The Model Dimensionality and Its Impacts on the Strategic and Policy Outcomes in IAMs the Findings from the RICE2020 Model

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Abstract

This paper studies the impacts of regional breakdowns or model dimensionality on the model’s optimal solutions. Using the United States (USA) and China (CHN) as the experimental subject, we test the various solutions related to USA and CHN, such as the Cournot-Nash equilibrium and the Lindahl equilibrium, in the RICE2020 model under three regional breakdowns. Their solutions’ invariance and variances across different model dimensionalities indicate that modeling dimensionality may play a role in the strategic interactions among the regions in GHG mitigation. The simulation results also point out the pitfalls of the model comparisons across IAMs for climate change.

Keywords Model comparison · IAM for climate change · Modeling methodologies · The Cournot-Nash equilibrium · The Lindahl equilibrium

1 Introduction

Integrated assessment modeling (IAM) for climate change is a mainstream analytical tool in climate change economics. Since the early 1990s, various IAMs for climate change have emerged (Yang, Wei, & Mi, 2018). Key results of the past IPCC Assessment Reports on mitigation (Working Group III) and climate impacts (Working Group II) have primarily relied on the IAM efforts from various research groups (IPCC, 1990, 1995, 2001, 2007, 2016). IAM research groups have frequently conducted cross-model comparisons as a common infrastructural modeling practice in the past three decades. An early example is Energy Modeling Forum EMF-14, led by Weyant (1994). Some subsequent comparison studies are summarized by Smith et al. (2015). Lessmann et al. (2015) illustrate a narrowly targeted comparative study across a small group of IAMs. Krey et al. (2019) is a recent comprehensive comparison study that involves many
contributors. Such comparative works always reveal some unreconcilable differences across different IAM efforts.

Different IAMs are constructed on diverse methodological approaches; some are scenario simulation models, some are computable general equilibrium (CGE) models, and others are dynamic optimization models. The models’ economic mechanisms of calculating endogenous variables, including the crucial projecting paths, are fundamentally heterogeneous. In addition, these IAMs also have different base data, time spans, regional breakdowns, and other parametric assumptions. Cross-model comparative studies often render conflicting variations. As a result, we see many “spaghetti”-like trajectories of the models’ endogenous variables in the literature when the results from different IAMs are presented together. As the lead user of IAMs for climate change, IPCC adopts a constructive approach to engaging the heterogeneous IAMs in its agenda. It designs the “scenarios” of typical GHG emissions and socio-economic projects in the future, such as in IPCC’s Special Report on Emissions Scenarios (2000) and “representative concentration pathways (RCPs)” in its AR5 (IPCC, 2014) that allows IAMs to find the solutions for those scenarios with their respective endogenous model drivers and mechanisms. Again, solutions of SRES and RCPs from various IAMs are a bundle of “spaghetti.” Nevertheless, decision-makers can conduct meta-data analysis based on such solutions and identify the “mean” or “consensus” of the simulation results from different models. Despite significant and often unspecified differences across IAMs, the individual model contributes to the collective wisdom on climate change.

It is no surprise that different IAMs lead to varied results under the standard simulation environment, such as SRES and RCPs. Nevertheless, investigating the sources of the differences is an essential methodological research task. Comparing the models under unified assumptions is a necessary first step. We should notice that IAMs are different in modeling the economic interactions they captured and the economic mechanisms they identified. These structural assumptions are fundamental differences across models. Many intrinsic differences across models cannot be removed through the “harmonization” of scenarios. As IAM modelers, scholars realize these deeply rooted differences in modeling philosophy. Some common phrases such as “optimal solutions” and “efficient outcomes” might be “apples and oranges” across models. To meet a specific mitigation target, different models’ policy designs are generally different, and the mitigation costs are thus varied. During the Covid-19 pandemic in 2020, we heard a quote by Dr. Fauci, a well-known American epidemiologist, on epidemiological models of virus spread: “A model is only as good as the assumptions you put into the model.” (This Week’ Transcript 3–15-20: Dr. Anthony Fauci, Secretary Steven Mnuchin—ABC News). Data-driven epidemiological models are so; mechanism-powered economic models are even more so. Dr. Fauci’s reminder should be a caution in our inspection of IAMs. To trace likely sources of model differences, we need to isolate such sources or model assumptions one at a time while controlling the other sources in the investigation.

This paper investigates a common and fundamental determinant of model differences in IAMs through controlled experiments. This determinant is the model dimensionality or regional breakdown. The experimental results show that even innocuous assumptions such as the number of regions in the model could lead to
different results. Various solution concepts and endogenous scenarios of the model play a vital role in such differences. Not all solutions are equally useful, and some are more appealing than others. If a useful solution concept is sensitive to the basic assumptions, such as model dimensionality or regional breakdown, we should focus on the economics behind the solution rather than the numerical differences. A logical extension of the set of experiments in this paper is that IAMs are irreconcilably different, and cross-model comparisons are often futile. All comparable parameters should be unified and focus on the solutions with shared economic rationales if comparisons deem necessary. Such practice would make model comparisons more meaningful.

