Superconducting pairing, and the collective magnetic excitation in the extended 2-dimensional \( t - J \) model

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To investigate the non-Fermi liquid behavior we consider the extended two-dimensional \( t - J \) model which includes additional hopping \( t'' \). In the regime \( t, J \ll t'' \) we were able to solve the model analytically. It has a very rich phase diagram including antiferromagnetic (AF) insulator and AF strange metal with different kinds of pseudospin-singlet superconducting pairings (p,d,g-waves). We also demonstrate a collective triplet excitation with energy below the superconducting gap.

1 Introduction

It is now widely accepted that superconductivity of cuprates is closely related to their unusual magnetic properties, and it is increasingly clear that magnetic pairing is the most realistic mechanism of cuprate superconductivity. However the mechanism of pairing as well as other unusual properties are far from completely understood. The problem has been attacked along several directions. First we have to mention the empirical or semi-empirical approach which allows one to relate different characteristics measured experimentally. This approach is to a large extent based on the Hubbard model. For a review see article [1]. In the low energy limit the Hubbard model can be reduced to the \( t - J \) model. Another approach to cuprates is based on numerical studies of the \( t - J \) model (see review [2]). Our studies are also based on this model. We used the ordered Neel state at zero doping as a starting point to develop the spin-wave theory of pairing [3]. The method we used was not fully satisfactory, since it violated spin-rotational symmetry, nevertheless it allowed us to calculate from first principles all of the most important properties including the critical temperature, the spin-wave pseudogap and the low energy spin triplet excitations [4].

A sharp collective mode with very low energy has been revealed in YBCO in spin polarized inelastic neutron scattering [5, 6, 7]. A number of theoretical explanations have been suggested for this effect [8, 9], all of these are based on the idea that the system is close to AF instability. However all known explanations use some uncontrolled approximations and assumptions.
In the present work we investigate close to half filling regime for the 2D $t-J$ model, where it can be solved analytically without any uncontrolled approximations. It can be done for the region of parameters where long-range AF order is preserved under doping. This is the regime where non Fermi liquid behavior can be studied in detail. We analyze the superconducting pairing in this regime and consider the spin triplet collective excitation. It is demonstrated that close to the point of AF instability energy of this excitation is very small. The excitation exists only at very small momenta. The idea of this work is somewhat similar to that of our previous paper [9], however here we investigate different regime.

2 Hamiltonian and single hole dispersion

Let us consider a $t-J-J''-V$ model defined by the Hamiltonian

$$H = -t \sum_{\langle ij \rangle \sigma} c_{i\sigma}^\dagger c_{j\sigma} - t'' \sum_{\langle ij \rangle \sigma} c_{i\sigma}^\dagger c_{j\sigma} + \sum_{\langle ij \rangle} J \left( S_i S_j - \frac{1}{4} n_i n_j \right) + V n_i n_j .$$  \hspace{1cm} (1)

$c_{i\sigma}^\dagger$ is the creation operator of an electron with spin $\sigma$ $(\sigma = \uparrow, \downarrow)$ at site $i$ of the two-dimensional square lattice. The $c_{i\sigma}^\dagger$ operators act in the Hilbert space with no double electron occupancy. The $\langle ij \rangle$ represents nearest neighbor sites, and $\langle ij \rangle$ represents next next nearest sites. The spin operator is $S_i = \frac{1}{2} \sum_{\alpha, \beta} c_{i\alpha}^\dagger \sigma_{\alpha\beta} c_{i\beta}$, and the number density operator is $n_i = \sum_{\sigma} c_{i\sigma}^\dagger c_{i\sigma}$. In addition to the minimal $t-J$ model (see Ref. [2]) we have introduced additional next next nearest hopping $t''$, and Coulomb repulsion $V$ at nearest sites. Note that we do not introduce next nearest neighbor hopping $t'$ (diagonal) because we do not need it for the purposes of this study.

