Spectral Element Modeling of Acoustic to Seismic Coupling Over Topography

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Abstract  Acoustic waves in the atmosphere are commonly recorded on seismometers as they couple into the ground. These signals, here called ground coupled airwaves, are not commonly considered in numerical modeling of infrasound propagation, which often assumes a rigid unmeshed boundary. Starting from an analytically-tractable spherical wave model, we analyze how the coupling of an acoustic wave into a planar elastic halfspace changes over a wide range of scenarios. We use energy admittance to quantify acoustic to seismic coupling over both a planar elastic halfspace and meshed topography. Our spectral element and analytic calculations have different maxima as a function of incidence angle, with very high admittance values for near-vertical incidence (maximum ≈78%). Energy admittance calculations at shallow incidence angles are much smaller (less than 1%). In simulations over the complex topography of Sakurajima Volcano, we attribute the variable spatial pattern of energy admittance to changes in earth parameters between each model. The observed pressure difference over the simulated 15 km region appears to be <2%. While this value is relatively small, the cumulative addition over 100s of km and multiple acoustic bounce points may be significant. Acoustic to seismic coupling along the propagation path may bias long distance yield estimates, particularly when infrasound propagates over regions with steep topography or particularly slow seismic velocities, such as alluvial planes.

Plain Language Summary When numerically modeling how low-frequency sound waves travel over the Earth's surface, it is usually assumed that the Earth does not react to the sound waves. This approximation can make the numerical modeling faster and easier, but it ignores the fact we often observe that sound waves convert into seismic waves when they encounter the Earth's surface. Here we estimate how much energy is moved from the sound wave into the seismic wave in order to better understand potential inaccuracies in previous modeling efforts. We find that the change is small, but now we have an estimated number. This kind of estimate is important when trying to analyze how a complex source like a volcanic eruption or a chemical explosion releases seismic and infrasound waves, which can in turn help inform future estimates of explosion size and other important characteristics.

1. Introduction

Acoustic waves in the frequency band below 20 Hz, called infrasound, are widely observed from natural and anthropogenic sources at local and global scales (e.g., Ceranna et al., 2009; Edwards, 2010; Fee & Matoza, 2013; Fuchs et al., 2019; Matoza et al., 2019). Due to its wide applicability, infrasound has joined seismology as a key monitoring technology at both volcano observatories (e.g., Lyons et al., 2019) and the Comprehensive Nuclear-Test-Ban Treaty Organization (Christie & Campus, 2010). The widespread adoption of both infrasound and seismology has led to observations of a variety of seismo-acoustic phenomena (Christie & Campus, 2010). For example, incident infrasonic waves can be observed on seismometers (Johnston, 1987), and seismic waves are known to produce infrasound through a variety of mechanisms (e.g., Kim et al., 2004). These developments have led many authors to consider a combined seismo-acoustic wavefield (Arrowsmith et al., 2010).

The coupling efficiency between acoustic waves in a fluid and seismic waves in a solid is described in terms of the density of the fluid, the density of the solid, the acoustic sound speed in the fluid, and the seismic wave speeds in the solid. The product of the density of a medium with its sound speed is called the characteristic impedance of the medium (Z). An elastic medium has separate characteristic impedances for P-wave (α) and S-wave (β) speeds.
the solid, an incident acoustic wave is more efficiently reflected. Conversely, when there is a smaller difference between the characteristic impedances of the fluid and the elastic solid, then an incident acoustic wave is less efficiently reflected and more acoustic energy is transmitted into the solid (Pierce, 2019). When sound waves are efficiently reflected from a solid, the boundary between the fluid and solid is often treated as a rigid boundary for mathematical simplicity (Ewing et al., 1957). A rigid boundary is a bounding surface between two media such that waves incident on the boundary from one medium produce no motion on the boundary, and no waves are transmitted into the second medium (Ewing et al., 1957). This boundary condition captures the reflections and diffractions of an incident sound wave, but it prohibits transmission into the Earth and deformation resulting from the acoustic wave loading the ground.

Waves traveling through fluid and elastic media are coupled by adding continuity conditions at the interface to their equations of motion (Dahlen & Tromp, 1998). These conditions result in transmitted body waves into the fluid or elastic solid and interface waves that propagate along the boundary between the two media. For horizontally layered media, the continuity of the horizontal motion across the boundary leads to Snell’s law, which relates the propagation directions (angles relative to the vertical axis) of incident and transmitted waves (e.g., Brekhovskikh, 1980). In the acoustics literature, the interaction of an incident acoustic wave with a bounding surface is approached by considering the specific acoustic impedance of the surface (e.g., Pierce, 2019). The specific acoustic impedance is the ratio of the pressure of a fluid at a surface to the normal component of the fluid velocity at the interface. This parameter combines the incidence angle of an impinging wave with the characteristic impedance. For mechanical waves incident on an elastic interface, seismic $P$-waves and $S$-waves have separate specific impedances. A rigid boundary has an infinite specific acoustic impedance by definition. For surfaces of finite impedance, analysis of planar or spherical waves incident on a planar boundary leads to scalar coefficients describing the reflected amplitude of the acoustic wave and the amplitudes of the transmitted seismic body waves (Aki & Richards, 2002; Brekhovskikh, 1980).

Observations of a seismic response to incident infrasound waves are somewhat common (e.g., Johnston, 1987). These ground coupled air waves (GCAs) have been recorded within 1 km to over 1,000 km from the acoustic source (Bonner et al., 2013; De Angelis et al., 2012; Fuchs et al., 2019; Hinzen, 2007; Matoza et al., 2018), and observations suggest that acoustic to seismic coupling of incident acoustic waves is frequency dependent (Bass et al., 1980; Ichihara et al., 2012; Matoza & Fee, 2014). The amplitude of the seismic response is also thought to be connected to the Rayleigh wave velocity in the Earth (Edwards et al., 2007; Madshus et al., 2005). Energy admittance, the ratio of seismic kinetic energy density to acoustic kinetic energy density, is one way to quantify the amount of acoustic to seismic of coupling that occurs in a GCA (Albert & Orcutt, 1989; Edwards et al., 2007). Notably, an approximately 2.13% energy admittance was calculated from seismic and infrasound measurements from the STARDUST return capsule (Edwards et al., 2007). This study is notable for two reasons: calculated energy admittance is much larger than previous estimates derived from earthquake and explosive analogs (e.g., McDonald & Goforth, 1969), and the estimated near surface shear wave speed at their experiment site ($\beta \approx 178$ m/s; $Z_s \approx 3.6 \times 10^5$ kg m$^{-2}$ s$^{-1}$) is comparable in magnitude to the near surface sound speed in the atmosphere ($c \approx 331$ m/s; $Z_s \approx 366$ kg m$^{-2}$ s$^{-1}$).

Previous studies using the rigid boundary condition have found that volcanic topography has a major effect on observations of infrasonic signals at local (<15 km) distances (Kim & Lees, 2011, 2014; Lacanna et al., 2014; Lacanna & Ripepe, 2013). However, both the compressional and shear wave speeds at the surface of a generic volcano velocity model are comparable to the atmospheric sound speed (Lesage et al., 2018). This smaller contrast between the acoustic sound speed and the seismic wave speeds may have implications for how much infrasound attenuation occurs through acoustic to seismic coupling (Godin, 2021). Fully coupled acoustic and seismic simulations can be used to examine the amount of acoustic to seismic coupling as well as inaccuracies when assuming a fully rigid boundary, which has implications for both amplitude based inversion methods and signal analysis methods. For example, waveform inversion with synthetics Green’s functions (Fee et al., 2017; Iezzi et al., 2019; Kim et al., 2012, 2015) might be affected by the coupling of acoustic waves into the earth along the propagation path (e.g., Iezzi et al., 2019). Additionally, the observation of GCAs motivated the development of signal analysis methods that combine vertical seismic data and infrasound pressure to detect acoustic sources (Fee et al., 2016; Ichihara et al., 2012; Matoza & Fee, 2014; Martire et al., 2020; Mendo Pérez et al., 2021; Nosovelsov et al., 2020) and estimate the signal back-azimuth from a singular seismometer and infrasound station.
Numerical methods that calculate acoustic to seismic coupling can be used to validate existing algorithms and perhaps develop new ways to combine observed acoustic and seismic data. Fully coupled acoustic and seismic solvers have previously been used to model different aspects of seismic and acoustic propagation with volcanic topography (Barrière et al., 2018; Haney et al., 2009; Matoza et al., 2009). Here we use SPECFEM3D Cartesian (Komatitsch & Tromp, 2002a,b) to examine a suite of earth models that is notably softer than those considered by previous numerical studies. Shear wave speeds of comparable magnitude to the adiabatic sound speed in the atmosphere have been noted in previous studies that examined acoustic-seismic coupling (Bass et al., 1980; Bonner et al., 2013; Edwards et al., 2007). SPECFEM3D Cartesian requires an ambient homogeneous atmosphere (Komatitsch et al., 2000), so it cannot model infrasound propagation through moving media. While atmospheric models with wind and temperature gradients are fundamental for accurate long range infrasound propagation (e.g., Blom, 2020), this limitation in our modeling does not greatly affect our study due to the local propagation distance (<10 km in distance) and emphasis on atmospheric to ground coupling. For completeness, we mention that SPECFEM-DG, a recently developed version of SPECFEM, can model seismic and acoustic waves with windy atmospheres (Brissaud et al., 2017; Martire et al., 2021).

Our study attempts to quantify the effect of the rigid boundary assumption on the linear propagation of infrasonic waves in the presence of topography. This assumption is commonly made but its validity has not been examined fully. To better relate our modeling efforts to previous infrasound modeling studies, we construct acoustic monopole and quadrupole models from a seismic moment tensor (Appendix A). Using the energy admittance (Albert & Orcutt, 1989; Edwards et al., 2007), we quantify the acoustic to seismic coupling from incident infrasound waves. We first use a series of halfspace models to examine the coupling as a function of the angle of incidence and various seismic phase speeds. We also compare simulated data and analytical expressions of coupling coefficients. Using the same methodology, we then perform a series of simulations over an example with large, complex topographic relief in order to approximate the acoustics in realistic scenarios. We find that diffraction from topography appears to create complex acoustic to seismic coupling patterns due to the change in incidence angle. Within approximately 10 km of the source, we note peak pressure differences across our models between 1–2%. Over a global scale with multiple bounce points, the accumulated coupling may be considerable.

