Observational Analysis of Small-scale Magnetic Flux Ropes from Ulysses In-situ Measurements

Yu Chen\textsuperscript{1}, Qiang Hu\textsuperscript{1,2}, Jakobus le Roux\textsuperscript{1,2}, Jinlei Zheng\textsuperscript{1}

\textsuperscript{1} Department of Space Science, The University of Alabama in Huntsville, USA
\textsuperscript{2} Center for Space Plasma and Aeronomic Research, The University of Alabama in Huntsville, USA

E-mail: \textsuperscript{1}yc0020@uah.edu
E-mail: \textsuperscript{2}qh0001@uah.edu

Abstract. Small-scale magnetic flux ropes, which have similar magnetic field configuration as their large-scale counterparts (i.e., magnetic clouds), but with different sizes and origin, constitute an important element of solar wind structures. They are also considered to be associated with local particle energization and other related processes. In this report, we apply the Grad-Shafranov (GS) reconstruction method to detect these small-scale flux ropes with a set of quantitative criteria by utilizing data from the Ulysses spacecraft measurements for the first time. We conduct full range automatic detection for years 1994, 1996, 2004 and 2005 during the solar minimum periods. Based on solar wind speed/helio-latitude ranges, these periods are categorized into two groups: one with high solar wind speeds at high latitudes (1994 and 1996) and the other with low solar wind speeds at low latitudes (2004 and 2005). Through mainly statistical analysis of the results from these four years worth of Ulysses data, we have obtained the following findings: (1) Alfvénic structures occur more frequently at higher latitudes or in high speed solar wind (1994 and 1996). (2) Small-scale flux ropes at lower latitudes tend to align with the nominal Parker spiral direction. (3) The scale sizes of small-scale flux ropes are in the same range for different heliocentric distances. Both scale size and duration distributions seem to obey power laws, similar to the analysis results at 1 astronomical unit (AU). (4) The waiting time distribution (WTD) is fitted well by an exponential function rather than a power law. (5) The power law fitting is applied to the wall-to-wall time distribution with the break point at \( \sim 200 \) min which is 3\textsuperscript{-}4 times the result at 1 AU.

1. Introduction
Magnetic flux ropes, with helical structures and winding magnetic field lines, can be categorized into two types: the large-scale magnetic flux ropes or magnetic clouds which have been extensively studied, and the small-scale ones or magnetic islands which are still under investigation. Compared to the former, the small-scale flux ropes have similar magnetic field configuration but different sizes (0.001 \textsuperscript{-}0.01 AU at 1 AU while large-scale ones have \( \sim 0.1 \) AU to 0.4 AU) and perhaps different origination and evolution processes. The existence of small-scale flux ropes in the solar wind was first introduced by Moldwin et al. [1]. They reported a small-scale magnetic flux rope with scale size \( \sim 0.05 \) AU observed by the Ulysses spacecraft at around 5 AU at low latitudes. With characteristics of high proton temperature, density, and plasma beta, etc., they interpreted this type of events as being generated from multiple magnetic reconnection events at the heliospheric current sheet (HCS). Later, Moldwin et al. [2] discovered
more small-scale flux ropes at 1 AU from the IMP 8 and Wind spacecraft measurements. Features of these small helical field structures including bimodal size distribution, different behavior of plasma parameters such as the proton temperature, and scale size similar to the HCS thickness were presented. Therefore, they concluded that these small helical field structures are signatures of magnetic reconnection across the HCS.

However, it is still debatable what factors triggered or formed these structures. Feng et al. [3] identified magnetic flux ropes including small- and intermediate-sized structures from 1995 to 2001 from the Wind spacecraft measurements. Based on their estimate of the energy contents of flux ropes, the authors suggested that the small- and intermediate-sized magnetic flux ropes are the interplanetary manifestations of small coronal mass ejections (CMEs), like magnetic clouds which are related to the large interplanetary coronal mass ejections (ICMEs). Moreover, Feng et al. [4] presented the annual number and distribution of the axial orientations of small- and intermediate-sized magnetic flux ropes and showed that these characteristics are similar to those of magnetic clouds. Based on these and other related features, they suggested that small- and intermediate-sized magnetic flux ropes have the same origin as magnetic clouds, i.e., solar eruptions. More supporting evidence that some small-scale flux ropes originate in the solar corona was also presented by investigating the associated counterstreaming suprathermal electron (CSE) signatures [5].

On the other hand, Cartwright and Moldwin [6] compared small-scale flux ropes with magnetic clouds by applying a semi-automated routine to 11 years of Wind data. The bimodal size distribution was confirmed again, and they concluded that small and large-scale flux ropes have different source mechanisms. Furthermore, they presented a comprehensive survey of small-scale flux ropes using multiple spacecraft measurements between 0.3 and 5.5 AU, including the Helios 1, Helios 2, IMP8 spacecraft missions, etc. By examining the statistical results, they found that the occurrence rate of small-scale flux ropes has a weak dependence on the solar cycle, and the generation of these structures from magnetic reconnection across the HCS is consistent with their results.

Small-scale flux ropes are thought to be connected to energization of particles in both theory, simulations and observations. Theory and simulations predict that merging and contraction of magnetic islands generate electric fields which accelerate quasi-trapped particles in a quasi-2D non-resonant interacting process [7; 8]. When suprathermal ions traverse a region of numerous contracting and merging magnetic islands, they can be accelerated to hard power-law spectra [8; 9]. When diffusive shock acceleration (DSA) of energetic particles is combined with acceleration by downstream contracting and merging magnetic islands, theoretical predictions suggest that energetic particle fluxes peak behind the shock instead of at the shock as predicted by DSA only [10; 11]. Thus, this provides an alternative, and novel explanation for downstream peaks in energetic particle fluxes, not solely relying on time-dependent DSA effects. Observational support for the latter theoretical prediction can be found in [10], [12], [13], and [14]. Figure 1 illustrates the magnetic reconnection associated with merging magnetic islands downstream of a shock.

