Towards a possible charge Kondo effect in optical lattices

M. S. Laad\(^1\)(*), L. Craco\(^2\) and A. Taraphder\(^3,4\)

\(^1\)Lehrstuhl für Theoretische Physik, Technische Universität Dortmund - 44221 Dortmund, Germany, EU
\(^2\)Max-Planck-Institut für Chemische Physik fester Stoffe - 01187 Dresden, Germany, EU
\(^3\)Department of Physics and Centre for Theoretical Studies, Indian Institute of Technology, Kharagpur, 721302 India
\(^4\)Max-Planck-Institut für Physik komplexer Systeme - 01187 Dresden, Germany, EU

received 25 August 2009; accepted 24 September 2009
published online 5 November 2009

PACS 03.75.Ss – Degenerate Fermi gases
PACS 71.10.Fd – Lattice fermion models (Hubbard model, etc.)
PACS 71.10.Hf – Non-Fermi-liquid ground states, electron phase diagrams and phase transitions in model systems

Abstract – The Kondo effect underpins a large body of recent developments in the physics of \(d\)- and \(f\)-band compounds. Although its charge analog is a rarity in solids, the recent observations of the charge Kondo effect and the consequent rise in superconducting \(T_c\) encourage a search for other accessible systems. Motivated by the possibility of wilfully tuning the sign of the inter-electronic interaction in optical lattices, we study conditions for the elusive charge Kondo liquid (CKL) state to manifest. We propose that a combination of Feshbach resonances and sequentially controlled laser pulses may produce the CKL. We show that the CKL is never a stable ground state, appearing only when the ordered ground states are destabilized. Finally, we discuss interesting analogies with nuclear matter.

The Kondo effect has historically played a monumental role in solid-state physics [1], where quasi-bound state (singlet) formation involving a local moment screened by the itinerant electron spins leads to a renormalized Fermi liquid (FL) with huge enhancements of quasiparticle masses, whence the name “heavy fermion”. Subsequently, more exotic, quadrupolar [2] and orbital [3] versions were invoked in other contexts. The destruction of FL behavior at quantum phase transitions near magnetic order in some \(f\)-electron systems is thought to be linked to “unbinding” of this Kondo quasi-bound singlet state [4]. However, the intriguing possibility of observing a charge analogue of the Kondo effect, requiring an attractive, local interaction, has not been addressed carefully. In practice, materials such as Ba\(_{1-x}\)(K, Pb)\(_x\)BiO\(_3\) are modelled by variants of the \(U < 0\) Hubbard model [5–8].

Recent experimental realization [9] of the charge Kondo effect (CKE), predicted theoretically some time back [10], and the consequent rise of superconducting \(T_c\) in doped PbTe [11] and SnTe [9], has re-ignited interest in the \(U < 0\) models. Here, chemical doping (\(\leq 2\%\)) generates a narrow impurity band in the semiconductor band gap, facilitating the use of \(U < 0\) lattice models in the intermediate- to-strong–coupling limit (\(|U|/W_{imp} > 1\)). The interesting issues for theory are thus: What are the precise conditions under which a charge Kondo Fermi liquid state (the CKL state) can appear? Is it ever a stable phase at zero temperature? Might a quantum phase transition (QPT), reminiscent of its well-studied spin analogue, occur, and if so, how? What instabilities might one generically expect to arise near such a QPT?

Advances in artificially engineered fermionic/bosonic optical lattices allowing wilful manipulation of parameters open up yet another route to test model Hamiltonian predictions. Such systems may hold more promise for CKE, given the paucity of real materials exhibiting \(U < 0\). In contrast to real materials, these systems are free of inhomogeneities. In this context, various interacting models have already been realized [12,13]. The physical conditions for this lie well within the regime of experimental techniques available. In practice, different hyperfine states of same or different atomic species (acting, as it may, as different fermionic/bosonic species) can be trapped and controlled independently. A particularly attractive feature is their ready tunability: it is even possible to choose the sign of the interaction in such systems; a mixture of two

\(*\)E-mail: laad@fkt.physik.uni-dortmund.de
fermionic species interacting via an attractive interaction is achieved by forcing a mixture of two such atomic spin states through a Feshbach resonance, whence a bound state appears in the two-particle problem. A p-wave Feshbach resonance [14] can even create a tunable asymmetry in the interactions, allowing one to access more complex “hidden” ordered states. Further, the periodic potential of each species of atoms was independently tuned in an optical lattice [15]. This could facilitate observation of the elusive CKL state.

