Non-perturbative gluon evolution, squeezing, correlations and chaos in jets

V.I. Kuvshinov, V.A. Shaparau
Institute of Physics, National Academy of Sciences of Belarus
F. Skaryna av. 68, 220072 Minsk, Belarus
E-mail: kuvshino@dragon.bas-net.by; shaporov@dragon.bas-net.by
V.V. Marmysh
Belorussian State University
F. Skaryna av. 4, 220050 Minsk, Belarus

Abstract

We study evolution of colour gluon states in isolated QCD jet at the non-perturbative stage. Fluctuations of gluons are less than those for coherent states under specific conditions. This fact suggests that there gluon squeezed states can arise. The angular and rapidity dependencies of the normalized second-order correlation function for present gluon states are studied at this stage of jet evolution. It is shown that these new gluon states can have both sub-Poissonian and super-Poissonian statistics corresponding to, respectively, antibunching and bunching of gluons by analogy with squeezed photon states.

We investigate the possibility of coexisting both squeezing and chaos using Toda criterion and temporal correlator analysis. It is shown that these effects may coexist under some conditions.

Key words: Squeezed states, non-perturbative stage, QCD jet, coherent states, correlation function, chaos.

PACS numbers: 42.50.Ar; 42.50.Dv; 05.45.Mt; 47.52.+j

1 Introduction

Many experiments at $e^+e^-, p\bar{p}, ep$ colliders are devoted to hadronic jet physics, since detailed studies of jets are important for better understanding and testing both perturbative and non-perturbative QCD and also for finding manifestations of new physics. Although the nature of jets is of a universal character, $e^+e^-$ annihilation stands out among hard processes, since jet events admit a straightforward and clear-cut separation in this process. In the reaction $e^+e^- \rightarrow$ hadron four evolution phases are recognized by various time and space scales. These are (I) the production of a quark-antiquark pair: $e^+e^- \rightarrow q\bar{q}$; (II) the emissions of gluons and quarks from primary partons — perturbative evolution of the quark-gluon cascade; (III) the non-perturbative evolution and the hadronization of quarks and gluons; (IV) the decays of unstable particles.

The second phase of $e^+e^-$ annihilation has been well understood and sufficiently accurate predictions for it have been obtained within the perturbative QCD (PQCD) [1, 2]. But predictions of the PQCD are limited by small effective coupling $\alpha(Q^2) < 1$ and third phase is usually taken into account either through a constant factor which relates partonic features with hadronic ones
(within local parton-hadron duality) or through the application of various phenomenological models of hadronization. As a consequence, theoretical predictions both for intrajet and for interjet characteristics remain unsatisfactory. For example, the width of the multiplicity distribution (MD) according to the predictions of PQCD is larger than the experimental one. The discrepancies between theoretical calculations and experimental data suggest that after perturbative stage the quark-gluon cascade undergoes non-perturbative evolution after that hadronization effects come into play. New gluon states, generated at the non-perturbative stage, contribute to various features of jets. For example, such a contribution to the multiplicity distribution can be in the form of the sub-Poissonian distribution [3, 4]. Therefore we must take into account both perturbative and non-perturbative stage of the jet evolution.

Calculations performed within PQCD [5, 6] show that multiplicity distribution at the end of the perturbative cascade is close to a negative binomial distribution. At the same time, gluon MD in the range of the small transverse momenta (thin ring of jet) is Poissonian [7]. Thus parton MD in the whole jet at the end of the perturbative cascade can be represented as a combination of Poissonian distributions each of which corresponds to a coherent state. Studying a further evolution of gluon states at the non-perturbative stage of jet evolution we obtain new gluon states that are squeezed states (SS) [8]-[10]. These states are formed as a result of non-perturbative self-interaction of the gluons expressed by nonlinearities of Hamiltonian. In this paper we prove that non-perturbative stage of jet evolution can be one of sources of a gluon SS by analogy with nonlinear medium for photon SS [11]-[14]. Squeezed states posses uncommon properties: they display a specific behaviour of the factorial and cumulant moments [15] and can have both sub-Poissonian and super-Poissonian statistics corresponding to antibunching and bunching of photons. Moreover oscillatory behaviour MD of photon SS is differentiated from Poissonian and Negative binomial distributions (NBD). Because of analogy between photon and gluon, MD of gluon SS must have oscillations and using Local parton hadron duality (LPHD) we can compare derived gluon MD with hadron MD. It is clear that in this case behaviour of hadron MD in jet events is differentiated from NBD and this fact is confirmed by experiments for $pp$, $p\bar{p}$-collisions [16]-[18].

In series of works [19]-[21] it was shown that presence of a chaos amplifies effect of squeezing. At the same time condition of appearance of chaos is sufficiently understood, basically, in classical systems [22]-[24]. Indeed, originally chaos was considered within classical and statistical physics. It was demonstrated that one of the causes of chaos is a local instability of a dynamical system [25] which can lead to mixing of trajectories in phase space and as result to non-regular behaviour of considered system [24].

Large progress in understanding of chaos was achieved in semi-classical limit of quantum mechanics [26, 27]. But not all criteria of the classical chaos are useful for quantum case. In particular, by virtue of Heisenberg uncertainty relation describing chaotic motion on the basis of exponentially separation of neighboring trajectories is not possible in quantum mechanics.

A keen interest to chaos in field theories is connected with the facts that all four fundamental particle interactions have chaotic solutions [28]. At the same time there are also some problems with definition of the chaos in the theory of field. In [29] it was shown that exponential decreasing of the Green function may serve as criterion of chaos both in the classical and quasiclassical theory of field. Therefore we assume that conditions of appearance of chaos for quantum field systems, where squeezing exists, can find by studying features of the correlators of the field operators, which are analogue of the Green functions.

Since the chaos phenomenon can be related with confinement [30] and with fractality for the factorial moments [31] then the question about condition of the appearance of the chaos in jet is important. In this connection we investigate SU(2)-jet model for the purpose of revealing of the local instability which can lead to chaos.
2 The colour evolution of gluon states

The solution of the Schrödinger evolution equation for small time $t$

$$|f\rangle \simeq |i\rangle - i t H_g |i\rangle$$

(1)

provides a possibility to observe an evolution of an initial state vector $|i\rangle$ for small time. Here the Hamiltonian $H_g$ has the form

$$\hat{H}_g = H_0 + V,$$

(2)

where $H_0 = \frac{1}{2} \int (E_a E_a + B_a B_a) d^3x$ is the Hamiltonian of the "free" gluons and the Hamiltonian of gluon self-interaction $V$ is

$$V = g \int f_{abc} E_a A^0_{bc} d^3x - \frac{g^2}{2} \int f_{abc} B_a [A_b A_c] d^3x + \frac{g^2}{2} \int (f_{abc} A^0_b A^0_c)^2 d^3x + \frac{g^2}{8} \int (f_{abc} [A_b A_c])^2 d^3x,$$

(3)

$$E_a = - \nabla A^\mu_a - \partial_0 A_a, B_a = [\nabla A_a], A^\mu_a - \mu$$

is the potential of gluon field ($\mu = 0, 4, a = 1, 8$). In the momentum representation the Hamiltonian of gluon self-interaction for the jet ring of thickness $d\theta$ can be represented in the form

$$V = \frac{k_0^4}{4(2\pi)^3} \left( 1 - \frac{q_0^2}{k_0^2} \right)^{3/2} g^2 \pi f_{abc} f_{ade} \left( 2 - \frac{q_0^2}{k_0^2} \right) \left[ a_{1212}^{bcde} + a_{1313}^{bcde} + a_{2323}^{bcde} + \frac{\sin^2 \theta}{2} \left( 1 - \frac{q_0^2}{k_0^2} \right) \left[ 2a_{1212}^{bcde} - a_{1212}^{bcde} - a_{1212}^{bcde} \right] \sin \theta d\theta. \right]$$

(4)

Here $a_{1212}^{bcde} = a_i^{b+} a_m^{c+} a_i^{a-} a_m^{a-} a_i^{a+} a_m^{a+} a_i^{a+} a_m^{a+} h.c., a_i^{b+} (a_i^{b+})$ is the operator annihilating (creating) a gluon of colour $b$ and vector component $l$, $q_0^2$ and $k_0$ are correspondingly the virtuality and energy of the gluon at the end of the perturbative cascade, $g$ is the coupling constant, $f_{abc}$ stands for the structure constants of the $SU_c(3)$ group, $\theta$ is the angle between the jet axis and the momentum $k$ ($0 \leq \theta \leq \theta_{\text{max}}, \theta_{\text{max}}$ is half of the opening angle of the jet cone).

