Quark Orbital Motion in the Nucleon

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Abstract

An unified scheme for describing both spin and orbital motion in symmetry-breaking chiral quark model is suggested. The analytic results of the spin and orbital angular momenta carried by different quark flavors in the nucleon are given. The quark spin reduction due to spin-flip in the chiral splitting processes is compensated by the increase of the orbital angular momentum carried by the quarks and antiquarks. The sum of both spin and orbital angular momenta in the nucleon is 1/2, if the gluons and other degrees of freedom are neglected. The same conclusion holds for other octet and decuplet baryons. Possible modification and application are briefly discussed.

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In the past decade, considerable experimental and theoretical progress has been made in determining the quark spin contribution in the nucleon \([1]\). The polarized deep-inelastic scattering (DIS) data \([2–4]\) indicate that the quark spin only contributes about one third of the nucleon spin. A natural and interesting question is where is the missing spin? The nucleon spin can be decomposed into three gauge-invariant pieces \([5]\):

\[
\frac{1}{2} = \frac{1}{2} \Delta \Sigma + < L_z >_{q+\bar{q}} + < J_z >_G
\]

Without loss of generality, the proton is chosen to be *longitudinal polarized* in \(z\) direction and has helicity of \(+\frac{1}{2}\). \(\frac{1}{2} \Delta \Sigma = \frac{1}{2} \sum (\Delta q + \Delta \bar{q})\) is the total spin from quarks and antiquarks. \(\Delta q \equiv q_\uparrow - q_\downarrow\) and \(\Delta \bar{q} \equiv \bar{q}_\uparrow - \bar{q}_\downarrow\), where \(q_\uparrow, q_\downarrow\) (\(\bar{q}_\uparrow, \bar{q}_\downarrow\)) are quark (antiquark) numbers of spin parallel and antiparallel to the nucleon spin, or more precisely, quark numbers of positive and negative helicities. \(< L_z >_{q+\bar{q}}\) denotes the total orbital angular momentum carried by *quarks and antiquarks*, and \(< J_z >_G\) is the gluon angular momentum. The smallness of \(\frac{1}{2} \Delta \Sigma\) implies that the missing part should be contributed by either the orbital motion or gluon angular momentum. Further separation of \(< J_z >_G\) into the spin and orbital pieces \(\Delta G\) and \(< L_z >_G\) is gauge dependent. Recently, it has been suggested that \(< J_z >_{q+\bar{q}} = \frac{1}{2} \Delta \Sigma + < L_z >_{q+\bar{q}}\) can be measured in the deep virtual compton scattering process \([6]\).

In the naive quark model \([7]\), all three quarks in the nucleon are assumed to be in S-states, so \(< L_z >_q = 0\) and the nucleon spin is entirely attributed to the quark spin. On the other hand, in the naive parton model \([8]\), all quarks, antiquarks and gluons are moving in the same direction, i.e. parallel to the proton momentum, there is no transverse momentum for the partons and thus \(< L_z >_{q+\bar{q}} = 0\) and \(< L_z >_G = 0\). This picture cannot be \(Q^2\) independent due to QCD evolution. In leading-log approximation, \(\Delta \Sigma\) is \(Q^2\) independent while the gluon helicity \(\Delta G\) increases with \(Q^2\). This increase should be compensated by the decrease of the orbital angular momentum carried by partons (see for instance earlier paper \([9]\) and later analysis \([10]\)). Similar situation occurs in the spin reduction case in the chiral quark model as we will show below. Recently, the leading-log QCD evolution of \(< L_z >_{q+\bar{q}}\) and \(< L_z >_G\) has been derived in \([11]\). The perturbative QCD can predict \(Q^2\) dependence of
the spin and orbital angular momenta but not their values at the renormalization scale $\mu^2$, due to their nonperturbative origin. The chiral quark model may provides some information on these quantities.

