Dark energy and the evolution of spherical overdensities

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ABSTRACT
We use the non-linear spherical model in cold dark matter (CDM) cosmologies with dark energy to investigate the effects of dark energy on the growth of structure and the formation of virialised structures. We consider dark energy models with a constant equation of state parameter $w$. For $-1 < w < -1/3$, clusters form earlier and are more concentrated in quintessence than in ΛCDM models, but they form later and are less concentrated than in the corresponding open model with the same matter density and no dark energy. We point out some confusion in the literature around the expression of the collapse factor (ratio of the radius of the sphere at virialisation to that at turn-around) derived from the virial theorem. We use the Sheth & Tormen extension of the Press-Schechter framework to calculate the evolution of the cluster abundance in different models and show the sensitivity of the cluster abundance to both the amplitude of the mass fluctuations, $\sigma_8$, and the $\sigma_8 - w$ normalisation, selected to match either the cosmic microwave background observations or the abundance of X-ray clusters.

Key words: cosmology: theory – galaxies: clusters: general – large-scale structure of universe – galaxies: formation

1 INTRODUCTION
The cosmological picture that has emerged from a host of recent observations is that of a spatially flat universe dominated by an exotic component with positive energy density and negative pressure, the dark energy, which is responsible for the observed accelerated cosmic expansion.

The nature of dark energy remains unknown. It could be Einstein’s cosmological constant or the manifestation of a scalar field slowly rolling down its potential, as in quintessence models (Peebles & Ratra 1988, Caldwell, Dave & Steinhardt 1998, see Peebles & Ratra 2003 for a review). The equation of state, $w = p_X/\rho_X$, where $p_X$ is the pressure and $\rho_X$ the density of the dark energy (hereafter referred to as the X-component), describes how the X-component evolves as the universe expands. Determining the equation of state will help reveal the nature of dark energy and discriminate among models, as the cosmological constant is characterised by a constant $w = -1$ whereas in quintessence models $w$ can be different from $-1$ and vary with cosmic time. $w$ is directly related to the shape of the potential of the quintessence field. It is a fundamental parameter that remains poorly constrained observationally, partly because of degeneracies with other parameters, mainly $\Omega_0$ and the r.m.s. of the amplitude of fluctuations within $8 h^{-1}$Mpc, $\sigma_8$ (e.g., Lokas, Bode & Hoffman 2004).

Dark energy not only affects the expansion rate of the background and the distance-redshift relation, but also the growth of structure. The collapse of overdense regions due to gravitational instability is slowed down by the Hubble drag due to the expansion. This effective friction depends on the expansion of the background. The formation rate of haloes, their evolution and their final characteristics are modified. Dark energy is therefore expected to have an impact on observables such as cluster number counts and lensing statistics due to intervening concentrations of mass on the line-of-sight of background sources.

The abundance of rich clusters of galaxies can be used to constrain the cosmological model and the properties of dark energy. This has been discussed first for pure cosmological constant models (e.g., Lahav et al. 1991, Liddle 1992, Lacey & Cole 1993, Viana & Liddle 1996, Eke, Cole & Frenk 1996) and then for models with dark energy where $w$ could be different from $-1$. In a seminal paper Wang & Steinhardt (1998) have discussed the evolution of the cluster abundance in general quintessence models with negative pressure. Following up on that paper, Lokas (2001), Basilakos (2003) have examined cluster formation in models with constant $w$. Mota & van de Bruck (2004) have selected particular potentials for the quintessence field. Battye & Weller (2003) have investigated the constraints...
on dark energy from future Sunyaev-Zeldovich surveys. Lokas, Bode & Hoffman (2004) have used N-body simulations in flat models with constant $w$ to measure cluster mass functions.

In this paper, we first discuss the linear evolution of perturbations in dark energy models with constant $w$. Then, we use the non-linear spherical model (“top-hat” model first developed by Gunn & Gott [1972]. Rather than using the approximation given by Wang & Steinhardt [1998] which is strictly valid for $w = -1$, we calculate explicitly the potential energy associated with the dark energy component inside the collapsing sphere and the ratio of the radius of the sphere at virialisation to the turn-around radius. We follow the evolution of overdense regions as they first expand with the background before collapsing and settling into virialised structures. We then calculate the abundance of massive clusters in the Press & Schechter [1974] framework. We use the Sheth & Tormen [1999] extension which has been shown to be in remarkable agreement with results from N-body simulations of hierarchical structure formation.

