Using Complex Surveys to Estimate the $L_1$-Median of a Functional Variable: Application to Electricity Load Curves

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Summary

Mean profiles are widely used as indicators of the electricity consumption habits of customers. Currently, in Électricité De France (EDF), class load profiles are estimated using point-wise mean profiles. Unfortunately, it is well known that the mean is highly sensitive to the presence of outliers, such as one or more consumers with unusually high-levels of consumption. In this paper, we propose an alternative to the mean profile: the $L_1$-median profile which is more robust. When dealing with large data sets of functional data (load curves for example), survey sampling approaches are useful for estimating the median profile avoiding storing the whole data. We propose here several sampling strategies and estimators to estimate the median trajectory. A comparison between them is illustrated by means of a test population. We develop a stratification based on the linearized variable which substantially improves the accuracy of the estimator compared to simple random sampling without replacement. We suggest also an improved estimator that takes into account auxiliary information. Some potential areas for future research are also highlighted.

Key words: Horvitz-Thompson estimator; k-means algorithm; post-stratification; stratified sampling; substitution estimator; variance estimation.

1 Introduction

The French electricity company, Électricité De France (EDF), uses extensively the customer class load profiles in distribution network calculation. Load profiles are also used to predict future loads in distribution network planning or to estimate the daily load curve of a new customer. The customer data usually includes information about the type of the electricity connection, the customer class, the consumption type, and some other additional information. The combination of the individual customer information and the class load profiles allows us to estimate its load curve.

At EDF, the mean profile is used as an indicator of the electricity consumption of the customers. Nevertheless, it is well known that the mean is highly sensitive to the presence of outliers, for instance consumers with high level of consumption. As Small (1990) states, “it suffices to have only one customer contaminating a data set and going off to infinity to send the mean curve
to infinity as well. By contrast, at least 50% of the data must be moved to infinity to force the median curve to do the same”. More precisely, the median is robust to punctually extreme electricity consumptions of some customers and from a practical point of view, this can help to manage the electricity supply. Moreover, in the context of the electricity open market, new customers join the EDF company while other ones leave it and it is important to know the amount of electricity demand. Since the load profiles are not known for new customers, it is more difficult to predict their impact on the global electricity demand. Based on individual customer information, new customers will belong to a specific class and will be allowed the synthetic profile that describes the consumption behaviour of its class. In these situations, robust profiles should be used and this is why, we suggest using the median profile besides the mean curve as a robust indicator for analyzing the population of electricity load curves.

The median of a sample of univariate observations is a natural and useful characteristic of central position. Multivariate data, on the other hand, have no natural ordering. There are several ways to generalize the univariate median to multivariate data and they all have their advantages and disadvantages (see Small (1990) for a survey of multidimensional medians). First uses of the multivariate median were limited to two-dimensional vectors and were motivated mainly by problems of quantitative geography (namely, of centro-graphical analysis) which were dealt with by the U.S.A. Bureau of census in the late 19th and early 20th century.

We focus here on the $L_1$-median, also called the geometric or spatial median. Early work on the spatial median is due to Hayford (1902) and Gini & Galvani (1929) among others. Its definition is a direct generalization of the real median proposed by Haldane (1948) and properties of the spatial median have been studied in details by Kemperman (1987). Iterative estimation algorithms have been developed by Gower (1974) and Vardi & Zhang (2000).

In the next few years, the French company EDF intends to install over 30 millions electricity smart meters, in each firm and household. A meter is an electronic device constructed for measuring the electricity consumption. These meters will be able to send individual electricity consumption measures on very fine time scales. The new smart electricity meters will provide accurate and up-to-date electricity consumption data that can be used to model distribution network loads. In view of this new setting, the interest variables such as the consumption curve for example, may be considered as realizations of functional variables depending on a continuous time index $t$ that belongs to $[0, T]$ rather than multivariate vectors. Kemperman (1987), Cadre (2001), and Gervini (2008) studied the properties of the median with functional data. We cite also the very recent work of Cardot et al. (2011).

Another important issue is data storage. The amount of load data will be enormous when all or almost all of the customers have smart meters. Collecting, saving, and analyzing all this information would be very expensive. For example, if measures are taken every 10 min during 1 year and if we are interested in estimating the total electricity consumption for residential customers, the data storage is of about 100 terabytes.

We suggest using survey sampling techniques in order to get estimates of the median profile that are as accurate as possible at reasonable cost. The reader is referred to Fuller (2009) for a very recent monograph on survey sampling theory. Nevertheless, the idea of selecting randomly a sample from a population of curves is rather new. Chiky & Hébral (2008) compare two approaches for estimating the mean curve. The first one consists in using signal compression methods for the whole population of curves and the second approach suggests taking a simple random sampling of the actual curves. Their conclusion is that the results are better in the latter situation even with rather simple sampling designs. Very recently, Cardot et al. (2010) developed the estimation of functional principal component analysis with survey data and Cardot & Josserand (2011) studied the properties of the mean curve estimator with stratified sampling. We cite also Chaouch & Goga (2010) who treated the estimation of geometric quantiles, the
generalization of the spatial median, with survey data. As far as we know nothing has been
done in the estimation of the $L_1$-median in a functional framework with survey data whereas
it might have great practical interest. This is why, we investigate in this paper the median
curve estimator when several sampling designs and estimators are used. It is worth mentioning
that the results presented in this paper may be applied for other functional data which are
not necessarily related to time as it was the case of the electricity data. Nowadays, functional
data may arise in various other domains such as chemometrics or remote sensing and then the
functional response variables depend on index $t$ that may be a frequency and not necessarily a time
index.

The paper is structured as follows: Section 2 gives the main results concerning the median
curve estimation with survey data. A weighted estimator for the median curve is suggested and
its asymptotic variance function is exhibited by means of the linearization technique developed
by Deville (1999). A variance estimator is also proposed. Section 3 gives the estimation of the
median curve and of its variance function for a firm population of $N = 18902$ load electricity
curves. We consider several sampling designs: the simple random sampling without replacement,
the systematic sampling, the stratified sampling with optimal and proportional allocation, and the
with replacement proportional-to-size design. In the case of the stratified sampling, we suggest
using the k-means algorithm to construct homogeneous strata with respect to the linearized
variables. We illustrate through simulations that a substantial reduction compared to simple
random sampling is obtained. We adapt to the functional framework the selection of a sample
when auxiliary information is used at the sampling stage as for the with replacement proportional-
to-size design. Finally, we improve the Horvitz-Thompson estimator of the functional median
by considering the post-stratified estimator.

