Computation of shock waves in media with an interphase boundary by the CIP-CUP method on an adaptive grid

T S Guseva
Institute of Mechanics and Engineering, Kazan Science Center, Russian Academy of Sciences, 420111 Kazan, Russian Federation
E-mail: ts.guseva@mail.ru

Abstract. A numerical technique of computing shock waves in compressible media with movable deforming interphase boundaries including those of the gas-liquid type has been realized. The approach without explicit separation of the interphase boundary is applied. The CIP-CUP method is used for integrating the equations of gas dynamics. An adaptive grid of special kind (the soroban-grid) is utilized. Some results of testing the technique using one- and two-dimensional problems are given. Results of computation of impact of a jet on a thin liquid layer on a wall are presented.

1. Introduction
Shock processes in media with deforming interphase boundaries arise, in particular, at high-speed impacts of jets and drops on the surfaces of bodies. In the case the interphase boundary is deformed quickly and strongly, the intensity of the shock waves is moderate the CIP-CUP method (Constrained Interpolation Profile - Combined Unified Procedure) is quite effective [1]. One feature of this method is an opportunity of calculating the dynamics of the media being in contact without explicit separation of the contact boundaries (including those of the gas-liquid type) [2]. A numerical technique based on the CIP-CUP method with the staggered Eulerian grids was realized and tested in [3]. In problems with shock waves and fast changing contact boundaries the adaptive grids are more preferable, and are sometimes necessary. However the CIP-CUP method has a number of features reducing its efficiency or complicating its application on the structured adaptive (non-uniform) grids. In [4], a new way of adaptive discretization of a computational domain using the soroban-grid ("soroban" in Japanese means an abacus), and also a corresponding modification of the CIP-CUP method were proposed. In the present paper, some results of computing shock waves in the liquid and gas media being in contact are given, which show possibility of effective application of this modification to calculating high-speed jet impact on the body surface including the case with a liquid layer covering it.

2. Mathematical model and calculation procedure
An example of two-dimensional soroban-grid is given in figure 1. The grid points are located on parallel straight lines and are not connected among themselves by any elements like cells. The use of staggered grids is complicated, but there are no restrictions which could be caused by deformations of the grid. On each time step a new grid not connected with the old one is constructed. The distance between the lines, their number, and also the distance between the grid points on each line and their number can change and be different for different lines.
Adaptation of the grid is determined by a monitor function which, in the one-dimensional case, can be written as 

$$M(x,t) = (1 + \alpha f_x^2)^{1/2} + \beta \left| f_{xt} \right|$$

where \( f \) is a parameter of the solution, for example, the density. In the two-dimensional case the monitor function is then written as 

$$M(x,y,t) = (1 + \alpha (f_x^2 + f_y^2))^{1/2} + \beta \left| f_{xt} \right|$$

Dynamics of the liquid and gas media being in contact without allowing for the effects of viscosity, heat conductivity and phase changes is described by a set of equations of the form

$$\rho \frac{D\mathbf{u}}{Dt} = \nabla \cdot (\nabla p - \rho \mathbf{u})$$

where \( \rho \) is the density, \( \mathbf{u} \) the velocity, \( p \) the pressure, \( \mathbf{c}_s = \mathbf{c}_{s1} + (1 - \varphi) \mathbf{c}_{s2} \) the sound speed, \( \gamma \) is the isentropic exponent, \( \beta = \beta_1 \) for the liquid and \( \beta = \beta_2 = 0 \) for the gas. The identifier \( \varphi = 1 \) in the liquid area and \( \varphi = 0 \) in the gas area.

