THE INCLINATION ANGLE AND EVOLUTION OF THE BRAKING INDEX OF PULSARS WITH PLASMA-FILLED MAGNETOSPHERE: APPLICATION TO THE HIGH BRAKING INDEX OF PSR J1640–4631

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ABSTRACT

The recently discovered rotationally powered pulsar PSR J1640–4631 is the first to have a braking index measured, with high enough precision, that is greater than 3. An inclined magnetic rotator in vacuum or plasma would be subject not only to spin-down but also to an alignment torque. The vacuum model can address the braking index only for an almost orthogonal rotator, which is incompatible with the single-peaked pulse profile. The magnetic dipole model with the corotating plasma predicts braking indices between 3 and 3.25. We find that the braking index of 3.15 is consistent with two different inclination angles, 18°.5 ± 3° and 56° ± 4°. The smaller angle is preferred given that the pulse profile has a single peak and the radio output of the source is weak. We infer the change in the inclination angle to be at the rate ~0°.23 per century, three times smaller in absolute value than the rate recently observed for the Crab pulsar.

Key words: pulsars: general – pulsars: individual (PSR J1640–4631) – stars: evolution

1. INTRODUCTION

The rotationally powered pulsar PSR J1640–4631 recently discovered by NuSTAR in X-rays may hold clues to the origins of pulsar spin-down. PSR J1640–4631 has a spin frequency of ν = 4.843 Hz and a spin-down rate of ˙ν = −2.28 × 10−11 s−2 (Archibald et al. 2016), implying a characteristic age of τc ≡ −ν/2 ˙ν = 3370 yr and spin-down power of 5.5 × 1051 erg s−1 (Gotthelf et al. 2014). Interestingly, it is the first pulsar to have a braking index (i.e., n ≡ ν/2 ˙ν) greater than 3, n = 3.15 ± 0.03 (Archibald et al., 2016), measured with high precision. This is unusual given the previous measurements (Lyne et al. 1993, 1996; Livingstone et al. 2007; Espinoza et al. 2011; Livingstone & Kaspi 2011; Roy et al. 2012; Antonopoulos et al. 2015; Archibald et al. 2015; Ferdman et al. 2015) for eight other pulsars with braking indices n < 3 (see Lyne et al. 2015, for a compilation).

A dipole rotating in vacuum is subject to torque due to magnetic dipole radiation (MDR) with spin-down and alignment components respectively given as

\[
I \frac{dΩ}{dt} = -\frac{2\mu^2\Omega^3}{3c^3} \sin^2 \alpha \tag{1}
\]

\[
I \frac{d\alpha}{dt} = -\frac{2\mu^2\Omega^2}{3c^3} \sin \alpha \cos \alpha \tag{2}
\]

(Davis & Goldstein 1970; Michel & Goldwire 1970) where Ω = 2πν is the angular velocity and μ is the magnetic moment of the star, α is the inclination angle between rotation and magnetic axes, and c is the speed of light. This model predicts a braking index of

\[ n = 3 + 2 \cot^2 \alpha \tag{3} \]

(Davis & Goldstein 1970; Michel & Goldwire 1970), which is always greater than 3 and diverges for small inclination angles. Such a model predicts rapid alignment within a spin-down timescale. According to this model of a vacuum magnetic dipole, a pulsar would stop spinning down when alignment is achieved, in obvious contradiction with observations. It was suggested by Goldreich (1970) that the progress of alignment would be slowed down by dissipative processes for a non-spherical pulsar. Therefore, the effect of the alignment torque given in Equation (2) has not been adequately appreciated in the literature. As a result it has been rarely used together with the spin-down torque given in Equation (1) although the alignment torque is an intrinsic component of the torque due to the magnetic dipole.

