Entropic force versus temperature force

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Abstract

We introduce the cavity enclosing a source mass $M$ to define the temperature force. Starting with the Tolman temperature in the stationary spacetime, we find a non-relativistic temperature $T_{\text{non}} = T_\infty(1 - \Phi/c^2)$ with the Newtonian potential $\Phi$. This temperature could be also derived from the Tolman-Ehrenfest effect, satisfying a relation of $T = T_\infty e^{-\Phi/c^2}$ with the local temperature $T$. Finally, we derive the temperature force $\vec{F}_{\text{tem}} = mc^2(\nabla \ln T)$ which leads to the Newtonian force law without introducing the holographic screen defined by holographic principle and equipartition law for entropic force.

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1 Introduction

Since the discovery of the laws of black hole thermodynamics [1], Bekenstein [2] and Hawking [3] have suggested a deep connection between gravity and thermodynamics, realizing black hole entropy and Hawking radiation. Later on, Jacobson [4] has demonstrated that Einstein equations of general relativity (describing relativistic gravitation) could be derived by combining general thermodynamic pictures with the equivalence principle. Padmanabhan [5] has observed that the equipartition law for horizon degrees of freedom combined with the Smarr formula leads to the Newton’s law of gravity. This may imply that the entropy is to link general relativity with the statistical description of unknown spacetime microscopic structure when the horizon is present.

Recently, Verlinde has proposed the Newtonian force law as an entropic force (non-relativistic version) by using the holographic principle and the equipartition rule in the absence of horizons [6]. If it is proven correct, gravity is not a fundamental interaction, but an emergent phenomenon which arises from the statistical behavior of microscopic degrees of freedom encoded on a holographic screen. In other words, the force of gravity is not something ingrained in matter itself, but it is an extra physical effect, emerging from the interplay of mass, time and space (information) through the entropy.

However, an urgent question is how one can construct a spherically holographic screen of radius $r$ which encloses a source mass $M$ located at the origin using the holographic principle. This is a very important issue [7] because the holographic screen (an exotic description of spacetime) originates from relativistic approaches to black hole [8, 9] and cosmology [10, 11]. Verlinde has introduced this screen by analogy with an absorbing process of a particle around the event horizon of black hole. Considering a smaller test mass $m$ located at $\Delta x$ away from the screen and getting the change of entropy on the screen, its behavior should resemble that of a particle approaching a stretched horizon of a black hole, as was described by Bekenstein [2]. It is clear that Verlinde has introduced the holographic screen as a basic input to derive the entropic force.

The next important question is why the equipartition rule could be applied to this non-relativistic screen to define the temperature without any justifications. For black holes, the equipartition rule becomes the Smarr formula of $E = NT/2 = 2ST$ when using $N = 4S = \frac{4\pi^3}{6h}$. Also it can be derived from the first law of thermodynamics $dE = TdS$ for the Schwarzschild black hole where the Komar charge is just the ADM mass $M$. Even though the equipartition rule is available for the classical thermodynamics, the holographic principle of $N = Ac^3/Gh$ is not guaranteed to apply to any non-relativistic situations. In this sense, this issue is closely related to the first one.

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If the above two questions are answered properly, one would make a further step to understand the origin of Newtonian force. However, there exists still a gap between non-relativistic approach (absence of horizons) and relativistic approach (presence of horizons). It is not legitimate to use the holographic screen defined by using the holographic principle and the equipartition rule in deriving a non-relativistic force law. It is shown that Verlinde has used some ideas for obtaining Einstein equations due to Jacobson’s derivation of Einstein equations. Also, it seems that he was using circular reasoning in his equations, by starting out with Einstein gravity.

In this work, we will show that the Newtonian force law could be derived from the temperature force on the cavity without referring to holographic principle and equipartition rule. This is a natural approach because it starts with the Tolman temperature on the stationary (black hole) spacetime and it is closely related to the Tolman-Ehrenfest effect which states that even for the Newtonian gravity, the temperature is not constant in space at equilibrium [12, 13].