We use dynamical mean-field theory (DMFT) to study realizable values for the hoppings and \( U \). Here, we show how a combination of the Hubbard model (driven by staggered periodic lattice potential) to Mott insulator (due to on-site interaction, \( U \)) transition, and possible emergence of correlated metallic phases sandwiched between them.

Optical lattices thus provide a unique tool to address interesting questions posed above. A suitable model for a two-component fermionic system (having an attractive two-body interaction), with temporally separated laser pulses simulating inter-site hybridization between the two spinless fermionic species \( (a, b) \) is described by the Hamiltonian

\[
H = \sum_{<ij>}(a_i^\dagger b_j + h.c.) - U_{ab} \sum_i n_{ia} n_{ib} + \Delta_0 \sum_i (n_{ia} - n_{ib}) + \sum_{i,a,b} \epsilon_a n_{ia}.
\]  

(1)

The hoppings are \( t_{aa} (t_{bb}) \) for \( a-a (b-b) \) hopping and \( t_{ab} \) for \( a-b \) hopping between nearest neighbors. \( \Delta \) is the “charge transfer” term related to on-site fermionic energies. In typical optical-lattice systems, the model parameters expressed in terms of the recoil energy are hoppings \( t = \frac{\hbar^2}{m \gamma_{\text{in}}^2} \left( \frac{\gamma_{\text{in}}}{\gamma_{\text{out}}} \right)^{3/4} e^{-2 \sqrt{\gamma_{\text{out}}/E_R}} \) and local correlation \( U \approx E_R \sqrt{8/\pi} a_{KL}(V_0/E_R)^{3/4} \) [12]. Here, the recoil energy, \( E_R = \hbar^2 k_L^2 / 2m \) is typically of order a few micro-kelvin. (\( k_L \) and \( V_0 \) are the wave vector and intensity of the laser, respectively). \( \gamma_{\text{in}} \) is the s-wave scattering length, and the fermionic band width, \( W \ll E_R \).

Typically, \( t/h \approx 1 \text{kHz} \) and \( U/h \approx 20-30 \text{kHz} \) [13]. So \( U/W \approx O(2-3) \) in three dimensions. We envisage comparable values for the hoppings and \( U_{ab} \) in our case. This is precisely the intermediate to strong-coupling regime of the \( U_{ab} < 0 \) “Hubbard-like” model we study below. Focussing solely on possible Kondo-like physics, we use dynamical mean-field theory (DMFT) to study the model. Its reliability for understanding quasi-local collective Kondo coherence in correlated metals is already established [16]. Here, we show how a combination of experimental manipulations in optical lattices facilitates observation of the elusive CKL state.

Equation (1) is a generalized negative-\( U \) Hubbard model (relabelling \( a \rightarrow c_1, b \rightarrow c_2 \)) with “spin”-dependent hoppings, and a “magnetic field”. Given the isomorphism between the \( U > 0 \) Hubbard model and the \( U < 0 \) Hubbard model in a magnetic field (seen by performing a particle-hole transformation), the possibility of the charge Kondo effect (CKE) involving pair pseudospin quenching by itinerance manifests itself (the pseudospin \( \tau_i^a = c_i^a \sigma_i^{ab} c_{ib} \)). On the other hand, lack of pseudospin \( SU(2) \) invariance of \( H \) (lowered to \( Z_2 \times U(1) \)) generically favors ordered ground states, either a charge density wave (CDW) or a superfluid (SF). A CKL state can then appear as a ground state only upon quantum melting of these ordered states, or for model parameters such that \( T_{\text{CKL}} > T_{\text{CDW, SF}} \).

