Charm quark and meson production in association with single-jet at the LHC

Rafał Maciuła\textsuperscript{1,∗} and Antoni Szczurek\textsuperscript{†,‡}

\textsuperscript{1}Institute of Nuclear Physics, Polish Academy of Sciences, Radzikowskiego 152, PL-31-342 Kraków, Poland

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Abstract

We discuss charm quark/antiquark and charmed meson production in association with one extra jet (gluon, quark, antiquark) at the LHC. The calculations are performed both in collinear and $k_T$-factorization approaches. Different unintegrated gluon distribution functions are used in the $k_T$-factorization approach. Several predictions for the LHC are presented. We show distributions in rapidity and transverse momenta of $c/\bar{c}$ (or charmed mesons) and the associated jet as well as some two-dimensional observables. Interesting correlation effects are predicted, e.g. in azimuthal angles $\varphi_{cc}$ and $\varphi_{c-jet}$. We have also discussed a relation of the $2 \to 2$ and $2 \to 3$ partonic calculations in the region of large transverse momenta of charm quarks/antiquarks as well as the similarity of the next-to-leading order collinear approach and the $k_T$-factorization approach with the KMR unintegrated parton distribution functions. Integrated cross sections for $D^0 +$ jet production for ATLAS detector acceptance and for different cuts on jet transverse momenta are also presented.

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\textsuperscript{†} also at University of Rzeszów, PL-35-959 Rzeszów, Poland

\textsuperscript{∗}Electronic address: rafal.maciula@ifj.edu.pl

\textsuperscript{‡}Electronic address: antoni.szczurek@ifj.edu.pl

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I. INTRODUCTION

Production of charm quarks/antiquarks or charmed mesons is interesting topic from the point of view of applications of quantum chromodynamics. At large energies and in particular at the LHC the gluon initiated processes dominate. Therefore the charm production processes can be very useful in testing and in extracting (hopefully in a future) gluon distribution in proton. Since charm quarks are relatively light one can get access to the region of rather small gluon momentum fractions where the QCD dynamics is not fully understood.

Usually inclusive distributions of single charmed mesons are presented by experimental collaborations [1–5]. Production of the D meson pairs and correlations between them were discussed so far only by the LHCb collaboration [6]. On the theoretical side the inclusive production is described by the collinear next-to-leading order (NLO) approach [7, 8]. An interesting alternative is the $k_T$-factorization approach. The latter approach is the only one which was successfully used to describe the correlation observables [9, 10].

The high luminosity already achieved at the LHC potentially allows to study more complicated final states such as D mesons and associated jets. Such final states are also accessible at present on theoretical side. The automatized methods (calculations) of multi-leg amplitudes are in this context very important. The main effort in this field was concentrated so far on multijet production [11–13] or production of Higgs boson or gauge bosons in association with a few jets [14–16]. Recently, also production of two charm quark-antiquark pairs has been carefully studied [17]. So far not much attention was devoted to similar case when jets are produced in association with charm. Even without any calculations one can expect large cross sections for associated production of charm and jets. The new situation (the new multi-leg methods, high-luminosity) opens new possibilities in testing dynamics of the pQCD processes. In our opinion it is a good time to explore the new possibility.

In the present paper we start the new investigation program limiting to production of charm quarks and/or D mesons associated with single-jet. We shall use the leading-order (LO) collinear approach as well as the $k_T$-factorization approach. The latter approach was successfully used both for $c\bar{c}$ production [9] and for inclusive jet [18], dijet [19] and even four-jet production very recently [13]. In our practical exploration we shall use
an unique tool - A Very Handy LIBrary (AVHLIB)\textsuperscript{1} prepared for calculating multi-leg processes, up to four-particle final states within both collinear and $k_T$-factorization frameworks. In the following paper we wish to compare results obtained with the two different approaches.

II. A SKETCH OF THE THEORETICAL FORMALISM

The diagrams under consideration relevant for the production of the $c\bar{c}$ pair in association with single-jet are shown schematically in Fig.1. We include three classes of the QCD $2 \rightarrow 3$ partonic subprocesses: $gg \rightarrow c\bar{c}g$, $gq(\bar{q}) \rightarrow c\bar{c}q(\bar{q})$ and $q(\bar{q})g \rightarrow c\bar{c}q(\bar{q})$, working in the $n_F = 3$ flavour scheme, where $q = u, d, s$ and $\bar{q} = \bar{u}, \bar{d}, \bar{s}$.

