The interaction of turbulence with parallel and perpendicular shocks

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Abstract. Interplanetary shocks exist in most astrophysical flows, and modify the properties of the background flow. We apply the Zank et al 2012 six coupled turbulence transport model equations to study the interaction of turbulence with parallel and perpendicular shock waves in the solar wind. We model the 1D structure of a stationary perpendicular or parallel shock wave using a hyperbolic tangent function and the Rankine-Hugoniot conditions. A reduced turbulence transport model (the 4-equation model) is applied to parallel and perpendicular shock waves, and solved using a 4th-order Runge Kutta method. We compare the model results with ACE spacecraft observations. We identify one quasi-parallel and one quasi-perpendicular event in the ACE spacecraft data sets, and compute various turbulent observed values such as the fluctuating magnetic and kinetic energy, the energy in forward and backward propagating modes, the total turbulent energy in the upstream and downstream of the shock. We also calculate the error associated with each turbulent observed value, and fit the observed values by a least square method and use a Fourier series fitting function. We find that the theoretical results are in reasonable agreement with observations. The energy in turbulent fluctuations is enhanced and the correlation length is approximately constant at the shock. Similarly, the normalized cross helicity increases across a perpendicular shock, and decreases across a parallel shock.

1. Introduction
Collisionless shocks exist in any astrophysical flows, and are important structures in the heliosphere. They are formed throughout the heliosphere due to the interaction of solar wind disturbances with the solar wind [1, 2]. Collisionless shocks change the characteristics of the solar wind when the solar wind passes through the shocks, through both heating and compression. Collisionless shocks play an important role in particle acceleration to high energies (several tens of $\sim$MeVs) [3-10] including cosmic rays [11],
which causes a significant hazard to satellite technology and human activity in space [7, 10, 12]. Shocks can also be responsible for geomagnetic activity [13, 14].

Interplanetary shocks (IP) can be formed either by the interaction of fast solar wind streams with slow preceding plasma or by fast interplanetary coronal mass ejections (ICMEs). However, to form a shock in the heliosphere, the difference in speed between two solar wind streams or the ICME and the preceding plasma should exceed the magnetosonic speed. The source of shocks depends on solar cycle, and during solar minimum most shocks are driven by stream interactions [15, 16], whereas during solar maximum most shocks are driven by ICMEs [15]. The acceleration characteristics for charged particles accelerated by ICME driven shocks and by stream interaction driven shocks is different for several reasons. The particle acceleration rate is governed by the magnetic field strength, shock strength, turbulence levels (also related to shock obliquity; Zank et al 2006 [17]), as discussed by Zank et al 2001 [5]. The acceleration is therefore determined in part by where the shock forms since the acceleration rate is largest near the Sun [5]. The time available for a particle to be accelerated also depends on where the shock forms, and so the maximum achievable particle energy to which a particle can be accelerated is determined by a balance of the particle acceleration rate and the dynamical time scale of the shocks ([5], [18], [19]). ICME driven shocks form near the Sun so that particles are accelerated for a longer time. Stream interaction driven shocks form at larger distances and so particles experience acceleration for a shorter time [20], and the particle acceleration rate is much slower [5]. Typically, coronal mass ejection driven shocks are faster and stronger, and show a larger distribution of shock parameters than stream interaction driven shocks [21].

The structure of a shock depends on its strength, the magnetic field geometry, and the plasma beta $\beta$. The strength is characterized by the magnetosonic Mach number and the compression ratio, and the geometry by the obliquity angle $\theta_{bn}$, where $\theta_{bn}$ is the angle between the magnetic field and the shock normal. Shocks are quasi-parallel if $\theta_{bn} \leq 45^\circ$, and quasi-perpendicular if $\theta_{bn} > 45^\circ$. With increasing Mach number, the shock profile changes and develops a foot and overshoot in association with ion reflection and gyration. [22, 23] found that $\theta_{bn}$ plays an important role in the shock geoeffectiveness in that most perpendicular shocks are more geoeffective than quasi-parallel shocks (see also [24]). The shock normal is another important property of IP shocks. The IP shocks propagate along the shock normal direction. Usually, the normal associated with ICMEs-driven IP shock is close to the Sun-Earth line when the shock passes 1 AU [2]. The shock normal for a stream interaction driven shock typically is at a large angle compared to the Sun-Earth line [25].

