Heat and mass transfer effects on nanofluid past a horizontally inclined plate

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Abstract: In this paper, the free convective flow past a nanofluid over an inclined plate with heat and mass transfer is investigated. Different kinds of nanofluid containing nano particles like Silver, Aluminum oxide and Copper were considered with water as a base fluid. The governing equations were transformed into the nonlinear equations. Then solutions are obtained using a crank Nicolson method; an efficient, tri-diagonal, iterative implicit finite difference method. The results are analyzed and presented through Graphs and tables for various parameters. Also, the results are in excellent agreement with the results in the open literature.

Keywords: Free Convection, Inclined plate, Nanofluid, Heat Transfer, Mass Transfer

1. Introduction

Nanofluid technical term introduced by Choi [1995]. Nanotechnology based fluid contains a suspended submicronic nano-scale particles has a main advantage that it enhances the base fluid thermal conductivity experienced by Masuda et al. [1993]. Since for, many researchers like Xuan and Li, Xue [2000, 2003] are interested to do research in nanofluids in the last few years, because of long-established fluids like water, ethylene, mineral oils have a low thermal conductivity compared with nanofluids, many publications are in search of understanding the behavior of nanofluids in industrial and engineering systems, Nuclear reactors, electronics, advanced nuclear systems and also in medicine too. The applications carried out currently and also to be used in the future, which involves nanofluids were given by Wong and Leon [2010].

A famous Rayleigh’s problem, dealt with the impulsively started infinite horizontal fluid through which the viscous incompressible fluid flow through got solved by Stokes [1851] using mixed explicit implicit finite difference method. Under uniform surface temperature and concentration, heat and mass transfer of free convective flow past a vertical plate was solved by Somers [1956] and Wilcox [1961] using an integral method. Followed them Gebhart and Pera [1971] studied the natural convection on a vertical plate with variable surface temperature and mass diffusion using similarity variables. Soundalgekar [1977] concerned with the problem of stoke’s for the vertical plate under the effect of MHD. Later Soundalgekar and Ganesan [1981] employed implicit finite difference method to explore the natural convection on an isothermal flat plate. Chen et al. [1986] Gave an important result of studying the heat transfer in horizontal, vertical and inclined plates with varying wall temperature and
heat flux. Chamkha and Al-Mudhaf [2005] numerically investigate the unsteady laminar heat and mass transfer. Mohammed Ibrahim and Baskar Reddy [2013] studied the Heat and Mass Transfer for natural convection over a moving vertical plate with internal heat generation and a convective boundary condition in the presence of thermal radiation viscous dissipation, and chemical reaction.

Upto the knowledge of the author, there is no research undertaken for the free convective nanofluid past an inclined plate under heat and mass transfer.

2. Mathematical Analysis

The present study considers free convective nanofluid flow along an inclined plate with MHD and heat generation/absorption effects. The schematic diagram and Cartesian coordinate system of the problem are shown in Figure 1 in which the x-axis is taken along the plate and the y-axis is taken normal to the plate. As shown in Figure 1, the inclined angle along the plate is assumed to be \( \phi \). \( T_\infty \), \( C_\infty \) is the temperature of both the fluid and the plate at the initial stage, i.e. at time \( t' \leq 0 \). Then at time \( t' > 0 \), the temperature of the plate is raised to \( T_w \), \( C_w \). The fluid taken into account here is a nanofluid containing nanoparticles like silver, aluminium oxide and copper. Also, the nanofluid considered to be an incompressible fluid.

All of the fluid physical properties are assumed to be constant except the density variations, which includes the buoyancy forces in the momentum equation. The governing boundary layer equations of continuity, momentum and energy under the Boussinesq approximation (Herman Schlichting [1969]) are as follows:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0
\]

\[
\rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = g \left( \rho \beta \right)_{nf} \cos \phi \frac{\partial}{\partial x} \int_{y}^{\infty} (T' - T_\infty) \, dy + g \left( \rho \beta \right)_{nf} \cos \phi \frac{\partial}{\partial x} \int_{y}^{\infty} (C' - C_\infty) \, dy
\]

