Towards black hole scattering

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Abstract: We study black holes in three-dimensional Chern-Simons gravity with a negative cosmological constant. In particular, we identify how the Chern-Simons interactions between a scattering particle and a black hole project the particle wavefunction onto a wavefunction in the black hole background. We also analyze the set of space-times that should be allowed in the theory and the way in which boundary conditions affect the spectrum of space-times.
1. Introduction

Three-dimensional gravity is a useful laboratory for studying physical phenomena that are universal to theories of gravity in any dimension. In particular, it is instructive to study three-dimensional gravity with a negative cosmological constant, since it allows for non-trivial black hole solutions (in contrast with three-dimensional gravity with zero cosmological constant). Since a lot of features of black hole physics are universal, it is reasonable to study black holes in a simplified context.

The Einstein-Hilbert action in three dimensions, with a negative cosmological constant, can be rewritten in terms of Chern-Simons theory with gauge group $SL(2,R) \times SL(2,R)$. In Chern-Simons theory, we can treat these black hole excitations in roughly the same way as we would treat particles in three-dimensional gravity with zero cosmological constant. Exact quantum scattering for particles in this context was studied in a metric approach and in the Chern-Simons formulation, with roughly equivalent results. In the theory with negative cosmological constant, we can revisit the problem, and try to study exact two-particle, two-black hole and black hole - particle scattering. This study and some of its surprising aspects are the subjects of our paper. But before we scatter sources we discuss some intriguing aspects of three-dimensional gravity itself.
2. Chern-Simons gravity

Three-dimensional gravity with a negative cosmological constant can be studied in terms of $SL(2,R) \times SL(2,R)$ Chern-Simons theory:\[^1^]

\[
S[A^+]_{CS} - S[A^-]_{CS} = \frac{k}{4\pi l} \int_M e_a e^{abc} R_{bc} + \frac{1}{3l^2} \epsilon^{abc} e_a e_b e_c + \frac{k}{4\pi l} \int_{\partial M} e^a \omega_a,
\]

\[
= \frac{k}{4\pi} \int_M Tr(A^+ dA^+ + \frac{2}{3} A^{+3}) - \frac{k}{4\pi} \int_M Tr(A^- dA^- + \frac{2}{3} A^{-3})
\]

where $A^\pm = \omega \pm \frac{\epsilon}{l}$ are two $SL(2,R)$ gauge fields and $e$ is the dreibein and $\omega$ is the spin connection one-form. The dimensionless constant $k = \frac{1}{4\pi G}$ (where $G$ is Newton’s constant) measures the radius of curvature associated to the cosmological constant $\Lambda = -1/l^2$ in units of the Plank length $l_p = G$. The $SL(2,R) \times SL(2,R)$ Chern-Simons action is equal to the Einstein-Hilbert action up to boundary terms. Our starting point for our discussion of the excitations in three-dimensional gravity will be the topological Chern-Simons theory on a line times a disk $R \times D$, where we puncture the disk by including a term in the action that couples a particle $\chi^\pm$ (that takes values in the gauge group) minimally to the Chern-Simons gauge field \[^9^][10^]. We briefly review the resulting picture of space-times. The field strengths of the Chern-Simons theory must be thought of as the curvature two-form and torsion two-form:

\[
\frac{F^+_a + F^-_a}{2} = d\omega_a + \frac{1}{2} \epsilon_{abc} (\omega^b e^c + \frac{1}{l^2} e^b e^c)
\]

\[
\frac{F^+_a - F^-_a}{2} = de_a + \frac{1}{2l} \epsilon_{abc} (\omega^b e^c + e^b \omega^c).
\]