![Diagram](image)

FIG. 1: A diagrammatic representation of the mechanisms considered for the $pp \rightarrow c\bar{c} + jet$ reaction.

The hadronic cross section for inclusive $pp \rightarrow c\bar{c} + jet$ reaction in the LO collinear approach can be written as:

$$d\sigma(pp \rightarrow c\bar{c} + jet) = \int dx_1 dx_2 \left[ g(x_1, \mu_F^2)g(x_2, \mu_F^2) \ d\hat{\sigma}_{gg \rightarrow c\bar{c}} + \sum_f q_f(x_1, \mu_F^2)g(x_2, \mu_F^2) \ d\hat{\sigma}_{gq \rightarrow c\bar{c}q} + g(x_1, \mu_F^2) \sum_f q_f(x_2, \mu_F^2) \ d\hat{\sigma}_{gq \rightarrow c\bar{c}q} + \sum_f \bar{q}_f(x_1, \mu_F^2)g(x_2, \mu_F^2) \ d\hat{\sigma}_{g\bar{q} \rightarrow c\bar{c}\bar{q}} + g(x_1, \mu_F^2) \sum_f \bar{q}_f(x_2, \mu_F^2) \ d\hat{\sigma}_{g\bar{q} \rightarrow c\bar{c}\bar{q}} \right],$$

(2.1)

where $g(x_{1,2}, \mu_F^2)$, $q_f(x_{1,2}, \mu_F^2)$ and $\bar{q}_f(x_{1,2}, \mu_F^2)$ are the standard collinear parton distribution functions (PDFs) for gluons, quarks and antiquarks, respectively, carrying $x_{1,2}$ mo-

\textsuperscript{1} available for download at https://bitbucket.org/hameren/avhlib
momentum fractions of the proton and evaluated at the factorization scale $\mu_F$. Here, $d\hat{\sigma}$ are the elementary partonic cross sections for a given $2 \rightarrow 3$ subprocess.

The elementary cross section, e.g. for the $gg \rightarrow c\bar{c}g$ mechanism has the following generic form:

$$d\hat{\sigma} = \frac{1}{2\hat{s}} |M_{gg\rightarrow c\bar{c}g}|^2 \frac{d^3p_1}{2E_1(2\pi)^3} \frac{d^3p_2}{2E_2(2\pi)^3} \frac{d^3p_3}{2E_3(2\pi)^3} (2\pi)^4 \delta^3(p_1 + p_2 + p_3 - k_1 - k_2),$$

(2.2)

where $M_{gg\rightarrow c\bar{c}g}$ is the partonic on-shell matrix element, $\hat{s}$ is the partonic center-of-mass energy squared, $p_1, p_2, p_3$ are four-momenta of final $c$ quark, $\bar{c}$ antiquark and gluon, respectively, and $k_1$ and $k_2$ are four-momenta of incoming gluons.

Switching to the $k_T$-factorization approach, the analogous formula to (2.1) takes the following form:

$$d\sigma(pp \rightarrow c\bar{c} + jet) = \int dx_1 \frac{d^2k_{1t}}{\pi} \frac{d^2k_{2t}}{\pi} \left[ F_g(x_1, k_{1t}^2, \mu_F^2) F_g(x_2, k_{2t}^2, \mu_F^2) d\hat{\sigma}_{g^*g^*\rightarrow c\bar{c}g} + F_q(x_1, k_{1t}^2, \mu_F^2) F_g(x_2, k_{2t}^2, \mu_F^2) d\hat{\sigma}_{q^*g^*\rightarrow c\bar{c}q} + F_\bar{q}(x_1, k_{1t}^2, \mu_F^2) F_g(x_2, k_{2t}^2, \mu_F^2) d\hat{\sigma}_{\bar{q}^*g^*\rightarrow c\bar{c}q} + F_{\bar{q}}(x_1, k_{1t}^2, \mu_F^2) F_\bar{q}(x_2, k_{2t}^2, \mu_F^2) d\hat{\sigma}_{\bar{q}^*\bar{q}^*\rightarrow c\bar{c}\bar{q}} \right].$$

(2.3)

Here, $k_{1,2t}$ are transverse momenta of incident partons (new degrees of freedom compared to collinear approach) and $F(x, k_t^2, \mu_F^2)$’s are transverse momentum dependent, so-called, unintegrated parton distribution functions (uPDFs). Within this framework the elementary partonic cross sections are defined in terms of off-shell matrix elements, that take into account that both partons entering the hard process are off-shell with virtualities $k_1^2 = -k_{1t}^2$ and $k_2^2 = -k_{2t}^2$.

