Application of the Kita-Sako model to soil-water characteristic curves of bentonite/sand mixture

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ABSTRACT

Bentonite/sand mixture is used as one of the materials of the engineered barrier included in the geological disposal for the high-level radioactive waste. It is expected that the swelling of the barrier will be occurred due to decrease in the suction with the increase in the degree of saturation during the post-closure site management of them. The thermo-hydraulic-mechanical (T-H-M) analysis is usually conducted to simulate the behavior of the engineered barrier. Soil-water characteristic curve (SWCC) of the Bentonite/Sand Mixture is one of the important parameters in the T-H-M analysis. In this paper, the Kita-Sako model is employed to derive the SWCC of the bentonite/sand mixture. To check the applicability of the model for the Bentonite/Sand Mixture is the purpose of this paper. The concept of the model is firstly introduced and then the applicability of the model for Bentonite/sand mixture is checked. Finally, the improvement of the model is discussed, comparing the calculation results with the soil test results.

Keywords: soil-water characteristic curve, bentonite/sand mixture, grain size distribution, pore size distribution

1 INTRODUCTION

The bentonite-based material is used as one of the materials of the engineered barrier included in the geological disposal for the high-level radioactive waste. The ground water and/or gas around the disposal facility and the heat generated from the waste material will infiltrate the engineered barrier made of the bentonite-based material during the long-term control of the disposal facility. The swelling of the bentonite-based material due to water absorption and the shrinkage of the bentonite-based material due to dehydration will be generated. The thermo-hydraulic-mechanical (T-H-M) analysis is usually conducted to estimate the stability of the facility (e.g. Yamamoto and Komine (2008)). Soil-water characteristic curve (SWCC) of the bentonite-based material is one of the important parameters in the T-H-M analysis. The water retention test on the bentonite-based material has the time-consuming problem.

This paper presents the applicability of a conceptual model to estimate the SWCC of the bentonite-based material. The Kita-Sako model (Sako and Kitamura, 2006) is employed to estimate the SWCC of the bentonite/sand mixture (Kunigel V1 bentonite : sand mixture (70% bentonite dry mass ratio)). The model can derive the SWCC using the general soil parameters (e.g. the grain size, the void ratio and so on). Firstly, the concept of the model will be introduced. The applicability and the improvement of the model will be discussed comparing the calculation results with the soil test results.

2 OUTLINES OF THE KITA-SAKO MODEL

Fig.1(a), (b) and (c) respectively show the triaxial specimen, the enlarged cube with about 1cm taken from the triaxial specimen by spoon and the cube with which is called the Elementary Particulate Body (EPB).

Fig.1. Population, sample population and sample for triaxial specimen.
The particulate soil structure in the EPB shown in Fig.2(a) is modelled by the spherical particles as shown in Fig.2(b). The diameter of the spherical particles \( D_s \) and the contact angle \( \beta \) at the contact point are adopted as the random variables. Then, Fig.2(c) show the modelling of pore structure by the Elementally Particle Model (EPM). The pores in the EPB are modelled by the pipe with the diameter \( D_v \) and the inclination angle \( \theta \). The soil particles in the EPB are modelled by the other impermeable parts. \( D_v \) and \( \theta \) are also adopted as the random variables. The probability density functions of \( D_v \), \( D_s \), \( \beta \) and \( \theta \) are used to estimate the arrangement of soil particles and pores with random shape and size. Additionally, it is assumed that the diameter finer than 10% in the cumulative grain size distribution, \( D_{10} \), is empirically adopted as the characteristic length \( D_{cha} \).

### 2.1 Grain size distribution

It is well known that the grain size distribution curve of most coarse-grained soil can approximately be expressed by the logarithmic normal distribution (e.g. Bagnold and Barndorff-Nielsen, 1980). Hence, the cumulative grain size distribution can be expressed by using the probability density function as follows:

\[
 f_s(D_v) = \frac{1}{\sqrt{2\pi \sigma_s^2 D_s}} \exp \left( -\frac{(\ln D_v - \lambda_s)^2}{2\sigma_s^2} \right) \tag{1} 
\]

where \( D_s \): diameter of soil particle assumed to be spherical, \( \lambda_s \): mean value of \( \ln D_s \), \( \sigma_s \): standard deviation of \( \ln D_s \).

\[
 \lambda_s = \ln \mu_s - \frac{1}{2} \sigma_s^2 \tag{2} 
\]

\[
 \sigma_s^2 = \ln \left( 1 + \frac{\sigma_s^2}{\mu_s^2} \right) \tag{3} 
\]

where \( \mu_s \): mean value of \( D_s \), \( \sigma_s \): standard deviation of \( D_s \), \( \lambda_s \) and \( \sigma_s \), which are the values of distribution parameters in Eq.(1), can be calculated by using Eqs.(2) and (3). \( \mu_s \) and \( \sigma_s \) are obtained from the laboratory test results of the grain size distribution using the nonlinear regression analysis.

