Chiral zero modes on intersecting heterotic 5-branes

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Abstract

We show that there exist two \(27\) and one \(\overline{27}\) of \(E_6\), net one chiral supermultiplet as zero modes localized on the intersecting 5-branes in the \(E_8 \times E_8\) heterotic string theory. A heterotic background is constructed by the standard embedding in the smeared solution, and the Dirac equation is solved explicitly on this background. It provides, after the compactification of some of the transverse dimensions, an interesting five-dimensional brane-world set-up in heterotic string theory.

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In this paper, we construct an intersecting 5-brane solution in the \(E_8 \times E_8\) heterotic string theory by the so-called standard embedding in the known smeared intersecting NS5-brane solution of type II theories. We then study the zero modes of the relevant Dirac operator on this background and show that there exist three localized chiral zero modes, two of which are in the \(27\) representation of \(E_6\), and one in the \(\overline{27}\) representation. They give rise to net one \(27\) of massless chiral fermions in the four-dimensional spacetime. Therefore, this is the first example of a brane set-up in heterotic string theory that supports four-dimensional chiral matter fermions as an \(E_6\) gauge multiplet\(^3\).

This paper is a concise version of [1], to which the reader is referred for further detail.

\(^3\) This corrects the statement made in the earlier version of this paper, in which it was erroneously conjectured that the three supermultiplets would be of the same chirality.
Table 1. Dimensions in which the 5-branes stretch.

|       | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
|-------|---|---|---|---|---|---|---|---|---|---|
| 5-brane1 | o | o | o | o | o | o | o | o | o | o |
| 5-brane2 | o | o | o | o | o | o | o | o | o | o |

where

$$h(x^1) = h_0 + \xi|x^1|.$$  

(2)

All other components of $H_{MNL}$ vanish. $h_0$ and $\xi$ are real constants. The prime $'$ denotes the differentiation with respect to $x^1$, and $|x^1'|$ is therefore a step function. This is a solution to equations of motion of the leading-order NSNS-sector Lagrangian in type II theories. The solution describes a pair of intersecting NS5-branes [3–5] stretching in dimensions as shown in table 1. These branes are delocalized in the $x^2, x^3, x^4, x^5$ and $x^6$ directions. Consequently, the solution depends only on $x^1$, and hence the name ‘smeared solution’.

Depending on the sign of $\xi$, the profile of the harmonic function $h(x^1)$ changes. $\xi$ is also related to the brane tension $V$ as [1]

$$\xi = -\kappa^2 V h_0^2.$$  

(3)

where $\kappa$ is the Newton constant. Since $\xi = 0$ implies that there is no brane, we consider the case when $\xi \neq 0$.

If $\xi > 0$, the brane tension is negative. In this case, the string coupling becomes stronger as one goes away from the intersection $x^1 = 0$, and then the weak-coupling supergravity analysis becomes less reliable (figure 1(a)). On the other hand, if $\xi < 0$, the brane has a positive tension. In this case, the string coupling $h(x^1)$ becomes convex upwards in $x^1$ and decreases linearly, until it necessarily crosses the $x^1$ axis where the string coupling becomes zero. Beyond that point, $h(x^1)$ becomes negative (figure 1(b)). One way to understand this point is to identify it as the ‘end of the world’; this means that we place a pair of negative tension branes at the positions $x^1 = \pm h_0/\xi$, and another pair of positive ones at $x^1 = \pm h_0/\xi$ and so forth, and identifying them periodically, so that the $x^1$ direction is compactified (figure 1(c)). In a new coordinate

$$z = -\text{sign}(x^1) \log \frac{h(x^1)}{h_0},$$  

(4)

the points $x^1 = \pm h_0/\xi$ are sent to $z = \pm \infty$, and the function $h(x^1)$ is expressed simply as

$$h = h_0 e^{-|z|}.$$  

(5)

(figure 1(d)).

Apparently, it looks similar to the Randall–Sundrum (RS) II model [6], but there is an important difference: these ‘end of the world branes’ are located at a finite physical distance, whereas there is no such ‘end of the world’ in the RS II model. These negative tension branes here may be regarded as a NS–NS analogue of orientifolds. Indeed, since the configuration (1) is also a solution for type IIB string theory, one may apply an S-duality transformation on this background to obtain an intersecting D5-brane system, in which the fluxes coming from D-branes in compact dimensions have nowhere to go, and are absorbed by orientifolds with negative tension. A related discussion can be found in [7]. Of course, a heterotic string is not invariant under the world sheet parity transformation, and the usual definition in terms of $\Omega$ projection does not extend to this case. While their microscopic definition is unknown at present and there remains a need for further investigation, the one-dimensional
smeared solution is very simple and easy to deal with, and can be directly compared with five-dimensional field theory models. In particular, we can use this solution to construct a heterotic background by the standard embedding, solve the Dirac equation and see whether any localized chiral modes exist near the peak of the profile. The analysis starting from a localized brane solution is also currently under investigation.

