A generalized framework for the quantum Zeno and anti-Zeno effects in the strong coupling regime

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It is well known that repeated projective measurements can either slow down (the Zeno effect) or speed up (the anti-Zeno effect) quantum evolution. Until now, studies of these effects for a two-level system interacting with its environment have focused on repeatedly preparing the excited state via projective measurements. In this paper, we consider the repeated preparation of an arbitrary state of a two-level system that is interacting strongly with an environment of harmonic oscillators. To handle the strong interaction, we perform a polaron transformation and then use a perturbative approach to calculate the decay rates for the system. Upon calculating the decay rates, we discover that there is a transition in their qualitative behaviors as the state being repeatedly prepared continuously moves away from the excited state and toward a uniform superposition of the ground and excited states. Our results should be useful for the quantum control of a two-level system interacting with its environment.

By subjecting a quantum system to frequent and repeated projective measurements, we can slow down its temporal evolution, an effect referred to as the quantum Zeno effect (QZE)¹–²⁴. Contrary to this effect is the quantum anti-Zeno effect (QAZE), via which the temporal evolution of the system is accelerated due to repeated projective measurements separated by relatively longer measurement intervals²⁵–³⁸. Both these effects have garnered great interest not only due to their theoretical relevance to quantum foundations but also due to their applications to quantum technologies. For example, the QZE has shown to be a promising resource for quantum computing and quantum error correction³⁹,⁴⁰. The QAZE, on the other hand, has interestingly been useful in, say, accelerating chemical reactions, suggesting the possibility of quantum control of a reaction⁴¹.

By and large, studies of the QZE and the QAZE have focused on population decay²⁵–³⁰,⁴²–⁴⁷ and pure dephasing models³¹. A few works have gone beyond these regimes. Reference⁴⁸, for instance, presents a general framework to calculate the effective decay rate for an arbitrary system–environment model in the weak coupling regime and finds it to be the overlap of the spectral density of the environment and a filter function that depends on the system–environment model, the measurement interval, and the measurement being performed. This approach, however, fails in the strong coupling regime where perturbation theory cannot be applied in a straightforward manner³¹. For a single two-level system coupled strongly to an environment of harmonic oscillators, Ref.⁴⁹ makes the problem tractable by going to the polaron frame and finding that for the excited state, the decay rate very surprisingly decreases with an increase in the system–environment coupling strength. This effect is further investigated in Ref.⁵⁰, which studies a two-level system coupled simultaneously to a weakly interacting dissipative-type environment and a strongly interacting dephasing-type one. It is found that even in the presence of both types of interactions, the strongly coupled reservoir can inhibit the influence of the weakly coupled reservoir on the central quantum system.

To date, the role of the state that is repeatedly prepared remains relatively unexplored, especially in the strong coupling regime. We emphasize that Ref.⁴⁹ considers the relatively simple case of the repeated measurement of the excited state only. As such, it remains unanswered whether increasing the coupling strength with a strongly coupled reservoir would lead to the decay rate decreasing for any general (or arbitrary) state. This forms the basis of our investigation in this paper. We work out the decay rates for a two-level system strongly interacting

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with a bath of harmonic oscillators and that is repeatedly prepared via a projective measurement in an arbitrary quantum state, that is, any arbitrary linear combination of the ground and excited states. To make the problem tractable, we first go to the polaron frame, where the system–environment coupling is effectively weakened, and, thereafter, use time-dependent perturbation theory to evolve the system state and find its decay rate. Compared to the treatment in Ref. 49, this is a far more complicated task since the polaron transformation also modifies the projection operators, thereby making the trace over the environment considerably more involved. From the decay rate, we are able to observe a stark difference when we perform projective measurements onto a uniform superposition of the excited and ground states. As we explain later, the qualitative variation of the decay rate with the system–environment coupling gets “inverted.” To describe these results, we coin the terms “z-type” and “x-type,” identifying the behavior displayed by Ref. 49 (where increasing the coupling strength leads to smaller decay rates) as the “z-type” behavior and the “inverted” behavior as the “x-type” one. It is found that projections onto the excited or ground states on the Bloch sphere exhibit “z-type” behavior whereas projections onto states lying close to the equatorial plane in the Bloch sphere exhibit “x-type” behavior. This provides the motivation for the names “z-type” and “x-type” since it is typical to take the excited and ground states as up and down along the z axis, respectively, and denote their superposition as being either up or down along the x axis. We also investigate the transition between these z and x behaviors. Our results should be useful in the study of open quantum systems in the strong coupling regime.

