Probing the equation of state of the early universe with a space laser interferometer

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We propose a method to probe the equation of state of the early universe and its evolution, using the stochastic gravitational wave background from inflation. A small deviation from purely radiation dominated universe ($w = 1/3$) would be clearly imprinted on the gravitational wave spectrum $\Omega_{GW}(f)$ due to the nearly scale invariant nature of inflationary generated waves.

Gravitational waves generated in the early universe are one of the most interesting targets in observational cosmology. Though direct detection of waves is not a simple task due to their extremely weak interaction, they could serve as an invaluable fossil of the early universe that can be hardly probed by other observational means.$^{1,2}$ The most realistic generation mechanism of the stochastic gravitational wave background in the early universe is quantum effects during inflation.$^{3}$ While the fact that we live in a large and nearly homogeneous universe alone is sufficient to prove the existence of an inflationary epoch in the early universe,$^{4}$ recent observations of the WMAP satellite$^{5}$ have provided a more sophisticated confirmation of inflation. This comes from that super-horizon-scale perturbations that were generated in the very early universe present a negative correlation between temperature anisotropy and E-mode polarization (TE).$^{6}$ Note that observation of nearly scale-invariant spectrum of curvature perturbation alone is insufficient as a proof of inflation because other causal mechanisms can also exist to generate a similar spectrum.$^{7}$ Such mechanisms, however, predict a positive correlation in TE and is now discriminated by WMAP observations.$^{6}$

Long wavelength components of the stochastic gravitational wave background generated during inflation can in principle be detected by the B-mode polarization of cosmic microwave background radiation (CMB),$^{8}$ and shorter-wave components with frequency $\sim$ Hz may be observed by a space laser interferometer. As for the former, although WMAP has obtained only an upper bound,$^{9}$ a number of observational projects are planned, and those include Planck$^{10}$ and BICEP$^{11}$ suggesting a strong possibility for detection of the stochastic background wave from inflation by CMB. On the other hand, most theoretical studies of the gravitational wave background of shorter wavelength
have emphasized on the detection itself so far. However a discussion with an anticipation for future diverse possibilities of gravitational wave cosmology, beyond the first detection, is worthwhile.

In the present paper, we argue that the amplitude of high-frequency stochastic gravitational wave background not only carries information on the inflationary regime, during which they are generated, but also serves as a probe of the equation of state in the early universe through which the waves propagate towards us. This double role played by primordial gravitational wave background in particle cosmology is similar to that played by high-redshift quasars in observational cosmology, which not only reflects the properties of high-redshift universe, at their location, but also carry line-of-sight information in their absorption spectra. In this sense, just as high-redshift quasars are regarded as lighthouses of the distant universe, we may regard the inflation-produced gravitational wave background as a lighthouse that can shed light on the early universe physics.

As is well-known, equation of state of the early universe, just after $t = 1$ sec and $T = 1$ MeV, has been accurately probed by big-bang nucleosynthesis (BBN) calculations and a comparison of their yield with observations, though we do not have any sensible means to probe it prior to this epoch. This is why we only impose a mild constraint that the universe should be dominated by radiation before BBN when we work on particle-physics model building of the early universe involving lately decaying particles. Thus, if we could observe the spectrum of gravitational waves from a source with a known property in the early universe, we could obtain important information and constrain high energy physics with which we describe the evolution of the universe.

We start with a brief review of properties of a gravitational wave generated quantum mechanically during inflation with the purpose of demonstrating our notation. We introduce tensor perturbations, $h_{ij}$, around a spatially flat Robertson-Walker metric as

$$ds^2 = -dt^2 + a^2(t) (\delta_{ij} + 2h_{ij}) dx^i dx^j,$$

with $a$ being the scale factor. Decomposing the tensor metric perturbation to Fourier modes as

$$h_{ij} = \sqrt{8\pi G} \sum_{A=+,-,\times} \int \frac{d^3k}{(2\pi)^{3/2}} \varphi^A_k(t)e^{ikx}e^{ij}_{A},$$

we find that the two independent degrees of freedom $\varphi^A_k$ behave as two massless minimally coupled scalar fields, where $e^{ij}_{A}$ represents polarization tensor with $e^{ij}_{A}e^{ij}_{A'} = \delta^{AA'}$ for $A, A' = +, -, \times$. Applying quantum field theory of a massless minimally coupled field in de Sitter spacetime, we find that the Fourier modes are characterized by the following vacuum correlation,

