Forecasting differences in life expectancy by education

Pieter van Baal1, Frederik Peters2, Johan Mackenbach2 and Wilma Nusselder2

1Erasmus University Rotterdam, 2Erasmus University Medical Center

Forecasts of life expectancy (LE) have fuelled debates about the sustainability and dependability of pension and healthcare systems. Of relevance to these debates are inequalities in LE by education. In this paper, we present a method of forecasting LE for different educational groups within a population. As a basic framework we use the Li–Lee model that was developed to forecast mortality coherently for different groups. We adapted this model to distinguish between overall, sex-specific, and education-specific trends in mortality, and extrapolated these time trends in a flexible manner. We illustrate our method for the population aged 65 and over in the Netherlands, using several data sources and spanning different periods. The results suggest that LE is likely to increase for all educational groups, but that differences in LE between educational groups will widen. Sensitivity analyses illustrate the advantages of our proposed method.

Keywords: life expectancy; Lee–Carter model; time series; educational inequalities

Introduction

Life expectancy (LE) has been increasing in most Western countries and is expected to continue to do so (Tuljapurkar et al. 2000; Oeppen and Vaupel 2002; White 2002; Bongaarts 2004; Christensen et al. 2009). It is a phenomenon with important implications for society because larger numbers of elderly people pose additional burdens on the healthcare and pension systems (Bongaarts 2004; Christensen et al. 2009). In many countries it has led to political debates about the statutory retirement age, and about how to finance the growing healthcare expenditures with public funds. Some Western European countries have explicitly linked their retirement age to the increase in LE (Ageing Working Group 2012), and the prospect of a continuing rise is reflected in the higher premiums now charged by the life insurance and annuity industry (Pitacco et al. 2009; De Waegenaere et al. 2010). But what are ignored by such measures are the great differences in length of life by socio-economic status (SES) (Mackenbach et al. 2008; Van Kippersluis et al. 2010). Those with fewer years of education have much shorter lives, and a growing number of studies report a widening of inequalities in LE between SES groups. In consequence the trend in the LE of the population as a whole becomes ever less informative about the life expectancies of different subgroups (Mackenbach et al. 2003). This means that forecasts of the life expectancies for these groups are needed if the political debate is to be adequately informed.

From about the 1980s, a growing number of techniques for forecasting LE became available (Booth and Tickle 2008). Although there have been exceptions, most approaches are based on time-series extrapolation models such as the Lee–Carter model (Lee 2000). Lee–Carter-based methods decompose time series of age-specific mortality rates into a latent time trend and an associated interaction with different age categories. The latent time trend is then forecast using ARIMA modelling (basically a random walk with drift) and serves as a basis for the derivation of future age profiles of mortality rates and corresponding projections of LE. Until now, Lee–Carter-based methods have been used to project mortality and LE of the general population (often stratified by sex) but have not been used to project LE of different SES groups. Although there have been population projections that accounted for the effect of changes in the educational distribution...
of mortality, these projections assumed only changes in projected overall mortality rates from compositional changes while keeping educational differences in mortality fixed (Kc et al. 2010; Kc and Lentzner 2010). To date there have been no forecasts of LE stratified by level of education.

The aim of this paper is to present a method of forecasting LE for different educational groups within a population. As a basic framework we use the Li–Lee model, which has been developed as an extension to the original Lee–Carter model in order to forecast mortality coherently for different groups, for example, countries or sexes (Li and Lee 2005). The rationale of the Li–Lee model is that because trends in mortality are, to some extent, similar in populations that have in common such features as their healthcare system and economic environment, it is unlikely that their future mortality patterns will diverge markedly. To date the Li–Lee model has been restricted to the projection of LE coherently for the different sexes within a country (Li and Lee 2005) and the projection of LE for countries within a group of countries (Janssen et al. 2013). To widen the applicability of the Li–Lee model, we have extended it in several ways. First, we have made it more flexible in order to incorporate more group-specific time trends, while retaining the idea that, to some extent, the different groups within a population have common trends. Second, the different subgroups included in the model can now be used to integrate data differing in quality and time span, allowing the combination of shorter survey-based time series of mortality disaggregated by SES with longer register-based time series on general mortality. This facility means that forecasts can now be made for particular subgroups even if the quality of their data is inferior, thus solving the problem that explains the absence of such subgroup forecasts in the literature to date. We will illustrate our method for the population aged 65 or over in the Netherlands and we will forecast LE by education group 30 years ahead for the years 2013–42. Although in the Netherlands mortality data by age and sex are routinely collected at the population level as part of national vital statistics, the data on mortality by level of education are gathered from smaller and more selective surveys (Kulhánová et al. 2014).

Our paper is structured as follows. First, we outline the Li–Lee model and pay special attention to the issue of extrapolating time trends within this framework. Second, we present the different kinds of data available for estimating education-specific mortality rates in the Netherlands. We then specify our model and demonstrate how the relatively short-term education-specific mortality trends can be combined with the overall and sex-specific data over a longer period. We make a base forecast for sex-specific and SES-specific LE in the Netherlands using a set of key assumptions about the future group-specific and common time trends. Finally, we demonstrate the sensitivity of our results to each of these key assumptions in four alternative sensitivity analyses.