In the paper [9] we analyzed the model defined by the Hamiltonian (1) in the limit $t, t'' \ll J$. In the present work we consider limit

$$t, J \ll t'' .$$  \hspace{1cm} (2)

It is well known that the $t-J$ model at half filling describes the Mott insulator. It is equivalent to the 2D Heisenberg model, and the ground state of the model has long range AF order. At small doping the holes are concentrated near the points $(\pm \pi/2, \pm \pi/2)$ where single hole dispersion has minima. In leading approximation the dispersion is of the form (we take energy at the minimum as a reference point)

$$\epsilon_k = \beta \left( \gamma_k^2 + (\gamma_k^-)^2 \right) ,$$  \hspace{1cm} (3)

$$\beta \approx 0.8 \times 8t'' = 6.4t'' ,$$

$\gamma_k = \frac{1}{2}(\cos k_x + \cos k_y)$, $\gamma_k^- = \frac{1}{2}(\cos k_x - \cos k_y)$. Calculation of the dispersion (3) is straightforward because it is due to hopping within the same magnetic sublattice. Coefficient 0.8 appears because of spin quantum fluctuations: $0.8 = 1 - 0.2$, where 0.2 is the spin flip probability in the Heisenberg model. Along with quasimomentum the hole in the AF background has an additional quantum number: pseudospin. We denote the hole creation operator by $h_{k\sigma}^\dagger$, where $\sigma = \pm 1/2$ is pseudospin. The relation between pseudospin and usual spin is discussed in the paper [10].
We will consider the case of very small doping, \( \delta \ll 1 \), with respect to half filling (total filling is \( 1 - \delta \)). In this case all holes are concentrated in small pockets around the points \( \mathbf{k}_0 = (\pm \pi/2, \pm \pi/2) \). Single hole dispersion (3) can be expanded near each of these points

\[
e_k = \frac{1}{2} \beta \mathbf{p}^2,
\]

where \( \mathbf{p} = (p_1, p_2) \) is deviation from the center of the pocket: \( \mathbf{p} = \mathbf{k} - \mathbf{k}_0 \), \( p_1 \) is orthogonal to the face of the magnetic Brillouin zone, and \( p_2 \) is parallel to the face (see Fig. 1). The Fermi energy and Fermi momentum for the holes equal \( \epsilon_F \approx \frac{1}{2} \pi \beta \delta \), \( p_F \approx (\pi \delta)^{1/2} \).

![Figure 1: Magnetic Brillouin zone and single hole dispersion](image)

3 Hole-spin-wave interaction and instability of the Neel state

Spin-wave excitations on an AF background are usual spin waves with dispersion \( \omega_q = 2J \sqrt{1 - \gamma_q^2} \approx \sqrt{2} J q \), at \( q << 1 \), see Ref. [11] for review. The hole-spin-wave interaction is well known (see, e.g. Ref.[12])

\[
H_{h,sw} = \sum_{\mathbf{k}, \mathbf{q}} g_{\mathbf{k}, \mathbf{q}} \left( h^\dagger_{\mathbf{k}+\mathbf{q}, \sigma} h^\dagger_{\mathbf{k}, \sigma} + h^\dagger_{\mathbf{k}, \sigma} h^\dagger_{\mathbf{k}+\mathbf{q}, \sigma} \beta_{\mathbf{q}} + \text{H.c.} \right),
\]

where \( h^\dagger_{\mathbf{k} \sigma} = c_{\mathbf{k}, -\sigma} \) is the hole creation operator with pseudospin \( \sigma \), \( \alpha^\dagger_{\mathbf{q}} \) and \( \beta^\dagger_{\mathbf{q}} \) are the spin wave creation operators for \( S_z = \pm 1 \), and \( U_q = \sqrt{\frac{2}{\omega_q}} \) and

\( V_q = -\text{sign}(\gamma_q) \sqrt{\frac{2}{\omega_q}} \) are parameters of the Bogoliubov transformation diagonalizing the spin-wave Hamiltonian, see Ref. [11]. Virtual spin wave emission gives a correction to the hole dispersion, see Fig.2. However this correction is small \( \delta \epsilon \sim t^2/t'' \) and therefore can be neglected compared with (3).
To describe renormalization of the spin wave under doping, it is convenient to introduce the set of Green’s functions:

\[
D_{\alpha\alpha}(t, q) = -i\langle T[\alpha_q(t)\alpha^\dagger_q(0)] \rangle,
\]

\[
D_{\alpha\beta}(t, q) = -i\langle T[\alpha_q(t)\beta_{-q}(0)] \rangle,
\]

\[
D_{\beta\alpha}(t, q) = -i\langle T[\beta^\dagger_{-q}(t)\alpha^\dagger_q(0)] \rangle,
\]

\[
D_{\beta\beta}(t, q) = -i\langle T[\beta^\dagger_{-q}(t)\beta_{-q}(0)] \rangle.
\]

In the present work we consider only the long-range dynamics: \( q \sim k \sim p_F \ll 1 \). In this limit all possible polarization operators coincide:

\[
P_{\alpha\alpha}(\omega, q) = P_{\alpha\beta}(\omega, q) = P_{\beta\alpha}(\omega, q) = P_{\beta\beta}(\omega, q) = \Pi(\omega, q),
\]

where \( \Pi(\omega, q) \) is given by the diagram presented at Fig. 3.

