Stability of Magnetic Equilibria in Radio Bubbles

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13 August 2020

ABSTRACT

Current-carrying flows, in the laboratory and in astrophysical jets, can form remarkably stable magnetic structures. Decades of experience shows that such flows often build equilibria that reverse field directions, evolving to an MHD Taylor state, which has remarkable stability properties. We model jets and the magnetic bubbles they build as reversed field pinch equilibria by assuming the driver current to be stiff in the MHD sense. Taking the jet current as rigid and a fixed function of position, we prove a theorem: that the same, simple MHD stability conditions guarantee stability, even after the jet turns off. This means that magnetic structures harboring a massive inventory of magnetic energy can persist long after the building jet current has died away. These may be the relic radio "fossils," "ghost bubbles" or "magnetic balloons" found in clusters. These equilibria under magnetic tension will evolve, retaining the stability properties from that state. The remaining fossil is not a disordered ball of magnetic fields, but a stable structure under tension, able to respond to the slings and arrows of outside forces. Typically their Alfvén speeds greatly exceed the cluster sound speed, and so can keep out hot cluster plasmas, leading to x-ray "ghosts." Passing shocks cannot easily destroy them, but can energize and light them up anew at radio frequencies. Bubbles can rise in the hot cluster plasma, perhaps detaching from the parent radio galaxy, yet stable against Rayleigh-Taylor and other modes.

Key words: plasma, stability, astrophysical jets, kink instability

1 INTRODUCTORY PHYSICS

Many physical configurations begin with a current-carrying flow propagating through a surrounding plasma, with an ambient magnetic field. This describes situations varying from Earthly lightning, to relativistic electron beams born in diodes and propagating in meter-long "drift" tubes, to plasma jets accelerated to high velocities by black hole accretion disks, eventually erecting magnetic structures millions of light years long.

Generally, primary current flows induce return currents in the ambient plasma (here assumed, for simplicity, to be initially uniform). Electric fields driven by induction induce currents proportional to the rate of change of the primary current, closing the circuit back to the source. This begins the process of magnetic confinement of jets, now an accepted view (Benford, 1978). The return current region is typically much larger than the primary current's (even for gas-pressure-confined situations, such as ordinary lightning), and for jets is termed the cocoon. Returning currents over larger cylindrical zones minimizes the kinetic energy cost to the inductive field. This follows from the general principle that processes minimize total energy in building equilibria. Also, such large cocoons increase the inductance, increasing endurance times against inductive decay. (Probably reconnection governs decay in such equilibria, in the region of toroidal field reversal; see below.)

Cocoons were invoked from the beginning of astrophysical jet analysis (the term is from the late Peter Scheuer). They are large regions surrounding jets, threaded by magnetic fields and sometimes seen in radiomaps by synchrotron emission—structures into which the jet has deposited much of its energy. Current-carrying jets must necessarily have their return currents carried in this volume, because it is energetically efficient to induce flows in many particles at low velocity (versus a jet, which has fewer particles at high velocity). This large, mass-loaded cocoon can preserve confinement of plasma through its self-organized magnetic configuration. Beyond the cocoon lies what we shall call the "shell," implying that plasma there is not magnetically confined (since the poloidal field has dropped, perhaps to zero). Its sole electrodynamic role is to form a conducting boundary in the sense of Figure 2. Magnetic fields there can respond passively to the evolution of the jet-cocoon system, and in environ-

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ments with pressure gradients can respond to the slow forces (buoyancy, ram pressures, etc.), independently of the jet-cocoon structure details. Numerical simulations of jet terminations show their hydrodynamical nature. Jets with strong toroidal magnetic fields do not develop substantial reverse current cocoons. Instead, the shocked jet plasma confined by magnetic stresses forms a “nose cone”-shaped head. This does not describe the cocoons seen in classical FR II radio sources, which do not appear to have such nose cones. This is because numerical simulations of strongly magnetized jets assume ideal MHD flows, whereas the current closure condition requires that a large fraction of magnetic fields to resistively dissipate, evolving to force-free jets (Lesch, Appl, & Camenzind, 1989; Appl and Camenzind, 1992). Plausibly this happens mostly in the very compact hot spots observed in radio lobes, but as the Figure suggests, jets spread radially and drive inductive return currents over large radii, building cocoons much larger than the jets (Lesch & Birk, 1998). This in turn creates reversed field pinches, as the structure evolves with immunity to pressure-driven instabilities. Like the magnetic well geometries of fusion plasmas, the Taylor state equilibria are not merely stable but have a finite margin of stability. Even if the kink instability occurs, kinks seem to lead to helicoidal equilibria with a redistribution of the current density rather than to disruption (Lery et al., 2000). Kinks evolve toward the relaxed Taylor state.

