Local effect of column flange flexibility on shear lag in steel box moment connections

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Abstract. This paper presents the shear lag phenomenon occurred in steel box moment connections by considering the local effect of column flange flexibility. The stress concentration due to shear lag in steel box member is one of the major concerns for steel box moment connections. When welding is utilized, the connections are susceptible to cracking or failure in the weld parts. Therefore, the maximum stress in the steel box moment connection is always checked during a preliminary design stage. In this study, the stresses due to shear lag were evaluated using least-work solution, considering the flexibility of column flange. The local flexibility of the connection, which caused by the column flange and diaphragms, is represented by an axial spring model, and contributed to stress distribution in the beam flange. Using the presumed longitudinal displacement functions for both the beam flange and web, the stress concentrations were evaluated to increase significantly compared with that obtained from a cantilever beam model. The finite element assessment was also conducted to check the validity of the manual prediction of the stress in various ranges of the beam and column section properties. The results are further summarized, discussed, and showed that more flexible column flange and diaphragm provided higher stress concentrations.

1. Introduction
A moment connection of an exterior column in framing system creates a conventional joint known as T-joint. The beam forces are generally transferred to one side of the exterior column. When box members are used, shear lag problem raised concern over the stress concentration localized at the junctions between the beam flange and web, which are able to cause cracking or failure of the weld parts in the joint region. For this reason, the stress concentration at the joint region is required to be checked during the preliminary design stage. Several solutions had been proposed to predict the shear lag stress in rectangular box members. One of the simplest methods called “least-work solution”, was initially delivered by Reissner [1,2], as concerned with aeronautical structures. The method was further developed to provide more accurate shear lag stresses in the box beams and girder bridges [3,4,5,6]. The method minimizes the energy of the systems corresponding to a presumed function of the longitudinal displacement of the beam flange. Moreover, shear lag can be also revealed using finite element method (FEM). However, this approach requires comprehensive modelling and costly computation time and memory sizes.

An existing study has revealed the cracks and brittle failures in the weld parts of the box-to-box connections in the steel bridge frame piers in Japan during a severe earthquake in 1995 [7]. Thereafter, numerous studies had been conducted to evaluate the stress excited in the connection using both FEM and experiment. The studies have proved that the existing manual prediction of shear lag stress given by Okumura [8] is not alternative and accurate for the design of the frame piers. Further investigation by Hwang et al. [9] revealed that the shear lag stress at the joint increases twice higher than the stress in Okumura’ equation. Nonetheless, the study had not yet reported the local effects of the column flange
flexibility and shear lag in the beam web on the stress distribution in the beam flange. The application of the box T-joint with diaphragms (continuity plates) has not been widely presented in most design codes.

The diaphragm improves the strength and stiffness of the connection and may affect the stress distribution in the beam flange. Hence, consideration of the local effect due to the column flange and diaphragm is vital to generate the accurate stress distribution in the beam flange. This study aims to indicate the effects of the column flange flexibility on the shear lag stress in the T-joints including the consideration of shear lag in the web of the beam. The maximum stress at the edges of the beam flange was simplified to serve for the design and check. The shear lag phenomenon occurs in box T-joint is illustrated in Figure 1.

2. Least-work solution
The least-work solution allows to minimize the energy of the systems using the presumed longitudinal deformation of the beam flange and web as be shown in equations, (1) and (2) to establish the governing differential equations. Hence, unknown parameters associated with the presumed deformations can be obtained and the stress distribution can be evaluated.

\[ U(x,y) = h \left[ w \left( 1 - \frac{y}{b} \right) u_1(x) \right] \]  

\[ U(x,z) = \left[ w'z + \left( \frac{z}{h} - \frac{z'}{h} \right) u_2(x) \right] \]  

where \( h \) and \( b \) express the height and haft width of the beam flange, as seen in Figure 1(b), respectively. \( w \) denotes the flexural displacement of the beam. \( x, y, \) and \( z \) denote the coordinate components. \( u_1(x) \) and \( u_2(x) \) represent the independent deformation density of the beam flange and web regarding to \( x \)-axis, respectively. Considering the identical symmetry of the top and bottom flanges, the total energy of the system maintains the energy of the load system, strain energy, and the energy of the deformed column flanges and diaphragm, and can be expressed as:

