GPS observables in general relativity

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I present a complete set of gauge invariant observables, in the context of general relativity coupled with a minimal amount of realistic matter (four particles). These observables have a straightforward and realistic physical interpretation. In fact, the technology to measure them is realized by the Global Positioning System: they are defined by the physical reference system determined by GPS readings. The components of the metric tensor in this physical reference system are gauge invariant quantities and, remarkably, their evolution equations are local.

In general relativity (GR), the “observables”, namely the physical quantities that we can predict and measure in real experiments, correspond to quantities of the theory that are invariant under coordinate transformations. It is not difficult to construct observables in concrete applications of the theory. Indeed, each physically meaningful number that we actually measure corresponds to one of these observables. For instance, in studying the general relativistic dynamics of the solar system, the distance of a planet from us, at given (solar) time (say, at midnight tonight) is an observable. On the other hand, most of the theoretical work is not done in terms of observables, but rather in terms of gauge dependent, that is, coordinate dependent, quantities. This course is the reason for which the same physical situation can be described in terms of different metrics tensors. The usual procedure in GR is indeed to develop the theoretical description of a certain physical situation in an arbitrary gauge, and then compute the value of a certain number of coordinate independent observables, which can be compared with observations.

This way of proceeding has an unsatisfactory aspect: we do not have at our disposal a complete set of observables, which we could imagine, in principle, to measure in a simple manner. A physical situation, of course, is characterized by an equivalence class under diffeomorphisms of solutions of the equations of motion; but we do not know how to effectively coordinatize the space of the equivalence classes in terms of realistically observable quantities. The individual observables that we use in concrete applications are just a small number, and are far from capturing the full gauge invariant physics. The difficulty of writing a complete class of observables is in fact well known (see references therein), and raises several well known problems. For instance it complicates the canonical analysis of the theory and it is a serious obstacle to quantization. Furthermore, it generates conceptual difficulties for the very physical interpretation of the theory.

Attempts to define a complete class of observables abound in the literature. It is easier to construct such observables in the presence of matter that in the context of pure GR. This is because the difficulty of writing observables is the consequence of the absence of absolute localization in a general relativistic theory. If there is matter we can localize things with respect to the matter. For instance, we can consider GR interacting with four scalar matter fields. Assume that the configuration of the fields is sufficiently nondegenerate. Then the components of metric field at points defined by given values of the matter fields are observable. The idea is clearly the same as defining an observable as the distance between us and a planet: instead of having just a few planets to define a few physical distances, we imagine to have a continuous of matter so that physical distances are defined everywhere. This idea has been developed in a number of variants, such as dust carrying clocks and others, by many authors, including the present one. See references therein. One succeeds in constructing a complete set of observables, but the extent to which the result is realistic or useful is certainly questionable. It is rather unsatisfactory to understand the theory in terms of fields that do not exist, or phenomenological objects such as dust, and it is questionable whether these procedures could make sense in the quantum theory, where the aim is to describe Planck scale. Other (earlier) attempts to write a complete set of observables are in the context of pure GR. The idea is to construct four scalar functions of the metric (say, scalar polynomials of the curvature), and use these to localize points. The value of a fifth scalar function in a point where the four scalar functions have a given value is an observable. This works, but the result is mathematically very intricate and physically extremely unrealistic. It is certainly possible, in principle, to construct detectors of such observables, but I doubt any experimenter would get founded for proposing to build such apparata. Do we thus have to declare defeat and, in a general relativistic context, give up the hope of having a complete set of well defined observables, which are realistic, easy to construct, and do not assume that the world is different from what it is?

In this paper, I’d like to argue that we do not have to declare defeat. I propose a simple way out, based on GR coupled with a minimal and very realistic amount
of additional matter. Indeed, the solution I discuss is so realistic that it is in fact real: it was inspired by an already existing technology, the Global Positioning System (GPS), the first technological application of GR, or the first large scale technology that needs to take GR effects into account. The other source of the ideas presented here is the Null Surface Formulation (NSF) approach to GR. What follows can be seen as a sort of pedestrian version of some NSF ideas. Ideas related to the ones presented here have been presented by Massimo Pauri and Luca Lusanna. See also [9].