Specifically, we conduct the controlled experiments within a single model – the RICE2020 model (Nordhaus & Yang, 2021). We compare the same region(s) results in different solution concepts and dimensionalities by simulating the different solution concepts under different model dimensionality or regional breakdowns. In these experiments, we observe invariance and variances of some key results of these regions. Such results indicate that model comparisons across IAMs are probably unable to identify the real sources of the differences. The conformation of the solution concepts is a prerequisite for meaningful model comparisons. Frequently, the terms such as “optimal” and “efficient” solutions carry different meanings across models; some make more economic sense than others. Knowing the qualitative differences in the solution concepts is one of the purposes of model comparison. The experiment in this paper highlights this point.

The following section provides further rationale for the experiment, introduces the experiment platform, i.e., the RICE2020 model, and outlines the experiment designs. Sect. 3 presents the simulation results in a comparative setting. Sect. 4 contains the comparative analysis of the simulation results and the implications of the model comparisons in IAMs for climate change. Sect. 5 is the concluding remarks.

2 An Analytical Consideration

Before introducing the RICE model, we justify our experiment ideas through a simple “thought experiment” first. A hypothetical multiple-agents economy of size G (G could be GDP, population, etc.) generates externalities, such as pollution. Suppose this economy has nine agents. Agent 1 is of size 0.2G, and the other eight agents are of size 0.1G each. Agents interact with one another through external effects. Several solution concepts can be identified over interactions, such as the efficient Lindahl equilibrium, utilitarian social optimum, and the inefficient Cournot-Nash equilibrium. Instead, suppose this 0.2G Agent 1 interacts with four agents of size 0.2G each or interacts with two agents of size 0.4G each. In general, agent 1 takes a fixed share of the total economy (20% here), and the remaining (n-1) agents have 80% of the economy. An interesting question is: Are the outcomes for this 0.2G Agent 1 the same or different in these three scenarios with varying numbers of agents? The following is a simple model of externality provision reflecting the above thought experiment.

Suppose each agent in the above economy has a Cobb–Douglas utility function with a private good and an externality as arguments. The agent’s utility maximization problem can be expressed as:
Max $U_i(x_i, B) = x_i B$

s.t. $x_i + b_i = G_i$, $\sum_{i=1}^{n} b_i = B$, $\sum_{i=1}^{n} G_i = G$

Here, the price of the private good is normalized to 1, and $b_i$ is the contribution to environmental amenity (a positive externality) in value terms. Also, $G_1 = 0.2G$, and agents 2 to $n$ are identical.

The optimal solutions for agent 1 under different $n$ are as follows (the derivations are in the appendix):

(1) The utilitarian social optimum:

$$b_1^* = \frac{G_1}{2}, \quad b_j^* = \frac{G_j}{2}, \quad j = 2, ..., n.$$  

The social optimum under equal social welfare weights leads to each agent’s contribution to the environmental amenity at half of the agent’s endowment. If agent 1 is always 20% of the economy, their contribution to the environmental amenity is invariant with respect to the number of agents in this model.

(2) The non-cooperative Cournot-Nash equilibrium:

$$b_1^N = \frac{G_1}{2} - \frac{n-1}{2} b_2^N, \quad b_2^N = \frac{G_2}{n} - \frac{b_1^N}{n}$$

In the above expression, $b_1^N$ is the environmental amenity contributed by agent 1 in a non-cooperative Cournot-Nash game and $b_2^N$ is that of agent 2’s (and agent 3 to agent $n$, as they are identical). We purposefully express agent 1’s contribution as the difference between utilitarian social optimum and “free-riding” reaction to other agents’ contributions if $n$ increases $b_2^N$ decreases. Nevertheless, the reaction of agent 1 to $b_2^N$ has a factor of $(n-1)$. Therefore, $b_1^N$ is not invariant with respect to the number of agents in the economy, but the reaction to the different numbers of agents is of second-order magnitude.

(1) The Lindahl equilibrium:

The analytical solution of the Lindahl equilibrium is given by the optimal solution of the social optimum under the Lindahl weights (Yang, 2020). The weights are given by:

$$\varphi_n^L = \frac{N}{\sum_{j=1}^{N} \left( \frac{1}{U^*(j) - U(j)} \right) \left( \frac{1}{U^*(n) - U(n)} \right)}, \quad \sum_{n=1}^{N} \varphi_n^L = N.$$  

The Lindahl weights are the harmonic mean of the Nash bargaining solution of the model ($U^*(n)$) with the payoffs of the Cournot-Nash equilibrium ($U(n)$) as the status
quanto point. Because the Cournot-Nash equilibrium is not invariant in (2), the Nash bargaining solution of the model is also variant with respect to the number of agents. Consequently, the Lindahl weight of agent 1 changes with respect to the number of agents. As a result, the Lindahl equilibrium, the most crucial solution concept in environmental externality provision, is variant for agent 1, even if they take a fixed share of the economy.

It should be cautious about extending the conclusions from a stylized and textbook-like model in general. The above simple results rely on the identical-agent assumption of \((n-1)\) agents. Climate change is a complicated externality phenomenon. IAMs are highly nonlinear and dynamic. Furthermore, regions are highly heterogeneous. The numerical simulations are proper methodologies to study the dimensionality issue. The subsequent experiments in the RICE model are to fulfill the above thought experiment ideas in empirical contexts.