Shocks are categorized as fast forward (FF) and fast reverse (FR) shocks with respect to the solar wind frame. Forward shocks propagate away from the Sun, and reverse shocks propagate towards the Sun. Generally, shocks associated with ICMEs are forward shocks. Most observed IP shocks at 1 AU are also found to be fast forward shocks. An increase in the magnitude of the magnetic field and other parameters is observed when a fast forward shock passes over the spacecraft in the spacecraft frame solar wind. On the other hand, a decrease in the magnitude of the magnetic field, the solar wind density and the temperature, and an increase in the solar wind speed are observed when a fast reverse shock passes over the spacecraft.
Shocks are also responsible for the generation of turbulence in the heliosphere. In this manuscript, we study the properties of several turbulent parameters in the upstream and downstream regimes of a shock. We present a turbulence transport model that describes the fluctuations of the solar wind parameters when interacting with quasi-parallel and quasi-perpendicular shocks. We introduce a steady state shock wave in the form of hyperbolic tangent function of \( x \) and solve the 1D steady state coupled turbulence transport equations. We compare the numerical solutions of the turbulence transport equations applied to the quasi-parallel shock of day 272 of year 2001 and the quasi-perpendicular shock of day 250 of year 2000, respectively. In so doing, we calculate turbulent parameters such as the energy in forward and backward propagating modes, the residual energy, their corresponding correlation lengths, the cross helicity, and the total turbulent energy from the upstream region to the downstream region of the shock. We also calculate the error associated with each turbulent quantity. We discuss the shock model equations in Section 2. Section 3 presents a comparison between the theoretical results and the observed values. Section 4 contains some conclusions. Finally, Appendix A presents an error analysis associated with several turbulent quantities.

2. Turbulence and Shock model equations
To study the interaction of turbulence with parallel and perpendicular shocks, we apply the Zank et al 2012 [26] (see also [27, 28]) turbulence transport model equations. We reduce the 1D six coupled equations of Zank et al 2012 model equations to four coupled equations. The reduction follows if we assume \( \lambda^+ = \lambda^- = \lambda \) and \( \lambda_D = 2\lambda \), where \( \lambda^+ \) and \( \lambda^- \) are the correlation length corresponding to backward and forward propagating modes, and \( \lambda_D \) is the correlation length of the residual energy. The Zank et al model equations can be reduced to four coupled equations for both the zero and nonzero cross helicity cases. Here we use a four coupled equation model that corresponds to the nonzero cross helicity case. The equations are expressed in a Cartesian coordinate system \( x \) for both parallel and perpendicular shocks. We adopt a coordinate system in which \( x \) is parallel to the shock normal.

2.1. Quasi-Parallel Shock Model Equations
For a parallel shock, the magnetic field \( B \) is parallel to the shock normal. The 1D steady state four coupled turbulence transport equations can be written as

\[
(U - V_A) \frac{\partial f}{\partial x} + \left[ \frac{f}{2} + \left( 2a - \frac{1}{2} \right) E_D \right] \frac{\partial U}{\partial x} + \frac{\partial V_A}{\partial x} (f - E_D) = -2 \frac{fg^{1/2}}{\lambda \cos \theta_{bn}}; \tag{1}
\]

\[
(U + V_A) \frac{\partial g}{\partial x} + \left[ \frac{g}{2} + \left( 2a - \frac{1}{2} \right) E_D \right] \frac{\partial U}{\partial x} - \frac{\partial V_A}{\partial x} (g - E_D) = -2 \frac{gf^{1/2}}{\lambda \cos \theta_{bn}}; \tag{2}
\]

\[
U \frac{\partial E_D}{\partial x} + \frac{1}{2} \frac{\partial U}{\partial x} \left[ (2a - \frac{1}{2})(f + g) + E_D \right] + \frac{V_A}{2\sqrt{fg}} \left[ f \frac{\partial g}{\partial x} - g \frac{\partial f}{\partial x} \right] + \frac{\partial V_A}{\partial x} \frac{f - g}{2} = -E_D \left[ \frac{f^{1/2} + g^{1/2}}{\lambda \cos \theta_{bn}} \right]; \tag{3}
\]
\[ U \frac{\partial \lambda}{\partial x} = \frac{f^{1/2} + g^{1/2}}{\cos \theta_{bn}}, \quad (4) \]

