Background-Oriented Schlieren (BOS) for Scramjet Inlet-isolator Investigation

Azam Che Idris 1,*, Mohd Rashdan Saad1, Kin Hing Lo2 and Konstantinos Kontis2

1Faculty of Engineering, National Defence University of Malaysia, Kem Sungai Besi, Kuala Lumpur 57000, Malaysia
2School of Engineering, University of Glasgow, James Watt South Building, Glasgow G12 8QQ, Scotland;

Corresponding email: *azam.che@alumni.manchester.ac.uk

Abstract. Background-oriented Schlieren (BOS) technique is a recently invented non-intrusive flow diagnostic method which has yet to be fully explored in its capabilities. In this paper, BOS technique has been applied for investigating the general flow field characteristics inside a generic scramjet inlet-isolator with Mach 5 flow. The difficulty in finding the delicate balance between measurement sensitivity and measurement area image focusing has been demonstrated. The differences between direct cross-correlation (DCC) and Fast Fourier Transform (FFT) raw data processing algorithm have also been demonstrated. As an exploratory study of BOS capability, this paper found that BOS is simple yet robust enough to be used to visualize complex flow in a scramjet inlet in hypersonic flow. However, in this case its quantitative data can be strongly affected by 3-dimensionality thus obscuring the density value with significant errors.

1. Introduction
The Background-Oriented Schlieren (BOS) technique, which has been gaining its popularity today, is particularly suitable for calculating the density increase across an oblique shock. The calculated density gradient can be used to estimate the overall performance of a scramjet inlet using the concept shown by Idris et al. [1]. Compared to their method where average isolator flow properties were inferred from the pressure map on sidewall, two-dimensional BOS technique would result in spanwise-integrated density field for the whole isolator [2]. Concurrently, a three-dimensional BOS which requires simple modifications from two-dimensional method could provide truly tomographic reconstruction of the isolator density-field along the spanwise direction [3][4]. Hence, the calculated isolator exit Mach number and subsequently the inlet-isolator performances by using BOS method will be more reliable and accurate.

The method was first presented by Dalziel et al. in late 1990s as the “synthetic schlieren” [5 – 7]. They demonstrated that the displacements of light-ray path from random dots set up on the background of a changing density volume can be processed by using cross-correlation software “borrowed” from Particle Image Velocimetry (PIV) methods. The results are in the form of light-ray path-integrated density-gradient field, which explains why they associated it with the classical
schlieren technique. At about the same time, Meier independently introduced and patented the concept of BOS in Germany [8].

This relatively new method has very simple setup, which requires only high resolution camera (typically Digital Single-Lens Reflex (DSLR) camera) and a suitable background image, thus making it adaptable for many applications. For example, Richard and Raffel visualised compressible vortices from spinning helicopter rotor blade using BOS [9]. They took images of a hovering helicopter with the camera positioned at an angle such that the tip of the rotor blade was set against background view of a grass field. The vortices from the blade tips were shown to be interacting with its engine exhaust plume.

Elsinga et al. compared this method with another quantitative schlieren technique called Calibrated Colour Schlieren (CCS) in analysing a two-dimensional wedge-plate model in flow Mach number of 1.94 [2][10]. BOS and CCS were shown to be effective in visualising shock and expansion wave. The resultant flow field properties calculated using BOS were close to the analytical value predicted from oblique shock relations and Pandtl-Meyer expansion theory for that particular geometry.

Venkatakrishnan and Meier applied this method to predict the flow around axisymmetric cone-cylinder geometry in a freestream Mach number of 2 [3]. Filtered back projection algorithm introduced in the article allows the authors to obtain slices of two-dimensional density-field around the model. The calculated density matched the published data of cone tables [11]. There are many other exciting and interesting examples of recent applications of BOS in flow-diagnostics, however, there has been no attempt yet to apply this method for scramjet inlet-isolator investigations.

2. Basic BOS Theory

The theory for BOS is relatively simple and can be described using figure 1 below. In the figure, x-axis is parallel to the freestream and starts at the model tip at half-symmetry plane, y-axis is parallel with the vertical plane and z-axis is along the line of sight. Random patterns of suitable shapes and sizes are printed onto single image and fixed on the background plane. The background plane is positioned at a distance \((Z_D)\) from the middle plane of density gradient volume in investigation. The lens of the image capture device is located at length \((Z_B)\) from the background plane. The notation \((Z_I)\) in the figure denotes the focal length of the lens, where the light rays from the background plane converged onto the camera chip. From the figure, consider that as the control volume changes in density, the light ray from a particular point on the background plane will shift by \((\Delta y)\) on the camera chip. This displacement can be calculated by using the PIV cross-correlation software using inputs of two still images of "wind-off" and "wind-on". The wind-off is the initial condition where the density field is known and is usually the wind-tunnel condition without flow. The wind-on, as the name implies, refers to the condition where the density field changes due to wind tunnel flow. This displacement is due to the light path being deflected vertically by angle \(\varepsilon_y\). The deflection angle is typically very small and can be obtained by:

\[
\sin(\varepsilon_y) \approx \tan(\varepsilon_y) \approx \varepsilon_y = \frac{\Delta y'}{Z_D}
\]

