Effect of changes in the crank radius on the kinematics of the pumping unit when regulating the dynamic level of the fluid in the well

A Kh Gabzalilova¹, N N Soloviev² and Z A Garifullina¹

¹Ufa State Petroleum Technological University, Branch of the University in the City of Oktyabrsky, 54a, Devonskaya St., Oktyabrsky, Republic of Bashkortostan, 452607, Russia
²LLC Gazprom VNIIGAZ
Staraya Basmanaya str., 20, b. 8, Moscow, Russia

E-mail: alfiragabzalilova@mail.ru

Abstract. The article deals with the effect of changes in the crank radius on the kinematics of the pumping unit when regulating the dynamic level of the fluid between the tubing strings and the casing in order to increase the well production rate or reduce the accident rate. To improve the quality of prediction, the movement of a polished rod over time is calculated using the Fourier variable separation method: the product of the rocker head of a pumping unit determined as movement of a slider crank mechanism along the straight line (as a circle chord), and the difference of this chord with a length of the circumference equivalent to the movement of the swivel joint of the balancer with a connecting rod. As a result, the accuracy of calculations is increased and the solution of the problem is simplified.

1. Introduction

With an increase in the crank radius, the stroke of the pump plunger increases which may be accompanied by an increase in the flow rate of the well. When decreasing, by reducing the stroke of the plunger of the submersible pump, the flow rate decreases, the amplitude of the longitudinal oscillations of the column decreases which helps reduce the accident rate for the column rod.

Non-linearity of the movement of the polished rod in time under uniform rotation of the crank can be determined using two methods:
- the analytical method based on the kinematics of the "crank - connecting rod - balancer - polished rod suspension cable" system changing in time when regulating the stroke length of the upper part of the rod string and the number of double rocking pump moves;
- the graphic method involving construction of positions of the main links of the rod drive mechanism, speed and acceleration plans for different angles of crank rotation (Figure 1) rotating at a constant angular speed rate.

Both methods have to account for geometrical dimensions of the main elements of the rod string drive.
Figure 1. Nonlinearity of the kinematics of the pumping unit rod drive (movements of polished rod \( h^\uparrow \), \( h^\downarrow \) and crack \( \beta^\uparrow \) and \( \beta^\downarrow \) ): a) general scheme for symmetrical (\( r = r_o, \theta = 0, \beta^\uparrow = \beta^\downarrow \) ) up and down movements; b) asymmetrical (crack radius \( r > r_o \); \( \theta < 0 \); \( \beta^\uparrow < \beta^\downarrow \) ) movement; c) asymmetrical (\( r < r_o, \theta > 0 \); \( \beta^\uparrow > \beta^\downarrow \) ) movement; 1 – crack; 2 – rod, 3 – balance; 4 – rope; 5 – polished rod.

Unfortunately, the methods used in field practice [1, 2] to account for the influence of the changes in the pumping crank radius on the fluid dynamics between the tubing and casing columns are rarely used.

2. Results and discussion

Figure 1 shows the kinematic diagram of the crank mechanism of the pumping unit (without an extension of the rod stroke) used in domestic fields. It characterizes the non-linearity of up \( h^\uparrow(t) \) and down \( h^\downarrow(t) \) movements of the polished rod for time \( t^\uparrow \) at the total value of the longitudinal movement \( H_{to} \) which \( T / 2 \) equivalent to crack rotation \( \beta^\uparrow \) at \( \pi \) corresponds. Figure 1 shows the effect of an increase (Figure 1, b) or a decrease (Figure 1, c) in the crank length (when adjusting the stroke length of a polished rod) on the change in the dependence (nonlinear in time) of the displacement angle \( \delta^\uparrow \) balance 3 from crack rotation angle \( \beta^\uparrow \) rotation at a constant angular speed rate

\[
\omega_{c-k} = \frac{2\pi n}{60},
\]

where \( n \) is the pump rate.

