\(\alpha+\alpha+t\) cluster structures and Hoyle-analogue states in \(^{11}\)B

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Abstract. The structures of \(^{11}\)B are studied with the \(2\alpha+t\) orthogonality condition model (OCM) for exploring Hoyle-analogue states in \(^{11}\)B.

Cluster picture as well as mean-field picture is important to understand the structure of light nuclei. The Hoyle state (the \(0^+_2\) state at \(E_x = 7.65\) MeV in \(^{12}\)C) is a typical cluster state with a \(3\alpha\) cluster structure \([1,2]\), which appears at only 0.38 MeV above the \(3\alpha\) threshold. This Hoyle state has been reinvestigated from the viewpoint of \(\alpha\) condensation \([3]\). The definitions and occurrences of the nuclear \(\alpha\)-particle condensation are discussed in detail in Ref. \([4]\). Many theoretical works have shown that the Hoyle state has a \(3\alpha\)-condensate-like structure, in which the \(3\alpha\) particles occupy an identical 0S-orbit with 70 % probability, forming a dilute \(\alpha\)-gas-like configuration with \((0S_{\alpha})^3\), i.e. the product state of three \(\alpha\)'s \([5,6,7]\).

Recently the \(4\alpha\) OCM (Orthogonality Condition Model) calculation \([8]\) succeeded for the first time in describing the structure of \(^{16}\)O up to \(E_x \approx 15\) MeV, and showed that the \(0^+_3\) state around the \(4\alpha\) threshold is a strong candidate of \(4\alpha\)-particle-condensate-like character, having a large \(\alpha\) condensate fraction of 90 \%. Similar dilute-gas states of \(\alpha\) clusters have been predicted around their alpha cluster disintegrated thresholds in self-conjugate \(A = 4n\) nuclei \([3,9]\). Besides the \(4n\) nuclei, one can also expect cluster-gas states composed of alpha and triton clusters (including valence neutrons etc.) around their cluster disintegrated thresholds in \(A \neq 4n\) nuclei, in which all clusters are in their respective 0S orbits, similar to the Hoyle state with \((0S_{\alpha})^3\). The states, thus, can be called \(\text{Hoyle-analogue}\) in non-self-conjugated nuclei. It is an intriguing subject to investigate whether or not Hoyle-analogue states exist in \(A \neq 4n\) nuclei, for example, \(^{11}\)B \((^{13}\)C\), composed of \(2\alpha\) and \(t\) clusters (3\(\alpha\) clusters and a valence neutron).

The purposes of the present study are to test the structure of \(^{11}\)B up to around the \(\alpha+\alpha+t\) and to explore the Hoyle-analogue state with the \(\alpha+\alpha+t\) structure \([10]\). This Hoyle-analogue states are expected to appear in \(1/2^+\) states because one can conjecture an appearance of the product state, \((0S_{\alpha})^2(0S_t)\), in \(J^\pi = 1/2^+\) of \(^{11}\)B \((1/2^+\) comes from the spin of triton\) from the similarity of \((0S_{\alpha})^3\) in the Hoyle state. The structure of the positive-parity states (isospin \(T = 1/2\)) in \(^{11}\)B so far have not been well discussed in the no-core shell model \([11]\) and AMD \([12]\). In the present study, we consider the \(\alpha+\alpha+t\) OCM with the Gaussian expansion method, the model space of which is large enough to cover the \(\alpha+\alpha+t\) gas, the \(^7\)Li+\(\alpha\) cluster, as well as the shell-model configurations.

The energy levels of \(3/2^-\) and \(1/2^+\) states (those of the positive-parity states) in \(^{11}\)B are shown in Fig. 1 (Fig. 2). The \(3/2^-\) state is the ground state with a shell-model-like structure. The
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**Figure 1.** Calculated energy levels of $3/2^-$ and $1/2^+$ states in $^{11}$B with respect to the $\alpha + t$ threshold together with the experimental data [10].

