Gauge invariant photon mass induced by vortex gauge interactions

M Cristina Diamantini\(^1\), Giuseppe Guarnaccia\(^2\) and Carlo A Trugenberger\(^3\)

\(^1\) NiPS Laboratory, INFN and Dipartimento di Fisica, University of Perugia, via A. Pascoli, I-06100 Perugia, Italy
\(^2\) Dipartimento di Fisica ‘E. R. Caianiello’, Università di Salerno, I-84084 Fisciano (Salerno), Italy
\(^3\) SwissScientific, chemin Diodati 10, CH-1223 Cologny, Switzerland

E-mail: cristina.diamantini@pg.infn.it, guarnacciagiuseppe@yahoo.it and ca.trugenberger@InfoCodex.com

Received 6 November 2013, revised 20 January 2014
Accepted for publication 24 January 2014
Published 17 February 2014

Abstract
We propose a vortex gauge field theory in which the curl of a Dirac fermion current density plays the role of the pseudovector charge density. In this field-theoretic model, vortex interactions are mediated by a single scalar gauge boson in its antisymmetric tensor formulation. We show that these long range vortex interactions induce a gauge invariant photon mass in the one-loop effective action. The fermion loop generates a coupling between photons and the vortex gauge boson, which acquires thus charge. This coupling represents also an induced, gauge invariant, topological mass for the photons, leading to the Meissner effect. The one-loop effective equations of motion for the charged vortex gauge boson are the London equations. We propose thus vortex gauge interactions as an alternative, topological mechanism for superconductivity in which no spontaneous symmetry breaking is involved.

Keywords: topological field theory, BF mechanism, topological superconductivity
PACS numbers: 11.10.—z, 11.15.Wx, 73.43.Nq, 74.20.Mn

Topology plays a major role in both field theory and condensed matter systems. It is by now well known that it is not necessary to spontaneously break the U(1) gauge symmetry to generate a photon mass: the famed topological Chern–Simons (CS) term in (2+1) dimensions \([1]\) accomplishes this in a gauge invariant manner, albeit breaking the \(P\) and \(T\) symmetries.
Since the CS term is the infrared-dominant term in the gauge field action it can describe new universality classes of topological matter [2] when the dual field strength is used to represent a conserved matter current.

An analogous mechanism can give photons a gauge invariant mass in (3+1) dimensions, and this time without breaking \( P \) and \( T \). This is achieved by another topological term, called the \( BF \) term and coupling the dual field strength \( \tilde{F}_{\mu\nu} \) to a second-rank antisymmetric pseudotensor \( B_{\mu\nu} \) [3]. This is a topological mechanism for superconductivity, in which the field equations for the antisymmetric tensor represent the London equations.

Due to a second-order gauge symmetry, leaving the action invariant under transformations \( B_{\mu\nu} \rightarrow B_{\mu\nu} + \partial_\mu \lambda_\nu - \partial_\nu \lambda_\mu \), the antisymmetric tensor field encodes actually a single scalar degree of freedom. Classically, this is dual to the phase of a \( U(1) \) order parameter and the superconductivity mechanism is dual to the Higgs mechanism in the limit of an infinitely heavy Higgs boson: in the \( BF \) mechanism, it is actually the scalar boson that ‘eats up’ the original massless photon to become a massive photon. Quantum mechanically, however, the theories are deeply different, since one can prove [3] that no spontaneous symmetry breaking is involved in the \( BF \) mechanism, contrary to the Higgs mechanism. The \( BF \) superconductivity mechanism is a topological mechanism for superconductivity, genuinely different from the Higgs mechanism.

It is well known that the CS term in (2+1) dimensions is radiatively induced at one-loop level [4] even if it is not present in the original Lagrangian. The generated CS coupling constant \( k \), in this case, is proportional to the sign of the fermion mass.

In this paper we show that, also in (3+1) dimensions the \( BF \) term is induced at one loop, if the vorticity field of charged fermions is coupled gauge-invariantly to the antisymmetric pseudotensor gauge field \( B_{\mu\nu} \). It is known that spin interactions, and particularly spin-orbit coupling, play an important role in the physics of topological insulators [5]. It has also been shown that collective excitations, like phonons, can induce long range spin-spin interactions [6]. Here we propose analogous vortex gauge interactions as an alternative, topological mechanism for superconductivity. As we will show, in this case, the BF coupling constant is proportional to \( \frac{\Delta \ln M}{m} = O(m) \), where \( m \) is the fermion mass and \( \Lambda \) an ultraviolet cutoff.

