Dynamics of entropic measurement-induced nonlocality in structured reservoirs

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We propose the entropic measurement-induced nonlocality (MIN) as the maximal increment of von Neumann entropy induced by the locally non-disturbing measurement, and study behaviors of it both in the independent and common structured reservoirs. We present schemes for preserving the MIN, and show that for certain initial states the MIN, including the quantum correlations, can even be enhanced by the common reservoir. Additionally, we also show that the different measures of MIN may give different qualitative characterizations of nonlocal properties, i.e., it is rather measure dependent than state dependent.

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I. INTRODUCTION

Nonlocality is a basic property of quantum states not available in classical world, and it has been the research interest for many years. The nonlocality can be studied in the context of Bell-type inequalities, it can also be investigated from other viewpoints, for example, from an information-theoretic perspective. On the other hand, the nonlocality is closely related with entanglement which plays a central role in quantum information processing, but they are not completely the same. There are a series of work concentrated on revealing the relationship between nonlocality and entanglement, for example, it is shown that while there are entangled states without Bell nonlocality, there also exists nonlocality without entanglement.

It is generally accepted that entanglement is a resource in quantum protocols and algorithms. Interestingly, recent progress shows that the separable but nonclassical states may be responsible for the speedup of a quantum algorithm. It ignites interests of people on studying, from various points of view, the quantumness of correlations and their relations with entanglement and nonlocality. Particularly, Luo and Fu introduced a geometric measure of nonlocality which they termed measurement-induced nonlocality (MIN). In comparison with Bell nonlocality which is featured by violation of Bell-type inequalities satisfied by any local hidden variable theory, the MIN is in some sense a more general kind of correlation which was introduced from the consideration that measurements in quantum mechanics usually causes disturbance. The extent of the maximum disturbance of locally non-disturbing measurements on the total state can reveal global effect of that state which cannot be accounted for locally and therefore can be exploited to quantify nonlocality.

The MIN provides an interesting classification scheme for the bipartite quantum states. In the original geometric quantification scheme, it is obtained by maximizing the Hilbert-Schmidt distance between the pre- and post-measurement states. For completeness of theoretical consideration, it is also desirable to quantify it from other perspectives so as to reveal more about its meaning and properties. This is one of our motivations in this paper. Moreover, we know that entanglement is fragile due to decoherence induced by inevitable interaction between the system and its environment. While the MIN can also be a powerful resource enabling fascinating quantum tasks as shown in, it is of practical significance to know the property of MIN, for example, to find ways to stabilize the MIN of a state or to minimize the devastating effects of the decohering environments. This is also a motivation of this work.

In this paper, we introduce the entropic measure of MIN, which is different from the corresponding geometric measure, and give an information-theoretic interpretation of it. We will obtain the conditions for non-vanishing MIN, and show that there is nonlocality without quantumness in the sense that the MIN may persist even for certain zero-discord states. We will also offer a comparative study of the relationships between MIN and different quantum correlation measures for the two-qubit case. We will show that when interacting with the structured reservoirs, the MIN can be frozen by introducing detuning or by performing local unitary operations on the initial state, particularly, the strength of MIN can even be enhanced by interacting the system with a common reservoir.

The paper is organized as follows. In Section II we first recall the concept of MIN and its quantification based on the Hilbert-Schmidt norm. Then we reformulate the MIN from an entropic way and list some general features of it. In Section III, we evaluate dynamics of MIN for a certain family of two-qubit states both in the independent and common reservoirs, and present schemes for preserving and enhancing the MIN. Finally, Section IV is devoted to a summary.
II. MEASURES OF THE MIN

The preform of MIN was first introduced by Luo and Fu [13] and further formulated by the same authors [17]. Consider a bipartite state $\rho$ shared by two parties $a$ and $b$, with marginal states $\rho^a$ and $\rho^b$, then the MIN is defined as follows

$$N^\ast(\rho) = \max_{\Pi^a} \|\rho - \Pi^a(\rho)\|^2,$$  

where the maximum is taken over the local von Neumann measurements $\Pi^a = \{\Pi^a_k\}$ satisfying $\sum_k \Pi^a_k \rho \Pi^a_k = \rho^a$, and $\|\cdot\|$ denotes the Hilbert-Schmidt norm with $\|X\|^2 = \text{tr}X^\dagger X$. Clearly, $\Pi^a$ contains in fact only the measurements that do not disturb $\rho^a$ locally, thus as pointed out in [17], the MIN is in some sense dual to the geometric measure of quantum discord (GMQD) [10] $D^\ast(\rho) = \min_{\Pi^a} \|\rho - \Pi^a(\rho)\|^2$, with $\Pi^a$ runs over all the local von Neumann measurements.