2. Background

2.1. Numerical Modeling

We use the SPECFEM3D software to model the propagation and coupling of acoustic and seismic waves with the spectral element method (Komatitsch et al., 2000; Komatitsch & Tromp, 2002a,b). SPECFEM3D solves the weak form of the conservation of momentum equation in the seismic domain. In the fluid domain, SPECFEM3D solves the weak form of the acoustic wave equation formulated using the displacement potential (Komatitsch et al., 2000; Luo et al., 2013). SPECFEM3D accommodates full seismic and acoustic coupling by maintaining continuity of traction and normal displacement at the interface between acoustic and seismic media. SPECFEM3D can use both structured and unstructured meshes and can include topography at the free surface and between interface layers.

We note that SPECFEM3D requires initialization with a moment tensor, the canonical seismic point source (Aki & Richards, 2002). In the infrasound community, an acoustic monopole is a widely used source model (e.g., Kim et al., 2012). The connection between acoustic and seismic sources has been examined previously (e.g., Aldridge, 2000; Haney et al., 2018; Lognonné et al., 1994; Lognonné et al., 2016). We initialize our models with an isotropic moment tensor, and we elaborate on how to construct an equivalent acoustic source in Appendix A. For mathematical simplicity, we use an error function source time function in our numerical modeling.

\[
K(t) = \frac{1}{2} \left[ 1 + \text{erf} \left( \frac{t - \mu_t}{\sqrt{2\sigma_t}} \right) \right],
\]

with \(\mu_t = 0.5\) s and \(\sigma_t = 3/(10\pi)\). These values were chosen to mimic band-limited behavior in the source; the 3σ value of the Gaussian mass rate function is at 5 Hz.
2.2. Seismo-Acoustic Coupling Models and Seismic Surface Waves

Models describing the conversion of incident planar acoustic waves into seismic waves at a planar elastic interface are well documented in the literature (Martire et al., 2020 and references therein), so we only briefly mention them here. In this case, transmission coefficients for converted P- and S-waves are parameterized with the angle of incidence that the normal vector to the impinging wavefront makes with the vertical axis and the phase speeds and densities of each medium (Figure 1a). The relationship between the acoustic wave speed in the atmosphere, \( c \), and the compressional and shear wave speeds of the elastic medium, \( \alpha \) and \( \beta \), define a series of critical angles (Table 1) that mark the transition between a homogeneous and an inhomogeneous transmitted wave as the incidence angle increases with respect to the vertical axis. Inhomogeneous plane waves are also called evanescent waves and exponentially decay in amplitude in at least one direction not aligned with the propagation direction (e.g., seismic surface waves, Woods et al., 2015).

![Figure 1](image)

**Figure 1.** Schematic diagrams showing the geometry of how an acoustic wave couples into a seismic wave at a planar atmosphere-earth interface. (a) A planar acoustic wave is denoted by a normal vector to the impinging wavefront (\( \vec{n}_{\text{incident}} \)) and the reflected wave (\( \vec{n}_{\text{reflected}} \)). The normal vector to the acoustic wavefront is resolved into vertical (\( n_z \)) and horizontal (\( n_x \)) components for reference. The transmitted compressional wave (\( P_{\text{transmitted}} \)) and shear wave (\( S_{\text{transmitted}} \)) are denoted by vectors normal to their wavefronts, which form angles \( \theta_L \) and \( \theta_S \) relative to the vertical axis, respectively. (b) A spherical acoustic wave in the fluid medium emitted from a source situated at a height \( H \) above the atmosphere-earth interface. To geometrically construct the reflected and refracted waves from a spherical source, an image source must be placed at a height of \(-H\) in the elastic medium (Morse & Ingard, 1969). The distances \( r \) and \( r_0 \) from the source and image source to a hypothetical receiver (denoted in red) are important for determining the arrival times of the acoustic and seismic waves (Equation 6).

Table 1 - Critical Angles for an Incident Acoustic Wave on an Elastic Halfspace

| Incidence angle (radians) | Description |
|---------------------------|-------------|
| \( \theta = 0 \) (normal incidence) | Only refracted P-waves are generated (no S-waves) (Brekhovskikh, 1980). |
| \( \theta = \arcsin\left(\frac{c}{\sqrt{\beta}}\right) \) | No refracted P-waves are generated (Brekhovskikh, 1980), and spherical pressure waves do not reflect (Roever et al., 1959). |
| \( 0 < \theta < \arcsin(c/\alpha) \) | Homogeneous transmitted P-waves are generated (Brekhovskikh, 1980). |
| \( 0 < \theta < \arcsin(c/\beta) \) | Homogeneous transmitted S-waves are generated (Brekhovskikh, 1980). |
| \( \theta = \arcsin(c/v_R) \) | Minimum in the plane wave reflection coefficient due to spatial resonance between the incident wave and the free Rayleigh wave (Roever et al., 1959; Woods et al., 2015). |

*Note.* All angles are measured from the vertical, i.e. the direction normal to the interface (\( \theta \) in Figure 1), and their values are expressed in terms of the acoustic sound speed in the fluid \( c \), the P-wave speed in the elastic medium \( \alpha \), the S-wave speed in the elastic medium \( \beta \), and the Rayleigh wave speed in the elastic medium \( v_R \). A critical angle marks a notable change in the character of the coupling - the prohibition of a refracted phase or a transition between the generation of homogeneous and inhomogeneous refracted waves.
As the incidence angle approaches grazing incidence (θ ≈ 90°), the compliance model of acoustic-seismic coupling becomes valid (e.g., Martire et al., 2020). One way to view this coupling mechanism is that the incident acoustic wave acts as a moving line load on the elastic interface (Dunkin & Corbin, 1970). However, for several of the earth models considered in this manuscript, we note that commonly applied transfer functions for horizontal (U) and vertical (W) ground velocity components in ground coupled airwaves (Ben-Menahem & Singh, 1981; Sorrells, 1971) are not strictly mathematically valid.

\[
\begin{align*}
\hat{U}(z = 0) &= -\frac{P_0\hat{c}}{2(\lambda + \mu)} e^{i(\lambda - \mu)\tau}, \\
\hat{W}(z = 0) &= \frac{2\hat{c}}{2(\lambda + \mu)} \left( \frac{\lambda + 2\mu}{\mu} \right) e^{i(\lambda - \mu)\tau},
\end{align*}
\]

(2)

where \(\lambda\) and \(\mu\) are Lamé parameters in the solid, and \(\hat{c} = c/\sin(\theta)\) is the trace velocity of the impinging acoustic wave. When \(\beta < \hat{c} < \alpha\), the seismic response functions have a compliance-related, non-propagating P-wave part, but a propagating S-wave part (De Bremaecker, 1967). The resulting motion is tilted and may be prograde for some combinations of \(\hat{c}\) and \(\beta\) (e.g., Langston, 2004). See Appendix B for more information.

When considering a spherical acoustic wave incident on a planar elastic surface, then the seismic response is more complicated than the planar case (Brekhovskikh, 1980; Roever et al., 1959). This problem can be approached with the Laplace transform method (Cagniard, 1962; Roever et al., 1959), where the source term is taken to be a delta function excited line source. In terms of \(u\), the sine of the incidence angle \(\theta\), the reflection coefficient is

\[
A(u) = \frac{\gamma R(u) - (\rho_f/\rho_s)K_2^2\chi}{\gamma R(u) + (\rho_f/\rho_s)K_2^2\chi},
\]

(3)

where

\[
R(u) = (K_2^2 - 2u^2)^2 - 4u^2 \gamma \psi,
\]

(4)

\(\gamma = \cos(\theta)\), \(K_1 = \frac{\gamma}{u}\), \(K_2 = \frac{\gamma}{\beta}\), \(\chi = \sqrt{K_1^2 - u^2}\), and \(\psi = \sqrt{K_2^2 - u^2}\). We note that Equation 4 is the Rayleigh equation, whose roots yield the Rayleigh wave speed of the elastic medium. Moreover, the denominator of \(A(u)\),

\[
S(u) = \gamma R(u) + (\rho_f/\rho_s)K_2^2 u
\]

(5)

is the equation governing the Stoneley wave speed of the halfspace (Strick & Ginzburg, 1956).

The horizontal and vertical velocity components (\(\hat{U}\) and \(\hat{W}\), respectively) of the transmitted seismic wave from an incident spherical acoustic wave are derived in reference to the arrival time of the reflected acoustic wave \((t_r = r/c)\) and the times of arrival of the critically reflected P-wave \((t_p)\) and S-wave \((t_s)\) (Figure 1b). Below are the equations for the case that the sound speed of the acoustic medium is greater than the shear wave speed of the elastic medium (Roever et al., 1959).

