Endogenous versus Exogenous Crashes in Financial Markets

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Abstract

In a series of papers based on analogies with statistical physics models, we have proposed that most financial crashes are the climax of so-called log-periodic power law signatures (LPPS) associated with speculative bubbles [Sornette and Johansen, 1998, Johansen and Sornette, 1999, Johansen et al., 1999, Johansen et al., 1999, Sornette and Johansen, 2001]. In addition, a large body of empirical evidence supporting this proposition have been presented [Sornette et al., 1996, Sornette and Johansen, 1998, Johansen et al., 1999, Johansen and Sornette, 2000, Johansen and Sornette, 2001b, Sornette and Johansen, 2001]. Along a complementary line of research, we have established that, while the vast majority of drawdowns occurring on the major financial markets have a distribution which is well-described by a stretched exponential, the largest drawdowns are occurring with a significantly larger rate than predicted by extrapolating the bulk of the distribution and should thus be considered as outliers [Johansen and Sornette, 1998, Sornette and Johansen, 2001, Johansen and Sornette, 2001b, Johansen, 2002]. Here, these two lines of research are merged in a systematic way to offer a classification of crashes as either events of an endogenous origin associated with preceding speculative bubbles or as events of an exogenous origin associated with the markets response to external shocks.

We first perform an extended analysis of the distribution of drawdowns in the two leading exchange markets (US dollar against the Deutsmark and against the Yen), in the major world stock markets, in the U.S. and Japanese bond market and in the gold market, by introducing the concept of “coarse-grained drawdowns,” which allows for a certain degree of fuzziness in the definition of cumulative losses and improves on the statistics of our previous results. Then, for each identified outlier, we check whether LPPS are present and take the existence of LPPS as the qualifying signature for an endogenous crash: this is because a drawdown outlier is seen as the end of a speculative unsustainable accelerating bubble generated endogenously. In the absence of LPPS, we are able to identify what seems to have been the relevant historical event, i.e., a new piece of information of such magnitude and impact that it is seems reasonable to attribute the crash to it, in agreement with the standard view of the efficient market hypothesis. Such drawdown outliers are classified as having an exogenous origin. Globally over all the markets analyzed, we identify 49 outliers, of which 25 are classified as endogenous, 22 as exogeneous and 2 as associated with the Japanese anti-bubble. Restricting to the world market indices, we find 31 outliers, of which 19 are endogenous, 10 are exogenous and 2 are associated with the Japanese anti-bubble. The combination of the two proposed detection techniques, one for drawdown outliers and the second for LPPS, provides a novel and systematic taxonomy of crashes further substantiating the importance of LPPS.
1 Introduction

The characterization of large stock market moves, and especially large negative price drops, is of profound importance for risk management and portfolio allocation. According to standard economic theory, the complex trajectory of stock market prices is the faithful reflection of the continuous flow of news that are interpreted and digested by an army of analysts and traders [Cutler et al. (1989)]. Accordingly, large market losses should result from really bad surprises. It is indeed a fact that exogenous shocks exist, as epitomized by the recent events of Sept. 11, 2001 and the coup in the Soviet Union on Aug. 19, 1991, which move stock market prices and create strong bursts of volatility. However, it could be argued that precursory fingerprints of even these events were known to some, suggesting the possibility that the action of these informed agents may have been reflected in part in stock market prices prior to the advent of the shock. This question is another formulation of the problem of market efficiency and to what degree is there some residual private information that is not fully reflected in prices [Fama, 1991]. Thus, a key question is whether large losses and gains are indeed slaved to exogenous shocks or on the contrary may result from an endogenous origin in the dynamics of that particular stock market. The former possibility requires the risk manager to closely monitor the world of economics, business, political, social, environmental ... news for possible instabilities. This approach is associated with standard “fundamental” analysis. The later endogenous scenario requires the investigation of signs of instabilities to be found in the market dynamics itself and could rationalize in part so-called “technical” analysis.

As a first step to address this question from a statistical viewpoint, we ask whether or not one can distinguish very large losses from the rest of the population of smaller losses. Of course, very large losses are naively distinct from the rest simply by their sheer size. The issue is not to quality naively a large loss simply by its magnitude, which would be a trivial and uninformative definition, but to ask whether there are distinctive statistical properties that distinguish big losses from the rest of the population of losses. According to the definition of the Engineering Statistical Handbook “An outlier is an observation that lies an abnormal distance from other values in a random sample from a population” [NIST]. In a sense, this definition leaves it up to a consensus process to decide what will be considered abnormal. However, before abnormal observations can be singled out, it is necessary to characterize normal observations. Hence, the question we want to address in this paper is whether the largest losses seen on the financial markets are merely “amplified small losses” or something entirely different. We do this by following two independent but complementary lines of investigation, namely a statistical analysis of drawdowns in various markets and a case-study analysis of speculative bubbles in the same markets. In our previous analysis and identification of outliers on the major financial markets [Johansen and Sornette, 1998, Johansen and Sornette, 2001b], drawdowns (drawups) were simply defined as a continuous decrease (increase) in the value of the price at the close of each successive trading day. Hence, a drawdown (drawup) was terminated by any increase (decrease) in the price no matter how small. In section 3, we will generalise the definition of drawdowns (drawups) and introduce the concept of “coarse-grained” drawdowns (drawups). Drawdowns (drawups) of the previous kind analyzed in [Johansen and Sornette, 1998, Johansen and Sornette, 2001b] will be referred to as “pure” drawdowns.

Analyzing a stock market index or a currency exchange rate using the distribution of drawdowns (or the complementary quantity drawups) rather than the more standard distribution of returns has the advantage that correlations of order two and higher are in part taken into account in this one-point statistics. These drawdowns (drawups) may identify transient bursts of dependences in successive returns. As our definition represents a “worst case scenario” of loss, see section 3, it is different from the one of Grossman & Zhou and others [Grossman and Zhou, 1993, Cviitanic and Karatzas, 1995, Chekhlov et al., 2000], where it is the present loss from the last maximum of the price. Furthermore, we will not address the portfolio allocation problem based on risks quantified by drawdowns, a very important problem which is left for a future publication.

By studying several variants of drawdowns (runs of cumulative losses with different degrees of fuzziness, see section 3), we show that the largest drawdowns in general are outliers to the vast majority (≥ 98%) of drawdowns and cannot be described by an extrapolation of the distribution of small and intermediate losses, a property that we refer to as the “king” effect [Laherrère and Sornette, 1998]. This analysis extends an increasing amount of evidence showing that the distribution of the largest negative market moves belongs to a population different from that of the smaller moves [Johansen and Sornette, 1998, Johansen and Sornette, 2001b, Johansen and Sornette, 2000, Sornette and Johansen, 2001, Lillo and Mantegna, 2000, Mansilla, 2001, Johansen, 2002].

The second step concerns the origin of this king effect: does it reflect the arrival of an anomalously serious piece of news or are very large drawdowns the outcome of a self-organized dynamical process of the stock market with its complex interactions between heterogeneous agents of varying sizes, all subjected to an incessant bombardment of news, each piece of news being insufficient by itself to explain the presence or absence of a shock? While it is a common practice to associate the large market moves and strong bursts of volatility with external
economic, political or natural events [White (1996)], there is simply no convincing evidence supporting it. The first indication that a combination of these two processes may be responsible for the creation of (drawdown) outliers in the stock market was obtained by [Johansen and Sornette, 1998], who found that for the DJIA in the last century two outliers were associated with a profound bull-market (the crashes of Oct. 1929 and Oct. 1987) and one outlier with a major historical event, namely the outbreak of WWI. Furthermore, this question has previously been addressed quantitatively for volatility shocks modeled quite accurately by the multifractal random walk model of stochastic volatility [Sornette et al, 2002]. It was shown that endogenous and exogenous shocks give rise to different precursory as well as relaxation dynamics [Sornette and Helmstetter, 2002]. In other words, the exogenous versus endogenous origins of a large volatility shock leave a sufficiently strong distinctive imprint in the price dynamics that one can distinguish two classes of signatures, a rather fast relaxation to normal volatility levels for exogenous shocks compared with an amplitude-dependent slower relaxation for endogenous shocks. Interestingly, most of the volatility shocks were found to be endogenous [Sornette et al, 2002]. Here, we study the problem of distinguishing between shocks of exogenous versus endogenous origin at larger time scales.

We stress that a shock of exogenous origin may very well be a large loss on another major stock market. By analyzing the time series of stock market prices prior to the occurrence of each of the qualified outliers, we show that for the large majority of cases, a distinctive structure in the price trajectories exists. Specifically, the majority of drawdown outliers are preceded by a (super-exponential) power law price appreciation decorated by accelerating perturbations in the form of news impacting the market. On the other hand, most of the crashes are found to be endogenous and can be seen as the natural deaths of self-organized self-reinforcing speculative bubbles giving rise to specific precursory signatures, specifically LPPS. For this, we generalize our previous analysis of drawdowns to drawdowns coarse-grained in amplitude (ε-drawdowns). Another generalization of drawdowns coarse-grained in time (τ-drawdowns) will be reported elsewhere. In relation with our previous works, we find that all of the crises (except the one in 1937) and associated bubbles previously analysed for the presence of LPPS are here causally linked with a drawdown outlier. This gives an additional reason to believe our previous results showing that LPPS is a strong discriminator of bubbles preceding strong corrections. It also strengthens the conclusion that LPPS, which are by construction transient structures, are almost uniquely associated with a speculative phase announcing a strong change of regime.

The emphasis of this paper thus lies first in the development of a systematic and objective definition of crashes and second in the systematic search for LPPS in the time series preceding the crash. This allows us to address previous criticisms of possible data-picking and to assess the robustness and significance of the previously proposed precursors. When an outlier has been detected, we search for both LPPS and for a major historical event which may be causally linked to the date of the outlier. We find in general that no clear piece of striking news can be associated with outliers preceded by LPPS while outliers without LPPS have in general been triggered by a great news surprise. By combining these two methods aimed at detecting anomalous events seen as outliers and LPPS qualifying endogenous speculative bubbles, respectively, we provide a new objective test of the hypothesis that the largest negative markets moves are special and form two distinct populations.

