Method for determining size of inhomogeneity localization region based on analysis of secondary wave field of second harmonic

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Abstract. The article describes the research of the method of localization and determining the size of heterogeneity in biological tissues. The equation for the acoustic harmonic wave, which propagates in the positive direction, is taken as the main one. A three-dimensional expression that describes the field of secondary sources at the observation point is obtained. The simulation of the change of the amplitude values of the vibrational velocity of the second harmonic of the acoustic wave at different coordinates of the inhomogeneity location in three-dimensional space is carried out. For the convenience of mathematical calculations, the area of heterogeneity is reduced to a point.

1 Introduction

In the process of localization of heterogeneity in the biological environment, an important role is played by determining the size of the localization area. The analysis of the secondary field of acoustic radiation passed through the medium can allow a quantitative assessment of the sizes of inhomogeneous inclusion. The laws of formation and propagation of the second harmonic of an acoustic wave passing through an inhomogeneous nonlinear biological medium are known [1, 2, 3].

2. Materials and methods

As the main equation of the wave running in the positive direction, let us use the expression [4, 5]:

$$u = \Phi \left[ x - \left( c_0 + \frac{y+1}{2} u \right) t \right]. \quad (1)$$

The three-dimensional scheme for equation analysis (1) is shown in figure 1.
Figure 1. The layout of heterogeneity in three-dimensional space

Figure 1 shows: M(x,y,z) – observation point, M0(x’,y’,z’) – secondary source location point.

Further, when carrying out the integration of equations describing the evolution of wave shape, limits will be the size of the inhomogeneity: x – from l1 to l2; y is between a1 and a2; z is from b1 to b2.

Let us consider the equation for a simple wave running in the positive direction (1) for the case of the second harmonic wave of the signal [5,6]:

\[ u = A_F \cos \left( t - \frac{r}{c_0 (1 + \frac{\partial u}{\partial x}/c_0) } \right), \]

where u – the value of the vibrational velocity at the observation point;

A_F – a function that reflects the law of change of amplitude parameters of the second harmonic of an acoustic wave;

c_0 – the phase velocity of sound;

r – a vector that connects points M(x’,y’,z’) and M (x,y,z).

3. Derivation of the equation for the field of secondary sources at the observation point

The structure of the secondary field of the past acoustic wave in three-dimensional space is particularly interesting.

The equation for a simple acoustic wave becomes more complicated when one passes to the accompanying coordinates due to the introduction of spatial vector \( \mathbf{R} \). The accompanying coordinate then takes the form of [7,8]:

\[ t - \frac{\mathbf{r}}{c_0} = t - \left( \frac{\mathbf{r}_0}{c_0} - \frac{x' \cos \theta}{c_0} - \frac{y' \sin \theta}{c_0} - \frac{z' \sin \varphi}{c_0} \right) \]

In this case, the second harmonic of the acoustic wave, which has the form represented by expression [9, 10] \( u = F_{2\omega} \cos(2\omega - k z \omega) \), was considered.

Then, at observation point M, the expression describing the second harmonic will take the form:

\[ A_F \cos(2\omega t - k z \omega) = \cos \left( 2\omega \left( t - \frac{\mathbf{r}_0}{c_0} + \frac{x' \cos \theta}{c_0} - \frac{y' \sin \theta}{c_0} - \frac{z' \sin \varphi}{c_0} - k z \omega \right) \right) = \]

\[ = \cos \left( 2\omega \left( t - \frac{\mathbf{r}_0}{c_0} + x' \left( 2\omega \frac{\cos \theta}{c_0} - k z \omega \right) + 2\omega \frac{\sin \theta}{c_0} + 2\omega \frac{z' \sin \varphi}{c_0} \right) \right) \]

(4)
where $k^2\omega = \frac{2\omega}{c^2}$.

For further mathematical transformations, let us assume:

\[ 2\omega \left( t - \frac{r_0}{c^2} \right) = \alpha; \]
\[ k^2\omega (\cos \theta - 1) = \beta; \]
\[ k^2\omega \sin \theta = \gamma; \]
\[ k^2\omega \sin \varphi = \eta. \]

Taking into account (5), equation (4) will take the form of:

\[ A_F \cos[\alpha + x'\beta k^2\omega z' \sin \varphi]. \]  

(6)

To determine the inclusion form, let us integrate the expression under the cosine:

\[ A_F \int_{y'} \cos(\alpha + x'\beta k^2\omega z' \sin \varphi) \, dx' \, dy' \, dz' = \]

\[ = A_F \int_{b/2}^{b/2} dy' \int_{l_1}^{l_1} dx' \int_{a_1}^{a_2} \cos(\alpha + x'\beta k^2\omega z' \sin \varphi) \, dz'. \]

(7)

Applying the basic trigonometric identity to expression (7), let us obtain:

\[ A_F \int_{b/2}^{b/2} dy' \int_{l_1}^{l_1} dx' \int_{a_1}^{a_2} \cos(\alpha) \cos(\beta x') \cos(\gamma y') \cos(\eta z') - \]

\[ - \cos(\alpha) \cos(\beta x') \sin(\gamma y') \sin(\eta z') - \cos(\alpha) \sin(\beta x') \cos(\gamma y') \sin(\eta z') - \]

\[ - \cos(\alpha) \sin(\beta x') \sin(\gamma y') \cos(\eta z') - \sin(\alpha) \cos(\beta x') \cos(\gamma y') \cos(\eta z') - \]

\[ - \sin(\alpha) \cos(\beta x') \sin(\gamma y') \cos(\eta z') \sin(\gamma y') \sin(\eta z') dy'. \]

As a result, there are eight components that need to be integrated. For convenience, let us consider each summand separately.

