, the Seifert
fibred homology spheres (Sfh-spheres). Both they and lens spaces are represented as Seifert
fibrations into circles $S^1$ over 2-sphere $S^2$ (a generalization of the well-known Hopf fibration).
Another description of a Seifert fibred manifold is as follows: a closed oriented 3-manifold
admitting a fixed point free action of $S^1$ or equivalently $U(1)$ [16]. This Seifert structure
permits us to build natural observables in terms of the Wilson loops, since a Seifert manifold
is represented as a union of pairwise disjoint simple closed curves called fibres.

The principal motivation for the present paper is to relate some results in low-dimensional
topology to BF-theory [20, 21] also known as topological gravity [22, 23]. Clearly speaking,
we are trying to realize an analogue of the classical Abelian BF-model on pV-cobordisms.
In addition to the usual characteristics of topological field theories (the absence of local
degrees of freedom, finite dimensionality of phase space, etc), in our model as analogues of
cosmological (coupling) constants there appear rational intersection forms (matrices) which
are the basic topological invariants of pV-cobordisms [17, 18, 24]. Thus in our model the
idea that the primary values of coupling constants (which correspond to a vacuum without any
local excitations) are to be looked for at the topological level, is realized.

The paper is organized as follows. Section 2 starts with a short review of the main
ideas concerning plumbed V-cobordisms and V-manifolds. We reproduce the definitions of
basic topological invariants of these spaces, specifically, the rational and integer intersection
matrices. We also formulate the important result [17] concerning reciprocity (the mutual
inverse property) of these matrices written in natural bases.

Section 3 is dedicated to the construction of Abelian BFE-models on plumbed
V-cobordisms. We show that these models possess an essentially new property in comparison
with the usual BF-theories: the role of vacuum (zero-level) coupling constants is played by
the topologically invariant intersection forms of plumbed V-cobordisms.

In section 4 we give a collection of non-trivial examples which realize the ideas proposed
in section 3. As a starting point, we take the primary sequence of three-dimensional Sfh-
spheres characterized by Seifert invariants constructed from the first nine prime numbers
$p_1 = 2, p_2 = 3, \ldots, p_9 = 23$. With the help of derivative operation acting on Sfh-spheres,
as introduced by us, we construct the basic 2-parametric family of Sfh-spheres $\Sigma_{n}^{(\theta)}$ which
by means of the well-known splicing operation are pasted together into \( \mathbb{Z} \)-homology spheres \( M(l)^n \). These latter turn out to be the boundary components of four-dimensional plumbed V-cobordisms \( X_{D^n}^{(l)} \) together with a specific collection of lens spaces. Just on this set of cobordisms we construct the respective BFE-systems \( S(l)^n \) with cosmological coupling matrices being, as we have proven, the representatives of rational intersection forms of \( X_{D^n}^{(l)} \). Namely, these V-cobordisms can be considered as construction units in gluing together the compound V-cobordisms which describe topology changes \( M(l)^n \rightarrow M(l)^{n+1} \) between \( \mathbb{Z} \)-homology spheres (this is sketched in section 5).

Further, we give the results of calculation of both rational and integer intersection matrices for a finite ensemble of plumbed V-cobordisms \( X_{D^n}^{(l)} \) with \( n, l = 0, \ldots, 4 \). So we find a relation between the diagonal elements of the integer intersection matrices and Euler numbers of the basic family of \( S^3 \)-spheres \( \Sigma_{1}^{(l)} \). These Euler numbers fairly well reproduce the hierarchy of the dimensionless low-energy coupling constants.

2. Plumbed four- and three-dimensional manifolds

In order to build a new version of the Abelian BF-theory on four-dimensional pV-manifolds and pV-cobordisms whose boundaries are \( \mathbb{Z} \)-homology spheres and lens spaces, we first give some necessary definitions essentially following the works of Saveliev [18, 25]. These ideas can be traced back to [14–16, 26].

2.1. Plumbing and splicing operations

A plumbing graph \( \Gamma \) is a graph with no cycles (a finite tree) each vertex \( v_i \) of which carries an integer weight \( e_i,i = 1, \ldots, s \). With each vertex \( v_i \) a \( D^2 \)-bundle \( Y(e_i) \) over \( S^2 \) is associated, whose Euler class (the self-intersection number of zero-section) is \( e_i \). If the vertex \( v_i \) has \( d_i \) edges connected to it on the graph \( \Gamma \), choose \( d_i \) disjoint discs in the base \( S^2 \) of \( Y(e_i) \) and call the disc bundle over the \( j \)-th disc \( B_{ij} = (D^2_j \times D^2) \). When two vertices \( v_i \) and \( v_k \) are connected by an edge, the disc bundles \( B_{ij} \) and \( B_{kl} \) should be identified by exchanging the base and fibre coordinates [15]. This pasting operation is called plumbing, and the resulting smooth 4-manifold \( P(\Gamma) \) is known as a plumbed 4-manifold. Its boundary \( M(\Gamma) = \partial P(\Gamma) \) is referred to as a plumbed 3-manifold.

Since the homology group \( H_1(P(\Gamma), \mathbb{Z}) = 0 \), the unique non-trivial homology characteristic is \( H_2(P(\Gamma), \mathbb{Z}) \) which has a natural basis (a set of generators) represented by the zero-sections of the plumbed bundles. All these sections are embedded 2-spheres \( z_a \) where \( a = 1, \ldots, r = \text{rank} \ H_2(P(\Gamma), \mathbb{Z}) \), and they can be oriented in such a way that the intersection (bilinear) form [25]

\[
A: H_2(P(\Gamma), \mathbb{Z}) \otimes H_2(P(\Gamma), \mathbb{Z}) \rightarrow \mathbb{Z} \tag{2.1}
\]

will be represented by the \( r \times r \)-matrix \( A(\Gamma) = (a_{ij}) \) with the following entries: \( a_{ij} = e_i \) if \( i = j \); \( a_{ij} = 1 \) if the vertex \( v_i \) is connected to \( v_j \) by an edge; and \( a_{ij} = 0 \) otherwise.

Let \( M \) be a Seifert fibred 3-manifold (\( S^1 \)-fibration) over \( S^2 \) with unnormalized Seifert invariants [26] \( (a_i, b_i), i = 1, \ldots, n, a_i > 1 \). It can be obtained as the boundary of the plumbed 4-manifold \( P(\Gamma) \) where \( \Gamma \) is a star graph shown in figure 1 [15].
The integer weights $t_{ij}$ in this graph are found from continued fractions $a_i/b_i = [t_{i1}, \ldots, t_{im}];$ here

$$[t_1, \ldots, t_k] = t_1 - \frac{1}{t_2 - \frac{1}{\ldots - \frac{1}{t_k}}}.$$ 

Note that among closed 3-manifolds $\mathbb{Z}$-homology spheres $M$ are characterized by $H_1(M, \mathbb{Z}) = 0.$

A Seifert fibred 3-manifold $M$ is a $\mathbb{Z}$-homology sphere (Sfh-sphere) iff the determinant of matrix $A(\Gamma)$ associated with the plumbing graph in figure 1, is $\pm 1$ which is equivalent to the condition

$$a \sum b_i/a_i = \mp 1 \quad (2.2)$$

where $a = a_1 \cdots a_n$. We fix an orientation on $M$ by choosing $+1$ in (2.2). From (2.2) it follows that $b_i a_i = 1 \mod a_i$ where $a_i = a/a_i$; thus the set of $a_i$ proves to be sufficient for the complete determination of any Sfh-sphere. This is why we use the standard notation $\Sigma(a_1, \ldots, a_n)$ for Sfh-spheres.

Lens spaces represent another case of Seifert fibred manifolds. Expanding $-p/q = [t_1, \ldots, t_n]$ into a continued fraction, we encounter $L(p, q)$ as a boundary of the 4-manifold obtained by plumbing on the chain $\Gamma$ shown in figure 2.

Note that this plumbing graph simultaneously represents the lens space $L(p, q^*)$ with $-p/q^* = [t_n, \ldots, t_1]$ where $qq^* = 1 \mod p$. This reflects the fact that $L(p, q)$ and $L(p, q^*)$ are homeomorphic.

In section 4 we shall build universe models which are plumbed 4-manifolds (cobordisms) with boundaries whose components are homeomorphic to lens spaces or to $\mathbb{Z}$-homology spheres, the latter being constructed by splicing certain set of Sfh-spheres with three exceptional fibres, i.e., $\Sigma(a_1, a_2, a_3)$. Therefore it is worth giving now the general definition of the splicing operation as well as the plumbed V-cobordism.

