Relationship between the dust-acoustic soliton parameters and the Debye radius

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Abstract. Based on the analysis of the dusty-acoustic soliton properties, a simple technique for estimating the Debye radius is proposed. It is shown that the Debye radius is approximately equal to 1–1/3 of the width of the soliton density profile, which can be easily determined from the analysis of photographs of a dust cloud.

1. Introduction
Dusty plasma contains a portion of charged microparticles in addition to electrons and ions [1,2]. The dust fraction expands the physical properties of the plasma, such as density ratio of various types of charged particles (including their profiles in a non-uniform configuration), the shape and position of strata in a glow discharge, etc. The fundamental importance of dusty plasma research includes the concept of active particles, gas discharge physics as well as plasma wave physics. In addition, the studies are important from a technological point of view. Despite the large number of works on this topic, the question of accurately determining the plasma parameters remains open. It becomes especially relevant in experiments on cryogenic discharge carried out in bulky thermostats and in conditions of a lack of available instruments. In these and other cases, new plasma diagnostic methods are needed. In this paper, the techniques of dusty plasma diagnostics based on the analysis of dust-acoustic solitons are developed. The lower limit of the frequency range of dusty-acoustic waves is in the range of 1–100 Hz, and fluctuations in the density of dust caused by a nonlinear wave can be easily recorded by a video camera. On the other hand, the connection of plasma parameters with wave parameters is determined by many theoretical models. The technique developed is designed to determine the Debye radius. There is no need to use probes, external fields that affect the plasma.

2. Main equations
Wave analysis in plasma can be divided into two main parts which are linear analysis and nonlinear one. Linear analysis consists in obtaining the dispersion relation under the assumption that all the main parameters can be represented as $A = A_0 + A_1$ and $A_1 = a_0 \exp(i\omega t - ikx)$,
where $\omega$ and $k$ are frequency and wave number respectively. The linear dispersion relation for dust-acoustic waves was first obtained in 1990 by Rao, Shukla and Yu [3].

An important feature of the dust-acoustic mode is the ultra-low frequency $\sim 100$ Hz, due to the large mass of charged dust particles. In this case, the main spatial scale is the ionic Debye radius. The dispersion relation allows us to obtain a relationship between the wave frequency and the wavelength as well as such plasma parameters as the Debye radius $\lambda_D$ and plasma frequency $\omega_d$. Linear analysis also allows one to find the increment (decrement) of plasma instabilities. An increment of dust-acoustic instability in the presence of an ion flux and the absence of dissipation was found in [4]. The dependence of the increment on $k$ has an extremum that determines the wavelength of the wave process. This allows determining the parameters of the ion flux. A more detailed linear analysis of dust-acoustic instabilities can be found in reviews [1,2]. It is important that the main parameters of the wave process in dusty plasma are available for measurement by simple optical methods. It is worth noting that the linear approximation is valid under the condition $n_0 \gg n_1$, where $n_0$ and $n_1$ are the initial and disturbed dust density. However, in most experiments, the density disturbance caused by the wave is quite significant [5–7]. This is clearly seen from the scattering of the laser light. Such waves, generally speaking, are nonlinear. In many cases [6, 7], the perturbation of the dust concentration has a soliton-like profile. Such waves can be described in the framework of a completely nonlinear wave theory [7].

Let us consider the properties of dusty acoustic solitons that can be used to dusty plasma diagnostics. We assume that the plasma consists of electrons, ions, charged particles of mass $m_d$ and a neutral buffer gas. In the framework of the hydrodynamic plasma model, the following one-dimensional equations can be written [2]:

$$
\frac{\partial N_d}{\partial t} + \frac{\partial N_d v_d}{\partial x} = 0, \quad (1)
$$

$$
\frac{\partial v_d}{\partial t} + v_d \frac{\partial v_d}{\partial x} = \frac{\partial \Phi}{\partial x}, \quad (2)
$$

$$
N_e(\Phi) \equiv \frac{n_e}{n_{e0}} = \exp \left( \frac{\Phi}{\beta \delta_1} \right), \quad (3)
$$

$$
N_i(\Phi) \equiv \frac{n_i}{n_{i0}} = \exp \left( - \frac{\Phi}{\beta \delta_1} \right), \quad (4)
$$

$$
\frac{\partial^2 \Phi}{\partial x^2} = \delta_e N_e - \delta_i N_i + N_d, \quad (5)
$$

