Experimental investigation on predictive models for motive flow calculation through ejectors for transcritical CO₂ heat pumps

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Abstract. Nowadays, air conditioning systems, especially those used in residential and office buildings, contribute largely to the energy consumptions and to the direct and indirect emissions of greenhouse gases. Carbon dioxide (CO₂) is an interesting option to replace traditional HFCs in vapor compression systems, due to its environmentally friendly characteristics: zero ODP and extremely low GWP. In the case of heat pumps, the use of ejection systems for the expansion phase can contribute to recovery a fraction of the mechanical energy otherwise dissipated as friction, bringing to significant benefits in terms of performance. Currently, at the laboratory DTE-PCU-SPCT of the research center ENEA (Casaccia) in cooperation with the Industrial Engineering Department of Federico II University of Naples, a project is in progress, in order to evaluate experimentally the effect of several ejectors geometries on the global performance of a CO₂ heat pump working with a transcritical cycle. As a part of this project, measurements of the motive flow mass flow rate have been carried out, in transcritical CO₂ conditions. The ejector sizing is a crucial point for the balancing of components and the correct operation of the CO₂ heat pump and therefore the availability of reliable calculation methods for the motive flowrate would be useful.

This paper presents the results obtained by a comparison between the new experimental data and the predictions of some predictive semi-empirical correlations available in the open literature for transcritical CO₂ conditions. Their predictions are analyzed as a function of the main physical parameters of the process to assess their reliability compared to the experimental data. Based on these indications and of the available experimental data, a new semi-empirical correlations and a calculation method based on the hypothesis of isentropic and choked two-phase flow are presented.

1. Introduction
Recently, an experimental campaign on a 30 kW invertible air-to-water heat pump, that uses CO₂ as a working fluid and designed for residential heating and cooling and hot water production, was
conducted at ENEA’s Casaccia Research Center. The choice of CO₂ as refrigerant is suggested by many positive features (non-flammability, non-toxicity, environmental compatibility), which make it a viable alternative to synthetic refrigerants banned by the latest F-gas regulation [1].

As it is well known, CO₂ heat pumps have significantly higher COPs than similar machines using synthetic refrigerants in domestic hot water production applications (characterized by low-gas-cooler inlet water temperatures and high water temperature lift). On the contrary, in residential heating and cooling applications, CO₂ heat pumps may have considerably lower performances than conventional machines [2], due to different gas-cooler operating conditions (high water temperature at gas-cooler inlet and lower temperature lift). In these conditions, the energy dissipated in the lamination phase increases, with consequent reduction of the COP. To reduce such losses, instead of the usual expansion devices, an ejector system can be used to recover part of the expansion work [3].

Many experimental studies on two-phase ejector expansion systems in transcritical carbon dioxide refrigeration cycles have been carried-out. Among others, Elbel et al. [4] made a comparison between a transcritical CO₂ ejector system and a conventional expansion valve system. Results show improvements of COP and cooling capacity of respectively 7% and 8% when using two-phase ejector. In transcritical refrigeration cycles, Lucas et Koehler [5] estimate an improvement up to 17% of the COP with the ejector cycle compared to the maximal COP with expansion valve cycle.

Unlike refrigeration machines, the heat pump works with highly variable operative conditions. For this reason, the use of a single ejector may not bring the desired benefits in all operating conditions. In this regard, the use of several ejectors, working in parallel and conveniently sized, can maximize the efficiency of the system in every operative condition.

The heat pump tested at ENEA facilities had a multi-ejector expansion pack as throttling device.

One of the objectives of the experimental campaign was the development of a thermofluidynamic simulation model calibrated on the experimental data to determine criteria and maps for the ejector sizing. In order to improve the model, correlations to predict efficiency and motive mass flow rates through the ejector are required.

In literature, some methods and semi-empirical correlations for the calculations of the motive mass flow rate were found: one is proposed by Lucas et al. [6]; another is based on the experimental investigation of a R744 multi-ejector by (Banasiak et al [7]; a third one is suggested by Martin et al. [8] for short tube orifices used as expansion devices in CO₂ refrigerant systems. In this paper, the experimental data have been used to calculate the mass flow rate through ejectors by the above mentioned semi-empirical correlations. Besides a new semi-empirical correlation was developed and evaluated by means of a comparison between the literature correlations [6-8] and the experimental data. Finally, an original correlation is proposed, based on the hypotheses of isentropic and choked two-phase flow.