To proceed, we “rotate” the \( a, b \)-fermions writing \( f_a = (u a + vb) / \sqrt{u^2 + v^2} \), \( f_b = (ua - ub) / \sqrt{u^2 + v^2} \) with \( u = \sqrt{\tau_{aa} / (\tau_{aa} + \tau_{bb})} \), \( v = \sqrt{\tau_{bb} / (\tau_{aa} + \tau_{bb})} \) on the parameter curve \( \tau_{ab} = \sqrt{\tau_{aa} \tau_{bb}} \) leads to

\[
H = t \sum_{\langle ij \rangle} (f_{ia} f_{ja} + h.c.) - U_{ab} \sum_i n_{ia} n_{ib} + \Delta \sum_i (f_{ia} f_{ib} + h.c.) + \sum_{i,a,b} \epsilon_a n_{ia},
\]  

(2)

where \( t = (\tau_{aa} + \tau_{bb}) \), \( \Delta = \Delta_0 - uv \) and \( \epsilon'_a = v^2 e_a, \epsilon'_b = u^2 e_b \). This is the \( U < 0 \) Falicov-Kimball model (FKM) with a local hybridization [17]. If \( \Delta \) is tuned to zero, this reduces to the pure FKM. Since the FKM can be exactly solved in \( D = \infty \) [18], we first solve \( H_{FKM} \) and then consider the effects of \( \tau_{ab} \neq \sqrt{\tau_{aa} \tau_{bb}} \).

We observe that \( n_{ia} f_{ia}, H_{FKM} \) = 0 for each \( i \), implying a local \( (1) \) invariance of \( H_{FKM} \). This is exactly the condition for degenerate, resonant charge fluctuations between local configurations with \( n_{ia} = 0 \). Elitzur’s theorem implies that both local excitonic \( (\langle f_{ia}^\dagger f_{ib} \rangle) \) and local pairing \( (\langle f_{ia}^\dagger f_{ib} \rangle) \) averages rigorously vanish. Choosing the \( f_a \)-band density of states (DOS) to be a Lorentzian with half-width \( W \), i.e., \( \rho_0(\epsilon) = W / [\pi (\epsilon^2 + W^2)] \), the \( f_a \)-band propagator within DMFT is exactly calculable as \( G_{f_a}(\epsilon) = \frac{\Delta}{\pi} \int \frac{d\omega}{\epsilon - \epsilon_0(\omega) - \Sigma_{f_a}(\omega)} \) [4].

\[
\eta = \int \frac{d\omega}{W} \frac{\rho_0(\omega)}{\epsilon_0(\omega)} \frac{\Sigma_{f_a}(\omega)}{\epsilon_0(\omega)} \frac{\epsilon_0(\omega)}{\epsilon_0(\omega) - \Sigma_{f_a}(\omega)} = \frac{\epsilon_0(\omega)}{\epsilon_0(\omega) - \Sigma_{f_a}(\omega)},
\]

where \( n_{fa} = (1/N) \sum_i (f_{ia} f_{ia}) \) with \( N \) the number of lattice sites. Correspondingly, the \( f_a \)-Fermion propagator exhibits an infra-red singular, branch cut form, \( G_{f_a}(\omega) \theta(\omega) |\omega| - (1 - \eta) \) at low energy. Here, \( \eta = \text{tan}^{-1}(U_{ab}/W) / \pi \) is the so-called \( s \)-wave phase shift of the \( f_a \)-band fermions induced by the strong-scattering term \( \Sigma_{f_a}(\omega) \) in \( H_{FKM} \). The \( f_a \)-fermion self-energy, \( \Sigma_{f_a}(\omega) \approx |\omega|^{1 - \eta} \), implying that the FL quasiparticle residue \( Z = 0 \), and the symmetry-unbroken metallic phase is a non-FL metal. The underlying cause for non-FL behavior is explicit in the impurity limit of \( H_{FKM} \), where recoil-less (infinitely heavy) \( f_a \)-fermions scatter the \( f_a \)-band fermions: it is the X-ray edge problem, where such an infra-red singularity is known to occur. This implies that the local pairing susceptibility (the analogue of the “excitonic” susceptibility for \( U > 0 \)) also diverges in the infra-red, \( \chi_{f_a f_b}(\omega) = \int d\omega \text{Re} \chi_{f_a f_b}(\omega) (f_{ia} f_{ib})^\dagger (f_{ia} f_{ia})^\dagger (f_{ia} f_{ia}) = |\omega|^{2(\eta - \eta_0)} \), coupling of \( f_a \)-band fermions to this singular pair susceptibility then destroys the FL theory.

