Feasibility study for a model independent measurement of $2\beta + \gamma$ in $B^0$ decays using $D^-K^0\pi^+$ final states.

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Abstract

In this paper we present a feasibility study for measuring the combination of the Unitarity Triangle angle $2\beta + \gamma$ with a time dependent Dalitz analysis in $B^0$ decays using $D^-K^0\pi^+$ final states following the method proposed in [1]. For this study we build a model for this decay using the latest experimental information and we investigate the possibility of fitting together with $2\beta + \gamma$ all the relevant strong amplitudes and phases of the decay model.

1 Introduction

The interference effects between the $b \to c$ and $b \to u$ decay amplitudes in the time-dependent asymmetries of $B$ decaying into $D^{(*)}\pi$ and $D^{(*)}\rho$ final states allow for the determination of $2\beta+\gamma$. Nevertheless the extraction of $2\beta+\gamma$ depends upon the knowledge of the absolute value of the ratio ($r$) of the amplitudes (e.g.: $r = |A(B^0 \to D^-\pi^+)|/|A(B^0 \to D^-\pi^-)|$) and of the strong phase difference, which have not been determined experimentally so far. Furthermore, the ratio $r$ is expected to be rather small, of the order of $\lambda |V_{ub}/V_{cb}| \approx 0.02$. In addition the extraction of $2\beta + \gamma$ from the two-body decays suffers from a eight-fold ambiguity. One can estimate the ratio $r$ by deducing for instance the branching fraction $Br(B^0 \to D^{(*)}\pi^-)$ from the measured branching fraction $Br(B^0 \to D^{(*)}_s\pi^-)$. This approach is valid up to a theoretical uncertainty related to SU(3) breaking effects and to the size of annihilation processes which do not contribute to $D^{(*)}_s\pi^-$ final states. Using all the experimental constraints [2],[3], a tentative extraction of $2\beta + \gamma$ has been made, attributing 100% uncertainty to these assumptions, and getting [4]: $2\beta + \gamma = (\pm 90 \pm 46)^\circ$ (the $\pm$ in front of the central value refers to the fact that $2\beta + \gamma$ is determined up to a $\pi$ ambiguity).

Another way of measuring $2\beta + \gamma$ is to perform a time dependent Dalitz analysis of the three-body decay : $B^0 \to D^-K^0\pi^+$, as proposed in [1]. This method allows in principle to overcome the disadvantages encountered in the determination of $2\beta + \gamma$.

$^{1}$Charge conjugation is implied throughout the paper unless otherwise stated.
from $D^{(*)}\pi$ or $D^{(*)}\rho$ decays. First of all, in the region of the Dalitz plane where most of the interference takes place the value of the ratio $r$ is expected to be of the order of 0.4 since it involves $b \to c$ and $b \to u$ colour suppressed diagrams. Furthermore $2\beta + \gamma$ can be measured with only a two-fold ambiguity as explained in [4]. In addition, this method could be theoretically clean, because the strong amplitudes and phases can be, in principle, measured from data.

Early studies can be found in [4]. In this paper we present a feasibility study of the analysis under realistic conditions. Unlike to previous studies, we have used a model for the Dalitz structure of the $B^0 \to D^-K^0\pi^+$ decays that uses the latest experimental measurements. We have also included realistic $B^0 - \bar{B}^0$ tagging performances and explored the possibility of fitting the strong amplitudes and phases of the most relevant interfering resonances by using both the tagged and untagged events. Finally we have studied the effect of background in the determination of $2\beta + \gamma$.

