Perturbative QCD Analysis of Exclusive Processes $e^+e^- \rightarrow VP$ and $e^+e^- \rightarrow TP$

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We study the $e^+e^- \rightarrow VP$ and $e^+e^- \rightarrow TP$ processes in the perturbative QCD approach based on $k_T$ factorization, where the $P,V$ and $T$ denotes a light pseudo-scalar, vector and tensor meson, respectively. We point out in the case of $e^+e^- \rightarrow TP$ transition due to charge conjugation invariance, only three channels are allowed: $e^+e^- \rightarrow a_2^{\pm}\pi^\mp$, $e^+e^- \rightarrow K_2^{\pm}K^\mp$ and the $V$-spin suppressed $e^+e^- \rightarrow K_2^{\mp}\bar{K}_2^{0} + K_2^{0}\bar{K}_2^{\mp}$. Cross sections of $e^+e^- \rightarrow VP$ and $e^+e^- \rightarrow TP$ at $\sqrt{s} = 3.67$ GeV and $\sqrt{s} = 10.58$ GeV are calculated and the invariant mass dependence is found to favor the $1/s^4$ power law. Most of our theoretical results are consistent with the available experimental data and other predictions can be tested at the ongoing BESIII and forthcoming Belle-II experiments.

I. INTRODUCTION

The exclusive processes of $e^+e^-$ annihilating into two mesons provide an opportunity to investigate various time-like meson form factors. The form factor dependence on the collision energy $\sqrt{s}$ sheds light on the structure of partonic constituents in the hadron [7, 8]. It means that these processes can be used to extract the relevant information on the structure of hadrons in terms of fundamental quark and gluon degrees of freedom. Another reason to study the $e^+e^-$ process is its similarity with annihilation contributions in charmless $B$ decays. In two-body charmless $B$ decays, annihilation diagrams are power-suppressed. However it has been observed that in quite a few decay modes annihilations are rather important [12, 68, 69]. Large annihilation diagrams will very presumably give considerable strong phases and as a consequence sizable CP asymmetries are induced [4, 5]. This fact has an important impact in the CP violation studies of $B$ meson decays.

The $e^+e^- \rightarrow VP, TP$ processes, where the $P,V, T$ denotes a light pseudo-scalar, vector and

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tensor meson, respectively, have the topology with annihilation diagrams in $B$ decays, and thus they can provide an ideal laboratory to isolate power correction effects.

It is anticipated that hard exclusive processes with hadrons involve both perturbative and non-perturbative strong interactions. Factorization, if it exists, allows one to handle the perturbative and non-perturbative contributions separately. The short-distance hard kernels can be calculated perturbatively. With the nonperturbative inputs determined from other sources, hard exclusive processes provide an effective way to explore the factorization scheme. The factorization theorem ensures that a physical amplitude can be expressed as a convolution of hard scattering kernels and hadron distribution amplitudes. However if one directly applies the collinear factorization to the $e^+e^− \rightarrow VP, TP$, the amplitude diverges in the end point region $x \rightarrow 0$. Here $x$ is the momentum fraction of the involved quark.

A modified perturbative QCD approach based on $k_T$ factorization, called PQCD approach for brevity, is proposed [1–6] and has been successfully applied to many reactions [9–25]. In this approach, the transverse momentum of partons in the meson is kept to kill endpoint divergences. Then the physical amplitude is written as a convolution of the universal non-perturbative hadronic wave functions and hard kernels in both longitudinal and transverse directions. Double logarithms, arising from the overlap of the soft and collinear divergence, can be resumed into Sudakov factor, while single logarithms from ultraviolet divergences can be handled by renormalization group equation (RGE). With Sudakov factor taken into account, the applicability of perturbative QCD can be brought down to a few GeV. In this work, we will study the $e^+e^− \rightarrow VP$ and $e^+e^− \rightarrow TP$ in the perturbative QCD (PQCD) approach [1–6] based on $k_T$ factorization.

The rest of this paper is organized as follows. In section II, we first collect the input parameters including decay constants and light-cone wave functions. Then we present the PQCD framework and give factorization formulas for the time-like form factors. Numerical results and detailed discussions are presented in section III. The last section contains the conclusion.

II. PERTURBATIVE QCD CALCULATION

A. Notations

We consider the $e^+e^− \rightarrow V(T)P$, in which $V(T)$ is a vector (tensor) meson with momentum $P_1$ and polarization vector $\epsilon_\mu$ (polarization tensor $\epsilon_{\mu\nu}$), and $P$ denotes a pseudoscalar meson with momentum $P_2$ in the center of mass frame. The collision energy is denoted as $Q = \sqrt{s}$. In the
standard model, such processes proceed through a virtual photon or a $Z^0$ boson. At low energy with $\sqrt{s} \sim$ a few GeV, the amplitude is dominated by a photon. In this case the hadron amplitude is parameterized in terms of a form factor:

$$\langle V(P_1, \epsilon_T) P(P_2) | j_{\mu}^{em} | 0 \rangle = F_{VP}(s) \epsilon_{\mu \alpha \beta} \epsilon_T^\alpha P_1^\alpha P_2^\beta. \quad (1)$$

Notice that in Eq.(1) the vector meson is transversely polarized. We have adopted the convention $\epsilon^{0123} = 1$ for the Levi-Civita tensor.

For a tensor meson, its polarization tensor $\epsilon_{\mu \nu}$ can be constructed via the polarization vector

$$\epsilon_{\mu}(0) = \frac{1}{m_T} (|\vec{P}_T|, 0, 0, E_T), \quad \epsilon_{\mu}(\pm) = \frac{1}{\sqrt{2}} (0, \mp 1, -i, 0). \quad (2)$$

Using the Clebsch-Gordan coefficients [26], one has

$$\epsilon_{\mu \nu}(\pm 2) = \epsilon_{\mu}(\pm) \epsilon_{\nu}(\pm), \quad \epsilon_{\mu \nu}(\pm 1) = \sqrt{\frac{1}{2}} [\epsilon_{\mu}(\pm) \epsilon_{\nu}(0) + \epsilon_{\mu}(0) \epsilon_{\nu}(\pm)], \quad \epsilon_{\mu \nu}(0) = \frac{1}{6} [\epsilon_{\mu}(+) \epsilon_{\nu}(-) + \epsilon_{\mu}(-) \epsilon_{\nu}(+)] + \sqrt{\frac{1}{3}} \epsilon_{\mu}(0) \epsilon_{\nu}(0). \quad (3)$$

In the calculation it is convenient to introduce a new polarization vector $\xi$:

$$\xi_{\mu}(\lambda) = \frac{\epsilon_{\mu \nu}(\lambda) q^\nu}{P_1 \cdot q} m_T, \quad (4)$$

where $q = P_1 + P_2$ is the four momentum of the virtual photon and $q^2 = s$. Then Eq.(3) becomes

$$\xi_{\mu}(\pm 2) = 0, \quad \xi_{\mu}(\pm 1) = \frac{1}{\sqrt{2}} \frac{Q^2 \eta}{2 m_T^2 + Q^2 \eta} \epsilon_{\mu}(\pm), \quad \xi_{\mu}(0) = \sqrt{\frac{1}{3}} \frac{Q^2 \eta}{3 m_T^2 + Q^2 \eta} \epsilon_{\mu}(0), \quad (5)$$

where $\eta = 1 - m_T^2 / Q^2$, with $m_T$ as the mass of the tensor meson. Here the mass of the pseudoscalar meson has been neglected. The new vector $\xi$ plays a similar role with the ordinary polarization vector $\epsilon$, regardless of some dimensionless constants.

