Numerical study of ubiquitous modes in tokamak plasmas in the presence of impurities

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Abstract
As an important branch of trapped electron modes (TEMs), the ubiquitous mode (UM) is a favored subject for the investigation of anomalous energy losses in magnetic fusion plasmas. In this paper, the numerical study of UMs was carried out in tokamak plasmas in the presence of impurities. The physical model and the gyrokinetic equations for TEM (and UM) instability is introduced, including the impurity effect. The numerical results show that impurity species impose significant impacts on the UM instability in many ways, one of which is that impurity effect on UMs is generally stabilizing, and on the whole the stabilizing effect is most pronounced for the case of \(-0.8 \lesssim L_{ez} < 0\) \((L_{ez} \equiv L_{ne}/L_{nz}, \text{with } L_{nz} = -\partial \ln n_z/\partial r)^{-1}\). As the impurity charge concentration \(f_z\) increases, or for the heavier or lower-ionized impurity ions, the UM instability is weaker. Compared to the ion and electron temperature gradient effects, impurity temperature gradient (expressed by \(h_z = L_{nz}/L_{Tz}\)) has only a minor effect, while the ratio of electron to impurity temperature \(T_e/T_z\) has a relatively larger effect on UMs, embodied in the mode linear growth rates and instability windows. The investigation of wavenumber threshold for UM showed that the mode was a fluid-like instability when \(b_i \equiv \frac{1}{2} k_i^2 \rho_i^2 \sim (n/n_{ci})^2\), and the presence of impurity resulted in the modification of the UM instability window. By surveying the parametric dependences of UMs in the presence of impurities, it is revealed that comparatively, the stabilizing effects of impurity are more pronounced in the regime of larger wavenumber, while the weakening effects of (i) decreasing the fraction of trapped electrons, (ii) increasing the magnetic shear, or (iii) decreasing the electron density gradient on UM instability are relatively more pronounced in the regime of smaller wavenumber. It also indicates that the stabilizing effect of impurity on UMs is owing to the non-resonant mechanism of the UM.

Keywords: trapped electron mode, ubiquitous mode, fluid-like instability, impurity effect

(Some figures may appear in colour only in the online journal)

1. Introduction

Drift wave driven turbulence is widely recognized to be an important candidate responsible for anomalous particle, energy and momentum transports in magnetically controlled fusion plasmas [1–4]. In particular, collisionless trapped electron modes (TEMs) [5–9] play an important role in
energy confinement in tokamaks. Typical TEMs (ty-TEMs) are characterized by moderate to long wavelengths, propagating in the electron diamagnetic drift direction. The ubiquitous mode (UM) \cite{10–12} is another important branch of TEMs with \( b_i \equiv \frac{1}{2} n_k^2 n_i^2 \gg 1 \), propagating in the direction of the ion diamagnetic drift velocity. Here, \( \rho_i = \sqrt{2T_e/m_i/\Omega_i} \) is the average sound speed ion gyro-radius, and \( \Omega_i \) is the ion gyro-frequency. The significance of UMs in magnetically confined plasma has been well recognized since UMs can lead to relatively strong anomalous diffusion. Recently, we extended \cite{13} the study of Coppi et al.\cite{10–12} and clarified many novel properties of UMs, employing the gyrokinetic model for low frequency drift instability on tokamaks.

As is well known, impurities are inevitable in tokamak plasmas which contain one or more other ion species besides hydrogen or deuterium ions. These additional impurity species include the deliberately injected impurity for reducing power load on in-vessel components, and the undesirable product of interaction between plasma and wall of device. The density of impurity ions is much lower than that of main ions. However, experiments have showed \cite{14, 15} that impurities have significant influence on plasma energy confinement because of the radiation losses and dilution of main ions. Therefore, effect of impurity is of crucial relevance for fusion plasmas.

In this respect, most of existing works have focused on understanding of impurity transport driven by ITG/TEM turbulence \cite{16–20}. The transports of light and heavy impurities with high and low charge number \( Z_i \) have been studied deliberately \cite{21–25}. Most of the researches address diffusive and convective process since the impurity source is mainly loaded in plasma edge.

The study of impurity transport involves trace impurities, which almost have no effect on the turbulence induced by ITG and TEM instabilities. Here, ‘trace impurities’ means a very small fraction of impurities in the plasma, e.g. the impurity charge concentration \( f_i \equiv Zn_T/n_i < 1\% \) for C\( ^{6+} \) impurity. The ion impurity must have significant impacts on ITG and TEM instabilities when impurity content is high enough and far beyond the validation of trace impurity treatment. In order to explore the influence of impurity on drift wave turbulence, in recent years, the impurity effects on ITG and TEM instability have been investigated \cite{26–32}. As an important branch of TEMs, UMs deserve more attentions in transport analysis due to the fact that although UMs cause less transport than ty-TEM, ITG, and ETG modes in general, under some specific parameters, UMs can cause strong heat diffusions, especially in the regime of fluid-like instability (FLI) \cite{12, 13}. Thereby, the effects of impurity ions on UMs have to be investigated. For this purpose, the results of the systematic analysis of the influence of impurity on UMs in tokamaks are present in this work. The nature of UM instability in the presence of impurities is addressed. This study provides an opportunity to further understand the characteristics of UM stability and its role in transport in tokamaks.

The rest of the paper is organized as the following. In next section, the physical model and relevant integral eigenmode equations will be introduced. The numerical results obtained will be brought in and analyzed systematically in section 3. The UM behaviors under impacts of impurity will be discussed there. Finally, we shall briefly summarize our conclusions and give some discussions in section 4.

