On scheduling quality of precise resource measure sets which consist of tasks of circular type in GRID systems

A E Saak\textsuperscript{1}, V V Kureichik\textsuperscript{1}

\textsuperscript{1} Southern Federal University, 105/42 Bolshaya Sadovaya Str., Rostov-on-Don, 344006, Russia

E-mail: aesaak@sfedu.ru

Abstract. To serve parallel tasks of multiple users of Grid systems require the development of efficient methods for allocating processors and time resources. Optimal resource management is practically unrealizable due to the exponential complexity. In the situation, when the Grid-system is modeled by the resource quadrant and the user's task is represented by the resource rectangle, heuristic polynomial algorithms are proposed with the quality assessment of scheduling by a Non-Euclidean heuristic measure. Using the example of arrays with an exact resource measure and induced by partridge linings, the adaptation of level polynomial algorithms is investigated, and a comparative analysis of the proposed algorithms is carried out.

1. Introduction
To serve parallel tasks of multiple users of grid systems requires the development of efficient methods for allocating processors and time resources. Optimal resource management is practically unrealizable due to the exponential complexity. The apparatus of the scheduling theory, the theoretical basis of the algorithmic support for scheduling of polynomial completeness, is the resource rectangles environment, proposed and developed in [1]. In [1] the circular, hyperbolic and parabolic types are defined for arrays from several tasks. The quadratic type of a single task was introduced in [2]. In the resource rectangles environment, operations on resource rectangles are introduced. Polynomial heuristic algorithms for resource allocation based on the introduced operations are proposed [1].

2. Materials and methods
The user's task, which demands service from the Grid-system scheduler, is a resource rectangle, the horizontal dimension of which is taken equal to the number of time resource units and the vertical dimension is taken to be equal to the number of processor resource units required to process the task [3]. Grid systems with a centralized scheduling system structure and the ability to simultaneously execute a multiprocessor task at several sites, which contain parallel systems, are represented by a resource quadrant [1]. The quality of heuristic algorithms scheduling is estimated by a non-euclidean heuristic measure that takes into account the area and shape of the occupied resource area [1].

In [4] an array of resource rectangles is defined, which is called as having a precise resource measure, for which the sum of the areas of the resource rectangles is equal to the square of an integer.

Scheduling of an array, which has the precise resource measure and consists of resource squares, was studied in [4] for level polynomial algorithms with level unattainability. Level polynomial algorithms
exceeding the level and with a minimum deviation are introduced in [2]. Resource square has a circular type, which follows from the definitions formulated in [2].

This article examines the adaptation of level polynomial algorithms with the exceeding of the level and with a minimum deviation for arrays of a precise resource measure which consist of circular type tasks.

3. Scheduling of a set of a precise resource measure with the tasks of circular type by level algorithms

Let us denote the array of the resource squares ordered by a decrease of their heights, which is induced by partridge tiling [5] with the partridge number $k$ of the square [6] equal to $k=11$ [7] as array I.

The array of resource squares induced by the partridge tiling with the partridge number of the square equal to $k=12$ [8], ordered by a decrease of squares’ heights, is denoted as array II.

The array of resource squares induced by the partridge tiling with the partridge number of the square equal to $k=13, 14$ [9], ordered by a decrease of squares’ heights, is denoted as arrays III and IV.

The results of packing of arrays I – IV by the level algorithm with the exceeding of the level are presented in Figures 1–4. Square’s side value is indicated in the center of a square.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Packing of array I by the level algorithm with the exceeding of the level}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig2.png}
\caption{Packing of array II by the level algorithm with the exceeding of the level}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3.png}
\caption{Packing of array III by the level algorithm with the exceeding of the level}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig4.png}
\caption{Packing of array IV by the level algorithm with the exceeding of the level}
\end{figure}
The heuristic measure values of the resource enclosures of the level algorithm with the exceeding of the level and the error in % relative to the optimal value of \( \frac{1}{2} \) are presented in Table 1.