2 The $B^0 \to D^-K^0\pi^+$ decays

2.1 Time dependence: general case

The measurement of $2\beta + \gamma$ can be achieved through a study of the time-dependent evolution of $B^0$ decay sensitive to that phase. This can only be performed on tagged events, that means events for which the original flavour of the reconstructed $B$ is known. In a $B$ factory this is possible through the determination of the flavour of the other $B$ in the event since the two $B$ mesons are produced in a coherent $J^{PC} = 1^{--}$ state. Thus if we determine that at a time $t$ we have a $B^0$, then the other $B$ in the event at the same time $t$ is a $\bar{B}^0$. In practice, one of the $B$ (the $B_{\text{tag}}$) is reconstructed in a flavour tagging state. The time-dependent decay rates taking into account CP violation for a $B^0 (P_+)$ or $\bar{B}^0 (P_-)$ tagging meson are given by:

$$P_\pm = \frac{N}{4\tau}e^{-t/\tau} (1 \mp C \cos(\Delta m_d t) \pm S \sin(\Delta m_d t))$$

where $\tau$ is the $B^0$ lifetime, $\Delta m_d$ is the $B^0 - \bar{B}^0$ mixing frequency and $N$ a normalisation factor. The parameter $S$ is non-zero if there is mixing-induced CP violation, while a non-zero value for $C$ would indicate direct CP violation.

2.2 The $B^0 \to D^-K^0\pi^+$ case

The final state $D^-K^0\pi^+$ can be reached through the diagrams shown in Figure 1 and considering the $B^0 - \bar{B}^0$ mixing. The parameters $S$ and $C$ of Eq. 1 will depend on the position in the $B$ Dalitz plot. Indeed, a given bin in the Dalitz plot gets contributions from amplitudes and strong phases of $b \to c$ and $b \to u$ transitions.

The model assumed for the decay parameterises the amplitude $A$ at each point $k$ of the Dalitz plot as a sum of two-body decay matrix elements and a non-resonant term according to the following expression:

$$A_{ck(u_k)}e^{i\delta_{ck}(u_k)} = \sum_j a_j e^{i\delta_j} BW_j^k(m, \Gamma, s) + a_{nr} e^{i\phi_{nr}}$$
Figure 1: Feynman diagrams describing the processes contributing to the decays $B^0 \rightarrow D^- K^0 \pi^+$. a) $B^0 \rightarrow D^- K^{*+} (K^0 \pi^+) \text{ and higher } K^{**} \text{ resonances}$, b) $B^0 \rightarrow D^{**0} (D^- \pi^+) K^0$, c) $B^0 \rightarrow \bar{D}^{**0} (\bar{D}^+ K^0) \pi^+ \text{ or higher } K^{**} \text{ resonances}$, d) $B^0 \rightarrow D^{**0} (D^+ \pi^-) K^0$. The processes in a,b (c,d) are $V_{cb}$ ($V_{ub}$) mediated.

where $c_k$ ($u_k$) indicates the Cabibbo allowed (suppressed) decay in each point $k$ of the Dalitz plot. Each term of the sum is parametrised with an amplitude ($a_j$ or $a_{nr}$) and a phase ($\delta_j$ or $\phi_{nr}$). The factor $BW_j^k(m, \Gamma, s)$ gives the Lorentz invariant expression for the matrix element of a resonance $j$ as a function of the position $k$ in the $B$ Dalitz plot; the functional dependence varies with the spin $s$ of the resonance according to the isobar model \[6\]. The total phase and amplitude are arbitrary. We have chosen amplitude unity and phase zero for the mode $K^{*+}(892)$ decaying into $K^0 \pi^+$.

Four transitions have to be considered associated to probabilities of connecting a $B^0$ or $\bar{B}^0$ initial state to a $D^+$ or $D^-$ final states. They are indicated in Table 1.

|            | $D^- K^0 \pi^+$ final state | $D^+ K^0 \pi^-$ final state |
|------------|------------------------------|------------------------------|
| $V_{cb}$   | $< D^- K^0 \pi^+ |T|B^0 > = A_c k e^{i \delta_{ck}}$ | $< D^+ K^0 \pi^- |T|B^0 > = A_c k e^{i \delta_{ck}}$ |
| $V_{ub}$   | $< D^- K^0 \pi^+ |T|\bar{B}^0 > = A_u k e^{i \delta_{uk} - \gamma}$ | $< D^+ K^0 \pi^- |T|\bar{B}^0 > = A_u k e^{i \delta_{uk} + \gamma}$ |

Table 1: Summary of the amplitudes involved in the $2\beta + \gamma$ analysis. The index $k$ refers to the position in the Dalitz plot. The third column is derived from the second one using CP transformation.

