THE ASTROPHYSICAL JOURNAL, 780:164 (14pp), 2014 January 10
© 2014. The American Astronomical Society. All rights reserved. Printed in the U.S.A.
doi:10.1088/0004-637X/780/2/164

THE LINK BETWEEN EJECTED STARS, HARDENING AND ECCENTRICITY GROWTH OF SUPER MASSIVE BLACK HOLES IN GALACTIC NUCLEI

LONG WANG\(^{1,2}\), PETER BERCZIK\(^{3,4,5}\), RAINER SPURZEM\(^{3,4,2}\), AND M. B. N. KOUWENHOVEN\(^{2,1}\)

\(^{1}\) Department of Astronomy, School of Physics, Peking University, Yiheyuan Lu 5, Haidian Qu, 100871 Beijing, China
\(^{2}\) Kavli Institute for Astronomy and Astrophysics, Peking University, Yiheyuan Lu 5, Haidian Qu, 100871 Beijing, China
\(^{3}\) National Astronomical Observatories of China, Chinese Academy of Sciences, 20A Datun Rd., Chaoyang District, 100012 Beijing, China
\(^{4}\) Astronomisches Rechen-Institut, Zentrum für Astronomie, University of Heidelberg, Mönchhofstrasse 12-14, D-69120 Heidelberg, Germany
\(^{5}\) Main Astronomical Observatory, National Academy of Sciences of Ukraine, 27 Akademika Zabolotnoho St., 03080 Kyiv, Ukraine

Received 2013 August 9; accepted 2013 November 17; published 2013 December 20

ABSTRACT

The hierarchical galaxy formation picture suggests that supermassive black holes (SMBHs) observed in galactic nuclei today have grown from coalescence of massive black hole binaries (MBHB) after galaxy merging. Once the components of an MBHB become gravitationally bound, strong three-body encounters between the MBHB and stars dominate its evolution in a “dry” gas-free environment and change the MBHB’s energy and angular momentum (semimajor axis, eccentricity, and orientation). Here we present high-accuracy direct \(N\)-body simulations of spherical and axisymmetric (rotating) galactic nuclei with order of \(10^6\) stars and two MBHs that are initially unbound. We analyze the properties of the ejected stars due to slingshot effects from three-body encounters with the MBHB in detail. Previous studies have investigated the eccentricity and energy changes of MBHs using approximate models or Monte Carlo three-body scatterings. We find general agreement with the average results of previous semi-analytic models for spherical galactic nuclei, but our results show a large statistical variation. Our new results show many more phase space details of how the process works, and also show the influence of stellar system rotation on the process. We detect that the angle between the orbital plane of the MBHBs and that of the stellar system (when it rotates) influences the phase-space properties of the ejected stars. We also find that MBHBs tend to switch stars with counter-rotating orbits into corotating orbits during their interactions.

Key words: black hole physics – galaxies: evolution – galaxies: kinematics and dynamics – galaxies: nuclei – methods: numerical

Online-only material: color figures

1. INTRODUCTION

The current galaxy-formation scenario based on the \(Λ\) cold dark matter (\(Λ\)CDM) cosmology model (Begelman et al. 1980; Volonteri et al. 2003) suggests that massive galaxies form from hierarchical merging and accretion of smaller galaxies. If the merging galaxies have massive black holes (MBHs) in their centers, the merging processes should also include coalescence of MBHs, since MBHs are commonly observed at the centers of most galaxies today (Greene & Ho 2009). Thus, understanding the process of MBH coalescence is significant for cosmological structure formation, galaxy formation, and MBH formation. An excellent review of the current observational situation is given by Kormendy & Ho (2013).

In the case of gas-poor galaxies merging, the two MBHs have three distinct evolution phases (Begelman et al. 1980). First, the dynamical friction exerted by the stars forces the two MBHs to sink toward the galactic center. This phase continues until the binary reaches the hard binary separation (Merritt 2001; Yu 2002). Second, a hard MBH binary (MBHB) orbit will continue to shrink due to slingshot ejection of surrounding stars. Third, when the MBHB reaches a separation at which the loss of orbital energy caused by gravitational wave (GW) emission becomes significant, the MBHB shrinks until the final coalescence. There is a challenging problem, often called the “Final Parsec Problem,” during this three-phase model in the merger of gas-poor galaxies. In a quasi-steady, spherical stellar environment, the slingshot efficiency decreases rapidly when the separation of the MBHB becomes much smaller than the average separation of neighboring stars around the MBHB (Quinlan & Hernquist 1997; Milosavljević & Merritt 2003; Berczik et al. 2005).

The hardening rate of the MBHB for large particle numbers, such as in real galactic nuclei, is too low, and the timescale of coalescence of the MBHB is larger than the Hubble time. Recent efforts suggest that if the stellar system has some degree of axisymmetry or triaxiality, this problem may be solved (Yu 2002; Merritt & Poon 2004; Berczik et al. 2006; Preto et al. 2011; Fiestas et al. 2012; Khan et al. 2012b, 2013). The simulations by Khan et al. (2013) show that even a mildly flattened stellar system with an axis ratio of 0.9 will result in an MBHB hardening rate that is eight times faster and that the MBHB evolution is independent of particle number when the axis ratio is 0.75. These results agree with the earlier work of Berczik et al. (2006). Also, Gualandris & Merritt (2012) argue that in post-merger galaxies the loss-cone is full and hardening does not depend on \(N\). Therefore, these studies show a solution for the “Final Parsec Problem” because the MBHB merges in a relatively short time. The final solution for this problem requires more work on the details of the slingshot phase.

