Three-body direct nuclear reactions: Nonlocal optical potential

A. Deltuva
Centro de Física Nuclear da Universidade de Lisboa, P-1649-003 Lisboa, Portugal
(Received February 5, 2009)

The calculations of three-body direct nuclear reactions with nonlocal optical potentials are performed for the first time using the framework of Faddeev-type scattering equations. Important nonlocality effect is found for transfer reactions like $d + ^{16}$O $\rightarrow p + ^{17}$O often improving the description of the experimental data.

PACS numbers: 24.10.-i, 21.45.-v, 25.55.Ci, 24.70.+s

Direct nuclear reactions dominated by three-body degrees of freedom provide an important test for models of nuclear dynamics. Extensively studied examples are deuteron ($d$) scattering from a stable nucleus ($A$) and proton ($p$) scattering from a weakly bound nucleus ($An$) consisting of a core $A$ and a neutron ($n$). The nucleon-nucleus ($NA$) interactions employed in three-body calculations are usually modeled by optical potentials (OP) with local central and spin-orbit parts. This local approximation yields a tremendous simplification in the practical realization of the continuum discretized coupled channels (CDCC) method [1], the distorted-wave Born approximation (DWBA), or various adiabatic approaches [2] that are widely used for the description of three-body nuclear reactions; to the best of our knowledge, the above mentioned methods using nonlocal optical potentials (NLOP) have never been attempted. However, NLOP can be included quite easily in the framework of exact three-body Fadddeev/Alt, Grassberger, and Sandhas (AGS) scattering theory [3, 4] if one uses the momentum-space representation. Its application to three-body nuclear reactions [5, 6] has become possible recently due to a novel implementation of the screening and renormalization method [7, 8, 9] for including the long-range Coulomb force between charged particles.

The aim of the present work is to calculate the observables of $d + A$ and $p + (An)$ reactions in a three-body model ($n, p, A$) using NLOP and its (nearly) equivalent local optical potential (LOP) in the framework of momentum-space AGS equations and thereby study the effect of the OP nonlocality.

The AGS equations [4] are Faddeev-like connected-kernel equations that provide an exact description of the quantum three-body scattering problem. In contrast to the Faddeev equations [3] formulated for the components of the wave function, the AGS equations,

$$ U_{\beta\alpha}(Z) = \tilde{\delta}_{\beta\alpha} G_0^{-1}(Z) + \sum_{\gamma} \tilde{\delta}_{\beta\gamma} T_\gamma(Z) G_0(Z) U_{\gamma\alpha}(Z), $$

are a system of coupled integral equations for the transition operators $U_{\beta\alpha}(Z)$ whose on-shell matrix elements are scattering amplitudes and therefore lead directly to the observables. In Eq. (1) $\tilde{\delta}_{\beta\alpha} = 1 - \delta_{\beta\alpha}$, $G_0(Z) = (Z - H_0)^{-1}$ is the free resolvent, and

$$ T_\gamma(Z) = v_{\gamma} + v_{\gamma} G_0(Z) T_\gamma(Z) $$

is the two-particle transition matrix with $Z = E + i\delta$, $E$ being the available three-particle energy in the center of mass (c.m.) system, $H_0$ the free Hamiltonian, and $v_{\gamma}$ the potential for the pair $\gamma$ in odd-man-out notation. We work in momentum space where local and nonlocal potentials are treated in the same manner. The AGS equations are solved numerically after partial-wave decomposition and discretization of momentum variables; more details on employed numerical techniques can be found in Refs. [10, 11].

The AGS equations were formulated originally for a short-range potentials $v_{\gamma}$. Nevertheless, the method of screening and renormalization [7, 8, 9] allows to include the long-range Coulomb force between charged particles using the AGS framework. The Coulomb-distorted short-range part of the transition amplitude is obtained by solving the AGS equations with nuclear plus screened Coulomb potentials; the convergence of the results with the screening radius has to be established. The method has been successfully applied to proton-deuteron [12, 13] elastic scattering and breakup and to three-body nuclear reactions involving deuterons or one-neutron halo nuclei [3, 6].

