MODELING MULTI-WAVELENGTH STELLAR ASTROMETRY. III. DETERMINATION OF THE ABSOLUTE MASSES OF EXOPLANETS AND THEIR HOST STARS

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ABSTRACT

Astrometric measurements of stellar systems are becoming significantly more precise and common, with many ground- and space-based instruments and missions approaching 1 \muas precision. We examine the multi-wavelength astrometric orbits of exoplanetary systems via both analytical formulae and numerical modeling. Exoplanets have a combination of reflected and thermally emitted light that causes the photocenter of the system to shift increasingly farther away from the host star with increasing wavelength. We find that, if observed at long enough wavelengths, the planet can dominate the astrometric motion of the system, and thus it is possible to directly measure the orbits of both the planet and star, and thus directly determine the physical masses of the star and planet, using multi-wavelength astrometry. In general, this technique works best for, though is certainly not limited to, systems that have large, high-mass stars and large, low-mass planets, which is a unique parameter space not covered by other exoplanet characterization techniques. Exoplanets that happen to transit their host star present unique cases where the physical radii of the planet and star can be directly determined via astrometry alone. Planetary albedos and day–night contrast ratios may also be probed via this technique due to the unique signature they impart on the observed astrometric orbits. We develop a tool to examine the prospects for near-term detection of this effect, and give examples of some exoplanets that appear to be good targets for detection in the \textit{K} to \textit{N} infrared observing bands, if the required precision can be achieved.

Key words: astrometry – planetary systems

Online-only material: color figures

1. INTRODUCTION

As part of a Space Interferometry mission (SIM) Science Study, in Coughlin et al. (2010a), hereafter referred to as Paper I, we examined the implications that multi-wavelength microarcsecond astrometry has for the detection and characterization of interacting binary systems. In Paper I we found that the astrometric orbits of binary systems can vary greatly with wavelength, as astrometric observations of a point source only measure the motion of the photocenter, or center of light, of the system. For systems that contain stellar components with different spectral energy distributions, the motion of the photocenter can be dominated by the motion of either component, depending on the wavelength of observation. Thus, with multi-wavelength astrometric observations it is possible to measure the individual orbit of each component, and thus derive absolute masses for both objects in the system. In Coughlin et al. (2010b), hereafter referred to as Paper II, we showed that multi-wavelength astrometry can also be used to directly measure the inclination and gravity darkening coefficient of single stars, as well as the temperature, size, and position of star spots.

Astrometry has long been used to measure fundamental quantities of binary stars, and more recently has been used to study extrasolar planets. Although no independently confirmed planet has yet been initially discovered via astrometry, many planets discovered via radial velocity (which only yields the planetary mass as a function of the system’s inclination and host star’s mass) have had follow-up astrometric measurements taken in order to determine their inclinations, and thus true planetary mass as a function of only the assumed stellar mass (McArthur et al. 2004, 2010; Benedict et al. 2006; Bean et al. 2007; Martioli et al. 2010; Röll et al. 2010; Reffert & Quirrenbach 2011). There are many ground- and space-based microarcsecond precision astrometric projects which are either currently operating or on the horizon. The proposed SIM Lite Astrometric Observatory, a redesign of the earlier proposed SIM PlanetQuest Mission, was to be a space-based 6 m baseline Michelson interferometer capable of 1 \muas precision measurements in \approx 80 spectral channels spanning 450–900 nm (Davidson et al. 2009), thus allowing multi-wavelength microarcsecond astrometry. Although the SIM Lite mission has been indefinitely postponed at the time of this writing, it has already achieved all of its technological milestones, and it, or another similar mission, could be launched in the future. The PHASES project obtained as good as 34 \muas astrometric precision of close stellar pairs (Muterspaugh et al. 2010). The CHARA array has multi-wavelength capabilities and can provide angular resolution to \approx 200 \muas (ten Brummelaar et al. 2005). PRIMA/VLTI is working toward achieving \approx 30–40 \muas precision in the \textit{K} band (van Belle et al. 2008), with GRAVITY/VLTI expected to obtain 10 \muas (Kudryavtseva et al. 2010). The Astra/KECK project will be able to simultaneously observe and measure the distance between two objects to better than 100 \muas precision. The Gaia mission will provide astrometry for \approx 10^8 objects with 4–160 \muas accuracy, for stars with \textit{V} = 10–20 mag, respectively, and does possess some multi-wavelength capabilities (Cacciari 2009). The MICADO instrument on the proposed E-ELT 40 m class telescope will be able to obtain better than 50 \muas accuracy at 0.8–2.5 \mu m (Tripple et al. 2010). Finally, the NEAT mission

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proposes to obtain as low as 0.05 μas astrometric measurements at visible wavelengths (Malbet et al. 2011). Thus, astrometric measurements of extrasolar planets are going to become significantly more common in the future.

