From QCD Lagrangian to Monte Carlo simulation

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Abstract

I discuss old and recent aspects of QCD jet-emission and describe how hard QCD results are used to construct Monte Carlo programs for generating hadron emission in hard collisions. I focus on the program HERWIG at LHC.

1 The status

LHC is a discovery machine, it is expected to tell us how to complete the unified theory of elementary interactions. New (heavy) particles are searched to indicate/confirm new symmetries. Events with heavy particles are expected to be accompanied by an intense emission of hadrons at short distances, and this is the domain of perturbative QCD. Therefore, to identify and understand non-standard events a quantitative knowledge of the characteristics of the hard radiation is strongly needed. In 1973 QCD was at the frontier of particle physics (discovery of asymptotic freedom [1] and beginning of quantitative QCD studies), now in 2007 QCD is at the center of particle studies. The Monte Carlo programs for jet emissions [2–4] are important instruments for analyzing standard and non-standard short distance events. They are the Summa of most QCD theoretical results and many present studies aim to improve their quantitative predictions. Thanks to the QCD factorization structure [5], Monte Carlo programs can be interfaced with hard cross sections involving also non-QCD processes (electroweak, supersymmetric, extra dimension, black holes, ...). In this way, Monte Carlo generators can describe both QCD and non-QCD events at short distances.

In this paper I describe the main QCD results which enter the construction of a Monte Carlo generator. They are so many that most of the key points will be recalled in a schematic way, but I hope that this short description could provide an idea of the reliability range of the Monte Carlo generators. For a more detailed description see [6]. Here, aiming to be simple and synthetic, I follow a personal point of view and the focus will be on the Monte Carlo event generator HERWIG [2]. Its general structure is similar to other important Monte Carlo generators [3, 4]. In section 2 I present the scheme of the operations performed by Monte Carlo codes for LHC. The fact that the generation of events can be subdivided into successive stages is physically based on QCD factorization properties. The theoretical basis are discusses/recalled in section 3. In section 4 I discuss...
the multi-gluon soft distributions and in section 5 I describe in detail a Monte Carlo code for soft emissions. Although important non-soft contributions included in a realistic Monte Carlo are here missed, it provides a simple example containing many important physical effects. In section 6 I discuss non-perturbative effects which enter the Monte Carlo generators. The last section contains final considerations.

2 Structure of Monte Carlo generator

I start describing schematically the way a Monte Carlo code is organized in order to generate hard QCD and non-QCD events at LHC. As a specific illustration I consider the emission of two jets with high $E_T$. This process is factorized into the elementary hard distribution, the parton densities (structure functions as in DIS) and the fragmentation functions (as in $e^+e^-$):

![Diagram](image)

Here are the necessary factorised steps:

• start form the hard elementary distribution $\hat{\sigma}_{ab \to cd}$ with $ab$ the incoming and $cd$ the two outgoing partons. This hard distribution corresponds to QCD jet emission at high $E_T$. Here one can substitute distributions for other QCD or non-QCD processes. There are many studies of hard distribution for processes relevant for LHC, see [7].

• generate the momenta of the hard incoming ($ab$) and outgoing ($cd$) partons (and possible non-QCD particles). Given the hard scales $E_T$ (and possible heavy masses), the momenta are generated (via important sampling) in computing the total cross section as convolutions of the elementary distribution and the parton densities (structure functions);

• use the initial state space-like evolution (which at the inclusive level gives the structure functions) to generate the “bremsstrahlung” of outgoing initial state partons $k_1', k_2', \cdots$. This requires imposing a minimal transverse momentum w.r.t. the collision direction;

• given the outgoing hard QCD partons $cd$ and $k_1', k_2', \cdots$, start the QCD shower (parton multiplication). First, from the set of these partons, identify their colour connections and reconstruct the set of the various primary $q\bar{q}$ dipoles. Here one works in the large $N_c$ approximation so that a gluon, from the colour point of view, can be represented as a pair of quark-antiquark lines, a gluon is then associated to two dipoles;
• generate, for each primary dipole, the multi-parton emission according to the coherent branching structure that will be illustrated in the following. This requires imposing a lower bound on the relative transverse momenta of final state partons (inside sub-jets);

• match with the exact high order calculation, if available. It consists in weighting the generated event by comparing [8] the Monte Carlo distribution and the exact square matrix element computed to higher order [7];

• given the system of all emitted partons, generate the final hadrons by using a hadronisation model making hadrons out of partons. Using hadronisation models based on colour connections and preconfinement [9], such a process should not substantially modify [10] the structure of the hadronic radiation with respect to the partonic one which has been obtained in the previous steps.

In the next sections I describe the QCD basis of these steps.

3 The long way to Monte Carlo

QCD has a dimensionless coupling but, even at large scale $Q$, when all masses can be neglected, the cross sections do not scale simply as powers of $Q^2$. This is due to the presence of ultraviolet, collinear and infrared divergences. Ultraviolet divergences are responsible for the presence of the fundamental QCD scale $\Lambda_{\text{QCD}}$ entering the running coupling. Collinear and infrared divergences are well known from QED [11]. Parton distributions can be computed only by fixing a resolution $Q_0$ (technically, a subtraction point) in the parton transverse momentum. Collinear and infrared divergences are responsible for large enhancements in these distributions which need to be resummed. Monte Carlo generators do actually perform these resummations as I discuss in the following.

The possibility to resum these enhanced terms is based on specific properties of the collinear and infrared singularities: they factories [5, 12, 13]. In this way one can formulate recurrence relations that lead to evolution equations. The fundamental one is the DGLAP evolution equation [14] resumming collinear singularities in parton densities and fragmentation functions. These are single-inclusive quantities, but to reach a complete description of an event one needs many-particle distributions so that the fully exclusive picture can be reconstructed (with given resolutions). The way to this is the jet-calculus formulated and constructed by Ken Konishi, Akira Ukawa and Gabriele Veneziano [15] as generalization of the DGLAP evolution equation. Therefore their work can be considered as the basis of the Monte Carlo parton multiplication. Jet calculus leads the way to the evolution equation for the generating functional [12, 13] of the multi-parton distributions and then to the branching probabilities for parton splitting in a way that could be implemented into Monte Carlo codes. The pioneering Monte Carlo codes [16–18] were resumming collinear singularities but only after the discovery of coherence of soft gluon radiation, both collinear and infrared enhanced logarithms where correctly resummed. The present Monte Carlo generators [2–4] fully resum not only the leading collinear and infrared singularities, but also relevant subleading contributions.