Until recently, small-scale flux rope identification in spacecraft data was hampered by a limited number of events and was narrow in scope by only focusing on temporal profiles. Accurate and automatic detection of small-scale flux ropes from in-situ spacecraft measurements is essential for better categorizing and understanding how they originate and interact with other processes. The Grad-Shafranov (GS) reconstruction technique, which can recover two-dimensional (2-D) magnetic field structures without assuming the force-free condition and axisymmetric cross sections from one-dimensional (1-D) in-situ spacecraft data, was developed and applied to study magnetic flux ropes [16–18]. It is based on the assumption of 2-D magnetohydrostatic equilibrium, and the solution of magnetic flux function over the cross section of a small-scale flux rope can be obtained by examining the calculated transverse pressure as
a function of the magnetic flux function using initial data from spacecraft measurements. This technique, therefore, enables us to develop the automated detection of small-scale flux ropes. A database containing over 70,000 small-scale flux ropes identified via the GS method from the Wind spacecraft data has been constructed [19]. By analyzing this database, Zheng and Hu [19] showed that the wall-to-wall time distribution and the non-Gaussian probability density function of the axial current density distribution correspond well to observations and numerical simulation results of magnetohydrodynamic (MHD) turbulence in the solar wind.

Although a number of small-scale flux ropes at 5 AU has been studied and hence resulted in suggestions of possible origination [1], quantitative modeling and analysis of small-scale flux ropes by applying the GS reconstruction to both the inner and outer heliosphere, and especially higher latitude regions, have not been attempted. In this study, we apply the Grad-Shafranov (GS) reconstruction method to the Ulysses *in-situ* spacecraft measurements and present statistical analysis results for four years during the solar minimum periods over a range of heliographic latitudes.

The *in-situ* data from the Ulysses spacecraft and the Grad-Shafranov (GS) equation will be introduced in section 2. The steps of the Grad-Shafranov (GS) reconstruction and a new set of criteria for the automated detection will be introduced as well. In section 3, we present the statistical results of small-scale flux ropes based on the automated detection from the Ulysses data. They include various properties of small-scale flux ropes, the waiting time distribution (WTD) analysis, wall-to-wall time distribution, partial variance of increments (PVI) statistic, the angular change ($\Delta \theta$) of magnetic field across flux rope boundaries, and the probability density function (PDF) of the axial current density. We also present two preliminary case studies of the particle energization signatures associated with the small-scale flux ropes. In the last section, we summarize the key findings of this study and discuss future work involving the GS reconstruction method.

Figure 1. Schematic of merging magnetic islands downstream of a shock. The heavy black wavy line is a shock. Alfvénic-like waves are shown by red wavy lines. Magnetic reconnection involving a pair of merging magnetic islands is illustrated in the right panel. Image credit: Zank et al. [10].
2. Data and Method

In this section, the in-situ data from the Ulysses spacecraft are introduced first. Then, the standard Grad-Shafranov equation is presented. With this equation, the Grad-Shafranov reconstruction method can be implemented, and the reconstruction steps are explained. The automated detection procedure of small-scale flux ropes, based on this technique, is described with a new set of criteria needed for analysis of Ulysses data.

2.1. Data from Ulysses In-situ Measurements

The analysis covers four years during solar minima. The solar wind flow had both high and low speed intervals during these times which were well separated by helio-latitude [20]. Besides, by combining flux-rope data at ~ 1 AU, mostly obtained in low latitudes near the ecliptic plane, with flux-rope events near the ecliptic at larger heliocentric distances, we are able to analyze the dependence of flux-rope characteristics on heliocentric distance. For this purpose, the years 1994, 1996, 2004 and 2005 were selected in the Ulysses database.

Ulysses has various instruments on board. In this study we use data from Vector Helium Magnetometer (VHM), Solar Wind Observations Over the Poles of the Sun (SWOOPS), Heliosphere Instrument for Spectra, Composition and Anisotropy at Low Energies (HI-SCALE), and Cosmic Ray and Solar Particle Investigation (COSPIN). The data files from the first two instruments contain date, time, magnetic field data with 1 min. resolution in the RTN (radial, tangential and normal) coordinates. They also provide the field magnitude, proton density, three components of solar wind velocity in the RTN coordinates, and the proton temperature in terms of $T_{\text{large}}$ and $T_{\text{small}}$. These two kinds of proton temperature are measured in two ways: determination of $T_{\text{large}}$ involves the integral of the proton distribution over all energy channels, which may lead to overestimation of temperature, while $T_{\text{small}}$ is calculated at a fixed energy resulting in underestimation for very cold plasma. Following most studies, $T_{\text{small}}$ is regarded as proton temperature in this study. Since the electron temperature is unavailable from the Ulysses spacecraft, the thermal pressure $p$ calculated in the first step of the GS reconstruction only contains proton temperature. The proton $\beta$, which partly defines a magnetic flux rope, is considered to be equivalent to the plasma $\beta$. Data files of the last two instruments which contain energetic proton flux measured from the low energy telescope (LET), and ion data from the low energy magnetic spectrometer at 30$^\circ$ relative to the spacecraft spin axis (LEMS30), are applied to case studies.