\[ U_{tot} = \int Mw'dx + \frac{1}{2} \int 2\mu \left( E \varepsilon^2 + G \gamma^2 \right) dx + \frac{1}{2} \int 2\mu \left( E \varepsilon^2 + G \gamma^2 \right) dy + \frac{1}{2} K \delta^2 \]  

where \( M \) is the bending moment and \( t_f \) denotes the thickness of the beam flange. \( E \) and \( G \) express the Young and shear modulus, respectively. \( K \) and \( \delta \) represent the stiffness and displacement of the column flange and diaphragm. The normal (\( \varepsilon \)) and shear (\( \gamma \)) strains of the beam flange and web are expressed in equations (4) and (5) below.
The total energy of the system can be expanded by substituting equations (4)-(5) into equation (3). Then, by minimizing the total potential energy, the moment equilibrium of the system can be evaluated as:

\[ M + Eh^x + E\left(\frac{i}{i+1} I_1 u_1' + \frac{2}{5h} I_s u_s^2\right) = 0 \]  

(6)

where \( I_s = 4bt_h^2 \), \( I_1 = \left(\frac{4}{3}\right)_h^3 \), and the third term in equation (6) represents the additional moment due to shear lags in the flange and web. Using the above equilibrium equation, the differential equations can be obtained as follows:

\[ u_1'' - k_1^2 u_1 - p_1^2 u_2 = u_{01} \]  

(7)

\[ u_2'' - k_2^2 u_2 - p_2^2 u_1 = u_{02} \]  

(8)

where the parameters \( k_1, p_1, u_{01}, k_2, p_2, \) and \( u_{02} \) are corresponding to the geometry, material and load properties. The general solutions of \( u_1 \) and \( u_2 \) can be obtained by solving 2nd-order ODE system of equations (7)-(8) above.

### 3. Stress distributions

Using the beam theory, the maximum stress at the joint can be determined by

\[ \sigma_{\text{max}} = \frac{M_{\text{Total}}}{I} \frac{h}{h} = \frac{M_B + M_A}{I} \frac{h}{h} \]  

(9)

where \( M_B \) represents the beam bending moment. \( M_A \) denotes the total additional moment due to shear lags in the flange and web. \( h \) and \( I \) denote the height of the beam flange and moment of inertia of the beam, respectively.

The determination of stress distribution given by equation (9) is much complicated for manual calculation because the procedure requires to solve the ODE system, and several parameters need to be determined ahead to receiving the stress equation. This investigation aimed to simplify the stress distribution in equation (9) in order to provide time saving during the preliminary design. The simplified stress distribution was proposed using the stress receiving from the shear lag in the flange alone and modified by a factor \( \beta \), which represents the shear lag in the beam web. The modification factor \( \beta \) was evaluated using a compatible solution in a parametric study between the stress in the cantilever beams given by equations (9) and (10) in relations with the web depth-to-flange width ratio \( d/2b \). The empirical chart of the factor \( \beta \) is depicted in Figure 2 below. The total stress distribution in the beam flange can be determined as:

\[ \sigma_L = Eh\left( w' - \beta\left(1 - \frac{v'}{b'} - \frac{i}{i+1}\frac{I_x}{I}\frac{p}{k} A_y + \frac{\delta \Delta}{h}\right) \right) \]  

(10)
where $P$ is the point load located at the free end of the beam, $\delta$ represents the displacement of the column flange, $A_n = \left( \frac{2i+1}{2i} \right)^n \frac{n}{EI}$, $n = \frac{1}{1 - \left( \frac{2i+1}{2i+2} \right)^n}$, and $k = \frac{1}{b} \sqrt{\frac{(i+1)(2i+1)}{2(2i-1)}} \frac{G}{E} n$.