The physical process by which the accretion disk drives the jet/lobe is thought to be magnetic helicity injection. Near a black hole, differential Keplerian disk rotation twists up a magnetic arcade above the disk, directly converting mechanical energy into magnetic energy. In the laboratory, a radial electric field imposed by coaxial electrodes faithfully simulates the same process by drawing current from a power supply to run through a plasma threaded by an externally imposed poloidal magnetic field. The toroidal magnetic flux is then injected by the current into the discharge chamber, driving a magnetic bubble expansion. Laboratory electrostatic helicity injection has found usage from spheroid formation to non-inductive current drive in spherical tori.

Much analysis has gone into the shock at the astrophysical jet head, and its injection of energetic particles into the cocoon, but often the electrodynamic necessity to return the current is neglected in equilibria. Yet jets seem often magnetically self-confined, which implies that cocoons are a major agency in transmitting external pressures to the jet through the intermediary magnetic fields. Laboratory experience echoes this, through of course many of the dimensionless parameters are very different. In the laboratory, cyclotron radii are significant, but not usually in astrophysics, except in the crucial physics of particle acceleration. However, general stability theorems transcend the differences, within the glade governed by MHD.

Our hypothesis is that generally these and similar situations can create eventually stable, magnetically confined (overall) equilibria that evolve to a Taylor state. Unstable beams or jets that do not lead to long-lived magnetic systems do not produce classic radio sources or well-controlled beams in plasma chambers, and are not of much interest.

The usual MHD description of these systems begins with an assumed equilibrium (usually very simple) and then considers perturbations affecting stability. But how to fix the equilibrium? This is harder than linear stability theory, since equilibria are fundamentally nonlinear solutions of the MHD equations, equating the $j \times B$ force to the pressure gradients.

We first argue, from much experience of the plasma physics community, that a particular class of equilibria, the reversed field pinch, should emerge. The reversed field pinch equilibrium in Fig. 2 has high plasma pressure and in the laboratory operates with small safety factors (Bellan, 2000). Both poloidal and toroidal fields are of similar magnitude and play important roles in radial pressure balance. A conducting wall at large radius is often used in laboratory pinches, and in jets this can be akin to the distant conducting plasma beyond formation of a cocoon, i.e., beyond the envelope of return current. The critical signature of the reversed poloidal field lends this class of equilibria its name, because in a qualitative sense the contained plasma is compressed between the outer poloidal tension and the inner toroidal pressure (Bellan, 2000). We conclude, then, that

![Figure 1](image1.png)

**Figure 1.** A propagating current $J_1$ produces a spatially- and time-varying magnetic field $B_0$ at its head, driving a return current over an area much broader than the equilibrium flow radius, $a_0$. The inductive $E_z$ drives the return current and time evolution proceeds with time $t$ increasing behind the head of the flow.

![Figure 2](image2.png)

**Figure 2.** The cylindrical reversed field pinch geometry. In plasma physics terms the confining poloidal field $B_0$ rises with radius $r$, while the toroidal field $B_0$ falls from the flow axis. Plasma pressure $p$ is flat throughout most of the region where it is confined. A conducting wall defines the boundary conditions.
many magnetic relics, sometimes called “ghosts” and fossils (Enßlin, 2003) may well be reversed field pinches. This means their later lives are governed by the magnetic tensions threading such structures. Particularly, then, it is no mystery that they are stable as they rise like balloons in the hot plasmas of clusters. Recall that ordinary balloons work because they have a surface tension and stable boundary equilibria, responding through their surface tension to outside pressures. While external fields may aid magnetic balloon stability, they are rugged already. This can influence much of our thoughts about cluster structures such as relics and ghosts (Enßlin & Gopal-Krishna, 2001; Jones & De Young, 2005; Eilek et al, 1999; Feretti, 2000; Giovannini, et al., 1993, 1999; Govoni, F., 2001). Such MHD-stable balloons in clusters can keep out hot cluster plasmas and if compressed and energized by passing shocks can lead to the observed radio “ghosts” (Enßlin, 1999).