2 Functional Median in a Survey Sampling Setting

Let us consider the finite population $U = \{1, \ldots, N\}$ of size $N$ and a functional variable $Y$
defined for each element $k$ of the population $U : Y_k(t)$, for $t \in [0, T]$, with $T < \infty$. Let $\langle \cdot, \cdot \rangle$
and $||\cdot||$, respectively the inner product, respectively the norm, defined on $L^2[0, T]$. The empirical
median trajectory calculated from $Y_1, \ldots, Y_N$ is defined as (Chaudhuri, 1996; Gervini, 2008)

$$m_N = \arg \min_{y \in L^2[0, T]} \sum_{k=1}^{N} ||Y_k - y||.$$  

(1)

Supposing that $Y_k$, for all $k = 1, \ldots, N$, are not concentrated on a straight line, the median exists
and is unique (Kemperman, 1987) and is the solution of the following estimating equation,

$$\sum_{k=1}^{N} \frac{Y_k - y}{||Y_k - y||} = 0$$  

(2)

provided that $m_N \neq Y_k$ for all $k = 1, \ldots, N$.

For $Y_1, \ldots, Y_N \in \mathbb{R}^d$, the median $m_N$ defined by the formula (1) arises as a natural
generalization of the well-known characterization of the univariate median (Koenker & Basset,
1978),

$$q = \arg \min_{\theta} \sum_{k=1}^{N} |Y_k - \theta|$$

and it was called the spatial median by Brown (1983) or the $L_1$-median by Small (1990).
Weber (1909) considered $m_N$ as a solution to a problem in a location theory in which the
Y₁, ..., Yₙ are the planar coordinates of N customers, who are served by a company that wants to find an optimal location for its warehouse. It is also known as the Fermat-Weber point. A geometrical interpretation of the median defined by (2) is that the centroid of the vectors \( \frac{Y_k - m_N}{||Y_k - m_N||} \) is the origin in \( \mathbb{R}^d \). With only three points and bidimensional data, the median \( m_N \) is known to be the Steiner point of the triangle \( Y_1Y_2Y_3 \). The spatial median has also origins in the early work during the 12 census in the United States in 1900 concerned by finding the geographical center of the population over time. Hayford (1902) proposed the point-wise median as the geographical center but explicitly noted the drawback of the fact that the point-wise median depends on the choice of the orthogonal coordinates and it is not equivariant under orthogonal transformations. Brown (1983) goes further with this idea and states that when dealing with spatial data where variables possess isometry and require statistical techniques that have rotational invariance, it is more appropriate to use a median that shares these properties. We recall that with observations \( Y_1, ..., Y_N \) that lie in \( \mathbb{R}^d \), the point-wise median is the \( d \)-dimensional vector of medians computed from the univariate components and for functional variables, the empirical point-wise median is obtained if the \( L^1 \) norm is used in (1) instead of the \( L^2 \) norm,

\[
\text{med}(t) = \arg\min_{y(t) \in \mathbb{R}} \sum_{k=1}^{N} |Y_k(t) - y(t)|, \quad \text{for all } t \in [0, T].
\]

To illustrate the mean curve and the point-wise median versus the spatial median, we plot in Figure 1 the three curves for the test population of \( N = 18902 \) companies considered in Section 3. The electricity consumption is measured every 30 min.

Chaudhuri (1996) shows that the geometric quantiles defined in formula (3) from below and which are a generalization of the median defined by (1) are equivariant under orthogonal transformations unlike the point-wise median. Moreover, Chaudhuri showed also that the spatial or the \( L^1 \) median is equivariant under any homogeneous scale transformation of the coordinates of the multivariate observations which is appropriate when one needs to standardize the coordinate variables appropriately before computing the median.
The main arguments that play in favor of the spatial median are the uniqueness (see, e.g., Chaudhuri (1996) for the $d$-dimensional case with $d \geq 2$ and Kemperman (1987) for the functional case) and the fact that it is a global and central indicator of the distribution of the data. More exactly, the spatial median takes into account all instants making the spatial median a central indicator of the distribution of the data while the point-wise median is a central indicator but only for each instant. Consider for example that we have consumption electricity data recorded during two weeks: one working week and one holiday week such as the Christmas week. We compute first the spatial, respectively, the point-wise median, by considering only the working week time measurements. Next, we consider the two week consumption electricity and we compute again the spatial, respectively, the point-wise median. It can be noticed that the coordinates of the point-wise median that correspond to the working week are the same in both situations while they are changed for the spatial median. It is due to the fact that the spatial median is computed by taking into account all the time measurements while the point-wise median is computed instant by instant.

Moreover, Brown (1983) shows that there is an asymptotic efficiency from using the spatial median instead of the point-wise median. In fact, one can see that the objective function is differentiable in the case of the spatial median while this property is not fulfilled for the point-wise median.

As noted by Serfling (2002), the median defined by (2) and $Y_1, \ldots, Y_N \in \mathbb{R}^d$, has a nice robustness property in the sense that $m_N$ depends only on its direction towards $Y_i$. More exactly, $m_N$ remains unchanged if the $Y_i$ are moved outward along these rays while it is obvious that the point-wise median will change.

REMARK. Chaudhuri (1996) extends the definition (1) to geometric quantiles by using the geometry of data clouds. In a functional setting, its definition indexes the quantiles by the elements $v \in L^2[0, T]$ with $||v|| < 1$,

$$Q(v) = \arg \min_{y \in L^2[0, T]} \sum_{k=1}^N (||Y_k - y|| + \langle Y_k - y, v \rangle).$$

In this way, functional quantiles are characterized by a direction and magnitude specified by $v \in L^2[0, T]$ with $||v|| < 1$. Nevertheless, except the case $v = 0$, it is difficult to interpret the functional quantile defined in this way. This is why, our discussion is limited to the case $v = 0$.