On the one hand, we identify exogenous crashes which can be attributed to extraordinary important external perturbations in the form of news impacting the market. On the other hand, most of the crashes are found to be endogenous and can be seen as the natural deaths of self-organized self-reinforcing speculative bubbles giving rise to specific precursory signatures, specifically LPPS. For this, we generalize our previous analysis of drawdowns to drawdowns coarse-grained in amplitude (ε-drawdowns). Another generalization of drawdowns coarse-grained in time (τ-drawdowns) will be reported elsewhere. In relation with our previous works, we find that all of the crashes (except the one in 1937) and associated bubbles previously analysed for the presence of LPPS are here causally linked with a drawdown outlier. This gives an additional reason to believe our previous results showing that LPPS is a strong discriminator of bubbles preceding strong corrections. It also strengthens the conclusion that LPPS, which are by construction transient structures, are almost uniquely associated with a speculative phase announcing a strong change of regime.
The outline of the paper is as follows. In the next section, we describe in detail using four concrete examples what we mean by exogenous and endogenous crashes and bubbles with LPPS. In section 3, we discuss how our drawdowns (drawups) are defined and explain the methodology of our analysis. Section 3.1 presents an analysis of the distribution of deviations of DJIA from a 4-year smoothed average obtained by using the Daubechies wavelet transform, which confirms independently the existence of anomalous large events in the tail of the distribution. In sections 4, 5, 6 and 7, we present our results on the FX market, major stock markets, the U.S. and Japanese bond market and the gold market, respectively. The last section concludes. The Appendix 8 lists the various markets and time periods we have analysed.

2 Exogenous and Endogenous Crashes and LPPS Bubbles

Two good examples of exogenous crashes, i.e., large declines in the price caused by external shocks, are the Nazi invasion of France and Belgium, Luxembourg and the Netherlands (Benelux) on May 10th 1940 as well as the resignation and the following controversial pardoning of president R. Nixon on August 8th and September 8th 1974, see the last entry in tables 3 and 4 for the parameters of the related drawdowns and figures 1 and 2 for the time series prior to these two exogenous crashes. In the two figures, the index show no signs of a bull-market, even less of an unsustainble bubble, and hence it seems reasonable to attribute the two large declines to these two historical events.

Two good examples of the endogenous class are the arch-typical crash of October 1929 and the “dot.com” crash of April 2000, see table 3 and 5 for the parameters of the related drawdowns (to be analyzed in detail below). These two crashes can be seen as nothing but efficient deflations \(^1\) of extended stock market bubbles in both cases funneled by heavy investment in so-called “new economy” companies\(^2\). Figures 3 and 4 show the bubbles preceding these two endogenous crashes with their LPPS. The solid lines are fits with a first order LPPS eq. (1), see figure captions for the fit parameters.

In a series of papers, the authors have documented that most speculative bubbles on the major stock markets as well as emergent markets present the following characteristics \[\text{Johansen et al., 1999, Johansen and Sornette, 1999, Johansen et al., 1999, Sornette and Johansen, 2001}\]: (i) a super-exponential overall acceleration of the price quantified by a power law in the time to the end of the bubble and (ii) oscillations with a frequency accelerating approximately in geometrical proportional to the distance to the (most probable) time \(t_c\) of the end of the bubble. These accelerating oscillations have been called log-periodic oscillations because they are quantified by the following formula

\[
I(t) = A + B(t_c - t)\gamma + C(t_c - t)\gamma \cos(\omega \ln(t_c - t) - \phi)
\]

Expression (1) has been proposed as the general signature of cooperative speculative behavior coexisting with rational behavior in a general mathematical theory of rational bubbles \[\text{Johansen et al., 1999, Johansen et al., 1999}\]. This formula (1) exemplifies the two characteristics of a bubble preceding an endogenous crash mentioned above:

(i) a faster-than-exponential growth of the price captured by the power law \((t_c - t)\gamma\) with \(0 < \gamma < 1\)

(ii) an accelerating oscillation decorating the accelerating price, described by the term \(\cos(\omega \ln(t_c - t) - \phi)\)

which leads to local periods converging to zero according to a geometrical progression with scale factor \(\lambda \equiv \exp(2\pi/\omega)\).

We stress that, of the 7 parameters present in eq. (1), the 3 linear parameters \(A, B\) and \(C\) are “slaved” in the fitting algorithm and calculated from the values obtained from the remaining 4 parameters \[\text{Johansen, 1997, Johansen et al., 1999}\]. Furthermore, \(A, B, C\) and \(\phi\) are simply units and carry no “structural” information and \(t_c\) is by definition event-specific. Hence, only the two variables \(\omega\) and \(\gamma\) quantifying the over-all acceleration in the price increase and the oscillations, respectively, carry any “structural” information about the market dynamics. This is expressed by the remarkable observation that the values for \(\omega\) and \(\gamma\) are found to be remarkably consistent for a large variety of speculative bubbles on different markets \[\text{Johansen, 2002}\]. Specifically, we have previously found that

\[
\omega \approx 6.36 \pm 1.56 \quad \gamma \approx 0.33 \pm 0.18
\]

for over thirty crashes on the major financial markets, as shown in figure 5 for the distribution of \(\omega\), figure 6 for the distribution of \(\gamma\) and figure 7 for the distribution of the complex exponent \(\gamma + i\omega\). Note the existence

\(^1\) resembling the impact of the famous “invisible hand” of Adam Smith, delayed over the long time scales of the speculative bubbles.

\(^2\) That the crash of Oct. 1929 caused the Great Depression is a part of financial folklore, but nevertheless untrue. For instance, using a regime switching framework, \[\text{Coe, 2002}\] finds that a prolonged period of crisis began not with the 1929 stock market crash but with the first banking panic of October 1930.
of a major peak at $\omega \approx 6.36 \pm 1.55$ in figure 5 and of a secondary peak centered on approximately twice this value, suggesting the relevance of a second harmonics of the main log-periodic angular log-frequency $\omega$. The reason for this is that in general, the best fit in terms of the r.m.s. is also the best solution in terms of estimating the structural variables $z$ and $\omega$ as well as the most probable time $t_c$ of the end of the bubble, see [Sornette and Johansen, 2001] for a more detailed discussion. However, for a few cases the two best fits are included in the statistics which explains the presence of the “second harmonics” around $\omega \approx 11.5$.

The consideration of the complex exponent $z + i\omega$ in figure 7 comes from the fact that the third term $C'(t_c - t)^z \cos(\omega \ln(t_c - t) - \phi)$ inequation (1) is nothing but the real part of the power law $C'(t_c - t)^{z+i\omega}$ where $C' = C e^{-i\phi}$, written with the complex exponent $z + i\omega$. Thus, the log-periodic formula (1) can be viewed simply as the generalization to the field of complex numbers of standard power laws [Sornette, 1998].

Such a narrow range of variation of a scaling index such as $\omega$ or $z+i\omega$ is in the physical sciences often attributed to a universal behavior reflecting a robust feature of the underlying mechanism of cooperative behavior between “units” leading to the observed critical pattern [Fisher, 1998]. We thus take the qualification of this pattern encoded in the mathematical formula (1) as the definition of an endogenous bubble ending in a drawdown outlier or “crash.” Reciprocally, the absence of such a pattern is taken as indicating that the shock was not the culmination of an endogenous speculative process and may have resulted from an exogenous source.

Here, we combine the search for log-periodic power law signatures (LPSS) embodied by eq. (1) with the outlier concept described below (which amounts to find significant deviations from eq. (3)), which were previously pursued independently. Before pursuing further, we need to clarify what financial quantity should be analyzed, in other words, what is the proxy $I(t)$ in eq. (1) to analyse. Specifically, for the bubble preceding the crash of Oct. 1929 (figure 3), we have chosen the market price index itself and, for the the bubble preceding the Nasdaq crash of April 2000, we have chosen the logarithm of the market price (figure 4). In fact, based on the rational bubble model of [Johansen et al., 1999, Johansen et al., 1999], it can be shown that, if the magnitude of the crash is proportional to the price increase only associated with the contribution of the bubble, then the correct proxy is the price itself; on the other hand, if the magnitude of the crash is proportional to the price, then the correct proxy is the logarithm of the price [Johansen and Sornette, 1999].

We stress that the physical framework underlying eq.(1) is invariant under an interchange of $t_c$ and $t$. This leads to the interesting property that eq. (1) with $t - t_c$ as its argument describes so-called “anti-bubbles”, see figure 8 for a specific example. Several cases of such anti-bubbles have been documented in many markets [Johansen and Sornette, 1999, Johansen and Sornette, 2000, Johansen et al., 1999, Johansen and Sornette, 2001b], most noteworthy that of the Japanese stock market since its collapse in 1990, see see figure 8. In fact, a very successful prediction based on the extension of the mathematical theory for bubbles to anti-bubbles was published in Jan. 1999 predicting the value of the Nikkei in Jan. 2000 with an accuracy of $\approx 1\%$ [Johansen and Sornette, 1999, Johansen and Sornette, 2000]. More recently, a similar anti-bubble has been detected in the behavior of the US S&P500 index from 1996 to August 2002 [Sornette and Zhou, 2002].

Last, it should be noted that eq.(1) is in fact a special solution of a first order “Landau-type” expansion where a general periodic function has been chosen as a cosine for simplicity. Including higher order expansions is in principal straightforward (see [Sornette and Johansen, 1997, Johansen, 1997, Johansen and Sornette, 1999] for works including second and third order terms), however the technical difficulties especially with respect to controlling the fitting algorithm are considerable. These extensions are not considered in the present work.

3 Methodology for coarse-grained drawdowns and drawups

3.1 First evidence of outliers at large time scales using wavelets

Before presenting the more detailed analysis of $c$-drawdowns, it is useful to give a broader view. For this, a smoothed version of the DJIA index in the last century using a wavelet filter has been constructed [Simonsen et al., 2002]. Specifically, a Daubechies’ wavelet of order 24 at a scale corresponding to a time window of 1024 trading days has been used to model long-term (meaning larger than the time window) trends in the DJIA [Press et al., 1992]. This is achieved by first transforming the data to the wavelet-domain, setting all wavelet coefficient corresponding to scales larger than 1024 trading days to zero and transforming back to the time domain again. Then, this approximating wavelet is subtracted from the original DJIA price series to obtain a residual time series. Figure 9 shows the histogram of the residues, that is, of the differences between the original DJIA index and the wavelet-filtered version at the scale of approximately 4 years.