By a link [16] $(\Sigma, K) = (\Sigma, S_1 \cup \cdots \cup S_n)$ we mean a pair consisting of an oriented $\mathbb{Z}$-homology sphere $\Sigma$ and a collection $K$ of disjoint oriented knots $S_1, \ldots, S_n$ in $\Sigma$. Empty links are just $\mathbb{Z}$-homology spheres. Note that the links $(S^3, K)$ where $S^3$ is an ordinary 3-sphere, are also allowed. If the link components $S_1, \ldots, S_n$ are fibres in $\Sigma$, then the link $(\Sigma, K)$ is called a Seifert link. Let $(\Sigma, K)$ and $(\Sigma', K')$ be links and choose components $S \in K$ and $S' \in K'. \quad$ Also let $N(S)$ and $N(S')$ be their tubular neighbourhoods, while $m, l \subset \partial N(S)$ and $m', l' \subset \partial N(S')$ are standard meridians and longitudes. The manifold $\Sigma'' = (\Sigma \setminus \text{int} N(S)) \cup (\Sigma' \setminus \text{int} N(S'))$ obtained by pasting along the torus boundaries by matching $m$ to $l'$ and $m'$ to $l$, is a $\mathbb{Z}$-homology sphere. The link $\left(\Sigma'', (K \setminus S) \cup (K' \setminus S')\right)$ is called the splice (splicing) of $(\Sigma, K)$ and $(\Sigma', K')$ along $S$ and $S'$. We shall use the standard notation $\Sigma'' = \Sigma_{372} \Sigma'$ or simply $\Sigma'' = \Sigma - \Sigma'$.
number of Seifert links by splicing is called a graph link. Empty graph links are precisely the plumbed (graph) $\mathbb{Z}$-homology spheres.

A plumbed graph $\Gamma$ with added arrowhead vertices, denoted as $\overline{\Gamma}$, represents a link $K$ in a (plumbed) 3-manifold $M = M(\overline{\Gamma})$ as follows: let $\Gamma$ be $\overline{\Gamma}$ with all the arrows deleted, and put $M = \partial P(\Gamma)$. Each arrowhead vertex $v_j$ of $\overline{\Gamma}$ is attached at some vertex of $\Gamma$, and with this arrow we associate a fibre $S_j$ of the bundle $Y(e_i)$ used in the plumbing [16, 18].

The splicing operation on graph links can be described in terms of plumbing graphs as follows. Suppose that two graph links are represented by their plumbing diagrams $\Gamma$ and $\Gamma'$ (see figure 3) with the arrows attached to vertices $e_n$ and $e'_m$, respectively. The corresponding plumbing diagram for a spliced link is shown in figure 4 where $a = \frac{\det A(\Gamma_0)}{\det A(\Gamma)}$, while $\Gamma_0$ is the plumbing graph $\overline{\Gamma}$ with the arrow deleted, and $\Gamma_0$ is a portion of $\Gamma$ obtained by removing the $n$th vertex weighted by $e_n$ as well as all its adjacent edges. Another integer $a'$ is similarly obtained from the graph $\Gamma'$ (for examples, see [16, 18]). The above description of splicing in terms of plumbing graphs makes it possible to treat splicing as an operation on the corresponding plumbed 4-manifold; moreover, $M(\overline{\Gamma}) = \partial P(\Gamma)$.

2.2. Plumbed V-cobordisms and V-manifolds

A plumbed V-cobordism (pV-cobordism) is called a plumbed 4-manifold when it is a cobordism between a plumbed 3-manifold and a disjoint union of lens spaces. pV-cobordisms may be considered as models of elementary topology changes. They are constructed as follows.

Let $P(\Gamma)$ be a plumbed 4-manifold corresponding to graph $\Gamma$, and $\Gamma^{ch}$ be a chain in $\Gamma$ of the form shown in figure 2. Plumbing on $\Gamma^{ch}$ yields a submanifold $P(\Gamma^{ch})$ of $P(\Gamma)$ whose boundary is a lens space $L(p, q)$. The closure of $P(\Gamma) \setminus P(\Gamma^{ch})$ is a smooth compact 4-manifold with oriented boundary $-L(p, q) \cup \partial P(\Gamma)$ [18] where $\cup$ denotes the disjoint sum operation. Starting with several chains $\Gamma_i^{ch}$ ($i = 1, I$) in $\Gamma$ (where $0, I$ is the integer number interval from 0 to $I$), one can introduce a cobordism $X_D$ between $M := \partial P(\Gamma)$ and the disjoint union $L = \bigsqcup_{i=1}^I L(p_i, q_i)$ of several lens spaces. The chains $\{\Gamma_i^{ch}\}$ must be disjoint in the following sense: no two chains should have a common vertex, and no edges of $\Gamma$ should have one endpoint on one chain and another endpoint on some other chain. Such cobordisms will be called pV-cobordisms. A pV-cobordism is always a smooth manifold. The notation ‘V-’ refers to the fact that each lens space $L(p_i, q_i)$ on the boundary of $X_D$ may be eliminated by pasting a cone $c_i L(p_i, q_i)$ over the lens space ($c_i$ is the vertex of this cone). This yields the
well-known V-manifold [13]
\[ X = X_D \bigsqcup_{i=1}^{l} c_i L(p_i, q_i) \] (2.3)
with isolated singular points. V-manifolds of the type X are called \textit{plumbed V-manifolds} (pV-manifolds). These topological spaces are also known as \textit{pseudo-free orbifolds} [24].

A pV-cobordism \( XD \) can be adequately represented by the so-called decorated plumbing graph \( \Gamma_D \) [18] (this explains the subindex \( D \)). Such graphs are decorated with extra ovals (or circles), each enclosing exactly one chain \( \Gamma_i^{ch} \) (or a vertex as a particular case of the chain). (The above conditions on the chains \( \Gamma_i^{ch} \) to be disjoint translate into the following conditions on the decorating ovals: any two 2-discs bounded by decorating ovals are disjoint, and no edge of \( \Gamma_D \) is intersected by more than one decorating oval.) Examples are considered in section 4.

From the viewpoint of physics, the cobordism \( XD \) with boundary \( \partial XD = -L \bigsqcup M \) can be treated as a topology change from the disjoint union \( L \) of lens spaces to the \( \mathbb{Z} \)-homology sphere \( M \). Then the V-manifold \( X \) may be interpreted as a topology change from the finite set \( \{ c_i | i \in [1,I] \} \) of singular points (say, big bangs) to the \( \mathbb{Z} \)-homology sphere \( M \). It means that to create a universe with the \( \mathbb{Z} \)-homology sphere as its spatial section, one needs at least three ‘big bangs’ described by cones of type \( c_i L(p_i, q_i) \).

### 2.3. Integral and rational intersection forms of pV-cobordisms

Let \( XD \) be a pV-cobordism with the boundary
\[ \partial XD = -L \bigsqcup M \]
where \( M \) is a \( \mathbb{Z} \)-homology sphere. In this case \( H^1(XD, \mathbb{Z}) = 0 \), and there exists the exact sequence [24, 19, 27]:
\[ 0 \rightarrow H^2(XD, \partial XD, \mathbb{Z}) \xrightarrow{i^*} H^2(XD, \mathbb{Z}) \xrightarrow{j^*} H^2(\partial XD, \mathbb{Z}) = \bigoplus_{i=1}^{l} \mathbb{Z}_{p_i} \rightarrow 0. \] (2.4)

Note that the relative cohomology group \( H^2(XD, \partial XD, \mathbb{Z}) \) is isomorphic to the \( H^2(X, \mathbb{Z}) \) where the pV-manifold is defined in (2.3).

As a consequence of the Poincaré–Lefschetz duality, the integral intersection form can be defined as
\[ \omega_{\mathbb{Z}} : H^2(XD, \partial XD, \mathbb{Z}) \otimes H^2(XD, \mathbb{Z}) \rightarrow \mathbb{Z} \] (2.5)
by means of the cup product pairing [19]
\[ \omega_{\mathbb{Z}}(b, e) = \langle b \cup e, [XD, \partial XD] \rangle \] (2.6)
for each \( b \in H^2(XD, \partial XD, \mathbb{Z}) \) and \( e \in H^2(XD, \mathbb{Z}) \). The operation \( \cup \) is known as the cup product, and \([XD, \partial XD]\) is the relative fundamental class [27, 24]. Note that the intersection form \( \omega_{\mathbb{Z}} \) becomes non-degenerate after factoring out the torsion subgroup \( \text{Tor}(H^2(XD, \mathbb{Z})) \) [19].

