BOUNDS ON $g^Z_5$ FROM PRECISION LEP MEASUREMENTS

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Abstract

The parity violating but CP conserving anomalous three-gauge-boson coupling $g^Z_5$ induces a universal contribution to the left-handed coupling of the Z boson to fermions. We find that the LEP measurements of the partial Z widths and lepton forward-backward asymmetries are sufficiently precise to place a bound of order $\lvert g^Z_5 \rvert$ less than $\sim 10\%$. This bound is significantly better than what can be obtained at present from rare K and B meson decays.
High precision measurements at the $Z$ pole at LEP combined with polarized forward backward asymmetries at SLC put stringent limits on any new physics beyond the standard model. The possible effects of new physics have typically been parameterized in terms of the “oblique” corrections to the standard model; those corrections involving the two-point functions of the $W$ and $Z$ gauge bosons. The ever increasing precision of the electroweak measurements leads to the question of whether limits can also be placed on the deviation of the three-point couplings from their standard model values. The tree-level effects of these couplings should be tested by future colliders. Here, we study their effect on LEP observables at the one-loop level.

The effective Lagrangian formalism is particularly useful for this study. In this approach, the low energy effects of potential new physics are parameterized by a few coupling constants multiplying the lowest dimension operators satisfying the symmetries of the standard model.

We have recently considered in some detail one of the next-to-leading operators in the effective Lagrangian, the one responsible for the “anomalous coupling” $g^{Z}_{5}$ in the conventional parameterization of the three-gauge-boson vertex. This is the only operator that is parity violating, but CP conserving. It contributes to the three and four gauge boson vertices $[1, 2]$. In this paper, we consider the bounds that can be placed on the coupling constant $g^{Z}_{5}$ from precision measurements at LEP.

These bounds arise because at the one-loop level this operator modifies the $Zf\bar{f}$ couplings. Because the operator modifies the gauge boson self-couplings, its one-loop effects on the $Z$ couplings to fermions affect both the flavor diagonal and the flavor changing vertices. Since the standard model does not contain flavor changing neutral vertices at tree level, one can bound the size of $g^{Z}_{5}$ from rare meson decays. Limits on the order $g^{Z}_{5} \leq O(1)$ have been derived from low energy decays such as $K_{L}\rightarrow \mu^{+}\mu^{-}$ $[1, 3]$. The flavor diagonal vertices have now been constrained by LEP to such levels that the limits obtained from precision measurements at LEP on $g^{Z}_{5}$ are more restrictive than those obtaining from rare meson decays.

As has been discussed at length in the literature, the minimal effective Lagrangian that describes the interactions of gauge bosons of an $SU(2)_{L} \times U(1)_{Y}$ gauge theory spontaneously broken to $U(1)_{EM}$ is given by:

$$L^{(2)} = \frac{v^{2}}{4} D^{\mu} \Sigma^{\dagger} D_{\mu} \Sigma - \frac{1}{2} \text{Tr} \left(W^{\mu\nu} W_{\mu\nu}\right) - \frac{1}{2} \text{Tr} \left(B^{\mu\nu} B_{\mu\nu}\right) + \Delta \rho \frac{v^{2}}{8} \left[ \text{Tr} \left(\tau_{3} \Sigma^{\dagger} D_{\mu} \Sigma\right)\right]^{2},$$

where $W_{\mu\nu}$ and $B_{\mu\nu}$ are the $SU(2)$ and $U(1)$ field strength tensors given in terms of $W_{\mu} \equiv W_{\mu}^{i} \tau_{i}$,

$$W_{\mu\nu} = \frac{1}{2} \left( \partial_{\mu} W_{\nu} - \partial_{\nu} W_{\mu} + i g [W_{\mu}, W_{\nu}] \right)$$

$$B_{\mu\nu} = \frac{1}{2} \left( \partial_{\mu} B_{\nu} - \partial_{\nu} B_{\mu} \right) \tau_{3}.$$  