For \(t_p \leq t \leq \frac{r}{c}\) or \(K_1 \leq u \leq \frac{x}{r}\):

\[
p'_{\nu}(x, z; \text{delta}) = \frac{2(\rho_f/\rho_s)cK_2^2P_0}{r\delta} \left[ \frac{\gamma(K_2^2 - 2u^2)^2\sqrt{u^2 - K_2^2}}{\gamma^2(K_2^2 - 2u^2)^2 - \chi^2(4u^2\gamma + (\rho_f/\rho_s)K_2^2)^2} \right];
\]

(6)

\[
\hat{U}(x, z = H; \text{delta}) = \frac{2\psi + (\rho_f/\rho_s)K_2^2}{K_2^2 - 2u^2} \left[ \frac{\gamma}{\gamma^2(K_2^2 - 2u^2)^2 - \chi^2(4u^2\gamma + (\rho_f/\rho_s)K_2^2)^2} \right];
\]

\[
\hat{W}(x, z = H; \text{delta}) = \frac{-\gamma}{\rho_f c} \frac{p'_{\nu}(\text{delta})}{\rho_f c};
\]

For the impulse response at the arrival of the direct wave, we can plot Equations 6 as a function of either distance or angle of incidence for a constant source height (Figure 2).
As described above, the equations describing Rayleigh and Stoneley waves appear to be closely connected to acoustic to seismic coupling mechanisms. Here we refer to a Stoneley wave as a guided seismic wave with a phase speed less than the phase speed in the overlying acoustic medium (Haney & Tsai, 2017), but we acknowledge that there appears to be some ambiguity about the name of this wave in the literature (Strick & Ginzbarg, 1956).

Here we review some basic properties of these surface waves and how they relate to the medium properties of the fluid layer and solid earth.

As a simplified case, we first consider the nondispersive Rayleigh wave, which always exists at the boundary between an isotropic elastic medium and a vacuum (Brower et al., 1979). The Poisson ratio of the elastic medium, $\eta$ (Table 2), is closely connected to the roots of the Rayleigh wave equation. If the Poisson ratio of the earth is less than $\approx 0.263$, then the Rayleigh equation (Equation 4) will have three real roots (Brower et al., 1979). The smallest of these roots will correspond to the Rayleigh wave phase speed ($v_R$), and the other two roots correspond to an exchange of polarizations. These roots correspond to angles for which an incident P-wave converts to an SV-wave and vice versa (Brekhovskikh, 1980). If $\eta > \approx 0.263$, then the Rayleigh equation will have one real root and two complex conjugate roots (Brower et al., 1979). The real root in this case corresponds to the Rayleigh wave phase speed, and the complex conjugate roots appear to contribute to the lateral or head wave in the solid (Harris & Achenbach, 2002).

For the Stoneley wave equation (Equation 5), which describes surface waves at the interface between an elastic medium and a fluid medium, the physical meaning of the roots depends on the ratio between the sound speed of the overlying fluid and the shear wave speed of the underlying elastic medium. The real root of the Stoneley wave equation always exists, and this root corresponds to the Stoneley wave speed (Brower et al., 1979). This wave is exponentially damped in both media, with attenuation normal to the propagation direction. We note that as the ratio of the fluid density to the solid density approaches zero, the Stoneley wave phase speed approaches either the sound speed in the overlying fluid ($c < v_R$) or the Rayleigh wave speed of the elastic medium ($c > v_R$) (Strick & Ginzbarg, 1956). The complex conjugate roots of the Stoneley wave equation correspond to the “leaky Rayleigh wave”, but this wave is not guaranteed to exist (Brower et al., 1979). The existence of the leaky Rayleigh wave is controlled by the radiation condition, and it exists for a range of density values $\rho_f/\rho_s$ for $c < \beta < \alpha$. Due to the loading of the overlying fluid, the leaky Rayleigh wave has a larger phase speed ($v_{RL}$) than a pure Rayleigh wave.

![Figure 2. Vertical component of seismic velocity from a spherical acoustic wave with peak pressure $P_0 = 1$ Pa, $c = 340$ m/s, $\alpha = 1,800$ m/s, and a fan of shear wave speeds $\beta$. The P-wave critical angle is denoted by the gray line, and the blue line denotes the approximate boundary that we can observe from our simulations. Note the large seismic response at near grazing angles for $\beta \approx 356.8$ m/s. In this case, the free Rayleigh wave speed ($v_R$) is approximately the same as $c$, the acoustic sound speed in the atmosphere. This large seismic response appears consistent with the modeling found in Madshus et al. (2005). See the Supporting Information S1 for a full table of values corresponding to each response function.](image-url)
wave, also called a free Rayleigh wave, but this phase speed is less than the shear wave speed of the elastic medium. As the ratio of \(\rho_s/\rho_f\) approaches 0, \(v_{RL}\) approaches \(v_g\) (Brower et al., 1979).

3. Methods

3.1. Energy Admittance

Energy admittance is the dimensionless ratio of seismic kinetic energy density to acoustic kinetic energy density (Albert & Orcutt, 1989; Edwards et al., 2007).

\[
EA = \frac{(1/2)\rho_s v_{\text{seismic}}^2}{(1/2)\rho_f v_{\text{acoustic}}^2}.
\]  

(7)

We use the calculated acoustic particle velocities directly from our numerical modeling instead of converting from the pressure with an impedance relationship. We place synthetic acoustic and seismic receiver pairs on both sides of the atmosphere-earth interface in each of our models, and we calculate energy admittance at each acoustic-seismic receiver pair, which we collectively describe as a receiver in this manuscript. For the halfspace modeling, we choose the peak values of the acoustic and seismic particle motion from the ground coupled airwave in the vertical (z) and horizontal (x) components. Our receivers are all located on the x-axis, directly east of the source, so particle motion in the north, y-component, is several orders of magnitude smaller than in the z- and x-components (Figure 4, for example). Peak particle velocities in all three components (x, y, z) for the ground coupled airwave were used for energy admittance calculations in our topography simulations.
3.2. Halfspace Modeling

Prior to modeling the complexities of acoustic to seismic coupling over topography, we first perform a more simple series of simulations over a homogeneous halfspace with no topography. In each simulation, the acoustic source is at a height of 1,500 m above the elastic interface in the acoustic medium. In our Cartesian coordinate system, the east component is along the positive x-axis, the north component is along the positive y-axis, and the vertical component is along the positive z-axis. Receivers are located in an east trending line starting at the source, with the acoustic receiver at 1 m above the interface and the seismic receiver at 1 m below the interface (Figure 4). Receivers were placed at the interface in order to geometrically obtain the angle of incidence for each ground coupled airflow.

The primary reason for choosing an elevated source height is so that spherical acoustic waves impinge into the solid earth over a range of incidence angles (0 to approximately 73°). Computational time and memory were also considered. A 10 km (x) by 6 km (y) by 4 km (z) region was meshed using 288, 192, and 63 elements, respectively. Our meshing corresponds to at least 5 grid points per dominant wavelength (Komatitsch et al., 1999), and a total of 3,483,648 elements were used. Raw waveforms were filtered between 0.1 and 4.0 Hz for all halfspace simulations except for Case 1, where waveforms were filtered between 0.1 and 2.8 Hz, and Case 6, where waveforms were filtered between 0.1 and 3.0 Hz.

To simplify our halfspace models further, we vary one earth parameter at a time while keeping the other variables constant. This structure allows us to separate the effect of the variable angle of incidence of the impinging acoustic wave from the coupling effects due to changing the seismic phase speeds between different earth models. In Case 1 (Table 2), we vary the shear wave speed $\beta$ with constant $\alpha$, $\rho_s$, $\rho_v$, and $c$. In Case 2 (Table 2), we change the compressional wave speed $\alpha$ with constant $\beta$, $\rho_s$, $\rho_v$, and $c$. The variations in Case 1 and 2 are equivalent to increasing the $S$-wave ($Z_s$) and $P$-wave ($Z_p$) impedances, respectively, while keeping the other impedances constant. In Case 3, we repeat one of the simulations in Case 2 ($\alpha = 563$ m/s, $\beta = 320$ m/s, $\rho = 1,492$ kg/m$^3$), but we increase the density to 2,000 kg/m$^3$ in order to briefly examine the coupling due to a change in density alone. In Case 4, we examine a more rigid, reflective model to approximate the coupling over an effectively rigid boundary. This model is our approximation to a rigid boundary for computational reasons; larger disparities between the acoustic medium parameters ($c$, $\rho$) and the earth parameters ($\alpha$, $\beta$, $\rho$) appear to cause some numerical instability when topography is added (Section 3.3).

Beyond these generic models, we examine two specific earth models previously described in the literature (Table 2). In Case 5, we use the surface layer of the generic volcano velocity model of Lesage et al. (2018) with a density estimated by the Gardner relationship (Brocher, 2005). Volcanic topography can be relatively complex and has been previously shown to strongly influence infrasound propagation within approximately 10 km from the source (e.g., Fee et al., 2017; Iezzi et al., 2019; Kim & Lees, 2011, 2014; Lacanna & Ripepe, 2013; Matteo et al., 2009). We note that the coupling mechanisms and interface waves described in Section 2 would not be simulated with modeling efforts that treat the topography surface as a rigid boundary. This model (Case 5) was chosen in order to investigate how acoustic to seismic coupling affects observed infrasound waveforms. See Section 3.3 for more information about our implementation of topography. In Case 6, we approximate the surface layer of the earth model described in Edwards et al. (2007). This model was chosen because of the notably slow shear wave speed ($\beta \approx 178$ m/s) compared to the atmospheric sound speed ($c \approx 331$ m/s). Additionally, energy admittance calculations from an observed atmospheric source (Edwards et al., 2007) allow us to compare our energy admittance calculations with those from observations.

3.3. Topography Modeling

To examine the effect of topography on the acoustic to seismic coupling, we use the well-studied Sakurajima volcano as an example (Fee et al., 2017; Kim & Lees, 2014; Yokoo et al., 2009). A 10 km (x) x 6 km (y) x 4.2 km (z) region around the volcano (Figure 3) is meshed with using 288, 192, and 72 elements, respectively. Our...
simulation domain meshes the topography at the interface between the homogeneous atmosphere (approximately 3.0 km thick) and homogeneous elastic region (approximately 1.2 km thick) in the structured mesh, where hexahedral mesh elements deform to match the topography on both sides of the interface. Our meshing corresponds to at least 5 grid points per dominant wavelength (Komatitsch et al., 1999), and a total of 3,926,016 elements were used. Raw waveforms were filtered between 0.1 and 3.0 Hz for all topography simulations. The original volcanic surface of our DEM was too steep for the internal mesher of SPECFEM3D (1,000 m of surface relief over the simulation domain). In order to mesh the topography, we smoothed the surface by removing its mean and scaling the surface by 0.78. This new topography retains the general features of the original topography such as the summit craters (Figure S1 in Supporting Information S1), but the new vertical relief is now approximately 700 m. In our meshed model, the acoustic source is placed in the atmosphere within the 75 m deep and 300 m wide Showa Crater on the southeast region of the volcano at an elevation of 400 m (see Figures 3 and 8).

