Double $\Delta(1232)$ excitation and the ABC effect

in the reaction $n + p \rightarrow ^2H(\pi\pi)$

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Abstract

The deuteron spectrum in $n+p \rightarrow ^2H(\pi\pi)$ at $k_n = 1.88$ GeV/c and $\theta_d = 0^\circ$ is explained by considering a $\Delta\Delta$ excitation as the dominant reaction mechanism for the $2\pi$ production. We present a new theoretical approach based on a coupled channel formalism which allows to include the residual interaction within the intermediate $\Delta\Delta$ and $\Delta N$ systems. The corresponding interaction potentials $V_{\Delta\Delta}$ and $V_{\Delta N}$ are adopted from a meson exchange model with $\pi$, $\rho$, $\omega$, and $\sigma$ exchange taken into account. The influence of the residual interaction on the deuteron spectrum is studied. We also predict the angular distribution of the two pions. It is shown that this distribution is closely connected to the spin structure of the $\Delta\Delta$ excitation.

13.75.-n, 25.10.+s, 14.20.Gk, 24.10.Eq
I. INTRODUCTION

The ABC effect, a mesonic structure of isospin $I = 0$, was first observed by Abashian, Booth and Crow in the reaction $p + ^2 H \rightarrow ^3\text{He} + (\pi\pi)$ [1–3]. Later on, it was also shown to appear in several other hadronic reactions, like e.g. in $n + p \rightarrow ^2\text{H} + X$ [4,5] and in $^2\text{H} + ^2\text{H} \rightarrow ^4\text{He} + X$ [6,7]. In all these cases, the ABC structure shows up as a cross section peak at missing masses of about 300 – 350 MeV, and with a position and width that varies quite rapidly according to the kinematic conditions. The only possible explanation for the effect seems to be an enhancement in the double pion production which is caused by a strongly energy dependent production amplitude.

Since the ABC effect is present in reactions of different type, it seems to be quite natural to assume that there exists a common underlying mechanism for the $2\pi$ production which should be able to explain all the different experiments. From a theoretical point of view, $n + p \rightarrow ^2\text{H} (\pi\pi)$ has to be the reaction to analyze first since it involves only two-nucleon states beside the pions and can thus be treated in the most accurate and direct way. For this reaction, the ABC enhancement corresponds to a beam momentum of approximately 2 GeV/c in the laboratory system and to a center of mass energy of $\sqrt{s} \approx 2M_\Delta$, i.e. twice the mass of the $\Delta(1232)$ resonance. This fact strongly suggests that the two pions are dominantly produced via a double $\Delta$ excitation. Indeed, as it has already been shown by Bar–Nir et al. [8,11], the main features of the experimental spectrum can be understood by assuming the $\Delta\Delta$ mechanism to be the most efficient one for $2\pi$ production in the energy region under consideration.

In the present work, we will present a new calculation of $\Delta\Delta$ and $\Delta N$ excitations in the $n + p \rightarrow ^2\text{H} (\pi\pi)$ reaction which explicitly includes the effects of residual interactions in the intermediate resonance states [12,13]. Our aim is to show that the $2\pi$ production may serve as an interesting tool for further examination of direct $\Delta\Delta$ and $\Delta N$ potentials which are not very well known. For this purpose, we will also examine how the spin structure of the $\Delta\Delta$ excitation influences the angular distribution of the two pions. Our model is
based on a coupled channel approach in a non–relativistic framework. The potentials of the residual interactions are adopted from a meson exchange model \[14,15\]. In this work, contributions from $\pi$, $\rho$, $\omega$, and $\sigma$ exchange are included. The $\Delta$ resonance is treated thereby as a quasi–particle with a given mass and an energy–dependent, intrinsic width.