In a certain gauge, and with an appropriate choice for the world-line time coordinate for the particle, the equations of motion for the gauge fields $A^\pm_0$ imply \[^1]^1:

\[
\frac{F^+_i + F^-_i}{2} = -\frac{\pi}{k} \left( (\chi^+ + \chi^-) + (\chi^- + \chi^-) \right) \delta
\]

\[
\frac{F^+_i - F^-_i}{2} = -\frac{\pi}{k} \left( (\chi^+ + \chi^-) - (\chi^- + \chi^-) \right) \delta,
\]

where $\delta$ is a two-dimensional spatial delta-function, and $\lambda^\pm$ specify the orbits of the gauge group on which the particle resides. We note that when the particle source-term is zero, the geometry has constant curvature and zero torsion (and negative cosmological constant $\Lambda = -1/l^2$). A solution to the equations of motion is given by the $AdS_3$ geometry, which is gauge trivial. When we include a source-term the topology becomes that of the real line times a disk $D$ with a puncture. We concentrate on sources that are associated to particles that rest at the unit element of the gauge group ($\chi^\pm = 1$). It is clear that a particle will manifest itself as non-trivial source terms for the curvature and torsion two-form. We argued in detail in \[^8^]^8 that we can associate hyperbolic weights $\lambda^\pm$ to generic BTZ space-times, a hyperbolic and a weight on the lightcone to extremal BTZ space-times.

[^1]: For our conventions and much more background on our set-up, see \[^3^]^3.
and two lightcone weights to the massless BTZ black holes \[2\] \[3\]. In fact, we can add one more space-time to the list, which is the well-defined geometry (self-dual under T-duality) identified in \[12\], which has trivial holonomy in one \(SL(2, R)\) factor, and hyperbolic weight in the other.

We also want to identify the sources for conical space-times (without angular momentum) explicitly (and more precisely than in \[8\]). The torsion of these space-times will be zero, and the curvature singularity can be shown to be \(R_{12} = 2\pi\beta\delta^{(2)}\), where \(2\pi\beta\) denotes the deficit angle of the conical geometry\(^2\). These statements show that classical sources associated to conical space-times without angular momentum have \(\lambda^+ = \lambda^-\) (because of zero torsion) and the weights are elliptic: \(\lambda^\pm = k\beta T_0^\pm\) (where \(T_0\) generates an elliptic subgroup of \(SL(2, R)\)). Upon quantization, these classical source terms give rise (see \[3\]) to discrete particle representation spaces specified by the parameter \(\tau^\pm = -\frac{k}{2}\beta - \frac{1}{2}\), when \(k > 0\) and \(\beta > 0\).\(^3\) In fact, we find that \(\tau^\pm = -1\), which is the lowest lying true discrete representation which can be obtained by quantizing orbits \[13\] is associated to a space-time with conical deficit angle \(2\pi\beta = 2\pi/k\).

**Mass**

With our definition for the bulk action in terms of Chern-Simons theory with no extra boundary terms, we obtain a conformal field theory on the boundary which consists of two chiral \(SL(2, R)\) Wess-Zumino-Witten models \[11\]. We propose to make use of the conformal symmetry of the boundary theory to canonically define a mass operator. A natural definition of space-time mass \(M_{CS}\) is given by the time translation generator in the boundary conformal field theory, normalized in such a way that it satisfies the standard Virasoro algebra (in the quantum theory):

\[
L_0 = -\frac{\tau^+(\tau^+ + 1)}{k - 2} + \text{osc}
\]

\[
\bar{L}_0 = -\frac{\tau^-(\tau^- + 1)}{k - 2} + \text{osc}
\]

\[
M_{CS} \equiv L_0 + \bar{L}_0.
\] (2.4)

Black hole space-times correspond to continuous representations with quadratic Casimir given in terms of the parameter \(\tau^\pm = -\frac{1}{2} + is^\pm\) (and \(s^\pm \in R\)) and consequently have a positive mass \(M_{CS}\) which is greater than \(M_{CS} = \frac{1}{2(k - 2)}\). The minimal mass which is associated to a CFT operator with positive conformal dimension is \(M_{CS} = 0\), which we obtain for \(\tau^\pm = 0\). It is the mass of the \(AdS_3\) space-time. We notice the counterintuitive fact that the space-time with conical deficit angle \(2\pi/k\) (associated to \(\tau^\pm = -1\)) also has zero space-time mass. Spaces with larger conical deficit angle have a negative space-time mass \(M_{CS}\).