2.1 The Design-Based Estimator for the Median $m_N$

Algorithms have been proposed to solve the equation (2) (Vardi & Zhang, 2000; Gervini, 2008) but they need important computational efforts especially when the number of time measurements is large. In this work, we suggest estimating the median curve $m_N$ by taking only a sample $s$ from $U$ according to a sampling design. A probability measure $p(\cdot)$ on the set of subsets of $U$, henceforth denoted $\mathcal{P}(U)$, is called a sampling design. Any random variable $S$ with values in $\mathcal{P}(U)$ and distribution $p$, is called a random sample associated to the sampling design $p$. Let $s$ be a realization of $S$. For any $k \in U$, the inclusion probability of $k$ is given by

$$\pi_k = \mathbb{P}(k \in S) = \sum_{k \in s} p(s),$$
where the sum is considered over all samples $s$ containing the individual $k$. If $k \neq l$ are two elements of $U$, the second-order inclusion probability of $k$ and $l$ is given by

$$\pi_{kl} = \mathbb{P}(k, l \in S) = \sum_{k, l} p(s),$$

where the sum is considered over all samples $s$ containing both $k$ and $l$.

In practice, a wide variety of selection schemes are used. We distinguish direct element sampling designs such as the simple random sampling without replacement (SRSWOR), stratified sampling (STRAT), or proportional-to-size sampling designs (with or without replacement). Most of these designs are used extensively in practice. However, such designs require having a sampling frame list identifying every population element which may be difficult, expensive, or even impossible to realize. In order to avoid it, more complex designs such as cluster or multi-stage designs can then be used. This is for example appropriate when the population is widely distributed geographically or may occur in natural clusters. Using such designs saves money and human efforts but entails a loss of efficiency. A detailed presentation of the survey sampling theory and many practical illustrations can be found in Korn & Graubard (1999), Lehtonen & Pahkinen (2004) and the reference book of Särndal et al. (1992).

The median $m_N$ given by (2) is a non-linear parameter of finite population totals defined by an implicit equation. In order to estimate $m_N$, we use the functional substitution approach proposed by Deville (1999) for multivariate variables $Y$ and extended to functional variables $Y$ by Cardot et al. (2010). Let $M$ be the discrete measure defined on $L^2[0, T]$ assigning the unity mass to each curve $Y_k$ with $k \in U$ and zero elsewhere, namely

$$M = \sum_{k \in U} \delta_{Y_k},$$

where $\delta_{Y_k}$ is the Dirac function in $Y_k$. The total mass of $M$ is $N$, the population size. Let $T$ be the functional with respect to $M$ and depending of $y$ as follows

$$T(M; y) = -\int \frac{\mathcal{Y} - y}{||\mathcal{Y} - y||} dM = -\sum_{k \in U} \frac{Y_k - y}{||Y_k - y||}. \quad (4)$$

Remark that $T$ defined in this way is the derivative with respect to $y$ of the objective function given in (1). The median $m_N$ is then defined as an implicit functional with respect to $M$,

$$T(M; m_N) = 0 \quad (5)$$

or equivalently,

$$\int \frac{\mathcal{Y} - m_N}{||\mathcal{Y} - m_N||} dM = 0. \quad (6)$$

Let $\widehat{M}$ be a weighted estimator of $M$ based on the sample $s$,

$$\widehat{M} = \sum_{k \in s} w_k \delta_{Y_k} = \sum_{k \in U} I_k w_k \delta_{Y_k}, \quad (7)$$

where $I_k = 1_{k \in s}$ is the sample membership indicator of element $k \in U$ (Särndal et al., 1992). In fact, $\widehat{M}$ is also a discrete and finite measure assigning the weight $w_k$ for each $Y_k$ with $k \in s$ and zero elsewhere. Usually, one take $w_k = 1/\pi_k$, the Horvitz-Thompson weights. In this case, we obtain the Horvitz-Thompson (1952) of $M$ which estimates unbiasedly the measure $M$ since $E_p(I_k) = \pi_k$, for all $k \in U$ for $E_p(\cdot)$ the expectation with respect to the sampling design $p(\cdot)$. The reader is referred to Cardot et al. (2010) and Cardot & Josserand (2011) for more details about the Horvitz-Thompson estimation with functional data. However, for $Y_1, \ldots, Y_N$ lying
in $\mathbb{R}^d$, weights that take into account auxiliary information have been suggested. We mention Deville (1999) for calibration weights or the very recent work of Goga & Ruiz-Gazen (2011) for non-parametric weights. Nevertheless, the extension to the functional case is not straightforward and it will be treated elsewhere. For the rest of the paper, we consider $w_k = 1/\pi_k$ and in Section 3.1, we suggest the post-stratified estimator of $M$.

Plugging $M$ into the functional expression of $m_N$ given by (5), yields the design-based estimator $\hat{m}_n$ of $m_N$. Hence, $\hat{m}_n$ verifies

$$T(\hat{M}; \hat{m}_n) = 0,$$

namely, $\hat{m}_n$ is the solution of the design-based estimating equation,

$$\sum_{k \in s} \frac{1}{\pi_k} \frac{Y_k - \hat{m}_n}{||Y_k - \hat{m}_n||} = 0. \quad (8)$$

Supposing now that for all $k \in s$, $Y_k \neq \hat{m}_n$ and that $Y_k$ are not concentrated on a straight line, we obtain that the solution $\hat{m}_n$ exists and is unique following the same arguments as in Kemperman (1987) or Chaudhuri (1996). The median estimator $\hat{m}_n$ is also called the substitution estimator of $m_N$ and it is defined by a non-linear implicit function of Horvitz-Thompson estimators. As a consequence, the variance as well as the variance estimator of $\hat{m}_n$ cannot be obtained directly using Horvitz-Thompson formulas. We will give in the next a first-order expansion of $\hat{m}_n$ in order to approximate $\hat{m}_n$ by the Horvitz-Thompson estimator for the finite population total of appropriate artificial variables.