We use five cosmological models, all with $H_0 = 70$ km s$^{-1}$ Mpc$^{-1}$: three dark energy models with the same energy content: $\Omega_\Lambda = 0.3$, $\Omega_{X,0} = 0.7$, but that differ in their value of $w$: $\Lambda$CDM with $w = -1$, a $w = -0.8$ model, a $w = -0.6$ model, and for comparison an Einstein-de Sitter model with $\Omega_0 = 1$ and an open model with $\Omega_0 = 0.3$.

## 2 THE BACKGROUND COSMOLOGY

We consider a cosmological model with two components: (1) a non-relativistic component which comprising all forms of matter (luminous and dark) that clusters under the action of gravity, and (2) a uniform X-component with negative pressure which does not cluster at the scales of interest. In general, quintessence models have a component that is spatially inhomogeneous but does not cluster on scales less than 100 Mpc. The Friedmann equation that describes the evolution of the scale factor $a$ with cosmic time is modified to include the effect of the X-component:

$$\left(\frac{H(a)}{H_0}\right)^2 = \Omega_0 \frac{\rho(a)}{\rho_0} + \Omega_{k} a^{-2} + \Omega_{X,0} a^{-n} \frac{\rho_{X}(a)}{\rho_{X,0}}$$

(1)

where $H(a) = \dot{a}/a$ is the Hubble parameter, $\Omega$ are the density parameters (ratios of the energy density to the critical density $\rho_{crit} = \frac{H^2}{2\pi^2 G}$) for the matter, the curvature ($\Omega_k = 1 - \Omega_0 - \Omega_{X,0}$) and the X-component respectively. The parameters are indexed by 0 when they refer to the present time.

The energy density of the X-component varies as

$$\frac{\rho_{X}(a)}{\rho_{X,0}} = a^{-n}$$

(2)

with $n = 3(1 + w)$.

Different $n$ and $w$ correspond to different types of energy densities, with $n = 0$ ($w = -1$) corresponding to the cosmological constant. Note that $n$ does not need to be either integer or constant. It may vary with cosmic time, as in quintessence models. In the following we consider only models with constant $n$ and $w$.

Note also that for $n = 2$ ($w = -1/3$), the dark energy term can be combined with the curvature term in the Friedmann equation; $\Omega_{X,0}$ cancels out and the expansion is the same as for a $\Omega_0$ universe. Thus a flat universe with $w = -1/3$ behaves like an open universe without dark energy and with the same $\Omega_0$. Cosmic acceleration (i.e., the existence of an inflexion point in the $a(t)$ curve) requires $n < 2$.

The Friedmann equation can be written more simply as

$$\dot{a} = \frac{H_0}{f(a)}$$

(3)

where

$$f(a) = a^{-1} \left[ \Omega_0 a^{-3} + (1 - \Omega_0 - \Omega_{X,0}) a^{-2} + \Omega_{X,0} a^{-n} \right]^{-1/2}$$

(4)

Integrating the Friedmann equation, one obtains the variations of the scale factor with cosmic time in different cosmologies (see Fig. 1). Universes with a dark energy component are older than with $\Omega_X = 0$ and give perturbations more time to grow. For the currently favoured parameters the evolution of the scale factor in quintessence models is intermediate between that of a cosmological constant model ($w = -1$) and of an open universe.

The density parameters vary with redshift $z = 1/a - 1$ as

$$\Omega(z) = \Omega_0 (1 + z)^3 \frac{E(z)}{E^2(z)}; \quad \Omega_X(z) = \Omega_{X,0} (1 + z)^n \frac{E(z)}{E^2(z)}$$

(5)

where $E(z) = \frac{H(z)}{H_0}$. In the case of the cosmological constant commonly denoted $\Lambda$,

$$\Omega_X = \lambda = \frac{\Lambda}{3H^2}$$

(6)

Figure 1. Evolution of the scale factor in the different models. In quintessence models, the evolution of the scale factor is intermediate between that in a $\Lambda$CDM model (continuous line) and that in an open model (dashed-dotted-dotted-dotted line).
will be stronger at early times in an open universe compared to a ΛCDM universe, with intermediate rates in quintessence models. But structure growth is reduced earlier in open and quintessence models, although it is still significantly higher than in ΛCDM models.