If we compare with the CS case in (2+1) dimensions we see that, in this case, the coupling constant is proportional to the fermion mass. The presence of the ultraviolet cutoff is due to the non-renormalizability of the model in (3+1) dimensions. Topological mass generation in (3+1)-dimensional QED (without the \( B_{\mu\nu} \) tensor field) has been also studied in [7] within a generalization to four dimensions of the Schwinger model.

First attempts to extend the gauge principle to vector-like charges [8] have considered as obvious candidates for a tensor gauge theory of spin the standard Dirac tensor and pseudotensor densities \( \bar{\psi} \sigma^{\mu\nu} \psi \) and \( \bar{\psi} \gamma^5 \sigma^{\mu\nu} \psi \), with \( \sigma^{\mu\nu} \equiv (i/2) [\gamma^\mu, \gamma^\nu] \) (we shall use units in which \( \hbar = 1 \), \( c = 1 \) throughout the paper, \( [\cdot] \) denotes commutators and \( \{\cdot,\cdot\} \) anticommutators). Both, however are not suitable for a gauge theory, since they are not conserved.

The spin density of Dirac fermions is given by the expression \( \bar{\psi} \Sigma \psi \) with \( \Sigma = \text{diag}(\sigma, \sigma) \). Using \( \sigma^{ij} = \epsilon^{ijk} \Sigma_k \), this can be embedded in the spin current \( S^{\alpha\beta\gamma\sigma} = (1/4) \bar{\psi} \{\gamma^\alpha, \Sigma^\beta \} \gamma^\gamma \psi \). This, however, is a third-order tensor. In order to obtain a second-order tensor we shall consider its derivative. Moreover, in order to maintain \( P \) and \( T \) conservation when coupling to the pseudotensor \( B_{\mu\nu} \), we shall add a \( i\gamma^5 \) matrix in the original spin current. We shall thus couple \( B_{\mu\nu} \) to the pseudotensor current

\[
J^{\mu\nu} = \frac{i}{2m} \partial_\alpha (\bar{\psi} \gamma^\alpha [\gamma^\mu, \sigma^{\mu\nu}] \psi) = \frac{-3}{2m} \partial_\alpha (\bar{\psi} \gamma^\alpha [\gamma^\mu, \gamma^\nu] \psi),
\]
where the symbol [...] in the exponent of [1] denotes total antisymmetrization of the indices. In the second representation of this current, its conservation is explicit. Thus, the coupling $B_{μν}J^{μν}$ is invariant under gauge transformations

$$B_{μν} \rightarrow B_{μν} + \partial_μ λ_ν - \partial_ν λ_μ.$$  

(2)

It is also invariant under $P$ and $T$ transformations.

In order to establish what is the pseudovector charge of this gauge theory we must consider the components $J^{0i}$ that couple to $B_{0i}$. By explicit computation we obtain $J^{0i} = -(1/2m)ε^{ijk} \partial_j ψ_i a^k ψ$. The expression $ψ_i a^k ψ$ is the velocity field of the Dirac fermion. Thus, in analogy to fluid dynamics, the pseudovector charge density $J^0$, given by the curl of the velocity field, represents the vorticity field of the fermion. The corresponding pseudovector charges $Φ^i = ∫_S d^2 x F^{0i}$ represent the vortex fluxes through the infinite planes $Σ$, with unit normals $n^i$. Since $∂_μ J^{μν} = 0$ these pseudovector charges are conserved.

Using the Gordon decomposition (for the purpose of illustrating the spin dependence of the current, we consider the Gordon decomposition of the non-interacting case)

$$ψ^i a^k ψ = \frac{i}{2m}[ψ \partial^k ψ - \partial^k ψ ψ] + \frac{1}{m} ψ ψ α^k μ ν, \quad (3)$$

one can separate the pseudovector charges into their orbital and spin contributions. In the non-relativistic limit, in which the lower components of Dirac spinors can be neglected for energies much lower than their mass, this reduces to [9]

$$(ψ^i a^k ψ)_{NR} \rightarrow \frac{i}{2m}[ψ \partial^k φ - \partial^k ψ φ] + \frac{1}{2m} ε^{kij} \partial_j φ ψ, \quad (4)$$

where the spinors $φ$ on the right-hand side are the two-dimensional Pauli spinor corresponding to the upper components of the four-dimensional Dirac spinors $ψ$. The non-relativistic spin contribution to the pseudovector charge density becomes thus

$$(J^μ_{quad})_{NR} = \frac{1}{2m^2} \nabla^2 (ψ^i σ^i ψ), \quad (5)$$

which is the Laplacian of the intrinsic magnetic moment density of the particle.

