Exclusive and Semi-inclusive $B$ Decays in QCD Factorization

Hai-Yang Cheng
Institute of Physics, Academia Sinica, Taipei, Taiwan 115, R.O.C.
E-mail: phcheng@ccvax.sinica.edu.tw

Abstract: Applications of QCD factorization to $B \to \phi K$, charmless $B \to VV$, $B \to J/\psi K(K^*)$ and semi-inclusive decays $B \to MX$ are discussed.

1. $B \to \phi K$ decay

Recently CLEO [1], Belle [2] and BaBar [3] have reported the results:

$$B(B^\pm \to \phi K^\pm) = \begin{cases} (5.5^{+2.1}_{-1.8} \pm 0.6) \times 10^{-6} & \text{CLEO}, \\ (7.7^{+1.6}_{-1.4} \pm 0.8) \times 10^{-6} & \text{BaBar}, \\ (11.2^{+2.2}_{-2.0} \pm 1.4) \times 10^{-6} & \text{Belle}, \end{cases}$$

and

$$B(B^0 \to \phi K^0) = \begin{cases} < 12.3 \times 10^{-6} & \text{CLEO}, \\ (8.1^{+3.1}_{-2.5} \pm 0.8) \times 10^{-6} & \text{BaBar}, \\ (8.9^{+3.4}_{-2.7} \pm 1.0) \times 10^{-6} & \text{Belle}. \end{cases}$$

The neutral mode $B^0 \to \phi K^0$ is a pure penguin process, while the charged mode $\phi K^-$ receives an additional annihilation contribution which is quark-mixing-angle suppressed. The predicted branching ratio is very sensitive to the nonfactorizable effects which are sometimes parameterized in terms of the effective number of colors $N_{\text{eff}}$; it falls into a broad range $(13 \sim 0.4) \times 10^{-6}$ for $N_{\text{eff}} = 2 \sim \infty$ [4]. Therefore, a theory calculation of the nonfactorizable corrections is urgently needed in order to have a reliable prediction which can be used to compare with experiment.

In QCD factorization approach, the branching ratio is predicted to be $B(B^\pm \to \phi K^\pm) = (4.0 \pm 0.8) \times 10^{-6}$ in the absence of annihilation contributions, where theoretical error comes from the logarithmic divergent term occurring in spectator interactions

$$X_H \equiv \int_0^1 \frac{dx}{x} = \ln \frac{M_B}{\Lambda_{\text{QCD}}}(1 + \rho_H), \quad \rho_H \leq 1$$
Power-suppressed annihilation is often treated to be negligible based on helicity suppression argument. However, annihilation diagrams induced by \((S - P)(S + P)\) penguin operators are not subject to helicity suppression. Including theoretical errors from both \(X_H\) for spectator interactions and \(X_A\) for weak annihilations, we obtain

\[
B(B^- \to \phi K^-) = (4.6^{+3.2}_{-1.5}) \times 10^{-6}, \quad B(B^0 \to \phi K^0) = (4.2^{+3.0}_{-1.3}) \times 10^{-6}.
\] (1.3)

Hence, the prediction is in agreement with data within experimental and theoretical errors.

Recently, calculations within the framework of pQCD are also available \([6]\). The pQCD results \(B(B^- \to \phi K^-) = (10.2^{+3.9}_{-2.1}) \times 10^{-6}\) and \(B(B^0 \to \phi K^0) = (9.6^{+3.7}_{-2.6}) \times 10^{-6}\) are large for two reasons. First, the relevant scale in the pQCD calculation is \(\mu \sim 1.5\) GeV and the relevant Wilson coefficient \(c_4(\mu)\) at this low scale increases dramatically as \(\mu\) decreases. However, such a “dynamic enhancement” does not exist in QCD factorization because the parameter \(a_4\) which contains the term \(c_4(\mu) + c_3(\mu)/3\) is formally renormalization scale and \(\gamma_5\) scheme independent after including \(O(\alpha_s)\) vertex-type and penguin-type corrections. Second, the contribution from the chromomagnetic dipole operator to \(a_4\), which is absent in the current pQCD calculation, is sizable but destructive. Therefore, a refined measurement of \(B \to \phi K\) decays will provide a nice ground for discriminating between the approaches of QCD factorization and pQCD.

2. Charmless \(B \to VV\) decays

It is known that the decay amplitude of a \(B\) meson into two vector mesons is governed by three unknown form factors \(A_1(q^2), A_2(q^2)\) and \(V(q^2)\) in the factorization approach. It has been pointed out in \([7]\) that the charmless \(B \to VV\) rates are very sensitive to the form-factor ratio \(A_2/A_1\). This form-factor ratio is almost equal to unity in the Bauer-Stech-Wirbel (BSW) model \([8]\), but it is less than unity in the light-cone sum rule (LCSR) analysis for form factors \([8]\). In general, the branching ratios of \(B \to VV\) predicted by the LCSR are always larger than that by the BSW model by a factor of \(1.6 \sim 2\) \([7]\). This is understandable because in the heavy quark limit, both vector mesons in the charmless \(B \to VV\) decay should have zero helicity and the corresponding amplitude is proportional to the form factor difference \((A_1 - A_2)\). These two form factors are identical at \(q^2 = 0\) in the BSW model.

