Dynamics of mathematical model interaction of living populations in making biogas as an alternative of renewable energy

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Abstract. Renewable energy which is often referred to alternative energy is needed as an alternative or substitute for conventional energy. Biogas is a type of energy source that comes from living things (plants and animals). Biogas is the most energy-efficient and environmentally friendly. Biogas as alternative energy has considerable potential to help reduce the use of fossil energy. One way to determine the effect of each aspect is to model it in mathematical language. Mathematical models represent the production of biogas by considering the interactions between living populations in making alternative renewable energy alternatives. The model is constructed as a nonlinear autonomous system with three dependent variables. After forming a mathematical model, dynamical behavior such as investigate the equilibrium point, a local stability analysis process is then carried out on the model. The numerical simulation will represent an illustration analysis results of the constructed model. The simulation results show the same stability analysis.

1. Introduction

The existence of energy in life is something absolute. Energy is the ability to do business (work), energy is not only present in humans but also in objects. Energy is abundant in both living and non-living things, such as petroleum, fossil materials, natural gas, the sun, wind, and water. Any object (living or non-living) that can produce energy is called an energy source. Based on the source, energy is divided into renewable and non-renewable energy [1].

Renewable energy comes from energy sources, such as wind, water, and the sun. Meanwhile, non-renewable energy is generally in fossil energy, such as coal, natural gas, and petroleum. The process of forming fossil energy which takes a very long time results in an imbalance between demand and availability. The decreasing supply of fossil energy from year to year encourages humans to look for alternative energy that comes from renewable sources. Fossil energy is also one of the causes of global warming. Thus, clean and renewable alternative energy is needed [2]. Some examples of alternative energy that have been developed include biogas, biodiesel, hydropower, ethanol, geothermal energy, and many more [3].

Biogas as alternative energy has great potential to help reduce the use of fossil energy. Biogas' benefits are vehicle fuel, a substitute for LPG gas, and a power plant [4,5]. Biogas is produced from a mixture of animal waste and agricultural waste decomposed anaerobically with the help of microorganisms, especially methane bacteria [6,7]. The gas produced is carbon dioxide (CO2), methane
gas (CH4) which has explosive properties, and other gases. The calorific value produced from biogas reaches 16-20 MJ / m3 [8]. The biogas production plant can be done near the sewage disposal site. The source of biogas which comes from the waste of living things allows its availability to be maintained [9]. However, a special strategy is needed to balance the numbers of animals and plants to ensure that biogas production continues.

Biogas production depends on the amount of animal waste which is the main raw material and is influenced by vegetable materials derived from agricultural waste. The process of making biogas produces waste that can be used as fertilizer for crops. Plants that grow can be a source of food for livestock. From this explanation, one can see the close relationship between animals, plants, and biogas. The existence of each other is interrelated and influential [6,7,9]. One way to determine the effect of each aspect is to model it in mathematical language.

Mathematical models can translate various phenomena that occur in everyday life so that the root of the problem can be found, the solution is obtained, and even future events can also be predicted [10-12]. To find out how the relationship between animals, plants, and biogas, it is necessary to create a dynamic model that involves these three variables.

Al Seadi [13] created a scheme of relationships between animals, plants, and biogas that show a relationship with one another. From this scheme, a mathematical model will be compiled to see how the existence of a variable affects the existence of other variables. The model formed is then analyzed to see its stability and equilibrium point. Thus, an overview of the conditions of the availability of each resource will be obtained. This information can be used as a reference in the biogas production process so that the quantity is maintained.

2. Method
The methods used in this research are as follows:

2.1 Literature study on Renewable Energy
At this stage, a literature study is conducted on various types of renewable energy, its benefits, and the process of its formation. The identification process of renewable energy sources, especially biogas which comes from plant remains and animal waste is carried out. At this stage, the effects of interaction between plants and anaerobic on the biogas production process were also observed. The types of digestion systems in the biogas production process are investigated and anaerobic (without oxygen) digestion systems are selected.

2.2 Dynamic Model Formation in Biogas Production
This research was started by constructing a mathematics model of the interaction of living populations on the progress of making biogas as the empowerment of energy sources. At this stage, a dynamic model is formed which represents the relationship between plants and decomposition in producing biogas. The mathematics model is in the form of a nonlinear differential equation consisting of three compartments.

