On dynamics of hard elastic scattering of hadrons

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Abstract. The main contribution to hard elastic scattering at high energies comes from components of wave functions of colliding hadrons that contain minimal number of partons. We discuss the details of such a mechanism in the regge and parton approaches and estimate the probability that colliding hadrons are in bare states containing only valent partons. The behavior of cross sections in this regime at various energies can give nontrivial information on high energy dynamics.

1. Introduction.

The elastic scattering of hadrons at high energies is reasonably well described in the regge approach. At small $t$ it has the diffractive nature and is connected with the pomeron exchanges (fig.1). As confirmed by both experimental data and QCD calculations the pomeron $P$ is supercritical, i.e. its intercept $\alpha_P = 1 + \Delta$ is higher than unity, $\Delta > 0$. For the supercritical pomeron the mean number of $P$ exchanges increases with energy as $\sim s^\Delta$. When energy increases it results in gradual change of energy behavior of total cross section from the power growth, $\sigma_{tot} \sim \exp(\Delta y)$ to the Froissart type behavior, $\sigma_{tot} \sim y^2/m^2$, where $y = \log s/m^2$ is full rapidity.

Fig.1: The reggeon diagram for single $P$ exchange and the general reggeon diagram for elastic amplitude, which becomes important at very high energy.

The width of the diffraction peak due to the single $P$ pole exchange, $\delta t \sim 1/R^2(y)$, is
The parton content of the pomeron can be realized as a gluonic ladder (fig.2) where gluons are uniformly distributed in rapidity. The mean rapidity gap between gluon steps, $\delta_y \sim 1/\alpha_s(k^2_{\perp})$, is determined by the value of QCD coupling constants on the scale of average transverse momenta of intermediate gluons $\langle k^2_{\perp} \rangle \sim -t$.

The mean number of intermediate gluons in the pomeron is $\sim y/\delta_y$. The distribution in number of gluons is the Poissonian one, so the probability to have no intermediate gluons is exponentially small, $\sim \exp[-y/\delta_y]$. With increase of $k^2_{\perp}$ the character of elastic scattering gradually changes from diffractive mechanism to the perturbative parton exchange. The trajectory $\alpha_P(t)$ of the QCD pomeron at $t = 0$ is higher than unity. It is nonlinear and at large $-t = k^2_{\perp}$, where the perturbative mechanisms dominate, must behave in perturbative way as $\alpha_P(k^2_{\perp}) \sim 1 + c/\log(k^2_{\perp})$.

In terms of the regge approach when $|t|$ increases at fixed energy the cut contributions with larger and larger number of pomeron exchanges dominate and the elastic amplitude can be approximately represented by the Orear type expression $\sim \exp[-k^2_{\perp}f(y)]$. In this case the large total transferred momentum $k^2_{\perp}$ is distributed more or less evenly among the exchanged pomerons, and this is connected with the approximate linearity of regge trajectories at low $t$ values.

For larger $k^2_{\perp}$ this uniform distribution of transferred momentum between pomeron lines changes, and the large transferred momentum $k^2_{\perp}$ is concentrated mainly on a single pomeron line. This is primarily related with the nonlinearity of the pomeron trajectory and its hard 1

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Footnote:

1 Note that, as can be seen from a comparison with the experimental data, the regge trajectories look linear up to rather large $k^2_{\perp} \approx 2 \div 5 GeV^2$, and only after that they can move to the perturbative values. The details of this phenomenon are not so well investigated, and it is, probably, related to the large nonperturbative contributions, essential up to sufficiently small distances $\sim 0.1 \div 0.2$ fm.

The structure of pomeron in QCD can be rather complicated and probably consists of a sequence of regge poles $P_n$ with intercepts $\sim 1 + c/n$. They have small $t$ slope $\sim 1/n^2$ and their internal virtualities $q^2_{\perp} > \Lambda^2$ grow like $\sim \exp n$. At high $-t$ all these satellite $P_n$ accumulate at their bare values like $\alpha_{nP}(k^2_{\perp}) \sim 1 + c/(n + c_1 \log(k^2_{\perp}/\Lambda^2))$. At high momentum transfer the contributions of satellites $P_n$ with high $n$ in the amplitude can be even dominant, so that the essential $n \sim \log(-t)$. But here we do not take into account these details because all $P_n$ trajectories move at high $-t$ to the same bare value, and the structure of exchange is determined by the minimal perturbative graph.
satellites at large $k_\perp$.