Here, \(\Delta y'\) denotes the virtual displacement of the point on background image and can be related to \(\Delta y\) displacement detected by the camera by:

\[
\frac{\Delta y'}{Z_D} = \frac{\Delta y}{Z_I}
\]

The deflection angle \(\varepsilon_y\) can be related to change in refractive index \(\frac{\delta n}{\delta y}\) of the volume by:

\[
\varepsilon_y = \frac{1}{n_0} \int_{Z_D+\Delta Z_D}^{Z_D}[\frac{\delta n}{\delta y}]_D dZ
\]
Here, $n_0$ denotes the initial refractive index of the flow, “wind-off” conditions prior to change in density field. The change in refractive index $\frac{\delta n}{\delta y}$ can be related to density gradient $\frac{\delta \rho}{\delta y}$ by partially-deriving the Gladstone-Dale equation:

$$\frac{\delta (n - 1)}{\delta y} = \frac{\delta (k \rho)}{\delta y} \quad \text{(4)}$$

$$\frac{\delta n}{\delta y} = k \frac{\delta \rho}{\delta y} \quad \text{(5)}$$

Here, $k$ is the Gladstone-Dale constant and its value depends on type of gas used.

Equation (1) to (5) can be similarly applied to consider the light path deflection on x-axis. Thus, the final density gradient at any point in the flow field is a product of $\frac{\delta \rho}{\delta x}$ and $\frac{\delta \rho}{\delta y}$.

![Figure 1. Deflection of light path from background image due to change in density gradient.](image)

### 3. Cross-correlation Theory

The background images of wind-off and wind-on can be discretized into a finite number of interrogation windows for cross-correlation analysis. The windows must be of the same size, $W$, and contains enough shape elements for analysis. Since a window could contain many elements that have different displacement magnitudes, it is best to have the smallest window size possible to have a good representative of average particles displacement (see figure 2). The cross-correlation function can be calculated by using equation shown below [12]:

$$\Phi(\Delta x, \Delta y) = \frac{\sum_{x,y} W (I_a(x,y) - \bar{I}_a)(I_b(x+\Delta x,y+\Delta y) - \bar{I}_b)}{\left(\sum_{x,y} W (I_a(x,y) - \bar{I}_a)^2 \sum_{x,y} W (I_b(x,y) - \bar{I}_b)^2\right)^{0.5}} \quad \text{(6)}$$

Here, $I_a$ denotes the pixel intensities at location (x,y) within the interrogation window in the wind-off image. Similarly, $I_b$ denotes the pixel intensities at location (x,y) within the interrogation window in the wind-on image. $\bar{I}_a$ and $\bar{I}_b$ are the average $I_a$ and $I_b$ for the whole window size. $\Phi(\Delta x, \Delta y)$ is the correlation function in terms of displacement in x-y axis, $\Delta x$ and $\Delta y$. The function reaches peak value at the coincident points of elements match between wind-off and wind-on images.
Depending on the interrogation window size, the computing requirements could be really expensive. Thus, Fast Fourier Transform (FFT) approximation is typically used to produce each displacement vector [13]. However, Pust had shown that Direct Cross Correlation (DCC), which directly solves the equation without approximation, is significantly better in producing realistic results in comparison to FFT [13].

4. Experimental Setup

The BOS setup for current scramjet inlet-isolator model was applied onto scramjet inlet model similar to Idris et al. [1]. The scramjet inlet was subjected to dry air flow with static pressure of 1220 Pa, static temperature of 62 K, total pressure of 645 kPa, total temperature of 370 K and Mach number of 5.