From the exactness of the sequence (2.4) and since \( H^2(\partial XD, \mathbb{Z}) \) is a pure torsion, for any \( e' \in H^2(XD, \mathbb{Z}) \) there exists \( k \in \mathbb{Z} \) such that \( i^*(ke') = 0 \). Hence \( ke' = j^*(b) \) for the unique \( b \in H^2(XD, \partial XD, \mathbb{Z}) \). Therefore it is natural to define the rational intersection form [24, 17]
\[ \omega_{\mathbb{Q}}(e', e) = \langle e' \cup e, [XD, \partial XD] \rangle := \frac{1}{k} \langle b \cup e, [XD, \partial XD] \rangle \] (2.7)
for any pair \( e, e' \in H^2(XD, \mathbb{Z}) \).
We shall use the cohomological version of proposition 4 from [17] which can be formulated as follows. Let $XD$ be a pV-cobordism with $H^1(X_D, \mathbb{Z}) = 0$. If we choose a basis $b_i$ of $H^2(X_D, \partial X_D, \mathbb{Z})$ and a dual basis $e^i$ of $H^2(X_D, \mathbb{Z})$ ($i = 1, \ldots, r$, $r = \text{rank } H^2(X_D, \mathbb{Z})$), dual in the sense of

$$\omega_Z(b_i, e^j) = \delta^j_i,$$

then the integral intersection matrix $g_{ij} = \omega_Z(b_i, b_j)$ for $H^2(X_D, \partial X_D, \mathbb{Z})$ is the inverse of the rational intersection matrix $\lambda_{ij} = \omega_Q(e^i, e^j)$ for $H^2(X_D, \mathbb{Z})$.

It is important that the just mentioned intersection forms (matrices) are the basic topological invariants of compact 4-manifolds. In the only cases considered here, namely those of pV-cobordisms (pV-manifolds) with vanishing first cohomology groups, all (co)homology information about these pV-cobordisms (pV-manifolds) is contained in the second (co)homology groups, and hence also in the intersection matrices $g_{ij}$ and $\lambda_{ij}$. These matrices are easily calculated by means of the procedure described in detail in [18], see also [16].

### 3. Abelian BFE-theory on pV-cobordisms

In this section, we propose a generalized version of topological BF-type theory (known as four-dimensional topological gravity) on pV-cobordisms and pV-manifolds. The generalization is related to the fact that ranks of the 2-cohomology groups of pV-cobordisms are in general greater than 1, i.e., $\text{rank } H^2(X_D, \partial X_D, \mathbb{Z}) = \text{rank } H^2(X_D, \mathbb{Z}) = r \geq 1$. Therefore the set of basic fields should consist of $r$ (Abelian) forms $B_a$, $a = 1, \ldots, r$, as well as $r$ 2-forms $E^a$ dual to $B_a$.

This model, which we call BFE-theory, will have the following important properties:

1. The role of vacuum (zero-level) coupling constants is played by the topologically invariant intersection forms (cosmological-constant matrices).

2. The space $\mathcal{N}$ of classical solutions (phase space) of the BFE-system will be a finite-dimensional vector space

$$\mathcal{N} = H^2(X_D, \mathbb{R}) \oplus H^2(X_D, \mathbb{R})$$

(3.1)

where $H^2(X_D, \mathbb{R})$ is de Rham-2-cohomology group of pV-cobordism $X_D$. (The possibility of defining the discrete phase space

$$\mathcal{N}_{\text{discr}} = H^2(X_D, \mathbb{Z}) \oplus H^2(X_D, \partial X_D, \mathbb{Z})$$

(3.2)

will be considered elsewhere.)

To realize a model with properties (1) and (2), we introduce a family of linearly independent elements $B_a$ of the cochain complex $C^2(X_D, \partial X_D, \mathbb{Z}) \otimes \mathbb{R}$. These elements (cochains) are represented by a set of Abelian 2-forms which we also denote as $B_a$. The equations of motion (3.9) and the gauge symmetries (3.8) of our model yield (see below) the property of 2-forms $B_a$ to be closed and defined up to exact forms $d\chi_a$. Thus we can assume that $B_a$ form a basis of the group $H^2(X_D, \partial X_D, \mathbb{Z}) \otimes \mathbb{R}$. Therefore the index $a$ runs from 1

3 Due to the group isomorphism $H^2(X_D, \partial X_D, \mathbb{Z}) \cong H^2(X, \mathbb{Z})$ (see section 2.3), the model we propose below can be constructed in the same way on pV-cobordisms and pV-manifolds. Thus we restrict ourselves to pV-cobordisms only.
to \( r = \text{rank } H^2(X_D, \partial X_D, \mathbb{Z}) \otimes \mathbb{R} \). Note that the number \( r \) of the basis elements \( B_a \) is also equal to the number of vertices exterior to decorated chains \( \Gamma_D^1 \) of the decorated graph \( \Gamma_D \) which determines the pV-cobordism \( X_D \) [18]; see also examples in section 4 and observation 3 below.

Let us introduce a suitable set of Abelian 1-forms \( A^a, a = 1, \ldots, r \), and a constant non-degenerate symmetric \( r \times r \)-matrix \( \Lambda^{ab} \). Then it is natural to write the action

\[
S_{BF}^{(r)} = \int_{X_D} \left\{ B_a \wedge F^a + \frac{1}{2} \Lambda^{ab} B_a \wedge B_b \right\}
\]

(3.3)

which is a direct generalization of the ordinary Abelian BF-theory action (for \( r = 1 \) and with the \( \Lambda \) term or cosmological constant) [20, 21, 28]

\[
S_{BF}^{(1)} = \int_{X_D} \left\{ B \wedge F + \frac{\Lambda}{2} B \wedge B \right\}.
\]

(3.4)

In (3.3), \( F^a = \text{d}A^a \), \( \text{d} \) is the exterior derivative, and in repeated indices the summation convention is applied. The constant matrix \( \Lambda^{ab} \) occupies the place of the cosmological constant, thus we can call it either a cosmological-constant matrix or a coupling constant matrix.

**Observation 1.** The action (3.3) has the appearance of the non-Abelian BF-theory action

\[
S_{GR} = \int \left\{ F^i \wedge F^i + \phi_{ij} B^i \wedge B^j + \frac{\Lambda}{2} B^i \wedge B^j \right\}
\]

which is equivalent to general relativity [29–31]. While in \( S_{GR} \) the traceless symmetric 0-form \( \phi_{ij} \) is a Lagrangian multiplier, our constant matrix \( \Lambda^{ab} \), as is shown below, coincides with the rational intersection form and is the basic topological invariant of the pV-cobordism \( X_D \). A comparison of (3.3) and \( S_{GR} \) shows that an immediate analogue of the cosmological constant \( \Lambda \) is the collection of diagonal elements of \( \Lambda^{aa} \), while the off-diagonal elements obviously describe interaction between different \( a \neq b \) Abelian fields \( B_a \) and \( B_b \). Moreover, the indices used in these two theories have different nature: in our model, \( a, b \) enumerate Abelian fields, and in the non-Abelian BF-theory, \( i, j \) are the gauge group indices.

However, one cannot use the model (3.3) in the case when the potential 1-forms \( A^a \) are defined globally on \( X_D \), so that \( F^a = \text{d}A^a \) are exact 2-forms. The action (3.3) yields the equations of motion

\[
\text{d}B_a = 0,
\]

(3.5)

\[
F^a + \Lambda^{ab} B_b = 0
\]

(3.6)

which lead in this case to exact 2-forms \( B_a \) (since the constant matrix \( \Lambda^{ab} \) is non-degenerate). Hence each of \( B_a \) merely represents the zero-class in \( H^2(X_D, \partial X_D, \mathbb{Z}) \otimes \mathbb{R} \), so \( B_a \) cannot form a basis of this group.

Nevertheless, one can attain a cohomological non-triviality of \( B_a \) as solutions of dynamical equations, as well as a realization of properties (1) and (2) mentioned in the beginning of this section. To this end it is sufficient to add a new set of 2-forms \( E^a \in C^2(X_D, \mathbb{Z}) \otimes \mathbb{R} \) to the fields used in (3.3). Thus we propose the following action,

\[
S_{BFE} = \int_{X_D} \left\{ (B_a + \Lambda_{ab} E^b) \wedge F^a + \frac{1}{2} \Lambda_{ab} (\Lambda^{ac} B_c - E^c) \wedge (\Lambda^{bd} B_d - E^d) \right\}
\]

\[
\equiv \int_{X_D} \left\{ B_a \wedge F^a + \frac{1}{2} \Lambda_{ab} B_a \wedge B_b \right\} + \Lambda_{ab} \left( E^a \wedge F^b + \frac{1}{2} E^a \wedge E^b \right) - B_a \wedge E^a
\]

(3.7)

where \( \Lambda_{ab} \) is the matrix inverse to \( \Lambda^{ab} \left( \Lambda_{ab} \Lambda^{bc} = \delta^b_c \right) \).
The Abelian gauge symmetries of this action (up to boundary terms [21]),

$$\delta A^a = d\phi^a, \quad \delta B_a = d\chi_a, \quad \delta E^a = \Lambda^{ab} d\chi_b$$  \hspace{1cm} (3.8)