2.2 Asymptotic Properties

The functional $T$ given by (4) is Fréchet differentiable (Serfling, 1980) with respect to the measure $M$ and $y$. Let $\Gamma$ be the Jacobian operator of $T$ with respect to $y$ and given by (Gervini, 2008)

$$\Gamma = \sum_{k \in U} \frac{1}{||Y_k - m_N||} \left[ I - \frac{(Y_k - m_N) \otimes (Y_k - m_N)}{||Y_k - m_N||^2} \right], \quad (9)$$

where $I$ is the identity operator defined by $Iy = y$ and the tensor product of two elements $a$ and $b$ of $L^2[0, T]$ is the rank one operator such that $a \otimes b(y) = \langle a, y \rangle b$ for all $y \in L^2[0, T]$. One can easily obtain that $\Gamma$ is a strictly positive operator, namely $\langle \Gamma y, y \rangle > 0$ and supposing that $N^{-1} \sum_{k \in U} ||Y_k - m_N||^{-1} < \infty$, we can get following the same arguments as in Cardot et al. (2011), that $\Gamma/N$ is a bounded operator, namely $||\Gamma/N||_\infty < \infty$ with $||\Gamma||_\infty = \sup_{||y||_2 \leq 1} ||\Gamma y||$. We recall that for the operator $\Gamma : L^2[0, T] \rightarrow L^2[0, T]$, we have

$$\Gamma y = \sum_{k \in U} \frac{1}{||Y_k - m_N||} \left( y - \frac{\langle Y_k - m_N, y \rangle}{||Y_k - m_N||^2} (Y_k - m_N) \right) \quad \text{for all } y \in L^2[0, T]$$

which gives

$$\Gamma y(t) = \sum_{k \in U} \frac{y(t)}{||Y_k - m_N||} - \int_0^T \gamma(r, t) y(r) dr$$

$$\gamma(r, t) = \sum_{k \in U} \frac{(Y_k(r) - m_N(r))(Y_k(t) - m_N(t))}{||Y_k - m_N||^2}.$$

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The median is defined by the implicit equation (6) and using then the implicit function theorem, we obtain that it exists a unique functional \( \tilde{T} \) such that

\[
\tilde{T}(M) = m_N
\]

and

\[
\tilde{T}(\hat{M}) = \hat{m}_N.
\]

Moreover, the functional \( \tilde{T} \) is also Fréchet differentiable with respect to \( M \) and the derivative of \( \tilde{T} \) with respect to \( M \) is called the influence function and defined, when it exists, as follows

\[
I \tilde{T}(M, \xi) = \lim_{\lambda \to 0} \frac{\tilde{T}(M + \lambda \delta_\xi) - \tilde{T}(M)}{\lambda}
\]

where \( \delta_\xi \) is the Dirac function at \( \xi \in L^2[0, T] \). Note that this definition suggested by Deville (1999) and extended to the functional case by Cardot et al. (2010) is slightly different from the usual definition of the influence function used in robust statistics (see, e.g. Hampel, 1974 or Serfling, 1980), which is based on a probability distribution instead of a finite measure. A non-standardized measure \( M \) is used in survey sampling because the total mass \( N \) may be unknown.

Under the asymptotic framework from Deville (1999), we may give a first-order von-Mises (1947) expansion of \( \tilde{T} \) in \( \hat{M}/N \) around \( M/N \),

\[
\tilde{T}\left(\frac{\hat{M}}{N}\right) = \tilde{T}\left(\frac{M}{N}\right) + \int I \tilde{T}\left(\frac{M}{N}, \xi\right) d\left(\frac{\hat{M}}{N} - \frac{M}{N}\right)(\xi) + o_p(n^{-1/2})
\]

(11)

which may be written in the equivalent form,

\[
\tilde{T}(\hat{M}) = \tilde{T}(M) + \int I \tilde{T}(M, \xi) d(\hat{M} - M)(\xi) + o_p(n^{-1/2})
\]

(12)

since \( \tilde{T} \) is a functional of degree zero, namely \( \tilde{T}(M/N) = \tilde{T}(M) \) and in this case, \( I \tilde{T}(\frac{M}{N}, \xi) = N \cdot I \tilde{T}(M, \xi) \) (Deville, 1999).

Let \( u_k \), for all \( k \in U \), be the linearized variables of \( \tilde{T}(M) = m_N \) and defined as the value of the influence function \( I \tilde{T} \) at \( \xi = Y_k \), namely

\[
u_k = I \tilde{T}(M, Y_k) = \Gamma^{-1}\left(\frac{Y_k - m_N}{||Y_k - m_N||}\right).
\]

(13)

We have used here the fact that for fixed \( y \), the functional \( T(M; y) = -\sum_{Y_k} \frac{Y_k - y}{||Y_k - y||} \) is a finite population total with influence function at \( Y_k \) given by \( I T(M, Y_k; y) = -(Y_k - y)/||Y_k - y|| \) (Deville, 1999). From the Riesz’s theorem, we have that for all bounded \( h \in L^2[0, T] \) there is a unique \( f \in L^2[0, T] \) such that \( \Gamma f = h \) and \( \Gamma f(g) = \langle h, g \rangle \) for all \( g \in L^2[0, T] \). This unique \( f \) will denote \( \Gamma^{-1} h \) for a given \( h \in L^2[0, T] \).

Hence, the expansion (12) becomes

\[
\hat{m}_n = m_N + \sum_{k \in \epsilon} \frac{u_k}{\pi_k} - \sum_{k \in U} u_k + o_p(n^{-1/2}).
\]

(14)

The above formula shows that the non-linear estimator \( \hat{m}_n \) may be approximated by the Horvitz-Thompson estimator for the total of the linearized variables \( u_k \). In this way, \( u_k \) is an artificial variable used to compute the approximative variance of \( \hat{m}_n \). Now, the linearized variable \( u_k \) is also a functional defined on \( L^2[0, T] \) and it is unknown since \( m_N \) and \( \Gamma \) are unknown. We
suggest estimating \( u_k \) by

\[
\hat{u}_k = \hat{\Gamma}^{-1}\left( \frac{Y_k - \hat{m}_n}{||Y_k - \hat{m}_n||} \right),
\]

where \( \hat{\Gamma} \) is given by

\[
\hat{\Gamma} = \sum_{k \in s} \frac{1}{\pi_k ||Y_k - \hat{m}_n||} \left[ I - \frac{(Y_k - \hat{m}_n) \otimes (Y_k - \hat{m}_n)}{||Y_k - \hat{m}_n||^2} \right].
\]

Using relation (14), one can obtain the asymptotic variance function of \( \hat{m}_n \) calculated under the sampling design,

\[
\text{var}(t) = \sum_{k \in U} \sum_{l \in U} (\pi_{kl} - \pi_k \pi_l) \cdot \frac{u_k(t)}{\pi_k} \cdot \frac{u_l(t)}{\pi_l} = \mathbf{u}'(t) \mathbf{\Delta} \mathbf{u}(t) \quad \text{for all} \quad t \in [0, T]
\]

where \( \mathbf{u}(t) = (u_k(t))_{k \in U} \) with \( u_k(t) \) is given by (13) and \( \mathbf{\Delta} = (\frac{\pi_{kl} - \pi_k \pi_l}{\pi_k \pi_l})_{k,l \in U} \). The variance is estimated by

\[
\hat{\text{var}}(t) = \sum_{k \in s} \sum_{l \in s} (\pi_{kl} - \pi_k \pi_l) \cdot \frac{\hat{u}_k(t)}{\pi_k} \cdot \frac{\hat{u}_l(t)}{\pi_l} = \mathbf{\hat{u}}'(t) \mathbf{\hat{\Delta}} \mathbf{\hat{u}}(t),
\]

where \( \mathbf{\hat{u}}(t) = (\hat{u}_k(t))_{k \in s} \) with \( \hat{u}_k \) given by (15) and \( \mathbf{\hat{\Delta}} = (\frac{\pi_{kl} - \pi_k \pi_l}{\pi_k \pi_l \pi_{kl}})_{k,l \in s} \).