Evolution of a single-gluon state vector $|a_i^b(t)\rangle$ with colour $b$ and vector component $l$ has the evident form

$$|a_i^b(t)\rangle \simeq |a_i^b(0)\rangle - i t C |a_i^b(0)\rangle,$$

(5)

where quantity $C$ depends from coupling constant, energy and virtuality of gluon. As a result of single-gluon state evolution the state vector with different colour is not appear. In this respect evolution of gluon coherent state $|a_i^b\rangle$ is more interesting. For example, evolution of gluon coherent state with colour $b = 1$ and vector component $l = 1$ in jet ring is the following

$$|a_1^1(t)\rangle \simeq \left\{ 1 - 2i t \pi \sin \theta \, d\theta \left( u_3 + u_4 | a_1^1 |^2 \right) |a_1^1(0)\rangle - 2i t \pi \sin \theta \, d\theta \, u_4 a_1^1 D(a_1^1) | a_1^1 \rangle \right\}$$

$$- 2i t \pi \, u_2 \left( 1 + u_1 - u_1^2 / 2 \sin^2 \theta \right) \sin \theta \, d\theta \, (a_1^1)^2 \sum_{k=2}^{7} \sum_{n=2}^{3} | a_1^1(0), 2a_k^b \rangle,$$

(6)

1Here we suggest that a motion equation is satisfied, i.e. $\partial_\mu F_{\mu}^{\nu} + g f_{abc} A_t^b F_{t c}^{\nu} = 0$.

2By performing integration with respect to $\theta$, it is easy to obtain the gluon Hamiltonian for the whole jet; this Hamiltonian differs from the original one only by a factor. Hence, all of the further conclusions about the existence of squeezed states remain valid in this case.
where $| a_i^b(0) \rangle$ is a single gluon vector, $D(\alpha) = \exp\{\alpha a^+ - \alpha^* a\}$ is the unitary displacement operator of amplitude $\alpha$,

$$
\sum_{k=2}^{7} \left( \sum_{k=2}^{7} + \frac{1}{4} \sum_{k=4}^{7} \right), \quad u_1 = \left( 1 - \frac{g_0^2}{k_0^2} \right), \quad u_2 = \frac{k_0^4}{4(2\pi)^3} \frac{g_0^2}{2} \sqrt{u_1^4},
$$

$$
u_3 = \frac{15}{2} k_0^3 \sqrt{u_1} \left( 1 + u_1 + \frac{g_0^4}{2k_0^4} \right) + 24 u_2(3 + 2u_1), \quad \left( \begin{array}{l}
\sum_{k=4} \sum_{k=7} \sum_{k=1} \sum_{k=3} \sum_{k=8} \sum_{k=3,8} \langle a_i^b \rangle \\
\sum_{k=4} \sum_{k=7} \sum_{k=1} \sum_{k=3} \sum_{k=8} \sum_{k=3,8} \langle a_i^b \rangle \\
u_4 = \frac{k_0^3}{2} \sqrt{u_1} \left[ \frac{g_0^4}{k_0^4} + \left( 1 + u_1 - \frac{g_0^4}{k_0^4} \right) \sin^2 \theta \right] + 6 u_2[2(1 + u_1) - u_1 \sin^2 \theta].
\end{array} \right)
$$

(7)

Analogously we can also investigate the evolution of gluon coherent states with any other colour charges and vector components.

As a result the following conclusions have been obtained:

1) for the initial vectors with the colour indices $b=1,2,3$ the vectors with another colour indices $k=4,7$ appear;
2) if the initial vectors have the colour indices $b=4,7$ then the new vectors with colour indices $k=1,2,3,8$ and the vectors with the combination of the colour indices $3,8$: $| a_i^b, a_i^3, a_i^8 \rangle$ appear;
3) as a result of the evolution of colour coherent state with $b=8$ the mixed colour states with colour indices $4,5,6,7$ appear.

It is clear that namely the difference among the structure constants of the $SU_c(3)$-group for different colour indices leads to the different evolution of the corresponding colours.

Since product of the gluon coherent states with different colour and vector indices $\prod_{c=1}^3 \prod_{l=1}^3 | a_i^c(t) \rangle$ corresponds to Poissonian distribution of the multimode gluon states in thin ring of jet, present state vector may be considered as initial state vector prepared by the perturbative stage. Therefore evolution problem of this initial state vector within a small interval of time $t$, which is defined according (1)

$$
|f\rangle = \prod_{c=1}^8 \prod_{l=1}^3 | a_i^c(t) \rangle \simeq \prod_{c=1}^8 \prod_{l=1}^3 | a_i^c(0) \rangle - itV \prod_{c=1}^8 \prod_{l=1}^3 | a_i^c(0) \rangle,
$$

(8)

is actual. In this case the time is reckoned from the beginning of non-perturbative stage and the Hamiltonian of the gluon self-interaction $V$ in the jet ring is determined by formula (4). The explicit form of the evolved state vector $|f\rangle$ is given in the Appendix A.

Because of producing of new evolved mixed state vectors the question, connected with identification of these states, arises, in particular, about fulfillment of squeezing condition for these new states.

### 3 Squeezed gluon states in QCD jet

The Hamiltonian of the gluon self-interaction $V$ in the jet ring includes the squares of the creation and annihilation operators for gluons with specified colour and vector indices. As is known from quantum mechanics and quantum optics, the presence of such structure in the Hamiltonian and, consequently, in the evolution operator is a necessary condition for emergence of squeezed states.

---

3For whole jet we have a superposition of these gluon coherent states with specific weights as initial state vector.
since the unitary squeezing operator involves quadratic combinations of the creation and annihilation operators:

\[ S(z) = \exp \left\{ \frac{z^*}{2} a^2 - \frac{z}{2} (a^+)^2 \right\}, \]  

(9)

where \( z = re^{i\delta} \) is an arbitrary complex number, \( r \) is a squeeze factor, phase \( \delta \) defines the direction of squeezing maximum \([13]\).

In order to verify whether the final gluon state vector describes the single-mode SS, it is necessary to introduce the phase-sensitive Hermitian operators \( (X^b_i) = \frac{[a^*_i + (a^b)^*]}{2} \) and \( (X^b_i) = \frac{[a^b_i - (a^b)^*]}{2i} \) by analogy with quantum optics and to establish conditions under which the variance of one of them can be less than the variance of a coherent state.

Mathematically, the condition of squeezing is expressed in the form of the inequalities \([14]\)

\[ \langle \left( \Delta(X^b_i) \right)^2 \rangle = \langle N \left( \Delta(X^b_i) \right)^2 \rangle + \frac{1}{4} < \frac{1}{4} \]

or

\[ \langle N \left( \Delta(X^b_i) \right)^2 \rangle < 0. \]  

(10)

Here \( N \) is the normal-ordering operator such as

\[ \langle N \left( \Delta(X^b_i) \right)^2 \rangle = \frac{1}{4} \left\{ \pm \left[ \langle a_i^b \rangle^2 - \langle a_i^b \rangle^2 \right] \pm \left[ \langle (a_i^b)^+ \rangle^2 - \langle (a_i^b)^+ \rangle^2 \right] \right\} \]

\[ +2 \left[ \langle a_i^b a_i^b \rangle - \langle (a_i^b)^+ \rangle \langle (a_i^b)^+ \rangle \right]. \]  

(11)

The expectation values of the creation and annihilation operators for gluons with specified colour and vector components is taken for the vector \( \mathbf{a}_l(t) \) \((A.1)\).

Let us consider the specific case where the colour index is \( b = 1 \) and the vector index \( l \) is arbitrary. Then we have

\[ \langle N \left( \Delta(X^1_i) \right)^2 \rangle = \pm 4\pi u_2 t \sin \theta d\theta \left\{ (1 + u_1) \left[ \delta_{11}(Z_{33} + Z_{22}) + (1 - \delta_{11})Z_{11} \right] + \left[ \delta_{12}Z_{22} + \delta_{13}Z_{22} \right] \right\} \]

\[ + u_1 \sin^2 \theta \left\{ \frac{1}{2} \left[ \delta_{11}(Z_{22} + Z_{33}) + \delta_{12}(Z_{33} - \frac{1}{2}Z_{11}) + \delta_{13}(Z_{22} - \frac{1}{2}Z_{11}) \right] \right\}. \]  

(12)

Here \( Z_{mn} = \sum_{k=2}^{7} \langle (X^k_m) \rangle \langle (X^k_n) \rangle \) \((m, n = 1, 2, 3)\). Since at small value of the squeeze factor we have

\[ r \cos \delta = \mp 2 \langle N \left( \Delta(X^1_i) \right)^2 \rangle, \]  

(13)

then taking into account formula \([12]\) the expression \([13]\) can be rewrite in the form

\[ r^1 \cos \delta = -8\pi u_2 t \sin \theta d\theta \left\{ (1 + u_1) \left[ \delta_{11}(Z_{33} + Z_{22}) + (1 - \delta_{11})Z_{11} \right] + \delta_{12}Z_{22} + \delta_{13}Z_{22} \right\} \]

\[ + u_1 \sin^2 \theta \left\{ \frac{1}{2} \delta_{11}(Z_{22} + Z_{33}) + \delta_{12}(Z_{33} - \frac{1}{2}Z_{11}) + \delta_{13}(Z_{22} - \frac{1}{2}Z_{11}) \right\}. \]  

