The suppression of direct collapse black hole formation by soft X-ray irradiation

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12 November 2014

ABSTRACT

The origins of supermassive black holes (SMBHs) in galactic nuclei is one of the major unsolved problems in astrophysics. One hypothesis is that they grew from \( \gtrsim 10^5 \, M_\odot \) black holes that formed in the ‘direct collapse’ of massive gas clouds that have low concentrations of both metals and molecular hydrogen (H\(_2\)). Such clouds could form in the early (\( z \gtrsim 10 \)) Universe if pre-galactic gas is irradiated by H\(_2\)-photodissociating, far-ultraviolet (FUV) light from a nearby star-forming galaxy. The key uncertainties with this scenario are (1) how strong the FUV flux must be to sufficiently suppress the H\(_2\) abundance to prevent fragmentation and ordinary star formation; and (2) whether the requisite conditions arise frequently enough in nature to account for the observed number density of SMBHs (luminous quasars) at high redshifts. In this work, we re-examine the critical FUV flux \( J_{\text{crit}} \) that is required to keep H\(_2\) photodissociated and lead to direct collapse. We show that \( J_{\text{crit}} \) could be much higher than previously believed if the same FUV sources also produce X-rays, which can work to offset H\(_2\) photodissociation by increasing the ionization fraction and promoting H\(_2\) formation via electron-catalyzed reactions. We stress that soft (\( \sim 1 \, \text{keV} \)) X-rays are far more effective at promoting H\(_2\) formation than hard (\( \sim 10 \, \text{keV} \)) X-rays. Further, we estimate how much soft X-rays can suppress the number density of direct-collapse black holes compared to previous calculations. We find that, even for conservative sets of assumptions, if \( J_{\text{crit}} \) is higher than \( 400 \) – \( 1000 \) then direct collapse would occur too rarely to explain the observed abundance of \( z > 6 \) quasars.

Key words: black hole physics, cosmology: theory, cosmology: dark ages, reionization, first stars, galaxies: formation, quasars: supermassive black holes

1 INTRODUCTION

Most nearby massive galaxies harbour a supermassive black hole (SMBH) in their nuclei. Empirical correlations between the masses of SMBHs and properties of their host galaxies suggest that SMBHs may play a key role in galaxy evolution, possibly during stages shining as luminous quasars (e.g. Magorrian et al. 1998; Ferrarese & Merritt 2000; Marconi & Hunt 2003; Hopkins et al. 2007; Kormendy & Ho 2013). Despite their apparent ubiquity and importance, when and how these cosmic behemoths formed remain poorly understood. Observations of luminous quasars at \( z \gtrsim 6 \) reveal that SMBHs with masses of \( \gtrsim 10^9 \, M_\odot \) were already in place 900 Myr after the Big Bang, and place strong constraints on possible formation scenarios (Fan et al. 2001; Fan 2006; Willott et al. 2011; Mortlock et al. 2011; Venemans et al. 2013; Banados et al. 2014).

One possibility is that the earliest SMBHs grew from ‘seed’ \( \sim 100 \, M_\odot \) BHs left behind by the first generation of stars (Population III or ‘Pop III’ stars) from \( z \gtrsim 30 \) via rapid gas accretion, aided by hierarchical BH mergers (Haiman & Loeb 2001; Madau & Rees 2001; Volonteri, Haardt & Madau 2003; Li et al. 2007; Tanaka, Perna & Haiman 2012; Tanaka, Li & Haiman 2013). To form the \( z > 6 \) quasar SMBHs, these seeds must

\[ \text{BH mergers play a secondary or minor role in BH growth.} \]

The gravitational recoil effect, while unlikely to prevent SMBH formation, suppresses the efficacy of mergers in assembling more massive BHs (Volonteri & Rees 2006; Tanaka & Haiman 2009).

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have accreted gas at a mean rate comparable to the Ed
ddington limit; a key uncertainty is whether Pop III remnant
BHs could have maintained such rates despite negative
radiative feedback in the shallow gravitational potentials
of their host protogalaxies (Alvarez, Wise & Abel 2003;
Milosavljević, Couch & Bromm 2009).

Alternatively, SMBHs could have originated as
\( \gtrsim 10^{-6} \, M_\odot \) BHs that formed via the `direct collapse` of
gas clouds with low abundances of metals and molecular hy-
drogen (H$_2$) (e.g., Loeb & Rasio 1994; Oh & Haiman 2002;
Bromm & Loeb 2003; Koushiappas, Bullock & Dekel 2004;
Begelman, Volonteri & Rees 2006; Lodato & Natarajan
2006). Theoretically, direct collapse can occur in primordial
gas clouds in massive dark-matter halos with virial tempera-
tures of \( \gtrsim 10^4 \) K, if H$_2$-line cooling is suppressed. The most
widely studied H$_2$-suppressing mechanism in this context is
photodissociation by strong far-ultraviolet (FUV) radiation in
the Lyman-Werner (LW) band \( (11.2-13.6 \, eV) \). In
such primordial gas without H$_2$ molecules, the gas loses
thermal energy primarily via atomic hydrogen transitions
( Lyα, two-photon, and H$^-$ free-bound emissions) and
collapses while maintaining a temperature of \( \sim 8000 \, K \)
(e.g., Omukai 2001). Recent numerical simulations have
suggested that the gas can collapse monolithically avoiding
the major-episode of fragmentation during the isothermal
phase (Shang, Bryan & Haiman 2010; Regan & Hachn
2009a,b; Latif et al. 2013; Inayoshi, Omukai & Tasker 2013;
Becerra et al. 2014).

After the collapse phase, a protostar with a
mass of \( \sim 1 \, M_\odot \) forms at the centre of the cloud and grows via
rapid gas accretion at the rate of \( \gtrsim 1 \, M_\odot \, yr^{-1} \) (Inayoshi, Omukai & Tasker 2014). The
protostar growing at such a high accretion rate evolves to a
supernova massive star within its lifetime
\( \sim 1 \, Myr \) overcoming the radiative feedback and
pulsation-driven mass loss (Hosokawa, Omukai & Yorke
2012; Inayoshi, Hosokawa & Omukai 2013; Hosokawa et al
2013; Schleicher et al. 2013) and finally forms a massive seed
BH by gravitational collapse due to general relativistic in-
stability (Chandrasekhar 1964; Zeldovich & Novikov 1971;
Shibata & Shapiro 2002). Compared to Pop III seed BHs,
the products of direct collapse (`direct collapse black holes,` henceforth
DCBH) require a somewhat lower (by \( \sim 10 \) to
20 per cent) mean accretion rate to grow to \( \gtrsim 10^9 \, M_\odot \) by
\( z \sim 6-7 \) (although the rate is still comparable to the Ed-
ddington limit; see e.g. Tanaka 2014 and references therein).

The most crucial question in the above scenario is how
large the LW intensity must be to keep H$_2$ dissociated. This
critical value, commonly called \( J_{\text{crit}} \), has been discussed
by many authors (Omukai 2001; Bromm & Loeb 2003;
Shang, Bryan & Haiman 2010; Inayoshi & Omukai 2011;
Wolcott-Green, Haiman & Bryan 2011; Latif et al. 2014b).
If the irradiating source has a thermal spectrum with a
brightness temperature \( T_* \), \( J_{\text{crit}} \approx O(10) \) (in units of \( 10^{-21} \)
\, erg \, s$^{-1}$ \, cm$^{-2}$ \, sr$^{-1}$ \, Hz$^{-1}$) for \( T_* = 10^4 \, K \) and \( J_{\text{crit}} \approx O(10^3) \)
for \( T_* = 10^5 \, K \). For example, Sugimura, Omukai & Inoue
(2014) recently obtained \( J_{\text{crit}} \approx 1.4 \times 10^5 \) and found that
this value does not change significantly between realistic
UV spectra of star-forming, low-metallicity galaxies. Several
studies have estimated the probability of forming DCBHs
via FUV fluxes \( J_{\text{LW}} > J_{\text{crit}} \), using Monte Carlo calcu-
lations (Dijkstra et al. 2008; Dijkstra, Ferrara & Mesinger
2014; hereafter, DFM14) and semi-analytic methods coupled
with N-body simulations (Agarwal et al. 2012). If \( J_{\text{crit}} \gtrsim 10^5 \),
then the expected number density of DCBHs is com-
parable to or lower than that of SMBHs with \( \gtrsim 10^9 \, M_\odot \) at
\( z \gtrsim 6 \) (\( \sim 1 \) comoving Gpc$^{-3}$).