Note that the mass gap (i.e. the gap in conformal dimension) between the \(SL(2, R)\) invariant state and the black hole continuum behaves as \(1/k\) for large \(k\) (i.e. in the classical

\(^2\)This can be shown using the theory of distributions, or by transforming the conical geometry to a conformally flat geometry with a singular conformal factor, and then regularizing the conformal factor.

\(^3\)The quadratic Casimir of a representation is given by \(c_2 = -\tau(\tau + 1)\) in our conventions.
Quantum mechanically, there is a “mass gap” and complementary representations (with \(-1 < \tau < 0 \) and \( \tau \neq -1/2 \)) are excitations on the vacuum that have mass smaller than the minimal black hole mass.

At this point we want to make remarks which clarify the relation between this theory of three-dimensional gravity and the metric theory of three-dimensional gravity. It is known that metric boundary conditions (which insist on the fact that the metric asymptotes to \( AdS_3 \) (see e.g.\([14]\))) gives rise to a Hamiltonian reduction of the boundary \( SL(2, R) \) conformal field theory. The Hamiltonian reduction is associated to a fixed value for an \( SL(2, R) \) current in a null-direction (see e.g.\([15]\)[16]). It gives rise to a Liouville conformal field theory on the boundary\([17]\). A natural definition of space-time mass in this metric theory is the sum of the left and right conformal dimensions, as measured in the Liouville theory by the standardly normalized Virasoro zero-modes. This is the definition adopted and analyzed in \([18]\). In this theory, the mass gap scales very differently. The mass gap scales as \( k \) with large \( k \), in the classical limit, and the allowed space-times include the full range of deficit angles from 0 to \( 2\pi \) \([18]\). It is clear then that these two theories of three-dimensional gravity are very different, and that the effect of the boundary conditions on the theory is very strong indeed.

We will further analyze the mapping between space-times and conformal field theory operators later on, but we first turn to the scattering problem.

3. Scattering two excitations

We discuss the scattering of two particles or black holes in the Chern-Simons formulation of gravity with a negative cosmological constant. In three-dimensional gravity with zero cosmological constant, the papers \([6][7]\) computed the quantum amplitude for the scattering of two particles. (See also \([21]\).) We will follow \([7]\) closely in the following, and we will generalize the analysis there to the case with negative cosmological constant. Since our results will closely parallel those obtained in \([7]\), we refrain from discussing many technical details. In fact, it is the conceptual assumptions that underlie the formalism which form the trickiest part of the computation, so we discuss them in some detail.

Base manifold scattering

When we think of two-particle space-time scattering, we usually think of a two-particle initial state at \( t = -\infty \) and a two-particle final state at \( t = +\infty \). But we should realize that we not only have the base manifold concept of time. Our Chern-Simons point-like particles also carry an “internal clock”, because their internal wave-function is a Hilbert space that is a representation of the \( SL(2, R) \times SL(2, R) \) gauge group. For instance, wave-functions that carry the same representation under the left and right \( SL(2, R) \) gauge group can simply be represented as wave-functions on the group manifold \( SL(2, R) \). The Casimir of the representation then specifies the (particle) mass of the wave-function, and the wave-function in a coordinate representation satisfies a wave-equation with that mass.

\(^4\)Note that the dimensionless ratio \( k = \frac{l}{4\pi} \) is the only coupling constant in the theory.
The wave-function represents internal degrees of freedom, but because of the non-compact nature of the gauge group, includes a time-like variable. In that sense each particle carries an “internal clock”.

