Inverse Compton Signatures of Gamma-Ray Burst Afterglows

H. Zhang¹*, I.M. Christie²†, M. Petropoulou³, J.M. Rueda-Becerril¹, & D. Giannios¹,4,5

1Department of Physics, Purdue University, 525 Northwestern Avenue, West Lafayette, IN, 47907, USA
2Center for Interdisciplinary Exploration & Research in Astrophysics (CIERA), Physics & Astronomy, Northwestern University, Evanston, IL 60208, USA
3Department of Astrophysical Sciences, Princeton University, 4 Ivy Lane, Princeton, NJ 08544, USA
4Department of Physics, Physics & Astronomy, University of Crete, Voutes, GR-70013, Heraklion, Greece
5Institute of Astrophysics, Foundation for Research and Technology Hellas, Voutes, GR-70013, Heraklion, Greece

Abstract

The afterglow emission from gamma-ray bursts (GRBs) is believed to originate from a relativistic blast wave driven into the circumburst medium. Although the afterglow emission from radio up to X-ray frequencies is thought to originate from synchrotron radiation emitted by relativistic, non-thermal electrons accelerated by the blast wave, the origin of the emission at high energies (HE; \(\gtrsim\) GeV) remains uncertain. The recent detection of sub-TeV emission from GRB 190114C by MAGIC raises further debate on what powers the very high-energy (VHE; \(\gtrsim\) 300 GeV) emission. Here, we explore the inverse Compton scenario as a candidate for the HE and VHE emissions and consider two sources of seed photons for scattering: synchrotron photons from the blast wave (synchrotron self-Compton or SSC) and isotropic photon fields external to the blast wave (external Compton). For each case, we compute the multi-wavelength afterglow spectra and light curves. We find that SSC will dominate particle cooling and the GeV emission, unless a dense ambient infrared photon field, typical of star-forming regions, is present. Additionally, considering the extragalactic background light attenuation, we discuss the detectability of VHE afterglows by existing and future gamma-ray instruments for a wide range of model parameters. Studying GRB 190114C, we find that its afterglow emission in the Fermi-LAT band is synchrotron-dominated for \(t \lesssim 10^3\) s, but it later becomes SSC-dominated. The late-time Fermi-LAT measurement (i.e., \(t \sim 10^4\) s) also sets an upper limit on the energy density of a putative external infrared photon field (i.e., \(\lesssim 7.5 \times 10^{-8}\) erg cm\(^{-3}\)). Finally, we predict that the VHE flux at \(10^4\) s, which is still SSC dominated, is \(3 \times 10^{-11}\) erg cm\(^{-2}\) s\(^{-1}\).

Key words: (stars:) gamma-ray burst: general – radiation mechanisms: non-thermal

1 INTRODUCTION

Gamma-ray bursts (GRBs) are short and intense pulses of gamma-rays that are produced by internal energy dissipation in collimated, relativistic plasma outflows launched by the collapse of massive stars (Woosley 1993; Paczyński 1998; MacFadyen & Woosley 1999) or the merger of compact objects (Goodman 1986; Paczynski 1986; Kochanek & Piran 1993). The prompt gamma-ray signal (\(\sim\)100 keV - 100 MeV) is followed by a broadband long-lasting emission, the so-called afterglow. This is thought to be produced by non-thermal radiative processes of particles accelerated at a relativistic blast wave that the outflow drives into the circumburst medium (Meszaros et al. 1994; Sari et al. 1998; Dermer & Chiang 1998; Chiang & Dermer 1999; Piran 2004; Fan et al. 2008).

Over the past decade the Fermi Large Area Telescope (LAT) has detected dozens of bursts at energies beyond 100 MeV, thus opening a new window to the electromagnetic GRB emission. The high-energy (100 MeV – 100 GeV) GRB emission usually rises quickly following the prompt keV-MeV component with a small (\(\sim\) second-long) delay (Omodei 2009; Ghisellini et al. 2010; Ghirlanda et al. 2010).
and decays with time as \( \propto t^{-\chi} \) with \( \chi \approx 1.2 \) (Ghisellini et al. 2010; Ackermann et al. 2013; Nava et al. 2014). The origin of the high-energy emission is still under debate, with possible interpretations including electron synchrotron radiation from the blast wave (Kumar & Barniol Duran 2009; Ghisellini et al. 2010), proton synchrotron radiation (Vietri 1997; Totani 1998; Asano & Inoue 2007; Razzaque et al. 2010), or proton-induced cascades (Dermer & Atoyan 2006; Asano & Inoue 2007; Asano et al. 2009, 2010; Murase et al. 2012; Petroponlou et al. 2014). Alternatively, gamma-ray photons can be produced by the inverse Compton scattering of low energy seed photons from relativistic electrons accelerated at the blast wave. The seed photons can be of synchrotron-self Compton (SSC) models, e.g. Dermer et al. 2000; Sari & Esin 2001; Zhang & Mészáros 2001; Nakar et al. 2009) or have an external origin (external Compton (EC) models, e.g. Beloborodov 2005; Fan et al. 2005; Fan & Piran 2006; Wang & Mészáros 2006; Giannios 2008; Beloborodov et al. 2014).

Long-duration GRBs (LGRBs), i.e. those with durations longer than \( \sim 2 \text{ s} \), are believed to be associated with the death of Wolf-Rayet (WR) stars (Woosley 1993; MacFadyen & Woosley 1999; Hjorth et al. 2003). Since its original proposition, this formation scenario has been supported by many multi-wavelength observations of LGRB host galaxies. More specifically, LGRBs are commonly found in the brighter inner regions of their hosts (e.g. Fruchter et al. 2006; Blanchard et al. 2016; Lyman et al. 2017). The ultraviolet (UV) light from young stellar populations (Massey & Hunter 1998; Crowther 2007) in the star-forming regions of the host galaxy can be absorbed by interstellar dust and re-emitted in the infrared (IR). If the galaxy contains copious amounts of dust (as is the case for massive and luminous galaxies), then nearly all of the UV starlight can be reprocessed into the IR/FIR (Casey et al. 2014). Studies of optically reddened or undetected bursts (i.e. “dark” GRBs) reveal that most of the host galaxies of those dust-obscured LGRBs are massive dusty star-forming galaxies (e.g. Krühler et al. 2011; Perley et al. 2013, 2017; Chirmes et al. 2018).

The presence of UV and/or IR ambient radiation fields at the explosion sites of LGRBs may have an impact on the high-energy afterglow emission. Giannios (2008) showed that the UV emission emitted by a massive star within the same star-forming region of the GRB progenitor, can be upscattered by the electrons accelerated in the external shock, thereby producing a powerful gamma-ray (i.e. \( 1 - 100 \text{ GeV} \)) event (see also Lu et al. 2015). Here, we generalize the model of Giannios (2008) by including the effects of EC scattering of an IR ambient photon field associated with the star-forming regions of the GRB host galaxy. By considering the IR photons, we predict more scatterings within the Thomson regime and more powerful \( \sim \text{TeV} \) emission, as opposed to the upscattering of UV photons. Taking into account the accompanying SSC emission, we explore the detectability of the combined Compton signals from GRB afterglows at high-energies by current and next-generation Cherenkov telescopes.

This paper is organized as follows. In Section 2, we determine the parameter regime in which the EC component dominates the high-energy afterglow emission while showing results of multi-wavelength afterglow spectra including synchrotron, SSC, and EC radiation. In Section 3, we discuss the high-energy light curves predicted by our analytical model for both SSC-dominated and EC-dominated regimes. In Section 4, we discuss the effects of the extragalactic background light (EBL) attenuation on the high-energy afterglow emission and present our model predictions for the detectability of GRB afterglows by the next-generation Cherenkov Telescopes Array (CTA). Finally, in Section 5, we discuss the recent MAGIC detection of GRB190114C in the context of Compton afterglow emission models. Our conclusions are provided in Section 6.