where \( f \equiv \langle z^+ \rangle \) and \( g \equiv \langle z^- \rangle \) are the energy in backward and forward propagating modes respectively, \( E_D \) is the residual energy, and \( \lambda \) is the correlation length. Here the parameters \( z^\pm = u \pm b/\sqrt{\mu_0 \rho} \) are Elsässer variables [29] for the turbulent fluctuations \( u \) and \( b \). The parameter \( U \) is the solar wind velocity, and \( V_A \) is the Alfvén velocity. The parameter \( a=1/2 \) or \( 1/3 \) for 2D or 3D mixing tensor ([26], [30]). Note that the magnetic field is outwardly directed. The parameter \( \theta_{bn} \) is the angle between the normal to an interplanetary shock front and the upstream magnetic field. The value of \( \theta_{bn} \) depends on the configuration of the interplanetary magnetic field through which the shock passes.

In this manuscript, we solve turbulence transport equations (1)–(4) for \( \theta_{bn} = 0 \).

2.2. Perpendicular Shock Model Equations

At a perpendicular shock, the magnetic field \( B \) is perpendicular to the shock normal. The 1D steady state four coupled equations in this case become

\[ U \frac{\partial f}{\partial x} + \left[ \frac{f}{2} + \left( 2a - \frac{1}{2} \right) E_D \right] \frac{\partial U}{\partial x} = -2 \frac{fg^{1/2}}{\lambda}; \quad (5) \]

\[ U \frac{\partial g}{\partial x} + \left[ \frac{g}{2} + \left( 2a - \frac{1}{2} \right) E_D \right] \frac{\partial U}{\partial x} = -2 \frac{gf^{1/2}}{\lambda}; \quad (6) \]

\[ U \frac{\partial E_D}{\partial x} + \frac{1}{2} \left[ \left( 2a - \frac{1}{2} \right) (f + g) + E_D \right] \frac{\partial U}{\partial x} = -E_D \left[ \frac{f^{1/2} + g^{1/2}}{\lambda} \right]; \quad (7) \]

\[ U \frac{\partial \lambda}{\partial x} = f^{1/2} + g^{1/2}. \quad (8) \]

The coupled turbulence transport equations (5)–(8) are simpler than (1)–(4) in that the Alfvén velocity is absent. The coupled equations (1)–(4) and (5)–(8) are solved using a 4th-order Runge Kutta method for different initial conditions, and the numerical solutions are then compared to ACE observations.

The other turbulent quantities can be calculated as [26]

\[ E_T = \frac{f + g}{2}; \quad E_C = \frac{g - f}{2}; \quad (9) \]

\[ \sigma_c = \frac{E_C}{E_T}; \quad \sigma_D = \frac{E_D}{E_T}; \quad (10) \]

\[ \langle u^2 \rangle = \frac{f + g + 2E_D}{2}; \quad \left\langle \frac{b^2}{\mu_0 \rho} \right\rangle = \frac{f + g - 2E_D}{2}; \quad (11) \]

where \( E_T \) is the total turbulent energy, \( E_C \) is the cross helicity, \( \sigma_c \) is the normalized cross helicity, \( \sigma_D \) is the normalized residual energy, \( \langle u^2 \rangle \) is the fluctuating kinetic energy, and \( \langle b^2 \rangle \) is the fluctuating magnetic energy.
2.3. Solar wind velocity and Alfvén velocity
To study the interaction of turbulence with shock waves, we introduce a stationary shock wave in the form of a hyperbolic tangent function of $x$ [31, 32]

$$U = \frac{1}{2}(U_1 + U_2) - \frac{1}{2}(U_1 - U_2) \tanh \left( \frac{x - x_0}{D_{sh}} \right),$$

(12)

where $U_1$ and $U_2$ are the upstream and downstream velocities of the shock. The parameter $x_0$ is the position of the shock, and $D_{sh}$ is the thickness of the shock. The thickness of a shock is determined by the appropriate dissipation mechanism. In the case of perpendicular shocks, the dissipation mechanism is due to reflected ions from the cross shock potential. The reflected ions form a foot in front of the ramp, which roughly determines the thickness of a shock. In the case of a parallel shock, the ions that reflect from the ramp propagate away from the shock as a beam in the upstream region. This introduces instabilities that generate waves that scatter the ions. Wave particle interactions leads to the formation of a diffuse ion population which eventually convepts through the shock. For parallel shocks, the dissipation mechanism occurs over an extended range. Hence, the thickness of parallel shock $D_{sh}$ is typically much larger than the thickness of perpendicular shock.