In this paper, we discuss the effect of ionization on
DCBH formation. Specifically, we consider the role of X-
rays from the same star-forming galaxies that are the
putative sources of H$_2$-dissociating FUV radiation. X-rays can
increase the hydrogen ionization fraction, promoting H$_2$
formation through the electron-catalyzed reactions
\begin{equation}
H + e^- \rightarrow e^- + \gamma, \\
H^- + e^- \rightarrow H_2 + e^-.
\end{equation}
By working to increase the H$_2$ fraction, X-rays work against
FUV photons and thus increase the effective value of \( J_{\text{crit}} \)—
that is, we should generally expect
\begin{equation}
J_{\text{crit}}^{(\text{UV+X})} > J_{\text{crit}}^{(\text{UV only})}
\end{equation}
if the irradiating galaxies produce both FUV and X-ray photons
(Inayoshi & Omukai 2011). As we show in this work,
soft (\( \sim 1 \) keV) X-rays are far more effective than hard
(\( \sim 10 \) keV) X-rays at promoting H$_2$ formation and increasing
\( J_{\text{crit}} \). Note that because higher-energy X-rays have longer
mean free paths, the dominant source of soft X-rays is ex-
pected to be nearby galaxies, whereas the hard X-ray flux
is dominated by a nearly isotropic cosmic background.
In other words, strong sources (star-forming galaxies) of H$_2$
-dissociating FUV photons are likely to also be strong sources
of H$_2$-forming soft X-rays. We show that if \( J_{\text{crit}}^{(\text{UV only})} \approx 10^5 \)
as suggested by most recent studies, then irradiation by
soft X-rays raises \( J_{\text{crit}} \) by several orders of magnitude: i.e. \( J_{\text{crit}}^{(\text{UV+X})} \gg J_{\text{crit}}^{(\text{UV only})} \).

We emphasize that the effective value of \( J_{\text{crit}} \) is a critical
quantity to which the abundance of potential DCBH formation
sites is extremely sensitive—for example, a difference in
a factor of 10 in \( J_{\text{crit}} \) can mean the difference of four orders of
magnitude in the number density of DCBHs (DFM14). We
follow DFM14 to estimate the maximum value of \( J_{\text{crit}} \) (ac-
tording to the effect of the X-rays that should accompany
the FUV radiation) that can explain the observed number density
of \( z > 6 \) quasars.

The rest of this paper is organised as follows. We de-
scribe in §2 our calculations of the critical LW intensity,
in particular our treatment of X-ray ionization. In §3 we
quantify the relation between LW and X-ray radiation from
star-forming galaxies in the early Universe, and arrive at a
relationship between the UV-only and X-ray-corrected values
of \( J_{\text{crit}} \) (\( J_{\text{crit}}^{(\text{UV only})} \) and \( J_{\text{crit}}^{(\text{UV+X})} \), respectively). In §4
we apply these results to arrive at the X-ray-corrected proba-
bility that an atomic-cooling halo can form a DCBH. We
estimate the number density of DCBHs as a function of
\( J_{\text{crit}} \) and redshift. Finally, we present our conclusions in
§5 and discuss the potential role of 21cm signatures and other
observations in placing empirical constraints on FUV-aided
DCBH formation.
2 EVALUATION OF $J_{\text{crit}}$

2.1 Thermal and chemical evolution

We consider the thermal evolution of primordial gas in a massive halo with a virial temperature of $\gtrsim 10^4$ K that is exposed to FUV radiation and X-rays from external sources. During the collapse of the self-gravitating cloud, its density profile approaches a self-similar form (Penston 1969; Larson 1964), consisting of a central core and an envelope with $\rho \propto r^{-2}$. We here adopt a one-zone model which approximates all the physical quantities to be uniform inside the central core, and solve for their temporal evolution (e.g., Omukai 2001).

The density of the central core increases on the free-fall timescale $t_{\text{ff}} = \sqrt{3\pi / 32G\rho}$ as

$$\frac{d\rho}{dt} = \frac{\rho}{t_{\text{ff}}}. \quad (4)$$

The energy equation of the gas is given by

$$\frac{de}{dt} = -p \frac{d}{dt} \left( \frac{1}{\rho} \right) - \frac{\Lambda - \Gamma_X}{\rho}, \quad (5)$$

where $e$ is the specific internal energy, $p$ the gas pressure, $\Lambda$ the cooling rate, and $\Gamma_X$ the heating rate due to the external X-rays. We consider the radiative cooling by atomic and molecular hydrogen species, as well as the cooling/heating associated with chemical reactions. As the collapse proceeds and the gas grows denser, the intensity of external radiation that reaches the central core is reduced. We estimate the optical depth by assuming the size of the central core to be the half of the Jeans length $\lambda_J$. At the collapsing central core, the column density of the $i$-th species is given by

$$N_i = n(i) \frac{\lambda_J}{2}, \quad (6)$$

where $n(i)$ is the number density of the species.

We solve the primordial chemical reactions among the following 9 species: H, H$_2$, $e^-$, H$^+$, H$^+_2$, H$^-$, He, He$^+$, and He$^{++}$. The chemical reactions we consider are the same as in Omukai (2001) but we have updated some reaction rate coefficients (Inayoshi, Omukai & Tasker 2014). We include the photoionization of H and He by X-rays.

The one-zone calculations start at $n = 0.1$ cm$^{-3}$ and $T = 160$ K, which corresponds to the gas in a halo virializing at $z_{\text{vir}} \simeq 10$ (Omukai, Schneider & Haiman 2008). We set the initial abundances of electrons, H$_2$, and He to $x_e = 10^{-4}$, $x_{H_2} = 10^{-6}$, and $x_{He} = 0.08$, respectively. These initial conditions are the same as in Inayoshi & Omukai (2011).

2.2 External FUV and X-ray radiation

We now discuss our treatment of FUV and X-ray radiation, and in particular how X-rays affect the effective value of $J_{\text{crit}}$. Below, we use the symbol $J_{\text{crit},0}$ to denote the value of $J_{\text{crit}}$ calculated without considering the effects of X-ray ionizations.

2.2.1 FUV radiation

We assume the FUV radiation to have a diluted thermal spectrum, $J_{\text{UV}}(\nu) \propto B_\nu(T_*), \quad (8)$

with the brightness temperature of $T_* = 10^4$ K. We normalise the intensity of the FUV radiation at the Lyman limit ($\nu_L = 13.6$ eV), and write this in conventional units: $J_{\text{UV},21} = J_{\text{UV}}(\nu_L) / (10^{-21} \text{erg s}^{-1} \text{cm}^{-2} \text{sr}^{-1} \text{Hz}^{-1})$. We also consider the H$_2$ self-shielding effect against external FUV radiation (Wolcott-Green, Haiman & Bruni 2011).

Let us briefly address the dependence of the critical FUV intensity on the FUV spectrum and the X-ray intensity. As many authors have shown (e.g., Omukai 2001; Inayoshi & Omukai 2011), the value of $J_{\text{crit},0}$ is well-approximated by a functional form $f(J_{\text{UV},T_*,x_0}, \text{i.e. it scales linearly with the electron fraction and the normalisation depends on the FUV spectrum. The primary role of the X-rays is to raise } J_{\text{crit}} \text{by increasing the equilibrium value of } x_0. \text{That is, } J_{\text{crit}}/J_{\text{crit},0} \text{can be approximated by the ratio of the values of } x_0 \text{obtained with and without X-ray ionizations. Thus, } J_{\text{crit}}/J_{\text{crit},0} \text{depends most sensitively on the X-ray intensity, and depends weakly on the choice of FUV spectrum.}$

2.2.2 X-rays

We assume that the X-ray mean intensity can be represented by a power-law spectrum

$$J_X(\nu) = J_{\text{X},21} \times 10^{-21} \left( \frac{\nu}{\nu_0} \right)^{-\alpha} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ sr}^{-1} \text{ Hz}^{-1}, \quad (7)$$

where $h\nu_0 = 1$ keV and $\alpha = 1.5$ (Glover & Brand 2003 and references therein). The ionization rates of H and He by direct X-ray photons are given by

$$\zeta_{X,p} = \int_{\nu}^{\nu_{\text{max}}} \frac{4\pi J_X(\nu)}{h\nu} e^{-\tau_{x_0}(\nu)} \tau_x(\nu) d\nu \quad (i = \text{H, He}), \quad (8)$$

$$\tau_x = N_{\text{H}_2} \sigma_{\text{H}_2}(\nu) + N_{\text{He}} \sigma_{\text{He}}(\nu), \quad (9)$$

where $\sigma_{\text{H}_2}(\nu)$ and $\sigma_{\text{He}}(\nu)$ are the cross sections of H and He to the ionizing photons (Verner et al. 1996; Yan, Sadeghpour & Dalgarno 1998, respectively) and $N_{\text{H}_2}$ and $N_{\text{He}}$ are the column densities of those species. Since the emitted electrons have large kinetic energy, they can ionise the surrounding gas (secondary ionization). We also estimate the secondary ionization and X-ray heating rates using the formulae in Shull & van Steenberg (1985), which are valid for X-ray photons with energies $\gg 0.1$ keV.