(14)
Evidently that \( \langle N \left( \Delta (X_l^h) \right)^2 \rangle \neq 0 \) if \( r \neq 0 \) and \( \delta \neq \frac{\pi}{2} \) \( - \frac{3\pi}{2} \). In final state being consideration fluctuations of one of the squared components of the gluon field, \( \Delta (X_m^l) \), are less than those in the initial coherent state under the following conditions: \( \langle (X_m^l) \rangle_1 < 0, \langle (X_m^l) \rangle_2 < 0 \) or \( \langle (X_m^l) \rangle_1 > 0 \) and \( \langle (X_m^l) \rangle_2 > 0 \) \( (k \neq 1, m \neq l) \) then, as following from (14), \( \frac{\pi}{2} < \delta < \frac{3\pi}{2} \). In this case we have phase-squeezed gluon states by analogy with quantum optics [12, 13]. If the conditions \( \langle (X_m^l) \rangle_1 > 0, \langle (X_m^l) \rangle_2 < 0 \) or \( \langle (X_m^l) \rangle_1 < 0 \) and \( \langle (X_m^l) \rangle_2 > 0 \) \( (k \neq 1, m \neq l, - \frac{\pi}{2} < \delta < \frac{\pi}{2}) \) are satisfied, fluctuations in another squared component of the gluon field, \( \Delta (X_l^l) \), will be less in the final state vector \( \sum_{c=1}^8 \sum_{l=1}^3 |\alpha_l^c(t)\rangle \) than in the coherent state. In this case we arrive at the amplitude-squeezed states (as in the case of photons [12, 13]).

Rewriting the expression (14) in terms of the amplitude and phase of the gluon coherent states \( \alpha_i^l = |\alpha_i^l| e^{i\gamma_i^l} \)

\[
\frac{r_1^l \cos \delta}{-8\pi u_2 t \sin \theta d\theta} \sum_{k=2}^7 \left\{ \left( 1 + u_1 - \frac{u_1}{2} \sin^2 \theta \right) \delta_{l1} \sum_{n=2}^3 |\alpha_n^k|^2 \sin(2\gamma_n^k) + \left( 1 - \delta_{l1} \right) |\alpha_1^k|^2 \sin(2\gamma_1^k) \right. \\
+ \left. \left( 1 + u_1 \sin^2 \theta \right) \delta_{l2} |\alpha_3^k|^2 \sin(2\gamma_3^k) + \delta_{l3} |\alpha_2^k|^2 \sin(2\gamma_2^k) \right\},
\]

(15)

we see that effect of the single-mode squeezing is absent \( (r_1^l \cos \delta = 0) \) then the initial gluon coherent fields are either real \( (\gamma_n^k = 0, n \neq l, k \neq 1) \) or imaginary \( (\gamma_n^k = \pi/2, n \neq l, k \neq 1) \). Similar conclusions will also be valid for a gluon field featuring other colour indices.

Thus, the vector \( \sum_{c=1}^8 \sum_{l=1}^3 |\alpha_l^c(t)\rangle \) describes the squeezed state of gluons that are produced at the non-perturbative stage of the jet evolution within a small interval of time \( t \). Here, the corresponding fluctuations of the squared components of the gluon field will be less than those in the case of the initial coherent state.

It should be noted that the Hamiltonian of the three-gluon self-interaction which in momentum representation has the next form

\[
V' = \frac{ig}{(2\pi)^{3/2}} f_{abc} \int \prod_{i=1}^3 \left( \frac{d^3k_i}{\sqrt{2k_{0i}}} \right) \left\{ \left[ \tilde{a}_a \tilde{b}_b(k_1 \tilde{a}_c) + \frac{k_{01}}{k_{03}k_{01}} \tilde{a}_a(k_3 \tilde{a}_c) \right] e^{-i(k_{01} + k_{02} + k_{03})t} \right. \\
\times \delta(k_1 + k_2 + k_3) + \left[ \tilde{a}_a \tilde{b}_b(k_1 \tilde{a}_c) + \frac{k_{01}}{k_{03}k_{01}} \tilde{a}_a(k_3 \tilde{a}_c) \right] e^{-i(k_{01} - k_{02} + k_{03})t} \delta(k_1 - k_2 + k_3) \\
+ \left[ \tilde{a}_a \tilde{b}_b(k_1 \tilde{a}_c) + \frac{k_{01}}{k_{03}k_{01}} \tilde{a}_a(k_3 \tilde{a}_c) \right] e^{-i(k_{01} - k_{02} - k_{03})t} \delta(k_1 - k_2 - k_3) \\
+ \left. \left[ \tilde{a}_a \tilde{b}_b(k_1 \tilde{a}_c) + \frac{k_{01}}{k_{03}k_{01}} \tilde{a}_a(k_3 \tilde{a}_c) \right] e^{-i(k_{01} - k_{02} - k_{03})t} \delta(k_1 - k_2 - k_3) \right\},
\]

(16)

don’t lead to squeezing effect. In fact, rewriting the expression for \( \langle N \left( \Delta (X_l^h) \right)^2 \rangle \) (14) for small time as

\[
\langle N \left( \Delta (X_l^h) \right)^2 \rangle = \mp \frac{i}{4} \left\{ \langle \alpha | [a_l^h (k), [a_l^h (k), V]] | \alpha \rangle - \langle \alpha | [[V, a_l^{n h} (k)]}, a_l^{n h} (k)] | \alpha \rangle \right\},
\]

(17)

it can be shown that \( [a_l^h (k), [a_l^h (k), V']] = 0, [[V', a_l^{n h} (k), a_l^{n h} (k)] = 0 \) since \( f_{hbb} = 0 \), that is the squeezing condition (14) does not hold.
4 Correlation functions for squeezed gluon states

The behaviour of a correlation function can serve one of a criterion of the existence of squeezed gluon states. It is common to define the normalized second-order correlation function as

\[
K_{(2)}(\theta_1, \theta_2) = \frac{C_{(2)}(\theta_1, \theta_2)}{\rho_1(\theta_1)\rho_1(\theta_2)},
\]

where

\[
C_{(2)}(\theta_1, \theta_2) = \rho_2(\theta_1, \theta_2) - \rho_1(\theta_1)\rho_1(\theta_2),
\]

with \(\rho_2(\theta_1, \theta_2)\) being the two-particle (single-particle) inclusive distribution. Then for gluons with a colour \(b\) and a vector component \(l\) we can write

\[
K_{l(2)}^{b}(\theta_1, \theta_2) = \frac{\rho_{l(2)}^{b}(\theta_1, \theta_2)}{\rho_{l(1)}^{b}(\theta_1)\rho_{l(1)}^{b}(\theta_2)} - 1.
\]

At the same time, we have

\[
\int_{\Omega} \rho_1(\theta) d\theta = \langle n \rangle = \langle a^+ a \rangle = \int_{\Omega} (f(\theta,t)|a^+ a|f(\theta,t))d\theta,
\]

where \(|f(\theta,t)\rangle\) is the final state vector. From (20) we find that the single- and two-particle inclusive distributions can be represented as

\[
\begin{align*}
\rho_1(\theta) &= \langle f(\theta,t)|a^+ a|f(\theta,t)\rangle, \\
\rho_2(\theta_1, \theta_2) &= \langle f(\theta_2, t), f(\theta_1,t)|a^+ a^+ a\ a\ a|f(\theta_1, t), f(\theta_2,t)\rangle.
\end{align*}
\]

By taking the expectation values over the vector \(\prod_{c=1}^{8} \prod_{l=1}^{3} |\alpha_c^c(\theta_1, t), \alpha_c^c(\theta_2, t)\rangle\), we obtain the explicit form of the normalized second-order correlation function for squeezed gluon states:

\[
K_{l(2)}^{b}(\theta_1, \theta_2) = -M_1(\theta_1, \theta_2)\{|\alpha_l^b|^4 - 2 |\alpha_l^b|^2 M_1(\theta_1, \theta_2) + M_2(\theta_1, \theta_2)\}.
\]

For the colour index \(b = 1\) and an arbitrary vector component \(l\) we have here

\[
M_1(\theta_1, \theta_2) = 24 t u_2 \pi \ |\alpha |^2 |\beta |^2 \sin \left(\frac{\delta + \frac{\pi}{2}}{2}\right) \left\{ (1 + \delta_{l1})(2 + u_1 - \delta_{l1})(\sin \theta_1 + \sin \theta_2) \\
- \frac{1}{2} u_1 (3\delta_{l1} - 1)(\sin^3 \theta_1 + \sin^3 \theta_2) \right\},
\]

\[
M_2(\theta_1, \theta_2) = 80 t u_2 \pi \ |\alpha |^3 |\beta |^3 \sin \left(\frac{\delta + \frac{\pi}{4}}{4}\right) \left\{ (1 + \delta_{l1})(2 + u_1 - \delta_{l1})(\sin \theta_1 + \sin \theta_2) \\
- \frac{1}{2} u_1 (3\delta_{l1} - 1)(\sin^3 \theta_1 + \sin^3 \theta_2) \right\}.
\]

In deriving these formulae, for simplicity we assumed that \(\alpha^1_l = |\alpha| e^{i\gamma_1}, l = \text{any}, \) and \(\alpha^b_l = |\beta| e^{i\gamma_2}\), when \(b \neq 1\) and an arbitrary \(l, \gamma_1 - \gamma_2 = \delta/2 + \pi/4\).

