In this paper we discuss the single diffractive production of open heavy flavor mesons and non-prompt charmonia in $pp$ collisions. Using the color dipole approach, we found that the single diffractive production constitutes 0.5-2 per cent of the inclusive production of the same mesons. In Tevatron kinematics our theoretical results are in reasonable agreement with the available experimental data. In LHC kinematics we found that the cross-section is sufficiently large and could be accessed experimentally. We also analyzed the dependence on multiplicity of co-produced hadrons and found that it is significantly slower than that of inclusive production of the same heavy mesons.

Keywords:

I. INTRODUCTION

In the kinematics of the Large Hadronic Collider (LHC), the diffractive events in $pp$ collisions constitute approximately twenty per cent of all inclusive events [1], and for this reason might be used as an additional tool for studies of the strong interactions. The characteristic feature of the diffractive events is the presence of rapidity gaps between hadronic products in the final state. In Quantum Chromodynamics (QCD) such rapidity gaps in high energy kinematics are explained by the exchange of pomerons in the $t$-channel. Since the structure of the pomeron is relatively well understood and largely does not depend on the process, the existence of rapidity gap allows to separate the strong interactions involving different hadrons. While conventionally diffractive production of mesons has been studied in $ep$ collisions, there are various theoretical suggestions to use $pp$ collisions for studies of the diffractive production of prompt quarkonia [2, 6], dijets [7], gauge bosons [8], Higgs bosons [9], heavy quarks [10, 11], quarkonia pairs [12] and Drell-Yan processes [13]. The possibility to measure diffractive production in $pp$ collisions has been demonstrated at the Tevatron [14, 15]. At the LHC some diffractive processes (e.g. single diffractive $pp \rightarrow pX$) have been measured with very good precision [1], although diffractive production of additional heavy hadrons so far has not been explored in depth (see however preliminary feasibility study [19]).

In this paper we are going to focus on single diffractive production of heavy mesons, $pp \rightarrow p + M X$, where $M$ is an open heavy flavor meson ($D$ or $B$) or a charmonium produced from decay of $B$-meson; we also assume that the recoil proton in the final state is separated by a rapidity gap from other hadrons. This process deserves special interest both on its own and because it could help to clarify the role of multipomeron contributions to the production of heavy quarks in general. The role of such mechanisms is not very clear at this moment. Usually it is believed that production of heavy quarks might be described perturbatively [20, 21] and is dominated by two-gluon (pomeron) fusion [22, 30]. However, this approach can hardly explain the recently measured dependence of the production cross-sections on the multiplicity of the charged hadrons co-produced together with a given heavy quarkonia [31, 36]. Potentially this discrepancy might indicate sizeable contributions of multigluon production mechanisms. At the same time, for $D$- and $B$-mesons such rapidly growing dependence was not observed [31]. On the other hand, theoretical studies [37-59] found that three-pomeron mechanism might give sizeable contribution and can explain the observed multiplicity dependence of quarkonia. For $D$-mesons it was found in the same framework that the three-pomeron correction is also pronounced and might constitute up to 40 percent of the result, although in the range of multiplicities available at present from the LHC it does not contribute to the observed multiplicity dependence due to partial cancellation with certain interference contributions [40]. Fortunately, it is possible to estimate the role of the three-pomeron fusion directly. The single diffractive production at the partonic level has a similar structure, and thus might provide independent estimate of the three-pomeron contribution. Since the single diffractive production amplitude includes only one cut pomeron which might contribute to the observed yields of co-produced hadrons, its cross-section might be used as a very clean probe of the multiplicity dependence of individual cut pomerons in high multiplicity events.

Earlier the single-diffractive production including heavy quarks has been studied in [10, 11] for the case of prompt production of quarkonia. As we will see below, the cross-sections of single diffractive production of $D$- and $B$-mesons is larger than that of the prompt charmonia and thus could be easier to study experimentally. The feasibility to measure such processes has been discussed in [14, 15, 19]. The study of rare events with large multiplicity requires better statistics, and for this reason we expect that such dependence could be measured during the High Luminosity Run 3 at the LHC (HL-LHC mode) [11, 33].

The paper is structured as follows. In Section 11 we develop the general framework for the evaluation of the open heavy meson production. We will perform our calculations within the color dipole framework, which describes
correctly the onset of saturation dynamics and thus might be used even for the description of high multiplicity events. In Section [III] we present our numerical results and make comparison with experimental data available from the Tevatron, as well as with other theoretical approaches. In Section [IV] we develop the framework for the description of multiplicity dependence in dipole framework and compare its predictions for multiplicity dependence with that of inclusive production. In Section [V] we discuss briefly the single diffractive process on nuclei, $pA \rightarrow p + MX$. Finally, in Section [VI] we draw conclusions.

II. SINGLE-DIFRACTIVE PRODUCTION IN COLOR DIPOLE FRAMEWORK

As was mentioned in the previous section, a defining characteristics of the single-diffractive production is the observation of the recoil proton separated by a large rapidity gap from other hadrons. In LHC kinematics the dominant contribution to such process stems from the diagrams which include the exchange of uncut pomeron between the proton and the other hadrons in the $t$-channel. The heavy mesons are produced predominantly near the edge of the rapidity gap, and for this reason a pomeron couples directly to the heavy quark loop, as shown in the Figure 1. In this paper we will focus on the production of open heavy-flavor $D$- and $B$-mesons, and will also discuss briefly the production of non-prompt charmonia from decays of $B$-meson. Previously, the single diffractive production for prompt charmonia production has been studied in [4–6]. In this last case the dominant contribution differs slightly from that of $D$- and $B$-mesons and is shown in the right panel of the Figure 1. In Section [III] we will use the results of [4–6] for comparison with our numerical results for non-prompt charmonia.

The cross-section of the heavy meson production might be related to the cross-section of the heavy quark production as [24–27],

$$\frac{d\sigma_M}{dy d^2p_T} = \sum_i \int_{x_Q}^1 \frac{dz}{z^2} D_i \left( \frac{x_Q(y)}{z} \right) \frac{d\sigma_{Q_iQ_i}}{dy^* d^2p_T^*}$$

where $y$ is the rapidity of the heavy meson ($D$- or $B$-meson), $y^* = y - \ln z$ is the rapidity of the heavy quark, $p_T$ is the transverse momentum of the produced $D$-meson, $D_i(z)$ is the fragmentation function, which describes the parton $i$ fragmentation into a heavy meson, and $d\sigma_{Q_iQ_i}$ is the cross-section of a heavy quark production with a rapidity $y^*$, discussed below in Subsection [IIA]. The dominant contribution to all heavy mesons stems from the $c$- and $b$-quarks (prompt and non-prompt mechanisms respectively), so the $d\sigma_{Q_iQ_i}$ might be evaluated in the heavy quark mass limit. The fragmentation functions for the $D$- and $B$-mesons, as well as non-prompt $J/\psi$ production, are known from the literature and for the sake of completeness are given in Appendix [B].