4 NON-LINEAR EVOLUTION OF A SPHERICAL OVERDENSITY

The equation of motion of a spherical shell in the presence of dark energy is:

\[
\frac{\ddot{r}}{r} = -\frac{4\pi G}{3} (\rho_{\text{cluster}} + \rho_{X,\text{eff}}) \tag{8}
\]

where \(\rho_{\text{cluster}}\) is the time-varying density inside the forming cluster, and

\[
\rho_{X,\text{eff}} = \rho_X + 3p_X = (n - 2)\rho_X \tag{9}
\]

is the effective energy density of the X-component. \(\rho_{X,\text{eff}}\) is constant for the cosmological constant; in a universe with a dark energy component with \(n = 2\) (\(w = -1/3\)), perturbations evolve in the same way as in a universe with the same \(\Omega_0\) and no dark energy. Note that for \(n \neq 0\) and \(n \neq 2\) (\(w \neq -1\) and \(w \neq -1/3\)), the density of the X-component inside the overdensity patch is dependent on the evolution of the background. This makes it impossible to integrate directly the equation of motion and cast it into a first-order energy equation. The second-order equation of motion has to be integrated taking into account the time variation of the dark energy.

We shall follow the evolution of a spherical overdense region with some initial overdensity. At early times, it expands along with the Hubble flow and density perturbations grow proportionally to the scale factor. If the initial overdensity exceeds a critical value, the overdense region will break away from the general expansion and go through three phases:

(i) expansion up to a maximum radius;
(ii) collapse;
(iii) virialisation.

We call \(z_{ta}\) the redshift at maximum expansion (turn-around) and \(z_{coll}\) the redshift at which the sphere suddenly virialises, \(r_{ta}\) and \(r_{vir}\) the corresponding radii of the sphere. Ideally, the sphere should collapse down to an infinitely small radius, but it is assumed that this is prevented by the growth of small inhomogeneities. Let us discuss in turn the different phases of evolution of a collapsing sphere.

4.1 Expansion up to \(r_{ta}\)

Let us calculate the overdensity of the forming cluster at turn-around

\[
\zeta = \left(\frac{\rho_{\text{cluster}}}{\rho_{bg}}\right)_{z = z_{ta}}. \tag{10}
\]

Let us define

\[
x = \frac{a}{a_{ta}} \quad \text{and} \quad y = \frac{r}{r_{ta}} \tag{11}
\]

where \(a_{ta}\) is the scale factor of the background when the perturbation reaches turn-around. For a flat universe, the
evolution of the background and that of the perturbation are governed by the two following equations:

\[ \dot{x} = H ta \Omega^{1/2} \left[ \Omega(x) x \right]^{-1/2} \]

\[ \ddot{y} = -H^2 \Omega ta \left[ \frac{\zeta}{y^2} + (n-2) \frac{\Omega_{X,ta} y}{\Omega ta} x^n \right] \]

where we have used the fact that the mass of the forming cluster is conserved:

\[ \rho_{\text{cluster}} r^3 = \rho_{\text{cluster,ta}} r_{\text{ta}}^3. \]

Note that the second term in eq. (13) vanishes in two cases: for perturbations in a \( n = 2 \) universe and for the ones in a universe with \( \Omega_X = 0 \). In the \( n = 0 \) (the cosmological constant) case also, the evolution of the perturbation is independent of that of the background.

\( \zeta \) can be determined by integrating the above differential equations using the boundary conditions \( (dy/dt)_{x=1} = 0 \) and \( (y)_{x=0} = 0 \). The variations of \( \zeta \) with collapse redshift are shown in Fig. 3 [Wang & Steinhardt (1998)] provide a fitting formula for \( \zeta \) as a function of \( \Omega_{\Lambda,ta} \) and \( w \) for a spatially flat Universe. Perturbations collapsing at a certain redshift are denser at turn-around relative to the background in a universe with quintessence than in a \( \Lambda \)CDM model. At high redshift, \( \zeta \) tends toward the fiducial value of 5.6 for an Einstein-de Sitter universe (as \( \Omega \) tends toward 1 and \( \Omega_X \) toward 0).