Ignoring this subtlety for a moment, we can compute the base manifold scattering amplitude of two particles by specifying an initial state and a final state for the two-particle system, and then sum over all paths with the appropriate phase. Since we work in a topological theory, we just have to sum over all topologically distinct paths. These are enumerated by specifying the number of times particle 1 winds around the world-line of particle 2. Summing the appropriate phases will give the amplitude to scatter two particles.\(^5\) We associate the following amplitude to this process:

\[
\mathcal{A} = \sum_{n=-\infty}^{\infty} \langle 1 | \otimes \langle 2 | B^n e^{i n \theta} | 1 \rangle \otimes | 2 \rangle
\]

(3.1)

The operator \( B \) is the braiding operation: it is the phase that the two-particle wave-function picks up when particle 1 winds around particle 2 once (in the clockwise direction say) due to the Chern-Simons interactions. The amplitude is a sum over all possible topological world-line histories, i.e. over all topologically distinct paths. We have introduced an angle \( \theta \) which specifies the phase which weighs the contributions to the path integral from the different topological sectors labeled by the winding number \( n \). We will specify the precise form of the operator \( B \) shortly.

**Internal free motion**

Another quantity that we might be interested in is the following. Suppose we want to study a two-particle state with particular initial conditions (at worldline time \( t_1 = -\infty \)) for particle 1 and similarly (at worldine time \( t_2 = -\infty \)) for particle 2. There is dynamics in these time-variables simply because we know that the wave-functions are \( SL(2, R) \otimes SL(2, R) \) matrix elements. (E.g. for zero spin particles, the internal dynamics is dictated by the Klein-Gordon equation with a mass squared given by minus the quadratic Casimir of the representation.) If we just study these wave-function by themselves, without referring to space-time, they evolve freely. There is dynamics, but it is trivial. The two particles do not interact as long as they do not move in space-time. They just “sit” in the base manifold and evolve according to their internal clock.

**Combination**

The way to reproduce a more intuitive concept of particle scattering in Chern-Simons theory, is to combine the above concepts. We study the wave-function for the two-particle system that evolves according to the internal clock, but we also demand that we take into account the fact that the two-particle wave-function should move from a given initial

\(^5\)We note that \( \otimes \) which indicates a tensor-product of two one-particle states, should not be confused with \( \otimes \) which will later indicate that a one-particle wave-function lives in a tensor product Hilbert space because of the product nature of the gauge group.
configuration to the same two-particle state at some final base manifold time $t$. As a consequence, we know that the final state is a superposition

$$|\psi\rangle = \sum_{n=-\infty}^{\infty} B^n e^{in\theta}|\psi_0\rangle$$

for some initial configuration $|\psi_0\rangle$ and that thus, it is invariant under the projection:

$$e^{i\theta} B|\psi\rangle = |\psi\rangle.$$ (3.3)

This will be our definition of scattering: we evaluate the internal two-particle wave-function, and project it out by the braiding operation, which represents the possibility of non-trivial space-time topology for the particle world-lines. This is the definition adopted in [7] and further analyzed in e.g. [19] [20] to which we refer for more details.

4. Projection and periodicity

After quantization, the source terms $J_i = \chi_i \lambda_i \chi_i^{-1}$ for the Chern-Simons bulk gauge field act as currents $J^i$ on the quantum-mechanical Hilbert space of the particles. The currents for each particle form an $SL(2, R) \times SL(2, R)$ algebra. It has been argued in detail in [7] that the braiding operator is given in terms of these current generators as:

$$B = \exp\left(\frac{2\pi}{k} Tr(J_1 \otimes J_2)\right).$$ (4.1)

where the trace $Tr$ is over the full gauge algebra, and the lower indices indicate the particle Hilbert space on which the currents act. The same result for the braiding operator was obtained from a detailed analysis of open Wilson lines in Chern-Simons theory in e.g. [22].