Unfortunately, the approximate simplified and specified [3] methods for accounting for the influence of the kinematics of the crank mechanism of the pumping unit do not take into account the effect of changes in the crank radius on a number of basic parameters of the rod string displacement. In addition, the approximate and specified methods of calculation are based on an exaggerated (for a rocking machine) mechanism of a crank-connecting rod mechanism with a slide moving along the straight line \( A_1A_2 \) rather than along the arch \( A_1A_2(\gamma) \) (Figure 1).

Crack rotation angles \( \beta^\uparrow \) correspond to angles of rotation of the balance \( \delta^\uparrow \). Depending on the displacement (inclination at a horizontal position of the balancer) of the lower end of the connecting rod 2 (point B) relative to the upper one (point A) to the right (Figure 1) or to the left of the vertical line, the total values of the angles of rotation of the crank, when the polished rod moves by \( H_{to} \) change within the limits (upper signs before \( \theta \) comply with conditions \( r > r_o \), lower ones - with \( r < r_o \) ):

- when moving the polished rod up

\[
\beta^\uparrow = 0^\circ \cdot \ldots \cdot (180^\circ \mp \theta),
\]

- when moving the polished rod down
\[ \beta^i = 0^\circ \ldots (180^\circ \pm \theta). \]

where \(\theta\) is the asymmetry of the movement of the balance relative to the horizontal caused by violation of its symmetrical movement (characterized by the crank drive mechanism operation according to figure 1, b and figure 1, c) as a result of changes in the crank radius when adjusting the stroke of the polished rod to the required plunger pump performance.

\[ \text{At} = r_o \theta = 0, \beta^i = \beta^i t , \text{ up and down movements of polished rod 5 (Figure 1, a) \(\beta^i\)} \]

become symmetrical with respect to rotation of crank 1. In this case, the synchronism of movement of the polished rod and the crank mechanism corresponds to a crank radius equal to \(r = r_o\). under the condition of equivalence of the movement of point A of the balance along the straight line

\[ \Delta_1 \Delta_2 = 2r_o = 2R \sin(\Delta^0/2). \] (1)

Therefore, when adjusting the crank radius, one should take into account the effect of pump performance and the well rate on the change in the level of produced formation fluid.

3. Experiment

Let us consider the dependence of the change in the movement of the polished rod (and, accordingly, in the upper part of the rod column) on the crank radius \(\text{atr} \neq r_o\).

Since the rotation of the balance head to the polished rod is transmitted through the cable suspension, to match the calculation results with a relatively more cumbersome specified solution (at \(r = r_o\)), the dependence of movements along the arc \(A_1A_2(\gamma)\) from the straight line \(A_1A_2\) (Figure 1) is calculated using the Fourier separation method.

The kinematic scheme for moving the main drive elements of the polished rod, and the upper part of the rod string presented in figure 1 is based on the uniform crack rotation in time. The angle of rotation of the crack after the beginning of the movement of the polished rod up or down [4] during time \(t\) is equal to

\[ \beta^i = \omega_{c-k} t = 2 \pi n t / 60, \] (2)

where \(\nu_{c-k}\) is the crack movement frequency; \(\omega_{c-k}\) is the circular frequency; \(n\) is the number of double strikes (up and down) of the pumping unit.

Up and down movements of the polished rod are determined by the dependence of the angles of rotation of the balance \(\delta_{up}^i\) (from points \(A_1\) or \(A_2\)) and crack \(\beta^i\) (from points \(B_1\) or \(B_2\)) corresponding to the beginning of up or down movement of the polished rod (Figure 1) and determined by the design of the pumping unit. The domestic field practice calculates the dependence \(\delta_{up}^i\) on \(\beta^i\) based on the sine theorem for variable deformation of triangle \(ABO\) in time (Figure 1) accounting for movements of point A of the balance arm joints \(O_1\) with radius \(R\) along the straight \(A_1A_2\) [5] rather than along the arc \(A_1A_2\). Up and down movements of point A and the polished rod \(\beta^i\) correspond to the angle of rotation of the balance over time by

\[ \Delta = \delta^i_{up} \bigg|_t = T/2, \]

where

\[ T/2 \approx 60/(2n). \] (3)

--time corresponding to the full up and down movement of the polished rod.