Then like Eq. (1), one can define the $TP$ form factor as

$$\langle T(P_1, \lambda) P(P_2) | j_{\mu}^{em} | 0 \rangle = F_{TP} \epsilon_{\mu \alpha \beta} \xi^\nu(\lambda) P_1^\alpha P_2^\beta. \quad (6)$$

in which the final state tensor meson is also transversely polarized.

Using the form factors in Eqs.(1,6), one can derive the cross sections for $e^+e^- \rightarrow VP, TP$

$$\sigma(e^+e^- \rightarrow VP) = \frac{\pi \alpha_{em}^2}{6} |F_{VP}|^2 \Phi^{3/2}(s), \quad (7)$$

$$\sigma(e^+e^- \rightarrow TP) = \frac{\pi \alpha_{em}^2}{3} \left( \frac{sn}{2m_T^2 + sn} \right)^2 |F_{TP}|^2 \Phi^{3/2}(s), \quad (8)$$

with the fine structure constant $\alpha_{em} = 1/137$, and

$$\Phi(s) = \left[ 1 - \frac{(m_{V(T)} + m_p)^2}{s} \right] \left[ 1 - \frac{(m_{V(T)} - m_p)^2}{s} \right]. \quad (9)$$
B. Decay constants and Light cone wave functions

Decay constants for a pseudoscalar meson and a vector meson are defined by:
\[
\langle P(p)|q_2\gamma_\mu\gamma_5q_1|0\rangle = -if^P_{p\mu},
\]
\[
\langle V(p,\epsilon)|q_2\gamma_\mu q_1|0\rangle = f_V m_V\epsilon_\mu, \quad \langle V(p,\epsilon)|q_2\sigma_{\mu\nu}q_1|0\rangle = -if^V_{\mu\nu}(\epsilon_\mu p_\nu - \epsilon_\nu p_\mu).
\]

Tensor meson decay constants are defined as [27]
\[
\langle T(P,\lambda)|j_{\mu\nu}(0)|0\rangle = f_T^2 m^2 T(\epsilon^{(\lambda)*}_\mu P_\nu - \epsilon^{(\lambda)*}_{\nu\delta} P_\mu).
\]

The interpolating currents are chosen as
\[
\begin{align*}
\tilde j_{\mu\nu}(0) &= \frac{1}{2} \left( \bar q_1(0)\gamma_\mu\gamma_5 \tilde D_\mu q_2(0) + \bar q_1(0)\gamma_\nu\gamma_5 \tilde D_\mu q_2(0) \right), \\
\tilde j_{\mu\nu}^{(\lambda)}(0) &= \bar q_2(0)\sigma^{\lambda\mu\nu} \tilde D_\delta q_1(0),
\end{align*}
\]
with the covariant derivative \( \tilde D_\mu = \tilde D_\mu - \tilde D_\mu \) with \( \tilde D_\mu = \tilde D_\mu + ig_\lambda A^\lambda_\mu \) and \( \tilde D_\mu = \tilde D_\mu - ig_\lambda A^\lambda_\mu \).

The pseudoscalar and vector decay constants can be determined from various reactions, \( \pi^- \rightarrow e^-\bar\nu, \tau^- \rightarrow (\pi^-, K^-\rho^-, K^{*+})\nu_\tau \) and \( V^0 \rightarrow e^+e^- \) [26]. For tensor mesons, their decay constants can be calculated in QCD sum rules [28, 29] and we quote the recently updated results from Ref. [27]. Results for decay constants are collected in Table I.

**TABLE I**: Decay constants of the relevant light mesons (in units of MeV)

| \( f_\pi \) | \( f_K \) | \( f_\rho \) | \( f_{\pi^0} \) | \( f_{K^0} \) | \( f_{K^*} \) | \( f_{\phi} \) | \( f_{\sigma} \) | \( f_{\rho_2} \) | \( f_{\omega} \) | \( f_{\eta} \) | \( f_{\eta_2} \) | \( f_{\pi^0} \) |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 131 | 160 | ± | 2 | 165 | ± | 9 | 195 | ± | 3 | 145 | ± | 10 | 217 | ± | 5 | 185 | ± | 10 | 231 | ± | 4 | 200 | ± | 10 | 107 | ± | 6 | 105 | ± | 21 | 118 | ± | 5 | 77 | ± | 14 |

The light-cone distribution amplitudes (LCDAs) are defined as matrix elements of non-local operators at the light-like separations \( z_\mu \) with \( z^2 = 0 \), and sandwiched between the vacuum and the meson state. The two-particle LCDAs of a pseudoscalar meson, up to twist-3 accuracy, are defined by [30]
\[
\langle P(p)|\bar q_2\gamma_5 z_\alpha q_1(0)|0\rangle = \frac{-i}{\sqrt{2}N_C} \int_0^1 dx e^{i2pz} \left[ \gamma_5 \phi^A_P(x) + m_0 \gamma_5 \phi^B_P(x) + m_0 \gamma_5 (\gamma_5 - 1) \phi^T_P(x) \right]_{\alpha\beta},
\]
where \( n, v \) are two light-like vectors. The final-state P meson is moving on the \( n \) direction with \( v \) the opposite direction. \( x \) is the momentum fraction carried by the quark \( q_2 \). The chiral enhancement
parameter $m_0 = m^2_P/(m_{q_1} + m_{q_2})$, is used in our work as $m^\pi_0 = 1.4 \pm 0.1$GeV, $m^K_0 = 1.6 \pm 0.1$GeV [31, 32].