For the modeling over topography, we again use the Case 4 and Case 5 models. Case 4 is our model that approximates a rigid interface, and Case 5 is our model that approximates the near surface of a volcano. Additionally, we consider an elastic model with exceptionally slow P-wave and S-wave speeds (Case 7) in order to estimate an upper bound on the energy admittance we would potentially observe from a volcanic source with steep topography. The acoustic medium has the same sound speed (340 m/s) and density (1.16 kg/m$^3$) for all simulations with topography.

During the analysis, we discovered that it was particularly difficult in the time domain to separate the ground coupled airwave from seismic waves generated beneath the source that also propagate outwards in the simulation. This coseismic shaking could potentially cause an error in estimating the proper seismic response, which would bias the energy admittance upwards. In the halfspace simulations (Section 3.2), we avoided this interference by elevating the source above the interface. To more clearly understand the effects of topography on the acoustic to seismic coupling, we complement our energy admittance results with two examples comparing the observed peak pressure values at each receiver. The acoustic medium is the same in each simulation, but propagation occurs...
over different earth models (Cases 4, 5, and 7, Table 2). Peak pressure values relative to Case 4 are presented in Figures 9 and 10. In only considering the infrasound data, we bypass the task of separating the ground coupled airwave from other parts of the seismic wave train. In comparing examples with varying levels of coupling, we estimate the amount of sound transmitted into the earth when full coupling is considered. Our assumption behind this method is that observed differences in peak pressure between different models results from varying amounts of acoustic to seismic coupling along the path from source to receiver. However, another result of full coupling is that it is possible that a seismic wave couples into the atmosphere (e.g., Arrowsmith et al., 2010). Case 4 is the only earth model we consider where the Rayleigh wave speed is larger than the acoustic sound speed in the atmosphere (Table 2), so it might leak energy into the acoustic medium (Brekhovskikh, 1980; Ichihara et al., 2012). Otherwise, acoustic wave generation from vertical seismic velocity is the main mechanism of seismic to acoustic coupling that we might see in the simulations.

4. Results

4.1. Halfspace Results

Energy admittance calculations involve measurements from both the acoustic and elastic media. With the exception of Case 6, the atmospheric phase speed ($c = 340 \text{ m/s}$) and atmospheric density ($\rho_f = 1.16 \text{ kg/m}^3$) are the same in all of our simulations. Here we briefly describe the characteristics of the infrasound propagation, which is highly similar across all of the halfspace models. Peak pressure amplitude appears to be spherically spreading (Figure S2 in Supporting Information S1). A secondary hemispherical pressure wave appears after the direct wave as a result of reflection from the ground surface (the vertical particle motion from the direct and reflected waves is visible in Figure 4), but the front of this wave appears unresolved from the direct wave due to the close proximity of the receivers to the ground surface.

The seismic response is the other component to the energy admittance calculations, and here we briefly describe trends in the peak vertical and radial components as a result of the coupled airwave. In Case 1 (varying $\beta$ from 160–356.8 m/s), we note that the peak vertical velocity at normal incidence (0°) decreases with increasing shear wave speed, but then the magnitude of the peak velocity increases beginning at approximately the critical angle for $P$-waves (Figure 5a, Table 1), which is the same for all simulations in Case 1. The location and magnitude of the peak vertical seismic velocity varies according to the shear wave speed of the medium (Figure 5a). An increasing shear wave speed causes the peak vertical velocity to increase, with the peak in our simulations occurring at $\beta = 200 \text{ m/s}$. As the shear wave speed further increases, the peak vertical velocity decreases and becomes a narrower peak. The peak horizontal velocity is zero for all models with an incident pressure wave at normal incidence (Table 1), and similar trends in the peak radial velocity values occur as in the peak vertical velocity. When the Rayleigh wave speed overlaps with the acoustic sound speed ($v_R = 340 \text{ m/s}$), we note that the magnitude of both the peak vertical and peak horizontal seismic velocities begins to increase as the angle of incidence approaches grazing incidence ($\theta \to 90^\circ$) at the end of our simulation domain. Trends in our numerical simulations are consistent with our analytical model (Figure 2).

The energy admittance for each earth model appear to have the largest value at vertical incidence (maximum $\approx 11.3\%$). A secondary peak occurs with increasing angle of incidence (Figure 5), with the magnitude and location of this secondary peak increasing with a decreasing shear wave speed. The largest value of the secondary peak occurs at $\beta = 181 \text{ m/s}$ and is approximately 0.74%. The location and magnitude of the secondary energy admittance peak for $\beta = 160 \text{ m/s}$ is undetermined from this modeling configuration due to our source to receiver geometry; our simulations only model angles of incidence up to a maximum of approximately 73°.

In contrast to Case 1, where a varied shear wave speed appears to primarily affect coupling beyond the $P$-wave critical angle (Figure 5), trends in the peak seismic velocity in Case 2 (varying $\alpha$ from 500 m/s to 1,800 m/s) appear to be broadly affected by a varied $P$-wave speed at all angles of incidence (Figure 6a). At vertical incidence, the magnitude of the peak vertical velocity increases as the $P$-wave speed decreases and the associated $P$-wave critical angle moves to shallower angles of incidence. After the $P$-wave critical angle, the magnitude of the peak vertical seismic velocity again increases before decreasing again as it approaches grazing incidence. As the $P$-wave speed speed (and Poisson ratio) increases, the secondary region of increased coupling develops as a growing, but narrowing, peak at steeper angles of incidence.
Figure 5. Peak values of the seismic velocity components and energy admittance as a function of angle for the Case 1 models in Table 2. Direct comparison of the peak seismic velocities for (a) the vertical component and (b) the horizontal component. Note the broadening and overall increasing peak size with a decrease in seismic shear wave speed. The dashed line shows the critical angle for \( P \)-waves: the angle at which transmitted \( P \)-waves convert from homogeneous to inhomogeneous (Table 1).

Figure 6. Peak values of the seismic velocity components and energy admittance as a function of angle for the Case 2 models in Table 2. Direct comparison of the peak seismic velocities for (a) the vertical component and (b) the horizontal component. Vertical dashed lines denote the \( P \)-wave critical angle for each velocity model. For comparison, the \( \alpha = 1,800 \) m/s model in this figure is the same as the \( \beta = 320 \) m/s model in Figure 5.
The largest energy admittance in Case 2 (≈78%) occurs for $\alpha = 500$ m/s at vertical incidence (Figure 6c), before rapidly decaying with increasing incidence angle and increasing $P$-wave speed. Secondary energy admittance peaks also occur in this suite of models that both increase in amplitude and arrive at steeper incidence angles as the $P$-wave speed and Poisson ratio $\eta$ increase (Table 2).

When only the density differs between two earth models, the peak seismic velocities and energy admittance from a ground coupled airwave undergo an approximate vertical translation that decreases with an increasing incidence angle (note the $\alpha = 563$ m/s and $\rho_s = 2,000$ kg/m$^3$ curves in Figures 6a and 6b). The $\alpha = 563$ m/s model in Case 2 has a density of 1,492 kg/m$^3$. The Case 3 model has the same compressional and shear wave speeds ($\alpha = 563$ m/s, $\beta = 320$ m/s), but it has a density of 2,000 kg/m$^3$, which results in larger the $P$-wave and $S$-wave impedances (Table 2). As expected, the more dense earth model has a smaller energy admittance and smaller peak velocities.

In our Case 4 model (Table 2), the maximum calculated energy admittance is approximately 1.7% at $\approx 0.48^\circ$ from vertical (the receiver closest to a vertical incidence angle in the simulation), and it rapidly falls to less than 0.01% at $\approx 7.1^\circ$ (Figure 7c). Therefore, for this scenario, most reflected acoustic waves have $\gg 99\%$ of the amplitude of the incident pressure wave, and we can then use this model to approximate a rigid boundary condition. We note that both $\alpha$ and $\beta$ are much larger than the acoustic sound speed $c$, so the assumptions for commonly used compliance models are applicable (e.g., Ben-Menahem & Singh, 1981). As the angle of incidence approaches grazing incidence, we see good agreement between simulated peak seismic velocities and predictions from Equation 2 (Figure 7; Martire et al., 2020).

Our Case 5 (Table 2) is a simplified estimate of the shallow subsurface of a general volcano (Lesage et al., 2018). This model has large energy admittance at shallow angles (maximum $\approx 67\%$), with a secondary peak at around $53^\circ$ ($\approx 0.25\%$, Figure 7c). This model can be considered as an instance of the suite of models in Case 1, and the results from this model are consistent with the trends from Case 1.

Figure 7. Peak values of the seismic velocity components and energy admittance as a function of angle for Cases 4, 5, and 6 in Table 2. Direct comparison of the peak seismic velocities for (a) the vertical component and (b) the horizontal component. The black dashed lines in (a) and (b) are the compliance model coupling predictions from Equations 2. Note how the black dashed lines and the orange lines (Case 4) appear to converge as the incidence angle increases. As the incidence angle approaches zero, then the assumptions behind the compliance model equations are no longer valid, and the predictions deviate from the numerical modeling.
Case 6 (Table 2) is the top layer of the velocity model discussed in Edwards et al. (2007), and it represents a slow sedimentary layer at the Earth's surface. Our simulation results again show large energy admittance at shallow angles (≈13%), with a broad, secondary peak at around 64° (≈0.69%). We can view this model as an example of the models discussed in Case 2 (Table 2), and the results from this model are consistent with the trends from Case 2.