**Remark.** It is worth mentioning that the linearized variable \( u_k \) plays a central role for the estimation of the median. More exactly, the efficiency of any sampling design used for estimating the median curve depends on how well it estimates the total of the linearized variable \( u_k \). For example, a stratified strategy will be efficient if the strata are homogeneous with respect to \( u_k \) as it will be showed below. Nevertheless, to put in practice such a design requires knowing all \( u_k \) which may not be readily available.

**Remark.** In practice, we observe the curves \( Y_k \) at \( D \) discretized points, say \( 0 \leq t_1 < t_2 < \cdots < t_D \leq T \) that we suppose to be the same for all the curves. When the discretization points vary to one curve to another, methods described in Ramsay & Silverman (2005) may be employed. In order to compute numerical approximations to integrals and inner products, quadrature rules are used.

With discretized points, curves may be viewed as multidimensional vectors, in our case, \( \mathbf{Y}'_k = (Y_k(t_1), \ldots, Y_k(t_D)) \) and \( \mathbf{\hat{u}}'_k = (\hat{u}_k(t_1), \ldots, \hat{u}_k(t_D)) \). For each \( k \in s \), we need to compute the estimated linearized variable in points \( t_1, \ldots, t_D \). Let \( \mathbf{\hat{u}}_k = (\mathbf{\hat{u}}'_k)_{k \in s} \) be the sample vector of estimated linearized variables which can be derived by solving the \( D \times n \) dimensional system

\[
\mathbf{\hat{\Gamma}} \mathbf{\hat{u}}_k = \left( \frac{\mathbf{Y}_1 - \hat{m}_n}{||\mathbf{Y}_1 - \hat{m}_n||}, \ldots, \frac{\mathbf{Y}_n - \hat{m}_n}{||\mathbf{Y}_n - \hat{m}_n||} \right),
\]

where \( \mathbf{\hat{\Gamma}} \) given by (16) is replaced by a \( D \times D \) symmetric matrix. The variance estimator is then derived directly using (18).

### 3 Application to the EDF Load Curves

#### 3.1 General Settings

The volume of data treated and analyzed by Électricité De France is increasing greatly. In fact, in the next few years Electricité De France plans to install millions of smart electricity...
meters that will be able to send, on request, electricity consumption measurements every second. Obviously, it will be difficult to store and analyse online all these information. The statistician’s challenge is to find a strategy, meaning indicators and estimation methods, capable to give a good description of data and to use it for forecasting. While working with huge data, methods not being time-consuming are highly desirable.

Our proposal consists in considering the median curve as a robust indicator of the data and estimating it with probability sampling designs. As Lohr stated in Sampling: Design and Analysis (1999): “If a probability sampling design is implemented well, an investigator can use a relatively small sample to make inferences about an arbitrarily large population.”

Let \( U \) be a population of \( N = 18\,902 \) electricity meters installed in small and large companies sending every 30 min the electricity consumption during a period of two weeks. We aim at estimating the median curve of the electricity consumption during the second week whereas the consumption recorded during the first week will be used as auxiliary information. This means that we have 336 time measures. So, our study population of curves is a set of \( N = 18\,902 \) vectors \( Y_k = (Y_k(t_1), \ldots, Y_k(t_D)) \) with \( D = 336 \). Let \( X_k \) be the consumption curve for the \( k \)-th firm and recorded during the first week. The consumption curves present low peaks corresponding to night time measurements and high peaks corresponding to middle day measurements. The electricity consumption decreases roughly around the 250th time measurement which corresponds to the beginning of weekend time. The mean and median curves present the same effect as we can see in Figure 1.

We consider several strategies of fixed size \( n = 2\,000 \) and we compare them through simulations. We distinguish two kinds of sampling designs whether they use or do not use auxiliary information. If auxiliary information is used at the sampling stage, some changing are needed because the variables involved now are curves. On the opposite situation, the selection of the sample is realized from the sampling frame list as for classical multivariate surveys. Finally, the frame list of French firms is well constructed being very often updated and most of the designs considered below are usually used in practice.

3.1.1 Simple random sampling without replacement (SRSWOR)

The SRSWOR sampling is a very simple design easy to put into practice. Every possible subset of \( n \) units in the population has the same chance to be the sample. In a functional framework, the selection of a sample of \( n = 2\,000 \) curves is performed as for the multivariate surveys, namely \( n \) labels are drawn from the list of \( N \) companies. The estimation of the median curve with SRSWOR is obtained from equation (8) for \( \pi_k = n/N \), namely \( \hat{m}_n \) is the unique solution of the following equation

\[
\sum_{k \in s} \frac{Y_k - \hat{m}_n}{||Y_k - \hat{m}_n||} = 0. \tag{19}
\]

The asymptotic variance function is equal to

\[
\text{var}_{\text{SRSWOR}}(t) = N^2 \left( \frac{1}{n} - \frac{1}{N} \right) S_{u(t),U}^2, \quad \text{for all } t
\]

where \( S_{u(t),U}^2 = \sum_{k \in U} (u_k(t) - \bar{u}(t))^2/(N - 1) \) with \( \bar{u}(t) = \sum_{k \in U} u_k(t)/N \). This variance is estimated by

\[
\hat{\text{var}}_{\text{SRSWOR}}(t) = N^2 \left( \frac{1}{n} - \frac{1}{N} \right) S_{\hat{u}(t),s}^2, \quad \text{for all } t \tag{20}
\]
where \( S^2_{\hat{u}(t),s} = \sum_{k \in s} (\hat{u}_k(t) - \bar{u}_s(t))^2 / (n - 1) \) with \( \bar{u}_s(t) = \sum_{k \in s} \hat{u}_k(t) / n \) and \( \hat{u}_k(t) \) given by (15).