Experimental results for the $n+p\rightarrow {}^2H(\pi\pi)$ cross section in the ABC energy region have been obtained by Plouin et al. \[4\] at $k_n = 1.88$ GeV. In order to allow for a direct comparison with our theoretical results, the contribution from one pion production has been subtracted from the spectra. It has been stressed that there might be some additional background from $\eta$ production \[14\] due to momentum spread of the beam, but since there is no microscopic model for this contribution available, we decided not to perform any further corrections. One should keep in mind, however, that not all of the uncertainties have been included in the experimental error bars.

In section II, we will present the theoretical framework of our model. The results of our calculation are presented and discussed in section III. The paper concludes with a summary in section IV.

II. THEORETICAL FRAMEWORK

A. Cross section

We are interested here in the calculation of the deuteron spectrum for the $n+p\rightarrow {}^2H(\pi\pi)$ reaction, i.e. in the double differential cross section $d^2\sigma/dk_d d\Omega_d$. Using relativistic kinematics, the cross section is given as

$$d\sigma = \frac{2E_p E_n}{\sqrt{\lambda(s, M_p, M_n)}} \frac{M_n}{E_n} \frac{M_p}{E_p} \frac{d^3 k_d}{(2\pi)^3} \frac{d^3 k}{(2\pi)^3(2E_a)(2E_b)} \delta(E_p + E_n - E_d - E_{\pi\pi}) |M|^2$$

(1)

where $\vec{k} = \frac{1}{2}(\vec{k}_a - \vec{k}_b)$ denotes the relative momentum of the two pions and $E_{\pi\pi} = E_a + E_b$ is the sum of their energies. The indices $n(p)$ and $d$ refer to the neutron projectile (proton target) and the deuteron ejectile, respectively. The center of mass momentum of the $2\pi$ system $\vec{K} = \vec{k}_a + \vec{k}_b$ is fixed by momentum conservation and equals $\vec{k}_n + \vec{k}_p - \vec{k}_d$. As usual,
the function \( \lambda \) is defined to be 
\[
\lambda(s, M_p^2, M_n^2) = [s - (M_p + M_n)^2] [s - (M_p - M_n)^2].
\]
Finally, \( |\mathcal{M}|^2 \) denotes the transition amplitude for the reaction; in the case of the unpolarized cross section, an average over the initial and a sum over the final spin orientations has to be performed.

From eq. (1) one easily obtains the deuteron spectrum

\[
\frac{d^2\sigma}{dk_d d\Omega_d} = \frac{1}{(2\pi)^5} \frac{M_n}{k_{n\text{lab}}^2} k_d^2 \int \frac{k^2}{4E_aE_b} \frac{dk}{dE_{\pi\pi}} d\Omega |\mathcal{M}|^2
\]

Since we are going to calculate the transition matrixelement in a non–relativistic approach, the above expression has to be evaluated in the deuteron rest frame (drf) where the deuteron is well described by the usual non–relativistic wave function \([15]\). Afterwards, the transformation of the spectrum into the laboratory frame (lab) is performed by using the simple relation

\[
\left( \frac{d^2\sigma}{dk_d d\Omega_d} \right)_\text{lab} = \frac{E_d^{\text{drf}}(k_{\text{lab}}^d)^2}{E_d^{\text{lab}}(k_{\text{drf}}^d)^2} \left( \frac{d^2\sigma}{dk_d d\Omega_d} \right)_\text{drf}.
\]

**B. Reaction mechanism and evaluation of the transition amplitude**

In our microscopic model for the \( n + p \rightarrow ^2\text{H} (\pi\pi) \) reaction, we take into account two different reaction mechanisms corresponding to the Feynman diagrams presented in fig. [1]. The first mechanism (a) involves the excitation of a double Delta (\( \Delta\Delta \)) intermediate state where both \( \Delta \) resonances subsequently decay into a pion and a nucleon. In the case of the second mechanism (b), each of the two pions couples to the same \( \Delta \), hence only one resonance (i.e. a \( \Delta N \) system) is excited.