($\phi^a$ are 0-forms and $\chi_a$, 1-forms) combined with equations of motion following from (3.7),

$$dB_a = 0 = dE^a, \quad F^a = 0, \quad E^a = \Lambda^{ab} B_b$$  \hspace{1cm} (3.9) (3.10) (3.11)

tell us that the space $\mathcal{N}$ of classical solutions (phase space) of the BFE-system is

$$\mathcal{N} = \left[ H^2(X_D, \partial X_D, \mathbb{Z}) \otimes \mathbb{R} \right] \oplus \left[ H^2(X_D, \mathbb{Z}) \right] \oplus \left[ H^1(X_D, \mathbb{Z}) \otimes \mathbb{R} \right]$$  \hspace{1cm} (3.12)

where

$$B_a \in H^2(X_D, \partial X_D, \mathbb{Z}) \otimes \mathbb{R} \cong H^2(X_D, \partial X_D, \mathbb{R}),$$

$$E^a \in H^2(X_D, \mathbb{Z}) \otimes \mathbb{R} \cong H^2(X_D, \mathbb{R}),$$

$$A^a \in H^1(X_D, \mathbb{Z}) \otimes \mathbb{R} \cong H^1(X_D, \mathbb{R}).$$  \hspace{1cm} (3.13)

Due to the exact sequence (2.4) the following de Rham groups are isomorphic:

$$H^2(X_D, \partial X_D, \mathbb{R}) \cong H^2(X_D, \mathbb{R}).$$  \hspace{1cm} (3.14)

Taking into account this fact as well as the triviality of the group $H^1(X_D, \mathbb{R})$, we come to the phase space (3.1) of the BFE-system described by (3.7). Note that equation (3.10) means that all connections $A^a$ are flat (1-forms $A^a$ are closed) and cohomologically trivial since $H^1(X_D, \mathbb{R}) = 0$. Moreover, two sets of closed 2-forms $B_a$ and $E^a$ defined up to exact forms may be considered as a family of fundamental solutions of the BFE-system (3.7). These 2-forms determine two bases of $H^2(X_D, \mathbb{R})$ related by the non-degenerate transformation (3.11) which is nothing more than constraint equations following from the action (3.7).

Let us impose on the bases $B_a$ and $E^a$ the duality condition of the type (2.8) in the de Rham representation (cf [32])

$$\frac{1}{4\pi^2} \int_{X_D} B_a \wedge E^b = \delta_a^b.$$  \hspace{1cm} (3.15)

This condition fixes the gauge 1-forms $\chi_a$ introduced in (3.8) which automatically determine the boundary terms. Then in this gauge the constraints (3.11) yield

$$\Lambda^{ab} = \frac{1}{4\pi^2} \int_{X_D} E^a \wedge E^b,$$  \hspace{1cm} (3.16)

and

$$\Lambda_{ab} = \frac{1}{4\pi^2} \int_{X_D} B_a \wedge B_b.$$  \hspace{1cm} (3.17)

Following [17, 33], note that the right-hand sides in (3.16) and (3.17) are rational and integer intersection forms, respectively, and they are represented by the mutually inverse intersection matrices $\Lambda^{ab}$ and $\Lambda_{ab}$ in the respective bases $E^a$ and $B_a$. In analogy with the Yang–Mills theory [33], the diagonal elements of the matrix $\Lambda^{ab}$ could be called topological charges. However, due to relation (4.12) between Euler numbers and absolute values of the diagonal elements of the inverse matrix $\Lambda_{ab}$, it is natural to consider as topological charges the inverse values of the latter ones. Thus any Abelian BFE-theory is characterized by the set of topological charges $1/|\Lambda_{aa}|, a \in 1, r$.4

---

4 Due to the isomorphism (3.14) one could make no distinction between the levels of indices of $B_a$ and $E^a$. Initially, these different levels reflected a relation of these 2-forms to the groups dual in the Poincaré–Lefschetz sense; below we would not like to overlook these hereditary features.
These intersection forms are basic topological invariants of pV-cobordisms of type $X_D$ [15, 16, 18]. Thus in our BFE-model the coupling (cosmological) constant matrix $\Lambda^{ab}$ is the basic topological invariant of spacetime (of Euclidean signature) described by the cobordism $X_D$, and it is the rational intersection form in the natural basis $E^a$, so that our BFE-system (3.7) does possess property (1) stated in the beginning of this section.

When for the field strength 2-forms $F^a$ there are no globally defined potential 1-forms $A^a$, we may return to the simpler action (3.3), see an analysis of this case for $r = 1$, i.e., of the action (3.4), in [21]. (Note that the model (3.7) is also valid in this case for any rank $r$.)

Imposing the duality condition, now as

$$\frac{1}{4\pi^2} \int_{X_D} B_a \wedge F^b = \delta^b_a, \quad (3.18)$$

we come to the relations

$$\Lambda^{ab} = \frac{1}{4\pi^2} \int_{X_D} F^a \wedge F^b \quad \text{and} \quad \Lambda_{ab} = \frac{1}{4\pi^2} \int_{X_D} B_a \wedge B_b, \quad (3.19)$$

which are analogous to (3.16) and (3.17).

4. BFE-systems on the 2-parametric family of pV-cobordisms as a set of cosmological models

In this section, we construct a set of cosmological models with a sequence of topology changes of three-dimensional sections of spacetime (in the Euclidean regime). Each elementary topology change is represented by a specific pV-cobordism $X_D^{(l)}_n$ which corresponds to the decorated graph $\Gamma^{(l)}_n$ (the origin of two non-negative integer parameters $n, l \in \mathbb{Z}^+$ will be explained below). On each pV-cobordism $X_D^{(l)}_n$ is defined its individual Abelian BFE-system of type (3.7) which is a pure topological ‘gravity’ with a ‘cosmological term’ represented by a rational intersection matrix $\Lambda^{ab}_n$.

4.1. The basic set of Seifert fibred homology spheres

We construct the cobordisms $X_D^{(l)}_n$ using basic structure elements which are plumbed 4-manifolds $P(\Gamma)$ with Seifert fibred homology spheres (Sfh-spheres) $\Sigma(a_1, a_2, a_3)$ (with only three exceptional fibres) as boundaries: $\partial P(\Gamma) = \Sigma(a_1, a_2, a_3)$. Let us recall the definition of these Sfh-spheres: $\Sigma(a) := \Sigma(a_1, a_2, a_3)$ is a smooth compact 3-manifold obtained by intersecting the complex algebraic Brieskorn surface $z_1^{a_1} + z_2^{a_2} + z_3^{a_3} = 0$ (with $z_i \in \mathbb{C}$) with the unit five-dimensional sphere $|z_1|^2 + |z_2|^2 + |z_3|^2 = 1$, where $a_1, a_2, a_3$ are pairwise coprime integers, $a_i > 1$. There exists a unique Seifert fibration of this manifold with unnormalized Seifert invariants [26]: $(a_i, b_i)$ subject to $e(\Sigma(a)) = \sum_{i=1}^3 b_i/a_i = 1/a$, where $a = a_1a_2a_3$ and $e(\Sigma(a))$ is the well-known topological invariant of a Sfh-sphere, its Euler number. This Seifert fibration is defined by the $S^1$-action which reads $t(z_1, z_2, z_3) = (t^{a_1}z_1, t^{a_2}z_2, t^{a_3}z_3)$, where $t \in S^1$, and $a_i = a_i/a_i$.

To construct our model of universe we need a specific family of Sfh-spheres which would be defined in the following way. First, we define the derivative of a Sfh-sphere $\Sigma(a)$ as a Sfh-sphere

$$\Sigma^{(1)}(a) := \Sigma(a_1, a_2a_3, a + 1) \equiv \Sigma(a_1^{(1)}, a_2^{(1)}, a_3^{(1)}). \quad (4.1)$$
for a product of three Seifert invariants $q_i$ where

$$a(l) = a_1^{(l)} a_2^{(l)} a_3^{(l)} = a(a + 1).$$

By induction, we define the derivative $\Sigma^{(i)}(q) = \Sigma(a_1^{(l)}, a_2^{(l)}, a_3^{(l)})$ of $\Sigma(q)$ of any order $l$. In particular, the recurrent relation

$$a^{(l)} = a^{(l-1)}(a^{(l-1)} + 1)$$

for a product of three Seifert invariants $a^{(l)} = a_1^{(l)} a_2^{(l)} a_3^{(l)}$ holds. Second, we define a sequence of Sf-spheres which we shall call the primary sequence. Let $p_i$ be the $i$th prime number in the set of the positive integers $\mathbb{N}$, e.g., $p_1 = 2, p_2 = 3, \ldots$. The primary sequence of Sf-spheres is defined as

$$\{\Sigma(q_i, p_{i+1}, p_{i+2}) | i \in \mathbb{N}\}$$

where $q_i = p_1 \cdots p_i$. Finally, to construct our model of the universe, we include in this sequence as its first two terms the usual three-dimensional spheres $S^3$ with Seifert fibrations (Sf-spheres) determined by the mappings $h_{pq} : S^3 \to S^3$ in their turn defined as $h_{pq}(z_1, z_2) = z_1^p z_2^q$, $p, q \in \mathbb{N}$ [34]. Recall that $S^3 = \{z_1, z_2 | |z_1|^2 + |z_2|^2 = 1\}$ and $z_1^p z_2^q \in \mathbb{C} \cup \{\infty\} \cong S^3$. We denote these two Sf-spheres as $\Sigma(1, 1, 2)$, $p = 1$, $q = 2$ and $\Sigma(1, 2, 3)$, $p = 2$, $q = 3$. In this notation, we use an additional third number (unit) which corresponds to an arbitrary regular fibre. This will enable us to take derivatives of Seifert fibrations on $\Sigma(1, 1, 2)$ and $\Sigma(1, 2, 3)$ by the same rule (4.1) as for other members of the sequence (4.3).

