Convective boundary mixing in a post-He core burning massive star model

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ABSTRACT
Convective boundary mixing (CBM) in the advanced evolutionary stages of massive stars is not well understood. Structural changes caused by convection have an impact on the evolution as well as the subsequent supernova, or lack thereof. The effects of convectively driven mixing across convective boundaries during the post He core burning evolution of 25$M_\odot$, solar-metallicity, non-rotating stellar models is studied using the\textit{MESA} stellar evolution code. CBM is modelled using the exponentially decaying diffusion coefficient equation, the free parameter of which, $f_{\text{CBM}}$, is varied systematically throughout the course of the stellar model’s evolution with values of (0.002, 0.012, 0.022, 0.032). The effect of varying this parameter produces mass ranges at collapse in the ONe, Si, Fe cores of (1.82$M_\odot$, 4.36$M_\odot$), (1.67$M_\odot$, 1.99$M_\odot$) and (1.46$M_\odot$, 1.70$M_\odot$) respectively, with percent differences from the model with minimal CBM as large as 86.3%. At the presupernova stage, the compactness of the stellar cores from O’Connor & Ott (2011), $\xi_M$, exhibit a range of (0.120, 0.354), suggesting that the extent of CBM in the advanced burning stages of massive stars is an important consideration for the explodability and type of compact remnant. The nucleosynthetic yields from the models, most notably C, O, Ne, Mg and Si are also significantly affected by the CBM assumptions, showing non-linear trends with increased mixing. The simulations show that interactions between convective C, Ne and O shells produce significant non-linear changes in the evolution, whereas from the end of Si burning, the structural changes attributed to the CBM are dominated by the growth of the convective C shell. Progenitor structures for all the models are available from HERE (link and DOI to appear).

Key words: stars - massive, stars - evolution: stars - interiors: convective boundary mixing: overshooting, compactness, convection

1 INTRODUCTION
The predictions of stellar evolution are subject to a number of uncertainties, both from a modelling and physical perspective. The effect of rotation and magnetic fields on the evolution of massive stars have been studied extensively in the past (e.g. Heger, Woosley & Spruit 2005). Farmer et al. (2016) analyzed the uncertainty introduced by different assumptions on the size of the nuclear network, which can have significant influence on the evolution leading up to the presupernova stage. More recently Sukhbold, Woosley & Heger (2017) investigated the effect of mass loss on the presupernova structure and explodability of a wide range of massive stars. Another significant source of uncertainty in stellar evolution is the treatment of convective boundaries. The effects that different convective boundary mixing (CBM) strengths have on the post-He core evolution of massive stars has not been investigated.

At the boundaries of convective regions in the star, fluid instabilities allow for the mixing of material between convectively stable and unstable regions. Simulations by Freytag, Ludwig & Steffen (1996); Meakin & Arnett (2007); Woodward, Herwig & Lin (2015); Viallet et al. (2015); Jones et al. (2016) have shown that the mixing at convective boundaries is driven by the largest-scale turbulent convective flows. In low-mass stars, CBM has been shown to explain the obser-
vation of carbon stars (Herwig 2000; Bertolli et al. 2013). CBM may facilitate the interaction of convective shells in the cores of massive stars (Meakin & Arnett 2006; Jones et al. 2016), and influence the light curve and nucleosynthesis of novae (Denissenkov et al. 2012). Young et al. (2005) showed that considering hydrodynamically-motivated mixing at convective boundaries had a profound impact on the presupernova structure of massive stars and, hence, their explosive nucleosynthesis. The influence of CBM on the advanced burning stages of massive stars is the subject of this paper.

Massive stars are typically considered to be stars with a zero age main sequence (ZAMS) mass of $M_{\text{ZAMS}} \gtrsim 8 - 9 M_\odot$ that end their lives as type-II or pair instability supernova. The lower limit is set by the critical core mass for neon ignition (Nomoto 1984; Jones et al. 2013; Jones, Hirschi & Nomoto 2014; Doherty et al. 2015; Woosley & Heger 2015), which depends on the initial metallicity of the star, extent of CBM of the H and He burning cores (Eldridge & Tout 2004), super-AGB star evolution (Poelarends et al. 2008) and rotation (Farmer, Fields & Timmes 2015). The internal structure of a massive star in its final evolutionary state consists of an inert core of Fe surrounded by shells burning Si, O, Ne, C, He and H (Woosley, Heger & Weaver 2002). These shells can be convectively unstable, and are then generally separated by radiative regions composed of the ash from the shell burning above.

Mixing mechanisms such as shearing, penetrative convection, gravity waves and boundary layer separation, collectively referred to as CBM, act to mix material from the stable layers into the convection zones and vice versa. Material mixed from above into the convection zone may reach the deeper and hotter layers of the convection zone, producing regions of convective reactive nucleosynthesis (e.g. Herwig et al. 2011; Ritter et al. 2017). The core burning stages of a massive stars are sensitive to the CO core mass left behind from He core burning, influencing the timing, extent and luminosity, amongst other evolutionary characteristics (e.g. Sukhbold & Woosley 2014).

This paper presents the results of a numerical experiment examining the sensitivity of the structure of one dimensional (1D) stellar evolution simulations of massive stars with respect to CBM strength in the advanced burning stages (post-He core burning evolution). Section 2 outlines the modelling assumptions and Section 3 describes the simulations and the key findings, which are summarized and discussed in Section 4.
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2 METHODS

Spherically symmetric (1D) stellar models with $M_{\text{ZAMS}} = 25M_\odot$ and initial metallicity $Z = 0.02$ were computed using the MESA stellar evolution code (Paxton et al. 2011, 2013, 2015), revision 7184. The relative abundances of the metals from Grevesse & Noels (1993) was used.

2.1 Input physics and modelling assumptions

The MESA models were based on those from Jones et al. (2015), to which a few modifications were made.

2.1.1 Mixing assumptions

The Ledoux criterion (Ledoux 1947; Sakashita & Hayashi 1959) was used to determine convective stability instead of the Schwarzschild criterion\(^1\) (Schwarzschild & H"arm 1958), i.e. the influence of the mean molecular weight $\mu$ on the stability was considered. The effect of semi-convective mixing (mixing on the thermal diffusion time scale in shells that have a sub-adiabatic temperature gradient only owing to a stabilizing gradient in mean molecular weight) is also considered, and uses the formulation of Langer, Fricke & Sugimoto (1983) who derived an effective diffusion coefficient from the solution to Kato’s equation for overstable, oscillatory convection in a $\mu$-stratified medium (Kato 1966). The efficiency parameter of semiconvection, with which the corresponding effective diffusion coefficient is multiplied, is taken to be $\alpha_{\text{semi}} = 0.1$ (Langer, Fricke & Sugimoto 1983), and is considered to be fast semiconvection (Woosley & Heger 2007, see also Section 6.2 of Maeder 2009 for a discussion about the fidelity of this approximation).

CBM is taken into account by the exponentially decaying diffusion model (Freytag, Ludwig & Steffen 1996; Herwig et al. 1997). In this model, the diffusion coefficient across the convective boundary and into the formally stable layer is given by

$$D_{\text{CBM}}(r) = D(r_0) \exp \left\{ -\frac{2|r - r_0|}{f_{\text{CBM}} H_p(r_0)} \right\}$$  \hspace{1cm} (1)$$

\(^1\) Comparing the template simulation (Section 2.2) to the MESA results of Jones et al. (2015), the CO core mass is larger with a percent difference of 1.61% (Table 1), the $^{12}\text{C}/^{16}\text{O}$ at the end of He core burning is smaller at 0.334, with a percent difference of 7.22%, and the main sequence and He core burning lifetimes were found to decrease to 6.911Myr and 0.635Myr with percent differences of 0.12% and 2.5% respectively.