The time dependent evolution of these probabilities can be obtained from the resolution of the Schroedinger equation. This leads to a new formulation of Eq. 1 in which the first argument within the parentheses refers to the production state of the $B$ while the second one indicates the reconstructed final state\(^2\).

\(^2\)If one neglects $b \rightarrow u$ transitions, the charges of the particles in the final state tag the $B$ decay
flavour and one is left with the standard mixing formulae : 

\[
P(B^0, D^+ K^0 \pi^-) = \frac{A^2_{c_k} + A^2_{u_k}}{2} e^{-\Gamma B t} \left\{1 - C^k \cos(\Delta m_t) + S^k_+ \sin(\Delta m_t)\right\} \\
P(B^0, D^+ \pi^-) = \frac{A^2_{c_k} + A^2_{u_k}}{2} e^{-\Gamma B t} \left\{1 + C^k \cos(\Delta m_t) - S^k_+ \sin(\Delta m_t)\right\} \\
P(B^0, D^- K^0 \pi^+) = \frac{A^2_{c_k} + A^2_{u_k}}{2} e^{-\Gamma B t} \left\{1 - C^k \cos(\Delta m_t) + S^k_- \sin(\Delta m_t)\right\} \\
P(B^0, D^- \pi^+) = \frac{A^2_{c_k} + A^2_{u_k}}{2} e^{-\Gamma B t} \left\{1 - C^k \cos(\Delta m_t) - S^k_- \sin(\Delta m_t)\right\} \\
\]

with :

\[
C^k = \frac{A^2_{c_k} - A^2_{u_k}}{A^2_{c_k} + A^2_{u_k}} ; \quad S^k_+ = \frac{2 Im(A_{c_k} A_{u_k} e^{i(2\beta + \gamma) + i(\delta_{c_k} - \delta_{u_k})})}{A^2_{c_k} + A^2_{u_k}} \quad \text{and} \quad S^-_k = \frac{2 Im(A_{c_k} A_{u_k} e^{i(2\beta + \gamma) - i(\delta_{c_k} - \delta_{u_k})})}{A^2_{c_k} + A^2_{u_k}}
\]

Using these relations, and because of the presence of the terms $BW^2_k(m, \Gamma, s)$ which vary over the Dalitz plot, we can fit the amplitudes ($a_j$) and the phases ($\delta_j$) of Eq. \ref{eq:fitting} together with $2\beta + \gamma$ with only a two-fold ambiguity.

### 2.3 The $B^0 \rightarrow D^- K^0 \pi^+$ model

We now discuss the hadronic model for the $B^0 \rightarrow D^- K^0 \pi^+$ decay. The resonances taken into account are listed in Table\ref{table:resonances} for both $V_{cb}$ and $V_{ub}$ mediated modes and shown in Figure\ref{fig:hadronic_model}. The Dalitz plot can be modelled in terms of the following resonances:

\[
\begin{align*}
D^- - X & \quad X \rightarrow K^0 \pi^+ \quad (K^*(892)^\pm, K_1^*(1430)^\pm, K_2^*(1430)^\pm, K^*(1680)^\pm) \\
K^0 - Y & \quad Y \rightarrow D^- \pi^+ \quad (D_0^*(2400)\text{^0}, D_2^*(2460)\text{^0}) \\
\pi - Z & \quad Z \rightarrow D^- K^0 \quad (D_{s,2}(2573)^\pm)
\end{align*}
\]

The $K^0 \pi^+$ resonances ($K^*$ like) can only come from $V_{cb}$ mediated processes, while both $V_{cb}$ and $V_{ub}$ transitions contribute to $D^- \pi^+$ resonances ($D^{*+}$ like). Finally the $D^- K^0$ resonances ($D^{s*+}$-like) come only through $V_{ub}$ mediated processes. For the final state under study, “non resonant” contributions could come from higher $K$ excited states or from higher excited $D^{**}$ states. In the first case only $V_{cb}$ processes can contribute while in the second both $V_{cb}$ and $V_{ub}$ processes are contributing.