2. Basic equations

In this work, a toroidal geometry with circular cross section is considered for the tokamak magnetic configuration. The dynamics of low frequency electrostatic perturbation in inhomogeneous plasmas is described by the following quasi-neutrality condition

\[ \vec{n}_i = \vec{n}_i + Z\vec{\eta}_z, \]

(1)

here, \( Z \) is the charge number of the impurity ions; \( \vec{n}_i \) and \( \vec{n}_z \) denote perturbed densities of main and impurity ions, respectively. In addition, \( \vec{n}_i = \vec{n}_{iq} + \vec{n}_{iz} \), with \( \vec{n}_{iq} \) and \( \vec{n}_{iz} \) being the perturbed densities of passing and trapped electrons, respectively. In our physical model, the kinetic characteristics of ions, such as Landau resonance, magnetic drift and finite Larmor radius are all taken into account. The full ion transit \( k_i \nu_i \) and toroidal drift effects are retained. The passing electron response is assumed adiabatic, and the finite Larmor radius effects of trapped electrons are neglected. The perturbed density of ions \( \vec{n}_i (s = i, z) \) is given as

\[ \vec{n}_i = \int f_i d^3\nu, \]

(2)

with

\[ f_i = \frac{q_i F_{Mi}}{T_i} \vec{\phi} + h_i J_0(\delta_i), \]

(3)

where \( \vec{\phi} \) is the perturbed electrostatic potential. The non-adiabatic response \( h_i \) is determined by solving the gyrokinetic equation in ballooning space

\[ \frac{\nu_{||}}{R q} \frac{\partial}{\partial \theta} \nu_s = (\omega - \omega_{DA}) J_0(\delta_s) F_{Ms} \frac{q_i n_{0i}}{T_i} \vec{\phi}(\theta), \]

(4)

here, \( \vec{\phi}(k) \) is the extended Fourier component (in ballooning space) of \( \vec{\phi}(r) \), with \( k = \delta (r_0) k_0 \theta \); and

\[ \omega_{DA} = 2 \nu_s \omega_n (\cos \theta + \sin \theta \delta) \left( \frac{\nu_s^2}{2} + \nu_{||}^2 \right), \]

\[ \omega_{DST} = \omega_{DA} \left[ 1 + \frac{\nu_s^2}{3} \left( \frac{\nu_s^2}{2} - \frac{3}{2} \right) \right], \]

\[ F_{Ms} = (\pi \nu_s^2)^{-3/2} \exp(-\delta^2), \]

\[ \omega_{Ds} = c k_0 T_i / \nu_i B L_{ms} \]

is the diamagnetic drift frequency, the velocity variable with sharp sign ‘\(^{\pm}\)’ denotes normalization to \( \nu_{||} = \sqrt{2T_i/m_i} \), \( J_0 \) is the Bessel function of zeroth order and \( \delta = \nu_i (2b_2)^{1/2} \), \( 2b_2 = k_0^2 \nu_i^2 / \Omega_i^2 \), and \( \Omega_i = q_i B / m_i c \) is the gyrofrequency of ion species \( s \).
Equation (4) can be integrated with the boundary condition $h_i(\theta) = 0$ as $\theta \to \pm \text{sgn}(v_i) \cdot \infty$. Thus, we have

$$\sum_{\phi=LZ} q_i \hat{h}_i = -\left[\tau_i(1 - f_e) + \tau_e Z f_{ek} \right] \hat{\phi}(k) + \int_{-\infty}^{+\infty} \frac{dk}{\sqrt{2\pi}} K(k, k') \hat{\phi}(k').$$

The kernel of equation (5) is

$$K(k, k') = -i \int_{-\infty}^{0} \omega_{pe} e^{i\omega \tau} \sqrt{2} e^{-i\omega\tau} \times \left[1 - f_e \right] \exp\left[-\frac{(k^2 + k'^2)}{4\lambda} \right] \times \left(\frac{\omega}{\omega_{pe}} \tau_i + L_{ei} \right) \frac{2\eta_i \tau_i + L_{ei}}{1 + a} \times \left[1 - \frac{k^2 + k'^2}{2(1 + a) \tau_i} \frac{k \cdot k'}{(1 + a) \tau_i} \right] + \frac{\eta_i \tau_i}{4D_e} \left\{ f_e \exp\left[-\frac{(k^2 + k'^2)}{4\lambda} \right] \left(\frac{\omega}{\omega_{pe}} \tau_g + L_{ee} \right) \frac{3}{2} \frac{\eta_e \tau_g + L_{ee}}{1 + a_e} \times \left[1 - \frac{(k^2 + k'^2)\mu}{2(1 + a_e) \tau_{ge}} \frac{k \cdot k'}{(1 + a_e) \tau_{ge}} \right] + \frac{\eta_e \tau_{ge} \tau_i}{4D_e} \right\} \hat{\phi}(k', \lambda') \right) \hat{\phi}(k', \lambda'),$$

where

$$\lambda = \frac{\tau^2}{\tau_i} \frac{\omega_{pe} e^{-i\omega\tau}}{q}, \quad \lambda_x = \frac{\tau^2}{\tau_x} \frac{\omega_{pe} e^{-i\omega\tau}}{q}, \quad a = 1 + \frac{i2\epsilon a \omega_{pe} e^{-i\omega\tau}}{\tau_i} \times (\delta + 1) \sin \theta \cdot \sin \theta' \times \left(\frac{\delta - (\perp \theta \cos \theta - \perp' \cos \theta')}{\perp \theta - \perp' \theta'}, \right)$$

and $a_z = 1 + \frac{i2\epsilon a \omega_{pe} e^{-i\omega\tau}}{\tau_z \tau_e} \times (\delta + 1) \sin \theta \cdot \sin \theta' \times \left(\frac{\delta - (\perp \theta \cos \theta - \perp' \cos \theta')}{\perp \theta - \perp' \theta'}, \right)$

$$\Gamma_0 = I_0(k \cdot k' \tau_i) \exp\left[-\frac{(k^2 + k'^2)}{2(1 + a) \tau_i} \right]$$

and

$$\Gamma_{ee} = I_0(k \cdot k' \mu \tau_{ge}) \exp\left[-\frac{(k^2 + k'^2)\mu}{2(1 + a_e) \tau_{ge}} \right].$$