Often in astrophysical models the magnetic field is tangled on scales that are much smaller than the scales of fluid motions. Therefore, the plasma of magnetic fields and relativistic particles is confined to small ‘bubbles’ intermixed with the non-relativistic, thermal plasma. The size of these bubbles is set by the tangling scale of the magnetic fields. Thus the two fluids are separated on microscopic scales (see Brunetti, et al., 2001, and references therein). This picture does not apply to coherently built magnetic structures, so is antithetical to our approach. (Jones & De Young, 2005; Enßlin, 2003) Cluster plasmas should have the wide range of values of magnetic field (microGauss), density, coherence lengths of the fields, etc. to allow an ideal MHD approach. Classical (Spitzer) conductivity in the 10 keV plasma yields magnetic diffusion times ∼ gigayear over scales of interest. Only in the turbulent setting-up of equilibria by jets should resistivity matter at the working jet head. We thus exploit the advantage of a general stability method over numerical simulations, which depend critically on parameter sizes.

2 EVOLUTION OF CURRENT-CARRYING JETS AND THEIR MAGNETIC BALLOONS

After a time longer than either the rise time of the jet current, or the Alfvén crossing time (whichever is longer), self-confined magnetic configurations can evolve by the production of return currents in the ambient plasma, and the interaction between the primary and return currents. The constellation of ideas regarding such evolving long-term structures centrally invokes the concept of magnetic helicity,

\[ K = \int \vec{A} \cdot \vec{B} \, d^3r \]  

(1)

here \( \vec{A} \) is the vector potential of the magnetic field \( \vec{B} \).

Evolution of magnetic structures built by current-carrying flows, if they produce magnetically confined beams or astrophysical jets, should follow three concepts developed in the study of laboratory plasma confinement. The guiding principles gained from laboratory experience are:

(i) For time scales less than the resistive diffusion time of the system, \( K \) is conserved. For sizable jets, this can mean essentially forever, since the diffusion time scales with \( A \), where \( A \) is the system cross section normal to current flow. (We assume here a generally cylindrical geometry, with current along the axis.)

(ii) The twist of a magnetic field cannot be too large, or it will be unstable to a variety of modes, particularly the kink. For a current-flow pattern of size \( L \), instability occurs if

\[ \mu_0 I > \psi L \]  

(2)

where \( I \) is the total current along a flux tube \( \psi \) and \( \mu_0 \) is the mks magnetic constant.

(iii) \( K \) is much better conserved than magnetic energy for microscopic dissipative processes.

These principles imply that long-term structures of large size can evolve by accumulating twist (helicity), and then suffer disruption that sheds some helicity, returning to a stable state for a while. Magnetic flux and energy are not conserved during helicity buildup or shedding. Dissipation of field energy (during reconnection, principally) can heat plasma (which has pressure \( P \)) and accelerate electrons, provoking emission of electromagnetic radiation (often, synchrotron). Jets and beams can build the long-lived magnetic structures, following the above three principles. In a sense these are like leaky thermodynamic systems that have temperature gradients and non-uniform fluxes. Magnetic systems will have gradients in the scale parameter \( \lambda \) and a non-uniform helicity flux (Bellan, 2000).

As an astrophysical jet source (presumably a collapsed rotating object with an accretion disk acting as a dynamo) delivers helicity to the magnetic volume, little flows back to the source; the two agencies are weakly coupled. This implies a gradient in \( \lambda \). Only if there was little helicity dissipation in the magnetic structure will the gradient in \( \lambda \) be small, and so \( \lambda \) will be nearly uniform.

