A generalised eigenvalue reweighting covariance matrix estimation algorithm for airborne STAP radar in complex environment

Hao Xiao1 | Tong Wang1 | Cai Wen2 | Bing Ren1

1National Laboratory of Radar Signal Processing, Xidian University, Xi'an, China
2School of Information Science and Technology, Northwest University, Xi'an, China

Abstract
To improve the space-time adaptive processing (STAP) performance of airborne radar in complex environments, a generalised eigenvalue reweighting covariance matrix estimation algorithm called GERCM is proposed here. First, the interference plus noise (IPN) covariance matrix of cell under test (CUT) data is estimated by the selected target-free training samples around the CUT with the sample covariance matrix method. Then, with the component decompositions of the selected training samples and the assumption of approximately equal subspace, the IPN covariance matrix of CUT data is reformulated by the eigenvector matrix, eigenvalue matrix, and the eigenvalue reweighting vector. Subsequently, based on the modified covariance matching estimation criterion, the eigenvalue reweighting vector is estimated by solving the redesigned convex optimisation problem with the Lagrange dual method. Finally, the STAP weight vector is calculated to process the CUT data. The proposed algorithm can obtain a relatively accurate IPN covariance matrix of CUT data by sufficiently utilising the non-homogeneous training samples and can effectively protect the moving targets in CUT data, which can be applied to airborne radar with arbitrary array structure and antenna configuration. Simulation results and performance analyses based on the multi-channel airborne radar measurement data demonstrate the effectiveness of the proposed GERCM algorithm.

1 INTRODUCTION

For an advanced airborne surveillance radar system, ground clutter returns are spread in Doppler because of platform motion. Compared with the ground-based radar, the airborne radar faces a more severe clutter problem in detecting small and slow moving targets. Space-time adaptive processing (STAP) is a powerful two-dimensional filtering technique for airborne phased-array radar to detect these moving targets. To test the existence of a target in the cell under test (CUT), the optimal STAP filter is calculated using the optimal interference plus noise (IPN) covariance matrix [1], which is usually estimated by the target-free training samples that are independent and identically distributed (i.i.d.) with the clutter signal in CUT data. According to the famous RMB criterion [2], the number of required target-free i.i.d. training samples is at least twice the system degrees of freedom (DoF) for obtaining less than 3dB average performance loss compared with the optimal filter. However, as a result of complex terrains, different land covers, moving targets, and geometric configuration, the sample requirements for effectively training the STAP filter cannot be met for airborne STAP radar working in a heterogeneous environment. This seriously degrades the clutter suppression performance of airborne STAP radar.

To improve the clutter suppression performance of airborne radar in a heterogeneous environment, STAP researchers proposed many different algorithms. Earlier approaches include the auxiliary channel receiver (ACR) [3], the multi-stage wiener filter (MWF) [4, 5], and the knowledge-aided (KA) algorithm [6, 7].
Although these algorithms can reduce the training requirements of an STAP filter, they suffer from several drawbacks. For example, it is hard for the ACR algorithm to determine the appropriate DoF for different range cells in an actual environment. Additionally, the traditional ACR algorithm cannot solve the clutter suppression problem of airborne STAP radar with non-side-looking configuration. In a heterogeneous environment, it is difficult for the MWF algorithms to select the suitable clutter rank. If the selected rank is lower than the best one, then the clutter component in CUT data cannot be effectively cancelled. On the contrary, if the selected rank is higher than the best one, then the noise will be enhanced. These will greatly reduce the clutter suppression performance of the MWF algorithms. In essence, KA algorithms are the clutter suppression algorithms that are based on the parameterised model or priori knowledge. This leads the performances of KA algorithms to be heavily dependent on the model precision and the accuracy of the priori knowledge. When the model mismatch happens or the priori knowledge is not accurate, the KA algorithms have poor clutter suppression performance. We note that the ACR, MWF, and KA algorithms only partially reduced the sample requirements for training the STAP weight vector. In terms of the increasingly complex work environment confronted by airborne radar, it is very important to further reduce the training sample requirements. To this end, many STAP algorithms with lower sample requirements have been developed. These developments include direct data domain (D3) [8, 9], and sparse recovery (SR) [10, 11]. These mentioned algorithms can reduce the training requirements to a single or several snapshot data but they also have some shortcomings. As the space-time sliding window processing of CUT data and the target protection is based on the direct cancellation in the element-pulse domain, D3 algorithms have an inherent problem of aperture loss and high minimum detectable velocity (MDV). In recent years, many STAP algorithms based on the clutter sparse recovery (SR) technique have been proposed successively. Utilising the sparseness of ground clutter in the angle-Doppler plane, STAP algorithms based on the SR technique (SR-STAP) can be usually summarised as a two-step procedure: first, using a certain SR algorithm to obtain the clutter space-time profile of CUT data; second, constructing the IPN covariance matrix with the obtained clutter profile and calculating the STAP weight vector to process the CUT data. Here, some typical SR-STAP algorithms are developed, such as iterative adaptive algorithm (IAA) [12], fast converging sparse Bayesian learning (FCSBL) [13], robust knowledge-aided sparse recovery (RKASR) [14], and parameter-searched orthogonal matching pursuit (PSOMP) [15]. Conventional SR-STAP algorithms can drastically decrease the training sample requirements, but they usually cannot thoroughly solve the off-grid problem even if grid mismatch correction is done [15] or a gridless technique [16] is used. As all these SR-STAP algorithms are fully space-time algorithms, they often have huge computational workload when the space-time DoF of airborne STAP radar is high. According to the latest research finding presented in [17], SR-STAP algorithms cannot accurately estimate the IPN covariance matrix using only CUT data. So, a feasible approach is to estimate the IPN covariance matrix with the sufficient employment of the non-homogeneous training samples in a complex environment, such as the sample reweighted (SRW) algorithms [18, 19]. In a partially heterogeneous environment, SRW algorithms can achieve lower clutter residual than in the traditional sample covariance matrix (SCM) method, but they cannot adequately suppress the strong ground clutter in the complex environment. In other words, SRW algorithms still have the problem of a lack of fully qualified training samples. To increase the number of qualified training samples in the complex environment, the persymmetric extend factor approach (PerEFA) [20] is proposed. As the PerEFA algorithm can increase the additional i.i.d. training samples, it can obtain better clutter suppression performance than the SRW algorithms. Nevertheless, the PerEFA algorithm relies heavily on the array structure, and it is sensitive to the cross-term matrix when the antenna array has an array error or the local oscillator system has phase noise, which results in the poor performance of the PerEFA algorithm.

To overcome the problems mentioned above, a generalised eigenvalue reweighting covariance matrix estimation algorithm called GERC, which can sufficiently utilise the non-homogeneous training samples and reduce the influences of the cross-term matrix in the PerEFA algorithm, is proposed for airborne STAP radar working in complex environments. The proposed GERC algorithm mainly contains the following key steps: first, estimating the initial IPN covariance matrix of CUT data by using the selected target-free training samples with the SCM method; second, taking the eigenvalue decomposition to the initially estimated IPN covariance matrix of CUT data and reformulating the IPN covariance matrix of CUT data with the eigenvector matrix, eigenvalue matrix, and the eigenvalue reweighting vector; third, estimating the eigenvalue reweighting vector by solving the redesigned convex optimisation problem with the Lagrange dual method; and finally, calculating the STAP weight vector combined with the obtained eigenvector matrix, eigenvalue matrix, and the eigenvalue reweighting vector to process the CUT data.

The main contributions of this work are listed as follows:

1. The cross-term matrix problem of the traditional PerEFA algorithm is qualitatively analysed and a new covariance matrix estimation algorithm for airborne STAP radar working in complex environments is proposed in this work.
2. By the modified covariance matching estimation criterion, a redesigned convex optimisation problem which can be solved by the Lagrange dual method is formulated to estimate the eigenvalue reweighting vector, and we derived the analytical solution of the estimated eigenvalue reweighting vector.
3. Performance advantages over other STAP methods and computational complexity of the proposed GERC algorithm are evaluated.
4. Simulation results and performance analyses based on the measured multi-channel airborne radar measurement
(MCARM) data are given to demonstrate the effectiveness of the proposed GERCM algorithm.

