Reflection Coefficients of Plane Waves at a Seawater-magnetorheological Fluid Interface

Min Shen¹ and Bai Cong²

School of Mechanical Engineering and Automation, Wuhan Textile University, 1Fangzhi Road, Wuhan, Hubei Province, 430073, China.

min_shen18@163.com

Abstract. The Magnetorheological fluid (MRF) is a class of smart material with tunable acoustic properties and could be used as active sound coating material on underwater structure. This paper presents an acoustic model to investigate the reflection of a plane wave incidence at seawater-MRF interface. The acoustic model is developed by modelling the MRF as Biot’s type porous material. In this paper, the effect of three major parameters of MRF material on the reflection coefficients are introduced. It is concluded that reflection coefficients can achieve low values by increasing shear modulus, particle volume fraction and decrease permeability.

1. Introduction

Magnetorheological fluids (MRF) consist of micrometric magnetic particles suspended in a viscous or viscoelastic fluid[1]. The MR fluids exhibit variable acoustic and rheology properties by applying an external magnetic field. Recently, MRF materials are proposed to as active sound barriers or acoustical metamaterial [2-3]. Submarine are often covered with sound absorbing or perforated materials called anechoic coatings. Hence, the MRF material can be used as active sonar anechoic material in submarines.

The MRF behaves as a Newtonian fluid in the absence of magnetic field and the particles are scattered randomly in the liquid carrier. The fluid shows a non-Newtonian behaviour in the presence of an external magnetic field and the dispersed magnetic particles make chain-like structures that align in the direction of the applied field. The transition between these two phases is highly reversible, which provides a unique feature of magnetic field controllability of the acoustic properties of the MRF. Hence, the structure of the MRF is considered as a skeletal frame filled with a base fluid, the acoustic mechanics of MRF is complicate because MRF is coupled multiphase medium: the solid phase of ferromagnetic grain and the fluid phase of the pore fluid.

A series of acoustic experiments have been conducted for comparing sound speed in MRF. Lo´pez et al.[4] have studied the effect of particle volume fraction and magnetic field intensity on the velocity and attenuation in MRF using ultrasonic techniques. Bramantya et al.[5] have experimentally studied the sound speed in some commercial MRF as function of the anisotropy, intensity and uniformity of the applied field. Shen et al.[6] have theoretically analysed the dependence of magnetic fields strength on sound velocity and attenuation in MRF based on Biot’s model.

In order to reduce the reflection of underwater structure, it is necessary to study the reflectivity of MRF material. However, very limited number of researches has so far been studied underwater acoustic properties of MRF as active underwater acoustic coating. Hence, this paper introduced the reflection coefficients of plane wave incidence at the interface of seawater and MRF material.
2. Acoustic model of plane wave incident at a seawater-MRF interface

The particles in MRF suspension are interconnected and small cavities generated in the presence of a magnetic field. Hence, the MRF is considered as a porous material and composed of the skeletal frame filled with silicon oil fluid. It suggests an attempt of modelling its mechanical behaviour by the Biot[7-8] theory for porous elastic media saturated by a compressional fluid. According to Biot’s theory[7-8], the wave propagating in the porous material has three independent components: a shear wave and two dilatational waves. The derivation of sound speed propagation in porous material can be found in several papers [9-10]. Firstly, the fast \( V_{p1} \) and slow dilatational wave velocities \( V_{p2} \) and shear wave velocities \( V_s \) have been calculated based on Biot’s theory.

The wave number of fast dilatational wave \( k_1 \) and the slow dilatational wave \( k_2 \) can be written by

\[
k_i = \frac{\omega}{V_i}
\]

\( i = 1, 2 \)

\[ k_1 = \frac{\omega}{V_{p1}} \quad \text{and} \quad k_2 = \frac{\omega}{V_{p2}} \quad (1) \]

The wave number of shear wave \( k_s \)

\[ k_s = \frac{\omega}{V_s} \quad (2) \]

\[ \phi_i, \phi_r, \phi_f \]

\[ \phi_i \text{ Transmitted fast P-wave} \quad \psi_s, \psi_f \text{ Transmitted S-wave} \quad \phi_t \text{ Transmitted slow P-wave} \]

**Figure 1.** Reflection and refraction of plane wave at seawater-MRF interface

We consider a plane wave incident at an angle \( \theta \) to the steady, laminar and incompressible flows of MRF and seawater at \( z = 0 \), where the externally imposed magnetic field may be spatially non-uniform as shown in Figure 1. There exit incident (\( \phi_i \)) and reflected waves (\( \phi_r \)) in the seawater, and three refracted waves that the fast P-wave (\( \phi_f \)) and the slow P-wave (\( \phi_s \)) of dilatational waves, and the shear wave (\( \psi_f \)) in the MRF material.