For the explicit calculation of the two matrix elements, we take advantage of the principle of detailed balance. This allows us to assume that the deuteron is in the initial state and absorbs the two pions, which finally will lead to a \( n + p \) state. The absorption of only one pion with momentum \( \vec{k}_\pi \) is described by the operator

\[
F_\pi^\dagger(\vec{k}_\pi) = e^{i\vec{k}_\pi \cdot \vec{r}/2} \frac{f_{\pi N\Delta}}{m_\pi} (\vec{S}_1 \cdot \vec{k}_\pi) T_1^\dagger + (1 \leftrightarrow 2).
\]
The exponential term represents the plane wave of the pion field. The structure of the πΔN coupling follows from the usual interaction lagrangian [15] in the non-relativistic reduction. \( \vec{S}^\dagger (\vec{T}^\dagger) \) is the spin (isospin) transition matrix [17] for the Δ excitation, and \( \vec{r} = \vec{r}_1 - \vec{r}_2 \) is the relative coordinate of the two nucleons in the deuteron. The symbol \( 1 \leftrightarrow 2 \) in eq. (4) shall indicate that the operator has to be properly symmetrized with respect to the nucleonic coordinates, i.e. each of the two nucleons in the deuteron may be excited to a Δ resonance.

Following refs. [18,14], the quantum mechanical state

\[
|\rho_{\Delta N}\rangle = F^\dagger_\pi (\vec{k}_\pi) |\psi_d \rangle
\]

is called the uncorrelated source function for the ΔN intermediate system, with \( |\psi_d \rangle \) being the deuteron wave function. After propagation of the ΔN system and absorption of a second pion, one obtains the source function for the ΔΔ state,

\[
|\rho_{\Delta\Delta}\rangle = F^\dagger_\pi (\vec{k}_a) G_{\Delta N} F^\dagger_\pi (\vec{k}_b) |\psi_d \rangle.
\]

In our model, the full propagator \( G_{\Delta N} \) is given by

\[
G_{\Delta N} = \frac{1}{\epsilon_{\Delta N} + \frac{i}{2} \Gamma_{\Delta}(s_\Delta) - T_{\Delta N} - V_{\Delta N}}.
\]

It contains the excitation energy \( \epsilon_{\Delta N} \), the energy–dependent width \( \Gamma_{\Delta}(s_\Delta) \) of the resonance [19], the operator of the kinetic energy \( T_{\Delta N} \) and the interaction potential \( V_{\Delta N} \) of the intermediate ΔN system. \( \epsilon_{\Delta N} \) and \( s_\Delta \) are fixed by imposing energy conservation at the πNΔ vertex. The potential \( V_{\Delta N} \) is constructed in a meson exchange model with \( \pi, \rho, \omega \) and \( \sigma \) exchange taken into account. A diagrammatic representation of the residual ΔN interaction is given in fig. 2 (a,b). For more details, we refer the reader to refs. [14,13] where also explicit expressions for \( V_{\Delta N} \) can be found.

In eq. (4) we have not yet considered that the ΔΔ source function has to be symmetric under exchange of the pions \( a \) and \( b \). According to the value \( T \) of total isospin in the \( \pi\pi \) system, we define the symmetrized operator

\[
F^\dagger_{\pi\pi}(\vec{k}_a, \vec{k}_b) = \frac{1}{\sqrt{2}} \left\{ F^\dagger_\pi (\vec{k}_b) G_{\Delta N} F^\dagger_\pi (\vec{k}_a) \pm F^\dagger_\pi (\vec{k}_a) G_{\Delta N} F^\dagger_\pi (\vec{k}_b) \right\}
\]
with \[
\left\{ \begin{array}{ll}
+ \text{ if } & |T_M T\rangle = |00\rangle = \frac{1}{\sqrt{3}} |\pi^+\pi^- - \pi^0\pi^0 + \pi^-\pi^+\rangle \\
- \text{ if } & |T_M T\rangle = |10\rangle = \frac{1}{\sqrt{2}} |\pi^+\pi^- - \pi^-\pi^+\rangle 
\end{array} \right.
\]