2. Literature survey of existing predictive methods specifically developed for CO₂

In a CO₂ heat pump, the ejectors typically operate with inlet pressures higher than critical value (73.77 bar); moreover, the outlet pressures depend on the evaporator pressure and are usually lower than 40 bar. In this situation, it is reasonable to suppose that the motive flow is choked, Lucas [6], Martin [8]; consequently, the ejector throat pressure is equal to the pson and the mass flow rate only depends on the input conditions.

The accurate prediction of the two-phase mass flow rate in sonic condition is very difficult due to incomplete knowledge of the complex thermal-fluid dynamic phenomena that occur between the two phases. In particular, the sound velocity in two-phase flow can change quickly with the vapour quality [9]. Its calculation is still a matter of discussion M.-S. Chung [10]. Moreover, specific behaviors, issues as metastable conditions and large slip ratio should be taken into account. The equation for calculating the dischargeable mass flow rate “Gₐ” through a throttling device having a geometric seat area “A” is usually defined as:

\[ G_t = k_d G_t A \]
where $G_t$ is the theoretical max flux in an ideal (isentropic) nozzle and $k_d$ is the two-phase “discharge coefficient”.

Reliable methods for the calculation of mass flow rate when the inlet conditions are supercritical are not available in the literature. This essentially depends on the fact that the available methods use physical quantities related to the vapour state, which are not present in supercritical conditions; e.g., the ISO method [11], derived from the previous HEM [12,13] method, is not recommended when the input pressure is greater than 50% of the critical thermodynamic pressure of the fluid.

In the open literature there are only a few semiempirical correlations based on experimental data to calculate $G$ for CO$_2$ in supercritical inlet conditions.

Lucas et al [6], propose a method based on the data of an experimental campaign [16] on a multi-ejector heat pump using CO$_2$ as refrigerant.

Lucas method assumes that the expansion in the converging driving nozzle is isentropic and, consequently, the driving mass flow rate can be calculated by equations:

$$u_{th} = \sqrt{2 \cdot (h_{in} - h_{th})}$$

$$G_{th} = A_{th} \cdot p_{th} \cdot u_{th}$$

The density and the enthalpy at the throat can be estimated knowing the throat entropy (equal to the inlet entropy for isentropic flow) and the pressure at the throat.

Correlating experimental data with eq. (2) and (3) with an iterative method, Lucas suggests calculating the ratio $\eta_e$ between the throat pressure and the inlet pressure with the next equation:

$$\eta_e = \frac{P_{th}}{P_{in}} = 0.0871942 \cdot \frac{P_{in}}{p_{cr}}^{0.9519907} \cdot \frac{P_{in}}{\rho_{cr}}^{2.348013} + \frac{P_{in}}{\rho_{cr}} + 0.39387$$

For the kind of data set used by Lucas (e.g. inlet pressure supercritical, evaporation pressure lower than 40 bar, inlet temperature equal or higher than the CO$_2$ critical temperature (30.98 °C)), $p_{th}$ corresponds to the choked pressure [8].

Banasiak et al [7] investigated the performance of a multi-ejector for R744 vapour compression units too; they proposed the following correlation for calculation of mass flow rate

$$G = \frac{\pi}{4} \phi^2 \left[ A \phi \rho_{in}^2 + B \phi_{in} + C \phi_{in} \frac{P_{in}}{p_{cr}} + D \phi_{in} \frac{P_{in}}{p_{cr}} + E \right]$$

where $A,B,C,D,E$ are coefficients adjusted for every ejector.

For example, for a $\phi_{th} = 0.7$ mm, these coefficients are [7]: $A = 1.71938 \cdot 10^{-1}$ m$^4$ kg$^{-1}$ s$^{-1}$, $B = -6.06326 \cdot 10^{-1}$ m$^4$ s$^{-1}$, $C = 4.55787 \cdot 10^1$ kg m$^{-2}$ s$^{-1}$, $D = 4.98027 \cdot 10^4$ m$^2$ kg s$^{-1}$, and $E = -5.46798 \cdot 10^4$ m$^3$ kg s$^{-1}$.

Martin et al. [8] conducted an experimental investigation on short tube orifices used as expansion devices in CO$_2$ refrigerant systems; they suggested a model with dimensionless numbers:

$$G = \phi^3 \sqrt{\rho_{in} \phi_{in} A \left( L \phi \right)^B \left( \frac{p_{in}}{p_{cr}} \right)^C \left( \frac{T_{in}}{T_{cr}} \right)^D \left( \frac{\phi}{\phi_{sha}} \right)^E}$$

where $A,B,C,D,E$ are parameters optimized to reach a minimal deviation between measurement and calculation. In the article the values of the coefficients obtained from experimental tests for a short tube are reported, with $\phi = 0.01$ mm and $L = 0.02$ m: $A = 0.36, B = -0.015, C = 0.9, D = 5, E = 0.36$.