Here, $L_{ef} = -(\text{dln}(n_e/\text{dr})^{-1}$ (j = e, i, z) is the density gradient scale length. The mode frequency $\omega(\equiv \omega_i + \iota \gamma)$ is normalized to electron diamagnetic drift frequency $\omega_{de} = \text{c}k_0 \text{T}_e/e\text{BL}_{de}$. The wavenumbers k, k’, and k0 are all normalized to $\rho_{i0} = k0T_e/\sqrt{\text{c}\text{T}_e \mu_i}$. Also, all the symbols have their usual meanings, i.e., $L_{T_i}$ is the temperature gradient scale lengths, $q$ is the safety factor, $\int d\sigma/q d\sigma$ is the magnetic shear. $I_m(m = 0, 1)$ is the modified Bessel function of the order m. In addition, $m_i$ and $T_i$ (i = e, i, z) are the ion species’ mass and temperature, respectively.

On the other hand, the dispersion relation for the dissipative trapped electron instability is derived by Adam, Tang and Rutherford [33], in the usual way by solving the Vlasov–Maxwell equations in the electrostatic limit. There collisions are described by an energy dependent Krook model [34]. The characteristic mode frequency lies between the thermal ion and electron bounce frequencies, i.e., $\omega_{e0} < \omega < \omega_{de}$. From linear theory, the approximate form of perturbed density of the trapped electrons is given as the following in the limit of small $\omega/\omega_{de}$.

$$\bar{n}_{et} = \frac{\sqrt{2\pi}}{\omega_{de}} \left(\frac{\omega - \omega_{de}}{\omega - \omega_{de}} \right) \hat{\phi} \times \omega - \omega_{de} + \text{i} \text{b}(E/T)^{1/2},$$

where $\text{b} = \frac{\mu a \epsilon e^{-i\omega\tau}}{q}, \text{e} = \nu_e/e, \nu_e$ is electron collision frequency. The bounce average of magnetic curvature and gradient drift frequencies of trapped electrons is given by

$$\omega_{de}(E, \kappa) = \omega_{de} E \frac{G(\hat{s}, \kappa)}{R T_e},$$

where $G(\hat{s}, \kappa) = \left(2\kappa(\kappa_0) - 1\right) + 4\kappa \left(\kappa(\kappa - 1) + 1\right)$ and $K$ is the complete elliptic integrals of the first and second kind, respectively; $\kappa = \sin(\theta_0/2), \theta_0$ is the azimuthal coordinate of the turning point of trapped electrons.

Considering the collisionless case only ($\nu = 0$), we rewrite equation (7) as

$$\hat{n}_{et} = \frac{\sqrt{2\pi}}{\omega_{de}} \int_0^{\infty} d\sigma q^{\perp} \int_{-\infty}^{+\infty} \frac{\omega - \omega_{de}}{\omega - \omega_{de} + \text{i} \text{b}(E/T)^{1/2}} \times \text{d}k^2/4\kappa(\kappa),$$

with $t = E/T_e$. Besides, the adiabatic response of passing electrons is $\hat{n}_{ep} = \frac{\epsilon a \hat{\phi}}{\tau_i \hat{\phi}}$. 

\[ \text{Equation (4)} \text{ can be integrated with the boundary condition } h_i(\theta) = 0 \text{ as } \theta \to \pm \text{sgn}(v_i) \infty. \text{ Thus, we have} \]

\[ \sum_{\phi=LZ} q_i \hat{h}_i = -[\tau_i(1 - f_e) + \tau_e Z f_{ek}] \hat{\phi}(k) + \int_{-\infty}^{+\infty} \frac{dk}{\sqrt{2\pi}} K(k, k') \hat{\phi}(k'). \]

\[ \text{The kernel of equation (5) is} \]

\[ K(k, k') = -i \int_{-\infty}^{0} \omega_{pe} e^{i\omega \tau} \sqrt{2} e^{-i\omega\tau} \times \left[1 - f_e \right] \exp\left[-\frac{(k^2 + k'^2)}{4\lambda} \right] \times \left(\frac{\omega}{\omega_{pe}} \tau_i + L_{ei} \right) \frac{2\eta_i \tau_i + L_{ei}}{1 + a} \times \left[1 - \frac{k^2 + k'^2}{2(1 + a) \tau_i} \frac{k \cdot k'}{(1 + a) \tau_i} \right] + \frac{\eta_i \tau_i}{4D_e} \left\{ f_e \exp\left[-\frac{(k^2 + k'^2)}{4\lambda} \right] \left(\frac{\omega}{\omega_{pe}} \tau_g + L_{ee} \right) \frac{3}{2} \frac{\eta_e \tau_g + L_{ee}}{1 + a_e} \times \left[1 - \frac{(k^2 + k'^2)\mu}{2(1 + a_e) \tau_{ge}} \frac{k \cdot k'}{(1 + a_e) \tau_{ge}} \right] + \frac{\eta_e \tau_{ge} \tau_i}{4D_e} \right\} \hat{\phi}(k', \lambda') \hat{\phi}(k', \lambda'), \]