The complete matrix element for the $\Delta\Delta$ mechanism can now be written as

\[ M_{\Delta\Delta} = \langle \psi_n | \langle \psi_p | V_{\Delta\Delta \rightarrow NN} G_{\Delta\Delta} F_{\pi\pi}^\dagger(\vec{k}_a, \vec{k}_b) | \psi_d \rangle \]  \(9\)

Here, $\psi_n(p)$ denote the distorted waves of the projectile neutron (target proton) which are calculated in eikonal approximation. In analogy to eq. (7), the full propagator for the $\Delta\Delta$ system is given by

\[ G_{\Delta\Delta} = \frac{1}{\epsilon_{\Delta\Delta} + \frac{i}{2} [\Gamma_{\Delta a}(s_{\Delta a}) + \Gamma_{\Delta b}(s_{\Delta b})] - T_{\Delta\Delta} - V_{\Delta\Delta}}. \]  \(10\)

Especially, this propagator includes the residual interaction $V_{\Delta\Delta}$ in the excited $\Delta\Delta$ system, see also fig. 2(c). $V_{\Delta\Delta}$, as well as the transition potential $V_{\Delta\Delta \rightarrow NN}$ between the intermediate $\Delta\Delta$ system and the $NN$ state, are both constructed within the same meson exchange model as $V_{\Delta N}$. To the $\Delta\Delta \rightarrow NN$ transition potential, of course, only the $\pi$ and $\rho$ meson exchanges contribute. The $\Delta\Delta$ excitation can thus be separated into a spin–longitudinal part (with $\pi$–like coupling to $NN$) and a spin–transverse part (with $\rho$–like coupling to $NN$). As we will demonstrate later, this spin–structure of the $\Delta\Delta$ excitation is important for the angular distribution of the pions.

The matrix element corresponding to the second mechanism where only one $\Delta$ is excited, fig. 1(b), is found to be

\[ M_{\Delta N} = \langle \psi_n | \langle \psi_p | \bar{F}_{\pi\pi}^\dagger(\vec{k}_a, \vec{k}_b) | \psi_d \rangle \]  \(11\)

Here, the operator $\bar{F}_{\pi\pi}^\dagger$ is obtained from $F_{\pi\pi}^\dagger$ by simply replacing for the second pion $\vec{S}$ and $\vec{T}$ with their hermitian adjungates $\vec{S}$ and $\vec{T}$, respectively.

In order to obtain the $2\pi$ production amplitude, the coherent sum of the two contributions of eqs. (9, 11) has to be taken. Due to the fact that the residual interactions $V_{\Delta N}$ and $V_{\Delta\Delta}$ couple states of different quantum numbers, the concrete evaluation of the matrix elements leads to a system of coupled integro–differential equations. It turns out that they can be
solved in a very effective way with the so–called Lanczos method \cite{20}. This procedure has already been described in ref. \cite{18} and can easily be applied to the present calculation.

### III. RESULTS AND DISCUSSION

#### A. Parameters of the model

Input parameters of the model are the meson and baryon masses, coupling constants, and formfactor cutoffs at each vertex. The values we used in our calculations are given in table I. Most of these quantities, of course, are physical observables that are already determined from other experiments, e.g. from $NN$ scattering \cite{21}. Only the sigma meson and cutoff parameters remain to be fixed. They have been adjusted to fit the experimental data for the one pion production $N + N \rightarrow ^2H \pi$. We made sure that not only the total cross section but also the angular distributions and analyzing powers are correctly reproduced in the $\Delta$ resonance energy region \cite{22}. After this consistency check, we may now continue and discuss our results for the $2\pi$ production $n + p \rightarrow ^2H (\pi\pi)$.

#### B. The deuteron spectrum

In fig. 3, we present the experimental deuteron spectrum $d^2\sigma/dk_d d\Omega_d$ as it is measured in the reaction $n + p \rightarrow ^2H (\pi\pi)$ at a beam momentum $k_n = 1.88$ GeV and in forward direction $\theta_d = 0^\circ$. The production of both neutral ($\pi^0\pi^0$) and charged ($\pi^+\pi^-$) pion pairs is possible. The available phase space for the $2\pi$ production is indicated by the dashed line.

