Test of an Analytic, Finite, Non-Perturbative, Gauge-Invariant QCD formulation against elastic pp scattering at the ISR energies.

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Abstract. A recent QCD formulation that is non-perturbative, finite, gauge-invariant, exact emerged from Schwinger’s Generating Functional. A first test of the validity of this formulation is provided here against elastic proton-proton scattering at the Intersecting Storage Rings (ISR) energies. Extension to LHC energies is currently underway.

INTRODUCTION

A recent formulation by H.M. Fried1, Y. Gabellini, T. Grandou1, R. Hofmann1, Y.M. Sheu, P.H. Tsang1, obtained analytic, finite, gauge-invariant, non-perturbative equations for QCD processes, [1]-[6]. Starting with the Schwinger Generating Functional,

\[ Z_{QCD}[j, \bar{q}, q] = \mathcal{N} e^{-\frac{i}{2} \int d^4x \bar{q} \gamma^\mu \partial_\mu q} \cdot e^{-\frac{i}{2} \int d^4x \frac{\partial}{\partial A^\mu} \cdot \left( A^\mu \right)} \cdot e^{-\frac{i}{2} \int d^4x \bar{q} G_{[A]} q \cdot L[A] \cdot A^{-1} \int d^4x \cdot j} \]  

(1)

where the quark line, \( G_A \), \( [x, y] = [m + \gamma \cdot (\delta - igA\tau)]^{-1} \) and the virtual quark loop, \( L[A] = \ln[1 - iA\tau_0] \), using a reciprocity relation [1]-[6], the \( F^2 \) field strength is then replaced with Halpern’s linear expression in \( F \), \( e^{-\frac{i}{2} \int d^4x \frac{\partial}{\partial A^\mu} \cdot \left( A^\mu \right)} = F \int d^4x e^{\frac{i}{2} \int d^4x \frac{\partial}{\partial A^\mu} \cdot \left( A^\mu \right)} \) [17]. Fradkin’s gaussian representation for \( G[A] \) and \( L[A] \) is used [15][16]. Experimentally, quarks do not have static transverse coordinates, and thus, the quark coordinates are to be written as a distribution, \( \int d^4x \bar{q}(x)\gamma_\mu x^\mu A_\mu(x)q(x) \), where \( \int d^4x \bar{q}(x)\gamma_\mu A_\mu(x)q(x) \), with \( a(x - x') \) real and symmetric, and \( x'_\mu = (x'_x, x'_y, x'_z, x'_t) \). The probability of finding two quarks separated by a transverse (or impact parameter) distance is then \( f(b) = \int \frac{d^4x}{D^4} \frac{e^{ib \cdot \xi}}{\pi^2} \), where we choose \( f(b) = f(0) e^{-\frac{ib \cdot \xi}{2}} \) with deformation parameter \( \xi \) real and small².

All QCD processes can now be expressed as gaussian differential operations acting upon gaussian potentials in \( A \), [1]-[6],

\[ e^{DA} F_1[A] F_2[A] = \mathcal{N} e^{\frac{i}{2} \int \frac{\delta}{\delta A} \cdot \left( (f \cdot \chi)^{-1} \right) \cdot \frac{\delta}{\delta A} + \frac{1}{2} \text{Tr} \ln(-(f \cdot \chi))} \cdot \frac{\delta}{\delta A} \cdot \left( (f \cdot \chi)^{-1} \right) \cdot \frac{\delta}{\delta A} - \int \frac{\delta}{\delta A} \cdot \left( (f \cdot \chi)^{-1} \right) \cdot \frac{\delta}{\delta A} \]  

(2)

where \( DA = \exp[-\frac{i}{2} \int \frac{\delta}{\delta A} D^\mu \frac{\delta}{\delta A}] \). As can be seen, QCD processes calculated from this formulation is gauge-invariant, non-perturbative, finite and exact. A new important property of Effective Locality³ emerged and will be discussed in detail by T. Grandou. All gluons exchanged, called the Gluon Bundle (GB), between two quarks are summed as \( (f \cdot \chi)^{-1} \). \( G[A], L[A] \) and \( (f \cdot \chi)^{-1} \), become the elements out of which all QCD processes can be calculated.