Within DMFT, this is an incoherent pair liquid state, where pair correlations diverge in the infra-red without SC order. We dub this state the incoherent Bose metal (IBM). The IBM is characterized by resonant charge fluctuations as
The term $\Delta$ number commutes with $(even at a)$ and "heavy" $(b)$ transitions ends in a second-order quantum critical point (QCEP) at $T = 0$ [21].

Away from $\Delta = 0$ or $t_{ab} = \sqrt{t_{aa}t_{bb}}$, the terms, $\Delta \sum_i (f_{ia}^\dagger f_{ib} + h.c.)$ (first case) or $(t_{aa} - t_{bb}) \sum_{i,j} (f_{ia}^\dagger f_{jb} + h.c.)$ (second case) change the local $U(1)$ invariance of $H_{FKM}$ to a global $U(1)$ invariance: only the total fermion number commutes with $H$, $[n_f + n_{\downarrow}, H] = 0$. This global symmetry can indeed be (and is) spontaneously broken, and, away from "half-filling", gives a color singlet superfluid (CSS) at low $T$, characterized by $\Delta_{ab} = \sum e^{i \phi_{ab}} (f_{ia}^\dagger f_{ib}^\dagger) > 0$ below $T_{SC} = f(U_{ab}/t, \Delta, n)$. This is readily checked by studying $H$ within the generalized Hartree-Fock random phase approximation [6], which is valid now, since there is no local constraint for any $\Delta \neq 0$. The enhancement of the superconducting $T_c$ in doped PbTe and SnTe [9] is now rationalized [11] as a manifestation of the proximity to the resonant charge fluctuation regime driven by strong quantum fluctuation between the two degenerate valence states, $n_{f\uparrow} = 0.1$, envisaged in the original prediction [10], for $\Delta \approx 0$ as above. The respective transition temperatures can be computed from DMFT [22] over the full range of $|U_{ab}|/(t_{aa}, t_{bb}, t_{ab})$, and yields a smooth BCS-BEC crossover with increasing $|U_{ab}|$.

This implies that the charge Kondo screening can never produce a CKL ground state. However, above the ordering scales, relevance/irrelevance of the $f_a - f_b$ hybridization determines whether the IBM or a correlated FL metal will result. Consider the symmetry unbroken phase for $\Delta > 0$. The term $\Delta \sum_i (f_{ia}^\dagger f_{ib} + h.c.)$ hybridizes the "infinitely heavy" (for $\Delta = 0$ above) $f_b$-fermion with the $f_a$-band. This is precisely the $U_{ab} < 0$ periodic Anderson model (PAM) with a local hybridization between the "light" ($a$) and "heavy" ($b$) fermions. To access the physics in this regime, we solve this PAM within DMFT.

We choose a Gaussian unperturbed DOS for the $a$-fermions with $W/\hbar = 12$ kHz and $|U_{ab}|/\hbar = 20$ kHz as representative values. To solve the impurity problem of DMFT, we use the iterated perturbation theory (IPT) as an impurity solver. Prior work carried out for the $U > 0$ periodic Anderson model with additional $f-d$ interaction [23] has demonstrated its accuracy vis-à-vis the more "exact" QMC solver [22]: the IPT spectral functions closely match those extracted from QMC at considerably less numerical cost. Additionally, self-energies can be readily extracted from IPT in contrast to the difficulties encountered with QMC in this context.