For stability of the system the condition (Stoner criterion)

\[
\omega_q + 2\Pi(0, q) > 0
\]

must be fulfilled. Otherwise the Green’s functions would possess poles with imaginary \( \omega \). Considering holes as a “normal Fermi liquid” one can easily calculate the polarization operator at \( q \ll p_F \): \( \Pi(0, q) \approx -4t^2\sqrt{2q/\pi\beta} \). Ref. [14]. Relatively weak pairing, which we consider below, does not influence this result. Then the condition of stability can be rewritten as

\[
\beta = 6.4t'' > \frac{8t^2}{\pi J}.
\]

To provide stability of the AF order we have to choose

\[
t'' > t_c'' \approx 0.4t^2 / J.
\]

If \( t < J \) or \( t \sim J \) the stability condition is automatically fulfilled since in the present work we consider \( t, J \ll t'' \). However at \( t \gg J \) one can violate the condition (8). In this case we will assume that \( t'' > t_c'' \). If \( t'' \) is close to \( t_c'' \) it is convenient to introduce the parameter \( \eta \)

\[
\eta^2 = 1 - \frac{8t^2}{\pi J\beta} = \frac{(t'' - t_c'')/t''}{t''}
\]

as a measure of this closeness. The criterion (6) is proportional to this parameter.
4 Spin-singlet p-wave pairing caused by the short-range attraction

It is not convenient to consider the superconducting pairing in the magnetic Brillouin zone with four half-pockets (see Fig. 1). Because of this we translate the picture to the shifted zone with two whole pockets, Fig. 4. We stress that this is question of convenience only, the representations are absolutely equivalent because of the translational invariance.

There are two mechanisms for the superconducting pairing: short-range attraction and long-range attraction. First we consider the short-range effect. Attraction between holes at nearest sites (short-range) is due to the reduction in number of missing AF links. The value of this attraction immediately follows from eq.(1)

\[ U = J \langle S_i S_j - \frac{1}{4} + V \rangle \approx -0.58 J + V. \]  

(11)

Strong enough Coulomb repulsion \((V > 0.58J)\) kills this mechanism. In the momentum representation the interaction \((11)\) can be rewritten as

\[ H_U = 8U \sum_{k_1,k_2,k_3,k_4} \gamma_{k_1-k_3} h_{k_4}^\dagger h_{k_2} h_{k_1}^\dagger \delta_{k_1+k_2,k_3+k_4}. \]  

(12)

For scattering inside a hole pocket the interaction is practically momentum independent because \(k_1 \approx k_2 \approx k_3 \approx k_4 \approx (\pi/2, \pi/2)\), and hence \(\gamma_{k_1-k_3} \approx 1\). Such interaction gives “s-wave pairing” with the gap without nodes at the Fermi surface. The value of the superconducting gap one can easily find using the results of papers [16, 17]. This gives

\[ \Delta = Ct'' \sqrt{\delta e^{\pi\beta/4U}} = Ct'' \sqrt{\delta e^{-5t''/(0.58J-V)}}, \]  

(13)

where \(C \sim 10\) is some dimensionless constant. The solution is valid only if \(V < 0.58J\), for stronger Coulomb repulsion the pairing disappears. It is important to stress the peculiar
symmetry properties of the above pairing. This peculiarity comes from the presence of long-range AF order. As we already mentioned, the gap has no nodes at the Fermi surface and from this point of view it is “s-wave pairing”. However we remind that we have considered the pairing in the shifted zone and in this zone it is not easy to classify the states by parity. For well defined parity we have to return to the magnetic Brillouin zone, so we have to translate the outside parts of the Fermi surface by the inverse vector of the magnetic lattice $G = (\pi, \pi)$, see Fig. 5.

Figure 5: Translation from the shifted zone to the magnetic Brillouin zone. The superconducting gap has no nodes at the Fermi surface. The gap changes sign under this translation.