In Figure 1 we also included colored oval blobs, which stand schematically for the secondary interactions which potentially could fill the large rapidity gap in the final state. The general framework for the evaluation of the rapidity gap survival factors (i.e. the probability that no particles will be produced in a rapidity gap) has been developed in [44–48], and is briefly discussed below in Section [IIB].
The single diffractive production of on-shell heavy quark pair in the reference frame of the recoil proton might be viewed as a fluctuation of the incoming virtual gluon into a heavy $Q\bar{Q}$ pair, with subsequent elastic scattering of the $Q\bar{Q}$ dipole on the target proton. In perturbative QCD the dominant contribution to such process is given by the diagram which includes exchange of a single pomeron between $Q\bar{Q}$ and a recoil proton, in the spirit of the Ingelman-Schlein model [49] (see Figure 2 for details). In LHC kinematics the typical light-cone momentum fractions $x_{1,2}$ carried by gluons are very small ($\ll 1$), so the gluon densities are enhanced in this kinematics. This enhancement modifies some expectations based on the heavy quark mass limit. For example, there could be sizeable corrections from multiple pomeron exchanges between the heavy dipole and the target. For this reason instead of hard process on individual partons it is more appropriate to use the color dipole framework (also known as CGC/Sat) [50–58]. At high energies the color dipoles are eigenstates of interaction, and thus can be used as the universal elementary building blocks automatically accumulating both the hard and soft fluctuations [59]. The light-cone color dipole framework has been developed and successfully applied to phenomenological description of both hadron-hadron and lepton-hadron collisions [60–67]. Another advantage of the CGC/Sat (color dipole) framework is that it allows a relatively straightforward extension for the description of high-multiplicity events, as discussed in [26, 68–74]. The cross-section of the single diffractive process, shown in Figure 2, in the dipole approach is given by

$$
\frac{d\sigma_{Q\bar{Q}, Q}(y, \sqrt{s})}{dy \, d^2p_T} = \int d^2k_T \frac{d^2r_1}{4\pi} \int d^2r_2 \frac{d^2z}{4\pi} e^{i(r_1-r_2)\cdot k_T} \Psi^\dagger_{Q\bar{Q}}(r_2, z, p_T) \Psi_{Q\bar{Q}}(r_1, z, p_T) \times N_{M}^{(SD)}(x_2(y); \vec{r}_1, \vec{r}_2) + (x_1 \leftrightarrow x_2),
$$

where $y$ and $p_T$ are the rapidity and transverse momenta of the produced heavy quark, in the center-of-mass frame of the colliding protons; $k_T$ is the transverse momentum of the heavy quark; $g(x_1, p_T)$ in the first line of (2) is the unintegrated gluon PDF; $\Psi_{g\rightarrow Q\bar{Q}}(r, z)$ is the light-cone wave function of the $Q\bar{Q}$ pair with transverse separation between quarks $r$ and the light-cone fraction of the momentum carried by the quark $z$. For $\Psi_{g\rightarrow Q\bar{Q}}(r, z)$ we use
standard perturbative expressions \[3\]

\[
\Psi_T^I (r_2, z, Q^2) \Psi_T (r_1, z, Q^2) = \frac{\alpha_s N_c}{2\pi^2} \left\{ \epsilon^2 r_1 (\epsilon_f r_1) K_1 (\epsilon_f r_2) \right\} \left[ e^{i\eta_1 z^2} + e^{-i\eta_1 (1-z)^2} \right] \tag{4}
\]

\[
+ m_f^2 K_0 (\epsilon_f r_1) K_0 (\epsilon_f r_2) \right],
\]

\[
\Psi_L^I (r_2, z, Q^2) \Psi_L (r_1, z, Q^2) = \frac{\alpha_s N_c}{2\pi^2} \left\{ 4Q^2 z^2 (1-z)^2 K_0 (\epsilon_f r_1) K_0 (\epsilon_f r_2) \right\}, \tag{5}
\]

\[
e^{2} = z(1-z) Q^2 + m_f^2 \tag{6}
\]

\[
\left| \Psi^{(f)} (r, z, Q^2) \right|^2 = \left| \Psi^{(f)} (r, z, Q^2) \right|^2 + \left| \Psi^{(f)} (r, z, Q^2) \right|^2 \tag{7}
\]

The meson production amplitude \(N_M\) depends on the mechanism of the \(Q\bar{Q}\) pair formation. For the case of the single-diffractive production, as we demonstrate in the Appendix \[A\] the contribution to the cross-section is given by

\[
N_M^{(SD)} (x, z, r_1, r_2) \approx \int d^2 b \left[ N_+ (x, z, r_1, b) \left( \frac{N_c}{4} \right) + N (x, r_1, b) \left( \frac{N_c^2 - 4}{4N_c} + \frac{1}{6} \right) \right] \times
\]

\[
\times \left[ N_+ (x, z, r_2, b) \left( \frac{N_c}{4} \right) + N (x, r_2, b) \left( \frac{N_c^2 - 4}{4N_c} + \frac{1}{6} \right) \right]. \tag{8}
\]

where

\[
N_+ (x, z, r, b) \equiv 2N (x, zr, b) + 2N (x, \bar{z}r, b) - \frac{1}{2}N (x, r, b), \tag{9}
\]

and \(N (x, r, b)\) is the color singlet dipole cross-section with explicit dependence on impact parameter \(b\).

In the heavy quark mass limit the main contribution to the integrals in \[2\] comes from small dipoles of size \(r \lesssim m_Q^{-1}\). In widely used phenomenological dipole parametrizations \[75-78\] it is expected that the \(b\)- and \(r\)-dependence factorize in this limit,

\[
N (x, r, b) \approx N (x, r) T(b), \tag{10}
\]

where the transverse profile \(T(b)\) is normalized as \(\int d^2 b T(b) = 1\), and \(N (x, r)\) is the dipole cross-section integrated over impact parameter. In this approximation we may rewrite \[6\] as

\[
N_M^{(SD)} (x, z, r_1, r_2) \approx \kappa \left[ N_+ (x, z, r_1) \left( \frac{N_c}{4} \right) + N (x, r_1) \left( \frac{N_c^2 - 4}{4N_c} + \frac{1}{6} \right) \right] \times
\]

\[
\times \left[ N_+ (x, z, r_2) \left( \frac{N_c}{4} \right) + N (x, r_2) \left( \frac{N_c^2 - 4}{4N_c} + \frac{1}{6} \right) \right], \tag{11}
\]

where

\[
N_+ (x, z, r) \equiv \int d^2 b N_+ (x, z, r, b) = 2N (x, zr) + 2N (x, \bar{z}r_1) - \frac{1}{2}N (x, r), \tag{12}
\]

\[
\kappa = \int d^2 b T^2 (b). \tag{13}
\]

As could be seen from the structure of \[6\], it is a higher twist (\(\sim O (r^2)\)) contribution compared to the amplitude of inclusive production, and thus should have stronger suppression at large \(p_T\).

The \(p_T\)-integrated cross-section gets contributions only from dipoles with \(r_1 = r_2 = \bar{r}\) in the integrand. For this case it is possible to show that the gluon uPDF \(x_1 g (x_1, p_T - k_T)\) is replaced with the integrated gluon PDF \(x_g G (x_g, \mu_F)\) taken at the scale \(\mu_F \approx 2m_Q\). In the LHC kinematics at central rapidities this scale significantly exceeds the saturation scale \(Q_s (x)\), which justifies the dominance of the three-pomeron approximation. However, in the small-\(x\) kinematics there are sizeable nonlinear corrections to the evolution in the dipole approach. In this
kinematics the corresponding scale \( \mu_F \) should be taken at the saturation momentum \( Q_s \). The gluon PDF \( x_1 G(x_1, \mu_F) \) in this approach is closely related to the dipole scattering amplitude \( N(x, r) = \int d^2b N(x, r, b) \) as \[ 68, 79 \]

\[
\frac{C_F}{2\pi^2 \alpha_S} N(x, r) = \int \frac{d^2k_T}{k_T^2} \phi(x, k_T) \left( 1 - e^{ik_T \cdot r} \right); \quad xG(x, \mu_F) = \int_0^{\mu_F} \frac{d^2k_T}{k_T^2} \phi(x, k_T),
\]

Eq. (14) can be inverted and gives the gluon uPDF in terms of the dipole amplitude,

\[
xG(x, \mu_F) = \frac{C_F \mu_F}{2\pi^2 \alpha_S} \int d^2r \frac{J_1(r \mu_F)}{r} \nabla_r^2 N(x, r).
\]

The corresponding unintegrated gluon PDF can be rewritten as \[ 80 \]

\[
x g(x, k^2) = \left. \frac{\partial}{\partial \mu_F} xG(x, \mu_F) \right|_{\mu_F^2 = k^2},
\]

which allows to express the single diffraction cross-section in terms of only the dipole amplitude. The expression (16) will be used below in Section IV for extension of our results to high-multiplicity events.