In this calculation, we set the maximum energy of the X-rays to $h\nu_{\text{max}} = 10$ keV. The following results do not depend on the choice of $\nu_{\text{max}}$ as long as $h\nu_{\text{max}} \gg 10$ keV.

The X-ray minimum energy is the more important quantity. The comoving mean free path of a X-ray photon with $h\nu$ can be written as

$$\lambda_X \simeq 9.1 \bar{x}_H^{-1} \left( \frac{1 + z}{11} \right)^{-2} \left( \frac{h\nu}{0.3 \text{ keV}} \right)^3 \text{ cMpc} \quad (10)$$

where $\bar{x}_H$ is the mean neutral fraction (Furlanetto, Oh & Briggs 2006). From the condition that $\lambda_X$ is longer than the Hubble horizon, hard X-ray

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\[ \text{Suppression of DCBH formation by soft X-rays} \hspace{0.5cm} 3 \]

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3 Previously, Inayoshi & Omukai (2011) adopted the cross sections by Rybicki & Lightman (1979) (for H) and Osterbrock (1989) (for He). These works overestimate the cross sections at X-ray energies, and hence in this work we adopt the more recent cross sections referenced above.
The ratio $J_{\text{crit}}/J_{\text{crit,0}}$, the relative increase in the requisite FUV intensity to photodissociate H$_2$ molecules inside an atomic-FUV cooling halo, when X-rays are added to the radiating spectrum. The thick (red) line shows the case for $h\nu_{\text{min}} = 1$ keV, and the thin (blue) line shows the case for $h\nu_{\text{min}} = 2$ keV. (The result for $h\nu_{\text{min}} = 0.5$ keV is nearly identical to that for $h\nu_{\text{min}} = 1$ keV and we do not show it here.) The rise in $J_{\text{crit}}/J_{\text{crit,0}}$ as a function of $J_{X,21}$ is well-described by the fitting function $J_{\text{crit}} = J_{\text{crit,0}}(1 + J_{X,21}/a)^b$, $(a, b) = (9.3 \times 10^{-4}, 0.57)$ for $h\nu_{\text{min}} \leq 1$ keV and $(a, b) = (1.1 \times 10^{-2}, 0.57)$ for $h\nu_{\text{min}} = 2$ keV. At $J_{X,21} \gtrsim 0.1$ ($\gtrsim 1$) for $h\nu_{\text{min}} \leq 1$ keV (2 keV), $J_{\text{crit}}/J_{\text{crit,0}}$ deviates from these fits and saturates at a value of $\approx 30$. A colour version of this figure is available in the online version.

2.3 X-ray enhancement of the critical LW flux

In Fig. 1 we present the ratio of $J_{\text{crit}}/J_{\text{crit,0}}$ (FUV+X to FUV only) as a function of the X-ray intensity $J_{X,21}$ for $h\nu_{\text{min}} \leq 1$ keV (red) and 2 keV (blue). For weak X-ray intensities, the value of $J_{\text{crit}}/J_{\text{crit,0}}$ converges to a constant value of $\approx 1$. For both cases, the critical LW intensity increases with the X-ray intensity at $J_{X,21} \gtrsim 10^{-3}$ ($\gtrsim 10^{-2}$) for $h\nu_{\text{min}} \leq 1$ keV ($h\nu_{\text{min}} = 2$ keV). This is because the electron fraction rises through X-ray ionization and thus the H$_2$ formation rate through the reactions in equation 11 and 2 increases (see also Inayoshi & Omukai 2011, in details). For $J_{X,21} \lesssim 0.1$ ($\lesssim 1$), the functional form of $J_{\text{crit}}$ can be fitted as $J_{\text{crit}}/J_{\text{crit,0}} = (1 + J_{X,21}/a)^b$, where $(a, b) = (9.3 \times 10^{-4}, 0.57)$ for $h\nu_{\text{min}} \leq 1$ keV and $(a, b) = (1.1 \times 10^{-2}, 0.57)$ for $h\nu_{\text{min}} = 2$ keV. With $J_{X,21} \gtrsim 0.1$ ($\gtrsim 1$) for $h\nu_{\text{min}} \leq 1$ keV ($h\nu_{\text{min}} = 2$ keV), the value of $J_{\text{crit}}/J_{\text{crit,0}}$ deviates from these fits and approaches a constant value $\approx 30$. At these high X-ray intensities, the electron fraction increases up to $x_e \sim 0.1$ and ionization rate, which is dominated by secondary ionizations, saturates (Shull & van Steenberg 1993). If $J_{\text{crit,0}} \approx 10^3$ (e.g., Sugimura, Omukai & Inoue 2014), then $J_{\text{crit}}$ has a maximum value of $\approx 3 \times 10^4$. Such a high value would suppress DCBH formation to levels where DCBHs alone cannot account for the observed abundance of $z \gtrsim 6$ quasars (see §1).

We study the dependence of the critical LW intensity $J_{\text{crit}}$ on the choice of $h\nu_{\text{min}}$. In the case of $h\nu_{\text{min}} = 2$ keV, the critical X-ray intensity is larger by one order of magnitude than that for our fiducial model ($h\nu_{\text{min}} = 1$ keV) because the cross sections of H and He against photons with energies $\gtrsim 2$ keV is small and thus the X-ray ionization is also less important.

Recently, Latif et al. (2014a) studied the effect of X-rays on DCBH formation assumed hard X-rays ($h\nu_{\text{min}} = 2$ keV) considering the X-ray background radiation. In their result, the critical FUV intensity begins to increase at $J_{X,21} \gtrsim 10^{-2}$ and boosts by a factor of $\approx 2$ at $J_{X,21} \gtrsim 10^{-1}$. As seen the blue line in Fig. 1 our result is $J_{\text{crit}}/J_{\text{crit,0}} \approx 4$ for $J_{X,21} \gtrsim 10^{-1}$, which is larger by a factor of $\approx 2$ than their three-dimensional simulation. A minimum energy of $h\nu_{\text{min}} = 2$ keV is a reasonable assumption for the cosmic X-ray background, as soft X-rays with 0.5 (1) keV would be absorbed at separations of $\approx 40$ (300) Mpc in the intergalactic medium. However, the nearby halos that putatively enable DCBH formation through large FUV fluxes should also irradiate their immediate environments with soft ($\sim 1$ keV) X-rays. We emphasize the key point that soft X-rays are far more effective at promoting H$_2$ formation through electron-catalyzed reactions.

On the other hand, the result for $h\nu_{\text{min}} = 0.5$ keV does not change from our fiducial model ($h\nu_{\text{min}} = 1$ keV). The value of $J_{\text{crit}}$ is determined by the electron fraction at $n_H \sim 10^5$ cm$^{-3}$ and $T \approx 8000$ K (Inayoshi & Omukai 2011). The photons with $h\nu_{\text{min}} \lesssim 1$ keV can ionise the gas easily and thus are absorbed at $n_H < 10^5$ cm$^{-3}$. We conclude that the value of $J_{\text{crit}}$ is sensitive to the intensity of X-ray photons at $\approx 1$ keV but not to that of softer X-rays at energies $\lesssim 1$ keV.

We here assume a simple power-law spectrum with $J_X(\nu) \propto \nu^{-1.5}$. However, the spectral energy distributions of observed high-mass X-ray binaries (HMXBs) are more complex (e.g., Gierliński et al. 1997, Gierliński et al. 1999). The spectral shapes are characterised by a power-law with $\alpha \approx 1.6$–1.8 (low-hard state) and by a bright thermal component with a peak temperature of $\approx 0.5$ keV having a soft power-law tail with $\alpha \gtrsim 2$ (hard-soft state). We note that the resulting value of $J_{\text{crit}}/J_{\text{crit,0}}$ for $h\nu_{\text{min}} \lesssim 1$ keV depends very weakly on the choice of the X-ray power-law index in the range 1.5 $\leq \alpha \leq 2.0$ because ionization soft X-rays ($\sim 1$ keV) increases $J_{\text{crit}}$ significantly. In any case, our choice $\alpha = 1.5$ is conservative, as the soft X-ray intensity at $\sim 1$ keV increases for steeper power-laws (see §1).