The recent observational evidence suggesting an increasing inclination angle of the Crab pulsar (Lyne et al. 2013; see also Ge et al. 2016) implies that the magnetospheric torques can dominate the dissipative processes invoked by Goldreich (1970). Yet the increasing inclination angle of the Crab pulsar may require a further ingredient such as the presence of return currents in the magnetosphere (Beskin & Nokhrina 2007) or precession (Arzamasskyi et al. 2015; Zanazzi & Lai 2015). But an orthogonal rotator with plasma-filled force-free magnetosphere (Philippov et al. 2014) requires a much larger current than used in Beskin & Nokhrina (2007), so that the minimum spin-down energy losses correspond to that of an aligned rotator in the end. There is also statistical evidence that the inclination angle of pulsars tends to achieve alignment in the long term (Lyne & Manchester 1988; Young et al. 2010).

The pulsar is unlikely to be rotating in vacuum; its magnetosphere is expected to be filled by a corotating plasma formed through charged particles ripped off from the surface of the neutron star (Goldreich & Julian 1969) and thereafter accelerated by a rotation-induced electric field along curved magnetic field lines to give an excess of electron–positron pair discharges (Philippov & Spitkovsky 2014). As shown by recent simulations (Chen & Beloborodov 2014; Philippov et al. 2015), only such a configuration is capable of keeping a pulsar active. The spin-down (Spitkovsky 2006) and alignment (Philippov et al. 2014) torques in the presence of a corotating plasma also predict alignment, but at a slower pace compared to that of the vacuum model. Moreover, the model predicts a braking index
of \( n = 3\)–3.25 depending on the inclination angle (Arzamasskiy et al. 2015). This obviates any necessity to invoke other assumptions to address the measured braking index of PSR J1640–4631, \( n = 3.15 \pm 0.03 \) (Archibald et al. 2016).

We show within the framework of the plasma-filled magnetosphere model that there are two possible solutions for the inclination angle of PSR J1640–4631. We infer the magnetic dipole moment and rate of change of the inclination angle of the pulsar, and provide the implications for the evolution of pulsars on the \( P - \dot{P} \) diagram. In Section 2 we describe the details of the model and its consequences. In Section 3 we discuss the results in view of braking indices less than 3.

2. EVOLUTION OF PULSARS WITH PLASMA-FILLED MAGNETOSPHERES

The spin-down (Spitkovsky 2006) and alignment (Philippov et al. 2014) torques in the presence of a corotating plasma are

\[
\frac{d\Omega}{dt} = -\frac{\mu^2 \Omega^3}{c^3} \left( 1 + \sin^2 \alpha \right) \tag{4}
\]

\[
\frac{d\alpha}{dt} = -\frac{\mu^2 \Omega^2}{c^3} \sin \alpha \cos \alpha. \tag{5}
\]

Although these equations seem to be only slightly different from the equations of the vacuum model given in Equations (1) and (2), they predict a slower alignment (Philippov et al. 2014) and imply the spin-down of the pulsar even when alignment is achieved as originally predicted by Goldreich & Julian (1969).

Using only Equation (4), the braking index in this model is

\[
n = 3 - 4\dot{\alpha} \tau_c u \tag{6}
\]

where

\[
u(\alpha) = \frac{\sin \alpha \cos \alpha}{1 + \sin^2 \alpha}. \tag{7}
\]

Using Equation (5) in Equation (6), the braking index can be calculated as

\[
n = 3 + 2\nu^2 \tag{8}
\]

(Arzamasskiy et al. 2015), which implies \( 3 < n < 3.25 \) depending on the inclination angle, and it does not diverge as its vacuum counterpart for small inclination angles. In Figure 1 we show this prediction of the plasma-filled magnetosphere model (Spitkovsky 2006; Philippov et al. 2014) together with that of the vacuum model (Davis & Goldstein 1970; Michel & Goldwire 1970) given in Equation (3). Accordingly, the plasma-filled model can explain the braking index of \( 3.15 \pm 0.03 \) observed from PSR J1640–4631 (Archibald et al. 2016) for two different inclination angles, \( 18.5 \pm 3^\circ \) and \( 56^\circ \pm 4^\circ \). Of these we favor the smaller value because the pulse profile shows a single peak (Archibald et al. 2016).

From the measured braking index of PSR J1640–4631 (Archibald et al. 2016) we find \( u = \sqrt{(n - 3)/2} = 0.274 \pm 0.025 \) by using Equation (8). Using this result in Equation (6), the rate of decrease of the inclination angle can be found as

\[
\dot{\alpha} = -(0.23 \pm 0.05)^\circ \text{ century}^{-1}. \tag{9}
\]

This value is about three times smaller (in absolute value) than the measured increasing inclination angle of the Crab pulsar (Lyne et al. 2013).