2 THE MULTI-WAVELENGTH AFTERGLOW EMISSION

In the following, we generalize the treatment of Sari & Esin (2001) for the synchrotron and SSC afterglow emission by computing the Compton scattering of an ambient monochromatic photon field with constant energy density \( \nu_{\text{ext}} \). In this section, we determine the parameter regime in which the EC component dominates the high-energy afterglow emission while leaving a detailed derivation of the EC afterglow spectrum in Appendix A. We also show the analytical results of the multi-wavelength afterglow spectra for the synchrotron, SSC, and EC radiation.

2.1 General Considerations

We begin by considering a relativistic, adiabatic blast wave, which has relaxed into a self-similar structure, propagating through an external medium of constant number density \( n \).

The energy \( E \) of the blast wave is constant in time and is given by \( E = 16\pi \Gamma^2 R^2 m_p c^2 / 17 \) (Blandford & McKee 1976; Sari 1997), where \( R \) and \( \Gamma \) are the radius and bulk Lorentz factor of the blast wave, \( m_p \) is the proton mass, and \( c \) is the speed of light. Henceforth, we focus on the deceleration phase of the blast wave, where \( \Gamma \propto R^{-3/2} \).

Photons produced when the blast wave has reached a radius \( R = \) are received by an observer at time \( t = (1+z) R / \Gamma^2 c \) after the GRB trigger. From the expression of the blast wave energy \( E \) and the previous expression for the observer time \( t \), one may solve for \( R \) and \( \Gamma \) as

\[
R(t) = \left( \frac{17E t}{4\pi m_p c n (1+z)} \right)^{1/4},
\]

and

\[
\Gamma(t) = \left( \frac{17E (1+z)^3}{1024 \pi m_p c^5 n t^4} \right)^{1/8}.
\]

As the blast wave drives a relativistic shock into the circumburst medium, particles crossing the shock front are accelerated into a non-thermal distribution. Particle acceleration at relativistic shocks has been extensively studied by analytical and numerical means (Kirk et al. 2000; Ackerberg et al. 2001; Spitkovsky 2008, see also Sironi et al. 2015) for a recent review). In general, the accelerated non-thermal electron distribution can be modeled as a power-law extending between a minimum Lorentz factor \( \gamma_{\text{min}} \) and a maximum one \( \gamma_{\text{max}} \) (see Giannios & Spitkovsky 2009)

\[
N_{\text{ej}}(\gamma') \propto \gamma'^{-P} \quad \text{for} \quad \gamma_{\text{min}} < \gamma' < \gamma_{\text{max}}.
\]
We note here that all quantities measured in the co-moving frame of the blast wave are denoted with a prime. Assuming that $\gamma_{\text{max}} \gg \gamma_{\text{min}}$ and $p > 2$, the minimum Lorentz factor $\gamma_{\text{min}}$ of the non-thermal particle distribution can be estimated by

$$\gamma_{\text{min}} \approx \left( \frac{p - 2}{p - 1} \right) \frac{\eta p \Gamma}{m_e c},$$

where $\epsilon_e$ is the fraction of the shock energy transferred into relativistic electrons (Sari et al. 1998). The maximum Lorentz factor $\gamma_{\text{max}}$ can be determined by balancing the acceleration and synchrotron loss rates (Dermer & Menon 2009)

$$\gamma_{\text{max}} = \left( \frac{6 \pi \epsilon_e \epsilon_{\text{acc}}}{\sigma_T B^2} \right)^{1/2},$$

where $\epsilon_{\text{acc}} \leq 1$ is the ratio of acceleration rate to the maximum possible particle energy-gain rate (i.e. assuming Bohm diffusion). In this work, we fix $\epsilon_{\text{acc}} = 0.35$.

The energy loss rates of a single electron with Lorentz factor $\gamma' \gg 1$ due to synchrotron, SSC, and EC radiation are (Rybicki & Lightman 1986)

$$P_{\text{syn}}' (\gamma') = \frac{4}{3} \sigma_T c \gamma'^2 U_B',$$

$$P_{\text{SSC}}' (\gamma') = \frac{4}{3} \sigma_T c \gamma'^2 U_{\text{syn}}',$$

and

$$P_{\text{EC}}' (\gamma') = \frac{4}{3} \sigma_T c \gamma'^2 U_{\text{ext}}',$$

where eqns. (7)–(8) are valid in the Thomson regime and $U_B'$, $U_{\text{syn}}'$, and $U_{\text{ext}}' \equiv 1/2 U_{\text{ext}}$ (Dermer 1995) are the energy densities of the magnetic field, synchrotron photons, and ambient external photons in the shocked fluid frame, respectively. The magnetic field strength in the co-moving frame of the blast wave is written as

$$B' = \left( \frac{32 \pi m_0 e n}{n_0} \right)^{1/2} \Gamma c,$$

where $\epsilon_B$ is the fraction of the shocked fluid energy that is carried by the magnetic field.

The characteristic cooling timescale of an electron, with Lorentz factor $\gamma'$, due to synchrotron, SSC, and EC radiation is given by

$$\tau_{\text{c}}' \approx \frac{\gamma' m_e c^2}{P_{\text{EC}}' + P_{\text{syn}}' + P_{\text{SSC}}'},$$

while the expansion time of the blast wave is written as

$$\tau_{\text{exp}}' \approx \frac{5 R}{8 \Gamma c},$$

By equating the two aforementioned timescales, we can estimate the characteristic cooling Lorentz factor as

$$\gamma_c' = \left( \frac{6 \pi \epsilon_e \epsilon_{\text{acc}}}{5 \sigma_T (U_B' + U_{\text{syn}}' + U_{\text{ext}}')} \right)^{1/8},$$

which can be more conveniently expressed as

$$\gamma_c' = \frac{\gamma_{\text{syn}}'}{1 + x + y}. \quad (13)$$

where $x \equiv U_{\text{syn}}' / U_B'$, $y \equiv U_{\text{ext}}' / U_B'$, and the synchrotron cooling Lorentz factor is given by

$$\gamma_{\text{syn}}' = \frac{6 \pi m_e c^2}{5 \sigma_T R U_B'} \approx 1800 c_{B,-2} E_{54}^{1/8} \epsilon_n^{1/8} (\frac{t_d}{1 + \gamma})^{1/8}. \quad (14)$$

Henceforth, we adopt the notation $Q_x = Q/10^x$ in cgs units and $t_d \equiv t/1$ day.