We shall henceforth consider a fully relativistic vortex gauge model of massive (mass $m$) charged ($e$) Dirac fermions with the following Lagrangian density

$$\mathcal{L} = ̄ψ γ^μ (i∂_μ - eA_μ) ψ - m ̄ψ ψ + \frac{i}{9} ψ_μ B^μ_{μν} - \frac{1}{4} F^{μν} F_{μν} + \frac{1}{12} H_{μνσ} H^{μνσ}, \quad (6)$$

where $H_{μνσ} \equiv ∂_μ B_{νσ} - ∂_ν B_{μσ}$ is the Kalb–Ramond [10] gauge invariant field strength for the antisymmetric tensor and $g$ is the dimensionless vortex coupling constant (the factor $1/9$ has been introduced to simplify later numerical factors). It is interesting to note that, using the gamma matrix relation $γ^μ γ^ν γ^σ = 16 ε^{μνσ} γ_σ$, one can rewrite the vortex interaction $i^4 g B_{μν} J^{μν}$ in (6) as:

$$i^4 g B_{μν} J^{μν} = - \frac{g}{2m} F_μ J^μ, \quad (7)$$

where $F_μ = (1/2ε_{μνσ}∂^ν B^{σμ})$ is the dual Kalb–Ramond field strength and $J^μ$ is the usual Dirac fermion current. Formally, thus, the dual Kalb–Ramond field strength plays the role of a gauge field in the Lorenz gauge (note however that the corresponding charge $g/2m$ has the dimensions of an inverse mass).

Due to the structure of the Kalb–Ramond kinetic term, the mixed components $B_{0i} = -B_{i0}$ play the role of non-dynamical Lagrange multipliers, leaving three dynamical degrees of freedom in the tensor $B_{μν}$. The gauge invariance equation (2) eliminates, however, two of
these, since there are two independent gauge parameters $\lambda_i$ (the other one being eliminated by the equivalence $\lambda_i \equiv \lambda_i + \partial_i \eta$), leaving thus one overall degree of freedom. The long-range vortex gauge interaction is thus mediated by a single massless scalar boson.

The theory defined by the Lagrangian density (6) is non-renormalizable due to the mass scale in the denominator of the current density $J_{\mu\nu}$. It is thus to be considered as a low-energy field theory, valid only for energies smaller than the Dirac mass scale $m$. This is, however, exactly the regime in which we are interested, the induced photon mass having the scale $egm \ll m$ in the perturbative regime $e, g \ll 1$. In other words we are adopting a hydrodynamic approximation in which we describe long-scale collective phenomena of a relativistic quantum plasma without addressing the high-energy physics on the scale of the individual electrons (positrons) composing it. The vorticity fields $J^0_i$ that we couple gauge-invariantly to the single scalar embedded in $B_{\mu\nu}$ are typical variables of this regime. As we now show, the induced photon mass is exactly one of the collective phenomena arising at these low energies.

In order to compute the one-loop effective action for the fields $A_\mu$ and $B_{\mu\nu}$, we rewrite the interaction term between the Kalb–Ramond tensor field and the fermions as:

$$L_{\text{int}} = \frac{ig}{3m} \bar{\psi} \gamma^5 \Gamma^{\rho\mu\nu} \psi \partial_\rho B_{\mu\nu},$$

where we have performed an integration by parts and used the antisymmetry of $B_{\mu\nu}$; the tensor $\Gamma^{\rho\mu\nu}$ is defined as

$$\Gamma^{\rho\mu\nu} = (\gamma^\rho \gamma^\mu \gamma^\nu + \gamma^\nu \gamma^\rho \gamma^\mu + \gamma^\mu \gamma^\nu \gamma^\rho).$$

Integrating over the fermion fields we obtain:

$$i\Gamma_{\text{eff}}(A, B) = \text{Tr} \ln \left(1 - eA + \frac{ig}{3m} \gamma^5 \Gamma^{\rho\mu\nu} \partial_\rho B_{\mu\nu} B_{\mu\nu}\right).$$