$$\langle \varphi^A_k(t)\varphi^{A'}_{k'}(t) \rangle = \frac{H^2}{2k^3}\delta^3(k-k')\delta^{AA'},$$

so that the amplitude per logarithmic frequency interval is given by

$$h^2_F(f) \equiv 2 \langle h_{ij}h^{ij}(f) \rangle = 4 \times 8\pi G \left(\frac{H(\phi)}{2\pi}\right)^2 = \frac{8}{\pi} \left(\frac{H(\phi)}{M_{Pl}}\right)^2,$$
where $M_{Pl} = G^{-1/2}$ denotes the Planck scale. Here $H(\phi)$ denotes the Hubble parameter during inflation to be evaluated when the mode with comoving frequency $f$ left the Hubble radius during inflation. It is expressed as

$$H(\phi) = \sqrt{\frac{8\pi}{3} \frac{V[\phi(f)]^{1/2}}{M_{Pl}}}$$

in terms of the inflaton potential $V[\phi]$, where $\phi(f)$ denotes the value of the inflaton field when the tensor mode $f$ crosses the horizon during the inflationary epoch.

In this paper, we use the present frequency $f$ to characterize the comoving wavelength of gravitational waves, and the scale factor $a$ for the cosmic time $t$. For the spectrum of gravitational wave background, it is convenient to use a dimensionless quantity $\Omega_{GW}(f, a)$ defined by

$$\Omega_{GW}(f, a) = \frac{\rho_{GW}(f, a)}{\rho_{cr}(a)} = \frac{\rho_{GW}(f, a)}{\rho_{tot}(f, a)} \left( \frac{2\pi f a_0}{a} \right)^2,$$

where $\rho_{GW}(f, a)$ is the energy density of gravitational waves per unit logarithmic frequency interval around $f$, $a_0$ is the current value of the scale factor, and $\rho_{cr}(a)$ is the critical density of the universe.

In the early universe we can identify the critical density $\rho_{cr}(a)$ to the total density of the universe $\rho_{tot}(a)$, as the spatial curvature is negligible.

In a power-law background $a(t) \propto t^p$ with $p < 1$, each Fourier mode behaves as

$$h(f, a) \propto a(t)^{1-3p} J_{3p-1} \left( \frac{p}{1-p} \frac{k}{a(t)H(t)} \right), \quad k = 2\pi f a_0,$$

with Bessel function $J_n(x)$. Such that the amplitude of gravitational wave takes a constant value, $h(f, a) = h_F(f)$, until a mode reenters the Hubble radius $H^{-1}$ at $a = 2\pi f a_0/H \equiv a_{in}(f)$, where the normalized energy density reads

$$\Omega_{GW}(f, a_{in}(f)) = \frac{4}{3\pi} \left( \frac{H(\phi)}{M_{Pl}} \right)^2 = \frac{32}{9} \frac{V[\phi(f)]}{M_{Pl}^2}.$$  

Evolution of the relative energy density of the tensor modes within the Hubble horizon depends on the equation of state of the universe as we see below. From the definition of $\Omega_{GW}$, we have

$$\frac{d\ln \Omega_{GW}(f, a)}{d\ln a} = \frac{d\ln \rho_{GW}(f, a)}{d\ln a} - \frac{d\ln \rho_{tot}(f, a)}{d\ln a}.$$  

From eqs. (6) and (7), the tensor modes well within the horizon behave simply as radiation,

$$\frac{d\ln \rho_{GW}(f, a)}{d\ln a} = -4,$$

and evolution of the total energy density is given by

$$\frac{d\ln \rho_{tot}(f, a)}{d\ln a} = -3 \left[ 1 + w(a) \right],$$

where $w$ is defined by $w \equiv P/\rho_{tot}$ with pressure of the universe, $P$. From eqs. (9-11) we have

$$\frac{d\ln \Omega_{GW}(f, a)}{d\ln a} = 3w(a) - 1.$$
\[ \ln \Omega_{GW}(f, a_0) = \int_{a_{in}(f)}^{a_0} [3w(a) - 1] d \ln a + \ln \Omega_{GW}(f, a_{in}(f)). \] 

(13)