3 The RICE2020 Model and Experiment Design

The RICE model originated in the mid-1990s (Nordhaus & Yang, 1996). It has been an influential IAM and a part of Nordhaus’ 2018 Nobel Prize contribution (Nobel committee, 2018). Over the past two decades, the RICE model has experienced several updates and has been used by many scholars. The model is undergoing a new round of revision and updating by Nordhaus and Yang (2021). We label it the “RICE2020 model” here. Identical to its predecessors, the RICE2020 model is a multi-region optimal growth model that treats climate change as a negative externality that impacts every region. The model has incorporated several useful solution concepts; some are new in this round of updates. The programming platform of the RICE2020 model is the GAMS language.

One of the new features of the RICE2020 model is its flexibility in regional breakdowns. This feature allows us to conduct the comparative study here conveniently. The full-scale RICE2020 model has 16 regions. We also obtain a 6-region version (the same regional breakdown as in the original RICE model by Nordhaus and Yang (1996)) and a 12-region version (compatible with the RICE2000 model by Nordhaus and Boyer (2000)) of the RICE2020 model. This study calls them RICE2020-R06, RICE2020-R12, and RICE2020-R16, respectively. The mathematical expressions of the RICE2020 model and the names of specific regional breakdowns are provided in the Appendix.

In the RICE2020-R06, R12, and R16 models, the United States (USA) and China (CHN) appear as stand-alone entities while other regions or countries have different demarcations. The base period data and parametric assumptions of USA and CHN are identical in R06, R12, and R16. Therefore, these two countries are invariant entities across three regional breakdowns of the RICE2020 model. In contrast, the base period data of other regions may be different due to the inclusion of varied countries in each version. Still, all regions are added to the same world aggregate in the base period. The parametric assumptions related to exogenous growth trends are calibrated.

1 The derivation of the Lindahl weights is complicated. Please refer Yang (2020) for details.
based on the information from the literature. They are heterogeneous across different regional breakdowns. Still, the main results of aggregate variables, such as global GHG emissions in the “Business as Usual” (BaU) scenario (to be explained shortly), are numerically identical. In sum, the R06, R12, and R16 are identical except for regional breakdowns. We use USA and CHN, the invariant entities across the three versions, to test the effects of regional breakdowns on the experiment’s simulation results.

The RICE2020 model is a climate externality model. Climate change is a long-lasting externality phenomenon faced by all regions. There are many interesting solution concepts for this type of economic-environment interaction, such as the non-cooperative Cournot-Nash equilibrium and the cooperative Lindahl equilibrium (Yang, 2020). These solutions can help answer some pressing policy issues in climate change. Specifically, in the RICE2020 model, we can solve many cooperative (efficient) solutions and the non-cooperative (inefficient) Cournot-Nash equilibrium. The RICE model also provides many optimal solutions under exogenous policy constraints in addition to the above two categories.

This study examines the solution trajectories of USA and CHN in RICE2020-R06, R12, and R16 under several solution concepts. More specifically, we execute and solve the following scenarios for RICE2020-R06, RICE2020-R12, and RICE2020-R16:

1. The BaU solution by setting the control variable “GHG emission control rate”, i.e., \( \mu(n, t) = 0 \);
2. The Non-cooperative Cournot-Nash equilibrium;
3. The social optimum under the utilitarian social welfare weights (equal weights);
4. The social optimum under the Lindahl weights;
5. The social optimum with a binding upper bound of temperature increase of 6 °C (under the utilitarian and the Lindahl weights).

Scenarios (1) to (4) represent significant benchmarks of the RICE model. Scenario (1) (BaU) here is for calibration purposes, and it is a standard solution in IAMs. Scenario (2) is the non-cooperative Cournot-Nash quantity game of emission control solution. It is the most significant outcome of strategic interactions among the regions. In the RICE model, the game is defined in Part II of the Appendix. Scenario (3) is the solution of the RICE model (Part I in the Appendix) under the equal social welfare weights \( \phi_n = 1 \). It is also a standard solution sought in other IAMs. Scenario (4) is the Lindahl equilibrium outcome. In the RICE model, the Lindahl equilibrium is the social optimum (Part I in the Appendix) under the Lindahl social welfare weights. The Lindahl equilibrium is the essential efficient solution concept in an economy with externality. As we mentioned, the RICE model treats climate change as an externality phenomenon—the RICE model pioneers the algorithms to identify the Cournot-Nash equilibrium and the Lindahl equilibrium in IAMs. Lastly, scenario (5) is an example of the wide-ranging “second-best” policy scenarios. Many policy simulations in IAMs are the solutions under various global restrictions imposed through policies. From a modeling perspective, they are optimization problems under additional boundary constraints. Therefore, scenario (5) tests the effects of model dimensionality under a boundary constraint.
The algorithm used in (2) was developed by Nordhaus and Yang (1996); the algorithms for identifying the Lindahl weights in (4) are given in Yang (2020). Due to their complexities, we do not describe them in detail here. The algorithmic programming codes in the GAMS language are available upon request. In all, we have 3 (R06, R12, R16) \(\times\) 6 (scenarios) = 18 sets of solution ((5) have two sets of runs). The initial period of the model is 2015, and the model’s time horizon in all versions is \(T = 50\) in 5 years a step. We truncate the results at \(t = 39\) or year 2200 for a concise and clear presentation of the results.

