Analysis of Data on Low-Energy $\gamma p$ Scattering
and Determination of Proton Polarizabilities

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Abstract

For the first time an analysis of all experimental data on the differential cross section of elastic $\gamma p$ scattering at photon energies $\omega < 150$ MeV is performed in order to determine the electric ($\alpha_p$) and magnetic ($\beta_p$) polarizabilities of the proton. A fit of these data with the two free parameters, $\alpha_p$ and $\beta_p$, embedded into a theoretical cross section obtained on the basis of finite-energy $s$-channel dispersion relations gives the following world-average values of the proton polarizabilities (in units of $10^{-4}$ fm$^3$):

$$\alpha_p^{exp} = 11.7 \pm 0.8 \text{ (stat. + sys.)} \pm 0.7 \text{ (model)}, \quad \beta_p^{exp} = 2.3 \pm 0.9 \text{ (stat. + sys.)} \pm 0.7 \text{ (model)},$$

where the first error takes into account statistical and systematic errors of the experimental cross sections and the second error does model uncertainties in the theoretical cross section.

I. Introduction

Experimental studies of elastic $\gamma p$ scattering began in the middle of 50’s years [1-6], but for the first time the proton polarizabilities $\alpha_p$ and $\beta_p$ were extracted from experimental cross sections at photon energies $\omega \leq 110$ MeV in the works [3,6]. Results of new experiments [7-10] on $\gamma p$ scattering and new determinations of the values of $\alpha_p$ and $\beta_p$ were published only in 90’s. However, statistical and systematic errors in the experimental values of the proton polarizabilities (especially in $\beta_p$ and $\alpha_p + \beta_p$) from the works [7-10] are yet not small.

This talk presents some results of our analysis concerning a compatibility of all data on elastic $\gamma p$ scattering at photon energies $\omega < 150$ MeV and the values of $\alpha_p$ and $\beta_p$ determined from different sets of the data.

II. Analysis of data and results

All experimental data on the differential cross section of elastic $\gamma p$ scattering at photon energies $\omega < 150$ MeV from FIAN, MAMI, SAL and other centres are split into two sets: 46 early data points of 1955-1974 [1-6] and 48 recent points of 90’s [7-10]. In our analysis we take into account that authors of three works [5-7] made corrections to their data afterwards. A compilation of these data, with corrections, and a description of the calculation of the theoretical cross section which we use for fitting the experimental data points are given in works [11,12].

An important point of our analysis is a combined use of the statistical and systematic experimental errors in a special way that was proposed in our earlier work [13]. The function $\chi^2$ used for fitting experimental cross sections is written as

$$\chi^2 = \sum_{j=1}^{N^{exp}} \left\{ \sum_{i=1}^{n_j} \left( \frac{k_j \sigma^{exp}_{ij} - \sigma^{th}_{ij}(\alpha, \beta)}{k_j \Delta^{exp}_{ij}} \right)^2 + \left( \frac{k_j - 1}{k_j \delta_j} \right)^2 \right\} \quad (1)$$
where \( j \) is the experiment number, \( i \) is the experimental point number, \( \sigma_{ij}^{\exp} \) and \( \sigma_{ij}^{\th} (\alpha, \beta) \) are the experimental and theoretical cross section at the photon energy \( \omega_i \) and the scattering angle \( \theta_i \), \( \Delta \sigma_{ij}^{\exp} \) is the statistical error of an individual point, \( \delta_j \) is the systematic error for the \( j \)-th experiment, \( k_j \) is a normalization factor to be found for the \( j \)-th experiment, \( \alpha \) and \( \beta \) are the proton polarizabilities to be found.

The first step of our analysis was in separate fits of experimental data from each of the quoted works. Results are shown in Table 1. Since experimental data in the works [4,8] are taken at fixed angle, two polarizabilities cannot be found and we fit those data using fixed \( \alpha_p + \beta_p = 14.0 \pm 0.5 \) [12] calculated from a dispersion sum rule.

The confidence levels, shown in the Table 1, are sufficiently high \( (P \geq 12\%) \). For this reason no experiment can be excluded from our following analysis. Below we assume the possibility to combine the data of different experiments [1-10] taking into account the errors and a spread in the found values of \( \alpha_p \) and \( \beta_p \).

