Abstract

We consider dark energy cosmology in a de Sitter universe filled with quantum conformal matter. Our model represents a Gauss-Bonnet model of gravity with contributions from quantum effects. To the General Relativity action an arbitrary function of the GB invariant, $f(G)$, is added, and taking into account quantum effects from matter the cosmological constant is studied. For the considered model the conditions for a vanishing cosmological constant are considered. Creation of a de Sitter universe by quantum effects in a GB modified gravity is discussed.
There is a growing interest towards various studies of the reported acceleration of our observable Universe \cite{1}. This is motivated by recent astrophysical data analyses, which hint to such a behavior. In order to explain it, the simplest possibility is to introduce a dark energy component, whose origin remains however uncertain.

The existence of such an energy, with almost uniform density distribution and a substantial negative pressure which completely dominates all other forms of matter, is inferred from recent astronomical observations \cite{1}. In particular, according to recent astrophysical analysis, this dark energy seems to behave like a cosmological constant, and is responsible for the accelerating expansion of the universe. And there are reasons to believe that the answer to this question has much to do with the possibility to explain the physics of the very early Universe.

Models of dark energy are abundant. One of the proposed candidates for it is the phantom, called so because it implies a negative energy field. The peculiar properties of a phantom scalar (with negative kinetic energy) in a space with non-zero cosmological constant have been discussed in an interesting paper by Gibbons \cite{2}. As indicated there, phantom properties bear some similarity with quantum effects \cite{3}. An important property of the investigation in \cite{2} is that it is easily generalizable to other constant curvature spaces, such as the Anti-de Sitter (AdS) space. There is presently considerable interest in such spaces, coming in particular from the AdS/CFT correspondence. According to it, the AdS space may have cosmological relevance \cite{4}, e.g. by increasing the number of particles created on a given subspace \cite{5}. It could also be used to study a cosmological AdS/CFT correspondence \cite{6}: the study of a phantom field in AdS space may give us a hint about the origin of such a field via the dual description. In the supergravity formulation, one may think of the phantom as a special RG flow for scalars in gauged AdS supergravity. (Actually, such an RG flow may correspond to an imaginary scalar.)

Another candidate for dark energy is the tachyon \cite{4,8}. This is an unstable field. The interest of models exhibiting a tachyon is motivated by its role in the Dirac-Born-Infeld (DBI) action as a description of the D-brane action \cite{9,10,11}. In spite of the fact that the tachyon represents an unstable field, its role in cosmology is still considered useful as a source of dark matter \cite{8,12} and, depending on the form of the associated potential \cite{13,14,15,16,17}, it can lead to a period of inflation. On the other hand, it is important to realize that a tachyon with negative kinetic energy (yet another type of phantom) can be introduced \cite{18}. In that phantom/tachyon model the thermodynamical parameter $w$ is naturally negative. In this case the late time de Sitter attractor solution is admissible, and this is one of the main reasons why it can be considered as an interesting model for the dark energy \cite{18}. Moreover, in order to understand the role of the tachyon in cosmology it is necessary to study its effects on other backgrounds, as in the case of an anti-de Sitter background \cite{7}.

On the other hand the origin of the so-called dark energy could be related with the problem of the cosmological constant. One of the most interesting approaches to this question is the modified gravity. Actually, it is not absolutely clear why standard General Relativity should be trusted at large cosmological scales. One may assume that, considering minimal modifications, the gravitational action contains some additional terms growing slowly in the
presence of decreasing curvature \[19, 20, 21, 22, 23\] (for a review, see \[24\]), which could be responsible for the current acceleration. In such a scenario one of the most accepted approaches is the model of Modified Gauss-Bonnet Gravity. In these models these additional terms in the gravitational action are introduced by adding to the action a function depending on the scalar curvature \(R\) and the Gauss-Bonnet invariant \(G\) \[25\]. In this way it is possible to demonstrate that such models lead to a plausible effective cosmological constant, quintessence, or a phantom era. From these results one may conclude that as concerns the role of a gravitational alternative for DE, the modified GB gravity may be a very important candidate \[26\].

In the present paper we consider a Gauss-Bonnet model of gravity with contributions from quantum effects. To the General Relativity action an arbitrary function of the GB invariant, \(f(G)\), is added. Taking into account quantum effects from matter the cosmological constant is studied. The conditions for vanishing of the cosmological constant are studied, and their effects on the stability of the de Sitter universe via quantum effects are discussed.