3.1.2 Systematic sampling (SYS)

We consider the systematic design in its basic form (Särndal et al., 1992). The inclusion probabilities are \( \pi_k = n / N \), so the median estimator is obtained according to the same equation (19). It is well known that the systematic sampling may be very inefficient compared to the SRSWOR sampling if the systematic samples are homogeneous. One way to improve the efficiency of SYS sampling is to order the sampling frame list according to an auxiliary variable highly correlated with the variable of interest. In this way, adjacent elements tend to be more similar than elements that are farther apart. In our study, we ordered the frame according to the mean electricity consumption during the first week, namely the variable \( \hat{X}_k = \sum_{d=1}^{D} X_k(t_d) / D \), for all \( k \in U \) and \( D \) is the number of discretization points in \([0, T]\). Another trade-off for the simplicity of SYS sampling is that there is no unbiased estimator of the design variance function since \( \pi_{kl} = 0 \) for all \( k \) and \( l \) not belonging to the same systematic sample. However, using the ordering according to the variable \( \hat{X}_k \), the SYS is at least as good as the SRSWOR sampling. So, we might use the variance estimator appropriate for the SRSWOR design given in (20).

Systematic sample is really a special case of cluster sampling, so it is often used when it is difficult to construct a sampling frame in advance.

3.1.3 Stratified sampling with simple random sampling without replacement within strata (STRAT)

In this case, the population is divided into \( H \) non-overlapping strata denoted \( U_h \) and a simple random sample without replacement is selected independently in each stratum. Let \( n_h \) be the sample size within stratum \( h \) and \( N_h \) be the stratum size. To obtain the median estimator, we solve the estimation equation

\[
\sum_{h=1}^{H} \sum_{s_h} \frac{Y_k - \hat{m}_n}{\pi_k^n ||Y_k - \hat{m}_n||} = 0,
\]

where \( s_h = s \cap U_h \) and \( \pi_k^n = n_h / N_h \). It is well known that stratification may substantially improve the quality of estimates compared to simple random sampling without replacement and systematic sampling if the strata are well constructed. More exactly, the more homogeneous the strata are the more efficient the stratification is. It is worth mentioning that improving the estimation of the median curve means constructing strata homogeneous with respect to the linearized variables, \( u_k \). Indeed, relation (17) gives us that the asymptotic variance function of \( \hat{m}_n \) with STRAT is

\[
\text{var}_{\text{STRAT}}(t) = \sum_{h=1}^{H} N_h^2 \left( \frac{1}{n_h} - \frac{1}{N_h} \right) S^2_{u(t),U_h},
\]

where \( S^2_{u(t),U_h} \) is the population variance within stratum \( h \) of \( u(t) = (u_k(t))_{k \in U_h} \). That is, the lower the variation of the linearized variable within stratum, the lower the asymptotic variance of \( \hat{m}_n \). The variance estimator is the sum of variance estimators (20) computed within each stratum.

Usually, one builds the stratification using a variable known on the whole population and strongly correlated with the variable of interest. In our case, we suggest two stratification variables computed using the first week: the first one is the linearized variable \( u_k \) and the second
one is the consumption $Y_k$ (Cardot & Josserand, 2011). The following two sample allocations are used:

- the proportional allocation (PROP): $n_h = n N_{h}/N$ for all $h = 1, \ldots H$.
- the $u^{(1)}$-optimal allocation ($u^{(1)}$-OPTIM) as suggested by Cardot & Josserand (2011) and computed here with respect to the variance $S_{u^{(1)}(t), U_h}^2$ of the linearized variable computed during the first week and denoted by $u^{(1)}_k$:

\[
n_h = n \frac{N_h}{\sum_{h=1}^{H} N_h} \sqrt{\int_0^T S_{u^{(1)}(t), U_h}^2 dt} \quad h = 1, \ldots, H.
\]

The $u^{(1)}$-optimal allocation is similar to the Neyman optimal allocation but computed using $u^{(1)}$ instead of $u$. The $x$-optimal allocation is obtained when the consumption during the first week $X_k$ is used.

(a) **Stratification based on the linearized variable during the first week.** The proposed strategy can be split into two steps:

**Step 1:** we calculate the linearized variables $u^{(1)}_k$ for all $k \in U$ during the first week.

**Step 2:** we stratify the population $U$ using the k-means clustering algorithm with the euclidean distance and applied to the linearized variables $u^{(1)}_k$ for $k \in U$. According to within cluster variance considerations, we decide to keep $H = 4$ different clusters. The strata sizes as well as both the proportional and $u^{(1)}$-optimal allocation are given in Table 1.

We plot in Figure 2(a), the mean of $u_k$ computed during the second week and within the $H = 4$ strata. Differences among the strata means are noticeable accounting for a significant gain in efficiency if the proportional allocation is used. Now, to better see what kind of consumers the four strata are built of, we plot in Figure 2(b) the mean of the consumption $Y_k$. We remark that the stratification based on $u^{(1)}_k$ induces a stratification for the consumption curves also.

(b) **Stratification based on the consumption curve during the first week.** Cardot & Josserand (2011) suggested taking $H = 4$ strata corresponding to the maximum level of consumption during the first week $X_k$ and based on quartiles so that all strata have the same size. We denote the allocation obtained in this way by $x$-OPTIM. The strata sizes as well as both the proportional and the $x$-optimal allocation are given in Table 2.

We plot in Figure 3(b), the consumption mean within strata and during the second week. We notice that the stratum 4 corresponds to consumers with high global levels of consumption.
Figure 2. Stratification based on the linearized variable: (a) Mean of linearized variables $u_k$ within each stratum. (b) Mean of the consumption curve $Y_k$ within each stratum.

Table 2
Strata sizes, proportional and $x$-optimal allocations for $n = 2000$.

| Stratum number | 1   | 2   | 3   | 4   |
|----------------|-----|-----|-----|-----|
| Stratum size $N_k$ | 4725 | 4726 | 4725 | 4726 |
| PROP allocation   | 500 | 500 | 500 | 500 |
| $x$-OPTIM allocation | 126 | 212 | 333 | 1329 |

whereas stratum 1 corresponds to consumers with low global of consumption. Figure 3(a) gives the mean curves of the linearized variable within strata and computed for the second week. As for the first stratification, the population of the linearized variable curves is also stratified.