In this paper, we examine the multi-wavelength astrometric signature of exoplanets. A star–planet system is a specialized case of a binary system with extreme mass and temperature ratios, and thus the findings of Paper I apply to exoplanets. Specifically, an extrasolar planet has a combination of reflected and thermally emitted light that causes the photocenter to be displaced from the center of mass of the star. Since the planet’s temperature is very different from that of the host star, the amount of photocenter displacement due to the planet will greatly vary with wavelength. Although the luminosity ratio between a star and planet is extreme, the planet also lies a much farther distance from the barycenter of the system compared to the star, and thus it has a large “moment-arm” with which to influence the photocenter. While conventional single-wavelength astrometric measurements can yield the inclination which to influence the photocenter. While conventional single-wavelength astrometric measurements can yield the inclination, the wavelength-independent amplitude of the angular astrometric reflex motion of a system, α0, is

$$\alpha_0 = \arctan \left( \frac{r_p}{D} \right) = \arctan \left( \frac{a \cdot q}{D \cdot (q + 1)} \right),$$

(4)

where $D$ is the distance to the system from Earth, and $a$, via Kepler’s third law, is

$$a = \left( \frac{G(M_\star + M_p)}{P^2} \right)^{1/3} \left( \frac{P}{2\pi} \right)^{2/3},$$

(5)

where $G$ is the gravitational constant and $P$ is the orbital period of the system.

When the planet’s luminosity is not negligible, in order to determine the wavelength-dependent value of $\alpha$, the location of the system’s photocenter, which varies with wavelength, must be determined. We define $s_\star$ and $s_p$, to be the distance to the system’s photocenter from the star and planet, respectively, as shown in Figure 1, where the photocenter is marked with a “×” symbol. We define the luminosity ratio at a given wavelength, $L_r$, as

$$L_r = \frac{L_p}{L_\star},$$

(6)

where $L_p$ is the luminosity of the planet and $L_\star$ is the luminosity of the star. Thus, similar to the previously presented derivations, the values for $s_\star$ and $s_p$ are

$$s_\star = \frac{a \cdot L_r}{L_\star + 1},$$

(7)

$$s_p = \frac{a}{L_\star + 1},$$

(8)

where by definition $s_\star + s_p = a$. The observed astrometric motion results from the movement of the system’s photocenter around the system’s barycenter. Thus, taking into account light from both the star and planet,

$$\alpha = \arctan \left( \frac{r_p - s_\star}{D} \right) = \arctan \left( \frac{s_p - r_p}{D} \right)$$

(9)
and thus
\[ \alpha = \arctan \left( \frac{a \cdot (q - L_r)}{D \cdot (q + 1) \cdot (L_r + 1)} \right). \]  

where we have defined \( \alpha \) so that \( \alpha > 0 \) signifies that the star dominates the observed reflex motion, i.e., \( L_r < q \), and \( \alpha < 0 \) signifies that the planet dominates the observed reflex motion, i.e., \( L_r > q \). Note that when the barycenter and photocenter are at the same point, i.e., \( L_r = q \), and thus \( \alpha = 0 \), no reflex motion is observable.

We now estimate the value of \( L_r \) based upon the values of readily measurable system parameters. Light emitted from the planet consists of both thermally emitted light as well as incident stellar light reflected off the planet. Thus,
\[ L_r = \frac{L_E + L_A}{L_\ast} = \frac{L_E}{L_\ast} + \frac{L_A}{L_\ast}, \]

where \( L_E \) is the luminosity of the planet from thermal emission, \( L_\ast \) is the luminosity of the star, and \( L_A \) is the luminosity of light reflected off the planet. To estimate the thermal component, we assume that both the star and planet radiate as blackbodies, and thus
\[ \frac{L_E}{L_\ast} = \frac{R_p^2}{R_\ast^2} \cdot \frac{\exp \left( \frac{hc}{\lambda kT_\ast} \right) - 1}{\exp \left( \frac{hc}{\lambda kT_p} \right) - 1}. \]

