Determination of return period for flood frequency analysis using normal and related distributions

Wan Husna Aini Wan Deraman¹, Noor Julailah Abd Mutalib¹ and Nur Zahidah Mukhtar²

¹Faculty of Electrical and Automation Engineering Technology, TATI University College, Teluk Kalong, 24000 Kemaman, Terengganu, Malaysia
²Faculty of Manufacturing Engineering Technology, TATI University College, Teluk Kalong, 24000 Kemaman, Terengganu, Malaysia
E-mail: husna@tatiuc.edu.my

Abstract. Flood frequency analyses are used to predict design floods for sites along a river. The technique involves using the observed annual maximum rainfall depths data to calculate statistical information such as mean values, standard deviations, skewness, and recurrence intervals. This research is to determine the return period for annual maximum rainfall depths data for two sites in Kemaman, which Jabor and Air Putih. All the data at two sites of study area are yearly data and the data of rainfall depths are taken starting from 1985 until 2015. The distributions used in this study are Normal, two-parameter Lognormal (LN(2)) and three-parameter Lognormal (LN(3)) distributions. The first step is to find the parameter estimates of rainfall depths in these two sites for each distribution. The parameters are estimated using the Method of Moments (MOM) and Probability Weighted Moments (PWM). Then, the best methods for each distribution in each site are obtained by using the performance measurements. Based on the best method for each distribution, the comparisons for the best method for each site are made. The results show that the best distribution that fits the characteristics of performance indicators for Jabor and Air Putih are two-parameter Lognormal distribution and three-parameter Lognormal distribution, respectively. Then, the last step is to determine the return period for the best distribution for 2, 5, 10, 50, and 100 years. Lastly, for the return period of 2, 5, 10, 50, and 100 years, it can be concluded that Jabor has the higher annual maximum rainfall depths than Air Putih.

1. Introduction
In many parts of the world, floods due to sudden snow melt and heavy rain storms cause loss of life and property. In Malaysia, floods are regular natural disasters which happen nearly every year during the monsoon seasons. The recent monsoon flood which has occurred in December 2014 was the largest floods to hit Malaysia in recent decades, with more than 100,000 flood victims evacuated from their homes (Reuters, 2014). Floods are also reported as the natural disaster that gives the highest percentage of economic damages compared to others (Baskar and Baskar, 2009). With 189 water basins in Malaysia and an average rainfall of over 2000mm per year, Malaysia is prone to flooding. Flooding has been a larger concern today due to rapid development in the river catchment area which increases the river runoff and decreasing the river capacity.

A return period is known as a recurrence interval which is an estimate of the interval of time between events like an earthquake, flood or river discharge flow of a certain intensity or size (Rakhecha and Singh, 2009). It is a statistical measurement denoting the average recurrence interval
over an extended period of time and to dimension structures so that they are capable of withstanding an event of a certain return period (with its associated intensity). The terms "10 year", "50 year", "100 year" and "500 year" floods are used to describe the estimated probability of a flood event happening in any given year (Watson and Adams, 2010). Using historic weather, experts derive the estimated rate of flow or discharge of a river or creek. A 10 year flood has a 10 percent probability of occurring in any given year, a 50 year event a 2% probability, a 100 year event a 1% probability, and a 500 year event a 0.2% probability. While unlikely, it is possible to have two 100 or even 500 year floods within years or months of each other.

Flood frequency analysis is the procedure for estimating the frequency of occurrence (return period) of a hydrological event such as flood. The technique involves using observed annual peak flow discharge data to calculate statistical information such as mean values, standard deviations, skewness and recurrence intervals. These statistical data are then used to construct frequency distributions, which are graphs and tables that tell the likelihood of various discharges as a function of recurrence interval or exceedence probability. Though the nature of most hydrological events such as rainfall is erratic and varies with time and space, it is commonly possible to predict return periods using various probability distributions (Singh and Yadava, 2003). In particular, analysis of annual maximum rainfall of different return periods (typically 2 to 100 years) is a basic tool for safe and economic planning and design of small dams, bridges and drainage work as well as for determining drainage coefficients (Bhakar, Bansal, & Chhajed, 2008).

There are many research were carried out for flood frequency analysis. Seckin et al., (2009) have studied about flood frequency analysis in Ceyhan River basin in May 2006. In that research, they compared the probability weighted moments (PWM) method and maximum likelihood (ML) method in order to find the best method in estimating the parameters. The longer the period of the observed flood peak series, the more realistic the results of the flood frequency analysis, because the parameters of the probability distribution functions estimated from longer sample series tend to be close to their population values. Parameters of four probability distribution functions which are Log-Pearson Type 3 (LP3), three-parameter Lognormal (LN3), Generalized Extreme Value (GEV) and Wakeby distribution functions were used to find the frequency relationship between the peak flood discharges and return periods. Kolmogorov-Simirnov, Cramer von Misses and Chi-Square goodness-of-fit (GOF) tests were used to evaluate the performances of PWM methods and ML method. Lastly, the result showed that LN3 was the best distribution that fits the data by using PWM method.