For our analysis it is convenient to use the following expression that relates the present energy density \( \Omega_{GW}(f, a_0) \) at different frequencies \( f_1 \) and \( f_2 \),

\[ \ln \Omega_{GW}(f_2, a_0) - \ln \Omega_{GW}(f_1, a_0) = \int_{a_{in}(f_1)}^{a_{in}(f_2)} [3w(a) - 1] d \ln a + \ln \Omega_{GW}(f_2, a_{in}(f_1)). \] 

(14)

The first term represents the effect caused by deviation of the equation of state from \( w = 1/3 \) and the latter one shows the difference of the intrinsic amplitudes \( \Omega_{GW}(f, a_{in}(f)) \) generated at the inflationary epoch. Using the slow-roll equation of motion,

\[ 3H \dot{\phi} = -V'[\phi], \] 

(15)

we can write down the latter term as

\[ \ln \frac{V[\phi(f_2)]}{V[\phi(f_1)]} \simeq -\frac{M_{Pl}^2}{8\pi} \left( \frac{V'}{V} \right)^2 \ln \left( \frac{f_2}{f_1} \right), \] 

(16)

with \( V'[\phi] \equiv dV[\phi]/d\phi \). Here the coefficient \( M_{Pl}^2/8\pi (V'/V)^2 \) is evaluated around \( \phi \sim \phi(f_1) \) and would be much smaller than unity, which is one of the slow-roll parameters.

As is well known, under the assumption that fluctuations are governed by a single inflaton field \( \phi \), we can obtain the magnitude and the first derivative of the inflaton potential \( V[\phi] \) from the quadrupole anisotropy of the cosmic microwave background radiation (CMB). Denoting the scalar and the tensor contributions by \( S \) and \( T \), respectively, we find

\[ S_{l=2} = 2.2 \left( \frac{V}{M_{Pl}^4} \right)^{(V'/V)}, \quad T_{l=2} = 0.61 \left( \frac{V}{M_{Pl}^4} \right), \] 

(17)

where \( V \) or \( V' \) is evaluated at \( \phi(f) \) with mode \( f \) corresponding to the present horizon \( 2\pi f = H_0 \), and the values of the numerical coefficients are those in the case of the Einstein de Sitter universe.\(^{13,14}\)

As for the spectral indices \( n_S \) and \( n_T \) for these two modes, there are deviations from the results \( (n_S = 1 \text{ and } n_T = 0) \) for purely de-Sitter inflation due to variation of the potential \( V[\phi(f)] \) along with the modes \( f,^{13,14} \)

\[ n_S - 1 = -\frac{M_{Pl}^2}{8\pi} \left( \frac{V'}{V} \right)^2 + \frac{M_{Pl}^2}{4\pi} \left( \frac{V'}{V} \right), \quad n_T = -\frac{M_{Pl}^2}{8\pi} \left( \frac{V'}{V} \right)^2. \] 

(18)

With eqs. (17) and (18) the coefficient \( M_{Pl}^2/8\pi (V'/V)^2 \) is expressed in a well-known form

\[ -\frac{M_{Pl}^2}{8\pi} \left( \frac{V'}{V} \right)^2 = n_T = -\frac{r}{7.0}, \quad r \equiv \frac{T_{l=2}}{S_{l=2}}, \] 

(19)

However, the factor 7.0 in the above expression should be replaced by 5.0 for realistic values of the cosmological parameters in the concordance model.\(^{15}\) WMAP has obtained an upper bound on the
coefficient in eq. (16), as $|M^2_{Pl}/8\pi (V'/V)^2| \sim r/5 \lesssim 0.1$. Thus even in the frequency difference of an order of magnitude, $\ln(f_2/f_1) \sim 2$, the second term of eq. (14) would be very small. If we could measure the difference of $\ln \Omega_{GW}(f_2, a_0) - \ln \Omega_{GW}(f_1, a_0)$, for example to be $\sim 0.5$, it would mainly be due to the first term. Namely, we could detect a curious behavior of equation of state $w \neq 1/3$ in the early universe.