3.1.4 Proportional-to-size sampling (PPS)

Unequal probability designs are widely used in practice because they are usually more efficient than the equal probability designs. In PPS sampling, the sampling is with-replacement and the probability $p_k$ with which the individual $k$ is selected is proportional to a positive measure $X_k$, where $X_k$ is an auxiliary variable roughly proportional to the study variable $Y_k$. The probability of selection has the expression

$$p_k = \frac{X_k}{\sum_{k \in U} X_k}.$$  

In our situation, the study variable is a curve and so is the auxiliary information. To cope with this problem, we suggest using $p_k$ proportional to the mean of $X_k(t)$ over all $t = 1, \ldots, D$ where $D$ is the number of discretization points in the interval $[0, T]$. This means that

$$p_k = \frac{\bar{X}_k}{\sum_{k \in U} \bar{X}_k},$$
where $\tilde{X}_k = \sum_{t=1}^{D} X_k(t)/D$. For our study, we consider again $X_k$ as being the electricity consumption for the $k$-th firm recorded during the first week. The inclusion probabilities are given by $\pi_k = 1 - (1 - p_k)^n$. The Horvitz-Thompson estimator of the median is obtained by solving the equation

$$\sum_{k \in \tilde{s}} \frac{Y_k - \hat{m}_n}{\pi_k||Y_k - \hat{m}_n||} = 0,$$

where $\tilde{s}$ is the set of distinct elements of $s$. In with-replacement designs, one may use the Hansen & Hurwitz (1943) estimator which presents the advantage that the variance formula is easier (no double sums are needed).

### 3.1.5 Post-stratification (POST)

Let consider now the post-stratification which is one of the simplest way to take into account auxiliary information in order to improve the Horvitz-Thompson estimator of the median. We suppose that the population is partitioned into sub-populations $U_1, \ldots, U_G$ according to a given classification principle. These sub-populations are called post-strata since they do not serve for performing stratified sampling as described before. Practical considerations may favor some other (perhaps simpler or less costly) designs, such as SRSWOR from the whole population $U$. After the sample selection, $Y_k$ is observed for the elements $k \in s$ and the sampling frame is used to establish the group each individual belongs to. Remark that group memberships may be unknown before the sample selection which makes impossible to perform the stratified sampling. Nevertheless, the group membership totals $N_g$ are known for all $g = 1, \ldots, G$ and this auxiliary information may be used to construct an improved estimator of $m_N$. The weights used in this case are given by

$$w_{ks} = \frac{N_g}{\hat{N}_g \pi_k} \quad \text{for all} \quad k \in s_g = s \cap U_g$$
where \( \hat{N}_g = \sum_{k \in s_g} 1/\pi_k \). Hence, the post-stratified estimator of \( m_N \) is obtained by solving the following equation

\[
\sum_{g=1}^{G} \sum_{k \in s_g} \frac{N_g}{N_g \pi_k} \frac{Y_k - \hat{m}_n}{||Y_k - \hat{m}_n||} = 0. \tag{22}
\]

It is important to notice that the weights used here depend on the sample and are more general than the Horvitz-Thompson weights used before in the sense that they include the auxiliary information given by the group size \( N_g \). In this case, the auxiliary information is used at the estimation stage and not at the design stage.

With SRSWOR sampling from \( U \), the post-stratified weights become \( w_k = N_g/n_g \) for all \( k \in s_g \) and \( n_g \) the size of \( s_g \). The median estimator verifies

\[
\sum_{g=1}^{G} \sum_{k \in s_g} \frac{N_g}{n_g} \frac{Y_k - \hat{m}_n}{||Y_k - \hat{m}_n||} = 0. \tag{23}
\]

One should remark that in the case of the post-stratification, the samples sizes \( n_g \) are random. The post-stratified estimator of the median given by (23) cannot be computed if some sample sizes \( n_g \) are equal to zero. However, if the total sample size \( n \) is large enough, and if no group accounts for a very small proportion of the whole population, then the probability of having \( n_g = 0 \) is very small. Särndal et al. (1992) suggest aggregating small groups in order to guarantee that \( n_g \) are at least 20.

The post-stratified estimator is a calibrated estimator (Deville & Särndal, 1992) when the auxiliary information is the group memberships with known totals. Using the same arguments as in Deville (1999), the expansion given in (14) remains valid and as a consequence, the asymptotic variance of \( \hat{m}_n \) is equal to the variance of residuals \( E_k = u_k - \bar{u}_g \), \( \bar{u}_g = \sum_{k \in U_g} u_k / N_g \) and with SRSWOR, we obtain

\[
\text{var}(t) = N^2 \left( \frac{1}{n} - \frac{1}{N} \right) \sum_{g=1}^{G} \frac{N_g - 1}{N - 1} S^2_{u(t),U_g},
\]

where \( S^2_{u(t),U_g} \) is the group variance. We can remark that the variance agrees very nearly with the asymptotic variance for the stratified sampling with proportional allocation.

We can conclude that the SRSWOR with post-stratification is essentially as efficient as STRAT sampling with proportional allocation unless the sample is very small. This result is well known for multivariate variables \( Y_k \) (see, e.g. Särndal et al. 1992). We have developed above a strategy based on the linearized variables to obtain well-constructed strata. One drawback with that method is that the so-constructed strata reduce the variance for the median estimator but could be inefficient for many other variables. Therefore, using SRSWOR with post-stratification will often improve overall efficiency.

**Consistency of \( \hat{m}_n(t) \) from survey data.** Focus is now on the estimation of the median curve of the consumption recorded during the second week. We consider for that the following sampling designs of size \( n = 2000 \) with the Horvitz-Thompson estimator: SRSWOR, STRAT based on the two stratification variables and with proportional and optimal allocation, SYS, PPS. We
consider for our study the POST estimator also. Figure 4 shows the estimation of the median curve computed from one sample selected according to four sampling designs.

In order to compare these designs, we made 500 replications and considered the following loss criteria,

$$R(\hat{m}_n) = \int_0^T |\hat{m}_n(t) - m_N(t)|dt.$$ 

Since in our study we have equally spaced discretized time measurements, the above loss criterion is approximated due to quadrature rules by 

$$\frac{1}{D} \sum_{d=1}^D |\hat{m}_n(t_d) - m_N(t_d)|.$$ 

Basic statistics, respectively boxplots, for the estimation errors of the median function estimator are given in Table 3, respectively Figure 5. The stratification variable used here is the one based on the linearized variables $u_{(1)}^k$. First, we can observe that clustering the space of functions by performing stratified sampling leads to an important gain in terms of accuracy of the estimators, dividing by at least two times the mean error compared to simple random sampling without replacement. We note that the post-stratification gives results similar to those given by stratified sampling.