The idea here is simple. Consider a general covariant system formed eucaby GR coupled with four small bodies. These are taken to have negligible mass; they will be considered as point particles for simplicity, and called “satellites”. Assume that the four satellites follow timelike geodesics; that these geodesics meet in a common (starting) point O; and, that at O they have a given (fixed) speed—the same for the four—and directions as the four vertices of a tetrahedron. The theory might include any other matter. Then (there is a region R of spacetime for which) we can uniquely associate four numbers sα, α = 1, 2, 3, 4 to each spacetime point p as follows. Consider the past lightcone of p. This will (generically) intersect the four geodesics in four points pα. The numbers sα are defined as the distance between pα and O. We can use the sα’s as physically defined coordinates.

The components gαβ(s) of the metric tensor in these coordinates are observables quantities. They are invariant under four-dimensional diffeomorphisms (because, of course, these deform the metric as well as the satellites’ worldlines). They define a complete set of observables for the region R.

The physical picture is simple, and its realism is transparent. Imagine that the four “satellites” are in fact satellites, each carrying a clock that measures the proper time along its trajectory, starting at the meeting point O. Imagine also that each satellite broadcasts its local time with a radio signal. Suppose I am at the point p and have an electronic device that simply receives the four signals and displays the four readings. See Figure 1. These four numbers are precisely the four physical coordinates sα defined above. Current technology permits to perform these measurements with accuracy well within the relativistic regime. If I then use a rod, and a clock and measure the physical distances between sα coordinates points, I am directly measuring the components of the metric tensor in the physical coordinate system. In the terminology of Ref. [11], the sα’s are partial observables, while gαβ(s) are complete observables.

As shown below, the physical coordinates sα have nice geometrical properties; they are characterized by

\[ g^{\alpha\beta}(s) = 0, \quad \alpha = 1, \ldots, 4. \]  

(1)

Surprisingly, in spite of the fact that they are defined by what looks like a rather nonlocal procedure, the evolution equations for gαβ(s) are local. These evolution equations can be written explicitly using the Arnowitt-Deser-Misner (ADM) variables. Lapse and Shift turn out to be fixed local functions of the three metric.

![FIG. 1. s1 and s2 are the GPS coordinates of the point p. \( \Sigma \) is a Cauchy surface with p in its future domain of dependence.](image-url)

In what follow, we begin for simplicity by introducing the GPS coordinates sα in Minkowski space. This allows us to introduce some tools in a simple context. Then we go over to a general spacetime. We take the speed of light to be one, signature \([+,-,-,-]\), and we assume the Einstein summation convention only for couples of repeated indices that are one up and one down. Thus \( \alpha \) is not summed over in (1). While dealing with Minkowski spacetime, the spacetime indices \( \mu, \nu \) are raised and lowered with the Minkowski metric. We write an arrow over three- as well as four-dimensional vectors.

Consider a tetrahedron in three-dimensional euclidean space. Let its center be at the origin and its four vertices \( \vec{v}^\alpha \), where and \( \vec{v}^\alpha \cdot \vec{v}^3 = -1/3 \) for \( \alpha \neq \beta \), have unit length \( ||\vec{v}^\alpha||^2 = 1 \). Here \( \alpha = 1, 2, 3, 4 \) is an index that distinguishes the four vertices, and should not be confused with vector indices. With a convenient orientation, these vertices have polar coordinates \( u = (r, \theta, \phi) \)

\[ v^{1\alpha} = (1, 0, 0) \]  

(2)

\[ v^{2\alpha} = (1, \alpha, 0) \]  

(3)

\[ v^{3\alpha} = (1, \alpha, 2\pi/3) \]  

(4)

\[ v^{4\alpha} = (1, \alpha, -2\pi/3) \]  

(5)

with \( \cos \alpha = -1/3 \), and cartesian coordinates \( (a = 1, 2, 3) \)

\[ v^{1a} = (0, 0, 1) \]  

(6)

\[ v^{2a} = (2\sqrt{2}/3, 0, -1/3) \]  

(7)

\[ v^{3a} = (-\sqrt{2}/3, \sqrt{2}/3, -1/3) \]  

(8)

\[ v^{4a} = (-\sqrt{2}/3, -\sqrt{2}/3, -1/3) \]  