In the following I describe the main theoretical points corresponding to the Monte Carlo steps recalled in the previous section.
3.1 Asymptotic freedom and physical coupling

At short distance the theory becomes free [1] and here the use of perturbation theory is justified. At two loops one has

\[ \alpha_s(Q) \simeq \frac{4\pi}{\beta_0 L} \left( 1 - \frac{2\beta_1}{\beta_0^2} \ln \frac{Q^2}{\Lambda_{QCD}^2} + \ldots \right), \quad L = \ln \frac{Q^2}{\Lambda_{QCD}^2} \gg 1, \]

with \( \beta_0 = 11 - \frac{2}{3}n_f, \beta_1 = 51 - \frac{19}{3}n_f \) and \( n_f \) the number of light flavours.

To account for high order effects one needs to start from the scheme for the definition of the running coupling. A physical definition [19] is given by the strength of the coupling, the transverse momentum \( k_t \) relative to the emitting dipole, is obtained by using dispersive methods [12, 22] or, directly, by two loop calculations [23]. In order to accurately describe soft emissions, the physical coupling with the argument in \( (2) \) is used in the Monte Carlo generators.

3.2 Coherence of soft gluons and colour connection

Successive soft gluon emission takes place into angular ordered regions with intensities related to the colour charges. In the large \( N_c \) limit these regions are identified by the parton colour connections. To explain this one starts from the emission of a soft gluon \( k \) off a colour singlet pair of a massless quark and antiquark of momenta \( p, \bar{p} \). It is given by

\[ dw_{pp}(k) = C_F \frac{\alpha_s(k)}{\pi k_t^3} \frac{d^3k}{2\pi k_t} \left( 2 \frac{(pk)(k\bar{p})}{(p\bar{p})} \right), \]

and corresponds to the coupling associated to the Wilson loop cusp anomalous dimension [20]. The relation to the \( \overline{\text{MS}} \) coupling is known at three loops [21]. The argument of the coupling, the transverse momentum \( k_t \) relative to the emitting dipole, is obtained by using dispersive methods [12, 22] or, directly, by two loop calculations [23]. In order to accurately describe soft emissions, the physical coupling with the argument in \( (2) \) is used in the Monte Carlo generators.

\[ dw_{p\bar{p}}(k) = \frac{(p\bar{p})}{(pk)(k\bar{p})} = \frac{1}{k^2} \left( \Psi_{pp}^\alpha(k) \frac{\xi_{pk}}{\xi_{k\bar{p}}} + \Psi_{p\bar{p}}^\alpha(k) \right), \quad \Psi_{p\bar{p}}^\alpha(k) = \frac{1}{2} \left( 1 + \frac{\xi_{pp} - \xi_{pk}}{\xi_{p\bar{p}}} \right), \]

and similarly for the function \( \Psi_{pp}^\alpha(k) \) associated to the singularity for \( \xi_{k\bar{p}} = 0 \). Performing the integration of \( \Psi_{p\bar{p}}^\alpha(k) \) over the azimuthal angle around \( a \) one has

\[ \int \frac{d\phi_{ak}}{2\pi} \Psi_{p\bar{a}}^\alpha(k) = \Theta(\xi_{pp} - \xi_{ak}), \quad a = p, \bar{p}. \]

This shows that the soft dipole distribution is made up of two collinear pieces, the one singular for \( k \) collinear to \( a \) (\( \xi_{ak} = 0 \)) is (upon azimuthal averaging) bounded to a cone around \( a \) with opening half-angle \( \theta_{p\bar{p}} \). Since the \( q\bar{q} \) dipole is a colour singlet system, the \( p \) and \( \bar{p} \) colour lines are “connected”.

This coherent structure can be generalized to the soft emission of a gluon \( k \) off a colour singlet system made of any number of partons. Consider a \( q\bar{q}g \) colour singlet of momenta \( p, \bar{p} \) and \( q \) respectively. The distribution is given by (for simplicity we take also the gluon \( q \) to be soft)

\[ w_{p\bar{q}g}(k) = w_{pq}(q) \cdot \left( w_{pq}(k) + w_{qp}(k) - \frac{1}{N_c} w_{pp}(k) \right). \]
Splitting all dipole distributions as in (3) one can classify all collinear singularities in successive emissions within corresponding angular regions. One finds that the piece which is singular for \( k \) collinear to \( a \) (with \( a = p, \bar{p} \) or \( q \)) is bounded to a cone around \( a \) with opening half-angle \( \theta_{ab} \) with \( b \) the parton colour connected to \( a \) (recall that in the planar limit the gluon is equivalent to a quark-antiquark pair).

This angular ordered structure associated to colour connections at large \( N_c \) has been extended [24] to the \( 2 \rightarrow 2 \) QCD hard processes needed for LHC and used in [2]. Beyond large \( N_c \), the structure of soft radiation off the \( 2 \rightarrow 2 \) hard QCD is quite more complex; it involves [25] rotation in the colour space for the hard matrix elements and includes Coulomb phase contributions. This is a very interesting contribution and would be nice if it could be included in a future Monte Carlo generator.

The distribution of a soft gluon \( k \) emitted off a colour singlet pair of massive quark and antiquark \( P \) and \( \bar{P} \) is given by

\[
W_{P\bar{P}}(k) = -\frac{1}{2} \left( \frac{P}{(Pk)} - \frac{\bar{P}}{(kP)} \right)^2 \frac{(P\bar{P})(kP) - \frac{1}{2} P^2}{(kP)^2} - \frac{1}{2} \frac{\bar{P}^2}{(kP)^2} \tag{6}
\]

with \((ij) = E_i E_j (1 - v_i v_j \cos \theta_{ij}) \) and \( v_i = \sqrt{1 - m^2_i/E^2_i} \). While in the massless case the distribution is collinear singular for \( k \) parallel to the emitting charges, in the heavy quark case the collinear singularities are screened: distribution vanishes for \( k \) parallel to the heavy quark (or antiquark) \( P_a \) and the radiation is suppressed [26, 27] in the cone \( \cos \theta_{ak} > v_a \).

The heavy quark screening is included into the Monte Carlo generators. One needs to avoid sharp cutoff around the heavy quark which, taken together with the angular limitations, would leave a \textit{dead cone}, a phase space region without radiation.