Note that fitting of a subset of data from the work [10] obtained only with the tagged photons leads to too large values of \( \alpha_p \) and \( \beta_p \) and their errors (the last line in Table 1). Consequently, experiments with the bremsstrahlung photon beam have a certain advantage for a determination of \( \alpha_p \) and \( \beta_p \) as compared with tagged photon experiments, at least at the present level of statistical errors in the differential cross section of elastic \( \gamma p \) scattering.

The second step of the analysis was fitting two sets, all the early 46 points and all the recent 48 points. After that, a fit was done of all 94 points. The corresponding global-average polarizabilities are shown in Table 2. One can see that there is a satisfactory agreement between the early and recent global averages at the existing level of errors. Consequently, there are no reasons to ignore the early experiments, and it is better to combine all the experimental points. The averaged values of the proton polarizabilities show an appreciable decrease of their errors in comparison with individual errors given in the Table 1.

The third step of the analysis was in a variation of model parameters in the theoretical cross section obtained on the basis of finite-energy s-channel dispersion relations and in estimation of model errors in the extracted values of \( \alpha_p \) and \( \beta_p \). The most important contributions to the model errors at \( \omega < 150 \text{ MeV} \) (and at \( \omega < 100 \text{ MeV} \)) are shown in Table 3. Summing these contributions in quadrature we get approximate total model errors

\[
\Delta \alpha_p^{\mod} \approx \Delta \beta_p^{\mod} \approx 0.7 \quad \text{at } \omega < 150 \text{ MeV}
\]

and

\[
\Delta \alpha_p^{\mod} \approx \Delta \beta_p^{\mod} \approx 0.3 \quad \text{at } \omega < 100 \text{ MeV}.
\]

Finally, taking results from Table 2 (for all data) and adding the model errors we get the world-average values of the proton polarizabilities:

\[
\alpha_p^{\exp} = 11.7 \pm 0.8 \text{ (stat. + syst.)} \pm 0.7 \text{ (model)}, \quad (2)
\]

\[
\beta_p^{\exp} = 2.3 \pm 0.9 \text{ (stat. + syst.)} \pm 0.7 \text{ (model)}. \quad (3)
\]

It is seen that the experimental and model errors are comparable. The values (2) and (3) may be recommended for a general use and for PDG publications. Note that the current PDG values of the proton polarizabilities are based on results of only 4 recent experiments [7-10] and, moreover, they are obtained at the fixed sum \((\alpha_p + \beta_p)^{\th} = 14.2 \pm 0.5\) in order to lower real experimental errors.
In conclusion we would like to emphasize that in order to reduce the experimental and model errors in the extracted values of $\alpha_p$ and $\beta_p$ it is necessary to make new and more complete measurements of the differential cross section of $\gamma p$ scattering with the statistical and systematic errors $\sim 1-2\%$ \cite{14} in the energy region $\omega < 100$ MeV where the model errors are essentially lower.

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Table 1. Polarizabilities of the proton in units of $10^{-4}$ fm$^3$ extracted from data of different experiments [1-10]. $n$ is the number of experimental points. Uncertainties take into account both statistical and systematic errors of the experimental cross sections of $\gamma p$ scattering. $P$ is the confidence level for the shown $\chi^2$. The last line shows values extracted from a subset of data of [10] obtained only with the tagged photons.