The remaining sections are organised as follows: Section 2 describes the problem formation of airborne pulsed-Doppler radar. In Section 3, analyses of the disadvantage of the Per-EFA algorithm are provided first. Then, we introduce the key theory of the proposed GERCM algorithm. Finally, we summarise the performance advantages and computational complexity of the proposed GERCM algorithm. Simulation results and performance analyses are given in section 4. Conclusions are drawn in section 5.

**Notations:** $(\cdot)^T$, $(\cdot)^H$, and $(\cdot)^{-1}$ are the transpose, conjugate-transpose, and matrix inverse operators, respectively; $\| \cdot \|$ and $\| \cdot \|_F$ are the 2-norm of a vector and Frobenius norm of a matrix, respectively; $\text{diag}(\cdot)$ and $\succ$ are the diagonalisation and element-wise greater-than operators, respectively; $\mathbb{R}^{K \times L}$ and $\mathbb{C}^{K \times L}$ are the sets of real and complex matrices of dimension $K \times L$, respectively; $I_K$, $0_K$, and $1_K$ are the identity matrix of order $K$, $K$-dimensional all-zero and all-one column vectors, respectively; $\text{Re}(\cdot)$ and $\text{Im}(\cdot)$ are the real and imaginary parts of a complex vector, respectively; $\otimes$ and $\odot$ are the Kronecker and Khatri-Rao product operators, respectively; $\text{Span}(\cdot)$ denotes the generated space spanned by a matrix;

### 2 | PROBLEM FORMATION

The platform geometry of the airborne radar is presented in Figure 1. Without loss of generality, a pulsed-Doppler radar system consisting of a uniform linear array of $N$ elements and spacing $d$ is under consideration. This radar system with wavelength $\lambda_c$ moves along the $Y$ axis at an altitude $H$ and constant velocity $V_P$. $M$ pulses are transmitted at a constant pulse repetition frequency (PRF) $f_s$ in one coherent processing interval (CPI).

According to the platform geometry shown in Figure 1, the normalised spatial frequency $f_s$, and normalised Doppler frequency $f_d$ of the clutter patch $S$ at a certain range cell can be expressed as

\[
f_s = \frac{d}{\lambda_c} \cos \phi \cos \theta \quad (1)
\]

\[
f_d = \frac{2V_P}{\lambda_c f_s} \cos \phi \cos(\theta + \alpha) \quad (2)
\]

where $\phi$ and $\theta$ are the elevation and azimuth angles, respectively, of the clutter patch $S$. $\alpha$ is the installation angle between antenna axis and flight direction. The spatial steering vector $\mathbf{v}$ and temporal steering vector $\mathbf{v}'$ of the clutter patch $S$ can be defined as

\[
\mathbf{v} = [1, \exp(j2\pi f_s), \ldots, \exp(j2\pi(N-1)f_s)]^T \quad (3)
\]

\[
\mathbf{v}' = [1, \exp(j2\pi f_d), \ldots, \exp(j2\pi(N-1)f_d)]^T \quad (4)
\]

![Figure 1](image_url) Platform geometry of airborne radar

Considering the range-ambiguous clutter, the received clutter data that come from the $r$th range cell can be expressed as

\[
\mathbf{c}_r = \sum_{u=1}^{N_u} \sum_{k=1}^{N_c} a_{k, r, u} \mathbf{v}_k \cdot r, u \otimes \mathbf{v}_k \cdot r, u \quad (5)
\]

where $a_{k, r, u}$, $\mathbf{v}_k \cdot r, u$, $\mathbf{v}_k \cdot r, u$, and $\mathbf{v}_k \cdot r, u$ are the complex amplitude, spatial steering vector, temporal steering vector, and the spatial-temporal steering vector, respectively, of the $k$th clutter patch at the $u$th ambiguous range cell. $N_u$ and $N_c$ are the number of range-ambiguous and independent clutter patches, respectively.

To correctly decide whether the target exists or not [21, 22], the core of the STAP algorithm is to estimate the optimal IPN covariance matrix of the $r$th range cell, that is, $R_r^{\text{Opt}}$. Assuming that the clutter and noise components are mutually independent, $R_r^{\text{Opt}}$ can be given by

\[
R_r^{\text{Opt}} = \sum_{u=1}^{N_u} \sum_{k=1}^{N_c} p_{k, r, u} (\mathbf{v}_k \cdot r, u) (\mathbf{v}_k \cdot r, u)^H + \sigma_n^2 I_{MN} \quad (6)
\]

where $p_{k, r, u}$ denotes the clutter patch power of the $k$th clutter patch at the $u$th ambiguous range cell, and $\sigma_n^2$ is the white noise power.

In light of the linearly constrained minimum variance criterion, the well-known optimal STAP weight vector of the $u$th Doppler bin at the $r$th range cell can be written as
\[ w_s(l) = \left( K_s^{\text{opt}} \right)^{-1} s_{T,l} s_{T,l}^T \left( K_s^{\text{opt}} \right)^{-1} s_{T,l} \]

where \( s_{T,l} \) represents the target spatial-temporal steering vector of the \( l \)th Doppler bin. Since \( K_s^{\text{opt}} \) is unknown in advance, it is usually estimated by the selected target-free homogeneous training samples around the CUT. However, due to the limitations of many practical factors in a complex clutter environment, it is impossible to obtain sufficient homogeneous training samples for effectively estimating the optimal IPN covariance matrix \( K_s^{\text{opt}} \). Thus, the STAP algorithm, which can sufficiently utilise the non-homogeneous training samples, is urgently needed for future airborne radar.

3 | PROPOSED METHOD

3.1 | Shortcoming analyses of the traditional PerEFA algorithm

To sufficiently utilise the training samples in a complex environment, based on the persymmetric structure of the IPN covariance matrix of CUT data the \( Q_s \) selected target-free training samples are used by the PerEFA algorithm [20] to estimate the IPN covariance matrix of CUT data, that is,

\[ R_{\text{cut}}^{\text{PerEFA}} = \frac{1}{4Q_s} \left( XX^H + X_sX_s^H + X_sX_t^H + X_sX_{st}^H \right) \]

where \( X = [x_1, x_2, \ldots, x_{Q_s}] \in \mathbb{C}^{D \times Q_s} \) denotes the original training sample matrix and \( x_q \in \mathbb{C}^{D \times 1} \) is the selected \( q \)th training sample. \( D = 3N \) is chosen as the reduced-dimension system DoF given in Ref. [20].

\[ X_s = (I_M \otimes J_s) (T_{\text{d}}^H X)^* \]
\[ X_t = (T_{\text{d}}^H (J_t \otimes I_N) X)^* \]
\[ X_{st} = T_{\text{d}}^H (J_t \otimes J_s) X)^* \]

are the spatial, temporal, and spatial-temporal permutation matrices, respectively, which can be seen in Ref. [20]. \( T_{\text{d}} \) is the reduced-dimension matrix of the traditional EFA algorithm [1].

In a complex environment, if the selected \( Q_s \) training samples are homogeneous with the clutter signal in the CUT, the number of i.d. training samples additionally increased by the PerEFA algorithm is \( 3Q_s \), which leads to the super performance of the PerEFA algorithm. However, the PerEFA algorithm can only be applied to the clutter suppression problem of airborne STAP radar with uniform sampling in the space-time dimension, ideally calibrated antenna array and an extremely stable local oscillator system. When the airborne STAP radar is not uniformly sampling in space-time dimension, the antenna array has an array error or the local oscillator system has phase noise, all the increased \( 3Q_s \) training samples will have different statistical properties with the clutter component in CUT data. Thus, the PerEFA algorithm has the perturbation problem of the cross-term matrix. To analyse the cross-term matrix’s problem, we only consider the influences of the array error and phase noise here.