The strong phases are not known experimentally and have to be chosen arbitrarily. For the values of the amplitudes we can use some information available on branching fractions \cite{7,8}. So far we have some information which can help defining the model for the $b \rightarrow c$ part. The Dalitz plot of this decay has been partially measured in \cite{6,9}. Two measurements are available to partially define the $D^{**}$ part of the Dalitz plot:

\[
\begin{align*}
Br(B^0 \rightarrow D^- K^{*+}) & = (4.6 \pm 0.6 \pm 0.5) \times 10^{-4} \\
Br(B^0 \rightarrow D_{s,2}^{*+} K^+) \times Br(D_{s,2}^{*+} \rightarrow \bar{D}^0 \pi^-) & = (1.8 \pm 0.4 \pm 0.3) \times 10^{-5}
\end{align*}
\]

flavour and one is left with the standard mixing formulae : $P(B^0, B^0) = P(\bar{B}^0, \bar{B}^0) \propto (1 - \cos(\Delta m_t))$ and $P(B^0, \bar{B}^0) = P(\bar{B}^0, B^0) \propto (1 + \cos(\Delta m_t))$.
Figure 2: Various resonances contributing to the $B^0 \to D^-K^0\pi^+$ Dalitz plot. From top to bottom, left to right: $K^*(892)^\pm$, $K_0^*(1430)^\pm$, $K_2^*(1430)^\pm$, $K^*(1680)^\pm$, $D_0^*(2400)^0$, $D_2^*(2460)^0$ and $D_{s,2}(2573)^\pm$
The production of the charged $D^{**}$ through the process indicated in the second equation (Eq. 3) involves only $V_{ub}$ transitions with diagrams at the tree level (T) (similar to the diagram in Figure 1(a), with a $D_{sJ=2}^{**}$ produced in the lower part and a $K^+$ emitted from the W), while we are interested in the neutral $D^{**}$ production which proceed via colour suppressed decays (C) (see Figure 1(b)). For our numerical evaluation of the branching fractions of interest for this analysis we have considered that the ratio $|C/T|$ could vary between 0.3 and 0.5. Nevertheless we are conscious that it is not straightforward to link production of specific hadronic states mediated by T and C suppressed processes. Furthermore, only $2^+$ final states have been measured so far. We make the hypothesis that the $0^+$ states decaying into $D\pi$ modes are as abundant as the $2^+$ states. The possible existence of a non resonant contribution is considered below.

For the $b \to u$ counterpart we use the hypothesis that $r$ is equal to 0.4 (where $r$ in this case is defined as $r = \frac{B^0 \to D^{*-0} K^0}{B^+ \to D^{*-0} K^+}$). The value of 0.4 is motivated by the fact that $r = \frac{|V_{ub}|}{|V_{cb}|} \times \frac{A(D^{*-0} K^0)}{A(D^{*-0} K^+)} = \sqrt{\beta^2 + \eta^2} \times \frac{A(D^{*-0} K^0)}{A(D^{*-0} K^+)}$. The term $\sqrt{\beta^2 + \eta^2} = 0.408 \pm 0.016$ [5].

The term $\frac{A(D^{*-0} K^0)}{A(D^{*-0} K^+)}$ can be expressed as the ratio of two colour suppressed diagrams involving $V_{ub}$ and $V_{ub}$ processes. This ratio is expected to be around unity with an error which is difficult to estimate. For studying the impact of the $r$ parameter on $2\beta + \gamma$ sensitivity we consider a variation for this parameter in a relatively wide range : between 0.3-0.5.