¹presenting at this conference.
²H.M.Fried’s talk in this conference will go into details.
³T. Grandou’s talk in this conference will explain Effective Locality in details.
Renormalization and sum of all physical processes

In order to make comparisons with experiments, Gluon Bundles (GBs) and closed-quark-loops (CQLs) need to be renormalized. The $\delta$ inside the Fradkin representation where a gluon bundle connects to a closed-quark-loop are to vanish while each closed-quark-loop is UV log divergent, or $\ell \to \infty$. Renormalization becomes a problem of finding a value for $n$ such that $\delta^n \cdot \ell$ becomes finite. As shown in Figure 1, in the first column where $n = 1$, all subsequent structures composed using $n = 1$ results in graphs identical to 0. In the third column where $n = 3$, graphs

![Figure 1: The only choice for $n$ in $\delta^n \cdot \ell = \text{finite}$ that results in physical structures is $n = 2$. This case is the second column, where two Gluon Bundles (GBs) and connected to a single closed-quark-loop (CQLs).](image)

FIGURE 1: The only choice for $n$ in $\delta^n \cdot \ell = \text{finite}$ that results in physical structures is $n = 2$. This case is the second column, where two Gluon Bundles (GBs) and connected to a single closed-quark-loop (CQLs).

with fewer than 3 vertices are divergent while graphs with more than 3 vertices are exactly 0. All graphs at the 4th

![Figure 2: Renormalization with $n = 2$ where $\delta^n \cdot \ell \equiv \kappa = \text{finite}$. Only linear chain-loop-terms become physical and non-zero. All other graphs are explicitly zero, 0.](image)

FIGURE 2: Renormalization with $n = 2$ where $\delta^n \cdot \ell \equiv \kappa = \text{finite}$. Only linear chain-loop-terms become physical and non-zero. All other graphs are explicitly zero, 0.
column or beyond will only result in divergent quantities. Only at \( n = 2 \), the second column, results in physical amplitudes. It is with this choice \( n = 2 \) for \( \delta^2 \cdot \ell = \kappa \) finite that we proceed to compare with experiments.

The cluster expansion in Figure 2 is used to express the summation of all configurations of GBs and CQL, [4]. With our renormalization scheme of \( \delta^2 \cdot \ell = \kappa \) where \( \kappa \) is finite, we can see that for each term of the cluster expansion \( Q_i \), only the chain-loop-terms (in bold) are non-zero. Thus, the entire amplitude becomes a finite geometric sum of all the chain-loop-terms as shown in Figure 2.

At this stage, a two-body approximation instead of the full six-quark problem is used for ease of computation. In the two-body approximation, a phenomenological energy dependence of \( (m_{\text{ext}}/E)^{2p} \), is used, with the understanding that it can be derived in the full six-body case. The resulting differential cross-section from summing all the GBs and chain-loop-terms in Figure 2 is then,

\[
\frac{d\sigma}{dt}(E, q^2) = K \left[ g^2 \beta \left( \frac{m_{\text{ext}}}{E} \right)^{2p} \right]^2 \left[ \frac{1}{4\pi} (9 \times 3 \times 4) \left( \frac{m_{\text{ext}}}{E} \right)^2 e^{-\left(3q^2/8m_{\text{ext}}^2\right)} - \frac{(9 \times 3 \times 8) A_{\text{ext}}(q^2)}{1 + \beta^2 g^2 A_{\text{int}}^2(q^2)} \right]^2
\]

where \( A_{\text{int/ext}}(q^2) = \kappa (q^2/m_{\text{int/ext}}^2) e^{-\left(q^2/4m_{\text{int/ext}}^2\right)} \). The coefficients \((9 \times 3 \times 4)\) and \((9 \times 3 \times 8)\) are for all possible ways two Gluon Bundles, and chain-loop-terms, respectively, can connect between the 3 quarks from one proton to the 3 quarks from the second proton, while preserving color charge. \( \beta \) is the sum over the \( SU(3) \) angles, \( g \) is the coupling constant, \( K \) is the conversion factor from GeV\(^2\) to millibarns.

**Comparing theory with ISR experiments**

![Graphs showing differential cross section for different energies](image)

**FIGURE 3:** Differential cross section of elastic pp scattering at ISR energies from 24 GeV to 63 GeV

The first term on the R.H.S of Equation 3 results from Gluon Bundles exchanged between the quark of one proton with the quark of the other proton. This contributes to the left part (left of the diffraction dip) of the ISR curves, Figure 3. The second term on the R.H.S. of Equation 3 results from the summation of all chain-loop-terms.

We list the values of the fixed parameters of Equation 3: \( K = 0.44 \text{ mb GeV}^{-2}, \ g = 7.6, \ \beta = 0.30, \ m_{\text{ext}} = 0.28 \text{ GeV} \approx 2m_{\pi}, \ m_{\text{int}} = 0.44 \text{ GeV} \approx 3m_{\pi}, \ p = 0.14 \) and \( \kappa = 5.22 \times 10^{-6} \).

We expect that, with the exception of \( K \) and \( \kappa \), these parameters may have a slight dependence on energy, as it increases from ISR to LHC values, and higher; and that such changes would be due to our two-body approximation of this six-quark scattering reaction.
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