The X-ray spectra of HMXBs in high-z galaxies could have an excess due to the thermal emissions from the power-law component at $\sim 1$–10 keV (Fragos et al. 2013). We note that the value of $J_{\text{crit}}$ begins to increase for X-ray intensities smaller than $J_{X,21} \sim 10^{-3}$ in a case with thermal soft X-ray components.
3 LW AND X-RAY SOURCES IN THE EARLY UNIVERSE

Having laid out above the general effect of X-ray fluxes on the quantity $J_{\text{SFR}}$, we now turn to the discussion of X-ray and LW sources in the $z \gtrsim 10$ Universe.

3.1 FUV and X-ray intensities

3.1.1 X-ray flux

We first estimate the X-ray intensities from thestar-forming galaxies in the $z \sim 20 – 10$ Universe. According to the most recent cosmological simulations, Pop III stars could be born as massive stars with $\sim 10 – 100 M_\odot$ (Hosokawa et al. 2013; Stacy, Greif & Bromm 2012; Hirano et al. 2014). Moreover, the efficiency of forming binary systems could be as high as $\sim 50\%$ (Stacy & Bromm 2013; Susa, Hasegawa & Tominaga 2014). Thus, we can consider HMXBs as X-ray sources in the early Universe.

From observations of local starburst galaxies, we can obtain a good correlation between the X-ray luminosities and their star formation rate (SFR). The X-ray emission is dominated by HMXBs, which is considered to be good tracers of the SFR because of their short lifetime. The bolometric X-ray luminosity ($2–10\text{ keV}$) is given by

$$L_{2–10\text{ keV}} \simeq 6.4 \times 10^{39} \left( \frac{\text{SFR}}{M_\odot \text{ yr}^{-1}} \right) \text{ erg s}^{-1}, \quad (11)$$

(e.g. Glover & Brand 2003). Many observations in various X-ray bands also have suggested the same $L_X$–SFR relation within a factor of $2 – 3$ (e.g. Grimm, Gilfanov & Sunyaev 2003; Lehmer et al. 2010; Mineo, Gilfanov & Sunyaev 2012). Furthermore, the dispersion of $L_X$/SFR is at most $\sim 0.4$ dex (Mineo, Gilfanov & Sunyaev 2012). Assuming a simple power-law spectrum with $L_X(\nu) \propto \nu^{\alpha-1}$, the luminosity density at $1\text{ keV}$ is given by

$$L_X(\nu_0) = 3.4 \times 10^{22} \left( \frac{\text{SFR}}{M_\odot \text{ yr}^{-1}} \right) \text{ erg s}^{-1} \text{ Hz}^{-1}. \quad (12)$$

We note that our choice of $\alpha = 1.5$ is conservative because $L_X(\nu_0)$ would be larger for $\alpha > 1.5$ by a factor of 1.5 for $\alpha = 1.8$ and by 2.0 for $\alpha = 2.0$. We can estimate the X-ray flux at $1\text{ keV}$ (in units of $10^{-21} \text{ erg s}^{-1} \text{ cm}^{-2} \text{ sr}^{-1} \text{ Hz}^{-1}$) as

$$\tilde{J}_{X,21} \simeq 2.3 \times 10^{-4} \left( \frac{\text{SFR}}{M_\odot \text{ yr}^{-1}} \right) \left( \frac{d}{10 \text{ kpc}} \right)^{-2}. \quad (13)$$

Several studies have investigated the redshift evolution of the $L_X$–SFR relation using empirical data. The linear relation observed in local star-forming galaxies does not change significantly up to $z \lesssim 2$ (Grimm, Gilfanov & Sunyaev 2003; Lehmer et al. 2008; Mineo et al. 2014). The Chandra Deep Field-South suggests that the ratio increases as $\alpha (1+z)$ out to $z \sim 4$ (Bass et al. 2013). Furthermore, the existence of the unresolved soft X-ray background places a constraint on its evolution at higher redshifts: $d \log(L_X/$SFR$)/d \log(1+z) \lesssim 1.3$ (Dijkstra et al. 2012).

As noted above, the latest simulations suggest that Pop III stars tend to form with large ($> 10 M_\odot$) masses in binary or multiple systems. This strongly implies that the incidence of HMXBs is higher in the first galaxies than in the local Universe. Thus, we expect more X-ray binaries in the high-$z$ Universe than in the local galaxies. Although the properties of Pop III binaries remain highly uncertain, Hummel et al. 2014 estimate that they produced an X-ray background intensity $J_{X,21} \sim 0.03$ at $z \sim 20$. This value is a few hundred times larger than the $L_X$/SFR of low-$z$ galaxies. Similarly, population synthesis models of Fragos et al. 2013 estimate that $L_X$/SFR at $z \sim 10$ is higher than the local value by an order of magnitude.

To keep our results and discussions conservative, we here adopt the X-ray intensity using the $L_X$–SFR relation obtained from observations of low-$z$ galaxies. We set $J_{X,21} = f_X J_{X,21}$, where $f_X = 1$ is defined in equation (13), and we treat $f_X$ as a parameter set fiducially to unity (note that $f_X$ has an empirical dispersion of $0.4$ dex in low-$z$ galaxies (Mineo, Gilfanov & Sunyaev 2012)). As we describe above, both observations and theoretical works suggest $f_X \gtrsim 1 – 10$.

We note that our fiducial model is in close agreement with Mesinger, Ferrara & Spiegel 2013, who used found $f_X \approx 1$ based on the number of X-ray photons per stellar baryon $N_X \approx 0.2$ and the fraction of baryons converted into stars $f_* \approx 0.1$.

3.1.2 FUV radiation

Next, we estimate the LW intensities from star-forming galaxies consisting of Pop II ($Z = 10^{-3}$) and Pop III stars ($Z = 0$). We adopt the Salpeter initial mass function ($1 \lesssim M_* \lesssim 100 M_\odot$). The number flux of LW photons is estimated as $Q_{\text{LW}} = 5.25 \times 10^{63} \text{ s}^{-1} (\text{SFR}/M_\odot \text{ yr}^{-1})$ for the Pop II (III) case (Schaerer 2003). To estimate the mean intensity at $13.6 \text{ eV}$, we here consider two types of spectral models of star-forming galaxies: (1) a thermal spectrum with the effective temperature of $\gtrsim 10^8 \text{ K}$ and (2) a flat spectrum, which may be expected because of the superposition of radiation from low-mass and massive stars (Inoue 2011). We find

$$J_{\text{LW,21}} \approx \left\{ \begin{array}{l} 0.54 \left( \frac{\text{SFR}}{M_\odot \text{ yr}^{-1}} \right) \left( \frac{d}{10 \text{ kpc}} \right)^{-2} \quad \text{for } T_* = 10^4 \text{ K}, \\ 1.3 \left( \frac{\text{SFR}}{M_\odot \text{ yr}^{-1}} \right) \left( \frac{d}{10 \text{ kpc}} \right)^{-2} \quad \text{for } T_* = 10^5 \text{ K} \end{array} \right. \quad (14)$$

where the first two values correspond to cases for thermal spectra with $T_* = 10^4, 2 \times 10^4 \text{ K}$ and $10^5 \text{ K}$, and the fourth value to the flat spectrum (Note that the value for $T_* = 10^5 \text{ K}$ is the Pop III case). The actual effective temperatures of Pop II galaxies is hotter than $10^4 \text{ K}$ (Inoue 2011), and so the $T_* = 10^4 \text{ K}$ model represents a minimum case for $J_{\text{LW,21}}$. We will call the dimensionless factor in the curly brackets as $f_{\text{LW}}$, and consider a range of values $0.5 \lesssim f_{\text{LW}} \lesssim 2$.

3.1.3 Relation between X-rays and FUV radiation

Combining the expressions for $J_X$ and $J_{\text{LW}}$ above, we obtain

$$\frac{J_{X,21}}{J_{\text{LW,21}}} \simeq 3.8 \times 10^{-6} \left( \frac{f_X}{f_{\text{LW}}} \right). \quad (15)$$

Note: If you're referring to another document, please provide the page number and section number for accurate citation.
theoretical estimates of $J$ valid for $J_{\text{crit},0} = 10^3$ (red), $3 \times 10^3$ (green), and $10^4$ (blue). Black lines show the relation between the LW and X-ray intensities given by equation (15). A colour version of this figure is available in the online version.

Above, $f_X \gtrsim 1 - 10$ is the normalization of the X-ray intensity with respect to low-z star-forming galaxies (§3.1.1), and $0.5 \lesssim f_{\text{LW}} \lesssim 2$ is the dependence of the J LW normalization on the galaxy FUV spectrum (§3.1.2). In what follows, we consider a wide range $0.1 \leq f_X / f_{\text{LW}} \leq 10$ in order to present a conservative discussion.