Here, the density, the initial density and the normalized density of electron, ion and dust particles are denoted $n_{e,i,d}, n_{e0,i0,d0}, N_{e,i,d} = n_{e,i,d}/n_{e0,i0,d0}$. In addition, $\beta = T_e/T_i$, $\delta_{e,i} = n_{0e,i0}/(Z n_{0d})$, $Z = q_d/e$, $q_d$ is the charge of the dust particle. Variables $t$ and $x$ are normalized to $\omega_d^{-1}$ and $\lambda_D$, respectively, where $\omega_d = (4\pi Z^2 e^2 n_{0d}/m_d)^{1/2}$ is dust plasma frequency, $\lambda_D = [T_i/(4\pi e^2 n_{0i})]^{1/2}$ is Debye radius. $\Phi = eZ \phi/(C_d^2 m_d)$ is potential, $C_d = [Z^2 n_{0d} T_i/(m_d n_{0d})]^{1/2}$ is dust-acoustic speed. Dust particle velocity, $v_d$, is normalized to $C_d$.

Using a single variable $X = x - M t$, where $M = V/C_d$ is the Mach number, $V$ is soliton speed, we can write from (1), (2)

$$
N_d(\Phi) = \frac{M}{\sqrt{M^2 + 2\Phi}}. \quad (6)
$$

Now, substituting expressions (3), (4), (6) into equation (5) and considering that $\partial/\partial x = \partial/\partial X$, we can obtain

$$
\frac{\partial^2 \Phi}{\partial X^2} = \delta_e \exp \left( \frac{\Phi}{\beta \delta_1} \right) - \delta_i \exp \left( - \frac{\Phi}{\beta \delta_1} \right) + \frac{M}{\sqrt{M^2 + 2\Phi}}. \quad (7)
$$
3. Numerical results

Various methods for solving equation (7) can be found in [2]. We will use the numerical Runge–Kutta method. We set the parameters traditional for laboratory discharge plasma $T_e = 6$ eV, $T_i = 0.03$ eV, $Z = 10^3$, $n_{0e} = 5 \times 10^8$ cm$^{-3}$, $n_{0i} = 5.4 \times 10^8$ cm$^{-3}$, $n_{0d} = 3.7 \times 10^4$ cm$^{-3}$, $m_d = 8.3 \times 10^{-13}$ g. Then $\beta = 200 \gg 1$, $\delta_e = 13.5$, $\delta_i = 14.5$, $\lambda_D = 5.2 \times 10^{-3}$ cm, $\omega_d = 358$ s$^{-1}$, $C_d = 1.9$ cm/s. The numerical solutions of equation (7) for the indicated parameters and different values of $M$ are presented in figure 1.

As one can see, the solution has a well-known appearance. The speed of the soliton exceeds the dust-acoustic speed (see, e.g., [8]). With increasing velocity, the soliton amplitude increases and its width decreases, figure 1(a, b). For large amplitudes, the width of the soliton is a few Debye radii. In most experiments, the disturbance of the dust density is clearly visible. And so, suppose that the density in the center of the soliton increases at least 2 times in comparison with $N_{0d}$. In this case, the interparticle distance decreases by $2^{1/3}$ times. As can be seen from figure 1(c), the width of the dust density profile is less than 3 at $N_{d_{max}} > 2$ (to the right of the vertical dashed line), where $N_{d_{max}}$ is maximum of the $N_d(X)$ profile. This means that, by measuring the width of the dusty acoustic soliton, we can get an estimate for the Debye radius:

$$3\lambda_D > \Delta_{Nd} > \lambda_D,$$

or

$$\Delta_{Nd} > \lambda_D > \frac{1}{3}\Delta_{Nd},$$

where $\Delta_{Nd}$ is the width of the dust density profile. The lower limit for $\Delta_{Nd}$ is $1\lambda_D$, where strong electrostatic repulsion forces arise.

The Debye radius is the main plasma parameter that relates temperatures and density of charged particles. It should be remembered that the Debye radius is determined by the electron population, in the discharge regions with super sonic ion flux [9]. It is easy to show that in this case our technique will also be applicable. Close results are obtained when analyzing models with a self-consistent charge of dust particles [2,10,11]. It was shown in [11] that for $n_{0e} \approx n_{0i}$, taking into account self-consistency of the dust charge weakly affects soliton solutions.

4. Conclusion

In a one-dimensional hydrodynamic model of dusty plasma, soliton solutions describing solitary dust-acoustic waves are obtained and analyzed. Unlike many other works, dust density profiles are studied in detail. The relationship between the density profile width and the Debye radius
is established. A simple estimation method was developed for the Debye radius, based on the analysis of dust cloud images only. There is no need to use probes, spectrometry, etc. In a sense, solitons and strongly nonlinear waves can be considered as “universal candles” for estimating the Debye radius.

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