Sensitivity of CPT Tests with Neutral Mesons

V. Alan Kostelecký

Physics Department, Indiana University, Bloomington, IN 47405, U.S.A.

(preprint IUHET 377, November 1997, published in Phys. Rev. Lett. 80, 1818 (1998))

The sensitivity of experiments with neutral mesons to possible indirect CPT violation is examined. It is shown that experiments conventionally regarded as equivalent can have CPT reaches differing by orders of magnitude within the framework of a minimal CPT- and Lorentz-violating extension of the standard model.

Neutral-meson interferometry is a powerful tool for investigating the discrete symmetry CPT. This product of charge conjugation C, parity reflection P, and time reversal T is known to be an invariance of local relativistic quantum field theories of point particles in flat spacetime [1]. Among the various tests of CPT [2], the sharpest published bounds are obtained with the neutral kaon system. As an example, the CPT figure of merit $r_K \equiv |m_K - m_{\bar{K}}|/m_K$ has recently been constrained to $r_K < 1.3 \times 10^{-18}$ at the 90% confidence level by the experiment E773 at Fermilab [3].

In neutral-meson interferometry, bounds on CPT violation are extracted using a phenomenological description of the meson time evolution. Denote by $P^0$ any of the possible neutral mesons $K^0$, $D^0$, $B^0_d$, $B_s^0$ produced using the strong interaction, and combine the Schrödinger wave functions of $P^0$ and its opposite-flavor antiparticle $\bar{P}^0$ into a two-component object $\Psi$. Then, the time evolution of $\Psi$ is governed by a $2 \times 2$ effective hamiltonian $\Lambda$ through the equation $i\partial_t \Psi = \Lambda \Psi$. Off-diagonal components of $\Lambda$ drive flavor oscillations between $P^0$ and $\bar{P}^0$.

Two possible kinds of CP violation can be studied within this formalism. The one usually considered involves T violation with CPT invariance and is controlled by a parameter $\epsilon_P$. In the kaon system, for example, a nonzero value of $\epsilon_K$ is well established [2]. The other involves CPT violation with T invariance. It is controlled by a complex parameter $\delta_P \approx \Delta \Lambda/\Delta \lambda$, where $\Delta \lambda \equiv (\Lambda_{11} - \Lambda_{22})/2$ is half the diagonal-element difference in $\Lambda$ and $\Delta \lambda$ is the eigenvalue difference. In the kaon system, a bound on $r_K$ constrains $\delta_K$.

The parameter $\delta_P$ can be bounded experimentally whether or not a nonzero value has any theoretical basis. However, a framework for CPT violation based on conventional quantum field theory does exist. The idea is that apparent low-energy CPT and Lorentz breaking might arise spontaneously within a more fundamental theory that is otherwise CPT and Lorentz invariant [4]. Any apparent breaking at the level of the standard model would then merely reflect a feature of the vacuum rather than a fundamental property of the theory [4]. Potentially observable effects within this general framework have been studied in neutral-meson systems [5] [6], in QED [7], and in baryogenesis [8].

Apparent Lorentz and CPT violation of this type can be incorporated in a general extension of the minimal SU(3) × SU(2) × U(1) standard model that preserves gauge invariance and renormalizability [12]. In the underlying theory, the spontaneous breaking generates constant background expectation values as usual, but the fields involved are Lorentz tensors instead of Higgs scalars. In the standard-model extension, nonzero expectation values appear as coupling constants with Lorentz indices. For example, an expectation value $a_\mu$ would allow a CPT- and Lorentz-violating term $-a_\mu \bar{\psi} \gamma^\mu \psi$ for a fermion $\psi$.

The present work investigates the sensitivity of neutral-meson experiments to indirect CPT-violating effects produced in the standard-model extension [12]. Most of the theoretical considerations apply to any of the four neutral-meson systems. For definiteness, in the discussion of CPT tests some emphasis is placed on the E773 experiment mentioned above. The results are of immediate interest for CPT tests because at present the framework of the standard-model extension seems to be the only available consistent theoretical basis for a nonzero $\delta_P$ within conventional quantum field theory [13].

