A possible interpretation of Λ baryon spectrum with pentaquark components

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Abstract. The Λ baryon spectrum is studied within the SU(3) flavor symmetry in a constituent quark model. We found that it is rather difficult to accommodate some negative-parity Λ resonances as single $q^2s$ ($q = u, d$ quarks) states in the conventional three-quark picture. The ground $q^3s$̅̄$q$ pentaquark mass spectrum is evaluated and a possible interpretation is proposed in the work: the observed $\Lambda(1405)\frac{1}{2}^−$, $\Lambda(1670)\frac{1}{2}^−$ and $\Lambda(1800)\frac{1}{2}^−$ are three-state mixtures of two $p$-wave $q^2s$ states and one ground $q^3s$̅̄$q$ pentaquark state, so are the $\Lambda(1520)\frac{3}{2}^−$, $\Lambda(1690)\frac{3}{2}^−$ and $\Lambda(2050)\frac{3}{2}^−$ resonances.

Keywords: Λ baryon mass spectrum, pentaquark mass spectra, three-body mixture

1. Introduction

In recent decades, new experimental approaches have been applied to study the Λ resonances. Experimental data are mainly from $K^-p$ invariant mass spectra of hyperons below 1.7 GeV [1-3], partial wave analyses of $K^-p$ reactions with energies from 1480 to 2100 MeV [4] and photoproduction of $\Lambda(1405)$ from JLab [5, 6]. The Λ resonances are listed in table [1, 7].

Recent data analyses have confirmed most $3^*$ and $4^*$ Λ states, but derived rather different information of some little known states. Recent couple channel analyses from the Kent group (KSU) and JLab have extracted more hyperons [8-15]. The Kent group found some new states like $\Lambda(1710)\frac{1}{2}^+$ and $\Lambda(2050)\frac{3}{2}^−$, which were listed in Particle Data Group (PDG) [7] with one star, by fitting partial-wave amplitudes using multichannel parametrization [8]. A similar couple-channel fit has been done by JPAC group [11], where $\Lambda(2050)$ is confirmed by this analysis but $\Lambda(1800)$ is not seen. A dynamical coupled-channel model applied by the Osaka group [9, 10] results in no clear existence of $\Lambda(1710)$, $\Lambda(1800)$, $\Lambda(2000)$ and $\Lambda(2050)$. Bonn-Gatchina (BnGa) group has
A possible interpretation of $\Lambda$ baryon spectrum with pentaquark components

recently reported $\Lambda(1710)1/2^+$, $\Lambda(2070)3/2^+$ and $\Lambda(2080)5/2^-$, suggesting all the states together with $\Lambda(1405)1/2^-$ as SU(3) singlet states \[12\] \[15\]. The experimental data from $K^-p$ induced reactions have made a great contribution to our knowledge of the $\Lambda$ baryon spectrum, but it is still insufficient to reveal the nature of all $\Lambda$ resonances.

The $\Lambda$ spectra have been studied in various quark models theoretically, for example, the Isgur-Karl model \[18\] \[20\] which includes the one-gluon exchange quark-quark interaction, models taking into account the Goldstone boson exchange interaction \[21\], models considering the relativistic instanton-induced quark-quark interaction \[22\], and relativistic quark models with quark-diquark interactions \[23\] \[24\].

The traditional constituent quark models always have difficulties in describing the low mass of $\Lambda(1405)$ \[25\] in the $q^3$ picture since a heavier strange quark is contained. The $\Lambda(1405)$ resonance, since discovered in the $\pi\Sigma$ invariant mass spectrum \[26\] in 1960s, has been an interesting state in the hyperon spectrum, the nature of the resonance is still under debate. $\Lambda(1405)$ was first generated dynamically as a $N\bar{K}$ quasi-bound molecular state \[27\] \[28\], then followed a large number of studies in chiral unitary approach (ChUA) \[29\] \[33\] and other chiral dynamics on this resonance \[34\] \[46\].