As a result, the $t$-behavior of elastic scattering amplitude is determined by the pomeron vertices, and one can expect the power behavior $A \sim 1/k_\perp^{2\nu}$ of the full amplitude, where $\nu$ depends on the minimal number of exchanges needed to scatter all parton components of hadron on the same angle.

In this mechanism (when the $P$ trajectory is already frozen) the main contribution to the hard scattering amplitude comes from components of wave functions of the colliding hadrons with minimal possible number of partons. This high $k_\perp$ scattering mechanism contains two main ingredients:

- The first is the amplitude of hard scattering of valent (bare) constituents. It is determined by the perturbative QCD, and can be estimated even on the dimensional grounds.
- The other is a probability to find the hadron in such a “bare” state with the minimal number of partons, and it is determined mainly by nonperturbative physics.

What one needs first of all is to know the dependence of this probability on the hadron energy and its transverse resolution. This is what we consider below in the regge and parton approaches.

2. Cross sections

Consider the contribution to the elastic scattering amplitude $A(s, t)$ from rather general reggeon diagrams of Fig.3 at high $|t| \gg 1/R^2$, but such that $|t| \ll s$. Because (as we have assumed) the pomeron trajectory is nonlinear, and freezes at high $-t$, the $P$ exchange amplitude $\nu \sim ig^2(t)s^{\alpha(t)}$ behaves at high $-t$ in power-like manner. As a result in this case the main contribution to $A$

\[ A \sim 1/k_\perp^{2\nu} \]

comes from diagrams of Fig.3a in which almost whole transverse momentum is transferred through a single $P$ line. The full diagram for $A$ can be symbolically represented as in Fig.3c

$\nu$ \[ \sim ig^2(t)s^{\alpha(t)} \]

The picture closely corresponds to the quark counting rules [1] and its development [2]-[3]. The power behavior of elastic cross-sections at large $-t$ is approximately seen in experiments.
The cross-section for the hard $P$ -exchange $d\sigma(y,t)/dt \sim g_1^2(t) g_2^2(t)$ is mainly determined by the behavior of $P$-vertices $g_1(t)$, because at large $-t$ the purely reggeon part

$$s^{\alpha(t)} \sim \exp( c y \Delta_P / \log(-t) ) , \quad c \sim 1$$

grows very slowly with $s$. In perturbative QCD the hard part of the pomeron can be approximately represented as the BFKL ladder with mean interval between rapidities of emitted gluons of order of $\sim 1/\alpha_s(t)$. Such a BFKL type $P$ exchange has almost the same form as the direct $2n$ gluon exchange; the $n > 1$ contributions can arise because the BFKL pomeron contains also the multigluon t-channel contributions due to gluon reggeization \(^3\). On this way we come finally to the quark counting type model.

\(^3\) At high $-t$ not only the pomeron but also the other multigluon QCD reggeons can equally contribute to the
The behavior of $g_s^2(t)$ at large $|t|$ in QCD can be estimated even quasi-classically, and this approach leads finally to the $t$ dependence:

$$d\sigma(y, k_\perp)/dk_\perp^2 \sim (\alpha_s(k_\perp))^\nu/(k_\perp^2)^N,$$

(4)

with

$$\nu = n_1 + n_2 + |n_1 - n_2|, \quad N = \frac{1}{2} \left(3(n_1 + n_2) + |n_1 - n_2|\right) - 1,$$

(5)

where numbers $N$ and $\nu$ depend on the number of valent constituents $n_1$ and $n_2$ in colliding hadrons (in fact the fast quarks), and on a configuration in which the hard scattering of constituents takes place. In (4) we have neglected the regge factor (3), which at high $-t$ grows very slowly with $s$, and have also neglected the possible Sudakov type suppression factors – they can even approximately compensate one another.