The point is that under such translation the superconducting gap changes the sign as it is shown at Fig. 5. This property follows from the fact that the coefficient in the interaction (12) changes sign under such translation: $\gamma_{k_1 - k_3 + G} = -\gamma_{k_1 - k_3}$ (for details see paper [18]).

Thus in reality we have negative parity pairing which is usually called p-wave. The above consideration was relevant to the hole pocket centered at $(\pi/2, \pi/2)$. Similar construction is valid for another pocket centered at $(\pi/2, -\pi/2)$. Existence of two solutions corresponds to the double degeneracy of the E-representation of the $C_{4v}$ group. Taking linear combinations of the single pocket solutions we find two degenerate solutions for the entire Brillouin zone with lines of nodes $k_x = 0$ or $k_y = 0$ well outside the Fermi surface. We would like to stress that we have considered the spin-singlet (more exactly pseudospin-singlet) pairing! This situation is very much different from the usual one when p-wave pairing implies spin triplet. We repeat that the peculiarity is due to the presence of long-range AF order.

5 D- and g-wave pairings caused by the long-range attraction

The long range attraction comes from the spin-wave exchange shown on Fig. 6. In this exchange the typical spin-wave momenta are $q \sim p_F \sim \sqrt{\delta}$, and hence the typical distances are $r \sim 1/q \sim 1/\sqrt{\delta} \gg 1$. 
Figure 6: Spin-wave exchange mechanism of attraction. Solid line corresponds to the hole and dashed line corresponds to the spin wave. The arrow shows the hole pseudospin.

Similarly to the previous section, it convenient to consider first the pairing inside a hole pocket, say centered at \((\pi/2, \pi/2)\), see Fig. 4. This pairing has been considered in detail in our previous work \[3\]. It has been shown that for the case of “isotropic” dispersion \[4\] the only solution is the one with a single node line in the pocket. The gap at the Fermi surface \((\epsilon_F = \frac{1}{2} \pi \beta \delta)\) is of the form

\[
\Delta(\phi) = \Delta_0 \sin \phi, \tag{14}
\]

\[
\Delta_0 = C \epsilon_F e^{-\pi J \beta / 2t} \approx 10 C t'' \delta e^{-10 J'' / t^2},
\]

where \(\sin \phi = p_2 / p_F\), \(p_F^2 = p_1^2 + p_2^2\), and \(C \sim 1\) is some constant.

The eqs.\[14\] describe pairing within a single pocket of the shifted zone. Translation of this solution to the magnetic Brillouin zone is shown at Fig. 7. This is absolutely identical to what we did in the previous section (change of sign at the translation).

Figure 7: Translation from the shifted zone to the magnetic Brillouin zone. The superconducting gap has line of nodes. The gap changes sign under the translation.

There are effectively two pockets in the Brillouin zone, see Fig. 4. Taking symmetric and antisymmetric combinations between the pockets, we get the d- and g-wave pairings respectively. The symmetries of the corresponding superconducting gaps are shown at Fig. 8. It is clear that the d-wave belongs to the \(B_1\) representation of the \(C_4\) group and the g-wave belongs to the \(A_2\) representation.

Both solutions originate from \[14\], therefore they are close in energy. Nevertheless the constant \(C\) in eq. \[14\] is smaller for the g-wave. This is the price for additional lines of nodes \((k_x = 0\) and \(k_y = 0\)). The above consideration did not include short range interaction \[12\]. This is absolutely correct for g-wave pairing which is not sensitive to the
interaction (12) at all. However the d-wave is sensitive. Therefore at \( V < 0.58J \) the d-wave pairing is enhanced because of (12), while, on the contrary, at larger Coulomb repulsion \( V > 0.58J \) the d-wave is suppressed and can even disappear. To avoid misunderstanding we stress that in the limit under consideration (\( t'' \gg t, J \)) the short range interaction (\( 12 \)) is too weak (even at \( V = 0 \)) to produce d-wave pairing without spin-wave exchange. However the short-range interaction influences the dimensionless constant \( C \) (see eq. (14)) which arises in spin-wave exchange mechanism.