Phenomenologically, long before the EMC experimental data published [2], using the Bjorken sum rule and low energy hyperon $\beta$-decay data, [12] shown that nearly 40% of the nucleon spin arises from the orbital motion of quarks and rest 60% is attributed to the spin of quarks and antiquarks. Most recently [13] shown that to fit the baryon magnetic moments and polarized DIS data, a large collective orbital angular momentum $<L_z>$, which contributes almost 80% of nucleon spin, is needed. Hence the question of how much of the nucleon spin is coming from the quark orbital motion remains. This paper will discuss this question within the chiral quark model.

The chiral quark model was first formulated by Manohar and Georgi in [14] and describes successfully the nucleon properties in the scale range between $\Lambda_{QCD}$ ($\sim 0.2-0.3$ GeV) and $\Lambda_{\chi_{SB}}$ ($\sim 1$ GeV). The dominant interaction is coupling among the constituent (dressed) quarks and Goldstone bosons, while the gluon effect is expected to be small. This model was first employed by Eichten, Hinchliffe and Quigg in [15] to explain both the sea flavor asymmetry and the smallness of $\Delta\Sigma$ in the nucleon. The model has been improved by introducing U(1)-breaking [16] and kaonic suppression [17]. A complete description with both SU(3) and U(1)-breakings was developed in [18], (similar version was given in [19], another version with $\lambda_8$-breaking was given in [20]), and has been reformed into an one-parameter scheme in [21].

The effective Lagrangian describing interaction between quarks and the octet Goldstone bosons and singlet $\eta'$ is

$$L_I = g_8 \bar{q} \begin{pmatrix} 1 \sqrt{2} \pi^0 + \sqrt{6} \eta + \sqrt{3} \eta' \pi^- & \frac{1}{\sqrt{2}} \pi^0 + \sqrt{6} \eta + \sqrt{3} \eta' \sqrt{\epsilon} K^+ & \pi^+ & \sqrt{\epsilon} K^0 \end{pmatrix} \begin{pmatrix} \sqrt{\epsilon} K^- \ \pi^- \ \sqrt{\epsilon} K^- \ \sqrt{\epsilon} K^- \end{pmatrix} q, \quad (2)$$

where breakings are explicitly included. $a \equiv |g_8|^2$ denotes the transition probability of chiral fluctuation or splitting $u(d) \rightarrow d(u) + \pi^+(-)$, and $\epsilon a$ denotes the probability of $u(d) \rightarrow$
The basic assumptions of the chiral quark model we used are: (i) the nucleon flavor, spin and orbital contents are determined by its valence quark structure and all possible chiral fluctuations \( q \rightarrow q' + \text{GB} \), (ii) the coupling between the quarks and Goldstone bosons is rather weak, one can treat the fluctuation \( q \rightarrow q' + \text{GB} \) as a small perturbation \( (a \sim 0.10 - 0.15) \) and the contributions from the higher order fluctuations can be neglected \( \left( a^2 << 1 \right) \), and (iii) the valence quark structure is assumed to be \( \text{SU}(3)_{\text{flavor}} \otimes \text{SU}(2)_{\text{spin}} \). Possible modification on the third assumption will be discussed later.

The important features of the chiral fluctuation are that: (i) Due to the pseudoscalar coupling between the quarks and GB’s, a quark flips its spin and changes (or maintains) its flavor by emitting a charged (or neutral) Goldstone bosons. The light quark sea asymmetry \( \bar{u} < \bar{d} \) is attributed to the existing flavor asymmetry of the valence quark numbers, two valence \( u \)-quarks and one valence \( d \)-quark, in the proton. (ii) The quark spin reduction is due to the spin-flip in the chiral splitting processes \( q_\uparrow \rightarrow q_\downarrow + \text{GB} \). (iii) Most importantly, since the quark helicity flips in the fluctuations with GB emission, hence the quark spin component changes one unit of angular momentum, \( (s_z)_f - (s_z)_i = +1 \) or \( -1 \), the angular momentum conservation requires the same amount change of the orbital angular momentum but with opposite sign, i.e. \( (L_z)_f - (L_z)_i = -1 \) or \( +1 \). This induced orbital motion distributes among the quarks and antiquarks, and compensates the spin reduction in the dilution, and restores the angular momentum conservation. This is the starting point to calculate the orbital angular momenta carried by quarks and antiquarks in the chiral quark model.