We use the following form for leading twist LCDAs derived from the conformal symmetry:

$$
\phi^A_P(x) = \frac{3f_P}{\sqrt{2N_C}}x(1-x)[1 + a_1^P C^{3/2}_1(t) + a_2^P C^{3/2}_2(t)],
$$

where $N_C = 3$ and $t = 2x - 1$. $C^{3/2}_i(i=1,2)$ are Gegenbauer polynomials, with the definition

$$
C^{3/2}_1(t) = 3t, \quad C^{3/2}_2(t) = \frac{3}{2}(5t^2 - 1).
$$

The Gegenbauer moments at $\mu = 1$GeV are used as [31, 32]:

$$
a_1^\pi = 0, \quad a_1^K = 0.06 \pm 0.03, \quad a_2^{\pi,K} = 0.25 \pm 0.15.
$$

In this paper, we will study the collision at $\sqrt{s} = 3.67$GeV and 10.58GeV, and then it is plausible to adopt the asymptotic forms for twist-3 DAs for simplicity:

$$
\phi^P_P(x) = \frac{f_P}{2\sqrt{2N_C}}, \quad \phi^T_P(x) = \frac{f_P}{2\sqrt{2N_C}}(1-2x).
$$

As for the $\eta - \eta'$ mixing, we use the quark flavor basis with the mixing scheme [33, 34]:

$$
\begin{pmatrix}
\eta \\
\eta'
\end{pmatrix}
= U(\phi)
\begin{pmatrix}
\eta_q \\
\eta_s
\end{pmatrix}
= \begin{pmatrix}
\cos \phi & -\sin \phi \\
\sin \phi & \cos \phi
\end{pmatrix}
\begin{pmatrix}
\eta_q \\
\eta_s
\end{pmatrix}.
$$

The mixing angle is $\phi = 39.3^\circ \pm 1.0^\circ$[33, 34] and

$$
\eta_q = \frac{1}{\sqrt{2}}(u\bar{u} + dd), \quad \eta_s = ss.
$$

Their decay constants are defined as:

$$
\langle 0|\bar{n}\gamma^\mu\gamma_5 n|\eta_\mu(P)\rangle = \frac{i}{\sqrt{2}}f_n P^\mu, \quad \langle 0|\bar{s}\gamma^\mu\gamma_5 s|\eta_\mu(P)\rangle = if_s P^\mu.
$$

In the following calculation, we will assume the same wave functions for the $n\bar{n}$ and $s\bar{s}$ as the pion’s wave function, except for the different decay constants [33, 34] and the chiral scale parameters [35]:

$$
f_n = (1.07 \pm 0.02)f_\pi, \quad f_s = (1.34 \pm 0.06)f_\pi, \quad m^n_0 = 1.07$GeV, \quad m^s_0 = 1.92$GeV.
$$

Similar with pseudoscalar mesons, the two-particle LCDAs for transversely polarized vector mesons up to twist-3 are parameterized as [36, 37]:

$$
\langle V(p, \epsilon_T)|\bar{q}_{2\beta}(z)q_{1\alpha}(0)|0\rangle = \frac{1}{\sqrt{2N_C}} \int_0^1 dx e^{ipz} \left[ \bar{t}_T^\mu \phi^T_V(x) + m_V \epsilon_T^\mu \phi^V_V(x) + m_V i\epsilon_{\mu\nu\rho\sigma} \gamma^\nu \epsilon^\rho_{\nu} \phi^V_V(x) \right]_{\alpha\beta}.
$$
The twist-2 LCDA can be expanded as:

\[
\phi_T^T(x) = \frac{3f_V}{\sqrt{2N_C}}x(1-x)[1 + a_T^1 C_1^{3/2}(t) + a_T^2 C_2^{3/2}(t)],
\]

with Gegenbauer moments at \(\mu = 1\text{GeV} [38, 39]\):

\[
\begin{align*}
a_{1K^*}^+ &= 0.04 \pm 0.03, & a_{1p}^+ &= a_{1\omega}^+ = a_{1\phi}^+ = 0, \\
a_{2K^*}^+ &= 0.11 \pm 0.09, & a_{2p}^+ &= a_{2\omega}^+ = 0.15 \pm 0.07, & a_{2\phi}^+ &= 0.06^{+0.09}_{-0.07}. 
\end{align*}
\]

As for the twist-3 LCDAs, we will also use the asymptotic forms:

\[
\begin{align*}
\phi_T^V(x) &= \frac{3f_V}{8\sqrt{2N_C}}[1 + (2x - 1)^2], & \phi_T^p(x) &= \frac{3f_V}{4\sqrt{2N_C}}(1 - 2x). 
\end{align*}
\]

For a generic tensor meson, the LCDAs up to twist-3 can be defined as [20]:

\[
\langle T(p, \pm1)|q_2\beta(z)q_1\alpha(0)|0\rangle = \frac{1}{\sqrt{2N_C}} \int_0^1 e^{ixp\cdot z} \\
\times \left[ T_T T_T \phi_T^T(x) + m_T T_T \phi_T^V(x) + m_T i\epsilon_{\mu\nu\rho\sigma} \gamma^\mu \xi_T \eta_{\nu\rho\sigma} \phi_T^\alpha(x) \right]_{\alpha\beta}.
\]

These LCDAs are related to the ones given in [27]:

\[
\begin{align*}
\phi_T^T(x) &= \frac{f_T^2}{2\sqrt{2N_C}} \phi_\perp(x), & \phi_T^V(x) &= \frac{f_T}{2\sqrt{2N_C}} g_\perp^{(v)}(x), & \phi_T^p(x) &= \frac{f_T}{8\sqrt{2N_C}} \frac{d}{dx} g_\perp^{(a)}(x). 
\end{align*}
\]

The asymptotic forms will be used in the calculation:

\[
\begin{align*}
\phi_{\perp,\perp}(x) &= 30x(1-x)(2x-1), \\
g_\perp^{(a)}(x) &= 20x(1-x)(2x-1), & g_\perp^{(v)}(x) &= 5(2x-1)^3.
\end{align*}
\]

In the above, we have only discussed the longitudinal momentum distributions. It is reasonable
that the transverse momentum also plays an important role. Thus we will include the transverse
momentum dependent parton distributions (TMDs) of the final-state light mesons. Following Ref.
[7], we assume no interference between the longitudinal and transverse distributions, and thus one
can use the following Gaussian forms to factorize the wave functions [40, 41]:

\[
\begin{align*}
\psi(x, b) &= \phi(x) \times \exp \left( -\frac{b^2}{4\beta^2} \right), \\
\psi(x, b) &= \phi(x) \times \exp \left[ -\frac{x(1-x)b^2}{4a^2} \right].
\end{align*}
\]

In the above equation \(\phi(x)\) is the longitudinal momentum distribution amplitude, and the exponential
factor describes the transverse momentum distribution. The parameters \(\beta\) and \(a\) characterize
the shape of the transverse momentum distributions. The parameter \(\beta\) is expected at the order
of \(\Lambda_{\text{QCD}}\) and related with the root of the averaged transverse momentum square \((k_T^2)^{1/2}\). If we
choose \((k_T^2)^{1/2} = 0.35\text{GeV}, \quad \beta^2 = 4\text{GeV}^{-2}\). According to Ref. [41], the size parameter \(a\) follows
\(a^{-1} \simeq \sqrt{8\pi} f_M\), where \(f_M\) is the decay constant of the related hadron.
C. PQCD Calculation