2 Entropic force

In this section, we briefly review how the Newtonian force law emerges from entropic considerations [6]. Explicitly, when a test particle with mass $m$ is located near a holographic screen with distance $\Delta x$, the change of entropy on a holographic screen may take the form

$$\Delta S = 2\pi k_B \Delta x \frac{mc}{\hbar}. \quad (1)$$

Considering that the entropy of a system depends on the distance $\Delta x$, an entropic force $F_{\text{ent}}$ could be arisen from the thermodynamical conjugate of the distance as

$$F_{\text{ent}} \Delta x = T_{hs} \Delta S \quad (2)$$

which may be regarded as an indication that the first law of thermodynamics is realized on the holographic screen. Plugging (1) into (2) leads to an important connection between the entropic force and temperature on the screen

$$F_{\text{ent}} = \frac{2\pi k_B m c}{\hbar} T_{hs}. \quad (3)$$

One uses mainly this connection to derive the entropic force, only after setting the temperature $T_{hs}$ on the holographic screen.

Introducing the Unruh temperature [14] as the holographic screen temperature

$$T_{hs} \rightarrow T_U = \frac{\hbar}{2\pi k_B c} a, \quad (4)$$
one may find the second law

$$F_{\text{ent}} = ma.$$  

(5)

However, the Unruh temperature has originated at the relativistic quantum field theory. It seems that the Unruh effect remains a theoretical phenomena with some restrictions \[15\] and this effect is not available for a non-relativistic case. Therefore, it is problematic to use the Unruh temperature as the screen temperature.

In order to derive the holographic screen temperature $T_{hs}$, let us suggest that the energy $E$ is distributed on a spherical screen with radius $r$ and the source mass $M$ is located at the origin of coordinates. Then, we assume that the equipartition rule \[16, 5\], the equality of energy and mass, and the holographic principle, respectively, hold as

$$E = \frac{1}{2} N k_B T_{hs}, \quad E = M c^2, \quad N = \frac{Ac^3}{G\hbar} = 4S$$  

(6)

with the area of a holographic screen $A = 4\pi r^2$. Importantly, these should be combined to put the temperature on the screen

$$T_{hs} = \frac{\hbar}{2\pi k_B c} \frac{GM}{r^2}.$$  

(7)

Substituting (7) into (3), one obtains the Newtonian force law as the entropic force

$$F_{\text{ent}} = \frac{GmM}{r^2} = mg,$$  

(8)

where the Galilean acceleration of the Newtonian gravity is given by

$$g = \frac{GM}{r^2} = \frac{2\pi k_B c}{\hbar} T_{hs}.$$  

(9)

We summarize “how the entropic force is realized”: as a test particle with mass $m$ approaches the holographic screen, its own entropy bits begin to transfer to the holographic screen, and it is this increase in the screen entropy that generates an attractive force on the test mass. We must emphasize that as is shown in (3), the role of holographic screen temperature $T_{hs}$ is essential for deriving the entropic force. Unless the Newtonian force is emergent as the entropic force, the choice of holographic screen temperature seems to be a major flaw of Verlinde’s idea. Instead, we use the Tolman-Ehrenfest effect which states that for the Newtonian gravity, the temperature is not constant in space at equilibrium. Thus, we could define the local temperature on any place where the test particle is located.
3 Temperature force on the cavity

In the presence of horizons, it is natural to define the horizon temperature as the surface gravity of event horizon for black hole and apparent horizon for cosmology. In the absence of horizons, Verlinde has attempted to introduce the holographic screen temperature instead of horizon temperature [6]. However, as was emphasized previously, the usage of the holographic screen is not guaranteed to describe a non-relativistic case of a source mass $M$. Hence, if the Newtonian gravity is not a fundamental force, it would be better to describe the Newtonian force law without imposing the holographic principle and equipartition rule.