We have tried to find a new semi-empirical correlation to obtain better mass flow rate predictions, using the experimental data available. In particular, in addition to dimensionless parameters already
used in the previous correlations, a new geometric parameter was introduced to take into account the ejector geometries.

$$G = \phi^2 \frac{\pi}{4} k \left( A \left( \frac{p_m}{p_e} \right)^B \left( \frac{\rho_m}{\rho_e} \right)^C \left( \frac{T_m}{T_e} \right)^D \left( \frac{L}{\phi_m} \right)^E + F \right)$$ (7)

A, B, C, D, E and F are calculated by a mathematical solver that reach the minimum MSE value and $k=10^5$ is a constant used to optimize the solver process.

Another correlation is proposed, considering that the saturation pressure, calculated with the isentropic expansion hypothesis from the input conditions, could be an indicator of the ejector output conditions. In fact:

- during the expansion at pressures lower than 40 bar in the convergent of the motive nozzle, saturation conditions are often reached [8]
- it can be assumed that, in these situations, the sonic velocity is reached at low vapour quality [10]
- the flow through orifices and nozzles can be assumed isentropic

The pressure thus calculated, called $p_{ise}$, has been correlated to the measured flow rate; the investigations have allowed the identification of a simple correlation expressing the mass flow rate as function of the parameter $\eta_{ise} = \frac{p_{ise}}{p_m}$ and the ejector diameter $\phi_{ej}$:

$$G = \phi_{ej}^2 \frac{\pi}{4} \left[ a + b \eta_{ise} \right]$$ (8)

Where $a$ and $b$ are function of $\phi_{ej}$ and are determined by experimental data.

$$a = \left( A + B \phi_{ej} + C \phi_{ej}^2 \right)$$ (9)

$$b = \left( D + E \phi_{ej} + F \phi_{ej}^2 \right)$$ (10)

### 3. Experimental setup

#### 3.1 Test facility

The experimental data used in this paper were obtained testing a 30 kW CO$_2$ air-water heat pump having a multi-ejector expansion pack as throttling device. The tests were performed using the experimental facility “Calorimetro Enea” at ENEA (Casaccia) research center. This climatic chamber allows testing air-to-water reversible heat pump, with thermal capacity until 50 kW, according to UNI EN 14511/2011 [15].

The experimental set consists of an alternative semi-hermetic compressor (CP) driven by an inverter, a plate heat exchanger (GC), a finned coil (EVAP), a plate internal heat exchanger (IHE), an electronic valve (EEV) and a multi-ejector expansion pack (EJEC). Figure 1 shows the multi-ejector CO$_2$ system including four different ejector geometries, with throat diameters from 0.7 mm to 2.0 mm.
The heat pump control system can active each ejector independently, depending on boundary conditions, with 15 different configurations; an ejector schematic is shown in figure 2.

Temperature and pressure sensors are installed at the inlet and outlet of each component. K-type and J-type thermocouples were placed to measure the temperatures and piezoelectric transmitters to measure the pressure. An electromagnetic transmitter measures the water volumetric flow rate. The electrical power is measured by a wattmeter. CO₂ mass flow rate is measured with a Coriolis flow meter. All measurement instruments are characterized by high accuracy, according to [15].

Table 1 summarizes the instrument specifications and their uncertainty, as indicated by the manufactures.

| Measurement                        | Range/Unit          | Uncertainty          |
|------------------------------------|---------------------|----------------------|
| Temperature (K-type)               | 0/150 °C            | ± 1.1 °C             |
| Temperature (J-type)               | -40/80 °C           | ± 1.1 °C             |
| Pressure                           | 0-60/0-100/0-160 bar| 0.08%                |
| Water volumetric flow rate         | 0/3.33x10⁻³ m³/s    | 0.02% of reading     |
| Electrical power                   | 0/25 kW             | Precision class 0.5  |
| CO₂ mass flow meter                | 0/0.95 kg/s         | ± 0.10% of reading   |
3.2 Experimental procedure

All tests were run according to [15]. The boundary conditions in terms of air temperature, relative humidity and water temperature and mass flow rate were fixed and kept constant during the tests. In addition, it was possible to set the outlet evaporator and inlet compressor superheating. Furthermore, it was possible to set manually the chosen ejectors configuration; tests here analyzed were done with ejectors 2, 3, 4 worked one by one. The trends of the thermodynamic measured variables were monitored via a software designed specifically. The main thermodynamic parameters were recorded and processed using Matlab software. The instabilities generated by the control system regulation are limited in accordance to [15].