In the PQCD scheme, a form factor can be written as the convolution of a hard scattering kernel with universal hadron wave functions. In small-\(x\) region, the parton transverse momentum \(k_T\) is at the same order with the longitudinal momentum. Once \(k_T\) is introduced in the hard kernel, a transverse momentum dependent (TMD) wave function is requested. Then the form factor is factorized as:

\[
F(Q^2) = \int_0^1 dx_1 dx_2 \int d^2k_T_1 d^2k_T_2 \Phi_{M_1}(x_1, k_T_1, P_1, \mu) H(x_1, x_2, k_T_1, k_T_2, Q, \mu) \Phi_{M_2}(x_2, k_T_2, P_2, \mu) \\
= \int_0^1 dx_1 dx_2 \int \frac{d^2b_1}{(2\pi)^2} \frac{d^2b_2}{(2\pi)^2} \mathcal{P}_{M_1}(x_1, b_1, P_1, \mu) H(x_1, x_2, b_1, b_2, Q, \mu) \mathcal{P}_{M_2}(x_2, b_2, P_2, \mu). \tag{34}
\]

Eq.(34) is the Fourier form in the impact parameter \(b\) space. Here \(\Phi_{M_1}(x_1, k_T_1, P_1, \mu)\) and \(\mathcal{P}_{M_i}(x_i, b_i, P_i, \mu)\) are both the hadron wave functions, relying on \(k_T\) and \(b\) respectively.

Double logarithms arising from the overlap of soft and collinear divergences, can be resumed into Sudakov factor [42, 43]:

\[
\mathcal{P}_{M_i}(x_i, b_i, P_i, \mu) = \exp[-s(x_i, b_i, Q) - s(1 - x_i, b_i, Q)] \mathcal{P}_{M_i}(x_i, b_i, \mu). \tag{35}
\]

The Sudakov factor \(s(\xi, b, Q), \xi = x_i \text{ or } 1 - x_i\), is given as [44, 45]:

\[
s(\xi, b, Q) = \frac{A^{(1)}}{2\beta_1} \hat{q} \ln \left(\frac{\hat{q}}{\hat{b}}\right) + \frac{A^{(2)}}{4\beta_1^2} \left(\frac{\hat{q}}{\hat{b}} - 1\right) - \frac{A^{(1)}}{2\beta_1^2} \hat{q} \beta_2 \left[\frac{\ln(2\hat{b}) + 1}{b} - \frac{\ln(2\hat{q}) + 1}{q}\right]\]
\[\nonumber - \left[\frac{A^{(2)}}{4\beta_1^3} - \frac{A^{(1)}}{4\beta_1} \ln \left(\frac{2\gamma - 2}{2}\right)\right] \ln \left(\frac{\hat{q}}{\hat{b}}\right) + \frac{A^{(1)}}{8\beta_1^2} \left[\ln^2(2\hat{q}) - \ln^2(2\hat{b})\right], \tag{36}\]

where the notations have been used:

\[
\hat{q} \equiv \ln \left[\frac{\xi Q}{\sqrt{2\Lambda_{QCD}}}\right], \quad \hat{b} \equiv \ln \left[\frac{1}{b\Lambda_{QCD}}\right]. \tag{37}\]

The running coupling constant is given as

\[
\frac{\alpha_s}{\pi} = \frac{1}{\beta_1 \log(\mu^2/\Lambda_{QCD}^2)} - \frac{\beta_2}{\beta_1^3} \ln^2(\mu^2/\Lambda_{QCD}^2), \tag{38}\]

and the coefficients \(A^{(i)}\) and \(\beta_i\) are

\[
\beta_1 = \frac{33 - 2n_f}{12}, \quad \beta_2 = \frac{153 - 19n_f}{24}, \quad A^{(1)} = \frac{4}{3}, \quad A^{(2)} = \frac{67}{9} - \frac{\pi^2}{3} - \frac{10}{27} n_f + \frac{8}{3} \beta_1 \ln \left(\frac{e\gamma_E}{2}\right), \tag{39}\]

Here \(n_f\) is the number of the quark flavors and \(\gamma_E\) is the Euler constant.
Apart from the double logarithms, single logarithms from ultraviolet divergence emerge in the radiative corrections to both the hadronic wave functions and hard kernels. These are summed by the renormalization group (RG) method:

\[
\left[ \mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} \right] P_{M_i}(x_i, b_i, P_i, \mu) = -2\gamma_q P_{M_i}(x_i, b_i, P_i, \mu),
\]

\[
\left[ \mu \frac{\partial}{\partial \mu} + \beta(g) \frac{\partial}{\partial g} \right] H(x_1, x_2, b_1, b_2, Q, \mu) = 4\gamma_q H(x_1, x_2, b_1, b_2, Q, \mu).
\]

Here the quark anomalous dimension is \(\gamma_q = -\alpha_s/\pi\). In terms of the above equations, we can get the RG evolution of the hadronic wave functions and hard scattering amplitude as

\[
P_{M_i}(x_i, b_i, P_i, \mu) = \exp \left[ -2 \int_{1/b_i}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu})) \right] \times P_{M_i}(x_i, b_i, 1/b_i),
\]

\[
H(x_1, x_2, b_1, b_2, Q, \mu) = \exp \left[ -4 \int_{\mu}^{t} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu})) \right] \times H(x_1, x_2, b_1, b_2, Q, t),
\]

where \(t\) is the largest energy scale in the hard scattering. Then from equations (35) and (42), the large-\(b\) behavior of \(P\) can be summarized as

\[
P_{M_i}(x_i, b_i, P_i, \mu) = \exp[-S(x_i, b_i, Q, \mu)] P_{M_i}(x_i, b_i, 1/b_i),
\]

with

\[
S(x_i, b_i, Q, \mu) = s(x_i, b_i, Q) + s(1 - x_i, b_i, Q) + 2 \int_{1/b_i}^{\mu} \frac{d\bar{\mu}}{\bar{\mu}} \gamma_q(\alpha_s(\bar{\mu})).
\]

Furthermore, QCD loop corrections for the electromagnetic vertex can induce another type of double logarithms \(\alpha_s \ln^2 x_i\). They are usually factorized from the hard amplitude and resummed into the jet function \(S_t(x_i)\) to further suppress the end-point contribution. It should be pointed out that Sudakov factor from threshold resummation is universal and independent on the flavors of internal quarks, twist and topologies of hard scattering amplitudes and the specific process [46–50]. The following approximate parametrization is proposed in [51] for the convenience of phenomenological applications

\[
S_t(x, Q) = \frac{2^{1+c} \Gamma(3/2 + c)}{\sqrt{\pi} \Gamma(1 + c)} (x(1 - x))^c,
\]

in which the \(c\) is a parameter depending on \(Q\). Ref. [52] proposed a parabolic parametrization of the \(Q^2\) dependence:

\[
c(Q^2) = 0.04 Q^2 - 0.51 Q + 1.87,
\]

The threshold resummation modifies the end point behavior of the hadron wave functions, rendering them vanish faster in this region.
FIG. 1: Feynman diagrams for $e^+e^- \rightarrow VP, TP$. In the first four panels, a hard momentum transfer occurs through the highly virtual gluon. In the last two panels, the neutral vector meson is generated by a photon.

