SUSY Ward identities in $N=1$ SYM theory on the lattice

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The SUSY Ward identities (WIs) for the $N=1$ SU(2) SUSY Yang Mills theory discretized on the lattice with Wilson fermions (gluinos) are considered. The study is performed in the framework of a Monte Carlo simulation of the model with light dynamical gluinos. The renormalization and mixing constants of the lattice SUSY current $Z_S$ and $Z_T$ and the additively renormalized gluino mass $m_S$ are unknown parameters of the SUSY WIs. Using suitable on-shell combinations of the WIs, the ratios $Z_T/Z_S$ and $m_S/Z_S$ are determined non-perturbatively at one value of the coupling constant $g_0$ and two values of the hopping parameter $\kappa$.

1. INTRODUCTION

The non-perturbative regime of SUSY gauge theories is highly interesting, since among other things it may provide a possible mechanism for dynamical SUSY-breaking. An increased understanding of the non-perturbative phenomena of SUSY gauge theories can be achieved in the framework of the lattice regularization. An immediate difficulty arises however because the lattice regularized theory is not supersymmetric, since the Poincaré invariance (which is a sector of the superalgebra) is lost.

The present computational resources allow now to approach the simplest of the SUSY gauge theories, the $N=1$ supersymmetric Yang Mills (SYM) models, where the $N^2-1$ gluons are accompanied by neutral fermionic partners (gluinos) in the same adjoint representation of the color group. The Wilson discretization of the SYM models, proposed by Curci and Veneziano [1], explicitly breaks SUSY by the Wilson term and, softly, by a gluino Majorana mass term. SUSY is only recovered upon tuning the hopping parameter to the massless gluino limit and by taking the continuum limit.

An extensive analysis of the low-energy behavior of the $N=1$ SU(2) SYM model with Wilson fermions was performed by this collaboration in a Monte Carlo simulation with light dynamical gluinos, see [2,3] and references therein. The issue of the restoration of SUSY in the massless gluino limit is now addressed, as proposed in [4], by the non-perturbative analysis of the on-shell lattice SUSY WIs. The same issue, from a perturbative point of view, has been dealt with in another contribution [5].

An approach to the SYM models with domain wall fermions was recently considered in [6].
2. LATTICE FORMULATION

In the Curci-Veneziano approach \(\mathbb{1}\), which we adopt, the standard Wilson discretization is applied to the SYM model, where SUSY and the (anomalous) \(U(1)\) chiral symmetry of the continuum theory are explicitly broken by the Wilson term and a gluino Majorana mass term; SUSY is also broken by the space-time lattice. The issue of the recovering of the appropriate symmetry in the continuum limit is addressed through the discussion of the relevant WIs. The gluonic action is the standard plaquette

\[
S_g = \beta \sum_{pl} \left( 1 - \frac{1}{N_c} \text{Re} \, Tr \, U_{pl} \right),
\]

where the bare gauge coupling is given by \(\beta \equiv 2N_c/g_0^2\). The fermionic action is (with Wilson parameter \(r=1\))

\[
S_f = \sum_x \text{Tr} \left[ \frac{1}{2} \bar{\lambda} (-1 + \gamma_\mu) U_{x,\mu} \lambda_{x+\hat{\mu}} U_{x,\mu} \right. \\
+ \frac{1}{2} \bar{\lambda}_{x+\hat{\mu}} (-1 - \gamma_\mu) U_{x,\mu} \lambda_{x} U_{x,\mu}^\dagger \\
+ \left. (m_0 + 4) \bar{\lambda}_x \lambda_x \right],
\]

\(\lambda \equiv \sum_T T_{\lambda} \lambda^T\) being a Majorana field (gluino field) transforming according to the adjoint representation of the gauge group (\(T_\lambda\) are the generators).

2.1. SUSY WARD IDENTITIES

The effect of the explicit breaking of the chiral symmetry by the Wilson fermions is well understood in the framework of lattice QCD \(\mathbb{1}\), where the WIs of the continuum can be recovered through renormalization of the chiral currents and the quark masses. The lattice SUSY WIs of the SYM models are derived in \(\mathbb{1}\) using the same theoretical setup. Specifically, the SUSY current \(S_\mu(x)\), considered in on-shell correlation functions, undergoes multiplicative renormalization and mixing with a gauge-invariant current \(T_\mu(x)\); the renormalized current reads

\[
\hat{S}_\mu(x) = Z_S S_\mu(x) + Z_T T_\mu(x),
\]

which, for a gauge invariant local operator \(O(y)\) \((y \neq x)\), satisfies the integrated WIs

\[
\sum_x (\nabla_o S_\mu(x) O(y)) = m_S \sum_x (\chi(x) O(y)) + O(a)
\]

(4)