### 3.2 Critical LW intensity

In Fig. 2 we show how the critical LW intensity $J_{\text{crit}}$ increases when accounting for X-ray ionizations. This figure shows the relationship between the X-ray-corrected value ($J_{\text{crit}}$) and the value calculated assuming a UV-only spectrum ($J_{\text{crit},0}$). We have found that the critical LW intensity $J_{\text{crit}}$ increases in the presence of an X-ray flux. For $h_{\text{min}} = 1$ keV, the X-ray-corrected value can be fit by the following simple formula (12):

$$J_{\text{crit}} = J_{\text{crit},0} \left(1 + \frac{J_{\text{X},21}}{9.3 \times 10^{-4}}\right)^{0.57}. \tag{16}$$

where $J_{\text{crit},0}$ is the value calculated without considering X-ray ionizations at all. Using equation (15), the actual critical LW intensity $J_{\text{crit}}$ can be written in terms of the original critical LW intensity (i.e., no X-ray flux) as

$$J_{\text{crit}} \simeq 7 \times 10^4 \left(\frac{J_{\text{crit},0}}{10^3}\right)^{2.3} \left(\frac{f_X}{f_{\text{LW}}}\right)^{1.3}, \tag{17}$$

which corresponds to an intersection of the solid curve and dashed line in Fig. 2. This equation is approximately valid for $J_{\text{crit},0} \gtrsim 400$ ($f_X / f_{\text{LW}}$). We therefore conclude that the X-ray ionization affects DCBH formation when $f_X / f_{\text{LW}} \gtrsim 0.14 (J_{\text{crit}}/7 \times 10^3)^{-1}$. Above, we described calculations showing that $f_{\text{LW}} \sim 0.5 - 2$ for several irradiation spectra, and $f_X \sim 1 - 10$ for $z \gtrsim 10$ galaxies. Taken together, those results suggest $f_X / f_{\text{LW}} \gtrsim 0.5$, and that soft X-ray irradiation can steeply raise $J_{\text{crit}}$ in all but the lowest theoretical estimates of $J_{\text{crit},0}$.

Equation (17) shows that $J_{\text{crit}}$ is highly sensitive to the value of $J_{\text{crit},0}$. The higher the value of $J_{\text{crit},0}$, the critical LW flux evaluated without taking into account X-ray ionizations, the higher the value of the effective value $J_{\text{crit}}$. The relative enhancement is roughly proportional to the FUV intensity itself.

The critical LW intensity evaluated without consideration of external ionizations, $J_{\text{crit},0}$, has been investigated using a one-zone model (e.g. Omukai 2001; Inayoshi & Omukai 2011 and three-dimensional numerical simulations (e.g. Bromm & Loeb 2003; Shang, Bryan & Haiman 2014; Wolcott-Green, Haiman & Bryan 2011; Latif et al. 2014) (SOI14)).

We summarize the results of these previous studies in Table 3. For a soft spectrum with $T_s = 10^4$ K, the values of $J_{\text{crit}}$ are $20 - 40$ (one-zone models) and $30 - 10^3$ (3D simulations). For harder spectra with $T_s > 2 \times 10^4$ K, the values of $J_{\text{crit}}$ are $>10^3$ (one-zone models) and $>10^4$ (3D simulations) (Sugimura, Omukai & Inoue 2014). Shaum found that $J_{\text{crit}}$ does not change significantly for $T_s \gtrsim 2 \times 10^4$ K because the typical Pop II galaxies have spectra that are flatter and harder than the thermal spectrum with $T_s = 10^4$ K (Inoue 2011). The values of $J_{\text{crit}}$ estimated from the 3D simulations tend to be larger by one order of magnitude than that derived in one-zone models because of ~20% spatial variation in the temperature inside the collapsing gas clouds. The temperature fluctuations produce a large difference in the H$_2$-collisional dissociation rate by one order of magnitude (Shang, Bryan & Haiman 2010). In light of this effect, it is reasonable to expect the actual value of $J_{\text{crit},0}$ to be $>10^3$.

### 4 DCBH FORMATION PROBABILITY AND NUMBER DENSITY

Now, we turn to the probability that a massive halo is exposed to $J_{\text{crit}}$ by a neighbouring galaxy, and use this quantity to estimate the number density of $z \sim 10$ seed BHs formed through FUV-aided direct collapse. The methods and calculations presented here follow those developed by (DFM14).

We summarize the requisite calculations below, and refer the reader to that paper for further details.

Metal pollution by galactic outflows could suppress the

| Authors          | $J_{\text{crit},0}$ ($\times 10^3$) | $T_s$ (K) | Method |
|------------------|-----------------------------------|-----------|--------|
| SBH10            | 39                                | $10^4$    | one-zone |
| IO11             | $30 - 300$                        | $10^4$    | 3D     |
| L14              | $400 - 1500$                      | $10^4$    | one-zone |
| SOI14            | $25$                              | $10^4$    | one-zone |
| SOI14            | $\gtrsim 1400$                    | $>2 \times 10^4$ | one-zone |

| Authors          | $J_{\text{crit},0}$ ($\times 10^3$) | $T_s$ (K) | Method |
|------------------|-----------------------------------|-----------|--------|
| SBH10            | $1.2 \times 10^4$                 | $10^5$    | one-zone |
| IO11             | $10^4 - 10^5$                     | $10^4$    | 3D     |
| WHB11            | $2 - 4 \times 10^4$               | $10^5$    | 3D     |

References:  
Shang, Bryan & Haiman (2010) (SBH10); Inayoshi & Omukai (2011) (IO11); Wolcott-Green, Haiman & Bryan (2011) (WHB11); Latif et al. (2014) (SOI14).

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probability of DCBH formation because the metal cooling induces efficient gas fragmentation. We here consider the case incorporating the metal-enriching wind model of DFM14 as well as the case without winds.

We briefly describe how to calculate the PDF of $J_{LW}$. We define potential DCBH sites as forming atomic-cooling halos with virial temperature $T_{\text{vir}} = 10^4$ K, corresponding to a mass $M_{\text{ac}}(z) = 8.1 \times 10^7 M_\odot ((1+z)/11)^{-3/2}$. A nearby star-forming galaxy can act as an FUV source for keeping $H_2$ photodissociated (Dijkstra et al. 2008). The differential probability distribution of finding an FUV source with mass $M$ at a distance $r$ is simply written

$$dP_{2}(M, r, z) = 4\pi r^2 (1 + z)^3 [1 + \xi(M_{\text{ac}}, M, r, z)] \frac{dn_{ST}}{dM},$$

where $\xi$ is the non-linear bias function, which represents the clustering of the two halos (i.e. the excess probability of finding another halo at a distance $r$; Iliev et al. 2003) and $dn_{ST}/dM$ is the Sheth-Tormen halo mass function (Sheth, Mo & Tormen 2001). Furthermore, we assume a lognormal distribution for the distribution of the LW luminosity of the source galaxies

$$dP_2(L_{\text{LW}}, M, z) = \frac{1}{\sqrt{2\pi \sigma_{\text{LW}}}} \times \exp \left[ -\frac{(\log L_{\text{LW}} - \log \langle L_{\text{LW}} \rangle)^2}{2\sigma_{\text{LW}}^2} \right],$$

$^5$ We estimate $\xi(z)$ using the formulae in Iliev et al. 2003, for all values of $z$. This differs from the approach of DFM14, who assumed $\xi(z) = \xi(z=10)[D(z)/D(z=10)]^2$. Our calculations show that $\xi(z)$ increases as a function of redshift, whereas it decreases in the formulation of DFM14. As a result, our PDFs for $z > 10$ are larger than those computed by DFM14.