By

| Particle    | Status | $J^P$ | $M_{exp}^{BW}$ MeV | $\Gamma_{exp}^{BW}$ MeV |
|-------------|--------|-------|--------------------|--------------------------|
| $\Lambda(1116)$ | ****   | $\frac{1}{2}^+$ | 1116 | - |
| $\Lambda(1600)$ | **** | $\frac{3}{2}^+$ | 1570-1630 | 150-250 |
| $\Lambda(1710)$ | *      | $\frac{1}{2}^+$ | 1700-1726 | 138-222 |
| $\Lambda(1810)$ | ***    | $\frac{1}{2}^+$ | 1740-1840 | 50-170 |
| $\Lambda(1820)$ | ****   | $\frac{3}{2}^+$ | 1815-1825 | 70-90 |
| $\Lambda(1890)$ | ****   | $\frac{5}{2}^+$ | 1870-1910 | 80-160 |
| $\Lambda(2070)$ | *      | $\frac{3}{2}^+$ | 2046-2094 | 320-420 |
| $\Lambda(2085)$ | **      | $\frac{5}{2}^+$ | 2000-2040 | 130-190 |
| $\Lambda(2110)$ | ***    | $\frac{7}{2}^+$ | 2050-2130 | 200-300 |
| $\Lambda(2350)$ | ***    | $\frac{9}{2}^+$ | 2340-2370 | 100-250 |
| $\Lambda(1380)$ | **      | $\frac{1}{2}^-$ | 1310-1340 | - |
| $\Lambda(1405)$ | ****   | $\frac{1}{2}^-$ | 1404-1406 | 48.5-52.5 |
| $\Lambda(1520)$ | ****   | $\frac{3}{2}^-$ | 1518-1520 | 15-17 |
| $\Lambda(1670)$ | ****   | $\frac{3}{2}^-$ | 1670-1678 | 25-35 |
| $\Lambda(1690)$ | ****   | $\frac{5}{2}^-$ | 1685-1695 | 60-80 |
| $\Lambda(1800)$ | ***    | $\frac{7}{2}^-$ | 1750-1850 | 150-250 |
| $\Lambda(1830)$ | ****   | $\frac{9}{2}^-$ | 1820-1830 | 60-120 |
| $\Lambda(2000)$ | *      | $\frac{1}{2}^-$ | 2000-2060 | 100-150 |
| $\Lambda(2050)$ | *      | $\frac{3}{2}^-$ | 2034-2078 | 432-554 |
| $\Lambda(2080)$ | *      | $\frac{5}{2}^-$ | 2069-2095 | 152-210 |
| $\Lambda(2100)$ | ****   | $\frac{7}{2}^-$ | 2090-2110 | 100-250 |
| $\Lambda(2325)$ | *      | $\frac{9}{2}^-$ | 2307-2347 | 120-200 |
solving the coupled channel Lippmann-Schwinger equation with a chiral meson-baryon
Lagrangian at the next-to-leading order, the $\Lambda(1405)$ resonance as a quasi-bound state of $N\bar{K}$ described the low-energy $K^-p$ scattering data well \cite{34}. Limited to the lowest-order chiral Lagrangian with coupled channel method, $\Lambda(1405)$ was identified as quasi-bound states of $N\bar{K}$ using the N/D method and dispersion relations \cite{35}. The two-poles structure of $\Lambda(1405)1/2^-$ was firstly proposed in the complex energy plane for the different Riemann sheets using the ChUA in 2001 \cite{29}. Then a variety of investigations on the two-poles structure of the $\Lambda(1405)$ resonance were carried out. The contributions of the two-pole structure to the lineshape of $\Lambda(1405)$ were shown in Refs. \cite{30,36}. After the SIDDHARTA kaonic hydrogen result \cite{16,17} was reported, different works \cite{37,42} revealed the real and imaginary parts of the two poles of $\Lambda(1405)1/2^-$. New fits including the CLAS photoproduction data were carried out in the works \cite{31,33,43} and the origin of two poles of $\Lambda(1405)1/2^-$ were discussed as well. In recent years, the first high-mass pole of $\Lambda(1405)1/2^-$ was shown to be mainly a $\bar{K}N$ molecular, the second pole was interpreted as a composite state of mainly $\pi\Sigma$ with tiny component $\bar{K}N$ in Refs. \cite{34,46}, and the influence of SU(3) flavor symmetry was analyzed in the last work \cite{46}. Exotic configurations for $\Lambda(1405)$ have also been proposed, such as a mixed pentaquark and three-quark state \cite{47}, a hybrid baryon \cite{48}, a mixed strange hybrid/three-quark baryon \cite{49} which all applied the method of the QCD sum rule. A mixed three-quark and meson-baryon molecular states for the $\Lambda(1405)$ resonance are studied in Refs. \cite{50,51} in quark models. The electromagnetic form factors of the $\Lambda(1405)$ resonance were calculated in Lattice QCD for the first time to reveal the inner structure of $\Lambda(1405)$ \cite{52,53}.

In the present work we extend the non-relativistic quark model employed in the works \cite{54,55} to study the $\Lambda$ mass spectra including light $q^3s\bar{q}$ pentaquark components. The lowest two pairs of negative-parity $\Lambda$ resonances are reproduced by mixing up the three-quark $\Lambda^*_8$ (flavor-singlet), $\Lambda_8$ (flavor-octet) states with the ground $q^3s\bar{q}$ pentaquark states. The paper is organized as follows. In section 2 we briefly introduce the Hamiltonian for multi-quark systems including the spin-orbit couplings. The masses of low-lying $q^2s$ states are also calculated in section 2. In section 3 we derive the mass spectra of ground $q^3s\bar{q}$ pentaquark states and propose an interpretation of the lowest two pairs of negative-parity $\Lambda$ resonances by introducing the ground $q^3s\bar{q}$ pentaquark components in the three-state mixtures. A discussion and a short summary are given in sec. 4.