Detection of GWs is the most important expected observation in the near future to give direct proof of general relativity. GWs will also provide a new window to understanding the universe independent of electromagnetic observations. The coalescence of MBHs is expected to be the strongest GW source to be measured with future space-satellite detectors, such as the Laser Interferometer Space Antennae (LISA/e-LISA/ALIA; see, e.g., Gong et al. 2011). This encourages researchers to concentrate on the final orbital parameters and the merging rate of MBHs to predict the GW signals to be measured.

It is very important to understand the eccentricity growth of the MBHB during phase 2 since it strongly influences both the coalescence time (Peters 1964) and the angular momentum...
evolution. Quinlan (1996) used three-body scattering experiments to study the properties of slingshot effects. He found the eccentricity growth is small unless the MBHB already forms with a large eccentricity. Preto et al. (2011); Khan et al. (2011) carried out a large number of $N$-body simulations of equal-mass MBHBs in an axisymmetric and triaxial stellar environment after a galaxy merger. They found that the initial eccentricities when MBHs become bound are very high—about $e = 0.95$ on average. This is also consistent with previous work (Aarseth 2003; Berentzen et al. 2008, 2009; Preto et al. 2009; Li et al. 2012). On the other hand, Khan et al. (2013) find low eccentricities for some of their elliptical galaxy models. The reason for this may be that they model a nonrotating stellar system. However, none of the these studies examined the detailed scattering processes near the MBHB; it is desirable to study the eccentricity evolution in detail in the full $N$-body simulation as a counterpart and to check the validity of the Quinlan (1996) three-body Monte Carlo work.

A galaxy merger can result in a rotating stellar system. Therefore, simulations of MBHBs in rotating star clusters are interesting (Berczik et al. 2006; Preto et al. 2011; Khan et al. 2011, 2012b). Several restricted studies of MBHB evolution in rotating star clusters were carried out by Sesana et al. (2011) and Gualandris et al. (2012). They studied very high mass ratios of the MBHB ($1/64$) and a small stellar system surrounding it (less than the mass of the MBHB). Under these restrictions, Sesana et al. (2011) found that the eccentricity evolution of MBHB in a rotating stellar cusp can be significantly influenced by the
fraction of corotating stars. However, larger N-body simulations may give different results, because many interactions occur with stars from unbound regions. Thus to understand the effect of stellar rotation further, it is necessary to carry out a deeper analysis of large N-body systems with different mass ratios for the MBHB.

It is interesting that triple black holes during galaxy mergers can also exist if one of the progenitor MBHBs does not coalesce before the next galaxy with a third MBH falls in. In such cases, very extreme eccentricities for the inner MBHB have been found \( (e \sim 0.99–0.999) \); Amaro-Seoane et al. 2010).

Yu & Tremaine (2003) studied the dynamical processes of hyper-velocity stars ejected by the MBHB in the Galactic center and predicted the rate of ejections. Lu et al. (2010) used analytical arguments and numerical simulations to study the distribution of hyper-velocity stars ejected by an MBH in the Galactic center. They found most of these are ejected antiparallel to the injecting direction of their progenitors.

In this paper, Section 2 provides the N-body simulations and the method to select ejected stars from the simulation results. In Section 3, we list the initial conditions. In Section 4, we discuss the eccentricity growth of MBHBs, angular momentum properties of MBHBs, and ejected stars. Finally, Section 5 contains our results and conclusions.

2. METHODS

In this work, we use the direct N-body code called \( \varphi \)-GPU (Berczik et al. 2011, 2013b) to do all the simulations. The full set of runs and the parameters will be described in more detail in a forthcoming publication (P. Berczik et al. 2014, in preparation).

The code is a direct N-body simulation package, with a high-order Hermite integration scheme and individual block time steps (the code supports time integration of particle orbits with fourth, sixth, and even eighth order schemes). A direct N-body code evaluates in principle all pairwise forces between the gravitating particles, and its computational complexity scales asymptotically with \( N^2 \); however, it is not to be confused with a simple brute force shared time step code due to the block time steps. We refer more interested readers to a general discussion about N-body codes and their implementation in Spurzem et al. (2011a, 2011b, 2012), and Berczik et al. (2013a).

The \( \varphi \)-GPU code is fully parallelized using the MPI library, and for each MPI process, GPU accelerator hardware is used to compute gravitational forces between particles. It is based on an earlier C version for GRAPE6a clusters (Fukushige et al. 2005). The new code is written from scratch in C++ and based on an earlier CPU serial N-body code (YEBISU; Nitadori & Makino 2008). The MPI parallelization was done in the same “j” particle parallelization mode as in the earlier \( \varphi \)-GRAPE code (Hafst et al. 2007).

The present version of the \( \varphi \)-GPU code uses native GPU support and direct access to the GPUs with only the NVIDIA native CUDA library. Multi-GPU support is achieved through MPI parallelization. More details as well as the \( \varphi \)-GPU public version are presented in Berczik et al. (2011), Spurzem et al. (2012), and Berczik et al. (2013a).

The present code is well tested and already used to obtain important results in our earlier large-scale few-million-body simulation (Khan et al. 2012a).