(9)
Let us now go to a four dimensional Minkowski space. Consider four timelike 4-vectors \( \vec{W}^\alpha \), of length one, \(|\vec{W}^\alpha|^2 = 1\), representing the normalized 4-velocities of four particles moving away from the origin in the directions \( \vec{v}^\mu \) at a common speed \( v \). Their Minkowski coordinates \( \mu = 0, 1, 2, 3 \) are

\[
W^{\alpha \mu} = \frac{1}{\sqrt{1 - v^2}} (1, \, v v^\alpha).
\] (10)

We fix the velocity \( v \) by requiring the determinant of the matrix \( W^{\alpha \mu} \) to be one one. (This choice fixes \( v \) at about half the speed of light; a different choice changes only a few normalization factors in what follows.) The four by four matrix \( W^{\alpha \mu} \) plays an important role in what follows. Notice that it is a fixed matrix whose entries are certain given numbers.

Consider one of the four 4-vectors, say \( \vec{W} = \vec{W}^1 \). Consider a free particle in Minkowski space that starts from the origin with 4-velocity \( \vec{W} \). Call it a “satellite”. Its world line is \( \vec{x}(s) = s \vec{W} \). Since \( \vec{W} \) is normalized, \( s \) is precisely the proper time along the world line. Consider now an arbitrary point \( p \) in Minkowski spacetime, with coordinates \( \vec{X} \). We want to compute the value of \( s \) at the intersection between \( l \) and the past light cone of \( p \). The calculation is particularly easy in a Lorentz frame in which the satellite stays still, namely has vanishing 3-velocity, and \( p \) is in the \((t, x)\) plane, with coordinates \((T, X)\). Assume \( X > 0 \). In this frame, the equation of \( l \) is

\[
x = 0,
\] (11)

the proper time is \( s = t \), and the equation of the (intersection with the \((t, x)\) plane of the) past light cone of \( p \) is

\[
(X - x) = \pm (T - t), \quad t < T.
\] (12)

Taking all this together gives

\[
s = T - X.
\] (13)

Notice now that in this frame \(|\vec{X}|^2 = T^2 - X^2 \) and \( \vec{X} \cdot \vec{W} = T \), so that we can write

\[
T - X = T - \sqrt{T^2 - (T^2 - X^2)}
\]

\[
= \vec{X} \cdot \vec{W} - \sqrt{(\vec{X} \cdot \vec{W})^2 - |\vec{X}|^2}.
\] (14)

From the last two equations we obtain

\[
s = \vec{X} \cdot \vec{W} - \sqrt{(\vec{X} \cdot \vec{W})^2 - |\vec{X}|^2}.
\] (15)

But this equation is Lorentz covariant and therefore it is true in any Lorentz system. (A direct full four dimensional calculation in an arbitrary system gives the same result.)

Let us now consider four satellites, moving out of the origin at 4-velocity \( \vec{W}^\alpha \). If they radio broadcast their position, an observer at the point \( p \) with Minkowski coordinates \( \vec{X} \) receives the four signals \( s^\alpha \)

\[
s^\alpha = \vec{X} \cdot \vec{W}^\alpha - \sqrt{(\vec{X} \cdot \vec{W}^\alpha)^2 - |\vec{X}|^2}.
\] (16)

We introduce (non-Lorentzian) general coordinates \( s^\alpha \) on Minkowski space, defined by the change of variables (16). These are the coordinates read out by a GPS device in Minkowski space. The Jacobian matrix of the change of coordinates is given by

\[
\frac{\partial s^\alpha}{\partial x^\mu} = W^\alpha_\mu - \frac{W^\alpha_\mu (\vec{X} \cdot \vec{W}^\alpha) - X_\mu}{\sqrt{(\vec{X} \cdot \vec{W}^\alpha)^2 - |\vec{X}|^2}};
\] (17)

where \( W^\alpha_\mu \) and \( X_\mu \) are \( W^{\alpha \mu} \) and \( X^\mu \) with the spacetime index lowered with the Minkowski metric. This defines the \((co-)tetrad field \( E^\alpha_\mu(s) \) of \( s^\alpha \). (18)

The contravariant metric tensor is given by \( g^{\alpha \beta}(s) = E^\alpha_\mu(s) E^{\alpha \beta}(s) \). Using \(|\vec{W}^\alpha|^2 = 1\), a straightforward calculation shows that

\[
g^{\alpha \alpha}(s) = 0, \quad \alpha = 1, \ldots, 4.
\] (19)

