Comparison of assessing the filtration coefficient from laboratory tests with correlation results determined from CPTu test for coarse-grain soil

Nikola Dudek and Irena Bagińska

Wrocław University of Science and Technology, Faculty of Civil Engineering, Wyb. Wyspińskiego 27, 50-370 Wrocław, Poland

nikoladudek@gmail.com, irena.baginska@pwr.edu.pl

Abstract. Filtration coefficient is a basic parameter to assess hydrogeological features of foundation soil and also the material to be built into geotechnical structures. It can be determined from laboratory testing, empirical formulae based on simple laboratory tests, and also from direct or indirect field tests. Each of these method may lead to different values of filtration coefficient. The paper is focused on determining filtration coefficient of coarse-grain soil with several techniques. Direct results from laboratory testing were referred to the indirect evaluation based on grain size analysis and on static sounding CPTu test.

1. Introduction – determination of filtration coefficient (k)

Filtration coefficient (k) is the parameter which determines the ability of soil medium to conduct water through it. In other words, it is defined as average flow velocity which will occur through the total cross-section area of soil at unit hydraulic gradient [1]. The value of the coefficient depends of such soil features as: grain size distribution, porosity, moisture content, structure and discontinuities or particle and grain system [2]. The methods of determining the coefficient can be divided into three groups:

- calculations based on formulae, both analytical and empirical ones,
- numerical, mathematical and physical modelling the water flow in soil,
- soil field tests and laboratory testing.

Somewhat different classification is given in Appendix S to the Eurocode 7: Geotechnical design – Investigations and testing the foundation soil [PN – EN 1997 – 2], where four testing methods of filtration coefficient are distinguished:

- field tests, i.e. pumping and testing the permeability in borehole,
- laboratory testing,
- estimating from oedometric examinations,
- empirical correlations from soil grain size composition (from grain size distribution curve) [3].

It is worth emphasising that the methods based on physical flow of water through soil (field and laboratory methods) most accurately consider the effect of grains on the value of coefficient k, hence provide the results closest to the real ones [4]. Field tests are valued for direct measurements taking into account local dislocations, fissures or cracks in natural soil medium. Laboratory testing are frequently more complicated and time consuming [1, 5, 6]. Filtration coefficient (k) values are also
needed for the numerical model of filtration flow. In this case, the value of the filtration coefficient \( k \) for a specific region can be assumed by calibrating the model, taking into account the distribution of the filtration coefficient [7].

Bibliography provides also some indirect techniques to evaluate the coefficient \( k \) by means of correlation relationships between values recorded in CPTu static sounding and filtration coefficient [8, 9]. The value of filtration coefficient \( k \) can be indirectly estimated from the values of cone resistance \( q_c \) and sleeve friction resistance \( f_s \) from CPTu tests, and calculating the vertical stresses in soil [8].

The paper is focused on determining the filtration coefficient for coarse grain soil using several methods. Direct results from laboratory testing are compared with indirect evaluation from grain size analysis and with indirect correlations based on records from CPTu static sounding.

2. Methods used to determine soil filtration coefficient

2.1. Laboratory test – testing of filtration at stable hydraulic gradient

![Figure 1](image1.png)  ![Figure 2](image2.png)

**Figure 1.** Scheme of filtration testing at constant hydraulic gradient

**Figure 2.** Scheme of sieve arrangement for grain size analysis

The water permeability coefficient can be determined with a set of vessels in laboratory. This testing can be made either at constant of variable hydraulic gradient [10]. To observe the test procedure, soil – in compaction corresponding to that existing at area of its deposition – should put into a tube. Then, the tube is secured in a stand and is slowly filled in with water. When the water level stabilizes, the test can be initiated. Measurement is made of water flow through the soil. Constant water supply should be kept during testing to maintain constant hydraulic gradient. Several measurements should be made to increase the accuracy of results [10]. Testing scheme is shown in Fig. 1.