The Alfvén velocity can also be expressed in the form of a hyperbolic tangent function of $x$. The Alfvén velocity can be written as

$$V_A = \frac{B}{\sqrt{\mu_0 \rho}} = \frac{B}{\sqrt{\mu_0 m}} \sqrt{U},$$

(13)

using mass conservation $\rho U = m$. Here $B$ is the magnetic field, $\mu_0$ is the permeability of free space, $\rho$ is the density, and $m$ is a constant. Equation (13) shows that the Alfvén velocity is proportional to the product of the magnetic field $B$ and the square root of the solar wind velocity. Note that for a parallel shock, the magnetic field is constant across a shock, and for a perpendicular shock the magnetic field in the downstream region is larger than the magnetic field in the upstream region (by the product of the upstream $B$ with the compression ratio $r$). We use Equations (12) and (13) in Equations (1)–(4) and (5)–(8), and solve the coupled turbulence transport equations.

3. Data Analysis
In this section, we compare our theoretical results with ACE observations. We selected two shock events, the quasi-parallel shock of day 272 of year 2001 and the quasi-perpendicular shock of day 250 of year 2000. We calculate various turbulent values such as the fluctuating magnetic energy, the fluctuating kinetic energy, the energy in forward and backward propagating modes, the total turbulent energy, and the normalized residual energy in the upstream and downstream region of a shock [27, 28, 30, 33]. The turbulent quantities are calculated from 64 second resolution plasma data sets. In this analysis we use a criterion that the ratio of fluctuations of the field in an interval and the mean field should be smaller than 1 i.e., $\langle \delta x^2 \rangle / X^2 \ll 1$. Here $\langle \delta x^2 \rangle$ represents the fluctuations of the field in each interval and $X$ is the average field in the upstream and downstream region. The average value $X$ is constant in the upstream and downstream
region of the shock, and is determined from the Rankine Hugoniot values obtained from the ACE shock database (http://www.ssg.sr.unh.edu/mag/ace/ACElists/obs-list.html). The purpose of the $\langle \delta x^2 \rangle/X^2 << 1$ criterion is to exclude large quasi-periodic downstream (and upstream) fluctuations that may correspond to overshoots and undershoots associated with shock structure that are not accounted for in the R-H analysis. The turbulent quantities are calculated in each 5 minute interval in the case of the quasi-parallel shock of day 272 of year 2001, and in each 4 minute interval in the case of the quasi-perpendicular shock of day 250 of year 2000.

We also calculate the error corresponding to each of the observed values, and show them in the plots with an error bar. At first, we assume 1% uncertainty in the R, T, and N components of the solar wind velocity and the magnetic field. Then we calculate the error associated with each observed values using equations presented in Appendix A. We fit further the observed values by a least squares method [34]. Also, we fit the data using a Fourier series function,

$$y(t) = a_0 + \sum_{i=1}^{N} \left( a_i \cos(i \omega t_i) + b_i \sin(i \omega t_i) \right)$$  \hspace{1cm} (14)

where the Fourier coefficients $a_0$, $a_i$ and $b_i$ are given by,

$$a_0 = \frac{\sum_{i=1}^{N} y_i}{N} \hspace{1cm} a_k = \frac{2}{N} \sum_{i=1}^{N} y_i \cos(k \omega t_i) \hspace{1cm} b_k = \frac{2}{N} \sum_{i=1}^{N} y_i \sin(k \omega t_i)$$ \hspace{1cm} (15)

where $t$ is time, $\omega = 2\pi/T$ is an angular frequency and $T$ is a time period of the oscillation.

3.1. Comparison to the quasi-parallel shock of day 272 of 2001
Here we compare the numerical solution of equations (1)–(4) to the quasi-parallel shock of day 272 of 2001 from 9 hours upstream to 9 hours downstream of the shock. The coupled turbulence transport equations are solved for initial conditions: $f = 33.76 \text{ km}^2\text{s}^{-2}$, $g = 116.39 \text{ km}^2\text{s}^{-2}$, $E_D = -14.3 \text{ km}^2\text{s}^{-2}$, and $\lambda = 2.1 \times 10^6$ km. The initial conditions for $f$, $g$, and $\sigma_D$ are calculated by averaging the observed values in the upstream region of the shock, based on the mean fields derived from the Rankine-Hugoniot fitting procedure [35].