2. $\Lambda$ resonances as $q^2s$ states

We calculate the mass of low-lying $q^2s$ states in the Hamiltonian, which is an extend of the model in Ref. \cite{54} by including the spin-orbit interactions, $$H = H_0 + H_{\text{OGE}} + H_{\text{SO}},$$
A possible interpretation of $\Lambda$ baryon spectrum with pentaquark components

\[ H_0 = \sum_{k=1}^{N} \left( m_k + \frac{p_k^2}{2m_k} \right) + \sum_{i<j}^{N} \left( \frac{3}{8} \lambda_i^C \cdot \lambda_j^C \right) (A_{ij} r_{ij} - \frac{B_{ij}}{r_{ij}}), \]
\[ H_{OGE}^{hyp} = -C_{OGE} \sum_{i<j} \frac{\lambda_i^C \cdot \lambda_j^C}{m_i m_j} \vec{\sigma}_i \cdot \vec{\sigma}_j, \]
\[ H_{SO} = 2k_i d_j \vec{L} \cdot \vec{S}. \]

(1)

where $A_{ij}$ and $B_{ij}$ are mass-dependent coupling parameters, taking the form

\[ A_{ij} = a \sqrt{\frac{m_{ij}}{m_u}}, \quad B_{ij} = b \sqrt{\frac{m_u}{m_{ij}}}. \]

(2)

with $m_{ij}$ being the reduced mass of $i$th and $j$th quarks, defined as $m_{ij} = \frac{m_i m_j}{m_i + m_j}$. The hyperfine interaction, $H_{OGE}^{hyp}$ includes only one-gluon exchange contribution (OGE), where $C_{OGE} = C_m m_u^2$, with $m_u$ being the constituent $u$ quark mass and $C_m$ a constant. $\lambda_i^C$ in the above equations are the generators of color SU(3) group. The three model coupling constants and two constituent quark masses are taken from the previous work [55],

\[ m_u = m_d = 327 \text{ MeV}, \quad m_s = 498 \text{ MeV}, \]
\[ C_m = 18.3 \text{ MeV}, \quad a = 49500 \text{ MeV}^2, \quad b = 0.75 \]

(3)

$H_{SO}$ in equation (1) is the spin-orbit interaction, taking the simplified form as in Refs. [18-20], where $L$ and $S$ are the total orbital angular and spin operators of baryon states. $d_j$ are $SU(3)_F$ flavor multiplet-dependent constants, taking the values,

\[ d_1 = 2/3, \quad d_8 = 1, \quad d_{10} = 1/3 \]

(4)

$k_i$ are $SU(6)_{SF}$ spin-flavor supermultiplet-dependent constants, which are determined by fitting the theoretical results to the mass splitting of spin-orbit pairs in the $\Lambda$, $\Sigma$ and $\Sigma^*$ spectra in the pure $q^3$ picture. The mass difference between the two $\Lambda(70, 4^8, 1, 1^-)$ states $\frac{1}{2}^- \Lambda(1800)$ and $\frac{3}{2}^- \Lambda(1830)$ leads to $k_1 \approx 4$ MeV while the gap between the $\frac{3}{2}^- \Sigma(1580)$ and $\frac{1}{2}^- \Sigma(1620)$ states results in $k_1 \approx 3$ MeV. The $(70, 1^-)$ and $(20, 1^+)$ supermultiplets have a direct relation in the lowest harmonic oscillation approximation, which results in $k_2 \approx 1.5 \ k_1 \approx 6$ MeV. $k_3$ is determined about $-14$ MeV by the $\Lambda(56, 4^8, 2, 2^+)$ states $\Lambda(1820)$ and $\Lambda(1890)$, and about $-5$ MeV by the $\Sigma(56, 4^8, 2, 2^+)$ states $\Sigma(1915)$ and $\Sigma(1940)$. The mass gap between the $\Lambda(70, 4^8, 2, 2^+)$ states $\Lambda(2110)$ and $\Lambda(2085)$ leads to $k_4 \approx -14$ MeV, and the one between the $\Sigma(70, 4^8, 2, 2^+)$ states $\Sigma(2070)$ and $\Sigma(2030)$ leads to $k_4 \approx -6$ MeV. In this work, we use averaged values,

\[ k_1((70, 1^-)) = 4 \text{ MeV}, \ k_2((20, 1^+)) = 6 \text{ MeV}, \]
\[ k_3((56, 2^+)) = -10 \text{ MeV}, \ k_4((70, 2^+)) = -10 \text{ MeV} \]

(5)

The positive-parity and negative-parity $\Lambda$ resonances of the first and second excitation bands are studied in the Hamiltonian in equation (1) in the three-quark picture. The color-orbital-spin-flavor wave functions of $q^2 s$ baryon states in the $SU(6)_{SF}$
Table 2. Theoretical Λ resonances matched with data up to the second excitation band. Λ* stands for the flavor-singlet states. Γ and D stands respectively for the SU(6)_{SF} and SU(3)_{F} multiplets, N, S, L^P and J^P are the principle quantum number, total spin, orbital and angular momentum with P being the parity.