The first step is to obtain an explicit expression for $\delta_P$ within the standard-model extension. A key point is that the parameter $\delta_P$ must be C violating but P and T preserving. This is because the strong-interaction states $P^0$, $\bar{P}^0$ are eigenvectors of parity with the same eigenvalue, so the linear combinations forming the physical eigenstates $P_S$, $P_L$ of $\Lambda$ are parity eigenstates too. Parity is therefore preserved during the time evolution of a neutral-meson state, so any CP violation appearing in $\Lambda$ is really C violation with P invariance.

In the lagrangian $\mathcal{L}$ for the standard-model extension, the parameters controlling the Lorentz and CPT violation are assumed suppressed by the (small) dimensionless ratio of the relevant light energy scale to the Planck scale $\Lambda_{\text{Pl}}$. Thus, only contributions linear in these parameters could produce observable CPT violation in experiments with neutral mesons. Also, since $\Delta \lambda$ is flavor-diagonal, any term in $\mathcal{L}$ with both CPT breaking and flavor changing would affect $\delta_P$ at most as the square of a small parameter and hence can be disregarded.

Remarkably, an inspection shows that only one type of term in the standard-model extension is flavor diagonal while violating C but preserving P and T. For each quark field $q$ it has the form $-a_0^q \bar{\psi} \gamma^0 \psi$, where $a_0^q$ is the zeroth component of a background expectation value $a_0^q$ that
varies with the flavor $q$ (cf. the usual Yukawa couplings). Thus, at first order in perturbation theory the diagonal elements of $A$ depend only on $a_q^0$.

This result is of particular interest both because no other CPT- and Lorentz-violating expectation values appear and because to date no other experiments sensitive to $a_q^0$ have been identified. Note that higher-order corrections from conventional interactions do not change the result. Pure strong or electromagnetic corrections preserve $C$, $P$, $T$ and therefore at most can modify the magnitude of the contributions proportional to $a_q^0$. Any weak-interaction corrections violating $C$ and $P$ while preserving net flavor would be suppressed by several orders of magnitude. Also, possible CPT violation in the gauge sector is expected to be small and in any case must appear as a higher-order correction here.

As a result of CPT violation, the parameter $a_q^0$ for the valence quark $q_1$ in the $P^0$ meson contributes with opposite sign to $a_q^0$ for the antiquark $\overline{q}_2$. This means that $\Delta L \times \Delta a_0 \equiv (a_q^0 - a_q^0)$ at leading order. The proportionality constant can be found in perturbation theory and is approximately one, so from the definition of $\delta_P$ one finds

$$\delta_P \approx i \sin \phi \exp(i\phi) \Delta a_0 / \Delta m,$$

where $\Delta m \equiv m_L - m_S$ and $\Delta \gamma \equiv \gamma_S - \gamma_L$ are the mass and decay-rate differences, respectively, between $P_L$ and $P_S$, and where $\phi \equiv \tan^{-1}(2\Delta m/\Delta \gamma)$. A subscript $P$ is understood on all these quantities.

The result (1), valid at leading order in all Lorentz-violating parameters in the standard-model extension, holds for $P^0$ mesons at rest in the (oriented) inertial frame in which $\Delta a_0$ is specified. In considering the effects of rotations and boosts on this result, one must keep distinct transformations of the observer (laboratory) frame from changes of the momentum or orientation of a particle within a given observer frame. The former are conventional Lorentz transformations, and full covariance is maintained because the background fields are perceived as changed when the observer frame changes. In contrast, changes of the particle momentum or orientation leave unaffected the background values, producing (small) apparent changes in the intrinsic properties of the particle.

Equation (1) shows that the result of a CPT test with mesons at rest in the laboratory is explicitly rotationally invariant. However, no experiments with mesons at rest have been performed. An approximation is provided by the Cornell CLEO experiment, which involves (correlated) $B$ mesons traveling at only about 6% of lightspeed. A measurement of $\delta_B$ in this experiment might therefore approximately bound $\Delta a_0$ via Eq. (1).