| Experiment  | $n$ | $\alpha_p$ | $\beta_p$ | $\alpha_p + \beta_p$ | $\chi^2/N_f$ | $P(\%)$ |
|------------|----|------------|-----------|---------------------|--------------|--------|
| Oxl58 [1]  | 4  | 17.0±8.1   | -6.7±3.7  | 10.2±9.2            | 4.2/2        | 12     |
| Hym59 [2]  | 12 | 13.9±5.6   | -4.7±7.2  | 9.2±6.1             | 0.6/10       | 100    |
| Gol60 [3]  | 5  | 10.1±7.8   | 9.0±5.0   | 19.1±10.2           | 2.3/3        | 52     |
| Ber61 [4]  | 2  | 11.4±2.9   | 2.6±2.9   | -             | 0.7/1        | 41     |
| Fri67 [5]  | 16 | 14.2±4.0   | 5.6±4.2   | 19.8±4.3           | 2.4/14       | 100    |
| Bar74 [6]  | 7  | 11.4±1.4   | -4.7±2.5  | 6.7±3.3            | 8.0/5        | 15     |
| Fed91 [7]  | 16 | 13.7±3.7   | 2.1±3.1   | 15.9±4.4           | 17.3/14      | 24     |
| Zie92 [8]  | 2  | 10.0±1.4   | 4.0±1.4   | -             | 0.1/1        | 73     |
| Hal93 [9]  | 12 | 9.1±1.7    | 3.7±1.5   | 12.7±2.0          | 5.9/10       | 82     |
| Mac95 [10] | 18 | 12.2±1.7   | 3.3±1.8   | 15.5±3.1          | 7.4/16       | 97     |
| Mac95(tagged) [10] | 8  | 18.3±5.7   | 13.2±7.2  | 31.5±12.3         | 2.2/6        | 90     |
Table 2. The global-average proton polarizabilities in units of $10^{-4}$ fm$^3$ extracted from early data [1-6], from recent data [7-10], and from all the data. Errors take into account statistical and systematic errors of the experimental cross sections.

| Exp. | n  | $\alpha_p$  | $\beta_p$  | $\alpha_p + \beta_p$ | $\alpha_p - \beta_p$ | $\chi^2/N_f$ |
|------|----|-------------|-------------|-----------------------|-----------------------|--------------|
| 50-70’s [1-6] | 46  | 12.8±1.1   | -0.3±1.6   | 12.5±2.2             | 13.0±1.7             | 33.1/44      |
| 90’s [7-10]    | 48  | 10.8±1.0   | 3.2±1.0    | 14.0±1.6             | 7.7±1.2              | 33.7/46      |
| all [1-10]     | 94  | 11.7±0.8   | 2.3±0.9    | 14.0±1.3             | 9.5±1.0              | 73.1/92      |

Table 3. Uncertainties in the values of extracted polarizabilities (in units of $10^{-4}$ fm$^3$) related with a model dependence of the theoretical cross section. Changes shown in the extracted $\alpha_p$ and $\beta_p$ emerge from using different values of pion photoproduction amplitudes near threshold (SAID and HDT), from increasing the value of the resonance pion-photoproduction amplitude $M_{1+}$, from ignoring contributions of pion-pair photoproduction in dispersion integrals, from changing other parameters of the dispersion theory: $M_1$ (“$\sigma$-meson mass”), $g_{\pi NN}$, $\gamma_\pi^{(non-\pi^0)}$. Total model uncertainties are shown in the last line. The model uncertainties shown in parentheses are related to the case when only experimental data at $\omega < 100$ MeV are used.

| Changes | $\delta\alpha_p$ | $\delta\beta_p$ | $\delta(\alpha_p + \beta_p)$ | $\delta(\alpha_p - \beta_p)$ |
|---------|------------------|------------------|-------------------------------|-------------------------------|
| SAID $\rightarrow$ HDT | -0.44            | -0.08            | -0.51                         | -0.36                         |
| $M_{1+} \rightarrow +2\%$ | +0.18            | -0.11            | +0.06                         | +0.29                         |
| without $\pi\pi$ photoproduction | +0.00            | -0.09            | -0.09                         | +0.09                         |
| $M_1 = 500$ MeV $\rightarrow 700$ MeV | -0.35            | +0.53            | +0.19                         | -0.88                         |
| $| g_{\pi NN} F_{\pi\gamma\gamma} | \rightarrow +4\%$ | -0.14            | +0.10            | -0.04                         | -0.24                         |
| $\gamma_\pi^{(non-\pi^0)} = 5.5 \rightarrow 7.3$ | +0.40            | -0.45            | -0.05                         | +0.84                         |
| Total model uncertainty | 0.72             | 0.72             | 0.56                          | 1.33                          |

(0.34) | (0.29) | (0.17) | (0.62)