We now construct an intersecting solution in the $E_8 \times E_8$ heterotic string theory by the standard embedding. The (generalized) spin connections \[8–10\] of the neutral intersecting background are identified as the gauge connection

$$A_{\mu} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix},$$

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where $\lambda_i$’s ($i = 1, \ldots, 8$) are the Gell–Mann matrices and $1 \equiv (1_1^{1}), s \equiv i\sigma_2 = (-1_1)$. The result is

$$A_{\mu=1} \, \lambda_i = 0,$$

$$A_{\mu=2} \, \lambda_i = \frac{h'}{h} \begin{pmatrix} -s \\ \frac{1}{2} s \\ \frac{1}{2} s \end{pmatrix} = \frac{h'}{h} \begin{pmatrix} \frac{3\lambda_3 + \sqrt{3}\lambda_5}{4} \end{pmatrix} \otimes s,$$

$$A_{\mu=3} \, \lambda_i = \frac{h'}{2h^2} \left( \begin{array}{c} 1 \\ -1 \end{array} \right) = \frac{h'}{2h^2} \left( -i\lambda_2 \right) \otimes 1,$$

$$A_{\mu=4} \, \lambda_i = \frac{h'}{2h^2} \left( \begin{array}{c} 1 \\ -1 \end{array} \right) = \frac{h'}{2h^2} \left( -\lambda_1 \right) \otimes s,$$

$$A_{\mu=5} \, \lambda_i = \frac{h'}{2h^2} \left( \begin{array}{c} 1 \\ -1 \end{array} \right) = \frac{h'}{2h^2} \left( -i\lambda_3 \right) \otimes 1,$$

$$A_{\mu=6} \, \lambda_i = \frac{h'}{2h^2} \left( \begin{array}{c} 1 \\ -1 \end{array} \right) = \frac{h'}{2h^2} \left( -\lambda_4 \right) \otimes s,$$

$$(6)$$

$$A_{\mu} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix},$$

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$$A_{\mu=6} \, \lambda_i = \frac{h'}{2h^2} \left( \begin{array}{c} 1 \\ -1 \end{array} \right) = \frac{h'}{2h^2} \left( -\lambda_4 \right) \otimes s,$$
The explicit expressions of $\omega_\pm$ show [1] that both are $SU(3)$ connections. Since we have embedded $\omega_+$ into the gauge connection $A$, the Bianchi identity is reduced to $dH = 0$, and solution (1) is consistent with it. An $SU(3)$ piece of the $E_8 \times E_8$ gauge connection is given a nonzero expectation value. On the other hand, the fact that $\omega_- \in SU(3)$ implies that the Killing spinor equations for the gravitino as well as gaugino variations have a common single Killing spinor. It can be checked that this Killing spinor also satisfies the equation for the dilatino SUSY variation to lowest order. Thus, the background (1) together with (6) preserves 1/4 of supersymmetries.

In the case of a single symmetric 5-brane [5], the connection $\omega_+$ embedded was in $SU(2)$, and the unbroken gauge symmetry was the centralizer $E_7$. The relevant decomposition $E_8 \supset E_7 \times SU(2)$ is

$$248 = (133, 1) \oplus (56, 2) \oplus (1, 3),$$

and therefore $56 \times 2 + 3 = 115$ broken generators, plus 5 (4 translations and 1 size), are the moduli. This number 120 is four times the dual Coxeter number of $E_8$ ($= 30$). In fact, for any compact simple gauge group $G$, the instanton moduli are correctly counted in this way.

Recalling that the instanton of the heterotic 5-brane is required by anomaly cancelation (that is, the Green–Schwarz mechanism), one may say that the 115 moduli arise from the choice of how the $SU(2)$ spin connection is embedded into the $E_8$ gauge connection [1].