It seems plausible that the governing, evolved equilibrium of jet-driven, long-lived magnetic structures will be a reversed field pinch (Taylor, 1963; Bellan, 2000). This axisymmetric configuration has field components \( B_z \sim B_r \sim (\mu_0 p)^{1/2} \) in an MHD equilibrium made stable by optimally efficient radial profiles demanding a minimum of \( B_z \). (\( B_0 \) is built by the jet current, \( I \); Figure 2.)

Stable, high-plasma pressure (\( \beta \)) reversed field pinch radial profiles have general several properties:

(i) A \( B_z \) field that reverses near the outside of the confinement region. This is the crucial shear that stabilizes interchange modes and prevents formation of kink (\( m = 1 \)) current-driven waves.

(ii) To suppress “sausage” modes driven by pressure, a value of \( \beta < \frac{1}{2} \).

(iii) A conducting “wall” close enough to the plasma core to suppress the kink, \( m = 1 \), current-driven internal kink modes. This also completely damps all external kink modes.

(iv) A pressure profile \( p(r) \) that is hollow or very flat in \( r \), to suppress interchange modes near the magnetic axis.

The fundamental theory describing evolution to a reversed field pinch state is due to Taylor (Taylor, 1963). We envision a (reversed field) pinch evolving adiabatically through a sequence of minimum-energy states, as the jet current drives expansion of the structure. Relaxation occurs behind the head of the jet, where current variations are slow.

Ohmic dissipation alone cannot yield a reversed field...
pinch. Some turbulence must maintain reversal by anomalous transport and plasma convection. Yet the turbulence cannot by definition be so disruptive as to disallow a long-term stable structure; such cases we would not see.

Radial pressure balance and overall force balance in the entire structure, including the return currents, are similar to those for a simple Z-pinch. Though the reversed field pinch pressure profile constraint is significant, these arise naturally in experiments with nearly force-free Ohmic discharges of high current (Bellan, 2000, p. 354).

The simplest requirement for overall stability against interchange of flux lines is that the average curvature of the magnetic geometry be positive. This is valid for general three-dimensional closed-line systems of arbitrary \( \beta \), i.e., \( p/B^2 \). This is a necessary, and quite general, condition.

We now turn to our major result. We model the evolution of reversed field pinch magnetic structures as a “rigid” (slowly varying, high inertia) jet, often relativistic, which by induction drives return currents in the surrounding plasma, constructing the entire return current structure and especially the “cocoon” which immediately surrounds the luminous jet. If a “rigid” (slowly varying) current in the jet generates the magnetic structure, what stable configuration does it make? And what happens to the stability of the structure after the jet current ceases, as it must?

### 3 THEOREM: STABILITY OF MAGNETIC WELL EQUILIBRIA WITH A RIGID CURRENT

If a jet ebbs, how can we track the stability of the changing equilibria? This is a vast problem and plasma theory has few tools to attack it. Solving the time-dependent fluid equations is hopelessly complicated. Ordinary stability theory identifies the failure modes, with growth rates, but offers little counsel about how the system responds. One method emphasizes “marginal stability”—gradual readjustments of gradients or other equilibrium parameters to make linear growth rates evolve to zero. This method has limited use; one still does not know how the system adjusts globally.

Here we use a different approach, taking the perfect conductivity energy principle analysis to assess stability. For very large structures, perfect conductivity is a plausible approximation because the scales over which an equilibrium adjusts allow no significant role to the diffusion time, even though the jets that set up the equilibrium need dissipation to bring about the return current circuit that establishes the structure.

This energy principle method sets general conditions on the equilibrium fields (Johnson et al., Kulcsrud, 1969). Rather than tracking individual modes, we think globally about how structures evolve. (Bernstein et al., 1958; Taylor, 1963) The perfect conductivity assumption is essential for energy principle analysis, because resistivity implies a steady draining of energy by Ohmic dissipation, an extra-erous effect outside stability analysis. We neglect resistivity because in large structures resistive zones lie typically where reconnection proceeds. In reversed field pinches, this is usually near where the axial field reverses—i.e., deep in the structure, where particles get energized—and so largely beside the point of overall stability. The global resistive decay times are enormously long, and so negligible.