Table 3

| Estimation errors for $m_N$. The bold values indicate the best strategy. |
|---------------------------------|-----------------|-----------------|-----------------|-----------------|
|                                 | Mean            | 1st Quartile    | Median          | 3rd Quartile    |
| SRSWOR                          | 2.531           | 1.322           | 1.982           | 3.351           |
| SYS                             | 2.625           | 1.355           | 2.412           | 3.087           |
| PROP                            | 1.060           | 0.8498          | 1.017           | 1.234           |
| $u_{(1)}$-OPTIM                 | **1.000**       | **0.7946**      | **0.9552**      | **1.142**       |
| POST                            | 1.041           | 0.8275          | 0.9785          | 1.203           |
| PPS                             | 7.1410          | 2.7880          | 6.1370          | 9.5600          |

Figure 4. One sample estimation of the median trajectory for $n = 2000$ with four sampling strategies (SRSWOR, SYS, STRAT.OPT, and STRAT.PROP).
with proportional allocation and that the SYS design with ordering on the first week mean consumption is almost as good as the SRSWOR design. A rather surprising result is obtained with PPS sampling. Simulations results not reported here show that this design performs very well for estimating the mean consumption curve, being as good as the stratified sampling but fails for the median curve. We believe that this fact is due to numerical problems encountered in the resolution of the implicit equation (21) when a large number of small probabilities of selection $p_k$ are used to estimate $m_N$. More research is needed to better clarify this issue and to find a way to improve it.

We also performed simulations for the second stratification based on the first week consumption $X_k$ and the results are given in Table 4.

We can remark that STRAT with proportional allocation and stratification based on $\mu^{(1)}$ gives better results than STRAT with $x$-optimal allocation stratification based on $X$. This result is not surprising since in the latter case, the strata have been constructed taken into account the consumption variable and the optimal allocation has been computed by minimizing the variance for the mean estimator while we are interested here in estimating the median curve. This is why, the proportional allocation is usually advisable with multipurpose surveys. Moreover, if we compare the two stratifications, we remark that the stratification based on the consumption

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Table 4

|          | Mean  | 1st Quartile | Median | 3rd Quartile |
|----------|-------|--------------|--------|--------------|
| PROP     | 1.7370| 1.0470       | 1.4860 | 2.2480       |
| $x$-OPTIM| 2.2940| 1.4660       | 1.9790 | 2.7830       |

---

Figure 5. Comparison of the distribution of estimation errors of the median curve for SRSWOR, PROP, OPTIM, and SYS.
variable is less efficient than the stratification based on the linearized variable but it remains still better than the SRSWOR or SYS designs.

Both stratifications used in this paper need the consumption curve $Y_k$ computed during the first week for all the individuals from the population. Sometimes, this can be too costly to obtain or even impossible because of storage or confidentiality constraints. In such situations, some other stratification variables may be considered such as for example, the electricity power given by the subscribed contract between one firm and EDF.

**Consistency of the variance function estimation from survey data.** We analyze in the following the estimator for the variance function $\text{var}(t)$ when the SRSWOR and STRAT designs are used. To judge the quality of the estimators, we use the following criterion

$$R(\text{var}) = \int_0^T |\text{var}(t) - \text{var}(t)| \, dt.$$ 

We give in Table 5 statistics for the estimation errors of the variance function estimation with SRSWOR and stratified sampling with proportional and $u^{(1)}$-optimal allocations. Figure 6 gives the theoretical standard deviation function curves of $\hat{\mu}_n$, $\sqrt{\text{var}(t)}$, with the considered designs.
One can remark that the theoretical variance is much smaller, at all instants $t$, for the stratified sampling with optimal allocation rule. The stratified sampling with optimal allocation gives more accurate estimation of $\text{var}(t)$ than the other strategies. We can observe that clustering the space of functions by performing stratified sampling may leads to a considerable gain in terms of accuracy of the estimators of the variance function, dividing by ten the mean error compared to simple random sampling without replacement. Moreover, there is also a difference between proportional and optimal allocations rules, for example the third quartile in optimal case is lower than the median loss in the proportional case.

4 Conclusion and Perspectives

In this paper, we have developed a survey sampling approach for estimating the median of a functional variable. From a practical point of view, an appealing consequence of the new methodology is that the proposed estimators are faster to calculate. The experimental results on a test population of electricity consumption curves confirm that even with high dimensional data, stratification associated with the optimal allocation rule leads to important reduction of the variance estimators. Having appropriate strata is the key for getting more accurate estimators and the $k$-means algorithm is well adapted in this situation. Nevertheless, choosing the stratification variables is a rather complex issue and more work is needed in this direction.

A challenging future research avenue concerns the use of auxiliary information at the estimation stage. While, in this paper, we have concentrated on the estimation of the median using the Horvitz-Thompson estimator or the post-stratified estimator, more complex estimators using functional regression models can be developed. For example, it is possible to set a linear functional model which explains the functional variable $Y_k$ using a scalar $X_k$ and to develop a regression estimator for the median curve. Developing a general framework for regression estimators for the median curve is left for future studies.

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Les courbes de profil moyen sont largement utilisées comme indicateurs du comportement de consommation d’électricité des clients. À Électricité de France (EDF), les profils de consommation de chaque catégorie de clients sont estimés par des courbes de charge moyennes. Malheureusement, la moyenne est très sensible à la présence des valeurs atypiques. Dans ce travail, nous proposons une alternative à la courbe moyenne : la L1-médiane, qui est plus robuste. En présence de grandes bases de données de type courbes (les courbes de charge, par exemple), des approches par sondages sont une alternative intéressante pour estimer la courbe médiane en évitant le stockage de courbes. Nous proposons ici plusieurs stratégies d’échantillonnage et plusieurs estimateurs de la courbe médiane. Une comparaison entre les différentes stratégies est réalisée sur une population-test. Nous développons une stratification basée sur les variables linéarisées qui améliore de façon substantielle l’estimateur par rapport au sondage aléatoire simple sans remise. Nous suggérons également un estimateur qui prend en compte l’information auxiliaire. Plusieurs directions de recherche sont envisagées.

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