2.2. The Grad-Shafranov Equation

Using the assumption of two-dimensional (2D) magnetohydrostatic equilibrium and four field line invariants, i.e., the thermal pressure $p$, the transverse current density $j_t$, the axial component of the magnetic field $B_z$, and the magnetic flux function $A$, the Grad-Shafranov (GS) equation, describing non-force-free magnetohydrostatic structures in space plasmas, is given by:

$$\frac{\partial^2 A}{\partial x^2} + \frac{\partial^2 A}{\partial y^2} = -\frac{\mu_0}{\partial A} \frac{dP_t(A)}{dA} = -\mu_0 j_z(A).$$

(1)

Here $j_z$ is the axial component of the current density, and $P_t$ is defined as the total transverse pressure $P_t = p + (B_z^2/2\mu_0)$ which is a single-variable function of $A$ only.

From the solution to the standard GS equation (1), we obtain the magnetic flux function $A(x,y)$, i.e., the $x$-$y$ plane, of a cross section and the axial current density $j_z$ over the cross section of a small-scale flux rope. Each flux surface or field line is of a distinct $A$ value.

2.3. The Grad-Shafranov (GS) Reconstruction

As aforementioned, the GS reconstruction is a tool that employs 1-D data from in-situ spacecraft measurements to recover 2-D magnetohydrostatic or magnetic field structures, i.e., the cross
Table 1. The criteria of small-scale magnetic flux rope detection for Ulysses.

| Duration (minutes) | $R_{dif}$ | $R_{fit}$ | Walén test slope |
|--------------------|-----------|-----------|------------------|
| 45 ~ 2165          | $\leq 0.12$ | $\leq 0.14$ | $\leq 0.5$ |

section of a small-scale flux rope.

Following Hu [21], for example, the basic steps of the GS reconstruction are:

1. Select a data interval with magnetic field $B$ and plasma parameters including the proton temperature $T_p$, proton number density $N$, the electron temperature $T_e$ if available, and the solar wind velocity $V_{SW}$. The thermal pressure $p$, therefore, can be computed by

$$p = Nk(T_e + T_p).$$

2. In order to satisfy the requirement of quasi-static equilibrium from the GS equation, we have to find the co-moving frame of reference. The deHoffmann-Teller (HT) frame is usually adopted [18]. Next, the frame velocity $V_{HT}$ is determined by minimizing the residual electric field in a frame moving with unknown velocity $V_F$. The residual electric field will reach its minimum when $V_F = V_{HT}$.

3. The minimum-variance analysis of magnetic field vector data (MVAB) provides three eigenvalues: the minimum, the intermediate and the maximum variance and their corresponding eigenvectors. We take the intermediate eigenvector as the initial trial $z$-axis and perform the trial-and-error process which is based on the requirement that $P_t(x,0)$ versus $A(x,0)$ be single-valued to find the optimal $z$-axis.

4. Once the optimal $z$-axis is determined, the reconstruction frame can be established. The $x$-axis is projected along the spacecraft path, and following the right-handed orthogonal coordinate system, the direction of the $y$-axis is determined.

5. With the spacecraft data of magnetic field along the $x$-axis as initial value, the magnetic flux function $A(x,0)$ along the $x$-axis can be calculated by integrating $-B_y$:

$$A(x,0) = \int_0^x \frac{\partial A}{\partial \xi} d\xi = \int_0^x -B_y(\xi,0) d\xi,$$

where $d\xi = -V_{HT} \cdot \hat{x} dt$, obtained from the previous steps, and $dt$ is the sampling time increment. By far, $P_t(x,0) = p(x,0) + B_y^2(x,0)/2\mu_0$ versus $A(x,0)$ is ready.

6. Having determined $P_t(x,0)$ versus $A(x,0)$ for data points along the $x$-axis, the transverse pressure as a function of the magnetic flux, $P_t(A)$, can be obtained by minimizing $\sum_i [P_t(x_i,0) - P_t(A(x_i,0))]^2$, where $P_t(x,0)$ is calculated from spacecraft measurements while $P_t(A(x_i,0))$ is found from a functional fitting.

7. With the determined $P_t(A)$ function and the initial data provided by the spacecraft, we can solve the GS equation and finally obtain the solution of $A(x, y)$ which describes the magnetic field configuration of a flux rope, together with the axial field $B_z(A)$.

2.4. Automated Detection

Using the Grad-Shafranov (GS) reconstruction, Zheng and Hu [19] completed the automated detection of small-scale flux ropes utilizing data from the Wind spacecraft in the vicinity of 1 AU. We applied their method to Ulysses data using a new set of appropriate detection criteria which is presented in Table 2.1.

The basic search criterion is based on the double-folded behavior of $A(x,0)$ along the spacecraft path. Since the spacecraft passes through the same set of magnetic field lines from
one edge to the center as it passes from the center to the other edge but in reversed order, we
can split this path into two branches: the first and the second halves which let $P_t(A)$ curve be
double-folded with an inflection or turning point. Then we apply the set of metrics given in
Table 2.1 to filter event candidates:

1. Duration: In order to find candidate events fulfilling the criteria quantitatively and
   qualitatively, multiple iterations are adopted for detecting flux ropes: the searching window
time interval is varied between (45, 80), (70, 105), to an upper bound of 2165 minutes. Taking
(45, 80) min as an example, event candidates which fall within the limits of the width of sliding
window set at 80 minutes maximum and the lower limit 45 minutes while exhibiting a double-
folded $P_t(A)$ are recorded by the detection algorithm. Moreover, we allow slight overlaps, 10
min, to exist between adjacent search windows in order to identify the maximum number of
qualified candidate events.