The distribution of residuals is made of two parts. The bulk part of small and intermediate size residuals is well-fitted by an exponential distribution, qualified as the straight lines in this log-linear representation. The
largest negative residuals deviate significantly from the extrapolation of the exponential distribution by shaping a secondary peak. This qualifies the existence of a second regime of "outliers" for the distribution of large negative drops. Interestingly, there are no outliers for the positive residuals. Thus, this wavelet analysis substantiates the asymmetry previously established for drawdowns and drawups [Johansen and Sornette, 2001b] as well as the outlier concept for drawdowns, however, for surprisingly long time scales.

This analysis is complementary to the drawdown analysis presented below which focuses on short time scales, i.e., days. We stress that figure 9 has been obtained without the need for any parametric adjustment, except for the choice of the scale in the wavelet. This analysis provides an extension of the outlier concept at larger time scales than the daily scale used in the \( \epsilon \)-drawdown analysis presented below and hence quantifies the existence of outliers on time scales much longer than those belonging to crashes and runs. In figure 10, we see that all the negative residue outliers belongs to the Great Depression in the 1930ties, which not surprisingly is the only outlier on time scales longer than \( \approx 4 \) years in the last century. A more thorough outlier analysis of the relevant time scales is left for a future publication [Johansen, Simonsen and Sornette, 2002].

### 3.2 Definitions of coarse-grained drawdowns and drawups

In our previous analysis and identification of outliers in the major financial markets [Johansen and Sornette, 1998, Johansen and Sornette, 2001b], drawdowns (drawups) were simply defined as a continuous decrease (increase) in the value of the price at the close of each successive trading day (daily close). Hence, a drawdown (drawup) was terminated by any increase (decrease) in the price no matter how small. However, this definition of drawdowns (drawups) is sensitive to noise, i.e., to random uncorrelated as well as correlated fluctuations in the price. Simulations adding noise to the time series analysed have already showed that the distributions of drawdowns are robust to i.d.d. noise of "reasonable" magnitude [Johansen and Sornette, 2001b] (we will comment further on this below). We now investigate this question in more detail by filtering the data before the distribution of drawdowns (drawups) is calculated.

There are two straightforward ways for defining such coarse-grained drawdowns (drawups). First, we may ignore increases (decreases) below a certain fixed magnitude (absolute or relative to the price). This is called price coarse-grained drawdowns or "\( \epsilon \)-drawdowns", where \( \epsilon \) is the minimum (relative or absolute) positive (negative) fluctuation which will terminate the drawdown (drawup). If \( \epsilon = 0 \), we call the drawdown (drawup) "pure". For a non-zero \( \epsilon \), we let the drawdown (drawup) continue if the amplitude of the fluctuation between consecutive daily closes is smaller than \( \epsilon \). In the present paper, the emphasis will be on these \( \epsilon \)-drawdowns, where the \( \epsilon \) used is the relative threshold or the minimum relative positive (negative) fluctuation which will terminate the drawdown (drawup). Second, we may ignore increases (decreases) that occur over a time horizon smaller than some horizon regardless of magnitude. This is called temporally coarse-grained drawdowns or "\( \tau \)-drawdowns", where \( \tau \) is the horizon. For \( \tau = 0 \) this is equivalent to pure drawdowns. Specifically, when a movement in the reverse direction is identified, the drawdown (drawup) is nevertheless continued if the index within a horizon of \( \tau \) days contains a continuation of the downward (upward) trend. This defines a \( \tau \)-drawdown and will not be discussed further in this paper. A thorough analysis using \( \tau \)-drawdowns will be left for a future publication [Johansen, 2002].

### 3.3 Distribution of reference for drawdowns and drawup and methodology for qualifying an outlier

Our previous analysis [Johansen and Sornette, 1998, Johansen and Sornette, 2001b] of the cumulative distributions of pure drawdowns in the U.S. stock market indexes (DJIA, SP500 and NASDAQ) as well in other western and emergent market indexes as well as individual stocks and in the DM/US$ exchange rate have shown that they are well-parameterized by a minor generalisation of an exponential function called the stretched exponential

\[
N(x) = Ae^{-bx^z},
\]

where it is found empirically that \( z \approx 0.8 - 1.1 \). The case \( z = 1 \) recovers an exponential distribution which is justified in the case where the returns are sequentially uncorrelated and the marginal distributions are exponential or super-exponentially distributed \(^3\). Taking the logarithm of expression (3) is convenient and efficient for the numerical fits to the data and will be used in our subsequence analysis.

The specific functional form of eq. (3) has not particular importance as long as it fits well the bulk of the distribution of drawdowns and our goal is not to defend this parameterization more than any other one. However,

\(^3\)that is, the convergence of the left and right tails of the cumulative distribution to 0 and 1 respectively is no slower than exponential.
we need a good model of the bulk of the distribution of drawdown in order to be able to ask the question about the existence or absence of outliers. We find expression (3) convenient for this purpose as it is the simplest extension of the most natural null-hypothesis of an exponential function. However, an objective quantitative definition of an outlier based on eq. (3) not straightforward. Hence, we will use the intuitively very appealing, though not quite objective, definition that an outlier is a drawdown that does not fit in the continuation of the statistics obtained from the bulk of the distribution containing at least 95% of the distribution. Thus, the detection of an outlier will boil down to the detection of a significant deviation from the fit with (3) in the tail of the distribution. We shall see that, in general, a “break” and/or “gap” in the distribution can easily be identified around a drawdown amplitude of 10 − 15%. The label “outlier” is then reserved for drawdowns with a magnitude well above this “break” or “gap”.

3.4 Price coarse-graining algorithm

The algorithm used for calculating price coarse-grained drawdowns (drawups) is defined as follows. We first identify a local maximum (minimum) in the price and then look for a continuation of the downward (upward) trend ignoring relative movements in the reverse direction smaller than $\epsilon$. Specifically, when a movement in the reverse direction is identified, the $\epsilon$-drawdown ($\epsilon$-drawup) is nevertheless continued if the relative magnitude of this reverse move is less than $\epsilon$. Those very few drawdowns (drawups) initiated by this algorithm which end up as drawups (drawdowns), i.e., as the complementary quantity, are discarded.

The amplitude $\epsilon$ of the coarse-graining is expressed in units of the volatility $\sigma$ defined by

$$\sigma^2 = N^{-1}\sum_{i=1}^{N}[r_{i+1} - \text{E}[r]]^2. \quad (4)$$

where $r_{i+1} = \log p(t_{i+1}) - \log p(t_i)$ is the return from time $i$ to $i+1$ and $\text{E}[r]$ is its historical average.

As the large drawdowns contribute to $\sigma$, we can expect that $\epsilon$ should be taken smaller than $\sigma$ in order not to destroy the possible bursts of dependence we are looking for. We find below that the choices $\epsilon = \sigma/4$ to $\sigma/2$ give reasonable and robust results, improving on the definition of drawdowns with $\epsilon = 0$. Since we know little about the nature of the noise present in financial data, we adopt the procedure of using systematically $\epsilon = 0, \sigma/4, \sigma/2$ and $\sigma$, which allows us to test for the robustness of the results. For larger $\epsilon \geq \sigma$, the larger $\epsilon$ is, the closer to a Gaussian distribution is the distribution of $\epsilon$-drawdowns, and thus the exponent $\alpha$ of (3) obtained from a fit to the distribution of $\epsilon$-drawdowns increases above 1. This is expected since, in the limit $\epsilon$ very large, the $\epsilon$-drawdowns are nothing but the returns over long stretches of price time series, which by the central limit theorem converge in distribution to the Gaussian law in the limit of long time intervals.

4 $\epsilon$-drawdown outliers of foreign exchange markets

4.1 Historical introduction

The foreign exchange market (FX) is by far the largest market in terms of volume. Of all possible pairs of currencies, we analyze the two pairs that account for more than 80% of the total trading volume, namely the US$ against the Deutch Mark (DM) and against the Yen acknowledging the leading role of the US$, see below.

A short account of the history of the FX market after World War II is appropriate here. Under the Bretton Woods Treaty (New Hampshire, 1944), the price of gold was set at $35 per ounce. Only the dollar had a direct gold parity. The gold content of the other currencies was established only indirectly, by means of a fixed parity with the dollar. The fixed parities were intended to be permanent: a pound sterling was to be worth 2.80 against the dollar, a dollar was to be 625 lire, 360 yen, and so on. Fluctuations were to be confined to a plus or minus 1% band. These fixed parities provided the inestimable benefit of price predictability: they meant that international traders would know that dollar bills of exchange used in international import-export transactions could be expected to vary no more than plus or minus 1% over their three month or six month lifetimes. The United States was expected to buy and sell gold in settlement of international transactions. If the United States ran a payments deficit with the rest of the world, then the rest of the world might ask for settlement in gold at the rate of $35 per ounce. Due to international pressure, on December 17, 1971, the dollar was formally and officially devalued with respect to gold by 8.57%, while Germany upvalued the mark by 13.57% and Japan the yen by 16.9%. Sweden and Italy each devalued by 1% with respect to gold. New bands of fluctuation of $\pm 2.25%$ were applied to all countries. However, within less than fourteen months and despite a second devaluation of the dollar, this agreement was swept away by international money flows and, by 1973, the Bretton Wood system collapsed completely. Hence, the turmoil seen on the FX markets in the first half of 1973 is most likely related to the collapse of the Bretton Wood system and will be described and classified as “BW shocks”.