### 3.3 Sudakov form factor and jets

An important element in Monte Carlo generator is the probability that, in a hard process, a parton is not radiating within a given resolution, the Sudakov form factor. To introduce this quantity, consider the \textit{inclusive distributions} (no particle momenta are measured but only energy flows) which are free from collinear and infrared singularities. Classical examples in \( e^+ e^- \) are the jet-shapes distributions \( \Sigma(Q, V) \) with

\[
V = \sum_i v(k_i). \tag{7}
\]

Here the sum runs over all particles in the final state (hadrons in the measurements and partons in the calculations). For \( v(k) \) \textit{linear} in the particle momentum, such jet-shape observables are collinear and infrared safe. Actually individual Feynman diagrams for real emitted partons and virtual corrections are divergent but they are summed in such a way that, order by order, the infinities cancel [11] leaving finite results.

Collinear and infrared safe jet-shape distributions \( \Sigma(Q, V) \) have a perturbative expansion with finite coefficients

\[
\Sigma(Q, V) = \Sigma_0(Q, V)(1 + \alpha_s(Q) c_1(V) + \alpha_s^2(Q) c_2(V) + \cdots), \quad Q \gg \Lambda_{\text{QCD}} \tag{8}
\]

with \( \Sigma_0(Q, V) \) the Born distribution and \( c_i(V) \) finite functions of \( V \) expressed in terms of the quark, \( C_F \), or gluon, \( C_A \), colour charges. Actually, by inhibiting the radiation by taking \( V \ll 1 \), these coefficients are enhanced by powers of \( \ln V \). A clever reshuffling
of PT series, based on universal nature of soft and collinear radiation (factorization) results [12, 13] in the exponentiated answer of the Sudakov form factor $S(Q, V)$

$$
\Sigma(Q, V) = \Sigma_0(Q, V) \cdot S(Q, V), \quad S(Q, V) = e^{-R(Q, V)},
$$

$$
R(Q, V) = \sum_{n=1}^{\infty} \alpha_s^n(Q^2) \left( d_n \ln^{n+1} V + s_n \ln^n V + \cdots \right).
$$

The $d_n$ series is referred to as double logarithmic (DL) and $s_n$ as single logarithmic (SL). Reliable predictions for these distributions require the matching [28] of the exact finite order calculation (8) for finite $V$ and the Sudakov resummation (9) for small $V$.

It is instructive to discuss the emergence of the powers of $\ln V$ in the Sudakov form factor $S(Q, V)$. They result from the incomplete cancellation of real and virtual effects. For $V \ll 1$ the real parton production is inhibited, one has $v(k) < V \ll 1$. Since the virtual PT radiative contributions remain unrestricted, the divergences do cancel in the region $v(k) < V$ leaving only virtual contributions for $v(k) > V$ which produce finite but logarithmically enhanced leftovers. The DL contributions originate from the fact that each gluon emission brings in at most two logarithms (one of collinear, another of infrared origin). This explains the first term $d_1 \ln^2 V$ while the rest of the DL series is generated simply by the presence of the running coupling (1). The SL contributions, are necessary to set the scale of the logarithms ($\ln^n c V = \ln^n V + n \ln c \ln^{n-1} V + \cdots$).

In conclusion, the Sudakov form factor $S(Q, V)$ corresponds to the probability that in $e^+e^-$ the primary quark-antiquark pair remains without accompanying radiation up to resolution $Q_0 = VQ$ for small $V$.

To obtain the result (9) one uses the fact that the collinear and/or infrared enhanced contributions factories and are resummed by linear evolution equations of the DGLAP type. Therefore, after factorization of collinear and infrared singularities (including soft gluon coherence) QCD radiation appears as produced by “independent” gluon emission (bremsstrahlung). Gluon branching (into two gluons or quark-antiquark pair) enters only in reconstructing the running coupling (1) as function of transverse momentum. The fact that here the branching component does not contribute (within SL accuracy) can be understood as a result of real-virtual cancellations of singularities. Indeed, in the collinear limit, the transverse momentum of an emitted gluon is equal to the sum of transverse momenta of its decay products. Therefore, if one measures the total emitted transverse momentum, as in broadening for instance, it is enough to consider the contributions of primary bremsstrahlung gluons. Further branching does not contribute due to unitarity (real-virtual cancellation).

### 3.4 Structure and fragmentation functions

Moving to less inclusive measurements one faces infinities. The simple case involves fixing (measuring) momentum of a hadron, e.g. that of the initial proton in DIS (structure function) or of a final hadron (fragmentation function), they are functions of the Bjorken and Feynman variables respectively

$$
x_B = -\frac{q^2}{2(Pq)}, \quad x_F = \frac{2(Pq)}{q^2}.
$$

In DIS $q$ is the large space-like momentum transferred from the incident lepton to the target nucleon $P$. In $e^+e^-$ annihilation $q$ is the time-like total incoming momentum and $P$ the momentum of the final observed hadron.
In perturbative calculation, replacing the hadron with a parton, one has infinities, real and virtual contributions do not cancel. Soft divergences still cancel but collinear ones do not, making such observables not calculable at the parton level. These effects, however, turn out to be universal and, given a proper technical treatment, can be factored out [5] as non-perturbative inputs. What remains under control then is only the $Q^2$-dependence (scaling violation pattern). This fact is realized in the DGLAP evolution equation which needs, in order to be solved, an initial condition at a low virtuality $Q_0$. This corresponds to a parton resolution (or a factorized subtraction point), which absorbs all large distance divergences. Such “initial condition” cannot be computed by perturbative means and has to be provided by low scale experimental data.

3.5 DGLAP evolution equation for DIS and $e^+e^-$

To derive the DGLAP evolution equation [14] one needs to study the phase space region leading to collinear singularities. The same Feynman diagrams are involved in the case of structure function (space-like) and fragmentation function (time-like). Therefore they can be studied simultaneously. First note that the Bjorken and Feynman variables (10) are mutually reciprocal: after the crossing operation $P \rightarrow -P$ one $x$ becomes the inverse of the other (although in both channels $0 \leq x \leq 1$ thus requiring the analytical continuation).