4. Column flange flexibility
The least-work solution requires to settle the boundary conditions in order to obtain the specific solution for the longitudinal displacement of the beam flange. Hence, the flexibility of the column flange, which creates an initial displacement was seriously concerned about the stress distribution. The column flange is much deformable at the mid-width of the flange and becomes fixed at the edge of the flange. The displacement of the column flange can be determined using the plate theory, excited by the effective load transmitted from the beam flange. A relevant study on the deflection of the rectangular plate of infinite length with simply-supported edges can be found in Timoshenko’s publication [10]. The deflection of the column flange was represented by a fixed-supported plate of infinite length under the effective load, and can be generally simplified to an expression corresponding to a flexibility coefficient $\alpha_f$ as shown in equation (11).

$$\beta_s = 1.3463(d/2b) + 0.0816$$
$$R^2 = 0.9886$$

**Figure 2.** Modification factor $\beta_s$

where $\beta_s$ represent the modification factor, $\beta_s = 1.3463(d/2b) + 0.0816$, $R^2 = 0.9886$.

A box T-joint may provide insufficient strength as required to achieve a moment connection. One of the trendy option for improving the performance of the box T-joint is to introduce the diaphragms into the connection [11]. The internal diaphragms play an important role to contribute the strength and stiffness to the connection. The internal diaphragms enable providing high restraint against the deflection of the column flange. Moreover, the internal diaphragm brings a significant strength increase in the connection when the width of the beam flange mismatches to the column flange. The internal diaphragms have also been seen in use for decades in box knee- and T-joints of bridge frame piers [12]. The reality of the connection is that the internal diaphragm and column flange can move slightly under the transmitted force by the beam flange, leading to generate an initial displacement to the system. A parallel spring model in Figure 3 was proposed for predicting the displacement of the column flange and diaphragm. The model represents the column flange and diaphragm, which attached to the fix sides (leftmost and rightmost) and a rigid body at the middle, subjected to a tension force transmitted from the beam flange. The axial stiffness and displacement of the system can be written as follows.
The 10 mm thick diaphragm, and end beam was subjected to a static load reached 147 kN. As observed in equation (10), the stress degraded gradually according to the smaller bending moment of the beam at the joint. Therefore, using a cantilever beam model for the box T-joint provided an under-predicted stress distribution.

5. Numerical investigations
5.1. Finite element modelling and validation

The performance of the box T-joints was numerically assessed to evaluate the displacement of the column flange and stress distribution in the beam flange. The column flange flexibility was predicted using a parametric study on the plate-to-box column connections. This numerical procedure used the finite element method (FEM) which assisted by a computer software Abaqus [13]. More importantly, the FEM requires comprehensive modelling which is able to assure the reliability of the results. A 3D solid 8-node element (C3D8-R) was used in this numerical investigation, as unveiled by Serrano-López et al. [14] and Moazed et al. [15] that the solid element provides more reliable results. The triple symmetric geometry allowed the plate-to-box column connection to be modeled as one eighth of the actual connection. However, the full connection model was carried out for the box T-joints. The pin supports were used for both ends of the column. The stress-strain curve of the steel was characterized as multi-linear isotropic behavior with strain hardening in accordance with JSCE [16], as shown in Figure 4a. The combined isotropic-kinematic hardening was also considered for cyclic loading analysis. Young’s modulus of the steel was assumed to be 205 GPa. A mesh size of one-half and one time of the element thickness was employed to the thickness and the considerable portions of the connection’s components.

A validation of the FEM modelling was carried out to compare with a test of a built-up stiffened box T-joint (test No. 1) by Sasaki et al. [17]. The test No. 1 composed of a beam with the length of 1350 mm, which connected to a column with the length of 2900 mm. The 10 mm-thick stiffeners were used and located at the mid-width of the flange and web of the beam and column. The beam was subjected to a pre-compression load of 294 kN and a lateral point load placed at the beam free end (cyclic). The yield strength of 288 MP, 293 MPa, and 299 MPa was utilized for the stiffener, the flange, and the web, respectively. The analytical results were compared with the test in terms of the static load-displacement relations and stress distribution at 20 mm from the column face when the lateral load reached 147 kN. As observed in Figures 4b, the FEM model maintained fairly matched load-
displacement relations compared with the experiment. Moreover, the stress distribution at 20 mm from the column face given by FEM and experiment (Figure 4c) were well comparable to settle a generalized numerical investigation.