Table 2 shows the motive nozzle throat section diameters and the available experimental data subdivided by the inlet temperature range. All tests are with inlet pressure higher than critical. The outlet pressures are between 20÷40 bar for all the tests and they are determined by typical operating conditions of heat pumps in heating applications.

| Ejectors [#] | nozzle throat diameter [mm] | Test |
|--------------|----------------------------|------|
|              |                            | T<\text{cr} | T>\text{cr} | Tot |
| 1            | 0.7                        | -           | -           | -   |
| 2            | 1.0                        | 18          | 0           | 18  |
| 3            | 1.4                        | 15          | 4           | 19  |
| 4            | 2.0                        | 7           | 12          | 19  |

Figure 3 shows the tests input conditions in the T-s diagram. Most of the ejector 4 tests have T>T\text{cr}, while the ejectors 2 and 3 tests are almost all with T<T\text{cr}.

4 Results and discussion

The mass flow rate calculated applying Lucas's correlation (2), (3), (4) with the coefficients proposed in [5] for p\text{th} calculation, figure 4, are affected by high errors (MSE =37.5 \times 10^{-5}). Using the solver to optimize coefficients of (4) respect to experimental data, predictions are improving, but maintaining a high value of MSE =21.18 \times 10^{-5}.
Figure 4. \(G_c\) error\% vs \(p_m\) - Lucas (2,3,4)

The recalculated coefficients in (4) are: 0.86535, -0.50933, -0.26973 and the constant term is 0. Figure 5 shows mass flow rate perceptual errors as a function of the ejector \(p_m\): predictions are almost always underestimated, but for ejector 4 are closest to the optimal values in comparison with the predictions for the other ejectors. Figure 6 depicts the percentage errors for Banasiak correlation (5) optimized with new experimental data \((A=0.07561, B=30.66, C=-3468.23, D=47016.37, E=-53737.9)\); MSE improves to \(13.11 \times 10^{-5}\) and error values are between -20 and 20\%. The three ejector predictions are almost parallel and errors increase by passing ejector 2 to ejector 4, remarking an uncaptured effect of the size of the system in this type of correlation. To understand these results we would like to point out that, in the spirit of this study, we have tried to use this correlation for the three ejectors simultaneously while the author has suggested that the coefficients for each geometry should be defined. Indeed, using the correlation individually for each ejector, as indicated by Banasiak, the obtained MSEs for ejectors 2 and 3 are good, about 2 and 4 \(10^{-6}\) respectively, and the calculated coefficients are close together. Instead, prediction for ejector 4 showed a relatively high MSE, \(9.8 \times 10^{-5}\) and very different coefficients.

Figure 6. \(G_c\) error\% vs \(p_m\) - Banasiak (5) with coefficients recalculated by solver

In Martin’s correlation (6), developed for short tubes, there are two terms related to geometry. To fit the ejector geometry, the term \(\phi/\phi_m\) related to \(E\) coefficient, was excluded and the other, \(L/\phi\), was calculated by setting \(L=\)nozzle length and \(\phi=\) average nozzle diameter. Martin’s correlation predictions
are good with $\text{MSE} = 4.83 \times 10^{-5}$ with $A=1.04901$, $B=0.72643$, $C=1.50484$, $D=-1.11549$; $E$ was not calculated. Almost all errors are between -10 and 10% and there are no particular differences for the three ejectors (figure 7).

Figure 8 depicts the predictions of the new correlation (7) where the parameter $L/\phi$ is the same introduced for Martin’s correlation and coefficient are: $A=2.7415$, $B=0.4084$, $C=0.0590$, $D=0.05048$, $E=-0.2360$ and $F=-2.3640$. MSE value is $4.08 \times 10^{-5}$ and most errors are between -7% and 6% and do not seem to be particularly affected by the ejector geometry.

Fig. 9 shows the relationship between the specific mass flow rate and the $\eta_{\text{in}}$ for the three ejectors with the relative linear fit function.

By interpolating the terms of the three linear functions, we obtained the coefficients $A=-60017$, $B=329680$, $C=-112530$, $D=203090$, $E=-473400$, $F=155440$ to be inserted into (9) and (10) to complete the correlation (8), that expresses the flow rate according to the $\eta_{\text{in}}$ and the ejector diameters.

