Quantum Corrections to the Schwarzschild and Kerr Metrics: Spin 1

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Abstract

A previous evaluation of one-graviton loop corrections to the energy-momentum tensor has been extended to particles with unit spin and speculations are presented concerning general properties of such forms.
1 Introduction

In an earlier paper, we described calculations of the graviton-loop corrections to the energy-momentum tensor of a charged spinless or a spin 1/2 particle of mass $m$ and we focused on the nonanalytic component of such results[1]. This is because such nonanalytic pieces involve singularities at small momentum transfer $q$ which, when Fourier-transformed, yield—via the Einstein equations—large distance corrections to the metric tensor. In particular, for both a spinless field and for a spin 1/2 field the diagonal components of the metric were shown to be modified from their simple Schwarzschild or Kerr forms—in harmonic gauge

\[
\begin{align*}
    g_{00} & = 1 - \frac{2Gm}{r} + \frac{2G^2m^2}{r^2} + \frac{7G^2m\hbar}{\pi r^3} + \ldots \\
    g_{ij} & = -\delta_{ij}[1 + \frac{2Gm}{r} + \frac{G^2m^2}{r^2} + \frac{14G^2m\hbar}{15\pi r^3} - \frac{76G^2m\hbar}{15\pi r^3}(1 - \log \mu r)] \\
         & \quad - \frac{r_ir_jG^2m^2}{r^2} + \frac{76G^2m\hbar}{15\pi r^3} + \frac{76G^2m\hbar}{5\pi r^3}(1 - \log \mu r)]
\end{align*}
\]

where $G$ is the gravitational constant. (Note that the dependence on the arbitrary scale factor $\mu$ can be removed by a coordinate transformation.) The classical—$\hbar$-independent—pieces of these modifications are well known and can be found by expanding the familiar Schwarzschild (Kerr) metric, which describes spacetime around a massive (spinning) object[2]. On the other hand, the calculation also yields quantum mechanical—$\hbar$-dependent—pieces which are new and whose origin can be understood qualitatively as arising from zitterbewegung[1].

In the case of a spin 1/2 system there exists, in addition to the above, a nonvanishing off-diagonal piece of the metric, whose one-loop corrected form, in harmonic gauge, was found to be

\[
g_{ij} = (\vec{S} \times \vec{r})_i \left( \frac{2G}{r^3} - \frac{2G^2m}{r^4} + \frac{3G^2\hbar}{\pi r^5} + \ldots \right)
\]

Here the classical component of this modification can be found by expanding the Kerr metric[3], describing spacetime around a spinning mass and once again there exist quantum corrections due to zitterbewegung[1].

Based on the feature that the diagonal components were found to have an identical form for both spin 0 and 1/2, it is tempting to speculate that
the leading diagonal piece of the metric about a charged particle has a universal form—indepdendent of spin. Whether the same is true for the leading off-diagonal—spin-dependent—component cannot be determined from a single calculation, but it is reasonable to speculate that this is also the case. However, whether these assertions are generally valid can be found only by further calculation, which is the purpose of the present note, wherein we evaluate the nonanalytic piece of the graviton-loop-corrected energy-momentum tensor for a particle of spin 1 and assess the correctness of our proposal. In the next section then we briefly review the results of the previous paper, followed by a discussion wherein the calculations are extended to the spin 1 system. Results are summarized in a brief concluding section.

2 Lightning Review

Since it important to the remainder of this note, we first present a brief review of the results obtained in our previous paper [1]. In the case of spin 0 systems, the general form of the energy-momentum tensor is

\[ < p_2 | T_{\mu\nu}(x) | p_1 >_{S=0} = \frac{e^{i(p_2-p_1)\cdot x}}{\sqrt{4E_2E_1}} \left[ 2P_{\mu}P_{\nu}F_{1,S=0}^{(S=0)}(q^2) + (q_{\mu}q_{\nu} - q^2\eta_{\mu\nu})F_{2,S=0}^{(S=0)}(q^2) \right] \]

where \( P = \frac{1}{2}(p_1 + p_2) \) is the average momentum while \( q = p_1 - p_2 \) is the momentum transfer. The tree level values for these form factors are

\[ F_{1,\text{tree}}^{(S=0)} = 1 \quad F_{2,\text{tree}}^{(S=0)} = -\frac{1}{2} \]
while the leading nonanalytic loop corrections from Figure 1a and Figure 1b were determined to be

\[
F_{1,\text{loop}}^{(S=0)} (q^2) = \frac{G q^2}{\pi} \left( -\frac{3}{4} L + \frac{1}{16} S \right)
\]

\[
F_{2,\text{loop}}^{(S=0)} (q^2) = \frac{G m^2}{\pi} \left( -2L + \frac{7}{8} S \right)
\]

where we have defined

\[
L = \log \left( \frac{-q^2}{m^2} \right) \quad \text{and} \quad S = \pi^2 \sqrt{\frac{m^2}{-q^2}}.
\]

Such pieces, which are singular in the small-q limit, come about due to the presence of two massless propagators in the Feynman diagrams[4] and can arise even in electromagnetic diagrams when this situation is present[5]. Upon Fourier-transforming, the component proportional to S is found to yield classical ($\bar{h}$-independent) behavior while the term involving L yields quantum mechanical ($\bar{h}$-dependent) corrections. The feature that the form factor $F_1^{(S=0)} (q^2 = 0)$ remains unity even when graviton loop corrections are included arises from the stricture of energy-momentum conservation[1]. There exists no restriction on $F_2^{(S=0)} (q^2 = 0)$.