Let us perform a comparative analysis of the correlation function (22) for gluon squeezed states and the corresponding function for photon squeezed states, which was thoroughly studied in quantum optics.

\footnote{That this vector also describes squeezed gluon states can be proven by verifying the squeezing condition (10).}
In quantum optics, the normalized second-order correlation function is defined as [14]

\[
K_{l(2)} = g_l^{(2)} - 1 = \frac{\langle \hat{a}_l^+ \hat{a}_l^+ \hat{a}_l \hat{a}_l \rangle}{\langle \hat{a}_l^+ \hat{a}_l \rangle^2} - 1,
\]

(25)

where the expectation values are taken over the evolved state vector at the instant \(t\). If the correlation function is positive, occurs photon bunching (super-Poissonian distribution); otherwise \((K_{l(2)} < 0)\), we have photon antibunching (sub-Poissonian distribution) [12, 14]. For a coherent field obeying Poissonian statistics, the normalized second-order correlation function vanishes \((K_{l(2)} = 0)\).

For photon squeezed states whose state vector is defined as \(|\alpha, z\rangle = S(z)|\alpha\rangle\), the corresponding correlation function has the form (at small values of the squeezing parameter \(r_l\))

\[
K_{l(2)} = -\frac{r_l[\alpha_l^2 e^{-i\delta} + (\alpha_l^*)^2 e^{i\delta}]}{|\alpha_l|^4 - 2r_l |\alpha_l|^2 [\alpha_l^2 e^{-i\delta} + (\alpha_l^*)^2 e^{i\delta}]}.
\]

(26)

In contrast to the correlation function for squeezed photon states, \(K_{l(2)}\), the corresponding function for the squeezed gluon states, \(K_{l(2)}^{gluon}\), includes, as follows from (22), the \(M_2(\theta_1, \theta_2)\) which appears because the Hamiltonian \(H\) of the gluon self-interaction involves a nonlinear combination of the creation and annihilation operators of gluons with different colours and vector components.

The angular dependence of the correlation function for squeezed gluon states (with colour \(b = 1\)) that are formed at the non-perturbative stage after a lapse of \(t = 0.001\) is investigated graphically at the following parameter values: \(\theta_2 = 0\); \(g^2 = 4\pi\) because \(\alpha_s = \frac{g^2}{4\pi} \sim 1\); \(g_o^2 = 1\) GeV\(^2\) that corresponds to the gluon virtuality at the beginning of the non-perturbative stage; \(k_o = \frac{\sqrt{s}}{2\langle n_{gluon}\rangle}\) corresponds to a gluon energy in the case of 2-jet events; \(\sqrt{s} = 91\) GeV and \(\langle n_{gluon}\rangle = |\alpha|^2 + 7|\beta|^2\).

If the amplitude \(|\alpha|\) of the gluon field being considered is equal to the amplitudes \(|\beta|\) of the cophased gluon fields having different colours and vector components, then the values of the correlation function lie in the negative region (Fig.1a), and there occurs the antibunching of gluons with the corresponding sub-Poissonian distribution. In this case, the correlation function tends to a constant \((K_{l(2)}^{gluon}(\theta_1, \theta_2 = 0) = -2.80094)\) as the angle \(\theta_1\) increases. The behaviour of the angular correlations of the cophased squeezed gluon states \((\delta = 0)\) is similar to correlations of the analogous photon states at small values of the squeezing parameter [11]. If amplitude of selected gluon field with the colour \((b = 1)\) \(\alpha\) begins to dominate in relation to the amplitudes of other colour fields \((b \neq 1)\), that is, \(\alpha > \beta\), then correlation function involves a singularity (Fig.1b) at \(\theta_1 \approx 1.208725 \times 10^{-11}\) \((|\alpha|^2 = 3, |\beta|^2 = 1)\).

![Figure 1: The angular dependence of the cophased \((\delta = 0)\) squeezed gluon correlation function at: a) \(|\alpha|^2 = 1, |\beta|^2 = 1\), b) \(|\alpha|^2 = 3, |\beta|^2 = 1\).](image)

For antiphased squeezed states of the gluons \((\delta = \pi)\) correlation function lies in positive region and there occurs gluon bunching with the corresponding super-Poissonian distribution. In this case
correlation function grows fast at small angles $\theta_1$ and tends to a constant irrespective of the values of the amplitudes $\alpha$ and $\beta$.

By using the transformation

$$\sin \theta = \sqrt{1 - \tanh^2 y \frac{u_1}{u}}$$

we can rewrite the correlation function for squeezed gluon states in terms of rapidities

$$K^{b}_{l(2)}(\theta_1, \theta_2) = -M_1(y_1, y_2)/[|\alpha|^4 - 2 |\alpha|^2 M_1(y_1, y_2) + M_2(y_1, y_2)]$$

Rapidity correlations of the cophased gluon squeezed states (Fig.2) fall within the region of negative values and have a minimum in the center ($K^{b}_{l(2)}(y_1 = y_2 = 0) = -0.0267894$ at $|\alpha|^2 = 1$). For $|\alpha| > |\beta|$ the correlation function has a less pronounced minimum in the center $K^{b}_{l(2)}(y_1 = y_2 = 0) = -0.00887147$ at $|\alpha|^2 = 3$.

Figure 2: The rapidity dependence of the cophased ($\delta = 0$) gluon squeezed correlation function at $y_2 = 0$: $|\alpha|^2 = 1, |\beta|^2 = 1$—solid line, $|\alpha|^2 = 3, |\beta|^2 = 1$—dotted line.

Rapidity correlations of the antiphased gluon squeezed states fall within the region of positive values and have a maximum in the center ($K^{b}_{l(2)}(y_1 = y_2 = 0) = 0.0241966$ at $|\alpha|^2 = 1$, $K^{b}_{l(2)}(y_1 = y_2 = 0) = 0.0080179$ at $|\alpha|^2 = 3$).

It should be noted that behaviour of the cophased gluon squeezed correlation function $K^{b}_{l(2)}(y_1, y_2)$ at $\sqrt{s} = 35$ GeV (Fig.3) is similar to hadron correlations with distinctive minimum at $y_1 - y_2 = 0.45$.

Figure 3: The rapidity dependence of the cophased gluon squeezed correlation function at $\sqrt{s} = 35$ Gev $|\alpha|^2 = 1, |\beta|^2 = 1$: a) $-1 \leq y_1 + y_2 \leq 0$, b) $0 \leq y_1 + y_2 \leq 1$.

Thus the behavior of rapidity correlations for the cophased gluon squeezed states under investigation suggests that, at the non-perturbative stage of evolution of a QCD jet, there exists the effect of the gluon antibunching with the corresponding sub-Poissonian statistics.
5 Chaos in jets

There are a lot of evidences that phenomenon of a chaos may exist in high-energy physics (fractality for the factorial moments [31]), in nuclear physics (energy spacing distributions [24]), in quantum mechanics (chaos assisted quantum tunnelling [32]).

There is also direct relation between chaos and squeezing in some quantum-mechanical systems [19]-[21]. Let us verify this statement by the example of the next Hamiltonians:

1) \[ H_1 = \frac{p_1^2}{2} + \frac{p_2^2}{2} + \frac{g^2}{2} q_1^2 q_2^2, \] (29)

2) \[ H_2 = \frac{p_1^2}{2} + \frac{p_2^2}{2} + \frac{q_1^2}{2} + \frac{q_2^2}{2} + \frac{g}{2} (p_1 q_1 + q_1 p_1 + p_2 q_2 + q_2 p_2), \] (30)

3) \[ H_3 = g(p_1 q_2 + q_1 p_2). \] (31)

Hamiltonian \( H_1 \) is a mechanical analogue of Hamiltonian of the Yang-Mills field for SU(2) gauge in the case of spatially homogenous fields (\( \partial_i A_j = 0 \)) and \( A_1^2 = A_1^3 = 0, A_3^3 = 0 \) [33]. Hamiltonians \( H_2 \) and \( H_3 \) correspond to Hamiltonians described degenerate and non-degenerate parametric amplifiers in the interaction picture [12].

The local stability of the dynamical system is determined by the eigenvalues of the stability matrix \( G \) which has the next form [34]

\[
G = \begin{pmatrix}
-\frac{\partial^2 H}{\partial p_i \partial q_j} & -\frac{\partial^2 H}{\partial q_i \partial q_j} \\
-\frac{\partial^2 H}{\partial q_i \partial p_j} & -\frac{\partial^2 H}{\partial q_i \partial p_j}
\end{pmatrix}
\] (32)

If at least one of the eigenvalues of the matrix \( G \) is real then the separation of the trajectories grows exponentially and the motion is unstable. Imaginary eigenvalues correspond to stable motion. It was showed for \( H_1 \) [35] that the stability matrix \( G \) has real eigenvalues. Therefore the motion of such systems is unstable.