\(\text{C} \hspace{1cm} \text{ONe} \hspace{1cm} \text{Si}\)

\(\log_{10}(\tau - t \text{[yr]})\)

\(\text{Mass} \ [M_\odot] \ | \ \boxed{\text{nuc}} \ | \ \boxed{\text{f}} \ | \ [\text{erg g}^{-1} \text{s}^{-1}]\)

$C \hspace{1cm} \text{ONe} \hspace{1cm} \text{Si}$

$\log_{10}(\tau - t \text{[yr]})$

$\text{Mass} \ [M_\odot]$

$\boxed{\text{nuc}} \ | \ \boxed{\text{f}}$

$[\text{erg g}^{-1} \text{s}^{-1}]$

Figure 2. The regions of the MESA simulation under investigation. The Kippenhahn diagram is taken from the template simulation. Grey areas represent regions that are convectively unstable and the blue contours are regions of positive energy generation where $\epsilon_\nu$ is the specific energy loss rate due to neutrino production. The x-axis is given in log of the time until the star collapses, where $\tau$ is the stars age at collapse. The solid blue line marks the H free core ($X_H < 10^{-2}$), the dashed green line is the CO core boundary ($X_{^{12}\text{C}} < 10^{-2}$), the dash-dotted red line is the ONe core ($X_{^{12}\text{C}} < 10^{-2}$), the dash-dotted light blue line is the Si core ($X_{^{16}\text{O}} < 10^{-2}$) and the dash-dotted magenta line is the Fe core ($X_{^{28}\text{Si}} < 10^{-2}$). The orange lines mark the beginning of core burning stages which are the points where the CBM is increased for each of the respective run sets (C, ONe, Si).
Figure 3. A diagram of the simulations computed for this study. Each solid line represents a simulation. The colours, separated by dashed lines, represent different core burning stages and the label at the end of each line is the run index (Table 1). The template simulation was run to collapse with minimal $f_{\text{CBM}}$ (0.002) and all subsequent runs use a radial profile of the structure from the template as a staring position.

where $r_0$ is the radius of the convective boundary, $H_p = dr/d\ln P$ is the pressure scale height, and $D(r_0)$ is the diffusion coefficient at the convective boundary given by $D(r_0) = \frac{2}{3} \nu_{\text{MLT}} (r_0) \alpha_{\text{MLT}} H_p(r_0)$. In this expression $\nu_{\text{MLT}}$ is the convective velocity and $\alpha_{\text{MLT}}$ is the mixing length parameter. $f_{\text{CBM}}$ is the free parameter of the model and the parameter of interest for this study. In MESA revision 7184, $f_{\text{CBM}}$ can be specified for H, He and metal burning convection zones. A metal burning convection zone is considered to be a convection zone whose peak nuclear energy generation does not come from H or He burning. The consequences of varying the $f_{\text{CBM}}$ parameter for the metal burning convection zones is the focus of this study (see Section 2.2).

2.1.2 Nuclear reaction network

A smaller nuclear reaction network is used in this work compared to Jones et al. (2015), who used a network consisting of 171 isotopes. This work uses a moderate-sized reaction network from H to Si up until the start of Ne and O burning, where the network is extended through the alpha-elements up to Fe (see Figure 1). A larger network is used for core Ne and O burning than main sequence and He core burning to more accurately capture the nuclear reactions involving heavier species found there. For core Si burning, the network is reduced to follow just 21 species to ease the computational burden. While the 21-species network approximates explicitly tracks the abundances of 21 species, $(\alpha, p)$ and $(p, \gamma)$ reactions (and their inverses) are also included by assuming a steady-state of the intermediate isotopes (Weaver, Zimmerman & Woosley 1978). One of the down-sides of switching networks is that outside of the Si burning shells the electron fraction $Y_e$ will be reset to 0.5 because neutron excesses are only introduced via burning of Si into Fe-group elements and electron captures by $^{56}$Ni and $^{56}$Fe. The (de-)leptonization rate in the Fe core is set by the electron capture rates by $^{56}$Ni and free protons and beta-decay and positron-capture by free neutrons (using reaction rates from Langanke & Martínez-Pinedo 2000). Farmer et al. (2016) have investigated the effects of network size on presupernova structure which gives an idea of the uncertainty in the present work with respect to the reaction network.

2.2 Outline of the $f_{\text{CBM}}$ parameter study

Following the extinction of convective core He burning\textsuperscript{2}, the model was evolved to the onset of iron core collapse\textsuperscript{3} using $f_{\text{CBM}} = 0.002$ for the metal burning convection zones. This simulation is referred to throughout this manuscript as the template simulation.

The template simulation was then branched at three evolutionary stages (as indicated in Figure 2):

(i) the ignition of central C burning (C series), when the central temperature exceeds $T_C \approx 7.59 \times 10^9 \text{K}$
(ii) the ignition of central Ne burning (ONe series), when the central temperature exceeds $T_C \approx 1.41 \times 10^9 \text{K}$
(iii) the extinction of core O burning (Si series), when the central temperature exceeds $T_C \approx 2.45 \times 10^8 \text{K}$.

At each branching point, three models were generated, each assuming a different value for $f_{\text{CBM}}$ (0.012, 0.022, 0.032), which was held constant across all of the convection zones inside the CO core. Figure 3 gives a schematic diagram of the branching points for each simulation and Table 1 gives the $f_{\text{CBM}}$ values for each burning stage of each simulation. The resolution of the resulting simulations consisted of $\approx 3000$ spatial points during the post He burning core evolution and $\approx 300,000$ temporal point over the lifetime.

\textsuperscript{2} The end of the core He burning phase is taken to be when the central He mass fraction falls below $10^{-5}$.

\textsuperscript{3} The onset of core collapse was defined to be when the infall velocity of the core reaches $1000 \text{ km s}^{-1}$.
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Table 1. Selected values for each simulation. The columns labeled \( f_{\text{CBM}} \) followed by a core burning stage, are the CBM parameters implemented during that burning stage. The column labeled \( M_{\text{CO}} \) is the CO core mass at the end of He core burning and is the same for each simulation. \( M_{\text{ONe}} \) is the mass of the ONe core when the simulations begin to burn Ne in the core (Section 2.2). \( M_{\text{Si}} \) is the mass of the Si core when convection stops in the convective Si core. \( M_{\text{Fe}} \) is the mass of the Fe core at \( \log_{10}(\tau-t) = -6 \). The dashes in the core mass columns represent values that are the same as the \text{TEMPLATE} simulation. The bottom row contains the range which is defined to be the absolute difference between the highest and lowest values. Also included is the percent differences (\% diff) for the smallest and largest core mass values compared to the \text{TEMPLATE} simulation. Calculating a percent difference with the \text{TEMPLATE} simulation is not meant to imply that the \text{TEMPLATE} value is the accepted value.