In (10) we can separate out the trace of the field-independent term $(i\bar{\psi} - m)$ and rewrite the resulting logarithm as:

$$i\Gamma_{\text{eff}}(A, B) = \text{Tr} \ln \left(1 + \frac{1}{i\bar{\psi} - m} \left(-eA + \frac{ig}{3m} \gamma^5 \Gamma^{\rho\mu\nu} \partial_\rho B_{\mu\nu}\right)^{-1}\right)$$

$$= \sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n} \text{Tr} \left(\frac{1}{i\bar{\psi} - m} \left(-eA + \frac{ig}{3m} \gamma^5 \Gamma^{\rho\mu\nu} \partial_\rho B_{\mu\nu}\right)^n\right).$$

The first term in (11) is the sum of the tadpole diagrams whereas the second term is the relevant quadratic term in the fields $A_\mu$ and $B_{\mu\nu}$:

$$i\Gamma_{(1\text{-loop})}^{\text{eff}}(A, B) = \frac{-1}{2} \text{Tr} \left[\frac{1}{i\bar{\psi} - m} \left(-eA + \frac{ig}{3m} \gamma^5 \Gamma^{\rho\mu\nu} \partial_\rho B_{\mu\nu}\right)^2\right]$$

$$\times \frac{1}{i\bar{\psi} - m} \left(-eA + \frac{ig}{3m} \gamma^5 \Gamma^{\rho\mu\nu} \partial_\rho B_{\mu\nu}\right).$$

The quadratic term (12) contains three contributions: the term quadratic in $A_\mu$, that corresponds to the standard vacuum polarization and gives rise to the renormalization of the electric charge; the quadratic term in $B_{\mu\nu}$ that gives rise to the corresponding vacuum polarization for the tensor field and the interaction term between $A_\mu$ and $B_{\mu\nu}$ that, as we now show, gives rise to the BF term generated at one loop.

In the following we shall use Pauli–Villars regularization introducing a cutoff $\Lambda = O(m)$. Since our theory is an effective theory at long distances, in the calculation we will consistently consider the limit of small momenta $k^2/m^2 \ll 1$.
The term quadratic in $A_\mu$ is given by:
\[ \Gamma^{(1\text{-loop})}_{AA} = -\frac{ie^2}{2} \text{Tr} \left( \frac{1}{i\gamma - m} \gamma^5 \frac{1}{i\gamma - m} \right) \]
\[ = \frac{1}{2} \int \frac{d^4k}{(2\pi)^4} \Pi_{\mu\nu}(k) \Pi^{\mu\nu}(k) \tag{13} \]
where $\Pi^{\mu\nu}$ is the usual vacuum polarization tensor of QED,
\[ \Pi^{\mu\nu}(k) = (g^{\mu\nu} k^2 - k_\mu k_\nu) \Pi(k^2) \]
\[ \Pi(k^2) = \frac{e^2}{12\pi^2} \ln \frac{\Lambda^2}{m^2}. \tag{14} \]
for $k^2 \ll \Lambda^2$. We obtain thus
\[ \Gamma^{(1\text{-loop})}_{AA} = \frac{1}{4} \frac{e^2}{12\pi^2} \ln \frac{\Lambda^2}{m^2} \int d^4xF_{\mu\nu}F^{\mu\nu}. \tag{15} \]

The computation of the quadratic term in the Kalb–Ramond field is straightforward but more lengthy and tedious since it involves the trace of eight $\gamma$ matrices. It is given by:
\[ \Gamma^{(1\text{-loop})}_{B\mu} = \frac{i\gamma^2}{9m^2} \text{Tr} \left( \frac{1}{i\gamma - m} \gamma^5 \Gamma^{\mu\nu} \partial_\nu B_{\mu\nu} \frac{1}{i\gamma - m} \gamma^5 \Gamma^{\rho\sigma} \partial_\rho B_{\rho\sigma} \right) \]
\[ = \int \frac{d^4k}{(2\pi)^4} B_{\mu\nu}(k)B_{\rho\sigma}(-k)Q^{\mu\nu\rho\sigma}(k), \tag{16} \]
with $Q^{\mu\nu\rho\sigma}(k)$:
\[ Q^{\mu\nu\rho\sigma} = \frac{i\gamma^2}{18m^2} k_\kappa k_\beta \int \frac{d^4p}{(2\pi)^4} \text{Tr} \left( \frac{1}{p^2 - m^2} \gamma^5 \Gamma^{\mu\nu} \frac{1}{(p^2 - k^2 - m^2)} \gamma^5 \Gamma^{\rho\sigma} \right). \tag{17} \]