In the CIP-CUP method equations (1) are split into the advection and non-advection parts. The CIP method belonging to semi-Lagrangian ones is applied to calculating each of the advection equations making up the convective part. On the new step \( n+1 \) the solution of the advection equation at the grid point of the new grid with coordinates \( x^{n+1} \) can be approximately presented as

$$f^n = f^n \left( x^{n+1} - \mathbf{u}(x^{n+1}) \right)$$

where the argument is a departure point, i.e. the location of the "Lagrangian particle" which during \( \Delta t^n \) will arrive at the grid point \( x^{n+1} \). If the location of the departure point does not coincide with the grid point \( x^n \) of the old grid at which \( f^n \) is already known, interpolation is applied. Thus, the displacement of the grid points and recalculation of the values from the old grid to the new one are combined in one procedure. Monotonicity, small numerical diffusion and dispersion of the CIP method are determined by effective interpolation [2].

The non-advection part of system (1) is reduced to the equation

$$\frac{p^{n+1} - p^n}{\rho^2 \Delta t^n} = \nabla \cdot \left( \nabla \cdot \frac{\mathbf{u}}{p^2} \right) - \nabla \cdot \mathbf{u}^n$$

which is solved by the SOR method in this case. The CIP interpolation is applied to determining the values of parameters at the nodes of the stencil of the finite difference scheme if they do not coincide with the grid points.

In solving problems with shock waves the CIP-CUP method is used in combination with the artificial viscosity [4]

$$\mathbf{u}^{n+1} = \mathbf{u}^{n} - \Delta t^n \nabla q_x / \rho^n, \quad p^{n+1} = p^{n} - \Delta t^n (\kappa - 1) q_x \nabla \cdot \mathbf{u}^n, \quad q_x = \rho^2 \left( -c_{s1} \Delta \mathbf{U} + c_{s2} 0.5(\kappa + 1) \Delta \mathbf{U}^2 \right), \quad \mathbf{U} = \min \left( 0, \lambda \nabla \cdot \mathbf{u}^n \right),$$

\( c_{s1}, c_{s2} \) are the coefficients of the linear and quadratic viscosity of the media, \( \kappa = \gamma_1 \) in the liquid and \( \kappa = \gamma_2 \) in the gas, \( \lambda = (\Delta x \Delta y)^{1/2} \), \( \Delta x, \Delta y \) are the local spatial grid steps.
3. Computational results

3.1. Riemann problem with discontinuity on the gas-liquid boundary

At the initial moment the gas ($\gamma = 1.4$) and the liquid ($\Gamma = 7.15$, $B = 3072$ bar) are at rest, while the pressure is discontinuous on the contact boundary at $x_0 = 360$ m. In the liquid area ($x > x_0$): $\varphi = 1$, $p = 14088$ bar, $\rho = 1.216\rho_0$ ($\rho_0 = 10^3$ kg/m$^3$), in the gas area ($x < x_0$): $\varphi = 0$, $p = p_0 = 1$ bar, $\rho = 10^{-3}\rho_0$. Figure 2 presents comparison of the results of calculation on the grid with $\Delta x_{\min} = \Delta x/2$ and $\Delta x_{\max} = 2\Delta x$ ($\Delta x = 5.7$ m, $\alpha = 3$, $\beta = 0.5$) with the exact solution. The calculations were carried out with $c_{v1} = c_{v2} = 0$ in the liquid and with $c_{v1} = c_{v2} = 0.7$ in the gas.

![Figure 2](image)

**Figure 2.** Spatial distributions of the pressure (a), density (b) and velocity (c) at $t = 0.35$ s in the Riemann problem with discontinuity on the gas-liquid boundary, $u_0 = 540$ m/s. The exact solution is given by solid lines, the numerical solution is shown by symbols.

3.2. Cylindrical explosion in a liquid

At the initial moment inside of the cylindrical domain $(x - x_0)^2 + (y - y_0)^2 < R_c^2$ ($x_0 = y_0 = 1500$ m, $R_c = 297$ m) there is gas: $\varphi = 0$, $\gamma = 1.4$, $p = 10^4$ bar, $\rho = 720$ kg/m$^3$, and outside it there is liquid: $\varphi = 1$, $\Gamma = 7.15$, $B = 3072$ bar, $p = p_0 = 1$ bar, $\rho_0 = 10^3$ kg/m$^3$, the both media are at rest.

