Coherent production of the long-lived pionium
nP states in relativistic nucleus–nucleus collisions

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Abstract

The coherent production of the $n\bar{p}$ states of the $\pi^+\pi^-$ atoms ($A_{2\pi}$) in relativistic nucleus–nucleus collisions is considered as a possible source of the $A_{2\pi}(nP)$ beam for the pionium Lamb-shift measurement. A general expression for estimation of the $A_{2\pi}(nP)$ yield is derived in the framework of the equivalent photon approximation.

1 Introduction

The DIRAC experiment [1, 2] aims to observe and study hydrogen-like atoms formed by pairs of $\pi^+\pi^-$ and $\pi^+K^-$ mesons ($A_{2\pi}$ and $A_{\pi K}$ atoms, respectively) that produced in their ground-states in inclusive processes with the use of the 24 GeV proton beam at PS CERN. The ground-state lifetime measurement of these atoms allows to obtain in a model independent way the difference of $\pi\pi$ ($a_0 - a_2$) and $\pi K$ ($a_{1/2} - a_{3/2}$) scattering lengths in $S$ state. The final measurements of the ground-states $A_{2\pi}$ and $A_{\pi K}$ lifetimes gave the values $\tau = 3.15 \cdot 10^{-15}$ s and $\tau = 2.5 \cdot 10^{-15}$ s, respectively [1].

The lifetimes of the dimesoatom $nP$ states are 3–5 orders of magnitude higher in comparison with the ground-state lifetimes, since for these states the strong interaction is suppressed. As a result, both the lifetime and mean path of such states are a few order higher compared to the ground state. Observation of such long-lived (meta-stable) states of $A_{2\pi}$ opens the possibility to measure the Lamb-shift in $A_{2\pi}$ and also to obtain the other combination the $\pi\pi$ scattering lengths: $2a_0 + a_2$. DIRAC plans to study the Lamb shift in $A_{2\pi}$ atoms and then to extract the latter combination of the scattering lengths.

The measurement the energy splitting between levels $nS$ and $nP$ coupled with the lifetime measurement would provide a determination of $a_0$ and $a_2$ separately. The values of these scattering lengths can be rigorously calculated in Chiral Perturbation Theory (ChPT)
Thus, these measurements provide an experimental test of the low-energy QCD predictions.

Some possibilities for measuring the pionium Lamb shift \( \Delta E_n = E_{nS} - E_{nP} \) based on observation of the interference between \( nS \) and \( nP \) (\( m = 0 \)) states in the external electro-magnetic fields are discussed in the papers [5]. For this aim the beams of relativistic \( A_{2\pi}(nP) \) are needed. One way of obtaining of such beam is regarded in [6]. An another way was proposed in [7].

In the present work, we consider the coherent production of the long-lived pionium \( nP \) states in relativistic nucleus–nucleus collisions as the possible source of \( A_{2\pi}(nP) \) beam for the pionium Lamb-shift measurements.

## 2 Primary idea

Let us consider the production of \( nP \) states of pionium atoms in the coherent collisions of the projectile \( A_P \) and target \( A_T \) nuclei

\[
A_P + A_T \rightarrow A_P + A_T + A_{2\pi}(nP). \quad (1)
\]

Since the quantum numbers of pionium in \( nP \) states are the same as photons ones, the coherent photoproduction process

\[
\gamma + A_T \rightarrow A_{2\pi}(nP) + A_T \quad (2)
\]

is quite intensive in GeV region. Another advantage of the coherent reaction (2) is the sharp angular distribution the produced pionium, comparable with angular acceptance of experimental setup.

The source of effective photons could be provided by projectile nucleus \( A_P \) in the spirit of EPA (Equivalent Photon Approximation) [8].

## 3 Reaction amplitude

The amplitude of (2) can be calculated as a projection of the amplitude of the reaction

\[
\gamma + A_T \rightarrow \pi^+ + \pi^- + A_T \quad (3)
\]
on the \( nP \) state of pionium.

The last one may be represented by following Feynman diagrams:
Here, the boxes represent the amplitudes of the \( \pi^\pm A_T \) scattering. The latter amplitude reads

\[
M\left( \gamma + A_T \to \pi^+ + \pi^- + A_T \right) = e\bar{\epsilon} p \left[ \frac{M_{\pi^+ A_T}(\bar{q})}{D_1} + \frac{M_{\pi^- A_T}(\bar{q})}{D_2} \right],
\]

where \( e \) is the elementary charge, \( \bar{\epsilon} \) is the photon polarization vector, and \( k_{1T} \) and \( k_{2T} \) are the transverse momenta of \( \pi^- \) and \( \pi^+ \), respectively.

For targets with zero value of isotopic spin (\( T = 0 \)) \( M_{\pi^- A_T} = M_{\pi^+ A_T} \). In general case, \( M_{\pi^- A_T} \) and \( M_{\pi^+ A_T} \) differ only slightly. So, we put

\[
M_{\pi^- A_T} = M_{\pi^+ A_T} \equiv M_{\pi A_T}.
\]

In this approximation, the final result reads

\[
\frac{d\sigma}{d\omega} \left( A_P + A_T \to A_P + A_T + A_{2\pi}(nP) \right) = n_{\text{eff}} \left( \frac{\omega}{Z_P} \right) \cdot \alpha^6 \cdot \sigma_{\pi A_T}^\text{el} \left( \frac{1}{n^3} - \frac{1}{n^5} \right),
\]

where \( n_{\text{eff}} \) is the number of effective photons with the energy \( \omega = E_\gamma = E_{A_{2\pi}} \) produced by projectile nucleus with charge \( Z_P \), and \( n \) is the principal quantum number of the pionium \( nP \) state. This expression can be used for the estimation of the \( A_{2\pi}(nP) \) yield.

We consider this paper as a preliminary report on the obtained results.
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