We now have all the tools to analyze the scattering of two excitations in the Chern-Simons theory of gravity with negative cosmological constant. We will first discuss the case where particle 2 can be treated as a classical source (i.e. we take the weights $\lambda_2^\pm$ to be large compared to $\lambda_1^\pm$) that we fix to reside at the origin of the gauge group (i.e. $\chi_2^\pm = 1$ where 1 is the unit element in the gauge group).

To analyze the action of the braiding/projection operator, we first discuss in more detail the Hilbert space associated to excitation 1. We start out by sketching the familiar picture for compact groups and then adapt the picture to our non-compact gauge group. For compact groups $G$ the space of quadratically integrable functions decomposes into a sum of tensor-products of irreducible Hilbert spaces:

$$\mathcal{L}^2(G) = \sum_{\lambda \text{irr}} \mathcal{H}_\lambda \otimes \mathcal{H}_\lambda$$ (4.2)

where the sum is over all irreps (labeled by a highest weight $\lambda$) of $G$. For a non-compact group, a similar statement holds involving the representations that occur in the left/right regular representation and the summation becomes an integral with a Plancherel measure [13].
Note moreover that when we solve the Laplace/Klein-Gordon equation on the group manifold in the space of quadratically integrable functions with given eigenvalue \(-c_2(\lambda)\), then the solution space will span a representation of \(G \otimes G\) which is \(\mathcal{H}_\lambda \otimes \mathcal{H}_\lambda\). The wavefunction with spin zero and mass squared \(-c_2(\lambda)\) can be identified with a matrix element in the representation labeled by \(\lambda\), which is a vector in the \(\mathcal{H}_\lambda \otimes \mathcal{H}_\lambda\) representation space. We concentrate on a spin zero particle probe 1 and we can thus work with a wavefunction which is a solution to the Klein-Gordon equation, or in other words, a matrix element of an irrep.

**Scattering off a black hole**

We are now ready to show how the Chern-Simons interaction mediates scattering off a target black hole. If we treat the target particle 2 classically, we can treat the associated currents \(J_2 = \chi_2 \lambda_2 \chi_2^{-1}\) classically. If we assume that the target particle is at rest at the unit of the group manifold, then we can equate the classical weight \(\lambda_2\) with the current \(J_2\). For a black hole target, we have the classical weights \(\lambda_2^\pm = k \sqrt{M \pm J}/lT_1\). Next, we parametrize the particle wavefunction of particle 1 in terms of a function on the group manifold: 

\[
\begin{align*}
\cosh^2 \rho &= \frac{r^2 - r_2^2}{r_+^2 - r_-^2} \\
u &= \frac{r_+ - r_-}{l}(t + \phi) \\
v &= \frac{r_+ + r_-}{l}(\phi - t),
\end{align*}
\]

(4.3)
gives rise to the usual BTZ metric, but, most importantly, on the group the coordinate \(\phi\) is not identified modulo \(2\pi\).

The braiding projection operator can be represented in the quantum Hilbert space of particle 1 by left and right multiplication of the argument of the wavefunction on the group manifold:

\[
B\psi_1(g) = e^{i\theta} \psi_1(e^{2\pi i \sigma_3 (r_+ - r_-)/2l} g e^{2\pi i \sigma_3 (r_+ + r_-)/2l}).
\]

(4.4)

The simple computation yields an important result. When we have a spinless probe particle 1, the projection condition on the wavefunction induced by the Chern-Simons interaction is exactly the condition that the wavefunction is periodic in the coordinate \(\phi\) with period \(2\pi\) (up to a possible phase given by a \(\theta\)-angle). Thus the wavefunction of particle 1 is interpreted as the wavefunction on a BTZ black hole background, after we implement the projection mediated by the gravitational interactions. Note that this fact does not depend on the mass of the spinless probe particle. In fact, from the algebras of the geometry of BTZ black holes \[3\], and particle excitations (see section \[2\] and \[8\]), it is clear that the reasoning holds quite generally for any type of probe or target particle. (The literature \[23\] \[24\] \[25\] contains a detailed analysis of the resulting projected wave-functions)
in a different context.) It would be very interesting to analyze the non-trivial dynamics that occurs when both excitations, their currents, and their Hilbert spaces are treated quantum mechanically. We again stress that we explicitly showed that the quantum-gravitational scattering off a black hole allowed us to reconstruct the semi-classical wavefunction for a particle in a black hole geometry.