For the $np$ interaction we take the realistic CD Bonn potential [14], in contrast to a simple Gaussian $np$ potential used in CDCC, DWBA, and adiabatic calculations. For the hadronic part of the $NA$ interaction we take the NLOP of Giannini and Ricco [15] which in configuration space has the form

$$ v_{\gamma}(r', r) = H(x) [V(y) + i W(y)] + \sigma \cdot L H_s(x) V_s(y) $$

with $x = |r' - r|$, $y = |r' + r|/2$, $H(x) = (\pi \beta^2)^{-3/2} \exp(-x^2 / \beta^2)$, $H_s(x) = (\pi \beta_s^2)^{-3/2} \exp(-x^2 / \beta_s^2)$; $V(y)$, $W(y)$, and $V_s(y)$ are the real volume, imaginary surface, and real spin-orbit parts, respectively, that are parametrized in the standard way using Woods-Saxon functions. The approximately equivalent LOP is taken over from Ref. [12] as well. Both NLOP and LOP are slightly modified: we adjust the parameter $W_N$, determining the strength of the imaginary part, to improve the description of the experimental $NA$ scattering data in the considered nucleon lab energy range from 10 to 40 MeV as well as the agreement between the $NA$ predictions of NLOP and LOP. The adjusted values for $W_N$ are given in Table II the other parameters are taken from Ref. [15].
In contrast to the NLOP, the LOP is energy-dependent owing to the equivalence transformation \[ \text{Eq. } (15) \]. Though energy-dependent potentials were used recently \[ \text{Ref. } 16 \] in Faddeev-type calculations, we refrain from doing so in the present work. We follow the standard procedure of fixing the energy-dependent parameters of the two-body OP in three-body calculations and use two types of Hamiltonians: (a) In \( d+A \) elastic scattering both \( nA \) and \( pA \) OP parameters are taken at half deuteron lab energy. (b) In \( p+(An) \) elastic scattering and transfer to \( d+A \) the parameters of the \( pA \) OP are taken at the proton lab energy whereas the \( nA \) potential has to be real in order to support an \( (An) \) bound state. Since the observables in \( p+(An) \) reactions are rather insensitive to the \( nA \) potential, provided it reproduces the spectrum of bound states, we take over the local real \( nA \) potential from Ref. \[ 16 \] that supports a number of bound states corresponding to the ground and excited single-particle states of the \( (An) \) nucleus, while all Pauli-forbidden states are removed; the potential parameters and the resulting binding energies are given in Ref. \[ 16 \] for \( ^{13}\text{C} \) and \( ^{17}\text{O} \) nuclei. The standard Hamiltonian (a) with the \( nA \) potential being complex does not support \( (An) \) bound states and therefore does not allow for calculations of \( d+A \rightarrow p+(An) \) reactions while most of the available transfer data come from this type of reactions. However, since the \( d+A \rightarrow p+(An) \) and \( p+(An) \rightarrow d+A \) reactions are related by time reversal provided the energy in the c.m. system is the same, we calculate the latter one using the standard Hamiltonian (b) where the nucleon \( (An) \) can be in its ground or excited state and apply the time reversal to obtain the observables for the former one. This is equivalent to using the Hamiltonian (b) in the \( d+A \) scattering, a nonstandard choice.

The interaction between \( np \), \( nA \), and \( pA \) pairs is included in partial waves with pair orbital angular momentum \( L \leq 3, 10 \), and 20, respectively, and the total angular momentum is \( J \leq 40 \); depending on the reaction some of these quantum numbers cutoffs can be safely chosen significantly lower, leading, nevertheless, to well converged results. The \( pA \) channel is more demanding than the \( nA \) channel due to the screened Coulomb force, where the screening radius \( R \approx 8 \) to 10 fm for the short-range part of the scattering amplitude is sufficient for convergence.

In Fig. \[ 1 \] we use \( p+^{16}\text{O} \) elastic scattering at proton lab energy \( E_p = 35.2 \text{ MeV} \) as an example to illustrate the achieved quality in fitting the two-body data and the approximate NLOP-LOP equivalence. We show also the predictions of the OP by Watson \[ et al. \] \[ 18 \] to demonstrate that the adjusted NLOP and LOP describe the \( NA \) data at least as good as the traditionally used potentials. An agreement of similar quality is found in all considered cases.