Let us describe the integration of the expression for each coordinate in more detail.

Let us carry out the integration on $y'$:

\[ \int_{a_1}^{a_2} \cos(\alpha) \cos(\beta x') \cos(\gamma y') \cos(\eta z') \, dy' = \]

\[ = \cos(\alpha) \cos(\beta x') \cos(\gamma \frac{a_1+a_2}{2}) \sin(\gamma \frac{a_2-a_1}{2}). \]

(9)

Let us carry out the integration on $x'$:

\[ \int_{l_1}^{l_1} \cos(\gamma \frac{a_1+a_2}{2}) \sin(\gamma \frac{a_2-a_1}{2}) \, dx' = \]

\[ = \cos(\gamma) \cos(\frac{a_1+a_2}{2}) \sin(\frac{a_2-a_1}{2}) \beta \cos(\frac{l_1+l_2}{2}) \sin(\frac{l_2-l_1}{2}). \]

(10)

Let us carry out the integration on $z'$:

\[ \int_{b_1}^{b_2} \cos(\eta \frac{b_2-b_1}{2}) \sin(\eta \frac{b_2+b_1}{2}) \, dz' = \]

\[ = \cos(\eta) \cos(\frac{b_2+b_1}{2}) \sin(\frac{b_2-b_1}{2}) \beta \cos(\frac{l_1+l_2}{2}) \sin(\frac{l_2-l_1}{2}). \]

(11)

In the same way, let us integrate each term of expression (7). As a result, let us get the following expression:

\[ A_F \cos(\alpha) \frac{2}{\beta} \cos(\frac{l_1+l_2}{2}) \beta \sin(\frac{l_2-l_1}{2}) \beta \cos(\frac{a_2+a_1}{2}) \sin(\frac{a_2-a_1}{2}) \eta \frac{2}{\eta} \cos(\frac{b_2-b_1}{2}) \eta \times \]

\[ \times \sin(\frac{b_2-b_1}{2}) = A_F \cos(\alpha) \frac{2}{\beta} \sin(\frac{l_2-l_1}{2}) \beta \cos(\frac{a_2+a_1}{2}) \beta \sin(\frac{a_2-a_1}{2}) \eta \times \]

\[ \times \sin(\frac{b_2-b_1}{2}) \eta \sin(\frac{b_2+b_1}{2}) \eta \sin(\frac{b_2-b_1}{2}) \eta - A_F \cos(\alpha) \frac{2}{\beta} \sin(\frac{l_2-l_1}{2}) \beta \times \]

\[ \times \sin(\frac{a_2-a_1}{2}) \eta \sin(\frac{b_2+b_1}{2}) \eta \sin(\frac{b_2-b_1}{2}) \eta - A_F \cos(\alpha) \frac{2}{\beta} \sin(\frac{l_2-l_1}{2}) \beta \times \]

\[ \times \sin(\frac{a_2-a_1}{2}) \eta \sin(\frac{b_2+b_1}{2}) \eta \sin(\frac{b_2-b_1}{2}) \eta \]
Expression (12) can be simplify and reduced to two terms $A_F \cos(\alpha) \cos(\beta)\sin(\gamma)$ and $A_F \sin(\alpha) \cos(\beta)\sin(\gamma)$

(13)

Thus, for the second harmonic, the expression describing the secondary field at the point $M(x,y,z)$ will look like:

(14)

The resulting expression describes the field of secondary sources concentrated in the volume of the heterogeneities. An ideal case was considered with the surrounding biological tissues, which have a homogeneous structure. Additional amendments are required to take account of the environmental parameters surrounding heterogeneity.

4. Mathematical modeling
For the mathematical modeling, heterogeneity was taken as a point in three-dimensional space. Mathematical calculation for different positions of a point at different values of a nonlinear parameter is carried out. The results of the calculations are presented in figure 2.

**Figure 2.** Change in the value of the vibrational velocity of the second harmonic wave at observation point M, relative to changes in its position in space along coordinate x

Also calculations for a group of points in space during the change of two coordinates for different biological environments are carried out. Three types of biological tissues were considered: 1 – blood, 2 – liver and 3 – fat. The calculation results are shown in figure 3.

**Figure 3.** Change in the value of the vibrational velocity of the second harmonic wave at observation point M, relative to changes in its position in space along coordinates x and y

5. **Conclusion**
The obtained calculations showed that the change in the parameters of the secondary acoustic field at the observation point depends both on the parameters of the medium and on the location of the secondary radiation sources.
Knowing the values of the parameters of the substance distributed over the investigated volume, it is possible to fix the change in the acoustic field at the observation point by varying the sizes. A set of field parameter values at different observation points can also be used as an acoustic projection of an object.

6. Acknowledgments
The research had been performed with the financial support of the RFBR within the framework of Project No. 16-07-00374/16.

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