In fig. 1, we show the results obtained within DMFT for the PAM for two cases: i) $\epsilon'_a = 0$, and ii) $\epsilon'_a \neq 0$. The first case has particle-hole symmetry, while the second does not. For a half-filled $a$-band, in case i), we find a hybridization-induced Kondo insulator, where the hybridization gap opens up after local dynamical correlations have generated a charge pseudospin quenched analogue of the Kondo screened state. The appearance of the high-energy Hubbard bands is an explicit manifestation of relevance of local dynamical correlations. At lower energies, hybridization opens up a (renormalized) band insulating gap. Interestingly, this is a new type of insulating state, which we dub a "charge Kondo insulator" (CKI).

In case ii), the lack of particle-hole symmetry, i.e. $\epsilon'_a \neq 0$, effectively leads to partially filled $a,b$ bands, inducing a correlated metallic state within DMFT. In this case, the $T = 0$ spectral function of the $(\text{"heavy"}) (a)$ fermion shows a sharp quasiparticle resonance near $\omega = 0$, and examination of the self-energy shows that the metallic state is a renormalized Fermi liquid (FL). Related signatures of the CKI to CKL transition are also clearly visible in the "conduction" $(a)$ band DOS. Thus, DMFT yields a correlated FL metal due to dynamical quenching of the...
charge pseudospin degree of freedom by itinerance of the b-fermion (due to “hybridization”, \( \Delta \), in eq. (1)). Emergence of this charge Kondo FL (CKL) within DMFT for the \( U_{ab} < 0 \) PAM is understood as follows. In the impurity version of the lattice problem, we have an X-ray edge problem with recoil (eq. (2)) for \( \Delta \neq 0 \). Within DMFT, the infra-red singularities described above for \( \Delta = 0 \) are immediately cut off below a low energy scale associated with this recoil. Expressing the low-energy part of \( H \) in terms of collective, bosonic variables [25], this coherence scale, below which correlated FL behavior sets in, is estimated to be \( k_B T_{coh} = Z \exp(U_{ab}^2 \text{ln}(\gamma)/(1 - \gamma^2)) \) with \( \gamma = m_{fa}/m_{fb} \), the mass ratios of the \( f_{a,b} \)-fermions. In our DMFT calculation, we find \( Z \approx 0.3 \) from the real part of the self-energy (not shown). Hence, with \( t_b = 0.3t_a \), we obtain \( T_{coh} \approx 0.07T_F \). With a slightly smaller value of \( U_{ab}/W \), \( T_{coh} \) can be increased to 0.17\( T_F \), which would lie in the lower range of temperatures observable in current experimental realizations [13]. Above \( T_{coh} \), the irrelevance of the “hybridization” (\( \Delta \)) implies that the system crosses over smoothly to the IBM phase found for \( \Delta = 0 \). Away from \( t_{ab} = \sqrt{t_{aa}t_{bb}} \), the \( f_{a,b} \)-fermions once again acquire a finite mass, giving correlated FL behavior below \( T_{coh} \), or broken symmetry ground states.

Only if \( T_{coh} > T_{SC,CDW} \) will the CKL state be observable in a small window in parameter space. The above analysis suggests a way to observe the CKL state experimentally in optical lattices:

a) force a mixture of two atomic states via a Feshbach resonance (creating attractive interaction),

b) apply sequential, time-synchronized laser excitation to simulate the “hybridization”, and

c) “tune” \( \Delta \) to relevance in the RG sense, by manipulating the one-body potential [15].

The change in the Fermi surface \( (FS) \) as the system is tuned across \( t_{ab} = \sqrt{t_{aa}t_{bb}} \), \( \Delta = 0 \) could be detected by time-of-flight measurements [26] for \( T > T_{SC,CDW} \). At \( t_{ab} \), \( \Delta = 0 \), a single smeared FS, corresponding to itinerant \( f_{a,b} \) quasiparticles, should be seen, while, for \( t_{ab} > \sqrt{t_{aa}t_{bb}} \) and/or \( \Delta \neq 0 \), two well-defined FS sheets (k-space eigenvalues of the free part of \( H \) in eq. (1), corresponding to coherent quasiparticle propagation, should be observable. Since the self-energy is strictly local in DMFT, the FS will be unaffected by interactions, which then have the sole effect of critically (or overcritically) damping the FS. Given that the CKL state involves local, dynamical renormalization, designing systems where the actual, measured FS agrees well with that computed without interactions will show improved chances of experimentally observing the CKL state.