Results

Effective decay rate for strong system–environment coupling. We start from the paradigmatic spin-boson model with the system–environment Hamiltonian written as (we work in dimensionless units with $h = 1$ throughout)

$$H_L = \frac{\varepsilon}{2} \sigma_z + \frac{\Delta}{2} \sigma_x + \sum_k \omega_k b_k^\dagger b_k + \sigma_z \sum_k \left( g_k^e b_k + g_k^i b_k^\dagger \right),$$

(1)

where $H_{S,L} = \frac{\xi}{2} \sigma_x + \frac{\Delta}{2} \sigma_x$ is the system Hamiltonian, $H_B = \sum_k \omega_k b_k^\dagger b_k$ is the environment one, and $V_L = \sigma_z \sum_k \left( g_k^e b_k + g_k^i b_k^\dagger \right)$ is the system–environment coupling. Note that $L$ denotes the lab frame, $s$ is the energy splitting of the two-level system, $\Delta$ is the tunneling amplitude, and the $\omega_k$ are frequencies of the harmonic oscillators in the harmonic oscillator environment interacting with the system. The creation and annihilation operators of these oscillators are represented by the $b_k^\dagger$ and $b_k$, respectively. The term ‘tunneling amplitude’ for $\Delta$ is especially appropriate since it is this term that leads to transitions between the ground and excited states; if $\Delta = 0$, the excited state remains the excited state and the ground state remains the ground state. In the strong system–environment interaction regime, we cannot treat the interaction perturbatively. Moreover, the initial system–environment correlations are significant and thus cannot be neglected to write the initial state as a simple product state. To make the problem tractable thus, we perform a polaron transformation, which yields an effective interaction that is weak. More precisely, the transformation to the polaron frame is given by $H = U_P H_L U_P^\dagger$, where $U_P = e^{\frac{\chi}{2} \sigma_z}$ and $\chi = \sum_k \left( \frac{\omega_k^e}{\omega_k} b_k^\dagger - \frac{\omega_k^i}{\omega_k} b_k \right)$. We then get the transformed Hamiltonian

$$H = \frac{\varepsilon}{2} \sigma_z + \sum_k \omega_k b_k^\dagger b_k + \frac{\Delta}{2} \left( \sigma_x e^\chi + \sigma_- e^{-\chi} \right).$$

(2)

For future convenience, we define $H_0 = \frac{\xi}{2} \sigma_x + \sum_k \omega_k b_k^\dagger b_k$. Now, if $\Delta$ is taken as being small (that is, much smaller than the other energy scales such as $\varepsilon$ and $g_k$), the system and environment interact effectively weakly in the polaron frame despite interacting strongly in the lab frame. Let $|0\rangle$ represent the excited state of our two-level system and $|1\rangle$ its ground state. Then, writing an arbitrary initial state of the two-level system as $|\psi\rangle = \zeta_1 |0\rangle + \zeta_2 |1\rangle$ with $\zeta_1 = \cos \left( \frac{\phi}{2} \right)$ and $\zeta_2 = e^{i\beta} \sin \left( \frac{\phi}{2} \right)$, where $\theta$ and $\phi$ are the standard angles on the Bloch sphere, we find the time-evolved density matrix by means of time-dependent perturbation theory. It is important to note that while the initial system–environment state cannot simply be taken as a simple uncorrelated product state in the lab frame, we can do so in the polaron frame since the system and its environment interact effectively weakly in it. We subsequently perform repeated measurements after time intervals of duration $\tau$ to see if the system state is still $|\psi\rangle$. The survival probability at time $\tau$ is $s(\tau) = Tr_{S,B} \left\{ \rho_{\psi} e^{-iH\tau} \right\}$, where $\rho(\tau)$ is the combined density matrix of the system and the environment at time $\tau = \tau$ in the polaron frame just before a projective measurement, $P_\psi = U_P^\dagger |\psi\rangle \langle \psi| U_P$, and $S$ and $B$ denote traces over the system and the bath of harmonic oscillators respectively. The survival probability can then be written as