| (Γ, 2S+1D, N, L^P) | Status | J^P | M^{exp}(MeV) | M^{cal}(MeV) |
|---------------------|--------|-----|--------------|--------------|
| Λ(56, 2S, 2, 0^+)   | ****   | 1/2^- | Λ(1116)      | 1112         |
| Λ(56, 2S, 2, 1^+)   | ****   | 3/2^- | Λ(1600)      | 1689         |
| Λ(56, 2S, 2, 2^+)   | ****   | 5/2^- | Λ(1820)      | 1825         |
| Λ(56, 2S, 2, 3^+)   | ****   | 5/2^+ | Λ(1890)      | 1875         |
| Λ(20, 2S, 2, 1^+)   | -      | 1/2^- | missing       | 1927         |
| Λ(20, 2S, 2, 2^+)   | -      | 3/2^- | missing       | 1945         |
| Λ*(20, 3S, 2, 1^+)  | -      | 1/2^- | missing       | 2162         |
| Λ*(20, 3S, 2, 2^+)  | -      | 3/2^- | missing       | 2174         |
| Λ*(20, 3S, 2, 3^+)  | -      | 5/2^- | missing       | 2194         |
| Λ(70, 2S, 2, 0^+)   | ***    | 1/2^- | Λ(1810)      | 1821         |
| Λ(70, 2S, 2, 1^+)   | *      | 3/2^- | Λ(2070)      | 2081         |
| Λ*(70, 3S, 2, 0^+)  | -      | 1/2^- | Λ(1710)      | 1838         |
| Λ*(70, 3S, 2, 1^+)  | -      | 3/2^- | ?Λ(1890)     | 1922         |
| Λ*(70, 3S, 2, 2^+)  | -      | 5/2^- | ?Λ(1820)     | 1872         |
| Λ(70, 4S, 2, 0^+)   | -      | 1/2^- | missing       | 2242         |
| Λ(70, 4S, 2, 1^+)   | -      | 3/2^- | missing       | 2212         |
| Λ(70, 4S, 2, 2^+)   | **     | 5/2^- | Λ(2110)      | 2142         |
| Λ(70, 4S, 2, 3^+)   | **     | 5/2^- | Λ(2085)      | 2092         |
| Λ*(70, 3S, 2, 2^+)  | -      | 5/2^- | missing       | 1929         |
| Λ*(70, 4S, 2, 2^+)  | -      | 5/2^- | missing       | 1896         |
| Λ(70, 5S, 1, 1^-)   | -      | 1^-   | 1579          |              |
| Λ(70, 5S, 1, 1^-)   | -      | 1^-   | 1591          |              |
| Λ(70, 5S, 1, 1^-)   | ***    | 1^-   | Λ(1800)      | 1810         |
| Λ(70, 5S, 1, 1^-)   | ****   | 1^-   | Λ(1830)      | 1842         |
| Λ(70, 5S, 1, 1^-)   | -      | 1^-   | missing       | 1822         |
| Λ*(70, 3S, 2, 1^-)  | -      | 1^-   | 1582          |              |
| Λ*(70, 4S, 2, 1^-)  | -      | 1^-   | 1590          |              |

representations, which are applied in the study, are listed in Appendix A. The theoretical masses and tentative matchings between data and the theoretical results are shown in table 2.

In the second excitation band shown in table 2, the positive-parity Λ resonances belonging to the 20-supermultiplet for both the singlet and octet flavors are missing. Λ(1820) and Λ(1890) are matched with the spin 5/2 and 3/2 states in the 56-multiplets respectively, but it can not be ruled out that the two resonances are matched with the 70-multiplet 5/2 and 3/2 states. The 3-star Λ(1810) state is assigned as a Λ(70, 2S, 2, 0^+) state in this work. This resonance Λ(1710) reported by KSU in 2013 is interpreted
as the $\Lambda^*(70,^31/2,0^+)$ singlet state in Ref. [15], but it can’t be described by the relativistic interacting quark-diquark models [23,24]. Though a higher mass is predicted for the flavor-singlet state $\Lambda^*(70,^31/2,0^+)$ in the work, we may tentatively assign the 1-star $\Lambda(1710)$ resonance as the $\Lambda^*(70,^31/2,0^+)$ state, considering the huge width of the $\Lambda(1710)$ resonance. The resonances $\Lambda(2070)^3_2^+$ reported by BnGa group in 2019 [13] is assigned as the $\Lambda^*(70,^31/2,2^+)$ state in Ref. [15], but this work prefers it being the $\Lambda(70,^48,2,0^+)$ state. And the resonance $\Lambda(2110)^5_2^+$ may be assigned as the $\Lambda(70,^48,2,2^+)$ state.