Let us begin with the action for the modified gravity in the following form \[26\]:

\[
S = \int d^4x \sqrt{-g} \left( F(G, R) + \mathcal{L}_m \right) .
\]  

(1)

Here \(G\) is the GB invariant:

\[
G = R^2 - 4R_{\mu\nu}R^{\mu\nu} + R_{\mu\nu\xi\sigma}R^{\mu\nu\xi\sigma} .
\]  

(2)

Varying over \(g_{\mu\nu}\) we get

\[
0 = T^{\mu\nu} + \frac{1}{2} g^{\mu\nu} F(G, R) \\
-2F_G(G, R)RR^{\mu\nu} + 4F_G(G, R)R^{\mu}_{\rho}R^{\rho\nu} \\
-2F_G(G, R)R^{\mu\rho\sigma\tau}R_{\rho\sigma\tau} - 4F_G(G, R)R^{\mu\rho\sigma\tau}R_{\rho\sigma\tau} \\
+2 (\nabla^{\mu}\nabla^{\nu} F_G(G, R)) R - 2g^{\mu\nu} (\nabla^2 F_G(G, R)) R \\
-4 (\nabla_\rho \nabla^{\mu} F_G(G, R)) R^{\mu\rho} - 4 (\nabla_\rho \nabla^{\nu} F_G(G, R)) R^{\mu\rho} \\
+4 (\nabla^2 F_G(G, R)) R^{\mu\nu} + 4g^{\mu\nu} (\nabla_\rho \nabla_\sigma F_G(G, R)) R^{\rho\sigma} \\
-4 (\nabla_\rho \nabla_\sigma F_G(G, R)) R^{\mu\rho\sigma} \\
-F_R(G, R)R^{\mu\nu} + \nabla^{\mu}\nabla^{\nu} F_R(G, R) \\
-2R^{\mu\nu} \nabla^2 F_R(G, R) ,
\]  

(3)

\(T^{\mu\nu}\) being the matter-energy momentum tensor, and where the following expressions are used:

\[
F_G(G, R) = \frac{\partial F(G, R)}{\partial G} , \quad F_R(G, R) = \frac{\partial F(G, R)}{\partial R} .
\]  

(4)

The spatially-flat FRW universe metric is chosen as

\[
ds^2 = -dt^2 + a(t)^2 \sum_{i=1}^{3} (dx^i)^2 .
\]  

(5)
Quantum effects will be taken into account by including the contributions from the conformal anomaly

\[ T = b \left( F + \frac{2}{3} \nabla^2 R \right) + b' G + b'' \nabla^2 R, \]  

(6)

where \( T \) is the trace of \( T^{\mu \nu} \) and \( F \) is the square of 4-D Weyl tensor,

\[ F = R_{\mu \nu \alpha \beta} R^{\mu \nu \alpha \beta} - 2 R_{\mu \nu} R^{\mu \nu} + \frac{1}{3} R^2. \]  

(7)

Note that such a conformal anomaly may be related to a bulk de Sitter space (see [27]).

Taking the trace of (3), we get the following equation

\[ 0 = T + 2F(G, R) - \frac{1}{3} R^2 F_G(G, R) - R(\nabla^2 F_G(G, R)) - 3 \nabla^2 F_R(G, R) - R F_R(G, R). \]  

(8)

We are interested in the de Sitter type solutions where the Ricci scalar, the Gauss-Bonnet invariant, and the square of 4-D Weyl tensor are constants:

\[ R = R_0, \quad G = G_0 = \frac{1}{6} R_0^2, \quad F = 0. \]  

(9)

Assuming the maximally symmetric metric solution, we get

\[ 0 = T + 2F(G_0, R_0) - \frac{1}{3} R_0^2 F_{G_0}(G_0, R_0) - R(\nabla^2 F_{G_0}(G_0, R_0)) - 3 \nabla^2 F_R(G_0, R_0) - R_0 F_R(G_0, R_0). \]  

(10)

For the CA we get

\[ T = b' G_0, \]  

(11)

and Eq. (10) takes the form

\[ \frac{1}{2} R_0 F_R(G_0, R_0) = F(G_0, R_0) - G_0 F_{G_0}(G_0, R_0) + b' G_0. \]  

(12)

As an example let us consider case when \( F(G, R) = R + f(G) \), (here \( 2\kappa^2 = 16\pi G = 1 \)); then one has

\[ G_0 f'(G_0) - f(G_0) - b' G_0 = \frac{R_0}{2}. \]  

(13)

In general, solving Eq. (12) in terms of \( R_0 \), one can rewrite the maximally symmetric solution as

\[ R^0_{\mu \nu} = \frac{R_0}{4} g^0_{\mu \nu} = \Lambda_{eff} g^0_{\mu \nu}. \]  

(14)
which defines an effective cosmological constant. For the considered example, when \( F(G, R) = R + f(G) \), one has

\[
\Lambda_{\text{eff}} = \frac{1}{2} \left( G_0 f'(G_0) - f(G_0) - b G_0 \right). \tag{15}
\]

In particular by taking \( F(G, R) = R \), we recover the well known result \[29\]

\[
R_0^2 = \frac{12}{b'} \Lambda_{\text{eff}}, \quad R_0 = -\frac{3}{b'}. \tag{16}
\]

It is important to note that the obtained result (Eq. (16)) leads us to the inflationary solution (for positive \( H \)) obtained first by Starobinsky in Ref. \[28\] using the renormalized EMT of conformal matter on the right-hand side of Einstein’s equations. This result shows the possibility of creation of a de Sitter inflationary universe from quantum effects in the way discussed in \[29, 30, 31, 32, 33, 34\].