4 The Simulation Results

Scenario (1) shows the consistency of benchmark calibration across three versions. Despite different regional breakdowns, the aggregate (global) outcomes without regional mitigation efforts (\(\mu (n, t) = 0\)) should roughly be the same across R06, R12, and R16. Figure 1 shows the aggregate (global) GHG emissions in three versions. The aggregate (global) GDP shows the same pattern (reducing redundancy, not presented). We see that the trajectories coincide with one another. The numerical discrepancies appear in the third decimal point in the original outputs. Such numerical discrepancies are due to the calibration inaccuracies in a complicated nonlinear dynamic model. Therefore, R06, R12, and R16 are identical except for the minuscule numerical differences in BaU (scenarios (1)). The RICE2020-R06, R12, and R16 models fit the comparative studies set in this study.

The critical decision (control) variable in the RICE model is \(\mu (n, t) (0 \leq \mu (n, t) \leq 1)\), the regional paths of GHG emission control rate. The control variables’ differences lead to state variables’ variations, such as GHG emission levels and global temperature increases. Therefore, we only plot the trajectories of optimal emission control rates \(\mu\).
(“USA,” t) and $\mu$ (“CHN,” t) for scenarios (2) to (5). These results are in Figs. 2, 3, 4, 5 and 6. To reduce redundancy, we plot six trajectories of $\mu$ (“USA,” t) and $\mu$ (“CHN,” t) in R06, R12, and R16 for each scenario in one figure. We should point out that the trajectories of both “USA” and “CHN” in Figs. 2 and 3 are very close in R06, R12, and R16; they “merge” into a single line. To illustrate the numerical values of the outputs, the emission control rates of USA and CHN in the Cournot-Nash equilibrium, or a portion of Fig. 2, are presented in Table 1.

![Fig. 2 Emission control rates in Cournot-Nash equilibriums](image1)

![Fig. 3 Emission control rates in utilitarian social optimums](image2)
To show the scenarios’ impacts on global emissions and other regions’ optimal choices, we plot global GHG emissions in the Cournot-Nash equilibrium in Fig. 7 and those in the Lindahl equilibrium in Fig. 8. Again, for comparison purposes, we put the trajectories of R06, R12, and R16 in one graph. The global GHG emissions in the utilitarian social optimum are similar to Fig. 8. To reduce redundancy, the figure is not presented. All scenarios’ complete output files are available, but most are irrelevant to the dimensionality study in this paper.

Lastly, we highlight that the simulation results are numerically consistent with the conclusions drawn from the analytical consideration in Sect. 2. Namely, the outcomes
**Fig. 6** Emission control rates in Lindahl equilibrium (6 °C boundary)

**Table 1** GHG control rates in Cournot-Nash equilibrium

| Year | USA  |     |     | CHN  |     |     |
|------|------|-----|-----|------|-----|-----|
|      | R06  | R12 | R16 | R06  | R12 | R16 |
| 2020 | 0.035452 | 0.035462 | 0.035458 | 0.040346 | 0.040342 | 0.040342 |
| 2025 | 0.035908 | 0.035925 | 0.035922 | 0.041325 | 0.041317 | 0.041314 |
| 2030 | 0.036251 | 0.036257 | 0.036265 | 0.042467 | 0.042462 | 0.042455 |
| 2035 | 0.036536 | 0.036522 | 0.036532 | 0.043703 | 0.043689 | 0.043677 |
| 2040 | 0.036787 | 0.036766 | 0.036756 | 0.044974 | 0.044952 | 0.044943 |
| 2045 | 0.037013 | 0.036994 | 0.036953 | 0.046272 | 0.046234 | 0.046225 |
| 2050 | 0.037215 | 0.037196 | 0.037131 | 0.047547 | 0.047518 | 0.047499 |
| 2055 | 0.037391 | 0.037352 | 0.037294 | 0.048819 | 0.048784 | 0.048754 |
| 2060 | 0.037541 | 0.037481 | 0.037444 | 0.050074 | 0.050024 | 0.049996 |
| 2065 | 0.037661 | 0.037593 | 0.037565 | 0.051293 | 0.051236 | 0.051213 |
| 2070 | 0.037754 | 0.037770 | 0.037665 | 0.052491 | 0.052424 | 0.052379 |
| 2075 | 0.037823 | 0.037803 | 0.037739 | 0.053652 | 0.053578 | 0.053528 |
| 2080 | 0.037872 | 0.037893 | 0.037789 | 0.054768 | 0.054693 | 0.054649 |
| 2085 | 0.037907 | 0.037938 | 0.037816 | 0.055856 | 0.05575 | 0.055683 |
| 2090 | 0.037924 | 0.03797 | 0.037821 | 0.056876 | 0.056756 | 0.05673 |
| 2095 | 0.03793 | 0.037945 | 0.037813 | 0.057848 | 0.057718 | 0.057648 |
| 2100 | 0.037904 | 0.037884 | 0.037782 | 0.058781 | 0.058634 | 0.058578 |
related to the fixed agents (USA and CHN here) do not change in the utilitarian social optimum. Still, they vary in the Lindahl equilibrium and the Cournot-Nash equilibrium. The comparative analysis in the next section focuses on climate mitigation strategies, as they are the purpose of IAMs.