$$s(\tau) = Tr_{S,B} \left\{ P_\psi e^{-iH\tau} P_\psi \frac{e^{-\beta H_0}}{Z} P_\psi e^{iH\tau} \right\}$$

(3)

with $Z = Tr_{S,B} \{ P_\psi e^{-iH\tau} P_\psi \}$ being a normalization factor and $\beta$ representing the inverse temperature. Until now, the treatment of the survival probability is completely general. Ref. 49 proceeds by considering only the simplest case where $|U_P |\psi\rangle \langle \psi| U_P^\dagger = 0$, which means that the polaron transformation leaves the projector $|\psi\rangle \langle \psi|$ untouched. This is only true for the states $|0\rangle$ and $|1\rangle$. The assumption that $|U_P |\psi\rangle \langle \psi| U_P^\dagger = 0$ greatly simplifies the subsequent calculation since the projection operator $P_\psi$ in the polaron frame contains no environment operators. In the more general case of $|\psi\rangle = \zeta_1 |0\rangle + \zeta_2 |1\rangle$ that we are considering in the current paper with arbitrary $\zeta_1$ and $\zeta_2$, the presence of the additional environment operators in $P_\psi$ makes the calculation far more complicated. The
The environment spectral density is given by \( C(t_2 - t_1) = e^{-\Phi(t_2 - t_1)} \Phi C(t_2 - t_1) - \Phi D_1(t_2 - t_1) \Phi D_2(t_2 - t_1) \Phi C(t_2 - t_1) \). Furthermore, in order to numerically investigate how the decay rate varies with the probability at time \( t \), we get the trace over the environment is much more complicated. Here, we present the results for the case \( \frac{1}{2} \pi \leq \theta \leq \frac{1}{2} \pi \), meaning that we reproduce the expression given in Ref.49. Here, \( C(t_2 - t_1) \) is the environment correlation function given by \( C(t_2 - t_1) = e^{-\Phi(t_2 - t_1)} \Phi C(t_2 - t_1) - \Phi D_1(t_2 - t_1) \Phi D_2(t_2 - t_1) \Phi C(t_2 - t_1) \).

For this case, we get \( \Gamma(t) = -\frac{1}{2} \ln s(t) \). Expanding \( \ln(s(t)) \) up to second order in \( \Delta \), we see that the decay rate works out to be \( \frac{1}{1 + |s(t)|^2} \). Furthermore, in order to numerically investigate how the decay rate varies with the measurement interval \( \tau \), we model the spectral density as \( J(\omega) = G(\omega_0 \zeta + i \epsilon / \omega) \), where \( G \) is a dimensionless parameter characterizing the strength of the system–environment coupling, \( \omega_0 \) is the cut-off frequency, and \( s(t) \) is the so-called Ohmicity parameter. Throughout, we present results for a super-Ohmic environment with \( s = 2 \).