Our analysis is based on the stars ejected via the slingshot effect of MBHBs. The method used to select ejected stars depends on the individual total energy change from the beginning to the end of the simulations \( (\Delta E_\ast) \) because ejected stars usually gain a lot of energy during the interaction and this energy gain should be much larger than the energy fluctuations caused by perturbations from other stars. The next step is to check the two-dimensional histogram of the initial energy of each star, \( E_0 \) versus log\((-\Delta E_\ast/E_0)\), for each model and determine a critical value, \( \delta_\ast \), to select ejected star candidates that satisfy log\((-\Delta E_\ast/E_0)\) > \( \delta_\ast \), where \( \delta_\ast \) is obtained by the number density gap shown in Figure 1. For both rotating and nonrotating models, these energy features are different, but we still can select a \( \delta_\ast \) that works well. This procedure also functions as an operational definition for the term “ejected star,” which here just means it leaves after a strong encounter with the MBHB with significantly higher energy than before. For the purpose of our paper, it is not important whether the ejected star has enough energy to become unbound from the MBHB, the central stellar cluster, or even the entire galaxy.

The next step is to check the energy evolution of each ejected star candidate and the MBHB. The final samples of stars are the method to select ejected stars from the simulation results. It is based on the stars ejected via the slingshot effect of MBHBs. The method used to select ejected stars depends on the individual total energy change from the beginning to the end of the simulations \( (\Delta E_\ast) \) because ejected stars usually gain a lot of energy during the interaction and this energy gain should be much larger than the energy fluctuations caused by perturbations from other stars. The next step is to check the two-dimensional histogram of the initial energy of each star, \( E_0 \) versus log\((-\Delta E_\ast/E_0)\), for each model and determine a critical value, \( \delta_\ast \), to select ejected star candidates that satisfy log\((-\Delta E_\ast/E_0)\) > \( \delta_\ast \), where \( \delta_\ast \) is obtained by the number density gap shown in Figure 1. For both rotating and nonrotating models, these energy features are different, but we still can select a \( \delta_\ast \) that works well. This procedure also functions as an operational definition for the term “ejected star,” which here just means it leaves after a strong encounter with the MBHB with significantly higher energy than before. For the purpose of our paper, it is not important whether the ejected star has enough energy to become unbound from the MBHB, the central stellar cluster, or even the entire galaxy.

The next step is to check the energy evolution of each ejected star candidate and the MBHB. The final samples of stars are chosen from each candidate energy change \( \Delta E_\ast > M_{\ast,i} \) during its ejection time \( t_\ast \), where \( M_{\ast,i} \) is the mass of a star in N-body units and \( i \) indicates the index of ejected stars. In our simulation, \( M_{\ast,i} \) is the same order of magnitude of the individual star’s
energy in $N$-body units. Thus it can be used to select the events with an obvious energy jump.

With the ejected-star samples, we calculate the angular momentum, $L_i$, of each ejected-stars and $L_0$ of the MBHB at $t_e$. We compare the angular momentum before and after ejection of each star.

We carry out all data reduction and analysis using the open source software ROOT.

3. UNITS AND INITIAL CONDITIONS

We scale the numerical units of our initial models, applying the standard $N$-body normalization (Aarseth et al. 1974), by setting both the gravitational constant, $G$, and the total mass of the stellar system to unity. The total energy of the system is scaled to $E = -1/4$. Our simulations are purely gravitational and thus scale-free, but for convenience we define one example of the scaling in physical units below. The initial conditions of our $N$-body simulations presented here are based on the ones used in Berczik et al. (2006) and Berentzen et al. (2009). The initial stellar galactic nucleus follows a distribution function (see, e.g., Einsel & Spurzem 1999 and references therein). It provides rigid rotation inside the half-mass radius and quickly decreasing differential rotation outward. After the MBHs settle into the galactic center, the density and velocity dispersion adjust to the MBH’s gravity inside its influence radius. The concentration and rotation parameters are set to $W_0 = 6$ and $\omega_0 = 1.8$, respectively, in all rotating models. We also simulate the nonrotating King models ($\omega_0 = 0$) for comparison. The total angular momentum vector of the stellar nucleus is aligned with the $z$-axis of our coordinate frame. We place the two MBHB members in the $z = 0$ midplane with initial coordinate components $x_{1,2} = 0$ and $y_{1,2} = \pm 0.3$, where the subscripts denote the two black hole particles.

The full set of models (with different field particle numbers and sets of the MBH’s initial velocities) and the MBH’s orbital evolution are presented elsewhere (P. Berczik et al. 2014, in preparation). Here we analyze only the subset of our runs (which include seven models) with a fixed stellar particle number of $N = 10^6$. The total integration time for these models was 150 $N$-body time units. The differences are the MBH masses, which are given in Table 1. The nonrotating models are indicated by the suffix -nonrot hereafter. Our work will focus on the rotating models; thus nonrotating models will only be shown in some parts. The initial $x$-velocities of the MBHs in these simulations have been chosen to be $v_x = \pm v_{\text{circ}}$, where $v_{\text{circ}}$ is the circular velocity within the stellar background model. With our choice of initial values, the circular velocity in $N$-body units is $v_{\text{circ}} = 0.7$ at the initial distance of the MBHs from the center.

Our models are scale free and can be applied to a range of real astrophysical systems. Here we give an example for the case of an MBH mass of 0.01 in $N$-body units i (see Table 1); if the black hole mass is, e.g., $10^7 M_\odot$, and the black hole separation $y_{1,2} = \pm 0.3$ is, e.g., $\pm 300$ pc, then one $N$-body time unit is about 15 Myr and one $N$-body velocity unit is about 65.6 km s$^{-1}$.