Such a reciprocity property can be extended to the Feynman diagrams for the two processes and, in particular, to the contributions from mass-singularities. Consider, for DIS (S-case) and $e^+e^-$ annihilation (T-case), the skeleton structure of Feynman graphs in axial gauge and the kinematical relation leading to the mass-singularities

$$q_{k_0}$$

$$k_n, k_{n-1}, \cdots, k_1$$

$$\frac{|k_i^2|}{k_{i,+}} = \frac{|k_{i-1}^2|}{k_{i-1,+}} + \frac{k_i^{\prime 2}}{k_{i,+}} + \frac{k_i^{\prime} k_i^\prime}{k_{i-1,+}} \left( \frac{k_{it}^\prime}{k_{i,+}} - \frac{k_{it}^\prime}{k_{i,+}} \right)^2$$

**DIS or $e^+e^-$ skeleton graphs**

Here $k_1', \cdots, k_n'$ are the outgoing parton systems (sub-jets). For space-like (S: $q^2 < 0$, $k_0$ entering) and time-like (T: $q^2 > 0$, $k_0$ outgoing) one has

$$S: \quad \frac{k_{i,+}}{k_{i-1,+}} \equiv z_i \quad \text{and} \quad T: \quad \frac{k_{i,+}}{k_{i-1,+}} \equiv z_i^{-1}. \quad (11)$$

The virtuality $k_i^2$ enters the denominators of the Feynman diagrams. In order for the transverse momentum integration produce a logarithmic enhancement, the conditions must be satisfied

$$\frac{|k_{i-1}^2|}{k_{i-1,+}} \ll \frac{|k_i^2|}{k_{i,+}} \Rightarrow k_{i-1}^2 \ll |k_i^2| z_i^\sigma,$$

(12)

with $\sigma = -1$ for DIS and $\sigma = 1$ for $e^+e^-$. The same Feynman graphs are contributing and, going from S- to T-channel, the mass singularities are obtained by reciprocity: change $z$ into $1/z$ and the momentum $k$ from space-like to time-like. This fact is at the origin of the Drell-Levy-Yan relation [29] and Gribov-Lipatov [30] reciprocity which has been largely used in order to obtain the time-like anomalous dimensions from the space-like ones [31, 32]. The ordering (12) in the inverse fluctuation time $k^2/k_+$ is well known, see for instance [33].
To make the Gribov-Lipatov reciprocity more clear, use the ordering (12) in the computation of the probability $D_\sigma(x, Q^2)$ to find a parton with longitudinal momentum fraction $x$ and virtuality $|k^2|$ up to $Q^2$ with $\sigma = -1$ for the S-case and $\sigma = +1$ for the T-case. This ordering gives rise to the following reciprocity respecting equation [34]

$$Q^2 \partial_{Q^2} D_\sigma(x, Q^2) = \int_0^1 \frac{dz}{z} P(z, \alpha_s) D_\sigma \left( \frac{x}{z}, Q^2 z^\sigma \right), \quad \sigma = \pm 1,$$

with the same parton splitting kernel $P(z, \alpha_s)$ in the S- or T-channel. This equation, derived simply from kinematical considerations, has been (partially) tested at two [31] and three loop [21, 34, 35].

The reciprocity respecting equation (13) is non-local since the derivative of $D_\sigma(x, Q^2)$ in the l.h.s. involves the distribution in the r.h.s. with all virtualities larger or smaller than $Q$ for $\sigma = -1$ or $\sigma = +1$ respectively. For the use in a Monte Carlo generator one needs to formulate (13) in terms of a local evolution equation, a Markov process. Formally this is easy to do: as a hard scale for the parton densities replace $Q^2 = x^2 Q^2$ in the T-case and, by reciprocity, with $Q^2 = x^{-1} Q^2$ in the S-case. The physical meaning of these two different hard scales is well known from the studies of soft gluon coherence [12, 13, 33, 36]: in the T-case is related to the branching angle and in the S-case to the transverse momentum.

It is interesting to illustrate this. The fact that, in the T-case, the ordering variable is not the inverse fluctuation time $k^2/k_+$ (12) but rather the angle $k^2/k_+^2 \simeq k_+^2/k_+^2 \simeq \theta_k^2$, originates from cancellations [36] due to destructive interference in the region

$$T-case: \quad z_1^2 k_1^2 < k_{i-1}^2 < z_i k_i^2,$$

thus leaving the angular ordered region $k_{i-1}^2 < z_1^2 k_1^2$. Using reciprocity ($z_i \to z_i^{-1}$) one has that in the S-case the canceling region (14) becomes

$$S-case: \quad |k_i^2| < |k_{i-1}^2| < z_i^{-1} |k_i^2|,$$

thus leaving the transverse momentum ordering $k_{i-1}^2 < k_i^2$. This agrees also, at small $x$, with the BFKL [37] leading order multi-parton kinematical region.

The cancellation in the region (15) has a well known physical basis for small $x$. Consider (see the skeleton graph) the successive emissions $k_{i-2} \to k_{i-1} + k_{i-1}'$ and $k_{i-1} \to k_{i} + k_i'$ in the region $k_{i+} \ll k_{i-1,+} \ll k_{i-2,+}$ giving the leading contribution for small $x$. These cancellations result from taking into account the emission of $k_i$ off the partons $k_{i-2}$ and $k_{i-1}'$ in the region (15). Physically, the process can be viewed upon as an inelastic diffraction of the incident particle $k_{i-2}$ in the external gluon field of transverse size of order $k_{i-2}$. In the kinematical region (15) the transverse size of the parton fluctuation $k_{i-2} \to k_{i-1} + k_{i-1}'$ is smaller than the resolution power of the probe, $k_{i-2}^2$. In these circumstances the destructive interference between $k_i$ interacting with the initial ($k_{i-2}$) and with the final state ($k_{i-1} + k_{i-1}'$) comes onto the stage. The cancellation under discussion is then equivalent to the general physical observation, due to V.N.Gribov, that inelastic diffraction vanishes in the forward direction.

To deal with very small $x$ one needs to resum at least all terms $\alpha_s^n \ln^n x$ as given by the BFKL equation [37] which cannot be accounted for by the collinear singularities resummation performed in the Monte Carlo codes. However, the evolution equation in [38] resums leading collinear and $\ln x$ terms (by enlarging the phase space and adding a non-Sudakov form factor) and allows Monte Carlo simulations [39] with the cost of generating events which need to be weighted.
4 Multi-gluon soft distributions

Collinear and infrared pieces of the multi-parton QCD distributions factories and can be reproduced by recurrence relations which can be formulated as a Markov branching process. This can be implemented into a Monte Carlo code and the simulation provides a “complete” description of the multi-parton emission in hard process.