For this case, we get \( \Phi_R = 4G \left( 1 - \frac{1}{1 + |s(t)|^2} \right) \) and \( \Phi_I = \frac{4G \epsilon t}{|\epsilon|^2} \), and we choose to work at zero temperature for the sake of simplicity. We thus obtain

\[
\Gamma(t) = \frac{\Delta^2}{2 \tau} \frac{1}{2} \left( \int_0^t dt_1 \int_0^{t_1} dt_2 e^{-i\omega(t_1 - t_2)} e^{-4G \left( 1 - \frac{1}{1 + |s(t)|^2} \right) \tau} e^{-\frac{4G t_1}{|\epsilon|^2}} \right),
\]

and note that the same decay rate is found if we repeatedly prepare the ground state instead (that is, we set \( \zeta = 0 \) and \( \zeta_2 = 1 \) instead in our general expression).

Now, consider the repeated preparation of an arbitrary quantum state of the two-level system. As noted before, for this case, the projection operator, in the polaron frame, contains environment operators. Consequently, taking the trace over the environment is much more complicated. Here, we present the results for the case \( \zeta_2 = \frac{1}{2} \pi \) that is, \( \psi(t) = \frac{1}{\sqrt{2}} (|0\rangle + e^{i\pi}|1\rangle) \), although we emphasize that our developed formalism allows us to work out the effective decay rate for any arbitrary state—see the “Methods” section and the supplementary information. Working at zero temperature again, we find the effective decay rate to be

\[
\Gamma(t) = \frac{1}{\tau} \left[ \frac{1}{4} \frac{1}{|\epsilon|^2} + 2Re \left\{ e^{i\epsilon t} e^{i\epsilon t} e^{-2\Phi(t_1)} \right\} + 2Re \left\{ e^{-i\epsilon t} e^{-i\epsilon t} e^{-2\Phi(t_1)} \right\} + 2Re \left\{ e^{i\epsilon t} e^{-i\epsilon t} e^{-2\Phi(t_1)} \right\} + 2Re \left\{ e^{-i\epsilon t} e^{i\epsilon t} e^{-2\Phi(t_1)} \right\} \right]
\]

where \( W \) and \( W' \) comprise further bath correlation terms and are given in the supplementary information. In Eqs. (5) and (6), what we have essentially found are the decay rates for initial states corresponding to two extremes characterized by the positions of the states on the Bloch sphere. Whereas Eq. (5) gives the decay rate for the state \( |\psi\rangle \) on a pole of the Bloch sphere, Eq. (6) does so for a state that is the farthest from the poles. Note that the effective decay rate is independent of \( \phi \) when \( \theta \) is chosen to be \( \frac{1}{2} \pi \). Eq. (6) may be seen as catering to all states lying in the equatorial plane of the Bloch sphere. The states \( |0\rangle \) and \( \frac{1}{2} \pi |0\rangle + \frac{1}{2} |1\rangle \) were chosen as simple representatives of the poles and the equatorial plane, respectively. These regions comprise the said extremes since we find that the respective variations of the effective decay rate with the system–environment coupling in these regions are markedly opposite. While increasing the coupling strength leads to a decrease in the decay rate for states on the poles, the opposite occurs when the coupling strength is...
increased for states lying in the equatorial plane. To make this claim concrete, we work out the integrals in Eqs. (5) and (6) numerically and plot the effective decay rate $\Gamma(\tau)$ in Fig. 1 for various system–environment coupling strengths. As Fig. 1a clearly shows, increasing the coupling strength effects a general increase in the decay rate if the initial state is $\frac{1}{\sqrt{2}}|0\rangle + \frac{1}{\sqrt{2}}|1\rangle$. Precisely the opposite is seen in Fig. 1b; that is, increasing the system–environment coupling decreases the effective decay rate when the initial state is $|0\rangle$. It might therefore be said that uniform superpositions essentially “invert” the inhibiting effect that an increase in the coupling strength has on the decay rate in Fig. 1b. The behavior of $\Gamma(\tau)$ as a function of $\tau$ also allows us to identify the Zeno and anti-Zeno regimes. The Zeno regime is the region where decreasing $\tau$ leads to a decrease in $\Gamma(\tau)$, whereas we observe only the QZE for $G = 1$ in Fig. 1b, we also see the QAZE for $G = 2$ and $G = 3$. In Fig. 1a, however, we see both the QZE and the QAZE for all the coupling strengths shown. Increasing $G$ thus bears forth a significant qualitative change in the QZE/QAZE behavior of the central quantum system if the initial state is on the poles of the Bloch sphere, but the same is not true for uniform superpositions. This also tells us that the aforementioned two regions on the Bloch sphere are characterized by significantly different variations of the decay rate with the system–environment interaction. Finally, it is worth noting that for uniform superpositions, the decay rates are generally much higher than the ones corresponding to the ground or excited states (see Fig. 1a,b). This result makes sense because in the strong coupling regime, the system–environment coupling acts as a protection for its eigenstates, meaning that the eigenstates of the interaction term actually benefit from an increased coupling with the environment in that they remain alive for longer times. This protection, however,
is lost as we move away from a pole on the Bloch sphere and toward the equatorial plane as is apparent in Fig. 2, where we plot the decay rates against $\pi/\theta$ for varying polar angles.