**B. Gap survival factors**

The rapidity gap between the recoil proton and the produced heavy meson might be filled potentially by products of various secondary processes, as shown schematically by the colored vertical and inclined ovals in Figure 1. As was demonstrated in \[ 44, 47 \], the effect of these factors is significant at high energies and might decrease the observed yields (i.e., probability of non-observation of particles in the gap) by more than an order of magnitude \[ 47, 48 \]. This suppression is due to soft interactions between the colliding protons and thus is not related to the particles produced due to hard interactions. The evaluation of this suppression conventionally follows the ideas of Good-Walker \[ 81 \], which are usually implemented in the context of different models (see for review \[ 82–85 \]). Technically, all these approaches perform evaluations in eikonal approximation, and predict that the observables, which include large rapidity gaps, are suppressed by a so-called gap survival factor,

\[
\langle s^2 \rangle = \frac{\int d^2b \left| \mathcal{M}(b, s, ...) \right|^2 \exp \left( -\hat{\Omega}(b, s) \right)}{\int d^2b \left| \mathcal{M}(b, s, ...) \right|^2},
\]

where \( \mathcal{M}(b, s, ...) \) is the amplitude of the hard process, \( b \) is the impact parameter, and \( \Omega \) is the opacity or optical density. In a single-channel eikonal model the opacity \( \Omega \) is directly related to the cross-sections of total, elastic and inelastic processes \[ 83 \]. It is expected that the energy dependence of the function \( \Omega \) is controlled by the Pomeron intercept, \( \Omega \sim e^{\sigma_{Pom}} \), so the factor (17) decreases as a function of energy. The single-channel model is very simple, yet its predictions are at tension with experimental data \[ 48 \]. More accurate description of data is achieved in multichannel extensions of these models, which assume that after interaction with a soft Pomeron the proton might convert into additional \( N_D - 1 \) diffractive states. In this basis, the soft pomeron interaction amplitude \( \Omega \) should be considered as an \( N_D \times N_D \) matrix. As was discussed in \[ 82, 84 \], for a good description it is sufficient to choose \( N_D = 2 \), with the common parametrization for the matrix \( \Omega_{ik} \) given in \[ 86 \] and briefly summarized for the sake of completeness in Appendix C. For the single diffractive scattering the exponent in the expression (17) should be understood as a matrix element between \( pp \) and \( pX \) states \[ 87, 88 \]. If \( \Phi_1 \) and \( \Phi_2 \) are eigenvalues of \( \Omega_{ik} \) with eigenvalues \( \Omega_1 \) and \( \Omega_2 \), then the matrix \( \exp \left( -\hat{\Omega}(b, s) \right) \) reduces in this basis to a linear combination of factors \( \sim e^{-\Omega_{1,2}(s,b)} \), in which the coefficients can be fixed by projecting the proton and diffractive states onto the eigenstates \( \Phi_1, \Phi_2 \) of the scattering matrix. For the single diffractive production the algorithm for evaluation of the survival factor was introduced earlier for the \( pp \rightarrow pX \) process in \[ 88 \], yielding

\[
\exp \left( -\hat{\Omega}(b, s) \right) \rightarrow S^2 (s_{pp}, b) = \frac{1}{4(1 + \lambda^2)} \left( (1 + \lambda)^3 e^{-(1+\lambda)^2 \Omega} + (1 - \lambda)^3 e^{-(1-\lambda)^2 \Omega} + 2 (1 - \lambda^2) e^{-(1-\lambda^2) \Omega} \right),
\]

where parameter \( \Omega \) is related to eigenvalues \( \Omega_{1,2} \) of the matrix \( \Omega_{ik} \) as

\[
\Omega = \frac{\Omega_1 + \Omega_2}{2},
\]
and the parameter $\lambda$ stands for the ratio of the production amplitude of diffractive state $X$ to the amplitude of elastic proton scattering of the incident proton on a pomeron (see Appendix C for more details). In this paper we are interested only in events without charged particles, produced at pseudorapidity $\eta < y$ (rapidity gap between the recoil proton and heavy quarks), whereas the evaluation of the survival factor in (17) was performed under the assumption that there are no co-produced particles in the whole rapidity range $\eta \in (-y_{\text{max}}, y_{\text{max}})$, which is much stricter than needed in this problem. For this reason we need to correct the estimate (18), using probabilistic considerations. In what follows we’ll use notations $P_A$ and $P_B$ for the probabilities to emit at least one charged particle in the intervals $\eta < y$ and $\eta > y$ due to soft interaction of the colliding protons; while $P_A \equiv 1 - P_A$ and $P_B \equiv 1 - P_B$ are the probabilities not to emit any particles in these intervals (the gap survival factors on these intervals). We will also use the notation $P_{A\cup B}$ for the probability not to produce particles in any of the intervals. The relation between the probabilities $P_{A\cup B}$ and $P_A, P_B$ depends crucially on possible correlations between particles from different rapidity intervals. Such correlations have been studied in the literature [89–91], and it is known that they are small when the separation between the bins is larger than 1-2 units in rapidity. If we neglect completely such correlations, the probabilities are related as $P_{A\cup B} = P_A P_B$, which implies that the survival factor should scale with the length of the rapidity bin as $S^2(\Delta \eta) \sim \text{const}^{\Delta \eta}$. For the single diffractive production of heavy mesons we require that no particles are produced with $\eta < y$, although we do not impose any conditions for $\eta > y$ (so we do not need to introduce the gap survival factor in this region). This implies that the overall survival factor (18) should be adjusted as

$$S^2 \to S^2(s_{pp}, b) = (S^2)^{\frac{\Delta y}{y_{\text{max}}} \geq S^2},$$

where $\Delta y$ is the width of the rapidity gap interval, and $y_{\text{max}} = -\frac{1}{2} \ln(m_Q^2 x/s)$ is the largest possible rapidity of heavy quarks. This factor $S^2(s_{pp}, b)$ should be included into the expressions (20) from the previous Section (IIA).

In the heavy quark mass limit the dipoles are small, $r \lesssim m_Q^{-1}$, and we may use a factorized approximation (10). The convolution of $S^2(s_{pp}, b)$ with impact parameter dependent cross-section can be simplified in this limit and yields for the suppression factor a much simpler expression

$$\langle S^2 \rangle \approx \frac{\int d^2b T^2(b) S^2(s_{pp}, b)}{\int d^2b T^2(b)},$$

which depends only on the energy (Mandelstam variable) $s_{pp}$ of the collision, but does not depend on masses nor kinematics of the produced heavy quarks.