The value of movement along \(A_1A\) characterizing the relation between the crack rotation angle and movement of the polished rod for \(r = r_o\), based on the assumption that \(r/R \to 0\), is calculated by the formula according to which path length \(S_{A1}\) from point \(A\) to crack rotation angle \(\beta^i\) is equal to the segment \(A_1A\), i.e. (Figure 1) the value of movement of the polished rod [6] is

\[ h^i \to A_1A \times A_2 = S_{B} = (r + L_x) - A_2 - A_1 = (r + L_x) - r \cos \beta^i \cos \gamma_{up} = r(1 - \cos \beta^i) + L_x (1 - \cos \gamma_{up}). \] (4)

As far as the value of formula (4) corresponds to \(h^i(t)\) movement of the end of the balance along the straight line \(A_1A_2\), to go the arc length \(A_1A_2\) (i.e. \(toh^i(t) = h^i(\gamma)\)) using the Fourier method of separation of variables, we will proceed from the geometric differences in determining the difference between the lengths of \(A_1A_3(\gamma)\) and straight line \(A_1A_2\) (Figure 1, a). Based on this
condition and given that $\Delta h_{|\beta| = \Delta / 2} << H$, to calculate the value of movement along the arc we have the following

$$h^{\uparrow\downarrow}(t) = h^{\uparrow\downarrow} (\gamma) = \frac{\Delta h}{\sin(\Delta / 2) \cdot \sin(\omega_{c-k} t^{\downarrow})} \cdot \sin(\Delta / 2) \cdot \sin(\Delta / 2)$$

or the variant of the formula is chosen depending on the problem to be solved

$$h^{\uparrow\downarrow}(t) = h^{\uparrow\downarrow} (\gamma) = \frac{\Delta h}{\sin(\Delta / 2) \cdot \sin(\omega_{c-k} t^{\downarrow})} = \frac{\Delta h}{\sin(\Delta / 2) \cdot \sin(2 \pi n t^{\downarrow})}.$$

For the general case (Figure 1, b and figure 1, c)

$$h^{\uparrow\downarrow} = \frac{(\pi / \Delta) \gamma^{\uparrow\downarrow} = 0 ... 90^\circ},$$

$$\gamma^{\uparrow\downarrow}(t) = \omega_{c-k} t^{\downarrow} \theta = 2 \pi n c-k t^{\downarrow} \theta = (2 \pi n t^{\downarrow} / 60) \neq 0,$$

$v_{c, k}$ – crank rotation speed; $\omega_{c-k}$ – circular frequency; $n$, $n$ – the number of double strikes of the pumping unit.

To analyze the changes in loads on a polished rod in time $t$, we will consider the process of moving the joint point A of the connecting rod AB with the crank $O_{2}B$ (Figure 1) along the arc $A_{1}A_{2}$ arc ($\gamma$) for a symmetric (at $r = r_{o}$, figure 1, a) mode of operation of the pumping unit. Let us present expression (6) in a more visual and convenient form based on the method of separation of Fourier variables [5]:

$$h^{\uparrow\downarrow} = \frac{\Delta h^{\uparrow\downarrow}}{\pi \Delta} \cdot 2 \pi t^{\downarrow} \cdot A_{1} A_{2}(\gamma).$$

where

$$h^{\uparrow\downarrow} = \frac{\Delta h^{\uparrow\downarrow}}{\pi \Delta} = 0 ... [2 R_{z} \sin(\Delta / 2)]$$

corresponds to the approximate solution (6.25), while

$$\Delta h^{\uparrow\downarrow}(\gamma) = \frac{\Delta A_{1} A_{2}(\gamma)}{\pi \Delta} = 0 ... [2 \pi n R_{z} (\Delta / 180^\circ) / \pi t^{\downarrow}]$$

Is a coefficient accounting for changes of variable $h^{\uparrow\downarrow}$, corresponding to line $(A_{1} A_{2})$ moving along $\arctan[A_{1} A_{2} / \sin(\Delta)]$ (by analogy with the discrete Fourier transform used in the harmonic analysis of functions set on a discrete set of points[7]).