By noting that any $m \times n$ bipartite state $\rho$ can always be represented as

$$\rho = \frac{1}{\sqrt{mn}} \left[ \begin{array}{c} a \end{array} \right] \left[ \begin{array}{cc} \Pi^a \otimes \frac{\mathbb{I}^b}{\sqrt{n}} & b \end{array} \right] + \frac{\mathbb{I}^a}{\sqrt{m}} \left( \sum_{j=1}^{n^2-1} y_j Y_j + \sum_{i=1}^{m^2-1} \sum_{j=1}^{n^2-1} r_{ij} X_i \otimes Y_j \right),$$

Luo and Fu derived an analytical formula of MIN for the $2 \times n$ dimensional states, which is given by [17]

$$N^\ast(\rho) = \left\{ \begin{array}{lc} \|R\|^2 - \frac{1}{\|x\|^2} x^T R R^T x & \text{if } x \neq 0, \\ \|R\|^2 - \lambda_{\min} & \text{if } x = 0, \end{array} \right.$$  

(3)

where $\{X_i\}$ and $\{Y_j\}$ are traceless Hermitian operators satisfying the conditions $X_i X_j = \delta_{ij}$, and $Y_j Y_j' = \delta_{jj'}$. Moreover, $\lambda_{\min}$ is the smallest eigenvalue of $RR^T$ with $R$ being a real $3 \times 3$ matrix with elements $r_{ij}$, and $x = (x_1, x_2, x_3)^T$ with $\|x\|^2 = \sum_{i=1}^{3} x_i^2$. Here the superscript $T$ denotes transpose of vectors or matrices.

Although the definitions of MIN and GMQD are different, they may assume equal values under certain circumstances. To see this, consider the two-qubit X state $\rho_X$ (i.e., the $4 \times 4$ density matrix with nonzero elements only along the main diagonal and anti-diagonal), for which one can obtain

$$x = \frac{1}{2} \left( 0, 0, \rho_{11} + \rho_{22} - \rho_{33} - \rho_{44} \right)^T,$$

$$\frac{1}{\|x\|^2} x^T R R^T x = \frac{\left( \rho_{11} - \rho_{22} - \rho_{33} + \rho_{44} \right)}{4},$$

(4)

and the eigenvalues of $RR^T$ as

$$\lambda_{1,2} = \left( \frac{\|\rho_{11} \pm \rho_{22}\|}{2} \right)^2,$$

$$\lambda_3 = \left( \frac{\|\rho_{11} - \rho_{22} - \rho_{33} + \rho_{44}\|}{4} \right)^2.$$  

(5)

Moreover, it has been shown by Dakić et al. [10] that for the two-qubit quantum states the GMQD can be presented explicitly as

$$D^\ast(\rho) = \|x\|^2 + \| R \|^2 - k_{max}.$$  

where $k_{max}$ is the largest eigenvalue of the matrix $K = \bar{x}^T + RR^T$. For $\rho_X$, the eigenvalues of $K$ can be obtained analytically as

$$k_{1,2} = \lambda_{1,2}, \quad k_3 = \frac{1}{2} \sum_{n=1}^4 \rho_{nn}^2 - (\rho_{11}^2 + \rho_{22}^2 + \rho_{33}^2 + \rho_{44}^2).$$  

(7)

Eqs. (4) and (7) imply that $\|x\|^2 + \frac{1}{\|x\|^2} x^T R R^T x = k_3$, so for nondegenerate $\rho_X^a$ (i.e., $x \neq 0$) we have $N^\ast(\rho_X) = D^\ast(\rho_X)$ when $k_{max} = k_3$. In fact, the eigenvector of $K$ corresponding to the eigenvalue $k_3$ is $|e_3\rangle = (0, 0, 1)^T$, hence the optimal measurement for obtaining the GMQD is $\Pi_{1,2}^a = \frac{1}{2}(\| e \pm \sigma \rangle \langle e \pm \sigma |)$ [10]. Moreover, if $\rho_X^a$ is nondegenerate, then the unique von Neumann measurement leaving $\rho_X^a$ invariant is $\Pi_{1,2}^a = \frac{1}{2}(\| e \pm \sigma \rangle \langle e \pm \sigma |)$ [17]. The optimal measurements for obtaining the MIN and GMQD are completely the same, thus it is understandable that they assume equal values under the condition $k_{max} = k_3$.