6 The phase diagram

The phase diagram of the model under consideration is given on Fig. 9. To be specific we present the case of the not too strong Coulomb repulsion at the nearest sites: \( V < 0.58J \). At stronger \( V \) the p-wave superconductivity disappears, see eq. (13). Comparing eqs. (13) and (14) we see that the p-wave pairing is stronger at \( t < t_c \), while at \( t > t_c \) the d-g-wave pairing dominates. At \( V = 0 \) the critical value is \( t_c \approx J \). In the p-wave phase the gap, as well as the critical temperature, is proportional to square root of the hole concentration: \( \Delta \sim T_c \propto \sqrt{\delta} \). But in the d-g-wave phase they are proportional to the first power of concentration: \( \Delta \sim T_c \propto \delta \).

According to eq. (9) at \( t < t_{cN} \approx 1.6\sqrt{t''}J \), the long range AF order at zero temperature is preserved under doping, so we have coexistence of the superconductivity and the Neel order. At \( t > t_{cN} \) the Neel order is destroyed by the doping and one gets a transition into the quantum disordered phase. However as soon as the magnetic correlation length is larger than the superconducting correlation length the mechanism of pairing is valid and one still has the d-g-wave superconductor. At a temperature higher than the critical one the system behaves as a metal with very strong scattering of mobile holes on spin-wave excitations. Following the tradition we call this state “strange metal”.

Figure 8: Symmetry of the superconducting gap corresponding to the d- and g-wave pairings.
Figure 9: The phase diagram of the extended 2D $t-t''-J-V$ model. $t_c$ is transition point from the p-wave to the d-g-wave superconductor. $t_{cN}$ is transition point from the Neel state to the spin quantum disordered state.

7 The spin-wave collective excitation

We will see that the spin-wave collective excitation has nontrivial behaviour in the vicinity of the quantum phase transition from the Neel to the disordered phase. Therefore we study this excitation only in the d-g-wave superconducting phase at $T=0$. The energy spectrum and Bogoliubov parameters are given by the usual BCS formulas

$$E_k = \sqrt{(\epsilon_k - \epsilon_F)^2 + \Delta_k^2},$$

$$u_k^2, v_k^2 = \frac{1}{2} \left( 1 \pm \frac{\epsilon_k - \epsilon_F}{E_k} \right)$$

with gap $\Delta_k$ from eq. (14). The spin wave polarization operator due to mobile holes is given by diagram on Fig. 3 plus a similar diagram with anomalous fermionic Green’s functions. Straightforward calculation gives (see e.g. Ref.[4])

$$\Pi(\omega, q) = \sum_{k,k_0} g_{k_0q}^2 \frac{2(E_k + E_{k+q})}{\omega^2 - (E_k + E_{k+q})^2} \left( u_{k+q}^2 + u_k v_k u_{k+q} v_{k+q} \right).$$

This equation includes summation over pockets $k_0 = (\pi/2, \pm \pi/2)$. In these pockets the vertex (4) is $g_{k_0q} \approx 2^{5/4} t(q_x \pm q_y)/\sqrt{q}$. Let us consider the case of very small momenta and frequencies: $v_F q < \Delta_0$, and $\omega < \Delta_0$. In this limit one can put $q = 0$ in eq. (16) everywhere except at the vertex and therefore the polarization operator can be evaluated
analytically

\[ \Pi(\omega, q) = -\frac{4t^2\omega_q}{\pi J\beta} \left(1 + i\frac{\pi\omega}{8\Delta_0}\right) \]  \hspace{1cm} (17)

Note that the imaginary part is nonzero even at \( \omega < 2\Delta_0 \) because the gap (14) has a line of nodes. Any of the Green’s functions (13) have a denominator \( \omega^2 - \omega_q^2 - 2\omega_q\Pi(\omega, q) \), see e.g. Refs. [13, 4]. The zero of this denominator gives the energy and width of the spin-triplet collective excitation. Using eqs. (17) and (11) we find

\[ \omega_q = \eta \omega_q, \]
\[ \Gamma_q = \frac{\pi}{8} \frac{1 - \eta^2 \omega_q}{\eta \Delta_0} \omega_q. \] \hspace{1cm} (18)

In essence this is the renormalized spin-wave. Far from the point of AF instability the parameter \( \eta \approx 1 \), therefore the renormalization is relatively weak and the decay width is small. The situation is different when approaching the point of instability \( t \to t_c N \approx 1.6\sqrt{t''/J} \). Here, according to eq. (11), \( \eta \to 0 \) and therefore the energy of the renormalized spin wave is much smaller than the energy of the bare spin-wave, \( \omega_q/\omega_q = \eta \ll 1 \). Moreover this collective excitation exists as a narrow peak only at very small \( q \), when

\[ \pi \omega_q/8\eta \Delta_0 < 1. \] \hspace{1cm} (19)

At higher \( q \) the width is larger than its frequency because of decay to particle-hole excitations. We stress that the closer to the point of instability, the smaller is \( \eta \), and therefore the smaller is the region of \( q \) where the excitation exists.