Meanwhile, Ibrahim & Isiguzo, (2009) from Kwara State, Nigeria have studied about the flood frequency analysis of Gurara River catchment at Jere, Kaduna State, Nigeria. The analysis was based on seventeen years daily discharge data converted from gauge height readings. Extreme Value Type 1, Normal, Exponential and Pearson Type 3 distributions were used in the study. The objectives of this research were to know the best distribution function for the data and the return period of the rainfall depths in that study area. Lastly, of the four probability distributions employed, they came up with Pearson Type 3 distribution as the best distribution that describes the data. Barkotulla et al., (2009) have studied about characterization and frequency analysis of consecutive days maximum rainfall at Boalia, Rajshahi and Bangladesh. Chi-square goodness-of-fit were used to determine the best probability distribution among Normal, Lognormal, and Gamma distributions. In this study, the results showed that the Lognormal distribution was the best fit probability distribution for one day and two to seven consecutive days annual maximum rainfall for the region.

Vivekanandan, (2015) have studied about frequency analysis of annual maximum flood discharge using Method of Moments and Maximum Likelihood Method of Gamma and Extreme Value Family of Probability Distribution. Method of Moments and Maximum Likelihood Method are used for determination of parameters of six probability distributions. Chi Square and Kolmogorov-Smirnov were used to test for the adequacy of fitting the best probability distributions. The results showed that the best distributions at Dedtalai and Ghala were exponential distribution (using MLM) and gamma distribution (using MLM), respectively. Omran et al., (2014) have studied about analysis of rainfall records of some Iraqi Eteorological Stations. Data of annual rainfall depth for four meteorological
stations in Iraq are analyzed to determine the characteristic of the observed frequency distributions. The distributions used are Normal, Log Normal, Log Normal type III and Gamma distribution. The Chi-Square and Kolmogorov-Smirnov indices and the graphical goodness of fit tests are applied to compare each of the theoretical distributions. Gumbel’s extreme value distribution, the Normal and the Log Normal distributions are used to fit the annual extremes rainfall data with 5, 10, 15 and 50 years return periods.

2. Material and methods

2.1 Study area
The yearly rainfall depths data starting from 1985 until 2015 was provided by the National Hydrological Network Management System. The data was collected from two sites in Kemaman which is Jabor and Air Putih.

![Figure 1](image1.png) The location of Kemaman in Peninsular Malaysia.

![Figure 2](image2.png) The location of two sites in Kemaman.

2.2 Probability distribution and parameter estimators
This research is to determine the return period of rainfall depths based on the best distribution that fits the data. The distributions used in this research are Normal, two-parameter Lognormal (LN(2)) and three-parameter Lognormal (LN(3)) distributions. All the parameters of the distributions are estimated using the Method of Moments (MOM) and Probability Weighted Moments (PWM). Table 1 shows the probability density function and the parameter estimators.
Table 1. Probability density function (PDF) and its parameter estimators.