Figure 10 depicts the $G$ errors of this method as function of pin for the three ejectors. The MSE=4.02 $10^{-5}$ is almost equal to that of the previous correlation. The most errors are between -5% and 11% and do not seem to be particularly affected by the ejector geometry.
In keeping with eq. (1), calculations have shown how correlations without a parameter related to ejector geometry have worse predictions.

The correlations analyzed, except for (8), use the transcritical conditions \((p_{cr}, \rho_{cr}, T_{cr})\) to look for representative parameters of the mass flow rate or to compute \(\eta_c(4)\). Also the reliability of calculation method (8) depends on input conditions because they may affect the validity of its initial hypotheses.

In these situations, different input conditions, even considering that expansion is exiting from the transcritical zone, may be a reason for the different reliability of correlations. By going to check the test input conditions (figure 3 and figure 11) we noticed that almost all the tests performed with the ejector 4 were characterized by different values of some physical inlet quantities compared to the other ejectors; the most of experimental points relative to ejectors 2 and 3 are almost in the same range of entropy \((1.16\div1.28 \text{ kJ/kg})\) and temperature \((23\div32 \degree C)\) while tests of ejector 4 have \(s=1.2\div1.43 \text{ kJ/kg}\) and \(T_{in}=23\div43 \degree C\).

This could explain because:

- the correlation (5), used individually for each ejector, has good predictions for ejectors 2 and 3 and the calculated coefficients are close together while its reliability is much worse for ejector 4.
- the three tests circled in the figure 11, have worse predictions for that geometry (ejector 3) with every correlations.

5 Conclusions

In order to improve the knowledge about ejector performance, experimental tests were carried-out at the DTE-PCU-SPCT laboratory of the ENEA (Casaccia) research center. A complete heat pump system with multi-ejector pack has been tested in a climatic chamber. Transcritical motive nozzle inlet conditions, with pressure range between 74 bar and 98 bar, have been investigated. Therefore, new experimental data in terms of motive nozzle mass flow rate have been obtained. The results have been elaborated using three existing semi-empirical correlations, Lucas et al. [5], Banasiak et al. [7] and Martin et al [8] in order to find the best methods to assessing the motive nozzle mass flow rate. Moreover, a new semi-empirical correlation and a new calculation method have been proposed and evaluated. We observed that:

- The semi-empirical correlations available in literature seem to be very dependent on experimental test from which they have been deduced (Lucas, Banasiak) and require dedicated coefficients for each specific geometry.
- Semi-empirical correlations (Martin, new correlation) that provide a parameter related to the ejector geometry can be used with the same coefficients for different ejectors.
- The method calculating the mass flow rate as a function of \(\eta_{inc}\) and \(\phi_{th}\) of ejectors is interesting and has comparable performance to the new correlation.
- The semi-empirical correlation of Martin, the new correlation and the isentropic method are able to predict the ejector 4 mass flow rate with an acceptable accuracy, while the correlation of Banasiak, which is optimal for the individually treated ejectors 2 and 3, has significantly worse predictions for ejector 4. Instead, Lucas's correlation has the best performance for ejector 4.
- Elaborations performed on the new correlation showed that the introduction of the geometric parameter involves an improvement of more than 50% of the MSE.
- There is a dependence on the reliability of the isentropic method from the test input conditions.

The objectives pursued in the future of the research will concern:

- The identification and study of phenomena occurring during expansion and their correlation with the evolving mass flow rate.
- The improvement of the semi-empirical correlations and the isentropic method.
In particular, it will be evaluated the possibility of identifying methods and/or correlations to be used with reference to CO2 input conditions.

**Nomenclature**

| Symbol | Description | Units |
|--------|-------------|-------|
| A      | area        | (m²)  |
| G      | mass flow rate | (kg/h) |
| h      | specific enthalpy | (J/kg) |
| k_d    | discharge coefficient | (-) |
| T      | temperature | (°C) |
| u      | velocity    | (m/s) |
| s      | specific entropy | (kJ/kg K) |
| p      | pressure    | (bar) |
| L      | length      | (m)  |

**Subscripts**

- c: calculated
- cr: thermodynamic critical conditions
- cha: chamfer
- ise: isentropic
- in: inlet conditions
- m: average value
- r: actual
- s: specific
- t: theoretical

**Greek Letters**

- φ: diameter | (m) |
- ρ: density  | (kg/m³) |
- η: pressure ratio | (-) |
- th: throat conditions
- sn: sonic (choked) flow condition

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