If $\Gamma(\tau)$ is plotted against $\tau$ for different values of $\theta$, it is found that for any coupling strength, all the initial states have decay rates with one maximum. If we now assume two different coupling strengths, say, $G_1$ and $G_2$, and we consider $G_2 > G_1$ without loss of generality, we notice that the decay rates exhibit either "z-type" or "x-type" behavior, depending on the initial state of the quantum system. For states we term as having "z-type" behavior, the maximum of $\Gamma(\tau)$ corresponding to $G_1$ is greater as is characteristic of the state $|0\rangle$ in Fig. 1b. Similarly, we term states as showing "x-type" behavior if the maximum of $\Gamma(\tau)$ corresponding to $G_1$ is smaller. Hence, for any $G_1$ and $G_2$, there has to exist a value of the angle $\theta$ at which we see a transition between these two behaviors. To show the existence of this critical value of $\theta$, which we label as $\theta_c$, we plot the difference between the respective maxima of decay rates corresponding to $G_1$ and $G_2$ against $\theta$ (see Fig. 3) and find the value of $\theta$ such that this difference becomes approximately zero. To show that $\theta_c$ is actually the said critical value of $\theta$, we plot the decay rates against $\tau$ for values of $\theta$ less than $\theta_c$, equal to $\theta_c$, and greater than $\theta_c$ as illustrated in Fig. 4. It is clear that when $\theta = \theta_c$ (approximately $\pi/225$ for the case chosen), the peaks of the curves corresponding to $G_1$ and $G_2$ are at the same height above the $\tau$ axis. When $\theta < \theta_c$, the peak for $G_2$ wins, showing that the "x-type" behavior dominates, and when $\theta > \theta_c$, the peak for $G_1$ wins, showing that the "z-type" behavior dominates. It is interesting to note that $\theta_c$ is $\phi$ dependent. While we have presented our analysis with $\phi = 0$, we have found that it is always possible to find $\theta_c$ regardless of the value of $\phi$.

**Modified decay rates for strong and weak system–environment coupling.** In investigating the effect of changing the initial state on the QZE and the QAZE, we used the complete Hamiltonian so far. This means that the evolution of the system state depends on the system Hamiltonian as well as the system–environment interaction. However, if we intend to study solely the effect of the dephasing reservoir on the QZE and the QAZE via its interaction with the system, we would like to remove the evolution due to the system Hamiltonian. We can do so by performing, just before each projective measurement, a reverse unitary time evolution due to the system Hamiltonian on the fully time-evolved density matrix as has also been done in Refs. 31, 48, 49, 63—such a reverse unitary time evolution can be realized by applying suitable control pulses to the central two-level system. 