For a spin-up valence \( u \)-quark, the allowed fluctuations are

\[
\begin{align*}
    u_\uparrow \rightarrow d_\downarrow + \pi^+, & \quad u_\uparrow \rightarrow s_\downarrow + K^+, & \quad u_\uparrow \rightarrow u_\downarrow + (\text{GB}_+)^0, & \quad u_\uparrow \rightarrow u_\uparrow, \\
\end{align*}
\]

the \( (\text{GB}_\pm)^0 \) denotes \( \pm \pi^0/\sqrt{2} + \sqrt{2} \eta^0/\sqrt{6} + \zeta^0 \eta^0/\sqrt{3} \). Similarly, one can write down the allowed fluctuations for \( u_\downarrow, d_\uparrow, \) and \( d_\downarrow \). Considering the valence quark numbers in the proton

\[
\begin{align*}
n^{(v)}_p(u_\uparrow) = \frac{5}{3}, & \quad n^{(v)}_p(u_\downarrow) = \frac{1}{3}, & \quad n^{(v)}_p(d_\uparrow) = \frac{1}{3}, & \quad n^{(v)}_p(d_\downarrow) = \frac{2}{3}.
\end{align*}
\]
the spin-up and spin-down quark (or antiquark) contents, up to first order chiral fluctuation, can be written as

\[ n_p(q'_{\uparrow, \downarrow}, \text{or } \bar{q}'_{\uparrow, \downarrow}) = \sum_{q=u,d} \sum_{h=\uparrow, \downarrow} n_p^{(q)}(q_h) P_{q_h}(q'_{\uparrow, \downarrow}, \text{or } \bar{q}'_{\uparrow, \downarrow}) \] (5)

where \( P_{q_{\uparrow, \downarrow}}(q'_{\uparrow, \downarrow}) \) and \( P_{\bar{q}_{\uparrow, \downarrow}}(\bar{q}'_{\uparrow, \downarrow}) \) are the probabilities of finding a quark \( q'_{\uparrow, \downarrow} \) or an antiquark \( \bar{q}'_{\uparrow, \downarrow} \) from all chiral fluctuations of a valence quark \( q_{\uparrow, \downarrow} \), and can be obtained from the effective Lagrangian (2). They are listed in Table I, where \( f \equiv \frac{1}{2} + \frac{\epsilon}{6} + \frac{\zeta'^2}{3} \), \( A \equiv 1 - \zeta' + \frac{1-\sqrt{\epsilon}}{2} \), and \( B \equiv \zeta' - \sqrt{\epsilon} \). Using (4), (5) and Table I, we can obtain all quark (antiquark) flavor and spin contents in the proton [21]. Especially,

\[ \Delta u_p = \frac{4}{5} \Delta_3 - a, \quad \Delta d_p = -\frac{1}{5} \Delta_3 - a, \quad \Delta s_p = -\epsilon a, \] (6)

where \( \Delta_3 \equiv \frac{5}{3} [1 - a(\epsilon + 2f)] \).

The discussion of the orbital angular momentum contents is somewhat different from above, because only quark spin-flip fluctuations can induce change of the orbital angular momentum. For a spin-up valence \( u \)-quark, these fluctuations are first three processes in (3). The last process, \( u_{\uparrow} \rightarrow u_{\uparrow} \) (means no chiral fluctuation) makes no contribution to the orbital motion, and will be disregarded. We assume that the orbital angular momentum produced from the splitting \( q_{\uparrow} \rightarrow q'_{\downarrow} + \text{GB} \) is equally shared by all quarks and antiquarks, and introduce a partition factor \( k \), which depends on the numbers of final state particles and interactions among them. If the Goldstone boson has a simple quark structure, i.e. each boson consists of a quark and an antiquark, one has two quarks and one antiquark (total number is three) after each splitting. Hence up to first order chiral fluctuation, one has \( k = 1/3 \), if the interactions between the fluctuated quark and spectator quarks are neglected.