### III. NUMERICAL RESULTS

For our numerical evaluations here and in what follows we will use the impact parameter $(b)$ dependent “bCGC” parametrization of the dipole cross-section [77, 78]

$$N(x, r, b) = \begin{cases} N_0 \left( \frac{r Q_s(x)}{2} \right)^{2 \gamma_{\text{eff}}(r)}, & r \leq \frac{2}{Q_s(x)} \frac{1}{s}, \\ 1 - \exp(-A \ln(B r Q_s)), & r > \frac{2}{Q_s(x)} \frac{1}{s}, \end{cases}$$

$$A = -\frac{N_0^2 \gamma_s^2}{(1 - N_0)^2 \ln(1 - N_0)}; \quad B = \frac{1}{2} (1 - N_0)^{-\frac{1 - N_0}{\gamma_s}},$$

$$Q_s(x, b) = \left( \frac{x_0}{x} \right)^{\lambda/2} T_G(b), \quad \gamma_{\text{eff}}(r) = \gamma_s + \frac{1}{\kappa \lambda Y} \ln \left( \frac{2}{r Q_s(x)} \right);$$

$$\gamma_s = 0.66, \quad \lambda = 0.206, \quad x_0 = 1.05 \times 10^{-3}, \quad T_G(b) = \exp \left( -\frac{b^2}{2 \gamma_s B_{\text{CGC}}} \right).$$

In Figures 3 and 5 we show the production cross-sections of the $D$-mesons, $B$-mesons and non-prompt $J/\psi$ mesons. We can see that in the small-$p_T$ region, which encompasses most of the events, the single diffraction production constitutes approximately one per cent of the inclusive cross-section. In the large-$p_T$ region the contribution from the single diffractive production is strongly suppressed since it is formally a higher twist effect.

To the best of our knowledge there is no direct experimental data for the cross-sections of the suggested process. The diffractive production of $B$-mesons has been studied earlier by the CDF collaboration in [15], although the results are only available for the ratio of the integrated cross-sections of diffractive and inclusive processes,

$$R_{bb}^{(\text{diff.})}(s) \equiv \frac{\sigma_{BB}^{\text{diff}}(s)}{\sigma_{BB}^{\text{incl}}(s)}.$$
Figure 3: The cross-section $d\sigma/dp_T$ of the single diffractive production of $D^+$-mesons. Integration over the rapidity bin $|y| < 0.5$ is implied. Left plot: Comparison with inclusive production in the LHC kinematics for $\sqrt{s} = 7$ TeV (theory and experiment). The curves with labels “SD, prompt” and “SD, non-prompt” correspond to single diffractive contributions to $D$-meson yields from the fragmentation of the $c$ and $b$ quarks respectively. The curves marked “2-pomeron inclusive” and “3-pomeron inclusive” stand for the contributions of 2- and 3-pomeron fusion mechanisms to inclusive $D$-meson yields respectively (see a short overview in Appendix A2 and more detailed discussion in [10]). The experimental data are for inclusive production from [92]. Right plot: $\sqrt{s}$-dependence of the data in the kinematics of LHC and the planned Future Circular Collider (FCC) [93]. For other $D$-mesons the $p_T$-dependence has a very similar shape, yet differs numerically by a factor of two.

Figure 4: Cross-section for the single diffractive $B^\pm$-mesons production. Left plot: Comparison of single diffractive predictions with inclusive cross-sections (experimental and theoretical results). The theoretical curves marked “2-pomeron incl.” and “3-pomeron incl.” stand for the additive contributions from 2- and 3-pomeron fusion mechanisms respectively (see [10] and a short discussion in Appendix A2). The experimental data are for inclusive production from CMS [94] (“$|y| < 2.1$’ data points) and ATLAS [95] (“$|y| < 0.5$’ data points). For some experimentally measured results bin-integrated cross-sections $d\sigma/dp_T$ was converted into $d\sigma/dp_T dy$ dividing by the width of the rapidity bin (this is justified since in LHC kinematics at central rapidities $y \approx 0$ the cross-section is flat). Right plot: The $p_T$-dependence of the cross-section $d\sigma/dy dp_T$ for several energies $\sqrt{s}$.

For energy $\sqrt{s} = 1.8$ TeV it was found that

$$R_{bb}^{(\text{diff.})}(\sqrt{s} = 1.8 \text{ TeV}) = (0.62 \pm 0.19 \pm 0.16)\%.$$

(27)

In the Table 4 we present our theoretical expectations for this value. For Tevaton kinematics the model prediction $R_{bb}^{(\text{diff.})} \approx 0.4\%$ agrees with (27), within uncertainty of experimental data (27). As we can see from the same Table 4 in LHC kinematics the ratio (26) is approximately of the same order. The smallness of the values in the Table 4 is due to the fact that the production of heavy quark in single diffraction events is formally a higher twist effect, and thus has an additional suppression by the factor $\sim (\Lambda_{QCD}/m_Q)^7$. While the absolute cross-sections of single diffractive and inclusive production increase as a function of energy, the ratio (27) slowly decreases due to energy dependence of
In Figure 5 we compare our predictions with earlier results from [11] obtained in the framework of Ingelman-Schlein model. As we can expect, the non-prompt mechanism is smaller than the prompt contribution, although the ratios are similar for other choices of $p_T$. For the sake of definiteness we considered $J/\psi$-mesons production. Left plot: Comparison of single diffractive and inclusive productions cross-sections, as defined in (26), in Tevatron and LHC kinematics. The second and the third columns correspond to the additive contributions from 2- and 3-pomeron fusion mechanisms respectively (see [40] and a short discussion in Appendix A2). The experimental data are for inclusive production from CMS [96]. Right plot: The $p_T$-dependence of the cross-section $d\sigma/dydp_T$ for several energies $\sqrt{s}$.

$$\sqrt{s} \begin{array}{ccc} R^{(\text{diff})}_{J/\psi} & R^{(\text{diff})}_{J/\psi} \end{array} \begin{array}{c} R^{(\text{incl})}_{J/\psi} \end{array}$$

| $\sqrt{s}$   | $R^{(\text{diff})}_{J/\psi}$ | $R^{(\text{incl})}_{J/\psi}$ |
|--------------|-------------------------------|-------------------------------|
| 1.8 TeV      | 2.20%                         | 0.40%                         |
| 7 TeV        | 1.87%                         | 0.33%                         |
| 13 TeV       | 1.50%                         | 0.30%                         |

Table I: Values of the ratio of single diffractive and inclusive productions cross-sections, as defined in (26), in Tevatron and LHC kinematics. The second and the third columns correspond to the additive contributions from 2- and 3-pomeron fusion mechanisms respectively. The last column $R^{(\text{diff})}_{J/\psi}$ is for the non-prompt $J/\psi$ production.

We extended the definition (28) and analyzed the ratio of differential cross-sections,

$$R_M^{(\text{diff})}(s, y, p_T) \equiv \frac{d\sigma_M^{\text{diff}}/dydp_T}{d\sigma_M^{\text{incl}}/dydp_T}, \quad M = D^\pm, B^\pm, ...,$$

which presents a novel observable. In Figure 6 we show this ratio as a function of $p_T$ for $D$-mesons, both for prompt and non-prompt mechanisms. For the sake of definiteness we considered $D^\pm$ mesons, although the results for the ratio (28) are almost the same for other choices of $D$-mesons. In Figure 7 we show the same ratio for the $B$-mesons ($B^+$ for definiteness) and non-prompt $J/\psi$. We can see that the ratio is smaller than for $D$-mesons, and decreases quite fast at large $p_T$. This behavior agrees with our earlier observation that the single-diffractive mechanism is formally a higher twist effect compared to the dominant two-gluon fusion mechanism, in the case of inclusive production. As expected, at small $p_T$ the ratios are similar for $B$-mesons and non-prompt $J/\psi$; for larger $p_T$ the results differ due to differences in fragmentation functions (see Appendix B for details).