The off-shell matrix elements are known only in the LO and only for limited types of QCD $2 \rightarrow 2$ processes (see e.g. heavy quarks [21], dijet [19], Drell-Yan [22]). Some first steps to calculate NLO corrections in the $k_T$-factorization framework have been done only very recently for diphoton production [23, 24]. Here, we extend the standard scope of mechanisms usually studied in the $k_T$-factorization approach by analysing the three-particle $c\bar{c} + jet$ final state. Moving on to higher final state parton multiplicities, it is possible to generate relevant amplitudes analytically applying suitably defined Feynman rules [25] or recursive methods, like generalised BCFW recursion [26], or numerically with the help of methods of numerical BCFW recursion [20] as implemented in AVHLIB. The lat-
ter method was already successfully applied even for $2 \to 4$ production mechanisms in the case of $c\bar{c}c\bar{c}$ \cite{17} and four-jet \cite{13} final states.

In this Fortran library, scattering amplitudes are calculated numerically as a function of the external four-momenta via Dyson-Schwinger recursion generalized to tree-level amplitudes with off-shell initial-state particles. This recursive method exists in several explicit implementations with on-shell initial-state particles (see e.g. Ref. \cite{27}). AVH-LIB allows for various choices of the representation of the external helicities and colors. The library includes a full Monte Carlo program with an adaptive phase-space generator \cite{28, 29} that deals with the integration variables related to both the initial-state momenta and the final-state momenta. The program can also conveniently generate a file of un-weighted events, which approach was used in the analysis presented in this paper.

In the numerical calculations below, we set charm quark mass $m_c = 1.5$ GeV and renormalization/factorization scales $\mu = \mu_R = \mu_F = \sqrt{m_t^2 + m_{c,\bar{c}}^2 + p_{T,\text{jet}}^2}$, where $m_t = \sqrt{p_T^2 + m_c^2}$ is the transverse mass of charm quark or antiquark. We use running strong-coupling $\alpha_S$ at next-to-leading order as implemented in the MMHT2014 set of PDFs \cite{30}.

III. NUMERICAL RESULTS

A. Collinear approach

Here we start presentation of our results within the collinear approach which is a good reference point for further calculations in the $k_T$-factorization. In this subsection we shall show results in the full phase space at the parton level. In Figs. 2, 3, 4 and 5, we show distributions in $c/\bar{c}$ transverse momentum, rapidity, diparton invariant masses as well as in relative azimuthal angle between different outgoing partons. In these calculation we used MMHT2014nlo set of PDFs \cite{30} as an example. Here by jets we understand partons (gluons, quarks, antiquarks) with transverse momenta $p_{T,\text{jet}}^c > 20$ GeV. We find that for the full phase space the contribution of processes $gg \to c\bar{c}g$ is 3-4 times larger than that for combined $q(\bar{q})g \to c\bar{c}q(\bar{q})$, $gq(\bar{q}) \to c\bar{c}q(\bar{q})$ partonic processes. However, the processes with light quarks/antiquarks start to dominate at large jet (pseudo)rapidities $|\eta| > 4.5$. We observe a broad plateau in transverse momentum distribution of $c/\bar{c}$ quarks/antiquarks for $p_{T}^c < 20$ GeV. The effect is a consequence of the cut on jet transverse momentum $p_{T,\text{jet}}^c > 20$
GeV. For larger cuts the plateau would be even broader. The distributions in $\phi_{c\text{-jet}}$ and $\phi_{c\bar{c}}$ are particularly interesting. While the first one has rather typical dependence with the maximum at the back-to-back configuration, the second one has maximum at $\phi_{c\bar{c}} \sim 0$, rather different than in the case of inclusive $c\bar{c}$ production [9].

In the next subsection we show similar results obtained in the $k_T$-factorization approach.

FIG. 2: Transverse momentum distribution of c-quark (left panel) and associated jet (right panel) in the collinear approach.