In (7) at $\Delta^{\uparrow\downarrow} = \Delta / 2$ the maximum values of relations are

$$\frac{\Delta A_{1} A_{2}}{H_{z}} = \frac{\sin(\Delta / 2)}{\pi \Delta} \cdot 180^\circ.$$

Due to the cable between the balance head [8] and the polished rod, the movement of the upper part of the rod string $h^{\uparrow\downarrow}(t)$ is equal to the arc length $R_{z}$ at the balance rotation angle $\beta^{\uparrow\downarrow}$. In this case, when rotating the crank with a constant angular speed $\omega_{c-k}$, of the crank mechanism of the pumping unit (according to Figure 1), the ratio

$$h^{\uparrow\downarrow} / H_{z} = \frac{\Delta t^{\downarrow}}{(T / 2)} = \frac{\beta^{\uparrow\downarrow}}{\pi A},$$

is typical where

$$a \tau > r_{o} (Figure 1, a)$$

$$\theta = 0, \quad t^{\downarrow} = t^{\uparrow}, \quad B^{\uparrow\downarrow} = 180^\circ$$

$$a \tau < r_{o} (Figure 1, b)$$

$$\theta = 0, \quad t^{\downarrow} = t^{\uparrow} - \Delta t, \quad B^{\uparrow\downarrow} = 180^\circ / \Delta t$$

$$(a \tau = r_{o} (Figure 1, c)$$

$$\theta = 0, \quad t^{\downarrow} = t^{\uparrow} + \Delta t, \quad B^{\uparrow\downarrow} = 180^\circ / \Delta t$$

According to Figure 1, a and (1) at $\tau = r_{o}$ the value

$$r_{o} = \frac{\Delta A_{1} A_{2}}{H_{z} / 2} = \frac{R_{z} \sin(\Delta / 2)}{2 \pi R_{z}}$$

The maximum distance of arc $A_{1}A_{2}$ from line $A_{1}A_{2}$ by balance radius $R_{z}$ is $\Delta R_{z, \text{max}}$. The maximum distance at $\Delta^{\uparrow\downarrow} = \Delta / 2$ is
\[ \Delta R_{\text{max}} = \Delta R|_{\delta = \Delta / 2} = R_z[1 - \cos(\Delta / 2)], \]  

(15)

As for real pumping units, \( \Delta R_{\text{max}} < \hbar \), at an arbitrary moment of crank rotation (Figure 1), we have

\[ \beta \downarrow \uparrow = 180^\circ, \ t \downarrow \uparrow \cdot \nu_{c-k} = 0 \ldots (180 \mp \theta) \] and \( \sin \delta \downarrow \uparrow |_{\Delta R_{\text{max}} << \hbar} \approx \sin \omega_{c-k} t. \]  

(16)

The value \( \hbar \uparrow \downarrow / \hbar \downarrow \downarrow \) is approximated accounting for expression (11) per one semi-revolution of the crack (at \( \Delta R_{\text{max}} << \hbar \)) in the form (at \( r \approx r_0 \))

\[ \hbar \downarrow \uparrow(t) / \hbar \downarrow \uparrow(t) = [( \bar{A}_1 \ A_2) / H_2][1 - \sin \omega_{c-k} \downarrow \downarrow t] = [( \bar{A}_1 \ A_2) / H_2][1 - \sin \delta \downarrow \uparrow(t)] = \]

\[ = \{(R_z / R) \ r[(1 - \cos \omega_{c-k} \downarrow \downarrow t + r / 2 \ l_\omega \ \sin^2(\omega_{c-k} t \downarrow \downarrow)] / H_2\}[1 - \sin \omega_{c-k} t]. \]  

(17)

where \( \bar{A}_1 / \bar{A}_2 = \hbar \downarrow \uparrow(t) \) is determined by approximated formula (4) or expression

\[ \hbar \downarrow \uparrow(t) = \pi R_z \ (\delta \downarrow \uparrow / 180^\circ)_{r = r_0}. \]  

(18)

The approximation error (13) in calculating \( \hbar \downarrow \uparrow \) does not exceed several percent when determining intensity of fatigue failure of rods. If necessary, \( \delta \downarrow \uparrow \) in (13) can be replaced by a more accurate trigonometric series.