| Distribution | Probability density function | Parameter estimators | MOM | PWM |
|--------------|------------------------------|----------------------|-----|-----|
| Normal       | $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-x')^2}$ | $\hat{\mu} = m'_1$ | $\hat{\mu} = l_1$ | $\hat{\sigma} = \sqrt{m_2}$ | $\hat{\sigma} = \sqrt{l_2}$ |
|              |                              | $\hat{\sigma} = \sqrt{m_2}$ |                              |                              |                              |
| Two-Parameter Lognormal (LN(2)) | $f(x) = \frac{1}{x\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(\log(x)-\mu)^2}$ | $\hat{\sigma}_y = \left( \log \left( \frac{m_2}{m'_1} \right)^{1/2} + 1 \right)$ |                              | $\hat{\sigma}_y = 2erf^{-1} \left( \frac{l_2}{l_1} \right)$ |
|              |                              | $\hat{\mu}_y = \log m'_1 - \frac{\hat{\sigma}_y^2}{2}$ |                              | $\hat{\mu}_y = \log l_1 - \frac{\hat{\sigma}_y^2}{2}$ |
| Three-Parameter Lognormal (LN(3)) | $f(x) = \frac{1}{(x-a)\sigma\sqrt{2\pi}} \left\{ e^{-\frac{1}{2\sigma^2} \left[ \log(x-a)-\mu \right]^2} \right\}$ | $\hat{\sigma}_y = \left[ \left\{ 1 + \frac{1}{2} \left( g_1^2 + G \right) \right\}^{1/3} + \left\{ 1 + \frac{1}{2} \left( g_1^2 - G \right) \right\}^{1/3} \right]^{1/2}$ | $\hat{\sigma}_y = 0.999281z$ | $\hat{\sigma}_y = -0.006118z^3 + 0.000127z^5$ |
|              |                              | $\hat{\mu}_y = \frac{1}{2} \log \left[ \frac{m_2}{e^{\hat{\phi}_2} (e^{\hat{\phi}_2} - 1)} \right]$ |                              |                              |
|              |                              | $\hat{\mu}_y = \log \left[ l_2 / erf \left( \frac{\hat{\sigma}_y}{2} \right) \right] - \frac{\hat{\sigma}_y^2}{2}$ |                              |                              |
|              |                              | $a = m'_1 - e^{-\frac{\hat{\phi}_2}{2}}$ | $a = l_1 - e^{-\frac{\hat{\phi}_2}{2}}$ | $G = g_1 \left( 4 + g_1^2 \right)^{1/3}$ |
|              |                              | $\hat{\mu}_y = \frac{1}{2} \log \left[ \frac{m_2}{e^{\hat{\phi}_2} (e^{\hat{\phi}_2} - 1)} \right]$ |                              |                              |
|              |                              | $a = m'_1 - e^{-\frac{\hat{\phi}_2}{2}}$ |                              |                              |
|              |                              | $\hat{\mu}_y = \frac{1}{2} \log \left[ \frac{m_2}{e^{\hat{\phi}_2} (e^{\hat{\phi}_2} - 1)} \right]$ |                              |                              |
|              |                              | $a = l_1 - e^{-\frac{\hat{\phi}_2}{2}}$ |                              |                              |

Notation: $m'_1 = \bar{x} = \text{sample mean}$, $\sqrt{m_2} = s = \text{standard deviation of sample data}$, $l_1 = \text{first L-moments}$, $l_2 = \text{second L-moments}$, $t_3 = \text{L-skewness}$, $a = \text{lower bound}$
2.3 Performance indicators
This study used six performance indicators to determine the best distribution to represent the data. The error measures used in this study is Normalized Absolute Error (NAE), Root Mean Square Error (RMSE) and Mean Absolute Error (MAE). The model is deemed to be the best model as the value of error measures is approaching 0. Meanwhile, the accuracy measures are predictive accuracy (PA), coefficient of determination ($R^2$), and Index of Agreement (IA). The accuracy value is between 0 and 1 and as the value approaches 1, the model is appropriate. Table 2 lists the performance indicators and their formula used in this study.

| Indicators                  | Equations                                                                 |
|-----------------------------|---------------------------------------------------------------------------|
| Normalized absolute error   | \( NAE = \frac{\sum_{i=1}^{n}|P_i - O_i|}{\sum_{i=1}^{n}O_i} \)          |
| Root mean square error      | \( RMSE = \sqrt{\frac{\sum_{i=1}^{n}(P_i - O_i)^2}{n-1}} \)              |
| Mean absolute error         | \( MAE = \frac{\sum_{i=1}^{n}|P_i - O_i|}{n} \)                         |
| Prediction accuracy         | \( PA = \sum_{i=1}^{n} \frac{(P_i - \bar{P})(O_i - \bar{O})}{(n-1)\sigma_P\sigma_O} \) |
| Coefficient of determination| \( R^2 = \left( \frac{\sum_{i=1}^{n}(P_i - \bar{P})(O_i - \bar{O})}{nS_P S_O} \right)^2 \) |
| Index of accuracy           | \( IA = 1 - \frac{\sum_{i=1}^{n}(P_i - O_i)^2}{\sum_{i=1}^{n}(|P_i - \bar{O}| + |O_i - \bar{O}|)^2} \) |

Notation: \( n \) = Number of observations, \( P_i \) = Predicted values, \( O_i \) = Observed values, \( \bar{P} \) = Mean of the predicted values, \( \bar{O} \) = Mean of the observed values, \( S_P \) = Standard deviation of the predicted values, \( S_O \) = Standard deviation of the observed values
2.4 Return period

Table 3 shows the return period for each distribution.