In the case of spin $1/2$ there exists an additional form factor—$F_3^{(S=\frac{1}{2})} (q^2)$—associated with the presence of spin—

\[
\langle p_2 | T_{\mu\nu}(x) | p_1 \rangle >_{S=\frac{1}{2}} = \frac{e^{i(p_2-p_1) \cdot x}}{\sqrt{E_1 E_2}} \bar{u}(p_2) \left[ P_\mu P_\nu F_1^{(S=\frac{1}{2})} (q^2) \right. \\
+ \frac{1}{2} (q_\mu q_\nu - q^2 \eta_{\mu\nu}) F_2^{(S=\frac{1}{2})} (q^2) \\
\left. - \left( \frac{i}{4} \sigma_{\mu\lambda} q^\lambda P_\nu + \frac{i}{4} \sigma_{\nu\lambda} q^\lambda P_\mu \right) F_3^{(S=\frac{1}{2})} (q^2) \right] u(p_1)
\]

In this case, the tree level values for these form factors are

\[
F_{1,\text{tree}}^{(S=\frac{1}{2})} = F_{2,\text{tree}}^{(S=\frac{1}{2})} = 1 \quad F_{3,\text{tree}}^{(S=\frac{1}{2})} = 0
\]

while the nonanalytic loop corrections from Figure 1a and Figure 1b were
determined to be

\begin{align*}
F_{1,\text{loop}}^{(S=\frac{1}{2})}(q^2) &= \frac{Gq^2}{\pi}(-\frac{3}{4}L + \frac{1}{16}S) \\
F_{2,\text{loop}}^{(S=\frac{1}{2})}(q^2) &= \frac{Gm^2}{\pi}(-2L + \frac{7}{8}S) \\
F_{3,\text{loop}}^{(S=\frac{1}{2})}(q^2) &= \frac{Gq^2}{\pi}(\frac{1}{4}L + \frac{1}{4}S)
\end{align*}

In this case both $F_{1,\text{loop}}^{(S=\frac{1}{2})}(q^2 = 0)$ and $F_{3,\text{loop}}^{(S=\frac{1}{2})}(q^2 = 0)$ retain their value of unity even in the presence of graviton loop corrections. That this must be true for $F_{1,\text{loop}}^{(S=\frac{1}{2})}(q^2 = 0)$ follows from energy-momentum conservation, as before, while the nonrenormalization of $F_{3,\text{loop}}^{(S=\frac{1}{2})}(q^2 = 0)$ is required by angular-momentum conservation\[1\]. An interesting consequence is that there cannot exist an anomalous gravitomagnetic moment. The universality of these radiative corrections is suggested by the results

\begin{align*}
F_{1,\text{loop}}^{(S=0)}(q^2) &= F_{1,\text{loop}}^{(S=\frac{1}{2})}(q^2) \quad \text{and} \quad F_{2,\text{loop}}^{(S=0)}(q^2) = F_{2,\text{loop}}^{(S=\frac{1}{2})}(q^2)
\end{align*}

but, of course, the spin-dependent gravitomagnetic form factor $F_{3,\text{loop}}^{(S=\frac{1}{2})}(q^2)$ has no analog in the spin 0 sector.

The connection with the metric tensor described in the introduction arises when these results for the energy-momentum tensor are combined with the (linearized) Einstein equation\[7\]

\begin{equation}
\Box h_{\mu\nu} = -16\pi G \left( T_{\mu\nu} - \frac{1}{2} \eta_{\mu\nu} T \right)
\end{equation}

where we have defined

\begin{equation}
g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}
\end{equation}

and

\begin{equation}
T \equiv \text{Tr} T_{\mu\nu}
\end{equation}

Taking Fourier transforms, we find—for both spin 0 and spin 1/2—the diag-
\[ h_{00}(\vec{r}) = -16\pi G \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k} \cdot \vec{r}} \frac{1}{k^2} \left( \frac{m}{2} - \frac{Gm^2\pi|\vec{k}|}{8\pi} \log \frac{k^2}{m^2} \right) - \frac{43G^2mh}{15\pi r^3} \]