The $3/2^-$ state appears at $E_x = 8.2$ MeV ($E = -2.9$ MeV referring to the $2\alpha + t$ threshold). The radius of $3/2^-$ is 3.00 fm. This value is by about 30% larger than that of the ground state of $^{11}$B, and the rms distance between $\alpha$ and $\alpha$ (between $^8$Be($2\alpha$) and $t$) is 4.47 fm (3.49 fm). Thus, $3/2^-$ has a $2\alpha + t$ cluster structure. A characteristic feature of $3/2^-$ is that the isoscalar monopole transition rate $B(E0:1S)$ is as large as $96 \pm 16$ fm$^4$ [13], comparable to that of the Hoyle state ($120 \pm 9$ fm$^4$). The present model (92 fm$^4$) reproduces well the data. It is interesting to study whether $3/2^-$ possesses an $\alpha$ condensate nature like the Hoyle state. In order to see it, we study the single-cluster orbits and their occupation probabilities in $3/2^-$ by solving the eigenvalue equation of the single-cluster density matrices [4].

**Figure 2.** Calculated energy levels of the positive-parity states in $^{11}$B with respect to the $\alpha + \alpha + t$ threshold together with the experimental data [10].

The $3/2^-$ state appears at $E_x = 8.2$ MeV ($E = -2.9$ MeV referring to the $2\alpha + t$ threshold). The radius of $3/2^-$ is 3.00 fm. This value is by about 30% larger than that of the ground state of $^{11}$B, and the rms distance between $\alpha$ and $\alpha$ (between $^8$Be($2\alpha$) and $t$) is 4.47 fm (3.49 fm). Thus, $3/2^-$ has a $2\alpha + t$ cluster structure. A characteristic feature of $3/2^-$ is that the isoscalar monopole transition rate $B(E0:1S)$ is as large as $96 \pm 16$ fm$^4$ [13], comparable to that of the Hoyle state ($120 \pm 9$ fm$^4$). The present model (92 fm$^4$) reproduces well the data. It is interesting to study whether $3/2^-$ possesses an $\alpha$ condensate nature like the Hoyle state. In order to see it, we study the single-cluster orbits and their occupation probabilities in $3/2^-$ by solving the eigenvalue equation of the single-cluster density matrices [4].

Figure 3 shows the occupation probabilities of the $n$-th $L$-wave single-$\alpha$-particle (single-$t$-particle) orbits in the $3/2^-$ and $3/2^-$ states. In $3/2^-$, the occupation probabilities of $\alpha$ orbits spread over in several orbits, and those of $t$ orbits concentrate mainly on two orbits. These results originate from the SU(3) nature of the $3/2^-$ state as mentioned above. On the other hand, in $3/2^-$, there is no concentration of the $\alpha$ occupation probability on a single orbit. This result is in contrast with those of the Hoyle state (see Fig. 2(d)). Consequently the $3/2^-$ state could not be identified as the analogue of the Hoyle state. The reason why the $3/2^-$ state is not of the Hoyle-analogue is given as follows: The $3/2^-$ state is bound by 2.9 MeV with respect to the $2\alpha + t$ threshold, while the Hoyle state is located by 0.38 MeV above the $3\alpha$ threshold. This extra binding energy of $3/2^-$ with respect to the $2\alpha + t$ threshold suppresses strongly the growth of the gas-like $2\alpha + t$ structure.

As for the $1/2^+$ states, the $1/2^+$ state appears as a bound state at $E_x^{\text{exp}} = 6.79$ MeV around the $^7$Li+\alpha threshold. This low excitation energy indicates that $\alpha$-type correlations should play an important role in the state. In fact, we found that the $1/2^+$ state with the nuclear radius of 3.14 fm has a $^7$Li(g.s)+\alpha structure with $P$-wave relative motion, although the $^7$Li(\alpha + t) part is rather distorted in comparison with the ground state of $^7$Li. Since the $3/2^-$ state has the largest $S^2$ factor for the $^7$Li(g.s)+\alpha channel with $S$-wave relative motion, the $1/2^+$ and $3/2^-$ states of

\begin{align*}
\text{EXP} & \quad \text{CAL} \\
3/2^- & \quad 3/2^- \\
1/2^+ & \quad 1/2^+
\end{align*}
$^{11}\text{B}$ can be interpreted as the parity-doublet partners of each other.