I illustrate in detail the case in the leading soft approximation. Although important non-soft contributions that are included in a realistic Monte Carlos are here neglected, many important physical effects are well described, in particular, large angle soft emission (without collinear approximation). Moreover, in this approximation the path from multi-gluon soft amplitudes to Monte Carlo is simple to explain. The scheme of the presentation involves the following steps:

- multi-gluon soft distributions. They are computed in the leading soft approximation and in the planar approximation;
- recurrence relation for the multi-gluon soft distributions. This is obtained by introducing the generating functional for all multi-gluon distributions [12] and deriving the evolution equation. From the generating functional one computes observables as it will be discussed in subsection 4.2. For collinear and infrared safe observables such as jet-shape distributions the cutoff contributes only with power corrections;
- Markov process and Monte Carlo implementation. Here one needs to include proper cutoff for collinear and infrared singularities. This will be discussed in the next section 5;
- from parton to hadron emission. This will be discussed in section 6.

The starting point is the amplitude for the emission of \( n \) soft gluons \( q_1, \ldots, q_n \) off a primary colour singlet \( q\bar{q} \) pair of momentum \( p, \bar{p} \). It is represented as a sum of Chan-Paton factors with the coefficients given by colour-ordered amplitudes. We consider the contribution with a single Chan-Paton factor (topological expansion [40]):

\[
\mathcal{M}_n(p\bar{p}q_1 \cdots q_n) = \sum_{\pi_n} \{ \lambda^{a_1} \cdots \lambda^{a_n} \}_{\beta\bar{\beta}} \mathcal{M}_n(pq_1 \cdots q_{i_n}\bar{p}) ,
\]

(16)

the sum is over the permutation \( \pi_n \) of colour indices, \( \lambda^a \) are the \( SU(N_c) \) matrices in the fundamental representation. The softest emitted gluon \( q_m \) factorizes and one has [12, 42]

\[
M_n(\cdots \ell m \ell' \cdots) = g_s M_{n-1}(\cdots \ell' \cdots) \cdot \left( \frac{q^\mu_\ell}{q\ell q_m} - \frac{q^\mu_{\ell'}}{q\ell' q_m} \right) .
\]

(17)

The softest gluon is emitted by the two partons neighbouring in colour space. This approximation is accurate in the soft limit without any collinear approximation. From this factorized structure one deduces a recurrence relation and computes all colour-amplitudes in the soft limit. Summing over the polarization indices, the squared averaged colour-amplitude is given, for the fundamental colour permutation, by

\[
|M_n(pq_1 \cdots q_n\bar{p})|^2 = |M_0|^2 (2g_s^2)^n W_{p\bar{p}}(q_1 \cdots q_n) , \quad W_{p\bar{p}}(q_1 \cdots q_n) = \frac{(p\bar{p})}{(pq_1) \cdots (q_n\bar{p})} .
\]

(18)

This very simple result for the square amplitude is valid for any energy ordering and depends only on the colour ordering. Note that here one takes the square of the same
colour-ordered amplitude. Indeed $M_n(\pi'_n)M^*_n(\pi_n)$ with $\pi_n$ and $\pi'_n$ two different colour permutations cannot be expressed in a closed form for any $n$. On the other hand contributions from different permutations enter the calculation of the averaged squared amplitude $|\mathcal{M}_n|^2$. A close expression for this distribution for any $n$ is obtained only in the planar approximation \cite{41}. To see this observe that

$$\text{Tr}(\lambda_{\pi_n} \lambda_{\pi'_n}) = 2C_F \left(\frac{N_c}{2}\right)^n \left(1 - \frac{1}{N_c}\right)^{n-1}$$

(19)

with $\lambda_{\pi_n} = \{\lambda^{a_1} \cdots \lambda^{a_n}\}$ and $\lambda_{\pi'_n} = \{\lambda^{a_n} \cdots \lambda^{a_1}\}$. Taking instead two different colour permutations one has that $\text{Tr}(\lambda_{\pi_n} \lambda_{\pi'_n})$ is suppressed at least by $1/N_c^2$. Therefore, only in the planar approximation one can use the simple result in \cite{18} and obtains \cite{12}

$$|\mathcal{M}_n|^2 = \frac{\sigma_0}{n!} (N_c g_s^2)^n \sum_{\pi_n} W_{pp}(q_1 \cdots q_n)$$

(20)

where $\sigma_0 = 2C_F |M_0|^2$ and symmetrisation has been taken into account.

The distributions \cite{18} contain the leading infrared singularities: for any colour permutation one has $W_{pp} \sim (\omega_1 \cdots \omega_n)^{-2}$ with $\omega_i$ the energy of gluon $q_i$. They contain also the leading collinear singularities for $\theta_{ij} = 0$ with $ij$ two partons neighbouring in colour (thus there are up to $n$ collinear singularities).

An alternative way to obtain the the multi-gluon colour amplitude is based on the helicity techniques \cite{43}. For $q\bar{q}$ with $+\text{ and } -$ polarization, the leading soft contribution is obtained when all gluons have $+\text{ helicities and the recurrence relation (17) reads (for opposite helicities the result is the complex conjugate one)}$

$$M_n(\cdots \ell m \ell' \cdots) = g_s M_{n-1}(\cdots \ell' \cdots), \quad \frac{\langle qq'\rangle}{\langle q\ell q'\rangle \langle \ell \ell' \cdots \rangle}, \quad \langle qq'\rangle = \sqrt{2qq'}e^{i\phi_{qq'}},$$

(21)

with $q_m$ the softest gluon, $z$ the longitudinal direction and the phase

$$e^{i\phi_{qq'}} = \sqrt{\frac{q_+ q'_+}{2qq'}} \left(\frac{q_t - q_{t'}}{q_+} \frac{q_t}{q'_+}\right), \quad q_t = q_x + iq_y.$$

(22)

The solution of this recurrence for the amplitude is very simple; it is the same for any energy ordering and depends only on the colour ordering. For the fundamental permutation one has

$$M_n(pq_1 \cdots q_n p) = g^n M_0 \frac{\langle pp\rangle}{\langle pq_1 \cdots q_n p\rangle},$$

(23)

with squared amplitude given by \cite{18}. This shows the well known result that non-planar contributions, obtained from $M_n(\pi_n)M^*_n(\pi'_n)$ for two different colour orderings, have the same soft singularities but reduced number of collinear singularities.

### 4.1 Virtual correction, generating functional and evolution

To compute observables one needs to supplement the multi-gluon soft distributions \cite{20} with the related virtual corrections. For infrared and collinear safe observables, such as jet-shape distributions, the infrared and collinear singularities in \cite{18} has to be canceled by corresponding singularities in virtual corrections. One way to compute the virtual corrections, at the same level of accuracy in the soft limit as for real emission contribution,
consists of performing the integration over the virtual gluon energy by the Cauchy method and then taking the soft limit for the virtual gluon. This way one also regularizes the ultraviolet divergences by neglecting the divergent contribution from the contour at the infinity of the complex energy plane. By properly choosing a constant this regularization corresponds to the physical scheme in \([11]\). The virtual corrections so computed can be included into the generating functional for the multi-gluon soft distributions. The result of this study not only gives the relevant virtual corrections but, due to the simple structure of (20) in the planar approximation, gives the branching structure of multi-gluon soft emission leading to the Monte Carlo generator.