The simplest configuration is such that before scattering all constituents in both hadrons are gathered in the small transverse region of size $\sim 1/k_\perp$ and then these bunches scatter on one another at mean transverse distance $\sim 1/k_\perp$. In this case we have

$$\nu = n_1 + n_2 + |n_1 - n_2|, \quad N = 2n_1 + 2n_2 - 2,$$

(6)

and we have from (4) the behavior of $\sim 1/|t|^{10}$ type for $pp$ and to $\sim 1/|t|^8$ for $\pi p$ cases.

There is also another initial transverse configuration of constituents, in which the scattering index $N$ entering Eq.(4) is less then in (6). It corresponds to a scattering in a state in which the hadron constituents are arranged (with an accuracy $\sim 1/k_\perp$) in the transverse plane on the line perpendicular to a scattering plane $^4$. In such a case we have

$$N \to \tilde{N} = \frac{5}{4} |n_1 + n_2| + \frac{3}{4} |n_1 - n_2| - \frac{1}{2},$$

(7)

which leads to the $\sim 1/|t|^7$ behavior for $pp$ and to $\sim 1/|t|^{13/2}$ for $\pi p$ cases.

Note that the bare hadron cross section $d\sigma/dk_\perp^2$ given by (4) can be approximately interpreted as the cross section of specific quasi-elastic process, where in the final state we have two hadrons with high transverse momenta $\simeq \pm k_\perp$, and all other particles have small $k_\perp^2 \sim \langle k_\perp^2 \rangle$. The particles which have large transverse momenta $k_\perp^2 \gg \langle k_\perp^2 \rangle$ and take more than a half of the total energy originate from hard interactions of leading partons. All other final particles are soft and appear in configurations typical for mean inelastic events at the same energy. These particles are created by the standard hadronization of soft parton cloud, which follows after “removing” the leading partons in a hard interaction.

Such events also resemble the final state containing two high energy jets in special configurations, in which all the jet energy is concentrated on one fast particle (pion or nucleon). The probability of jet coming in such “empty” state very likely contains the same damping factors $d\sigma/dk_\perp^2$ and $S_0(s)$.

4 In fact all what is needed and what leads to expression (7) is the following: all valent constituents must scatter at very close angles $\theta \simeq k_1/k_\perp$ such that all relative transverse momenta of scattered partons be $\lesssim m$. Also one must fulfill the condition that after scattering all the constituent partons should be located in the same packet of longitudinal size $\sim 1/k_\perp$. It leads to the condition that hadrons predominately scatter in specific initial configurations, when their partons are arranged on the lines in transverse planes. These lines must be perpendicular to the scattering plane, but the relative separation of partons on these lines can be arbitrary. Such a picture of hard elastic scattering picture looks in a sense similar to the Landshoff mechanism [3].
It can be interesting to detect similar events in high-energy heavy ion interactions. For example, in $A_1A_2$ nucleus collisions those are the events with two nucleons of high transverse momenta $\pm k_\perp$ (but without jets) and all other hadrons with transverse momenta typical for soft $A_1A_2$ collisions. The cross-section of such a process is  
\[ d\sigma/dk_\perp^2 \simeq A_1A_2 \cdot d\hat{\sigma}/dk_\perp^2, \]
because the nuclei are transparent for the bare nucleons.

3. The estimation of $|S_0|^2$ in parton models

Thus, to understand the large $-t$ behavior of $d\sigma/dt$ it is essential to estimate the behavior of $|S_0|$, which gives the probability for fast hadron to be in a bare state, i.e. to contain only valent components and no additional parton cloud. One can expect that this probability is small because at all accessible energies the fast hadron is basically accompanied by the soft parton cloud related to the bare state is finite and does not decrease with the hadron energy.

The $\mathbb{P}$-ladder corresponds, roughly speaking, to a soft parton (gluon) cascading with some mean step in rapidity $\delta_y$. Because the cascading steps are almost independent, the probability to have no cascade (i.e. to realize a state without additional partons) is of Poisson type  
\[ \sim \exp(-y\Delta), \]
where $\Delta = c/\delta_y$, $c \sim 1$. It corresponds to the fact that at the rapidity boost the mean number of low energy partons $n(y)$ is defined by the linear equation $\partial n(y)/\partial y = \Delta n(y)$.