The matrix $\Sigma \equiv \exp(i \vec{\omega} \cdot \vec{\tau}/v)$, contains the would-be Goldstone bosons $\omega_{i}$ that give the $W$ and $Z$ their masses via the Higgs mechanism and the $SU(2)_{L} \times U(1)_{Y}$ covariant
derivative is given by:

\[ D_\mu \Sigma = \partial_\mu \Sigma + \frac{i}{2} g W^\mu \tau^i \Sigma - \frac{i}{2} g' B_\mu \Sigma \tau^3. \]  \hspace{1cm} (3)

The first term in Eq. (3) is the \( SU(2)_L \times U(1)_Y \) gauge invariant mass term for the \( W \) and \( Z \). The physical masses are obtained with \( v \approx 246 \text{ GeV} \). This non-renormalizable Lagrangian is interpreted as an effective field theory, valid below some scale \( \Lambda \lesssim 3 \text{ TeV} \). The lowest order interactions between the gauge bosons and fermions are the same as those in the minimal standard model.

It is useful to classify the operators in the effective Lagrangian into those that respect a “custodial” \( SU(2)_C \) symmetry and those that do not. In Eq. (3) the last term breaks the \( SU(2)_C \) symmetry. The next-to-leading order effective Lagrangian has also been discussed at length in the literature. It contains five terms which conserve the custodial \( SU(2) \) (up to hypercharge couplings) and CP, six terms which conserve CP, but break the custodial \( SU(2) \), and three terms which violate CP and the custodial \( SU(2) \) \cite{4,5}. In this letter we will consider only one of these terms, the one corresponding to a parity violating but CP conserving operator. This operator also breaks the custodial \( SU(2) \) symmetry. Because of the parity violating nature of this term, it can lead to observable signals at LEPII via \( e^+e^- \rightarrow W^+W^- \) \cite{1} and at high energy \( e\gamma \) colliders \cite{1,2}. The operator is:

\[ \mathcal{L}^{(4)} = g_\alpha^2 \frac{v^2}{\Lambda^2} \epsilon^{\alpha\beta\mu\nu} Tr \left( \tau_3 \Sigma^\dagger D_\mu \Sigma \right) \left( W_\alpha \Sigma \Sigma^\dagger \right), \]  \hspace{1cm} (4)

where \( \Lambda \) is the scale of the new physics responsible for this operator, and our normalization is such that the coupling \( \hat{\alpha} \) is naturally of order \( \mathcal{O}(1) \) if the fundamental theory does not have a custodial \( SU(2) \) or of order \( \mathcal{O}(\Delta \rho) \) if there is an underlying custodial symmetry. In terms of the conventionally used coupling:

\[ g_5^Z = \frac{g^2 c_\theta^2}{\hat{\alpha}} \frac{v^2}{\Lambda^2} = \frac{4M_Z^2}{\Lambda^2} \hat{\alpha}. \]  \hspace{1cm} (5)

A term of this form is induced at the one-loop level in the minimal standard model through fermion loops \cite{1}. Beyond the standard model it can occur, for example, in technicolor models through techni-fermion loops \cite{5}. The operator of Eq. 4 modifies the minimal standard model \( W^+W^-Z \) and \( W^+W^-Z\gamma \) interactions. It does not modify the \( W^+W^-\gamma \) interaction. We have presented the Feynman rules for this operator in unitary gauge in Ref. \cite{1}.