In this section, we wish to define a temperature on the cavity enclosing the source mass $M$ located at the origin without imposing the holographic screen. First we start with a stationary spherically symmetric spacetime which may include a black hole

$$ds^2_{\text{stat}} = g_{00}(\vec{x})dx^0 dx^0 - g_{ij}(\vec{x})dx^i dx^j.$$  \hspace{1cm} (10)

The Tolman temperature observed by an observer located on the cavity is defined by [17]

$$T_T(r) = \frac{T_\infty}{\sqrt{g_{00}}}, \quad g_{00}(r) = 1 + \frac{2\Phi(r)}{c^2}$$ \hspace{1cm} (11)

where $T_\infty$ is the temperature measured by observer at infinity and the denominator of $\sqrt{g_{00}}$ is considered as the redshift factor. $\Phi$ is the Newtonian potential. The situation becomes clear when introducing a Schwarzschild black hole [18]:

$$g_{00} = 1 - \frac{2MG}{c^2 r} = 1 + \frac{2\Phi}{c^2}, \quad T_\infty = T_H = \frac{1}{8\pi GM},$$ \hspace{1cm} (12)

where $T_H$ is the Hawking temperature observed at the infinity. It is worth to mention a non-relativistic case of $c \to \infty$ for Newtonian gravity

$$T_T \simeq \frac{T_\infty}{1 + \frac{2\Phi}{c^2}} \simeq T_\infty \left(1 - \frac{\Phi}{c^2}\right) = T_{\text{non}},$$ \hspace{1cm} (13)

where the last relation defines the non-relativistic temperature $T_{\text{non}}$. On the other hand, from the Tolman-Ehrenfest effect [13], equilibrium between two systems happens when the total entropy is maximized as

$$dS = dS_1 + dS_2 = 0.$$ \hspace{1cm} (14)

If a heat element $dE_1$ enters the second system, then it leads to the condition of thermal equilibrium

$$\frac{dS_1}{dE_1} - \frac{dS_2}{dE_2} = 0 \to T_1 = T_2.$$ \hspace{1cm} (15)
Figure 1: Cavity with temperature $T$ is placed at a distance of $r$ from the source mass $M$ at the origin. A test mass with $m$ is on the cavity. Solid and dotted cavities are introduced to realize the Tolman-Ehrenfest effect such as Eq.(16).

However, if two systems are at different Newtonian potentials, the amount of energy $dE_1$ leaving the first one does not equal the amount of energy entering the second one. This is because $E = mc^2$ and the equality of inertial and gravitational mass imply that any form of energy has a gravitational mass and thus, falls in a gravitational field. In this case, as is shown in Fig. 1, it is very natural to define the local temperature $T(r)$ on the cavity located at $r$ from the source mass $M$. $dE_2$ is $dE_1$ increased by the potential energy $m\Delta\Phi$:

$$
dE_2 = dE_1 \left(1 + \frac{\Delta\Phi}{c^2}\right).$$

(16)

Then, making use of (15) and (16), one finds

$$
\frac{1}{T_2} \frac{dS_2}{dE_2} = -\frac{dS_1}{dE_1} = \frac{1}{T_1} \frac{1}{1 + \frac{\Delta\Phi}{c^2}}
$$

(17)
Figure 2: Non-relativistic temperature $T_{\text{non}}(r)$ (dotted curve) and local temperature $T(r)$ (solid curve) are depicted in the upper plane. $T_\infty$ is the undetermined temperature observed by the distant observer within the Newtonian gravity. The Newtonian potential $\Phi(r)$ is drawn as function of $r$ in the lower plane.

which implies

$$\frac{\Delta \Phi}{c^2} = \frac{T_2}{T_1} - 1 \rightarrow \frac{\nabla \Phi}{c^2} = - \frac{\nabla T}{T} = - \frac{\vec{g}}{c^2}. \quad (18)$$

Here we have made replacements: $T_1 \rightarrow T$ and $T_2 - T_1 \rightarrow -\Delta T$. Then, the above leads to an important relation between temperature and Newtonian potential

$$- \Phi = c^2 \ln[T/T_\infty] \quad (19)$$

which provides the local temperature as a function of Newtonian potential

$$T(r) = T_\infty e^{-\Phi/c^2} \simeq T_\infty \left(1 - \frac{\Phi}{c^2}\right). \quad (20)$$

As is depicted in Fig. 2, the non-relativistic temperature $T_{\text{non}}$ is the first order approximation of the local temperature $T(r)$. Two are different at small $r$, while two are nearly the same for large $r$ and approach $T_\infty$ at infinity. The connection to the Newtonian potential is shown explicitly.