5 4-2 drawdown outliers of foreign exchange markets
Table 1: List and properties of outliers identified in the distribution of \( \epsilon \)-drawdowns for \( \epsilon = 0 \) (left part) and \( \epsilon = \sigma/4 \) (right part) of the DM/US$ exchange data. The outliers are ranked by decreasing amplitudes. The term “shock” refers to an outlier which has been triggered by an event exogenous to the market. BW Shock refers to the collapse of the Bretton Wood system. The term “bubble” embodies the idea that the corresponding outlier corresponds to a crash ending a speculative endogenous LPPS bubble. The event with a * has already been studied in previous publications. Bubbles identified by the outlier analysis presented here, i.e., outliers which have prior log-periodic bubbles not identified in previous works will be marked by †.

| \( \epsilon \) | Date   | Size   | Duration | Class     | \( \epsilon \) | Date   | Size   | Duration | Class     |
|-------|--------|--------|----------|-----------|-------|--------|--------|----------|-----------|
| 0     | 1973.485 | 11.4%  | 8 days   | BW Shock  | \sigma/4| 1973.469 | 11.9%  | 12 days  | BW Shock  |
| 0     | 1973.108 | 9.4%   | 2 days   | BW Shock  | \sigma/4| 1973.089 | 9.7%   | 7 days   | BW Shock  |
| 0     | 1985.716* | 8.4%  | 6 days   | Bubble   | \sigma/4| 1985.716* | 8.4%  | 6 days   | Bubble   |
| 0     | 1981.697† | 7.1%  | 6 days   | Bubble   | \sigma/4| 1981.688† | 8.4%  | 9 days   | Bubble   |
| 0     | 1995.169 | 6.1%   | 4 days   | No Outlier | \sigma/4| 1991.103 | 7.8%   | 13 days  | No Outlier |

4.2 The DM and the Yen against the US$

The two data sets analysed here spans from January, 4, 1971 to May, 19, 1999. The last date ensures the absence of any possible distortion due to the conversion of the DM to the Euro, which is not analyzed due to its still short existence. Figures 11 and 15 present the cumulative distribution of \( \epsilon \)-drawdowns of the DM/US$ and of the Yen/US$ respectively, with \( \epsilon \) taking the values 0, \( \sigma/4 \) and \( \sigma \), where \( \sigma \) is the volatility, defined as the standard deviation of the daily log-return eq. (4). For \( \epsilon = 0 \) and \( \sigma/4 \), respectively four and three \( \epsilon \)-drawdowns lie above a naive visual extrapolation of the rest of the bulk of the cumulative distribution. The break in the cumulative distribution lies approximately around an \( \epsilon \)-drawdown of 6.5%. In contrast, for \( \epsilon = \sigma \), all \( \epsilon \)-drawdowns seem to belong to the same distribution as to be expected.

To make this more quantitative, we fit the three cumulative distributions over the whole range of \( \epsilon \)-drawdowns excluding those five (for \( \epsilon = 0 \)) and four (for \( \epsilon = \sigma/4 \)) presumed outliers to the stretched exponential function (3). The fits as well as the parameters of the fits are shown in figures 11 and 15. These fits of the cumulative distributions of \( \epsilon \)-drawdowns for \( \epsilon = 0 \) and \( \sigma/4 \) confirm the visual impression that there are four outliers, the fifth one being only present for \( \epsilon = 0 \) is less clear-cut. The dates, sizes and durations (in number of trading days) of these outliers are given in Tables 1 and 2.

Having identified four or five outliers, the next question is whether we can classify them and pinpoint their origin. For this, the time series preceding and including the four outliers of the US$ in DM are in figure 12 for the two events of 1973.5 and 1973.1, in figure 13 for the event of 1985.7 and in figure 14 for the event of 1981.7. In addition, the US$ expressed in Swiss Franc and the S&P500 index are also shown in figure 13, to stress the similarity between the three time series. The three time series of figure 13 and the time series of figure 14 before the \( \epsilon \)-drawdown outliers are all preceding by a pattern which has been found previously to be characteristic of speculative LPPS bubbles [Johansen et al., 1999, Johansen and Sornette, 1999, Johansen et al., 1999, Sornette and Johansen, 2001]. The fits with eq. (1) of the three time series of figure 13 and the time series of figure 14 preceding the \( \epsilon \)-drawdown outliers are shown as smooth continuous lines and the parameters obtained from the fits are given in the figure captions.

According to the fits presented in figures 13 and 14, the two FX events of the DM/US$ of 1985.7 and 1981.7 qualify as the end of a speculative unsustainable bubble that became unstable and burst into an endogenous crash. The 1985.7 event is particularly noteworthy as the dollar crashed against most other currencies after reaching record-heights under the Reagan administration. In contrast, the two events of 1973.1 and 1973.5 shown in figure 12 have occurred without any of the qualifying structures described previously for the other events quantified by expression (1). Recalling the historical account of the FX market given above, it is reasonable to attribute these two events to the collapse of the Bretton Wood system. Last, the two events ranked fifth for \( \epsilon = 0 \) and \( \epsilon = \sigma/4 \), respectively do not classify as outliers but as “normal” events since they belong to the bulk of the distribution and have been included for illustration purposes only. For \( \epsilon = \sigma/4 \), we have the 1995.2 event which was caused by the crises in Mexico and subsequent devaluation of the Peso. As for the event of 1981.1, the authors have

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The US$ was vulnerable to this crisis because of the NAFTA agreement between USA, Canada and Mexico, a fact well-illustrated by the bail-out by the Clinton administration that followed the Mexican crises.
Table 2: Same as table 1 but for the Yen/US$ exchange data. The event with a ∗ has already been studied in previous publications.

| ϵ | Date      | Size | Duration | Class     | ϵ | Date      | Size | Duration | Class |
|---|-----------|------|----------|-----------|---|-----------|------|----------|-------|
| 0 | 1998.749* | 14.7%| 7 days   | Bubble    | σ/4| 1998.749* | 14.7%| 7 days   | Bubble |
| 0 | 1973.103  | 12.3%| 5 days   | BW Shock  | σ/4| 1973.097  | 12.5%| 7 days   | BW Shock |
| 0 | 1985.716  | 12.2%| 11 days  | Shock     | σ/4| 1985.716  | 12.2%| 11 days  | Shock  |
| 0 | 1990.756  | 8.7% | 12 days  | Shock     | σ/4| 1990.740  | 9.6% | 15 days  | Shock  |

not found any qualifying historical event.

Similar results are found for the Yen/US$ exchange data, for which for ϵ-drawdown outliers are identified and listed in table 2. The event ranked fourth is borderline but has again been included for illustration purposes. Figure 16 shows that the largest outlier qualifies as an endogenous crash terminating a speculative bubble phase. It is difficult not to associate this bubble with the prolonged bubbles in western stock markets fueled by heavy investments in the emergent market of Russia and Eastern Europe, which were followed by a general crash triggered by the default of the ruble and by severe economic problems in Russia. This 1998 crash ending a speculative bubble have previously been studied in [Johansen et al., 1999] and we refer to that paper for further details. The evolution of the Yen/US$ exchange rate preceding its second largest ϵ-drawdown outlier is shown in figure 17 and does not exhibit the qualifying structure described by expression (1). Recalling the historical account of the FX market given above, it is reasonable to attribute this event to the collapse of the Bretton Wood system. Figure 18 shows the Yen/US$ exchange rate preceding the 1985.7 outlier. Here, the case is ambiguous, since the date of the largest drawdown is approximately 6 month after the date of the end of the rising trend identified as the date of the maximum. As with the DM, we can observe a signature of an increasing trend starting in early 1984 and ending in early 1985, however rather smoothly as if a speculative bubble has been aborted in its main course before its full ripening. The crash does not occur until 1985.7, as we also saw in table 1, which suggests that it was triggered by the crash of the US$ against the other main currency, namely the DM. This event is thus intermediate between a genuine endogenous bubble destabilized into a crash and an exogenous event. We refer to it as “Shock” in table 2 as no LPPS can be established. The same scenario can be used to explain the borderline event dated 1990.765. It is causally linked with two large drawdowns in the Nikkei dated 1990.622 and 1990.723, see table 8, and one in the Nasdaq dated 1990.622, see table 5. Hence, it is quite likely an exogenous event caused by the collapse of the Japanese stock market and should be classified as a “Shock” in table 2.

In sum, we have identified three clear-cut cases of endogenous speculative bubbles leading to an ϵ-drawdown outlier (the 1981.7 and 1985.7 events on the DM/US$ exchange rate and the 1998.7 event on the Yen/US$ exchange rate). The three events in 1973.1 and 1973.5 can be associated with the breakdown of the Bretton Wood system (two for the DM/US$ exchange rate and one for the Yen/US$ exchange rate) and thus qualify as exogenous. The 1985.7 Yen/US$ event can be associated with the collapse of the US$ against the other major currency DM in 1985.7. The 1990.8 borderline Yen/US$ event likewise can be associated with the collapse of the Japanese real-estate market and the subsequent events on the Japanese stock market in 1990.6 and 1990.7 and on NASDAQ in 1990.6. Hence, the seven clear outliers found on the FX market (DM and Yen against US$) contains three LPPS bubbles and four shocks of which three could be linked with the collapse of the Bretton-Woods system in 1973 and one (borderline) with the collapse of the Japanese real-estate and two outliers on the Nikkei stock market as well as one on the NASDAQ. Last, the 1985.7 Yen/US$ event can be linked with the collapse of the US$ against the DM and the CHF. This means that the majority (namely four) of outliers found here on the FX market can be classified as exogenous, whereas three have prior LPPS bubbles and should be classified as endogenous. As we shall see in the next section, this situation is reversed for the stock markets, where the majority of outliers are of endogenous origin.

5 ϵ-drawdown outliers of major stock markets

In the previous section on the FX markets, we found that the selection of ϵ-drawdown outliers is robust with respect to different degrees of filtering quantified by the amplitude of ϵ in the range ϵ = 0 − σ/2. As we shall see below, the situation is not quite as simple for the major stock markets, where not only the rank of say the largest five events is permutated by changing ϵ but the duration and/or size of the ϵ-drawdowns may be drastically
| \( \epsilon \) | Date       | Size   | Duration | Class     | \( \epsilon \) | Date       | Size   | Duration | Class     |
|-------|------------|--------|----------|-----------|-------|------------|--------|----------|-----------|
| 0     | 1987.786*  | 30.7%  | 4 days   | Bubble    | \( \sigma \) | 1914.374  | 32.7%  | 64 days  | Shock     |
| 0     | 1914.579   | 28.8%  | 2 days   | Shock     | \( \sigma \) | 1987.786* | 30.7%  | 4 days   | Bubble    |
| 0     | 1929.818*  | 23.6%  | 3 days   | Bubble    | \( \sigma \) | 1929.810* | 29.5%  | 6 days   | Bubble    |
| 0     | 1933.549   | 18.6%  | 4 days   | Depression| \( \sigma \) | 1940.261  | 23.7%  | 44 days  | Shock     |

Table 3: Same as table 1 but for the DJIA stock market index. The events with a \( \star \) have already been studied in previous publications.

modified. Two good illustrations are found in table 3: 1) the crash caused by the outbreak of World-War I (WWI) is prolonged from 4 days to 64 days using a non-zero \( \epsilon \), however not adding significantly to its size; 2) a 11.4\% drop over 2 days caused by the Nazi invasion of France, Belgium, Luxembourg and the Netherlands on May, 10th 1940 is amplified to a 23.7\% drawdown over 44 days. This suggests the need for more sophisticated measures than pure size to quantify drawdowns. An alternative and perhaps more appropriate ordering could be obtained by defining a combined measure of drawdown size and “drawdown velocity” equal to drawdown size divided by crash duration. This will not be pursued further here.

