Multi-agent Consensus Problem Based on Observer and Considered Lur’e Nonlinearity in Directed Topology

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Abstract. This paper studies the multi-agent systems consensus problem considering Lur’e-type nonlinearity under directed network. Compared with the conclusions of the existing undirected communication topology, the conclusions obtained have greatly reduced the communication topology requirements and are more general. This paper designs a control protocol based on the observer, which solves the problem that the system state information is unknown. By decomposition of a specific form of the Laplacian matrix, consensus problem is converted into stability problem for low-dimensional systems. In this paper, the method of solving the control protocol is given and proved by the piecewise Lyapunov function. The simulation results show that the designed control protocol can solve the consistency problem.

1. Introduction

In recent years, the collaborative control of multi-agent systems has received extensive attention and research, especially in the field of group behavior such as swarming, assembly and formation control [1-4]. As a fundamental problem in the control of multi-agent systems, consistency-related research has achieved a lot of results. Literature [5] studies the consistency control problem of multi-agent systems with first-order dynamics model. It is concluded that the communication topology map needs to contain a spanning tree if the system is agreed upon. In the literature [6], the controller of the second-order time-delay multi-agent system in the directional topology is designed. By using a multi-leader framework, the condition of the formation controller of the system to achieve a fixed formation is obtained. Literature [7] studies multi-agent system with linear time-invariant dynamics. The literature [8] proposes a unified framework for dealing with the consistency of nonlinear agent networks that satisfy certain sector conditions. In literature [9], a class of nonlinear agent systems is standardized by feedback linearization method, then a consistency protocol is designed, and the performance index of the protocol is analyzed according to the properties of the lower triangular matrix.

Actual control systems often need to be modeled as nonlinear systems. Lur’e type nonlinearity is a typical nonlinear link and has a wide range of applications in many fields such as aircraft control, hydraulic servo control, and chaotic synchronization. The literature [10] studies the control problem of Lur’e type nonlinear link system under undirected topology and gives the global synchronization region. The literature [11-12] studies the robust consistency problem with leaders in an undirected topology.

In many practical applications, full state information cannot be used for controller design. Similarly, in a multi-agent system, a single agent often cannot effectively obtain relative state information of neighbor nodes, and even cannot obtain its own measurement output. In response to this situation, the
introduction of an observer is generally used to solve such problems [13-15]. In literature [16], a finite-time consistency control algorithm based on sliding mode observer is proposed for the multi-agent system with undetectable speed of the agent. The literature [17] designs a robust consistency control protocol for the nonlinear multi-agent system with unknown interference using the state estimation information of the observer.

This paper studies the multi-agent consistency control based on observer-based Lur'e-type nonlinearity under directional topology. The existing literature has relaxed the topological conditions of the undirected topology, making the application conditions of the research more general. This paper proposes a consistency controller based on the observer type. Different from literature [18-20] need to solve the algebraic Riccati equation (ARE) to solve the controller parameters. In this paper, the consistency control problem is transformed into several sets of linear matrix inequality (LMI) solving problems, which makes the problem solving more convenient and reduces the conservativeness of the results.

2. Problem Formulation

The symbol $\mathbb{R}^{m \times n}$ and $\mathbb{C}^{m \times n}$ respectively represent $n \times n$ dimensional real matrix and complex matrix. $\text{Re}(\mu)$ represent the real of $\mu \in \mathbb{C}$. $I_n$ $n \times n$ dimensional represent unit matrix. $A \otimes B$ represent the Kronecker of $A$ and $B$. $A > B$ and $A \geq B$ respectively represent $A - B$ is positive or semi-positive.

$\mathcal{G}=(\mathcal{V}, \mathcal{E}, \mathcal{A})$ represent the directed topology during the agents, where $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$ is node set of topology. $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is edge set of topology. $\mathcal{A} = [a_{ij}]_{N \times N}$ is adjacency matrix of topology and $a_{ij} \geq 0$. $a_{ij}$ is the connection weight of note $v_j$ and $v_i$, and it represent the note $v_i$ can receive information from $v_j$, otherwise $a_{ij} = 0$. If there is a note $v_i$, from this point, along the directed edge, it can reach any other point in the graph, we call the graph $\mathcal{G}$ contain a directed spanning tree, and $v_i$ is root node. When connection weight $a_{ij} = 1$, $N_i = \{j \in \mathcal{V} : a_{ij} = 1\}$ represent the neighbor of $v_i$.

