Learning Dynamical System for Grasping Motion
Xiao Gao, Miao Li and Xiaohui Xiao

Abstract—Dynamical System has been widely used for encoding trajectories from human demonstration, which has the inherent adaptability to dynamically changing environments and robustness to perturbations. In this paper we propose a framework to learn a dynamical system that couples position and orientation based on a diffeomorphism. Different from other methods, it can realise the synchronization between position and orientation during the whole trajectory. Online grasping experiments are carried out to prove its effectiveness and online adaptability.

Index Terms—Imitation learning, Dynamical System, Diffeomorphism, Grasping motion

I. INTRODUCTION

Generating robot motion is one of the most important parts in robotics. Imitation learning [1] can be adopted to encode human demonstrations and transfer skills to robots, which offer a convenient way for robot motion planning. In this paper, we focus on the time-invariant dynamical system, which can be robust to temporal and spatial perturbation and can adapt to dynamically changing environments.

Many dynamical systems for reaching movements have been proposed based on imitation learning, which encode human demonstration trajectories as a nonlinear system \( \dot{x} = f(x) \), where \( x \) is the robot state and \( \dot{x} \) is its velocity. In order to make sure any initial point in the system can converge to the only attractor (equilibrium point), Khansari et al. [2] proposed the Stable Estimator of Dynamical System (SEDS) algorithm to learn a global asymptotically stable DS. They used a quadratic Lyapunov function as constraints when learning SEDS by Gaussian mixture models. Due to the Lyapunov function, learning a global asymptotically stable DS is designed to encode pose trajectories to receive simple trajectories in latent spaces. Then a global asymptotically stable DS is designed to encode pose trajectories in one latent space, which is mapped back to the original demonstration for generating the required DS. Our main contribution is as follows:

1) a framework for the coupled DS with synchronization between position and orientation;
2) guarantee of the global asymptotically stability for the coupled DS;
3) inherent robustness and adaptability for both position and orientation.

II. METHOD

Assuming that there is a human demonstration trajectory \( B = \{x_i^*, q_i^*\}_{i=1}^N \), which is plotted in Fig. 1(c) as red the curve for position and red arrows (x-axis direction) for orientation. Here we use quaternion representation. The end pose of trajectory \( B \) is as \( 0, q_l \), also as the attractor. \( q_l = [1, 0, 0, 0]^\top \) is the identity quaternion. A simple trajectory \( A \) in space A (Fig. 1(b)) is generated by a linear interpolation of the start point of \( B \) as:

\[
A = \{x_i, q_i\}_{i=1}^N = \{tx_i^*, q_i^*\}, t = \frac{N-1}{N-1}(i = 1, 2, ..., N),
\]

(1)

Blue arrows in Fig. 1(b) show the orientation of \( A \). In Fig. 1(c), trajectory \( D = \{x_i, q_i\}_{i=1}^N \), where the position is the same as \( A \) and orientation is constant at identity quaternion.

Based on our previous diffeomorphic mapping framework for motion mapping [3], and set an arbitrary pose \( N = \{x, q\} \) in space A, we build a diffeomorphism \( \Phi_1 \) between the space A and B to map the trajectory \( A \) to \( B \), and another diffeomorphism \( \Phi_2 \) between the space D and A to map the trajectory \( D \) to \( A \). The two diffeomorphisms are as follows:

\[
\Phi_1: \begin{cases}
x' = h_1(x), 
q' = g_1(x) \ast q,
\end{cases} \quad \Phi_2: \begin{cases}
x = h_2(x_D) = x_D, 
q = g_2(x_D) \ast q_l = g_2(x),
\end{cases}
\]

(2)
where \((x_D, q_1)\) is the corresponding pose of \(\Phi_2^{-1}(x, q)\), and \(g_1(x), g_2(x) \in S^3\) are unit quaternions, and also functions of position \(x\).

To visualize the two diffeomorphisms, we set equal-spaced grid points in space \(D\) (gray lines in Fig. 1(a)), with identify quaternions. Under the two diffeomorphisms \(\Phi_1\) and \(\Phi_2\), grid points are distorted in the space \(A\) and \(B\), where when grid points are close to trajectories \(A\) or \(B\), the orientations of the points are also close to the corresponding orientation direction (Fig. 1(b)–(c)).

The position DS in space \(A\) is as:
\[
\dot{x} = f_1(x) = \gamma_1(x)P \dot{x}, \quad \gamma(x) > 0.
\] (3)

where the symmetric negative definite matrix \(P\) is designed based on \(A\). \(\gamma_1(x)\) is to adjust the velocity. For orientation, we set the angular velocity as:
\[
\dot{\omega} = \gamma_2(q) \beta \log(q \ast \tilde{g}_2(x)) + \omega_r, \quad \gamma_2(q) > 0, \quad \beta < 0 (4a)
\]
\[
\omega_r = -2q \ast \frac{\partial g_2(x)}{\partial x} \ast \dot{x} \ast g_2(x) \ast \dot{q} \quad (4b)
\]

which includes a feedback term and a feedforward term. The goal is to track the desired orientation \(g_2(x)\). So, the angular velocity \(\omega\) is a function of current position \(x\) and orientation \(q\). The stability can be proved by the quadratic Lyapunov functions for both position and orientation. Finally, we can use the diffeomorphism \(\Phi_1\) to compute the coupled DS in space \(B\).

III. EXPERIMENTS

Experimental setup is shown in Fig. 2. Six markers of Vicon motion tracking system are attached on a bottle. The goal is to grasp it with the UR5e robot and the Robotiq 85 gripper. The coupled DS is generated by the method in Section II.

Fig. 2 shows the snapshots of the experiments. The robot/bottle position and orientation are plotted in Fig. 3 and 4. We can see that the robot pose was tracking the pose of the bottle, while the pose of bottle was changing due to the movements of the user’s right hand. When \(t = 28-31\) s (Fig. 2(c)), a disturbance was applied on the robot by the user’s left hand. The robot can also moved to the desired grasping pose.

IV. CONCLUSION

In this paper, we propose a framework of coupled DS for the generation of linear velocity and angular velocity synchronously. The dynamical grasping experiments show that it can adapt to dynamical environments and can be robust to perturbations.

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