Table 2. Summary of how the critical FUV intensity for DCBH formation ($J_{\text{crit}}$) and the corresponding formation probability ($P_{\text{DCBH}}$) increase when accounting for X-ray ionization. We show how these quantities change for three representative values of the X-ray-uncorrected value $J_{\text{crit,0}}$ ($400, 10^3$ and $2 \times 10^3$); for $z = 10$ and $z = 20$; with and without metal enrichment by galactic winds (DFM14 model). These calculations assume $f_X/f_{\text{LW}} = 1$.

| $J_{\text{crit,0}}$ | UV only | UV + X |
|-------------------|---------|--------|
| $z = 10$ w/wind | $3.9 \times 10^{-10}$ | $5.8 \times 10^{-12}$ |
| no wind | $3.7 \times 10^{-8}$ | $7.5 \times 10^{-11}$ |
| $z = 20$ w/wind | $1.9 \times 10^{-6}$ | $7.3 \times 10^{-9}$ |
| no wind | $1.2 \times 10^{-3}$ | $1.7 \times 10^{-5}$ |

where $\langle L_{\text{LW}} \rangle$ is the mean LW luminosity (see equation 6 and 8 in DFM14) and $\sigma_{\text{LW}} = 0.4$ is the dispersion. Using the relation $L_{\text{LW}} = 16\pi^2 r_{\text{min}}^2 J_{\text{LW}}$, we obtain the PDF of a given atomic-cooling halo being exposed to a LW flux $J_{\text{LW}}$:

$$dP_{\text{DCBH}}(J_{\text{LW}}, z) = \int_{M_{\text{min}}}^{\infty} dM \int_{r_{\text{min}}}^{\infty} dr \frac{dP_1}{dM} \frac{dP_2}{dJ_{\text{LW}}},$$

where $M_{\text{min}} = M_{\text{ac}}$. For the case with metal pollution by galactic winds, we apply $r_{\text{min}} = \max(r_{\text{vir}}(M_{\text{ac}}) + r_{\text{ac}}(M), r_{\text{vir}}(M))$, where $r_{\text{ac}}(M)$ is the distance from the sources within which the gas is polluted by metals and DCBH formation is quenched (see equation 5 in DFM14). For the case without metal-enriching winds, we simply set $r_{\text{min}} = r_{\text{vir}}(M_{\text{ac}}) + r_{\text{vir}}(M)$. Note that DFM14 set $r_{\text{min}} = 2r_{\text{vir}}(M_{\text{ac}})$.

In Fig. 3 we show the PDF of $J_{\text{LW}}$ for two redshifts of $z = 10$ (blue) and 20 (red). The solid and dashed lines represent the cases with and without metal pollution by galactic outflows, respectively. With (without) metal pollution, $dP_{\text{DCBH}}/d\log J_{\text{LW}} \sim 8 \times 10^{-11} (9 \times 10^{-10})$ for $J_{\text{LW}} = 10^3$ at $z \approx 10$. The PDF rapidly falls off with increasing $J_{\text{LW}}$. For $10^3 \leq J_{\text{LW}} \leq 10^4$, this behavior roughly follows a power law $\propto J_{\text{LW}}^{-\beta}$ with $\beta \approx 5$.

$^6$ Both values are smaller than the results shown in Fig. C1 of DFM14. We have determined that this difference is due to our choice of $r_{\text{min}} = r_{\text{vir}}(M_{\text{ac}}) + r_{\text{vir}}(M)$, which is greater than the choice $r_{\text{min}} = 2r_{\text{vir}}(M_{\text{ac}})$ adopted by DFM14. Larger $r_{\text{min}}$ results in lower probabilities of being exposed to a given $J_{\text{LW}}$, since the requisite luminosity $L_{\text{LW}} \propto r_{\text{min}}^2 J_{\text{LW}}$ follows a log-normal PDF. Note that this also means that $dP_{\text{DCBH}}/d\log J_{\text{LW}}$ decreases with redshift since $r_{\text{vir}}(M_{\text{ac}}) \propto (1 + z)^{-3/2}$ and $r_{\text{vir}}(M) \propto M^{1/2}/(1 + z)^{-1}$. 

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that a given halo has star formation) to be unity for simplicity. Note that since the formation probability (per halo): \[ J \]

The steep dependence of \( dP_{\text{DCBH}}/d \log J_{\text{f}} \) has a dramatic effect on the probability of DCBH formation. Using the approximation above that \( dP_{\text{DCBH}}/d \log J_{\text{f}} \propto J_{\text{f}}^{-\beta} \) with \( \beta \approx 5 \), and combining this with equation (17), we can write how the X-ray-corrected PDF depends on \( J_{\text{crit},0} \):

\[
dP_{\text{DCBH}}(z = 10) \approx \left\{ \frac{2 \times 10^{-15}}{3 \times 10^{-14}} \right\} \times \left( \frac{J_{\text{crit},0}}{10^3} \right)^{-12} \left( \frac{f_X}{f_{\text{f}}/f_{\text{LW}}} \right)^{-6.7}
\]

where the two values inside the curly brackets correspond to cases with (top) and without (bottom) metal-enrichment. In Table 2, we summarize the values of the integrated probability \( P_{\text{DCBH}}(\geq J_{\text{crit}}, z) \) for \( z = 10 \) and \( z = 20 \). As we showed in Fig. 3, X-ray ionizations can increase \( J_{\text{crit}} \) by a factor \( \sim 3-30 \) if the X-ray-uncorrected value is \( J_{\text{crit},0} \gtrsim 400 \left( f_X/f_{\text{f}}/f_{\text{LW}} \right)^{-1} \). This decreases \( P_{\text{DCBH}} \) by several orders of magnitude.

Above, we showed that soft X-rays can increase the critical FUV intensity to \( J_{\text{crit}} \sim 7 \times 10^5 \). This results in a DCBH formation probability (per halo): \( P_{\text{DCBH}} \sim 1.7 \times 10^{-16} \left( 2.7 \times 10^{-15} \right) \) at \( z \sim 10 \) with (without) metal enrichment by galactic winds (Note that we have not accounted for enrichment via in situ star formation.). These values are 4-5 orders of magnitude lower than the result calculated without considering soft X-rays: \( P_{\text{DCBH}} \sim 5.8 \times 10^{-12} \left( 7.5 \times 10^{-11} \right) \).

Finally, the number density (comoving) of forming DCBHs in an atomic cooling halo with mass \( M_{\text{ac}} \) is given by

\[
n_{\text{DCBH}}(z) = \int_{M_{\text{ac}}}^{\infty} \frac{dM_{\text{ac}}}{dM} P_{\text{DCBH}}(\geq J_{\text{crit}}, z),
\]

where we have followed the approximations taken by [DFM14] except that we have taken the factor \( P_{\text{gen}} \) (the probability that a given halo has not been metal-enriched by in situ star formation) to be unity for simplicity. Note that since \( P_{\text{gen}} \leq 1 \), the estimate given by equation (22) is conservative.

In Fig. 4 we show \( n_{\text{DCBH}} \) as a function of \( z \) for \( J_{\text{crit}} = 10^3 \) (thick red) and \( 7 \times 10^3 \) (thick blue). The solid and dashed lines represent the cases with and without metal-enrichment by galactic winds, respectively. At \( z \sim 10 \), the number density of DCBHs is smaller than that of \( \gtrsim 10^9 \) \( M_\odot \) SMBHs observed at \( z \gtrsim 6 \) (\( \sim 10^{-9} \) cMpc\(^{-3} \); [Willott et al. 2010] shown as a horizontal line in the figure), for all cases considered here. For \( J_{\text{crit}} = 7 \times 10^3 \), which corresponds to \( J_{\text{crit},0} \approx 10^3 \) (no X-ray ionization) for \( f_X/f_{\text{f}}/f_{\text{LW}} = 1 \), even the case without metal enrichment cannot exceed \( 10^{-9} \) cMpc\(^{-3} \) for all redshift.

Note that for combinations of high \( J_{\text{crit}} \) and lower redshifts (e.g. \( J_{\text{crit}} \gtrsim 10^4 \) and \( z \lesssim 15 \)), the characteristic halo mass where \( P_{\text{DCBH}} \) is highest can increase to \( M \gg M_{\text{ac}} \). This is because \( r_{\text{min}} \) increases toward low \( z \) as described above. Since the required LW luminosity for a neighbouring halo to enable direct collapse is \( L_{\text{crit}} > 16\pi^2 r_{\text{crit}}^2 \), for cases of large \( J_{\text{crit}} \) and large \( r_{\text{min}} \) (low \( z \)) only massive neighbouring halos are able to provide such a flux. In such cases, the probability is limited not by the number of atomic-cooling halos, but by the number of massive FUV sources, and equation (22) should be evaluated using the abundances of the latter.