We also use the measurement of the following ratio [5]:

$$\frac{B^0 \to D^- K^{*+}(892)(K^0 \pi^+)}{B^0 \to D^- K^0 \pi^+} = 0.66 \pm 0.08$$  (8)

to distribute the rest of events in the Dalitz plot. It has to be considered that no information on how to distribute these events between different excited K states ($K_0^*(1430), K_2^*(1430), K^*(1680)$) is available. The contribution from the $D_{sJ=2}^{**}$ resonance in $b \to u$ transitions is difficult to evaluate. By analogy we can consider a similar decay mode: $B^0 \to D^{*-} D^{*-0} K^0$. These decays are mediated by tree diagrams, however the contributions from $B^0 \to D^{*-} D^{*-0}$ final states are small [10]. This result would indicate the predominance of non resonant or very large states. A conservative choice has been made not to include them in the present model. The contributions from the different resonances are fixed at the values given in Table 2. Finally we make the conservative hypothesis to have, all over the Dalitz, a small non resonant contribution and in agreement with the Standard Model we assume $2\beta + \gamma = 2$ radians (see for example [5]). All these parameters will be varied to evaluate the impact of the chosen model on the sensitivity of $2\beta + \gamma$.

The number of $B^0 \to D^- K^0 \pi^+$ signal events is estimated using the measured branching fraction and observed yield [8]. We have generated 250 signal events per unit of 100 fb$^{-1}$.

### 2.4 Sensitivity study for $B^0 \to D^- K^0 \pi^+$ decays

In order to show which are the regions in the Dalitz plot that contribute most in the determination of $2\beta + \gamma$, we have evaluated, on an event by event basis, the second derivative with respect to $2\beta + \gamma$ of the log-likelihood $\frac{\partial^2 \log L}{\partial (2\beta+\gamma)^2}$ constructed according to Eq.2 (this likelihood is signal only and does not consider resolution effects). A very high statistics Monte-Carlo sample of signal events has been generated according to the nominal model described in Section 2.3. The result is shown in Figure 3 where each
Table 2: List of mass, widths and quantum numbers of the resonances considered in our model, as taken from PDG2004. The last four columns present the chosen values of the coefficients \( a_j \) and \( \delta_j \) in Eq. 2 for the Cabibbo allowed and Cabibbo suppressed decays respectively. Note that the choices for the phases are arbitrary and are not indicated in the Table. In the numerical analysis we have evaluated the effect of a different choice of strong phases.

The choice of this weight is motivated by the fact that the uncertainty on \( 2\beta + \gamma \) is equal to \( \sqrt{\sum \frac{1}{\partial^2(2\beta + \gamma)}} \). Figure 3 shows the distribution of the Monte Carlo events in the \( M_{D_s K^0}^2 \) versus \( M_{K^0 \pi^-}^2 \) plane (black dots) and the corresponding sensitivity weights superimposed (coloured squares). The regions with interference between \( B^0 \rightarrow D^{*+} K^0 \) and \( B^0 \rightarrow D^{*0} K^0 \) colour suppressed processes show the greater sensitivity to \( 2\beta + \gamma \). A particularly sensitive zone is at the intersection between \( B_0 \rightarrow D^{-} K^{*+} \) and the colour suppressed \( B^0 \rightarrow D^{*0} K^0 \). In the same figure we can also notice that some sensitivity is also present because of the interference of the \( D_{s,2}(2573)^\pm \) with the excited K resonances \( K_0^*(1430)^\pm, K_2^*(1430)^\pm, K^*(1680)^\pm \). As expected, we observe no sensitivity in the regions where there is only one path \( (b \rightarrow c \ or \ b \rightarrow u) \) to reach the \( DK^0 \pi^- \) final state and a maximal sensitivity when there is an overlap between resonances due to both \( b \rightarrow c \) and \( b \rightarrow u \) transitions.