**Background**

**The Li–Lee model**

The Li–Lee model is based on the Lee–Carter model, which is the most popular model for forecasting mortality rates and LE (Lee and Carter 1992; Lee 2000; Koissi et al. 2006). The Lee–Carter model postulates that mortality rates can be modelled as a function of three sets of parameters: age-specific constants; a time-varying index; and interaction terms between time and age (Lee and Carter 1992):

\[
\log [m(a, t)] = \alpha(a) + [\beta_1(a) \times \kappa_1(t)] + \epsilon(a, t) \quad (1)
\]
where \( m \) stands for mortality rate and \( a, t \) are indices for age and time (calendar year). The \( \alpha(a) \) parameters indicate the time-averaged log mortality rate stratified by age; \( \kappa_1(t) \) refers to an age-independent latent time trend in mortality that is shared by all ages; while \( \beta_1(a) \) can be interpreted as the interaction between each age category and the general time trend. The \( \beta_1(a) \) parameters indicate at which ages mortality declines or increases more rapidly or more slowly in response to changes in \( \kappa_1(t) \). The \( \kappa_1(t) \) values can be treated as a time series, and forecasts of mortality rates can be made by forecasting \( \kappa_1(t) \) and substituting values of these forecasts of \( \kappa_1(t) \) into equation (1). Lee and Carter proposed that specifying a time series model of a random walk with drift parameter describes \( \kappa_1(t) \) best. Extensions of the Lee–Carter model focused on alternative estimation techniques (Currie et al. 2004; Koissi et al. 2006), how to select the optimal fitting period (Booth et al. 2002), and how to account for parameter uncertainty when forecasting mortality rates (Booth et al. 2002; Booth and Tickle 2008).

A drawback to forecasting LE for different, but related, populations (e.g., neighbouring countries) by fitting a separate Lee–Carter model for each population is that the forecasts usually diverge significantly in the long run. This was recognized by Li and Lee, who developed the Li–Lee model in response to this problem (Li and Lee 2005). The central idea behind the Li–Lee model is that related populations share a common time trend in the long run, but that there may be population-specific deviations in the short run. Common trends in the mortality of related populations may be the result of similarities in environmental causes of childhood disease, dietary patterns, and lifestyle (White 2002). Common trends may also result from breakthroughs in health technology that quickly spread (Papageorgiou et al. 2007). Li and Lee did not precisely define ‘related population’, but mentioned the two sexes in a country and different countries with similar levels of development. Li and Lee proposed to extend the Lee–Carter model in the following manner:

\[
\log[m(a, t, g)] = \alpha(a, g) + [\beta_1(a) \times \kappa_1(t)] \\
+ [\beta_2(a, g) \times \kappa_2(t, g)] \\
+ \epsilon(a, t, g)
\]

(2)

where \( g \) is an index for subgroups (i.e., different countries or different sexes); \( \kappa_2(t, g) \) indicate the deviations of the subgroup from the common time trend \( \kappa_1(t) \); and \( \beta_2(a, g) \) are the subgroup-specific age interactions with these subgroup-specific time trends. As in the basic Lee–Carter model, \( \alpha(a, g) \) equal the average log mortality rates by age and now also subgroup. Here \( \kappa_1(t) \) is the common time trend for all subgroups and \( \beta_1(a) \) the common age interactions with the common time trend. Li and Lee proposed forecasting values of \( \kappa_2(t, g) \) using a mean-reverting process, such as an AR(1) process.

### Time trends in the Li–Lee model

The assumption of a mean-reverting process for the subgroup-specific kappa parameter prevents a large divergence of forecasts between subgroups, since the forecasts of \( \kappa_2(t, g) \) return to the values observed in the period used to fit the model. In other words, in the long run, the subgroup-specific time trends revert to their mean deviation from the common trend. This provision ensures that the ratio of age-specific mortality rates for different

![Figure 1](image_url)  

Lee–Carter alpha parameter estimates by age, sex, and educational attainment in our base model specification.
subgroups eventually returns to values observed in the data. (This is equivalent to assuming that the age-specific hazard ratios for mortality are constant in the long run.) Consequently, forecasts of life expectancies for both the overall population and the subgroups are coherent in the sense that they do not diverge in the long run.

A drawback of assuming a mean-reverting process is that if widening mortality rates between subgroups have been observed in the fitting period, these will...
automatically become smaller in the future (and vice versa). In cases where the subgroup-specific time trends are difficult to characterize as a mean-reverting process, Li and Lee advised that mortality rates for those subgroups should be modelled separately.

