A Comparative Study of Thermal Performance of Different Nanofluids: An Analytic Approach

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Abstract: The purpose of this study was to determine an exact solution for the fluid flow and heat transfer of laminar steady magnetohydrodynamics (MHD) nanofluid flow over a stretching/shrinking surface. Appropriate similarity transformations were used to transform the governing partial differential equations into coupled nonlinear ordinary differential equations. The current study showed good correspondence with previously published work. The solution was deduced from the solution of the flow field and temperature field. Furthermore, the dimensionless skin friction coefficient and Nusselt number were derived. The solution of the temperature field was deduced in terms of the generalized Laguerre polynomial. The value of the generalized Laguerre polynomial was calculated using the “LaguerreL” command in MuPAD. The impact of different physical parameters of the symmetry on the thermal performance, including the nanoparticle volume fraction parameter, magnetic parameter, mass suction/injection parameter and stretching/shrinking parameter, is discussed in detail for different nanoparticles. Furthermore, the effect of nanoparticle type on the fluid velocity component, temperature distribution, skin friction coefficient and Nusselt number was studied in detail. Four different nanoparticles were considered in this study. This work reveals that the nanoparticles within the base fluid have the potential to increase the heat transfer ability of many liquids. The results indicate that silver and titanium oxide nanoparticles had the largest and lowest skin friction coefficients, respectively, in the shrinking surface case, exhibiting opposite behavior in the stretching surface case among all the nanoparticles considered. The results also indicate that silver and titanium oxide nanoparticles had the largest and lowest Nusselt numbers, respectively, for both the stretching and the shrinking surface cases. It is suggested silver nanoparticles are not used for optimum heat transfer.

Keywords: nanofluid; MHD flow; Nusselt number

1. Introduction

Common fluids do not have very high thermal conductivity. Nanofluids were introduced to improve the heat conduction of fluids. A nanofluid is a fluid that contains a suspension of nanosolid particles with dimensions of less than 100 nm. Nanoparticles have the properties of both a solid and liquid, i.e., they are actually liquids but have strong thermal properties akin to solids. They find large use in industry as fuel cells, biomedicines, nuclear reactors, etc. [1]. Nanofluids were first introduced by Choi [2]. Choi et al. [3] revealed that nanofluids have a good rate of heat transfer. A nanofluid is a combination of nanoparticles and a base fluid. Comparative studies of different nanofluids have
gained much more importance in recent decades. The heat transfer rate for nanofluids can be increased by up to 40% upon adding only a small amount (1–5% by volume) of nanoparticles [4]. Eastman et al. [5] increased the thermal transmissivity of CuO – H$_2$O nanofluid by 60% upon adding only 5% nanoparticles. Eastman et al. [6] increased the thermal conductivity by up to 40% using a small amount (0.3%) of copper nanoparticles in the ethylene glycol base fluid. Due to the different conductivity of different nanoparticles and base fluids, the study of different terminologies and the comparative study of different nanoparticles and base fluids have gained much importance in the past decade.