The eigenvalues of \( G \) for systems described by \( H_2 \) and \( H_3 \) are \( \lambda_{1,2} = \pm \sqrt{g^2 - 1} \), \( \lambda = g \) correspondingly, where we take into account that the interaction constant \( g \) is real and positive. Obviously, the local nonstability for \( H_2 \) exists if \( g^2 > 1 \) and in the case \( H_3 \) — for an arbitrary \( g \). Thus the motion of considered physical systems can be chaotic.

Investigating the variances \( (\triangle p_i)^2 \) and \( (\triangle q_i)^2 \) we can show that in the case of the Hamiltonian \( H_1 \) SS are not appear. At the same time from QO it is well known that SS appear for systems described by \( H_2 \) and \( H_3 \). It should be noted, if for two-mode degenerate parametric amplifier squeezing exists for single mode, so in the case of non-degenerate parametric amplifier we have two-mode squeezed states and the squeezing does not exist in the single mode [12].

Hamiltonians \( H_2 \) and \( H_3 \), for which the squeezing exists, have terms proportional to the product of the canonical momentum and coordinate \( (p q) \). Indeed, let consider the next expression of the variance \( (\triangle p_i)^2 \)

\[
(\triangle p_i)^2 = \langle f | p_i^2 | f \rangle - \langle f | p_i | f \rangle^2,
\] (33)

where the evaluate vector \( | f \rangle \) is a solution of the Schrödinger equation for small time

\[
| f \rangle \simeq | i \rangle - it \langle i | H | i \rangle
\] (34)
Thus we have

$$(\triangle p_i)^2 = <i|p_i^2|i> - <i|p_i|i>^2 = -i t(<i|[p_i^2, H]|i> - 2 <i|p_i|i><i|[p_i, H]|i>). \quad (35)$$

We choose coherent state as initial state vector $|i> = |\alpha>$ because it is most similar to the classical case, any vector may be decompose on these basis vectors and these states are widely used in QO. Then as followed from (35) term which is proportional to time $t$ is a measure of departure from initial value of the variance

$$-i t(<\alpha|[p_i^2, H]|\alpha> - 2 <\alpha|p_i|\alpha><\alpha|[p_i, H]|\alpha>) = -t <\alpha|[q_i, [p_i, H]]|\alpha> \quad . \quad (36)$$

As considered Hamiltonians are to polynomial like of $p$ and $q$ we may write (36) in the next general form

$$-t <\alpha|[q_i,[p_i, H]]|\alpha> = -t <\alpha|\frac{\partial^2 H}{\partial p_i\partial q_i}|\alpha> \quad . \quad (37)$$

Squeezed states exist when $(\triangle p_i)^2 \neq <\alpha|[p_i^2]|\alpha> - <\alpha|p_i|\alpha>^2$ that is as followed from (35),(37)

$$<\alpha|\frac{\partial^2 H}{\partial p_i\partial q_i}|\alpha> \neq 0. \quad (38)$$

Analogously it can be shown that the multimode SS exist when

$$<\alpha|\frac{\partial^2 H}{\partial p_i\partial q_j}|\alpha> + <\alpha|\frac{\partial^2 H}{\partial q_i\partial p_j}|\alpha> \neq 0, \quad (i \neq j). \quad (39)$$

This means that if components $\frac{\partial^2 H}{\partial p_i\partial q_i}$ or sum of the components $\frac{\partial^2 H}{\partial p_i\partial q_j}$ and $\frac{\partial^2 H}{\partial q_i\partial p_j}$ from stability matrix $G \quad (32)$ are not equal zero then for corresponding quantum system the squeezing exists.

One of criteria of the appearance chaos is a fast exponential decreasing of the Green function \quad (29) which is analogy of the correlator of the field operators. Therefore in order to find conditions of appearance the non-stable motion in physical systems $(H_2 \text{ and } H_3)$ for which the squeezing exists we investigate the next temporal correlators of the field operators

$$R_{ij} = <a_i^+(t)a_j(0)> \quad . \quad \quad (40)$$

Here averaging is made over initial coherent states $|\alpha>$, $a_i^+(t)$ is a solution of the Heisenberg equations in the interaction picture.

For two-mode degenerate parametric amplifier we have \quad (12)

$$a_1^+(t) = a_1^+ \cosh(gt) + a_1 \sinh(gt),$$

$$a_2^+(t) = a_2^+ \cosh(gt) + a_2 \sinh(gt). \quad \quad (41)$$

Substituting these expressions in (40) we obtain the explicit forms of the correlators

$$R_{ij}(t) = \frac{\alpha_i}{2} (\alpha_i^* + \alpha_i) e^{gt} + \frac{\alpha_i}{2} (\alpha_i^* - \alpha_i) e^{-gt}, \quad i, j = 1, 2. \quad (42)$$

As follows from (42) the correlators $R_{ij}(t)$ decrease exponentially at equal phases $\varphi_1 = \varphi_2 = \frac{\pi}{2}, \frac{3\pi}{2}$

$(\alpha_i = |\alpha_i| e^{i\varphi_i})$. In this case we have

$$R_{ij}(t) = |\alpha_i||\alpha_j| e^{-gt}, \quad i, j = 1, 2. \quad (43)$$
As squeezing does not depend on $\alpha_1$ and $\alpha_2$ \[12\] then in this case the effects of squeezing and chaos coexist.

Using the solutions of the evolutions equations for $a_1(t)$ and $a_2(t)$ in the case non-degenerate parametric amplifier \[12\] we can obtain explicit expression for corresponding correlators

\[
R_{ii}(t) = \frac{1}{2}(|\alpha_i|^2 + \alpha_1\alpha_2)e^{gt} + \frac{1}{2}(|\alpha_i|^2 - \alpha_1\alpha_2)e^{-gt}, \quad i = 1, 2,
\]

\[
R_{ij}(t) = \frac{\alpha_i}{2}(\alpha_i^* + \alpha_j)e^{gt} + \frac{\alpha_j}{2}(\alpha_i^* - \alpha_j)e^{-gt}, \quad (i \neq j), \quad i, j = 1, 2.
\]

Obviously, these correlators are exponentially reduced at $|\alpha_1| = |\alpha_2| \equiv |\alpha|, \varphi_1 = \varphi_2 = \frac{\pi}{2}, \frac{3\pi}{2}$ and then $R_{ii}(t) = R_{ij}(t), \quad R_{ij}(t) = |\alpha|^2 e^{-gt}.
(45)

This clearly demonstrates the possibility of existence of chaos when the squeezing condition is fulfilled for any $\alpha$.

The problem of the existence of a chaos in model SU(2)-jet merits special attention. Here we take the case of SU(2)-group only for simplification of calculations. Respective Hamiltonian of the interaction $V_{\text{int}}$ is obtained from the Hamiltonian for SU(3)-jet \[4\] by replacing structure constants of the SU(3)-group the corresponding SU(2)-group constant, that is $f_{abc} \rightarrow \varepsilon_{abc}$ ($a, b, c = 1, 3$), and take into account that $\varepsilon_{abc}\varepsilon_{ade} = \delta_{bd}\delta_{ce} - \delta_{be}\delta_{cd}$

\[
V_{\text{int}} = \frac{k_0^4}{4(2\pi)^3} g^2 \pi \left( \frac{1 - \frac{q_0^2}{k_0^2}}{1} \right)^{3/2} \left( \frac{2 - \frac{q_0^2}{k_0^2} - \frac{\sin^2 \theta}{2} \left( 1 - \frac{q_0^2}{k_0^2} \right)}{2} \right) \left[ a_{1212}^{bcbc} + a_{1313}^{bcbc} - a_{1212}^{bcb} - a_{1313}^{bcb} \right]
\]

\[
+ \left( 1 + \sin^2 \theta \left( 1 - \frac{q_0^2}{k_0^2} \right) \right) \left[ a_{2323}^{bcbc} - a_{2323}^{bcb} \right] \sin \theta d\theta.
\]

(46)

The local instability for the Hamiltonian \[16\] has been verified by Toda criterion. Analysis was made numerically accordingly the next algorithm:

1. we come to classical Hamiltonian by keeping the order of operators $a^+, a$ and consider them as $c$-numbers ($\alpha^*, \alpha$);

2. we have 18 variables and calculate the instability matrix $18 \times 18$ for this case;

3. next step is calculation of its eigenvalues to find out whether they are real or imaginary.

As a result the following conclusions have been obtained:

1) If all variable $\alpha$ and $\alpha^*$ are real or imaginary then the system of gluons described by the above mentioned Hamiltonian \[16\] is strictly ordered and effect of the squeezing is absent.

2) If at least one of $\alpha$ or $\alpha^*$ is imaginary and other are real or at least one of $\alpha$ and $\alpha^*$ is real and other are imaginary — we have the local instability, which can lead to chaotical system.

6 Conclusion

Investigating of the gluon fluctuations we have proved theoretically the possibility of existence of the gluon single-mode SS at non-perturbative stage of the QCD jet evolution. The emergence of
such remarkable states becomes possible owing to the self-interaction of gluons with different colour indices.