Moreover, it is direct to check that $\|x\|^2 - k_3 = -\lambda_3$, thus if $\rho_X^a$ is degenerate with the addition of $k_{max} = k_3$ and $\lambda_{min} = \lambda_3$, we still have $N^\ast(\rho_X) = D^\ast(\rho_X)$.

Besides the Hilbert-Schmidt norm which measures the MIN from a geometric perspective [17], it is also desirable to quantify it from other perspectives. In particular, if we accept that the quantum mutual information (QMI) is a good measure of total correlations in a bipartite state $\rho$, then it is natural to quantify MIN by the maximal discrepancy between the QMI of the pre- and post-measurement states as

$$N^\nu(\rho) = I(\rho) - \min_{\Pi^a} I[\Pi^a(\rho)],$$  

where $I(\rho) = S(\rho^a) + S(\rho^b) - S(\rho),$ represents the QMI, and the minimum is taken only over the local von Neumann measurements leaving $\rho^a$ invariant. This measure of MIN quantifies in fact, the maximal loss of total correlations under locally invariant (non-disturbing) measurements. Since $I[\Pi^a(\rho)] = S(\rho^a) - S(\rho[\Pi^a(\rho)])$ [8] represents the average information gained about the system $b$ conditioned on the measurements $\{\Pi^a\}$, $N^\nu(\rho)$ is in some sense dual to the quantum discord (QD) $D^\nu(\rho) = I(\rho) - \max_{\Pi^a} I[\Pi^a(\rho)]$ [8]. But the maximum in $D^\nu(\rho)$ runs over all the local von Neumann measurements. Moreover, the measure of MIN introduced here is also quite different from the quantity $M(\rho) = I(\rho) - I[\Pi^a(\rho)]$, which was introduced by Luo [8] and has been termed measurement-induced disturbance (MID), because when defining $M(\rho)$ the locally invariant measurements $\Pi = \{\Pi^a \otimes \Pi^b\}$ are performed on both parties of $a$ and $b$.

Since the projective measurement $\Pi^a$ leaves $\rho^a$ invariant ($\rho^b$ is obviously also invariant), $N^\nu(\rho)$ defined above
is equivalent to
\[ N^v(\rho) = \max_{\Pi^v} S[\Pi^v(\rho)] - S(\rho), \tag{9} \]
from which one can obtain \( N^v(\rho) \geq 0 \) because projective measurements always increase entropy (Theorem 11.9 of [18]). Eq. (9) also indicates that \( N^v(\rho) \) quantifies in fact the maximal increment of von Neumann entropy induced by the locally invariant measurements. This result thus establishes a connection between increment of entropy and nonlocality in the quantum domain. Moreover, from an information-theoretic perspective, the entropy measures how much uncertainty there is in the state of a physical system \( \mathcal{S} \), \( N^v(\rho) \) can therefore also be interpreted as the maximal increment of our uncertainty about that system induced by the locally invariant measurements.

The post-measurement state can always be written as \( \Pi^v(\rho) = \sum_i p_i \rho_i^v \) (11), combination of this with the joint entropy theorem [18] gives rise to \( S[\Pi^v(\rho)] = H(p_i) + \sum_i p_i S(\rho_i') \), where \( H(p_i) \) represents the Shannon entropy. Furthermore, from Theorem 11.10 of [18] one can obtain \( H(p_i) + S(\rho_i') \geq H(p_i) + \sum_i p_i S(\rho_i') \geq S(\rho_i') \), and from [18] we have \( \sum_i p_i S(\rho_i') \leq S(\rho) \), thus \( N^v(\rho) \) is lower bounded by \( N^v(\rho) \geq -S(a(b)) \) and upper bounded by \( N^v(\rho) \leq \min\{I(\rho), S(\rho')\} \). With \( S(a(b)) = S(\rho) - S(\rho') \) being the conditional von Neumann entropy for \( \rho \), and \( -S(a(b)) \) gives the lower bound of the one-way distillable entanglement. But both the lower and the upper bounds are non-trivial only when \( S(a(b)) < 0 \), because we always have \( N^v(\rho) \geq 0 \) and \( N^v(\rho) \leq I(\rho) \).