4. RESULTS

4.1. Coordinate System and Angles

In our simulations, we have initially defined rectangular, Cartesian coordinates with $x$, $y$, and $z$ axes (Section 3). Here, we define three angles: $\alpha$, $\delta$, and $\theta$. When we use the spherical coordinate system instead, the radius $r$ denotes the distance to the origin and the two angles $\alpha$ and $\delta$ define the direction of a vector. The transformation from $(x, y, z)$ to $(r, \alpha, \delta)$ can be

![Figure 3. Eccentricity growth rate, $K$, vs. eccentricity, $e$, for all rotating models. The points are calculated results using Equation (4). The circles with crosses are the average $K$'s from $t = 81$ $N$-body units to the end of simulation. The solid and dashed curves are fitted functions for $K_1$ from Quinlan (1996). Here $v$ represents $v/V_{\text{bis}}$ in Quinlan (1996).](image-url)
described as:

\[ x = r \cos \delta \cos \alpha, \]
\[ y = r \cos \delta \sin \alpha, \]
\[ z = r \sin \delta. \tag{1} \]

We will later use these spherical coordinates to define the angular momentum vectors of both a star, \( L_s \), and the MBHB, \( L_b \). The angle, \( \theta \), between \( L_b \) and \( L_s \) is defined as

\[ \theta = \arccos \frac{\overrightarrow{L_b} \cdot \overrightarrow{L_s}}{|L_b| |L_s|}. \tag{2} \]

Hereafter we use the suffix “b” to denote MBHBs and “s” to denote ejected stars.

Figure 4. Evolution of each component—\( L_x \) (red), \( L_y \) (green), and \( L_z \) (black)—of the MBHB’s angular momentum (Section 4.1). The last two panels are nonrotation models for comparison. Due to the inclination between the MBHB’s orbit and the stellar system’s rotational symmetry plane, models 1001, 1002, and 2020 are classified as I-models; all others are classified as P-models.

(A color version of this figure is available in the online journal.)
For each angle of individual ejected stars or MBHBs, we also have two values: the one before ejection time $t_i$ (hereafter denoted with the suffix “BE”) and the one after ejection time $t_o$ (hereafter denoted with the suffix “AE”). Due to our simulation output time resolution, the interval time between $t_i$ and $t_o$ is one $N$-body time unit.

4.2. Ejected Stars Sample Selection

Using the method discussed in Section 2, we successfully find most ejected stars for the MBHB models. The numbers of ejected stars and $\delta_e$ (defined in Section 2) for all rotating models are listed in Table 2.

To ensure our samples are convincing, we calculate the integrated energy change, $\Delta E_i(t)$, of all ejected stars during their ejections and compare it to the MBHB’s binding-energy loss, $\Delta E_b(t)$, during each time unit (Figure 2). If the residual energy change $\Delta E_r(t) = \Delta E_b(t) - \Delta E_i(t)$ is zero, the ejected star sample is complete; its deviation from zero gives information about how many ejected stars we may have missed, since there is no other significant mechanism in our simulations through which the MBHB can lose energy. Figure 2 shows that $\Delta E_r(t)$ in both the rotating model 2020 and the nonrotating model 2020-nonrot is almost zero after the binary formation ($t > 40$). This result holds for all models with large black hole masses. This indicates our method for selecting ejected stars is reliable. The only exceptions are the two models 0110 and 0210 with low MBHB masses (one is shown in Figure 2).

There are two possibilities that may cause incomplete samples for low-mass MBHBs. One is that low-mass MBHBs probably generate more ejected stars with low $\Delta E_i$, which cannot be distinguished from energy fluctuations caused by other mechanisms; therefore we cannot select these ejected stars. Another reason is that the low-mass MBHBs become gravitationally bound at a later time than the massive ones—slingshot effects dominate the energy loss of MBHBs only after the binary formation.

4.3. Eccentricity Growth Rate of MBHBs

The specific angular momentum, $J$, of an MBHB can be described by

$$J = \frac{L}{\mu} = \sqrt{G M a (1 - e^2)},$$

where $L$ is the standard angular momentum, $\mu$ is the reduced mass, $a$ is the semimajor axis, $M = m_1 + m_2$ is the total mass of the binary components, $e$ is the eccentricity, and $G$ is the gravitational constant. The hardening process of an MBHB provides energy to the ejected stars and thus increases its binding energy and reduces its semimajor axis, $a$. As a result of Equation (3), the angular momentum of the MBHB will also be reduced, even if $e$ remains constant (which is generally not the case—see below). Any eccentricity growth will lead to an additional decrease of $L$. In phase 2, the ejected stars dominate the energy and angular momentum evolution of the MBHB. Thus to understand the properties of ejected stars, it will help to know how they carry away both the energy and the angular momentum from the MBHB.

Quinlan (1996) defined an eccentricity growth rate, $K$, as

$$K = \frac{\Delta e}{\Delta \ln(1/a)}$$

and then derived $K_1$, a numerical expression of $K$ with the assumption that all stars have an identical velocity, $v$. Note
that, for the special case of constant specific angular momentum ($\Delta J = 0$), we have

$$K = \frac{\Delta e}{\Delta \ln(1/a)} = \frac{-1 - e^2}{2e}.$$  

(5)

Quinlan also carried out Monte Carlo models (three-body scattering experiments of single stars with the MBHB), the results of which we can compare with our data. In our work, we also calculate $K$ by using Equation (4) directly from the MBHB’s measured changes, $\Delta e$ and $\Delta \ln(1/a)$. To compute the differences, we use the eccentricity, $e$, and semimajor axis, $a$, of the MBHB averaged over time spans of one $N$-body time unit (for good statistics).