### 2.2 Pore size distribution

In this model, it is assumed that the pore size distribution in soil can also be expressed by the logarithmic normal distribution. Then the following equation is used.

\[
 f_v(D_v) = \frac{1}{\sqrt{2\pi \sigma_v^2 D_v}} \exp \left[ -\frac{(\ln D_v - \lambda_v)^2}{2\sigma_v^2} \right] \tag{4} 
\]

where \( D_v \): diameter of pore assumed to be circular, \( \lambda_v \): mean value of \( \ln D_v \), \( \sigma_v \): standard deviation of \( \ln D_v \).

\[
 \lambda_v = \ln \mu_v - \frac{1}{2} \sigma_v^2 \tag{5} 
\]

\[
 \sigma_v^2 = \ln \left( 1 + \frac{\sigma_v^2}{\mu_v^2} \right) \tag{6} 
\]

where \( \mu_v \): mean value of \( D_v \), \( \sigma_v \): standard deviation of \( D_v \).

Furthermore, it is assumed that the coefficient of variation \( \delta \) is same for grain size distribution and pore size distribution. It means that the grain size distribution is parallel to the pore size distribution. The equation of the coefficient of variation is expressed as follows.

\[
 \delta = \frac{\sigma_s}{\mu_s} = \frac{\sigma_v}{\mu_v} \tag{7} 
\]

\( \mu_s \) and \( \sigma_s \) are obtained from the grain size distribution, and then \( \delta \) is also derived from Eq.(7). The only unknown parameter in Eq.(4) becomes either \( \mu_v \) or \( \sigma_v \) through Eqs.(5) and (6). The details on the determination method of \( \mu_v \) or \( \sigma_v \) are mentioned later.

### 2.3 Distribution of contact angle and inclination angle

The probability density function of contact angle is assumed to be pentagonal shape as shown in Fig.3 is expressed as follows.

\[
 f_\beta(\beta) = \frac{2}{\pi^2 \delta^2} \beta^2 + \frac{2}{\pi} \beta \delta \tag{8a} 
\]

\[
 f_\theta(\theta) = \frac{2}{\pi^2 \delta^2} \theta^2 + \frac{2}{\pi} \beta \delta \tag{8b} 
\]

where \( \delta \): distribution parameter which describes the pentagonal shape.
Eq. (8) means that the tangential plane at the contact point tends to horizontal under gravity field. Here the ratio of height at $\theta = \pm \pi/2$ to $\theta = 0$ is tentatively assumed to be 1:3 referring to the measuring results of contact angle by the microscope (Oda (1972)). Then the distribution parameter $\zeta$ is calculated to be the value of 0.159.

It is considered that the water flow in EPB shown in Fig.2(a) has the predominant direction which may be parallel to the tangential plane shown in Fig.2(b). Then the direction of predominant flow expressed by the pipe with the inclination angle $\theta$ in Fig.2(b) is same as $\theta$ in Fig.2(c). Therefore the probability density function for the predominant flow direction is expressed as follows, referring to Eq.(8).

For $\frac{-\pi}{2} \leq \theta \leq 0$

$$f_\theta(\theta) = \frac{2}{\pi} \sin \theta + \frac{\zeta}{\pi}$$

(9a)

For $0 \leq \theta \leq \frac{\pi}{2}$

$$f_\theta(\theta) = -\frac{2}{\pi} \sin \theta + \frac{\zeta}{\pi}$$

(9b)

### 2.4 Derivation of void ratio, volumetric water contents and suction

The volume of elementary particulate model (EPM) and the volume of pipe in EPM by $V_{EPM}$ and $V_{EPM,p}$ shown in Fig.4 can be expressed by the functions of two random variables $D_v$ and $\theta$, as follows:

$$V_{EPM}=D_v \left( \frac{D_v}{\sin \theta} + \frac{D_{cha}}{\tan \theta} \right)$$

$$V_{EPM,p}=\pi \left( \frac{D_v}{2} \right)^2 \frac{D_{cha}}{\sin \theta}$$

(10)

(11)

The volume of the solid part in EPM shown in Fig.4 is obtained using Eqs.(10) and (11) as follows:

$$V_{EPM,s} = \phi_{EPM}(D_v,\theta) - \phi_{EPM,p}(D_v,\theta)$$

(12)

where $V_{EPM,s}$: volume of solid part in EPM.