5.1 The U.S. markets

Figures 19, 20 and 21 show the \( \epsilon \)-drawdown distributions for the DJIA, SP500 and NASDAQ index, respectively, obtained by the same analysis as for the FX market. The time series extend from 01/01 1900 to 17/07 2000 for the DJIA, from 29/11 1940 to 17/07 2000 for SP500 and from 05/02 1971 to 17/07 2000 for the NASDAQ.

Tables 3, 4 and 5 show the four, five and six largest events for the DJIA, SP500 and NASDAQ respectively. The three largest \( \epsilon \)-drawdown outliers both for \( \epsilon = 0 \) and \( \epsilon = \sigma \) in table 3 are associated with three well-known crashes of the past century. The fourth ranked outlier for \( \epsilon = 0 \) belongs to the Great Depression while the fourth ranked outlier for \( \epsilon = \sigma \) is a drawdown related to WWII, specifically the Nazi invasion of France, Belgium, Luxembourg and the Netherlands on May 10th 1940.

The 1987.8 and 1929.8 outliers are nothing but the famous crashes of Oct. 1929 and 1987, already analyzed in depth in [Sornette et al., 1996, Sornette and Johansen, 1997]. The price time series preceding these two crashes have been fitted with (1) over more than two years of data and over almost 8 years with a simple extension based on a second-order expansion of the mathematical theory of critical crashes [Sornette and Johansen, 1997]. These two crashes thus qualify as endogenous events following self-organized speculative LPPS bubbles.

The 1914.5, 1933.5 and 1940.3 events are characterized by preceding time series that can absolutely not be represented by formula (1). According to our classification, they are exogenous. Indeed, they are associated with external shocks, respectively the outbreak of WWI, the political maneuvering of president F. D. Roosevelt 5 upsetting the financial markets and the Nazi invasion of France, Belgium, Luxembourg and the Netherlands on May 10th 1940.

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Table 4 gives the list of the five largest \( \epsilon \)-drawdowns in the SP500 index since 1940. All outliers except the 1974.7 event have price time series preceding them which can be fitted well with formula (1) and thus qualify as endogenous crashes. All these events are marked with a \( \star \) to remind us that they have already been identified and analyzed in previous studies. The 1987.7 and 1987.8 events are two big drops associated with the crash of October 1987. The 1998.6 event is the crash associated with the ruble crisis and Russian default which was already discussed in our above analysis of the FX and has been analyzed specifically in [Johansen and Sornette, 1999]. The 1962.3 event is the (rather slow) crash ending the "tronic" boom of the early 1960s and was discovered in a blind search [Johansen, 1997, Johansen et al., 1999]. The event 1946.6 is also preceded by a log-periodic power law structure described by formula (1) and has been analyzed in [Sornette and Johansen, 2001]. [Sornette and Johansen, 2001] has in addition identified a strong accelerating log-periodic signal in the price time series ending in 1937 in a slow crash, which is not detected by our selection of \( \epsilon \)-drawdown outliers.

The 1974.7 event is a drawdown of 11.2\% which does not qualify as endogenous as it can not be fitted by formula (1). Actually, it is associated with a well-known external shock, namely the political turmoil caused by the resignation and the pardoning of president R. Nixon on August 8th and September 8th 1974.

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5Roosevelt New Deal policy included the passing of the Securities Acts of 1933 and 1934 as well as going of the Gold standard in 1933.
Table 4: Same as table 1 but for the SP500 stock market index. The events with a * have already been studied in previous publications.

| ε  | Date       | Size  | Duration | Class    | ε  | Date       | Size  | Duration | Class |
|----|------------|-------|----------|----------|----|------------|-------|----------|-------|
| 0  | 1987.784*  | 28.5% | 4 days   | Bubble   | σ/2| 1987.784*  | 28.5% | 4 days   | Bubble |
| 0  | 1962.370*  | 13.7% | 9 days   | Bubble   | σ/2| 1946.636*  | 16.2% | 9 days   | Bubble |
| 0  | 1998.649*  | 12.4% | 4 days   | Bubble   | σ/2| 1962.370*  | 13.7% | 9 days   | Bubble |
| 0  | 1987.805*  | 11.8% | 3 days   | Bubble   | σ/2| 1998.649*  | 12.4% | 4 days   | Bubble |
| 0  | 1974.721*  | 11.2% | 9 days   | Shock    | σ/2| 1987.805*  | 11.9% | 3 days   | Bubble |

Table 5: Same as table 1 but for the NASDAQ stock market index. The events with a * have already been studied in previous publications. The bubbles identified by the outlier analysis presented here, i.e., the outliers which have prior LPPS not identified in previous works are marked by †.

| ε  | Date       | Size  | Duration | Class    | ε  | Date       | Size  | Duration | Class |
|----|------------|-------|----------|----------|----|------------|-------|----------|-------|
| 0  | 2000.268*  | 25.3% | 5 days   | Bubble   | σ/2| 1987.762†  | 27.7% | 11 days  | Bubble |
| 0  | 1987.784†  | 24.6% | 5 days   | Bubble   | σ/2| 2000.268*  | 25.3% | 5 days   | Bubble |
| 0  | 1987.805†  | 17.0% | 5 days   | Bubble   | σ/2| 1998.630†  | 19.2% | 9 days   | Bubble |
| 0  | 1998.649†  | 16.6% | 4 days   | Bubble   | σ/2| 1998.724†  | 18.6% | 9 days   | Bubble |
| 0  | 2000.374†  | 14.9% | 5 days   | Bubble   | σ/2| 1987.806†  | 17%   | 5 days   | Bubble |
| 0  | 1990.622†  | 12.5% | 6 days   | Shock    | σ/2| 1978.753†  | 16.6% | 21 days  | Bubble |

Table 5 gives the characteristics of the six largest ε-drawdowns found in the NASDAQ time series. As with the SP500 index, all events but one (the 1990.6 event) have preceding price time series inflating as in a speculative bubble that can be very well fitted by formula (1) and qualify as endogenous. The crashes of 1987 and 1998 were also identified by the analysis of the DJIA and SP500 and are hence classified as “Bubbles”. The other crashes listed and preceded by a log-periodic power law bubble are the crash of April 2000 (of which we are still suffering from its aftermath [Sornette and Zhou, 2002]) which was analyzed in depth in [Johansen and Sornette, 2000] and the crash of Oct. 1978 (unpublished until now) and shown in figure 22. The value of ω = 4.5 found in the log-periodic fit of the speculative bubble preceding it is rather small compared with previous values in the range given by our previous results bracketed in (2) but still within two standard deviations from the mean.

The 1990.6 event was not preceded by any LPPS bubble as can be seen from figure 25. A comparison with table 8 shows that the Nasdaq crash of 1990 coincides with the largest ε-drawdown of the Japanese Nikkei index for ε = σ/2 as already mentioned in section 4. As documented previously [Johansen and Sornette, 1999], this ε-drawdown is part of the decay of almost 50% during the year 1990 of the Japanese stock market after it reached its all-time peak on Dec. 29th 1989. Hence, it is quite likely an reaction to the outlier event on the Japanese market and thus exogenous and should be classified as a shock.

Fig. 26 shows the NASDAQ time series since 1976 with arrows pointing the endogenous crashes detected by the log-periodic formula (1) fitted to the time series preceding them. All except the one in 1980 also show up as ε-drawdown outliers. The crash in 1980 is rank 10 and 14 for ε = 0 and ε = σ/2, respectively, and its amplitude is 11.3.

The cumulative evidence from analyzing the DJIA, SP500 and NASDAQ indexes is that all LPPS bubbles on the US-markets ending in a crash previously published by the authors [Sornette and Johansen, 2001, Johansen and Sornette, 1999, Johansen et al., 1999, Johansen et al., 1999, Johansen and Sornette, 2000] are recovered as the largest outliers of the distribution of ε-drawdowns. The sole exception is the 1937.2 crash previously published in [Sornette and Johansen, 2001], which qualifies only as an ≈ 6% pure drawdown and an ≈ 8% ε-drawdown using ε = σ. Conversely, all of the identified ε-drawdown outliers can be linked to either the crash of a LPPS bubble or to a historical event of major proportion playing the role of an external shock. To the first class belong the events in 1929, 1946, 1962, 1978, 1987, 1998 and 2000 which all had prior LPPS previously published [Sornette and Johansen, 2001, Johansen and Sornette, 1999, Johansen et al., 1999, Johansen et al., 1999, Johansen and Sornette, 2000]. To the second class belong the events of 1914 (WWI), 1933 (New Deal etc.), 1940 (Nazi invasion of France and Benelux), 1974 (resignation and pardonning of R. Nixon) and
Table 6: Same as table 1 but for the FTSE stock market index. Bubbles identified by the outlier analysis presented here, i.e., outliers which have prior LPPS not identified in previous works are marked by †.

1990 (burst of Japanese real estate bubble and the anti-bubble that followed with sharp declines) which were all due to external shocks represented by historical events.

5.2 London stock exchange

Figure 27 shows the distribution of ε-drawdowns for the FTSE index of the London stock exchange. The time series extends from 02/03 1984 to 13/07 2000. Table 6 shows that the outliers identified for \( \epsilon = 0 \) and \( \epsilon = \sigma/2 \) are identical except for a reshuffling of the ranks 3 and 4. All these four drawdown outliers are preceded by a LPPS bubble well-parametrised by (1) as shown in figures 28, 29 and 30 for the fits and in the captions for the fit parameters. Except for the 1998 outlier, the obtained values of the log-frequencies and of the exponents are in good agreement with (2) and with figures 5 and 6. For the 1998 outlier, the rather larger value \( \omega \approx 8.6 \) is probably due to the fact that the fit picks up the “dip” associated with the presence of the 1997 outlier.