2. Residues: In order to check the double-folding quality, the difference residue and the
   fitting residue are both used [18; 22]. Separated by the turning or inflection point,$P_t(A)$ values
   of the two branches are calculated to obtain the difference residue $R_{diff}$ while the fitting residue
   $R_{fit}$ is determined through the deviation of $P_t$ values between the measurements and the fitting
   function. Because $P_t$ may vary from one individual flux rope to another, normalization by
   the difference between the maximum and minimum is necessary. Therefore, the smaller these
   residues are, the more accurately the $z$-axis is determined:

   $$R_{diff} = \left[ \frac{1}{2N} \sum_{i=1}^{N} ( (P_t)^{1st}_i - (P_t)^{2nd}_i )^2 \right]^{\frac{1}{2}} / \max(P_t) - \min(P_t)$$  \hspace{1cm} (4)

   and

   $$R_{fit} = \left[ \frac{1}{L} \sum_{i=1}^{L} (P_t(x_i,0) - P_t(A(x_i,0)))^2 \right]^{\frac{1}{2}} / \max(P_t) - \min(P_t)$$  \hspace{1cm} (5)

   where the maximum $P_t$ value is usually reached in the center of flux ropes.

3. Walén slope threshold: Walén slope is the slope of linear regression between the remaining
   flow $V - V_{HT}$ and the local Alfvén velocity $V_A$. A small slope threshold excludes Alfvénic
   structures or waves and therefore ensures the satisfaction of the requirement of quasi-static
   equilibrium. In this study, a value 0.5 is set as the threshold in order to achieve sufficient event
   statistics while fulfilling the quasi-static assumption.

3. Results
3.1. Statistical Results of Occurrence Rate
Figure 2 presents the polar plot of solar wind speed as a function of helio-latitude for the four
years: 1994, 1996, 2004 and 2005. Both 1994 and 1996 data were in high speed solar wind
conditions, and for most of the time during these two years, the Ulysses was at higher latitudes.
The Ulysses was at lower latitudes or near the ecliptic plane for most of 2004 and 2005. Also, the
average solar wind speed is below 500 km/s for these two years. Therefore, we can categorize
these four years of data into two groups based on the aforementioned information: Group 1
(1994 and 1996) corresponds to higher solar wind speed/latitudes, and Group 2 (2004 and 2005)
refers to lower solar wind speed/latitudes, respectively.

Table 3.1 illustrates the parameters for these four years along with the flux-rope occurrence
rates for different Walén slope thresholds. The data files of average solar wind speed, heliocentric
distance and HelioGraphic Inertial (HGI) latitude are provided by Ulysses from COHOweb 1
hour merged data products. Occurrence rates or counts of small-scale flux ropes with different
Walén slope thresholds are obtained from automated detection as described in the last section.
Because this slope represents the ratio between the remaining flow speed and the local Alfvén
speed, the smaller Walén slope threshold we set, the more Alfvénic structures and waves will
Figure 2. Polar plot of solar wind speed as a function of latitude during 1994 (blue line), 1996 (orange line), 2004 (gold line) and 2005 (purple line). HelioGraphic Inertial (HGI) latitude and plasma flow speed in RTN coordinates are provided by Ulysses from COHOweb 1 hour merged data files.

Table 2. The statistical results of small-scale flux ropes from Ulysses measurements. From left to right the columns are: Year, average solar wind speed in units of km/s, heliocentric distance (AU), latitude, counts with Walén slope (WS) \( \leq 0.5 \), counts with Walén slope \( \leq 0.3 \), and counts with Walén slope \( \leq 0.1 \).

| Year | \(<V_{SW}>>| Distance (AU) | Latitude (°) | WS \( \leq 0.5 \) | WS \( \leq 0.3 \) | WS \( \leq 0.1 \) |
|------|--------------|---------------|--------------|----------------|----------------|----------------|
| 1994 | 762          | 1.56 \sim 3.83 | -80.2 \sim -44.2 | 1585           | 1418           | 609            |
| 1996 | 712          | 3.05 \sim 4.71 | 19.1 \sim 53.1  | 1729           | 1706           | 1171           |
| 2004 | 445          | 5.31 \sim 5.41 | -15.7 \sim 2.6  | 1166           | 1164           | 1108           |
| 2005 | 494          | 4.48 \sim 5.31 | -37.1 \sim -15.7 | 1204           | 1199           | 1161           |

be excluded. According to the results in Table 3.1, it is clear that with decreasing threshold values, the counts of small-scale flux ropes significantly decrease in Group 1, but not in Group 2. Therefore, we can conclude that Alfvénic structures (or waves) occur more frequently at higher latitudes under higher speed solar wind speed conditions, while the quasi-static small-scale flux ropes appear more dominantly at lower latitudes where lower solar wind speeds prevail.

3.2. Statistical Properties of Small-scale Flux Ropes

The histogram of average solar wind speed \( V_{SW} \) is shown in Figure 3. Group 1 data (years 1994 and 1996) is represented by the blue curve. A peak occurs near \( V_{SW} = 760 \) km/s close to the median \( V_{SW} = 753 \) km/s. Group 2 data (years 2004 and 2005) is denoted by the red curve. It also has high solar wind speed, such as the maximum \( V_{SW} = 798 \) km/s. But Group 2 can still be regarded as low solar wind speed since the mode is 429 km/s and the median is 462 km/s, which are all less than 500 km/s. Note that Ulysses went to lower latitudes (down to 19.1° north) during year 1996 and higher latitudes (up to -37.1° south) during year 2005. Thus, there is an intersection of two groups and the minimum solar wind speed of Group 1 is 482 km/s which is really close to the mean value of Group 2, i.e., 480 km/s, and the difference between two maximum values is just 36.51 km/s. Fortunately, because low speed solar wind only occurred
Figure 3. Histogram of average solar wind speed with 10 km/s as bin size.

for part of year 1996 (Figure 2), our categorization of two groups based on solar wind speed is still valid.

