Non-equilibrium dynamics in a 3d spin-glass

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Non-equilibrium dynamics in a Ag(11%Mn) spin-glass has been studied by low frequency ac-susceptibility and magnetic relaxation experiments. The results unequivocally show that spin structures that memorize the cooling process are imprinted in the system. These imprinted structures disclose themselves through dramatic changes of the dynamics on re-heating the spin-glass through the temperatures where intermittent stops or changes of the cooling rate have been imposed. We can qualitatively interpret our results in terms of the droplet spin-glass model developed by Fisher and Huse [Phys. Rev. B 38 (1988) 373; 386].

I. INTRODUCTION

A successful real space theory for the dynamics of 3d spin-glass systems is the droplet model developed a decade ago by Fisher and Huse. Particular experimental characteristics such as the aging phenomenon and the non-existence of a phase transition in a magnetic field are inherent properties of the droplet spin-glass model. However, the validity of this model has been questioned: e.g. a compact domain growth picture is not consistent with the multiple memory effect recently found in 3d spin-glass systems. Fig. 1 shows a low frequency ac-susceptibility experiment on a Cu(Mn) spin-glass where this suggestive effect is illustrated. In this paper, extended measurements of some associated non-equilibrium features of the ac-susceptibility and the dc-magnetic relaxation on the 3d spin-glass material Ag(11at% Mn) are presented. The results can qualitatively be incorporated in the original droplet model without apparent contradictions and we do not here try to interpret our results on aging in spin-glasses in terms of non real space models.

II. THE DROPLET MODEL

The results of this study are interpreted in the spirit of the droplet model using relevant parts as summarized in the following points;

i) The spin-glass ground state is unique but two-fold degenerate by its spin reversal symmetric state.

ii) Chaos with temperature - a small temperature shift changes the ground state spin configuration completely on long enough length scales.

iii) The length scale, $l$, up to which no essential change in spin configuration is observed after a temperature change $\Delta T$, introduces the concept of an overlap length $l(\Delta T)$.

iv) In the non-equilibrium case, the development towards the ground state is governed by the growth of domains belonging to either of the two degenerate ground states. The typical domain size after a time, $t_w$, at a constant temperature $T$, is

$$R \propto \left( \frac{T \ln(t_w/\tau_0)}{\Delta(T)} \right)^{1/\psi}$$  (1)

where $\tau_0$ is a microscopic time of order $10^{-13}$ s. $\Delta(T)$ sets the free energy scale and $\psi$ is a barrier exponent. This length scale $R$, that we often will refer to as a typical domain size, should not be taken literally, but rather be considered as a typical measure of the smallest separation between domain walls remaining after the wait time $t_w$.

In order to discuss the relaxation towards equilibrium, a 3d Ising spin-glass quenched to a temperature $T_1 < T_g$ is considered. The state at $t = 0$ is hence characterized by spins of random direction. To further simplify the following argumentation it is also assumed that the spins are situated on a regular lattice. In the droplet model