In Figure 8 we compare our results for non-prompt production of $J/\psi$ with the predictions for prompt production from [5] (color octet contributions + gluon fragmentation, dominant at large $p_T$) and from [4] (color evaporation model). As we can expect, the non-prompt mechanism is smaller than the prompt contribution, although the qualitative behavior is similar in both cases.

In Figure 9 we compare our predictions with earlier results from [11] obtained in the framework of Ingelman-Schlein model. We can see that in the region $p_T \lesssim 5$ GeV, where a majority of heavy mesons are produced, both approaches give comparable contributions. At larger $p_T$ the discrepancy between the two approaches increases.

Finally, we would like to stop briefly on the ratio $R^{(\text{diff})}_{J/\psi}$ of single diffractive and inclusive contributions. It was predicted in [6] that for the prompt contributions $R^{(\text{diff, prompt})}_{J/\psi} \approx 0.65 \pm 0.15\%$, although later the CDF collaboration [14] found a value twice larger

$$R^{(\text{diff, CDF})}_{J/\psi} \approx 1.45 \pm 0.25\%$$

(29)
Figure 6: The ratio of single diffractive to inclusive production cross-sections, as defined in (28). The left plot corresponds to the prompt production (from $c \rightarrow D$ fragmentation), and the right plot is for the non-prompt mechanism (from $b \rightarrow D$ fragmentation). For the sake of definiteness we considered $D^+$ mesons; for other $D$-mesons the results have a very similar shape.

Figure 7: The ratio of single diffractive to inclusive production cross-sections, as defined in (28). The left plot is for the $B$ mesons, the right panel is for non-prompt production of $J/\psi$-mesons.

This mismatch might be explained by sizeable non-prompt contributions: combining $R_{J/\psi}^{(\text{diff, prompt})}$ with $R_{J/\psi}^{(\text{diff, non-prompt})}$ from the first line in Table I, we get $R_{J/\psi}^{(\text{diff, prompt+nonprompt})} \approx 1.22\%$, in reasonable agreement with the experimental value (29).

\section*{IV. MULTIPLICITY DEPENDENCE}

According to the Local Parton Hadron Duality (LPHD) hypothesis \cite{97,98}, the multiplicity of produced hadrons in a given event is directly related to the number of partons produced in a collision. For this reason the study of multiplicity dependence of different processes presents an interesting extension, which allows to understand better the onset of the saturation regime in high energy collisions. A feasibility to measure such processes was demonstrated for inclusive channels by the STAR \cite{32,100} and ALICE \cite{31,101} collaborations. The extension of these experimental measurements to single diffractive production is quite straightforward, since their detectors have the capability to detect simultaneously both the rapidity gaps and the charged particles outside of the rapidity window. Since the cross-section of single diffractive production is significantly smaller than that of inclusive production, and the probability of events with large multiplicity is exponentially suppressed \cite{101}, each measurement will require larger integrated luminosity.

In order to get rid of a common exponential suppression at large multiplicities, for a comparison of the multiplicity
In the color dipole (CGC/Sat) approach, the framework for description of the high-multiplicity events has been developed in \cite{26,68–74}. In this picture the observation of enhanced multiplicity signals that a larger than average number of partons is produced in a given event. Nevertheless, we still expect that each pomeron should satisfy the

dependence in different channels it is widely accepted to use a self-normalized ratio \cite{102}

\[
\frac{dN}{d\eta} = \frac{\langle w (N_M) \rangle}{\langle w (N_M) \rangle} \times \frac{\langle w (N_{ch}) \rangle}{\langle w (N_{ch}) \rangle} \times \frac{d\sigma_M (y, \eta, \sqrt{s}, n) / dy}{d\sigma_M (y, \eta, \sqrt{s}, n = 1) / dy} / \frac{d\sigma_{ch} (\eta, \sqrt{s}, Q^2, n) / d\eta}{d\sigma_{ch} (\eta, \sqrt{s}, Q^2, n = 1) / d\eta}
\]

(30)

where $\langle N_{ch} \rangle = \Delta \eta dN_{ch}/d\eta$ is the average number of particles detected in a given pseudorapidity window $(\eta – \Delta \eta/2, \eta + \Delta \eta/2)$, $n = N_{ch}/\langle N_{ch} \rangle$ is the relative enhancement of the number of charged particles in the same pseudorapidity window, $w (N_M) / \langle w (N_M) \rangle$ and $w (N_{ch}) / \langle w (N_{ch}) \rangle$ are the self-normalized yields of heavy meson $M$ ($M = D, B$) and charged particles (minimal bias events) in a given multiplicity class; $d\sigma_M (y, \sqrt{s}, n)$ is the production cross-sections for heavy meson $M$ with rapidity $y$ and $N_{ch} = n \langle N_{ch} \rangle$ charged particles in the pseudorapidity window $(\eta – \Delta \eta/2, \eta + \Delta \eta/2)$, whereas $d\sigma_{ch} (y, \sqrt{s}, n)$ is the production cross-sections for $N_{ch} = n \langle N_{ch} \rangle$ charged particles in the same pseudorapidity window. Mathematically the ratio \cite{30} gives a conditional probability to produce a meson $M$ in a single diffractive collision in which $N_{ch}$ charged particles are produced.

In the color dipole (CGC/Sat) approach, the framework for description of the high-multiplicity events has been developed in \cite{26,68,74}. In this picture the observation of enhanced multiplicity signals that a larger than average number of partons is produced in a given event. Nevertheless, we still expect that each pomeron should satisfy the

dependence in different channels it is widely accepted to use a self-normalized ratio \cite{102}.

\[
\frac{dN}{d\eta} = \frac{\langle w (N_M) \rangle}{\langle w (N_M) \rangle} \times \frac{\langle w (N_{ch}) \rangle}{\langle w (N_{ch}) \rangle} \times \frac{d\sigma_M (y, \eta, \sqrt{s}, n) / dy}{d\sigma_M (y, \eta, \sqrt{s}, n = 1) / dy} / \frac{d\sigma_{ch} (\eta, \sqrt{s}, Q^2, n) / d\eta}{d\sigma_{ch} (\eta, \sqrt{s}, Q^2, n = 1) / d\eta}
\]
nonlinear Balitsky-Kovchegov equation. The bCGC dipole amplitude \( \Omega \) was constructed as an approximate solution of the latter, and for this reason it should maintain its form, although the value of the saturation scale \( Q_s \) might be modified. As was demonstrated in [68–70], the observed number of charged multiplicity \( dN_{ch}/dy \) of soft hadrons in \( pp \) collisions is proportional to the saturation scale \( Q_s^2 \) (modulo logarithmic corrections), for this reason the events with large multiplicity might be described in dipole framework by simply rescaling \( Q_s^2 \) as a function of \( n \) [68–74],

\[
Q_s^2(x, b; n) = n Q_s^2(x, b).
\]

It was demonstrated in [26] that the error of the approximation (31) is less than 10% in the region of interest \( n \lesssim 10 \), and for this reason we will use it for our estimates. While at LHC energies it is expected that the typical values of saturation scale \( Q_s(x, b) \) fall into the range 0.5–1 GeV, from (31) we can see that in events with enhanced multiplicity this parameter might exceed the values of heavy quark mass \( m_Q \) and lead to an interplay of large-\( Q_s \) and large-\( m_Q \) limits. The expression (31) explicitly illustrates that the study of the high-multiplicity events gives us access to a new regime, which otherwise would require significantly higher energies.