Taking into account all the above ingredients, one can obtain the analytic results of the first four diagrams in Fig. 1 in $k_T$ factorization:

$$F_a = 16\pi C_F Q \int_0^1 dx_1 dx_2 \int_0^\infty b_1 b_2 b_2 E(t_a) h(\bar{x}_1, x_2, b_1, b_2) S_t(x_2)$$
$$\times \left\{ r_1 \left[ \phi_1^{p(a)}(x_1, b_1) - \phi_1^v(x_1, b_1) \right] \phi_2^A(x_2, b_2) \right\},$$

(48)

$$F_b = 16\pi C_F Q \int_0^1 dx_1 dx_2 \int_0^\infty b_1 b_2 b_2 E(t_a) h(x_2, \bar{x}_1, b_1, b_2) S_t(\bar{x}_1)$$
$$\times \left\{ r_1 \bar{x}_1 \left[ \phi_1^{p(a)}(x_1, b_1) + \phi_1^v(x_1, b_1) \right] \phi_2^A(x_2, b_2) - 2r_2 \phi_1^T(x_1, b_1) \phi_2^P(x_2, b_2) \right\},$$

(49)

$$F_c = -16\pi C_F Q \int_0^1 dx_1 dx_2 \int_0^\infty b_1 b_2 b_2 E(t_c) h(\bar{x}_2, x_1, b_1, b_2) S_t(x_1)$$
$$\times \left\{ r_1 x_1 \left[ \phi_1^{p(a)}(x_1, b_1) - \phi_1^v(x_1, b_1) \right] \phi_2^A(x_2, b_2) + 2r_2 \phi_1^T(x_1, b_1) \phi_2^P(x_2, b_2) \right\},$$

(50)

$$F_d = -16\pi C_F Q \int_0^1 dx_1 dx_2 \int_0^\infty b_1 b_2 b_2 E(t_d) h(x_1, \bar{x}_2, b_1, b_2) S_t(\bar{x}_2)$$
$$\times \left\{ r_1 \left[ \phi_1^{p(a)}(x_1, b_1) + \phi_1^v(x_1, b_1) \right] \phi_2^A(x_2, b_2) \right\},$$

(51)
where $E(t_i)$ and $h$ are given as

$$E(x_1, x_2, b_1, b_2, Q, t_i) = \alpha_s(t_i) \exp[-S_1(x_1, b_1, Q, t_i) - S_2(x_2, b_2, Q, t_i)],$$  \hspace{1cm} (52)$$

$$h(x_1, x_2, b_1, b_2, Q) = \left(\frac{i\pi}{2}\right)^2 H_0^{(1)}(\sqrt{x_1 x_2 Q b_1}) [\theta(b_1 - b_2) H_0^{(1)}(\sqrt{x_2 Q b_2}) + \theta(b_2 - b_1) H_0^{(1)}(\sqrt{x_2 Q b_1})], \hspace{1cm} (53)$$

where $J_0$ and $H_0^{(1)}$ are both Bessel functions. We take $\bar{x} = 1 - x$ for short and define $r_i = m_i/Q$, with the index $i = 1, 2$ for the cases of final state meson is vector(tensor) or pseudoscalar meson. The factorization scale $t$ is chosen as the largest mass scale involved in the hard scattering:

$$t_a = \max(\sqrt{x_2 Q}, 1/b_1, 1/b_2), \quad t_b = \max(\sqrt{x_1 Q}, 1/b_1, 1/b_2),$$

$$t_c = \max(\sqrt{x_1 Q}, 1/b_1, 1/b_2), \quad t_d = \max(\sqrt{x_2 Q}, 1/b_1, 1/b_2). \hspace{1cm} (54)$$

If the final state meson is not a strange meson, the distribution amplitudes are completely symmetric or antisymmetric under the interchange of the quark and antiquark’s momentum fraction $x$ and $1 - x$. Then one can obtain

$$F_a(VP) = F_d(VP), \quad F_b(VP) = F_c(VP); \hspace{1cm} (55)$$

$$F_a(TP) = -F_d(TP), \quad F_b(TP) = -F_c(TP). \hspace{1cm} (56)$$

The contributions from a photon radiated from the interaction point into a vector meson, shown as the last two panels in Fig. 1, might be sizable. Although these diagrams are suppressed by $\alpha_{em}$, they are enhanced by the almost on-shell photon propagator $(1/m_V^2)$ compared with the gluon propagator in the first four diagrams ($\sim 1/s$) [53–56]. These two amplitudes can be calculated in collinear factorization due to the absence of endpoint singularities in these two diagrams. In particular, they are equal after integrating out the momentum fractions:

$$F_e = F_f = \frac{12\pi\alpha^2_{em} f_P f_V}{m_V s}(1 + a_2^P). \hspace{1cm} (57)$$

Finally, the form factors for the explicit channels of $e^+ e^- \rightarrow VP$ process are combinations of
the six amplitudes $F_{a-f}$:

$$F_{\rho^+\pi^-} = F_{\rho^-\pi^+} = \frac{1}{3} [F_a(\rho\pi) + F_b(\rho\pi)]$$

$$F_{\rho^0\rho^0} = \frac{1}{3} [F_a(\rho\pi) + F_b(\rho\pi)] + \frac{1}{6} [F_e(\rho\pi) + F_f(\rho\pi)]$$

$$F_{K^{++}K^-} = \frac{2}{3} [F_a(K^*K) + F_b(K^*K)] - \frac{1}{3} [F_e(K^*K) + F_d(K^*K)]$$

$$F_{K^{--}K^+} = -\frac{1}{3} [F_a(K^*K) + F_b(K^*K)] + \frac{2}{3} [F_e(K^*K) + F_d(K^*K)]$$

$$F_{K^{*0}K^{*0}} = F_{K^{*0}K^{*0}} = -\frac{1}{3} [F_a(K^*K) + F_b(K^*K)] - \frac{1}{3} [F_e(K^*K) + F_d(K^*K)]$$