Fig. 4 shows the evolution of a perturbation collapsing now \( (z_{coll} = 0) \) in different cosmological models. Perturbations reach turn-around and collapse earlier in the quintessence model than in the \( \Lambda \)CDM model, and even earlier in the OCDM model.

4.2 Collapse and virialisation

The virial theorem can be expressed in the general form \( T = pU/2 \) for a system with a potential energy of the form \( U \propto R^p \) and kinetic energy \( T \) [Landau & Lifshitz, 1960]. For the cosmological constant and the quintessence, the potential energy is proportional to \( R^2 \), and we can therefore use the virial theorem. Combining the virial theorem and the conservation of energy one obtains a relation between the potential energies of the collapsing sphere at turn-around and at the time it virialises:

\[ U_{G,ta} + U_{X,ta} = \frac{1}{2} U_{G,vir} + 2 U_{X,vir} \]

where \( U_G \) is the potential energy of the matter \( (U_G = -\frac{1}{2} \Omega M^2 R^2) \) and \( U_X \) is the potential energy of the quintessence inside the collapsing sphere.

The potential energy associated with the quintessence \( U_X \) can be obtained from the Poisson equation with the pressure term:

\[ \Delta \Phi_X = 4\pi G (\rho_X + 3p_X) \]

which yields: \( \Phi_X = (n-2)\rho_X \frac{2\pi G}{10} R^2 \) and

\[ U_X = (n-2)\rho_X \frac{4\pi GM}{10} R^2. \]

Replacing the energies by their expressions in Eq. (15), we can obtain a relation between the collapse factor, defined as

\[ \eta = \frac{r_{vir}}{r_{ta}} \]

and the magnitude of the dark energy at turn-around. Following [Iliev & Shapiro, 2001], let us define a quantity \( \theta \) to
describe the importance of the X-component relative to the gravitational attraction of the matter at maximal expansion:

\[ \theta = \left( \frac{\rho_{X,\text{eff}}}{2\rho} \right)_{\text{ta}} \]

where \( \rho_{X,\text{eff}} \) is defined in eq. \( \text{eq. 19} \).

Noting that

\[ \left( \frac{\rho_{X,\text{eff}}}{\rho_{\text{vir}}} \right) = 2\theta \eta^3 \left( \frac{\rho_{\text{coll}}}{\rho_{\text{ta}}} \right)^{-n} \]

we obtain the following relation between \( \eta \) and \( \theta \):

\[ 4\theta \left( \frac{\rho_{\text{coll}}}{\rho_{\text{ta}}} \right)^{-n} \eta^3 - 2(1 + \theta)\eta + 1 = 0. \]  

We would like to point out that there is some confusion in the literature around the exact expression of the cubic equation \( \text{eq. 20} \). Some authors (e.g., 
Iliev & Shapiro 2001, Battye & Welche 2003) have derived the same expression as ours, also using the Poisson equation with the pressure term for the dark energy. Other authors, however, (e.g., Lahav et al. 1991, Baldi 2003, Weinberg & Kamionkowski 2003) seem to have used the approximation of \( r_{\text{vir}}/r_{\text{ta}} \) given by Wang & Steinhardt (1993), which is exact for the cosmological constant case, but differs significantly from the analytical expression when \( w \neq -1 \). Rewriting equation \( \text{eq. 21} \) using the same notations as Wang & Steinhardt (1993), we have:

\[ 2\eta \eta^3 - 2 \left( 1 + \frac{n}{2} \right) \eta + 1 = 0 \]

with

\[ \{ \eta_t = -\frac{(n-2) \Omega_{X,\text{ta}}}{\Omega_{\text{vir}}} \}
\]

\[ \eta_w = -\frac{(n-2) \Omega_{X,\text{vir}} (1 + z_{\text{eff}})^3}{\Omega_{\text{vir}} (1 + z_{\text{ta}})^3}. \]