Thus the crucial quantity is $\Delta \simeq \delta_y^{-1}$, where $\delta_y$ is the mean rapidity step between essential degrees of freedom in the pomeron ladder. This gives an estimate $S_0(y) \simeq \exp(-y/\delta_y)$.

These arguments can be presented in a slightly different way. To scatter at large $k_\perp$ both hadrons should have the minimal parton clouds. It means that colliding particles must fluctuate to a state with small transverse size $r(y)$. The probability $W$ of such a fluctuation is  
\[ \sim (r(y))^{2(\nu-1)}, \]
where $\nu$ is a number of the valent constituents. In this case, the mean rapidity interval between steps in parton ladder would be $\delta_y \sim 1/\alpha_s(r(y))$. From the condition $\delta_y \simeq y$ (no partons except of valent ones in the whole rapidity interval) we find that $r(y) \sim \exp(-c_1y)$, and we come to the same type of the exponential dependence $S_0(y) \sim w(y) \sim \exp(-2c_1(\nu-1)y)$ as before.

Additional arguments for exponential behavior of probability $S_0(y)$ come from the requirement of boost invariance of hard collision description in partonic terms. Let us consider this process in an arbitrary longitudinal frame, when the colliding particles have rapidities $y_1$ and $y_2$. Then the quantity $S_0(y)$, which is the amplitude that both colliding particles are in a bare state must have the multiplicative form  
\[ S_0(y) = S_0(y_1) \cdot S_0(y_2), \]
where $y = y_1 + y_2$. Because at boost $y_1 \rightarrow y_1 + \eta$, $y_1 \rightarrow y_1 - \eta$, $y \rightarrow y$, the only functional form of $S_0(y)$ that satisfies this condition in arbitrary frame is the exponential one $S_0(y) \sim \exp(-cy)$.

So to better estimate the behavior of $S_0(y)$ we need to know the mean number of steps in the pomeron ladder. Two possibilities can be emphasized, and they correspond to a different choice of partons – independent degrees of freedom in the Fock wave function of fast hadron.

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5. Evidently this is true only for supercritical pomeron. If $\alpha_\perp(0) < 1$ the probability to find the hadron in the “bare” state is finite and does not decrease with the hadron energy.

6. This type of reasoning can remind the approach [6] to high energy scattering used in dual models embedded in the $AdS_5$ space.
In one case we can consider the pomeron ladder as constructed from colorless particles (for example, from $\pi$ and $\rho$ mesons), as it is usually done in various multiperipheral type models. In this case the steps (parton) density in rapidity is $\sim 0.5 \rho \sim 1$, where $\rho$ is the mean density of produced hadrons at not too high energies, when single pomeron exchange dominates.

In other case one can identify the soft pomeron ladder as a sparse gluon ladder with large steps $\delta_y \sim \Delta_T^{-1}$, where $\Delta_T \sim 0.1 \div 0.2$ is the “experimental” soft pomeron intercept $^7$.

I guess the second possibility is theoretically more preferable, because the colorless hadron-partons do not represent the independent degrees of freedom, and therefore they are not completely appropriate to be used in the Fock space Hamiltonian.

If hard partons were also essential in the Fock wave function then at first sight instead of $\sim \exp(-y/\delta_y)$ one can expect a more complicated behavior of $|S_0(y)|$. But the hard gluons are mainly taken into account in the parton w.f. as constituents of soft partons. We come to the same conclusion from a different way by remarking that the hard gluon spectra “measured” in deep-inelastic reactions can be successfully described $^5$ as coming from renormalization group rescaling of the soft parton component.

At asymptotic energies the saturation of soft and even more hard partons can become fully essential and as a result the saturation scale $Q^{(sat)}(y)$ in transverse momenta can also become large. Then one should take into account in $|S_0|^2$ also more hard partons. Note that even in the asymptotical Froissart regime, where the hard saturation probably dominates, the behavior of $|S_0|$ can be estimated in the same way – as the probability that the valent components do not emit any primary gluons in the corresponding ranges of energy and transverse momenta ($q_\perp < \langle k_\perp \rangle \sim Q^{(sat)}(y)$). This is quite enough, because all other partons are emitted by these primaries. This evidently leads to a mean number of the primary partons

$$\bar{n}(y, k_\perp) \sim \int d\omega/\omega \int^{k_\perp} dq_\perp^2 \alpha_s(q_\perp)/q_\perp^2 \sim y \ln \ln(k_\perp/m),$$

and to Sudakov-type factor for the no-emission probability $W(y, k_\perp) \sim \exp(-\bar{n})$, and, eventually, to the estimate $|S_0(y, k_\perp)| \sim W(y_1, k_\perp)W(y_2, k_\perp)$, which is again boost-invariant, and has almost the same behavior.