The results subject to analysis according to Darcy law which assumes that the volume of water flowing through porous medium per unit of time is proportional to the hydraulic gradient, cross-section area of the medium affected by filtration and the filtration coefficient [11]. This can be expressed by the formula:

\[
Q = k_t \cdot t \cdot A \cdot \frac{\Delta H}{l}
\]

where: \( Q \) – water volume taken in a time \( t \) [cm\(^3\)], \( k_t \) – filtration coefficient for water at temperature \( T \) [cm/s], \( t \) – test time [s], \( A \) – horizontal cross-sectional area of sample [cm\(^2\)], \( \Delta H \) – water level difference [cm], \( l \) – sample thickness [cm].
The value of filtration coefficient should be then converted to that valid for water temperature $T$ equal to 10 °C ($k_{10}$) according to the formula:

$$
    k_{10} = \frac{k_t}{0.7 + 0.03 \cdot T}
$$

where: $T$ – water temperature [°C], $k_t$ – filtration coefficient for $T \neq 10$ °C calculated acc. to formula (1) [cm/s]

2.2. Indirect estimation based on results from grain size analysis

Granulometric (sieve) analysis enable to identify the soil in terms of the fractions included and their contents. Examination consists in sieving dry sample of known mass $m_s$ through sieves located on a shaker. At the end, the soil remaining on particular sieves $m_{si}$ and that under the sieve with the smallest mesh subject to weighing. When soil masses on particular sieves are found, the percentages of particular fractions $z_i$ are determined from the relationship:

$$
    z_i = \frac{m_{si}}{m_s} \cdot 100\%
$$

where: $m_{si}$ – the mass of soil remaining on particular sieves, $m_s$ – total mass of sample.

The results of sieve analysis are most often shown in cumulative curve called the screening curve or particle size curve, which provides percentage values of undersize mass of particular sieves with respect to the total mass of the sample. The system of used sieves is shown in Fig. 2. [12]. Basing on granulometric analysis, and empirical formulae, the filtration coefficient for water temperature $T = 10$ °C, i.e. the coefficient $k_{10}$ was estimated. Calculations were made in compliance with [13], where the relations between the filtration coefficient $k_{10}$ and the granulation measures and porosity of coarse – grain soils are given. One method consists in using the Beyer table [13], which can be applied when the effective grain size is $d_{10} = 0.06$÷0.6 mm and the soil graining heterogeneity index is $U=1.0÷20.0$.

The other method, taken from bibliography [14] consists in application of empirical formulae:
- A so called American formula USBSC based on effective grain size $d_{20}$, where $d_{20}$ is the diameter of grains, which – together with the smaller ones represent 20% of the mass of the soil under investigation. The formula (4) may be used when $0.01 \text{ mm} \leq d_{20} \leq 2.0 \text{ mm}$ [14].

$$
    k_{10} = 0.36 \cdot d_{20}^{2.3} \left[ \text{cm/s} \right]
$$

- Seelheim formula (5), which can be used to determine water-permeability of all detached rocks:

$$
    k_{10} = 0.357 \cdot d_{50}^{2} \left[ \text{cm/s} \right]
$$

where $d_{50}$ is the grain diameter, which – together with the smaller ones – represent 50% of the mass of the soil under investigation [mm] [14].

It is a quite problematic issue to use empirical formulae as the resultant values of $k$ may differ each other significantly. This is outlined in the paper [15]. Comparison of filtration coefficients calculated from USBSC formula (4), from Seelheim formula (5) and from Beyer table for fine sand, medium sand and sandy gravel is presented in [13].