![Graph](image-url)  
FIG. 1. The out-of-phase component of the ac susceptibility of a Cu(13.5 at% Mn) ($T_g \approx 68$ K) spin-glass measured on continuous heating after the sample has been subjected to two intermittent stops of order $10^4$ s at 40 and 50 K during the cooling procedure. The two pronounced dips in the curve occur at the temperatures of the stops at constant temperature during cooling. $f = 1.7$ Hz.
there exists only two degenerate ground states, \(\Psi\) and its spin reversal counterpart \(\tilde{\Psi}\), it is thus possible to map all spins to either of the two desired ground states immediately after a quench. Citing Fisher and Huse\cite{FisherHuse1986} “This results in an interpenetrating network of regions of the two states”. In fact, the domain walls of one lattice spacing, which separates the two states \(\Psi\) and \(\tilde{\Psi}\) at \(t = 0\) after the quench should have a fractal surface giving rise to fractal domains of all sizes. The subsequent equilibration process at \(t > 0\) is governed by droplet excitations yielding a domain growth to a typical length scale which depends on temperature and wait time, \(t_{w1}\), according to Eq. \[1\]. After this time, fractal structures typically smaller than \(R(T_1, t_{w1})\) have become equilibrated, but, viewing the system on a larger length scale, it still looks intact. The implication of this argument is that large enough fractal structures which were obtained at \(t = 0\) after the quench are virtually unaffected after the wait time \(t_{w1}\). If the system thereafter is quenched to a considerably lower temperature \(T_2\), a new fractal domain structure can be mapped onto the equilibrium configuration at \(T_2\). The short length scale limit is now set by the overlap length, \((L_1 - L_2)\), and fractal structures will be washed out with increasing wait time, \(t_{w2}\), starting from this length scale and ending at the size \(R(T_2, t_{w2})\). Structures on longer length scales are again intact. If the system is further quenched and then instantly re-heated to \(T_2\), the domain structure at \(T_2\) of course remains unaffected, but also, when instantaneously heated back to \(T_1\), the re-structuring on short length scales, \(R(T_2, t_{w2}) \ll R(T_1, t_{w1})\), that occurred at \(T_2\) is rapidly washed out and the system is effectively left with only the original large length scale domain structure which was imprinted at the first quench to \(T_1\). If the droplet model is relevant, these basic properties of the logarithmically slow (eternal) equilibration process should be reflected in the dynamic response of a real 3d spin-glass subjected to a corresponding thermal history. In a real experimental situation, the system can only be cooled and heated at a controlled but finite rate. The domain growth is in such processes governed by the cooling/heating rate and limited by an interplay between the domain growth (Eq. \[1\]), the chaotic nature of the spin glass phase and the overlap between states on short enough length scales. Accounting for this complication, we have designed a series of dynamic magnetization and susceptibility experiments that should mirror the discussed development of a domain structure in an equilibrating 3d spin-glass.

Droplet excitations occur on time limited length scales and are in fact responsible for the equilibration process of the quenched spin-glass, also giving rise to the domain growth function Eq. \[1\]. If a weak magnetic field is applied on the spin-glass at time \(t_{obs} = 0\), the time dependent response, \(m(t_{obs})\), is due to a continuous magnetization process governed by polarization of droplets of size, \(L(t_{obs})\):

\[
L \propto \left( \frac{T \ln(t_{obs}/\tau_0)}{\Delta(T)} \right)^{1/\psi}
\]

Since \(L\) grows with the same logarithmic rate as \(R\), the relevant droplet excitations and the actual domain size become comparable large at time scales \(t_w \approx t_{obs}\). For \(t_{obs} \ll t_w\) the relevant excitations occur mainly within equilibrated regions while for \(t_{obs} \gg t_w\) these excitations occur on length scales of the order of the growing domain size and yield a non-equilibrium response. A crossover occurs in the intermediate region, \(t_w \approx t_{obs}\) which is characterized by a maximum in the relaxation rate \(S(t) = 1/H \partial m(t)/\partial \ln(t)\). This aging behavior is also reflected in low frequency ac-susceptibility measurements. Different from dc-relaxation measurements is that the observation time is kept constant by the probing frequency, \(\omega/2\pi\), according to \(t_{obs} = 1/\omega\) and hence, in an experiment at constant temperature, sets the associated probed length scale, \(L(T, 1/\omega)\), fixed. In this study we are primarily interested in the processes that affects the response at the observation time of the experiment, the consequences of these are best exposed in plots of the relaxation rate \(S\) vs. \(t_{obs}\) for different wait times, the out-of-phase component of the ac-susceptibility vs. time at constant temperature or vs. temperature at different cooling/heating rates. Therefore, the dc-relaxation measurements are primarily presented in the form of the relaxation rate, \(S\), and the ac-susceptibility is represented by its imaginary part \(\chi''\), these quantities can be related through the relation\[2\]

\[
\chi'' \approx \frac{\pi}{2} S(t_{obs}) , \quad t_{obs} = 1/\omega
\]

\[3\]

### III. EXPERIMENTAL

The sample was prepared by melting pure Ag and Mn together at \(T = 1000^\circ\text{C}\) in an evacuated atmosphere. After annealing the sample at \(850^\circ\text{C}\) for 72h it was water quenched to room temperature.