For the $L = 1$ first excitation band, we may interpret the $\Lambda(1800)^1_2^-$ and $\Lambda(1830)^5_2^-$ resonances as three-quark states in the $\Lambda(70,^48,1,1^-)$ multiplet, as shown in table 2. The spin $^3_2^-$ state, the spin-orbit coupling partner of the $\Lambda(1800)^1_2^-$ and $\Lambda(1830)^5_2^-$ resonances has not been reported. The two predicted flavor-singlet and two flavor-octet $\Lambda$ states of spin $^1_2^-$ and $^3_2^-$ lay rather far from the observed lowest negative-parity $\Lambda$ resonance states, thus we may not make any matching for these four negative-parity $\Lambda$ resonances in the pure 3q picture.

### 3. Pentaquark components in negative-parity $\Lambda$ resonances

As shown in the previous section, most $\Lambda$ resonances are reasonably described in the three-quark picture, except for the two pairs of the lowest negative-parity $\Lambda$ resonances. The flavor-singlet and flavor-octet $\Lambda$ states may be mixed to give the mass of $\Lambda(1405)$ and other hyperon resonances, as shown in Refs. [15,49,56]. In the present model, however, the masses of the $\Lambda_1$ singlet-flavor state and $\Lambda_8$ octet-flavor state are so close that one

| $J^P$ | $q^3s\bar{q}$ configurations | $(S^q\bar{s}, S, S)$ | $M(q^3s\bar{q})$ |
|-------|-----------------------------|---------------------|----------------|
| $^1_2^-$ | $\Psi_{[211]}c[31]f_{p,s}[31]f_{s}[4]s_{s}(q^3s\bar{q})$ | (2,1/2,5/2) | 2408 |
| $^3_2^-$ | $\Psi_{[211]}c[31]f_{p,s}[31]f_{s}[4]s_{s}(q^3s\bar{q})$ | (1,1/2,3/2) | 2392 |
| | $\Psi_{[211]}c[31]f_{p,s}[31]f_{s}[4]s_{s}(q^3s\bar{q})$ | (2,1/2,3/2) | 1966 |
| | $\Psi_{[211]}c[31]f_{p,s}[31]f_{s}[3]s_{s}(q^3s\bar{q})$ | (1,1/2,3/2) | 2407 |
| | $\Psi_{[211]}c[31]f_{p,s}[31]f_{s}[3]s_{s}(q^3s\bar{q})$ | (1,1/2,3/2) | 2116 |
| | $\Psi_{[211]}c[31]f_{p,s}[22]f_{s}[31]s_{s}(q^3s\bar{q})$ | (1,1/2,3/2) | 2229 |
| $^5_2^-$ | $\Psi_{[211]}c[31]f_{p,s}[4]s_{s}[31]f_{s}(q^3s\bar{q})$ | (1,1/2,1/2) | 2650 |
| | $\Psi_{[211]}c[31]f_{p,s}[31]f_{s}[4]s_{s}(q^3s\bar{q})$ | (1,1/2,1/2) | 2162 |
| | $\Psi_{[211]}c[31]f_{p,s}[31]f_{s}[22]s_{s}(q^3s\bar{q})$ | (0,1/2,1/2) | 2314 |
| | $\Psi_{[211]}c[31]f_{p,s}[21]f_{s}[31]s_{s}(q^3s\bar{q})$ | (1,1/2,1/2) | 1742 |
| | $\Psi_{[211]}c[31]f_{p,s}[21]f_{s}[22]s_{s}(q^3s\bar{q})$ | (0,1/2,1/2) | 2052 |
| | $\Psi_{[211]}c[31]f_{p,s}[22]f_{s}[31]s_{s}(q^3s\bar{q})$ | (1,1/2,1/2) | 1894 |
may make the mixture unreasonable for deriving the mass of the lowest negative-parity Λ resonances. In this work, we assume that the $l = 1$ negative-parity Λ states may mix with the $q^3 s \bar{q}$ ground pentaquark states which are also of negative-parity.

The $q^3 s \bar{q}$ pentaquarks are studied in the Hamiltonian in equation (1), where we have considered the couplings among different configurations due to the one-gloun-exchange (OGE) hyperfine interaction. Listed in table 3 are the theoretical masses of $q^3 s \bar{q}$ ground state pentaquarks. There are one 5/2, five 3/2 and six 1/2 states in the spectra. As shown in Ref. [34], the color and total spin-flavor parts of pentaquark states are always in the [211]$_C$ and [31]$_{SF}$ configurations. The $q^3 s$ [31] flavor configurations can be [4], [31] and [211], where the two possible Weyl tableaux $\begin{array}{cc}
u & u \\ s & d \end{array}$ and $\begin{array}{cc}
u & u \\ s & d \end{array}$ for the $q^3 s$ [31] flavor configuration correspond respectively to isospin $I = 3/2$ and $I = 1/2$ $q^3 s$ components while the flavor configuration [4] leads to isospin $I = 1$ and 2 pentaquark states. We focus only on isospin $I = 0$ pentaquark states which may mix with negative-parity $q^2 s$ states.