\[ \text{where} \]

\[ \lambda = \frac{\tau^2}{\tau_i} \frac{\omega_{pe} e^{-i\omega\tau}}{q}, \quad \lambda_x = \frac{\tau^2}{\tau_x} \frac{\omega_{pe} e^{-i\omega\tau}}{q}, \quad a = 1 + \frac{i2\epsilon a \omega_{pe} e^{-i\omega\tau}}{\tau_i} \times (\delta + 1) \sin \theta \cdot \sin \theta' \times \left(\frac{\delta - (\perp \theta \cos \theta - \perp' \cos \theta')}{\perp \theta - \perp' \theta'}, \right) \]

and $a_z = 1 + \frac{i2\epsilon a \omega_{pe} e^{-i\omega\tau}}{\tau_z \tau_e} \times (\delta + 1) \sin \theta \cdot \sin \theta' \times \left(\frac{\delta - (\perp \theta \cos \theta - \perp' \cos \theta')}{\perp \theta - \perp' \theta'}, \right)$

\[ \Gamma_0 = I_0(k \cdot k' \tau_i) \exp\left[-\frac{(k^2 + k'^2)}{2(1 + a) \tau_i} \right] \]

and

\[ \Gamma_{ee} = I_0(k \cdot k' \mu \tau_{ge}) \exp\left[-\frac{(k^2 + k'^2)\mu}{2(1 + a_e) \tau_{ge}} \right]. \]

\[ k = k_0 \theta \theta', \quad k' = k_0 \theta \theta', \quad k^2 = k_0^2 + k^2, \quad k'^2 = k_0^2 + k'^2, \quad \varepsilon_n = \frac{L_{ne}}{R}, \quad \eta_e = \frac{L_{ne}}{L_{Te}}, \quad \eta_i = \frac{L_{ei}}{L_{Ti}}, \quad \eta_x = \frac{L_{ex}}{L_{Tx}}, \quad f_z = \frac{Zn_{et}}{n_{de}}, \quad \mu = \frac{m_z}{m_i}. \]
Finally, the quasi-neutrality condition equation (1) yields the integral dispersion equation as,
\[ 1 + \tau_z (1 - f_e) + \tau_i Z_f \Phi(k) = \int_{-\infty}^{+\infty} \frac{dk'}{2\pi} K(k', k') \Phi(k') \]
\[ + \left[ \frac{2e}{\pi} \int_0^{+\infty} d\epsilon \sqrt{\epsilon} e^{-\epsilon} \int_0^1 \frac{\omega - \omega_{pe}}{\omega - \omega_{De}} \times \frac{d\epsilon^2}{4F(\epsilon)} \right] \sum_{j=-\infty}^{+\infty} g(\theta - 2\pi j, \kappa) \int_{-\infty}^{+\infty} d\theta' g(\theta', \kappa) \Phi(\theta' - 2\pi j). \]  
(9)

Here we should notice again that \( k = \delta(r_0)k_0\theta \). HD7 code [35] is a numerical tool specially used to solve this integral dispersion equation equation (9).

In equation (9), the fraction of trapped electrons is \( f_e = \sqrt{2e} \) with \( \epsilon = r/R \), and \( r \) and \( R \) are the minor and major radii of plasma column, respectively. ITG or impurity modes (IMs) are obtained by solving this equation when \( \epsilon = 0 \). It is worth noting that IM is a low-frequency electrostatic drift mode propagating in ion diamagnetic drift direction, which can be excited in the case of \( L_{ci} \leq 1 \sim -0.8 \) even in the absence of ion temperature gradient [30, 36]. The solutions of equation (9) may be ITG, IMs or TEMs as \( \epsilon > 0 \). In the solutions of TEMs, besides the one (namely ty-TEM) resonating with the bounce averaged magnetic drift of the trapped electrons, i.e. \( \omega_{pe,\text{avg}} > 0 \), there is another branch that avoids the resonance, where \( \omega_{De,\text{avg}} < 0 \) and thus \( \omega_{pe,\text{avg}} < 0 \). Such two solutions can be obtained by solving equation (9), and the latter is so-called UM.

Clearly, impurity effects have also been included in equation (9). The key parameters related to impurity effects include: (i) impurity charge concentration \( f_i \); (ii) mass number of impurity ions \( \mu_i \); (iii) impurity charge number of impurity ions \( Z_i \); (iv) ratio of density gradient scale lengths of electrons to impurity ions \( L_{ce}/L_{ci} \), representing the peaking direction of impurity density relative to the electron and main ion densities, which is an important quantity that may influence the properties of impurity effect; (v) temperature gradient parameter of impurity ions \( \eta_i (=L_{ni}/T_{ni}) \), and (vi) temperature ratio of electrons to impurity ions \( \tau_i (=T_e/T_i) \).

3. Numerical results

Employing HD7 code [35], solving equation (9) in specific parameter regimes gives the results that could completely describe the properties of mode branches of TEMs. Typically, we take the following parameter: \( \eta_i = \eta_e = \eta_n = 0 \), \( \delta = 0.5 \), \( q = 2 \), \( \epsilon_0 = 0.2 \), \( \tau_i = T_e/T_i = 1 \), \( \epsilon = 0.25 \), \( k_0 \rho_i = 4 \), and \( L_{ci} = 1 \), unless otherwise specified. Carbon (C), oxygen (O), and lithium (Li) would be considered as impurity ions.