\[ h_{ij}(\vec{r}) = -16\pi G \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k} \cdot \vec{r}} \frac{1}{k^2} \left[ \frac{m}{2} \delta_{ij} - \delta_{ij} \left( \frac{Gm^2\pi|\vec{k}|}{32} - \frac{3Gm\vec{k}^2}{8\pi} \log \frac{k^2}{m^2} \right) \right] \]

\[ + \left( k_i k_j + \frac{1}{2} \vec{k}^2 \delta_{ij} \right) \left( \frac{7Gm^2\pi}{16|\vec{k}|} - \frac{Gm}{\pi} \log \frac{k^2}{m^2} \right) \]

\[ + 4G^2m^2 \left( \frac{\delta_{ij}}{r^2} - 2 \frac{r_ir_j}{r^4} \right) + \frac{G^2mh}{15\pi r^3} (\delta_{ij} + 44 \frac{r_i r_j}{r^2}) \]

\[ - \frac{44G^2mh}{15\pi r^3} (\delta_{ij} - 3 \frac{r_i r_j}{r^2})(1 - \log \mu r) \] (13)

while in the case of the spin 1/2 gravitomagnetic form factor we find the off-diagonal term

\[ h_{0i}(\vec{r}) = -16\pi G \frac{i}{2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{k^2} \left( 1 - \frac{Gm\pi|\vec{k}|}{4\pi} - \frac{G\vec{k}^2}{4\pi} \log \frac{k^2}{m^2} \right) (\vec{S} \times \vec{k})_i \]

\[ + \frac{21G^2h}{5\pi r^5} (\vec{S} \times \vec{r})_i \] (14)

Evaluating the various Fourier transforms, we find the results quoted in the introduction.\[^8\]

The purpose of the present note is to study how these results generalize to the case of higher spin. Specifically, we shall below examine the graviton-loop corrections to the energy-momentum tensor of a massive spin 1 system.

### 3 Spin 1

A neutral spin 1 field \( \phi_\mu(x) \) having mass \( m \) is described by the Proca Lagrangian\[^9\]

\[ \mathcal{L}(x) = -\frac{1}{4} U_{\mu\nu}(x) U^{\mu\nu}(x) + \frac{1}{2} m^2 \phi_\mu(x) \phi^\mu(x) \] (15)

\[^1\]Here the \( r \)-dependent corrections proportional to \( h \) arise from the graviton vacuum polarization correction, while those independent of \( h \) arise from corrections to the linear Einstein equation.\[^1\]
where

\[ U_{\mu\nu}(x) = i \partial_\mu \phi_\nu(x) - i \partial_\nu \phi_\mu(x) \]  

(16)

is the spin 1 field tensor. Having the Langrangian for the interactions of a spin-1 system, we can calculate the matrix elements which will be required for our calculation. Specifically, the general single graviton vertex for a transition involving an outgoing graviton with polarization indices \( \mu\nu \) and four-momentum \( q = p_1 - p_2 \), an incoming spin one particle with polarization index \( \alpha \) and four-momentum \( p_1 \) together with an outgoing spin one particle with polarization index \( \beta \) and four-momentum \( p_2 \) is

\[
V_{\beta,\alpha,\mu\nu}^{(1)}(p_1, p_2) = \frac{i\kappa}{2} \left\{ (p_1\mu p_2\nu + p_1\nu p_2\mu)\eta_{\alpha\beta} + \eta_{\mu\nu} p_1\beta p_2\alpha 
- p_1\beta (p_2\mu \eta_{\alpha\nu} + p_2\nu \eta_{\alpha\mu}) - p_2\alpha (p_1\mu \eta_{\nu\beta} + p_1\nu \eta_{\beta\mu}) 
+ (p_1 \cdot p_2 - m^2)(\eta_{\mu\alpha} \eta_{\nu\beta} + \eta_{\mu\beta} \eta_{\nu\alpha} - \eta_{\mu\nu} \eta_{\alpha\beta}) \right\} 
\]  

(17)

where \( \kappa = \sqrt{32\pi G} \) is the gravitational coupling, while the two-graviton vertex with polarization indices \( \mu\nu \) and \( \rho\sigma \), an incoming spin one particle with polarization index \( \alpha \) and four-momentum \( p_1 \) together with an outgoing spin
The triple graviton vertex function is given by