Another interesting case is obtained by choosing \( f(G) = -\alpha G^\beta \). As a result one has

\[
\Lambda_{\text{eff}} = \frac{1}{2} \alpha G_0^\beta (1 - \beta + b') = \frac{R_0}{4}, \tag{17}
\]

and the solution for \( R_0 \) has the following form

\[
R_0 = \left[ 2\alpha (1 - \beta + b') \left( \frac{1}{6} \right)^\beta \right]^{\frac{1}{1-2\beta}}. \tag{18}
\]

Let us now consider several cases for solutions depending on the values of the parameter \( \beta \) in the function \( f(G) \).

First of all let us consider the situation where \( \beta \) is small. Then,

\[
R_0 \approx 2\alpha (1 + b'). \tag{19}
\]

Neglecting \( b' \) in the above result, i.e. if quantum effects are omitted, we get \( R_0 \sim \alpha \). This result coincides with that obtained in \[26\] for modified Gauss-Bonnet gravity without quantum effects. Furthermore, since \( b' \) is negative from the above results we see that the solution for \( R_0 \) and therefore for \( \Lambda_{\text{eff}} \) is lesser thanks to quantum effects for \( b' > -1 \). For \( b' = -1 \) the cosmological constant vanishes. In this way we conclude that the creation of the inflationary de Sitter universe occurs only when \( b' > -1 \).

Let us consider now the case where \( \beta = -\frac{1}{2} \). Then one obtains for \( R_0 \) the following solution,

\[
R_0 = (6)^{\frac{3}{4}} \sqrt{\alpha (3 + 2b'}). \tag{20}
\]

From Eq. \[20\] one sees that quantum creation of the de Sitter universe occurs when \( \alpha > 0 \) only for \( b' > -\frac{3}{2} \) and the effective cosmological constant becomes

\[
\Lambda_{\text{eff}} = \frac{1}{4} (6)^{\frac{3}{4}} \sqrt{\alpha (3 + 2b')} \tag{21}
\]
However one can see that de Sitter universe may also occur even if $b' < -\frac{3}{2}$ but in such a case $\alpha$ must be negative.

It is also of interest to see the behavior of the cosmological function depending on the value of the parameter $\beta$. One possibility is to evaluate this function taking, for the parameters $b'$ and $\alpha$, the values -0.5 and 1 respectively. As a result, from figure 1 it is clear that the cosmological constant is stable for any negative value of $\beta$ and vanishes asymptotically when $\beta$ goes to 0.5.

The combination of quantum effects with modified gravity may thus solve the cosmological constant problem bringing the effective cosmological constant to an extremely small value.

As a final point, let us consider again the case $f(G) = -\alpha G^\beta$. As noted above, $G_0 = \frac{1}{6} R_0^2$ for the de Sitter solution. If we now assume somewhat more generally that $G$ is proportional to some power of $R$, we can write the action as

$$S = \int d^4x \sqrt{-\bar{g}} \left( R - \alpha' R^\beta + \mathcal{L}_m \right),$$

(22)

$\alpha'$ and $\beta'$ being new constants. Constructing the equations of motion by the variational procedure, putting $T^{\mu\nu}$ on the right hand side as usual, one finds that the covariant divergence of the left-hand side is equal to zero. Energy-momentum conservation is accordingly a consequence of the field equations, just as in Einstein’s theory. This important property was demonstrated explicitly by Koivisto [35]. The form of action in Eq. (22) was also considered in [36] in connection with viscous modified gravity. This action means that we end up with the same energy-conservation equation as in Einstein’s theory:

$$\nabla^\nu T_{\mu\nu} = 0.$$  

(23)
It means that we are permitted to use the generalized form of energy-momentum tensor corresponding to a viscous fluid,

\[ T_{\mu\nu} = \rho U_{\mu}U_{\nu} + (p - \zeta \theta) h_{\mu\nu}, \]  

(24)

where \( \zeta \) is the bulk viscosity, \( \theta \) the scalar expansion, and \( h_{\mu\nu} = g_{\mu\nu} + U_{\mu}U_{\nu} \) the projection tensor (the shear viscosity is omitted). Physically, this means that we are working to the first order in deviation from thermal equilibrium. This property of the formalism supports the general consistency of the modified gravity theory.

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