The $\eta'/N$ interaction from a chiral effective model and $\eta'/N$ bound state

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Abstract The $\eta'$ mass reduction in the nuclear medium is expected from the degeneracy of the pseudoscalar-singlet and octet mesons when chiral symmetry is manifest. In this study, we investigate the $\eta'/N$ 2body interaction which is the foundation of the in-medium $\eta'$ properties using the linear sigma model as a chiral effective model. The $\eta'/N$ interaction in the linear sigma model comes from the scalar meson exchange with $U_A(1)$ symmetry effect and is found to be fairly strong attraction. Moreover, the $\eta N$ transition is included in our calculation, and is important for the imaginary part of the $\eta'$-optical potential. The transition amplitude of $\eta'/N$ to the $\eta N$ channel is relatively small compared to that of elastic channel. From the analysis of the $\eta'/N$ 2body system, we find a $\eta'/N$ bound state with the binding energy $12.3 - 3.3i\text{MeV}$. We expect that this strongly attractive two body interaction leads to a deep and attractive optical potential.

Keywords $\eta'/N$ system · chiral symmetry · linear sigma model
1 Introduction

The study of the meson properties in nuclear medium is one of the most exciting topics in the hadron physics. The meson property is strongly related to the non-perturbative natures of Quantum Chromodynamics (QCD). Especially the $\eta'$ meson has a strong connection to the $U_A(1)$ anomaly and the spontaneous breaking of chiral symmetry. In the ordinary explanation, the large mass of the $\eta'$ meson should be attributed to the explicit breaking of the $U_A(1)$ symmetry due to the anomaly. However, the chiral symmetry breaking is also responsible for the generation of the $\eta'$ mass [1,2,3]. In Refs. [1,2,3,4], they have argued the degeneracy of the pseudoscalar singlet and octet mesons when chiral symmetry is fully restored in three flavor system independently of the $U_A(1)$ symmetry. Thus, both the $U_A(1)$ anomaly and the chiral symmetry breaking are essential for the generation of the $\eta'$ mass. The effective restoration of the $U_A(1)$ symmetry is also pointed out from the viewpoint of the instanton dynamics [5]. The in-medium properties of the $\eta'$ meson concerned with the in-medium $U_A(1)$ symmetry is interested in for a long time (see, for example, Refs. [6,7]).

One of the recent interest of the in-medium $\eta'$ property is related with the partial restoration of chiral symmetry in the nuclear medium [4]. The partial restoration of chiral symmetry means the reduction of the quark condensate in low density systems. The in-medium quark condensate $\langle \bar{q}q \rangle_\rho$ is given as

\begin{equation}
\langle \bar{q}q \rangle_\rho = \langle \bar{q}q \rangle_{\rho=0} \left( 1 - \frac{\sigma_\pi N}{f_\pi m_\pi} \rho \right),
\end{equation}

in small density systems [5]. Some experiments of the pion-nucleus system suggest about 35% reduction of the quark condensate at the normal nuclear density [9,10]. Assuming small change of the $\eta$ meson mass in the nuclear medium, we expect large mass reduction of the $\eta'$ mass in the nuclear medium. In the finite temperature and density system, there are some model calculations focusing on the $\eta'$ mass [7,11,12,13,14,15]. Furthermore, the interpretation of the mass reduction suggests possible $\eta'$-mesonic nuclei as the attractive optical potential in the nuclear medium [16].

There are some analyses of the experimental data related to the $\eta'$ properties. From the analysis of the transparency ratio, the optical potential of the $\eta'$ meson is estimated [17]. The $\eta'N$ scattering length is extracted from the $pp \to pp\eta'$ reaction [18,19].

The purpose of our work is to study the $\eta'$-optical potential. In this study, we analyze the $\eta'N$ system with the linear sigma model. Advantages to use the linear sigma model in this study are as follows; it shares common symmetry feature to QCD, such as the three flavor chiral symmetry and the $U_A(1)$ anomaly. It is easy to demonstrate the spontaneous chiral symmetry breaking and its partial restoration in the nuclear medium. Here we assume that the chiral symmetry is partially restored at the normal nuclear density with 35% reduction of the quark condensate. The nucleon degree of freedom can be implemented into the linear sigma model as a fundamental field.
In the following sections, we explain the model setup and show the results of our analysis of the \( \eta'N \) system.