Obviously, the cross section does not follow the phase space but shows a characteristic structure which three pronounced peaks. The two outer maxima correspond to a kinematical situation where the invariant mass $M_{\pi\pi}$ of the $2\pi$ system is minimal, i.e. where $M_{\pi\pi} = 2m_\pi$. The broad central maximum, on the other hand, is located at the position where $M_{\pi\pi}$ has its maximal value (511 MeV in the present case). In the CMS, this situation will be realized if all the kinetic energy is taken by the two pions and the deuteron remains at rest.
As can be seen from fig. 4, our model is able to explain the characteristic structure of the experimental spectrum. The solid line represents the full result with all intermediate interactions included. The $\Delta\Delta$ mechanism, for which the Feynman diagram was presented in fig. (a), is in fact the only important contribution to $2\pi$ production in the energy range under consideration. The cross section contribution due to the $\Delta N$ mechanism of fig. (b) is almost negligible. Since the optimal energy for this second mechanism would be $\sqrt{s} = M_{\Delta} + M - m_\pi$, its suppression in the ABC energy region is not surprising.

With focus on the $\Delta\Delta$ excitation and following the arguments of ref. [11], the origin of the different maxima can be understood quite easily. Let us define

$$K = p_a + p_b \quad \text{and} \quad k = p_a - p_b ,$$

(12)

where $p_a, p_b$ denote the four–momenta of the two pions. The excitation of a $\Delta\Delta$ system is most effective if the invariant masses of the two $\Delta$’s are approximately equal (and hence both close to the resonant mass). In this case we have

$$s_{\Delta a} = \frac{1}{4} (p_d + K + k)^2 = \frac{1}{4} (p_d + K - k)^2 = s_{\Delta b} ,$$

(13)

and since $K \cdot k = 0$ this is equivalent to

$$p_d \cdot k = 0 .$$

(14)

This condition can be fullfilled in two ways:

1. $k = 0$ and therefore $M_{\pi\pi} = \sqrt{K^2} = 2m_\pi$, or

2. $\vec{p}_d = 0$ in the CMS of the two pions (which means that the deuteron restframe and the $2\pi$ restframe are identical). In the latter case, the two pions carry the whole kinetic energy, and $M_{\pi\pi} = \text{max}$.

The first situation leads to the outer maxima and corresponds to the parallel decay of the two $\Delta$’s because of $p_a \approx p_b$, i.e. the relative momentum is very small. The second situation explains the central maximum and corresponds to the antiparallel decay of the
$\Delta \Delta$ excitation for which the relative momentum is maximal. In the deuteron restframe, the different kinematical configurations for the three maxima can be visualized as depicted in fig. 5.

C. Angular distribution of the pions

The kinematical situations leading to the three maxima of the deuteron spectrum are also clearly visible in the angular distribution of the pions. In fig. 6, we show the triple differential cross section $d^3\sigma/dk_{d}d\Omega_{d}d\Omega_{\pi}$ for different deuteron laboratory momenta. It is plotted as a function of $\theta_{\pi}$ which is the angle between the relative momentum $\vec{k}$ of the two pions and the beam axis, as given in the deuteron rest frame (see also fig. 5).

For the two outer maxima ($k_{d}^{\text{lab}} = 1.1$ GeV and $k_{d}^{\text{lab}} = 1.9$ GeV), the pion momenta are nearly parallel and hence the relative momentum is dominantly perpendicular to the beam axis. Consequently, the differential cross section reaches its largest value at $\cos \theta_{\pi} = 0$.

For the central maximum ($k_{d}^{\text{lab}} = 1.5$ GeV), however, the pion momenta are antiparallel, and all angles $\theta_{\pi}$ are kinematically possible. If there was no spin dependence of the excitation and the residual interaction, an isotropic distribution would be the result. The observed angular variation of the cross section is thus reflecting the spin structure of the process which mainly follows from the $\pi$ and $\rho$ exchange contributions to the $NN \rightarrow \Delta \Delta$ transition potential.