This equation has the following nice geometrical interpretation. Fix \( \alpha \) and consider the one-form field \( \omega_\alpha = ds^\alpha \). In \( s^\alpha \) coordinates, this one-form has components \( \omega_\beta^\alpha = \delta_\beta^\alpha \), and therefore “length” \(|\omega_\alpha|^2 = g^{\alpha \beta} \omega_\beta^\alpha \omega_\alpha^\beta = g^{\alpha \alpha} \). But the “length” of a 1-form is proportional to the volume of the (infinitesimal, now) 3-surface defined by the form. The 3-surface defined by \( ds^\alpha \) is the surface \( s^\alpha = \text{constant} \). But \( s^\alpha = \text{constant} \) is the set of points that read the GPS coordinate \( s^\alpha \), namely that receive a radio broadcasting from a same event \( p_\alpha \) of the satellite \( \alpha \), namely that are on the future light cone of \( p_\alpha \). Therefore \( s^\alpha = \text{constant} \) is a portion of this light cone, therefore it is a null surface, therefore its volume is zero, therefore \(|\omega|^2 = 0\), therefore \( g^{\alpha \alpha} = 0 \).

Since the \( s^\alpha \) coordinates define \( s^\alpha = \text{constant} \) surfaces that are null, we denote them as “null GPS coordinates”. It is useful to introduce another set of GPS coordinates as well, which have the traditional timelike and spacelike character. We denote these as \( s^\mu \), call them “timelike GPS coordinates”, and define them by

\[
s^\alpha = W^\alpha_\mu s^\mu.
\] (20)

This is a simple algebraic relabeling of the names of the four GPS coordinates, such that \( s^\mu = 0 \) is timelike and \( s^{\mu = \alpha} \) is spacelike. In these coordinates, the gauge condition (19) reads

\[
W^\alpha_\mu g^{\mu \nu}(s) = 0.
\] (21)
This can be interpreted geometrically as follows. The (timelike) GPS coordinates are coordinates $s^\alpha$ such that the four 1-forms

$$\omega^\alpha = W^\alpha_\mu ds^\mu$$

(22)

are null.

Let us now jump from Minkowski space to full GR. Consider GR coupled with four satellites of negligible mass that move geodesically and whose world lines emerge from a point $O$ with directions and velocity as above. Locally around $O$ the metric can be taken to be Minkowskian; therefore the details of the initial conditions of the satellites worldlines can be taken as above. The phase space of this system is the one of pure GR plus 10 parameters, giving the location of $O$ and the Lorentz orientation of the initial tetrahedron of velocities. The integration of the satellites’ geodesics and of the light cones can be arbitrarily complicated in an arbitrary metric. However, if the metric is sufficiently regular, there will still be a region $\mathcal{R}$ in which the radio signals broadcasted by the satellites are received. (In case of multiple reception, the strongest one can be selected. That is, if the past light cone of $p$ intersects $\mathcal{R}$ more than once, generically there will be one intersection which is at shorter luminosity distance.) Thus, we still have well defined physical coordinates $s^\alpha$ on $\mathcal{R}$. Equation (21) holds in these coordinates, because it depends just on the properties of the light propagation around $p$. We define also timelike GPS coordinates $s^\mu$ by $(21)$, and we get the condition $(21)$ on the metric tensor.

To study the evolution of the metric tensor in GPS coordinates it is easier to shift to ADM variables $N, N^\alpha, \gamma_{ab}$. These are functions of the covariant components of the metric tensor, defined in general by

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu$$

$$= N^2 dt^2 - \gamma_{ab}(dx^a - N^a dt)(dx^b - N^b dt).$$

(23)

Equivalently, they are related to the contravariant components of the metric tensor by

$$g^{\mu\nu}v_\mu v_\nu = -\gamma^{ab}v_a v_b + (n^\mu v_\mu)^2,$$

(24)

where $\gamma^{ab}$ is the inverse of $\gamma_{ab}$ and $n^\mu = (1/N, N^\alpha/N)$. Using these variables, the gauge condition $(21)$ reads

$$W^\alpha_a W^\mu_b \gamma_{ab} = (W_\mu^\alpha n^\mu)^2.$$  

(25)