The ratio $U_{\text{syn}}'/U_B'$ can be written as

$$y = \frac{U_{\text{syn}}'}{U_B'} = \frac{U_{\text{ext}} - U_{\text{syn}}}{U_B} \epsilon_{B,-2} \epsilon_{n,0}^{-1}.$$ (15)

and remains constant at all stages of the blast wave evolution. The ratio $U_{\text{syn}}'/U_{\text{ext}}'$, which is a measure of the SSC to synchrotron losses, can be written as (see also Sari & Esin 2001)

$$x = \frac{U_{\text{syn}}'}{U_{\text{ext}}'} = \frac{\eta U_{\text{ext}}'}{U_{\text{ext}} (1 + x + y)} = \frac{\eta \epsilon_e}{\epsilon_{B}(1 + x + y)}. \quad (16)$$

Here, $U_{\text{syn}}'$ is the kinetic energy density of relativistic electrons and $\eta$ is the radiative efficiency, namely the fraction of the electron energy radiated away via synchrotron, SSC, and EC processes. The latter can be written as

$$\eta = \left( \frac{1}{\gamma_{\text{c}}'^{2-p}} \frac{\gamma_{\text{c}}'}{\gamma_{\text{c}}'^{2-p}} \right)^{\gamma_{\text{c}}' < \gamma_{\text{c}}'} \quad (17)$$

where $\gamma_{\text{c}}'^{2-p}$ and $\gamma_{\text{c}}'$ are given in eqns. (4) and (14), respectively, while $t_0$ is the transition time from the fast cooling (i.e. $\gamma_{\text{c}}' > \gamma_{\text{c}}'$) to the slow cooling (i.e. $\gamma_{\text{c}}' < \gamma_{\text{c}}'$) regime (considering only synchrotron losses)

$$t_0 \approx 1.2 \left( \frac{p - 2}{p - 1} \right)^2 \frac{c_{B,-2} E_{54}^{2} n_0}{(1 + z)} d. \quad (18)$$

Substitution of eqn. (16) to eqn. (17) yields

$$x(1 + x + y) = \frac{\epsilon_e}{\epsilon_B} \gamma_{\text{c}}'^{2-p} \gamma_{\text{c}}' > \gamma_{\text{c}}', \quad (19)$$

$$x(1 + x + y)^{3-p} = \frac{\epsilon_e}{\epsilon_B} \left( \frac{t}{t_0} \right)^{2-p} \gamma_{\text{c}}'^{2-p} \gamma_{\text{c}}' < \gamma_{\text{c}}'. \quad (20)$$

Depending on the ordering of $x$ and $y$, one can define two regimes of particle cooling and Compton emission:

- SSC-dominated, for $x \gg y > 1$ (see Petropoulou & Moustaki-Adi 2009, for numerical results). Here, $x$ is given by

$$x \approx \left[ \frac{\gamma_{\text{c}}'}{\gamma_{\text{c}}'} \right]^{\gamma_{\text{c}}' > \gamma_{\text{c}}'} \left( \frac{\epsilon_e}{\epsilon_B} \right)^{1-p} \left( \frac{t}{t_0} \right)^{2-p} \gamma_{\text{c}}'^{2-p} \gamma_{\text{c}}' < \gamma_{\text{c}}'. \quad (21)$$

- EC-dominated, for $y \gg x > 1$. Here, $x$ is given by

1. The case of $\gamma_{\text{max}} \gg \gamma_{\text{min}}$ has been discussed in Petropoulou et al. (2011).
increasing the ratio of EC-dominated and SSC-dominated cases, respectively. The synchrotron and SSC spectra have been extensively discussed in the literature (see, e.g. Sari et al. 1998; Sari & Esin 2001, for details). Analytical expressions for the EC component dominates (i.e. \( y \gg x \)). Small values of \( U_{\text{ext}}/n \) (e.g. \( U_{\text{ext}}/n \lesssim 10^{-3} \) erg for \( B = 10^{-4} \)) corresponding to an SSC-dominated cooling scenario (i.e. \( x \gg y \)). Below the horizontal dashed-dotted line, synchrotron radiation dominates the particle cooling (i.e. \( x, y < 1 \)). A coloured version of this plot is available online.

In both the SSC-dominated and EC-dominated cooling regimes, we find that \( x \) is independent of time in the fast cooling regime, but it decreases gradually once the system enters the slow cooling regime (this is valid for \( p = 2.1 - 2.5 \)).

Fig. 1 shows the dependence of \( x \) and \( y \) on \( U_{\text{ext}}/n \) for different values of \( B \) according to eqns. (15) and (19). For illustration purposes, we consider only the fast cooling regime while noting that the temporal dependence of \( x \) in the slow cooling regime is weak for \( p = 2 \). SSC dominates electron energy losses (i.e. \( x > y \)) in the fast cooling regime, if the following condition is satisfied

\[
\epsilon_x/\epsilon_y \ll n_0 U_{\text{ext}}^{-1} \gtrsim 5.4 \times 10^{-3}. \tag{22}
\]

2.2 Multi-Wavelength Afterglow Spectra

The synchrotron and SSC spectra have been extensively discussed in the literature (see, e.g. Sari et al. 1998; Sari & Esin 2001, for details). Analytical expressions for the EC emission of the afterglow are provided in Appendix A. For the following illustrative examples, we consider an external monochromatic photon field of energy \( \epsilon_0 \sim 8 \times 10^{-3} \) eV, as expected from dust heated to \( T \approx 90 \) K (Wilson et al. 2014; Scoville et al. 2015; Yoast-Hull et al. 2015; Perley et al. 2017; Yoast-Hull et al. 2017). All other parameters are listed in Table 1.

Multi-wavelength spectra, including synchrotron, SSC, and EC emission, are shown in Fig. 2 for an observer time \( t = 10^5 \) s. Panels (a) and (b) show examples of the EC-dominated and SSC-dominated cases, respectively. The transition from the latter to the former regime is achieved by increasing the ratio \( U_{\text{ext}}/n \) (see also eqn. 22) by two orders of magnitude. This effectively results in an increase of the EC flux by a factor of \( \sim 20 \) (see eqn. A10). For a summary of the parameters values used in Fig. 2, see Table 1.

| Parameters and units | EC-dominated | SSC-dominated |
|----------------------|--------------|--------------|
| \( n \) [cm\(^{-3}\)] | 0.1          | 1            |
| \( U_{\text{ext}} \) [erg cm\(^{-3}\)] | 7.5 \times 10^{-7} | 7.5 \times 10^{-8} |
| \( \epsilon_0 \) [eV] | 0.02         | 0.02         |
| \( E \) [erg] | 10^{54}       | 10^{54}       |
| \( \epsilon_x \) | 0.1          | 0.1          |
| \( \epsilon_B \) | 10^{-3}       | 10^{-3}       |
| \( \epsilon_{\text{acc}} \) | 0.35         | 0.35         |
| \( p \) | 2.2          | 2.2          |
| \( d_h \) [cm] | \( 9 \times 10^{27} \) | \( 9 \times 10^{27} \) |

We define two characteristic observed frequencies of the synchrotron spectra, namely

\[\nu_\text{min} \equiv \Gamma y_{\text{min}}^2 \frac{eB'}{2\pi mc^2}\]

and

\[\nu_\epsilon \equiv \Gamma y_{\epsilon}^2 \frac{eB'}{2\pi mc^2}.\]

For the EC-dominated case (Fig. 2a), we find \( \nu_{\text{min}} \approx 1.8 \times 10^{-3} \) eV and \( \nu_{\epsilon} \approx 12 \) eV, while for the SSC-dominated case (Fig. 2b), the peak of the synchrotron spectrum occurs at \( \nu_{\epsilon} \approx 7 \) eV; for the adopted parameter values (see Table 1), the minimum synchrotron frequency is the same as in the EC-dominated case.

In both panels, we show our computed spectra from our analytical model\(^2\). The Klein-Nishina (KN) suppression of the Compton scattering cross section is not included in our analytical treatment, but it is expected to become important at the part of the spectrum highlighted with dotted lines. In the SSC spectrum, the KN effects become important above \( 2\nu_{\epsilon} y_{\text{KN}}^2 \) (see Sec. 4 and eqn. 50 in Nakar et al. 2009), where \( y_{\text{KN}} \) is the Lorentz factor of electrons which can upscatter photons with \( \nu_{\epsilon} \) in the KN regime, i.e. \( \nu_{\epsilon} y_{\text{KN}}^2 / \nu_{\epsilon} \approx mc^2 \) (see Eqn. 6 in Nakar et al. 2009). For the parameter values used in Fig. 2, we estimate that the KN effects on the SSC spectra at that time become apparent above \( \sim 5 \) TeV. For the EC spectrum, the \( \nu_{\epsilon} \) cutoff becomes relevant at even higher energies (here, \( \sim 10 \) TeV) – see also eqn. (A19).