We have analyzed \(B \to VV\) decays within the framework of QCD factorization \([10]\). We see from Table I that the first observed charmless \(B \to VV\) mode, \(B \to \phi K^*\), recently measured by CLEO \([1]\), Belle \([2]\) and BaBar \([3]\), clearly favors the LCSR over the BSW model for \(B \to V\) transition form factors. Contrary to phenomenological generalized factorization, nonfactorizable corrections to each partial-wave or helicity amplitude are not the same; the effective parameters \(a_i\) vary for different helicity amplitudes. The leading-twist nonfactorizable corrections to the transversely polarized amplitudes vanish in the chiral limit and hence it is necessary to take into account twist-3 distribution amplitudes of the vector meson in order to have renormalization scale and scheme independent predictions. Owing to the absence of \((S - P)(S + P)\) penguin operator contributions to \(W\)-emission amplitudes, tree-dominated \(B \to VV\) decays tend to have larger branching ratios than the
| Decay          | LCSR | BSW | Expt.                  |
|---------------|------|-----|------------------------|
| \( B^- \to K^{*-} \phi \) | 9.30 | 4.32 | \( 9.7_{-3.4}^{+4.2} \pm 1.7 \) (BaBar) |
|               |      |     | \( 10.6_{-3.9}^{+4.4}^{+1.8} \) (CLEO) |
|               |      |     | \(< 18 \) (Belle)       |
| \( B^+ \to K^{*0} \phi \) | 8.71 | 4.62 | \( 8.6_{-2.4}^{+2.8} \pm 1.1 \) (BaBar) |
|               |      |     | \( 11.5_{-3.7}^{+4.5}^{+1.8} \) (CLEO) |
|               |      |     | \( 13.0_{-5.2}^{+6.4} \pm 2.1 \) (Belle) |

Table 1: Branching ratios (in units of \( 10^{-6} \)) for \( B \to K^{*}\phi \) modes. Two different form-factor models, the LCSR and the BSW models, are adopted and the unitarity angle \( \gamma = 60^\circ \) is employed. Experimental results are taken from [1, 2, 3].

penguin-dominated ones [10]. For example, \( B^0 \to \rho^+ \rho^- \) has a branching ratio of order \( 4 \times 10^{-5} \).

3. \( B \to J/\psi K(K^*) \) decays

The hadronic decays \( B \to J/\psi K(K^*) \) are interesting because experimentally they are a few of the color-suppressed modes which have been measured, and theoretically they are calculable by QCD factorization even the emitted meson \( J/\psi \) is heavy. That is, this is the only color-suppressed mode that one can compute and compare with experiment.

To leading-twist contributions from the light-cone distribution amplitudes (LCDAs) of the mesons, vertex corrections and hard spectator interactions including \( m_c \) effects imply \( |a_2(J/\psi K)| \sim 0.11 \) vs. \( 0.25 \) by experiment [11]. Hence, the predicted branching ratio is too small by a factor of 5; the nonfactorizable corrections to naive factorization to leading-twist order are small. We study the twist-3 effects due to the kaon and find that the coefficient \( a_2(J/\psi K) \) is largely enhanced by the nonfactorizable spectator interactions arising from the twist-3 kaon LCDAs \( \phi_K \), which are formally power-suppressed but chirally, logarithmically and kinematically enhanced. Therefore, factorization breaks down at twist-3 order. It is found in [11] that \( a_2(J/\psi K) = 0.19^{+0.14}_{-0.12} \) for \( |\rho_H| \leq 1 \) and that twist-2 as well as twist-3 hard spectator interactions are equally important.

Recently, the spin amplitudes \( A_0 \), \( A_\parallel \) and \( A_\perp \) for \( B \to J/\psi K^* \) decays in the transversity basis and their relative phases have been measured by Belle [12] and BaBar [13]. The decay \( B \to J/\psi K^* \) is currently analyzed within the framework of QCD factorization [14] and it is found that the effective parameters \( a_2^h \) for helicity \( h = 0, +, - \) states receive different nonfactorizable contributions. Contrary to the \( J/\psi K \) case, \( a_2^0 \) in \( B \to J/\psi K^* \) does not receive twist-3 contributions and it is dominated by twist-2 hard spectator interactions.

4. Semi-inclusive \( B \) decays

The semi-inclusive decays \( B \to M + X \) that are of special interest originate from the quark level decay, \( b \to M + q \). They are theoretically cleaner compared to exclusive decays and have distinctive experimental signatures [13, 14]. The theoretical advantages are: (i) A very
important theoretical simplification occurs in the semi-inclusive decays over the exclusive decays if we focus on final states such that $M$ does not contain the spectator quark of the decaying $B(B_s)$ meson as then we completely by-pass the need for the transition form factor for $B(B_s) \to M$. (ii) There is no troublesome infrared divergent problem occurred at endpoints when working in QCD factorization, contrary to the exclusive decays where endpoint infrared divergences usually occur at twist-3 level, and (iii) As for $CP$ violation, contrary to the exclusive hadronic decays, it is not plagued by the unknown soft phases. Consequently, the predictions of the branching ratios and partial rate asymmetries for $B \to M X$ are considerably clean and reliable. Since these semi-inclusive decays also tend to have appreciably larger branching ratios compared to their exclusive counterparts, they may therefore be better suited for extracting CKM-angles and for testing the Standard Model.