\[ V_{\beta_1\alpha_1,\mu_1,\sigma_1}^{(2)}(p_1, p_2) = -\frac{i}{4} \left\{ [p_1 \beta_1 p_2 \alpha - \eta_{\alpha_1}(p_1 \cdot p_2 - m^2)](\eta_{\mu_1}\eta_{\sigma_1} + \eta_{\alpha_1}\eta_{\mu_1} - \eta_{\mu_1}\eta_{\sigma_1}) + \eta_{\mu_1}\eta_{\beta_1}(p_1 \nu p_2 + p_1 \nu p_2) - \eta_{\alpha_1} p_1 \beta_1 p_2 \alpha - \eta_{\alpha_1} p_1 \beta_1 p_2 \alpha + (p_1 \cdot p_2 - m^2)(\eta_{\alpha_2}\eta_{\beta_1} + \eta_{\alpha_1}\eta_{\beta_1}) + \eta_{\mu_1}\eta_{\beta_1}(p_1 \nu p_2 + p_1 \nu p_2) - \eta_{\alpha_1} p_1 \nu p_2 \alpha - \eta_{\beta_1} p_1 \nu p_2 \alpha + (p_1 \cdot p_2 - m^2)(\eta_{\alpha_1}\eta_{\beta_1} + \eta_{\alpha_1}\eta_{\beta_1}) + \eta_{\alpha_1} p_1 \nu p_2 \alpha - \eta_{\beta_1} p_1 \nu p_2 \alpha + (p_1 \cdot p_2 - m^2)(\eta_{\alpha_1}\eta_{\beta_1} + \eta_{\alpha_1}\eta_{\beta_1})\right\}\]

The triple graviton vertex function is given by \[11\]
where we have defined

\[ I_{\alpha\beta,\mu\nu} = \frac{1}{2} (\eta_{\alpha\mu} \eta_{\beta\nu} + \eta_{\alpha\nu} \eta_{\beta\mu}) \]  

(20)

and

\[ P_{\alpha\beta,\mu\nu} = I_{\alpha\beta,\mu\nu} - \frac{1}{2} \eta_{\alpha\beta} \eta_{\mu\nu} \]  

(21)

The final ingredient which we need is the harmonic gauge graviton propagator

\[ D_{\alpha\beta,\mu\nu}(q) = \frac{i}{q^2 + i\epsilon} P_{\alpha\beta,\mu\nu} \]  

(22)

The leading component of the on-shell energy-momentum tensor between charged vector meson states is then found, from Eq. 17, to be

\[ < k_2, \epsilon_B | T^{(0)}_{\mu\nu} | k_1, \epsilon_A > = (k_{1\mu} k_{2\nu} + k_{1\nu} k_{2\mu}) \epsilon^*_B \cdot \epsilon_A - k_1 \cdot \epsilon^*_B (k_{2\mu} \epsilon_{A\nu} + k_{2\nu} \epsilon_{A\mu}) - k_2 \cdot \epsilon_A (k_1 \eta^*_B \eta^*_A + k_1 \epsilon^*_B \epsilon^*_A) + (k_1 \cdot k_2 - m^2) (\epsilon^*_B \epsilon_{A\nu} + \epsilon^*_A \epsilon_{B\mu}) - \eta_{\mu\nu} [(k_1 \cdot k_2 - m^2) \epsilon^*_B \cdot \epsilon_A - k_1 \cdot \epsilon^*_B k_2 \cdot \epsilon_A] \]  

(23)

and the focus of our calculation is to evaluate the graviton loop corrections to Eq. 23 via the diagrams shown in Figure 1 and keeping only the leading nonanalytic terms, details of which are described in the appendix. Note that due to conservation of the energy-momentum tensor—\( \partial^\mu T_{\mu\nu} = 0 \)—the on-shell matrix element must satisfy the gauge invariance condition

\[ q^\nu < k_2, \epsilon_B | T_{\mu\nu} | k_1, \epsilon_A >= 0 \]  

(24)

In our case, the leading order contribution satisfies this condition

\[ q^\mu < k_2, \epsilon_B | T^{(0)}_{\mu\nu} | k_1, \epsilon_A >= 0 \]  

(25)

and, in addition, the contributions of both diagrams 1a or 1b are independently gauge-invariant

\[ q^\mu Amp[a]_{\mu\nu} = q^\mu Amp[b]_{\mu\nu} = 0 \]  

(25)

and these strictures serve as an important check on our result.
Because of this gauge invariance condition, the results of these calculations are most efficiently expressed in terms of spin 1 form factors. Indeed, due to covariance and gauge invariance the form of the matrix element of $T_{\mu\nu}$ between on-shell spin 1 states must be expressible in the form

$$< p_2, \epsilon_B | T_{\mu\nu}(x) | p_1, \epsilon_A > = -\frac{e^{i(p_2-p_1) \cdot x}}{\sqrt{4E_1E_2}} [2P_{\mu}P_{\nu}\epsilon^*_B \cdot \epsilon_A F_1^{(S=1)}(q^2)$$

$$+ (q_{\mu}q_{\nu} - \eta_{\mu\nu}q^2)\epsilon^*_B \cdot \epsilon_A F_2^{(S=1)}(q^2)$$

$$+ [P_{\mu}(\epsilon^*_B \cdot \epsilon_A \cdot q - \epsilon_A \epsilon^*_B \cdot q) + P_{\nu}(\epsilon^*_B \epsilon_A \cdot q - \epsilon_A \epsilon^*_B \cdot q)] F_3^{(S=1)}(q^2)$$