Excluding the spin $\frac{5}{2}^-$ state, we have only four spin $\frac{3}{2}^-$ and five $\frac{1}{2}^-$ $q^3 s \bar{q}$ ground pentaquark states which may mix up with the two pairs of the low $p$-wave $q^2 s$ states shown in the lower block of table 2. We mix the two $q^2 s$ three-quark states and one $q^3 s \bar{q}$ pentaquark state,

$$
\begin{pmatrix}
\psi_1 \\
\psi_2 \\
\psi_3
\end{pmatrix}
= U
\begin{pmatrix}
\Lambda_1 \\
\Lambda_8 \\
q^3 s \bar{q}
\end{pmatrix}
$$

(6)

with $U$ takes the form as,

$$
\begin{pmatrix}
\cos(\theta) \cos(\psi) \cos(\phi) + \sin(\psi) \sin(\phi) & \cos(\psi) \sin(\phi) - \cos(\theta) \sin(\psi) \cos(\phi) & \sin(\theta) \cos(\phi) \\
\sin(\psi) \cos(\phi) - \cos(\theta) \cos(\psi) \sin(\phi) & \cos(\theta) \sin(\psi) \sin(\phi) + \cos(\psi) \cos(\phi) & -\sin(\theta) \sin(\phi) \\
-\sin(\theta) \cos(\psi) & \sin(\theta) \sin(\psi) & \cos(\theta)
\end{pmatrix}
$$

where $\psi_1, \psi_2, \psi_3$ are the resulting three negative-parity physical states from low to high energies corresponding to the observed negative-parity Λ resonances, and $\Lambda_1$, $\Lambda_8$ and $q^3 s \bar{q}$ stand for the flavor-singlet and flavor-octet $q^2 s$ states in table 2 and the ground $q^3 s \bar{q}$ pentaquark states in table 3 respectively. The $3 \times 3$ unitary matrix $U$ is parameterized as in Ref. [57], which is made of three rotation matrices of the angles $\theta$, $\psi$ and $\phi$. The mixing angles are in the domain $0 \leq \theta \leq \pi$, $-\pi < \phi \leq \pi$ and $0 \leq \psi < \pi$. Through the unitary transformation of the mass matrices, one get the mass relation

$$
M_{\Lambda_1} + M_{\Lambda_8} + M_{q^3 s \bar{q}} = M_{\psi_1} + M_{\psi_2} + M_{\psi_3}
$$

(7)

where $M_{\Lambda_1}$, $M_{\Lambda_8}$ and $M_{q^3 s \bar{q}}$ are the masses of the $\Lambda_1$ flavor-singlet state, the $\Lambda_8$ flavor-octet state and the ground $q^3 s \bar{q}$ pentaquark state. $M_{\psi_3}$ and the mixing angles are determined by adjusting the $J = 1/2$ lower states $\psi_1$ and $\psi_2$ respectively to $\Lambda(1405)$ and $\Lambda(1670)$, and the $J = 3/2$ lower states $\psi_1$ and $\psi_2$ to $\Lambda(1520)$ and $\Lambda(1690)$. Listed in
Table 4. The three-state mixtures of $q^2 s$ singlet $\Lambda_1^*$, octet $\Lambda_8$ and ground $q^3 s \bar{q}$ pentaquark states. The $\Lambda_1^*$ states, $\Lambda_8$ states for spin $1/2^-$ and $3/2^-$ take the theoretical values from table 3. All the possible $1/2^-$ and $3/2^-$ pentaquark states and masses (in MeV) are from table 4.

| $\psi_1$ State | $\psi_2$ State | $\psi_3$ State | $J^P$ | $\Lambda_1$ State | $\Lambda_8$ State | $\theta$ | $\phi$ | $\gamma$ | $q^3 s \bar{q}$ states |
|----------------|----------------|----------------|------|-------------------|-------------------|--------|-------|--------|----------------------|
| 1405           | 1670 (Λ(1670))| 2248           | 1/2- | 1582              | 1579              | 157.3° | 135.0° | 90.4°  | 2162                  |
| 1405           | 1670 (Λ(1670))| 2400           | 1/2- | 1582              | 1579              | 159.9° | 135.0° | 90.3°  | 2314                  |
| 1405           | 1670 (Λ(1670))| 1828 (Λ(1800))| 1/2- | 1582              | 1579              | 132.8° | 133.0° | 94.4°  | 1742                  |
| 1405           | 1670 (Λ(1670))| 2138           | 1/2- | 1582              | 1579              | 154.6° | 134.9° | 90.6°  | 2052                  |
| 1405           | 1670 (Λ(1670))| 1980           | 1/2- | 1582              | 1579              | 148.2° | 134.9° | 90.6°  | 1894                  |
| 1510           | 1660 (Λ(1690))| 1977 (Λ(2050))| 1/2- | 1590              | 1591              | 169.3° | 136.6° | 88.0°  | 1966                  |
| 1510           | 1660 (Λ(1690))| 2418           | 1/2- | 1590              | 1591              | 173.1° | 137.0° | 87.6°  | 2407                  |
| 1510           | 1660 (Λ(1690))| 2127           | 1/2- | 1590              | 1591              | 171.2° | 137.3° | 87.3°  | 2116                  |
| 1510           | 1660 (Λ(1690))| 2240           | 1/2- | 1590              | 1591              | 172.1° | 137.6° | 87.0°  | 2229                  |