In order to examine this spin structure in more detail, we will neglect for the moment the residual interaction and the possibility of spin–flips. Then, the spin–longitudinal $\pi$ exchange leads to an operator proportional to

$$\left[ (\vec{S}_{1} \cdot \vec{q}) (\vec{S}_{1}^{\dagger} \cdot \vec{k}_{a}) \right] \left[ (\vec{S}_{2} \cdot \vec{q}) (\vec{S}_{2}^{\dagger} \cdot \vec{k}_{b}) \right] \sim (\vec{q} \cdot \vec{k}_{a}) (\vec{q} \cdot \vec{k}_{b}),$$

where $\vec{k}_{a}, \vec{k}_{b}$ are the pion momenta and $\vec{q}$ is the momentum transfer from the neutron projectile to the proton target. If both the pion momenta are parallel (or antiparallel) to $\vec{q}$, the corresponding cross section will be largest. This results in a maximum at $\cos \theta_{\pi} = 1$ in the spin–longitudinal channel.
In complete analogy, the spin–transverse $\rho$ exchange has the operator structure

$$\left[(\vec{S}_1 \times \vec{q}) \cdot \vec{k}_a \right] \left[(\vec{S}_2 \times \vec{q}) \cdot \vec{k}_b \right] \sim (\vec{q} \times \vec{k}_a) \cdot (\vec{q} \times \vec{k}_b).$$

Therefore, the maximum in the spin–transverse channel is reached if both the pion momenta are perpendicular to $\vec{q}$, i.e. if $\cos \theta_\pi = 0$.

Indeed, the theoretical curves in fig. 7 exactly reflect the expected shapes. We conclude that a measurement of the pion angular distribution would be very helpful in order to reveal the spin structure of the interaction.

D. Influence of the residual interaction

Effects of residual interactions can be examined in both the $\Delta\Delta$ and the $\Delta N$ system since these are the intermediate configurations in our model. The influence of the corresponding interaction potentials on the theoretical spectrum is shown in fig. 8 for the case of forward scattering ($\theta_d = 0^\circ$).

The dash–dotted line was calculated without any residual interaction, i.e. with $V = 0$ for the intermediate states. The typical ABC structure with its three peaks as discussed in section III B is clearly present. Our result comes quite close to the fully relativistic calculation of Bar–Nir et al. [8] which also does not include the residual interactions.

By choosing $V = V_{\Delta N}$ but still neglecting the direct $\Delta\Delta$ interaction, we obtain the dashed line. Here, the maxima are even more pronounced. The enlargement of the peak cross sections is due to the reduction of the excitation energy for the $\Delta N$ system which is caused by the attractive $\Delta N$ potential. For $\sqrt{s} < 2M_\Delta$ as in the present case, the whole system is thus closer to the resonance energy. The relative position of the two invariant $\Delta$ masses remains however unchanged since in our model they are fixed independently of $V_{\Delta N}$.

On the other hand, the $\Delta\Delta$ potential can directly influence the relative wave function of the $\Delta\Delta$ system via a redistribution of momentum within the two particle system. The solid line in fig. 8 has been calculated with the full residual interaction $V = V_{\Delta N} + V_{\Delta\Delta}$. 
The inclusion of $V_{\Delta\Delta}$ results in a transfer of strength from the maxima to the regions between which are kinematically less favoured. This transfer takes place because the optimal configuration of equal $\Delta$ masses can now be reached even if the initial energy distribution of the two pions was asymmetric. As a consequence, the cross section is enhanced between the peaks and simultaneously the magnitude of the peaks is reduced. This reduction is more prominent for the outer maxima since they are related to a small relative momentum of the $\Delta\Delta$ system. For the central maximum, the relative momentum is large and therefore only a slight influence of the $\Delta\Delta$ potential is observed.