Most experiments involve relativistic mesons. This has implications for the CPT reach because the $\Lambda$ formalism is obtained from nonrelativistic quantum mechanics, and so for a boosted meson $\delta_P$ is defined in the comoving frame. To obtain the analogue of Eq. (1) valid for relativistic mesons, one can take advantage of the covariance of the standard-model extension under observer boosts. Suppose a particle at rest in the laboratory frame is described by a lagrangian including the term $-a_q^0 \vec{P} \gamma^\mu q$, as before. Then, another particle with momentum related to the first by an inverse boost $(L^{-1})^\mu_\nu$, is described in the same frame by $-a_q^0 (L^{-1})^\mu_\nu \vec{P} \gamma^\nu q$, since the background value $a_q^0$ is fixed. The lagrangian describing this boosted particle in a comoving observer frame is obtained by performing an observer boost with $L^\mu_\nu$, under which both the background values and the fields transform. The result is a term $-a_q^0 \vec{P} \gamma^\mu q$, where $a_q^0 \equiv L^\mu_\nu a_q^0$. Thus, in the comoving frame in which the $\Lambda$ formalism is valid and $\delta_P$ is defined, the CPT-violating physics is controlled by $a_q^0$ instead of $a_q^0$. For a 4-velocity $\beta^0 \equiv \gamma(1, \vec{\beta})$, one has $a^0_q = \beta^0 a^0_q$ and hence

$$\delta_P \approx i \sin \phi \exp(i\phi) \Delta a_0 / \Delta m, \quad (2)$$

where $\Delta a \equiv \vec{a}^0 - \vec{a}^0$.

Since the expressions for $\delta_P$ in Eqs. (1) and (2) have the same phase, the real and imaginary parts of $\delta_P$ are scaled in the same way when a meson is boosted in the laboratory. However, Eq. (2) shows that there is an overall multiplicative factor of $\gamma$ acting to enhance the CPT-violating effect. Moreover, the observed CPT violation for relativistic mesons depends not only on $\Delta a_0$ but also on the angle $\alpha$ between $\vec{\beta}$ and $\Delta a$ and on the magnitudes $\beta$ of $\vec{\beta}$ and $\Delta a$ of $\Delta a$. Since $|\beta \cos \alpha| < 1$, the factor involving $\Delta a$ is always suppressed relative to $\Delta a_0$, although the combination $\Delta a_0 - \beta \cdot \Delta a$ may be larger or smaller than $\Delta a_0$. Note that if spontaneous Lorentz breaking generates only 0-component expectation values as seen in the laboratory frame, then $\Delta a = 0$ for a meson at rest and so the value of $\delta_P$ is enhanced exactly by a factor $\gamma$ for a boosted meson. If instead no pure 0-component expectation values are generated, then $\Delta a_0 = 0$ for a meson at rest, $\delta_P$ depends on $\gamma \beta \Delta a \cos \alpha$, and a meson boost is necessary to observe any effect. Finally, if $\Delta a > \Delta a_0$, then there is a hyperbola in $\beta \cos \alpha$ space along which CPT violation is exactly cancelled and near which significant suppression could reduce the CPT reach of certain experiments.

In an ideal case, increasing the statistics by a factor $N$ would improve the CPT reach by a factor $\sqrt{N}$. However, the variation of $\delta_P$ with $\gamma$ provides an alternative possibility. Suppose for simplicity that data at fixed $\alpha$ are obtained from an experiment in a regime where $\delta_P \propto \gamma$ holds to a good approximation. Then, the CPT reach of the experiment can be doubled either by quadrupling the statistics or by doubling the boost.

One implication is that the bounds on $\delta_P$ reported from different experiments may be inequivalent. Experiments with comparable statistical precision can involve mesons with very different boosts and hence may have
very different CPT sensitivity. In some experiments, the boost factor is large. For example, the $B$ mesons in the proposed LHC-B experiment at CERN are expected to have $\gamma \approx 15$. Similarly, the E773 experiment at Fermilab involves kaons with mean value $\gamma \approx 10^2$. The figure of merit $r_K$ bounded in this experiment is proportional to $\delta_K$, so in the above scenario the attainable sensitivity to CPT violation is about two orders of magnitude better than the limit on $r_K$ would naively suggest.

Many experiments with neutral mesons involve a distribution of momenta. If CPT violation were indeed detectable in such cases, then the predicted momentum dependence of $\delta_P$ would provide a striking signal. The presence of a momentum spectrum also has implications for the extraction of a CPT bound. Consider first an experimental asymmetry $A$ that is directly sensitive to a linear combination of the real and imaginary parts of $\delta_P$. An example is the fully integrated asymmetry $A_L$ for uncorrelated neutral-$B$ mesons described in Ref. [9]. In a regime where $\delta_P \propto \gamma$ holds and $\alpha$ is constant, $\delta_P$ and hence $A$ both scale linearly with $\gamma$. The mean value $\bar{A}$ of the asymmetry is then proportional to $\bar{\gamma}$, determined by the form of the normalized meson-momentum spectrum. For example, $\gamma \propto p$ approximately for large momentum magnitudes $p$, so in this case the effective meson momentum determining the CPT reach is just the mean momentum $\bar{p}$ of the distribution [10].