In a two-component material, such as seawater-MRF material, two vectors, \( u \) and \( U \) represents the displacements of the frame and fluid, respectively.

\[
u = \nabla \phi_f + \text{rot} \psi_f
\]

\[
U = \nabla \phi_f + \text{rot} \psi_f
\]

The fluid displacement relative to the frame is introduced as:

\[
w = -\beta(U - u)
\]

where \( \beta \) is porosity.

The incident (\( \phi_i \)) and reflected waves (\( \phi_r \)) in the seawater will have the displacement potentials

\[
\phi_i = A_i \exp[i(\alpha t - k_w \cos \theta z - k_w \sin \theta x)]
\]

\[
\phi_r = A_r \exp[i(\alpha t - k_w \cos \theta z - k_w \sin \theta x)]
\]

where \( k_w = \omega/c_w \), \( c_w \) is the sound velocity in seawater. \( A_i \) is the amplitude of incident wave, \( A_r \) is the amplitude of reflected wave and three refracted wave potentials.
In the MRF material, the scalar and vector potentials defined in Eqs.(3) and (4) are
\[
\phi_i = A_i \exp[i(\alpha x - k_{1r}z - k_{1t}x)] + \bar{A}_i \exp[i(\alpha x + k_{1r}z - k_{1t}x)]
\]
\[
\phi_j = A_j \exp[i(\alpha x - k_{2r}z - k_{2t}x)] + \bar{A}_j \exp[i(\alpha x - k_{2r}z + k_{2t}x)]
\]
\[
\psi_i = A_i \exp[i(\alpha x - k_{3r}z - k_{3t}x)]
\]
\[
\psi_j = A_j \exp[i(\alpha x - k_{4r}z - k_{4t}x)]
\]
where \( k_{nr} = k_n \sin \Theta \ (n = 1,2,3,4) \), \( k_{1r} \) and \( k_{2r} \) are \( z \) direction component of the fast and slow longitudinal wavenumbers, \( k_{1t} \) and \( k_{2t} \) are \( x \) direction component of the fast and slow longitudinal wavenumbers, \( k_{3r} \) and \( k_{4r} \) are \( x \) direction component of the shear wavenumbers. \( A_{i} \) is the refraction coefficient of fast dilatational wave, \( A_{s} \) is the refraction coefficient of slow dilatational wave and \( A_{r} \) is the shear wave refraction coefficient in porous solid respectively. \( A_{f1} \) is the fast dilatational wave, \( A_{f2} \) is the slow dilatational wave and \( A_{f} \) is the shear wave reflection coefficient in porous fluid medium, respectively.

The following four boundary conditions are required at a seawater-MRF material interface [11]

1. (1) For continuity of fluid movement
   \[
   \frac{\partial \phi_i}{\partial z} + \frac{\partial \phi_j}{\partial z} = \frac{\partial \psi_i}{\partial z} - \frac{\partial \psi_j}{\partial z}
   \]

2. (2) For equilibrium of normal traction
   \[
   H\left(\frac{\partial^2 \phi_i}{\partial x^2} + \frac{\partial^2 \phi_i}{\partial z^2} - \frac{\partial^2 \psi_i}{\partial x^2} - \frac{\partial^2 \psi_i}{\partial z^2}\right) - c_i\left(\frac{\partial^2 \phi_i}{\partial x^2} + \frac{\partial^2 \psi_i}{\partial x^2}\right) = \rho_f\left(\frac{\partial^2 \phi_i}{\partial t^2} + \frac{\partial^2 \psi_i}{\partial t^2}\right)
   \]

3. (3) For equilibrium of fluid pressure
   \[
   M\left(\frac{\partial^2 \phi_i}{\partial x^2} + \frac{\partial^2 \phi_i}{\partial z^2} - \frac{\partial^2 \psi_i}{\partial x^2} - \frac{\partial^2 \psi_i}{\partial z^2}\right) - c_i\left(\frac{\partial^2 \phi_i}{\partial x^2} + \frac{\partial^2 \psi_i}{\partial x^2}\right) = -\rho_f\left(\frac{\partial^2 \phi_i}{\partial t^2} + \frac{\partial^2 \psi_i}{\partial t^2}\right)
   \]

4. (4) For equilibrium of tangential traction
   \[
   2\mu\frac{\partial^2 \phi_i}{\partial x \partial z} - \mu\left(\frac{\partial^2 \psi_i}{\partial x^2} - \frac{\partial^2 \psi_i}{\partial z^2}\right) = 0
   \]