$$F_{\omega\pi^0} = [F_a(\omega\pi) + F_b(\omega\pi)] + \frac{1}{18} [F_e(\omega\pi) + F_f(\omega\pi)]$$

$$F_{\phi\pi^0} = \frac{\sqrt{2}}{18} [F_e(\phi\pi) + F_f(\phi\pi)]$$

The form factors for $e^+e^- \to V(T)\eta^{(t)}$ are mixtures of the $\eta_q$ and $\eta_s$ components:

$$F_{V(T)\eta} = \cos \theta F_{V(T)\eta_q} - \sin \theta F_{V(T)\eta_s}$$

$$F_{V(T)\eta'} = \sin \theta F_{V(T)\eta_q} + \cos \theta F_{V(T)\eta_s}$$

where $V = \rho^0, \omega, \phi$ and

$$F_{\rho\eta_q} = \frac{1}{3} [F_a(\rho\eta_q) + F_b(\rho\eta_q)] + \frac{5}{18} [F_e(\rho\eta_q) + F_f(\rho\eta_q)]$$

$$F_{\rho\eta_s} = -\frac{\sqrt{2}}{6} [F_e(\rho\eta_s) + F_f(\rho\eta_s)]$$

$$F_{\omega\eta_q} = \frac{1}{3} [F_a(\omega\eta_q) + F_b(\omega\eta_q)] + \frac{5}{54} [F_e(\omega\eta_q) + F_f(\omega\eta_q)]$$

$$F_{\omega\eta_s} = -\frac{\sqrt{2}}{18} [F_e(\omega\eta_s) + F_f(\omega\eta_s)]$$

$$F_{\phi\eta_q} = -\frac{5\sqrt{2}}{54} [F_e(\phi\eta_q) + F_f(\phi\eta_q)]$$

$$F_{\phi\eta_s} = -\frac{2}{3} [F_a(\phi\eta_s) + F_b(\phi\eta_s)] - \frac{1}{27} [F_e(\phi\eta_s) + F_f(\phi\eta_s)]$$

Similarly, based on Eq.(56), form factors of the $e^+e^- \to TP$ channels can be written as:

$$F_{a_2^+}\pi^- = -F_{a_2^-}\pi^+ = [F_a(a_2\pi) + F_b(a_2\pi)]$$

$$F_{K_2^{*+}K^-} = \frac{2}{3} [F_a(K_2^*K) + F_b(K_2^*K)] - \frac{1}{3} [F_e(K_2^*K) + F_d(K_2^*K)]$$

$$F_{K_2^{--}K^+} = -\frac{1}{3} [F_a(K_2^*K) + F_b(K_2^*K)] + \frac{2}{3} [F_e(K_2^*K) + F_d(K_2^*K)]$$

$$F_{K_2^{*0}K^{*0}} = F_{K_2^{*0}K^{*0}} = -\frac{1}{3} [F_a(K_2^*K) + F_b(K_2^*K)] - \frac{1}{3} [F_e(K_2^*K) + F_d(K_2^*K)]$$

The abbreviations $a_2, K_2^*$ correspond to the tensor meson $a_2(1320)$ and $K_2^*(1430)$, respectively.
III. NUMERICAL RESULTS AND DISCUSSIONS

Using Eqs.(48)-(51), and other input parameters, we can calculate cross sections for the processes $e^+e^- \rightarrow VP$ and $e^+e^- \rightarrow TP$. In Tab. II, we have collected the results for cross sections at $\sqrt{s} = 3.67\text{GeV}$, together with the experimental data from CLEO-c collaboration [58, 59] (see Ref. [57] for BES measurements), and the results at $\sqrt{s} = 10.58\text{GeV}$, together with the data measured by Belle [60] and Babar [61] collaborations. As we have discussed before, three different types of transverse momentum distribution functions were used, denoted as $S_1$, $S_2$ and $S_3$ respectively. $S_1$ denotes the calculation without intrinsic transverse momentum distribution, $S_2$ and $S_3$ are obtained with the distributions in Eqs.(32) and (33), respectively. Theoretical errors are obtained by varying $\Lambda_{QCD} = (0.25 \pm 0.05)\text{GeV}$, and the factorization scale $t$ from 0.75$t$ to 1.25$t$ (without changing $1/b_t$).

TABLE II: Cross sections of $e^+e^- \rightarrow VP,TP$ at $\sqrt{s} = 3.67\text{GeV}$ and $\sqrt{s} = 10.58\text{GeV}$. $S_1$ denotes the calculation without intrinsic transverse momentum distribution, $S_2$ and $S_3$ are obtained with the distributions as Eqs.(32) and (33). The experimental measurements from Refs. [58–61] are also shown. Theoretical errors are obtained by varying $\Lambda_{QCD} = (0.25 \pm 0.05)\text{GeV}$, and the factorization scale $t$ from 0.75$t$ to 1.25$t$ (without changing $1/b_t$).