In this case, according to (17) and (9), an additional decrease in the value of movements of the polished rod \( \hbar \downarrow \downarrow \) during initial up and down movements can be presented at \( r \approx r_0 \) as

\[ \hbar \downarrow \uparrow |_{r = r_0 << l_\omega} = ( \bar{A}_1 \ A_2) = 2 \ R_z \sin (\Delta / 2), \]  

(19)

\[ \hbar \downarrow \uparrow(\gamma) = [A_1 A_2(\gamma)] = 2 \pi R_z (\Delta / 180^\circ). \]  

(20)

According to (11) and figure 1

\[ \hbar(t) \uparrow / H_2 = \delta(t) \downarrow \uparrow / (\Delta \theta) = \beta(t) \downarrow \uparrow / 180^\circ = t \downarrow \uparrow \cdot \nu_{c-k}, \]  

(21)

i.e. dependencies of relative movement values \( \hbar(t) \uparrow \), \( \hbar(t) \downarrow \downarrow \), \( \delta(t) \downarrow \uparrow \), \( \beta(t) \downarrow \uparrow \) of the elements of the pumping unit are synchronous in time. The equivalence sign in expressions (11) and (21) means the need to take into account the dependencies of movements of the polished rod \( \hbar(t) \uparrow \) and balance rotation angle \( \gamma(t) \uparrow \) on crack rotation angle \( \beta(t) \uparrow \) accounting for the design of the pumping unit.

The period of formation of the longitudinal wave [9] \( \hbar(t) \uparrow \uparrow \) can be determined by the dynamogram of loads on the polished rod or based on the period between the beginning of a change in the influence of pressure on the dynamic fluid level between the tubing string and casing (after closing the injection valve of the plunger pump or opening the suction) and the end of the transfer of pressure to the wellhead. The period of transmission of the pressure change from the pump plunger to the wellhead is determined by the time of movement of this change in the liquid column from the pump plunger to the wellhead [10, 11]. This is due to the fact that the speed of movement of the longitudinal wave along the liquid column is less than that of the rod string [5]. According to (18), (11), and (14), balance rotation angle \( \delta(t) \downarrow \uparrow \) corresponding to the period of formation of the longitudinal wave is determined by expression

\[ \delta \downarrow \uparrow(t)|_{r = r_0} = [h \downarrow \uparrow(t) / (\pi R_z)] 180^\circ. \]  

(22)

Using \( \delta \downarrow \uparrow(t)|_{r = r_0} \)(22), we can determine the ration between time \( \hbar(t) \downarrow \downarrow(\gamma) \) for motion of the longitudinal wave [12] after closing the discharge valve of the plunger pump or opening the suction and the value of the longitudinal movement of the polished rod \( \hbar(t) \uparrow \) (Figure 1).

The difference between the values of the longitudinal movement of the polished rod (Figure 1)

- along line \( A_1 \ A_2 \) equal to

\[ \hbar \downarrow \uparrow |_{r = r_0 << l_\omega} = ( \bar{A}_1 \ A_2) = 2 \ R_z \sin (\Delta / 2); \]  

(23)

- along arc \( A_1 A_2(\gamma) \) equal to

\[ \hbar \downarrow \uparrow(\gamma) = [A_1 A_2(\gamma)] = 2 \pi R_z (\Delta / 360^\circ). \]  

(24)

At \( \delta = \Delta / 2 \) the difference between them is

\[ \Delta h|_{\delta = \Delta / 2} = h \downarrow \uparrow(\gamma) / 2 - h \downarrow \uparrow / 2 = R_z[(\pi (\Delta / 360^\circ) - \sin (\Delta / 2)]. \]  