Thus, the spectral steepening due to KN effects can be safely neglected at energies relevant to Fermi-LAT observations (denoted by the shaded regions in Fig. 2) for most parameters.

3 GAMMA-RAY LIGHT CURVES

The high energy emission (100 MeV - 100 GeV) of GRB afterglows has been found to peak after the prompt keV-MeV component within seconds, and then decays as \( \propto t^{-\chi} \), with \( \chi \approx 1.2 \) (Ghisellini et al. 2010; Ghirlanda et al. 2010; Ackermann et al. 2013). Here, we explore the temporal trends...

\(^2\) The SSC spectrum appears smooth due to numerical integration of the Compton emissivity over the seed synchrotron photon spectrum.
predicted in our analytical model for both SSC and EC dominated regimes.

As an indicative example, we show in Fig. 3 the 1 GeV and 1 TeV light curves for the same parameters used in Fig. 2 (see also Table 1). The flux at a fixed frequency decays as a single power law in time (i.e., \( F_{\nu} \propto t^{-\alpha} \)), as long as it is produced by a single emission mechanism (either EC or SSC). The broken power-law light curve obtained at 1 GeV (solid blue line) is the result of the transition from a synchrotron-dominated to an EC-dominated emission at \( t \sim 10^5 \) s. An early-time flat -GeV light curve may, therefore, be a signature of an EC dominant component.

For the adopted value of \( p = 2.2 \) for the electron power-law index, we find decay slopes \( \chi \sim 0.9 - 1.15 \), which are similar to those observed in Fermi-LAT GRB light curves. Interestingly, for \( p \gtrsim 2 \), the predicted values of \( \chi \) do not seem to depend either on the cooling regime or the origin of seed photons for Compton scattering. Our results suggest that the gamma-ray light curve alone may not be sufficient to distinguish between EC and SSC processes, and multi-wavelength spectral and temporal information is thereby required to identify the dominant mechanism.

To further expand upon this, we present parametric scalings of the observed flux on the model parameters. In the EC dominated regime, the flux scales as (see also eqs. A14 and A18 in Appendix A)

\[
F_{\nu} \propto \begin{cases} 
U_{\text{ext}} E^{\frac{n+3}{2}} \epsilon^{n+1} & \text{if } \nu_{\text{EC}} < \nu < \nu_{\text{SSC}} \\
E^{\frac{n+1}{2}} \epsilon^{n+1} & \text{if } \nu > \nu_{\text{SSC}}
\end{cases}
\]

where the EC cooling frequency is defined as \( \nu_{\text{EC}} = \frac{4}{3} \epsilon^2 \nu_{\gamma}^* \nu_0 \) and \( \nu_{\text{SSC}} = \frac{2}{3} \epsilon^2 \nu_{\gamma}^* \nu_0 \), with \( \nu_0 \) being the frequency of monochromatic external photons. Similarly, the scaling for the SSC-dominated case reads

\[
F_{\nu} \propto \begin{cases} 
E^{\frac{n+1}{2}} \epsilon^{n+1} & \text{if } \nu_{\text{SSC}} < \nu < \nu_{\gamma}^* \\
E^{\frac{n+1}{2}} \epsilon^{n+1} & \text{if } \nu > \nu_{\gamma}^*
\end{cases}
\]

where \( \nu_{\gamma}^* = 2 \epsilon^2 \nu_{\gamma}^* \nu_{\text{min}} \), \( \nu_{\gamma}^* = 2 \epsilon^2 \nu_{\gamma}^* \nu_0 \) (see eqn. 13), \( \nu_{\text{min}} \) is the minimum synchrotron frequency as defined in Sari et al. (1998), and \( \nu_{\gamma}^* \) is the cooling synchrotron frequency given by eqn. (24).

Nava et al. (2014) considered the GeV light curves of ten GRBs detected by Fermi-LAT and found that all decay as a power-law with a similar slope, i.e., \( F_{\nu} \propto t^{-1.2} \). After re-normalizing the integrated LAT luminosity to the burst’s total isotropic prompt emission energy, Nava et al. (2014) showed that the light curves of all GRBs in their sample...
overlapped. They argued that this result supports the interpretation of the LAT emission as synchrotron radiation from external shocks.

Here, we examine the dependence of inverse Compton emission on the total energy of the burst. In our model, the dependence of SSC and EC fluxes on $E$ is given by eqns. (25) and (26). For instance, when $p = 2.2$, eqns. (25) and (26) show that the flux is proportional to $E^{1.3}$ (for $v < v_{\text{SSC}}^c$) and $E^{1.5}$ (for $v > v_{\text{SSC}}^c$) for EC emissions and $E^{1.7}$ (for $v < v_{\text{SSC}}^c$) and $E^{0.95}$ (for $v > v_{\text{SSC}}^c$) for SSC. We therefore find an almost linear dependence of the flux on $E$ if the LAT emission is attributed to EC scattering (independent of the cooling break) or to SSC for $v > v_{\text{SSC}}^c$.

4 DETECTABILITY OF AFTERGLOW EMISSION AT VERY HIGH ENERGIES

A very high-energy (VHE; $E \gtrsim 100$ GeV) detection of a GRB afterglow can be used to probe the extragalactic background light (EBL). From the far-infrared to the visible and UV wavelengths, the EBL is thought to be dominated by starlight, either through direct emission or through absorption and re-radiation by dust. These low-energy ambient photons interact with VHE photons from extragalactic sources to produce electron-positron pairs (Gould & Schrédér 1967; Puget et al. 1976). If the redshift and the intrinsic VHE spectrum of the source are both known, then the observed spectrum can be used to constrain different EBL models.

Fig. 4 shows the instantaneous VHE afterglow spectrum computed at $t = 0.5$ hr with (coloured lines) and without (solid grey line) EBL absorption, for two fiducial redshifts ($z = 0.5$ and 1) and for the parameters used in our EC-dominated model (see Fig. 2a and Table 1). For the attenuation of VHE photons, we considered several EBL models, as noted in the inset legend. The attenuated flux is compared against the 0.5 hr differential sensitivity curves of the next-generation Cherenkov telescope array, i.e. CTA South (Hassan et al. 2017) and two currently operating VHE telescopes, namely VERITAS and MAGIC. For a burst located at $z = 0.5$, the EBL affects the spectrum already at energies $\gtrsim 100$ GeV, while the photon-photon absorption optical depth rises rapidly between 100 GeV and 1 TeV. High-quality spectra in this energy range can be used, in principle, to differentiate between EBL models, as shown in the top panel of Fig. 4. For $z = 1$, the flux at $\sim 1$ TeV is strongly attenuated for all the EBL models we considered. Still, CTA will be sensitive enough to detect emission up to $\sim 300$ GeV from that burst for almost all EBL models considered here.

We next discuss the detectability of the combined Compton (SSC and EC) signal at 100 GeV by CTA, for a fiducial burst located at $z = 0.5$ and different model parameters (e.g. $E$, $\epsilon_0$, and $\eta_0$). Using eqn. (A5) from Sari & Esin (2001) and eqn. (A7), we calculate the average Compton flux at 100 GeV\(^4\) over an interval of $T = 0.5$ hr starting from $t = 0.5$ hr, namely \(\langle F_C \rangle = T^{-1} \int_0^T dt'[F_{\text{SSC}}(t') + F_{\text{EC}}(t')]\), and compare it against the 0.5 hr CTA-South sensitivity at 100 GeV (see Fig. 4). We define a burst as detectable, if $\langle F_C \rangle$ exceeds the 0.5 hr CTA sensitivity. Our results are presented in Fig. 5.