5. A broader picture

Although the probe-target approximation used in the previous section leads to intuitively plausible results for the scattering of a particle excitation of a black hole, there are important counterintuitive features of the formalism. These counterintuitive features can be seen to arise from the following observation. The space-time mass that we defined in section 2 is, when we ignore boundary oscillatory excitations, proportional to minus the mass squared that appears in the Klein-Gordon equation for the internal (spinless) particle wave-function. More precisely, the correspondence between space-time mass and KG mass squared is as follows. The space-time black hole mass spectrum corresponds to KG masses that violate the Breitenlohner-Freedman bound. When we concentrate on particle excitations with positive space-time mass, and which lie below the black hole mass spectrum, we find that these have internal wave-functions corresponding to quadratic Casimirs $0 \leq \tau(\tau + 1) \leq -1/4$, which implies that they are “stable tachyons”, that is, they have a negative mass squared which is above the Breitenlohner-Freedman bound. When the space-time has negative mass $M_{CS}$, the internal KG mass squared is positive.

It is crucial to ask which space-times we should allow for in our theory of quantum gravity. From the perspective of the CFT on the boundary, we may want to restrict to space-times with positive mass (or, perhaps more appropriately put, positive conformal dimension). That demand excludes space-times with a deficit angle larger than $2\pi/k$. The only true discrete representations that would be allowed would be $D^{\pm}_{1,1}$, corresponding to $2\pi/k$ deficit angle space-times. On the other hand, we know that the complementary representations are not obtained from quantizing an orbit using the path integral method. We could nevertheless define the quantum dynamics in these representations by algebraically extending (e.g. the braiding operation) to the complementary representation Hilbert spaces. It is not clear whether we should embrace these truly quantum Hilbert spaces that seem to have no corresponding classical geometry, or whether we should just exclude them. From the boundary CFT point of view, certainly the continuous representations are acceptable and these correspond to the black hole space-times. They do lead to tachyonic internal wave-functions. We note here on the one hand the clear analogy with Liouville theory (see e.g. [27]), and on the other hand with the corresponding picture in three-dimensional gravity with zero cosmological constant (see e.g. [28]).

We further remark that, since the tachyonic instability is clearly associated with the space-time itself (and not with the particle scattering off the space-time), the phenomenon of having a tachyonic wave-function for a BTZ black hole is reminiscent of the instability of

\[6\text{It is intriguing to note that this naturally seems to give rise to } 2k \text{ sectors in the multi-particle Hilbert space, which was argued for on entirely different grounds in [28].}\]
time-dependent orbifolds (see e.g. [29] and references there to) in string theory (where one, as in the BTZ geometry, also identifies space-time under a boost). There, in most scenarios, the space-time background itself is unstable against collapse when a single excitation is added to the space-time. Lastly, we mention the intriguing fact that it has been advocated recently that space-like (i.e. tachyonic) geodesics can be used in the AdS/CFT context to probe regions of space-time behind the event horizon of the black hole [30].

Note that we have often ignored oscillatory excitations on the boundary in the above. We will start amending that omission in the following.