Due to the sensitivity of BOS system the model should be located halfway between the background and camera lens (i.e $Z_D = 0.5Z_B$) [3][14]. Hargather and Settles demonstrated that as the model is gradually moved towards the background, the sensitivity of system drops significantly [14]. The problem with this requirement is that with such small isolator height of only 6.8 mm, a sufficient compromise of camera focusing could not be found. If the lens is focused on the background image, then isolator section would appear heavily blurred, rendering the resultant image useless. On the other hand, if the focusing is made onto the middle plane of the isolator section, then the patterns on the background image would be too out of focus to be detected by cross-correlations software. Compromise was made by applying the background image onto one sidewall, while the other sidewall provided the optical access (see figure 3). We believe that sensitivity was not significantly compromised since the shockwaves inside the isolator section are strong and can be detected easily.
The setup for BOS experiments is shown in figure 4. The camera used was Canon DSLR EOS-600D with lens of focal length 135 mm. The camera was positioned such the lens touched the test-section window. This was done in order to increase the ratio of $Z_0/Z_B$ and thus increasing the sensitivity in measurement. The camera was set to capture a wind-off image after the test section was vacuumed right before the tunnel starting. The camera was also set to capture a series of image at 4 fps during the tunnel run, and the best image was chosen for wind-on. The two images chosen were then processed together to obtain the desired density gradient field.

![Figure 4](image-url)

**Figure 4.** Top view schematic of BOS experimental setup

Two red-head lamps with power of 650 W each were directed towards a large white screen. The reflective lights from the screen gave uniform illumination to the background image printed on sidewall. The experiments were conducted in total darkness except for the two light sources to obtain the best contrast for easy detection of particles deflection.

The pattern for the background image was made using open source general PIV post-processing software, PIVMat© [15] written in MATLAB® environment. The software was set to draw 700,000 black particles of diameter 0.15 mm each in random pattern on an A4 sized image. Effectively, there were 11.22 particles/mm² and the filled surface ratio was 0.198. The image was then printed on projector transparency sheet (A4 size) to allow for background illumination of the particles. A small rectangular strip of size $55 \times 6.8$ mm² was cut and pasted onto the inner face of sidewall as shown in figure 4.

5. Results and Discussion

The raw pre-processed images of wind-off and wind-on conditions are shown in figure 5. Any changes in particles cluster shapes and locations cannot be detected by naked eye. The model also experienced slight clockwise rotation in figure 5(b) due to heavy downwards force on the compression ramps. Even though the rotation was very subtle and cannot be observed with naked eyes, it is still of significant magnitude in comparison to each particle deflection value and it can influence their final value as calculated by cross-correlation software. Thus, it must first be corrected prior to processing. The model movement was corrected using ‘cp2tform’ function in MATLAB 7.10.0 (R2010a), where control points were selected on both figures and the algorithm automatically applied rotation and translation on figure 5(b), so that the control points on both figures agree with each other.
Figure 5. Pre-processed images of (a) wind-off condition; (b) wind-on

Both figures were processed by using open-source PIVLab® cross-correlation software written by Thielicke and Stamhuis [16]. The software’s capability to cross-correlate PIV data has been demonstrated in papers by Mahanti et al.[17], Mirsepassi and Dunn-Rankin [18] and Ryerson and Schwenk [19]. The software was chosen since because of its robustness despite being freely available. Commercial software for PIV processing typically has more features but tends to be very expensive.

The small isolator size prohibited the use of small interrogation window size. This is due to requirements of sufficient particle elements in each interrogation window. Using smaller particles size and higher particles density in the background image might circumvent this issue, but they were limited by the printing capabilities. Thus, to overcome this problem, interrogation window of the size $192 \times 192$ pixels was chosen, with overlap value of 96%. Spatial resolution was improved significantly with high overlap value, but it comes with large computing cost. If using DCC scheme, rendering all displacement vectors took about 7 hours on a Dell M6300 mobile workstation with Core2Duo Extreme X7900 (2.8 Ghz) processor and 4 GB of RAM. The images in figure 6 were also processed using FFT (Fast Fourier Transform) cross-correlation scheme, and the results came at much faster of less than 5 minutes on the same laptop. Comparison of the density gradient using standard colour schlieren (taken from Idris et al. [1]), DCC and FFT are given in the Fig. 6 below.

From figure 6(a), we can observe that a large flow separation at shoulder produced a very strong separation shock that propagates downstream in the form of a shock-train. The shock-train structure is very complex and identifying each shock location was not straightforward. There are at least two more separations that can be detected, one at just downstream of cowl-tip and another one at further downstream. Each separation would produce two shockwaves of their own, a separation shock and a re-attachment shock. All background shock-waves reflected as they propagated downstream and interacted with each other creating an overall complex system of shocks.