Note that those equations differ from those of Wang & Steinhardt (1993) where the term \(-(n-2)\) in the expressions above of \( \eta_t \) and \( \eta_w \) is replaced by 2. The formulae are identical for \( n = 0 \) (the cosmological constant case) but they differ when \( n \neq 0 \). This results in different values of the density of the virialised clusters, as we shall see. In particular, using the approximation of Wang & Steinhardt (1993) in the \( w = -1/3 \) case yields to erroneous results. Because of the contribution of the pressure term in the Poisson equation, a factor \((n-2)\) appears in the cubic equation, which means that for \( n = 2 \) \((w = -1/3)\) one should recover the same expression for density contrast at virialisation as for an open universe with the same \( \Omega_0 \). This is not the case if the \((n-2)\) term is omitted.

4.2.1 The collapse factor versus \( \theta \) and \( \frac{2\rho_{\text{coll}}}{\rho_{\text{ta}}} \)

Let us solve eq. \( \text{eq. 21} \) to express the collapse factor \( \eta \). For \( \theta = 0 \) (which corresponds to \( \rho_X = 0 \) or to \( n = 2 \)), \( \eta = 1/2 \). This is a well-known result for models without dark energy: the final radius of a virialised sphere is half the turn-around radius. For \( \theta \neq 0 \) one obtains a cubic equation in \( \eta \) that can be solved analytically to express the collapse factor as a function of \( \theta \) and \( \frac{2\rho_{\text{coll}}}{\rho_{\text{ta}}} \) for different values of \( n \). Here the discriminant of the cubic equation is negative and there are three unequal real solutions. The first solution is always greater than 1 for the range of \( \theta \) of interest, while the second solution is negative. The third solution tends to \( 1/2 \) for \( \theta \to 0 \) and it can be expressed as:

\[ \eta = 2 \sqrt{1 + \theta} \left( \frac{\rho_{\text{coll}}}{\rho_{\text{ta}}} \right)^{n/2} \cos \left( \frac{\phi + 4\pi}{3} \right) \]

(23)

with

\[ \phi = \arccos \left( -\frac{(6\theta)^{3/2}}{8\theta(1+\theta)^{3/2}} \left( \frac{2\rho_{\text{coll}}}{\rho_{\text{ta}}} \right)^{-n/2} \right). \]

(24)

For \( w = -1 \) (the cosmological constant), \( \eta \) is independent of \( \frac{2\rho_{\text{coll}}}{\rho_{\text{ta}}} \) and it can be calculated readily, as was first done by Lahav et al. (1991), who also gave a simple approximated solution valid for a first-order expansion of \( r_{\text{vir}}/r_{\text{ta}} \) around \( 1/2 \) (the value for \( \lambda = 0 \)).

In quintessence models, \( \eta \) depends on \( \frac{2\rho_{\text{coll}}}{\rho_{\text{ta}}} \), which will be calculated below. Fig. 5 shows the ratio of the density of the virialised sphere to the density at maximum expansion versus \( \theta \), the strength of the dark energy at maximum expansion, for different cosmological models. In order to reach an equilibrium in the presence of dark energy, the sphere has to achieve a higher density as \( \theta \) increases. The final density of the virialised object is even higher in quintessence models than in \( \Lambda \)CDM models, and it increases with increasing \( w \). The curves corresponding to quintessence models are intermediate between those of the \( \Lambda \)CDM model and the open model.

The minimum radius of the virialised sphere is reached for \( \theta = 0.5 \). In a universe with a cosmological constant, the minimum radius is \( \eta_{\text{min}} = 0.366 \). In quintessence models, the radius of the virialised sphere tends toward a minimum \( \eta_{\text{min}} = 0.333 \) as the ratio \( \frac{2\rho_{\text{coll}}}{\rho_{\text{ta}}} \) increases. The maximum ratio of the density of the virialised sphere to the density at turn-around is thus 8 for models without dark energy, 20 for \( \Lambda \) models, and 27 in quintessence models (when \( w \to -1/3 \)).