The experiments have been performed in a non-commercial low-field SQUID magnetometer\[4\]. The dc magnetic field is generated by a small superconducting solenoid always working in persistent mode during measurements. The ac-field was generated by a copper coil directly wound on the sample which is shaped as a 5 mm long cylinder, 2.5 mm in diameter. The magnetic response of the sample, subtracted by the ac-field from a compensating coil, was recorded with a set of pick-up loops positioned to form a third order gradiometer. At the position of the sample, the resulting rms value of the ac-field was 0.1 Oe and the background field was less than 1 mOe. All ac-susceptibility measurements were performed at a frequency of 1.7 Hz. A sapphire rod was used to provide a good thermal contact between the heater, the thermometer and the sample.
IV. RESULTS AND DISCUSSION

A. Aging characteristics

Figs. 2 and 3 introduce the overall behavior of the dynamic susceptibility of our Ag(11 at% Mn) sample and expose two classical manifestations of the aging phenomenon in spin-glasses. In the inset to Fig. 2 both components of the ac-susceptibility, $\chi'(T) = \chi'(\omega) + \chi''(\omega)$ at the frequency $\omega/2\pi = 1.7$ Hz, are plotted vs. temperature at a cooling rate of 0.25 K/min. The maximum in $\chi'(T)$, defining the freezing temperature, is accompanied by the onset of an out-of-phase component of the susceptibility. In the main frame of Fig. 2 the influence of aging on $\chi''$ is illustrated by a plot of $\chi''$ vs. time elapsed at 23 K, where the continuous cooling was interrupted. Fig. 3 exemplifies the results from an ordinary zero field cooled (ZFC) aging experiment at $T_m = 27$ K, in (a) the magnetization and in (b) the corresponding relaxation rate is plotted vs. the observation time. The sample is probed in a dc-field of 1 Oe after the wait times $t_w = 100$ s (open symbols) and $t_w = 3\,000$ s (filled symbols) at $T_m$. The signatures of aging in spin-glasses: an inflection point in $m(t)$ and a corresponding maximum in $S(t)$ vs. $\log t_{\text{obs}}$ are reproduced in the figure.

Some words about time and non-equilibrium dynamics: experimentally we are confined to time scales set by the cooling/heating rates and the time it takes to thermally equilibrate the sample to a constant temperature. Although spin-glass dynamics occurs on all time scales ranging from $10^{-13}$ s to infinity, the shortest observation times where consequences of non-equilibrium dynamics can be observed experimentally are confined to $t_{\text{obs}} \geq 10^{-3}t_{\text{aeff}}$ where $t_{\text{aeff}}$ is an effective age of the system set by the cooling/heating rate or the time spent at constant temperature. I.e. when $t_{\text{obs}} \ll t_{\text{aeff}}$ no aging is observed and the dynamic response of the system appears stationary. In practice this applies to most ac-susceptibility measurements at frequencies larger than 20 Hz ($t_{\text{obs}} = 1/\omega \approx 10^{-2}$ s). On the other hand, in a dc magnetic relaxation experiment spanning some decades in time, the observation time of the experiment continuously increases and $\log t_{\text{obs}}$ unavoidably approaches $\log t_{\text{aeff}}$ at long enough observation times and effects of the aging process becomes discernible. The different times necessary to discuss the aging behavior are related according to: $t_{\text{aeff}} \approx t_w + t_{\text{obs}} = t_a$, where $t_a$ is the total time spent at constant temperature i.e. the age of the system.

We will continue by discussing results from ac-susceptibility experiments using different cooling rates and employing intermittent stops at different temperatures during the cooling of the sample, and interpret this cooling behavior as well as the subsequent heating curves (always recorded at one and the same heating rate) in terms of an assumed domain structure imprinted during cooling. This discussion will be complemented by results from dc-magnetic relaxation experiments subsequent to cooling and heating procedures that closely mimic those of the ac-experiments.

![FIG. 2](image1.png)

**FIG. 2.** Relaxation of $\chi''(t)$ with time at constant temperature after the sample has been cooled from a temperature above $T_g$ to $T = 23$ K. Inset, both components of the ac-susceptibility vs. $T$ measured at a cooling rate of 0.25 K/min. $f = 1.7$ Hz, $h_{\text{ac}} = 0.1$ Oe, Ag(11 at% Mn).