3.1. Overall features of the UM

The overall features of TEMs, including UM and ty-TEM, are depicted in figure 1, where the normalized growth rate \( \gamma' \) and real frequency \( \Omega' \) is given as a function of normalized wavenumber \( k_0\rho_i \) for the cases with or without \( O^{+1} \) impurity, respectively. Here, \( \gamma' \equiv \gamma \times k_0 \rho_i / \omega_{pe} = \gamma L_n / \sqrt{2} c_s \) and \( \Omega' \equiv \omega_i \times k_0 \rho_i / \omega_{pe} = \omega_i L_n / \sqrt{2} c_s \), with \( c_s \) being the ion sound speed. It is seen that the TE mode is divided into two parts. The left one has positive real frequency, and the right one is characterized by negative real frequency, corresponding to ty-TEM and UM, respectively. The mode growth rate is generally larger in UM regime. As \( k_0 \rho_i \sim 1 \) for the case without impurity (figure 1(a)), and \( k_0 \rho_i \sim 4.75 \) for the case with \( O^{+1} \) impurity (figure 1(b)), the mode growth rates can reach the approximate maximum value, where the mode enters the regime of so-called FLI, which occurs in the transitional region, around \( \omega_i = 0 \), between the ty-TEM and the UM.

Comparing figures 1(a) with (b), it demonstrates that the ranges of the wavenumber spectra, growth rates and real frequencies significantly change in UMs due to the presence of impurity, which partly indicates the significance of impurity effects on UMs.

Figure 2 displays the typical eigen-function \( \Phi(\theta) \) of ty-TEM (a), (b) and UMs (c), (d), in the cases without and with \( O^{+1} \) impurity ions. It suggests that both the two TEM branches are even modes and are highly localized in the regime of \( [-\pi, \pi] \). And for UMs, obviously, there is also a similar mode structure for the cases with and without impurity, indicating that both of them belong to the same type of electrostatic drift modes.

In the following we will survey the effects of impurity parameters on the UM or the parametric dependences of the UM under the action of impurity effects, in order to address the nature of UMs in the presence of impurity. Because of the particularity of UMs, we have to focus on the wavenumber spectra of TEMs, so that we can observe the properties of ty-TEM, UM and FLI under various plasma parameters at the same time.

3.2. Impurity concentration effect

The impurity concentration \( f_i \) effect on UMs is demonstrated in figure 3. We notice that in figure 3(a), the right side of the line for \( (k_0 \rho_i) \text{abs} \), the normalized wavenumber at \( \omega_i = 0 \), is the UM-dominated regime, and the left one is the regime occupied by the ty-TEM. It is evident that the normalized growth rate appreciably decreases as the impurities are presented. If \( f_i \) increases, the growth rate of UM will decrease. At the same time the real frequency becomes less negative, and the point \( \omega_i = 0 \) moves right (as shown in figure 3(b)), meaning that the ty-TEM instability window becomes wider and thus the UM instability window becomes narrower. The above results suggest that impurity has stabilizing effects on UMs. The stabilizing effect of impurities is due to the dilution of the main ions by the addition of impurities when the impurity content is far beyond the ‘trace’ level. Besides, the impurity ions are heavier and the magnetic drift effect is very weak, which leads to the weaker UM instability in plasma.
function of \( f \), at \( r = q \)

it confirms the stabilizing effect of \( f \) on UM.

### 3.3. Impurity density gradient effect

\( L_{ez} \) is the ratio of electron (\( L_{ne} \)) to impurity (\( L_{nz} \)) density scale lengths. As is known, if \( L_{ez} > 0 \), it means that the impurity density peaks inwardly; otherwise, it peaks opposite to the electron and ion density. The \( L_{ez} \) effects of impurities on TEMs, including on UM, are shown in Figure 4. As shown in Figure 4(a), it demonstrates that when \( k_0 \rho_s > (k_0 \rho_s)_{thr} \sim 1.5 \), the linear growth rate of the UM increases as \( L_{ez} \) decreases from 1 to 0.5. For the negative \( L_{ez} \) case (\( L_{ez} = -0.4 \)), we see that the growth rate decreases dramatically (shown in figure 4(a)) although the ty-TEM instability window becomes narrower and UM instability window becomes relatively wider, compared to the case without impurities (shown in figure 4(b)). It is clear that for negative \( L_{ez} \) cases, impurity has much stronger stabilizing effect in the region of wavenumber higher than the threshold \( (k_0 \rho_s)_{thr} \), compared to that for positive \( L_{ez} \) cases. In the lower wavenumber regime, the magnitude of the decrease of the mode growth rate is similar to that of positive \( L_{ez} \) cases.

- **Figure 1.** The normalized growth rate \( \gamma(\equiv \gamma(k_0 \rho_s/\omega_{\text{ce}})) \) and real frequency \( \omega_r(\equiv \omega_r(k_0 \rho_s/\omega_{\text{ce}})) \) as a function of \( k_0 \rho_s \) in the plasma without (a) or with \( O^{8+} \) impurity (b). The parameters are \( \delta = 0.5 \), \( \varepsilon = 0.25 \) and \( \varepsilon_s = 0.2 \). The specific parameters are \( f_z = 0.25 \) and \( L_{ez} = 2 \) for the case of (b). The black region shows the domain of fluid-like instability (FLI).

- **Figure 2.** The typical eigenfunction \( \Phi(\theta) \) of ty-TEM (a), (b) at \( k_0 \rho_s = 0.5 \) and UM (c), (d) at \( k_0 \rho_s = 5 \), respectively, in a plasma without (\( f_z = 0 \)) (a), (c) and with \( O^{8+} \) impurity (b), (d). The other parameters are the same with figure 1.
to each other for positive and negative $L_{ez}$ cases. In summary, negative $L_{ez}$ could offer preferable stabilizing effect on UMs in tokamaks on the whole. However, we should limit our study in the range of $L_{ez} > -0.8$, since as $L_{ez} < -0.8$, an IM may occur [30, 36].