Figure 3 shows the comparison between our result and that from Quinlan (1996). There is a stochastic variation of $K$ (which creates positive and negative $\Delta e$’s in individual encounters) due to the three-body encounters. The average growth rate of $e$ (see circles with crosses in Figure 3) is positive and agrees fairly well with previous semi-analytic work. The dash-dotted line shows the predicted growth rate when $J$ of the MBHB is conserved. If $J$ of the MBHB decreases as the MBHB hardens, $K$ should be located above this line. Our results indicate that during the hardening of the MBHB, the MBHBs with lower $e$ always lose $J$, and the MBHBs with higher $e$ also lose $J$ in most cases.

4.4. Angular Momentum Exchange between Stars and MBHBs

4.4.1. Angular Momentum Evolution

Figure 4 shows how the three components of the MBHB’s angular momentum decrease over time. The $L_\phi$ of the MBHBs in models 0110, 0210, and 2020 have the same or smaller magnitude as $L_x$ and $L_y$. Thus their MBHBs have an orbital plane that is tilted with respect to the $x$-$y$ plane, which is the symmetry plane of the rotating stellar cluster. The inclination angle between these two planes is far from zero; hereafter we
call these models I-Models. In contrast, the MBHBs in all the other models have an orbital plane almost parallel to the $x$–$y$ plane (hereafter we call these P-Models).

The angular momentum of individual ejected stars, $L_{r, AE}$, as compared to that before ejection, $L_{r, BE}$, shows an increasing trend in all models. Figure 5 provides evidence for this trend. If one of the ejected stars gains $L_r$ during its ejection, it is located in the top-left region of the density map, and vice versa. We see that there is both gain and loss of the $L_r$ of ejected stars. However, the total $L_r$ gain of the ejected stars is larger than the loss since the density peaks are located in the top-left region for all models. This indicates that the ejected stars will carry away net angular momentum from the MBHB. As an additional effect, they also carry away and redistribute some of the angular momentum of the stellar system.

Two more effects are shown in Figure 5. One is that, independent of the mass of the MBHBs, the stellar angular momentum after the encounter, $L_{r, AE}$, is approximately constant (it is actually a distribution where the highest level contour lines are nearly flat, parallel to the horizontal axis, which is the angular momentum before the encounter, $L_{r, BE}$). This means that stars of any incoming angular momentum get a typical angular momentum after the encounter which is independent of its initial value and is only determined by the properties of the MBHB. The value of such a post-encounter angular momentum becomes smaller for larger MBHB mass. In the case of the nonrotating stellar system, the effect is not visible in the plot; the angular momentum after the encounter scatters in a more symmetric distribution around the line of equality with the initial angular momentum.

4.4.2. Distribution of Angular Momentum Direction

The direction of incoming and ejecting orbits of ejected stars viewed in a rectangular coordinate system is influenced by the rotational planes of both the whole stellar system and the...
The Astrophysical Journal, 780:164 (14pp), 2014 January 10

Wang et al.

MBHB. The distribution of $\alpha_s$ and $\delta_s$ (see Section 4.1) is similar before and after ejection (Figure 6).

For I-Models, the MBHBs have a stable $\alpha_b$ and $\delta_b$ (e.g., $\alpha_b \approx 3.8$ and $\cos(\delta_b) \approx 0.15$ in model 2020; Figure 6), i.e., the direction of the angular momentum of the MBHB (or its orbital plane) does not change much during all the stellar encounters. The $\alpha_s$ and $\delta_s$ distribution concentrates either around the same angles as the MBHB or $\pm \pi$ both before and after ejection. This indicates that the MBHB interacts preferentially with stars having the same orbital plane. The rotational direction of the stars may be the same as or opposite to that of the MBHBs (co- or counter-rotating), and $\cos(\delta_s)$ also has a concentration toward 0. Thus ejected stars are oriented with their orbital plane to that of the MBHB, which is perpendicular to the stellar-system rotational symmetry plane (the $x$-$y$ plane).

For P-Models, the MBHB’s orbital plane is close to the $x$-$y$ plane, aligned with the stellar system’s rotational symmetry plane; $\alpha_b$ cannot be defined well. Therefore there is an extended distribution of $\alpha_b$ and we expect to see no special trend of $\alpha_s$ related to $\alpha_b$ (like model 4020 in Figure 6). The $\cos(\delta_s)$ distribution also has a strong concentration close to 0 and less concentration around $\cos(\delta_s)$. The orbits of the ejected stars, which prefer $\cos(\delta_s) \approx 0$, are now not correlated with the MBHB’s orbital plane but rather with the stellar system’s rotational symmetry plane. Therefore we see in this case an effect of the rotation of the stellar system, which dominates the orbit direction of ejected stars in P-Models and overrides the MBHB’s rotational effect that we see in the I-Models.