2.3. Estimation basing on correlations from CPTu testing

The method enabling to find the filtration coefficient from CPTu sounding is available in bibliography. This method, updated by Robertson i Wride [8], estimates $k$ by determining the soil behaviour type index $I_c$ given by formula:
where: \( Q_{ctn} = [(q_t - \sigma_v)/p_n](\sigma_v/\sigma_{vd})^n \), \( F_r = [f_s/(q_t - \sigma_v)] \cdot 1.099 \), \( q_t \) — CPT corrected total cone resistance [kPa], \( f_s \) — CPT sleeve friction [kPa], \( \sigma_v \) — total vertical stress [kPa], \( \sigma_{vd} \) — effective vertical stress [kPa], \( (q_t - \sigma_v)/p_n \) — dimensionless net cone resistance, \( (p_n/\sigma_{vd})^n \) — stress normalization factor

\( n \) — stress exponent that varies with SBT, \( p_n \) — atmospheric pressure in same units as \( q_t \) and \( \sigma_v \).

In this paper, according to recommendations from [16], it was assumed \( n=1 \). Finally, when calculating \( I_c \), the filtration coefficient \( k \) can be found acc. to the formula:

\[
I_c = [(3.47 - \log Q_{ctn})^2 + (\log F_r + 1.22)^2]^{0.5} \tag{6}
\]

The equations (7) and (8) are used to estimate the filtration coefficient and to demonstrate its variability with CPT sounding depth. However, please note that the parameters \( Q_{ctn} \) and \( F_r \) are dependent on the soil loading history and on many other variables, hence the relation between \( k \) and \( I_c \) should be considered as rough estimate only [8].

3. Determination of filtration coefficient for non-cohesive soil with laboratory method and on the basis of CPTu sounding – case study

3.1. Testing in Kamienski tube at constant hydraulic gradient

A sample of coarse-grain soil (Fig. 3) was taken from area where CPTu sounding have been performed. This sample was used to determine the filtration coefficient with the method of communicating vessels at constant hydraulic gradient. The method used was described in section 2.1 of this paper as illustrated in Figs. 1 and 4.

![Figure 3. Soil used for testing](image1)

![Figure 4. Testing set for coarse-grain soil](image2)

The testing was carried out at constant thickness of soil sample (7 cm) for five (5) different hydraulic gradients: 1.71, 2.43, 3.14, 4.57 and 7.14. Water volume during filtration was recorded in three different time periods: 120 s, 180 s, 300 s. The testing revealed that water volume rises with increasing the hydraulic gradient. In total 15 measurements were taken. Average value of filtration coefficient from all measurements was \( k_{10} = 6.82 \cdot 10^{-4} \) [cm/s] at standard deviation of \( 8.25 \cdot 10^{-5} \) [cm/s] and coefficient of variation 0.121. The results from individual measurements are summarized in Table 1.
Table 1. Results from determination of coefficient $k$ at constant hydraulic gradient for all cases under investigation

| Test No. | Hydraulic gradient | Average filtration speed $k_{10}$ [cm/s] | Standard deviation SD[cm/s] | Coefficient of variation COV |
|----------|--------------------|-----------------------------------------|-----------------------------|-------------------------------|
| 1        | 1.71               | 8.11-10$^{-2}$                          | 4.81-10$^{-3}$              | 0.059                         |
| 2        | 2.43               | 7.17-10$^{-2}$                          | 4.86-10$^{-3}$              | 0.068                         |
| 3        | 3.14               | 6.46-10$^{-2}$                          | 2.20-10$^{-3}$              | 0.034                         |
| 4        | 4.57               | 6.02-10$^{-2}$                          | 1.74-10$^{-3}$              | 0.029                         |
| 5        | 7.14               | 6.34-10$^{-2}$                          | 2.90-10$^{-3}$              | 0.046                         |

3.2. Application of empirical formulae basing on grain size analysis

The soil, after being dried, was subjected to grain size analysis according to the method outlined in section 2.2 or this paper. According to the results, the grain size curve was determined and using it, respective effective grain sizes $d_{10}$, $d_{30}$, $d_{50}$, $d_{60}$ were found. The soil under testing is the unsorted silty sand with $U=d_{60}/d_{10}=4.11$. The basic effective grain sizes were $d_{10}=0.1512$ [mm], $d_{20}=0.2412$ [mm], $d_{50}=0.5008$ [mm], $d_{60}=0.6217$ [mm]. The grain size curve is shown in Fig. 5.