(the exact expressions for \(S_\mu(x), T_\mu(x), \chi(x)\) are given below). The quantity \(m_S\) is obtained from \(m_0\) by additive renormalization, \(m_S \equiv m_0 - m_S(m_0, g_0)\). As it is evident from the WIs \(\mathbb{1}\), the renormalizations \(Z_S, Z_T\) and \(m_S\) reabsorb the main lattice artifact of the explicit SUSY breaking; the residual \(O(a)\) breaking terms on the r.h.s. of (4) imply indeed effects vanishing exponentially with \(g_0\). If these are negligible, the condition \(m_S=0\) ensures the restoration of SUSY on the lattice. When \(m_S=0\), also the renormalized gluino mass multiplying the soft breaking term in the lattice chiral WIs vanishes \(\mathbb{1}\) as expected.

For the numerical analysis of the SUSY WIs we consider two possible lattice forms of the SUSY and mixing currents; a point-like discretization:

\[
S_\mu^{(1)}(x) = -\sum_{\rho \sigma} \sigma_{\rho \sigma} \gamma_\mu \text{Tr} \left[ P^{(cl)}_{x,\rho \sigma} \lambda_x \right]
\]

\[
T_\mu^{(1)}(x) = 2 \sum_\nu \gamma_\nu \text{Tr} \left[ P^{(cl)}_{x,\nu \mu} \lambda_x \right],
\]

(5)

and a point-split one \(\mathbb{3}\):

\[
S_\mu^{(2)}(x) = -\frac{1}{2} \sum_{\rho \sigma} \sigma_{\rho \sigma} \gamma_\mu \text{Tr} \left[ P^{(cl)}_{x,\rho \sigma} U^\dagger_{x,\rho \sigma} \lambda_{x+\hat{\mu}} U_{x,\mu} \right]
\]

\[
+ P^{(cl)}_{x+\hat{\mu},\rho \sigma} U_{x,\mu} \lambda_{x+\hat{\mu}} U^\dagger_{x,\mu} \\
T_\mu^{(2)}(x) = \sum_\nu \gamma_\nu \text{Tr} \left[ P^{(cl)}_{x,\nu \mu} U^\dagger_{x,\nu \mu} \lambda_{x+\hat{\mu}} U_{x,\mu} \right]
\]

\[
+ P^{(cl)}_{x+\hat{\mu},\nu \mu} U_{x,\nu \mu} \lambda_{x+\hat{\mu}} U^\dagger_{x+\hat{\mu},\mu}. \]

(6)

The values of \(Z_S\) and \(Z_T\) are expected to be different for these two choices. Consistently with parity and time-reversal, \(P^{(cl)}_{x,\mu \nu}\) is defined as a clover-symmetrized lattice field tensor; moreover, the lattice derivative in (4) is defined, for the discretization \(\mathbb{1}\): \(\nabla^{(1)}_o f(x) \equiv 1/2(f(x + \hat{\mu}) - f(x - \hat{\mu}))\); in the case \(\mathbb{3}\): \(\nabla^{(2)}_o f(x) \equiv f(x) - f(x - \hat{\mu})\).

Finally, the lattice form of the operator \(\chi(x)\) in the soft-breaking term is

\[
\chi(x) = \sum_{\rho \sigma} \sigma_{\rho \sigma} \text{Tr} \left[ P^{(cl)}_{x,\rho \sigma} \lambda_x \right].
\]

(7)
3. NUMERICAL ANALYSIS

We use two sets of gauge configurations at $\beta=2.3$ on a $12^3 \times 24$ lattice, at $\kappa=0.1925$ and at $\kappa=0.194$ with a statistics of 4212 and 2034 configurations, respectively. The second set is closer to the critical point of the chiral phase transition $\kappa_c=0.1955(5)$ as determined in [2]. All configurations were obtained by applying the two-step dynamical-fermion multi-bosonic algorithm, see [3] and references therein for details.

We consider for our study the WIs obtained by inserting in (4) the two gauge invariant operators

$$O^{(1)}(x) = \sum_{i<j} \sigma_{ij} \text{Tr}[P_{x,ij}\lambda_x]$$

(8)

$$O^{(2)}(x) = \sum_i \sigma_i \text{Tr}[P_{x,0i}\lambda_x]$$

(9)

with the clover and the simple-plaquette definition of the lattice field tensor $P_{x,\mu\nu}$ (the latter only for the operator (8)). The above operators undergo a (common) multiplicative renormalization and the form (4) of the WIs is consequently not spoiled. Since spatial integration breaks hypercubic symmetry in (6) these two operator insertions amount to independent choices.