Arash Karimipour et al. [7] employed the lattice Boltzmann method to analyze the laminar forced convective heat transfer in slip flow for Cu – H$_2$O nanofluid in a microchannel. Tasawar Hayat et al. [8] studied the magnetohydrodynamic (MHD) mixed convection flow across a curved stretching sheet for Ag – H$_2$O and Cu – H$_2$O nanofluids. Tasawar Hayat et al. [9] analyzed the hybrid convective flow of an MHD nanofluid with nonlinear thermal radiation for Ag – H$_2$O and Cu – H$_2$O nanofluids. Tasawar Hayat et al. [10] studied the nanofluid flow initiated by a rotating disc through a homogeneous–heterogeneous reaction for Fe$_3$O$_4$ – H$_2$O nanofluid. Tasawar Hayat et al. [11] analyzed the partial slip effect in the magnetic nanofluid flow across two parallel extendable discs for Fe$_3$O$_4$ – H$_2$O nanofluid. Kumar and Kumar [12] explored the nanofluid flow across a steady convective natural boundary layer. They analyzed the heat transfer using a stretching sheet through a transverse uniform magnetic field, considering three different nanoparticles and two different base fluids as nanofluids. Tasawar Hayat et al. [13] considered nanofluid flow in a non-Darcy permeable medium across two stretching and rotating disks for Ag – H$_2$O and Cu – H$_2$O nanofluids. Sheikholeslami and Ganji [14] used the control-volume-based finite element method (FEM) to study the impact of an external magnetic source on the free convection of nanofluid for Fe$_3$O$_4$ – H$_2$O nanofluid. Hatami and Jing [15] employed the FEM and response sheet method (RSM) to analyze the wavy direct absorber solar collector (WDASC) for Al$_2$O$_3$ – H$_2$O nanofluid. Biglarian and Gorji [16] investigated unsteady MHD nanofluid flow and heat transfer across parallel plates as a function of the normal motion of the permeable upper plate. They considered four different water-based nanoparticles for this study. Bhuvnesh Sharma et al. [17] employed the homotopy analysis method (HAM) to analyze unsteady MHD nanofluid flow. They considered three different nanoparticles based on sodium alginate as nanofluids. Nor Fadhilah Dzulkifli et al. [18] analyzed the stability and heat transfer in dual solutions of unsteady stagnation point nanofluid flow through a permeable exponential stretching/shrinking sheet with a slip velocity impact. Sheikholeslami and Shehzad [19] determined the effect of a nonuniform magnetic field on nanofluid flow in a penetrable cavity for Fe$_3$O$_4$ – H$_2$O nanofluid. Khan and Ahmad [20] conducted a relative performance analysis of different water-based nanofluids utilized in automobile radiators. Asker and Gadanya [21] investigated the performance of flat-plate solar collectors utilizing five different water-based nanoparticles. Imtiaz et al. [22] studied the melting heat transfer for MHD nanofluid flow using a rotating disc with homogeneous–heterogeneous reactions for Ag – H$_2$O and Cu – H$_2$O nanofluids. Harkirat Kaur Sandhu et al. [23] analyzed the solidity of different nanofluids utilizing two different nanoparticles and two different base fluids. Sravanthi [24] used HAM to study the impact of nonlinear thermal radiation on nanofluid flow through a rotating disc with erratic thickness across a nonuniform heat source/sink for Ag – H$_2$O and Cu – H$_2$O nanofluids. Tanveer Sajid et al. [25] investigated the rotating Prandtl nanofluid for certain parameters. The governing partial differential equations (PDEs) were converted to ordinary differential equations (ODEs) using the similarity transformations, which were then solved numerically using the shooting technique. They concluded that the temperature and mass profile increased upon enhancing the temperature convection boundary and species diffusivity parameter, respectively. Davood et al. [26] numerically investigated H$_2$O – Al$_2$O$_3$ nanofluid through a micro-concentric annulus. They considered two models of a double-pipe heat exchanger (DPHE) to study convective heat transfer. They used an FEM for simulation and concluded that model B presented a higher heat transfer rate as compared to the other
considered models. Amier et al. [27] investigated the single- and two-phase approaches in a vertical channel to study the impact of radiation on the natural convection of a nanofluid. They used SIMPLEC and PRESTO methods and computed the results in terms of various parameters. Hamed et al. [28] numerically simulated heat transfer and turbulent flow in a three-dimensional (3D) microchannel with certain variations in the wavelength. They used the SIMPLEC method to compute the results for different values of the Reynolds number \((Re)\), concluding that \(Re = 7500\) provided the best heat transfer rate with a nanofluid volume fraction of 3%. Ramin et al. [29] performed computational fluid dynamics (CFD) analysis of the hydrodynamic and thermal properties of a hybrid nanofluid in a sinusoidal double-layered heat sink (SDLHS). They employed the finite volume method (FVM) to study certain variations in \(Re\) and concluded that \(Re = 700\) is not recommended for this type of study. Hossein et al. [30] used a nanofluid to numerically enhance the hydrothermal performance of SDLHS according to variations in \(Re\) and Darcy numbers \((Da)\). They used ANSYS FLUENT to compute the results, and they recommended not using a higher \(Da\) under local thermal equilibrium (LTE).

In this way, the study of different nanoparticles with different base fluids and comparative study are significantly more important. By analyzing the rate of heat transfer phenomenon of different nanofluids, it will become easier to choose a nanofluid for a certain purpose. In past studies, there were some specific nanoparticles used. However, most researchers study nanofluids with specific nanoparticles. Therefore, the thermal conductivity of four different nanofluids was calculated and compared in this study. The current study used four different nanofluids based on four different nanoparticles. By analyzing the four different nanofluids, we could select an optimum model for certain applications. This article concentrates on the exploration of heat transfer for different nanofluids. A conclusion was reached based on variations in different inclusive parameters and the Nusselt number and the skin friction coefficient.

The current study primarily concentrated on the comparison of heat transfer for four different nanofluids. In this study, water (H\(_2\)O) was considered a base fluid. Copper (Cu), silver (Ag), magnetite (Fe\(_3\)O\(_4\)) and titanium oxide (TiO\(_2\)) were the considered nanoparticles. An analytic solution was procured for both the velocity distribution and temperature distribution for four different nanofluids, i.e., Cu – H\(_2\)O, Ag – H\(_2\)O, Fe\(_3\)O\(_4\) – H\(_2\)O and TiO\(_2\) – H\(_2\)O and plotted for certain variations in inclusive parameters, i.e., nanoparticle volume fraction parameter \((\phi)\), mass suction/injection parameter \((f_w)\), stretching/shrinking parameter \((\lambda)\) and magnetic parameter \((M)\). Additionally, the values of the Nusselt number \((Nu)\) and the skin friction coefficient \((C_f)\) were computed and plotted to obtain conclusions. The current study was based on an analytic technique. It can further be extended by applying some numerical techniques.