$$+ [(\epsilon_A \epsilon^*_B + \epsilon^*_B \epsilon_A)q^2 - (\epsilon^*_B q_{\mu} + \epsilon_B q_{\mu})\epsilon_A \cdot q$$

$$+ (\epsilon_A q_{\mu} + \epsilon_A q_{\mu})\epsilon^*_B \cdot q + 2\eta_{\mu\nu} \epsilon_A \cdot q \epsilon^*_B \cdot q] F_4^{(S=1)}(q^2)$$

$$+ \frac{2}{m^2}P_{\mu}P_{\nu}\epsilon_A \cdot q \epsilon^*_B \cdot q F_5^{(S=1)}(q^2)$$

$$+ \frac{1}{m^2}(q_{\mu}q_{\nu} - \eta_{\mu\nu}q^2)\epsilon^*_B \cdot q \epsilon_A \cdot q F_6^{(S=1)}(q^2)]$$

(26)

Using the feature that in the Breit frame for a nonrelativistic particle the spin operator can be defined via

$$i(\hat{\epsilon}^*_B \times \hat{\epsilon}_A)_k = < 1, m_f | S_k | 1, m_i >$$

(27)

we observe that $F_1^{(S=1)}(q^2), F_2^{(S=1)}(q^2), F_3^{(S=1)}(q^2)$ correspond exactly to their spin 1/2 counterparts while $F_4^{(S=1)}(q^2), F_5^{(S=1)}(q^2), F_6^{(S=1)}(q^2)$ represent new forms unique to spin 1.

In terms of these definitions, the tree level predictions can be described as

$$F_{1,\text{tree}}^{(S=1)} = F_{3,\text{tree}}^{(S=1)} = 1$$

$$F_{2,\text{tree}}^{(S=1)} = F_{4,\text{tree}}^{(S=1)} = -\frac{1}{2}$$

$$F_{5,\text{tree}}^{(S=1)} = F_{6,\text{tree}}^{(S=1)} = 0$$

(28)

while the results of the one loop calculation can be expressed as
a) Seagull loop diagram (Figure 1a)

\[
F_{1,\text{loop }a}(q^2) = \frac{GLq^2}{\pi} (0 + 3 - 1 - \frac{1}{2}) = \frac{3GLq^2}{2} \\
F_{2,\text{loop }a}(q^2) = \frac{GLm^2}{\pi} (-5 + 2 - 2 + 4) = -\frac{GLm^2}{\pi} \\
F_{3,\text{loop }a}(q^2) = \frac{GLq^2}{\pi} (0 + 3 - 1 - \frac{1}{2}) = 0 \\
F_{4,\text{loop }a}(q^2) = \frac{GLm^2}{\pi} (0 + 1 - 1 + \frac{3}{2}) = \frac{3GLm^2}{2} \\
F_{5,\text{loop }a}(q^2) = \frac{GLm^2}{\pi} (0 - 3 + 0 + 0) = -\frac{3GLm^2}{\pi} \\
F_{6,\text{loop }a}(q^2) = \frac{GLm^2}{\pi} (-5 - \frac{1}{2} + 0 + 3) = -\frac{5GLm^2}{2} \\
\]

(29)

b) Born loop diagram (Figure 1b)

\[
F_{1,\text{loop }b}(q^2) = \frac{Gq^2}{\pi} [L(\frac{1}{4} - 3 + 2 - \frac{3}{2}) + S(\frac{1}{16} - 1 + 1 + 0)] = \frac{Gq^2}{\pi} (\frac{1}{16} S - \frac{9}{4} L) \\
F_{2,\text{loop }b}(q^2) = \frac{Gm^2}{\pi} [S(\frac{7}{8} - 1 + 2 - 1) + L(1 - 3 + 4 - 3)] = \frac{Gm^2}{\pi} (\frac{7}{8} S - L) \\
F_{3,\text{loop }b}(q^2) = \frac{Gq^2}{\pi} [S(0 - \frac{1}{2} + \frac{1}{2} + \frac{1}{4}) + L(\frac{1}{6} - \frac{5}{4} + \frac{3}{4} + \frac{7}{12})] = \frac{Gq^2}{\pi} (\frac{1}{4} S + \frac{1}{4} L) \\
F_{4,\text{loop }b}(q^2) = \frac{GLm^2}{\pi} (0 - 1 + \frac{1}{2} - \frac{3}{2} + \frac{Gq^2}{\pi} \left[L(-\frac{17}{8} + \frac{3}{8} - \frac{1}{2} + \frac{7}{8}) + S(-\frac{41}{128} + \frac{3}{16} - \frac{1}{4} + \frac{1}{16})\right] = -\frac{3GLm^2}{2} - \frac{Gq^2}{\pi} (\frac{11}{8} L + \frac{41}{128} S) \\
F_{5,\text{loop }b}(q^2) = \frac{GLm^2}{\pi} (0 + 3 + 0 + 0) + \frac{Gq^2}{\pi} \left[S(\frac{5}{128} + \frac{3}{16} + 0 - \frac{3}{16}) + L(0 + \frac{3}{4} + 0 - \frac{1}{2})\right] = 3\frac{GLm^2}{\pi} + \frac{Gq^2}{\pi} (\frac{5}{128} S + \frac{1}{4} L) \\
F_{6,\text{loop }b}(q^2) = \frac{Gm^2}{\pi} \left[S(\frac{43}{64} - \frac{1}{8} + \frac{1}{4} - \frac{1}{8}) + L(\frac{13}{3} + \frac{1}{2} + \frac{1}{2} - \frac{7}{3})\right] \\
= \frac{Gm^2}{\pi} (3L + \frac{43}{64} S) \\
\]