In all panels, the coloured regions indicate the parameter space of detectable bursts and the colour denotes the contribution of EC (red) and SSC (blue) to the total ob-

\(^3\) To obtain the 0.5 hr sensitivity curves of MAGIC and VERITAS, we scaled the publicly available curves for 50 hr, respectively, assuming that the sensitivity increases as $T^{-1/2}$, where $T$ is the observation time.

\(^4\) The EBL attenuation is taken into account. Here, we used the EBL model of Finke et al. (2010).
erved 100 GeV flux. Different panels show results for different combinations of $\epsilon_e$ and $\epsilon_B$. When EC makes only a small fraction of the total flux, we find that only rather powerful blasts may be detectable through their afterglow emission at high energies. For example, $E \gtrsim 5 \times 10^{53}$ erg is required for an SSC dominated GRB at $z = 0.5$ to be detectable by CTA at 0.5 hr after the trigger (see upper right panel of Fig. 5). However, when a dense ambient radiation field is present in the vicinity of a GRB, EC can significantly increase the production of 100 GeV − 1 TeV photons. As a result, the detectability requirements on the blast isotropic equivalent energy are greatly reduced. This is illustrated by the extension of the red-coloured region towards lower $E$ values, if $U_{\text{ext}} > 10^{-8} - 10^{-7}$ erg cm$^{-3}$. Especially for $\epsilon_e = 0.1$ and $\epsilon_B = 10^{-5}$ (see lower left panel of Fig. 5), the lower limit of $E$ is reduced by two orders of magnitude when $U_{\text{ext}}$ increases from $\sim 10^{-8}$ erg cm$^{-3}$ to $10^{-6}$ erg cm$^{-3}$. The region of the parameter space lying above the dashed horizontal line is unrealistic, as it implies energy densities exceeding that of a black-body photon field with temperature $T = 100$ K, i.e. $U_{\text{ext}} = 7.5 \times 10^{-6}$ erg cm$^{-3}$, which is expected for hot dust (Wilson et al. 2014; Scoville et al. 2015; Yoast-Hull et al. 2015; Perley et al. 2017; Yoast-Hull et al. 2017).

The parameter space of detectable events is also strongly dependent upon $\epsilon_e$. A larger value of $\epsilon_e$ suggests that more of the shock energy is transferred into relativistic electrons, therefore producing more powerful Compton emission (either via SSC or EC). This, in turn, relaxes the requirements on the blast wave energy. The fraction of shocked fluid energy carried by the magnetic field, $\epsilon_B$, affects only the detectability of SSC-dominated bursts (e.g. compare the top left and bottom left panels in Fig. 5). With all other parameters fixed, a larger value of $\epsilon_B$ increases the density of synchrotron photons that serve as targets for Compton scattering and, as a result, the SSC flux (see e.g. eqn. 26). Thus, a smaller value of $\epsilon_B$ indicates weaker SSC emissions, which will strengthen the requirements for a larger value of $E$ for VHE photons to be detected. This can be seen when transitioning from the top to bottom panels in Fig. 5.

5 MAGIC DETECTION OF GRB 190114C

GRB 190114C (at redshift $z = 0.42$, Selsing et al. 2019) is the first gamma-ray burst detected at sub-TeV energies by the MAGIC Cherenkov telescope (Mirzoyan 2019). After the Swift-BAT trigger, the MAGIC detector showed a significance $> 20\sigma$ in the first 20 minutes of observations for energies $> 300$ GeV. This VHE emission extended to
Flux [erg cm$^{-2}$ s$^{-1}$]

extinction by the host galaxy (assuming model flux for the optical and EBL absorption (Franceschini et al. 2008) is considered. The repository and the LAT data are taken from Ajello et al. (2019). The X-ray data are retrieved from Swift-XRT GRB light curve (https://www.swift.ac.uk/xrt_curves/00883832/).

Figure 6. Modeling of the afterglow light curves of GRB 190114C. The optical data are taken from Laskar et al. (2019), the X-ray data were retrieved from Swift-XRT/GRB light curve repository and the LAT data are taken from Ajello et al. (2019). The expected VHE light curve is shown by the solid orange line and EBL absorption (Franceschini et al. 2008) is considered. The model flux for the optical $R$ band has been modified due to the extinction by the host galaxy (assuming $A_V = 3.6$ mag). The model-predicted light curves are displayed at times after the end of the coasting phase ($t = 30$ s), where we assumed that the initial bulk Lorentz factor is $\Gamma_0 = 400$. In the inset plot we show the average spectrum from 50 s to 100 s. The bowtie shows the 1σ contour of the power-law model fitted to the Fermi-LAT data (Ajello et al. 2019), while different types of lines show the model spectra from various processes (for details, see inset legend). The synchrotron flux at $> 40$ MeV is multiplied by a factor of $\sqrt{e/e_0}$ to account for the KN effects on the cooling of the radiating electrons. The parameters used here are: $E = 9 \times 10^{51}$ erg, $n = 0.1$ cm$^{-3}$, $e_0 = 0.12$, $e_p = 4 \times 10^{-5}$, $p = 2.6$, $\nu_0 = 0.02$ eV, $U_{\text{ext}} = 7.5 \times 10^{-3}$ erg cm$^{-3}$. A coloured version of this plot is available online.

> 300 GeV provides a unique opportunity to test existing GRB afterglow models. Several studies aiming to interpret the VHE of GRB 190114C have already been presented. Ravasio et al. (2019), for instance, argue that the afterglow energy at energies between 10 keV and 30 GeV should be produced by a single mechanism, either synchrotron or inverse Compton. Others propose that the SSC emission of GRB 190114C dominates over the synchrotron component at GeV energies (e.g. Fraija et al. 2019; Wang et al. 2019). In this section, we will demonstrate that synchrotron radiation can explain the sub-GeV/GeV emissions for $t \lesssim 10^5$ s ($t$ is the time after the Swift-BAT trigger, $T_0$), while at later times inverse Compton scattering contributes mostly to the sub-GeV/GeV and VHE fluxes. We will also estimate the upper limit on the energy density of a putative ambient photon field using the LAT measurement at $\sim 10^8$ s after the trigger.

Fig. 6 shows the optical (Laskar et al. 2019), Swift-XRT X-ray, and Fermi-LAT gamma-ray (Ajello et al. 2019) observations together with the optical, X-ray and gamma-ray light curves of GRB 190114C (colored lines) as obtained from our analytical model described in Sec. 2 (for the parameters used, see figure caption). As we are not considering the coasting phase of the blast wave in our model, we only show results for times larger than the deceleration time $t_{\text{dec}} \approx [E/(\pi m_p c^2 \Gamma_0^2)]^{1/3}(1+z)$, where $\Gamma_0$ is the initial bulk Lorentz factor. Here, we adopt $\Gamma_0 = 400$, which results in $t_{\text{dec}} \approx 20$ s.

In order to compare the effects of synchrotron and inverse Compton scattering on the electrons cooling, we estimate the values of $x$ and $y$ (for details, see Sec. 2.1). For this choice of the parameters, $x$ decreases with time from $x \approx 45$ at $t = 50$ s to $x \approx 3$ at $10^5$ s while $y$ remains constant ($y \approx 5$). This indicates that SSC will dominate the cooling during most of the blast wave’s deceleration phase.