The lowest order (standard model) coupling of a \( Z \) to a fermion pair can be written as

\[ \mathcal{L} = -\frac{g}{4c_\theta} Z^\mu \gamma_\mu \left( R_f (1 + \gamma_5) + L_f (1 - \gamma_5) \right) f \]  \hspace{1cm} (6)

where \( R_f = -2s^2_\theta Q_f, \) \( L_f = T_3 - 2s^2_\theta Q_f, \) \( (T_3 = \pm 1) \) and \( s^2_\theta \equiv \sin^2 \theta_W. \)

When we abandon the renormalizable minimal standard model in favor of a non-renormalizable effective field theory, we must in general allow for additional
gauge-boson fermion interactions. The next-to-leading terms have been considered in Ref. 6. One of the possible terms listed in Ref. 6 is:

\[ L_f' = \frac{g}{2c_{\theta}} \bar{f}_i \gamma_{\mu} (\kappa_{L}^{NC})_{ij} f_j Z^\mu. \] (7)

The notation is that of Ref. 6. The index \( L \) indicates that this operator involves only the left-handed fermions (there are analogous operators involving the right-handed fermions but we will not concern ourselves with those). The \( i, j \) are generation indices indicating that in general this operator modifies the tree-level coupling \( L_f \) and also introduces flavor changing neutral couplings to the \( Z \). The limits on flavor changing neutral currents have been very stringent for a long time. Those limits, as well as low energy measurements of \( L_f \), taken from Ref. 7 led the authors of Ref. 6 to conclude that the \( (\kappa_{NC}^{L})_{ij} \) had to be less than 1%.

Operators like Eq. 4 induce \( (\kappa_{NC}^{L})_{ij} \) at the one-loop level, and thus at a natural size of a few percent or less. These one-loop contributions to \( (\kappa_{NC}^{L})_{ij} \) are formally of next-to-next-to-leading order. To reach precise conclusions from the study of these effects one would need a complete effective field theory analysis at next-to-next-to-leading order where the number of free parameters becomes too large to be useful. However, we can bound three-gauge-boson coupling constants like \( g_5^Z \) from their contribution to \( (\kappa_{NC}^{L})_{ij} \) under the naturalness assumption that no cancellation will take place amongst the different contributions that can occur at the same order.

We thus proceed to evaluate the diagram of Fig. 1 to find the effect of \( g_5^Z \) on the \( Z\bar{f}f \) coupling. In order to preserve gauge invariance we use dimensional regularization. Since, as discussed above, we will not consider all the possible couplings that occur at next-to-next-to-leading order (that would act as counterterms for our loop calculations) we base our analysis on the leading non-analytic contributions from the loop diagrams. (This has become common practice in chiral perturbation theory whenever a complete calculation with all possible counterterms is not practical 8.)

We thus compute only the terms that go like \( \log(\mu) \) for the diagram shown in Fig. 1. These terms are easily found from the coefficient of the divergent part of the integral (which is dropped along with all other finite parts). This gives us an estimate for the size of the new physics effects if we choose the scale \( \mu \) in such a way that the \( \log(\mu/M_W) \) is of order one. We find that this type of new physics affects only the left handed coupling of the \( Z \) to fermions and its effects can be incorporated by modifying the tree level coupling in the form:

\[ L_f \rightarrow L_f + \eta c_{\theta}^2 / s_{\theta}^2 \]  

where

\[ \eta = \frac{3\alpha}{2\pi} g_5^Z \log \left( \frac{\mu}{M_W} \right). \] (8)

We have also neglected the mass and momentum of the external fermions compared to the \( Z \) mass. This modification of the lowest order vertices, \( \delta L_f \), does not grow with \( M_t \) and is therefore a universal contribution to all left-handed fermion interactions.

In addition to this direct contribution of \( g_5^Z \) to the \( Z\bar{f}f \) vertex we must consider indirect effects due to renormalization. In particular, the operator of Eq. 4 also
modifies the $W^\pm \to \ell^\pm \nu$ coupling, contributing in this way to muon decay and thus introducing a renormalization of $G_F$. At the one loop-level (and always working to lowest order in $g_Z$) we find from the diagrams in Figure 2 the following Lagrangian for the effective $W^\pm \to \ell^\pm \nu$ coupling:

$$\mathcal{L} = -\frac{g}{2\sqrt{2}} \left(1 - \frac{\eta}{2}\right) W^\nu \gamma_\mu (1 - \gamma_5) \nu.$$  \hspace{1cm} (9)