Using Eqs.(4), (9), (10), (11) and (12), the equation of void ratio can be obtained:

$$e = \frac{V_v}{V_s} = \frac{V_{\psi,i}}{V_{\psi,i}}$$

(13)

As the void ratio $e$ on the left side of Eq.(13) is obtained from the laboratory test, the unknown parameter, $\mu_v$ or $\sigma_v$ in the probability density function shown by Eqs.(4), (5), (6) and (7) can be back-calculated numerically.

In the drying process of saturated soil, the entry of air into pores filled with water begins in larger pores and inversely in the wetting process of unsaturated soil the entry of water into pores filled with air begins in smaller pores. Considering these behaviors in the elementary particulate model, it can be assumed that the pipe with the range of diameter $0 < D_v \leq d_w$ is filled with water and the pipe with the range of diameter $d_w < D_v < \infty$ is filled with air. Then the following equation can be derived by using Equations (4), (9), (10), (11) and (13).

$$w_v = \frac{V_w}{V} = \frac{1}{1+e^E \left( \frac{V_{\psi,i}}{V_{\psi,i}} \right)}$$

(14)

As the volumetric water content $w_v$ on the left side of Eq.(14) is obtained, the unknown $d_w$, limit of the integral range which corresponds to the boundary between liquid phase and gas phase can be numerically back-calculated.

The suction can be obtained from the phenomena of the capillary rise in the narrow glass tube as shown in Fig.5. Assuming that the diameter of the narrow glass tube, $d$, is equal to the maximum diameter of pipe filled with water, $d_w$, derived by Eq.(14), the equation of suction can be expressed as follows:

$$s_u = \frac{4T_c \cos \alpha}{d_w}$$

(15)

where $s_u$: suction, $d_w$ maximum diameter of pipe filled with water, $T_c$: surface tension of water, $\alpha$: contact angle of meniscus ($\equiv 0$).

### 2.5 Procedure to obtain the SWCC

Fig.6 shows the procedure to obtain the SWCC by the Kita-Sako model. Firstly, the pore size distribution expressed by Eq.(4) is back-calculated using the grain size distribution and void ratio obtained from the
laboratory tests and Eq.(13). Then, $d_w$ can be derived by Eq.(14) and a value of the volumetric water content. Substituting $d_w$ into Eq.(15), the relationship between the volumetric water content and the suction (i.e. SWCC) can be calculated.

3 COMPARISON OF NUMERICAL EXPERIMENTS WITH SOIL TEST RESULTS

The Kita-Sako model is employed to estimate the SWCC of the bentonite/sand mixture (Kunigel V1 bentonite : sand mixture (70% bentonite dry mass ratio)). The soil tests have been conducted by Sato et al. (2017) and Yamamoto et al. (2017) in order to obtain the SWCC and the microscopic pore structure of compacted bentonite/sand mixture. The soil water characteristic curves were obtained using a dew-point mirror psychrometer (WP4 Dewpoint potentiometer, WP4-T, Decagon Devices, USA) and the pore size distribution were measured by the mercury intrusion porosimetry test.

3.1 Outlines of the experiments on the SWCC

The soil water characteristic curve of the bentonite/sand mixture was obtained by the WP4-T. The measurement range is from -300MPa to 0Mpa. A total suction is measured by the WP4-T. The osmotic suction is obtained by measuring the potential value of the saturated specimen, and the matric suction is obtained by the difference between the total suction and the osmotic suction. The size of specimens is 20mm in diameter and 7.5mm in height. The dry density of specimens is 1.6Mg/m³. The specimens with four kinds of initial water content, 17.0, 17.3, 24.0, 26.9% were prepared to conduct the water retention tests. The measurement results are shown in Fig.6. The osmotic suction at the saturated condition was 1.00Mpa in the experiments.

3.2 Input parameters for the Kita-Sako model

Table 1 shows the list of input parameters for the Kita-Sako model. The total density, the void ratio, the grain size distribution and so on are used as the input parameters in the model. Because the grain size distribution used in the soil test was not obtained, the grain size distribution simulated by bentonite/Toyoura sand mixture is used as the input parameter. The calculation results derived using the input parameters are shown and discussed in next section.