Recently, the quasiparticle dispersion and the spectral function of strongly interacting fermions exhibiting a BCS-BEC crossover in an optical trap has been experimentally deduced by Stewart et al. [27]. We suggest that similar experiments carried out below \( T_{coh} \approx 0.07T_F \) should exhibit a relatively sharp low-energy peak in \( A_k(\omega) = (-1/\pi)\text{ln}[1/(\omega - \epsilon_k - \Sigma_0(\omega))]^{-1} \), this would constitute a characteristic signature of the CKL state. The CKL state in the symmetric limit above could also be probed using this technique: one should observe two \( (a \text{ and } b \text{ bands}) \) dispersive features below and above \( E_F \), with a finite hybridization gap at \( E_F \). At higher \( T \) currently accessible, we predict an incoherent pseudogap structure in \( A_k(\omega) \) at low energy. Achieving lower \( T \) may be feasible in the near future: theoretical suggestions to do this by efficient shaping of the optical-lattice profile is predicted to lower the entropy per particle by a factor of 10 [28].

Our study also permits drawing an analogy with the phase diagram of nuclear matter. In quantum chromodynamics (QCD), where three quarks form a trion bound state, a baryon in the \( N = 3 \) (color) theory, one also has, as a function of chemical potential, “color superfluid” (CSS) and high-\( T \), quark matter phases. In the cold-atom setting, in a pioneering work, such a trion gas phase has theoretically been shown to be separated from a color superfluid phase by a second-order transition [29]. Let us now study the possibility of realizing analogous physics in our case. Our model corresponds to the “\( N = 2 \)” version of QCD, where the CSS (p-p order) phase is separated from the excitonic (p-h order) and incoherent IBM phases by a line of first-order transitions, ending in a second-order end point, generically at finite \( T \). Interestingly, the IBM phase has co-existing single-particle (“quark”) and pair states at low energy; it is an incoherent state where “quarks” and “pairs” continuously transmute into each other, their equilibrium fraction being determined by the ratio \( U_{ab}/t \), \( \Delta \), and \( T \). The confinement-deconfinement transition, familiar in the QCD context, can be illuminated in our case by employing bosonization of the “impurity model” of DMFT. At \( \Delta = 0 \), the impurity version of \( H_{FK} \) is readily bosonized [30] into a collection of radial bosonic models about the “impurity”.

This reads \( L_0 = v_F \int \{I[2](r) + (\tilde{\partial}_t \phi(r))^2\} dr \), where \( v_F = 2ta \) is the carrier Fermi velocity. For small \( \delta = (t_{ab} - \sqrt{t_{aa}t_{bb}}) \neq 0(<0) \), the \( f_a - f_b \) hybridization is small enough that the two-particle hopping terms (second order in \( \delta \)) are more relevant. These terms are \( H_{res} \approx -\frac{\delta^2}{U_{ab}} \sum_{i,j} f^\dagger_i a^\dagger_i f^\dagger_j b^\dagger_j f^\dagger_j a^\dagger_j \), which can be decomposed in two ways in a Hartree-Fock decoupling (exact in \( D = \infty \), and in the corresponding impurity model, this corresponds to two, p-h and p-p pairing fields), generating either excitonic (particle-hole) order (in the embedded impurity problem of DMFT) with order parameter \( \Delta_{ph} = \sum_i (f^\dagger_i a^\dagger_i + f^\dagger_i b^\dagger_j) > 0 \) or CSS (particle-particle) order, i.e., \( \sum_i e^{i\beta}(f^\dagger_i a^\dagger_i f^\dagger_j b^\dagger_i) > 0 \). Bosonization then yields a dual cosine quantum sine Gordon (QsG) model:

\[
L = L_0 - g_1 \int \cos(\beta_1 \phi(r)) dr - g_2 \int \cos(\beta_2 \theta(r)) dr
\]  