2. Problem Formulation

A nanofluid flow was taken through a sheet. The flow was considered to be a steady MHD laminar and the sheet was considered to be permeable. There were four different nanofluids used, with water as a base fluid, as described in Figure 1. Nanoparticles were considered to be thermally stable with a no-slip condition. The induced magnetic field was ignored as it had a very low value. The upper half-plane in rectangular coordinates was occupied by the fluid. The sheet was stretched/shrunk along the \(x\) axis and the \(y\) axis was perpendicular to it. The sheet was extended in the \(x\) direction with a fixed origin. \(u_w(x) = bx\) and \(v_w\) are the velocities of the sheet in the \(x\) and \(y\) directions, respectively. \(T_\infty\) and \(T_w\) are the temperature of the ambient fluid and the sheet, respectively. The temperature of the sheet varied linearly by \(x\). For the above assumption, the 2-D continuity, momentum and energy equations can be scripted as [31]

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]
\[ 
\rho_{nf} \left( u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = \mu_{nf} \frac{\partial^2 u}{\partial y^2} - \sigma B^2 u 
\]

where the velocity components in the \( x \) and \( y \) directions are \( u \) and \( v \), correspondingly, \( \rho_{nf} \) and \( \mu_{nf} \) are the density and dynamic viscosity of the nanofluid, respectively, where \( \mu_{nf} \) was suggested by Brinkman [32], \( T \) is the temperature of the nanofluid, \( \sigma \) represents the electrical conductivity, \( B \) is the magnetic field applied along the \( y \) axis, \( (\rho c_p)_{nf} \) represents the heat capacitance, and \( k_{nf} \) shows the phenomenon of efficient thermal conductivity of the nanofluid [33].

\[ 
(\rho c_p)_{nf} \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k_{nf} \frac{\partial^2 T}{\partial y^2} 
\]

Figure 1. Geometrical representation of the problem: (a) stretching sheet, (b) shrinking sheet.

The nanofluid constants are

\[ 
(\rho c_p)_{nf} = (1 - \varphi) (\rho c_p)_f + \varphi (\rho c_p)_s, \quad \rho_{nf} = (1 - \varphi) \rho_f + \varphi \rho_s, \quad \frac{k_{nf}}{k_f} = \frac{k_s + 2k_f}{k_s + 2k_f} - 2\varphi \left( \frac{k_f - k_s}{k_s + 2k_f} \right) + \varphi \left( \frac{k_f - k_s}{k_s + 2k_f} \right), \quad \mu_{nf} = \frac{\mu_f}{(1 - \varphi)^2} \]

where \( \varphi \) represents the nanoparticle volume fraction parameter and the subscripts “s” and “f” represent the solid- and fluid-state properties, respectively. It is necessary to mention that the approximated values for \( k_{nf} \) are only for the spherical coordinates and are not valid for any other shapes of the nanoparticles. Some thermal characteristics of different nanoparticles and the base fluid (water) are presented in Table 1 [33–38].
Table 1. Thermo-physical properties of different nanoparticles and base fluid.

| Thermal Properties | Base Fluid (Water) | Cu   | Ag   | Fe₃O₄ | TiO₂ |
|--------------------|-------------------|------|------|-------|------|
| c_p (J/kgK)        | 4076.4            | 385  | 235  | 670   | 686.2|
| ρ (kg/m³)          | 997.8             | 8933 | 10,500| 5180  | 4250 |
| k (W/mK)           | 0.60475           | 401  | 429  | 9.7   | 8.9538|

The corresponding boundary conditions are [31]:

\[
\begin{align*}
u &= u_w(x) = bx, \quad T = T_w(x) = T_\infty + ax, \quad v = v_w, \text{ at } y = 0 \\
T &\to T_\infty, \quad u \to 0, \quad \text{as } y \to \infty
\end{align*}
\]

(5)

The dimensionless functions \( f(\eta), \theta(\eta) \) and similarity variables are defined as [31]:

\[
\eta = \left(\frac{a}{v_f}\right)^{1/2} y, \quad \varphi(x,y) = \left(\frac{v_f a}{v_f}\right)^{1/2} x f(\eta), \quad \theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}
\]

(6)

where \( \varphi(x,y) \) represents the stream function as was used by [39] in a basic paper. It fulfills the continuity equation as [24]:

\[
\begin{align*}
u &= \frac{\partial \varphi}{\partial y} = ax f'(\eta), \\
v &= -\frac{\partial \varphi}{\partial x} = -\left(\frac{v_f a}{v_f}\right)^{1/2} f(\eta)
\end{align*}
\]

(7)

This procedure has been known since [40]. After inputting the values of similarity variables and dimensionless functions, we obtain

\[
\frac{1}{(1 - \phi)^{2.5}} f'''(\eta) - \left(1 - \phi + \phi \frac{\rho_s}{\rho_f}\right) \left[f''(\eta) - f(\eta) f''(\eta)\right] - M f''(\eta) = 0
\]

(8)

\[
-Pr \left[f(\eta) \theta'(\eta) - f'(\eta) \theta(\eta)\right] = \frac{k_{sf}/k_f}{1 - \phi + \phi \left(\frac{\rho c_p}{\rho c_p}\right)} \theta''(\eta)
\]

(9)

where differentiation of \( \eta \) is represented by prime, the magnetic parameter is \( M = \sigma B_0^2 / \alpha \rho_f \), and the Prandtl number (Pr) is \( Pr = v_f / \alpha_f \).

The transformed boundary conditions are:

\[
\begin{align*}
f(\eta) &= f_w, \quad f'(\eta) = \lambda, \quad \theta(\eta) = 1, \quad \text{at } \eta = 0 \\
f'(\eta) &\to 0, \quad \theta(\eta) \to 0, \quad \text{as } \eta \to \infty
\end{align*}
\]

(10)

where the mass suction/injection parameter is \( f_w = -v_w / \sqrt{\nu_f} \) and the stretching/shrinking parameter is \( \lambda = b/a \). Its positive and negative values refer to the stretching and shrinking sheet, respectively.