where \( R_p \) is the radius of the planet, \( \lambda \) is a given wavelength, \( h \) is Planck’s constant, \( c \) is the speed of light, \( k \) is Boltzmann’s constant, \( T_\ast \) is the effective temperature of the star, and \( T_p \) is the effective temperature of the planet. To derive \( T_p \) we first assume that the planet is in radiative equilibrium and has perfect heat re-distribution, i.e., a uniform planetary temperature, and thus
\[ T_p = T_\ast \cdot \left( \frac{1 - A_B}{4a^2} \right)^{1/4}, \]

where \( T_\ast \) is the temperature of the star, \( A_B \) is the planetary Bond albedo, and \( R_\ast \) is the radius of the star.

To estimate the contribution due to reflected light, we first note that the flux received at the planet’s surface is \( L_\ast \) divided by the surface area of a sphere at a distance \( a \), i.e., \( 4\pi a^2 \). The planet intercepts and reflects this light on only one of its hemispheres, which has effective cross-sectional area of \( \pi R_p^2 \) with an efficiency equal to the albedo. Combining these terms and rearranging to obtain the luminosity ratio due to reflected light yields
\[ \frac{L_A}{L_\ast} = \frac{A_B R_p^2}{4a^2}. \]
surveys are most sensitive to close-in planets. As can be seen, the top candidates for detecting $\alpha < 0$, and thus measuring the absolute mass of the planet, are WASP-12 b in the $K$ band with $\alpha = -0.05 \mu m$, HD 209458 b in the $L$ and $M$ bands with $\alpha = -0.23$ and $-0.66 \mu m$, respectively, and HD 189733 b in the $N$ band with $\alpha = -3.04 \mu m$. It is interesting that three low-mass Neptune and sub-Neptune mass planets, 55 Cnc e, Gliese 436 b, and GJ 1214 b, also make the list, illustrating that this technique can “favor” the characterization of low-mass planets.

3. NUMERICAL MODELING VIA REFUX

In order to provide a check on our analytical formulae, better illustrate the multi-wavelength astrometric orbits of exoplanet systems, and probe some more subtle effects, we use the REFLUX\(^6\) code (Coughlin et al. 2010a), which computes the flux-weighted astrometric reflex motion of binary systems at multiple wavelengths, to model a couple of known exoplanet systems. We discussed the code in detail in Papers I and II, but in short, it utilizes the Eclipsing Light Curve (ELC) code, which was written to compute light curves of eclipsing binary systems (Orosz & Hauschildt 2000). ELC includes the dominant physical effects that shape a binary system’s light curve, such as non-spherical geometry due to rotation and tidal forces, gravity darkening, limb darkening, mutual heating, reflection effects, and the inclusion of hot or cool spots on the stellar surface. The

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\(^6\) REFLUX can be run via a web interface from http://astronomy.nmsu.edu/jlcough/reflux.html. Additional details as to how to set up a model are presented there.
ELC code represents the surfaces of two stars, or a star–planet system, as a grid of individual luminosity points, and calculates the resulting light curve given the provided systemic parameters. REFLUX takes the grid of luminosity points at each phase and calculates the flux-weighted astrometric photocenter location at each phase, taking into account the system’s distance from Earth. Although ELC is capable of using model atmospheres, for this paper we set the code to calculate luminosities assuming both the star and planet radiate as blackbodies.

We choose to model Wasp-12, HD 209458, and HD 189733, as they are all well-studied systems, and have the most negative \( \alpha \) values for the \( K, L, M, \) and \( N \) bandpasses presented in Table 1. For each system we set the values for \( M_\star, R_\star, M_p, R_p, P, D, \) and rotation period of the star to those in the Exoplanets.org database, and set the rotation period of the planet to the orbital period of the system, i.e., assume the planet is tidally locked, and assume a circular orbit. We assume that the spin axes of both the star and planet are perfectly aligned with the orbital axis. We employ the use of spots in the ELC code to simulate a day/night-side temperature difference, by assuming a uniform day-side temperature for the planetary hemisphere facing the star, and a uniform night-side temperature for the planetary hemisphere facing away from the star. We employ the values for the day- and night-side temperatures derived by Cowan & Agol (2011), which were 2939 K for the day side of Wasp-12 b, 1486 and 1476 K for the day and night sides, respectively, of HD 209458 b, and 1605 and 1107 K for the day and night sides, respectively, of HD 189733 b. We adopted a temperature of 1470 K for the night side of Wasp-12 b, 1486 and 1476 K for the day and night sides, respectively, of HD 209458 b, and 1605 and 1107 K for the day and night sides, respectively, of HD 189733 b.