Figure 4 presents the angular axial orientations of small-scale magnetic flux ropes in the RTN coordinate system. The blue and yellow bars represent two different groups: Group 1 data is associated mainly with high solar wind speed while Group 2 data refers mainly to relatively low solar wind speed as mentioned in Table 3.1. In the histogram of polar angle $\theta$, the angle between the $z$ axis (given in the RTN coordinates) and $N$ direction, the largest count of small-scale flux rope occurrence is at 80 $\sim$ 90$^\circ$ bin which indicates that most cases for both groups tend to lie parallel to the local RT plane. The histogram of the azimuthal angle $\phi$, the angle between the projected flux-rope $z$-axis and the T direction of Group 2 shows that two peaks are at 100 $\sim$ 120$^\circ$ and 280 $\sim$ 300$^\circ$ which are separated by 180$^\circ$ from each other. It indicates that the projection of the $z$ axis onto the RT plane tends to align with the nominal Parker spiral direction on the ecliptic plane. However, Group 1 does not have clear tendency, although there might be two peaks located at 160 $\sim$ 170$^\circ$ and 300 $\sim$ 310$^\circ$.

Figure 5 presents the histogram of duration of small-scale flux ropes for these four years. The gold line represents the entire event set. The counts for Group 1 and 2 are shown by blue and red lines, respectively. Based on the criteria of automated detection, the minimum duration investigated is 45 minutes for both data groups, which guarantees the minimum duration of double-folding $P_t(A)$. The largest search window limits are between (1450, 2165) minutes. The distributions of both groups obey a power law, similar to the results for flux ropes identified at 1 AU, although the slopes may be different. Plus, as shown by both lines, the same tendencies indicate that there is no significant difference in duration between the high and low latitudes among the identified flux rope events except for the slopes in the power-law distributions. Both the Group 1 and Group 2 data exhibit the same trend of plateaus for flux-rope durations less than $\sim$ 80 min, and a negative power-slope for durations more than $\sim$ 100 minutes. However, Group 1 data forms a steeper power-law with a lower mean duration, which is partly due to Group 1 data being associated with higher solar wind speeds.

Figure 6 shows the distribution of scale sizes which are the spatial extent of the cross section of small-scale flux ropes along the spacecraft path. The maximum flux rope scale size is $\sim$ 0.43 AU at least while the minimum is within the range of scale sizes observed at 1 AU. Similarly, the distributions of scale sizes of two groups seem to be close to each other which also indicates that small-scale flux ropes occurring at high or low latitudes or with different solar wind speed
Figure 4. The top panel is histogram of the polar angle $\theta$ of the flux rope axial orientation with bin size 10°. In our searching process, the theta angle has been limited to be within 0 ∼ 90°. That is if the theta angle exceeds 90°, we simply flip the z axis. The bottom panel is histogram of azimuthal angle $\phi$ with 20° bin size. Blue and yellow bars represent Group 1 and 2, respectively.

maintain similar characteristics. Again, the distributions appear to follow power laws but with different power-law indices.

In addition, we compare the distributions of scale size at different heliocentric distances in the ecliptic plane using Group 2 (2004 and 2005) Ulysses data at ∼ 5 AU and Wind data at ∼ 1 AU for the same time periods. The occurrence rate from Wind observations was obtained for a duration range ∼ 9 - 361 minutes [19]. In order to make this comparison more consistent, we set the duration range from 5 to 355 minutes for the automated detection for Ulysses. The results are presented in Figure 7. One can see that two lines representing results from different spacecraft have a similar trend in that more event counts occur at smaller scale size and there are larger scale sizes continuously distributed through a range that may be defined as intermediate-scale. Although the scale size of the cross section of small-scale flux ropes is not dramatically different at different latitudes (Figure 6), the differences are somewhat more pronounced at different heliocentric distances. The largest count of scale size in the Wind database occurs at ∼ 0.002 AU which is twice that from the Ulysses data. Moreover, the difference between counts
Figure 5. The histogram of duration of small-scale flux ropes in minutes which corresponds to the length of each flux rope interval. The range is from 45 minutes to 2165 minutes with 20 minutes bin size.

Figure 6. The histogram of scale size of small-scale flux ropes cross section with bin size 0.0055 AU.

of small-scale flux ropes between these two spacecraft measurements becomes larger at scale sizes greater than the line intersection value of $\sim 0.01$ AU based on the power law fitting curves. Beyond the intersection point, Ulysses data tend to yield more events with relatively larger scale sizes.

In Figure 8 the histograms of average magnetic field magnitude, maximum magnetic field magnitude, proton temperature and plasma $\beta$ are displayed. $T_{\text{small}}$ of proton temperature data is used as in previous studies. The distribution of proton temperature peaks at higher temperature for Group 1 compared to Group 2. This indicates that small-scale flux ropes with lower solar wind speed are inclined to have low proton temperature and high proton temperature persists in high speed solar wind flux ropes, consistent with results at 1 AU. The histogram of average magnetic field shows that Group 1 has relatively higher peak values than Group 2.
Figure 7. The histogram of scale size of small-scale flux ropes with data from Ulysses (blue line) and Wind (red line) in years 2004 and 2005.

Figure 8. From the top to the bottom panels: the histograms of average magnetic field magnitude with bin size 0.07 nT, the maximum magnetic field with 0.08 nT bin size, the proton temperature $T_{\text{small}}$ with $0.0035 \times 10^6$ K bin size, and the plasma $\beta$ with 0.05 as bin size.

Except for the minimum value, the maximum, mean, median and mode of average magnetic field are all greater under high speed solar wind conditions.

Since the plasma $\beta$ is the ratio between the plasma pressure $NkT_p$ and magnetic pressure $B^2/2\mu_0$, its distribution must contain features of aforementioned parameters. Beyond the value $0.07 \times 10^6$ K on the distribution of proton temperature plot, small-scale flux ropes under high speed solar wind conditions dominate since the gold line which represents the entire data set, then overlaps the blue line of Group 2 data. This tendency is also clear in distribution of plasma $\beta$. Most $\beta$-values are less than 1 and the occurrence of flux ropes drops strongly with increasing
The waiting time distribution (WTD), which is defined as the separation time between the starting times between adjacent flux-rope structures (dotted curves). The WTDs are fitted both by an exponential (solid curves) and a power law (dash-dotted curves).

