Simple proof of the robustness of Gaussian entanglement in bosonic noisy channels

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The extremality of Gaussian states is exploited to show that Gaussian states are the most robust, among all possible bipartite continuous-variable states at fixed energy, against disentanglement due to noisy evolutions in Markovian Gaussian channels involving dissipation and thermal hopping. This proves a conjecture raised recently in [M. Allegra, P. Giorda, and M. G. A. Paris, Phys. Rev. Lett. 105, 100503 (2010)], providing a rigorous validation of the conclusions of that work. The problem of identifying continuous variable states with maximum resilience to entanglement damping in more general bosonic open system dynamical evolutions, possibly including correlated noise and non-Markovian effects, remains open.

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Continuous-variable (CV) systems such as light modes and ultracold atomic ensembles [11] provide advantageous resources to achieve unconditional implementations of quantum information processing [2], ranging from teleportation protocols [8] to quantum key distribution [4] and one-way quantum computation [5]. Gaussian states and Gaussian operations, that represent respectively the most natural and easily controllable light states as well as the set of manipulations efficiently realizable by linear optics, have traditionally occupied a privileged role in all such implementations. Furthermore, by virtue of their mathematical simplicity compared to general states and their realizable role in all such implementations.

Among all the possible input states, the two-mode squeezed state \( |\psi\rangle \) is the only Gaussian instance (up to symplectic transformations [17]), that allows to formulate valuable bounds on suitable entanglement measures and entropic dephasing channels, is provided in detail.

We consider general pure two-mode CV states \( \varrho_{12}(0) \) with fixed initial mean energy \( \bar{n}_0 = \text{Tr}[\varrho_{12}(0)(a_1^\dagger a_1 + a_2^\dagger a_2)] \), where \( a_j, a_j^\dagger \), are the ladder operators of mode \( j = 1, 2 \) satisfying the canonical commutation relations \( [a_i, a_j^\dagger] = \delta_{ij} \).

Among all the possible input states, the two-mode squeezed state \( \varrho_{12}^{(0)}(0) = |\psi\rangle\langle\psi| \), where \( |\psi\rangle = \sum_n \lambda^n \sqrt{1 - \lambda^2} |n, n\rangle \) and \( \bar{n}_0 = \lambda^2/(1 - \lambda^2) \), is the only Gaussian instance (up to local unitary operations). We let each possible initial state undergo a dissipative evolution through the channel described by the following master equation [17],

\[
\dot{\varrho}_{12}(t) = \sum_{j=1,2} \frac{\Gamma}{2} N_j L[a_j^\dagger] \varrho_{12}(t) + \frac{\Gamma}{2} (N_j + 1) L[a_j] \varrho_{12}(t),
\]

that encompasses losses and thermal hopping in the presence of nonclassically fluctuating local environments. The dot denotes time-derivative and the Lindblad superoperator is defined as \( L[O]_\varrho = 2 O \varrho O^\dagger - O^\dagger O \varrho - \varrho O^\dagger O \). Here \( \Gamma \) is a
tant ingredient we need is the extremality of Gaussian states.