We define \( <L_z>_{q'/q_{\uparrow}} \) (or \( <L_z>_{q'/q_{\uparrow}} \)) as the orbital angular momentum carried by the quark \( q' \) (antiquark \( \bar{q}' \)), arises from a valence spin-up quark fluctuates into all allowed final states except for no emission case. Considering the quark spin component changes one unit of angular momentum in each splitting, we can obtain all \( <L_z>_{q'/q_{\uparrow}} \) and \( <L_z>_{q'/q_{\uparrow}} \) for \( q = u, d \) in the proton. Note that \( <L_z>_{q'/q_{\uparrow}} = - <L_z>_{q'/q_{\uparrow}} \), and similar relation holds.
for $q'$. The total orbital angular momentum carried by a specific quark flavor, for instance $u$-quark in the proton, is

$$< L_z >^p_u = \sum_{q=u,d} [n_p^{(v)}(q_{\uparrow}) - n_p^{(v)}(q_{\downarrow})] < L_z >_{u/q_{\uparrow}}$$

(7)

where $\sum$ summed over valence quarks $u$ and $d$ in the proton. Similarly, one can obtain the $< L_z >^p_d$, $< L_z >^p_s$, and corresponding quantities for the antiquarks, they are listed in Table II.

Defining $< L_z >^p_q$ ($< L_z >^p_{\bar{q}}$) as the total orbital angular momentum carried by all quarks (antiquarks), and we finally obtain

$$< L_z >^p_q = 2ka(1 + \epsilon + f), \quad < L_z >^p_{\bar{q}} = ka(1 + \epsilon + f)$$

(8a)

$$< L_z >^p_{q+\bar{q}} = < L_z >^p_q + < L_z >^p_{\bar{q}} = 3ka(1 + \epsilon + f)$$

(8b)

On the other hand, from (6), one has

$$\frac{1}{2} \Delta \Sigma^p = \frac{1}{2} - a(1 + \epsilon + f)$$

(9)

The sum of (8b) and (9) gives

$$< J_z >^p_{q+\bar{q}} = \frac{1}{2} - a(1 - 3k)(1 + \epsilon + f)$$

(10)

Taking $k = 1/3$, we obtain $< J_z >^p_{q+\bar{q}} = 1/2$. This result shows that in the chiral fluctuations, the missing part of the quark spin is transferred into the orbital motion of quarks and antiquarks. The amount of quark spin reduction $a(1 + \epsilon + f)$ in (9) is exactly canceled by the same amount increase of the quark orbital angular momentum in (8b), and the total angular momentum of nucleon is unchanged. This conclusion is independent of the probabilities of specific chiral fluctuations. In addition, although the orbital angular momentum carried by quarks (or antiquarks) $< L_z >^p_q$ (or $< L_z >^p_{\bar{q}}$) depends on the the chiral parameters, the ratio $< L_z >^p_q / < L_z >^p_{\bar{q}} = 2$ is independent of the probabilities of chiral fluctuations. This is originated from the mechanism of the chiral splitting: there are two quarks and one antiquark after the splitting, and they equally share the orbital angular momentum
produced in the splitting process. The total loss of quark spin $a(1 + \epsilon + f)$ appeared in (9) is due to the fact that there are three splitting processes (for instance see (3)), which flip the quark spin, the probabilities of these fluctuations are $a$, $\epsilon a$, and $fa$ respectively. The results for the proton hold for the neutron as well. The description given in this paper has been extended to other octet and decuplet baryons. The result shows that the loss of quark spin due to spin-flip in the chiral splitting processes is compensated by the gain of the orbital angular momentum carried by the quarks and antiquarks for \textit{all baryons}. The sum of both spin and orbital angular momenta in the baryon is $1/2$, if the gluons and other degrees of freedom are neglected. The detail results will be presented in a forthcoming paper.