The optical and X-ray fluxes consist mainly of synchrotron emission at all times. Given the adopted parameters, we find that $v_{\text{min}}$ (given by eqn. 23) decreases from 200 eV at 50 s to $10^{-3}$ eV at $10^5$ s. For $t \lesssim 10^3$ s, the peak of the synchrotron spectrum (in $F_\nu$ units) lies beyond the $R$ band (i.e. $v_{\text{min}} > \nu_{\text{R}}$), while at $t \gtrsim 10^3$ s we find $v_{\text{min}} < \nu_{\text{R}}$. The crossing of $v_{\text{min}}$ through the $R$ band causes a break at $\sim 10^3$ s in the optical light curve. Note, however, that the model falls short in explaining the observed optical flux at $t \sim 30$ s. The bright early time optical emission might be produced by the reverse shock, not considered here (see, e.g. Laskar et al. 2019). We also estimate the cooling break of the synchrotron spectrum using eqn. (24) and find that $\nu_c$ decreases only slightly from 700 eV at 50 s to 200 eV at $10^6$ s. This demonstrates that the X-ray flux corresponds to the fast cooling segment of the synchrotron spectrum (i.e. $\nu > \nu_c$) for all times considered here, and is consistent with the constant decay rate of the observed X-ray light curve. When electron cooling is dominated by SSC, as is the case here, then the observed decay rate of the X-ray flux can only be explained by the propagation of a blast wave in a constant density medium (see eqns. B9 & C6 in Panaitescu & Kumar 2000). In contrast, if electron cooling was synchrotron-dominated, then both the constant and the wind-like density profiles would lead to similar temporal decay rates (Panaitescu & Kumar 2000; Ajello et al. 2019).

The Fermi-LAT gamma-ray flux in the 0.1 $-$ 1 GeV energy range is dominated by synchrotron radiation (dotted blue line), while at later times both the electron cooling and inverse Compton scattering contribute significantly to the gamma-ray flux (dotted green line for SSC). At early times, the gamma-ray light curve, similar to the X-ray and optical ones, can be explained by synchrotron emission of electrons accelerated at the external shock wave. However, the electrons emitting in gamma-rays cannot cool efficiently through inverse Compton scattering due to KN effects. As a result, the number of electrons with $\gamma > \gamma_{\text{KN}}$ is larger than in the case of Compton cooling in the Thomson regime. Nakar et al. (2009) studied the KN effects on synchrotron and SSC spectra and found that KN scattering can boost the high energy part of the synchrotron spectrum by a factor of $\sqrt{e/e_0}$ when the Comptonization parameter of the corresponding electrons becomes $Y \sim 1$ (for more details, see Nakar et al. 2009). For the adopted parameters, the synchrotron flux produced by electrons with a Lorentz factor greater than $\gamma_0$ will be boosted due to the suppression of SSC cooling by KN effects (see Sec. 2.2). We can also estimate $\gamma'_0 \sim 6 \times 10^6$ at $t \sim 50$ s and $7 \times 10^5$ at $t \sim 10^6$ s.

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\[^{5}\text{https://www.swift.ac.uk/xrt_curves/00883832/}\]

\[^{6}\gamma_0\text{ is defined such that } Y(\gamma_0) = 1.\]
The characteristic synchrotron photon energies produced by electrons with $\gamma_{0}^s$ are, respectively, $h\nu(\gamma_{0}^s) \sim 40$ MeV and 2 MeV. Thus, the effect of KN scatterings can increase the synchrotron flux of the LAT gamma-ray band by a factor of $\sqrt{\epsilon_{\nu}/\epsilon_{\nu}} \sim 50$ at all times considered here (i.e. $t > 50$ s). However, KN effects hardly affect the synchrotron flux of the optical and X-ray bands, since they both lie well below $\nu(\gamma_{0}^s)$. The synchrotron-emitting electrons in the optical and X-ray bands can still cool via inverse Compton emission in the Thomson regime.

It has also been suggested that the GeV emission at early times (i.e. $t \lesssim 10^3$ s) could originate from inverse Compton scatterings (Fraija et al. 2019). However, neither EC nor SSC is likely to be the process powering the early GeV emission of this burst, as we explain below. If EC dominated the GeV afterglow emission at all times, this would require a small value of $\epsilon_{\nu}$ (see eqn. 22). For typical long-GRB afterglow parameters, i.e. $\epsilon_{\nu} \simeq 0.1$ and $n_0 \simeq 1$ cm$^{-3}$, the requirement for an EC-dominated case would translate to $1^1/2_{-2} U_{\text{ext}}^{\text{EC}} < 5.4 \times 10^{-3}$. Since the expected energy density of a blackbody radiation field with $T \simeq 100$ K is $U_{\text{ext}} \sim 10^{-5}$ erg cm$^{-3}$, then $\epsilon_{\nu} \lesssim 3 \times 10^{-7}$. With such a small value of $\epsilon_{\nu}$, it is difficult to simultaneously explain the high flux in the X-ray and sub-TeV bands. Alternatively, the LAT flux at $t \lesssim 10^3$ s could be attributed to SSC afterglow emission. However, it is difficult to make the SSC emission within the LAT energy band peak at times as early as $\sim 10$ s for typical parameter values, as synchrotron photons at these times are typically up-scattered by electrons into the sub-TeV or TeV bands, and the light curve would rise instead of decay under this condition. For example, the peak of SSC can be estimated as $2\gamma^2_{\text{max}}\nu_{\text{min}} \simeq 300$ GeV at $t \simeq 50$ s for this particular case. For these reasons, synchrotron radiation is the most likely mechanism for producing the sub-GeV and GeV afterglow emission at $t \lesssim 10^3$ s (see dashed blue line in Fig. 6).

Although the emission in the LAT energy band is initially dominated by synchrotron radiation, inverse Compton scattering can dominate at later times. For the adopted parameter values, this happens at $t \gtrsim 10^3$ s (see dashed-dotted blue line in Fig. 6). The late-time measurement of the LAT flux (i.e. at $\sim 10^4$ s) is crucial for constraining the parameters related to the inverse Compton scattering process, namely $n$ and $U_{\text{ext}}$. The Fermi-LAT light curve from $t \gtrsim 10^5$ s to $10^6$ s can be described by a single power law. In our model, the transition from synchrotron-dominated to Compton-dominated emission in the LAT energy band occurs in between the two Fermi-LAT data points (i.e. at $t \sim 10^5$ s). Given that the synchrotron, SSC, and EC light curves show similar temporal power-law decays (see blue curves in Fig. 6), neither the SSC nor the EC light curves at their peak can be much brighter than the synchrotron flux at that time. Hence, we can calculate the maximum energy density of the external field $U_{\text{ext}}^{\text{max}} \simeq 7.5 \times 10^{-8}$ erg cm$^{-3}$.

SSC emission can also help in constraining the maximum density of the circumburst medium $n$. Assuming that SSC dominates the electron cooling, the synchrotron flux$^7$ $F_{\nu} \propto n^{(p-1)/32-2p}$ or $\propto n^{-0.1}$ for $p = 2.6$. The SSC flux is written as $F_{\nu}^{\text{SSC}} \propto n^{(p^2-14p+24)/(32-8p)}$ or $\propto n^{-0.5}$ for $p = 2.6$ (see eqn. 26). The SSC flux dominating the Fermi-LAT at $t > 500$ s is more sensitive to $n$, whereas the synchrotron flux which explains the X-ray and early Fermi-LAT emission is almost independent of $n$. As a result, the observed Fermi-LAT flux at $\sim 10^4$ s provides constraint on $n$, which can be estimated as $n_{\text{max}} \simeq 0.1$ cm$^{-3}$.