3. Method and Solution

The analytical solution of Equation (8) is supposed to be

\[
f(\eta) = a + be^{-\alpha \eta} \quad (a > 0)
\]

with boundary conditions

\[
\begin{align*}
f(0) &= f_w, \quad f'(0) = \lambda, \quad f'(\infty) = 0
\end{align*}
\]

(12)

By using Equations (11) and (12), the solution of Equation (8) is obtained as:

\[
f(\eta) = f_w + \frac{\lambda}{\alpha} (1 - e^{-\alpha \eta})
\]

(13)
where

\[
\alpha = \frac{1}{2} \left( f_w \left( 1 - \varphi + \varphi \left( \frac{\rho_s}{\rho_f} \right) \right) \left( 1 - \varphi \right)^{3.5} \mp \sqrt{4 \left( M + \lambda \left( 1 - \varphi + \varphi \left( \frac{\rho_s}{\rho_f} \right) \right) \right) \sqrt{1 - \varphi} - f_w^2 \left( 1 - \varphi + \varphi \left( \frac{\rho_s}{\rho_f} \right) \right) \left( -1 + \varphi \right)^3 \left( -1 + \varphi \right)^2} \right)
\]

Differentiation of Equation (13) gives the solution for the flow field for both cases, i.e., the stretching and shrinking sheet.

\[
f' (\eta) = \lambda e^{-\alpha \eta}
\]

(14)

For the occurrence of a physical solution, \( \alpha \) must be positive. The solution domain of \( \alpha \) for \( \lambda \) as a function of \( f_w \) and various values of \( M \) as a function of \( \lambda \) are shown in Figures 2 and 3, respectively, for four different nanofluids. The positive and negative values of \( \lambda \) refer to the stretching and shrinking sheet, respectively.

**Figure 2.** The domain for the solution of \( \alpha \) for certain values of \( \lambda \) as a function of \( f_w \); red, green, blue and cyan solid lines represent the Cu – H₂O, Ag – H₂O, Fe₃O₄ – H₂O and TiO₂ – H₂O nanofluids, respectively.

**Figure 3.** The domain for the solution of \( \alpha \) for certain values of \( M \) as a function of \( \lambda \); red, green, blue and cyan solid lines represent the Cu – H₂O, Ag – H₂O, Fe₃O₄ – H₂O and TiO₂ – H₂O nanofluids, respectively.
Now consider \( \zeta = -e^{-a\eta} \) so that Equation (9) will take the form

\[
\zeta \theta''(\zeta) + \left(1 - \frac{p}{\Lambda} - \frac{P_{r^*}\lambda}{\Lambda} \right) \theta'(\zeta) + \frac{P_{r^*}\lambda}{\Lambda} \theta(\zeta) = 0
\]  

(15)

with the boundary conditions \( \theta(-1) = 1, \ \theta(0) = 0 \), where 

\[
\Lambda = \frac{k_{nf}/k_f}{1 - \phi + \phi\left(\frac{\rho_p}{\rho}, \frac{c_p}{c_{pf}}\right)},
\]

\[
P_{r^*} = \frac{\nu}{aw^2}, \ p = P_{r^*}(\lambda + f_{w\lambda}).
\]

The solution for Equation (15) with the above-mentioned boundary conditions can be obtained in terms of the generalized Laguerre polynomial function [41,42].

\[
\theta(\zeta) = (-1)^{1-p} e^{-\alpha\eta} L \left[ \frac{1 - \frac{p}{\Lambda}, \frac{p}{\Lambda}, \frac{P_{r^*}\lambda}{\Lambda} \zeta}{1 - \frac{p}{\Lambda}, \frac{p}{\Lambda}, -\frac{P_{r^*}\lambda}{\Lambda}} \right]
\]  

(16)

where \( L[a, b, z] \) is the generalized Laguerre polynomial, and its value is calculated by using the “LaguerreL” command in MuPAD, which is a symbolic toolbox of MATLAB.

One of the most significant physical parameters is the skin friction coefficient. The general formula is:

\[
C_{f_{x}} = \frac{\tau_w}{\rho_f u_w^2}
\]

where \( \tau_w = \mu_{nf} \frac{\partial u}{\partial y}\bigg|_{y=0} \)

Using Equation (13), the skin friction coefficient in a dimensionless form for the current problem can be obtained as:

\[
C_{f_{x}} \text{Re}_{x}^{1/2} = -\left(\frac{1}{1 - \phi}\right)^{2.5} \alpha \lambda
\]  

(17)

Another important physical quantity for the current study is the Nusselt number. The general formula for the Nusselt number is:

\[
Nu_{x} = \frac{xq_{w}}{k_f (T_{f} - T_{\infty})}
\]

where \( q_{w} = -k_{nf} \frac{\partial T}{\partial y}\bigg|_{y=0} \)

Using Equation (16), the dimensionless Nusselt number for the present problem can be obtained as:

\[
Nu_{x}/Re_{x}^{1/2} = k_{nf} \left(\frac{\alpha \rho L}{\lambda} + \frac{P_{r^*}\alpha \lambda L \left[-\frac{p}{\Lambda}, 1 + \frac{p}{\Lambda}, -\frac{P_{r^*}\lambda}{\Lambda}\right]}{\Lambda L \left[1 - \frac{p}{\Lambda}, \frac{p}{\Lambda}, -\frac{P_{r^*}\lambda}{\Lambda}\right]} \right)
\]  