A possibility, one noted by Li and Lee themselves, is that fitting a separate Lee–Carter model for each education group might result in a large divergence between LE forecasts for them. Both options (modelling all subgroups separately, or modelling them simultaneously but assuming mean-reverting processes) thus have clear disadvantages. Li and Lee did not propose formal tests to decide whether the subgroup-specific trends (i.e., \( \kappa_2(t, g) \)) should be modelled using a mean-reverting process, or whether each subgroup should be modelled separately. Instead, they proposed applying measures of goodness of fit of equations (1) and (2), and the estimates of the AR(1) model, in order to decide between these modelling options. (Li and Lee introduced the concept of an ‘explanation ratio’ which is a measure of goodness of fit that compares the contributions of the different time trends in the Li–Lee model for a defined subgroup.)
Methods

Below we present our strategy for forecasting education differences in LE. We will illustrate the strategy for the case of the Netherlands, where data on mortality by education level (available for the years from 1996 to 2012) are combined with data on overall and sex-specific mortality for the years 1973–2012. Before specifying our model, we first describe the different data. This background information will help the reader better understand how the education-specific mortality rates are combined with the overall and sex-specific mortality rates in our model.

Data

In the Netherlands there is no single data source that contains information on mortality by sex, age, and education. While deaths and births by age and sex have been recorded in the country since the nineteenth century, information on education level is not included in vital statistics. We created a time series for mortality by education level for the years 1996–2012, using individual-level data from the Dutch Labour Force Survey (LFS) linked to the municipal population registries (the LFS has been known as the GBA since 1997) (see the Appendix for details of the construction of these time series).

In our analysis, educational attainment was categorized as follows: low: primary education (basisonderwijs); middle: pre-vocational education (Vmbo, mbo 1, mavo); high: secondary education and tertiary education (Havo, Vwo, Mbo 2, 3, 4, Hbo, Wo).

In this paper we focus on the remaining LE at age 65 for four reasons. First, socio-economic differentials in mortality are at least as large at older ages as at younger ages. Mortality differentials by education at older ages have been found in a wide range of countries (Huisman et al. 2004). Second, age 65 has, until 2012, been the official retirement age in the Netherlands (in many other Western countries the official retirement age is also around 65). Many countries have linked the age at pension entitlement to LE, and are considering raising the future retirement age in line with the increase in LE (OECD 2011). Third, because most healthcare is consumed by the elderly, the results are also relevant to the debate regarding growing healthcare expenditure. Finally, by focusing on those aged 65 and over we could estimate mortality trends by education more reliably, given the concentration of deaths in the elderly.
where \( g \) respectively. The parameters \( k \) (categorical classes within each sex-specific group) mortality rates by age, sex, and education. First, to estimate \( \beta_3(a, g) \) and \( \kappa_2(t, g) \), the basic Lee–Carter model from equation (1) is estimated for the total population, that is, it is not specified by sex and education. After estimating equation (1), \( \beta_3(a, g) \) and \( \kappa_2(t, g) \) can be estimated using the Singular Value Decomposition by inserting the estimates obtained from equation (1) into equation (2):

\[
\log [m(a, t, g)] = \alpha(a, g) - [\beta_1(a) \times \kappa_1(t)] \\
= [\beta_2(a, g) \times \kappa_2(t, g)] + e(a, g, t).
\]

To estimate equations (3) and (4), we used data on overall and sex-specific mortality for the period 1973–2012. We chose this period because LE for both men and women has been increasing since 1973. In earlier years, the trends in LE between men and women differed starkly. This choice of period is consistent with previous research, which indicated that the optimal time period for the Lee–Carter model using data from the Netherlands starts in the 1970s (Stevens et al. 2010; Janssen et al. 2013). Furthermore, as our goal was to forecast LE 30 years ahead, our choice of historical period accords with a general recommendation that the historical period should be at least as long as the projection horizon (Janssen and Kunst 2007).

After estimating equation (4), \( \beta_3(a, g, e) \) and \( \kappa_3(t, g, e) \) can be estimated by inserting the estimates obtained in equations (1) and (2) into equation (3):

\[
\log [m(a, t, g, e)] - \alpha(a, g, e) - [\beta_1(a) \times \kappa_1(t)] \\
= [\beta_2(a, g) \times \kappa_2(t, g)] + e(a, g, e, t).
\]

Because the estimation of the Li–Lee model is iterative, it allows the use of time series of different lengths. In our case, this meant we could use longer time series (1973–2012) to model the overall trend and sex-specific trends, while using shorter time series (1996–2012) to model deviations from these trends for the different education groups. Note that only values for the \( \kappa_1 \) and \( \kappa_2 \) parameters for the

### Table 4 Differences in life expectancy (LE) in the Netherlands at age 65 in 2042 in base projection and several sensitivity analyses (95 per cent prediction intervals given in parentheses)

|                      | Base projection | Sensitivity analysis A | Sensitivity analysis B | Sensitivity analysis C | Sensitivity analysis D |
|----------------------|-----------------|------------------------|------------------------|------------------------|------------------------|
| Men vs. women        | 2.7 (2.1/3.5)   | 2.7 (2.3/3.1)          | 4.2 (0.5/8.7)          | 2.1 (1.8/2.4)          | 1.4 (–0.2/3.1)         |
| High vs. low educated | 4.9 (4.3/5.7)   | 2.8 (2.2/3.4)          | 4.8 (3.4/6.1)          | 5.4 (4.8/6.0)          | 4.8 (3.4/6.2)          |
| High vs. low educated | 4.5 (3.3/5.8)   | 2.7 (2.3/4.0)          | 4.9 (2.9/7.0)          | 4.9 (4.5/5.4)          | 4.9 (2.9/6.9)          |

Table 1 shows estimates of LE at age 65 calculated from the combined data. The LE at age 65 increased more for men than for women between 1996 and 2012, although LE was still longer for women. Education differences in LE at retirement age were more than 2.5 years for both men and women in 1996, and have since widened for both. The implication is that the less educated enjoy fewer years of retirement than the more educated. From Table 1 it can be seen that, overall, LE improved over time, probably because the distribution of educational attainment changed in a positive manner.