Table 4 are all possible three-state mixtures of the ground $q^3 s \bar{q}$ pentaquark states with $J^P = \frac{1}{2}^-$ and $\frac{3}{2}^-$ in table 3 with the $q^2 s$ singlet $\Lambda_1$ and octet $\Lambda_8$ pairs. And the mixing matrices in equation (8) show the composition of the physical states in terms of two $q^2 s$ states and the lowest pentaquark state. Since it is impossible to derive $M_{\psi_1} = 1520$ MeV and $M_{\psi_2} = 1690$ MeV in the present model, we let 1510 MeV and 1660 MeV for $\Lambda(1520)$ and $\Lambda(1690)$ respectively.

4. Discussion and Summary

In this work, the masses of low-lying $q^2 s$ states and ground $q^3 s \bar{q}$ states are evaluated. Most positive-parity $\Lambda$ resonances and some negative-parity resonances are well reproduced in the three-quark picture. A tentative interpretation of the low-lying negative-parity $\Lambda$ resonances $\Lambda(1405)$, $\Lambda(1520)$, $\Lambda(1670)$ and $\Lambda(1690)$ is proposed, that is, the resonances might be the mixture of the $p$-wave $q^2 s$ states and ground $q^3 s \bar{q}$ pentaquark states. The theoretical results in equation (8) have shown that $\Lambda(1405)1/2^−$ and $\Lambda(1520)3/2^−$ are mainly $q^2 s$ states, but $\Lambda(1670)1/2^−$ and $\Lambda(1690)3/2^−$ contain considerable $q^3 s \bar{q}$ pentaquark contributions. Except for the $\Lambda(1380)1/2^−$ resonance which can not be interpreted either as a three-quark or the mixture of three-quark and pentaquark states, we have reproduced the whole $\Lambda$ baryon spectrum for $N \leq 2$ bands.

As shown in table 4, the lowest $J = 1/2$ and $J = 3/2$ $M_{\psi_3}$ states are close to the $\Lambda(1800)1/2^−$ and $\Lambda(2050)\frac{3}{2}^−$ resonance, respectively. The 3-star $\Lambda(1800)1/2^−$ resonance of $M = 1800 \pm 50$ MeV and $\Gamma = 200 \pm 50$ MeV is tentatively assigned in the $q^2 s\Lambda(70, 4^8s, 1, 1^-)$ multiplet in Section 2. However, the much larger BW width of the $\Lambda(1800)1/2^−$ resonance than the spin-orbit pair state $\Lambda(1830)\frac{5}{2}^−$ and the fact that Osaka group [9][10] and JPAC [11] analysis found no evidence of this state make it debatable that the $\Lambda(1800)1/2^−$ resonance is a pure three-quark state. One may make
A possible interpretation of $\Lambda$ baryon spectrum with pentaquark components

A bold guess that the $\Lambda(1800)^{1-}_{\frac{3}{2}}$ may be composed of two resonances, one is the $q^2s$ $\Lambda(70, ^4S_1, 1, 1^-)$ state and another the highest state of the three-state mixture of two lowest negative-parity $q^2s$ states and one ground $q^3s\bar{q}$ pentaquark state.

$$
\begin{pmatrix}
\Lambda(1405) \\
\Lambda(1670) \\
\Lambda(1800) \\
\Lambda(1520) \\
\Lambda(1690) \\
\Lambda(2050)
\end{pmatrix} =
\begin{pmatrix}
0.69 & -0.72 & 0.06 \\
-0.52 & -0.44 & 0.73 \\
-0.50 & -0.54 & -0.68 \\
0.71 & -0.70 & -0.01 \\
-0.69 & -0.70 & 0.19 \\
-0.14 & -0.13 & -0.98
\end{pmatrix}
\begin{pmatrix}
\Lambda_1^s \\
\Lambda_8 \\
q^3s\bar{q}
\end{pmatrix}
\tag{8}
$$

The $\Lambda(2050)^{3-}_{\frac{3}{2}}$ resonance of $M = 2056 \pm 22$ MeV and $\Gamma = 493 \pm 61$ MeV was first reported by the KSU group [8], and the bump was confirmed by JPAC analysis [11] in 2016, with a smaller width $\Gamma = 269 \pm 35$ MeV. The fact that the $P_c(4450)$ state was turned out to be two much narrower $P_c$ states in the more precise LHCb experiment [58] has inspired one to guess that there may be more resonance states in the pole mass region of $\Lambda(2050)^{3-}_{\frac{3}{2}}$ since the width of this resonance is much larger than all other $\Lambda$ resonances in PDG.