(18)

4. Results and Discussion

The nonlinear ODEs presented in Equations (8) and (9) were evaluated considering the boundary conditions (10). This was realized by presenting a closed-form result for the flow field along with the temperature field by employing the Laguerre polynomial. Both the solutions were calculated for some values of different parameters, including \( M, \phi, f_{w} \) and \( \lambda \). The results were calculated for four different nanofluids, i.e., Cu – H\(_2\)O, Ag – H\(_2\)O, Fe\(_3\)O\(_4\) – H\(_2\)O and TiO\(_2\) – H\(_2\)O. All nanofluids are considered in the figures in this study, focusing on the effect of engineering parameters such as \( Nu \) and \( C_{f_{x}} \). For the present work, \( Pr = 6.587 \) (for water at 20 °C) [36]. For both cases, a closed-form solution exists for different conditions. For confirmation of our results, we compared the results of the stretching sheet case for the values of \(-f'(0)\) with the Turkyilmazoglu results [43] for \( \phi = 0, f_{w} = 2 \) and \( \lambda = 1 \). Additionally, we compared the values of \(-\theta'(0)\) with the
Abolbashari results [44] for \( \varphi = M = f_w = 0 \) and \( \lambda = 1 \). These comparisons are presented in Tables 2–5. It can be observed in the following tables that the current results are much closer to the above-stated references and are more accurate in some cases. So, the current study was more accurate in comparison to the previous results. Tables 3–5 show the \( f''(0) \) and \( \theta'(0) \) for certain values of two key parameters, i.e., \( f_w \) and \( \varphi \), for stretching as well as shrinking sheet cases. It was described earlier that there exist two solutions for \( \lambda = -1 \). Both solutions are presented in Table 5.

Table 2. Comparative results of \(-f''(0)\) for the current work and [43] for \( \varphi = 0, f_w = 2 & \lambda = 1 \).

| \( M \) | Current Result | Turkyilmazoglu [43] | Freidoonimehr [35] |
|-------|----------------|----------------------|--------------------|
| 0     | 2.414214       | 2.41421356           | 2.39871456         |
| 2     | 3.000000       | 3.00000000           | 2.98555535         |
| 5     | 3.645751       | 3.64575131           | 3.61168892         |

Table 3. Results of \( f''(0) \) for several variations in \( f_w \) and \( \varphi \) for \( \lambda = 1 \) and \( M = 3 \).

| \( f_w \) | \( \varphi = 0 \) | \( \varphi = 0.1 \) | \( \varphi = 0.2 \) |
|---------|-----------------|-----------------|-----------------|
| -2      | -1.23606798    | -0.98434789    | -0.84071464    |
| -1      | -1.56155281    | -1.34999009    | -1.19502776    |
| 0       | -2              | -1.91959518    | -1.78891346    |
| 1       | -2.56155281    | -2.72953534    | -2.67793892    |
| 2       | -3.23606798    | -3.74343838    | -3.80653696    |

Table 4. Results of \( f''(0) \) for certain variations in \( f_w \) and \( \varphi \) for \( \lambda = -1 \) and \( M = 3 \).

| \( f_w \) | \( \varphi = 0 \) | \( \varphi = 0.1 \) | \( \varphi = 0.2 \) |
|---------|-----------------|-----------------|-----------------|
| -2      | 0.73205081     | 0.30238824      | 0.07702940      |
|         | [-2.73205081]  | [-3.06147873]   | [-3.04285172]   |
| -1      | 1               | 0.49409453      | 0.14406428      |
|         | [-2]            | [-1.87369798]   | [-1.62697544]   |
| 0       | 1.41421356     | 0.96216172      | 0.48413742      |
|         | [-1.41421356]  | [-0.96216172]   | [-0.48413742]   |
| 1       | 2               | 1.87369798      | 1.62697544      |
|         | 2               | 3.06147873      | 3.04285172      |
| 2       | 2.73205081     | 3.06147873      | 3.04285172      |

Table 5. Comparison of current values of \(-\theta'(0)\) for certain variations in the \( Pr \) for \( M = f_w = \varphi = 0 \) and \( \lambda = 1 \).

| \( Pr \) | Current Value | Abolbashari [44] | Freidoonimehr [35] |
|---------|---------------|------------------|--------------------|
| 0.72    | 0.80863135    | 0.80863135       | 0.80863135         |
| 1.00    | 1.00000000    | 1.00000000       | 1.00000000         |
| 3.00    | 1.92368259    | 1.92368259       | 1.92368259         |
| 7.00    | 3.07225021    | 3.07225021       | 3.07225021         |
| 10.00   | 3.72067390    | 3.72067390       | 3.72067390         |

The values presented in the above tables were computed for Cu – H2O nanofluid. The values for the other three nanofluids are presented directly in graphical representations.

5. Stretching Sheet Case

For the MHD Newtonian flow, based on the above assumptions, there exists an analytical solution for a stretching sheet case, which was solved utilizing Equations (13) and (16). In the case of a diminishing sheet (\( \lambda > 0 \)), there exists only one solution, which is presented in Figures 2 and 3. So, the acceptable form of \( a \) involves the positive square root.