Figure 1 shows the comparison between the numerical solutions of Equations (1)–(4) and the observed values corresponding to the quasi-parallel shock of day 272 of 2001 as a function of $X/X_{sh}$ or $T/T_{sh}$. The parameters $X_{sh}$ and $T_{sh}$ are the spatial and temporal positions of the shock. In Figure 1, the solid black curves represent the theoretical results, the blue “+” symbols the observed values, and the red “*” symbols have error bars associated with the observed values. Similarly, the solid straight red lines indicate a least square fitting to the observed values in the upstream and downstream region. The green sinusoidal curves are a fitting to the observed values using Fourier series function. To fit the observed values by a Fourier series function, we set $T = 40$ minute and $i=1$ in the Fourier series function (14) i.e., we consider only the lowest frequency of any potentially periodic behavior. In region immediately downstream of the shock,
we exclude the very large amplitude fluctuations that are associated shock structure
overshoots and undershoots.

Figure 1a illustrates the square root of the fluctuating magnetic energy \( \langle b^2 \rangle^{1/2} \) as a function of \( X/X_{sh} \) or \( T/T_{sh} \). The theoretical \( \langle b^2 \rangle^{1/2} \) decreases slightly in the upstream region of the shock, increases at the shock, and then decreases gradually downstream of the shock. The least square and Fourier series fittings to the observed \( \langle b^2 \rangle^{1/2} \) values in the upstream and downstream region show that the theoretical and observed \( \langle b^2 \rangle^{1/2} \) are reasonably consistent with each other.

In Figure 1a, the theoretical \( \langle b^2 \rangle^{1/2} \) and the observed \( \langle b^2 \rangle^{1/2} \) both exhibit an enhancement at the shock i.e., turbulence is both amplified and generated at the shock in the following sense. From equations (1)–(4), flow and Alfvén gradients are essentially zero and so the turbulence intensity \( (f, g \equiv \langle z^\pm 2 \rangle) \) simply decays as spectral transfer leads to the dissipation of fluctuating energy. At the shock transition itself, the gradient terms are large and act as source terms in the abrupt deceleration region. This effectively leads to a compression of upstream turbulence as it transitions the shock larger. Notice too that the residual energy \( E_D \) changes in response to the flow, magnetic field, and density gradients leading to an amplification and redistribution of kinetic and magnetic fluctuating energy. This is discussed below.

Figures 1b–1f describe the fluctuating kinetic energy \( \langle u^2 \rangle \), the energy in backward propagating modes \( f \), the energy in forward propagating modes \( g \), the total turbulent energy \( E_T \), and the normalized residual energy \( \sigma_D \) as a function of \( X/X_{sh} \) or \( T/T_{sh} \), respectively. The theoretical results show that all the turbulent energies decrease slightly in the upstream region, are enhanced at the shock, and then gradually decrease downstream of the shock. The theoretical \( \sigma_D \) shows that the energy in the fluctuations is dominated by the magnetic energy in the upstream and downstream regions of the shock. However, the kinetic and magnetic energy become approximately equipartitioned at the shock. The least square and Fourier series function fitting indicate that the theoretical energies \( \langle u^2 \rangle \), \( f \), \( g \), \( E_T \), and \( \sigma_D \) and the observed energies agree reasonably well to each other.

Figure 1g shows the normalized cross helicity \( \sigma_c \) as a function of \( X/X_{sh} \) or \( T/T_{sh} \). The theoretical \( \sigma_c \) increases gradually in the upstream region, decreases at the shock, and again increases slowly in the downstream region. However, there appears to be a dip in the observed \( \sigma_c \) downstream of the shock followed by a recovery and slow increase. With the exception of the dip, the observed and theoretical values seem to be quite consistent. Figure 1h illustrates the normalized correlation length as a function of \( X/X_{sh} \). It shows that the correlation length in the upstream and downstream regions increases, with a slight flattening at the shock.

