The mechanism of the onset of turbulent structures under the ion temperature gradient drift instability conditions

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Abstract. A possible mechanism of the appearance of turbulent structures in plasma is considered as a result of the development of unstable ion temperature gradient (ITG) drift waves, both with and without a sheared flow. A description of several stages of the drift wave evolution is proposed. The condition for limiting the growth of the wave amplitude is discussed. A qualitative agreement of the results with the theoretical and experimental results of other authors is shown. The motion of particles in the wave and vortex structures is studied on the basis of the proposed mechanism and ideas of other authors.

1. Introduction

At the present time, very few works are devoted to the study of the onset of turbulence and description of turbulent structures in plasma. However, a large number of works on the development of turbulence exist for a classical fluid at critical Reynolds numbers. This regime is accompanied by intermittency, i.e. an alternation of laminar and turbulent flows, and significantly affects the macroscopic parameters of the flow.

It is likely that similar processes of intermittency occur in plasma. The justifying experimental data was obtained on the linear TAU-1 device [1]. In this experiment, a periodic appearance and increase in a drift oscillation signal was observed under certain conditions. In other words, a transition between regular oscillations and chaotic turbulent fluctuations was recorded.

Since plasma turbulence is usually associated with the presence of various types of instabilities, presumably, regular oscillations that increase with time may arise in the plasma, which was observed in experiment [1]. Such regular oscillations are directly related to the instabilities. In the present work, we consider the ITG turbulence caused by the ITG (ion temperature gradient) drift instability, which is a type of instability that is the most fully studied to date [2–4].

In the H-mode, it is often said that the ITG turbulence is partially suppressed in the region of the transport barrier. The suppression consists of the reduction of the level of turbulent density and temperature fluctuations due to the shear effects caused by the shear flow. In Refs. [5–7], which study the effects of the shear flow, an already developed turbulence was considered, and the development of turbulent structures was not discussed. Knowing how these structures appear, it becomes possible to more accurately estimate the resulting level of turbulent fluctuations depending on both the velocity shear and the plasma parameters.

Popular theoretical models [5–6] for calculating the level of fluctuations are based on an analysis of the decorrelation processes. The decrease in the fluctuation level is associated with the splitting of
the vortex structures due to the shear effects, which was observed by the two-dimensional gas-puff imaging (GPI) diagnostic on the TEXTOR tokamak [7]. This paper examines whether the shear affects the turbulent structures at the stage of their appearance, and not at the developed stage.

2. Model of the emergence of turbulent structures
A possible process of the appearance of turbulent structures under ITG instability conditions was described in articles [8–10]. The object of this work is the transverse drift wave. Several stages of its development are highlighted, and the possibility of their cyclic renewal is not excluded.

A small-amplitude drift wave arises in unperturbed plasma. Due to the development of an ITG instability, the amplitude of the wave increases exponentially at a given growth rate $\gamma = \text{const}$. The growth rate is calculated using the works of other authors, in which the ITG instability is studied in the linear approximation. The amplitude of the wave cannot increase infinitely, and therefore at some point, the plasma transitions to a new state in which the conditions for the ITG instability are not fulfilled.

The analysis carried out in [10] established that the local values of the density and temperature gradients directed along the wave increase the fastest. The limitation of the wave amplitude growth is based on these parameters. At some time they become comparable with the specified external (radial or transverse) density and temperature gradients, and it is assumed that at this moment, the amplitude of the wave stops growing, and the final amplitude of the wave is related to the level of density fluctuations observed in experiments.

Further, complex processes of the diffusion of the developed fluctuations begin in the plasma, accompanied by the disappearance of the wave. In our opinion, the described wave evolution process leads to a developed turbulent state of the plasma, which was described in works [5–7].

The results of the calculations [5, 6] and [10] agree qualitatively despite the different approaches to determining the level of turbulent density fluctuations. For the velocity shear $\gamma_s < 2\gamma$ (where $\gamma_s$ is the shear and $\gamma$ is the growth rate of the ITG instability), which characterizes the shear flow, the wave is not overturned. Under this condition, the limitation of the growth of the wave amplitude is equally applicable for plasmas both without and with a shear flow.