Notice now that his can be solved for the Lapse and Shift as a function of the three-metric (recall that $W^\alpha_\mu$ are fixed numbers), obtaining

$$n^\mu = W^\mu_\alpha q^\alpha$$

(26)

where $W^\mu_\alpha$ is the inverse of the matrix $W^\alpha_\mu$ and

$$q^\alpha = \sqrt{W^\alpha_a W^\alpha_b \gamma_{ab}}.$$

(27)

Or, explicitly,

$$N = \frac{1}{W^\mu_0 q^\mu}, \quad N^\alpha = \frac{W^\alpha_0 q^\alpha}{W^\mu_0 q^\mu}. $$

(28)

The geometrical interpretation is as follows. We want the 1-form $\omega^\alpha$ defined in $(22)$ to be null, namely its norm to vanish. But in the ADM formalism this norm is the sum of two parts: the norm of the pull back of $\omega^\alpha$ on the constant time ADM surface, which is $q^\alpha$, given in $(27)$, and depends on the three metric; plus the square of the projection of $\omega^\alpha$ on $n^\mu$. We can thus obtain the vanishing of the norm by adjusting the Lapse and Shift. We have four conditions (one per each $\alpha$) and we can thus determine Lapse and Shift out of three metric. In other words, whatever is the three metric, we can always adjust Lapse and Shift so that the gauge condition $(21)$ is satisfied.

But in the ADM formalism, the arbitrariness of the evolution in the Einstein equations is entirely captured by the freedom in choosing Lapse and Shift. Since here Lapse and Shift are uniquely determined by the three metric, evolution is determined uniquely if the initial data on a Cauchy surface are known. Therefore the evolution in the GPS coordinate $s^0$ of the GPS components of the metric tensor, $g_{\mu\nu}(s)$, is governed by deterministic equations: the ADM evolution equation with Lapse and Shift determined by equations $(27–28)$, $(21)$. Notice also that evolution is local, since the ADM evolution equations, as well as the equations $(27–28)$, are local.

To understand what is going on in this gauge, consider an arbitrary coordinate system $x^\mu$, and an arbitrary coordinate transformation $s^\alpha = s^\alpha(x)$ such that the transformed metric tensor satisfies $(19)$. This is equivalent to imposing one differential equation on each of the four functions $s^\alpha(x)$, as follows

$$g^{\mu\nu}(x)\partial_\mu s^\alpha(x)\partial_\nu s^\alpha(x) = 0.$$  

(29)

Equivalently, the problem is to find four independent exact null 1-forms $\omega^\alpha = ds^\alpha$, integrable over a finite region. Or, equivalently again, the problem is to find four nowhere-parallel foliations of Minkowski space with families of null surfaces. The last is precisely the main ingredient of the Null Surface Formulation of GR. A 10 parameter family of solutions of this problem is given by the construction above, namely by the light fronts emerging from the geodesics of the satellites, and labelled with the proper time along these geodesics. Equation $(23)$ has of course more solutions than these, since there are more families of null foliations than the ones constructed here. A null surface in a solution of $(29)$, for instance, does

*This does not imply that the full set of equations satisfied by $g_{\mu\nu}(s)$ must local, since initial conditions on $s^0 = 0$ satisfy four other constraints besides the ADM ones.
not need to focus in a point. Even less, the surfaces of the family need to focus on a line which is a geodesic, and so on. But if we require that the surfaces focus on a point, that the points are along a timelike geodesic, that the label of the foliation is given by the proper time along this geodesic, and that the four geodesics meet at a single point and with the prescribed angles, then the space of solutions of (29) is reduced down to a finite 10 parameter space. The 10 parameters are the location of the origin O and the Lorentz orientation of the tetrahedron of initial velocities. (By pushing O all the way to past infinity, one obtains the asymptotic NSF coordinates.)