As one of identification criterion of existence of gluon SS can served correlation function. Therefore we have analyzed the behaviour of angular and rapidity correlations and have compared our results with the corresponding correlation function for photon squeezed states, which was comprehensively investigated in quantum optics. The form of the normalized correlation function $K_{1(2)}^b$ for cophased squeezed states specifies the gluon antibunching effect if the amplitudes of all gluon fields (with various colour and vector components) are equal to one another. Such behaviour of angular correlations is analogous to the behaviour of the corresponding correlations of the photon squeezed states at small values of the squeezing parameter. At the same time, there is distinction between them: in contrast to the normalized correlation function known in quantum optics, the correlation function of the gluon SS has a singularity if the amplitude for the fixed-colour gluon field being studied is greater than the amplitudes for gluon fields with other colour indices. The correlations of the cophased squeezed states specifies the presence of the gluon antibunching effect, whereas the gluon bunching occurs for antiphased squeezed states. Hence, non-perturbative gluon evolution makes a contribution to the parton distribution prepared by the perturbative stage of jet evolution in the form of a sub-Poissonian (cophased squeezed states) or a super-Poissonian (antiphased squeezed states) distributions.

Thus, the behaviour of the two-particle angular and rapidity correlations can serve as one of the criteria of the existence of squeezed gluon states. At the same time for a comparison of our results with experimental data, we must take into account the contribution of the perturbative stage of jet evolution and hadronization effects. This can be done by using Monte-Karlo methods and will be the subject of our further investigations.

In this paper we investigate the possibility of coexistence both condition of squeezing and chaos for some physical systems: mechanical model of Yang-Mills field for SU(2) gauge, two-mode degenerate and non-degenerate parametric amplifiers, SU(2)-jet model. Using Toda criterion we check the local instability in the corresponding classical systems and determine conditions of the coexistence of this effect and squeezing at small time. It was shown that the squeezing exists at small time under condition that either components $\frac{\partial^2 H}{\partial p_i \partial q_i}$ or sum of the components $\frac{\partial^2 H}{\partial p_i \partial q_j}$ and $\frac{\partial^2 H}{\partial q_i \partial p_j}$ from stability matrix G are not equal zero.

The possibility of existence of squeezing and chaos in corresponding quantum systems is studied by using correlators of the field operators. The correlators of the field operators for degenerate and non-degenerate parametric amplifiers decrease exponentially independently from amplitude values of the initial coherent fields $|\alpha_1|, |\alpha_2|$ if phases $\varphi_1$ and $\varphi_2$ coincide and equal $\frac{\pi}{2}$ or $\frac{3\pi}{2}$. Therefore in this case chaos may exist. Moreover for these systems squeezing does not depend on value of amplitude of initial coherent field.

Under investigation the local instability within SU(2)-jet model it was numerically shown that this effect existences under condition if at least one of amplitude of the coherent fields $\alpha$ or $\alpha^*$ is imaginary and other are real and vice versa.

Thus for degenerate and non-degenerate parametric amplifiers and also for SU(2)-jet model the effects of the squeezing and chaos can coexist under some conditions.
Appendix A

\[
\prod_{c=1}^{8} \prod_{l=1}^{3} \alpha_i^c(t) \simeq (1 - 2i\pi u_3 \sin \theta d\theta) \langle i \mid - 2i\pi u_4 \sin \theta d\theta \sum_{k=1}^{8} \alpha_i^k a_1^{k+1} \mid i \rangle - 2i\pi u_5 \sin \theta d\theta \sum_{k=1}^{8} \sum_{m=2}^{3} \alpha_i^k a_m^{k+1} \mid i \rangle \\
- 2i\pi u_2 \sin \theta d\theta \{(1 + u_1) [B_{1212} + B_{2211} + B_{1133} + B_{3311} - B_{1212} - B_{2121} - B_{1313} - B_{3131}] \\
+ [B_{2233} + B_{3322} - B_{2323} - B_{3232}] + u_1 \sin^2 \theta [B_{2233} + B_{3322} - B_{2323} - B_{3232}] \\
- \frac{1}{2} (B_{1122} + B_{2211} + B_{1133} + B_{3311} - B_{1212} - B_{2121} - B_{1313} - B_{3131}) \} \\
+ 2(1 + u_1) [C_{1122} + C_{2211} + C_{1133} + C_{3311} + D_{1212} + D_{2121} + D_{1313} + D_{3131}] \\
+ 2 [C_{2233} + C_{3322} + D_{2323} + D_{3232}] + 2u_1 \sin^2 \theta [C_{2233} + C_{3322} + D_{2323} + D_{3232}] \\
- \frac{1}{2} (C_{1122} + C_{2211} + C_{1133} + D_{1212} + D_{2121} + D_{1313} + D_{3131}) \} \rangle, \tag{A.1}
\]

where \[ \langle i \rangle = \prod_{c=1}^{8} \prod_{l=1}^{3} |\alpha_i^c(0)\rangle, \quad u_5 = \frac{k_0^3}{2} \sqrt{u_1} \left\{ 1 + u_1 - \frac{1}{2} \left( 1 + u_1 - \frac{a_0^4}{k_0^4} \right) \sin^2 \theta \right\} + 6u_2 \left\{ 2 + u_1 + \frac{1}{2} u_1 \sin^2 \theta \right\} , \]

\[ B_{ij} = A_{11}^{11} \sum_{k=2}^{7} \alpha_i^{kk} + A_{22}^{11} \sum_{k=1}^{7} \alpha_i^{kk} + A_{33}^{11} \sum_{k=1}^{7} \alpha_i^{kk} + \frac{1}{4} A_{44}^{11} \left[ \sum_{k=1}^{7} \alpha_i^{kk} + 3 \sum_{k=5,8} \alpha_i^{kk} + \sqrt{3}(\alpha_i^{38} + \alpha_i^{83}) \right] \]

\[ + \frac{1}{4} A_{55}^{11} \left[ \sum_{k=1}^{7} \alpha_i^{kk} + 3 \sum_{k=4,8} \alpha_i^{kk} + \sqrt{3}(\alpha_i^{38} + \alpha_i^{83}) \right] + \frac{1}{4} A_{66}^{11} \left[ \sum_{k=1}^{7} \alpha_i^{kk} + 3 \sum_{k=7,8} \alpha_i^{kk} - \sqrt{3}(\alpha_i^{38} + \alpha_i^{83}) \right] \]

