The absolute calibration of high-precision optical flats across a wide range of spatial frequencies

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Abstract. We analyse the applicability of the two-flat-test calibration method across wide spatial frequency range based on its calculated accuracy. A number of simulations have been performed to determine the accuracy of the absolute calibration. The simulation results show that the two-flat-test calibration method is applicable to surfaces within the spatial frequency range from $1.67 \times 10^{-1}$ to $1.67 \times 10^{-3}$ mm$^{-1}$.

1. Introduction

The surface form of large optical flats is critical to the performance of powerful laser systems and optical telescopes. The measurement of the surface topography and optical aberrations of these optics with nanometre resolution is a challenging problem [1-3]. Usually, the Fizeau interferometer [4] is used to measure the surface accuracy of optical flats and the result of this measurement is a difference between the test and the reference flat. In the case of highly precise optics, the topography of the reference flat must be established through absolute calibration methods and this calibration is subsequently used to accurately quantify the surface topography of the test flat.

Early absolute calibration methods for optical flats used liquid mirrors [5-8], but these methods have problems associated with vibration and the capillary effect near the wall of the vessel that holds the liquid. Moreover, mirrors must be calibrated in horizontal orientation, which results in a sag error caused by gravity and which must be compensated for in some way. Later, a three-flat-test method was developed [9-12], which compares three flat surfaces using the Fizeau interferometer. This method determines only a linear profile of the test flat along one diameter. To obtain the three-dimensional topography of the test flat, extended methods of the three-flat-test have been proposed [13-30]. These methods involve additional measurements with rotation or translation of at least one surface. The most widely used methods are based on Zernike polynomials [13-17], or even and odd functions as described by Ai and Wyant [18-23], or rotation and mirror symmetry [24-28]. Methods with a horizontally mounted reference flat are also used [29, 30]. However, none of these methods enable the measurement of the topography of large aperture optical components across a wide spatial frequency range (e.g. from $4 \times 10^{-4}$ to $1.67 \times 10^{-3}$ mm$^{-1}$) with desired accuracy.

In this study we analyse the applicability of the two-flat-test calibration method (designed for French project Laser Megajoule [31]) over a wide spatial frequency range based on its calculated accuracy. Numerical simulations of the surface reconstruction utilizing surface translations in horizontal ($x$) and vertical ($y$) directions are used to compute the accuracy of the method. These
simulation results show that the two-flat-test calibration method is applicable to surfaces within a spatial frequency range from $1.67 \times 10^3$ to $1.67 \times 10^5$ mm$^{-1}$.

The remainder of this paper is organized as follows: in Section 2 we introduce the general aspects of the two-flat-test calibration method in application to our experimental setup; in Section 3 simulation results are presented of the two-flat-test calibration method for the specific spatial frequency ranges of interest; and our conclusions are presented in Section 4.

2. Method description

The two-flat-test calibration method was used with a Fizeau interferometer plus an additional collimator in the form of a spherical mirror. The spherical mirror was used to illuminate an aperture of a large optical flat of about 1000mm diameter. A reference flat was designed as an optical wedge to reflect the beam on the spherical mirror. A simplified layout of the experimental setup is presented in Figure 1.

![Figure 1. Layout of the large aperture interferometer, showing major components: light source; spherical mirror, extending the beam; reference flat, performed by the optical wedge; and the test flat.](image)

The two-flat-test calibration method is based on the translation of the test flat with respect to the reference flat on a distance T along the vertical and horizontal axes (see Figure 2). The measurements may be represented mathematically in matrix form (following Küchel [24]):

$$
\begin{bmatrix}
  m_1(x, y) \\
  m_2(x, y) \\
  m_3(x, y) \\
  m_4(x, y)
\end{bmatrix} =
\begin{bmatrix}
  1 & 1 & 0 & 0 \\
  1 & 0 & 1 & 0 \\
  1 & 0 & 0 & 1 \\
  1 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
  h_{\text{reference}}(-x, y) \\
  h_{\text{test}}(x + T / 2, y) \\
  h_{\text{test}}(x - T / 2, y) \\
  h_{\text{test}}(x, y + T / 2) \\
  h_{\text{test}}(x, y - T / 2)
\end{bmatrix},
$$

where $h_{\text{ref}}(-x, y)$ and $h_{\text{test}}(x, y)$ are the surface height distributions of the reference and test flats, respectively.
Figure 2. Four configurations of the two-flat-test method. The test flat (solid line) moves with respect to the reference flat (dashed line) along horizontal axis over $T/2$ left ($m_1(x,y)$) and over $T/2$ right ($m_2(x,y)$) along vertical axis over $T/2$ up ($m_3(x,y)$) and over $T/2$ down ($m_4(x,y)$).

The surface height distribution of the test flat $h_{test}(x,y)$ is computed as follows:

$$h_{test}(x,y) = \frac{1}{2\pi(\nu_x^2 + \nu_y^2)} \left[ \nu_x FT \left[ \iint 2i\pi \nu_x \nu_y FT(h_{test}(\nu_x,\nu_y)) e^{2i\pi(\nu_x x + \nu_y y)} d\nu_x d\nu_y \right] + \nu_y FT \left[ \iint 2i\pi \nu_x \nu_y FT(h_{test}(\nu_x,\nu_y)) e^{2i\pi(\nu_x x + \nu_y y)} d\nu_x d\nu_y \right] \right] e^{2i\pi(\nu_x x + \nu_y y)} d\nu_x d\nu_y,$$

where $FT[ ]$ is the Fourier Transform operator, $\nu_x$ and $\nu_y$ are the spatial frequencies.