We choose as our standard model input parameters: $G_F$ as measured in muon decay, $\alpha_s(M_Z^2) \approx 1/128.8$ and the physical $Z$ mass. We then use a $s_\theta^2$ defined by the relation:

$$s_\theta^2 c_\theta^2 \equiv \frac{\pi \alpha_s}{\sqrt{2} G_F M_Z^2}. \hspace{1cm} (10)$$

Because of the structure of the $g_5^Z$ interaction (the epsilon tensor), there are no one-loop contributions linear in $g_Z^2$ to any of the two point functions and hence no wavefunction or $M_Z$ renormalization linear in $g_Z^2$. There is also no renormalization of $\alpha_s$ proportional to $g_Z^2$ since the $W^+ W^- \gamma$ vertex is unaffected by the interaction of Eq. 4.

We are now in a position to compute the contribution of the $g_5^Z$ term to the measured observables at LEP.

The extraordinary agreement between the standard model predictions and the measurements at LEP allows us to put very stringent bounds on the existence of the coupling $g_5^Z$. We begin by considering the partial decay widths. For $Z \to f \bar{f}$ we write:

$$\Gamma(Z \to f \bar{f}) = \Gamma^{SM}_f + \delta \Gamma^5_f \equiv \Gamma^{SM}_f \left(1 + \frac{\delta \Gamma^5_f}{\Gamma^{(0)}_f}\right), \hspace{1cm} (11)$$

where we have factored out the standard model result including radiative corrections, $\Gamma^{SM}_f$. Since we drop higher order terms involving both a standard model radiative correction and a correction due to $g_5^Z$, we normalize the corrections introduced by $g_5^Z$, $\delta \Gamma^5_f$, to the tree level partial width:

$$\Gamma^{(0)}(Z \to f \bar{f}) = N_c (R_f^2 + L_f^2) G_F M_Z^3 \frac{12 \pi \sqrt{2}}{s_\theta^2 c_\theta^2}. \hspace{1cm} (12)$$

where $N_c = 3$ (1) for quarks (leptons). At the one-loop level the correction to the partial widths introduced by the operator Eq. \[ is then:

$$\frac{\delta \Gamma^5_f}{\Gamma^{(0)}_f} = \eta \left[ \frac{2 L_f}{L_f^2 + R_f^2} \frac{c_\theta^2}{s_\theta^2} + \left(1 + \frac{2 R_f (L_f + R_f)}{L_f^2 + R_f^2} \frac{c_\theta^2}{s_\theta^2 - c_\theta^2}\right) \right]. \hspace{1cm} (13)$$

As we said, we find two contributions to $\delta \Gamma^5_f$. The first term in Eq. 13 is the direct contribution from the induced $Z f \bar{f}$ vertex, and the second term in Eq. 13 comes from the indirect contribution due to the renormalization of the tree-level parameters in the lowest order result.

\[1\]We are grateful to W. Marciano for bringing this point to our attention.
We compare the standard model predictions, $\Gamma_{SM}$, including the one loop QED and QCD radiative corrections with the most recent results from LEP. We use the theory numbers of Langacker [13] which use the global best fit values for $M_t$ and $\alpha_s$ with $M_H$ in the range 60 – 1000 GeV. They include errors from the uncertainty in $M_Z$ and $\Delta r$, $M_t$ and $M_H$, and from the uncertainty in $\alpha_s$. We add this theoretical uncertainty and the experimental error in quadrature. In all cases we will use $\mu = 1$ TeV as the typical scale in Eq. (8). We present our results as 90% confidence level intervals for the allowed values of $g_5^Z$ in Table 1. The sensitivity of a particular observable to $g_5^Z$ depends on the combination of couplings in Eq. [13]. The most sensitive observable listed in the table is $R_h$.