### 3 Feasibility study

#### 3.1 Dependence on the Dalitz model of the \( 2\beta + \gamma \) error.

The first study is to consider the dependence on the determination of \( 2\beta + \gamma \) from the Dalitz structure of the decay model. For this study we have used the time evolution equations defined in Eq. 4 without including the effect of time resolution. Preliminary studies using a realistic resolution indicate an increase on \( 2\beta + \gamma \) uncertainty of about
Figure 3: a) Dalitz distribution of the very high statistics Monte-Carlo sample of signal events. Each event is entering into the plot with a weight given by the value of the second derivative with respect to $2\beta + \gamma$ of the log-likelihood. The black points correspond to the same events with weight equal to unity. b) Distribution similar to Figure-a except that we have fixed a maximum value for the weight to be plotted in order to see in a finer way the structure of the weights over the Dalitz.
Figure 4: Average absolute uncertainty obtained on the parameter $2\beta + \gamma$ as a function of the integrated luminosity for different models. The magenta thick stars refers to the model in table 2 while the thin stars are obtained by varying the strong phases values; the red (dark) full circles and the green (clear) open square are obtained multiplying the $D^{*+0}$ amplitudes by a factor 1.3 and 0.7 respectively. The red full (dark) and green (clear) open triangles refer to previous models considering $r=0.5$ and $r=0.3$ respectively. Finally the blue crosses refers to the model in table 2 where the non-resonant component is set to zero. The cyan (clear) squared are just given as reference and correspond to the simplified model presented in [4] in which all $V_{ub}$ processes are considered through non-resonant amplitudes.

10%. Instead, we have included realistic $B^0 - \overline{B^0}$ tagging performances in terms of purities and efficiencies. The effect of the background is studied in the following section. For this study we just fit $2\beta + \gamma$ fixing all the other parameters describing the Dalitz decay model. The impact on the error on $2\beta + \gamma$ by varying the decay model is studied as a function of the luminosity and shown in Figure 4.

From this study the first conclusion is that the present measurements [8],[9] are not sufficient to fix the Dalitz model and give a precise indication on the error which can be obtained on $2\beta + \gamma$. In fact, the precision on $2\beta + \gamma$ strongly depends on the variation of the branching fractions of neutral $B$ into $D^{*+}$ states within the measured errors. There is also some dependence on the presence or not of non resonant contributions and on the
### Table 3: Average absolute uncertainty obtained on the parameter $2\beta + \gamma$ as a function of the fit configuration for an integrated luminosity of 1 $ab^{-1}$. The configurations are explained in the text. See also Figure 5.

| $2\beta + \gamma$ | Parameter fitted | Configuration |
|-------------------|------------------|---------------|
| 0.63              | $2\beta + \gamma$ | signal only   |
| 0.69              | $2\beta + \gamma$ + $V_{cb}$ amp./phases | signal only tagged only |
| 0.65              | $2\beta + \gamma$ + $V_{cb}$ amp./phases | signal only tag + untag |
| 0.79              | $2\beta + \gamma$ + $V_{cb}$ amp./phases + $V_{ub}$ phases, r-fix | signal only tag + untag |
| 0.84              | $2\beta + \gamma$ + $V_{cb}$ amp./phases + $V_{ub}$ phases, amp. | signal only tag + untag |
| 0.82              | $2\beta + \gamma$ | signal + 50% back. |
| 0.85              | $2\beta + \gamma$ + $V_{cb}$ amp./phases | signal + 50% back. tag + untag |
| 1.00              | $2\beta + \gamma$ + $V_{cb}$ amp./phases + $V_{ub}$ phases, r-fix | signal + 50% back. tag + untag |
| not conv.         | $2\beta + \gamma$ + $V_{cb}$ amp./phases + $V_{ub}$ phases, amp. | signal + 50% back. tag + untag |
Figure 5: Average absolute uncertainty obtained on the parameter $2\beta + \gamma$ as a function of the fit configuration. The magenta thick stars refers to fit with no background while red dots and blue square correspond to fit where the level of background is set at 50% and 30%. The background has been simulated flat over the Dalitz plot. The fit configuration are explained in the text.
values of strong phases for the most relevant resonances. We also show the impact of the variation of \( r = \frac{B^0 \to D^{\ast 0} K^0}{B^0 \to D^{\ast 0} K^0} \) from 0.3 to 0.5.

The model using only a non resonant contribution for the \( V_{ub} \) component is also given as a reference since it was taken as example in the feasibility study presented in [4]. The corresponding error on \( 2\beta + \gamma \) is a factor 3-6 better with respect the one obtained using a set of realistic decay models. This model has not been further considered as it is contradiction with present data [7].