FIG. 3: Rapidity distribution of c-quark (left panel) and associated jet (right panel) in the collinear approach.
B. \( k_T \)-factorization approach

In this subsection we show similar results but in the \( k_T \)-factorization approach (see Figs. 6, 7, 8 and 9). Here we include only the dominant \( gg \)-initiated processes. We show results for a few different unintegrated gluon distribution functions (uGDFs) from the literature. In the case of inclusive production of \( cc \) usually the Kimber-Martin-Ryskin (KMR) \([31, 32]\) distribution gives the best description of the experimental data \([9]\). For consistency, we use MMHT2014 collinear PDFs in calculation of the KMR transverse momentum dependent distributions. In the case of the KMR uGDF we show also result when limiting transverse momenta of initial gluons (this will be discussed below). The
comparison of corresponding results show that the contribution of large initial gluon transverse momenta (virtualities) is rather large. For the KMR distribution it is very probable that more than one jet (two or three) are produced. A rigorous eliminating of such cases in the $k_T$-factorization approach is not an easy and obvious task. We think that the cut on initial gluon transverse momenta $k_T < p_{T,min}^{jet}$ takes this into account in an approximate way. This procedure is reliable only for the KMR uGDFs when the range of (pseudo)rapidity coverage is large (as for ATLAS or CMS). For other uGDFs it strongly depends on their construction. For comparison, we also use here the JH2013 set2 [33] and the Jung setA0 [34] CCFM-based uGDFs.

Compared to the LO collinear factorization approach no plateau at $p_T^{c} < 20$ GeV can be observed for the $k_T$-factorization approach (see left panel in Fig. 6).

In Fig. 10 we show two-dimensional distributions in transverse momenta of both initial gluons. Once again we observe that large transverse momenta (virtualities) of the initial gluons are involved into production of our final state.

The KMR method allows to construct not only uGDFs but also unintegrated quark/antiquark distributions. Therefore we can compare the contributions of the different subprocesses as it was done in the previous subsection for the collinear case. In Figs. 11, 12, 13 and 14 we again show distributions in $c/\bar{c}$ transverse momentum, rapidity, diparton invariant masses as well as in relative azimuthal angle between different outgoing partons. The results obtained here for the $k_T$-factorization approach are very
FIG. 7: Rapidity distribution of c-quark (left panel) and associated jet (right panel) in the $k_T$-factorization approach with different uGDFs.

FIG. 8: Distribution in invariant mass of the c-quark-jet (left panel) and $c\bar{c}$ system (right panel) in the $k_T$-factorization approach with different uGDFs.

similar to those obtained in the collinear approach.

C. Comparison of the two approaches

In Figs. 15, 16, 17 and 18 we again show distributions in $c/\bar{c}$ transverse momentum, rapidity, diparton invariant masses as well as in relative azimuthal angle between different outgoing partons simultaneously for the collinear and $k_T$-factorization approaches with the KMR uGDF and extra cut to effectively eliminate cases of more than one jet in the final state. In general, the results are rather similar. The $k_T$-factorization approach gives slightly larger cross section. However, the shapes of differential distributions are rather
FIG. 9: Distribution in azimuthal angle between $c$-quark and jet (left panel) and between $c$-quark and $\bar{c}$-antiquark (right panel) in the $k_T$-factorization approach with different uGDFs.

FIG. 10: Two-dimensional distributions in transverse momenta of initial gluons for three different uGDFs specified in the figure caption.

similar. Dividing differential distributions for the $k_T$-factorization by corresponding ones for the collinear approach one could get phase-space-point dependent $K$-factor. Rough inspection of the figures shows that the $K$-factor is only weakly dependent on kinematical variables. Approximately it is about $K \sim 1.5$, which is a typical value for NLO pQCD calculations for processes with initial gluons.

The one-dimensional distributions in both approaches are rather similar. In Fig. 19 and 20 we compare also several two-dimensional distributions. In all cases the result of the $k_T$-factorization approach with the KMR uGDF and the correction for exclusion of extra (more than one) jets gives rather similar results as those in the collinear approach. Other uGDFs may give slightly different results. In the LO collinear approach jet-$p_T$ is balanced by transverse momenta of $c$ and $\bar{c}$. Therefore the sharp cut $p_T^{\text{jet}} > 20$ GeV generates an excluded region of the triangle shape at small $p_T^c$ and $p_T^{\bar{c}}$. In contrast, there
FIG. 11: Transverse momentum distribution of c-quark (left panel) and associated jet (right panel) in the $k_T$-factorization approach with the KMR uGDF and the extra cut on initial gluon transverse momenta.

FIG. 12: Rapidity distribution of c-quark (left panel) and associated jet (right panel) in the $k_T$-factorization approach with the KMR UGDF and the extra cut on initial gluon transverse momenta.

is no such excluded region in the $k_T$-factorization. Clearly detailed studies of such two-dimensional distributions would be an important test for uGDFs.