We cannot follow the jet decay, which would demand a full time-dependent analysis. Instead, one can model in snapshot fashion. We take the reversed field pinch in the jet-on state, applying energy condition stability criteria, and then compare with the final, jet-off state. It will turn out that the conditions on magnetic field equilibria are the same. When the jet is on we take the driver current to be stiff in the MHD sense. This seems appropriate for jet flows that are either dense or relativistic, constituting a quasi-rigid current system, essentially unaffected by the slow reaction of the far larger surrounding plasma. With high kinetic energy, the jet resists any surrounding magnetic fields. Our strategy is then to examine what the powerful energy principle method says about the long term, once the jet current dies. This seems plausible because by the time the jet builds magnetic equilibria on scales larger than galaxies, the inductive decay time far exceeds the Hubble time. When the jet current dies, the equilibrium adjusts slowly, so there is no quick perturbing pressure to make it unstable. The entire equilibrium adjusts from the jet, radially outward. End-state analysis can frame the stability issue without following the intricate intermediate evolution. We take two “snapshots” during and after the jet lifetime and find that they have identical stability requirements.

The effect of the initial current driver should then be included only in the equilibrium-generating \( \nabla \times \vec{B} \) equation,

\[
\nabla \times \vec{B} = \vec{j}_p + \vec{j}_e, \tag{3}
\]

where \( \vec{B} \) = magnetic flux density (in rationalized units, with \( \mu_0 = 1 \)), \( \vec{j}_p \) = plasma current, and \( \vec{j}_e \) = driver current. The \( \vec{j}_e \) term is treated as a fixed function of position \( \vec{j}_e = \vec{j}_e(\vec{x}) \).

Apart from this, the usual magnetohydrodynamic (MHD) equations apply, with the sole proviso that the \( \vec{j} \) appearing in the \( \vec{j} \times \vec{B} \) term of the MHD equation of motion is \( \vec{j}_p \).

Quite generally the usual energy condition holds, with some modifications (Bernstein et al., 1958).

Here we consider the analogous problem for a tensor-pressure equilibrium of the “mod-B” type with pressure \( p \perp \) perpendicular to the field and \( p \parallel \) parallel to it (Northrop & Whiteman, 1964)—i.e., when both pressure components are functions of \( B(\vec{B}) \):

\[
p \perp = p \perp (\vec{B}), \quad p \parallel = p \parallel (\vec{B}). \tag{4}
\]

We assume that all flux lines ultimately intersect a plasma boundary, and for unbounded cases (such as jets) this must mean some distant conducting plasma that imposes a boundary condition on the entire equilibrium. Here we are concerned with axisymmetric geometries, the same conclusions follow trivially on grounds of symmetry. Such equilibria highly favor containment and stability.

We now show that this remains true in the presence of an additional \( \vec{j}_e \) and therefore that the equilibria built by laboratory beams or astrophysical jets will have the same stability properties after the source current turns off. Specifically, we show that the well-known Hastie-Taylor criterion applies (Hastie and Taylor, 1964).

Take the equilibrium condition:

\[
\vec{j}_p \times \vec{B} = \nabla \left[ p \perp + \vec{B} \cdot \nabla B^{-2}(p \parallel - p \perp) \right]
\]

\[
= (\nabla \times \vec{B}) \times \vec{B} - \vec{j}_e \times \vec{B}. \tag{5}
\]
Taking the parallel component, one easily finds
\[ Bp_\parallel = p_1 - p_\perp, \tag{6} \]
where the “prime” denotes differentiation with respect to \( B \). In the case of interest (beam- or jet-generated magnetic structures), both components are decreasing functions of \( B \), since the magnetic “well” confines the plasma. For a reversed field pinch, this confinement occurs in the broad region near the reversal in sign of the toroidal magnetic field (Fig. 2). This generally implies
\[ p_\perp > p_\parallel. \tag{7} \]

In the absence of a \( \vec{j}_e \), it follows that
\[ \vec{j} = -\left(1 + \frac{p_\perp - p_1}{B^2}\right)^{-1}\left(\frac{p_\perp - p_0}{B^2}\right) \nabla \vec{B} \times \vec{B}, \tag{8} \]
and from this it follows directly that
\[ \vec{j} \cdot \vec{B} = 0. \tag{9} \]