4.2. Topography Results

We apply a similar methodology used in the halfspace modeling to quantify patterns of acoustic to seismic coupling over an approximation to Sakurajima Volcano topography. Compared to the halfspace simulations (Figure 4), waveforms from acoustic propagation over topography are more complex (Figure 8) and feature more clearly differentiated seismic phases.

Figure 9 shows a horizontal transect through the topography with the components of the particle motion shown from the simulation at 2.66 s. The location of this transect in map view is shown in Figure 10b. From the original particle motion directions (Figure 9a), we can rotate the coordinate axes into so that the horizontal component is locally tangent to the topography slope in the transect (Figure 9b). We then determine the angle of incidence by projecting the resulting particle motion vector (green) onto the local vertical component (Figure 9c). We see that the incidence angle of the propagating acoustic wave is greater than 60° for all receivers shown here, but we note the. With the exception of some local variation, there is a general trend that this angle becomes more grazing as the wave propagates down the volcano slope. These angles are within 1 degree of each other for the Cases 4, 5, and 7 models.
Figure 9d shows an estimate of the energy admittance at each receiver. Since we use the topography slope along the transect, only the $x$-component and $z$-component particle motions were used for these calculations. We do see a general trend that the energy admittance decreases as the acoustic particle motion approaches grazing incidence. These results are complemented by Figure 9e, which shows the difference in the peak pressure between the receivers in Case 5 and Case 7 and Case 4. Again, the slope of this difference decreases as the acoustic particle motion approaches grazing incidence.

Figure 10a shows the magnitude of the topography gradient vector with the color scheme saturated at 0.8. Using the DEM spatial sampling, we constructed this figure by calculating a gradient vector for the topographic relief and taking its magnitude. More gradually sloping terrain has a smaller gradient and smaller gradient magnitude. Our color scheme has the effect that more gradual terrain with a gradient magnitude $\leq 0.8$ plots in the same color, so that the regions of steeper topography (and larger gradient magnitude) are highlighted. We note two regions of rapid topographic change: the summit crater shaded in yellow immediately west of the source and the region shaded in blue to the northwest of the source.

Figure 10b shows energy admittance calculations for Case 7 in map view. Case 7 was chosen to have slow longitudinal and shear wave speeds in order to simulate a model where we expect large acoustic to seismic coupling. Here we see that larger energy admittance values (light colors) primarily appear to the south and west of the source. Moving out in radial distance from the volcano, the energy admittance appears to decrease and go to near zero, which roughly corresponds to a shallower, grazing angle of incidence (Figures 5, 6 and 9).
We note that as we increased the shear wave speed in our modeling with topography, it was increasingly difficult to separate the ground coupled airwaves in the seismic traces from surface waves propagating outwards from beneath the source location. As a result, we complement our energy admittance results (Figure 10b) with two figures showing the relative difference in observed peak pressure between Case 5 and Case 4 (Figure 10c) and Case 7 and Case 4 (Figure 10d). In only considering the infrasound data, we bypass the task of separating the ground coupled airwave from other parts of the seismic wave train. Case 4 was chosen as the reference model because it well approximates a rigid boundary condition (Figure 7). Case 5 represents an approximation to a generic volcano velocity model (Lesage et al., 2018) and Case 7 was chosen in order to estimate an upper bound on the energy admittance. Both Figures 10c and 10d show a similar magnitude of relative difference, but they have different spatial patterns. The largest pressure difference in Figure 10c occurs in a broad region west of the volcano (blue region in Figure 10a). The largest pressure change in Figure 10d occurs directly west of the source, which aligns with the summit crater (yellow region in Figure 10a). Despite the complexity of this data, we do confirm that the elastic model in Case 7 absorbs more energy than the Case 4 and Case 5 models (Figure 11).

We note for completeness that we observe P-SV to SH conversions in the seismic data (Figure 8), which appears to be the result of internal reflections in the volcano topography (Ripperger et al., 2003).

5. Discussion

5.1. Halfspace Modeling

To explain trends in our halfspace simulations, we accompany our numerical modeling with an analytical description of how a spherical acoustic wave couples into the earth (Figure 2). This is a form of Lamb’s problem (Aki & Richards, 2002), and it appears to be decidedly more complex than an analysis of plane wave transmission (Brekhovskikh, 1980; Roever et al., 1959). From the geometry of the simulations, we cannot simulate values at...
angles of incidence greater than approximately ~73°, but trends in Figures 5a and 5b still show good agreement with trends from the analytical model (Roever et al., 1959) in Figure 2.

At normal incidence to the interface, the peak horizontal component of the velocity is zero (Figures 5 and 6), which corresponds to the lack of refracted S-waves (Table 1). In Case 1 (varying β), the P-wave critical angle (Table 1) is the same for all simulations in this set of data. The magnitude of the peak vertical and horizontal components of the seismic velocity decreases with increasing S-wave speed at near vertical incidence, but it transitions from increasing to decreasing with increasing S-wave speed at incidence angles exceeding the P-wave critical angle (Figures 5a and 5b). In Case 2 (varying α), the magnitude of the peak vertical and horizontal components of the seismic velocity decreases with increasing P-wave speed at angles less than the P-wave critical angle, where the transmitted P-wave is homogeneous. However, beyond the P-wave critical angle, where the transmitted P-wave is inhomogeneous, the secondary velocity peak grows and moves to shallower angles as the P-wave speed increases. Taken together, Figures 5 and 6 illustrate the coupling differences when the transmitted P-wave is homogeneous or inhomogeneous. An inhomogeneous P-wave and a SV-wave couple to form a surface wave, specifically a leaky Rayleigh wave (De Bremaecker, 1967), so our velocity observations also depict the transition from separately propagating transmitted body waves to the coupling of body waves to generate a leaky surface wave. As the incidence angle approaches grazing incidence for the β = 357 m/s model in Case 1 (Figure 5), the peak vertical and horizontal components both increase. At grazing incidence for this model, the transmitted P and SV waves are inhomogeneous, and they couple to form a Rayleigh wave (Aki & Richards, 2002). From the analytical models, we suspect that peak velocity values will rapidly decrease as the angle of incidence approaches 90° (Figure 2).

As expected from their mathematical relationship, the energy admittance calculations and the peak seismic velocities show some similar trends. Due to the increase in impedance with increasing seismic speed, the energy
admittance at vertical incidence decreases with increasing compressional and shear wave speeds across all cases (Figures 5c, 6c and 7c). At the $P$-wave critical angle, the transmitted $P$-wave travels horizontally along the interface (e.g., Hudson, 1962), and we see a minimum in the energy admittance curves at approximately this angle (Figures 5c, 6c and 7c). Beyond the $P$-wave critical angle, where the transmitted inhomogeneous $P$-wave and SV-wave couple to form a leaky surface wave, we again see a rise in the energy admittance. We find that the maximum amplitude of this secondary energy admittance peak increases with increasing Poisson ratio (Table 2 and Figures 5 and 6c), which is a nonlinear function of compressional and shear wave speeds. The Poisson ratio increases in Case 1 as the shear wave speed $\beta$ decreases (for a given $\alpha$), but it increases in Case 2 as the the compressional speed $\alpha$ increases (for a given $\beta$). We recall that the Poisson ratio is closely related to the roots of the Rayleigh wave speed in the elastic medium (Brower et al., 1979). After this secondary peak, the energy admittance again decreases (and approaches zero) as the incident wave can no longer perform work on the boundary as it approaches grazing incidence (Hudson, 1962).

Our energy admittance calculations for Case 6 at the angle specified in Edwards et al. (2007) (38.2°) is 0.3%, which is approximately 86% smaller than than their estimated value of $\approx$2.13% (Figure 7). We think a combination of two key factors could explain this apparent discrepancy. First, we note that our simulations inherently propagate a spherical wave, whereas the incident infrasonic wave in Edwards et al. (2007) is likely to be nearly planar due to both the relatively large propagation distance and its source geometry - a shock front from an approximately cylindrical source at great elevation (Edwards, 2010). A spherical wave can be represented as a superposition of plane waves, and each individual plane wave will couple into the Earth independently (Aki & Richards, 2002; Brekhovskikh, 1980). Furthermore, we also note a difference in how the energy admittance values were calculated in this manuscript and in Edwards et al. (2007). In this manuscript, direct particle motion values were used, while the measured infrasonic pressure wave was converted to particle velocity using a planar impedance relationship and compared with the vertical component of the seismometer in Edwards et al. (2007). This may mean that studies using energy admittance near a spherical acoustic source might underestimate the acoustic particle velocity if a plane wave impedance relationship is used (Pierce, 2019). Second, we note that the earth model in our simulations may be too simple an approximation for good agreement. The model used in this manuscript is based on the top layer ($\approx$6 m) of the estimated velocity model in Edwards et al. (2007). Using the acoustic sound speed (331 m/s) and $P$-wave speed (1344 m/s) from Edwards et al. (2007), we estimate from Equation 6 that a shear wave speed of approximately 570 m/s would be required to create strong coupling at the reported incidence angle (38.2°). In a layered earth model, Rayleigh wave dispersion would result in frequency dependent coupling (Matoza & Fee, 2014), with faster Rayleigh wave speeds occurring at lower frequencies. Density stratification in the seabed has previously been shown to dampen hydroacoustic normal modes orders of magnitude more than an unstratified seabed through larger coupling of the propagating acoustic waves into shear waves on the ocean floor (Godin, 2021).