![FIG. 3](image2.png)

**FIG. 3.** Zero field cooled magnetization vs. observation time at 23 K. a) $m(t)$ vs. $\log t$ b) relaxation rate $S(t)$. $H_{\text{dc}} = 1$ Oe, Ag(11 at% Mn).
B. ac-susceptibility: cooling rate dependence and memory

A simple memory effect in the low frequency ac-susceptibility is shown in Fig. 4 after the following sequence has been performed. The sample is cooled to 23 K while measuring every 0.25 K (filled circles). At 23 K the sample is subjected to a wait time \( t_w = 7 \, 200 \, s \) during which \( \chi'' \) decays (cf. Fig. 3). After this wait time, the sample is further cooled to 20 K and then immediately re-heated, the data on heating are indicated by open circles in the figure. Both the cooling and the heating is made with the same rate 0.25 K/min. The results from identical cooling and a heating procedures, but without the wait time at 23 K, are indicated by the full lines in Fig. 4 for reference. The effect of the relaxation during the wait time is clearly visible in the cooling curve as a dip in \( \chi''(T) \), followed by a rather rapid increase and approach to the reference curve at temperatures below 23 K on the continued cooling and, as expected from the droplet model, it also reappears as a corresponding dip in \( \chi''(T) \) centered around \( T = 23 K \) in the heating curve. The merging with the cooling reference curve on continued cooling can be interpreted as an effect of chaos with temperature and a region of overlapping states, \( \Delta T_s \). The dip on re-heating is a consequence of the imprinted spin configuration at \( T=23 K \) on large length scales. I.e. when returning to this temperature all domain growth at lower temperatures have occurred on length scales \( \ll R(23 K, 7 \, 200 \, s) \) and these are here washed out on shorter time scales leaving a system with an effectively equivalent domain configuration to the one originally obtained during the wait time at \( T=23 K \).

An immediate consequence of the chaos and overlap concepts is that the ac-susceptibility depends on the cooling rate. Larger domains have time to grow within the overlap region \( \Delta T_s \) when cooling the spin-glass at a slower rate. Thus, the observed magnitude of \( \chi''(T) \) will be closer to the equilibrium level in a slow cooling process than in a fast. This prediction is confirmed in the inset to Fig. 4, where \( \chi''(T) \) at two different cooling rates, 0.25 and 0.005 K/min. are shown. The overlap concept is further supported by the fact that when suddenly at 25 K the cooling rate is increased from 0.005 to 0.25 K/min., the cooling curve within a distance of \( \Delta T_s \approx 1 \, K \) merges with the fast cooling curve. Assuming that the domain size, \( R(T, t_c) \), at all temperatures in the cooling process is limited by a characteristic time \( t_c \), which is set by the cooling rate and an interplay between chaos and the overlap length, obviously this time \( t_c \) must increase with decreasing cooling rate. In the \( \chi''(T) \) experiments, droplet excitations on the length scale \( L(T, 1/\omega) \) govern the response, these excitations includes a large amount on the size of the domains if \( 1/\omega \) is of order \( t_c \), but such excitation rapidly becomes rare with an increasing \( t_c \). This implies as mentioned above that the measured susceptibility decreases with decreasing cooling rate and that

![FIG. 4. \( \chi''(T) \) vs. \( T \) measured on cooling (solid circles) and heating (open circles) at a rate of 0.25 K/min. The sample was intermittently kept at \( T=23 K \) for \( t_w = 7 \, 200 \, s \) during cooling. Solid lines show the corresponding curves without the interrupted cooling at 23 K. The inset shows \( \chi''(T) \) using two different cooling rates: 0.25 K/min. (open circles) and 0.005 K/min. (solid circles). The slow cooling rate is changed to 0.25 K/min. at 25 K. \( f = 1.7 \, Hz, h_{ac} = 0.1 \, Oe, Ag(11 \, at \% \, Mn) \).](image-url)
temperature in a continuous cooling process. This implies that the domain structure on re-heating the sample has become partially reconstructed also on length scales of order $R(T, t_c)$, i.e. the system appears less equilibrated and the susceptibility attains a comparably larger magnitude. The behavior on changing from cooling to heating occurs at $t = 20 K$ resulting in a heating reference curve which initially is smaller in magnitude than the cooling reference curve. This is explained by the fact that the sample spends the longest time in the temperature interval within the overlap region $(20 K \pm \Delta T^*)$ first during cooling and then in the subsequent heating.