5 A Comparative Analysis

Three trajectories of global GHG emission in Fig. 1 overlap into one. The pattern shows that R06, R12, and R16 are the same model except for different regional breakdowns.
Therefore, the RICE2020 model sets up a good platform for the experiment. USA and CHN are invariant entities across all three regional breakdowns in this platform. Under such controlled conditions, the sole reason for their optimal emission control rates’ differences or stationarity is the regional breakdowns or model’s dimensionality. The patterns in Figs. 2, 3, 4, 5 and 6 show some predictable and surprising results.

First, the global GHG emissions in the Cournot-Nash equilibrium are significantly different across R06, R12, and R16 in Fig. 7. The emissions in R06 are lower than in R12 and R16. The emissions in R12 are lower than in R16.2 The graph shows that the more regions in the model, the higher the aggregate GHG emissions. This result is consistent with economic theory related to externality provision in a non-cooperative setting: more agents lead to more inefficiency due to more substantial “free-riding” incentives. Nevertheless, the emission control rates of USA and CHN in the Cournot-Nash equilibrium are stable across R06, R12, and R16, as shown in Fig. 2. The actual emission control rates vary insignificantly, taking place in the 4th or 5th decimal point. The emission control rates have the order R06 > R12 > R16 marginally (see Table 1). Again, the pattern is consistent with economic theory: more agents lead to less mitigation intensity due to the “free-riding” tendency. Because the changes across R06, R12, and R16 are so insignificant, we can conclude that a region’s responses to the different number of players in a Cournot-Nash game of strategic GHG emissions are invariant when the total number of players is small (single-digit to a couple of dozens). In a non-cooperative game of GHG mitigations, the number of players (the model dimensionality) plays a role in the equilibrium outcome. However, the same player reacts similarly when interacting with the different numbers of players, as long as the range of players’ numbers is not too wide.

Both the utilitarian social optimum and the Lindahl equilibrium fully internalize the climate externality and thus are efficient. These scenarios are the two most important benchmarks for efficient outcomes. Figures 3 and 4 show that the implications of dimensionality on USA and CHN are different in these two scenarios.

The utilitarian social optimums of R06, R12, and R16 are unsurprisingly uniform. The global GHG emissions are numerically identical in three regional breakdowns (the figure is not shown, and the pattern is like in Fig. 8). Both USA and CHN’s optimal emission control rates are numerically identical across R06, R12, and R16, as shown in Fig. 3. In the objective function of the RICE model (A-1), each region’s payoff function is multiplied by its population. Therefore, the “shares” of USA and CHN in the utilitarian social welfare function are the same across different regional breakdowns, regardless of how many regions come up to the same global aggregate. Based on the simulations in this scenario, we conclude that the same entity across different IAMs should be comparable under the utilitarian assumption. If a clearly defined entity, say the USA, performs differently across IAMs under the utilitarian assumption, the divergence in models’ mechanisms and structures causes such difference, and the model dimensionality does not. Many IAMs in the literature do not treat climate change as an externality phenomenon, and their modeling methodologies are fundamentally heterogeneous. Even under the utilitarian assumption, cross-model comparisons are

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2 The trajectories of R12 and R16 seem to coincide with each other. However, the numerical values of R16 are consistently higher than R12 at the first decimal point.
often futile, and the sources of differences are tough to trace without going into the models’ structural details.

Climate change is an environmental externality, and the IAMs for climate change should treat it as so, just like the RICE model. When climate change is modeled as an externality, the utilitarian social optimum is a flawed solution concept (Yang, 2020). One of its deficiencies is that some regions are worse off in the utilitarian social optimum than their Cournot-Nash equilibrium in the RICE model (and other IAMs). The Lindahl equilibrium without transfers is the most crucial solution concept in the IAMs that treat climate change as an externality phenomenon. The Lindahl equilibrium is the solution of the weighted social optimum in the RICE model. The endogenous Lindahl social welfare weights are calculated through a series of algorithmic procedures, and they are built in the RICE2020 programming codes. The Lindahl equilibrium reflects the “willingness to contribute” to GHG migration by all regions collectively. The Lindahl equilibrium is the most desirable efficient outcome in which every region is better off than its respective Cournot-Nash equilibrium payoffs. 3

As Fig. 8 shows, the global GHG emissions’ trajectories in the Lindahl equilibrium are numerically identical across R06, R12, and R16. The outcomes show that the Lindahl equilibrium’s aggregate GHG emission trajectories (the flows of climate externality) are stable with respect to model dimensionality. Such invariance across the model dimensionality is a characteristic of the Lindahl equilibrium revealed by our experiment.

On the other hand, the endogenous Lindahl social welfare weights vary across different regional breakdowns. In R06, the weights of USA and CHN are 1.457791827182 and 0.571238011070, respectively (the weights of all regions add to 6); in R12, they are 1.469395011608 and 0.614877933229, respectively (the weights of all regions add to 12); in R16, they are 1.272584470918 and 0.549274969109, respectively (the weights of all regions add to 16). Due to different social welfare weights, the individual GHG emission control rates are different in R06, R12, and R16. In Fig. 5, both USA and CHN’s optimal emission control rates are significantly different across R06, R12, and R16. Both have the monotonic pattern of R06 < R12 < R16. The pattern indicates that when more regions form the global economy with climate externality, the invariant entities, such as USA and CHN, are more proactive in achieving the social optimum or efficiency given by the Lindahl equilibrium. When more regions are involved, global cooperation becomes more difficult. In such situations, the invariant and substantial entities (USA and CHN are the top two GHG emitters worldwide) are willing to do more to hold the global collaboration under the Lindahl principle. In this most crucial scenario, the regional breakdowns or model’s dimensionality makes a difference.