![Figure 3](image_url) Plots of the reflex motion amplitude, \( \alpha \), versus the wavelength of observations, \( \lambda \), for an Earth-like planet (\( M_p = 1.0 M_\oplus \) and \( R_p = 1.0 R_\oplus \)) around F0V, G2V, and M0V stars at 10 pc (left, middle, and right columns, respectively), at periods of 1, 10, 100, and 1000 days (top to bottom rows, respectively). The solid, dashed, dotted, and dash-dotted lines represent planetary albedos of 0.0, 0.25, 0.5, and 0.75, respectively.
Table 1
Currently Known Exoplanets with the Most Negative \( \alpha \) Values

| Name         | \( D \) (pc) | \( M_p \) (\( M_\odot \)) | \( R_p \) (\( R_\odot \)) | \( T_p \) (K) | \( M_* \) (\( M_\odot \)) | \( R_* \) (\( R_\odot \)) | \( P \) (Days) | \( \alpha \) (\( \mu \)as) |
|--------------|--------------|-----------------|-----------------|-------------|---------------|-----------------|-------------|-------------|
| WASP-12 b   | 427.128      | 1.63            | 6300            | 1.35        | 1.79          | 1.091           | 1.091       | 0.05        |
| WASP-19 b   | 250.093      | 0.99            | 5500            | 1.11        | 1.39          | 0.789           | 0.789       | 0.05        |
| WASP-33 b   | 115.150      | 1.44            | 7430            | 2.05        | 1.50          | 1.220           | 1.220       | 0.00        |
| 55 Cnc e    | 12.096       | 0.96            | 5234            | 0.03        | 0.19          | 0.737           | 0.737       | 0.01        |
| CoRoT-1 b   | 480.955      | 1.11            | 5950            | 1.03        | 1.49          | 1.509           | 1.509       | 0.01        |

K Band (2.19 \( \mu \)m)

| Name         | \( D \) (pc) | \( M_p \) (\( M_\odot \)) | \( R_p \) (\( R_\odot \)) | \( T_p \) (K) | \( M_* \) (\( M_\odot \)) | \( R_* \) (\( R_\odot \)) | \( P \) (Days) | \( \alpha \) (\( \mu \)as) |
|--------------|--------------|-----------------|-----------------|-------------|---------------|-----------------|-------------|-------------|
| HD 209458 b | 49.113       | 1.16            | 6065            | 0.69        | 1.36          | 3.525           | 3.525       | 0.23        |
| WASP-33 b   | 115.150      | 1.44            | 7430            | 2.05        | 1.50          | 1.220           | 1.220       | 0.20        |
| WASP-19 b   | 250.093      | 0.99            | 5500            | 1.11        | 1.39          | 0.789           | 0.789       | 0.15        |
| WASP-17 b   | 300.119      | 1.20            | 6550            | 0.49        | 1.51          | 3.735           | 3.735       | 0.11        |
| WASP-12 b   | 427.128      | 1.63            | 6300            | 1.35        | 1.79          | 1.091           | 1.091       | 0.10        |

L Band (3.45 \( \mu \)m)

| Name         | \( D \) (pc) | \( M_p \) (\( M_\odot \)) | \( R_p \) (\( R_\odot \)) | \( T_p \) (K) | \( M_* \) (\( M_\odot \)) | \( R_* \) (\( R_\odot \)) | \( P \) (Days) | \( \alpha \) (\( \mu \)as) |
|--------------|--------------|-----------------|-----------------|-------------|---------------|-----------------|-------------|-------------|
| HD 209458 b | 49.113       | 1.16            | 6065            | 0.69        | 1.36          | 3.525           | 3.525       | 0.66        |
| HD 189733 b | 19.081       | 0.76            | 5040            | 1.14        | 1.14          | 2.219           | 2.219       | 0.30        |
| WASP-33 b   | 115.150      | 1.44            | 7430            | 2.05        | 1.50          | 1.220           | 1.220       | 0.29        |
| WASP-19 b   | 250.093      | 0.99            | 5500            | 1.11        | 1.39          | 0.789           | 0.789       | 0.21        |
| WASP-17 b   | 300.119      | 1.20            | 6550            | 0.49        | 1.51          | 3.735           | 3.735       | 0.19        |