**Model specification for base projection**

To extrapolate mortality rates we used the Li–Lee model as the starting point, and extended it in several ways. First of all, we extended the Li–Lee model by distinguishing two different subgroups—sex and education—instead of just one. This means that there would be a common time trend shared by all groups (\( \kappa_1(t) \)), a time trend shared by all educational classes within each sex-specific group (\( \kappa_2(t, g) \)), and a time trend specific for each educational class by sex (\( \kappa_3(t, g, e) \)). For each of these time-trend parameters (\( 1 + 2 + 2 \times 3 = 9 \) in total) there is also a set of age-specific interaction terms, which leads to the following model specification:

\[
\log [m(a, t, g, e)] = \alpha(a, g, e) + [\beta_1(a) \times \kappa_1(t)] \\
+ [\beta_2(a, g) \times \kappa_2(t, g)] \\
+ [\beta_3(a, g, e) \times \kappa_3(t, g, e)] \\
+ e(a, t, g, e)
\]

where \( g \) and \( e \) are indices for sex and education, respectively. The parameters \( \kappa_3(t, g, e) \) reflect the latent subgroup-specific time trend by education, class, and sex, and \( \beta_3(a, g, e) \) the education and sex-specific interactions with that time trend. Equation (3) can be estimated in a stepwise manner given that \( \log [m(a, g, e)] \) equals the time-averaged log
period 1996–2012 were used in equation (5). After fitting the model in the steps described above, nine time series of $k_1(t)$, $k_2(t, g)$, $k_3(t, g, e)$ values were retained. Forecasts of mortality rates could then be made by forecasting $k_1(t)$, $k_2(t, g)$, $k_3(t, g, e)$, and substituting the values of these forecasts into equation (3). Crucial to forecasting LE is the choice of a model to extrapolate the different kappa parameters: $k_1(t)$, $k_2(t, g)$, $k_3(t, g, e)$.

We think there is always an advantage in modelling common trends if there are theoretical reasons to assume that common determinants affect trends in mortality in related populations. If there are clear indications that subgroup-specific time trends in equation (3) do not revert to the average, one cannot conclude that the mortality rates are unrelated to the overall time trend. Even if the subgroup-specific kappa parameters were not mean reverting, the influence of these subgroup-specific time trends will lessen (as will also the problem of divergence/convergence), if part of the time trend is modelled using a common trend. Also, when considering the prediction intervals, there is a clear benefit in modelling common

**Figure 4**  Forecasts of life expectancy (LE) in the Netherlands at age 65 in Sensitivity analysis A [assuming convergence]. Overall population including 95 per cent prediction intervals (upper left graph) and forecasts of LE at age 65 for the different education groups (middle left and bottom left graphs for men and women, respectively). Forecasts of differences in LE at age 65 between men and women (upper right graph), high and low educated men (middle right graph), and high and low educated women (bottom right graph) including 95 per cent prediction intervals.
trends since this generates the expected positive correlation between the forecasts for the different subgroups. In our specific application, modelling common trends also allowed us to strengthen forecasts of LE by education by using longer time series for the overall and sex-specific time trends. This mixture of a common time trend with potentially deviating subgroup-specific time trends follows a broader literature that highlights the importance of common unobserved factors in time-series data of separate groups (Breitung and Pesaran 2008).

Figure 5 Forecasts of life expectancy (LE) in the Netherlands at age 65 in Sensitivity analysis B [no common trends]. Overall population including 95 per cent prediction intervals (upper left graph) and forecasts of LE at age 65 for the different education groups (middle left and bottom left graphs for men and women, respectively). Forecasts of differences in LE at age 65 between men and women (upper right graph), high and low educated men (middle right graph), and high and low educated women (bottom right graph) including 95 per cent prediction intervals.

To avoid more or less arbitrary judgements and to consider a broader category of time-series models to forecast all kappa parameters, we favour using a criterion-based method to selecting optimal time-series models for all kappa parameters. Therefore, to forecast values for the kappa parameters for the different models, we selected optimal ARIMA models by comparing the BIC values of different ARIMA models. We prefer this option over the alternative of assuming a random walk to model overall mortality and imposing the condition that the sex-specific and SES-specific time trends be mean reverting.
Furthermore, since the time series by education are rather short, we prefer to select forecasting models this way because small samples generally make testing procedures difficult. It is important to note that our method allows the identification of the same time-series models for the kappa parameters as Li and Lee’s method, if the data warrant it.