In Fig. 5 we show contour plots of \( J_{\text{crit}} \) (including X-ray ionization) in the two-dimensional parameter space \((J_{\text{crit},0}, f_X/f_{\text{f}}/f_{\text{LW}})\). The shaded regions show the regions in this parameter space where the number density of DCBHs falls below the value of \( \sim 10^9 \) \( M_\odot \) SMBHs observed at \( z \sim 6 \). The lightly (darkly) shaded region shows this space for \( n_{\text{DCBH}} \) evaluated at \( z = 10 \) (20), corresponding to \( J_{\text{crit}} \gtrsim 700 \) (\( 3 \times 10^3 \)). Cross symbol indicates our fiducial model \( J_{\text{crit},0} = 10^3 \) and \( f_X/f_{\text{f}}/f_{\text{LW}} = 1 \).
values are conservative, because they ignore metal enrichment by in situ star formation, as well as the decrease in the comoving number density that results from hierarchical merging of DCBH-forming halos. We therefore argue that the enhancement of $J_{\text{crit}}$ by soft X-ray irradiation rules out this region in the physical parameter space. Most of the theoretical expectations discussed above (regarding $f_X$, $f_{\text{UV}}$ and $J_{\text{crit,0}}$; especially point to $J_{\text{crit}}$ being $> 10^3$).

Furthermore, we note that the requirement that the DCBH number density at $z \sim 10$ should be greater than $\sim 1 \text{ Gpc}^{-3}$ is itself also very conservative (in addition to the assumptions made regarding $P_{\text{gas}}$). This is because this value reflects only the abundance of $\sim 10^3 \text{ M}_\odot$ SMBHs at $z \sim 6$. If one stipulates that the FUV DCBH scenario must also account for the observed $\sim 10^8 \text{ M}_\odot$ SMBHs in the same redshift range, then the number density of DCBH must be at least $\sim 100 \text{ Gpc}^{-3}$ ($\text{Willett et al.} 2014$). All of these factors strongly put into question the viability of the FUV-aided DCBH scenario in explaining the $z \sim 6$ quasar observations.

5 DISCUSSION AND CONCLUSIONS

In this paper, we investigated the effect of X-ray irradiation on direct collapse black hole (DCBH) formation via far-ultraviolet (FUV) irradiation. X-ray ionization promotes the H$_2$ formation because H$_2$ molecules are produced by the electron-catalyzed reactions (equation 2). Thus, X-ray irradiation increases the critical FUV flux $J_{\text{crit}}$ required to suppress the H$_2$ formation and cooling. We stress that this effect is strongest for soft X-rays ($\sim 1 \text{ keV}$), which should accompany any FUV flux from close-proximity star-forming galaxies; we predict that harder X-rays (with longer mean-free paths) that comprise the cosmic X-ray background will play only a secondary role in promoting H$_2$ formation ($\text{Latif et al.} 2014$). Soft X-ray irradiation raises the actual value of $J_{\text{crit}}$ (FUV + X-ray) by a factor of $\sim 10 - 30$ compared to the case with FUV only. The number density of potential DCBH formation sites drops precipitously, by several orders of magnitude, placing new challenges on this SMBH seed formation scenario to explain the observed high-redshift quasar population.

5.1 Constraints from 21 cm observations

We have shown that X-ray ionizations may play a powerful role in suppressing FUV-aided DCBH formation. As shown in Figs. 4 and 5, the predicted number density of DCBHs at $z \sim 10 - 20$ for $f_X \gtrsim 1$ can be much less than that of the observed high-z QSOs with $M < 10^9 \text{ M}_\odot$, $\sim 1$ comoving Gpc$^{-3}$ (or $\sim 100$ comoving Gpc$^{-3}$ for $M > 10^9 \text{ M}_\odot$ SMBHs). For $f_X \gtrsim 0.1$, if $J_{\text{crit,0}} \gtrsim 3 \times 10^3$ (the value found in three-dimensional simulations for thermal spectra with $T_e \gtrsim 2 \times 10^5$ K (e.g., Wolcott-Green, Haiman & Bryan 2011), FUV-DCBH formation is ruled out as it cannot make enough seed BHs at $10 < z < 20$ to account for the observed $z \sim 6$ quasar population. However, the possibility that $f_X$ is much smaller than 0.1 is not excluded by current observational data.

The 21-cm line transition of neutral hydrogen is one of the most promising observations to probe the thermal history of the intergalactic medium before the cosmic reionization. Future observations of 21-cm signals from the high-$z$ Universe could give a lower limit on $f_X$ and thus help to further test the viability of the FUV-DCBH scenario.

The power spectrum of the brightness temperature of the 21-cm line at the scale of $\sim 0.1$ Mpc$^{-1}$ has three peaks as a function of redshift (e.g., Pritchard & Furlanetto 2007, Mesinger, Ferrara & Spiegel 2013, Christian & Loeb 2013). The location and amplitude of the second peak is sensitive to the value of $f_X$. As the X-ray intensity is weaker than that from star-forming galaxies at the low-$z$ universe ($f_X \ll 0.1$), the peak position shifts toward lower-redshift and its amplitude becomes larger. In this case, the 21-cm signal can be observed by 1st-generation interferometers; e.g. the Low Frequency Array (LOFAR, van Haarlem et al. 2013) and Murchison Wide Field Array (MWA, Tingay et al. 2013). On the other hand, for stronger X-ray intensities ($f_X \gg 0.1$), the spin temperature approaches the CMB temperature due to X-ray heating at higher redshift. Then, the peak of 21-cm signal moves to higher redshift and becomes smaller.

Second generation interferometers such as the Square Kilometre Array (SKA, Stieffler et al. 2013) will be required to observe the signal for larger $f_X$. However, near-future observations will be able to impose a lower limit of X-rays in the early Universe around $f_X \sim 0.1$ (e.g., Christian & Loeb 2013, Mesinger, Ewall-Wice & Hewitt 2014). The same observations should also be able to constrain the efficacy of SMBH growth through rapid growth of Pop III remnants (Tanaka, OLeary & Perna, in prep.).

5.2 Co-production of X-ray and FUV radiation

In this paper, we assume that star-forming galaxies emit X-rays as well as FUV radiation. This assumption seems reasonable when star formation occurs continuously during the cosmic time at the high-$z$ Universe ($\sim 300$ Myr for $z \sim 15$), because the lifetime of massive stars is shorter than $\sim 10$ Myr. The lifetime $t_{\text{H}_{\text{II}}}$ of massive stars with mass of $15/25/40/200 \text{ M}_\odot$ is $10/6.5/3.9/2.2$ Myr (Schaerer 2002). Observations of star-forming galaxies at lower redshift also support this assumption (e.g., Mineo, Gilfanov & Sunyaev 2012 and references therein). The co-production of X-rays with FUV photons is also seemingly unavoidable when a galaxy or an atomic-cooling halo undergoes an intense and short burst of star formation. Because some fraction of the newly formed stars will die and form HMXBs, there is a very narrow window (at most $t_{\text{H}_{\text{II}}} \sim 1 \text{ Myr}$) inside which a DCBH forming halo is irradiated by FUV radiation but not X-rays.

Let us also consider the gas properties inside a DCBH-forming halo. The gas density at the central core, before radiative cooling operates, is $\lesssim 10^{-2} \text{ cm}^{-3}$ $(1+z)/(11)^3$ (Visbal, Haiman & Bryan 2014). After the virial temperature reaches $\gtrsim 8000$ K, H atomic cooling causes the gas to undergo gravitational collapse. Since the density increases on the free-fall timescale, it will take $t_{\text{coll}} \gtrsim 20$ Myr ($(1+z)/(11)^{-3/2}$ for the gas density to exceed $\sim 10^3 \text{ cm}^{-3}$, the value where H$_2$ molecules can be collisionally dissociated instead of by FUV irradiation. Then it follows that X-ray irradiation must accompany any strong FUV intensity as long as $t_{\text{H}_{\text{II}}} < t_{\text{coll}}$.

Recently, Visbal, Haiman & Bryan (2014) proposed a
new DCBH formation scenario, which considers synchronised pairs of pristine atomic cooling halos having a small separation \( \lesssim 0.5 \) kpc. If one of the halos reaches the atomic cooling threshold (i.e., \( T_{\text{vir}} \gtrsim 8000 \) K) just after star formation occurs in another halo, the first halo can be irradiated with the critical FUV intensity due to a small separation. To be viable, this scenario must keep the gas free of ionizing X-rays and metal-enriching winds for a collapse timescale \( \sim 20 \) Myr \( (1+z)/11^{-3/2} \). As argued above, the FUV-producing massive stars can become X-ray sources on a significantly shorter timescale \( t_{\text{wind}} \). Thus, we argue that X-ray irradiation can also suppress this “synchronised pair” scenario. (Also note that the timescale on which winds can reach and pollute the gas is also comparable to \( t_{\text{coll}} \).) However, these arguments (lifetime of massive stars, photo-evaporation, and metal pollution) depend on a number of uncertain parameters (e.g. initial mass function, star formation efficiency, clumping factor of the intergalactic medium, and wind velocity) and a finer knowledge of these details will be required to better understand the impact of various photon sources inside such close, synchronised halo pairs.