Table 1: 90% confidence level intervals for $g_5^Z$ from different LEP observables.

| Observable | Experiment [12] | SM prediction [13] | 90% c.l. interval for $g_5^Z$ |
|------------|-----------------|--------------------|-----------------------------|
| $\Gamma_{ee}$ | $(83.98 \pm 0.18$ MeV | $(83.87 \pm 0.1)$ MeV | $(-0.10, 0.05)$ |
| $\Gamma_{\nu}$ | $(499.8 \pm 3.5$ MeV | $(501.9 \pm 0.91)$ MeV | $(-0.08, 0.04)$ |
| $R_h$ | $(20.795 \pm 0.040)$ | $20.782 \pm 0.031$ | $(-0.07, 0.10)$ |
| $\Gamma_Z$ | $(2497.4 \pm 3.8$ MeV | $(2496 \pm 4.4)$ MeV | $(-0.9, 1.2)$ |

The limits presented in Table 1 are more than an order of magnitude better than the limit $g_5^Z < 0(1)$ obtained from rare $K$ and $B$ decays [14]. They are comparable to the limits that could be placed in future high energy $e^-\gamma$ colliders [4].

We have not included in Table 1 the $Z \to b\bar{b}$ width. Presented as a ratio to the total hadronic width of the $Z$, the latest experimental result is $R_{bb} = 0.2202 \pm 0.0020$ [12]. The theoretical prediction is $\delta_{bb}^{new} = 0.022 \pm 0.011$ [13], where $\delta_{bb}^{new} = [\Gamma(Z \to b\bar{b}) - \Gamma(Z \to b\bar{b}^{SM})]/\Gamma(Z \to b\bar{b}^{SM})$, which falls outside the experimental 90% c.l. range. A negative value of $g_5^Z$ would push the theoretical prediction up and we can use this to obtain $|g_5^Z| \leq 0.7$ at 90% c.l. which is not competitive with the limits presented in Table 1.

We can also extract a limit on $g_5^Z$ from forward backward asymmetries at LEP. The new interaction proportional to $g_5^Z$ modifies the asymmetries in a way most easily parameterized by:

$$g_V/g_A = \left(\frac{g_V}{g_A}\right)_{SM} \left[1 + \left(\frac{2 R_\ell}{\sqrt{R_\ell^2 - L_\ell^2 S_\ell^2}} \frac{c_\theta^2}{s_\theta^2} + \frac{2 R_\ell}{R_\ell + \sqrt{L_\ell^2 s_\ell^2 - c_\theta^2}}\right)\right]$$  \hspace{1cm} (14)$$

where again the first correction term comes from the direct $Zf\bar{f}$ vertex and the second one from the renormalization of the tree level parameters. With the current theoretical results for the effective $g_V/g_A$, and experimental value, the bounds placed on $g_5^Z$ are not as good as those coming from the partial $Z$ widths. Using the numbers in Ref. [14] for example, we find the 90% c.l. interval for $g_5^Z$: $-0.3 \leq g_5^Z \leq 0.1$.

In conclusion we have found that precision measurements of the partial $Z$ widths at LEP constrain the anomalous three-gauge-boson coupling $g_5^Z$ to be of order $|g_5^Z|$ less than $\sim 10%$. This is significantly better than the bounds that can be placed on
from rare $K$ and $B$ meson decays. It is comparable to the bounds that one will be able to place on this coupling at future $e\gamma$ colliders. These limits, however, should be taken only as approximate indications of the sensitivity of the LEP experiments to the presence of the $g_5^Z$ coupling. More precise statements would require a complete effective field theory calculation at next-to-next-to leading order.

Acknowledgments
The work of S. Dawson is supported by the U.S. DOE under contract DE-AC02-76CH00016. We are grateful for conversations with W. Marciano and A. Sirlin.

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FIGURE CAPTIONS
1. One-loop diagram giving a contribution from the anomalous three-gauge-boson coupling $g_5^Z$ to the $Zf\bar{f}$ vertex.

2. One-loop diagrams giving a contribution from the anomalous three-gauge-boson coupling $g_5^Z$ to the $W^{\pm}\ell^{\pm}\nu$ vertex.
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