5.3 Frankfurt stock exchange (DAX)

The time series for the Frankfurt stock exchange (DAX) goes from 02/01 1970 to 13/07 2000. Beside the well-known crashes of 1987 and 1998 whose preceding time series for the stock markets visited so far are well-fitted by the log-periodic power law formula (1), the ε-drawdown outliers allow us to identify another familiar date, namely the 1990.7 event already discussed for the NASDAQ. As with the NASDAQ event of the same date, no prior bubble can be qualified by a fit with (1), as shown in figure 32 and as with the NASDAQ, we conclude that the origin of this large drawdown is exogenous and most likely due to the burst of the Japanese real estate bubble and the “anti-bubble” that followed with sharp declines [Johansen and Sornette, 1999, Johansen and Sornette, 2000]. In figure 34, we see the DAX prior to the 1987 outlier and we must conclude that no LPPS is present and that the crash was of exogenous origin, specifically caused by the collapse of the U.S. markets. In figure 33, we see the DAX prior to the 1998 outlier and we must conclude that this outlier is of endogenous origin with LPPS. Furthermore, the values obtained for \( \omega \) and \( z \) are in good agreement with (2).

The analysis of the ε-drawdown outliers identifies two new dates, 1970.3 and 1989.8. Unfortunately, the data at our disposal is not sufficient to decide whether the 1970.3 event was caused by a LPPS bubble or not. However, this slow crash coincides with the so-called “liquidity crisis” of May 1970\(^6\). With respect to the 1989.8 crash, we see from figure 32 that this event was definitely not caused by a LPPS bubble. Historically, it is tempting to relate this event to the unification of Germany. In fact, the exact date of the crash is 16th October 1989 which is also the date where the Central Committee of the SED took control and forced general secretary Honecker to resign from his office as head of state and party leader. That the financial system of then West-Germany reacted in panic on these events is no great surprise. This means that with the exception of the slow crash or “liquidity crisis” of May 1970, all large drops in the DAX in the period 02/01 1970 to 13/07 2000 was either caused by an historically important exogenous shock or an endogenous speculative bubble.

5.4 Tokyo stock exchange

The analyzed time series goes from 05/01 1973 to 10/03 2000. Among the 3 events listed in table 8, we find the well-known date of the crash of 1987. In figure 37, we see that, as with the outlier on the DAX of the same

\(^6\)Between 1965 and 1968, government spending in the US increased from $118 billion to $179 billion of which half was related to defense. As a result, the inflation rate increased to nearly 6% by 1970. By May 1970, this excessive demand for money led to a liquidity crisis in which interest rates rose to new heights. The largest railways in America, Penn Central Railway, went bankrupt as a consequence. Confidence was restored when Congress passed the Securities Investor Protection Act of 1970.
Table 7: Same as table 1 but for the DAX stock market index. Bubbles identified by the outlier analysis presented here, i.e., outliers which have prior LPPS not identified in previous works are marked by †

| $\epsilon$ | Date       | Size | Duration | Class       | $\epsilon$ | Date       | Size | Duration | Class       |
|------------|------------|------|----------|-------------|------------|------------|------|----------|-------------|
| 0          | 1987.830   | 19.7%| 7 days   | Shock       | 0          | 1987.830   | 19.7%| 7 days   | Shock       |
| 0          | 1970.345   | 17.3%| 13 days  | ?           | 0          | 1970.345   | 17.3%| 13 days  | ?           |
| 0          | 1990.699   | 15.3%| 9 days   | Shock       | 0          | 1989.773   | 15.3%| 5 days   | Shock       |
| 0          | 1998.743†  | 14.9%| 4 days   | Bubble      | 0          | 1990.699   | 15.3%| 9 days   | Shock       |
| 0          | 1989.781   | 14.1%| 2 days   | Shock       | 0          | 1998.743†  | 14.9%| 4 days   | Bubble      |

Table 8: Same as table 1 for the Nikkei stock market index. The events with a * have already been studied in previous publications. See text for an explanation of the class name “Anti-Bubble.”

| $\epsilon$ | Date       | Size | Duration | Class       | $\epsilon$ | Date       | Size | Duration | Class       |
|------------|------------|------|----------|-------------|------------|------------|------|----------|-------------|
| 0          | 1987.786   | 17.8%| 4 days   | Shock       | 0          | 1990.699   | 19.8%| 12 days  | Anti-Bubble |
| 0          | 1990.622⋆   | 15.6%| 6 days   | Anti-Bubble | 0          | 1987.786   | 17.8%| 4 days   | Shock       |
| 0          | 1990.723⋆   | 15.0%| 5 days   | Anti-Bubble | 0          | 1990.622   | 15.6%| 6 days   | Anti-Bubble |

The remaining two events are Aug. and Sept. 1990. Figure 36 shows the Nikkei index from 1989.9 to 1992.3. The Nikkei index reached its all-time high on the last trading day of 1989 (29th of Dec.) and has since then followed a downward trend punctuated by decelerating oscillations with large amplitudes. In two prior publications [Johansen and Sornette, 1999, Johansen and Sornette, 2000], we have shown that the behavior of the Nikkei index since the beginning of 1990 could be understood as the symmetric to a speculative bubble, which we termed an “anti-bubble”, reflecting also imitation and herding processes leading to positive feedbacks similarly to what occurs during speculative bubbles. The difference is that the positive feedbacks reinforce the speculative bearish phase rather than a bullish phase. The degree of symmetry, after the critical time $t_c$ corresponding to the all-time high, is characterized by a power law decrease of the price (or of the logarithm of the price) during the anti-bubble as a function of time $t > t_c$ and by decelerating/expanding log-periodic oscillations. Another good example is found for the gold future prices after 1980, after its all-time high. The Russian market prior to and after its speculative peak in 1997 also constitutes a remarkable example where both bubble and anti-bubble structures appear simultaneously for the same $t_c$. This is however a rather rare occurrence, probably because accelerating markets with log-periodicity often end-up in a crash, a market rupture that thus breaks down the symmetry ($t_c − t$ for $t < t_c$ into $t − t_c$ for $t > t_c$). A remarkable similarity in the behavior of the US S&P500 index from 1996 to August 2002 and of the Japanese Nikkei index from 1985 to 1992 (11 years shift) has also been found recently, with particular emphasis on the structure of the bearish phases [Sornette and Zhou, 2002].

The 1990.6 and 1990.7 events (Aug. and Sept. 1990) occurred during the descending phase of the Nikkei decelerating log-periodic oscillation as shown in figure 8. We thus propose to see them as a consequence of the anti-bubble regime in its first stage where the power law decay and the log-periodic oscillations combine to create large drops in the descending phases of the oscillations. We thus capture this concept by the term “Anti-Bubble” in table 8.

5.5 Hong-Kong stock exchange

The analyzed Hong-Kong stock exchange time series goes from 24/11 1969 to 13/07 2000. The Hong-Kong stock market is the most volatile of the markets considered here and, for example, is twice as volatile as the NASDAQ index in the considered historical period. Using the analysis of the distribution of $\epsilon$-drawdowns, we identify 8 drawdown outliers, which are listed in table 9.

In chronological order, the 1973.2 (March and April 1973) events have a time series preceding them that is very well fitted by formula (1) (see [Sornette and Johansen, 2001]). This qualifies these two events as endogenous and part of the same “crash.”
Table 9: Same as table 1 for the Heng-Seng stock market index. The events with a * have already been studied in previous publications.

The 1973.75 event is not preceded by an accelerating log-periodic power law which puts it in the exogenous class. Indeed, it seems that it can be attributed to the Arab-Israeli “Yom Kippur” War and the subsequent Arab Oil Embargo which both occurred in Oct. 1973.

The 1974.6 event identified as an $\epsilon$-drawdown outlier with $\epsilon = \sigma/2$ is not preceded by an accelerating log-periodic power law which puts it in the exogenous class. However, no historical event has been found which this event could casually be linked to. However, as with the event just below, it could be due to the Crown colony’s special situation, e.g., its complete dependence on oil import as well as its sensitivity to political events in mainland China.

The 1982.7 event is not preceded either by an accelerating log-periodic power law and this suggests also that this event is exogenous and resulted from an external shock. Indeed, we attribute it to the failure of the negotiations between British prime minister Margaret Thatcher and Deng Xiao-Peng and Zhao Zhiyang in Beijing, in Sept. 1982. Deng rejected Thatcher’s proposal for continued British administration in Hong-Kong after 1997. The strong reaction of the market to this decision may be attributed to the large sensitivity of the Hong-Kong market to the activities of China proper, before the return of the colony to China. Since this political event coincided very much in time with this very large $\epsilon$-drawdown, we identify this political shock with the external source.

The 1987.8 event is preceded by a very nice accelerating log-periodic power law time series (see [Johansen and Sornette, 2001b]), qualifying it as an endogenous event.

The 1989.4 event is also preceded by an accelerating log-periodic power law time series (see [Sornette and Johansen, 2001]), qualifying it as an endogenous event.

The 1997.8 and 1997.99 events are preceded by a very neat accelerating log-periodic power law time series (see [Johansen and Sornette, 2001b]), qualifying them as an endogenous events.

Compared with our previously published analysis, we recover the endogenous events preceded by speculative bubbles of 1973.2, 1987.8, 1989.4 and 1997. However, the methodology using $\epsilon$-drawdown outliers misses the speculative bubbles ending in 1971.7, 1978.7, 1980.9 and 1994 (the Asian crisis) that we have previously identified based on the quality of the fits with formula (1). The reason can be seen from an examination of the price time series after these four events (see [Sornette and Johansen, 2001] and [Johansen and Sornette, 2001b]): the accelerating log-periodic power laws found to fit the time series indeed signal the end of speculative bubbles, which do not lead to real crashes but rather to large corrections or to a transition to a bearish regime lasting many months to years. Such a behavior will not qualify as a large $\epsilon$-drawdown outlier. Nevertheless, these events (1971.7, 1978.7, 1980.9 and 1994) are significant in that they signal a strong shift of regime in the Hong-Kong stock market. These examples show the limits of our analysis which misses important change of regimes where the amplitude is more to be found in the duration of the shift rather than in the fast loss of the market. The subtle definition of strong and durable regime shifts are until now eluded our attempts for a rigorous definition, except for the recognition that such strong regime shifts are detected by their log-periodic signatures, as are crashes. This is a weakness of our methodology that we hope to improve upon in the future.
Table 10: Same as table 1 for the U.S. Government T-bond. Bubbles identified by the outlier analysis presented here, i.e., outliers which have prior log-periodic bubbles not identified in previous works are marked by †.