M Band (4.75 \( \mu \)m)

| Name         | \( D \) (pc) | \( M_p \) (\( M_\odot \)) | \( R_p \) (\( R_\odot \)) | \( T_p \) (K) | \( M_* \) (\( M_\odot \)) | \( R_* \) (\( R_\odot \)) | \( P \) (Days) | \( \alpha \) (\( \mu \)as) |
|--------------|--------------|-----------------|-----------------|-------------|---------------|-----------------|-------------|-------------|
| HD 189733 b | 19.081       | 0.76            | 5040            | 1.14        | 1.14          | 2.219           | 2.219       | 0.30        |
| HD 209458 b | 49.113       | 1.16            | 6065            | 0.69        | 1.36          | 3.525           | 3.525       | 1.53        |
| Gliese 436 b| 10.045       | 0.46            | 3684            | 0.07        | 0.38          | 2.644           | 2.644       | 0.95        |
| WASP-34 b   | 120.101      | 0.93            | 5700            | 0.58        | 1.22          | 4.318           | 4.318       | 0.66        |
| GJ 1214 b   | 12.016       | 0.21            | 3026            | 0.02        | 0.24          | 1.580           | 1.580       | 0.59        |

N Band (10.0 \( \mu \)m)

The presence of the primary transit and secondary eclipse is clearly visible in all three cases, with the primary transit dominating the maximum amplitude of the astrometric shift for the visible wavelengths, particularly in the \( Y \)-direction. As no limb darkening was assumed in these models, the variation in the primary and secondary eclipse signatures with wavelength is due to the relative flux of the star and planet in those passbands. As noted by Gaudi (2010), measuring the astrometric shift of the primary transit directly yields the angular radius of the host star, and if the distance to the system is precisely known, one can directly derive the physical radius of the star. Additionally, if the density of the star is directly determined from the photometric light curve (Seager & Mallén-Ornelas 2003), then one can also directly derive the mass of the star. We also note, for the first time, that measuring the astrometric signature of the primary transit and, if observing at longer wavelengths, the secondary eclipse, specifically the duration of ingress and egress, similarly directly yields the angular radius of the planet. Since one may directly determine the surface gravity of the planet from the photometric light and radial-velocity curves alone (Southworth et al. 2007), one may also directly determine the mass of the planet. Thus, for transiting planets, multi-wavelength astrometric measurements yield two independent methods of measuring the physical stellar and planetary masses.

4. DISCUSSION AND SUMMARY

We have shown that the multi-wavelength astrometric measurements of exoplanetary systems can be used to directly determine the masses of extrasolar planets and their host stars, in addition to the inclination and spatial orientation of their orbital axis. If the planet happens to transit the host star, then the angular radius of both the star and planet can be directly determined, and when combined with the trigonometric parallax of the system, the absolute radii of the planet and host star can be directly determined via astrometry alone. We found that this technique is best suited to, though is certainly not limited to, large, low-mass planets that orbit large, high-mass stars, and thus covers a unique parameter space not usually covered by other exoplanet characterization techniques.
Figure 4. Plots of the multi-wavelength astrometric orbit for the Wasp-12 system. Left: the $X$ and $Y$ components of motion versus phase. Right: the sky-projected, $X$–$Y$, orbit. The point $(X, Y) = (0, 0)$ corresponds to the system’s barycenter, and the projected orbital rotation axis is parallel to the $Y$-axis. Phase 0.0 corresponds to the primary transit, when the planet passes in front of the star and is closest to the observer, and phase 0.5 corresponds to the secondary eclipse, when the planet passes behind the star and is farthest away from the observer.

(A color version of this figure is available in the online journal.)

Figure 5. Plots of the multi-wavelength astrometric orbit for the HD 209458 system. Left: the $X$ and $Y$ components of motion versus phase. Right: the sky-projected, $X$–$Y$, orbit. The point $(X, Y) = (0, 0)$ corresponds to the system’s barycenter, and the projected orbital rotation axis is parallel to the $Y$-axis. Phase 0.0 corresponds to the primary transit, when the planet passes in front of the star and is closest to the observer, and phase 0.5 corresponds to the secondary eclipse, when the planet passes behind the star and is farthest away from the observer.