The impact of \( \varphi \) on the velocity field for \( \lambda = 1 \) is presented in Figure 4. The boundary layer thickness of the velocity was reduced due to the presence of solid nanoparticles. It
can be noticed that the value of the velocity field was highest for TiO$_2$ – H$_2$O among all the considered nanofluids. Therefore, when using TiO$_2$ – H$_2$O, the thermal boundary layer thickness was the highest among all the considered nanofluids.

![Figure 4](image1.png)

**Figure 4.** Influence of $\phi$ on the velocity field for certain nanofluids.

The impact of $\varphi$ on the temperature field for certain nanofluids is presented in Figure 5. It can be noted that the thermal boundary layer thickness was higher for higher values of the nanoparticle volume fraction parameter. This is the basic reason for the employment of nanofluids. The results show that the temperature field had the highest value for the Ag – H$_2$O nanofluid among all the considered nanofluids.

![Figure 5](image2.png)

**Figure 5.** Influence of $\varphi$ on the temperature field for different nanofluids.

The impact of the magnetic parameter on the velocity field for various nanoparticles for $\lambda = 1$ is displayed in Figure 6. It can be noted that the velocity field value was high for TiO$_2$ – H$_2$O for a higher value of the magnetic parameter. Three different cases can be considered for $f_w$: (a) $f_w > 0$ represents the mass suction case, (b) $f_w < 0$ represents the mass injection case, and (c) $f_w = 0$ depicts the wall without permeability. For the stretching sheet, suction extracted a specific amount of fluid into the surface, and subsequently,
the hydrodynamic boundary layer became diluted and the thermal boundary layer was depressed with an enhancement in the mass suction parameter. The opposite trend was observed for the mass injection case.

Figure 6. Influence of \( M \) on velocity field for different nanofluids.

Figure 7 presents the impact of higher magnetic parameters on the temperature field for \( \lambda = 1 \). Lorentz force was generated through the imposition of a vertical magnetic field to a fluid that is electrically conducting. Due to this phenomenon, the momentum boundary layer thickness and velocity were reduced. In the case of the stretching sheet, this force retarded the motion of the fluid. This illustrates that the traverse magnetic field resisted the transfer. So, for a higher value of the vertical magnetic field, the resistance applied to nanoparticles was also enhanced. So, heat was produced in the fluid. In short, by increasing \( M \), a slight enhancement in the temperature field was induced.

Figure 7. Influence of \( M \) on temperature field for certain nanofluids.

Figure 8 presents the impact of the mass injection/suction on the temperature field. The case of mass suction is considered here. By employing the suction to the stretching sheet, a specific amount of fluid was drawn toward the sheet, resulting in reducing the
hydrodynamic boundary layer thickness. By enhancing $f_w$, the value for the thermal boundary layer was depressed. There were different behaviors in the case of mass injection. On applying mass injection to the stretching sheet, more fluid was introduced to the sheet, which resulted in the enhancement of the hydro-dynamic boundary layer along with the temperature field.

![Graph](image1.png)

**Figure 8.** Influence of $f_w$ on temperature field for certain nanofluids.

Figures 9 and 10 represent the influence of enhancement in the value of the stretching/shrinking effect on the velocity and temperature field, respectively, for the stretching sheet. The initial approximation for the velocity field was dependent on the stretching/shrinking effect and it fulfilled the criteria for the bounding sheet. The velocity distribution was higher for higher values of the stretching/shrinking parameter. Additionally, with an enhancement in the velocity of the stretching sheet, the breadth of the thermal boundary layer was reduced.

![Graph](image2.png)

**Figure 9.** Influence of $\lambda$ on the velocity field for different nanofluids.
The existence of two solution branches depends upon two factors: presented in Figures 11 and 12, respectively. For both cases, the same trends were observed, parameters, as shown in Figures 2 and 3. These solution branches are related to the positive values of $\phi$ for the upper solution branches, the lower solution branch showed a greater boundary layer thickness for higher values of $\phi$, but there was one difference in the occurrence of two solutions for the shrinking sheet. For higher values of $\lambda$, the thermal boundary layer increased, and the hydrodynamics velocity boundary layer decreased. Therefore, the temperature profile was dominant for higher values of $\phi$. In comparison to the upper solution branches, the lower solution branch showed a greater boundary layer thickness for the velocity and temperature field.

6. Shrinking Sheet Case

The case of the shrinking sheet, flow was very engaging as compared to the stretching sheet case. In this case, dual solutions exist for the specific approximations of the governing parameters, as shown in Figures 2 and 3. These solution branches are related to the positive and negative signs of the value of $\alpha$. Dual solutions exist only in the case of mass suction. The existence of two solution branches depends upon two factors:

- The sign for $\alpha$;
- The symbol for the inclusive term within the square root sign in $\alpha$.

The impact of the enhanced value of $\varphi$ on the velocity and the temperature field is presented in Figures 11 and 12, respectively. For both cases, the same trends were observed, but there was one difference in the occurrence of two solutions for the shrinking sheet. For higher values of $\varphi$, the thermal boundary layer increased, and the hydrodynamics velocity boundary layer decreased. Therefore, the temperature profile was dominant for higher values of $\varphi$ and the velocity profile was dominant for smaller values of $\varphi$. In comparison to the upper solution branches, the lower solution branch showed a greater boundary layer thickness for the velocity and temperature field.
Figure 12. Influence of $\phi$ on the temperature field for certain nanofluids.