The accuracy of the two-flat-test method depends on the translation along each axis. The translations lead to filtering of the measured spatial frequency region. The Modulation Transfer Function of filtration for different translations is shown in Figure 3. It can be seen from the figure that the larger translations allow the reconstruction only of low frequencies because of this filtering. At the same time, smaller translations provide us with a large bandwidth of the spatial frequencies where noise limits the reconstruction.

3. Results

We have applied the two-flat-test method in numerical simulations. Initially, the accuracy of the method was estimated using the surface topography of a test flat modeled as a two-dimensional sine function called harmonic and stored as a 1000×1000 matrix. During the simulation, harmonics from 1 to 240 order were retrieved. This range of the harmonics corresponds to the spatial frequency range from $1.67 \times 10^{-3}$ to $4 \times 10^{-1}$ mm$^{-1}$. Two model apertures were adopted in these initial simulations: a 600mm diameter aperture for a spatial frequency range of $1.67 \times 10^{-3}$ – $3.0 \times 10^{-2}$ mm$^{-1}$ (denoted Region I); and a 33mm diameter aperture for a spatial frequency range of $3.0 \times 10^{-2}$ – $4 \times 10^{-1}$ mm$^{-1}$ (denoted Region II). We defined these two ranges of spatial frequency based upon the technical characteristics. The upper bound of the first range was defined by the aperture of the flat under the measurement and the lower bound of the first range was defined by the size of the polishing tool. The lower bound of the second region was defined by the Nyquist frequency.
Figure 3. Modulation Transfer Function for different translation T values.

Each modeled surface topography was translated by T in both x and y directions. Having four surface height distributions (see Equation (1)), the two-flat-test method was applied. Finally, the reconstruction error was computed as follows:

\[ E = \frac{\text{RMS}_{\text{test}} - \text{RMS}_{\text{retrieved test}}}{\text{RMS}_{\text{test}}}, \] (3)

where \( \text{RMS}_{\text{test}} \) is a root mean square value of the surface height distribution of the test flat, and \( \text{RMS}_{\text{retrieved test}} \) is a root mean square value of the surface height distribution of the retrieved test flat.

We are looking for a translation T value which guarantees a reconstruction error of less than 5%, over the defined spatial frequency regions. The results of the simulations, including the spatial frequency regions, translation T values and computed reconstruction errors are presented in Table 1.

| Spatial frequency range | Harmonic band | Translation, T (mm) | Error (%) |
|-------------------------|--------------|---------------------|-----------|
| Region I                | 1-18         | 48 - 4.8            | 4.95      |
| Region II               | 18-87        | 4.8 – 1.2           | 5         |
| Region II               | 240          | 1.2                 | 33.88     |

The results of the first simulation showed the theoretical limitations of the spatial frequency range of the two-flat test method. As can be seen from Table 1 the Region I wasn’t totally retrieved.

Further simulations of the two-flat-test method were applied to actual surface data. Surface topography was obtained by measurements of a test flat with aperture 100mm in diameter and a surface accuracy about \( \lambda/40 \) using an Intellium H2000 interferometer [32]. Measurements were obtained with reduced sensitivity to vibration. The measured surface topography of the test flat is presented in Figure 4. The measured topography was band-pass filtered in the spatial frequency range from \( 1.67 \times 10^{-3} \) to \( 3.0 \times 10^{-2} \) mm\(^{-1} \) (see Figure 5).
Figure 4. Initial surface topography.

Figure 5. Initial surface topography with filtration over $1.67 \times 10^{-3} - 3.0 \times 10^{-2}$ mm$^{-1}$ region.

It can be seen from the results presented in Figures 4 and 5 that the filtered topography has been successfully reconstructed by the two-flat-test method. The estimation of the reconstruction accuracy was computed using (Equation (3)). The reconstruction error and the translation T values are presented in Table 2.

Table 2. Results of simulations on actual topography data.

| Spatial frequency region | Translation, T (mm) | Error (%) |
|--------------------------|---------------------|-----------|
| Region I                 | 0.21                | 5.3805    |
| Region I                 | 0.43                | 5.4184    |
| Region I                 | 0.87                | 5.507     |

The results of the second simulation showed the allowed translations for the two-flat-test calibration in chosen spatial frequency region with minimum reconstruction error.

4. Conclusion

In this paper we have analysed the applicability of the two-flat-test calibration method to measuring surface topography of the optical flats across a wide range of spatial frequencies.

To determine the accuracy of the method a number of numerical simulations have been performed. The simulation results show the translation T value which guarantees the quality of the reconstruction for the $1.67 \times 10^{-3} - 3.0 \times 10^{-2}$ mm$^{-1}$ spatial frequency region to be 4.8mm, and to be 1.2mm for the $3.0 \times 10^{-2} - 4 \times 10^{-1}$ mm$^{-1}$ region. Consequently, the two-flat-test method is valid for absolute calibration, in terms of desired accuracy, for spatial frequencies from $1.67 \times 10^{-3}$ to $5.0 \times 10^{-2}$ mm$^{-1}$ within the simulation setup used.

The results for reconstruction of an actual surface, obtained using the experimental setup with an Intellium H2000 interferometer, show the accurate reconstruction of surface topography for a spatial frequency region $1.67 \times 10^{-3} - 3.0 \times 10^{-2}$ mm$^{-1}$ with translation T value more than 0.2mm.

The practical importance of our study is the measurement of the surface topography and optical aberrations of large aperture optics with nanometre resolution. The results showed the possibility to apply the two-flat-test method to the measurement in a certain spatial frequency region with a great precision.
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