In Fig. 6, we also show the model prediction for the VHE light curve (orange lines) and the time-averaged spectrum for $68 \sim 110$ s, taking into account the EBL attenuation based on the model of Finke et al. (2010). The VHE flux is mostly explained by inverse Compton emission, for the photons energy is much greater than the cutoff energy of synchrotron spectrum, which is $\sim 10$ GeV at $\sim 10^5$ s. The detection of high energy photons by MAGIC will help to understand the underlying mechanism of this GRB and test existing EBL models.

Here, we discussed the synchrotron and inverse Compton emission from a forward shock propagating into a constant density circumstellar environment, but it is also possible that a wind-like density profile can explain the afterglow emissions. Ajello et al. (2019) showed that the synchrotron model in a wind-like circumstellar environment works well in explaining the X-ray and sub-GeV/GeV gamma-ray afterglow light curves. However, the authors assumed that electrons are cooling mainly via synchrotron radiation, while inverse Compton cooling was neglected, which may not be a valid assumption, especially at later times, when both EC and SSC are in Thomson regime and electrons can cool via inverse Compton scattering. A detailed study of the multi-wavelength afterglow emission for a wind-like density profile could be the topic of a future publication, following the release of the MAGIC data.

6 SUMMARY AND DISCUSSION

In this paper, we perform a systematic study of the Compton emission in GRB afterglows, with the inclusion of a narrow-band ambient radiation field as a source of scattering. We calculated synchrotron, SSC, and EC spectra and light curves produced by a power-law distribution of electrons accelerated at the relativistic shock during its deceleration phase, as it sweeps up matter from a constant-density circumstellar medium. Similar to the synchrotron radiation, we find that the flux at the peak of EC remains constant in both slow and fast cooling regimes for adiabatic hydrodynamic evolution of the blast wave, while the peak of the SSC component decreases with time.

The calculations of inverse Compton scattering indicate that either EC or SSC can explain the high energy emission at energies beyond 100 MeV. We find that SSC may dominate the cooling of electrons over EC, except when there is a dense ambient IR radiation (as observed in some star-forming galaxies) or a low-density circumburst medium (see eqn. 22).

We also discuss the detectability of VHE afterglow emission by existing and future gamma-ray instruments when the EBL attenuation is considered. When a dense ambient radiation field is present in the vicinity of a GRB, EC scattering can significantly increase the emission of 100 GeV–1 TeV photons. As a result, the detectability requirements

\footnote{Substituting $\nu_{i}$ in eqn. 7 from Sari et al. (1998) with the value calculated by eqn. 13.}
on the blast isotropic equivalent energy are greatly reduced. Being about one order of magnitude more sensitive than current Cherenkov telescopes, CTA should be capable of detecting sub-TeV and TeV photons with flux as low as $\nu F_{\nu} \sim 10^{-12}$ erg cm$^{-2}$ s$^{-1}$ (with an observation time 0.5 h). This also means that a burst may be detectable with CTA even at very late times, assuming a power-law decay of the flux $\propto \nu^{p-3}$ for SSC-dominated cases or $\propto \nu^{-(p-2)/3}$ for EC-dominated ones. In the CTA era, we expect more detections of GRB afterglows in GeV and TeV bands in host galaxies with regions of dense IR radiation. We apply our analytical afterglow emission model to the GRB 190114C, the first gamma-ray burst detected at sub-TeV energies by the MAGIC Cherenkov telescope. We find that the optical and X-ray light curves can be explained by synchrotron emission of particles accelerated in a power-law energy spectrum with slope $p = 2.6$ at a relativistic adiabatic blast wave of energy $E = 10^{54}$ erg propagating in a cirumburst medium with density $n = 0.1$ cm$^{-3}$. We also find that the early-time ($t \leq 10^3$ s) Fermi-LAT light curve is synchrotron dominated, but becomes Compton dominated at later times (i.e. $t > 10^3$ s). The Fermi-LAT measurement at $10^4$ s after trigger is crucial for setting an upper limit on the energy density of a putative IR photon field (i.e. $U_{\text{ext}} \lesssim 7.5 \times 10^{-8}$ erg cm$^{-3}$). We also predict that the VHE afterglow emission is SSC-dominated till $\sim 10^5$ s, while EC scattering may take over at even later times, if a strong ambient photon field is present. The predicted 0.3–3 TeV flux at $\sim 10^2$ s after trigger is $\sim 10^{-8}$ erg cm$^{-2}$ s$^{-1}$ and decays as $t^{-1.4}$, indicating that the 300 GeV–3 TeV flux of GRB 190114C could have been detectable until $\sim 10^4$ s after the trigger (see Fig. 6).

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In the fast cooling regime (i.e. $\nu < \nu_{EC}^{\min}$) the low-energy part of the net spectrum (i.e. $\nu < \nu_{EC}^{\min}$) is the sum of the low-energy tails of the single-particle Compton spectrum from all electrons, and as such $F_{\nu}^{EC} \propto \nu$. The remaining part of the spectrum can be calculated according to the relationship

$$F_{\nu}^{EC} \propto P_{EC}(\gamma')|N(\gamma')d\gamma'|,$$  

(A6)

where $P_{EC}(\gamma') \equiv 1/2\nu_{EC}^{\nu}e'_{\nu} \gamma'^2$ is the EC power in the observer frame and is determined using eqn. (8).

The total spectrum in the slow cooling regime can be written as

$$F_{\nu}^{EC} = F_{\nu_{v,\max}}^{EC} \times \begin{cases} \left(\frac{\nu}{\nu_{EC}^{\min}}\right)^{-p/2} & \nu < \nu_{EC}^{\min} \\ \left(\frac{\nu}{\nu_{EC}^{\min}}\right)^{-\nu_{EC}^{\nu}e'_{\nu}/\sigma_{T}\Gamma(U_{\text{ext}})} & \nu_{EC}^{\min} < \nu < \nu_{EC}^{\nu} \\ \left(\frac{\nu}{\nu_{EC}^{\nu}}\right)^{(p-1)/2} & \nu > \nu_{EC}^{\nu} \end{cases},$$

(A7)

while in the fast cooling regime it is given by

$$F_{\nu}^{EC} = F_{\nu_{v,\max}}^{EC} \times \begin{cases} \frac{\nu}{\nu_{EC}^{\nu}} & \nu < \nu_{EC}^{\nu} \\ \left(\frac{\nu}{\nu_{EC}^{\nu}}\right)^{-1/2} & \nu_{EC}^{\nu} < \nu < \nu_{EC}^{\nu_{v,\min}} \end{cases},$$

(A8)

where $F_{\nu_{v,\max}}^{EC}$ is the observed peak flux

$$F_{\nu_{v,\max}}^{EC} \equiv \frac{P_{\nu_{v,\max}}^{EC}}{4\pi d_{L}^2 (1+z)}.$$

(A9)

Here, $d_{L}$ is luminosity distance of the source and $L_{EC}^{\nu_{v,\max}} \equiv (4/3)\pi R^2 n_{e}T_{\nu_{v,\max}}^{EC}$ is the maximum spectral luminosity.

In the EC-dominated regime (for details, see Sec. 2.1), we obtain simple expressions for the peak flux, minimum, and cooling frequencies of the EC spectrum:

$$F_{\nu_{v,\max}}^{EC} = 6.1 \times 10^{-3} \xi_{0,\nu_{v}} U_{\text{ext}}^{-6} E_{54}^{\nu} d_{L,28}^{-2} (1+z) [\text{nJy}],$$

(A10)

$$\nu_{EC}^{\min} = 0.64 \left(\frac{p-2}{p-1}\right)^2 \eta_{0,\nu_{v}}^{-1/2} \nu_{EC}^{\nu} e_{\nu}^{-1/2} E_{54}^{1/2} n_{0,\nu}^{1/2} n_{e}^{3/2} (1+z)^{1/2} [\text{GeV}],$$

(A11)

$$\nu_{EC}^{\nu} = 1.1 \times 10^{4} \xi_{0,\nu_{v}} U_{\text{ext}}^{-6} E_{54}^{\nu} d_{L,28}^{-2} (1+z)^{-1/2} [\text{GeV}],$$

(A12)

where $\eta_{0,\nu} = \eta_{0}/[1\text{ eV}]$ and $d_{L}$ is the time in the observer frame normalized to 1 day.