By dividing Equation (4) by Equation (5) and then integrating, one finds that \( \cos^2 \alpha / P \sin \alpha \) is a constant throughout the evolution of the pulsar (Philippov et al. 2014): as \( P = 2\pi / \Omega \) increases, \( \cos^2 \alpha / \sin \alpha \) should also increase, which is achieved for small values of \( \alpha \). Thus we define

\[
A = \frac{\cos^2 \alpha_0}{P_0 \sin \alpha_0} = \frac{\cos^2 \alpha}{P \sin \alpha} \tag{10}
\]

where \( P_0 \) is the initial spin period and \( \alpha_0 \) is the initial inclination angle. For \( P = 0.2065 \text{ s} \) and \( \alpha = 18.5^\circ \) we find \( A = 13.72 \) for this pulsar. Solving \( \sin \alpha \) from the above equation gives

\[
\sin \alpha = \sqrt{1 + \left( \frac{AP}{2} \right)^2} - \frac{AP}{2}. \tag{11}
\]

Accordingly the derivative of the period is related to the period by

\[
\dot{P} = \frac{(2\pi)^2 \mu^2}{Ic^3} \left( 1 + \sin^2 \alpha \right) \tag{12}
\]

where \( \sin \alpha \) is from Equation (11) and \( A \) is from Equation (10). We infer the magnetic dipole moment of the pulsar in units of \( 10^{30} \text{ G cm}^3 \) as \( \mu_{30} = 11.2 \) from the measured spin frequency and its derivative from Equation (4), employing \( \alpha = 18.5^\circ \) and \( I = 10^{45} \text{ g cm}^2 \). The position of the pulsar is shown in the \( P - \dot{P} \) diagram in Figure 2.
3. DISCUSSION

We have shown that the braking index greater than 3 of the recently discovered pulsar PSR J1640–4631 (Archibald et al. 2016) is readily explained by the plasma-filled magnetic dipole model (Goldreich & Julian 1969; Spitkovsky 2006; Philippov et al. 2014; Arzamasskiy et al. 2015). The only free parameter of the model is the inclination angle and there are two possible solutions: $18.5^\circ \pm 3^\circ$ and $56^\circ \pm 4^\circ$. The smaller inclination angle is favored because of the single peak in the pulse profile (Rankin 1983, 1990; Weltevrede & Johnston 2008; Hankins & Rankin 2010). The relatively small radio output of this object, with an upper limit of $0.018 \text{ mJy}$ at 1.4 GHz (Archibald et al. 2016), may also be a consequence of this small inclination angle of the pulsar.

Note that the vacuum model as employed by Archibald et al. (2016) and also seen in Figure 1 predicts an almost orthogonal rotator. As this is not compatible with the single-peaked pulse profile, Archibald et al. (2016) disfavored the idea that the alignment of the rotation and magnetic axes, i.e., the decrement in the inclination angle, is the cause of the braking index greater than 3 of PSR J1640–4631 and suggested that the quadrupole magnetic moment of this object could be important. The model with plasma-filled magnetosphere, on the other hand, does not require an orthogonal rotator to produce the observed braking index of PSR J1640–4631 as seen in Figure 1. Given that the presence of charged particles around pulsars is well established (see, e.g., Goldreich & Julian 1969), we conclude that the evolution of the inclination angle of PSR J1640–4631 toward an aligned rotator, within the framework of the plasma-filled magnetosphere model (Spitkovsky 2006; Philippov et al. 2014), is a simpler and better understood explanation of the observed braking index.

Another possibility for explaining the braking index greater than 3 could be an anomalous $\nu$ resulting from a glitch (Alpar & Baykal 2006). We note that $n = 3.15 \pm 0.03$ is not an anomalous braking index within the framework of the plasma-filled magnetosphere model (Arzamasskiy et al. 2015).