D. $p_T$ distributions of $c\bar{c}$ associated with jets and for the inclusive case

In this subsection we wish to discuss how the transverse momentum distributions of $c$ or $\bar{c}$ quarks/antiquarks associated with jet compare to the so-called inclusive charm distributions (see e.g. Ref. [9]). In Fig. 22 we make such a comparison for the collinear-
FIG. 13: Distribution in invariant mass of the $c$-quark-jet (left panel) and $c\bar{c}$ system (right panel) in the $k_T$-factorization approach with the KMR uGDF and the extra cut on initial gluon transverse momenta.

FIG. 14: Distribution in azimuthal angle between $c$-quark and jet (left panel) and between $c$-quark and $\bar{c}$-antiquark (right panel) in the $k_T$-factorization approach with the KMR uGDF and the extra cut on initial gluon transverse momenta.

For the collinear case we show both LO (dotted line) and NLO \cite{35} (dashed line) results. The NLO distribution is much larger than the LO distribution especially for large transverse momenta of $c$ or $\bar{c}$. The corresponding $K$-factor is therefore strongly dependent on $p_T$. For comparison we show the result for the associated $c\bar{c} + \text{jet}$ production (solid line). We can see that the corresponding $c$ or $\bar{c}$ distribution almost coincides with that for the NLO inclusive case for transverse momenta $p_T > p_{T,\text{min}}^{\text{jet}} = 20$ GeV. This shows that the NLO distribution at large $p_T$ is practically always associated with a (mini)jet.
FIG. 15: Comparison of transverse momentum distributions of c-quark (left panel) and associated jet (right panel) for the collinear approach (dotted) and the $k_T$-factorization approach with the KMR uGDF and the extra cut on initial gluon transverse momenta (solid).

FIG. 16: Comparison of rapidity distribution of c-quark (left panel) and associated jet (right panel) in the collinear approach (dotted) and the $k_T$-factorization approach with the KMR uGDF and the extra cut on initial gluon transverse momenta (solid).

The same is true for the $k_T$-factorization with the KMR uPDFs. The $2 \to 2$ and $2 \to 3$ results coincide at large $p_T^c$. This clearly shows that the KMR uPDFs already in the $2 \to 2$ case effectively include higher-order corrections related to associated jet production ($2 \to 3$ and even $2 \to 4$ cases). We get slightly lower cross section for $2 \to 3$ case when imposing the extra cut $k_{1,T}, k_{2,T} < p_{T,min}^{jet}$ that in our opinion restricts the calculations to the case of production of the single-jet coming from hard-interaction and does not allow for additional jets hidden in the KMR uPDFs (and not controlled in the calculation).

The $k_T$-factorization $2 \to 3$ result with the KMR uPDFs and the extra cut on $k_{1,T}$ and
FIG. 17: Comparison of distribution in invariant mass of the c-quark-jet (left panel) and c\bar{c} system (right panel) in the collinear approach (dotted) and in the $k_T$-factorization approach with the KMR uGDF and the extra cut on initial gluon transverse momenta (solid).

$\kappa_T$ almost coincides with the collinear NLO result at large $p_T$’s as can be inferred by comparing the left and right panels. However, there is a difference between the two approaches for $c\bar{c} + jet$ predictions at small transverse momenta $p_T < p_{T,min}^{cut} = 20$ GeV. Certainly a measurement of $D^0$ ($\bar{D}^0$) mesons in association with jets would be a valuable option at the LHC.
FIG. 19: Two-dimensional distribution in transverse momenta of the outgoing $c$-quark and $\bar{c}$-antiquark for the collinear and $k_T$-factorization approaches. The details are specified in the figure captions.

E. Predictions for $D^0 + \text{jet}$ production at the LHC

Finally we present first measurable predictions for the LHC for the $D^0 + \text{jet}$ (including $\bar{D}^0 + \text{jet}$) production. As an example we consider the case of the ATLAS apparatus. We assume that $D^0$ or $\bar{D}^0$ are registered by the main ATLAS tracker and the jets are produced in the interval $|y_{\text{jet}}| < 4.9$. Similar experimental conditions for the considered reaction can be also achieved by the CMS experiment. The distribution in azimuthal angle between $D^0$ and the jet is shown in Fig. 23. This is a new possibility to test uGDFs. We show result for different uGDFs as well as for the collinear result.

Another interesting option is to look at $D^0-\bar{D}^0$ correlations. This is a bit more complicated and will be discussed elsewhere. ALICE and LHCb has smaller capability (smaller range of pseudorapidities) to measure jets and therefore we leave corresponding calculations for a dedicated studies.