In addition to the $1/2^+_1$ state, the $1/2^+_2$ state appears as a resonant state at $E_x = 11.95$ MeV ($\Gamma = 190$ keV) around the $2\alpha + t$ threshold with the complexed-scaling method. The large radius ($R_N = 5.98$ fm) indicates that the state has a dilute cluster structure. The analysis of the single-cluster properties showed that this state has as main configuration $(0S_\alpha)^2(0S_t)$ with about 65% probability. Thus, the $1/2^+_2$ state can be called the Hoyle-analogue with $(0S_\alpha)^2(0S_t)$. Recently, the $1/2^+_1$ (3/2 $^+_1$) state at $E_x = 12.56$ MeV with $\Gamma = 210 \pm 20$ keV (located at 1.4 MeV above the $2\alpha + t$ threshold) was observed in the $\alpha + ^6\text{Li}$ decay channel [14]. The energy and width of the $12.56$-MeV state are in good correspondence to the present study. The Hoyle-analogue state in $^{11}\text{B}$, thus, may be assigned as the $12.56$-MeV state. It should be reminded that $1/2^+_2$ is located by 0.75 MeV above the $\alpha + \alpha + t$ threshold, while $1/2^+_1$ is bound by 4.2 MeV with respect to the three cluster threshold. The latter binding energy leads to the suppression of the development of the gas-like $\alpha + \alpha + t$ structure in $1/2^+_1$, whereas the gas-like structure with a large nuclear radius develops in the $1/2^+_2$ state because it appears above the three-body threshold.

In $^{13}\text{C}$, the Hoyle-analogue states with the dominant configuration of $[(0S_\alpha)^3s_{1/2}(n)]$ are expected to appear in $1^+_2$ states, because the three $\alpha$ particles (the valence neutron) are in $S$ orbit ($s_{1/2}$). The structure of $^{13}\text{C}$ has been studied with the $3\alpha+n$ OCM [15], the model space of which can cover the $3\alpha+n$ gas, the $^{12}\text{C}+n$ cluster, the $^9\text{Be}+\alpha$ cluster, the $^8\text{Be}+^5\text{He}$ cluster, as well as the shell-model-like configurations. This model succeeded in reproducing the lowest four $1/2^-$ states and three $1/2^+$ states up to about the $3\alpha+n$ threshold ($E_x = 12.5$ MeV), including the shell-model-like ground state (1/2$^+_1$) [15]. We found that the 1/2$^+_2$ state around the $3\alpha+n$ threshold (see Fig. 5) has the nuclear radius of 5.40 fm with a dilute $\alpha$ condensate feature, in which the $3\alpha$ particles occupy an identical OS orbit with 55% probability and the valence neutron is in the $s_{1/2}$ state. This means that the $1/2^+_2$ state is a candidate of the Hoyle-analogue

![Figure 3](image-url)

**Figure 3.** Occupation probabilities of the single-$\alpha$-particle and single-$t$-particle orbits for the (a) $3/2^-_1$ (g.s.) and (b) $3/2^-_3$ states in $^{11}\text{B}$ [10]. We also show those of the single-$\alpha$-particle orbits for the (c) $0^+_1$ (g.s.) and (d) $0^+_2$ (Hoyle) states in $^{12}\text{C}$ calculated by the $3\alpha$ OCM [6].
state in $^{13}$C. The reason why the $1/2^+_{1,2}$ states in Fig. 5 have no Hoyle-analogue character is that the two states are bound with respect to the $3\alpha+n$ threshold. This extra binding energy suppresses the development of the $3\alpha+n$ gas-like structure in the $1/2^+_{1,2}$ states. From Fig. 5, one notices as a common feature that the Hoyle-analogue states in $^{16}$O, $^{11}$B, and $^{13}$C appear around their cluster disintegrated threshold, like the Hoyle state ($0^+_2$ in $^{12}$C).

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