![Figure 9](image1)

**Figure 9.** The waiting time distribution (WTD), which is defined as the separation time between the starting times between adjacent flux-rope structures (dotted curves). The WTDs are fitted both by an exponential (solid curves) and a power law (dash-dotted curves).

The distribution of wall-to-wall time which is defined as the separation between boundaries of small-scale flux ropes.

![Figure 10](image2)

**Figure 10.** The distribution of wall-to-wall time which is defined as the separation between boundaries of small-scale flux ropes.

β-values beyond $\beta \simeq 1$, thus confirming that small-scale flux ropes are non-linear structures often associated with strong magnetic fields in force balance with significant plasma pressure gradient. An abnormally large $\beta$-values of 90.46 was detected in the Group 1 data set, which may be due to strong current sheet structure.

The waiting time distribution (WTD), which indicates if discrete events occur independently, is presented by Figure 9. The WTD in this study is defined by the separation time between the starting times of adjacent flux ropes. The minimum therefore has to be $\sim 45$ min. The WTD of each group of data is fitted both by an exponential function and a power law function, respectively. It is clear that the former yields a better fit to both WTDs.
3.3. Wall-to-wall Time Distribution Analysis
Figure 10 illustrates the distribution of wall-to-wall time. We assume that small-scale flux ropes are bounded by current sheets or magnetic field jumps with negligible thickness. Thus, current sheets, i.e., walls, exist at the start and end points of each flux rope interval. A power law fit is applied to each wall-to-wall time distribution. Each distribution can be thought of as double power law with a breaking point separating the two slopes, indicating separation between the inertial range (harder slope) and the energy containing range (softer range) of flux rope cross sections. The breaking point is located at ~ 200 min which is 3 - 4 times the observational result at 1 AU (not shown). Compared to Greco et al. [23] who observed a power law index \(-1.23 \pm 0.03\) for 1 AU in-situ observations in the inertial range, we find an index of \(-1.57\) below the breaking point and \(-2.1\) beyond the breaking point for Group 1 data, and for Group 2 data, these indices are \(-1.33\) and \(-1.78\). So the results from Group 2 are closer to those at 1 AU of Greco et al. [23], within the inertial range. Since Group 1 may contain a lot of Alfvénic structures or waves (Table 3.1), the result of wall-to-wall time analysis at higher latitudes or in high speed solar wind might be affected, and is different from the result in low latitudes.

3.4. Partial Variance of Increments (PVI) Statistics
The partial variance of increments (PVI), according to Greco et al. [24; 25], is defined as

$$\Im = \frac{|\Delta B|}{\sqrt{<|\Delta B|^2>}} = \frac{|B(t+\tau) - B(t)|}{\sqrt{<|B(t+\tau) - B(t)|^2>}}.$$  \hspace{1cm} (6)

Any sharp changes in the magnetic field will be detected via large PVI values. This is a quick method to locate the boundaries of structures and also can be used for comparing signatures of observations with results of simulations.

Values of calculated partial variance of increment (PVI) for the selected time periods are illustrated in Figure 11. The first panel is Group 1 data with two separate lines for 1994 and 1996, respectively. There are no apparent peaks for most high latitudes during mostly high speed solar wind conditions. But peaks occur for the last 4 months in 1996 when Ulysses was
transitioning into low latitudes and low speed solar wind streams. Such peaks are also visible in the predominantly low latitude, slow solar wind Group 2 data in the second panel. Group 2 data has about at least two strong peaks per month, in addition to relatively weak peaks that occur more often throughout the year. During solar minimum, the heliospheric current sheet (HCS) occurs more frequently in the equatorial plane while tending to occur more frequently at high latitudes at solar maximum [26]. All four years of data analyzed in this study were observed during solar minima. Therefore, it is reasonable that Group 2 data near the ecliptic plane contains more recurring peaks probably corresponding to HCS crossings.

3.5. $\Delta \theta$: Angle Across Small-scale Flux Rope Boundaries

The distribution of deflection angle across small-scale flux rope boundaries is presented with exponential fits in Figure 12. Miao et al. [28] identified about 28,000 current sheets in Ulysses 1-2 second resolution magnetic field measurements, and showed a breaking point around $\Delta \theta = 72^\circ$. The largest count for the deflection angle occurred at around $30^\circ$. In this study, Group 2 data can be compared directly with the results of Miao et al. [28] since in both cases the observations were made near ecliptic plane or at low latitudes. The distributions were fitted with double exponential functions with characteristic decay values of $1/8.17^\circ$ and $1/16.83^\circ$, respectively, separated by a break point around $\Delta \theta = 30^\circ$. Although the main features of our distributions seem agree with those of Miao et al. [28], both two characteristic exponential decay values and location of the breaking point differ substantially. Note that Miao et al. [28] detected 28214 events and their analysis used much higher resolution data while in this study we consider only about 4 years of data with much less events. Therefore, our statistical results may be affected by the relative lack of sample size and data resolution. Moreover, we did not impose a limit on the magnetic field magnitude as one of the criteria for the automated detection, which means our statistical result might contain small fluctuations and other structures, which needs to be investigated further.

The axial electric current density can be determined by the derivative of the transverse pressure with respect to the magnetic flux function $A$: $j_z = dP_t/dA$. The distribution of $j_z$ is shown in Figure 13 normalized to the standard deviation of $j_z$, $\sigma = 3.2 \times 10^{-12} \, A/m^2$. 

