The dipole form of the gluon part of the BFKL kernel *

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Abstract

The dipole form of the gluon part of the colour singlet BFKL kernel in the next-to-leading order (NLO) is obtained in the coordinate representation by direct transfer from the momentum representation, where the kernel was calculated before. With this paper the transformation of the NLO BFKL kernel to the dipole form, started a few months ago with the quark part of the kernel, is completed.

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1 Introduction

The BFKL equation [1] allows to find the Green’s function for the scattering of two Reggeized
 gluons, which determines the high-energy behaviour of QCD amplitudes. It is an integral
equation in the space of momenta transverse to the momenta of colliding particles. The kernel
of the BFKL equation is known now in the next-to-leading order (NLO) both for the forward
scattering [2], i.e. for \( t = 0 \) and color singlet (Pomeron) in the \( t \)-channel, and for any fixed (not
growing with energy) momentum transfer \( t \) and any possible color state in the \( t \)-channel [3, 4, 5].

The Pomeron channel is the most important for phenomenological applications. However,
from the theoretical point of view the color octet case seems to be even more important
because of the gluon Reggeization. Indeed, high-energy QCD can be reformulated in terms of a
gauge-invariant effective field theory for Reggeized gluon interactions [6], so that the primary
Reggeon in QCD is not the Pomeron, but the Reggeized gluon. The main property of the octet
BFKL kernel which is required by the Reggeization is that the gluon trajectory turns out to
be its eigenfunction. This property can be verified in the momentum representation, i.e. the
representation in which the BFKL approach was originally developed. On the contrary, a re-
markable property of the colour singlet BFKL kernel in the leading approximation [7] is entirely
concerned with the coordinate representation. It is the famous conformal invariance, which is
extremely important for finding solutions of the equation.

In the NLO the coordinate representation of the colour singlet BFKL kernel is also very
interesting. First, it reveals conformal properties of the kernel. Evidently, the conformal invar-
iance is violated by renormalization. One may wonder, however, whether the renormalization
is the only source of the violation. If so, one can expect the conformal invariance of the NLO
BFKL kernel in supersymmetric extensions of QCD.

Another important reason for transferring the colour singlet NLO BFKL kernel in the co-
dordinate representation is to make possible the comparison with the color dipole approach for
high-energy processes [8]. This approach gives a clear physical picture of high-energy pro-
cesses and can be naturally extended from the regime of low parton densities to the saturation
regime [9], where the evolution equations of parton densities with energy become nonlinear.
In general, there is an infinite hierarchy of coupled equations [10, 11]. In the simplest case of
a large nucleus as a target, this set of equations is reduced to the BK (Balitsky-Kovchegov)
equation [10].

It is affirmed [8, 10] that in the linear regime the color dipole framework gives the same
results as the BFKL one for the color singlet channel. Before the advent of the dipole approach,
the leading order color singlet BFKL kernel was investigated in the coordinate representation
in detail in Ref. [7]. More recently, the relation between BFKL and color dipole approaches was
analyzed in the leading order in Ref. [12]. The extension of the analysis to the NLO was started
a few months ago. In Ref. [13] the quark contribution to the color dipole approach at large
number of colors $N_c$ has been transferred from the coordinate to the momentum representation and it has been verified that the resulting contribution to the NLO Pomeron intercept agrees with the well-known result of Ref. [2]. In Refs. [14, 15] the “non-Abelian” (leading in $N_c$) and “Abelian” parts of the quark contribution to the non-forward BFKL kernel in the momentum representation have been transformed to the coordinate one and it has been found that the dipole form of the quark contribution agrees with the result obtained in Ref. [16] by the direct calculation of the quark contribution to the dipole kernel in the coordinate representation.

Evidently, the main and the most important part of the BFKL kernel is given by the gluon contribution. The aim of this work is to consider this part of the kernel in the NLO and to find its dipole form.

The paper is organized as follows: in Section 2 we give basic definitions, fix our notations and recall the main results of the papers in Refs. [14, 15]; in Section 3 we describe the decomposition of the kernel into “planar” and “symmetric” parts; in Section 4 we present the NLO gluon trajectory in a convenient form; in Section 5 we show a convenient representation for the real part of the colour octet kernel entering into the “planar” part; in Section 6 we discuss the cancellation of the infrared singularities and give the resulting form of the colour octet kernel; in Section 7 we give the “symmetric” part of the kernel in a form suitable for the subsequent transformation; in Section 8 we present the general structure of the dipole form of the kernel; in Section 9 we describe the procedure to transfer the “planar” piece of the gluon part of the kernel from the momentum to the coordinate representation; in Section 10 we outline this procedure for the “symmetric” piece; in Section 11 we present our final result; in Section 12 we draw our conclusions. Appendices contain all necessary integrals.

2 Basic definitions and notation

We use the same notation as in Ref. [14]: $\vec{q}_i, \vec{q}_i', i = 1, 2$, represent the transverse momenta of Reggeons in initial and final $t$-channel states, while $\vec{r}_i, \vec{r}_i'$ are the corresponding conjugate coordinates. The state normalization is

$$\langle \vec{q} | \vec{q}' \rangle = \delta(\vec{q} - \vec{q}') , \quad \langle \vec{r} | \vec{r}' \rangle = \delta(\vec{r} - \vec{r}') , \quad (1)$$

so that

$$\langle \vec{r} | \vec{q} \rangle = \frac{e^{i\vec{q} \cdot \vec{r}}}{(2\pi)^{1+\epsilon}} , \quad (2)$$

where $\epsilon = (D - 4)/2$; $D - 2$ is the dimension of the transverse space and it is taken different from 2 for the regularization of divergences. We will also use the notation $\vec{q} = \vec{q}_1, \vec{q}_2, \quad \vec{q}' = \vec{q}_1', \vec{q}_2', \quad \vec{k} = \vec{q}_1 - \vec{q}_1' = \vec{q}_2' - \vec{q}_2$ and for brevity we will usually write $\vec{p}_{ij} = \vec{p}_i - \vec{p}_j$. 


The BFKL kernel in the operator form is written as

$$\hat{K} = \hat{\omega}_1 + \hat{\omega}_2 + \hat{K}_r,$$

where

$$\langle \vec{q}_i | \hat{\omega}_i | \vec{q}_i' \rangle = \delta(\vec{q}_i - \vec{q}_i')\omega(-\vec{q}_i^2),$$

with $\omega(t)$ the gluon Regge trajectory, and $\hat{K}_r$ represents real particle production in Reggeon collisions. The s-channel discontinuities of scattering amplitudes for the processes $A + B \rightarrow A' + B'$ have the form

$$-4i(2\pi)^{D-2}\delta(\vec{q}_A - \vec{q}_B)\text{disc}_s A'_{AB}' = \langle A'\bar{A}|e^{Y\hat{K}}\frac{1}{q_1^2 q_2^2}|B'B\rangle.$$ 

In this equation $Y = \ln(s/s_0)$, $s_0$ is an appropriate energy scale, $q_A = p_A - p_A$, $q_B = p_B - p_B'$, and

$$\langle \vec{q}_1, \vec{q}_2 | \hat{K} | \vec{q}_1', \vec{q}_2' \rangle = \delta(\vec{q} - \vec{q}')\frac{1}{q_1^2 q_2^2}\mathcal{K}(\vec{q}, \vec{q}'; \vec{q}),$$

$$\langle \vec{q}_1, \vec{q}_2 | B'B \rangle = 4p_B^\pm\delta(\vec{q}_B - \vec{q}_1 - \vec{q}_2)\Phi_{B'B}(\vec{q}_1, \vec{q}_2),$$

$$\langle A'\bar{A}|\vec{q}_1, \vec{q}_2 \rangle = 4p_A^\pm\delta(\vec{q}_A - \vec{q}_1 - \vec{q}_2)\Phi_{A'A}(\vec{q}_1, \vec{q}_2),$$

where $p^\pm = (p_0 \pm p_z)/\sqrt{2}$; the kernel $\mathcal{K}(\vec{q}_1, \vec{q}_1'; \vec{q})$ and the impact factors $\Phi$ are expressed through the Reggeon vertices according to Ref. [17]. Note that the appearance of the factors $(\vec{q}_1^2 \vec{q}_2^2)^{-1}$ in (5) and $(\vec{q}_1^2 \vec{q}_2^2)^{-1}$ in (6) cannot be explained by a change of the normalization (11). We have used a freedom in the definition of the kernel. Indeed, one can change the form of the kernel (in any representation) performing the operator transformation

$$\hat{K} \rightarrow \hat{O}^{-1}\hat{K}\hat{O}, \quad \langle A'\bar{A}| \rightarrow \langle A'\bar{A}|\hat{O}, \quad \frac{1}{q_1^2 q_2^2}|B'B\rangle \rightarrow \hat{O}^{-1}\frac{1}{q_1^2 q_2^2}|B'B\rangle,$$

which does not change the discontinuity (5). In (9) $\hat{O}$ is an arbitrary nonsingular operator. The kernel $\hat{K}$ in (6) is related with the one defined in Ref. [17] by such transformation with $\hat{O} = (\vec{q}_1^2 \vec{q}_2^2)^{1/2}$. The reason for this choice is that in the leading order the kernel which is conformal invariant and simply related to the dipole kernel is not the kernel defined in Ref. [17], but just the kernel $\hat{K}$ in [6] [7] [12]. Note that after the choice of the operator $\hat{O}$ in the leading order, additional transformations with $\hat{O} = 1 - \hat{O}$, where $\hat{O} \sim g^2$, are still possible. At the NLO after such transformation we get

$$\hat{K} \rightarrow \hat{K} - [\hat{K}^{(B)}, \hat{O}],$$

where $\hat{K}^{(B)}$ is the leading order kernel.
In the NLO the BFKL kernel contains both gluon and quark contributions. Refs. [14, 15] were devoted to the quark contribution. In Ref. [14] the “non-Abelian” part of the quark contribution [3] was transformed to the coordinate representation. The transformation was carried out in the most general way: at arbitrary $D$ and for the case of arbitrary impact factors. Generally speaking, the BFKL and dipole kernels are not equivalent. But in case of scattering of colorless objects, besides the freedom in the definition of the kernel discussed above, there is an additional freedom related to the “gauge invariance” (vanishing at zero Reggeized gluon momenta) of the impact factors [7, 12]. In this case the kernel $\langle \vec{r}_1 \vec{r}_2 | \hat{K} | \vec{r}_1' \vec{r}_2' \rangle$ can be written in the dipole form (see below). Owing to the possibility of the operator transformations (10), the dipole form is not unique. It was shown in Ref. [14] that after the transformation (10) with a suitable operator $\hat{O}$ the “non-Abelian” part of the quark contribution [3] to the BFKL kernel, transferred to the dipole form, agrees with the result obtained recently in Ref. [16] by direct calculation of the quark contribution to the BK kernel in the coordinate representation.

In Ref. [15] the “Abelian” part of the quark contribution [3] was considered and its dipole form was found. Since the “Abelian” part is known only in the limit $\epsilon \to 0$, the approach adopted in Ref. [15] was analogous to the one used in Section 5 of Ref. [14], i.e. the starting point was the renormalized BFKL kernel, which was then simplified by the cancellation of the infrared singularities between virtual and real contributions and presented at $D = 4$. After that its dipole form was found, which turned out to coincide with the corresponding part of the quark contribution to the BK kernel calculated in Ref. [16] and to be evidently conformal invariant.

Here we consider the gluon part of the kernel (in the following sometimes for brevity we will call this part simply “kernel”) and find its dipole form. Evidently this is the main and the most important part. Since the gluon part is known only in the limit $\epsilon \to 0$, we adopt the same strategy as in Section 5 of Ref. [14] and in Ref. [15].