Obviously, the residual interactions play quite an important role in the $2\pi$ production and significantly influence the deuteron spectrum. The inclusion of these effects in the calculation clearly improves the agreement between theory and experiment.

E. Angular and energy dependence of the deuteron spectrum and the total cross section

Studying the dependence of the deuteron spectrum on the scattering angle $\theta_d$, we find that the theoretical cross section for $\Delta\Delta$ excitation is decreasing too fast. Exemplatory we present in fig. 9 our calculations for $\theta_d = 4.5^\circ$ and $\theta_d = 7.5^\circ$. The experimental data are underestimated by a factor $2 \sim 3$. Other microscopic calculations, e.g. assuming a two nucleon exchange as the dominant reaction mechanism, yield a comparable angular dependence and are also not able to describe the data [4,5]. The reason for this behaviour is not understood. Maybe more than only one production mechanism has to be taken into account in order to solve this problem.

Fig. 10 is demonstrating the energy dependence of the deuteron spectrum in our model. One recognizes that the central maximum is getting more pronounced if the total energy increases. We remind the reader that this peak corresponds to the $\Delta\Delta$ excitation with maximal relative momentum, which already gives the natural explanation of the observed effect.
After integration over deuteron momentum and scattering angle, the total cross section for the \( n + p \rightarrow ^2H(\pi\pi) \) reaction is obtained. In fig. [4], our theoretical result is compared to the experimental data [4, 23, 24]. In the \( \Delta\Delta \) resonance energy region, i.e. for a neutron momentum of \( k_n \approx 2 \text{ GeV} \), the agreement is quite good. This confirms our assumption that the \( 2\pi \) production in the regime of the ABC effect is dominated by the \( \Delta\Delta \) excitation mechanism. On the other hand, one recognizes that the experimental cross section close to the \( 2\pi \) production threshold \((k_n = 1.19 \text{ GeV})\) is underestimated by nearly two orders of magnitude. In this case, the total energy of \( \sqrt{s} = M_d + 2m_\pi \ll 2M_\Delta \) is simply not sufficient for the excitation of a \( \Delta\Delta \) system, and other production mechanisms will be more important. To mention just two possibilities, the pions could be produced by s–wave rescattering or via excitation of the \( N^*(1440) \) resonance, as recently discussed in [25]. Since there is also experimental evidence [24] that the ABC structure completely disappears when approaching the \( 2\pi \) production threshold, further theoretical investigations of the low energy region would be of high interest.

IV. SUMMARY AND CONCLUSIONS

To summarize we have shown that the \( \Delta\Delta \) excitation is the dominant reaction mechanism in the \( n+p \rightarrow ^2H(\pi\pi) \) two pion production at \( k_n = 1.88 \text{ GeV}/c \). It is able to explain the ABC effect as observed in the experimental deuteron spectrum. Hereby, the residual interactions in the intermediate \( \Delta\Delta \) and \( \Delta N \) states play an important role. We obtain good agreement with experimental data at forward scattering but too fast a decrease of the cross section for higher scattering angles of the deuteron. The total cross section is also underestimated close to the \( 2\pi \) production threshold where other reaction mechanisms get more important. Furthermore, we found that the spin structure of the \( \Delta\Delta \) excitation directly influences the angular distribution of the two pions. We conclude that the \( 2\pi \) production in the ABC energy region may well serve as a tool for closer examination of direct \( \Delta\Delta \) interaction and transition potentials.
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TABLES

TABLE I. Parameters of the model.

|          | \( f_{\alpha NN}^2 / 4\pi \) | \( f_{\alpha N\Delta}^2 / 4\pi \) | \( f_{\alpha \Delta\Delta}^2 / 4\pi \) | \( \Lambda_\alpha \) [GeV] | \( m_\alpha \) [MeV] |
|----------|-------------------------------|----------------------------------|----------------------------------|-----------------|-----------------|
| \( \pi \) | 0.081                         | 0.32                             | 0.0031                           | 1.1             | 138             |
| \( \rho \) | 5.4                          | 21.6                             | 0.286                            | 1.4             | 770             |
| \( \omega \) | 8.1 a                         | —                                | 8.1 a                            | 1.7             | 783             |
| \( \sigma \) | 6.9 a                        | —                                | 6.9 a                            | 1.6             | 580             |