Scenario (5) enforces a binding global upper bound (the temperature increase ≤ 6 °C) in the RICE2020 model. Under the utilitarian weights and the Lindahl weights in scenario (4), the optimal emission control rates of USA and CHN demonstrate interesting patterns, as shown in Figs. 5 and 6. Under the utilitarian weights, USA and CHN move close to the upper bound of emission control rate in late periods. Their trajectories in R06, R12, and R16 are almost identical (see Fig. 5). In contrast, USA

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3 Yang (2020) contains the analytical properties of the Lindahl equilibrium without transfers in the context of environmental externalities.
and CHN’s optimal emission control rates are less stringent under the Lindahl weights. Implicitly, other regions must take more responsibilities to meet the binding global constraint.

Furthermore, the optimal trajectories are significantly different in R06, R12, and R16 in the bounded constraint Lindahl equilibrium (see Fig. 6). Like their unconstrained counterpart in scenario (4), the optimal control rates have the pattern of R06 < R12 < R16. In response to a binding global constraint, the significant entities, such as USA and CHN, are more proactive when more regions are involved in the global cooperation under the Lindahl equilibrium. Again, the regional breakdowns or the model’s dimensionality plays a role in the Lindahl equilibrium solution of an IAM that treats climate change as an externality.

Through the tightly controlled simulations in scenarios (1) to (5) in the RICE2020 model, we identified what may change and what stays stable in the optimal solutions under the different regional breakdowns. The results are either non-surprising or unexpected; they are stable or varied with the model’s dimensionality. The regional breakdowns affect the aggregate GHG emissions in the non-cooperative Cournot-Nash equilibrium. The more regions are involved, the higher the aggregate emission level because market failure is more severe. The outcome is consistent with our understanding of the Cournot-Nash equilibrium when externalities are present. However, the relatively stable emissions control rates of USA and CHN across R06, R12, and R16 are somewhat surprising. Conventional wisdom suggests they may contribute less to GHG mitigation when more regions interact strategically. But more and smaller regions (from R06 to R16) contribute less to the global GHG mitigation. The results of the utilitarian social optimum, in contrast, are uneventful. Lastly, the optimal emission control rates of USA and CHN in the Lindahl equilibrium shift across R06, R12, and R16. This phenomenon is largely unknown in the literature. We conclude that the pattern is consistent with the strategical cooperation to internalize climate externality through the analysis in this section.

The exercise here also has significant implications for the model comparisons among IAMs for climate change. If the model mechanisms and solution concepts are different, the comparisons are “apple and orange,” and even the models’ key parameters are unified. As we see here, an innocuous assumption of model dimensionality can cause variations in the optimal solutions. Many other unknown factors in IAMs may lead to diverse performances. The more productive approach is to open the respective models, from mathematical structure to modeling codes. We learn from one another through transparent model structures. Comparisons of “black boxes” leave inquiries in the dark.

6 Concluding Remarks

Integrated assessment models (IAMs) for climate change have a common goal: to analyze the various aspects of climate change from socio-economic perspectives. Diverse IAM efforts adopt varied methodological approaches, and the colorful acronyms of IAMs around the world convey different purposes of the models. Despite the structural differences of models, modelers compare and double-check the IAM outcomes
The comparative studies contribute collectively to our understanding of climate change. Nevertheless, the comparisons could not trace the sources of the differences across IAMs generally.

This paper conducted a tightly controlled experiment of model comparison: the RICE model compares itself under different regional breakdowns. We have found both expected and surprising results: the utilitarian social optimums are invariant across regional breakdowns as expected. It is a surprise that the regional strategic responses in a noncooperative Cournot-Nash game of GHG mitigation only shift marginally across regional breakdowns, and the regional contributions to the Lindahl equilibrium vary considerably. A seemingly innocuous assumption of regional breakdowns or model dimensionality may cause significant repercussions.

However, we must emphasize that IAMs fundamentally differ in modeling philosophies and solution concepts. The model dimensionality is merely one of many possible differences across IAMs. Here we provide a reliable conclusion on its role in the difference or stationarity of model results. Model comparisons across IAMs are helpful in modeling and policy assessment. The models’ transparency and clearly defined solution concepts are pre-conditions for a meaningful comparison. Ceteris Paribus is a principle to be followed verbatim in comparative studies of IAMs. We hope that the exercise here provides a valuable example.

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**Declarations**

**Conflict of interest** The authors have not disclosed any competing interests.

**Appendix**

The mathematical description of the RICE2020 model in discrete form.