3 Decomposition of the gluon contribution

The real part of the gluon contribution to the BFKL kernel in the colour singlet channel is written [5] as

$$\hat{K}_r = 2 \hat{K}^{(8)}_r + \frac{1}{2} \hat{K}^{(s)}_{GG},$$

where $\hat{K}^{(8)}_r$ is the real part of the colour octet kernel, concerned with the gluon Reggeization, and $\hat{K}^{(s)}_{GG}$ is the so called “symmetric” part of the two-gluon contribution to the non-forward BFKL kernel. The first of them was calculated in Ref. [4], and the second in Ref. [5]. It is convenient to consider them separately. The use of the decomposition (11) is due to the fact that all infrared singularities turn out to be located in $\hat{K}^{(8)}_r$, whereas the “symmetric” part $\hat{K}^{(s)}_{GG}$...
is infrared finite. Moreover, in the momentum representation \( \hat{K}_{r}^{(8)} \) looks much simpler than \( \hat{K}_{GG}^{(s)} \). The origin of the relative simplicity of \( \hat{K}_{r}^{(8)} \) is that only planar diagrams do contribute to it. The “symmetric” part includes contributions from both planar and non-planar diagrams, the latter being the most complicated.

At fixed non-zero \( \vec{k}^2 \), when the term \( (\vec{k}^2/\mu^2)^\epsilon \) can be expanded in powers of \( \epsilon \), the piece \( \langle \vec{q}_{1}', \vec{q}_{2}' | \hat{K}_{r}^{(8)} | \vec{q}_{1}, \vec{q}_{2} \rangle \) is finite at \( \epsilon = 0 \). But this part is singular at \( \vec{k}^2 = 0 \), therefore the region of \( \vec{k}^2 \) so small that \( \epsilon |\ln(\vec{k}^2/\mu^2)| \sim 1 \) and the expansion of \( (\vec{k}^2/\mu^2)^\epsilon \) cannot be done, is important. Moreover, terms of order \( \epsilon \) must be taken into account in the coefficient of the expression divergent at \( \vec{k}^2 = 0 \), in order to take all contributions non-vanishing in the limit \( \epsilon \to 0 \) after the integration over \( \vec{k} \).

The total BFKL kernel in the singlet case must be free from singularities. Therefore, the infrared singularities of \( 2\hat{K}_{r}^{(8)} \) must cancel the singularities of the “virtual” contribution \( \hat{\omega}_{1} + \hat{\omega}_{2} \) in (3). In the following we join these parts. The experience of the transformation of the “non-Abelian” part of the quark contribution \([14]\) teaches us that the kernel in the coordinate representation can be simplified by the operator transformation \((10)\) with an appropriate \( \hat{O} \). Here we will use the transformed kernel with

\[
\hat{O} = - \frac{11\alpha_s(\mu) N_c}{24\pi} \ln \left( \frac{\hat{q}_{1}^2 \hat{q}_{2}^2}{\hat{q}_{1}^2 \hat{q}_{2}^2} \right),
\]

and call

\[
\hat{K}_{p} = \hat{\omega}_{1} + \hat{\omega}_{2} + 2\hat{K}_{r}^{(8)} + \frac{11\alpha_s(\mu) N_c}{24\pi} \left[ \hat{K}^{(B)}, \ln \left( \frac{\hat{q}_{1}^2 \hat{q}_{2}^2}{\hat{q}_{1}^2 \hat{q}_{2}^2} \right) \right]
\]

“planar” part. We put also \( \hat{K}_{GG}^{(s)} = 2\hat{K}_{s} \), call \( \hat{K}_{s} \) “symmetric” part of the kernel and write the kernel as

\[
\hat{K} = \hat{K}_{p} + \hat{K}_{s}.
\]

The “symmetric” part \( \langle \vec{q}_{1}, \vec{q}_{2} | \hat{K}_{s} | \vec{q}_{1}', \vec{q}_{2}' \rangle \) is finite in the limit \( \epsilon = 0 \). Moreover, it does not give terms divergent in \( \epsilon = 0 \) by action of the kernel, since it has no non-integrable singularities in the limit \( \epsilon = 0 \). Hence, we can consider the “symmetric” part in the physical space-time dimension \( D = 4 \) from the beginning.

4 The NLO gluon trajectory

We use the representation of the gluon trajectory in the form of integral in the transverse momentum plane \([18]\). Taking into account that the bare coupling \( g \) in pure gluodynamics is connected with the renormalized coupling \( g_{\mu} \) in the \( \overline{\text{MS}} \) scheme through the relation

\[
g = g_{\mu} \mu^{-\epsilon} \left[ 1 + \frac{11 \hat{g}_{\mu}^2}{3} \right]^{14} \left[ 1 + \frac{11 \hat{g}_{\mu}^2}{2\epsilon} \right], \quad \hat{g}_{\mu}^2 = \frac{g_{\mu}^2 N_c \Gamma(1-\epsilon)}{(4\pi)^{2+\epsilon}},
\]

\[(15)\]
where \( \Gamma(x) \) is the Euler gamma-function, we have

\[
\omega(-q_i^2) = -\frac{g_\mu^2 q_i^2}{\pi^{1+\epsilon} \Gamma(1-\epsilon)} \int \frac{d^22+2\epsilon k \mu^{-2\epsilon}}{k^2 (k-q_i)^2} \left[ 1 + g_\mu^2 \left( \frac{11}{3\epsilon} - f(\vec{k}, \vec{k} - \vec{q}_i) + f(\vec{k}, 0) + f(0, \vec{k} - \vec{q}_i) \right) \right],
\]

(16)

Here \( \psi(x) = \Gamma'(x)/\Gamma(x) \) and

\[
f(\vec{k}_1, \vec{k}_2) = \frac{\vec{k}_{12}^2}{\pi^{1+\epsilon} \Gamma(1-\epsilon)} \int \frac{d^22+2\epsilon q \mu^{-2\epsilon}}{(\vec{k}_1 - q)(\vec{k}_2 - q)^2} \left( \ln \left( \frac{\vec{k}_{12}^2}{q^2} \right) + a_D \right),
\]

(17)

where

\[
a_D = -2\psi(D-3) - \psi \left( 3 - \frac{D}{2} \right) + 2\psi \left( \frac{D}{2} - 2 \right) + \psi(1) + \frac{2}{(D-3)(D-4)} + \frac{D-2}{4(D-1)(D-3)}.
\]

(18)

The expressions (17) and (18) are exact in \( \epsilon \). We need to keep the terms of zero order in \( \epsilon \) in the trajectory (16), that means the terms of order \( \epsilon \) in \( f(\vec{k}_1, \vec{k}_2) \) and of order \( \epsilon^2 \) in \( a_D \). With the required accuracy

\[
a_D = -\frac{1}{\epsilon} - \frac{11}{6} + \epsilon \left( \frac{67}{18} - \zeta(2) \right) - \epsilon^2 \left( \frac{202}{27} - 7\zeta(3) \right),
\]

(19)

where \( \zeta(n) \) is the Riemann zeta-function. The integral entering \( f(\vec{k}, 0) = f(0, \vec{k}) \) is known at arbitrary \( D \) (see for instance Eq. (B.16) in the second of Refs. [4]):

\[
\frac{\vec{k}^2}{\pi^{1+\epsilon} \Gamma(1-\epsilon)} \int \frac{d^22+2\epsilon q \mu^{-2\epsilon}}{q^2 (\vec{k} - q)^2} \ln \left( \frac{\vec{k}^2}{q^2} \right) = \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} \left( \frac{\vec{k}^2}{\mu^2} \right)^\epsilon \left( \frac{1}{2\epsilon} - \psi(1) + \psi(1-\epsilon) - \psi(1+\epsilon) + \psi(1+2\epsilon) \right).
\]

(20)

The integral for \( f(\vec{k}_1, \vec{k}_2) \) was calculated with the required accuracy in Ref. [20] (see Eq. (A.13)):

\[
\frac{\vec{k}_{12}^2}{\pi^{1+\epsilon} \Gamma(1-\epsilon)} \int \frac{d^22+2\epsilon q \mu^{-2\epsilon}}{(\vec{k}_1 - q)(\vec{k}_2 - q)^2} \ln \left( \frac{\vec{k}_{12}^2}{q^2} \right) = \frac{\Gamma^2(\epsilon)}{2\epsilon \Gamma(2\epsilon)} \left( 2 \left( \frac{\vec{k}_{12}^2}{\mu^2} \right)^\epsilon - \left( \frac{\vec{k}_1^2}{\mu^2} \right)^\epsilon - \left( \frac{\vec{k}_2^2}{\mu^2} \right)^\epsilon \right) + \ln \left( \frac{\vec{k}_{12}^2}{\vec{k}_1^2} \right) \ln \left( \frac{\vec{k}_{12}^2}{\vec{k}_2^2} \right) - 8\epsilon\zeta(3).
\]

(21)

We get then

\[
\omega(-q_i^2) = -\frac{g_\mu^2 q_i^2}{\pi^{1+\epsilon} \Gamma(1-\epsilon)} \int \frac{d^22+2\epsilon k \mu^{-2\epsilon}}{k^2 (k-q_i)^2} \left( 1 + g_\mu^2 f_\omega(\vec{k}, \vec{k} - \vec{q}_i) \right),
\]

(22)
where, with the required accuracy,

\[
\begin{align*}
\omega(k_1, k_2) &= \frac{11}{3\epsilon} - f(k_1, k_2) + f(k_1, 0) + f(0, k_2) = \frac{11}{3\epsilon} - \frac{67}{9} + 2\zeta(2) \\
\end{align*}
\]

\[
+ \varepsilon \left( \frac{404}{27} - \frac{11}{3} \zeta(2) - 6\zeta(3) \right) \left[ \left( \frac{k_2}{\mu^2} \right)^\epsilon - \left( \frac{k_1}{\mu^2} \right)^\epsilon \right] - \ln \left( \frac{k_{12}^2}{\mu^2} \right) \ln \left( \frac{k_{12}^2}{k_2^2} \right). \tag{23}
\]

The representation (22)-(23) is extremely convenient, since it permits to get easily the known expression for the trajectory in the limit \( \epsilon \to 0 \) \( [19] \). But its main advantage is that it gives the possibility to perform explicitly the cancellation of the infrared singularities and to write the kernel at the physical space-time dimension \( D = 4 \), as it will be shown in Section 6.

5 The real part of the colour octet kernel

The real part of the colour octet kernel (see Eq. (68) of Ref. [4]) expressed through the renormalized coupling constant \( g_\mu \) has the form

\[
\langle \vec{q}_1, \vec{q}_2 | \hat{K}_r^{(8)} | \vec{q}_1', \vec{q}_2' \rangle = \delta(\vec{q} - \vec{q}') \frac{g_\mu^2 \mu^{-2\epsilon}}{2\pi^{1+\epsilon} \Gamma(1-\epsilon)q_1^2 q_2^2} \left\{ \left( \frac{\vec{q}_1^2 q_2^2 + \vec{q}_1'^2 q_2'^2}{k^2} - \frac{1}{k^2} \right) \right. \\
\times \left( 1 + g_\mu^2 \left[ \left( \frac{11}{3\epsilon} \right) \left( \frac{k_2^2}{\mu^2} \right)^\epsilon - \left( \frac{67}{9} - 2\zeta(2) + \epsilon \left( \frac{404}{27} + 14\zeta(3) + \frac{11}{3} \zeta(2) \right) \right) \right] \right)
\]

\[
+ g_\mu^2 \left[ \left( \frac{11}{3} \ln \left( \frac{q_1^2 q_2^2}{q_1'^2 q_2'^2 k^2} \right) + \frac{1}{2} \ln \left( \frac{q_1^2}{q_1'^2} \right) - \frac{1}{2} \ln \left( \frac{q_2^2}{q_2'^2} \right) + \frac{1}{2} \ln \left( \frac{q_2^2}{q_2'^2} \right) - \frac{1}{2} \ln \left( \frac{q_1^2}{q_1'^2} \right) \left( \frac{11}{3} - \frac{1}{2} \ln \left( \frac{q_1^2}{q_1'^2} \right) \right) \right)
\]

\[
+ \left[ \frac{q_2^2 (k^2 - q_1^2 - q_1'^2)}{k^2} + 2q_1^2 q_1'^2 - q_1^2 q_2'^2 - q_2^2 q_1'^2 + \frac{q_2^2 q_2'^2 - q_2^2 q_1'^2}{k^2} (q_1^2 - q_1'^2) \right] I(q_1^2, q_1'^2, k^2) \right\} \\

\]

\[
+ (\vec{q}_1 \leftrightarrow \vec{q}_2, \ \vec{q}_1' \leftrightarrow \vec{q}_2'), \tag{24}
\]

where

\[
I(q_1^2, q_1'^2, k^2) = \int_0^1 \frac{dx}{q_1^2 (1-x) + q_1'^2 x - k^2 x (1-x)} \ln \left( \frac{q_1^2 (1-x) + q_1'^2 x}{k^2 x (1-x)} \right). \tag{25}
\]
Note that $I(a, b, c)$ is a totally symmetric function of the variables $a$, $b$ and $c$ \[21\], as it is obvious from the representation

\[
I(a, b, c) = \int_0^1 \int_0^1 \int_0^1 dx_1 dx_2 dx_3 \delta(1 - x_1 - x_2 - x_3) \frac{1}{(ax_1 + bx_2 + cx_3)(x_1 x_2 + x_1 x_3 + x_2 x_3)}.
\] (26)

We will use also the representation

\[
I(a, b, c) = \int_0^1 \int_0^1 \int_0^1 dx dx_1 dz \frac{1}{cx(1-x)z + (b(1-x) + ax)(1-z)}.
\] (27)

The leading singularity in (24) is $1/\epsilon$. It turns again into $\sim 1/\epsilon^2$ after subsequent integrations of the kernel because of the singular behaviour at $\vec{k}^2 = 0$. The additional singularity arises from the region where $|\epsilon| \ln |k^2/\mu^2| \sim 1$. For this reason, we have not expanded in $\epsilon$ the term $(k^2/\mu^2)^\epsilon$. The terms $\sim \epsilon$ are taken into account in the coefficient of the expression divergent at $k^2 = 0$ in order to save all non-vanishing contributions in the limit $\epsilon \to 0$ after the integrations.