3.3 Calculation results

Fig.7 shows the grain size distributions and pore size distributions. The grain size distribution (solid line in Fig.5) can be derived by the logarithmic normal distribution shown in Eq.(1). Although the grain size distribution of the bentonite/sand mixture (circle points in Fig.5) has step-like curve, the calculation results cannot simulate the curve.

The pore size distribution is derived by Eq.(4). It is assumed that the coefficient of variation $\delta$ is same for the grain size distribution and the pore size distribution. Hence, the grain size distribution is parallel to the pore size distribution as shown in Fig.7. The unknown parameter, $\mu_v$ or $\sigma_v$ in the probability density function shown by Eqs.(4), (5), (6) and (7) was back-calculated numerically comparing the test results of void ratio with Eq.(13). Using the pore size distribution derived by Eq.(4) is used to calculate the volumetric curve and the suction.

| Grain size distribution back-calculated by Eq.(4) |
|-----------------------------------------------|
| Grain size distribution back-calculated by Eq.(14) |
| Suction (Eq.(15)) |

Fig.6. Procedure to obtain the SWCC.

Fig.7. Grain size distribution and pore size distribution.

| Table 1. Input parameters for the Kita-Sako model |
|--------------------------------------------------|
| Grain size distribution (mm) | Percentage finer by mass (%) |
|-----------------------------|-----------------------------|
| 0.800 | 100.00 |
| 0.425 | 99.99 |
| 0.250 | 72.91 |
| 0.106 | 71.00 |
| 0.075 | 70.02 |
| 0.0457 | 69.49 |
| 0.0205 | 66.01 |
| 0.0119 | 62.54 |
| 0.0084 | 59.06 |
| 0.0012 | 48.64 |
The pore size distribution back-calculated from the experimental data of the SWCC are shown as the square points in Fig. 7. In this model, the pore size distribution can be recalculated by the change of the value of $\mu_v$ comparing with the experimental data of the SWCC. The recalculated result of the pore size distribution is drawn in Fig. 7. Fig 9 shows the SWCC derived using the recalculated grain size distribution. It seems that the recalculated result gets close to the experimental data.

3.4 A consideration on the pore size distribution

Yamamoto et al. (2017) have conducted the mercury intrusion porosimetry test to obtain the pore size distribution on the bentonite/sand mixture. The pore size distributions under initial condition ($S_r=65\%$), saturated condition, dehydrated condition ($s_u=1.3\text{MPa}$) were obtained as shown in Fig. 10. Fig. 10 shows the change in the pore size distribution with the change in the degree
of saturation.

The size of the diameter of pipe as shown in Fig.11, which is used in the recalculation, is smaller than the pore size distribution obtained from the mercury intrusion porosimetry test as shown in Fig.10. The pore size distribution derived by Eq.(4) is defined as the distribution of the number of the pipe with the diameter of $D_v$. On the other hand, the pore size distribution obtained from the mercury intrusion porosimetry test is defined as the relationship between the pore size and the pore volume. It seems that the difference of the definition of pore size distribution generates the difference of the value of the pore size distribution.

The probability density functions of the experimental data have two peaks in the curve because the grain size distribution of bentonite/sand mixture shows the step-like curve as shown in Fig.7. On the other hand, Fig.11 shows the recalculated result of the probability density function of pore size. In this calculation, two peaks in probability density function of the experimental data are not appeared in the calculation results as shown in Fig.11 because the grain size distribution of the bentonite/sand mixture were approximated by the logarithmic normal distribution. It is necessary to propose the approximation method for mixture materials.

4 CONCLUSIONS

The applicability of the conceptual model to estimate the SWCC of the bentonite-based material was discussed in this paper. The application of the Kita-Sako model was tried to estimate the SWCC of the bentonite/sand mixture (Kunigel V1 bentonite:sand mixture (70% bentonite dry mass ratio)) although the model was proposed for the coarse-grained soil. The concept of the model was explained in detail, and the comparison with the numerical results and the soil test results was conducted. Using the Kita-Sako model, the SWCC of the bentonite/sand mixture can be derived by the general soil parameters (e.g. grain size distribution, void ratio and so on). However, it is necessary to consider the definition of the pore size distribution in order to check the validity of the numerical results, and to propose the approximation method of the grain size distribution for the bentonite/sand mixture.

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