Using Feynman parametrization [11] equation (17) can be rewritten as:
\[ Q^{\mu\nu\rho\sigma} = -\frac{i\gamma^2}{18m^2} k_\kappa k_\beta \int_0^1 dx \int \frac{d^4p}{(2\pi)^4} \frac{N^{\mu\nu\rho\sigma}}{(p^2 - k^2 - m^2)^2}, \tag{18} \]
with $\Delta = m^2 - k^2 x(1 - x)$, and
\[ N^{\mu\nu\rho\sigma} = \text{Tr}(\gamma^5 \Gamma^{\mu\nu} (p^2 - k^2 + m^2) \gamma^5 \Gamma^{\rho\sigma}), \tag{19} \]
that is, the sum of two terms with eight $\gamma$ matrices and one term with six $\gamma$ matrices. Computing the traces we obtain:
\[ Q^{\mu\nu\rho\sigma}(k) = -\frac{g^2}{12\pi^2} \frac{k^2}{m^2} \ln \left( \frac{\Lambda^2}{m^2} \right) \tilde{Q}^{\mu\nu\rho\sigma}(k), \tag{20} \]
where $\tilde{Q}^{\mu\nu\rho\sigma}(k)$ is the Kalb–Ramond kernel
\[ \tilde{Q}^{\mu\nu\rho\sigma}(k) = [k^2 (g^{\mu\rho} g^{\nu\sigma} - g^{\mu\sigma} g^{\nu\rho}) - (g^{\mu\rho} k^\kappa k^\sigma - g^{\mu\sigma} k^\kappa k^\rho) - (g^{\nu\rho} k^\kappa k^\sigma - g^{\nu\sigma} k^\kappa k^\rho)]. \tag{21} \]

We first notice that, as expected, gauge invariance prevents a mass term for $B_\mu$, to be generated at one-loop level, contrary to the result [8], where the Kalb–Ramond tensor field is coupled to a non-gauge-invariant current. This result is crucial, since we are seeking a mechanism for a mass that is topologically generated through the BF mechanism for the photon in analogy to the gauge invariant mass generation in Chern–Simons theories. The term equation (20), that is generated at the one-loop level is, however, a term of higher order in derivatives with respect to the Kalb–Ramond kernel $\tilde{Q}^{\mu\nu\rho\sigma}(k)$, due to the non-renormalizability of our model. This term is suppressed in the low-energy limit in which we are interested, where $m^2 \gg k^2$. The coupling constant $g$ is not renormalized at one-loop.
Let us now compute the interaction term between $A_\mu$ and $B_{\mu\nu}$:

$$\Gamma_{\text{BF}}^{\text{1-loop}} = \frac{ieg}{3m} \text{Tr} \left( \frac{1}{i\not\!D - m} \frac{1}{i\not\!D - m} \gamma^5 \Gamma^{\mu\nu\rho\sigma} \partial_\rho B_{\mu\nu} \right)$$

$$= \frac{ieg}{3m} \int \frac{d^4k}{(2\pi)^2} A_\sigma(k) B_{\mu\nu}(-k) G^{\mu\nu\sigma}(k),$$

(22)

with

$$G^{\mu\nu\sigma}(k) = \int \frac{d^4p}{(2\pi)^4} k_\rho \text{Tr} \left( \frac{1}{p - m} \gamma^\sigma \frac{1}{(p - k) - m} \gamma^5 \Gamma^{\mu\nu\rho\sigma} \right).$$

(23)

Again, using Feynman parametrization, we have:

$$G^{\mu\nu\sigma}(k) = \int_0^1 dx \int \frac{d^4p}{(2\pi)^4} \frac{N^{\mu\nu\sigma}}{((p - kx)^2 - \Delta)^2},$$

(24)

where:

$$N^{\mu\nu\sigma} = k_\rho \text{Tr} \left( (\not\!p + m)\gamma^\sigma ((\not\!p - \not\!k) + m)\gamma^5 \Gamma^{\mu\nu\rho\sigma} \right)$$

$$= -12ie^2k_\rho \epsilon^{\mu\nu\sigma\rho\sigma}.$$

(25)

Using equations (24) and (25) we obtain the one-loop effective action contribution:

$$\Gamma_{\text{BF}}^{\text{1-loop}} = \frac{egm}{4\pi^2} \ln \frac{\Lambda^2}{m^2} \int d^4x B_{\mu\nu}(x) \epsilon^{\mu\nu\rho\sigma} \partial_\rho A_\sigma(x).$$

(26)

Equation (26) is exactly the BF topological interaction [3], which, as the Chern–Simons term in [1], is generated at one loop by radiative corrections.