6 Cancellation of the infrared singularities

Let us introduce the cut-off $\lambda \to 0$, making it tending to zero after taking the limit $\epsilon \to 0$, and divide the integration region in the integral representation of the trajectory (22) into three domains. In two of them either $\vec{k}^2 \leq \lambda^2$, or $(\vec{k} - \vec{q}^i)^2 \leq \lambda^2$, and in the third one both $\vec{k}^2 > \lambda^2$ and $(\vec{k} - \vec{q}^i)^2 > \lambda^2$. Then in the third domain we can take the limit $\epsilon = 0$ in (23) and put $f_\omega(\vec{k}_1, \vec{k}_2) = f_\omega^{(0)}(\vec{k}_1, \vec{k}_2)$, where

\[
f_\omega^{(0)}(\vec{k}_1, \vec{k}_2) = \frac{67}{9} - 2\zeta(2) - \frac{11}{3} \ln \left( \frac{\vec{k}_1^2 \vec{k}_2^2}{\mu^2 k_{12}^2} \right) - \ln \left( \frac{k_{12}^2}{k_1^2} \right) \ln \left( \frac{k_{12}^2}{k_2^2} \right).
\] (28)

In the first domain we have

\[
f_\omega(\vec{k}, \vec{k} - \vec{q}^i) = \frac{11}{3\epsilon} - \left( \frac{\vec{k}^2}{\mu^2} \right)^\epsilon \left[ \frac{11}{3\epsilon} - \frac{67}{9} + 2\zeta(2) + \epsilon \left( \frac{404}{27} - \frac{11}{3} \zeta(2) - 6\zeta(3) \right) \right],
\] (29)

and in the second one we have the same expression with the substitution $\vec{k}^2 \to (\vec{k} - \vec{q}^i)^2$. Writing the real part of the colour octet kernel as

\[
\langle \vec{q}_1, \vec{q}_2 | \hat{K}_r^{(8)}(\vec{q}_1', \vec{q}_2') \rangle = \langle \vec{q}_1, \vec{q}_2 | \hat{K}_r^{(8)}(\vec{q}_1', \vec{q}_2') \rangle \theta(\lambda^2 - \vec{k}^2) + \langle \vec{q}_1, \vec{q}_2 | \hat{K}_r^{(8)}(\vec{q}_1', \vec{q}_2') \rangle \theta(\vec{k}^2 - \lambda^2),
\] (30)
and comparing (24) with (22) we see that in the “planar” kernel (13) the first term in the R.H.S. of (30) cancels almost completely the contributions of the regions $k^2 \leq \lambda^2$ and $(k - q)^2 \leq \lambda^2$ in the trajectories $\omega(-q^2)$. The only piece which remains uncancelled in each of the trajectories for $\epsilon \to 0$ is

$$\frac{\tilde{g}_\mu^4}{\pi^{1+\epsilon}(1 - \epsilon)} \int \frac{d^2k}{k^2} \frac{\tilde{k}^2 - \lambda^2}{\tilde{k}^2 - \lambda^2} \theta(\lambda^2 - \tilde{k}^2) = \frac{\alpha_s^2(\mu) N_c^2}{2\pi^2} \zeta(3). \tag{31}$$

On account of this cancellation and using the equality

$$\int \frac{d^2k}{4\pi} \frac{q^2}{k^2(\tilde{k} - q)^2} \ln \left( \frac{k^2}{q^2} \right) \ln \left( \frac{(\tilde{k} - q)^2}{q^2} \right) = \zeta(3), \tag{32}$$

we can put

$$\langle q_1, q_2 | \omega_1 + \omega_2 + 2\tilde{k}r^{(8)} | q_1', q_2' \rangle = -\delta(q_1')\delta(q_2') \frac{\alpha_s(\mu) N_c}{4\pi^2}$$

$$\times \left( \int d^2k \left( \frac{2}{k^2} + \frac{2\tilde{k}(\tilde{k} - k)}{k^2(\tilde{k} - k)^2} + \frac{\alpha_s(\mu) N_c}{\pi} \left( V(\tilde{k}) + V(\tilde{k}, \tilde{k} - \tilde{k}) \right) \right) \right)$$

$$+ \delta(q - q') \frac{\alpha_s(\mu) N_c}{4\pi^2 q_1^2 q_2^2} \left\{ \left( \frac{q_1^2 q_2^2 + q_1'^2 q_2'^2}{k^2} - q^2 \right) \left( 1 + \frac{\alpha_s(\mu) N_c}{4\pi} \left[ \frac{11}{3} - \frac{1}{2} \ln \left( \frac{k^2}{\mu^2} \right) + \frac{67}{9} - 2\zeta(2) \right] \right) \right\}$$

$$+ \frac{1}{2} \frac{\ln (\frac{q_1^2}{q_1'^2})}{k^2} - \frac{q_1^2 q_2^2 + q_1'^2 q_2'^2}{k^2} \ln (\frac{q_1^2}{q_1'^2}) + \frac{q_1'^2 q_2'^2 - q_2'^2 q_1'^2}{k^2} \ln (\frac{q_1^2}{q_1'^2}) + \frac{q_1'^2 q_2'^2}{k^2} \ln (\frac{q_1^2}{q_1'^2})$$

$$+ \left[ q^2 (k^2 - q_1^2 - q_1'^2) + 2q_1^2 q_1'^2 - q_1^2 q_2'^2 - q_2^2 q_1'^2 + \frac{q_1^2 q_1'^2}{k^2} (q_1^2 - q_1'^2) I(q_1^2, q_1'^2, k^2) \right]$$

$$+ (q_1 \leftrightarrow q_2, \ q_1' \leftrightarrow q_2') \right), \tag{33}$$

where

$$V(\tilde{k}) = \frac{1}{2\tilde{k}^2} \left( \frac{67}{9} - 2\zeta(2) - \frac{11}{3} \ln \left( \frac{\tilde{k}^2}{\mu^2} \right) \right), \tag{34}$$

9
\[ V(\vec{k}, \vec{q}) = \frac{\vec{k} \cdot \vec{q}}{2 \vec{k}^2 \vec{q}^2} \left( \frac{11}{3} \ln \left( \frac{\vec{k}^2 \vec{q}^2}{\mu^2 (\vec{k} - \vec{q})^2} \right) - \frac{67}{9} + 2\zeta(2) \right) - \frac{11}{12 \vec{k}^2} \ln \left( \frac{\vec{q}^2}{(\vec{k} - \vec{q})^2} \right) \]  

(35)

Thus we have obtained this piece of the kernel at \( D = 4 \). Of course, the infrared singularities in (33) must be regularized either by limitations on the integration regions as discussed above or in an equivalent way.

7 The “symmetric” part of the kernel

The “symmetric” part of the kernel was found in the momentum representation in Ref. [5]. It contains neither ultraviolet nor infrared singularities and therefore it does not require regularization and renormalization. For this reason one can use from the beginning physical space-time dimension \( D = 4 \) and renormalized coupling constant \( \alpha_s(\mu) \). Nevertheless, in the momentum representation the “symmetric” part is the most complicated piece of the gluon contribution to the kernel. In this respect the “symmetric” part is analogous to the “Abelian” part of the quark contribution which was considered in Ref. [15] and turned out to be surprisingly simple in the coordinate representation. But contrary to the “Abelian” part, the “symmetric” part contains the subtraction term which makes its transformation more complicated and its form in the coordinate representation considerably intricate.

As well as for the “Abelian” part it is better to start from the expressions for the “symmetric” part before integration over the momenta of the produced particles. The starting point is Eq. (3.43) of Ref. [5], where a symmetrization operator appears. The use of this symmetrization operator is inconvenient, because our definition of the kernel [5] differs from the definition of Ref. [5]. Instead, we reconstruct the explicit symmetry by restoring the term represented by the last line in Eq. (3.40) of Ref. [5], which was omitted in Eq. (3.43). As a result, we decompose the “symmetric” part into two pieces:

\[ \hat{K}_s = \hat{K}_{s1} + \hat{K}_{s2} \]  

(36)

where

\[ \langle \vec{q}_1, \vec{q}_2 | \hat{K}_{s1} | \vec{q}_1', \vec{q}_2' \rangle = \delta(\vec{q} - \vec{q}') \frac{1}{\vec{q}_1^2 \vec{q}_2^2} \frac{\alpha_s^2(\mu) N_c}{2 \pi^3} \int_0^1 dx \int \frac{d^2 k_1}{2 \pi} \left( \frac{F_s(k_1, k_2)}{x(1-x)} \right) \]  

(37)

and

\[ \langle \vec{q}_1, \vec{q}_2 | \hat{K}_{s2} | \vec{q}_1', \vec{q}_2' \rangle = -\delta(\vec{q} - \vec{q}') \int \frac{d^2 k_1}{4 \vec{q}_1^2 \vec{q}_2^2} \frac{\hat{K}_r(\vec{q}_1, \vec{q}_1 - \vec{k}_1; \vec{q}) \hat{K}_r(\vec{q}_2 - \vec{k}_1, \vec{q}_1'; \vec{q})}{(\vec{q}_1 - \vec{k}_1)^2 (\vec{q}_2 + \vec{k}_1)^2} \ln \left( \frac{\vec{q}^2}{\vec{k}^2} \right) \]  

(38)
Here $\bar{k}_1 + \bar{k}_2 = \bar{k} = \bar{q}_{11'}$, the subscript $+$ means
\[
\left( \frac{f(x)}{x(1-x)} \right)_+ \equiv \frac{1}{x} [f(x) - f(0)] + \frac{1}{1-x} [f(x) - f(1)],
\] (39)
the function $F_s(k_1, k_2)$ is defined in Eqs. (3.44), (3.45) and (4.1) of Ref. [5], and $K^B_r$ is the real part of the leading order kernel,
\[
K^B_r(q_1, q_1'; q) = \frac{\alpha_s(\mu)N_c}{2\pi^2} \left( \frac{q_1^2 q_1'^2 + q_1'^2 q_2^2}{k^2} - q^2 \right).
\] (40)

8 Dipole form of the kernel

With our normalizations, the kernel $\hat{K}$ in the coordinate representation is given by
\[
\langle \bar{r}_1 \bar{r}_2 | \hat{K} | \bar{r}_1' \bar{r}_2' \rangle = \int \frac{d^2q_1}{2\pi} \frac{d^2q_2}{2\pi} \frac{d^2q_1'}{2\pi} \frac{d^2q_2'}{2\pi} \langle q_1, q_2 | \hat{K}_r + \hat{K}_s | q_{1'}, q_{2'} \rangle e^{i(q_1 \bar{r}_1 + q_2 \bar{r}_2 - q_{1'} \bar{r}_1' - q_{2'} \bar{r}_2')}.
\] (41)