with \( \beta_1 = \sqrt{8\pi K} \) and \( \beta_2 = \sqrt{8\pi K} \). \( K = [1 - (U/2\pi v_F)]^{-1} \) in the weak-coupling limit for our negative-\( U \) model.
exactly at the FK point in the impurity model. A
renormalization group analysis of $L$ shows two different
phases, depending upon the sign of $\delta g = (g_1 - g_2)$ [31]:

i) when $g_1 > g_2$, the $\phi(r)$ field acquires a finite expectation
value, giving $p-h$ excitonic order. In QCD lore,
this translates into $\langle \psi_a \psi_\alpha \rangle > 0$, with $\alpha = a,b$, i.e., to
a chiral meson phase, and,

ii) when $g_2 > g_1$, the $\theta(r)$ field acquires a finite expectation
value, giving CSS order. This corresponds to
$\langle \psi_a \psi_b \rangle > 0$, i.e., to a color singlet superfluid in QCD.

iii) Interestingly, these phases are separated by a critical
curve $g_1 = g_2$ or $K = 1$, where the QsG model becomes
self-dual. This is exactly soluble, and the correlation functions
are characterized by continuously varying,
$K$-dependent exponents.

The above meson and CSS phases arise from the
IBM (bosonic Gaussian model) via Kosterlitz-Thouless
transitions (driven by relevance of either of the cosine
terms for $K < 1$ or $K > 1$), and are separated from each other
by a line of first-order transitions, ending in a
second-order end point at finite $T = T^*$. For $T > T^*$,
both $\langle \psi \psi \rangle$, $\langle \psi_\alpha \psi_\beta \rangle = 0$, corresponding to “quark matter” in
QCD [32]. Of course, this analogy is only suggestive: in
our case the
$a,b$ particles have very different masses,
and, quarks have very different masses,
and, $u,d$ quarks in QCD. Nevertheless, the analogy is interesting, and
generalization to $N = 3$ is possible. We will extend
this qualitative analogy in more depth in future.

In conclusion, from a DMFT calculation, we propose that
a combination of wilfully tunable Feshbach resonances and sequentially applied laser pulses in optical
turulence to tune $U_{ab}/t_{ab}, \Delta$ through a desired range,
could facilitate the observation of the elusive CKL state.
Time-of-flight measurements can, by probing the “Fermi
surface”, provide valuable guidance on the experimental
conditions favoring observation of the CKL state. Our
analysis also shows interesting analogies with the “phase diagram” of nuclear matter. Extensions to asymmetric
interaction models [14], allowing for realization of more
exotic ordered phases and anisotropic CKL states within
the DMFT approach, is also foreseen.

***

MSL and AT thank R. Møssner and M. Haque for
discussions. We acknowledge helpful comments from T. V.
Ramakrishnan and G. V. Pai, and thank C. Kollath
for pointing out refs. [27,28]. Financial support and
hospitality from the MPIPKS, Dresden, are gratefully
acknowledged.