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Note that the equations differ from those of Wang & Steinhardt (1993) where the term \(-(n-2)\) in the expressions above of \( \eta_t \) and \( \eta_w \) is replaced by 2. The formulae are identical for \( n = 0 \) (the cosmological constant case) but they differ when \( n \neq 0 \). This results in different values of the density of the virialised clusters, as we shall see. In particular, using the approximation of Wang & Steinhardt (1993) in the \( w = -1/3 \) case yields to erroneous results. Because of the contribution of the pressure term in the Poisson equation, a factor \((n-2)\) appears in the cubic equation, which means that for \( n = 2 \) \((w = -1/3)\) one should recover the same expression for density contrast at virialisation as for an open universe with the same \( \Omega_0 \). This is not the case if the \((n-2)\) term is omitted.
Figure 6. Overdensity $\Delta_{\text{vir}}$ versus collapse redshift for different cosmological models. Quintessence models are intermediate between the $\Lambda$CDM model and the open model.

Expanding $\eta$ around $1/2$, its value for $\theta = 0$, one obtains a first-order approximation:

$$\eta = \frac{1 - \theta (\frac{a_{\text{coll}}}{a_{\text{ta}}})^{-n}}{2 - \theta (3 (\frac{a_{\text{coll}}}{a_{\text{ta}}})^{-n} - 2)}.$$  \hspace{1cm} (25)

4.2.2 The $\frac{a_{\text{coll}}}{a_{\text{ta}}}$ ratio

The $\frac{a_{\text{coll}}}{a_{\text{ta}}}$ ratio can be derived from the integration of the Friedmann equation to calculate the scale factor as a function of time, using the fact that the collapse time is twice the turn-around time:

$$\frac{k_{\text{coll}}}{k_{\text{ta}}} = 2.$$  \hspace{1cm} (26)

$a_{\text{coll}}$ and $a_{\text{ta}}$ are related by:

$$\int_{0}^{a_{\text{coll}}} f(a)da = 2 \int_{0}^{a_{\text{ta}}} f(a)da.$$  \hspace{1cm} (27)

Although $\frac{a_{\text{coll}}}{a_{\text{ta}}}$ is constant for an Einstein-de Sitter universe (equal to $(\frac{a_{\text{coll}}}{a_{\text{ta}}})^{2/3} = 0.587$), it varies in other cosmologies, decreasing with increasing $z_{\text{coll}}$, and converges toward the Einstein-de Sitter value at high redshifts.

4.3 Density contrast at virialisation

In Sect. 4.1, we have calculated the overdensity of the forming cluster at turn-around: $\zeta = (\rho_{\text{cluster}}/\rho_{bg})_{\text{ta}}$. In Sect. 4.2 we have used the virial theorem to express $\rho_{\text{vir}}/\rho_{\text{cluster,ta}}$ as a function of $(\rho_{X}/\rho_{\text{cluster}})_{\text{ta}}$. We are now able to calculate the density contrast of the virialised cluster as a function of the collapse redshift:

$$\Delta_{\text{vir}}(z_{\text{coll}}) = \frac{\rho_{\text{vir}}}{\rho_{bg,\text{vir}}} = \eta^{-3} \left(1 + \frac{z_{\text{ta}}}{1 + z_{\text{coll}}} \right)^{3}.$$  \hspace{1cm} (28)

Figure 7. Decrease of $\sigma_{8}$ with increasing $w$, using the X-ray cluster normalisation or the CMB normalisation.

5 ABUNDANCE OF RICH CLUSTERS

Let us now calculate the comoving number density of virialised clusters in a certain mass range as a function of redshift:

$$n(M, z, w) dM = \rho_{0} \frac{d\nu(M, z, w)}{dM} f(\nu) dM$$  \hspace{1cm} (29)

where $\rho_{0}$ is the background density at redshift zero, and

$$\nu(M, z, w) = \frac{\delta_{c}(z, w)}{\sigma(M, z, w)}.$$  \hspace{1cm} (30)

where $\delta_{c}$ is the linear overdensity of a perturbation collapsing at redshift $z$ and $\sigma$ is the r.m.s. of the mass fluctuation in spheres containing the mass $M$. $\delta_{c}$ depends weakly on the cosmological model and is close to the Einstein-de Sitter value of 1.68. We have taken the fit provided by Weinberg & Kamionkowski (2003) to $\delta_{c}$ as a function of $z$ and $w$.