In Figs. \[ 2 \] and \[ 3 \] we show the results of the three-body calculation for elastic proton scattering from \( ^{17}\text{O} \) and \( ^{13}\text{C} \) nuclei around \( E_p = 35.2 \text{ MeV} \). The mutual agreement between NLOP and LOP predictions and with the data is as good as in the two-body case shown in Fig. \[ 1 \]. This is not surprising since the differential cross section and the proton analyzing power in \( p+(An) \) elastic scattering are known to correlate strongly with the corresponding observables in \( p+A \) elastic scattering. Therefore, the effect on the OP nonlocality is tiny; even the small differences between NLOP and LOP predictions seen in \( p+^{16}\text{O} \) cross section at large angles are reproduced well in \( p+^{17}\text{O} \) elastic scattering.

The Hamiltonian (b) used to calculate \( p+^{13}\text{C} \) elas-
perfect of the OP nonlocality becomes significant in the simultaneous. In contrast to elastic scattering, the effect of the OP nonlocality becomes significant in the $p + ^{13}\text{C}$ elastic scattering at $E_p = 35$ MeV. Curves as in Fig. 2 and the experimental data are from Ref. 19.

![Graph](image1)

**FIG. 3:** (Color online) Differential cross section divided by Rutherford cross section and proton analyzing power for for $p + ^{13}\text{C}$ elastic scattering at $E_p = 35$ MeV. Curves as in Fig. 2 and the experimental data are from Ref. 19.

![Graph](image2)

**FIG. 4:** (Color online) Differential cross section for $p + ^{13}\text{C} \rightarrow d + ^{12}\text{C}$ transfer at $E_p = 35$ MeV. Curves as in Fig. 2 and the experimental data are from Ref. 20.

Anomalous elastic scattering in Fig. 3 allows for the transfer to $d + ^{12}\text{C}$ as well; the AGS equations for both reactions are solved simultaneously. In contrast to elastic scattering, the effect of the OP nonlocality becomes significant in the $p + ^{13}\text{C} \rightarrow d + ^{12}\text{C}$ transfer cross section at angles $\Theta_{c.m.} > 30$ deg as demonstrated in Fig. 4 although it does not improve the description of the experimental data. We therefore expect the OP nonlocality to be important also in the inverse reactions, i.e., $d + A \rightarrow p + (An)$, where more data exist with the final nucleus ($An$) being in its ground or excited state; the observables are calculated using the Hamiltonian (b) as described above. We start in Fig. 5 with the $d + ^{12}\text{C} \rightarrow p + ^{13}\text{C}$ reaction at deuteron lab energy $E_d = 30$ MeV which in the $p + ^{13}\text{C} \rightarrow d + ^{12}\text{C}$ case corresponds to $E_p = 30.6, 27.3$, and $26.5$ MeV for $^{13}\text{C}$ states $1/2^-, 1/2^+$, and $5/2^+$, respectively. For the transfer to the $^{13}\text{C}$ ground state $1/2^-$ the shape of the experimental data and theoretical predictions are similar to the ones of $p + ^{13}\text{C} \rightarrow d + ^{12}\text{C}$ reaction at $E_p = 35$ MeV in Fig. 4 as expected from the detailed balance, given the small difference in energy. Thus, except for forward angles, the transfer reactions involving the $^{13}\text{C}$ ground state $1/2^-$ are described rather unsuccessfully much like it was found in Ref. 10. In the case of the transfer to $^{13}\text{C}$ excited states $1/2^+$ and $5/2^+$ the OP nonlocality is again important, and the predictions of the NLOP account for the data quite successfully, in contrast to the ones of the LOP and Ref. 10. The differential cross section for the transfer to $^{13}\text{C}$ $1/2^+$ state is increased by the NLOP at forward angles and decreased at $\Theta_{c.m.} > 20$ deg while for the $5/2^+$ state it is significantly decreased at $\Theta_{c.m.} > 35$ deg such that the data is slightly overestimated by almost a constant factor that may be associated with the spectroscopic factor.

The results for the $d + ^{16}\text{O} \rightarrow p + ^{17}\text{O}$ transfer cross sections at $E_d = 25.4$ and $36.0$ MeV are presented in Figs. 6 and 7. The corresponding proton lab energy in the inverse reactions $p + ^{17}\text{O} \rightarrow d + ^{16}\text{O}$ is $E_p = 25.9$ and 35.9.
FIG. 6: (Color online) Differential cross section for \( d^{+}\text{O} \rightarrow p^{+}\text{O} \) transfer to the ground state \( \frac{5}{2}^{+} \) of \( \text{O} \) at \( E_d = 25.4 \) and 36.0 MeV. Curves as in Fig. 4. The experimental data are from Ref. 22.