For the two-qubit X states \( \rho_X \) with nondegenerate \( \rho_X^v \), the only von Neumann measurement leaving \( \rho_X^v \) invariant is \( \Pi^v = \{11, \Pi_2^v\} \) with \( \Pi_2^v = \frac{1}{2} (\mathbb{1} \pm \sigma_z) \), which gives rise to the post-measurement state as \( \Pi^v(\rho_X) = \sum_k (\Pi_k^v \otimes \mathbb{1}) \rho_X(\Pi_k^v \otimes \mathbb{1}) = \text{diag}\{\rho_{11}, \rho_{22}, \rho_{33}, \rho_{44}\} \). For \( \rho_X \) with nondegenerate \( \rho_X^v \) one can check directly that \( \Pi(\rho_X) = \sum_k (\Pi_k^v \otimes \mathbb{1}) \rho_X(\Pi_k^v \otimes \mathbb{1}) = \text{diag}\{\rho_{11}, \rho_{22}, \rho_{33}, \rho_{44}\} \). Thus for the special circumstance here we always have \( N^v(\rho_X) = M(\rho_X) \). But if \( \rho_X^v \) is degenerate, then \( N^v(\rho_X) \geq M(\rho_X) \) in general.

Now we discuss conditions under which the MIN disappears. First, it is clear that the MIN is strictly positive for any state \( \rho \) with non-vanishing quantum correlations. This can be shown directly from the definitions of MIN and QD based on the Hilbert-Schmidt distance or the discrepancy between QMI of the pre- and post-measurement states, for which we always have \( N^v(\rho) \geq D^v(\rho) \) and \( N^v(\rho) \geq D^v(\rho) \). Thus one only need to consider the classical-quantum state \( \rho_{\text{CQ}} = \sum_k p_k k\langle k | \otimes \rho_k^b \) (10). If the reduced state \( \rho_{\text{CQ}}^b = \sum_k p_k k\langle k | \otimes \rho_k^b \) is nondegenerate (i.e., \( p_k \neq p_l \)), then the MIN disappears because for this case the only von Neumann measurement leaving \( \rho_{\text{CQ}}^b \) invariant is \( \Pi^v = \{11, \Pi_2^v = |k\rangle\langle k|\} \), which yields \( \Pi^v(\rho_{\text{CQ}}) = \rho_{\text{CQ}} \) and thus there is no MIN [17]. If \( \rho_{\text{CQ}}^b \) is degenerate (i.e., \( p_k = p_l \)), then we need to consider two different cases. The first one is that \( \rho_k^b = \rho_l^b \) for all \( k \) and \( l \). For this case, we always have \( \rho_{\text{CQ}} = \rho_{\text{CQ}}^b \otimes \rho_{\text{CQ}}^b \) and there is still no MIN. Second, if \( \rho_k^b \neq \rho_l^b \), then one can always find a complete projective measurement \( \Pi^v = \{k', k''\} \) so that \( \rho_{\text{CQ}} = \rho_{\text{CQ}}(k', k'') \), and for this two special cases one can always choose corresponding complete projective measurements \( \Pi^v = \{11, \Pi_2^v\} \) so that \( I(\rho_{\text{CQ}}) = 1/4 \) and \( S[\Pi^v(\rho_{\text{CQ}})] = 2 \). In fact, the classical correlation in \( \rho_{\text{CQ}} \) is completely destroyed by the projective measurement \( \Pi^v \) because \( I(\rho_{\text{CQ}}) = 0 \). For \( \rho_{\text{Bell}} \), however, the classical correlation is undisturbed because the optimal measurement \( \Pi^v \) (may be different from that for \( \rho_{\text{CQ}} \) for obtaining the MIN gives \( I(\Pi^v(\rho_{\text{Bell}})) = 1 \), equals to the classical correlation present in \( \rho_{\text{Bell}} \)). Moreover, we would like to point out that for \( \rho_{\text{CQ}} \) with non-vanishing MIN, the classical correlation is partially destroyed by the optimal measurement \( \Pi^v \).

The above discussions also indicate that the different measures of MIN may impose different orderings of quantum states, and in particular, the maximum \( N^v(\rho) = 1 \) does not always indicates that the state \( \rho \) is maximally entangled, which is in sharp contrast to that of MIN measured by \( N^v(\rho) \) or quantum correlations measured by QD, GMQD and MID [8,10].