In Table I we present the integrated cross section for the ATLAS acceptance (for $D^0$ mesons and jets) for different uGDFs and for the collinear case. Clearly measurable cross
FIG. 20: Two-dimensional distribution in transverse momenta of the outgoing $c$-quark and jet for the collinear and $k_T$-factorization approaches. The details are specified in the figure captions.

TABLE I: The calculated cross sections in microbarns for inclusive $D^0 + \text{jet (plus } \bar{D}^0 + \text{jet)}$ production in $pp$-scattering at $\sqrt{s} = 13$ TeV for different cuts on transverse momentum of the associated jet. Here, the $D^0$ meson is required to have $|y^{D^0}| < 2.5$ and $p_{T}^{D^0} > 3.5$ GeV and the rapidity of the associated jet is $|y^{\text{jet}}| < 4.9$, that corresponds to the ATLAS detector acceptance.

| $p_{T,\text{min}}^{\text{jet}}$ cuts | collinear $p_{T}^{\text{jet}}$, min | $k_T$-factorization approach $k_T < p_{T,\text{min}}^{\text{jet}}$ Jung setA0 |
|----------------------------------|-----------------------------|---------------------------------|
| $p_{T}^{\text{jet}} > 20$ GeV | 22.36 | 49.20 |
| $p_{T}^{\text{jet}} > 35$ GeV | 3.70 | 9.60 |
| $p_{T}^{\text{jet}} > 50$ GeV | 1.14 | 3.32 |

sections of the order of a few to tens of $\mu$b (depending on the jet-$p_T$ cuts) are obtained.
FIG. 21: Two-dimensional distribution in relative azimuthal angle between $c$-quark and $\bar{c}$-antiquark and jet transverse momentum for the collinear and $k_T$-factorization approaches. The details are specified in the figure captions.

FIG. 22: Transverse momentum distribution of $c$-quarks in the collinear (left panel) and $k_T$-factorization (right panel) approaches. We compare results of $2 \to 2$ and $2 \to 3$ partonic calculations. More detailed discussion is given in the main text.
FIG. 23: Azimuthal angle correlation between $D^0$ meson and jet for the collinear and $k_T$-factorization approaches. The cuts are specified in the figure caption.
IV. CONCLUSIONS

In the present paper we have presented first theoretical study related to associated production of charm and single-jet. The related calculations were performed both in the collinear and $k_T$-factorization approaches. The related matrix elements were obtained with the help of the AVHLIB events generator.

We have performed first phenomenological study for $\sqrt{s} = 13$ TeV. In most cases we have limited to parton-level and full phase space. Different uGDFs have been used in the $k_T$-factorization approach calculations. We have compared results of the collinear approach with those within the $k_T$-factorization. The results, one-dimesional and two-dimensional distributions, with the KMR uGDF and a practical correction to exclude production of more than one jet are very similar as those obtained within the collinear approach.

We have discussed in addition how the transverse momentum distributions of $c/\bar{c}$ associated with jets relate to the inclusive charm case both for collinear LO/NLO calculation and for the $k_T$-factorization with the KMR uPDFs. We have shown that the production of $c$ (or $\bar{c}$) at large transverse momenta is unavoidably related to an emission of extra parton (jet). We have shown that the KMR uPDFs in the case of inclusive charm distributions effectively include higher-order corrections related to associated jet production. The distributions of $c/\bar{c}$ associated with jets obtained within the $k_T$-factorization with the KMR uPDFs, when done without extra conditions, contain effectively the situations with more than one jet (two or even three). This can be approximately eliminated by imposing extra cut(s) on transverse momenta of the initial partons. Then such a result almost coincides with the NLO collinear result for inclusive charm production at large $c/\bar{c}$ transverse momenta.

We have also performed first feasibility studies of $D^0 +$ jet production for ATLAS (and/or CMS) cuts. We have obtained rather large cross sections. We hope our first phenomenological studies will be an inspiration for experimental groups to perform corresponding or similar analysis.

Associated production of charm (charmed mesons) and single-jet is only a first step. In principle, production of charm and two jets is equally interesting and can be done within considered here framework. The program of activity sketched here may be very
important in detailed testing of pQCD dynamics for more complicated processes. The large luminosity at the LHC now available opens such a possibility. So far most of the efforts at the LHC concentrated on inclusive charm production. However, a study of correlation observables (some examples have been discussed here) may be very interesting and important in this context.

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