**Sensitivity analyses**

To investigate the sensitivity of our forecasts to several key assumptions, and to illustrate the advantages of our proposed methodology, we also forecast LE in the following sensitivity analyses:

- **Sensitivity analysis A** [assuming convergence]: in this sensitivity analysis we imposed a random walk with drift for the common trend $k_1(t)$, and an AR(1) process for all sex and education-specific time trends. This sensitivity analysis is similar to the original Li–Lee model specification in which all subgroup-specific trends are mean reverting.
- **Sensitivity analysis B** [no common trends]: in this sensitivity analysis we fit a Lee–Carter model for each group separately. As in the base projection, we used data for the period 1973–2012 for the overall and sex-specific mortality, and data for the period 1996–2012 for the education-specific mortality. For all Lee–Carter models we select ARIMA models to extrapolate the kappa parameters by optimizing the BIC criterion.
- **Sensitivity analysis C** [shorter historical period]: in this sensitivity analysis we again used data for the period 1996–2012 for overall and sex-specific mortality. Everything else was the same as in our baseline model specification.
- **Sensitivity analysis D** [shorter historical period without common trends]: in this sensitivity analysis we forecast mortality by fitting a Lee–Carter model for each education and sex group separately using data for 1996–2012 only. To extrapolate the kappa parameters we again selected the optimal ARIMA models.

Sensitivity analysis A mimics the original Li–Lee model by imposing a mean-reverting process on the subgroup-specific trends. By also forecasting LE in Sensitivity analyses B and D we can investigate the benefits of our proposed methodology because it allows a comparison between our base forecasts with the separate Lee–Carter forecasts for each subgroup. Sensitivity analysis C allows us to investigate the value of using a longer time series to model common trends. It is interesting to compare Sensitivity analysis D with C as it allows a straightforward comparison of separate Lee–Carter models and our modelling strategy using time series which are all of equal length. Note that to avoid jump-off bias, in each series we used the last observed mortality rate as a starting point for our forecasts in all analyses.

**Results**

Figure 1 graphs the estimates of the $a$ parameters of equation (3), which are simply the time-averaged log mortality rates (for the period 1996–2012) by age, sex, and education. From the two graphs it can be seen that there is a clear education gradient in mortality rates for both men and women.

Figure 2 graphs the estimates of the kappa and beta parameters as described in equation (3). From the upper left graph in Figure 2 we can see a clear downward trend in overall mortality over time as illustrated by the decreasing $k_1$ values. The deviations from the overall time trend for men and women are also displayed in the same graph. For men, the increasing $k_2$ values from 1973 to about 2000 suggest that mortality for men has been decreasing at a less rapid pace than overall mortality, while the reverse is true for women. From about 2000 onwards, this pattern reversed. However, the $k_2$ parameters are difficult to interpret in isolation because they interact with the $\beta_3$ parameters, which are negative for some ages. At ages from 65 to about 77, men have negative $\beta_3$ values, while at ages above 77 they have positive $\beta_2$ values indicating opposite trends in mortality at these ages. The fact that $k_3$ values increase over time in the middle left graph in Figure 2 shows that the overall decline in mortality has been slower for the less educated men. It should be noted that changes in the $k_2(t, g)$ and the $k_3(t, g, e)$ values over time are much smaller than those in $k_1(t)$ as many of the changes over time in mortality have already been captured by the common trend.

Table 2 displays the optimal ARIMA models selected using the BIC criterion used to forecast values for the different kappa parameters. From this table we can observe that the overall time trend $k_1(t)$ is, similar to that in previous studies, best modelled using a random walk with drift. Although the sex-specific trends $k_2(t, g)$ do not contain a drift term, the time trends for men and women are not mean reverting. All education-
specific time trends are also not mean reverting. While for women both the high and middle educated time trends are modelled best as a random walk without drift, the less educated time trend does contain a drift term. For men, the time trends for all educational groups contain a drift term. However, it should be kept in mind that changes in the $k_2(t, g, e)$ values over time are much smaller than changes in $k_2(t, g)$ and $k_1(t)$ values over time, so that the ‘drift magnitude’ is much smaller.

Figure 3 displays trends and forecasts of LE (left-hand graphs) and differences in LE between different subgroups (right-hand graphs). From this figure it can be seen that LE is predicted to increase for all educational classes for both men and women but that LE increases less for the less educated. The difference in LE between the more and the less educated increases at the same pace as observed in the period 1996–2012 for both men and women. Furthermore, although differences in LE between men and women are expected to decrease, the rate of this decrease is slower than has been observed in the last decade. Also noteworthy are the prediction intervals that increase over time and the fact that the trends in the subgroups are rather similar as a result of modelling the common time trends.