The observation of enhanced multiplicity in the process shown in the left diagram of Figure 1 implies that unintegrated gluon density \( g(x, k_{\perp}, n) \) in (2) is also modified. This change might be found taking into account the relation of gluon density with the dipole amplitude \( N(x, r, b) \) given by (16). For the sake of simplicity below we’ll focus on the multiplicity dependence of the \( p_T \)-integrated cross-section, which is easier to measure experimentally. For this case the cross-section (2) simplifies considerably, since, after integration over \( p_T \), the multiplicity dependent (integrated) gluon density factorizes and contributes to the result as a multiplicative factor. For this reason the ratio \( \Omega \) reduces to a common factor

\[
\frac{dN_M}{dy} = \frac{\int d^2r \frac{J_\gamma(x, y, b)}{r} \nabla^2 N(y, r, b)}{\int d^2r \frac{J_\gamma(x, y, b)}{r} \nabla^2 N(y, r, 1)}
\]

the same for all mesons. In Figure 10 we show the multiplicity dependence of the ratio (32). At very small \( n \), when saturation effects are small, the size of the dipole is controlled by the mass of heavy quark \( \sim 1/m_Q \), and thus the dipole amplitude \( N(y, r, n) \) might be approximated as \( N(y, r, n) \sim (r Q_s(y, n))^{\gamma} \), where \( \gamma \approx 0.63 - 0.76 \) is a numerical parameter. In view of (31) this translates into the multiplicity dependence

\[
\frac{dN_M}{dy} \sim n^{\gamma},
\]

as shown in the same Figure 10 with red dotted line. At larger values of \( n \), due to saturation effects, the curve deviates from the small-\( n \) asymptotic behavior. As we can see from the right panel of the same Figure 10 this behavior is different from the dependence seen by ALICE for inclusive the production [31], as well as from our theoretical result for inclusive production from [10]. This happens because in single diffractive production the co-produced hadrons stem from only one cut pomeron, whereas in inclusive production, in the setup studied in [31], at least two pomerons can contribute to the observed multiplicity enhancement. Since each cut pomeron gives a factor \( \sim n^{\gamma} \) in multiplicity dependence, this explains the predicted difference between the single diffractive and inclusive processes.

V. NUCLEAR EFFECTS

The study of the single diffractive production on nuclear collisions is appealing because its cross-section grows rapidly with atomic number \( A \), and thus is easier to measure experimentally. The \( AA \) collisions are not suitable for this purpose due to formation of hot Quark-Gluon Plasma at later stages [103–109]. For this reason we will focus on \( pA \) collisions and in the kinematics when the scattered proton in the final state is separated by large rapidity gap from the produced heavy meson and nuclear debris.

In CGC framework the nucleus differs from the proton by larger size \( R_A = A^{1/3} R_p \) and larger values of the saturation scale \( Q_{sA}^2 \). As was found in [110] [111] from analysis of the experimental data, the dependence of \( Q_{sA}^2 \) on atomic number \( A \) might be approximated by

\[
Q_{sA}^2(x) \approx Q_s^2(x) A^{1/35}, \quad \delta \approx 0.79 \pm 0.02.
\]

The value \( \delta < 1 \) indicates that the saturation scale grows faster than \( A^{1/3} \) expected from naive geometric estimates. In single diffractive process the nucleus contributes in (2) only through the unintegrated gluon density \( g(x, k) \). Currently the latter is poorly defined experimentally [112], for this reason we will estimate it from the dipole
Figure 10: Left plot: Multiplicity dependence of open heavy flavor meson production cross-sections with single diffractive mechanism (the same for all mesons, see the text for explanation). The red dotted line corresponds to the asymptotic expression for small multiplicities, as explained in the text. Right plot: comparison of multiplicity dependence for non-prompt $J/\psi$ mesons. The experimental points are from ALICE [31] for inclusive production, the theoretical curve for inclusive production is from [40].

The magnitude of nuclear effects is conventionally expressed in terms of the normalized ratio of the cross-sections on the nucleus and proton,

$$R_A(y) = \frac{d\sigma_{pA \to pMX}/dy}{A d\sigma_{pp \to pMX}/dy}.$$ (35)

For the sake of simplicity we’ll focus on the $p_T$-integrated cross-section. In this case the dependence on the gluon PDF factorizes, and thus the ratio (35) reduces to a common prefactor

$$R_A(y) \approx \frac{g_A(x_1(y), \mu_F)}{g_N(x_1(y), \mu_F)} = \frac{1}{A} \int d^2b \int d^2r \frac{J_1(r \mu_F)}{r} \nabla^2 r N_A(y, r, b/A^{1/3}),$$ (36)

where $N_A(y, r, b)$ is a nuclear dipole amplitude with adjusted saturation scale [34], and the rescaling of the impact parameter $b$ in the numerator reflects the increase of the nuclear radius. In the Figure 11 we have shown the ratio (35) as a function of the atomic number $A$. We can see that due to nuclear (saturation) effects the cross-section decreases by up to a factor of two for very heavy nuclei. This finding is in agreement with expected suppression of nuclear gluon densities found in [112] from global fits of experimental data.

Finally, from comparison of (32) and (36) we may obtain the relation between the nuclear suppression factor $R_A$ and the multiplicity dependence of the proton cross-section (32),

$$A R_A(y, A) = \left. \frac{dN_M/dy}{\langle dN_M/\rangle_y} \right|_{n=(Q_A^2/Q^2)},$$

which might be checked experimentally.

VI. CONCLUSIONS

In this paper we studied single diffractive production of open heavy-flavor mesons. We analyzed in detail the production of $D$- and $B$-mesons, as well as non-prompt production of $J/\psi$ mesons. While in general diffractive events constitute up to 20 per cent of inclusive cross-section [1], we found that for heavy mesons production the single diffractive events constitutes only 0.4-2 per cent of all inclusively produced heavy mesons. This happens because the leading order contribution to single diffractive production is formally a higher twist effect (compared to leading order inclusive diagrams) and thus includes additional suppression $\sim (\Lambda_{QCD}/m_Q^2)$. Similarly, the observed
Figure 11: The nuclear suppression factor $R_A$ defined in (35) as a function of the atomic number $A$ for the $p_T$-integrated cross-section (the same for all mesons, see the text for explanation).

Figure 12: The diagrams which contribute to the heavy meson production cross-section in the leading order perturbative QCD. The contribution of the last diagram (c) to the meson formation might be also viewed as gluon-gluon fusion $gg \rightarrow g$ with subsequent gluon fragmentation $g \rightarrow QQ$. In CGC parametrization of the dipole cross-section approach each “gluon” is replaced with reggeized gluon (BK pomeron), which satisfies the Balitsky-Kovchegov equation and corresponds to a fan-like shower of soft particles.

suppression at large transverse momentum $p_T$ of the produced heavy meson agrees with expected pattern of higher twist suppression. Nevertheless, we believe that the cross-sections are sufficiently large and thus could be measured with reasonable precision at the LHC.

We also analyzed the dependence on multiplicity of co-produced hadrons, assuming that these are produced only on one side of the heavy meson. We found that the dependence on multiplicity is mild, in contrast to the vigorously growing multiplicity seen by ALICE [31] for inclusive production. Our evaluation is largely parameter-free and relies only on the choice of the parametrization for the dipole cross-section (22).

We expect that suggested processes might be studied by the CMS (see their recent feasibility study in [19]), ALICE [31, 101] and STAR collaborations.