When considered four boundary conditions at the interface \( z = 0 \), the following four linear equations can be obtained,
\[
\begin{bmatrix}
  a_{11} & a_{12} & a_{13} & a_{14} & \vdots & A_i & 1 \\
  a_{21} & a_{22} & a_{23} & a_{24} & \vdots & A_i & -\rho_f \\
  a_{31} & a_{32} & a_{33} & a_{34} & \vdots & A_i & \rho_f \\
  a_{41} & a_{42} & a_{43} & a_{44} & \vdots & A_i & 0
\end{bmatrix}
\]
\[
A_i = \text{the complex amplitude of seawater and MRF material. Therefore, the reflection coefficient}
R = |A_i / A_i| \text{ can be obtained by solving Eq.(16), once } A_i \text{ is specified. The solution of reflection coefficients can be obtained straight by applying Grammer's rule. The components of Eq.(16) are listed in Appendix A.}

3. Results and Discussion
In this section, the reflection coefficients at seawater-MRF interface are derived. The MRF is named MRHCCS4-A manufactured by the UK LIQUID RESEARCH company. The parameters of the model for the MRF are listed in Table 1. All parameters of MRF in braces are either directly supplied
by the manufacturer or have been derived by Carmignani et al.[12]. For the MRF description, the fluid bulk modulus can be assumed elastic, because in our case the base fluid is low-viscosity silicone oil.

Table 1. The Parameters of MRHCCS4-A by UK LIQUID REASEARCH Corporation

| Parameters                      | Symbol(Unit)     | MRF  |
|---------------------------------|------------------|------|
| Solid density                   | \( \rho_s (kg/m^3) \) | 2980 |
| Fluid density                   | \( \rho_f (kg/m^3) \) | 820  |
| Grain bulk modulus              | \( k' (N/m^3) \)  | \( 1.26 \times 10^8 \) |
| Fluid bulk modulus              | \( k_f (N/m^3) \)  | \( 2.6 \times 10^8 \) |
| Grain diameter                  | \( d_s (\mu m) \)  | 3 – 10 |
| Fluid viscosity                 | \( \eta (Pa/s) \) | 0.013 |
| Frame shear modulus             | \( G_s (N/m^3) \) | \( 0 - 7 \times 10^5 \) |
| Shear specific loss             | \( \delta_{\gamma} \) | 0 – 0.5 |
| Frame bulk modulus              | \( k_f (N/m^3) \) | \( 0 - 4 \times 10^8 \) |
| Volumetric specific loss        | \( \delta_{\psi} \) | 0 – 0.5 |
| Particle volume fraction        | \( \phi \)        | 0.01 – 0.35 |
| Structure factor                | \( \alpha \)      | 1.4  |
| Permeability                    | \( \kappa (m^2) \) | \( 4 \times 10^{-12} \) |

![Figure 2](image-url) Normal reflection coefficient as a function of frequency for the sand

![Figure 3](image-url) Normal reflection coefficient as a function of frequency for three different storage modulus \( \mu \)

To refine the correctness of this model, the reflection coefficients at a seawater-sediment interface for a typical kind of marine sediment sand is shown in Figure 2. The physical parameters of sand were taken from Kimura[11]. In this calculation, sound velocity in seawater is assumed to be 1500m/s. In order to analyse the effects of different frequency, we suppose that the plane wave is normal incidence.

Figure 3. shows the magnitude of the reflection coefficient as the frequency for three different values of the storage modulus \( \mu = 10^3 \), \( \mu = 10^4 \) and \( \mu = 10^5 \). The intensity of the magnetic field is 100mT, 200mT and 500mT, respectively. It is seen from the Figure 3. that the reflection coefficients are obviously improved when the shear modulus increase from 10kPa to 100kPa. The reason is that an external magnetic field induces chain-like particulate structures in the direction of the field. The new
structure alters the shear resistance and fluid viscosity. The fluid viscosity and shear modulus increase with the increasing of the strength of magnetic field. The more sound energy is dissipated by viscoelastic properties and fluid viscosity. Hence, the reflection coefficient is greatly reduced by applying a small strength of magnetic field.

Figure 4. shows the magnitude of the reflection coefficient as the frequency for three different values of the particle volume fraction, $\phi=0.1$, $\phi=0.2$, and $\phi=0.3$, respectively. The particle volume fraction is a parameter related to the suspension itself and independent of the external field strength. It is seen from Figure 4. that the reflection coefficients decrease with the increasing of the volume fraction. Because the iron particle is more connected formed a frame with high volume fraction ($\phi=0.3$), which is weak-link regime. The contribution of the solid part is more important than fluid when the particle volume fraction is increased.

![Figure 4. Reflection coefficient as a function of frequency for three different particle volume fraction $\phi$.](image)

Figure 5. shows the magnitude of the reflection coefficient as a function of frequency for three different values of permeability $\kappa=4 \times 10^{10}$, $\kappa=4 \times 10^{11}$ and $\kappa=4 \times 10^{12}$. It is seen from Figure 5. that reflection coefficients are obviously decrease with the permeability decrease over a wide range of frequency. Permeability as a function of porosity, fluid viscosity and pore distribution. The permeability also is affected by imperfect pore structures and non-uniformity in particle geometry. It is a complicated parameter influenced the acoustic performance of MRF.