Consider the soft distribution \(d\sigma_{ab}^{(n)}\) for the emission of \(n\) gluons off a colour singlet dipole \(ab\) (thus one generalizes the primary dipole \(pp\) to a general dipole with \(a\) and \(b\) in arbitrary directions). For each emitted soft gluon \(q_i\) one introduces a source function \(u(q_i)\) and defines the *generating functional* as

\[
G_{ab}[E, u] = \sum_n \frac{1}{n!} \int \frac{d\sigma_{ab}^{(n)}}{\sigma_{ab}^{\text{tot}}} \prod_i u(q_i), \tag{24}
\]

with \(E = Q/2\) the hard scale. This functional depends on the directions \(a\) and \(b\) of the primary dipole. By setting all \(u(q_i) = 1\) one has \(G_{ab}[E, 1] = 1\). Using (20) one has the *real emission* contribution for the generating functional

\[
G_{ab}^{\text{real}}[E, u] = \sum_n \int \prod_i \left\{ \alpha_s u(q_i) \frac{d\Omega_{q_i}}{4\pi} \omega_i d\omega_i \Theta(E - \omega_i) \right\} \cdot W_{ab}(q_1 \cdots q_n), \tag{25}
\]

with \(\alpha_s = N_c \alpha_s / \pi\). Here one neglects \(1/N_c^2\) corrections (planar limit) and uses the soft approximation for the phase space \(\omega_i \ll E\). Symmetry of the phase space is used. The condition \(G_{ab}[E, 1] = 1\) must be satisfied only after including the virtual corrections. To include them we construct the evolution equation for the generating functional. To this end we use the fact that the very simple expression \([18]\) has the following factorization property

\[
W_{ab}(q_1 \cdots q_n) = w_{ab}(q_\ell) \cdot W_{ad}(q_1 \cdots q_{\ell-1}) \cdot W_{eb}(q_{\ell+1} \cdots q_n), \tag{26}
\]

with \(q_\ell\) one of the soft gluons and \(w_{ab}(q)\) the dipole distribution \([3]\). Taking \(q_\ell\) as the hardest (soft) gluon and differentiating (25) with respect to \(E\), thus setting \(\omega_\ell = E\), one obtains \([44]\)

\[
E \partial_E G_{ab}[E, u] = \int \frac{d\Omega_q}{4\pi} \frac{\alpha_s}{\xi_{aq} \xi_{qb}} \left\{ u(q) G_{aq}[E, u] \cdot G_{qb}[E, u] - G_{ab}[E, u] \right\}, \tag{27}
\]

with \(\xi_{ij} = 1 - \cos \theta_{ij}\). The negative term in the integrand originates from the virtual corrections obtained via Cauchy integration as mentioned before. Since they are evaluated within the same soft approximation used for the real contributions, at the inclusive level they cancel against the real contributions giving the correct constraint \(G_{ab}[E, 1] = 1\). Both the real emission (first term in the integrand) and the virtual correction (second term) are collinear and infrared singular. For inclusive observables, (i.e. for suitable sources \(u(q)\)) these singularities cancel. This evolution equation accounts for coherence of soft gluon radiation \([12, 13]\).

### 4.2 Observables in the soft limit

Using \(G_{ab}[E, u]\) one obtains all inclusive distributions in the soft limit. No collinear approximations are involved in (20), therefore the functional \(G_{ab}[E, u]\) gives quantities
which involves also large angle soft emission. Let me first recall some observables which are collinear singular around the primary partons $a$ and $b$.

**Collinear observables.** The simplest one is the multiplicity of soft gluons with resolution $Q_0$. Taking $u(q) = u$ this observable is defined as, see (24),

$$n_{ab}(E) = \partial_u G_{ab}(E, u)\bigg|_{u=1} = \sum_n n^{(n)} G_{ab}^{(n)}(E, u).$$

(28)

It is easy to derive from (27) the well known result [36] for the multiplicity

$$n_{ab}(E) \simeq n_{ab}^{(0)} \exp \left\{ \frac{4\pi}{\beta_0} \sqrt{\frac{2N_c}{\pi\alpha_s(E)}} \right\},$$

(29)

with $n_{ab}^{(0)}$ the non-perturbative initial condition. Similarly one derives the fragmentation function $D_{ab}(x, E)$ by taking the source $u(q) = u(x)$ with $x$ the soft gluon energy fraction

$$D_{ab}(x, E) = \frac{\delta}{\delta u(x)} G_{ab}[E, u]\bigg|_{u(x)=1}.\quad (30)$$

Soft gluon coherence here is shown by a depletion of radiation [12, 13] at small $x$.

**Observables at large angle.** The simplest case is the distribution discussed in [45] of heavy systems of mass $M$ emitted in $e^+e^-$ at large angle $\rho = \frac{1}{2}(1 - \cos \theta)$ and small velocity. The heavy system (typically a heavy $q\bar{q}$ system) originates from a gluon in the cascade. The collinear singularities are screened by $M$ so this distribution is finite and given by a function of the SL quantity

$$\tau = \int_{\mathcal{M}}^E dq_t \alpha_s(q_t).$$

(31)

It is interesting that this distribution $I(\rho, \tau)$ satisfies an equation with a structure similar to the BFKL equation [45] and then its asymptotic behaviour in $\tau$ involves the BFKL characteristic function. One has

$$I(\rho, \tau) \sim \frac{e^{4\ln^2 \tau}}{\tau^{3/2}} \cdot \frac{\ln \rho_0/\rho}{\sqrt{\rho}} e^{-\frac{\ln^2 \rho_0/\rho}{2D\rho}} \cdot D = 28 \zeta(3).$$

(32)

The functional $G_{ab}[E, u]$ is suited to give the distributions in the energy emitted away from jets. Such distributions do not have collinear singularities, but only infrared ones. An example in $e^+e^-$ is the distribution in energy recorded outside a cone $\theta_{in}$ around the thrust (this is a typical “non-global” jet observable [46]):

$$\Sigma(E, E_{out}) = \sum_n \int_0^E \frac{d\sigma_n(E)}{\sigma_{tot}} \Theta \left( E_{out} - \sum_{k_{ti}} E_{k_{ti}} \right).$$