Acknowledgements

We thank our colleagues at UTFSM university for encouraging discussions. This research was partially supported by the project Proyecto Basal FB 0821 (Chile) and Fondecyt (Chile) grant 1180232. Also, we thank Yuri Ivanov for technical support of the USM HPC cluster, where some evaluations were performed.
so the functions For very small dipoles, the dipole cross-section is related to the gluon uPDF as \[115\] of the high energy processes it is possible to express the exclusive amplitude or inclusive cross-section as a linear combination interactions of \(\bar{\gamma}\) shown in the Figure (13). As was explained at the beginning of this appendix, in the heavy quark mass limit the strong coupling \(\alpha_s(m_Q)\) is small, which allows to consider the interaction of a heavy \(QQ\) dipole with gluons perturbatively and discuss them similar to the treatment of the \(k_T\)-factorization approach. At the same time we tacitly assume that each such gluon should be understood as a parton shower ("pomeron").

In the high-energy eikonal picture, the interaction of the quarks and antiquark with a color singlet dipole cross-section might be found in \[50–58\]. In the heavy quark mass limit the strong coupling \(\alpha_s(m_Q)\) is small, which allows to consider the interaction of a heavy \(QQ\) dipole with gluons perturbatively and discuss them similar to the treatment of the \(k_T\)-factorization approach. At the same time we tacitly assume that each such gluon should be understood as a parton shower ("pomeron").

\[\Delta\sigma(x, r) \equiv \sigma(x, \infty) - \sigma(x, r) = \frac{1}{8} \int d^2b |\gamma(x, b - zr) - \gamma(x, b + \bar{z}r)|^2 \]

where \(r\) is the transverse size of the dipole, and \(z\) is the light-cone fraction of the dipole momentum carried by the quarks. The equation (A1) might be rewritten in the form

\[\frac{1}{8} \int d^2b \gamma(x, b) \gamma(x, b + r) = \frac{1}{2} \sigma(x, r) + \int d^2b |\gamma(x, b)|^2 - \frac{1}{2} \sigma(x, \infty). \]

For very small dipoles, the dipole cross-section is related to the gluon uPDF as \[115\]

\[\sigma(x, \vec{r}) = \frac{4\pi\alpha_s}{3} \int \frac{d^2k_\perp}{k_\perp^2} F(x, k_\perp) (1 - e^{ik\cdot r}) + O\left(\frac{\Lambda_{\text{QCD}}}{m_c}\right), \]

so the functions \(\gamma(x, r)\) might be also related to the unintegrated gluon densities. With the help of (A2), for many high energy processes it is possible to express the exclusive amplitude or inclusive cross-section as a linear combination of the color singlet dipole cross-sections \(\sigma(x, r)\) with different arguments. While in the deeply saturated regime we can no longer speak about individual gluons (or pomerons), we expect that the relations between the dipole amplitudes and color singlet cross-sections should be valid even in this case.

For the case of single-diffractive heavy quark pair production, the leading-order contribution is given by the diagrams shown in the Figure (13). As was explained at the beginning of this appendix, in the heavy quark mass limit the interactions of \(QQ\) with gluons become perturbative, which implies that the \(t\)-channel pomeron might be considered as a color singlet pair of gluons. Taking into account all the diagrams shown in the Figure (13) and properties of the \(SU(N_c)\) structure constants, we may express the amplitude of the single diffractive process as

\[A^{(3)}(x, \vec{r}_Q, \vec{r}_{\bar{Q}}) = \left[\frac{N_c}{4} \gamma_+^2 (x, \vec{r}_Q, \vec{r}_{\bar{Q}}) + \left(\frac{N_c^2 - 4}{4N_c} + \frac{1}{6}\right) \gamma_+^2 (x, \vec{r}_Q, \vec{r}_{\bar{Q}})\right] t_a
\equiv a(x, \vec{r}_Q, \vec{r}_{\bar{Q}}) t_a.\]
The three-pomeron contributions (diagram (a)) contribute at the same order in $\alpha_s$ as the interference of LO and NNLO diagrams (diagram (b)). In both plots the vertical dashed line is a unitary cut, lower blob is a target (proton), and all possible connections of pomerons (thick wavy lines) to the heavy $Q,\bar{Q}$ quark lines are implied. Note that in diagram (a) both pomerons are cut, whereas in case of the interference contribution one of the pomerons is uncut.

where

$$
\begin{align*}
\gamma_+ (x, \vec{r}_1, \vec{r}_2) &= \gamma (x, \vec{r}_1) + \gamma (x, \vec{r}_2) - 2\gamma \left( x, \frac{\vec{r}_1 + \vec{r}_2}{2} \right), \\
\gamma_- (x, \vec{r}_1, \vec{r}_2) &= \gamma (x, \vec{r}_1) - \gamma (x, \vec{r}_2),
\end{align*}
$$

$a$ is the color index of the incident (projectile) gluon, and $\vec{r}_Q, \vec{r}_{\bar{Q}}$ are the coordinates of the quarks. For evaluation of the $p_T$-dependent cross-section we need to project the coordinate space quark distribution onto the state with definite transverse momentum $p_T$, so we have for the evaluate the additional convolution

$$
\sim \int d^2 r_1 d^2 r_2 e^{ip_T \cdot (\vec{r}_1 - \vec{r}_2)},
$$

where

$$
\begin{align*}
\vec{r}_1, \vec{r}_2 &= \text{the coordinates of the quark in the amplitude and its conjugate, viz:} \\
\left| A^{(3)} (p_T) \right|^2 &= \left( 1 + \eta^2 \right) \int d^2 x_Q \int d^2 y_Q e^{ip_T \cdot (x_Q - y_Q)} \left( A^{(3)} (\vec{x}_i) \right)^* A^{(3)} (\vec{y}_i) \big| \vec{x}_Q = \vec{y}_Q \right. (A4)
\end{align*}
$$

As discussed earlier, at high energies we may apply iteratively the relation (A1) and express the three-pomeron dipole amplitude in terms of the color singlet dipole cross-sections, as given in (8). In the frame where the momentum of the primordial gluon is not zero, we should take into account an additional convolution with the momentum distribution of the incident (“primordial”) gluons, as shown in (2), and was demonstrated in (27).

### 2. Inclusive production

In Section [3] we compared predictions for single-diffractive production of heavy quarks with those of the inclusive production of the same mesons. For the sake of completeness, in this Appendix we would like to mention briefly the main expressions used for evaluation of the cross-sections for the latter case. A detailed discussion of inclusive production, as well as comparison with experimental data might be found in [10]. The evaluation of the cross-section follows the steps outlined in the previous Appendix [A1]. The leading order contribution in the inclusive case is due to a standard fusion of two gluons (pomerons). In the evaluation of the three-pomeron we should take into account that there are two complementary mechanisms, shown schematically in Figure [14]. In what follows we’ll refer to the contribution shown in the diagram (a) as genuine three-pomeron corrections, whereas the contribution of the diagram (b) is the interference term. The two diagrams differ by number of cut pomerons, and for this reason they have a different multiplicity dependence. As we discussed in [10], both twist-three corrections give sizeable contributions at small $p_T \lesssim 5$ GeV. For $D$-mesons the two corrections together contribute up to 40-50 per cent of the leading order result, whereas for $B$-mesons these contributions are of order 10% even for $p_T \sim 0$, in agreement with the heavy mass limit.
Both the leading order cross-section and the higher twist correction might be written as

\[
\frac{d\sigma_{pp \to Q,Q+X}}{dy d^2 p_T} = \int d^2 k_{T,x_1} g(x_1, p_T - k_T) \int_0^1 dz \int_0^1 dz' \frac{1}{4\pi} \int \frac{d^2 r_1}{4\pi} \int \frac{d^2 r_2}{4\pi} e^{i(r_1 - r_2) \cdot k_T} \Psi^\dag_{QQ} (r_2, z, p_T) \Psi^\dag_{QQ} (r_1, z, p_T) \times N_M (x_2(y); r_1, r_2) + (x_1 \leftrightarrow x_2),
\]