3.2 Model Independent fit

An important feature of this method is the possibility to have a model independent determination of \( 2\beta + \gamma \) by fitting the parameters of the decay model. We consider a sample corresponding to 1 ab\(^{-1}\) of collected statistics. The reference error is obtained as previously by fitting only \( 2\beta + \gamma \) and by fixing all the other parameters. We try to progressively release the other parameters which characterise the decay model.

First we relax the \( D^{\ast 0} \) and the non-resonant \( V_{cb} \) components. We managed to fit the three amplitudes and phases and the error on \( 2\beta + \gamma \) increases by 10%. So far we have used only tagged events since they are the only one carrying information on \( 2\beta + \gamma \). Nevertheless also untagged events could play an important role in fitting the decay model parameters. In practice we can add the following term to the likelihood given in Eq.4:

\[
P_{\text{UNTAG}}(B^0 \to D^- K^0 \pi^+) = \frac{A_{ck}^2 + A_{uk}^2}{2} \tag{10}\]

The same fit is repeated with untagged events included in the sample. The error on \( 2\beta + \gamma \) decreases by about 10%. In the following untagged events are used.

The following test is to leave free the strong phases of the \( D^{\ast 0} \) and the non-resonant \( V_{ub} \) components. In this case the error on \( 2\beta + \gamma \) increases by about 20%. An extra increase of 10% is obtained if the amplitudes of those components are further left free to vary in the fit.

One can also try to estimate the effect of the presence of the background in the Dalitz plot. For simplicity we take a flat background over the Dalitz plot, being aware that reality could be different. We assume that the discrimination between signal and background events is performed using additional variables (e. g. the reconstructed \( B \) mass) and we fix the signal over background ratio in the CP violation fit. We run our simulation by considering a signal over background (S/B) ratio of 30% or 50%. The result is that the error on \( 2\beta + \gamma \) increases by 25% and 50%, respectively. Furthermore if S/B is equal to 50% it seems difficult to perform a fit where all \( V_{cb} \) and \( V_{ub} \) \( D^{\ast 0} \) and the non-resonant components are left free to vary in the fit.

As by-product of this analysis, the Dalitz model can be fitted from the data if the level of background is not too large. The \( V_{cb} \) \( D^{\ast 0} \) related amplitudes and phases can be fitted with precisions lying between 20-30%. Furthermore both the phases and the amplitudes for \( D^{\ast 0} \) from \( V_{ub} \) processes are determined with errors which can go up to 50%. In addition the excited K resonances (\( K^*_0(1430)^\pm, K^*_2(1430)^\pm, K^*(1680)^\pm \)) which concerns only \( V_{cb} \) processes can be precisely determined from data with relative uncertainties better than 25%.
4 Conclusions and perspectives

We have performed a full feasibility study for measuring $2\beta + \gamma$ with time dependent Dalitz analysis using $D^- K^0 \pi^+$ final states. For this study a realistic decay model, based on the available experimental information has been elaborated.

We conclude that the error on $2\beta + \gamma$ strongly depends on the decay model and that the currently available experimental information is not enough for a precise estimate. By varying the different parameters in the currently allowed range, the error on $2\beta + \gamma$ can vary within a factor of about 2.5. Only a complete Dalitz analysis could tell us where we stand. This is possible on data.

In fact we have shown that if the level of background is not too large (< 50%) the full decay model can be obtained by fitting the Dalitz plot using both tagged and untagged events.

All the amplitudes and strong phases can be fitted and, in case of the Dalitz decay model given in Table 2, the precision on $2\beta + \gamma$ could be of about 50% with 500 fb$^{-1}$ and should not be limited by the systematical uncertainties. This implies that the precision on $2\beta + \gamma$ could be reduced at few percent level in case this analysis is performed at a high luminosity B factory [11].

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[http://www.slac.stanford.edu/BFROOT/www/Organisation/1036_Study_Group/](http://www.slac.stanford.edu/BFROOT/www/Organisation/1036_Study_Group/)
and related documents