FIG. 7: (Color online) Differential cross section for \( d^{+}\text{O} \rightarrow p^{+}\text{O} \) transfer to the excited state \( \frac{1}{2}^{+} \) of \( \text{O} \) at \( E_d = 25.4 \) and 36.0 MeV. Curves as in Fig. 5. The experimental data are from Ref. 22.

MeV for \( \text{O} \) ground state \( \frac{5}{2}^{+} \) and \( E_p = 25.0 \) and 35.0 MeV for \( \text{O} \) excited state \( \frac{1}{2}^{+} \), respectively. The effect of the OP nonlocality is again sizable and qualitatively very similar to the one observed in \( d + ^{12}\text{C} \rightarrow p + ^{13}\text{C} \) reactions involving \( ^{13}\text{C} \) excited states \( \frac{5}{2}^{+} \) and \( \frac{1}{2}^{+} \). The differential cross section for the transfer to the \( \text{O} \) ground state \( \frac{5}{2}^{+} \) is decreased at very forward angles and at \( \Theta_{c.m.} > 30 \) deg. Although in particular narrow angular regimes the predictions of the LOP are closer to the data, the NLOP provides a better overall description of the data, especially of its shape. In the case of the transfer to the \( ^{17}\text{O} \) excited state \( \frac{1}{2}^{+} \) the NLOP reproduces the data almost perfectly.

Next we consider deuteron-nucleus elastic scattering that is usually described using the Hamiltonian of type (a). We show in Fig. 3 the results for \( d^{+}\text{O} \) elastic cross section at \( E_d = 25.4 \) MeV. The effect of the OP nonlocality is visible at larger angles \( \Theta_{c.m.} > 30 \) deg and moves the predictions towards the data up to \( \Theta_{c.m.} = 60 \) deg; beyond 60 deg the data is strongly underpredicted by both NLOP and LOP results. This indicates that the Hamiltonian (a) is too absorptive in the considered case where negative energies in the two-body \( \text{NA} \) subsystem
have a large weight in the three-body scattering equations as discussed in Ref. [10]; it was found there that using energy-dependent potential that becomes real and therefore less absorptive at negative N A energies significantly increases the cross section at $\Theta_{c.m.} > 60$ deg. For curiosity in Fig. 8 we present also results of the Hamiltonian (b) which is less absorptive since the nA potential is real. Surprisingly, with this choice both the NLOP and the LOP roughly account for the data also at large angles. Thus, NLOP and LOP with the Hamiltonian of type (b) describe elastic $d + ^{16}O$ and $p + ^{12}O$ data with similar quality while for the transfer reactions the predictions of NLOP is clearly superior.

Finally in Fig. 9 we present observables for $d + ^{16}O$ and $d + ^{40}Ca$ elastic scattering at $E_d = 56$ MeV calculated with the Hamiltonian of type (a). Visible differences between NLOP and LOP show up at large angles $\Theta_{c.m.} > 60$ deg. The predictions of NLOP are closer to the data but show too sharp oscillations; a similar feature is present already in the $p + A$ and $p + (An)$ observables in Figs. 1—4 but is less pronounced there. Both NLOP and LOP fail in reproducing small-angle deuteron vector analyzing power $A_\theta$ much like other potentials as found in Ref. [24]. The effect of the OP nonlocality is of comparable size also for deuteron tensor analyzing powers, but it is unable to resolve the discrepancy [24] in the large-angle $A_{\xi \xi}$. The results for deuteron-$^{12}C$ elastic scattering exhibit similar features as those for deuteron-$^{16}O$ and are not shown separately.

To be sure that the observed OP nonlocality effect is not a consequence of only approximate NLOP-LOP equivalence, we performed the following test calculations. We varied the parameter $W_N$ by $\pm 1$ MeV thereby inducing changes in the N A observables of a size comparable to the NLOP-LOP difference. As a consequence, similar changes occur in $p + (An)$ elastic observables, but for $d + A$ elastic scattering and especially for all transfer reactions the induced changes are considerably smaller than the observed NLOP-LOP difference. Thus, the imperfection in the NLOP-LOP equivalence does not affect our conclusions on the importance of the OP nonlocality. A possible reason for an overall better description of the three-body observables by NLOP may be that NLOP fits the N A data over a broader energy range compared to LOP which, although being energy-dependent, has to be chosen at a fixed energy.