\[ + \frac{1}{4} A_{77}^{11} \left[ \sum_{k=1}^{6} \alpha_i^{kk} + 3 \sum_{k=6,8} \alpha_i^{kk} - \sqrt{3}(\alpha_i^{38} + \alpha_i^{83}) \right] + \frac{3}{4} A_{88}^{11} \sum_{k=4}^{7} \alpha_i^{kk} \tag{A.2} \]
\[ C_{ij mn} = A_{mn}^{12} \left\{ -a_{ij}^{21} + \frac{1}{4} \left[ a_{ij}^{76} - a_{ij}^{67} + a_{ij}^{45} - a_{ij}^{54} \right] \right\} + A_{mn}^{13} \left\{ -a_{ij}^{31} + \frac{1}{4} \left[ a_{ij}^{64} - a_{ij}^{46} + a_{ij}^{75} - a_{ij}^{57} \right] \right\} \\
= \frac{1}{2} A_{mn}^{14} \left\{ a_{ij}^{25} - a_{ij}^{36} + \frac{1}{2} \left[ a_{ij}^{52} - a_{ij}^{63} - a_{ij}^{41} - \sqrt{3} a_{ij}^{68} \right] \right\} \\
= \frac{1}{2} A_{mn}^{15} \left\{ a_{ij}^{24} + a_{ij}^{37} + \frac{1}{2} \left[ a_{ij}^{42} + a_{ij}^{73} + a_{ij}^{51} + \sqrt{3} a_{ij}^{78} \right] \right\} \\
= \frac{1}{2} A_{mn}^{16} \left\{ a_{ij}^{34} - a_{ij}^{27} + \frac{1}{2} \left[ a_{ij}^{43} - a_{ij}^{72} - a_{ij}^{61} - \sqrt{3} a_{ij}^{48} \right] \right\} \\
= \frac{1}{2} A_{mn}^{17} \left\{ a_{ij}^{35} + a_{ij}^{26} + \frac{1}{2} \left[ a_{ij}^{53} + a_{ij}^{62} - a_{ij}^{71} - \sqrt{3} a_{ij}^{58} \right] \right\} + \frac{\sqrt{3}}{4} A_{mn}^{18} \left\{ a_{ij}^{46} + a_{ij}^{64} + a_{ij}^{57} + a_{ij}^{75} \right\} \\
= \frac{1}{2} A_{mn}^{23} \left\{ -a_{ij}^{32} + \frac{1}{4} \left[ a_{ij}^{65} - a_{ij}^{56} + a_{ij}^{47} - a_{ij}^{74} \right] \right\} + \frac{1}{2} A_{mn}^{24} \left\{ a_{ij}^{37} - a_{ij}^{15} + \frac{1}{2} \left[ a_{ij}^{73} - a_{ij}^{42} - a_{ij}^{51} + \sqrt{3} a_{ij}^{78} \right] \right\} \\
= \frac{1}{2} A_{mn}^{25} \left\{ a_{ij}^{14} - a_{ij}^{36} + \frac{1}{2} \left[ a_{ij}^{41} - a_{ij}^{63} - a_{ij}^{52} - \sqrt{3} a_{ij}^{78} \right] \right\} \\
= \frac{1}{2} A_{mn}^{26} \left\{ a_{ij}^{35} + a_{ij}^{17} + \frac{1}{2} \left[ a_{ij}^{53} + a_{ij}^{71} - a_{ij}^{62} + \sqrt{3} a_{ij}^{58} \right] \right\} \\
= \frac{1}{2} A_{mn}^{27} \left\{ a_{ij}^{16} + a_{ij}^{34} + \frac{1}{2} \left[ a_{ij}^{72} + a_{ij}^{43} + a_{ij}^{61} - \sqrt{3} a_{ij}^{48} \right] \right\} + \frac{\sqrt{3}}{4} A_{mn}^{28} \left\{ a_{ij}^{56} + a_{ij}^{65} - a_{ij}^{47} - a_{ij}^{74} \right\} \\
= \frac{1}{2} A_{mn}^{34} \left\{ a_{ij}^{16} - a_{ij}^{27} + \frac{1}{2} \left[ a_{ij}^{61} - a_{ij}^{43} - a_{ij}^{72} + \sqrt{3} a_{ij}^{48} \right] \right\} \\
= \frac{1}{2} A_{mn}^{35} \left\{ a_{ij}^{26} + a_{ij}^{17} + \frac{1}{2} \left[ a_{ij}^{71} + a_{ij}^{62} - a_{ij}^{53} - \sqrt{3} a_{ij}^{58} \right] \right\} \\
= \frac{1}{2} A_{mn}^{36} \left\{ a_{ij}^{25} + a_{ij}^{14} + \frac{1}{2} \left[ a_{ij}^{52} + a_{ij}^{41} + a_{ij}^{63} - \sqrt{3} a_{ij}^{68} \right] \right\} \\
= \frac{1}{2} A_{mn}^{37} \left\{ a_{ij}^{24} - a_{ij}^{15} + \frac{1}{2} \left[ a_{ij}^{42} - a_{ij}^{51} - a_{ij}^{73} + \sqrt{3} a_{ij}^{78} \right] \right\} + \frac{\sqrt{3}}{4} A_{mn}^{38} \left\{ a_{ij}^{44} + a_{ij}^{55} - a_{ij}^{66} - a_{ij}^{77} \right\} \\
= \frac{1}{4} A_{mn}^{45} \left\{ a_{ij}^{12} - a_{ij}^{21} + a_{ij}^{64} - a_{ij}^{67} - a_{ij}^{76} - 4 a_{ij}^{54} \right\} \\
= \frac{1}{4} A_{mn}^{46} \left\{ a_{ij}^{31} - a_{ij}^{13} + a_{ij}^{75} - a_{ij}^{64} + 4 a_{ij}^{57} + \sqrt{3} \left( a_{ij}^{18} + a_{ij}^{81} \right) \right\} \\
= \frac{1}{4} A_{mn}^{47} \left\{ a_{ij}^{23} - a_{ij}^{32} - a_{ij}^{65} - a_{ij}^{74} - 2 a_{ij}^{56} - \sqrt{3} \left( a_{ij}^{28} + a_{ij}^{82} \right) \right\} + \frac{\sqrt{3}}{4} A_{mn}^{48} \left\{ a_{ij}^{34} + a_{ij}^{16} - a_{ij}^{27} + \sqrt{3} a_{ij}^{84} \right\} \\
= \frac{1}{4} A_{mn}^{56} \left\{ a_{ij}^{32} + a_{ij}^{74} - a_{ij}^{23} - a_{ij}^{65} - 2 a_{ij}^{47} + \sqrt{3} \left( a_{ij}^{28} + a_{ij}^{82} \right) \right\} \\
= \frac{1}{4} A_{mn}^{57} \left\{ a_{ij}^{64} + a_{ij}^{31} - a_{ij}^{13} - a_{ij}^{75} + 2 a_{ij}^{46} + \sqrt{3} \left( a_{ij}^{18} + a_{ij}^{81} \right) \right\} - \frac{\sqrt{3}}{4} A_{mn}^{58} \left\{ a_{ij}^{17} + a_{ij}^{26} + a_{ij}^{35} + \sqrt{3} a_{ij}^{85} \right\} \\
= \frac{1}{4} A_{mn}^{67} \left\{ a_{ij}^{54} - a_{ij}^{15} + a_{ij}^{75} - 4 a_{ij}^{76} \right\} - \frac{\sqrt{3}}{4} A_{mn}^{68} \left\{ a_{ij}^{14} + a_{ij}^{25} - a_{ij}^{36} + \sqrt{3} a_{ij}^{86} \right\} \\
- \frac{\sqrt{3}}{4} A_{mn}^{78} \left\{ a_{ij}^{15} - a_{ij}^{24} - a_{ij}^{37} + \sqrt{3} a_{ij}^{87} \right\}, \quad \text{(A.3)} \]
\[
D_{ij \ mn} = A_{mn}^{12} \{2 \alpha_{ij}^{12} - \alpha_{ij}^{21} + \frac{3}{4} [ \alpha_{ij}^{45} - \alpha_{ij}^{54} + \alpha_{ij}^{76} - \alpha_{ij}^{67}] \} + A_{mn}^{13} \{2 \alpha_{ij}^{13} - \alpha_{ij}^{31} + \frac{3}{4} [ \alpha_{ij}^{64} - \alpha_{ij}^{46} + \alpha_{ij}^{75} - \alpha_{ij}^{57}] \} \\
+ \frac{1}{4} A_{mn}^{14} \{3 (\alpha_{ij}^{63} - \alpha_{ij}^{52}) + \sqrt{3} (2 \alpha_{ij}^{86} - \alpha_{ij}^{68}) - \alpha_{ij}^{41} + 2 \alpha_{ij}^{14} \} \\
+ \frac{1}{4} A_{mn}^{15} \{3 (\alpha_{ij}^{42} + \alpha_{ij}^{73}) + \sqrt{3} (2 \alpha_{ij}^{87} - \alpha_{ij}^{78}) - \alpha_{ij}^{51} + 2 \alpha_{ij}^{15} \} \\
+ \frac{1}{4} A_{mn}^{16} \{3 (\alpha_{ij}^{72} - \alpha_{ij}^{43}) + \sqrt{3} (2 \alpha_{ij}^{84} - \alpha_{ij}^{48}) - \alpha_{ij}^{61} + 2 \alpha_{ij}^{16} \} \\
- \frac{1}{4} A_{mn}^{17} \{3 (\alpha_{ij}^{53} + \alpha_{ij}^{62}) - \sqrt{3} (2 \alpha_{ij}^{85} - \alpha_{ij}^{58}) + \alpha_{ij}^{71} - 2 \alpha_{ij}^{17} \} - \sqrt{3} A_{mn}^{18} \{\alpha_{ij}^{46} + \alpha_{ij}^{47} + \alpha_{ij}^{57} + \alpha_{ij}^{75} \} \\
+ A_{mn}^{23} \{2 \alpha_{ij}^{23} - \alpha_{ij}^{32} + \frac{3}{4} [\alpha_{ij}^{56} - \alpha_{ij}^{57} + \alpha_{ij}^{74} - \alpha_{ij}^{47}] \} \\
+ \frac{1}{4} A_{mn}^{24} \{3 (\alpha_{ij}^{51} - \alpha_{ij}^{73}) - \sqrt{3} (2 \alpha_{ij}^{87} - \alpha_{ij}^{78}) - \alpha_{ij}^{42} + 2 \alpha_{ij}^{21} \} \\
+ \frac{1}{4} A_{mn}^{25} \{3 (\alpha_{ij}^{63} - \alpha_{ij}^{41}) + \sqrt{3} (2 \alpha_{ij}^{86} - \alpha_{ij}^{68}) - \alpha_{ij}^{52} + 2 \alpha_{ij}^{25} \} \\
- \frac{1}{4} A_{mn}^{26} \{3 (\alpha_{ij}^{53} + \alpha_{ij}^{71}) - \sqrt{3} (2 \alpha_{ij}^{85} - \alpha_{ij}^{58}) + \alpha_{ij}^{62} - 2 \alpha_{ij}^{26} \} \\
+ \frac{1}{4} A_{mn}^{27} \{3 (\alpha_{ij}^{43} + \alpha_{ij}^{61}) - \sqrt{3} (2 \alpha_{ij}^{84} - \alpha_{ij}^{48}) - \alpha_{ij}^{72} + 2 \alpha_{ij}^{27} \} - \sqrt{3} A_{mn}^{28} \{\alpha_{ij}^{56} + \alpha_{ij}^{57} - \alpha_{ij}^{47} - \alpha_{ij}^{74} \} \\
+ \frac{1}{4} A_{mn}^{29} \{3 (\alpha_{ij}^{72} - \alpha_{ij}^{61}) + \sqrt{3} (2 \alpha_{ij}^{84} - \alpha_{ij}^{48}) - \alpha_{ij}^{43} + 2 \alpha_{ij}^{24} \} \\
- \frac{1}{4} A_{mn}^{30} \{3 (\alpha_{ij}^{62} + \alpha_{ij}^{71}) - \sqrt{3} (2 \alpha_{ij}^{85} - \alpha_{ij}^{58}) + \alpha_{ij}^{53} - 2 \alpha_{ij}^{35} \} \\
+ \frac{1}{4} A_{mn}^{31} \{3 (\alpha_{ij}^{52} + \alpha_{ij}^{41}) - \sqrt{3} (2 \alpha_{ij}^{86} - \alpha_{ij}^{68}) - \alpha_{ij}^{63} + 2 \alpha_{ij}^{36} \} \\
+ \frac{1}{4} A_{mn}^{32} \{3 (\alpha_{ij}^{51} - \alpha_{ij}^{42}) - \sqrt{3} (2 \alpha_{ij}^{87} - \alpha_{ij}^{78}) - \alpha_{ij}^{73} + 2 \alpha_{ij}^{37} \} - \sqrt{3} A_{mn}^{33} \{\alpha_{ij}^{44} + \alpha_{ij}^{55} - \alpha_{ij}^{66} - \alpha_{ij}^{77} \} \\
+ A_{mn}^{34} \{2 \alpha_{ij}^{45} - \alpha_{ij}^{54} + \frac{3}{4} [\alpha_{ij}^{12} - \alpha_{ij}^{21} + \alpha_{ij}^{76} - \alpha_{ij}^{47}] \} \\
+ \frac{1}{4} A_{mn}^{35} \{3 (\alpha_{ij}^{31} - \alpha_{ij}^{13} - \alpha_{ij}^{75}) - \sqrt{3} (\alpha_{ij}^{18} + \alpha_{ij}^{81}) - \alpha_{ij}^{64} + 2 \alpha_{ij}^{46} \} \\
+ \frac{1}{4} A_{mn}^{36} \{3 (\alpha_{ij}^{23} - \alpha_{ij}^{32} + \alpha_{ij}^{65}) + \sqrt{3} (\alpha_{ij}^{28} + \alpha_{ij}^{82}) - \alpha_{ij}^{74} + 2 \alpha_{ij}^{47} \} \\
+ \sqrt{3} A_{mn}^{37} \{2 (\alpha_{ij}^{61} - \alpha_{ij}^{72} + \alpha_{ij}^{43}) - \alpha_{ij}^{16} + \alpha_{ij}^{27} - \alpha_{ij}^{34} + \sqrt{3} (2 \alpha_{ij}^{48} - \alpha_{ij}^{84}) \} \\
+ \frac{1}{4} A_{mn}^{38} \{3 (\alpha_{ij}^{32} - \alpha_{ij}^{23} + \alpha_{ij}^{74}) - \sqrt{3} (\alpha_{ij}^{28} + \alpha_{ij}^{82}) - \alpha_{ij}^{65} + 2 \alpha_{ij}^{56} \} \\
+ \frac{1}{4} A_{mn}^{39} \{3 (\alpha_{ij}^{31} - \alpha_{ij}^{13} - \alpha_{ij}^{64}) - \sqrt{3} (\alpha_{ij}^{18} + \alpha_{ij}^{81}) - \alpha_{ij}^{75} + 2 \alpha_{ij}^{57} \} \\
+ \sqrt{3} A_{mn}^{40} \{2 (\alpha_{ij}^{71} + \alpha_{ij}^{62} + \alpha_{ij}^{53}) - \alpha_{ij}^{17} - \alpha_{ij}^{26} - \alpha_{ij}^{35} + \sqrt{3} (2 \alpha_{ij}^{58} - \alpha_{ij}^{85}) \} \\
+ A_{mn}^{41} \{2 \alpha_{ij}^{67} - \alpha_{ij}^{76} + \frac{3}{4} [\alpha_{ij}^{21} - \alpha_{ij}^{12} - \alpha_{ij}^{45} + \alpha_{ij}^{54}] \} \\
+ \frac{1}{4} A_{mn}^{42} \{2 (\alpha_{ij}^{41} + \alpha_{ij}^{52} - \alpha_{ij}^{63}) - \alpha_{ij}^{14} - \alpha_{ij}^{25} + \alpha_{ij}^{36} + \sqrt{3} (2 \alpha_{ij}^{68} - \alpha_{ij}^{86}) \} \\
+ \frac{1}{4} A_{mn}^{43} \{2 (\alpha_{ij}^{51} - \alpha_{ij}^{42} - \alpha_{ij}^{73}) - \alpha_{ij}^{15} + \alpha_{ij}^{24} + \alpha_{ij}^{37} + \sqrt{3} (2 \alpha_{ij}^{78} - \alpha_{ij}^{87}) \}, 
\] 
(A.4)
\[ A^{bc}_{mn} = a^{b+}_m a^{c-}_n, \quad \alpha^{bc}_{ij} = \alpha^b_i \alpha^c_j. \]