The analogous relations in the present case are
\[ (1 + \frac{p_\perp - p_1}{B^2}) \vec{j}_p + \left(\frac{p_\perp - p_1}{B^2}\right) \vec{j}_e \]
\[ = -\left(\frac{p_\perp - p_0}{B^2}\right) \nabla B \times \vec{B}, \tag{10} \]
and
\[ (B^2 + p_\perp - p_0) \vec{j}_p \cdot \vec{B} + (p_\perp - p_0) \vec{j}_e \cdot \vec{B} = 0. \tag{11} \]

To derive the stability criterion, we can use the paper of Taylor and Hastie (1964), making appropriate modifications wherever necessary. We note first that their Eq. (4) continues to apply in the present case, if \( \vec{j}_p \) is substituted for \( \vec{j} \) in the second term.

This can be shown by going through the original derivation of the guiding-center energy principle (Kruskal and Oberman, 1958) and making sure that whenever using the \( \nabla \times \vec{B} \) equation, the \( \vec{j}_e \) term is properly included. In Taylor and Hastie’s subsequent calculations, the \( \vec{j} \cdot \vec{B} = 0 \) condition was explicitly used in several places. If we use Eq. (9) instead, however, we find that \( \vec{j}_e \) terms cancel out in the end, thus leading to no change in the final result [their Eq. (1)].

Now we seek general conditions for stability. In its final form, the integrand of the energy integral can be written as \((B + p_\perp)\) times a positive term, plus \((B - p_\parallel)\) times a positive term, plus a positive term. Moreover, by an appropriate choice of trial functions, the following conditions can be satisfied simultaneously: (1) the third term is negligible everywhere. (2) The first and second terms are negligible everywhere except in the immediate neighborhood of a single arbitrarily chosen point \( P \). (3) The ratio of the first two terms in the neighborhood of \( P \) can be made either arbitrarily large or arbitrarily small. Under these circumstances, it is necessary and sufficient for stability if both the following conditions are satisfied for all values of \( B \):
\[ B - p_\parallel > 0, \quad B + p_\perp > 0. \tag{12} \]

The first condition is always satisfied in the case of interest (magnetic confinement), and the second sets and upper limit of \( \lambda \left(\frac{B^2_{\text{max}}}{B^2_{\text{min}}} - 1\right) \) on the maximum of \( p_\perp \) at the center of the well. Here \( p_{\perp, \text{max}} = p_\parallel (B_{\text{min}}); \quad p_\perp (B_{\text{max}}) = p_\parallel (B_{\text{max}}) = 0 \).

These are the same as Hastie and Taylor’s conclusions, which have become standard wisdom in fusion plasma physics. The point is that they are unaffected by the presence of a nonvanishing \( \vec{j}_e \). This implies that a wide range of equilibria available to propagating currents, achieved by driving return currents in the larger surrounding plasma, can be set up and will then persist after the driver current tapers away (as a beam shuts off, or a black hole jet dies) because its stability is ensured.

Laboratory experiments provide much of our lore about reversed field pinches, and these are usually low pressure devices (“low beta”, where \( \beta = [p_{\perp, \text{max}}/(B^2_{\text{max}})] \)). However, our energy principle stability analysis does not depend upon this critical stability parameter \( \beta \) being very small, only \( \beta < 1 \). Taylor argued that in all systems with \( \beta < 1 \) magnetic reconnection conserves global helicity, allowing stable geometries to evolve without losing stability. “Unfreezing” the magnetic field lines from plasma demands resistivity and thus some form of magnetic energy dissipation, but helicity can be preserved during this, as is the case for dissipation by reconnection. Thus \( \beta \sim 1 \) may occur in long-lived systems if helicity may be shed through turbulent processes other than reconnection.

4 CONCLUSIONS: THE EVOLUTION OF MAGNETIC BUBBLES

We have argued that long-lived magnetic structures generated by current-carrying flows evolve into Taylor states, and stay that way for quite long times, probably \( > \) a billion years. Our principal assertion is that one can model reversed field pinch equilibria by taking the early-age driver current to be stiff in the MHD sense. Stability conditions of the minimum-B variety, familiar from fusion plasma studies, apply in vast radio structures while current drivers are on. The stability condition so reached will then also obtain for the later, “cooling down” state without an active current source, when magnetic equilibria will persist against dissipation of magnetic energy.