To elaborate further on the cooling/heating rate dependence, we show in Fig. 5 three curves obtained at one and the same heating rate, but recorded after cooling the sample at 4 K/min., 0.25 K/min. and 0.005 K/min. The differences between the curves are significant and in accord with the discussion above that a domain structure on long length scales $R(T, t_c)$ is imprinted but also reinitialized during continuous cooling. We thus expect that the curve recorded at a cooling rate of 4 K/min. should always yield $t_h > t_c$ and the system should appear least equilibrated and have the largest magnitude of $\chi''(T)$. The curve obtained after cooling at 0.005 K/min. has an imprinted spin structure characterized by $t_c \gg t_h$ and $\chi''(T)$ should have a substantially smaller magnitude. The curve measured after cooling at 0.25 K/min. is expected to be found in-between the other two, just as is seen in the figure.

For visual clarity in the following figures, all presented ac-susceptibility data are subtracted by either the cooling or the heating reference curve. These two reference curves are obtained using a cooling/heating rate of 0.25 K/min. (see Fig. 4). Since the ac-susceptibility always decreases towards the equilibrium value when the system ages at constant temperature, data points which are positive in such plots indicate a state further away from equilibrium than the corresponding points on the reference curve. For negative values the data points are closer to equilibrium. Furthermore, in all these experiments, the dc-field is zero and the cooling is always initiated at $T = 40 K$ where the system is paramagnetic.

As has been experimentally observed in a recent study on memory effects in spin-glasses and was reproduced in Fig. 1, the single memory experiment presented in Fig. 4 can be extended to include two (or more) wait times at well separated temperatures during the cooling of the sample. A procedure, which in the subsequent heating process results in dips in the ac-susceptibility positioned at each of these temperatures. For our current sample, this intriguing behavior is illustrated in Fig. 6 where the sample has been subjected to two wait times of 30 000 s duration, first at $T = 24 K$ and then at $T = 21 K$. Except at these particular temperatures the sample is cooled and heated with the usual rate of 0.25 K/min. As can be seen from the open circles in Fig. 6, the equilibration at each temperature gives rise to corresponding dips when re-heating the sample.

Evidently from the results discussed above, it is possible to imprint a spin configuration at a temperature where the spin-glass has been allowed to age and recover this state when returning to the same temperature, and even to imprint and recover two or more equilibrated spin configurations at well separated temperatures. This ability excludes, as pointed out recently, that new compact domains grow at each aging temperature. In other words, assuming that $|T_1 - T_2| > \Delta T^*$, it is impossible to imagine that compact $T_1$ domains coexist with compact $T_2$ domains. However, by assuming that the equilibration process does not correspond to the growth of compact domains but rather, as discussed above, a removal of fractal domain wall structures up to the length scale $R$, the...
temperatures, so that the spin-glass appears less equilibrated in a temperature interval $T = (25 \pm \Delta T_\ast) \pm \Delta T$. At higher temperatures, the spin configuration appears less equilibrated than the original slow cooling curve, but still closer to equilibrium than the reference heating curve (which is measured at the same heating rate, 0.25 K/min.). This behavior mirrors both the continuously imprinted equilibrated spin configurations obtained during slow cooling and the simultaneous partial reinitialization of these configurations which occurs at lower temperatures. It must hence also be concluded that the equilibrated length scales $R(T, t_c)$ obtained at each temperature in the slow cooling process are partly preserved during the whole experiment. For comparison we have in Fig. 7 included a single memory curve recorded at a cooling/heating rate of 0.25 K/min. and waiting 7 200 s at 25 K during cooling. Filled squares show the cooling curve and open squares the heating curve. It may be noticed that this dip is centered around $T = 25$ K as expected.