8 Conclusions

We have considered a close to half filling \( t - t'' - J - V \) model at \( t'' \gg t, J \). We restrict our consideration to the case of small doping \( \delta \ll 1 \). It is demonstrated that at \( t < t_{cN} \approx 1.6\sqrt{t''/J} \) the Neel order is preserved under the doping, and at \( t > t_{cN} \) the order is destroyed and the system undergoes a transition to the quantum spin disordered phase, see phase diagram at Fig. 9.

If the hole-hole Coulomb repulsion at nearest sites is not too strong \( (V < 0.58J) \), then at small \( t \) the model has pseudospin-singlet p-wave superconductivity. As \( t \) increases, at the some point \( t_c \) (at \( V = 0 \) the critical point is \( t_c \approx J \)) the system undergoes a phase transition from the p-wave to the d-g-wave superconductor, see Fig. 9. Which state is realized (d- or g-wave) crucially depends on the Coulomb repulsion \( V \). If \( V \) is small the d-wave is preferable, while at larger \( V \) the g-wave superconductivity is realized.

In the Neel state we found the collective spin triplet excitation (renormalized spin wave). In the vicinity of the quantum phase transition to the spin disordered state the excitation exists as a narrow mode only at very small momenta and its energy is substantially below the energy of the bare spin wave.
References

[1] A. V. Chubukov and D. K. Morr, Phys. Rep. 288, 355 (1997) and reference therein.

[2] E. Dagotto, Rev. Mod. Phys. 66, 763 (1994).

[3] V. V. Flambaum, M. Yu. Kuchiev, and O. P. Sushkov, Physics C 227, 267 (1994); V. I. Belinicher, A. L. Chernyshev, A. V. Dotsenko, and O. P. Sushkov, Phys. Rev. B 51, 6076 (1995).

[4] O. P. Sushkov, Phys. Rev. B 54, 9988 (1996).

[5] J. Rossat-Mignot et al., Physica C 185-189, 86 (1991).

[6] P. Dai et al., Phys. Rev. Lett., 70, 3490 (1993); 77, 5425 (1996).

[7] H. F. Fong et al., Phys. Rev. Lett., 75, 316 (1996); 78, 713 (1997).

[8] E. Demler ans S. C. Zhang, Phys. Rev. Lett., 75, 4126 (1995); D. Z. Liu et al., ibid. 75, 4130 (1995); I. I. Mazin and V. M. Yakovenko, ibid. 75, 4134 (1995); V. Barzykin and D. Pines, Phys. Rev. B 52, 13585 (1995); G. Blumberg et al., ibid. 52, R15741 (1995); N. Bulut and D. J. Scalapino, ibid. 53, 5149 (1996).

[9] D. Marinaro and O. P. Sushkov. Phys. Rev. B 58, 14934 (1998).

[10] O. P. Sushkov, G. A. Sawatzky, R. Eder, and H. Eskes, Phys. Rev. B 56, 11769 (1997).

[11] E. Manousakis, Rev. Mod. Phys. 63, 1 (1991).

[12] C. L. Kane, P. A. Lee, and N. Read, Phys. Rev. B 39, 6880 (1989).

[13] J. Igarashi and P. Fulde, Phys. Rev. B 45, 12357 (1992).

[14] O. P. Sushkov and V. V. Flambaum, Physica C 206, 269 (1993).

[15] “Normal Fermi liquid” is a somewhat misleading term because we consider a system which is not a normal Fermi liquid from the common point of view. In this case “Normal Fermi liquid” means that we consider an ideal gas of Fermions with dispersion $E(k)$. At zero temperature all states inside the semicircles at the Fig. 1 are occupied.

[16] M. Randeria, J.-M. Duan, and L.-Y. Shieh, Phys. Rev. Lett., 62, 981 (1989).

[17] M. Yu. Kuchiev and O. P. Sushkov, Phys. Rev. B 53, 443 (1996).

[18] M. Yu. Kuchiev and O. P. Sushkov, Physica C 218, 197 (1993).