How can the evolution of the quantities \( g_{\mu\nu}(s) \) be local? The conditions on the null surfaces described in the previous paragraph are nonlocal. Coordinate distances yield typically to nonlocality: Imagine we define physical coordinates in the solar system using the cosmological time \( t_c \) and the spatial distances \( x_S, x_E, x_J \) (at fixed \( t_c \)) from, say, the Sun the Earth and Jupiter. The metric tensor \( g_{\mu\nu}(t_c, x_S, x_E, x_J) \) in these coordinates is observable, but its evolution is highly non local. To see this, imagine that in this moment (in cosmological time), Jupiter is swept away by a huge comet. Then the value of \( g_{\mu\nu}(t_c, x_S, x_E, x_J) \) here changes instantaneously, without any local cause: the value of the coordinate \( x_J \) has changed because of an event happened far away. What’s special about the GPS coordinates that avoids this nonlocality? The answer is that the value of a GPS coordinate in a point \( p \) does in fact depend on what happens “far away” as well. Indeed, it depends on what happens to the satellite. However, it only depends on what happened to the satellite when it was broadcasting the signal received in \( p \), and this is in the past of \( p \) ! If \( p \) is in the past domain of dependence of a partial Cauchy surface \( \Sigma \), then the value of \( g_{\mu\nu}(s) \) in \( p \) is completely determined by the metric an its derivative on \( \Sigma \), namely evolution is causal, because the entire information needed to set up the GPS coordinates is in the data in \( \Sigma \). See Figure 1. Explicitly, the \( s^a = \text{constant} \) surfaces around \( \Sigma \) can be uniquely integrated ahead all the way to \( p \). They certainly can, as they represent just the evolution of a light front! This is how local evolution is achieved by these coordinates.

Let us summarize. We have introduced a set of physical coordinates, determined by certain material bodies. Geometrical quantities such as the components of the metric tensor expressed in physical coordinates are of course observable. This procedure is well known. The novelty here is that we have shown that in order to obtain physical coordinates, there is no need, as usually assumed, to introduce a large unrealistic amount of matter or to construct complicated and unrealistic physical quantities out of the metric tensor. Indeed, four particles are sufficient to coordinatize a (region of a) four-geometry. Furthermore, the coordinatization procedure is not artificial: it is the real one utilized by existing technology.

Notice that the degrees of freedom of the theory we have considered are still only two per point (plus the 10 parameters of the initial condition of the satellites.) This is why initial data on \( s^0 \) for \( g_{\mu\nu}(s) \) must satisfy additional constraints. We leave the study of these and the explicit construction of the associated Hamiltonian formalism for future investigations.

We conclude with some simple comments on observability. The components of the metric tensor in (time-like) GPS coordinates can be measured in principle as follows. Take a rod of physical length \( L \) (small with respect to the distance along which the gravitational field changes significantly) with two GPS devices at its ends (reading timelike GPS coordinates). Orient the rod (or search among recorded readings) so that the two GPS devices have the same reading \( s \) of all coordinates except for \( s^1 \). Let \( \delta s^1 \) be the difference in the two \( s^1 \) readings. Then we have along the rod

\[
d s^2 = g_{11}(s) \delta s^1 \delta s^1 = L^2. \tag{30}
\]

Therefore

\[
g_{11}(s) = (L/\delta s^1)^2. \tag{31}
\]

Non-diagonal components of \( g_{ab}(s) \) can be measured by simple generalizations of this procedure. The \( g_{ab}(s) \) are then algebraically determined by the gauge conditions.

In a thought experiment, a spaceship could travel in a spacetime region and compose a map of values of the GPS components of the metric tensor.

To avoid confusion, it may be useful to recall here the distinction between partial and complete observables [1]. A partial observable is a quantity to which a measuring procedure can be associated. A complete observable is an observable quantity that can predicted by the theory, or, equivalently, whose knowledge provides information on the state of the system. For instance, in the theory of a single harmonic oscillator \( q(t) \) the time \( t \) and the position of the oscillator \( q \) are partial observables, while the position of the oscillator at a given time, or at the initial time, is a complete observable. The GPS coordinates are partial observables: we can associate them a measuring procedure (this is what has been done in this paper), but we can of course not “predict” \( s^1 \). Equivalently, there is no sense in which the metric field of the universe is characterized by a certain value of \( s^1 \). The complete observables, or true observables, are the quantities \( g_{\mu\nu}(s) \), for any given value of the coordinates \( s^\mu \). These quantities are diffeomorphism invariant, are uniquely determined by the initial data and in a canonical formulation are represented by functions on the phase space that commute with all constraints.

Finally, notice that the observables we have defined are a straightforward generalization of Einstein’s “point coincidences”[1]. In a sense, they are precisely Einstein’s

\[† \] All our space-time verifications invariably amount to a
point coincidences. Einstein’s “material points” are just replaced by photons (light pulses): the spacetime point $s^\alpha$ is characterized as the meeting point of four photons designated by the fact of carrying the radio signals $s^\alpha$.

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