**Figure 12.** The histogram of the angular change in $B$ across the small-scale flux rope boundaries, which is calculated with increment of 2 minutes before and after each boundary point. Data and fitting curve by an exponential function for Group 1 are marked in blue, while the red dots, red dotted line and the dash-dotted line are for Group 2.
The distribution is non-Gaussian distribution with pronounced tails in agreement with the 1 AU observational result (not shown), thus confirming the accuracy and unique outcome of combining the Grad-Shafranov reconstruction method with automated detection in small-scale flux rope identification and characterization.

3.6. Applications: Small-scale Flux Ropes and Particle Energization

Observation evidence of particle acceleration near the HCS indicated local acceleration related to merging and contraction of small-scale magnetic islands [12]. Particles, magnetically confined inside cavities, may experience additional acceleration up to $1 \sim 1.5 \text{ MeV}$ if they are pre-accelerated [15]. They can also be trapped after acceleration via magnetic reconnection during the initial process and re-accelerated up to $\sim 5 \text{ MeV}$ via small-scale magnetic flux rope dynamics [13].

The Grad-Shafranov reconstruction method has been applied to various data of spacecraft missions, such as the Wind, Advanced Composition Explorer (ACE) and Ulysses, to detect small-scale magnetic flux ropes automatically. Analysis of results and further applications are presented in the following case studies. Case 1 is an analysis combining flux-rope and energetic ion data from the ACE and the Solar and Heliospheric Observatory (SOHO) spacecraft. Case 2 is an application based on the detection result of flux rope and energetic ions in Ulysses data. The idea is to look for evidence that small-scale flux-ropes coincide with enhancements in energetic ion fluxes, thus proving potential evidence that these structures are responsible for ion acceleration in the solar wind.

3.6.1. Case 1: Observations During January 10 - 11, 2002.

Case 1 consists of data from both SOHO and ACE since SOHO provides more channels of particles energies up to 22 MeV while the highest energy of ions is 4.8 MeV from ACE/EPAM. Because of their proximity in location, it is considered feasible for double-checking the same structures in both ACE and SOHO spacecraft data. We applied our automated detection to both data sets to analyze structures in the vicinity of interplanetary shocks with a special emphasis on the occurrence of small-scale flux ropes downstream of these shocks. Figure 14 shows the automatic identification of small-scale flux ropes downstream (up to 15 hours after the shock transition) of the shock at January 10, 2002 that occurred just before 16:00 UT. The presence of flux ropes coincides with particle flux...
Figure 14. Example of small-scale flux-rope detection during the period from January 10, 2002, 13:00 UT to January 11, 2002, 06:00 UT. An interplanetary shock is shown by the solid vertical line and 20 identified flux rope intervals are shaded in grey. The top two panels are proton intensity data for the energy range, 1.33 MeV to 22.4 MeV, and its flux amplification normalized at the shock from SOHO. The third and fourth panels are the corresponding ion data from ACE EPAM (LEMS 120; 5 minute averages). The magnetic field data in the GSE coordinates are shown in the next three panels. The plasma parameters including the proton $\beta$, density, temperature, and the solar wind bulk speed are shown in the last two panels.

enhancements for flux rope intervals suggesting energetic ion energization by flux ropes during these intervals. The particle flux enhancements were observed at both spacecraft, but what flux-rope acceleration mechanism is responsible for the continuous acceleration associated with the flux enhancement starting from around January 10, 2002, 22:00 UT and persisting for quite a long period of at least 8 hours far downstream, still needs to be investigated.

3.6.2. Case 2: Observations During February 12 - 25, 2004. Case 2 represents our detection results based on Ulysses data. PVI analysis is applied, and different energy levels of proton flux from the COSPIN and LEMS30 telescopes are presented. Ulysses provides three RTN coordinate components of solar wind velocity. We present the radial component $V_r$ only since the other two components are relatively small compared to $V_r$. We figure that the second continuous particle flux enhancement which occurs at around late February 16th to early 17th (day 47-48), may be associated with small-scale flux ropes, but this preliminary conclusion needs further
Figure 15. Example of small-scale flux ropes application from February 12, 2004, 00:00 UT to February 25, 2004, 060:00 UT at Ulysses (courtesy of Lingling Zhao). The total number of identified flux rope (intervals shaded in grey) is 43. All data are provided by the Ulysses in-situ measurements. The panels are, from the top to the bottom: calculated PVI values (same method as Figure 11) with time, proton flux from 0.9 to 8 MeV from COSPIN, proton flux from 214 to 5000 keV from LEMS30, magnetic field in RTN coordinate system and field magnitude (black line), plasma beta, the radial component of solar wind velocity, the proton number density and the proton temperature $T_{\text{small}}$.

4. Summary and Discussion

We have applied a combination of Grad-Shafranov reconstruction with automated detection to Ulysses in-situ measurements at high helio latitudes and low latitudes at $\sim$ 5 AU to perform a statistical analysis of small-scale flux ropes for years 1994, 1996, 2004 and 2005. The results are compared for some statistical properties with the corresponding 1 AU observational results. The main results of this study are summarized as follows:

1. Significant drop of occurrence rate of small-scale flux ropes with decreasing Walén slope threshold indicates that at higher latitudes or under high speed solar wind conditions during 1994 and 1996, there are considerably more Alfvénic structures (or waves) than at low latitudes. This tendency may affect the statistical results of small-scale flux ropes during 1994 and 1996.

2. Most small-scale flux ropes identified in this study tend to lie parallel to the local RT plane, and in low speed solar wind near the ecliptic plane, they are likely aligned with the nominal Parker spiral magnetic field direction.
3. Statistical properties of small-scale flux ropes, such as the distributions of duration, scale size, proton temperature, etc., remain similar for both high and low latitudes. The scale sizes for different heliocentric distances in the ecliptic plane are also in the same range. The distributions are mainly power laws with different slopes at high latitudes as compared to low latitudes.