2.3 Stability Analysis and Simulation.
At this stage, the process of investigating the model's equilibrium point is carried out then analysing the model's stability around the equilibrium point. The point of equilibrium is obtained when the growth rate is unchanged or equal to zero. The stability analysis is determined by linearizing the system so that the Jacobi matrix is obtained. The condition stability shows whether the equilibrium point is stable or not. The model that has been investigated for its stability and equilibrium is then simulated with mathematical software. The simulation results are analyzed and concluded as a whole

2.4 Conclusion
Based on the results of the previous stages, conclusions can be drawn as well as providing useful suggestions for improvement and further research development.
3. Result and Discussion

3.1 Model Formulation in Biogas Production

The formation of a mathematical model the interaction of living organisms in the progress of making biogas as empowerment of energy sources refers to Figure 1. Figure 1 is an illustration of the steps for recycling wet organic waste against pollution to the environment [13]. The system considers the following model assumptions

- $E(t)$: Concentration on the formation of biogas as an alternative to renewable energy
- $P(t)$: Concentration of processing in digestion (Anaerobic) which can obtain biogas
- $T(t)$: Concentration of plant that are influenced by the surrounding environment

![Figure 1. Schematic representation of the sustainable cycle of anaerobic co-digestion of animal manure and organic wastes [13]](image_url)

The growth rate of processing in decomposition is influenced by animal manure which is taken for decomposition (animal manure) indicated by a parameter ($\alpha_1$). Falling leaves which can become plant waste ($\nu_1$) and industrial or domestic waste that can be decomposed can affect the growth of the digestion process ($I$). Furthermore, the concentration of the decomposition product which can be used as fertilizer for plants ($p$) and the concentration of the decomposition product which is a source of renewable energy (biogas) are the effects of the reduced rate of degradation ($\epsilon$).

The rate of growth of renewable energy is influenced by the concentration of decomposition products which is a source of renewable energy (biogas) ($e$) as well as the concentration of oxygen needed to utilize renewable energy ($o$). The concentrations of ($CO_2$)($c$) and ($H_2O$)($h$) come out as a result of the use of energy that has been used, then plants are used for photosynthesis and become the influence of the growth rate of plants. The concentration of the decomposition products which can be used as fertilizer for plants also affects the growth rate of the plants. The rate of reduced plant growth is influenced by the concentration of animals that eat plants for their survival ($\alpha$), the concentration of aging plants or the process of rejuvenation ($\nu_2$), and the concentration of oxygen released from photosynthesis.
To obtain a model of the interaction of living populations on the progress of making biogas as empowerment of energy sources as follows:

\[
\begin{align*}
\frac{dP}{dt} &= a_1 + l + v_1 T - (p + e)P, \\
\frac{dE}{dt} &= eP - (c + h)E + oT, \\
\frac{dT}{dt} &= (c + h)E + pP - aT - (v_2 + o)T,
\end{align*}
\]  

(1)

where \( P(0) \geq 0, E(0) \geq 0, T(0) \geq 0 \).

3.2 Equilibrium Points and Stability Analysis

The equilibrium point is obtained from the solution of the system solution

\[
\frac{dP}{dt} = \frac{dE}{dt} = \frac{dT}{dt} = 0,
\]

The system has the following equilibrium points \( E_0(T^*, P^*, E^*) \), where

\[
T^* = \frac{a_1 + l}{a},
\]

\[
P^* = \frac{1}{(e + p)} \left( v_1 + a \left( \frac{a_1 + l}{a} \right) \right),
\]

\[
E^* = \frac{1}{(c + h)} \left( \frac{a_1 + l}{a} \right) \left( \frac{e}{(e + p)} \right) (v_1 + a) + o.
\]

The stability analysis was carried out by linearizing the system in order to obtain the Jacobi matrix as follows,

\[
J = \begin{bmatrix}
-(p + e) & 0 & v_1 \\
0 & -(c + h) & 0 \\
p & (c + h) & -a - (v_1 + o)
\end{bmatrix}.
\]

The stability of \( E_0 \) can be determined by analyzing eigenvalues, it is found eigenvalues for \( E_0 \) are the solution of polynomial equation, \( A^3 + a_1 A^2 + a_2 A + a_3 = 0 \), where

\[
a_1 = p + e + h + a + v_1 + o + c \\
a_2 = pc + ph + ap + op + ec + ae + v_1 e + oe + ac + v_i c + ah + v_i h + eh \\
a_3 = apc + aph + eca + eha
\]