Most experiments constrain $\delta_P$ either through fits to time-dependent asymmetries or from the measurement of other quantities. For example, the E773 experiment measures $\Delta m$, the $K_S$ lifetime $\tau_S$, and the usual two phases $\phi_{+-} \propto \Delta m$. $\Delta \phi \equiv \phi_{00} - \phi_{+-}$ related to ratios of amplitudes for $2\pi$ decays: $A(K_L \to \pi^+ \pi^-)/A(K_S \to \pi^+ \pi^-) = |\eta_{+-}| \exp(i\phi_{+-})$, and similarly for $2\pi^0$ decays. Defining $\chi \equiv \phi_{+-} - \Delta \phi/3$, a bound on $|\Delta m|$ can be extracted using the established value $|\eta_{+-}|$ of $\eta_{+-}$, along with a bound on $r_K \approx 2\Delta m|m_K sin \phi|$. It can be shown that for small CPT violation a scaling of $\delta_P$ by $\gamma_2/\gamma_1$ occurring when the meson boost is changed from $\gamma_1$ to $\gamma_2$ would arise largely from a corresponding additive change to $\phi_{+-}$ by an amount $[(\gamma_2/\gamma_1) - 1]|\chi|$, while the other measured quantities remain essentially unchanged. In this case, CPT violation would manifest itself as a momentum dependence in the observed value of $\phi_{+-}$.

Since precision constraints on $\delta_P$ require high statistics, experiments are performed over many days. For example, the E773 data were collected over several months [8]. Effects of the Earth’s motion on the meson velocities and orientations must therefore be considered.

The parameter $\Delta a_{\mu}$ is constant in any special-relativistic inertial frame. In a frame in the solar neighborhood, the velocity of a laboratory on the Earth’s surface is nonrelativistic, so any associated effects can be neglected. However, the rotation of the Earth about its axis introduces a time variation in the apparent orienta-

The parameter $\Delta a_{\mu}$ is constant in any special-relativistic inertial frame. In a frame in the solar neighborhood, the velocity of a laboratory on the Earth’s surface is nonrelativistic, so any associated effects can be neglected. However, the rotation of the Earth about its axis introduces a time variation in the apparent orienta-
typically different because they travel in opposite directions. Also, the effect of the Earth’s rotation must again be allowed for in the analysis. In contrast, the boosted case (asymmetric factory) produces a meson-momentum spectrum, so a combination of momentum and angular dependence can govern the CPT-violating effects.

This paper has considered the boost and orientation dependences appearing in the CPT-violating parameter \( \delta_P \) within the context of the Lorentz-violating standard-model extension. A related analysis for the usual CP-violating parameter \( \epsilon_P \) shows that it too could acquire contributions from Lorentz-violating terms in the standard-model extension that involve C and T violation but preserve P (and CPT). However, the effect would require second-order flavor-changing contributions, and hence it is expected to be suppressed relative to the contributions to \( \delta_P \) discussed above. Moreover, unlike the case of \( \delta_P \), conventional contributions to \( \epsilon_P \) from low-energy physics can arise, so any Planck-scale effect may be masked. It therefore appears unlikely that observable boost or orientation dependence for \( \epsilon_P \) would arise in the context of the present framework [21].

This work is supported in part by the Department of Energy under grant number DE-FG02-91ER40661.

[1] The discrete symmetries C, P, T are discussed, for example, in R.G. Sachs, *The Physics of Time Reversal* (University of Chicago Press, Chicago, 1987).

[2] See, for example, R.M. Barnett et al., *Review of Particle Properties*, Phys. Rev. D 54 (1996) 1.

[3] B. Schwingenheuer et al., Phys. Rev. Lett. 74 (1995) 4376; R.A. Briere, Ph.D. thesis, University of Chicago, June, 1995; B. Schwingenheuer, Ph.D. thesis, University of Chicago, June, 1995.

[4] This bound holds for negligible direct CPT violation in decay amplitudes. See Ref. [21] and E. Shabalina, Phys. Lett. B 369 (1996) 335.