| Name   | \( f_{\text{CBM}}(\text{H}, \text{He}) \) | \( f_{\text{CBM}}(\text{C}) \) | \( f_{\text{CBM}}(\text{Ne}, \text{O}) \) | \( f_{\text{CBM}}(\text{Si}) \) | \( M_{\text{CO}}[M_\odot] \) | \( M_{\text{ONe}}[M_\odot] \) | \( M_{\text{Si}}[M_\odot] \) | \( M_{\text{Fe}}[M_\odot] \) |
|--------|----------------------------------|------------------|------------------|------------------|-------------------|-------------------|-------------------|-------------------|
| \text{TEMPLATE} | 0.002 | 0.002 | 0.002 | 0.002 | 6.93 | 1.77 | 1.51 | 1.59 |
| C1     | 0.002 | 0.012 | 0.012 | 0.012 | - | 1.74 | 1.41 | 1.47 |
| C2     | 0.002 | 0.022 | 0.022 | 0.022 | - | 1.77 | 1.60 | 1.56 |
| C3     | 0.002 | 0.032 | 0.032 | 0.032 | - | 1.86 | 1.82 | 1.60 |
| \text{BHe1} | 0.002 | 0.002 | 0.012 | 0.012 | - | - | 1.46 | 1.46 |
| \text{BHe2} | 0.002 | 0.002 | 0.022 | 0.022 | - | - | 1.61 | 1.54 |
| \text{BHe3} | 0.002 | 0.002 | 0.032 | 0.032 | - | - | 1.68 | 1.52 |
| S1     | 0.002 | 0.002 | 0.002 | 0.012 | - | - | - | 1.62 |
| S12    | 0.002 | 0.002 | 0.002 | 0.022 | - | - | - | 1.54 |
| S13    | 0.002 | 0.002 | 0.002 | 0.032 | - | - | - | 1.43 |

| range \([M_\odot]\) | 0.12 | 0.41 | 0.19 |
| \% diff \(|(\text{1.96}, 5.08)| (6.62, 20.5)| (10.1, 1.89)\)

The values of \( f_{\text{CBM}} \) were chosen to span CBM strengths ranging from the lowest value such that the simulation would converge without numerical smoothing \((f_{\text{CBM}} = 0.002)\), to a large value of \( f_{\text{CBM}} = 0.032 \). The value of \( f_{\text{CBM}} = 0.032 \) has been deduced from idealized high-resolution 3D hydrodynamic simulations of an O burning shell in a 25\(M_\odot\) star at the upper convective boundary (Jones et al. 2016). Note, however, that the implementation presented here is for the upper and lower convective boundaries. Additionally, in the analysis of Jones et al. (2016), the effective diffusion coefficient was decayed from a distance \( f_{\text{CBM}} H_R \) inside of the Schwarzschild convective boundary. This parameter is generally denoted by \( f_0 \) and represents a linear shift of the CBM model, from the convective boundary, into the convection zone. In this study the value of \( f_0 = 0.002 \) for all simulations. The value of \( f_0 \) was fixed in order to only test the effect of \( f_{\text{CBM}} \) on the simulations.

Currently there is no convincing model for how CBM should depend on the physical properties of the plasma (e.g. its thermodynamic state) and the flow characteristics (see Arnett et al. 2015; Viallet et al. 2015, for the current status in CBM modelling for stellar evolution).

Additionally, applying different values of \( f_{\text{CBM}} \) to each type of convection zone (C burning, Ne burning, etc.), would significantly increase the parameter space of the study (number of \( f_{\text{CBM}} \) values to the fourth power for C, Ne, O and Si), which is not the intention of this work \(^4\). In view of the considerable uncertainties when adopting the \( f_{\text{CBM}} \) parameter, this numerical experiment is primarily designed to study the sensitivity of the MESA simulations’ stellar structure with respect to CBM strength, at different times in the later stages of evolution.

\(^4\) However, let it be noted that a number of similar uncertainty studies of a more statistical nature have recently been published (Farmer, Fields & Timmes 2015; Fields et al. 2016; Farmer et al. 2016), the latter of which is concerned with the presupernova structure of massive stars and its sensitivity to numerical resolution and nuclear reaction network size.

3 RESULTS

In this section, the effect of varying the \( f_{\text{CBM}} \) parameter in the post He core burning phases of a 25\(M_\odot\) stellar model is examined. The implications for core structure, presupernova compactness and nucleosynthesis are presented. More detailed information on the structure of each simulation can be found in Davis (2017)\(^5\).

3.1 Convective structure

Changing the CBM during the late stage evolution changes the structure of the simulations significantly. The C simulations have different numbers of convective C shells which change the ONe core mass and have consequences for the later evolution. The Ne simulations experience Ne core-C shell interactions that change the number of C shells and change the Ne and O core evolution. Enhanced CBM in both of these post He core evolutionary stages change the structure near collapse.

The \text{TEMPLATE} simulation has two large convective C shells during its evolution (Figure 4, 5). This simulation’s second convective C shell experiences limited entrainment from below so that the ONe core mass, after this shell has developed, remains roughly constant at about 2.34\(M_\odot\) (Figure 5, Table 2). In the C1 simulation, which experiences three convective C shells throughout its evolution, the second convective C shell starts before Ne core burning. After the convective O core forms, the bottom convective boundary of the second C shell moves further out in mass, decreasing the mass contained by the convection zone (Figure 5). During O shell burning, the ONe core mass increases to a maximum value of about 2.1\(M_\odot\) before convective Si core burning begins and a third convective C shell forms. The bottom boundary of this shell begins to entrain material and the ONe core mass drops to around 1.9\(M_\odot\) before the end of

\(^5\) http://hdl.handle.net/1828/8054
Figure 4. Kippenhahn diagrams for the C simulations with increasing values of $f_{\text{CBM}}$. The diagrams show the evolution of the first C shell up to the beginning of the convective Ne core. The x-axis is the time from the point of maximum luminosity due to C burning, $L_C$, in the convective C shell. The star’s life (Table 2). For the C2 simulation, four convective C shells develop during the evolution. O core burning between the second and third convective C shells pushes the ONe core boundary up to $\approx 2.7M_\odot$ (Figure 5, 6). When the third convective C shell forms, entrainment from the bottom is large enough to reach the ash left behind by radiative Ne burning, in the form of $^{16}\text{O}$, $^{24}\text{Mg}$ and $^{28}\text{Si}$ (Figure 6). This C shell reaches a depth of $2.27M_\odot$ at the black dotted line in Figure 6. Although the convective boundary only reaches the top of the Ne ash deposit, because the convective C shell spans a large portion of the star, the $^{16}\text{O}$, $^{24}\text{Mg}$ and $^{28}\text{Si}$ that is mixed into the convection zone is brought up to the top, at around $6.5M_\odot$. The C3 simulation also develops.

Figure 5. Kippenhahn diagrams for the C simulations with increasing values of $f_{\text{CBM}}$. The diagrams show the evolution from Ne core burning to $\approx 1\text{hr}$ before collapse. The evolution here is a continuation from Figure 4. The dashed black lines refer to Figure 6, 7, 8 and 11, and the solid black lines refer to Figure 19. Definitions for the core boundaries can be found in Figure 2.
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Figure 6. Kippenhahn diagram and abundance profile from the C2 simulation as the third C shell bottom boundary approaches the Ne burning ash below. The upper convection zone in the figure is the bottom of the C shell, a radiative Ne burning region is on the left and the convection zone on the bottom right is an O shell. The thin dash-dotted red line is the ONe core boundary. The dotted black line is the point where the convective boundary reaches it deepest in mass. The dashed black line is the point where the abundance profile is taken. This is the same dashed black line as in Figure 5. The abundance profile shows the bottom boundary of the convective C shell mixing in Ne ash from below (grey shaded area).

Figure 7. Kippenhahn diagram and abundance profile from the C3 simulation. The Kippenhahn diagram shows the merger of the Ne (bottom) and C (top) shells. The red dash-dotted line is the ONe core boundary, and the dashed black line is the point where the abundance profile is taken and is the same dashed line as in Figure 5. The grey shaded regions on the abundance profile are the convection zones in the Kippenhahn diagram.

ops four C convection zones during its evolution (Figure 4, 5). Where the C2 simulation experiences something like a classical dredge-up (convection entrains ash from a previous burning stage), in the C3 simulation the convective Ne and C shells merge after convective Si core burning (near the dotted line in Figure 5, panel 4). During Si core burning, a radiative Ne shell forms at about 2.6M⊙. At this point in time, C starts burning radiatively at the ONe core boundary, at ≈ 3.4M⊙. The C shell becomes convective and begins to erode the ONe core (Figure 7). After this, the Ne shell also starts to convect, mixing material in from the top. The difference in entropy between the Ne and C shell is relatively small in this case, at Δs/N₄kerg ≈ 0.2 (Figure 8). In the template simulation, a Ne shell forms under the second

C shell with an entropy difference of Δs/N₄kerg ≈ 2. The convective boundaries of the two shells meet and form one convection zone spanning 4M⊙ (Figure 9). Relative to the C shell in the template simulation, the abundances of 24Mg and 28Si increase as the ash from form Ne burning is mixed into the convective C shell.