Now we form the family of manifolds corresponding to the first nine primary Sf-spheres and their derivatives up to the fourth order,

$$\{\Sigma^{(i)}(q_1, p_2, p_{i+1}) | i \in \{0, 3\}, l \in \{0, 4\}\}$$

(4.4)

Note that the subfamily corresponding to $i = 0, 1$ is built from the ordinary spheres $S^3$ with fixed Seifert fibrations. In order to include the Sf-spheres $\Sigma(1, 1, 2)$ and $\Sigma(1, 2, 3)$ in this family, one has to put $q_1 = q_0 = p_0 = 1$. For example, for the well-known Poincaré homology sphere $\Sigma(p_1, p_2, p_3) = \Sigma(2, 3, 5, 7)$, the sequence of derivatives is

$$\Sigma^{(1)}(2, 3, 5) = \Sigma(2, 15, 31),$$

$$\Sigma^{(2)}(2, 3, 5) = \Sigma(2, 465, 931),$$

$$\Sigma^{(3)}(2, 3, 5) = \Sigma(2, 342915, 865831),$$

$$\Sigma^{(4)}(2, 3, 5) = \Sigma(2, 374831, 227365, 749662, 454731).$$

The calculation results for Euler numbers of Seifert structures of Sf- and Sf-spheres in the family (4.4) are given in Table 1. We find that for the subfamily

$$\{\Sigma^{(l)} = \Sigma^{(i)}(q_{2l-1}, p_{2l}, p_{2l+1}) | l \in \{0, 4\}\}$$

(4.5)

| $i$ | $l$ | 0 | 1 | 2 | 3 | 4 |
|-----|-----|---|---|---|---|---|
| 0   | 0.5 | 0.166 | 2.38 × 10^{-2} | 5.53 × 10^{-4} | 3.06 × 10^{-7} | 
| 1   | 0.166 | 2.38 × 10^{-2} | 5.53 × 10^{-4} | 3.06 × 10^{-7} | 9.39 × 10^{-14} | 
| 2   | 3.33 × 10^{-2} | 1.07 × 10^{-3} | 1.15 × 10^{-6} | 1.33 × 10^{-12} | 1.78 × 10^{-24} | 
| 3   | 4.76 × 10^{-3} | 2.26 × 10^{-5} | 5.09 × 10^{-10} | 2.59 × 10^{-19} | 6.73 × 10^{-38} | 
| 4   | 4.33 × 10^{-4} | 1.87 × 10^{-7} | 3.51 × 10^{-14} | 1.23 × 10^{-27} | 1.52 × 10^{-54} | 
| 5   | 3.33 × 10^{-5} | 1.11 × 10^{-9} | 1.23 × 10^{-18} | 1.51 × 10^{-36} | 2.29 × 10^{-72} | 
| 6   | 1.96 × 10^{-6} | 3.84 × 10^{-12} | 1.47 × 10^{-23} | 2.17 × 10^{-46} | 4.70 × 10^{-92} | 
| 7   | 1.03 × 10^{-7} | 1.06 × 10^{-14} | 1.13 × 10^{-28} | 1.28 × 10^{-56} | 1.63 × 10^{-112} | 
| 8   | 4.48 × 10^{-9} | 2.01 × 10^{-17} | 4.04 × 10^{-44} | 1.64 × 10^{-67} | 2.66 × 10^{-134} |
Spatial sections homeomorphic to Seifert fibrations were considered in [36]). To this end, we note that it is characterized only by the Fermi constant (if effects of mixing different fundamental particles are not taken into account). In this sense, each interaction given in table 2 is really characterized by a single coupling constant and is universal, though only at low energies \( E_{\text{low}} \ll M_W \approx 80 \text{ GeV} \). At higher energies a unification of interactions takes place, and the collection of fundamental interactions changes. The collection of coupling constants changes too. In our model, this situation can be found in table 3; cf also section 4.2.

### Table 2. Euler numbers versus experimental DLEC constants.

| \( l \) | \( \epsilon(\Sigma_n^{(l)}) \) | Interaction | \( \alpha_{\text{exper}} \) |
|---|---|---|---|
| 0 | 0.5 | Strong | 1 |
| 1 | \( 1.07 \times 10^{-3} \) | Electromagnetic | \( 7.20 \times 10^{-3} \) |
| 2 | \( 3.51 \times 10^{-14} \) | Weak | \( 3.04 \times 10^{-12} \) |
| 3 | \( 2.17 \times 10^{-46} \) | Gravitational | \( 2.73 \times 10^{-46} \) |
| 4 | \( 2.70 \times 10^{-134} \) | Cosmological | \( < 10^{-120} \) |

**Notes.** (1) The dimensionless strong interaction constant is \( \alpha_s = G/\hbar c \), where \( G \) characterizes the strength of the coupling of the meson field to the nucleon. (2) The fine structure constant (electromagnetic) is \( \alpha_e = e^2/\hbar c \). (3) The dimensionless weak interaction constant is \( \alpha_{\text{weak}} = (G_F/\hbar c) (m_e/\hbar)^2 \), with \( G_F \) being the Fermi constant \( (m_e \) is the mass of an electron). (4) The dimensionless gravitational coupling constant is \( \alpha_g = G_N m_e^2/\hbar c \), with \( G_N \) being the Newtonian gravitational constant. (5) The cosmological constant \( \Lambda \) multiplied by the squared Planckian length is \( \alpha_{\text{cosm}} = \Lambda G_N \hbar/c^3 \). The mentioned dimensionless constants (except the cosmological one) are also known as Dyson numbers.

### Table 3. The Euler number of \( (n, t) \)-family of Sf- and Sfh-spheres.

| \( n \) | 4 | 3 | 2 | 1 | 0 | −1 | −2 | −3 | −4 |
|---|---|---|---|---|---|---|---|---|---|
| 0 | | | | | | | | | |
| 1 | | | | | | | | | |
| 2 | 5.0 \( \times 10^{-4} \) | 1.7 \( \times 10^{-1} \) | 2.3 \( \times 10^{-2} \) | 5.5 \( \times 10^{-4} \) | 3.1 \( \times 10^{-7} \) |
| 3 | 3.3 \( \times 10^{-2} \) | 1.1 \( \times 10^{-3} \) | 1.2 \( \times 10^{-4} \) | 1.3 \( \times 10^{-6} \) | 1.8 \( \times 10^{-8} \) |
| 4 | 4.3 \( \times 10^{-4} \) | 1.9 \( \times 10^{-7} \) | 3.5 \( \times 10^{-14} \) | 1.2 \( \times 10^{-27} \) | 1.5 \( \times 10^{-54} \) |
| 5 | 2.0 \( \times 10^{-6} \) | 3.8 \( \times 10^{-12} \) | 1.5 \( \times 10^{-31} \) | 2.2 \( \times 10^{-46} \) | 4.7 \( \times 10^{-92} \) |
| 6 | 4.5 \( \times 10^{-9} \) | 2.0 \( \times 10^{-17} \) | 4.0 \( \times 10^{-34} \) | 1.6 \( \times 10^{-62} \) | 2.7 \( \times 10^{-134} \) |

The Euler numbers (the boldface numbers) reproduce fairly well the experimental hierarchy of dimensionless low-energy coupling (DLEC) constants of fundamental interactions, see table 2.