Fig. 6 and 7 show that a long wait time at constant temperature also affects the ac-response at surrounding temperatures, so that the spin-glass appears less equilibrated than in the reference procedure at temperatures separated more than $\Delta T_\ast$ from the temperature where the aging took place. As discussed above, in the continuous cooling case the typical equilibrated domain size $R(T, t_c)$ is governed by the effective cooling time $t_\ast$, which also sets an effective age and thus the magnitude of $\chi''(T, t_\ast)$. When the cooling is stopped at $T_m$, the domains can grow unrestricted, and after the wait time they have reached a size $R(T_m, t_w)$. When cooling is recaptured, this new domain size is adequate as long as the overlap length, $l(T_m - T) > R(T_m, t_w)$, however, at temperatures just outside $T = (T_m - \Delta T_\ast)$ the typical domain size is determined by the overlap length
l(T_m − T) ≪ R(T, t_w). (The overlap length, l(ΔT), is a very rapidly decreasing function with ΔT). At lower temperatures, the cooling time again becomes the governing parameter. However, just outside the overlap region, T_m − ΔT_a, the actual cooling time, t_ac, i.e. the time the sample has been kept within a region where the growing domain size overlaps with the domain size achieved at temperatures just above, is of course considerably shorter than the cooling time, t_c, of the continuous reference cooling procedure and thus R(T, t_ac) < R(T, t_c). During the continued cooling, t_ac increases, but a temperature decrease at least of order ΔT_a is needed to regain t_ac ≈ t_c and a typical domain size equivalent to that of the continuous cooling procedure. These arguments suggest, in accord with the experiments, that the disturbed χ''(T)-curve at temperatures below T_m first should increase towards and even cross the reference level as the temperature is decreased beyond ΔT_a, go through a maximum deviation and then slowly approach χ''_ref further away from T_m. Similar arguments as discussed above can also be applied to the heating curve at temperatures above T_m, i.e. the dip should be succeeded by a temperature region outside T_m + ΔT_a where the relative susceptibility should become positive. The longer time the sample is kept at T_m, the deeper the dip and the larger the positive levels outside the overlap region will become. This conclusion is further elucidated in an experiment where the sample is cooled, either with the rate 0.25 K/min. or 4 K/min., to the temperature 23 K where it is subjected to different wait times before it is heated with the rate 0.25 K/min. In Fig. 8 the ac-susceptibilities, measured on heating, are indicated by filled symbols after the wait time, t_w = 3 600 s. Open symbols indicate the corresponding curves after a wait time, t_w = 30 000 s. To separate the two cooling rates, squares are used to symbolize the faster one while circles are used for the slower. Also included in Fig. 8 is a reference curve obtained during heating after cooling to 20 K with the fast rate, 4 K/min. As can be seen, the system appears more reinitialised, at high enough temperatures, after a waiting time of 30 000 s. The effect is most pronounced in the case where a slow cooling rate was used. A wait time of 3 600 s is apparently too short realize a measurable reinitialisation.

C. Relaxation rate experiments

The usefulness of this fractal domain picture can be further examined by means of relaxation measurements where the observation time spans several decades. A zero-field-cooled relaxation experiment can be designed to give complementary information on the reinitialisation observed in the ac-susceptibility curves of Fig. 7. In these measurements, as in the ac-experiments, the sample is cooled with a rate of 0.25 K/min. to 23 K where it is subjected to a wait time, t_w. The sample is then heated at the rate 0.25 K/min. but only to 25 K where the sample is kept at constant temperature and after a time of 10 s a magnetic field of 1 Oe is applied. The subsequent relaxation rate is shown in Fig. 9 for the three different wait times, t_w = 0 (filled squares), 3 600 s (open circles), and 30 000 s (filled circles). For reference purpose, the corresponding rate obtained after cooling the sample at 4 K/min. directly to 25 K and waiting there 10 s before the field is applied, is included and indicated by open squares. H_{dc} = 1 Oe, Ag(11 at% Mn).
measurement where the sample is directly cooled to 25 K using a cooling rate of 0.25 K/min. gives a relaxation rate curve that is closely equal to the curve where a zero second wait time at 23 K is used (filled squares). The prime aging characteristic of the magnetic relaxation in spin-glasses is an inflection point in the relaxation rate at an observation time closely equal to the wait time before the magnetic field is applied, \( t_w \). A rather slow cooling rate, like 0.25 K/min., yields a broad and ill-defined maximum in the relaxation rate at an observation time that may be called effective wait time or age. The three curves taken after a cooling rate of 0.25 K/min. in Fig. 9 show a continuous development towards the rapidly cooled (4K/min.) reference curve with increasing wait time at 23 K. In terms of the droplet model and in agreement with the discussions above, the behavior may be interpreted as follows; a longer wait time at \( T = 23 \) K should create equilibrated regions on larger and larger length scales, length scales that should approach the typical domain size achieved during cooling at higher temperatures. At 25 K, which on our experimental time scales is out of the overlap region, these regions looks random, i.e. with increasing wait time at 23 K, the system should appear more and more reinitialized at \( T = 25 \) K, just as Fig. 9 shows.