2 The linear sigma model and the \( \eta'N \) interaction

In this section, we explain the setup to evaluate the \( \eta'N \) interaction \[15\].

The Lagrangian used in our calculation given as follows;

\[
L = \frac{1}{2} \text{tr} \partial_{\mu} M \partial^{\mu} M^\dagger - \frac{\mu^2}{2} \text{tr} M M^\dagger - \frac{\lambda}{4} \text{tr} (M M^\dagger)^2 - \frac{\lambda'}{4} \left[ \text{tr} (M M^\dagger) \right]^2 \\
+ \text{Atr} (\chi M^\dagger + \chi^\dagger M) + \sqrt{3}B(\text{det} M + \text{det} M^\dagger) \\
+ \bar{N}(i\bar{\psi} - m_N)N - \bar{N}g \left( \frac{\bar{\sigma}_0}{\sqrt{3}} \mathbf{1} + \frac{\bar{\sigma}_8}{\sqrt{6}} \right) N \\
- \bar{N}ig\gamma_5 \left( \frac{\mathbf{\tau} \cdot \mathbf{\tau}}{\sqrt{2}} + \frac{\bar{\eta}_0}{\sqrt{3}} \mathbf{1} + \frac{\bar{\eta}_8}{\sqrt{6}} \right) N, \quad (2)
\]

where \( \tau_a \) and \( \lambda_a \) are the Pauli and Gell-Mann matrices normalized as \( \text{tr} \lambda_a \lambda_b = \text{tr} \tau_a \tau_b = 2\delta_{ab} \), respectively, and

\[
M = \sum_{a=0}^{8} \frac{\sigma_a \lambda_a}{\sqrt{2}} + i \sum_{a=0}^{8} \frac{\pi_a \lambda_a}{\sqrt{2}}, \quad (3)
\]

\[
N = \left( \begin{array}{c} \bar{p} \\ \eta \end{array} \right), \quad \chi = \sqrt{3} \left( \begin{array}{c} m_u \\ m_d \\ m_s \end{array} \right) = \sqrt{3} \left( \begin{array}{c} m_q \\ m_q \\ m_s \end{array} \right), \quad (4)
\]

\[
\bar{\sigma}_0 = -\langle \sigma_0 \rangle, \quad \bar{\sigma}_8 = -\langle \sigma_8 \rangle, \quad m_N = \frac{g}{\sqrt{3}} \left( \langle \sigma_0 \rangle + \frac{\langle \sigma_8 \rangle}{\sqrt{2}} \right). \quad (5)
\]

This Lagrangian is constructed to be invariant under the chiral transformation of hadron fields with the explicit breaking due to the quark mass. The baryon field is assumed to belong to the \((3,3) \oplus (3,\bar{3})\) representation of the \(\text{SU}(3)_L \otimes \text{SU}(3)_R\) group and here we show only the relevant nucleon field and its coupling to the non-strange mesons. In Eq. (4), we assume the isospin symmetry with \( m_q = m_u = m_d \), and introduce the explicit flavor symmetry breaking with \( m_q \neq m_s \). Due to the flavor symmetry breaking, the observed \( \eta \) and \( \eta' \) are linear combinations of the \(\text{SU}(3)\) eigenstate, \( \eta_0 \) and \( \eta_8 \). The free parameters contained in the Lagrangian, \( \mu^2, \lambda, \lambda', A, B, m_q, m_s \), are determined to reproduce the observed meson masses and decay constants and the 35% reduction of the quark condensate at the normal nuclear density. The nuclear medium is introduced with the nucleon mean-field approximation.