We consider two additional models for analysis, a relatively rigid/reflective model that we consider to approximate a rigid boundary in this set of simulations (Case 4) and Case 5, a homogeneous approximation from the surface values of a generic volcano velocity model (Lesage et al., 2018). Energy admittance results for our Case 4 model (Figure 7) are a fraction of 1% for all angles of incidence, which is in good agreement as an approximation to a rigid boundary. Energy admittance results from Case 5 have a large amount of coupling at steep incidence angles (67%), which falls to less than 1% for incidence angles angles less than approximately 10° (Figure 7). We hypothesize that the large coupling at steep angles is a result of the slower compressional wave speed ($\alpha = 540$ m/s) in Case 5. The full model from (Lesage et al., 2018) is layered, so Rayleigh wave dispersion, and hence, frequency dependent coupling, would also occur. We note that beginning at approximately 27° from the vertical, the Case 6 model, which has a higher $P$-wave speed but a lower $S$-wave speed, has the larger energy admittance. While these homogeneous elastic models may be too coarse of an approximation for exact comparison to the original models in the literature, they do help us build intuition and quantitative model estimates of acoustic to seismic coupling in both flat ground and topography.

5.2. Topography Modeling

Our energy admittance calculations for the Sakurajima topography (Figure 10b) are the largest to the west of the volcanic vent. Our source model is isotropic, and the earth and atmospheric models are homogeneous, so we attribute the anisotropy observed in the coupling to topographic influence on the incident infrasound waves.
Specifically, we hypothesize that the steep slopes of the volcano and diffraction over the topography causes the acoustic waves to refract into the ground with a complex distribution of incidence angles (Figures 8 and 9), and therefore energy admittance values. Notably, the estimated incidence angles from the acoustic particle motion are within $\pm 1^{\circ}$ for the acoustic domain in the Cases 4, 5, and 7 models. As we saw in the halfspace cases, changes in the angle of incidence can greatly affect the amount of coupling.

In Figure 10c, the largest relative pressure difference appears to be to the northwest and southwest of the source. We note some similarity between the pattern shown here and previous descriptions of acoustic diffraction patterns over the topographic surface (Ishii et al., 2020; Kim & Lees, 2014). Notably, the western edge of the northern crater is particularly steep (the blue shaded region in Figure 10a). We propose that the acoustic waves propagating to the region southwest of the source first passed through the summit crater and its steep crater walls (yellow region in Figure 10a). Due to the apparent similarity in the acoustic incidence angles between Cases 4 and 5 (Figure 9), we interpret this pattern to be the result of enhanced acoustic to seismic coupling due to diffraction over the volcano peak. For two synthetic stations, located in the summit crater, the peak pressure of the incident wave is actually larger in Case 4 than in Case 5. Due to the location of these receivers within summit crater (Figure 8), we suspect that the Case 4 measurement either captured a refracted seismic wave from the steep sides of the volcanic crater (Green et al., 2009) or an additional, coincidental, pressure perturbation from a leaking Rayleigh wave generated under the source (e.g., Ichihara et al., 2012; Matoza et al., 2009).

In Figure 10d, the largest relative pressure difference appears to be directly west of the source, which we interpret as the result of increased coupling into either the walls of Showa crater (the source crater, Figure 8). Due to the steep slope of the crater wall (Figure 8), we hypothesize that the incidence angle of the traversing infrasound wave is nearly perpendicular, so large coupling occurs (Figure 7). As with the energy admittance calculations, we interpret the difference between Figures 10c and 10d to also be the result of angles of incidence.

It is unclear why the patterns in Figures 10b and 10d are not more similar, but we suspect this may be due to the propagating seismic wave potentially biasing our energy admittance calculation in Figure 10b. While we think the results in Figure 10d are useful as a starting point for interpreting acoustic to seismic coupling in regions of steep topography, we think that a code that uses an impedance relationship to incorporate the acoustic to seismic coupling may be the most ideal for further investigations of how coupling relates to the earth model parameters and diffraction effects. This analysis would avoid potential contamination in the analysis by both seismic body waves refracting into the atmosphere and leaking surface waves.

Over local distances to the volcano (<10 km), our numerical modeling suggests that acoustic to seismic coupling has implications for methods that use infrasound amplitudes for source parameter estimates (e.g., Fee et al., 2017). An energy admittance of approximately 5% means that the reflected wave has an amplitude approximately 2% lower than if it was perfectly reflected (Figure 10). We note that the energy admittance calculations for our simulations with topography were not as large as the largest values from the halfspace simulation with the same earth model. This suggests that acoustic waves in our Sakurajima example do not arrive at near vertical incidence angles (Figure 9). Perhaps different volcanoes and source-receiver geometries will exhibit a different range of incidence angles than depicted by the modeling over topography shown here.

At farther distances from the source, acoustic to seismic coupling may have a non-negligible affect on observed amplitudes over some propagation paths. Ground coupled airwaves have been observed over 100 km from the source (e.g., De Angelis et al., 2012; Fuchs et al., 2019; Hinzen, 2007; Hedlin & Drob, 2014; Johnson and Malone, 2007; Matoza et al., 2018), and infrasound waves typically experience multiple “bounces” on the ground as they propagate at long distances. Using trace velocity ($\hat{c}$) estimates from tropospheric ($\approx 343$ m/s), stratospheric ($\approx 330–350$ m/s), and thermospheric arrivals ($\approx 400$ m/s), we estimate that infrasound waves arriving from those ducts would have incidence angles (measured from the vertical axis) of approximately $88^{\circ}$, $76–90^{\circ}$, and $58^{\circ}$, respectively (Fee & Matoza, 2013; Matoza et al., 2011). These numbers were calculated using the formula $c = \sin(\theta)\hat{c}$ with a reference acoustic sound speed ($c$) of 340 m/s. Recent propagation modeling in the geometrical acoustics approximation has shown that infrasound waves interact with extended topography along the propagation path (Blom, 2020), which could introduce a suite of incidence angles at steeper incidence. Even if acoustic-seismic coupling is small (<1%), a stratospherically ducted wave recorded at 2,000 km distance would interact with the ground approximately eight times, so the cumulative energy loss may be considerable.
Acoustic to seismic coupling along the propagation path would affect long distance yield estimates (Blom et al., 2018; Ceranna et al., 2009). From energy conservation, the reflection coefficient is related to the energy admittance (in the limit the distance the wavenumber $k$ is much less than 1) as $\sqrt{1 - EA}$. Over $N$ bounces, the peak pressure will fall by $(1-\text{EA})^{N/2}$ as a result of coupling along the propagation path alone. Thus, for a 0.5% energy admittance (e.g., Figure 5), the peak amplitude will fall by approximately 2% over 8 bounces. In numerical models of long-range infrasound propagation (e.g., Assink, 2012; Blom & Waxler, 2012; Waxler et al., 2008), finite ground impedance has previously been used to incorporate the effect of coupling into ground along the acoustic propagation path on observed pressure amplitudes. For computational efficiency, future work might first examine how a range of finite impedance models would affect observed infrasound (low-frequency) amplitudes. Our work suggests that acoustic to seismic coupling is the most significant with steep incidence angles and particularly slow seismic speeds (Figures 5–7). Future work would clarify the effect of coupling over repeated bounces as infrasound propagates over regions with extended topography (potentially providing steeper angles of incidence) or regions with slower seismic speeds such as alluvial planes (e.g., Hinzen, 2007). These estimates would better inform how propagation over ground with finite impedance would affect source parameter estimates.

5.3. Approximations and Future Work

There are some notable approximations to the acoustic and elastic domains in our modeling that limit our analysis. The atmosphere in the simulations is 1D and lacks wind, which may affect local acoustic-seismic coupling in addition to general infrasound propagation. If wind is included, then the atmospheric acoustic sound speed is directionally perturbed upwards or downwards relative to the direction of the wind velocity vector. This change would perturb the critical angles for a given atmosphere and earth model pair, since they are all a function of the acoustic sound speed $c$, albeit the perturbation should be small. Additionally, if $c < v_R$, then the Stoneley wave speed in an elastic medium has the acoustic sound speed in the atmosphere as an upper bound (Strick & Ginzburg, 1956). If the atmospheric acoustic sound speed is anisotropic due to winds, then the resulting Stoneley wave propagation might also show minor anisotropy (Mikhailenko & Mikhailov, 2013), even if the elastic halfspace was isotropic.

Due to meshing limitations in our simulations, the elastic domain in all of our simulations was homogeneous and purely elastic. As previously mentioned in our discussion about Case 6 (Figure 7), this configuration removes frequency dependence from the surface wave propagation. In a layered elastic medium with an increase in seismic speed with increasing distance into the medium, Rayleigh surface wave propagation is dispersive with lower frequencies propagating with faster speeds. Coupling between the incident acoustic wave and the earth model is large when the horizontal phase speed of the acoustic wave overlaps with the Rayleigh wave phase speed (e.g., Madshus et al., 2005). This trend implies that a steeper angle of incidence will be required for comparable coupling at lower frequencies (Figure 2). Likewise, calculations of the acoustic to seismic gain from observations shows an increase with increasing frequency (Matoza & Fee, 2014). Therefore, under the halfspace approximation shown here, we may be underestimating the coupling from a particular incidence angle at higher frequencies and overestimating the coupling at lower frequencies.

In the halfspace case, we note that the acoustic particle velocity measurements used in the energy admittance may be biased from the reflected spherical wave. Additionally, our geometric conversion from distance to incidence angle in the halfspace cases may possibly be biased by the spherical wave propagation. We note that this convention is applied across the analytical and numerical analyses (Figure 2).

We also note that while energy admittance is intuitive as a measure of acoustic to seismic coupling, a better measure perhaps exists. As applied in this manuscript, energy admittance is a point measurement, and we compare the peak amplitudes from simulated acoustic and seismic waves. This choice allows us to specifically pick the ground coupled airwave in what may be a complex train of seismic waves. However, this instantaneous calculation ignores differences in other parts of the waveform (e.g., rarefactions). Alternatives to be considered in future work might include calculating an energy flux vector (Woods et al., 2015) or computing the ratio of root mean square acoustic particle velocity and seismic velocity terms, which bears some resemblance to the volcano seismic acoustic ratio (Johnson & Aster, 2005). Wavelet misfit methods are notable for capturing localized differences in waveform shape (Kristeková et al., 2006).
Future work applying complementary sets of modeling efforts might further elucidate the effects of acoustic to seismic coupling beyond what we have shown here. Simulations that can convert between applying a rigid boundary condition and full coupling might be useful to observe pressure differences across the full waveform. Comparison with codes that solve the equations of motion with viscosity would allow for an estimation of the relative effect of attenuation due to acoustic to seismic coupling with intrinsic attenuation. A frequency domain method might better discern incident acoustic waves and surface waves generated near the source. Such an approach might be better suited to view the interaction of incident acoustic waves with porous media and dispersive Rayleigh and Stoneley waves as well. Finally, we note that a code like SPECFEM-DG (Brissaud et al., 2017; Martire et al., 2021) or Elac (Petersson & Sjögren, 2018), which can model both seismic waves in the solid earth and acoustic waves in a moving atmosphere, might be used to further inspect the effects of a moving, 3-D atmosphere on acoustic to seismic coupling and surface wave propagation.