In the present intersecting case, the connection embedded into $E_8$ is in $SU(3)$, and therefore the unbroken gauge symmetry is $E_6$. The adjoint representation of $E_6$ is decomposed into

$$248 = (78, 1) \oplus (27, 3) \oplus (27, \bar{3}) \oplus (1, 8)$$

as representations of the subalgebra $E_6 \times SU(3)$. Since the $E_8 \times E_8$ gauge field $A_M$ has by construction a vev in $SU(3)$, the latter three-gauge rotations are the moduli (figure 2).

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Figure 2. Broken generators which give rise to zero modes.
(This figure is in colour only in the electronic version)

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On the coordinate patch containing the point $x^1 = 0$ of the periodically identified $x^1$ coordinate. Note that on the other patch containing the opposite point $x^1 = \frac{1}{2} h_0 |\xi|$, the sign of $H$ is flipped, and consequently $\omega_+$ is replaced with $\omega_-$ and vice versa. Since $\omega_+$ and $\omega_-$ belong to different $SU(3)$ subgroups, supersymmetries are broken globally.

For example, the moduli of the $F_4$ instanton is $36k - 52$, where $k$ is the instanton number. This instanton number dependence can be obtained as follows: $F_4$ contains $Sp(6) \times SU(2)$ as a subgroup and is decomposed as $52 = (21, 1) \oplus (14, 2) \oplus (1, 3)$. The number of broken generators is $14 \times 2 + 1 \times 3 = 31$. Adding 4 + 1 translations and scale transformation, we obtain the correct number of proportionality 36.

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On the coordinate patch containing the point $x^1 = 0$ of the periodically identified $x^1$ coordinate. Note that on the other patch containing the opposite point $x^1 = \frac{1}{2} h_0 |\xi|$, the sign of $H$ is flipped, and consequently $\omega_+$ is replaced with $\omega_-$ and vice versa. Since $\omega_+$ and $\omega_-$ belong to different $SU(3)$ subgroups, supersymmetries are broken globally.
Let us focus on the $E_6$ non-singlet moduli. The spontaneously broken generators in $(27, 3) \oplus (\overline{27}, 3)$ give rise to Nambu–Goldstone bosons, each of which has one bosonic degree of freedom. On the other hand, since a $D = 4$, $N = 1$ chiral supermultiplet needs two bosonic degrees of freedom, the Nambu–Goldstone bosons which transform as 27 and $\overline{27}$ must be combined to form a single $N' = 1$ chiral supermultiplet. That is, the $E_6$ non-singlet moduli form three chiral supermultiplets in the 27 (or $\overline{27}$, but not both) representation of $E_6$.

At first sight, one might think that the argument above would be contradictory to the well-known fact in Calabi–Yau compactifications that the number of chiral generations are determined by the Dirac index, in which the same decomposition (8) is used and one triplet of zero modes together corresponds to one supermultiplet, and is not counted as three. Of course, it is not a contradiction, because what we consider here is not the fermionic zero modes of the Dirac operator, but bosonic zero modes of the gauge fields. They are not removed by gauge transformations, and necessarily exist to cancel the anomaly inflow into each of the two intersecting 5-branes [1]. Each of small gauge rotation generators in $(27, 3) \oplus (\overline{27}, 3) \oplus (1, 8)$ is an independent generator and gives rise to an independent zero mode. We also recall that exactly the same way of counting was done in the parallel symmetric 5-brane case, and was indeed consistent with the index analysis [11].

We should emphasize that, in [1], the anomaly inflow argument is used not to establish the net chirality in four dimensions, but only to confirm that the gauge rotation moduli are certainly not absorbed by the Higgs mechanism, and to demonstrate that the bosonic moduli on a (six-dimensional) 5-brane are indeed correctly counted by the decomposition $E_8 \supset E_7 \times SU(2)$. This fact suggests that one may also count the moduli in the intersecting case by the decomposition $E_8 \supset E_6 \times SU(3)$, as we do in this paper. Of course, the four-dimensional chirality must be confirmed by another argument, as we check below.