![Figure 3](image)

**Figure 3.** The problem of cylindrical explosion. At the time moment $0.22$ s: the grid (a), the density contours (b), the density-profiles in the section $y = y_0$ (solid line) and the one-dimensional axisymmetric solution (symbols) (c).

The problem was solved in the two-dimensional planar statement, and also in the one-dimensional statement with axial symmetry. The initial distribution of the parameters in the both cases was set on the uniform grid with the spacing $\Delta = 7.5$ m. The coefficients of artificial viscosity in the liquid are $c_{v1} = 1.2$, $c_{v2} = 0.4$. Figure 3 shows some results of computation on the soroban-grid with $\Delta_{\min} = \Delta/4$, $\Delta_{\max} = 8\Delta$, $\Delta = 30$ m ($\alpha = 1.1$, $\beta = 0.73$).

3.3. Impact of a jet on a liquid layer on a wall

Impact of an axisymmetric liquid jet on a rigid wall covered with a thin layer of the same liquid is considered (figure 4a). In the liquid: $\varphi = 1$, $\Gamma = 7.15$, $B = 3072$ bar, in the gas: $\varphi = 0$, $\gamma = 1.33$. The end of the jet is hemispherical, the velocity of the jet is $V = 250$ m/s, its radius is $R = 12.7$ $\mu$m, the liquid and
gas pressure is $p=1.25$ bar, the liquid density is $\rho=10^3$ kg/m$^3$, the gas density is 0.34 kg/m$^3$, the thickness of the liquid layer on the wall is $6.25\times10^{-3}R$.

Figure 4. Impact of a jet on a liquid layer on a wall: the scheme (a), the interphase surface (bold lines), the pressure contours (thin lines), the velocity vectors at moments 2 and 4 in the half of the axial section in the vicinity of the end of the jet (b, c); the pressure profiles on the wall (d), $p_{wh}=5$ kbar is the waterhammer pressure under the given conditions. Moments 1–6 correspond to $t/\tau=0.01, 0.017, 0.029, 0.039, 0.049, 0.065$, $\tau=R/C_{S1}=8.6$ ns.

Figure 4 presents the results of computation on the grid with the minimum step $\Delta_{\text{min}}=4.4\times10^{-5}R$ and the maximum step $\Delta_{\text{max}}=6.3\times10^{-3}R$. In the liquid $c_{v1}=0.6$, $c_{v2}=0.3$, in the gas $c_{v1}=c_{v2}=0.9$. During impact of the jet on the layer two shock waves arise: one moves up the jet, another propagates in the layer to the wall and reflects from it (figure 4b). At the moment the second wave reaches the wall the pressure in the center of the area of its reflection ($r=0$) is approximately equal to the waterhammer pressure $p_{wh}$ and then it decreases. On the boundary of the reflection area the pressure profiles have a growing maximum due to the growth in the pressure on the edge of contact between the jet and the layer (figure 4d). Interaction of the shock wave with the wall is separated into the stages of regular (figure 4b) and irregular (figure 4c) reflection.

4. Conclusion
A numerical technique of computing shock waves in media with moving boundaries of the gas-liquid type has been realized. The approach without explicit separation of the interphase boundary, the CIP-CUP method and the adaptive sorban-grid were applied. The one- and two-dimensional tests confirm the reliability of the created technique. Some results of computation of impact of a jet on a liquid layer on a wall are given.

Acknowledgments
The author would like to thank Prof. Alexander A Aganin for useful discussions. This work was supported by the Russian Foundation for Basic Research (project No 14-01-97004 ru povolzhje a).

References
[1] Yabe T and Wang PY 1991 J. Phys. Soc. Japan 60 2105–08
[2] Yabe T, Xiao F and Utsumi T 2001 J. Comput. Phys. 169 556–93
[3] Aganin A A and Guseva T S 2012 Uchenye Zapiski Kazanskogo Universiteta. Seriya Fiziko-Matematicheskie Nauki 154 74–99 (in Russian)
[4] Takizawa K, Yabe T, Tsugawa Y, Tezduyar T E and Mizoe H 2007 Comput. Mech. 40 167–83