Table 2. Summary of coefficient $k_{10}$ estimated by empirical formulae

| Formula used | Coefficient $k_{10}$ [cm/s] |
|--------------|------------------------------|
| Beyer table  | 2.100-10$^{-2}$              |
| USBSC (4)    | 1.367-10$^{-2}$              |
| Seelheim (5) | 8.954-10$^{-2}$              |
3.3. Application of empirical correlations basing on CPTu sounding

While running the CPTu testing, three quantities were recorded, namely $q_{ct}$, $f_s$, and $u_2$, which enabled to found the type of natural soil, i.e. the soil profile. The analysis was carried out with three methods: with Robertson nomograms from 1968 (Fig. 8) and from 1990 (Fig. 9) [17], but also basing on variability of soil behaviour type index Ic z 2009 r. (Fig. 6) [8].

The coarse – grain soil interbedding was analysed in detail at the depth from 2.22 m to 2.42 m below ground level as this soil was examined in laboratory (sections 3.1, 3.2 of this paper). Therefore, it can be concluded that the soil under testing is sand and silty sand/sand (SBT 8/9) acc. to Robertson classification of 1986 (Fig. 6); sands – clean sands to silty sands (SBTn 6) acc. to Robertson classification of 1990 (Fig. 7); sandy silt to clayey silt (SBTn 6) acc. to classification as per Ic of 2009 [8] (Fig. 8). Such identification is confirmed by grain size analysis shown in section 3.2.

**Figure 6.** Soil classification acc. to Robertson of 1986r. [17]

**Figure 7.** Soil classification acc. to Robertson of 1990r. [8]

**Figure 8.** Profiles of $q_{ct}$, $f_s$, $u_2$ recorded in CPTu testing and variability of Ic with division to SBTn zones acc. to classification of 2009
Precise calculations were made for the coefficient at the depth from 2.22 m to 2.42 m below ground level according to section 2.3. The results are in Table 3. As \( I_c \) is in the range \( 1.0 < I_c < 3.27 \), the formula (7) was used. Then, indirect results (for above depth, with step of 0.02 m) were presented on the diagram of filtration coefficient \( k \) versus soil behaviour type index \( I_c \) (Fig. 9). These points are situated within the range determined for SBTn zone 6 (sand) on the straight line defined acc. to formulae determined for SBT [8]. The values of \( I_c \) are from the range 1.31 – 1.77, which is also the value determined for soil of sand type and is included within the range 1.31 < \( I_c < 2.05 \) given in paper [18].

The values of filtration coefficient obtained fall within the range \( 3.61 \times 10^{-5} – 6.53 \times 10^{-4} \) m/s and are within the range \( 1.0 \times 10^{-5} – 1.0 \times 10^{-3} \) m/s given in paper [8] as characteristic for sands.

**Table 3.** Filtration coefficient \( k \) on the basis of index \( I_c \) for the depth from 2.22 m and 2.42 m below ground level

| Depth [m] | Index \( I_c \) | Filtration coefficient \( k \) [m/s] |
|-----------|-----------------|-------------------------------------|
| 2.22      | 1.75            | 4.26 \times 10^{-5}                 |
| 2.24      | 1.64            | 9.59 \times 10^{-5}                 |
| 2.26      | 1.53            | 1.98 \times 10^{-4}                 |
| 2.28      | 1.48            | 2.91 \times 10^{-4}                 |
| 2.30      | 1.42            | 4.37 \times 10^{-4}                 |
| 2.32      | 1.37            | 6.49 \times 10^{-4}                 |
| 2.34      | 1.32            | 8.82 \times 10^{-4}                 |
| 2.36      | 1.30            | 1.01 \times 10^{-3}                 |
| 2.38      | 1.36            | 6.53 \times 10^{-4}                 |
| 2.40      | 1.59            | 1.30 \times 10^{-4}                 |
| 2.42      | 1.77            | 3.61 \times 10^{-5}                 |