3.4. Impurity ion mass and charge number effects

In this paper, $\mu$ denotes impurity ion mass, and $Z$ is impurity charge number. Figures 5(a) and (b) show the $\mu$ and $Z$ effects on UMs, respectively. Li$^{2+}$, C$^{3+}$, O$^{3+}$, or C$^{2+}$, C$^{4+}$ and C$^{6+}$ are chosen as impurity ions. Comparing the normalized growth rates among the cases with different impurities, it is seen that in the UM regimes, the greater the impurity mass, the smaller the growth rate is (shown in figure 5(a)); and that for the same kind of impurity ions, the linear growth rate is smaller as the impurity charge number is higher, as demonstrated in figure 5(b). These indicate the stronger stabilizing effects of heavier (larger $\mu$) or higher-ionized (larger $Z$) impurities on UMs, compared to lighter (smaller $\mu$) or lower-ionized (smaller $Z$) impurities.

3.5. Temperature gradient effects

Plotted in figure 6(a) is the electron and ion temperature gradient effects on ty-TEMs and UMs. It is clear that $\eta_i$ and $\eta_e$ has little influence on the FLI, as indicated with the dashed lines in the figure; nevertheless, in the other points of the UM domain, the linear growth rates almost all decrease with simultaneously increasing of both $\eta_i$ and $\eta_e$. Certainly, the stabilizing effect of impurity on UMs is also observed in both regimes. These suggest that simultaneously increasing $\eta_i$ and $\eta_e$ may make the UM instability window wider, and make the growth rate smaller, indicating a way to stabilize the bulk of UMs by letting $\eta_i$ and $\eta_e$ increase simultaneously. Obviously, impurities may significantly enhance this stabilizing effect.

Now we address the impurity ion temperature gradient effect. As shown in figure 6(b) and compared with coupled $\eta_i$, $\eta_e$ effects illustrated in figure 6(a), it is clear that $\eta_i$ has a weaker effect in varying the linear growth rate of TEMs under the same impurity conditions. However, it can make the UM instability window narrower. This is due to the small share of impurities in the plasma so that the influence of impurity temperature gradient on the UM stability is much weaker than that of $\eta_i$ and $\eta_e$.

3.6. Impurity temperature effect

The ratio of electron to impurity ion temperature is defined with $\tau_e$. Impurity ion temperature effect is shown in figure 7. Evidently, $\tau_e$ has a relatively greater impact on UMs compared to $\eta_e$. That is, the mode growth rate decreases
In figure 9(b), the stabilizing effect of $f_e$ on FLI is shown. It is also demonstrated that when $\hat{s}$ increases, the normalized growth rate of FLI, $\gamma (k_0\rho_s/\omega_{ei})_{FLI}$, first increases gradually until $\hat{s} = 0.5$, and then decrease with $\hat{s}$. For the latter case ($\hat{s} \geq 0.5$), as $\hat{s}$ increases, the mode growth rate would decrease and the UM instability window becomes narrower, as demonstrated in figure 9(a). It can be understood that the occurrence of the UM impose a limitation on the value of $\hat{s}$, and the smaller value of $\hat{s}$ is more preferable for exciting the UM [13]. Physically, this is due to the suppressing effect of the magnetic shear on the UM. Besides, comparing the case of $\hat{s} = 0.5$ in a ‘pure’ plasma (black solid line) with the case of $\hat{s} = 2$ in a ‘pure’ plasma (red line with diamonds), or with the case with $C^{6+}$ impurity and $\hat{s} = 0.5$ (blue line with triangles) in figure 9(a), we see that in the larger wavenumber region ($k_0\rho_s > 3$ here), the stabilizing effect of impurity becomes more pronounced while the weakening effect of increasing magnetic shear (here $\hat{s} > 0.5$ is needed) on UM instability becomes weaker; however, the influence of magnetic shear on UMs is more pronounced in the smaller wavenumber region.

3.7. The fraction of trapped electrons parametric dependence

The $\varepsilon$ dependence of the UM indicates the influence of the fraction of trapped electrons on UMs, which is plotted in figure 8. The parameters are $L_{\text{ae}} = 1$, $f_e = 0.2$, $\varepsilon_n = 0.12$. It is demonstrated that as $\varepsilon$ increases, the linear growth rate increases, and the point $\omega_i = 0$ moves to left, meaning that the UM instability window becomes wider, both of which indicate the driving role of trapped electrons for UM instability. In addition, comparing the case of $\varepsilon = 0.2$ in a ‘pure’ plasma (red lines with triangles) with the case with $C^{6+}$ impurity and $\varepsilon = 0.2$ (green line with diamonds), or with the case of $\varepsilon = 0.06$ in a ‘pure’ plasma (black solid line) in figure 8(a), it is interesting to see that in comparison, the stabilizing effect of impurity is still strong while the weakening effect of decreasing the value of $\varepsilon$ on UM instability becomes weaker in the regime of the larger wavenumber ($k_0\rho_s \gtrsim 4$), however, in the regime of smaller wavenumber ($0 < k_0\rho_s < 4$) the latter is much stronger than the former.

Shown in figure 8(b) is the normal growth rates at the point $\omega_i = 0$, $\gamma (k_0\rho_s/\omega_{ei})_{FLI}$, varying with $\varepsilon$. It displays the following characteristic features. The mode linear growth rate of FLI increases with growing $\varepsilon$. Besides, as $f_e$ increases, the FLI is suppressed, so the effect of UM on the electron thermal transport is weakened.