Figure 6(b) and (c) shows that all part of the shock-train can be visualized using both the DCC and the FFT calculation scheme of BOS. Even better, the BOS method detected two weak shocks that are barely visible in the colour schlieren image, at the end of the isolator section. However, since BOS detected only density gradient, and by definition a shockwave is a density gradient; region without a
shockwave should only have colour ‘dark blue’ as per the legend in figure 6. This is not the case, such as that in figure 6(b) and 6(c) show readings of density gradient between successive shockwaves. The density gradient field calculated by the FFT method (see figure 6(c)) suffer less error than the one calculated by DCC (see figure 6(b)). This is because the FFT ignored small movement vectors inside each interrogation window in order to decrease its computational time thus making its signal to noise ratio more favourable.

Figure 6. Schlieren images of baseline scramjet inlet-isolator model; (a) Colour schlieren (taken with permission from Idris et al.; (b) schlieren image rendered from density gradient calculated using Direct Cross Correlation (DCC) of BOS data; (c) schlieren image rendered from density gradient calculated using Fast-Fourier Transform Cross Correlation (FFT) of BOS data

The density of the flow exiting the isolator section can be calculated easily by integrating the density field and inserting the boundary condition. The boundary condition is the value of density at the start of isolator section and this is a known value of 0.29 kg/m$^3$[1]. If we do a line-integration at the middle of the field, from upstream towards the exit of the isolator, the density at the exit is 1.52 kg/m$^3$ for DCC and 1.01 kg/m$^3$ for FFT. Both values are off by a significant margin from the actual density calculated using equation of state with known value of pressure and temperature, which is about 0.39 kg/m$^3$[1]. We believe that the error is not from the BOS setup per-se, instead, we argue that using the BOS method is not suitable for the current case. This is because, shock-train in a scramjet isolator is typically very complex and any schlieren based measurement, be it traditional, colour or even BOS will be presented with a big problem due to the smearing of shock waves as viewed by an observer. As explained by Raffel, the BOS is a line-of-sight integration technique thus if the flow is not fully 2-dimensional then the density-gradient will be distorted [20]. The flow inside the isolator suffered from 3-dimensionality effect close to the sidewall region [1]. Thus the shock waves appeared to be thicker than their theoretical size and their width was wrongly accounted into when the density gradient field was integrated.
6. Conclusion
An exploratory study of applying BOS method to characterize a scramjet inlet-isolator has been performed. The setup could not be simpler, requiring only a DSLR camera and suitable background image. The processing software needed for rendering the density-gradient field is open-source and easily available. Using FFT scheme of calculating cross-correlation is more beneficial in term of computational time and noise rejection. However, using schlieren-based density-measurement for scramjet inlet-isolator investigation would not produce an accurate reading of density since the shockwaves would appeared smeared to the observer.

References
[1] Idris A C, Saad M R, Zare-Behtash H and Kontis K 2014 Sensors 14(4) 6606
[2] Elsinga G, Van Oudheusden B, Scarano F and Watt, D 2004 Experiments in Fluids 36 309
[3] Venkatakrishnan L and Meier G 2004 Experiments in Fluids 37 237
[4] Goldhahn E, Alhaj O, Herbst F and Seume J 2009 Imaging Measurement Methods for Flow Analysis 135
[5] Dalziel S, Hughes G O and Sutherland B R 2000 Fluids 28 322
[6] Dalziel S B, Hughes G O, and Sutherland B R 1998 Proc. 8th Int. Symp. on Flow Visualization
[7] Dalziel S B 2000 Proc. of the 5th Intern. symp. on stratified flows
[8] Meier G 1999 Deutsche Patentanmeldung 856
[9] Richard H and Raffel M 2001 Measurement Science and Technology 12 1576
[10] Elsinga G, Van Oudheusden B, Scarano F and Watt D 2003 Proceedings of PSFVIP-4
[11] Sims J L 1964 Tables for supersonic flow around right circular cones at zero angle of attack NASA SP-3004
[12] Willert C and Gharib M 1991 Experiments in fluids 10 181
[13] Pust O 2000 Proceedings of the 10th International Symposium on Applications of Laser Techniques to Fluid Mechanics 27.
[14] Hargather M J and Settles G S 2011 HVAC Research 17 771
[15] Moisy F, Rabaud M and Pinsolle E 2008 ISFV13-13th International Symposium on Flow Visualization, and FLUVISU12-12th French Congress on Visualization in Fluid Mechanics Paper No. 326
[16] Thielicke W and Stamhuis E J 2014 Journal of Open Research Software 2(1):e30
[17] Mahanti P, Keebaugh M and Weiss N 2014 Proceedings of the COMSOL Conference
[18] Mirsepassi A and Dunn-Rankin D 2012 16th Int Symp on Applications of Laser Techniques to Fluid Mechanics
[19] Ryerson W and Schwenk K 2012 Journal of Experimental Zoology 127–140
[20] Raffel M 2015 Exp Fluids 56(60)