![Figure 5. Reflection coefficient as a function of Frequency for three different permeability $\kappa$.](image)

4. Conclusions
The MRF could be used as active sound coating material on underwater structure. In order to reduce the reflection of underwater structure, this paper presents an acoustic model to investigate the reflection of a plane wave incidence at seawater-MRF interface. We highlighted the effect of the major parameter on reflection coefficient in the presence of an external magnetic field.

(1) We find that applying appropriate magnetic density could improve the reflective coefficient of MRF and there is no obvious change when the strength of magnetic field is below 200mT.

(2) The reflection coefficients decrease with the increasing of particle fraction volume from 0.1 up to 0.3. The contribution of the solid part is more important than fluid if the value of particle volume fraction is above 0.3.

(3) The reflection coefficients are obviously decrease with the permeability decrease over a wide range of frequency.
Acknowledgments
This work was financially supported by the Natural Science Foundation of Hubei Province of China (No.2014CFB766), China National Science Foundation (No.51505344), Scientific Research Foundation for Returned Scholars, Ministry of Education of China.

References
[1] Ghaffari A, Hashemabadi S H, and Ashtiani M, A review on the simulation and modeling of magnetorheological fluids, Journal of Intelligent Material Systems and Structures, 26(2015)881-904.
[2] Szary M, The analytical model of rheological fluid for vibration and noise control, Proceedings of Meetings on Acoustics, Montreal, Canada, 19(2013)1-8.
[3] Donskoy D M and Malinovsky V S, Broadband acoustic metamaterials with electro-magnetically controlled properties, Proceedings of Meetings on Acoustics, Montreal, Canada, 19(2013)1-8.
[4] Lo´pez J R, Castro P, Elvira L and Espinosa F M D, Study of the effect of particle volume fraction on the microstructure of magnetorheological fluids using ultrasound: Transition between the strong-link to the weak-link regimes, Ultrasonics, 61(2015)10-14.
[5] Bramantya M A, Motozawa M and Sawada T, Ultrasonic propagation velocity in magnetic and magnetorheological fluids due to an external magnetic field, Journal of Physics:Condensed Matter, 22(2010)1-5.
[6] Shen M and Huang Q B, Acoustic velocity and attenuation coefficient of magnetorheological fluids under electromagnetic fields, Applied Acoustics, 107(2016)27-33.
[7] Biot M A, Theory of propagation of elastic waves in a fluid-saturated porous solid, Part I:Low frequency range, Journal of the Acoustical Society of America, 28(1956)168-178.
[8] Biot M A, Theory of propagation of elastic waves in a fluid-saturated porous solid, Part II:Higher frequency range, Journal of the Acoustical Society of America, 28(1956)179-191.
[9] Stoll R D and Bryan G M, Reflection of acoustic waves at a water-sediment interface, Journal of the Acoustical Society of America, 70(1981)149-156.
[10] Wu K, Xue Q and Adler L, Reflection and transmission of elastic waves from a fluid-saturated porous solid boundary, Journal of the acoustical society of America, 990,87:2349-2358.
[11] Kimura M, Reflection of plane acoustic waves at a Seawater-marine sediment interface, Journal Applied Physics, 35(1996)2948-2951.
[12] Carmignani C, Forte P and Rustighi E, Design of a novel magnetorheological squeeze-film damper, Smart Materials and Structures, 15(2006)164-170.

Appendix A
\[
\begin{align*}
    a_{11} &= 1; & a_{12} &= (1 - r_1)c_w \cos \theta_2 / (c_2 \cos \theta); \\
    a_{13} &= (1 - r_2)c_w \cos \theta_2 / (c_2 \cos \theta); & a_{14} &= (1 - r_3)c_w \sin \theta_2 / (c_2 \cos \theta); \\
    a_{21} &= \rho_f; & a_{22} &= (r_1C - H)/c^2_1 + 2\mu \sin^2 \theta / c^2_w; \\
    a_{23} &= (r_2C - H)/c^2_2 + 2\mu \sin^2 \theta / c^2_w; & a_{24} &= -2\mu \sin \theta \cos \theta_2 / c_w c_1; \\
    a_{31} &= -\rho_f; & a_{32} &= (C - M r_1)/c^2_1; & a_{33} &= (C - M r_2)/c^2_2; & a_{34} &= 0; \\
    a_{43} &= 2 \sin \theta \cos \theta_2 / c_w c_2; & a_{41} &= 0; & a_{42} &= 2 \sin \theta \cos \theta_2 / c_w c_1; & a_{44} &= \sin^2 \theta / c^2_2 - \cos^2 \theta / c^2_1;
\end{align*}
\]