The survival probability then becomes

$$s(\tau) = Tr_{S,B} \left\{ P_\psi U_{S,I}^\dagger(\tau) U_{S,0}(\tau) U_0(\tau) U_I(\tau) P_\psi e^{\beta H} P_\psi U_I^\dagger(\tau) U_{0}(\tau) U_{S,0}(\tau) U_{S,I}(\tau) \right\}, \tag{7}$$

**Figure 4.** Transitory behavior in the effective decay rates. (a) Graph of $\Gamma(\tau)$ (at zero temperature) with the initial state corresponding to $\theta = \pi/200$ for $G = 1$ (solid magenta curve) and $G = 3$ (dashed, red curve). (b) Same as (a) with the initial state corresponding to $\theta_c = \pi/225$, showing critical behavior. (c) Same as (a) except that $\theta = \pi/250$. 

**Figure 3.** Difference between the maxima of the effective decay rates corresponding to $G_1$ and $G_2$ against $\pi/\theta$. 
where \(U_{S0}(\tau) = e^{-iH_{S}\tau}\) and \(H_{S} = \frac{\epsilon}{2}\sigma_{z} + \frac{\gamma}{2}\sigma_{x}\). As before, we can simplify this further by operating in the polaron frame (see the “Methods” section for details). This procedure yields the decay rate

\[
\Gamma_n(\tau) = \Gamma(\tau) + \Gamma_{\text{mod}}(\tau).
\]  

Equation (8) shows that upon removing the system evolution, the decay rate works out to contain both the earlier found effective decay rate and some additional terms represented by \(\Gamma_{\text{mod}}(\tau)\). This follows from the further application of the perturbative approach. We present in the supplementary information a general expression for the modified decay rate for an arbitrary state \(|\psi\rangle\). Using this expression, we show the behavior of the modified decay rate as a function of the measurement interval for different states in Fig. 5. We again find that the decay rate increases as we move toward the circle of uniform superpositions on the Bloch sphere. Moreover, Fig. 6a shows that increasing the system–environment coupling strength generally increases the decay rate for \(\theta = \frac{\pi}{2}\). As such, the removal of the system evolution does not change the qualitative behavior of the decay rates in any significant way, and we can confidently say that the primary contribution to the decay rate comes from the system–environment interaction. We arrive at a similar conclusion upon plotting the decay rate corresponding to the initial state \(|0\rangle\) in Fig. 6b, that is, the qualitative behavior remains the same as before and increasing the coupling strength now generally decreases the decay rate. We again emphasize that we could have chosen any states other than \(\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)\) and \(|0\rangle\) since our general expression for the modified decay rate works for any arbitrary state.

To complete our analysis, we can, as before, numerically sample through decay rates corresponding to different points on the Bloch sphere to identify a transitory stage, or \(\theta_{c}\), and find similar transitions as found before (see Fig. 7). As we move toward the poles (see Fig. 8a–c), we observe the shift from the “x-type” behavior to the “z-type” behavior.
\[ \chi_{\sigma} \left( \sigma \right) \sum_{\sigma} \theta e^{i \theta} = \chi \sum_{\sigma} \theta e^{i \theta} \]

This gives the polaron transformed Hamiltonian.

