Response function beyond mean field of neutron-rich nuclei

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The damping of single-particle and collective motion in exotic isotopes is a new topic and its study may shed light on basic problems of nuclear dynamics. For instance, it is known that nuclear structure calculations are not able, as a rule, to account completely for the empirical single-particle damping. In this contribution, we present calculations of the single-particle self-energy in the case of the neutron-rich light nucleus $^{28}$O, by taking proper care of the continuum, and we show that there are important differences with the case of nuclei along the valley of stability.

1. Introduction

A useful approximation to describe atomic nuclei is in terms of an average potential in which neutrons and protons move independently of each other. This average potential is not a static one, and undergoes conspicuous fluctuations which give rise to collective oscillations of the system. The coupling of the particle motion to the collective modes is at the origin of the energy dependence of the average potential (cf., e.g., Refs. [1, 2]). The empirical energy-dependent average potential is also able to account (at positive energy) for the behavior of a particle impinging on the nucleus in a scattering experiment, as it includes an imaginary part which corresponds to the absorptive scattering channels. Within the so-called nuclear structure approach, the Hartree-Fock (HF) mean field can thus be viewed as a starting point to which corrections can be added, the leading ones being associated with processes in which a nucleon excites 1 particle-1 hole (1p-1h) pairs – preferably correlated pairs, that is, nuclear collective vibrations. Such corrections have been extensively studied for nuclei along the valley of stability. As a rule one finds that this approach is able to account only for about 50% of the empirical damping of single-particle motion [3]. This is still an unresolved issue and points to the limits of accuracy of our understanding of the nuclear dynamics.

In keeping with these facts, it is quite natural to ask the question whether in the recently discovered exotic nuclei (for instance, in systems with larger neutron excess compared to the stable isotopes) the situation described above is changed or not. This is one motivation for the present work. An even stronger motivation comes from the fact that, as it is well known [3], the coupling of the single-particle motion to nuclear vibrations is responsible for most of the spreading widths of giant resonances in stable nuclei. In the case of exotic nuclei, the importance of the coupling with the continuum, that is, of the escape width of both the single-particle states and collective states has been stressed in many works.
On the other hand, not much work has been done to calculate the spreading width due to coupling with more complicated configurations (2p-1h for single-particle states, 2p-2h for giant resonances).

In a previous work \[4\], we have calculated the quadrupole response in the neutron-rich nucleus $^{28}$O by taking into account both the escape and spreading width. We have reached the conclusion that the spreading width is as important as the escape width, at least in the example considered. Some of the results of Ref. \[4\] were considered to be model-dependent, so that a more refined calculation is highly desirable. In fact, so far one has been able to take properly into account the continuum part of the single-particle spectrum in calculations at the 1p-1h level and “more refined calculation” means therefore, within the present context, a proper treatment of the continuum also at the 2p-2h level. With this purpose we performed calculations in which the self-energy of particles (holes) is obtained by coupling with 2p-1h (2h-1p) configurations and for the first time in the study of exotic nuclei, we properly treat the continuum at this 2p-1h (2h-1p) level.

Preliminary results are reported in this contribution. In a previous work \[5\], considerations based on analytical expressions had been presented and we confirm here the soundness of these considerations by means of some numerical findings.

2. Imaginary part of the single-particle self-energy in exotic nuclei

The imaginary part of the single-particle self-energy felt by a nucleon in the presence of an $A$-particle bound system has been one of the main concerns of many studies in nuclear structure, both experimental and theoretical. When the nucleon is at positive energy and its wave function has ingoing wave boundary conditions, this imaginary part corresponds to the imaginary part of the optical potential. Whereas for heavy ions the dominant contribution to the optical potential comes from transfer phenomena and this can be accurately calculated \[6\], in the case of nucleon scattering the theoretical understanding of the optical potential is still a rather open problem. In fact, there is not a clear-cut explanation why, for instance, the optical potential calculated by starting from nuclear matter studies and using a local density approximation to deal with finite nuclei, reproduces more or less the strength of the empirical optical potential; and why, on the other hand, within the above mentioned nuclear structure approach, only about one half of the potential strength can be accounted for.