**Observation 2.** Usually, one calls such an interaction, which is essentially characterized by only one coupling constant, fundamental (universal). For example, the weak interaction is universal since it is characterized only by the Fermi constant \( G_F \). We eliminate the Sf- and Sfh-spheres, see section 4.2 and [35] (compact locally homogeneous universes with spatial sections homeomorphic to Seifert fibrations were considered in [36]). To this end, we primarily have to reduce and reparametrize the family (4.4). First, in accordance with (4.5), we eliminate the Sf- and Sfh-spheres with odd numbers \( i \) introducing a new parameter \( n \in [0,4] \) related to \( i \) as \( i = 2n \). Then (in certain cases) it is also convenient to use another parameter \( t = n - l, t \in [-4,4] \). The resulting family of Sf- and Sfh-spheres is

\[
\Sigma_n^{(l)} = \sum (a_1^{(l)}, a_2^{(l)}, a_3^{(l)}) \quad n \in [0,4], l \in [0,4]
\]

\[
:= \{ \Sigma_n^{(l)} (q_{2n-l}, p_{2n}, p_{2n+l}) \mid n \in [0,4], l \in [-4,4] \},
\]

which contains (4.5) as a subset for \( l = 0 \), i.e., when \( n = l \). The Euler numbers of this family of Sfh-spheres are given in table 3.
Topological gravity on plumbed V-cobordisms

\[
P(\Gamma_{\alpha(l)}^0) \Rightarrow P(\Gamma_{\alpha(l)}^1) \Rightarrow \ldots \Rightarrow P(\Gamma_{\alpha(l)}^n) \Rightarrow P(\Gamma_{\alpha(l)}^\pm_1)
\]

Figure 5. The decorated graph \(\Gamma_{D_n}^{(l)}\).

Parameter \(t\) in our model is the discrete cosmological ‘time’, \(t = 0\) labelling the present state of the universe where an observer can determine the DLEC constants \(\alpha_{(l)}\) of the five \((l \in 0, 4)\) fundamental interactions (see table 2; remember that in this case \(l = n\)). Relation (4.2) readily yields good estimates of the DLEC constants \(\alpha_{(l)}^{(l)} = e(\Sigma_n^{(l)}) \approx (q_{2l+1})^{-2}\). Remember that \(q_{2l+1} = p_1 \cdots p_{2l+1}\) is a product of the first \(2l + 1\) prime numbers in \(\mathbb{N}\). Note that for \(l = 5\) there would be \(\alpha_5^{(3)} \approx 1.4 \times 10^{-357}\) which is too small to be identified with a certain experimentally determined coupling constant, thus we impose the restriction \(n, l \in 0, 4\) (see, however, [35] where this restriction is lifted when the open discrete cosmological models are described). Our hypothesis is that also in other \((t \neq 0)\) columns of table 3, the Euler numbers can be related to the coupling constants as \(e(\Sigma_n^{(l)}) \sim \alpha_{(l)}^{(l)}\). In the next subsection, we discuss the realization of this hypothesis in our BFE-model and introduce a new concept of \((n, l)\)-pre-interactions which play the role of fundamental interactions in our purely topological approach. In the framework of our model, any low-energy \((n, l)\)-pre-interaction (which can be labelled, or better baptized, on the basis of the coupling constants hierarchy, see table 2) can be traced to its counterparts at \(t \neq 0\) when one of the parameters \((n, l)\) does not change. For example, the counterparts of the ‘cosmological’ \((4, 4)\)-pre-interaction are \((4, l)\)-pre-interactions with \(l \in 0, 4\), and \((n, 4)\)-pre-interactions \((n \in 0, 4)\).

Though this approach leads to a hierarchy of the DLEC constants, it yields neither a description of the spacetime structure of universe, nor its other features, therefore we pass to framing a more constructive universe model in terms of four-dimensional plumbed cobordisms with boundary components glued by splicing \(Sf\)- and \(Sfh\)-spheres.

4.2. Construction of \(pV\)-cobordisms and respective BFE-systems

Let \(\Gamma_{D_n}^{(l)}\) be the decorated graph shown in figure 5.

This graph corresponds to the result of plumbing elementary manifolds \(P(\Gamma_n^{(l)})\) with boundaries homeomorphic to \(Sfh\)-spheres \(\Sigma_n^{(l)}\), \(m \in 0, n\) minus the plumbed manifolds \(P(\Gamma_n^{(l)})\) corresponding to decorated chains \(\Gamma_m^{(l)}\), \(i \in \mathbb{T}\). The notation \(P(\Gamma_n^{(l)})\) in figure 5 means subtraction of the 4-manifold \(P(\Gamma_n^{(l)})\) with the boundary \(L(p_i, q_i)\) from the 4-manifold \(P(\Gamma_n^{(l)})\) where \(\overline{\Sigma}^{(l)}_{n}\) is the graph \(\Gamma_{D_n}^{(l)}\) without decoration of chains. In other words, the graph \(\Gamma_{D_n}^{(l)}\) determines the \(pV\)-cobordism

\[
X_{D_n}^{(l)} = P(\Gamma_{D_n}^{(l)}) = P(\Gamma_n^{(l)}) \bigcup_{i=1}^{l} P(\Gamma_i^{(l)})
\]

(4.7)

describing the topology change between the \(Z\)-homology sphere

\[
\Sigma_n^{(l)} = \Sigma_0^{(l)} \Sigma_1^{(l)} \ldots \Sigma_n^{(l)}
\]

(4.8)

of Abelian forms of the (3
\text{vertex} v_a \text{ of endpoint is}
\text{observation 3.}
The calculation of
\text{integer}
\text{where the relative cycles (mod } \partial X_{D_n^{(l)}} \text{) } \varsigma_{n ab}^{(l)} \text{ are representatives of cohomological classes}
\text{Here rank } H^2(X_{D_n^{(l)}} , \mathbb{R}) = \text{rank } H_2(X_{D_n^{(l)}} , \partial X_{D_n^{(l)}} , \mathbb{R}) = n + 1. \text{ Hence the set of Abelian forms of the (3.7)-type BFE-system corresponding to the cobordism } X_{D_n^{(l)}} \text{, is}
\text{Definition 3.}
\text{An integer intersection matrix (2.1) corresponding to the graph } \Gamma. \text{ The summation}
\Lambda_{n ab}^{(l)} \text{ was given in practical terms by Saveliev in [18]. A vertex } v_a \text{ and a decorating oval (or circle) are called adjacent if the oval intersects an edge one of whose endpoints is } v. \text{ The generators of the cohomology group } H^2(X_{D_n^{(l)}} , \mathbb{R}) \text{ correspond to the vertices of } \Gamma_{D_n^{(l)}} \text{ outside all decorating ovals. Given such a vertex } v_a \text{ weighted by an integer } e^a, \text{ we have (see section 4.1 for basic definitions)}
\Lambda_{n ab}^{(l)aa} = e^a - \sum d_i, \quad \det A \left( \Gamma_{n}^{\text{pch}} \right) / \det A \left( \Gamma_{D_n^{(l)}}^{\text{pch}} \right).
portion of the chain $\Gamma_i^{ch}$ obtained by removing the vertex of $\Gamma_i^{ch}$ adjacent to $v_a$ and deleting all its adjacent edges. (Note that the determinant of an empty graph is equal to 1.) For any two generating vertices $v_a$ and $v_b$ connected by an edge inside $\Gamma_D^{(j)}$ away from the decorating ovals $\Lambda_n^{(j)ab} = 1$. If these two vertices are adjacent to the same decorating oval enclosing a chain $\Gamma_k^{ch}$, we have

$$\Lambda_n^{(j)ab} = 1/\det \Lambda(\Gamma_k^{ch})$$

Note that we use unnormalized Seifert invariants ([26] and section 2.1), thus all generating vertices $v_a$ have in our diagrams weights $e^a = 0$, see figure 6.

To begin with, we consider as an example the subset of rational intersection matrices $\Lambda_n^{(j)ab}$ corresponding to $n = l$, i.e., the matrices $\Lambda_l^{(j)ab}$, $l \in 0, 4$, $a, b \in 0, l$, calculated with the use of Saveliev’s algorithm [18] (see also [16]):

$$\Lambda_0^{(0)} = (\Lambda_{strong}) = [0.5],$$

$$\Lambda_1^{(1)} = (\Lambda_{elmag}) = 
\begin{bmatrix}
9.5 \times 10^{-2} & -1.3 \times 10^{-2} \\
-1.3 \times 10^{-2} & 6.1 \times 10^{-4}
\end{bmatrix},$$

$$\Lambda_2^{(2)} = (\Lambda_{weak}) = 
\begin{bmatrix}
9.8 \times 10^{-3} & -1.3 \times 10^{-4} & 0 \\
-1.3 \times 10^{-4} & 1.6 \times 10^{-6} & -6.7 \times 10^{-12} \\
0 & -6.7 \times 10^{-12} & 3.9 \times 10^{-17}
\end{bmatrix},$$

$$\Lambda_3^{(3)} = (\Lambda_{grav}) = 
\begin{bmatrix}
1.9 \times 10^{-4} & -1.8 \times 10^{-8} & 0 & 0 \\
-1.8 \times 10^{-8} & 1.8 \times 10^{-12} & -1.4 \times 10^{-21} & 0 \\
0 & -1.4 \times 10^{-21} & 1.2 \times 10^{-27} & -2.2 \times 10^{-40} \\
0 & 0 & -2.2 \times 10^{-40} & 4.1 \times 10^{-53}
\end{bmatrix},$$

$$\Lambda_4^{(4)} = (\Lambda_{cosm}) = 
\begin{bmatrix}
1.0 \times 10^{-7} & -4.9 \times 10^{-16} & 0 & 0 \\
-4.9 \times 10^{-16} & 2.4 \times 10^{-24} & -5.5 \times 10^{-41} & 0 \\
0 & -5.5 \times 10^{-41} & 1.5 \times 10^{-54} & -1.2 \times 10^{-76} \\
0 & 0 & -1.2 \times 10^{-76} & 4.7 \times 10^{-92} & -6.9 \times 10^{-119} \\
0 & 0 & 0 & -6.9 \times 10^{-119} & 1.0 \times 10^{-145}
\end{bmatrix}.$$ 

Note that all numbers in these matrices are rational; they are given here up to two significant digits.