III. MIN DYNAMICS IN STRUCTURED RESERVOIRS

A. The model

We consider a system consists of two identical atoms
with lower and upper levels denoted by $|0\rangle$ and $|1\rangle$, and is coupled to two typical structured reservoirs, i.e., the independent and common zero-temperature bosonic reservoir [20]. For the former case, the two atoms interact with their own reservoir with $H = H_1 + H_2$, and the single “qubit+reservoir” Hamiltonian reads

$$H_n = \omega_0 \sigma^+_n \sigma^-_n + \sum_k \omega^+_k b_k^{\dagger} b_k^n + \sum_k (g_k \sigma^+_n \sigma^+_k + h.c.), \quad (10)$$

where $\omega_0$ denotes the transition frequency of the atoms, and $\sigma^+_n (n = a, b)$ are the Pauli raising and lowering operators. The index $k$ labels the reservoir field mode with frequency $\omega^+_k$, with $b_k^{\dagger}$ and $b_k^n$ being the bosonic creation and annihilation operators and $g_k^n$ the coupling strength.

We will assume that the two reservoirs are completely the same and denote $\omega_{1,2}^k \equiv \omega_k$ and $g_{1,2}^k \equiv g_k$.

For the latter case, the two atoms are coupled to the same reservoir and the Hamiltonian reads

$$H = \omega_0 \sum_n \sigma^+_n \sigma^-_n + \sum_k \omega^+_k b_k^{\dagger} b_k + \sum_k (g_k \sigma^+_n \sigma^+_k + h.c.). \quad (11)$$

In the following, we consider the structured reservoir with the spectral density having the Lorentzian form [20]

$$J(\omega) = \frac{1}{2\pi} \frac{\gamma_0 \lambda^2}{(\omega - \omega_0)^2 + \lambda^2}, \quad (12)$$

where $\lambda$ denotes the spectral width of the reservoir and is related to the reservoir correlation time $\tau_B$ by the relation $\tau_B \approx \lambda^{-1}$, and $\gamma_0$ is related to the decay time scale $\tau_R$ of the excited state of the atom in the Markovian limit of flat spectrum via $\tau_R \approx \frac{1}{\gamma_0}$. $\lambda > 2\gamma_0$ corresponds to the Markovian regime, while $\lambda < 2\gamma_0$ corresponds to the non-Markovian regime. Moreover, $\omega_0 = \omega_0 - \delta$ is the central frequency of the reservoir detuned from the transition frequency $\omega_0$ by an amount $\delta$.

For the independent reservoir, we work in the interaction picture and thus the Hamiltonian of Eq. (10) can be rewritten as $H_{n,t}(t) = \sum_k g_k \sigma^+_n e^{i(\omega_0 - \omega_k)t} + h.c.$. We suppose the initial single “qubit+reservoir” state to be $|\psi(0)\rangle = c_0|00\rangle + c_1|10\rangle$ ($|0\rangle$ represents vacuum state of the reservoir), then by using the Schrödinger equation $i\hbar \frac{d}{dt}|\psi(t)\rangle = H_{n,t}(t)|\psi(t)\rangle$ with $|\psi(t)\rangle = c_0|00\rangle + c_1|10\rangle + \sum_k c_k(t)|01_k\rangle$ ($|01_k\rangle$ represents the reservoir with one excitation in mode $k$), one can obtain

$$\dot{c}_1(t) = -i \sum_k g_k c_k(t) e^{i(\omega_0 - \omega_k)t}, \quad \dot{c}_k(t) = -i g_k^* c_1(t) e^{-i(\omega_0 - \omega_k)t}. \quad (13)$$