| Distribution                       | Quantile Estimates |
|------------------------------------|--------------------|
| Normal                             | \( \hat{x}_r = \hat{\mu} + u\hat{\sigma} \) |
| Two-Parameter Lognormal (LN(2))    | \( \hat{x}_r = e^{\hat{\beta}_1 u + \hat{\beta}_0} \) |
| Three-Parameter Lognormal (LN(3))  | \( \hat{x}_r = a + e^{\hat{\beta}_1 u + \hat{\beta}_0} \) |

where the standard normal variate, \( u \) can be calculated by using the following formula,

\[
u = \frac{1}{1 + d_1 W + d_2 W^2 + d_3 W^3 + \varepsilon(P)}
\]

\[
C_0 = 2.515517, \quad d_1 = 1.432788
\]

\[
C_1 = 0.802853, \quad d_2 = 0.189269
\]

\[
C_2 = 0.010328, \quad d_3 = 0.001308
\]

\[
W = \left( -2 \log(P) \right)^{1/2} \quad \text{for } P < 0.5
\]

\[
, P = 1 - F \quad \text{and} \quad F = 1 - T^{-1}.
\]

3. Results and discussions

Table 4 shows the summaries of annual maximum rainfall depths at Jabor and Air Putih. By comparing these two sites, it can be seen that Jabor has the lower minimum value of rainfall depths which is 0.0 mm and has the higher maximum value of rainfall depths which is 614.5 mm. From the table, Jabor shows the higher mean of annual maximum rainfall depths compared to Air Putih that is 73.25 mm. These two sites have positive values for skewness showing that the distributions of rainfall depths are skewed to the right.

| Sites                      | Jabor | Air Putih |
|----------------------------|-------|-----------|
| No. of Observations       | 372   | 372       |
| Min value                 | 0.0   | 0.50      |
| Max value                 | 614.5 | 320.5     |
| Mean                      | 73.25 | 65.561    |
| Standard deviation        | 75.8213 | 50.6757 |
| Kurtosis                  | 22.4342 | 8.7218   |
| Skewness                  | 3.6278 | 2.0747    |

The parameter estimates as well as the performance indicators of the three distributions for the two sites by using MOM and PWM method respectively are given in Table 5 and Table 6. All the estimates have been obtained by using these two methods as discussed earlier. By looking at the
performance indicator, the best method that fits the observed distribution can be chosen. For both sites of the study area, it can be seen that PWM is the better method that suit the Normal distribution compared to MOM method. Then, for two-parameter Lognormal distribution and three-parameter Lognormal distribution, it shows that MOM is the better method compared to PWM method.

Table 5. Parameter estimates and performance indicators using MOM.

| Stations  | Distributions | Performance Indicators | NAE  | RMSE   | MAE   | PA     | R²    | IA     |
|-----------|---------------|-------------------------|------|--------|-------|--------|-------|--------|
| Jabor     | Normal        | µ 73.25                 | 0.3918 | 44.7337 | 28.6974 | 0.8266 | 0.6795 | 0.9054 |
|           |               | σ 75.8213               |       |         |        |        |       |        |
|           | LN(2)         | µ 3.9298                | 0.0615 | 13.0943 | 4.5025  | 0.985  | 0.965  | 0.9924 |
|           |               | σ 0.8534                |       |         |        |        |       |        |
|           | LN(3)         | µ 4.08                  |       |         |        |        |       |        |
|           |               | σ 0.7932                | 0.0635 | 13.4419 | 4.652   | 0.9842 | 0.9635 | 0.992  |
|           |               | a -7.7594               |       |         |        |        |       |        |
| Air putih | Normal        | µ 65.561                | 0.2447 | 22.1456 | 16.0432 | 0.9048 | 0.8143 | 0.9501 |
|           |               | σ 50.6757               |       |         |        |        |       |        |
|           | LN(2)         | µ 3.9488                | 0.0481 | 7.4712  | 3.1511  | 0.9891 | 0.9731 | 0.9945 |
|           |               | σ 0.6844                |       |         |        |        |       |        |
|           | LN(3)         | µ 4.0595                |       |         |        |        |       |        |
|           |               | σ 0.5658                | 0.0594 | 6.7422  | 3.8949  | 0.9912 | 0.9771 | 0.9956 |
|           |               | a -16.9314              |       |         |        |        |       |        |

Table 6. Parameter estimates and performance indicators using PWM.