(A color version of this figure is available in the online journal.)

Figure 6. Plots of the multi-wavelength astrometric orbit for the HD 189733 system. Left: the $X$ and $Y$ components of motion versus phase. Right: the sky-projected, $X$–$Y$, orbit. The point $(X, Y) = (0, 0)$ corresponds to the system’s barycenter, and the projected orbital rotation axis is parallel to the $Y$-axis. Phase 0.0 corresponds to the primary transit, when the planet passes in front of the star and is closest to the observer, and phase 0.5 corresponds to the secondary eclipse, when the planet passes behind the star and is farthest away from the observer.

(A color version of this figure is available in the online journal.)
We have provided analytical formulae and numerical models to estimate the amplitude of the photocenter motion at various wavelengths. We found that, for some systems, the planet can dominate the motion of the system’s photocenter at wavelengths as short as ~2 μm, though the amplitude of the effect is only ~0.05 μas. If one is able to obtain astrometric measurements at wavelengths up to 10 μm, then the motion of the photocenter due to the planet could be as high as several microarcseconds, and can often be of a much larger magnitude than seen at optical wavelengths when the photocenter motion is due solely to stellar motion.

We performed numerical modeling of several exoplanet systems via the REFLUX code and found it to be consistent with the predictions of our analytical model. The numerical modeling revealed that, even at shorter wavelengths where α > 0, the planet has a visible impact on the observed astrometric orbit of the system. As well, deviations from pure sinusoidal motions due to day–night flux differences are clearly visible, and thus multi-wavelength astrometry could probe planetary properties of albedo and heat redistribution efficiency.

One caveat when working to extract the planetary and stellar masses from actual observations is that one will likely need to either precisely know the luminosity ratio of the system, or make assumptions about the luminosity of the planet, e.g., it radiates as a blackbody and is in thermal equilibrium. It may be possible that other observations could yield this information, such as the secondary eclipse depth if the planet happens to transit. The remaining parameters of the system’s distance and period should be well determined via other methods such as microarcsecond precision parallax and radial-velocity or photometric light curves.

For the prospects of detection, it is clear that this effect will probably not be detected in the very near term. Although astrometric measurements are approaching 1 μas accuracy, they have not yet been performed. Much of the ground-based work is being focused on the optical and K bands, where in the latter the effect is just barely detectable. The development of microarcsecond precision astrometric systems in the mid-infrared, or sub-microarcsecond precision in the near-infrared, is clearly needed, and the methods presented here will serve to preselect the best planetary system candidates to be observed by those systems.

The work presented in this paper assumed that both the star and planet radiate as blackbodies, however it is known that both can significantly deviate from that assumption, especially in the near-infrared (e.g., Gillon et al. 2009; Rogers et al. 2009; Gibson et al. 2010; Croll et al. 2011; de Mooij et al. 2011; Coughlin & López-Morales 2012). At the extreme end, Swain et al. (2010) and Waldmann et al. (2012) recently found evidence for a very large non-LTE emission feature around 3.25 μm in the atmosphere of HD 189733 b. Although via blackbody approximations we calculate that the planet-to-star flux ratio should be 8.3 × 10−4, Swain et al. (2010) and Waldmann et al. (2012) measure the 3.25 μm emission feature to be ~8.5 × 10−3 times the stellar flux, or about 10 times greater than expected. Assuming blackbody emission, the expected value for α for this system at visible wavelengths is 2.15 μas and at 3.25 μm is 0.83 μas. If the emission feature is real however, the expected value for α at 3.25 μm is a very large ~11.3 μas, dominated due to the planetary motion. Thus, the key in performing these types of observations may be to select particular wavelengths where the planets are unusually bright.

Finally, although we did not assume any limb darkening in our models since we were examining near- to mid-infrared wavelengths, limb darkening will be significant when observed at different optical bandpasses. The astrometric signature of transiting planets will vary greatly due to limb darkening in the optical regime, and thus multi-wavelength astrometry of transiting planets may be used to explore the limb-darkening profiles of stars, or vice versa, stellar limb darkening may need to be precisely understood in order to extract planetary and stellar parameters of interest.

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