From the second expression of the above equation one can derive $c_k(t) = -i g_k^* \int_0^t dt_1 c_1(t_1) e^{-i(\omega_0 - \omega_k)t_1}$, substituting of which into the first expression we obtain

$$\dot{c}_1(t) = -i \int_0^t dt_1 f(t - t_1)c_1(t_1). \quad (14)$$

where the function $f(t - t_1) = \int d\omega J(\omega) e^{-i(\omega_0 - \omega)(t - t_1)} = \frac{\gamma_0}{2} e^{-(\lambda - i\delta)(t - t_1)}$ in the limit of large number of modes $k$. Then by performing the Laplace transformation of Eq. (14) one can obtain $c_1(t) = p(t)c_1(0)$ with

$$p(t) = e^{-\frac{t}{2}(\lambda - i\delta)} \left[ \cosh \frac{dt}{2} + \frac{\lambda - i\delta}{d} \sinh \frac{dt}{2} \right], \quad (15)$$

where $d = \sqrt{(\lambda - i\delta)^2 - 2\gamma_0\lambda}$. If we further define $|1\rangle = e^{-\frac{t}{2}(\lambda - i\delta)} \left( \cosh \frac{dt}{2} + \frac{\lambda - i\delta}{d} \sinh \frac{dt}{2} \right) |0\rangle + c(0)|1\rangle$, then $|\psi(t)\rangle = c_0|00\rangle + c_1(t)|10\rangle + c(1)|01\rangle$, according to which the single-qubit reduced density matrix can be determined as

$$\rho^S(t) = \begin{pmatrix} \rho^S_{11}(0) |p(t)|^2 & \rho^S_{10}(0)p(t) \\ \rho^S_{01}(0)p^*(t) & 1 - \rho^S_{11}(0) |p(t)|^2 \end{pmatrix}, \quad (16)$$

in the standard basis $\{|1\rangle, |0\rangle\}$. For the two-qubit system considered in this work its reduced density matrix can be obtained by the procedure presented in Ref. [21].

If the two atoms are coupled to the common reservoir, then their dynamics can be described by the following pseudome mode master equation [22 25]

$$\frac{\partial \tilde{\rho}}{\partial t} = -i [V, \tilde{\rho}] + \lambda (2b\tilde{\rho}b^d - b^d b\tilde{\rho} - \tilde{\rho}b^d b), \quad (17)$$

where $\tilde{\rho}$ is the density matrix for the atoms and the pseudome, and the interaction Hamiltonian $V$ of the atomic system and the pseudome can be written as

$$V = \sqrt{\frac{\gamma_0}{2}} (\sigma^+_a + \sigma^+_b) b + h.c., \quad (18)$$

We assume throughout this work that the total system contains at most two excitations, then Eq. (17) turns out to be set of 64 differential equations, which can be solved by using different methods [23 27]. Particularly, for the initial extended Werner-like (EWL) states, analytical solutions of Eq. (17) were obtained by using the Laplace transform which turns the differential equations to the polynomial equations [24] (note that there may be some misprints in [24]). First, the $12\Omega^2$ in Eq. (A.21) should be $12\Omega^2$, and $\alpha\sqrt{1 - \alpha^2}$ should be multiplied by $e^{-it}$. Second, the solution of $\tilde{\rho}_{-1}(t)$ in Eq. (A.22) should be $\tilde{\rho}_{-1}(t) = \frac{1}{4\pi}$.}

### B. MIN dynamics

Now we begin our discussion about MIN dynamics in structured reservoirs. We suppose the two atoms are prepared initially in the EWL states

$$\rho_0^L = r^L |\Xi\rangle\langle\Xi| + \frac{1 - r^L}{4} I, \quad (19)$$

with $|\Xi\rangle = |\Psi\rangle$ or $|\Phi\rangle$, and $|\Psi\rangle = \alpha|00\rangle + e^{i\theta} \sqrt{1 - \alpha^2} |11\rangle$, $|\Phi\rangle = \alpha|10\rangle + e^{i\theta} \sqrt{1 - \alpha^2} |01\rangle$. For the case of independent reservoir, $\rho_0^L$ and $\rho_0^\Phi$ yield qualitatively similar MIN dynamics, which are independent of the phase factor $e^{i\theta}$. In Fig. 2 we plotted $N^L(\rho)$...
versus $\alpha^2$ and $\gamma_0 t$ with the two atoms prepared in different initial states and coupled resonantly (i.e., $\delta = 0$) to the reservoirs. It is clear that they attain certain maxima at $\alpha^2 = 0.5$ and vanish at $\alpha^2 = 0$ or 1. For fixed $\alpha^2$, MIN decreases exponentially with increasing $\gamma_0 t$ for the Markovian reservoir [see Figs. 2(a) and 2(b)], while there are revivals of MIN for the non-Markovian reservoir [see Figs. 2(c) and 2(d)]. Since there are neither direct nor reservoir-mediated interactions between the two atoms, the revivals of MIN after a certain minimum may be induced by the non-Markovian effects.