6. Conclusions

We investigate and quantify the effect of acoustic to seismic coupling from incident infrasonic waves. We use SPECFEM3D to calculate full coupling between our acoustic source and a variety of solid earth models. To better understand trends in our simulations, we also consider an analytical model describing the coupling of a spherical acoustic wave into a planar elastic halfspace.

We first examine halfspace scenarios without topography in order to build intuition, and then we apply similar methods to topography modeled after Sakurajima volcano. For the earth models considered here, halfspace models show high energy admittance values in the tens of percentage points at vertical incidence (the largest was 78%). These results suggest that very large energy admittance can occur, but over a limited spatial area. Additionally, a second, smaller energy admittance peak (generally below 1%) was observed at shallower incidence angles. This secondary peak changes in amplitude, width, and location as a function of the P-wave and S-wave speeds in the elastic medium. Particularly, we find that the maximum amplitude of this secondary energy admittance peak increases with increasing Poisson ratio. We recall that the Poisson ratio is closely related to the Rayleigh wave phase speed of the elastic medium. While our simulated energy admittance values for an approximation to the model in (Edwards et al., 2007) was 86% smaller than reported in the literature, energy admittance values in this range might still have significant impact for long-range propagation.

When examining the effect of topography on energy admittance, we see that diffraction from the volcanic craters appears to affect the amount of acoustic to seismic coupling due to a change in incidence angle. Complex patterns in both energy admittance and relative pressure changes arise that are related to the compressional wave (α) and shear wave (β) speeds in the earth. However, the overall effect of coupling appears small when considering the relative amplitudes of observed pressure waves (≤ 2%). Our modeling framework still does not fully address coupling between acoustic waves and seismic waves. Modeling efforts with options for a rigid boundary, an impedance boundary, and full coupling with the same numerical scheme would greatly benefit future work. Additionally, future work could further quantify acoustic to seismic coupling with layered elastic media and coupling over regional to global scales.

Appendix A: Acoustic and Seismic Sources

A1. The Seismic Moment Tensor and Stress-Momentum Glut

For wavelengths much longer than the source size, we can approximate a seismic source as point source. The canonical seismic point source, the moment tensor ($\mathbf{M}$), can be described as the volume integral of the stress glut ($\mathbf{\Sigma}$) over the source region $V_{\text{src}}$:

$$\mathbf{M} = \int_{V_{\text{src}}} \mathbf{\Sigma} \, dV. \quad (A1)$$

The stress glut describes a perturbation between the model stress in a source region and the actual source stress in the region (Dahlen & Tromp, 1998). This difference in stresses arises from a failure of the linear constitutive equation, here Hooke's Law, to accurately describe the anelastic strain in the source region. In seismology, stress glut occurs on the rupturing fault along which tangential slip occurs.
A generalization of the stress glut concept was introduced into acoustics by Lognonné et al. (1994, 2016), where the stress-momentum glut is defined in Cartesian coordinates as

$$\Pi = (p_{\text{true}} I + \rho_f \vec{v} \vec{v}) - p I.$$

(A2)

The source moment tensor for the fluid medium is the integral of Equation A2 over the source region. We note that a connection between the seismic moment tensor and a classical quadrupole acoustic source was also derived previously using radiated power considerations by Haney et al. (2018). Below we connect the stress-momentum glut tensor with the aerodynamic description of sound from turbulent flows (Lighthill, 1978).

### A2. A Momentum Source for the Acoustic Wave Equation

Consider turbulent fluid motion throughout some small source region in an overall unbounded acoustic medium (Morse & Ingard, 1969). The region can be bounded by some spherical surface with radius $a$. We consider this exterior medium to be a uniform, isentropic acoustic medium at rest, so that the hydrostatic pressure is the only source of stress. Density fluctuations in the source region arise from differences between the true stresses in the turbulent flow and stresses in the uniform acoustic medium at rest (Lighthill, 1978).

$$\Pi = (B + \rho_f \vec{v} \vec{v}) - p I.$$  \hspace{1cm} (A3)

where the stress-momentum tensor $\Pi$ is the difference between the source stress terms $B$ and $\rho_f \vec{v} \vec{v}$ and the pressure from linearized acoustics, $-pI$. The tensor $B$ describes the total hydrostatic and viscous stresses in the source region, and the tensor $\rho_f \vec{v} \vec{v}$ is the momentum flux tensor (Morse & Ingard, 1969). We note the similarity between Equations A2 and A3.

We can define an acoustic wave equation

$$\nabla^2 p - \frac{1}{c^2} \frac{\partial^2 p}{\partial t^2} = -S(t) + \nabla \cdot \vec{F}(t) - \nabla \cdot \Pi \cdot \nabla,$$

(A4)

where the source terms on the right-hand side are zero outside of the source region and describe the monopole source term, the force term, and the stress-momentum source term, respectively (Morse & Ingard, 1969). For simplicity, we now only consider the stress-momentum term in this analysis. Converting to the frequency domain, the Green’s function solution $G(\omega, \vec{x}_s | \vec{x})$ of Equation A4 for a receiver at $\vec{x}$ from a source at $\vec{x}_s$ is

$$p(\omega) = \iiint V_{src} \left[ (B + \rho_f \vec{v} \vec{v}) - p I \right] dV,$$

(A5)

where the volume integral is taken over the entire domain. For a source region much smaller than the wavelengths $k$ of interest ($ka \ll 1$), we obtain a quadrupole pressure wave

$$p(\omega) \approx \sum_{i<j=1}^3 Q_{ij}(\omega) - \frac{\partial^2}{\partial x_i \partial x_j} G(\omega, \vec{x}_s | \vec{x}).$$

(A6)

The acoustic quadrupole is the integral of Equation A3 over the source region,

$$\vec{Q} = \iiint_{V_{src}} \left[ (B + \rho_f \vec{v} \vec{v}) - p I \right] dV.$$  \hspace{1cm} (A7)

We note that we follow the convention of Pierce (2019) instead of Morse and Ingard (1969) to describe the monopole source strength in order to maintain consistency in the definition of the quadrupole source with the equivalent definition based on sums of spatially distributed monopole point sources.

In writing Equation A4, we used the following definition of the acoustic sound speed ($c$):

$$c^2 = \frac{K_s}{\rho_f}.$$  \hspace{1cm} (A8)
where \( K \) is the adiabatic bulk modulus of the fluid. Thus, we see that the linearized pressure value in an acoustic medium at rest (Equation A3) is the Hooke's law pressure for the acoustic medium. This definition connects the stress-momentum glut (Lognonné et al., 1994, 2016) with the aerodynamic description of sound (Lighthill, 1978). It follows that a generalized seismic moment tensor in a fluid is an acoustic quadrupole in the fluid.

A3. The Acoustic Monopole and Isotropic Moment Tensor

In the case of an isotropic moment tensor, the diagonal components \( M_{11}, M_{22}, M_{33} \) are equal to the seismic moment \( M_{0} \), and other values are zero. For simplicity, we assume our source is in a free space, so the frequency domain Green's function takes the form

\[
G_\omega(\vec{x}_\text{src}|\vec{x}) = \frac{e^{i\omega r}}{r},
\]

where \( r \) is the distance from the source to the receiver. From Equation A6, the pressure field from a source with seismic moment \( M_0 \) and source time function \( \dot{K}(\omega) \) is

\[
\hat{p}(\omega) = \frac{(-i\omega)^2 M_0 \dot{K}(\omega)}{c^2} \frac{e^{i\omega r}}{r} .
\]

Converting back to the time domain, we see that

\[
p(t) = \frac{M_0}{4\pi c^2} \dot{K} \left( t - \frac{r}{c} \right) .
\]

The pressure at any point is omnidirectional, so we identify the resulting field as a monopole source field (Morse & Ingard, 1969). The pressure from a conventional monopole source in a free space is typically expressed as

\[
p(t) = \frac{1}{4\pi r} \dot{S} \left( t - \frac{r}{c} \right) ,
\]

where \( \dot{S}(t) \) is the mass acceleration (Lighthill, 1978). Comparing Equations A11 and A12, we see that

\[
\frac{M_0}{c^2} \dot{K} \left( t - \frac{r}{c} \right) = \dot{S} \left( t - \frac{r}{c} \right) .
\]

For numerical modeling, Equation A13 allows us to construct an equivalent acoustic monopole source from an isotropic moment tensor source with seismic moment \( M_0 \). A similar relationship to Equation A13 has been suggested by a few authors previously (e.g., Aldridge, 2000).

Appendix B: Prograde Particle Motion From Ground Coupled Airwaves

An acoustic wave impinging on an elastic halfspace can be viewed as a line load moving across the surface of the halfspace (Dunkin & Corbin, 1970; Sorrells, 1971). We can derive transfer functions that describe the horizontal (U) and vertical (W) displacements from an impinging pressure wave that moves with a horizontal velocity \( \dot{c} = c/\sin \theta \) across the elastic halfspace. Our axis convention has the vertical (\( z \)) axis pointing upwards and the horizontal (\( x \)) axis pointing to the right.