In Figure 7, we use the angles $\alpha_s$ and $\delta_s$ to illustrate the relation between the stellar orbit before (BE) and after (AE) the encounters. Particularly interesting are the concentrations near the lines $\alpha_s,AE - \alpha_s,BE = \pm \pi$, which indicate that initially counter-rotating stars (with respect to the MBHB orbital plane) become corotating after the encounter and ejection. But the $\alpha_s,BE$ and $\alpha_s,AE$ in both I-Models and P-Models also show concentrations near the line without change, which means that many ejected stars preserve their rotational direction during the encounter, independent of that of the MBHB. We can also see that $\alpha_s$ concentrates near $\alpha_b$ with a small change before and after ejection in Model 2020. The $\delta_s$’s have a wider change than the $\alpha_s$’s and concentrate near $\delta_s = \pi/2$ with a small change.

The information about the relation between incoming and ejected stellar orbital planes, relative to that of the MBHB, can be more easily analyzed by looking at a single angle $\theta$, which is the inclination angle between ejected stars and the MBHB (Figure 8). For I-Models, the distribution of $\cos \theta$ is flat (models 1001 and 1002) with a little increase around $\cos \theta = \pm 1$ (model 2020). This indicates that ejected stars show a preference for co- and counter-rotation with respect to the MBHBs’ rotation, which is consistent with the results discussed above.

For P-Models, $\cos(\theta_{BE})$ and $\cos(\theta_{AE})$ show clear concentrations near $\pm 1$ and 0 (slight bias for $\cos(\theta_{AE})$ in model 4040). It means that the ejected stars tend to have incident and ejecting orbits parallel or perpendicular to the MBHB’s rotational orbit in the P-Models, which is consistent with Figure 6.

There is also a slight trend showing that ejected stars prefer to corotate with MBHBs, since the fraction of positive $\cos \theta$ is larger than the negative fraction.

The distributions of $\cos(\theta)$ in models 4040 and 4020 also show a significant difference before and after ejection (Figure 8). In these two cases, the MBHBs tend to switch the orbits of incident stars from counter-rotating with the MBHBs to corotating. For nonrotating models, both $\cos(\theta_{BE})$ and $\cos(\theta_{AE})$ show no trend of concentrations near $-1$ and 0 but do show a strong concentration around 1. This indicates that for nonrotating models, ejected stars prefer to have incident and ejecting orbits corotating with the MBHB’s rotational orbit; it also confirms that the concentration near 0 is an effect of stellar system rotation. There is also the trend that incident counter-rotating stars are transformed to co-rotating orbits for rotating models.

5. CONCLUSIONS

High-accuracy direct N-body models of spherical and axisymmetric (rotating) star clusters in galactic nuclei have been presented here, which consist of one million stars and two MBHBs that are initially unbound. We study the evolution of an MBHB
forming during its detailed interactions (superelastic scatterings) with single stars. The two MBHs have three evolutionary phases: the dynamical friction phase, the three-body encounter phase, and a final GW radiation phase. The MBHs will sink toward the galactic center, forming a binary whose orbit shrinks through superelastic three-body encounters until they final coalesce under strong emission of GWs.

Some authors have reported a “Final Parsec Problem” for this three-phase MBHB-merging scenario based purely on stellar dynamical processes. The timescale of MBHB merging would be too long compared to the evolutionary time scale of galactic nuclei and galaxy mergers, which are the origin of MBHBs. The MBHB would stall at a separation of about a parsec with an empty loss cone; no further hardening (orbit shrinking) occurs and relativistic energy losses are too small (Begelman et al. 1980).

Currently, it seems that the “Final Parsec Problem” only occurs under unphysical, idealized conditions such as a strictly spherical stellar system. Under more general conditions, such as some degree of rotation or triaxiality (bars or tidal fields) or the presence of gas, there is no problem in bringing an MBHB to complete relativistic coalescence in a few gigayears (Berczik et al. 2005, 2006; Preto et al. 2011; Khan et al. 2011, 2012b, 2012a, 2013).

In this work, we have studied the details of the interactions between single stars and the MBHB with unprecedented detail and statistical quality due to the large particle number in our simulations (obtained with the $\psi$GPU code on large GPU-accelerated

**Figure 6.** Density maps of $\delta$ vs. $\alpha$ of ejected stars and MBHBs with $t > 50$ N-body time units before (left) and after (right) ejection. The top two panels represent model 2020 (I-model) and the bottom two panels represent model 4020 (P-model). The colors of the contours represent ejected stars and both black points and lines (near the center in the top panels and near the top edge in the bottom panels) represent the MBHBs.

(A color version of this figure is available in the online journal.)
supercomputers in China and Germany). The detailed evolution of the energy and angular momentum of the MBHB during a large number of slingshot interactions with stars is analyzed. Also the effect of a large-scale rotation of the stellar cluster surrounding the MBHB is taken into account.

The nuclear stellar cluster surrounding the MBHB is simulated with a direct high-accuracy N-body simulation (Hermite scheme, ϕGPU (Berczik et al. 2011) code) with up to 10^6 equal-mass stars. A parameter study is presented with different mass ratios of the black holes to each other and to the single stars (see Table 1). We build an efficient method to select the ejected stars from the simulations in order to understand the detailed properties of ejected stars and how they change the eccentricity of the MBHBs when slingshot effects dominate the hardening of the MBHBs. About 0.08%–8% of stars are ejected by MBHBs in our 150 N-body time unit simulations (see Table 2; if we use the scale factor as discussed in the last part of Section 3, 150 time units is about 2.25 Gyr).