### Table 1. The heuristic measure values of resource enclosures in the level algorithm with the exceeding of the level

| Array’s number | Heuristic measure value | \( \Delta, \% \) |
|----------------|-------------------------|-----------------|
| I              | 0.60                    | 20              |
| II             | 0.58                    | 16              |
| III            | 0.58                    | 16              |
| IV             | 0.57                    | 14              |

We could see that heuristic measure values of resource enclosures in the level algorithm with the exceeding of the level don’t exceed the value of \( \frac{1}{2} + 0.1 \).

The results of packing of sets I – IV by the level algorithm with minimal deviation are presented in Figures 5-8.
The heuristic measure values of the resource enclosures of the level algorithm with minimal deviation and the error in % relative to the optimal value of $\frac{1}{2}$ are presented in Table 2.

| Set’s number | Heuristic measure value | $\Delta$, % |
|--------------|-------------------------|-------------|
| I            | 0.57                    | 14          |
| II           | 0.58                    | 16          |
| III          | 0.57                    | 14          |
| IV           | 0.55                    | 10          |

We could see that resource enclosures’ heuristic measure values of the level algorithm with minimal deviation don’t exceed the value of $\frac{1}{2} + 0.08$.

The graphs of the heuristic measure values of resource enclosures, which were obtained with the use of the level algorithm with the exceeding of the level and the one with minimal deviation when scheduling sets I to IV, are presented in Figure 9. The results of the level algorithm with level unattainability [4] were presented for the sake of comparison completeness of the algorithms.

![Graph](image)

**Figure 9.** The heuristic measure values of resource enclosures in level algorithms

We could see that the level algorithm with level unattainability had the smallest maximum value of the heuristic measure equaled to $\frac{1}{2} + 0.17$ when considering tested arrays of resource squares, which have the precise resource measure and are induced by partridge tiling.

The expediency of using the proposed algorithms for scheduling by arrays, which have the precise resource measure, is confirmed by a deviation value not exceeding 20% of the optimal value.

The running time of the optimal algorithm exceeds 13 days and 8 hours, while the 3.3GHz Intel Xeon single-core computer has allocated an array of 105 squares corresponding to $k = 14$ [9], whereas the level algorithm with a minimum deviation gives a solution with an error of 10% almost instantly.

4. **Conclusion**

For tested arrays that had the precise resource measure and were induced by partridge tiling, the resource enclosures’ heuristic measure values of the proposed level algorithms were calculated. The adaptation potential of the developed polynomial algorithms was shown. Therefore we can recommend the algorithms for scheduling in grid systems for arrays classes of user tasks, which were studied in the paper.
Acknowledgments
The reported study was funded by the Russian Foundation for Basic Research according to research project No. 19-01-00059.

References
[1] Saak A E 2013 Polynomial algorithms of resource distribution in Grid systems based on quadratic typification of task sets Journal of information technologies 7 32.
[2] Saak A E 2016 Scheduling of tasks of the circular-type and hyperbolic type in Grid systems Journal of information technologies 22 323-32.
[3] Caramia M, Giordani S and Iovanella A 2004 Grid scheduling by on-line rectangle packing Networks 44 2 106-119.
[4] Saak A E 2015 Scheduling of tasks of the total resource measure equal to the square of an integer Journal of information technologies 21 675-679.
[5] Friedman E 2003 Reptiles, Partridges, and Golden Bees: Tiling Shapes with Similar Copies. http://www2.stetson.edu/~efriedma/papers/reptiles.ppt
[6] Hamlyn P, Friedman E 2003 Partridge numbers Geombinatorics Quarterly XIII 1 10-18.
[7] Simonis H, O'Sullivan B 2008 Search strategies for rectangle packing. Retrieved from: https://pdfs.semanticscholar.org/8092/b93269fcb1628e778ae4399e9adec70dcac65.pdf
[8] Ågren M, Carlsson M, Beldiceanu N, Sbihi M, Zampelli S, Truchet C 2009 Six ways of integrating symmetries within non-overlapping constraints SICS Technical Report T2009:01 Retrieved from: https://www.diva-portal.org/smash/get/diva2:1042511/FULLTEXT01.pdf
[9] Hougardy S 2012 A Scale Invariant Exact Algorithm for Dense Rectangle Packing Problems. Research Institute for Discrete Mathematics University of Bonn Report No: 101020 Retrieved from: http://www.or.uni-bonn.de/~hougardy/paper/PerfectRectanglePacking.pdf