3.2. Comparison to the quasi-perpendicular shock of day 250 of 2000

We compare the numerical solution of the turbulence transport equations (5)–(8) with the quasi-perpendicular shock of day 250 of 2000 for a period from 3 hours upstream to 3 hours downstream of the shock. The coupled equations (5)–(8) are solved using upstream initial conditions: \( f = 76.77 \text{ km}^2\text{s}^{-2}, g = 22.01 \text{ km}^2\text{s}^{-2}, E_D = -16.82 \text{ km}^2\text{s}^{-2}, \) and \( \lambda = 1.2 \times 10^6 \text{ km} \). Figure 2 illustrates the comparison between the theoretical results and observed values. In Figure 2, the solid black curves represent the theoretical results,
the blue scattered "*" symbols with error bars identify the observed values and the associated error, the solid straight red lines and the green oscillatory curves correspond to a least square and Fourier series function fitting, respectively. To fit the observed values in the upstream and downstream region of the shock by the Fourier series function (14), we consider only the lowest frequency and set \( i = 1 \) and \( T = 14 \) minute.

Figure 2a shows the fluctuating magnetic energy \( \langle b^2 \rangle^{1/2} \) as a function of \( X/X_{sh} \) or \( T/T_{sh} \). Evidently, both the theoretical and observed values in the downstream region are larger than in the upstream region. The least square and the Fourier series fitting to the observed values in the upstream and downstream region show that the theoretical \( \langle b^2 \rangle^{1/2} \) is consistent with the observed \( \langle b^2 \rangle^{1/2} \).

Figure 2b shows the fluctuating kinetic energy \( \langle u^2 \rangle \) as a function of \( X/X_{sh} \) or \( T/T_{sh} \). The theoretical \( \langle u^2 \rangle \) exhibits similar trends to that observed in upstream and downstream of the shock. Figures 2c, 2d, and 2e illustrate the energy in backward propagating modes \( f \), the energy in forward propagating modes \( g \), and the total turbulent energy \( E_T \) as a function of \( X/X_{sh} \) or \( T/T_{sh} \). We find that the turbulent energies \( f \), \( g \), and \( E_T \) are approximately constant in the upstream region, are enhanced at the shock, and decrease gradually in the downstream of the shock, consistent with the least square and Fourier series function fitting to the observed quantities.

Figure 2f plots the normalized residual energy \( \sigma_D \) as a function of \( X/X_{sh} \) or \( T/T_{sh} \). The theoretical normalized residual energy is approximately constant in upstream and downstream of the shock, and is enhanced at the shock. Comparison of the theoretical \( \sigma_D \) and the observed \( \sigma_D \) shows reasonable consistency. The theoretical and observed \( \sigma_D \) are negative upstream and downstream of the shock, indicating that the magnetic energy dominates the fluctuations. Figure 2g is a plot of the normalized cross helicity \( \sigma_c \) and Figure 2h the normalized correlation length \( \lambda/L_C \) as a function of \( X/X_{sh} \) or \( T/T_{sh} \). The theoretical \( \sigma_c \) is reasonably consistent with the observed \( \sigma_c \). The normalized cross helicity \( \sigma_c \) is negative upstream and downstream of the shock, indicating that the intensity in backward modes is greater than in forward modes upstream and downstream of the shock. The normalized correlation length increases upstream and downstream of the shock, and flattens very slightly at the shock.

4. Conclusions
To study the interaction of turbulence with parallel and perpendicular shocks, we derived four coupled turbulence transport equations appropriate to parallel and perpendicular shocks, using the turbulence transport equations of Zank et al 2012. We solved the four coupled turbulence transport equations for parallel and perpendicular shocks, and compared the numerical solutions to the quasi-parallel shock of day 272 of year 2001, and the quasi-perpendicular shock of day 250 of year 2000. We assumed a 1% uncertainty in the R, T, and N components of the solar wind velocity and the magnetic field, and calculated uncertainties corresponding to various turbulent quantities using the results presented in Appendix A. We fitted the observed values in the upstream and downstream of the shock using a least square method and a low frequency Fourier series function fitting. The least square and Fourier series function fitting to the observed values, and the comparison of the theoretical results with the observed values indicates that the theoretical results show similar trends to the observed values. We can summarize the
findings of this work as,
1) all the turbulent energies are enhanced in transmission across parallel and perpendicular shocks, and decrease monotonically in the downstream region;
2) the correlation length increases upstream and downstream of a shock, with a slight flattening at the shock;
3) the cross helicity decreases at the shock in the parallel shock case, and increases at the shock in the perpendicular shock case, and
4) according to the theoretical model for the normalized residual energy, the energy in the fluctuations is dominated by the magnetic energy in the perpendicular shock case. For a parallel shock, the magnetic energy dominates the fluctuations in the upstream and downstream regions, whereas the two energies are approximately equipartitioned at the shock.