Our results (see Figure 4) exhibit two different classes of systems based on the MBHB’s rotational-axis direction at the time it becomes gravitationally bound: I-models (where the inclination angle between the MBHB’s orbit and stellar-system rotational-symmetry plane is large) and P-models (where the MBHB’s orbital plane is nearly parallel to the stellar system’s rotational-symmetry plane). I-models and P-models lead to different characteristics of the angular momentum distribution of ejected stars (Figures 6, 7, and 8). The histogram reflects both the rotation of the surrounding star cluster as well as that of the MBHB; there are maxima both at the angular momentum perpendicular to the MBHB orbit and another one aligned with

---

**Figure 7.** Left: density map of α_{AE} vs. α_{BE} of ejected stars and MBHBs. Right: similar map for δ. The models here (2020 and 4020) are the same as in Figure 6. The colored contour indicates ejected stars and the black contour shows MBHBs. (A color version of this figure is available in the online journal.)
the stellar system (Figure 8). If the stellar system is spherically symmetric, we only see the maximum at ejected stellar orbits corotating with the MBHB (see the last two histograms in Figure 8).

Besides, the larger mass MBHBs have a stronger rotational correlation with ejected stars. If the both the black hole’s and the stellar system’s rotational symmetry plane are similar, the effect is even stronger. For the $P$-model, the stellar system’s rotational symmetry plane dominates the concentration features of ejected stars (see Figures 6 and 8).

Finally, we find that MBHBs (models 4040 and 4020 in Figure 8) in both rotating and nonrotating galactic nuclei deplete corotating stars because relatively more ejected stars are corotating with the MBHB. This agrees with the models of Zier & Biermann (2001, 2002), Iwasawa et al. (2011), Meiron & Laor (2013), and Cui & Yu (2014). Iwasawa et al. (2011) carried out simulations with small mass ratios (1/100) and they only considered bound stars and a nonrotating stellar system. They argue that nonaxisymmetric perturbations by the secondary black hole create an effect, which is also seen in our simulations with rotating models containing an MBHB and with nonrotating models (see Figure 8): initially counter-rotating stars become corotating after being scattered. Similar results have been found by Madigan & Levin (2012). Our results show this effect for much more general conditions (up to an equal-mass ratio for the MBHB and including mostly unbound scattered stars).

The average eccentricity changes of our MBHBs agree fairly well with early Monte Carlo predictions (Quinlan 1996; see Figure 3), but the scatter for individual events is quite large. This means that our interactions, detected in the numerical simulations, cover a different (and we think more realistic) range of encounters than that used in the early investigation of Quinlan (1996). This could be due to different distributions of relative velocities, impact parameters, or the movement of the MBHB. This effect is more pronounced for the few cases where we have low eccentricity, while for the large-$e$ cases our results follow the same trend as Quinlan (1996). Our data show that the eccentricity grows in a stochastic way where positive and negative $K$ occur all the time, but there is an average trend toward higher eccentricity. The relativistic post-Newtonian evolution of the MBHB and its GW emission in the final phase before coalescence depends on highly detailed orbital evolution simulations, which is one reason why we need simulations like ours and others for a correct assessment of gravitational radiation from MBHBs in the universe (see, e.g., Preto et al. 2011; Khan et al. 2011, 2013). We obtain, on average, higher eccentricities as compared to Khan et al. (2013); the reason for this is probably that we have rotating models while their galaxy models are nonrotating.

We thank the anonymous referee for constructive comments that helped to improve the paper.
We acknowledge support by the Chinese Academy of Sciences through the Silk Road Project at NAOC; through the Chinese Academy of Sciences Visiting Professorship for Senior International Scientists, grant No. 2009S1 − 5 (R.S.); and through the “Qianren” special foreign experts program of China.

L.W. and R.S. acknowledge support and hospitality through research visits at the Max-Planck Institute for Astronomy (MPA) in Garching and helpful discussions with Thorsten Naab and Hao Wei. L.W. also acknowledges support through the University of Heidelberg (SFB881), and through the European Gravitational Wave Observatory (EGO), VESF grant EGO-DIR-50-2010 to attend a school in Rome, Italy.

The special GPU-accelerated supercomputer laohu at the Center of Information and Computing at National Astronomical Observatories, Chinese Academy of Sciences, funded by Ministry of Finance of People’s Republic of China under the grant ZDY/2008 − 2, has been used for some of the largest simulations. We also used smaller GPU clusters titan,
hydra and kepler, funded under the grants I/80041-043 and I/84678/84680 of the Volkswagen Foundation and grants 823.219-439/30 and /36 of the Ministry of Science, Research and the Arts of Baden-Württemberg, Germany. Some code development was also done on the Milky Way supercomputer, funded by the Deutsche Forschungsgemeinschaft (DFG) through the Collaborative Research Center (SFB 881) “The Milky Way System” (subproject Z2), hosted and cofunded by the Jülich Supercomputing Center (JSC).

P.B. acknowledges the special support by the NASU under the Main Astronomical Observatory GRID/GPU computing cluster project.

M.B.N.K. was supported by the Peter and Patricia Gruber Foundation through the PPGF fellowship, by the Peking University One Hundred Talent Fund (985), and by the National Natural Science Foundation of China (grants 11010237, 11050110414, 11173004). This publication was made possible through the support of a grant from the John Templeton Foundation and National Astronomical Observatories of Chinese Academy of Sciences. The opinions expressed in this publication are those of the author(s) and do not necessarily reflect the views of the John Templeton Foundation or National Astronomical Observatories of Chinese Academy of Sciences. The funds from the John Templeton Foundation were awarded in a grant to The University of Chicago, which also managed the program in conjunction with the National Astronomical Observatories, Chinese Academy of Sciences.