The increased spatial training sample matrix \( \tilde{X}_s \), temporal training sample matrix \( \tilde{X}_t \), and the spatial-temporal training sample matrix \( \tilde{X}_{st} \) can be expressed as

\[ \tilde{X}_s = (I_M \otimes J_s) \left( T_{\text{d}}^H T_{\text{a}} X \right)^* \]
\[ \tilde{X}_t = \left( T_{\text{d}}^H (J_t \otimes I_N) T_{\text{a}} X \right)^* \]
\[ \tilde{X}_{st} = T_{\text{d}}^H (J_t \otimes J_s) T_{\text{a}} X \]

where \( T_{\text{a}} = \text{diag} (e_a) \otimes \text{diag} (e_a) \) is the space-time taper matrix, \( e_a \in \mathbb{C}^{N \times 1} \) and \( e_b \in \mathbb{C}^{M \times 1} \) are the array errors modelled in [23] and the phase noise introduced in [24], respectively. Then, the IPN covariance matrix of CUT data estimated by the PerEFA algorithm can be expressed as

\[ R_{\text{cut}}^{\text{PerEFA}} = \frac{1}{4Q_s} \left( \tilde{X} \tilde{X}^H + \tilde{X}_s \tilde{X}_s^H + \tilde{X}_t \tilde{X}_t^H + \tilde{X}_{st} \tilde{X}_{st}^H \right) \]

where

\[ \tilde{X} = T_{\text{d}}^H T_{\text{a}} X \]

\[ \tilde{X}_s = T_{\text{d}}^H (J_t \otimes J_s) T_{\text{a}} X_{Q_s} \]

Because of the complex environment confronted by the airborne STAP radar, the non-homogeneous samples are inevitably selected to estimate the IPN covariance matrix of CUT data by a certain non-homogeneous detector, such as the generalised inner product (GIP) criterion. Then, the practically \( q \)th training sample \( \tilde{x}_q \in \mathbb{C}^{D \times 1} \) can be decomposed into

\[ \tilde{x}_q = \tilde{x}_q^{\text{Ho}} + \tilde{x}_q^{\text{He}} + \tilde{n}_q \]

where \( \tilde{x}_q^{\text{Ho}} \) is the homogeneous component with the same statistical property as the clutter component in CUT data, \( \tilde{x}_q^{\text{He}} \) is the heterogeneous component with statistical properties that are different from the clutter component in CUT data, and \( \tilde{n}_q \) is the noise data. Then, the cross-term matrix problem of the PerEFA algorithm can be qualitatively illustrated by the following expression:

\[ R_{\text{cut}}^{\text{PerEFA}} = \frac{1}{4Q_s} \sum_{q=1}^{Q_s} \left( \tilde{x}_q^{\text{Ho}} + \tilde{x}_q^{\text{He}} + \tilde{n}_q \right) \left( \tilde{x}_q^{\text{Ho}} + \tilde{x}_q^{\text{He}} + \tilde{n}_q \right)^H + R_{\text{cross}}^{\text{PerEFA}} \]

where
where

\[
K_{\text{cross}}^{\text{PerFMA}} = \frac{1}{4Q_s} \sum_{q=1}^{Q_s} \left( \tilde{x}_q^{\text{Ho}} + \tilde{n}_q \right) \left( \tilde{x}_q^{\text{He}} \right)^T
+ \left( \tilde{x}_q^{\text{He}} \right)^T \tilde{x}_q^{\text{Ho}} + \tilde{x}_q^{\text{He}} + \tilde{n}_q \left( \tilde{x}_q^{\text{He}} \right)^T
+ \frac{1}{4Q_s} \left( \tilde{x}_1^{\text{He}} + \tilde{x}_2^{\text{He}} + \tilde{x}_3^{\text{He}} \right)^T
\]

is the cross-term matrix of the PerFMA algorithm. In a complex environment, \(K_{\text{cross}}^{\text{PerFMA}}\) will seriously damage the structure of matrix \(K_{\text{cross}}^{\text{PerFMA}}\), thus incurring a high residual clutter in the PerFMA algorithm.

### 3.2 Key theory of the GERCM algorithm

For the sake of convenient description, we first introduce some definitions and analyses, which can be used to derive the proposed GERCM algorithm.

Considering the array error and phase noise, the practically received clutter data come from the \(l\)th and \((l+1)\)th range cells, respectively, can be expressed as

\[
\tilde{c}_l = \sum_{u=1}^{N_u} \sum_{k=1}^{N_c} \tilde{a}_{k, l, u} T_{\text{ua}} \mathbf{v}_{k, l, u} = T_{\text{ua}} V_l \tilde{a}_l
\]

\[
\tilde{c}_{l+1} = \sum_{u=1}^{N_u} \sum_{k=1}^{N_c} \tilde{a}_{k, l+1, u} T_{\text{ua}} \mathbf{v}_{k, l+1, u} = T_{\text{ua}} V_{l+1} \tilde{a}_{l+1}
\]

where

\[
V_l = \left[ \mathbf{v}_{1, l, 1}, \ldots, \mathbf{v}_{N_c, l, 1}, \ldots, \mathbf{v}_{1, l, N_u}, \ldots, \mathbf{v}_{N_c, l, N_u} \right]
\]

\[
V_{l+1} = \left[ \mathbf{v}_{1, l+1, 1}, \ldots, \mathbf{v}_{N_c, l+1, 1}, \ldots, \mathbf{v}_{1, l+1, N_u}, \ldots, \mathbf{v}_{N_c, l+1, N_u} \right]
\]

\[
\tilde{a}_l = \begin{bmatrix} \tilde{a}_{1, l, 1}, \ldots, \tilde{a}_{N_c, l, 1}, \ldots, \tilde{a}_{1, l, N_u}, \ldots, \tilde{a}_{N_c, l, N_u} \end{bmatrix}^T
\]

\[
\tilde{a}_{l+1} = \begin{bmatrix} \tilde{a}_{1, l+1, 1}, \ldots, \tilde{a}_{N_c, l+1, 1}, \ldots, \tilde{a}_{1, l+1, N_u}, \ldots, \tilde{a}_{N_c, l+1, N_u} \end{bmatrix}^T
\]

As there is a correlation between \(\mathbf{v}_{k, l, u}\) and \(\mathbf{v}_{k, l+1, u}\) [19] and \(T_{\text{ua}}\) is not related with the range cells, the practical clutter subspace of the \(l\)th range cell, \(\Phi_l^c\), is approximately equal to the clutter subspace of the \((l+1)\)th range cell, \(\Phi_{l+1}^c\), that is

\[
\Phi_{l+1}^c = \text{Span}(T_{\text{ua}} V_{l+1}) \approx \text{Span}(T_{\text{ua}} V_l) = \Phi_l^c
\]

As \(T_{\text{d}}\) is a full column rank matrix in the context of conventional EFA algorithm, we have

\[
\tilde{\Theta}_{l+1}^c = \text{Span}(T_{\text{d}}^H T_{\text{ua}} V_{l+1}) \approx \text{Span}(T_{\text{d}}^H T_{\text{ua}} V_l) = \tilde{\Theta}_l^c
\]

where \(\tilde{\Theta}_l^c\) and \(\tilde{\Theta}_{l+1}^c\) are the reduced-dimension clutter subspaces of the \(l\)th and \((l+1)\)th range cells, respectively. Define \( \tilde{C}_q = [\tilde{C}_{c, q}, \tilde{C}_{n, q}] (q = 1, 2, \ldots, Q_s)\) as the basis matrix of clutter plus noise data in the training sample \(\tilde{x}_q (q = 1, 2, \ldots, Q_s)\), where \(\tilde{C}_{c, q}\) and \(\tilde{C}_{n, q}\) are the corresponding basis matrices of the clutter and noise subspaces, respectively. As the heterogeneous component \(\tilde{x}_q^{\text{He}}\) also falls in the clutter subspace of \(\tilde{x}_q\), \(\tilde{x}_q\) can be represented as

\[
\tilde{x}_q = \tilde{x}_q^{\text{Ho}} + \tilde{x}_q^{\text{He}} + \tilde{n}_q = \tilde{C}_{c, q} \tilde{z}_q^{\text{Ho}} + \tilde{C}_{c, q} \tilde{z}_q^{\text{He}} + \tilde{C}_{n, q} \tilde{z}_q^{\text{He}}
\]