The Laplacian of $\mathcal{G}$ is $\mathcal{L} = [L_{ij}]_{N \times N}$, where $L_{ii} = \sum_j a_{ij}L_{ij} = -a_{ii}, i \neq j$.

Lemma 1 [1] The Laplacian of $\mathcal{G}$ a least have one zero eigenvalue, other non-zero eigenvalues have positive real parts. If the directed topology $\mathcal{G}$ have a directed spanning tree, then 0 is simple eigenvalue of $\mathcal{L}$, and $1_\mathcal{V}$ is the feature vector of $\mathcal{L}$.

Lemma 2 [21] For any given $x, y \in \mathbb{R}^r$ and the matrix with compatible dimensions $P > 0, D$ and $S$. We can get that

$$2x^T DSy \leq x^T DPD^T x + y^T S^T P^{-1} S y$$

Lemma 3 [22] Shur complement lemma of matrix

Assume $S(P)$ is $n \times n$ dimensional, its block is expressed as

$$S(P) = \begin{bmatrix} S_{11}(P) & S_{12}(P) \\ S_{21}(P) & S_{22}(P) \end{bmatrix}$$ (1)

Where, $S_{11}(P)$ is $r \times r$ dimensional, if $S_{11}(P)$ is non-singular, then $S_{22}(P) - S_{21}(P)S_{11}^{-1}(P)S_{12}(P)$ is the Shur complement of $S_{11}(P)$ in $S(P)$. Then the following three conclusions are equivalent

1) $S(P) < 0$;
2) $S_{11}(P) < 0$, $S_{22}(P) - S_{21}(P)S_{11}^{-1}(P)S_{12}(P) < 0$;
3) $S_{22}(P) < 0$, $S_{11}(P) - S_{12}(P)S_{22}^{-1}(P)S_{21}(P) < 0$.

Lemma 4 [23] For the Laplacian $\mathcal{L} \in \mathbb{R}^{N \times N}$, there is a matrix $M \in \mathbb{R}^{N \times N}$, which let $\mathcal{L} = ME$, where $E \in \mathbb{R}^{(N-1) \times N}$.

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Further, if the graph contains a directed spanning tree, then $M$ is full rank and the eigenvalues of $EM$ are the eigenvalues of $\mathcal{L}$. So, $\text{Re}\{\lambda(EM)\} > 0$.

Lemma 5 [24] If the graph contains a directed spanning tree, then there is a symmetric positive definite matrix $\mathcal{Q}$ and a positive scalar parameter $\alpha$

Let $(EM)^T Q + QEM > \alpha Q$, where $0 < \alpha < 2 \min \{\text{Re}\{\lambda(EM)\}\}$

Consider the following $n$ multi-agent systems with nonlinear terms, the model of the $i$ agent is

\[
\begin{align*}
\dot{x}_i(t) &= Ax_i(t) + Bu_i(t) + Df(y_i(t)) \\
y_i(t) &= Cx_i(t)
\end{align*}
\]

where $x_i(t)$ is the status of agent $i$, the control input of agent $i$ is $u_i(t) \in \mathbb{R}^m$, the output of agent $i$ is $y_i(t)$. $A, B, C, D$ are system matrix, control matrix, and output matrix. Nonlinear function $f(y_i(t)) = (f(y_{i1}), ..., f(y_{iq}))^T$ need slope condition $[0, \Delta]$, which is

\[
0 \leq (f(b) - f(a)) \leq \delta_i(b - a), f_i(0) = 0, \forall b = a, i = 1, ..., q.
\]

For general slope conditions $[\Delta_i, \Delta_j]$, we can converted to a slope condition $[0, \Delta]$ by a reasonable loop transformation, so we only need consider slope condition $[0, \Delta]$.