Within this latter approach \[7, 8\], the optical potential is calculated by adding to the HF mean field ($V_{HF}$) the contribution $\Delta V$ coming from the excitation of 2p-1h states (mainly correlated, that is, 1p-1 collective vibration). Only $\Delta V$ gives rise to an imaginary part in the single-particle potential. In particular, only the upper two diagrams associated with $\Delta V$ and shown in Fig. 1, are relevant. We report the analytic expression for the upper left diagram, contributing to the particle self-energy, which reads

$$Im \Delta V_{l,j}(r, r'; \omega) = \sum_{l', j', \lambda} \int d\omega_\lambda \left(-\pi \right) v(r) v(r') \tilde{u}_{l'j'}^{(\omega-\omega_\lambda)}(r) \tilde{u}_{lj}^{(\omega-\omega_\lambda)}(r') \times$$

$$\times \delta g^{(\omega_\lambda)}(r) \delta g^{(\omega_\lambda)}(r') \frac{(l_j || Y_{lj} || l'_j)^2}{2j + 1}.$$

The quantum numbers $l, j$ ($l', j'$) refer to the initial (intermediate) state, $\tilde{u}(r)$ are radial
wave functions of the particles, $\delta g(r)$ are transition densities of 1p-1h pairs coupled to multipolarity $\lambda$ and $v(r)$ is the p-h interaction derived from the effective force from which the HF mean field is determined (in our case, we have used the Skyrme force SIII). The link between Eq. (1) and the empirical imaginary part of the optical potential $W$ is as follows,

$$W(r, r'; \omega) = \sum_{ij} \frac{2j + 1}{4\pi} Im \Delta V_{ij}(r, r'; \omega).$$  

The real part is obtained in a similar way, or through dispersion relation techniques (see Ref. [1, 2] and references therein). A procedure exists to connect the non-local potential with its local equivalent, which is the quantity to be compared with the empirical parametrizations or to be given as an input in a reaction calculation, e.g., of DWBA type.

The novel feature of Eq. (1), in comparison with what has been used in the past, is that we treat properly the continuum also at the level of the 2p-1h doorway states. In the calculations performed in the seventies in well-bound nuclei like $^{208}$Pb, discrete particle states were employed as the whole system was set in a box and an averaging parameter was employed in Eq. (1) to ensure the match of the initial 1p and intermediate 2p-1h energies (see Ref. [1] and references therein). This procedure is satisfactory for well-bound systems as we have been able to confirm by making use of Eq. (1) for a test calculation in $^{208}$Pb where we have reproduced essentially the results of Ref. [8]. For systems with loosely bound nucleons, where the correct treatment of the continuum is more important, the use of discrete particle states is not able to reproduce the results that we are going to illustrate in Sec. 3.

Figure 1. The diagrams corresponding to the leading terms of the particle (left column) or hole (right column) self-energy.
Before that, we use Eq. (1) to get some insight in the qualitative low-energy behavior of the imaginary part of $\Delta V$ (by “low-energy” we mean for values of the energy $\omega$ close to the particle emission threshold $\omega_{th}$). We use the fact that in the case of nuclei with loosely bound neutrons, the low-energy part of the (1p-1h) multipole response is characterized by a pronounced “threshold effect”, that is, by a sudden increase of the strength function $S(\omega)$ above $\omega_{th}$ [4, 5]. On ground of simple arguments, it is expected that

$$S(\omega) \sim (\omega - \omega_{th})^{l'+1/2},$$

where $l'$ is the orbital angular momentum of the particle states contributing to $S(\omega)$.

Let us consider a single term of the sum appearing in eq. (1) and fix $r = r'$. A plane-wave approximation for $\bar{u}^{(\omega-\omega_{\lambda})}_{l'j'}(r)$ suggests that it contributes with a factor $k_{part}^{l'+1/2}$ if $k_{part} = \hbar^{-1}\sqrt{2m(\omega - \omega_{\lambda})}$ is close to zero. Adding the condition on the normalization of the radial transition densities given by

$$| \int dr \ r^{2+\lambda} \delta^{(\omega_{\lambda})}(r) |^2 = S(\omega),$$

with the asymptotic behaviour of the strength recalled in Eq. (3), we obtain

$$Im\Delta V \sim \int_{\omega_{th}}^{\omega} d\omega_{\lambda} \ (\omega - \omega_{\lambda})^{l'+1/2} (\omega_{\lambda} - \omega_{th})^{m'+1/2}.$$ (4)

If $\omega$ is close to $\omega_{th}$, i.e., $\omega = \omega_{th} + \delta$ and $\omega_{\lambda} = \omega_{th} + \delta/2$, we find that the imaginary part of the optical potential behaves approximately like $\delta^{l'+m'+2}$, that is, a fast increase just after threshold which has no counterpart in stable nuclei (where $Im\Delta V \sim (\omega - \varepsilon_{F})^{n}$, with $1 \leq n \leq 2$ [4]) and has consequences on the qualitative features of scattering experiments with exotic beams. A realistically calculated $Im\Delta V$ follows indeed this asymptotic behaviour and this is shown in the next Section.