Regression, regardless of the specific form of the states and of their evolution, over any other CV state with the same energy, and maintains its advantage throughout the whole noisy evolution. Given that (as soon as $N_1, N_2 \neq 0$) the channel in Eq. (1) takes a finite “separation time” $\tau_{[g_{12}(0)]}$ to erase all the entanglement in any initial quantum state $g_{12}(0)$ [17], the entanglement in the Gaussian state must be the last one to vanish, i.e., $\tau_{[g_{12}(0)]} \geq \tau_{[g_{12}(0)]}$ for all CV states $g_{12}(0)$ with the same mean energy as $g_{12}(0)$. This proves the robustness of Gaussian entanglement conjecture of Ref. [17], for continuous and strongly superadditive bipartite entanglement monotones, in the general case of $N_1 \neq N_2$ and without the need to restrict $g_{12}(0)$ to be in the form of a photon-number-entangled-state (i.e., a two-mode state whose one-mode marginal density matrices are diagonal in the Fock basis). Actually, being essentially a consequence of extremality, the present result holds for any set of, not necessarily pure, iso-energetic input states $g(0)$ of an arbitrary number of modes that include a Gaussian monogamy for any set of, not necessarily pure, iso-energetic input states $g(0)$ of an arbitrary number of modes that include a Gaussian monogamy for any set of, not necessarily pure, iso-energetic input states $g(0)$ of an arbitrary number of modes that include a Gaussian monogamy for any set of, not necessarily pure, iso-energetic input states $g(0)$ of an arbitrary number of modes that include a Gaussian monogamy for any set of, not necessarily pure, iso-energetic input states $g(0)$ of an arbitrary number of modes that include a Gaussian monogamy for any set of, not necessarily pure, iso-energetic input states $g(0)$ of an arbitrary number of modes that include a Gaussian monogamy for any set of, not necessarily pure, iso-energetic input states $g(0)$ of an arbitrary number of modes that include a Gaussian monogamy for any set of, not necessarily pure, iso-energetic input states $g(0)$ of an arbitrary number of modes that include a Gaussian monogamy for any set of, not necessarily pure, iso-energetic input states $g(0)$ of an arbitrary number of modes that include a Gaussian monogamy for any set of, not necessarily pure, iso-energetic input states $g(0)$ of an arbitrary number of modes that include a Gaussian monogamy for any set of, not necessarily pure, iso-energetic input states $g(0)$ of an arbitrary number of modes that include a Gaussian monogamy for any set of, not necessarily pure, iso-energetic input states $g(0)$ of an arbitrary number of modes that include a Gaussian monogamy for any set of, not necessarily pure, iso-energetic input states $g(0)$ of an arbitrary number of modes that include a Gaussian monogamy for any set of, not necessarily pure, iso-energetic input states $g(0)$ of an arbitrary number of modes that include a Gaussian monogamy for any set of, not necessarily pure, iso-energetic input states $g(0)$ of an arbitrary number of modes that include a Gaussian monogamy for any set of, not necessarily pure, iso-energetic input states $g(0)$ of an arbitrary number of modes that include a Gaussian monogamy for any set of, not necessarily pure, iso-energetic input states $g(0)$ of an arbitrary number of modes that include a Gaussian monogamy for any set of, not necessarily pure, iso-energetic input states $g(0)$ of an arbitrary number of modes that include a Gaussian monogamy for any set of, not necessarily pure, iso-energetic input states $g(0)$ of an arbitrary number of modes that include a Gaussian monogamy for any set of, not necessarily pure, iso-energetic input states $g(0)$ of an arbitrary number of modes that include a Gaussian monogamy for any set of, not necessarily pure, iso-energetic input states $g(0)$ of an arbitrary number of modes that include a Gaussian monogamy for any set of, not necessarily pure, iso-energetic input states $g(0)$ of an arbitrary number of modes that include a Gaussian monogamy for any set of, not necessarily pure, iso-energetic input states $g(0)$ of an arbitrary number of modes that include a Gaussian monogamy for any set of, not necessarily pure, iso-energetic input states $g(0)$ of an arbitrary number of modes that include a Gaussian monogamy for any set of, not necessarily pure, iso-energetic input states $g(0)$ of an arbitrary number of modes that include a Gaussian monogamy for any set of, not necessarily pure, iso-energetic input states $g(0)$ of an arbitrary number of modes that include a Gaussian monogamy for any set of, not necessarily pure, iso-energetic input states $g(0)$ of an arbitrary number of modes that include a Gaussian monogamy for any set of, not necessarily pure, iso-energetic input states $g(0)$ of an arbitrary number of modes that include a Gaussian monogamy for any set of, not necessarily pure, iso-energetic input states $g(0)$ of an arbitrary number of modes that include a Gaussian monogamy for any set of, not necessarily pure, iso-energetic input states $g(0)$ of an arbitrary number of modes that include a Gaussian monogamy for any set of, not necessarily pure, iso-energetic input states $g(0)$ of an arbitrary number of modes that include a Gaussian monogamy for any set of, not necessarily pure, iso-energetic input states $g(0)$ of an arbitrary number of modes that include a Gaussian monogamy for any set of, not n...
ity. In these cases, the Gaussian nature of transmitted states is not preserved, and the state evolved from $\rho_{12}(0)$ immediately loses its privileged role for $t > 0$, entering the competition among the $\rho_{12}(t)$’s as a peer. This is the case, for instance, if purely local phase diffusive decoherence channels are considered, which are non-Gaussian channels modeled by a master equation of the form

$$\dot{\rho}_{12}(t) = \sum_{j=1,2} \frac{\Gamma}{2} L [a_j^\dagger a_j] \rho_{12}(t).$$

(4)

For such noise models, the coherences in an initial two-mode squeezed state are degraded undergoing a non-Gaussian evolution, eventually driving the two modes into an uncorrelated product of thermal states, each corresponding to the reduced one-mode density matrix of the input two-mode state. In this scenario, there is no apparent hierarchy regulating the dynamical comparison between Gaussian and non-Gaussian initial states $\rho_{12}(0)$, when energy or alternative resources (i.e., the squeezing degree) are fixed at time $t = 0$. More complicate is the situation when a two-mode dephasing channel, with correlated noises affecting each mode, is taken into account; in that case, not even the marginal states are preserved.

To formulate an outlook, we feel that even more challenging, yet surely important, would be to investigate the case of more realistic channels where dissipation, thermal hopping, as well as phase diffusion and possibly memory effects and correlated noise may all simultaneously take place. Some of these mechanisms are currently investigated, for instance, in the context of unveiling quantum coherence effects and noise-assisted energy transport phenomena in biological systems [25]. The problem of entanglement evolution and robustness turns thus into a highly non-trivial arena, and since the powerful extremality results [14] are of little or no use at all in these more general cases, a numerical approach along the lines of Ref. [17] should be the preferable avenue to pursue. This will be the subject of further work. The main result of this Brief Report should then be recovered in the limit of negligible dephasing, uncorrelated noises and memoryless channels, but we feel that no reliable prediction can be made at this stage about which type of state, in the uncountable arena of CV states, might gain the crown of the most robust state of light in the intermediate regime where all the diverse noise effects are comparable.

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