![Steel stress-strain curve](image1)
(a) Steel stress-strain curve

![Static load-displacement curve](image2)
(b) Static load-displacement curve

![Stress distribution](image3)
(c) Stress distribution

**Figure 4.** Steel material modelling and FEM validation

5.2. Evaluation of column flange flexibility

The column flange flexibility, as described in section 4, is defined by the coefficient $\alpha_f$, which generally obtained based on the presumed effective width of the beam flange. Nonetheless, the presumed effective width may deliver an invalidated deflection of the column flange. In this FEM investigation, the column flange flexibility in the box T-joint was evaluated using a simplified plate-box column model. The two matched plates were connected to a box in the opposite directions. Each plate was subjected to tension loading. The plate-to-box column connection creates a triple symmetric geometry which allows to simulate the connection as one eighth of the actual connection. The procedure for evaluating the column flange flexibility was to construct the load-displacement relations of the column flange under the tension force of the plate. The study included 35 connection models in which the column flange width and thickness were respectively varied from 100 to 1000 mm and 4 to 40 mm, in accordance with the width-to-thickness ratio ($2\gamma = \frac{b_c}{t_c}$) from 15 to 50. Using equation (11) for compatibility with the FEM results, the flexibility coefficient of the column flange ($\alpha_f$) can be established corresponding to the column width-to-thickness ratio ($2\gamma$), as shown in Figure 5 below. As observed, the flexibility of the column flange ($\alpha_f$) degraded relatively to the greater column width-to-thickness ratio ($2\gamma$).

![Flexibility coefficient of column flange](image4)

**Figure 5.** Flexibility coefficient of column flange

5.3. Evaluation of stress distribution

The stress distribution was evaluated and compared with the hand calculation. Hence, the static performances of the box T-joints, in which the beam depth was changed, were carried out. A 300x300x8 mm size was used for the column with the length of 2 m. The beam size was matched to the column size but the depth of the beam was varied as 200 mm, 300 mm, and 450 mm. A point load equivalent to a bending stress of 100 MPa was applied at the free end of the beam ($L_0 = 1.6$ m). The von Mises stress...
contours of the box T-joints in this study are depicted in Figure 6. The stress concentrated at the edges of the beam flange and downgraded to a minimum at the mid-width of the flange. More importantly, it was also observed that the stress increased significantly accordingly to the deep web of the beam.

The stress distribution was recorded along the beam flange at the vicinity of the column flange (next to welding). The stress distribution was plotted and compared with equation (10) and shown in Figure 7. In equation (10), the stress was also calculated using 4 and 6-order parabolic curves of the deformation of the beam flange. The results showed that for all T-joints, the peak stresses given by FEM and manual prediction in equation (10) provided well matched values. However, stress distribution for the connection with the unequal web depth-flange width was slightly different at the mid-width of the beam flange. This circumstance admitted that the simplified approach given by equation (10) for the stress distribution is suitable for the peak stress but turns into a slight difference at the mid-width of the beam flange when the beam web depth unequal to the flange width. However, since the stress at the mid-width of the beam flange is not necessary for design and check, the manual approach is alternative enough in use to predict the peak stress using either 4 or 6-order parabolic curve.

6. Concluding remarks
The flexible column flange in the box T-joint was very critical in producing high stress concentration at the beam flange-web junctions. This circumstance can be enhanced by utilizing the thicker diaphragms. The column flange flexibility is represented by a flexibility coefficient $a_f$ and can be empirically calculated based on the FEM parametric study. Moreover, the peak stress of the beam flange in the box T-joint with diaphragms was also influenced by the web depth-to-flange width ratio of the beam ($d/2b$). The peak stresses increased corresponding to the deep web. To provide a convenient calculation, a simplified method for stress distribution was provided in this study. The method requires to calculate the stress obtained by considering that shear lag presented in the beam flange alone, and multiply it to a modification factor ($\beta_s$). Considering the effects of column flange and diaphragm flexibility including...
the shear lag in the web, the simplified approach provided well predicted peak stress, as well compared with the FEM results for both 4 and 6-order parabolic functions of the longitudinal displacement of the beam flange.

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