\[
+ g \left( \rho \beta \right)_{nf} \sin \phi (T' - T_\infty) + g \left( \rho \beta \right)_{nf} \sin \phi (C' - C_\infty) + \mu_{nf} \frac{\partial^2 u}{\partial y^2}
\]
\[
\begin{align*}
\left( \rho c_p \right)_{nf} \left( \frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) &= k_{nf} \frac{\partial^2 T'}{\partial y^2} \\
\left( \frac{\partial C'}{\partial t} + u \frac{\partial C'}{\partial x} + v \frac{\partial C'}{\partial y} \right) &= D \frac{\partial^2 C'}{\partial y^2}
\end{align*}
\]

(3)

(4)

Initial and boundary conditions are

\[ i' \leq 0, u = 0, v = 0, T' = T_{\infty}', C' = C_{\infty}' \text{ for all } x \text{ and } y \]

\[ i' > 0, u = 0, v = 0, T' = T_{v}', C' = C_{v}' \text{ at } x = 0 \]

\[ u = 0, v = 0, T' = T_{w}', C' = C_{w}' \text{ at } y = 0 \]

\[ u \to 0, v \to 0, T' \to T_{s}', C' \to C_{s}' \text{ as } y \to \infty \]

(5)

Non-dimensional quantities used to dimensionalize the basic equations are

\[ X = \frac{x}{L}, Y = \frac{y}{L}, Gr = \frac{g \beta L}{\nu^2} \frac{(T_{w}' - T_{\infty}')}{(T_{s}' - T_{w}')}, U = \frac{u L}{\nu}, T' = \frac{v L}{\nu}, t = \frac{v t L}{\nu}, T = \frac{T' - T_{\infty}'}{T_{\infty}' - T_{s}'} \]

\[ C = \frac{C'}{C_{\infty}' - C_{w}'} - Gr = \frac{g \beta L}{\nu^2} \frac{(T_{w}' - T_{\infty}')}{(T_{s}' - T_{w}')}, Gr^{*} = \frac{g \beta L}{\nu^2} \frac{(C_{w}' - C_{\infty}')}{(C_{s}' - C_{w}')}, N = \frac{Gr^{*}}{Gr} \]

\[ Pr = \frac{\nu}{L}, \text{ or } Pr = k_{j} \frac{\nu (\rho C_{p})_{f}}{k_{j}}, Sc = \frac{\nu}{D} \]

(6)

For nanofluids, the expressions of density \( \rho_{nf} \), thermal expansion coefficient \( \beta \), and heat capacitance \( \rho C_{p} \), are given by

\[ \rho_{nf} = (1 - \varphi) \rho_{f} + \varphi \rho_{s} \]

\[ (\rho \beta)_{nf} = (1 - \varphi)(\rho \beta)_{f} + \varphi(\rho \beta)_{s} \]

\[ (\rho C_{p})_{nf} = (1 - \varphi)(\rho C_{p})_{f} + \varphi(\rho C_{p})_{s} \]

\[ \mu_{nf} = \frac{\mu_{f}}{(1 - \varphi)^{2.5}} \]

\[ k_{nf} = k_{f} \left[ \frac{k_{s} + 2k_{f} - 2\varphi(k_{f} - k_{s})}{k_{s} + 2k_{f} + \varphi(k_{f} - k_{s})} \right] \]

(7)
Equations in Non-dimensional form after the simplification will be

\[
\frac{\partial U}{\partial X} + \frac{\partial U}{\partial Y} = 0
\]  \hspace{1cm} (8)

\[
\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = \frac{1 - \theta + \frac{\phi (\rho \beta_s)}{\rho_f}}{1 - \theta + \frac{\rho_f}{\rho_f}} \left[ Gr \left( \frac{1}{\cos \phi} \int_T dY \right) \right] + \frac{NC \sin \phi + \frac{1}{(1 - \theta) \frac{1}{2}} \frac{1}{1 - \theta} \frac{\partial^2 U}{\partial Y^2}}{1 - \theta + \frac{\rho_f}{\rho_f}} \hspace{1cm} (9)
\]

\[
\frac{\partial T}{\partial t} + U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} = \frac{1}{Pr} \frac{1}{k} \frac{\partial^2 T}{\partial Y^2} \hspace{1cm} (10)
\]

\[
\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial Y} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Y^2} \hspace{1cm} (11)
\]