(see the Section II for notations and definitions). For the leading order contribution, the amplitude \(N_M\) is given by \[27, 40\]

\[
N_M^{(2)} (x, \vec{r}_1, \vec{r}_2) =
\]

\[
= -\frac{1}{2} N (x, \vec{r}_1 - \vec{r}_2) - \frac{1}{16} [N (x, \vec{r}_1) + N (x, \vec{r}_2)] - \frac{9}{8} N (x, \bar{z} (\vec{r}_1 - \vec{r}_2))
\]

\[
+ \frac{9}{16} [N (x, \bar{z} \vec{r}_1 - \vec{r}_2) + N (x, \bar{z} \vec{r}_2 - \vec{r}_1) + N (x, \bar{z} \vec{r}_1 + \vec{r}_2) + N (x, \bar{z} \vec{r}_2)].
\]

Similarly, the three-pomeron contribution shown in the diagram (a) of the Figure 14 may be rewritten as

\[
N_M^{(3)} (x, z, \vec{r}_1, \vec{r}_2) \approx \frac{1}{8\sigma_{\text{eff}}} \left[ N_+^2 (x, z, \vec{r}_1, \vec{r}_2) \left( \frac{3N_c^2}{8} \right) + N_+^2 (x, \vec{r}_1, \vec{r}_2) \left( \frac{43N_c^4 - 320N_c^2 + 720}{72N_c^2} \right) \right.
\]

\[
+ \left. \frac{(N_c^2 - 4)}{2} N_+ (x, z, \vec{r}_1, \vec{r}_2) N_-(x, \vec{r}_1, \vec{r}_2) \right].
\]

where

\[
N_- (x, \vec{r}_1, \vec{r}_2) \equiv -\frac{1}{2} [N (x, \vec{r}_2 - \vec{r}_1) - N (x, \vec{r}_1) - N (x, \vec{r}_2)]
\]

\[
N_+ (x, z, \vec{r}_1, \vec{r}_2) \equiv -\frac{1}{2} [N (x, \vec{r}_2 - \vec{r}_1) + N (x, \vec{r}_1) + N (x, \vec{r}_2)] + N (x, \bar{z} \vec{r}_1 - \vec{r}_2) + N (x, \bar{z} \vec{r}_1)
\]

\[
+ N (x, -\bar{z} \vec{r}_2 + \vec{r}_1) + N (x, -\bar{z} \vec{r}_2) - 2N (x, \bar{z} (\vec{r}_1 - \vec{r}_2))
\]

and \(\sigma_{\text{eff}} \approx 20\text{mb}\) is a numerical parameter. Finally, for the interference term shown in the diagram (b) of the Figure 14 we may get in a similar way

\[
N_M^{(\text{int})} (x, z, \vec{r}_1, \vec{r}_2) = -\frac{3}{16\sigma_{\text{eff}}} \left[ 2 N_+ (x, z, \vec{r}_1, \vec{r}_2) \bar{N}_+ (x, z, \vec{r}_2) \left( \frac{3N_c^2}{8} \right) + \right.
\]

\[
- N_- (z, \vec{r}_1, \vec{r}_2) \bar{N}_- (x, \vec{r}_2) \left( \frac{43N_c^4 - 320N_c^2 + 720}{72N_c^2} \right) +
\]

\[
+ \left. \left( \frac{N_c^2 - 4}{2} \right) \left( N_+ (z, \vec{r}_1, \vec{r}_2) \bar{N}_- (x, \vec{r}_2) + \bar{N}_+ (x, \vec{r}_2) N_- (z, \vec{r}_1, \vec{r}_2) \right) \right].
\]

**Appendix B: Fragmentation functions**

For the sake of completeness, in this appendix we briefly summarize the fragmentation functions used in our evaluations. Since the fragmentation functions are essentially nonperturbative and cannot be evaluated from first principles, currently their parametrization is extracted from the phenomenological fits of e+e− annihilation data. For the \(B\)-mesons the dominant contribution comes from the fragmentation of \(b\)-quarks, and for the fragmentation function of this process we used the parametrization from [21]

\[
D_{b \to B} (z, \mu_0) = N z^\alpha (1 - z)^\beta,
\]

(B1)
where $N = 56.4$, $\alpha = 8.39$, $\beta = 1.16$. The shape of parametrization (B1) is close to another widely used parametrization from [113]

$$D_{b \to B}(z, \mu_0) = \frac{N}{z \left(1 - \frac{1}{1 - \frac{\epsilon}{1 - z}}\right)^2}, \quad \epsilon \approx 0.0126 \quad (B2)$$

The production of non-prompt charmonia which stem from decays of the B-mesons might also be described using a fragmentation function, which is related to that of B-mesons as [25]

$$D_{b \to J/\psi}(z, \mu) = \int_z^1 dx D_{b \to B}(x, \mu^2) \times \frac{1}{\Gamma_B} \frac{d\Gamma}{dz}(z, P_B)$$

where $\Gamma_B \equiv 1/\tau_B$ is the total decay width of the B-meson, and the function $d\Gamma(z, P_B)/dz$ was evaluated in detail in [25]. In the Figure 13 we compare the fragmentation functions $D_{b \to B}$ and $D_{b \to J/\psi}$. These two functions differ by the branching fraction $Br_{B \to J/\psi} \approx 0.8\%$, and for this reason in order to facilitate comparison, we plotted the fragmentation functions normalized to unity, $\hat{D}(z) = D(z)/\int_0^1 dz D(z)$. As we can see, the distribution $D_{b \to J/\psi}$ is significantly wider than $D_{b \to B}$ and has a peak near smaller values of $z \approx 0.5$.

The $D$-mesons might be produced either from fragmentation of $c$-quarks (prompt mechanism) or from $b$-quarks (non-prompt mechanism). The fragmentation functions for both cases are available from [114],

$$D_{i \to D}(z, \mu_0) = N_i z^{-\left(1+\gamma^2\right)} \left(1 - z\right)^{\alpha} \exp \left(-\gamma^2/z\right), \quad i = b, c \quad (B4)$$

with parameters given in the Table II. Though the parameters for $D^+$ and $D^0$ in the table differ significantly, their fragmentation functions have very similar shapes and differ only by a factor of two in normalization.

### Appendix C: Parametrization for the matrix $\Omega_{ik}$

In this appendix we briefly summarize the parametrization of the soft pomeron scattering amplitude $\Omega_{ik}$ used in Section II.B. In the two-channel model it is assumed that in addition to proton there is another diffractive state $X$,
which might be produced instead of proton in inelastic processes (e.g., single diffractive, double diffractive). The matrix $\Omega_{ik}$ is thus a $2 \times 2$ matrix in the subspace which includes a proton and the diffractive state $X$.

For our evaluations we used a parametrization from [15], which has a form

$$\Omega_{ik}(b, s) = \int \frac{d^2q}{4\pi^2} e^{iq \cdot \tilde{\Omega}_{ik}(t = -q^2, s)}$$

and has been fitted using recent LHC data on elastic, single diffractive and double diffractive scattering.

\[\text{(C1)}\]

\[\text{(C2)}\]

\[\text{(C3)}\]

\[\text{(C4)}\]

\[\text{(C5)}\]

\[\text{(C6)}\]

\[\text{(C7)}\]

\[\text{(C8)}\]

\[\text{(C9)}\]

and has been fitted using recent LHC data on elastic, single diffractive and double diffractive scattering.

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