where \(\tilde{z}_q^{\text{Ho}} \in \mathbb{C}^{q_1 \times 1}, \tilde{z}_q^{\text{He}} \in \mathbb{C}^{q_2 \times 1}\), and \(\tilde{z}_q^{\text{He}} \in \mathbb{C}^{(\nu_2 - q_2) \times 1}\) are the representation coefficients of the \(\tilde{x}_q^{\text{Ho}}, \tilde{x}_q^{\text{He}},\) and \(\tilde{n}_q\) components, respectively. \(\nu_2\) denotes the effective clutter rank [1]. As the practical radar antenna has an array error and the local oscillator system has phase noise, the persymmetric structure of the IPN covariance matrix of CUT data is not on hold. Thus, the training sample matrices \(\tilde{X}_l, \tilde{X}_s, \) and \(\tilde{X}_a\) cannot be used to estimate the IPN covariance matrix of CUT data. Keeping these facts in mind, the initially estimated IPN covariance matrix of the reduced-dimension CUT data \(\tilde{x}_{\text{cut}} \in \mathbb{C}^{D \times 1}\) by the SCM method, that is, \(\tilde{K}_{\text{cut}}^{\text{SCM}}\), can be represented by the underlying form

\[
\tilde{K}_{\text{cut}}^{\text{SCM}} = \frac{1}{Q_s} \sum_{q=1}^{Q_s} \left( \tilde{x}_q^{\text{Ho}} + \tilde{x}_q^{\text{He}} + \tilde{n}_q \right) \left( \tilde{x}_q^{\text{Ho}} + \tilde{x}_q^{\text{He}} + \tilde{n}_q \right)^T
\]

\[
= \frac{1}{Q_s} \sum_{q=1}^{Q_s} \left( \tilde{C}_{c, q} \tilde{z}_q^{\text{Ho}} \right) \left( \tilde{C}_{c, q} \tilde{z}_q^{\text{Ho}} \right)^T
+ \frac{1}{Q_s} \sum_{q=1}^{Q_s} \left( \tilde{C}_{c, q} \tilde{z}_q^{\text{He}} \right) \left( \tilde{C}_{c, q} \tilde{z}_q^{\text{He}} \right)^T
+ \left( \tilde{C}_{n, q} \tilde{z}_q^{\text{He}} \right) \left( \tilde{C}_{n, q} \tilde{z}_q^{\text{He}} \right)^T
\]

where

\[
\tilde{z}_q^{\text{Ho}} = \begin{bmatrix} \tilde{z}_q^{\text{Ho}} \end{bmatrix}
\]

\[
\tilde{z}_q^{\text{He}} = \begin{bmatrix} \tilde{z}_q^{\text{He}} \end{bmatrix}
\]

\[
\tilde{z}_q^{\text{He}} = \begin{bmatrix} \tilde{z}_q^{\text{He}} \end{bmatrix}
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\[
\tilde{z}_q^{\text{He}} = \begin{bmatrix} \tilde{z}_q^{\text{He}} \end{bmatrix}
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\tilde{z}_q^{\text{He}} = \begin{bmatrix} \tilde{z}_q^{\text{He}} \end{bmatrix}
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\tilde{z}_q^{\text{He}} = \begin{bmatrix} \tilde{z}_q^{\text{He}} \end{bmatrix}
\]

\[
\tilde{z}_q^{\text{He}} = \begin{bmatrix} \tilde{z}_q^{\text{He}} \end{bmatrix}
\]
Let $\tilde{C}$ represent the basis matrix of the clutter plus noise data in $\tilde{x}_{\text{cut}}$. Additionally, we define $\tilde{Z}^H = \frac{1}{Q^2} \sum_{q=1}^{Q^2} (\tilde{z}_q^1) (\tilde{z}_q^1)^H$ and $\tilde{Z}^{He} = \frac{1}{Q^2} \sum_{q=1}^{Q^2} (\tilde{z}_q^1) (\tilde{z}_q^2)^H + (\tilde{z}_q^2) (\tilde{z}_q^2)^H$ as the two auxiliary matrices. By inspecting Equation (17), we note that the clutter subspaces in the selected target-free training samples are approximately identical with that in $x_{\text{cut}}$. Meanwhile, the noise subspaces in the selected training samples are the same as that in $\tilde{x}_{\text{cut}}$. Thus, Equation (19) can be expressed as

$$
\tilde{R}_{\text{cut}}^{\text{SCM}} = \tilde{C} \tilde{Z}^H \tilde{C}^H + \tilde{C} \tilde{Z}^{He} \tilde{C}^H
$$

$$
= \tilde{C} \left( \tilde{Z}^H + \tilde{Z}^{He} \right) \tilde{C}^H
$$

$$
= \tilde{C} \tilde{Z}^H \tilde{C}^H + \tilde{R}_{\text{cross}}^{\text{SCM}}
$$

(21)

where $\tilde{Z} = \tilde{Z}^H + \tilde{Z}^{He}$ and $\tilde{R}_{\text{cross}}^{\text{SCM}} = \tilde{C} \tilde{Z}^{He} \tilde{C}^H$ denote the cross-term matrix of the traditional SCM method. According to the expression shown in Equation (21), since the positive semidefinite matrix $\tilde{R}_{\text{cut}}^{\text{SCM}}$ has a unique eigenvalue decomposition [25], $\tilde{C}$ and $\tilde{Z}$ will be equal to the eigenvector and eigenvalues matrices, respectively, of $\tilde{R}_{\text{cut}}^{\text{SCM}}$. By checking $\tilde{R}_{\text{cross}}^{\text{SCM}} = \tilde{C} \tilde{Z}^{He} \tilde{C}^H$, we find that the cross-term matrix $\tilde{R}_{\text{cross}}^{\text{SCM}}$ has the same eigenvector matrix as that of $\tilde{R}_{\text{cut}}^{\text{SCM}}$. Thus, if an unknown indefinite matrix $\Delta \tilde{R}_{\text{cut}}$ with the same eigenvector matrix as that of $\tilde{R}_{\text{cut}}^{\text{SCM}}$ is used to compensate the cross-term matrix in $\tilde{R}_{\text{cut}}^{\text{SCM}}$, then the sum of $\tilde{R}_{\text{cut}}^{\text{SCM}}$ plus $\Delta \tilde{R}_{\text{cut}}$ can be expected to be close to the optimal IPN covariance matrix of CUT data $\tilde{x}_{\text{cut}}$, that is,

$$
\tilde{R}_{\text{cut}}^{\text{SCM}} + \Delta \tilde{R}_{\text{cut}} = \tilde{R}_{\text{cut}}^{\text{SCM}} + \tilde{C} \Delta \tilde{Z} \tilde{C}^H \rightarrow \tilde{R}_{\text{cut}}^{\text{Opt}}
$$

(22)

where $\Delta \tilde{R}_{\text{cut}} = \tilde{C} \Delta \tilde{Z} \tilde{C}^H$ is the eigenvalue decomposition of $\Delta \tilde{R}_{\text{cut}}$ and $\Delta \tilde{Z}$ is the resultant eigenvalue matrix. Substitute $\tilde{R}_{\text{cut}}^{\text{SCM}} \approx \tilde{C} \tilde{Z} \tilde{C}^H$ given in Equation (21) into Equation (22). Then, Equation (22) can be rewritten as

$$
\tilde{C} \left( \tilde{Z} + \Delta \tilde{Z} \right) \tilde{C}^H \rightarrow \tilde{R}_{\text{cut}}^{\text{Opt}}
$$

(23)