In order to have a reliable study of semi-inclusive decays both theoretically and experimentally, we will impose two cuts. First, a momentum cutoff imposed on the emitted light meson $M$, say $p_M > 2.1 \text{ GeV}$, is necessary in order to reduce contamination from the unwanted background and ensure the relevance of the two-body quark decay $b \to M q$. Second, it is required that the meson $M$ does not contain the spectator quark in the initial $B$ meson and hence there us no $B - M$ transition form factors. Under these two cuts, we argue that the factorization formula for exclusive decays can be generalized to the semi-inclusive decay:

$$
\langle MX|O|B\rangle = \int_0^1 du T^I(u)\Phi_M(u) + \int_0^1 d\xi du T^{II}(\xi,u)\Phi_B(\xi)\Phi_M(u) .
$$

However, this factorization formula is not as rigorous as the one for the exclusive case. To the order $O(\alpha_s)$, there are two additional contributions besides vertex corrections: the bremsstrahlung process $b \to M q g$ ($g$ being a real gluon) and the process $b \to M q g^* \to M q q' q'$. The bremsstrahlung subprocess could potentially suffer from the infrared divergence. However, the vertex diagram in which a virtual gluon is attached to $b$ and $q$ quarks is also infrared divergent. This together with the above-mentioned bremsstrahlung process will lead to a finite and well-defined correction. This finite correction is expected to be small as it is suppressed by a factor of $\alpha_s/\pi \approx 7\%$. In the presence of bremsstrahlung and the fragmentation of the quark-antiquark pair from the gluon, the factorizable configurations $\langle X_1 M | j_1 | 0 \rangle \langle X'_1 | j_2 | B \rangle$ and $\langle X_2 | j_1 | 0 \rangle \langle X'_2 M | j_2 | B \rangle$ with $X_1 + X'_1 = X$ and $X_2 + X'_2 = X$ are allowed. In general, one may argue that these configurations are suppressed since the momentum cut $p_M > 2.1 \text{ GeV}$ favors the two-body quark decay $b \to M q$ and low multiplicity for $X$. However, it is not clear to us how rigorous this argument is. Therefore, we will confine ourselves to vertex-type and penguin-type corrections as well as hard spectator interactions so that the factorization formula (4.1) is applicable to semi-inclusive decays at least as an approximation.

Some highlights of the present analysis are [17]:

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1. Important theoretical simplification occurs in the semi-inclusive decays over the exclusive decays if we focus on final states such that $M$ does not contain the spectator quark of the decaying $B(B_s)$ meson as then we completely by-pass the need for the transition form factor for $B(B_s) \to M$.
2. There is no troublesome infrared divergent problem occurred at endpoints when working in QCD factorization, contrary to the exclusive decays where endpoint infrared divergences usually occur at twist-3 level.
3. As for $CP$ violation, contrary to the exclusive hadronic decays, it is not plagued by the unknown soft phases.

Consequently, the predictions of the branching ratios and partial rate asymmetries for $B \to M X$ are considerably clean and reliable. Since these semi-inclusive decays also tend to have appreciably larger branching ratios compared to their exclusive counterparts, they may therefore be better suited for extracting CKM-angles and for testing the Standard Model.

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Some highlights of the present analysis are [17]:

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• Though phase-space and power suppressed, hard spectator interactions are extremely important for color-suppressed modes, e.g. $(\pi^0, \rho^0, \omega)X_\bar{s}$, $\phi X$ and $J/\psi X_\bar{s}, J/\psi X$. This is because the relevant hard spectator correction is color allowed, whereas $b \to Mq$ for these modes are color-suppressed.

• The prediction $\mathcal{B}(B \to J/\psi X_\bar{s}) = 9.6 \times 10^{-3}$ is in agreement with experiments: $(8.0 \pm 0.8) \times 10^{-3}$ from CLEO and $(7.89 \pm 0.10 \pm 0.40) \times 10^{-3}$ from BaBar.

• $\bar{B}_s^0 \to (\pi^0, \rho^0, \omega)X_\bar{s}$, $\rho^0 X_\bar{s}$, $\bar{B}^0 \to (K^- X, K^{*-} - X)$ and $B^- \to (K^0 X_\bar{s}, K^{*0} X_\bar{s})$ are the most promising ones in searching for direct CP violation: they have branching ratios of order $10^{-6} - 10^{-4}$ and CP rate asymmetries of order $(10 - 40)\%$. With $1 \times 10^7$ $\bar{B}B$ pairs, the asymmetry in $K^{*-}$ channel starts to become accessible. With about $7 \times 10^7$ $\bar{B}B$ events, the PRA’s in other modes mentioned above will become feasible.

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