3.8. Magnetic shear $\hat{s}$ variation

The $\varepsilon_n$ dependence of the UM indicates the influence of the fraction of trapped electrons on UMs, which is plotted in figure 8. The parameters are $L_{\text{ae}} = 1$, $f_e = 0.2$, $\varepsilon_n = 0.12$. It is demonstrated that as $\varepsilon$ increases, the linear growth rate increases, and the point $\omega_i = 0$ moves to left, meaning that the UM instability window becomes wider, both of which indicate the driving role of trapped electrons for UM instability. In addition, comparing the case of $\varepsilon = 0.2$ in a ‘pure’ plasma (red lines with triangles) with the case with $C^{6+}$ impurity and $\varepsilon = 0.2$ (green line with diamonds), or with the case of $\varepsilon = 0.06$ in a ‘pure’ plasma (black solid line) in figure 8(a), it is interesting to see that in comparison, the stabilizing effect of impurity is still strong while the weakening effect of decreasing the value of $\varepsilon$ on UM instability becomes weaker in the regime of the larger wavenumber ($k_0\rho_s \gtrsim 4$), however, in the regime of smaller wavenumber ($0 < k_0\rho_s < 4$) the latter is much stronger than the former. Shown in figure 8(b) is the normalized growth rates at the point $\omega_i = 0$, $\gamma (k_0\rho_s/\omega_{ei})_{FLI}$, varying with $\varepsilon$. It displays the following characteristic features. The mode linear growth rate of FLI increases with growing $\varepsilon$. Besides, as $f_e$ increases, the FLI is suppressed, so the effect of UM on the electron thermal transport is weakened.

3.9. Electron density gradient parametric dependence

$\varepsilon_n$ denotes the normalized electron density scale length. Shown in figure 10 is the impact of electron (and ion) density gradient parameter $\varepsilon_n$ on UMs in the absence or the presence of $C^{6+}$ impurity. Here, it is worth noting that the ion density gradient is assumed as the same as the electron density gradient in this study. For the case of $L_{\text{ae}} = 1$, as $\varepsilon_n$ increases, the maximum growth rate of the TEM decreases, as shown in figure 10(a); and the threshold $(k_0\rho_s)_{\text{thu}}$ increases, as shown in figure 10(b), indicating that the increase of $\varepsilon_n$ is beneficial to weaken UM instability. This shows that the electron density gradient is a driving factor of UM. It is also seen that the larger the $\varepsilon_n$, the larger the magnitude of $(k_0\rho_s)_{\text{thu}}$ varies, as shown in figure 10(b).

On the other hand, as shown in figure 10(a), it is clear that in the regime of the larger wavenumber, the stabilizing effect of impurity is almost the same as the weakening effect of increasing the value of $\varepsilon_n$ (here $\varepsilon_n > 0.08$ is needed) on UM instability; however, the latter is obviously stronger in the regime of smaller wavenumber, as comparing the case of $\varepsilon_n = 0.08$ in a ‘pure’ plasma (black solid line) with the case with $C^{6+}$ impurity and $\varepsilon_n = 0.08$ (blue line with circles), or with the case of $\varepsilon_n = 0.2$ in a ‘pure’ plasma (red line with triangles).
3.10. The thresholds for the UM

Figures 11(a) and (b) show the thresholds of normalized wavenumbers \( k_\theta \rho_s \) and the critical magnetic shear \( \dot{s} \) for the onset of UMs as functions of \( e \), respectively. We know from figure 11(a) that the mode is a FLI when \( \dot{\theta} > \dot{\theta}_b \), is \( \hat{\theta}^1_1 \), \( \hat{\theta}_2 \), \( \hat{\theta}_3 \), and \( \hat{\theta}_4 \) are taken as impurity ions. Here, \( L_{ei} = 1, f_\delta = 0.2, \dot{s} = 1.5 \), and the other parameters are the same as those in figure 3.

Figures 11(b) and (b) depict the\( \dot{\theta} \) effect by plotting the normalized growth rates as functions of \( k_\theta \rho_s \) when \( \dot{\theta}_2 = 0.1 \). Here, \( L_{ei} = 1, f_\delta = 0.2, \) and the other parameters are the same with those in figure 3. \( \hat{\theta}_3 \) is taken as impurity ion.

Figures 11(a) and (b) show the thresholds of normalized wavenumbers \( (k_\theta \rho_s)_{\text{th}} \) and the critical magnetic shear \( \dot{s} \) for the onset of UMs as functions of \( e \), respectively. We know from figure 11(a) that the mode is a FLI when \( b_i \equiv \frac{1}{2} k_\theta^2 \rho_s^2 \sim (n_i/n_e)^3 \), and the presence of impurity can adjust the specific value of threshold \( (k_\theta \rho_s)_{\text{th}} \) (and then \( (b_i)_{\text{th}} \)) for various cases. For example, the fitting results of the three cases: (i) \( f_\delta = 0 \), and (ii) \( f_\delta = 0.2 \) and \( L_{ei} = 1 \), and (iii) \( f_\delta = 0.2 \) and \( L_{ei} = -0.4 \), are \( y \equiv (k_\theta \rho_s)_{\text{th}} = 0.62 \times \left( \frac{1}{2\xi^3} \right)^{1.4}, 0.74 \times \left( \frac{1}{2\xi^3} \right)^{1.66} \) and \( 0.53 \times \left( \frac{1}{2\xi^3} \right)^{1.38} \), respectively. This confirms that impurity effect involves the adjustment of the UM instability windows.