Table 11: Same as table 1 for the Japanese Government Bonds.

6 \( \varepsilon \)-drawdown outliers of Bond markets

6.1 The U.S. T-bond market

The analyzed time series of the U.S. T-bond market extends from 27/11 1980 to 21/09 1999. Besides the date of the Oct. 1987 crash, the dates of the largest drawdowns in the market for the U.S. T-bond do not correlate with those of the largest drawdowns in the stock markets, see section 5.1, nor to those in the FX markets, see section 4. Figure 40 shows that, prior to the T-bond crash of 1987, the bond market also experienced a log-periodic bubble similar to that of the stock market, qualifying it as endogenous. With respect to the \( \varepsilon \)-drawdown of 1980.9, we do not have enough data to decide whether this crash was caused by a bubble or not. However, it is tempting to relate this event with the strong decline in the Gold price after its crash in early 1980, see table 12. The large drawdowns of 1982.7 and 1986.1 are part of a very large slow decline, as can be seen in figure 41 and should qualify as exogenous. However, we have not been able to identify a “smoking gun” for the external sources except that the 1982.7 event could again be linked to the decline in the Gold price. Last, we show an example of another log-periodic bubble in the T-bond market, see figure 42, which did not end in a large drawdown but rather in a large slow price-slide starting around 1984.4. The case of the T-bond market is thus the least convincing of all the markets examined until now in terms of the existence of endogenous speculative bubbles: only one case (1987.8) is convincingly associated with an \( \varepsilon \)-drawdown outlier and another one (1984.4) also exhibits the accelerated log-periodic signature but is associated with a change of regime rather than a larger drawdown outlier.

6.2 The Japanese Bond market

The analyzed time series of the Japanese bond market extends from 01/01 1992 to 22/03 1999. We find that one of the three large drawdown outliers in the Japanese bond-market is correlated with a similar event in the stock market, namely the crash of Aug. 1998. Figure 44 shows the price of the Japanese Government Bonds in the period from 1998.2 to 1999.2. All three drawdowns listed in table 11 are clearly visible. In the absence of any reasonable fit with formula (1), we qualify these three events as exogenous. It is probable that the driving source of the 1998.67 event was the synchronous crash on the stock market itself being associated with the Russian crisis. We also observe the occurrence of a large drawup in figure 44 preceding the two large drawdowns. Such behavior is observed in synthetic time series of the GARCH(1,1) model calibrated to this time series.

7 \( \varepsilon \)-drawdown outliers of the Gold Market

The analyzed time series of the Gold market goes from 2/1 1975 to 24/7 1998. The number of large drawdowns not following the fit with eq. (3) is surprisingly larger for Gold than for all previous markets, with up to 20 events qualifying as “outliers”, see figure 45. Table 12 lists only the four largest ones.
Table 12: Same as table 1 for the Gold market. The events with a ⋆ have already been studied in previous publications.

| ε  | Date       | Size   | Duration | Class       | ε  | Date       | Size   | Duration | Class       |
|----|------------|--------|----------|-------------|----|------------|--------|----------|-------------|
| 0  | 1980.057   | 18.2%  | 2 days   | Bubble      | σ/4| 1980.057   | 18.2%  | 2 days   | Bubble      |
| 0  | 1980.199   | 17.8%  | 4 days   | Bubble      | σ/4| 1980.199   | 17.8%  | 4 days   | Bubble      |
| 0  | 1981.057   | 14.6%  | 6 days   | Anti-bubble | σ/4| 1981.480   | 14.7%  | 11 days  | Anti-bubble |
| 0  | 1983.149   | 14.0%  | 2 days   | ?           | σ/4| 1980.113   | 14.7%  | 7 days   | Bubble      |

The two largest events are the crash of the Gold price in 1980 and a related aftershock already analyzed in [Johansen and Sornette, 1999]. A second aftershock turns up for ε = σ/4. Two events, 1981.1 for ε = 0 and 1981.5 for ε = σ/4, are related to the anti-bubble following the crash of 1980 previously published in [Johansen and Sornette, 1999], see also figure 46. With respect to the event of 1983.1 for ε = 0, figure 47 shows that its preceding time series cannot be fitted convincingly with formula (1), which qualifies it as exogenous.

8 Conclusion

By combining the statistical evidence of two methods, that is, the analysis of financial data from the point of view of detecting (i) anomalous events seen as outliers to the parameterization (3) and (ii) accelerating log-periodic power laws quantified by eq.(1) with values of ω and z satisfying (2) qualifying endogenous speculative bubble, we have provided an objective test of the hypothesis that the largest negative markets moves are special and form two distinct populations. The exogenous crashes can be attributed to extraordinary important external perturbations and/or news impacting the market. The endogenous crashes can be seen as the natural deaths of self-organized self-reinforcing speculative bubbles giving rise to specific precursory signatures in the form of log-periodic power laws accelerating super-exponentially.

The generalization of the analysis of drawdowns to drawdowns coarse-grained in amplitude (ε-drawdowns) has strengthened the evidence for the presence of anomalously large drops/corrections/crashes in the financial markets. A particularly remarkable result is that a large majority of the crashes and associated bubbles previously studied for the presence of strong LPPS have been identified with an outlier or extremely large drawdown. Furthermore, several new cases of LPPS have been identified on the basis of our generalised outlier analysis, most notably for the FTSE where all outliers were found to have LPPS. This confirm previous tests [Johansen et al., 1999] that showed that LPPS is a strong discriminator of bubbles preceding strong corrections. It also strengthen the conclusion that LPPS (which is by construction transient) is almost uniquely associated with a speculative phase announcing a strong change of regime. The analysis reported here can thus be seen as refuting possible criticism concerning data picking of a few cases. Here, we have developed a systematic approach which compares the occurrence of the largest drawdowns with the existence of log-periodicity and super-exponential growth (LPPS). Doing this, we have identified novel occurrences of log-periodicities which were unnoticed before.

Globally over all the markets analyzed, we identify 49 drawdown outliers, of which 25 are classified as endogenous, 22 as exogenous and 2 as associated with the Japanese anti-bubble. Restricting to the world market indices, we find 31 outliers, of which 19 are endogenous, 10 are exogenous and 2 are associated with the Japanese anti-bubble. For the FX market, we identify 7 outliers (3 endogenous with LPPS and 4 exogenous shocks); for the US markets, we find 12 outliers (7 endogenous with LPPS and 5 exogenous shocks); in the London stock market, we identify 4 outliers, all of them qualified endogenous with LPPS: In the German market, we find 4 outliers (1 endogenous with LPPS and 3 exogenous shocks); for the Japanese market, we find 3 outliers (1 endogenous with LPPS, and 2 shocks associated with the anti-bubble phase starting in Jan. 1990); in the Hong-Kong market, we find 8 outliers (6 endogenous LPPS and 2 exogenous shocks); In the US T-Bond market, we find 4 outliers (1 endogenous with LPPS and 3 exogenous shocks); in the Japanese bond market, we identify 3 outliers, all of them being exogenous shocks; finally in the gold market, we find 4 outliers (2 endogenous with LPPS and 2 exogenous shocks).

Two cases with exogenous shocks stand out, in the sense that these external shock were not due to political or economic news but rather to the impact of a crash in the US market and in most of the other stock markets in the world [Barro et al., 1989]. These two drawdown outliers are the 1987 drawdown outlier on the German DAX index and the 1987 drawdown outlier on the Japanese Nikkei index. They have been qualified as due to exogenous
shocks because of the absence of any LPPS in the time series preceding them, implying that these markets did not crash by an instability associated with a developing bubble. These two phenomena are reminiscent of the contagion literature (see [Claessens et al. (2001), Forbes and Rigobon (2002), Malevergne and Sornette, 2002] for reviews), which refer to manifestations of propagating crises resulting from an increase in the correlation (or linkage) across markets during turmoil periods. Our present analysis suggest new avenues for research to establish the role of the presence and absence of instabilities ripening in local markets in the propagation and amplitude of contagions.

Notwithstanding the overall positive results presented in this work, there are still cases that are unclear. For instance, as shown by the example of the T-bond bubble ending in 1984, shown in fig. 42, we can observe a strong log-periodic signal with super-exponential acceleration of the price without the occurrence of a crash ending this spell. True, there is a noticeable change of regime but it occurs rather smoothly over a long time scale. This calls for more work to characterize this change of regime or regime switch, possibly by combining techniques of Markov-switching models [Kim and Nelson, 1999] with those developed here.

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Appendix: Data

The markets and corresponding time periods analyzed in this paper are:

- The foreign exchange market (FX) in terms of the exchange between the US$ and the German Mark (DM) and the exchange between the US$ and the Japanese Yen. The data are from 04/01 1971 to 19/05 1999 for both exchange rates.

- The U.S. stock market in terms of the Dow Jones Industrial Average (DJIA), the SP500 and the NASDAQ. The data are from 01/01 1900 to 17/07 2000 for the DJIA, 29/11 1940 to 17/07 2000 for SP500 and 05/02 1971 to 17/07 2000 for the NASDAQ.

- The London stock market index FTSE. The data are from 02/03 1984 to 13/07 2000.

- The Frankfurt stock market index DAX. The data are from 02/01 1970 to 13/07 2000.

- The Tokyo stock market index Nikkei. The data are from 28/03 1986 to 13/07 2000.

- The Hong-Kong stock market index Hang-Seng. The data are from 24/11 1969 to 13/07 2000.

- T-bonds and Japanese Government Bonds (JGB). The data are from 27/11 1980 to 21/09 1999 for the T-bonds and 01/01 1992 to 22/03 1999 for the JGB.

- The Gold market. The data are from 2/1 1975 to 21/08 1998.

Many other markets, especially the emergent markets of Asia and Latin America, as well as individual stocks have been analysed previously using pure drawdowns [Johansen and Sornette, 2001b]. The main reason for not analyzing German Government Bonds is the special historical situation that Germany has been in since the unification on 3. Oct. 1990.
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Figure 1: The 23.7% $\epsilon$-drawdown in the DJIA in May 1940 caused by the Nazi invasion of France and Benelux, see table 3.

Figure 2: The 11.2% pure drawdown in the SP500 in Sep. 1974 caused by the resignation and subsequent controversial pardoning of president R. Nixon, due to the Watergate scandal, see table 4.