In our model, this subset of matrices is associated with the contemporary ($t = n - l = 0$) stage of universe, and with the low-energy sector of fundamental interactions (see the boldface column in table 3). Note that the matrix elements $\Lambda^{(j)ab}_{l,l-1} = \Lambda^{(j)l-1l}_{l-1}$ (boldface numbers) are subject to a hierarchy similar to that of the Euler numbers $e(\Sigma^{(j)}_l)$. Thus we shall name the $(l, l)$-pre-interaction (corresponding to $\Lambda^{(j)ab}_{l,l}$) in the same way as the fundamental interaction labelled by the same parameter $l$ in table 2. It is clear that the rational intersection matrices $\Lambda^{(j)ab}_{n,l}$ contain more numerical information about $(n, l)$-pre-interactions than the Euler numbers $e(\Sigma^{(j)}_n)$. To disentangle this information, it is worth passing to the inverse matrices $\Lambda^{(j)}_{n,ab}$ being
integer intersection matrices:

$$(A^{(0)}_4)^{-1} = \begin{bmatrix}
-2 & 10 & -660 & 2.4 \times 10^6 & -1.6 \times 10^{12} \\
10 & -30 & 1980 & -7.1 \times 10^6 & 4.9 \times 10^{12} \\
-660 & 1980 & -2310 & 8.3 \times 10^6 & -5.7 \times 10^{12} \\
2.4 \times 10^6 & -7.1 \times 10^6 & 8.3 \times 10^6 & -5.1 \times 10^5 & 3.5 \times 10^{11} \\
-1.6 \times 10^{12} & 4.9 \times 10^{12} & -5.7 \times 10^{12} & 3.5 \times 10^{11} & -2.2 \times 10^8
\end{bmatrix},$$

$$(A^{(1)}_4)^{-1} = \begin{bmatrix}
-6 & 124 & -2.8 \times 10^5 & 1.4 \times 10^{11} & -3.2 \times 10^{19} \\
124 & -930 & 2.1 \times 10^6 & -1.1 \times 10^{12} & 2.4 \times 10^{20} \\
-2.8 \times 10^5 & 2.1 \times 10^6 & -5.3 \times 10^6 & 2.7 \times 10^{12} & -6.1 \times 10^{20} \\
1.4 \times 10^{31} & -1.1 \times 10^{12} & 2.7 \times 10^{12} & -2.6 \times 10^{11} & 5.8 \times 10^{19} \\
-3.2 \times 10^{19} & 2.4 \times 10^{20} & -6.1 \times 10^{20} & 5.8 \times 10^{19} & -5.0 \times 10^{16}
\end{bmatrix},$$

$$(A^{(2)}_4)^{-1} = \begin{bmatrix}
-42 & 11172 & -1.9 \times 10^6 & 2.2 \times 10^{17} & -2.1 \times 10^{28} \\
11172 & -8.7 \times 10^5 & 1.5 \times 10^{11} & -1.7 \times 10^{19} & 1.6 \times 10^{30} \\
-1.9 \times 10^9 & 1.5 \times 10^{11} & -2.9 \times 10^{13} & 3.2 \times 10^{31} & -3.1 \times 10^{32} \\
2.2 \times 10^{37} & -1.7 \times 10^{19} & 3.2 \times 10^{21} & -6.8 \times 10^{22} & 6.6 \times 10^{34} \\
-2.1 \times 10^{28} & 1.6 \times 10^{30} & -3.1 \times 10^{32} & 6.6 \times 10^{33} & -2.5 \times 10^{33}
\end{bmatrix},$$

$$(A^{(3)}_4)^{-1} = \begin{bmatrix}
-1806 & 7.3 \times 10^7 & -7.2 \times 10^{16} & 4.0 \times 10^{29} & -7.4 \times 10^{45} \\
7.3 \times 10^7 & -7.5 \times 10^{11} & 7.4 \times 10^{20} & -4.1 \times 10^{33} & 7.6 \times 10^{59} \\
-7.2 \times 10^{16} & 7.4 \times 10^{20} & -8.1 \times 10^{26} & 4.5 \times 10^{39} & -8.3 \times 10^{55} \\
4.0 \times 10^{39} & -4.1 \times 10^{33} & 4.5 \times 10^{59} & -4.6 \times 10^{45} & 8.6 \times 10^{61} \\
-7.4 \times 10^{45} & 7.6 \times 10^{69} & -8.3 \times 10^{55} & 8.6 \times 10^{61} & -6.14 \times 10^{66}
\end{bmatrix},$$

$$(A^{(4)}_4)^{-1} = \begin{bmatrix}
-3.3 \times 10^6 & 2.7 \times 10^{15} & -8.8 \times 10^{31} & 1.2 \times 10^{54} & -7.8 \times 10^{80} \\
2.7 \times 10^{15} & -5.6 \times 10^{23} & 1.8 \times 10^{40} & -2.4 \times 10^{62} & 1.6 \times 10^{89} \\
-8.8 \times 10^{31} & 1.8 \times 10^{40} & -6.6 \times 10^{53} & 8.7 \times 10^{75} & -5.9 \times 10^{102} \\
1.2 \times 10^{54} & -2.4 \times 10^{62} & 8.7 \times 10^{75} & -2.2 \times 10^{91} & 1.5 \times 10^{118} \\
-7.8 \times 10^{60} & 1.6 \times 10^{69} & -5.9 \times 10^{102} & 1.5 \times 10^{118} & -3.8 \times 10^{133}
\end{bmatrix}.$$
that in section 3, the inverses of the matrices $\Lambda^{(i)}_{ab}$ were called cosmological- (or coupling) constant matrices. Now this name acquires justification not only in the framework of the generalized BF-theory, but also from the viewpoint of a comparison with the experimental low-energy coupling constants. In fact, the boldface elements of the matrices $\Lambda^{(i)}_{ab}$ correspond to the coupling constants at $t = n - l = 0$ in the sense that $a^{(i)}_{ij} \sim e^{(\Sigma^{(i)})} = 1 / |\Lambda^{(i)}_{ii}|$. Hierarchically, the next near-diagonal elements $\Lambda^{(i)}_{l,l-1} = \Lambda^{(i)}_{l-1,l}$ are also related to them.

These relations permit us to conclude that our rather exotic cosmological model may be associated with the really existing universe. To put it more strictly, the experimentally observed coupling constants’ hierarchy may be determined by topological invariants (intersection matrices) of cobordisms $X^{(0)}_{Dn}$ which we relate to Euclidean spacetime regions describing topology changes.

It is now worth mentioning this evolutionary scheme in more detail (to the extent admissible in our model). Insofar as the BFE-system (3.7) does not possess local degrees of freedom, the evolution ought to be understood as a sequence of topological (phase) transitions resulting in changes of the set of topological invariants $\Lambda^{(i)}_{n} := (\Lambda^{(i)}_{n}ab, \Lambda^{(i)}_{n}ab)$ as well as of the BFE-system of forms $S^{(i)}_{n} := (\Lambda^{(i)}_{n}ab, B^{(i)}_{n}ab, E^{(i)}_{n}ab)$. We consider the BFE-theory as a topological analogue of gravitation with a cosmological constant (but without other (phenomenological) kinds of matter), thus our model is purely cosmological: one may imagine a table similar to table 3; now each cell characterized by the parameters $(n, t)$ or $(n, l = n - t)$, will be related to a cosmological model which involves the respective $\mathbb{Z}$-homology spheres $M_{n}^{(i)}$ as spacelike sections, and the BFE-system $S^{(i)}_{n}$. Instead of the Euler numbers $e^{(\Sigma^{(i)})} = 1 / a^{(i)}_{n}$, in this new table there should appear the intersection (cosmological-constant) matrices $\Lambda^{(i)}_{n}$.