The real part of the kernel $\langle q_1, q_2 | \hat{K}_r | q_{1'}, q_{1'}' \rangle$ vanishes when any of the $q_i$'s or $q_i'$'s tends to zero. This property, together with the “gauge invariance” property of the impact factors of colorless “projectiles” $\Phi_{A'A}(0, q) = \Phi_{A'A}(\bar{q}, 0) = 0$, permits us to change the “target” impact factors in [5] so that they acquire the “dipole” property
\[
\langle \bar{r}, \bar{r}| (\hat{q}_1^2 \hat{q}_2^2)^{-1} | \bar{B}'B \rangle_d = 0.
\] (42)

Further, adding the terms independent either of $\bar{r}_1$ or of $\bar{r}_2$ (that is possible due to the “gauge invariance”) one can change the kernel $\langle \bar{r}_1 \bar{r}_2 | \hat{K} | \bar{r}_1' \bar{r}_2' \rangle$ so that it conserves the “dipole” property. Thereafter, terms proportional to $\delta(\bar{r}_1' - \bar{r}_2')$ in $\langle \bar{r}_1 \bar{r}_2 | \hat{K} | \bar{r}_1' \bar{r}_2' \rangle$ can be omitted (see Ref. [14] for details). We call the remaining part $K_d$ “the dipole form of the BFKL kernel”. In the LO, discussed in detail in Ref. [14], it coincides with the dipole kernel:
\[
\langle \bar{r}_1 \bar{r}_2 | \hat{K}_d^{LO} | \bar{r}_1' \bar{r}_2' \rangle = \frac{\alpha_s(\mu)N_c}{2\pi^2} \int d\bar{\rho} \frac{\bar{r}_{12}^2}{\bar{r}_{12}^2} \left[ \delta(\bar{r}_{11'})\delta(\bar{r}_{22'}) + \delta(\bar{r}_{1'2})\delta(\bar{r}_{22'}) - \delta(\bar{r}_{11'})\delta(\bar{r}_{22'}) \right].
\] (43)

Here $r_{i\rho} = \bar{r}_i - \bar{\rho}$ and $r_{i\rho}' = \bar{r}_i' - \bar{\rho}'$.

Note that the integrand in (43) contains ultraviolet singularities at $\bar{\rho} = \bar{r}_1$ and $\bar{\rho} = \bar{r}_2$ which cancel in the sum of the contributions with account of the “dipole” property of the “target” impact factors. The coefficient of $\delta(\bar{r}_{11'})\delta(\bar{r}_{22'})$ is written in the integral form in order to make
the cancellation evident. The singularities do not permit us to perform the integration in this coefficient.

In the NLO the dipole form can be written as

\[
\langle \vec{r}_1\vec{r}_2|\hat{\mathcal{C}}_{d}^{NLO}|\vec{r}_1'\vec{r}_2'\rangle = \frac{\alpha_s^2(\mu)N_c^2}{4\pi^3} \left[ \delta(\vec{r}_1')\delta(\vec{r}_2') \int d\vec{p} \, g^0(\vec{r}_1, \vec{r}_2; \vec{p}) + \delta(\vec{r}_1'\vec{r}_2)g(\vec{r}_1, \vec{r}_2; \vec{r}_1') + \frac{1}{\pi} g(\vec{r}_1, \vec{r}_2; \vec{r}_1', \vec{r}_2') \right], \tag{44}
\]

with the functions \( g \) turning into zero when their first two arguments coincide.

The first three terms in the R.H.S. of (44) contain ultraviolet singularities which cancel in their sum, as well as in the LO, with account of the “dipole” property of the “target” impact factors. The coefficient of \( \delta(\vec{r}_1')\delta(\vec{r}_2') \) is written in the integral form in order to make the cancellation evident.

In the next Sections we will find the functions \( g \).

9 Transformation of the “planar” part

Using (13) and (33) and omitting terms with \( \delta(\vec{r}_1'\vec{r}_2') \) in the coordinate space, we reduce the NLO piece of the “planar” part to the form:

\[
\begin{align*}
\langle \vec{q}_1, \vec{q}_2|\hat{\mathcal{C}}_{p}^{NLO}|\vec{q}_1', \vec{q}_2'\rangle &\to \frac{\alpha_s^2(\mu)N_c^2}{4\pi^3} \left[ -\delta(\vec{q}_1')\delta(\vec{q}_2') \left( \int d\vec{k} \right) (V(\vec{k}) + V(\vec{k} - \vec{q}_1)) - 3\pi\xi(3) \right] \\
&+ \delta(\vec{q} - \vec{q}') \left\{ V(\vec{k}) + 2V(\vec{k}, \vec{q}_1) + \frac{(\vec{q}_1\vec{q}_2)}{4\vec{q}_1^2\vec{q}_2^2} \left[ \ln \left( \frac{\vec{q}_1^2}{\vec{q}_2^2} \right) \ln \left( \frac{\vec{q}_2^2}{\vec{q}_1^2} \right) + \ln^2 \left( \frac{\vec{q}_1^2}{\vec{q}_2^2} \right) - \frac{1}{2k^2} \ln^2 \left( \frac{\vec{q}_1^2}{\vec{q}_2^2} \right) \right] \right. \\
&+ \left[ \frac{(\vec{q}_1\vec{k})^2}{\vec{q}_1^2k^2} - 1 - \frac{(\vec{q}_1 + \vec{k})\vec{q}_2}{\vec{q}_2^2} \right] \frac{(\vec{k}\vec{q}_2)}{\vec{k}^2} + \frac{(\vec{k}\vec{q}_1)}{\vec{k}^2} + \frac{(\vec{q}_1\vec{q}_2)}{\vec{q}_2^2} \right) \frac{(\vec{k}\vec{q}_2)}{\vec{k}^2} \right] I(\vec{q}_1, \vec{q}_1', \vec{k}) \\
&+ \frac{(\vec{k}\vec{q}_1)}{2k^2\vec{q}_1^2} \left[ \ln^2 \left( \frac{\vec{q}_1^2}{\vec{q}_2^2} \right) + \frac{1}{2} \ln \left( \frac{\vec{q}_1^2}{\vec{q}_2^2} \right) \ln \left( \frac{\vec{q}_2^2\vec{q}_1}{k^4} \right) + \ln^2 \left( \frac{\vec{q}_1^2}{\vec{q}_2^2} \right) - \frac{1}{2} \ln \left( \frac{\vec{q}_1^2}{\vec{q}_2^2} \right) \ln \left( \frac{\vec{q}_1^2\vec{q}_2^2}{k^4} \right) \right] \\
&+ \frac{1}{4\vec{q}_1^2} \left\{ \ln \left( \frac{\vec{q}_1^2}{\vec{q}_2^2} \right) \ln \left( \frac{\vec{q}_2^2\vec{q}_1^2}{\vec{q}_2^4} \right) + \ln \left( \frac{\vec{q}_2^2}{\vec{q}_1^2} \right) \ln \left( \frac{\vec{q}_1^2\vec{q}_2^2}{\vec{q}_1^4} \right) \right\} \right\} + (\vec{q}_1 \leftrightarrow \vec{q}_2; \ \vec{q}_1' \leftrightarrow \vec{q}_2') \tag{45}
\end{align*}
\]
Contributions to the coefficient of \( \delta(\vec{r}_{11'}) \delta(\vec{r}_{22'}) \) in the square brackets of (411) come only from the terms with \( V(\vec{k}) \) and \( \zeta(3) \) in (45). The terms with \( V(\vec{k}) \) contribute to this coefficient as the integral

\[
\tilde{V}(\vec{r}_{12}) = 2 \int d\vec{k} V(\vec{k}) \left( e^{i\vec{k}\cdot\vec{r}_{12}} - 1 \right),
\]

with ultraviolet divergence. The divergence appears as a result of the separation of the ultraviolet non-singular sum \( V(\vec{k}) + V(\vec{k}, \vec{k} - \vec{q}_1) \) into two pieces. We have to represent the contribution of the terms with \( V(\vec{k}) \) in the integral form in order to make evident the cancellation of ultraviolet singularities of separate terms in (411), as it was already mentioned in Section 8. We use the same trick as in Section 5 of Ref. 14 and obtain with the help of the integrals (75) and (76) of Appendix A

\[
\tilde{V}(\vec{r}_{12}) = - \int \frac{d\vec{q} d\vec{q}'}{2(2\pi)^2} \frac{(\vec{q} \cdot \vec{k})}{\vec{q}^2 \vec{k}^2} \left( \frac{67}{9} - 2\zeta(2) - \frac{11}{6} \ln \left( \frac{\vec{k}^2 \vec{q}^2}{\mu^4} \right) \right) \left( 2e^{i[q \cdot (\vec{r}_{1\rho} + \vec{q} \cdot \vec{r}_{2\rho})]} \right.
\]

\[-e^{i(\vec{k}+\vec{q})r_{1\rho}} - e^{i(\vec{k}+\vec{q})r_{2\rho}} \biggr) = - \frac{11}{12} \int d\rho \left[ \frac{\vec{r}_{12}^2}{r_{1\rho}^2 r_{2\rho}^2} \ln \left( \frac{\vec{r}_{12}^2}{r_{1\rho}^2 r_{2\rho}^2} \right) + \left( \frac{1}{r_{2\rho}^2} - \frac{1}{r_{1\rho}^2} \right) \ln \left( \frac{r_{2\rho}^2}{r_{1\rho}^2} \right) \right],
\]

where

\[
\ln r_{1\rho}^2 = 2\psi(1) - \ln \frac{\mu^2}{4} - 3 \left( \frac{67}{9} - 2\zeta(2) \right).
\]

For uniformity we also contribute the contribution of the term with \( \zeta(3) \) in integral form via the representation

\[
\zeta(3) = \frac{1}{4\pi} \int d\rho \frac{\vec{r}_{12}^2}{r_{1\rho}^2 r_{2\rho}^2} \ln \left( \frac{\vec{r}_{12}^2}{r_{1\rho}^2 r_{2\rho}^2} \right) \ln \left( \frac{r_{2\rho}^2}{r_{1\rho}^2} \right).
\]

Using the subscript \( \rho \) to denote the contributions of the “planar” part we have

\[
g_0^\rho(\vec{r}_1, \vec{r}_2; \vec{\rho}) = \frac{3}{2} \frac{r_{12}^2}{r_{1\rho}^2 r_{2\rho}^2} \ln \left( \frac{r_{1\rho}^2}{r_{12}^2} \right) \ln \left( \frac{r_{2\rho}^2}{r_{12}^2} \right) + \frac{11}{12} \left[ \frac{r_{12}^2}{r_{1\rho}^2 r_{2\rho}^2} \ln \left( \frac{r_{1\rho}^2}{r_{12}^2} \right) + \left( \frac{1}{r_{2\rho}^2} - \frac{1}{r_{1\rho}^2} \right) \ln \left( \frac{r_{2\rho}^2}{r_{1\rho}^2} \right) \right].
\]

The contribution to \( g(\vec{r}_1, \vec{r}_2; \vec{\rho}') \) comes from all terms which do not depend on \( \vec{q}_1 \) but depend on \( \vec{k} \) and \( \vec{q}_2 \). The transformation of these terms into the coordinate representation by means of the integrals (75) - (85) and (87) of Appendix A gives

\[
\tilde{g}_\rho(\vec{r}_1, \vec{r}_2; \vec{r}_1'') = - \frac{11}{6} \frac{1}{r_{12}^2} \ln \left( \frac{r_{12}^2}{r_{1\rho}^2} \right) + \frac{11}{6} \frac{r_{12}^2}{r_{22'} r_{12'}} \ln \left( \frac{r_{12}^2}{r_{1\rho}^2} \right) + \frac{11}{6} \left( \frac{1}{r_{22'}^2} - \frac{1}{r_{12'}^2} \right) \ln \left( \frac{r_{12'}^2}{r_{22'}^2} \right)
\]

\[+ \frac{1}{4} \left( \frac{r_{22'} r_{12'}}{r_{22'} r_{12'}} - \frac{2}{r_{22'}^2} \right) \ln^2 \left( \frac{r_{12}^2}{r_{12'}^2} \right) + \frac{1}{4} \left( \frac{2}{r_{22'}^2} - \frac{1}{r_{12'}^2} \right) \ln \left( \frac{r_{12}^2}{r_{22'}^2} \right) \ln \left( \frac{r_{12}^2}{r_{12'}^2} \right).
\]

13
We could obtain \( g_p(\vec{r}_1, \vec{r}_2; \vec{r}_1', \vec{r}_2') \) subtracting from \( \tilde{g}_p \) the half-sum of its values with \( \vec{r}_2 \) changed into \( \vec{r}_1 \) and vice versa. Since \( g_p \) is not unique, we prefer to construct a shorter form for it. We have

\[
g_p(\vec{r}_1, \vec{r}_2; \vec{r}_1', \vec{r}_2') = \tilde{g}_p(\vec{r}_1, \vec{r}_2; \vec{r}_1', \vec{r}_2').
\]

Note that \( g_p(\vec{r}_1, \vec{r}_2; \vec{r}_1', \vec{r}_2') \) has non-integrable ultraviolet singularities at \( \vec{r}_2' = \vec{r}_2 \) and \( \vec{r}_2' = \vec{r}_1 \). The former singularity cancels the corresponding singularity in \( g_p(\vec{r}_1, \vec{r}_2; \rho) \) at \( \rho = \vec{r}_2 \), while the latter is unessential because of the “dipole” property of “target” impact factors.