Next we examine the fermionic zero modes. The ten-dimensional heterotic gaugino equations of motion reads

$$D(\omega - \frac{1}{3} H, A) \chi - \Gamma^M \chi \partial_M \phi + i \frac{1}{2} \Gamma^M \gamma^{A\bar{B}} (F_{A\bar{B}} + \tilde{F}_{A\bar{B}})(\psi_M + \frac{1}{2} \Gamma_M \lambda) = 0,$$

where

$$D(\omega - \frac{1}{3} H, A) \chi \equiv (\partial_M + i \frac{1}{4} (\omega_M^{A\bar{B}} - \frac{1}{2} H_M^{A\bar{B}}) \Gamma_{A\bar{B}} + \text{ad} A_M) \chi.$$  

Here $\chi$ is the adjoint 248 representation of $E_6$, and ad $A_M \cdot \chi \equiv [A_M, \chi]$. If we set $\psi_M = 0$, $\lambda = 0$ and $\tilde{\chi} \equiv e^{-\phi} \chi$, then the equation is simplified to

$$D(\omega - \frac{1}{3} H, A) \tilde{\chi} = 0.$$  

Since there are no nontrivial backgrounds for the four-dimensional $i = 0, 7, 8, 9$ directions,

$$\Gamma^\mu \partial_\mu \tilde{\chi} + \Gamma^\mu D_\mu (\omega - \frac{1}{3} H, A) \tilde{\chi} = 0.$$  

If $\tilde{\chi} = \tilde{\chi}_{AD} \otimes \tilde{\chi}_{6D}$, the second term is regarded as the mass term for the four-dimensional spinor $\tilde{\chi}_{4D}$. We are interested in the zero modes of this Dirac operator $\Gamma^\mu D_\mu (\omega - \frac{1}{3} H, A)$.

We fix the $SO(6)$ gamma matrices as

$$\gamma_1 = \sigma_2 \otimes 1 \otimes 1, \quad \gamma_2 = \sigma_1 \otimes \overline{\sigma}_1 \otimes 1,$$

$$\gamma_3 = \sigma_1 \otimes \sigma_2 \otimes 1, \quad \gamma_4 = \sigma_1 \otimes \sigma_3 \otimes \overline{\sigma}_1,$$

$$\gamma_5 = \sigma_1 \otimes \overline{\sigma}_2 \otimes \sigma_2, \quad \gamma_6 = \sigma_1 \otimes \overline{\sigma}_3 \otimes \sigma_3.$$  

The six-dimensional chiral operator is $\gamma_5' \equiv -i \gamma_1' \gamma_2' \ldots \gamma_6' = \sigma_3 \otimes 1 \otimes 1$. For $SO(9, 1)$ gamma matrices, we take $\Gamma^a = \gamma_5'^a \otimes 1$s ($a = 0, 7, 8, 9$), $\Gamma^a = \gamma_5'^{a'} \otimes \gamma_6' (a = 1, \ldots, 6)$, where
\(\gamma_{\alpha}^4\)'s \((\alpha = 0, 7, 8, 9)\) are the ordinary \(SO(3, 1)\) gamma matrices in the chiral representation.

The ten-dimensional chirality is \(\Gamma_{11} \equiv \gamma_{\alpha}^4 \otimes \gamma_5 = (\sigma_3 \otimes 1) \otimes (\sigma_3 \otimes 1 \otimes 1)\).

Now we consider the Dirac equation

\[
\Gamma^\mu D_\mu (\omega - \frac{1}{2} H, A) \tilde{\chi} = 0. \tag{14}
\]

The 16-component \(SO(9, 1)\) (Majorana-)Weyl spinor \(\chi\) (or \(\tilde{\chi}\)) is decomposed in terms of \(SO(3, 1)\) and \(SO(6)\) spinors as \(16 = (2_4, 4_2) \oplus (2_{12}, 4_{-12})\), where the subscripts are the \(SO(3, 1)\) and \(SO(6)\) chiralities, \(\gamma_{4D}^\alpha\) and \(\gamma_8\), respectively. Since \(\tilde{\chi}\) is Majorana (but complex in this representation), the \((2_4, 4_2)\) and \((2_{12}, 4_{-12})\) components are not independent but are transformed each other by a charge conjugation.

As \(\Gamma^\mu D_\mu (\omega - \frac{1}{2} H, A)\) is \(SO(3, 1)\) diagonal, it is enough to consider

\[
\gamma^\alpha D_\mu (\omega - \frac{1}{2} H, A) \tilde{\chi}_{6D} = 0, \tag{15}
\]

with the understanding that each component of \(\tilde{\chi}_{6D}\) is accompanied by a two-component \(SO(3, 1)\) Weyl spinor with a correlated chirality (\(\gamma_5 \gamma_{4D}^\alpha = +1\)).

On the other hand, we are interested in the gaugino zero modes in \((27, 3)\) or \((\bar{27}, \bar{3})\) in the decomposition \(E_8 \supset E_6 \times SU(3)\) of \(248\). The gauge connections \(A_\mu\) take only nonzero values in the \(SU(3)\) subalgebra, and we look for the zero modes \(\tilde{\chi}_{6D}\) transforming as a triplet, either \(3\) or \(\bar{3}\), of \(SU(3)\).