We present next expressions for the temporal evolution of the EC flux, assuming $p = 2.2$, in both cooling regimes. For the slow cooling regime, we find

$$F_{\nu}^{EC}(t)_{\nu_{EC}^{\nu}-\nu_{EC}^{\min}}^{\nu_{EC}^{\nu}} = e_{\nu}^{-1} U_{\text{ext}}^{-6} E_{54}^{\nu} n_{0,\nu}^{1/2} n_{e}^{-1/2} (1+z)^{1/2} [\text{nJy}],$$

(A13)

$$F_{\nu}^{EC}(t)_{\nu_{EC}^{\nu}}^{\nu_{EC}^{\nu_{v,\min}}} = e_{\nu}^{1.2} U_{\text{ext}}^{-6} E_{54}^{\nu} n_{0,\nu}^{-0.3} n_{e}^{-0.9} (1+z)^{1.15} [\text{nJy}],$$

(A14)

$$F_{\nu}^{EC}(t)_{\nu_{EC}^{\nu_{v,\max}}}^{\nu_{EC}^{\nu}} = e_{\nu}^{1.2} U_{\text{ext}}^{-6} E_{54}^{-0.05} n_{0,\nu}^{-0.05} n_{e}^{-1.1} (1+z)^{1.15} [\text{nJy}],$$

(A15)

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where $v_{\text{KN}} \equiv v/(2.4 \times 10^{23} \text{ Hz})$. Accordingly, the expressions for the fast cooling regime are

$$F_{\text{EC}}(t)_{v>v_{\text{EC}}}^{\text{EC}} = \frac{t_{\text{ext}}^{3}}{1.4 \times 10^{-3}(1+z)^{1.3}[\text{nJy}]} = 6^{1.5} \nu_{0}^{-0.5} \gamma_{\text{GeV}} d_{L,28}^{-2}.$$  
(A16)

$$F_{\text{EC}}(t)_{\nu>\nu_{\text{min}}}^{\text{EC}} = \frac{5.75 \nu_{0}^{0.25} t_{d}^{-0.25} v_{0}^{-0.5} \gamma_{\text{GeV}} d_{L,28}}{4.5(1+z)^{0.75}[\text{nJy}]}.$$  
(A17)

$$F_{\text{EC}}(t)_{\nu>\nu_{\text{min}}}^{\text{EC}} = \frac{8.2 \times 10^{-2}(1+z)^{1.05}}{\epsilon_{e,1} t_{d,1}^{1.05} \nu_{0}^{-0.05} \nu_{0}^{-0.05} \nu_{0}^{-1.15} \nu_{0}^{-1.15} \gamma_{\text{GeV}} d_{L,28}}.$$  
(A18)

All expressions derived so far are valid for scatterings occurring in the Thomson limit (see eqn. A2). Electrons with Lorentz factor greater than $\gamma_{\text{th}}^{v_{\text{th}}}$ scatter $h\nu_{0}$ photons into the KN regime, where the scattering cross section is proportional to $\sigma_{\text{c}}$. Electrons with a Lorentz factor greater than $\gamma_{\text{th}}^{v_{\text{th}}}$ scatter $h\nu_{0}$ photons into the Thomson limit (see eqn. A2). Electrons with Lorentz factor greater than $\gamma_{\text{th}}^{v_{\text{th}}}$ scatter $h\nu_{0}$ photons into the KN regime, where the scattering cross section is proportional to $\sigma_{\text{c}}$. Electrons with Lorentz factor greater than $\gamma_{\text{th}}^{v_{\text{th}}}$ scatter $h\nu_{0}$ photons into the Thomson limit (see eqn. A2). Electrons with Lorentz factor greater than $\gamma_{\text{th}}^{v_{\text{th}}}$ scatter $h\nu_{0}$ photons into the KN regime, where the scattering cross section is proportional to $\sigma_{\text{c}}$.

A direct effect of this is the suppression of high-energy photon production, which happens to the observed photons with energies above

$$h\nu_{\text{KN}} \sim \Gamma_{\text{th}}^{v_{\text{th}}} \approx \frac{(mc_{\text{e}}^{2})^{2}}{\epsilon_{0}} \approx 3 \text{ TeV} \left( \frac{0.1 \text{eV}}{\epsilon_{0}} \right).$$  
(A19)

As long as $v < \nu_{\text{KN}}$, one can safely use the analytical expressions for $F_{\nu}(t)$ presented here.

Fig. A1 shows EC spectra of GRB afterglows computed using eqns. (A7)–(A8) for different parameters. In the upper left panel, we fix all parameters except $n$ and compare spectra at a given time. A less dense ISM results in the shock taking longer time to slow down. So for the given time, the shock in the less dense ISM has a larger Lorentz factor, indicating a higher peak frequency $v_{\text{ECPeak}}$. We also notice that the break frequency of the EC spectrum $v_{\text{ECPeak}}$ increases with $n$ when $n < 1 \text{ cm}^{-3}$, but $v_{\text{ECPeak}}$ shrinks significantly for $n \gtrsim 1 \text{ cm}^{-3}$. This is because when $n > 1 \text{ cm}^{-3}$, SSC starts to dominate the cooling of electrons. Therefore, the EC emission at 100 GeV and above drops significantly. In EC-dominated cases, the flux of 100 GeV to 1 TeV photons depends on $n$ weakly, which may provide a method to estimate $E$ (see Eqn. A15).

The upper right panel of Fig. A1 demonstrates the dependence of flux on the isotropic energy $E$. It is noticed that a more energetic burst will not only produce a larger flux in all bands, but also increase the break frequency $v_{\text{min}}$ and $v_{\text{ECPeak}}$, since it can accelerate particles to higher energy.

The lower left panel of Fig. A1 illustrates the influence of the energy density of the external field $U_{\text{ext}}$ on the EC spectrum. We notice that a stronger ambient photon field will increase the EC emission and significantly enhance the VHE flux. In our model the hydrodynamics of the shock is independent of the external photon field, and $v_{\text{ECPeak}}$ does not change for different values of $U_{\text{ext}}$. But electrons cool faster due to a stronger photon field. Thus, the break frequency of EC spectrum, $v_{\text{ECPeak}}$, drops as $U_{\text{ext}}$ increases.

The lower right panel of Fig. A1 shows the time-dependent EC spectra. As time evolves, the external shock gradually decelerates, and particles becomes less energetic. Hence, the peak frequency of scattered photons $v_{\text{ECPeak}}$ decreases. We find it interesting that the peak flux $F_{\text{ECPeak}}$ remains unchanged as time evolves.
Figure A1. External Compton spectra computed using eqns. (A13)-(A15) for different model parameters. From top left and in clockwise order we vary the number density of the circumburst medium $n$, the initial blast wave energy $E$, the observer time $t$, and the external photon field energy density $U_{\text{ext}}$. The parameters used, unless otherwise specified, are: $p = 2.2$, $n = 0.1 \text{ cm}^{-3}$, $E = 10^{55} \text{ erg}$, $\epsilon_e = 0.1$, $\epsilon_0 = 0.02 \text{ eV}$, $d = 10^{28} \text{ cm}$, $z = 0.5$, and $t = 0.5 \text{ h}$. A coloured version of this plot is available online.