The ONe1 and ONe2 simulations both experience three convective C shells throughout their evolution as compared to the template which has two (Figure 4, 10). In many of the simulations in this study (template, C1, C3, ONe1, ONe2, and ONe3), convective Ne core burning is followed by the formation and growth of a convective C shell. In these simulations, near the end of Ne core burning, convection in the C shell begins near the ONe core boundary, at the composition profile imprint left by the previous convective C shell. As the Ne core burns out, material is mixed through the top and bottom boundaries of the C shell, as can be seen in Figure 4 and 5. The amount of CBM at the C shell boundaries determines the growth of this shell, and in the C3, ONe2 and

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\textbf{Figure 8.} Kippenhahn diagram from the \textit{template} simulation and entropy profiles from the \textit{template} and C3 simulations. The Kippenhahn diagram shows a Ne shell underneath a convective C shell. The entropy profile shows the large entropy gradient between the two convection zones in the \textit{template} simulation and is taken at the dashed black line in the Kippenhahn diagram (same dashed black line as in Figure 5). In the C3 simulation, the entropy gradient is much lower before the shell merger. This profile is taken at the dashed black line in Figure 5 for the C3 simulation. Convective regions in the entropy plot are illustrated by light grey shading for the \textit{template} and dark grey for the C3 simulation.

\textit{ONe3} simulations, can delay the formation of the convective O core (Figure 4, 10). In the \textit{template} simulation this does not happen. The second convective C shell remains until collapse with a bottom boundary at \( \approx 2.35M_\odot \). The convective O core follows Ne core burning with a relatively short delay of 0.04yr or about 15 days. The \textit{ONe1} simulation is similar in that the delay from convective Ne core burning to convective O core burning is also relatively short, 0.07yr (26 days). Although, for this simulation, the second convective C shell does not last to collapse, it is extinguished before the end of convective O core burning at a depth of \( \approx 1.77M_\odot \) (Figure 10). The \textit{ONe2} simulation has a delay of 1.07yr before convective O core burning. In this case the CBM in the second C shell pushes the bottom boundary to a depth of 1.5\( M_\odot \). After the bottom boundary of this C shell recedes, O core convection starts (Figure 10). The \textit{ONe3} simulation is similar to \textit{ONe2} with a delay of 1.26yr before convective O core burning. The convective C shell then recedes after reaching \( \approx 1.55M_\odot \) and convection in the O core begins (Figure 10). The O core formation delay in the C3 simulation is shorter than both the \textit{ONe2} and \textit{ONe3} simulations at \( \approx 0.45yr \) or about 164 days. The C shell reaches a depth of 1.47\( M_\odot \) before the O core becomes convective.

Relatively small changes in the shape of each zone, as a result of the enhanced CBM, produce large changes in the structure overall. Even with a small difference in CBM strength, as in the C simulations, dredge-ups and shell mergers change the size of convective shells and the elemental distribution within them. Enhanced CBM in the ONe core can create a delay between the core Ne and core O burning on the order of \( \approx 1/10 \) of the Ne nuclear burning time scale, causing differences in the C shell formation above. Changing the CBM changes the structure of the simulations significantly, producing simulations with different elemental distribution, convective structure and time evolution.

\subsection{3.2 Dredge-ups, Shell Mergers and Core Masses}

The two main mechanisms by which CBM changes the core structure are \textit{dredge-ups} and \textit{shell mergers}. Either one of these events alone do not change the structure significantly, although the many interacting dredge-ups and shell mergers over the evolution will. One of the main consequences of increased CBM in the advanced burning stage of massive stars is to alter the relative masses of the burning shells in the stellar core.

A dredge-up occurs when entrainment at the bottom convective boundary mixes ash from a previous burning stage into the convection zone as in Figure 11. The Kippenhahn diagram shows the third C shell of the C1 simulation entraining C ash from below. Entrainment from below mixes Ne ash, from the previous Ne burning region, into the C shell. In this case, because the C shells that develop in
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The late stage evolution of the star are large, spanning a few solar masses, the Ne ash can be mixed much closer to the surface. A shell merger occurs when a radiative region between two convection zones is eroded by entrainment, providing an opportunity for the two shell boundaries to meet and form one convection zone. Figure 7 shows a C-Ne shell merger found in the C3 simulation. This shell merger is between the third Ne shell, which forms at the end of the second O shell, and the fourth C shell. After the two shells merge, C can be transported to the bottom of the Ne burning region and Ne ash can be transported upward closer to the surface. During the evolution of the simulations, dredge-ups and shell mergers interact and cause the largest changes in the simulations. Although in some cases, one dredge up may not change the structure drastically, the cumulative effects of many interacting dredge-ups and shells mergers throughout the evolution create the large differences seen in these simulations.

Table 1 gives the CO, ONe, Si and Fe core masses of the 10 simulations, along with a summary of the $f_{CBM}$ val-
ues used to compute them. The same information is represented graphically in Figure 12. An example of the differences caused by the C shell dredge-ups can be seen in the C simulations (Figure 12, upper panel). In the C simulations, once the first convective C shell forms, it grows as material from above and below is mixed into the convection zone. Entrainment from above mixes $^{12}$C, $^{20}$Ne and $^{16}$O into the convection zone where the $^{12}$C can burn. From below, entrainment erodes the ONe core formed by previous radiative C burning, mixing $^{20}$Ne, $^{24}$Mg and $^{25}$Mg into the convection zone. At the bottom boundary, the amount of entrainment increases for larger values of $f_{\text{CBM}}$ deepening the C shell in mass coordinate. This increased entrainment can reduce the ONe core mass experienced by the next convective C shell, and subsequently, the ONe core mass at Ne ignition in the TEMPLATE and C1 simulations. In the C2 and C3 cases, a second C shell develops before the onset of core Ne burning, which increases the ONe core mass at Ne ignition (Figure 4). Although the C simulations have a difference in ONe core masses of $\Delta M_{\text{ONe}} = 0.12M_\odot$, the cumulative effects of increased CBM create a $\Delta M_{\text{Si}} = 0.41M_\odot$.

Even though the effect of one dredge-up or shell merger produces relatively small changes to the simulations in most cases, the interaction of many of these events throughout the evolution can produce simulations that are unrelated in the convective structure, having different convection zones at different mass coordinates.

3.3 Nucleosynthesis

During a Type II supernova explosion, the supernova remnant in the form of a black hole or neutron star, will be composed of material that previously made up the core of the star. This material consists of some of the metals produced in the core, and will not be ejected by the supernova explosion (see, e.g. Colgate & White 1966; Fryer 2009; Janka 2012; Ugliano et al. 2012; Ertl et al. 2016, and references therein). This means that a significant amount of the material the star produces will not be ejected and used for further star formation.