From this Lagrangian, we can obtain the \( \eta'N \) scattering amplitude. At the tree level, the diagrams contributing to the scattering amplitude are shown in Fig. [1]. The diagram (a) is the contribution from the scalar meson exchange, and the diagrams (b) and (c) are the Born terms.
Especially in the chiral limit, the $\eta'N$ scattering amplitude $V_{\eta'N}$, and $\eta'N$ transition amplitude to $\eta N$ channel $V_{\eta'N \rightarrow \eta N}$ are given as

$$V_{\eta'N} = \frac{6gB}{\sqrt{3}m_{\sigma_0}^2},$$  \hspace{1cm} (6)$$

$$V_{\eta'N \rightarrow \eta N} = \frac{6gB}{\sqrt{6}m_{\sigma_8}^2}. \hspace{1cm} (7)$$

Here, $m_{\sigma_0}$ and $m_{\sigma_8}$ are the singlet and octet scalar meson masses, respectively. The parameter $B$ is the coefficient of the determinant term, so it reflects the $U_A(1)$ anomaly. The $\eta'N$ transition to the $\eta N$ channel is relatively suppressed by the larger octet scalar-meson mass compared with $V_{\eta'N}$.

Evaluating the amplitude at the $\eta'N$ threshold, we find that the interaction is comparably strong to the Weinberg-Tomozawa interaction in the $\bar{K}N$ channel with $I=0$ where there exists the $\Lambda(1405)$ as a quasi-bound state \[20\].

3 The analysis of the $\eta'N$ two-body system

In this section, we explain the analysis of the $\eta'N$ system and show the result of our calculation.

We analyze the $\eta'N$ system with the analogous method to the $\bar{K}N$ system \[20\]: the scattering amplitude from the tree-level chiral perturbation theory is used as the interaction kernel. The divergence of the loop integral is renormalized with the natural renormalization scheme \[21\]. This means that we eliminate the dynamics other than the $\eta'$ and nucleon with this scheme.

The $T$ matrix obeys the scattering equation,

$$T_{\alpha\beta}(P) = V_{\alpha\beta} + V_{\alpha\gamma}G_\gamma(P)T_{\gamma\beta}(P)$$ \hspace{1cm} (8)

where the subscript $\alpha, \beta, \gamma$ are the label of the channel and they can be $\eta'N$, $\eta N$, or $\pi N$, and $P$ is the total momentum of the two particles. $G(P)$ is the two-body Green’s function and its divergence is renormalized with the natural renormalization scheme. Now, we use the scattering amplitude obtained from the linear sigma model at the $\eta'N$ threshold as the interaction kernel. The interaction kernel is momentum independent, so the scattering equation can be solved in the algebraic way.

In Table 1, we present the values obtained from the analysis of the $T$ matrix of the $\eta'N$ channel \[22\]. From the search of the pole in the complex energy
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| \( \eta'N \) binding energy [MeV] | \( \eta'N \) scattering length [fm] | \( \eta'N \) effective range [fm] |
|-----------------|----------------|-----------------|
| 12.3-3.3i       | -1.91+0.24i    | 0.24-7.6\times10^{-3}i |

Fig. 2 The absolute values of the T matrices [22]. The solid, dotted, and dashed lines represent the channel of the \( \eta'N \) to \( \eta'N \), \( \eta'N \) to \( \eta N \), and \( \eta N \) to \( \eta N \), respectively. The threshold of the \( \eta'N \) channel is 1896.7 MeV.

plane, we find a pole corresponding to the \( \eta'N \) bound state with the binding energy 12.3 – 3.3i MeV. The scattering length and effective range of the \( \eta'N \) system are \(-1.91 + 0.24i \) fm and \( 0.24 - 7.6 \times 10^{-3}i \) fm, respectively. The scattering length is the repulsive sign due to the existence of the \( \eta'N \) bound state. Compared with the scattering length suggested from the analysis of the experimental data which are less than 1 fm [18,19], the obtained scattering length is large value.

The plot of the absolute value of the T matrix is shown in Fig. 2 [22]. One can see the sub-threshold peak coming from the \( \eta'N \) bound state in Fig. 2.

4 Summary

Using the linear sigma model, we investigate the \( \eta'N \) 2body interaction with is essential for the discussion of the in-medium \( \eta' \) property.

In the linear sigma model, the \( \eta'N \) interaction is generated by the anomaly-induced scalar meson exchange, which is quite different origin from the ordinary NG boson-nucleon interaction. The transition to the \( \eta \) mesons is relatively suppressed by the large mass of the octet scalar meson compared with the \( \eta'N \) elastic channel.

From the analysis of the \( \eta'N \) system, we find a bound state with the binding energy 12.3 MeV and the half width 3.3 MeV. The obtained scattering length is about \(-1.91 + 0.24i \) fm whose real part has the repulsive sign.
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