For $\lambda = -1$, Figures 13 and 14 present the influence of certain values of $M$ on the velocity field and the temperature, respectively. The thickness of thermal boundary layers and hydrodynamic velocity was lowered for greater $M$ values. For the lower solution branch, the horizontal velocity at one point was reduced due to a higher value of $M$, and the opposite was true for the solution of the upper branch. For the first solution, increasing the value of $M$ for a minor region close to the sheet reduced the temperature. The degree of reverse cellular flow over the sheet was reduced with an enhancement in the value of the magnetic effect. Therefore, the fluid velocity advection of the velocity of fluid above the sheet was affected by the temperature field.

Figure 13. Effect of $M$ on the velocity field for different nanofluids.
Figure 14. Impact of $M$ on the temperature field for certain nanofluids.

Figure 15 represents the influence of higher values of the mass suction/injection effect on the temperature field for $\lambda = -1$. In the case of suction, with higher $f_w$, velocity values, the thermal boundary layers’ thickness was reduced. In short, the value of the $Nu$ was enhanced due to the phenomenon of suction. Because of the mass suction, the fluid restrained the vorticity diffusion and came near the sheet. Further, the thermal boundary layer thickness for the lower solution branch was higher than that for the upper solution branch.

Figure 15. Impact of $f_w$ on temperature field for different nanofluids.

Figures 16 and 17 present the influence of the stretching/shrinking effect on velocity and temperature fields, respectively, for the shrinking sheet case. In the case of the shrinking sheet, the thickness of thermal boundary layers and the velocity were enhanced as the effect of the velocity ratio increased. The boundary conditions presented in Equation (10) were also fulfilled asymptotically. So, we validated the occurrence of dual solutions and the results of analytical solutions.
Due to this, the polarization of nanoparticles decreased against the magnitude of the magnetic field. For the nanoparticle minimum value for absolute values of nanoparticles examined. So, Ag provided the maximum value and TiO\textsubscript{2} maximized and minimized the values of the density among the various kinds of nanoparticles. TiO\textsubscript{2} provided the maximum value and TiO\textsubscript{2} provided the minimum value for absolute values of nanoparticles. Nu\textsubscript{S} was highest for the silver nanoparticle due to its greater thermal conductivity among all the considered nanoparticles. For the nanoparticle TiO\textsubscript{2}, the minimum heat transfer rate was obtained because of the ascendancy of the conduction mode of heat exchange and its lower conductivity among all the considered nanoparticles. The same behavior was obtained by Freidoonimehr [43].

7. Physical Quantities

The influences of some inclusive effects will now be discussed in detail. The impact of some parameters, such as \( M \), \( \varphi \), \( f_w \) and \( \lambda \), are presented in Figures 18–24 for the cases of \( \lambda = 1 \) and \( \lambda = -1 \) against \( C_f \) and \( Nu \). For \( \lambda = 1 \), \( C_f \) was enhanced with an increase in \( f_w \) on temperature field for different nanofluids.

![Figure 16. Impact of \( \lambda \) on velocity field for different nanofluids.](image)

![Figure 17. Impact of \( \lambda \) on temperature field for different nanofluids.](image)
increased. The value of $N_u$ was reduced with an enhancement in $M$, as presented in Figure 19.

**Figure 18.** The variations in $C_{f_x}$ for broad variations in $M$ in cases of stretching and shrinking sheets for different nanofluids.

**Figure 19.** The variations in $N_u$ for certain values of $M$ for $\lambda = 1$ different nanofluids.

**Figure 20.** The variations in $C_{f_x}$ for certain values of $\lambda$ for both the cases of stretching and shrinking sheets for different nanofluids.

In Figure 22, $f_w$ within the boundary layer enhanced the velocity, and $C_{f_x}$ was reduced due to this effect. Similarly, an increase in $f_w$ across boundary layer led to a reduced $C_{f_x}$. Figure 23 presents an enhancement in $N_u$ with an enhancement in $f_w$. The main cause of this phenomenon was the superiority of conduction heat exchange to convection heat exchange.
Figure 20. The variations in $C_{f\lambda}$ for certain values of $\lambda$ for both the cases of stretching and shrinking sheets for different nanofluids.

Figure 21. The variations in $Nu$ for some variations in $\lambda$ for broad variations in $M$ across the stretching sheet for different nanofluids.

Figure 22. The fluctuations in $C_{f\lambda}$ for broad values of $f_w$ for the stretching and shrinking sheets for different nanofluids.

Figure 23. The variations in $Nu$ for a broad range of $f_w$ across the stretching sheet for different nanofluids.
An increase in $C_{f\lambda}$ was observed with the reduction in thermal conductivity and enhanced molecular weight of the nanoparticle volume fraction as presented in Figure 26. Additionally, it is shown in Figure 27 that $Nu$ was reduced for certain kinds of nanoparticles for a broad variety of volume fractions.

**Figure 24.** The variation in $C_{f\lambda}$ for broad values of $f_w$ in the cases of stretching and shrinking sheets for different nanofluids.

For $\lambda = 1$, $C_{f\lambda}$ was reduced with an increase in the number of nanoparticles in the occurrence of a constant magnetic field, as presented in Figure 18. The polarization of the nanoparticles and the lowered adhesive coefficient caused this effect. The reduction in $C_{f\lambda}$ occurred with an enhancement in $M$. Due to this, the polarization of nanoparticles increased. The value of $Nu$ was reduced with an enhancement in $M$, as presented in Figure 19.