(30)

where we have divided each contribution into the piece which arises from the
first four bracketed pieces of the triple graviton vertex above.\(^2\)

The full results of this calculation can then be described via:

\[
egin{align*}
F_{1}^{(S=1)}(q^2) &= 1 + \frac{Gq^2}{\pi}(-\frac{3}{4}L + \frac{1}{16}S) + \ldots \\
F_{2}^{(S=1)}(q^2) &= -\frac{1}{2} + \frac{Gm^2}{\pi}(-2L + \frac{7}{8}S) + \ldots \\
F_{3}^{(S=1)}(q^2) &= 1 + \frac{Gq^2}{\pi}(\frac{1}{4}L + \frac{1}{4}S) + \ldots \\
F_{4}^{(S=1)}(q^2) &= -\frac{1}{2} + \frac{Gq^2}{\pi}(\frac{11}{8}L + \frac{41}{128}S) + \ldots \\
F_{5}^{(S=1)}(q^2) &= \frac{Gq^2}{\pi}(\frac{1}{4}L + \frac{5}{128}S) + \ldots \\
F_{6}^{(S=1)}(q^2) &= \frac{Gm^2}{\pi}(\frac{1}{4}L + \frac{43}{128}S) + \ldots
\end{align*}
\]

and we note that \(F_{1,2,3,loop}^{(S=1)}(q^2)\) as found for unit spin agree precisely with the forms \(F_{1,2,3,loop}^{(S=\frac{1}{2})}(q^2)\) determined previously for spin 1/2 and with \(F_{1,2,loop}^{(S=0)}(q^2)\) in the spinless case. It is also interesting that the loop contributions to the "new" form factors \(F_{4,loop}^{(S=1)}(q^2)\), \(F_{5,loop}^{(S=1)}(q^2)\) which have no lower spin analog, vanish to order \(q^0\) even though there exist nonzero contributions from both loop diagrams individually. Of course, the nonrenormalization of \(F_{1}^{(S=1)}(q^2 = 0)\) and \(F_{3}^{(S=1)}(q^2 = 0)\) required by energy-momentum and angular momentum conservation is obtained, as required, meaning that, as noted above, there exists no anomalous gravitomagnetic moment.

However, there is a new feature here that deserves notice. Working in the Breit frame and assuming nonrelativistic motion, we have the kinematic constraints

\[
\begin{align*}
\epsilon_A^0 &\simeq \frac{1}{2m} \hat{\epsilon}_A \cdot \hat{q}, \quad \epsilon_B^0 \simeq -\frac{1}{2m} \hat{\epsilon}_B \cdot \hat{q} \\
\hat{\epsilon}_B \cdot \epsilon_A &\simeq -\hat{\epsilon}_B \cdot \hat{\epsilon}_A - \frac{1}{2m^2} \hat{\epsilon}_B \cdot \hat{q} \hat{\epsilon}_A \cdot \hat{q}
\end{align*}
\]

\(^2\)There exists no contribution to the nonanalytic terms from the pieces in the fifth bracket since the intermediate gravitons are required to be on-shell.
we find that
\[
<p_2, \epsilon_B | T_{00}(0) | p_1, \epsilon_A > \cong m \left\{ \hat{\epsilon}_B^* \cdot \hat{\epsilon}_A F_1^{(S=1)}(q^2) + \frac{1}{2m^2} \hat{\epsilon}_B^* \cdot \vec{q} \hat{\epsilon}_A \cdot \vec{q} \right\} \times \left[ F_1^{(S=1)}(q^2) - F_2^{(S=1)}(q^2) - 2(F_4^{(S=1)}(q^2) + F_5^{(S=1)}(q^2) - \frac{q^2}{2m^2} F_6^{(S=1)}(q^2)) \right] + \ldots
\]

\[
<p_2, \epsilon_B | T_{0i}(0) | p_1, \epsilon_A > \cong - \frac{1}{2} [(\hat{\epsilon}_B^* \times \hat{\epsilon}_A) \times \vec{q}]_i F_3^{(S=1)}(q^2) + \ldots
\]