\(^a\) \( g_{\alpha NN}^2 / 4\pi \) resp. \( g_{\alpha \Delta\Delta}^2 / 4\pi \) is given.
FIG. 1. Feynman diagrams of $2\pi$ production via $\Delta$ excitations in the $n + p \rightarrow ^2\text{H}(\pi\pi)$ reaction: (a) $\Delta\Delta$ excitation, (b) $\Delta N$ excitation. The shaded areas symbolize the residual interaction in the intermediate $\Delta\Delta$ and $\Delta N$ states, respectively.

FIG. 2. Residual interactions in the meson exchange picture. (a) Direct and (b) exchange contribution of the $\Delta N$ potential, (c) $\Delta\Delta$ potential. The mesons taken into account are $\pi$, $\rho$, $\omega$, and $\sigma$. 
FIG. 3. Experimental deuteron spectrum $d^2\sigma/dk_d d\Omega_d$ of the reaction $n+p \rightarrow ^2\text{H} (\pi\pi)$, measured at $k_n = 1.88$ GeV and $\theta_d = 0^\circ$. Data have been taken from [4]. The dashed line is the phase space for the $2\pi$ production (in arbitrary units), and $M_{\pi\pi}$ is the invariant mass of the $2\pi$ system.

FIG. 4. Theoretical result in comparison with the experimental spectrum of fig. 3. Solid line: full calculation with $\Delta\Delta$ and $\Delta N$ mechanism included; dashed line: only $\Delta N$ excitations. (see also fig. [4]).
FIG. 5. Kinematical configurations in the deuteron rest frame $\vec{p}_d = \vec{0}$. Right and left: Momentum configuration for the outer maxima of the deuteron spectrum; middle: momentum configuration for the central maximum.

FIG. 6. Angular distribution $d^3\sigma/dk_d d\Omega_d d\Omega_\pi$ of the pions, for $k_n = 1.88$ GeV and $\theta_d = 0^\circ$. $\theta_\pi$ is the angle between the relative pion momentum and the beam axis in the deuteron rest frame. The different deuteron laboratory momenta of 1.1, 1.5 and 1.9 GeV correspond to the left, central, and right maximum of the deuteron spectrum.
FIG. 7. Contributions of the spin–longitudinal (LO) and spin–transverse (TR) component of the transition potential $NN \rightarrow \Delta \Delta$ to the differential cross section at $k_d = 1.5$ GeV (without residual interaction).

FIG. 8. Influence of the residual interaction on the deuteron spectrum, for $k_n = 1.88$ GeV and $\theta_d = 0^\circ$. Solid line: full calculation with both $\Delta N$ and $\Delta \Delta$ potential; dashed: with $\Delta N$ potential but without $\Delta \Delta$ potential; dash–dotted: without any residual interaction.
FIG. 9. Deuteron spectra in the reaction $n + p \rightarrow ^2\text{H}(\pi\pi)$ at $k_n = 1.88$ GeV and $\theta_d = 4.5^0$ and $7.5^0$, respectively. The solid line is the theoretical result. Experimental data from ref. [4].

FIG. 10. Theoretical deuteron spectra in our model for forward scattering $\theta_d = 0^0$ at beam momenta $k_n = 2.5$ GeV and $k_n = 1.46$ GeV, respectively.
FIG. 11. Total cross section for $n + p \rightarrow ^2\text{H(}\pi\pi)$ as a function of the neutron momentum.

Experimental data from [4,5,23,24]. For $k_n = 2.09$ GeV, we have $\sqrt{s} = 2M_\Delta$. The threshold for $2\pi$ production is at $k_n = 1.19$ GeV.