By the Routh-Hurwitz criterion [14], all eigenvalues have negative real part if and only

\[
a_1 > 0, a_2 > 0, a_3 > 0 \text{ and } a_1a_2 - a_3 > 0.
\]

Therefore, the equilibrium point \( E_0(T^*, P^*, E^*) \), is locally asymptotically stable if only if inequalities (2) are satisfied, or

\[
(p + e + h + a + v_1 + o + c)(pc + ph + ap + op + ec + ae + v_1 e + oe + ac + v_i c + ah + v_i h + eh) > \]

\[
+ apc + aph + eca + eha.
\]
3.3 Numerical Simulations

In this section, we study the result of numerical simulations the dynamics of system (1) to illustrate the analysis results. The parameter values are assume that $a_1 = 0.26, l = 0.3, v_1 = 0.5, p = 0.4, e = 0.2; c = 0.35; h = 0.3, o = 0.1, p = 0.5, a = 0.54,$ and $v_2 = 0.7$.

Based on these parameters, we found one exist equilibrium point, namely $E_0(1.34,0.53,0.76)$.

Based on analysis, we have $a_1 = 0.49 > 0, a_2 = 1.679 > 0, a_3 = 0.25 > 0, a_1a_2 - a_3 = 3.93 > 0$, it means $E_0(1.34,0.53,0.76)$ locally asymptotically stable. As shown in Fig. 2, all solutions of system (1) converge to the equilibrium point $E_0(1.34,0.53,0.76)$.

![Figure 2. Simulations to system (1) with $a_1 = 0.26; l = 0.3; v_1 = 0.5; p = 0.4; e = 0.2; c = 0.35; h = 0.3; o = 0.1; p = 0.5; a = 0.54$ and $v_2 = 0.7$](image)

We have shown interaction of living populations in making biogas as an alternative of renewable energy. It is found that the model has one equilibrium points, namely $E_0$. Based on analytical result, $E_0$ are locally asymptotically stable under some conditions. Numerical simulation results show the same result with analysis.

4. Conclusion

We have managed to construct a model of the interaction of living populations on the progress of making biogas as empowerment of energy sources. This model is formed considering a schematic representation of the sustainable cycle of anaerobic co-digestion of animal manure and organic wastes. From the stability analysis process, we can ensure that the model is locally asymptotically stable if some conditions are fulfilled. We have also examined the interactions of living populations in making biogas as an alternative of renewable energy using numerical simulations by defining some parameters.

References

[1] Mahjabeen, SZA Shah, Chughtai S, Simonetti 2020 Energy Strategy Rev. Elsevier. 29:100484.
[2] DO Hall, HE Mynick, and RH Williams 1991 Nature. 353 11
[3] Rathore NS, Panwar NL 2007 Renewable Energy Sources for Sustainable Development (New Delhi, India: New India Publishing Agency)

[4] M Berglund, P Borjesson 2006 Biomass and Bioenergy 30 254

[5] I Angelidaki, HV Hendriksen, IM Mathrani, JE Schmidt, AH Sørensen, BK Ahring 1996 The biogas process. Lecture notes for: Energy from biomass (6362).

[6] Gupta, Yadvika, Santosh, TR Sreekrishnan, Kohli, Sangeeta, Rana, and Vineet 2004 rev. Bioresour. technol. 95 1

[7] Achinas S, Euverink GJW 2016 Resource-Efficient Technol. 3 143

[8] Kurchania AK, Panwar NL, Pagar SD 2010 Int. J. Sustain. Energy 29 116

[9] Deublein D and Steinhauser A 2011 Biogas from Waste and Renewable Resources (Wiley-VCH, Weinheim, Germany)

[10] Njuguna M, Anthony & Mohamed, Belaid & Seodigeng, Tumisang & Ngila J 2016 Mathematical Modelling for Biogas Production (University of Johannesburg: South Africa)

[11] Benyahia B, Sari T, Cherki B, Harmand J 2010 IFAC Proceedings Volumes 43(6)371

[12] Delgadoillo Mirquez L, Machado Higuera M, Hernández Sarabia M 2018 Int. J. Eng. Syst. Model. Simul. 10(2)97.

[13] Al Seadi T 2002 IEA Bioenergy, Task 24-Energy from Biological Conversion of Organis Waste (www. IEA-Biogas. Net)

[14] Murray JD 2002 Mathematical Biology I : An Introduction Third Edition (Springer-Verlag Berlin Heidelberg, New York)

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