If these equilibria with jets present cannot shed helicity, they can go kink unstable. After there is no jet, the problem vanishes–kink stability can be achieved by expansion of the equilibrium radius, so that the unstable wavelengths become longer than the structure length. Generally, the large magnetic structures built by current flows from compact sources can be studied using energy principle methods and invoking the Taylor logic learned from laboratory cases. This means that observed MHD-stable bubbles in hot clusters can keep out hot cluster plasmas, leading to radio “ghosts.” Stability conditions assume that magnetic fields exert non-isotropic stresses, as is critical in flows. Helical strong fields appear to keep jets from widening in numerical simulations (Punsley, 2001, Enßlin et al, 1997), and we should expect this helical field structure for the jet and inner cocoon. The observed similar axial ratios (width to length) in cocoons of FRII sources suggests a self-similar evolution, and magnetic confining structures can satisfy this demand.

How do these ideas apply to magnetic “balloons” built by jets? The reversed field pinch is endangered by the current-driven global kink instability; this seems to be the most likely way for structures to fail when the current source is on. If a jet can survive the current-driven era, late stability seems more probable, as loss of the jet removes
a source of free energy. But suppose the system fails to shed helicity \( K \) as dissipation of magnetic energy proceeds; this is well known to lead to kink instability. Recent detections of several ghost cavities in galaxy clusters (Enßlin and Heinz, 2002; Soker et al., 2002) – often, but not always, radio-emitting – suggest that the cluster hot plasma stays separated from the bulk of the relativistic plasma on a timescale of 100 Myr. Some leakage of higher energy particles is not excluded by these observations, of course.

What sort of equilibria are plausible? Dunn and Fabian (2004) found limits on \( k/f \), where \( k \) is the ratio of the total relativistic particle energy to that in electrons radiating between 10 MHz to 10 GHz and \( f \) is the volume filling factor of the relativistic plasma. None of their bubbles had a simple equipartition between the pressures from the relativistic particles and the magnetic field. Further, \( k/f \) had no strong dependence on any physical parameter of the host cluster, and though at first there seemed to be two populations – \( k/f \) values around 2, another bunch around 300 – this did not hold up (Dunn, Fabian and Taylor, 2005). The apparent bimodality of the \( k/f \) distribution could have been explained as arising from two kinds of jets–electron-positron, giving a low value for \( k \), and electron-proton. If protons are the extra particles needed to maintain pressure equilibrium, but unseen in the radio emission, \( k \) is high. Also, bimodality could be caused by either a non-uniform magnetic field, or a filamentary structure in the lobes. Both possibilities are consistent with a reversed field equilibrium, since fields vary, and especially in the lobes there are dissipative processes afoot, which do not smooth out structures. Later, thermal plasma entrained during bubble formation would reduce the volume filling factor and provide extra particles, yielding the calculated values. Variations in re-acceleration, which may occur in the field reversal volume from reconnection events, could also affect the \( k/f \) measure. At this point we know too little to infer much. The important lesson is that constraints on the magnetic field obtained by comparing the synchrotron cooling time to the bubble age show that no bubbles in the sample are in equipartition. In a few years measurements of the rotation measures from sources behind clusters using EVLA might reveal correlations of the upper limits on \( k/f \) with magnetic field. This could test whether the older a bubble is, then the larger its value of \( k/f \), from aging of the relativistic electrons.

Plainly, stable “balloon” fossils of earlier jets can influence cluster evolution by rising as bubbles, conveying energy, and hastening vertical mass mixing. Rising magnetic balloons can detach from their host galaxy by reconnection near their foot points—so they typically should be larger, the farther they are from the center. This can be checked as a general tendency, once we can resolve many such “ghosts.” Heating of clusters can come from the shifting of such balloons, allowing gas to infall and warm. Many magnetic balloons, small and large, can contribute—rather than, say, one huge structure from the central galaxy, which seems energetically difficult. Magnetic balloons that resist the incursion of cluster plasma may help explain evolution of the cluster plasma over long times.

**Acknowledgements**

I thank D. Buote, L. Feretti, G. Giovanninni, P. Bellan and J. Eilek for useful discussions.