To estimate the effects of CBM on the chemical yields from massive stars during the advanced burning stages, a mass cut is used. The mass cut is an estimate of the Lagrangian coordinate separating the compact remnant from the ejected material. Here, the mass cut is taken from the formula of Fryer et al. (2012) for the delayed explosion scenario. This formula gives a remnant mass of $5.7M_\odot$ for the $25M_\odot$ solar-metallicity model considered in this work. Had the rapid explosion scenario been considered, the entire star would fall back into the remnant, and only the material from the envelope that has been ejected by winds would enrich subsequent star formation. The value of $5.7M_\odot$ puts the mass cut near the top of the C shell for all simulations. Because the C shell spans a large range of masses, and is convective in most simulations, small variations to this mass cut (on the order of a solar mass or more) would not significantly change the chemical signatures found in the ejected material. The work by Sukhbold et al. (2016); Farmer et al. (2016), and the work here show that the final state of stellar simulations is dependent on more than just ZAMS mass and metallicity. Therefore, determining the mass cut in this way is an approximation and is meant to illustrate how the nucleosynthetic signatures closer to the surface are affected by the CBM in the later stages of evolution (see Section 4).

In order to investigate the net effects of CBM on the nucleosynthesis of each simulation, the ejected mass of each element is determined from the stellar evolution model by the presupernova overproduction factors, $\Theta_i$. These overproduction factors (Pignatari et al. 2016; Ritter et al. 2017) are given by

$$\Theta_i = \frac{1}{M_{\text{sol.i}}} \int_{M_{\text{delay}}}^{M_{\text{surf}}'} Z'_i(m) dm,$$

Figure 12. ONe, Si and Fe core masses at Ne-ignition, Si-depletion and about 30 seconds before collapse, respectively. Refer to Table 1 for more details of these values. All of the core masses show a non-linear dependence on the convective boundary mixing parameter $f_{\text{CBM}}$. 

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where the primes denote values taken at collapse, and $Z_i$ is the abundance of element $i$. $M_{\text{delay}}$ is the mass cut, $M'_{\text{surf}}$ is the mass of the simulation at collapse and $M_{\text{sim},i}$ is the mass of the element from the initial abundance, given by

$$M_{\text{sim},i} = \int_{M_{\text{delay}}}^{M'_{\text{surf}}} Z_i(m) \, dm = Z_i(M'_{\text{surf}} - M_{\text{delay}}),$$

for a uniform initial composition. The values for $\Theta_i$ give the mass of the element $i$ above the mass cut normalized to the initial composition of the model. Note that the $\Theta_i$ values have not been processed by the supernova shock or include any contributions from the winds. Figure 13 gives $\Theta_i$ for the C, ONe and Si simulation sets. The greatest deviation from the template simulation is found in the C simulations. In the C set, the amount of C does not significantly change between the C simulations, where as the template simulation is enhanced by a factor of 10. Ne, Mg and Si also show large nonlinear variation due to the complexity of the Ne and C shell interactions (Section 3.1 and 3.2). The C3 simulation produces the least amount of Ne whereas as the C1 simulation produces the most, with a variation of less than one order of magnitude.

Figure 14. Kippenhahn diagrams for the Si simulations. Starting from the top, the diagrams are for the template, Si1, Si2 and Si3 simulations. The black dashed lines are where the entropy profiles are taken in Figure 15.
Figure 15. Entropy for the S11, S12 and S13 simulations for the bottom of the C shell. The profiles are taken at the black dashed lines in Figure 14.

Figure 16. The ONe, Si and Fe core masses for the S1 simulations. The x-axis is log of the time left until collapse and the y-axis is the enclosed mass from the core. The plot shows the differences in the core masses with respect to the amount of CBM implemented during core Si burning. The small differences in the lines before core Si burning is due to differences in the evolutionary time for each simulation.

Table 2. Presupernova core masses and compactness parameter \( \xi_{2.5} \). The core masses are taken when the infall velocity in the Fe core reaches 1000 km/s. The core masses are defined in terms of lower limits of key abundances, these definitions can be found in the caption of Figure 2. The presupernova compactness, \( \xi_{2.5} \), taken at \( \log_{10}(\tau - t) = -6 \), the values do not change significantly past this point. The last two rows give the range and percent difference (% diff.) of the smallest and largest values compared to the \textsc{template} simulation. The range is defined to be the absolute difference between the highest and lowest values.

| Name    | \( M_{\text{ONe}}^{\text{pres}} [M_\odot] \) | \( M_{\text{Si}}^{\text{pres}} [M_\odot] \) | \( M_{\text{Fe}}^{\text{pres}} [M_\odot] \) | \( \xi_{2.5} \) |
|---------|--------------------------------|--------------------------------|--------------------------------|---------|
| \textsc{template} | 2.34 | 1.97 | 1.69 | 0.272 |
| C1      | 1.95 | 1.76 | 1.50 | 0.172 |
| C2      | 4.36 | 1.93 | 1.67 | 0.304 |
| C3      | 3.95 | 1.99 | 1.70 | 0.354 |
| ONe1    | 2.03 | 1.73 | 1.55 | 0.159 |
| ONe2    | 2.21 | 1.91 | 1.66 | 0.249 |
| ONe3    | 2.03 | 1.80 | 1.60 | 0.152 |
| S11     | 2.20 | 1.80 | 1.62 | 0.217 |
| S12     | 2.01 | 1.90 | 1.54 | 0.162 |
| S13     | 1.82 | 1.67 | 1.46 | 0.120 |
| range [M_\odot] | 2.54 | 0.32 | 0.24 | 0.234 |
| % diff   | (22.2, 86.3) | (15.2, 0.01) | (13.6, 0.06) |

3.4 Presupernova core masses and compactness parameter

The core masses of the models at the presupernova stage are shown in Figure 17 and given in Table 2. The \( C \) simulations show a large increase in the ONe core mass for the C2 and C3 simulations (Figure 5). This ONe core mass increase corresponds to the large entrainment of Ne burning ash into the C shell. In the C2 simulation, this happens during the C shell dredge up that takes place after radiative Ne shell burning (see Section 3.1, Figure 6). In the C3 simulation, this increase happens after the Ne-C shell merger (see Section 3.1, 3.2, Figure 7). The Si and Fe core masses of the

value of \( \Theta_{\text{Si}} = 9.7 \), compared to \( \Theta_{\text{Si}} = 1.8 \) for the \textsc{template}. The \( \Theta_{C} \) for the S1 simulations have the least deviation from the \textsc{template}. Figure 13 shows that \( \Theta_{\text{C}} \) decreases for increasing values of \( f_{\text{CBM}} \), where as Ne and Ne ash both increase. Unlike the ONe simulations, the Si simulations don’t show large dredge-ups of Ne ash into the C shell (Figure 10, 14). In the S11, S12 and S13 simulations, only a limited amount of mixing between the C shell and the underlying Ne shell is possible due to the large entropy gradients found between the shells (see Section 3.5, Figure 15). In these simulations, \( \Theta_{\text{C}} \) shows the largest variation from that of the \textsc{template}. During core Si burning, Ne and O shells form under the C shell and promote C shell dredge-ups of C ash (Figure 14). This decreases the ONe core mass and the depth of the C shell boundary (Figure 16), increasing the temperature at the bottom of the C shell.