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Appendix A. Error associated with various turbulent quantities
Here we derive uncertainty relations for the energy in forward and backward propagating modes, the total turbulent energy, the residual energy, the normalized residual energy, the normalized cross helicity, the fluctuating kinetic and magnetic energy. To derive the uncertainty associated with the observations, we start from an error of propagation equation

$$\sigma_\xi^2 = \sum_{i=1}^{N} \left( \frac{\partial \xi}{\partial x_i} \right)^2 \sigma_i^2,$$  \hspace{1cm} (A.1)

where $\sigma_\xi$ is the error associated with the $\xi$ variable and $\sigma_i$ is the error associated with $x_i$. Following Equation A.1, if a function $\xi = x \pm y$ and $\sigma_x$ and $\sigma_y$ are the errors associated with $x$ and $y$, then the error associated with $\xi$ i.e., $\sigma_\xi$, is given by,

$$\sigma_\xi^2 = \sigma_x^2 + \sigma_y^2.$$  \hspace{1cm} (A.2)

Similarly, if $\xi = x/y$, then

$$\sigma_\xi^2 = \xi^2 \left( \frac{\sigma_x^2}{x^2} + \frac{\sigma_y^2}{y^2} \right).$$  \hspace{1cm} (A.3)

If $\xi = \sum_{i=1}^{N} x_i/N$ and the $\sigma_i$ are the errors associated with $x_i$, then the error associated with $\xi$ is given by

$$\sigma_\xi^2 = \frac{1}{N^2} \sum_{i=1}^{N} \sigma_i^2.$$  \hspace{1cm} (A.4)
If $V = 1/(N - 1) \sum_{i=1}^{N} (x_i - \bar{x})^2$ is a variance and $\sigma_i$ are the errors associated with $x_i$, the error associated with the variance $\sigma_V$, following Equation (A.1), can be written in the form

$$\sigma_V^2 = \left( \frac{2}{N - 1} \right) \sum_{i=1}^{N} (x_i - \bar{x})^2 \sigma_i^2.$$  \hspace{1cm} (A.5)

Using equations (A.1)–(A.5), we can derive the error for various turbulent quantities. For the residual energy $E_D = \langle u^2 \rangle - \langle b^2 / \mu_0 \rho \rangle$, the error $\sigma_{E_D}$ can be written in the form

$$\sigma_{E_D}^2 = \sigma_{\langle u^2 \rangle}^2 + \frac{\sigma_{\langle b^2 \rangle}^2}{\mu_0^2 \rho^2} + \frac{\langle b^2 \rangle^2 \sigma_{\rho}^2}{\mu_0^2 \rho^4},$$  \hspace{1cm} (A.6)

where $\sigma_{\langle u^2 \rangle}$, $\sigma_{\langle b^2 \rangle}$, and $\sigma_{\rho}$ are errors associated with the fluctuating kinetic energy $\langle u^2 \rangle$, the fluctuating magnetic energy $\langle b^2 \rangle$, and the mean density $\rho$, respectively. Similarly, for the fluctuating solar wind velocity $\mathbf{u} = \mathbf{U} - \langle \mathbf{U} \rangle$ and the fluctuating magnetic field $\mathbf{b} = \mathbf{B} - \langle \mathbf{B} \rangle$, where $\mathbf{U}$ and $\mathbf{B}$ are the fields, and $\langle \mathbf{U} \rangle$ and $\langle \mathbf{B} \rangle$ are the mean fields, the errors $\sigma_{\mathbf{u}}$ for $\mathbf{u}$ and $\sigma_{\mathbf{b}}$ for $\mathbf{b}$ are given by

$$\sigma_{\mathbf{u}}^2 = \sigma_{\mathbf{U}}^2 + \sigma_{\langle \mathbf{U} \rangle}^2; \quad \sigma_{\mathbf{b}}^2 = \sigma_{\mathbf{B}}^2 + \sigma_{\langle \mathbf{B} \rangle}^2,$$  \hspace{1cm} (A.7)

where $\sigma_{\mathbf{U}}$ and $\sigma_{\mathbf{B}}$ are errors associated with background fields, and $\sigma_{\langle \mathbf{U} \rangle}$ and $\sigma_{\langle \mathbf{B} \rangle}$ are the errors associated with background mean fields.