To conclude, we have extended the investigation of the QZE and the QAZE for a two-level system interacting strongly with a harmonic oscillator environment. The transformation is given by the unitary operator \( U_P = e^{\frac{\theta}{2} H} \) such that \( H = e^{\frac{\theta}{2} H} H e^{-\frac{\theta}{2} H} \), where \( \chi = \sum_k \left( \frac{2g_k}{\omega_k b_k^+ b_k} - \frac{2g_k^*}{\omega_k b_k^+ b_k} \right) \). Making use of \( e^{2g_k Y e^{-g_k X}} = Y + \theta [X, Y] + \frac{\theta^2}{2!} [X, [X, Y]] + \cdots \), we first evaluate \( \frac{\theta}{2} \sum_k \omega_k b_k^+ b_k \), \( \frac{\theta}{2} \sum_k \omega_k b_k^+ b_k \), \( \frac{\theta}{2} \sum_k \omega_k b_k^+ b_k \), and all the higher-order commutators. We find that \( \frac{\theta}{2} \sum_k \omega_k b_k^+ b_k = -\sigma_z \sum_k \left( g_k^* b_k + g_k b_k^+ \right) \) and \( \frac{\theta}{2} \sum_k \omega_k b_k^+ b_k = \left( g_k^* b_k + g_k b_k^+ \right) \right) = -2 \sum_k \frac{|g_k|^2}{m} \). Since the latter is only a constant, the higher-order commutators are zero. Moreover, since the tunneling term could be written in the form \( \frac{\xi}{2} \sigma_z = \frac{\xi}{2} \left( \sigma_+ + \sigma_- \right) \) and \( \frac{\theta}{2} \sum_k \sigma_+ = \chi \sigma_+ \), while \( \frac{\theta}{2} \sum_k \sigma_- = -\chi \sigma_- \), the tunneling term in the polaron frame is \( \frac{\xi}{2} \left( \sigma_+ e^\xi + \sigma_- e^{-\xi} \right) \). Using these commutators, we get \( e^{\theta \sigma_z/2} \left[ \frac{\xi}{2} \sigma_z + \frac{\xi}{2} \sigma_x + \sum_k g_k b_k^+ b_k + \sigma_z \sum_k \left( g_k^* b_k + g_k b_k^+ \right) \right] e^{-\theta \sigma_z/2} \equiv \frac{\xi}{2} \sigma_z + \sum_k \omega_k b_k^+ b_k + \frac{\xi}{2} \left( \sigma_+ e^\xi + \sigma_- e^{-\xi} \right) \). This gives the polaron transformed Hamiltonian.

**Effective decay rate for a strongly interacting environment.** We now describe the procedure for deriving the decay rate from the survival probability stated in Eq. (3). To do so, we first work out the time-evolved density matrix of the composite system, that is, \( \rho(\tau) = e^{-iH\tau} \rho \sum_{\sigma} e^{i\theta} \rho \sum_{\sigma} e^{iH\tau} \). Since we are in the
polaron frame and we take Δ as being small, the effective system–environment interaction may be treated perturbatively. Now, ρ(τ) = U0(τ)U1(τ)ρ(0)U† 1(τ)U0 2(τ), where U0(τ) is the unitary time-evolution operator corresponding to the system Hamiltonian HS = 1 2 σ2 and the environment Hamiltonian H2 = ∑k ak bk†bk whereas U1(τ) is the unitary evolution due to the system–environment interaction. The survival probability thus becomes

\[ s(τ) = \text{Tr}_S,\mathcal{S} \left\{ \rho_0 U(τ) U(τ) P_\psi U(τ) \right\} = \frac{1}{2} \sigma_Z \rho_0 U(τ) U(τ) P_\psi U(τ) \right\} . \]

Using cyclic invariance, we absorb the system time evolution into the projector \( P_\psi \) and evolve it to \( P_\psi = U_1^2(τ) P_\psi U_1^2(τ) \). We then find U1(τ)P^z = U^z 2 P_\psi U_1^2(τ). Recalling that the interaction Hamiltonian is \( H_I = \frac{1}{2}(\sigma_x e^x + \sigma_y e^{-x}) \) in the polaron frame and writing \( V_2(t) = e^{\theta B(t)} H_I(t) e^{-\theta B(t)} \), we get \( V_1(t) = \frac{1}{2} \sum_{ij} (\tilde{F}_{ij}(t) \otimes \tilde{B}_{ij}(t)) \), where \( \tilde{F}_0(t) = \sigma_x e^{-\theta B(t)} \), \( \tilde{F}_1(t) = \sigma_x e^{\theta B(t)} \), \( \tilde{B}_1(t) = e^{-\theta B(t)} \). This gives \( U_2(τ) = 1 - i \int_0^τ dt_1 V_1(t_1) - \int_0^τ \int_0^{t_1} dt_2 V_1(t_1) V_1(t_2) + \cdots \). Defining \( A_1(τ) = -i \int_0^τ dt_1 V_1(t_1) \) and \( A_2(τ) = -i \int_0^τ \int_0^{t_1} dt_2 V_1(t_1) V_1(t_2) \), we find that \( ρ(τ) \) up to the second order is