Since \(\gamma^\alpha\)'s are in the form

\[
\gamma^1 = \begin{pmatrix}
-i 1 & -i 1 & i 1 & i 1 & i 1 & i 1
\end{pmatrix},
\]

\[
\gamma^\alpha = \begin{pmatrix}
\gamma^\alpha & \tilde{\gamma}^\alpha
\end{pmatrix} \quad (\alpha = 2, \ldots, 6),
\]

and \(\omega_{\mu}^{\alpha\beta}, H_\mu^{\alpha\beta}\) and \(A_\mu^{\alpha\beta}\) all vanish if \(\mu = 1\), \(15\) is reduced to two independent differential equations

\[
\frac{i}{\hbar} \frac{d}{dt} \tilde{\chi}_{6D}^{+} + M^{+} \tilde{\chi}_{6D}^{+} = 0, \tag{17}
\]

\[
\frac{i}{\hbar} \frac{d}{dt} \tilde{\chi}_{6D}^{-} - M^{-} \tilde{\chi}_{6D}^{-} = 0, \tag{18}
\]

where \(\tilde{\chi}_{6D}^{+}\), are the upper and lower components having definite chiralities: \(\tilde{\chi}_{6D} = (\tilde{\chi}_{6D}^{+}) \tilde{\chi}_{6D}^{+}\) \((\tilde{\chi}_{6D}^{-})\) is a \(4 \times 4 SO(6)\) Weyl spinor, and each of the four components is a triplet of \(SU(3)\). Thus, \(M^{+}\) \((M^{-})\) is a \((4 \times 3) = 12 \times 12\) matrix, given explicitly by

\[
\begin{pmatrix}
M^{+} & M^{-}
\end{pmatrix} \equiv \begin{pmatrix}
\frac{h'}{\hbar^2} & \begin{pmatrix}
0 & 0 & 0 & 0 & -\frac{3}{2} & -\frac{1}{2} & 0 & -\frac{1}{2}
0 & 0 & 0 & 0 & -\frac{1}{4} & -\frac{3}{4} & 0 & 0
0 & 0 & 0 & 0 & -\frac{1}{4} & -\frac{3}{4} & 0 & 0
\end{pmatrix} \\
\begin{pmatrix}
\frac{3}{2} & -\frac{3}{2} & 0 & -\frac{1}{2} & 0 & 0 & 0 & 0
\frac{1}{2} & \frac{3}{2} & -\frac{3}{2} & 0 & 0 & 0 & 0 & 0
\frac{1}{2} & \frac{3}{2} & -\frac{3}{2} & 0 & 0 & 0 & 0 & 0
\end{pmatrix} & \begin{pmatrix}
0 & 0 & 0 & 0 & \frac{3}{2} & \frac{3}{2} & 0 & 0
0 & 0 & 0 & 0 & \frac{3}{2} & \frac{3}{2} & 0 & 0
\end{pmatrix}
\end{pmatrix} \otimes 1_3
\]
\begin{equation}
\begin{pmatrix}
0 & 0 & 0 & 0 & -\frac{i\lambda_4}{2} & -\frac{s\lambda_1-\lambda_5}{4} & -\frac{2\lambda_2-s\lambda_3}{4} & 0 \\
0 & 0 & 0 & 0 & \frac{\lambda_1-s\lambda_1}{2} & \lambda_4 & 0 & -\frac{2\lambda_2-s\lambda_3}{4} \\
0 & 0 & 0 & 0 & \frac{2\lambda_2-s\lambda_3}{4} & 0 & \frac{\lambda_4}{2} & \frac{s\lambda_1+s\lambda_5}{2} \\
\frac{\lambda_1-s\lambda_1}{2} & \frac{s\lambda_1}{2} & 0 & -\frac{2\lambda_2-s\lambda_3}{4} & 0 & 0 & 0 & 0 \\
\frac{2\lambda_1-s\lambda_3}{4} & 0 & \frac{s\lambda_1}{2} & \frac{s\lambda_1+s\lambda_5}{2} & 0 & 0 & 0 & 0 \\
0 & \frac{2\lambda_2-s\lambda_3}{4} & \frac{s\lambda_1-s\lambda_5}{2} & -\frac{s\lambda_1}{2} & 0 & 0 & 0 & 0 \\
\end{pmatrix}\end{equation}

where \(\lambda_0 = 3\lambda_3 + \sqrt{3}\lambda_5\). In identifying the spin connection as an SU(3) gauge connection, \(s = (-1)^i\) can either be mapped to \(i\), or to \(-i\), and depending on this choice, the SU(3) gauge connection matrix becomes one in the \(3\) representation, or in the \(\bar{3}\) representation.