In some practical systems, considering the cost, often do not directly get the full state of the agent, this paper designs the following observer for each agent to observe the system status.

\[
\begin{align*}
\dot{\xi}_i(t) &= A\xi_i(t) + Bu_i(t) + G(y_i(t) - \xi_i(t)) \\
\dot{\xi}_i(t) &= C\xi_i(t)
\end{align*}
\]

Where $\xi_i(t)$ and $\xi_i(t)$ respectively represent the status and output of the observer $i$. $G$ is feedback matrix to be sought.

Assumption 1 Linear multi-agent system, $(A, B)$ is controllable, and $(A, C)$ is considerable.

So, for the agent $i$, based on the above observer relative output information, the following control protocol is designed.

\[
u_i = K \sum_{j \in N_i} a_{ij}(\bar{\xi}_j(t) - \bar{\xi}_i(t)),\text{ where } K \text{ is feedback matrix to be sought.}
\]

Agent state and observer observation error is $h_i = x_i - \xi_i$. So

\[
\begin{align*}
\dot{h}_i(t) &= Ah_i(t) - G(y_i(t) - \xi_i(t)) + Df(y_i(t)) \\
\dot{h}_i(t) &= (A - GC)h_i(t) + Df(y_i(t))
\end{align*}
\]

Then the system can be described as

\[
\begin{align*}
\dot{x}(t) &= (I \otimes (A - \mathcal{B}K))x(t) + (L \otimes \mathcal{B}K)h(t) + (I \otimes D)f(y) \\
\dot{h}(t) &= (I \otimes (A - GC))h(t) + (I \otimes D)f(y)
\end{align*}
\]

Definition 1 The system can get consensus, when for arbitrary finite initial state $x_i(0)$, there is control protocol $u_i(t)$ let $\lim_{t \to \infty} \|x_i(t) - x_j(t)\| = 0, \forall i, j = 1, 2, ..., N$.

Consider system consistency issues, let $x_i = x_i - x_{i+1}$, $i = 1, 2, ..., N - 1$

So we can get $\dot{x}(t) = (E \otimes I)x = [\dot{x}_1, \dot{x}_2, ..., \dot{x}_{N-1}]^T$.
The same reason we can get $\hat{h}(t) = (E \otimes I)h = [\hat{h}_1, \hat{h}_2, \ldots, \hat{h}_{N-1}]^T$, where $\hat{h}_i = h_i - h_{i+1}$, $i = 1, 2, \ldots, N-1$.

So we can get

$$\dot{x} = (I_{N-1} \otimes A - EM \otimes BKC)\dot{x} + (EM \otimes BKC)\hat{h} + (I_{N-1} \otimes D)\phi(y)$$

(7)

And $\lim_{i \to \infty} \|\dot{x}_i(t)\| = 0, \forall i = 1, 2, \ldots, N-1$ $\iff$ $\lim_{i \to \infty} \|x_i(t) - x_j(t)\| = 0, \forall i, j = 1, 2, \ldots, N$

3. Theoretical results

The control protocol design process is as follows

a. Select scalar parameter $0 < \alpha < 2 \min \{\Re(\lambda(EM))\}$, use lemma 5, we can get $Q$

b. Select the appropriate nonlinear slope parameter $\sigma_0$, solve linear matrix inequality (6), we can get $P_1$

$$\begin{bmatrix} A^TP_1 + P_1A - \frac{\alpha}{2} P_1BB^TP_1 & P_1D & C^T \\ D^TP_1 & -I & 0 \\ C & 0 & -\frac{1}{\sigma_0^2}I \end{bmatrix} < 0$$

(8)

c. solve linear matrix inequality (7), we can get $P_2$

$$\begin{bmatrix} A^TP_2 + P_2A + \frac{\alpha}{2} P_2BB^TP_2 & P_2D & C^T \\ D^TP_2 & -I & 0 \\ C & 0 & -\frac{1}{\sigma_0^2 - 2}I \end{bmatrix} < 0$$

(9)

Prove: Consider the following piecewise continuous quadratic Lyapunov function

$$V(t) = V_1(t) + V_2(t)$$

$$V_1(t) = \dot{x}(t)^T(Q \otimes P_1)\dot{x}(t)$$

$$V_2(t) = \hat{h}(t)^T(Q \otimes P_1)\hat{h}(t)$$

(10)

let $K = B^TP_1C^T(CC^T)^{-1}$, use lemma 5, we can get

$$V_1(t) = \dot{x}(t)^T(Q \otimes (A^TP_1 + P_1A) - (EM)^TQ + QEM \otimes P_1BB^TP_1)\dot{x}(t) + 2\dot{x}(t)^T(Q \otimes P_1D)\phi(y)$$