Because of the Dirac structure of the operators involved, Eq. (6) gives 16 (real) WIs for every time-separation $t = x_0 - y_0$ and a given insertion operator $O(x)$. Due to the Majorana nature of the field $\lambda_x$ and the discrete symmetries of the action (6), only two equations are however independent. Consequently, determinations of the ratios $m_S/Z_S$ and $Z_T/Z_S$ can be obtained, for a fixed $O(x)$ and for every $t$, by solving a two-by-two linear system. These determinations contain $O(a)$ effects.

In order to get a significant signal for the correlations, a combined APE smearing for the gluon field and Jacobi smearing for the gluino field was performed on the insertion operator. We use two sets of smearing parameters (set A and B) giving equally good results for the ratios.

In Figs. 1 and 2 the determinations of the ratios $m_S/Z_S$ and $Z_T/Z_S$ are reported, at $\kappa=0.1925$ as a function of the time-separation $t$, for the two discretizations of the SUSY and mixing currents (5) (6) and the two insertion operators (8) (9) (with clover field tensor). A plateau can be observed for $m_S/Z_S$ (Fig. 1) for $t \geq 3$ with the insertion operator $O^{(1)}(x)$ and for $t \geq 4$ with $O^{(2)}(x)$: in these regions of $t$ contact terms due to the time-extension of the operators entering the correlations are absent. The signal for $Z_T/Z_S$ (Fig. 2) is noisier.

The results of the global fit over the range of time-separations $t \geq 4$ are reported in Table 1.

Different discretizations of the SUSY or mixing current give different renormalizations $Z_S$ and $Z_T$.
Table 1
Results for $m_S/Z_S$ and $Z_T/Z_S$ with the discretizations 1 and 2 of the currents and the two insertion operators, with plaquette and clover field tensor and smearing parameters A and B.

| $\kappa$ | current | field tensor | smearing | $m_S/Z_S$ ($\mathcal{O}(1)$) | $m_S/Z_S$ ($\mathcal{O}(2)$) | $Z_T/Z_S$ ($\mathcal{O}(1)$) | $Z_T/Z_S$ ($\mathcal{O}(2)$) |
|----------|---------|--------------|-----------|-------------------------------|-------------------------------|-------------------------------|-------------------------------|
| 0.1925   | 1       | plaquette    | A         | 0.155(11)                     | 0.184(28)                     | 0.160(25)                     | 0.29(6)                       |
| 0.1925   | 1       | plaquette    | B         | 0.170(12)                     | 0.144(18)                     | 0.244(35)                     | 0.190(45)                     |
| 0.194    | 1       | clover       | A         | 0.118(17)                     | 0.118(17)                     | 0.132(16)                     | 0.11(7)                       |
| 0.194    | 1       | clover       | B         | 0.168(9)                      | 0.168(9)                      | 0.132(16)                     | 0.11(7)                       |
| 0.1925   | 2       | plaquette    | A         | 0.152(9)                      | −0.010(33)                    | 0.168(9)                      | 0.132(16)                     |
| 0.1925   | 2       | plaquette    | B         | 0.170(12)                     | 0.132(16)                     | 0.168(9)                      | 0.132(16)                     |

for any finite $g_0$. In particular, the discretization $\mathcal{O}(1)$ gives a mixing coefficient $Z_T$ compatible with zero according to our data, but in case of $\mathcal{O}(2)$ $Z_T$ seems to be small and non-zero.

Results from the two independent insertion operators $\mathcal{O}(1)(x)$ and $\mathcal{O}(2)(x)$ differ by $O(a)$ effects. The discrepancy appears to be comparable to the statistical uncertainty, even if the insertion operator $\mathcal{O}(2)(x)$ seems to give a lower value for $m_S/Z_S$ compared to $\mathcal{O}(1)(x)$. Observe that also results coming from different lattice forms of a given operator differ by discretization errors.

Theoretically, $m_S/Z_S$ should decrease when the hopping parameter approaches the expected critical point $\kappa_c$, and vanish at $\kappa_c$; $Z_T/Z_S$ should take its asymptotic value for massless gluino. Comparison between data at $\kappa = 0.1925$ and $\kappa = 0.194$ seems to confirm this expectation. The statistical indetermination still prevents an accurate extrapolation of $\kappa$ to massless gluino.

4. CONCLUSIONS

The present study shows that the extraction of the ratios $m_S/Z_S$ and $Z_T/Z_S$ from the on-shell SUSY Ward identities is technically feasible with the computing resources at hand. The main technical difficulty (related to SUSY) is that high-dimensional operators with a mixed gluonic-fermionic composition must be considered, introducing relatively large statistical fluctuations. This difficulty can be handled with an appropriate smearing procedure. The non-perturbative determination of the ratio $m_S/Z_S$ can be used for an independent extrapolation to the critical hopping parameter corresponding to massless gluinos. Our results for $m_S/Z_S$ and $Z_T/Z_S$ are consistent with the WIs (3,4) with $O(a)$ effects comparable to the statistical indetermination.

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