If the two atoms interact with the common reservoir, however, $\rho_0^\Psi$ and $\rho_0^\Phi$ will exhibit completely different MIN dynamics. Particularly, for the initial sub-radiant state $|\Psi^\ominus\rangle = (|00\rangle - |01\rangle)/\sqrt{2}$, the density matrix maintains its initial value (i.e., $\rho(t) = |\Psi^\ominus\rangle\langle\Psi^\ominus|$) [28] and thus the MIN does not decay. To see more about MIN dynamics for this case, we show in Fig. 3 the $N^\nu(\rho)$ versus $\alpha^2$ and $\gamma_0 t$ with different initial states. One can note that the MIN displays complicated behaviors. First, it is not a symmetric quantity with respect to $\alpha^2 = 0.5$ for the initial state $|\Psi\rangle$, and $N^\nu(\rho)$ can even does not behave as a monotonic function of $\gamma_0 t$ for the Markovian reservoir. As one can see from Figs. 3(a) and 3(b), there are creations of MIN for the initial state $|\Psi\rangle$ with $\alpha^2 = 0$ and for $|\Phi\rangle$ with $\alpha^2 = 0$ or 1. The underlying reason for this is the reservoir-mediated interaction because here the two atoms interact with a common reservoir. Second, there are still revivals of MIN for the non-Markovian case, but the combined effects of the non-Markovian effects and the reservoir-mediated interaction makes it more evident than those for the independent reservoirs.

When the two atoms interact with independent reservoirs, their entanglement may be trapped by detuning the central frequency of the reservoir from $\omega_0$ [28]. In Fig. 4(a) we display $N^\nu(\rho)$ versus $\gamma_0 t$ for the initial state $\rho^\Psi_0$ with various values of detuning $\delta$. The result shows that $N^\nu(\rho)$ may be enhanced by introducing $\delta$ and when $\delta = 5\gamma_0 t$ it begins to oscillating weakly around its initial value, from which it is reasonable to conjecture that up to the large detuning limit case $N^\nu(\rho)$ will maintains its initial value during the time evolution process and thus MIN trapping occurs. Experimentally, for two Rb Rydberg atoms with lifetime $T_{2\alpha} \approx 30$ ms and inside the Fabry-Pérot superconducting resonant cavities with quality factor $Q \approx 4.2 \times 10^{10}$, we have the atomic decay rate $\gamma_0 \approx 33.3$ Hz, and the spectral width of the reservoir $\lambda \approx 7$ Hz [29]. These values correspond to a good non-Markovian regime. Moreover, the detuning needed to freeze the MIN can be implemented by Stark-
versus \( \theta/\pi \)

FIG. 5: (Color online) Numerical results of \( \Delta \gamma \) thus a dimensionless shift with respect to field. The typical Stark shifts are different correlation measures, but \( \Delta C \) states (explicit example, we show in Fig. 4(b) the numerical results obtained from the initial local unitary equivalent states \( (U \otimes V)\rho_0^\phi (U^\dagger \otimes V^\dagger) \) with \( \alpha^2 = 1/2, r = 1/2 \) and \( \lambda = 0.1\gamma_0 \). It is clear that the strength of MIN at finite time \( t > 0 \) can be tuned among regions bounded approximately by those obtained for the initial states \( \rho_0^\phi \) with \( \theta = 0 \) and \( \theta = \pi \), respectively. Particularly, one can note that for the initial state \( \rho_0^\phi \) with \( \theta = \pi \) (the top red line), the amount of MIN can even be enhanced by subjecting the two atoms to a common reservoir.

In fact, for the initial state \( \rho_0^\phi \) there exists a threshold phase angle \( \theta_{th} \) (depends on the system parameters) beyond which the MIN as well as the quantum correlations measured by entanglement of formation (denoted by \( E \)) [31]. QD and GMQD can be enhanced to a larger value and further be frozen in the long time limit. In Fig. 5 we display numerical results of \( \Delta C = \langle C(t) \rangle - \langle C(0) \rangle \) versus \( \theta/\pi \) for \( \rho_0^\phi \) with \( \alpha^2 = 1/2, r = 1/2, \lambda = 0.1\gamma_0 \) and \( t = 5000/\gamma_0 \). Clearly, the threshold \( \theta_{th} \) are different for different correlation measures, but \( \Delta C \) always increases with increasing value of \( \theta \) and attains the maxima at \( \theta = \pi \). This provides an efficient method for enhancing and storing MIN for future usage in external environments. One can also note that the curves for \( N^e(\rho) \) and \( D^e(\rho) \) are overlapped when \( \theta < \theta_{th} \), this is because shifting the transition frequencies with a static electric field. The typical Stark shifts are \( \delta \approx 200 \) kHZ [30] and thus a dimensionless shift with respect to \( \gamma_0 \) of the order \( \delta/\gamma_0 \approx 6 \times 10^3 \) can be realized.