(11)

$$\leq \dot{x}(t)^T(Q \otimes (A^TP_1 + P_1A - \alpha P_1BB^TP_1))\dot{x}(t) + \dot{x}(t)^T(Q \otimes \alpha P_1BB^TP_1)\hat{h}(t) + 2\dot{x}(t)^T(Q \otimes P_1D)\phi(y)$$

Let $\bar{x}(t) = (U^T \otimes I)\dot{x}(t)$, $\bar{h}(t) = (U^T \otimes I)\hat{h}(t)$ We can get

$$V_1(t) \leq \bar{x}(t)^T(\Omega \otimes (A^TP_1 + P_1A - \alpha P_1BB^TP_1))\bar{x}(t) + \bar{x}(t)^T(\Omega \otimes \alpha P_1BB^TP_1)\bar{h}(t) + 2\bar{x}(t)^T(\Omega \otimes P_1D)(U^T \otimes I)\phi(y)$$

(12)

Use lemma 2, we can get
\[
2\tilde{x}(t)^T (\Omega \otimes P_D)\left(\Omega^T \otimes I\right)\phi(y) \\
\leq \tilde{x}(t)^T (\Omega \otimes P_D)\left(\Omega^T \otimes I\right)\phi(y) \\
\leq \tilde{x}(t)^T (\Omega \otimes P_D)\left(\Omega^T \otimes I\right)\phi(y)
\]  

(13)

Let \( R = \Omega^{-1} \), so

\[
\dot{V}_1(t) \leq \tilde{x}(t)^T \left( \Omega \otimes \left( A^T P_2 + P_2 A - \alpha P DBB^T P_1 + P_2 DD^T P_1 + \sigma_2 C^T C \right) \right) \tilde{x}(t) \\
+ \tilde{x}(t)^T \left( \Omega \otimes \alpha P DBB^T P_1 \right) \tilde{h}(t)
\]  

(14)

The same reason, we can get

\[
\dot{V}_2(t) = \tilde{h}(t)^T \left( Q \otimes (A - GC) + (A - GC)^T P_2 \right) \tilde{h}(t) + 2\tilde{h}(t)^T (Q \otimes P_D) \phi(y)
\]  

(15)

Let \( G = P_2^{-1} C^T \), so

\[
\dot{V}_2(t) = \tilde{h}(t)^T \left( \Omega \otimes (A^T P_2 + P_2 A - 2C^T C) \right) \tilde{h}(t) + 2\tilde{h}(t)^T (\Omega \otimes P_D) \left( U^T \otimes I \right) \phi(y)
\]  

(16)

So,

\[
\dot{V}_2(t) \leq \tilde{h}(t)^T \left( \Omega \otimes \left( A^T P_2 + P_2 A - 2C^T C + \sigma_2 C^T C + P_2 DD^T P_2 \right) \right) \tilde{h}(t)
\]  

(17)

We can get

\[
V(t) = V_1(t) + V_2(t) \leq \tilde{x}(t)^T \left( \Omega \otimes \left( A^T P_1 + P_1 A - \alpha P DBB^T P_1 + P_1 DD^T P_1 + \sigma_2 C^T C \right) \right) \tilde{x}(t) \\
+ \tilde{x}(t)^T \left( \Omega \otimes \alpha P DBB^T P_1 \right) \tilde{h}(t) + \tilde{h}(t)^T \left( \Omega \otimes \left( A^T P_2 + P_2 A - 2C^T C + \sigma_2 C^T C + P_2 DD^T P_2 \right) \right) \tilde{h}(t)
\]  

(19)

Let

\[
\zeta(t) = \begin{bmatrix} \tilde{x}(t) \\ \tilde{h}(t) \end{bmatrix}, V(t) \leq \zeta(t)^T H \zeta(t)
\]  

(20)

Where

\[
H_1 = A^T P_1 + P_1 A - \alpha P DBB^T P_1 + P_1 DD^T P_1 + \sigma_2 C^T C
\]
\[
H_2 = A^T P_2 + P_2 A - 2C^T C + \sigma_2 C^T C + P_2 DD^T P_2
\]
\[
H_s = \frac{\alpha}{2} P DBB^T P_1
\]  

(21)

Solve linear matrix inequality \( \), we can get \( H_1 + H_2 < 0 \) and \( H_1 < 0 \)

Solve linear matrix inequality \( \), we can get \( H_2 + H_3 < 0 \) and \( H_s < 0 \).