REFERENCES

[1] Hewson A. C., The Kondo Problem to Heavy Fermions,
(Cambridge University Press) 1997.
[2] Cox D. L., Phys. Rev. Lett., 59 (1987) 1240.
[3] Craco L., Laad M. S. and Müller-Hartmann E.,
Phys. Rev. Lett., 90 (2003) 237203; Kolesnychenko
O. Yu., Helin K. M. M., Zhuravlev A. K., de Kort
R., Katsnelson M. I., Lichtenstein A. I. and
van Kempen H., Phys. Rev. B, 72 (2005) 085456.
[4] Ji Q., Kotliar G. and Georges A., Phys. Rev. B, 46
(1992) 1261.
[5] Varma C. M., Phys. Rev. Lett., 61 (1988) 2713.
[6] Taraphder A., Krishnamurthy H. R., Pandit
R. and Ramakrishnan T. V., Phys. Rev. B, 52
(1995) 1368.
[7] Taraphder A., Pandit T., Krishnamurthy H. R. and
Ramakrishnan T. V., Int. J. Mod. Phys. B, 10
(1996) 863 and references therein.
[8] Wilson J. A. and Zahir A., Rep. Prog. Phys., 60
(1997) 941.
[9] Mathieshita Y., Bluhm H., Geballe T. H. and Fisher I. R.,
Phys. Rev. Lett., 94 (2005) 157002; Erickson A. S., Breenay N. P., Nowadnick
E. A., Geballe T. H. and Fisher I. R., preprint
arXiv:cond-mat/0809.0924 (2008); preprint arXiv:cond-
mat/0904.3359.
[10] Taraphder A. and Coleman P., Phys. Rev. Lett., 66
(1991) 2814.
[11] Dzero M. and Schmalian J., Phys. Rev. Lett., 94 (2005)
157003.
[12] Georges A., preprint arXiv:cond-mat/0702122.
[13] Jördens R., Strohmaier N., Günter K., Moritz H.
and Esslinger T., Nature, 455 (2008) 204.
[14] Regal C. A., Ticknor C., Bohn J. L. and Jin D. S.,
Phys. Rev. Lett., 90 (2003) 053201; Zhang J., van
Kempen E. G. M., Bourdel T., Khaykovich L.,
Cubizolles J., Chevy F., Teichmann M., Tarruell
L., Kokkelmans S. J. J. M. F. and Salomon C.,
Phys. Rev. A., 70 (2004) 030702; Chen Q., Stajic J., Tan S.
and Levin K., Phys. Rev., 412 (2005) 1.
[15] Mandel O., Greiner M., Widera A., Rom T.,
Hänsch T. W. and Bloch I., Phys. Rev. Lett., 91
(2003) 010407; Nature (London), 425 (2003) 937.
[16] Kotliar G., Savrasov S. Y., Haule K., Oudovenko
V. S., Parcollet O. and Marianetti C. A., Revs.
Mod. Phys., 76 (2006) 865.
[17] Craco L., Phys. Rev. B, 59 (1999) 14387.
[18] Brandt U. and Mielisch C., Z. Phys. B., 75 (1989)
365; Laad M. S. and Müller-Hartmann E., Phys. Rev.
Lett., 87 (2001) 246402.
[19] Das D. and Doniach S., Phys. Rev. B., 60 (1999) 1261;
Phillips P. and Dalidovich D., Science, 302 (2003)
243.
[20] van Dongen P., Majumdar K., Huscroft C. and
Zhong F.-C., Phys. Rev. B, 64 (2001) 195123.
[21] Watanabe S., Tsuruta A., Miyake K. and Flouquet
J., Phys. Rev. Lett., 100 (2008) 236401, for the $U\alpha > 0$
case; we expect similar behavior for $U\sigma < 0$ in the
symmetry-unbroken phase within DMFT.
[22] Jarrell M., Akhlaghpour H. and Pruschke Th.,
Phys. Rev. Lett., 70 (1993) 1670; Vidhyadhara N. S.,
Tahvildar-Zadeh A. N., Jarrell M. and Krishnamurthy H. R.,
Europhys. Lett., 49 (2000) 459.
[23] Craco L., Phys. Rev. B, 77 (2008) 125122.
[24] Portengen T., Östreich Th. and Sham L. J., Phys.
Rev. B, 54 (1996) 17452.
[25] Müller-Hartmann E. et al., Phys. Rev. B, 3 (1971) 1102.
[26] Bloch I., Dalibard J. and Zwerger W., Rev. Mod. Phys., 80 (2008) 885.
[27] Stewart J. T., Gaebler J. P. and Jin D. S., Nature, 454 (2008) 744.
[28] Bernier J.-S., Kollath C., Georges A., De Leo L., Gerbier F., Salomon C. and Köhl M., Phys. Rev. A, 79 (2009) 061601(R).
[29] Rapp A., Zaránd G., Honerkamp C. and Hofstetter W., Phys. Rev. Lett., 98 (2007) 160405.
[30] Schotte K. D. and Schotte U., Phys. Rev., 182 (1969) 479.
[31] Gogolin A., Nersesyan A. and Tsvelik A., Bosonization and Strongly Correlated Systems (Cambridge University Press, Cambridge) 1998.
[32] Hands S., Contemp. Phys., 42 (2001) 209.