To see how much of the proton spin is contributed by the orbital motions, we first estimate (8a-b). In full U(3) symmetry case, $1 + \epsilon + f = 3$, while in extreme SU(3)- and U(1)-breaking case ($\epsilon = \epsilon_\eta = \zeta'^2 = 0$), $1 + \epsilon + f = 1.5$. The reality is presumably in between. The detail analysis given in [21] leads to $1 + \epsilon + f \simeq 2.0$ and $a \simeq 0.15$, hence $< L_z >_q \simeq 0.20$ and $< L_z >\bar{q} \simeq 0.10$. The orbital motions shared by different quark flavors are listed in Table III, and compared with other models. Hence we have $< s_z >_{q+\bar{q}} / < J_z >_{q+\bar{q}} \simeq 2/5$, and $< L_z >_{q+\bar{q}} / < J_z >_{q+\bar{q}} \simeq 3/5$. i.e. nearly 60% of the proton spin is coming from the orbital motion of quarks and antiquarks, and 40% is contributed by the quark spin. The ratio of the spin to orbital angular momenta is $< s_z >_{q+\bar{q}} / < L_z >_{q+\bar{q}} \simeq 2/3$.

We have assumed that there are \textit{no gluons} and other degrees of freedom in the proton, hence $< J_z >_G = 0$. This is presumably a good approximation at very low $Q^2$. However, if $< J_z >_G$ is nonzero [22] and not small, the results given above should be modified. Taking $< J_z >_G (1 \text{ GeV}^2) \simeq 0.25 \pm 0.10$ given in [24], and \textit{assuming} the ratios derived from the chiral quark model still hold, one has $< L_z >_{q+\bar{q}} (1 \text{ GeV}^2) \simeq 0.15 \pm 0.07$ and $< s_z >_{q+\bar{q}} (1 \text{ GeV}^2) \simeq 0.10 \pm 0.07$, which is consistent with DIS data [3,4], and lattice QCD result [22].

One of important applications of our description is to study the baryon magnetic moments, which should depend on both spin and orbital motions of quarks and antiquarks. Assuming $\mu_u = -2\mu_d = -3\mu_s$, the ratio of the proton to neutron magnetic moments is
\[ \frac{\mu_p}{\mu_n} = -(3/2)[1 - (5a/6)(1 - 2\epsilon/3)/(1 - a(2\xi' - 2\epsilon/3 - 5/2))], \]

where \( \xi' \equiv (1 - k)(1 + \epsilon + f). \)

If the orbital motion is not included \((k = 0)\), one obtains \( \frac{\mu_p}{\mu_n} \simeq -1.33 \), while for \( k = 1/3 \), \( \frac{\mu_p}{\mu_n} \simeq -1.38 \) (data: \(-1.48\)). The agreement with data is improved. A detailed discussion of the baryon magnetic moments will be presented in another paper.

To summarize, we have developed a new and unified scheme for describing both spin and orbital motions of quarks and antiquarks in symmetry breaking chiral quark model. The orbital motions carried by different quark flavors in the proton are calculated. Extension, modification and application of this scheme will be presented elsewhere.

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Note added. – After this preprint was posted, the author learned that the basic feature of orbital angular momentum in the nucleon has been suggested before by Ta-Pei Cheng and Ling-Fong Li \cite{25}. However, our consideration is more general in the chiral quark model with SU(3) and U(1) breakings, and thus has different conclusions on the octet and decuplet baryons and their magnetic moments.
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TABLE I. The probabilities $P_{u^\uparrow}(q'_\uparrow, q'_\downarrow)$ and $P_{d^\uparrow}(q'_\uparrow, q'_\downarrow)$