6. Using CFT

In this section we want to indicate another approach to particle scattering which will allow us to make contact with another recent attempt to describe black hole scattering in three-dimensional gravity [18]. It will consist of making use more heavily of the connection between Chern-Simons theory on compact manifolds and two-dimensional conformal field theory [3]. Suppose we have $n$ punctures in the disk which represent the particles or black holes to be scattered. The boundary of the disk represents the space at infinity. We will need to specify boundary conditions to make the scattering problem well-defined. When we quantize the particle actions associated to the punctures (by integrating over the particle degrees of freedom), we obtain the expectation value of $n$ operators [9]:

$$Z_n(R \times D) = \int dA e^{iS_{CS}O_1 \ldots O_n}$$  \hspace{1cm} (6.1)

where the operators $O_i$ are given by [7]

$$O_i = \lambda^+_i \otimes \lambda^-_i \langle \text{init} | P e^{\int A} | \text{fin} \rangle.$$  \hspace{1cm} (6.2)

These are loop operators evaluated in the Hilbert space obtained by quantizing the respective point particle action associated to the weights $\lambda^+_i \otimes \lambda^-_i$. We need to specify an initial and final condition for the particle path integral and these give rise to the evaluation in an initial and final state. (We note that these open loop operators $O_i$ are gauge invariant since gauge transformations are assumed trivial in the infinite past and future.)

**Exact results**

At this point we note that we can evaluate formal aspects of the scattering problem exactly by reasoning as follows. Suppose we compactify the time in the base manifold, and moreover integrate over initial conditions which we put equal to the final conditions (effectively tracing over the Hilbert space). The resulting amplitude to evaluate is:

$$Z_n(S^1 \times D) = \int dA e^{iS_{CS}W_1 \ldots W_n}$$  \hspace{1cm} (6.3)

where the operators $W_i$ are now true Wilson loops in the representations $\lambda^+_i \otimes \lambda^-_i$. Now, if we moreover specify a boundary conditions at infinity by inserting a Wilson loop on
the boundary and gluing the disk to an “outer” disk over their boundaries\(^7\), we obtain a spatial two-sphere \(S^2\) with \(n + 1\) Wilson loop operators with a topology determined by the scattering process under study:

\[
Z_n(S^1 \times S^2) = \int dA e^{iSCS} W_1 \cdots W_n W_{n+1}^{bc}. \tag{6.4}
\]

Now, if we blindly analytically continue to the euclidean model and corresponding \(SL(2,C)/SU(2)\) conformal field theory, we can use our knowledge of the conformal field theory \(\text{(}[32])\) to evaluate these partition functions \(\text{([9])}\).

We note that this type of formal analysis makes contact with the proposal in \(\text{([18])}\), where Liouville amplitudes (closely connected to \(SL(2,C)/SU(2)\) amplitudes) are interpreted as relevant to the scattering of black holes in the three-dimensional theory of gravity with \(AdS_3\) boundary conditions on the metric. Of course, we have merely sketched the nature of the computation to be performed in this section. It would be interesting to flesh it out.

7. Conclusions

By applying known techniques in three-dimensional gravity to the case with negative cosmological constant, we found that Chern-Simons interactions can mediate gravity such as to reproduce scattering of particles off black holes. At the same time we have shown that once the internal black hole wave-functions is taken seriously, we run into interpretational difficulties (see discussion in section \(\text{[5]}\)). We have argued that more information can be found by carefully interpreting data of the conformal field theory on the boundary as relevant to black hole scattering. We have stressed the crucial different behaviors (as a function of the ratio of the cosmological radius and the Plank length) of the theory with boundary conditions such that the metric asymptotes to \(AdS_3\) and the theory with boundary conditions on the gauge connection consistent with the full current algebra. We end by remarking that an \(AdS/CFT\) interpretation of the Chern-Simons boundary conformal field theory would require copying the construction of \(\text{([33])}\) for still another space-time Virasoro algebra in the three-dimensional gravity context. It will be interesting to see whether this third choice of Virasoro algebra can help in resolving some of the counterintuitive features that we found above, by implementing a more conventional picture of holography (i.e. slightly closer to the string theoretic one \(\text{[34]}\)).

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\(^7\)We refer to the case of a compact gauge group to argue for this prescription \(\text{([1])}\).
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