\[ ρ(τ) = ρ(0) + A_1(τ) ρ(0) + A_2(τ) ρ(0) + ρ(0) A^*_1(τ) + ρ(0) A^*_2(τ) + A_1(τ) ρ(0) A^*_1(τ) \] (9)

It should be noted that ρ(0) as given in the Results section may be written as \( P_\psi e^{-θ H_0} P_\psi / Z = \sum_{nj} M_{nj} C^n_{j} E^n_{j} / Z \), where \( i, j, \) and \( n \) could be either 0 or 1, \( M_{ij} = |i⟩ ⟨j| \), and the \( C^n_{j} \) and \( E^n_{j} \) are given by the following tables:

Using Tables 1 and 2, we find it easy to see that Eq. (9) may be recast as \( ρ(τ) = \sum_{nj} T_{nj} M_{nj} C^n_{j} E^n_{j} / Z \), where \( T_{nj} = \sum_{mn} M_{mj} C^n_{j} E^n_{j} T_{mn} = T_{m1} A^*_1 T_{1j} \). Now, we only need to work out the density matrix after the system evolution has been removed, that is, \( U_1^2(τ) e^{-θ H_0} \left[ ρ_0 + A_1(τ) ρ_0 + A_2(τ) ρ_0 + ρ(0) A^*_1(τ) + ρ(0) A^*_2(τ) + A_1(τ) ρ(0) A^*_1(τ) \right] e^{-θ H_0} U_1^2(τ) \). Writing \( V_1(t) = e^{i H_0} H_I(t) e^{-i H_0} \), we get \( V_2(t) = \frac{1}{2} \sum_{ij} (\tilde{F}_{ij}(t) \otimes \tilde{B}_{ij}(t)) \), where \( \tilde{F}_0(t) = \sigma_x e^{-i H_0} \), \( \tilde{F}_1(t) = \sigma_x e^{i H_0} \), \( B_0 = e^B, \) and \( B_1 = e^{-B} \) as before. This leads to \( U_2(τ) = 1 - i \int_0^τ dt_1 V_1(t_1) - \int_0^τ \int_0^{t_1} dt_2 V_1(t_1) V_1(t_2) + \cdots \). Using \( A^{(1)} = -i \int_0^τ dt_1 V_1(t_1) \) and \( A^{(2)} = -i \int_0^τ \int_0^{t_1} dt_2 V_1(t_1) V_1(t_2) \) now, we can conveniently write the fully time-evolved density matrix with the system evolution removed as \( ρ(τ) = \left( 1 + A^{(1)} + A^{(2)} \right) e^{-θ H_0} \sum_{m=1}^N \int_0^τ t e^{i H_0} \left( 1 + A^{(1)} + A^{(2)} \right) \). We work this out to second order, apply the projection operator \( P_\psi \), and find the trace over the system and the environment in the same way as before. We then arrive at the survival probability that the modified decay rate could be found from. Details on its expression could be found in the supplementary information.

**Data availability**

The datasets used and/or analyzed during the current study are available from the corresponding author on reasonable request.
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Author contributions
A.Z.C came up with the basic idea behind this work. G.K., M.U.B., H.S., and I.J. carried out the calculations with G.K. contributing majorly. M.U.B. plotted the graphs. A.Z.C., H.S., and I.J. contributed toward the writing of the manuscript.

Competing interests
The authors declare no competing interests.

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