To calculate the contribution of the remaining terms in (45) we need the integrals (86) and (83) – (84) of Appendix A and (106) of Appendix B, in addition to the integrals used before. We get

\[
\tilde{g}_p(\vec{r}_1, \vec{r}_2; \vec{r}_1', \vec{r}_2') = \frac{1}{4\tilde{r}_{22}'} \left[ \frac{\tilde{r}_{11}'}{\tilde{r}_{12}'} \ln \left( \frac{\tilde{r}_{12}'}{\tilde{r}_{11}'} \right) + \frac{\tilde{r}_{12}'}{\tilde{r}_{22}'} \ln \left( \frac{\tilde{r}_{22}'}{\tilde{r}_{11}'} \right) \right] - \frac{\tilde{r}_{22}'}{2\tilde{r}_{12}'} \left[ \frac{\tilde{r}_{12}'}{\tilde{r}_{22}'} + \frac{\tilde{r}_{11}'}{\tilde{r}_{12}'} \frac{\partial}{\partial \tilde{r}_{11}'} - \frac{\tilde{r}_{12}'}{\tilde{r}_{11}'} \frac{\partial}{\partial \tilde{r}_{12}'} \right] I \left( r_{11}', r_{12}', r_{12}' \right)
\]

\[
+ \frac{\tilde{r}_{22}'}{2\tilde{r}_{12}'} \left[ \frac{\tilde{r}_{12}'}{\tilde{r}_{22}'} \ln \left( \frac{\tilde{r}_{12}'}{\tilde{r}_{22}'} \right) + \frac{\tilde{r}_{12}'}{\tilde{r}_{12}'} \ln \left( \frac{\tilde{r}_{12}'}{\tilde{r}_{22}'} \right) \right] + (1 \leftrightarrow 2).
\]

Here and hereafter \((1 \leftrightarrow 2)\) means both \( 1 \leftrightarrow 2 \) and \( 1' \leftrightarrow 2' \) substitutions. The derivatives in this equality can be calculated using the identity (96) of Appendix B. We find

\[
\tilde{g}_p(\vec{r}_1, \vec{r}_2; \vec{r}_1', \vec{r}_2') = \left[ \frac{\tilde{r}_{22}'}{\tilde{r}_{11}' r_{22}'} + \frac{\tilde{r}_{22}'}{\tilde{r}_{12}' r_{22}'} \left( \tilde{r}_{11}' r_{12}' \right) - \frac{\tilde{r}_{22}'}{2\tilde{r}_{12}'} \left( \tilde{r}_{12}' \right) \right] \ln \left( \frac{\tilde{r}_{22}'}{\tilde{r}_{11}'} \right) - \frac{\tilde{r}_{22}'}{2\tilde{r}_{12}'} \left( \tilde{r}_{12}' \right) \ln \left( \frac{\tilde{r}_{12}'}{\tilde{r}_{22}'} \right) + (1 \leftrightarrow 2).
\]
The dipole property of this term is explicit.

10 Transformation of the “symmetric” part

In the coordinate representation the piece $\hat{K}_{s1}$, see Eq. (37), is given by the integral

$$
\langle \vec{r}_1 \vec{r}_2 | \hat{K}_{s1} | \vec{r}_1' \vec{r}_2' \rangle = \frac{\alpha_s^2(\mu) N_c^2}{4\pi^4} \int_0^1 dx \int \frac{d\vec{q}_1}{2\pi} \frac{d\vec{q}_2}{2\pi} d\vec{k}_1 d\vec{k}_2 e^{i[\vec{q}_1 \vec{r}_{11} + \vec{q}_2 \vec{r}_{22} + \vec{k} \vec{r}_{12}]} \frac{F_s(k_1, k_2)}{q_1^2 q_2^2 x(1-x)} .
$$

(56)

Here $F_s(k_1, k_2)$ is given by Eqs. (3.44), (3.45) and (4.1) of Ref. [5] and $\vec{k} = \vec{k}_1 + \vec{k}_2$. We restrict ourselves to the dipole form of the kernel. Hence, we omit those terms in $F_s(k_1, k_2)$, which lead to $\delta(\vec{r}_{12}')$ in the coordinate representation. Decomposing the remaining terms and taking into account the symmetry of the integration measure in (56) with regard to the substitution $k_1 \leftrightarrow k_2$, we can make the replacement

$$
\frac{F_s(k_1, k_2)}{q_1^2 q_2^2} \rightarrow -2 \frac{k_1^i k_2^j}{k_1^2 k_2^2} x_2 a_2^{ij} - 2 \frac{x_1}{\sigma_{11}} a_1^{ij} \frac{k_1^i k_2^j}{k_1^2 k_2^2} + 2 \frac{x_1}{\sigma_{11}} a_1^{ij} x_2 a_2^{ij} ,
$$

where

$$
x_1 = x, \quad x_2 = 1 - x, \quad \sigma_{11} = (\vec{k}_1 - x_1 \vec{q}_1)^2 + x_1 x_2 \sigma_1^2, \quad \sigma_{22} = (\vec{k}_2 + x_2 \vec{q}_2)^2 + x_1 x_2 \sigma_2^2,
$$

and

$$
a_1^{ij} = \frac{\delta^{ij}}{2} x_2 \left( 1 - 2 \frac{q_1^i q_1^j}{q_1^2} \right) + \frac{x_2 k_1^i k_1^j}{x_1 q_1^2} - q_1^i (q_1 - k_1)^j \frac{\sigma_{22}}{q_1^2} + k_1^i (q_1 - k_1)^j \frac{\sigma_{11}}{q_1^2} ,
$$

$$
a_2^{ij} = \frac{\delta^{ij}}{2} x_1 \left( 1 + 2 \frac{q_2^i k_2^j}{q_2^2} \right) - \frac{x_1 q_2^i k_2^j}{x_2 q_2^2} - (q_2 + k_2)^i q_2^j \frac{\sigma_{11}}{q_2^2} - (q_2 + k_2)^i k_2^j \frac{\sigma_{22}}{q_2^2} .
$$

(58)
Since the first term in (57) does not depend on \( \vec{q}_1 \), the second is independent of \( \vec{q}_2 \) and the third one depends on all momenta, they contribute to \( g(\vec{r}_1, \vec{r}_2; \vec{r}_1', \vec{r}_2') \), \( g(\vec{r}_2, \vec{r}_1; \vec{r}_1', \vec{r}_2') \) and \( g(\vec{r}_1, \vec{r}_2; \vec{r}_1', \vec{r}_2') \) in (54), respectively. We denote these contributions with the \( s_1 \) subscript. For the first term we have

\[
-2 \frac{k_1^i k_2^j}{k_1^2 k_2^2} \left( \frac{x_2 a_2^{ij}}{\sigma_{22} x(1-x)} \right) = - \frac{k_1^i}{k_1^2} \left[ \frac{(k_2 + \vec{q}_2)^i}{x_1} \left( \frac{1}{q_2^2} - \frac{1}{k_2^2} \right) \left( \frac{1}{\sigma_{22}} - \frac{1}{(k_2 + \vec{q}_2)^2} \right) \right] - \frac{q_2^j}{x_2 q_2^2} \left( \frac{1}{\sigma_{22}} - \frac{1}{k_2^2} \right) \right].
\]

Note, that infrared singularities vanish here, as well as in other terms in the R.H.S. of (57), in a rather tricky way, namely by means of the \(+\) prescription. Indeed, each term in (57) contains non-integrable infrared singularities. But these terms do not depend on \( x \) so that the singularities vanish after the subtraction.

For the term in question it seems more convenient to perform first the integration over \( x \). We obtain

\[
-2 \int_0^1 dx \frac{k_1^i k_2^j}{k_1^2 k_2^2} \left( \frac{x_2 a_2^{ij}}{\sigma_{22} x(1-x)} \right) = - \frac{k_1^i}{k_1^2} \left[ \frac{(k_2 + \vec{q}_2)^i}{x_1} \left( \frac{1}{q_2^2} - \frac{1}{k_2^2} \right) + \frac{q_2^j}{q_2^2 k_2^2} \right] \ln \left( \frac{(k_2 + \vec{q}_2)^2}{k_2^2} \right).
\]

Then we use the integrals (89) and (90) of Appendix B to find the contribution of this term to \( g_{s_1}(\vec{r}_1, \vec{r}_2; \vec{r}_1', \vec{r}_2') \).

\[
g_{s_1}(\vec{r}_1, \vec{r}_2; \vec{r}_1', \vec{r}_2') = \left[ 1 - \frac{\left( \vec{r}_{22} \cdot \vec{r}_{12}' \right)^2}{\vec{r}_{22}^2 \vec{r}_{12}'^2} \right] I(\vec{r}_{12}, \vec{r}_{12}', \vec{r}_{22}') + \left( \frac{\vec{r}_{22} \cdot \vec{r}_{12}'}{\vec{r}_{22}^2 \vec{r}_{12}'^2} \right) \ln \frac{\vec{r}_{12}^2}{\vec{r}_{22}'^2} \ln \frac{\vec{r}_{12}^2}{\vec{r}_{12}'}.
\]

One can see that the second term in (57) equals the first one after the substitution \( \vec{q}_1 \to -\vec{q}_2 \), \( \vec{k}_1 \to -\vec{k}_2 \), \( x_1 \to x_2 \). Hence, we can construct its integrated form replacing \( \vec{r}_{22} \to -\vec{r}_{11} \), \( \vec{r}_{12} \to -\vec{r}_{12}' \), \( \vec{r}_{12}' \to \vec{r}_{12}' \) in (61). The third term in (57) is easier to integrate with respect to momenta before the convolution. We introduce

\[
\vec{l}_1 = \vec{k}_1 - x\vec{q}_1, \quad \vec{l}_2 = \vec{k}_2 + (1-x)\vec{q}_2, \quad \vec{p}_1 = \vec{q}_1 - \vec{k}_1, \quad \vec{p}_2 = \vec{q}_2 - \vec{k}_2.
\]

In this notation

\[
\left( \frac{2x_1 a_1^{ij} x_2 a_2^{ij}}{x(1-x)\sigma_{11}\sigma_{22}} \right) = 2 \frac{1}{\sigma_{11}} \left[ \delta_{ij} \frac{2}{x_2} \left( 1 - 2x_1 \right) - \frac{2\vec{q}_2^i \vec{l}_1^j}{\vec{q}_2^2} \right] + \frac{x_2 \vec{l}_2^i \vec{q}_2^j}{x_1 \vec{q}_1^2} + \frac{\vec{p}_1^i \vec{p}_2^j}{\vec{q}_1^2 \vec{k}_1^2}.
\]