With the standard rescalings $A_\mu \rightarrow eA_\mu$, $B_{\mu\nu} \rightarrow gB_{\mu\nu}$ we can write the one-loop effective action as:

$$\mathcal{L} = -\frac{1}{4e_{\text{ph}}^2} F_{\mu\nu} F^{\mu\nu} + \frac{M}{2\pi} B_{\mu\nu} \epsilon^{\mu\nu\rho\sigma} \partial_\rho A_\sigma + \frac{1}{12g^2} H_{\mu\nu\alpha} H^{\mu\nu\alpha},$$

(27)

where:

$$e_{\text{ph}}^2 = e^2 \left( 1 + \frac{e^2}{12\pi^2} \ln \frac{\Lambda^2}{m^2} \right),$$

(28)

is the renormalized charge (note that this is exactly as in QED since in (3+1) dimensions, contrary to the Maxwell–CS case in (2+1), the Maxwell term is marginal [13], and the $B_{\mu\nu}$ field gives no contribution to charge renormalization at this order) and

$$M = \frac{m}{2\pi} \ln \frac{\Lambda^2}{m^2} = O(m),$$

(29)

is a new mass scale of the order of the electron mass. The action equation (27) generalizes to four dimensions the Chern–Simons mechanism for topological mass. The mass is topological because it does not arises from spontaneous symmetry breaking, but is due to the presence of the BF term, independent of the metric.

The equations of motions are:

$$\partial_\mu F^{\mu\nu} = -\frac{e_{\text{ph}}^2 M}{6\pi} \epsilon^{\mu\nu\rho\beta} H_{\rho\beta},$$

$$\partial_\mu H^{\mu\rho\beta} = \frac{g^2 M}{2\pi} \epsilon^{\mu\rho\beta\nu} F_{\nu\mu},$$

(30)

from which one can derive [3, 12]:

$$\left[ \Box + \left( \frac{Me_{\text{ph}}g^2}{2\pi} \right)^2 \right] F_{\mu\nu} = 0.$$

(31)
Equation (31) shows that the field strength $F_{\mu\nu}$ satisfies the Klein–Gordon equation with mass given by $\frac{M_{\text{ph}}}{2\pi}$. By the same analysis also $H^{\mu\nu}$ satisfies the same equation of motion.

From equation (30) we also recognize that the dual of the Kalb–Ramond field strength acts as charged current for the photon field:

$$\partial_{\mu} F_{\mu\nu} = J^\nu, \quad J^\nu = -\frac{e_{\text{ph}}^2 M}{\pi} H^\nu = -\frac{e_{\text{ph}}^2 M}{6\pi} \epsilon^{\nu\mu\alpha\beta} H_{\alpha\beta}. \quad (32)$$

Substituting equation (32) in the second equation equation (30) we obtain:

$$\epsilon^{\mu\nu\alpha\beta} \partial_\alpha J_\beta = - \left( \frac{e_{\text{ph}}^2 M}{\pi} \right)^2 \tilde{F}_{\mu\nu},$$

$$\tilde{F}_{\mu\nu} = \frac{1}{2} \epsilon^{\mu\nu\alpha\beta} F_{\alpha\beta} \quad (33)$$

which is nothing else than a relativistic version of the London equations for superconductivity.

In conclusion, we have shown that electron vorticity gauge interactions mediated by a scalar boson in a hydrodynamic effective low-energy field theory generate the BF topological mass term at one-loop in the effective action. Photons acquire a gauge-invariant topological mass, the additional scalar boson acquires charge and the corresponding current satisfies the London equations, all the hallmarks of ‘gauge-invariant’ superconductivity with no spontaneous symmetry breaking involved.

Acknowledgment

We thank Professor H Hansson for a critical reading and very helpful comments on the manuscript.

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