The CBM during the later stage evolution of these simulations can affect the \( \Theta_{C} \) values by mixing material processed in metal burning shells into the C shell. Simulations that show large deviations from the \textsc{template} are those that have dredge-ups and shell mergers with the C shell, and have enough time left in their evolution to mix that material passed the mass cut.

magnitude. Simulations with the lowest Ne production (C2 and C3) produce the most Mg and Si where simulations with high Ne (\textsc{template} and C1) produce lower amounts of Mg and Si. This implies that in the C2 and C3 cases the abundances in the C shell shows evidence of Ne burning from below. For the ONe simulation set, the \textsc{template}, ONe1 and ONe2 simulations all show similar trends in \( \Theta_{C} \). C decreases for enhanced \( f_{\text{CBM}} \) and Ne, Mg and Si all increase, in part due to the depth of the third C shell (Section 3.1). The ONe3 simulation does not follow this trend. The \( \Theta_{\text{Ne}} \) value is less than that for the \textsc{template}, and Mg and Si are both higher. Si in particular is larger than that from the \textsc{template} with a
C and ONe simulations are non-monotonic with increasing $f_{\text{CBM}}$. This is due to the cumulative interaction of the C, Ne and O shells that each simulation experiences, changing the core structure of the star significantly before collapse (Section 3.2). In the Si simulations, due to the lack of time left in the evolution compared to the convective turn over time scales of shells, and the presence of a well established C shell, the convective shells do not interact as in the C and ONe simulations. Rather than being the result of many dredge-ups and shell mergers, the changes in the Si simulation core masses are dominated by the C shell dredge-ups that happen at the beginning of Si core burning (Figure 14). These dredge-ups are larger with increasing $f_{\text{CBM}}$ as can be seen in the presupernova ONe core masses (Table 2). In these simulations, both the presupernova ONe core masses and the Fe core masses are monotonically decreasing with increasing $f_{\text{CBM}}$. Similar effects of CBM on the core of the simulations can be seen in the compactness of the massive star core.

The compactness parameter of a massive star core is a measure of the depth of the gravitational potential well at the bounce phase of a core-collapse supernova. A correlation has been found between this parameter and whether or not the stellar model will produce a supernova explosion when exploded using one dimensional codes (O’Connor & Ott 2011; Ugliano et al. 2012; Ertl et al. 2016; Müller et al. 2016; Sukhbold et al. 2016; Sukhbold, Woosley & Heger 2017). Lower values of the compactness favour explosions while higher compactness favour weak or failed explosions that likely result in black hole formation (Ugliano et al. 2012; Ertl et al. 2016). O’Connor & Ott (2011) define the bounce compactness of a stellar core as

$$\xi_M = \frac{M}{R(M)/1000\text{km}}_{t=t_{\text{bounce}}}$$

where $M$ is the baryonic mass and $R(M)$ is the radius in which a mass of $M$ is enclosed at the time that the infalling core material bounces off the proto-neutron star ($t_{\text{bounce}}$). To evaluate the compactness, the mass is generally set to $M = 2.5\mathcal{M}_\odot$, i.e. the relevant mass scale for black hole formation (O’Connor & Ott 2011). Alternatively, evaluating the compactness at the presupernova stage, when the infall velocity of the iron core reaches 1000km$^{-1}$, provides a reasonable estimate for the value of $t_{\text{bounce}}$. Sukhbold & Woosley (2014) found that, among other things, the non-monotonicity of the presupernova compactness with respect to the progenitor’s ZAMS mass is strongly dependent on the behaviour of the C and O burning shells (more recently Sukhbold, Woosley & Heger (2017)). In this study it is found that the position and timing of these shells are, in turn, dependent on the strength of the CBM used to compute them.

It is important to note that the bounce compactness given by O’Connor & Ott (2011) is not a definitive determination of the explodability of massive star simulations. Other effects such as asymmetries, turbulence and even the dimension in which the explosion is calculated all affect the explodability (Dolence et al. 2013; Radice et al. 2017; Müller et al. 2016; Ertl et al. 2016). Never the less, calculating the value of $\xi_{2.5}$ allows for comparison to the work of Sukhbold, Woosley & Heger (2017) and Farmer et al. (2016).

Investigating how the compactness changes throughout the evolution of the star with respect to CBM provides more insight into the non-monotonicity found by Sukhbold.
& Woosley (2014). The compactness throughout the core evolution, the *evolutionary compactness*, is plotted in Figure 18 and the quantity $\xi_{2.5}$ at the presupernova stage is given in Table 2. Each stellar lifetime plotted in Figure 18 shows spikes in the evolutionary compactness where the value increases rapidly, followed by dips where the value decreases. The spikes are caused by contractions within 2.5$M_\odot$ as a convective burning event ends. This decreases the radius at which 2.5$M_\odot$ is enclosed, and increases the compactness. At the end of a core contraction, a burning phase begins, whether that be a shell or the core. The convective regions that result from the burning expand in radius pushing 2.5$M_\odot$ further out and decreasing the compactness value.

Changing the CBM strength changes the value of $\xi_{2.5}$ significantly. For the simulations branched at C burning, the value of compactness ranges from 0.17 to 0.32. The C1 simulation, with the smallest values of $f_{\text{CBM}}$, has the largest deviation from the presupernova compactness of the TEMPLATE simulation, with $\xi_{2.5} = 0.17$ for C1, and $\xi_{2.5} = 0.27$ for the TEMPLATE. Both the C2 and C3 simulations have increas-
Convective boundary mixing in a post-He core burning massive star model

The ONe simulations show more non-monotonicity than that in the C simulations. The values of $\xi_{2.5}$ range from 0.15 to 0.27 with the template simulation being the largest and the ONe3 simulation being the smallest. In this case, the ONe1 and ONe3 simulations have similar values, with $\xi_{2.5} = 0.16$ for the ONe1 simulation, while the ONe2 simulation has a smaller value. The bottom convective boundary mass location of C burning (or C, Ne and O burning) convection zones near 2.5$M_\odot$ shows that simulations with similar presupernova compactness values have similar ONe core mass values. In the ONe simulation set, at the end of the evolution, the ONe core mass corresponds to the bottom boundary mass of these large convection zones. The template and ONe2 simulations have a bottom convective boundary mass of 2.34$M_\odot$ and 2.21$M_\odot$, respectively (with $\xi_{2.5}$ of 0.27 and 0.25). The ONe1 and ONe3 simulations both have bottom convective boundary masses of 2.03$M_\odot$, with $\xi_{2.5}$ of 0.16 and 0.15. In this case, similar to the C simulations, the more material that is contained in the large convection zone around 2.5$M_\odot$, the lower the value of $\xi_{2.5}$ will be due to the expansion of material in that convection zone. The effect of expansion and contraction of the core on the compactness can be seen in the evolutionary compactness plot for the ONe3 simulation. This simulation experiences a sharp spike in compactness around $\log_{10}(\tau-t) \approx -3$ ($\log_{10}(\nu_0-t) \approx -1.6$ in Figure 10). Leading up to this spike in the compactness evolution, the only significant energy generation within the inner 6$M_\odot$ of the CO core comes from convective Si core burning which extends to $\approx 1.3M_\odot$. Above the convective Si core, the material is contracting, increasing the evolutionary compactness. The Si core burning ends abruptly and both C and Ne begin to burn radiatively in the layers above and then transition into convection, expanding the material and decreasing the compactness. Once the C and Ne shells merge, the resulting convection zone experiences a large dredge-up into Ne ash, dropping its convective boundary down to $\approx 2.03M_\odot$ and creating the large drop in evolutionary compactness.

For the Si simulations, as $f_{\text{CBM}}$ increases, the evolutionary compactness decreases, spanning a range of 0.12 to 0.27. The ONe core mass for these simulations is plotted in Figure 16 and shows that the depth of the dredge-up increases with increasing $f_{\text{CBM}}$. As the CBM increases, the dredge-ups of the C shell overlying the Si burning core deepen in mass, mixing in more material and decreasing the size of the ONe core (Figure 14, 16). All of these simulations have ONe core boundaries that correspond to the bottom of the convective shell above this boundary, all of which are below 2.5$M_\odot$. Similar to the C and ONe simulations, the Si simulations with a lower bottom convective boundary have smaller compactness.