As argued above, there are essentially two frequency bands which may be regarded as a serious target for observations. One is the ultra-low frequency band with $f \sim H_0$, which can be detected indirectly through B-mode polarization of CMB. There are some CMB projects aiming to measure the tensor fluctuations as a primary target. As explained earlier, BICEP, which is expected to start observations in 2004, has a sensitivity down to $r \sim 0.1$. CMBPOL, a space mission planned to be launched around 2012, has sensitivity down to $r \sim 0.0001$ which is close to the fundamental limitation for the detection of $r$ using CMB polarization. Planck, which is scheduled to be launched in 2007, will determine the index $n_S$ with error less than $\Delta n_S < 0.005$. With these future CMB projects, we will observationally obtain information related to the potential $V[\phi]$ at a large scale, $f \sim H_0$, and can use it to estimate $V[\phi]$ up to the second derivative using (18). Thus, although this frequency band is irrelevant to the evolution of the cosmic equation of state, the information obtained through these observations are invaluable that allow us to extrapolate the spectrum $\Omega_{GW}(f, a_0)$ up to much higher frequencies which can be used as a calibrator in order to investigate an anomalous change of the equation of state.

At present we have only an upper bound on tensor perturbations derived from COBE and WMAP, namely, $H(\phi(f = H_0)) < 2 \times 10^{-5} M_{Pl}$. Without taking a possible change in $V[\phi]$ during inflation into account, this corresponds to a constraint $\Omega_{GW}(f \sim 1\text{Hz}, a_0) < 8 \times 10^{-16}$. Thus WMAP data do not provide us direct information on the gravitational wave background. If we consider more specific inflationary models, however, we can predict a precise value of $\Omega_{GW}$ in the high-frequency region. Among many mechanisms of inflation, chaotic inflation proposed by Linde is the most attractive for its naturalness of initial condition. WMAP, however, has ruled out one of them, namely a model with a quartic potential $V[\varphi] = \lambda \varphi^4/4$. The only remaining renormalizable model with a massive scalar potential, on the other hand, has been shown to be not only in good agreement with WMAP data but also well motivated by particle physics and phenomenologically successful if we identify the inflaton with a sneutrino field. This model predicts $r = 0.16$ on large scales, which is within the reach of BICEP. We can also calculate $\Omega_{GW}$ in the observable frequency range by laser interferometers as $\Omega_{GW}(f \sim 1\text{Hz}, a_0) = 2.5 \times 10^{-16}$. Thus allowing a room of uncertainty in inflationary models, it is desirable to make a detector sensitive to $10^{-20} \lesssim \Omega_{GW} \lesssim 10^{-15}$.

The sensitivity of a laser interferometer is often represented by the dimensionless gravitational wave amplitude $h_c(f)$. For a detection of the stochastic wave background, correlation analysis using
multiple detectors is very powerful,\textsuperscript{1,2} and a detector with sensitivity \( h_c(f) \) can probe \( \Omega_{GW} \) up to
\[
\Omega_{GW}(f, a_0) \sim 10^{-16} X \left( \frac{h_c(f)}{10^{-24}} \right)^2 \left( \frac{f}{1 \text{Hz}} \right)^2 \left( \frac{\Delta f}{1 \text{Hz}} \right)^{-1/2} \left( \frac{\Delta T}{1 \text{yr}} \right)^{-1/2},
\]
where \( \Delta T \) is the observational period, \( \Delta f \) is the effective bandwidth, and the coefficient \( X \) represents the overlap of two detectors and is of order unity.\textsuperscript{1,2} The initial LIGO has a sensitivity corresponding to \( \Omega_{GW} \sim 5 \times 10^{-6} \) at \( f \sim 100 \text{Hz} \) with \( \Delta T = 4 \text{ months} \).\textsuperscript{2} The advanced LIGO can detect \( \Omega_{GW} \sim 5 \times 10^{-11} \) for the same observational period. As is apparent from eq. (20), a lower frequency band is more advantageous for the detection of gravitational waves with a nearly scale-invariant spectrum, \( \Omega_{GW} \simeq \text{constant} \). LISA, which is scheduled to be launched in 2011, has the best sensitivity at \( f \sim 10^{-3} \text{Hz} \).\textsuperscript{20} Although we cannot make correlation analysis with LISA due to the vanishing overlap of data streams whose noises do not correlate, we could attain the level of \( \Omega_{GW} \gtrsim 10^{-12} \).\textsuperscript{21} Unfortunately, however, the LISA band is severely contaminated by astrophysical sources and would hamper measurement of the primordial background with \( \Omega_{GW} \lesssim 10^{-11} \), because there are so many close white dwarf binaries that they cannot be resolved separately, which result in a confusion noise.\textsuperscript{22} Their contamination is expected to disappear around \( f \gtrsim 0.2 \text{Hz} \) due to their large radii. The frequency band \( f \sim 1 \text{Hz} \) is expected to be relatively clean,\textsuperscript{21,23} and this band could be observed by DECIGO.\textsuperscript{23} If its ultimately intended sensitivity is achieved, we could detect the stochastic background up to \( \Omega_{GW} \sim 10^{-20} \) at \( \sim 0.2 \text{Hz} \). Thus DECIGO would serve as an ideal detector to study the stochastic gravitational wave background from inflation.