3. Results

We have chosen as an example of neutron-rich nucleus, the isotope $^{28}$O. It is double magic and the neutron separation energy (about 1 MeV according to the HF calculation already mentioned, employing the SIII interaction) lies between the stable nuclei and the extreme cases of nuclei with a very extended halo like $^{11}$Li. We must recall that a number of experimental evidences have been recently accumulated which point to the non-existence of this isotope as a bound system [4]. Since essentially all calculations predict $^{28}$O to be bound, a strong point should be made about the necessity of improving our tools for nuclear structure studies, to face the outcome of the considerable experimental efforts made in the field of drip-line nuclei.

We have calculated, in the case of neutrons, the quantity $W$ defined by Eq. (2). First, we have solved the HF equations with the Skyrme interaction SIII on a radial mesh of 250 points with a 0.1 fm step. As a result, the levels shown in Fig. 1 of Ref. [4] are obtained. In particular, the highest occupied neutron orbital $d_{3/2}$ has a small binding energy of about 1.1 MeV while the protons are all bound by more than 30 MeV. This large asymmetry between the neutron and proton mean fields is clearly due to the large neutron excess. Its consequences on the multipole response are quite dramatic as remarked
Figure 2. Strength function for the isoscalar $2^+$, $3^-$ and isovector $1^-$, calculated within continuum-RPA.

first in Ref. [9]. In fact, we have calculated [4, 5] the strength function associated with isoscalar quadrupole, octupole and dipole operators, within the framework of continuum RPA in coordinate space [11]. This theory is able to treat properly the particle states in the continuum, because the one-body propagator has an exactly computable form if expressed as function of $\vec{r}$, $\vec{r}'$. We have kept 25 fm as the upper limit of the radial integrals while the radial mesh has a 0.5 fm step. The strength functions mentioned above are depicted in Fig. 2. Their main feature is that they reflect how “excess neutrons” and “core particles” degrees of freedom are decoupled in systems like the one at hand. The low-energy part, although characterized by a strong enhancement just above the particle threshold, is not associated to any collective effect. What is enhanced so much to form a well-defined bump, are the single-particle transitions from the $d_{3/2}$ neutron state to s-, p-, or d-states in the continuum: because of the small $d_{3/2}$ binding energy, the wave function of these neutrons is so extended that its overlap with continuum wave functions is large and this makes the matrix elements of the multipole operators quite large. On the other hand, the higher energy region of the strength functions is characterized by states to which more pf configurations partecipate (neutron excess states play a predominant role but neutron core states are not negligible). Transitions from proton states are completely decoupled and lie above 30 MeV.

From the continuum RPA calculation we can extract the radial transition densities $\delta g(\omega\lambda)(r)$ which appear in Eq. (1). For each multipolarity $\lambda = 2^+, 3^-, 1^-$, we take energy bins of 1 MeV for $\omega\lambda$ and we associate to each of them the appropriate $\delta g(r)$ so that the integral $\int d\omega\lambda$ can be performed. The continuum wave functions $\tilde{u}(r)$ are obtained by integrating the radial Schrödinger equation including the kinetic energy plus the HF
Figure 3. Optical potential $W$ as defined in Eq. (2), plotted in the case of $r = r' = 3.2$ fm, as a function of the energy $\omega$ close to $\omega_{th}$ (thick line). In this low-energy region, the approximation $W \sim (\omega - \omega_{th})^4$ is valid (thin line).

potential at positive energy $\omega - \omega_{th}$.

We first study the low-energy asymptotic behavior of $W$ (Eq. (2)), to see whether it meets the expectations from Eq. (4). In Fig. 3, we show (by means of thick line) the values of $W$ for $r = r'$ fixed approximately at the nuclear surface. The thin lines is a function of the type $C(\omega - \omega_{th})^4$, with $C$ constant and chosen to get the best fit to the microscopic calculation. The good agreement between the two lines confirms the soundness of the argument of the last Section, about the asymptotic behavior of the optical potential.

In Fig. 4, $W(r, r' = r; \omega)$ is plotted as a function of $r$ for three different values of the energy $\omega$. As in stable nuclei, this function is surface peaked. On the other hand, the width of the peak is rather large, as can be expected if the nucleus has extended radial wave functions. All these characteristics of the imaginary part of the self-energy in exotic nuclei will be studied in more details in a forthcoming work. We can also notice that for $\omega = 25$ MeV a second bump, at the interior of the nucleus, shows up. This can be interpreted as follows. In the calculation of the terms of the sum appearing in Eq. (2), one performs integrals of the type $\int_{\omega_{th}}^\omega d\omega_{\lambda}$. If $\omega$ is large, one includes, through the transition densities, contributions from protons and these are of course peaked elsewhere than on the nuclear surface, since the protons are concentrated in a much smaller region. In this respect, this interior peak is another example of the general statement according to which core particles and excess particles are decoupled in this light, exotic isotopes.

\[^1\text{Fig. 1 of Ref. \cite{5} shows that the presence of nucleons with binding energies of a few MeV give rise to a very diffuse surface. We have taken here a value of } 3.2 \text{ fm, where the density is approximately half of its central value.}\]

\[^2\text{Fig. 2 of Ref. \cite{5} shows } W(\pi, \pi; \omega) \text{ plotted.}\]
Figure 4. Optical potential $W$ as defined in Eq. (2), plotted in the case of $r = r'$, as a function of the radial distance for three different values of the energy $\omega$ (dotted line: 5 MeV; dashed line: 15 MeV; full line: 25 MeV).