Although the large changes in the evolutionary compactness for the C, ONe and Si simulations are dominated by the convective C shell growth around 2.5$M_\odot$, decreasing this mass to avoid including these convective shell interactions in the calculation of $\xi_{2.5}$ would not change the non-monotonicity found in the values. The values of $\xi_{2.5}$ change due to the expansions and contractions of the core caused by the interaction of burning regions within the core. If the mass used in the compactness where decreased to avoid the C shell entainment, the O shells, for example, may act in a similar way as the C shells.

Changes to the $f_{\text{CBM}}$ parameter affect the value of $\xi_{2.5}$, which spans a range of 0.12 $\leq \xi_{2.5} \leq 0.35$ for the simulations studied here. From the ZAMS mass-$\xi_{2.5}$ relation of Sukhbold & Woosley (2014), the ZAMS mass of 25$M_\odot$ lies at the edge of a relative maximum (see Sukhbold, Woosley & Heger (2017) for updated models). These maximums in the mass evolution of $\xi_{2.5}$ have been given the name, islands of non-explodability (Sukhbold & Woosley 2014). Because the ZAMS mass of the stellar models studied here are on the edge of one of these islands of non-explodability, and the variation in $\xi_{2.5}$ with respect to $f_{\text{CBM}}$ is similar to the height of the island of non-explodability near this mass, some of the variation found in $\xi_{2.5}$ with respect to CBM may be due to changes in the ONe core mass mimicking a different ZAMS mass.

3.5 Cases for 3D hydrodynamics Simulations

Although the 1D simulations presented here give the cumulative effects of CBM during the evolution, specific situations occur where 3D hydrodynamic simulations are necessary in order to understand the mixing. After the shell mergers and dredge-ups in the C simulations, $^4$He has a minimum abundance in the centre of the convection zones. Entropy differences prevent the merger of shells in the Si simulations although the CBM model allows some material to mix. Late time shell mergers can occur during the last few hours of the simulations, these may not affect the elemental distribution within the simulations as there is not enough time to do so. All of these events are highly dependent on the fluid dynamics that would be present and the 1D simulations may not capture these effects accurately.

The Si simulations all experience two significant burning regions in the large convective C shells, one at the top and the other at the bottom (Figure 5). Figure 19 shows the specific luminosity from $^{12}$C + $^{12}$C, $^{20}$Ne + $^\alpha$ and $^{24}$Mg + $^\alpha$. Peak energy generation at the top comes from $^{20}$Ne and $^{24}$Mg captures. As $^4$He can be mixed into the convection zone from above. Deeper in the convection zone, $^{12}$C + $^{12}$C dominates.

Initially these are C burning shells but later in the evolution, they can mix in C and Ne ash and begin to burn C, Ne and O.
the energy production from about 2.5\(M_\odot\) to 4.2\(M_\odot\). The C burning produces \(^4\)He in this region and this \(^4\)He is used in further \(\alpha\) capture on \(^{20}\)Ne and \(^{24}\)Mg at the bottom of the convection zone. Although the profiles for \(^{20}\)Ne and \(^{24}\)Mg are fairly flat in Figure 19, the profile of \(^4\)He has a minimum in the convection zone. Because the \(^4\)He profile is not flat, the time scale in which \(^4\)He is being consumed and produced is much faster then the convective turn over time scale, which is about 5hr. This implies that the mixing assumptions of MLT might not be valid. In 3D, a convective flow such as this would need to be treated as a convective reactive flow, where the fluid dynamics are coupled to the reactions. This is necessary because the local turbulent mixing in a given region would determine the concentration of each species and the energy from the reactions would feed back into the fluid dynamics.

During Si burning the Ne, O and Si convective shells form relatively close to each other in mass. Because the entropy gradients are smaller in this region than further out, there is a potential for nearby convective shells to merge. In the Si1, Si2 and Si3 simulations, after convective core Si burning, an O shell forms (dashed line in Figure 14) at \(log_\odot(\tau - t) \approx -3.5\). The TEMPLATE simulation forms an O shell later on in its evolution. In the Si1 and Si2 simulations, a convective Ne shell exists above the O shell during its formation. The three convective shells are only separated from the convective C shell by a very small mass, in some cases, \(< 0.05M_\odot\). Despite the proximity and enhanced CBM of these shells, they do not merge (Figure 20). Strong entropy gradients between these shells inhibit the mixing across the shell boundaries and prevent the shells from merging (Figure 15). Although these shells don’t merge, because of the enhanced CBM of the Si simulations and the small separation of the convective shells, some mixing still occurs between them. The exponential decay of the diffusion coefficient across the boundary allows for a region outside of the convection zone to mix with material from the convection zone. In this case, both convection zones are mixing into this small radiative region separating them (Figure 20). This configuration may be analogous to a double convective boundary, where the boundary between two convection zones is stable.

During Si and Fe core burning, C, Ne and O shells can exist separated by relatively small masses (Figure 14). Specifically, in the Si2 simulation, a convective Si shell approaches an O shell underlying the convective C shell (black solid line in Figure 14). These shells are in the process of merging, although a relatively slow mixing event such as this (compared to the nuclear time scale) would have little effect on the abundances in the C shell. This is because the convective turn over time scale of the C shell at this time is \(\approx 5\)hr, and the star has roughly the same amount of time left before collapse. Therefore, any mixing event that didn’t
significantly decrease the convective turn over time scale of the C shell may not be able to mix material high enough in time to surpass the fallback mass. The results given by the 1D MESA simulations do not model this event appropriately, as the mixing between the shells is most likely dependent on the fluid dynamics of convective mixing.

All of these situations are dependent on the dynamics of turbulent convective mixing between fluids over relatively short time scales, which can’t be captured by the diffusive approximation. The production and consumption of 4He in the in the C shells of the simulations is much faster then the convective flows within the shell. The CBM model allows for some mixing between convective shells that have not merged, which is similar to two convective shells separated by a stable boundary in 3D. The late time mixing between the shells which have convective turnover time scales on the order of the evolutionary life time may effect the final abundances. In order to understand how the mixing is happening in these situations, 3D hydrodynamic simulations are necessary.

4 SUMMARY AND DISCUSSION

In order to test the effects of CBM on the post He core evolution of a 25M⊙ solar metallicity stellar model, the CBM parameter, f_{CBM}, was varied at different evolutionary times (see Section 2, Figure 2). The simulations implemented values of f_{CBM} of 0.002, 0.012, 0.022 and 0.032 with the TEMPLATE simulation having the lowest value, acting as a control for the other simulations (Figure 3). The simulations were run with ≈ 3000 spatial zones and ≈ 300,000 time steps.

Enhanced CBM promotes dredge-ups and shell mergers which restructure the core significantly leading up to the end of the star’s life. The effects of enhanced CBM on a single convection zone can be seen in the first C shell of the C simulations (Section 3.1). CBM decreases the lifetime of the first C shell and mixes ash into the convection zone from below. The ash distribution left behind by the first C shell defines the starting point for the second C shell. Enhanced CBM also pushes the bottom convective boundary deeper in mass into the ONe core, burning C deeper in the star and mixing up C ash. The net effect is similar compared to the He-shell flash convection in AGB stars, in which CBM at the bottom of the convection zone drives the Lagrangian coordinate of the bottom boundary deeper into the underlying core and adding material from below to the He-shell flash convection (Herwig 2000). In that situation CBM can accommodate several observational properties of AGB (Herwig 2005) and post-AGB (Werner & Herwig 2006) stars. The situations is also reminiscent of the effect of CBM in simulations of nova (Denissenkov et al. 2012) where it causes models to have a fast rise time and enhancements of C and O in the ejecta, as observed.