The standard Press-Schechter mass function is of the form: $f(\nu) = \sqrt{\frac{2}{\pi}} \exp\left(-\nu^{2}/2\right)$. It provides a good general
representation of the observed distribution of clusters. However, it is known to predict too many low-mass clusters and too few high mass clusters, as well as too few clusters at high redshifts. We use the modified Press-Schechter function proposed by Sheth & Tormen (1999) that gives substantially better fits to simulated mass functions [Jenkins et al 2001]. This formula has been given a physical justification in terms of an ellipsoidal model for the collapse of perturbations [Sheth, Mo & Tormen 2001]. The mass function is

$$f(v) = \sqrt{\frac{2}{\pi}} \frac{0.2709(1 + 1.1096v^{0.6})}{v^2} \exp\left(-0.707v^2/2\right)$$  

(31)

We need to calculate the amplitude of the mass fluctuations $\sigma(M)$, as well as its derivative with respect to $M$. In a Gaussian density field, the variance depends on the scale $R$:

$$\sigma^2(R) = \frac{1}{2\pi^2} \int_0^\infty k^3 P(k) W^2(kR) \frac{dk}{k},$$  

(32)

where $R(M) = \left[\frac{3M}{4\pi}\right]^{1/3}$ is the Lagrangian radius of a halo of mass $M$ at the present time, $W(u) = 3(\sin(u) - u \cos(u))/u^3$ is the Fourier transform of a spherical top-hat filter with radius $R$, and $P(k)$ is the power spectrum of density fluctuations extrapolated to $z = 0$ according to linear theory. Assuming that the baryon density parameter $\Omega_B,0 < \Omega_{\text{CDM},0}$, the CDM power spectrum can be approximated by $P(k) \propto kT^2(k)$. As for the transfer function, we have used

$$T(k) = \frac{\ln(1+2.34q)}{2.34q} \times \left[1 + 3.89q + (16.1q)^2 + (5.46q)^3 + (6.71q)^4\right]^{-1/4}$$

with $q = k/[(\Omega_Bh^2\text{Mpc}^{-1})]$. [Bardeen et al. 1986].

We normalise the power spectrum by setting the value of the amplitude of the fluctuation within $8h^{-1}\text{Mpc}$ spheres in a $\Lambda$CDM model.

$\sigma_8$ can be constrained from observations of the cosmic microwave background (CMB), [Doran, Schwindt & Wetterich 2001] provide a useful approximation relating $\sigma_8$ in a universe with quintessence to the $\sigma_8$ in a corresponding $\Lambda$CDM universe with the same quantity of dark energy, and consistent with the COBE normalisation [Bunn & White 1992]. In the case of a dark energy component with a constant equation of state, as in the models we consider here, the expression is simple and precise to about 5%:

$$\sigma_8^X = (1 - \Omega_X,0)^{-(1+w)/3} \sqrt{\frac{\Omega_X,0}{\tau_0}}$$  

(33)

where $\tau_0 = \tau(a = 1)$ is the conformal age of the universe:

$$\tau(a) = \int_0^a \frac{da'}{a'^2 H(a')}.$$  

$\sigma_8$ can also be inferred from the observed abundance of X-ray clusters [Wang & Steinhardt 1998]:

$$\sigma_8 \simeq 0.5\Omega_0^{\gamma}(\Omega_0, w)$$  

(34)

where $\gamma$ is a function of $\Omega_0$ and $w$.

In Fig. 2 we show both normalisations of $\sigma_8$: the CMB and the X-ray cluster normalisation. We have taken $\sigma^X_8 = 0.95$. $\sigma_8$ has to decrease with increasing $w$ to meet the observational constraints. The CMB normalisation implies a steeper decrease of $\sigma_8$ with increasing $w$.

The cluster evolution is sensitive both to $\sigma_8$ and to $w$ and might provide a way to constrain those important parameters. However, the degeneracy between $w$ and $\sigma_8$ produces a near cancellation of the two effects. We have shown that, as $w$ increases, structures form earlier and are more concentrated. However, because of the accompanying decrease in $\sigma_8$, more clusters will form at a later epoch.