The established way to show the possibility for a spin-glass to simultaneously carry several characteristic length scale, at one temperature, is by temperature cycling experiments. The relaxation rate result presented in Fig. 10 is obtained by first cooling the sample to \( T_1 = 31 \) K where it is subjected to a wait time of 1 000 s. The temperature is thereafter changed to \( T_2 = 29.7 \) K where it is kept for \( t_{w2} = 3 \) 000 s. When returning to \( T_1 \) again after this temperature cycling, diluted regions of size \( R(T_2, t_{w2}) \) have been reinitialized due to the domain wall movements at \( T_2 \). The effect of a third wait time \( t_{w3} = 10 \) s at \( T_1 \) is a system which initially equilibrates in these reinitialized regions up to length scales \( R(T_1, t_{w3}) \). By choosing the temperature step and the wait times carefully it is possible to fulfill the condition \( R(T_1, t_{w3}) < R(T_2, t_{w2}) < R(T_1, t_{w1}) \). The two maxima in the relaxation rate in Fig. 10 then correspond to the non-equilibrium response due to the remaining two relevant characteristic length scales at \( T_1 \), \( R(T_1, t_{w3}) \) and \( R(T_1, t_{w1}) \).

It is also possible to show that several length scales can coexist in the spin-glass at different temperatures with relaxation measurements. Indicated by crosses in Fig. 11a is the relaxation rate for the temperature 27 K after a wait time of 3 000 s. Crosses, the sample was immediately cooled to 27 K, open circles, the sample was intermittently kept at 31 K for 3 000 s during cooling. b) Relaxation rate \( S \) vs. \( \log t \), measured at 31 K after having heated the sample back from 27 K and immediately applying the dc-field (open squares). Open circles shows a corresponding curve where the sample was kept at 23 K for 3 000 s instead of at 27 K. Solid circles shows the relaxation rate after cooling the sample directly to 31 K and wait 3 000 s before applying the field. \( H_{dc} = 1 \) Oe, Ag(11at\% Mn).
resolved between these two procedures. A length scale, caused by the wait time at 31 K, is however hidden in the second case. In order to reveal this, the sample is heated back to 31 K and immediately probed in a magnetic field. The subsequent relaxation rate is indicated by squares in Fig 11b. Also shown by open circles in this figure, is the corresponding relaxation rate but with the second aging at 23 K instead of 27 K. As a reference, the relaxation rate probed directly after the initial wait time at 31 K is shown as filled circles. Basically, all three curves show an age of the system equal to 3 000 s only differing in the magnitude of the rate. This experiment, similar to the ac double memory experiment, shows that a spin-glass always recalls what has happened during cooling when it is re-heated to (or through) the temperature of an interrupted cooling process.

V. CONCLUSIONS

The non-equilibrium behavior of the dynamic susceptibility of a 3d spin-glass has been measured after specific thermal procedures which have been designed to expose consequences of a corresponding domain growth as predicted in the Fisher and Huse droplet model of spin-glass dynamics. A good qualitative agreement between these experimental results and the model predictions is found, which give support for the relevance of this real space model. However, there remain important questions to be answered. One crucial point within the model is to obtain an understanding of the difference between the initial fractal domain configuration after a quench and a corresponding snap shot of the equilibrated system with droplet excitations on all length scales. Is it a difference of the fractal dimension of the instantaneous domain walls in the two cases that signifies the difference? A prime remaining issue is also to theoretically establish if the concept of fractal domains on many length scales can be valid for a model Ising as well as for a real 3d spin-glass.

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