Since \(\tilde{X}_{6D}\) and \(\tilde{Y}_{6D}\) are not independent, we have to solve only equation (17), and the solutions to (18) may then be obtained by a charge conjugation. To solve (17), we diagonalize \(M^*\) to obtain its eigenvalues. Let \(i\lambda\) be an eigenvalue of the constant matrix \(\left(\frac{\chi}{\bar{\chi}}\right)^{-1} M^*\), and and \(\psi_\lambda(x^1)\) be the corresponding eigenvector. Then equation (17) amounts to

\begin{equation}
\frac{i}{\hbar} \psi_\lambda' + i\lambda \frac{\hbar}{\hbar^2} \psi_\lambda = 0.
\end{equation}

This is solved to give

\begin{equation}
\psi_\lambda(x^1) = \text{const.} \, (h(x^1))^{-\lambda}.
\end{equation}

Therefore, for each eigenvalue of \(M^*\), there exists a zero mode of the Dirac operator.

The list of eigenvalues of \(\left(\frac{\chi}{\bar{\chi}}\right)^{-1} M^*\) is as follows: if \(s = +i\), the eigenvalues are

\begin{equation}
\left\{ -i, i, i, i, \frac{3i}{2}, \frac{3i}{2}, \frac{3i}{2}, \frac{7i}{2}, \frac{7i}{2} \right\},
\end{equation}

while if \(s = -i\), they are

\begin{equation}
\left\{ -i, -i, \frac{3i}{2}, \frac{3i}{2}, \frac{3i}{2}, 2i, 2i, 2i, 2i, 4i \right\}.
\end{equation}

We can clearly see an asymmetry between (22) and (23), in particular that the former has only one negative (times imaginary unit) eigenvalue, while the latter has two negative eigenvalues. Since the branes at \(x^1 = 0\) are assumed to have positive tension and \(h(x)\) has the profile shown in figure 1(b), these are the only modes whose profiles have a peak at \(x^1 = 0\) or \(z = 0\) in the coordinate \((4)\). Thus, we indeed find three localized fermionic zero modes near \(x^1 = 0\), one of which is in one (say, \((27,3)\)) representation, and the other two belong to its conjugate \((27,\bar{3})\) representation. This is consistent with the bosonic moduli counting we have done below equation (8). This is the main result.

The other modes either (in the original gaugino wavefunction \(\chi = h\bar{\chi}\)) (i) vanish at \(x^1 = 0\) and diverge at \(x^1 = \pm \frac{\hbar}{10} \) \((i\lambda = \frac{3}{2}, 2i, 2i, 4i)\) or (ii) are constant \((i\lambda = i)\). They do not have a peak at \(x^1 = 0\) and cannot be identified as modes localized near \(x^1 = 0\). On the other hand, taking into account the volume factor (the determinant of the string-metric vielbein), some of these modes turn out to have a finite norm and hence are also normalizable. Therefore, in order for such a periodically identified intersecting 5-brane system to be used to build a realistic particle physics model, one not only needs to generalize in some way to
realize three chiral matter generations (not just only the $(n_2 - 1 = 0)$ one we found in this paper), but also needs to consider how these modes are coupled (or decoupled) to the ones localized near $x^1 = 0$.

It will be interesting to consider what happens if the two smeared 5-branes are separated in the $x^1$ direction. We note that an obvious attempt to generalize the solution: $H_{234} = \frac{h_k}{2}$, $H_{256} = \frac{h_z}{2}$, etc, for two harmonic functions $h_1(x^1) = h_{10} + \xi_1|x^1|$ and $h_2(x^1) = h_{20} + \xi_2|x^1 - \Delta_1|$, and similar modifications of other fields do not solve equations of motion any more. Therefore, at least, there is no massless modulus of that form. Finally, it would also be interesting to examine whether the net chirality found in this paper can be understood in terms of $\chi(Z; V)$, where $Z$ is a complex 3-fold and $V$ a vector bundle with the $SU(3)$ structure group, as was similarly explained in the example of symmetric 5-branes.

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