Table 3 summarizes the estimates of LE in 2042 in the different sensitivity analyses, and Table 4 displays differences in LE between different groups in 2042. If we compare the predictions of the sensitivity analyses with the forecasts of the base analyses, we can make several observations. First of all, predictions of overall and sex-specific LE in Sensitivity analyses C and D, which are based on the period 1996–2012, are longer than those in the base projection. This is because in this period LE was increasing rather sharply. In Sensitivity analysis A, in which we imposed mean reversion, we can see that differences in LE between men and women, and between education classes, decline as a result.

From Table 4 we can see that the prediction intervals of differences in LE between subgroups increase if we model them without common trends as is done in Sensitivity analyses B and D. A definite advantage of using the Li–Lee approach is that the correlation between the predictions of LE for the different subgroups is taken into account by modelling a common trend. This results in a much smaller variation in the predicted differences in LE between subgroups in the base projection, and in Sensitivity analysis C compared with Sensitivity analyses B and D, in which we estimated a separate Lee–Carter model for each subgroup. The 95 per cent prediction interval of the difference in LE between men and women includes no difference, that is, it is zero, if we model no common trend in Sensitivity analysis D, which seems implausible. Also noteworthy from Tables 3 and 4 is that, by modelling common trends, the predictions of differences in LE between education groups are fairly similar, while the levels of the LE predictions may change as different time periods are chosen to model the common trends.

To understand the consequences of the key assumptions for the forecasts, we compare the results of Sensitivity analyses A and B in Figures 4 and 5. The right-hand side panels of Figure 4 clearly illustrate that assuming convergence (Sensitivity analysis A) for modelling education-specific time trends implies a clear break in the trend in the differences in LE between education classes, which seems implausible. Figure 5 shows that if separate Lee–Carter models are used (no common time trends), the forecasts for the different groups do not seem consistent with the forecasts of the overall group of which they are part. This is illustrated most prominently in the forecasts of LE for men. Thus, even in cases of diverging time trends that are sex-specific, education-specific, or both, there is still a benefit in modelling common underlying trends.

**Discussion**

We have demonstrated a novel approach to the forecasting of LE, by combining mortality trends at different strata in a population (overall, by sex, by education), available for time periods of different lengths. We have shown that even if sub-group-specific trends in mortality appear to be diverging, modelling a common trend can have benefits. This method has an impact not only on the mean forecasts, but also on the prediction intervals of the forecasts and the correlations between forecasts of different educational groups. We have illustrated the usefulness of the approach by projecting LE by level of education in the Netherlands up to 2042. Our base projection forecasts a general increase at all levels with a continuing convergence of LE of males and females, but a divergence of LE between the education classes, which was slightly stronger in men than in women. In our case study, we combined data from a long time series to estimate the time trend for the overall group with shorter time series of education-specific data. Thus, the shorter time series borrowed information from the longer time series.
Our model extends the existing literature on projection of mortality trends by education in several directions. First, we suggest a flexible framework allowing subgroup-specific trends to diverge, which allows a continuation of the widening or narrowing of socio-economic inequalities if signalled by the data. Second, we have augmented the Li–Lee model to include time series with different lengths and data quality without the ad hoc assumption of a future convergence of the subgroup-specific trends. This facility allows a convergence of LE between men and women and a divergence among educational groups at the same time. Both extensions enable a wide applicability of the model in other applications dealing with the projection of subgroup-specific differentials. Our model requires a certain degree of correlation between the different groups to allow the hierarchical approach to benefit from its strength. While LE was restricted in our illustrative example to age 65, the approach could readily be applied to projections of educational differences for LE at birth. Other possible applications may deal with subgroup-specific differentials among groups distinguished by ethnic origin, occupation, body mass index, or smoking. A particular merit of our model is its simplicity; because the determinants of mortality are not required, the model can be applied in a broad range of applications with minor computational effort and relatively modest data requirements. However, the absence of determinants can also be seen as a drawback since separate modelling of smoking-associated and non-smoking-associated mortality in the Netherlands has revealed that in the short run a further convergence of mortality of males and females is likely (Janssen et al. 2013). A logical next step would be to investigate possible ways of including such determinants when forecasting LE by education.

An important precondition of the applicability of our strategy is whether the trends observed at the higher levels in our hierarchical design are appropriate for the lower level. For example, we use the overall mortality trend of the population as the underlying trend for the subgroups of men and women, as well as for the different educational subgroups. This assumption eliminates the need to extrapolate temporary deviations caused, for example, by the impact of smoking on mortality trends that differ between men and women by education. Previous forecasting studies have focused on changes in overall mortality caused by compositional changes while keeping education differences in mortality fixed (Kc et al. 2010; Kc and Lentzner 2010). A drawback of our strategy is that while it avoids the assumption of fixed variations in mortality, it does not account for the impact of compositional changes on differential mortality trends by subgroup, either for the past or for the future. If such changes were indeed the main drivers of the observed trends, the outcomes of our model could be seriously biased if the compositional changes in the future differed fundamentally from those in the past. Moreover, if our approach is used to forecast compositional changes of the subgroups, consistency with the overall population size cannot be guaranteed. A drawback of the original Lee–Carter model, as well as the Li–Lee model, and our model, is that the age–time interactions are assumed constant. We checked whether a changing age profile could be incorporated by adding the second factor obtained from the Singular Value Decomposition. However, there was not a strong trend over time, and including the second factors in the forecasts only slightly increased the prediction intervals, and did not change the mean predictions. We also forecast LE assuming there were no education-specific time trends (this is equivalent to setting $\kappa(t, g, e)$ equal to zero in equation (3)) which led to a narrowing of inequalities in LE by education. This occurs because the models are fitted on the log scale and because absolute decreases in mortality are larger when mortality rates are higher. Furthermore, we also predicted LE in a sensitivity analysis in which we selected optimal ARMA models for the $\kappa_2(t, g), \kappa_3(t, g, e)$ parameters instead of the optimal ARIMA model. The results showed that differences in LE between subgroups were similar to those for Sensitivity analysis A in which we also assumed mean-reverting processes for the subgroup-specific trends.