As $\tilde{Z}$ and $\Delta \tilde{Z}$ are diagonal matrices, we set $\tilde{Z} = \text{diag}([\tilde{z}_i, \tilde{z}_2, \ldots, \tilde{z}_D])$ and $\Delta \tilde{Z} = \text{diag}([\Delta \tilde{z}_1, \Delta \tilde{z}_2, \ldots, \Delta \tilde{z}_D])$, where $\tilde{z}_i \geq 0 (i = 1, 2, \ldots, D)$ and $\Delta \tilde{z}_i (i = 1, 2, \ldots, D)$ are the $i$th diagonal entry of matrix $\tilde{Z}$ and $\Delta \tilde{Z}$, respectively. Based on the aforementioned definitions and analyses, a generalised eigenvalue reweighting covariance matrix estimation algorithm is proposed to estimate the optimal IPN covariance matrix of CUT data $\tilde{x}_{\text{cut}} \in \mathbb{R}^{D \times 1}$, that is,

$$
\tilde{R}_{\text{cut}}^{\text{GERCM}} = \tilde{C} \left( \tilde{Z} + \Delta \tilde{Z} \right) \tilde{C}^H
$$

$$
= \tilde{C} \left\{ \text{diag} \left( [\tilde{z}_1 + \Delta \tilde{z}_1, \tilde{z}_2 + \Delta \tilde{z}_2, \ldots, \tilde{z}_D + \Delta \tilde{z}_D] \right) \right\} \tilde{C}^H
$$

$$
= \tilde{C} \left\{ \text{diag} \left( [\tilde{z}_1 \times \tilde{u}_1, \tilde{z}_2 \times \tilde{u}_2, \ldots, \tilde{z}_D \times \tilde{u}_D] \right) \right\} \tilde{C}^H
$$

$$
= \tilde{C} \tilde{Z} \text{diag} (\tilde{u}) \tilde{C}^H
$$

(24)

where $\tilde{u} = [\tilde{u}_1, \tilde{u}_2, \ldots, \tilde{u}_D]$ is the positive eigenvalue reweighting vector which needs to be estimated, and $\tilde{u}_i (i = 1, 2, \ldots, D)$ is the $i$th entry of $\tilde{u}$. To estimate $\tilde{u}$, the underlying covariance matching estimation criterion is introduced as [26]

$$
f = \left\| \left( \tilde{R}_{\text{cut}}^{\text{GERCM}} \right)^{-1/2} (\tilde{x}_{\text{cut}} \tilde{x}_{\text{cut}}^H - \tilde{R}_{\text{cut}}^{\text{GERCM}}) \right\|_F^2
$$

(25)

Substitute the expression of $\tilde{R}_{\text{cut}}^{\text{GERCM}}$ into Equation (25). Then, Equation (25) can be recast as

$$
f = -2 \left\| \tilde{x}_{\text{cut}} \right\|^2_2 + \left\| \tilde{x}_{\text{cut}} \right\|^2_2 \tilde{R}_{\text{cut}}^{\text{GERCM}}^{-1} \tilde{x}_{\text{cut}}^H
$$

$$
+ \sum_{i=1}^{D} \tilde{u}_i \tilde{z}_i \left\| \tilde{c}_i \right\|^2_2
$$

(26)

where $\tilde{c}_i$ is the $i$th eigenvector in $\tilde{C}$. Then, we can estimate the eigenvalue reweighting vector $\tilde{u}$ by the undermentioned optimisation problem

$$
\min \tilde{x}_{\text{cut}}^H \tilde{R}_{\text{cut}}^{\text{GERCM}}^{-1} \tilde{x}_{\text{cut}}
$$

s.t. \begin{align*}
\sum_{i=1}^{D} \text{Re}(\tilde{u}_i) \tilde{z}_i &\geq \sum_{i=1}^{D} \tilde{u}_i \tilde{z}_i \\
\sum_{i=p+1}^{D} \text{Re}(\tilde{u}_i) \tilde{z}_i &\geq \sum_{i=p+1}^{D} \tilde{u}_i \tilde{z}_i
\end{align*}

(27)

where the first inequality constrains that the modified clutter-to-noise ratio (CNR) is greater than the unmodified CNR [27, 28], and the second inequality constrains that the entries of the eigenvalue reweighting vector $\tilde{u}$ are positive real numbers. As the objective function in Equation (27) is convex [29] and the feasible sets of Equation (27) are convex sets, Equation (27) is a convex optimisation problem, which can be iteratively solved by the canonical primal-dual interior-point algorithm [29]. Nevertheless, we find that $\tilde{u}$ is implied in the cost function of Equation (27). This will incur some inconveniences in solving Equation (27) with the help of the primal-dual interior-point algorithm. Additionally, the clutter rank $p_c$ is difficult to determine for the airborne STAP radar working in the complex
environment [30]. To obtain the close-form solution of \( \tilde{u} \), an alternative convex problem is redesigned to estimate the eigenvalue reweighting vector \( \tilde{u} \).

First, to avoid the matrix inverse and operations of taking the real and imaginary parts in Equation (27), we replace the cost function in Equation (27) with the following simplified form:

\[
\left\| \mathcal{G} - \tilde{\mathcal{H}} \text{diag}(\tilde{u}) \tilde{\mathcal{H}}^T \right\|_F^2 \quad \text{(28)}
\]

where

\[
\tilde{\mathcal{G}} = \text{Re} \left( \tilde{x}^* \tilde{x}^T \right) + \text{Im} \left( \tilde{x}^* \tilde{x}^T \right) \quad \text{(29a)}
\]

\[
\tilde{\mathcal{H}} = \text{Re} \left( \tilde{C} \tilde{Q} \right) + \text{Im} \left( \tilde{C} \tilde{Q} \right) \quad \text{(29b)}
\]

\[
\tilde{Q} = \text{diag} \left( \tilde{z}_1^{0.5}, \tilde{z}_2^{0.5}, \ldots, \tilde{z}_{D'}^{0.5} \right) \quad \text{(29c)}
\]

After the cost function in Equation (27) is replaced by Equation (28), Equation (27) can be rewritten as

\[
\min_{\tilde{u}} \left\| \mathcal{G} - \tilde{\mathcal{H}} \text{diag}(\tilde{u}) \tilde{\mathcal{H}}^T \right\|_F^2 \\
\text{s.t.} \quad \sum_{i=1}^{\rho_c} \tilde{u}_i \tilde{z}_i \geq \frac{\sum_{i=1}^{D'} \tilde{z}_i}{\sum_{i=p_t+1}^{D'} \tilde{z}_i} \\
\tilde{u}_i > 0, \quad i = 1, 2, \ldots, D' \quad \text{(30)}
\]

Next, to eliminate the obstacle of determining the reduced-dimension clutter rank \( \rho_c \) of the IPN covariance matrix of CUT data in the complex environment, the first inequality constraint in Equation (30) can be replaced by the underlying two inequality constraints, respectively,

\[
\sum_{i=1}^{D'} \tilde{u}_i \tilde{z}_i \geq \left\| \tilde{\mathcal{X}}_{\text{cut}} \right\|^2_2 \left( \sum_{i=1}^{D'} \tilde{z}_i \right) \quad \text{(33a)}
\]

\[
\sum_{i=1}^{D'} \tilde{u}_i \tilde{z}_i \leq \left\| \tilde{\mathcal{X}}_{\text{cut}} \right\|^2_2 \left( \sum_{i=1}^{D'} \tilde{z}_i \right)^2 \quad \text{(33b)}
\]

where Equations (33a) and (33b) are mainly used to guarantee the estimated accuracy of the IPN covariance matrix of CUT data and prevent the target self-nulling of the proposed GERCM algorithm. Define \( \tilde{r}_{\text{de}} \) as a column vector composed of the diagonal entries in \( \tilde{Z} \) with the descend order and \( \tilde{C}_{\text{de}} \) as the corresponding eigenvector matrix. Then, Equation (30) can be recast as the equivalent form

\[
\min_{\tilde{u}} \left\| \tilde{\mathcal{X}}_{\text{cut}}^{\Lambda} - \tilde{U}_{\text{de}}^{\Lambda} \tilde{u} \right\|^2_2 \\
\text{s.t.} \quad \tilde{u}^T \tilde{r}_{\text{de}} \geq \left\| \tilde{\mathcal{X}}_{\text{cut}} \right\|^2_2 \left( \tilde{I}_D^T \tilde{r}_{\text{de}} \right) \quad \text{(34)}
\]