| Channel | $\sqrt{s} = 3.67\text{GeV}$ | $\sqrt{s} = 10.58\text{GeV}$ |
|---------|----------------------------|----------------------------|
|         | $\sigma_{S1}(\text{pb})$ | $\sigma_{S2}(\text{pb})$ | $\sigma_{S3}(\text{pb})$ | $\sigma_{exp}(\text{pb})$ | $\sigma_{S1}(\text{fb})$ | $\sigma_{S2}(\text{fb})$ | $\sigma_{S3}(\text{fb})$ | $\sigma_{exp}(\text{fb})$ |
| $\rho^+\pi^+$ | 6.80 ± 1.18 | 3.38 ± 0.53 | 3.95 ± 0.63 | 4.8^{+1.5}_{-1.2}^{+0.5}_{-0.5} | 0.66 ± 0.10 | 0.53 ± 0.08 | 0.60 ± 0.09 |
| $\rho\rho^0$ | 3.38 ± 0.60 | 1.69 ± 0.27 | 1.99 ± 0.32 | 3.1^{+1.0}_{-0.9}^{+0.4}_{-0.4} | 0.25 ± 0.05 | 0.20 ± 0.04 | 0.23 ± 0.04 |
| $K^+K^-$ | 10.13 ± 0.91 | 5.27 ± 0.50 | 5.39 ± 0.35 | 1.0^{+1.1}_{-0.7}^{+0.5}_{-0.5} | 1.15 ± 0.10 | 0.94 ± 0.08 | 1.02 ± 0.08 | 0.18^{+0.14}_{-0.12} ± 0.02 |
| $K^0\bar{K}^0 + \bar{K}^0K^0$ | 61.94 ± 13.76 | 31.34 ± 6.15 | 35.85 ± 6.25 | 23.5^{+4.6}_{-4.3}^{+1.1}_{-1} | 6.65 ± 1.20 | 5.39 ± 0.93 | 5.88 ± 1.02 | 7.48 ± 0.67 ± 0.51 |
| $\omega\pi^0$ | 24.94 ± 4.59 | 12.41 ± 2.08 | 15.18 ± 2.59 | 15.2^{+2.8}_{-3.4}^{+4.7}_{-1.5} | 2.38 ± 0.40 | 1.90 ± 0.31 | 2.16 ± 0.35 |
| $\phi\pi^0$ | 1.2 ± 10^{-4} | 1.2 ± 10^{-4} | 1.2 ± 10^{-4} | < 2.2 | 2.2 ± 10^{-3} | 2.2 ± 10^{-3} | 2.2 ± 10^{-3} |
| $\rho^0\eta$ | 14.37 ± 2.10 | 7.21 ± 0.96 | 8.10 ± 1.06 | 10.0^{+2.2}_{-1.9}^{+0.0}_{-1.0} | 1.10 ± 0.13 | 0.89 ± 0.11 | 1.03 ± 0.12 |
| $\rho\rho'$ | 8.22 ± 1.19 | 4.10 ± 0.54 | 4.57 ± 0.59 | 2.1^{+4.7}_{-1.6}^{+0.2}_{-0.2} | 1.03 ± 0.11 | 0.83 ± 0.09 | 0.93 ± 0.10 |
| $\omega\eta$ | 1.31 ± 0.20 | 0.65 ± 0.09 | 0.77 ± 0.11 | 2.3^{+1.8}_{-1.0}^{+0.5}_{-0.5} | 0.10 ± 0.01 | 0.081 ± 0.011 | 0.094 ± 0.012 |
| $\omega\rho'$ | 0.75 ± 0.11 | 0.37 ± 0.05 | 0.43 ± 0.06 | < 17.1 | 0.094 ± 0.011 | 0.076 ± 0.009 | 0.086 ± 0.010 |
| $\phi\eta$ | 17.82 ± 3.34 | 9.21 ± 1.51 | 8.23 ± 1.32 | 2.1^{+1.4}_{-0.7}^{+0.7}_{-0.2} | 2.21 ± 0.30 | 1.75 ± 0.23 | 1.84 ± 0.25 | 2.9 ± 0.5 ± 0.1 |
| $\phi\rho'$ | 21.97 ± 4.13 | 11.36 ± 1.87 | 10.20 ± 1.65 | < 12.6 | 2.81 ± 0.42 | 2.31 ± 0.33 | 2.47 ± 0.35 |
| $a_2^+\pi^+$ | 43.88 ± 13.98 | 20.34 ± 6.59 | 28.96 ± 8.62 | 6.66 ± 1.73 | 4.96 ± 1.30 | 6.06 ± 1.58 |
| $K^+K^-$ | 60.57 ± 15.89 | 27.81 ± 7.45 | 33.81 ± 8.98 | 11.48 ± 2.45 | 8.48 ± 1.79 | 9.98 ± 2.15 | 8.36 ± 0.95 ± 0.62 |
| $K^0\bar{K}^0 + \bar{K}^0K^0$ | 3.2 ± 10^{-2} | 1.1 ± 10^{-2} | 1.3 ± 10^{-2} | 8.8 ± 10^{-3} | 6.0 ± 10^{-3} | 7.3 ± 10^{-3} | 1.65^{+0.86}_{-0.78} ± 0.27 |

A few remarks are in order.

- Results at different center of mass energy $\sqrt{s}$ can be used to study the $1/s^n$ dependence of cross sections. From our results at $\sqrt{s} = 3.67\text{GeV}$ and 10.58GeV, the averaged value is about
\[ n = 4.1 \text{ for } e^+e^- \rightarrow VP \text{ and } n = 3.9 \text{ for } e^+e^- \rightarrow TP \]  

This favors the \(1/s^4\) scaling, which is consistent with the constituent scaling rule \([62, 63]\). The fitted result from experimental data is \(n = 3.83 \pm 0.07\) and \(3.75 \pm 0.12\) for \(e^+e^- \rightarrow K^*(892)^0\bar{K}^0\) and \(\omega\pi^0\), respectively \([60]\).

- From Table II, we can see that, cross sections for many processes are large enough to be measured, such as the \(e^+e^- \rightarrow \rho\pi, \rho\eta, \omega\pi\) and \(a_2^{\pm}\pi^\mp\) at \(\sqrt{s} = 10.58\text{GeV}\), and \(e^+e^- \rightarrow a_2^{\pm}\pi^\mp, K_{2}^{\pm}\bar{K}^\mp\) at \(\sqrt{s} = 3.67\text{GeV}\). We suggest the experimentalists to measure these channels especially at BESIII \([64]\) and Belle-II in future.

- For the channels \(e^+e^- \rightarrow K^{*\pm}K^\mp\), there are very poor measurements from CLEO collaboration \([59]\), since the charged \(K^*\) meson is reconstructed by three–body decays: \(K^{*\pm} \rightarrow K^0\pi^\mp \rightarrow 3\pi\), with large systematic uncertainties. Our results are larger than the central of experimental data. We hope the future experimental measurements can clarify this difference more clearly.

- If we neglect the photon-enhanced amplitudes \(F_{e,f}\), and assume the flavor SU(3) symmetry, one has the relations for cross sections: \(\sigma(\omega\pi^0) : \sigma(\rho^\pm\pi^\mp) : \sigma(\rho^0\pi^0) : \sigma(K^{*\pm}K^\mp) : \sigma(K^*\bar{K}^0 + \bar{K}^*K^0) = 1 : 2/9 : 1/9 : 2/9 : 8/9\).

- At \(\sqrt{s} = 3.67\) GeV, we have \(\sigma(e^+e^- \rightarrow \rho^\pm\pi^\mp) = 2\sigma(e^+e^- \rightarrow \rho^0\pi^0)\), while the photon-enhanced contribution becomes more important at \(\sqrt{s} = 10.58\text{GeV}\), and the ratio \(\sigma(e^+e^- \rightarrow \rho^\pm\pi^\mp) / \sigma(e^+e^- \rightarrow \rho^0\pi^0)\) is approximately 2.5.

- In the SU(3) limit, we expect \(\sigma(\omega\pi^0)/\sigma(K^{*\pm}\bar{K}^0 + \bar{K}^*K^0) = 9/8 > 1\), however our calculation has indicated that the cross section \(\sigma(\omega\pi^0)\) is smaller than that for \(e^+e^- \rightarrow K^*\bar{K}^0 + \bar{K}^*K^0\) by a factor of 2 to 3. One reason arises from the fact that the decay constants \(f_\pi f_\omega\) is about 30% smaller than \(f_K f_{K^*}\). The chiral scale parameter \(m_0^K\) will further enhance the cross sections.