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Appendix A. Explicit $q^2s$ $\Lambda$ wave functions

In this Appendix the $q^2s$ color-orbital-spin-flavor wave functions with the principal quantum number $N \leq 2$ are listed in table A1 where $\chi_i$, $\Phi_j$, and $\phi^{N\prime}_{L'M'y}$ are the spin, flavor, and spatial wave functions, respectively. The $SU(3)_F$ decuplet states are excluded since $\Sigma$ and $\Sigma^*$ resonances are not discussed in the present work.

The explicit flavor wave functions of flavor-singlet $\Lambda_1$ and flavor-octet $\Lambda_8$ are shown below:

$$
\Phi_{\Lambda_1} = \frac{1}{\sqrt{6}}(uds - dus + sud - usd + dsu - sdu),
$$
$$
\Phi_{\Lambda_8} = \frac{1}{2}(sud + usd - dsu - sdu),
$$
$$
\Phi_{\rho} = \frac{1}{\sqrt{12}}(2uds - 2dus + sdu + usd - dsu - sdu)
$$
A possible interpretation of $\Lambda$ baryon spectrum with pentaquark components

Table A1. Explicit $q^2s$ color-orbital-spin-flavor wave functions for isospin $I=0$. All the color wave function is $\psi_{[111]}^c$ singlet.

| $N$ | Representations | $l^P$ | $SU(6)_{SF}$ $SU(3)_{F}$ singlet | $SU(3)_{F}$ octet |
|-----|-----------------|-------|----------------------------------|-------------------|
| 0   | 56              | 0$^+$ | $J^P = \frac{1}{2}^+$            | $\frac{1}{\sqrt{2}}\phi_{100}^A(\Phi_{\Lambda}\chi_{\Lambda} + \Phi_{\rho}\chi_{\rho})$ |
| 1   | 70              | 1$^-$ | $J^P = \frac{1}{2}^-$, $\frac{3}{2}^-$ | $\frac{1}{\sqrt{2}}(\chi_{\rho}\phi_{1m}\chi_{\lambda} - \chi_{\lambda}\phi_{1m}\chi_{\rho})\Phi_A$ |
|     |                 |       |                                  | $\frac{1}{\sqrt{2}}(\phi_{1m}(\Phi_{\Lambda}\chi_{\rho} + \Phi_{\rho}\chi_{\lambda}) + \phi_{1m}(\Phi_{\rho}\chi_{\lambda} - \Phi_{\lambda}\chi_{\rho}))$ |
|     |                 |       |                                  | $J^P = \frac{1}{2}^-$, $\frac{3}{2}^-$, $\frac{5}{2}^-$ |
|     |                 |       |                                  | $\frac{1}{\sqrt{2}}\chi_{S}(\phi_{1m}\Phi_{\Lambda} + \phi_{1m}\Phi_{\rho})$ |
| 2   | 56              | 0$^+$ | $J^P = \frac{1}{2}^+$            | $\frac{1}{\sqrt{2}}\phi_{000}^A(\Phi_{\Lambda}\chi_{\Lambda} + \Phi_{\rho}\chi_{\rho})$ |
| 2   | 20              | 1$^+$ | $J^P = \frac{1}{2}^+, \frac{3}{2}^+, \frac{5}{2}^+$ | $\frac{1}{\sqrt{2}}(\chi_{\rho}\phi_{2m}\chi_{\lambda} - \chi_{\lambda}\phi_{2m}\chi_{\rho})\Phi_A$ |
|     |                 |       |                                  | $\phi_{2m}(\Phi_{\Lambda}\chi_{\rho} + \Phi_{\rho}\chi_{\lambda})$ |
| 2   | 56              | 2$^+$ | $J^P = \frac{3}{2}^+, \frac{5}{2}^+$ | $\frac{1}{\sqrt{2}}\phi_{2m}^A(\Phi_{\rho}\chi_{\rho} + \Phi_{\lambda}\chi_{\lambda})$ |
| 2   | 20              | 2$^+$ | $J^P = \frac{3}{2}^+, \frac{5}{2}^+$ | $\frac{1}{\sqrt{2}}(\chi_{\rho}\phi_{2m}\chi_{\lambda} - \chi_{\lambda}\phi_{2m}\chi_{\rho})\Phi_A$ |
|     |                 |       |                                  | $\frac{1}{\sqrt{2}}(\phi_{2m}(\Phi_{\Lambda}\chi_{\rho} + \Phi_{\rho}\chi_{\lambda}) + \phi_{2m}(\Phi_{\rho}\chi_{\lambda} - \Phi_{\lambda}\chi_{\rho}))$ |
|     |                 |       |                                  | $J^P = \frac{1}{2}^+, \frac{3}{2}^+, \frac{5}{2}^+, \frac{7}{2}^+$ |
|     |                 |       |                                  | $\frac{1}{\sqrt{2}}\chi_{S}(\phi_{2m}\Phi_{\Lambda} + \phi_{2m}\Phi_{\rho})$ |

(A.1)

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