### 5.3 Other effects to enhance \( J_{\text{crit}} \) and suppress the DCBH formation

We here discuss the suppression of DCBH formation by other ionization effects. In the early universe, the promising ionizing radiations other than X-rays are cosmic rays (CRs) and EUV photons (\( \gtrsim 13.6 \) eV) from star-forming galaxies. These sources of radiation increase the ionization fraction of the gas in the atomic cooling halos where SMSs would be born. Thus, they should also increase the requisite FUV intensity for DCBH formation, in much the same way as the effect of X-ray ionizations discussed in this work.

The enhancement of \( J_{\text{crit}} \) by CR ionization operates when the ionization rate is larger than \( \sim 10^{-18} \) s\(^{-1} \) at H column densities of \( \sim 10^{22} \) cm\(^{-2} \), which corresponds to \( n \sim 10^3 \) cm\(^{-3} \) and \( T \sim 8000 \) K (Inayoshi & Omukai 2011). Assuming the CR energy distribution comprises a power-law spectrum of \( dN_{\text{CR}}/dE \propto E^{-2} \) with \( 10^6 \leq E \leq 10^{15} \) eV, the ionization rate increase to \( J_{\text{crit}} \) can be estimated as \( \zeta_{\text{CR}} \gtrsim 10^{-17} \) s\(^{-1} \), which is smaller than that observed in Milky Way. \( 10^{-17} \lesssim \zeta_{\text{CR}} \lesssim 10^{-15} \) s\(^{-1} \) (Hayakawa, Nishimura & Takayanagi 1961; Spitzer & Tomasko 1968; Webber 1998; McCall et al. 2001; Indriolo et al. 2007). According to theoretical estimate (e.g., Stacy & Bromm 2007; Inayoshi & Omukai 2011; Nakauchi, Inayoshi & Omukai 2014), the CR ionization rate is

\[
\zeta_{\text{CR}} \sim 2 \times 10^{-20} \text{ s}^{-1} \left( \frac{d}{10 \text{ kpc}} \right)^{-2} \left( \frac{\text{SFR}}{M_\odot \text{ yr}^{-1}} \right),
\]

where we assume that 10 percent of the supernovae explosion energy \( (E_{\text{SN}} = 10^{51} \) erg) converts to the CR acceleration and the Salpeter initial mass function as we assumed for the FUV intensity. Combining equation (23) and \( J_{\text{LW,21}} = f_{\text{LW}} J_{\text{LW,21}} \), then we can find

\[
\zeta_{\text{CR}} \sim 3 \times 10^{-19} \text{ s}^{-1} \left( f_{\text{LW}} J_{\text{LW,21}} \right),
\]

which is smaller than the ionization rate above which \( J_{\text{crit}} \) increases. However, the CR intensity in the early universe has uncertainties associated with magnetic fields, e.g., the confinement of CRs in star-forming galaxies and CR propagation in the intergalactic medium (e.g., Strong, Moskalenko & Ptuskin 2007, and references therein). Thus, more sophisticated models are required to better evaluate the impact of CR ionizations on DCBH formation.

EUV photons with \( \gtrsim 13.6 \) eV can easily absorbed by the intergalactic medium because their optical depth in neutral hydrogen is as large as

\[
\tau_H(\nu) \sim 10^4 \left( \frac{n}{1.0 \text{ cm}^{-3}} \right)^{1/2} \left( \frac{h\nu}{13.6 \text{ eV}} \right)^{-3}.
\]

Thus, EUV photons cannot penetrate into the dense and hot region \( (n \sim 10^3 \) cm\(^{-3} \) and \( T \approx 8000 \) K) in atomic cooling halos. As a result, the critical FUV intensity does not change because of self-shielding to EUV photons. However, EUV photons would suppress DCBH formation during and after cosmic reionization, only when the gas around atomic-cooling halos becomes ionised completely (Johnson et al. 2014).

### 5.4 Alternative models forming DCBHs

We discuss alternative scenarios of forming DCBHs which do not require strong FUV radiation. The relevant H\(_2\) dissociating process instead of FUV photodissociation is collisional dissociation (\( H_2 + H \rightarrow 3H \)). In atomic cooling halos \( (T_{\text{crit}} \gtrsim 10^5 \) K), this process works efficiently in that the gas density and temperature are \( n \gtrsim 10^4 \) cm\(^{-3} \) and \( T \gtrsim 6000 \) K, respectively. Once the primordial gas enter such a dense and hot region (so-called “zone of no return”), the H\(_2\) formation/cooling is quenched by the collisional dissociation even without FUV radiation enough for the gas to collapses keeping a high temperature \( (\sim 8000 \) K) by H-atomic cooling (Inayoshi & Omukai 2012).

One promising process for forming such a dense and hot gas is strong shock by collisions of cold accretion flows due to assembly of the first galaxies. Since the radiative cooling of the gas is efficient in the first galaxies, the gas can penetrate deep to the centre \( (\sim 0.1 R_{\text{vir}}) \) as dense filamentary inflows. If the cold flows jump into the zone of no return by shock heating, a supermassive star can form from the parent cloud in the post-shock region. However, supersonic filamentary flows are unlikely to be dense before experiencing shocks for weak-cooling case \( (T_{\text{vir}} \lesssim 8000 \) K; Fernandez et al. 2014) and no-cooling case (Visbal, Haiman & Bryan 2014a).

These two examples suggest that massive halos \( (T_{\text{vir}} \gtrsim 10^5 \) K) could be necessary for the gas to arrive in the zone of no return. To better understand the actual probability of a dense shocked gas cloud forming in this way, a large, statistical sample of numerical simulations of atomic-cooling halos is required.

A galaxy merger is another mechanism that can induce strong inflows and form an environment similar to the one made by the cold accretion shocks. Mayer et al. (2014) performed a numerical simulation of the merger between massive \( (\sim 10^{12} \) \( M_\odot \)) and metal-enriched \( (\sim Z_\odot) \) proto-galaxies at \( z \approx 6 \), assuming a simple polytropic equation of state (i.e., \( p \propto \rho^\gamma \)). After the merging, the gravitationally unstable disc is formed, where the non-axisymmetric structures (spiral arms and bars) transport the gas angular
momentum efficiently. In that case, strong inflows rapidly accumulate a mount of gas with \(10^6 \, M_\odot\) within the central pc scale. The average density of the nuclear region reaches \(\sim 10^4\,\text{cm}^{-3}\), at which point H\(_2\) molecules can remain collisionally dissociated. However, Ferrara, Haardt & Salvaterra (2013) have noted that the gas actually fragments into clumps with \(\lesssim 10^2\, M_\odot\) if one considers more realistic radiative cooling prescriptions, instead of a simple equation of state. Further research is required to determine whether the galaxy merger could produce massive clouds forming DCBHs, when fully accounting for the cooling and chemical reactions of primordial gas.

A third avenue for forming DCBHs without FUV radiation was recently proposed by Tanaka & Li (2014). The relative bulk streaming motion between baryons and dark matter left over from cosmic recombination (Tseliakhovich & Hirata 2010) has been shown to delay gas inflow and Pop III star formation in \(z \gtrsim 20\) halos with \(T_{\text{vir}} \sim 1000 - 2000\,\text{K}\) (Stacy, Bromm & Loeb 2011; Greif et al. 2011; Fialkov et al. 2012; Naoz, Yoshida & Gnedini 2013). Tanaka & Li (2014) noted that in rare combinations of particularly massive halos and exceptionally large streaming velocities, the delay in gas inflow may persist until the halo reaches \(T_{\text{vir}} \sim 8000\,\text{K}\). Gas falling into such halos would naturally shock to 8000 K (Stacy, Bromm & Loeb 2011; Greif et al. 2011; Fialkov et al. 2012) before ever forming stars. The gas will undergo direct collapse if it can reach sufficiently large densities to keep H\(_2\) collisionally dissociated (however, note the caveats and uncertainties discussed above). Tanaka & Li (2014) predicted that this this mechanism a characteristic redshift \(z \sim 30\), where the product of the atomic-cooling halo number density and the probability of having a sufficiently large streaming velocity (i.e. significant delay in gas inflow) is maximized.

ACKNOWLEDGEMENTS

We thank Mark Dijkstra, Zoltán Haiman, Eli Visbal, Kazuyuki Omukai and Kazuyuki Sugimura for fruitful discussions, as well as Jarrett L. Johnson, Muhammad Latif, Dominik Schleicher, and Marta Volonteri for comments on the manuscript. This work is partially supported by Grants-in-Aid from the Ministry of Education, Culture, and Science of Japan (to KI).

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