Then using the connections
\[
i \hat{\epsilon}_B^* \times \hat{\epsilon}_A = < 1, m_f | \vec{S} | 1, m_i > \\
\frac{1}{2} (\hat{\epsilon}_B^* \epsilon_A + \epsilon_A \hat{\epsilon}_B^*) - \frac{1}{3} \delta_{ij} \hat{\epsilon}_B^* \cdot \hat{\epsilon}_A = < 1, m_f | \frac{1}{2} (S_i S_j + S_j S_i) - \frac{2}{3} \delta_{ij} | 1, m_i >
\]

between the Proca polarization vectors and the spin operator \( \vec{S} \) we can identify values for the gravitoelectric monopole, gravitomagnetic dipole, and gravitoelectric quadrupole coupling constants
\[
K_{E0} = m F_1^{(S=1)}(q^2 = 0) \\
K_{M1} = \frac{1}{2} F_3^{(S=1)}(q^2 = 0) \\
K_{E2} = \frac{1}{2m} \left[ F_1^{(S=1)}(q^2 = 0) - F_3^{(S=1)}(q^2 = 0) - 2F_4^{(S=1)}(q^2 = 0) - 2F_5^{(S=1)}(q^2 = 0) \right]
\]

Taking \( Q_g \equiv m \) as the gravitational ”charge,” we observe that the tree level values—
\[
K_{E0} = Q_g \quad K_{M1} = \frac{Q_g}{2m} \quad K_{E2} = \frac{Q_g}{m^2}
\]

are unrenormalized by loop corrections. That is to say, not only does there not exist any anomalous gravitomagnetic moment, as mentioned above, but also there is no anomalous gravitoelectric quadrupole moment.

## 4 Conclusion

Above we have calculated the graviton loop corrections to the energy-momentum tensor of a spin 1 system. We have confirmed the universality
which was speculated in our previous work in that we have verified that
\[
F^{(S=0)}_{1,\text{loop}}(q^2) = F^{(S=\frac{1}{2})}_{1,\text{loop}}(q^2) = F^{(S=1)}_{1,\text{loop}}(q^2) \\
F^{(S=0)}_{2,\text{loop}}(q^2) = F^{(S=\frac{1}{2})}_{2,\text{loop}}(q^2) = F^{(S=1)}_{2,\text{loop}}(q^2) \\
F^{(S=\frac{1}{2})}_{3,\text{loop}}(q^2) = F^{(S=1)}_{3,\text{loop}}(q^2) \tag{37}
\]
The universality in the case of the classical (square root) nonanalyticities is not surprising and in fact is required by the connection to the metric tensor. In the case of the quantum (logarithmic) nonanalyticities it is not clear why these terms must be spin-independent. We also found additional form factors for the spin 1 system and have shown that in addition to the vanishing of the anomalous gravitomagnetic moment there cannot exist any anomalous gravitoelectric quadrupole moment. It is tempting to conclude that the graviton loop correction universality which we obtained holds for arbitrary spin. However, it is probably not possible to show this by generalizing the calculations above. Indeed the spin 1 result involves considerably more computation than does its spin 1/2 counterpart, which was already much more tedious than that for spin 0. Perhaps a generalization such as that used in nuclear beta decay can be employed\cite{12}. Work is underway on such an extension and results will be reported in an upcoming communication.

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### 5 Appendix

In this section we sketch how our results were obtained. The basic idea is to calculate the Feynman diagrams shown in Figure 1. Thus for Figure 1a we find\cite{13}

\[
Amp[a]_{\mu\nu} = \frac{1}{2!} \int \frac{d^4k}{(2\pi)^4} \frac{e^{\beta} V_{\beta,\alpha,\lambda\kappa,\rho\sigma}^{(2)}(p_2, p_1)\epsilon_\alpha^A P[\alpha; \lambda, \kappa] P[\gamma, \delta; \sigma, \rho] \tau_{\mu\nu}^{\alpha\beta, \gamma\delta}(k, q)}{k^2(k - q)^2} \tag{38}
\]
while for Figure 1b

\[ \text{Amp}[b]_{\mu \nu} = \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2(k-q)^2(k-p)^2-m^2} \times e_B^{\beta} V^{(1)}_{\beta,\delta,\lambda,\kappa}(p_2, p_1 - k) \left( -\eta^{\delta \xi} + \frac{(p_1 - k)^{\delta}(p_1 - k)^{\xi}}{m^2} \right) \times V^{(1)}_{\xi,\theta,\rho,\sigma}(p_1 - k, p_1) e_A^\theta P[\alpha \beta; \lambda \kappa] P[\gamma \delta; \sigma \rho] \tau^{\alpha \beta, \gamma \delta}_{\mu \nu}(k, q) \]  