Because this study represents the very first approach to forecasting LE by level of education/SES, it is impossible to compare our results with those of previous forecasts. However, we can compare our forecasts of overall LE and LE by sex to previous forecasts. The most recent official projection of Statistics Netherlands (CBS) projects LE at age 65 to be 22.2 years for men and 24.3 years for women in 2042 (van Duin et al. 2012). This is slightly longer than our projections, which estimated 21.1 years for men and 23.8 years for women in 2042. Given that Statistics Netherlands used a similar historical period (data from 1970 to 2011), the differences can be explained by the fact that they included the experience of other Western European countries in their variant of the Li–Lee model. Compared with the Netherlands, the mortality improvement was much greater in the other countries over
the whole historical period 1970–2011. Hence, adding a shared all-countries trend, assuming mean reversion, and using the Li–Lee model produced a larger improvement in Dutch LE. We believe that the assumption of mean reversion is justified, given the strong inter-depencies of the countries in economic development, technological progress, and lifestyle. The flexibility of our model would allow differentials in these other categories to be included of course but we focused in this paper mainly on the level of SES differentials for the purpose of illustration.

In our case study we employed the long-run overall time trends in mortality to assist the projection of educational differences, for which only a short time series was available. This design helped prevent implausible patterns that would arise were only data on education-specific mortality to be used for the long-term forecasts. Our model could not fully solve the problem of sparse data on mortality by education. Projecting convergences or divergences based on short time series always risks extrapolating tendencies that are, in fact, only temporary. To decide whether the outcomes are plausible or realistic, additional information needs to be taken into account, such as data on the underlying trends of the determinants of the differences in mortality among the subgroups. Additionally, longer time series on mortality by education would help identify more stable trend differentials. Furthermore, larger sample sizes to provide more reliable estimates of mortality by education would improve the model. Our data on mortality by education were based on record linkages between the Dutch LFS, a representative 1-per-cent sample of the Dutch population, and the death registry, a source of high-quality data (Bakker et al. 2006). Nevertheless, the usual caveats with survey data also apply to this source. Non-response rates of about 40 per cent might have resulted in a selective sample composition that excluded high-risk groups (Visscher 1997), though this problem is partly mitigated by the sampling weights used (Bruggink 2009). Also, it was demonstrated that relative mortality differentials in SES were much less affected by selective non-response than absolute mortality differentials in the Dutch LFS (Kulhánová et al. 2014).

The results of our forecasts indicate diverging trends of mortality between the higher and lower educated subgroups, which are larger for men than women. A recent study of trends in socio-economic inequalities in mortality has reported the first signs of a narrowing of the inequalities for men in several Western countries, while inequalities for women continue to widen (Mackenbach et al. 2014). Although this analysis did not include the Netherlands and applied to another age range (30–74), we must concede that ignoring underlying determinants of SES differentials in mortality, such as smoking or alcohol consumption, may have affected our forecasts. Perhaps the widening we found in inequalities in LE for men were too pessimistic, and actually a narrowing would have been a more plausible result. In countries with better data on education-specific LE and its determinants, that possibility could be investigated. In our data we found no indications of such a narrowing.

In general, educational attainment is related to health and vice versa through a variety of mechanisms (Smith 1999; Cutler and Lleras-Muney 2010). Nevertheless, a clear and causal effect of education on mortality has been demonstrated convincingly in a series of analyses of natural experiments, mostly compulsory schooling reforms (Van Kippersluis et al. 2011; Clark and Roayer 2013). Education influences health not only by affecting lifestyle-related risk factors—such as smoking, alcohol consumption, dietary patterns, and physical inactivity—but also by its impact on financial resources, housing and work conditions, and access to care. Despite great advances in medical treatment, a decrease in smoking prevalence, and programmes to tackle health inequalities, the large differentials in LE between SES groups have persisted, which suggests that more fundamental societal forces drive these inequalities (Meara et al. 2008; Phelan et al. 2010; Olshansky et al. 2012). Therefore, it is likely that SES disparities will endure in the future even if the precise mechanisms explaining the differentials change over time.