The comoving wave corresponding to \( f \sim 1 \text{Hz} \) today reentered the Hubble horizon around \( t \sim 3 \times 10^{-20} \) sec with the cosmic temperature \( T \sim 3 \times 10^6 \text{GeV} \) in the standard cosmology. Thus, if significant entropy production takes place around \( t \sim 10^{-20} \) sec, we can detect its imprint on the spectrum of \( \Omega_{GW} \) using DECIGO.

A typical mechanism of entropy production in this epoch is decay of massive particles, which we denote by \( \chi \), with the decay rate \( \Gamma_{\chi} \sim 10^{-7} - 10^{-5} \text{GeV} \). If this particle decays through gravitational interaction, its mass corresponds to \( m \sim (0.8 - 4) \times 10^{10} \text{ GeV} \). Evolution of the energy density of two components, the radiation (relativistic particles) and the particle \( \chi \) is solved with the following equations
\[
\frac{d\rho_{\text{rad}}(t)}{dt} = -4H\rho_{\text{rad}}(t) + \Gamma_{\chi}\rho_{\chi}(t), \quad \frac{d\rho_{\chi}(t)}{dt} = -3H\rho_{\chi}(t) - \Gamma_{\chi}\rho_{\chi}(t),
\]
where \( \Gamma_{\chi} \) is the decay rate of the particle \( \chi \) to radiation. These equations are solved as
\[
\rho_{\chi}(a) = \rho_{\chi}(a_i) \left( \frac{a}{a_i} \right)^{-3} e^{-\Gamma_{\chi}(t-t_i)}, \quad a = a(t),
\]
\[
\rho_{\text{rad}}(a) = \rho_{\text{rad}}(a_i) \left( \frac{a}{a_i} \right)^{-4} + \Gamma_{\chi} \left( \frac{a}{a_i} \right)^{-4} \int_{t_i}^{t} \frac{a(\tau)}{a_i} \rho_{\chi}(a_i)e^{-\Gamma_{\chi}(\tau-t_i)} d\tau,
\]
where \( a_i = a(t_i) \) and \( t_i \ll \Gamma_{\chi}^{-1} \) is an initial time with \( \rho_{\text{rad}}(a_i) \gg \rho_{\chi}(a_i) \).
Figure 1 depicts $\log \Omega_{GW}(f, a_0)$ in chaotic inflation model driven by a massive scalar field with various values of $\Gamma_\chi$ and the entropy increase factor $F$. The latter is defined by

$$F \equiv \frac{s(t_e)a^3(t_e)}{s(t_i)a_i^3}, \quad s = \frac{4\pi^2}{90} g_s T^3 = \frac{4 \rho_{rad}}{3} T,$$

(24)

with $s$ being the entropy density, and $g_s$ is the effective number of relativistic degrees of freedom. Here $t_i \gg \Gamma_\chi^{-1}$ is an arbitrary time in the radiation dominated regime after entropy production has terminated. In this figure, solid line represents the case with $\rho_\chi = 0$ from the beginning as a calibrator. The weak deviation from a flat straight line is due to the weak time dependence of the Hubble parameter during inflation as described by the second term in the right hand side of eq. (14). In the present model of a massive scalar inflaton, this curve is represented as $\Omega_{GW}(f, a_0) = 2.5 \times 10^{-16} [1 - 0.1 \ln (f/1\text{Hz})]$. In this figure, dash-dotted line at the bottom depicts the case with $\Gamma_\chi = 10^{-7} \text{GeV}$ with the entropy increase by a factor $F = 10^3$. Three intermediate curves, on the other hand, all correspond to the case $F = 10$ but with different values of $\Gamma_\chi$.