It can be observed in Figure 20 that for positive $\lambda$, $C_{f\lambda}$ was reduced by an enhancement in $\lambda$ due to a high flow rate, and thus the boundary layer was significantly strengthened and the skin friction coefficient decreased. High polarization of nanoparticles and a strong magnetic field did not influence this trend for any value of $\lambda$. However, the negative $\lambda$ changed the flow direction, which led to significant changes in the velocity and $C_{f\lambda}$. It can be noted in Figure 21 that for a positive value of $\lambda$, the $Nu$ was reduced with an enhancement in the value of $\lambda$.

In Figure 22, $f_w$ within the boundary layer enhanced the velocity, and $C_{f\lambda}$ was reduced due to this effect. Similarly, an increase in $f_w$ across boundary layer led to a reduced $C_{f\lambda}$. Figure 23 presents an enhancement in $Nu$ with an enhancement in $f_w$. The main cause of this phenomenon was the superiority of conduction heat exchange to convection heat exchange.

It can be observed in Figure 24 that the growth in the velocity changes caused an enhancement in the suction/injection effect and magnetic effect. There was an increase in $C_{f\lambda}$ for positive $\lambda$ and a reduction for negative $\lambda$. So, it can be observed in Figure 25 that $f_w$ had a greater influence on the value of the $Nu$ number relative to $M$. So, the most effective parameter in heat transfer is $f_w$.

An increase in $C_{f\lambda}$ was observed with the reduction in thermal conductivity and enhanced molecular weight of the nanoparticle volume fraction as presented in Figure 26. Additionally, it is shown in Figure 27 that $Nu$ was reduced for certain kinds of nanoparticles for a broad variety of volume fractions.
The main results of the study are:

- The variation in the velocity and temperature fields, skin friction coefficient indicates a large thickness of the boundary layer in comparison to the magnetic field effect.

Additionally, it is shown in Figure 27 that $Nu$ was reduced for certain kinds of nanoparticles for the shrinking sheet case and have the opposite behavior in the stretching sheet case. Four different nanofluids have been considered, i.e., $Cu - H_2O$, $Ag - H_2O$, $Fe_3O_4 - H_2O$, and $TiO_2 - H_2O$.

Figure 25. The variation in $Nu$ for broad values of $f_w$ across the stretching sheet for different nanofluids.

Figure 26. The variation in $C_{f_x}$ for certain nanofluids for $\lambda = 1, -1$.

Figure 27. The variation in $Nu$ for different nanofluids for $\lambda = 1, -1$. 
8. Conclusions

In this paper, the flow problem and heat exchange have been examined for steady laminar nanofluid flow caused by the stretching/shrinking sheet. Four different nanofluids have been considered, i.e., Cu – H₂O, Ag – H₂O, Fe₃O₄ – H₂O and TiO₂ – H₂O. The analytic solution for the temperature and velocity fields has been established and discussed. For the shrinking sheet, it was concluded that the lower side result for both the velocity and temperature fields indicates a large thickness of the boundary layer in comparison to the upper side solution. The effects of the four governing thermo-physical parameters, $M$, $\varphi$, $\lambda$ and $f_{w}$, on the velocity and temperature fields, skin friction coefficient, and local $Nu$ have been studied. The main results of the study are:

- $C_{fx}$ gives the maximum value for Ag – H₂O and minimum value for TiO₂ – H₂O nanofluids in the shrinking sheet case and have the opposite behavior in the stretching sheet case.
- The Nusselt number gives the maximum value for Ag – H₂O and minimum value for TiO₂ – H₂O nanoparticles for the shrinking sheet case and have the opposite behavior in the stretching sheet case. However, as silver is expensive, we suggest using Cu – H₂O.
- For $Nu$, $f_{w}$ is more efficient in comparison to the magnetic field effect.
- By decreasing thermal energy and increasing the molecular weight of nanoparticles, the ratio of $C_{fx}$ to the nanoparticle volume fraction is enhanced.
- The Nusselt number reduces the four different kinds of nanoparticles with a vast range of volume fractions.
- $f_{w}$ has a prominent and efficient effect on heat transfer.

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Nomenclature

| $u, v$ | Velocity components |
| $\mu_{nf}$ | Dynamic viscosity of nanofluid |
| $B_{0}$ | Magnetic field |
| $k_{nf}$ | Thermal conductivity of nanofluid |
| $\psi(x, y)$ | Free stream function |
| $Pr$ | Prandtl number |
| $\lambda$ | Stretching/shrinking parameter |
| $\tau_{w}$ | Surface shear stress |
| $Re_{x}$ | Local Reynolds number |
| $q_{w}$ | Surface heat flux |
| $Nu_{x}/Re_{x}^{1/2}$ | Dimensionless Nusselt number |

$\rho_{nf}$ Density of nanofluid
$\sigma$ Electrical conductivity
$(\rho c_{p})_{nf}$ Heat capacitance of nanofluid
$\varphi$ Nanoparticle volume fraction parameter
$M$ Magnetic parameter
$f_{w}$ Mass suction/injection parameter
$C_{fx}$ Generalized Laguerre polynomial
$Re_{x}$ Skin friction coefficient
$Nu_{x}$ Local Nusselt number

The subscripts “$\varphi$”, “$f$” and “$nf$” represent the solid-, fluid- and nanofluid-state properties, respectively.

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