where

\[
\tilde{\mathcal{X}}_{\text{cut}}^{\Lambda} = \text{Re} \left( \tilde{x}_{\text{cut}}^{\Lambda} \otimes \tilde{\mathcal{X}}_{\text{cut}} \right) + \text{Im} \left( \tilde{x}_{\text{cut}}^{\Lambda} \otimes \tilde{\mathcal{X}}_{\text{cut}} \right) \quad \text{(35a)}
\]

\[
\tilde{C}_{\text{de}}^{\Lambda} = \left( \tilde{C}_{\text{de}}^{\Lambda} \otimes \tilde{C}_{\text{de}} \right) \text{diag} \left( \tilde{r}_{\text{de}} \right) \quad \text{(35b)}
\]

\[
\tilde{U}_{\text{de}}^{\Lambda} = \text{Re} \left( \tilde{C}_{\text{de}}^{\Lambda} \right) + \text{Im} \left( \tilde{C}_{\text{de}}^{\Lambda} \right) \quad \text{(35c)}
\]

with the above information, the augmented Lagrange function of Equation (34) is given by
\[ L(\hat{u}, \alpha_d, \beta_d, t_d) = \tilde{u}^T \tilde{U}_{de} \tilde{u} - \tilde{u}^T \left( 2\tilde{U}_{de} \tilde{x}_{\text{cut}} + \alpha_d \tilde{r}_{de} - \beta_d \tilde{r}_{de} + t_d \right) + \alpha_d \|\tilde{x}_{\text{cut}}\|^2_2 \left( I_D^T \tilde{r}_{de} \right)^2 + \|\tilde{x}_{\text{cut}}\|^2_2 \] (36)

where \( \tilde{U}_{de} = \tilde{U}_{de}^T \tilde{U}_{de} \) are the Lagrange multipliers. Define \( g_d = [\alpha_d; \beta_d; t_d] \). Then, the augmented Lagrange dual function of Equation (34) can be expressed as

\[ D(\alpha_d, \beta_d, t_d) = \inf_{\hat{u} \in \text{dom}(\hat{u})} L(\hat{u}, \alpha_d, \beta_d, t_d) = \inf_{\hat{u} \in \text{dom}(\hat{u})} L(\hat{u}, g_d) = D(g_d) = g_d^T \Pi_d g_d + \omega_d^T g_d + \chi_d \]

where

\[ \Pi_d = \frac{1}{4} \begin{bmatrix} -\tilde{r}_{de}(\tilde{U}_{de}^-1 \tilde{r}_{de}) & \tilde{r}_{de}(\tilde{U}_{de}^-1 \tilde{r}_{de}) & -\tilde{r}_{de}(\tilde{U}_{de}^-1 \tilde{r}_{de}) \\ \tilde{r}_{de}(\tilde{U}_{de}^-1 \tilde{r}_{de}) & \tilde{r}_{de}(\tilde{U}_{de}^-1 \tilde{r}_{de}) & -\tilde{r}_{de}(\tilde{U}_{de}^-1 \tilde{r}_{de}) \\ -\tilde{r}_{de}(\tilde{U}_{de}^-1 \tilde{r}_{de}) & \tilde{r}_{de}(\tilde{U}_{de}^-1 \tilde{r}_{de}) & (\tilde{U}_{de}^-1 \tilde{r}_{de}) & (\tilde{U}_{de}^-1 \tilde{r}_{de}) \end{bmatrix} \]

As the inequality constraints in Equation (34) are affine, the Slater criterion is satisfied, which means that Equation (34) has strong duality [29]. This indicates that the eigenvalue reweighting vector \( \hat{u} \) can be estimated by the underlying problem

\[ \min_{\hat{u}} \tilde{u}^T \tilde{U}_{de} \tilde{u} - \hat{u}^T \left( 2\tilde{U}_{de} \tilde{x}_{\text{cut}} + \alpha_d^{\text{Opt}} \tilde{r}_{de} - \beta_d^{\text{Opt}} \tilde{r}_{de} + t_d^{\text{Opt}} \right) + \alpha_d^{\text{Opt}} \|\tilde{x}_{\text{cut}}\|^2_2 \left( I_D^T \tilde{r}_{de} \right)^2 + \|\tilde{x}_{\text{cut}}\|^2_2 \]

Solving Equation (41) by the minimisation method of unconstrained least squares problem, the estimated \( \hat{u} \) can be represented by

\[ \chi_d = (\tilde{x}_{\text{cut}}^A)^T \begin{bmatrix} I_{D^2} & -U_{de}^-1 & U_{de}^-1 \end{bmatrix} \] (38c)
\[
\tilde{u}^\Delta = \left( \tilde{U}_{dc}^\Lambda \right)^{-1} \left[ \left( \tilde{U}_{dc}^\Lambda \right)^T \tilde{x}_{\text{cut}} + 0.5(\sigma_{u,d}^{\text{opt}} - \beta^{\text{opt}}_{d}) \tilde{r}_{dc} + 0.5 \tilde{r}^{\text{opt}}_{d} \right]
\]

(42)

After the eigenvector matrix \( \tilde{C} \), eigenvalue matrix \( \tilde{Z} \), and the eigenvector reweighting vector \( \tilde{u}^\Delta \) are obtained, the filtered output of the CUT data \( \tilde{x}_{\text{cut}} \) at the \( l \)th Doppler bin, that is, \( \tilde{Y}_{\text{cut}}(l) \), can be written as

\[
\tilde{Y}_{\text{cut}}(l) = \left[ \hat{w}_{\text{cut}}^{\text{GERCM}}(l) \right]^H \tilde{x}_{\text{cut}}
\]

\[
= \frac{s_{T,l}^H \left( \hat{R}_{\text{cut}}^{\text{GERCM}} \right)^{-1} \tilde{x}_{\text{cut}}}{s_{T,l}^H \left( \hat{R}_{\text{cut}}^{\text{GERCM}} \right)^{-1} s_{T,l}}
\]

\[
= \frac{s_{T,l}^H \left( \tilde{C} \tilde{Z} \text{diag}(\tilde{u}^\Delta) \tilde{C}^H \right)^{-1} \tilde{x}_{\text{cut}}}{s_{T,l}^H \left( \tilde{C} \tilde{Z} \text{diag}(\tilde{u}^\Delta) \tilde{C}^H \right)^{-1} s_{T,l}}
\]

(43)

where \( \hat{w}_{\text{cut}}^{\text{GERCM}}(l) \) is the STAP weight vector of the \( l \)th Doppler bin calculated by the proposed GERCM algorithm.

### 3.3 Advantage analysis of the GERCM algorithm

Compared with the existing PerEFA algorithm, a relatively accurate IPN covariance matrix of CUT data can be estimated by the proposed GERCM algorithm by sufficiently utilising the non-homogeneous training samples, which leads to the better performance of airborne STAP radar working in complex environments. As the proposed GERCM algorithm is independent of the priori information on the array structure and antenna configuration, theoretically, the proposed GERCM algorithm can be applied to the clutter suppression problem of airborne radar with arbitrary array structure and antenna configuration. Since the reformulated IPN covariance matrix of CUT data based on the component decompositions of selected target-free training samples and the assumption of approximately equal subspace has nothing to do with the preprocessing of CUT data, the proposed GERCM algorithm derived under the framework of element-space post-Doppler can be generalised to other frameworks, such as beam-space post-Doppler. In addition, as the proposed GERCM algorithm is a sample-dependent statistical STAP algorithm, it also can suppress the range-ambiguous clutter.

### 3.4 Complexity analysis of the GERCM algorithm

To analyse the computational complexity of the proposed GERCM algorithm, the computational complexity [31] is measured in terms of the number of complex multiplications in this work. For the reduced-dimension system DoF \( D' \), the complexity of the eigenvalue decomposition is \( O(23D'^3) \); the complexity of obtaining the estimated eigenvector reweighting vector \( \tilde{u}^\Delta \) is \( O(23D'^3 + 8(D'^2 + 2MND')) \); and the complexity of the weight vector calculation for clutter suppression is \( O(23D'^3 + 8D'^3) \). Thus, the total complexity of the proposed GERCM algorithm is \( O(69D'^3 + 16(D'^2 + MND')) \).