The zone under testing, at the depth from 2.22 m and 2.42 m below ground level is recognized as uniform in geotechnical properties. The average values of \( I_c \) and \( k \) representative for the whole zone were calculated. Having the variability of \( I_c \) and \( k \) for the zone section (see Fig. 9), standard deviations and coefficients of variation of these quantities were calculated and shown in Table 4. We can found from these calculations that the coefficient of variation is substantially higher for calculated filtration coefficient \( k \) than for soil behaviour type index \( I_c \). This attests to low variability of \( I_c \), i.e. high lithologic homogeneity of the layer tested and to high variability of \( k \) within this layer.

**Table 4.** Average values of filtration coefficient \( k \) on the basis of index \( I_c \) for the whole layer under examination at depth from 2.22 m to 2.42 m below ground level

|                      | Average value | Standard deviation | Coefficient of variation |
|----------------------|---------------|--------------------|--------------------------|
| Index \( I_c \)      | 1.50          | 0.168              | 0.112                    |
| Coefficient \( k \)  | \(3.98 \times 10^{-4} \) m/s | \(3.44 \times 10^{-4} \) m/s | 0.865                    |
4. Summary and conclusions
The calculations completed show that depending on the technique used to determine the filtration coefficient, various final results are achieved. It should be supposed that the reference filtration coefficient for temperature of 10°C, \( k_{10} \), is the same coefficient which is denoted in literature as \( k \).

It can be seen that all values of filtration coefficient we have obtained fall within the literature-available range limits, as fixed for sand [8,17]. Figure 10 illustrates values of filtration coefficient determined from laboratory testing and from empirical formulae basing on grain size analysis and examination with CPTu static sounding.

![Figure 10. Summary of determined values of filtration coefficient plotted on the diagram from the paper [7]](image)

The largest value of \( k \) was reached with the formula (4): \( 1.367 \times 10^{-4} \) [m/s], while the smallest – with the formula (5): \( 8.954 \times 10^{-4} \) [m/s]. As we can observe on the diagram, the value taken from Beyer table [13] coincides with that from the formula (7), i.e. the method outlined in section 2.3 given in paper [8].

While considering the coefficient of variation for the values achieved, it is clear that it is the lowest for laboratory test 0.121 and the highest for the formula (7). It shows that result variation is low for laboratory test and high for CPTu testing. It results from low variability of external factors affecting laboratory test and from many factors affecting in situ testing. For in situ CPTu testing within one lithological layer, measurements of \( q_c \) and \( f_s \) are affected by adjacent layers [17]. As the result, we have no constant records of \( q_c \) and \( f_s \) within the whole sand layer at depth from 2.22 m to 2.42 m below ground level, which might be expected for homogenous soil, and consequently large variation of the coefficient \( k \) (Fig. 6 and 9). Searching for values of \( k \) varying with the depth, even within a single layer is important, for example, for determining leaks and water flow through interlocking joints of sheet piles which protect deep excavations [19].

Estimating the coefficient \( k \) on the basis of grain size analysis does not allow for determining the standard deviation and coefficient of variation for the quantity under assessment. Estimation on the basis of empirical formulae from CPTu testing \( (I_c) \) provided results consistent with empirical solutions based on grain size analysis (Beyer table, equation (4)) and satisfactory convergence with laboratory tests at constant hydraulic gradient. The results obtained confirm that it is worth to search constantly the correlation between direct in situ testing and laboratory tests. Further on, the authors would try to estimate the filtration coefficient on the basis of testing the dissipations of pore pressures \( u_2 \) as recorded during static CPTu sounding [20, 21].
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