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1As was explained in Ref.\[19\], $\vec{g}$ is the Galilean acceleration of gravity and $1/c^2$ is inserted as a slight relativistic effect. $\nabla \ln T = \vec{g}/c^2$ means that a vertical column of fluid at equilibrium is hotter at the bottom. For example, $\nabla \ln T = 10^{-18}/\text{cm}$ on the surface of the Earth with $g = 9.81 \text{m/s}^2$. We should note that even though the derivation of $\nabla \ln T = \vec{g}/c^2$ is different from here, it has already been derived in Ref.\[12\].
Table 1: Comparison between entropic force (EF) and temperature force (TF). HP(ER) denote holographic principle (Equipartition rule), and T-EE means Tolman-Ehrenfest effect. Finally, HS denotes holographic screen.

| principle | where | temperature | potential | force |
|-----------|-------|-------------|-----------|-------|
| EF        | HP and ER | HS          | $T = \frac{\hbar}{2\pi k_B c} g$ | $\Phi \propto S$ | $F = \frac{2\pi k_B mc}{\hbar} T = mg$ |
| TF        | T-EE    | cavity     | $\vec{\nabla} \ln T = \frac{\Phi}{c^2}$ | $\Phi \propto \ln T$ | $\vec{F} = mc^2 \vec{\nabla} \ln T = m\vec{g}$ |

Finally, the temperature force is defined as

$$\vec{F}_{\text{tem}} = mc^2 \left( \frac{\vec{\nabla} T}{T} \right) = mc^2 \vec{\nabla} \ln T = m\vec{g},$$

which leads to the Newtonian universal law. We wish to explain how the temperature force comes into play. In contrast to the entropy change in the holographic screen when a test mass approaches the screen, the energy change is essential in the cavity when a test approaches the cavity. This is because $E = mc^2$ and the equivalence principle (equality of inertial and gravitational mass) imply that any form of energy has a gravitational mass and thus, falls in a gravitational field. Hence, $dE_2$ is increased by the potential energy $m\Delta\Phi$, which generates an attractive force on the test mass. The key mechanism is different: the entropic force is realized as

$$\text{test mass } (m) \rightarrow \text{entropy increase in holographic screen},$$

while the temperature force is realized as

$$\text{test mass } (m) \rightarrow \text{energy increase in cavity}.$$  

Also, the entropic force depends on the screen temperature itself, while the temperature force depends on the gradient of logarithmic temperature. The Newtonian potential is realized by the entropy in the entropic force, whereas it is realized by the temperature in the temperature force. In this sense, we use the notion of temperature force instead of the entropic force. In defining the temperature force, $T_\infty$ is not uniquely determined, while $\hbar$ and $k_B$ do not appear, compared to the entropic force.

4 Discussions

It is fair to say that the origin of the gravity is not yet fully understood. If the gravity is not a fundamental force, it may be emergent from the other approach to gravity. Verlinde’s
idea was that Newtonian force law could be emergent from the equipartition rule and the holographic principle [6]. However, an important thing is to show that the holographic screen could be defined by enclosing a source mass $M$. This is unlikely possible to occur.

In this work, we have defined the local temperature on the cavity from the Tolman-Ehrenfest effect. It is a natural way to define the temperature without imposing the equipartition rule and the holographic principle. We have introduced the cavity enclosing a source mass $M$ to define the temperature force. Starting the Tolman temperature $T_T = T_\infty / \sqrt{g_{00}}$, we find a non-relativistic temperature $T_{\text{non}} = T_\infty (1 - \Phi / c^2)$. This temperature could be also derived from the Tolman-Ehrenfest effect, satisfying a relation $T = T_\infty e^{-\Phi / c^2} \simeq T_\infty (1 - \Phi / c^2)$. Finally, from the defining equation $\vec{F}_{\text{tem}} = mc^2 (\vec{\nabla} \ln T)$, we derive the temperature force which is identified with the Newtonian force law.

As is shown in Table 1, there are two kinds of forces which led to the same Newtonian universal law. One is based on the entropy of holographic screen, while the other is based on the local temperature on the cavity. Unless the Newtonian force is emergent as the entropic force, the choice of holographic screen temperature seems to be a major flaw of Verlinde’s idea. On the other hand, our idea on the temperature force came from the Tolman temperature which is a well-defined quantity in the relativistic description. Hence, we did not need to introduce an exotic spacetime of the holographic screen defined by holographic principle and equipartition rule.

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