| $q'$   | $P_{u^\uparrow}(q'_\uparrow)$ | $P_{d^\uparrow}(q'_\uparrow)$ |
|--------|--------------------------------|---------------------------------|
| $u^\uparrow$ | $1 - (\frac{1+\epsilon}{2} + f)a + \frac{a}{18}(3 - A)^2$ | $\frac{a}{18}A^2$ |
| $u^\downarrow$ | $(\frac{1+\epsilon}{2} + f)a + \frac{a}{18}(3 - A)^2$ | $a + \frac{a}{18}A^2$ |
| $d^\uparrow$ | $\frac{a}{18}A^2$ | $1 - (\frac{1+\epsilon}{2} + f)a + \frac{a}{18}(3 - A)^2$ |
| $d^\downarrow$ | $a + \frac{a}{18}A^2$ | $(\frac{1+\epsilon}{2} + f)a + \frac{a}{18}(3 - A)^2$ |
| $s^\uparrow$ | $\frac{a}{18}B^2$ | $\frac{a}{18}B^2$ |
| $s^\downarrow$ | $\epsilon a + \frac{a}{18}B^2$ | $\epsilon a + \frac{a}{18}B^2$ |
| $\bar{u}^\uparrow, \downarrow$ | $\frac{a}{18}(3 - A)^2$ | $\frac{a}{2} + \frac{a}{18}A^2$ |
| $\bar{d}^\uparrow, \downarrow$ | $\frac{a}{2} + \frac{a}{18}A^2$ | $\frac{a}{18}(3 - A)^2$ |
| $\bar{s}^\uparrow, \downarrow$ | $\frac{\epsilon a}{2} + \frac{a}{18}B^2$ | $\frac{\epsilon a}{2} + \frac{a}{18}B^2$ |

TABLE II. The orbital angular momentum carried by the quark $u$, $d$, and $s$, and antiquark $\bar{u}$, $\bar{d}$, and $\bar{s}$ in the proton.

| $< L_z >^p_{u^\uparrow}$ | $\frac{ka}{3} [4(1 + \epsilon + f) - 1 + \frac{4(3-A)^2}{9} - \frac{A^2}{9}]$ | $< L_z >^p_{\bar{u}^\uparrow}$ | $\frac{ka}{3} [-1 + \frac{4(3-A)^2}{9} - \frac{A^2}{9}]$ |
|----------------------|-------------------------------------------------|----------------------|-------------------------------------------------|
| $< L_z >^p_{d^\uparrow}$ | $\frac{ka}{3} [4 - (1 + \epsilon + f) + \frac{4A^2}{9} - \frac{(3-A)^2}{9}]$ | $< L_z >^p_{\bar{d}^\uparrow}$ | $\frac{ka}{3} [4 + \frac{4A^2}{9} - \frac{(3-A)^2}{9}]$ |
| $< L_z >^p_{s^\uparrow}$ | $\frac{ka}{3} [3\epsilon + \frac{B^2}{3}]$ | $< L_z >^p_{\bar{s}^\uparrow}$ | $\frac{ka}{3} [3\epsilon + \frac{B^2}{3}]$ |
TABLE III. Quark spin and orbital angular momenta in the chiral quark model and other models.

| Quantity        | Data [9] | This paper | Sehgal [12] | CS [13] | NQM |
|-----------------|----------|------------|-------------|----------|-----|
| $< L_z >^p_u$   |          | 0.136      | 0.237       |          | 0   |
| $< L_z >^p_{\bar{u}}$ |          | -0.002     | 0           |          | 0   |
| $< L_z >^p_d$   |          | 0.044      | -0.026      |          | 0   |
| $< L_z >^p_{\bar{d}}$ |          | 0.079      | 0           |          | 0   |
| $< L_z >^p_s$   |          | 0.026      | 0           |          | 0   |
| $< L_z >^p_{\bar{s}}$ |          | 0.026      | 0           |          | 0   |
| $< L_z >^p_{q+\bar{q}}$ |      | 0.31       | 0.21        | 0.39     | 0   |
| $\Delta u^p$   | 0.85 ± 0.04 | 0.85       | 0.91        | 0.78     | 2/3 |
| $\Delta d^p$   | -0.41 ± 0.04 | -0.40      | -0.34       | -0.48    | -1/6 |
| $\Delta s^p$   | -0.07 ± 0.04 | -0.07      | 0           | -0.14    | 0   |
| $\frac{1}{2} \Delta \Sigma^p$ | 0.19 ± 0.06 | 0.19       | 0.29        | 0.08     | 1/2 |