4. Real part of the self-energy

The off-shell diagrams which are shown in the lower part of Fig. 1 contribute to the real part of the self-energy, in addition to the two which appear above. It has been shown in the past (see Ref. [1] and references therein) that dispersion relation techniques can be used to determine the values of Re $\Delta V$, once Im $\Delta V$ has been calculated. We do not recall here but the basic idea, by using again the particle self-energy (diagrams in the left column of Fig. 1).

The imaginary part of the self-energy (Eq. (1)) can be obtained as a limit,

$$
\text{Im} \Delta V_{lj}(r, r'; \omega) = \text{Im} \lim_{\eta \to 0} \sum_{l', j', \lambda} \int d\omega_{\lambda} \int d\omega' \frac{f}{\omega - (\omega' + \omega_{\lambda}) + i\eta},
$$

where $f \equiv f_{lj', j', \lambda}(r, r'; \omega, \omega', \omega_{\lambda})$ denotes the product of the two particle-vibration vertices. The integral over $d\omega_{\lambda}$ runs from $\omega_{th}$ to $+\infty$. The integral over $d\omega'$ can be extended from $-\infty$ to $+\infty$ and then the contribution from the 2 particles-1 hole-1 phonon intermediate states (lower left diagram of Fig. 1) is also included. This is just a formal extension if one takes the imaginary part of the limit $\eta \to 0$, since this results in a delta-function $\delta(\omega' - (\omega - \omega_{\lambda}))$ which rules out the 2 particles-1 hole-1 phonon states contribution if $\omega$ is positive. But if we consider the real part in the limit $\eta \to 0$, we easily get the expression for the real part of the self-energy,

$$
\text{Re} \Delta V_{lj}(r, r'; \omega) = \text{Re} \lim_{\eta \to 0} \sum_{l', j', \lambda} \int d\omega_{\lambda} \int d\omega' \frac{f}{\omega - (\omega' + \omega_{\lambda}) + i\eta}.
$$
\[ \Delta E_{nlj} = \int dr dr' r u_{nlj}(r) \text{Re} \Delta V_{lj}(r, r'; \omega) r' u_{nlj}(r'), \]  

where \( \omega \) is usually fixed as the unperturbed (i.e., HF) energy \( E^{(0)}_{nlj} \) of the level under consideration. The expressions for \( \text{Re} \Delta V_{lj} \) in the case of hole states are analogous to those derived above for the case of particles. In the case of \( ^{28}\text{O} \) calculated within the SIII-HF procedure, no unoccupied particle states can be found at negative energy and the only particle resonance at positive energy is the \( f_{7/2} \), so that the particle spectrum consists essentially only of a smooth continuum. Therefore, we have calculated the shift \( \Delta E_{nlj} \) for neutron hole states, in particular for the loosely bound \( d_{3/2} \) orbital.

We find a positive shift of 330 keV, to be compared with \( E^{(0)} = -1.1 \text{ MeV} \). This makes the nucleus more close to being unbound and of course would in turn affect the strength function of the multipoles we have considered in Sec. 3. As a conclusion to this part, the following two statements about the role of the real part of the self-energy in neutron-rich nuclei can be made. (a) The particle shift of 330 keV obtained for the loosely bound \( d_{3/2} \) orbital forces us to ask questions about the reliability of simple mean field methods to predict the single-particle energies and consequently the position of drip lines, as well as the properties of the excited states. This is not the case for stable nuclei, since one usually obtains corrections of the order of less than 1 MeV to the energy of states which are usually 7-10 MeV bound. These corrections are important to give the correct density of states around the Fermi energy and effective mass but do not alter the predictive power of theories like the standard RPA in which these corrections are not taken into account.  
(b) The fact that the net result of 330 keV comes from a strong cancellation between the two diagrams of the right column of Fig. 1 (930 keV - 600 keV), one has to wonder about the use of second RPA (as we did in [4]) where only the first of the two diagrammatic contributions is included. In conclusion, we believe that further research along this line can address some basic problems of the present physics of nuclei with large neutron excess.

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