Now we want to calculate a more directly observable quantity, namely the number density of clusters above a certain mass per unit steradian and per redshift interval:

$$\frac{d^2N(z)}{dz \, d^3v} = \frac{d^2V_c}{dz \, d^3v} \int_{M_{\text{min}}}^{\infty} n(M, z) dM$$  

(35)

where the comoving volume is

$$\frac{d^2V_c}{dz \, d^3v} = \frac{d_A(z)^2 (1+z)^2}{H(z)}$$  

(36)

and $d_A$ is the angular diameter distance.

Fig. 6 shows that quantity as a function of redshift for the different dark energy models and using both the CMB and the cluster $\sigma_8 - w$ relations. As the threshold mass of the cluster catalog increases, the peak of the redshift distribution is shifted toward lower redshifts and the number density of clusters decreases. The amplitude of the curves is sensitive to the $\sigma_8 - w$ relation, but not the position of the peak in redshift. Note that we have assumed a constant limiting mass as a function of redshift. It is possible to carry out a similar calculation assuming a dependence of $M_{\text{min}}$ with $z$, appropriate for a given survey. For instance the expected yield of a survey of galaxy clusters using the Sunyaev-Zeldovich effect is sensitive to the mass threshold of detection (e.g., Holder et al. 2000). Also, such an approach of course assumes that the cluster masses are known. In practice, the total masses have to be inferred assuming some scaling relation with an observable (for instance the temperature of the X-ray emitting gas). The evolution of the mass-observable relation has to be well-calibrated. In principle, this can be achieved in dark energy models with constant $w$ if the survey is deep enough so that the clusters counts can be binned both in redshift and in the observable quantity [Hu 2003].

6 SUMMARY AND CONCLUSION

Using the top-hat model and the Sheth-Tormen refinement of the Press-Schechter mass function, we have examined the effect of the equation of state of dark energy on the evolution of spherical overdensities and the abundance of clusters. We have considered constant-$w$ models. We have used a general expression for the virial theorem in the presence of dark energy instead of the Wang & Steinhardt 1998 approximation widely used in the literature and valid mainly around $w = -1$. Using the general expression, one can show that a dark energy model with an equation of state $w = -1/3$ is equivalent to an open CDM model with the same matter density $\Omega_0$ and no dark energy. The Wang & Steinhardt 1998 approximation in that regime yields an overestimate of the overdensity of the virialised cluster $\Delta_{\text{vir}}$ by as much as 40%. A flat dark energy model with $w = -1/3$ differs, however, from the corresponding open CDM model because of the different curvature, implying a different distance-redshift
Figure 8. Number density of virialised clusters with mass larger than a given mass threshold per redshift interval and square degree in the three cosmological models with dark energy, using the X-ray cluster normalisation (thin lines) or the CMB normalisation (thick lines). The solid line shows the ΛCDM, the dashed line the $w = -0.8$ model and the dotted-dashed line the $w = -0.6$ model.

The number density of massive clusters is sensitive both to the amplitude of the mass fluctuations, $\sigma_8$, and to the $\sigma_8 - w$ relation. As $w$ increases, $\sigma_8$ has to decrease to match the observations of the cosmic microwave background and the observed abundance of X-ray clusters. As $w$ increases, clusters form earlier and are more concentrated. However, this effect is partly counteracted by the decrease of $\sigma_8$ with $w$. A recent analysis of the evolution of the X-ray temperature of clusters favours a low $\sigma_8$ and a higher $w$: $\sigma_8 = 0.66 \pm 0.16$ with $w = -(0.42 \pm 0.21)$ at 68% confidence level [Henry 2004]. A previous similar analysis with fewer distant clusters in a flat ΛCDM model had yielded a higher value of $\sigma_8$ (0.77 ± 0.15 [Henry 2000]). Breaking the $\sigma_8 - w$ degeneracy will require very sensitive observations of the evolution of the cluster abundance together with a good knowledge of the mass-temperature relation and the limiting mass of the survey.

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