As seen there, the ratio of the values of $\Omega_{GW}$ in the left and the right plateau depends only on the entropy increase factor (apart from the weak dependence of the solid line on $f$). The reason is simple. As the mode $f$ reenters the Hubble radius, $\Omega_{GW}(f, a)$ evolves as eq. (9) or

$$\Omega_{GW}(f, a) = \frac{\rho_{total}(a_e)a_e^4}{\rho_{total}(a)a^4} \Omega_{GW}(f, a_{in}(f)),$$

(25)

with the initial condition given in eq.(8). Let us consider $\Omega_{GW}(f, a)$ at $a = a_e = a(t_e)$ with $t_{in}(f) < t_e$. If $t_{in}(f) \gg \Gamma_\chi^{-1}$ the mode reenters the horizon after the entropy production has terminated, so the factor in the right hand side of eq. (25) is equal to unity and we find $\Omega_{GW}(f, a_e) = \Omega_{GW}(f, a_{in}(f))$ in this case, which corresponds to the left plateau in the figure 1. For the modes that reenter the horizon sufficiently early, we have $\rho_{total}(a_{in})a_{in}^4 = \rho_{rad}(a_{in})a_{in}^4$, so we find $\Omega_{GW}(f, a_e) = F^{-4/3} \Omega_{GW}(f, a_{in}(f))$, because $\rho_{total}$ is also equal to $\rho_{rad}$ at $a = a_e$. Here we have assumed that $g_s$ does not change between $t_{in}(f)$ and $t_e$. This region corresponds to the right plateau in the figure. Thus the ratio is given by $F^{-4/3}$ apart from the weak dependence on $V[\phi(f)]$.

If the entropy production occurs at a later epoch with $\Gamma_\chi \ll 10^{-7} \text{GeV}$, the entire frequency band observable with DECIGO lies in the right plateau region in the figure. We could still estimate the entropy increase factor $F$ in such a case if we could measure the Hubble parameter during inflation with CMB polarization and extrapolate it to a higher frequency band.

So far we have assumed that inflation-produced tensor perturbations are the only source of the stochastic gravitational wave background apart from possible contamination from Galactic and extra-Galactic stellar binaries. But, there are several proposed cosmological sources of backgrounds, such as cosmic strings and field rearrangement associated with a global phase transition. The former produces

$$\Omega_{GW}(1\text{Hz}, a_0) = 3 \times 10^{-8} \left( \frac{v_L}{10^{16} \text{GeV}} \right),$$

(26)
where $v_L$ is the symmetry breaking scale of string formation.\(^{26}\) This would exceed inflationary gravitational radiation if $v_L > 10^8$ GeV. On the other hand, global phase transition induces

$$\Omega_{GW}(f, a_{in}(f)) \simeq \frac{8\pi}{3} \frac{v_G^4}{M_{Pl}^4}, \quad (27)$$

which should be compared with eq. (8).\(^{25}\) Thus, in order to suppress the contribution related to the symmetry breaking scale, $v_G$ should be smaller than $H(\phi)$ during inflation. Even if these contributions surpass that of inflation, however, we can probe equation of state in the early universe with these gravitational waves because they also have intrinsically scale-invariant spectrum at the outset. (Note that eqs. (26) and (27) apply to the case phase transitions occur after inflation. If the symmetry breaking scales are so large that phase transitions take place in the early stage of inflation, they do not contribute to the stochastic gravitational wave background. Alternatively, if phase transitions occur in a late stage of inflation,\(^{27}\) they might leave an interesting imprint on the stochastic background observable with laser interferometers.)

In summary we have proposed to use the stochastic gravitational wave background from inflation to probe the equation of state of early universe and its evolution. Due to the nearly scale invariant nature of inflation, small deviations from a simple radiation dominated universe ($w = 1/3$) would be clearly imprinted in the spectrum of $\Omega_{GW}(f, a_0)$. We could obtain interesting information that can be hardly extracted by other methods.

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Fig. 1. Energy spectrum of the gravitational wave background, $\Omega_{GW}(f, a_0)$, in a chaotic inflationary model driven by a massive scalar field. Solid line represents the case with no entropy production. Dash-dotted line depicts the case with $\Gamma_\chi = 10^{-7}$ GeV and the entropy increase factor $F = 10^3$. Three intermediate curves are for $F = 10$ with $\Gamma_\chi = 10^{-5}$ GeV (short-dashed line), $\Gamma_\chi = 10^{-6}$ GeV (dotted line), and $\Gamma_\chi = 10^{-7}$ GeV (long-dashed line), respectively.