The errors $\sigma_{z^\pm}$ for the Elsässer variables $\mathbf{z}^\pm = \mathbf{u} \pm \mathbf{b} / \sqrt{\mu_0 \rho}$ can be written in the form

$$\sigma_{z^+}^2 = \sigma_{z^-}^2 = \sigma_{\mathbf{u}}^2 + \frac{\sigma_{\mathbf{b}}^2}{\mu_0 \rho} + \frac{b^2 \sigma_{\rho}^2}{4 \mu_0 \rho^3},$$  \hspace{1cm} (A.8)

where $\sigma_{\rho}$ is error associated with the density $\rho$. Using Equation (A.5), the errors associated with the variance of the Elsässer variables $\mathbf{z}^\pm$ can be written in the form

$$\sigma_{(z_i^+)}^2 = \left( \frac{2}{N - 1} \right) \sum_{i=1}^{N} (z_i^+ - \bar{z}_i^+)^2 \sigma_i^2; \quad \sigma_{(z_i^-)}^2 = \left( \frac{2}{N - 1} \right) \sum_{i=1}^{N} (z_i^- - \bar{z}_i^-)^2 \sigma_i^2,$$  \hspace{1cm} (A.9)

where $\sigma_i$ is the errors associated with $z_i^\pm$. The errors corresponding to the total energy $E_T$ and the cross helicity $E_C$ are the same, given by

$$\sigma_{E_T}^2 = \sigma_{E_C}^2 = \frac{1}{4} \left( \sigma_{(z_i^+)}^2 + \sigma_{(z_i^-)}^2 \right).$$  \hspace{1cm} (A.10)

Using Equation (A.3), the errors corresponding to the normalized cross helicity $\sigma_{c} = E_C / E_T$ and the normalized residual energy $\sigma_{D} = E_D / E_T$ can be written in the form

$$\sigma_{\sigma_c}^2 = \sigma_{c}^2 \left( \frac{\sigma_{E_C}^2}{E_C^2} + \frac{\sigma_{E_T}^2}{E_T^2} \right); \quad \sigma_{\sigma_D}^2 = \sigma_{D}^2 \left( \frac{\sigma_{E_D}^2}{E_D^2} + \frac{\sigma_{E_T}^2}{E_T^2} \right).$$  \hspace{1cm} (A.11)
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Figure 1. Comparison of solutions of turbulence transport equations (1)–(4) with the quasi-parallel shock of day 272 of 2001 as a function of $T/T_{sh}$ or $X/X_{sh}$. Figure 1a shows the square-root of the fluctuating magnetic energy $\langle b^2 \rangle^{1/2}$. Figure 1b illustrates the fluctuating kinetic energy $\langle u^2 \rangle$. Figures 1c and 1d the energy in backward $f$ and forward $g$ propagating modes. Figures 1e and 1f the total turbulent energy $E_T$ and the normalized residual energy $\sigma_D$. Figures 1g and 1h the normalized cross helicity $\sigma_c$ and the correlation length $\lambda$. The parameter $L_C = 0.03$ AU is the correlation length at 1 AU [36].
Figure 2. Comparison of solutions of the turbulence transport equations (5)–(8) with the quasi-perpendicular shock of day 250 of 2000 as a function of $T/T_{sh}$ or $X/X_{sh}$. Figure 2a plots the square-root of the fluctuating magnetic energy $\langle b^2 \rangle^{1/2}$; Figure 2b the fluctuating kinetic energy $\langle u^2 \rangle$; Figures 2c and 2d the energy in backward $f$ and forward $g$ propagating modes; Figures 2e and 2f the total turbulent energy $E_T$ and the normalized residual energy $\sigma_D$, and Figures 2g and 2h the normalized cross helicity $\sigma_c$ and the correlation length $\lambda$. The parameter $L_C = 0.03$ AU is the correlation length at 1 AU.