Implementation steps of the proposed GERCM algorithm are summarised in Table 1.

### 4 NUMERICAL EXPERIMENTS

Based on the MCARM data from flight 5, acquisition 151, in this section we assess the algorithm performance of the proposed GERCM algorithm by comparing with other existing STAP algorithms. The main parameters of the MCARM data are listed in Table 2. As shown in Figure 2, this simulation scene contains rich terrains (such as river, lake, mountain, island, and so on) and different land covers (such as forest, road, man-made building, and so on) [32]. These factors will result in the heterogeneity of the echo data received by the airborne STAP radar. As we can see from the processing result of conventional pulse-Doppler (PD) given in Figure 3, the echo data coming from different range cells show the obvious power heterogeneity under the influences of antenna pattern, slant range, complex terrains and different land covers.

Clearly, simulation results based on the measured MCARM data can sufficiently illustrate the clutter suppression performance of the proposed GERCM algorithm in complex environments. Here, the returns from only the top row of the MCARM array, consisting of 11 chosen channels from 22 available channels, are used for the following numerical experiments. Meanwhile, 66 target-free training samples are selected from 72 initial training samples by the sliding

| TABLE 1 GERCM |
|-----------------|
| **Input:** Selected the \( \text{Q}_{s} \text{ s} \) Target-free Training Samples |
| **Output:** IPN Covariance Matrix \( \hat{R}_{\text{cut}}^{\text{GERCM}} \) |
| **Step 1:** Estimating the initial IPN covariance matrix of CUT data by the SCM method with the selected target-free training samples to obtain the matrix \( \hat{R}_{\text{cut}}^{\text{GERCM}} \) |
| **Step 2:** Taking the eigenvalue decomposition to matrix \( \hat{R}_{\text{cut}}^{\text{GERCM}} \) for obtaining the eigenvector matrix \( \tilde{C} \) and eigenvalue matrix \( \tilde{Z} \) |
| **Step 3:** Calculating the estimated eigenvalue reweighting vector \( \tilde{u}^\Delta \) via Equation (42) |
| **Step 4:** Reconstructing the final IPN covariance matrix of CUT data with the obtained \( \tilde{C} \), \( \tilde{Z} \), and \( \tilde{u}^\Delta \) components by (24) |

**Abbreviations:** CUT, cell under test; GERCM, generalised eigenvalue reweighting covariance matrix; IPN, interference plus noise; SCM, sample covariance matrix.
### TABLE 2 Main parameters of MCARM data

| Parameters                        | Magnitude |
|----------------------------------|-----------|
| Azimuth channel number           | 11        |
| Elevation channel number         | 2         |
| Number of pulses per CPI         | 128       |
| Receiver bandwidth               | 0.8 MHz   |
| Radar wavelength                 | 0.242 m   |
| Platform altitude                | 3.59 km   |
| Platform velocity                | 125.3 m/s |
| Number of valid range cells      | 557       |
| Installation angle               | 0°        |
| PRF                              | 1984 Hz   |

Abbreviations: CPI, coherent processing interval; MCARM, multi-channel airborne radar measurement; PRF, pulse repetition frequency.

### Figure 2 Terrain map of simulation scene

### Figure 3 Result of conventional pulse-Doppler processing

Window GIP method, discarding two guard cells to prevent target self-nulling in all listed STAP algorithms.

To demonstrate the effectiveness of the proposed GERCM algorithm, the following comparative simulations including range-Doppler processing, averaged clutter power residual, target protection, and detection performance are taken into account to conduct the corresponding numerical experiments. As the radar antenna array inevitably has an array error and the optimal IPN covariance matrix of CUT data is unknown in advance, we do not conduct the comparative simulations individually in the case of an array error and plot the improvement factor curves in the underlying part of this section.

#### 4.1 Comparison simulations of range-Doppler processing

To demonstrate the clutter suppression performance of the proposed GERCM algorithm, we plot the range-Doppler processing results of the listed six STAP algorithms in Figure 4, which are the extended factored approach (EFA) [1], generalised eigenvalue reweighting covariance matrix extended factored approach (GERCM-EFA), PRI-staggered (PRIS) [1], generalised eigenvalue reweighting covariance matrix PRI-staggered (GERCM-PRIS) approach, persymmetric extended factored approach (PerEFA) [20], and the generalised eigenvalue reweighting covariance matrix persymmetric extended factored approach (GERCM-PerEFA). Here, 3 Doppler bins are chosen for the EFA and PRIS algorithms. As the phase noise can be ignored in a CPI [24], with the increased 66 target-free training samples obtained by the temporal transform, a total of 132 training samples are used in the context of PerEFA algorithm. Compared with the processing results shown in Figures 4(a), 4(c), and 4(e), we can see from Figures 4(b), 4(d), and 4(f) that the clear
zones in both the mainlobe and sidelobe Doppler regions are largely extended by the GERC-M-EFA, GERC-M-PRIS, and GERC-M-PerEFA algorithms. Due to the fact that the clutter power residuals of STAP algorithms are directly related to the estimated accuracy of the IPN covariance matrix of CUT data, Figures 4(b), 4(d), and 4(f) show that the proposed GERC-M algorithm can enhance the estimated accuracy of the IPN covariance matrix of the conventional EFA, PRIS, and PerEFA algorithms, resulting in better clutter suppression performance of airborne STAP radar.

4.2 Comparison simulations of averaged clutter power residual

To compare the averaged clutter suppression performance of the STAP algorithms listed in Figure 4, utilising the residual data taken from 1th to 557th range cells and 1th to 128th Doppler bins processed by these aforementioned algorithms, the averaged clutter power residual (ACPR) curves obtained by the average of range-dimension the average of Doppler-dimension are depicted in Figure 5 and Figure 6, respectively. Figure 5 shows that the GERC-M-EFA, GERC-M-PRIS, and GERC-M-PerEFA algorithms can significantly outperform the EFA, PRIS, and PerEFA algorithms in the mainlobe and sidelobe Doppler regions, which implies that the proposed GERC-M algorithm can effectively reduce the false alarm caused by the Doppler-spread ground clutter. In addition, Figure 6 shows that the GERC-M-EFA, GERC-M-PRIS, and GERC-M-PerEFA algorithms can achieve lower ACPR values than those of the EFA, PRIS, and PerEFA algorithms at most range cells. This means that the proposed GERC-M algorithm can effectively reduce the false alarm caused by the severe ground clutter coming from different range cells.

4.3 Comparison simulations of target protection

To examine the target protection performance of the proposed GERC-M algorithm, 22 moving targets are injected in the 200th and 550th range cells, respectively. In Figure 7, the target power values are plotted, where 'true value' denotes the original power values of injected moving targets. Figures 7a and 7b show that the target power values obtained from the filter-processing of the GERC-M-EFA, GERC-M-PRIS, and GERC-M-PerEFA algorithms are almost coincided with the true power values at the shown Doppler bins, which means that the proposed GERC-M algorithm can effectively protect the short-range and long-range moving targets with different size and Doppler frequencies.
4.4 | Comparison simulations of detection performance

To evaluate the detection performance of the proposed GERCM algorithm, as presented in Figure 8, we plot the probability of detection (Pd) versus input signal-to-noise ratio (SNR) curves, which are obtained by applying the cell average constant false alarm rate (CA-CFAR) detector to the filtered data processed by the EFA [1], PRIS [1], AIWCM [18], RAPR [19], and PerEFA [20] algorithms.

Here, assuming that the simulated targets are in boresight and are uniformly distributed from 101th to 300th range cells, we consider two groups of CUTs: a) 1th - 45th Doppler bins which simulate the relatively fast targets moving away from airborne radar and b) 84th - 128th Doppler bins which simulate the relatively fast targets moving towards airborne radar. Each group is conducted by the 9000 Monte Carlo simulations. In these simulations, the probability of false alarm (Pfa) is set to be \(10^{-6}\). Figures 8a and 8b show that the GERCM-EFA, GERCM-PRIS, and
GERCM-PerEFA algorithms have apparent performance improvements compared to those of the EFA, PRIS, PerEFA, AIWCM, and RAPR algorithms, which means that the proposed GERC M algorithm can help airborne STAP radar effectively detect relatively fast moving targets' fall in these regions.