Since the jet region is excluded, there are no collinear singularities to SL accuracy and the resummed PT contributions come from large angle soft emission. Here resummation is complex but informative. It brings information on the QCD radiation between jets, a region interesting for understanding colour neutralization among jets.
It is interesting to discuss this quantity in some detail since it illustrates the structure of (27). First observe that the distribution depends on $E$ and $E_{\text{out}}$ through the SL function $\tau$ given by (31) with $M \rightarrow E_{\text{out}}$. To obtain $\Sigma(\tau)$ from $G_{ab}[E, u]$ one takes $u(q) = 0$ away from jets and $u(q) = 1$ inside the jet region. From (27) one derives the evolution equation [44]

$$
\partial_\tau \Sigma_{ab}(\tau) = -s_{ab} \Sigma_{ab}(\tau) + \int_{\text{in}} \frac{d\Omega_q}{4\pi} \frac{\bar{\alpha}_s \xi_{ab}}{\xi_{aq} \xi_{qb}} \left\{ \Sigma_{aq}(\tau) \cdot \Sigma_{qb}(\tau) - \Sigma_{ab}(\tau) \right\},
$$

(33)

with $s_{ab}$ related to the Sudakov form factor

$$
S(\tau) = e^{-\tau s_{ab}}, \quad s_{ab} = \int_{\text{out}} \frac{d\Omega_q}{4\pi} \frac{\xi_{ab}}{\xi_{aq} \xi_{qb}} \sim \ln \theta^{-1}. 
$$

(34)

Equation (33) has a bremsstrahlung (first) and branching (second term) components:

![Bremsstrahlung Component](image1)

![Branching Component](image2)

The *bremsstrahlung component* resums contributions from gluons emitted in the recorded region outside the cone. These contributions are the only ones present for the global jet observables considered in the previous subsection. Here, since the collinear singularities are screened by the cone $\theta_{\text{in}}$, the Sudakov form factor is a SL function.

The *branching component* resums contributions from gluons emitted inside the jet region. These gluons need to branch in order to generate decay products entering the recorded region. Here real-virtual cancellation is incomplete and virtual enhanced contributions are dominating thus leading to a strong suppression of the distribution which asymptotically turns out to be Gaussian in $\tau$.

The Monte Carlo generator [2] resums only collinear singularities therefore it does not fully resum soft emissions at large angles although phenomenologically, it turns out [47] that the most important pieces are correctly reproduced due to soft gluon coherence.

## 5 Monte Carlo simulation for soft emission

The evolution equation (27) can be formulated as a Markov process and then numerically solved. This Monte Carlo procedure has been introduced in [46] to study non-global distributions. A similar procedure based on dipole branching is used in the Monte Carlo generator [4].

To construct a Monte Carlo generator from (27) one splits the real and virtual corrections. To do so it is necessary to introduce a cutoff $Q_0$ in transverse momentum (the argument of $\alpha_s$) giving the Sudakov form factor

$$
\ln S_{ab}(E) = -\int_{Q_0}^{E} \frac{d\omega_q}{\omega_q} \int \frac{d\Omega_q}{4\pi} \frac{\bar{\alpha}_s \xi_{ab}}{\xi_{aq} \xi_{qb}} \cdot \theta(q_{ab} - Q_0), \quad q_{ab}^2 = 2 \omega_q^2 \frac{\xi_{aq} \xi_{qb}}{\xi_{ab}},
$$

(35)

which is the solution of (27) with the real emission piece neglected. Here $q_{ab}$ is the transverse momentum of $q$ with respect to the $ab$-dipole. Then the evolution equation (27) can be integrated to give (the cutoff $Q_0$ dependence is implicit)

$$
G_{ab}[E] = S_{ab}(E, Q_0) + \int d\mathcal{P}_{ab}(E, \omega_q, \Omega_q) \ u(q) G_{aq}[\omega_q, u] \cdot G_{qb}[\omega_q, u],
$$

(36)
where one has introduced the probability for dipole branching: \((ab) \to (aq)(qb)\)

\[
dP_{ab}(E, \omega, \Omega_q) = \left\{ \frac{\omega_q}{\omega_q} S_{ab}(E) \right\} \frac{\Delta \Omega_q}{4\pi} \frac{\bar{\alpha}_s \xi_{ab}}{\xi_{aq} \xi_{qb}} \cdot \theta(q_{\text{tab}} - Q_0) .
\]  

(37)

To see how this could be used in a Monte Carlo simulation one writes \(dP_{ab}(E, \omega, \Omega)\) in the equivalent form (the bound \(q_{\text{tab}} > Q_0\) is implicit)

\[
dP_{ab}(E, \omega, \Omega) = dr_{ab}(E, \omega) \cdot dR_{ab}(\Omega) \quad (38)
\]

with

\[
\begin{align*}
    r_{ab}(E, \omega_q) &= \frac{S_{ab}(E)}{S_{ab}(\omega_q)}, & \int dr_{ab}(E, \omega_q) &= 1 - S_{ab}(E) \\
    dR_{ab}(\Omega_q) &= N_{ab} \frac{\Delta \Omega_q}{4\pi} \frac{\bar{\alpha}_s \xi_{ab}}{\xi_{aq} \xi_{qb}}, & \int dR_{ab}(\Omega_q) &= 1 .
\end{align*}
\]

(39)

The integral of the branching probability gives

\[
\int dP_{ab}(E, \omega, \Omega) = 1 - S_{ab}(E) ,
\]

(40)

and this shows that the Sudakov factor \(S_{ab}(E)\) gives the probability for not emitting a gluon within the resolution \(Q_0\) in \(q_{\text{tab}}\).

The probability distribution \(dP_{ab}(E, \omega, \Omega)\) can be used to generate Monte Carlo events distributed according to QCD in the soft and planar approximation. Using sets of random numbers \(0 < \rho < 1\) the procedure is the following:

1. take the \(ab\)-dipole with the energy scale \(E\) and compare the Sudakov factor \(S_{ab}(E)\) with \(\rho\). If \(\rho < S_{ab}(E)\) then the \(ab\)-dipole does not emit any soft gluon within the resolution. In the opposite case the dipole is emitting a soft gluon with energy \(\omega_q\) given by solving the equation \(\rho = r_{ab}(E, \omega_q)\);

2. obtain the direction \(\Omega_q\) by sampling the distribution \(dR_{ab}(\Omega_q)\). At this point, from the \(ab\)-dipole one has generated two dipoles: \(aq\) and \(qb\), both at the new energy scale \(\omega_q\);

3. repeat the procedure for each new generated dipole till no dipole emits any more within the resolution.