Archibald et al. (2016) favors the quadrupole structure of the magnetosphere (Petri 2015) for explaining the braking index greater than 3 of PSR J1640–4631. We show here that this process alone, with no assistance from the alignment torque, would require very strong quadrupole fields to increase $n$ up to 3.15. In this case the spin-down luminosity due to the quadrupole field has to be $(n - 3)/(5 - n) \approx 10\%$ of the dipole spin-down luminosity. This implies that the Poynting flux of the quadrupole field is 10% of that of the dipole field. This then requires the quadrupole field to be $\sqrt{0.1} \approx 0.3$ times the dipole field at the light cylinder, $R_L = c/\Omega \approx 10^9 \text{ cm}$. Assuming a stellar radius of $R_* = 10^6 \text{ cm}$, this implies a quadrupole field $\sim 300$ times stronger than the dipole field! Given that the dipole field at the equator is $B_0 = \mu/R^3 \approx 10^{13} \text{ G}$, we infer a quadrupole field of $B_0 \approx 3 \times 10^{15} \text{ G}$ for this object. The radio polarimetry measurements, for normal pulsars, indicate that the field is of dipolar form at heights $\lesssim 30R_*$ where radio emission is generated (Karastergiou & Johnston 2007). Hence such a discrepancy between the dipole and quadrupole fields at the...
surface is unusual for normal pulsars. Yet we cannot exclude the possibility that PSR J1640–4631 has superstrong quadrupole fields unlike those of ordinary pulsars. The presence of such superstrong quadrupole fields would render the object similar to the ~800 year-old pulsar PSR J1846–0258 located in the Kesteven 75 supernova remnant (Gotthelf et al. 2000), which emitted several magnetar-like bursts (Gavriil et al. 2008). The dipole field of PSR J1846–0258 is about $5 \times 10^{13}$ G, i.e., five times stronger than that of PSR J1640–4631, and is marginally beyond the quantum-critical limit, $B_q = 4.4 \times 10^{13}$ G. If PSR J1640–4631 also has such strong quadrupole fields near the surface it may be expected to show magnetar-like bursts likely associated with some glitch activity in the future. Such pulsars with superstrong quadrupole fields may be another manifestation of low-B magnetars (Rea et al. 2010) as argued by Perna & Pons (2011).

Given that the measured braking index greater than 3 of PSR J1640–4631 can be addressed by employing the conventional view of a pulsar with corotating magnetosphere, the question naturally arises why most other measured braking indices are less than 3. Many different models have been suggested to explain braking indices less than 3. Some models invoke an external torque similar to that from stellar winds (e.g., Ou et al. 2016) or from a putative fallback disk (e.g., Michel & Dessler 1981; Menou et al. 2001; Özşükän et al. 2014). Others modify the magnetic dipole model by suggesting that the magnetic dipole fields are increasing (Blandford & Romani 1988), e.g., due to diffusion following post-supernova field burial by accreted matter (e.g., Geppert et al. 1999; Espinoza et al. 2011; Güneyda & Ekşi 2013; Ho 2015) or by poloidal field growth at the expense of interior toroidal field decay through the Hall effect (Gourgouliatos & Cumming 2015). Another interesting suggestion is that the increasing inclination angles of pulsars are increasing in the long term (Beskin et al. 1984), as possibly observed in the Crab pulsar (Lyne et al. 2013). Yet the latter observation might result from precession (Arzamasskiy et al. 2015; Zanazzi & Lai 2015). Yet another modification of the dipole model emphasizes the finite-size effects for the dipole given the presence of a corotating plasma (Melatos 1997).

We showed that a magnetic dipole within a plasma-filled magnetosphere does produce braking indices greater than 3. Therefore, invoking a combination of mechanisms (e.g., wind and magnetic quadrupole moments assisting the plasma-filled magnetosphere) appears to be unnecessary to explain the braking index of PSR J1640–4631. It is particularly unclear how these physical mechanisms affect the inclination angle. Any process that could potentially account for the braking index less than 3 should start reducing the braking index from higher values (i.e., $n = 3–3.25$ as predicted by Arzamasskiy et al. 2015) rather than the canonical value $n = 3$ assumed by neglecting the alignment torque.

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