1. The RICE2020 model as a social planner’s problem (efficient solutions fully internalize climate externality):

   1. The objective function of the social planner that is subject to the constraints (2) to (17):

      \[
      \text{Max}_{\{I(n,t), \mu(n,t)\}} \quad W = \sum_{n=1}^{N} \sum_{t=0}^{T} \varphi_{n} L(n, t) U(c(n, t))(1 + \rho)^{-t}, \quad \sum_{n=1}^{N} \varphi_{n} = N \quad (1)
      \]

   **The economic module (regional Solow growth model with embedded economy-climate interaction):**

   Definition of utility function:
\[
U(c(n, t)) = \begin{cases} 
\frac{c(n, t)^{1-\alpha}}{1-\alpha}, & 0 \leq \alpha < 1 \\
\log(c(n, t)), & \alpha = 1 
\end{cases}
\]

(2)

Definition of per capita consumption:

\[
c(n, t) = \frac{C(n, t)}{L(n, t)}
\]

(3)

Aggregate production function without climate damage and mitigation cost:

\[
Q(n, t) = A(n, t) L(n, t)^\gamma K(n, t)^{1-\gamma}
\]

(4)

Aggregate production function including climate damage and mitigation costs:

\[
Y(n, t) = \Omega(n, t) Q(n, t)
\]

(5)

Net GDP identity:

\[
Y(n, t) = C(n, t) + I(n, t)
\]

(6)

The motion equation of Capital formation:

\[
K(n, t + 1) = I(n, t) + (1 - \delta_K) K(n, t), \quad K(n, 0) = K_{n,0} > 0, \quad 0 < \delta_K < 1
\]

(7)

**Carbon emission and economy-climate linkage module:**

Industrial carbon emission equation:

\[
E_{Ind}(n, t) = \sigma(n, t)(1 - \mu(n, t)) Q(n, t), \quad \mu(n, t) \geq 0.
\]

(8)

Total regional carbon emission:

\[
E(n, t) = E_{Ind}(n, t) + E_{Land}(n, t)
\]

(9)

Global carbon stock constraint:

\[
\sum_{t=0}^{T} \sum_{n=1}^{N} E_{Ind}(n, t) \leq CCum
\]

(10)

Climate to economy adjustment function:

\[
\Omega(n, t) = \frac{1 - MC(\mu(n, t))}{1 + CD(n, TAT(t))}, \quad 0 \ll \Omega(n, t) < 1
\]

(11)
Carbon cycle module:

\[
M_{AT}(t + 1) = \sum_{n=1}^{N} E(n, t) + \phi_{11} M_{AT}(t) + \phi_{21} M_{UP}(t) 
\]

\[
M_{UP}(t + 1) = \phi_{12} M_{AT}(t) + \phi_{22} M_{UP}(t) + \phi_{32} M_{LO}(t) 
\]

\[
M_{LO}(t + 1) = \phi_{23} M_{UP}(t) + \phi_{33} M_{LO}(t) 
\]

\[
F(t) = \eta\left \{ \log_2[M_{AT}(t)/M_{AT}(1750)] \right \} + F_{EX}(t) 
\]

\[
T_{AT}(t + 1) = T_{AT}(t) + \xi_1[F(t) - \xi_2 T_{AT}(t) - \xi_3[T_{AT}(t) - T_{LO}(t)]] 
\]

\[
T_{LO}(t + 1) = T_{LO}(t) + \xi_4[T_{AT}(t) - T_{LO}(t)] 
\]

II. The RICE2020 model as a non-cooperative Cournot-Nash game (inefficient and unique Cournot-Nash equilibrium):

\[
Max_{\{I(n,t), \mu(n,t)\}} \quad V^n = \sum_{t=0}^{T} U(c(n, t))(1 + \rho)^{-t}, \quad n = 1, 2, ..., N. 
\]

s.t. constraints (A-2) to (A-17) except for (A-12) is revised as:

\[
M_{AT}(t + 1) = E(n, t) + \sum_{j \neq n} E(j, t) + \phi_{11} M_{AT}(t) + \phi_{21} M_{UP}(t) 
\]

III. The RICE2020 model’s Nash bargaining problem and the determination of the Lindahl social welfare weights:

\[
W(n) = \sum_{t=0}^{T} L(n, t)U(c(n, t))(1 + \rho)^{-t}, 
\]

\[
Max_{\{I(n,t), \mu(n,t)\}} \quad V = \sum_{n=1}^{N} \log \left( W(n) - V^n \right) 
\]

s.t. (A-2) to (A-17).

\( \overline{V^n} \) is the payoff of problem II.
Calculation of the Lindahl weights:

\[
\varphi^L_n = \left(\sum_{j=1}^{N} \frac{1}{W^*(j) - V^*} \right) \left(\frac{1}{W^*(n) - V^*}\right), \quad \sum_{n=1}^{N} \varphi^L_n = N. \tag{21}
\]

\(W^*(n)\) is the regional payoff of problem III.

The Definition of notations and symbols in I and II:

Definitions of sets:

- \(n\) – regions; \(t\) – time (in discrete steps, 5-year a period).

Definition of the objective functions:

- \(W, V^n, V\) – in I, II, III, respectively.

Definitions of 7 control variables and state variables:

(a) Control variables (underneath “Max” in (A-1) and (A-18)):

- \(I(n, t)\) – investment of regions; (trillions of 2015 U.S. dollars).
- \(\mu(n, t)\) – carbon control rates of regions. (in percentage, maybe greater than 1 if carbon sequestration technology is allowed).