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\begin{align}
\times \frac{1}{\sigma_{22}} \left[ \frac{\delta_{ij}}{2 x_1} \left( (1 - 2x_2) + \frac{2 (\vec{q}_1 \cdot \vec{q}_2)}{q_2^2} \right) - \frac{\delta_{ij}}{x_2 \frac{q_j^2}{q_2^2} - \delta_{ij} \frac{k_j^2}{k_2^2}} \right] \\
+ \frac{2k_1^i q_1^j}{x_1 k_1^2 q_1^2} \left( \frac{q_2^j}{q_2^2} + \frac{k_2^j}{k_2^2} \right) \frac{(k_2 + q_2)^i}{(k_2 + q_2)^2} + \frac{2k_1^j q_1^i}{x_1 k_1^2 q_2^2} \left( \frac{q_1^i}{q_1^2} - \frac{k_1^i}{k_1^2} \right) \frac{(k_1 - q_1)^j}{(k_1 - q_1)^2} .
\end{align}

Then we use the integrals (91)–(93) of Appendix B to find

\begin{align}
\int \frac{d^2q_1}{2\pi} \frac{d^2q_2}{2\pi} \frac{d^2k_1}{2\pi} \frac{d^2k_2}{2\pi} \left( 2x_1 a_i^2 x_2 a_j^2 \right) e^{i[\vec{q}_1 \cdot \vec{r}_{1\nu} + \vec{q}_2 \cdot \vec{r}_{2\nu} + \vec{k}_1 \cdot \vec{r}_{1\nu}]} \\
= \frac{2}{d_1 d_2} \left[ \frac{\delta_{ij}}{2 x_1} \left( 1 + 2 \frac{(\vec{r}_{1\nu} \cdot \vec{r}_{1\nu}^{'} \nu)}{r_{1\nu}^2} \right) - \frac{\delta_{ij}}{x_2 \frac{r_{1\nu}^2}{r_{1\nu}^2} - \frac{r_{1\nu}^2}{r_{1\nu}^2}} \frac{r_{1\nu}^2}{r_{1\nu}^2} \right] \\
\times \int \frac{\delta_{ij}}{2 x_1} \left( 1 - 2 \frac{(\vec{r}_{2\nu} \cdot \vec{r}_{1\nu}^{'} \nu)}{r_{2\nu}^2} \right) + \frac{\delta_{ij}}{x_2 \frac{r_{2\nu}^2}{r_{2\nu}^2} - \frac{r_{2\nu}^2}{r_{2\nu}^2}} \frac{r_{2\nu}^2}{r_{2\nu}^2} \right] \\
+ \frac{2 \left( \vec{r}_{1\nu} \cdot \vec{r}_{2\nu}^{'} \nu \right) \left( \vec{r}_{2\nu} \cdot \vec{r}_{2\nu}^{'} \nu \right)}{x_1 \frac{r_{1\nu}^2}{r_{1\nu}^2} - \frac{r_{1\nu}^2}{r_{1\nu}^2}} + \frac{2 \left( \vec{r}_{2\nu} \cdot \vec{r}_{1\nu}^{'} \nu \right) \left( \vec{r}_{1\nu} \cdot \vec{r}_{1\nu}^{'} \nu \right)}{x_2 \frac{r_{2\nu}^2}{r_{2\nu}^2} - \frac{r_{2\nu}^2}{r_{2\nu}^2}} ,
\end{align}

where \(d_1 = x_1 r_{1\nu}^2 + x_2 r_{2\nu}^2\) and \(d_2 = x_1 r_{2\nu}^2 + x_2 r_{1\nu}^2\). The subsequent integration over \(x\) is straightforward. We obtain

\begin{align}
\tilde{g}_s(\vec{r}_1, \vec{r}_2; \vec{r}_{1\nu}^{'} \nu, \vec{r}_{2\nu}^{'} \nu) = g_s(\vec{r}_1, \vec{r}_2; \vec{r}_{1\nu}^{'} \nu, \vec{r}_{2\nu}^{'} \nu) - \frac{1}{r_{1\nu}^4} \left[ \frac{r_{1\nu}^2}{r_{1\nu}^2} \ln \frac{r_{1\nu}^2}{r_{1\nu}^2} + \frac{r_{2\nu}^2}{r_{2\nu}^2} \ln \frac{r_{2\nu}^2}{r_{2\nu}^2} - 2 \right] \\
+ \frac{r_{1\nu}^2}{r_{1\nu}^2} \ln \frac{r_{1\nu}^2}{r_{1\nu}^2} + \frac{r_{2\nu}^2}{r_{2\nu}^2} \ln \frac{r_{2\nu}^2}{r_{2\nu}^2} ,
\end{align}

Here

\begin{align}
g_s(\vec{r}_1, \vec{r}_2; \vec{r}_{1\nu}^{'} \nu, \vec{r}_{2\nu}^{'} \nu) = \frac{1}{r_{1\nu}^4} \left( \frac{r_{1\nu}^2}{d} \ln \frac{r_{1\nu}^2}{r_{1\nu}^2} \right) - 1 \right) + \frac{1}{d r_{1\nu}^2} \left[ \frac{(r_{1\nu}^2 r_{1\nu}^2) r_{1\nu}^2}{r_{1\nu}^2} - \frac{(r_{2\nu}^2 r_{2\nu}^2) r_{2\nu}^2}{r_{2\nu}^2} \right] \ln \frac{r_{2\nu}^2}{r_{2\nu}^2} \\
+ \frac{2(r_{1\nu}^2 r_{2\nu}^2)(r_{1\nu}^2 r_{2\nu}^2)}{r_{2\nu}^2} - \frac{2(r_{1\nu}^2 r_{1\nu}^2)(r_{1\nu}^2 r_{1\nu}^2)}{r_{1\nu}^2} + \frac{2(r_{2\nu}^2 r_{2\nu}^2)(r_{2\nu}^2 r_{2\nu}^2)}{r_{2\nu}^2} - 2\frac{r_{2\nu}^2}{r_{2\nu}^2} \ln \frac{r_{2\nu}^2}{r_{2\nu}^2} \\
+ \frac{1}{r_{2\nu}^2} \left( \frac{(r_{1\nu}^2 r_{2\nu}^2)}{r_{1\nu}^2} - \frac{(r_{2\nu}^2 r_{1\nu}^2)}{r_{2\nu}^2} \right) \ln \frac{r_{2\nu}^2}{r_{2\nu}^2} .
\end{align}

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where $d = \vec{r}_{12}^2 - \vec{r}_{21}^2$. One can see that $g_{s1}$ vanishes when $\vec{r}_1$ equals $\vec{r}_2$ while the remaining terms in (64) are independent of either $\vec{r}_1$ or $\vec{r}_2$, therefore we dropped them.

Now we turn to $\tilde{K}_{s2}$. We can rewrite this expression in a form convenient for the integration

$$
\langle \vec{r}_1, \vec{r}_2 | \tilde{K}_{s2} | \vec{r}_1', \vec{r}_2' \rangle = -\frac{\alpha^2(\mu) N_c^2}{4\pi^4} \int \frac{d\vec{q}_1}{2\pi} \frac{d\vec{q}_2}{2\pi} \frac{d\vec{k}_1}{2\pi} \frac{d\vec{k}_2}{2\pi} \ln \left( \frac{\vec{k}_2^2}{\vec{k}_1^2} \right) e^{[\vec{q}_1 \cdot (\vec{r}_1 + \vec{r}_2 + \vec{k}_1') + \vec{q}_2 \cdot (\vec{r}_1' + \vec{r}_2')]} \left[ \frac{1}{\vec{k}_1^2 \vec{k}_2^2} \left( \frac{\vec{k}_2^2}{\vec{k}_1^2} \frac{\vec{q}_2 + \vec{k}_1}{(\vec{q}_2 + \vec{k}_1)^2} - \frac{\vec{q}_2}{\vec{q}_1^2} \frac{\vec{q}_1 - \vec{k}_1}{(\vec{q}_1 - \vec{k}_1)^2} \right) \frac{\vec{k}_1^2}{\vec{k}_2^2} \frac{\vec{q}_1 + \vec{k}_1}{(\vec{q}_1 + \vec{k}_1)^2} - \frac{\vec{q}_1}{\vec{q}_2^2} \frac{\vec{q}_2 - \vec{k}_1}{(\vec{q}_2 - \vec{k}_1)^2} \right] - \frac{(\vec{q}_1 \cdot \vec{q}_2)}{\vec{q}_1^2 \vec{q}_2^2} \left[ \frac{\vec{k}_2^2}{\vec{k}_1^2} \frac{\vec{q}_2 + \vec{k}_1}{(\vec{q}_2 + \vec{k}_1)^2} - \frac{\vec{q}_2}{\vec{q}_1^2} \frac{\vec{q}_1 - \vec{k}_1}{(\vec{q}_1 - \vec{k}_1)^2} \right] - \frac{(\vec{q}_1 \cdot \vec{q}_2)}{\vec{q}_1^2 \vec{q}_2^2} \left[ \frac{\vec{k}_1^2}{\vec{k}_2^2} \frac{\vec{q}_1 + \vec{k}_1}{(\vec{q}_1 + \vec{k}_1)^2} - \frac{\vec{q}_1}{\vec{q}_2^2} \frac{\vec{q}_2 - \vec{k}_1}{(\vec{q}_2 - \vec{k}_1)^2} \right] \right].
$$

The first term in the square brackets vanishes in the integration. Again, we omit terms proportional to $\delta(\vec{r}_1' \vec{r}_2')$ in the coordinate representation. The remaining terms which do not depend on $\vec{q}_1$ contribute to $g_{s2}(\vec{r}_1, \vec{r}_2; \vec{r}_1', \vec{r}_2')$, the ones independent of $\vec{q}_2$ contribute to $g_{s2}(\vec{r}_2, \vec{r}_1; \vec{r}_1', \vec{r}_2')$ and the others to $g_{s2}(\vec{r}_1, \vec{r}_2; \vec{r}_1', \vec{r}_2')$. During the calculation we used the integrals (90) and (94)–(106) of Appendix B. Finally, after the integration we obtain

$$
g_{s2}(\vec{r}_1, \vec{r}_2; \vec{r}_1', \vec{r}_2') = \frac{1}{4\vec{r}_{12}^2} \ln \left( \frac{\vec{r}_{12}^2}{\vec{r}_{22}^2} \right) \ln \left( \frac{\vec{r}_{12}^2}{\vec{r}_{12}^2} \right) - \frac{\vec{r}_{12}^2}{4\vec{r}_{12}^2} \ln^2 \left( \frac{\vec{r}_{12}^2}{\vec{r}_{12}^2} \right),
$$

$$
\tilde{g}_{s2}(\vec{r}_1, \vec{r}_2; \vec{r}_1', \vec{r}_2') = \tilde{g}_p(\vec{r}_1, \vec{r}_2; \vec{r}_1', \vec{r}_2') - \left[ \frac{(\vec{r}_{22}^2 \vec{r}_{12}^2)}{\vec{r}_{12}^2 \vec{r}_{22}^2} + \frac{1}{\vec{r}_{12}^2 \vec{r}_{22}^2} \right] \ln \left( \frac{\vec{r}_{12}^2}{\vec{r}_{12}^2} \right) + (1 \leftrightarrow 2).
$$

Hence,

$$
g_{s2}(\vec{r}_1, \vec{r}_2; \vec{r}_1', \vec{r}_2') = g_p(\vec{r}_1, \vec{r}_2; \vec{r}_1', \vec{r}_2') - \left[ \frac{(\vec{r}_{22}^2 \vec{r}_{12}^2)}{\vec{r}_{12}^2 \vec{r}_{22}^2} \ln \left( \frac{\vec{r}_{12}^2}{\vec{r}_{12}^2} \right) + (1 \leftrightarrow 2) \right].
$$

Here we used the “dipole” form of $g_p$ and dropped the terms independent of $\vec{r}_1$ or of $\vec{r}_2$. 