| Stations  | Distributions | Performance Indicators | NAE  | RMSE   | MAE   | PA     | R²    | IA     |
|-----------|---------------|-------------------------|------|--------|-------|--------|-------|--------|
| Jabor     | Normal        | µ 73.25                 | 0.3011 | 42.8151 | 22.0525 | 0.8266 | 0.6795 | 0.8905 |
|           |               | σ 59.0265               |       |         |        |        |       |        |
|           | LN(2)         | µ 3.9282                | 0.0621 | 13.103  | 4.5523  | 0.985  | 0.965  | 0.9924 |
|           |               | σ 0.8552                |       |         |        |        |       |        |
|           | LN(3)         | µ 4.0595                |       |         |        |        |       |        |
|           |               | σ 0.7864                | 0.0513 | 13.6596 | 3.756   | 0.984  | 0.9631 | 0.9915 |
|           |               | a -5.6916               |       |         |        |        |       |        |
| Air putih | Normal        | µ 65.561                | 0.2133 | 21.625  | 13.9827 | 0.9048 | 0.8143 | 0.9458 |
|           |               | σ 44.2305               |       |         |        |        |       |        |
|           | LN(2)         | µ 3.9362                | 0.0513 | 8.0722  | 3.3626  | 0.9881 | 0.9712 | 0.9938 |
|           |               | σ 0.7025                |       |         |        |        |       |        |
|           | LN(3)         | µ 4.0862                |       |         |        |        |       |        |
|           |               | σ 0.6297                | 0.0492 | 6.8007  | 3.2276  | 0.991  | 0.9768 | 0.9955 |
|           |               | a -7.0057               |       |         |        |        |       |        |
After getting all the best methods for each distribution, then the comparison between all the best methods is made. From each value of performance indicators that was shown in each distribution, the best distribution that fits the data can be chosen. For all the data of annual maximum rainfall depths at Jabor, the results show that two-parameter Lognormal distribution is the best distribution among the three distributions that have been studied. Meanwhile, at Air Putih, the results show that three-parameter Lognormal distribution is the best distribution among all the distributions that have been studied. Table 7 shows the parameter estimates for the best distribution at Jabor and Air Putih.

Table 7. Parameter estimates for the best distribution at each station.

| Distributions      | Sites  | Jabor     | Air putih |
|--------------------|--------|-----------|------------|
| Best Method        |        | MOM       | MOM        |
| Best distribution  |        | LN(2)     | LN(3)      |
| Mean, $\mu$        |        | 3.9298    | 4.2526     |
| Standard deviation, $\sigma$ | | 0.8534   | 0.5658     |
| The amount of $x$ shifted, $a$ | | -        | -16.9314   |

Next, the return period of annual maximum rainfall depths at these two sites of study area for 2, 5, 10, 50, and 100 years later can be calculated. At Jabor, the return period will be discussed based on the best method for two-parameter Lognormal distribution while for Air Putih, the return period will be discussed based on the best method for three-parameter Lognormal distribution. Table 8 shows the return period for Jabor and Air Putih. The results in Table 8 can be seen more clearly when the data is given in the form of figure as shown in Figure 3.

Table 8: The return period for each station.

| Years | Sites  | Jabor | Air Putih |
|-------|--------|-------|------------|
| 2     |        | 51    | 53         |
| 5     |        | 104   | 96         |
| 10    |        | 152   | 128        |
| 50    |        | 294   | 208        |
| 100   |        | 371   | 245        |

From Figure 3, it can be seen that Jabor has the higher values of annual maximum rainfall depths for 5, 10, 50 and 100 years later compared to Air Putih. Even though the value of rainfall depths for 2 years later at Jabor is lower than Air Putih, in average, it can be concluded that Jabor is the site that has the higher values of rainfall depths for the return period that has been studied.
Figure 3. The return period for the best distribution at each station.

4. Conclusion
This study discussed about the return period of annual maximum rainfall depths for two sites in Kemaman, which are Jabor and Air Putih. Three distributions have been used in this study in order to determine the return period, which are Normal distribution, two-parameter Lognormal distribution, and three-parameter Lognormal distribution. The methods used to determine the parameter estimates are Method of Moments (MOM) and Probability Weighted Moments (PWM) method. The descriptive statistics show that Jabor has the higher mean of rainfall depths compared to Air Putih. The parameter estimates for each distribution were calculated by using MOM and PWM method. Then, the best method for each distribution was obtained by using the performance indicator. The results show that for these two sites of study area, PWM method is the best method for Normal distribution while MOM is the best method for two-parameter Lognormal distribution and three-parameter Lognormal distribution. By comparing all the data of the best method, the best distribution that fits the data for Jabor is two-parameter Lognormal distribution and for Air Putih, the best distribution is three-parameter Lognormal distribution. Lastly, the return period for 2, 5, 10, 50, and 100 years of these two study area were obtained by using the best distribution. From the result of the study, it can be concluded that Jabor has the higher annual maximum rainfall depths compared to Air Putih for the return period that were studied.

Acknowledgements
The authors would like to thank the National Hydrological Network Management System for providing the rainfall depths data in this study.

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