References

[1] Yu. L. Dokshitzer, et al., Rev. Mod. Phys. 60, 373 (1988).

[2] Yu. L. Dokshitzer, V. A. Khoze, A. H. Mueller, S. I. Troyan, Basics of Perturbative QCD (Frontières, France, 1991).

[3] V. I. Kuvshinov, Acta Phys. Pol. B 10, 19 (1979).

[4] E. S. Kokoulina, V. I. Kuvshinov, Acta Phys. Pol. B 13, 553 (1982).

[5] E. Malasa, B. Webber, Nucl. Phys. B 267, 702 (1986).

[6] I. M. Dremin, R. C. Hwa, Phys. Rev. D 49, 5805 (1994).

[7] S. Lupia, W. Ochs, J. Wosiek, Nucl. Phys. B 540, 405 (1999).

[8] S. Ya. Kilin, V. I. Kuvshinov, S. A. Firago, Proceed. of a Workshop on Squeezed states and Uncertainty relations, (Moscow, Russia, 1992), NASA, 301 (1993).

[9] V. I. Kuvshinov, V. A. Shaporov, Acta Phys. Pol. B 30, 59 (1999).

[10] V. I. Kuvshinov, V. A. Shaparau, Nonlinear Phenomena in Complex Systems 3, 28 (2000).

[11] O. Hirota, Squeezed light (Japan, Tokyo, 1992).

[12] D. F. Walls, G. J. Milburn, Quantum Optics (Springer-Verlag, NY., USA, 1995).

[13] M. O. Scully, M. S. Zubairy M.S, Quantum Optics (Cambridge University Press, 1997).

[14] S.Ya. Kilin, Quantum Optics (Minsk, 1990). (in Russian)

[15] I. M. Dremin, Phys. Lett. A 193, 209 (1994).

[16] G.J. Aher et al., (UA5 coll.) Phys. Rep. 154, 247 (1987)

[17] P. Abreu et al., (DELPHI coll.) Z. Phys. C 50, 185 (1991)

[18] P.D. Acton et al., (OPAL coll.) Z. Phys. C 53, 539 (1992)

[19] K.N. Alekseev, Opt. Commun. 116, 468 (1995) (preprint quant-ph/9808010).

[20] K.N. Alekseev, Peřina J., Phys. Rev. E 57, 4023 (1998) (preprint chao-dyn/9804041).

[21] K.N. Alekseev, D.S. Prijmak, JETP 113, 111 (1998).

[22] A.J. Lichtenberg, M.A. Lieberman, Regular and stochastic motion (Springer-Verlag, NY., 1983).

[23] H.G. Shuster, Deterministic chaos. An Introduction (Physic-Verlag, Weinheim, 1984).

[24] G.M. Zaslavsky, P.Z. Sagdeev, Introduction into nonlinear physics (Moscow, 1988). (in Russian)

[25] N.S. Krylov, Works on substantiation of statistical physics (Moscow-Leningrad, 1950). (in Russian).
[26] G.M. Zaslavsky, Phys. Rep. 80, 158 (1991).
[27] T. Prosen, M. Robnik, J. Phys. A27, 8059 (1994).
[28] G. Mandelbaum, Proc. of the Int. Conf. ”Chaos and Complexity” (Edition Frontieres, 1995).
[29] V.I. Kuvshinov, A.V. Kuzmin, Proceed. of the Int. Seminar NPCS’01 (Minsk, Belarus, 2001) 183.
[30] G. K. Savvidy, Phys. Lett. B 71, 133 (1977).
[31] E. A. De Wolf, I. M. Dremin, W. Kittel, Phys. Rep. 270, 1 (1996).
[32] W. A. Lin, L. E. Ballentine, Phys. Rev. Lett. 65, 2927 (1990).
[33] G. K. Savvidy, S.G. Matinyan, N.G. Ter-Ar.-Savvidy, JETP 80, 830 (1981).
[34] M. Toda, Phys. Lett. A 48, 335 (1974).
[35] L. Salasnich, preprint nucl-th/9707033.