Since the reservoir always acts in some basis and therefore changing the basis may change the strength of reservoir and thus affect the durability of correlations. Particularly, when the two identical atoms equally coupled to the common reservoir, there exists a sub-radiant state \( |\Psi^{-}\rangle \) which is decoupled from the field modes and thus does not decay with time [23]. This motivates us to enhance robustness of MIN by performing appropriate local unitary operations on the initial state, though the MIN itself is invariant under local unitary operations. As an explicit example, we show in Fig. 4(b) the numerical results obtained from the initial local unitary equivalent states \( (U \otimes V)\rho_0^\phi (U^\dagger \otimes V^\dagger) \) with \( \alpha^2 = 1/2, r = 1/2 \) and \( \lambda = 0.1\gamma_0 \). It is clear that the strength of MIN at finite time \( t > 0 \) can be tuned among regions bounded approximately by those obtained for the initial states \( \rho_0^\phi \) with \( \theta = 0 \) and \( \theta = \pi \), respectively. Particularly, one can note that for the initial state \( \rho_0^\phi \) with \( \theta = \pi \) (the top red line), the amount of MIN can even be enhanced by subjecting the two atoms to a common reservoir.

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FIG. 6: (Color online) Comparison of the geometric and entropic MIN in common reservoir with \( \alpha^2 = 1/2, r = 1/2 \) and \( \lambda = 0.1\gamma_0 \). The plot in (a) shows the case of the initial state \( \rho_0^\phi \), while (b) shows that of \( \rho_0^\pi \) with \( \theta = 0 \).

IV. SUMMARY

In this paper, we introduced the entropic measure of MIN based on the maximal difference between QMI of the pre- and post-measurement states. While the geometric measure of MIN [17] quantifies the maximal Hilbert-Schmidt distance between \( \rho \) and \( \Pi^a(\rho) \), the entropic measure of MIN introduced here quantifies the maximal loss of total correlations or equivalently, the maximal increment of von Neumann entropy of \( \rho \) by \( \Pi^a \). It seems naturally a definition from information-theoretical point of view. Based on this measure, we discussed conditions for nonvanishing MIN and consolidated the finding that there exists nonlocality without quantumness [9]. For the two-qubit case, we further compared MIN with different quantum correlation measures [8] [10], showing that some of them may assume equal values under certain special conditions, and a quantum state with the maximum \( N^v(\rho) \) does not always indicate that the state possess...
the maximum quantum correlation.

Since real physical system is inevitably coupled to its surroundings, in comparing with fragile entanglement, it is significant to study stability of MIN against decoherence. Here as a concrete example, we evaluated MIN dynamics for a family of two-qubit states in the Lorentzian structured reservoirs, and displayed their rich dynamical behaviors both for the Markovian and non-Markovian situations. We showed that the MIN can be frozen by increasing the detuning parameter, and for some initial states the strength of MIN as well as the quantum correlations can even be enhanced in the short time region and further be frozen in the long time limit by interacting the system with a common reservoir, the underlying reason for which may be the existent component of the decoherence-free state $|\Psi^\pm\rangle$.

Finally, we studied the issue related to the relativity of different MIN measures, just as that of different entanglement [32] and quantum correlation measures [33]. By evaluating the dynamics of $N^s(\rho)$ and $N^v(\rho)$ in an non-Markovian common reservoir, we showed that the two MIN measures do not necessarily imply the same ordering of quantum states. This is reminiscent of the cases for entanglement and quantum correlation measures and may indicate that the difference in nonlocality is rather measure dependent than state dependent. Experimentally, besides atomic systems [28–30], two-qubit quantum state with different reservoirs, such as Markovian and non-Markovian, can be realized [34], and different correlations such as quantum discord have been shown recently [35]. It will also be interesting to explore the entropic MIN of this paper in experiments.

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