(25)
In this case, according to the method of separation of Fourier variables, with separation of solutions for determining the movement of the polished rod along the straight line (A1 A2) and arc [A1A2(γ)] at t, the total value of movement of the rod when the balance rotation angles δ↑↑ and crack rotation angles β↑↑↑↑are equal at

\[ \delta^{↑↑}(t) = \omega_{c-k}t^{↑↑↑↑+0} = 2 \pi v_{c-k}t^{↑↑↑↑+0} = (t^{↑↑+0} \cdot 2 \pi n / 60)t, \]

can be presented as

\[ h^{↑↑}(\gamma) = \bar{h}^{↑↑} \cdot [1 + \Delta h|_{\delta = \Delta / 2} \cdot \sin \delta^{↑↑}(t)]. \tag{26} \]

or

\[ h^{↑↑}(\gamma) = (R_2/R)\bar{r}[(1 - \cos(\omega_{c-k}t)) + (r/2 L_\alpha \sin^2(\omega_{c-k}t)) \cdot [1 + \Delta h|_{\delta = \Delta / 2} \cdot \sin(\omega_{c-k}t)]. \tag{27} \]

For the more general case (\(\omega \neq r_0\) – Figures 1, bandc) replacing(\(\omega_{c-k}t\))by(\(\omega_{c-k}t \mp 0\)) and expanding (26) in relation to\(\Delta h|_{\delta = \Delta / 2}\) we have a more complete formula for non-linear movement of the polished rod in time:

\[ h^{↑↑}(\gamma) = (R_2/R)\bar{r}[(1 - \cos(\omega_{c-k}t \mp 0)) + (r/2 L_\alpha \sin^2(\omega_{c-k}t \mp 0)) \cdot [1 + R_2(\pi(\Delta / 360°) - \sin(\Delta / 2)) \cdot \sin(\omega_{c-k}t \mp 0)]. \tag{28} \]

4. Conclusion

The solution allows for assessment of the effect of changes in the radius of the crank of the pumping unit on the kinetostatic movement of the rod string and the pump plunger affecting the pump performance and the dynamic fluid level as well as a number of other well characteristics.

References

[1] Yakupov R F, Mukhametshin V Sh and Tyncherov K T Filtration model of oil coning in a bottom water-drive reservoir Periodico tche quimica 15(30) 725–33
[2] Mukhametshin V V 2018 Rationale for trends in increasing oil reserves depletion in Western Siberia cretaceous deposits based on targets identification Bull. of the Tomsk Polytechnic Univer. Geo Assets Engineering 329(5) 117–24
[3] Korn G A and Korn T M 1984 Mathematical Handbook for Scientists and Engineers: Definitions, Theorems, and Formulas for Reference and Review (Moscow: Nauka)
[4] Quan Hui et al 2017 Mathematical modeling for the evolution of the large- and meso-scale vortex in the screw centrifugal pump with the buoyancy effect Adv. Mech. Eng. 9(5)
[5] Yan Di et al 2017 Numerical modelling of twin-screw pumps based on computational fluid dynamics Proc. of the Institution of Mechanical Engineers Part C. J. of Mechanical Engineering Sci. 231(24) 4617–34
[6] Dennis G Z 2013 A First Course in Differential Equations (Los Angeles: Ricard Stratton)
[7] Yuksel S 2014 Differential Equations for Engineering Science (Canada: Queen’s University)
[8] Suleimanov R I, Gabdrakhimov M S, Khabibullin M Y, Zaripova L M and Vasilyeva E R 2018 The study of hydraulic hammer device in drilling tool assembly in hydraulic rotary drilling Int. J. of Engineer. and Technol. 7(2) 28–30
[9] Habibullin M Ya and Sidorkin D I 2016 Determination of tubing string vibration parameters under pulsed injection of fluids into the well SOCAR Proc. 3 27–32
[10] Almukhametova E M, Fattakhov D I, Zakirov A I and Safiullina A R 2018 The analysis of the hydraulic fracturing efficiency at the Potochnoe field facility AV1-2 IOP Conf. ser. Earth Env. 194(8) 082004
[11] Almukhametova E M, Zakirov A I, Fattakhov D I, Faizullin A A and Safiullina A R 2018 Analysis of the technology of intensifying oil production through the bottomhole formation zone treatment in the Potochnoe field IOP Conf. ser. Earth Env. 194(8) 082005
[12] Ibragimov N G, Fattakhov I G, Kuleshova L S, Kadyrov R R, Sakhapova A K and Khamidullina E R 2011 New dedicated software determines water production behavior Oil industry 7 48–9