(39)

Here the various vertex functions are listed in section 3, while for the integrals, all that is needed is the leading nonanalytic behavior. Thus we use

\[ I(q) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2(k-q)^2} = -\frac{i}{32\pi^2} (2L + \ldots) \]

\[ I_\mu(q) = \int \frac{d^4k}{(2\pi)^4} \frac{k_\mu}{k^2(k-q)^2} = \frac{i}{32\pi^2} (q_\mu L + \ldots) \]

\[ I_{\mu \nu}(q) = \int \frac{d^4k}{(2\pi)^4} \frac{k_\mu k_\nu}{k^2(k-q)^2} = -\frac{i}{32\pi^2} (q_\mu q_\nu \frac{2}{3} L - q^2 \eta_{\mu \nu} \frac{1}{6} L + \ldots) \]

\[ I_{\mu \nu \alpha}(q) = \int \frac{d^4k}{(2\pi)^4} \frac{k_\mu k_\nu k_\alpha}{k^2(k-q)^2} = -\frac{i}{32\pi^2} (-q_\mu q_\nu q_\alpha \frac{L}{2} + \frac{(\eta_{\mu \nu} q_\alpha + \eta_{\mu \alpha} q_\nu + \eta_{\nu \alpha} q_\mu)}{12} L q^2 + \ldots) \]  

(40)
for the "bubble" integrals and

\[
\begin{align*}
J(p, q) &= \int \frac{d^4k}{(2\pi)^4} \frac{1}{k^2(k-q)^2((k-p)^2 - m^2)} = -\frac{i}{32\pi^2 m^2}(L + S) + \ldots \\
J_\mu(p, q) &= \int \frac{d^4k}{(2\pi)^4} \frac{k_\mu}{k^2(k-q)^2((k-p)^2 - m^2)} = \frac{i}{32\pi^2 m^2} \\
	imes \left[ p_\mu((1 + \frac{1}{2} \frac{q^2}{m^2})L - \frac{1}{4} \frac{q^2}{m^2}S) - q_\mu(L + \frac{1}{2} S) + \ldots \right] \\
J_{\mu\nu}(p, q) &= \int \frac{d^4k}{(2\pi)^4} \frac{k_\mu k_\nu}{k^2(k-q)^2((k-p)^2 - m^2)} = \frac{i}{32\pi^2 m^2} \\
	imes \left[ -q_\mu q_\nu(L + \frac{3}{8} S) - p_\mu p_\nu \frac{q^2}{m^2}(\frac{1}{2} L + \frac{1}{8} S) \\
+ q^2 \eta_{\mu\nu} \left( \frac{1}{4} L + \frac{1}{8} S \right) + (q_\mu p_\nu + q_\nu p_\mu)(\frac{1}{2} + \frac{1}{2} \frac{q^2}{m^2})L + \frac{3}{16} \frac{q^2}{m^2} S \right] \\
J_{\mu\nu\alpha}(p, q) &= \int \frac{d^4k}{(2\pi)^4} \frac{k_\mu k_\nu k_\alpha}{k^2(k-q)^2((k-p)^2 - m^2)} \\
&= -\frac{i}{32\pi^2 m^2} \left[ q_\mu q_\nu q_\alpha \left( L + \frac{5}{16} S \right) + p_\mu p_\nu p_\alpha \left( -\frac{1}{6} \frac{q^2}{m^2} \right) \\
+ (q_\mu p_\nu p_\alpha + q_\nu p_\mu p_\alpha + q_\alpha p_\mu p_\nu) \left( \frac{1}{3} \frac{q^2}{m^2} L + \frac{1}{16} \frac{q^2}{m^2} S \right) \\
+ (q_\mu q_\nu p_\alpha + q_\nu q_\mu p_\alpha + q_\mu q_\alpha p_\nu) \left( \left( -\frac{1}{3} - \frac{1}{2} \frac{q^2}{m^2} \right) L - \frac{5}{32} \frac{q^2}{m^2} S \right) \\
+ (\eta_{\mu\nu} p_\alpha + \eta_{\mu\alpha} p_\nu + \eta_{\nu\alpha} p_\mu) \left( \frac{1}{12} q^2 L \right) \\
+ (\eta_{\mu\alpha} q_\nu + \eta_{\nu\alpha} q_\nu + \eta_{\nu\alpha} q_\mu) \left( -\frac{1}{6} q^2 L - \frac{1}{16} q^2 S \right) \right] + \ldots
\end{align*}
\]

(41)

for their "triangle" counterparts. Similarly higher order forms can be found, by either direct calculation or by requiring various identities which must be satisfied when the integrals are contracted with \( p^\mu, q^\mu \) or with \( \eta^{\mu\nu} \). Using these integral forms and substituting into Eqs. 38 and 39, one determines the results quoted in section 3.
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