4. The waiting time distribution (WTD) of flux ropes are best fitted by an exponential function for both high and low speed winds in Ulysses data while 1 AU statistical results have good fits both by a power law and an exponential function.

5. Fitting with a double power law worked best for the observed wall-to-wall time distribution of flux ropes, yielding a breaking point at \( \sim 200 \) min. The power law indices for Group 2 data (low latitude Ulysses data at \( \sim 5 \) AU) correspond approximately to the low latitude indices at 1 AU while for Group 1 high latitude data, containing more Alfvénic structures or waves, the power-law indices are distinctly different.

6. Preliminary results combining Grad-Shafranov reconstruction method with energetic ion fluxes data from ACE, SOHO and Ulysses provide the potential evidence that enhancements in energetic ion fluxes can be attributed partly to small-scale flux ropes.

In retrospect, the limit of magnitude of magnetic field applied in the previous study was not applied in this paper. Therefore, our statistical results might contain small magnetic field turbulences. As shown by Table 3.1, more Alfvénic structures could be included by using a larger Walén slope threshold, e.g., 0.5, especially for high speed solar wind in high latitudes. Thus, Grad-Shafranov reconstruction can be extended in a future study to take into account significant field-aligned flow [29], corresponding to cases with significant Walén slopes.

5. Acknowledgments
All of Ulysses data in this paper are from Coordinate Data Analysis Web (CDAWeb) of NASA GSFC. We would like to thank people behind this website and scientists working for Ulysses spacecraft. In addition, we acknowledge NASA grants NNX15AI65G and NNX17AB85G, subawards NRL N00173-14-1-G006 and SAO SV4-84017, and NSF grant AGS-1650854 for support. We thank our colleagues at CSPAR/UAH, and Dr. O. Khabarova for stimulating discussions.

References
[1] Moldwin M B, Phillips J L, Gosling J T, Scime E E, McComas D J, Bame S J, Balogh A and Forsyth R J 1995 *J. Geophys. Res.* **100** 19903–19910
[2] Moldwin M B, Ford S, Lepping R, Slavin J and Szabo A 2000 *Geophys. Res. Lett.* **27** 57–60
[3] Feng H Q, Wu D J and Chao J K 2007 *Journal of Geophysical Research (Space Physics)* **112** A02102
[4] Feng H Q, Wu D J, Lin C C, Chao J K, Lee L C and Lyu L H 2008 *Journal of Geophysical Research (Space Physics)* **113** A12105
[5] Feng H Q, Zhao G Q and Wang J M 2015 *Journal of Geophysical Research (Space Physics)* **120** 10
[6] Cartwright M L and Moldwin M B 2008 *Journal of Geophysical Research (Space Physics)* **113** A09105
[7] Drake J F, Swisdak M, Che H and Shay M A 2006 *443* 553–556
[8] Zank G P, le Roux J A, Webb G M, Dosch A and Khabarova O 2014 *Astrophys. J.* **797** 28
[9] le Roux J A, Webb G M, Zank G P and Khabarova O 2015 *Journal of Physics: Conference Series* **642** 012015 URL http://stacks.iop.org/1742-6596/642/i=1/a=012015
[10] Zank G P, Hunana P, Mostafavi P, Le Roux J A, Li G, Webb G M, Khabarova O, Cummings A, Stone E and Decker R 2015 *Astrophys. J.* **814** 137
[11] le Roux J A, Zank G P, Webb G M and Khabarova O V 2016 Astrophys. J. 827 47
[12] Khabarova O, Zank G P, Li G, le Roux J A, Webb G M, Dosch A and Malandraki O E 2015 Astrophys. J. 808 181 (Preprint 1504.06616)
[13] Khabarova O V and Zank G P 2017 Astrophys. J. 843 4
[14] Zheng J, Hu Q, Chen Y and le Roux J 2017 Astrophys. J. 900 012024
[15] Khabarova O V, Zank G P, Li G, Malandraki O E, le Roux J A and Webb G M 2016 Astrophys. J. 827 122
[16] Sonnerup B U Ö and Guo M 1996 Geophys. Res. Lett. 23 3679–3682
[17] Hu Q and Sonnerup B U Ö 2001 Geophys. Res. Lett. 28 467–470
[18] Hu Q and Sonnerup B U Ö 2002 Journal of Geophysical Research (Space Physics) 107 1142
[19] Zheng J and Hu Q 2018 Astrophys. J. Lett. 852 L23 (Preprint 1801.01771)
[20] McComas D J, Gosling J T and Skoug R M 2000 Geophys. Res. Lett. 27 2437–2440
[21] Hu Q 2017 Science China Earth Sciences 60 1466–1494 ISSN 1869-1897 URL https://doi.org/10.1007/s11430-017-9067-2
[22] Hu Q, Smith C W, Ness N F and Skoug R M 2004 Journal of Geophysical Research (Space Physics) 109 A03102
[23] Greco A, Matthaeus W H, Servidio S and Dmitruk P 2009 Physical Review 80 046401
[24] Greco A, Chuychai P, Matthaeus W H, Servidio S and Dmitruk P 2008 Geophys. Res. Lett. 35 L19111
[25] Greco A, Matthaeus W H, Servidio S, Chuychai P and Dmitruk P 2009 Astrophys. J. Lett. 691 L111–L114
[26] Richardson J D, Wang C and Burlaga L F 2004 Adv. Space Res. 34 150–156
[27] Greco A, Matthaeus W H, Perri S, Osman K T, Servidio S, Wan M and Dmitruk P 2018 Space Science Reviews 214 1
[28] Miao B, Peng B and Li G 2011 Annales Geophysicae 29 237–249
[29] Teh W L 2018 Earth, Planets and Space 70 34