To further demonstrate the detection performance of the proposed GERC M algorithm, we plot receiver operating characteristic curves in Figure 9. Here, the simulated targets are injected from 101th to 300th range cells, and the directions of arrival for all simulated targets are at the centre of the mainlobe. We also consider the two groups of CUTs: a) 46th - 63th Doppler bins which simulate the relatively slow targets moving away from airborne radar and b) 66th - 83th Doppler bins which simulate the relatively slow targets moving towards airborne radar. Each group is conducted by the 9000 Monte Carlo simulations. In these simulations, the SNRs of injected moving targets are set to be $-36$ dB. Figures 9a and 9b show that the GERC M-EFA, GERC M-PRIS, and GERC M-PerEFA algorithms have higher Pd values than those of the EFA, PRIS, PerEFA, AIWCM, and RAPR algorithms at the most shown Pfa values, which indicates that the proposed GERC M algorithm can largely improve the MDV performance of relatively slow moving targets that are slightly deviated from the main beam of the radar.
In this work, a novel generalised eigenvalue reweighting covariance matrix estimation algorithm for the clutter mitigation of airborne STAP radar working in complex environments is proposed. The proposed GERCM algorithm could obtain a relatively accurate IPN covariance matrix of CUT data by sufficiently utilising non-homogeneous training samples in a complex environment and effectively protect the moving targets with different size and Doppler frequencies in the CUT data, which could solve the clutter suppression problem of airborne radar with arbitrary array structure and antenna configuration. Although the proposed GERCM algorithm is derived in the context of element-space post-Doppler, theoretically, it can be generalised to other frameworks, such as beam-space post-Doppler. In addition, as it is a sample-dependent statistical STAP algorithm, it also can effectively eliminate range-ambiguous clutter. Simulation results and performance analyses based on the high-fidelity MCARM data demonstrated that the proposed GERCM algorithm could effectively suppress heterogeneous clutter and greatly improve the detection performance of moving targets in complex environments.

5 | CONCLUSION

![Figure 8](image.png)

**Figure 8** $P_d$ against input signal-to-noise ratio, (a) 1th - 45th Doppler bins, (b) 84th - 128th Doppler bins.
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ORCID
Hao Xiao https://orcid.org/0000-0002-3419-7177

REFERENCES
1. Ward, J.: Space-time adaptive processing for airborne radar. Lincoln Laboratory, Lexington (1994)
2. Reed, I.S., Mallet, J.D., Brennan, L.E.: Rapid convergence rate in adaptive arrays. IEEE Trans. Aerosp. Electron. Syst. 10(4), 853–863 (1974)
3. Klemm, R.: Adaptive airborne MTI: an auxiliary channel approach. IEEE Trans. Aerosp. Electron. Syst. Mag. 19(1), 19–35 (2004)
4. Goldstein, J.S., Reed, I.S., Scharf, L.L.: A multistage representation of the wiener filter based on orthogonal projections. IEEE Trans. Inform Theory. 44(7), 2943–2959 (1998)
5. Gold, J.S., Reed, I.S., Zulch, P.A.: Multistage partially adaptive STAP CFAR detection algorithm. IEEE Trans. Aerosp. Electron. Syst. 35(2), 645–662 (1999)
6. Gerard, T.C., et al.: Knowledge-based radar signal and data processing: a tutorial overview. IEEE Trans. Signal Process Mag. 23(1), 18–29 (2006)
7. Zhu, X.M., Li, J., Stoica, P.: Knowledge-aided space-time adaptive processing. IEEE Trans. Aerosp. Electron. Syst. 47(2), 1325–1336 (2011)
8. Yang, E., Adve, R., Chun, J.: Hybrid direct data domain sigma-delta space-time adaptive processing algorithm in non-homogeneous clutter. IET Radar, Sonar Navig. 4(4), 611–625 (2010)
9. Sun, K., et al.: Direct data domain STAP using sparse representation of clutter spectrum. Signal Process. 91, 2222–2236 (2011)
10. Yang, Z., et al.: On clutter sparsity analysis in space-time adaptive processing airborne radar. IEEE Geosci. Remote Sens. Lett. 10(5), 1214–1218 (2013)
11. Sen, S.: Low-rank matrix decomposition and spatio-temporal sparse recovery for STAP radar. IEEE J. Sel. Topics Signal Process. 9(8), 1510–1523 (2015)
12. Yang, Z.C., Li, X., Wang, H.Q.: Adaptive clutter suppression based on iterative adaptive approach for airborne radar. Signal Process. 93, 3567–3577 (2013)
13. Wang, Z.T., et al.: Clutter suppression algorithm based on fast converging sparse Bayesian learning for airborne radar. Signal Process. 150, 159–168 (2017)
14. Gao, Z.Q., Tao, H.H.: Robust STAP algorithm based on knowledge-aided SR for airborne radar. IET Radar, Sonar Navig. 11(2), 321–329 (2017)
15. Bai, G.T., Zhao, J., Bai, X.: Parameter-searched OMP method for eliminating basis mismatch in space-time spectrum estimation. Signal Process. 138, 11–15 (2017)
16. Su, Y.Y., et al.: A grid-less total variation minimisation-based space-time adaptive processing for airborne radar. IEEE Access. 8, 29334–29343 (2020)
17. Zhang, W., He, Z.S., Li, H.Y.: Linear regression based clutter reconstruction for STAP. IEEE Access. 6, 56862–56869 (2018)
18. Dai, B.Q., et al.: Adaptively iterative weighting covariance matrix estimation for airborne radar clutter suppression. Signal Process. 106, 282–293 (2015)
19. Jiang, L., Wang, T.: Robust non-homogeneity detector based on reweighted adaptive power residue, IET Radar, Sonar & Navig. 10(8), 1367–1374 (2016)
20. Tong, Y.L., Wang, T., Wu, J.X.: Improving EFA-STAP performance using persymmetric covariance matrix estimation. IEEE Trans. Aerosp. Electron. Syst. 51(2), 924–936 (2015)
21. Wang, Y.L., et al.: Reduced-rank space-time adaptive detection for airborne radar. Sci China Inf Sci. 57(8), 1–11 (2014)
22. Liu, W.J., et al.: Multichannel adaptive signal detection: basic theory and literature review. Sci China Inf Sci. 1(3), 1–41 (2021)
23. Yang, Z.C., Liu, W.J.: Sparsity-based STAP using alternating direction method with gain/phase errors. IEEE Trans. Aerosp. Electron. Syst. 50(3), 952–954 (2017)
24. Augusto, A., et al.: Radar phase noise modelling and effects part I: MTI filters. IEEE Trans. Aerosp. Electron. Syst. 52(2), 698–711 (2016)
25. Gene, H.G.: Matrix computations, 4th ed. Posts Telecom Press, Beijing (2014)
26. Rojas, C.R., Katselis, D., Hjalmarsson, H.: A note on the SPICE method. IEEE Trans Signal Process. 61(18), 4545–4551 (2013)
27. Jeon, H., et al.: Clutter covariance matrix estimation using weight vectors in knowledge-aided STAP. Electronic Letter. 53(6), 560–562 (2017)
28. Haimovich, A.: The eigencanceler: adaptive radar by eigenanalysis methods. IEEE Trans. Aerosp. Electron. Syst. 32(2), 532–542 (1996)
29. Boyd, S., Vandenberghe, L.: Convex optimisation. Cambridge University Press, Cambridge (2004)
30. Vijay, V., Jeffrey, L.K.: Joint space-time interpolation for distorted linear and bistatic array geometries. IEEE Trans Signal Process. 54(3), 848–860 (2013)
31. Arakawa, M.: Computational workloads for commonly used signal processing kernels. Lincoln Laboratory, Lexington (2003)
32. Wu, Y.F., Wang, T., Wu, J.X.: A knowledge-aided space time adaptive processing based on road network data. J Electron Inf Technol. 37(3), 613–618 (2015)

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