Initial and boundary conditions in non-dimensional form is

\[
i' \leq 0, U = 0, V = 0, T = 0, C = 0 \text{ for all } X \text{ and } Y\]

\[
i' > 0, U = 0, V = 0, T = 0, C = 1 \text{ at } X = 0\]

\[
U = 0, V = 0, T = 1, C = 1 \text{ at } Y = 0\]

\[
U \rightarrow 0, Y \rightarrow 0, T \rightarrow 0, C \rightarrow 1 \text{ as } Y \rightarrow \infty \hspace{1cm} (12)
\]
By substituting

\[
E_i = \frac{1}{(1-\varphi)^{2.5}} \frac{1}{1-\varphi + \varphi \frac{\rho_s}{\rho_f}}, \quad E_2 = \frac{1-\varphi + \varphi \frac{\rho_s}{\rho_f}}{1-\varphi + \varphi \frac{\rho_s}{\rho_f}}
\]

\[
E_3 = \frac{1}{Pr} \frac{1}{1-\varphi + \varphi \frac{\rho_c_p}{\rho_f}} \frac{k_{nf}}{k_f},
\]

(13)

In non-Dimensional equations, the equations become

\[
\frac{\partial U}{\partial X} + \frac{\partial U}{\partial Y} = 0
\]

(14)

\[
\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial X} + V \frac{\partial U}{\partial Y} = E_1 \frac{\partial^2 U}{\partial Y^2} + E_2 \left[ Gr^{-\frac{1}{2}} \cos \phi \left[ \frac{\partial}{\partial X} \left( T \right) \right] + N \frac{\partial}{\partial Y} \left( C \right) + NC \sin \phi \right]
\]

(15)

\[
\frac{\partial T}{\partial t} + U \frac{\partial T}{\partial X} + V \frac{\partial T}{\partial Y} = E_3 \frac{\partial^2 T}{\partial Y^2}
\]

(16)

\[
\frac{\partial C}{\partial t} + U \frac{\partial C}{\partial X} + V \frac{\partial C}{\partial Y} = \frac{1}{Sc} \frac{\partial^2 C}{\partial Y^2}
\]

(17)

Local and Average Skin Friction, Nusselt Number, Sherwood Number are

\[
\tau_x = Gr^{\frac{3}{2}} \frac{1}{(1-\varphi)^{2.5}} \left( \frac{\partial u}{\partial Y} \right)_{y=0}
\]

(18)

\[
\tau = Gr^{\frac{3}{2}} \frac{1}{(1-\varphi)^{2.5}} \int_0^1 \left( \frac{\partial u}{\partial Y} \right)_{y=0} dX
\]

(19)

\[
Nu_x = Gr^{\frac{1}{2}} \left( - \frac{k_{nf}}{k_f} \right) X \left( \frac{\partial T}{\partial Y} \right)_{y=0}
\]

(20)
3. Numerical solution and discussion

The set of non-linear coupled equations along with the initial and boundary conditions are solved using a most reliable, unconditionally stable implicit finite difference method of the Crank-Nicolson type. The resulting problem attained from the partial difference equations are solved using a tridiagonal system which is solved by using the Thomas Algorithm as described by Carnahan et al. [1969] and the integrals in the equation are solved using the Newton-Cotes formula.

The results are illustrated graphically to discuss interesting features of the problem.
4. Conclusions:

- On increment of the value of $N$, both Velocity and Concentration increases and temperature decreases.
- On increment of the value of $Gr$, Velocity decreases and the temperature and concentration has no effect.
- On increment of the value of Angle, Velocity increases and both the temperature and concentration decreases.
- On increment of the value of $Sc$, both Velocity and Concentration decreases and temperature increases.
- On increment of the value of $\phi$, Velocity, temperature and concentration increases and when the value of $\phi$ is small, copper shows the maximum reaction while the value is larger, Aluminum shows the maximum reaction.

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