The latter of course contain much more numerical information than a mere set of coupling constants $a^{(i)}_{n} \sim e^{(\Sigma^{(i)})}$. We shall call the collection $U^{(i)}_{n} = \{X^{(i)}_{Dn}, S^{(i)}_{n}, \Lambda^{(i)}_{n}\}$ the primary $(n, l)$-universe, or $(n, l)$-pre-universe (reminiscent of the pre-geometry of John A Wheeler).

From table 3 one can see that at $t = n - l = 0$ values of the Euler numbers $e^{(\Sigma^{(i)})}$ (boldface numbers) well represent the hierarchy of the DLEC constants (see also table 2). Since the information on the Euler numbers $e^{(\Sigma^{(i)})}$ is contained in the intersection matrices $\Lambda^{(i)}_{n}$, it is possible to conclude that the ensemble of $(n, l)$-pre-universes $\{U^{(i)}_{n} | n = l\}$ should correspond to the basic vacuum state of the present stage of the composite universe with five $(l \in 0, 4)$ BFE-systems. It is remarkable that this ensemble of pre-universes contains information about the hierarchy of dimensionless low-energy coupling constants of the real fundamental interactions (see boldface numbers in the intersection matrices given above). This hierarchy has in our model a purely topological origin, and it springs up before any local degrees of freedom are introduced. Thus the coupling constants which in most field theories have a (semi-)phenomenological character, in our model are topological invariants describing the global properties of the spacetime (at least, in the Euclidean regime).

For other values of $t \in -4, 4$, the ensembles of $(n, l)$-pre-universes

$$\{U^{(i)}_{n} | n - l = t\}$$  \hspace{1cm} (4.13)

describe the composite-universe states both of the ‘past’ ($t < 0$) and the ‘future’ ($t > 0$). Thus in our model, the ‘real’ universe is a superposition of $(n, l)$-pre-universes at any fixed discrete time parameter $t$. Let us identify the BFE-system $S^{(i)}_{n}$ with the unique ‘fundamental interaction’ $(n, l)$-pre-interaction acting in the $(n, l)$-pre-universe. Then from the modified version of table 3 (which we described above only verbally) it follows that the number of $(n, l)$-pre-interactions in the superposition of $(n, l)$-pre-universes (4.13) grows from 1 to 5
when \( t \) changes from \(-4\) to 0. The further growth of \( t \) from 0 to 4 results in the decrease of the number of \((n, l)\)-pre-interactions to 1. Thus in our model there exists the possibility of realizing the idea of unification of interactions, but in a rather unusual form (instead of successive symmetry breakdowns in the gauge theory, in our model a sequence of topology changes takes place).

The above-described scheme corresponds to a closed model of the universe. Some details of the universe evolution with ‘inflationary’ stages and a possible treatment of unification ideas in closed and open cosmological models, are given in [35] on the basis of the \( T_{2n}\)-discrete space approach.

5. Conclusions

Let us now summarize the basic features of our model and some pending problems.

The ensemble of pre-universes \( U_n(l) \) proposed in this paper involves BFE-systems \( S_n(l) \) which possess the basic characteristic features of ordinary BF-systems [20]; this means that all physical fields may be gauged away locally. Thus the phase spaces (sets of classical solutions) are finite-dimensional spaces (of type (3.1)) of zero-modes and have a purely topological sense. But BFE-models also involve the intersection matrices \( \Lambda_n(l) \) as analogues of coupling constants of fundamental interactions. The latter ones are, however, described at the purely topological level, thus we call them \((n, l)\)-pre-interactions. Since the intersection matrices are basic topological invariants of \( PV\)-cobordisms \( XD_n(l) \), in our version of BF-theory coupling constants lose their usual phenomenological character and acquire the status of topological invariants of the spacetime manifold on which the BFE-system \( S_n(l) \) is constructed. Insofar as the experimentally observed ‘running coupling constants’ do depend on local characteristics of interactions (such as energy density), these ‘constants’ have even in the low-energy case their values different from those calculated in our model (see table 2). However, already at the topological vacuum level (i.e., in the complete absence of local degrees of freedom of all fields including the gravitational one) the information about the hierarchy of (at least) the DLEC constants of real fundamental interactions is contained.

This situation may be interpreted as a generalization of the Mach principle in the sense of a determining influence of the global topological (cosmological) characteristics of the universe on the local properties of universal interactions ‘switched on’ in this universe. It is appropriate to note that in our model all (pre-)interactions bear ‘cosmological traces’, that is, each pre-interaction is forming a certain pre-universe \( U_n(l) \) where it is the only one which is switched on. The spacetime topology of this pre-universe is completely determined by the cosmological-constant matrix \( \Lambda_n(l) \) representing the rational intersection form of the \( PV\)-cobordism \( XD_n(l) \). The real universe involving several interactions, is treated as a superposition of pre-universes \( U_n(l) \) with \( n-l = t = \text{const} \). The problem still is how to determine ‘ordinary’ fields with their local degrees of freedom in conformity with the topological structure of this superposition.

The pre-interactions unification concept qualitatively differs from the usual scheme of unification accepted in gauge theories. For example, the set of \((n, l)\)-pre-interactions found for \( t = n-l \) is replaced by another set of \((n', l')\)-pre-interactions. If \( t \leq 0 \) and \( n'-l' = t-1 \), the latter set contains one pre-interaction less than the former. The number of pre-interactions decreases by a shift to the left from \( t = 0 \) in table 3 (or in the analogue of this table verbally described in section 4.2) as well.

Elementary \( PV\)-cobordisms \( XD_n(l) \) and \( XD_{n+1}(l) \) can be pasted into cobordisms describing topology changes between \( Z\)-homology spheres. This is accompanied by creation and annihilation of certain sets of disjoint lens spaces \( L_{\text{out}} \) and \( L_{\text{in}} \). Pasting of these \( PV\)-cobordisms
is naturally performed along sets of pairwise homeomorphic lens spaces \([37] L_n \subset \partial X D_n\)
and \(L_{n+1} \subset \partial X D_{n+1}\) yielding the pV-cobordism
\[ X D_n \cup X D_{n+1} = X D_n \cup X D_{n+1} \]
The boundary of this cobordism,
\[ \partial X D_n \cup X D_{n+1} = (M_n \cup L_{\text{in}}) \cup (M_{n+1} \cup L_{\text{out}}) \]
contains both \(\mathbb{Z}\)-homology spheres \(M_n \subset \partial X D_n\), \(M_{n+1} \subset \partial X D_{n+1}\) and sets of mutually non-homeomorphic lens spaces \(L_{\text{in}}, L_{\text{out}}\).

Thus the pV-cobordism \(X D_{n,n+1}\) describes the topology change
\[ M_n \cup L_{\text{in}} \longrightarrow M_{n+1} \cup L_{\text{out}}. \]

Here one confronts, however, the still open problem of the junction of the BFE-systems \(S^{(i)}_n\) and \(S^{(i)}_{n+1}\) which are defined on pV-cobordisms \(X D^{(i)}_n\) and \(X D^{(i)}_{n+1}\) respectively.

It is interesting that the intersection matrices \(\Lambda^{(i)}_n\) always have signature \((- ++ \cdots +\)) for any values of \(n\) and \(l\). This may hint at the possibility of constructing a discrete model of a spacetime based on Lorentz-signature lattices spanned on eigenvectors of intersection matrices, the dimensionality of any lattice being \(n + 1\). Realization of this approach should be based on a study of the discrete phase space (3.2) containing richer cohomological information about the pV-cobordisms \(X D^{(i)}_n\) than the real vector space (3.1).

Finally, note that in addition to the direct analogues of coupling constants (that is, topological charges (4.12)) the intersection matrices \(\Lambda^{(i)}_n\) contain a large amount of numerical information about \((n,l)-\)pre-universes, so that these matrices could be considered as their numerical ‘code’, maybe (see tables 2 and 3) a ‘code’ of our proper universe as well. This information is encoded in the topology of the ordinary 3-sphere’s ‘nearest relatives’, namely in topology invariants of \(\mathbb{Z}\)-homology spheres being spacelike sections of spacetime manifolds.

In fact, we are greatly baffled by the strange results which followed from an application of quite an abstract and fundamental part of mathematics, the algebraic topology, and we feel it to be appropriate to conclude this paper with comforting and reassuring words of Eugene P Wigner:

\[
\ldots \text{the mathematical formulation of the physicist’s often crude experience leads in an uncanny number of cases to an amazingly accurate description of a large class of phenomena. This shows that the mathematical language has more to commend it than being the only language which we can speak; it shows that it is, in a very real sense, the correct language} \ [38].
\]

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