In summary, we performed the calculations of three-body direct nuclear reactions with the NLOP for the first time. Exact scattering equations in the AGS form were solved and the Coulomb interaction was included using the method of screening and renormalization. The OP nonlocality effect is found to be very small for $p + (An)$ elastic scattering, moderate for $d + A$ elastic scattering at larger angles, and especially important for transfer reactions $p + (An) \to d + A$ and $d + A \to p + (An)$. In the latter case the NLOP is clearly more successful in accounting for the data in transfer reactions involving $^{13}C$ and $^{17}O$ nuclei in the states $1/2^+$ and $5/2^+$. We hope that the present work, demonstrating the feasibility of the calculations with NLOP and the importance of the nonlocality, will stimulate the development of new and more precise nonlocal nuclear interaction models.

The author thanks A. C. Fonseca for comments on the manuscript. The work is supported by the Fundação para a Ciência e a Tecnologia (FCT) grant SFRH/BPD/34628/2007.

[1] N. Austern, Y. Iseri, M. Kamimura, M. Kawai, G. Rawitscher, and M. Yahiyo, Phys. Rep. 154, 125 (1987).
[2] R. C. Johnson, J. S. Al-Khalili, and J. T. Costeini, Phys. Rev. Lett. 79, 2771 (1997); N. K. Timofeyuk and R. C. Johnson, Phys. Rev. C 59, 1545 (1999).
[3] L. D. Faddeev, Zh. Eksp. Teor. Fiz. 39, 1459 (1960) [Sov. Phys. JETP 12, 1014 (1961)].
[4] E. O. Alt, P. Grassberger, and W. Sandhas, Nucl. Phys. B2, 167 (1967).
[5] A. Deltuva, Phys. Rev. C 74, 064001 (2006).
[6] A. Deltuva, A. M. Moro, E. Cravo, F. M. Nunes, and A. C. Fonseca, Phys. Rev. C 76, 064602 (2007).
[7] J. R. Taylor, Nuovo Cimento B23, 313 (1974); M. D. Semon and J. R. Taylor, ibid. A26, 48 (1975).
[8] E. O. Alt and W. Sandhas, Phys. Rev. C 21, 1733 (1980).
[9] A. Deltuva, A. C. Fonseca, and P. U. Sauer, Annu. Rev. Nucl. Part. Sci. 58, 27 (2008).
[10] K. Chmielewski, A. Deltuva, A. C. Fonseca, S. Nemoto, and P. U. Sauer, Phys. Rev. C 67, 014002 (2003).
[11] A. Deltuva, K. Chmielewski, and P. U. Sauer, Phys. Rev. C 67, 034001 (2003).
[12] A. Deltuva, A. C. Fonseca, and P. U. Sauer, Phys. Rev. Lett. 95, 092301 (2005).
[13] A. Deltuva, A. C. Fonseca, and P. U. Sauer, Phys. Rev. C 71, 054005 (2005); 72, 054004 (2005); 73, 057001 (2006).
[14] R. Machleidt, Phys. Rev. C 63, 024001 (2001).
[15] M. M. Giannini and G. Ricco, Ann. Phys. (N.Y.) 102, 458 (1976).
[16] A. Deltuva and A. C. Fonseca, Phys. Rev. C 79, 014606 (2009).
[17] E. Fabbric, S. Micheletti, M. Pignanelli, F. G. Resmini, R. D. Leo, G. D’Erasmo, and A. Pantaleo, Phys. Rev. C 21, 844 (1980).
[18] B. A. Watson, P. P. Singh, and R. E. Segel, Phys. Rev. 182, 978 (1969).
[19] H. Ohnuma et al., Nucl. Phys. A456, 61 (1986).
[20] H. Toyokawa, H. Ohnuma, Y. Tajima, T. Nizeki, Y. Honjo, S. Tomita, K. Ohkushi, M. H. Tanaka, S. Kubono, and M. Yosoi, Phys. Rev. C 51, 2592 (1995).
[21] H. Ohnuma et al., Nucl. Phys. A448, 205 (1986).
[22] M. D. Cooper, W. F. Hornyak, and P. G. Roos, Nucl. Phys. A218, 249 (1974).
[23] N. Matsuoka et al., Nucl. Phys. A455, 413 (1986).
[24] A. Deltuva, arXiv:0901.3313.