Further we can get \( H_1 - H_1^{-1} H_1 H_2 < 0 \).

Use lemma 3, we can get \( H < 0 \), which is \( V(t) < 0 \). So, the system reach a consensus.

Note 1 Compare with literature [25], the method used in this paper is applicable to more common communication topologies, releasing the conditions of the undirected graph to the directed graph. By transforming, the Laplacian matrix of the system is similar to the approximate array, and it is impossible to perform coupling analysis on the multi-agent system with external disturbances. This paper uses a special variable substitution to successfully transform the consistency problem of multi-agent systems into the stability problem of dimensionality reduction systems. The method used in this paper has more intuitive physical meaning and a more concise derivation process.

Note 2 Compare with literature [11–12], the consensus problem of the leader without the leader in this paper. Compared with the non-navigate consistency problem, the consistency control problem of the leader can often define the tracking error of the follower and the leader. It's easier to turn it into a stability problem with an equivalent system. We can deal with mature M matrix theory.
Note 3 Inspired by the processing method of matrix M in the derivation process of the literature [26], it is necessary to introduce linear unitary transformation. In reference [24], the linear unitary transformation is not used in the derivation process. Although the correct conclusion can be obtained, the derivation process in this paper is more rigorous and reasonable.

4. Simulation results
This section demonstrates the validity of the conclusions obtained through a numerical experiment. Considering a multi-agent system consisting of a set of Chua's circuits, the Chua's circuit can be written in the form of a Lur'e type nonlinear system.

\[
\begin{bmatrix}
 x_1 \\
x_2 \\
x_3
\end{bmatrix} = \begin{bmatrix}
-a(m_2 + 1) & a & 0 \\
1 & -1 & 1 \\
0 & -b_0 & 0
\end{bmatrix} \begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix} + \begin{bmatrix}
0 \\
0 \\
d
\end{bmatrix}
\]

Nonlinear link \( f(y) = \frac{1}{2}(|x_i + 1| - |x_i - 1|) \), slope condition is [0, 1]. Select parameter \( a = 9, b_0 = 14, m_1 = \frac{3}{4}, m_2 = \frac{4}{3}, d = 10 \)

The directed topology is shown in Figure 1. Easy to verify, the topology contains a directed spanning tree. Let \( \phi = 1.9 \) Solve \( Q \) with the LMI toolbox. Further, the feedback matrix can be calculated.

\[
K = \begin{bmatrix}
-1.438 & -1.4750 & -0.1051
\end{bmatrix}
\]

\[
G = \begin{bmatrix}
1.4570 & 3.3999 & 26.1296
\end{bmatrix}^T
\]

\[
Q = \begin{bmatrix}
32.2857 & -0.2398 & 0.4793 \\
-0.2398 & 3.4716 & 3.2651 \\
0.4793 & 3.2651 & 3.3234
\end{bmatrix}
\]

Figure 1. Topology

The states of each agent in a multi-agent system are shown in Figure 2, Figure 3 and Figure 4 respectively. It can be seen from the graph that all three states of an agent can achieve consensus.

Figure 2. Agent status \( x_i, i = 1, 2, 3, 4 \)
5. Conclusions
In this paper, the consensus problem of multi-agent systems with Lur’e-type non-linear links is transformed into the stability problem of low-dimensional systems by decomposing and substituting the Laplacian matrix in a specific form under general communication topology. By constructing Laypunov function, this paper gives and proves the consistency control protocol of the system. The simulation results verify the effectiveness of the proposed method. Compared with the existing conclusions, this paper reduces the communication topology conditions of multi-agent systems, considering that a single agent can not effectively obtain the relative state information of neighbor nodes. It makes the conclusion of this paper more practical.

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