Because we focused on those aged 65 and over our LE forecasts have a clear relevance to the debate on retirement age and the demand for healthcare. In the Netherlands, the current policy is to link retirement age to LE (van Duin 2013), keeping the number of years in retirement more or less fixed. In consequence, according to our forecasts, the less educated will experience a decrease in the number of years in retirement because their forecast increase in LE is less than the average increase in LE. With respect to a possible increase in the demand for healthcare owing to increased longevity, our results suggest that this additional demand may be caused more by the more educated than the less educated. If the financing of healthcare and pension schemes does not change these differentials, the changes in LE imply a redistribution of wealth from the less to the more educated, because the latter will consume
more healthcare resources and will receive pension payments for a longer time. Therefore, these differences in LE should be taken into account in political decisions that affect solidarity issues between SES groups.

In conclusion, we believe that the extended Li–Lee model proposed in this paper provides a useful method for forecasting LE by education. Although our method cannot solve problems caused by poor data quality, it makes optimal use of the available data, and might also facilitate LE forecasts for other subgroups for which fewer data are available. We hope the method will be of interest to a variety of potential users, including national statistical offices and actuarial societies, and researchers in health, economics, and social science.

Notes

1 Pieter van Baal is at the Institute of Health Policy & Management, Erasmus University Rotterdam, The Netherlands. E-mail: vanbaal@bmg.eur.nl. Frederik Peters, Johan Mackenbach, and Wilma Nusselder are at the Department of Public Health, Erasmus University Medical Center, Rotterdam, The Netherlands.

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ORCID

Pieter van Baal © http://orcid.org/0000-0001-9913-2393

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Appendix: Estimation of mortality rates by education for the Netherlands, 1996–2012

Mortality rates for different educational classes for the years 1996–2012 were estimated by first estimating age and calendar-year-specific relative risk of mortality (denoted $\text{RR}(a, t, e)$, which is the mortality rate of educational class $e$, age $a$, year $t$ divided by the mortality rate of the reference educational class age $a$, year $t$). These relative risks were then used to decompose mortality rates for the total population by exploiting the following relationship:

$$m(a, t, e) = \text{RR}(a, t, e) \times \frac{m(a, t)}{\sum_e [\text{RR}(a, t, e) \times p(e|a, t)]}.$$  \hfill (A1)

Equation (A1) states that the mortality rate in a particular year at a particular age is the weighted average of the mortality rates of the different educational subgroups ($p(e|a, t)$ denotes the proportion the educational subgroup forms of the overall total for a particular age in a given year) and that the ratio of mortality rates between different subgroups can be expressed by the relative risk. Estimates of $\text{RR}(a, t, e)$ were made using data from the Labour Force Survey (LFS) linked to the death registry. The LFS is a rotating panel survey from Statistics Netherlands which started in 1987. The LFS is the largest data source in which information on education attainment is collected in the Netherlands, and consists of an annual sample of more than 60,000 households. From 1996 onwards it is possible to link persons that have participated in the LFS to the death registry. This makes it possible to quantify the relation between educational attainment and mortality. Values of $p(e|a, t)$ were calculated directly from the LFS. For mortality, we constructed a panel in which the annual number of deaths of all persons ever interviewed in the LFS was obtained from the death registry. The number of person-years-at-risk was estimated as the sum of the people surveyed in a particular year and the survivors from the previous year. To estimate $\text{RR}(a, t, e)$ we fitted a Poisson regression model with the person-years-at-risk as offset and the expected number of deaths by year, age, education class and year as outcome variable:

$$E(D|a, e, t, y) = \exp(\theta X)$$ \hfill (A2)

where $y$ denotes year, $X$ a vector of predictor variables, and $\theta$ the vector of coefficients that needs to be estimated. Predictor variables were dummy variables indicating education class and interactions with age and calendar year (both being continuous variables). To control for confounding, a set of dummy variables for each year and age were added to the model. Furthermore, a variable measuring the length of follow-up time in the LFS and an interaction with age were added to the model. This was intended to control for selection effects into the LFS. From the regression model we calculated $\text{RR}(a, t, e)$. Table A1 displays the estimates of the exponentiated coefficients derived from the regression model (coefficients for the year and age dummies are not shown).

### Table A1 Estimated coefficients for the regression models (low educated are the reference category)

| Education level | Men    | Women   |
|-----------------|--------|---------|
| Middle          | $0.677^{**}$ | $0.620^{**}$ |
| High            | $0.497^{**}$ | $0.487^{**}$ |
| Middle × age    | $1.055^{**}$ | $1.059^{**}$ |
| High × age      | $1.081^{**}$ | $1.081^{**}$ |
| Low × year      | $0.950^{**}$ | $1.111^{**}$ |
| Middle × year   | $0.956^{**}$ | $1.113^{**}$ |
| High × year     | $0.961^{**}$ | $1.108^{**}$ |
| Low × year × age| $1.011^{**}$ | $1.009^{**}$ |
| Middle × year × age | $1.007^{**}$ | $1.007^{**}$ |
| High × year × age | $1.006^{**}$ | $1.008^{**}$ |
| followuptime    | $0.978^{**}$ | $1.008^{**}$ |
| followuptime × age | $0.999$ | $0.991^{**}$ |

**Significant at 0.01.