At the end of this procedure one is left with a Monte Carlo event: a collection of emitted soft gluons \(q_1 \cdots q_n\) together with the primary partons \(a, b\). These events are distributed with the QCD probability so they can be used to compute any soft distribution as discussed in subsection 4.2.

Such a Monte Carlo simulation, based on evolution equation in energy, is then a successive emission of softer and softer gluons. Angles are given by the dipole distribution \(3\) so they are ordered (upon azimuthal average) and coherence is automatically implemented.

In order to obtain a realistic simulation one needs to overcome the soft approximation, that is, to take into account the recoil in the emission and the non-soft pieces of the gluon splitting function (only the singular pieces are present in \(27\))

\[
P_{g \to gg}(z) = N_c \left( \frac{1}{z} + \frac{1}{1-z} + z(1-z)-2 \right).
\]

(41)
Similarly, one needs to account also for the quark branching channels. All these points are accounted in the present realistic Monte Carlo generators. Their basis is an evolution equation in angle rather than in energy (as (27)). However this implies that one considers collinear approximations in the emission thus soft radiation at large angles are not fully accounted for.

6 From partons to hadrons

The above description of the Monte Carlo code refers to the generation of events with emission of partons (possibly together with non-QCD particles) which, due to the presence of collinear and infrared singularities, requires a cutoff $Q_0$. The main questions are then: how to go from partons to hadrons and how much a phenomenological hadronisation model affects and distorts the QCD radiation generated perturbatively. A suggestion on hadronisation models which do not substantially modify the perturbative radiation is provided by preconfinement [9].

Preconfinement. The basis is again the Sudakov function which suppress the probability of “non-emitting”. Consider, in the planar approximation, two colour connected partons emitted in a hard collision at scale $Q$ and with resolution $Q_0$. Colour connection means that the quark colour line of one parton ends into the antiquark colour line of the other parton (in the planar approximation a gluon could be, from the colour point of view, described as a pair of $q\bar{q}$ colour lines). Thus no gluons are emitted within the resolution $Q_0$ by this colour line and a Sudakov form factor arises which forces the two colour connected partons to form a system of mass of order $Q_0$ (even for very large $Q$). The system of the quark and antiquark in question forms a colour singlet of small mass. Although this is not yet an indication of confinement (the colour system should be local-ized in space), such a preconfinement property suggests that any hadronisation models that associates hadrons to colour connected partons would not distort the perturbative structure of the QCD radiation: parton and hadron flows are similar within the resolution $Q_0$. Preconfinement is then related to the property of local hadron-parton duality [10] which has been phenomenologically well tested.

Power corrections. Other non-perturbative effects are the power corrections to the observables. They result from the non-convergence the PT expansions even if the coefficients are finite as in (8) and (9). As a consequence all PT predictions are affected by corrections in powers of $\Lambda_{\text{QCD}}/Q$ with coefficients determined by NP effects. An important NP effect, present in short distance quantities, is that the running coupling is involved at any scale smaller than $Q$. For example, the average value of $V$ in (7) is given by an integral of the type

$$\langle V \rangle = \int_0^Q \frac{dk_t}{k_t} \alpha_s(k_t) \cdot \mathcal{V}(k_t/Q) = v_1 \alpha_s(Q) + v_2 \alpha_s^2(Q) + \cdots,$$

where the virtual momentum $k_t$ in the Feynman diagrams runs into the large distance region (although the observable is dominated by short distance physics). Since the observable is collinear and infrared finite, for $k_t \to 0$ the Feynman integrand is regular ($\mathcal{V}(k_t/Q) \sim k_t/Q$) so that the integral is finite, apart from the presence of $\alpha_s(k_t)$ which enters the confinement region. Mathematically this is reflected into the fact that, although
all PT coefficients in $\alpha_s(Q)$ are finite, the expansion is non-convergent [48] (renormalon singularity).

The fact that the running coupling entering the NP region is at the origin of the leading power correction can be checked phenomenologically. From the study of jet-shape observable one finds [49] that, within $10-20\%$, the power corrections are described by the same parameter accounting for the running coupling in the NP region. In the Monte Carlo generators one sets a cutoff $Q_0$ in the argument of the coupling and this does bring in these physically relevant power corrections at the perturbative — parton — stage. Instead, power behaving contributions to jet shapes arise at the hadronisation level [51].

**Underlying event.** Another important NP component in the Monte Carlo for LHC is the presence of radiation besides the one emitted in the hard event. This is typically around the beams as for the peripheral interactions (events at low $E_T$). Perturbative QCD does not provide indication for this component. Thus there are various models which needs to be studied [50] at the Tevatron together with the extrapolation at LHC.

### 7 Conclusion

What I have discussed shows that the Monte Carlo generators involves the entire *Summa* of hard QCD results and provide a framework for many future QCD and non-QCD studies. The general attempts to improve the Monte Carlo generators go in the directions of making the quantitative predictions both more reliable (by adding new theoretical QCD results and phenomenological studies) and more general (by including also electroweak and beyond the standard model physics). As far as the first direction, I have mentioned the works made to include in the Monte Carlo generator the known exact higher order distributions [8]. As also mentioned, it is interesting to include into the present generators reliable predictions on large angle soft emission (see subsection 4.2). This would require also the need to account for non-planar corrections by studying colour rotations involved in the colour structure of ensembles of more than three hard partons (see [25]).

The three key elements in a Monte Carlo generator for jet emissions are the QCD factorization properties, the branching algorithm and the procedure for converting partons into hadrons. As I have mentioned, Gabriele Veneziano has either contributed to or started each of these three key developments: The Monte Carlo generators are based on factorization of QCD collinear singularities [5]. Jet calculus [15] leads to the evolution equation for the generating functional for multi-parton distributions which can be formulated as a Markov process. Moreover, the preconfinement property [9] is at the basis of hadronisation models that do not destroy the QCD radiation structure.

### Acknowledgements

In addition to Gabriele, I am grateful to the many colleagues which shared with me the beauty of QCD and in particular to Bryan Webber, we undertook the risk of conveying incomplete theoretical concepts and results into an event generator, and to Marcello Ciafaloni, Yuri Dokshitzer and Al Mueller, for many discussions during the construction of the original Monte Carlo generator.
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