The effect of CBM on the convection zones compound during Ne, O and Si core burning for both the C and ONe simulations. In these simulations enhanced CBM interacts with the C, Ne and O shells by promoting dredge-ups and shell mergers, acting to restructure them. The dredge-ups of the C shells are of particular significance (Section 3.2). During the growth of a new convective C shell, dredge-ups can mix Ne ash from radiative Ne burning into the C shell, decreasing the location in mass of the bottom boundary. These mixing events can transport Ne ash found under the C shells to the tops of the convection zones, provided there is sufficient time to do so before the end of the stars life. Similarly, in the C3 simulation, a convective Ne shell merges with the newly formed C shell having a similar effect. Because these mixing events happen around 3.5d before collapse, the material can be mixed throughout the C shell, as the convective turn over time scale of the C shell is ≈ 5hr. In contrast, the C shells of the TEMPLATE simulation do not experience any significant dredge-up or shell merger with the underlying material.

Using the fallback prescription given by Fryer et al. (2012) with a mass cut of M_{\text{delay}} = 5.7M⊙, the presupernova overproduction factors, \( \Theta_i \), from these simulations show large variations in C, Ne, Mg and Si (Section 3.3). The largest deviations from the TEMPLATE occur in simulations that experience enhanced CBM during C, Ne and O core burning (C and ONe) due to the dredge-ups and shell mergers that occur in those simulations. These mixing events happen early enough in the evolution of the core that the Ne and O ash can be mixed to the top of the C shell, above the mass cut. Although the S1 simulations have tightly packed convective shells near the end of their evolution (Section 3.5), they do not merge due to the large entropy gradients between them. Changes in \( \Theta_i \) for these simulations are dominated by the C shell dredge-ups found earlier in the evolution. With a convective turnover time scale of ≈ 5hr during Si shell burning, the C shell needs a large increase in luminosity to decrease the convective turnover time scale to a value less then the time left till collapse (≈ 30min). Dynamic events such as dredge-ups and shell mergers in the late time of evolution could potentially provide the luminosity, but events energetic enough to do this are not seen in the simulations at this time.

The mass cut, M_{\text{delay}} = 5.7M⊙, lies under the upper convective boundary of the C shell for all simulations. It is ≈ 1.5M⊙ from the top of the C shell and ≈ 3.5M⊙ from the bottom for the TEMPLATE. The C shell is convective near collapse for these simulations and the abundances of the elements plotted Figure 13 are fairly mixed when the shell is not experiencing a dredge-up or shell merger. Therefor variations to the mass cut on the order of a solar mass or more do not have a large effect on the \( \Theta_i \) distribution. The amount of ejected material changes but the relative quantities are insensitive to variations of the mass cut within the C shell. If the mass cut were to vary within this mass range, in order for \( \Theta_i \) to change, material from below will still need to be mixed into the C shell. The mass cut of M_{\text{delay}} = 5.7M⊙ was taken as an approximation in order to determine the \( \Theta_i \) values for the simulations and illustrate the nucleosynthetic effects of CBM mechanisms on the late stage structure. The overproduction factors assume different evolutionary outcomes than that given by the compactness. For example, a simulation that collapses to a black hole as determined by the compactness, should, but will not have an overproduction factor showing zero mass ejected. These two diagnostics are meant to be used to investigate the CBM in the stellar cores rather than provide nucleosynthetic yields or determine the explodability of a particular model.

The compactness of each simulation is dependent upon the strength of the CBM (Section 3.4). The values of \( \xi_{2.5} \)
range from 0.12 to 0.35 when taken 30 seconds before collapse. Simulations which experience enhanced CBM during the Ne and O core burning stages (C and ONe) show non-monotonicity of $\xi_{2.5}$ with respect to $f_{\text{CBM}}$, whereas $\xi_{2.5}$ is a monotonically decreasing function of $f_{\text{CBM}}$ in the SI simulations. Both the C and ONe sets diverge from the TEMPLATE significantly during C and O burning as dredge-ups and shell mergers change the CO core structure, creating different numbers of convective shells. This means that in these simulations, the non-monotonicity in $\xi_{2.5}$ primarily comes from the C, Ne and O core and shell interactions in the form of dredge-ups and shell mergers during those core burning stages, but not during the end of Si core and shell burning. The ONe core mass of the SI simulations decrease with increasing CBM and do not show the non-monotonic deviation for the TEMPLATE. During the late stages of evolution in the SI simulations, the main effect of the CBM is to decrease the bottom C shell boundary. Because this boundary is below 2.5$M_\odot$ the values of $\xi_{2.5}$ represent the amount of material that is in the C shell rather than any intricate shell interactions. This means that the variation found in $\xi_{2.5}$ is due to the interaction of the C, Ne and O convective regions during those core burning stages where as during Si and Fe core, the mixing events have little effect. This is mainly because the time scales over which these mixing mechanisms can change the structure are longer than the time remaining until collapse. This result is somewhat consistent with Sukhbold, Woosley & Heger (2017) who found that the non-monotonic behaviour of $\xi_{2.5}$ with respect to ZAMS mass around the islands of non-explodability is mainly due to the formation of the C and O burning shells.

The variation in $\xi_{2.5}$ of $\Delta\xi_{2.5} = 0.23$ is due to two simulations with the highest values of $f_{\text{CBM}}$ (S13 and C3). If only the simulations implementing $f_{\text{CBM}}$ values of 0.012 and 0.022 are considered, the variation in $\xi_{2.5}$ decreases to 0.15, but the non-monotonicity found in the evolutionary compactness is still present. This means that the impact of CBM on the compactness is a cumulative effect of convective shell interactions and is not just limited to the cases where large amounts of mixing change the structure.

The progenitor structures for all models presented in this work are available for download HERE (link and DOI to appear).

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REFERENCES

Arnett W. D., Meakin C., Viallet M., Campbell S. W., Lattanzio J. C., Mocak M., 2015, ApJ, 809, 30
Bertolli M. G., Herwig F., Pignatari M., Kawano T., 2013, ArXiv e-prints

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—, 2015, ApJS, 220, 15
Pignatari M. et al., 2016, ApJS, 225, 24
Poelarends A. J. T., Herwig F., Langer N., Heger A., 2008, ApJS
Radice D., Abdikamalov E., Ott C. D., Moesta P., Couch S. M., Roberts L. F., 2017, ArXiv e-prints
Ritter C., Andrásy R., Côté B., Herwig F., Woodward P. R., Pignatari M., Jones S., 2017, eprint arXiv:1704.05985
Ritter C., Herwig F., Jones S., Pignatari M., Fryer C., Hirschi R., 2017, ArXiv e-prints
Sakashita S., Hayashi C., 1959, Progress of Theoretical Physics, 22, 830
Schwarzschild M., Här r R., 1958, ApJ, 128, 348
Sukhbold T., Ertl T., Woosley S. E., Brown J. M., Janka H.-T., 2016, The Astrophysical Journal, 821, 38
Sukhbold T., Woosley S., 2014, The Astrophysical Journal, 783, 10
Sukhbold T., Woosley S., Heger A., 2017, ArXiv e-prints
Ugliano M., Janka H.-T., Marek A., Arcones A., 2012, ApJ, 757, 69
Viallet M., Meakin C., Prat V., Arnett D., 2015, Astronomy & Astrophysics, 580, A61
Weaver T. A., Zimmerman G. B., Woosley S. E., 1978, ApJ, 225, 1021
Werner K., Herwig F., 2006, PASP, 118, 183
Woodward P. R., Herwig F., Lin P.-H., 2015, ApJ
Woosley S., Heger A., 2007, Physics Reports, 442, 269
Woosley S. E., Heger A., 2015, ApJ, 810, 34
Woosley S. E., Heger A., Weaver T. A., 2002, Reviews of modern physics, 74, 1015
Young P. A., Meakin C., Arnett D., Fryer C. L., 2005, ApJL, 629, L101

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