4. The estimation of $|S_0|^2$ in regge approach

The numerical values of $S_0 \equiv |\bar{S}_0(y, b = 0)|$ can be estimated directly from the experimental data on the behavior of the profile function $F(y, b) = 1 - \bar{S}(y, b)$ at $b = 0$, calculated by the Fourier transformation of $\sqrt{(d\sigma/dt)(exp)}$. For the $pp$ and $p\bar{p}$ scattering it ranges from values $S_0 \simeq 0.6$ at $\sqrt{s} \sim 50 GeV$ up to values $S_0 \simeq 0.01 \div 0.02$ at $\sqrt{s} \sim 2 TeV$.

In the regge type models the value of $S_0$ depends crucially on the relative weights of contributions of multipomeron exchanges to the elastic amplitude, and, in principle, can be extracted from “every” good model descriptions of $d\sigma/dt$ data at low $t$. But it is essential that in the most popular models one can not expect the functional behavior $S_0(y) \sim \exp(-yc)$ at large $y$, which seams very natural in the parton approach.

If $\bar{v}(y, b)$ is the single $P$ contribution to the amplitude, then the full sum of all pomeron diagrams (neglecting the pomeron interactions with one another) can be approximately

$^7$ Such type of model was proposed $^4$ long ago, and in this case (with large $\delta_y \sim 5$) it provides a simple explanation of unnaturally small values of some pomeron parameters, such as $\alpha_s^T$, $\Delta_T$, $\gamma_{pp}$, etc.
represented as some function of $v$ in the form
\[ \tilde{S}(y, b) = \tilde{S}[\tilde{v}] = \sum_{n=0}^{\infty} \frac{\gamma_n}{n!} (i\tilde{v})^n, \quad \gamma_0 = \gamma_1 = 1, \] (8)
where the quantities $\gamma_n \geq 1$ for $n \geq 2$, and their values reflect the weights with which the diffractive jets contribute to the corresponding multipomeron vertices $N_n$.

In the simplest eikonal case (no inelastic diffraction!), when all $\gamma_n = 1$, we have $\tilde{S} = \exp(i\tilde{v})$ and it leads to
\[ |\tilde{S}(y, b)| = \exp(-\text{Im} \tilde{v}(y, b)) \sim \exp(-c_1 \exp(\Delta y) ) \] (9)
Although such an eikonal-type amplitude (especially if properly adjusted \(^8\)) can lead to a reasonable description of many data, there can be the inconsistency with parton picture at very large $y$ in any case.

The purely exponential behavior of $\tilde{S}(y, b)$ can take place only if the eikonal coefficients $\gamma_n$ grow like $n!$ at large $n$. In this case we have
\[ |\tilde{S}(y, b)| \simeq (c + \text{Im} \tilde{v}(y, b))^{-1} \sim \exp(-\Delta y) \] (10)
at large $y$.

For a reasonable behavior of series (8) the coefficients $\gamma_n$ considered as function of $n$ should be analytic for $\text{Re} n \geq 0$. The asymptotics of $S[\hat{v}]$ for $v \to \infty$ is determined by the most right singularities of $\gamma_n$ in $n$. To have the asymptotics (10) the function $\gamma_n$ must have the most right singularity at $n = -1$. The fairly interesting case corresponds to the behavior $\gamma_n = \Gamma(n + 1)$ which corresponds to $c = 1$ in (10).