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11 Final result

In this Section we gather the contributions from the “planar” and “symmetric” parts of the kernel to the functions \( g \) defined in (44). We have

\[
g^0(\vec{r}_1, \vec{r}_2; \rho) = \frac{3}{2} \left( \frac{r_{12}^2}{r_{12}^2 r_{20}^2} \right) \ln \left( \frac{r_{12}^2}{r_{12}^2} \right) \ln \left( \frac{r_{20}^2}{r_{20}^2} \right) - \frac{11}{12} \left[ \frac{r_{12}^2}{r_{12}^2 r_{20}^2} \ln \left( \frac{r_{20}^2}{r_{20}^2} \right) + \left( \frac{1}{r_{20}^2} - \frac{1}{r_{20}^2} \right) \ln \left( \frac{r_{20}^2}{r_{20}^2} \right) \right],
\]

(70)

\[
g(\vec{r}_1, \vec{r}_2; \vec{r}_{12}') = \frac{11}{6} \left( \frac{r_{12}^2}{r_{12}^2 r_{20}^2} \right) \ln \left( \frac{r_{12}^2}{r_{12}^2} \right) + \frac{11}{22} \left( \frac{1}{r_{20}^2} - \frac{1}{r_{20}^2} \right) \ln \left( \frac{r_{20}^2}{r_{20}^2} \right)
\]

(71)

where

\[
\ln r_{12}^2 = 2\psi(1) - \ln \left( \frac{\mu^2}{4} - \frac{3}{11} \left( \frac{67}{9} - 2\zeta(2) \right) \right).
\]

One can see that both \( g^0(\vec{r}_1, \vec{r}_2; \rho) \) and \( g(\vec{r}_1, \vec{r}_2; \vec{r}_{12}') \) vanish at \( \vec{r}_1 = \vec{r}_2 \). Then, these functions turn into zero for \( \vec{r}_{12}^2 \rightarrow \infty \) faster than \( (\vec{r}_{12}^2)^{-1} \) to provide the infrared safety. The ultraviolet singularities of these functions at \( \vec{r}_{12} = \vec{r}_1 \) and \( \vec{r}_{12} = \vec{r}_2 \) cancel in the sum of the first three contributions in the R.H.S. of (44) on account of the “dipole” property of the “target” impact factors.

The last term in the R.H.S. of (44) is the most complicated one:

\[
g(\vec{r}_1, \vec{r}_2; \vec{r}_{12}', \vec{r}_{12}'') = \left[ \frac{(r_{12}' r_{12})}{r_{12}'^2 r_{20}^2 r_{12}''} - 2 \left( \frac{r_{12}'' r_{12}'}{r_{12}'^2 r_{20}^2 r_{12}''} \right) + 2 \left( \frac{r_{12}'' r_{12}'}{r_{12}'^2 r_{20}^2 r_{12}''} \right) \right] \ln \left( \frac{r_{12}'^2}{r_{12}'^2} \right)
\]

(73)

where \( d = r_{12}' r_{21}'' - r_{12}'' r_{21}' \).

This term also vanishes at \( \vec{r}_1 = \vec{r}_2 \), so that it possesses the “dipole” property. It has ultraviolet singularity only at \( \vec{r}_{12}'' = 0 \) and tends to zero at large \( \vec{r}_{12}' \) and \( \vec{r}_{12}'' \) sufficiently quickly in order to provide the infrared safety.
12 Conclusion

The coordinate representation of the BFKL kernel is extremely interesting, because it gives the possibility to understand its conformal properties and the relation between the BFKL and the color dipole approaches. Generally speaking, the BFKL kernel is not equivalent to the dipole one. Actually the first one is more general than the second. This is clear, because the BFKL kernel can be applied not only in the case of scattering of colourless objects. However, when applied to the latter case, we can use the “dipole” and “gauge invariance” properties of targets and projectiles and omit the terms in the kernel proportional to $\delta(\vec{r}_1/\vec{r}_2')$, as well as change the terms independent either of $\vec{r}_1$ or of $\vec{r}_2$ in such a way that the resulting kernel becomes conserving the “dipole” property, i.e. the property which provides the vanishing of cross-sections for scattering of zero-size dipoles. The coordinate representation of the kernel obtained in such a way is what we call the dipole form of the BFKL kernel. We have found the dipole form of the gluon contribution to the BFKL kernel in the NLO by the transfer of the kernel from the momentum representation where it was calculated before. This paper completes the transformation of the NLO BFKL kernel to the dipole form, started a few months ago with the quark part of the kernel [14, 15].

The striking result of [14, 15] was the simplicity of the dipole form of the quark contribution to the kernel. Moreover, it was shown that the dipole form agrees with the quark contribution to the BK kernel obtained in [16].

As it can be seen from the results of this paper, the dipole form of the gluon contribution to the kernel is also extremely simple in the coordinate representation. This holds especially for the “symmetric” part of the kernel, the momentum representation of which has a very complicated form [5].

We do not have the possibility to compare the results obtained for the NLO gluon contribution with the BK kernel, since the latter has not been obtained yet. Instead, our results can be used for finding the NLO BK kernel. In this respect, one has to bear in mind the ambiguity of the NLO kernel related to the operator transformation \( \hat{O} \) with an appropriate $\hat{\phi}$.

Note that in contrast to the LO, as well as to the quark contribution, the functions $g^0(\vec{r}_1, \vec{r}_2; \vec{p})$ and $g(\vec{r}_1, \vec{r}_2; \vec{p})$ in the dipole form (44) turned out to be unequal. Although the function $g^0(\vec{r}_1, \vec{r}_2; \vec{p})$ can be changed by adding any function with zero integral over $\vec{p}$, the inequality cannot be removed. On the other hand, according to [16] in the colour dipole approach these functions should be equal.

We have to say that our consideration was not completely rigorous. In particular, we did not regularize the ultraviolet singularities arising as a result of separation of the ultraviolet non-singular sum $V(\vec{k}) + V(\vec{k}, \vec{k} - \vec{q})$ in (45) into two pieces. Instead of the regularization we used the trick of representing the coefficient of $\delta(\vec{r}_1')\delta(\vec{r}_2'')$ in (44) in integral form. Note, however, that the trick is the same which was used in Ref. [14], where its validity was checked.
by completely rigorous calculations. This permits to rely on the results obtained here.

**Acknowledgments**

We would like to thank R.E. Gerasimov for calculating the integral (106).

## 13 Appendix A

Here we present a list of the integrals necessary to perform the Fourier transform of the planar part of the kernel. Most of them are calculated straightforwardly via the exponential representation:

\[
a^{-j} = \frac{1}{\Gamma(j)} \int_0^\infty d\alpha \alpha^{j-1} e^{-\alpha a}.
\]

We have

\[
\int \frac{d\vec{k}}{2\pi} e^{i\vec{k} \cdot \vec{r}} \frac{\vec{k}}{\vec{k}^2} = \frac{e^{i\vec{r} \cdot \vec{r}}}{\vec{r}^2}, \quad \int \frac{d\vec{k}}{2\pi} e^{i\vec{k} \cdot \vec{r}} \ln \vec{k}^2 = -\frac{2}{\vec{r}^2},
\]

\[
\int \frac{d\vec{k}}{2\pi} e^{i\vec{k} \cdot \vec{r}} \frac{\vec{k}^2}{\vec{k}^2} \ln \frac{\vec{k}^2}{\mu^2} = \frac{e^{i\vec{r} \cdot \vec{r}}}{\vec{r}^2} \left( 2\psi(1) - \ln \left( \frac{\vec{r}^2 \mu^2}{4} \right) \right),
\]

\[
\int \frac{d\vec{q}}{2\pi} \int \frac{d\vec{k}}{2\pi} e^{i\vec{q} \cdot \vec{r} + \vec{k} \cdot \vec{\rho}} \frac{(\vec{q} \cdot \vec{k})}{q^2 k^2} \ln \left( \frac{\vec{k}^2 + \vec{q}^2}{q^2} \right) = -\frac{(\vec{r} \cdot \vec{\rho})}{\vec{r}^2 \vec{\rho}^2} \ln \left( \frac{(\vec{r} - \vec{\rho})^2}{\vec{r}^2 \vec{\rho}^2} \right),
\]

\[
\int \frac{d\vec{q}}{2\pi} \int \frac{d\vec{k}}{2\pi} e^{i\vec{q} \cdot \vec{r} + \vec{k} \cdot \vec{\rho}} \frac{(\vec{q} \cdot \vec{k})}{q^2 k^2} \ln^2 \left( \frac{\vec{k}^2 + \vec{q}^2}{q^2} \right) = -\frac{(\vec{r} \cdot \vec{\rho})}{\vec{r}^2 \vec{\rho}^2} \ln^2 \left( \frac{(\vec{r} - \vec{\rho})^2}{\vec{r}^2 \vec{\rho}^2} \right),
\]

\[
\int \frac{d\vec{q}}{2\pi} \int \frac{d\vec{k}}{2\pi} e^{i\vec{q} \cdot \vec{r} + \vec{k} \cdot \vec{\rho}} \frac{1}{q^2} \ln^2 \left( \frac{(\vec{q} - \vec{k})^2}{\vec{k}^2} \right) = \frac{1}{\vec{\rho}^2} \ln^2 \left( \frac{(\vec{r} + \vec{\rho})^2}{\vec{r}^2} \right),
\]

\[
\int \frac{d\vec{q}}{2\pi} \int \frac{d\vec{k}}{2\pi} e^{i\vec{q} \cdot \vec{r} + \vec{k} \cdot \vec{\rho}} \frac{1}{q^2} \ln \left( \frac{(\vec{q} - \vec{k})^2}{\vec{k}^2} \right) = \frac{1}{\vec{\rho}^2} \ln \left( \frac{(\vec{r} + \vec{\rho})^2}{\vec{r}^2} \right). \tag{82}
\]
The integral (82) is calculated most easily after the decomposition $\ln a \ln b = \frac{1}{2}(\ln^2 a + \ln^2 b - \ln^2 \frac{a}{b})$.

\[
\int \frac{d{\vec q}_1}{2\pi} \int \frac{d{\vec q}_2}{2\pi} \int \frac{d{\vec k}}{2\pi} e^{i{\vec q}_1 \cdot {\vec r}_1 + i{\vec q}_2 \cdot {\vec r}_2 + i{\vec k} \cdot {\vec p}} \ln^2 \left( \frac{{\vec q}_1 + {\vec q}_2}{k^2} \right) = \frac{4}{r_1^2 r_2^2 \rho^2} \ln \left( \frac{r_1^2 r_2^2}{(r_1^2 - r_2^2)^2 \rho^2} \right), \quad (83)
\]

\[
\int \frac{d{\vec q}_1}{2\pi} \int \frac{d{\vec q}_2}{2\pi} \int \frac{d{\vec k}}{2\pi} e^{i{\vec q}_1 \cdot {\vec r}_1 + i{\vec q}_2 \cdot {\vec r}_2 + i{\vec k} \cdot {\vec p}} \ln^2 \left( \frac{{\vec q}_1^2}{k^2} \right) = \frac{4}{r_1^2 r_2^2 \rho^2} \ln \left( \frac{r_1^2 r_2^2}{\rho^2} \right). \quad (84)
\]

The following integrals contain $I$ defined in (25):

\[
\int \frac{d{\vec q}}{2\pi} \int \frac{d{\vec k}}{2\pi} e^{i{\vec q} \cdot {\vec r} + i{\vec k} \cdot {\vec p}} I({\vec q}^2, (\vec q - \vec k)^2, \vec k^2) = I(\vec p^2, (\vec p + \vec r)^2, \vec r^2), \quad (85)
\]

\[
\int \frac{d{\vec q}}{2\pi} \int \frac{d{\vec k}}{2\pi} e^{i{\vec q} \cdot {\vec r} + i{\vec k} \cdot {\vec p}} \frac{I(\vec q^2, (\vec q - \vec k)^2, \vec k^2)}{\vec k^2} = -\frac{1}{\vec r^2} I(\vec p^2, (\vec p + \vec r)^2, \vec r^2), \quad (86)
\]

\[
\int \frac{d{\vec q}}{2\pi} \int \frac{d{\vec k}}{2\pi} e^{i{\vec q} \cdot {\vec r} + i{\vec k} \cdot {\vec p}} \frac{I(\vec q^2, (\vec q - \vec k)^2, \vec k^2)}{\vec q^2 \vec k^2} = \frac{1}{\vec r^2 \vec p^2} I(\vec p^2, (\vec p + \vec r)^2, \vec r^2). \quad (87)
\]