REFERENCES

Aarseth, S. J. 2003, Ap&SS, 285, 367
Aarseth, S. J., Henon, M., & Wielen, R. 1974, A&A, 37, 183
Amaro-Seoane, P., Sesana, A., Hoffman, L., et al. 2010, MNRAS, 402, 2308
Begelman, M. C., Blandford, R. D., & Rees, M. J. 1980, Natur, 287, 307
Berczik, P., Merritt, D., & Spurzem, R. 2005, ApJ, 633, 680
Berczik, P., Merritt, D., Spurzem, R., & Bischof, H.-P. 2006, ApJL, 642, L21
Berczik, P., Nitadori, K., Zhong, S., et al. 2011, in High Performance Computing HPC-UA, Vol. 4, 8
Berczik, P., Spurzem, R., & Wang, L. 2013b, in Proc. of the Third Int. Conf., High Performance Computing, 52
Berczik, P., Spurzem, R., Zhong, S., et al. 2013a, in Proc. of 28th Intl. Supercomputing Conf. ISC 2013, ed. J. M. Kunkel, T. Ludwig, & H. E. Meuer (Lecture Notes in Computer Science, Vol. 7905; Berlin: Springer), 13

Berentzen, I., Preto, M., Berczik, P., Merritt, D., & Spurzem, R. 2008, AN, 329, 904
Berentzen, I., Preto, M., Berczik, P., Merritt, D., & Spurzem, R. 2009, ApJ, 695, 455
Cui, X., & Yu, Q. 2014, MNRAS, 437, 777
Einsel, C., & Spurzem, R. 1999, MNRAS, 302, 81
Fiestas, J., Porth, O., Berczik, P., & Spurzem, R. 2012, MNRAS, 419, 57
Fukushige, T., Makino, J., & Kawai, A. 2005, PASJ, 57, 1009
Gong, X., Xu, S., Bai, S., et al. 2011, CoGra, 28, 094012
Greene, J. E., & Ho, L. C. 2009, PASP, 121, 1167
Gualandris, A., Dotti, M., & Sesana, A. 2012, MNRAS, 420, L38
Gualandris, A., & Merritt, D. 2012, ApJ, 744, 74
Harfst, S., Gualandris, A., Merritt, D., et al. 2007, NewA, 12, 357
Iwasawa, M., An, S., Matsubayashi, T., Funato, Y., & Makino, J. 2011, ApJL, 731, L9
Khan, F., Holley-Bockelmann, K., Berczik, P., & Just, A. 2013, ApJ, 773, 100
Khan, F. M., Berentzen, I., Berczik, P., et al. 2012a, ApJ, 756, 30
Khan, F. M., Just, A., & Merritt, D. 2011, ApJ, 732, 89
Khan, F. M., Preto, M., Berczik, P., et al. 2012b, ApJ, 749, 147
Kormendy, J., & Ho, L. C. 2013, ARA&A, 51, 511
Li, S., Liu, F. K., Berczik, P., Chen, X., & Spurzem, R. 2012, ApJ, 748, 65
Lu, Y., Zhang, F., & Yu, Q. 2010, ApJ, 709, 1536
Madigan, A. M., & Levin, Y. 2012, ApJ, 754, 42
Meiron, Y., & Laor, A. 2013, MNRAS, 433, 2502
Merritt, D. 2001, ApJ, 556, 245
Merritt, D., & Poon, M. Y. 2004, ApJ, 606, 788
Milosavljević, M., & Merritt, D. 2003, ApJ, 596, 860
Nitadori, K., & Makino, J. 2008, NewA, 13, 498
Peters, P. C. 1964, PhRv, 136, 1224
Preto, M., Berentzen, I., Berczik, P., Merritt, D., & Spurzem, R. 2009, JPhCS, 154, 012049
Preto, M., Berentzen, I., Berczik, P., & Spurzem, R. 2011, ApJL, 732, L26
Quinlan, G. D. 1996, NewA, 1, 35
Quinlan, G. D., & Hernquist, L. 1997, NewA, 2, 533
Sesana, A., Gualandris, A., & Dotti, M. 2011, MNRAS, 415, L35
Spurzem, R., Berczik, P., Berentzen, I., et al. 2011a, in Large Scale Computing Techniques for Complex Systems and Simulations, ed. W. Dubitzky, K. Kurowski, & B. Schott (New York: Wiley), 35
Spurzem, R., Berczik, P., Hamada, T., et al. 2011b, CSRD, 26, 145
Spurzem, R., Berczik, P., Zhong, S., et al. 2012, in ASP Conf. Ser. 453, Advances in Computational Astrophysics: Methods, Tools, and Outcome, ed. R. Capuzzo-Dolcetta, M. Limongi, & A. Tornambè (San Francisco, CA: ASP), 223
Volonteri, M., Haardt, F., & Madau, P. 2003, ApJ, 582, 559
Yu, Q. 2002, MNRAS, 331, 935
Yu, Q., & Tremaine, S. 2003, ApJ, 599, 1120
Zier, C., & Biermann, P. L. 2001, A&A, 377, 23
Zier, C., & Biermann, P. L. 2002, A&A, 396, 91