- On the experiment side, the ratios \(R_{VP}\) and \(R_{TP}\) are introduced to explore the SU(3) symmetry breaking effect in the \(e^+e^- \rightarrow K^*K\) and \(e^+e^- \rightarrow K^*K\) processes, with the definition

\[
R_{VP} = \frac{\sigma(e^+e^- \rightarrow K^*(892)^0\bar{K}^0)}{\sigma(e^+e^- \rightarrow K^*(892)^-\bar{K}^+)} , \quad R_{TP} = \frac{\sigma(e^+e^- \rightarrow K_2^0(1430)^0\bar{K}^0)}{\sigma(e^+e^- \rightarrow K_2^0(1430)^-\bar{K}^+)} . \tag{77}
\]

\(^1\) We correct here the improper statement in Ref. [7].
In the PQCD framework, this ratio can be written as

$$R = \frac{(F_a + F_b) + (F_c + F_d)}{2(F_a + F_b) - (F_c + F_d)} = \left| \frac{1 + \frac{F_c + F_d}{F_a + F_b}}{2} \right|^2. \quad (78)$$

In SU(3) symmetry limit, the wave functions of $K, K^*$ and $K^*_2$ is symmetric or antisymmetric under the exchange of the momentum fractions of quark and antiquark, and thus the relations in Eq.(55) are obtained. Then one can drive $R_{VP} = 4$. One source of the SU(3) symmetry breaking is that the $s$ quark is heavier than $q(= u, d)$ quark and carries more momentum in the final state meson, therefore the gluon which generates $s\bar{s}$ is harder than the $\bar{q}q$ one. In this case, the coupling constant in the $s\bar{s}$ process is smaller. Consequently, the amplitude $|F_a + F_b|$ will be smaller than $|F_c + F_d|$, and thus $R_{VP}$ is expected larger than 4.

From Table II, one can obtain theoretical results for $R_{VP}$:

$$R_{VP}(\sqrt{s} = 3.67\text{GeV}) \simeq 5.99, \quad R_{VP}(\sqrt{s} = 10.58\text{GeV}) \simeq 5.76. \quad (79)$$

- At $\sqrt{s} = 3.67$ GeV, the CLEO-c collaboration [59] has measured the ratio:

$$R_{VP}^{Exp}(\sqrt{s} = 3.67\text{GeV}) = 23.5^{+17.1}_{-26.1} \pm 12.2, \quad (80)$$

with very large error-bar. Its central value is significantly larger, but within the errors it is consistent with our theoretical results. Belle collaboration gives the results at $\sqrt{s} = 10.52\text{GeV}, 10.58\text{GeV}$ and $10.876\text{GeV}$, respectively [60]

$$R_{VP}^{Exp} > 4.3, \quad 20.0, \quad 5.4. \quad (81)$$

Note that in the region near $10.58\text{GeV}$, Belle result is significantly larger than our expectation, which might come from the $\Upsilon(4S)$ resonance contribution. Off the $\Upsilon(4S)$ resonance, the experimental results are consistent with our theoretical calculations.

- Due to the charge conjugation invariance, we have the relations for the $e^+e^- \rightarrow TP$ transition amplitude given in Eq. (56). Thus only three channels are allowed: $e^+e^- \rightarrow a_2^\pm \pi^\mp, e^+e^- \rightarrow K_2^{\ast \pm}K^\mp$ and $e^+e^- \rightarrow K_2^{*0}\bar{K}^0 + \bar{K}_2^{*0}K^0$.

If one further assume V-spin symmetry, the process $e^+e^- \rightarrow K_2^{*0}\bar{K}^0 + \bar{K}_2^{*0}K^0$ is highly suppressed since $F_a + F_b \sim -(F_c + F_d)$. From Table II, one can obtain theoretical results for $R_{TP}$:

$$R_{TP} \lesssim 10^{-4}. \quad (82)$$
This is consistent with the Belle data [60]:

\[ R_{TP}^{Exp} < 1.1, \ 0.4, \ 0.6. \] (83)

- The theoretical uncertainties in our calculation are mainly from the uncertainties of the meson wave functions. The longitudinal distribution amplitudes in exclusive B decays will give about 10% - 20% uncertainties [41]. When the transverse momentum distribution functions are introduced in Eqs.(32) and (33), the contribution from the large-\(b\) region will be suppressed. This suppression makes the PQCD approach more self-consistent. Comparing the different results in Table II, one can observe severe suppressions especially at \(\sqrt{s} = 3.67\text{GeV}\): the suppression is about 50% for \(S2\) and about 40% for \(S3\). Since the results depend on the explicit form of transverse momentum distribution, more accurate transverse momentum dependent wave functions and more experimental results would be valuable.

- In this calculation, we have limited ourselves to the leading-order accuracy. The next-to-leading order (NLO) calculation is complicated [65–67] that will be presented in a future publication. As an estimation of the size of the NLO contribution, we vary \(\Lambda_{QCD}\) and the factorization scale \(t\) in Eq.(54): \(\Lambda_{QCD} = (0.25 \pm 0.05)\text{GeV}\), and changing the hard scale \(t\) from 0.75\(t\) to 1.25\(t\) (without changing 1/\(b_i\)). We find that our results are not sensitive to these variations. It implies that the NLO contributions are presumably not very large.

IV. CONCLUSION

Hard exclusive processes \(e^+e^- \rightarrow VP\) and \(e^+e^- \rightarrow TP\) at center of mass energy \(\sqrt{s} = 3.67\text{GeV}\) and 10.58GeV are investigated in the perturbative QCD framework in this work. For the wave functions of the light mesons involved in the factorization amplitudes, we have employed various models of transverse momentum dependence of wave functions. At the center of mass energy \(\sqrt{s} = 3.67\text{GeV}\), two different transverse momentum distribution functions can give about 50% and 40% suppressions, respectively. The value \(R_{VP}\) and \(R_{TP}\) obtained from our results are consistent with the experimental data. We found that our theoretical results favor the \(1/s^4\) scaling law for the cross sections. Most of our results are consistent with the experimental data and the others can be tested at the ongoing BESIII and forthcoming Belle-ILL experiments.
Acknowledgements

The authors are grateful to Jian-Ping Dai, Hsiang-nan Li, Cheng-Ping Shen and Yu-Ming Wang for valuable discussions. This work is supported in part by National Natural Science Foundation of China under Grant No.11575110, 11521505, 11655002, 11621131001, 11735010, Natural Science Foundation of Shanghai under Grant No. 15DZ2272100, by Shanghai Key Laboratory for Particle Physics and Cosmology, and by Key Laboratory for Particle Physics, Astrophysics and Cosmology, Ministry of Education.

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