As shown in figure 11(b), it is clear that for the same fraction of trapped electrons, the presence of impurity could make the \( \dot{s} \) change in a relatively large range. For the positive (negative) \( L_{ei} \) cases, the magnetic shear \( \dot{s} \) for the onset of UMs would decrease (increase), and it is not conflict to the stabilizing effect of impurity on UMs.
4. Conclusion and discussion

The numerical results of the systematic study of the UM instabilities in the presence of impurities were reported here. The investigations employed the gyrokinetic equations for low frequency drift waves, with impurity and trapped electron effects included. The results show that in general, impurity has stabilizing effect on UMs. The mode enters UM regime when \( b_i \sim (n/n_{i,T})^3 \), and the impurity effect can cause the modification of the UM instability window. With the presence of impurity, the UM instability window is narrower, and the mode growth rate decreases. And the results suggest that, as the impurity charge concentration \( f_z \) increases, or for the heavier or lower-ionized impurity ions, the UM instability becomes weaker. In particular, the stabilizing effect of impurity becomes much stronger on the whole when \( L_{\text{ex}} \) takes

![](image.png)

**Figure 8.** (a) Shows the normalized growth rate. (b) Illustrates \((\gamma k_0 \rho_s/\omega_{pe})_{\text{FLL1}}\) as a function of \( \varepsilon \). Here, \( f_z = 0.2 \), \( \varepsilon_n = 0.12 \), and \( L_{\text{ex}} = 1 \) if not specified, the other parameters are the same with figure 3.

![](image.png)

**Figure 9.** (a) Shows the normalized growth rates as functions of \( k_0 \rho_s \). The cases with \( \hat{\delta} = 0.5 \) and 2 are compared. (b) Depicts \((\gamma k_0 \rho_s/\omega_{pe})_{\text{FLL1}}\) as functions of \( \hat{\delta} \), respectively. \( L_{\text{ex}} = 1 \) if not specified, the other parameters are the same with figure 8.

![](image.png)

**Figure 10.** (a) The normalized growth rates as functions of \( k_0 \rho_s \). The cases with \( \varepsilon_n = 0.08 \) and 0.2 are compared. (b) Shows and \((k_0 \rho_s)_{\text{br}}\) as functions of \( \varepsilon_n \). Here, \( \varepsilon = 0.25 \), \( L_{\text{ex}} = 1 \) if not specified, the other parameters are the same with figure 8.
a small negative value at the rough range of $\approx (-0.8, 0)$. In addition, $\eta_{i}$ and $\tau_{e}$ has a minor and relatively greater effect on UMs, respectively. The second issue of this paper is the quantitative analysis of the parametric dependence of UM in the presence of impurity. The resultant evidences demonstrate that with the presence of impurity, the dependence of UMs on the main parameters, such as $\varepsilon$, $\hat{s}$, and $\varepsilon_{n}$, do not change qualitatively, but change a lot in quantity. In particular, comparatively, the stabilizing effects of impurity is more pronounced in the regime of larger wavenumber, while the weakening effects of (i) decreasing the fraction of trapped electrons (taking smaller $\varepsilon$), (ii) increasing magnetic shear ($\hat{s}$), or (iii) making the electron density more flatter (taking larger $\varepsilon_{n}$) on UM instability are relatively more pronounced in the smaller wavenumber region. Note that $\hat{s} > 0.5$ and $\varepsilon_{n} > 0.08$ is needed for the parameters referred here. Physically, the stabilizing effect of impurities on UMs is due to its heavier mass and the dilution of main ions, resulting in the change of the magnetic drift of the ions, which contributes to the non-resonant mechanism of UMs.

The above results probably give some implications in some considerations of experimentally stabilizing UMs. For example, in the core plasma in H-mode discharges [37–39], the broader flat density profile is quite beneficial to the stabilization of UMs. Besides, by simultaneously increasing the ion and electron temperature gradient, the bulk of UMs may be partly suppressed, or the UM instability window would become narrower. In addition, we could take advantage of the magnetic drift of the ions, which is effective by taking $L_{\text{ex}}$ within the domain of finite low negative values, here roughly estimated as $\gtrsim -0.8$. The significance of this issue on experiments is that a certain amount of impurities is suitable to peak in the edge region of the plasma. Of course, the negative gradient of impurities should not be too large, so as not to produce additional so-called IMs, which requires that the peak of impurity density profile should not be too steep; or say, the impurity density should rise slowly from the core to the edge plasma regimes.

Figure 11. (a) The thresholds of the normalized wavenumbers $(k_{0}\rho_{i})_{\text{th}}$ as functions of $\varepsilon$. The red solid, dashed and dotted lines correspond to the fitting results of the cases of (i) $f_{s} = 0$, and (ii) $f_{s} = 0.2$ and $L_{\text{ex}} = 1$, and (iii) $f_{s} = 0.2$ and $L_{\text{ex}} = -0.4$, respectively. The overall fitting result is $y \approx (k_{0}\rho_{i})_{\text{th}} \sim \left(\frac{L}{q_{\text{e}}(\varepsilon)}\right)^{2/5}$, corresponding to $(b)_{\text{th}} \sim \left(n/n_{\text{e}}\right)^{1/3}$. (b) The magnetic shear $\hat{s}_{i}$ for the onset of UMs as functions of $\varepsilon$, where $f_{s} = 0.2$, $k_{0}\rho_{i} = 2$. C$^{6+}$ is taken as impurity ion, $\eta_{i} = \eta_{e} = 0$, the other parameters are the same with those in figure 3.

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