This integral can be expressed through (86) and simpler integrals with the help of the identity

\[
(\vec k \cdot \vec q) = \frac{(1 - x(1 - z))(\vec k \cdot \vec q)}{2(1 - x)(1 - z)} + \frac{(z + x(1 - z))(\vec q^2)}{2x(1 - z)} - \frac{(\vec q^2 - \vec k^2)(1 - z) + (\vec k^2)(1 - x)}{2(1 - x)x(1 - z)} z.
\]

14 Appendix B

Here we present a list of the integrals necessary to perform the Fourier transform of the symmetric part of the kernel:

\[
\int \frac{d{\vec q}}{2\pi} \int \frac{d{\vec p}}{2\pi} e^{i{\vec p} \cdot {\vec r} + i{\vec q} \cdot {\vec p}} \ln \frac{\vec p^2}{\vec q^2 \vec p^2} = -\frac{i}{2} \left[ I(\vec p^2, \vec r^2, (\vec p - \vec r)^2) \right]
\]

\[
+ \frac{r}{\vec r^2} \left[ I(\vec p^2, \vec r^2, (\vec p - \vec r)^2) + \frac{1}{2} \ln \left( \frac{\vec p^2}{(\vec p - \vec r)^2} \right) \ln \left( \frac{\vec r^2}{\vec p^2} \right) \right]. \quad (89)
\]

\[
\int \frac{d{\vec q}}{2\pi} \int \frac{d{\vec k}}{2\pi} e^{i\vec q \cdot \vec r} \ln \frac{\vec q^2}{\vec k^2} = \frac{i\vec p}{4\vec p^2} \ln \left( \frac{\vec r^2}{\vec r^2} \right) \ln \left( \frac{\vec p^4}{\vec r^1 \vec r^2} \right), \quad (90)
\]
\[
\int \frac{d\vec{q} \, d\vec{l}}{2\pi \, 2\pi \, \vec{l}^2 + x(1-x)\vec{q}^2} \, e^{i\vec{q} \cdot \vec{r} + i\vec{q} \cdot \vec{\rho}} = \frac{1}{\vec{r}^2 + x(1-x)\vec{\rho}^2}, \tag{91}
\]
\[
\int \frac{d\vec{q} \, d\vec{l} \, q_i l_j}{2\pi \, 2\pi \, \vec{q}^2 \, \vec{l}^2 + x(1-x)\vec{q}^2} \, e^{i\vec{q} \cdot \vec{r} + i\vec{q} \cdot \vec{\rho}} = \frac{-r_i \rho_j}{\vec{r}^2(\vec{r}^2 + x(1-x)\vec{\rho}^2)}, \tag{92}
\]
\[
\int \frac{d\vec{q} \, d\vec{k} \, q_i k_j}{2\pi \, 2\pi \, \vec{k}^2 \, (1-x)\vec{k}^2 + x\vec{q}^2} \, e^{i\vec{q} \cdot \vec{r} + i\vec{q} \cdot \vec{\rho}} = \frac{-r_i \rho_j}{\vec{r}^2(\vec{r}^2 + x\vec{\rho}^2)}, \tag{93}
\]
\[
\int \frac{d\vec{k}}{2\pi} \left( e^{i\vec{k} \cdot \vec{r}_1} - e^{i\vec{k} \cdot \vec{r}_2} \right) \frac{1}{\vec{k}^2} = \frac{1}{2} \ln \left( \frac{\vec{r}_2^2}{\vec{r}_1^2} \right), \tag{94}
\]
\[
\int \frac{d\vec{q}}{2\pi} \int \frac{d\vec{k}}{2\pi} e^{i(\vec{q} \cdot \vec{r} + \vec{q} \cdot \vec{\rho})} \frac{(\vec{q} \cdot \vec{k}) (\vec{q} \cdot \vec{k})}{\vec{q}^2 \vec{k}^2 (\vec{q} + \vec{k})^2} = -i \frac{1}{4} \left( \frac{\vec{r}}{\vec{r}^2} \ln \frac{\vec{\rho}^2}{(\vec{r} - \vec{\rho})^2} + \frac{\vec{\rho}}{\vec{\rho}^2} \ln \frac{\vec{r}^2}{(\vec{r} - \vec{\rho})^2} \right), \tag{95}
\]
For calculating the following integrals we use the identities
\[
\frac{\partial I(a, b, c)}{\partial a} = \frac{1}{(c-a-b)^2 - 4ab} \left( (c+b-a)I(a, b, c) + 2 \ln \frac{a}{c} + \frac{(c-a-b)}{a} \ln \frac{b}{c} \right), \tag{96}
\]
\[
\int_0^1 \frac{2c \cdot x \, dx}{ax + b(1-x) - cx(1-x)} \ln \left( \frac{ax + b(1-x)}{cx(1-x)} \right) = I(a, b, c)(c + b - a) + \text{Li}_2 \left( 1 - \frac{b}{a} \right) - \text{Li}_2 \left( 1 - \frac{a}{b} \right) + \frac{1}{2} \ln \frac{a}{b} \ln \frac{ab}{c^2}, \tag{97}
\]
where
\[
\text{Li}_2(x) = -\int_0^1 \frac{dt \ln(1-xt)}{t}. \tag{98}
\]
\[
\int \frac{d\vec{q}}{2\pi} \int \frac{d\vec{k}}{2\pi} e^{i(\vec{q} \cdot \vec{r} + \vec{q} \cdot \vec{\rho})} \frac{(\vec{q} \cdot \vec{k}) (\vec{q} \cdot \vec{k})}{\vec{q}^2 \vec{k}^2 (\vec{q} + \vec{k})^2} \ln \vec{k}^2 = \frac{i}{4} \left\{ \frac{\vec{r}}{\vec{r}^2} \ln \left( \frac{\vec{\rho}^2}{(\vec{r} - \vec{\rho})^2} \right) + \frac{\vec{\rho}}{\vec{\rho}^2} \ln \left( \frac{\vec{r}^2}{(\vec{r} - \vec{\rho})^2} \right) \right\} \left( -2\psi(1) + \ln \left( \frac{(\vec{r} - \vec{\rho})^2}{4} \right) \right)
\]
\[
+ \frac{\vec{\rho}}{\vec{\rho}^2} \left( \frac{1}{2} \ln \left( \frac{\vec{r}^2}{(\vec{r} - \vec{\rho})^2} \right) - (\vec{\rho} \cdot \vec{r})I(\vec{r}^2, \vec{\rho}^2, (\vec{r} - \vec{\rho})^2) \right), \tag{99}
\]

23
\[
\int \frac{d^2q}{2\pi} \int \frac{d^2k}{2\pi} e^{i(q\cdot r - \tilde{\rho}\cdot \bar{\rho})} \frac{q^i(q + k)^j}{q^2(\bar{q} + \bar{k})^2} \ln \bar{k}^2
\]
\[
= \frac{1}{4r^2\tilde{\rho}^2} \left( \delta^{ij}(\bar{r} \cdot \bar{\rho}) + \rho^i r^j - \rho^j r^i \right) \ln \left( \frac{(\bar{r} - \bar{\rho})^4}{r^2\tilde{\rho}^2} \right) + \frac{\rho^i r^i}{2r^2\tilde{\rho}^2} \left( 4\psi(1) - \ln \left( \frac{\tilde{\rho}^2}{16} \right) \right)
\]
\[
+ \frac{1}{2(\bar{r} - \bar{\rho})^2} \left( \frac{\rho^i \rho^j}{\tilde{\rho}^2} - \frac{r^i r^j}{r^2} + \frac{(\bar{r}^2 - \bar{\rho}^2)}{2r^2\tilde{\rho}^2} \left( \delta^{ij}(\bar{r} \cdot \bar{\rho}) - \rho^i r^j - \rho^j r^i \right) \right) \ln \left( \frac{\tilde{\rho}^2}{r^2} \right).
\]

We simplified the tensor structure of this integral via the identity
\[
\delta^{ij} = \frac{\rho^i \rho^j r^2 + r^i r^j \bar{\rho}^2 - (\bar{r} \cdot \bar{\rho}) (\rho^i r^j + \rho^j r^i)}{r^2\tilde{\rho}^2 - (\bar{r} \cdot \bar{\rho})^2}.
\]
The integral
\[
J = \int \frac{d\bar{q}_1}{2\pi} \int \frac{d\bar{q}_2}{2\pi} \int \frac{d\bar{k}}{2\pi} e^{i(\bar{q}_1 \cdot \bar{r}_{1\rho} + \bar{q}_2 \cdot \bar{r}_{2\rho} + \bar{k} \cdot \bar{r}_{1\rho}')} \frac{(\bar{q}_1 \cdot \bar{q}_2)}{\bar{q}_1^2\bar{q}_2^2} \ln \left( \frac{\bar{q}_1 - \bar{k}}{\bar{q}_1 + \bar{k}} \right) \ln \left( \frac{\bar{q}_2 + \bar{k}}{\bar{q}_2 - \bar{k}} \right)
\]
appearing in the “planar” part can be written as
\[
\int \frac{d\tilde{\rho}}{(2\pi)^2} d\bar{q}_1 d\bar{q}_2 \delta \left( \bar{q}_1 - \bar{k} - \bar{q}_1' \right) e^{i\bar{r}_{2\rho} [\bar{q}_1 + \bar{q}_1' - \bar{q}_2']} = 1
\]
into the integrand in (102) and then integrating over momenta with the help of (75). Using the same trick we obtain
\[
\int \frac{d\bar{q}_1}{2\pi} \int \frac{d\bar{q}_2}{2\pi} \int \frac{d\bar{k}}{2\pi} e^{i(\bar{q}_1 \cdot \bar{r}_{1\rho} + \bar{q}_2 \cdot \bar{r}_{2\rho} + \bar{k} \cdot \bar{r}_{1\rho}')} \frac{(\bar{q}_1 \cdot \bar{q}_2)}{\bar{q}_1^2\bar{q}_2^2} \ln \left( \frac{\bar{q}_1 - \bar{k}}{\bar{q}_1 + \bar{k}} \right) \ln \left( \frac{\bar{q}_2 + \bar{k}}{\bar{q}_2 - \bar{k}} \right) = \frac{r_{1\rho}^2 r_{2\rho}^2}{8} J
\]
for the integral appearing in the “symmetric” part. One can calculate \(J\) by means of complex variables
\[
a_+ = a_x + i a_y, \quad a_- = a_x - i a_y, \quad (\bar{a} \cdot \bar{c}) = \frac{a_+ c_- + a_- c_+}{2}, \quad \bar{a}^2 = a_+ a_-
\]
Shifting \(\tilde{\rho} \rightarrow \tilde{r}_1 - \tilde{\rho}\), introducing \(z = e^{i\phi}\), using
\[
\rho_+ = \rho z, \quad \rho_- = \frac{\rho}{z}, \quad d\phi = \frac{dz}{iz},
\]
and integrating over \(z\) via residues, one gets only trivial integrals over \(\rho\) to perform. They yield
\[
J = \frac{2}{r_{1\rho}^2 r_{2\rho}^2} \left( \frac{(\bar{r}_{1\rho} \cdot \bar{r}_{2\rho}')}{r_{1\rho}^2 r_{2\rho}^2} \ln \left( \frac{\bar{r}_{2\rho}^2 r_{1\rho}^2}{r_{1\rho}^2 r_{2\rho}^2} \right) + \frac{(\bar{r}_{2\rho} \cdot \bar{r}_{1\rho}')}{r_{2\rho}^2 r_{1\rho}^2} \ln \left( \frac{\bar{r}_{1\rho}^2 r_{2\rho}^2}{r_{1\rho}^2 r_{2\rho}^2} \right) \right).
\]
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