3. About turbulent structures and particle trajectories
Existing works [5-7] consider turbulent two-dimensional vortex structures by analogy with the classical theory of turbulence of fluids. Without background flows, the turbulent vortex has a certain coherence length, which can be approximately considered equal to the size of the vortex. Due to the ITG instability, one of the dimensions (cross or radial) of the vortex increases. In the presence of a shear flow, the vortex inclines and lengthens. It is usually assumed that the size of the vortex in the direction perpendicular to the shear flow decreases, and its longitudinal size increases (see Figure 1). As a result, the decrease in the transverse (radial) size of the vortices decreases the turbulent transfer. At high values of $\gamma_s$, the shear flow tilts and breaks up the vortices.

![Figure 1. Scheme of the vortex distortion: a) instability effect, b) shear flow effect.](image-url)
According to the wave model described in [10], in the absence of background flows, the wave particles oscillate in the transverse (radial) direction as 
\[ x = x_0 \sin(\omega t), \]
where \( x_0 \) is the displacement amplitude and \( \omega \) is the frequency. The shear velocity is \( v_y = \gamma_s x \). After substitution and integration
\[ v_y = \gamma_s x \rightarrow \frac{dy}{dt} = \gamma_s x_0 \sin(\omega t) \rightarrow y = \frac{\gamma_s x_0}{\omega} \cos(\omega t). \]

Under the initial condition \( y(t = 0) = \frac{\gamma_s x_0}{\omega} \), an elliptical trajectory of the particle is obtained:
\[ \frac{y^2}{\left(\frac{\gamma_s x_0}{\omega}\right)^2} + \frac{x^2}{x_0^2} = 1. \]

Similar calculations at the exponential growth of the displacement amplitude, \( x = x_0 \exp(\gamma t) \sin(\omega t) \), show that the wave particle moves in a spiral. Figure 2 shows the calculated trajectories of the particles.

![Figure 2. The perturbed trajectories: a) without background flows, b) shear flow, c) shear flow with exponential growth of the wave](image)

Let us consider the trajectory of a particle in the vortex with a center in the origin of coordinates. In the absence of background flows, the particle moves in a circle with a radius \( \rho \) according to equations
\[ x(t) = \rho \cos(\omega t), \quad y(t) = \rho \sin(\omega t) \]
and its velocity components are defined as
\[ v_{x,\rho}(t) = -\rho \omega \sin(\omega t), \quad v_{y,\rho}(t) = \rho \cos(\omega t). \]

Let us assume that the shear flow leads only to a change in the velocity \( v_{y,\rho} \). Then the particle velocity components will become
\[ v_x(t) = v_{x,\rho}(t) = -\rho \omega \sin(\omega t), \]
\[ v_y(t) = v_{y,\rho}(t) + v_{y,\gamma}(t) = \rho \cos(\omega t) + \gamma_s x. \]

The initial condition is \( x = \rho \) and \( y = 0 \) at \( t = 0 \). After integration, we obtain
\[ x(t) = \rho \cos(\omega t), \quad y(t) = \rho (1 + \gamma_s/\omega) \sin(\omega t). \]

Again, an elliptical particle trajectory is obtained.

In this approximation, simple estimates show that the vortex narrows or expands only in the longitudinal direction. The proposed solution is only the first approximation and is valid in a small
range of $\gamma_s$. Estimates indicate that a more careful analysis of the processes within the plasma vortex is necessary due to the shear effects.

4. Conclusions
The results obtained using the wave model described in [10] are in qualitative agreement with the works [5, 6] of other authors. Despite the different approaches, the idea of this work is that the study of the possible mechanisms of the generation of turbulent structures in plasma is necessary in order to better understand the physical effects caused by the shear flow. In contrast to existing works, the effect of the shear flow on the formation of turbulent structures at the stage of their generation is shown. The reason for the cessation of the growth of the turbulent fluctuations due to the development of the ITG instability is established. Estimates of turbulent fluctuations depending on plasma parameters (density, temperature, and their gradients) are obtained in an explicit form. In our opinion, the data of visual diagnostics used in experiments shows the residual formations of the drift wave according to the theory described in [10], which confirms the validity of our assumptions.

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