(b) State variables:

- \(U(c(n, t))\) – instantaneous payoff functions of regions; (in scalable and nontransferable “utils”).
- \(C(n, t)\) – consumptions of goods and services of regions; (trillions of 2015 U.S. dollars).
- \(c(n, t)\) – per capita consumptions; (thousands of U.S. dollars per person).
- \(K(n, t)\) – capital stocks of regions; (trillions of 2015 U.S. dollars).
- \(Q(n, t)\) – output of goods and services of regions, gross of abatement and damages; (trillions of 2015 U.S. dollars).
- \(Y(n, t)\) – output of goods and services of regions, net of abatement and damages; (trillions of 2015 U.S. dollars).
- \(E_{\text{Ind}}(n, t)\) – industrial carbon emissions by regions; (billions of metric tons C per period).
- \(E(n, t)\) – total carbon emissions by regions; (billions of metric tons C per period).
- \(MC(\mu(n, t))\) – mitigation cost of regions; (trillions of 2015 U.S. dollars).
- \(CD(n, T_{AT}(t))\) – climate damage of regions; (trillions of 2015 U.S. dollars).
- \(\Omega(n, t)\) – gross-net output converting factor.
- \(M_{AT}(t), M_{UP}(t), M_{LO}(t)\) – carbon concentration in atmosphere, upper oceans and deep oceans respectively; (billions of metric tons C at the beginning of each period).
- \(T_{AT}(t), T_{LO}(t)\) – global mean surface temperature and temperature of upper oceans (°C from 1900).
- \(F(t)\) – total radiative forcing; (watts per m² from 1900).

Definitions of exogenous trends:

- \(A(n, t)\) – total factor productivity of regions; (productivity units).
- \(L(n, t)\) – population and proportional to labor inputs of regions; (billions).
- \(E_{\text{Land}}(n, t)\) – carbon emissions from land use; (billions of metric tons C per period).
\[ \sigma(t) - \text{ratio of uncontrolled industrial emissions to output; (a proxy for AEEI).} \]
\[ F_{EX}(t) - \text{exogenous radiative forcing; (watts per m}^2 \text{ from 1900).} \]

Definitions of parameters:
- \( \phi_n \) – social welfare weights;
- \( \alpha \) – elasticity of marginal utility of consumption; (pure number).
- \( \rho \) – pure rate of social time preference (per year).
- \( CCum \) – maximum consumption of fossil fuels (billions metric tons carbon).
- \( \gamma \) – elasticity of output with respect to capita (pure number).
- \( \delta_K \) – rate of depreciation of capital (per period).
- \( \eta \) – temperature-forcing parameter \(^°C\) per watts per meter squared.
- \( \phi_{11}, \phi_{21}, \phi_{32}, \phi_{12}, \phi_{33}, \phi_{23} \) – parameters of the carbon cycle (flows per period).
- \( \xi_1, \xi_2, \xi_3, \xi_4 \) – parameters of climate equations (flows per period).
- \( \psi_1, \psi_2 \) = parameters of damage function.

The regional breakdowns of the RICE2020-R06, R12, and R16 models:
- R06: USA, JPN, EEC, CHN, FU, ROW.
- R12: USA, EUS, JPN, OHI, RUS, EEC, CHN, IND, MDE, SSA, ROW.
- R16: USA, CAN, EUS, JPN, BRA, CHN, IND, RSA, RUS, CIS, LA, MDO, SEA, SSA, ROW, USP.

Derivations of the analytical results in Sect. 2:

If agent 1 takes a fixed share of the economy \( (G_1 = 20\% \text{ share here}) \) and the remaining \((n-1)\) identical agents take the remaining share \((80\% \text{ here})\), the utilitarian social optimum is:

\[
\text{Max } W = \sum_{i=1}^{n} U_i(x_i, B) = \sum_{i=1}^{n} x_i B = \left[ \sum_{i=1}^{n} (G_i - b_i) \right] \sum_{i=1}^{n} (b_i) \\
= [G_1 - b_1 + (n - 1)(G_2 - b_2)] (b_1 + (n - 1)b_2)
\]

The first-order conditions (FOCs) are:

\[
\frac{\partial W}{\partial b_1} = -2b_1 + G_1 + (n - 1)(G_2 - 2b_2) = 0, \quad \frac{\partial W}{\partial b_2} = -2b_2 + G_2 + \frac{G_1 - 2b_1}{n - 1} = 0.
\]

Solve the FOCs; we get the solution for the utilitarian social optimum.

The non-cooperative Cournot-Nash game faced by agent 1 is given by:

\[
\text{Max } U_1 = x_1 B = (G_1 - b_1)(b_1 + (n - 1)b_2)
\]

Its reaction function in response to \((n-1)\) identical agents (agent 2) is the FOC:

\[
\frac{dU_1}{db_1} = -2b_1 + G_1 - (n - 1)b_2 = 0 \Rightarrow b_2 = \frac{G_1}{2} - \frac{(n - 1)b_2}{2}
\]

Similar to agent 1, we can obtain the representative agent 2’s reaction function:

\[
\text{Max } U_2 = x_2 B = (G_2 - b_2)(b_2 + (n - 2)b_2 + b_1)
\]
\[
\frac{dU_2}{db_2} = -2b_2 + G_2 - (n - 2)\bar{b}_2 - \frac{b_1}{n} = 0 \Rightarrow b_2 = \frac{G_2 - \frac{b_1}{n}}{n} (\bar{b}_2 = b_2)
\]

Agent 1’s reaction function is related to \( n \). The simultaneous solution of the reaction functions is the Cournot-Nash equilibrium of the model.

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