Figure 3: The 23.6% pure drawdown in the DJIA in Oct. 1929. The fit is eq. (1) where $A \approx 571$, $B \approx -267$, $C \approx -14.3$, $z \approx 0.45$, $t_c \approx 1930.22$, $\phi \approx 4$, $\omega \approx 7.9$.

Figure 4: The 25.3% pure drawdown in the Nasdaq in April 2000. The two fits are eq. (1) where $A \approx 9.5; 8.8$, $B \approx -1.7; -1.1$, $C \approx 0.06; 0.06$, $z \approx 0.27; 0.39$, $t_c \approx 2000.339; 2000.247$, $\phi \approx -0.14; -0.814$, $\omega \approx 7.0; 6.5$. 
Figure 5: Empirical distribution of the log-periodic angular frequency $\omega$ in eq. (1) for over thirty case studies. The fit with a Gaussian distribution gives $\omega \approx 6.36 \pm 1.55$. The smaller peak centered on $11 - 12$ suggests the existence of a second discernable harmonics at $2\omega \approx 12$.

Figure 6: Empirical distribution of the exponent $z$ of the power law in eq. (1) for over thirty case studies. The fit with a Gaussian distribution gives $\beta \approx 0.33 \pm 0.18$.

Figure 7: Empirical distribution of the norm $|z + i\omega|$, of the complex exponent $z + i\omega$, where $z$ and $\omega$ are defined in eq. (1). The fit with a Gaussian distribution gives $|z + i\omega| \approx 6.04 \pm 1.85$.

Figure 8: The anti-bubble in the Nikkei starting in 1990. The fit is eq. (1) with argument $t - t_c$, i.e., time reversed. The parameters of the fit are $A \approx 10.7$, $B \approx -0.54$, $C \approx -0.11$, $z \approx 0.47$, $t_c \approx 1989.99$, $\phi \approx -0.86$, $\omega \approx 4.9$. 
Figure 9: Histogram of the differences between the DJIA and the approximating wavelet (window of 1024 days) shown in figure 10.

Figure 10: The history of DJIA and the approximating wavelet (window of 1024 days).
Figure 11: Logarithm of the cumulative distribution of ϵ-drawdowns in the DM/US$ exchange rate using an ϵ of 0, σ/4 and σ, where σ = 0.0065 has been obtained from the data.

Figure 12: DM/US$. The exchange rate between DM and US$ in the period 1971.5 to 1974. The two BW crashes of the US$ in 1973.1 and 1973.5 are easily identified by eye.

Figure 13: The US$ bubble ending in 1985. For the DM the fit is eq. (1) where A ≈ 3.88, B ≈ −1.2, C ≈ 0.08, z ≈ 0.28, t_c ≈ 85.20, φ ≈ −1.2, ω ≈ 6.0. For illustration purposes, we also show the fit for the CHF/US$ exchange rate and that of the SP500 prior to the Oct. 1987 crash.

Figure 14: The DM/US$ bubble ending in 1981. The fit is eq. (1) where A ≈ 2.70, B ≈ −0.90, C ≈ 0.04, z ≈ 0.58, t_c ≈ 81.65, φ ≈ −1.2, ω ≈ 7.5
Figure 15: Yen/US$. Logarithm of the cumulative distribution of $\epsilon$-drawdowns in the Yen/US$ exchange rate using an $\epsilon$ of 0, $\sigma/4$ and $\sigma$, where $\sigma = 0.0065$ has been obtained from the data.

Figure 16: The Yen/US$ bubble ending in 1998. The fit is eq. (1) where $A \approx 182, B \approx -61, C \approx 2.9, z \approx 0.27, t_c \approx 98.76, \phi \approx 1.7, \omega \approx 6.8$

Figure 17: The exchange rate between Yen and US$ in the period 1972 to 1974.5. The BW crash of the US$ in 1973.1 is easily identified by eye.

Figure 18: The Yen/US$ exchange rate in the period 1983.5 to 1986. The crash of the US$ in 1985.7.1 is easily identified by eye.
Figure 19: Logarithm of the cumulative distribution of \( \epsilon \)-drawdowns in the DJIA using an \( \epsilon \) of 0, \( \sigma \) and 2\( \sigma \), where \( \sigma = 0.011 \) has been obtained from the data.

Figure 20: Logarithm of the cumulative distribution of \( \epsilon \)-drawdowns in the SP500 using an \( \epsilon \) of 0, \( \sigma/2 \) and 2\( \sigma \), where \( \sigma = 0.009 \) has been obtained from the data.

Figure 21: Logarithm of the cumulative distribution of \( \epsilon \)-drawdowns in the NASDAQ using an \( \epsilon \) of 0, \( \sigma/2 \) and 2\( \sigma \), where \( \sigma = 0.010 \) has been obtained from the data.

Figure 22: The NASDAQ bubble ending in 1978. The fit is eq. (1) where \( A \approx 154, B \approx -57, C \approx 5.8, z \approx 0.35, t_c \approx 78.71, \phi \approx 0.8, \omega \approx 4.5. \)
The fit is eq. (1) where $A \approx 460$, $B \approx -92$, $C \approx -34$, $z \approx 0.68$, $t_c \approx 87.67$, $\phi \approx -1.6$, $\omega \approx 6.1$.

The fit is eq. (1) where $A \approx 2051$, $B \approx -535$, $C \approx 39$, $z \approx 0.68$, $t_c \approx 98.64$, $\phi \approx -0.4$, $\omega \approx 6.0$. 

Figure 23: The NASDAQ bubble ending in 1987.

Figure 24: The NASDAQ bubble ending in 1998.
Figure 25: NASDAQ in the period from 1988 to 1991.5. The crash in 1990.6 is clearly visible.

Figure 26: NASDAQ time series since 1976 with arrows pointing the endogenous crashes detected by the log-periodic formula (1) fitted to the time series preceding them. All except the one in 1980 also show up as $\epsilon$-drawdown outliers.

Figure 27: Logarithm of the cumulative distribution of $\epsilon$-drawdowns in the FTSE using an $\epsilon$ of 0, $\sigma/2$ and $2\sigma$, where $\sigma = 0.010$ has been obtained from the data.

Figure 28: The FTSE bubble ending in 1997. The fit is eq. (1) where $A \approx 8212$, $B \approx -4108$, $C \approx 74$, $z \approx 0.18$, $t_c \approx 97.93$, $\phi \approx -1.2$, $\omega \approx 7.6$. 
Figure 29: The FTSE bubble ending in 1987. The fit is eq. (1) where $A \approx 2884$, $B \approx -1309$, $C \approx 106$, $z \approx 0.30$, $t_c \approx 87.55$, $\phi \approx 1.1$, $\omega \approx 5.1$.

Figure 30: The FTSE bubble ending in 1998. The fit is eq. (1) where $A \approx 13122$, $B \approx -7847$, $C \approx 136$, $z \approx 0.18$, $t_c \approx 98.99$, $\phi \approx 3.6$, $\omega \approx 8.6$. Note that the value for $\omega$ is rather high compared with the distribution shown in figure 5. This is due to the outlier shown in figure 28 since the fit picks up this “dip” in the time series.
Figure 31: Logarithm of the cumulative distribution of $\epsilon$-drawdowns in the DAX using an $\epsilon$ of 0, $\sigma/2$ and $\sigma$, where $\sigma = 0.011$ has been obtained from the data.

Figure 32: DAX in the period from 1989 to 1991. The crashes in 1989.8 and 1990.7 are clearly visible.

Figure 33: The DAX bubble ending in 1998. The fit is eq. (1) where $A \approx 8343$, $B \approx -4553$, $C \approx 257$, $z \approx 0.28$, $t_\epsilon \approx 98.61$, $\phi \approx 0.2$, $\omega \approx 5.7$.

Figure 34: DAX in the period from 1986.5 to 1988. The crash of Oct. 1987 is clearly visible.
Figure 35: Logarithm of the cumulative distribution of $\epsilon$-drawdowns in the Nikkei using an $\epsilon$ of 0, $\sigma/2$ and $\sigma$, where $\sigma = 0.015$ has been obtained from the data.

Figure 36: Nikkei in the period from 1989.8 to 1992.3. The crashes of Aug. and Sept. 1990 are clearly visible.

Figure 37: Nikkei in the period from 1987.1 to 1988. The crashes of Oct. 1987 is clearly visible.
Figure 38: Logarithm of the cumulative distribution of $\epsilon$-drawdowns in the Hang-Seng using an $\epsilon$ of 0, $\sigma/2$ and $\sigma$, where $\sigma = 0.021$ has been obtained from the data.

Figure 39: Logarithm of the cumulative distribution of $\epsilon$-drawdowns in the price of the T-bond using an $\epsilon$ of 0, $\sigma/4$ and $\sigma$, where $\sigma = 0.012$ has been obtained from the data.

Figure 40: The T-bond bubble ending in 1987. The fit is eq. (1) where $A \approx 14.8, B \approx -7.3, C \approx -0.34, z \approx 0.16, t_s \approx 87.83, \phi \approx -1.5, \omega \approx 5.5$.

Figure 41: The price of the T-bond in the period from 1982.5 to 1986.3. The large drawdowns of Oct. 1982 and Feb. 1986 are clearly visible.
Figure 42: The T-bond bubble ending with “a soft landing” in 1984. The fit is eq. (1) where $A \approx 14.5$, $B \approx -3.5$, $C \approx 0.36$, $z \approx 0.33$, $t_c \approx 84.4$, $\phi \approx 1.4$, $\omega \approx 4.6$.

Figure 43: Logarithm of the cumulative distribution of $\epsilon$-drawdowns in the price of the Japanese Government Bond using an $\epsilon$ of 0, $\sigma/4$ and $\sigma$, where $\sigma = 0.013$ has been obtained from the data.

Figure 44: The price of the Japanese Government Bond in the period from 1998.2 to 1999.2. The large drawdowns of early 1999 are clearly visible.

Figure 45: Logarithm of the cumulative distribution of $\epsilon$-drawdowns in the price of Gold using an $\epsilon$ of 0, $\sigma/4$ and $\sigma$, where $\sigma = 0.013$ has been obtained from the data.
Figure 46: The price of Gold in the period from 1980.0 to 1981.6. The large drawdowns of early 1980 as well as early and mid-1981 are clearly visible.

Figure 47: The price of Gold in the period from 1982.5 to 1983.3. The large drawdown of early 1983 is clearly visible.