Note that such a grow of multi P contributions, in comparison with the eikonal case, can essentially affect the form of the tail of multiplicity distribution. In the eikonal case the behavior of the tail of multiplicity distribution is roughly $\sigma_n \sim \sigma_{\text{inel}} / \Gamma(1 + n/\bar{n})$. In the case (10) the tail is more flat: $\sigma_n \sim \sigma_{\text{inel}} \exp(-cn/\bar{n})$. Note that experimentally the tail has the similar form $\sigma_n \sim \exp(-c_1 n/\bar{n})$, $c_1 \simeq 1.5$, where $\bar{n}$ is the mean multiplicity.

The experimental data on the behavior of $d\sigma/dt$ at high $|t|$ are not rich, especially at high $s$. The old data [3] on $pp$ show the universal behavior of $d\sigma/dt \simeq 0.1 |t|^{-8}$ for $2 \lesssim |t| \lesssim 15$ GeV\(^2\) in the energy range $\sqrt{s} \simeq 30 - 60$ GeV. At such energies the $S_0$ is still rather large, and the decrease of $S_0$ with $s$ can in fact be compensated by the small growth (3) of $d\sigma/dt$. New data at higher $s$ and $|t|$ are needed for better understanding the behavior of $S_0$.

5. Conclusion

We end with a few remarks.

- The value of elastic S-matrix at zero impact parameter $S_0(y) = |\tilde{S}(y, b = 0)|$ gives a probability that the fast hadron with energy $\sim m \exp(y)$ has no soft parton cloud. This quantity can be extracted from the behavior of elastic cross section $d\sigma/dt$ at relatively low $|t|$ (see Section 4).
- The same quantity also enters as a factor in the expression (2) for the elastic cross section $d\sigma/dt$ at relatively large $|t| > 2 \div 5$ GeV\(^2\) (far outside of the diffraction peak), because hard scattering occurs mainly in states with a minimal number of partons.

\(^8\) The “minimal” form of corrections to the simple eikonal is the quasi-eikonal picture, in which one explicitly introduces the bare parton state. A bit better and more consistent are the multichannel (matrix) eikonal models, but they also lead to the same rapid decrease of $S$ as in (9).
• The existing data on $d\sigma/dt$ show that the value of $S_0(y)$ is not so small, as one can expect at first sight from the large multiplicity of secondary hadrons at the same energies. It indicates that the high energy hadron wave function contains a relatively small number of partons in average. This also means that most of secondary soft hadrons are created only after collision (they can come from the decay of nonperturbative QCD tubes or minijets), and so are not directly represented by the corresponding degrees of freedom in the incoming parton wave function. In most models of multiperipheral type the opposed picture is usually supposed. Therefore all new data on elastic $d\sigma/dt$ are very interesting also from this point of view, especially at maximally large (LHC) energies.

• Comparison of two different approaches, one of which is based on the parton model and the other on the Regge theory to estimation of the $S_0(y)$ suggests that the pure eikonal-type unitarization (when $\tilde{S}(y, b) = \exp(i\tilde{v})$) can be not suitable at very high energies. The most simple form, giving the same type of answer for $S_0$ as in the parton case, is given by the expression $\tilde{S}(y, b) = (1 + ic\tilde{v})^{-1}$, and it corresponds to the case when the eikonal coefficients grow ($\sim n!$) due to large contribution of diffractive jets in multipomeron vertices $N_n$.

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[1] S.J.Brodsky and G.R.Farrar Phys.Rev.Lett 31 (1973) 1153.
V.A.Matveev, R.M.Muradyan and A.N.Tavkhelidze, Lett.Nuovo Cim. 7 (1973)719
[2] G.Sterman, arXiv:1008.4122.
M.G.Sotiropoulos and G.Sterman, hep-ph/9401237, Nuclear Physics B 425 (1994) 489
[3] A. Donnachie and P.V. Landshoff, Nucl.Phys. B244 (1984) 322;
[4] V.A.Abramovsky and O.V.Kancheli, Pisma Zh.Eksp.Teor.Fiz.32:498-501,1980.
[5] A.Capella, E.G.Ferreiro, C.A.Salgado, A.B.Kaidalov, Nucl.Phys. B593 (2001) 336,
Phys.Rev. D63 (2001) 054010
[6] J.Polchinski and M.J.Strassler, JHEP 0305:012,2003.
R.C.Brower, J.Polchinski, M.J.Strassler, C.Tan, JHEP 0712:005,2007