[5] For example, this might occur in the context of string theory. See V.A. Kostelecký and S. Samuel, Phys. Rev. Lett. 63 (1989) 224; *ibid.*, 66 (1991) 1811; Phys. Rev. D 39 (1989) 683; *ibid.*, 40 (1989) 1885; V.A. Kostelecký and R. Potting, Nucl. Phys. B 359 (1991) 545; Phys. Lett. B 381 (1996) 89.

[6] An analogy is the behavior of an electron in a crystal, which reflects rotational (Lorentz) breaking although the underlying dynamics is invariant.

[7] V.A. Kostelecký and R. Potting, in D.B. Cline, ed., *Gamma Ray–Neutrino Cosmology and Planck Scale Physics* (World Scientific, Singapore, 1993) hep-th/9211116; Phys. Rev. D 51 (1995) 3923.

[8] D. Colladay and V.A. Kostelecký, Phys. Lett. B 344 (1995) 259; Phys. Rev. D 52 (1995) 6224.

[9] V.A. Kostelecký and R. Van Kooten, Phys. Rev. D 54 (1996) 5585.

[10] OPAL Collaboration, R. Ackerstaff et al., Z. Phys. C 76 (1997) 401; DELPHI Collaboration, M. Feindt et al., preprint DELPHI 97-98 CONF 80 (July 1997).

[11] R. Bluhm, V.A. Kostelecký and N. Russell, Phys. Rev. Lett. 79 (1997) 1432; Phys. Rev. D 57 (1998) 3952.

[12] O. Bertolami et al., Phys. Lett. B 395 (1997) 178.

[13] D. Colladay and V.A. Kostelecký, Phys. Rev. D 55 (1997) 6760; preprint IUHET 359 (1997), Phys. Rev. D, in press [hep-ph/980952].

[14] Direct CPT violation in decay amplitudes is neglected here because it is highly suppressed in the standard-model extension and would be unobservable.

[15] For the neutral-kaon system, it has been suggested that an unconventional quantum mechanics in which the Schrödinger equation is replaced with a density-matrix formalism might generate extra CPT-violating terms. However, the parameter \( \delta_K \) is unaffected. See J. Ellis et al., Phys. Rev. D 53 (1996) 3846.

[16] The appearance of the difference \( \Delta m_0 \) in the physical observable is a general feature. The parameters \( \alpha_i^u \) can be shifted by a constant using a field redefinition, so only their differences are observable. Note also that the velocities and spins of the valence quarks are irrelevant here because their expectations vanish in the wave function for the \( P^\pi \) meson at rest.

[17] Although \( \phi \) and \( \Delta m \) are indirectly affected by the boost, this is at most second order in the small CPT-violating parameters and hence is unobservable. Note also that the appearance of the combination \( \beta \cdot \Delta a \) in Eq. (2) is compatible with the requirement that \( \delta_P \) be C violating but P and T preserving.

[18] Since the CPT reach improves linearly with \( \gamma \) but only as the square root of the statistics, it may be possible to increase the experimental sensitivity by examining only a (high-momentum) subset of the data available. For the simple case of a constant distribution in \( p \) in the regime where \( \delta_P \propto p \), the gain is at most about 10%. For peaked distributions, such as the Malensek-type distribution of the E773 experiment, the loss of statistics at high momenta typically offsets any gain from the \( \gamma \) factor.

[19] The actual E773 experiment extracts \( \phi_{+} \) from an interference pattern involving also a regeneration phase \( \phi_{\rho} \), which is unaffected to leading order by CPT violation. Due to averaging effects, some sensitivity to CPT violation might be lost using a procedure for extracting \( \phi_{+} \) that disregards the momentum dependence, although it is unlikely that the net CPT-violating effect would vanish. Particular care is required in determining \( \Delta b \) because the experimental acceptances differ for \( 2\pi^0 \) and \( \pi^+\pi^- \) decays, and so differences occur in the reconstructed momentum spectra used to extract \( \phi_{00} \) and \( \phi_{+-} \).

[20] The variation is approximately diurnal, with a seasonal drift due to the orbital motion.

[21] Various authors have investigated the possibility that the usual observed CP violation might be boost dependent: J.S. Bell and J.K. Perring, Phys. Rev. Lett. 13 (1964) 348; S.H. Aronson et al., Phys. Rev. D 28 (1983) 495.