The horizontal (\( R_x \)) and vertical (\( R_z \)) transfer functions are

\[
R_x = \frac{-i k}{\mu} \left( \frac{(2k^2 - k''^2) \exp(\xi z) - 2\xi k' \exp(\xi' z)}{(2k^2 - k''^2) - 4k^2 \xi' \xi''} \right) ,
\]

\[
R_z = \frac{1}{\mu} \left[ \frac{(2k^2 - k''^2) \exp(\xi z) - 2k^2 \exp(\xi' z)}{(2k^2 - k''^2) - 4k^2 \xi' \xi''} \right] .
\]

\[
\frac{(2k^2 - k''^2) \exp(\xi z) - 2k^2 \exp(\xi' z)}{(2k^2 - k''^2) - 4k^2 \xi' \xi''} \].

\[
\frac{(2k^2 - k''^2) \exp(\xi z) - 2\xi k' \exp(\xi' z)}{(2k^2 - k''^2) - 4k^2 \xi' \xi''} \].
where $\mu$ is the shear modulus of the earth, $\beta$ is the seismic shear wave speed, $\alpha$ is the seismic compressional wave speed, $k = \frac{\omega}{c}$ is the wavenumber of the incident acoustic wave, $k_p = \frac{\omega}{\beta}$ is the wavenumber of the transmitted shear wave,

$$\xi = \sqrt{k^2 - \frac{\omega^2}{\alpha^2}}$$  \hspace{1cm} (B2)

and

$$\xi' = \sqrt{k^2 - \frac{\omega^2}{\beta^2}}.$$  \hspace{1cm} (B3)

We note that the denominator in Equation B1 is the Rayleigh wave equation (Equation 4). We assume the incident acoustic wave is planar and is described in the time and spatial domains as $A_A(t - \bar{x}/\bar{c})$. In the frequency and wavenumber domains, we can express the pressure wave as

$$P(\omega, k) = 2\pi P(\omega) \delta(k - \omega/\bar{c}).$$  \hspace{1cm} (B4)

The resulting horizontal and vertical elastic particle motions are

$$U(\omega, x, z) = \frac{-i}{2\pi} \int_{-\infty}^{\infty} R_A(\omega, k) e^{-i\xi z} dk,$$  \hspace{1cm} (B5)

$$W(\omega, x, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_Z(\omega, k) e^{-i\xi' z} dk.$$  \hspace{1cm} (B6)

In the case $\beta < \bar{c} < \alpha$, we can write Equation B2 as

$$\xi = \frac{\omega}{\bar{c}} \sqrt{1 - \left(\frac{\bar{c}}{\alpha}\right)^2}$$  \hspace{1cm} (B7)

and Equation B3 as

$$\xi' = \frac{\omega}{\bar{c}} \sqrt{\left(\frac{\bar{c}}{\beta}\right)^2 - 1}.$$  \hspace{1cm} (B8)

Equation B7 implies that the “P-wave” part of the transfer function (described by $\exp(\xi z)$) is exponentially damped with depth, but Equation B8 implies that the “S-wave” part of the transfer function (described by $\exp(\xi' z)$) is freely propagating (Langston, 2004). We are primarily interested in the particle displacement at the surface due to the relatively shallow burial depths of seismometers, so we let $z = 0$ in Equation B1. For notational convenience, we define the auxiliary variables

$$Y = \sqrt{\left(\frac{\bar{c}}{\beta}\right)^2 - 1};$$

$$L = \sqrt{1 - \left(\frac{\bar{c}}{\alpha}\right)^2};$$

$$M = 2 - \left(\frac{\bar{c}}{\beta}\right)^2;$$

$$D = M^4 + 16Y^2 L^2.$$  \hspace{1cm} (B9)

Substituting Equations B7 and B8 into Equations B1 and splitting the resulting transfer functions into complex and real parts, we obtain the equations

$$R_A = \frac{-2\bar{c}}{\mu\omega D} ML Y \left(\frac{\bar{c}}{\beta}\right)^2 + \frac{i\bar{c}}{\mu\omega D} (M^4 + 8Y^2 L^2),$$

$$R_Z = \frac{\bar{c}}{\mu\omega D} \left(\frac{\bar{c}}{\beta}\right)^2 LM^2 + \frac{i4\bar{c}}{\mu\omega D} \left(\frac{\bar{c}}{\beta}\right)^2 (YL^2).$$  \hspace{1cm} (B10)
Due to the complex-valued transfer functions, a real pressure perturbation can result in elastic displacement with a complex amplitude. We now examine the real part of the horizontal and vertical components of the particle displacement. We define the particle displacement matrix as

$$
\begin{pmatrix}
\hat{c} & 0 & 0 \\
-2\mu M \nu \left(\frac{\hat{c}}{\gamma}\right)^2 & \left(\mu M + 8\nu^2\mu\right) & -4\nu\mu^2 \left(\frac{\hat{c}}{\gamma}\right)^2 \\
LM^2 \left(\frac{\hat{c}}{\gamma}\right)^2 & -4\nu L^2 \left(\frac{\hat{c}}{\gamma}\right)^2 & \left(\nu L + 2\mu\right)^2 - \nu L M\nu 
\end{pmatrix}
$$

(B11)

The real-valued elastic particle displacements can then be expressed in matrix form as

$$
\begin{pmatrix}
X \\
Z
\end{pmatrix} = P(\omega) \overline{A} \begin{pmatrix}
\cos(\omega t) \\
\sin(\omega t)
\end{pmatrix},
$$

(B12)

where $X = \text{Re}(U(\omega, z, x))$ and $Z = \text{Re}(W(\omega, z, x))$. Unlike the similar matrix for a free Rayleigh wave (Aki & Richards, 2002), our matrix $A$ is not diagonal, so the resulting particle motion ellipse is tilted (i.e., rotated) in the $xz$ plane.

The eigenvalues ($\lambda_i$) of matrix $\bar{A}$ (Equation B11) can be used to determine the polarity of the particle motion.

$$
\lambda_{1,2} = -\frac{\hat{c}}{\mu\omega D} \left(\frac{\hat{c}}{\gamma}\right)^2 YL(M + 2L) \pm \frac{\hat{c}}{\mu\omega D} \left(\frac{\hat{c}}{\gamma}\right) \sqrt{\left(\frac{\hat{c}}{\gamma}\right)^2 Y^2L^2(M + 2L)^2 - LMD}. \quad (B13)
$$

For our axis convention, the particle motion is retrograde when both eigenvalues have the same sign, and the particle motion is prograde when they have the opposite sign (Figure B1).

We can view the retrograde-prograde particle motion transition through Snell’s Law.

$$
\frac{\sin \theta}{c} = \frac{1}{\sqrt{2}} \frac{\beta}{\hat{c}} = \sin \theta_x, \quad (B14)
$$
where the angle $\theta_s$ describes the angle that the transmitted SV-wave makes with respect to the vertical axis (Figure 1). The particle motion is retrograde if $\theta_s > \pi/4$ and prograde if $\theta_s < \pi/4$.

Data Availability Statement

The parameter files used to create the SPECFEEM3D runs used in this study are available at https://github.com/jwbishop/Bishop_et_al_JGR_Spectral_Element_Modeling_of_Acoustic_to_Seismic_Coupling_over_Topography.git via https://doi.org/10.5281/zenodo.5761912 with a MIT license.

Notation

- $\alpha$: P-wave/Longitudinal wave speed in the solid earth (m/s)
- $\beta$: S-wave/Shear wave speed in the solid earth (m/s)
- $c$: Acoustic sound speed in the atmosphere (m/s)
- $\tilde{c}$: Horizontal sweep speed or trace velocity of an incident acoustic wave (m/s)
- $\bar{T}(t)$: A force vector function (N)
- $K(t)$: Moment tensor source time function
- $S(t)$: Mass acceleration (kg/m$^3$)
- $\rho_f$: Fluid density (kg/m$^3$)
- $\rho_s$: Solid density (kg/m$^3$)
- $p$: Pressure (Pa)
- $P_0$: Seismic moment (N − m)
- $M$: Moment tensor ($N − m$)
- $\bar{S}$: Stress glut ($N/m^2$)
- $\bar{\Pi}$: Stress-Momentum glut ($N/m^2$)
- $\bar{\bar{T}}$: Stress ($N/m^2$)
- $\bar{\bar{B}}$: Total hydrodynamic and viscous stress ($N/m^2$)
- $\bar{Q}$: Acoustic quadrupole ($N − m$)
- $M_0$: Seismic moment ($N − m$)
- $\delta$: Delta function
- $x$: Spatial coordinate (m)
- $x_{src}$: Source location (m)
- $r$: The distance from the source (m)
- $G_{ut}$: Greens function (m$^{-1}$)
- $t$: Time (s)
- $\theta$: Incidence angle, measured from the vertical axis (degree)
- $\theta_S, \theta_L$: Transmitted angles measured from the vertical axis (degree) for S-waves and P-waves, respectively
- $u$: $\sin(\theta)$
- $\gamma$: $\cos(\theta)$
- $A(u)$: The reflection coefficient for a spherical acoustic wave
- $R(u)$: Rayleigh wave equation
- $S(u)$: Stoneley wave equation
- $\chi/\psi$: Auxiliary terms
- $K_1$: Velocity parameter - $c/\alpha$
- $K_2$: Velocity parameter - $c/\beta$
- $K_s$: Adiabatic bulk modulus ($N/m^2$)
- $W$: Vertical component of seismic velocity
- $U$: Horizontal component of seismic velocity
- $t_r$: Time of arrival for direct acoustic wave (s)
- $t_p$: Time of arrival for critically reflected P-wave (s)
- $t_s$: Time of arrival for critically reflected S-wave (s)
- $\eta$: Poisson ratio
- $EA$: Energy admittance
- $v_R$: Rayleigh wave phase speed (m/s)
- $v_{RL}$: Leaky Rayleigh wave phase speed (m/s)
- $Z_j$: Characteristic impedance ($kg \ m^{-2} s^{-1}$), where $j = c, \alpha, \beta$ is the reference phase speed.
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